Alfvénic tornadoes

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It is shown that a three-dimensional (3D) modified-kinetic Alfvén waves (m-KAWs) can propagate in the form of Alfvénic tornadoes characterized by plasma density whirls or magnetic flux ropes carrying orbital angular momentum (OAM). By using the two fluid model, together with Ampère’s law, we derive the wave equation for a 3D m-KAWs in a magnetoplasma with \( m_e/m_i \ll \beta \ll 1 \), where \( m_e (m_i) \) is the electron (ion) mass, \( \beta = 4\pi n_0 k_B (T_e + T_i)/B_0^2 \), \( n_0 \) the unperturbed plasma number density, \( k_B \) the Boltzmann constant, \( T_e (T_i) \) the electron (ion) temperature, and \( B_0 \) the strength of the ambient magnetic field. The 3D m-KAW equation admits solutions in the form of a Laguerre-Gauss (LG) Alfvénic vortex beam or Alfvénic tornadoes with plasma density whirls that support the dynamics of Alfvén magnetic flux ropes.
1. Introduction

The discovery of the Alfvén wave [1942] had significantly changed the landscape of collective phenomena in magnetized plasmas that frequently occur in interstellar media. Shukla and Dawson [1984] and in large laboratory plasma devices [Lennert et al., 1999; Kletzing et al., 2010; Gekelman et al., 2011]. In the Alfvén wave, the restoring force comes from the pressure of the magnetic fields, and the ion mass provides the inertia.

The dispersion relation for the low-frequency (in comparison with the ion gyrofrequency) Alfvén wave is deduced from the magnetohydrodynamic (MHD) equations [Alfven, 1942; Cramer, 2001], which do not account for the wave dispersion effects. The dispersion [Stefant, 1970; Brodin and Stenflo, 1990; Morales et al., 1994; Morales and Maggs, 1997; Shukla and Stenflo, 1999; Dastgeer and Shukla, 2009; Damino et al., 2009] to the Alfvén wave comes from the finite frequency (ω/ωci) and the magnetic field-aligned electron inertial force in cold magnetoplasmatics, as well as from the finite ion gyroradius effect and the gradient of the electron pressure perturbation in a warm magnetoplasma.

Due to non-ideal effects Brodin and Stenflo [1990]; Morales et al. [1994]; Morales and Maggs [1997]; Shukla and Stenflo [1999]; Damino et al. [2009], Alfvén waves thus couple to fast and slow magnetoacoustic waves, the inertial and kinetic Alfvén waves [Stefant, 1970; Morales et al. [1994]; Morales and Maggs [1997]; Dastgeer and Shukla [2009]; Damino et al. [2009] in a uniform magnetoplasma. The magnetic field-aligned phase speed (ω/kz) of the inertial (kinetic) Alfvén wave is much larger (smaller) than the electron thermal speed. Accordingly, the inertial (kinetic) Alfvén wave arises in a plasma with β ≪ me/mi (me/mi ≪ β ≪ 1). The dispersive shear Alfvén wave plays a significant role...
role in the auroral Chaston et al. [2008a, b], magnetospheric Sundkvist et al. [2005],
solar Tripathi and Gekelman [2010]; Salem et al. [2012], astrophysical and laboratory
[Gekelman et al., 2011, 2012] magnetoplasmas with regard to acceleration and heating of
the plasma particles Shukla et al. [1998], reconnection of magnetic field lines, wave-wave
and wave-particle interactions [Hasegawa and Chen, 1976; Carter et al., 2006; Shukla
et al., 2007], and the formation of nonlinear Alfvénic structures [Sundkvist et al., 2005;
Stasiewicz et al., 2003; Shukla et al., 2012]. Specifically, it is likely that the dispersive
Alfvén waves power the auroral activities Chaston et al. [2008a, b] and the solar coronal
heating [Tomczyk et al., 2007; Jess et al., 2009; McIntosh et al., 2011], as well as may be
responsible for the formation of Alfvénic tornadoes Widemeyer-Böhlm et al. [2012] which
can transport magneto-convective energy from one region to another in solar plasmas.

In this Brief Report, it is shown that a 3D m-KAW can propagate as a twisted vortex
beam in a plasma with \( \frac{m_e}{m_i} \ll \beta \ll 1 \). The present work thus complements a recent
investigation Shukla [2012] that focused on examining a 3D twisted inertial Alfvén wave.
Twisted m-KAWs can be related with magnetic flux ropes or Alfvénic tornadoes of differ-
ent scalesizes in observational data from laboratory experiments Gekelman et al. [2012],
and also from the solar plasmas Widemeyer-Böhlm et al. [2012]; Salem et al. [2012].

2. Mathematical description

We consider a magnetized electron-ion plasma in the presence of low-frequency (in com-
parison with the electron gyrofrequency \( \omega_{ce} = \frac{eB_0}{m_e c} \), where \( e \) is the magnitude of the
electron charge, \( B_0 \) the strength of the external magnetic field \( \hat{z}B_0 \), \( m_e \) the electron mass,
and \( c \) the speed of light in vacuum, \( \hat{z} \) the unit vector along the z-axis in a Cartesian coor-
m-KAWs with the electric and magnetic fields \( \mathbf{E} = -\nabla \phi - \left(1/c\right)\hat{z} \partial A_z/\partial t \) and \( \mathbf{B}_\perp = \nabla A_z \times \hat{z} \), respectively, where \( \phi \) and \( A_z \) are the scalar and magnetic field-aligned vector potentials, respectively. In the m-KAW fields, the electron density perturbation is obtained from

\[
\frac{\partial n_{e1}}{\partial t} + \frac{c}{4\pi e} \frac{\partial \nabla_\perp^2 A_z}{\partial z} = 0, \tag{1}
\]

where we have used the electron fluid velocity

\[
\mathbf{u}_e \approx \frac{c}{B_0} \hat{z} \times \nabla \phi - \frac{c k_B T_e}{B_0 n_0} \hat{z} \times \nabla n_{e1} + \hat{z} \frac{c}{4\pi e n_0} \nabla_\perp^2 A_z, \tag{2}
\]

which uses Ampère’s law that relates \( A_z \) and the magnetic field-aligned electron fluid velocity \( u_{ez} \). Here \( n_{e1} (\ll n_0) \) is a small electron density perturbation in the equilibrium density \( n_0 \).

Since the parallel phase speed of the m-KAWs is much smaller than the electron thermal speed \( V_{Te} = (k_B T_e/m_e)^{1/2} \), one can neglect the electron inertia and obtain from the magnetic field aligned electron momentum equation

\[
0 = e \frac{\partial \phi}{\partial z} + e \frac{\partial A_z}{\partial t} - \frac{k_B T_e}{n_0} \frac{n_{e1}}{\partial z}, \tag{3}
\]

which dictates that the parallel electric force \(-n_0 e E_z\) and \( \partial p_1/\partial z \) are in balance, where the magnetic field-aligned electric field \( E_z = -\partial \phi - c^{-1} \partial A_z/\partial t \) and \( p_1 = k_B T_e n_{e1} \) is the electron pressure perturbation.

We can now eliminate \( A_z \) from Eqs. (1) by using (3), obtaining
where \( \lambda_{De} = \left(\frac{k_B T_e}{4 \pi n_0 e^2}\right)^{1/2} \) is the electron Debye radius.

The perpendicular (to \( \hat{z} \)) component of the ion fluid velocity \( u_{i\perp} \) is determined from

\[
\left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) u_{i\perp} = \frac{c \omega_{ci}^2}{B_0} \hat{z} \times \nabla \phi - \frac{c \omega_{ci} \partial \nabla \phi}{B_0} \frac{\partial \phi}{\partial t},
\]

which is obtained by manipulating the perpendicular component of the ion momentum equation, where \( \omega_{ci} = eB_0/m_ic \) is the ion gyrofrequency and \( m_i \) the ion mass. We have assumed that the m-KAW phase speed is much larger than the ion thermal speed, and therefore neglected in Eq. (5) the contribution of the ion pressure gradient. The ions are assumed to be confined in a two-dimensional \( (x - y) \) plane perpendicular to \( \hat{z} \). The magnetic field-aligned ion dynamics is unimportant for the m-KAWs, since the phase speed of the latter is much larger than the ion-sound speed.

From the linearized ion continuity equation and Eq. (5), we readily obtain

\[
\left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) n_{i1} - \frac{n_0 c \omega_{ci}}{B_0} \nabla_{\perp}^2 \phi = 0,
\]

where \( n_{i1}(\ll n_0) \) is a small ion number density perturbation.

Invoking the quasi-neutrality condition \( n_{e1} = n_{i1} = n_1 \), we eliminate \( n_{i1} \) from Eq. (6) by using Eq. (5), obtaining the wave equation for a 3D m-KAW after elementary calculation

\[
\left[ \frac{\partial^2}{\partial t^2} - C_A^2 \partial^2 + \omega_{ci}^2 \right] n_1 = 0,
\]
where \( C_A = B_0 / \sqrt{4 \pi n_0 m_e} \) is the Alfvén speed, and \( \rho_s = C_s / \omega_{ci} \) is the sound gyroradius, with \( C_s = (k_B T_e / m_i)^{1/2} \) being the ion-sound speed. The effect of the ion temperature can be incorporated by re-defining \( \rho_s = (C_s / \omega_{ci})(1 + 3 T_i / T_e)^{1/2} \).

Within the framework of a plane-wave approximation, assuming that \( n_1 \) is proportional to \( \exp(-i \omega t + i \mathbf{k} \cdot \mathbf{r}) \), where \( \omega \) and \( \mathbf{k}(= \mathbf{k}_\perp + \hat{z} k_z) \) are the angular frequency and the wave vector, respectively, we Fourier analyze (8) to obtain the frequency spectra Shukla and Stenflo [1999] of the modified (by the \( \omega / \omega_{ci} \) effect) KAWs

\[
\omega^2 = \frac{k^2 V_A^2 (1 + k^2 \rho_s^2)}{1 + k^2 \lambda_i^2},
\]  

(8)

where \( \mathbf{k}_\perp \) and \( k_z \) are the components of \( \mathbf{k} \) across and along \( \hat{z} \), and \( \lambda_i = c / \omega_{pi} \) the ion inertial scale length, with \( \omega_{pi} = (4 \pi n_0 e^2 / m_i)^{1/2} \) being the ion plasma frequency. The \( k_z \lambda_i \)-term in Eq. (8) comes from the \( \omega / \omega_{ci} \) effect.

Let us now study the property of a twisted m-KAW. We seek a solution of Eq. (7) in the form

\[
n_1 = N(r) \exp(i k_z z - i \omega_{k} t),
\]  

(9)

where \( N(r) \) is a slowly varying function of \( z \). Here \( r = (x^2 + y^2)^{1/2} \) and \( k \) is the propagation wave number along the axial (the \( z \)-axis) direction. By using Eq. (10) we can write Eq. (7) in a paraxial approximation (viz. \( |\partial^2 N / \partial z^2| \ll k_z^2 N \)) as

\[
\left( 2i \frac{\partial}{\partial Z} + \nabla^2_\perp \right) N = 0,
\]  

(10)
where \( \omega_k = k_z V_A/(1 + k_z^2 \lambda_i^2)^{1/2} \) is the angular frequency of the magnetic field-aligned KAW. Here \( Z = (1 + k_z^2 \lambda_i^2) k_z z \) and \( \nabla_\perp \) is in unit of \( \rho_s \). Furthermore, we have denoted the operator \( \nabla_\perp^2 N = (1/r)(\partial/\partial r)(r\partial N/\partial r) + (1/r^2)\partial^2 N/\partial \theta^2 \), and introduced the cylindrical coordinates with \( \mathbf{r} = (r, \theta, z) \).

The solution of Eq. (10) can be written as a superposition of Laguerre-Gaussian (LG) modes [Allen et al., 1992, 2003; Mendonça et al., 2009], each of them representing a state of orbital angular momentum, characterized by the quantum number \( l \), such that

\[
N = \sum_{pl} N_{pl} F_{pl}(r, Z) \exp(ial\theta),
\]

where the mode structure function is \( F_{pl}(r, z) = H_{pl} X^{|l|} L^{|l|}_p(X) \exp(-X/2) \), with \( X = R^2/W^2(Z), \) \( r = r/\rho_s \), and \( W = 2\pi(1 + 4\pi^2 \lambda_i^2/L_z^2)z/L_z, \) \( L_z = 2\pi/k_z \), and \( W(Z)/L_z \) representing the normalized width of a twisted Alfvénic vortex beam. The normalization factor \( H_{pl} \) and the associated Laguerre polynomial \( L^{|l|}_p(X) \) are, respectively,

\[
H_{pl} = \frac{1}{2\sqrt{\pi}} \left[ \frac{(l+p)!}{p!} \right]^{1/2},
\]

and

\[
L^{|l|}_p(X) = \frac{\exp(X)}{Xp!} \frac{dp}{dX^p} \left[ X^{l+p} \exp(-X) \right],
\]

where \( p \) and \( l \) are the radial and angular mode numbers of the DSAW orbital angular momentum state. In a special case with \( l = 0 \) and \( p = 0 \), we have a Gaussian beam.
The LG solutions, given by Eq. (11), describe the structure of a twisted 3D Alfvénic vortex beam carrying OAM. In a twisted m-KAW vortex beam, the wavefront rotates around the beam’s propagation direction (viz. the $z$-axis) in a spiral that looks like fusilli pasta (or a bit like a DNA double helix), creating a vortex and leading to the m-KAW vortex (KAWV) beam with zero intensity at its center. A twisted m-KAWV beam (or an Alfvén tornado) can be created with the help of two oppositely propagating 3D m-KAWs that are colliding in a magnetoplasma. Twisting of 3D m-KAWs occurs because different sections of the wavefront bounce off different steps, introducing a delay between the reflection of neighboring sections and, therefore, causing the wavefront to be twisted due to an entanglement of the wavefronts. Thus, due to angular symmetry, Noether theorem guards OAM conservation for a m-KAWV beam that is accompanies the parallel electric field, sheared magnetic field, and finite density perturbations.

3. Summary and discussions

In summary, we have shown that 3D m-KAWs in a uniform magnetoplasma can propagate in the form of Alfvénic tornadoes which have axial and radial extents of the order of the ion inertial length $c/\omega_{pi}$ and the ion sound radius $\rho_s$, respectively. The Alfvénic swirls can be identified as observational signatures of rapidly rotating magnetic field ropes [Tripathi and Gekeklman, 2010; Gekelman et al., 2012] (or magnetic field tornadoes), which can provide an alternative mechanism for channelling electromagnetic energy and heat fluxes from the surface of the Sun into the corona Widemeyer-Böhm et al. [2012] through Alfvénic tornadoes/whirls. Furthermore, the present investigation of twisted m-
KAW vortex beams can also be useful for diagnostic purposes, since the m-KAW beam frequency can be a fraction of the ion gyrofrequency.

**Acknowledgments.** This research was partially supported by the Deutsche Forschungsgemeinschaft (DFG), Bonn, through the project SH21/3-2 of the Research Unit 1048.

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