Luminous and Dark Matter in the Milky Way

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\textbf{ABSTRACT}

Axisymmetric models of the Milky Way exhibit strong interrelations between the Galactic constants (the Sun’s distance to the Galactic centre ($R_0$), and the local rotation speed ($\Theta_0$)), the local stellar column density ($\Sigma_* (R_0)$) and the shortest-to-longest axis ratio of the dark matter halo ($q$). In this paper we present simple analytical approximations that allow for an efficient search through the vastness of parameter space, and apply this approximation to investigate the consequences of the uncertain gaseous velocity dispersion ($\sigma_g$) on the constraints imposed by the thickness of the Milky Way’s gas layer. The extra degree of freedom does not significantly alter the conclusions drawn in a previous paper on the shape of the Milky Way’s dark matter halo. A significant contribution to the total gas pressure by cosmic rays and magnetic fields beyond the optical disk is thus ruled out. We find that the Milky Way’s dark halo is close to spherical if $R_0 \gtrsim 7.1$ kpc, while a significantly flattened dark matter halo is only possible if our distance to the Galactic centre is smaller than $\sim 6.8$ kpc.

Thus, if $R_0$ is larger than $\sim 7$ kpc, or $\Theta_0 \gtrsim 170$ km s$^{-1}$, we can rule out two dark matter candidates that require a highly flattened dark matter halo: 1) decaying massive neutrinos; and 2) a disk of cold molecular hydrogen.

It is only possible to construct a self-consistent axisymmetric model of the Galaxy based on the IAU-recommended values for the Galactic constants ($R_0 = 8.5$ kpc, $\Theta_0 = 220$ km s$^{-1}$) in the unlikely case that the effective gaseous velocity dispersion is $\sim 19\%$ larger than observed, \textit{and} if the local stellar column density is less than about 18 $M_\odot$ pc$^{-2}$. If we assume that the halo is oblate and a value of $\Sigma_*$ of 35 $M_\odot$ pc$^{-2}$ \cite{KuijkenGilmore1989}, we can rule out Galactic models with $R_0 \gtrsim 8.0$ kpc and $\Theta_0 \gtrsim 200$ km s$^{-1}$.

Combining the best kinematical and star-count estimates of $\Sigma_*$, we conclude that $\Sigma_*$ probably lies between 25 and 45 $M_\odot$ pc$^{-2}$. We find that Kuijken & Gilmore’s (1991) determination of the column density of matter within 1.1 kpc of the plane is robust and valid over a wide range of Galactic constants.

Our mass models show that, largely due to the uncertainty in the Galactic light distribution, the dark matter density in the Galactic centre is uncertain by up to three orders of magnitude. In the Solar neighbourhood this uncertainty is much reduced: our models imply a dark matter density of some 0.42 GeV/c$^2$ per cubic centimetre, or $(11 \pm 5) mM_\odot$ pc$^{-3}$—roughly 15% of the total mass density.

\textbf{Key words:} Galaxy: structure - Galaxy: kinematics and dynamics - Galaxy: solar neighborhood - Galaxy: fundamental parameters - Galaxy: stellar content

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1 INTRODUCTION

The observational fact that rotation curves of external galaxies are rather flat in their outer parts (Rubin, Ford & Thonnard 1983; Bosma 1981; Kent 1987; Egemen 1987; Lacerenza & van Gorkom 1991; Broeils 1992; Persic, Salucci & Stel 1996) indicates the presence of unseen matter in those galaxies. A problem which is specific for the Milky Way is that we do not know the shape of the Galactic rotation curve (RC). The slope of the RC depends on the assumed values of the Galactic constants (Olling & Merrifield 1998b, 1998c). But dark matter is required whatever the values of the Galactic constants. A fit to the observed rotation curve can be used to yield the stellar mass-to-light ratio $\Upsilon_\odot$, and the dark halo parameters, albeit with large uncertainties. Rather than using this fitting procedure, we adopt an analytical approximation in which there is only one free parameter: the degree to which the disk is maximal ($\gamma$), from which $\Upsilon_\odot$ and the dark halo parameters follow (see Olling 1995 for details). The parameter $\gamma$ is preferred for dynamical modeling purposes as it is bound between 0 and 1 for a zero mass and a full-hedged disk, respectively. For example, in the popular “maximum-disk” hypothesis (van Albada & Sancisi 1986), the amplitude of the stellar rotation curve equals $85\pm10\%$ of the observed rotation speed (Sackett 1997), so that $\gamma = 0.85$. In contrast, Bottema (1993) used stellar velocity dispersion measurements to “weigh” stellar disks, and concluded that they are sub-maximal, with $\gamma = 0.63\pm0.1$. The situation is similarly indeterminate for the Milky Way: the Kuijken & Gilmore (1989, hereafter referred to as KG89) model implies $\gamma \sim 0.5$, while more recent models with lower rotation speed and shorter scale-lengths can be close to maximal ($\gamma = 0.85\pm0.1$; Sackett 1997). As a result of the uncertain stellar mass-to-light ratio, the dark halo parameters are very ill determined for most galaxies (e.g., van Albada & Sancisi 1986; Lake & Feinswog 1989; Olling 1997). The Milky Way is no different: the combined uncertainty in the local disk mass and the stellar scale-length introduces an uncertainty of over three orders of magnitude in the central dark matter (DM) density of the Milky Way, and about an order of magnitude uncertainty in its core radius ($R_0$; Dehnen 1998). In the Solar neighborhood the situation is less dramatic, although no consensus exists on the local volume density of dark matter [$\rho_{DM}$ or the Oort limit; see e.g. and Crézé et al. (1998) for a recent review]. However, the local dark matter density is important for many astrophysical problems. For example, if the dark matter comprises elementary particles like neutralinos, axions, neutrinos, gravitinos etc., their expected detection rate is proportional to the DM density. Likewise, if the dark halo is made up of massive compact halo objects (MACHOs), the event rate for gravitational lensing depends on the integrated dark matter density along the line of sight towards the lens. Thus, observational

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 signatures of the Milky Way’s dark matter distribution like micro-lensing time scales and optical depths (Gates, Gyuk & Turner 1993), and expected neutralino annihilation rate (Bergstrom, Ullio & Buckley 1998) depend on the Galactic dark matter density distribution and hence the assumed values for the Galactic constants.

In a previous paper (Olling & Merrifield 2000, henceforth Paper I) we determined the dark matter density in the Solar neighbourhood at $R=R_0$ and around $R\sim2R_0$ to infer the minor to major axial ratio ($q = a/c$), or shape, of the dark matter halo of the Milky Way. In the present paper we investigate the reliability of several of the assumptions made in Paper I, and find that relaxing these assumptions does not greatly change the conclusion of Paper I: the shape of the dark matter halo of the Milky Way is probably rather round. Before going into more detail, let us review some of the difficulties which arise when one tries to determine $\rho_{DM}$.

First, large values of the local Galactic rotation speed ($\Theta_0$) result in large DM densities, while low rotation speeds require small DM densities. Second, since the shape of the Galactic rotation curve depends on the value of our distance to the Galactic center (Olling & Merrifield 1998a), the amount of dark matter increases with $R_0$ at constant $\Theta_0$. And finally, more highly flattened DM halos have larger midplane densities (Olling 1995).

We use two sets of observations to constrain the midplane dark matter density. First, stellar kinematical data provides a measure of the total column density within 1.1 kpc of the plane ($\Sigma_{col}$); cf. Kuijken & Gilmore 1991. The dark matter density follows after subtracting the luminous components and dividing by the scale-height. This method yield ambiguous results because uncertainties in the surface density of stellar mass ($\Sigma_*$) translate into a similar uncertainty of the DM density. Thus, low values of $\Sigma_*$ require more dark matter and hence a more highly flattened dark halos at constant $R_0$ and $\Theta_0$.

Second, the rate at which the thickness of the gas layer increases with radius (“flaring”) is a measure of $\rho_{DM}$. Assuming, for now, a hydrostatic balance between internal pressure and gravity, it follows that an increasing gas layer width is evidence for a decreasing midplane density. Large dark matter densities result in thin gas layers, while low densities yield a thicker gas disk. A larger DM density, due to either a larger $\Theta_0$ and/or $R_0$ or a smaller q, results in a thinner gas layer. However, such a thinner gas layer would also occur if the actual gas pressure—or equivalently, the gaseous velocity dispersion, $\sigma_g$—is smaller than assumed. Hence the significant correlations between the assumed values of the Galactic constants, $\Sigma_*$ and $\sigma_g$ and the inferred shape of the Milky Way’s dark matter halo referred to above.

In practice, the stellar kinematical data impose correlations between $\Theta_0$ and $q$ at the Solar circle, while the observed H I flaring does so at $R \sim (2 \pm 0.25)R_0$. At these large radii the stellar disk has vanished so that the poten-

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† See McGaugh & de Blok (1998) for a review of the alternative hypothesis that the law of gravity has to be modified instead.

‡ Throughout this paper we use cylindrical coordinates with $R$ the Galactocentric distance, and $z$ the distance of the plane.
tial is dominated by the gaseous self-gravity and the dark matter.

Unfortunately, the Galactic constants are ill-determined (Kerr & Lynden-Bell 1984; Reid 1993). Olling & Merrifield 1998a, 2000), and this has consequences for analyses that depend on \( R_0 \) and \( \Theta_0 \). In an ideal world, parameters such as the Galactic constants, the rotation curve \([\Theta(R)]\), the scale-length of the optical disk \( (h_d) \), the local stellar column density, the gaseous velocity dispersion, and so forth, are not only measured, but also have normal errors. In that case one could determine quantities that depend on these parameters, such as the DM density or the halo’s shape, by comparing the model and observed parameters in a \( \chi^2 \) sense. Unfortunately, the quoted errors on \( R_0 \) and \( \Theta_0 \) are not normal, and the values themselves are not averages in the statistical sense, but are rather consensus values with consensus errors. These arguments lead to the conclusion that the values of the Galactic constants and their errors can not be used in analyses like maximum-likelihood estimates of a property \( X \) which depends on the Galactic constants. Instead we urge researchers to investigate the dependence of their results on the assumed values of the Galactic constants and present their conclusions as functions of \( R_0 \) and \( \Theta_0 \). Given the tremendous increase in computing power, such an approach is currently more feasible than in the past. Practicing what we preach, we follow this approach in previous papers (Olling & Merrifield 1998c, 2000) as well as in the current article.

In Paper I we used both the constraints set by the flaring of the H I layer as well as the boundary conditions imposed by the local stellar kinematics to infer the halo’s flattening. The procedure works as follows: 1) pick values for \( R_0 \) and \( \Theta_0 \) and determine the corresponding rotation curve, 2) create mass models with as ingredients the stellar bulge and disk, the interstellar medium (ISM) and a dark halo of varying degrees of flattening, 3) pick values for the observationally ill-determined scale-length and mass of the stellar disk, 4) select a value for \( \sigma_g \) and calculate model flaring curves (Olling 1995), 5) select the model with flattening \( q_{HI} \) such that the observed H I flaring at \( R \sim 2R_0 \) is reproduced [this flattening will be independent of the distribution of the stellar mass], 6) now vary the stellar mass \( (\Sigma_*) \) at \( R_0 \) to keep track of the dark matter column density \( \Sigma^{1.1} \) that needs to be added to \( \Sigma_* \) to match \( \Sigma^{1.1} \), and finally, 9) select the model that identical \( q_{HI} \) and \( q_{1.1} \) values. This method is equivalent to finding the zero-point of the \([q_{HI}(\Sigma_*) - q_{1.1}(\Sigma_*)]\) function. Graphically, this process can be represented as determining the intersection of the \( q_{HI}(\Sigma_*) \) and \( q_{1.1}(\Sigma_*) \) curves (see Paper I, figure 3). Note that the \( q_{1.1}(\Sigma_*) \) relation hardly depends on the disk scale-length.

In this manner we construct self-consistent mass models, with specific values for \( h_d \), \( \Sigma_* \) and \( q_0 \), for the selected combination of the Galactic constants. Possibly the weakest link in this procedure is the assumed value for \( \sigma_g \). In the present paper we will remedy this shortcoming of Paper I by repeating the procedure outlined above for many values of the gaseous velocity dispersion. Picking a value for \( \sigma_g \) in the outer Galaxy is related to the question as to the relevance of non-thermal pressure support (due to magnetic fields and cosmic ray energy density) of the gas layer. We investigate these issues in detail in section 3.

The surface density of stars in the Solar neighbourhood provides a significant constraint on the determination of the halo’s shape. We therefore review recent determinations in section 3.1. Kuijken & Gilmore (1991, henceforth KG91) claim that the column density within 1.1 kpc of the plane is much better determined than the values of the individual components. However, their Galactic constants lie at the high end of the range we use. Since \( \Sigma^{1.1}_\text{tot} \) provides us with such an important constraint, we consider it prudent to check KG91’s assertion. We indeed confirm KG91’s finding that \( \Sigma^{1.1}_\text{tot} \) is relatively well determined over a large range in Galactic constants, \( \Sigma_* \) and \( h_d \) (Appendix A). In the following section we describe our mass models in more detail and determine the density of dark matter in the Solar neighbourhood. We summarize and conclude in 5.

2 MASS MODELS

In order to interpret the available data we build axisymmetric Milky Way mass models for which we determine the model values of the total disk mass, the H I flaring, and other parameters. It is well-known that the Milky Way deviates from azimuthal symmetry. However, as we have shown in Paper I, and will do so again below, the current observational constraints are only barely good enough to rule-out the most extreme combinations of Galactic constants. Thus, it would be unrealistic to try to incorporate fine-structure in the stellar mass distribution due to a bar and/or spiral structure, especially since these features are not terribly well determined themselves. This situation may dramatically change with the advent of future astrometric satellites (e.g., DIVA, FAME, GAIA, SIM) that could determine the stellar column density over large parts of the Galactic disk.

A comparison between the observations and the models yields the size, mass and shape of the Milky Way. Our models include: a stellar bulge and disk, a gas layer, and a dark halo. Some of the relevant parameters of our models are tabulated in Table 1. We consider models with a range of Galactic constants, disk exponential scale-length, stellar disk mass, total column density, and halo flattening. To calculate the exact vertical force law \((K_a(R,z))\) at every point \((R,z)\) in the Galaxy we integrate over the full mass distribution:

\[
K_a(R,z) = G \int_{z}^{\infty} r d\rho(r,0) \int_{\infty}^{\infty} dw \rho(r,w) \int_{-\pi}^{\pi} \frac{d\phi}{\pi} \frac{d\phi}{2\pi} \text{ with}
\]

\[\mathcal{S} = \{r,w\}, \mathcal{S} = \{R,z\}, \rho \text{ the total mass density at } (R,z), \text{ and } G \text{ Newton’s constant of gravity (Olling 1995). Since the calculation of } K_a \text{ and the model flaring curve is rather expensive, we perform these detailed calculations only for the limited subset of models listed in Table 2. We determine}\]

\[\text{The calculation of model gas layer widths for a given combination of } R_0, \Theta_0, h_d \text{ and bulge and disk mass-to-light ratios takes about 1.3 hours per } q \text{-values, on a SPARC-10 processor.}\]
Table 1. Some parameters used in the model calculations.

| Parameter       | Value          |
|-----------------|----------------|
| $d_K$           | 3.53 pc        |
| $q_K$           | 0.61 pc        |
| $h_b$           | 667 pc         |
| $\zeta_b$       | 10 pc          |
| $\rho_b(0)$     | 15.24 $L_{\odot,K}$ pc$^{-3}$ |
| $L_{b,K,tot}$   | $1.5 \times 10^{10} L_{\odot,K}$ |

The bulge is a modified spheroid luminosity distribution. We base our model for the bulge on Kent’s (1992) K-band exponential disk with central K-band surface brightness exponentially beyond 3 kpc. We use disk mass-to-light ratios ($\Upsilon_{b,K}$) such that Kuijken & Gilmore’s (1989b) 2-σ range for the local stellar column density is spanned. The stellar disk is truncated at $(R_0 + 4.5)$ kpc (Robin, Crézé & Mohan 1993; Freudenreich 1996, 1998).

2.1.2 Interstellar Medium Components

We now turn to the contribution to the mass models from the interstellar medium. Our location inside the Milky Way means that distances to diffuse components like the ISM are based on a kinematical model. Thus, important properties like the full width at half maximum (FWHM), volume density, and total mass of the ISM depend on the Galactic constants and the Milky Way’s rotation curve. Thus, we re-determine the atomic and molecular gas distributions for each choice of $R_0$ and $\Theta_0$. While the columndensities at fractional radius $R/R_0$ are independent of the choice of $R_0$ & $\Theta_0$, the number density, the thickness and the total gas mass of the Galaxy are not. Observationally, we can currently do no better than determining the gaseous column-density ($\Sigma_g$) at fractional radius $R/R_0$. Likewise, since the thickness measurement returns an angular size, we can only determine FWHM/$R_0$ at fractional radius $R/R_0$ [FWHM$/R_0 = \zeta/R/R_0$, (Binney & Merrifield 1998), their figure 9.25]. Physical length-scales for the thickness and Galactocentric radius can be assigned after choosing a value for $R_0$.

For the inner Galaxy, the H I columndensity was determined from the midplane volume density (Burton 1988) and the observed thickness of the layer (Malhotra 1994). The H I columndensities for $R \geq R_0$ were taken from Wouterloot et al. (1990). The H2 columndensities for the inner and outer Galaxy were copied from Bronfman et al. (1988) and Wouterloot et al. (1990), respectively.

For the radial profile, from Binney & Merrifield’s (1998) equation 9.12 it follows that the distance-dependent function that appears in the exponential term only depends on $r = R/R_0$, so that one can rewrite the observed brightness temperature at line-of-sight velocity $v_{los}$ as $R_0 \int dr F(r ... n(r)H(u_{los}, r, ...)$ with $F$ a function that results from the transformation from line-of-sight distance to $r$, and $H$ the velocity-dependent function. If $H$ varies more quickly with $r$ than the number density $n(r)$, then the product $R_0 \times n(r)$ is approximately constant. Thus, as $R_0$ increases, the volume density goes down. Because the gas-layer width increases with $R_0$, the vertical column-density at scaled distance $R/R_0$ is independent of the Galactic constants. See also Bronfman et al. (1988).

A graphical representation of the derived gaseous surface density distributions can be found in a related paper on the Galactic...
files of the ISM we neglect the column density due to the other phases of the ISM. However, in order to properly take into account all baryonic contributions to the local column density, we include 1.4 $M_\odot$ pc$^{-2}$ of ionized hydrogen (Kalnajs & Eilles 1983) at the Solar position only. We further include 23.8% helium by mass (Olive & Steigman 1994) to compute the surface densities.

As in our previous papers, we do not distinguish between the various phases of the atomic Hydrogen but rather assume that the warm neutral medium has a kinetic temperature equivalent to the “temperature” associated with the bulk motions of the clumped, cold neutral medium. Note that the cold medium is likely to be absent beyond the stellar disk (Paper 1; Braun 1997)

2.1.3 The Dark Matter Component

As is commonly done (e.g., van Albada & Sancisi 1986; Kent 1987; Begeman 1989; Lake & Feinswog 1989; Sackett & Sparke 1990; Broeils 1992; Olling 1992; Olling 1996b), we model the dark matter density distribution as a non-singular isothermal spheroid with flattening $q$, and a density distribution given by:

$$\rho_0(R, z; q) = \rho_0(q) \left( \frac{R_c^2(q)}{R_c^2(q) + R^2 + (z/q)^2} \right),$$

where the halo’s core radius ($R_c$) and central density ($\rho_0$) depend on the flattening in such a way that the family of density distributions $\rho_0(q)$ have a rotation curve that is essentially independent of $q$ (Olling 1995). Figure 1 shows the radial distribution of dark matter for three values of $R_0$, for the range in model parameters as listed in Table 2. Each choice for $h_\lambda$ and $\Sigma_\lambda$ results in a different distribution. The largest DM densities are obtained for large $h_\lambda$’s and small $\Sigma_\lambda$’s. Even though the central DM density is uncertain by over three orders of magnitude, the local dark matter density is determined rather well: we find

$$\frac{\rho_{DM}(R_0, \Theta_0)}{10^{-3} M_\odot \text{pc}^{-3}} = \frac{11.5 + 3.8 \times (R_0 - 7.8) \pm 2}{q (26.7/\Theta_0)^2},$$

with $\Theta_0 = \Theta_0/R_0$, and where $R_0$ is in kpc, and $\Theta_0$ in km s$^{-1}$. The $(\sim 25/q)\%$ uncertainty arises as a result of the uncertainty in $\Sigma_\lambda$ and $\Sigma_c$. For example, taking $R_0 = 7.1$ kpc, $q = 0.71$, and using the parameters tabulated in Table 2, we find $\rho_{DM}(R_0) = 10.5 M_\odot$ pc$^{-3}$ ($1 M_\odot$ pc$^{-3} = 10^{-3} M_\odot$ kpc$^{-3}$). Thus, the $R_0 = 7.1$ model value for the DM matter density compares well with the values given by equation (1) as well as with the observational determination (3).

& Oort constants (Olling & Merrifield 1998). The H$_2$ column densities have been re-scaled from the original sources assuming $N(H_2)/W(CO) = 2.3 \times 10^{20}$ cm$^{-2}$ (K km s$^{-1}$)$^{-1}$. We present the radial increase of the thickness of the gas layer in Paper 1. The thickness and column density of the H I and H$_2$ are tabulated in Appendix [3].

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Figure 1. The dark matter midplane density distribution calculated using equation (1) for three values of $R_0$ and $q$. The densities scale approximately as $1/q$ (Olling 1995). The error bars represent the full range for the case corresponding to $\Sigma_\lambda = 35 \pm 5 M_\odot$ pc$^{-2}$, and $h_\lambda = 2$ to 3 kpc. Notice that although the DM density at the Solar circle is relatively well determined, the central density is uncertain by more than three orders of magnitude. Note that we have offset the radial coordinates of the $R_0 = 7.1$, and 8.5 kpc models from their true positions (i.e., the $R_0 = 7.8$ kpc points) to avoid overlapping symbols.

2.2 Other Dark Matter models

Of course, other mass models can be constructed which represent the radial dark matter density distribution (e.g., van Albada & Sancisi 1986; Navarro, Frenk & White 1996; Dehnen & Binney 1998). However, all viable mass models must share the property that they reproduce the Galactic rotation curve. For a round halo the vertical force approximates $z/R$ times the radial force. Since the radial force is approximately the same for all models which reproduce the observed rotation curve, the ensemble of possible models also have approximately equal vertical forces, independent of the exact radial DM density distribution (Olling 1992). In a flattened halo with the same rotation curve, the DM densities are roughly proportional to $1/q$, independent of the radial mass distribution [cf. eqn. (3)]. Thus, our analysis will not be seriously compromised by restricting ourselves to one particular DM density distribution.
Table 2. Disk & halo parameters. A log for a representative sample of models for which exact flaring curves are calculated. The meaning of the columns is as follows: 1) $\gamma$, the degree to which the stellar disk is maximal; 2) scale-length of stellar disk (kpc); 3&4) bulge & disk K-band mass-to-light ratios; 5&6) stellar column density & total column density within 1.1 kpc of the plane ($\Sigma$) and central density ($\Sigma_0$). The errors for the Milky Way are similar to those for extra-galactic systems.

| $\gamma$ | $h_0$ | $\Upsilon_{b,k}$ | $\Upsilon_{d,k}$ | $\Sigma_0$ | $\Sigma_{1,1}^1$ | $R_0$ | $\rho_0(=1)$ | $q_{HI}$ | $q_{1,1}$ |
|----------|-------|-----------------|-----------------|-----------|--------------|------|------------|--------|--------|
| 2.0      | 0.55  | 0.422           | 30.21           | 62.48     | 2.39         | 73.8 | 0.630 ± 0.107 | 1.14   | 1.14 ± 0.382 |
| 2.0      | 0.55  | 0.488           | 34.92           | 66.02     | 3.54         | 35.6 | 0.687 ± 0.113 | 0.664  | 0.226  |
| 2.0      | 0.55  | 0.558           | 39.96           | 68.72     | 6.09         | 14.6 | 0.699 ± 0.120 | 0.753  | 0.295  |
| 2.5      | 0.55  | 0.327           | 29.58           | 63.56     | 1.32         | 246.3| 0.687 ± 0.152 | 0.620  | 0.183  |
| 2.5      | 0.55  | 0.392           | 35.39           | 68.84     | 1.86         | 125.1| 0.717 ± 0.134 | 0.826  | 0.268  |
| 2.5      | 0.55  | 0.449           | 40.63           | 73.40     | 2.45         | 73.0 | 0.674 ± 0.127 | 1.061  | 0.371  |
| 3.0      | 0.55  | 0.306           | 30.21           | 64.34     | 0.70         | 853.9| 0.740 ± 0.181 | 0.651  | 0.194  |
| 3.0      | 0.55  | 0.351           | 34.68           | 68.6      | 1.00         | 421.0| 0.770 ± 0.159 | 0.820  | 0.261  |
| 3.0      | 0.55  | 0.412           | 40.70           | 74.18     | 1.44         | 201.9| 0.762 ± 0.137 | 1.114  | 0.382  |

| $R_0=7.1$, $\Theta_0=182$ | $\Theta_0=189$ |
|---------------------------|-----------------|
| 2.0                       | 0.45            | 0.482           | 29.71     | 61.94     | 2.73         | 62.7 | 0.625 ± 0.132 | 0.544  | 0.165  |
| 2.0                       | 0.45            | 0.568           | 35.00     | 65.66     | 4.53         | 25.5 | 0.624 ± 0.116 | 0.636  | 0.214  |
| 2.0                       | 0.45            | 0.647           | 39.87     | 66.74     | 9.77         | 8.6  | 0.679 ± 0.104 | 0.603  | 0.239  |
| 2.5                       | 0.55            | 0.370           | 29.66     | 62.60     | 1.13         | 340.1| 0.727 ± 0.159 | 0.581  | 0.173  |
| 2.5                       | 0.55            | 0.441           | 35.39     | 67.76     | 1.66         | 158.2| 0.738 ± 0.161 | 0.765  | 0.250  |
| 2.5                       | 0.55            | 0.506           | 40.55     | 72.26     | 2.25         | 86.4 | 0.823 ± 0.223 | 0.999  | 0.356  |
| 3.0                       | 0.55            | 0.333           | 29.73     | 61.82     | 0.20         | 9884.5| 0.735 ± 0.181 | 0.550  | 0.168  |
| 3.0                       | 0.55            | 0.395           | 35.28     | 67.10     | 0.55         | 1343.6| 0.703 ± 0.158 | 0.734  | 0.243  |
| 3.0                       | 0.55            | 0.421           | 37.63     | 69.32     | 0.71         | 802.8| 0.786 ± 0.210 | 0.843  | 0.290  |

| $R_0=7.8$, $\Theta_0=207$ | $\Theta_0=227$ |
|---------------------------|-----------------|
| 2.5                       | 0.60            | 0.670           | 30.66     | 78.68     | 6.81         | 39.1 | 2.922 ± 0.863 | 1.169  | 0.252  |
| 2.5                       | 0.60            | 0.761           | 34.83     | 80.43     | 8.34         | 28.9 | 2.512 ± 0.608 | 1.228  | 0.279  |
| 2.5                       | 0.60            | 0.857           | 39.26     | 81.28     | 11.10        | 19.9 | 1.931 ± 0.366 | 1.256  | 0.306  |
| 3.0                       | 0.85            | 0.486           | 27.23     | 76.53     | 5.09         | 60.2 | 1.424 ± 0.156 | 1.165  | 0.193  |
| 3.0                       | 0.85            | 0.627           | 35.11     | 82.49     | 6.20         | 43.2 | 1.498 ± 0.183 | 1.364  | 0.213  |
| 3.0                       | 0.85            | 0.785           | 44.00     | 88.53     | 8.07         | 28.9 | 1.475 ± 0.197 | 1.641  | 0.159  |

3 CONSTRaining THE DARK MATTER DENSITY

In this section we present two simple analytical models to illustrate the existing correlations between $R_0$, $\Theta_0$, $\Sigma_{1,1}$, $q_0$, and $\sigma_0$ outlined in the Introduction. These models can be used to show how the local stellar kinematics and the H I flaring constrain the dark matter density in the Solar neighbourhood and at $\sim 2 R_0$. But first we investigate how well the local stellar column density is determined and how accurately this determination constrains the dark matter density in the Solar neighbourhood.

3.1 The local stellar column density: a constraint?

The Milky Way is a unique galaxy in that we can, at least in principle, determine the local column density of stars directly. Once $\Sigma_0$ is accurately determined, it is possible to establish to what degree the Milky Way disk is maximal, which would provide an important benchmark for external galaxies. Unfortunately this benchmark is not yet available,
as the values for $\Sigma_*$ reported in the literature range from 26 to 145 $M_\odot$ pc$^{-2}$.

Two basic techniques have been employed to determine $\Sigma_*$. The direct method involves converting star counts as a function of Galactic coordinates and magnitude to insitu mass densities. This method is somewhat hampered by uncertainties in the conversion from luminosity to mass, completeness problems in the Solar neighborhood, and binary corrections at large distances. Gould, Bahcall & Flynn (1997; hereafter referred to as GBF97) used deep HST star counts of M-dwarfs at great heights above the plane in combination with a local normalization to infer a stellar column-density of only 25.8 \pm 3.8 $M_\odot$ pc$^{-2}$. The second, kinematical, method employs the interrelationship between the potential, the vertical density distribution, and the variation of the velocity dispersion with $z$. Many authors have employed this method, yielding a large range in inferred values for $\Sigma_*$. For example, Bahcall (1984b) found that the inferred value of $\Sigma_*$ depends significantly on the assumed vertical distribution of the dark matter: \sim 40, \sim 52, \sim 68, and \sim 145 $M_\odot$ pc$^{-2}$ for matter distributions that resemble the gaseous disk, an isothermal halo, the thin stellar disk, or the thick stellar disk, respectively. Other authors find values as low as 35 $M_\odot$ pc$^{-2}$ (Kuijken & Gilmore 1989) and 68, and \sim 25.8 $M_\odot$ pc$^{-2}$ (Flynn & Fuchs 1994).

Methods that combine the two primary methods exist as well (Bienaymé, Robin & Crézé 1987; Crézé, Robin & Bienaymé 1987). In Appendix A we simulate the kinematic determination of the stellar column density with the aid of Galaxy models – taken from Table 2 – for which we know the exact force law. Our results are in complete agreement with the findings of previous authors: the “vertical disk-halo conspiracy” can only be resolved if high-$z$ stellar kinematical data are included and/or if additional assumptions are made (e.g., a “reasonable” value for the DM density). Typical values for the mass of the stellar disk are 52 (Bahcall 1984), 35 \pm 5 (Kuijken & Gilmore 1989), and 37 \pm 13 $M_\odot$ pc$^{-2}$ (Flynn & Fuchs 1994).

Considering the uncertainties in the kinematical estimates of the total mass of the disk, it might be preferable to use the direct star count method to determine $\Sigma_*$. The latest results by GBF97 imply $\Sigma_* = 25.8 \pm 3.8 M_\odot$ pc$^{-2}$. However, the local space density of stars found by these authors \([33.2 \pm 8.6] M_\odot$ pc$^{-2}\) is somewhat lower than the values reported in the literature \([43 \pm 15] M_\odot$ pc$^{-3}\). This suggests that GBF97 may have underestimated $\Sigma_*$ by a factor of 1.4 \pm 0.2, and so we find $\Sigma_* = 36 \pm 5 M_\odot$ pc$^{-2}$; the star count method yields a value for $\Sigma_*$ that is remarkably close to the kinematical estimates.

On the other hand, the total matter density in the Solar neighborhood as inferred from recent Hipparcos data (Crezé et al. 1998) of 76 \pm 15 $M_\odot$ pc$^{-3}$ favours a local stellar volume density which is even smaller than reported by GBF97. After subtracting the density of the ISM (see Table 3 and the local DM density [eqn. (3)] we find a local stellar density of \sim 21 \pm 15 $M_\odot$ pc$^{-3}$. Note that even lower values for $\rho_*$ result if large values for the Galactic constants and/or $\Omega_0$ are chosen, while smallish Galactic constants and $\Omega_0$ increase the local stellar density.

To summarize, a “reasonable” value for $\Sigma_*$ might be 35 $M_\odot$ pc$^{-2}$, and we suggest a “consensus” error of 10 $M_\odot$ pc$^{-2}$. However, since the differences between the various $\Sigma_*$ estimates are not random but systematic, one should not interpret the reasonable $\Sigma_*$ value and its error in a statistical sense. That is to say, one can not construct likelihood contours based on a reasonable average value and a consensus error bar. Any attempt to do so would be an overinterpretation of the available data.

A “reasonable” value for the local dark matter column can be obtained by subtracting the column-densities of the stellar and ISM distributions; $\Sigma_h = (24.5 \pm 11) M_\odot$ pc$^{-2}$ within 1.1 kpc of the plane. This dark matter column amounts to an average local dark matter density of 11 \pm 5 $M_\odot$ pc$^{-3}$.

### 3.2 The connection between dark and luminous matter in the Solar neighbourhood

In the previous section we have seen that the contribution of the stellar and dark matter components to the local disk mass cannot be clearly segregated. Thus the observed value for $\Sigma^{1.1}_{\text{tot}}$ implies that $\Sigma_*$ and $\Sigma_h$ are highly correlated: a low stellar column implies a large amount of dark matter, and vice versa. Since the dark matter density depends on the amplitude of the rotation curve and the halo’s flattening, these parameters are in turn related to $\Sigma_*$. Below we investigate the relations between $\Sigma^{1.1}_{\text{tot}}$, $\Theta_0$, $\Sigma_*$ and $q$ in some detail. These relations are independent of the H I flaring.

Integrating equation (4) with respect to $z$ and applying equation (A4) from Olling (1995), we find the column-density of dark matter within $z$ of the Galactic plane as a function of $q$:

$$\Sigma^1_{\text{tot}}(q) = \frac{2\pi\rho_h(q)R_h^2(q)}{\sqrt{R^2(q) + R^2}} \arctan \left( \frac{z}{\sqrt{R^2(q) + R^2}} \right)$$

$$= \frac{V_{h,\infty}^2}{2\pi G R^2(q) + R^2} \arctan \left( \frac{z}{\sqrt{R^2(q) + R^2}} \right),$$

with $V_{h,\infty}$ the asymptotic rotation velocity of the round dark halo. For each of our self-consistent mass models we can calculate $\Sigma^{1.1}_{\text{tot}}(q)$ and find that it obeys a simple power law relation:

$$\Sigma^{1.1}_{\text{tot}}(q) = \Sigma^1_{\text{tot}}(q) + \Sigma_* + \Sigma_h$$

$$\approx \Sigma^{1.1}_{\text{tot}}(q = 1) \times q^{-p}.$$
and equating

These examples show that the $\Sigma_{1.1}^q$ equation of hydrostatic equilibrium:

in the ISM, the thickness of the gas layer follows from the non-thermal pressure gradients.

section we will expand our analysis to include the effects of... adopted the same assumption in the analysis below. In this

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the observed FWHM of the gas layer can be used to con-

In an idealized picture, the equilibrium thickness of the gas

3.3 Constraints from the thickness of the gas layer

In an idealized picture, the equilibrium thickness of the gas

in the Milky Way depends only on the gas “temperature” and

the form of the potential in which it has settled. Thus, the

observed FWHM of the gas layer can be used to con-

strain the potential of the Galaxy. In Paper I we present ev-

dence that the interstellar medium beyond the optical disk

comprises only a single, iso-thermal component. We there-
fore adopt the same assumption in the analysis below. In this

section we will expand our analysis to include the effects of

non-thermal pressure gradients.

If the gas layer is in a steady state and we assume that only “thermal” motions of the gas contribute to the pressure in the ISM, the thickness of the gas layer follows from the equation of hydrostatic equilibrium:

$$
\frac{d}{dz} \sigma_g^2 \rho_g(z) = \rho_g(z) K_2(z),
$$

where $\sigma_g$, $\rho_g(z)$, and $K_2$ are the gaseous velocity dispersion

each combination of the Galactic constants, the stellar disk mass and scale-length $p$ can be determined from a fit to equation (10), where we determine $\Sigma_{1.1}^q(q)$ for several models with varying $q$’s. Note that we also use equation (10) to extrapolate into the prolate regime, with the exponent $p$ as determined for the oblate models. The reader can verify this by plotting the $q_{1.1}$ values as a function of $\Sigma_*$ from table 2.

and volume density, and the vertical force per unit mass, respectively. The vertical force is commonly determined using a “local approximation” which $K_2$ follows from the the Poisson equation and the local mass densities and the radial gradient of the rotation curve. However, this approach is known to fail in regions where either the mass densities or the rotation curve, or both, have steep gradients. Further, the local approach neglects any variation of the circular speed with height above the plane. These problems can be overcome by calculating $K_2$ from the global mass distribution (the “global approach,” Olling 1995). As mentioned in section 2, we combine the exactness of the global approach with the speed of the local approximation for optimal results.

If we employ the local approximation the dependencies between the various parameters of the model become apparent. For example, when the potential is dominated by mass component $i$, equation (10) can be solved for the thickness of the gas layer:

$$
\text{FWHM}_i \propto \frac{\sigma_g}{\sqrt{4\pi\rho_i}},
$$

where the proportionality constant depends on the vertical density distribution of the dominant contributor to the local vertical potential. We copy some relations from Olling (1995). In case the gas is fully self-gravitating, the width is given by:

$$
\text{FWHM}_g \approx 0.15 \frac{\sigma_g^2}{\Sigma_g}. \tag{11}
$$

If the potential is dominated by an iso-thermal stellar disk with sech$^2$ scale-height $z_0$ ($z_0 = 2z_c$), the thickness of a gas layer would be:

$$
\text{FWHM}_s \approx 0.51 \sigma_{g,2} \sqrt{\frac{1.0}{\Sigma_s 35}}. \tag{12}
$$

or 510 parsec at the Solar circle ($\sigma_g$, $z_0$ and $\Sigma_s$ are expressed in units of $9.2 \text{ km s}^{-1}$, 0.6 kpc and 35 $M_{\odot} \text{ pc}^{-2}$; see also, van der Kruit 1988). And if the dark matter halo dominates the potential, we find:

$$
\text{FWHM}_h \approx 1.4 \sigma_g \sqrt{\frac{R_{c1}}{R_{c1}^2 + R^2}}, \tag{13}
$$

where $R_{c1}$ is the core radius of the equivalent round halo (Olling 1995). All distances and widths are in kpc, and all velocities in km s$^{-1}$. The thickness of the gas layer in the combined potential of several mass components can be solved analytically in the form of an integral equation, but requires an iterative solution procedure (Olling 1995, Appendix C). However, a further approximation is possible. Following Olling (1995, Appendix D) we use:

$$
\frac{1}{\text{FWHM}_i} \approx \sum_i \frac{w_i}{\text{FWHM}_i^i}, \tag{14}
$$

where the weighting factors $w_i$ reflect the relative impor-
Solving equation (14) for $q_{HI}$, taking $V_{h,\infty} \approx \Theta_0$ and neglecting $Rc,1$, we find:

$$q_{HI}(2R_0) \approx \frac{1.4}{24\left(\frac{1}{(2R_0)^2} - \left(\frac{\Sigma_g w_g}{\Sigma_g} \right)^2 \left(\frac{2R_0}{w_0} \frac{\sigma_g}{\Theta_0} \right)^2\right) - 1},$$  

(15)

with $\zeta(R) = \text{FWHM}_{\text{HI}}/R \approx 0.07$ for the outer Milky Way, and where $\Sigma_g, w_g$, and $w_0$ have to be evaluated at $R = 2R_0$. From this equation we infer that the contribution of the self-gravity of the gas depends strongly on the gaseous velocity dispersion: low $\sigma_g$ values result in a more negative self-gravity contribution to the denominator, and hence leads to a larger inferred $q_{HI}$. As we have already mentioned, the self-gravity of the gas is an important component as it reduces the width of the gas layer by approximately 45%. A further simplification can be made by neglecting the gaseous self-gravity as well, we find:

$$q_{HI} \approx \frac{1.4\zeta(R)\Theta_0}{13.5\sigma_g^2 - \zeta(R)\Theta_0^2},$$  

(16)

$$q_{HI} \propto \frac{\Theta_0^2}{\sigma_g^2},$$  

(17)

where the second line arises because the first term in the denominator dominates. The errors in the halo flattening and rotation speed are related through equation (C1), where we have neglected the contribution of the velocity dispersion error. The dependence of $q_{HI}$ on $\Theta_0$ and $\sigma_g$ are indeed as outlined in the Introduction. Also note that, due to the quadratic nature of the proportionality, the observed flaring and the small allowed range of $q_{HI}$ constrain $\Theta_0$ and $\sigma_g$ rather tightly.

Comparing equations (16) and (17) we see that the constraints on the halo shape arising from the local stellar kinematics and the H I flaring the have rather different $R_0$ and $\Theta_0$ dependencies. Thus, it is indeed possible to learn more about the Galactic dark matter distribution by combining these two constraints. Further, unlike the stellar kinematical method, the constraints from the H I flaring is independent of $\Sigma_*$ since it arises at $R \sim 2R_0$, whereas the truncation of the stellar disk.

Equations (15)-(17) only serve to illustrate the dependence of $q_{HI}$ on the model parameters since several important aspects are treated too simplistic. First, neglecting $Rc$ and equating $V_{h,\infty}$ with $\Theta_0$ is only seldomly warranted. Second, depending on the slope of the rotation curve, an error of order ±20% is made in the inferred $q_{HI}$. We therefore do not recommend using equations (15)-(17) “as is” to determine $q_{HI}$ directly. In all our calculations we employ equation (14) where we take all mass components fully into account, without any further simplifications. For many combinations of model parameters, a round halo is too dense to explain the observed flaring (cf. Fig. 2). Albeit not entirely correct, we will determine the halo’s contribution to the potential using equation (14) in those cases.

4 THE EFFECTS OF A LARGER GASEOUS VELOCITY DISPERSION

Equations (15)-(17) show that $q_{HI}$ is a function of $\sigma_g, \Theta_0$ and $R_0$. On the other hand, the halo flattening inferred from the local stellar kinematics is a function of the Galactic constants and $\Sigma_*$. In a self-consistent model, $q_{HI} \equiv q_{HI}^\star$, so that strong inter-relationships are imposed among the currently ill-determined values of $R_0, \Theta_0, \Sigma_*$ and $\sigma_g$.

Below we investigate how the inferred halo flattening and stellar column density of a self-consistent model depend upon our choice of $\sigma_g$. The value of $\sigma_g$ can have significant effects on the inferred values of $q$ and $\Sigma_*$. For example, in Paper I we assumed that the true velocity dispersion equals 9.2 km s$^{-1}$, and found an upper limit to the local rotation speed of about 190 km s$^{-1}$. Further, models with the IAU recommended Galactic constants have prolate dark halos ($q \sim 1.9$) and require a rather high local stellar column density of $\sim 55$ M$_\odot$ pc$^{-2}$. Increasing $\sigma_g$ would bring $q$ and $\Sigma_*$ down to more acceptable levels, and it might thus be worthwhile to treat $\sigma_g$ as a free rather than as a fixed parameter.

However, observations of external galaxies imply that the velocity dispersion declines slightly in the radial range over which the Milky Way’s flaring has been measured, by a factor 1.12 ± 0.12 (van der Kruit & Shostak 1984, Dickey, Hanson & Helou 1990). Kamphuis 1993, Côté 1995, Olling 1996a, Sicking 1997). Furthermore, there is some evidence that $\sigma_g$ in the outer Galaxy equals the value inside the Solar circle (Blitz & Spergel 1991). Also, one would expect that a change in gaseous velocity dispersion would be reflected in a change of the residual motions of young stars with respect to the mean streaming field. Such changes have not been observed, neither in B stars nor in Cepheids (Brand & Blitz 1993, Font, Quelez, Bratchi & Mayor 1997). Thus, an increase in gaseous velocity dispersion beyond the Solar circle is not likely.

4.1 Non-thermal pressure terms

In Paper I we argued that the ISM beyond the optical disk comprises a single, iso-thermal component (the warm neutral medium). We now investigate the possibility that non-thermal pressures have to be included in the hydrostatic balance.

It is estimated that in the Solar neighbourhood thermal motions, cosmic rays, and magnetic fields contribute about

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equally to the interstellar pressure \( P \) and the kinetic pressure terms are proportional to the kinetic pressure \( P \) with that there exists significant non-thermal pressure support 

\[ P_0 = \sigma_B^2 \rho_0(z) \]

with \( P_0 = \sigma_B^2 \rho_0(z) \) the thermal gas pressure, \( P_B = B^2 / 8\pi \) the magnetic field pressure, and \( P_C = 1 / 3UC \) is the pressure due to cosmic rays with energy density \( U \). Taking the scale-heights of the kinetic, magnetic and cosmic ray energy density to be \( z_G, z_B, \) and \( z_C \), we write the pressure gradients as \( \frac{dP_0}{dz} = P_0, \frac{dP_B}{dz} = P_B, \) and \( \frac{dP_C}{dz} = P_C \).

### 4.2 Gas-Layer Support at \( R_0 \)

In the Solar neighborhood the scale-heights \( z_B \) and \( z_C \) are \( \kappa_B \) and \( \kappa_C \) times larger than the gaseous scale-height \( \kappa_B \sim 10, \kappa_C \sim 4 \). Following Spitzer (1978) we assume that the non-thermal pressure terms are proportional to the kinetic pressure: \( P_0 = \alpha_B P_0 + P_C = \alpha_C P_C \), with \( \alpha_B \approx 0.25 \) and \( \alpha_C \approx 0.4 \) in the Solar neighborhood. Using these parameterizations we find:

\[
\rho_g(z) \kappa_s(z) = \frac{dP_0}{dz} \quad \text{and} \quad \frac{dP_B}{dz} = \frac{dP_C}{dz} = \frac{dP_0}{dz} \approx \sigma_g^2 \left( 1 + \frac{\alpha_B}{\kappa_B} + \frac{\alpha_C}{\kappa_C} \right) \frac{dP_0}{dz}
\]

\[
\approx (\sigma_g^2)^2 \frac{dP_0}{dz} \approx \sigma_g^2 \left( \frac{dP_0}{dz} \right)
\]

where we have lumped all terms contributing to the hydrostatic balance into a single unknown, the effective velocity dispersion \( \sigma_g \). With the above simplifications and equations \( (\ref{eq:14}) \) and \( (\ref{eq:16}) \), it becomes possible to estimate the effects of non-thermal pressure support on the thickness of the gas layer and the inferred shape of the dark matter halo:

\[
\begin{align*}
\text{FWHM}_{GBC} & \approx \frac{\sigma_g}{\sigma_B} \approx \sqrt{1 + \frac{\alpha_B}{\kappa_B} + \frac{\alpha_C}{\kappa_C}} \approx 1.06 \quad (\ref{eq:21}) \\
\frac{q_{GBC}}{q_G} \bigg|_{R_0} & \approx \left( \frac{\sigma_g}{\sigma_B} \right)^2 \approx 0.89, \quad (\ref{eq:22})
\end{align*}
\]

where the subscripts GBC indicates that the gaseous, magnetic and cosmic ray terms are taken into account in the hydrostatic balance of the gas layer. Thus, neglecting the non-thermal contribution to the pressure balance in the local ISM leads to a gas layer width that is under-estimated by a few percent only. Similarly, a slightly more flattened halo is required if non-thermal pressure support were important in the Solar neighborhood.

### 4.2.1 Non-Thermal Support, or Not?

The above statements are at odds with the current paradigm [cf. Binney & Merrifield (1998), problem 9.7] which states that there exists significant non-thermal pressure support of the gas layer in the Solar neighborhood. The argument in support of this paradigm is that models without non-thermal pressure support under-predict the observed gas-layer widths. However, the model predictions depend sensitively on the values of the Galactic constants. Our models with small values for \( R_0 \) and \( \Theta_0 \) show only a small discrepancy between the model and observed widths, while a large width difference exists for models with IAU-standard Galactic constants (Paper I, figure 5). Furthermore, the gas-layer width depends sensitively on the local stellar column density as well as the functional form of the vertical density distribution. For example, the gas layer is almost 40% thinner in the Solar neighborhood than the gaseous scale-height is.

In fact, if the potential in the Solar neighborhood were fully determined by the stellar disk, and the H I were truly isothermal, then the predicted width via equation \( (\ref{eq:12}) \) overpredicts the observed width (cf. table \( \ref{tab:1} \)). The other contributors to the potential decrease the model width slightly (cf. eqn. \( \ref{eq:14} \)), in better agreement with the observed width. We conclude that the paradigm of significant non-thermal pressure support in the Solar neighborhood is inaccurate and that the observed H I width at the Solar circle is consistent with “thermal” pressure support only (cf. equation \( \ref{eq:23} \)).

### 4.3 Gas-Layer Support at \( 2R_0 \)

Since we only employ the H I thickness measurements at large radii as a constraint in our mass models, let us try to guess the value of \( \sigma_g^2 \) at \( \sim 2R_0 \).

The gas density in the outer Galaxy is about four times smaller than at \( R_0 \) as a result of the decrease in column-density (factor 2) and the increase in thickness (factor 2). To make a conservative estimate as to the importance of the non-thermal terms at large distances, we assume that the magnetic field strength, the cosmic ray energy density and their vertical gradients equal the values in the Solar neighbourhood. Furthermore, we assume that \( \sigma_g \) remains unchanged. With these assumptions, the \( \alpha \) values at \( 2R_0 \) are four times larger than at \( R = R_0 \), while the \( \kappa \) values are decreased by a factor two. Thus, the effective velocity dispersion and the inferred halo shape are strongly affected:

\[
\begin{align*}
\frac{\sigma_g}{\sigma_B} \bigg|_{2R_0} & = \sqrt{1 + \frac{4\alpha_B(R_0)}{\kappa_B(R_0)/2} + \frac{4\alpha_C(R_0)}{\kappa_C(R_0)/2}} \sim \sqrt{2} \quad (\ref{eq:23}) \\
\frac{q_{GBC}}{q_G} \bigg|_{2R_0} & \approx \left( \frac{\sigma_g}{\sigma_B} \right)^2 \sim 0.5. \quad (\ref{eq:24})
\end{align*}
\]

On the basis of this over-estimation, it appears that the gas layer is significantly out of thermal equilibrium at large radii. Do we expect this to be the case? Two effect reduce the importance of the non-thermal contribution. First, the \( B \)-field is probably co-spatial, “frozen in”, with the ionized part of the ISM which’ scale-height has most likely doubled at \( \sim 2R_0 \) (like the H I). Thus, the Solar neighbourhood value for \( \kappa_g(2R_0) \) should be used in equation \( (\ref{eq:23}) \). Second, radio-continuum measurements of external galaxies suggest
that high cosmic ray fluxes are closely associated with the sites of star formation (Bikay & Helou 1990). Since active star-formation ceases to exist beyond \( \sim 1.5 R_0 \), the \( \alpha_C \)-term in eqn. (23) vanishes. Thus, in a more realistic treatment of the non-thermal pressure terms, their effect on the vertical equilibrium of gas at large radii is much reduced:

\[
\frac{\sigma'_{g}}{\sigma_{g}} \bigg|_{2R_0} = \sqrt{1 + \frac{4\alpha_B (R_0)}{\kappa_B (R_0)}} = \sqrt{1.1} \sim 1.05 \quad (25)
\]

\[
\frac{q_{\text{GC}}}{q_B} \bigg|_{2R_0} \approx \left( \frac{\sigma_B}{\sigma_{g}} \right)^2 \sim 0.9 . \quad (26)
\]

Further, if the magnetic field strength decreases with Galactocentric radius, the above estimates should be even closer to unity. Thus realistic estimates of the importance of non-thermal pressure terms in the equation of hydrostatic equilibrium indicate that the thickness of the H I layer is not much affected, neither in the inner, nor in the outer Galaxy.

And finally it is worth mentioning that there exist both theoretical and empirical evidence that non-thermal pressure support is small even when simplified calculations indicate that they may dominate \( dP_C/dz \). From a theoretical perspective one finds that the vertical balance is only affected if the magnetic field is horizontally stratified, a configuration that is unstable (Parker 1966), and which is thus not expected to exist on a global scale in the Milky Way. In fact, 3D simulations of the growth of the Parker instability show that the gas is almost entirely supported by thermal pressure within the first four scale-heights (Kim, Hong, Ryu & Joo, 1998). On the observational side, Rupen (1991) presents evidence that the non-thermal pressure gradients equal 400% of the thermal pressure gradient in NGC 891, and 50% in NGC 4565. Since the luminous mass distributions in these galaxies are very similar, one would expect very different gas layer widths if non-thermal effects were indeed important. In fact, these two galaxies have almost indistinguishable flaring curves, and we can conclude that thermal pressure gradients dominate (Rupen 1991, Olling 1996a).

To summarize, simplified theoretical considerations indicate that non-thermal pressure support could dominate the hydrostatic balance if extreme assumptions about the magnetic field geometry and the cosmic ray energy density are made. More realistic assumptions regarding the vertical gradients of \( P_B \) and \( P_C \), as well as observational data lead to the opposite conclusion.

In the next paragraph we will investigate the effects of significant non-thermal pressure support by treating the gaseous velocity as an unknown, notwithstanding the indications that non-thermal pressure support is actually small in the outer Galaxy. Based on the arguments presented above, we expect \( \sigma'_{g} \) to lie between \( \sim 0.85 \) and \( \sqrt{2} \) times the default value of 9.2 km s\(^{-1}\), while a more realistic upper limit to \( \sigma'_{g}/\sigma_{g} \) might be 1.05 [cf. eqn. (25)].

### 4.4 Constraints on the pressure term

In this section we address the question as to how the interrelations between the Galactic constants, \( \Sigma_{\ast} \) and \( q \) are affected when assuming that \( \sigma_{g} \) is unknown. We follow the procedure to determine the halo flattening as outlined in the Introduction (and Paper I) for several trial values of \( \sigma_{g} \) (for \( \sigma'_{g} = 8.0, 8.6, 9.2, 9.9, 10.5, 12 \) and 14 km s\(^{-1}\), and on the same \( R_0 - \Theta_{0} \) grid as in Paper I. As predicted by equation (4), the inferred halo flattening and stellar column-density are rather sensitive to the adopted value of \( \sigma'_{g} \). In figure 3 we present \( q \) (dotted lines) and \( \Sigma_{\ast} \) (long dashed lines) as calculated for increasing values of \( \sigma'_{g} \), from left to right, and from top to bottom. The oblate region of parameter space, below the heavy dash-dotted line, is strongly restricted if \( \sigma'_{g}/\sigma_{g} < 1 \) (lower-left panel), while essentially the whole range of the Galactic constants is allowed if the effective dispersion is as large as 12 km s\(^{-1}\) (upper-right panel). From these figures we can also infer that the mass of the stellar disk, if measured accurately, constrains the allowed range of \( \sigma'_{g} \).

Values for \( \sigma'_{g} \) that are as much as 30% larger than the default value (upper right panel of Fig. 3) are completely excluded because of the very small stellar disk masses required. On the other hand, lower velocity dispersions need lower density (rounder) halos and hence a more massive stellar disk, for given Galactic constants. This is indeed what is observed in the lower-left panel of Fig. 3 for \( \Theta_{0} \gtrsim 175 \) km s\(^{-1}\). However, below \( \Theta_{0} \sim 175 \) km s\(^{-1}\), the situation is much more chaotic (extreme negative values for \( \Sigma_{\ast} \) and \( q \)), which is the result of the fact that the H I flaring and the stellar kinematical constraints are mutually exclusive (Appendix B). In fact, an effective velocity dispersion \( \lesssim 8 \) km s\(^{-1}\) is essentially ruled out.

### 4.5 Other Constraints

In order to take all constraints properly into account, we should present a three dimensional plot with \( R_0 \), \( \Theta_{0} \) and \( \sigma'_{g} \) as the axes. Because this is a little tedious, we opt for to determine the halo flattening and \( \Sigma_{\ast} \) along lines of constant \( \Omega_0 \). We select \( \Omega_0 \) values which bracket the uncertainty of the proper motion of Sgr A\(^{\ast} \) (Reid et al. 1999). In each of the three panels of Figure 3 (\( \Omega_0 = 29.9, 27.4, 24.9 \) km s\(^{-1}\) kpc\(^{-1}\), from top to bottom) we present contours of constant halo flattening by dotted lines as a function of \( R_0 \) and \( \sigma'_{g} \). The heavy dashed-dotted line is the round-halo contour. The hashed parts of the diagram depict regions of parameter space where \( \Sigma_{\ast} = 35 \pm 5 \) M\(_{\odot}\) pc\(^{-2}\) (heavy horizontal hash) and \( \Sigma_{\ast} = 27.8 \pm 3.8 \) M\(_{\odot}\) pc\(^{-2}\) (light vertical hash). These column-density ranges correspond to the stellar disk mass as determined by KPG89 and GBF97, respectively. In figure 3 we also plot our estimate to the upper limit of the non-thermal pressure support (\( \sigma'_{g} \sim 1.05 \sigma_{g} \)) as the thick horizontal line. The cross at \((R_0, \sigma'_{g}) = (7.2, 9.2)\) corresponds to the value of \( R_0 \) derived from the Oort constant (Olling & Merrifield 1998a) and the standard value of the gaseous velocity dispersion.
Figure 2. Here we present the *mutually consistent* set of Galactic constants, stellar column density in the Solar neighborhood \((\Sigma_\ast; \text{long dashed lines})\) and halo flattening \((q; \text{dotted lines})\). The individual panels show the results of the calculation for several values of \(\sigma'_g/\sigma_g\): 0.85 \((\sigma'_g=8 \text{ km s}^{-1}; \text{lower left})\), 1.0 \((\sigma'_g=9.2 \text{ km s}^{-1}; \text{lower right})\), 1.15 \((\sigma'_g=10.5 \text{ km s}^{-1}; \text{upper left})\), and 1.30 \((\sigma'_g=12 \text{ km s}^{-1}; \text{upper right})\). The oblate-prolate boundary is indicated by the heavy dash-dotted line. The heavy full line and the heavy dashed line correspond to KG89’s determination of \(\Sigma_\ast\), and the \(\pm 1\sigma\) values. The upper limit of GBF97’s determination of the stellar column density corresponds to the \(\Sigma_\ast \sim 30 \, M_\odot \, \text{pc}^{-2}\) contour. Both the halo flattening and stellar column are determined to \(\sim 6\%\) accuracy (see also Appendix C). Because the Galactic constants, \(\Sigma_\ast\), and \(q\) have to be mutually consistent one cannot arbitrarily choose the four parameters. Fixing one of the four parameters severely restricts the other three. Any two parameters follow immediately from any choice of the other two, for a given \(\sigma_g\). In the shaded region of parameter space, the mass of the stellar disk is as measured by KG89 (dark shading) and GBF97 (light shading).

A general feature of these diagrams is that flatter halos are found in regions with large values of \(\sigma'_g\). This arises naturally from the fact that a stronger gravitational pull is needed to constrain a gas layer with additional pressure support, for a given observed thickness [see also eqn. (13)]. These figures also clearly show that regions with large DM densities, due to either large \(\Theta_0\) or small \(q\), have low stellar column density in the Solar neighbourhood. We can see that significant non-thermal pressure support, which we define here as having \(\sigma'_g = \sqrt{2}\sigma_g \sim 13 \text{ km s}^{-1}\), requires very
Figure 3. In this figure we present the interrelations between the adopted value of the gaseous velocity dispersion \( \sigma_g' \) and the Galactic constants. The line coding is the same as in Figs. 2. This figure was generated by extracting the halo flattening and \( \Sigma^* \) values along the lines \( \Theta_0/R_0 = 24.9, 27.4, \) and 29.9 (from top to bottom), for several values of the effective gaseous velocity dispersion. The fat cross represents our best estimate for the Galactic constants \( R_0 = 7.1 \pm 0.4 \) kpc, \( \Theta_0 = 184 \pm 8 \) km s\(^{-1} \), (Olling \& Merrifield 1998c), and the gaseous velocity dispersion \( \sigma_g = 9.2 \pm 1 \) km s\(^{-1} \), (Malhotra 1995). The thick horizontal line represents our theoretical expectation as to the maximum value of \( \sigma_g' \) (eqn. 25). Large effective velocity dispersions require flattened halos so that the model gas layer widths are as thin as observed. Round, and even prolate, halos are found for small \( \sigma_g' \) values. In the shaded region of parameter space, the mass of the stellar disk is as measured by KG89 (horizontal dark shading) and GBF97 (vertical light shading).
highly flattened dark matter halos and very small stellar column densities in the Solar neighbourhood. Such strong non-thermal pressure support is thus ruled out.

Several other generic conclusions can be drawn from figure 3. First, highly flattened halos are only possible for realistic values of $\Sigma_*$ if $R_0 < 7$ kpc, and only for small values of $\Omega_0$. Second, large values for $R_0$, an acceptable stellar column density, and an oblate halo occur only if the angular velocity of the Milky Way is small, whatever the value of the effective velocity dispersion. Third, the extra degree of freedom associated with $\sigma_g$ does not greatly influence the inferred halo flattening if a good constraint on $\Sigma_*$ can be used. And finally, the combined constraints set by the observed stellar column density and the halo’s oblateness severely restrict the allowed range for $\sigma_g$, in particular for $R_0 \geq 7$ kpc.

More specific results follow if we are willing to make more restrictive assumptions. For example, if we assume that $R_0 \geq 7$ and $\sigma_g \leq 10.2$, we find $q \gtrsim 1.0 (\Omega_0 = 29.9), q \gtrsim 0.65 (\Omega_0 = 27.4), \text{and } q \gtrsim 0.2 (\Omega_0 = 24.9)$. In case $R_0 \geq 8$ kpc, $q \gtrsim 2, q \gtrsim 1.3$ and $q \gtrsim 0.8$ for the same angular velocities. Alternatively, when choosing particular values for the Galactic constants, simple relations between the remaining three parameters of the mass model follow. For example, taking the IAU-recommended values for $R_0$ and $\Theta_0$ (8.5 kpc, and 220 km s$^{-1}$) we find $q \sim 2.26 - 0.90 \times \sigma_g + 0.11 \times \sigma_g^2$ and $\Sigma_* \sim 65.6 - 35.2 \times \sigma_g + 4.8 \times \sigma_g^2$, with $\sigma_g \sim 9.2$. Also, with the standard value for the gaseous velocity dispersion we find: $\Theta_0 (q \leq 1) \sim 187 + 5 \times \sigma_g$ and $\Sigma_r (0.5 \leq q \leq 1) \sim 37.5 + 18.4 \times \sigma_g + 8.5 \times \sigma_g^2$, with $\sigma_g \sim 9$. (Fig. 3, the lower-right panel).

An inspection of the lower two panels of figure 3 reveals that rather tight constraints can be placed on the halo shape, the local angular velocity and the effective velocity dispersion if the halo is oblate and $30 \lesssim \Sigma_* \lesssim 40 \ M_\odot \text{ pc}^{-2}$ and $R_0 \geq 7$ kpc. In that case we find: $24.9 \lesssim \Omega_0 \lesssim 27.4 \text{ km s}^{-1} \text{ kpc}^{-1}, 0.5 \lesssim q \lesssim 1$, and $8.6 \lesssim \sigma_g \lesssim 10.3 \text{ km s}^{-1}$. If we impose the additional constraint that the non-thermal pressure support is limited to 5% as derived in equation (23), it follows that the Sun’s distance to the Galactic centre is less than 8 kpc, and that the rotation speed of the Milky Way is less than 200 km s$^{-1}$ at the Solar circle.

The stellar disk mass provides a strong constraint on the effective velocity dispersion of the gas: a low disk mass requires more dark matter, which would lead to a thinner gas layer in the outer Galaxy if $\sigma_g$ were not increased. For example, if the stellar disk mass exceeds $22 \ M_\odot \text{ pc}^{-2}$, figure 3 shows that the effective velocity dispersion has an upper bound of about 10.5 km s$^{-1}$, so that the non-thermal pressure support can not exceed 14%.

5 SUMMARY AND CONCLUSIONS

In Paper I we showed that the constraints on the dark matter density in the Solar neighbourhood and the outer Galaxy place tight limits on the choice of the Galactic constants, the mass of the stellar disk, and the dark halo’s flattening. The internal errors for this procedure are of order 6% for both the halo flattening and the local stellar column density (see Appendix 3).

In this paper we present the analytical tools that allow for an efficient search through parameter space. Employing these tools, we extend the analysis of Paper I by investigating the effects of the ill-determined contribution of non-thermal pressure support on the H I flaring, and the consequences for the inferred $R_0, \Theta_0, \Sigma_*, q$ values. The strongest constraints available are the observed $\Theta_0/R_0$ ratio, the mass of the stellar disk and the fact that the dark halo is almost certainly oblate. Taken together, these constraints rule out substantial contributions to the support of the H I layer by cosmic ray pressure or magnetic fields if the Sun’s distance to the Galactic centre is greater or equal than 7 kpc, in agreement with theoretical predictions. We find that the Milky Way’s dark matter halo is close to spherical for all but the smallest values of $R_0$ or $\Theta_0$. A dark matter halo as flattened as $q = 0.2$ is only possible if our distance to the Galactic centre is smaller than about 6.8 kpc.

It is possible to construct a self-consistent oblate model of the Galaxy with $R_0 = 8.5$ kpc and $\Theta_0 = 220 \text{ km s}^{-1}$, but only if the local stellar column density is less than about 18 $M_\odot \text{ pc}^{-2}$, and $\sigma_g \gtrsim 11 \text{ km s}^{-1}$.

Kuijken & Gilmore’s (1991) determination of the column-density of matter within 1.1 kpc of the plane $(71 \pm 6 \ M_\odot \text{ pc}^{-2})$ is robust and valid over a wide range of Galactic constants and disk scale-lengths. For the stellar contribution to this total mass we suggest a consensus average of $\Sigma_* = 35$ and a consensus error of 10 $M_\odot \text{ pc}^{-2}$.

If $R_0 \geq 7$ kpc, then the dark halo of the Milky Way is fairly close to spherical, independent of the amount of non-thermal pressure support. Such a round halo argues against dissipational baryons as a viable dark matter candidate. Further, since all other baryonic dark matter candidates have already been observationally excluded (Hegyi & Olive 1986), we must conclude that the dark matter halo of the Milky Way is most likely made up of something altogether more exotic (MACHOS, neutrinos, axions, neutralinos ...). Even in the Solar neighborhood, there are non-negligible quantities of this material: our proposed dark halo models imply that it amounts to some 0.42 GeV/c$^2$ per cubic centimetre or $(11 \pm 5) \text{ mM}_\odot \text{ pc}^{-3}$. The direct detection of this material remains a challenge for experimental physicists.

Employing the local disk mass and the flaring of the Galactic H I layer, we find strong correlations between the parameters of axisymmetric mass models. We presented several examples in the previous section. Since we make strong predictions as to the values of $R_0, \Theta_0, \Sigma_*, q$ and $\sigma_g$ (Figs. 3 and 4), our models can be subjected to experimental verification. For example, all parameters but $\sigma_g$ could be determined using astrometric data from future astrometric space missions such as FAME, SIM and GAIA. Such high precision data are ideally suited to support, or falsify, the models we propose here. It would also be worthwhile to investigate the effects of deviations from axisymmetry on the inferred halo shape and rotation speed. Whatever the outcome, we will learn a great deal more about the structure and dynamics of the Milky Way galaxy.
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APPENDIX A: MORE ON THE VERTICAL DISK-HALO CONSPIRACY

In section 3.1 we reviewed the stellar kinematical route to determine the local stellar columndensity. In this appendix we try to emulate and check this method in some detail. In order to do so we calculate the exact vertical force law \( K_{z,\text{exact}} \) for a limited number of Galaxy models listed in
Table 2 using the global approach (§ 3.3, cf. Olling 1995). We also compute an approximation to $K_{s,\text{exact}}$ from the local mass distribution (§ 3.3, cf. Olling 1995), which follows from an integration of the Poisson equation:

$$K_{s,\text{local}}(z) = -4\pi G \int_0^z dz' \left[ \rho_m(z') + \rho_{rot} \right],$$  \hspace{1cm} (A1)

with $\rho_m$ the matter density, $\rho_{rot}(= -2\sqrt{\frac{V_{rot}^2}{R}} \frac{dV_{rot}}{dr})$ a pseudo density which arises from gradients in the rotation curve, and $K_s$ the vertical force per unit mass.

In figure A1 we present true vertical force (filled circles) and the local approximation (open circles) for one particular neighborhood and close to the plane, the local approximation is good to the true vertical force law. Significant deviations from the true $K_s$ only occur above approximately 800 pc. We also include the observationally determined $K_s$ [crosses with error bars; KG89’s equation (1)] with $D=250$ pc.

Now we investigate the possibility that the actual dark matter density law differs from the distribution in our models [eqn. (1)]. Choosing a different DM distribution while keeping the observed $K_s$ fixed requires a different luminous mass distribution. As a simple test case, we plot a model where the DM density is multiplied by 1.92 (labeled “dGR+Hx1.92” in Fig. A1). In this case, we have to decrease the stellar column-density to the GBF97 value (25.8 $M_\odot$ pc$^{-2}$) in order to keep $K_s$ approximately unchanged. Next we consider DM distributions inspired by Bahcall’s “P” models (1984b), where the DM density is assumed to be a linear combination of the known components:

$$\rho_{\text{err}} = X_d \times \rho_d + X_g \times \rho_g + X_h \times \rho_h + X_{rot} \times \rho_{rot},$$  \hspace{1cm} (A2)

where the subscripts $d, g, h$ denote the disk, gas, and halo components, respectively. Using this formalism, we can calculate the columndensities of all components (ISM, stars, DM and $\Sigma_{\text{tot}}^{1.1}$) which arise from a particular choice of the $X_i$-values. From such an erroneous density distributions ($\rho_{\text{err}}$) we determine the erroneous vertical force ($K_{s,\text{err}}$) using equation (A1). We present the $K_{s,\text{true}}/K_{s,\text{err}}$ ratio, averaged in several z-height ranges, in Table A1 (columns 5-7). In this table we also list the $X_i$, $\Sigma_*$ and $\Sigma_{\text{tot}}$ values for some models. In addition to the P-models, we also evaluate models in which we keep the ISM and $\rho_{rot}$ contributions fixed while varying the stellar and dark matter densities such that the force at 400 pc above the plane equals the true value. From table A1 we see that it is possible to decrease $\Sigma_*$ by more than an order of magnitude and simultaneously increase $X_d$ without altering $K_s(z) \leq 800$ pc substantially. It seems that the vertical distribution of dark and stellar matter conspires in such a way that their relative contributions can not be easily separated. Thus, this “vertical disk-halo conspiracy” is much like the classical disk-halo conspiracy which arises in considerations of the radial distribution of luminous and dark luminous matter derived from galaxy rotation curves.

From the data presented in Table A1 it is clear that, without additional assumptions, stellar kinematical data extending to a few hundred parsec are not sufficient to determine the mass of the stellar disk. However, $\Sigma_{\text{tot}}^{1.1}$ is much better determined (rightmost column of Table A1), in agreement with Kuijken & Gilmore (1991). In order to discriminate between models, it is essential to incorporate a rotation curve constraint to limit the possible DM densities. Kuijken & Gilmore (1989) used such a constraint explicitly, while Bahcall (1984b) and Flynn & Fuchs (1994) used “reasonable” values for the density of non-disk-like dark matter to rule out extreme values for the mass of the stellar disk. Furthermore, if data at larger $z$ are considered, the differences between the force laws shown in figure A1 start to become apparent and can be used to limit the allowed range for $\Sigma_*$ (Flynn & Fuchs 1994).

Although the model parameters we have used to construct Figure A1 and Table A1 lie at the extreme end of the models investigated by Kuijken & Gilmore, we find that other models, within the 1-$\sigma$ range of $\Theta_0/R_0$, $h_0$, $\Sigma_*$, and $\Sigma_{\text{tot}}$, yield similar results. Thus, mass models for the Milky Way have to conform to the constraint $\Sigma_{\text{tot}}^{1.1} = (71 \pm 6)$  

| $X_d$ | $X_g$ | $X_{rot}$ | $X_h$ | $\Sigma_*$ | $\Sigma_{\text{tot}}^{1.1}$ |
|------|------|-----------|------|-----------|------------------|
| 1.25 | 1.25 | 1.00      | 1.03 | 0.99      | 45.4             |
| 1.29 | 1.29 | 0.00      | 0.95 | 1.04      | 46.9             |
| 2.05 | 0.00 | 0.00      | 0.85 | 1.03      | 74.5             |
| 1.95 | 0.00 | 1.95      | 0.85 | 1.03      | 70.8             |
| 0.00 | 3.49 | 0.00      | 1.35 | 0.90      | 50.9             |

For the ensemble of models for which the parameters differ by $\leq 1$-$\sigma$ from the model presented here, the approximations based on equation (A1) reproduce the true $K_s$ typically to within 75%: sometimes models that include the rotation curve gradient term perform best, sometimes not. The same is true for the models listed in Table 2 with other Galactic constants than those presented in figure A1.
At the Solar circle, and for $z = 1.1$ kpc, equation (6) can be rearranged to read:

$$\frac{\Sigma_h^1(q)}{\Sigma_h^1(1)} \approx 1.1 R_0 + R_0 (0.9 - R_0) q \quad q \lesssim 0.15 \quad (B1)$$

At the Solar circle, and for $z = 1.1$ kpc, equation (6) can be rearranged to read:

$$\frac{\Sigma_h^1(q)}{\Sigma_h^1(1)} \approx q^{-13/20}, \quad q \geq 0.15 \quad (B2)$$

where the core radius has been set to $R_0$. In this appendix we use —for the sake of convenience— the symbol $q$ for the halo flattening derived from the local stellar kinematics constraint, and $q_{H1}$ for the halo’s shape derived from the H I flaring. Taking the fiducial values for $\Sigma_{col}^1$, $\Sigma_*$, and $\Sigma_g$ to be 71, 35, and 14.5 $M_\odot$ pc$^{-2}$, respectively, and employing equation (6) we determine the fiducial value for $\Sigma_h^1$ to be 21.5 $M_\odot$ pc$^{-2}$. Further, defining $\Sigma_* = 35 + \delta \Sigma_*$, use the fiducial column densities defined above, and apply equations (B1) and (B2), we can relate the required halo flattening to $\delta \Sigma_*$:

$$q \approx \frac{21.5 - R_0 \Sigma_h^1(1) - \delta \Sigma_*}{0.9 R_0^2 \Sigma_h^1(1)} \quad (B3)$$

APPENDIX B: STELLAR KINEMATICS AND THE RELATION BETWEEN THE MODEL’S PARAMETERS

In section 3.2 we described how the observationally determined $\Sigma_{col}^1$-value imposes correlations between luminous and dark matter in the Solar neighbourhood. In this section we present some specific examples to illustrate what we can learn about the structure of the Milky Way by treating $\Sigma_*$ as a parameter which is only constrained by $\Sigma_{col}^1$ [cf. eqn (5)]. Note that these correlations are independent of any H I flaring constraints.

To summarize the above, in order to determine the stellar column density from stellar kinematics, the existence of the vertical disk-halo conspiracy requires that one has to include self-consistent rotation curve constraints and/or sample the region above 800 pc.
\[
\approx \left( \frac{\Sigma_{h}^{1.1}(1)}{21.5} \right)^{20/13} \left[ 1 + \frac{20/13}{21.5} \delta \Sigma_{*} \right] \]  

(B4)

for the same \( q \)-ranges as in Eqs. \((\text{B1})\) and \((\text{B2})\). Since \( \Sigma_{h}^{1.1}(q = 1) \) depends on \( \Theta_{0} \) [eqn. \((\text{B1})\)], the solutions of equations \((\text{B3})\) and \((\text{B4})\) depend on \( R_{0}, \Theta_{0} \), and \( \delta \Sigma_{*} \). We use our mass models to determine \( \Sigma_{h}^{1.1}(q = 1; 1 \equiv 1, \Theta_{0}) \) and rewrite equations \((\text{B3})\) and \((\text{B4})\) for a few interesting cases. For \( \delta \Sigma_{*} = 0 \) we find:

\[
q \approx 0.07 \pm 0.05 \quad (\Theta_{0} = 165) \quad (\text{B5})
\]

\[
q \approx 0.4 - 0.1(R_{0} - 7.1) \pm 0.1 \quad (\Theta_{0} = 175) \quad (\text{B6})
\]

\[
q \approx 0.8 - 0.2(R_{0} - 7.1) \pm 0.2 \quad (\Theta_{0} = 185) \quad (\text{B7})
\]

Thus, lower rotation speeds and larger \( R_{0} \)'s require more highly flattened halos. Furthermore, for any fixed value of \( R_{0} \), the slope of the \( q - \delta \Sigma_{*} \) relation depends strongly upon \( \Theta_{0} \). For example, with \( R_{0} = 7.1 \) kpc we find:

\[
q \approx 0.06 \pm 0.005 \delta \Sigma_{*} \quad (\Theta_{0} = 165) \quad (\text{B8})
\]

\[
q \approx 0.400 \pm 0.030 \delta \Sigma_{*} \quad (\Theta_{0} = 175) \quad (\text{B9})
\]

\[
q \approx 0.800 \pm 0.060 \delta \Sigma_{*} \quad (\Theta_{0} = 185) \quad (\text{B10})
\]

Equations \((\text{B3})\) through \((\text{B10})\) clearly show that small values of the Galactic rotation speed imply a highly flattened dark matter halo, whatever our distance to the Galactic centre, and whatever the mass of the stellar disk. We also see that the last relations constrain \( \Sigma_{*} \) rather tightly: for models with \( \Theta_{0} \approx 185 \) (175) km s\(^{-1}\), an increase in the stellar column density of \( \sim 3 \) (20) \( M_{\odot} \) pc\(^{-2}\) covers the whole allowed range for \( q \).

In section \((\text{D})\) and figure \((\text{A})\) we have seen the stellar kinematics and H I flaring constraints provide mutually exclusive constraints on \( \Sigma_{*} \) and \( q \) if both the rotation speed and the effective velocity dispersion are small. This can be understood as follows: equations \((\text{B3})\)-\((\text{B10})\) show that the \( \Sigma_{tot}^{1.1} \) constraint implies small \( q \)-values for low \( \Theta_{0} \)'s. Likewise, the H I flaring constraint yields small \( q \)'s for small \( \Theta_{0} \)'s, but only if the velocity dispersion does not decrease by too much [cf. eqn. \((\text{B4})\)]. Furthermore, because the slope of the \( q(\Sigma_{*}) \) relation becomes shallower with decreasing \( \Theta_{0} \), the accessible range for \( q_{1} \) decreases with \( \Theta_{0} \). If the flaring analysis leads to a \( q_{H} \sim 1 \) halo, extreme values for \( \Sigma_{*} \) are required to match \( q_{1} \) with \( q_{H} \). Obviously, if the required stellar column exceeds \( \Sigma_{tot}^{1.1} \), it is not possible to construct a self-consistent model for that particular combination of Galactic constants and gaseous velocity dispersion. In these circumstances it is possible that our procedure to determine \( \Sigma_{*} \) and \( q \) yields extreme values for \( \Sigma_{*} \) and \( q \), and sometimes even negative values.

### APPENDIX C: ERROR ESTIMATION

At this point it is also possible to estimate the accuracy to which the various parameters in figure \((\text{A})\) are determined. For example, as we have seen before, the halo flattening inferred from the H I flaring has only a slight dependency on the mass of the stellar disk (cf. figure 3 of Paper I). This allows us to estimate the flattening of the halo by averaging the \( \delta q_{H} \) values from the various model runs at a given \( R_{0} \) and \( \Theta_{0} \). For example, the nine \( \delta q_{H} \) errors tabulated in table \((\text{B})\) yield \( \delta q_{H} \approx 0.06 \) for the two low \( R_{0} \) values at \( q \approx 0.1 \). Employing \( q \propto \Theta_{0}^{2} \) [cf. eqn. \((\text{B3})\)] we find:

\[
\frac{\delta \Theta_{0}}{\delta q_{H}} = \frac{2 \delta q_{H}}{\Theta_{0}^{2}} \quad (\text{C1})
\]

The \( \delta q_{H} \) values as determined from the flaring measurements are thus precise to about 6\%, while the value of the local Galactic rotation speed we derive from a halo measurement has an estimated accuracy of 3\%.

To estimate how well the local stellar column density is determined from the flaring and the observed \( \Sigma_{h}^{1.1} \) value of the total column density we re-write equation \((\text{B14})\) to read:

\[
\delta(\Sigma_{*}) \sim \frac{q}{c_{1}c_{2}} \sqrt{(\frac{\delta q}{q})^{2} + (\frac{\delta c_{1}}{c_{1}})^{2}},
\]

where \( c_{1} = \Sigma_{h}^{1.1}(1)/21.5 \), \( c_{2} = (20/13)/21.5 \) and \( q/(c_{1}c_{2}) \sim 11.2 \) for \( q \approx 0.8 \) and \( \delta \Sigma_{*} \sim 21.5 \). If assume that \( c_{1} \) is without error, the second term in eqn. \((\text{B14})\) vanishes, and we arrive at a lower limit for the error in \( \Sigma_{*} \) of about 0.7 \( M_{\odot} \) pc\(^{-2}\). If we assign the full error in the observed total column density (6 \( M_{\odot} \) pc\(^{-2}\)) to \( \Sigma_{h}^{1.1}(1) \), we have \( c_{1} \sim 1 \pm 0.43 \), so that \( \delta(\Sigma_{*}) = 11.2 \sqrt{(0.06)^{2} + (0.43)^{2}} \sim 4.8 M_{\odot} \) pc\(^{-2}\). We thus estimate that our method of deriving the stellar column density has an accuracy somewhere between 0.7 and 4.8 \( M_{\odot} \) pc\(^{-2}\) or about 2 to 14 percent.

The error estimates above are close to the errors derived from our detailed modeling procedure.

### APPENDIX D: THE ISM OF THE MILKY WAY TABULATED

In this appendix we present a tabulated version of the radial variation of the atomic and molecular hydrogen, as well as their widths. We present these data in a manner which is independent of the values of the Galactic constants as well as the shape of the rotation curve. The H I column density at the Solar position includes 1.4 \( M_{\odot} \) pc\(^{-2}\) of ionized hydrogen. No ionized hydrogen is included at any other radius. The column densities listed do not include the contribution due to Helium. In our model calculations of the potential we increase the listed columndensities to include 23.8\% Helium. We have brought the data from the sources listed in section \((\text{B})\) onto a common distance scale defined by Merrifield's (1992) determination of the \( W(R/R_{0}) = v_{rad} / (\sin \ell \cos b) \) curve. Here \( v_{rad} \) and \( b \) are the radial velocity and the Galactic coordinates, respectively. In practice this rescaling works as follows: 1) from the rotation curve \( \Theta'(R') \) in the original reference, determine \( R' / R_{0} \) and \( W'(R'/R_{0}) \equiv R_{0}^{2} \Theta'(R') / R' - \Theta_{0} / R_{0} \) [the primed quantities refer to the values assumed in the reference], 2) for each property \( X' \) (gas layer width and volume density), determine \( X'(W') \) from \( X'(R) \) and \( W'(R') \), 3) find Merrifield's \( R/R_{0} \) values for which \( W = W' \), 4) the property \( X' \) re-gridded onto Merrifield’s distance scale is now given by \( X(R/R_{0}) \).

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Table D1. The surface density and thickness of the atomic and molecular hydrogen components in the Milky Way as a function of scaled Galactocentric radius, $R/R_0$. The H I and H$_2$ column densities have units $M_\odot$ pc$^{-2}$. The widths listed are full widths at half maximum (FWHM) and have units of $R_0$. For $R > R_0$, the widths of the H I layer is the average of several works, see Paper I for details. Some entries have been left blank: we were unable to derive reliable widths at these radii. The column densities at these radii are from Wouterloot et al. (1990).

| $R/R_0$ | $\Sigma_{\text{HI}}$ | $\Sigma_{\text{H}_2}$ | $W_{\text{HI}}$ | $\delta W_{\text{HI}}$ | $W_{\text{H}_2}$ |
|---------|-----------------|-----------------|--------------|-----------------|---------|
| 0.218   | 0.59            | 1.08            | 0.0330       | 0.0014          | 0.0151  |
| 0.276   | 1.04            | 1.54            | 0.0320       | 0.0031          | 0.0158  |
| 0.309   | 1.23            | 1.47            | 0.0323       | 0.0017          | 0.0134  |
| 0.343   | 1.30            | 1.67            | 0.0306       | 0.0014          | 0.0119  |
| 0.376   | 1.29            | 2.67            | 0.0294       | 0.0008          | 0.0131  |
| 0.409   | 1.31            | 4.29            | 0.0268       | 0.0045          | 0.0140  |
| 0.440   | 1.41            | 4.91            | 0.0290       | 0.0031          | 0.0142  |
| 0.467   | 1.59            | 5.02            | 0.0313       | 0.0028          | 0.0143  |
| 0.499   | 1.85            | 5.57            | 0.0278       | 0.0011          | 0.0153  |
| 0.530   | 2.10            | 6.23            | 0.0240       | 0.0017          | 0.0165  |
| 0.559   | 2.30            | 5.87            | 0.0301       | 0.0014          | 0.0168  |
| 0.586   | 2.48            | 5.48            | 0.0346       | 0.0014          | 0.0163  |
| 0.612   | 2.90            | 5.18            | 0.0344       | 0.0023          | 0.0152  |
| 0.641   | 3.72            | 4.62            | 0.0415       | 0.0034          | 0.0139  |
| 0.668   | 4.36            | 3.94            | 0.0504       | 0.0017          | 0.0129  |
| 0.692   | 4.24            | 3.73            | 0.0525       | 0.0014          | 0.0120  |
| 0.717   | 3.60            | 3.97            | 0.0447       | 0.0031          | 0.0115  |
| 0.742   | 3.17            | 4.27            | 0.0452       | 0.0039          | 0.0122  |
| 0.764   | 3.23            | 4.33            | 0.0485       | 0.0130          | 0.0136  |
| 0.786   | 3.68            | 4.09            | 0.0495       | 0.0101          | 0.0152  |
| 0.806   | 4.22            | 3.60            | 0.0565       | 0.0039          | 0.0161  |
| 0.828   | 4.72            | 2.95            | 0.0530       | 0.0048          | 0.0159  |
| 0.848   | 4.90            | 2.44            | 0.0570       | 0.0034          | 0.0144  |
| 0.865   | 4.80            | 2.06            | 0.0572       | 0.0011          | 0.0134  |
| 0.882   | 4.52            | 1.70            | 0.0509       | 0.0014          | 0.0136  |
| 0.897   | 4.41            | 1.45            | 0.0525       | 0.0020          | 0.0146  |
| 0.913   | 4.75            | 1.30            | 0.0523       | 0.0036          | 0.0158  |
| 0.926   | 5.39            | 1.26            | 0.0652       | 0.0017          | 0.0167  |
| 0.938   | 6.13            | 1.29            | 0.0730       | 0.0020          | 0.0174  |
| 1.000   | 9.25            | 1.80            | 0.0577       | 0.0042          | 0.0199  |
| 1.058   | 8.57            | 1.80            |              |                 |         |
| 1.089   | 7.77            | 1.28            | 0.0611       | 0.0044          | 0.0238  |
| 1.193   | 6.09            | 0.44            |              |                 |         |
| 1.332   | 6.56            | 1.62            | 0.0734       | 0.0061          | 0.0385  |
| 1.457   | 7.45            | 1.47            |              |                 |         |
| 1.586   | 7.62            | 1.10            | 0.0801       | 0.0062          | 0.0458  |
| 1.721   | 6.02            | 0.54            |              |                 |         |
| 1.848   | 5.30            | 0.37            | 0.1108       | 0.0099          | 0.0742  |
| 1.995   | 4.36            | 0.20            |              |                 |         |
| 2.101   | 3.41            | 0.11            | 0.1391       | 0.0115          | 0.0767  |
| 2.177   | 3.31            | 0.14            |              |                 |         |
| 2.262   | 2.84            | 0.12            |              |                 |         |
| 2.355   | 2.36            | 0.09            | 0.1666       | 0.0143          |         |
| 2.443   | 2.01            | 0.05            |              |                 |         |
| 2.532   | 1.66            | 0.01            |              |                 |         |