Chiral symmetry and open-charm mesons\textsuperscript{1}

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Abstract
Pseudo-scalar and scalar $D$ mesons are considered within the QCD sum rule approach. We present an analysis of the mass splitting of the pseudo-scalar $D - \bar{D}$ mesons and the relation to QCD condensates. Weinberg type sum rules are derived for chiral partners which highlight the role of the chiral condensate.

1. Introduction
Chiral symmetry and its breaking pattern represents an important aspect of strong interaction physics governing to a large extent the structure of hadrons. The spontaneous breaking of chiral symmetry is signalled by the large value of the chiral condensate, $\langle \bar{q}q \rangle_0$, which is one quantity describing the QCD vacuum. There is, furthermore, the breaking of the dilatation invariance by quantum fluctuations leading to the non-zero value of the gluon condensate. Further nonzero vacuum expectation values of QCD operators characterize, among other quantities, the QCD ground state and are of relevance for the hadron spectrum. At nonzero density and temperature these condensates may change and modify the hadronic excitations. For instance, at baryon density $n$ and small temperature, the chiral condensate behaves as $\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 (1 - \frac{\sigma_N n}{(m_u + m_d) \langle \bar{q}q \rangle_0})$ with $\sigma_N$ being the nucleon sigma term. Accordingly, one is looking for observables being sensitive to ”chiral restoration” meaning effects indicating the drop of $\langle \bar{q}q \rangle$ with increasing density.

\textsuperscript{1}Dedicated to the late Prof. Dr. I. Iori.
Various experiments have been performed with the goal to seek for modifications of hadron properties and to assign them to changes of the QCD vacuum. Among currently running experiments are those by the HADES collaboration \cite{1} investigating the dilepton emissivity of strongly interacting matter which is suspected to have some relation to chiral symmetry \cite{2}. It happens, however, that the relation is not as direct as earlier conjectured \cite{3, 4, 5}. One may look, therefore, for other potential observables with more direct relations to the chiral condensate. In \cite{6}, open charm mesons are envisaged as suitable hadrons depending strongly on the chiral condensate. Following this line of arguments we consider here QCD sum rules for pseudoscalar $D$ mesons (section 2) and address briefly sum rules for chiral partners (section 3). Such considerations are related to the charm programmes at the forthcoming experiments CBM \cite{7} and PANDA \cite{8} at FAIR.

2. Spectral moments for $D - \bar{D}$ mesons

QCD sum rules \cite{9} represent a direct tool to relate moments of hadronic spectral functions with QCD condensates via the operator product expansion (OPE). In \cite{10}, the following moments have been defined

$$S_n(M^2) \equiv \int_{s^-}^{s^+} d s \, s^n \Pi(s) e^{-s^2/M^2},$$

where $\Pi$ denotes the hadronic spectral function and $s^\pm_0$ are the so-called continuum thresholds bracketing the $D$ and $\bar{D}$ strengths. Thereby, new quantities $\Delta m$ and $m$ may be defined which encode the combined mass-width properties of the particles under consideration:

$$\Delta m \equiv \frac{1}{2} \frac{S_1 S_2 - S_0 S_3}{S_1^2 - S_0 S_2}, \quad m \equiv -\frac{S_2}{S_1} - \frac{S_3}{S_2},$$

and $m^2 = \Delta m^2 + m_+ m_-$. For the often employed pole + continuum ansatz, these quantities allow for an interpretation as mass splitting $2\Delta m$ and mass centroid $m$.

Employing the current operators $j_{D^+} = i\bar{d}\gamma_5 c$ and $j_{D^-} = j_D^+ = i\bar{c}\gamma_5 d$, we obtain for the OPE evaluation of the current-current correlator $\Pi(q) = i \int d^4x \, e^{iqx} \langle \Omega | [j(x)j(0)] | \Omega \rangle$ up to mass dimension 5, in the rest frame of nuclear matter $v = (1, \vec{0})$ ($v$ stands for the medium four-velocity), in the limit of a light $d$ quark mass $m_d \to 0$, for sufficiently large charm-quark pole mass $m_c$, and for the $D$ mesons at rest the infrared-safe result \cite{10} for the
even \((e)\) and odd \((o)\) Borel transformed correlators

\[
\tilde{\Pi}^e(M^2) = \frac{1}{\pi} \int_{m_c^2}^{\infty} ds \ e^{-s/M^2} \text{Im} \Pi_{D^+}^{per}(s, \vec{q} = 0)
\]

\[
+ e^{-m_c^2/M^2} \left\{ -m_c \langle \bar{d}d \rangle + \frac{1}{2} \left( -\frac{m_c^3}{2M^4} + \frac{m_c}{M^2} \right) \langle \bar{d}g\sigma G d \rangle + \frac{1}{12} \frac{\alpha_s}{\pi} G^2 \right\} \\
+ \left[ \left( \frac{7}{18} + \frac{1}{3} \ln \frac{\mu^2 m_c^2}{M^4} - \frac{2}{3} \frac{M^4}{3 M^2} \right) \left( \frac{m_c^2}{M^2} - 1 \right) - \frac{2 m_c^2}{3 M^2} \right] \frac{\alpha_s}{\pi} \left( \frac{(v G)^2}{v^2} - \frac{G^2}{4} \right) \\
+ 2 \left( \frac{m_c^2}{M^2} - 1 \right) \langle d^i D_0^i d \rangle + 4 \left( \frac{m_c^3}{2M^4} - \frac{m_c}{M^2} \right) \left[ \langle \bar{d}D_0^2 d \rangle - \frac{1}{8} \langle \bar{d}g\sigma G d \rangle \right] \right],
\]

\[
\tilde{\Pi}^o(M^2) = e^{-m_c^2/M^2} \left\{ \langle \bar{d}^i d \rangle - \frac{2m_c^2}{M^4} \langle d^i D_0^i d \rangle - \frac{4}{M^2} \langle d^i D_0^2 d \rangle - \frac{\langle \bar{d}^i g\sigma G d \rangle}{M^2} \right\},
\]

where \(\alpha_s = g^2/4\pi\) and the perturbative spectral function \(\text{Im} \Pi_{D^+}^{per}(s)\) is according to [11]. We stress the occurrence of the term \(m_c \langle \bar{d}d \rangle\), where the large charm-quark mass acts as an amplifier of the genuine chiral condensate \(\langle \bar{d}d \rangle\).

Results of the numerical analysis of (2) with \(S_0 = \tilde{\Pi}^o\), \(S_1 = \tilde{\Pi}^e\) etc. are exhibited in Fig. 1 for condensates listed in [10] with the exception of the poorly known condensate \(\langle q^i g\sigma G q \rangle\) for which we employ here the opposite sign to get a measure of its importance. (This sign change also affects \(\langle q^i D_0^2 q \rangle\) due to \(\frac{1}{12} \langle q^i g\sigma G q \rangle - \langle q^i D_0^2 q \rangle \approx 0.031 \text{ GeV}^2\).) While the mass splitting is fairly robust, the centroid mass shift is fragile under variation of the continuum threshold parameters. The actual value of the mass splitting indeed depends sensitively on the sign of \(\langle q^i g\sigma G q \rangle\); the other condensates in (3, 4) (cf. [10]) are fairly known.

3. Chiral partners with open charm

With the same technique one may derive difference QCD sum rules for pseudo-scalar \((P)\) and scalar \((S)\) chiral partners, say \(D(J^P = 0^-)_{1864}\) and \(D(J^P = 0^+)_{2400}\), with the result

\[
\frac{1}{\pi} \int_{-\infty}^{\infty} ds \ s^i \left[ \Pi_{P-S}(s) \right] = \langle O_i \rangle
\]

with \(\langle O_i \rangle = -2m_c \langle \bar{q}q \rangle\) for \(i = 1\), \(-2m_c^3 \langle \bar{q}q \rangle + m_c \langle \bar{q}g\sigma G q \rangle\) for \(i = 3\) and \(-m_c^5 \langle \bar{q}q \rangle + \frac{3}{2} m_c^3 \langle \bar{q}g\sigma G q \rangle + \cdots\) for \(i = 5\), where \(\cdots\) stand for condensates of mass dimension 7 and their Wilson coefficients. These Weinberg type QCD sum rules clearly exhibit how the condensates drive the difference between moments of spectral functions.
4. Summary

In summary we have considered the QCD sum rule approach to pseudo-scalar and scalar mesons composed of a light and heavy quark. For the chiral sum rules the role of the chiral condensate is highlighted: Dropping in-medium condensates, in particular $\langle q\bar{q}\rangle$, cause the degeneracy of the spectral moments of chiral partners.

Acknowledgements: The work is supported by BMBF 06DR136 and GSI-FE.

References

[1] HADES Collaboration (G. Agakishiev et al.), Phys. Rev. Lett. 98 (2007) 052302, Phys. Lett. B 663 (2008) 43, arXiv:0902.3478 [nucl-ex].

[2] R. Rapp et al., Adv. Nucl. Phys. 25 (2000) 1; arXiv:0901.3289 [hep-ph].

[3] G.E. Brown, M. Rho, nucl-th/0509002, nucl-th/0509001

[4] T. Hatsuda, S.-H. Lee, Phys. Rev. C 46 (1992) 34.

[5] R. Thomas, S. Zschocke, B. Kämpfer, Phys. Rev. Lett. 95 (2005) 23230.

[6] R. Thomas, T. Hilger, B. Kämpfer, Prog. Part. Nucl. Phys. 61 (2008) 297.

[7] http://www.gsi.de/fair/experiments/CBM/index_e.html

[8] http://www-panda.gsi.de/auto/phy_home.htm

[9] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 385.

[10] T. Hilger, R. Thomas, B. Kämpfer, Phys. Rev. C 79 (2009) 025202.

[11] T.M. Aliev, V.L. Eletsky, Sov. J. Nucl. Phys. 38 (1983) 936;
S. Narison, QCD as a theory of hadrons, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology 17, Cambridge 2004.