EFFECT OF STRONG CORRELATION ON THE ELECTRON-PHONON INTERACTION

LILIA BOERI
Dipartimento di Fisica, Università “La Sapienza”, P.le Aldo Moro 2
Roma, 00185, Italy
and INFM, Unità Roma1

EMMANUELE CAPPELLUTI
Dipartimento di Fisica, Università “La Sapienza”, P.le Aldo Moro 2
Roma, 00185, Italy
and INFM, Unità Roma1

CLAUDIO GRIMALDI
École Polytechnique Fédérale, Département de microtechnique IPM
Lausanne, CH-1015, Switzerland
and

LUCIANO PIETRONERO
Dipartimento di Fisica, Università “La Sapienza”, P.le Aldo Moro 2
Roma, 00185, Italy
and INFM, Unità Roma1

High-$T_c$ superconductors are usually described as strongly correlated electronic systems. This feature deeply affects the one-particle and two-particle properties of the system. In particular, a large incoherent background develops on the top of a narrow quasi-particle peak in the one-electron spectral function. We schematize this structure with a simple phenomenological form. The corresponding Green’s function is employed to calculate the charge response of the system taking into account in a proper way strong correlation effects. The effective charge interaction acquires a structure in the exchanged momentum space with a predominance of forward scattering, in agreement with previous numerical calculations. The consequences of the momentum dependence of the interaction are discussed in the framework of the nonadiabatic theory of superconductivity proposed for the high-$T_c$ materials.

1. Introduction

High Temperature Superconductors (HTSC) show a highly complex phenomenology, as it is evident from the extremely rich phase diagram; this issue, however, is not surprising since in this class of materials the electronic bands are very narrow. Kinetic energies are accordingly quite small, so that the system is sensitive to any kind of instability which may give rise to visible effects. In this situation, it is clear that, in order to provide an effective description of the properties of HTSC, electronic correlation may not be neglected.
Strongly correlated systems show some common characteristic features. On one hand the spectral weight associated with the coherent part of the one-particle spectral function is reduced with respect to the uncorrelated case and meanwhile a broad incoherent background arises. In addition, the electronic dispersion of the coherent peak is narrowed by correlation effects. As a result, in a correlated system electronic kinetic energies, characterized by the Fermi energies $E_F$, may become comparable with phonon frequencies $\omega_{ph}$, leading to the breakdown of the adiabatic hypothesis (i.e. $\omega_{ph}/E_F \ll 1$) on which conventional superconductivity theory rests.

In addition to the modification of one-particle properties, electronic correlation affects as well the two-particle ones of the system. In particular different analytical approaches point out to a predominance of the forward scattering (small $q$) in the charge density response function. This feature, together with the above mentioned bandwidth reduction, makes Migdal’s theorem unapplicable.

In this paper we investigate the effects of electronic correlation on the electron-phonon coupling. The modulation of the electron-phonon scattering induced by correlation results to be particularly important in the Cooper channel leading to new feature of the superconductivity properties.

2. The Model

Systems of correlated electrons are usually described in terms of a Hubbard Hamiltonian, which can be considered a good paradigmatic model:

$$\hat{H} = \sum_{i,j,\sigma} t_{i,j} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}. \quad (1)$$

The behaviour of the system can easily be determined in the two limit cases $(t/U) \to \infty$ and $(t/U) \to 0$. In the first case the Hubbard Hamiltonian reduces to the free-electron one: the system is a perfect Fermi gas, with energy levels $\varepsilon_k$ and Fermi energy $E_F$. In the second case the most favorable configuration is that in which the electrons are all “localized”, i.e. no hopping between sites is possible. In the intermediate region of parameters one expects a coexistence of the two behaviours.

In terms of the spectral function $A(k,\omega)$ the situation can be described as follows: when $(t/U) = \infty$, $A(k,\omega)$ is a delta-function centered at the energy $\varepsilon_k$, fulfilling the sum rule $\sum_k \int d\omega A(k,\omega)e^{i\omega 0^+} = N$, where $N$ is the total number of particles. As $(t/U)$ decreases, the delta-function, which represents itinerant electrons, will ‘lose weight’, i.e. the coherent spectral weight will be such that $\sum_k \int d\omega A_{co} (k,\omega)e^{i\omega 0^+} = NZ$, with $Z < 1$. The spectral weight which is lost in the coherent part forms an incoherent (i.e. independent of $k$) background of states which represents localized electrons and is such that $\sum_k \int d\omega A_{inc} (k,\omega)e^{i\omega 0^+} = (1-Z)N$. Even though Hubbard model cannot be solved exactly, there are many approximate methods of solution which can be used to determine the dependence of $Z$ on “physical” quantities, such as the doping $\delta$ or $U$. The above description retains its validity.
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regardless of the particular approximation chosen.

In particular, without any loss of generality, the Green’s function of the system can be written as $G = G_{co} + G_{inc}$. Traditional analytical approaches usually focus on either of the two components, according to which properties they want to underline: itinerant properties (mean field slave-bosons, Gutzwiller) or insulating ones (Hubbard I approximation). Interplay between the two components can be recovered at a higher order level of approximation.

In order to deal with this problems we have recently introduced an approximation based on a modified mean field solution within the slave boson technique, which allows us to treat both components on the same footing.

The Green’s function can be expressed in a particularly simple form:

$$G(k, \omega) = Z \tilde{G}(k, \omega) + (1 - Z) \sum_k \tilde{G}(k, \omega),$$

where $\tilde{G}$ is the standard mean-field slave boson solution describing only the coherent part. It is clear that $Z$ gives a parameter to estimate the degree of correlation of the system ($Z = 1$ uncorrelated, $Z = 0$ maximally correlated).

The above result can be quite easily proved by splitting the generic Green’s function in a local and non-local part:

$$G(i, j; t) = -i \langle T \hat{c}_i(t) \hat{c}_j^\dagger \rangle [1 - \delta_{i,j}] - i \langle T \hat{c}_i(t) \hat{c}_j^\dagger \rangle \delta_{i,j}$$

In the limit of infinite $U$ the slave bosons technique decomposes the real electron into two operators:

$$c_{i,\sigma} = b_i^\dagger f_i,\sigma.$$

The standard mean-field approximation [$b_i(t) \to b \equiv \langle b_i(t) \rangle$] is now applied only to the non-local part of the Green’s function, while in the local part the no-double occupancy constraint can be exactly implemented by the relation

$$b_i b_i^\dagger f_i^\dagger(t) f_i = f_i^\dagger(t) f_i.$$

After some simple algebra, we obtain therefore:

$$G(i, j; t) = -ib^2 \langle T \hat{f}_i,\sigma(t) \hat{f}_j^\dagger,\sigma(t) \rangle - i[1 - b^2] \langle T \hat{f}_i,\sigma(t) \hat{f}_i^\dagger,\sigma(t) \rangle \delta_{i,j},$$

which, written in $k$, gives eq. (3) with $Z = b^2$.

It can be shown that the retarded Green’s function of this form preserves the total spectral weight ($\sum_k \int d\omega \text{Im}G_R(k, \omega) = N$). As a consequence Luttinger’s theorem is naturally fulfilled, contrary to what happens with other approximations. This function can then be used as a basic ingredient to build up a diagramatic theory.

In this context we focus on the study of how charge-density response (namely electron-phonon interaction) is affected by electronic correlation.
3. Electron-phonon interaction: predominance of forward scattering

We wish to study how the effective electron-phonon coupling function $g^2(q, \omega)$, is affected by electronic correlation. This issue has already been addressed with different analytical techniques focusing on charge fluctuations around mean-field solutions. The results are in substantial agreement and show that electronic correlation induces a structure of the electron-phonon coupling which is strongly peaked around $q = 0$ leading to a predominance of forward scattering. This structure can be thought of as resulting by the poorer screening provided by the correlated electron system with respect to a normal metal, where highly itinerant electrons effectively screen out any charge modulation.

In this work we introduce an alternative and more intuitive way to describe correlation effects in an electron-phonon system based on the use of the Green’s function in eq. (2).

Let us consider the screening of the bare electron-phonon interaction $g_0$ by the electron-electron scattering. According the standard RPA approximation, the renormalized $g^2(q)$ can be written as:

$$g^2(q, \omega) = \frac{g_0^2(q, \omega)}{1 - V(q)\Pi(q, \omega)},$$

where $V(q)$ is the Coulomb repulsive potential between electrons and $\Pi(q, \omega)$ is the charge density “bubble” of the correlated system:

$$\Pi(q, \omega) = \sum_k \int_{-\infty}^{+\infty} \left( \frac{d\omega'}{2\pi} \right) G(k, \omega')G(k + q, \omega + \omega').$$

In a normal metal $\Pi(0) = -N(E_F)$, where $N(E_F)$ is the density of states at the Fermi level, indicating that all the electrons at the Fermi surface contribute to the screening. The total response can be express in term of the “Thomas-Fermi” cut-off $k_{TF}$, which is usually larger than the Brillouin zone, and the resulting screened electron-phonon interaction is essentially $q$-independent.

Things are different in a correlated system. By using the expression of Green’s function in eq. (2), $\Pi(q, \omega)$ can be rewritten in a coherent and incoherent part:

$$\Pi(q, \omega) = Z\Pi_{co}(q, \omega) + \frac{(1 - Z^2)}{Z}\Pi_{inc}(\omega),$$

where, just as for $G$, $\Pi_{inc}(\omega) = \sum_q \Pi_{co}(q, \omega)$.

The two components contribute in a different way to the screening of the external charge. On a physical ground we expect that the part of the electrons which exhibits itinerant behaviour will be screening the external charge with a characteristic Thomas-Fermi cut-off which scales with $Z$. On the other hand the localized states will provide just a residual dynamical screening.

In this picture, the coherent part, which describes itinerant quasi-particles, can be described as a non-interacting renormalized system, by means of the standard
Thomas-Fermi approximation: $\Pi_{co}(q, \omega) = -N(E_F)$. The factor $Z$ which multiplies $\Pi_{co}(q, \omega)$ is due both to band renormalization and spectral weight reduction. The incoherent part, which describes localized states, exhibits a more complex behaviour as a function of the parameter $Z$ and of the exchanged frequency $\omega$. When $\omega$ is comparable with the effective kinetic energies of the electrons $Z E_F$, namely when the system is strongly correlated ($Z \to 0$), the charge response of the localized states is in counterphase, yielding a negative screening.

The resulting $g^2(q)$ can then be studied as a function of the degree of correlation of the system $Z$; in an intuitive picture $Z$ can be related to the hole-doping $\delta$ of the system through several approximations; in the infinite-$U$ limit it can be shown that $Z \sim \delta$. In Fig. 1 is plotted the renormalized electron-phonon interaction as function of the exchanged momentum $q$ for two different $Z$’s. The Thomas-Fermi constant $k_{TF}$ has been chosen $k_{TF} = 2k_F$. As shown in the figure, $g^2(q)$ presents a sharp peak in the small-$q$ region, which corresponds to forward scattering. A similar structure provides a cut-off $q_c$ for electron-phonon interactions in momentum-space which depends on the “degree of correlation” of the system.

Predominance of forward scattering has important consequences in the context of the electron-phonon theory of superconductivity. In particular, it is clear that in this situation Migdal’s theorem, which relies on the small parameter $(\omega_{ph}/v_F q)$, cannot be justified. Vertex corrections, usually neglected in virtue of Migdals’ theorem, need therefore to be explicitly included. In the past years, the extension of the theory of superconductivity in the so-called “nonadiabatic” regime, where first corrections beyond Migdal’s theorem are relevant, has been largely studied. A major role is played by the momenta structure of the electron-phonon interaction: for small $q$’s the resulting effects of the vertex corrections is mainly positive, leading to an enhancement of the electron-phonon coupling and to an increase of the superconducting critical temperature $T_c$ (see fig. 2).
4. Conclusions

In this work we present a rather simple and compact approach to the problem of electronic correlation in HTSC. It is based on the observation that the main features of strongly correlated electron systems described by Hubbard-type models are the reduction of the spectral weight associated with the itinerant part of the spectral function and the onset of an incoherent background of states. Through the introduction of a phenomenological parameter $Z$ which describes the “degree of correlation” of the system we were able to treat the two parts of the spectral function on the same footing. We applied this model to the computation of the electron-phonon coupling function $g^2(q)$ and found a structure which is strongly peaked in the small-$q$ region and exhibits a strong dependence on $Z$.

The peaked structure of $g^2(q)$ provides an upper cut-off in momentum-space for electron-phonon interactions, which leads to the breakdown of the adiabatic hypothesis in the momentum space ($\omega_{ph}/v_F q$). This points to the necessity of including electron-phonon vertex corrections in Migdal-Eliashberg equations, as predicted by the non-adiabatic theory of superconductivity, which shows that $T_c$’s of HTSC are enhanced by vertex corrections when the momentum $q$ exchanged in electron-phonon interactions is small.

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