On Active and Reactive Power Control of Synchronous Generators: A Port-Hamiltonian Approach

Elham Tajik, Thordur Runolfsson

School of Electrical and Computer Engineering, University of Oklahoma, Norman, Ok.

E-mail: elhamtajik@ou.edu
E-mail: runolfsson@ou.edu

Abstract. In this paper a general port-Hamiltonian model of a synchronous generator is presented and the effects of deviations and disturbances of the terminal signals on generator control laws are discussed. We discuss how control laws should be designed to minimize the effect of small variations in the generator's terminal conditions, i.e. local control that does not have access or knowledge to a model of the rest of the system. For analysis purpose we present a linearized generator model and apply a full state feedback control consisting of both regulation of the generator flux and rotational dynamics.

1. Introduction

Traditionally, adjusting active and reactive power of a Synchronous Generator (SG) is used to regulate its frequency and terminal voltage. Active and reactive power can be controlled by either scalar or vector-based techniques. Excitation control is necessary in providing dynamical stability of power systems. The main object of field voltage control is to make the generator terminal voltage tracks its operating point value.

In [1] Mielczarski and Zajaczkowski derive a linearized state feedback for a reduced-order model of generator and investigate the behavior of the closed-loop system during and post fault. The control structure in [1] includes two feedback loops, an inner for feedback linearization, and an outer linear loop for locating the system poles at the desired positions in the complex plane, considering the limitations by the inner loop and system dynamical characteristics. Liu and Luo in [1] review various control strategies for electrical machines such as Field Oriented Control (FOC), Direct Torque Control (DTC) and Model Predictive Control (MPC). FOC is mainly based on decoupling among the components of stator current, flux and torque. The three-phase system is transferred into two-phase rotating coordinates [2]. In direct FOC the flux magnitude and angle feedback signals are directly calculated by voltage and current measurements. In [3] the flux and rotor position in an induction motor are estimated by Luenberger and PI observer, respectively.

In [5] a vector control based on Field-Orientation principle for a synchronous generator with excitation and damping windings is suggested. By neglecting the stator resistance, active power and reactive power are controlled by the quadrature and direct component of field-oriented stator current, respectively. To control active and reactive power flow through a short or medium length transmission line connected system, [4] proposes independent P-Q control equations. The control scheme in [4] is based on decoupling active and reactive output power of a synchronous generator by rotational
transformation of the synchronous generator’s terminal signals. DTC techniques do not include current control loops and are based on directly controlling the torque and stator flux [2], [6]. There are various schemes that apply DTC and the common attribute among them is using the torque and stator flux deviation from their reference value as the control variables. In [7] a simplified synchronous generator model connected to an infinite bus is presented in a Port-Hamiltonian formulation and a passivity-based design for excitation control of the generator is proposed. This passivity-based controlling technique is based on shaping the energy of the system and it is shown to produce an increase in critical clearing time compared to classical control schemes. However, the synchronous generator model in [7] is very simplified and the effects of load and network dynamics are neglected.

In most of the above papers simplifying assumptions are made about the dynamics of the synchronous generator. In this paper we present a general port-Hamiltonian model for a synchronous generator and discuss the effects of deviations and disturbances of the terminal conditions on generator control laws. For analysis purpose we present a linearized generator model and apply a full state feedback control consisting of both regulation of the generator flux and rotational dynamics and relate the general control law to classical control structures.

2. Preliminaries

1.1. Port Hamiltonian Systems
A system of the form
\[ \dot{x} = (J(x) - R(x))\nabla H(x) + B(x)u \]
\[ y = B^T(x)\nabla H(x) \]
where \( J(x) = -J^T(x), R(x) = R^T(x) \geq 0 \) and \( \nabla H(x) \) is the gradient of the Hamiltonian \( H(x) \) is called a port Hamiltonian system. The input \( u \) and output \( y \) are the port variables that connect the system to the external world (environment) [10]. It is easy to see that
\[ \frac{d}{dt} H(x(t)) = -\nabla H^T(x(t))R(x(t))\nabla H(x(t)) + y^T(t)u(t) \]
Thus, \( H(x(t)) = H(x(0)) - \int_0^t \nabla H^T(x(s))R(x(s))\nabla H(x(s))ds + \int_0^t y^T(s)u(s)ds \).

The term \( \int_0^t \nabla H^T(x(s))R(x(s))\nabla H(x(s))ds \) is the dissipated energy and \( \int_0^t y^T(s)u(s)ds \) is the exchanged energy with the environment thorough the input-output (port) variables \( u \) and \( y \). Consider two port Hamiltonian systems with states \( x_1, x_2, \) Hamiltonian functions \( H_1, H_2, \) system matrices \( J_1, R_1, B_1, \) and inputs and outputs \( u_i, y_i, i = 1, 2. \) Assume we connect the ports of the two systems so that \( u_1 = y_2 \) and \( y_1 = -u_2. \) Typically \( u_1 \) and \( y_1 \) would be potentials and \( y_2 \) and \( u_2 \) flows with the − sign in the second connection relationship representing a flow out of one of the systems and into the other. Then the total system satisfies the differential equation
\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} J_1(x_1) & B_1(x_1)B_1^T(x_2) \\ -B_2(x_2)B_2^T(x_1) & J_2(x_2) \end{bmatrix} \begin{bmatrix} R_1(x_1) & 0 \\ 0 & R_2(x_2) \end{bmatrix} \nabla H(x_1, x_2) \]
(1)
where \( H(x_1, x_2) = H_1(x_1) + H_2(x_2). \) We note that \( \begin{bmatrix} J_1(x_1) & B_1(x_1)B_1^T(x_2) \\ -B_2(x_2)B_2^T(x_1) & J_2(x_2) \end{bmatrix} \) is skew symmetric and \( \begin{bmatrix} R_1(x_1) & 0 \\ 0 & R_2(x_2) \end{bmatrix} \) positive semi definite and thus the combined system is a Hamiltonian system.

Finally note that if we add an additional input terms \( E_i(x)v_i \) to the two port Hamiltonian systems we get the non autonomous combined system corresponding to (1),
\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} J_1(x_1) & B_1(x_1)B_2^T(x_2) \\ -B_2(x_2)B_1^T(x_1) & J_2(x_2) \end{bmatrix} \begin{bmatrix} R_1(x_1) \\ 0 \end{bmatrix} - \begin{bmatrix} H(x_1,x_2) + \begin{bmatrix} E_i(x_1) \\ 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \] (2)

While the port variables represent energy flow between the two systems the inputs would typically be control and/or disturbance inputs.

1.2. Dynamics and Stability of Interconnected Power System Components

The Hamiltonian model of the electrical dynamics of a synchronous generator has the form

\[ \dot{x} = (J(\omega_e) - R)\nabla H_g(x_g) - M(\omega_e)\bar{v} - \bar{v} \]

\[ y_g = \begin{bmatrix} \bar{v} \\ i_f \end{bmatrix} \]

where \( x_g = \bar{x},aw^T = [\bar{v}_f^T, v_f] \), \( \bar{v} \), and \( i_f \) are the terminal voltage and terminal current, respectively and \( v_f \) and \( i_f \) are the field voltage and current, \( \bar{x} = \begin{bmatrix} \lambda_t \\ \lambda_f \end{bmatrix}, \bar{x} \) is the vector of armature flux and \( \lambda_f \) is the field flux. Here the Hamiltonian function has the form \( H_g(\bar{x}, \omega, \Delta \theta) = \frac{1}{2} \bar{x}^T \bar{L}^{-1} \bar{x} \).

The network and load that is connected to the synchronous generator can be represented by a total Hamiltonian function \( H_n \) that is typically a quadratic function of network and load inductance's fluxes and capacitance's

\[ \dot{y}_n = g_n(x_n)^T \nabla H_n(x_n) + g_n(x_n)u_n + e_n(x_n)v_n \]

where \( y_n = \bar{v}, u_n = -\bar{v} \) are the port variables and \( v_n \) is an external input. The system consisting of the synchronous generator and the network and load port-Hamiltonian model has the overall Hamiltonian function \( H(x_g,x_n) = H_g(x_g) + H_n(x_n) \) and dynamic equations of the overall system is

\[ \begin{bmatrix} \dot{x}_g \\ \dot{x}_n \end{bmatrix} = (J_g(x_g,x_n) - R_g)\nabla H(x_g,x_n) + M_g(x_g)\bar{v} + g_i u_i + E(x_n)v_n. \]

While each individual component \( H_g \) and \( H_n \) may be a stable system in isolation (with constant boundary conditions) the interconnected system may not be stable for all the operating conditions and stability can only be achieved by an appropriate control design such as Interconnection and Damping Assignment (IDA) \([8, 9]\) where a desired Hamiltonian function is assigned by proper control law design. This requires complete knowledge of \( H_n \) which in reality may not be available and thus alternative methods such as robust IDA may be needed. Such problem will be studied in future research.

3. Generator Model

A simple steady state model of a synchronous generator, is a voltage source behind a reactance where the voltage source magnitude is proportional to the (constant) excitation current and its frequency is the synchronous frequency \( \omega_e \). In reality, each generator can be considered as a complex controlled dynamical system consisting of several components. A port Hamiltonian dynamic model of the synchronous generator that with sufficient details to capture the dominant mechanical and electrical dynamics of the system is presented here. This port Hamiltonian model can be used for controls analysis and design. Corresponding dynamic models of other generator types, such as DFIG, will be considered elsewhere.

The swing equations for a generator have the form

\[ J_\omega \dot{\omega} = T_m - T_e \] (3)
where \( T_m \) is the mechanical torque produced by the prime mover and \( T_e \) is the electrical torque produced by the generator and \( \omega_m \) is the angular frequency of rotation. In the steady state situation, there is a balance between the mechanical and electrical torques and thus \( \omega_m = \text{const} \) where the \( \text{const} \) should be determined the network synchronous frequency \( \omega_n \). As a starting point we adopt a standard three axis model with armature coils \( a, b, c \) and field coil \( f \) (see e.g. [10,11]). The armature flux linkages satisfy

\[
\lambda_{\alpha\beta\gamma} = (L_\alpha + M_j) i_\alpha + M_f i_f C(\theta_f)
\]

(4)

where \( L_\alpha, M_\alpha \) are the armature self-inductance and armature mutual inductance, respectively, \( C(\theta_f) = \begin{bmatrix} \cos(\theta_f) & \cos(\theta_f - \frac{2\pi}{3}) & \cos(\theta_f - \frac{4\pi}{3}) \end{bmatrix}^T \) and \( L_{sf} = M_f \cos(\theta_f) \) mutual inductance between the field coil and armature coil \( a \), etc. In the above model \( i_a \) is the armature current in coil \( a \) and \( i_f \) is the field current. If coil \( a \) has resistance \( R_a \) then the voltage drop \( v_a \) from the terminal of \( a \) to neutral satisfies

\[
d\lambda_a = -R_a i_a - L_a i_a
\]

(5)

The other armature coils satisfy a corresponding relationship. The field circuit satisfies a corresponding (under balanced conditions)

\[
v_f = R_f i_f + \frac{d\lambda_f}{dt}
\]

(6)

where the field flux linkage is given by

\[
\lambda_f = L_g i_f + M_f \cos(\theta_f) i_a + M_f \cos(\theta_f - \frac{2\pi}{3}) i_b + M_f \cos(\theta_f - \frac{4\pi}{3}) i_c
\]

(7)

and \( L_g \) is the field coil self-inductance. Combining (4) and (7) as well as (5) and (6) gives \( \dot{\lambda} = L(\theta_f) i \) and \( \dot{\lambda} = -R_i - v \), where

\[
L(\theta_f) \begin{bmatrix} (L_\alpha + M_j) & M_f \cos(\theta_f) \\ M_f \cos(\theta_f - \frac{2\pi}{3}) & L_f \end{bmatrix} M(\theta_f) = \begin{bmatrix} \lambda_a \\ \lambda_f \end{bmatrix}
\]

\[
R = \begin{bmatrix} R_a & 0 \\ 0 & R_f \end{bmatrix}, \dot{\lambda} = \begin{bmatrix} \lambda_a \\ \lambda_f \end{bmatrix}, v = \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_f \end{bmatrix}, i = \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}
\]

If we define the Hamiltonian, \( H(\lambda) = \frac{1}{2} \lambda^T L^{-1}(\theta_f) \lambda \) the system equations take the port Hamiltonian form (with input matrix \( B = -I \))

\[
\dot{\lambda} = -R \nabla H(\lambda) - v = -R \nabla H(\lambda) + B v
\]

(8)

where the output is the negative of the current

\[
y = -i = B^T \nabla H(\lambda)
\]

(9)

Note that \( -i^T v = -P_e + i_f v_f \) where \( i_f v_f \) is the power supplied to the rotor (field) circuit and \( P_e = v_a i_a + v_b i_b + v_c i_c \) is the generator power output. Consequently, \( i_e^T v \) is the generator net power output. When we combine (8) and the rotational dynamics (3) using the relationship

\[
T_e = \frac{P_e}{\omega}, \quad \dot{T}_e = \frac{P_e}{\omega} \omega_m, \quad \frac{P_e}{\omega} = \frac{v}{\omega} i_a + \frac{v_b}{\omega} i_b + \frac{v_c}{\omega} i_c
\]

and \( \dot{\theta}_d = \frac{P_e}{2} - \dot{\theta}_m = \frac{P_e}{2} \omega_m \) we have the complete dynamics of the generator. We now map the
generator dynamics into coordinates defined by a rotating reference frame at the system reference frequency $\omega_R$. Indeed, let $T(\theta_R)$ be the (modified) Park transformation defined by

$$T(\theta_R) = \begin{bmatrix}
\frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\cos(\theta_R) & \cos(\theta_R - \frac{2\pi}{3}) & \cos(\theta_R - \frac{4\pi}{3}) \\
\sin(\theta_R) & \sin(\theta_R - \frac{2\pi}{3}) & \sin(\theta_R - \frac{4\pi}{3}) \\
0 & 0 & 0
\end{bmatrix}$$ (10)

and consider a change of coordinates $\vec{x} = T(\theta_R)x$ where the reference angle is defined as $\theta_R = \omega_R t$ and $x, \vec{x}$ are any of the stator related variables, e.g. in the new coordinates, so-called dq0 coordinates, the armature flux becomes $[\lambda_d \lambda_q \lambda_0]^T = T(\theta_R)[\lambda_d \lambda_q \lambda_0]^T$. We note that these are the dq0 coordinates defined here are with the respect to the reference frame, not with respect to the rotational reference frame of the machine itself (at angle $\theta_R$). The field quantities are not transformed, and we define a total coordinate change by $Y(\theta_R) = \begin{bmatrix} T(\theta_R) & 0 \\ 0 & 1 \end{bmatrix}$.

It is not hard to see that $Y^T(\theta_R)Y(\theta_R) = I$. Note that in terms of the new coordinates

$$\vec{\lambda} = \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} = Y(\theta_R)\lambda$$

the Hamiltonian takes the form

$$H(\vec{\lambda}) = \frac{1}{2} \vec{\lambda}^T \vec{L}^{-1}(\theta_d, \theta_R) \vec{\lambda}$$ (11)

where $\vec{L}(\theta_d, \theta_R) = Y(\theta_R)L(\theta_d)Y^T(\theta_R)$. It is easy to see that with $\Delta \theta = \theta_R - \theta_d$ we have

$$\vec{L}(\theta_d, \theta_R) = \vec{L}(\Delta \theta), \vec{L}(\Delta \theta) = \begin{bmatrix} (L_s + M_s)I & \vec{M}(\Delta \theta) \\ \vec{M}(\Delta \theta)^T & L_{ff} \end{bmatrix}$$

where

$$\vec{M}(\Delta \theta) = \begin{bmatrix} \frac{2}{\sqrt{3}} M_f \cos(\Delta \theta) \\ \frac{2}{\sqrt{3}} M_f \sin(\Delta \theta) \\ 0 \end{bmatrix}. \text{Note that for constant speed } \Delta \theta = \theta_R - \theta_d = (\omega_R - \omega)t - \theta_{d0}, \text{i.e.}$$

when $\omega_R - \omega = 0$ matrix $\vec{L}(\Delta \theta)$ depends only on the "initial" generator angle $\theta_{d0}$.

A straightforward calculation shows that

$$\frac{d}{dt} \vec{\lambda} = N(\omega_R)\vec{\lambda} + (-R\vec{\lambda} - \vec{v})$$

where $\vec{\lambda} = Y(\theta_R)v$, $\vec{v} = Y(\theta_R)v$ and we have used the fact that $Y(\theta_R)R Y^T(\theta_R) = R$ and

$$\left( \frac{d}{dt} Y(\theta_R) \right) Y^T(\theta_R) = \omega_R$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = N(\omega_R)$$

Furthermore, since $\vec{\lambda} = \vec{L}(\Delta \theta)\vec{\lambda}$ we
have
\[ \frac{d}{dt} \lambda = N(\omega_R) \dot{\lambda} + (-R \dot{\nu} - \dot{v}) = N(\omega_R) \dot{\lambda} - RL^{-1}(\Delta \theta) \dot{\lambda} - \dot{v} = (N(\omega_R) \dot{\lambda} - R) \nabla H(\lambda) - \dot{v} \]
where \( H(\lambda) \) is given by (11). We note that \( H(\lambda) \) is dependent on the angle difference \( \Delta \theta = \theta_R - \theta_d \).

Furthermore,
\[
N(\omega_R)L(\Delta \theta) = \omega_R \left[ \begin{array}{cccc}
0 & -(L_s + M_s) & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array} \right] + \omega_R \left[ \begin{array}{cccc}
\frac{-3}{2} M_f \sin(\Delta \theta) & 0 & 0 & 0 \\
0 & \frac{3}{2} M_f \cos(\Delta \theta) & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array} \right] = J(\omega_R) + M(\omega_R, \Delta \theta)
\]

Thus
\[
\frac{d}{dt} \lambda = (N(\omega_R)L(\Delta \theta) - R) \nabla H(\lambda) - \dot{v} = (J(\omega_R) - R) \nabla H(\lambda) + m(\omega_R, \Delta \theta) i_f - \dot{v}
\]
(12)

where \( m(\omega_R, \Delta \theta) \) is the last column of \( M(\omega_R, \Delta \theta) \).

With input term, \( Bu = v \), i.e. \( B = -I \) and \( \nu = \nabla \). We define the output equation in the usual way as
\[
\nabla^T \nu = -\nabla^T = -T
\]
(13)

Then \( \nabla^T \nu = -T^T \nu = -iY^T(\theta_R)Y(\theta_R)v = -i^T v = -P_e + i_f v_f \) as before. Furthermore, after some manipulations \( \frac{dh}{dt} = \nabla H(\lambda)^T \frac{d}{dt} \lambda = -T^T \dot{R} + \omega_R \sqrt{2} M_f i_f (\cos(\Delta \theta)i_q - \sin(\Delta \theta)i_d) - P_e + i_f v_f \).

We note that under steady state conditions the generator delivers power \( P_e \) to the network.

Consequently, \(-i^T \dot{R} + \omega_R \sqrt{2} M_f i_f (\cos(\Delta \theta)i_q - \sin(\Delta \theta)i_d) + i_f v_f = 0 \) in steady state.

Consider again the rotational dynamics (in terms of \( \dot{\theta}_d \) and \( \omega \)). We note that the generator model (12), (13) depends on the angle difference \( \Delta \theta = \theta_d - \theta_R \). Define \( \dot{\omega} = \frac{d\Delta \theta}{dt} = \omega - \omega_R \) (since \( \dot{\theta}_R = \omega_R t \)).

Then we have
\[
\frac{d}{dt} \Delta \theta = \Delta \omega
\]
\[
\dot{J} \frac{d}{dt} \Delta \omega = T_m - T_e
\]
(14)

where we used the fact that \( \omega_R \) is constant and \( J = \frac{2J}{p} \).

1.1. Generator Control

When we consider the synchronous generator system as a controlled system the field voltage \( v_f \) and the mechanical torque \( T_m \) are control inputs. In most "classical" applications the control objective is to regulate the terminal voltage, the generator frequency and the power output (real and reactive). The most common approach is to regulate the frequency and real power output is by adjusting the mechanical power (torque) input \( T_m \) (this requires a model of the prime mover that generates the mechanical torque). The terminal voltage and reactive power are regulated by adjusting the field voltage.

For a fixed value of \( \omega \) say \( \omega = \omega_R \), and rotor phase angle \( \Delta \theta_0 \) define \( A = A(\Delta \theta_0) = (N(\omega_R) - RL^{-1}(\Delta \theta_0)) \) and consider the steady state flux equation \( 0 = A \lambda - \nabla \), along with the design constraints.
where $v_t$ is the terminal voltage part of $v_s$, $td = L^{-1}(\theta_d)\lambda_d$ and $v_w, P_{ed}$ and $Q_{ed}$ are the design values for the terminal voltage and terminal real and reactive power. We can rewrite the steady state equations as

$$0 = A(\Delta \theta) \hat{\lambda} - \hat{v} = A(\Delta \theta) \tilde{L}(\Delta \theta) \bar{i}_s - \hat{v} = A(\Delta \theta) \tilde{L}(\Delta \theta) \begin{bmatrix} i_{ts} \\ i_{fs} \end{bmatrix} - \begin{bmatrix} I_o \\ 0 \end{bmatrix} v_{fs}$$

**Proposition:** For given values of the steady state rotor speed and phase angle, $\omega = \omega_R, \Delta \theta = \Delta \theta_0$, as well as terminal voltage, $v_t = v_{td}$, and terminal real and reactive powers, $P_e = P_{ed}, Q_e = Q_{ed}$ there exists a (unique up to phase shift) steady terminal current, $\bar{i}_t$, and field voltage input $v_{fi}$. The steady state generator flux is given by $\hat{\lambda} = \tilde{L}(\Delta \theta) \begin{bmatrix} i_{ts} \\ i_{fs} \end{bmatrix}$ where $\bar{i}_f = \frac{v_{fs}}{R_f}$. 

**Proof:** From the power design constraints and the terminal voltage constraint we can (completely) characterize $\bar{\lambda}$. Indeed, the steady state terminal voltage has the form $v_t = \begin{bmatrix} \cos \phi_t & \sin \phi_t & 0 \end{bmatrix} V$ and, similarly, the steady state terminal current has the form $\bar{i}_t = \begin{bmatrix} \cos \phi_t & \sin \phi_t & 0 \end{bmatrix} I$. From (15) we get $\cos(\phi_f) = -\frac{v_{fs}}{v_t}$ and $\sin(\phi_f) = -\frac{v_{fs}}{v_t}$ and thus we see that we can solve for the unknown $I$ and $\phi_f$. Consider the equation (12) in steady state i.e. $0 = (f(\omega_R) - R)\nabla H(\hat{\lambda}) + m(\omega_R, \Delta \theta_0)\hat{i}_f - \hat{v}$

Noting that $J(\omega_R) = R = \begin{bmatrix} \omega_L(L_s + M_s)N - R_s I \\ 0 \\ -R_f \end{bmatrix}$ and $\nabla H(\tilde{\lambda}) = \bar{i}_t$ we easily get

$$0 = (\omega_R(L_s + M_s)N - R_s I)\tilde{i}_s + \hat{m}(\omega_R, \Delta \theta_0)\hat{i}_f - \hat{v}_s$$

We finally note that the steady state flux is given by $\tilde{\lambda}_s = \tilde{L}(\Delta \theta_0)\bar{i}_s$ where $\Delta \theta_0 = \theta_R - \Delta \theta_d$ is the steady state angle.

The control problem can now be formulated as the problem of regulating the state $\tilde{\lambda}$ to the steady state value $\tilde{\lambda}_s$ and the rotor speed and angle to the design values $\omega_R, \Delta \theta_0$.

Consider the overall dynamics of the system on the form

$$\frac{d}{dt} \tilde{\lambda} = (N(\omega_R) - RL^{-1}(\Delta \theta))\tilde{\lambda} - \hat{v}$$

$$\frac{d}{dt} \Delta \omega = \frac{1}{J} \left( T_m - \frac{P_e}{\omega_R + \Delta \omega} \right)$$

Linearizing (16) given by (15) gives $\frac{d}{dt} x = Fx + Gu + Hd$, where
\( F = \begin{bmatrix} A(\Delta \theta_0) & Rl(\Delta \theta_0, \lambda_s) & 0 \\ -a(\tilde{v}_t, \Delta \theta_0) & b(\tilde{v}_t, \Delta \theta_0, \lambda_s) & -c(\tilde{v}_t, \Delta \theta_0, \lambda_s) \end{bmatrix} \), \( G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \), \( H \)

where the control input is \( u = \begin{bmatrix} \Delta \nu_f \\ \Delta T \end{bmatrix} \) while \( d = \tilde{v}_t \) is a disturbance input and

\[
\begin{align*}
\delta \theta &= (\Delta \theta - \Delta \theta_0), \\
\Delta T &= T_m - T_{es}, \\
a(\tilde{v}_t, \Delta \theta_0) &= \frac{\tilde{v}_t^T[I - 0]l(\Delta \theta_0, \lambda_s)}{\omega_R} \\
b(\tilde{v}_t, \Delta \theta_0, \lambda_s) &= \frac{l(I \tilde{\lambda})}{\omega_R} \\
c(\tilde{v}_t, \Delta \theta_0, \lambda_s) &= \frac{P_{ed}}{\omega_R} d(\lambda_s, \Delta \theta_0) \frac{\delta l(\Delta \theta_0)}{\delta \theta} \frac{l(I \tilde{\lambda})}{\omega_R} \\
l(I \tilde{\lambda}) &= -l^{-1}(\Delta \theta_0) \frac{\delta l(\Delta \theta_0)}{\delta \theta} l^{-1}(\Delta \theta_0) \tilde{\lambda}_s.
\end{align*}
\]

The equilibrium point of the linearized system is at the origin, i.e. when \( x = \) we have \( \lambda = \lambda_s \) and \( \omega = \omega_g \). We note that when \( u = 0 \) we have \( T_{es} = T_m \). The control objective is to regulate the state to zero for all values of the disturbance input. If we apply a state feedback law \( u = -Kx \) to this system we get the closed loop system \( \frac{dx}{dt} = (F - GK)x + Hd \).

Applying the Laplace transform to this equation we obtain
\[
x(s) = (sI - (F - GK))^{-1}Hd(s) = (sI - (F - GK))^{-1}Hd(s)
\]

Clearly, in order to reject the disturbance input \( d \) the feedback matrix \( K \) should be chosen so that the effect of the \( d \) on \( x \) should be minimized, i.e. \( G_{at}(s, K) = (sI - (F - GK))^{-1}H \) should be "small". The first component of the control law is

\[
\Delta \nu_f(s) = [1 \ 0]u(s) = -[1 \ 0]Kx(s) = -[1 \ 0]KG_{dx}(s, K)d(s) = C(s)d(s)
\]

This control law has the familiar form \( \Delta \nu_f(s) = C(s)d(s) = C(s)(\tilde{v}_t - \tilde{v}_td) \), i.e. exciter control where the objective is to select the exciter control input so as to regulate the terminal voltage to the zero. Since the terminal voltage is not a state in the generator system but rather an external input we see that the exciter control attempts to reject deviations of the terminal voltage from the steady state reference value.

The second component of the control law has the form

\[
\Delta T = [0 \ 1]u(s) = -[0 \ 1]Kx(s) = -[0 \ 1]KG_{dx}(s, K)d(s) = D(s)d(s)
\]

Consequently, deviations in the terminal voltage will result in deviations in the net torque and thus the terminal power output.

As we discussed in Section 2.2 the stability of the overall power system depends on the interconnection of the system components. This is a well studied problem for the swing dynamics of interconnected generators and various stability conditions exists for synchronous operation. Stability conditions for systems where the time scale separation between the swing and electrical dynamics is no longer valid has not been studied much. A notable exception is ~\cite{Haml-Fiaz-ortega} where a generator connected to a simple load was considered and clearly this an open area of research.
4. Conclusion

In this paper we presented a port-Hamiltonian model for a synchronous generator and discussed how control laws should be designed to minimize the effect of small variations in the generator's terminal conditions, i.e., local control that does not have access or knowledge to a model of the rest of the system.

5. References

[1] W. Mielczarski and A.M. Zajaczkowski, ¨Nonlinear field voltage control of a synchronous generator using feedback linearization,"Automatica, VOL. 30, NO. 10, 1994.
[2] C. Liu and Y. Luo, ¨Overview of advanced control strategies for electric machines,"Chinese Journal of Electrical Engineering, VOL. 3, NO. 2, pp. 53-61, 2017.
[3] B. H. Mouna and S. Lassaad, ¨Direct Stator Field Oriented Control of Speed Sensorless Induction Motor,"2006 IEEE International Conference on Industrial Technology, pp. 961-966, 2006.
[4] H. A. Khan and P. Bargiev and V. Sreeram and H. H. C. Iu and T. L. Fernando and Y. Mishra,¨Active and reactive power control of synchronous generator for the realization of a virtual power plant,"IECON 2012 - 38th Annual Conference on IEEE Industrial Electronics Society, pp. 1204-1210, 2012.
[5] M. Imecs, I. Iov Incze, C. Szabó, ¨Stator-Field Oriented Control of the Synchronous Generator: Numerical Simulation,"12th International Conference on Intelligent Engineering Systems, 2008.
[6] C. Lascu, and A. M. Trzynadlowski, ¨Combining the principles of sliding mode, direct torque control, and space-vector modulation in a high performance sensorless AC drive" IEEE Trans. Ind. Appl., vol. 40, no. 1, pp. 170-177, Jan./Feb. 2004.
[7] M. Galaz, R. Ortega, A. S. Bazanella and A. M. Stankovic, ¨An energy-shaping approach to the design of excitation control of synchronous generators," Automatica , VOL. 39, NO. 1, pp. 11-119, 2003.
[8] R. Ortega, A. van der Schaft, A. Astolfi, ¨Control by Interconnection and Standard Passivity-based Control of Port-Hamiltonian Systems," IEEE Transaction on Automatic Control, VOL. 53, NO. 11, pp. 2527-2542, Dec 2008.
[9] R. Ortega, A. van der Schaft, B. Maschke and G. Escobar, ¨Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems," Automatica, VOL. 38, NO. 4, pp. 585-596, 2002.
[10] T. Runolfsson, ¨On the dynamics of three phase electrical energy systems," 2016 American Control Conference (ACC), pp. 6827-6832, July 2016.
[11] D. P. Kothari, I. J. Nagrath, ¨Modern Power System Analysis,"McGraw-Hill Science, 2008.
[12] S. Fiaz, D. Zonetti, R. Ortega, J.M.A. Scherpen, A.J. van der Schaft, ¨A port-Hamiltonian approach to power network modeling and analysis," European Journal of Control, Vol. 19, pp. 477-485, 2013.