ON THE ODDERON INTERCEPT IN QCD

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The odderon singularity is studied in perturbative QCD in the framework of the Bartels-Kwieciński-Praszalowicz (BKP) equation. Arguments for the odderon intercept being exactly equal to unity are given. Besides, a variational method based on a complete system of one-gluon functions is presented. For the odderon, the highest intercept calculated by this method is $1 - (N_c\alpha_s/\pi)0.45$. Comparison to other calculations is shown.

\footnote{Talk given by N. Armesto at the Madrid Workshop on low x Physics (Miraflores de la Sierra, Spain, June 18th-21st 1997).}
1 Introduction

Regge theory [1] has been used during the last 30 years to describe strong interaction at high energies and low transferred momenta. The amplitude for the reaction \( p_a p_b \rightarrow p'_a p'_b \), in the limit \( s \gg m^2 \approx -t \), can be expressed as a sum over Regge trajectories \( j(t) = 1 + \omega(t) \) exchanged in the \( t \)-channel:

\[
A(s, t) = \sum_{j,p=\pm} \xi^p_{j,p(t)} s^{j_p(t)} g^p_1(t) g^p_2(t), \tag{1}
\]

with \( s = (p_a + p_b)^2 \) and \( t = (p_a - p'_a)^2 \) the Mandelstam variables and \( \xi^p_j = i - (\cos \pi j + p) / \sin \pi j \) the signature factor.

The amplitude can be decomposed as a sum over parts with definite signature, i.e. definite behaviour under the exchange \( s \rightarrow -s \):

\[
A(s, t) = A^+(s, t) + A^-(s, t), \quad A^\pm(s, t) = \pm A^\pm(-s, t). \tag{2}
\]

As the total cross section can be related to the amplitude via the optical theorem, \( \sigma_{tot} \propto \text{Im} A(s, 0)/s \), the behaviour at high energies will be determined by those Regge trajectories exchanging vacuum quantum numbers and with highest intercept \( \omega(0) + 1 = j(0) \). For positive signature (contributing equally both to \( pp \) and \( p\bar{p} \) scattering) the trajectory with the highest intercept is called the pomeron and its intercept has been determined [2] to be supercritical (i.e. \( > 1 \)): \( \omega(t) \approx 0.08 + (0.25 \text{ GeV}^{-2}) t \). The trajectory with negative signature (contributing to the difference between \( pp \) and \( p\bar{p} \)) has been called the odderon [3]; phenomenological fits to soft data seem to indicate that its contribution to soft interactions at high energies is negligible, although this is still a matter of debate [4].

There have been several attempts to relate Regge theory to QCD. Within the framework of perturbative QCD, in the limit \( \alpha_s \ln s \sim 1 \) (leading-log approximation in \( s \)), the so-called Balitsky-Fadin-Kuraev-Lipatov [5] (BFKL) pomeron appears as a fixed cut in the \( j \)-plane with intercept \( 1 + \omega_{BFKL}(0) = 1 + (N_c \alpha_s/\pi)4 \ln 2 \) (\( \approx 1.5 \) for \( N_c = 3 \) and \( \alpha_s \approx 0.2 \)); it corresponds to the bound state of two \( t \)-channel reggeized
gluons. Under the same approximation the odderon appears as the bound state of three gluons in a symmetric colour configuration, given by the solution of the BKP equation.

Attempts to solve the odderon problem have gone in different directions. On the one hand two-dimensional conformal techniques have been applied [7,8]. On the other hand variational methods have been used, both with conformal invariant [9,10] and polynomial [11] trial functions. This constitutes a first step towards the unitarization [12] of QCD at high energies, which, together with next-to-leading-log corrections [13], are expected to give full consistency to the whole approach [14].

In this contribution we will treat the following aspects [11]: In Sect. 2 arguments will be presented for the odderon intercept being exactly unity. In Sect. 3 a variational method will be proposed. Finally in Sect. 4 some numerical results and comparison to other calculations will be shown.

2 Argument for intercept equal to unity

The BKP equation for three gluons with transverse momenta \(q_1, q_2, q_3\) can be written in a Hamiltonian form with \(E = 1 - j\):

\[
H\psi = E\psi, \quad H = T_1 + T_2 + T_3 + U_{12} + U_{23} + U_{31}. \tag{3}
\]

In units of \(N_c\alpha_s/\pi\),

\[
T_1 = -\omega(q_1) = \frac{\eta(q_1)}{4\pi} \int \frac{d^2q'_1}{\eta(q'_1)\eta(q_1 - q'_1)}, \tag{4}
\]

with \(\eta(q) = q^2 + m^2\), is the gluon Regge trajectory. In (4) the dependence on \(m\) vanishes, so the BKP equation is infrared stable [3,13].

The \(U_{ik}\)'s are integral operators which act with measure

\[
d\mu = d^2q_1d^2q_2d^2q_3\delta^{(2)}(q_1 + q_2 + q_3 - q)/[\eta(q_1)\eta(q_2)\eta(q_3)],
\]

\[
U_{12}(q_1, q_2, q_3|q'_1, q'_2, q'_3) = \eta(q_3)\delta^{(2)}(q_3 - q'_3)V_{12}(q_1, q_2|q'_1, q'_2). \tag{5}
\]
Here the $V_{ij}$'s are BFKL interaction kernels for 2 gluons in a vector colour state (a factor $1/2$ with respect to the vacuum channel appears):

$$V_{12}(q_1, q_2|q'_1, q'_2) = -\frac{1}{4\pi} \left[ \eta(q_1)\eta(q'_2) + \eta(q'_1)\eta(q_2) \right] - \eta(q_1 + q_2).$$  \hfill (6)

Due to the bootstrap identity [16]:

$$\int \frac{d^2 q'_1}{\eta(q'_1)\eta(q_1 + q_2 - q'_1)} V_{12}(q_1, q_2|q'_1, q'_2) = \omega(q_1) + \omega(q_2) - \omega(q_1 + q_2),$$  \hfill (7)

for $q = q_1 + q_2 + q_3 = 0$, $\psi_B(q_1, q_2, q_3) = \psi_0$ is a solution with maximal symmetry which gives $E = 0$. This solution does not fulfill the gauge invariance requirement $\psi_B(q_i = 0) = 0$ and for $m \to 0$ it decouples from the physical spectrum, but still offers a lower bound for the energy (as the state with $|n| = 1, \nu = 0$ for the $|n| = 1$ sector in the BFKL pomeron [17]). More elaborated mathematical arguments can be found in [11].

3 Variational method

Setting $m = 0$, and redefining $\psi \rightarrow \prod_{i=1}^3 q_i^2 \psi$, we have

$$H \psi = E \prod_{i=1}^3 q_i^2 \psi, \quad H = (1/2)(H_{12} + H_{23} + H_{31})$$  \hfill (8)

and, in a mixed representation ($C$ is Euler’s constant),

$$H_{ik} \psi = \prod_{j=1}^3 q_j^2 (\ln q_i^2 q_k^2 + 4C) \psi + \prod_{j=1, j \neq i, k}^3 q_j^2 \left[ q_i^2 \ln(r_{ik}^2/4) q_k^2 + (i \leftrightarrow k) \right] \psi$$

$$+ 2(q_i + q_k)^2 \psi (r_i - r_k = 0).$$  \hfill (9)

As usually, the variational approach provides us with an upper bound for the ground state energy ($\equiv$ lower bound for the intercept) and consists in finding a minimum of the functional

$$\Phi = \int \prod d^2 q_i \psi^* H \psi,$$

$$\int \prod d^2 q_i \psi^* \prod_{j=1}^3 q_j^2 \psi = 1.$$  \hfill (10)
As trial function we choose a linear combination of one-gluon functions:

$$\psi(r_1, r_2, r_3) = \sum_{\alpha_1, \alpha_2, \alpha_3} c_{\alpha_1, \alpha_2, \alpha_3} \prod_{i=1}^{3} \psi_{\alpha_i}(r_i), \quad (11)$$

with

$$\int d^2r \psi_{\alpha}^* q^2 \psi_{\alpha'} = \delta_{\alpha, \alpha'}, \quad \sum_{\alpha_1, \alpha_2, \alpha_3} |c_{\alpha_1, \alpha_2, \alpha_3}|^2 = 1 \quad (12)$$

and $c_{\alpha_1, \alpha_2, \alpha_3}$ fully symmetric in $\alpha_1, \alpha_2, \alpha_3$.

Finally all is reduced to diagonalize the matrix $E_{\alpha_1, \alpha_2, \alpha'_1, \alpha'_2}$ defined by

$$\Phi = \frac{3}{2} \sum_{\alpha_1, \alpha_2, \alpha'_1, \alpha'_2} c_{\alpha_1, \alpha_2, \alpha_3}^* c_{\alpha'_1, \alpha'_2, \alpha_3} E_{\alpha_1, \alpha_2, \alpha'_1, \alpha'_2}, \quad (13)$$

multiplied by the identity for the third gluon and symmetrized in $(1, 2, 3)$ and $(1', 2', 3)$.

Our concrete choice of trial basis are the harmonic oscillator eigenfunctions:

$$\psi_\alpha(r) = \psi_{k,l}(z) \exp il\phi, \quad z = \ln r^2, \quad \psi_{k,l}(-\infty) = 0, \quad (14)$$

with $H_k(z)$ the Hermite polynomials.

### 4 Numerical results

From our numerical experience the best results are obtained with $|l| \leq l_{\text{max}}$, $k = 0, 1, \ldots, (l_{\text{max}} + 1 = r)$. They are shown in Table 1.

These results are related to the corresponding intercepts as

$$1 + \omega_{\text{BFKL}}(0) = 1 - \frac{N_c\alpha_s}{\pi} \epsilon_2, \quad 1 + \omega_{\text{odd}}(0) = 1 - \frac{N_c\alpha_s}{\pi} \frac{3}{2} \epsilon_3. \quad (15)$$

The dimension of the matrix grows, for the three-gluon case, from 12 ($r = 2$) to 3368 ($r = 6$). For $r = 1$ the potential energy $U_{ik}$ vanishes and one gets the kinetic energy per gluon, i.e. $\epsilon_2 = \epsilon_3$ in this case. It can be seen that the convergence of the method is quite slow.

In Table 2 we compare our results to other calculations. Our lower bound is compatible with our previous argument of $\epsilon_3 = 0$ but weaker than that of [10], which gives
Table 1: Lowest eigenvalues of \((8),(9)\) for the two- \((\epsilon_2)\) and three-gluon \((\epsilon_3)\) bound states, using our trial functions labelled by \(r\). \(-2.773 \simeq -4 \ln 2\) is the exact value for \(\pi \omega_{BFKL}(0)/(N_c\alpha_s)\).

| \(r\) | \(\epsilon_2\) | \(\epsilon_3\) |
|------|----------------|----------------|
| 1    | 0.968          | 0.968          |
| 2    | 0.022          | 0.605          |
| 3    | 0.475          | 0.454          |
| 4    | 0.743          | 0.379          |
| 5    | 0.912          | 0.331          |
| 6    | 1.032          | 0.298          |
| \(\infty\) | 0.912 | 0.331 |

a supercritical odderon; this discrepancy may have a numerical origin. As to the results of [8], they are in principle exact but contain some semiclassical approximation whose reliability is difficult for us to quantify. So our conclusion is that the odderon intercept lies in the interval 0.91 ÷ 1.79 (for \(N_c = 3\) and \(\alpha_s = 0.2\)).

To quantify the existing uncertainty in more practical terms, let us consider the process \(\gamma p \rightarrow \eta c p\), i.e. diffractive photoproduction of \(\eta_c\). The corresponding cross-section has been estimated [18] to be \(\sigma(\gamma p \rightarrow \eta c p) = D \times (47 \div 100)\) pb, \(D = (\overline{\tau})^{-2\omega_{odd}(0)}\). From the previous considerations, \(D\) (for \(N_c = 3\) and \(\alpha_s = 0.2\)) lies in the range 0.3 ÷ 55000 for HERA (\(\overline{\tau} \simeq 10^{-3}\)). Clearly this reaction, in case it could be studied experimentally, is very sensitive to the value of the odderon intercept and offers a good opportunity to measure it.

In conclusion we have studied the odderon singularity as solution of the BKP equation for three colour-symmetric gluons and given an argument for the odderon intercept
Table 2: Comparison of our results to other calculations of the odderon intercept.

|                  | Our result | Ref. [10] | Ref. [8] |
|------------------|------------|-----------|---------|
|                  | $\geq -0.45$ | $\geq 0.37$ | $2.41$ |
|                  | $\geq 0.91$  | $\geq 1.07$  | $1.46$  |
| Upper bound from [9] | $\leq 4.16$ | $\leq 1.79$ |         |

$-(3/2)\epsilon_3 1 + \omega_{odd}(0)$ for $N_c = 3, \alpha_s = 0.2$

to be equal to unity. Besides a variational method to compute it has been presented and its results compared to other calculations. The odderon intercept lies in the interval $1 - (N_c\alpha_s/\pi)0.45 \leq j_{odd}(0) = 1 + \omega_{odd}(0) \leq 1 + (N_c\alpha_s/\pi)4.16$.

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