Mode Selection and Single-mode Lasing by Active Transformation Optics

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By linearizing about a multimode solution of the semiclassical lasing equation, we propose a simple and systematic method to achieve selective excitation of modes that would have a high threshold with spatially uniform pumping. The key of this method is incorporating the control of gain saturation by means of trial and error, and hence are computationally straightforward: by choosing a pump profile that overlaps strongly in space with the desired modes, these modes will have a better utilization of the pump power. If the desired modes have the highest quality ($Q$) factors, then their already low thresholds can be further reduced; if the desired modes have relatively low $Q$-factors (i.e., “higher-order” modes) and hence relatively high thresholds with uniform pumping, they can become the modes of the lowest thresholds, instead of the higher-$Q$ ones.

The motivation for this latter scenario can be, for example, a flexible scheme to switch the laser frequency and output direction via the spatial modulation of the pump power, or to enhance the output power via the excitation of optimally outcoupled modes [8]. We note that here the “order” of the modes does not refer to their frequencies or certain quantum number, but rather the sequence of lasing when the pump power is increased uniformly in the laser cavity.

This intuitive procedure to selectively excite the higher-order modes, however, works only when they overlap weakly in space with higher-$Q$ modes, such as in a deformed microcavity supporting distinct short periodic orbits [9] [10] or in a random laser in the localized regime [11] [12]. Otherwise the intuitively chosen pump profile still has a significant overlap with the non-targeted, higher-$Q$ modes and the latter lase before the target mode. In this mode overlapping regime, brutal-force optimizations of the spatial pump profile have been performed to select higher-order modes [5] [7], but they require many iterations of trial and error, and hence are computationally intense and non-ideal, especially for optical switching applications.

In this work we show that in the mode overlapping regime, a general method of mode selection, especially of the higher-order ones, can be found by active transformation optics, utilizing an arbitrary nonlinear multimode solution. More specifically, all nonlinear lasing modes, at a given pump power $D_0$ and with an arbitrary pump profile $f(\vec{r})$ (taken to be uniform for simplicity), can be treated as the linear solutions of the saturated gain media, the spatial distribution of which is denoted by $f_s(\vec{r}; D_0)$. With the original $f(\vec{r})$, these modes may have very different thresholds if the pump power $D_0$ is high, and the desired higher-order mode $\mu$ has very low output power. If we apply, instead of $f(\vec{r})$, a pump profile that matches the saturated gain profile $f_s(\vec{r}; D_0)$, then all the modes in the original nonlinear solution have exactly the same threshold, given by $D_0$ reduced by a common normalization factor. Thus the disadvantage of the desired higher-order mode $\mu$ is eliminated, and it will lase at the lowest threshold, together with the lowest-order modes and having much increased output power. This is the first step of our proposal. Furthermore, if we reduce the self-saturation of the desired higher-order mode $\mu$ and/or enhance it for the others in the transformed pump profile $f_s(\vec{r}; D_0)$, then the resulting new pump profile can further favor the desired mode and selectively excite it, leading to a significant pump range of single-mode excitation. This is the second step of our proposal.

We discuss this two-step method in detail using the steady-state ab-initio laser theory (SALT) [13] [14], which finds the steady-state solutions of the semiclassical laser equations with good accuracy and without lengthy time-dependent simulations [15] [16]. In a steady state, the electric field is multi-periodic in time, i.e.

$$E^+(\vec{r}, t) = \sum_{\mu=1}^{N} \Psi_\mu(\vec{r}) e^{-i\Omega_\mu t},$$  

where $N$ is the number of lasing modes. At a given pump power $D_0$, measured by the population inversion of the gain medium it creates in the absence of the electric field, the nonlinear lasing modes $\Psi_\mu(\vec{r})$ and the laser frequencies $\Omega_\mu$ can be obtained by solving the following set of coupled Helmholtz equations [14]

$$\{\nabla^2 + [\epsilon_r(\vec{r}) + \epsilon_g(\vec{r}; D_0)] \Omega^2_\mu \} \Psi_\mu(\vec{r}; D_0) = 0,$$

$$\left(\nabla^2 + [\epsilon_r(\vec{r}) + \epsilon_s(\vec{r}; D_0)] \Omega^2_\mu \right) \Psi_\mu(\vec{r}; D_0) = 0.$$
in which we have taken the speed of light in vacuum to be unity. \( \Psi_\mu(\vec{r}) \) here is dimensionless, measured in its natural units of \( \epsilon_c = \hbar / \sqrt{\gamma_\parallel \gamma_\perp} / 2g \), where \( \gamma_\parallel \) and \( \gamma_\perp \) are the inversion and polarization relaxation rates and \( g \) is the dipole matrix element between the energy levels of lasing transition.

\( \epsilon_c(\vec{r}) \) in Eq. (2) is the “passive” part of the cavity dielectric function, given by \( n_2^2(\vec{r}) \) in terms of the refractive index. \( \epsilon_s(\vec{r}; D_0) \) captures the “active” part of the effective dielectric function, i.e.

\[
\epsilon_s(\vec{r}; D_0) = \frac{\gamma_\perp}{\Omega_\mu - \omega_n + i\gamma_\perp} \frac{D_0 f(\vec{r})}{1 + \sum_{\nu=1}^N \Gamma_{\nu}|\Psi_\nu(\vec{r}; D_0)|^2}, \tag{3}
\]

which contains the nonlinear spatial hole-burning interactions exactly. \( \omega_n \) here is the atomic transition frequency, \( \Gamma_{\nu} = \gamma_\perp^2 / [\gamma_\perp^2 + (\Omega_\nu - \omega_n)^2] \) is the Lorentzian gain curve evaluated at the lasing frequency \( \Omega_\nu \), and \( f(\vec{r}) \geq 0 \) is the spatial pump profile, which is normalized by \( \int_{\text{cavity}} f(\vec{r}) d\vec{r} = S \), where \( S = \int_{\text{cavity}} d\vec{r} \) is the length (area) of the cavity in one (two) dimension.

To select a certain high-order mode \( \mu \), the pump profile \( f(\vec{r}) \) needs to make its threshold the lowest among all possible lasing modes. Instead of comparing their actual thresholds \( D_{0,\text{int}}^{(\mu)} \) in which the nonlinearity interactions plays an important role, it is more convenient to work with the noninteracting thresholds \( D_0^{(\mu)} \), defined by

\[
\left[ \nabla^2 + \left( \epsilon_c(\vec{r}) + \frac{\gamma_\perp D_0^{(\mu)} f(\vec{r})}{\Omega_\mu - \omega_n + i\gamma_\perp} \right) \Omega_\mu \right] \Psi_\mu(\vec{r}; D_0) = 0. \tag{4}
\]

They serve our purpose just as well, since at the lowest threshold the semiclassical laser intensity is zero and \( D_0^{(\mu)} = D_{0,\text{int}}^{(\mu)} \). Unless a mode is very lossy, the reduction of its threshold \( D_0^{(\mu)} \) due to a non-uniform \( f(\vec{r}) \) is given approximately by the pump overlapping factor \( s \)

\[
r_\mu = \frac{\int_{\text{cavity}} f(\vec{r})|\Psi_\mu(\vec{r}; D_0)|^2 d\vec{r}}{\int_{\text{cavity}} |\Psi_\mu(\vec{r}; D_0)|^2 d\vec{r}}, \tag{5}
\]

which becomes one for uniform pumping by definition (i.e., \( r_\mu = 1 \) for \( f(\vec{r}) = 1 \)). Suppose that there are two modes \( \mu, \nu \) with distinct spatial profiles and that mode \( \mu \) has a higher threshold with uniform pumping. By focusing the pump on the intensive part(s) of mode \( \mu \) (for example, with \( f(\vec{r}) \propto |\Psi_\mu(\vec{r}; D_0)|^2 \)), \( r_\mu \) can become much larger than 1 while \( r_\nu \) necessarily becomes much less than 1, which can then invert their order of lasing and achieve the selective excitation of the higher-order mode \( \mu \). When modes \( \mu, \nu \) overlap strongly in space however, usually one finds \( r_\mu \sim r_\nu \) for a given pump profile, and \( D^{(\mu)}_0 \rightarrow D^{(\mu)}_0 / r_\mu \) is still higher than \( D^{(\nu)}_0 \rightarrow D^{(\nu)}_0 / r_\nu \), i.e., the non-targeted mode \( \nu \) is still the first one to lase when the pump power is increased. Optimizations based on the generic algorithm have been employed to find a suitable pump profile to excite higher-order modes in this regime \( \text{[5][7]} \), which however requires many iterations of trial and error and is hence computationally expensive and inconvenient for real-time applications.

In contrast, our method is deterministic and hence very efficient. To understand how it works, we first note that the nonlinear equation (2) is equivalent to the linear equation (4) by the transformation

\[
f(\vec{r}) \rightarrow f_s(\vec{r}; D_0) = \frac{f(\vec{r})}{1 + \sum_{\nu=1}^N \Gamma_{\nu}|\Psi_\nu(\vec{r}; D_0)|^2}. \tag{6}
\]

Since this transformation is about the active part of the dielectric function, it can be viewed as an example of active transformation optics. Eq. (6) can be applied to all nonlinear lasing modes at \( D_0 \) with the original pump profile \( f(\vec{r}) \) (which we simply choose to be uniform), and it is straightforward to see that they all have exactly the same threshold

\[
D_0^{(\mu)} = \frac{\int_{\text{cavity}} f_s(\vec{r}; D_0) d\vec{r}}{S} D_0 \tag{7}
\]

after the transformation (6). We have used the tilde to distinguish the thresholds with the transformed pump profile \( f_s(\vec{r}; D_0) \) from their values with the original pump profile \( f(\vec{r}) \). Since \( \int_{\text{cavity}} f(\vec{r}) d\vec{r} = S \) and \( f_s(\vec{r}; D_0) \leq f(\vec{r}) \), we find that \( D_0^{(\mu)} \) given by Eq. (7) is always lower than \( D_0 \), the pump power at which the transformation is performed. This is the first step in our approach, which levels up the threshold of the mode to be selected with all the lower-order modes, and hence eliminates its disadvantage due to its lower \( Q \)-factor.

We note that before the transformation (6) is applied, the uniformly pumped laser is at a pump power above the lowest threshold, and the gain distribution is saturated by the finite amplitudes of the lasing mode; after the transformation with the new pump profile, the laser is at its lowest threshold, with unsaturated gain and zero amplitudes for all lasing modes. The intensities of the lasing modes in the transformation (6) are those of the original nonlinear solution and can be modified in the transformation freely, which we will exploit in the next step; they are not the intensities of the modes with the new pump profile, which are determined physically by nonlinearity.

In the second (and final) step, we modify \( f_s(\vec{r}; D_0) \) given by Eq. (6) such that it favors the target mode \( \mu \). We will refer to the resulting pump profile as \( f_\mu(\vec{r}; D_0) \), and it can be chosen, for example, by increasing the self-saturation of the non-targeted modes in Eq. (6), i.e., by increasing the amplitudes of \( |\Psi_\nu| \neq |\Psi_\mu| \) \( (\text{"approach 1"}) \), which further suppresses these modes. Admittedly, this procedure may also increase the threshold the target mode \( \mu \) due to cross saturation, but as we will show, its effect is much less dramatic than the increased self-saturation. Another option ("approach 2") is to reduce
FIG. 1: (Color online) Threshold analysis in a 1D cavity before and after applying active transformation optics. Open and filled dots show the actual thresholds of mode 1 and 2 with uniform pumping. The leftmost data points on the solid and dashed lines show their leveled thresholds after the transformation $f_2(\vec{r}; D_0)$ at $D_0 = 1.88D_0^{(1)}$ in step 1. In step 2, we either (a) suppress mode 1 (red dashed line) by increasing its intensity in the original spatial hole burning interactions up to 3 times, or (b) favor mode 2 (black solid line) by decreasing its intensity (which is about 1/17 of that of mode 1) to negative 28 times. In (c) both approaches are combined. (d) Mode 2 is not selected using a naive attempt of pump focusing described in the text. Inset in (a): The cavity has refractive index $n_e = 3$ and a perfect mirror on the left side. The gain medium is characterized by $\omega_aL = 20$ and $\gamma_L = 2$.

The self-saturation of the target mode. Since this intensity can be freely set in the transformation as mentioned, we can even make it negative (as long as the pump profile $f_m(\vec{r}; D_0)$ is still non-negative everywhere), resulting in “spatial peak creation” that resembles $|\Psi_1(\vec{r}; D_0)|^2$ [see Fig. 2(a)], instead of “spatial hole burning” in which the gain is reduced at the intensity peaks of the lasing modes in space. To achieve the best result and extend the pump range of single-mode operation of the target mode, a combination of approach 1 and 2 can be applied.

We note that the first step of our proposal can be viewed as a special case of the second step, in which not only the self-saturation of the non-targeted modes but also that of the target mode $\mu$ are increased from zero in the uniform pump profile. Although the latter is not ideal and can be improved using the approach 2 in step 2, the fact that the very different thresholds of all the modes in the original nonlinear solution level up after the first step is already a confirmation of the effectiveness of our approach, the key of which is incorporating the control of gain saturation into the pump profile.

In Fig. 1(a) and (b) we compare approach 1 and 2 for a one-dimensional (1D) slab laser of length $L$. With uniform pumping, the frequency of the first mode is $\Omega_1L \simeq 20.5$ at its threshold $D_0^{(1)}$, and we want to select the second mode with $\Omega_2L \simeq 18.9$, the actual threshold of which is $1.78D_0^{(1)}$. We perform the transformation $f_2(\vec{r}; D_0)$ at a slightly higher pump power $D_0 = 1.88D_0^{(1)}$, which levels up the thresholds of mode 1 and 2 [see the leftmost data points in Fig. 1(a) and (b)]. We then modify this $f_2(\vec{r}; D_0)$ by gradually increasing the intensity of mode 1 in it [see Fig. 1(a)] or decreasing the intensity of mode 2 in it [see Fig. 1(b)]. Both approaches can create a large enough difference between $\tilde{D}_0^{(2)}$ and $\tilde{D}_0^{(1)}$, which is required for an extended pump range of single-mode operation for the target mode 2 (see Fig. 2 for example). We find that approach 2 is more favorable, since it leads to a threshold that is even lower than the lowest threshold (of mode 1) with uniform pumping. Fig. 1(c) shows the combination of approach 1 and 2 in (a) and (b). Although the resulting threshold of mode 2 is higher than that in approach 2, it creates the largest difference between the noninteracting thresholds of the first two modes, which is beneficial to extend the pump range of single-mode operation. In all three cases modes other than 1 and 2 have higher thresholds and can be safely neglected [see also Fig. 2(d)].

If one attempts a naive choice of pump focusing in this

FIG. 2: (Color online) Reduced threshold and single-mode lasing using active transformation optics. (a) Pump profile $f_2(\vec{r})$ (purple thin solid line) that corresponds to the rightmost data in Fig. 1(b). Also shown are the normalized mode profiles $|\Psi_1(\vec{r})|^2$ (red dashed line) and $|\Psi_2(\vec{r})|^2$ (black solid line) at $D_0 = 1.88D_0^{(1)}$ with uniform pumping. (b) Intensities at the right end of the cavity. With $f_2(\vec{r})$ in (a), the target mode 2 (black solid line) is the only lasing mode in the pump range shown. Red dashed line and black squares show the intensities of mode 1 and 2 with uniform pumping, respectively. (c) shows the frequencies of the lasing modes in (b), and the left end of each curve marks its threshold. (d) Modal gains for the first four modes with $f_2(\vec{r})$ in (a).
example, the desired mode 2 cannot be selected exclusively. For example, with $f_2(\vec{r}) \propto |\Psi_2(\vec{r})|^2$, the threshold of mode 2 is reduced by 32%, but it is still slightly (1.2%) higher than that of mode 1. Even with the help of step 1, for example, by using $f_2(\vec{r}; D_0) = f_s(\vec{r}; D_0)(1 + \alpha|\Psi_2(\vec{r})|^2)$ ($\alpha > 0$) at $D_0 = 1.88D_0^{(1)}$, the resulting threshold changes of modes 1 and 2 are almost identical [see Fig. 1(d)], and they lase almost simultaneously above the lowest threshold.

Henceforth we drop the $D_0$-dependence of $f_6(\vec{r})$ to avoid confusion between (i) the power of uniform pumping at which the transformation (6) is performed and (ii) the pump power of this transformed pump profile when we study the nonlinear behaviors of the selected mode. In Fig. 2 we compare the nonlinear lasing solutions with uniform pumping and with $f_2(\vec{r})$ that corresponds to the rightmost data in Fig. 1(b). Not only is the threshold of the target mode 2 reduced to 0.77$D_0^{(1)}$ with this $f_2(\vec{r})$, mode 2 is also the only lasing mode in the whole pump range shown in Fig. 2(b). The latter observation can be confirmed by calculating the modal gain [13]: a mode becomes lasing if its modal gain reaches unity from below, which then stays at unity unless the mode is killed [13, 14]. Indeed all the non-targeted modes have a modal gain below unity in this pump range, as shown in Fig. 2(d). We also note that the power slope of mode 2 with $f_2(\vec{r})$ is higher than that of both mode 1 and 2 with uniform pumping.

Having demonstrated the principles of the active transformation optics for mode selection in the simple 1D slab cavity, next we tackle a more complicated laser, the two-dimensional (2D) diffusive random laser [13]. As Fig. 3(a) shows, there are six modes lasing at $D_0 = 1.6D_0^{(1)}$ in this example, and we will target the sixth mode with frequency $\Omega_0 R \approx 30.30$ to test the robustness of our method. We perform the transformation (6) at this pump power, after which all the six lasing modes have the same threshold [the leftmost data points in Fig. 3(b)]. Next we follow approach 2 when modifying $f_6(\vec{r})$, by decreasing the intensity of mode 6 in the spatial hole burning interactions to −10 times. As a result, the threshold of the target mode 6 is reduced to below $D_0^{(1)}$ and significantly lower than the other five modes. If we choose $f_6(\vec{r})$ that corresponds to the rightmost data points in Fig. 3(b), the target mode 6 becomes the only lasing mode until the pump power is 35% above $D_0^{(1)}$ [Fig. 3(c)], with slightly shifted frequency and more than ten-fold power increase [Fig. 3(d)], while mode 1 is suppressed in the pump range shown.

The same procedure has also been applied to select mode 2 to 5, and they all yield a significant range of single-mode operation for the target mode. These results highlight the generality of mode selection based on active transformation optics, which should apply to all lasers with strong spatial hole burning interactions, as well as other nonlinear media such as exciton-polariton condensates [19, 22]. As for experimental realizations, the pump profile can be shaped via a spatial light modulator [3, 23] for optically pumped lasers and a pixelated contact for electrically pumped lasers. We thank Hui Cao, Douglas Stone, Seng Fatt Liew, and Hakan Türeci for helpful discussions. This project is supported by PSC-CUNY 45 Research Grant.

FIG. 3: (Color online) Mode selection by active transformation optics in a 2D diffusive random laser. (a) Intracavity intensity for the first six modes with uniform pumping. The black line shows the sixth mode to be selected. Inset: The system is modeled as a disk region of radius $R$ containing random scatterers of refractive index $n = 1.2$ and a background index $n = 1$. The gain medium is characterized by $\omega_n R = 30$ and $\gamma_L R = 2$. (b) Noninteracting thresholds of all six modes in (a) after the transformation (6) (the leftmost data points) and decreasing the intensity of the target mode 6 in the spatial hole burning interactions. Inset: False-color intensity plot of mode 6. (c) Same as (a) but with the pump profile $f_6(\vec{r})$ shown in the inset [the rightmost data points in (b)]. (d) Spectra at $D_0 = 1.6D_0^{(1)}$ with uniform pumping (upper panel) and $f_6(\vec{r})$ in (c) (lower panel).

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