Infall near clusters of galaxies: comparing gas and dark matter velocity profiles

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ABSTRACT
We consider the dynamics in and near galaxy clusters. Gas, dark matter and galaxies are presently falling into the clusters between approximately 1 and 5 virial radii. At very large distances, beyond 10 virial radii, all matter is following the Hubble flow, and inside the virial radius the matter particles have on average zero radial velocity. The cosmological parameters are imprinted on the infall profile of the gas, however, no method exists, which allows a measurement of it. We consider the results of two cosmological simulations (using the numerical codes RAMSES and Gadget) and find that the gas and dark matter radial velocities are very similar. We derive the relevant dynamical equations, in particular the generalized hydrostatic equilibrium equation, including both the expansion of the Universe and the cosmological background. This generalized gas equation is the main new contribution of this paper. We combine these generalized equations with the results of the numerical simulations to estimate the contribution to the measured cluster masses from the radial velocity: inside the virial radius it is negligible, and inside two virial radii the effect is below 40%, in agreement with the earlier analyses for DM. We point out how the infall velocity in principle may be observable, by measuring the gas properties to distance of about two virial radii, however, this is practically not possible today.

Key words: galaxies: clusters: general – galaxies: halos – galaxies: intergalactic medium

1 INTRODUCTION
The massive galaxy clusters are still in the process of accreting material, so the gas, dark matter and galaxies just outside the virial radius are presently infalling (Gunn & Gott 1972). This effect is most visible when observing ongoing mergers, however, it may also be visible in the smooth accretion of material (Rines & Diaferio 2006). The details of the infall profile depend on the cosmological parameters (Silk 1974, Regos & Geller 1989, Zu & Weinberg 2013), which makes it particularly interesting to observe.

For dark matter this infall is already known to depend on the cluster mass (Privato et al. 2006, Cuesta et al. 2008), and this also implies that the standard mass determination is affected outside the virial radius (Wojtak et al. 2005, Falco et al. 2013A). The gas is known to shock near the virial radius, and at larger radii the gas is expected to be free-falling onto the cluster together with the dark matter. At distances beyond 5 virial radii, both gas and dark matter is swept away with the Hubble expansion.

In this paper we will consider some of the details of the transition region near the galaxy clusters. To this end we will consider the results of 2 numerical cosmological simulations. We will compare the dark matter and gas velocity profiles. We will also derive the relevant equations (one for gas, and one for dark matter and galaxies) including the effect of radial velocities. These equations turn out to be very similar, despite the very different nature of the particles involved and hence the different derivations.

Improving the mass profile reconstruction at and beyond the virial radius is becoming relevant, as the sensitivity of Sunyaev-Zeldovich and X-ray observations are becoming good enough to measure the gas properties at these large radii. Under the assumption of hydrostatic equilibrium, the observed gas temperature and density gives the total mass profile (Sarazin 1986). However, magnetic fields, turbulence, and other velocity terms like bulk motion and infall will induce extra terms in the hydrostatic equilibrium equation, the so-called mass excess terms. Most other studies aiming...
at improving the mass modelling of clusters focus on including such non-thermal pressure components within the virial radius (see e.g. Lau et al. [2009]; Fang et al. [2009]; Suto et al. [2013]; Shi et al. [2015, 2016]; Biffi et al. [2016]) or non-radially symmetric contributions [Skielboe et al. [2012]; Svensmark et al. 2015]. Rasia et al. (2006) considered the sum of the Hubble expansion and a peculiar velocity contribution.

For the gas particles, we will use the notation

\[ v_r = v_p + v_H, \]

where \( v_r \) represents the radial velocity of a fluid particle.

The motion of any particle can at any radius be considered the sum of the Hubble expansion and a peculiar velocity [Peebles 1976; Gunn 1978]

\[ v_r = v_p + v_H, \]

where \( v_H = H(t)r \).

For the gas particles, we will use the notation

\[ v_r^{\text{gas}} = v_{\text{gas}} + v_H. \]

The acceleration due to expansion is the time derivative of the Hubble law

\[ \frac{dv_H}{dt} = -\frac{r}{2} H^2 q, \]

where \( q \) is the deceleration parameter, \( q = \Omega_m/2 - \Omega_L \).

The background density must be included in the gravitational potential, whereby one gets [Peebles 1993; Falco et al. 2013A]

\[ \frac{\partial \Phi(r)}{\partial r} = \frac{GM_{\text{tot}}(r)}{r^2} + 4\pi G \rho_b r + \frac{1}{3} \Lambda r, \]

(4)

where \( \Lambda \) is the cosmological constant, \( \Omega_L = \Lambda/3H^2 \), and \( M_{\text{tot}}(r) \) is the total gravitating mass inside radius \( r \). This can be rewritten as

\[ \frac{\partial \Phi}{\partial r} = \frac{GM_{\text{tot}}(r)}{r^2} + rH^2 q. \]

(5)

After cancellation of a few terms the generalized Euler equation becomes

\[ GM_{\text{tot}}(r) = -k_T v^{\text{gas}} \frac{\partial n_{\text{gas}}}{\partial r} + \frac{\partial n_{T_{\text{gas}}}}{\partial r} - r^2 \left( \frac{\partial v_{\text{gas}}}{\partial t} + H v_{\text{gas}} + Hr \frac{\partial v_{\text{gas}}}{\partial r} + v_{\text{gas}} \frac{\partial v_{\text{gas}}}{\partial r} \right), \]

(6)

which can be written as

\[ GM_{\text{tot}}(r) = GM^{\text{HE}}(r) - r^2 \tilde{S}(v_{\text{gas}}), \]

(7)

where \( M^{\text{HE}}(r) \) represents the standard hydrostatic equilibrium terms. The extra term, \( \tilde{S}(v_{\text{gas}}) \), vanishes for \( v_{\text{gas}} \) going to zero.

### 3 GENERALIZED JEANS EQUATION

Dark matter and galaxies are treated as collisionless, and therefore the fluid equations do not apply. Instead one must start from the collisionless Boltzmann equation.

By integrating over the velocities one obtains the Jeans equations. Falco et al. [2013A] included both the expansion of the universe and the background cosmology, and obtained the generalized Jeans equation

\[ GM_{\text{tot}}(r) = -\sigma_r^2 r \left( \frac{\partial n_p}{\partial r} + \frac{\partial n_{\sigma^2}}{\partial r} + 2\beta \right) - r^2 \left( \frac{\partial v_{\text{dim}}}{\partial t} + H v_{\text{dim}} + Hr \frac{\partial v_{\text{dim}}}{\partial r} + v_{\text{dim}} \frac{\partial v_{\text{dim}}}{\partial r} \right), \]

(8)

where \( v_{\text{dim}} \) is defined similarly to the gas peculiar velocity \( v_{\text{gas}} \). The motion of the DM, \( M_{\text{tot}}(r) \), includes both gas and DM, and the velocity anisotropy, \( \beta \), measures the departure from an isotropic velocity distribution of the DM velocities

\[ \beta(r) \equiv 1 - \frac{\sigma_r^2 + \sigma_\theta^2}{2\sigma_r^2}. \]

(9)

The generalized Jeans equation can also be written as

\[ GM_{\text{tot}}(r) = GM^{\text{HE}}(r) - r^2 \tilde{S}(v_{\text{dim}}). \]

(10)

where \( M^{\text{HE}}(r) \) represents the standard Jeans equation terms. The extra term, \( \tilde{S}(v_{\text{dim}}) \), vanishes for \( v_{\text{dim}} \) going to zero.

It is important to keep in mind, that \( v_{\text{dim}} \) here represents an average velocity of the individual collisionless particles, which differs from the fluid velocity \( v_{\text{gas}} \) in Eq. (6). Despite the very different derivations (and fundamentally different assumptions) the generalized Jeans equation and the generalized hydrostatic equation look remarkably similar. In the outer region where the gas collisions are still not important the equations are naturally expected to look similar since
both equations essentially represent momentum conservation. In the inner cluster region the similarity is more surprising, since the collisionless DM has no equation of state, and therefore the concept of pressure and temperature are not well defined for the DM.

4 NUMERICAL SIMULATIONS

In order to study the new terms in the generalized hydrostatic and Jeans equations, we first need to find the peculiar velocity in and near clusters. To this end we consider numerical simulations of structure formation in a cosmological setting.

To have our results be fairly general, we chose to include two different cosmological simulations, generated using both an Adaptive Mesh Refinement (AMR) code, and a Smoothed Particle Hydrodynamics (SPH) code. These represent two very different approaches to solving the fluid dynamics of the gas component (see e.g. Agertz et al. (2007); Hel & Springel (2012)).

We are using two samples of simulated cluster haloes from Martizzi et al. (2014) and Bonafede et al. (2011). We refer the reader to those papers for details, but summarize the most important properties of the simulations below for completeness.

The AMR code RAMSES (Teyssier 2002) simulates structure formation in a flat Universe with cosmological constant density parameter ΩΛ = 0.728, matter density parameter Ωm = 0.272 of which the baryonic density parameter is Ωb = 0.045, power spectrum normalization σ8 = 0.809, primordial power spectrum index ns = 0.963, and current epoch Hubble parameter H0 = 70.4 km/s/Mpc. The simulation was initially run as a dark-matter-only simulation with comoving box size 144 Mpc/h and particle mass m_dm = 1.5 · 10^8 M⊙/h. Here h is the dimensionless Hubble parameter, defined as h = H0 / 100 km/s/Mpc. After the dark matter only simulation was run, 51 cluster sized haloes with total masses above 10^14 M⊙ were identified. These regions were then resimulated with a baryonic component, with dark matter particle mass m_dm = 1.62 · 10^7 M⊙/h and baryonic component mass resolution of 3 · 10^7 M⊙. The 51 resimulation runs implemented models of radiation, gas cooling, star formation, metal enrichment, supernovae feedback as well as AGN feedback. The simulations were evolved to the present day epoch. A detailed description of the initial simulation can be found in Bonafede et al. (2011) while a detailed description of the physics included in the resimulations can be found in Ragone-Figueroa et al. (2013); Planelles et al. (2014); Munari et al. (2013). From the 29 resimulated clusters the same data was extracted as for the RAMSES simulation. The data of this simulation was averaged in and extracted from shells linearly distributed according to the same physical radii in different cluster haloes. For this reason the data for different haloes does not extend to the same normalized radius.

Whereas most of the haloes from the Gadget-3 simulation have a total mass above 10^15 M⊙, only a few of the haloes from the RAMSES simulation have a total mass above this threshold.

5 PECULIAR VELOCITY FROM NUMERICAL SIMULATIONS

The peculiar velocity of dark matter is shown in figure 1, as a function of radius. The value of v_200 used to normalize the profiles has been found for each cluster as \( \sqrt{G M_{200} / r_{200}} \). The difference between the two simulations arises mainly from the different cluster masses considered, and to a much smaller extend from different cosmological parameters used. For different cosmological models the turn around radius (where the average radial velocity is zero) changes, for instance for a larger cosmological constant the turn around radius is at smaller radii, as can be seen from the spherical collapse model (Pavlidou & Tomaras 2014). For different cluster masses the detailed infall profile changes significantly (Pivato et al. 2006; Cuesta et al. 2008). Recently Lee & Yepes (2016) tried to quantify this effect, and fitted
the peculiar velocities to the form suggested by Falco et al. (2013A)

\[ v_{dm} = -a \left( \frac{r}{r_{200}} \right)^{-b}. \]  
(11)

They found that \( b \) is about 0.26 for structures of masses \( \sim 4 \times 10^{13} M_\odot \), and increases to \( b = 0.43 \) for masses above \( 10^{14} M_\odot \).

In figure 2 we show the ratio between the DM and the gas infall profiles. From this figure it is clear, that the both AMR and SPH simulations give very similar infall profiles for the gas and the DM. For a given cosmology and cluster mass, it is expected that the gas and DM infall profiles should agree outside the radius where gas cooling becomes important, as discussed in section 3. The similarity of gas and DM infall profiles is expected to break down for smaller structures like galaxies with masses \( \sim 10^{12} M_\odot \) as shown by Wetzel & Nagai (2015). The gas cooling (and star formation and feedback) has very little effect for the large masses considered here, which is seen by the infall profiles essentially agreeing between the two different numerical simulations in the innermost regions.

Having found the peculiar velocity, we can now consider the extra mass terms in the generalized hydrostatic equilibrium and Jeans equations.

The two mass excess terms differ only in that \( S(r,t) \) depends on the average peculiar radial velocity of dark matter particles, \( v_{dm} \), while \( \tilde{S}(r,t) \) depends on the peculiar radial fluid velocity of the gas, \( v_{gas} \).

In order to calculate the mass excess terms, \( S(r,t) \) and \( \tilde{S}(r,t) \), \( \frac{\partial v_{dm}}{\partial r} \) and \( \frac{\partial v_{gas}}{\partial r} \) must in principle be determined for each simulated cluster. In Falco et al. (2013A) the authors used 7 different methods to attempt to estimate \( \frac{\partial v_{gas}}{\partial r} \). Each of the methods was either based on theoretical calculations or on knowledge of the growth rate of clusters in simulations. However, the results were not entirely conclusive. Since the effect of \( \frac{\partial v_{gas}}{\partial r} \) is expected to be very similar for the gas and the DM infall, we will ignore that mass excess term for the present analysis. See appendix A and B in Falco et al. (2013A) for an extended discussion on this point.

The mass excess therefore becomes simply

\[ S(r) = \frac{H_0 v_{dm} + H_0 \partial_r v_{dm}}{H_0} + v_{dm} \frac{\partial v_{dm}}{\partial r}. \]  
(12)

with a similar definition for the ICM mass excess \( \tilde{S}(r) \). Since the time evolution of \( S \) has been dropped, and since both the simulations used are evolved to the present epoch, the time dependent Hubble parameter \( H(t) \) has been reduced to its current value of \( H_0 \).

We chose to fit the infall profile. In the central part one has \( \nabla \times v = 0 \), giving \( v_{dm} = -H_r \), so that the fitting function should be linear as \( r \to 0 \). At large radii there is almost no braking due to the low density there, and all matter is therefore in free fall towards the cluster center. The peculiar kinetic energy of particles is thus simply the negative value of the gravitational potential, \( 1/2 v_{dm}^2 = -\Phi \), if they are assumed to have fallen in from infinity. The density at this radius is a tiny fraction of its central value, so the mass as a function of radius is very nearly a constant \( M(r) \approx M_{total} \).

This means that if the background density of the universe is neglected, the potential is the Keplerian potential, \( \Phi \approx -GM_{total}/r \). The peculiar radial velocity should therefore be \( v_{dm} \approx -\sqrt{GM_{total}/r} \propto r^{-1/2} \), and the fitting function should then go as \( r^{-1/2} \) in the outer part. The precise shape of this part of the curve is not crucial, as it is almost entirely outside the region that is analyzed.

We use the form

\[ \frac{v_{dm}}{v_{200}} = f \left( \frac{r}{r_{200}} \right) = -\alpha \left( \frac{r}{r_{200}} \right)^{-b} \left[ 1 + C \left( \frac{r}{r_{200}} \right)^{2} \right]^{1/2} - D, \]  
(13)

where \( \alpha \), \( b \), \( C \), and \( D \) are positive parameters. This is a slightly modified version of eq. (22) in Falco et al. (2013A).

It can be seen that for \( r \to 0 \) the function approaches a linear shape, \( f \to -\alpha \frac{r}{r_{200}} \), while for \( r \to \infty \) it goes to \( f \to -\frac{2}{C} \left( \frac{r}{r_{200}} \right)^{-1/2} \), as wished. \( D \) determines the shape of the function in the intermediate region where braking occurs. The parameter \( \alpha \) can either be chosen to act as a free parameter, or be given the value \( \alpha = \frac{H_{200}}{r_{200}} \), for which the condition of a static central region will be fulfilled. An example of such a fit to the RAMSES simulated profiles gives parameters \( \alpha = 0.137 \pm 0.0069 \), \( b = 16.8 \pm 24 \), \( C = 8.9 \times 10^{-8} \pm 1.6 \times 10^{-6} \) and \( D = 0.314 \pm 0.022 \), which in figure 1 is seen to provide an acceptable fit. For the Gadget simulation the fit to \( \alpha \) is approximately 25% larger, and the other parameters are within the error-bars quoted above. The shown fit has a \( \chi^2 \) of 1.11 with respect to the data, meaning that the reduced \( \chi^2 \) is 0.25. The reason why the uncertainties on the parameters \( b \) and \( c \) are so exceptionally large, is because they are very strongly correlated in the inner part of the clusters, where data is available. If further studies should choose to include the free-falling outer regions of the clusters, then it should be possible to break the degeneracy and better constrain these two parameters. The turn-around is barely visible when considering only radii within two virial radii, which means the variable \( C \) to be consistent with zero. If we fix \( C = 0 \), i.e. having no turn-around in velocity, then the best fit parameters are \( \alpha = 0.143 \) and \( D = 0.289 \) for the RAMSES simulation.

Using this fit, we can calculate the resulting mass excess
It is fair to remind ourselves why we would be interested in measuring the magnitude of the infall velocity profile. The infall velocity is determined mainly by the cluster mass and cosmology, as described in section 5 (for earlier discussions on this point, see e.g. Silk (1974); Regos & Geller (1989); Zu & Weinberg (2013)). For instance the value of the cosmological constant will affect the position of the turn around radius, which is the radius where the infall velocity exactly cancels the Hubble expansion (Pavlidou & Tomaras 2014). Similarly, various modified gravity models have predictions for the magnitude of the infall velocity, which differs from those of ΛCDM (Lee & Li 2016).

The infall velocity leads to a mass excess term, as shown in eq. (6). It is clear from the generalized hydrostatic equilibrium, eq. (6), that if one can simultaneously measure accurately the total mass (for instance from lensing) and gas density and temperature (for instance from either X-ray or SZ), then one can directly get the mass excess term, and hence estimate the mass excess contribution from the velocity infall.

This is, however, rather non-trivial, since a combination of observational techniques always requires a careful control of systematic effects. We will instead here entertain the possibility of using only one method to measure gas parameters, and not include any measure of the total mass. The X-ray method is very well established (Sarazin 1986), however, it is also clear that the effect of clumping may affect the precision of the mass determination, in particular in the outer regions (see Battaglia et al. 2015 for a recent overview). Only few X-ray observations have today observed to the virial radius (Urban et al. 2014).

The SZ effect, on the other hand, is linear in both temperature and density (Sunyaev & Zeldovich 1972), which makes it possible to measure the cluster temperature (Pointecouteau et al. 1998; Hansen et al. 2002) without concerns about clumpiness. Future SZ observatories may in principle deproject the spectra to get the full temperature and density profiles (Hansen 2004).

In principle the full velocity profile may be measured, but for clarity we will here treat it as a one parameter search. This could be the normalization α in eq. (5). We therefore consider the situation where the gas temperature and density have been measured accurately to large radii (e.g. two virial radii) and that α is unknown.

Let us clarify how to measure the magnitude of the infall velocity. Pick a value of α. From the generalized hydrostatic equilibrium, eq. (5) one can now calculate everything on the r.h.s. This gives us the total mass, $M_{\text{tot}}(r)$. By subtracting the gas mass, we can now derive the dark matter density, which is one of the parameters of the generalized Jeans equation, eq. (5).

Using numerical simulations, the ratio of the gas temperature and the dark matter “temperature” (i.e. the mean non-translationlal kinetic energy of dark matter particles) can be parametrized. This is a number which today is believed to be fairly close to unity (Host et al. 2009), and future numerical simulations will determine this ratio more accurately. Thereby we get $\sigma_t^2$ in the generalized Jeans equation, eq. (6). The dark matter velocity anisotropy, $\beta$, is known to follow the density profile of the dark matter (Hansen & Moore 2006), or alternatively it can be parametrized from numerical simulations (see e.g. Rasia et

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**Figure 3.** The resulting mass excess divided by the total mass. Upper panel: Inside the virial radius there is no effect of the combination of Hubble flow and infall. Upper panel: At radii between 1 and 2 virial radii, we see that the radial velocity affects the mass estimates up to 40%, both for gas and dark matter. Lower panel: Each of the 3 new terms in the generalized hydrostatic equilibrium.

The mass excess of the dark matter found this way is in good agreement with Falco et al. (2013A), where the mass excess of the galaxy component in simulated clusters was estimated and found to be between 20% and 40% inside 2 $r_{200}$.

6 MEASURING THE MAGNITUDE OF THE VELOCITY INFALL

This technical section comes with the following warning: the section will conclude that whereas it in principle would be possible to measure a mass excess term coming from the infall profile, in practice it is very difficult. Some readers may therefore prefer to go straight to the concluding section.
ical simulations is still far too low for this method to be used
out that the precision of the needed calibration with numer-
sionless DM, and it effectively has an equivalent form. We
generalized equation has previously been derived for colli-
40%. Inside the virial radius it is negligible. The similar
the infall velocity induces a mass excess which is less than
4 equilibrium equation, and we find that within two virial radii
2% near the virial radius (Avestruz et al. 2016; Planelles et
of clumpiness, which could induce mass excess of the order
would be possible to measure a mass excess term coming
the SZ effect to measure the temperature and density. Fur-
othermore, there could be bulk rotation or turbulence. These
effects would be very difficult to remove from the data. And
finally, the entire analysis gets even more convoluted (or im-
We have hereby shown that whereas it in principle
would be possible to measure a mass excess term coming
from the infall profile, in practice it is very difficult. Further-
more, there will be other effects which also induce a mass excess: When using X-ray temperatures there is the problem of clumpiness, which could induce mass excess of the order
20% near the virial radius (Avestruz et al. 2016, Planelles et
This problem could potentially be avoided by using
the SZ effect to measure the temperature and density. Fur-
thermore, there could be bulk rotation or turbulence. These
effects would be very difficult to remove from the data. And
finally, the entire analysis gets even more convoluted (or im-
possible) when considering that structures usually depart
from sphericity, see for instance the discussion in Samsung
et al. (2012); Suto et al. (2016); Vega-Ferrero et al. (2016).
7 CONCLUSION
We compare the infall velocity near galaxy clusters from
cosmological simulations, and we find that the gas and dark
matter infall profiles are very similar. We derive the relevant
gas equation, which is a generalization of the hydrostatic equilibrium equation, and we find that within two virial radii
the infall velocity induces a mass excess which is less than
40%. Inside the virial radius it is negligible. The similar
generalized equation has previously been derived for colli-
sionless DM, and it effectively has an equivalent form. We
suggest how future detailed observations in principle may be
used to measure this infall profile. However, we also point
out that the precision of the needed calibration with numerical simulations is still far too low for this method to be used
in practice.

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