Testing an ansatz for the leading secular loop corrections from quantum gravity during inflation

S Basu and R P Woodard

Department of Physics, University of Florida, Gainesville, FL 32611, USA
E-mail: shinjinibasu@ufl.edu and woodard@phys.ufl.edu

Received 9 June 2016
Accepted for publication 1 September 2016
Published 21 September 2016

Abstract
It is widely believed that the leading secular loop corrections from quantum gravity can be subsumed into a coordinate redefinition. Hence the apparent infrared logarithm corrections to any quantity would be just the result of taking the expectation value of the tree order quantity at the transformed coordinates in the graviton vacuum. We term this the transformation ansatz and we compare its predictions against explicit one loop computations in Maxwell + Einstein and Dirac + Einstein on de Sitter background. In each case the ansatz fails.

Keywords: inflation, loop corrections, secular growth

1. Introduction

Inflationary perturbations [1, 2] represent the first recognized quantum gravitational phenomena [3–5] and provide our most powerful tool for reconstructing the mechanism of primordial inflation [6–8]. These perturbations derive from 0-point fluctuations of gravitons and (in the simplest models) minimally coupled scalars on the background

\[ ds^2 = -dt^2 + a^2(t)dx^i \cdot dx^i \implies H(t) \equiv \frac{\dot{a}}{a} > 0, \quad \epsilon(t) \equiv -\frac{H}{H^2} < 1. \] (1)

The tensor and scalar mode functions, \( u(t, k) \) and \( v(t, k) \), are initially oscillating and red-shifting like those of normal particles [9]

1 Author to whom any correspondence should be addressed.
Two related, and often confused, loop phenomena are infrared divergences while the nonlocal correction factors, \( (\text{I}) \)\( \), Here the mode functions approach constants of the form \( \theta^\prime (t) \), when in the evolution equations

\[
\ddot{u} + 3H\dot{u} + \frac{k^2}{a^2} u = 0, \quad \ddot{v} + \left( 3H + \frac{\dot{a}}{a} \right) \dot{v} + \frac{k^2}{a^2} v = 0.
\]

That both mode functions approach constants of the form \( \text{[10, 11]} \)

\[
k \ll H(t) a(t) \implies |u(t, k)|^2 \rightarrow \frac{H^2(t_k)}{2k^3} \times C(\epsilon(t_k)) \times C(k),
\]

\[
k \ll H(t) a(t) \implies |v(t, k)|^2 \rightarrow \frac{H^2(t_k)}{2k^3} \epsilon(t_k) \times C(\epsilon(t_k)) \times S(k).
\]

Here the (monotonically decreasing) slow-roll correction factor is

\[C(\cdot) \equiv \frac{1}{\pi} \Gamma^2 \left( \frac{1}{2} + \frac{1}{1 - \epsilon} \right) [2(1 - \epsilon)]^{\frac{1}{2\epsilon}} \implies C(0) = 1 \geq C(\epsilon) > C(1) = 0,\]

while the nonlocal correction factors, \( C(k) \) and \( S(k) \), are unity for \( \dot{\epsilon} = 0 \) and depend upon conditions only a few e-foldings before and after \( t_k \). This transition from the ultraviolet, early-time form (2) to the infrared, late-time form (4) and (5) is known as freezing-in. It is how primordial tensor and scalar perturbations fossilize so that they can survive to the current epoch.

The generation of perturbations is a tree order phenomenon but it has clear implications for loop corrections, because the same tensor and scalar mode functions appear in their respective propagators

\[
i\Delta_{\theta}(x; x') = \int \frac{dk}{(2\pi)^3} e^{ik (x - x') \cdot \theta^\prime(t - t') } \{ \theta(t - t') u(t, k) u^\dagger(t', k) + (t \leftrightarrow t') \},
\]

\[
i\Delta_{\psi}(x; x') = \int \frac{dk}{(2\pi)^3} e^{ik (x - x') \cdot \theta^\prime(t - t') } \{ \theta(t - t') v(t, k) v^\dagger(t', k) + (t \leftrightarrow t') \}.
\]

Two related, and often confused, loop phenomena are infrared divergences and secular growth. Infrared divergences arise because an infinite number of very long wavelength modes obey \( k \ll H(t) a(t) \) even at the time \( t_k \) when inflation begins. Hence these modes begin life in the ‘saturated’ form (4) and (5). From the fact that \( H(t) \) typically falls as inflation proceeds, and \( \epsilon(t) \) typically grows, one can see that both \( u(t, k) u^\dagger(t', k) \) and \( v(t, k) v^\dagger(t', k) \) diverge more strongly than \( 1/k^2 \) at \( k = 0 \), so the mode sums (7) and (8) diverge. Note that infrared divergence is due to the small \( k \) behavior of the mode functions, and is present even at the beginning of inflation. However, it would not occur if one formulated inflation on a spatially closed manifold such as \( T^3 \times R \) [12], or if one assumed that the initially super-horizon modes were in less infrared singular state than (4) and (5) [13]. Infrared divergences were first noted in 1977, for \( i\Delta_{\psi}(x; x') \) on constant \( \epsilon(t) \) backgrounds on \( R^3 \times R \), by Ford and Vilenkin [14].

In contrast, secular growth arises because the progression of inflation causes more and more initially ultraviolet modes to make the transition from the oscillatory ultraviolet form (2) to the saturated, infrared form (4) and (5). This endows the propagators (7) and (8) with secular growth from the constructive interference of the ever-larger number of super-horizon modes.
Unlike infrared divergences, secular growth occurs on spatially compact manifolds, and without regard to assumptions about initially super-horizon modes. It really has nothing to do with infrared divergences, except for the fact that the late time form (4) and (5) happens to be the same as the small $k$ limiting form. Secular growth of $i\Delta_h(x; \ x')$ was first noted in 1982, on de Sitter background ($\epsilon(t) = 0$), by Vilenkin and Ford [15], by Linde [16], and by Starobinsky [17]

$$\epsilon(t) = 0 \implies i\Delta_h(x; \ x')_{\text{secular}} = \frac{H^2}{4\pi^2} \ln \left( \frac{a(t)}{a(t')} \right).$$

In 1987 Allen and Folacci demonstrated that the very same secular growth occurs on the full de Sitter manifold, which is spatially compact [18].

The first proof that secular growth affects loop amplitudes came in 2002 with a fully dimensionally regulated and renormalized evaluation of the expectation value of the stress tensor for a massless, minimally coupled (MMC) scalar with a quartic self-interaction on a nondynamical de Sitter background [19, 20]. It was subsequently shown that one and two loop corrections to the scalar mode functions of the same theory also experience secular growth [21, 22]. For (MMC) scalar quantum electrodynamics on nondynamical de Sitter background secular growth has been seen in one loop corrections to the photon wave function [23, 24], in one loop corrections to electrodynamic forces [25], as well as in the two loop expectation value of the stress tensor [26]. And secular growth was demonstrated as well for Dirac fermions Yukawa-coupled to a MMC scalar on nondynamical de Sitter in the one loop correction to the fermion mode function [27, 28], and in the one loop expectation value of the stress tensor [29].

Each of these MMC scalar results can be understood using the stochastic formalism of Starobinsky [30], which has been proved to capture the leading secular effects of scalar potential models at each order in the loop expansion [31–33]. The Starobinsky formalism has been extended to scalar quantum electrodynamics [34] and Yukawa theory [29]. It even provides a nonperturbative resummation of the leading secular effects for those cases in which a time independent limit is approached [35].

Making quantum gravitational computations is vastly more difficult, but fully dimensionally regulated and BPHZ renormalized\(^2\) results have been obtained on de Sitter background for one graviton loop corrections to MMC scalars, photons and fermions. MMC scalar mode functions experience no secular corrections at one loop because the scalar only couples to the metric through its kinetic energy, which red-shifts to zero [40, 41]. In contrast, photons carry spin, which allows them to continue interacting with inflationary gravitons to arbitrarily late times [42]. As a consequence, dynamical photons experience a secular enhancement from inflationary gravitons [43], as do certain electrodynamic forces [44]. The spin–spin coupling between fermions and gravitons also gives rise to a persistent interaction [45]. Hence inflationary gravitons also induce a secular enhancement of fermions [46, 47].

No comparably explicit calculations have been performed on realistic inflationary backgrounds (which means $\epsilon \neq 0$) because the mode functions and propagators are unknown.

\(^2\) The four initials stand Bogoliubov, Parasiuk, Hepp and Zimmermann [36, 37] and Zimmermann [38, 39], who developed the standard technique of subtracting divergences with local counterterms, even for nonrenormalizable theories like quantum gravity.
Even working out the interactions of the gauge-fixed and constrained theory has only been done to 3-point [48] and 4-point orders [49–51]. In spite of these limitations, an important theorem by Weinberg establishes that loops corrections to the power spectra can grow no faster than powers of the logarithm of the scale factor [52, 53]. There has also been a convincing demonstration by Giddings and Sloth that infrared divergences from the graviton propagator (7) affect the inflationary power spectra at one loop order [54].

The technique of Giddings and Sloth has much to do with why so many people concede the existence of secular effects from MMC scalars but deny that they can occur from gravitons. It helps to change the temporal variable from $t$ to conformal time $\eta$ with $d\eta = dt/a(t)$, so that the background metric takes the form $a^2\eta_{\mu\nu}$. Now conformally transform the full metric by the scale factor and express the conformally transformed metric in terms of the graviton field $h_{\mu\nu}$:

$$g_{\mu\nu}(\eta, \vec{x}) \equiv a^2\tilde{g}_{\mu\nu}(\eta, \vec{x}) \equiv a^2[\eta_{\mu\nu} + \kappa h_{\mu\nu}(\eta, \vec{x})], \quad \kappa^2 \equiv 16\pi G. \tag{12}$$

At linearized order the graviton field can be expressed as a mode sum over spatial plane waves and polarizations whose precise form is known [55] but not relevant for our discussion

$$h_{\mu\nu}(\eta, \vec{x}) = \int \frac{d^4k}{(2\pi)^4} \sum_{\lambda} \{u(\eta, k, \lambda)e^{i\vec{k} \cdot \vec{x}} \epsilon_{\mu\nu\lambda}(\vec{k}, \lambda)\} \alpha(\vec{k}, \lambda) + \text{c.c.} \}. \tag{13}$$

Because the mode functions $u(\eta, k, \lambda)$ of the super-horizon ($k < H(t) a(t)$) wavelengths freeze in to constant values, the super-horizon part of the mode sum behaves as a classical stochastic random field [15, 30, 56–58]. This is still an operator by virtue of the factors of $\alpha(\vec{k}, \lambda)$ and $\alpha(\vec{k}, \lambda)$ but, if we neglect the very small residual time dependence of $u(\eta, k, \lambda)$, it commutes with its time derivative and its value in any particular state does not change [59]. This means we can treat it as a constant. Of course a local observer would choose coordinates so as to absorb this constant. So the long wavelength modes have no effect in these new coordinates—because the long wavelength modes are not even present—and their apparent effect in the original coordinates $x^\mu = (\eta, \vec{x})$ is just the result of evaluating tree order results at the transformed coordinates.

This nice insight by Giddings and Sloth [54, 60], which was anticipated by Urakawa and Tanaka [61–64], is valid for the case of infrared divergences because the modes which cause them are in the saturated state from the beginning of inflation. It is not clear that one can apply the same insight to the case of secular dependence because the modes responsible for that were initially sub-horizon, with nontrivial spacetime dependence, and they only later experienced freeze-in. An example of the potential problems was given in equations (41)–(43) and the associated discussion of [65]. However, enthusiasm over progress on the very tough problem of computing loop corrections to the power spectra made it inevitable that such applications would appear [66–73]. Related, and perhaps contradictory, claims have also been made that the power spectra of single scalar inflation show no secular corrections at all once changes in the perturbative background are properly incorporated [74–76].

The various authors who deny the existence of secular corrections from inflationary gravitons are focussed narrowly on the special case of the power spectra for single-scalar inflation. However, there seems nothing about the key argument which restricts its applicability, either as regards the model of inflation or the quantity under study. We shall therefore formalize their belief in the transformation ansatz: that secular loop corrections to any quantity from inflationary gravitons are the result of evaluating the tree order quantity at the transformed coordinates which would render the metric (12) conformal to $\eta_{\mu\nu}$ for an exactly constant $\tilde{g}_{\mu\nu}(\eta, \vec{x})$. 


The purpose of this paper is to test the transformation ansatz by working out its consequences for photons and fermions at one loop order on de Sitter background, and then comparing with the exact results of dimensionally regulated and renormalized computations \[42, 43, 46, 47\]. If the ansatz is correct then the leading secular effects will agree. In section 2 we work out the coordinate transformation and the associated, local Lorentz transformation, which would carry a truly constant \(g_{\mu\nu}\) back to \(\eta_{\mu\nu}\). In section 3 we apply this transformation to the free photon wave function, and then compute the leading secular corrections. Section 4 does the same thing for the free fermion mode function. Our conclusions comprise section 5.

2. Constructing the transformations

The purpose of this section is to construct the general coordinate transformation, and the associated local Lorentz transformation, which would carry the metric \(g_{\mu\nu} = a^2 \tilde{g}_{\mu\nu}\) to the pure de Sitter form \(h_{\mu\nu} = \eta_{\mu\nu}\) under the (false) assumption that \(\tilde{g}_{\mu\nu}\) is a spacetime constant. We begin by giving the vierbein in Lorentz-symmetric gauge, which plays a prominent role in the construction. Then the general coordinate transformation is derived. The section closes by working out the Dirac spinor Lorentz transformation this general coordinate transformation induces. To simplify the discussion we commit a small abuse of our earlier notation by considering the de Sitter scale factor to be a function of the conformal time \(a(\eta) \equiv -1/H\eta\).

2.1. The vierbein in symmetric gauge

When considering theories with half-integral spin coupled to gravity it is convenient to introduce a fictitious local Lorentz gauge symmetry under which the spinor indices transform. The place of the metric is taken by the vierbein \(e_{\mu a}(x)\), with vector index \(\mu\) and local Lorentz index \(a\). The metric follows by Lorentz contracting two vierbeins, \(g_{\mu\nu}(x) = e_{\mu b}(x) e_{\nu d}(x) \eta^{bd}\).

For our conformally transformed metric \(\tilde{g}_{\mu\nu}\) the associated vierbein would be \(\tilde{e}_{\mu a}\),

\[
\tilde{g}_{\mu\nu}(x) = \tilde{e}_{\mu a}(x) \tilde{e}_{\nu b}(x) \eta^{ab}, \quad \tilde{e}_{a \mu}^a \tilde{e}_{\mu b} = \eta_{ab}.
\]  

The fictitious nature of local Lorentz symmetry is evidenced by its failure to obey the famous rule of van Nieuwenhuizen (for real symmetries) that ‘gauge fixing counts twice.’ Once local Lorentz symmetry has been gauge fixed, the associated constraint equations are automatically obeyed, instead of imposing nontrivial relations between the surviving fields. That is how the 16 components of the vierbein reduce to the usual two graviton degrees of freedom

\[
(2 \text{ gravitons}) = (16 \text{ fields}) - (4 \text{ coordinate gauges}) - (6 \text{ Lorentz gauges}) - (4 \text{ coordinate constraints}) - (0 \text{ Lorentz constraints}).
\]

Had local Lorentz symmetry been real, the counting would have produced the absurd result \(16 - 4 - 6 - 4 - 6 = -4\)! Giving trivial constraints is one way to recognize when compensating fields have been introduced to make a theory appear to possesses some symmetry it really does not have \[77\].

Although local Lorentz invariance is fictitious, its gauge fixing still induces a Faddeev–Popov determinant, which can complicate calculations. Lorentz symmetric gauge \((\tilde{e}_{\mu a} = \tilde{e}_{a \mu})\) is a particularly nice condition for which the Faddeev–Popov determinant is unity \[78\]. Any local Lorentz gauge allows one to solve for the vierbein in terms of the metric. For Lorentz symmetric gauge with \(\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\) this solution is \[78\].
The inverse vierbein is

\[ \delta^\mu_a \equiv \bar{g}^{\mu\nu} \bar{e}_\nu = \delta^\mu_a - \frac{1}{2} \kappa \delta^\mu_a \phi + \frac{3}{8} \kappa^2 \delta^\mu_a h_{\nu\lambda} - \cdots. \]  

(17)

### 2.2. The general coordinate transformation

A general coordinate transformation \( x^\mu \rightarrow x'^\mu(x) \) carries the metric to

\[ g'_{\mu\nu}(x') = \frac{\partial x^\sigma(x')}{\partial x'^\mu} \frac{\partial x^\tau(x')}{\partial x'^\nu} g_{\sigma\tau}(x) \iff g'_{\mu\nu}(x) = \frac{\partial x^\sigma(x)}{\partial x'^\mu} \frac{\partial x^\tau(x)}{\partial x'^\nu} g_{\sigma\tau}(x). \]

(18)

Under the (false) assumption that \( g_{\mu\nu} \) is constant in space and time it is clear that we seek a linear transformation

\[ x'^\mu = \Omega^\mu_\nu x^\nu \iff x^\mu = \omega^\mu_\nu x'^\nu, \quad \Omega^\mu_\nu \omega^\nu_\rho = \delta^\mu_\rho = \omega^\mu_\rho \Omega^\rho_\nu. \]

(19)

We require that the transformation makes the metric conformal

\[ g'_{\mu\nu}(x) = \omega^\rho_\mu \omega^\sigma_\nu g_{\rho\sigma}(\omega x) = a^2 (\omega^\rho_\mu x^\rho) \eta_{\rho\sigma} \quad \Rightarrow \quad \bar{g}_{\mu\nu} \omega^\rho_\mu \omega^\sigma_\nu = \eta_{\mu\nu}. \]

(20)

and also that the proportionality factor depends only on conformal time, although it may have a different Hubble constant

\[ \omega^0_\mu = \frac{H'}{H} \times \delta^0_\nu. \]

(21)

The first condition (20) is achieved by the symmetric gauge vierbein

\[ \bar{g}_{\mu
u} \bar{e}^\rho_a \bar{e}^\sigma_b = \eta_{ab}. \]

(22)

However, the vierbein does not generally obey the condition (21). To enforce this without disturbing (20) we concatenate a Lorentz transformation

\[ \omega^\mu_\nu = \bar{e}^\mu_a \Lambda^a_\nu. \]

(23)

The desired transformation takes the form of a boost whose \( 3 + 1 \) decomposition can be expressed in terms of a 3-velocity \( \beta^i = \beta^i \beta^j \) with \( \gamma = 1/\sqrt{1 - \beta^2} \)

\[ \Lambda^\mu_\nu \equiv \left( \begin{array}{cc} \Lambda^0_0 & \Lambda^0_n \\ \Lambda^m_0 & \Lambda^m_n \end{array} \right) = \left( \begin{array}{cc} \gamma & -\beta^i \beta^m \\ -\beta^i \beta^m & \delta^{mn} + (\gamma - 1) \beta^m \beta^n \end{array} \right). \]

(24)

The \( 3 + 1 \) expression for the full transformation is

\[ \left( \begin{array}{c} \omega^0_0 \\ \omega^0_n \\ \omega^m_0 \\ \omega^m_n \end{array} \right) = \left( \begin{array}{c} \bar{e}^0_0 \\ \bar{e}^0_i \\ \bar{e}^m_0 \\ \bar{e}^m_i \end{array} \right) \times \left( \begin{array}{cc} \Lambda^0_0 & \Lambda^0_n \\ \Lambda^m_0 & \Lambda^m_n \end{array} \right). \]

(25)

---

3 We follow the usual \( 3 + 1 \) convention of making no distinction between upper and lower indices for intrinsically spatial quantities such as \( \beta^i = \beta^i \) and \( \delta^{mn} = \delta^{mn} \).
\[
\begin{pmatrix}
\varepsilon^0_{\mu \alpha} + \varepsilon^0_{\alpha i} & \varepsilon^0_{\mu \beta} + \varepsilon^0_{\beta i} \\
\varepsilon^m_{\mu \alpha} + \varepsilon^m_{\alpha i} & \varepsilon^m_{\mu \beta} + \varepsilon^m_{\beta i}
\end{pmatrix}
\]  
(26)

Condition (21) requires \( \omega^0_n = 0 \), which implies

\[
\begin{align*}
\beta^j &= \frac{\varepsilon^0_j}{\sqrt{\varepsilon^0_j \varepsilon^0_j}}, \\
\beta^i &= \frac{\varepsilon^0_i}{\varepsilon^0_0}, \\
\gamma &= \frac{\varepsilon^0_0}{\sqrt{-g^{00}}}. 
\end{align*}
\]  
(27)

The time–time component of the transformation gives us the multiplicative change in the Hubble constant

\[
\frac{H'}{H} = \omega^0_0 = \sqrt{-g^{00}} = 1 + \frac{1}{2} \kappa h_{00} + \frac{3}{8} \kappa^2 h_{00}^2 - \frac{1}{2} \kappa^2 h_{00} h_{00} + \cdots. 
\]  
(28)

The space–time component can be expressed in terms of the metric

\[
\omega^m_0 = -\frac{\sqrt{-g^{0m}}}{\sqrt{-g^{00}}} = -\kappa h_{m0} - \frac{1}{2} \kappa^2 h_{00} h_{0m} + \kappa^2 h_{00} h_{im} + \cdots. 
\]  
(29)

The space–space component has a superficially complicated form that can be recognized as the inverse of the three-dimensional dreibein

\[
\omega^m_n = \varepsilon^m_n - \frac{\varepsilon^m_0 \varepsilon^0_n}{\sqrt{-g^{00}}} + \varepsilon^m_j \varepsilon^0_n \left( \frac{\varepsilon^0_j}{\sqrt{-g^{00}}} - 1 \right), 
\]  
(30)

\[
= \delta_{mn} - \frac{1}{2} \kappa h_{mn} + \frac{3}{8} \kappa^2 h_{mn} h_{mn} + \cdots. 
\]  
(31)

2.3. The associated Dirac spinor transformation

We stress that local Lorentz symmetry is completely fictitious. When one fixes it by imposing some local Lorentz gauge condition then a general coordinate transformation will generally disrupt the condition. One defines the associated local Lorentz transformation by requiring it to restore the gauge condition. This is how spinor indices are transformed in general relativity.

Using expression (23) one can see that our general coordinate transformation takes the vierbein to

\[
\tilde{e}^\mu_{\mu a} = \omega^\rho_{\mu a} \tilde{e}^\rho = \frac{\varepsilon^{ab} \Lambda^b_{\mu}}{\eta_{ab} \Lambda^b_{a \mu}}. 
\]  
(32)

From the Lorentz invariance of \( \eta_{ab} \) we see that the additional, local Lorentz transformation, needed to restore \( \tilde{e}^\mu_{\mu a} \) to symmetric gauge is the very same one (24) with parameters (27)

\[
\tilde{e}^\mu_{\mu a} = \eta_{ab} \Lambda^b_{a \mu}. 
\]  
(33)

It remains to construct the Dirac spinor representation which corresponds to the boost (27). The general spinor Lorentz transformation takes the form

\[ \footnote{It is amusing to note that the order \( \kappa^4 \) contribution implies secular back-reaction at two loop order, which is something else which the skeptics disbelieve [79, 80].} \]
\[ \Lambda_{ij} = \exp \left[ -\frac{i}{2} \theta_{ij} J^{ab} \right], \quad [J]^{ab} = \frac{i}{4} [\gamma^a, \gamma^b]_{ij}, \]  

(34)

where the gamma matrices are \( \gamma^i \). The boost (27) is achieved by choosing the infinitesimal parameters as

\[ \theta^{ij} = -\tilde{\beta}^j \tanh^{-1}(\beta), \quad \theta^{ij} = 0. \]  

(35)

The spinor transformation takes the 2-component form

\[ \Lambda = \begin{pmatrix} B & 0 \\ 0 & B^{-1} \end{pmatrix}, \quad B = \frac{1}{\sqrt{2(1 + \gamma)}} \left( I + \frac{\gamma}{\sqrt{2(1 + \gamma)}} \cdot \tilde{\sigma} \right). \]  

(36)

where \( I \) is the 2 \( \times \) 2 unit matrix and \( \tilde{\sigma} \equiv (\sigma_1, \sigma_2, \sigma_3) \) are the Pauli matrices. Expanding the 2 \( \times \) 2 matrix \( B \) gives

\[ B = \left[ 1 + \frac{9\kappa^2}{32} h_{00}^2 - \frac{\kappa^2}{4} h_{0i} h_{0i} + \cdots \right] I + \left[ \frac{\kappa}{4} h_{0i} + \frac{\kappa^2}{16} h_{00} h_{00} - \frac{3\kappa^2}{8} h_{ij} h_{ij} + \cdots \right] \sigma_i. \]  

(37)

3. The photon polarization vector

The purpose of this section is to compare exact one loop results with the predictions of the transformation ansatz for the leading secular corrections to the photon polarization vectors from quantum gravity at one loop (\( \kappa^2 \)) order. We begin by summarizing the exact one loop computation [42, 43]. Then we use the results of the previous section to derive the prediction of the transformation ansatz.

3.1. The exact computation

The computation was performed in two steps. First, dimensional regularization and BPHZ renormalization were employed to evaluate the diagrams of figure 1 giving the one graviton loop contribution to the vacuum polarization \( -i\Pi_{\mu\nu}(x; x') \) on de Sitter background [42]. In the second step, the linearized Schwinger–Keldysh effective field equations were solved for plane wave photons \( A_\mu(x) = \epsilon_\mu(x, \kappa^\nu \tilde{k}_\nu) \times e^{i\delta} \) [43]

\[ \partial_\nu [\sqrt{-g} \ g^{\rho\sigma} F_{\rho\sigma}(x)] + \int d^4x' [\Pi^\nu(x; x')] A_\nu(x') = 0. \]  

(38)

Because four-dimensional photons are conformally invariant, the tree order polarization vector on de Sitter is identical (in conformal coordinates) to the usual flat space one,
\[ \epsilon_{\mu}^{(0)}(\eta, \vec{k}, \lambda) = \frac{e^{-i k \phi}}{\sqrt{2k}} \begin{pmatrix} 0 & i \lambda \phi \hat{\eta} \\ \hat{\eta} & i \lambda \phi \end{pmatrix} \equiv \begin{pmatrix} \epsilon_{\mu}^{(0)}(\eta, \vec{k}, \lambda) \\ \epsilon_{\mu}^{(0)}(\eta, \vec{k}, \lambda) \end{pmatrix}, \]

where \( \hat{\eta} \) and \( \tilde{\phi} \) are the other orthogonal unit vectors in the system where the momentum \( \vec{k} = k \hat{r} \) is radial. At late times the one loop correction takes the form

\[ \epsilon_{\mu}^{(1)}(\eta, \vec{k}, \lambda) \longrightarrow \frac{\kappa^2 H^2}{8 \pi^2} \frac{ik \ln(a)}{H a} \times \epsilon_{\mu}^{(0)}(\eta, \vec{k}, \lambda). \]

Although the one loop polarization vector (40) actually falls off with respect to tree order result (39), its approach to zero is slower than the latter’s approach to a constant. Hence the one loop electric field strength grows relative to the tree result [43]

\[ F_{\mu}^{(1)}(\eta, \vec{x}) \longrightarrow \frac{\kappa^2 H^2}{8 \pi^2} \ln(a) \times F_{\mu}^{(0)}(\eta, \vec{x}), \]

\[ F_{\nu}^{(1)}(\eta, \vec{x}) \longrightarrow \frac{\kappa^2 H^2}{8 \pi^2} \frac{ik \ln(a)}{H a} \times F_{\nu}^{(0)}(\eta, \vec{x}). \]

This growth must eventually lead to a breakdown of perturbation theory. The physical interpretation of (41) seems to be that the photon’s physical 3-momentum redshifts like \( k/a \), so 3-momentum tends to be added by scattering with the ensemble of inflationary gravitons, whose peak 3-momenta remains at about \( H \) due to continual production.

### 3.2. Prediction of the transformation ansatz

The graviton propagator in the gauge which was used for the explicit computations [42, 43, 46, 47] takes the form of a sum of scalar propagators times constant tensors [81, 82]

\[ i[\mu \nu \Delta_{\mu \nu}](x; x') = \sum_{I=A,B,C} i \Delta_I(x; x') \times [\mu \nu T_{\mu \nu}]. \]

The \((D\text{-dimensional})\) tensor factors are expressed in terms of the purely spatial Lorentz metric \( \eta_{\mu \nu} \equiv \eta_{\mu \nu} + \delta^0_\mu \delta^0_\nu \),

\[ [\mu \nu T_{\mu \nu}^A] = \eta_{\mu \rho} \eta_{\nu \sigma} + \eta_{\mu \sigma} \eta_{\nu \rho} - \frac{2}{D - 3} \eta_{\mu \rho} \eta_{\nu \rho}, \]

\[ [\mu \nu T_{\mu \nu}^B] = -\delta^0_\mu \eta_{\nu \rho} \delta^0_\sigma - \delta^0_\nu \eta_{\mu \rho} \delta^0_\sigma - \delta^0_\nu \eta_{\mu \sigma} \delta^0_\rho - \delta^0_\mu \eta_{\nu \sigma} \delta^0_\rho, \]

\[ [\mu \nu T_{\mu \nu}^C] = \frac{2}{(D - 3)(D - 3)} [(D - 3) \delta^0_\mu \delta^0_\nu + \eta_{\mu \nu}] (D - 3) \delta^0_\mu \delta^0_\nu + \eta_{\mu \nu}). \]

The \(A\)-type, \(B\)-type and \(C\)-type propagators are those of minimally coupled scalars with masses

\[ m_A^2 = 0, \quad m_B^2 = (D - 2) H^2, \quad m_C^2 = 2(D - 3) H^2. \]

Their full spacetime dependence is well known [81, 82] but the only thing we require for this analysis is their coincidence limits

\[ i \Delta_A(x; x) = \text{Constant} + \frac{H^2}{4 \pi^2} \ln(a), \quad i \Delta_B(x; x) = \text{Constant} = i \Delta_C(x; x). \]

The leading secular growth comes from just the logarithm in \( i \Delta_A(x; x) \) which, by the way, agrees with the results of Vilenkin and Ford [15], Linde [16], Starobinsky [17] and Allen and Folacci [18]. For our purposes the expectation value of two gravitons is therefore
The transformation ansatz asserts that the leading secular growth in the solution of (38) comes from taking the expectation value, in the graviton vacuum, of the transformed tree order solution

\[ \langle \Omega | \kappa h_{\mu}(\chi) \kappa h_{\nu}(\chi) | \Omega \rangle \rightarrow \frac{\kappa^2 H^2 \ln(a)}{4\pi^2} \times \left[ \bar{\eta}_{\mu\nu} \bar{\eta}_{\alpha\beta} + \bar{\eta}_{\mu\alpha} \bar{\eta}_{\nu\beta} - 2\bar{\eta}_{\mu\nu} \bar{\eta}_{\alpha\beta} \right]. \]  

(49)

The matrix \( \omega^\mu_\nu \) is the one we constructed in section 2.2, with the graviton fields evaluated at \( x^\mu = (\eta, \bar{x}) \). Of course the transformation derived in section 2.2 was only valid for spacetime constant graviton fields. When the spacetime dependence of the graviton field becomes significant there is no transformation which can re-impose de Sitter background. We therefore expect the transformation ansatz to disagree with the exact computation (40), but it is worth making the comparison in order to demonstrate that secular growth is neither the same as infrared divergence, nor is it pure gauge.

Expression (49) shows that only the purely spatial components of the graviton field contribute secular growth factors. We can therefore drop the temporal components from series expansions (28), (29) and (31)

\[ \omega^\mu_\nu \equiv \begin{pmatrix} \omega^0_0 & \omega^0_n \\ \omega^n_0 & \omega^n_n \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 \\ \delta_{m n} - \frac{\kappa}{2} h_{m n} + \frac{3\kappa^2}{8} h_{m i} h_{n i} + \cdots \end{pmatrix}. \]  

(51)

There are two factors in (50), the spatial plane wave

\[ e^{i k_x^\mu x_\nu} = e^{i \tilde{k}_x^\mu \tilde{x}_\nu} \left\{ 1 - \frac{ik_x}{2} h_{m n} k^m k^n + \frac{3\kappa^2}{8} h_{m i} h_{n i} k^m k^n \right\} \]  

(52)

and the (purely spatial) vector transformation

\[ \omega^i_j e^{(0)}_j = e^{(0)}_j - \frac{\kappa}{2} h_{ij} e^{(0)}_j + \frac{3\kappa^2}{8} h_{ik} h_{jk} e^{(0)}_j + \cdots. \]  

(53)

Multiplying (53) by (52) and taking the expectation value using (49) gives

\[ A^T_A(x) = e^{(0)}_i(\eta, \tilde{k}, \lambda) e^{i \tilde{k}_x \tilde{x}} \]

\[ + \frac{\kappa^2 H^2 \ln(a)}{16\pi^2} \{ (3 + i\tilde{k} \cdot \tilde{x} - k^2x^2) + (\tilde{k} \cdot \tilde{x})^2 \} e^{(0)}_i(\eta, \tilde{k}, \lambda) e^{i \tilde{k}_x \tilde{x}} + O(\kappa^3). \]  

(54)

As expected, the prediction (54) of the transformation ansatz disagrees with the exact one loop computation (40). Not only is (54) stronger by a (huge) scale factor, it also contains some strange factors of the 3-momentum and the spatial position.

4. The fermion wave function

The purpose of this section is to compare exact one loop results with the predictions of the transformation ansatz for the leading secular corrections to the fermion mode functions from quantum gravity at one loop (\( \kappa^2 \)) order. We begin by summarizing the exact one loop computation [46, 47]. Then we use the results of the previous section to derive the prediction of the transformation ansatz.
4.1. The exact computation

Like the electromagnetic analog (which was actually performed afterwards) the computation was made in two steps. First, dimensional regularization and BPHZ renormalization were employed to evaluate the diagrams of figure 2 to give the one graviton loop contribution to the self-energy $\Sigma_{\text{xx}}^{(0)}(s, x')$ of massless Dirac fermions on de Sitter background [46]. In the second step, the linearized Schwinger–Keldysh effective field equations were solved for (conformally rescaled) plane wave fermions $\Psi(x) = u_{i}(\eta, \vec{k}, s) \times e^{i x \cdot x} [47]$

$$i \gamma_{ij}^\mu \partial_{\alpha} \Psi_{\alpha}(x) - \int \! d^{4}x' [\Sigma_{ij}(s; x')] \Psi_{\alpha}(x') = 0.$$  \hspace{1cm} (55)

Because massless fermions are conformally invariant, the tree order mode function on de Sitter is identical to the usual flat space one

$$u_{i}^{(0)}(\eta, \vec{k}, s) = \frac{e^{-i \delta_{i}}}{\sqrt{2}} \begin{bmatrix} I - \vec{k} \cdot \tilde{\sigma} \xi(s) \\ I + \vec{k} \cdot \tilde{\sigma} \xi(s) \end{bmatrix},$$  \hspace{1cm} (56)

where $I$ is the $2 \times 2$ unit matrix, $\tilde{\sigma} = (\sigma_{1}, \sigma_{2}, \sigma_{3})$ are the Pauli matrices and $\xi(s)$ is a 2-component spinor. At late times the one loop correction takes the form [47]

$$u_{i}^{(1)}(\eta, \vec{k}, s) \longrightarrow \frac{17}{2} \frac{k^{2}H^{2}}{2^{\eta} \pi^{2}} \ln(a) \times u_{i}^{(0)}(\eta, \vec{k}, s).$$  \hspace{1cm} (57)

The physical interpretation seems to be that the ensemble of inflationary gravitons which pervades space scatters the propagating fermion by an amount which grows larger the farther in space (and hence the longer in time) the fermion propagates. One fascinating consequence of the secular growth evident in (57) is that perturbation theory must eventually break down.

A vertex-by-vertex examination of the computation revealed that the secular growth factors of $\ln(a)$ in (57) all derive from the singly differentiated graviton fields of the spin connection [45]. This is consistent with the physical insight that the spin–spin coupling between fermions and gravitons allows even a highly redshifted fermion to continue interacting with inflationary gravitons. That makes good sense but it is completely at odds with the basic assumption of the transformation ansatz that there is no distinction between infrared divergences and secular effects, and that spacetime constant field configurations (which would be annihilated by differentiation) are responsible for both phenomena. We therefore expect that the transformation ansatz will fail to recover the result (57) of explicit computation.

4.2. Prediction of the transformation ansatz

The transformation ansatz asserts that the leading secular corrections to the fermion mode function come from taking the expectation value, in the graviton vacuum, of the transformed
tree order solution
\[ \Psi^{TA}(x) \equiv \langle \Omega | \Lambda_{ij} u^{(0)}_{i} (\omega^{\mu}_{\rho} x^{\rho}, \vec{k}, s) e^{i k \cdot x} | \Omega \rangle. \] (58)

Here \( \omega^{\mu}_{\rho} \) is the general coordinate transformation which was worked out in section 2.2, and \( \Lambda_{ij} \) is the associated local Lorentz transformation constructed in section 2.3. In both cases the graviton fields are evaluated at the same point \( x^{\mu} = (\eta, \vec{x}) \) as the mode function. As with the photon case, this is invalid because the transformations were constructed under the assumption of constant graviton fields. Indeed, there is no transformation which would restore de Sitter for spacetime dependent graviton fields. However, we evaluate (60), and compare it with the exact result (57), to demonstrate that secular graviton corrections are neither the same as infrared divergences, nor are they pure gauge.

Because only spatial graviton fields engender secular dependence (49), we can neglect any graviton fields with temporal components from \( \omega^{\mu}_{\rho} \) and \( \Lambda_{ij} \). The result of doing this for \( \omega^{\mu}_{\rho} \) was given in expression (51). From (36), and the expansion (37), we see that dropping the temporal graviton fields makes a dramatic simplification in the Lorentz transformation,
\[ \Lambda_{ij} \rightarrow \delta_{ij}. \] (59)

It follows that we need only the spatial plane wave factor already expanded in (52). Taking the expectation value using (49) gives
\[ \Psi^{TA}(x) = u^{(0)}_{i} (\eta, \vec{k}, s) e^{i \vec{k} \cdot \vec{x}} \]
\[ + \frac{\kappa^{2} H^2 \ln(a)}{16 \pi^2} \left[ 3i \vec{k} \cdot \vec{x} - k^2 x^2 + (\vec{k} \cdot \vec{x})^2 \right] u^{(0)}_{i} (\eta, \vec{k}, s) e^{i \vec{k} \cdot \vec{x}} + O(\kappa^4). \] (60)

This possesses the same exotic spatial dependence as the transformation ansatz prediction for the photon (54), and it is equally discordant with the exact computation (57).

5. Discussion

The cosmological perturbations generated by primordial inflation arise because the mode functions of MMC scalars and gravitons freeze in to nonzero constants (4) and (5) after their physical wave lengths have red-shifted beyond the Hubble radius. The fact that more and more modes reach this saturated limit as inflation proceeds is responsible for the appearance of secular loop corrections. This inevitable consequence of freezing-in has occasioned much angst and scepticism within the very same community which hails the tree order effect—the generation of primordial perturbations—as a triumph of inflation theory. This curiously contradictory attitude is often justified by arguing that because the spacetime dependence of very long wavelength metric perturbations cannot be discerned by a local observer, they should be subsumed into a coordinate redefinition. The argument runs that secular loop corrections are merely the effect of evaluating tree order, noninvariant correlators at these transformed coordinates. We refer to this belief as the transformation ansatz.

There are two reasonable approaches to demonstrating the reality of secular loop corrections:
1. Show that they appear even in the expectation values of invariant operators [83, 84]; and
2. Evaluate a tree order, noninvariant correlator at the appropriately transformed coordinates and compare the result with secular loop corrections computed in an exact computation.

We have here followed the second approach, taking as our points of comparison two exact one loop computations of the quantum gravitational corrections to the photon [42, 43].
and fermion [46, 47] wave functions on de Sitter background. In neither case did the leading secular dependence implied by the transformation ansatz—expressions (54) and (60)—agree with the exact computation—expressions (40) and (57). The unavoidable conclusion would seem to be that the transformation ansatz is incorrect. That is not surprising because the ansatz was based on ignoring the spacetime dependence of the graviton field whereas the phenomenon of secular growth derives precisely from the continual passage of modes from the spacetime dependent, sub-horizon form to the spacetime constant, super-horizon form.

It is conceivable that some other ansatz can be devised to explain away secular dependence as the sceptics wish. There have been many, many expressions of the belief that it is pure gauge, and we confess to some frustration in extracting explicit and testable assertions from them. Much of the relevant literature seems based on confusing infrared divergences (which really should be pure gauge) with secular growth (which seems to be a physical effect). It is also characterized by imprecision about approximations (small is not the same as zero, $\epsilon = 0$ is not the same as $\epsilon \neq 0$, $\dot{\epsilon} = 0$ is not the same as $\dot{\epsilon} \neq 0$), and by poorly articulated principles which occasionally shade into mysticism. For example, why are time dependent but spatially constant quantities unobservable? How can any local field become constant in an interacting, four-dimensional quantum field theory? Why do alleged proofs of these assertions begin by specializing to unphysical models of inflation which exclude the normal fields of the standard model and are guaranteed not to experience sufficient reheating? And why would it be a tragedy for the power spectra to be time dependent in their 8th significant figure?

The degree of scepticism towards secular effects from quantum gravity is sometimes difficult to fathom. For example, no one doubts that a homogeneous ensemble of gravitational radiation on flat space background would scatter light by an amount which increases the further (and hence the longer in time) the light propagates. This is the basis for pulsar timing measurements of gravitational radiation. How can there be any doubt that inflationary gravitons have the same effect?

In formulating the transformation ansatz we have done our best to extract a clear and explicit enunciation of the gauge artifact belief which can be checked. However, it is possible that those sceptical about secular loop effects have some other analytic realization in mind. If so, we apologize for having misinterpreted their work, we invite them to enlighten us as to its true meaning, and we propose that they check it against the explicit one loop results (40) and (57).

Acknowledgments

We are grateful for conversations and correspondence with S Giddings, G Pimentel and M Sloth. This work was partially supported by NSF grant PHY-1506513 and by the Institute for Fundamental Theory at the UF.

References

[1] Starobinsky A A 1979 JETP Lett. 30 682
Starobinsky A A 1979 Pis’ma Zh. Eksp. Teor. Fiz. 30 719
[2] Mukhanov V F and Chibisov G V 1981 JETP Lett. 33 532
Mukhanov V F and Chibisov G V 1981 Pis’ma Zh. Eksp. Teor. Fiz. 33 549
[3] Woodard R P 2009 Rep. Prog. Phys. 72 126002
[4] Ashoorioon A, Bhupal Dev P S and Mazumdar A 2014 Mod. Phys. Lett. A 29 1450163
[5] Krauss L M and Wilczek F 2014 Phys. Rev. D 89 047501
[6] Mukhanov V F, Feldman H A and Brandenberger R H 1992 Phys. Rep. 215 203
[7] Liddle A R and Lyth D H 1993 Phys. Rep. 231 1
[8] Lidsey J E, Liddle A R, Kolb E W, Copeland E J, Barreiro T and Abney M 1997 Rev. Mod. Phys. 69 375
[9] Woodard R P 2014 Int. J. Mod. Phys. D 23 1430020
[10] Brooker D J, Tsamis N C and Woodard R P 2016 Phys. Rev. D 93 043503
[11] Brooker D J, Tsamis N C and Woodard R P 2016 Phys. Rev. D 94 044020
[12] Tsamis N C and Woodard R P 1994 Class. Quantum Grav. 11 2969
[13] Vilenkin A 1983 Nucl. Phys. B 226 527
[14] Ford L H and Parker L 1977 Phys. Rev. D 16 245
[15] Vilenkin A and Ford L H 1982 Phys. Rev. D 26 1231
[16] Linde A D 1982 Phys. Lett. B 116 335
[17] Starobinsky A A 1982 Phys. Lett. B 117 175
[18] Allen B and Folacci A 1987 Phys. Rev. D 35 3771
[19] Onemli V K and Woodard R P 2002 Class. Quantum Grav. 19 4607
[20] Onemli V K and Woodard R P 2004 Phys. Rev. D 70 107301
[21] Brunier T, Onemli V K and Woodard R P 2005 Class. Quantum Grav. 22 59
[22] Kahya E O and Onemli V K 2007 Phys. Rev. D 76 043512
[23] Prokopec T, Tornkvist O and Woodard R P 2002 Phys. Rev. Lett. 89 101301
[24] Prokopec T, Tornkvist O and Woodard R P 2003 Ann. Phys. 303 251
[25] Degueldre H and Woodard R P 2013 Eur. Phys. J. C 73 2457
[26] Prokopec T, Tsamis N C and Woodard R P 2008 Phys. Rev. D 78 043523
[27] Prokopec T and Woodard R P 2003 J. High Energy Phys. JHEP10(2003)059
[28] Garbrecht B and Prokopec T 2006 Phys. Rev. D 73 064036
[29] Miao S P and Woodard R P 2006 Phys. Rev. D 74 044019
[30] Starobinsky A A 1986 Lecture Notes Phys. 246 107
[31] Tsamis N C and Woodard R P 2005 Nucl. Phys. B 724 295
[32] Finelli F, Marozzi G, Starobinsky A A, Vacca G P and Venturi G 2009 Generation of fluctuations during inflation: comparison of stochastic and field-theoretic approaches Phys. Rev. D 79 044007
[33] Finelli F, Marozzi G, Starobinsky A A, Vacca G P and Venturi G 2010 Stochastic growth of quantum fluctuations during slow-roll inflation Phys. Rev. D 82 064020
[34] Prokopec T, Tsamis N C and Woodard R P 2008 Ann. Phys. 323 1324
[35] Starobinsky A A and Yokoyama J 1994 Phys. Rev. D 50 6357
[36] Bogoliubov N N and Parasiuk O S 1957 Acta Math. 97 227
[37] Hepp K 1966 Commun. Math. Phys. 2 301
[38] Zimmermann W 1968 Commun. Math. Phys. 11 1
[39] Zimmermann W 1969 Commun. Math. Phys. 15 208
[40] Zimmermann W 2000 Lecture Notes Phys. 558 217
[41] Kahya E O and Woodard R P 2007 Phys. Rev. D 76 124005
[42] Kahya E O and Woodard R P 2008 Phys. Rev. D 77 084012
[43] Leonard K E and Woodard R P 2014 Class. Quantum Grav. 31 015010
[44] Wang C L and Woodard R P 2015 Phys. Rev. D 91 124054
[45] Glavan D, Miao S P, Prokopec T and Woodard R P 2014 Class. Quantum Grav. 31 175002
[46] Miao S P and Woodard R P 2008 Class. Quantum Grav. 25 145009
[47] Miao S P and Woodard R P 2006 Class. Quantum Grav. 23 1721
[48] Miao S P and Woodard R P 2006 Phys. Rev. D 74 024021
[49] Maldacena J M 2003 J. High Energy Phys. JHEP05(2003)013
[50] Seery D, Lidsey J E and Sloth M S 2007 J. Cosmol. Astropart. Phys. JCAP01(2007)027
[51] Jarnhus P R and Sloth M S 2008 J. Cosmol. Astropart. Phys. JCAP02(2008)013
[52] Xue W, Gao X and Brandenberger R 2012 J. Cosmol. Astropart. Phys. JCAP06(2012)035
[53] Weinberg S 2005 Phys. Rev. D 72 043514
[54] Weinberg S 2006 Phys. Rev. D 74 023508
[55] Giddings S B and Sloth M S 2011 J. Cosmol. Astropart. Phys. JCAP01(2011)023
[56] Tsamis N C and Woodard R P 1992 Phys. Lett. B 292 269
[57] Rey S J 1987 Nucl. Phys. B 284 706
[58] Sasaki M, Nambu Y and Nakao K I 1988 Nucl. Phys. B 308 868
[59] Winitzki S and Vilenkin A 2000 Phys. Rev. D 61 084008
[59] Tsamis N C, Tzetzias A and Woodard R P 2010 J. Cosmol. Astropart. Phys. JCAP09(2010)016
[60] Giddings S B and Sloth M S 2010 J. Cosmol. Astropart. Phys. JCAP07(2010)015
[61] Urakawa Y and Tanaka T 2009 Prog. Theor. Phys. 122 779
[62] Urakawa Y and Tanaka T 2010 Prog. Theor. Phys. 122 1207
[63] Urakawa Y and Tanaka T 2010 Phys. Rev. D 82 121301
[64] Urakawa Y and Tanaka T 2011 Prog. Theor. Phys. 125 1067
[65] Miao S P and Woodard R P 2012 J. Cosmol. Astropart. Phys. JCAP1207(2012)008
[66] Giddings S B and Sloth M S 2011 Phys. Rev. D 84 063528
[67] Giddings S B and Sloth M S 2012 Phys. Rev. D 86 083538
[68] Tanaka T and Urakawa Y 2011 J. Cosmol. Astropart. Phys. JCAP1105(2011)014
[69] Urakawa Y 2011 Prog. Theor. Phys. 126 961
[70] Tanaka T and Urakawa Y 2013 Prog. Theor. Exp. Phys. 2013 083E01
[71] Tanaka T and Urakawa Y 2013 Prog. Theor. Exp. Phys. 2013 063E02
[72] Tanaka T and Urakawa Y 2013 Class. Quantum Grav. 30 233001
[73] Tanaka T and Urakawa Y 2014 Prog. Theor. Exp. Phys. 2014 073E01
[74] Senatore L and Zaldarriaga M 2013 J. High Energy Phys. JHEP1301(2013)109
[75] Senatore L and Zaldarriaga M 2012 J. Cosmol. Astropart. Phys. JCAP1208(2012)001
[76] Pimentel G L, Senatore L and Zaldarriaga M 2012 J. High Energy Phys. JHEP1207(2012)166
[77] Tsamis N C and Woodard R P 1986 Ann. Phys. 168 457
[78] Woodard R P 1984 Phys. Lett. B 148 440
[79] Garriga J and Tanaka T 2008 Phys. Rev. D 77 024021
[80] Tsamis N C and Woodard R P 2008 Phys. Rev. D 78 028501
[81] Tsamis N C and Woodard R P 1994 Commun. Math. Phys. 162 217
[82] Woodard R P 2006 Deserfest: a Celebration of the Life & Works of Stanley Deser ed J T Liu, M J Duff, K S Stelle and R P Woodard (Hackensack: World Scientific) 339-51
[83] Tsamis N C and Woodard R P 1992 Ann. Phys. 215 96
[84] Tsamis N C and Woodard R P 2013 Phys. Rev. D 88 044040