Cranked shell model and isospin symmetry near $N = Z$

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A cranked shell model approach for the description of rotational bands in $N \approx Z$ nuclei is formulated. The isovector neutron-proton pairing is taken into account explicitly. The concept of spontaneous breaking and subsequent restoration of the isospin symmetry turns out to be crucial. The general rules to construct the near yrast-spectra for rotating nuclei are presented. For the model case of particles in a $j$-shell, it is shown that excitation spectra and the alignment processes are well described as compared to the exact shell model calculation. Realistic cranked shell model calculations are able to describe the experimental spectra of $^{75,74}$Kr and $^{81}$Rb isotopes.

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I. INTRODUCTION

The classification of rotational bands as quasiparticle configurations in a rotating mean-field has led to an understanding of the yrast- spectra of rapidly rotating nuclei. This popular approach referred to as the cranked shell model (CSM) has been employed quite extensively and is reviewed in refs. [3,4]. These high-spin studies have provided new insights into the nature of the pair-correlations among identical particles. The CSM in its traditional form assumes that there are no proton-neutron (nn-) pair correlations. Modern γ-ray detector arrays will allow to study high-spin states for nuclei near the $N = Z$ line in the mass $\sim 50$ and $80$ regions and these possibilities will be greatly enhanced with the availability of the radioactive beams. For these nuclei strong pn-pair correlations are expected. By studying the rotational bands one may obtain new information about the pn-pairing in deformed nuclei. For this purpose, it is necessary to reformulate the CSM in such a way that the pn-pairing is included.

The main motivation of the present work is to develop a quasiparticle CSM approach near $N = Z$ which includes the pn-pairing effects. As will be shown, the isospin symmetry plays a central role, permitting to derive the basic structure of the rotational spectra in terms of quasiparticle configurations in a $T = 1$ pair-field. General rules for constructing the quasiparticle excitation spectra in the presence of np-pairing will be provided. We shall study a model problem of protons and neutrons in a deformed $f_{7/2}$ shell interacting with a $\delta$-force, which is solved by exact numerical diagonalization. The suggested CSM approach will be tested using these exact results. We have used the $f_{7/2}$ shell for the simplicity in carrying out the exact deformed shell model (SM) calculations, expecting the CSM predictions to improve in larger shells as the mean-field becomes more dominant with increasing number of particles.

It will be demonstrated that the suggested CSM with good isospin can be cast into a form that permits calculations by means of a conventional CSM approach. Taking advantage of this, the yrast-spectra of even-even, odd-odd $N = Z$ and odd-$A$ $N = Z \pm 1$ nuclei near $A = 72$ are constructed and compared with the experimental data. The meager experimental information available is consistent with the suggested dominance of a $T = 1$ monopole pn-pair field.

II. DEFORMED SHELL MODEL

As a case study for the CSM to be developed, we use the deformed shell model hamiltonian which consists of a cranked deformed one-body term, $h'$ and a scalar two-body delta-interaction $V$. The one-body term is the familiar cranked Nilsson mean-field potential which takes into account of the long range part of the nucleon-nucleon interaction. The residual short range interaction is specified by the delta-interaction. In many of the high spin studies, the residual interaction is in the form of a monopole interaction. However, it has been demonstrated that the higher multipoles can be important. Hence, in our model study we consider all possible multipole components of the delta-interaction and also all the possible interaction terms, proton-proton (pp), neutron-neutron (nn) and neutron-proton (np). The SM hamiltonian employed is given by

$$H' = h' - g\beta(\hat{r}_1 - \hat{r}_2)$$

where,

$$h' = -4\kappa\sqrt{4\pi}Y_{20} - \omega J_x.$$  \hspace{1cm} (2)

We use $G = g \int R_{J\alpha}^4 r^2 dr$ as our energy unit and the deformation energy $\kappa$ is related to the deformation parameter $\beta$. For the case of $f_{7/2}$ shell, $\kappa = 1.75$ approximately corresponds to $\beta = 0.16$. In order to solve the eigenvalue problem exactly, we are limited to a small configuration space. As in the previous work, the model space in the present analysis consists of a single $j$ shell. We have diagonalized the hamiltonian exactly for neutrons and protons in the $f_{7/2}$ shell. As the strengths of nn-, pp-
and np-parts are identical, the hamiltonian is invariant with respect to rotations in isospace, i.e.
\[ RH'R^{-1} = H', \tag{3} \]
where \( R \) defines a rotation in isospace, generated by the isospin operators \( T_x, T_y \) and \( T_z \). Furthermore, the hamiltonian \( (1) \) is invariant with respect to a spatial rotation about the x-axis by an angle of \( \pi \). As a consequence, the signature \( \alpha \) is a good quantum number [4], which implies that the SM solutions represent states with the angular-momentum \( I = \alpha + 2n \) \((n \text{ integer})\).

III. MEAN-FIELD APPROACH

As mentioned in the introduction, the motivation of the present work is to develop a mean-field approach in the presence of the pn-pairing. The mean-field CSM approach in the case of identical particles has been quite successful to describe the rotational properties of medium and heavy mass nuclei. In the simplest form of this approach, the mean-fields in the form of the pair-gap and the deformation are held fixed as a function of the rotational frequency. In a fully selfconsistent Hartree-Fock-Bogolubov (HFB) calculation, the mean-field (most important the pair-field) often shows rapid changes as a function of rotational frequency that are due to the broken symmetries in the intrinsic frame of reference. The rapid changes are smeared out by projecting out the wavefunction with good quantum numbers (most important the particle number). In the CSM approach the problem of the “phase transition” is avoided, because the mean-fields are held fixed. This leads, often, to a better description than a selfconsistent HFB calculation [13]. However, we would like to caution here that this approach has obvious limitations. It holds below a phase transition and permits a fair description of the lower part of the extended transition region. With the appropriate choice of the mean-field, the same is true for the upper region. Thus used with care, the CSM is a robust and a simple tool to analyze the near-yrast spectra.

In subsection II A, we briefly present the HFB approach in the presence of the pn-pairing. There is an extensive literature on this subject that has been reviewed in ref. [14], to which we refer concerning the previous work. The HFB equations are solved selfconsistently in subsection II B. It will be found that for low-frequency, \( \omega \) the pair-field has isovector character. Using this result and other studies [13,15], which also suggest the dominance of isovector pairing at low-\( \omega \), we proceed with the assumption that for realistic nuclei above mass=40, the pair-field has isovector character. The CSM for such a scenario is developed in subsections II C and II D, where the general rules for constructing the quasiparticle spectra are given.

A. Hartree-Fock-Bogolubov equations

In the development of the CSM with np-pairing, we have employed the HFB method, details of which can be found, for instance, in refs. [14,19]. The HFB equations are given by
\[ \mathcal{H}' \left( \begin{array}{c} U \\ V \end{array} \right) = e'_i \left( \begin{array}{c} U \\ V \end{array} \right) \tag{4} \]
where
\[ \mathcal{H}' = \left( \begin{array}{cc} h'_{ij} + \Gamma_{ij} & -(\lambda + \lambda_{\tau_1})\delta_{ij} \\ -\Delta_{ij} & -h'_{ij} - \Gamma_{ij} + (\lambda + \lambda_{\tau_1})\delta_{ij} \end{array} \right) \tag{5} \]
\[ \Gamma_{ij} = \sum_{kl}(ik|j\rangle \rho_{kl}, \tag{6} \]
\[ \Delta_{ij} = \frac{1}{2} \sum_{kl}(ij|k\rangle \delta_{kl} t_{kl}, \tag{7} \]
\[ \rho = V^*V^T, \tag{8} \]
\[ t = V^*U^T. \tag{9} \]
The quantities in the brackets \( \langle ij \rangle \) in \( (6) \) and \( (7) \) are the antisymmetric uncoupled matrix elements of the interaction. In Eq. (5), we have introduced the isospin label \( \tau = 1, -1 \) for neutrons and protons, respectively and rearranged the chemical potentials \( \lambda_n \) and \( \lambda_p \), which constrain \( N \) and \( Z \), into \( \lambda = (\lambda_n + \lambda_p)/2 \) and \( \lambda_{\tau} = \lambda_n - \lambda_p \) which fix mass \( A \) and the isospin projection \( T_\tau \), respectively. The HFB solutions are obtained by solving the equations (5) - (9) selfconsistently.

The pairs of states \( \{ij\} \) that define the pair-field (6) can be rewritten in a coupled representation as \( \{t, t_z, \alpha, \beta\} \), which explicitly indicates the isospin \( t \) and \( t_z \) and \( \alpha, \beta \) denote all other quantum numbers except the isospin. For the single-\( j \) shell model, \( \alpha, \beta = J, M \), where \( J, M \) are the angular-momentum and its projection, which are sufficient to fix the quantum numbers. If \( t = 0 \), the pair-field is an isoscalar and for \( (t = 1, t_z) \) it is an isovector. The pp-pair-field has \( (t = 1, t_z = 1) \) and the np- has \( (t = 1, t_z = 0) \). There are two pn-pair-fields with \( (t = 1, t_z = 0) \) and \( (t = 0, t_z = 0) \). We use the lower case letters \( t \) and \( t_z \) for the isospin of the pair-field in order to avoid confusion with the total isospin of the states, which we denote by \( T \) and \( T_\tau \).

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No symmetry has been imposed on the HFB wavefunction since it is known that this may lead to exclusion of particular correlations [4].
B. HFB solution for the $j$-shell

We find that the pair-field is either $t = 1$ or $t = 0$. This coincides with earlier results of refs. [2,14], where in contrast to the present work, additional symmetries were imposed. Hence, the mutual exclusiveness of the $t = 1$ and $t = 0$ pair-fields is not a consequence of these symmetries. The lack of a mixed phase can be interpreted qualitatively in the following way: Since the interaction conserves isospin, it scatters pairs only into pairs with the same $t$. The correlation energy is of the order of $n^2$, with $n$ being the number of states the pairs can be scattered into. Since the $t = 1$ pairs block partially the $t = 0$ phase space, and vice versa, the pure field has a larger correlation energy.

For the ($N = Z = 4$) system we find a $t = 1$ pair-field at low-rotational frequency $\omega$. Fig. 1 shows that there is a rapid alignment at $\omega = 0.45G$, where the self consistent solution changes to a $t = 0$ pair-field. For the ($N = Z = 3$) system we also find a $t = 1$ pair-field at low- $\omega$, which changes to a $t = 0$ field at a higher frequency. The change is associated with the crossing of the $T = 1$ g-band with an aligned $T = 0$ odd-spin band, which will be discussed below. The change from a $t = 1$ to a $t = 0$ field has also been found in the HFB calculation of ref. [14] when changing the $t = 0$ strength of the np-interaction. In the case of the $t = 1$ solution, the $J \neq 0$ fields are found to be small as compared to the monopole pair-field, $J = 0$. In the remainder of the paper, we shall consider the various aspects of the $t = 1$ HFB solutions. The $t = 0$ solutions have been studied separately [24,25]. We only mention that the $J = 1$ and 7 components of the pair-fields dominate [24].

As compared to the SM calculation, the fully selfconsistent HFB solution shows a much too early alignment and has a wrong behavior after the crossing. This is related to the fact that $t = 1$ pair-field vanishes at the crossing point. The crossings frequency in even-even nuclei is determined by competition between the $t = 1$ pair-field and the Coriolis force. In the exact calculations, the $t = 1$ pair-correlations persist to very high frequencies [24].

C. Spontaneous breaking of the isospin symmetry by the $t = 1$ pair-field

Before discussing the symmetry breaking by the isovector pair-field, it is useful to state the familiar case of spontaneous breaking of the spatial isotropy by a mean-field solution with a deformed density distribution (c.f. ref. [10,21]). Since the two-body hamiltonian is isotropic, this symmetry is broken spontaneously. There is a family of mean-field solutions with the same energy which correspond to different orientations of the density distribution. All represent one and the same intrinsic quasiparticle configuration, which is not an eigenfunction of the total angular-momentum. Any of these solutions can be chosen as the intrinsic state. The principal axes of its density distribution define the body-fixed coordinate system. The states of good angular momentum are superpositions of these states of different orientations, the weight being given by the Wigner $D$-functions. Thus, the relative importance of the different orientations is fixed by restoring the angular momentum symmetry. At the simplest level of the cranking model, which is valid for sufficiently strong symmetry breaking, the energy of the good angular momentum state is given by the mean-field value.

Let us now consider a $t = 1$ HFB solution found for the $N = Z$ system (for example, the one discussed in the previous subsection). The $t = 1$ pair-field $\hat{\Delta}$ is a vector that points in a certain direction in isospace, breaking the isospin symmetry. Since the two body hamiltonian is isospin invariant, the symmetry is a spontaneously broken and all orientations of the isovector pair-field:

$$\Delta_{J,M,t=1} = \pm \Delta_{J,M,t=1} \sin \theta \exp \mp i\phi/\sqrt{2}$$

$$\Delta_{J,M,t=1} \sin \theta \exp \mp i\phi/\sqrt{2}$$

are equivalent. Fig. 2 illustrates this family of HFB solutions, the energy of which does not depend on the orientation of the pair-field. In particular, the cases of a pure pn-field ($\theta = 0$, z-axis) and pure pp- and nn-pair fields ($\theta = \pi/2$, $\phi = 0$, y-axis) represent the same intrinsic state. Hence, on the mean-field level the ratio between the strengths of pp-, nn- and pn-pair-fields is given by the orientation of the pair-field, which is not determined by the HFB procedure. The relative strengths of the three types of pair-correlations becomes only definite when the isospin symmetry is restored. The symmetry breaking by the isovector pair-field has been discussed before in [24,25], where references to earlier work can be found.

D. Intrinsic excitations with $T = 0$

Let us first discuss the states with total isospin $T = 0$. They are isotropic superpositions of all the orientations of the pair-field, which corresponds to an equal amount of pn-, pp- and nn-correlation energy.

Like in the case of spatial rotation, the intrinsic excitations are constructed from the quasiparticles (qps) belonging to one of the orientations of pair-field. We choose the $y$-direction, $\Delta_{nn} = \Delta_{pp}, \Delta_{np} = 0$. This is a particularly convenient choice because it permits to reduce the construction of the qp-excitation spectrum to the familiar case with no pn-pairing [4]. The choice of the qp operators is not unique [28]. We choose them to be pure quasineutrons or quasiprotons and denote their creation operators by $\beta_{t,\ell}^+ \epsilon \ell$, where $\ell$ indicates the isospin projection. They are pairwise degenerate, i.e. the qp
Only these linear combinations represent \( T = 0 \) two-qp excitations in the odd - odd system. The combinations with the plus sign must be discarded.

There is an alternative way to construct the \( T_y = 0 \) configurations of good number parity. We start with qp operators \( \alpha^+_{\pm, k} \) of good \( T_y \), which are given by the linear combinations

\[
\alpha^+_{\pm, k} = \frac{1}{\sqrt{2}} (\beta^+_{\pm, k} \pm i \beta^+_{- \pm, k}).
\]

Configurations of good number parity are constructed with the help of the relations

\[
e^{-i \pi N} \alpha^+_{\pm, k} e^{i \pi N} = \mp i \alpha^+_{\mp, k},
\]

\[
e^{-i \pi Z} \alpha^+_{\pm, k} e^{i \pi Z} = \pm i \alpha^+_{\mp, k}.
\]

The \( T_y = 0 \) excitation \( \alpha^+_{\pm, k} \alpha^+_{- \pm, k} \) is odd under (23). Thus, it represents a \( T = 0 \) state in the odd - odd nucleus. It is identical with \( \beta^+_{\pm, k} \beta^+_{- \pm, k} \). Using (23) one sees that

\[
e^{-i \pi N} \alpha^+_{\pm, k} e^{i \pi N} = \alpha^+_{\mp, k} \alpha^+_{- \pm, k}.
\]

and that the \( T_y = 0 \) excitations \( \frac{1}{\sqrt{2}} (\alpha^+_{\pm, k} \alpha^+_{- \pm, k} \pm \alpha^+_{\mp, k} \alpha^+_{- \pm, k} \mp \alpha^+_{\pm, k} \alpha^+_{\mp, k} \mp \alpha^+_{\mp, k} \alpha^+_{\pm, k}) \) are even and odd under (23), respectively. They are identical with the combinations appearing in the eqs. (15) and (21), as can be seen by means of direct evaluation using eqs. (22).

An important \( T_y = 0 \) four qp configurations in the even-even system is

\[
\alpha^+_{\pm, k} \alpha^+_{- \pm, k} \alpha^+_{\mp, k} \alpha^+_{- \pm, k} |0\rangle = \beta^+_{\pm, k} \beta^+_{- \pm, k} \beta^+_{\mp, k} \beta^+_{- \pm, k} |0\rangle,
\]

which appears in the double-alignment process.

It is noted that the number parity and \( T_y \) do not commute,

\[
e^{-i \pi N} T_y e^{i \pi N} = e^{-i \pi Z} T_y e^{i \pi Z} = - T_y.
\]

As a consequence of eq. (26), only the eigenvalue \( T_y = 0 \) is possible for states with good number parity.

E. States with \( T > 0 \)

The \( T > 0 \) bands are found by “cranking in isospace”, employing the analogy between angular momentum and
isospin. The “frequency” $\lambda_T$ is chosen such that \( \langle T_z \rangle = T \). The corresponding configurations are interpreted as the states with maximal projection $T_z = T$. The states with $T_z < T$ are the isobaric analogs, which are generated by multiple application of $T_-$ on the configuration \( [T, T_z = T] \), generated by cranking in isospace. They have the same energy as the state \( [T, T] \).

Since the isospin symmetry is broken spontaneously, any infinitesimal value $\lambda_T$ in the hamiltonian fixes the orientation of the pair-field perpendicular to the $z$-axis, i.e. $\Delta_{np} = 0$. This explains, why in previous HFB studies solutions with $\Delta_{np} \neq 0$ are found only for $\lambda_T = 0$.

The pn-pair-correlations appear via the isospin symmetry. The state \( [T, T] \) can be interpreted as the pair-field being oriented in the direction of no pn-correlation. The field executes zero-point oscillations around this equilibrium orientation that represent the pn-pair correlations, because any rotation away from the y-direction introduces a pn-component in $\Delta$. The amplitude of these oscillations quickly decreases with $T_z$ (or $\lambda_T$), which explains the rapid decrease of the pn-pair correlations found in the good isospin calculations of ref. [1]. The RPA theory of these oscillations has been worked out in refs. [22,28]. It is shown there that these oscillations contribute the term $\lambda_T/2$ to the energy. The appearance of this term is well known for spatial rotation, which obeys the same group SU$_2$. It corresponds to the familiar “frequency” $\lambda_T$, which corresponds to an exchange of a proton by a neutron, is zero in contrast to the case $T = 0$. Nevertheless, for small $T_z$ the amplitude of the oscillations of the pair-field is substantial. This will lead to interactions between the quasiproton and quasineutron states, which will be discussed in section VI.

F. Cranked Shell Model

In order to simplify the analysis of the excitation spectrum we shall employ the CSM approximation. It consists in solving the HFB equations for the ground-state configuration at $\omega = 0$. The states with finite angular momentum are calculated from the qp routhians, which are the eigenvalues $\epsilon'_i(\omega)$ of the routhian keeping the mean-fields $\Gamma$ and $\Delta$ fixed to their values at $\omega = 0$. The energy of the configuration with the qps $\{i, j, \ldots\}$ excited, is calculated as

\[
E'(\omega) = E'_o + \epsilon'_i(\omega) + \epsilon'_j(\omega) + \ldots,
\]

where $E'_o$ is the energy of the reference configuration, the choice of which is discussed in several reviews [34,35] in detail. In the present analysis, it is calculated as the HFB energy with the $\omega = 0$ mean-field, i.e.

\[
E'_o = Tr[(h'(\omega = 0) + 1/2\Gamma)\rho + 1/2\Delta t^\dagger] + \lambda_T r/2.
\]

The fields $\Gamma$ and $\Delta$ are the ones found from selfconsistency at $\omega = 0$ and $\Delta$ is taken in y-direction. The density matrix $\rho$ and the pair-tensor $t$ are constructed from the eigenfunctions of the qp hamiltonian (22) using Eqs. (8) and (9). The last term is the above discussed correction for the conservation of isospin.

IV. COMPARISON OF THE CRANKED SHELL MODEL WITH THE EXACT RESULTS

We have calculated the exact energies by diagonalizing the SM hamiltonian (1) for $(Z + N = 3 + 3)$, $(3+3)$ and $(4+4)$ particles in the $f_{7/2}$ shell. The results are shown in the upper panels of figs. [1,2]. The states are classified with respect to the isospin and the signature. We have solved the HFB equations selfconsistently for $(4+4)$ at $\omega = 0$. The fields $\Gamma$ and $\Delta_{nn} = \Delta_{pp}$ determined thus are kept fixed for all other values of $\omega$. They are also used to describe the $(3+3)$ and $(3+4)$ systems, for which only $\lambda$ and $\lambda_T$ are adjusted to have \( \langle N \rangle = N \) and \( \langle Z \rangle = Z \) at $\omega = 0$.

Fig. 8 shows the quasiparticle routhians $\epsilon'_i(\omega)$ for the $(4+4)$ system. All the quasiparticle routhians are two-fold degenerated, corresponding to a quasiproton and a quasineutron, which are labeled, respectively, by a, b, c, ... and A, B, C, ..., adopting the popular CSM letter convention. The degeneracy is lifted for $\lambda_T \neq 0$.

A. Zero-quasiparticle configuration

The configurations are constructed by the standard qp occupation scheme, described in ref. [3]. The vacuum $|0\rangle$ corresponds to all negative qp orbitals filled. It has signature $\alpha = 0$, even-$N$ and -$Z$ and $T'_o = 0$. It represents the even-spin $T = 0$ yrast-band of the $(N = Z = 4)$
system. The AB-crossing at $\omega = 0.6$ corresponds to the simultaneous alignment of a proton- and a neutron-pair (because the routhians are degenerate). Since each $f_{7/2}$ pair carries 6 units of alignment, the total gain amounts to 12 units. The double-alignment as a specific feature of even-even $N = Z$ nuclei has first been noticed in ref. [13] for the case of one kind of particles in a $j$-shell. It is important to take into account of the renormalization of the single particle levels by the interaction ($\Gamma$ in eq. (5)) to obtain the agreement. As seen in fig. 3, the calculation where the fields are determined self-consistently for all $\omega$ shows a much too early alignment. A similar result has been found in ref. [13] for one kind of particles in a $j$-shell. Thus, for the model case of a single $j$-shell, the CSM approximation mocks up some of the corrections to the mean-field approximant.

B. One-quasiparticle configurations

The simplest configurations are generated by exciting one-quasiparticle to the lowest routhians. They correspond to the $T = 1/2$ bands in the odd-A nuclei. Fig. 4 shows the case ($Z = 3, N = 4$). The lowest bands correspond to the one-quasiproton configurations $[a]$, $[b]$, $[c]$, $[d]$. Their excitation energies agree rather well with the exact SM calculation for the lowest bands, which have, as expected, $T = 1/2$ and signature $\alpha = -1/2, 1/2, -1/2$ and $1/2$, respectively.

All bands show a crossing with three qp bands that contain the pair $[AB]$ of aligned quasineutrons. In Fig. 4, only the configurations $[aAB]$ and $[bAB]$ are included before the crossing. It is seen that the crossings occur systematically at a higher frequency in the SM calculation than in the CSM. This effect will be discussed in section VI.

C. Two-quasiparticle excitations in the odd-odd system

The lowest two-qp excitation is generated by putting one-quasiproton and one-quasineutron on the lowest routhian. We denote this configuration by $[A, a]_0$. As discussed in section III D, it has $T_y = 0$ and thus correspond to a $T = 0$ band. The subscript indicates the isospin $T$ of the configuration. The total signature is $\alpha = 1$ and corresponds to an odd-spin band. The particle numbers $N$ and $Z$ must be odd, because exciting one-quasineutron changes $N$ from even to odd or from odd to even and the same holds for the quasiprotons. Thus $[A, a]$ is the lowest $T = 0$ odd-spin band in the odd-odd $N = Z$ system. Fig. 6 shows the CSM estimate for this band, which is obtained choosing $\lambda$ such that $\langle N \rangle = \langle Z \rangle = 3$ but using the $\omega = 0$ mean-field parameters calculated self consistently for $\langle N \rangle = \langle Z \rangle = 4$. It is important to note that the vacuum $|0\rangle$ obtained for the same $\lambda$ does not represent a physical state, because it corresponds to even-$N$ and even-$Z$. The configuration $[B, b]$ is the second odd-spin $T = 0$ band and $[A, b]$ the first even-spin $T = 0$ band in the odd-odd system. As discussed in section III D, the configuration $[a, B]$ does not generate a new state, because isospin $T = T_y = 0$ corresponds to the superposition $\langle [A, b] - [a, B] \rangle / \sqrt{2}$. To keep the notation simple, we label the configuration as $[A, b]$. But it is understood that the superposition is meant. These are the three lowest $T = 0$ bands.

The lowest $T = 1$ band is found by “cranking in isospace”. We calculate the isobaric analog state $|T = 1, T_z = 1\rangle$ by adjusting $\lambda$ and $\lambda_z$ such that $\langle Z \rangle = 2$ and $\langle N \rangle = 4$. According to CSM ideology, the other mean-field parameters are kept to values found for the $\langle N \rangle = \langle Z \rangle = 4$ system. The qp spectrum looks like fig. 6 except that the quasineutron routhians (A, B, ...) and quasiproton routhians (a, b, ...) are no longer degenerate. The vacuum, which we denote by $|0\rangle$, starts as the g-configuration, being crossed first by $|ab\rangle$ and then by $|AB\rangle$. It represents the even-spin $T = T_z = 1$ yrast-band of the even-even $Z = N = 2$ system. The configuration $|0\rangle$ appears as isobaric analog band in the odd-odd $Z = N$ system, where it represents the lowest even-spin $|T = 1, T_z = 0\rangle$ band.

The comparison with the SM calculation in fig. 5 demonstrates that this simple procedure of “cranking in isospace” reproduces well the position of the $T = 1$ even-spin band relative to the three lowest $T = 0$ bands, the relative position of which is also well reproduced by the CSM. The appearance of the $T = 1$ even-spin band below the $T = 0$ bands is a specific feature of the CSM. The three lowest $T = 0$ states at $\omega = 0$ has the consequence that the $T = 1$ even-spin band is crossed by the aligned odd-spin $T = 0$ band. This crossing has been observed in $^{74}$Rb [24]. The similar energy of the $T = 1$ and $T = 0$ states at $\omega = 0$ appears as a cancellation between the pair-gap and the “iso-rotational” energy. Relative to the $\lambda_r = 0$ qp vacuum, the configuration $|Aq\rangle_0$ is shifted by $2\Delta_r$. The configuration $|0\rangle_1$ is shifted by $T(T + 1)/2\mathcal{J} = 1/\mathcal{J} = \lambda_r/(T_2)$. Both quantities are nearly equal. This is not a special feature of our $j$-SM, but a quite general phenomenon, as discussed in ref. [25].

The CSM reproduces SM value for the energy difference between the $T = 0$ and $T = 1$ at $\omega = 0$ rather well. The correction $\lambda_r/2 = 0.88$ in the ground state energy considerably improves the agreement. The crossing frequency between the $T = 1$ and $T = 0$ bands is somewhat underestimated by the CSM, because it overestimates the alignment of the odd-spin band (for $\omega = 0.3G$
the CSM gives \( J_x = 5.7 \) and the SM \( J_x = 4.7 \).

**D. Two-quasiparticle excitations in the even-even system**

Fig. 6 shows the energies of the lowest bands in the system (4+4). Most of them have \( T = 0 \). In order to have even-\( N \) and -\( Z \) one must excite two quasiprotons or two quasineutrons. The lowest excitation is \([AB]\) which has signature \( \alpha = 0 \) and is degenerate with \([aB]\). Out of these two, only the combination \(([a,B] + [a,b])/\sqrt{2}\) has \( T_y = 0 \) and represents a \( T = 0 \) even-spin band. In order to keep the notation simple, we denote this band \([AB]\). Again it is understood that the superposition is meant. The next two \( T = 0 \) configurations are the \( \alpha = 1 \) (odd-spin) band \([AC]\) and the \( \alpha = 1 \) (even-spin) band \([BC]\). These bands are crossed by the \( T = 0 \) four qp band \([abAB]\), which causes the double-alignment in the yrast-line. This structure is clearly correlated with the sequence of \( T = 0 \) bands in the SM calculation. The CSM underestimates the excitation energy of \([AB]\) and \([abAB]\), which has the consequence that the alignment in the yrast-band comes too early.

There is a discrepancy at low-\( \omega \). In the CSM, the \([AB]\) configuration is separated from \([AC]\) and \([BC]\), which become a signature doublet and degenerate with the next doublet \([AD]\) and \([BD]\). For the SM, the lowest bands at small \( \omega \) are two signature doublets with a finite energy difference. We have not analyzed this discrepancy. It is possible that for small \( \omega \) the assumption of stable rotation about the x-axis is violated and the system translates to a tilted rotational axis \((31)\). This would result in substantial changes of the spectrum.

The lowest \( T = 1 \) configurations are obtained by "cranking in isospace". In the CSM spirit, we take the \((4+4)\) mean-field and adjust \( \lambda \) and \( \lambda_\gamma \) such that \((Z) = 3 \) and \((N) = 5 \) and include the correction \( \lambda_\gamma/2 = 1.01 \) in \( E_o \). The configurations must have odd \( N \) and odd \( Z \), because the isobaric analog state \((11)\) belongs to the odd-odd \((3+5)\) system. The lowest configuration is the \( \alpha = 1 \) configuration \([aA]\) and the next the \( \alpha = 0 \) configuration \([aB]\). They represent the odd-spin band \([aB]\) and even-spin band \([AB]\), respectively. They are nearly degenerate, only at high \( \omega \) there appears some signature splitting, favoring the odd-spin band. In the SM calculation there is an odd- and an even-spin band with \( T = 1 \) at about the right energy. They show a small signature splitting. Hence, the CSM also accounts well for the lowest excited states in the even-even system.

**V. CRANKED SHELL MODEL ANALYSIS OF NUCLEI WITH \( A \approx 72 \)**

The analysis of realistic nuclei mainly follows the CSM procedure as described in ref. 31. We will point out the modifications ensued by the isospin conservation. The modified harmonic oscillator potential with the standard set of Nilsson parameters as given in ref. 32 and the deformations \( \varepsilon = 0.3 \), \( \gamma = 0 \) and \( \varepsilon_1 = 0 \) and the monopole pair-fields with \( \Delta_\lambda = \Delta_\sigma = 1.1MeV \), corresponding to about 80% of the experimental even-odd mass differences, are used. Calculations of equilibrium deformations within the shell-correction method \( \text{(23, 24)}\) show that the deformations at low-frequency are characterized by a coexistence of prolate and oblate shapes and softness with respect to the triaxiality parameter \( \gamma \). At larger frequencies all nuclei tend to take on a near prolate shape with \( \varepsilon = 0.3 \). Since we are interested in the qualitative structure of the yrast-spectra of the \( N \approx Z \) nuclei at large spin, the deformation is kept fixed at \( \varepsilon = 0.3 \).

The experimental routhians are calculated along the lines of ref. 2 as

\[
\omega = \frac{1}{2}(E(I) - E(I - 2)) \sqrt{1 - \frac{K}{I - \frac{1}{2}}}^2, \tag{29}
\]

\[
E' = \frac{1}{2}(E(I) + E(I - 2)) - (I - \frac{1}{2})\omega. \tag{30}
\]

The frequency has the meaning of the 1-component of angular velocity. The symmetry axis is 3 and \( K \) the component of the angular momentum along this axis, which is kept constant 4. A common Harris reference

\[
E_o' = -\frac{\omega^2}{2} J_o - \frac{\omega^4}{4} J_1 + \frac{1}{8J_o} \tag{31}
\]

is subtracted from all the experimental routhians. The parameters \( J_o = 13MeV^{-1} \) and \( J_1 = 8MeV^{-3} \) are fitted to the experimental yrast-energies in \( ^{72}\text{Kr} \). Likewise, a common Harris reference is also subtracted from the calculated routhians. The parameters \( J_o = 8.4MeV^{-1} \) and \( J_1 = 25MeV^{-3} \) are fitted to the calculated routhian of the ground \((g-)\) band in \( ^{72}\text{Kr} \).

Fig. 8 shows the quasineutron routhians for \( N = 36 \). The quasiproton routhians are nearly identical (The slight deviations are due to differences of the Nilsson potential for protons and neutrons.) The standard letter coding is used to label the qp routhians. The use of \( A, a, ... \) indicates that the diagram \( \text{(7)} \) is relevant for both neutrons and protons.

\[A. \texts{\textit{72}}\text{Kr}\]

Fig. 8 shows the experimental and CSM routhians for \( ^{76}\text{Kr} \). The low-frequency part consists of the one-quasineutron configurations \([A], [B], [E], [F], ...\).... The calculated routhians slightly deviate from fig. 8 because \( \lambda_\gamma \)

\[\footnote{Eq. 26 represents a slight modification of the expressions given in 23 and is more accurate near the bandhead} \]
is adjusted to have $\langle N \rangle = 37$. The relative position of the trajectories $A$, $E$, $F$ are reasonably well reproduced. At $\omega = 0.5\text{MeV}$ the calculated routhians bend downwards as a consequence of $g_0/2$ alignments. In the case of $[A]$ and $[B]$, it is the proton $s$-band $|ab\rangle$ that crosses. In the case of $[E]$ and $[F]$, it is the double $s$-band $|AB\rangle$. The negative parity bands become yrast because the neutron alignment $|AB\rangle$ is blocked in $[A]$ and $[B]$. The CSM under estimates the frequency of these band crossings. There are the $p$-correlations which are not included in the CSM, that systematically delay the crossing. The systematic investigation of these shifts in ref. [3] indicates that for $[A]$ the crossing should be similarly delayed as in $^{72}\text{Kr}$, where the double $s$-bands cross at $\omega = 0.75$. It will be interesting to see where the double $s$-band crosses in the case of $[E]$ and $[F]$. If it will cross earlier than in $^{72}\text{Kr}$, this would be evidence that the pairing channel is important for the correlations, because substantial blocking is expected due to $E$ and $F$. The absence of the alignments in the data seems to indicate a substantial delay for both parities, but definite conclusions cannot be drawn before the alignments have been observed.

B. $^{74}\text{Rb}$

Fig. [1] displays the spectrum of $^{73}\text{Rb}_{37}$. The upper panel also shows the $T_z = 1$ bands measured in $^{73/2}\text{Kr}_{38}$. They are isobaric analog to the $T = 1$ bands in $^{74}\text{Rb}$ and should give a good estimates of these bands. Since the ground states belong to an isobaric triplet, we set the energy $E_\nu$ of $^{74}\text{Kr}$ equal to the one of $^{74}\text{Rb}$. As seen, the routhians of the $4^+ \rightarrow 2^+$ and $2^+ \rightarrow 0^+$ transitions in both nuclei are nearly identical. The experimental verification of the expectation that $T_z = 1$ bands are rather good estimates of the $T_z = 0$ bands, which are not yet measured, and the study of the fine differences will shed new light on the breaking of the isospin conservation by the Coulomb potential.

The lowest $T = 0$ configurations are generated by exciting a quasiproton and a quasineutron. The first is the positive parity odd-spin band $|Aa\rangle_0$. Next, $|Ab\rangle_0$ and $|Ae\rangle_0$ are expected. As discussed in section [11A], the condition $T_y = 0$ permits only one linear combination of the two excitations, obtained by exchanging the quasi proton with the quasineutron, which we arbitrarily label by only one of the terms in order to keep the notation simple. Analog to $^{73}\text{Kr}$, $|Ae\rangle_0$ crosses $|Ab\rangle_0$, because $b$ blocks the $ab$-alignment, but $e$ does not.

The lowest $T = 1$ bands are generated by cranking in isospace. Thus, $\lambda_\nu$ and $\lambda_\lambda$ are fixed to have $\langle Z \rangle = 36$ and $\langle N \rangle = 38$ at $\omega = 0$ and kept fixed for the other $\omega$ values. The isospin correction energy $\lambda_{\nu}/2$, which is in this case $0.39\text{MeV}$ is included in $E_\nu$. The lowest band is the vacuum $|0\rangle_1$. It is crossed by the $T = 0$ band $|Aa\rangle_0$, which has a large alignment. (In the CSM calculation it is about 7 and in experiment with the chosen reference it is changing from 3 at the bottom to 7 for the highest observed transition). The crossing frequency is fairly well reproduced. Thus it seems, that this crossing is a phenomenon belonging to the realm of $t = 1$-pair-correlations. As already discussed for the $f_{5/2}$ model case, the close energies of the lowest $T = 0$ and $T = 1$ states result from the near cancelation of two large energies: The pair-energy $2\Delta$, by which the $T = 0$ state with two-qp character is shifted and the iso-rotational energy $T(T+1)/2\mathcal{J} = \lambda_T/T_z^2$ by which the $T = 0$ state with zero-qp character is shifted. The latter is somewhat smaller than the former (numbers are given in subsection [VC]). An accurate estimate of the crossing frequency cannot be expected from our the CSM calculation, because it simply assumes that $\Delta$ is $80\%$ of experimental even-odd mass difference, which is pretty rough. The estimate of $\mathcal{J}$ (the level density) is also rough, because the common deformation of $\varepsilon = 0.3$ and $\gamma = 0$ is assumed for all the considered nuclei. Optimizing the shape will reduce the level density near the Fermi surface and, thus, make $\mathcal{J}$ smaller and push up the $T = 1$ band.

The transition $3^+ \rightarrow 2^+$ observed in $^{74}\text{Rb}$ is of $M1$ type, if the tentative spin and parity assignments are correct. Since the M1-transition operator is predominantly isovector, it favors transitions $T = 0 \rightarrow T = 1$. A measurement of the lifetime would be quite interesting, because the difference in aligned angular momentum (about $3$ for the frequency of the observed transition) makes the $B(M1)$ value sensitive to isospin impurities.

C. $^{72}\text{Kr}$

Fig. [10] displays the spectrum of $^{72}_{36}\text{Kr}_{36}$. The upper panel shows also the $T_z = 1$ bands measured in $^{73}_{35}\text{Br}_{37}$. They are expected to be isobaric analog. Since the ground state of $^{72}\text{Kr}$ has $T = 0$, the energy difference between the lowest states of each isospin must be estimated. We use the expressions discussed in ref. [30],

$$E_{symm} = \frac{T(T+1)}{A}[134.4 - 203.6A^{-1/3}]\text{MeV}$$

which can be considered as an phenomenological expression for the iso-rotational energy, fitting the experimental binding energies, and

$$\Delta = 5.39A^{-1/3}\text{MeV} \quad (N = Z + 2),$$
$$\Delta = 6.24A^{-1/3}\text{MeV} \quad (N = Z) .$$

The ground state of $^{72}\text{Br}$ is then at $E_{symm} + 2\Delta$. The numbers for $A = 72$ are $E_{symm} = 2.37\text{MeV}$ and $2\Delta = 2.59$. As discussed in sect. [VIB], the difference $(E_{symm} - 2\Delta)$ gives the distance between the lowest $T = 1$ and $T = 0$ states in the odd-odd nuclei. For $^{72}\text{Rb}$, one finds $E_{symm} = 2.32\text{MeV}$ and $2\Delta = 2.97\text{MeV}$. The difference of 0.65MeV can be compared with the ex-
experimental difference of 0.57 MeV between the intrinsic energies of the $T = 1$ and $T = 0$ states in $^{74}$Rb.\footnote{The intrinsic energy of the $T = 1$ band is estimated assuming that the rotational energy is given by the strong coupling expression $(I(I + 1) - K^2)/J_{rot}$ with $K = 2$ and fitting $J_{rot}$ to the $(3^+) \rightarrow (2^+)$ transition.}

The yrast band is the $T = 0$ configuration $[0]_0$. It is crossed by $[ABab]_0$. This double-alignment is observed in the experimental yrast sequence at a substantially higher frequency than in the CSM. The next bands are $[ab]_0$, $[ac]_0$ and $[af]_0$, where we use again the short hand notation for the $T_y = 0$ linear combinations. The $J_{7/2}$ SM study suggests that the configuration $[ab]_0$ is predicted too low by the CSM (c.f. section V B). The lowest $T = 1$ configurations are $[Aa]_1$, $[Ea]_1$ and $[F a]_1$. They are experimentally seen as the isobaric analog bands in $^{72}$Br. Using these energies and the above described estimate for the relative energies of the $T = 0$ and $T = 1$ ground states the “experimental” $T = 1$ bands in $^{72}$Kr lie about 1 MeV higher than our CSM estimate. As already discussed in section V B, the CSM estimates for $2\Delta$ and $E_{symm}$, which determine the relative energy of the $T = 0$ and $T = 1$ bands are pretty rough in our CSM calculation.

\section{TRS calculation for $^{74}$Rb}

The CSM assumptions of fixed deformation and pairing are too inaccurate for the high frequency region of the considered nuclei. Here we present the spectrum of $^{74}$Rb as an example that the concepts suggested in this paper can be combined with more sophisticated mean-field calculations.

The TRS calculations presented in ref. \cite{34} describe rather well the rotational bands in $^{74}$Kr. The deformed Woods-Saxon potential with the “universal parameters” is used. Only pp- and nn- pairing is considered, but in addition to the monopole a quadrupole pair-field is taken into account. For each configuration and frequency $\omega$, the deformation parameters are individually optimized. The details of the calculation are described in ref. \cite{53}. The calculations of \cite{34} for the yrast sequence in $^{74}$Kr are used for the configuration $[0]_1$ and the results of an analogous TRS calculation \cite{34} for $^{74}$Rb are used for the configurations $[Aa]_0$ and $[Ae]_0$. The relative energy of the $T = 0$ and $T = 1$ bands is calculated by setting at $\omega = 0$ the energy difference between the configurations $[0]$ in $N = 38$, $Z = 36$ and $N = 37$, $Z = 37$ equal to the expression \cite{32} for the iso-rotational energy. The same Harris reference as used for the experimental Routhians is subtracted from the calculated ones.

\footnote{The TRS calculations have been carried out by W. Satula. The authors would like to express their gratitude for making these results available to them.}

It seen by comparing fig. \cite{11} with fig. \cite{8} that the calculated spectrum now agrees much better with the data at high $\omega$. As compared to the CSM calculation based on the Nilsson potential, the main differences are: i) different (and probably better) positions of the spherical single particle levels, ii) different deformations of the $T = 0 (\beta \approx 0.31)$ and $T = 1$ bands ($\beta \approx 0.39$) and iii) inclusion of the quadrupole pair-fields. The expression \cite{32} places $[0]_1$ somewhat too high relative to the $T = 0$ bands. The reason is that the TRS calculation gives an excitation energy of 2.5 MeV for the configuration $[Aa]$ relative to $[0]$. The fits \cite{32} which rather well reproduce the experimental difference between the $T = 1$ and 0 bands in odd-odd nuclei (cf. section V C and ref. \cite{34}), give the larger value of $2\Delta = 3.0$ MeV for $N = Z$.

\section{VI. THE ROLE OF $T = 1$ PROTON-NEUTRON PAIR-CORRELATIONS}

Although the construction of the intrinsic configurations within the CSM frame looks as if there was no pn-pairing, it is implicitly included. It is the spontaneous breaking of the isospin symmetry that permits to choose the orientation of the $t = 1$ pair-field such that the pn-component of the field disappears. However, in order to restore the isospin symmetry of the total wave function the pn-pairing is absolutely necessary. Its strength is completely determined by the isospin symmetry. Hence, the $t = 1$ pn-pairing manifests itself by the isospin symmetry of the states. In this section we are going to elucidate this important point further.

One important consequence of the isospin symmetry is that in $N = Z$ nuclei, the $T = 0$ configurations like $[AB]_0$ or $[Ab]_0$ appear only once. The additional configurations obtained by exchanging quasiprotons with quasineutrons, which would have the same energy if there was no pn-pairing, do not appear. More accurately one must say that only the $T_y = 0$ combinations $([AB] + [ab])/\sqrt{2}$ and $([Ab] + [aB])/\sqrt{2}$ are pushed to large energy by the pn-pair-correlations. This symmetry restriction can be considered as a special case of a more general feature of the pn-correlations. It is a consequence of the fact that the total wavefunction has for $T = 0$ a constant probability for all orientations of $\vec{A}$. For small $T$, the total wave function $D_{T \beta}^T$ still corresponds to a substantial probability for an orientation of $\vec{A}$ different from the $y$-direction, i.e. to a substantial pn-pair field. As a consequence, one of the two linear combinations is energetically favored over the other. Only for large $T$, when the wavefunction becomes concentrated near small angles $\theta$, the proton and neutron excitation become independent. This has been demonstrated in ref. \cite{11} by a systematic SM study states with the character $([AB] \pm [ab])/\sqrt{2}$, which represent the two s-bands in the $h_{11/2}$ shell. It is demonstrated that for
\[ N \approx Z \] one of the bands is pushed up relative to the other by the pn-interaction and the wave function is a linear combination of the proton and neutron excitations. On the other hand, for \[ Z \ll N \] the pn-interaction does not change the relative position of the two bands very much, which are almost pure proton or neutron excitations. As discussed in \[ [11] \], the data on the alignment of \( h_{11/2} \) particles in the mass 120-130 region seems to support this prediction of the theory. Clearly, more detailed measurements of the rotational spectra in nuclei near \( N \approx Z \) are necessary to test this signature of the \( t = 1 \) pn-pair correlations.

Another consequence of the pn-interaction is the systematic enlargement of the rotational frequency where first alignment (ab and/or AB) appears when the intruder shell becomes more symmetrically filled. It has been demonstrated in ref. \[ [39] \] that this effect is generated by the \( T = 1 \) components of the pn-interaction. Fig. 1 illustrates this point showing a SM calculation where we took off all the \( t = 0 \) components of the \( \delta \)-interaction. The crossing shows up at almost the same frequency as in the calculation with the full interaction. Hence the possible \( t = 0 \) correlations cannot influence the crossing in an important way. The CSM calculation predicts the crossing earlier than the SM. Generally, the comparison with the exact SM calculations in section IV shows that the CSM approximation systematically underestimates the frequency of the alignment processes. Since the protons and neutrons are assumed to independently move in a fixed rotating field, the CSM can also not reproduce the delay of the the (ab) crossing caused by the presence of an A quasineutron, which is found in the full SM calculations and in the experiment as well \[ [39] \]. The mechanism of this effect, which seems to be of the same origin as the late alignment in \( N \approx Z \) nuclei, is still an open question. It is possible that it is caused by the part of \( t = 1 \) pn-pair correlations that are not taken into account by the CSM approximation.

In order to check this conjecture and to disentangle it from other mechanisms (shape changes, for example) causing similar modifications of the alignment processes, a careful treatment of the isospin symmetry and the angular momentum dependence of the shape and pairing degrees of freedom is necessary. The example in section V D demonstrates that combining the concept of spontaneous breaking of isospin symmetry with careful mean-field calculations by means of codes that take only the pp- and nn-pairing into account is expected to provide a good description of \( N \approx Z \) nuclei with a strong \( t = 1 \) pair field. Such calculations may serve as a bench mark to look for \( t = 0 \) pair correlations.

The present paper also sheds light on the results of the recent analysis of high spin data in nuclei with \( T_\pi = 1/2 \) and \( 1 \) \[ [34,35] \] by means of the conventional mean-field approach that does not explicitly take into account the proton-neutron pairing. It is stated there that “the agreement between theory and experiment can be considered as very good...” These results are consistent with our suggestion that in nuclei with \( 70 < A < 80 \) there is strong \( t = 1 \) pairing and we want to point out once more our most important message: The fact that the mean-field theory without an explicit pn-pair field works well does by no means imply that there is no \( t = 1 \) pn-field. On the contrary, it must have a strength comparable with the pp- and nn-fields in order to restore the isospin symmetry. Hence the other statement of the paper \[ [24] \], “we do not find clear evidence for collective pn-pairing...” must be taken literally. Many features of the rotational spectra are insensitive to the presence of the pn-field, which manifests itself in a rather subtle way via isospin conservation. The discussed absence of pairs of two quasiparticle excitations (two quasiproton and two quasineutron), which are expected if there was no pn-pair field, is one consequence of the pn-pairing. The available data on \( N = Z \) nuclei do not permit stringent tests of these CSM predictions so far.

In refs. \[ [12] \] the consequences of the pn-pair correlations are studied by introducing a pn-pair fields, the strength of which is varied. Concerning the \( t = 1 \) pair fields, a free variation is inconsistent with the isospin conservation which fixes the ratios between the pn-, pp- and nn-fields.

VII. CONCLUSIONS

The cranked shell model mean-field approach has been extended to include \( t = 1 \) proton-neutron pairing in order to describe nuclei near \( N = Z \). The central concept encountered is the spontaneous breaking of the isospin invariance by the isovector pair-field. All orientations of the pair-field represent one and the same intrinsic state. One particular choice is the direction with no pn-pair field. On the mean-field level this permits to treat the \( N \approx Z \) nuclei essentially as if there was no pn-pairing. Of course, the pn-field exists and it is strong. However, it is completely determined by the isospin symmetry. It comes into play when interpreting the mean-field solutions as intrinsic states of the total wavefunction, with good isospin. As in the analogous case of spatial rotation and deformed mean-field solutions, the isospin symmetry may be restored with different accuracy. In this paper we have focused on the simplest possibility, the limit of strong symmetry breaking. In this limit it is well known from spatial rotations how to connect the symmetry breaking mean-field solutions with the quantal states of good \( I \) or \( T \), respectively. The resulting scheme is very simple. The spectrum is generated from quasiparticle excitations with no pn-pairing in the standard way. Thus, an odd number of quasineutrons means odd \( N \) and the same holds for protons. The following additional rules have to be applied for \( N \approx Z \):

1) The isospin is fixed by “cranking in isospace”, i.e. \( \langle T_\pi \rangle = T \). For \( T = T_\pi \) states, this amounts to the ordinary constraints in particle number that fix the chemical
potentials $\lambda_p$ and $\lambda_n$. The energy of states $T_2 < T$ is taken to be the same as the isobaric analog states $T = T_2$.

2) The lowest quasiparticle excitations for the given values of $\lambda_p$ and $\lambda_n$ are the lowest states of the corresponding value of $T = (T_2)$.

3) A term $(\lambda_n - \lambda_p)/2$ is added to the energies. It is a consequence of isospin conservation and, hence, a manifestation of the pn-pairing.

4) For the $T = 0$ states in $N = Z$ nuclei, only quasiparticle configurations with $T_V = 0$ are permitted. This additional symmetry restriction excludes certain configurations that are permitted in nuclei with large $T$.

We have exactly solved the SM problem for the model system of protons and neutron in an $f_{7/2}$ shell interacting via a $\delta$-force and exposed to an rotating external quadrupole potential. The spectra for the half filled shell, $Z = 3,4$ and $N = 3,4$ have been calculated as a function of the rotational frequency $\omega$. The rotational bands generated in this way are compared to the mean-field theory. The mean-field calculations are carried out in the spirit of the cranked shell model approximation. The mean-field is determined for the $(N = Z = 4)$ system at zero-frequency by solving the Hartree-Fock-Bogolubov equations. The solution turns out to be an isovector pair-field with a dominating monopole component but substantial contributions from the higher multipoles. Keeping this mean-field constant, states with finite angular momentum and isospin are generated by changing $\omega$, $\lambda_n$ and $\lambda_p$. The structure of the exact SM excitation spectra is reproduced by this CSM procedure, where the modifications 1 - 4) turn out to be important. A fair quantitative agreement is also obtained. In particular, the relative position of $T = 0$ and $T = 1$ bands is well reproduced. The most conspicuous discrepancy is that the CSM predicts the rotational alignment of quasiparticle pairs at a too low-frequency and is not able to describe the delay of the neutron alignment by the presence of odd protons in the same shell (and vice versa).

With the aforementioned restrictions, the suggested CSM appears to be quite capable to describe the yrast-spectra of rotating nuclei near $N = Z$. One expects the CSM to be better for realistic nuclei than for our $f_{7/2}$ model case, because the larger number of nucleons favors the mean-field approximation. Realistic Hartree-Fock-Bogolubov calculations [13] and SM calculations [15,18] point to the existence of a $t = 1$ pair-field at low-spin in the mass 70 region. Hence we have applied our CSM to $^{72}_{36}$Kr, $^{73}_{36}$Kr, $^{73}_{37}$Rb, and $^{74}_{37}$Rb. Satisfactory agreement with the data is found and the assumption of a $t = 1$ pair-field in $N \approx Z$ nuclei in the mass 70-80 region is consistent with the existing measurements. In particular, the recently observed crossing between the $T = 1$ even-spin ground band and a $T = 1$ odd-spin band in $^{72}$Rb can be well explained within the scenario of a $t = 1$ pair-field. The close energy of the two bandheads, which is the reason for the crossing, is a consequence of the near cancelation of two larger energies, twice the pair-gap, by which the the $T = 0$ band is pushed up, and the “iso-rotational” energy (symmetry + Wigner energy), by which the $T = 1$ band is pushed up. This compensation is a quite general feature of the odd-odd $N = Z$ nuclei, as pointed out in ref. [19].

Finally, it should be pointed out that isospin is not exactly conserved. The Coulomb potential breaks this symmetry. This has the interesting consequence that the isovector pair-field does not completely rotate freely in isospace (i.e. the wavefunction is not exactly a $D$ function), but it feels the Coulomb field like an external potential, which prefers a certain orientation. This effect and other manifestations of isospin symmetry in the rotational spectra are very interesting subjects for future studies. In particular, measuring the differences between the moments of inertia and alignment processes of isobaric analog bands will shed new light on the old question of isospin purity in heavier nuclei.

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FIG. 1. Angular momentum expectation value $\langle J_x \rangle$ for the yrast-band in the $(Z = N = 4)$ system. The full SM result is denoted by SM, the SM result with a modified two-body interaction leaving out the $T = 0$ components of the $\delta$-force by SM $T=1$, the fully selfconsistent HFB calculation by HFB and the CSM approximation by CSM.
FIG. 2. The isovector pair-field $\vec{\Delta}$. 
FIG. 3. Quasiparticles in the $f_{7/2}$ shell as function of the rotational frequency $\omega$. The chemical potential corresponds to a half filled shell $\langle Z \rangle = \langle N \rangle = 4$. The mean-field is kept fixed to the values calculated by solving the HFB equations (1) for $\omega = 0$. Full drawn and dashed dotted lines denote the favored and unfavored signature ($\alpha = -1/2$ and $1/2$ for $f_{7/2}$), respectively.
FIG. 4. Total routhians for the \((Z = 3, N = 4)\) system. The upper panel shows the SM results and the lower the CSM approximation. Full lines correspond to signature -1/2 and dashed ones to 1/2. The labeling of the quasiparticle configurations is explained in the text.
FIG. 5. Total routhians for the $(Z = N = 3)$ system. The upper panel shows the SM results and the lower the CSM approximation. Full lines correspond to even-spins and dashed ones to odd spins. The labeling of the quasiparticle configurations is explained in the text.
FIG. 6. Total rothians for the \( (Z = N = 4) \) system. The upper panel shows the SM results and the lower the CSM approximation. Full lines correspond to even-spins and dashed as well as dashed dotted ones to odd spins. The labeling of the quasiparticle configurations is explained in the text.
FIG. 7. Quasiparticles for ($N = Z = 36$) as function of the rotational frequency $\omega$. The mean-field is the modified oscillator with the deformations $\varepsilon = 0.3$, $\varepsilon_4 = 0$ and $\gamma = 0$ and $\Delta_n = \Delta_p = 1.1 MeV$. The diagram is relevant for both protons and neutrons. Full drawn and dashed dotted lines denote positive and negative parity, respectively. The signature is indicated by the letters: $\alpha = 1/2$ for $A,E$ and $\alpha = -1/2$ for $B,F$. 
FIG. 8. Total routhians for the $^{73}\text{Kr}_{37}$. The upper panel shows the experimental routhians and the lower the CSM approximation. The parity and signature assignments ($\pi, \alpha$) are: Full lines (+,1/2), dashed (+,-1/2), dashed dotted (-,1/2) and dotted (-,-1/2). A Harris reference is subtracted.
FIG. 9. Total routhians for $^{74}_{37}$Rb$_{37}$. The upper panel shows the experimental routhians [29] and the lower the CSM approximation. For $T = 1$ also the isobaric analog $T_z = 1$ bands in $^{74}_{36}$Kr$_{38}$ [30] are shown. The parity and signature assignments ($\pi, \alpha$) are: Full lines (+,0), dashed (+,1), dashed dotted (-,0) and dotted (-,1). A Harris reference is subtracted.
FIG. 10. Total routhians for the $^{72}$Kr$_{36}$. The upper panel shows the experimental routhians and the lower the CSM approximation. For $T = 1$ also the isobaric analog $T_z = 1$ bands in $^{72}$Br$_{35}$ are shown. The text explains how the energy of the $T = 1$ bands relative to the energy of the $T = 0$ ground state is fixed. The parity and signature assignments ($\pi, \alpha$) are: Full lines (+,0), dashed (+,1), dashed dotted (-,0) and dotted (-,1). A Harris reference is subtracted.
FIG. 11. Total routhians for $^{74}_{37}\text{Rb}_{37}$ calculated by means of the deformation optimized Woods Saxon Strutinsky method. The text explains how the energy of the $T = 1$ bands relative to the energy of the $T = 0$ ground state is fixed. The parity and signature assignments ($\pi, \alpha$) are: Full lines (+,0), dashed (+,1) and dotted (-,1). A Harris reference is subtracted.