Model independent approaches to reionization in the analysis of upcoming CMB data

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**ABSTRACT**

**Aims.** On large angular scales, CMB polarization depends mostly on the evolution of the ionization level of the IGM during reionization. In order to avoid biasing parameter estimates, an accurate and model independent approach to reionization is needed when analyzing high precision data, like those expected from the Planck experiment. In this paper we consider two recently proposed methods of fitting for reionization and we discuss their respective advantages.

**Methods.** We test both methods by performing a Monte Carlo Markov Chain analysis of simulated Planck data, assuming different fiducial reionization histories. We take into account both temperature and polarization data up to high multipoles, and we fit for both reionization and non reionization parameters.

**Results.** We find that while a wrong assumption on reionization may bias $\tau_e$, $A_s$ and $r$ by $1 - 3$ standard deviations, other parameters, in particular $n_s$, are not significantly biased. The additional reionization parameters introduced by considering the model independent methods do not affect the accuracy of the estimates of the main cosmological parameters, the biggest degradation being of order $\sim 15\%$ for $\tau_e$. Finally, we show that neglecting Helium contribution in the analysis increase the bias on $\tau_e$, $r$ and $A_s$ even when a general fitting approach to reionization is assumed.

**Key words.** Cosmology: cosmic microwave background – Cosmology: cosmological parameters

1. Introduction

The upcoming measurements of Cosmic Microwave Background (CMB) by the Planck mission will allow for an unprecedented accuracy in the determination of the CMB angular power spectra. Due to its full sky coverage and sensitivity, Planck will provide an accurate characterization of $E$–mode polarization autocorrelation power spectrum, $C^{EE}_l$, at large angular scales, and either detect or significantly improve the current limits on the $B$–mode polarization power spectrum, $C^{BB}_l$. While other CMB polarization are currently planned (e.g., [Taylor et al. 2004] Yoon et al. 2006 [MacTavish et al. 2007] Samtleben 2008), none of them is expected to provide a measurement of the lowest $C^{EE}_l$ multipoles with an accuracy better than Planck. To a first approximation, the average power of $C^{EE}_l$ on these scales depends mostly on the optical depth to Thomson scattering due to reionization, $\tau_e$. The value of $\tau_e$ also determines the suppression of the intermediate to high multipoles of the temperature power spectrum, $C^{TT}_l$. Current data by the Wilkinson Microwave
Anisotropy Probe (WMAP) imply a value $\tau_e = 0.087 \pm 0.017$, with variations of $\Delta \tau_e \approx 0.01$ depending on the details of the analysis procedure and data sets considered (Dunkley et al. 2008). These constraints assume that reionization is a sharp transition occurring at a given redshift $z_r$.

However, theoretical and numerical studies suggest that reionization is a fairly complex process, possibly resulting from the sum of several contributions occurring over different time frames (e.g. Barkana & Loeb 2001, Venkatesan et al. 2003, Wyithe & Loeb 2003, Cen 2003, Haiman & Zolder 2003, Shull & Venkatesan 2007). In addition, observations of Lyα emitters in the redshift range $6 < z < 7$, show a rapid evolution of the neutral Hydrogen fraction of the intergalactic medium (IGM) (Ota et al. 2007). In the context of a sharp reionization, a reionization redshift $z \approx 7$ implies $\tau_e \approx 0.04$, and WMAP 5–year data rule out such scenario at more than 3.5σ significance level. In order to represent our ignorance of the reionization process, it is then necessary to relax the hypothesis on reionization, and consider more complex reionization histories.

In this case, the low $C^{EE}_l$ and $C^{BB}_l$ multipoles depend not just on $\tau_e$ but also on the detailed redshift evolution of the (assumed homogeneous) number density of free electrons in the IGM, $x_e(z)$, expressed in units of the Hydrogen atoms number density. For fixed values of $\tau_e$ and all other relevant cosmological parameters, differences in $x_e(z)$ affect the shape of the polarization power spectra up to multipoles $l \approx 40 - 50$. An incorrect ansatz on reionization may lead to a strong bias in the determination of $\tau_e$ (Kaplinghat et al. 2003, Holder et al. 2003, Colombo et al. 2005). In turn a bias on $\tau_e$ may result in errors on related parameter, such as the normalization of the primordial power spectrum of density fluctuations, $A_s$, and the tensor–to–scalar ratio $r$. At the sensitivity level of current WMAP data, such bias is a fraction of the experimental error, and current constraints on the optical depth can be considered safe. In turn, this implies that constraints on the other main cosmological parameters, in particular on $n_s$, are not strongly dependent on the value of $\tau_e$ (Dunkley et al. 2008). Planck sensitivity, however, will be ~ 10 times better than WMAP 5–year data, making an accurate and model independent approach to reionization a requirement for correct determination of $\tau_e$ and the other cosmological parameters.

One such approach is to simply divide the redshift interval relevant for reionization in a number of bins and try to directly constrain the averaged value of $x_e(z)$ in each bin (Lewis et al. 2006). The implementation of the method is straightforward and allows to easily take into account direct constraints on $x_e(z)$ (e.g. from 21cm measurements, Tashiro et al. 2008). However, the choice of bins characteristics is not obvious, and allowing for a fine redshift resolution implies the addition of a significant number of strongly correlated parameters.

A principal component (PC) approach (Hu & Holder 2002, Mortonson & Hu 2007a) is a possible alternative. The reionization history is decomposed over a set of eigenmodes, which encode the effects of a change in $x_e(z)$ on $C^{EE}_l$. The amplitude of each eigenmode is left as a free parameter to be determined from the data. The advantage of the method lies in that a reduced number ($\sim 5$) eigenmodes is sufficient to approximate the effects of a generic reionization history on the $C^{EE}_l$’s. Using a Monte Carlo Markov Chains (MCMC) approach, Mortonson & Hu (2007a, 2007b) showed that PC analysis allows to correctly recover the value of $\tau_e$, also avoiding the introduction of spurious effects on $r$. These results considered only the $l < 100$ polarization multipoles, and assumed that the remaining cosmological parameters were fixed to their correct value. However, actual data

\footnote{Comparing the nominal single channel WMAP sensitivity with the specifications for Planck 143GHz channel.}
analysis needs to include also temperature data and high multipoles, and simultaneously fit for the whole set of cosmological parameters.

CMB data allow to probe a large number of different parameters and Planck is expected to measure the basic cosmological parameters with high accuracy \cite{PlanckBlueBook}, providing reference values for other kinds of measurements which probe only a subset of the parameter space (e.g., SNIa data) and/or cover different redshift ranges and scales (e.g., galaxy surveys, Ly\alpha measurements). However, estimates of Planck performances typically take into consideration only the basic sharp reionization model, which can be accurately described by one parameter. Introduction of new (reionization) parameters in the model may give rise to new degeneracies, which in turn may bias the estimates of the other parameters and worsen the accuracy of their determinations. In addition, degeneracies also decrease the efficiency of the parameter estimation procedure. In the light of the upcoming Planck data, it is then relevant to compare how these methods affect the whole parameter estimation process, i.e., considering also TT and TE spectra and high-\ell's, and including also non–reionization parameters, under the same set of conditions.

Moreover, previous studies did not take into account Helium reionization (see, e.g., \cite{Shull2004, Furlanetto2007} and references therein). Helium reionization has been often neglected in CMB studies, as it contributes at most 10% of the total optical depth. However, the Planck satellites is expected to measure \( \tau_e \) with a precision of a few percent \cite{PlanckBlueBook} and it is interesting to study whether Helium contribution must be explicitly accounted for in the modeling of reionization. In addition to the physical aspects of reionization modeling and their impact on parameter estimation, the computational aspects of the problem need to be factored in. The analysis of current and future experiments require significant numerical resources. Choosing an inappropriate parametrization can greatly decrease the efficiency of MCMC methods, even more so when including a large number of parameters poorly constrained by data. In this paper we perform a comparison of the performances of the two approaches by simulating future Planck data, corresponding to different fiducial reionization histories both with and without Helium contribution, and analyze them assuming either sharp reionization or the two methods outlined below. We consider in the analysis polarization and temperature data up to multipoles \( l = 2000 \), and fit simultaneously for the main cosmological parameters. We discuss the the advantages of each methods, both in terms of the effects on the recovered parameters and in terms of computational cost.

The outline of the paper is as follow. In Section 2 we briefly review the proposed model independent methods. In the following Section 3 we discuss the fiducial reionization histories considered and our simulations of experimental data and MCMC analysis. We present our results in Section 4 and we draw our conclusions in Section 5.

\section{2. Model Independent Approaches to Reionization}

\subsection{2.1. Binning The Reionization History}

We consider the redshift set \( z_0 < z_1 < z_2 < \ldots < z_N \), dividing the interval \((z_0, z_N)\) into \( N \) bins, so that

\begin{equation}
 x^e_z = x_{e,i} \quad z_{i-1} < z < z_i, \quad i = 1, \ldots, N.
\end{equation}
In modeling the reionization history, we neglect Helium reionization and assume \( x_e(z) = 1 \) for \( z < z_0 \) while for \( z > z_N \) we match \( x_e(z) \) to the small residual ionization level from incomplete recombination. In particular, according to data on Ly\( \alpha \) emitters (Ota et al. 2007) and quasar spectra (Fan et al. 2006), we assume \( z_0 = 6 \). Fixing \( z_N = 30 \) allows for the contributions of the first stars and/or early black holes (Ricotti et al. 2005) to \( \tau_e \); we ignore here the possible X-ray emission from high–\( z \) dark–matter interactions (e.g. Hansen & Haiman 2004, Mapelli et al. 2006). The interval \( (z_0, z_N) \) is then divided into \( N = 6 \) equal bins.

To avoid instabilities during numerical integration, we in practice enforce an analytical expression for \( x_e(z) \):

\[
x_e(z) = \sum_{i=1}^{N} x_{e,i} \chi_i(z)
\]

(2)

\[
\chi_i(z) = \frac{1}{2} \left\{ \tanh \left[ \alpha \frac{\eta(z) - \eta(z_{i-1})}{\eta(z_i) - \eta(z_{i-1})} \right] - \tanh \left[ \alpha \frac{\eta(z) - \eta(z_i)}{\eta(z_i) - \eta(z_{i-1})} \right] \right\}
\]

(3)

where \( \eta(z) \) is the conformal time at redshift \( z \) and \( \alpha \) governs the sharpness of the transition. Following CAMB (http://www.camb.info), we usually take \( \alpha = 150 \). We also assume a flat prior on the \( x_{e,i} \). As pointed out by Lewis et al. (2006) constraints on the \( x_{e,i} \) may depend significantly on the details of the binning kernels and the priors, if the data are poor. In addition, results for adjacent bins will usually be strongly correlated.

### 2.2. Principal Component Analysis

Following Mortonson & Hu (2007a, 2007b) we divide the interval \( (z_0, z_N) \) in \( N \) equal bins of width \( \Delta z = 0.25 \), and consider a fiducial binned reionization history \( \{x_{e,i}\}, i = 1, 2, ..., N \). We take \( z_0 = 6 \) and \( z_N = 30 \) and define \( x_e(z) \) outside this interval as we did in the previous section. An estimate of the accuracy with which an experiment can measure the \( x_{e,i} \) is given by the Fisher matrix. Since we are interested in the effects of \( x_e(z) \) on CMB spectra, we can approximate the Fisher matrix as:

\[
F_{i,j} \sim \sum_{l=2}^{l_{\text{max}}} \sum_{l'=2}^{l_{\text{max}}} \frac{\partial C_{\ell}^{EE}}{\partial x_{e,i}} \frac{\partial C_{\ell'}^{EE}}{\partial x_{e,j}} \text{Cov}(C_{\ell}^{EE}, C_{\ell'}^{EE})^{-1}
\]

(4)

For a full–sky noise–free experiment, the covariance \( \text{Cov}(C_{\ell}^{EE}, C_{\ell'}^{EE}) = \frac{2}{2\ell+1} (C_{\ell}^{EE})^2 \delta_{l'l} \), and the main contribution to the Fisher matrix comes from the \( l \lesssim l_{\text{max}} = 100 \) multipoles of \( C_{\ell}^{EE} \). Contributions from \( TT \) and \( TE \) modes are negligible with respect to those from \( E–mode \) polarization and we do not include them into the definition of the Fisher matrix. In the following we will test if this approximation is still adequate when considering also high–\( l \) \( TT \) data.

The principal components of \( x_e(z) \) are defined as the eigenvectors, \( S_n(z_i) \), of the Fisher matrix:

\[
F_{i,j} = \frac{1}{N^2} \sum_{n=1}^{N} S_n(x_i) S_n(x_j) \chi_n^2 S_n(z_i) ;
\]

(5)

which satisfy the orthonormalization conditions:

\[
\sum_{i=1}^{N} S_n(z_i) S_m(z_i) \Delta z = (z_N - z_0) \delta_{nm} ,
\]

(6)

\[
\sum_{n=1}^{N} S_n(z_i) S_n(z_j) = N \delta_{ij} ,
\]

(7)
In the limit $\Delta z \to 0$, the first relation can be replaced with an integral over $z$, and a generic $x_e(z)$ can be written as:

$$x_e(z) = x_{e,fid}(z) + \sum_{n=1}^{N} \mu_n S_n(z). \quad (8)$$

In this representation, we replaced the $N$ values $\{x_{e,i}\}$ defining a generic $x_e(z)$ with respect to our choice of binning, with the $N$ mode amplitudes $\{\mu_n\}$. Thus, in principle, the number of parameters required to characterize a generic reionization history has not changed. However Mortonson & Hu (2007a) showed that most information needed to determine CMB features is contained in the $\sim 5$ eigenmodes corresponding to the highest eigenvalues, $\lambda_n^2$, thus allowing for a significant compression of information. When truncating the sum in Equation (8), care must be taken that the resulting $x_e(z)$ be consistent with the definition of the Hydrogen ionization fraction, i.e. $0 < x_e(z) < 1$.

From an operative point of view, when analyzing the synthetic data, we define $S_n(z)$ in analogy with Equation (2). In addition, we take flat priors on the $\{\mu_n\}$ and check that the resulting $x_e(z)$ does not have unphysical values.

3. Analysis of Simulated Data

3.1. Reference Models

In order to test the effects on parameter estimation of the model independent approaches discussed in the previous section, we consider an ideal experiment with instrumental characteristics like the nominal performance of the 143GHz Planck channel (Planck Blue Book): Gaussian beam of width $\theta_{\text{FWHM}} = 7.1'$, temperature and polarization sensitivities of $\sigma_T = 42\mu K\cdot\text{arcmin}$ and $\sigma_P = 80\mu K\cdot\text{arcmin}$, respectively, and assuming a sky coverage $f_{\text{sky}} = 0.80$. The actual Planck performance will exceed this specification, both due to the availability of more frequency channels and to an actual noise level which is better than the nominal requirements cited here. However, real data will require a significant foreground removal, and possibly not all channels will be available for cosmological analysis. While the actual Planck data analysis will therefore need to incorporate more subtleties, the main aim of this work is a comparison of different approaches to reionization modeling.

We then generate simulated data corresponding to different fiducial reionization histories: 1) a sharp reionization model with $\tau_e = 0.085$; 2) a model with the same $\tau_e$ but:

$$x_e(z) = \begin{cases} 1 & z < 6 \\ 0.15 & 6 < z < 30 \end{cases}; \quad (9)$$

notice that this is the same reionization model used to define the eigenmodes; 3) a model with the same $\tau_e$ and:

$$x_e(z) = \begin{cases} 1.158 & z < 3 \\ 1.079 & 3 < z < 6 \\ 1 & 6 < z < 10.65 \end{cases}. \quad (10)$$

The values of the remaining cosmological parameters are unchanged between the models: the physical baryon and cold dark matter densities $\omega_b = 0.0224$ and $\omega_c = 0.112$, respectively; the
Fig. 1. Fiducial models. Top left: $x_e(z)$ for a sharp reionization (solid line), two–step reionization (short–dashed) and Helium reionization (long–dashed). All models have $\tau_e = 0.085$, and the same values of the other cosmological parameters. The other panels show the corresponding angular power spectra for $TE$ (top right), $EE$ (bottom left) and $BB$ (bottom right). The dotted line shows the assumed Planck noise power spectrum.

The slope and amplitude of the primordial power–law spectrum of density fluctuations $n_s = 0.95$ and $\mathcal{A}_s = \log_{10}(10^{10}A_s) = 3.135$; the Hubble parameter $H_0 = 72\text{Km/s/Mpc}$; $r(k = 0.05\text{Mpc}^{-1}) = 0.1$ and $Y_{\text{He}} = 0.24$ is the Helium mass fraction. The spectral index of tensor modes is fixed according to the consistency relation for slow–roll inflation: $n_T = -r/8 = -.0125$. Even though the effects of reionization models considered here are restricted to the $l < 100$ (see figure [1]) multipoles of $E$ and $B$–mode power spectra, we take into account also temperature data, as we are interested in assessing the effects of the different parametrizations also on non–reionization parameters. Each models is in turn analyzed: 1) assuming a sharp reionization, 2) considering $N = 6$ bins of $\Delta z = 4$ between $z_0 = 6$ and $z_N = 30$, 3) using the principal components method. Besides fitting for the reionization parameters, we take also take $\{\omega_b, \omega_c, n_s, \mathcal{A}_s, H_0, r\}$ as free parameters, while $n_T$ and $Y_{\text{He}}$ are fixed to the input values.

We notice here that complete ionization in the fiducial sharp–reionization model happens at $z_r \approx 11$. Thus, the corresponding reionization history can not be correctly modeled neither by the binning scheme we selected nor by a small (i.e $\leq 5$) number of eigenmodes. Conversely, the two–step fiducial model can be accurately parametrized by both model independent approaches. The third model has been chosen as a toy model of Helium reioniza-
tion. Numerical studies suggest that Helium singly ionizes at about the same time as Hydrogen (Venkatesan et al. 2003, Shapiro et al. 2004), due to the closeness of the respective ionization energies. The simplest way to account for Helium reionization would be to assume a single reionization event after which \( x_e \approx 1.08 \). However, as discussed above, comparison of WMAP measurements and Ly\( \alpha \) observations suggest an extended reionization process in which ionization of the IGM begins at \( z_r \sim 20 \) and is completed by \( z_\sim 6 − 7 \). In this kind of scenario, Helium contribution allows \( x_e \) to exceed unity only for \( z \lesssim 6 \). We follow here a conservative approach and consider doubly ionized Helium for \( z < 3 \) and singly ionized Helium for \( 3 < z < 6 \); Helium contribution to the total optical depth is then \( \Delta \tau_e \sim 0.004 \).

### 3.2. Likelihood approximation

A set of CMB measurements can be represented by a set of vector coefficients \( \mathbf{a}_{lm} = \{a_{T,lm}, a_{E,lm}, a_{B,lm}\} \), with covariance matrix \( C_l \equiv \langle \mathbf{a}_{lm} \mathbf{a}_{l'm}' \rangle \delta_{ll'} \delta_{mm'} \) (e.g. Percival & Brown 2006, Hamimeche & Lewis 2008); the corresponding quadratic estimator is given by:

\[
\hat{C}_l = \frac{1}{2l+1} \sum_{m=-l}^{l} \mathbf{a}_{lm} \mathbf{a}_{lm}^\dagger .
\]

(11)

For a full–sky, noise–free experiment with infinite resolution, the \( \hat{C}_l \) are distributed according to a Wishart distribution. Using Bayes theorem, the corresponding log–likelihood, normalized so that \( \mathcal{L} = 1 \) for \( C_l = \hat{C}_l \), is given by:

\[
-2 \ln \mathcal{L}(\{C_l\}|\{\hat{C}_l\}) = \sum_{l=2}^{l_{\max}} (2l + 1) \left[ \text{Tr}(\hat{C}_l C_l^{-1}) - \ln(|\hat{C}_l C_l^{-1}|) - 3 \right] .
\]

(12)

In the presence of white isotropic noise and assuming a perfect Gaussian beam, the above expression is still valid if we replace \( C_l \) with \( C_l + N_l \), where \( N_l = \text{diag}(N_{T,l}, N_{E,l}, N_{P,l}) \) is the noise correlation matrix, while the noise power spectrum \( N_{T,l} = \sigma^2 T \exp[l(l+1)\theta^2_{\text{FWHM}}/(8 \ln 2)] \) and similarly for polarization. An analogous substitution is required for the estimator \( \hat{C}_l \).

If full–sky measurements are not available, the spherical harmonics coefficients for the cut sky, \( \tilde{a}_{lm}^X \), are a linear combination of true spherical harmonics coefficients corresponding to different modes and multipoles,

\[
\tilde{a}_{lm}^X = \sum_Y \sum_{l'm'} K_{lml'm'}^{XY} a_{l'm'}^Y
\]

(13)

where the kernels \( K_{lml'm'}^{XY} \) encodes the effect of non uniform sky coverage [Hivon et al. 2002]. In this case Equation (12) is no longer valid, although for azimuthal symmetric cuts an analytic expression for the likelihood can still be evaluated [Lewis et al. 2002], although at the cost of a significantly increased computational time. Here, instead, we suppose that the mode coupling resulting from incomplete sky coverage can be accounted for by multiplying Equation (12) by a factor \( f_{sky}^2 \). Although this approximation does not correctly account for mode mixing, in particular \( E–B \) mixing, in this way the likelihood functions still peaks at the full sky value, therefore any bias we find in our results is due to the modeling of reionization rather than the likelihood approximation. In addition, while errors on parameters may not be correctly estimated by this approximation, a correct assessment of errors would need to take into account the actual details of the data analysis pipeline, including tod filtering, map making and foreground removal, which are beyond the scope of this paper.
Fig. 2. Results for MCMC analysis assuming a sharp reionization for three different fiducial models: sharp reionization (solid lines), two–step reionization model (short–dashed) and Helium reionization (long–dashed). The reference value of the input parameters, shown by the vertical dotted lines, are the same in all fiducial models.

We then perform a MCMC analysis of the simulated data using a version of the CosmoMC package (http://cosmologist.info/cosmomc/) modified to take into account different reionization models. We also fix \( l_{\text{max}} = 2000 \). We determine convergence of our chains by requiring that the Gelman & Rubin ratio be \( R - 1 < 0.05 \) and simultaneously checking the stability of the 95% confidence limit on all parameters. In practice, this latter criterion leads to \( R - 1 \sim 0.02 - 0.03 \) for the converged chains.

4. Results

First of all, in order to identify how parameters can be affected by an incorrect assumption on reionization, we can compare the results of analyzing the three reference models assuming a sharp reionization. In Figure 2 we show the resulting marginalized distribution. When an incorrect assumption on reionization is made the estimate of \( \tau_c \) is biased by 1 or more standard deviations, depending on the fiducial reionization history, in agreement with previous findings (Kaplinghat et al. 2003, Holder et al. 2003, Colombo et al. 2005, Mortonson & Hu 2007a). For the models considered here, the bias is more relevant in the case of the two–step fiducial model, as in this scenario reionization starts significantly earlier than in either of the other models considered; thus the range of multipoles affected is greater. The corresponding numerical values are summarized by the third columns of tables 1 through 3 for the sharp, two–step and Helium reionization fiducial models described in Section 3.1, respectively. For each parameter, we report the mean estimated by the chains.

In turn, an incorrect determination of \( \tau_c \) biases the value of the amplitude of the primordial power spectrum according to \( A_s e^{-2\tau_c} = \text{const} \), and also results in a wrong determination of \( r \), as pointed out by Mortonson & Hu 2007b. Notice that here we fixed the value of the tensor spectral index; leaving \( n_T \) as a free parameter would slightly increase the error on \( r \), reducing the significance of the discrepancy. Other parameters are mostly unaffected. In particular, it is interesting to notice that the distribution for \( n_s \) does not depend on the reionization priors, even though in-
### Table 1. Parameter estimates for a two–step reionization fiducial model with \( \tau_e = 0.085 \) and other parameters as specified in the text, assuming: a sharp reionization (SR), a binned reionization (BR), either 3 (E3) or 5 (E5) principal components eigenmodes. Bold faced entries show when the bias between the input and recovered value exceeds half the associated error.

| Parameter | SR          | BR          | E3         | E5         |
|-----------|-------------|-------------|------------|------------|
|           | fiducial    | value       | value      | value      |
| \( \Omega_c h^2 \) | 0.224       | 2.240 ± 0.015 | 2.240 ± 0.015 | 2.242 ± 0.015 |
| \( \omega_e \)  | 0.112       | .1119 ± .0013 | .1118 ± .0013 | .1117 ± .0013 |
| \( \tau_e \)   | .085        | .0848 ± .0046 | .0853 ± .0053 | .0830 ± .0052 |
| \( n_s \)      | 0.950       | 0.9506 ± 0.0039 | 0.9509 ± 0.0039 | 0.9505 ± 0.0040 |
| \( A_s \)      | 3.135       | 3.136 ± 0.009 | 3.137 ± 0.010 | 3.132 ± 0.009 |
| \( r \)        | .10         | .104 ± .026   | **.118 ± .027** | **.120 ± .029** |
| \( H_0 \)      | 72          | 72.03 ± 0.66  | 72.14 ± 0.66  | 72.12 ± 0.63  |

### Table 2. Same as Table 1 but for a sharp reionization fiducial model analyzed assuming: a sharp reionization (SR), a binned reionization (BR), 5 (E5) principal components eigenmodes.

| Parameter | SR          | BR          | E5         |
|-----------|-------------|-------------|------------|
|           | fiducial    | value       | value      |
| \( \Omega_c h^2 \) | 0.224       | 2.240 ± 0.015 | 2.240 ± 0.015 |
| \( \omega_e \)  | 0.112       | .1119 ± .0013 | .1116 ± .0013 |
| \( \tau_e \)   | .085        | .0816 ± .0044 | .0831 ± .0049 |
| \( n_s \)      | 0.95        | 0.9506 ± 0.0039 | 0.9512 ± 0.0039 |
| \( A_s \)      | 3.135       | **3.129 ± 0.009** | 3.132 ± 0.010   |
| \( r \)        | .10         | .108 ± .028   | **.124 ± .027** |
| \( H_0 \)      | 72          | 72.05 ± 0.68  | 72.18 ± 0.65  |

### Table 3. Same as Table 1 but for a Helium reionization fiducial model analyzed assuming: a sharp reionization (SR), a binned reionization (BR), 5 (E5) principal components eigenmodes.

| Parameter | SR          | BR          | E5         |
|-----------|-------------|-------------|------------|
|           | fiducial    | value       | value      |
| \( \Omega_c h^2 \) | 0.224       | 2.240 ± 0.015 | 2.240 ± 0.015 |
| \( \omega_e \)  | 0.112       | .1119 ± .0013 | .1116 ± .0013 |
| \( \tau_e \)   | .085        | .0816 ± .0044 | .0831 ± .0049 |
| \( n_s \)      | 0.95        | 0.9506 ± 0.0039 | 0.9512 ± 0.0039 |
| \( A_s \)      | 3.135       | **3.129 ± 0.009** | 3.132 ± 0.010   |
| \( r \)        | .10         | .108 ± .028   | **.124 ± .027** |
| \( H_0 \)      | 72          | 72.05 ± 0.68  | 72.18 ± 0.65  |
Fig. 3. Results for MCMC analysis for two–step reionization fiducial model. Model has been analysed assuming a sharp reionization (solid lines), a binned reionization (long–dashed) and using 3 (short–dashed) or 5 (dot–dashed) principal components.

Fig. 4. Constraints on values of $x_e(z)$ in bins of width $\Delta z = 4$ between $z_0 = 6$ and $z_N = 30$ for a two–step reionization fiducial model. The dashed line show the input value of $x_e(z)$ in each bin.

formation from polarization is critical to break the $\tau_e$–$n_s$ degeneracy present in $TT$ spectra. This is due to the fact that at the sensitivity level considered here it is possible to get information on $n_s$ from the $l > 100$ multipoles; on these scales the $n_s$ affect the $l$–scaling of the power spectrum, while the reionization history affect the $C_l$'s with an overall suppression depending on the value of $\tau_e$. With Planck $l$–leverage, it is possible to disentangle the effects of $n_s$ and $\tau_e$, and recast any uncertainty on $\tau_e$ on the value of $A_s$. Since deviation of $n_s$ from unity allow to place constraints on the shape of inflation potential, we can conclude that such constraints will be safe even if an incorrect assumption on reionization is made.

We next turn to the model independent approaches (Lewis et al. 2006, Mortonson & Hu 2007a, Mortonson & Hu 2007b). In particular, we assess the effect of introducing these additional parameters on the whole analysis procedure, on the same set of data. As a first case, we consider a fiducial two–step reionization model. In principle, fitting this reionization would require a low number of reionization parameter, e.g. a single value of $x_e(z)$ for $6 < z < 30$, or a single eigenmode. Here in-
stead, we run MCMC assuming a \( N = 6 \) redshift bins, or either 3 or 5 eigenmodes. By considering more reionization parameters than effectively needed, we can study whether a bias is introduced or if error estimates are affected.

Figure 3 shows the resulting marginalized likelihoods. For reference purpose, we also repeat results for the sharp reionization history. Notice that for the binned and the PC analysis, \( \tau_e \) is a derived parameter. It is clear that in this case the modeling of reionization does not bear a strong impact on the estimates of the various parameters. Estimates of \( \tau_e \) shift by \( \sim 0.2\sigma \) depending on the methods considered, while the corresponding error for the 3 eigenmodes analysis is \( \sim 10\% \) smaller than in the other cases. There are no other significant differences between the 3 and 5 eigenmodes, or between eigenmodes and bins results. Both the binned reionization and principal components methods slightly overestimate \( \tau_e \), however the difference between input and recovered values are below half a standard deviation and are compatible with the statistical uncertainty. In fact, the Gelman and Rubin convergence diagnostic \( R \) roughly translates into \( R \approx 1 + r_x \), where \( r_x \) is the ratio between the variance of the sample mean and the variance of the target distribution (Dunkley et al. 2005), so that a value \( R - 1 \sim 0.05 \) corresponds to an uncertainty on the mean of about 25\% of the measured standard deviation.

Reconstruction of the reionization history, on the other hand, is not particularly accurate. Figure 4 shows constraints on the value of the ionization fraction \( x_i \) in the different bins. Only weak upper limits are found: the target model could be clearly described by just a single value of \( x_e \) and the data do not allow to significantly constrain the additional parameters. However, it is reassuring that this does not have any adverse effect on the accuracy with which data can constrain \( \tau_e \) and other relevant parameters. This is possibly because, while \( x_e \) bins are highly correlated (Lewis et al. 2006), we do not find significant degeneracies between the reionization parameters and the remaining cosmological ones. A qualitatively similar conclusion holds for the eigenvalue method: even though we add a significant number of poorly constrained parameters, the accuracy on the reconstruction of the main cosmological parameters is unaffected. In general, we find that the different model independent approaches considered lead to shift in the estimates of \( 0.1 - 0.2\sigma \); in the case of non–reionization parameters, this holds also for the sharp \( \tau_e \) analysis. Error estimates increase at most by 10\% – 15\% over the sharp reionization value.

As a second case, we consider a fiducial sharp reionization model, analysed with either \( N = 6 \) redshift bins or 5 PC eigenmodes (see Figure 5). This case is conceptually opposite to the previous one, as the fiducial reionization history cannot be accurately described by either modeling we considered. In this case, we do not consider the 3 PC model, as in general 3 eigenmodes are not enough to accurately recover the reionization parameters even when all other parameters are kept fixed (Mortonson & Hu 2007a).

Also in this case, both parametrizations considered measure \( \tau_e \) without any relevant bias, although the 5 PC method slightly underestimate it. Again, the discrepancy is compatible with the statistical uncertainty. When either model independent approach is assumed, the error on \( \tau_e \) increase by just \( \sim 15\% \) over the error that would be obtained by analyzing the data assuming a sharp reionization. It is interesting to note that both modeling overestimate \( r \) by \( \sim 0.6 - 0.7 \) standard deviations. While this does not represent a significant bias, it could be an hint that the modeling of reionization considered is not fully adequate to the underlying data and more eigenmodes, or a
different binning scheme, need to be included in the analysis. In general, checking that results are consistent between different parametrizations allow to minimize these spurious effect.

We next consider the impact of Helium reionization. Let us recall that both the bins and PC approaches we implemented here assume that \( x_e(z) = 1 \) at \( z \leq 6 \), therefore Helium reionization is not accounted for in the modeling using for data analysis. Comparing results for this case with those for the sharp reionization fiducial history, then, allows to establish whether Helium reionization needs to be included in the modeling. Results of the analysis are summarized in Table 3 and Figure 6.

At the sensitivity level considered, we find that the estimated value of \( \tau_e \) is consistent with the fiducial value at the 1\( \sigma \) level, regardless of the assumptions on reionization. More in detail, assuming a sharp reionization or using 5 PC, \( \tau_e \) is biased by \( 0.6 - 0.8\sigma \), while using bins \( \tau_e \) is recovered within 0.5\( \sigma \) from the input value. In addition, \( r \) is overestimated by \( 0.9\sigma \), both using bins and PC, while assuming sharp reionization \( r \) is recovered without a significant bias. This is due to the fact that, for fixed \( \tau_e \), Helium contribution alters the reionization history at \( z \leq 6 \) and therefore increases the power in \( EE \) and \( BB \) spectra at multipoles \( l < 5 \), with respect to a sharp reionization model. On the other hand, in extended reionization scenarios, power is shifted from \( l < 5 \) multipoles to \( 10 < l < 30 \) multipoles, as reionization starts earlier than in a sharp reionization scenario with the same \( \tau_e \) [Kaplinghat et al. 2003, Colombo et al. 2005]. For the sharp reionization fiducial model, instead, the model independent approaches overestimated \( r \) by \( 0.6 - 0.7\sigma \) and \( \tau_e \) was recovered to within half a standard deviation, regardless of the assumptions on reionization (see Table 2). This suggests that Helium reionization must be explicitly taken into account in our modeling, more so if we consider that the actual Planck performance is likely to exceed the conservative specifications assumed here, and Helium contribution is probably higher than that of our conservative approach. However, a more in depth study of the impact of Helium reionization in CMB data analysis is required.

Finally, we briefly discuss the computational costs of the different approaches. For the fiducial histories considered, we found that chains assuming 6 \( x_e \) bins take 30-50% more time to converge than those assuming 5 PC eigenmodes. In principle, under ideal condition, MCMC methods scale linearly with the number of parameters, so we can expect the chains for the binned analysis to take \( 10\% \) more, simply due to the different number of parameters in the two models. However, reaching this theoretical limit is significantly dependent on an efficient proposal distribution, i.e. on an accurate covariance matrix in the case of Gaussian proposal densities [Dunkley et al. 2005]. In this work, for each model we run a preliminary set of chains of 60000 points (total) to determine a starting covariance matrix. The difference in convergence times we found here suggests that the orthogonality of the eigenmodes allow for a more efficient exploration of the parameter space, even though such orthogonality holds properly only when the target reionization history is near the fiducial model used to define the eigenmodes [Mortonson & Hu 2007a]. A full assessment of this point would, however, require more simulations than those performed in this work, and is likely to depend on the details of the actual reionization history assumed as a fiducial model.
Fig. 5. Results for MCMC analysis for a sharp reionization fiducial model. Model has been analysed assuming a sharp reionization (solid lines), a binned reionization (long–dashed) and using 5 principal components (short–dashed).

Fig. 6. Results for MCMC analysis for the Helium reionization fiducial model. Model has been analysed assuming a sharp reionization (solid lines), a binned reionization (long–dashed) and using 5 principal components (short–dashed).

5. Conclusions

With the advent of high precision CMB polarization measurements, a detailed modeling of reionization becomes of great relevance, both to better constrain the detail of reionization itself and to avoid biases on the cosmological parameters, in particular those related to inflation. If the actual reionization history is not a single, quick transition, assuming a sharp reionization while analyzing data may lead to bias of 1 or more standard deviations on parameters like $\tau_e$, $A_s$ and $r$. However, a full theoretical understanding of reionization is still lacking and consensus on a physically motivated parametrization of the effects of reionization on CMB has not aroused yet. Thus, two model independent parametrizations have been recently proposed: using a binned reionization history (Lewis et al. 2006) and principal component approach (Hu & Holder 2003, Mortonson & Hu 2007a).
In this work we have considered both approaches and applied them to simulated Planck data in order to assess their accuracy and to find out any side–effect on the estimation of the other cosmological parameters. We considered fiducial models with the same values of all cosmological parameters, but with different reionization histories, and we analyzed these models assuming a sharp reionization or using both model independent approaches. In our analysis, we included $TT$, $TE$, $EE$ and $BB$ spectra up to multipoles $l_{\text{max}} = 2000$, and we fitted both for the reionization parameters and for the remaining cosmological parameters. In agreement with previous results considering only $EE$ data and/or low multipoles [Kaplinghat et al. 2003, Colombo et al. 2005, Mortonson & Hu 2007a], we found that the sharp reionization analysis give accurate results only when the fiducial model is not significantly different from a sharp reionization history, while in general biases of order 1–3 standard deviations can be expected on $\tau_e$, $A_s$ and $r$.

On the other hand, we found that both model independent methods are able to correctly recover the various parameters; none of the approaches provide a significant advantage over the other in term of accuracy of the recovered parameters. More in detail, the correct value of $\tau_e$ and $A_s$ are recovered to better than half a standard deviation. The additional parameters, either for bin reionization or for principal components, increase the error in $\tau_e$ by $\sim 15\%$, but do not affect the error on the other parameters. However, when the target model is not accurately described by the adopted parametrization, we noticed a residual bias on $r$, of order $0.6 – 0.7\sigma$. While this level of bias can be considered safe, it nonetheless indicates that our parametrization can be refined. More in general, to further reduce this bias, it is helpful to include as much external information on reionization than can be available, e.g. using 21 cm measurements. These external constraints can be directly implemented into a binned reionization approach, while for PC analysis additional work is required.

It is worth noticing that estimates of the remaining parameters, such as $\omega_b$, $\omega_c$ and $n_s$, are largely unaffected by the assumptions on reionization. This is valid not only when a model independent description of reionization is adopted, but also when the modeling of reionization assumed for data analysis is not an adequate description of the actual reionization history. In particular, estimates of $n_t$ from current and future experiments can be considered safe, regardless of the details of reionization.

We also considered a toy model of Helium reionization, in order to assess whether such contribution must be explicitly accounted for. We assumed a fiducial model with Helium contributing to the ionization fraction at redshifts $z < 6$, however both the bin reionization and the principal component method used for analyzing the simulated data assumed $x_e(z) = 1$ for $z \leq 6$. A comparison of the results of this analysis, with those for a sharp reionization fiducial model with the same $\tau_e$, show that the discrepancy between fiducial and recovered value increase by $\sim 0.2 – 0.5\sigma$ for $\tau_e$, $A_s$ and $r$, in the case of the Helium reionization fiducial model. However, in no case we detected a bias of $1\sigma$ or more. Even though Helium contributes to the total optical depth for $\Delta \tau_e = 0.004$, compared to an expected error $\sigma(\tau_e) \approx 0.005$, these results suggest that Helium reionization need to be explicitly taken into account in the analysis of future data.

Finally, we point out that, while we did not found significant differences between the model independent approaches considered in terms of the accuracy of the recovered parameters, in order to reach convergence a bin reionization approach requires $\sim 30 – 50\%$ more time than a principal component method. Since the analysis of current and future CMB data sets require significant
computational resources, this latter aspect needs to be taken into account when choosing a modeling of reionization.

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