Implications of MAX for CDM

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ABSTRACT

We analyze the Gamma Ursae Minoris (GUM) and Mu Pegasi (MuP) scans of the Millimeter-wave Anisotropy eXperiment in the context of cold dark matter (CDM) models of structure formation, paying particular attention to the two-dimensional nature of the GUM scan. If all of the structure in the (foreground subtracted) data is attributed to cosmic microwave background anisotropy, then there is a detection in each scan. For a standard CDM model, the amplitudes of the signals are individually compatible with the COsmic Background Explorer measurement, but marginally inconsistent with each other.

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1. Introduction

There have recently been many results quoted for anisotropies in the cosmic microwave background (CMB) on degree scales. Of these, the detection of a large and significant signal in the Gamma Ursae Minoris (GUM) scan by the Millimeter-wave Anisotropy eXperiment (MAX) [1] (see also [2]) and the strong upper limit placed by the same experiment in their Mu Pegasii (MuP) scan [3] are the most puzzling.

In this Letter we examine in detail the implications of this data for cold dark matter (CDM) models of structure formation, paying particular attention to the two-dimensional nature of the GUM scan in our analysis. We find that the conventional approach to calculating theoretical predictions must be substantially modified when the scan (or scans) to be modelled cover a two-dimensional patch of the sky. The important point is that scanning and chopping introduce a fundamental asymmetry into the two-dimensional autocorrelation function for the data points.

The MAX experiment provides a good test of CDM models, since its filter function is well-matched to the Doppler peak of the (radiation) power spectrum (see e.g. [4]). Our results indicate that if all of the structure in the GUM scan is attributed to CMB fluctuations, the results are consistent with the normalization measured by the COsmic Background Explorer satellite (COBE) [5,6]. However, they are marginally inconsistent with the results from the MuP scan. It is possible that some of the GUM signal is not primordial, and/or that some CMB signal was removed from MuP data by the foreground subtraction procedure; there is no way to tell from the present data alone. This leads us to speculate that all these results can be accommodated within the standard CDM theory.

2. Modelling MAX-GUM

The GUM data set consists of 165 temperatures, from the co-added 6 cm$^{-1}$ and 9 cm$^{-1}$ wavebands, in a “bow-tie” pattern of bins covering a patch of the sky roughly 6° $\times$ 1° near Gamma Ursae Minoris. During each flight the MAX telescope is scanned back and forth, tracking on GUM, while taking data in several frequency bands. The beam is “chopped” parallel to the scan direction at a frequency of $\nu = 6$ Hz to define the temperature “difference” assigned to each point. The bow-tie pattern arises due to the effect of sky rotation during the flight. The scan pattern is shown in Fig. 1, in a coordinate system in which
GUM is at the origin. At each point, the scan and chop directions are toward (for \( y < 0 \)) or away (for \( y > 0 \)) from the origin.

In order to compare theory and experiment it is necessary to calculate the predicted temperature autocorrelation function \( C_{\text{th}} \) for the scan. To begin let \( T(\theta, \phi) \) denote the temperature at a point on the sky:

\[
T(\theta, \phi) = Q \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi),
\]

where the \( a_{\ell m} \)'s are random variables whose distribution must be specified by a model and \( Q \equiv \langle Q_{\text{RMS}}^2 \rangle^{0.5} = Q_{\text{rms-PS}} [5,6] \) defines the overall normalization. In general, rotational invariance implies that

\[
\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_{\ell} \delta_{\ell \ell'} \delta_{mm'},
\]

where the angular brackets denote an ensemble average over the probability distribution for the \( a_{\ell m} \)'s, and \( C_{\ell} \) is normalized so that \( C_2 = 4\pi/5 \). For a pure Sachs–Wolfe, \( n = 1 \) spectrum, \( C_\ell^{-1} \propto \ell(\ell + 1) \). We compute the \( C_{\ell} \)'s for CDM models using power spectra provided by Sugiyama (e.g. [7]), which are essentially identical with those computed by Bond and Efstathiou (e.g. [8]). We restrict our study to standard CDM models with \( \Omega_0 = 1 \) and \( H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \), but allow a range of values for \( \Omega_\text{b} \).

It has become conventional in degree-scale experiments to express the sensitivity of the experiment to the underlying power spectrum in terms of a “window” or “filter” function \( W_\ell \). The autocorrelation function is then a sum of the \( C_{\ell} \) weighted with the \( W_\ell \) (see e.g. [9,4]). This approach is perfectly adequate for experiments in which the data are taken along a single linear scan, such as the SP91 data of Gaier et al. [10] or the MuP scan of MAX. However, it is not well suited to analyzing experiments in which the data span two dimensions, such as the GUM scan of MAX or multiple scans of SP91. The problem is that the chopping strategies used by these experiments define a position-dependent, preferred direction in the sky plane. Points which are separated by a vector parallel to the chop direction are generally more correlated than those which are separated by a vector perpendicular to it; furthermore these stronger correlations are negative when the separation angle is near the peak-to-peak chop angle [11]. This results in a strongly anisotropic autocorrelation matrix. This is illustrated in Fig. 2 where we show the autocorrelation function on the sky [as given by Eq. (7) below] for an ideal experiment in which the chop direction is held fixed (parallel to the \( y \)-axis) for simplicity.
The anisotropy makes it difficult to derive an analytic expression for the autocorrelation matrix, but it is easy to compute numerically. To proceed we define the beam-smoothed temperature in the direction \( \hat{n} \) as

\[
\Theta(\hat{n}) = Q \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n}) \exp[-1/2(\ell + 1/2)^2 \sigma^2],
\]

where \( \sigma = 0.425 \times 0.5 \) is the gaussian beam-width of the MAX antenna. The autocorrelation function for \( \Theta \) is then

\[
C_\sigma(\hat{n}_1, \hat{n}_2) \equiv \langle \Theta(\hat{n}_1) \Theta(\hat{n}_2) \rangle
\]

\[
= \frac{Q^2}{4\pi} \sum_{\ell=2}^\infty (2\ell + 1) C_\ell P_\ell(\hat{n}_1 \cdot \hat{n}_2) \exp[-(\ell + 1/2)^2 \sigma^2],
\]

which specifies the predicted correlation between two beams instantaneously separated by an angle \( \theta = \cos^{-1}(\hat{n}_1 \cdot \hat{n}_2) \) on the sky. Technically one should also include the contribution from the \( \ell = 1 \) term in this sum; however, due to the chopping of the beam, ignoring it introduces a negligible error in our final results. Including the effects of the beam chopping, we can write the temperature assigned by the experiment to the direction \( \hat{n} \) as

\[
\tilde{T}(\hat{n}, \hat{i}) \equiv \int_0^{1/\nu} dt \kappa(t) \Theta(\hat{n} \cos \alpha(t) + \hat{i} \sin \alpha(t)).
\]

Here \( \alpha(t) = \alpha_0 \sin(2\pi \nu t) \) accounts for the chopping motion of the beam; \( \alpha_0 = 0.65 \) for MAX. Also, \( \hat{i} \) is a unit vector lying along the chop direction (which implies that \( \hat{n} \cdot \hat{i} = 0 \)), and \( \kappa(t) \) is the weighting factor for the beam chop. For SP91, \( \kappa(t) = 2\nu \text{sign}(t) \). For MAX, \( \kappa(t) = k \nu \sin(2\pi \nu t) \), where \( k \) is determined as follows [12]. Take the temperature pattern in Eq. (1) to be \( T(\theta, \phi) = T_0 \) for \( 0 < \theta < \pi/2 \) (the northern hemisphere) and \( T(\theta, \phi) = 0 \) for \( \pi/2 < \theta < \pi \) (the southern hemisphere). Then for any direction vector \( \hat{\rho} \) in the \( x-y \) (equatorial) plane, Eq. (5) should give \( \tilde{T}(\hat{\rho}, \hat{z}) = T_0 \). This condition results in

\[
k^{-1} = \frac{1}{\pi} \int_0^{\pi/2} dr \sin r \text{erf}(\gamma \sin r) = \left( \frac{\gamma}{2\pi^{1/2}} \right)_{1}F_{1}(1/2, 2; -\gamma^2),
\]

where \( \gamma = \alpha_0 / \sqrt{2}\sigma \), erf is the error function, and \( _1F_1 \) is the confluent hypergeometric function. For \( \sigma = 0, k = \pi \); for the MAX values of \( \sigma \) and \( \alpha_0, \gamma = 2.16 \) and \( k = 3.34 \).

Now we can write the autocorrelation function for the MAX temperature “differences” as

\[
Q^2 C_{\theta h}(\hat{n}_1, \hat{i}_1; \hat{n}_2, \hat{i}_2) \equiv \left\langle \tilde{T}(\hat{n}_1, \hat{i}_1) \tilde{T}(\hat{n}_2, \hat{i}_2) \right\rangle
\]

\[
= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

\[
= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

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= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

\[
= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

\[
= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

\[
= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

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\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

\[
= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

\[
= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

\[
= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]

\[
= \int_0^{1/\nu} dt_1 \int_0^{1/\nu} dt_2 \kappa(t_1) \kappa(t_2) \times
\]

\[
C_\sigma(\hat{n}_1 \cos \alpha(t_1) + \hat{i}_1 \sin \alpha(t_1), \hat{n}_2 \cos \alpha(t_2) + \hat{i}_2 \sin \alpha(t_2))
\]
This integral has several symmetries which can be exploited when numerically evaluating the 165 × 165 autocorrelation matrix, $C_{th,ij}$, for the GUM scan.

3. Maximum Likelihood Analysis

We now turn to the limits which can be placed on $Q$, the normalization of the fluctuation spectrum. In accord with standard CDM models we consider underlying cosmological fluctuations with a gaussian probability distribution. We also assume that the experimental errors $\sigma_i$ for each $\tilde{T}_i$ are uncorrelated and gaussian distributed. The 165-point data set is binned finely enough that correlations introduced by the binning process should not be important. The unnormalized likelihood function for $Q$ is then given by

$$L(Q) \propto \frac{1}{\sqrt{\det K}} \exp \left[ -\frac{1}{2} \tilde{T}_i (K^{-1})_{ij} \tilde{T}_j \right],$$

where the matrix $K$ is

$$K_{ij} = Q^2 C_{th,ij} + \sigma_i^2 \delta_{ij}. \quad (9)$$

Eq. (8) assumes that the temperatures have no systematic errors. In fact, MAX has a possible systematic offset which has been removed from each of the 11 azimuthal scans of 15 data points. This offset removal then requires a modification of $K$. The simplest way to implement the constraint that the data in each azimuthal scan have zero weighted mean is to first change to a new basis $\tilde{T}_a' = R_{ai} \tilde{T}_i$, where $\tilde{T}_1', \tilde{T}_2', \ldots, \tilde{T}_{11}'$ are the weighted means of each of the 11 scans, and $R_{ai}$ is a matrix whose linearly independent rows are chosen so that each of the first 11 is orthogonal to each of the last 154. Then integrating over the 11 weighted means in Eq. (8) is equivalent to replacing Eq. (8) by

$$L(Q) \propto \frac{1}{\sqrt{\det M}} \exp \left[ -\frac{1}{2} \tilde{T}_a' (M^{-1})_{ab} \tilde{T}_b' \right],$$

where $M_{ab} = R_{ai} K_{ij} R_{jb}^{T'}$, and $a$ and $b$ each run only over the range $12, \ldots, 165$; that is, over the subspace orthogonal to the one spanned by the 11 weighted means.

Before we can use Eq. (10) to find the distribution of $Q$ predicted by the data, we must choose a prior distribution for $Q$; that is, we must decide whether equal intervals of $\log Q$, $Q$, $Q^2$, or some other monotonic function $f(Q)$, are equally likely a priori. Alternatively, one could decide that we actually have some prior information and use the COBE
measurement of $Q$ to fix the prior distribution, but we will not consider that approach in this Letter.

Since the allowed range of $Q$ is from zero to infinity, the conventional Bayesian choice (based on scale invariance arguments [13]) is to assume a prior distribution $f(Q) = \log Q$. For any data set with non-vanishing errors, $\sigma_i$, this choice causes the likelihood $L(Q)$ to diverge for $Q \ll \sigma_i$. This divergence can be removed by imposing a lower limit on $Q$. For the GUM data set we find that the final results are totally insensitive to the lower limit imposed for $Q$ once it is less than a few micro-Kelvin (but still non-zero).

Alternatively, one could take a maximum-likelihood approach and consider prior distributions which are uniform in some scaling variable. The usual choice is $f(Q) = Q$ (the “bias” parameter $b_\rho$ is proportional to $Q^{-1}$, so that $dQ = db_\rho/b_\rho^2$). However, there is no compelling reason to assume a prior distribution that is uniform in $Q$. It is just as natural, for example, to assume that the prior distribution is uniform in the power spectrum normalization $Q^2$. Note that for “good” data, the choice of prior should make only a small difference, so “prior dependence” gives us a handle on the constraining power of the data. In the context of small-scale CMB experiments, and assuming that the signal-to-noise ratio exceeds unity, we expect the data to be “good” when the solid angle on the sky which is covered by the experiment is much larger than that subtended by the correlation angle of the theoretical autocorrelation function [14]. Comparing Figs. 1 and 2, we see that this condition is fulfilled by the GUM data set.

In Fig. 3, we show the normalized likelihood $L(Q) df/dQ$ for four different choices of $\Omega_B$ and for $f(Q) = \log Q$. Additionally, for $\Omega_B = 0.01$ we show curves for $f(Q) = Q$ and $f(Q) = Q^2$. Clearly our expectation was correct: the effect of the prior distribution is not very significant, and is in fact smaller than the effect of varying $\Omega_B$ in the CDM model. In Fig. 3 we also show two lines corresponding to the COBE measurement of $Q = 17 \pm 5 \mu$K (1\(\sigma\) errors). In Table 1, we list the upper and lower limits at the 95\% confidence level for the four values of $\Omega_B$ with $f(Q) = \log Q$. We see that the GUM lower limit is within the COBE 1\(\sigma\) range for all values of $\Omega_B$, and reaches the COBE mean value for $\Omega_B \gtrsim 0.06$.

We also wish to compare CDM with the results of the MuP scan of the MAX experiment [3]. This scan consists of 21 data points taken in a line. We have used temperatures provided by the MAX group which correspond to the second component of a two-component fit, with the first component consisting of emission from 18 K dust [15].
These temperatures were the ones used in [3] to set upper limits on the observed CMB anisotropy. An offset has been removed from this data, and we have used an appropriately modified autocorrelation matrix. The resulting likelihood curves are also shown in Fig. 3, and the corresponding 95% confidence limits in Table 1. Note that the small $Q$ divergence in the likelihood function for $f(Q) = \log Q$ is more pronounced for the MuP data. For the purposes of computing confidence levels we have imposed a lower cutoff of 0.25 $\mu$K (where the likelihood function begins to rise significantly) on $Q$. As expected from the general considerations of [14], the MuP results are much more sensitive to the prior distribution than are the GUM results. Furthermore the MuP likelihood curves have larger widths than the GUM curves when measured in units of the corresponding mean value of $Q$. Thus the GUM results are more statistically robust than the MuP results. For $f(Q) = \log Q$ or $Q$, the 95% confidence upper limits on $Q$ from the MuP scan are just below the 95% confidence lower limits from the GUM scan. For $f(Q) = Q^2$, these limits overlap.

The most striking feature of Fig. 1 is that the MuP and GUM results nicely bracket the COBE normalization. In fact, if we compute likelihoods for the combined MuP+GUM data set, we find that the mean values of $Q$ always lie in the COBE $1\sigma$ range. We note again that the apparent inconsistency between the MuP and GUM results, when analyzed separately, could be resolved if there is some foreground contamination in the GUM data, and/or the two-component fit to the MuP data removed some of the actual CMB signal. This point of view must be considered speculative, since there is no clear candidate for a source of foreground contamination near GUM [1], and since the nonzero correlation between the two components in the fit to the MuP data could mean that part of the second component is in fact due to dust emission rather than CMB fluctuations [3]. Further data will be needed to resolve these issues.

4. Conclusions

We have analyzed data from the MAX experiment in the context of cold dark matter models of structure formation. An important feature of our analysis was the incorporation of the strong anisotropy due to the experimental chopping strategy in our theoretical autocorrelation function $C_{th}$ for the GUM scan. If we assume that all of the structure in the GUM data can be attributed to primordial microwave background fluctuations, then the data are consistent with CDM models normalized by the COBE results at the 95%
confidence level. However, the GUM results are marginally inconsistent with the MuP results for our preferred prior distribution and low $Q$ cutoff. This could be explained if there is a small amount of foreground contamination in the GUM data, and/or the two-component fit to the MuP data removed some of the actual CMB signal.

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| $\Omega_B$ | 95% CL GUM | 95% CL MuP | $C_{th}(0)$ |
|------------|-------------|-------------|-------------|
| 0.01       | 20, 40      | 5, 17       | 14.7        |
| 0.03       | 19, 37      | 4, 16       | 17.5        |
| 0.06       | 17, 34      | 4, 15       | 20.8        |
| 0.10       | 15, 31      | 3, 13       | 26.0        |

Table 1: The 95% confidence level lower and upper limits (90% enclosed) for $Q$ ($\mu$K), from the MAX-GUM and MAX-MuP scans for a range of $\Omega_B$, assuming $f(Q) = \log Q$ (see text). We also give $C_{th}(0)$ to show the scaling of the CDM predictions with $\Omega_B$.  

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Figure Captions

Fig. 1 The MAX-GUM scan pattern. The X’s indicate the locations where the data were binned (into 15 positions along 11 scans), in a coordinate system where Gamma Ursae Minoris (GUM) is at the origin, and the units are degrees on the sky. The scan and chop direction point either toward (for \( y < 0 \)) or away (for \( y > 0 \)) from the origin. For comparison, the circle has a radius equal to the Gaussian width \( \sigma \) of the beam, and the vertical line shows the size of the peak-to-peak beam chop.

Fig. 2 An illustrative plot of the theoretical autocorrelation function, \( C_{th}(\hat{n}_1, \hat{y}; \hat{n}_2, \hat{y}) \), where \( \hat{n}_1 \) points towards \((0,0)\) (at the back of the plot) and \( \hat{n}_2 \) points towards \((x, y)\); \( x \) and \( y \) are given in degrees on the sky. Note the strong anisotropy. For this plot only, the beam is always chopped in the positive \( y \) direction. \( C_{th} \) was computed assuming a CDM model with \( \Omega_B = 0.06 \).

Fig. 3 The likelihood function \( \mathcal{L}(Q) \, df/dQ \) vs. \( Q (\mu K) \), for the MuP and GUM scans of MAX, assuming a CDM model with \( \Omega_B = 0.10 \) (long dashed), 0.06 (short dashed), 0.03 (dotted), 0.01 (solid), and using the prior distribution \( f(Q) = \log Q \) (see text). Also shown are the likelihoods for \( \Omega_B = 0.01 \) with \( f(Q) = Q \) and \( f(Q) = Q^2 \) (the extra solid curves furthest right for each scan). The two vertical lines bracket the COBE preferred value (\( \pm 1\sigma \)).
