Performance analysis of decode and forward cooperative diversity networks with threshold scheme

Shweta Rajoria, Anjana Jain and Prakash D Vyavahare

S. G. S. Institute of Technology and Science, Indore, INDIA

E-mail: shwtrajoria@gmail.com,jain.anjana@gmail.com

Abstract—Cooperative diversity technique is used for achieving high data rate and low BER with low transmit power. The paper proposes a new scheme for selection of first relay having SNR above a predefined threshold level in cooperative diversity network, based on geometric distribution. Closed form expression for outage probability is derived for independent and identically distributed channels with decode and forward protocol. Numerical results show that the proposed scheme not only minimizes resource requirements but also improves the overall outage probability over previously proposed schemes for cooperative diversity network.

1. Introduction

In cooperative diversity networks, cooperation among relay stations help to create spatial diversity in the wireless network even if the individual nodes do not use antenna array for transmission and reception [1], thus improving availability of wireless system. Outage probability analysis for various relaying strategies at high SNR over a Rayleigh fading channel is derived in [2]. The paper proposes, a cooperative network with M relay nodes in which M+1 channels are needed to ensure orthogonal transmission which reduces bandwidth efficiency. To overcome this drawback new cooperative scheme was proposed in [3] that requires C (C< M) relays to forward source information. Further in [4, 5], best relay and Nth best relay selection scheme were proposed in order to improve performance and to utilize channel resources efficiently. In best relay selection scheme and Nth best relay selection scheme there is excess processing delay at each relay to find best (or Nth) best relay. Recently, Al-Tous and Barhumi [6] have proposed a decode and forward cooperative diversity scheme which investigated sharing of two order best relay. This paper proposes a scheme for selection of a relay having SNR above specified threshold level in cooperative diversity network with decode and forward protocol. It is based on geometric distribution to find the first relay having SNR above threshold. The proposed scheme results in reduced processing time and in saving time slots/ channel in the system.

The paper is organized as follows: Section 2 introduces the proposed system model. Section 3 presents the analysis of outage performance of the proposed scheme. Section 4 demonstrates numerical results and section 5 concludes the paper.

2. System model

Source S communicates with destination D directly and with the help of M other cooperating relay nodes via channel with flat Rayleigh fading coefficients as shown in figure 1. The Rayleigh fading
channels from source to node and node to destination are assumed to be independent and identically distributed. Consider a Time Division Multiple Access arrangement with M+1 time slots to ensure orthogonal transmission. In the first time slot, the source node broadcasts its signal to the destination node and to the set of M relay nodes $R_1, R_2, ... R_M$. A minimum SNR threshold level is defined such that any relay node having SNR above threshold has ability to successfully decode the source message.

![Diagram of cooperative diversity network with threshold scheme.](image)

In the second time slot, first relay having SNR above threshold is only allowed to process and forward source information. Here, geometric distribution is used for the selection of first relay having SNR above threshold. Maximum combining ratio technique is used at the destination to combine the direct link signal and the relay signal.

### 3. Outage probability analysis

Mutual information between source and cooperative node $R_i = 1, 2, 3, ... M$ is given by [4]

$$I_i = \frac{1}{2} \log(1 + \gamma \hat{x}_i)$$  \hspace{1cm} (1)

Here, factor $\frac{1}{2}$ indicates that only two time slots are needed to forward source signal instead of M+1 channels/timeslots as in [3]. $\hat{x}_i = |h_{s,i}|^2$ where $|h_{s,i}|$ is channel gain between source S and relay station i, defined as a zero-mean circularly symmetric Gaussian random variable. Therefore, $\hat{x}_i$ is exponentially distributed with parameter $\hat{\lambda}_i$. And $\gamma$ is the average transmit SNR.

According to the scheme, it is assumed that $K^{th}$ number relay is the first relay having instantaneous SNR above threshold. Mutual information between source and destination using threshold scheme is

$$I_{DF} = \frac{1}{2} \log(1 + \gamma (x_0 + \frac{x_K}{\sum_{K=1,2,3,M}}))$$  \hspace{1cm} (2)

The outage probability $P_{out}$ for required transmission rate, $R$ is given by

$$P_{out} = [I_{DF} \leq R] = P \left( x_0 + \frac{x_K}{\sum_{K=1,2,3,M}} \leq (2^{2R} - 1)/\gamma \right)$$  \hspace{1cm} (3)

Where, $x_0 = |h_{s,d}|^2$ and $x_K = |h_{K,d}|^2$ are exponentially distributed with parameter $\lambda_0$ and $\lambda_K$. As the channel is i.i.d, therefore, $\hat{\lambda}_i = \lambda_0 = \lambda_K = \lambda$.

Let Z be an another random vector, such that, $Z = x_0 + Y$, where $Y = x_1, x_2, x_3, ..., x_M$.

Then, equation (3) of outage probability can be rewritten as
\[ P_{\text{out}} = P\left[ Z \leq \frac{(2^R - 1)}{\gamma} \right] \]  

From the above equation it can be visualized that outage probability is CDF of \( Z \) at \( \frac{(2^R - 1)}{\gamma} \) as described below.

PDF of \( Z \)

PDF of \( Z \) is the convolution of the PDF of \( Y \) and PDF of \( x_0 \). For the Rayleigh fading channel, PDF of \( x_0 \) is given by

\[ f_{x_0}(x) = \lambda \exp(-\lambda x) \]

By using total probability theorem and having the knowledge of geometric distribution, PDF of \( Y \) can be written as

\[ f_Y(x) = \sum_{k=1}^{M} \lambda \exp(-\lambda x).p.(1-p)^{K-1} \]

Where, \( p \) is the probability that randomly selected relay has SNR above threshold value, which is given by

\[ p = P[x_k \geq \gamma_{th}] = \exp(-\frac{\lambda \gamma_{th}}{\gamma}) \]

After convolving PDF of \( x_0 \) and PDF of \( Y \), PDF of \( Z \) can be written as:

\[ f_Z(x) = \sum_{k=1}^{M} \lambda^2 x \exp(-\lambda x).p.(1-p)^{K-1} \]

and \( P_{\text{out}} \), which is CDF of \( Z \), is given by

\[ P_{\text{out}} = F_Z(x) = \sum_{k=1}^{M} \lambda^2 \exp(-\frac{\lambda \gamma_{th}}{\gamma}).[1 - \exp(-\frac{\lambda \gamma_{th}}{\gamma})]\{1-(1+\lambda x) \exp(-\lambda x)\}^{K-1} \]

4. Numerical results

In this section numerical evaluation and results for the outage probability of proposed threshold scheme for i.i.d links is being presented. Figure 2 shows results for outage probability as a function of total number of cooperative relay nodes in the system for \( R = 1/2 \text{ bit/sec/Hz} \), \( \gamma_{th} = 1, \lambda = 1 \). It may be noted that outage probability performance improves as number of relays in the system increase.

| Scheme                        | SNR(5 dB) | SNR(10 dB) |
|-------------------------------|-----------|------------|
| Proposed Scheme               | \(10^{-3}\) | \(10^{-4}\) |
| \textit{N}th best relay       | \(10^{-1}\) | \(10^{-2}\) |
| Selection scheme              |           |            |
Furthermore, it is seen that even at the low SNR, proposed scheme gives better results as compared to the previously proposed schemes. Finally, comparison of results of proposed scheme with the results given in [5] is summarized in Table 1. It is seen that for a system with three relays, plot shows negligible outage probability after 15 dB.

![Plot](image)

**Figure 2.** Outage probability performance for $R = 1$ bit/sec/Hz, $\gamma_{th} = 2$, $\lambda = 1$

5. Conclusion

In this paper, decode and forward cooperative diversity network with threshold scheme over Rayleigh fading channels is being presented and analyzed. Closed form expression for the outage probability is determined for relay links with i.i.d Rayleigh distribution. The main advantage of proposed scheme over the recently presented scheme is that it reduces delay, transmit power and resource. It is observed that for a system with two relay nodes, proposed scheme shows ten percent reduction in the outage probability than earlier proposed scheme at the same SNR of 15 dB.

References

[1] Sendronaris, Erkip E and Aazhang B 2003 User cooperation diversity part 1: system description *IEEE Trans on Comm*, vol 51, pp 1927-38.
[2] Laneman N, Tse D N C and Wornell G W 2004 Cooperative diversity in wireless network: efficient protocol and outage behavior *IEEE Trans on Information Theory* vol 50 pp 3062-80.
[3] Beaulieu C and Hu J 2000 A closed-form expression for the outage probability of decode and forward relaying in dissimilar Rayleigh fading channels *IEEE Comm Lett* vol 10 pp 813-15.
[4] Ikki S S and Ahmed M H 2010 Performance analysis of adaptive decode and forward cooperative diversity networks with best relay selection scheme *IEEE Trans on Comm* vol 58 pp 68-72.
[5] Ikki S S and Ahmed M H 2010 On the performance of cooperative diversity networks with the Nth best relay selection scheme *IEEE Trans on Comm* vol 58 pp 3062-69.
[6] Hanan Al-Tous and Imad Barhumi 2012 Performance analysis of relay selection in cooperative networks over Rayleigh flat fading vol 58 pp 2-16.
[7] Simon M K and Alouini M S 2000 *Digital Communication Over Fading Channels* (John Wiley and Sons).
[8] Papoulis A 1991 *Random Variables, and Stochastic Processes* (New York: McGraw-Hill Education).