Bulk viscosity, $r$-modes, and the early evolution of neutron stars

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ABSTRACT. We discuss the effect of nonlinear bulk viscosity and the associated reheating on the evolution of newly born, rapidly rotating neutron stars with $r$-modes destabilized through the Chandrasekhar-Friedman-Schutz (CFS) mechanism. Bulk viscosity in these stars is due to the adjustment of the relative abundances of different particle species as the density of a fluid element is perturbed. It becomes nonlinear when the chemical potential difference $\delta \mu$, measuring the chemical imbalance in the fluid element, becomes larger than the temperature $T$, which is generally much smaller than the Fermi energy. From this scale on, the bulk viscosity increases much faster with $\delta \mu$ than predicted by the usual, linear approximation. This provides a potential saturation mechanism for stellar oscillation modes at a small to moderate amplitude. In addition, bulk viscosity dissipates energy, which can lead to neutrino emission, reheating of the star, or both. This is the first study to explicitly consider these effects in the evolution of the $r$-mode instability. For stars with little or no hyperon bulk viscosity, these effects are not strong enough to prevent the $r$-modes from growing to amplitudes $\alpha \sim 1$ or higher, so other saturation mechanisms will probably set in earlier. The reheating effect makes spin-down occur at a higher temperature than would otherwise be the case, in this way possibly avoiding complications associated with a solid crust or a core superfluid. On the other hand, stars with a substantial hyperon bulk viscosity and a moderate magnetic field saturate their mode amplitude at a low value, which makes them gravitational radiators for hundreds of years, while they lose angular momentum through gravitational waves and magnetic braking.

1. Introduction

$R$-modes in inviscid, rotating, relativistic stars (in which the Coriolis force acts as the restoring force to produce periodic fluid motions) are generically unstable to the emission of gravitational waves (Andersson 1998; Friedman & Morsink 1998), whose back-reaction produces a growth of the modes by the mechanism first studied by Chandrasekhar (1970) and by Friedman & Schutz (1978a,b; “CFS instability”).

In neutron stars at low temperatures $T$, the long mean free path of the fluid particles allows rapid momentum exchange over macroscopic length scales (shear viscosity), impeding the growth of the modes (Lindblom et al. 1998). At high $T$, the successive compression and expansion of a fluid element (a small effect for $r$-modes, unless the stellar rotation rate, $\Omega$, is extremely fast; e.g., Lindblom & Owen 2002) leads to a departure from the chemical equilibrium conditions, inducing reactions (typically weak interactions) which tend to restore the equilibrium state. Since these reactions occur out of equilibrium, they constitute irreversible processes that dissipate energy and create entropy, extracting mechanical energy from the oscillation modes while increasing the temperature of the fluid and/or leading to the emission of neutrinos from the star.

The competition between the destabilizing effect of the gravitational radiation reaction, which is strongly dependent on $\Omega$, and the damping effect of the temperature-dependent shear and bulk viscosity generically defines one or two “instability regions” for low-amplitude $r$-modes at high $\Omega$ and intermediate $T$, as shown in Fig. 1. In these regions, the mode amplitude grows exponentially, until it is choked by cooling or feeds back on $\Omega$ and $T$ (and itself) through heating or gravitational-radiation torque.

Studies done so far of the evolution of young neutron stars under this instability (e.g., Owen et al. 1998; Lindblom & Owen 2002) include amplification and damping of $r$-modes
by the above-mentioned effects in the linear regime, spin-down due to gravitational waves, and passive cooling, ignoring the nonlinear terms and the heating associated with the bulk viscosity. These effects are included in the present work, in addition to a probably more realistic treatment of angular momentum loss, motivated by the model case studied by Levin & Ushomirsky (2001a).

2. Nonlinear bulk viscosity and reheating

In the simplest case, namely neutron star matter composed only of neutrons \( n \), protons \( p \), and electrons \( e \), the equilibrium state is set by \( \mu_n = \mu_p + \mu_e \), where \( \mu_i \) is the chemical potential of particle species \( i \). In the simplest approximation, we consider non-interacting particles, of which the neutrons and protons are non-relativistic, satisfying

\[
\mu_i \approx \frac{p_{Fi}^2}{2m_i} \propto n_i^{2/3},
\]

where \( p_{Fi} \) and \( n_i \) are the respective Fermi momenta and number densities, and the electrons are extremely relativistic, with \( \mu_e \approx p_{Fe}c \propto n_e^{1/3} \). If the total density \( \rho \) is perturbed by an amount \( \delta \rho \), at constant (charge-neutral) composition, the equilibrium state is perturbed by

\[
\delta \mu \equiv \mu_p + \mu_e - \mu_n \approx -\mu_e \delta \rho / (3 \rho).
\]

The equilibrium state can be restored by the “modified Urca reactions” (hereafter \( mUrca \)), \( n + N \to p + N + e + \bar{\nu}_e \) and \( p + N + e \to n + N + \nu_e \), where \( \nu_e \) and \( \bar{\nu}_e \) are the electron neutrino and antineutrino, and \( N \) is a “bystander nucleon” whose identity is not changed, but whose function is to absorb momentum in order to allow for overall momentum conservation in the reaction, given that the neutron Fermi momentum is much larger than that of all the other particles involved.

The net rate \( \Gamma_m \) of these reactions (net lepton number emitted per unit volume per unit time) is limited by the available phase space, increasing both with temperature, \( T \), and with \( \delta \mu \). The associated energy dissipation rate is
\begin{equation}
\Gamma_m(\delta\mu, T)\delta\mu = \epsilon_m(0, T) \frac{14680u^2 + 7560u^4 + 840u^6 + 24u^8}{11513},
\end{equation}

where \(u \equiv \delta\mu/(\pi T)\), and the neutrino emissivity (energy emitted in the form of neutrinos and antineutrinos per unit time per unit volume) takes a similar form,

\begin{equation}
\epsilon_m(\delta\mu, T) = \epsilon_m(0, T) \left(1 + \frac{22020u^2 + 5670u^4 + 420u^6 + 9u^8}{11513}\right),
\end{equation}

with the equilibrium value \(\epsilon_m(0, T) \propto T^8\) (see Reisenegger 1995). Studies of neutron star evolution so far have considered only the lowest-order term in \(u\) in each of these expressions, which yield the linear bulk viscosity and equilibrium neutrino cooling of the star, respectively. Here, we consider the full expression for both. Equation (1) shows that, at high amplitudes, the dissipation increases much more strongly than predicted by linear theory, potentially providing a saturation mechanism that impedes the unlimited growth of the amplitude. In addition, we note that the net heating of the neutron star is given by the difference \(\Gamma_m\delta\mu - \epsilon_m\), a fourth-order polynomial in \(u^2\) whose zeroth and first-order terms are negative whereas the three higher powers are positive. Thus, for small-amplitude oscillations, the star undergoes net cooling (somewhat faster than in equilibrium at the same \(T\)), whereas at high amplitudes it is heated more and more strongly.

In the inner core of a neutron star, additional particles may appear, such as the (baryonic) \(\Lambda^0\) and \(\Sigma^-\) hyperons. Their presence changes the equilibrium abundances, increasing the proton-to-neutron ratio and probably allowing the much faster “direct Urca reactions” (hereafter dUrca), \(n \rightarrow p + e + \bar{\nu}_e\) and \(p + e \rightarrow n + \nu_e\), to take place, conserving momentum without the need of a “bystander particle”. For these, the quantitative details are slightly different from the mUrca case (see Reisenegger 1995 for the explicit expressions), but the qualitative properties discussed above still hold.

In addition, the hyperons give rise to the additional reactions \(\Sigma^- + p \leftrightarrow n + n\) and \(\Lambda^0 + p \leftrightarrow p + n\), which contribute to the bulk viscosity, substantially reducing the unstable region in the \(\Omega - T\) plane, probably to two small and disjoint “windows” (see Fig. 1 and Lindblom & Owen 2002). These reactions emit no neutrinos, and therefore always have a heating effect. The rates of these reactions have only been calculated in the small-amplitude (“linear”) limit (Jones 2001; Lindblom & Owen 2002), but we extrapolate them into the nonlinear regime by using the expressions obtained by Madsen (1992) for the analogous processes in quark matter, \(u + d \leftrightarrow s + u\).

3. Angular momentum loss and neutron star evolution

The evolution of the star is modeled by coupled first-order differential equations for \(\Omega\), \(T\), and the dimensionless mode amplitude \(\alpha\). The evolution of \(T\) is determined by the heating and cooling processes discussed in the previous section, and that of \(\alpha\) by the mentioned competition between the destabilizing gravitational radiation reaction and the stabilizing viscous effects. The evolution of \(\Omega\) deserves some additional comments.

In the instability conditions derived by Friedman & Schutz (1978b), a crucial role is played by the so-called “canonical angular momentum” associated with the mode, a quadratic functional of the displacement field which is related (though possibly not identical) to the physical angular momentum of the mode. In previous studies of the evolution of neutron stars under this instability (e.g., Owen et al. 1998), it was assumed that the total angular momentum of the star is the sum of a term corresponding to a rigid rotation, \(I\Omega\), where \(I\) is the star’s (constant) moment of inertia, plus the canonical angular momentum of the mode. Furthermore, it was assumed that only the latter term is modified by the gravitational radiation reaction force, whereas the stellar rotation rate...
4. Results

We have tracked the evolution of a young neutron star for three different scenarios, which are determined by the instability boundaries shown in Fig. 1. Our numerical simulations start with the same initial conditions as in Owen et al. (1998), i.e., an initial temperature $T_0 = 10^{11}$K and an initial rotation rate $\Omega_0 = \frac{2}{9} \sqrt{\pi G \bar{\rho}}$, about the maximum rotation the star can sustain without disrupting due to the centrifugal force, where $\bar{\rho}$ is the mean density of the star.

Our first scenario is a neutron star composed only of neutrons, protons and electrons, where its bulk viscosity is caused by $m\text{Urca}$ reactions (Fig. 2). Non-linear bulk viscosity is unable to saturate the $r$-modes on its own, so they can grow to amplitudes $\alpha \geq 1$, spinning down the star within an hour. Reheating is weak compared to neutrino emission, leading only to a delay in the cooling of the star, but not to a net temperature change only by viscous dissipation of the modes, which would transfer angular momentum from one term to the other.

For one specific model problem (a thin spherical shell), Levin & Ushomirsky (2001a) were able to show that these assumptions are incorrect. For this case, the $r$-modes have no physical angular momentum, i.e., do not contribute to the total angular momentum of the star, which is exactly $I\Omega$. Furthermore, $I\Omega$ is changed by the gravitational waves, as there is no other angular momentum available for them to carry away. There is no proof that these results can be extrapolated to the much more complicated case of a complete neutron star. However, given that the previously made assumptions are rather special and have been disproved in the only available model case, we choose to base our evolutionary model on the results of Levin & Ushomirsky, taking the total angular momentum to be given by $I\Omega$ and its derivative to be equal to minus the angular momentum carried away by the gravitational waves.

Fig. 2. Evolution of a non-hyperonic neutron star. The solid line corresponds to the case without artificial saturation. The segmented lines represent the evolution with artificial saturation of the $r$-modes at different amplitudes $\alpha$, as indicated.
Fig. 3. Evolution of a neutron star with large hyperon bulk viscosity (solid line) when it gets to the second unstable region, whose boundary is shown for reference (dashed line).

increase. Since other saturation mechanisms can be active (Arras et al. 2002), we have saturated the r-modes artificially (as in Owen et al. 1998) by an ad hoc damping force whose work is directly dissipated into heat. This leads to a rapid and substantial temperature increase, depending on the saturation amplitude, until the star is hot enough for neutrino cooling to become stronger. For saturation amplitudes \( \alpha < 10^{-2} \), as for the unsaturated case, the heat gain is not enough to raise the temperature and only delays the cooling. However, the spin-down timescale becomes much longer, \( t_{sd} \sim \alpha^{-2} \) hours. This mechanism can in principle explain the inexistence of very rapidly rotating, young neutron stars. However, with our current understanding of the microphysics, the instability boundary restricts the final rotation periods (after spin-down through this mechanism) only to \( P > 5 \) ms, not to the observed \( P > 15 \) ms.

In our second scenario (not plotted), we consider the presence of hyperons in the inner core of the neutron star, contributing the relatively low bulk viscosity estimated by Jones (2001) and allowing \( d\text{Urca} \) processes to take place. The evolution is similar to our first scenario, but happens at lower temperatures (\( \sim 10^{9} \text{K} \)), allowing some temperature increase to occur once the mode amplitude is large.

Our third scenario is basically the same as the second one, but the contribution of hyperon bulk viscosity is taken to be large (as suggested by Lindblom & Owen 2002), so as to split the unstable region in two (Fig. 1). Due to the rapid \( d\text{Urca} \) cooling, the high-temperature unstable region is passed through in seconds, and initially small-amplitude r-modes do not have enough time to grow significantly. It takes about a year for the neutron star to cool down to the second unstable region, which is avoided due to the large spin-down if the magnetic field of the star is large, \( B \sim 10^{13} \) G or greater. If the magnetic field is weaker, the star enters this unstable region, and the r-modes grow until the heat released by viscous dissipation is large enough to raise the temperature, leading the star back to the stable region where it came from. Then, the r-modes are damped until neutrino cooling moves the star back to the unstable region. The cycle repeats until an equilibrium between amplitude-dependent heating and neutrino cooling.
is achieved and the star stabilizes with a fixed amplitude ($\alpha \sim 10^{-6}$) very close to the instability boundary (Fig. 3). The star then evolves along the instability boundary, spinning down due to the combination of magnetic dipole braking (which dominates for magnetic fields larger than $\sim 10^9$ G, as inferred for “classical”, young pulsars) and gravitational radiation, providing a persistent source of gravitational radiation for $\sim 200$yr. This is qualitatively similar to the evolution found by Andersson et al. (2002) for strange stars, but with an important difference. Since those authors do not consider reheating nor a magnetic field, the timescale for the evolution along the instability boundary is set by the star’s cooling process rather than the magnetic torque.

5. Conclusions

We have found that the non-linear bulk viscosity terms cause no dramatic changes in the evolution of a young neutron star. They can decrease the maximum amplitude achieved by the $r$-modes, but cannot completely saturate the modes. The reheating makes the rapid spin-down phase take place at higher temperatures, possibly simplifying the models by avoiding the formation of a solid crust and a superfluid core. The most important consequence of reheating is that, with strong bulk viscosity at sufficiently low temperatures, it can balance neutrino cooling, keeping the star close to the instability boundary, and turning it into a persistent source of gravitational radiation for hundreds of years.

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