Neutron Beta Decay in a Left-Right Symmetric Model

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Abstract

We start from a Left-Right Symmetric Model and we analyze the beta decay of the free neutron \( n \rightarrow p + e^- + \bar{\nu}_e \). We applied this model to incorporate the right currents, whereby we propose an amplitude whose leptonic part contains the parameter \( \lambda \) defined as left-right asymmetry parameter which measures the parity violation. The analysis consists of seeing how the spectrum of energy of the electrons, the total rate of decay, and the lifetime \( \tau \) of the neutron are affected by the left-right asymmetry parameter, besides taking into account corrections of mass, that is, \( m_e \neq 0 \), \( m_p \neq m_n \), and the recoil of the proton.

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I. INTRODUCTION

Matter is composed of atoms which, although they are of very different classes, contain the same basic constituents: the proton, the neutron and the electron. Of these three, two (the proton and the neutron) are only in the nucleus and because of this they are usually called nucleones.

The neutron is a particle with charge zero, mass in rest of $939.565330 \pm 0.000038 \text{MeV}$, and spin $\hbar/2$. The neutron is a fermion, which decays or disintegrates for weak interaction in: $n \rightarrow p + e^- + \bar{\nu}_e$ \cite{1,2}, that is to say, the neutron becomes a proton emitting an electron plus an antineutrino of the electron. The experimental confirmation of the discovery of the neutron was made in 1932 by James Chadwick \cite{2}.

The free neutron is unstable and this instability results in that the neutron decays. However, in the nuclei it can remain without disintegrating. This is due to the fact that inside the nucleus, the protons and neutrons are connected by the nuclear forces, that is to say, by means of a strong interaction.

The process of beta decay of the neutron is: $n \rightarrow p + e^- + \bar{\nu}_e$. The theory of Fermi of beta decay is used currently, although with some changes in weak processes to low energy, now called the theory V-A. For weak processes to high energy, the correct theory is the electroweak theory of Weinberg, Salam and Glashow \cite{3}, of which the theory V-A is a limit.

One of the important characteristics of the decay of the neutron is that this process has current interest in nuclear physics \cite{4} and in cosmological theories \cite{4,5,6} since it is necessary to have complete theoretical results of the total rate of decay and thus, more and more precise experimental results of the lifetime $\tau$ of the neutron \cite{1,5,6,7}.

The total rate of decay of the neutron is an important quantity for its cosmological impact on the synthesis of light elements \cite{4,5} and because, combined with the asymmetry of the electron, it provides in the standard model a direct determination of the coupling constants vectorial semileptonic ($F_V$) and axial-vector ($G_A$) \cite{1,5,6}.

The mensurations of the beta decay of the neutron provide important information about
the strong and electroweak interactions. The decay of the free neutron is distinguished among the beta decay, because (i) it is theoretically quite pure (the theoretical uncertainties are small); (ii) the free neutrons can produce abundantly without expensive accelerators, and the detectors of the products of decay of the neutron are also relatively economic.

The experiments of the decay of the free neutron make possible the precise and exact determination of the following important parameters of the standard model of the elementary particles: the constants of coupling weak vector (\(F_V\)) and axial-vector (\(G_A\)), and the elements \(V_{ud}\) of the matrix of Kobayashi-Maskawa (KM). The value of \(F_V\) is important for the test of the conservation of the hypothesis of the vectorial current; \(V_{ud}\) is necessary to prove the unitary nature of the matrix KM; the reason \(\alpha = G_A/F_V\) helps to understand the fundamental interaction of the first quarks generation. \(\alpha\) can be determined for diverse forms and independently of mensurations of the decay of the neutron, and the comparison of the several results of \(\alpha\) provides an important test of the phenomenological model V-A (the approach to low energy of the part of the standard model of the weak interaction of the charged current). The precise value of \(\alpha\) is important for the calculation of cross sections of the weak hadronic interaction, for applications in astrophysics, cosmology big bang, solar physics and neutrinos detection (see the articles [5,6,8–10] and references therein).

There are many extensions of the standard model that predict measured effects of deviations of the standard model in decays of the neutron. One of the most popular is the model with left-right symmetry, with the norm group \(SU(2)_L \times SU(2)_R \times U(1)\) and currents charged with right helicity \([11,12]\).

The purpose of this work is to carry out an analysis of the decay of the free neutron, in the context from a model with left-right symmetry \([13]\), for this we start from an extension of the electroweak model applied to the baryons decay \([14]\). This model contains the parameter \(\lambda\) defined as parameter of left-right asymmetry, which measures the parity violation. We apply this theory to incorporate the right currents, for which we propose an amplitude whose leptonic part is \(V + \lambda A\), with \(\lambda = -1\) for left currents and \(\lambda = 1\) for right currents. We also consider the mass of the electron different from zero \((m_e \neq 0)\), the recoil of the proton and
the difference of masses among the proton and the neutron \((m_p \neq m_n)\).

The central point of ours result is that we have considered an interaction point of four fermions, and we have considered the proton and the neutron as if they were simply point particles. The most realistic thing is to consider that the proton and the neutron possess structure which is coupled to the weak boson \(W\), and to express the transition amplitude in terms of several form factors whose structure is limited by the covariance of Lorentz. However, this case goes beyond the purpose of this work and thus is not considered here.

This work is structured in the following way: In Section II, we consider the model with left-right symmetry, which is the point of departure of this work. Section III contains our analytic calculations of the decay of the neutron, and finally, Section IV contains our results and conclusions. An appendix is also included, which contains the calculations of the total rate of decay for the case when the mass of the electron is different from zero, that is, \(y = \frac{m_e}{m_n} \neq 0\).

II. THEORETICAL MODEL

In the theory of Fermi V-A the transition amplitude for the beta decay of the neutron is given by the expression

\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} [\bar{u}_p \gamma^\alpha (F_V + G_A \gamma_5) u_n] [\bar{u}_e \gamma_\alpha (1 - \gamma_5) v_\nu],
\]

where \(F_V\) and \(G_A\) are produced by the strong interaction of the proton and of the neutron.

Experimentally it is found that the coefficient \(F_V\) of the vector it is \(F_V = 1\) and the coefficient \(G_A\) of the axial-vector is \(G_A = 1.2670 \pm 0.0035\).

Although the experiments carried out up to now indicate in a definitive manner that the emitted neutrinos are of mainly negative helicity (which is known as left currents), there is not a fundamental theoretical reason for the same happening to higher energy.

The electroweak model takes as a fact that there are only neutrinos of negative helicity and, as it was commented previously, their limit to low energy is the theory V-A. Other
physicists extend the electroweak theory building a model with left-right symmetry [13], that is, incorporating neutrinos of positive helicity (which is known as right currents).

The focus that we use here to include right currents is exposed by R. Huerta in the Ref. [13], and later is used by A. García et al. [14], in semileptonic decay of baryons.

We take as starting point the amplitude

\[ \mathcal{M} = \frac{G}{\sqrt{2}} \left[ a J_L^L J_L^h + b (J_L^R J_R^L + J_R^L J_L^R) + c J_R^R J_R^h \right], \]  

(2)

where \( a, b, \) and \( c \) contain the parameters of the electroweak model with left-right symmetry, which is not dealt with here, but we can say that it is the version to low energy of the electroweak symmetrical model.

The amplitude (2) is for the decay \( A \rightarrow B + e^- + \bar{\nu}_e \) where \( A \) and \( B \) are baryons. Using the notation: \(|1>\) for the neutrino, \(|2>\) for the electron and \(|A>, |B>\) for the baryons, it has that the left leptonic part is:

\[ J_L^L = <2|V - A|1>, \]

which contains the neutrinos of negative helicity; the right leptonic part is

\[ J_L^R = <2|V + A|1>, \]

while the left baryonic part is:

\[ J_L^h = <B|F_L V + G_L A|A>, \]

and the right baryonic part

\[ J_R^h = <B|F_R V + G_R A|A>, \]

where \( F_L, F_R, G_L, G_R \) are induced form factors for the strong interaction. Substituting the expressions of the currents in (2)

\[ \mathcal{M} = \frac{G_F}{\sqrt{2}} \left[ a <2|V - A|1><B|F_L V + G_L A|A> 
+b <2|V - A|1><B|F_R V + G_R A|A>
+b <2|V + A|1><B|F_L V + G_L A|A>
+c <2|V + A|1><B|F_R V + G_R A|A> \right], \]  

(3)
and after regrouping appropriately and defining

\[ \mathcal{P} = aF_L + bF_R + bF_L + cF_R, \]
\[ Q = -aF_L - bF_R + bF_L + cF_R, \]
\[ R = aG_L + bG_R + bG_L + cG_R, \]
\[ S = -aG_L - bG_R + bG_L + cG_R, \]

(4)

we obtain

\[ \mathcal{M} = \frac{G_F}{\sqrt{2}} \langle < 2 | V + \frac{Q}{\mathcal{P}} A | 1 > < B | \mathcal{P} V | A > + < 2 | V + \frac{S}{R} A | 1 > < B | R A | A > \rangle, \]

(5)

where

\[ \lambda = \frac{Q}{\mathcal{P}} = \frac{-aF_L - bF_R + bF_L + cF_R}{aF_L + bF_R + bF_L + cF_R}, \]
\[ \lambda' = \frac{S}{\mathcal{R}} = \frac{-aG_L - bG_R + bG_L + cG_R}{aG_L + bG_R + bG_L + cG_R}. \]

(6)

As the strong interaction is invariant under parity, we can suppose that

\[ F_L = F_R, \quad G_L = G_R \] and then \[ \lambda = \lambda' = \frac{-a + c}{a + 2b + c}, \]

and furthermore define

\[ F \equiv \mathcal{P} = aF_L + bF_R + bF_L + cF_R = (a + 2b + c)F_L, \]
\[ G \equiv \mathcal{R} = aG_L + bG_R + bG_L + cG_R = (a + 2b + c)G_L. \]

Substituting these expressions and ordering appropriately, we obtain the amplitude of decay for the model with left-right symmetry

\[ \mathcal{M} = \frac{G_F}{\sqrt{2}} \langle < 2 | V + \lambda A | 1 > < B | FV + GA | A > \rangle. \]

(7)

In this amplitude, the effects of the currents are already included in \( \lambda \) and in the form factors \( F \) and \( G \).
III. DECAY OF THE NEUTRON

As we already mentioned in the introduction, the neutron is a particle with charge zero, spin $\hbar/2$ and mass in rest of $939.565330 \pm 0.000038 \text{ MeV}$. This particle decays or disintegrates for weak interaction in:

$$n \rightarrow p + e^- + \bar{\nu}_e,$$

as indicated in the diagram of Feynman, Fig. 1.

A. Amplitude of Transition

The amplitude of decay of the neutron is

$$M = \frac{G_F}{\sqrt{2}} [\bar{u}_p \gamma^\alpha (1 - \rho \gamma_5) u_n] [\bar{u}_e \gamma^\beta (1 + \lambda \gamma_5) v_\nu]. \quad (9)$$

The coefficient $\lambda$ that appears in the leptonic part of the Eq. (9), is what we call the coefficient of left-right asymmetry, which contains information on the effects of the theory V+A (right currents). This coefficient does not appear in the common literature and it is introduced to incorporate the theory V-A and part of V+A, and, as already commented, is the starting point of this work.

To determine the square of the amplitude of transition of our process, we should determine the conjugated complex firstly of (9)

$$M^* = \frac{G_F}{\sqrt{2}} [\bar{v}_\nu \gamma_\beta (1 + \lambda \gamma_5) u_e] [\bar{u}_n \gamma^\alpha (1 - \rho \gamma_5) u_p]. \quad (10)$$

Now, of the Eqs. (9) and (10), and after applying some of the theorems of traces of the Dirac matrix, we have that the square of the amplitude of transition is

$$\sum_s |M|^2 = \frac{G_F^2}{2} [32(1 + \rho^2)(1 + \lambda^2) \{(p_p \cdot p_e)(p_n \cdot p_\nu) + (p_p \cdot p_\nu)(p_n \cdot p_e)\}$$

$$+ 32m_\nu m_e(1 + \rho^2)(1 - \lambda^2)(p_n \cdot p_e) + 128\lambda \rho \{(p_p \cdot p_\nu)(p_n \cdot p_e) - (p_p \cdot p_e)(p_n \cdot p_\nu)\}$$

$$- 32m_n m_p(1 - \rho^2)(1 + \lambda^2)(p_\nu \cdot p_e) - 64m_\nu m_e m_p m_n(1 - \rho^2)(1 - \lambda^2)]. \quad (11)$$
To simplify the calculations, we move to the system center of masses of the neutron where $p_n = 0$, so that the four-momentum products are:

$$p_p \cdot p_e = E_p E_e - \mathbf{p}_p \cdot \mathbf{p}_e,$$

$$p_n \cdot p_\nu = m_n E_\nu,$$

$$p_p \cdot p_\nu = E_p E_\nu - \mathbf{p}_p \cdot \mathbf{p}_\nu,$$

$$p_n \cdot p_e = m_n E_e,$$

$$p_n \cdot \mathbf{p}_p = m_n E_p,$$

$$p_\nu \cdot p_e = E_\nu E_e - \mathbf{p}_\nu \cdot \mathbf{p}_e.$$

(12)

Up until now, there are no precise results on the mass of the neutrino of the electron, it is only known that it is very small compared with the mass of the electron ($m_\nu << m_e$) so that we can suppose that $m_\nu = 0$, and of the energy-momentum relativistic relationship for the neutrino we have that $E^2_\nu = m^2_\nu + p^2_\nu$ where $E_\nu = p_\nu$, which simplifies the calculations.

To simplify even more the expression (11), we define the following which will be later of great utility

$$\beta_e = \frac{\left| \mathbf{p}_e \right|}{E_e} , \quad (\hat{\mathbf{p}}_e \cdot \hat{\mathbf{p}}_\nu) = x_{e\nu} = x, \quad A_\rho = (1 + \rho^2),$$

$$A_\lambda = (1 + \lambda^2), \quad B_\rho = -2 \rho, \quad B_\lambda = 2 \lambda, \quad C_\rho = (1 - \rho^2),$$

thus, we have finally that the square of the transition amplitude for our process is

$$\sum_s | \mathcal{M} |^2 = 16 G^2_{F} m_n [M_1 E_\nu + M_2 E^2_\nu + (N_1 E_\nu + N_2 E^2_\nu)(\hat{\mathbf{p}}_e \cdot \hat{\mathbf{p}}_\nu)].$$

(13)

This expression contains the details of the interaction, that is, it describes the dynamics of the process.

In (13) we have defined

$$M_1 = A_\rho A_\lambda (2 E_e m_n - E^2_e - m^2_e) - A_\lambda C_\rho E_e m_p + B_\rho B_\lambda (E^2_e - m^2_e),$$

$$M_2 = (-A_\rho A_\lambda - B_\rho B_\lambda) E_e,$$

$$N_1 = (A_\rho A_\lambda E_e + A_\lambda C_\rho m_p - B_\rho B_\lambda E_e) E_e \beta_e,$$

$$N_2 = (A_\rho A_\lambda + B_\rho B_\lambda) E_e \beta_e.$$

(14)
B. Differential Decay Rate

Our following step, now that we know the square of the transition amplitude Eq. (13), is to calculate the differential rate of decay of the neutron.

The expression for the differential rate of decay for a particle that decays in three is expressed by [1]:

\[
\frac{d}{dt} = (2\pi)^4 \frac{1}{2E_A} \delta^4(p_A - p_a - p_b - p_c) \frac{d^3p_a d^3p_b d^3p_c}{(2\pi)^4 2E_a (2\pi)^3 2E_b (2\pi)^3 2E_c}, \quad (15)
\]

where \(| A >\) is the initial state of the system, \(< abc | \) is the final state and \(\tau\) is the operator that makes the transition. For our case \(< p e^-\bar{\nu} | \tau | n > = \mathcal{M}\).

Applying the expression (15) to our process we have that

\[
\frac{d}{dt} = \sum_s | \mathcal{M} |^2 \frac{d^3p_e d^3p_\nu d^3p_p}{E_e E_\nu E_p} \delta(E_n - E_e - E_\nu - E_p) \delta^3(p_n - p_e - p_\nu - p_p), \quad (16)
\]

in this expression, the square of the transition amplitude \(\sum_s | \mathcal{M} |^2\) provides information on the details of the interaction, the Dirac delta function implies conservation of energy-momentum, and the other terms describe the kinematics of the process, which are standard for all the reactions.

The order in which (16) should be integrated will depend on the physical quantities wanted, as well as on the ease of carrying out the integrations.

The first aspect that we want is to calculate the spectrum of energy of the electron, reason for which we will integrate firstly with regard to the moment of the proton \(p_p\):

\[
\frac{d}{dt} = \sum_s | \mathcal{M} |^2 \frac{d^3p_e d^3p_\nu}{E_e E_\nu E_p} \delta(E_n - E_e - E_\nu - E_p). \quad (17)
\]

Now we integrate with regard to the moment of the neutrino; for this we use spherical coordinates, and from the energy-momentum relativist relationship, we have that

\[
d^3p_\nu = p_\nu E_\nu d\Omega_\nu dE_\nu,
\]

then
\[ d\Gamma = \frac{\sum_s |\mathcal{M}|^2}{16(2\pi)^5 m_n} \frac{d^3\mathbf{p}_e d\mathbf{p}_\nu}{E_e E_p} d\Omega_\nu \delta(E_n - E_e - E_\nu - E_p) \, dE_\nu. \]  \hspace{1cm} (18)

To integrate the Eq. (18) with regard to the energy of the neutrino \( E_\nu \), we use the following property of the delta of Dirac:

\[
\int G(E_\nu) \delta[f(E_\nu)] \, dE_\nu = \frac{G(E^0_\nu)}{|f'(E^0_\nu)|},
\]

where we have defined

\[
G(E_\nu) = \frac{\sum_s |\mathcal{M}|^2}{16(2\pi)^5 m_n} \frac{d^3\mathbf{p}_e d\mathbf{p}_\nu}{E_e E_p} d\Omega_\nu,
\]

\[
f(E_\nu) = E_n - E_e - E_\nu - E_p, \quad f(E^0_\nu) = E_n - E_e - E_\nu - E_p = 0,
\]

Here, \( E^0_\nu \) corresponds to the real energy of the neutrino of the electron, energy for which the function \( f(E_\nu) \) is made zero. Then we have that the Eq. (18) takes the form

\[ d\Gamma = \frac{G^2}{(2\pi)^4 m_n} \frac{d^3\mathbf{p}_e}{E_e} \frac{p_\nu}{(a + bx)^3} [M_1 E_\nu + M_2 E_\nu^2 + (N_1 E_\nu + N_2 E_\nu^2) x] \, dx. \]  \hspace{1cm} (19)

To integrate with regard to the variable \( x \) we should express \( E_\nu = p_\nu \) as a function of \( x \), which is achieved using energy-momentum conservation. We have this way that

\[ d\Gamma = \frac{G^2}{(2\pi)^4 m_n} \frac{d^3\mathbf{p}_e}{E_e} [M_1 \frac{(E_m - E_e)^2}{(a + bx)^3} + M_2 \frac{(E_m - E_e)^3}{(a + bx)^4} + N_1 \frac{(E_m - E_e)^2}{(a + bx)^3} x + N_2 \frac{(E_m - E_e)^3}{(a + bx)^4} x] \, dx, \]  \hspace{1cm} (20)

where

\[
E_\nu = p_\nu = \frac{(E_m - E_e)}{(a + bx)}, \quad E_m = \frac{m_n^2 - m_p^2 + m_e^2}{2m_n},
\]
and the last equation corresponds to the expression for the maximum energy of the electron.

The following is integrated with regard to the variable $x$:

$$d\Gamma = \frac{G_F^2}{(2\pi)^4 m_n} \frac{d^3 p_e}{E_e} [M_1(E_m - E_e)^2 I_1 + M_2(E_m - E_e)^3 I_2 + N_1(E_m - E_e)^2 I_3 + N_2(E_m - E_e)^3 I_4],$$

(21)

where

$$I_1 = \int_{-1}^{1} \frac{dx}{(a + bx)^3}, \quad I_2 = \int_{-1}^{1} \frac{dx}{(a + bx)^4}, \quad I_3 = \int_{-1}^{1} \frac{x dx}{(a + bx)^3}, \quad I_4 = \int_{-1}^{1} \frac{x dx}{(a + bx)^4},$$

and we also define

$$y = \frac{m_e}{m_n}, \quad z = \frac{m_p}{m_n}, \quad \epsilon = \frac{E_e}{m_n}, \quad \epsilon_m = \frac{(1 - z^2)}{2}, \quad \epsilon'_m = \frac{E_m}{m_n},$$

$$\eta = \frac{p_e}{m_n} = \sqrt{\epsilon^2 - y^2}, \quad a_0 = 1 + y^2, \quad b_0 = -2, \quad c_0 = \frac{a_0}{b_0} = \beta.$$

After evaluating the integral explicitly and rewriting in terms of the quantities defined above, as well as the coefficients $M_1, M_2, N_1, N_2$, we obtain

$$I_1 = \frac{2(1 - \epsilon)}{(a_0 + b_0 \epsilon)^2}, \quad I_2 = \frac{2(3 - 6 \epsilon + 4 \epsilon^2 - y^2)}{3(a_0 + b_0 \epsilon)^3}, \quad I_3 = -\frac{2 \eta}{(a_0 + b_0 \epsilon)^2}, \quad I_4 = -\frac{8(1 - \epsilon) \eta}{3(a_0 + b_0 \epsilon)^3},$$

where now:

$$M_1 = [A_B A_\lambda (2 \epsilon - \epsilon^2 - y^2) - A_\lambda C_B \epsilon + B_B B_\lambda (\epsilon^2 - y^2)] m_n^2,$$

$$M_2 = [-A_B A_\lambda \epsilon - B_B B_\lambda \epsilon] m_n,$$

$$N_1 = [A_B A_\lambda \epsilon + A_\lambda C_B \epsilon - B_B B_\lambda \eta] m_n^2,$$

$$N_2 = [A_B A_\lambda + B_B B_\lambda] \eta m_n.$$

To integrate with regard to the moment of the electron, we express the differential of volume $d^3 p_e$ in spherical coordinates

$$d^3 p_e = p_e^2 dp_e d\Omega \Rightarrow p_e^2 dp_e 4\pi = m_n^3 \eta \epsilon \epsilon 4\pi,$$

obtaining the following
\[ d\Gamma = \frac{4G^2m_n^5(\epsilon'_m - \epsilon)^2}{(2\pi)^3(a_0 + b_0\epsilon)^3} \eta d\epsilon \left[ (-A_pA_\lambda - B_\rho B_\lambda)a_0y^2 + A_\lambda C_\rho za_0y^2 + \frac{4}{3}(A_pA_\lambda + B_\rho B_\lambda)\epsilon'_m y^2 \right. \\
+ \left. \{(-A_pA_\lambda - B_\rho B_\lambda)(b_0 - a_0)y^2 + (2A_pA_\lambda - A_\lambda C_\rho z)a_0 + \frac{1}{3}(-A_pA_\lambda - B_\rho B_\lambda)(3 - y^2)\epsilon'_m \right. \\
+ A_\lambda C_\rho zb_0y^2 + (A_pA_\lambda - B_\rho B_\lambda)a_0y^2 + \frac{4}{3}(-A_pA_\lambda - B_\rho B_\lambda)(1 + \epsilon'_m)y^2 \} \epsilon \\
+ \{(-A_pA_\lambda + B_\rho B_\lambda)b_0y^2 + (2A_pA_\lambda - A_\lambda C_\rho z)(b_0 - a_0) + (-A_pA_\lambda + B_\rho B_\lambda)a_0 \\
+ 2(A_pA_\lambda + B_\rho B_\lambda)\epsilon'_m + \frac{1}{3}(A_pA_\lambda + B_\rho B_\lambda)(3 - y^2) - A_\lambda C_\rho za_0 + (A_pA_\lambda - B_\rho B_\lambda)b_0y^2 \\
+ \frac{4}{3}(-A_pA_\lambda - B_\rho B_\lambda)(\epsilon'_m - y^2)\} \epsilon^2 + \{-2A_pA_\lambda - A_\lambda C_\rho z\}b_0 + (-A_pA_\lambda + B_\rho B_\lambda)(b_0 - a_0) \\
+ \frac{1}{3}(-A_pA_\lambda - B_\rho B_\lambda)(6 + 4\epsilon'_m) - A_\lambda C_\rho zb_0 + (-A_pA_\lambda + B_\rho B_\lambda)a_0 \\
+ \frac{4}{3}(-A_pA_\lambda - B_\rho B_\lambda)(-1 - \epsilon'_m)\} \epsilon^3 \].

As a first approach, we will take as zero the mass of the electron in the previous expression, that is, \( y = m_e/m_n = 0 \) since, compared with other terms, their contribution is very small. With this approach and after making some pertinent arrangements, we find that the expression for the spectrum of energy of the electrons is

\[ \frac{d\Gamma}{d\epsilon} = \frac{2G^2m_n^5}{(2\pi)^3} \frac{\eta}{b_0^3(c_0 + \epsilon)^3} \sum_{n=1}^{5} F_n \epsilon^n, \]  

(23)

where:

\[ F_1 = Q\epsilon^2, \]

\[ F_2 = Re^2 - 2Q\epsilon_m, \]

\[ F_3 = S\epsilon^2 - 2R\epsilon_m + Q, \]

\[ F_4 = -2S\epsilon_m + R, \]

\[ F_5 = S, \]

with

\[ Q = 2A_pA_\lambda - A_\lambda C_\rho z - (A_pA_\lambda + B_\rho B_\lambda)\epsilon_m, \]

\[ R = -6A_pA_\lambda + 2B_\rho B_\lambda + 2A_\lambda C_\rho z + \frac{2}{3}(A_pA_\lambda + B_\rho B_\lambda)\epsilon_m, \]

\[ S = \frac{16}{3}A_pA_\lambda - \frac{8}{3}B_\rho B_\lambda. \]
C. Total Decay Rate

To determine the total rate of decay of the neutron we integrate the expression (23) with regard to the variable \( \epsilon \), that is,

\[
\Gamma = \frac{2G_Fm_n^5}{(2\pi)^3b_0^3} \sum_{n=1}^{5} F_n I_n, \tag{24}
\]

where

\[
I_n = \int_{\epsilon_0}^{\epsilon_1} \eta \left( \frac{\epsilon^n}{\epsilon_0 + \epsilon} \right)^3 d\epsilon, \quad n = 1, 2, 3, \ldots, 5, \quad \epsilon_0 = 0, \quad \epsilon_1 = \epsilon_m,
\]

This type of integral is not difficult to solve, since with an appropriate change of variables, these are solved quickly.

Explicitly the sum of the Eq. (24) is

\[
\sum_{n=1}^{5} F_n I_n = \left( \frac{1}{4} F_1 + \frac{1}{8} F_2 + \frac{1}{16} F_3 + \frac{1}{32} F_4 + \frac{1}{64} F_5 \right) J_{-3} + \left( F_1 + \frac{3}{4} F_2 + \frac{1}{2} F_3 + \frac{5}{16} F_4 + \frac{3}{16} F_5 \right) J_{-2} + \left( F_1 + \frac{3}{2} F_2 + \frac{3}{4} F_3 + \frac{5}{4} F_4 + \frac{15}{16} F_5 \right) J_{-1} + \left( F_2 + 2F_3 + \frac{5}{2} F_4 + \frac{5}{2} F_5 \right) J_0 + \left( F_3 + \frac{5}{2} F_4 + \frac{15}{4} F_5 \right) J_1 + \left( F_4 + 3F_5 \right) J_2 + F_5 J_3 = \sum_{n=-3}^{3} G_n J_n,
\]  

where the \( J_n \) are the aforementioned integrals, which are

\[
J_{-3} = \int_{\alpha_0}^{\alpha_1} \frac{1}{\alpha^3} d\alpha = \frac{2\epsilon_m^2 - 2\epsilon_m}{(\epsilon_m - 1/2)^2}, \quad J_1 = \int_{\alpha_0}^{\alpha_1} \alpha d\alpha = \frac{1}{2}(\epsilon_m^2 - \epsilon_m),
\]

\[
J_{-2} = \int_{\alpha_0}^{\alpha_1} \frac{1}{\alpha^2} d\alpha = -\frac{2\epsilon_m}{(\epsilon_m - 1/2)}, \quad J_2 = \int_{\alpha_0}^{\alpha_1} \alpha^2 d\alpha = \frac{1}{3} \epsilon_m^3 - \frac{1}{2} \epsilon_m^2 + \frac{1}{4} \epsilon_m,
\]

\[
J_{-1} = \int_{\alpha_0}^{\alpha_1} \frac{1}{\alpha} d\alpha = \ln(1 - 2\epsilon_m), \quad J_3 = \int_{\alpha_0}^{\alpha_1} \alpha^3 d\alpha = \frac{1}{4} \epsilon_m^4 - \frac{1}{3} \epsilon_m^3 + \frac{3}{8} \epsilon_m^2 - \frac{1}{8} \epsilon_m,
\]

\[
J_0 = \int_{\alpha_0}^{\alpha_1} d\alpha = \epsilon_m
\]

and their respective coefficients \( G_n \):
As we already have explicitly the integrals $J_n$ with their respective coefficients $G_n$, the following is to substitute both results in the Eq. (24), and after a simple algebra we obtain the total rate of decay of the neutron:

\[
\Gamma = \frac{G^2 \pi m^5}{60 \pi^3 \epsilon_m^5} (1 + \lambda^2)(1 + \epsilon_m + \ldots) + z(1 - \rho^2)(-\frac{1}{2} - \epsilon_m + \ldots). \tag{27}
\]

This result is consistent with those of the common literature, those that only contain left currents. The new contribution in the Eq. (27) is the parameter $\lambda$ that, as we have already mentioned, gives us information on those effects of the right currents ($V + A$), and $\epsilon_m$ (where $\epsilon^2_m$ and $\epsilon^3_m$ have been rejected because their contribution is very small) which is the most important term of correction. The numeric value of this term is presented in the part corresponding to the results and conclusions.

An immediate consequence of the Eq. (27) is that we can calculate immediately the lifetime $\tau$ of the neutron, that is, the required time for a certain sample of particles or nuclei to disintegrate until decreasing by half.

**IV. RESULTS AND CONCLUSIONS**

In this part we present our results and conclusions of the decay of the free neutron. Firstly, we have the corresponding result for the spectrum of energy of the electrons that,
as is known, during the beta disintegration, the energy spectra of the electrons are always continuous.

The physical meaning of the Eq. (23), that corresponds to the spectrum of energy of the electrons, is that this expression determines the total number of electrons that leave with a certain energy (or the total probability of emission of an electron).

An important point here is to see the influence of the parameter of left-right asymmetry $\lambda$ in the spectrum or distribution of energy of the electrons, thus, a group of figures for different values of this parameter is presented.

Fig. 1 corresponds to $\lambda = -1$, that is to say, for left currents, Fig. 2, for $\lambda = 1$ right currents, while in Fig. 3, several curves for different values of the parameter of asymmetry $\lambda$ are superimposed.

Comparing these figures, it is clear that the parameter of asymmetry $\lambda$ does not modify the curve of energy distribution, and only drops or goes up depending on the value of $\lambda$, as is observed more clearly in Fig. 3; but we consider that this parameter is important in the calculation of the lifetime of the neutron.

Another important result is the total rate of decay Eq. (27). This expression determines the total probability that the particle decays, and as we can see, it depends of the constant of Fermi $G_F$, of the parameter of left-right asymmetry $\lambda$, of $\rho$ that is originated by the strong interaction, and of $\epsilon_m$.

As it was already commented, the parameter $\lambda$ does not alter the form of the spectrum of energy, but we believe that this does play an important role in the calculation of the lifetime of the neutron, since in the Eq. (27) the factor $(1 + \lambda^2)$ appears, that whether $\lambda = \pm 1$ would have a factor of 2, that would be considerable for the numeric calculation of the lifetime, besides $\rho$ and $\epsilon_m$. Now we present the calculation of the lifetime of the neutron

$$\tau = \frac{1}{\Gamma} = 1060.85 \text{ seg.}$$

(28)

Although this result is very far from the most recent experimental value about the lifetime of the neutron $[\overline{2}]$, this calculation illustrates the ease of determining the lifetime once the
expression for the total rate of decay is obtained. Furthermore, we stress the importance implied in determining more precise analytical expressions on the total rate of decay and as a consequence of the lifetime, which plays a central role in the theory of the weak interaction, since it provides a mensuration of the rate of the constants of coupling $g_V/g_A$. On the other hand, to have a reliable value of the lifetime of the neutron is also of great importance in astrophysics in relation to the problem of the abundance of helium in the universe.

The numeric result of the correction term $\epsilon_m$ of the Eq. (27), which is originated when considering the recoil of the proton and the difference of masses between the proton and the neutron is:

$$\epsilon_m = \frac{1}{2}(1 - z^2) = 0.0013755 \pm 5.9477 \times 10^{-7}.$$  \hspace{1cm} (29)$$

Comparing our result with that corresponding to the radiative corrections [16], we find that both results are of the same order of magnitude and they coincide up to the fourth decimal digit, which implies that our result is good, since it compares an approach to first order with recoil of the proton and difference of masses ($m_p \neq m_n$), with a radiative correction. The other correction terms in the Eq. (27) have rejected their contribution since it is very small.

Modifying the Eq. (27) by means of the approaches that commonly are made, that is, to take the limit $V - A$ and to make the masses of the proton and of the neutron the same $m_p = m_n$, we obtain:

$$\Gamma = \frac{G_F^2 m_n^5 \epsilon_m^5}{60 \pi^3} (1 + 3\rho),$$  \hspace{1cm} (30)$$

which is the result that commonly appears in the literature, which implies that our calculations are consistent with the theory.
Appendix

This appendix contains the calculations corresponding to the total decay rate, for the case when the mass of the electron is different from zero, that is, \( y = \frac{m_e}{m_n} \neq 0 \) which corresponds to a more complete analysis. We start from the Eq. (22) but now conserving all the terms with \( y \).

After a simple algebra, it is found that the new expression for the spectrum of energy of the electrons is:

\[
\frac{d\Gamma}{d\epsilon} = \frac{2G_F^2m_n^5}{(2\pi)^3} \frac{\eta}{b_0^3(c_0 + \epsilon)^3} \sum_{n=0}^{5} F'_n \epsilon'^n, 
\]

where:

\( F'_0 = P' \epsilon'^2 \),
\( F'_1 = Q' \epsilon'^2 - 2P' \epsilon'_m \),
\( F'_2 = R' \epsilon'^2 - 2Q' \epsilon'_m + P' \),
\( F'_3 = S' \epsilon'^2 - 2R' \epsilon'_m + Q' \),
\( F'_4 = -2S' \epsilon'_m + R' \),
\( F'_5 = S' \),

\( P' = (-A_{\rho}A_{\lambda} - B_{\rho}B_{\lambda} + A_{\lambda}C_{\rho}z)y^2 + (-A_{\rho}A_{\lambda} - B_{\rho}B_{\lambda} + A_{\lambda}C_{\rho}z)y^4 
+ \frac{4}{3}(-A_{\rho}A_{\lambda} + B_{\rho}B_{\lambda})\epsilon'_m y^2 \),
\( Q' = 2A_{\rho}A_{\lambda} - A_{\lambda}C_{\rho}z + (-A_{\rho}A_{\lambda} - B_{\rho}B_{\lambda})\epsilon'_m + \left( \frac{14}{3}A_{\rho}A_{\lambda} + \frac{2}{3}B_{\rho}B_{\lambda} - 3A_{\lambda}C_{\rho}z \right)y^2 
+ 2A_{\rho}A_{\lambda}y^4 + (-A_{\rho}A_{\lambda} - B_{\rho}B_{\lambda})\epsilon'_m y^2 \),
\( R' = -6A_{\rho}A_{\lambda} + 2B_{\rho}B_{\lambda} + 2A_{\lambda}C_{\rho}z + \frac{2}{3}(A_{\rho}A_{\lambda} + B_{\rho}B_{\lambda})\epsilon'_m + (-6A_{\rho}A_{\lambda} + 2B_{\rho}B_{\lambda})y^2; \)

\( S' = \frac{16}{3}A_{\rho}A_{\lambda} - \frac{8}{3}B_{\rho}B_{\lambda} \).

As it can be observed, it has increased the number of terms in (31), which makes a more laborious algebra.
What follows is to determine the total rate of decay; for this we integrate the Eq. (31) with regard to the variable $\epsilon$:

$$\Gamma = \frac{2G^2m_n^5}{(2\pi^3b_0^2} \sum_{n=0}^{5} F'_n J'_n,$$

where

$$I_n = \int_{\epsilon_0}^{\epsilon_m} \frac{\epsilon^n}{(c_0 + \epsilon)^3} d\epsilon, \quad n = 0, 1, 2, ..., 5, \quad \epsilon_0 = m_e/m_n = y, \quad \epsilon'_m = \frac{1}{2}(1 + y^2 - z^2).$$

To evaluate this type of integral, we carry out the following variable change:

we define

$$\alpha = c_0 + \epsilon, \quad d\alpha = d\epsilon \quad y \quad \alpha_0 = -\frac{1}{2}(1 - y^2), \quad \alpha_1 = -\frac{1}{2}z^2,$$

so that we obtain

$$I'_n = \int_{\alpha_0}^{\alpha_1} \frac{1}{\alpha^3}(\alpha - c_0)^n \sqrt{R(\alpha)}d\alpha,$$

with

$$R = C\alpha^2 + B\alpha + A, \quad C = 1, \quad B = -2c_0, \quad A = c_0^2 - y^2.$$

Following a similar procedure as in the subsection C

$$\sum_{n=1}^{5} F'_n J'_n = \sum_{n=-3}^{2} G'_n J'_n,$$

where the new integrals $J'_n$ are:

$$J'_{-3} = \int_{\alpha_0}^{\alpha_1} \frac{1}{\alpha^3} \sqrt{R(\alpha)}d\alpha = \frac{2p}{q^2(1 + y^2 - 2\epsilon'_m)} - \frac{2}{(1 + y^2 - 2\epsilon'_m)^2} \sqrt{R} + \frac{4y^2}{q^3}L_2,$$

$$J'_{-2} = \int_{\alpha_0}^{\alpha_1} \frac{1}{\alpha^2} \sqrt{R(\alpha)}d\alpha = \frac{2}{(1 + y^2 - 2\epsilon'_m)} \sqrt{R} - \frac{p}{q}L_2 + L_1,$$

$$J'_{-1} = \int_{\alpha_0}^{\alpha_1} \frac{1}{\alpha} \sqrt{R(\alpha)}d\alpha = \sqrt{R} - \frac{1}{2}qL_2 + \frac{1}{2}pL_1,$$

$$J'_0 = \int_{\alpha_0}^{\alpha_1} \sqrt{R(\alpha)}d\alpha = \frac{1}{2}\epsilon'_m \sqrt{R} - \frac{1}{2}y^2L_1,$$

$$J'_1 = \int_{\alpha_0}^{\alpha_1} \alpha \sqrt{R(\alpha)}d\alpha = \left[-\frac{1}{3}y^2 - \frac{1}{4}\epsilon'_m + \frac{1}{3}\epsilon'_m^2 - \frac{1}{4}y^2\epsilon'_m\right] \sqrt{R} + \frac{1}{4}py^2L_1,$$

$$J'_2 = \int_{\alpha_0}^{\alpha_1} \alpha^2 \sqrt{R(\alpha)}d\alpha = \left[\frac{1}{3}y^2 + \frac{1}{2}y^4 + \frac{1}{3}\epsilon'_m - \frac{1}{3}\epsilon'_m^2 + \frac{1}{4}\epsilon'_m^3 + \frac{1}{8}y^2\epsilon'_m + \frac{1}{8}y^4\epsilon'_m - \frac{1}{3}y^2\epsilon'_m\right] \sqrt{R} - \frac{1}{32}(5p^2 - q^2)y^2L_1.$$
To solve this type of integral is more laborious than those obtained in the subsection C. In these new integrals we define the following
\[ p = 1 + y^2, \quad q = 1 - y^2, \quad \epsilon'_m = \epsilon_m + \frac{1}{2}y^2, \quad z^2 = 1 + y^2 - 2\epsilon'_m, \quad L_1 = \log \frac{p - z^2 + 2\sqrt{R}}{2y}, \]

\[ L_2 = \log \frac{q^2 - z^2p + 2q\sqrt{R}}{2yz^2}, \quad \sqrt{R} = \frac{1}{2}\sqrt{z^4 - 2z^2p + q^2} = \sqrt{\epsilon'_m - y^2}. \]

The corresponding coefficients \( G'_n \) for this case are:

\[ G'_{-3} = \frac{1}{24}(1 + y^2 - 2\epsilon'_m)^3(1 - y^2)^2(A_\rho A_\lambda + B_\rho B_\lambda), \]

\[ G'_{-2} = (1 + y^2 - 2\epsilon'_m)[(A_\rho A_\lambda + B_\rho B_\lambda)((1 - y^2)(\frac{1}{6} + \frac{1}{6}y^2 + \frac{1}{6}y^4 - \frac{5}{12}\epsilon'_m)
+ \frac{1}{6}\epsilon'_m - \frac{1}{4}y^2\epsilon'_m - \frac{2}{3}y^4\epsilon'_m) + \frac{1}{3}y^6 - \frac{2}{3}y^6\epsilon'_m + \frac{1}{3}y^6\epsilon'_2] + \frac{1}{4}A_\lambda C_\rho z(1 - y^2)(1 + y^2 - 2\epsilon'_m)], \]

\[ G'_{-1} = A_\rho A_\lambda(\frac{2}{3} + 2y^2 + 2y^4 + \frac{2}{3}y^6 - \frac{5}{2}\epsilon'_m + 2\epsilon'_2 + \frac{2}{3}\epsilon'_3 - 5y^2\epsilon'_m - \frac{5}{2}y^4\epsilon'_m + 2y^2\epsilon'_2)
+ B_\rho B_\lambda(-\frac{1}{3} - y^2 - \frac{1}{3}y^4 + \frac{3}{2}\epsilon'_m - 2\epsilon'_2 + \frac{2}{3}\epsilon'_3 + 3y^2\epsilon'_m + \frac{3}{2}y^4\epsilon'_m - 2y^2\epsilon'_2)
+ A_\lambda C_\rho z(\frac{2}{3} + y^2 - \frac{1}{2}y^4 - 4\epsilon'_m + 2\epsilon'_2), \quad (34) \]

\[ G'_0 = A_\rho A_\lambda(\frac{10}{3} + \frac{22}{3}y^2 + \frac{10}{3}y^4 - 9\epsilon'_m + 4\epsilon'_2 - 9y^2\epsilon'_m)
+ B_\rho B_\lambda(-\frac{8}{3} - \frac{14}{3}y^4 - 8\epsilon'_m - 4\epsilon'_2 + 7y^2\epsilon'_m) + A_\lambda C_\rho z(3 + y^2 - 4\epsilon'_m), \]

\[ G'_1 = A_\rho A_\lambda(\frac{22}{3} + \frac{22}{3}y^2 - 10\epsilon'_m) + B_\rho B_\lambda(-\frac{14}{3} - \frac{14}{3}y^2 + 6\epsilon'_m) + 2A_\lambda C_\rho z, \]

\[ G'_2 = \frac{16}{3}A_\rho A_\lambda - \frac{8}{3}B_\rho B_\lambda. \]

The following step is to carry out the product of the integrals with their respective coefficients to determine explicitly (33), and after carrying out the pertinent adjustments and substituting this result in (32), the new expression for the total decay rate is:

\[ \Gamma = \frac{G'_2m^5}{4\pi^3}(1 + \lambda^2)[(1 + \rho^2)[(1 - 3\epsilon'_m + \frac{4}{3}\epsilon'_2 + \frac{2}{3}\epsilon'_3 + y^2(\frac{1}{6} - \frac{1}{3}\epsilon'_m + \frac{2}{3}\epsilon'_m)]
+ \frac{y^4}{6} - \frac{7}{6}\epsilon'_m - \frac{1}{12}y^6)\sqrt{R} + \{-\frac{1}{2} + 2\epsilon'_m - 2\epsilon'_2 + y^4(\frac{1}{2} - 2\epsilon'_m + 2\epsilon'_2)\}L_2
+ \{\frac{1}{2} - 2\epsilon'_m + 2\epsilon'_2 + y^4(-\frac{1}{2} - 2\epsilon'_m + 2\epsilon'_2)\}L_1]\]

\[ + z(1 - \rho^2)[(2 - 4\epsilon'_m + \frac{2}{3}\epsilon'_2 + y^2(-\frac{5}{6} + \frac{5}{3}\epsilon'_m) - \frac{1}{3}y^4)\sqrt{R} \quad (35) \]
\[
+\{1 + 3\epsilon_m - 2\epsilon_m^2 + y^2\epsilon_m(1 - 2\epsilon_m) + y^4(-\frac{1}{2} + \epsilon_m)\}L_2
+\{1 - 3\epsilon_m^2 + 2\epsilon_m^2 + y^2(-1 + 2\epsilon_m) + y^4(\frac{1}{2} + \epsilon_m)\}L_1\}.
\]

Comparing this last equation for the total decay rate with that corresponding to the case when the mass of the electron is zero, that is, \(y = \frac{m_e}{m} = 0\), we can see the simple and compact form that it leads to taking \(y = 0\) in the Eq. (27), however, the Eq. (35), although it does not present a very aesthetic form, is very complete.

An important observation of (35) is that when we consider \(y = 0\), we obtain exactly the Eq. (27), which means that both results are consistent with the theory.

It is necessary to mention that to be able to analyze how much the powers of \(y\) will contribute, it is necessary to rewrite (35) in a simpler form, for this we must express \(\sqrt{R}\), \(L_1\), and \(L_2\) as a development in series of powers. But we should take care when carrying out these developments since they are very sensitive to any change, which is why they should be analyzed carefully.

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FIGURES

Fig. 1 Diagram of Feynman for the decay of the neutron $n \rightarrow p + e^- + \bar{\nu}_e$.

Fig. 2 Spectrum of energy of the electrons for $\lambda = 1$ (right currents).

Fig. 3 Spectrum of energy of the electrons for $\lambda = -1$ (left currents).

Fig. 4 Spectrum of energy of the electrons for four values different from the parameter of asymmetry $\lambda = 1, -1, 0.9, -0.9$. 
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