Yukawa Unification and Neutralino Dark Matter in $SU(4)_c \times SU(2)_L \times SU(2)_R$

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Abstract

We consider a left-right symmetric $SU(4)_c \times SU(2)_L \times SU(2)_R$ (4-2-2) model with gravity mediated supersymmetry breaking. We find that with 4-2-2 compatible non-universal gaugino masses, $t - b - \tau$ Yukawa coupling unification is consistent with neutralino dark matter abundance and with constraints from collider experiments (except $(g - 2)_\mu$). The gluino mass lies close to that of the lightest neutralino, so that the gluino co-annihilation channel plays an important role in determining the neutralino relic abundance. By relaxing the Yukawa unification constraint we find stau and stop masses as low as $200 - 220$ GeV. We highlight some benchmark points for these cases with $40 \leq \tan \beta \leq 58$.

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1 Introduction

As a maximal subgroup of Spin(10) (commonly known as SO(10)), the gauge symmetry $SU(4)_c \times SU(2)_L \times SU(2)_R$ (4-2-2) \cite{1} captures many salient features exhibited by its covering group. Even as a stand alone symmetry group, 4-2-2 implements electric charge quantization, albeit in units of $\pm e/6$, rather than $\pm e/3$ \cite{2, 3}. It explains the standard model quantum numbers of the quark and lepton families by assigning them in bi-fundamental representations and it also predicts the existence of right handed neutrinos \cite{1}. However, there are some important differences between models based on SO(10) and 4-2-2 which, in principle, can be experimentally tested. For instance, in 4-2-2 the lightest magnetic monopole carries two quanta of Dirac magnetic charge \cite{4}. (In SO(10) the lightest monopole carries one quantum of Dirac magnetic charge, unless SO(10) breaks via 4-2-2.) By the same token, 4-2-2 predicts the existence of SU(3) color singlet states carrying electric charges $\pm e/2$ \cite{3, 5}. Finally, gauge boson mediated proton decay is a characteristic feature of SO(10) which is absent in the 4-2-2 framework.

While these different experimental signatures can help distinguish SO(10) from 4-2-2, they mostly rely on physics operating at superheavy scales. A major motivation for this paper is to highlight some important differences in the low energy predictions of supersymmetric SO(10) and 4-2-2 models, stemming from the Higgs and sparticle sectors of these models. An exciting new feature is that these predictions can be sufficiently different so that they can be compared at the LHC.

Supplementing 4-2-2 with a discrete left-right (LR) symmetry \cite{11, 6}(more precisely C-parity) \cite{7} reduces from three to two the number of independent gauge couplings in supersymmetric 4-2-2. In combination with Yukawa unification \cite{8}, this has important implications for low energy Higgs and sparticle spectroscopy which we will explore in this paper, and compare with the corresponding predictions from an SO(10) model.

In 4-2-2 the matter fields are unified into three generations of $\psi$ (4, 2, 1), and the antimatter fields are in three generations of $\psi_c$ (4, 1, 2). If the MSSM electroweak doublets come from the bi-doublet H(1, 2, 2), the third family Yukawa coupling $H\psi_c\psi$ yields the following relation valid at the GUT scale ($M_{GUT}$), namely

$$Y_t = Y_b = Y_\tau = Y_{Dirac}. \quad (1)$$

We will assume that due to C-parity the soft mass terms, induced at $M_{GUT}$ through gravity mediated supersymmetry breaking \cite{9}, are equal in magnitude for the scalar squarks and leptons of the three families. The asymptotic MSSM gaugino masses, on the other hand, can be non-universal from the following consideration. From C-parity, we can expect that the gaugino masses at $M_{GUT}$ associated with $SU(2)_L$ and $SU(2)_R$ are the same. However, the asymptotic $SU(4)_c$ and consequently $SU(3)_c$ gaugino masses can be different. With the hypercharge generator in 4-2-2 given
by \( Y = \sqrt{\frac{2}{5}}(B - L) + \sqrt{\frac{3}{5}}I_{3R} \), where \( B - L \) and \( I_{3R} \) are the diagonal generators of \( SU(4)_c \) and \( SU(2)_R \), we have the following asymptotic relation between the three MSSM gaugino masses:

\[
M_1 = \frac{3}{5}M_2 + \frac{2}{5}M_3.
\]

(2)

The supersymmetric 4-2-2 model with C-parity thus has two independent parameters \((M_2, M_3)\) in the gaugino sector.

In this paper we wish to explore whether Yukawa coupling unification in 4-2-2 is compatible with recent observations of the dark matter relic abundance and other collider-based experimental constraints. A similar analysis for \( SO(10) \), which we closely follow, has been carried out by Baer et al. [10]. Solutions consistent with \( SO(10) \)Yukawa unification have been obtained in [10] only for very special values of the fundamental parameters. Furthermore, it turns out to be quite difficult in this model to reconcile the lightest neutralino primordial abundance with the observed dark matter densities.

By introducing non-universality in the gaugino sector, we can allow the neutralinos in 4-2-2 to be closely degenerate in mass with the gluino, which is not possible in \( SO(10) \). This opens up, in particular, the bino-gluino co-annihilation channel [11], which turns out to be an essential difference between the 4-2-2 and \( SO(10) \) models. In order to make Yukawa coupling unification compatible with radiative electroweak symmetry breaking (REWSB), one needs to implement some splitting in the Higgs sector, with \( m_{H_u}^2 < m_{H_d}^2 \). Such a splitting may be introduced via a \( D \)-term contribution to all scalar masses [12], or it can be generated via GUT scale threshold corrections related to a large Dirac neutrino Yukawa coupling [13]. It has been noted [14] that a splitting just in the Higgs soft terms, as opposed to splitting in all scalar masses, yields better Yukawa unification, and so we focus on this approach. Since one of our goals is a comparison of 4-2-2 and \( SO(10) \) models, we follow the same notation as in [10]. We parameterize the Higgs soft mass splitting by \( m_{H_{u,d}}^2 = m_{10}^2 \mp 2M_D^2 \), where \( m_{10}^2 \) is the MSSM universal Higgs soft mass\(^2 \) term. The supersymmetric 4-2-2 model we are discussing thus has the following fundamental parameters:

\[
m_{16}, m_{10}, M_D, M_2, M_3, A_0, \tan \beta, \text{sign } \mu.
\]

(3)

Thus, compared to the \( SO(10) \) model of [10], we have one additional parameter in 4-2-2 which plays a crucial role in realizing Yukawa unification consistent with the desired neutralino relic density.

The outline for the rest of the paper is as follows. In Section 2 we summarize the scanning procedure and the experimental constraints that we have employed. We present the results from our scan in Section 3, where we compare the 4-2-2 and \( SO(10) \) models and then proceed to highlight some of the predictions of the 4-2-2 model. Our conclusions are summarized in Section 4.
2 Phenomenological constraints and scanning procedure

We employ ISAJET 7.78 package [15] to perform random scans over the parameter space. In this package, the weak scale values of gauge and third generation Yukawa couplings are evolved to $M_{\text{GUT}}$ via the MSSM renormalization group equations (RGEs) in the $\overline{\text{DR}}$ regularization scheme, where $M_{\text{GUT}}$ is defined to be the scale at which $g_1 = g_2$. We do not enforce an exact unification of the strong coupling $g_3 = g_1 = g_2$ at $M_{\text{GUT}}$, since a few percent deviation from unification can be assigned to unknown GUT-scale threshold corrections [16]. At $M_{\text{GUT}}$, the boundary conditions are imposed and all the SSB parameters, along with the gauge and Yukawa couplings, are evolved back to the weak scale $M_Z$. The effect of the neutrino Dirac Yukawa coupling in the running of the RGEs has been shown in [17] to be significant for coupling values $\sim 2$. In the 4-2-2 model with $t - b - \tau$ unification, the asymptotic neutrino Dirac Yukawa coupling has the same value as $y_t(M_{\text{GUT}})$ which is relatively small ($\sim 0.5$). Thus, in the following discussion we will ignore it.

In the evaluation of Yukawa couplings the SUSY threshold corrections [18] are taken into account at the common scale $M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_L}m_{\tilde{t}_R}}$. The entire parameter set is iteratively run between $M_Z$ and $M_{\text{GUT}}$ using the full 2-loop RGEs until a stable solution is obtained. To better account for leading-log corrections, one-loop step-beta functions are adopted for gauge and Yukawa couplings, and the SSB parameters $m_i$ are extracted from RGEs at multiple scales $m_i = m_i(m_i)$. The RGE-improved 1-loop effective potential is minimized at an optimized scale $M_{\text{SUSY}}$, which effectively accounts for the leading 2-loop corrections. Full 1-loop radiative corrections are incorporated for all sparticle masses.

The requirement of radiative electroweak symmetry breaking (REWSB) [19] puts an important theoretical constraint on the parameter space. Another important constraint comes from limits on the cosmological abundance of stable charged particles [20]. This excludes regions in the parameter space where charged SUSY particles, such as $\tilde{\tau}_1$ or $\tilde{t}_1$, become the lightest supersymmetric particle (LSP). We accept only those solutions for which one of the neutralinos is the LSP.

We have performed random scans for the following parameter range:

\[
\begin{align*}
0 \leq & \quad m_{16} \quad \leq 20 \text{ TeV}, \\
0 \leq & \quad M_2 \quad \leq 1 \text{ TeV}, \\
0 \leq & \quad M_3 \quad \leq 1 \text{ TeV}, \\
-3 \leq & \quad A_0/m_{16} \quad \leq 0, \\
0 \leq & \quad M_D/m_{16} \quad \leq 0.95, \\
0 \leq & \quad m_{10}/m_{16} \quad \leq 1.5, \\
40 \leq & \quad \tan \beta \quad \leq 58,
\end{align*}
\]

with $\mu > 0$, and $m_t = 172.6$ GeV [21].
We first collected 150,000 points for both the $SO(10)$ and 4-2-2 models. All of these points satisfy the requirement of REWSB with the neutralino being the LSP in each case. Furthermore, all of these points satisfy the constraint $\Omega_{CDM} h^2 \leq 10$. This is done so as to collect more points with a WMAP compatible value of cold dark matter relic abundance. Once we identify good regions in parameter space, we perform a random scan focused around those regions for the 4-2-2 case. After collecting the data, we use the IsaTools package [22] to implement the following phenomenological constraints:

\[
\begin{align*}
    m_{\tilde{\chi}^\pm} &\geq 103.5 \text{ GeV} \quad [20], \\
    m_h &\geq 114.4 \text{ GeV} \quad [23], \\
    m_{\tilde{\tau}} &\geq 86 \text{ GeV} \quad [20], \\
    m_{\tilde{g}} &\geq 220 \text{ GeV} \quad [20], \\
    BR(B_s \to \mu^+\mu^-) &< 5.8 \times 10^{-8} \quad [24], \\
    2.85 \times 10^{-4} &\leq BR(b \to s\gamma) \leq 4.24 \times 10^{-4} \quad (2\sigma) \quad [25], \\
    \Omega_{CDM} h^2 &\approx 0.111^{+0.028}_{-0.037} \quad (5\sigma) \quad [26], \\
    3.4 \times 10^{-10} &\leq \Delta a_{\mu} \leq 55.6 \times 10^{-10} \quad (3\sigma) \quad [27].
\end{align*}
\]

We apply the experimental constraints successively on the data that we acquire from ISAJET. As a first step we apply the constraints from $BR(B_s \to \mu^+\mu^-)$, $BR(b \to s\gamma)$, the WMAP upper bound on the relic density of cold dark matter, and the (s)particle mass bounds. We then apply the WMAP lower bound on the relic density of dark matter, followed by the constraint on the muon anomalous magnetic moment $a_{\mu} = (g - 2)\mu/2$ at the $3\sigma$ allowed region. The data is then plotted showing the successive application of these constraints.

### 3 Results

Following Baer et al. [10] we introduce a parameter $R$ to quantify Yukawa unification. Namely, $R$ is the ratio,

\[
R = \frac{\max(y_t, y_b, y_\tau)}{\min(y_t, y_b, y_\tau)},
\]

so that $R = 1$ corresponds to perfect unification and a higher value of $R$ signifies a larger deviation from unification.

We next present the results of the random scan. We first compare the $SO(10)$ model with the 4-2-2 model in Figures [1] and [2] following the treatment in [10]. In Figure [1] we plot the results in the $(R, m_{16})$, $(R, \tan \beta)$ and $(\Omega h^2, R)$ planes for $SO(10)$ (left panel) and 4-2-2 (right panel). All of these points satisfy the theoretical requirement of REWSB and correspond to a neutralino LSP.
In addition, these points satisfy the various experimental constraints listed earlier. The light blue points satisfy the constraints from $BR(B_s \to \mu^+\mu^-)$, $BR(b \to s\gamma)$, the Higgs, chargino, gluino and stau mass bounds, and the upper bound on the relic density of dark matter from WMAP. Shown in dark blue are points that also satisfy the lower bound on $\tilde{\chi}_1^0$ dark matter abundance. In Figure 2 we similarly present results in the $(m_{10}/m_{16}, R)$, $(M_D/m_{16}, R)$ and $(A_0/m_{16}, R)$ planes for $SO(10)$ (left) and 4-2-2 (right). It is quite obvious from the results that, as expected, using just a random scan it is quite difficult to realize acceptable Yukawa unification in $SO(10)$ consistent with the experimental constraints. Ref. [10] employs a modified scanning algorithm based on Markov Chain Monte Carlo (MCMC) to search the parameter space more efficiently. It is shown there that they show that only the $h$-resonance (light Higgs) channel is available to bring the neutralino dark matter density in the right (WMAP) ball park. While this channel does yield acceptable Yukawa unification consistent with WMAP, it is more or less ruled out by the lower bound of 114.4 GeV on the the SM Higgs mass.

In the initial sweep of the $SO(10)$ model around 150,000 points were identified, consistent with REWSB and the requirement that is a LSP neutralino. Yukawa unification consistent with the experimental data was found to be no better than 40%, even if we ignore the constraint from $\Delta a_\mu$. The 4-2-2 model yields ‘good’ solutions with Yukawa unification to better than 10%. More concentrated searches around such ‘good’ points have yielded ‘near perfect’ unification. Such concentrated searches were not performed for the $SO(10)$ model as they have already been reported in [10] with the conclusion that a narrow, almost excluded, light Higgs funnel region is the only one that is viable from the point of view of Yukawa unification and dark matter relic density.

We now focus on the 4-2-2 model, which does much better than the $SO(10)$ model in terms of Yukawa unification and most of the experimental constraints, including the WMAP bounds on dark matter abundance. The constraint from $(g - 2)_\mu$ is found to be largely incompatible with Yukawa unification (Yukawa unification is worse than 35% if one insists on $(g - 2)_\mu$). From Figures 1 and 2 we find that the following parameter values are preferred:

$$m_{16} \gtrsim 7 \text{ TeV},$$

$$46 \lesssim \tan \beta \lesssim 48 \text{ and } 50 \lesssim \tan \beta \lesssim 52,$$

$$0.6 \lesssim m_{10}/m_{16} \lesssim 0.8 \text{ and } m_{10}/m_{16} \approx 1.1,$$

$$0.3 \lesssim M_D/m_{16} \lesssim 0.5,$$

$$A_0 \approx -2m_{16} \text{ and } A_0 \approx -2.5m_{16}.$$

In Table 1 we show a few benchmark points that are consistent with Yukawa unification. Point 1 displays the spectrum corresponding to essentially perfect unification ($R = 1.00$). Point 2 gives a ‘light’ gluino ($\sim 265$ GeV) consistent with ‘good’ unification ($\sim 9\%$). Point 3 has the lightest stop ($1911$ GeV), again consistent with respectable Yukawa unification ($\sim 7\%$). Note that most of
the sparticles are rather heavy as a consequence of requiring Yukawa unification. Note that for all three benchmark points the lightest neutralino (LSP) relic abundance is compatible with the WMAP dark matter bounds. This comes about because of the relatively small mass splitting between the neutralino (essentially bino-like) and gluino which leads to efficient co-annihilation [11].

Figure 3 shows plots in the \((M_3,m_{16})\), \((R,M_2/M_3)\), \((M_3,m_{10}/m_{16})\), \((M_3,M_D/m_{16})\), \((M_3,\tan\beta)\) and \((M_3,A_0/m_{16})\) planes for the 4-2-2 model. Color coding is essentially the same as in Figure 1 except that we now also show in red points that are consistent with all experimental constraints (except \((g-2)_\mu\)) and have Yukawa unification better than 10%. It appears that the points with Yukawa unification seem to favor a non-universal gaugino sector, with \(M_2 \gtrsim 10 M_3\). This ratio is higher still if we also require these solutions to satisfy constraints from experiments. This, of course, does not mean that solutions with \(M_2 \approx M_3\) do not exist, as the latter have been reported in [10]. However, this does suggest a statistical preference for solutions with a significant splitting in the gaugino sector.

The PAMELA experiment has reported an excess in the observed positron flux with no corresponding anti-proton excess [28]. It may be possible to explain this ‘excess’ in the context of SUSY with the lightest neutralino as the dark matter candidate. One explanation invokes a neutralino of mass around 300 GeV decaying into positrons via ‘tiny’ \((\sim 10^{-13})\) R-parity violating couplings [29]. This scenario is consistent with Yukawa unification as we can see in Figure 4.

We have stressed that Yukawa unification seems incompatible with the current experimental bound on \((g-2)_\mu\). If we do not insist on Yukawa unification in 4-2-2, we can find a much lighter MSSM spectrum, which is consistent with all experimental constraints (including \((g-2)_\mu\)). This can be seen from Figures 4 and 5. In Figure 4 we show plots in the \((m_\tilde{t},m_{\tilde{\chi}_1^\pm})\), \((m_\tilde{\tau},m_{\tilde{\chi}_1^0})\), \((m_{\tilde{\chi}_1^\pm},m_{\tilde{\chi}_1^0})\) and \((m_{\tilde{\chi}_1},m_{\tilde{\chi}_1^0})\) planes, with the same color coding as in Figure 3. We also show the unit slope line in each plot, thus highlighting the stop co-annihilation region, the stau co-annihilation region, and the mixed bino-wino dark matter region. In Figure 5 we show similar plots in the \((m_H,m_{\tilde{\chi}_1^0})\), \((m_{A},m_{\tilde{\chi}_1^0})\), \((m_{h},m_{\tilde{\chi}_1^0})\) and \((m_{\tilde{e}_R},m_{\tilde{\chi}_1^0})\) planes. We indicate the A-funnel region with the line \(m_A = 2m_{\tilde{\chi}_1^0}\). In Table 2 we present points corresponding to the lightest spectrum found in our investigation (disregarding Yukawa unification, but consistent with all experimental constraints). Points 1 through 5 respectively display the spectrum corresponding to the lightest chargino (133 GeV), CP-odd Higgs (284 GeV), gluino (268 GeV), stau (198 GeV) and stop (226 GeV).

4 Conclusion

The 4-2-2 gauge symmetry, supplemented by left-right symmetry (C-parity) captures many attractive features exhibited by the simplest \(SO(10)\) models. One of these features happens, in some models, to be Yukawa unification. We have shown that by relaxing in 4-2-2 the assumption of universal gaugino
masses, the resulting MSSM models have rather distinctive mass spectra which can be tested at the LHC. Moreover, the primordial abundance of the lightest neutralino in this case is consistent with the WMAP dark matter limits, something which is difficult to achieve in $SO(10)$ with $t-b-\tau$ Yukawa unification. We have also studied the implications of relaxing the Yukawa unification condition and identified several additional benchmark points which also can be explored at the LHC. Finally, we wish to note the recent observation that the little hierarchy problem can be largely resolved in the 4-2-2 framework [30]. The implication of this for sparticle spectroscopy will be discussed elsewhere.

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Figure 1: Plots in the \((m_{16}, R)\), \((\tan \beta, R)\) and \((\Omega h^2, R)\) planes for SO(10) (left panels) and 4-2-2 (right panels). Gray points are consistent with REWSB and \(\tilde{\chi}_1^0\) LSP. Light blue points satisfy the WMAP upper bound on \(\tilde{\chi}_1^0\) abundance and various constraints from colliders \(BR(B_s \to \mu^+\mu^-), BR(b \to s\gamma)\), and (s)particle mass bounds). Dark blue points also satisfy the lower bound on \(\tilde{\chi}_1^0\) density. Green points, additionally, satisfy the constraint from \((g - 2)_\mu\).
Figure 2: Plots in the \((m_{10}/m_{16}, R)\), \((M_D/m_{16}, R)\) and \((A_0/m_{16}, R)\) planes for SO(10) (left) and 4-2-2 (right). Color coding is the same as in Figure 1.
Table 1: Sparticle and Higgs masses in 4-2-2 model (in units of GeV), with $m_t = 172.6$ GeV and $\mu > 0$. Point 1 corresponds to exact Yukawa unification ($R = 1.00$) while points 2 (3) shows the spectrum corresponding to the lightest stop (gluino) with Yukawa unification of 10% or better. Note that in each case gluino co-annihilation plays as essential role.
Figure 3: Plots in the $(M_3, m_{16})$, $(R, M_2/M_3)$, $(M_3, m_{10}/m_{16})$, $(M_3, M_D/m_{16})$, $(M_3, \tan \beta)$ and $(M_3, A_0/m_{16})$ planes for 4-2-2. Gray points are consistent with REWSB and $\tilde{\chi}_1^0$ LSP. Light blue points satisfy the WMAP upper bound on $\tilde{\chi}_1^0$ abundance and various constraints from colliders ($BR(B_s \to \mu^+\mu^-)$, $BR(b \to s\gamma)$, and $(s)$particle mass bounds). Dark blue points also satisfy the lower bound on $\tilde{\chi}_1^0$ primordial abundance. Green points, additionally, satisfy the constraint from from $(g-2)_\mu$. Points in red represent a subset of dark blue ones that is consistent with 10% or better Yukawa unification.
Figure 4: Plots in the $(m_\tilde{t}, m_{\tilde{\chi}^0_1})$, $(m_\tau, m_{\tilde{\chi}^0_1})$, $(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}^0_1})$ and $(m_{\tilde{q}}, m_{\tilde{\chi}^0_1})$ planes for 4-2-2. Color coding same as in Figure 3. Also shown is the unit slope line in each plane.
Figure 5: Plots in the \((m_h, m_{\tilde{\chi}^0_1}), (m_A, m_{\tilde{\chi}^0_1}), (m_{\tilde{b}}, m_{\tilde{\chi}^0_1})\) and \((m_{\tilde{e}_L}, m_{\tilde{\chi}^0_1})\) planes for 4-2-2. Color coding same as in Figure 3. In the \((m_A, m_{\tilde{\chi}^0_1})\) case we also show the line \(m_A = 2m_{\tilde{\chi}^0_1}\).
Table 2: Sparticle and Higgs masses (in units of GeV), with $m_t = 172.6$ GeV and $\mu > 0$. Points 1 through 5 correspond to the lightest chargino, CP-odd Higgs, gluino, stau and stop for the 4-2-2 parameter space given in Eq. (4). Note that these points are not consistent with Yukawa unification ($R > 2.3$), but they satisfy all experimental constraints including the one from $(g - 2)_\mu$.

|        | Point 1 | Point 2 | Point 3 | Point 4 | Point 5 |
|--------|---------|---------|---------|---------|---------|
| $m_{16}$ | 1529.4  | 1038.5  | 1402.5  | 958.4   | 1469.8  |
| $M_2$   | 158.0   | 630.1   | 736.5   | 607.6   | 630.1   |
| $M_3$   | 467.9   | 122.0   | 79.43   | 117.8   | 103.6   |
| $\tan \beta$ | 56.4    | 57.2    | 46.8    | 54.5    | 46.2    |
| $M_{D/m_{16}}$ | 0.2185  | 0.2085  | 0.0721  | 0.1732  | 0.0276  |
| $m_{10}/m_{16}$ | 0.459   | 0.339   | 0.317   | 0.291   | 0.059   |
| $A_{10}/m_{16}$ | -1.485  | -1.976  | -1.434  | -2.063  | -2.45   |
| $m_h$   | 119     | 118     | 117     | 119     | 120     |
| $m_H$   | 940     | 284     | 448     | 297     | 468     |
| $m_A$   | 934     | 284     | 445     | 299     | 472     |
| $m_{H^\pm}$ | 946     | 302     | 458     | 315     | 491     |
| $m_{\tilde{\chi}^+_1}$ | 133,1545| 526,1070| 620,1238| 505,1015| 541,1676|
| $m_{\tilde{\chi}^0_2}$ | 121,132 | 186,526 | 208,620 | 178,504 | 186,539 |
| $m_{\tilde{\chi}^{0,\pm}_{1,2}}$ | 1543,1543| 1065,1068| 1232,1236| 1010,1014| 1675,1676|
| $m_{\tilde{g}}$   | 1176   | 368     | 268     | 354     | 335     |
| $m_{\tilde{u}_{L,R}}$ | 1784,1784| 1135,1064| 1475,1406| 1056,986| 1527,1481|
| $m_{\tilde{t}_{1,2}}$ | 1148,1392| 409,764 | 777,1080| 319,709 | 226,857 |
| $m_{\tilde{d}_{L,R}}$ | 1785,1790| 1138,1066| 1477,1404| 1059,987| 1530,1479|
| $m_{\tilde{b}_{1,2}}$ | 1331,1497| 613,799 | 972,1115| 560,743 | 739,987 |
| $m_{\tilde{e}_{L,R}}$ | 1528,1540| 1118,1055| 1483,1413| 1039,974| 1527,1478|
| $m_{\tilde{\mu}_{1,2}}$ | 878,1261| 200,864 | 960,1299| 198,808 | 586,1192|
| $\mu$         | 1555   | 1077    | 1247    | 1020    | 1685    |
| $\Omega_{LSP}h^2$ | 0.079  | 0.076   | 0.074   | 0.114   | 0.127   |