Light-emitting current of electrically driven single-photon sources

David M.-T. Kuo

Department of Electrical Engineering, National Central University, Chung-Li, Taiwan, 320, Republic of China

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Abstract

Time-dependent tunnelling current arising from the electron-hole recombination of the exciton state is theoretically studied using the nonequilibrium Green’s function technique and the Anderson model with two energy levels. The charge conservation and gauge invariance are satisfied in the tunnelling current. Apart from the classical capacitive charging and discharging behavior, interesting oscillations superimpose on the tunnelling current for the applied rectangular pulse voltages.
I. INTRODUCTION

Recently, much effort has been devoted to the study of single-photon sources (SPSs) made from quantum dots (QDs), which have potential applications in quantum cryptography and quantum computing$^{1-3}$). The second order correlation function of light emitted from such devices must be measured to determine their functionality as SPSs, that is, the light source antibunching feature must be examined. The antibunching features of SPSs were demonstrated in Ref. [1,2], where electrons and holes in the QD are excited by optical pumping. Nevertheless, only a few experiments have employed electrical pumping is to demonstrate the antibunching behavior of SPSs. From a practical point of view, it is more efficient to use the electrically driven SPSs. A prototype SPS made from individual QDs embedded in a semiconductor p-n junction was suggested by Imamoglu and Yamamoto$^4$. In addition to antibunching, enhancing the spontaneous emission rate is necessary, because it is crucial to SPS performance.

For small size QDs, the strong three dimensional confinement effect creates large energy separations among the low-lying confined levels. Consequently, it is possible to inject electrons and holes into the corresponding ground state energy levels of a single QD and generate photons via electron-hole recombination in the exciton, positive/negative trion or biexciton states in the QD. To fabricate a single of nanometer-sized QD at a specific location is one of the most challenging processes in the realization of electrically driven SPSs. Self-assembled quantum dots (SAQDs) combined with a selective formation method may overcome such a difficulty$^5$. Even though, some theoretical studies have been devoted to electrically driven SPS devices$^6$–$^8$ the dynamic properties of an electrically driven SPS are still not clear. In ref. 4, the Monte Carlo method was used to simulate junction dynamics, where time-dependent junction voltage is calculated. Other studies emphasized steady state characteristics such as the exciton assistant tunnelling current$^6$, Purcell effect on the tunnelling current$^7$ and electrode effects on the exciton complexes$^8$.

The main purpose of this study is to investigate the time-dependent tunnelling current (or spontaneous emission rate) arising from electron-hole recombination of exciton state by using nonequilibrium Green’s function technique$^9$–$^{11}$, which has been applied to several different systems.$^{12}$–$^{15}$ This study
attempts to clarify how the carrier tunnelling process, applied modulation voltage and temperature influence the spontaneous emission rate in an electrically driven SPS. The schematic band diagram describing the studied system is shown in Fig. 1. Only the ground states of conduction and valence band of the QD are considered. The energy level of $E_2$ is 20 meV below the Fermi energy level of left electrode $E_{F,e} = 50$ meV. On the other hand, the exciton state denoted by $E_{ex}$ is 15 meV above the Fermi energy of right electrode $E_{F,h} = 50$ meV. $\Gamma_{2(1)}$ denotes the tunnelling rate from the QD to the left (right) electrode. The emitted photons will be observed when the periodic square bumplike modulation $v(t)$ added into the right electrode injects holes into the exciton resonant level.

II. HAMILTONIAN

An Anderson model with two energy levels in terms of electron picture is used to describe the system as shown in Fig. 1,

$$H = \sum_k \epsilon_{k,L} a_k^\dagger a_k + \sum_k \epsilon_{k,R} b_k^\dagger b_k + \sum_i \epsilon_i d_i^\dagger d_i$$

$$+ \lambda_{12} d_1^\dagger d_2 + \lambda_{21}^* d_2^\dagger d_1 + \sum_{k,1} t_{k,1} b_k^\dagger d_1 + h.c$$

$$+ \sum_{k,2} t_{k,2} a_k^\dagger d_2 + h.c$$

where $a_k^\dagger (a_k)$ and $b_k^\dagger (b_k)$ create (destroy) an electron of momentum $k$ in the left and right system electrodes, respectively. The free electron model is used in the electrodes in which electrons have frequency-dependent energies $\epsilon_{k,L} = \epsilon_k - \omega/2$ and $\epsilon_{k,R} = \epsilon_k + \omega/2 + v(t)$. Time-dependent modulation $v(t)$ denotes the time-dependent applied voltage in the right electrode. $d_i^\dagger (d_i)$ creates (destroys) an electron inside the QD with orbital energy $\epsilon_i = E_i - (-1)^i \omega/2$. In this study $i = 1$ and $i = 2$ represent, respectively, the ground states of valence band and conduction band of individual QDs. The fourth and fifth terms describe the coupling of the QD with electromagnetic field of frequency $\omega$. $\lambda = -\mu_\tau \mathcal{E}$ is the Rabi frequency, where $\mu_\tau = \langle f | r | i \rangle$ is the matrix element for the optical transition and $\mathcal{E}$ is the electric field per photon. $t_{k,i}$ describes the coupling between the band states of electrodes and energy levels of the QD. Note that a unitary transformation, $S(t) = e^{i \mathcal{E} \omega t/2 (\sum_k (b_k^\dagger a_k - a_k^\dagger b_k) + d_1^\dagger d_1 - d_2^\dagger d_2)}$, has been used to obtain Eq. (1) via
\[ H = S^{-1} H(t) S - i S^{-1} \frac{\partial}{\partial t} S. \]

For small semiconductor QDs, the particle correlation is strong. This indicates that the intralevel Coulomb interactions of the QD, \( U_{11} \) and \( U_{22} \), can not be ignored. In order to simplify problem, it is assumed that a regime of the applied voltage can not overcome the charging energies \( U_{11} \) and \( U_{22} \). To investigate the exciton assistant process, the interlevel Coulomb interaction \( U_{12} \) is taken into account in Eq. (1)

\[ H_U = U_{12} d_1^\dagger d_1 d_2^\dagger d_2, \quad (2) \]

which is invariant under unitary transformation.

III. FORMALISM

Although a spontaneous emission process of photons is the quantum effect of an electromagnetic field, the electromagnetic field is still treated as a semiclassical field in Eq. (1). The approach detailed in ref.[11] is employed to study the spontaneous emission process of photons. The optical susceptibility of QDs not only provides the absorption coefficient and the refractive index, but also determines the spontaneous emission rate. Therefore, the optical susceptibility of individual QDs is calculated using the lesser Green’s function defined as \( G_{i,j}^< (t, t') = i \langle d_i^\dagger(t') d_j(t) \rangle \). Based on Keldysh’s equation, we have

\[ G_{i,j}^< (t, t) = \int dt_1 \int dt_2 G_{i,n}^r (t, t_1) \Sigma_{n,m}^< (t_1, t_2) G_{m,j}^a (t_2, t), \quad (3) \]

where \( \Sigma_{n,m}^< \), \( G_{i,n}^r \) and \( G_{m,j}^a \) are the lesser self-energy, the retarded Green’s function and the advanced Green’s function, respectively. The Einstein summation index is used in Eq. (3). The spontaneous emission rate will be suppressed if tunnelling rates \( \Gamma_2 \) and \( \Gamma_1 \) are smaller than spontaneous emission rate \( R_{eh} \). The detailed expression of \( R_{eh} \) is given later. To avoid the suppression of the spontaneous emission rate, the device shown in Fig. 1 favors the condition of \( \Gamma_2 > \Gamma_1 >> R_{eh} \) or \( \Gamma_1 > \Gamma_2 >> R_{eh} \) to function as a light emitting source. The condition of \( \Gamma_2 > \Gamma_1 >> R_{eh} \), \( \lambda_{ij} \) will be regarded as a...
small parameter in the comparison with $t_{k,i}$. Consequently, the lesser self-energy of Eq. (3) is mainly attributed to tunnelling

$$\Sigma_{<,i,i}(t, t') = i \int \frac{de}{2\pi} \Gamma_i f_i(e)e^{-ie(t-t')},$$

(4)

where the tunnelling rates are $\Gamma_i = \sum_k t_{k,i}^1 t_{k,i}^\ast \delta(\epsilon - \varepsilon_k)$, and $f_i(e) = 1/(e^{\epsilon-\mu_{F,i}} - 1)$ is the Fermi distribution function of the electrodes, in which the chemical potential is given by $\mu_{F,i} = E_{F,i} - (1)i\omega/2$.

It is worth noting that energy-independent tunnelling rates are assumed in Eq. (4) for the sake of simplicity. Inserting Eq. (4) into Eq. (3), we obtain two off-diagonal lesser Green’s functions

$$G_{2,1}^<(t, t) = i \int \frac{de}{2\pi} \{ \Gamma_1 f_1(e)A_{21}^r(\epsilon, t)A_{11}^a(\epsilon, t)$$

$$\quad + \Gamma_2 f_2(e)A_{22}^r(\epsilon, t)A_{21}^a(\epsilon, t) \},$$

(5)

and

$$G_{1,2}^<(t, t) = i \int \frac{de}{2\pi} \{ \Gamma_2 f_2(e)A_{12}^r(\epsilon, t)A_{22}^a(\epsilon, t)$$

$$\quad + \Gamma_1 f_1(e)A_{11}^r(\epsilon, t)A_{12}^a(\epsilon, t) \},$$

(6)

as well as two diagonal lesser Green’s functions

$$G_{1,1}^<(t, t) = i \int \frac{de}{2\pi} \{ \Gamma_1 f_1(e)A_{11}^r(\epsilon, t)A_{11}^a(\epsilon, t)$$

$$\quad + \Gamma_2 f_2(e)A_{12}^r(\epsilon, t)A_{21}^a(\epsilon, t) \},$$

(7)

and

$$G_{2,2}^<(t, t) = i \int \frac{de}{2\pi} \{ \Gamma_2 f_2(e)A_{22}^r(\epsilon, t)A_{22}^a(\epsilon, t)$$

$$\quad + \Gamma_1 f_1(e)A_{21}^r(\epsilon, t)A_{12}^a(\epsilon, t) \},$$

(8)

where $\int dt_1 e^{ie(t-t_1)}G_{ij}^r(t, t_1) = A_{ij}^r(\epsilon, t)$ and $\int dt_1 e^{-ie(t-t_1)}G_{ij}^a(t, t_1) = A_{ij}^a(\epsilon, t)$. The expression of $A_{ij}^{r(a)}(_i\epsilon, t)$ depends on the detailed form of retarded and advanced Green’s functions. Eqs. (5) and (6) determine the optical susceptibility of QDs. Eqs. (7) and (8) determine the electron occupation numbers. Once $G_{21}^<(t, t)$ is determined, $G_{12}^<(t, t)$ is also obtained as a result of $G_{21}^<(t, t) = (G_{12}^<(t, t))^\dagger$. 


To solve $G_{21}^<(t, t)$, some approximations are considered in the following derivation due to the complicated Hamiltonian of Eq. (1). Because the energy level of $i=2$ emerges into the band width of the left electrode and $\Gamma_2$ is larger than $\Gamma_1$, the retarded and advanced Green’s functions for $i=2$ can be regarded as a steady state solution. That is $G_{22}^{r(a)}(t, t') = \mp i\theta(t - t')e^{-i(\epsilon_1 \mp \Gamma_2/2)(t-t')}$, which gives $A_{22}^{r(a)}(\epsilon, t) = 1/(\epsilon - \epsilon_2 \pm i\Gamma_2/2)$. As for $G_{i,j}^{r(a)}(t, t')$, the off-diagonal Green’s function ($i \neq j$) are solved using Dyson’s equation,

$$G_{ij}^{r(a)}(t, t') = \int dt_1 G_{i,i}^{r(a)}(t, t_1)\lambda_{i,j} G_{j,j}^{r(a)}(t_1, t').$$

(9)

Due to $\lambda_{i,j} < \lambda_{k,i}$ and $G_{ij}^{r(a)}(t, t')$ in terms of the first order parameter $\lambda$, the Green’s functions $G_{ii}^{r(a)}$ in Eq. (9) are $\lambda$-independent functions. Consequently, $A_{21}^{r(a)}(\epsilon, t) = \lambda_{21}A_{22}^{r(a)}(\epsilon)A_{11}^{r(a)}(\epsilon, t)$ and Eq. (5) is rewritten as

$$G_{2,1}^<(t, t) = \lambda_{21}\int d\epsilon \frac{d}{2\pi} \{f_1(\epsilon)\Gamma_1 |A_{11}^{r(a)}(\epsilon, t)|^2 A_{22}^{r(a)}(\epsilon)
+ f_2(\epsilon)\Gamma_2 |A_{22}^{r(a)}(\epsilon)|^2 A_{11}^{r(a)}(\epsilon, t)\}.$$

(10)

To obtain the spontaneous rate resulting from the imaginary part of $G_{2,1}^<(t, t)$, the imaginary part on the both sides of Eq. (10) is used

$$ImG_{2,1}^<(t, t) = \lambda_{21}\int d\epsilon \frac{d}{2\pi} \{f_1(\epsilon)\Gamma_1 |A_{11}^{r(a)}(\epsilon, t)|^2 \frac{1}{2}[A_{22}^{r(a)}(\epsilon) - A_{22}^{a(a)}(\epsilon)]
+ f_2(\epsilon)\Gamma_2 |A_{22}^{r(a)}(\epsilon)|^2 \frac{1}{2}[A_{11}^{a(a)}(\epsilon, t) - A_{11}^{r(a)}(\epsilon, t)]\}.$$

(11)

Employing $G_i^r - G_i^a = G_i^> - G_i^<$, where the greater Green’s function is given by

$$G_{i,j}^>{\gamma}(t, t) = \int dt_1 \int dt_2 G_{i,n}^>{\gamma}(t, t_1)\Sigma_{n,m}^>(t_1, t_2)G_{m,j}^a(t_2, t)$$

(12)

with the greater self-energy

$$\Sigma_{i,j}^>(t, t') = i\int \frac{d\epsilon}{2\pi} \Gamma_i(1 - f_i(\epsilon))e^{-i\epsilon(t-t')}$$

(13)

Eq. (11) can be simplified as
\[
\text{Im} G_{e,h}^\leq(t, t) = -\lambda_{eh} \int \frac{d\epsilon}{2\pi} (1 - f_e(\epsilon) - f_h(\epsilon)) \Gamma_h |A_{hh}^r(\epsilon, t)|^2 \Gamma_e |A_{ee}^a(\epsilon)|^2.
\]

In Eq. (14) the electron and hole picture are employed to label \(i=2\) and \(i=1\), respectively. Note that Eq. (14) contains a factor of \((1 - f_e(\epsilon) - f_h(\epsilon))\). It is always possible to write \(\lambda_{eh} \text{Im} G_{e,h}^\leq(t, t) = \mathcal{X}_e(\omega, t) - \mathcal{X}_a(\omega, t)\), where \(\mathcal{X}_a(\omega, t)\) and \(\mathcal{X}_e(\omega, t)\) are, respectively, in proportion to \((1 - f_e(\epsilon))(1 - f_h(\epsilon))\) and \(f_e(\epsilon)f_h(\epsilon)\). \(\mathcal{X}_a(\omega, t)\) and \(\mathcal{X}_e(\omega, t)\) denote the absorption and emission spectra, respectively. We only focus on the emission spectrum, which was frequently reported in experiments,

\[
\mathcal{X}_e(\omega, t) = \lambda_{eh}^2 \int \frac{d\epsilon}{2\pi} \{f_e(\epsilon)f_h(\epsilon)\Gamma_h |A_{hh}^r(\epsilon, t)|^2 \Gamma_e |A_{ee}^a(\epsilon)|^2\}.
\]

By reference to ref. 11, the current arising from the electron-hole recombination of the exciton state may be written as

\[
J_{sp}(t) = e\alpha \int d\omega \omega^3 \int \frac{d\epsilon}{\alpha^2} f_e(\epsilon)f_h(\epsilon)\Gamma_h |A_{hh}^r(\epsilon, t)|^2 \mathcal{I}m[A_{ee}^a(\epsilon)]
\]

with \(\alpha = 4n_r^2\mu^2/(6\epsilon_0^3c^3\epsilon_0)\), where \(n_r\) and \(\epsilon_0\) are the refractive index and static dielectric constant of system, respectively. \(e\) denotes the electron charge. Note that Eq. (15) uses \(\Gamma_e |A_{ee}^a(\epsilon)|^2 = 2 \mathcal{I}m[A_{ee}^a(\epsilon)]\) which is valid only for the steady state. It should be noted that \(A_{hh}^r(\epsilon, t)\) satisfies the condition of \(\Gamma_h |A_{hh}^r(\epsilon, t)|^2 = -2 \mathcal{I}m[A_{hh}^r(\epsilon, t)] - \frac{d|A_{hh}^r(\epsilon, t)|^2}{dt}\) resulting from total charge conservation and gauge invariance. \(\rho(\omega) = \omega^2\) of Eq. (15) arises from the density of states of photons. \(J_{sp}(t)\) is determined by the time-dependent interband joint density of states and the factors of \(f_e(\epsilon)f_h(\epsilon)\). According to the expressions of \(\mathcal{X}_e(\omega, t)\) and \(J_{sp}\), we prove that the intensity of emission spectrum is linear variation with respect to \(J_{sp}\).

To solve the spectral function of holes \(A_{hh}^r(\epsilon, t)\), the retarded Green’s function of holes \(G_{hh}^r(t, t_1)\) is derived to obtain

\[
G_{hh}^r(t, t_1) = (1 - N_e)g_h^r(\epsilon_h, t, t_1) + N_e g_h^r(\epsilon_h + U_{eh}, t, t_1)
\]

with
where \( \epsilon_h = -E_h + \frac{\omega}{2} \), and \( N_e \) is the electron occupation number at steady state. Two branches exist in Eq. (16); one corresponds to the resonant energy level of \( E_h \) with a weight of \((1 - N_e)\), and the other corresponds to the exciton resonant level of \( E_{ex} = E_h - U_{eh} \) with a weight of \( N_e \). Consequently, holes injected into the energy levels of QDs depend not only on the applied voltage of the right electrode, but also on the electron occupation number \( N_e \), which is given by 
\[
N_e = \int \frac{d\varepsilon}{\pi} f_e(\varepsilon) \text{Im} \frac{1}{\varepsilon - E_e - i\Gamma_e/2}.
\]
As long as the applied voltage is insufficient to inject holes into the resonant energy level of \( E_h \), it is only necessary to consider the exciton branch of \( N_e g_h^r(\epsilon_h + U_{eh}, t, t_1) \) in Eq. (16). The detailed expression of \( G_{hh}^r(t, t_1) \) depends on the applied voltage \( v(t) \), considering a rectangular pulse with the duration time of \( \Delta_s \) and amplitude \( \Delta \) as \( v(t) \).

\[
g_h^r(\epsilon_h, t, t_1) = -i\theta(t - t_1)e^{-i(\epsilon_h - i\Gamma_h/2)(t-t_1)} - i \int_t^{t_1} dt_2 v(t_2),
\]

(17)

For \( t \geq \Delta_s \),

\[
A_h^r(\epsilon, t) = \frac{N_e}{\epsilon - \epsilon_h - \Delta + i\Gamma_h/2} \frac{\Delta}{(1 - e^{i(\epsilon_h - \Delta + i\Gamma_h/2)(t-t_0)})}.
\]

(18)

The time-dependent tunnelling current associated with the spontaneous radiative transition in individual quantum dots is next discussed based on Eqs. (15), (18) and (19).

**IV. RESULTS AND DISCUSSION**

Due to current conservation, the time-dependent tunnelling current is given by \( J(t) = J_{sp}(t) = e\Gamma_s(t) \), where \( \Gamma_s(t) \) denotes the time-dependent spontaneous emission rate. Owing to small tunnelling rates \( (\Gamma_e/E_{F,e} << 1 \text{ and } \Gamma_h/E_{F,h} << 1) \), Eq. (15) can be reduced to
\[ J_{sp}(t) = 2e \ast R_{eh} f_e(T) A_h(T, t), \]  
\tag{20}

with

\[ A_h(t) = \int \frac{d\varepsilon}{\pi} f_h(\varepsilon) \Gamma_h |A_h^*(\varepsilon, t)|^2. \]  
\tag{21}

In Eq. (20) we define the time-independent spontaneous emission rate \( R_{eh} = \alpha \Omega_{ex}^3 \), where \( \Omega_{ex} = E_g + E_c + E_h - U_{eh} \). According to Eq. (20), the time-dependent feature of \( J_{sp}(t) \) is determined by the hole occupation number of Eq. (21).

To reveal the time-dependent behavior of \( J_{sp}(t) \), Eq. (20) is solved numerically, and shows \( J_{sp}(t) \) for various duration times at zero temperature in Fig. 2 for the tunnelling rate \( \Gamma_h = 0.5 \) meV and \( \Delta = 20 \) meV; the solid line (\( \Delta s = t_0 \)), the dashed line (\( \Delta s = 3 t_0 \)) and the dotted line (\( \Delta s = 5 t_0 \)). The current almost reaches saturation at \( t = 5 t_0 \). Apart from the classical capacitive charging and discharging behavior (exponential growth and decay)\(^{17}\), the interesting oscillations superimpose on the tunnelling current not only for \( t \leq \Delta s \) but also for \( t > \Delta s \). However, the amplitude of oscillation for \( t \leq \Delta s \) is small. In particular, the oscillation period for \( t > \Delta s \) does not depend on the magnitude of duration time. The oscillatory current is yielded from the particle coherent tunnelling between the electrodes and the QD. Such a coherent tunnelling was also pointed out as the mechanism of quantum ringing for electrons tunnelling current through the single dot embedded in an n-i-n structure\(^{18}\). In ref. 18, oscillatory current is not observed as \( t > \Delta s \). According to the result of quantum interference between the outgoing wave (leaving the QDs) and the wave reflected from the barrier, this oscillatory behavior could be observed in the discharging process\(^{19}\).

To examine if the oscillation currents shown in Fig. 2 depend on applied bias strength, Figure 3 illustrates \( J_{sp}(t) \) for different applied biases with duration time \( \Delta s = 3 t_0 \) at zero temperature and \( \Gamma_h = 0.5 \) meV. The solid line denotes \( \Delta = 15 \) meV, and the dashed and dotted lines denote, \( \Delta = 20 \) meV and \( \Delta = 25 \) meV, respectively. For amplitude \( \Delta = 15 \) meV, the Fermi energy of the right electrode just reaches the alignment with the resonant exciton level. \( J_{sp}(t) \) displays a strong oscillation characteristic. If the pulse amplitude is increased to \( \Delta = 20\)meV, the resonant exciton level is covered by the right electrode reservoir, and therefore the magnitude of \( J_{sp}(t) \) increases. When
the pulse amplitude to $\Delta = 25 \text{ meV}$, $J_{sp}(t)$ becomes saturated. The results shown in Fig. 3 clearly indicate that the oscillation feature of $J_{sp}(t)$ depends on the magnitude of $\Delta$. Due to the emitted photon numbers in proportion to $J_{sp}(t)$, the features of emitted photon numbers as functions of time will correspond to the results shown in Fig. 3.

In Figs. (2) and (3) hole tunnelling rate is set as $\Gamma_h = 0.5 \text{ meV}$, it is interesting to understand how the tunnelling rate influences $J_{sp}(t)$. $J_{sp}(t)$ for different hole tunnelling rates is shown in Fig. 4; the solid line ($\Gamma_h = 0.5 \text{ meV}$), the dashed line ($\Gamma_h = 0.75 \text{ meV}$) and the dotted line ($\Gamma_h = 1 \text{ meV}$). Increasing the tunnelling rate, $J_{sp}(t)$ reaches exponential growth quickly. This also indicates that electrically driven SPSs can quickly reach the maximum photon emission efficiency within a shorter operating time compared to optically driven SPSs with a phonon bottleneck. To clarify, the relation between the photon number $N_s$ and $J_{sp}(t)$ should be constructed. Because $\frac{dN_s(t)}{dt} = \Gamma_{sp}(t)$, the time-dependent photon number $N_s(t) = \int_0^t dt \Gamma_{sp}(t)$ is obtained. For the step-like pulse ($\Delta s = \infty$), $A^r_h(\epsilon, t)$ is given by Eq. (18). Due to the small oscillatory amplitude in the charging process, Eq. (18) is approximated as $A_h(t) = f_h(T)(1 - \exp^{-\Gamma_h t})$ for $t \geq 0$. Consequently, $N_s(t) = R_{eh} f_e(T) f_h(T) t (1 + (\exp^{-\Gamma_h t} - 1)/(\Gamma_h t))$. We found that $N_s(t)/t = R_{eh} f_e(T) f_h(T)$ as $\Gamma_h t \gg 1$. This indicates that a higher tunnelling rate allows an electrically driven SPS to quickly reach the maximum photon emission efficiency.

Finally, the finite temperature effect on $J_{sp}(t)$ is shown in Fig. 5; the solid line ($k_B T = 0$), the dashed line ($k_B T = 1 \text{ meV}$), the dotted line ($k_B T = 2 \text{ meV}$) and the dot-dashed line ($k_B T = 3 \text{ meV}$). Fig. 5 shows that the spontaneous photon emission rate $\Gamma_{sp}(t)$ is suppressed by temperature effects resulting from a factor of $f_e(T) f_h(T)$ and $N^2_e(T)$ (see Eqs. (18) and (19)). $N^2_e(T)$ is yielded by the electron-hole interaction. From Fig. 2 to Fig. 5, $J_{sp}(t)$ is in units of $J_0 = 2e R_{eh}$. For InAs QDs, the typical value of $R_{eh}$ is on the order of $1 \mu eV/(1/\text{ns})$. This indicates that $J_{sp}$ results from spontaneous photon emission on the order of nA, which can be readily measured.
V. SUMMARY

The expression of tunnelling current $J_{sp}(t)$ arising from spontaneous radiative transition in individual quantum dots is obtained by using the nonequilibrium Green’s function method. $J_{sp}(t)$ is found to be functions of the spontaneous emission rate $R_{eh}$, the tunnelling rates $\Gamma_h(\Gamma_e)$ and the factor of $f_e(T)f_h(T)$. In addition to the exponential growth and decay features corresponding to the charging and discharging process, the oscillatory behavior of $J_{sp}(t)$ as a function of time is observed as a result of particle coherent tunnelling between the electrodes and QDs, which is suppressed by the temperature effects.

In this study, resonant tunnelling carriers are used to yield the triggered single-photon sources, in contrast to the captured carriers typically used in an optically driven SPS. Due to the phonon bottleneck effect, it is predicted that the QD capture rate of electrons will be low. This could reduce the performance of SPS devices, which use captured carriers as the source of single-photon generations. Using carriers injected via the resonant tunnelling process can prevent such problems.

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Figure Captions

Fig. 1. The schematic band diagram for the single quantum dot embedded in a p-i-n junction. The ground state energy level of electrons is 20 meV below the Fermi-energy of left-electrode, and the exciton state for holes is 15 meV above the Fermi-energy of right-electrode. The periodic square bump-like modulation $v(t)$ supplies holes into the quantum dot.

Fig. 2. Time-dependent current arising from the electron-hole recombination $J_{sp}(t)$ for different duration times of applied rectangular pulse voltage with amplitude $\Delta = 20$ meV at zero temperature. Current is given in units of $J_0 = 2e R_{eh}$, where $R_{eh}$ denotes the time and temperature-independent spontaneous emission rate, and time is in units of $t_0 = h/meV$.

Fig. 3. Time-dependent current arising from the electron-hole recombination $J_{sp}(t)$ for different applied voltages with duration time $\Delta s = 3 \ t_0$ at zero temperature. Current is given in units of $J_0 = 2e R_{eh}$, where $R_{eh}$ denotes the time and temperature-independent spontaneous emission rate, and time is in units of $t_0 = h/meV$.

Fig. 4. Time-dependent current arising from the electron-hole recombination $J_{sp}(t)$ for different tunnelling rates of holes at zero temperature and the applied voltage with duration time $\Delta s = 3 \ t_0$ and amplitude $\Delta = 20$ meV. Current is given in units of $J_0 = 2e R_{eh}$, where $R_{eh}$ denotes the time and temperature-independent spontaneous emission rate, and time is in units of $t_0 = h/meV$.

Fig. 5. Time-dependent current arising from the electron-hole recombination $J_{sp}(t)$ for different temperatures at the applied voltage with duration time $\Delta s = 3 \ t_0$ and amplitude $\Delta = 20$ meV. Current is given in units of $J_0 = 2e R_{eh}$, where $R_{eh}$ denotes the time and temperature-independent spontaneous emission rate, and time is in units of $t_0 = h/meV$. 

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