A numerical analysis on slender columns for flat-plate structures using finite element method (FEM) technique

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Abstract. The research carries out a parametric study to analyse slenderness effect in column design of reinforced cement concrete (RCC) flat-plate structures based on Indian standard (IS) codes namely, IS 456: 2000 and IS 875. This study is conducted on 36 columns at three locations (i.e. corner, edge and inner columns) in 12 flat-plate RCC structural models. First the methods describe in the Indian standard code for designing slender column is reviewed manually for one structural model and then FEM models are generated by using ETABS software. Parametric study is achieved by considering: four different length of column range from 3m to 6m using an increment of 1m and three slab panel’s size 4.5m x 4.5m, 6m x 6m and 7.5m x 7.5m with three panels in both ways. Result shows that corner and edge column has most slenderness effect. Column height increase on and beyond 5m is substantially vulnerable for sway effect. If column length increases from one particular stage to another then steel ratio increases. In many cases, a column is not slender by considering now-sway frame but very slender by considering sway frame. For this reason, a designer should check a column for both non-sway and sway conditions.

Keywords: Finite element method (FEM), ETABS, Slenderness effect, Sway frame, Non-sway frame, Flat plate structure, slender column.

1. Introduction
If a beam element is under compressive load and the length of this element are larger than its other dimension such a beam is called as a column [1]. Basically, I have seen two type of column which is based on the material and cross section like short column as well as slender column. In brief, the column for which strength is controlled entirely by strength of the material and cross section is called as short column. On the other hand, the column for which the strength is controlled by material, cross section as well as length of the column is called as slender column. Two types of frames are usually seen in structures like sway frame and non-sway frame. The frame that is braced against sideways is termed as non-sway frame and that is not braced against sideways known as sway frame [2].
A slender column of the building contains less strength than a short column of the same cross sectional area. Hence, it can transmit the lesser load as compared to short column [3]. If the slenderness effect increases with increasing length then buckling effect of the column is developed under gravity load [4]. Hence, the assessment of slender column involves the column length in addition to its cross section. Slender column shows deflections due to applying eccentric loading on it. Due to the increase in eccentricity by the amount of transverse deflection (Δ), additional flexural stresses is developed which is called as the $P - \Delta$ effect or secondary moment. This $P - \Delta$ effect decreases the axial load capacity of slender column [5]. If the entire moment (including the secondary moment) reaches the ultimate capacity of column then column would be failed by the self-weight or material failure.

In India, the scarcities of undeveloped areas are exist and the cost of property is increase due to huge population day by day. Therefore, vertically extend building has now become essential. For the above reasons, a lot of high rise buildings are being constructed in smart cities of India. Most of the buildings are RCC beam-column or flat-plate frame structure. The most significant problems in high rise structures are the reduction of the cross section of the column and increasing in the length of the column without proper care of slenderness effect. So, this is the important point to attention otherwise, this could make severe design fault [6].

2. Objectives and methodology
The main objective is to understand the influence of slenderness in corner, edge and inner column by carrying out a parametric study on flat-plate structures only.

The guidelines of the slender column design has been described in section 25 of IS code 456: 2000 which is performing for this research. Commercial Finite Element Method (FEM) modelling software called ETABS is used for this study. All possible loads (Wind, Earthquake load etc.) are considered. A typical model of RCC high-rise building is developed by ETABS where the different parameters are changed. The parameters such as length of column, dimension of column and span length will be considered and varied in normal range. Results of the parametric study should indicate the important parameters which need to be considered and should indicate their significant influence on the column design.

3. Slender column design procedure
A short description of slender column design is given in this segment. Details of computing slenderness of column can be found elsewhere [7]. As it mentioned earlier, building structure could be braced or un-braced. If Stability Index is less than 0.05 then story can be considered as braced and can be calculated by using Eq. (1).

$$Q = \frac{\sum P_u \Delta_0}{\sum V_u l_c}$$

Where $\sum P_u$ is the total factored vertical load in the story, $V_u$ is the total factored story shear in the story, $l_c$ is the length of the column measured center-to-center of the joints in the frame and $\Delta_0$ is the first-order relative deflection between the top and the bottom of the story due to $V_u$.

Primarily slenderness ratio $(k l/r)$ is determined for each column. For non-sway frame, a designer can neglect slenderness effect if $k l_u/r \leq 34 - 12 M_1/M_2$ and for sway frame slenderness effect can be neglected if $k l_u/r < 22$, where $k$ is the effective length factor; $l_u$ is the unsupported length, $r$ is the radius of gyration of cross section of column associated with axis about which bending is occur; $M_1$ is the value of smaller end moment on the column which are calculated by calculated from a conventional first-order elastic analysis (positive if the member is bent in single curvature and negative if bent in double curvature); and $M_2$ is the value of the larger factored end moment on the compression number, always positive.
If the column is slender, it will fail by buckling into the shape of a sine wave when the load reaches a particular value of $P_c$, called the Euler load or critical load which is mention in Eq. (2).

$$P_c = \frac{n^2EI_{min}}{(kl)^2}$$  \hspace{1cm} (2)

Where, $E$ is the elastic modulus of column and $I_{min}$ is the minimum moment of inertia of column. It is seen that the buckling load decreases rapidly with increasing slenderness ratio. If a column falls under either non-sway or sway frame and crossed the limiting value then a moment magnification factor is needed to calculate moment considering slenderness effect. Steps used to calculate non-sway moment magnification factor ($\delta_{ns}$) are shown in Eq. (3) to (6).

$$EI = \frac{0.4E_il_g}{1+\beta_d}$$  \hspace{1cm} (3)

$$\beta_d = \frac{\text{Factored dead load within a story}}{\text{Total factored shear in the story}}$$  \hspace{1cm} (4)

$$\delta_{ns} = C_m \left(1-\frac{P_u}{0.75P_c}\right)$$  \hspace{1cm} (5)

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \geq 0.40$$  \hspace{1cm} (6)

Where, $E_c$ is the elastic modulus of column and $I_g$ is the effective moment of inertia of girder, which is equals to $0.35I_{gross}$, $I_{gross}$ is the gross moment of inertia of girder, $P_u$ is the ultimate load of column, and $C_m$ is a factor which is a function of column moments. The column original moments are then multiplied by the $\delta_{ns}$ to get the moment required for calculation of steel ratio for slender column. $\delta_{ns}$ value could be less than 1.0 and for that case it is assumed as 1.0. If $\delta_{ns}$ is greater than 1.0 then the column is considered have slenderness effect. However, a column could have $\delta_{ns}$ value less than 1.0 but the $kl/r$ crossed the limiting value or vice-versa. For those cases, considering a column a slender column would give some safety factor in the design.

Steps required to calculate sway moment magnification factor ($\delta_s$) are shown in Eq. (7).

$$\delta_s = \frac{1}{(1-\frac{\Sigma P_u}{0.75\Sigma P_c})}$$  \hspace{1cm} (7)

Calculate $P_c$ for each column in the story of the column being designed and then calculate $\Sigma P_c$ for the given story. Similarly, calculate $P_u$ for each column in the story of the column and then calculate $\Sigma P_u$ or the given story.

4. FEM model development

4.1. Model development

The ETABS FEM software is selected for the parametric study. Entire flat-plate models had 10 stories and a square shape building. Every floor consists of three panels in each direction and a shear is placed at the middle of the building. The foundations for columns and shear walls are provided as fixed support. The ground floor is increased from 3m to 6m height with an increment of 1m which is described in details in the next section. The other story height is 3m and kept unchanged in all structures. The floor slabs are 200mm thick that confirms the minimum thickness for flat slab with periphery beam. No drop panels or column capitals are provided in flat-plate model.

The plan view of ground floor of one model is considered as 4.5m x 4.5m, 6m x 6m and 7.5m x 7.5m slab panels which are shown in Figure 1, Figure 2 and Figure 3 respectively. The tube shape shear wall is 200mm thick. The clear cover of concrete column is 40mm.
4.2. Mechanical characteristics of construction materials

Any structure’s strength can be influenced by the material properties, for this purpose; the material properties are identified by Indian standard. Table 1 represents the mechanical properties of concrete and Table 2 represents the mechanical properties of steel reinforcement.

Table 1. Mechanical properties of concrete (M25).

| Properties                                  | Values          |
|---------------------------------------------|-----------------|
| Concrete cube compressive strength, $f_{ck}$ | 25 MPa          |
| Weight per unit volume                      | 25 kN/m³        |
| Modulus of elasticity, $E_C$                | 25000 MPa       |
| Coefficient of thermal expansion, $A_c$     | 0.0000055 1/C   |
| Poisson’s ratio, $\nu_c$                    | 0.2             |

![Figure 1. 4.5m x 4.5m floor panel.](image1)

![Figure 2. 6m x 6m floor panel.](image2)

![Figure 3. 7.5m x 7.5m floor panel.](image3)
Shear Modulus, $G_c$  

**Table 2.** Mechanical properties of reinforcement (Fe500).

| Properties                        | Values   |
|-----------------------------------|----------|
| Minimum yield strength, $f_y$     | 415 MPa  |
| Minimum Tensile strength, $f_u$   | 450 MPa  |
| Weight per unit volume            | 76.9729 kN/m$^3$ |
| Modulus of elasticity, $E_s$      | 210000 MPa |
| Coefficient of thermal expansion, $A_s$ | 0.0000117 1/C |

### 4.3 Loading considerations

Two types of loads are considered: gravity load and environmental load. Gravity loads come from self-weight of building and live loads on building. The seismic loads are calculated by IS 1893 (Part 1): 2002 which are given below in detail:

#### 4.3.1 Gravity load

The dead load from the slabs, beams, and columns are calculated automatically by the software using the unit weight of concrete. The unit weight of concrete is taken as 25 kN/m$^3$. Live load (LL) is caused by occupancy of the building and do not include in dead, construction land environment load. It may also change location [3]. Table 3 represent dead load data and Table 4 represent live load data which are used in the analysis of the building.

**Table 3.** Dead load details.

| Dead load       | Values   |
|-----------------|----------|
| Boundary wall   | 1.4 kN/m$^3$ |
| Floor finish    | 1 kN/m$^3$ |
| False ceiling   | 0.6 kN/m$^3$ |

**Table 4.** Live load details.

| Live load for all floors | Values |
|--------------------------|--------|
| Live load for all floors | 3 kN/m$^2$ |

#### 4.3.2 Earthquake load

Table 5 represents the earthquake load data.

**Table 5.** Earthquake load details.

| Parameters                   | Values |
|------------------------------|--------|
| Seismic zone                 | IV     |
| Zone factor, $Z$             | 0.24   |
| Importance factor, $I$       | 1      |
| Soil type                    | II     |
| Response modification factor, $R$ | 5      |
| Function damping ratio       | 0.05   |
4.3.3. Load combinations. These combinations are considered according to IS 1893 (Part 1): 2002. Table 6 represents load combinations which are considered in the building analysis.

Table 6. Load combination details.

| Numbers | Combinations                   |
|---------|-------------------------------|
| DCon1   | 1.5 DL                        |
| DCon2   | 1.5 DL + 1.5 LL               |
| DCon3   | 1.2 DL + 1.2 LL + 1.2 EQ      |
| DCon4   | 1.2 DL + 1.2 LL - 1.2 EQ      |
| DCon5   | 1.5 DL + 1.5 EQ               |
| DCon6   | 1.5 DL - 1.5 EQ               |
| DCon7   | 0.9 DL + 1.5 EQ               |
| DCon8   | 0.9 DL - 1.5 EQ               |
| DCon9   | 1.2 DL + 1.2 LL + 1.2 EQX     |
| DCon10  | 1.2 DL + 1.2 LL + 1.2 EQY     |
| DCon11  | 1.5 DL + 1.5 EQX              |
| DCon12  | 1.5 DL + 1.5 EQY              |
| DCon13  | 0.9 DL + 1.5 EQX              |
| DCon14  | 0.9 DL + 1.5 EQY              |

4.4. Parametric study

Table 7 presents the floor panel size, column position, column length and size and periphery beam size which are used for the parametric study. In this parametric study, 12 models (3 models for each floor panel size with 4 varying column length) are generated for flat-plate structures with a tube shape shear wall in core of the structure. So, among these 12 flat plates structures total 36 ground floor columns are considered for slender column behavior analysis.

Table 7. Parameters which are used for parametric study.

| Floor panel size | Column position | Column length @ ground level | Column size | Periphery beam size |
|------------------|-----------------|------------------------------|-------------|---------------------|
| 4.5m x 4.5m      | Corner column   | 3m to 6m @ 1m increment (3m, 4m, 5m and 6m) | 350mm x 350mm | 250mm x 450mm m |
|                  | Edge column     |                              | 400mm x 400mm |                     |
|                  | Inner column    |                              | 450mm x 450mm |                     |
| 6.0m x 6.0m      | Corner column   | 3m to 6m @ 1m increment (3m, 4m, 5m and 6m) | 400mm x 400mm |                     |
|                  | Edge column     |                              | 500mm x 500mm |                     |
|                  | Inner column    |                              | 550mm x 550mm |                     |
| 7.5m x 7.5m      | Corner column   |                              | 450mm x 450mm |                     |
|                  | Edge column     |                              | 600mm x 600mm |                     |
|                  | Inner column    |                              | 650mm x 650mm |                     |
5. Results and discussion
The analyses of 12 models that include different dimension of columns and panels have been performed by using ETABS. The parameters such as floor panel sizes, column length, column position and column sizes are to be studied.

5.1. Corner column
Based on clause 25.1.2 of IS 456: 2000, if the value of Slenderness Ratio \( (k_l/r) \) is greater than 12, the column should be treated as slender column for non-sway frame. Figure 4 is showing that \( k_l/r \) is above or equal to 40 in 6 columns among 15 corner column. In case of three panels, for column 5m to 6m, the \( P_c \) value decreases around 25.0%. In general, \( P_c \) value decreases with increase in column length. Figure 5 represents the ratio of column design load \( (P_u) \) to critical buckling load \( (P_c) \). The corner column showed the slender behaviour while \( P_u \) increased about 40% of \( P_c \).

![Figure 4](attachment:Critical_buckling_load_variation.png)
**Figure 4.** Critical buckling load variation.

![Figure 5](attachment:Ultimate_load_to_critical_buckling_load_ration_variation.png)
**Figure 5.** Ultimate load to critical buckling load ration variation.

Figure 6 is showing that for 4.5m x 4.5m slab panel size, \( \delta_{ns} \) value increased about 67.0% when column length increased from 5m to 6m. \( \delta_{ns} \) value increased about 25% when column length increased from 4m to 5m. For 6m x 6m slab panel size, \( \delta_{ns} \) value increased about 132.0% for same increment of column length (5m to 6m), and for 7.5m x 7.5m panel size \( \delta_{ns} \) increased about 244.0%. So, there is a drastically change observed when column length increased from 5m to 6m for all three slab sizes. Figure 7 showed \( \delta_s \) for 15 columns of 4 different lengths and 3 different slab panels. It is observed that the value of \( kl/r \) is always greater than 12 for 15 corner columns in flat-plate frame structure. Therefore, the columns which are neglected due to lower value (less than 1.0) of \( \delta_{ns} \) must be consider carefully for sway moment effect even the value is low. There is not sudden increase observed in the variation in \( \delta_s \) values of different columns. For 7.5m x 7.5m slab panel and 6m column length, the \( kl/r \) is greater than 100. That means this column is a very slender and need second order computer analysis. Hence, this could conclude that for flat-plate structure slenderness effect occurred in all cases for corner column of various extents and the effect is vulnerable.
5.2. Edge column

Critical Buckling Load versus Slenderness Ratio for edge column is shown in Figure 8. In case of three panels, the $P_c$ value decreased around 25.0% for column 5m to 6m. Figure 9 represents the ratio of column design load to critical load of column with slenderness ratio. The edge column showed slender behaviour while the design load increased about 40% from critical buckling load. Figure 10 is showing that ten column experience slenderness effect. $\delta_{ns}$ ranges from 1.02 to 5.84. Out of five columns, four columns of different height of 4.5m x 4.5m panel experience slenderness effect. The same situation occurred in 6m x 6m panel size. The $\delta_{ns}$ value increases about 164.0% when column length increases from 5m to 6m for 6m x 6m slab panel. It is seen that the trend lines for 6m x 6m slab panel and for 4.5m x 4.5m slab panel are much extended and steeper when it reached from 5m to 6m compared to the curve for 7.5m x 7.5m. Figure 11 represents the $\delta_s$ with Slenderness Ratio of 15 columns in 3 panels. In edge column, all the members of different height of different panel have sway effect like corner columns.
5.3. Inner column

Figure 12 presents the critical buckling load variations for inner column with slenderness ratio. In case of three panels, for column 5m to 6m the \( P_c \) value decreases around 25.0%. Figure 13 represents the ratio of column design load to critical buckling load verses slenderness ratio. Columns showed slenderness behaviour while the design load increases about 40% compared to the Critical Buckling Load. According to Figure 14, the \( \delta_{ns} \) value is not significantly higher for inner column in 7.5m x 7.5m panel compared with the other two panels. The \( \delta_{ns} \) value increased about 44.0% when column length increased from 5m to 6m for 4.5m x 4.5m panel size. The \( \delta_{ns} \) value increased about 38.0% for column length 5m to 6m for 6.0m x 6.0m panel size. The \( \delta_{ns} \) value increased about 16.0% for 7.5m x 7.5m panel. Comparing to other two locations of column, i.e. corner and edge, the value of \( \delta_{ns} \) is not much higher in inner column for all the panel sizes. So, the inner column slenderness behaviour is not much considerable in flat-plate frame structure. According to Figure 15, the values of \( \delta_s \) for inner columns are less than corner and edge columns as given in Figure 7 and Figure 11. The curves are very regular in shape. Increment in Slenderness Ratio is higher in larger span but \( \delta_s \) is higher in smaller span.

![Figure 10](image10.png) **Figure 10.** Non-sway moment magnification factor variation.

![Figure 11](image11.png) **Figure 11.** Sway moment magnification factor variation.

![Figure 12](image12.png) **Figure 12.** Critical buckling load variation.

![Figure 13](image13.png) **Figure 13.** Ultimate load to critical buckling load ration variation.
6. Conclusion
From the outcomes of the research, it can be concluded that column in flat-plate structures are sensitive to slenderness effects. From location, corner and edge column shows maximum slenderness values and should carefully judge at the time of calculating magnification factors. Certainly edge column of flat-plate structure shows maximum slenderness values. Most of the inner columns do not show any slender effect or sway condition. In many cases, a column is not slender by considering slenderness ratio only but could have slenderness effect if non-sway or sway moment magnification factor is calculated. In addition, a column could have shown non-slenderness behaviour of non-sway frame is considered but showed significant slenderness if sway frame is considered. For this reason, a designer should check a column for both non-sway and sway frame and determine both non-sway and sway moment magnification factor.

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**Acknowledgement**

The Authors would like to gratefully acknowledge the Gautam Buddha University for supporting this work.