Conformal symmetry limit of QED and QCD and the identities between the concrete perturbative contributions to deep-inelastic scattering sum rules

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Abstract

Conformal symmetry based relations between the concrete perturbative QED and QCD approximations of the polarized Bjorken, the Ellis-Jaffe, the Gross-Llewellyn Smith sum rules and of the Adler functions of the axial vector and vector channels are derived. They are based on application of the operator product expansion to three triangle AVV Green functions, constructed from the non-singlet axial vector-vector-vector currents, the singlet axial-vector and two non-singlet vector currents and the non-singlet axial-vector-vector and singlet vector currents, in the limit when the conformal symmetry of gauge models with fermions is unbroken. We specify the conditions when the conformal symmetry is valid in the $U(1)$ and $SU(N_c)$ models. The identity between perturbative approximations of the Bjorken, Ellis-Jaffe and the Gross-Llewellyn Smith sum rules, which follow from this theoretical limit, is proved. The expressions for the $O(\alpha_s^3)$ and $O(\alpha^3)$ conformal symmetry based contributions for these sum rules and for the NS Adler function are considered. The differences between these terms and the similar corrections, obtained recently within the phenomenologically oriented generalization of the BLM-approach, which is based on the Principle of Maximal Conformality, are discovered. The necessity of careful study of the origin of this difference is discussed.

Keywords: Conformal Symmetry; Deep Inelastic Scattering.
1 Introduction

As was originally proved in the coordinate space-time representation, in the conformal symmetry limit of quantum field massless gauge models with fermions, which corresponds to the case when the renormalized coupling constants are equal to the bare ones (i.e. when $\alpha = \alpha B$ or $\alpha_s = \alpha_s B$), the multiloop expression for the three-point Green function of flavour non-singlet (NS) axial vector-vector (AVV) currents coincide with the lowest-order 1-loop triangle graph [1]. Considering this three-point function in the momentum space-time representation [2], it is possible to rewrite the result of Ref. [1] as

$$T_{\mu \alpha \beta}^{abc}(p, q) = i \int <0| T A_\mu^a(y) V_\alpha^b(x) V_\beta^c(0)|0 > e^{ipx+iqy} dx dy = A_{\mu \alpha \beta}^{abc}(p, q) .$$ (1)

Here $A_\mu^a(y) = \bar{\psi}(y) \gamma_\mu (\lambda^a / 2) \gamma_5 \psi(y)$, $V_\alpha^b(x) = \bar{\psi}(x) \gamma_\mu (\lambda^b / 2) \psi(x)$ are the NS axial vector and vector currents, $A_{\mu \alpha \beta}^{abc}$ is the symmetric structure constant of the $SU(N_c)$ group and $A_{\mu \alpha \beta}^{abc}(p, q)$ is the 1-loop contribution to the triangle Green function. In Ref. [3] the validity of this non-renormalization property was explicitly demonstrated at the two-loop level within differential regularization and the related differential renormalization prescription, both proposed in Ref. [4]. Within dimensional regularization approach of Refs. [5, 6, 7, 8, 9] and in the $\overline{\text{MS}}$-scheme [10], formulated in more detail in [11], the cancellation of the one-loop internal corrections to the AVV three-point function if Eq.(1) was rediscovered in Ref. [12]. The agreement of this result with the outcomes of two-loop calculations, performed in Ref. [3] within differential regularization and renormalization approaches, is not accidental. Indeed, it was shown in Ref. [13] that the differential renormalization can be straightforwardly related to the dimensional regularization and the $\overline{\text{MS}}$-scheme.

In the present work we will also study two extra AVV three-point Green functions, which are closely related to the one of Eq.(1). The first of them is constructed from one singlet (SI) axial vector current and two NS vector currents

$$\tilde{T}_{\mu \alpha \beta}^{ab}(p, q) = i \int <0| T A_\mu^a(y) V_\alpha^b(0) V_\beta^c(0)|0 > e^{ipx+iqy} dx dy$$ (2)

where the SI axial vector fermion current is defined as $A_\mu(y) = \bar{\psi}(y) \gamma_\mu \gamma_5 \psi(y)$. This type of Green function was considered previously in Ref. [14]. The second one is the three-point Green function, constructed from the NS axial-vector, the NS vector and SI vector fermion currents, namely

$$\tilde{T}_{\mu \alpha \beta}^{ab}(p, q) = i \int <0| T A_\mu^a(x) V_\alpha^b(0) V_\beta^c(y)|0 > e^{ipx-iqy} dx dy$$ (3)

where $V_\alpha(0) = \bar{\psi}(0) \gamma_\alpha \psi(0)$ is the SI vector current.

The important consequence of the study of the three-point function of Eq.(1) in the conformal-invariant limit is the discovery of the relations between the $\pi \to \gamma \gamma$ decay constant, the Bjorken sum rule of the polarized deep-inelastic scattering (DIS) process and the $e^+ e^-$ annihilation to hadrons ratio in the Born approximation [15]. Other relations, which follow from application of the operator product expansion approach to this AVV Green function in different kinematic regimes, were derived in the Born approximation in Ref. [16]. As the result the basic Crewther relation of Ref. [15] was generalized to the case of conformal invariant approximation of QED at the $O(\alpha^2)$-level.

These QED studies were further continued in the works of Refs. [17, 18] at the higher orders of perturbation theory. Moreover, the derived in Ref. [16] Born relation between the coefficient functions of the polarized Bjorken sum rule, the Gross-Llewellyn Smith sum rule of the neutrino-nucleon DIS and the $e^+ e^-$ annihilation Adler function was extended to the case of renormalized $SU(N_c)$ model with fermions in the work of Ref. [19]. These relations are based essentially on the
first direct QCD discovery of the generalized Crewther relation in the \( \overline{\text{MS}} \)-scheme at the \( \alpha_s^3 \) level [21]. The existence of the closed analytical form of this generalization of Crewther relation with the factorized renormalization-group (RG) \( \beta \)-function [21] was confirmed by the direct analytical calculations of the \( \alpha \)-corrections to the Bjorken sum rule, the Gross-Llewellyn Smith sum rule and the Adler \( D \)-function, performed in the works of Refs. [22, 23], and improved by theoretical foundation of the additional SI-type \( \alpha_s^4 \) contribution to the polarized Bjorken sum rule [24], which is not yet confirmed by the direct diagram-by-diagram evaluation. The detailed analysis of the current knowledge on the relations between \( \overline{\text{MS}} \)-scheme \( SU(N_c) \)-generalizations of the Crewther relation for the products of different DIS sum rules and the NS part and total expression for the \( e^+e^- \)-annihilation Adler function with order \( \alpha_s^4 \) SI-type corrections taken into account, including the discussions of the existing explicit proofs of the property of factorization of the QCD \( \beta \)-function [25, 26] (see Refs. [2, 27] as well) will be considered elsewhere.

This work is devoted to the analysis of the results obtained within conformal invariant limit of the \( U(1) \) and \( SU(N_c) \) gauge models with fermions. For the three-point functions of Eq. (2) and Eq. (3) this limit is realized when not only the coupling constant of the gauge model under consideration, but the external SI axial vector current \( A_\mu \) and SI vector current \( V_\mu \) are not renormalized as well. This happens when \( \alpha = \alpha^B, \alpha_s = \alpha^B \), \( A_\mu(x) = (A_\mu(x))^B, V_\mu(x) = V_\mu^B \) are the bare un-renormalized quantities. In this case two NS vector currents in the three-point function of Eq. (2) and the SI vector current in Eq. (3) are conserved, while the SI axial vector operator \( (A_\mu(x))^B \) and the SI vector operator \( (V_\mu)^B \) are not renormalized by construction. As will be discussed below, in the case of perturbative QED, i.e. in the \( U(1) \) abelian model with fermions, these requirements can be formulated on the diagrammatic language, while for the \( SU(N_c) \) non-abelian colour gauge group with fermions they can be realized in the non-physical imaginable approximation only. However, this approximation is quite useful for deriving the relations between concrete analytical scheme-independent NS and perturbative contributions to the DIS sum rules and the \( e^+e^- \)-annihilation Adler \( D \)-function. In the case of \( U(1) \) model they are already confirmed by direct analytical calculations, while in the \( SU(N_c) \) group their determinations, made in Refs. [28, 29], form the basis of all-order generalization of the BLM-scale fixing approach [30] can be used in theoretical and phenomenological QCD studies [31].

In the conformal-invariant limit Eq. (2) and Eq. (3) takes the form, similar to the non-renormalized expression of Eq. (1), namely

\[
\tilde{t}_{\mu\alpha\beta}(p, q) = \delta^{ab} \Delta_{\mu\alpha\beta}^{1-l}(p, q) 
\]

(4)

\[
\tilde{v}_{\mu\alpha\beta}(p, q) = \delta^{ab} \Delta_{\mu\alpha\beta}^{1-l}(p, q) 
\]

(5)

with the same 1-loop term \( \Delta_{\mu\alpha\beta}^{1-l}(p, q) \) in the r.h.s. of Eq. (1), Eq. (4) and Eq. (5).

In this limit, when the expression of Eq. (1) is valid, the further application of the operator-product expansion approach allows to derive following massless QCD analog of the Crewther relation

\[
C_{Bjp}(a_s) \times C_D^{NS}(a_s) = 1 \ .
\]

(6)

It is valid in the case of Born approximation [15] and within conformal-invariant limit of perturbative \( U(1) \) model (see e.g. [16]) and in within definite representation of perturbative \( SU(N_c) \) colour gauge model with fermions [29], which was formulated in Ref. [28] and called (\( \beta \))-expansion formalism, with (\( \beta \))-denoting here the set of the coefficients of the RG \( \beta \)-function.

In general the coefficient function \( C_{Bjp}(a_s) \) is proportional to the perturbative expression for the Bjorken sum rule \( S_{Bjp} \) of the polarized lepton-nucleon DIS process and \( C_D^{NS}(a_s) \) is defined through the Adler \( D \)-function of the NS axial vector currents

\[
S_{Bjp}(a_s(Q^2)) = \int_0^1 (g_1^{jp}(x, Q^2) - g_1^{ln}(x, Q^2))dx = \frac{1}{6} g_A C_{Bjp}(a_s(Q^2)) 
\]

(7)
\[ D_{NS}(a_s(Q^2)) = -12\pi^2 Q^2 \frac{d\Pi^{NS}(Q^2)}{dQ^2} = dR C_D^{NS}(a_s(Q^2)) \quad . \] (8)

In Eq.\( 8 \), \( g_A \) is the axial nucleon coupling constant, while \( \Pi^{NS}(Q^2) \) in Eq.\( 8 \) enters the expression for the correlator of two NS axial-vector currents

\[ i \int <0|T(A^{(a)}_\mu(x)A^{(b)}_\rho(0))|0> e^{i\mathbf{q} \cdot \mathbf{x}} d^4x = \delta^{\mu\rho}(q_\mu q_\rho - g_\mu g_\rho Q^2) \Pi^{NS}(a_s(\mu^2), Q^2/\mu^2) \quad . \] (9)

The coefficient functions of Eq.\( 7 \) and Eq.\( 8 \) obey the following RG equations

\[ \mu^2 \frac{\partial}{\partial \mu^2} + \beta(a_s) \frac{\partial}{\partial a_s} C_{Bjp}(a_s(\mu^2), Q^2/\mu^2) = 0 \quad (10) \]

\[ \mu^2 \frac{\partial}{\partial \mu^2} + \beta(a_s) \frac{\partial}{\partial a_s} C_D^{NS}(a_s(\mu^2), Q^2/\mu^2) = 0 \quad (11) \]

where \( Q^2 = -q^2 \) is the Euclidean momentum transfer and \( \mu^2 \) is the scale parameter of the \( \overline{MS} \)-scheme. Fixing \( Q^2 = \mu^2 \) we re-write the solution of the RG equations of Eq.\( 10 \) and Eq.\( 11 \) through the power series in \( a_s = \alpha_s/\pi \), fixed at \( \mu^2 = Q^2 \), where \( \alpha_s \) is the coupling constant of the \( SU(N_c) \) gauge theory with the dimension of the quark representation of the \( SU(N_c) \) gauge group, which is defined as

\[ \beta(a_s) = \mu^2 \frac{\partial a_s}{\partial \mu^2}(a_s) \mu \text{ fixed} = - \sum_{k \geq 0} \beta_k a_s^{k+2} \quad . \] (12)

where the scheme-dependent three-loop coefficient \( \beta_2 \) was analytically evaluated in the \( \overline{MS} \)-scheme in Ref.\( 32 \) (for details see Ref.\( 33 \)) and confirmed in Ref.\( 34 \). The four-loop term \( \beta_3 \) was calculated in the analytical form in the work of three authors \( 35 \) and confirmed later on in the independent work of one author \( 36 \).

Consider now the case of QED. Apart of Born approximation, in \( U(1) \) theory another widely used perturbative approximation, which is called the perturbative quenched QED (pqQED) model, is widely used.

The latter one is described by the set of QED graphs without internal vacuum polarization insertions to various multiloop diagrams. Among these diagrams is the photon-electron-positron vertex, which is defining the renormalized charge of QED. In the pqQED model the the external photon line of this photon-electron-positron vertex is renormalized by the multiloop photon vacuum polarization function with the single external fermion loop only, while the the internal photon propagators of this vacuum polarization insertion do not contain any other fermion loop insertions. The related renormalization constant \( Z_3 \) and the renormalization-group \( \beta \)-function are defined by the photon vacuum polarization graphs with one \textit{external} fermion loop only. Within pqQED model with \( N \)-number of leptons the expression for the \( \beta \)-function can be written down as

\[ \beta_{pqQED}(a) = \mu^2 \frac{\partial a}{\partial \mu^2} = \sum_{k \geq 0} \beta_k^{[1]} N a^{k+2} \quad (13) \]

where \( a = \alpha/\pi \), \( \alpha \) is the renormalized coupling constant of the pqQED model, and the coefficients \( \beta_k^{[1]} \) do not depend from the number of leptons \( N \). At the 4-loop level these results follow the analytical calculations of Ref.\( 37 \). The first, but indirect, confirmation of the quenched part of the 4-loop QED results of Ref.\( 37 \) came from the demonstration that the original Crewther relation of Eq.\( 6 \) is explicitly valid at the 4-loop pqQED approximation \( 21 \). Then the 4-loop analytical expression for Eq.\( 13 \) was directly confirmed by means of the typical pqQED studies, performed in Ref.\( 38 \).
The result of the analytical evaluations of the 5-loop coefficient $\beta_4^{[1]}$ was announced in Ref. [39] and published later on in Ref. [22] after performing the outlined in Ref. [17] calculating cross-check. It was based on the proposal to demonstrate the validity of the original Crewther relation of Eq. (6) by the direct analytical 5-loop order calculations of $C_{Bjp}(a)$ in pqQED. Note, that since there are no sub-divergences in the total pqQED expression for the photon vacuum polarization function, the $\beta$-function of pqQED, as defined in Eq. (13), does not depend from the choice of the subtractions scheme in all orders of perturbation theory. Therefore, pqQED approximation is the example of the model, where only the scale dependence is remaining and is manifesting itself in the perturbative expressions for the related massless Green functions.

2 Specification of the conformal invariant limit in QED and QCD

Let us move one more step below and define the conformal invariant limit of perturbative QED. It is realized when there is no external scale in the theory, which is introduced by charge renormalization, namely in the approximation when $a = a^B$. Within the language of renormalization constants this happens when $Z_3 = 1$. On the diagrammatic language this property is fulfilled when in pqQED model the set of graphs, which are responsible for the renormalization of charge, namely the graphs with single-fermion loop photon vacuum polarization insertions in the external photon line of the photon-lepton-antilepton vertex, are not taken into account. In this approximation we arrive to the condition $Z_3 = 1$, $a = a^B$ and $\beta(a) = \beta(a^B) = 0$.

These conditions specify conformal-invariant limit of QED on the diagrammatic language. It differs from the theoretical language, which realizes the proposals of Ref. [40] to restore conformal symmetry of the renormalizable QED by proper modification of its Lagrangian in an arbitrary number of the dimensions.

Thus in our case the conformal invariant limit of the standard perturbative QED exists

1. in the approximation, which is formulated by taking into account the contributing to Green functions Feynman diagrams without fermion loop insertions into internal photon lines;
2. this approximation should be combined with the theoretical requirement that in the concrete perturbative expansions one should not running, but fixed parameter $a$;
3. the latter property holds when in the photon-lepton-antilepton vertex diagrams the photon vacuum polarization insertions (even with single fermion loop) are not considered.

This means that in the conformal invariant limit of QED the photon vacuum polarization function of pqQED model the two-point Green function of vector-vector currents is taken into account, while in the vertex Green functions for the dressed photon-lepton-antilepton vertex they are neglected.

In the work of [18], [19] the specified above conformal invariant limit of QED was used to outline the derivation of all-order identity between special contributions to the NS and SI coefficient functions for the Ellis-Jaffe sum rule of the polarized lepton-nucleon DIS. In the $\overline{MS}$-scheme its QCD expression is defined as

$$EJ^{lp(n)}(Q^2) = \int_0^1 g_1^{lp(n)}(x, Q^2) dx = C_E^{NS}(a_s(Q^2)) (\pm \frac{1}{12} a_3 + \frac{1}{36} a_8)$$

$$+ C_E^{SI}(a_s(Q^2)) \exp(\int_{a_s(\mu^2)}^{a_s(Q^2)} \frac{\gamma_S(x)}{\beta(x)} dx) \frac{1}{9} \Delta \Sigma(\mu^2) .$$

Here $a_3 = \Delta u - \Delta d = g_A$, $a_8 = \Delta u + \Delta d - 2\Delta s$, $\Delta \Sigma = \Delta u + \Delta d + \Delta s$ and $\Delta u$, $\Delta d$ and $\Delta s$ are the polarized parton distributions, while the subscripts $lp(n)$ indicate the structure functions $g_1(x, Q^2)$.
of polarized DIS of charged leptons (l) on protons (p) and neutrons (n). Note, the polarized gluon distribution $\Delta G$, introduced in QCD in Refs. [41], [42] does not contribute to the $\overline{MS}$-scheme expressions for Eq. [14] [13].

The NS and SI coefficient functions $C_{EJ}^{NS}(a_s)$ and $C_{EJ}^{SI}(a_s)$ enter the operator product expansion of the T-product of two NS vector currents as

$$i \int T(A_\mu^a(x)V_0^b(0)e^{ipx}dx|_{p^2\to\infty} = \delta^{ab}(p_\alpha p_\beta - g_{\alpha\beta}p^2)\Pi^{NS}(a_s(\mu^2), P^2/\mu^2) + id^{abc}\epsilon_{\alpha\beta\gamma\rho}p^\gamma C_{EJ}^{NS}(a_s(\mu^2), P^2/\mu^2) A_\rho^b(0) + i\delta^{ab}\epsilon_{\alpha\beta\gamma\rho}p^\gamma C_{EJ}^{SI}(a_s(\mu^2), P^2/\mu^2) A_\rho^b(0) + \text{higher twist terms}$$

where $A_\rho = \bar{\psi}\gamma_\rho \gamma_5 \psi$ is the SI axial vector current and $P^2 = -p^2$ is the Euclidean transferred momentum and $\mu^2$ is the renormalization scale parameter. The second term of the r.h.s. of Eq. [16] is also defining the coefficient function $C_{Bjp}(a_s)$ of the Bjorken polarized sum rule [40], which was introduced above in Eq. (7). Therefore, one has $C_{EJ}^{NS}(a_s) = C_{Bjp}(a_s)$. The analytical expressions for the $a_s^2$- and $a_s^3$-corrections to $C_{Bjp}(a_s)$ were evaluated in the $\overline{MS}$-scheme in the works of Ref. [47] and Ref. [48] respectively, while the corresponding $a_s^4$ contributions from the diagrams with NS-type structure were evaluated in Ref. [22]. The concrete analytical results of these calculations can be also found in Ref. [29]. As was already mentioned above, the additional, but not yet confirmed by direct calculations, SI type analytical $a_s^4$ contributions to the Bjorken sum rule was found in Ref. [21]. This contribution does not affect the expression for the perturbative coefficient function of the Gross-Llewellyn Smith sum rule of neutrino-nucleons DIS, which in the $\overline{MS}$-scheme is defined through the OPE of the NS axial-vector and vector currents (see e.g. [46]) as

$$i \int T A_\mu^a(x)V_0^b(0)e^{ipx}dx|_{p^2\to\infty} = i\delta^{ab}\epsilon_{\mu\rho\beta\gamma}p^\beta P^2 C_{GLS}(a_s) V_\rho(0) + \text{higher twist terms}$$

and is related to the QCD expression for this sum rule as

$$S_{GLS}(a_s) = \frac{1}{2} \int_0^1 F_{3p+\overline{p}p}(x, P^2)dx = 3C_{GLS}(a_s)$$

The order $a_s^2$-corrections to this sum rule were evaluated in Ref. [47] and confirmed in Ref. [49], the $a_s^3$ corrections were evaluated in the work of Ref. [48] and the analytical for the $a_s^4$ corrections to $C_{GLS}(a_s)$ are presented in Ref. [23].

Let us discuss now in more detail the RG evolution of the SI coefficient function $C_{EJ}^{SI}(a_s)$ of the Ellis-Jaffe sum rule, defined in Eq. [16]. It satisfies the following RG equation

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(a_s) \frac{\partial}{\partial a_s} + \gamma_{SI}(a_s)\right) C_{EJ}^{SI}(a_s(\mu^2), P^2/\mu^2) = 0$$

The anomalous dimension of the SI axial current $A_\mu$ is defined as

$$\gamma_{SI}(a_s) = \mu^2 \frac{\partial \ln Z_{SI}(a_s)}{\partial \mu^2} = \sum_{l\geq0} \gamma_l a_s^{l+1}$$

where $A_\mu = Z_{SI}(a_s)(A_\mu)^R$. This anomalous dimension enters the four-loop calculations of Ref. [44], though its analytical expression in the $\overline{MS}$-scheme is known at the three-loop level only [45] and can be re-written as

$$\gamma_{SI}(a_s) = -3\overline{C}_F T_F N_F a_s^2 + \left(\beta_0 \frac{71}{44} C_F T_F N_F - \frac{5}{11} C_F (T_F N_F)^2 + \frac{9}{16} C_F^2 (T_F N_F)\right) a_s^3 + O(a_s^4)$$
In Eq. \((20)\) \(\beta_0 = (11/12)C_A - T_F N_F/3\) is the first coefficient of the QCD RG \(\beta\)-function, defined by Eq. \((12)\). \(\gamma_0\) is zero due to the fulfilment of the Ward identities for the SI axial vector current, \(C_F\) and \(C_A\) are the Casimir operators, \(N_F\) counts the number of flavours, \(T_F = 1/2\) is the normalization factor. In QED the diagram-by-diagram representation of this form for Eq. \((20)\) is rather clear.

Taking now formally \(C_F = 1\), \(C_A = 0\), \(T_F = 1\), \(N_F^k = 0\) for \(k \geq 2\) in Eq. \((20)\) we get the non-zero pqQED analog of the \(O(a^3)\) approximation of the anomalous dimension \(\gamma_{SI}(a)\). In order to put it to zero as the whole and thus move to the case of conformal symmetric approximation of perturbative QED, it is necessary to add to the introduced above requirements \((1)\)-(3), which are specifying the the conformal invariant limit of perturbative QED, the additional condition of non-renormalizability of the SI axial-vector current, i.e. the condition \(A_\mu(x) = (A_\mu(x))^B\). In this case the correlator of two SI bare axial-vector currents contains single external lepton loop and has the transverse form, namely

\[
i \int \langle 0| T(A_\mu(x))^B(A_\mu(0))^B |0 \rangle e^{iqx} d^4x = (g_\mu\rho q^2 - q_\mu q_\rho) \Pi^{SI}(a^B, Q^2/\mu^2) . \tag{21}\]

The corresponding multiloop approximation of the formfactor \(\Pi^{SI}(a^B, Q^2/\mu^2)\) does not contain the diagrams with triangle contributions to the external bare vertex, and therefore the anomalous dimension \(\gamma_{SI}(a)\), discussed in the related QED studies of Ref.\([14]\), is absent. Moreover, in this conformal invariant limit of perturbative QED the formfactor \(\Pi^{SI}(a^B, Q^2/\mu^2)\) coincide with the conformal-invariant approximation for the formfactor \(\Pi^{NS}(a^B, Q^2/\mu^2)\) of two vector currents, which is defined by the QED version of the first line in Eq. \((16)\) and do not contain SI-type perturbative contributions with two “light-by-light-type” scattering external loops, connected by three photon propagators. Note, that in general these type of graphs are contributing to the photon vacuum polarization function staring from the 4-loop order \([37]\).

In the case of perturbative QCD or, more generally in \(SU(N)\) gauge model with fermions, it is still unclear how to specify the conformal invariant limit on the diagrammatic language. However, it is possible to restore the conformal symmetry of non-renormalized massless gauge models by fixing \(a_s = a_s^B\) and \(A_\mu(x) = (A_\mu(x))^B\). Using these definitions one gets identically zero expressions for the RG functions \(\beta(a_s) = 0\) and \(\gamma^{SI}(a_s) = 0\), which on the renormalized language can be obtained within perturbation theory in the imaginable world only. This step back to the non-renormalized quantities will allow us to derive the identities between concrete perturbative contributions to the NS and SI coefficient functions of the Ellis-Jaffe sum rule and to the Gross-Llewellyn Smith sum rule.

3 Conformal symmetry governed contributions to the coefficient functions of the DIS sum rules in QED and QCD

3.1 Theoretical considerations

Let us compare the application of the OPE approach to the AVV three-point Green functions of Eq. \((1)\), constructed from the flavour NS axial vector-vector-vector fermion currents, with the similar results, which will be obtained from the AVV three-point Green function of Eq. \((9)\), constructed from the SI axial-vector and two NS vector fermion currents.
As was shown in Ref. [2] (see Ref. [50] as well), in the kinematic regime \((pq) = 0\) the l.h.s. of the AVV three-point Green function of Eq. (11) can be expressed through three form-factors as

\[
T_{\mu\alpha\beta}(p,q) = \xi_1(p^2, q^2) \epsilon_{\mu\alpha\beta\tau} q^\tau + \xi_2(p^2, q^2)(q_\alpha \epsilon_{\mu\beta\rho\tau} p^\rho q^\tau - q_\beta \epsilon_{\mu\alpha\rho\tau} p^\rho q^\tau) + \xi_3(p^2, q^2)(p_\alpha \epsilon_{\mu\beta\rho\tau} p^\rho q^\tau + p_\beta \epsilon_{\mu\alpha\rho\tau} p^\rho q^\tau)
\]  

(22)

where the first form-factor is not renormalized due to the non-renormalization of the axial anomaly, while the second and third form-factors contain additional additive contributions, proportional to the factor \((\beta(a_s)/a_s)\) (for details see Ref. [2]). This perturbative factor is responsible for the violation of the conformal symmetry of the non-renormalized gauge models with massless fermions by the procedure of renormalizations. Its appearance was theoretically guessed in Ref. [51] and theoretically proved in the independent works of Refs. [52, 53, 54, 55]. Note, that the recent analytical calculations of the three-loop corrections to the AVV Green functions, performed in the \(\overline{MS}\)-scheme, in Ref. [56], explicitly indicated to the manifestation of the factor \((\beta(a_s)/a_s)\) starting from the three-loop level. Indeed, these calculations of Ref. [56] revealed the appearance of the proportional to \(\beta a_s^2\) corrections in the transverse form-factors of this AVV Green function.

In the conformal-invariant limit one should fix \(\beta(a_s) = 0\). This property takes place when the coupling constant is considered as the bare non-renormalized one, namely when \(a_s = a_s^B\). Let us now use the operator product expansion approach to the triangle function of Eq. (11) in this case. It corresponds to taking first the T-product of Eq. (10) at \(p^2 \rightarrow \infty\), and then fixing the T-product of two NS axial-vector currents through Eq. (9). In this case we get the following for the non-renormalized 1-loop expression of the second form-factor in Eq. (22).

\[
\xi_2^{1\text{-loop}}(q^2, p^2)|_{|p^2| \gg |q^2|} = \frac{1}{p^2} C_{EJ}^{NS}(a_s^B)\Pi^{NS}(a_s^B, Q^2/\mu^2).
\]

(23)

The similar expression in the SI channel can be derived from the AVV function of Eq. (22) when the SI axial vector current is considered as the bare one, namely when \(A_\mu(x) = (A_\mu(x))^B\). Indeed, using the T-product of Eq. (9) and the definition for the correlator of two bare SI axial -vector currents from Eq. (21), we get the following expression for the l.h.s. of Eq. (23).

\[
\xi_2^{1\text{-loop}}(q^2, p^2)|_{|p^2| \gg |q^2|} = \frac{1}{p^2} C_{EJ}^{SI}(a_s^B)\Pi^{SI}(a_s^B, Q^2/\mu^2).
\]

(24)

Considering now the three-point function of Eq. (3) in the case when one vector current is fixed as the bare one, namely when \(V_\alpha(0) = (V_\alpha(0))^B\), using Eq. (16) and the definition for the correlator of two bare vector currents

\[
i \int <0|TV_\beta(y)^B V_\rho(0)^B|0 > e^{iqy} d^4 y = (g_{\beta\rho} q^2 - q_\beta q_\rho)\Pi^V(a_s^B, Q^2/\mu^2).
\]

(25)

we get the following identity

\[
\xi_2^{1\text{-loop}}(q^2, p^2)|_{|p^2| \gg |q^2|} = \frac{1}{p^2} C_{GLS}(a_s^B)\Pi^V(a_s^B, Q^2/\mu^2).
\]

(26)

Taking the ”weighted” derivative \(-Q^2(d/dQ^2)\) with respect to the Euclidean transferred momentum \(Q^2 = -q^2\) in the l.h.s. of Eq. (23), Eq. (24) and Eq. (26) and keeping in mind the one-loop character of the l.h.s. these equations, we get the analogs of the reported in Ref. [19] \(SU(N_c)\) generalizations of the results of Ref. [18], namely the following Crewther-type identities

\[
C^{SI}(a_s) \times C^{SI}_D(a_s) = 1
\]

(27)

\[
C^{NS}(a_s) \times C^{NS}_D(a_s) = 1
\]

(28)

\[
C^{GLS}(a_s) \times C^{V}_D(a_s) = 1.
\]

(29)
It is possible to understand, that in the conformal-invariant limit the following identities takes place in all orders of perturbation theory

\[ C_{SI}^{D}(a_{s}) \equiv C_{NS}^{D}(a_{s}) \equiv C_{V}^{D}(a_{s}). \]  \( (30) \)

Keeping this in mind and comparing the l.h.s. of Eq.(27) with the l.h.s. of Eq.(28) and Eq.(29), we get the following relation

\[ C_{NS}(a_{s}) \equiv C_{SI}(a_{s}) \equiv C_{GLS}(a_{s}) \mid \text{conformal invariant limit} \]  \( (31) \)

where \( a_{s} \) does not depend on the choice of the scale and is considered as the bare parameter. This identity is valid in all orders of the perturbation theory in the conformal-invariant limit of \( SU(N_{c}) \) gauge models with fermions and in the conformal invariant limit of QED as well, as discussed first in Ref [18].

Note, that in the conformal symmetry limit the ratios of the corresponding approximations for the Ellis-Jaffe and Bjorken sum rules give us the following relation

\[ \frac{E_{Jlp}(Q^{2})}{B_{jp}(Q^{2})} = \pm \frac{1}{2} + \frac{a_{8}}{6a_{3}} + \frac{2\Delta \Sigma}{3a_{3}} \]  \( (32) \)

where \( a_{8} = 3a_{3} - 4D \), \( a_{3}, a_{8} \) and \( \Delta \Sigma \) were defined above through the polarized parton distributions. and \( D \) is the hyperon decay constant. These relations coincide with the ones, obtained within massless quark-parton model and can be rewritten as

\[ \frac{E_{Jlp}(Q^{2})}{B_{jp}(Q^{2})} = 1 + \frac{2(\Delta \Sigma - D)}{3a_{3}} \quad ; \quad \frac{E_{Jln}(Q^{2})}{B_{jp}(Q^{2})} = -\frac{2(\Delta \Sigma - D)}{3a_{3}} \]  \( (33) \)

They lead to the standard quark-parton model definition of the Bjorken sum rule through the Ellis-Jaffe sum rules, namely

\[ B_{jp} \equiv E_{Jlp} - E_{Jln}. \]  \( (34) \)

This gives us confidence in the self-consistence of the considerations presented above.

### 3.2 Analytical and numerical results

Let us present now some concrete expressions for the scheme-independent approximations of the several coefficient functions, discussed in this work. The first one is expression for the coefficient function of ND Adler D-function, obtained in the conformal-invariant approximation of QED. It follows from the results of direct analytical 5-loop calculations, presented first in the work of Ref.[39], discussed in detail in the work of Ref.[17] and published in the work of Ref.[22].

For the purposes of the comparisons with the expansions, given recently for the generalization of the BLM approach of Ref.[30] within Principle of Maximal Conformality, which is in the process of systematical studies (see Refs.[57, 58, 59] and the works of Refs. [29, 31] as well), the existing analytical results are supplemented by the numerical expressions of the available coefficients:

\[ C_{NS}^{D} = 1 + \frac{3}{4}a - \frac{3}{32}a^{2} - \frac{69}{128}a^{3} + \left( \frac{4157}{2048} + \frac{3}{8} \zeta_{3} \right) a^{4} + O(a^{5}) \]  \( (35) \)

Notice the difference with the results of the applications of PMC approach in the works of Refs.[57, 58]:

\[ C_{NS}^{D} = 1 + 0.24a(Q_{1}) - 0.08a_{c}(Q_{2})^{2} - 0.13a_{c}(Q_{3})^{3} + 0.05a_{c}(Q_{4})^{4} + O(a_{c}^{5}) \]  \( (36) \)
where scales of the QED expansion parameter are fixed using the ideas of all-orders generalization of the BLM method of Ref.\cite{28}, or the PMC approach. These ideas are rather similar to define the scales, of the expansion parameter the terms, proportional to the coefficients of the QED $\beta$ function, which are known at present in different schemes at the 5-loop order (see Refs.\cite{60, 60, 62}). The most surprising point is that that this difference is manifesting itself in the coefficient of the first term already. For sure, the origin of this difference deserve detailed clarification.

In the case of $SH(N_c)$ model the similar extression is analytically known at the $O(a_s^3)$-order from the analysis of Ref.\cite{28} (see the work Ref.\cite{29} as well). The results read

$$C_D^{NS}(a_s) = 1 + 3C_Fa_s + \left( -\frac{3}{32}C_F^2 + \frac{1}{16}C_FC_A \right)a_s^2$$

(37)

$$+ \left( -\frac{69}{128}C_F^2 + \frac{71}{64}C_F^2C_A + \left( \frac{253}{768} - \frac{27}{8}\zeta_3 \right)C_FC_A^2 \right)a_s^3 + 0(a_s^4)$$

(38)

where the analytical and numerical results of Eq.\cite{38} should are given for QCD, namely for $SU(3)$ gauge theory with $C_F = 4/3$, $C_A = 3$.

Notice again the difference with the results, presented in the works of Refs.\cite{57, 58}:

$$C_D^{NS}(a_s) = 1 + a_s(Q_1) + 1.84a_s(Q_2)^2 - 1.00a_s(Q_3)^3 - 11a_s(Q_4)^4$$

(39)

Here the most surprising for us point is that the value of the coefficient before $O(a_s^3)$ correction is drastically different in the approximations of Eq.\cite{38} and Eq.\cite{39}. To our point of view this coefficient should coincide with the the coefficient of the BLM approach of Ref.\cite{30}, which was reproduced within one-scale generalization of the BLM approach, studied at the $a_s^3$-level in the work of Ref.\cite{63}, and this is true for the result of Eq.\cite{38}.

On the conformal-invariant limit of the perturbative QED and $SU(N_c)$ with fermions the results for the coefficient functions of DIS sum rules can be obtained from the Crewther relations of Eq.\cite{27}-Eq.\cite{17}

$$C_{BiJ}(a) = C_{EJ}^{SI}(a) = C_{GLS}(a) = 1/C_D^{NS}(a_s)$$

(40)

In the case of conformal-invariant limit of QED the explicit expression has the following form

$$C_{BiJ}(a) = C_{EJ}^{SI}(a) = C_{GLS}(a) = 1 - \frac{3}{4}a + \frac{21}{32}a^2 - \frac{3}{128}a^3 - \left( \frac{4823}{2048} + \frac{3}{8}\zeta_3 \right)a^4 + O(a^5)$$

(41)

It is possible to demonstrate explicitly the validity of Eq.\cite{31} in the case of the conformal invariant limit of QED at the level of third order corrections \cite{18}. In the process of these studies the following $O(a^3)$ pqQED expressions were used: the $O(a^3)$ approximation for $C^{NS}(a_s)$, available from the analytical calculations, consequently performed at the next-to-leading-order (NLO) and at the next-to-next-to-leading order (NNLO) in Ref.\cite{17} and Ref.\cite{18} and the $O(a^3)$ expression for $C^{SI}(a_s)$ \cite{18}. The expression the $O(a^4)$ QED correction to these functions, first given in Ref.\cite{17} is in agreement with the one, which follows from the direct analytical evaluation of Ref.\cite{22} one can use the pqQED expression of the analytical result from \cite{22}. Let us remind once more, that the parameter $a$ in is fixed, and therefore, this expression is scale-independent. This, in turn, means that all coefficients in these conformal-symmetry motivated approximations of the perturbative series do not depend from the choice of the normalization scheme.
In the case of $SU(N_c)$ and $SU(3)$ models the analogous expression takes the following form

$$C_{B,jp}(a_s) = C_{GLS}^{SI}(a_s)$$

$$= 1 - \frac{3}{4} C_F a_s + \left( \frac{23}{32} C_F^2 - \frac{1}{16} C_F C_A \right) a_s^2$$

$$+ \left( -\frac{3}{128} C_F^3 - \frac{65}{64} C_F^2 C_A - \left( \frac{523}{768} - \frac{27}{8} C_F C_A^2 \right) a_s^3 + O(a_s^4) \right)$$

$$= 1 - a_s + \frac{11}{12} a_s^2 + \left( -\frac{7859}{576} + \frac{81}{2} \right) a_s^3 + O(a_s^4) \quad (42)$$

$$= 1 - a_s + 0.91667 a_s^2 + 35.038928 a_s^3 + O(a_s^4) \quad (43)$$

Note, that the numerical expression for the $O(a_s^2)$-coefficient coincides with the result, obtained in Ref. [64] within one-scale generalization approach of Ref. [63]. It will be of interest to compare the expressions obtained in Ref. [29] with the PMC approximations, discussed in Ref. [57, 58] and to understand the origin of disagreement, if any.

4 Conclusions

In this work using the language of bare and unrenormalized parameters of QED and QCD and the properties of the absence of radiative corrections of the three AVV three-point Green functions in the case when conformal symmetry of gauge models remain valid, we clarify how it is possible to formulate the conformal-invariant limit of perturbative QED and QCD approximations for the two-point Green functions. We also prove the existence of all-order identities between different coefficient functions of DIS sum rules, check their validity at the level of $a_3$ corrections for the conformal-invariant limit of QED. As the concrete applications the comparisons with the existing variant of Principle of Maximal Conformality approach is discussed and the necessity to clarify the origin of discrepancies between the values of the coefficients of the PMC series and those analytical ones, which explicitly follow from the principles of the conformal symmetry in QED and QCD is discussed.

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