PROBING THE ISOSCALAR EXCITATIONS OF $^{12}$C WITH INELASTIC ALPHA SCATTERING

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The robust (spin and isospin zero) α-particle remains one of the best projectiles to probe the nuclear isoscalar excitations. In the present work, a microscopic folding model analysis of the $α+^{12}$C inelastic scattering to the $2^+$ (4.44 MeV), $0^+$ (7.65 MeV), $3^-$ (9.64 MeV), $0^+$ (10.3 MeV) and $1^-$ (10.84 MeV) states in $^{12}$C has been performed using the 3-α resonating group method wave functions. The isoscalar transition strengths of these states were carefully studied based on the coupled-channel analysis using the microscopic folded form factors. A correlation between the weak binding and/or short lifetime of the excited state and absorption in the exit channel of inelastic scattering has been established.

1. Introduction

Given the well established $α$-cluster structures of some excited states of $^{12}$C, the low-lying isoscalar (IS) excitations of $^{12}$C become a subject of significant interest recently. In particular, the isoscalar $0^+_2$ state at 7.65 MeV in $^{12}$C (known as the Hoyle state) has been studied extensively due to its vital role in the stellar synthesis of Carbon. Although this state was identified long ago in the inelastic $α+^{12}$C scattering and inelastic electron scattering as an isoscalar $E0$ excitation, our knowledge about its unique structure is still far from complete. Since the Hoyle state lies slightly above the $α$-decay threshold of 7.27 MeV, its wave function turns out to have a dominant three $α$ clusters component, which has been confirmed, e.g., by the Resonating Group Method (RGM) calculations. Quite interesting is the condensate structure suggested in Refs. where the three $α$ clusters were shown to condense into the lowest s-state of their potential and form, therefore, a Bose-Einstein condensate (BEC). Such a BEC structure of the Hoyle state was shown to be mixed also with the molecular $^8$Be+$α$ configuration.

In general, to validate conclusion made in the structure calculation, the wave functions must be carefully tested in the study of nuclear reactions. Since the spin-
and isospin zero $\alpha$-particle is a very good projectile to excite the nuclear IS states, the 3-$\alpha$ RGM wave function of the Hoyle state obtained by Kamimura\cite{8} has been recently used in a folding model analysis\cite{12} of the inelastic $\alpha^{+12}$C scattering and realistic $E0$ transition was deduced. In the present work we extend this approach to study also other IS excitations of $^{12}$C like $2^+$ (4.44 MeV), $3^-$ (9.64 MeV), $0^+$ (10.3 MeV) and $1^-$ (10.84 MeV) states, using the RGM wave functions\cite{8}.

2. Double-folding calculation and coupled-channel scheme

In general, the $\alpha$-nucleus form factor (FF) contains all structure information of the nuclear state and it is, therefore, desirable to have FF evaluated in the microscopic folding model using an appropriate effective nucleon-nucleon (NN) interaction and realistic wave functions for the $\alpha$-particle and target nucleus, respectively. The main ingredients of the folding approach and coupled-channel (CC) formalism for elastic and inelastic nucleus-nucleus scattering can be found, e.g., in Refs.\cite{13,14}.

In this work, we have used the complex density dependent CDJLM interaction constructed\cite{12} based on the Brueckner Hartree-Fock results for the nucleon optical potential (OP) in symmetric nuclear matter. The nuclear wave functions obtained in the RGM method by Kamimura\cite{8} have been used to generate nuclear densities for the folding calculation of the complex OP and transition FF for the $\alpha^{+12}$C system. These RGM wave functions were proven to give consistently a realistic description of the compact structure of the ground state $0^+$ and the first $2^+$ and $3^-$ states, as well as the dilute structure of the $0^+_2$ (Hoyle state) and $1^-$ state (see Table 1). For the Hoyle state, the RGM wave function has been shown\cite{10,15} to be very close to the BEC wave function. To take into account coupling between different scattering channels, the RGM densities were constructed for both the diagonal and nondiagonal transitions between states under study, including the $E2$ reorientation of the $2^+$, $3^-$ and $1^-$ states. In total, 23 diagonal and nondiagonal densities were used to calculate the $\alpha$-nucleus OP and inelastic FF for the CC equations, with

| $J^\pi$ | $E_{\text{cal}}$ (MeV) | $E_{\text{exp}}$ (MeV) | $B(EJ)_{\text{cal}}$ $^*$ ($e^2\text{fm}^{J+2}$) | $B(EJ)_{\text{exp}}$ $^*$ ($e^2\text{fm}^{J+2}$) | $\tau_{\text{exp}}$ (sec) | RMS$_{\text{cal}}$ (fm) | $S_\rho$ | $S_W$ |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------|-------|
| $2^+$  | 2.77            | 4.44            | 46.5            | $40 \pm 4$      | $6 \times 10^{-14}$ | 2.38            | 1.00  | 1.00  |
| $0^+$  | 7.74            | 7.65            | 6.62            | $5.4 \pm 0.2$   | $8 \times 10^{-17}$ | 3.47            | 0.55  | 3.27  |
| $3^-$  | 8.14            | 9.64            | 872             | $610 \pm 90$    | $2 \times 10^{-20}$ | 2.77            | 0.75  | 1.75  |
| $0^+$  | 14.0            | 10.3            | 6.33            | $2 \times 10^{-22}$ | 3.14            | 0.45            | 4.85  |
| $1^-$  | 10.8            | 10.8            | 4.16            | $2 \times 10^{-21}$ | 3.39            | 0.20            | 8.60  |

$^*$ $M(EJ)$ in $e \text{ fm}^{J+2}$ for $0^+$ and $1^-$ states.

Table 1. Characteristics of the IS states of $^{12}$C under present study. Calculated values are those taken from the RGM calculations\cite{8}. $S_\rho$ is the scaling factor of the complex inelastic FF used in the renormalized DWBA calculation (see Fig. 2). $S_W$ is the ratio of the absorption strength in the exit channel to that in the entry channel of inelastic $\alpha^{+12}$C scattering.
Fig. 1. Coupling scheme used in the CC equations for elastic and inelastic $\alpha+^{12}\text{C}$ scattering.

the coupling scheme shown in Fig. 1. One can see from Fig. 1 that the two-step excitation of the IS states are treated in equal footing with the direct excitation. In particular, the two-step excitation $(0^+_1 \rightarrow 2^+ \rightarrow 0^+_2)$ of the Hoyle state is the exact inverse of transitions taking place in the stellar Carbon production. Since the $E\lambda$ transition strengths for some states are well determined from experiment, we have slightly scaled the RGM densities to match the calculated $B(E\lambda)$ values with the data for $B(E2, 0^+_1 \rightarrow 2^+), B(E2, 2^+ \rightarrow 0^+_2)$ and $B(E3, 0^+_1 \rightarrow 3^-)$, so that the corresponding inelastic FF can be reliably used in the CC analysis of the inelastic $\alpha+^{12}\text{C}$ scattering. For the $0^+_2$ and $1^-_1$ states, given a good description of the $(e,e')$ data by the corresponding RGM transition densities, we kept them unchanged in the folding calculation. All the CC calculations have been performed using the code ECIS97 written by Raynal.

3. Results and discussion

Although many experiments on inelastic $\alpha+^{12}\text{C}$ scattering were done (see, e.g., Refs. [2,3] in the last century, inelastic scattering to the $0^+_1$ (10.3 MeV) and $1^-_1$ (10.8 MeV) states have been measured with high precision only recently at the $\alpha$ energy of 240 MeV [4]. We concentrate, therefore, on the 240 MeV data in the present work. To fine tune the strength of complex CDJLM interaction, the real and imaginary elastic folded potentials were slightly renormalized by an optimal fit to the elastic scattering data. The same renormalization factors were then used to scale the real and imaginary inelastic folded FF for the inelastic scattering study either in the distorted wave Born approximation (DWBA) or in one- and two-step CC schemes. The DWBA and CC results are compared with the data in Fig. 2. Except for the $2^+$ state, the calculated cross sections for other states significantly overestimate
Fig. 2. Inelastic $\alpha$+$^{12}$C scattering data measured at $E_{\text{lab}} = 240$ MeV for the IS states of $^{12}$C under study in comparison with the DWBA and CC results obtained with the folded OP and inelastic FF. The transition momenta (or probabilities) and fractions of the EWSR are deduced from the renormalized DWBA calculation.

The data, especially for the $0^+_3$ and $1^-_1$ states. The CC effect by one- and two-step coupling could not help to solve this problem. To fit the data in the conventional DWBA or CC methods, the complex inelastic FF needs to be scaled by a factor $S_\rho$ (see Table 1 and the renormalized DWBA results plotted in Fig. 2). Since the RGM transition densities were used to reproduce quite well the experimental $(e, e')$ form factors of these states, such a scaling leads naturally to a “suppression” of the IS transition strength, in terms of $B(E\lambda)$ probability or fraction of IS energy weighted sum rule (EWSR) shown in Fig. 2. It can be seen from Table 1 that the IS strengths deduced from the renormalized DWBA results strongly disagree with the experimental values. For example, the renormalized DWBA implies about 6.9% of the IS monopole EWSR for the Hoyle state, while this fraction should be around 15% as deduced from the $(e, e')$ data. Since the $(e, e')$ data measured by Strehl covers only low momentum transfers, we compare in Fig. 3 the $(e, e')$ form factors for the Hoyle state given by the RGM transition density with a set of high precision $(e, e')$ data taken up to high momentum transfers. It is easy to deduce from Fig. 3 that the $E0$ transition strength of the Hoyle state exhausts about 23% of the IS monopole EWSR, more than twice the sum rule fraction found from the conventional DWBA analysis of inelastic $\alpha$+$^{12}$C scattering. We have found that an enhanced absorption in the exit channel (due to the short lifetime and weakly bound structure of the Hoyle state) leads directly to a suppression the
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Fig. 3. Inelastic electron scattering FF for the Hoyle state given by the original RGM transition density [5] (which exhausts 22.8% of the IS monopole EWSR) and that scaled to exhaust 15.2% (as deduced from $(e,e')$ data at low momentum transfers [5]) and 6.9% (as deduced from the renormalized DWBA) of the monopole EWSR. The data are taken from Refs. [17,18].

Fig. 4. The same as Fig. 2 but with the enhanced absorption in the exit $\alpha+^{12}$C* channels. The transition momenta (or probabilities) and fractions of the EWSR are given by the RGM transition densities used in the folding calculation. See more details in text.

monopole strength in the inelastic $\alpha+^{12}$C scattering. To explore similar effect for other IS states of $^{12}$C, we have added to the microscopic folded imaginary OP for
each exit $\alpha+^{12}\text{C}^*$ channel a surface term $\Delta W$ with parameters determined from the best DWBA and CC fit to the inelastic data. In this way, consistent descriptions of the inelastic $\alpha+^{12}\text{C}$ scattering and $(e, e')$ data could be reached using the RGM nuclear transition densities (see Fig. 4). Excepting for the $2^+$ state, the imaginary OP in the exit $\alpha+^{12}\text{C}^*$ channel turned out, as expected, to be much stronger than that in the entrance $\alpha+^{12}\text{C}_{g.s.}$ channel (see the ratio $S_W$ of the volume integral of the imaginary OP in the exit channel to that in the entrance channel in Table 1).

By comparing the nuclear RMS radii, mean lifetimes $\tau$ and ratios $S_W$ for the IS states of $^{12}\text{C}$ under study, we conclude that there exists a correlation between the diluteness, lifetime (or weak binding) of the excited state and absorption in the exit channel. Since most of the candidates for $\alpha$-cluster states are weakly bound, this absorption effect must be taken into account in the future studies of cluster states excited in the inelastic $\alpha$-nucleus or nucleus-nucleus scattering.

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