Strongly Coupled Semi-Direct Mediation of Supersymmetry Breaking

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Abstract

Strongly coupled semi-direct gauge mediation models of supersymmetry breaking through massive mediators with standard model charges are investigated by means of composite degrees of freedom. Sizable mediation is realized to generate the standard model gaugino masses for a small mediator mass without breaking the standard model symmetries.
1 Introduction

Supersymmetry (SUSY) \[1\] is expected to be a crucial ingredient of basic laws in Nature. It is an attractive possibility that SUSY is broken at low energy within the experimental reach in the near future. Among others, low-energy dynamics with gauge mediation between a hidden sector of SUSY breaking and the visible sector of SUSY standard model may be phenomenologically viable \[2\]. In particular, the gauge interactions are flavor blind, so that the unwanted flavor-changing processes are naturally suppressed.1

The pivot of gauge mediation consists of messenger fields that are charged under the standard model gauge symmetries. We can assume messengers of three types \[3\]: minimal, direct, and semi-direct \[4, 5\].2 Let \( X = m + \theta^2 F \) be a representative spurion for R and SUSY breaking (presumably with dynamical origin), where \( \theta \) denotes the superspace coordinate. The minimal gauge mediation is given by a superpotential term \( X q \tilde{q} \) with a standard model vector-like pair \( q \) and \( \tilde{q} \) of chiral superfields as the messengers. The direct gauge mediation is more economical class which is given by a superpotential term \( X Q \tilde{Q} \) with standard model vector-like pairs \( Q \) and \( \tilde{Q} \) of chiral superfields as the mediators with hidden gauge interaction charges (whose dynamics cause SUSY breaking encoded in the \( X \) value). The semi-direct mediation which, as we shall see shortly, can overcome one of the difficulties of the direct gauge mediation is given by a superpotential mass term \( \mu Q \tilde{Q} \) with \( \mu \) a constant and a representative term \( X \Phi \tilde{\Phi} \) with a hidden gauge interaction vector-like pair \( \Phi \) and \( \tilde{\Phi} \) of standard-model singlet chiral superfields.3

An important difference of the semi-direct gauge mediation from the direct mediation is that the messengers do not play important roles in dynamical SUSY breaking. Thus, in the semi-direct mediation models, the rank of gauge group required in the SUSY breaking sector can be smaller than that in the direct mediation models. Hence, in the semi-direct mediation model, we can ameliorate the Landau pole problem that is often encountered in the direct mediation models at low-energy scale.

In the semi-direct mediation, however, it seems problematic that the gaugino masses

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1The SUSY flavor problem is also discussed in \[1\].

2Some other ingredients than messenger fields may well be present such as additional nonabelian messenger gauge interaction \[4, 6\], which we do not consider in this paper.

3This by no means gives a complete classification of gauge mediation models. For example, the models in Ref.\[7\] contain massive messengers without hidden gauge interaction charges.
in the SUSY standard model vanish to the leading order in $F$ (the gaugino screening [8]). This is because the supersymmetric masses of the messengers do not lead to the SUSY breaking in the holomorphic standard-model gauge coupling and the leading contribution only occurs by the wavefunction renormalization at the higher loop order. On the other hand, the scalar masses emerge at the leading order in $F$. Thus, the gaugino masses are suppressed at least by $|F/m^2|^2$ compared with the scalar masses, which leads to a little hierarchy between the gaugino masses and the scalar masses without careful tuning between the sizes of $F$ and $m$. If this hierarchy exists, we cannot set the gaugino masses and the scalar masses at the order 1 TeV at the same time, which causes the standard model hierarchy problem again.

In this paper, we consider a possible amelioration of the little hierarchy between the gaugino masses and the scalar masses in the semi-direct mediation while keeping the indirectness of the messengers–SUSY breaking interaction. For that purpose, we introduce a new mechanism of mediating the SUSY breaking effect to the SUSY standard model, that is, we consider a model of strongly coupled semi-direct gauge mediation where the mass of the mediators are small compared with the dynamical scale in the SUSY breaking sector. In the strongly coupled model, the semi-direct messengers make bound states due to the strong dynamics in the SUSY breaking sector and obtain non-vanishing SUSY breaking vacuum expectation values, which make it possible for the gaugino masses in the SUSY standard model to have the leading contribution in $F$.

In the next section, we provide our model of strongly coupled semi-direct gauge mediation and derive its effective theory description. In section 3, the SUSY-breaking vacuum of the model is investigated in terms of the effective theory. Section 4 is devoted to estimating the standard model gaugino masses to see the scale of visible soft masses. The final section concludes the paper with a few comments.

2 The model

We adopt hidden SUSY $SU(N)$ gauge theory with a superpotential

$$ W = X(\Phi \bar{\Phi}) + \mu \text{tr}Q\bar{Q} + \kappa (\text{tr}Q\bar{Q})^2, $$

(1)

4The $|F/m^2| \lesssim 1$ is required for the messenger sectors not to have tachyonic modes.
where the spurion $X$ is given by $X = m + \theta^2 F$, $\Phi$ and $\bar{\Phi}$ denote a vector-like pair of the hidden gauge interaction, and $Q$ and $\bar{Q}$ are (anti-)fundamentals with $N_f = N + 1$ massive flavors. The subgroups of the flavor symmetry, $SU(N_f)$, are eventually identified with the gauge groups in the standard model. In particular, $N = 4$ implies $N_f = 5$ with $SU(N_f)$ grand unification structure of the visible sector. In the above superpotential, $\mu$ denotes a mass parameter of $Q$’s which is assumed to be smaller than the dynamical scale of $SU(N)$ gauge theory, $\Lambda_H$, i.e. $\mu \ll \Lambda_H$, and $\kappa$ denotes a coupling constant with a negative mass dimension.

Let us first integrate out the hidden matter $\Phi, \bar{\Phi}$, assuming $m \gg \Lambda_H$, which results in SUSY QCD-like theory of $N_f$ massive flavors with mass $\mu$ and spurious dynamical scale $\Lambda^b = X^a \Lambda_H^{b-a}$, where $b = 3N - N_f = 2N - 1$ and $X$ is again $X = m + \theta^2 F$. (For example, $a = N_\Phi$ for $N_\Phi$ pairs of (anti-)fundamentals $\Phi$ and $\bar{\Phi}$.) For later convenience, we define

$$
\Lambda = \Lambda_0 (1 + \theta^2 R) = m \Lambda_H^{1 - \delta} \left( 1 + \theta^2 \frac{a F}{b m} \right).
$$

This ratio $R \sim F/m \gtrsim 100\text{ TeV}$ turns out to be a characteristic mass scale of gauge mediation.

Further, we proceed to integrating out the gauge and matter degrees of freedom to obtain the effective theory of meson $M$ and baryon $B, \bar{B}$ chiral superfields. Under the standard model gauge groups, the mesons behave as the singlet and adjoint representations, while the baryons behave as the (anti-)fundamental representations. The superpotential and Kähler potential of those composite fields are given by

$$
W_{\text{eff}} = \frac{1}{\Lambda^b} (BM \bar{B} - \det M) + \mu \text{tr} M + \kappa (\text{tr} M)^2,
$$

and

$$
K_{\text{eff}} = \left( \frac{\alpha^2 |M|^2}{|\Lambda|^2} + \frac{\beta^2 |B|^2}{|\Lambda|^{2(N-1)}} + \frac{\beta^2 |\bar{B}|^2}{|\Lambda|^{2(N-1)}} \right) K,
$$

where $\alpha$ and $\beta$ are positive constants and

$$
K(M/\Lambda^2, B/\Lambda^N, \bar{B}/\Lambda^N, \Lambda/X, \mu/\Lambda, \kappa \Lambda) = 1 + \cdots.
$$

The meson $M$ is an $N_f \times N_f$ matrix and the baryons $B, \bar{B}$ are $N_f$-component vectors.
Here, the ellipsis denotes the higher contributions to the Kähler potential in the arguments.

In the supersymmetric limit, \( F = 0 \), with \( \mu, \kappa > 0 \), the vacuum expectation values are given by \( \langle M \rangle \simeq \sqrt[\lambda]{\mu \Lambda_0 b} \) and \( \langle B \rangle = \langle \tilde{B} \rangle = 0 \), where we have assumed the coupling constant \( \kappa \) to be so small as \( \kappa \sqrt{\mu \Lambda_0 b} \ll \mu \). We presume the case \( |F| \ll m^2 \) and \( \mu \ll \Lambda_0 \) to adopt an approximation \( K \simeq 1 \) in the following sections.

### 3 The SUSY-breaking vacuum

Now, let us investigate how the original SUSY breaking effect in \( X \) is propagated into the mediator fields. For small SUSY breaking \( |R| \ll \mu \), we still expect \( \langle B \rangle = \langle \tilde{B} \rangle = 0 \) and diagonal \( \langle M \rangle \), and hence, the standard model gauge symmetries are not broken at the vacuum. The (soft) masses of the composite degrees of freedom may be seen from Eqs.\( 11 \) and \( 12 \) for retrospective justification of these vacuum expectation values.

To see the SUSY breaking effects on composite fields explicitly, let us define the normalized chiral superfields:

\[
M = \alpha M/\Lambda_0, \quad B = \beta B/\Lambda_0^{N-1}, \quad \tilde{B} = \beta \tilde{B}/\Lambda_0^{N-1}. \tag{6}
\]

With the aid of

\[
\frac{1}{\Lambda^b} = \frac{1}{\Lambda_0^b} \left( 1 - \theta^2 bR \right) = \frac{1}{\Lambda_0^b} \left( 1 - \theta^2 \frac{aF}{m} \right), \tag{7}
\]

we obtain

\[
W_{\text{eff}} = (1 - \theta^2 bR) \left( \lambda B M \tilde{B} - \frac{\gamma}{\Lambda_0^{N-2}} \det M \right) + \tilde{\mu} \Lambda_0 \text{tr} M + \tilde{\kappa} \Lambda_0^3 (\text{tr} M)^2, \tag{8}
\]

where \( \lambda = \alpha^{-1} \beta^{-2}, \gamma = \alpha^{-\left(N+1\right)}, \tilde{\mu} = \alpha^{-1} \mu, \tilde{\kappa} = \alpha^{-2} \kappa \) and

\[
K_{\text{eff}} \simeq |1 - \theta^2 R|^2 |M|^2 + |1 - \theta^2 (N - 1) R|^2 (|B|^2 + |	ilde{B}|^2). \tag{9}
\]

The effective scalar potential is obtained by performing the integrals over the Grassmann coordinates in the above effective superpotential and the Kähler potential. Then, minimizing this effective potential, we see

\[
\frac{\mathcal{M}_1}{\mathcal{M}_0} \simeq \frac{bR}{N} \left( 1 + \frac{2\kappa \langle \text{tr} M \rangle}{N \mu} \right) \propto F, \tag{10}
\]

\[5\]
for the vacuum expectation value $\langle \mathcal{M} \rangle = (\mathcal{M}_0 + \theta^2 \mathcal{M}_1) \mathbb{I}$ to the leading order of the SUSY-breaking parameter $R$ of gauge mediation, where we abuse the notation $\langle \text{tr} M \rangle$ for its lowest component as $\langle \text{tr} M \rangle \simeq N_f \sqrt{\mu \Lambda_0}$.

Therefore, we found that the diagonal component of the meson fields, $\mathcal{M} \propto \mathbb{I}$, obtains a non-vanishing SUSY breaking $F$-term vacuum expectation value. As we will see below, the $F$-term vacuum expectation value of the diagonal component plays a crucial role to generate the gaugino mass in the SUSY standard model at the leading order in $F$.

4 Visible gaugino masses

Now, we evaluate the standard model gaugino masses in terms of the vacuum expectation value discussed above. In the SUSY-breaking vacuum, we obtain

$$W_{\text{eff}} \simeq (1 - \theta^2 bR) \left( \lambda B \langle \mathcal{M} \rangle \bar{B} + \frac{1}{2} \frac{\gamma}{\Lambda_0^{N-2}} (\mathcal{M}_0 + \theta^2 \mathcal{M}_1)^{N-1} \text{tr} \tilde{M}^2 \right),$$

where $\tilde{M} = \mathcal{M} - \langle \mathcal{M} \rangle$, and

$$K_{\text{eff}} \simeq |1 - \theta^2 bR|^2 |\tilde{M}|^2 + |1 - \theta^2 (N - 1) R|^2 (|B|^2 + |\bar{B}|^2),$$

as the leading approximations to the quadratic terms of the composite fields charged under the standard model gauge symmetries.

There are two main contributions to the gaugino masses. One is from the threshold corrections which emerge when we integrate out the mesons $\mathcal{M}$ and the baryons $B$ and $\bar{B}$ which obtain SUSY breaking scalar masses from the $F$-term vacuum expectation value of the diagonal $\mathcal{M}$ via the superpotential in Eq. (11). The other contribution at the leading order comes from the threshold corrections which emerge when we integrate out the heavier modes than the mesons and the baryons in the strong dynamics. Although, it is highly difficult to calculate the threshold corrections from those heavy modes in the strong dynamics in general, it is possible to extract the gaugino mass with the help of the anomaly matching conditions of the global symmetries [10].

First, let us calculate the gaugino mass contribution from the mesons and baryons by analytic continuation of the gauge coupling renormalization factors into superspace [11].

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6The SUSY breaking effects arising from the Kähler potential, while present, are subleading in $F$ and therefore will not be considered in this paper.
Table 1: the charge assignments in the microscopic theory.

| $Q$ | $\bar{Q}$ | $\mu$ | $\kappa$ | $\Lambda^b$ |
|-----|---------|-------|---------|------------|
| $N$ | $\bar{N}$ | 1     | $2\frac{N}{N_f}$ | 0         |
| $N_f$ | $\bar{N}_f$ | $1 - \frac{N}{N_f}$ | $2\frac{N}{N_f}$ | $2N_f$ |

Table 2: the charge assignments in the macroscopic theory.

| $B$ | $\tilde{B}$ | $M$ |
|-----|-------------|-----|
| $N_f$ | $\bar{N}_f$ | $\text{adj} + 1$ |
| $N - \frac{N^2}{N_f}$ | $N - \frac{N^2}{N_f}$ | $2 - 2\frac{N}{N_f}$ |

By using the superpotential Eq. (11) and the vacuum expectation value of $F$-term in Eq. (10), we obtain the gaugino mass contribution from mesons and baryons,

$$\delta m_{1/2} \simeq \frac{\alpha_{\text{sm}}}{2\pi} b R \left( \frac{\kappa \langle \text{tr} M \rangle}{\mu} - 1 \right) \propto F,$$

(13)

where $\alpha_{\text{sm}}$ indicates the standard model gauge couplings (squared over $4\pi$) and $b$ and $R$ are defined in Eq. (2) and just above it.

The other leading contribution to the gaugino masses, i.e., the threshold effects of the heavier modes in the strong dynamics, is estimated in the following way. Let us first recapitulate the gauge and global symmetries of the model, whose charge assignments to the matter fields in the microscopic and the macroscopic theory are shown in tables 1 and 2, respectively. We also assign charges to the coupling constants, regarding them as background chiral superfields. The anomalous $U(1)_A$ global symmetry can be treated as a symmetry by rotating the dynamical scale $\Lambda$.

The $U(1)_A - SU(N_f)^2$ anomaly matching condition implies that we further need an effective operator of the form $(\ln \Lambda^b) W^a W_a$, where $W$ denotes the field strength superfield of the standard model gauge multiplets. Hence we conclude that the standard model
gauge masses are given by

\[ m_{1/2} \simeq \frac{\alpha_{\text{sm}}}{2\pi} b R \left( \frac{\kappa \langle \text{tr} M \rangle}{\mu} - 1 \right) + \frac{\alpha_{\text{sm}}}{2\pi} b R = \frac{\alpha_{\text{sm}} a \kappa F}{2\pi \mu m} \langle \text{tr} M \rangle \propto F, \tag{14} \]

where the \( \kappa \)-independent contributions cancel out and the results are proportional to the dynamical scale \( \Lambda_0 \) of the theory in accord with the perturbative gaugino screening. As a result, we find that the gaugino mass in the standard model can be generated at the leading order in \( F \) which comes from the non-vanishing \( F \)-term expectation value of the diagonal component of the composite meson.

5 Conclusion

In this paper, we proposed a possible solution of the gaugino screening problem in the semi-direct mediation by introducing the strongly coupled messenger sector. We investigated the model Eq. (11) of strongly coupled semi-direct gauge mediation and its effective theory given by Eqs. (11) and (12). Sizable mediation is realized to generate the standard model gaugino masses Eq. (14) without breaking the standard model symmetries. That is, the leading contribution to the gaugino masses does not vanish and is proportional to the dynamical scale of the model with a small mediator mass. We note that similar analyses can be performed for more general gauge theories, though we have definitely considered the model of hidden \( SU(N) \) gauge theory with \( N_f = N + 1 \).

Finally let us mention a few obvious issues to be investigated. One is the scale of the gravitino mass. It depends on the origin of SUSY-breaking spurion, which we have not specified in this paper. Another is the SUSY-breaking scale itself. We have restricted ourselves to the case of small SUSY breaking, \( |F| \ll m^2 \), though large SUSY-breaking case seems interesting to explore, which may achieve lowest-scale gauge mediation with a cosmologically favorable tiny gravitino mass.

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