Stellar angular momentum distribution linked to galaxy morphology

Sarah M. Sweet,1,2,3⋆ Karl Glazebrook,2 Danail Obreschkow,4 Deanne B. Fisher,2 Andreas Burkert,5,6 Claudia D. P. Lagos,3,4 Juan M. Espejo Salcedo2

1School of Mathematics and Physics, University of Queensland, Brisbane, QLD 4072, Australia
2Centre for Astrophysics and Supercomputing, Swinburne University of Technology, PO Box 218, Hawthorn, VIC 3122, Australia
3ARC Centre of Excellence for All Sky Astrophysics in 3 Dimensions (ASTRO 3D)
4International Centre for Radio Astronomy Research, University of Western Australia, 7 Fairway, Crawley, WA 6009, Australia
5Ludwig-Maximilians University Munich, University Observatory, Scheinerstr. 1, D-81679 München, Germany
6Max-Planck Institute for Extraterrestrial Physics, Giessenbachstr. 1, D-85748 Garching, Germany

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
We study the spatially-resolved stellar specific angular momentum $j_*$ in a high-quality sample of 24 CALIFA galaxies covering a broad range of visual morphology, accounting for stellar velocity and velocity dispersion. The shape of the spaxel-wise probability density function of normalised $s = j_*/j_{\text{mean}}$, PDF$(s)$, deviates significantly from the near-universal initial distribution expected of baryons in a dark matter halo and can be explained by the expected baryonic effects in galaxy formation that remove and re-distribute angular momentum. Further we find that the observed shape of the PDF$(s)$ correlates significantly with photometric morphology, where late-type galaxies have a PDF$(s)$ that is similar to a normal distribution, whereas early types have a strongly-skewed PDF$(s)$ resulting from an excess of low-angular momentum material. Galaxies that are known to host pseudobulges ($b_{1rb} < 2.2$) tend to have less skewed bulge PDF$(s)$, with skewness $(b_{1rb}) < 0.8$. The PDF$(s)$ encodes both kinematic and photometric information and appears to be a robust tracer of morphology. Its use is motivated by the desire to move away from traditional component-based classifications which are subject to observer bias, to classification on a galaxy’s fundamental (stellar mass, angular momentum) properties. In future, PDF$(s)$ may also be useful as a kinematic decomposition tool.

Key words: galaxies: bulges — galaxies: evolution — galaxies: fundamental parameters — galaxies: kinematics and dynamics — galaxies: spiral — galaxies: elliptical and lenticular, cD

1 INTRODUCTION
During the protogalactic growth of structure, large-scale tidal torques in the cosmic density field spin up the growing dark matter haloes (Peebles 1969; Doroshkevich 1970; White 1984). The angular momentum of the haloes provides a seed for the angular momentum of the galaxies that form at the centres of the haloes (White & Rees 1978; Zavala et al. 2016). Although the precise connection between the angular momenta of the galaxies and their haloes remains a topic of active research (e.g. Jiang et al. 2019; Posti et al. 2019), it is well established that the angular momentum of galaxies plays a fundamental role for their global appearance, such as their size and density (Mo et al. 1998). The co-evolution of a galaxy’s mass and angular momentum thus influences local (Toomre 1964) and global (Ostriker & Peebles 1973) stability modes, which is a likely reason for angular momentum to correlate with Hubble morphology (Fall 1983; Romanowsky & Fall 2012; Obreschkow & Glazebrook 2014), disk thickness and colour (Hernandez & Cervantes-Sodi 2006), star formation efficiency (Moster et al. 2010) and cold gas fraction (Huang et al. 2012; Obreschkow et al. 2016).

The relationship between stellar mass $M_*$, stellar angular momentum $J_*$ and morphology has been explored since Fall (1983). Since the angular momentum of a mechanical system is proportional to its mass (if keeping lengths and velocities fixed), it is common to remove this obvious mass-dependence by introducing the specific angular momentum $j_* = J_*/M_*$. Any residual dependence between $j_*$...
and $M_\ast$, then requires a non-trivial explanation. Empirically, more massive galaxies have higher angular momentum, with $j_\ast = qM_\ast^2$. The morphology-dependent scale factor $q$ indicates a lower angular momentum for bulge-dominated galaxies (Romanowsky & Fall 2012; Posti et al. 2018b; Fall & Romanowsky 2018), and the roughly constant exponent $\alpha = 2/3$ is consistent with the corresponding exponent for haloes in a scale-free cold dark matter (CDM) universe (Mo et al. 1998). However, at fixed bulge-to-total light ratio $\beta$, the data for spiral galaxies is consistent with $\alpha = 1$ (Obreschkow & Glazebrook 2014; Cortese et al. 2016; Sweet et al. 2018). In particular, galaxies that host rotation-supported pseudobulges follow along a tight track in $\beta - j_\ast/M_\ast$, consistent with $\alpha = 1$, whereas galaxies that host dispersion-supported classical bulges have larger bulges for a given $j_\ast/M_\ast$ (Sweet et al. 2018), consistent with $\alpha = 2/3$ (Fall & Romanowsky 2018).

Conversely, turbulent, clumpy galaxies thought to be the local analogues of high-redshift galaxies have smaller bulges at fixed $j_\ast/M_\ast$ than described by the ‘pseudobulge track’, possibly indicating that their clumps may form future pseudobulges as they lose angular momentum and migrate to the centres of these galaxies (Sweet et al. 2019).

Early toy models of disks (e.g. Fall & Efstathiou 1980) assumed that the specific angular momentum of the baryons in the disk $j_\ast$ follows that of the DM halo, $j_\ast$, since the DM halo spinning up in the early universe is subject to the same tidal torques as the baryons it contains. This assumption has provided a working empirical description (e.g. since Fall 1983). However, the connection is not well understood: while it is reasonable that the baryons in the halo intially share the same $j$ as the DM, it does not necessarily follow that the baryons that collapse into a disk also share the same $j$. Since only a fraction of the baryons collapse into the disk, one naively expects that it is those with low $j$ that collapse first (Navarro & Steinmetz 1997; Dutton & van den Bosch 2012). This is indeed what early simulations showed, where disks in smoothed-particle hydrodynamics simulations historically had an excess of low-angular momentum material, manifested as larger bulges and thicker disks than observed (Governato et al. 2010; Agertz et al. 2011). This “angular momentum catastrophe” is somewhat ameliorated by including appropriate feedback mechanisms to remove low-angular momentum material (e.g. Genel et al. 2015). However, this fine tuning does not explain the broad correspondence between $j_\ast$ and $j_\ast$, wherein the ratio $j_\ast/j_\ast$ must vary with $(M_\ast/M_\ast)^{2/3}$ to give $j_\ast \sim M_\ast^{2/3}$ (Posti et al. 2018a).

While the mean $j_\ast$ and $j_\ast$ agree within a factor of two, the internal angular momentum distribution of the baryons is substantially affected by the physical processes that rearrange and remove angular momentum and stellar mass in the disk. For example, stellar and AGN feedback remove low-angular momentum material, and tidal stripping preferentially removes high-angular momentum material (Governato et al. 2010; van den Bosch 2001). Viscosity, resulting from bars and star-forming clumps, transports angular momentum in the disk and impacts the angular momentum distribution (van den Bosch et al. 2001; Elmegreen & Struck 2013). The angular momentum distribution is also affected by galaxy mergers. Lagos et al. (2018) used the EAGLE cosmological hydrodynamical simulations (Schaye et al. 2015) to quantify the effect. They found that gas-poor (dry) mergers significantly redistribute the stellar $j_\ast$ in a way that the internal parts become largely $j_\ast$-deficient, while gas-rich (wet) mergers can have the opposite effect, of increasing $j_\ast$ in the central regions. The angular momentum distribution thus promises a stronger tracer of a galaxy’s evolutionary history than mean kinematic properties such as $j_\ast$ or global spin parameter $\lambda$.

The angular momentum distribution is thought to be near-universal for DM haloes owing to the balance between cosmic torques imparting angular momentum and the mass profile of the material to which it is imparted (van den Bosch et al. 2001; Bullock et al. 2001; Liao et al. 2017). This universality has provided clear predictions for the distribution of angular momentum in haloes and inspired much effort to link to observed angular momentum, using various methods to quantify angular momentum and its distribution. A simple quantification is found in specific angular momentum $j = J/M = \int dM/r \times v/M$, which is defined and measurable both for stars in the disk ($j_\ast$) and for the halo ($j_\ast$).

The distribution of $j_\ast$ has been studied as early as Crampin & Hoyle (1964), who found the mass distribution of $j_\ast$, for eight Sb and Sc spiral galaxies to be consistent with the mass distribution of a uniformly-rotating spheroid. Fall & Efstathiou (1980) then explored the formation of model disk galaxies embedded in dark matter haloes and also found the angular momentum of the disk to be consistent with that of an isothermal sphere. However, it was later shown that the distribution of $j_\ast$ in the baryons ought to be further affected by nonlinear torques (Barnes & Efstathiou 1987), so the link between the the angular momentum distributions for baryonic disk and DM halo is likely more complicated than first appeared. Bullock et al. (2001) used a shape parameter $\mu$ to parametrize the cumulative mass distribution of $j$ in haloes, and found a narrow distribution around mean $\log(\mu - 1) = -0.6 \pm 0.4$, having little or no correlation with mass or morphology; $\mu$ is larger for the (gas) disk than for the DM halo (Chen et al. 2003). van den Bosch et al. (2001) showed for the same DM haloes in Bullock et al. (2001) that the mean probability density function (PDF) of normalised $s_\ast = j/M_\ast$ is described by a distribution that peaks at $j = 0$ with a tail to high $j$, with the exact functional form depending on $\mu$. Later study of the misalignment between gas and DM demonstrated a component of negative angular momentum material contributing to a bulge and leaving behind an exponential density distribution (van den Bosch et al. 2002). Exponential disks have also been shown to form ubiquitously from stellar scattering off of star-forming clumps (Elmegreen & Struck 2013), and radial migration (Hernquist et al. 2017). van den Bosch et al. (2001) were among the first to remark that rotating disk galaxies (in their case, dwarfs) are deficient both in high- and low-$j_\ast$ material when compared with their parent DM halo. In order to describe both centrally-peaked halo PDFs and exponential disk PDFs with a single functional form, Sharma & Steinmetz (2005) parametrized $s$ by a Gamma distribution function. They found a mean shape parameter $\alpha_0 = 0.83$ for haloes and $\alpha_0 = 1.41$ for model exponential disks embedded in a NFW halo (Sharma et al. 2012). However, since the Gamma function diverges, such fits are unstable near the critical value at $\alpha_0 = 1$ and

\[1 \text{ Referred to here with subscript so as not to be confused with slope of the } M_\ast - j_\ast \text{ relation.} \]
this parametrization is consequently not usefully robust for most observations.

Regardless of the parametrization, the PDF(s) of the stars encodes much physical information about the evolution of the baryonic material after becoming decoupled from the DM halo. Outflowing material due to AGN- or stellar feedback carries away inner, dispersion-dominated gas with low s, leading to lower angular momentum of the stars that later form from that gas (Dutton 2009; Brook et al. 2011; Sharma et al. 2012), with low-mass galaxies losing the bulk of such material but high-mass galaxies potentially causing a galactic fountain, where the outflowing material falls back onto the disk and is redistributed (Brook et al. 2012). Conversely, tidal stripping by interactions with the intergalactic medium removes outer, rapidly-rotating, high-s material (van den Bosch et al. 2001). Angular momentum is also redistributed by star-forming clumps, spiral arms, bars, mergers and viscosity (Mo et al. 1998). Biased collapse of preferentially low-angular momentum material (Posti et al. 2018a) is another mechanism for differences between the stellar and halo angular momentum distribution. Given the near-universality of halo PDF(s) and the fact that the distributions of the halo and baryonic j are initially linked due to common tidal torques, observed differences between the distributions trace the physical processes that affect the baryons and produce the observed morphologies (van den Bosch et al. 2001). In addition, the observable stellar PDF(s) combines both photometric (surface density) and kinematic information in a spatially-resolved manner and may be useful as a tracer of morphology and as a bulge-disks decomposition tool.

The motivation for this paper is that there is no sample of nearby galaxies spanning morphological types E0 to Sd for which PDF(s) has been measured, yet quantitative measures of PDF(s) can provide a new, more physically-motivated way to compare simulations and data, where additional (kinematic) information is added to visual morphology. Accordingly, in this paper we present stellar specific angular momentum distributions for 24 nearby galaxies with high-quality observations in the Calar Alto Legacy Integral Field Area survey (CALIFA; Sánchez et al. 2012; Husemann et al. 2013; Walcher et al. 2014; Sánchez et al. 2016), and we develop and provide statistical parametrizations of the shapes that can be easily compared with those from simulations. Section 3 describes the data and the observations. Our methods for computing spatially-resolved specific angular momentum are described in Section 3. Parametrizations of PDF(s) are presented in Section 4. In Section 5 we investigate the PDF(s)-morphology relation. Implications for disk evolution and the utility of PDF(s) as a kinematic tracer of morphology and decomposition tool are discussed in Section 6. Section 7 concludes the paper.

We assume a cosmology where \( H_0 = 70 \text{km s}^{-1} \text{Mpc}^{-1} \), \( \Omega_M = 0.27 \) and \( \Omega_\Lambda = 0.73 \), and quote comoving coordinates throughout.

## 2 SAMPLE & OBSERVATIONS

The CALIFA survey presented stellar kinematic and surface density maps for 300 nearby galaxies in Falcón-Barroso et al. (2017). The maps are based on the V1200 grating, with wavelength range 3750–4550Å, instrumental resolution of approximately 72 km s\(^{-1}\), and 1″ spaxels. The spatial resolution is limited in the galaxy centres by the 2.7′-wide fibres and at the outskirts by the Voronoi binning (Cappellari & Copin 2003) used to increase signal-to-noise per bin above 20. The authors employed penalised pixel fitting pPXF (Cappellari & Emsellem 2004), using spectral templates from Vazdels et al. (2004), to obtain Gaussian line-of-sight velocities and velocity dispersions. In Sweet et al. (2018) we inspected the integrated angular momentum \( j_{i,<}(r) \) for CALIFA galaxies observed to the highest multiples of effective radius \( r_e \), and found that \( j_{i,(r)} \) for those galaxies typically reaches \( >0.99j_\star \) at a radius \( r \sim 3r_e \). We thus selected galaxies observed to at least \( 3r_e \), which resulted in a sample of 50 galaxies. For this work we focus on galaxies without bars in order to simplify our analysis of the relation between PDF(s) and morphology. After discarding barred galaxies we are left with 26 regular local galaxies with high-quality kinematics. We also discard two galaxies with skewness \( b_1 \) or \( b_{1,p} \) (as defined in Section 3) more than 15\( \sigma \) from the sample mean. Bulge-to-total light ratios \( \beta \) are taken from bulge-disk decompositions given in Méndez-Abreu et al. (2017), who use a Levenberg-Marquardt algorithm to fit components to the 2D surface brightness distribution of Sloan Digital Sky Survey (SDSS, Abazajian et al. 2009) images. We use the \( r \)-band decompositions in this work. We obtain Hubble types \( T \) using the methods of Fisher & Drory (2008), namely combining high-resolution imaging from HST with wide-field ground-based imaging, and accounting for a different mass-to-light ratio in the bulge versus the disk. These \( T \) are robust to within an uncertainty of \( \Delta T = \pm 0.5 \). The \( M_{B} - j_{\star} \) relation is shown in Figure 1. SDSS postage stamps for each galaxy in our sample are shown in Appendix A. The final sample of 24 galaxies spans bulge-to-total ratios \( 0.02 \leq \beta \leq 0.67 \) and Hubble types from Sd to E/S0.

## 3 METHODS

Specific angular momentum is measured in a spatially-resolved manner as follows. First, in every spaxel \( i \) within the observed map of \( n \) spaxels, we calculate the \( z \)-component (along the rotation axis) of the mean stellar specific angular momentum,

\[
 j_{z,i} = (r_i \times v_i)_z, \tag{1}
\]

where \( v_i \) is the stellar velocity in that spaxel and \( r_i \) is the radius of that spaxel from the galaxy centre. Since only the line-of-sight component of \( v_i \) and the transverse components of \( r_i \) can be measured, we recover the remaining vector components by assuming that all galaxies are flat circular disks of radius-independent inclination and position angle derived from a fit to the CALIFA \( B \)-band surface brightness maps.

Note that each spaxel \( i \) contains a large number of unresolved particles (stars) on different orbits, as evidenced by the measurable line-of-sight velocity dispersion \( \sigma_r \). The velocity distribution is assumed to be isotropic and normal. In this case, the distribution of specific angular momenta in each spaxel is also normally distributed, with a mean given by Eq. 1 and standard deviation \( |r_i| \sigma_r \). Formally, each spaxel \( i \) gives rise to a normal \( j_{z,i} \)-distribution expressed as

\[
 PDF(j_z) = N(j_z; \mu = j_{z,i}, \sigma^2 = |r_i| \sigma_r^2), \tag{2}
\]
The total PDF($j_s$) of the full stellar disk can then be computed as the mass-weighted average of PDF$_i$(${j}_i$),

$$PDF(j_s) = \frac{\sum_{i=1}^{n} m_{s,i} \cdot PDF_i(j_i)}{M_s}, \quad (3)$$

where $m_{s,i}$ is the stellar mass in the $i$-th spaxel and $M_{s,i} = \sum_{i=1}^{n} m_{s,i}$ is the total stellar mass. Upon assuming a constant mass-to-light ratio ($M/L$) across the galaxy, the actual value of $M/L$ is irrelevant, since it appears equally in the numerator and denominator of Eq. 3 and hence cancels out.

It is convenient to introduce the normalised specific angular momentum $s = j_s / j_{s,\text{mean}}$, where

$$j_{s,\text{mean}} = j_s / M_{s} = \frac{\sum_{i=1}^{n} m_{s,i} \cdot (r_i \times v_i)_z}{\sum_{i=1}^{n} m_{s,i}} = M_{s}^{-1} \sum_{i=1}^{n} m_{s,i} \cdot j_{s,i}. \quad (4)$$

From here on, PDF$(s)$ is defined as Eq. 3, where the argument is $j_s = s \cdot j_{s,\text{mean}}$.

The probability density in a finite interval $[s, s + \Delta s]$ can be computed as

$$P(s, s + \Delta s) = \int_{s}^{s + \Delta s} PDF(s) \, ds = \frac{1}{2M_s} \sum_{i=1}^{n} m_{s,i} \cdot \text{erf} \left( \frac{s + \Delta s - s_i}{\sqrt{2} \sigma_{s,i}} \right) - \text{erf} \left( \frac{s - s_i}{\sqrt{2} \sigma_{s,i}} \right), \quad (5)$$

where $s_i = j_{s,i} / j_{s,\text{mean}}$ and $\sigma_{s,i} = |r_i| \sigma_j / j_{s,\text{mean}}$. Discretised probability densities can thus be obtained as $P(s, s + \Delta s) / \Delta s$.

For plotting purposes, we will choose $\Delta s$ such that the number of histogram bins equals the number of seeing elements along the major axis of the galaxy, but all our statistical analyses rely on much smaller (numerically converged) bins.

In addition to the dispersion-broadened PDF$(s)$ described above, we also derive a circular-only PDF$(s)$, that is, setting the dispersion to zero. The motivation for this kind of PDF$(s)$ is to provide a benchmark comparison with samples that do not have dispersion measurements e.g. observations of gas kinematics-only, at low spectral resolution, and/or at high-redshift. Equation 5 is undefined when dispersion is zero, so the circular-only PDF$(s)$ is calculated by setting the dispersion to be small (in practise, $\sigma_{s,i} \sim v_i / 1e4$); this is appropriate when the number of spaxels across the galaxy is correspondingly large.

We can analyse the shape of PDF$(s)$, for instance by analysing the moments of this continuous distribution. The mean $\mu = E(s) \equiv 1$ by definition of $s$. The $n$-th moment is given by

$$\mu_n = \int (s - \mu)^n PDF(s) \, ds. \quad (6)$$

We evaluate this integral numerically between $\pm A$, where $A = \max\{j_{s,i} + 3|\sigma_j|/j_{s,\text{mean}}\}$. The standard deviation $\sigma = \sqrt{E[(s - \mu)^2]}$ is the square root of the second central moment of the distribution. The third standardised moment about the mean quantifies skewness$^2$:

$$b_1 = \frac{\mu_3}{\sigma^3} = \frac{E[(s - \mu)^3]}{\sigma^3}. \quad (7)$$

Distributions that are symmetric about the mean have $b_1 = 0$. Higher $b_1 > 0$ indicates more probability density in the tail to the right of the mean (positively skewed), and $b_1 < 0$ indicates more probability density in the left tail (negatively skewed).

The fourth standardised moment about the mean quantifies kurtosis:

$$b_2 = \frac{\mu_4}{\sigma^4} = \frac{E[(s - \mu)^4]}{\sigma^4}. \quad (8)$$

Kurtosis $b_2 = 3$ corresponds to a Gaussian-like (mesokurtic) distribution. Higher $b_2 > 3$ indicates that there is more probability density in the peak and tails rather than the shoulders of the distribution compared to a normal distribution (leptokurtic). Lower $b_2 < 3$ indicates that there is less probability density in the peak and tails (platykurtic).

Uncertainties are estimated by conducting a Monte-Carlo simulation where the noise term added to $v_i$ is randomly drawn from a normal distribution whose width is the stellar circular velocity 1-$\sigma$ uncertainty map; the noise added to $\sigma_j$ is similarly drawn from the velocity dispersion 1-$\sigma$ uncertainty map.

Results and properties of the sample of CALIFA galaxies are given in Table 1. Plots of the PDF$(s)$ are shown in Figure 2.

---

$^2$ The notation $b_1, b_2$ refers to the sample statistic. We use this notation rather than population $\beta_1, \beta_2$ to avoid confusion with bulge-to-total ratio $\beta$ as commonly used in angular momentum studies.
Table 1. Properties of CALIFA galaxies.

| Name         | RA           | Dec          | $z$  | $M_*$  | $\Delta M_*$ | $\beta$ | $\Delta \beta$ | $n_0$ | $\Delta n_0$ | $T$  |
|--------------|--------------|--------------|------|--------|--------------|---------|---------------|-------|--------------|------|
| IC1151       | 239.6347351  | 17.4414577   | 0.009 | 9.85   | 0.15         | 0.02    | 0.10          | 0.6   | 0.0          | Sc   |
| MCG-02-02-030 | 7.5304556    | -11.1136293  | 0.011 | 10.37  | 0.15         | 0.08    | 0.10          | 1.2   | 0.1          | Sb   |
| NGC5001      | 1.8160844    | 27.7080822   | 0.015 | 10.80  | 0.15         | 0.46    | 0.10          | 2.8   | 0.2          | Sb   |
| NGC2253      | 199.024364   | 65.2063751   | 0.013 | 10.52  | 0.15         | 0.06    | 0.10          | 1.1   | 0.1          | Sbc  |
| NGC2592      | 126.7853646  | 25.9703159   | 0.008 | 10.62  | 0.15         | 0.54    | 0.10          | 3.3   | 0.1          | E7S07 |
| NGC2639      | 130.9086151  | 50.2055397   | 0.012 | 11.17  | 0.15         | 0.20    | 0.10          | 2.0   | 0.1          | Sb   |
| NGC2906      | 143.0259904  | 8.4417688    | 0.008 | 10.39  | 0.15         | 0.06    | 0.10          | 1.4   | 0.0          | Sbc  |
| NGC3815      | 175.4137115  | 24.8004990   | 0.014 | 10.35  | 0.15         | 0.04    | 0.10          | 0.7   | 0.1          | Sab  |
| NGC3961      | 196.4481812  | 27.7341385   | 0.010 | 9.68   | 0.15         | 0.02    | 0.10          | 0.7   | 0.1          | Sb   |
| NGC5480      | 211.5989948  | 50.7250633   | 0.008 | 10.14  | 0.15         | 0.07    | 0.10          | 1.4   | 0.1          | Sc   |
| NGC5971      | 239.0307476  | 56.4616890   | 0.016 | 10.32  | 0.15         | 0.67    | 0.10          | 4.4   | 0.4          | Sa   |
| NGC5980      | 235.3768463  | 15.7876863   | 0.016 | 10.72  | 0.15         | 0.07    | 0.10          | 0.9   | 0.1          | Sbc  |
| NGC6063      | 241.8041382  | 7.9879910    | 0.011 | 10.14  | 0.15         | 0.03    | 0.10          | 3.6   | 0.3          | Sc   |
| NGC6247      | 265.9108276  | 25.4939384   | 0.013 | 10.75  | 0.15         | 0.36    | 0.10          | 2.3   | 0.3          | S0    |
| NGC6762      | 286.4045140  | 63.9410877   | 0.011 | 10.38  | 0.15         | 0.32    | 0.10          | 3.5   | 0.3          | S0    |
| NGC7311      | 338.5283203  | 5.5703239    | 0.015 | 11.07  | 0.15         | 0.27    | 0.10          | 1.6   | 0.1          | Sab  |
| NGC7623      | 350.1259305  | 8.3957691    | 0.013 | 10.98  | 0.15         | 0.47    | 0.10          | 2.0   | 0.2          | S0    |
| NGC7653      | 351.2056860  | 15.2756601   | 0.014 | 10.50  | 0.15         | 0.33    | 0.10          | 3.3   | 0.2          | Sc   |
| NGC7671      | 351.8305664  | 12.4674091   | 0.014 | 10.96  | 0.15         | 0.26    | 0.10          | 2.1   | 0.1          | S0    |
| NGC7683      | 352.2659302  | 11.4451685   | 0.012 | 11.02  | 0.15         | 0.46    | 0.10          | 2.4   | 0.1          | S0    |
| NGC7716      | 354.1310425  | 0.2972720    | 0.009 | 10.39  | 0.15         | 0.14    | 0.10          | 1.8   | 0.2          | Sbc  |
| NGC7824      | 1.275986  | 6.9201469    | 0.020 | 11.25  | 0.15         | 0.42    | 0.10          | 2.3   | 0.2          | Sab  |
| UCG00987     | 21.3810329   | 32.1362724   | 0.015 | 10.61  | 0.15         | 0.24    | 0.10          | 3.9   | 0.4          | Sa   |
| UCG09476     | 220.3834534  | 44.5172710   | 0.013 | 10.21  | 0.15         | 0.03    | 0.10          | 1.2   | 0.1          | Sc   |

Columns: (1) galaxy identifier; (2) right ascension (J2000) [deg]; (3) declination (J2000) [deg]; (4) redshift; (5) base 10 logarithm of stellar mass [log($M_*$)]; (6) measurement uncertainty in $M_*$ [dex]; (7) $r$-band bulge-to-total light ratio; (8) measurement uncertainty in $\beta$; (9) bulge Sérsic index; (10) measurement uncertainty in $n_0$; (11) Hubble type; (12) stellar specific angular momentum [kpc km s$^{-1}$]; (13) measurement uncertainty in $j_*$ [kpc km s$^{-1}$]; (14) area under ‘bulge’ curve from normal + log-normal fit (* denotes fit failed); (15) skewness of PDF(s); (16) measurement uncertainty in $b_{1c}$; (17) kurtosis of PDF(s); (18) measurement uncertainty in $b_2$; (19) skewness of PDF(s) without including velocity dispersion; (20) measurement uncertainty in $b_{1c}$; (21) skewness of radius-selected bulge PDF(s); (22) measurement uncertainty in $b_{1\rho}$; (23) radius of ‘bulge’ component [$r_\rho$]. (7-10) from CALIFA; (11-23) this work.
4 RESULTS

In this Section we describe the probability density function of normalised specific angular momentum PDF(s) and investigate methods for quantifying its shape, demonstrating that parametrisation is not the best method for describing PDF(s).

4.1 Shape of PDF(s)

Figure 2 illustrates the range of PDF(s) that characterises local CALIFA galaxies. For galaxies with earlier types and larger bulge-to-total ratio $\beta$, the PDF(s) visually tends to have a sharper peak near $s = 0$ and exponential decline, corresponding to a positively-skewed PDF(s) with $b_1 > 1.2$. On the other hand, the later type galaxies with smaller $\beta$ visually have a broader, more normal distribution and peak nearer $s = 1$, have more symmetric PDF(s) with $b_1 \sim 0.8$.

In Fig. 3 we illustrate a spanning set of local examples highlighted in Fig. 2. NGC 2906 is a late-type spiral galaxy with low bulge-to-total light ratio $\beta = 0.06$. Its PDF(s) is broad and symmetric, and peaks near $s = 1$. NGC 2592 is an early-type galaxy with large $\beta = 0.54$; its PDF(s) is strongly-skewed and peaks near $s = 0$. In between these two extremes, the PDF(s) for NGC 7311 ($\beta = 0.27$) has characteristics of both disk and bulge.

Figure 2. Probability density functions of normalised specific angular momentum (PDF(s)) for CALIFA galaxies, plotted in bulge-to-total light ratio $\beta$ - Hubble type $T$ space. An offset of up to $\pm 0.5\Delta T$ is added in the y-direction for clarity. Each PDF(s) has ticks at $s=0, 1, \text{and } 2$, and is labelled with $b_1$. Highlighted with black outlines are three examples examined further throughout the paper: 1) late-type spiral NGC 2906 with low $\beta = 0.06$ has a broad, symmetric PDF(s) centred on $s = 1$. 2) early type NGC 2592 with high $\beta = 0.54$ has a strongly-skewed PDF(s) which peaks nearer $s = 0$. 3) NGC 7311 with moderate $\beta = 0.27$ has a PDF(s) which is intermediate between the two extremes. The shape of PDF(s) is quantified by skewness $b_1 = \mu_3/\sigma_3$ as defined in the text. More normal, symmetric distributions ($b_1 \sim 0.8$) are coloured blue and strongly-skewed ($b_1 > 1.2$) distributions are coloured red. Galaxies with earlier types and larger $\beta$ tend to have skewed PDF(s), while the later types with smaller $\beta$ have symmetric PDF(s).
Figure 3. PDF(s) for three local galaxies in the CALIFA sample, with models and fits overlaid. The blue histogram denotes observed s; the black solid curve gives the best-fitting model normal+log-normal function; black dotted curves indicate the normal and log-normal components to that fit; the red short-dashed, long-dashed and short-long-dashed curves represent the measurement for the dark matter halo in three NIHAO galaxies; the orange dot-dashed curve is the analytic solution for an exponential disk. The s < 0 measurements arise from the inclusion of the dispersion term and are physically meaningful. Neither the models nor the fits adequately describe the observations.

In Figure 4 we show PDF(s) derived using circular-only velocity, that is, calculated without the dispersion term. The circular PDF(s) for late-type NGC 2906 is symmetric, centred near s = 1. For early-type NGC 2592 the circular PDF(s) peaks nearer s = 0.5 and is positively skewed, while NGC 7311 has skewness intermediate between them. In all cases the effect of omitting the dispersion term is to recover more structure in the PDF(s); in other words, including the dispersion term (as in the dashed histograms and Figure 3) has the effect of smoothing the PDF(s).

Figure 5 demonstrates the tight correlation between circular PDF(s) skewness $b_1$ and dispersion-broadened PDF(s) skewness $b_1$, with correlation coefficient $R = 0.94$ and scatter $\sigma = 0.07 \pm 0.01$. Circular PDF(s) may thus provide a useful comparison in the cases where dispersion measurements are unavailable. The offset between $b_1$ and $b_1c$ is a function of Hubble type, with early-type galaxies (darker shades of red) tending to have smaller offsets ($b_1 - b_1c$) and late-type galaxies (lighter shades) having larger offsets, with $b_1 > b_1c$. For early-type galaxies, the velocity dispersion profile $\sigma(<r)$ tends to decrease with radius (Falcón-Barroso et al. 2017) and thus also with $j_*$, so that the effect of including dispersion to calculate $b_1$ has the greatest magnitude at low s. Since the bulk of the probability mass is at low s, the
skewness \( b_1 \) is not much larger than \( b_1c \), for the earliest-type galaxies. Conversely, for late-type galaxies \( \sigma(< r) \) tends to increase with radius and \( j_\alpha \), so that the effect of including dispersion has the greatest magnitude at high \( s \). The result is that \( b_1 \) is appreciably larger than \( b_1c \), for late types. However, we point out that conversion between \( b_1c \) and \( b_1 \) can be performed using the coefficients given in Table 2, without needing to measure Hubble type or \( \sigma(< r) \).

### 4.2 Quantifying PDF(s)

In this subsection we demonstrate and explain that simple models and various functional forms do not well describe PDF(s).

Motivated by the fact that the naive model where \( j_\text{p}(\text{disk}) \sim j_\text{h}\text{alo} \) works surprisingly quite well, we also explore a naive model where PDF(s) \( \sim \) PDF(\( j_\text{p} \)). Our DM controls come from the NIHAO simulations (Wang et al. 2015), where halo PDF(\( j_\text{p} \)) is measured for the DM inside the virial radius at \( z = 0 \), and the \( z \)-direction is defined as the direction of the mean angular momentum vector of all matter inside the virial radius. The individual particle values were used in order to align with our observational method of accounting for the dispersion. Three NIHAO galaxies were selected to approximately span the mass range of our CALIFA sample and to qualitatively demonstrate that the NIHAO PDF(\( j_\text{p} \)) tends to be rather self-similar, but different to the observed PDF(s) for CALIFA galaxies. NIHAO galaxy g8.26e11 is a Milky-Way-like galaxy, with virial mass \( 1.02 \times 10^{12} M_\odot \) and stellar mass \( 4.68 \times 10^{10} M_\odot \); Galaxy g7.55e11 has virial mass \( 8.93 \times 10^{10} M_\odot \) and stellar mass \( 3.11 \times 10^{10} M_\odot \); Galaxy g1.06e11 is a dwarf galaxy, with virial mass \( 1.18 \times 10^{10} M_\odot \) and stellar mass \( 5.67 \times 10^9 M_\odot \) at \( z = 0 \) (Wang et al. 2015). See Wang et al. (2018) for further analysis of g8.26e11. The PDF(\( j_\text{p} \)) for these galaxies is shown by the red, short-dashed, long-dashed and short-long-dashed lines in Fig. 3. It is clear that the halo curves do not at all describe late-type NGC 2906, intermediate NGC 7311, or even early type NGC 2592. In all cases there is less baryonic material compared with the DM halo at low \( s (\lesssim 0) \), more at average \( s (\sim 1) \), and less at high \( s (\gtrsim 3) \).

The analytic solution for an exponential disk with constant rotation velocity is

\[
p(s) = 4 \pi e^{-2s},
\]

plotted in Fig. 3 as orange dot-dashed lines. As expected, this model fails to describe the galaxies in our sample, even the almost-pure disks such as NGC 2906 which has \( \beta = 0.06 \), since the rotation velocities are not constant but decrease towards the galaxy centres.

In attempt to better parametrize the PDF(s), Sharma & Steinmetz (2005) fitted a Gamma distribution function

\[
P(j) = \frac{1}{\Gamma(\alpha)} j^{\alpha-1} e^{-j/\mu},
\]

where

\[
\mu = \frac{j_{\text{mean}}}{\alpha},
\]

which is described by a single shape parameter \( \alpha \). The function goes to zero at \( s = 0 \) when \( \alpha > 1 \) and to \( \infty \) when \( \alpha < 1 \). A value of \( \alpha = 2 \) describes an exponential disk embedded in an isothermal halo with constant rotation velocity. Sharma & Steinmetz (2005) showed that gas has higher \( \alpha \) than the halo because the bulk of its mass rotates more quickly in the inner parts for the ‘particle method’ of analysis, though no significant difference was found between the gas and halo for their other methods. However, since the fit is critically unstable at \( \alpha = 1 \) and is strongly biased by the few bins around \( s \sim 0 \) and the choice of bin width, the fit often fails to converge for galaxies in our sample, and is consequently not plotted in Fig. 3.

Instead we attempt to fit the PDF(s) with a simple, two-component model intended to describe the dispersion-dominated material (that is, bulge, thick disk, clumps) and the rapidly-rotating thin disk. The random motion of the dispersion-dominated material can be approximated with a normal distribution that peaks at \( s = 0 \), allowing a component of negative velocity material as demonstrated in van den Bosch et al. (2002). Ideally the thin disk would be represented by Equation 11, which assumes an exponential disk surface brightness with scale length \( r_d \) and a constant rotation velocity \( v_{\text{max}} \); alternatively a more typical rotation curve could be assumed, e.g. \( v(r) = v_{\text{max}} (1 - \exp(-r/r_d)) \). We found that the combined normal + Eqn. 11 fits frequently failed to converge in our sample as they do not well describe even the high-\( s \) part of the distributions. The typical rotation curve model is less simple, since it varies with both \( r_d \) and \( r_{\text{halo}} \); it therefore introduces additional variation with disk concentration \( r_d/r_{\text{halo}} \) which is not constant with stellar mass (Ohreschkow & Glazebrook 2014). Consequently, the corresponding analytic solution \( p(s) \) is not a simple function of \( s \) alone but also of covariant parameters \( v_{\text{max}} \) and \( r_{\text{halo}} \). Such fits also failed to converge for our sample. We thus settle on a normal + log-normal distribution. We perform these fits using a non-linear least squares approach on the unbinned data, using the Levenberg-Marquardt algorithm.

---

3 This comparison is similar to the van den Bosch et al. (2001) analysis, except that was concerned with the norm of \( j \) rather than the \( z \)-component we show in this work:

\[
p(s) = \frac{\xi \mu (\mu - 1)}{\xi + \mu - 1},
\]

where \( \mu \) is a parameter describing the mass distribution of \( j/j_{\text{max}} \) and

\[
\xi = \frac{j_{\text{mean}}}{j_{\text{max}}} = 1 - \mu \ln \left( \frac{\mu}{\mu - 1} \right). \tag{10}
\]

They assumed \( \mu = 1.25 \), the average found by Bullock et al. (2001), though we note that Eqn 9 is peaked at \( s = 0 \) for any value of \( \mu \). They investigated a sample of dwarf galaxies and similarly found a lack of low-\( s \) material, as well as of high-\( s \) material. Indeed, Sharma & Steinmetz (2005) found that Equation 9 does not well describe an NFW halo with an embedded exponential disk, regardless of choice of parameter \( \mu \), or model concentration, virial radius, scale length or mass fraction.

4 Note that the Sharma & Steinmetz (2005) work was also concerned with the norm of \( j \) rather than the \( z \)-component.

5 If one considers that the dispersion-dominated material may be skewed towards positive \( x \), the normal distribution could be replaced with a second log-normal distribution, but we found that such fits were not well constrained.
Figure 4. Example PDF(s) for local galaxies in the CALIFA sample (blue, filled histogram), using circular velocity (not including the dispersion term). The dispersion-broadened PDF(s) from earlier figures is shown as a black, dashed, open histogram.

(Moré 1978). The normal + log-normal distribution fits are overlaid in black on the examples in Figure 3. These distributions tend to fit the PDF(s) reasonably well at $s > 1$ but less so at $s < 1$. For example, for NGC 7311 the peak near $s \sim 0$ is not well captured by the normal curve. Further, one could define the area under the normal, ‘bulge’ component curve to be $A_B$, and the area under the normal + log-normal curve as $A$. One would then expect the ratio $A_B/A = P_B$, to be related to bulge-to-total ratio $\beta$. However, Figure 6 illustrates that $P_B$ does not correlate with $\beta_{\text{CALIFA}}$, so we conclude that the normal + log-normal fit is not a useful method for quantifying the PDF(s) and its relationship with galaxy photometric morphology.

In sum, neither the analytical CDM fits nor the normal+lognormal model fits describe the data well for the purposes of this work. Instead it seems that the best method for quantifying PDF(s) is with the statistics of the underlying distribution as defined in Section 3, particularly skewness $b_1$. These statistics are robust against choice of bin size and are model-independent, so are our choice for quantifying PDF(s) in the remainder of the paper.

5 THE PDF(s) - MORPHOLOGY RELATION

In this section we analyse the relation between the shape of PDF(s), quantified by skewness $b_1$, and galaxy morphology, quantified by bulge-to-total ratio $\beta$ and Hubble type $T$. We also investigate PDF(s) calculated without dispersion, and bulge-disk decomposition.

Recall that Figure 2 illustrates a correlation between...
PDF(s) shape, $\beta$ and $T$, where late-type galaxies with low $\beta$ have symmetric PDF(s) with skewness $b_1 \sim 0.8$, while early-type galaxies with high $\beta$ have strongly positively-skewed PDF(s) with $b_1 > 1.2$. In Figure 7 we show the correlation between $T$ and $b_1$, and between $\beta$ and $b_1$; the corresponding fit parameters and correlation coefficients (derived using hyper.fit from Robotham & Obreschkow 2015) are given in Table 2. The Hubble type units are such that the difference between two tick-marks $\Delta T = Sd - Scd = 1$. We find that earlier-type galaxies that have bigger bulges tend to have PDF(s) that are more strongly-skewed. There is a moderate correlation between $T$ and $b_1$, with Pearson correlation coefficient $R = 0.58$ and significant scatter $\sigma = 0.20 \pm 0.03$. The correlation between $\beta$ and $b_1$ is stronger, with $R = 0.66$ and $\sigma = 0.09 \pm 0.02$. The fact that the $\beta - b_1$ correlation is stronger than the $\beta - T$ relation may reflect that Hubble type classification is more subjective than bulge-to-total light ratio. We suspect that the reason behind the PDF(s) - morphology relation is that the processes that cause the build-up, ejection and rearranging of angular momentum are those that contribute to the build-up of the bulge, effecting morphological change towards earlier types. The skewness of PDF(s) can therefore be used as a quantitative morphological tracer that is based on fundamental properties.

We do not find a strong correlation between morphology and kurtosis $b_2$, so have not shown the corresponding fits here.

### 5.1 Bulge-disk decomposition

In Figure 9 we show PDF(s) for NGC 2906, NGC 2592 and NGC 7311 decomposed into ‘bulge’ and ‘disk’ components. We use CALIFA $\beta$ to determine the fraction of light to assign to each component under a radius-based selection. We sum up the spaxels representing the cylinder of material nearest to the galaxy centre, increasing the radius of the cylinder until its mass (surface density) as a fraction of the total equals $0.5$. These spaxels are then assigned to the radius-selected bulge. Their PDF(s) is coloured orange in this figure and its skewness is denoted $b_{1rb}$. The remaining spaxels are assigned to the radius-selected disk; their PDF(s) is coloured red. The component distributions are normalised to the mean $j_i$ of the galaxy rather than the mean $j_i$ of each component. The radii of the bulges selected in this man-

Figure 5. Correlation between skewness of the bulge PDF(s) using the method described in Section 3 which accounts for velocity dispersion ($b_1$) vs. circular-only velocities ($b_{1c}$). Points are coloured by Hubble type, with redder colours indicating earlier-type galaxies. The three CALIFA example galaxies presented in Figure 3 are highlighted with black circles and labels. The black, dashed line gives the 1:1 relation. The two methods are tightly correlated, with circular-only PDF(s) being on average less skewed than the dispersion-broadened PDF(s), particularly for late-type galaxies which have increasing $\sigma(<r)$ profiles.

Figure 6. Comparison between fractional area under the Gaussian curve illustrated in Fig. 3, against bulge-to-total ratio from CALIFA. Three example galaxies presented in Fig. 3 are highlighted with black circles. A typical error bar is shown. The fitted $P_\beta$ does not correlate with CALIFA $\beta$.

Figure 8 illustrates that the skewness of circular-velocity PDF(s) $b_{1c}$ correlates with Hubble type $T$ and bulge-to-total ratio $\beta$. The correlations are similar in strength to the corresponding correlations with $b_1$. At first glance, the fact that $b_{1c}$ correlates with morphology may seem unsurprising given the tight correspondence between $b_{1c}$ and $b_1$, demonstrated earlier in Figure 5 and Table 2. However, it is non-trivial that omitting the fundamental morphological history that is encoded in the stellar velocity dispersion does not strongly affect the correlation with morphology. Likewise it is noteworthy that if dispersion measurements are unavailable (e.g. at low spectral resolution, of gas kinematics only, and/or at high redshift), then one can still use circular velocities to obtain a meaningful PDF(s)-based measure of morphology.
Figure 7. Correlation between morphology and shape of PDF(s) quantified by skewness $b_1$. The three CALIFA example galaxies presented in Figure 3 are highlighted with black circles and labels. Left: PDF(s) skewness $b_1$ is positively correlated with Hubble type $T$. Right: $b_1$ is positively correlated with bulge-to-total light ratio $\beta$. Earlier-type galaxies with bigger bulges have PDF(s) that are more strongly-skewed.

Figure 8. Correlation between morphology and shape of circular PDF(s) quantified by skewness $b_{1c}$. The three CALIFA example galaxies presented in Figure 3 are highlighted with black circles and labels. Left: PDF(s) skewness $b_{1c}$ is positively correlated with Hubble type $T$. Right: $b_{1c}$ is positively correlated with bulge-to-total light ratio $\beta$. Earlier type galaxies with bigger bulges have circular PDF(s) that are more strongly-skewed.
Table 2. Fit parameters for skewness of PDF(s) against Hubble type, bulge-to-total ratio, and bulge Sérsic index.

| x = a + bT | a | $\Delta a$ | b | $\Delta b$ | $\sigma$ | $\Delta \sigma$ | R |
|------------|---|-----------|---|----------|--------|-----------|---|
| y = $b_1$  | 0.82 | 0.10 | 0.06 | 0.02 | 0.20 | 0.03 | 0.58 |
| T (Fig. 7a) | 0.81 | 0.09 | 1.52 | 0.32 | 0.09 | 0.02 | 0.66 |
| y = $b_1c$ | -0.82 | 0.13 | 1.51 | 0.11 | 0.07 | 0.01 | 0.94 |
| $b_1$ (Fig. 5) | 0.47 | 0.16 | 0.08 | 0.03 | 0.31 | 0.05 | 0.53 |
| T (Fig. 8a) | 0.29 | 0.18 | 2.74 | 0.69 | 0.11 | 0.03 | 0.60 |
| y = $b_{1rb}$ | 0.43 | 0.10 | 0.03 | 0.02 | 0.17 | 0.03 | 0.41 |
| T (Fig. 10a) | 0.35 | 0.05 | 1.03 | 0.17 | 0.05 | 0.02 | 0.77 |
| $\beta$ (Fig. 10b) | 0.34 | 0.06 | 0.13 | 0.03 | 0.12 | 0.02 | 0.67 |

Clearly this is simply a first demonstration of such a PDF(s)-based decomposition, using photometric $\beta$ as a prior. A more sophisticated decomposition would ideally employ an iterative approach to arrive at a photokinematic $\beta$. This would allow to test to what extent $b_{1rb}$ could be used to classify bulge type. It would also be interesting in future work to explore the effect of spatial resolution.

6 DISCUSSION

We have shown that the spatially-resolved stellar specific angular momentum distribution, PDF(s), of a morphologically-broad sample does not match a selection of toy analytic solutions and simple functions. Instead, we characterise the shape of the PDF(s) with its skewness $b_1$, which correlates with photometric morphology. In this section we discuss the implications of our findings in the context of galaxy evolution.

In Figure 3 we demonstrated that the observed stellar PDF(s) differs significantly from the corresponding PDF(s) of constantly rotating exponential disks, as well as the PDF(s) expected for DM haloes. A normal plus log-normal distribution also fails to fit the range of PDF(s) shapes. Thus even though the baryonic $j_y$ is shown to be empirically similar to halo $j_y$ (Barnes & Efthaston 1987; Catelan & Theuns 1996a,b; van den Bosch et al. 2001; Posti et al. 2018a), which appears to be something of a coincidence, we find that the stellar PDF(s) is completely different to the halo PDF(s). The difference between the observed and predicted distributions shows that galaxy assembly is dominated by angular momentum transport processes experienced by the baryons after decoupling from the DM halo, and the fine tuning between $j_y$ and $j_y$ probably is a coincidence.

Compared with the DM halo models there is a dearth of low-$s$ ($s \lesssim 0$) baryonic material, an excess of average-$s \sim 1$ material, and a smaller lack of high-$s$ ($s \gtrsim 3$) material across the full range of morphological types. Processes that selectively decrease angular momentum within a galaxy include outflows and tidal stripping (van den Bosch et al. 2001). Low-$s$ material may be ejected by stellar- and AGN-feedback; such outflows preferentially have low angular momentum as predicted by Brook et al. (2011) and confirmed by Sharma et al. (2012). The amount of material lost due to outflows is dependent on stellar mass, since the deeper potential of high-mass systems may cause the outflowing material to fall back onto the galaxy in a galactic fountain and be redistributed throughout the disk (Brook et al. 2012). The large discrepancy between observed and predicted distributions in the low-s regime indicates that outflows may be important in the evolution of normal local galaxies such as in the CALIFA survey. High-s material can be stripped when galaxies interact with local tidal fields, removing rapidly-rotating material at the outskirts (van den Bosch et al. 2001). The fact that there is only a small difference between the observed and predicted PDF(s) at the high-s regime indicates that tidal stripping is not an important process for...
these galaxies; this is consistent with their typically isolated environment within the local universe.

However, neither outflows nor stripping can explain the apparent excess of baryonic material above that predicted at moderate $s$. Instead, the height of the distribution suggests that either the baryons experience a redistribution of angular momentum from the initial state when it was coupled with its DM halo, or the baryon fraction in the disk is small and the baryon distribution was initially higher than that of the DM. Angular momentum could be redistributed via viscosity, but that would cause low-$s$ material to fall to the centre and high-$s$ material to be pushed to the outskirts, increasing the amount of low- and high-$s$ material, which is not what is seen here. Alternative mechanisms include redistribution of angular momentum via high-dispersion components such as star-forming clumps, spiral arms and bars, or by merging with nearby galaxies. Simulations show that accreted gas tends to have higher $j$ than the DM halo (Stevens et al. 2017; Stewart et al. 2017; El-Badry et al. 2018). Modern theory predicts that disc contraction, or biased collapse, is also important in explaining the difference between angular momentum distributions of the DM halo and the baryons (Dekel & Burkert 2014; Posti et al. 2018a).

The shape of the PDF($s$) is parametrized by higher-order moments, where $b_1$ quantifies the skewness of the distribution. We showed that this parameter correlates with the photometric morphological tracers Hubble type $T$ and bulge-to-disk light ratio $\beta$. For each of these relations there is non-zero intrinsic scatter around a linear fit when accounting for uncertainties in both axes. Arguably, PDF($s$), which is computed from both kinematic and photometric data, encodes more physical information than photometry alone, so may be a more robust tracer of galaxy type. To test this suggestion would require 1) careful photometric classification...

Figure 9. Example PDF($s$) for local galaxies in the CALIFA sample (blue), with bulge (orange) and disk (red), using radius-based selection. See text for details of calculation.
Figure 10. Correlation between morphology and shape of bulge PDF(s) quantified by skewness $b_{1,rb}$. The three CALIFA example galaxies presented in Figure 3 are highlighted with black circles and labels. Left: bulge PDF(s) skewness $b_{1,rb}$ is correlated with Hubble type $T$. Right: $b_{1,rb}$ is correlated with bulge-to-total light ratio $\beta$. The angular momentum distributions of bigger bulges in earlier-type galaxies are more strongly skewed.

Figure 11. Correlation between CALIFA bulge Sérsic index $n_b$ and skewness of radius-selected bulge PDF(s) $b_{1,rb}$. The vertical, dashed line shows a pseudobulge cut, where $n_b < 2.2$ corresponds to galaxies that host pseudobulges, and $n_b \geq 2.2$ includes galaxies that may host pseudobulges or classical bulges. The horizontal, dashed line shows a similar cut in $b_{1,rb}$, where almost all of the galaxies that are known to host pseudobulges are below ($b_{1,rb} > 0.62$. Bulges with higher Sérsic index have more strongly skewed PDF(s).

Interestingly, while $b_1$ traces morphology, as does global or aperture $j_*$ (e.g. Fall 1983; Romanowsky & Fall 2012; Obreschkow & Glazebrook 2014; Cortese et al. 2016; Sweet et al. 2018), the same cannot be said for the cousin spin parameter, either global, aperture, or PDF. Here spin parameter $\lambda = J|E|^{1/2}/GM^{5/2}$ (Peebles 1969), with $J$ total angular momentum, $E$ energy, $M$ mass, and $G$ Newton’s gravitational constant, while the $Re$ subscript defines the effective radius aperture within which $\lambda_{Re}$ is measured. Spin parameter has become popular observationally for quantifying angular momentum, particularly in early-type galaxies (as in Emsellem et al. 2007; Burkert et al. 2016, and many other integral field spectroscopic works since). We note that it is straightforward to calculate $\lambda$ of the halo, but estimating the spin parameter of the observed galaxy requires further assumptions to derive total mass and energy of the system, including the coupling of the disk and halo spins (Scannapieco et al. 2009; Sharma et al. 2012; Teklu et al. 2015; Cortese et al. 2016). Indeed, the global theoretical value has low dynamic range (0.22 dex; Bullock et al. 2001) around $\lambda \sim 0.05$, and has no strong correlation with halo mass (Barnes & Efstathiou 1987) or morphology (traced by of a larger sample of galaxies, e.g. the SAMI Galaxy Survey (Bryant et al. 2015), with forthcoming bulge-disk decompositions using ProFit (Robotham et al. 2017) by Sarah Casura et al. (in prep), and Stefania Barsanti et al. (in prep), or 2) simulations such as Magneticum (Teklu et al. 2015; Schulze et al. 2018) or EAGLE (Schaye et al. 2015; Lagos et al. 2017, 2018), as far as these are well-matched to observations (van de Sande et al. 2019), which would enable investigation of the strength of correlation with other fundamental properties.
concentration parameter; Bullock et al. 2001), though some spin-morphology trend is seen observationally, particularly for low-Sérsic-index galaxies (Cortese et al. 2016; Fall & Romanowsky 2018). Falcón-Barroso et al. (2019) showed that aperture kinematic spin parameter $\lambda_{\text{pc}}$ does not monotonically correlate with photometric morphometry quantified by $n_b$ or $\beta$. Specifically, early-type galaxies in CALIFA have $\lambda_{\text{pc}}$ that are consistent with the literature, and high-mass, highly star-forming Sb galaxies have higher $\lambda_{\text{pc}}$ as expected, but low-mass Sc and Sd galaxies have lower $\lambda_{\text{pc}}$. Moreover, the spin profile $\lambda(R)$ is less sensitive to local features in the halo outskirts than the ratio of rotational velocity to dispersion support $V(R)/\sigma(R)$ (Wu et al. 2014). Recently, Zhu et al. (2018a,b) used orbit–superposition Schwarzschild models to fit stellar orbits of CALIFA galaxies and decomposed these into a continuum of cold, warm, hot and counter-rotating kinematic components based on the PDF of circularity $\lambda_c$. They found no correlation between Sérsic index and bulge type (as traced by the kinematically hot component), contrary to Fisher & Dromy (2008, 2016) who found Sérsic index to be the best predictor of bulge type. We note that although there are similarities between Zhu et al. (2018b) and this work, in that both assess the relation between a galaxy’s morphology and the PDF of its kinematic properties, our work uses measured specific angular momentum $j_\lambda$ rather than modelled circularity $\lambda_c$ and does not require fitting a functional form. Thus PDF($s$) may be preferred for morphological tracing.

A near-universal halo angular momentum distribution was predicted by van den Bosch et al. (2001); Bullock et al. (2001); Liao et al. (2017) which is consistent with linear tidal-torque theory or angular momentum transport from minor mergers. However, we have seen above that PDF($s$) is not universal for the baryons in general, whereby $b_1$ correlates with $T$ and $\beta$. One could consider that dispersion-supported (classical) bulges may be more likely to approximate the angular momentum distribution of dispersion-supported DM haloes than galaxies as a whole do, but a similar non-universality is seen for bulge components, in that the skewness of the radius-selected bulge PDF($s$) $b_{1b}$ traces $T$, $\beta$ and bulge Sérsic index $n_b$. We note that all of the galaxies known to host pseudobulges have $b_{1b} < 0.62$, and suggest that this line might represent a clear distinction in bulge type (that is, all pseudobulges with $b_{1b} < 0.62$ and all classical bulges $b_{1b} > 0.62$. Such a distinction would restrict classical bulges to the top-right quadrant of Figure 11, restricting the spread in $b_{1b}$ and thus indicating a more universal PDF($s$) for classical bulges.

A galaxy’s PDF($s$) is a byproduct of its evolution – the tidal torques experienced by the baryons and their dark matter halo, as well as the physical processes that affect the baryons after decoupling from the halo. At the peak of cosmic star formation, during $1 < z < 3$, stable, albeit clumpy, disks are starting to settle amongst the frequent mergers and irregular systems (Glazebrook et al. 1995; Driver et al. 1995; Abraham et al. 1996a,b; Conselice et al. 2000; Elmegreen et al. 2005; Yuan et al. 2017). The disks tend to have similar-to-low specific angular momentum for their mass (Obreschkow et al. 2015; Swinbank et al. 2017; Sweet et al. 2019; Gillman et al. 2019), and have high gas surface densities (Daddi et al. 2010; Tacconi et al. 2013; Burkert et al. 2016), fuelling high rates of star-formation (Bell et al. 2005; Juneau et al. 2005; Swinbank et al. 2009; Genzel et al. 2011) and enhanced turbulence (Förster Schreiber et al. 2009; Wisnioski et al. 2011; Wuyts et al. 2012; Fisher et al. 2014) within giant, star-forming clumps (Obreschkow et al. 2016; Romeo & Mogotsi 2018; Behrendt et al. 2019). These self-rotating clumps may remain bound while migrating towards the galaxy centre (Ceverino et al. 2012), contributing to growth of the (pseudo)bulge and moving the galaxy towards the $z = 0$ $\beta - j_\lambda/M_\text{B}$ ‘pseudobulge track’ (Sweet et al. 2019). Since the PDF($s$) encodes the evolutionary history of the galaxy, we suggest that it may be used to single out galaxy components and witness these key evolutionary stages. In a forthcoming paper (Espejo et al., in prep) we will illustrate that using circular-only velocity at high-redshift results in a PDF($s$) that is sensitive to a thin disk-like component, even in the presence of giant, star-forming clumps. This technique could thus trace the emergence of disks predicted at this epoch. Further, we have demonstrated a proof-of-concept separation of the bulge and disk PDF($s$), where we used a photometric $\beta$ prior to select ‘bulge’ material in a radius-based method. Future work will develop these methods with an iterative approach to find a photokinematic $\beta$, which may then yield a powerful, physically-grounded method for decomposing galaxies in a photokinematic manner, separating clumps, classical bulges and pseudobulges, thin and thick disks.

7 SUMMARY AND CONCLUSIONS

In this work we have shown, using a sample of normal local galaxies from the CALIFA survey, that a galaxy’s internal distribution of spatially-resolved stellar specific angular momentum, PDF($s$), is a tracer of its photometric morphology.

Our findings are summarised as follows:

(i) We confirm previous results that PDF($s$) is not well described by the corresponding angular momentum distribution of a dark matter halo in a ΛCDM universe, for any morphological type, even though $j_\lambda$ is empirically and coincidentally similar to $j_\text{B}$, depending on the assumed functional form of the stellar mass – halo mass relation (Barnes & Efstathiou 1987; Catelan & Theuns 1996a,b; van den Bosch et al. 2001; Posti et al. 2018a). The mismatch between stellar PDF($s$) and halo PDF($\sigma_0$) may be explained by physical processes that remove high- or low-angular momentum material and transport angular momentum throughout the disk, illustrating that galaxy assembly is dominated by such processes. PDF($s$) is also not well described by an analytic solution for an exponential disk with uniform rotational velocity, or by a normal + log-normal distribution fitting function.

(ii) The shape of PDF($s$) is characterised by skewness $b_1$, which advantageously does not depend on bin size since it is calculated on the underlying distribution, and does not require fitting as it is non-parametric.

(iii) PDF($s$) is not universal. Rather, $b_1$ is positively correlated with Hubble type $T$ and bulge-to-total light ratio $\beta$ with the following best-fitting relations:

$\beta_1 = 0.82(0.10) + 0.06(0.02)T$, with scatter $\sigma = 0.20(0.03)$;

$\beta_1 = 0.81(0.09) + 1.52(0.32)\beta$, with $\sigma = 0.09(0.02)$;

the quantities in parentheses represent 1–$\sigma$ uncertainties. That is, earlier-type, more bulge-dominated galaxies have more strongly-skewed PDF($s$).
We decompose the PDF into ‘bulge’ and ‘disk’ components using a photometric prior for $\beta$ and the radius-selection method.

$\beta_{1Tb}$ is correlated with Hubble type $T$ and bulge-to-total light ratio $\beta$ with the following best-fitting relations: $b_{1Tb} = 0.43 (0.10) + 0.03 (0.02) T$, with scatter $\sigma = 0.17 (0.03)$; $b_{1Tb} = 0.35 (0.05) + 1.03 (0.17) \beta$, with $\sigma = 0.05 (0.02)$.

Bulge PDF skewness $b_{1Tb}$ correlates with bulge Sérsic index $n_b$, with $b_{1Tb} = 0.34 (0.06) + 0.13 (0.03) n_b$ and $\sigma = 0.12 (0.02)$. Almost all galaxies that are known to host pseudobulges ($n_b < 2.2$) have bulge PDF skewed $b_{1Tb} \leq 0.62$.

In the absence of reliable dispersion measurements (e.g. at high redshift$^7$), the circular-velocity PDF can be calculated. Circular PDF skewness $b_{1c}$ underestimates dispersion-broadened skewness $b_1$ but the tight correlation (coefficient $R = 0.94$; scatter $\sigma = 0.07 \pm 0.01$) allows reliable conversion using the relation $b_{1c} = -0.82 (0.13) + 1.51 (0.11) b_1$.

The PDF traces photokinematic morphology of a galaxy. It encodes more physical information than photometry alone, and requires fewer assumptions than spin parameter $A$. At high redshifts, due to decreased angular size and surface brightness dimming, photometric morphology is even more dependent on who is doing the classification, and singling out a thin disk is even more difficult than for local galaxies. PDF of a product of the evolutionary history of the baryons in the galaxy, and thus provides a classifier-independent, physically-grounded tracer of galaxy morphology.

In forthcoming papers we will explore a sample of $z \sim 1.5$ galaxies in COSMOS (Espejo et al.) and further investigate the fundamental link between PDF(s) and morphology (Sweet et al.). As it matures, PDF will become a useful method to separate out kinematic components: thin disk from thick disk and bulge, clumps from bulges, and pseudobulges from classical bulges, facilitating observation of the emergence of the thin disk at high redshift and tracing the morphological evolution of clumps to today's bulges.

ACKNOWLEDGEMENTS

SMS, KG and CL have received funding from the ARC Centre of Excellence for All Sky Astrophysics in 3 Dimensions (ASTRO 3D), through project number CE170100013. SMS, KG, DO and DBF acknowledge support from ARC DP grant DP160102235. DBF acknowledges support from ARC Future Fellowship FT170100376. AB thanks the colleagues at Swinburne University of Technology for their hospitality and support during his visit.

This study uses data provided by the Calar Alto Legacy Integral Field Area (CALIFA) survey (http://califa.caha.es/). Based on observations collected at the Centro Astronómico Hispano Alemán (CAHA) at Calar Alto, operated jointly by the Max-Planck-Institut für Astronomie and the Instituto de Astrofísica de Andalucía (CSIC).

REFERENCES

Abazajian K. N., et al., 2009, ApJS, 182, 543
Abraham R. G., van den Bergh S., Glazebrook K., Ellis R. S., Santiago B. X., Surma P., Griffiths R. E., 1996a, ApJS, 107, 1
Abraham R. G., Tanvir N. R., Santiago B. X., Ellis R. S., Glazebrook K., van den Bergh S., 1996b, MNRAS, 279, L47
Ageorges, T., Tisserro, M., Moore B., 2011, MNRAS, 410, 1391
Barnes J., Efthymiou G., 1987, ApJ, 319, 575
Behrendt M., Schartmann M., Burkert A., 2019, MNRAS, 488, 306
Bell E. F., et al., 2005, ApJ, 625, 23
Brook C. B., et al., 2011, MNRAS, 415, 1051
Brook C. R., Stinson G., Gibson B. K., Roskar R., Wadsley J., Quinn T., 2012, MNRAS, 419, 771
Bryant J. H., et al., 2015, MNRAS, 447, 2857
Bullock J. S., Dekel A., Kolatt T. S., Kravtsov A. V., Klypin A. A., Porciani C., Primack J. R., 2001, ApJ, 555, 240
Burkert A., 2016, ApJ, 826, 214
Cappellari M., Copin Y., 2003, MNRAS, 342, 345
Cappellari M., Emsellem E., 2004, PASP, 116, 138
Catelan P., Theuns T., 1996a, MNRAS, 282, 436
Catelan P., Theuns T., 1996b, MNRAS, 282, 455
Ceverino D., Dekel A., Mangledner N., Bournaud F., Burkert A., Genzel R., Primack J., 2012, MNRAS, 420, 3490
Chen D. N., Jing Y. P., Yoshikawa K., 2003, ApJ, 597, 35
Conselice C. J., Bershady M. A., Jiangren A., 2000, ApJ, 529, 886
Cortese L., et al., 2016, MNRAS, 463, 170
Crampin D. J., Boyle F., 1964, ApJ, 140, 99
Daddi E., et al., 2010, ApJ, 713, 886
Dekel A., Burkert A., 2014, MNRAS, 438, 1870
Doroshkevich A. G., 1970, Astrophysics, 6, 320
Driver S. P., Windhorst R. A., Griffiths R. E., 1995, ApJ, 453, 48
Dutton A. A., 2009, MNRAS, 396, 121
Dutton A. A., van den Bosch F. C., 2012, MNRAS, 421, 608
El-Badry K., et al., 2018, MNRAS, 473, 1930
Elmegreen B. G., Struck C., 2013, ApJ, 775, L35
Elmegreen D. M., Elmegreen B. G., Rubin D. S., Schaffer M. A., 2005, ApJ, 611, 85
Emsellem E., et al., 2007, MNRAS, 379, 401
Fabricius M. H., Saglia R. P., Fisher D. B., Drory N., Bender R., Hopps U., 2012, ApJ, 754, 67
Falcón-Barroso J., et al., 2017, A&A, 597, A48

MNRAS 000, 1–18 (2019)
APPENDIX A: SDSS IMAGES

We show an SDSS image cutout of each of the galaxies in our sample in Figures A1 to A24. Each caption contains Hubble type $T_{\text{Hubble}}$ from our method, as well as $T_{\text{RC3}}$ from the Third Reference Catalogue of Bright Galaxies (RC3, de Vaucouleurs et al. 1991) for comparison. Each image is 205” on a side. N is up; E is left.

Stevens A. R. H., Lagos C. d. P., Contreras S., Croton D. J., Padilla N. D., Schaller M., Schaye J., Theuns T., 2017, MN-RAS, 467, 2066
Stewart K. R., et al., 2017, ApJ, 843, 47
Sweet S. M., Fisher D., Glazebrook K., Obreschkow D., Lagos C., Wang L., 2018, ApJ, 860, 37
Sweet S. M., et al., 2019, MNRAS,
Swinbank A. M., et al., 2009, MNRAS, 400, 1121
Swinbank A. M., et al., 2017, MNRAS, 467, 3140
Tacconi L. J., et al., 2013, ApJ, 768, 74
Teklu A. F., Remus R.-S., Dolag K., Beck A. M., Burkert A., Schmidt A. S., Schueler F., Steinborn L. K., 2015, ApJ, 812, 29
Toomre A., 1964, ApJ, 139, 1217
Valdes F., Gupta R., Rose J. A., Singh H. P., Bell D. J., 2004, ApJS, 152, 251
Walcher C. J., et al., 2014, A&A, 569, A1
Wang L., Dutton A. A., Stinson G. S., Macció A. V., Penzo C., Kang X., Keller B. W., Wadsley J., 2015, MNRAS, 454, 83
Wang L., et al., 2018, ApJ, 868, 93
White S. D. M., 1984, ApJ, 286, 38
White S. D. M., Rees M. J., 1978, MNRAS, 183, 341
Wise (2015, MNRAS, 454, 233)
Zavala J., et al., 2016, MNRAS, 460, 4466
Zhu L., et al., 2018a, Nature Astronomy, 2, 233
Zhu L., van de Ven G., Méndez-Abreu J., Obreja A., 2018b, MNRAS, 479, 945
de Vaucouleurs G., de Vaucouleurs A., Corwin Herold G. J., Buta R. J., Paturel G., Fouque P., 1991, Third Reference Catalogue of Bright Galaxies
van de Sande J., et al., 2019, MNRAS, 484, 869
van den Bosch F. C., 2001, MNRAS, 327, 1334
van den Bosch F. C., Burkert A., Swaters R. A., 2001, MNRAS, 326, 1205
van den Bosch F. C., Abel T., Croft R. A. C., Hernquist L., White S. D. M., 2002, ApJ, 576, 21

APPENDIX A: SDSS IMAGES

We show an SDSS image cutout of each of the galaxies in our sample in Figures A1 to A24. Each caption contains Hubble type $T_{\text{Hubble}}$ from our method, as well as $T_{\text{RC3}}$ from the Third Reference Catalogue of Bright Galaxies (RC3, de Vaucouleurs et al. 1991) for comparison. Each image is 205” on a side. N is up; E is left.

Stevens A. R. H., Lagos C. d. P., Contreras S., Croton D. J., Padilla N. D., Schaller M., Schaye J., Theuns T., 2017, MNRAS, 467, 2066
Stewart K. R., et al., 2017, ApJ, 843, 47
Sweet S. M., Fisher D., Glazebrook K., Obreschkow D., Lagos C., Wang L., 2018, ApJ, 860, 37
Sweet S. M., et al., 2019, MNRAS,
Swinbank A. M., et al., 2009, MNRAS, 400, 1121
Swinbank A. M., et al., 2017, MNRAS, 467, 3140
Tacconi L. J., et al., 2013, ApJ, 768, 74
Teklu A. F., Remus R.-S., Dolag K., Beck A. M., Burkert A., Schmidt A. S., Schueler F., Steinborn L. K., 2015, ApJ, 812, 29
Toomre A., 1964, ApJ, 139, 1217
Valdes F., Gupta R., Rose J. A., Singh H. P., Bell D. J., 2004, ApJS, 152, 251
Walcher C. J., et al., 2014, A&A, 569, A1
Wang L., Dutton A. A., Stinson G. S., Macció A. V., Penzo C., Kang X., Keller B. W., Wadsley J., 2015, MNRAS, 454, 83
Wang L., et al., 2018, ApJ, 868, 93
White S. D. M., 1984, ApJ, 286, 38
White S. D. M., Rees M. J., 1978, MNRAS, 183, 341
Wisnioski E., et al., 2011, MNRAS, 417, 2601
Wu X., Gerhard O., Naerth L., Martinez-Valpuesta I., Hilz M., Churazov E., Lyskova N., 2014, MNRAS, 438, 2701
Wuyts S., et al., 2012, ApJ, 753, 114
Yuan T., et al., 2017, ApJ, 850, 61
Zavala J., et al., 2016, MNRAS, 460, 4466
Zhu L., et al., 2018a, Nature Astronomy, 2, 233
Zhu L., van de Ven G., Méndez-Abreu J., Obreja A., 2018b, MNRAS, 479, 945
de Vaucouleurs G., de Vaucouleurs A., Corwin Herold G. J., Buta R. J., Paturel G., Fouque P., 1991, Third Reference Catalogue of Bright Galaxies
van de Sande J., et al., 2019, MNRAS, 484, 869
den Bosch F. C., 2001, MNRAS, 327, 1334
den Bosch F. C., Burkert A., Swaters R. A., 2001, MNRAS, 326, 1205
den Bosch F. C., Abel T., Croft R. A. C., Hernquist L., White S. D. M., 2002, ApJ, 576, 21

APPENDIX A: SDSS IMAGES

We show an SDSS image cutout of each of the galaxies in our sample in Figures A1 to A24. Each caption contains Hubble type $T_{\text{Hubble}}$ from our method, as well as $T_{\text{RC3}}$ from the Third Reference Catalogue of Bright Galaxies (RC3, de Vaucouleurs et al. 1991) for comparison. Each image is 205” on a side. N is up; E is left.
Figure A2. MCG-02-02-030. $T_{\text{Fisher}} = Sb$; $T_{\text{RC3}} = Sb$.

Figure A3. NGC0001. $T_{\text{Fisher}} = Sb$; $T_{\text{RC3}} = Sb$.

Figure A4. NGC2253. $T_{\text{Fisher}} = Sbc$; $T_{\text{RC3}} = Sc$.

Figure A5. NGC2592. $T_{\text{Fisher}} = E?S0?$; $T_{\text{RC3}} = E$.

Figure A6. NGC2639. $T_{\text{Fisher}} = Sa$; $T_{\text{RC3}} = Sa$.

Figure A7. NGC2906. $T_{\text{Fisher}} = Sbc$; $T_{\text{RC3}} = Sc$.

Figure A8. NGC3815. $T_{\text{Fisher}} = Sab$; $T_{\text{RC3}} = Sab$.

Figure A9. NGC4961. $T_{\text{Fisher}} = Sc$; $T_{\text{RC3}} = Sc$.

This paper has been typeset from a T\TeX/\LaTeX file prepared by the author.
Figure A10. NGC5480. $T_{\text{Fisher}} = \text{Sd}; \ T_{\text{RC3}} = \text{Sc}.$

Figure A11. NGC5971. $T_{\text{Fisher}} = \text{Sa}; \ T_{\text{RC3}} = \text{Sa}.$

Figure A12. NGC5980. $T_{\text{Fisher}} = \text{Sbc}; \ T_{\text{RC3}} = \text{Sbc}.$

Figure A13. NGC6063. $T_{\text{Fisher}} = \text{Sc}; \ T_{\text{RC3}} = \text{Sc}.$

Figure A14. NGC6427. $T_{\text{Fisher}} = \text{S0}; \ T_{\text{RC3}} = \text{E-S0}.$

Figure A15. NGC6762. $T_{\text{Fisher}} = \text{S0}; \ T_{\text{RC3}} = \text{S0-a}.$

Figure A16. NGC7311. $T_{\text{Fisher}} = \text{Sab}; \ T_{\text{RC3}} = \text{Sab}.$

Figure A17. NGC7623. $T_{\text{Fisher}} = \text{S0}; \ T_{\text{RC3}} = \text{S0}.$
Figure A18. NGC7653. $T_{\text{Fisher}} = \text{Sb}; T_{\text{RC3}} = \text{Sb}$.

Figure A19. NGC7671. $T_{\text{Fisher}} = \text{S0}; T_{\text{RC3}} = \text{S0}$.

Figure A20. NGC7683. $T_{\text{Fisher}} = \text{S0}; T_{\text{RC3}} = \text{S0}$.

Figure A21. NGC7716. $T_{\text{Fisher}} = \text{Sbc}; T_{\text{RC3}} = \text{Sb}$.

Figure A22. NGC7824. $T_{\text{Fisher}} = \text{Sab}; T_{\text{RC3}} = \text{Sab}$.

Figure A23. UGC00987. $T_{\text{Fisher}} = \text{Sa}; T_{\text{RC3}} = \text{Sa}$.

Figure A24. UGC09476. $T_{\text{Fisher}} = \text{Sc}; T_{\text{RC3}} = \text{SABc}$.