Abstract: One of the major problems in modern gas compressing stations is related to surge control of compression system with multi-compressor. Due to the coupled system dynamics, standard controllers can interfere with each unit’s performance. This article introduces a new robust decentralized model predictive control (MPC) scheme for a team of cooperating compressors. In the decentralized control strategy, the integration of $H_\infty$ control scheme into a robust tube MPC algorithm is used to solve the surge problem of a multi-compressor dynamical process subject to constraints in a computation efficient way. Nonlinear dynamic for each compressor that contains acoustic of compressor system is considered and brings up the effects of the station’s piping system on the compressor surge. Numerical simulations...
approve the capability of the proposed surge control method and guaranty the asymptotic stability of compression system in the presence of disturbances and coupling effects.

Subjects: Systems & Control Engineering; Technology; Systems & Controls;

Keywords: centrifugal compressor surge; decentralized model predictive control; $H_\infty$ control; piping acoustic; tube-MPC

1. Introduction

Compressors play a major role in modern industrial processes for the compressing and transporta-
tion of fluids and gases. They are so important in the operation of distinguished energy sectors like petroleum, gas, aerospace and power stations in order to increase pressure for transferring a fluid. Furthermore compressors are central components in heating, ventilation and air conditioning systems for houses and commercial buildings (Cohen, Rogers, & Saravanamuttoo, 1996; Gravdahl & Egeland, 1999; Whitfield & Baines, 1990).

Because of importance of these equipment, industrial compression systems are expensive devices. Among them, compressors are critical part of the entire system in which they operate. Therefore, the maintenance of this kind of systems becomes more and more important. Compressors have the ability to show instability under different operating conditions. A review of instabilities found in compression system is given by Greitzer (Greitzer, 1981). Surge is one of the main dynamic instability which limits the operation of centrifugal compressors and it is an axisymmetrical oscillation of flow through the compressor which is characterized by a limit cycle in the compressor characteristic. This phenomenon forces the flow back toward the compressors inlet and initiates the surge limit cycle which affects the entire system. As a result of high vibration and thermal loads, surge can cause extensive structural damage to the machine (Pampreen, 1993).

The required pressure rise is seldom realized with a single compressor so most industrial compressing systems contain multiple compressors in series, resulting in complicated mechanical and aerodynamic designs. In these cases, multiple compressors need to operate in series or parallel to realize the required pressure rise or flow, and therefore, potentially there will be a dynamic interaction in the systems that should be considered.

The first step in studying of the mentioned effects over surge instability is preparing the accurate model of a compressor system. Any element in such sophisticated system whether it is a plenum, pipeline, or a throttle valve, can influence to connected compressor. We can find many models developed over the years to describe the dynamic of the flow in references Gravdahl & Egeland (1999) and Longley (1994); for instance, the lumped parameter model introduced in Greitzer (1976a, 1976b) for axial compressors is referenced frequently and later it has shown that it was applicable to centrifugal compressors by Hansen et al. (Hansen, Jørgensen, & Larsen, 1981).

Furthermore, as mentioned in Van Helvoirt & Jager (2007), the Greitzer model is not adequate to predict the dynamics related to fluid flow in distributed systems, such as acoustic waves and flow pulsations in pipelines. Based on the findings of Brun and Spark (Brun & Kurz, 2013; Sparks, 1983), the entire piping system connected to the compressor can reinforce and speed up surge occurrence and limit the stability range of compressor. Therefore, a careful evaluation of acoustic and impedance effects over compression system should be performed to avoid operating range impacting of machine and to design the surge control system properly. The mathematical model for the compression system that captures the effect of the piping acoustics during both the stable and unstable operating conditions, along with the dynamics during the transition between these two states is explained in Yoon, Lin, Goyne, & Allaire (2011).
During the past two decades, many researches have been carried out on the development of active and passive methods for centrifugal compressor surge control. Passive methods are widely used in order to avoid surge in systems which have got good reliability but sacrifice the efficiency and operating range (Gravdahl & Egeland, 1999; Willems & De Jager, 1999). Almost, in all cases we need compression system dynamic model to develop an active surge controller. Not only active surge control systems don’t limit the operating range of the compressor but also stabilize surge itself by feedback control under several types of actuators as mentioned in Bartolini, Muntoni, Pisano, & Usai (2008); Gravdahl, Egeland, & Vatland (2002); Uddin & Gravdahl (2011); and Willems & De Jager (1998).

Although some important results have been achieved by employing advanced algorithms like LQR, the Lyapunov method, and sliding mode variable structure control, but none of these controllers did not consider the effects of pipe and dynamic interrelationship between different units which are negative points for stability of active controller. Optimization of control signal as well as restrictions on actuators and states are also important subjects which should be considered in the design of controller. Also as a result of disturbance in practical applications, the disturbance effects should be considered in design of the compressor system surge controller.

To solve these problems with respect to increasing approaches to design predictive active controllers (Bagha & Modak, 2017; Camacho & Bordons, 2015; Jones, Cortinovis, Mercangoez, & Ferreau, 2017; Malekshahi, Mirzoei, & Aghasizade, 2015, Rakhsan, Moula, Shabani-Nia, Safarinejadian, & Khorsheid, 2016, Stewart, Venkat, Rawlings, Wright, & Pannocchia et al., 2010), a new decentralized model predictive control (MPC) has been investigated for a special class of nonlinear discrete system with the bounded disturbance. The effects of subsystems are also considered in the large-scale system. A new decentralized predictive controller uses tube-MPC methods. $H_{\infty}$ method has also been used to find the appropriate yield in tube-MPC for avoiding the disturbances and effects of other subsystems in order to assure the system stability and finally a robust invariant set against other subsystems disturbances and effects is presented.

Therefore, this article is classified as follows. In the second part, the multiple-compressor system dynamic will be presented with respect to the effects of pipeline. In Section 3, a novel synthesis approach for robust decentralized MPC is proposed. In Section 4, the simulation results are presented. And finally the conclusions are drawn in Section 5.

2. System dynamics

The research focuses on multiple-compressor compression systems. In typical systems in the Gas processing in Iran, fluid is ducted to compression system to perform the required pressure increase. The compression system uses a bank of constant speed compressors to deliver the high pressure gas directly to transmission areas. Figure 1 is a schematic of an example system with three compressors that contains two parallel and one serial compressor with the same characteristics. In the compressing system, recycle valve is intended as an actuator for each compressor according to Uddin & Gravdahl (2015). Fluid into two parallel compressors is strengthened and then is sent to the third compressor. For greater certainty, Spill Back is anticipated as a safety valve for the system that causes interaction between compressors dynamics.

The block illustration for each compressor with pipeline dynamics is presented in Figure 2. The dynamics of each compressor are given in reference Imani, Jahed-Motlagh, Salahshoor, Ramezani, & MoarefiFar (2017). By attention to the characteristics of the Spill Back valve, the relation between the Spill Back flow ($\Phi_{SB}$) and third compressor throttle pressure ($\Psi_{th3}$) in the compressor system of Figure 1 is as follows

$$\Phi_{SB} = \gamma_{SB} \sqrt{\Psi_{th3}}$$  \hspace{1cm} (1)

where, $\gamma_{SB}$ is also the valve’s yield.
The resultant nonlinear system has the compressor mass flow ($\Phi_c$), the plenum pressure rise ($\Psi_p$), the throttle section pressure rise ($\Psi_{th}$), and the plenum mass flow rate ($\Phi_p$) as state variables. According to Figure 1, the state space equations for the two parallel compressors and one serial compressor are as follows:

$$\begin{bmatrix} 
\dot{\Phi}_c \\
\dot{\Psi}_p \\
\dot{\Psi}_{th} \\
\dot{\Phi}_p 
\end{bmatrix}_i = 
\begin{bmatrix} 
0 & -B_{oH} & 0 & 0 \\
\frac{m_o}{\pi} & 0 & 0 & -\frac{m_0}{\pi} \\
0 & 0 & 0 & \frac{2A_{21}A_{22}}{\rho \mu} \\
0 & \frac{B_{12}U}{\rho \mu} & -A_{21}U & A_{22}
\end{bmatrix}_i 
\begin{bmatrix} 
\Phi_c \\
\Psi_p \\
\Psi_{th} \\
\Phi_p 
\end{bmatrix}_i + 
\begin{bmatrix} 
B_{oH}(\Psi_c, ss) \\
0 \\
0 \\
0 
\end{bmatrix}_i 
+ 
\begin{bmatrix} 
0 \\
0 \\
0 \\
0 
\end{bmatrix}_i \frac{\rho P_0}{\rho P_1 U c} (A_{21} + B_{21})_i 
+ 
\begin{bmatrix} 
0 \\
0 \\
0 \\
0 
\end{bmatrix}_i g_i(x)$$

(2)

where $i = 1, 2, 3$ and $g_1 = \begin{bmatrix} 0 & \Phi_{SB} \\ 0 & 0 \\ \Phi_{SB} & 0 \\ 0 & 0 \end{bmatrix}$, $g_2 = \begin{bmatrix} 0 & \Phi_{SB} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $g_3 = \begin{bmatrix} 0 & \Phi_{SB} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$. Details on the calculation of the coefficient matrices $A_{ij}$ and $B_{ij}$ can be found in (Yoon, Lin, & Allaire, 2013). An Euler approximation of system (2) with sampling time $T_c$ is given by
The system is subject to constraints
$$\omega_f(x_i(k)) + B_{w(i)} w_i(k) + B_r u_i(k) + g_i(x(k))$$

which is a subset of nonlinear discrete systems in the form
$$x_i(k + 1) = A_i x_i(k) + f_i(x_i(k)) + B_{w(i)} w_i(k) + B_r u_i(k) + g_i(x(k))$$

3. Robust decentralized MPC

Consider a process plant $\Sigma$ consisting of $h$ interconnected nonlinear systems, each denoted as $s_i$, $i = 1, \ldots, h$ in the discrete time state space

$$s_i : x_i(k + 1) = A_i x_i(k) + f_i(x_i(k)) + B_{w(i)} w_i(k) + B_r u_i(k) + g_i(x(k))$$

where $x_i(k) \in R^{n_i}$ is the state of the system, $u_i(k) \in R^{r_i}$ is the control input. The signal $w_i(k) \in R^{w_i}$ is the exogenous disturbance or uncertainty, which is unknown but bounded, and lies in a compact set:

$$D_i = \{ w_i \in R^{w_i} \mid w_i \leq W_{\max(i)} \}$$

i.e., $w_i(k) \in D_i$, for all $k \geq 0$. The system is subject to constraints

$$x_i(k) \in X_i, \quad u_i(k) \in U_i \forall k > 0$$

The $A_i$, $B_i$, $B_{w(i)}$ matrices are known and constant. $f_i(x) : R^{n_i} \rightarrow R^{n_i}$ is continuous nonlinear function over the $x$ which satisfy Lipschitz condition.

$$f_i(x_1) - f_i(x_2) \leq L_i x_1 - x_2 \quad \forall x_1, x_2 \in X_i$$

$L_i$ are diagonal matrices with positive constant known numbers and $X_i$ is a convex, closed subset of $R^{n_i}$, containing the origin in its interior.

Also, $g_i(x(k)) : R^{n_i} \rightarrow R^{w_i}$ is interactive-bounded nonlinear function. By defining a nominal system as

$$\tilde{x}_i(k + 1) = A_i \tilde{x}_i(k) + f_i(\tilde{x}_i(k)) + B_r \tilde{u}_i(k)$$

The optimization problem which is solved online is formulated as follows:

$$\text{minimize} \quad J_i(k)$$

$$\bar{u}_i(k + j \mid k) j = 0, 1, 2, \ldots, N$$

s.t

$$x_i(k + 1) = A_i x_i(k) + f_i(x_i(k)) + B_r \bar{u}_i(k)$$

$$x_i(k) \in X_i$$

$$\bar{u}_i(k) \in U_i$$

and the nominal cost function is defined as:

$$J_i(k) = \sum_{j=0}^{N} (\tilde{x}_i(k + j \mid k) \tilde{Q}_i \tilde{x}_i(k + j \mid k) + \tilde{u}_i(k + j \mid k) \tilde{R}_i \tilde{u}_i(k + j \mid k))$$
Now, by selecting the
\[ u_i(k) = \hat{u}_i(k) + v_i(k) \] (12)

This controller has two components: a reference trajectory which shows the nominal trajectory of system (5) when there is not any interaction between subsystems or unknown disturbance in the dynamic model of compression system and an ancillary controller which maintains the state of the disturbed interconnected system, real model of compression system, close to the trajectory of the nominal system.

By defining error
\[ e_i(k) = x_i(k) - \bar{x}_i(k) \] (13)

The dynamics of the error can be achieved as follows:
\[ e_i(k + 1) = A_i e_i(k) + f_i(x_i(k)) - f_i(\bar{x}_i(k)) + B_{wi} w_i(k) + B_i v_i(k) + g_i(x(k)) \] (14)

Describing
\[ h_i(x_i, \bar{x}_i, e_i) = A_i e_i(k) + f_i(x_i(k)) - f_i(\bar{x}_i(k)) \] (15)

Finally, we have
\[ e_i(k + 1) = h_i(x_i, \bar{x}, e_i) + N_i \theta_i(k) + B_i v_i(k) \] (16)

where \( N_i = [B_{wi}]_{n_w \times n_w} \begin{bmatrix} I_{n_x \times 1} \end{bmatrix} \) and \( \theta_i(k) = \begin{bmatrix} w_i(k)_{n_w \times n_w} \\ g_i(x(k))_{1 \times n_x} \end{bmatrix} \). \( \theta_i \) is uncertainty matrix, \( N_i \) its coefficient and \( I \) is defined as \( I = [1 \ldots 1]_{n_x \times 1} \).

**Theorem.**

The control law \( v_i(k) \) can be obtained by solving \( H_{\infty} \) control problem for the system (16). To do this, a \((n_w + 1) \times (n_w + 1)\) square matrix, \( P_i \) and a symmetric matrix \( M_i \) must be defined.

\[ M_i = \begin{bmatrix} M_{11}(i) & M_{12}(i) \\ M_{21}(i) & M_{22}(i) \end{bmatrix} \]

\[ M_{11}(i) = B_i^T P_i B_i + R_i \]

\[ M_{12}(i) = M_{21}(i)^T = B_i^T P_i N_i \]

\[ M_{22}(i) = N_i^T P_i N_i - \gamma_i^2 I \] (17)

where \( \gamma_i \) is the designing parameter. The quadratic function is also defined as
\[ V_i(e) = e_i^T P_i e_i \] (18)

According to Magni, De Nicolao, Scattolini, & Allgower (2003), supposing there is a positive definite matrix \( P_i \) such that

(i) \( M_{11}(i) > 0 \), \( M_{22}(i) \) is semi negative definite.

(ii) \(-P_i + (A_i + L_i) P_i (A_i + L_i)^T + Q_i - (A_i + L_i)^T P_i B_i N_i M_i^{-1} [B_i \quad N_i] P_i (A_i + L_i) < 0 \)

Then, the control law \( v_i(k) \) becomes
\[ \begin{bmatrix} \kappa_i(x_i, \bar{x}_i, e_i) \\ \xi_i(x_i, \bar{x}_i, e_i) \end{bmatrix} = -M_i^{-1} [B_i \quad N_i] P_i h_i(x, \bar{x}, e) \] (19)
\( \gamma_i(x_i, \tilde{x}_i, e_i), \xi(x_i, \tilde{x}_i, e_i) \) are auxiliary variables that depend on the difference between the nominal value and the actual value. With \( \Omega_{c(i)} = \{ e_i : e_i^T P_i e_i \leq \alpha_i \} \) (20)

where \( \alpha_i \) is a finite positive constant. The matrix \( P_i \) can also be derived by solving a discrete time \( H_\infty \) algebraic Riccati equation as follows

\[
P_i = A_i (A_i + L_i) - \frac{P_i (A_i + L_i)^T}{|N|} |M_i|^{-1} [B_i N_i (A_i + L_i)^T P_i A_i + Q_i - A_i (A_i + L_i)] N_i |M_i|^{-1} [B_i N_i (A_i + L_i)^T P_i A_i + Q_i - A_i (A_i + L_i)] P_i A_i \]

(21)

**Proof.**

To demonstrate the stability of the system, in spite of the actions of the auxiliary controller and disturbances, we have first assumed that

\[
H_i(x_i, \tilde{x}_i, e_i, u_i, \theta_i) = V_i(e_i(k + 1)) - V_i(e_i(k)) = V_i(h_i + B_i v_i + N_i \theta_i) - V_i(e_i(k)) \leq \left( e_i^T Q_i e_i + \gamma_i^2 e_i^T \theta_i^2 \right) \]

(22)

\[
(h_i + B_i v_i + N_i \theta_i) - e_i^T P_i e_i + e_i^T Q_i e_i + \gamma_i^2 \theta_i^2 \theta_i \leq 0
\]

(23)

\[
h_i^T P_i e_i + e_i^T Q_i e_i + v_i^T \left( B_i P_i B_i + R_i \right) v_i \leq 2 \left( \ell_i \eta_i \right) \left( B_i P_i h_i \right) + 2 \ell_i \left( \eta_i \right) P_i h_i \leq 0
\]

(24)

\[
h_i^T P_i e_i + e_i^T Q_i e_i + \left[ v_i^T \theta_i \right] \left[ M_i \left[ \eta_i \theta_i \right] + \left[ v_i \theta_i \right] \left[ B_i P_i h_i \right] \right] < 0
\]

(25)

And computing (25) for (19)

\[
H_i(x_i, \tilde{x}_i, e_i, x_i(x_i, \tilde{x}_i, e_i), \xi(x_i, \tilde{x}_i, e_i)) = h_i^T P_i e_i + e_i^T Q_i e_i + v_i^T (B_i P_i B_i + R_i) v_i + \ell_i^2 \eta_i (N_i P_i N_i - \gamma_i^2) \theta_i + 2 \left[ v_i \theta_i \right] \left[ B_i P_i h_i \right] + 2 \ell_i \left[ v_i \theta_i \right] P_i h_i \leq 0
\]

(27)

According to (15)

\[
(A_i e_i(k) + f_i(x_i(k)) - f_i(\tilde{x}_i(k))) P_i (A_i e_i(k) + f_i(x_i(k)) - f_i(\tilde{x}_i(k)))
\]

\[
+ e_i^T (Q_i - P_i) e_i - (A_i e_i(k) + f_i(x_i(k)) - f_i(\tilde{x}_i(k))) P_i |B_i| (N_i |M_i|^{-1} [B_i N_i] P_i A_i)
\]

(28)

According to Lipschitz conditions (8)

\[
e_i^T \left( (A_i + L_i) P_i (A_i + L_i) + Q_i - P_i - (A_i + L_i) P_i |B_i| (N_i |M_i|^{-1} [B_i N_i] P_i (A_i + L_i)) \right) e_i \leq 0
\]

(29)

There are positive constants \( \varepsilon, \quad r_{1(i)} \) are followed from (ii) such that

\[
H_i(x_i, \tilde{x}_i, e_i, x_i(x_i, \tilde{x}_i, e_i), \xi(x_i, \tilde{x}_i, e_i)) \leq -\varepsilon e_i^2 \forall e_i \in \Omega_{c(i)} = \{ e_i : e_i^T \leq r_{1(i)} \}
\]

(30)

By Taylor expansion theorem
Apply the control $u_k$ during the sampling interval $k$. 

$$H_j(x_i, x_i, e_i, u_i, \theta_i) = H_i(x_i, x_i, e_i, \kappa_i(x_i, x_i, e_i), \zeta_i(x_i, x_i, e_i))$$
$$+ \frac{1}{2} [v_i - \kappa_i(x_i, x_i, e_i) \theta_i - \zeta_i(x_i, x_i, e_i)] R [v_i - \kappa_i(x_i, x_i, e_i) \theta_i - \zeta_i(x_i, x_i, e_i)]$$

(31)

where the first order term is evaluated and terms of order >2 are null. If the system is controlled by $v_i(k) = \kappa_i(x_i, x_i, e_i)$ then

$$H_j(x_i, x_i, e_i, u_i, \theta_i) = H_i(x_i, x_i, e_i, \kappa_i(x_i, x_i, e_i), \zeta_i(x_i, x_i, e_i))$$
$$+ \frac{1}{2} (\theta_i - \zeta_i(x_i, x_i, e_i)) M_{22(i)} (\theta_i - \zeta_i(x_i, x_i, e_i))$$

(32)

Because $M_{22(i)}$ is semi negative definite and (30)

$$H_j(x_i, x_i, e_i, \kappa_i(x_i, x_i, e_i), \theta_i) \leq H_i(x_i, x_i, e_i, \kappa_i(x_i, x_i, e_i), \zeta_i(x_i, x_i, e_i))$$
$$\leq -\epsilon e_i^2 \forall e_i \in \Omega_{2(i)} = \{ e_i : e_i \leq \epsilon (2(i)) \}$$

all $w_i \in D_i$

(33)

From (22) and (33) follow that

$$V_i (h_i + B_i v_i + N_i \theta_i) \leq V_i (e_i) - (e_i^T Q_i e_i + v_i^T R_i v_i - \gamma_i^2 \theta_i^2) - \epsilon e_i^2$$
$$\leq V_i (e_i) - \epsilon e_i^2$$

$\forall e_i \in \Omega_{2(i)} = \{ e_i : V_i (e_i) \leq \alpha_i \} \subset \Omega_{2(i)}$

all $w_i \in D_i$

(34)

Hence $\Omega_{2(i)}$ is an invariant set. This equation is fixed for each subsystem; therefore, for all subsystems of the overall system, the following equation will be established

$$\sum_{i=1}^{n} (V_i (h_i + B_i v_i + N_i \theta_i) \leq V_i (e_i) - (e_i^T Q_i e_i + v_i^T R_i v_i - \gamma_i^2 \theta_i^2) - \epsilon e_i^2 \leq V_i (e_i) - \epsilon e_i^2)$$

(35)

Finally, the following controlling algorithm is employed to stabilize the overall compressing system.

Algorithm 1

Step 0. At time $k = 0$, set $X_i(0) = X_i(0)$ in which $X_i(0)$ is the current state.

Step 1. At time $k$ and current states $(X_i(k), X_i(k))$, solve problem (30) to obtain the actual control $u_i(k) = u_i(k) + v_i(k)$.

Step 2. Apply the control $u_i(k)$ to the system (5), during the sampling interval $[k, k + 1]$.

Step 3. Measure the states $X_i(k + 1)$ at the next time instant $k + 1$ of the system (5) and compute the successor states $\bar{X}_i(k + 1)$ of the nominal system (9) under the nominal control $\bar{u}_i(k)$.

Step 4. Set $(\bar{X}_i(k), X_i(k)) = (\bar{X}_i(k + 1), X_i(k + 1))$, $k = k + 1$ and go to step 1.

4. Simulation results

In this section, to illustrate the effectiveness and performance of the proposed decentralized tube-based MPC in stabilizing the compressor, the control law of Algorithm 1 is applied to the model (5) of compressor system. The control objective is to avoid surge, i.e. stabilizing the system.

A 10% sinusoidal variation in mean flow is considered as disturbance. This flow fluctuation simulates the most horrible type of flow unsteadiness observed from vortex shedding or other periodic flow excitation (Brun & Kurs, 2013).
\[ d_{\phi_{ci}}(t) = (0.1 \sin(0.1t) + 0.1 \cos(0.4t)) \cdot \dot{\phi} \]

\[ d_{\phi_{pi}}(t) = (0.1 \sin(0.1t) + 0.1 \cos(0.4t)) \cdot \dot{\phi} \]

(36)

where \( \dot{\phi} \) represents the average flow in the system. Next, the existing constraints in the compressor systems should be incorporated in the optimization problem (10). According to control signal from recycle valve actuator, the control signal constraint is

\[ \bar{u}_j(k) \in U \triangleq \{u \mid |u(k)| \leq 0.2\}, \ i = 1, 2, 3 \]  \hspace{1cm} (37)

The next constraint is that the flow has some maximum and minimum values. This constraint should also be considered.

\[ \phi_{mi} \leq \phi_i(t) \leq \phi_{Choke}, \ i = 1, 2, 3 \]  \hspace{1cm} (38)

The cost function is defined by (11) with \( Q = I, R = 0.01 \). To demonstrate the various features of the proposed surge control method, a following scenario is performed on the compressing system with disturbance, change in throttle opening percentage and change in Spill Back valve gain. Compressors initially operate in steady state where the throttle valve openings equals to 20%. Spill Back valve gain is 0.1 and there is not disturbance in the system. At time \( t = 1s \), the throttle is closed to 10% that makes the compressors experience surge. Applying the disturbance at time \( t = 5s \) will be exacerbated the surge effects. At time \( t = 9s \), Spill Back valve gain is increased to 0.2 as the most difficult conditions that will cause changes in amplitude of surge. If the controller will not be activated, the compressors are interred into surge as shown in the Figures(3–5).

We should note that each of these changes, closing the throttle valves, applying the disturbance and variation in Spill Back valve gain, will be able to bring out the compressors from normal operating condition and enter them to the surge cycle, but as the worst case, all these changes have consecutively entered into the compression system. The states and control signal for compression system using proposed method are shown in Figure 6–11. Also, it is compared with nonlinear model predictive control (NMPC) to demonstrate the effectiveness of the proposed controller. As it can be seen, Figures (6–8) illustrate the robustness of the proposed controller for steering the each centrifugal compressor against change in throttle opening percentage, disturbance and change in Spill Back valve gain as the coupling effects of subsystems.

It should be noticed that by the control signal constraint, NMPC is unable to control the compressor system, but the results can be compared for the two controllers by reduction in control signal limitation.

**Figure 3.** States of compressor 1 when surge controller is inactivated.
Figure 4. States of compressor 2 when surge controller is inactivated.

Figure 5. States of compressor 3 when surge controller is inactivated.

Figure 6. States of compressor 1 when surge controller is activated.
Figure 7. States of compressor 2 when surge controller is activated.

Figure 8. States of compressor 3 when surge controller is activated.

Figure 9. Control signal of compressor 1.
As shown in Figures (6–8), using the proposed controller, the compressors operate at low flow rates and higher pressures away from the surge conditions. Also, according to Figures (9–11), the control signals have lower amplitude than NMPC controller, which indicate better optimization in the proposed controller. Also, Limited States oscillations should be considered in the proposed controller. We should notice, as long as the severe changes to the system will continue, regardless of the control signal, the oscillations will continue in steady state.

Simulation results show controller capability against disturbance and comprehending the effects of pipe, throttle and the whole of compressor system as mentioned in the scenario. Using of a decentralized tube-based predictive control and taking into account nonlinear dynamics of pipe and using the recycle valve as control actuator, we'll be able to stabilize the compressor system under the most difficult conditions to prevent from the surge occurrence and also enlarge the operational range of compressor.

5. Conclusion
This article investigates an approach for controlling interconnected large-scale compressor system surge instability using the robust decentralized MPC. The algorithm is developed for interconnected large-scale discrete nonlinear systems with constraints. The proposed decentralized MPC scheme is tube-based and uses $H_{\infty}$ control approach for solving an ancillary MPC problem, which serves to contain the trajectories of the actual system in a tube around the nominal trajectory. The scheme
is proven to guarantee robust stability satisfaction under the assumption of bounded disturbance and interactive functions. Even though the auxiliary controller is a robust one, it does not completely eliminate the effect of the disturbance. Also, the H infinity control signal is not a priori bounded and thus realistic input constraints cannot be enforced. The simulation results demonstrate that the closed-loop system is robust against the disturbances, the change in interactive functions and whole of compressor system.

Funding
The authors received no direct funding for this research.

Author details
Hashem Imani
E-mail: imani.hashem@gmail.com
Mohammad Reza Jahed-Motlagh
E-mail: jahedm@ust.ac.ir
Karim Salahshoor
E-mail: salahshoor@put.ac.ir
Amin Ramezani
E-mail: ramezani@modares.ac.ir
Ali Morefanipour
E-mail: morefanipour@srbiu.ac.ir

1 Department of Electrical Engineering, Science and Research branch, Islamic Azad University, Tehran, Iran.
2 School of Electrical Engineering, University of Science and Technology, Tehran, Iran.
3 Department of Automation and Instrumentation, Petroleum University of Technology (PUT), Tehran, Iran.
4 Department of Electrical Engineering, Tarbiat Modares University, Tehran, Iran.

Citation information
Cite this article as: Robust decentralized model predictive control approach for a multi-compressor system surge instability including piping acoustic, Hashem Imani, Mohammad Reza Jahed-Motlagh, Karim Salahshoor, Amin Ramezani & Ali Morefanipour, Cogent Engineering (2018), 5:1483811.

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