

Unification with and without Supersymmetry: Adjoint SU(5)

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Abstract. I present two new renormalizable grand unified theories where the neutrino masses are generated through the type I and type III seesaw mechanisms. These theories can be considered as the simplest (SUSY) renormalizable grand unified theories based on the SU(5) gauge symmetry. Several phenomenological and cosmological aspects of these proposals are discussed.

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1 Introduction

The unification of the electromagnetic, weak and strong interactions is one of the main motivations for physics beyond the Standard Model. The candidates which describe physics at the unification scale, $M_{\text{GUT}} \approx 10^{14-16}$ GeV, are called Grand Unified Theories (GUTs). The simplest theories based on SU(5) or SO(10) have been proposed a long time ago. However, we still do not know which is the best candidate for grand unification and all possibilities to test or rule out the simplest theories. Usually the most promising way to test grand unified theories is through proton decay.

Grand unified theories predict the unification of the Standard Model interactions at the GUT scale, the quantization of the electric charge, the value of $\sin^2 \theta_W(M_{\text{GUT}}) = 3/8$ at the GUT scale, the decay of the proton and the existence of leptoquarks. I will focus on the grand unified theories based on SU(5) since those are the simplest candidates for grand unification.

The simplest grand unified theory was proposed in reference [2]. This theory is based on the SU(5) gauge symmetry and one Standard Model family is partially unified in the anti-fundamental $\tilde{5}$ and antisymmetric $10$ representations. The Higgs sector is composed of $5_H$ and $\tilde{5}_H$, the GUT symmetry is broken down to the Standard Model by the vacuum expectation value of the Higgs singlet field in $24_H$, and the Standard Model Higgs resides in $5_H$. Unfortunately, this theory is ruled out since in this case the unification of gauge couplings is in disagreement with the values of $\alpha_s(M_Z)$, $\sin \theta_W(M_Z)$ and $\alpha_{em}(M_Z)$. Therefore, we have to look for realistic GUTs based on SU(5).

The minimal supersymmetric version of the Georgi-Glashow model was discussed for the first time in reference [3]. In this case one generation of matter of the Minimal Supersymmetric Standard Model (MSSM) is unified in two chiral superfields $\hat{5} = (\hat{D}^C, \hat{L})$ and $\hat{10} = (\hat{U}^C, \hat{Q}, \hat{E}^C)$, while the Higgs sector is composed of $\hat{5}_H = (\hat{T}, \hat{H}_1)$, $\hat{5}_H = (\hat{\tau}, \hat{H}_1)$, and $24_H$. The renormalizable version of this theory is ruled out since the relation between $Y_E$ and $Y_D$, $Y_F = Y_D^T$, is in disagreement with the experimental values of the fermion masses at the low scale and the neutrinos are massless if the so-called R-parity is conserved.

As we have mentioned above, the main and common problems of the Georgi-Glashow model [2] and minimal renormalizable SUSY SU(5) [3] are the relation between $Y_E$ and $Y_D$, $Y_F = Y_D^T$, and the absence of neutrino masses. In the context of SU(5) models there are two possible ways to obtain a consistent relation between the masses for down quarks and charged leptons: i) one introduces an extra Higgs in the 45 representation or ii) one takes into account the effect of higher-dimensional operators. Notice that in the first possibility we can write a consistent model and keep renormalizability, while in the second case it is difficult to know which is the effect of higher-dimensional operators in all sectors of the theory. Now, in order to generate neutrino masses at tree level in this context one can study the implementation of the Type I [8], Type II [9] or Type III [10] seesaw mechanisms. Different combinations of the needed mechanisms to solve the problem of fermion masses give us the possibility to write down several realistic grand unified theories which could be tested. Let us discuss the different possibilities:

- Renormalizable Non-SUSY SU(5) models. The Higgs sector of those models will be composed at least of $5_H, 24_H$ and $45_H$. Now, we can have three different

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1 See reference [4] for the most general constraints coming from unification and [5] for all possible dimension five contributions to the decay of the proton in this context.
ent scenarios:

- **Model with Type I seesaw**: In this case at least two extra fermionic singlets are included in the theory. See reference [11] for a recent study.

- **Model with Type II seesaw**: In this scenario the Higgs sector is extended adding a Higgs in the $15$ representation. See reference [11].

- **Model with Type III seesaw**: In order to generate neutrino masses a fermionic multiplet in the adjoint representation is added. In this talk I will discuss in detail this model. We refer to this model as “Renormalizable Adjoint $SU(5)$” [12].

- **Non-Renormalizable (NR) $SU(5)$ models**: In this case the Higgs sector is composed of $5_H$ and $24_H$, and the effect of higher-dimensional operators is taking into account.

- **NR Model with Type I seesaw**: This model is ruled out by unification as the original model proposed by Georgi and Glashow.

- **NR Model with Type II seesaw model**: This theory has been proposed in reference [13] and studied in detail in [14,15,16]. In this case it is possible to achieve unification in agreement with the experiments, the model predicts for the first time the existence of light scalar leptoquarks and the upper bound on the total proton decay lifetime [17] is $\tau_{\text{upper}} \leq 2 \times 10^{36}$ years. Therefore, we could have the possibility to test the idea of grand unification at future colliders since the light scalar leptoquarks, $\phi_H = (3, 2, 1/6)$, could be produced at the Large Hadron Collider (LHC) or at $e^+e^-$ colliders.

- **NR Model with Type III seesaw**: This model has been proposed in reference [18]. See also reference [19] for the predictions coming from the unification of gauge couplings in this context.

Now, let us discuss the different supersymmetric models based on $SU(5)$.

- **Renormalizable SUSY $SU(5)$ models**: In these models the Higgs sector is composed of $5_H$, $\tilde{5}_H$, $24_H$, $45_H$ and $\tilde{45}_H$, and one can have different scenarios:

  - **SUSY Model with Type I seesaw**: In this case we have to add at least two singlets superfields in order to generate neutrino masses.

  - **SUSY Model with Type II seesaw**: The Higgs sector is extended by adding two chiral superfields, $15_H$ and $\tilde{15}_H$. See reference [20] for the study of this scenario.

- **SUSY Model with Type III seesaw**: This model has been proposed in reference [21] and will be discussed in the next sections. We refer to this model as “Supersymmetric Adjoint $SU(5)$” [21].

We can also write different Non-Renormalizable SUSY $SU(5)$ models where the effect of higher-dimensional operators is considered and the neutrino masses is generated through the different seesaw mechanisms. We do not list here all possibilities since in this case we just replace the Higgs chiral superfields $45_H$ and $\bar{45}_H$ by the higher dimensional operators. See for example [22].

After this short review I will discuss two new grand unified theories based on the $SU(5)$ gauge symmetry. The first theory has been proposed in reference [12] and the fermion masses are generated with the minimal set of Higgs bosons, $5_H$ and $45_H$. The neutrino masses are generated through the type I [8] and type III [10] seesaw mechanisms using the fermionic $24$ representation. In the second section we will discuss the SUSY version of this model and we will show that these theories can be considered as the simplest (SUSY) renormalizable grand unified theories based on $SU(5)$.

2 **Renormalizable Adjoint $SU(5)$**

In this section we will discuss the simplest non-renormalizable $SU(5)$ model where the neutrino masses are generated through Type I and Type III seesaw mechanisms. In our model [12] the Higgs sector is composed of $24_H = (\Sigma_8, \Sigma_3, \Sigma_{(3,2)}, \Sigma_{(3,2)}, \Sigma_{24}) = (8,1,0) \oplus (1,3,0) \oplus (3,2,-5/6) \oplus (2,5/6) \oplus (1,1,0) \oplus (\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6, H_2) = (8,2,1/2) \oplus (6,1,-1/3) \oplus (3,3,-1/3) \oplus (3,2,-7/6) \oplus (3,1,-1/3) \oplus (3,1,4/3) \oplus (1,2,1/2) \oplus 5_H = (1,2,1/2) \oplus (3,1,-1/3)$. The field $45$ satisfies the following conditions: $(45)_{\alpha\beta} = -(45)_{\beta\alpha}; \Sigma_{\alpha=1} (45)_{\alpha\beta}^3 = 0$, and $v_{45} = (45)^{15} = (45)^{25} = (45)^{35}$. Now, in this model the Yukawa potential for charged fermions reads as:

\[
V_Y = 10 \bar{5} (Y_1 5_H + Y_2 45_H) + 10 10 (Y_3 5_H + Y_4 45_H) + h.c.
\] (1)

and the masses for charged leptons and down quarks are given by:

\[
M_D = Y_1 v_5^* + 2Y_2 v_{45}^*,
\] (2)

\[
M_E = Y_1^T v_5 - 6 Y_2^T v_{45}^*,
\] (3)

where $\langle 5_H \rangle = v_5$, $Y_1$ and $Y_2$ are arbitrary $3 \times 3$ matrices. Notice that there are clearly enough parameters in the Yukawa sector to fit all charged fermions masses. See reference [23] for the study of the scalar potential and [15] for the relation between the fermion masses at the high scale, which is in agreement with the experiment.
The SM decomposition of the needed extra multiplet for type III seesaw is given by: \( \hat{24} = (\rho_8, \rho_3, \rho_{(3, 2)} \ldots) = (9, 1, 0) \bigoplus (1, 3, 0) \bigoplus (3, 2, -5/6) \bigoplus (3, 2, 5/6) \bigoplus (1, 1, 0) \). In our notation \( \rho_3 \) and \( \rho_0 \) are the \( SU(2)_L \) triplet responsible for type III seesaw and the singlet responsible for type I seesaw, respectively.

The new relevant interactions for neutrino masses in this context are given by:

\[
V_{\nu} = c_5 \tilde{\nu}_5 24 5_H + p_4 \tilde{\nu}_4 45_H + \text{h.c.}
\]

Notice from Eq. (1) and Eq. (4) the possibility to generate all fermion masses, including the neutrino masses, with only two Higgses: \( 5_H \) and \( 45_H \). Using Eq. (4) the neutrino matrix reads as:

\[
M'_{ij} = \frac{a_i a_j}{M_{\rho_3}} + \frac{b_i b_j}{M_{\rho_0}},
\]

where

\[
a_i = c_i v_5 - 3 p_i v_{45},
\]

and

\[
b_i = \frac{\sqrt{15}}{2} \left( c_i v_5 + p_i v_{45} \right).
\]

Let us discuss the different contributions to proton decay. For a review on proton decay see [1]. In this model there are five multiplets that mediate proton decay. These are the superheavy gauge bosons \( V = (3, 2, -5/6) \bigoplus (\tilde{3}, 2, 5/6) \), the \( SU(3) \) triplet \( T, \tilde{\Phi}_3, \Phi_5 \) and \( \Phi_6 \). The least model dependent and the dominant proton decay contributions in non-supersymmetric scenarios are mediated by the gauge bosons. Its strength is set by \( M_V \) and \( \alpha_{GUT} \). In order to satisfy the experimental lower bounds on proton decay we must have \( M_V \geq (2 \times 10^{13}) \) GeV if we do (not) neglect the fermion mixings [17]. The different constraints coming from unification and proton decay issue have been studied in detail in reference [12].

### 3 Supersymmetric Adjoint \( SU(5) \)

Let us discuss in this section the supersymmetric version of this model. In the minimal supersymmetric \( SU(5) \) the MSSM chiral superfields are unified in \( \tilde{5}_H \) and \( \tilde{10}_H \), while its Higgs sector comprises \( \tilde{5}_H, 5_H \), and \( 24_H \). Now, in order to write down the supersymmetric version of the realistic grand unified theory discussed above we have to introduce three extra chiral superfields, \( 45_H, \tilde{45}_H \) and \( 24 \). Therefore, our Higgs sector will be composed of \( \tilde{5}_H, 5_H, 24 \). \( 45_H = (\tilde{\Phi}_1, \tilde{\Phi}_2, \tilde{\Phi}_3, \tilde{\Phi}_4, \tilde{\Phi}_5, \tilde{\Phi}_6, H_2) \), and \( \tilde{45}_H = (\tilde{\Phi}_1, \tilde{\Phi}_2, \tilde{\Phi}_3, \tilde{\Phi}_4, \tilde{\Phi}_5, \tilde{\Phi}_6, \tilde{T}_2) \). In this model the Yukawa superpotential for charged fermions reads as:

\[
W_0 = 10 \tilde{\Phi}_1 (Y_1 \tilde{5}_H + Y_2 45_H) + 10 \tilde{\Phi}_2 (Y_3 \tilde{5}_H + Y_4 45_H)
\]

where \( Y_2 \) is an arbitrary \( 3 \times 3 \) matrix. As it is well-known the relation between the masses of \( \tau \) lepton and \( b \) quark, \( m_b (M_{GUT}) = m_{\tau} (M_{GUT}) \), is in agreement with the experiment. Therefore, the \( Y_2 \) matrix must only modify the relation between the masses of quarks and leptons of the first and second generation.

Since in this section we are interested in the supersymmetric version of the model, a new matter chiral superfield has to be introduced only if we want to have the so-called matter parity as a symmetry of the theory. Matter parity is defined as \( M = (-1)^{3(B-L)} = (-1)^{28} R \), where \( M = -1 \) for all matter superfields and \( M = 1 \) for the Higgses and gauge superfields. In the case that matter parity is not conserved the neutrino masses can be generated through the M-parity violating interactions \( c_{i} \tilde{\Phi}_{i} \delta_5 H \) and \( \eta \tilde{\Phi}_{i} 24 H \delta_{5H} \). Particularly, in the second term we have an \( SU(2)_L \) fermionic triplet needed for type III seesaw mechanism. However, we want to keep matter-parity as a symmetry of the theory to avoid the dimension four contributions to the decay of proton coming from \( \lambda_{ik} \tilde{\Phi}_{j} \delta_{5H} \) and have the lightest neutralino as a good candidate for the cold dark matter of the universe.
The new superpotential relevant for neutrino masses in this context is given by:

\[ W_1 = c_i \tilde{\Phi}_i \tilde{\Phi}_i \tilde{24}_H + p_i \tilde{\Phi}_i \tilde{24}_H \tilde{24}_H \tilde{45}_H \]  

(14)

As in the non-supersymmetric model the Higgses in the 45 representation play a crucial role to generate masses for charged fermions and neutrinos as well.

There are also new relevant interactions between \( \tilde{24} \) and \( \tilde{24}_H \) in this model:

\[ W_2 = m_\Sigma \text{Tr} \tilde{24}_H^2 + \lambda_\Sigma \text{Tr} \tilde{24}_H^3 + m \text{Tr} \tilde{24}_H^2 \]

\[ + \lambda \text{Tr} \left( \tilde{24}_H^2 \tilde{24}_H \right) \]  

(15)

Notice that there are only two extra terms since matter parity is conserved. Our Higgs sector is composed of \( \tilde{5}_H, \tilde{5}_H, \tilde{45}_H, \tilde{25}_H \) and \( \tilde{24}_H \) and the additional interactions between the different Higgs chiral superfields in the theory are:

\[ W_3 = m_H \tilde{5}_H \tilde{5}_H + \lambda_H \tilde{5}_H \tilde{24}_H \tilde{5}_H \]

\[ + c_H \tilde{5}_H \tilde{24}_H \tilde{45}_H + b_H \tilde{25}_H \tilde{24}_H \tilde{5}_H \]

\[ + m_{45} \tilde{25}_H \tilde{45}_H + a_H \tilde{25}_H \tilde{45}_H \tilde{24}_H \]  

(16)

Notice the simplicity of the model. Unfortunately, the scalar sector of the non-supersymmetric grand unified theory proposed in reference \( \text{(5)} \) is not very simple since there are many possible interactions between \( \tilde{5}_H, \tilde{24}_H \) and \( \tilde{45}_H \).

In this model there are several multiplets that mediate proton decay. We have the usual gauge \( d = 6 \) contributions, the Higgs \( d = 6 \) contributions, and the dimension five contributions. The most important proton decay contributions are mediated by the fields: \( \tilde{T}, \tilde{T}, \tilde{\Phi}_3, \tilde{\Phi}_3, \tilde{\Phi}_5, \tilde{\Phi}_0, \) and \( \tilde{\Phi}_6 \). Let us discuss the different LLL and RRRR contributions. The so-called LLL effective operators, \( Q \bar{Q} Q \bar{L} \), are generated once we integrate out the fields \( \tilde{T}, \tilde{T}, \tilde{\Phi}_3, \tilde{\Phi}_3, \tilde{\Phi}_5, \tilde{\Phi}_0, \) and \( \tilde{\Phi}_6 \). The RRRR contributions, \( \tilde{U}^C \tilde{E} \tilde{C} \tilde{D} \tilde{C} \), are due to the presence of the fields \( \tilde{T}, \tilde{T}, \tilde{\Phi}_5, \tilde{\Phi}_5, \tilde{\Phi}_0, \) and \( \tilde{\Phi}_6 \). In reference \( \text{(21)} \) we have discussed how to suppress those contributions in order to satisfy the proton decay bounds. See also reference \( \text{(24)} \) for an alternative way to suppress nucleon decay in this scenario.

4 Summary and Outlook

In this talk I have discussed two new renormalizable grand unified theories based on \( SU(5) \): we refer to these models as Renormalizable Adjoint \( SU(5) \) \( \text{(12)} \) and Supersymmetric Adjoint \( SU(5) \) \( \text{(21)} \). In both models it is possible to generate all fermion masses, including the neutrino masses, with the minimal number of Higgses. These theories predict one massless neutrino at the tree level and the leptogenesis mechanism can be realized. The neutrino masses are generated through the type I and type III seesaw mechanisms. The contributions to proton decay have been discussed in detail. The models presented in this talk can be considered as the simplest renormalizable (SUSY) grand unified theory based on the \( SU(5) \) gauge symmetry.

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