Diffusive magnetotransport in a 2D Rashba system

S.G. Novokshonov\textsuperscript{1}, and A.G. Groshev\textsuperscript{2}

\textsuperscript{1}Institute of Metal Physics, Ural Division of RAS, Ekaterinburg, Russia
\textsuperscript{2}Physical-Technical Institute, Ural Division of RAS, Izhevsk, Russia

(Dated: November 14, 2018)

We present calculations of the conductivity tensor $\sigma$ of a 2D–system with the Rashba spin–orbit interaction (SOI) in an orthogonal magnetic field with allowance for electron elastic scattering by a Gaussian $\delta$–correlated random potential in the self–consistent Born approximation. The calculations are performed proceeding from the Kubo formula using a new exact representation of the one–particle Green function of the 2D–system with SOI in an arbitrary magnetic field. We have obtained the analytical expressions for the density of states and $\sigma$ which have a simple interpretation in terms of the two–subband model and hold good in a wide range from the classical magnetic fields ($\omega_c \tau \ll 1$) up to the quantizing ones ($\omega_c \tau \gtrsim 1$). The numerical analysis of the Shubnikov—de Haas oscillations of the kinetic coefficients and of their behavior in the classical fields region is performed.

PACS numbers: 71.10.Ej, 72.15.Gd, 73.20.At, 73.21.-b

I. INTRODUCTION

The growing interest in studying the spin–orbit interaction (SOI) in semiconductor 2D–structures is mostly due to its potential application to the spin–based electronic devices. There are two main types of SOI in the quantum well based on zinc–blende–lattice semiconductors. First, the Dresselhaus interaction\textsuperscript{2} that originates from the bulk inversion asymmetry (BIA): second, the Rashba interaction induced by structural inversion asymmetry (SIA) of the confined field of a quantum well. Both these interactions lead to the momentum–dependent spin splitting of the electron energy spectrum and to the formation of quantum states with the hard linked spatial and spin degrees of freedom of the electrons. They are responsible for many interesting effects in the transport phenomena like beatings in the Shubnikov—de Haas (SdH) oscillations, weak antilocalization, current–induced non–equilibrium spin polarization, spin Hall effect, and so on.

At present there are some sufficiently well developed theories of the kinetic and spin phenomena in 2D–systems with SOI in zero or classical weak ($\omega_c \tau \ll 1$) orthogonal magnetic fields. Here $\omega_c = |e|B/mc$ is the cyclotron frequency, and $\tau$ is the electron scattering time. As for theoretical studies of the considered systems in strong, and especially in quantizing ($\omega_c \tau \gtrsim 1$) magnetic fields, there is still no satisfactory analytical description of the kinetic phenomena even in the usual diffusive regime (without quantum corrections). The complex form of the eigenspinors and energy spectrum of an electron in the presence of SOI and a strong magnetic field\textsuperscript{12,13,15} is the main cause of such a situation. Direct employment of this basis forces one to proceed almost right from the start to the numerical analysis of very cumbersome expressions\textsuperscript{14,15,16,17}.

The strong magnetic field is however one of the most efficient tools for investigation of SOI\textsuperscript{14} and manipulation of the spin degrees of freedom in the semiconductor 2D–structures. Thus, a rather simple theoretical description of the kinetic phenomena in the 2D–systems with SOI in a strong orthogonal magnetic field becomes a necessity. In the present work we consider the problem of calculation of the longitudinal and Hall resistances of a disordered Rashba system in the self–consistent Born approximation (SCBA).

We have found the exact relation between the one–particle Green function (GF) of the Rashba 2D–electron in an arbitrary orthogonal magnetic field and the well known GF of an "ideal" electron, that is an electron with the ideal value of the Zeeman coupling ($g_0 = 2$) and without SOI. This allows one to obtain the analytical expressions for the density of states (DOS) and the conductivity tensor $\sigma_{ij}$ in the SCBA. These expressions hold good in a wide range, from the classically weak magnetic fields ($\omega_c \tau \ll 1$) up to the quantizing ones ($\omega_c \tau \gtrsim 1$). They have a simple physical interpretation in the framework of the two–subband conductor model. On the basis of these results, we perform the numerical analysis of the beatings of the SdH oscillations of the considered kinetic coefficients, as well as of their behavior in the classical magnetic fields region.

II. MODEL

Let us consider a two–dimensional ($OXY$) degenerate gas of electrons with effective mass $m$, and effective Zeeman coupling $g$ that moves in a Gaussian $\delta$–correlated random field $U(r)$ in the presence of an external orthogonal ($B||OZ$) magnetic field $B = \nabla \times A$. We assume the Rashba interaction to be the dominant mechanism of the energy spin splitting in the absence of a magnetic field. This situation occurs, for example, in the narrow–gap semiconductor heterostructures, such as InAs/GaSb\textsuperscript{18}, InGaAs/InAlAs\textsuperscript{19}. The one–particle Hamiltonian of the considered system has the form

$$\mathcal{H} + U = \frac{\mathbf{p}^2}{2m} + \alpha (\mathbf{\sigma} \times \mathbf{\pi}) \cdot \mathbf{n} + \frac{1}{4} g \omega_c \mathbf{\sigma}_z + U(r) \quad (1)$$
Here $\pi = p - eA/c = mv$ is the operator of the kinematic electron momentum; $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ is the vector formed by the Pauli spin matrices; $\alpha$ is the Rashba spin–orbit coupling; $g$ is the effective Zeeman coupling ($g$–factor).

In the gauge $A = (0, Bx, 0)$, the components of the eigenspinors of the Hamiltonian $H$ of a free ($U(r) = 0$) Rashba electron are expressed through the Landau wave functions $\psi_{n,X}(r)$ depending on the Landau level number $n = 0, 1, 2, \ldots$ and the $X$–coordinate of the cyclotron orbit centre $X = -k_y/m\omega_c^2$:

$$\Psi_\alpha(r) = \frac{1}{\sqrt{1 + C_{s,n}^2}} \left[ C_{s,n} \psi_{n-1,X}(r) \psi_{n,X}(r) \right], \quad \alpha = (s, n, X).$$

(2a)

The corresponding energy levels have the following form:

$$E_{s,n} = \left\{ \begin{array}{ll}
-\omega_c \delta, & n = 0, \ s = +1, \\
\omega_c (n + s\sqrt{\delta^2 + 2\gamma^2 n} - \delta), & n > 0, \ s = \pm 1.
\end{array} \right.$$  

(2b)

Here $C_{s,n} = \gamma \sqrt{2\pi} / \left( \sqrt{\delta^2 + 2\gamma^2 n} - \delta \right)$ is a normalizing coefficient; $\delta = (g - 2)/4$ is the relative deviation of the effective Zeeman coupling from its ideal value $g_0 = 2$ (for definiteness, it is assumed that $\delta < 0$ in these equations, but all the following results are valid for any sign of $\delta$); and, finally, $\gamma = \alpha \sqrt{m}/\omega_c$ is the dimensionless Rashba spin–orbit coupling.

The quantum number $s = \pm 1$ describes the helicity of the Rashba electron eigenstate as in the absence of a magnetic field\cite{11}. Indeed, it can be verified immediately that $s = \pm 1$ is the eigenvalue of the operator

$$\nu = \frac{[\alpha \sigma \times \pi + \omega_c \delta \sigma] \cdot \mathbf{n}}{\sqrt{2m\alpha^2 \omega_c^2 - \omega_c^2 \delta^2}},$$

(3)

that is diagonal in the basis (2a) and approaches the helicity operator $[\sigma \times \mathbf{n}] \cdot \mathbf{n}/|\mathbf{n}|$ as $B \to 0$. Here $\mathbf{n}$ is the unit normal vector to the considered 2D–system;

$$H_0 = \frac{1}{2m} \left( \frac{\hbar^2}{2m} + \frac{1}{2} \omega_c \sigma_z \right)$$

(4)

is the Hamiltonian of the "ideal" ($g_0 = 2$) electron in a magnetic field, which commutes with $\sigma \cdot \mathbf{n}$, $(\sigma \times \pi) \cdot \mathbf{n}$, and with $H$\cite{11}.

In spite of this analogy with the $B = 0$ case, we cannot say that the Rashba electron has in the states (2a) the spin projection $\pm 1/2$ onto the direction $\alpha \pi \times \mathbf{n} + \omega_c \delta \mathbf{n}$, because the components of the kinematic momentum operator $\pi$ are not commuting motion integrals. Nevertheless, this interpretation makes sense in the quasiclassical limit, when one can speak about the electron path in a magnetic field. Namely, the quantum number $s = \pm 1$ determines the value of the spin projection on the instant direction of $\alpha \pi \times \mathbf{n} + \omega_c \delta \mathbf{n}$ that changes along the quasiclassical electron path. Thus, the spin configurations of the Rashba electron states form vortices in the $XY$–plane with center at the origin.

The conductivity tensor $\hat{\sigma}$ of the considered system has just one independent circularly polarized component $\sigma = \sigma_x + i\sigma_y$. In the one–electron approximation, it has the form\cite{10}:

$$\hat{\sigma}^{XY} = \frac{e^2}{4\pi} \text{Tr} \left[ V_+ \left[ 2\Phi_{EE}^{RA} - \Phi_{EE}^{RR} - \Phi_{EE}^{AA} \right] \right]_{E = E_F} + \int_{E_F}^{E} \left[ \frac{\partial E}{\partial E'} \left[ \Phi_{EE}^{AA} - \Phi_{EE}^{RR} \right] \right]_{E' = E} dE.$$ 

(5)

Here $\Phi_{EE'}^{XY} = \langle \hat{G}^{X}(E) V_+ \hat{G}^{Y}(E') \rangle$ is the current–vertex operator; $V_+ = V_x + iV_y = v_x \pm 2i\alpha v_y$ are circularly polarized components of the full velocity operator [the corresponding components $\sigma$ are defined as $\sigma_{\pm} = (\sigma_x \pm i\sigma_y)/2$, where the last term occurs due to SOI\cite{11}]; $\hat{G}^{RA}(E) = 1/(E - H - U + i0)$ is the resolvent (retarded ($R$) or advanced ($A$)) of the Hamiltonian $\hat{H}$, and angular brackets $\langle \ldots \rangle$ denote the averaging over the random field $U$ configurations.

III. ONE–ELECTRON GREEN FUNCTION

By definition, the one–particle GF is the averaged resolvent of the Hamiltonian $\langle \hat{G}^{RA}(E) \rangle = 1/(E - H - U + i0)$. It is connected with the electron self–energy operator $\Sigma^{RA}(E)$ by the relation ($X = R, A$)

$$\langle \hat{G}^{X}(E) \rangle = \frac{1}{E - H - \Sigma^{X}(E)}. $$

(6)

The direct employment of the eigenspinors\cite{2} for cal-
The Hamiltonian of the free Rashba system
\[ \hat{G}(E) = \]
\[ \frac{1}{2\Omega_B} \sum_{s=\pm 1/2} \left[ \Phi_s - 2s \frac{\alpha(\pi \times n) \cdot \sigma}{\Omega_B} \right] \hat{G}(E + ma^2 + s\Omega_B). \]

We use here the same notation (\( \hat{G} \)) for the GF of the Rashba electron and for the GF of the "ideal" electron. However, this does not lead to confusion since the latter depends always on the energy arguments like \( E + ma^2 + s\Omega_B \) etc.

It is important that the same representation can be obtained for the averaged resolvent of the Rashba system in the SCBA. We restrict ourselves here to an approximation in which the electron self–energy operator is diagonal in the spin space. Then, the SCBA equation for \( \Sigma^N(E) \) has the following form
\[ \hat{\Sigma}(E) = \text{Sp} \hat{G}(E) = \left[ \frac{\Sigma_{\uparrow\uparrow}(E)}{0} \Sigma_{\downarrow\downarrow}(E) \right], \]

where Sp denotes the trace only over the spatial degrees of freedom; \( W \) is the mean–square fluctuation of the Gaussian random field \( \langle U(r)U(r') \rangle = W \delta(r - r') \). Therefore, it suffices to make everywhere in Eq. (10) the following substitutions
\[ E \to E - \Sigma_c(E) \quad g\omega_c \to g\omega_c + 4\Sigma_o(E) \]

to obtain the desired representations for the averaged GF’s in the SCBA. Here \( \Sigma_{c(o)}(E) = [\Sigma_{\uparrow\uparrow}(E) \pm \Sigma_{\downarrow\downarrow}(E)]/2 \) are the even and odd parts of the electron self–energy. The first \( \Sigma_{c} = \Delta_s \pm i/2\tau_o \) describes the perturbation (shift \( \Delta_s \) and broadening \( 1/\tau_o \)) of the one–electron energy levels by a random field. The real part of \( \Sigma_o = \Delta_s \pm i/2\tau_o \) defines the renormalization of the Zeeman coupling \( \Omega_B \), while its imaginary part \( \propto 1/\tau_o \) makes a contribution to the overall broadening of the one–electron energy levels. As a result, we obtain a expression like Eq. (10) for the averaged GF, where
\[ \hat{G}^{R(A)}(E + ma^2 + s\Omega_B) = \frac{1}{E + ma^2 + s\Omega_B - \hat{\Sigma}(E) \pm \frac{i}{2\tau_o}} \]
is the averaged retarded (advanced) GF of the ”ideal” electron, and
\[ \Omega_B = \frac{1}{2}(\Omega_B^R + \Omega_B^A), \]

The denominator of the right–hand side of Eq. (13) depends on the ”ideal” electron Hamiltonian alone. Expanding this expression into the partial fractions, we obtain the desired representation of the one–electron GF of the free Rashba system
\[ \hat{G}(E) = \]
\[ \frac{1}{2\Omega_B} \sum_{s=\pm 1/2} \left[ \Phi_s - 2s \frac{\alpha(\pi \times n) \cdot \sigma}{\Omega_B} \right] \hat{G}(E + ma^2 + s\Omega_B). \]

Here \( \nu \) is the helicity operator defined in Eq. (3), \( \mathcal{H}_0 \) is the Hamiltonian of the "ideal" electron (4), and, finally, to redefine the parameter \( \delta \to \delta_D = (g + 2)/4 \).

The denominator of the right–hand side of Eq. (13) depends on the "ideal" electron Hamiltonian alone. Expanding this expression into the partial fractions, we obtain the desired representation of the one–electron GF of the free Rashba system
\[ \hat{G}(E) = \]
\[ \frac{1}{2\Omega_B} \sum_{s=\pm 1/2} \left[ \Phi_s - 2s \frac{\alpha(\pi \times n) \cdot \sigma}{\Omega_B} \right] \hat{G}(E + ma^2 + s\Omega_B). \]

We use here the same notation (\( \hat{G} \)) for the GF of the Rashba electron and for the GF of the "ideal" electron. However, this does not lead to confusion since the latter depends always on the energy arguments like \( E + ma^2 + s\Omega_B \) etc.

It is important that the same representation can be obtained for the averaged resolvent of the Rashba system in the SCBA. We restrict ourselves here to an approximation in which the electron self–energy operator is diagonal in the spin space. Then, the SCBA equation for \( \Sigma^N(E) \) has the following form
\[ \hat{\Sigma}(E) = \text{Sp} \hat{G}(E) = \left[ \frac{\Sigma_{\uparrow\uparrow}(E)}{0} \Sigma_{\downarrow\downarrow}(E) \right], \]

where Sp denotes the trace only over the spatial degrees of freedom; \( W \) is the mean–square fluctuation of the Gaussian random field \( \langle U(r)U(r') \rangle = W \delta(r - r') \). Therefore, it suffices to make everywhere in Eq. (10) the following substitutions
\[ E \to E - \Sigma_c(E) \quad g\omega_c \to g\omega_c + 4\Sigma_o(E) \]

to obtain the desired representations for the averaged GF’s in the SCBA. Here \( \Sigma_{c(o)}(E) = [\Sigma_{\uparrow\uparrow}(E) \pm \Sigma_{\downarrow\downarrow}(E)]/2 \) are the even and odd parts of the electron self–energy. The first \( \Sigma_{c} = \Delta_s \pm i/2\tau_o \) describes the perturbation (shift \( \Delta_s \) and broadening \( 1/\tau_o \)) of the one–electron energy levels by a random field. The real part of \( \Sigma_o = \Delta_s \pm i/2\tau_o \) defines the renormalization of the Zeeman coupling \( \Omega_B \), while its imaginary part \( \propto 1/\tau_o \) makes a contribution to the overall broadening of the one–electron energy levels. As a result, we obtain a expression like Eq. (10) for the averaged GF, where
\[ \hat{G}^{R(A)}(E + ma^2 + s\Omega_B) = \frac{1}{E + ma^2 + s\Omega_B - \hat{\Sigma}(E) \pm \frac{i}{2\tau_o}} \]
is the averaged retarded (advanced) GF of the ”ideal” electron, and
\[ \Omega_B = \frac{1}{2}(\Omega_B^R + \Omega_B^A), \]

The denominator of the right–hand side of Eq. (13) depends on the "ideal" electron Hamiltonian alone. Expanding this expression into the partial fractions, we obtain the desired representation of the one–electron GF of the free Rashba system
\[ \hat{G}(E) = \]
\[ \frac{1}{2\Omega_B} \sum_{s=\pm 1/2} \left[ \Phi_s - 2s \frac{\alpha(\pi \times n) \cdot \sigma}{\Omega_B} \right] \hat{G}(E + ma^2 + s\Omega_B). \]
are the disorder–modified frequency of the spin precession \( \Omega \) and the inverse life time of an electron in the \( s \)-th spin–subband. As usual, we do not take explicitly into consideration in (13) the one–electron energy levels shift \( \Delta s \), that is absorbed by the normalization condition, but we mean here that the odd shift \( \Delta o \) is included in the definition of the effective \( g \)-factor in accordance with (12). The explicit allowance for the Zeeman coupling renormalization is particularly important in the SdH oscillations regime.

IV. DENSITY OF STATES AND SELF–ENERGY

We first consider the calculation of the DOS \( n(E) = \text{Im}(\text{Tr} \, G^A(E))/\pi \) using the above–obtained expression for the one–particle GF (14). Here, the symbol \( \text{Tr} \) denotes the trace over the spatial and spin degrees of freedom. For the sake of simplicity, we shall deal with the case of large filling numbers \( (E \gg \omega_c) \). Calculating the trace of resolvent (10) over the spatial and spin degrees of freedom, we obtain the following expression for the DOS

\[
n(E) = \sum_{s=\pm 1/2} \frac{m}{m} n^{(0)}[E + m\alpha^2 + s(\Omega_B \pm \omega_c)] = \sum_{s=\pm 1/2} \frac{m}{m} n_s^{(0)}(E) . \tag{15}
\]

Here, we take into account that the DOS of a spinless electron in an orthogonal magnetic field \( n^{(0)}(E) \) satisfies \( n^{(0)}(E) = n^{(0)}(E \pm \omega_c) \) at large filling factors \( (E \gg \omega_c) \). The sign before \( \omega_c \) is chosen in such a way as to ensure the right–hand limit \( s(\Omega_B \pm \omega_c) \rightarrow \pm s\omega_c/2 \), as the spin–orbit coupling approaches zero. The effective mass \( m_s \) in the \( s \)-th subband is defined as

\[
m_s = m \left( 1 + s \frac{4m\alpha^2}{\Omega_B} \right) = m \partial_E (E + s\Omega_B) . \tag{16}
\]

In the considered case this expression coincides with the usual definition of the transport and cyclotron effective masses in the isotropic nonparabolic band. The symbol \( \partial_E \) denotes the derivative with respect to energy \( E \).

In full accordance with the two–subband model, the DOS in Eq. (15) is presented as a sum of partial contributions. Using this expression for the DOS, we can obtain the analytical form of the equation for the electron concentration \( n = \int E \, n(E) \, dE \) that is the normalization condition for the Fermi level determination. For example, at \( B = 0 \) we have

\[
n = \frac{m}{\pi} (E_F + m\alpha^2) = \frac{m}{\pi} E_0 . \tag{17}
\]

Thus, the energy \( E_0 = E_F + m\alpha^2 \) corresponds to the Fermi level in the absence of SOI. Notice that the partial electron concentrations \( n_s = m (E_0 + s\Omega_B)/2\pi \) depend nonlinearly on the Fermi energy, in contrast to \( n \) (17). Of course, the difference between \( E_0 \) and \( E_F \) is small for weak SOI \( (m\alpha^2 \ll E_F) \), but it should be taken into account when analyzing the SdH oscillations that are very sensitive to the electron spectrum character.

The representation (15) allows one to obtain a simple analytical expression for the DOS that holds good up to the quantizing fields region \( (\omega_c/T \gtrsim 1) \). Indeed, the DOS of a spinless electron in the large filling factors region \( (E \gg \omega_c) \) has the form

\[
n^{(0)}(E) = \frac{m}{2\pi} \frac{\sin \pi}{\omega_c} \frac{\omega_c}{\omega_c} + \cos 2\pi \frac{E}{\omega_c} . \tag{18}
\]

Inserting Eq. (18) into Eq. (15), we obtain for the oscillating part of the DOS the following expression

\[
\Delta n(E_F) = \frac{2m}{\pi} \exp \left( -\frac{\pi}{\omega_c} \right) \left[ \cos \frac{2\pi}{\omega_c} \frac{E_0}{\omega_c} \cos \frac{\Omega_B}{\omega_c} - \frac{2m\alpha^2}{\Omega_B} \sin \frac{2\pi}{\omega_c} \sin \frac{\pi}{\omega_c} \Omega_B \right] . \tag{19}
\]

This limit was considered in Ref. 22. Unlike the results of that work, the above–obtained equations still stand in the case of strong SOI. In addition, we have taken into account the Zeeman splitting of the electron spectrum that allows to describe more correctly the oscillation pattern. For example, the Eq. (20) allows to determine both the spin–orbit \( \alpha \) and Zeeman \( g \) couplings by measured locations of two different nodes (see upper curve in Fig. 1(a)). On the other hand, the spin precession frequency \( \Omega_B \) approaches \( |\delta \omega_c| \) as the magnetic field \( B \) increases. Therefore, in this case a gradual transition from the beatings of the SdH oscillations to the familiar Zeeman splitting of the oscillating peaks should be observed. The beginning of this transition can be seen on the lower curve in Fig. 1(a).

Another important characteristic of the one–electron states of the \( 2D \)–Rashba system is the difference of the partial DOS’s with opposite spin projections onto the \( OZ \)-axis

\[
\delta n(E) = n^{(0)}(E) - n^{(0)}(E) = \frac{4\omega_c}{\Omega_B} \sum_{s=\pm 1/2} n_s(E) . \tag{21}
\]
V. CONDUCTIVITY

The general expression for the conductivity \( \sigma^{II} \) consists of two different terms. The first of them describes the contribution of the electrons at the Fermi level, the second one contains the contributions of all filled states below the Fermi level. We begin the calculation of the conductivity with the last term of (5) \( \sigma^{II} \). First of all, it is pure imaginary and, therefore, makes a contribution in the Hall conductivity alone. \( \text{St\u{e}reda}^{23} \) was first to show that, in the absence of SOI, this part of the conductivity is equal to

\[
\sigma^{II} = i|e|c \left( \frac{\partial n}{\partial B} \right)_E ,
\]

(23)
where $n$ is the electron concentration. It should be pointed out that Eq. (24) is exact, and with the thermodynamic Maxwell relation $\sigma^{II}$ can be expressed through $(\partial M/\partial \xi_E)_B$, where $M$ is the orbital magnetization of the electron gas. Detailed discussion of $\sigma^{II}$ and its physical interpretation can be found in survey\textsuperscript{24}. This result is extended immediately to the electron systems with SOI. Following St˘ reda’s argument, it can be shown that the part $\sigma^{II}$ of the of 2D Rashba system conductivity is expressed as

$$
\sigma^{II} = i|e|c \left[ \left( \frac{\partial n}{\partial B} \right)_E - \left( \frac{\partial M_p}{\partial E} \right)_B \right] = i|e|c \left[ \left( \frac{\partial n}{\partial B} \right)_E - \frac{g|e|}{4mc} \left[ n_{\downarrow}(E) - n_{\uparrow}(E) \right] \right] - i|e|nc \frac{1}{B} \left[ n - N_+ - N_- \right],
$$

(24)

where $M_p$ is the spin magnetization of the electron gas. The quantities

$$
N_s = \left[ E_0 + s \left( \Omega_B + \frac{2\omega_s^2\delta}{\Omega_B} \right) \right] n^{(0)} \left[ E_0 + s(\Omega_B - \omega_c) \right].
$$

(25)

Now, we turn to the first term in the conductivity \textsuperscript{19}. It is quite easy to show, by identical transformations, that

$$
-\frac{e^2}{4\pi} \text{Re} \text{Tr} V_+ \Phi^{RA}_{EE} = \frac{e^2}{2\pi m} \text{Re} \text{Tr} \langle \hat{G}^A \rangle = \frac{e^2}{2\pi m} \sum_s \text{Re} \text{Tr} \left[ \Phi^R_s \Phi^A_s \hat{G}^A(E_0 + s\Omega_B) + \Phi^R_s \Phi^A_s \hat{G}^A(E_0 - s\Omega_B) \right],
$$

(26)

where the averaged GF’s of the ”ideal” electron are defined in Eqs. \textsuperscript{13} and \textsuperscript{14}. In obtaining the last equality in \textsuperscript{26}, we used the immediately verified identities

$$
\Phi^R_s = \Phi^R_s \Phi^A_s + \Phi^R_s \Phi^A_s, \quad \Phi^A_s = \Phi^R_s \Phi^A_s + \Phi^R_s \Phi^A_s.
$$

(27)

The main contribution to the dissipative part of the conductivity is proportional to the current vertex $\Phi^{RA}_{EE}$ in Eq. \textsuperscript{5}. If we accept the SCBA \textsuperscript{11} for the electron self-energy $\Sigma$, we must evaluate the conductivity \textsuperscript{5} in the ladder approximation in order to satisfy the particle conservation law. But as show the calculations, the relative magnitude of the ladder correction to the conductivity is second–order in the small parameter $1/(k_Fl)$, and it can be neglected in comparison with the ”bare” part of the conductivity $(\Delta\sigma^{lad}/\sigma = 0.01 \div 0.001$ for typical values of $k_Fl = 10 \div 30$).

Thus, it suffices to calculate the ”bare” part of the conductivity that is obtained by replacement $\text{Tr} V_+ \Phi^{RA}_{EE} \rightarrow \text{Tr} V_+ \langle \hat{G}^R \rangle V_+ \langle \hat{G}^A \rangle$ in the first term in the right–hand side of Eq. \textsuperscript{5}. We drop the details of calculations that can be found in Appendix B and proceed to the results. The overall contribution of the three above–considered parts has the usual Drude — Boltzmann form

$$
\sigma = \frac{|e|}{B} \left[ n - \sum_s \frac{\tilde{N}_s}{1 - i\mu_s B} \right] + \Delta\sigma
$$

(28)

(it is meant here and below that $B \rightarrow B/c$). Here, the first two terms represent the sum of the partial conductivities of the electrons of two subbands with different mobilities $\mu_s = |e|\tau_s/m$, and effective concentrations

$$
\tilde{N}_s = N_s + \frac{4m\alpha^2}{\tau_s \Omega_B^2 (2\pi)^2} \frac{m}{(E - s\Omega_B)^2} \mu_s^2 B^2.
$$

(29)

This expression differs from $N_s$ (see Eq. \textsuperscript{25}) by the second term that originates from the principal values of the one–electron GF’s.

The last term in Eq. \textsuperscript{28} represents the small correction to the conductivity due to the electron intersubband transitions. In the leading approximation in powers of the smallness parameters $\omega_c/E$ and $\Omega_B/E$, it has the form

$$
\Delta\sigma = -\frac{|e|}{B} \frac{2m\alpha^2}{\Omega_B^2 \omega_c^2 \delta} \frac{m}{(1 - i\omega_c \tau_c)^2} \Omega_B^2 \tau_c.
$$

(30)
where \( n^{(0)} = \sum_{\sigma} \nu^{(0)}_{\sigma}(E) \) is the DOS at the Fermi level averaged over the electron subbands. The relative contribution of this correction to the full conductivity \( \sigma_{\text{SOI}} \) is of the order of magnitude \( (\omega_c/E)^2 \) and can be neglected in the large filling factors \( E \gg \omega_c \) region.

We emphasize that the contribution to the conductivity from intersubband transitions vanishes as the magnetic field approaches zero. This would be expected, because the conductivity tensor in the absence of a magnetic field is diagonal in the original spin space \( (\sigma_{\uparrow\uparrow} = \sigma_{\downarrow\downarrow} = 0) \) by virtue of the momentum parity of the GF’s, and the full conductivity is equal to \( \sigma = \sigma_{\uparrow\uparrow} + \sigma_{\downarrow\downarrow} \). In fact, the case in point concerns the time inversion symmetry. Using a unitary transformation, it can be turned into a matrix \( s \)-representation in which the one–electron GF’s are diagonal and, therefore, \( \sigma = \sigma_{\uparrow\uparrow} + \sigma_{\downarrow\downarrow} = \sigma_{s+} + \sigma_{s–} \) due to the trace conservation. Applying an external magnetic field breaks the above–mentioned symmetry that is responsible for the appearance of the intersubband transition–induced conductivity \( \Delta\sigma \).

VI. RESULTS AND DISCUSSION

First of all, let us summarize briefly the main results obtained in this work. We have shown that the eigenstates of the 2D Rashba electron in an orthogonal magnetic field are characterized by a special motion integral \( \hbar \) that generalizes the notion of helicity. Using this fact, we have found the relation \( (10) \) between the GF’s of the 2D Rashba electron and the “ideal” one that holds good for arbitrary orthogonal magnetic fields as well as for the strong spin–orbit coupling. With the help of this relation, we have obtained, in contrast to Refs. 14,15,17, the analytical SCBA expressions for the DOS \( (15) \) and magnetoconductivity \( (25) \) of the 2D Rashba system that are valid in a wide range from the classical magnetic fields up to the quantizing ones \( (\omega_c \tau \gtrsim 1) \). They permit a simple interpretation in the framework of the model of two types of current carriers with different concentrations and mobilities. The spin–orbit as well as the Zeeman splitting of the electron energy are properly allowed for in these expressions, unlike the results of Ref. 26. We have shown that the competition of the relaxation rates of the orbital and spin degrees of freedom in the total life time \( 1/\tau \) of the one–electron states in the \( s \)-th subband leads to the supression of beatings of the SdH oscillations as does the competition of the Rashba and Dresselhaus SOI’s. Finally, we have shown that the breaking of the time inversion symmetry in a magnetic field leads to the appearance of the intersubband term in the 2D Rashba system conductivity \( (25) \).

We start the discussion of the results with the conductivity in zero magnetic field. In this case, it follows immediately from \( (25) \) that

\[
\sigma = |e|(n_{+\uparrow} + n_{–\downarrow}) = \sigma_D \left[ 1 - 2 \left( \frac{m\alpha}{k_F} \right)^2 \right]. \quad (31)
\]

Thus, the Rashba spin–orbit interaction leads to a decrease in conductivity, and not to its increase, as it was claimed in Ref. 25. Let us note that the authors of that work ignored the difference in mobility between the electrons of different subbands. As a result, they obtained a correction to the conductivity of opposite sign compared to \( (31) \), which is actually unobservable, because it is absorbed by normalization condition \( (17) \).

Let us proceed now to the discussion of the magnetotransport in the 2D Rashba system. It is well known that in the two–subband conductors the classical positive magnetoresistance and the magnetic–field–dependent Hall coefficient are observed (see, for example 25). The considered system differs from a classical two–subband conductor in two points. Firstly, the mobilities and effective concentrations of current carriers \( (25) \) depend on the magnetic field. Secondly, the full conductivity of the 2D Rashba system in an orthogonal magnetic field is not an additive sum of the intrasubband contributions, but it contains a nonadditive intersubband term \( (25) \). However, all these factors lead to very slight magnetic field dependences of kinetic coefficients due to small differences between concentrations and mobilities of current carriers. For example, the relative magnitude of the classical magnetoresistive effect comes to only \( 1 \pm 2\% \) for typical values of parameters (see Fig. 3).

In discussing the SdH oscillations, we restrict ourselves to the consideration of the large filling factors \( (E \gg \omega_c) \) region, where the SCBA is applicable to the description of the one–electron states and kinetic phenomena. As usual, we extract in the linear approximation the oscillating parts of the conductivity that enter through partial DOS’s into the effective concentrations \( N_\uparrow \) and mobilities \( \mu_\uparrow \). Neglecting the small differences between concentrations and mobilities of current carriers in the smooth parts of conductivity, we obtain the expressions for the oscillating parts of the longitudinal \( \rho \) resistance

\[
\Delta \rho/\rho = (25) \text{ at fixed } g = 3.5, \text{ and } k_F l = 32, \text{ and different } \Omega \tau = 1.5, 1.0, 0.5 \text{ (up to down).}
\]

\[
\text{FIG. 3: The smooth magnetic–field dependence of the resistance calculated with Eqs. (25) at fixed } g = 3.5, \text{ and } k_F l = 32, \text{ and different } \Omega \tau = 1.5, 1.0, 0.5 \text{ (up to down).}
\]
spin precession frequency $\Omega$ agree with expressions for the longitudinal and Hall con-

tance (see Eq. (31)) and Hall coefficient in zero magnetic field due to allowing for Zeeman coupling. As result, the period (see, for example, (19)) depend on the magnetic field. Here, $\rho(0) = 1/\sigma(0)$ and $R_{\|}^0 \approx -1/|e| nc$ are is the resistance (see Eq. (31)) and Hall coefficient in zero magnetic field respectively. Results of numerical calculations of the SdH oscillations are performed using total expressions (28)–(31) are showed in Fig. 4.

Up to definition of the beatings period, the Eqs. (32) agree with expressions for the longitudinal and Hall conductivities obtained in Ref.22. As pointed out above, the spin precession frequency $\Omega = 3\mu B$ and, therefore, beatings period (see, for example, (13)) depend on the magnetic field due to allowing for Zeeman coupling. As result, the measurement of two different nodes location permits to determine both the spin–orbit $\alpha$ and Zeeman $g$ couplings using the Eq. (20).

![FIG. 4: Plots of the SdH oscillations of the longitudinal magnetoresistance (upper panel) and Hall coefficient (bottom panel) of the 2D Rashba system calculated with Eqs. 28–31 at fixed $g = 3.5$, and $k_F l = 32$, and different $\Omega \tau = 1.5; 1.0; 0.5$ (up to down).](image)

**APPENDIX A: SOME USEFUL IDENTITIES**

We obtain here several identities for the off–diagonal matrix elements $\langle G_{\uparrow \downarrow}^{\pm} \rangle$ of the one–particle GF that are necessary for calculation of the kinetic coefficients (see, for example, Eq. (12)). The $\uparrow$ matrix elements of Eq. (A1) has the form

$$
\begin{align}
E - \Sigma_{\uparrow} - \frac{\pi^2}{2m} - \frac{1}{4}g_\omega \sigma_z 
\langle \hat{G} \rangle - \alpha n \cdot (\sigma \times \pi) \langle \hat{G} \rangle = \hat{I},
\end{align}
$$

(A1)

where $\hat{I}$ is the unit $2 \times 2$–matrix. It is assumed that the odd part of the electron self–energy $\Sigma$ is included into the effective Zeeman coupling $g$ (see (12)). The $\uparrow$ matrix element of Eq. (A1) has the form

$$
\begin{align}
E - \Sigma_{\uparrow} - \frac{\pi^2}{2m} - \frac{1}{4}g_\omega \pi 
\langle \hat{G} \rangle - i\alpha \pi \langle \hat{G} \rangle = 1,
\end{align}
$$

(A2)

Analogous relations can be obtained from the conjugated Dyson equation (A1). From their comparison with (A2) it follows that

$$
\begin{align}
\pi_-(\hat{G}_\uparrow) = -\langle G_{\uparrow \downarrow}^\pm \rangle \pi_+, \quad \pi_+(\hat{G}_\uparrow) = -\langle G_{\uparrow \downarrow}^\pm \rangle \pi_.
\end{align}
$$

(A3)

Now, we replace in (A2) the matrix elements $\langle G_{\uparrow \downarrow}^{\pm} \rangle$ with their expressions through the one–particle GF of a “ideal” electron (see Eq. (10)). As a result, we have the following useful relations between $\langle G_{\uparrow \downarrow}^{\pm} \rangle$ and the one–particle GF of the “ideal” electron

$$
\begin{align}
\pi_\mp \langle G_{\uparrow \downarrow}^{\pm} \rangle = \pm i m \sum_{s=\pm1/2} E_0 + s\Omega_B G_{\uparrow \downarrow}^{\pm}(E_0 + s\Omega_B),
\end{align}
$$

(A4)

By combining these identities with the corresponding diagonal matrix elements of Eq. (10), we obtain

$$
\begin{align}
\pi_\mp \langle G_{\uparrow \downarrow}^{\pm} \rangle = \pm 2im \alpha \sum_{s=\pm1/2} sG_{\uparrow \downarrow}^{\pm}(E_0 + s\Omega_B).
\end{align}
$$

(A5)

**APPENDIX B: CALCULATION OF CONDUCTIVITY**

We first consider the derivation of the last equality in Eq. (24). The immediate differentiation of the electron
concentration with respect to the magnetic field gives the following result

\[
\frac{\partial n}{\partial B}_E = \frac{n}{B} - \frac{1}{\pi} \text{Im } \text{Tr} \langle \hat{G}^A \rangle \frac{\partial \mathcal{H}}{\partial B}. \tag{B1}
\]

Now we should calculate the last term in this expression. Using the representations (10), (13) for the averaged GF, we write down

\[
\text{Tr} \langle \hat{G} \rangle \frac{\partial \mathcal{H}}{\partial B} = \frac{1}{2} \sum_{s = \pm 1/2} \text{Tr} \hat{G}(E_0 + s \Omega_B) \left[ 1 + \frac{4s m c^2 - \alpha (\sigma \times \pi) \cdot n - \omega_c \delta \sigma_z}{\Omega_B} \right] \frac{\partial \mathcal{H}}{\partial B}, \tag{B2}
\]

Let us multiply together the expression in square brackets and the derivative of Hamiltonian \( \mathcal{H} \) \[3\], \[7\], keeping the terms diagonal in the spin space and neglecting the terms linear in \( \sigma_z \). These latter make contributions proportional to \( n^{(0)}(E) - n^{(0)}(E \pm \omega_c) \) and vanish in the magnetic field region of interest to us. As a result, Eq. \( \text{B2} \) takes the form

\[
\text{Tr} \langle \hat{G} \rangle \frac{\partial \mathcal{H}}{\partial B} = \frac{1}{2} \sum_{s = \pm 1/2} \times \text{Tr} \hat{G}(E_0 + s \Omega_B) \left[ \frac{\partial \mathcal{H}_0}{\partial B} - \frac{1}{B} \frac{4s m c^2 - \omega_c \delta_0}{\Omega_B} \right], \tag{B3}
\]

where \( \mathcal{H}_0 \) is the Hamiltonian of the "ideal" electron [4]. According to the well known theorem of quantum mechanics, there is the identity \( \partial \mathcal{H} / \partial \lambda \big|_{n} = \partial E_n / \partial \lambda \), where \( E_n \) is the \( n \)-th eigenvalue of the Hermitian operator \( \mathcal{H} \). This make it possible to perform the following substitution \( \partial \mathcal{H}_0 / \partial B \rightarrow \mathcal{H}_0 / B \) in Eq. \( \text{B3} \). Then, on simple rearrangements, Eq. \( \text{B1} \) takes the form

\[
\frac{\partial n}{\partial B}_E = \frac{n}{2\pi B} \sum_{s = \pm 1/2} \times \text{Im } \text{Tr} \left[ E_0 + s \Omega_B - \frac{4s m c^2 - \omega_c \delta_0}{\Omega_B} \right] \hat{G}^A(E_0 + s \Omega_B). \tag{B4}
\]

Eqs. \( \text{B1}, \text{B4} \) are derived immediately from this equation.

Now, let us proceed to the calculation of the dissipative part of the conductivity that is proportional to \( \text{Tr} \langle \hat{G}^R \rangle \text{V}_- \langle \hat{G}^A \rangle \) in the SCBA. Performing the trace over the spin degrees of freedom and taking into account the relations \( \text{A3} \), we write it down in the form

\[
\text{Tr} \text{V}_+ \langle \hat{G}^R \rangle \text{V}_- \langle \hat{G}^A \rangle = \frac{1}{m^2} \text{Sp} \left[ \pi_+ \langle G^R_{11} \rangle \pi_- \langle G^A_{11} \rangle + \pi_+ \langle G^R_{12} \rangle \pi_- \langle G^A_{12} \rangle - 4m^2 c^2 \langle G^R_{11} \rangle \langle G^A_{11} \rangle + 2 \langle G^R_{11} \rangle \langle G^A_{11} \rangle \right]. \tag{B5}
\]

Two last terms in the right-hand side of this equation are calculated using identities \( \text{B1}, \text{A3} \). We present a more detailed calculation of one of the two first terms in \( \text{B5} \).

For example, let us substitute, in the first term in Eq. \( \text{B5} \), the diagonal matrix elements \( \langle \uparrow \uparrow \rangle \) of \( \hat{G} \) for \( \langle \hat{G} \rangle \), and use the identity

\[
G^R_{11}(E_0 + s \Omega_B) \pi_- G^A_{11}(E_0 + s' \Omega_B) = \frac{\pi_- G^A_{11}(E_0 + s' \Omega_B) - G^R_{11}(E_0 + s \Omega_B) \pi_-}{\tau_c + \omega_c + s' \Omega_B - s \Omega_B}. \tag{B6}
\]

Then, after some simple but cumbersome algebra, we can rewrite the contribution of this term to the conductivity in the following form

\[
\sigma \rightarrow \frac{e^2}{\pi i m} \sum_s \Phi^R_{s, \uparrow} \Phi^A_{s, \uparrow} \left\{ \left( E_0 + s \Omega_B - \frac{\omega_c}{2} \right) \frac{\tau_s}{1 - i \omega_c \tau_s} \text{Im } \text{Sp} G^A_{11}(E_0 + s \Omega_B) - \frac{1}{2} \text{Re } \text{Sp} G^A_{11}(E_0 + s \Omega_B) \right\} + \frac{e^2}{2\pi i m} \sum_s \Phi^R_{s, \uparrow} \Phi^A_{s, \downarrow} \left\{ \left( E_0 + s \left( \Omega^R_B - \Omega^\prime_B \right) - \frac{\omega_c}{2} \right) \frac{\tau_c}{1 - i (\omega_c + 2s \Omega_B) \tau_c} \text{Sp} \left[ G^A_{11}(E_0 - s \Omega_B) - G^R_{11}(E_0 + s \Omega_B) \right] + \frac{1}{2i} \text{Sp} \left[ G^A_{11}(E_0 - s \Omega_B) + G^R_{11}(E_0 + s \Omega_B) \right] \right\}. \tag{B7}
\]
The last terms in curly brackets are cancelled exactly by the corresponding terms from Eq. (26). The contribution of the second term from Eq. (B5) can be transformed in a similar way.

Of course, it is necessary to perform a set of unwieldy some transformations to obtain Eqs. (28), (29), and (30). However, further calculations have a purely technical character and we omit them.

1. I. Zutić, J. Fabian, S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004).
2. G. Dresselhaus, Phys. Rev. 100, 580 (1955).
3. E.I. Rashba, Fiz. Tverd. Tela 2, 1224 (1960) [Sov. Phys. Solid. State 2, 1109 (1960)]; Yu.A. Bychkov, E.I. Rashba, Pis’ma ZhETF 39, 66 (1984) [JETP Lett. 39, 78 (1984)]; Yu.A. Bychkov, E.I. Rashba, J. Phys. C: Solid State Phys., 17, 6039 (1984).
4. J. Luo, H. Munekata, F.F. Fang, P.J. Stiles, Phys. Rev. B 38, 10142 (1988); Phys. Rev. B 41, 7685 (1990).
5. S.V. Iordanskii, Yu.B. Lyanda–Geller, G.E. Pikus, Pis’ma ZhETF 60, 199 (1994) [JETP Lett. 60, 206 (1994)].
6. F.G. Pikus, G.E. Pikus, Phys. Rev. B 51, 16928 (1995).
7. W. Knap, C. Skierbiszewski, A. Zdunia, et al., Phys. Rev. B 53, 3912 (1996).
8. L.E. Golub, Phys. Rev. B 71, 235310 (2005).
9. L.S. Levitov, Yu.V. Nazarov, G.M. Eliashberg, Zh. Eksp. Teor. Fiz., 88 229 (1985) [Sov. Phys. JETP 61, 133 (1985)].
10. A.G. Aronov, Yu.B. Lyanda–Geller, Pis’ma ZhETF, 50, 398 (1989) [JETP Lett., 50, 431 (1989)].
11. V.M. Edelstein, Solid State Commun. 73, 233 (1990).
12. M.I. Dyakonov, V.I. Perel’, Pis’ma ZhETF, 13, 467 (1971) [JETP Lett. 13, 467 (1971)].
13. S. Murakami, N. Nagaosa, S.C. Zhang, Science 301, 1348 (2003).
14. X.F. Wang, P. Vasilopoulos, Phys. Rev. B 67, 085313 (2003).
15. M. Langenbuch, M. Suhrke, U. Rössler, Phys. Rev. B 69, 125303 (2004).
16. N.S. Averkiev, M.M. Glazov, S.A. Tarasenko, Solid State Commun. 133, 543 (2005).
17. X.F. Wang, P. Vasilopoulos, preprint LANL cond-mat/0501214.
18. J. Nitta, T. Akazaki, H. Takayanagi, T. Enoki, Phys. Rev. Lett., 78, 1335 (1997); T. Koga, J. Nitta, T. Akazaki, H. Takayanagi, Phys. Rev. Lett., 89, 046801 (2002).
19. R.R. Gerhardt, Z. Phys. B 22, 327 (1975).
20. M.I. Dyakonov, V.I. Perel’, Fiz. Tverd. Tela, 13, 3581 (1971) [Sov. Phys. Solid State, 13, 3023 (1971)].
21. I.M. Tsidilkovskii Zonnaya struktura poluprovodnikov, . . . (1978); I.M. Tsidilkovskii, Band Structure of Semiconductors, Oxford etc.; Pergamon Press (1982).
22. S.A. Tarasenko, N.S. Averkiev, Pis’ma ZhETF, 75, 669 (2002) [JETP Lett. 75, 562 (2002)].
23. L. Smrčka, P. Středa, J.Phys. C: Solid State Phys., 10, 2153 (1977); P. Středa, J.Phys. C: Solid State Phys., 15, L717 (1982).
24. A.M.M. Pruisken, In The Quantum Hall Effect, ed. by R.E. Prange, and S.M. Girvin, Springer–Verlag, New York e.a. (1990).
25. J. Inoue, G.E.W. Bauer, L.W. Molenkamp, Phys. Rev. B 67, 033104 (2003); Phys. Rev. B 70, 041303 (2004).
26. J.M. Ziman, Principles of the Theory of Solids, Cambridge, University Press (1972).