Formation of New Fermi Surfaces in 3D Crystals at Ultra High Magnetic Field with Different Orientations

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In the tight-binding approximation the Harper like equation describing an electron in 3D crystal subject to a uniform magnetic field is obtained. It is supposed that the vector \( \mathbf{H} \) can be oriented along several directions in the lattice. The Fermi surfaces relevant to a magnetic flux \( p/q = 1/2 \) in a simple cubic lattice are built. The quantization rules in magnetic fields slightly distinguished from \( p/q = 1/2 \) are investigated.

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The effects of a high magnetic field on Bloch electrons are fascinating problem with very rich physics. This is not surprising since there are two fundamental periods in the problem – period of the potential and the period of the phase of the wave function – and the interplay of these two periods gives very interesting spectrum (Hofstadter butterfly) and eigenstate structure. Starting from classical works of Azbel [1], Hofstadter [2], Wannier [3] (see also [4-7]) this problem attracts the increasing attention. During the last decade

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the one electron quantum states in lateral superlattices in the presence of perpendicular magnetic field have been studied in a series of theoretical [8-10] and experimental [11,12] works. But unfortunately, all works mentioned above deals only with 2D crystals subject to a perpendicular magnetic field. Nowadays when in VNIIEF (Sarov) the magnetic field up to 28 MGs is reached [13] the idea of observation of details forecasted by the theory in the crystals with the lattice spacing of $a = 0.6 \text{ nm}$ and more does not seem to be unaccessible.

In this paper we shall consider electronic states in 3D crystals subject to ultrahigh magnetic field. We consider that the vector $H$ is parallel to arbitrary translation vector of a lattice. In a tight-binding approximation the wave function satisfying the generalized Bloch-Peierls conditions is obtained and Harper like equation for different orientations of a magnetic field is derived. As an example, the Fermi surfaces for magnetic subbands relevant to rational value of a magnetic flux $p/q = 1/2$ are built. The quantization rules for magnetic flux near this rational value of flux quanta are studied.

First of all in the tight-binding approximation let us derive Harper’s equation for the simple cubic lattice subject to uniform magnetic field. To be specific let the basis vectors on the $x, y$ plane be $\mathbf{a}_1$ and $\mathbf{a}_2$, and the magnetic field is applied along lattice basis vector $\mathbf{a}_3 \parallel 0z\ (|\mathbf{a}_1| = |\mathbf{a}_2| = |\mathbf{a}_3| = a)$. Let vector potential is be chosen in Landau gauge $\mathbf{A} = (0, Hx, 0)$. Then electron wave function must satisfy the generalized Bloch conditions (Peierls conditions) [14]

$$\psi_{\mathbf{k}}(\mathbf{r}) = \psi_{\mathbf{k}}(x + qa, y + a, z + a) \exp(-i k_x qa) \exp(-i k_y a) \exp(-i k_z a) \exp(-2\pi ipy/a), \quad (1)$$

due to the fact that the vector potential is not a periodic function of the coordinates. Here $p$ and $q$ are mutually prime integers which define the number of magnetic flux quanta per
two-dimensional square elementary cell

\[
p = \frac{\Phi}{\Phi_0} = \frac{|e|H|a_1 \times a_2|}{2\pi \hbar c},
\]

(2)

where \( \Phi_0 = h\varepsilon/|e| \) is the magnetic flux quanta. In a presence of a magnetic field the electron quantum states must be classified in accordance with irreducible representations of magnetic translation group

\[a_{mag} = nqa_1 + ma_2 + la_3, \quad (n, m, l \text{ are integer numbers}).\]

The quasimomentum \( k \) takes values in the magnetic Brillouin zone

\[-\frac{\pi}{qa} \leq k_x \leq \frac{\pi}{qa}, \quad -\frac{\pi}{a} \leq k_y \leq \frac{\pi}{a}, \quad -\frac{\pi}{a} \leq k_z \leq \frac{\pi}{a}.\]

(3)

In the tight-binding approximation the electron wave function which satisfies the conditions (1) can be written in the form [15]

\[
\psi_k(r) = \sum_{n,m,l} g_n(k) \exp(ia_nk) \exp(-2\pi i \frac{p(y - ma)}{qa} n) \psi_0(r - na, y - ma, z - la)
\]

(4)

where \( \psi_0(r - a_n) \) is the atomic function. The path-dependent geometric phase \( 2\pi \frac{p(y - ma)}{qa} n \) of the wave function is known to play a fundamental role in the problem.

By substituting of (4) into Schrödinger equation and evaluating the transfer integrals between neighbour sights we shall obtain a system of difference equations for coefficients \( g_n \). The magnetic field has a non-trivial influence on the transfer integrals between neighbour lattice sights in the \( x \) direction

\[
A = \int \exp\left( \pm 2\pi i \frac{p(y - ma)}{qa} n \right) \psi_0(r - a'_n)(V(r) - U(r - a_n))\psi_0(r - a_n) \, d\tau,
\]

(5)

where \( a'_n = ((n \pm 1)a, ma, la) \), \( V(r) \) is the crystal scalar potential with periodicity \( a \) in three dimensions and \( U \) is the atomic potential. Calculating the transfer integrals in the mean-value approximation we substitute \( y = ma \) and obtain \( A = E_0 \) where \( E_0 \) is the
transfer integral (5) in the absence of a magnetic field. In the \( y \) direction the transfer integral is

\[
B = \exp \left( \pm 2\pi i \frac{p}{q} \right) \int \psi_0 (r - a'_n) (V(r) - U(r - a_n)) \psi_0 (r - a_n) \, d\tau,
\]

(6)

where \( a'_n = (na, (m \pm 1)a, la) \) and in the \( z \) direction the transfer integral is equal to \( E_0 \). As a result we obtain the well-known Harper’s equation [4]

\[
\exp (ik_x a) g_{n+1} + \exp (-ik_x a) g_{n-1} + 2g_n \cos \left( 2\pi n \frac{p}{q} + k_y a \right) = \varepsilon (k_x, k_y, k_z) g_n,
\]

(7)

where

\[
\varepsilon (k_x, k_y, k_z) = \varepsilon_\perp (k_x, k_y) - 2 \cos k_z a
\]

(7a)

is the dimensionless energy measured in terms of \( E_0 \). The spectrum \( \varepsilon_\perp (k_x, k_y) \) in Eq.(7) consists of \( q \) bands and depend on one single parameter \( p/q \) counting the number of magnetic flux quanta per unit cell (Fig.1). If flux is irrational the spectrum is a singular-continuum – uncountable but measure zero set of points (Cantor set). Originally the Eq.(7) was derived by Harper [4] using Peierls substitution [14].

For some values of \( p/q = 1/2, 1/3, 1/4 \) it is possible to solve the Eq.(7) analytically. As a result one can obtain the following expressions for dispersion laws for \( q \) magnetic subbands: for \( p/q = 1/2 \) (two subbands)

\[
\varepsilon_\perp^{(1,2)} (k_x, k_y) = \pm 2 \sqrt{\cos^2 k_x a + \cos^2 k_y a},
\]

(8)

for \( p/q = 1/3 \) (three subbands)

\[
\varepsilon_\perp^{(1)} (k_x, k_y) = -2 \sqrt{2} \cos \left( \frac{2\pi}{3} + (\arctan \beta)/3 \right),
\]

\[
\varepsilon_\perp^{(2)} (k_x, k_y) = 2 \sqrt{2} \cos \left( \frac{\pi}{3} + (\arctan \beta)/3 \right),
\]

\[
\varepsilon_\perp^{(3)} (k_x, k_y) = -2 \sqrt{2} \cos \left( (\arctan \beta)/3 \right),
\]

(9)
where \( \beta = \sqrt{8 - \alpha^2/\alpha} \), \( \alpha = \cos 3k_xa + \cos 3k_ya \); and for \( p/q = 1/4 \) (four subbands)
\[
\varepsilon_{(1,2,3,4)}(k_x, k_y) = \pm \sqrt{4 \pm 2\sqrt{4 - \sin^2 2k_xa - \sin^2 2k_ya}}.
\]

Fig.2 shows the Fermi surfaces in the lowest subband for \( p/q = 1/2 \) obtained using (7a) and (8) in the first Brillouin zone \(-\pi/qa \leq k_x \leq \pi/qa, -\pi/qa \leq k_y \leq \pi/qa, -\pi/a \leq k_z \leq \pi/a\). Three surfaces for different representative energies are plotted. For an ellipsoid-type surface such as (1) (\( |k| \to 0 \)) the simple analytical expression for an energy spectrum \( \varepsilon(k) \) can be easily obtained with the help of (7a) and (8). The relevant effective masses are determined as
\[
m_{x,y}^* = \frac{\hbar^2}{\sqrt{2E_0a^2}}, \quad m_z^* = \frac{\hbar^2}{2E_0a^2}.
\]

The asymptotic spectrum near the point \( k_{x,y} \to \pi/2a \) ((2)-type surface) is \( \varepsilon(k_x, k_y, k_z) \sim \sqrt{k_x^2 + k_y^2 - 2\cos k_z} \).

Now we shall discuss the magnetic quantization rules at values of a flux slightly distinguished from \( p/q = 1/2 \):
\[
\frac{p}{q} = \frac{1}{2} + \frac{1}{q'}, \quad q' \gg 1.
\]

This corresponds to the magnetic field \( H = H(1/2) + \Delta H(1/q') \). As one can see from Fig.1 near value of \( p/q = 1/2 \) the spectrum represents a system of narrow subbands (practically discrete levels). Here it is possible to see an equidistant spectrum, points of level accumulation and ranges where \( \varepsilon_\perp \sim \sqrt{N} \), (\( N \) is the level number).

The spectrum \( \varepsilon_\perp(k_x, k_y) \) of Harper’s equation near one half of flux quanta can be studied analytically due to the self-similarity of its structure. In the case \( p/q = 1/2 \) two subbands and two subsystems in the eigenvector distribution can be observed. It may be derived analytically from Harper’s equation that every eigenvector function from a subsystem has an argument step \( \Delta n = 2 \).

The magnetic flux (12) gives us the diagonal term in Harper’s equation of the form
2 \cos(\pi n + 2\pi n/q')g_n = 2 \cos \pi n \cos(2\pi n/q')g_n. \text{ Now } q' \text{ is a new period of the equation.}

Since \cos \pi n = (-1)^n our system of equations (7) consists of two subsystems where only the sign varies. It is easy to construct an equation for each subsystem. After simple algebra we obtain \((k_x = k_y = 0)\):

\[
g_{n+2} + g_{n-2} + 2 \cos \frac{4\pi}{q'} n g_n = (\varepsilon_{\perp}^2 - 4) g_n.
\]  

(13)

Now \(\varepsilon_{\perp}^2 - 4 = \varepsilon'_{\perp}\) is the energy in Harper’s equation (7) and \(\varepsilon_{\perp} = \pm \sqrt{\varepsilon'_{\perp} + 4}\) describes the structure of the spectrum near one half of flux quanta and \(\cos k_z a = 0\). Two signs at the square root correspond to two groups of energy subbands. The energy \(\varepsilon_{\perp}\) describes all parts of the spectrum near \(p/q = 1/2\) (Fig.1). We can obtain from (13) the ”square root” dependence at \(\varepsilon_{\perp} = 0\), the clustering points \(\varepsilon_{\perp} = \pm 2\) and Landau levels near \(\varepsilon_{\perp} = \pm 2\sqrt{2}\). These peculiarities of a spectrum can be obtained with the help of Onsager-Lifshitz quasiclassical quantization rules on the Fermi surfaces (Fig.2a,b). It is easy to see that magnetic quasiclassical quantization on the (1)-type surface results to the equidistant energy spectrum at the bottom of the lowest magnetic subband. Accordingly, the magnetic Onsager-Lifshitz quantization near \(k_z \to 0\) on (2)-type surface yields the ”square root” spectrum. At last, it is clear that in magnetic field \(H \parallel 0z\) there are self-crossing orbits on (3)-type surface. It gives the accumulation point in the spectrum \(\varepsilon_{\perp}\) near \(p/q = 1/2\) (see Fig.1). It can be stressed that level spacing obtained from (13) has the same value that was derived using quasiclassical quantization.

In accordance with Harper’s equation (7) and (7a) the partial overlapping between \(q\) energy subbands is observed. When \(p/q = 1/2\) the overlapping takes place in the energy interval \((-2, 2)\). It is clear that when the electron concentrations leading to \(\varepsilon_F \in (-2, 2)\) in the magnetic field \(H = H(1/2) + \Delta H(1/q')\) the de Haas - van Alfen oscillations with different periods corresponding to the different energy level series (Fig.1) can be observed.
Further it is necessary to make the following relevant note. The wave function (4) describes non-homogeneous probability distribution on the lattice sights. This is due to the amplitudes $g_n(k)$ which give us non-uniform shapes. However, in real crystals due to Coulomb interaction the static electron density distribution must maintain its symmetry even in the presence of magnetic field.

Fortunately, the spectrum of Harper’s equation is $q$-fold degenerate since $\varepsilon(k_x, k_y + 2\pi j/qa, k_z) = \varepsilon(k_x, k_y, k_z)$, $j = 0, 1, 2, \ldots, q - 1$ and a wave function with homogenous electron density distribution can be written as a combination of (4). Let us introduce

$$\psi'_k(r) = \sum_{j=0}^{q-1} C_j(k) \hat{K}_j \hat{T}_ja \psi_k(r),$$

where $\psi_k(r)$ is from (4), $C_j(k) = \exp(-i k_x j a)$, $\hat{T}_ja \psi = \psi(x + ja, y, z) \exp(ief/\hbar c)$ ($f = -Hyja$), the operator $\hat{K}_j$ transforms $\psi_k$ with $k = (k_x, k_y, k_z)$ into $\psi'_k$ with $k' = (k_x, k_y - 2\pi pj/qa, k_z)$. After this procedure we can write (14) in the form

$$\psi'_k(r) = D(k) \sum_n \exp\left(-2\pi i \frac{p}{q} (y - ma) a n\right) \exp(i ka_n) \psi_0(r - a_n),$$

where $D(k) = \sum_{j=0}^{q-1} g_{n+j}(k)$ does not depend on $n$ due to the periodicity of the system. Now (15) describes homogeneous electron density distribution on the lattice sights and corresponds to the same energy as $\psi_k$.

Let us consider now the case of a simple cubic lattice subjected to a uniform magnetic field oriented along the diagonal of a square in the plane $(x, y)$. The magnetic field with amplitude $H$ has now the following Cartesian coordinates:

$$H = \frac{H}{\sqrt{2}} (1, 1, 0).$$

It is convenient to choose a new coordinate system making a rotation about the old $z$ axis by $\pi/4$ and defining the magnetic field orientation as $x_3$. This transformation is written
as following:

\[
x = \frac{x_3 - x_1}{\sqrt{2}},
\]

\[
y = \frac{x_3 + x_1}{\sqrt{2}},
\]

\[
z = x_2.
\]

We choose a new elementary cell: it is a rectangular parallelepiped based on the new coordinate system vectors: a square with the side \(\sqrt{2}a\) at the base in the plane \((x_1, x_3)\) and the height \(a\). This cell is non-primitive: it consists of two atoms (additional sights are located at the centers of top and bottom faces). In this coordinate system the magnetic field is oriented along one of the elementary cell basic vectors (namely, \(a_3\)). According to the general principles in this case the classification of electron states is possible [16]. Now the first Brillouin zone has an area two times less with respect to the initial value. The magnetic field in this coordinate system is written:

\[
H = H(0, 0, 1)
\]  
and it is convenient to choose a vector potential using Landau gauge:

\[
A = (-H x_2, 0, 0).
\]

The wavefunction in the tight-binding approximation can be written as following:

\[
\psi_k(r) = \sum_n g_n(k) \left\{ \exp \left( \frac{2\pi i p x_1 - m\sqrt{2}a}{q a/\sqrt{2}} n \right) \exp(ik\mathbf{a}_n)\psi_0(r - \mathbf{a}_n) + \exp \left( \frac{2\pi i p x_1 - (m + 1/2)\sqrt{2}a}{q a/\sqrt{2}} n \right) \exp(ik(\mathbf{a}_n + \mathbf{d}))\psi_0(r - (\mathbf{a}_n + \mathbf{d})) \right\}.
\]  

Here \(ka_n = k_1\sqrt{2}am + k_2an + k_3\sqrt{2}al\), \(\psi_0\) is an isolated atomic function in the presence of the magnetic field, the amplitudes \(g_n(k)\) are the subject of further research and \(g_{n+q} = g_n\). The indexes \((m, n, l)\) are taking all integer values independently, \(n\) numerates sights along \(x_2\) axis, perpendicular to the magnetic field. The sum is taken over all lattice
sights and two addendums in (22) correspond to the two existing crystalline subsystems. Each of them is a simple tetragonal lattice with the elementary cell being a rectangular parallelepiped with square of the side $\sqrt{2}a$ in its base and the height $a$. These subsystems are displaced on the vector $\mathbf{d}$ with new coordinates $(a/\sqrt{2}, 0, a/\sqrt{2})$. The magnetic translation group is constructed by the $q$ - time multiplication of the translation period along $x_2$ axis: $x_2 \to x_2 + qa$. The other translations are not changed. Their periods in the new coordinate system are $\sqrt{2}a$ which is the diagonal of a square with the side $a$. The classification of wavefunctions is possible when the number of flux quanta $\Phi/\Phi_0$ through the area $a^2/\sqrt{2}$ (being the minimal inter-sight area in the transversal to magnetic field projection) is a rational number: $\Phi/\Phi_0 = p/q$ where $p$ and $q$ are mutually prime integers.

The substitution of (19) into Schrödinger equation is performed and the transfer integrals are calculated. We will have the equation for the energy $\varepsilon$ and the amplitudes $g_n(\mathbf{k})$:

$$e^{ik_{2a}}g_{n+1} + e^{-ik_{2a}}g_{n-1} + 4\cos \left( \frac{k_{3a}}{\sqrt{2}} \right) \cos \left( 2\pi \frac{p}{q} n + \frac{k_{1a}}{\sqrt{2}} \right) g_n = \varepsilon g_n. \quad (20)$$

It should be mentioned that this equation may be obtained using Peierls substitution $\mathbf{k} \to \hat{k} - \frac{eA}{hc}$ with the initial spectrum at null magnetic field. In old coordinates it was written as $\varepsilon = 2(\cos k_x a + \cos k_y a + \cos k_z a)$, and in new coordinates $(x_1, x_2, x_3)$ it is

$$\varepsilon = 2 \cos k_2 a + 4 \cos \frac{k_{3a}}{\sqrt{2}} \cos \frac{k_{1a}}{\sqrt{2}}.$$

The equation (20) is a generalization of Harper’s equation for the case of magnetic field parallel to a diagonal of a square which is a face of the elementary cell cube. Keeping a three-diagonal structure (20) has some remarkable features with respect to standart Harper’s equation. First, (20) describes total (not ”transversal” $\varepsilon_\perp(k_1, k_3)$) energy as a function of quasimomentum $\mathbf{k}$: $\varepsilon = \varepsilon(k_1, k_2, k_3)$. Now $k_3 = const$ corresponds to the plane perpendicular to $\mathbf{H}$ and $k_1 = const$ describes the plane parallel to magnetic field. Second,
(20) corresponds to anisotropic Harper’s equation with anisotropy ratio

\[4 \cos \left( \frac{k_3 a}{\sqrt{2}} \right) = \lambda\]  \hspace{1cm} (21)

depending on the cross-section transversal to the magnetic field which is fixed by choosing the value of \(k_3\). It is well-known that Harper’s equation in the case \(\lambda \neq 2\) describes 2D square lattice with anisotropic transfer integrals [5,6]. According to (21) the persistent interval for \(\lambda\) is \(-4 \leq \lambda \leq 4\). In the quasiclassical limit it describes the situation where open-type trajectories are available. The contribution of open trajectories to the whole topological structure varies with the respect to a cross-section \(k_3 = const\) being chosen. For example, the cross-section \(k_3 = \pi \sqrt{2}/3a\) corresponds to \(\lambda = 2\) which is the case of isotropic Harper’s equation and only close-type orbits are available. This is illustrated in Fig.3 where Fermi surfaces (Fig.3a,b) and spectrum (Fig.3c) are shown for \(p/q = 31/60\). Then, the cross-section \(k_3 = \pi/\sqrt{2}a\) gives us \(\lambda = 0\) which is a ”full anisotropic” limit for Harper’s equation. Here as one can see in Fig.4a,b only opened-type orbits are available. It gives a continious energy spectrum (Fig.4c). For remaining values of \(\lambda\) both opened and closed orbits are present. In the last case the spectrum \(\varepsilon_{\perp}\) consists of a continious and quasi discrete (narrow subbands) parts.

As a conclusion, we suggested the expression for electron wave function which satisfies to Peierls conditions and obtained Harper’s equation from initial principles of the tight-binding model. The Fermi surfaces are built for different orientations of the magnetic field. In the following paper we plan to study the electron states in 3D crystals in the case of the magnetic field orientation parallel to the arbitrary translation vector of the lattice. The properties of de Haas - van Alfen effect in this problem will be studied.

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References

1. M.Ya.Azbel’, Zh. Exper. i Teor. Fiz. **46**, 929 (1964) [in Russian].

2. D.R.Hofstadter, Phys.Rev.B **14**, 2239 (1976).

3. F.H.Claro, G.H.Wannier, Phys.Rev.B **19**, 6068 (1979).

4. P.G.Harper, Proc.Phys.Soc.(London) A **68**, 874 (1955).

5. H.Hiramoto, M.Kohmoto, Int.Journ.of Modern Physics **6**, Nos. 3&4, 281 (1992).

6. M.Kohmoto, Y.Hatsugai, Phys.Rev.B **41**, 9527 (1990).

7. A.Barelli, R.Fleckinger, Phys.Rev.B **46**, 11559 (1992).

8. H.Silberbauer, J.Phys.: Condens. Matter **4**, 7355 (1992).

9. V.Ya.Demikhovskii, A.A.Perov, JETP **87**, 973 (1998)

10. V.Ya.Demikhovskii, A.A.Perov, Phys. Low-Dim. Structures **7/8**, 135 (1998).

11. D.Weiss, M.L.Roukes, A.Menschig, et al., Phys.Rev.Lett. **66**, 27 (1991).

12. T.Schlässer, K.Ensslin, J.P.Kotthaus, et al., Semicond. Sci. Technol. **11**, 1582 (1996).

13. B.A.Boyko, A.I.Bykov, M.I.Dolotenko et al, book of abstracts, VIIIth Int. Conference on Megagauss Magnetic Field Generation and Related Topics, p.149, Tallahassee, USA.

14. R.E.Peierls, Z.Phys. **80**, 763 (1933).
15. V.Ya.Demikhovskii, A.A.Perov, D.V.Khomitsky, Proc. of the VIIIth International Conference on Megagauss Magnetic Field Generation and Related Topics (MEGAGAUSS-VIII), Tallahassee, USA (1998) (in press).

16. J.Zak, Phys.Rev.A 134, 1602 (1964); Phys.Rev.A 134, 1607 (1964).
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Fig.1. Parts of the classical Hofstadter "butterfly" near $p/q = 0, 1/3, 1/2$.

Fig.2. The Fermi surfaces for the lower (a) and upper (b) magnetic subbands of 3D crystals with simple cubic lattice at $p/q = 1/2$, $\mathbf{H} \parallel \mathbf{a}_3$. (a): 1) $\varepsilon = -4.3$; 2) $\varepsilon = -2.0$; 3) $\varepsilon = 0.0$; (b): 1) $\varepsilon = 4.3$; 2) $\varepsilon = 2.0$; 3) $\varepsilon = 0.0$.

Fig.3. a), b) The parts of Fermi surfaces bounded by planes $k_3 = \pm \pi \sqrt{2}/3a$ ($\lambda = 2$) at $p/q = 1/2$, $\mathbf{H} \parallel \mathbf{a}_1 + \mathbf{a}_2$ for the upper subband; c) the energy spectrum $\varepsilon_\perp(k_1, k_2, k_3 = \pi \sqrt{2}/3a)$ at $p/q = 31/60$.

Fig.4. a), b) The Fermi surfaces in the first Brillouin zone at $p/q = 1/2$ and $\lambda = 0$ ($\mathbf{H} \parallel \mathbf{a}_1 + \mathbf{a}_2$) for the upper subband; c) the energy spectrum $\varepsilon_\perp(k_1, k_2, k_3 = \pi /a\sqrt{2})$ at $p/q = 31/60$. 