Robust entanglement generation by reservoir engineering

Christine A Muschik¹, Hanna Krauter², Kasper Jensen², Jonas M Petersen², J Ignacio Cirac³ and Eugene S Polzik²

¹ ICFO-Institut de Ciències Fotòniques, Mediterranean Technology Park, 08860 Castelldefels, Barcelona, Spain
² Niels Bohr Institute, Danish Quantum Optics Center QUANTOP, Copenhagen University, Blegdamsvej 17, 2100 Copenhagen, Denmark
³ Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, D-85748 Garching, Germany

E-mail: Christine.Muschik@icfo.es

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Abstract
Following a recent proposal (Muschik et al 2011 Phys. Rev. A 83 052312), engineered dissipative processes have been used for the generation of stable entanglement between two macroscopic atomic ensembles at room temperature (Krauter et al 2011 Phys. Rev. Lett. 107 080503). This experiment included the preparation of entangled states which are continuously available during a time interval of 1 h. Here, we present additional material, further-reaching data and an extension of the theory developed in Muschik et al (2011 Phys. Rev. A 83 052312). In particular, we show how the combination of the entangling dissipative mechanism with measurements can give rise to a substantial improvement of the generated entanglement in the presence of noise.

(Some figures may appear in colour only in the online journal)

1. Introduction
In a recent experiment [1, 2], a new technique for generating extremely robust and long-lived entanglement has been demonstrated following a proposal put forward in [3] (see also [4]). By means of reservoir engineering [5–10, 4, 11–26], entanglement has been produced purely dissipatively. Moreover, it has been shown how engineered dissipative processes in combination with continuous measurements can be used to create entanglement in a steady state. Using this method, entanglement between two macroscopic atomic ensembles⁴ has been maintained and verified for up to 1 h. This extends the time intervals during which event-ready entanglement of material objects can be provided by several orders of magnitude. In this paper, we give an extended description of this experiment and present additional supporting data. A detailed and rigorous derivation of the theoretical framework of the employed dissipative entangling mechanism can be found in [3]. Here, we also discuss an extension of this method which has been used in the experiment and includes measurements. We show how they can be used to improve the purely dissipative protocol and explain the basic working principle in detail.

The coupling of a quantum system to its environment, commonly referred to as dissipation, is traditionally considered to be a main problem impairing experiments involving quantum superposition states and the development of quantum technologies. Harnessing dissipative processes rather than aiming for eliminating their influence is a radically new concept and represents a paradigm shift in quantum information science. We show that even limited control of the coupling between the system and environment can enable one to turn a major problem into an asset. This change in perspective is not only of conceptual interest but also yields significant practical advantages. The protocol discussed here relies on engineered dissipation. More specifically, the coupling of a system with a reservoir is tailored such that the desired state is obtained as the steady state of the dissipative evolution. This way, the target state is reached independently of the initial conditions. Accordingly, this type

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⁴ Note that the term ‘entanglement between macroscopic objects’ does not refer to a macroscopic superposition since the generated entanglement corresponds to less than one e-bit.
of protocol does not require the precise initialization of the system in a well-defined state. The resulting quantum state can be maintained for long times since it is stabilized by the dissipative dynamics. This mechanism continuously drives the system into the desired entangled state, even in the presence of noise, which limit the coherence time of the quantum system. Thus, the use of engineered dissipation enables the realization of unlimited entanglement lifetimes, which is not achievable by traditional methods.

We consider here two macroscopic atomic ensembles at room temperature interacting with freely propagating coherent light. Quantum information can be encoded in collective atomic spin states which are unaffected by the thermal motion of the atoms. This system has been shown to provide an excellent platform for quantum memory schemes and the realization of light–matter interfaces [27–34]. In the protocol discussed here, two atomic ensembles are entangled by virtue of a dissipative mechanism which is induced by the application of a strong driving field. The generated entanglement can be accessed at any moment during an extended period of time, which makes it particularly useful for protocols, where it is not known in advance when the entangled state is needed (for example, if probabilistic subroutines are involved). If the entangled state is to be used, the driving field inducing the entangling mechanism is switched off before the actual protocol is run. Since entanglement is created between two ensembles, the resulting atomic state can either be used directly or be read out on demand using light–matter interface schemes [35, 36]. Another very interesting application is the use in continuous protocols, for example, in dissipative quantum repeater schemes [37]. This type of scheme requires continuous entanglement for establishing high-quality steady state entanglement over large distances.

The remainder of this paper is organized as follows. In section 2, we summarize the main results and explain the key features of the scheme. Section 3 is concerned with the creation of purely dissipative entanglement. We offer an intuitive explanation and data supporting this interpretation. We also compare the basic working mechanism to standard approaches and highlight the distinguishing features to the dissipative scheme discussed here. Thereafter, a hybrid method is described, where the dissipative mechanism is combined with continuous measurements on the light field. In section 4, we explain how monitoring of the scattered photons can lead to an improvement of the produced entanglement in the presence of noise. In section 5, we present additional experimental material and section 6 concludes the paper.

2. Overview and central results

In this section, we provide a brief overview to the method of dissipative entanglement generation put forward in [1, 2] and [3]. We explain the main idea, introduce the experimental setup and summarize our main results.

Key idea

As outlined in the introduction, the entangling mechanism employed here is invoked by reservoir engineering and drives the system into a unique inseparable steady state. In the absence of other decoherence mechanisms, the steady state of the system (corresponding to the reduced density matrix after tracing out the environment) can be a pure state. In our case, the reservoir consists of a continuum of electromagnetic vacuum modes, which provide a common environment for the two ensembles. The atomic system can be coupled to this environment in a controlled fashion by applying suitable laser fields. More specifically, we consider the setup shown in figure 1. Each atomic ensemble consists of a large number N of hydrogen-like atoms with an internal level structure with two ground states |↑⟩ and |↓⟩. Both ensembles are driven by a far-off-resonant y-polarized coherent field. This strong classical driving field induces effective ground state transitions |↑⟩ → |e⟩ → |↓⟩ and |↓⟩ → |e⟩ → |↑⟩, which involve the emission of y-polarized photons (cf. figure 1). This way, the classical driving field couples the atomic system to the bath of electromagnetic modes in y-polarization. The basic entangling mechanism can be understood by considering the y-polarized vacuum modes in the direction of the laser field with the wave-vector k\(_y\) and the rest of the modes separately. The latter give rise to the standard spontaneous emission and represent noise processes. The former are shared by both ensembles and therefore provide the desired common environment. In the setting considered here, emission into the forward direction is collectively enhanced for a large optical depth d [32]. Hence, these modes can successfully compete with all the others and the entangling processes happen on a faster time scale than the undesired ones.

In our case, the target state is a two-mode squeezed state which is entangled in the collective spin states of the two atomic ensembles |\(\Psi_{\text{EPR}}\rangle\). This state is reminiscent of the entangled quantum state introduced by Einstein, Podolski and Rosen (EPR) [38]. |\(\Psi_{\text{EPR}}\rangle\) is the simultaneous eigenstate with eigenvalue zero of two nonlocal operators A and B, \(A|\Psi_{\text{EPR}}\rangle = 0\), \(B|\Psi_{\text{EPR}}\rangle = 0\), where

\[
A = \mu J_{\gamma}^+ - \nu J_{\gamma}^- , \quad B = \mu J_{\gamma}^- + \nu J_{\gamma}^+ .
\]

\(J_{\gamma}^+\) and \(J_{\gamma}^-\) denote collective spin operators with \(J_x = \sum_{i=1}^N |\psi_i\rangle\langle\psi_i|\), \(\mu = \cosh(r)\) and \(\nu = \sinh(r)\), where r is the so-called squeezing parameter. This type of entangled state is the main working horse for applications in quantum information science with atomic ensembles and continuous variable systems in general [39]. Entanglement can be verified and quantified by the parameter \(\xi = \sum_{i,j} (2|\langle J_i|J_j\rangle|)^2 = (\mu - \nu^2)^2\), with \(\Sigma_i = \text{var}(J_{\gamma,1} - J_{\gamma,\Pi}) + \text{var}(J_{\gamma,1} - J_{\gamma,\Pi}), \xi < 1\) certifies the creation of an inseparable state [40, 41].

The existence of a steady state |\(\Psi_{\text{EPR}}\rangle\) with \(A|\Psi_{\text{EPR}}\rangle = B|\Psi_{\text{EPR}}\rangle = 0\) can be understood as an interference process where an (y-polarized) photon, which is emitted in the forward direction with a given frequency, could have been originated from either of the two ensembles. For a particular atomic state,

\[5\] In the famous example introduced by Einstein, Podolski and Rosen, the quantum state under consideration is a simultaneous eigenstate of the sum of positions \(x = x_1 + x_2\) and difference of momenta \(p = p_1 - p_2\) of two separate particles. This corresponds to an infinitely squeezed state with \(\text{var}(x_1) = \text{var}(p_1) = 0\), where the momenta/positions are perfectly correlated/anticorrelated.
ψ into two sideband modes centred around separation in frequency space (\omega_1 lower (red) sideband, respectively. For a sufficiently large transitions |↑⟩ ↔ |↓⟩ is the largest effective atomic transition rate for ground state which is applied along the \hat{x} direction (shown as wavy lines). The Zeeman shift \omega of the ground states leads to the emission of photons into two sideband modes centred around \omega_0 \pm \Omega, where \omega_0 is the frequency of the applied laser field. The vacuum modes in the \hat{x}-direction provide a common environment for the two atomic systems. Due to collective effects, the scattering of photons in this direction is enhanced.

It can be shown [3] that \rho_{EPR} = |\Psi_{EPR}\rangle \langle \Psi_{EPR}| is the unique steady state of this evolution. In order to obtain a two-mode squeezed state, the system is coupled to two reservoirs, which give rise to the jump operators A and B, respectively. To this end, the ensembles are placed in a homogeneous magnetic field which causes a Zeeman splitting of the atomic ground states \Omega. Due to the different ground state energies, photons are scattered into two different frequency bands centred around \omega_0 \pm \Omega, which we will refer to as the upper (blue) and lower (red) sideband, respectively. For a sufficiently large separation in frequency space (\Omega \gg \Gamma_{\text{Atomic}}, where \Gamma_{\text{Atomic}} is the largest effective atomic transition rate for ground state transitions |↑⟩ ↔ |↓⟩ [3]), the continua of modes in the lower and upper sideband can be treated as independent reservoirs (compare appendix A). The first (second) term in equation (2) is due to the interaction with the photons in the lower (upper) sideband. Note that the jump operators defined in equation (1) are nonlocal (i.e. involve atomic operators referring to both the first and the second ensemble) and can therefore give rise to an entangled steady state. As explained above, it originates from the interference between processes where a photon is emitted in the forward direction by the first or the second ensemble. Other processes can be included in the form of additional terms in the master equation \frac{d}{dt} \rho = \mathcal{L}_{\text{ent}}(\rho) + \mathcal{L}_{\text{noise}}(\rho), where \mathcal{L}_{\text{noise}}(\rho) summarizes undesired processes such as spontaneous emission, collisions and fluctuating magnetic fields. A key point lies in the fact that the rate of the entangling processes (\mathcal{L}_{\text{ent}}(\rho)) scales with the optical thickness (due to collective effects which originate from constructive interference involving each individual atom within a single ensemble), whereas the rate of the detrimental processes does not. Thus, for sufficiently optically thick samples, the creation of entanglement in a steady state is possible even in the presence of noise (see equation (B.1) in appendix B).

Setup and experimental results

The experiment is carried out using \(^{133}\text{Cs}\) vapour at room temperature. The two-level subsystem is encoded in the two outermost hyperfine levels of the 6S_{1/2} ground state within the manifold with total spin \( F = 4 \). We identify |↑⟩_I \equiv |F = 4, m_F = 4\rangle, |↓⟩_I \equiv |F = 4, m_F = 3\rangle and |↑⟩_II \equiv |F = 4, m_F = -3\rangle, |↓⟩_II \equiv |F = 4, m_F = -4\rangle (where \( m_F \) is the magnetic quantum number). The atoms are confined in cubic glass cells which are separated by a distance of approximately 0.5 m and have a spatial extent of 2.2 cm. Each cell contains \( 10^{12} \) atoms and is equipped with a paraffin-based spin-preserving coating. The experimental setup is sketched

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\[^{133}\text{Cs}\] is an alkali atom with nuclear spin \( I = 3/2\). The ground state is split into two manifolds with total spin \( F = I \pm 1/2\).
in figure 2(a). The two ensembles are prepared in oppositely oriented coherent spin states (CSS). This is achieved by optically pumping the atoms of the ensembles into $m_F = \pm 4$ in the $\hat{x}$-direction, respectively. The circularly polarized pump lasers are depicted in blue and figure 2(b) shows the atomic level structure, indicating laser frequencies and polarization. The strong probe beam which is initially polarized in the $\hat{y}$-direction transverses the atoms in the $\hat{z}$-direction. Behind the cells, the detection system is set up.

In [1], two types of results have been obtained. Firstly, entanglement has been created purely dissipatively, demonstrating that this type of process can be harnessed for tasks in quantum information science. In this series of experiments, entanglement has been maintained for a time span, which is an order of magnitude longer than the time intervals for which entanglement could be sustained in this system so far [32]. Entanglement has been obtained in a quasi-steady state rather than in a true steady state due to the multi-level structure of caesium. As opposed to the two-level model discussed above, atoms can leave the two-level system by undergoing transitions to other internal states. The entangling processes are fast compared to these undesired transitions. The desired dynamics with respect to the two-level subsystem reaches a steady state, but since it is superposed by the slow detrimental processes involving atom losses, entanglement disappears.

In a second series of experiments, strong pump and repump fields have been applied in order to transfer the atoms back, which left the two-level system. These incoherent fields lead to increased noise contributions, which prevents the generation of a purely dissipative steady state in this particular setting. However, by combining the dissipative mechanism with continuous measurements on the light field, a true steady state has been obtained. In the following, we consider the generation of entanglement by dissipation in more detail.

3. Purely dissipative entanglement

This section is concerned with the purely dissipative generation of entanglement. The basic mechanism is explained and further substantiated by additional experimental data. Moreover, the perspectives for obtaining steady state entanglement for multi-level systems are discussed.

Entanglement creation by virtue of interference

As mentioned earlier, the underlying mechanism can be understood as an interference effect in the dissipative channel. Due to destructive interference, no photon is scattered into the forward direction in the steady state. Accordingly, no measurement needs to be performed on the light field. Additional evidence that the entanglement is produced by the collective dissipation process is provided by its dependence on the dephasing between spin coherences of the two ensembles. To demonstrate this dependence, we introduce a detuning $\delta \Omega = \Omega_I - \Omega_{II}$ of the Larmor frequencies of the two ensembles by tuning the bias magnetic fields. Figure 3(a) shows that already at $\delta \Omega = 20 \text{ Hz}$ entanglement disappears. This can be understood as a consequence of the ‘which way’ information (or ‘which ensemble’ information in this case) provided by the distinguishability of the photons scattered from the two ensembles into the sideband modes $\omega_{LL} \pm \Omega_I$ and $\omega_{LL} \pm \Omega_{II}$. Hence, the atomic samples no longer share the same reservoir and entanglement disappears.
Note that the effect demonstrated in the experiment cannot be understood in terms of photons which are spontaneously emitted by the first ensemble interacting with the second one. Due to the large detuning ($\Delta = 850$ MHz), the interaction of a single photon emitted by the first ensemble with atoms in the second one is negligible in the parameter regime accessible in the experiment. Here, atomic transitions are to a very good approximation only induced by the classical driving field (accordingly, the input–output relations for atoms in the second sample are identical with those in the absence of the first ensemble).

**Comparison of dissipative entanglement generation with other methods**

There exist a large variety of methods for creating entanglement between two quantum systems. In particular, several methods have been devised and demonstrated for entangling atomic ensembles. Even though some schemes, which have been experimentally implemented share similarities with the ones described here, they are fundamentally different. In the following, we explain this in detail and highlight the features of the method realized here in comparison with previously demonstrated ones.

In standard approaches, which are based on a coherent interaction followed by a measurement [27, 42, 41, 43–50, 32], two atomic ensembles A and B are prepared in specific pure states $|a\rangle_A$ and $|b\rangle_B$. An additional system, E, which typically corresponds to certain modes of the electromagnetic field, is also initialized in a specific state, for example, the vacuum $|0\rangle_E$. For appropriately chosen external parameters such as the frequency and polarization of applied laser fields, the interaction of system E with A and B gives rise to an entangled state $|\Psi\rangle = U(a, b)_{AB}|0\rangle_E$. If the system E is measured, e.g., using a beam splitter and single-photon detectors, or by means of homodyne detection, the state of systems A and B is projected onto an entangled state $|\Phi(e)\rangle_{AB}$. This state depends on the outcome of the measurement e. If no measurement is performed (which corresponds to averaging with respect to the possible measurement outcomes) the resulting state is not entangled. For instance, the DLCZ protocol [42] yields a separable state if the photons emitted by the ensembles are not detected.

Dissipative methods can be described as follows. $|a\rangle_A$, $|b\rangle_B$ and $|0\rangle_E$ denote again the initial states of systems A, B and E, respectively. Due to the interaction of system E with A and B, the state $|\Psi(t)\rangle = U(t)|a, b\rangle_{AB}|0\rangle_E$ is created, where the dependence of the resulting quantum state on the time $t$ is explicitly indicated. Under ideal conditions, i.e., if systems A and B do not couple to other environments, the interaction of A and B with E can be engineered such that the atomic system evolves towards an entangled state. In contrast to the schemes described above, the implementation of this entangling dynamics does not require measurements on system E. This type of behaviour can occur if system E possesses an infinite number of degrees of freedom, such that a non-unitary dynamics drives the system towards a fixed state. Due to this property, E is typically referred to as the environment and the corresponding interaction with systems A and B is referred to as a dissipative process. Dissipative phenomena of this kind are best described by means of master equations. To this end, the environment is traced out and an equation for the reduced density operator of systems A and B, $\rho$, is derived as described in appendix A. In the presence of other environments, the dissipation induced by the coupling of A and B to system E can still create entanglement with a lifetime, which exceeds the decoherence times due to these extra noise sources significantly, if the corresponding (uncontrolled) coupling is sufficiently weak. Note further that typically, noise processes can be included in the master equation description as is done in this work.

In the experiment discussed here, entanglement induced by dissipation has been observed. In particular, in contrast to approaches which have been previously implemented, entanglement is obtained without using measurements on the quantum state of the environment. Furthermore, systems A and B remain entangled for a duration which is an order of magnitude longer than previously measured entanglement lifetime. In the absence of the dissipative process, the measured entanglement lifetime is limited to 2.5 ms due to the remaining noise sources such as collisions or inhomogeneities of the applied magnetic fields.

Dissipative methods exhibit another distinctive feature, which is present for an ideal two-level system (cf [3]), but not in the multilevel description of the experiment. For long times $t \to \infty$, systems A and B decouple from the environment E, $|\Psi(t)\rangle \to |\Phi\rangle_{AB}|E(a, b, t)\rangle_E$ under ideal conditions, i.e., in the absence of additional noise sources. Remarkably, the desired state $|\Phi\rangle_{AB}$ is reached irrespective of the initial state of systems A and B which can be highly mixed. Moreover, except for an initial waiting time, no special timing is required. This behaviour is again due to the fact that E possesses an infinite number of degrees of freedom, which guarantees that revival effects are not present. This way, entropy is transferred from the system to the environment, which drives A and B into a particular steady state, which depends only on the engineered coupling.

Using dissipative methods, a mixed but still entangled steady state can be reached even in the presence of additional noise sources, as long as the coupling of A and B to other environments is sufficiently weak compared to the engineered dissipative processes. This opens up the possibility of keeping systems A and B entangled for arbitrarily long times.

**Perspectives for creating purely dissipative steady state entanglement in multi-level systems**

Below, we investigate the possibility of generating purely dissipative steady state entanglement in atoms with multi-level ground states. The main reason why the system in

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7 A measurement yielding the result e, produces the quantum state $\rho_e$. The situation where no measurement is performed corresponds to the situation where the measurement result is unknown. The resulting mixed state is given by $\tilde{\rho} = \sum \rho(e)\rho_e$, where $\rho(e)$ is the probability to obtain the measurement result e and the sum $\sum$ runs over all possible measurement results.

8 In the experiment reported here, the light field is measured continuously due to technical reasons. However, the measurement results are not used in the first set of experiments, where entanglement is created purely dissipatively. This corresponds to a situation where the measurement outcomes are not known or, equivalently, no measurement is performed.
the experiment reported on in [1] does not display purely
dissipatively generated entanglement in a steady state is
the depopulation of the relevant two-level subsystem due to
spontaneous emission which transfers the atoms into other
Zeeman levels. The depopulation of the relevant levels can be
avoided by applying strong laser fields, which transfer atoms
back. However, these fields introduce additional decoherence
processes which inhibit the creation of entanglement in a
steady state. This problem can be circumvented by increasing
the optical depth of the atomic ensembles such that the
entangling dissipative process prevails over the noise processes
and dominates the dynamics.

We consider this scenario by including additional \( \sigma_{\pm} \)
polarized pump and repump fields, which induce resonant
transitions with \( \Delta n_F \pm 1 \) in the first/second ensemble in the
model. Pump fields drive transitions within the manifold of
atomic states with \( F = 4 \) and repump fields transfer atoms
from states with \( F = 3 \) back to \( F = 4 \). As explained in
appendix C, we estimate the effect of these fields using a
simplified model, which has been used in [1] to fit
the experimental data. Figure 3(b) shows the predicted time
evolution of entanglement for \( d = 55,100,150 \) in the
presence of both pump and repump fields (in [1], an optical
depth of \( d = 55 \) was used). The other parameters used in
this calculation take values close to the ones used in
[1]. The theory predicts that under the present maximal
optical depth \( d = 55 \), the steady state atomic variance is
just slightly above the separability criterion (this has also been
confirmed experimentally). However, the experimental
realization of purely dissipative steady state entanglement
should be feasible along two possible routes. Firstly, atoms
possessing two-level electronic ground states can be used, for
example, ytterbium\(^{171}\)Yb [51, 52]. In this case, the two-level
theory formulated here can be directly implemented, avoiding
additional dynamics which leads to the growth of \( \xi \) with time.
Alternatively, a true dissipatively generated steady state using
multi-level ground states can be achieved for higher optical
depths, which can be obtained, for example, by placing the
atoms inside a low finesse optical cavity.

In [1], we devised and implemented an alternative
approach, which enables the creation of entanglement
which persists for arbitrarily long times. This alternative
approach combines the dissipative mechanism with continuous
measurements as explained below.

4. Dissipative entanglement assisted by
measurements

In the following, we show by means of a simple model how
measurements on the light field can improve the generation
of entanglement under the dissipative dynamics described
above in the presence of noise sources. In the basic model
employed here to illustrate the relevant effects, the Holstein–
Primakoff approximation [53] is used to describe the atomic
system and noise is included in the form of decay of the
transverse spin components \( J_y \) and \( J_z \) at a rate \( \gamma_{\text{extra}} \)
(a more detailed discussion is to be published elsewhere).
In this subsection, the dissipative generation of entanglement
assisted by measurements is explained in terms of input–
output relations, since this approach is more illustrative than
the master equation formalism employed in section 2. Both
descriptions are equivalent and yield the same results.

For large, strongly polarized atomic ensembles, collective
spins can be described by bosonic modes in terms of the
quadratures

\[
X_{\text{int}} = J_{y,\text{int}}/\sqrt{|J_{x,\text{int}}|}, \quad P_{\text{int}} = \pm J_{z,\text{int}}/\sqrt{|J_{x,\text{int}}|}.
\]

Light propagates in the \( \hat{z} \)-direction (see figure 1) and
interacts with the atomic ensembles. We consider here a one-
dimensional model which includes only light scattered in the
forward direction. Processes corresponding to scattering of
photons into other directions enter in the form of noise. Light is
characterized in terms of spatially localized modes [54, 55]

\[
y(z) = \frac{1}{\sqrt{4\pi}} \int_b \, d\omega (a_\omega e^{i(\omega - \omega_0)z} + \text{h.c.}),
q(z) = -\frac{i}{\sqrt{4\pi}} \int_b \, d\omega (a_\omega e^{i(\omega - \omega_0)z} - \text{h.c.}),
\]

where \( c \) is the speed of light. \( b \) and \( \omega_0 \) are the bandwidth and
central frequency of the applied laser field. The operators for
spatially localized field modes \( y(z) \) and \( q(z) \) obey the canonical
commutation relation \( [y(z), q(z')] = i\hbar \delta(z - z') \), where the
deltafunction has a width of the order of \( c/b \). The spatial argument \( z \)
refers to the distance along the propagation direction \( \hat{z} \). Atoms and
light interact according to \( H = H_A + H_L + H_{\text{int}} \), where
\( H_A = \frac{\gamma}{2} (X_I^2 + P_I^2) - \frac{\gamma}{2} (X_{\text{int}}^2 + P_{\text{int}}^2) \) describes the Zeeman splitting
of the atomic ground states. Due to the applied magnetic field,
the transverse components of the collective spin described by
\( X_{\text{int}} \) and \( P_{\text{int}} \) precess at the Larmor frequency \( \Omega \). \( H_I \) represents
the free propagation of light \( \frac{\Delta}{2} y(z) = i[H_{\text{int}, y(z)}] \equiv -c \frac{\Delta}{2} y(z) \).
The interaction of the light field with two pointlike atomic
ensembles [56] located at \( z = 0 \) and \( z = R \) is given by

\[
H_{\text{int}} = \sqrt{2\gamma} \left( \frac{1}{Z} X_I y(0) + Z P_I q(0) \right)
\]

\[
+ \sqrt{2\gamma} \left( \frac{1}{Z} X_{\text{int}} y(R) + Z P_{\text{int}} q(R) \right),
\]

where \( Z = \mu + v \) and \( 1/Z = \mu - v \). We assume in the
following that the distance \( R \) between the ensembles can be
neglected, which is justified for \( \gamma_{\text{Atomic}} R \ll c \), where \( \Gamma_{\text{Atomic}} \)
is the effective rate at which transitions between ground states
occur, and \( L^2 k_L \gg R \), where \( L \) is the spatial extent of an atomic
ensemble and \( k_L \) is the wave vector of the driving field.
The former is a necessary condition to neglect retardation effects,
while the latter is used in the calculation of averaged emission
rates for fast moving atoms (compare appendix A and [3]).
Both conditions are well fulfilled for the parameter regime
of the experiment, where the effective decay rates are of the
order of few ms and \( R \) is on the order of a meter \( (L \) is about
2 cm and \( k_L \) of the order of \( 10^7 \text{ m}^{-1} \)). The resulting Heisenberg
equations can be solved by changing to a coordinate system
rotating at the Larmor frequency \( \Omega \) and performing the variable
transformation \( \xi = ct - z \) such that \( \tilde{y}(\xi, t) = y(ct - \xi, t) \). We
introduce exponential \( \cos(\Omega t) \) and \( \sin(\Omega t) \) modulated light functions

\[
\begin{align*}
y^\text{in}_{e,\pm} &= \frac{1}{N_e} \int_0^T e^{\pm \Omega t} \cos(\Omega t) \hat{y}(\xi, 0), \\
q^\text{in}_{e,\pm} &= \frac{1}{N_e} \int_0^T e^{\pm \Omega t} \sin(\Omega t) \hat{q}(\xi, 0),
\end{align*}
\tag{3}
\]

with \( N_e = \frac{2\sqrt{\gamma}}{\sqrt{1-\gamma^2}} \) and \( N_\pm = \frac{2\sqrt{\gamma}}{\sqrt{1-\gamma^2}} \). \( y^\text{in}_{e,\pm} \) and \( e^\text{in}_{e,\pm} \) are defined analogously. We assume that the Larmor precession is fast compared to the atomic evolution \( (\Omega T \gg 1) \), such that the operators describing the sin and cos-modulated light modes are canonical and independent, \( [y^\text{in}_{e,\pm}, q^\text{in}_{e,\pm}] = [y^\text{in}_{e,\pm}, q^\text{in}_{e,\pm}] = i \) and \( [y^\text{in}_{e,\pm}, q^\text{in}_{e,\pm}] = [y^\text{in}_{e,\pm}, q^\text{in}_{e,\pm}] = 0 \). This yields

\[
\begin{align*}
X^\text{out}_{e/s} &= e^{-\gamma T} X^\text{in}_{e/s} + Z \sqrt{1 - e^{-2\gamma T}} Q^\text{in}_{e/s}, \\
Q^\text{out}_{e/s} &= e^{-\gamma T} Q^\text{in}_{e/s} - \frac{1}{Z} \sqrt{1 - e^{-2\gamma T}} P^\text{in}_{e/s}, \\
y^\text{out}_{e/s} &= e^{-\gamma T} y^\text{in}_{e/s} + Z \sqrt{1 - e^{-2\gamma T}} P^\text{in}_{e/s}, \\
q^\text{out}_{e/s} &= e^{-\gamma T} q^\text{in}_{e/s} - \frac{1}{Z} \sqrt{1 - e^{-2\gamma T}} X^\text{in}_{e/s},
\end{align*}
\tag{4}
\]

where \( X_t = \frac{1}{\sqrt{2}} (X_t + X_{ti}) \), \( P_e = \frac{1}{\sqrt{2}} (P_t + P_{ti}) \) and \( X_t = \frac{1}{\sqrt{2}} (P_t - P_{ti}) \) used. As a next step, we include continuous measurements on the light field and consider the corresponding time evolution of the atomic state in the Schrödinger picture. The continuous interaction and measurement process shown in figure 1 is illustrated schematically in figure 4(a) in a discretized way. Spatially localized light modes correspond here to infinitesimally short pulses of duration \( \tau \approx 1/b \) (where \( b \) is the bandwidth of the incident laser field as explained above), which interact successively with the atomic system. Each of these spatially localized light modes is initially in the vacuum state, such that the quantum state at time \( t = n \tau \) is given by \( |\Psi(t)\rangle\rangle|0\rangle_{L_{n+1}} \), where \( |\Psi(t)\rangle \) denotes the atomic state at time \( t \). Then, atoms and light are subject to an entangling interaction resulting in the quantum state \( e^{-iH_T} |\Psi(t)\rangle \rangle|0\rangle_{L_{n+1}} \). Finally, the \( y \)-quadrature of the light field is measured, yielding the measurement outcome \( y_n \) such that \( |\Psi(t + \tau)\rangle \rangle = \int L_{n+1} |y_n| e^{-iH_T}|0\rangle_{L_{n+1}} |\Psi(t)\rangle \)\rangle, where \( P(y_n) \) is the probability to obtain the result \( y_n \). The resulting expression can be expanded up to first order in the parameter \( \tau \) yielding a differential equation for the time evolution of the atomic state. The atomic state obtained after the measurement depends on the measurement outcome \( y_n \). We consider here Gaussian quantum states, i.e. states with a Gaussian–Wigner function. These states are fully characterized by their first and second moments; prominent examples include coherent as well as two-mode squeezed states. The Gaussian character of a state is preserved under the evolution of Hamiltonians, which are at most quadratic in the system operators and under Gaussian measurements such as homodyne detection. Since we consider all interactions and measurements to be of this kind and all states to be Gaussian, the entanglement of the resulting state is completely determined by the atomic variance \( \text{var}(P_{e/s}) \), which does not depend on \( y_n \) \[57\]. Therefore, the resulting entanglement is independent of the measurement outcome. If the measurement results are traced out \( (\rho(t + \tau) = \sum_{y_n} P(y_n) M_{y_n}^f, \text{ where } M_{y_n}^f = L_{n+1} |y_n| e^{-iH_T}|0\rangle_{L_{n+1}}, \) and the resulting expression is evaluated to first order in \( \tau \), the master equation used in section 2 and appendix A is recovered if the spin operators are replaced by creation and annihilation operators within the Holstein–Primakoff approximation \[53\].

The whole process can be conveniently described by means of the Gaussian formalism, where atomic states are expressed in terms of their covariance matrix \( \Gamma_\alpha \) and displacement vector \( \mathbf{D} \), which display the second moments (variances and covariances) and first moments (mean values) of the system, respectively. In particular, this formalism allows one to easily calculate the variances of atomic quadratures after the Gaussian measurement of the \( y \)-quadrature of the light field \( \text{var}(P_{e/s}) \) at the end of each time step depending on the variance prior to the measurement

\[
\text{var}(P_{e/s})_{\text{cond}} = \text{var}(P_{e/s}) - \frac{(P_{e/s} y_{c/s} + y_{c/s} P_{e/s})^2}{4 \text{var}(y_{c/s})}, \tag{5}
\]

where \( y_{c/s} \) and \( q_{c/s} \) refer here to the localized light mode interacting with the ensemble in the \( n \)th time step and \( \gamma_T \ll 1 \) is assumed. This way, a differential equation for the squeezed atomic variances is derived. In the ideal case,

\[
\text{var}(P_{e/s})_{\text{cond}}(t + \tau) = \text{var}(P_{e/s})_{\text{cond}}(t) + \text{var}(P_{e/s})_{\text{cond}}(t)(1 - \text{var}(P_{e/s})(t) Z^2) \gamma_T \tau,
\]

which yields

\[
\text{var}(P_{e/s})_{\text{cond}}(t) = \frac{1}{e^{-\gamma_T^2} (\text{var}(P_{e/s})(0))^{-1} + Z^2(1 - e^{-\gamma_T^2})},
\]
whereas in the absence of measurements,
\[ \text{var}(P_{\epsilon}(t)) = e^{-\gamma t} \text{var}(P_{\epsilon}(0)) + \frac{1}{Z^2} (1 - e^{-\gamma t}). \]

Both time evolutions result in a steady state with \( \text{var}(P_{\epsilon}(t)) = 1/Z^2 = (\mu - \nu)^2 \), since atoms and light decouple for \( t \rightarrow \infty \). Accordingly, the steady state entanglement cannot be improved by means of measurements on the light field in the ideal case. The situation is different in the presence of noise sources, which prevent the decoupling of atoms and light. In this case, residual atom–light correlations persist in the steady state and measurements on the light field can be used to improve the entanglement. Here, we illustrate this effect by including atomic transverse decay at a rate \( \gamma_{\text{extra}} \). If the y-quadrature of the scattered light field is measured, one obtains
\[ \xi_{\text{cond.}} = \frac{1}{Z^2} \left( 1 - \frac{\gamma_{\text{extra}}}{\gamma} + \frac{1}{\gamma} \left( 1 - \frac{\gamma_{\text{extra}}}{\gamma} \right)^2 + 4Z^2 \frac{\gamma_{\text{extra}}}{\gamma} \right). \]

Figure 4(b) shows that for \( \gamma_{\text{extra}} > 0 \), the steady state entanglement described by this equation is higher than the steady state if no measurements are performed, which is given by
\[ \xi_{\infty} = \frac{1}{Z^2} \frac{\gamma}{\gamma + \gamma_{\text{extra}}}. \]

Note that, in principle, all measurement results \( y(t) \) obtained during the continuous measurement procedure could be used to perform feedback operations which stabilize the atomic state at a certain position in phase space. However, this is not necessary here, since the atomic quantum state at time \( t \) depends only on the recent history of the measurements, i.e. on \( y(t') \) for \( t_s \leq t' < t \), where \( t_s \) is the time it takes to reach the steady state. For the dissipative processes considered here, the atomic state \( \rho(t) \) is memoryless regarding events which occurred in a time interval longer than \( t_s \). Thus, only measurement results obtained during a fixed time interval \( t_s \), which is independent of \( t \), are needed to localize the atomic state in phase space.

5. Experimental atomic state reconstruction

The experimental setup is depicted in figure 2 and described in the caption and the surrounding text. This section is focussed on the characterization of the atomic state via measurements on light which has interacted with the atomic ensembles.

The light observable of interest is the Stokes operator \( S_2 \) given by the difference of the number of photons polarized in \( \pm 45^\circ \). In the specific setting discussed here, the probe beam is strongly polarized in the \( \hat{y} \)-direction and the \( \hat{x} \)-polarization represents the polarization mode of interest for measurement as discussed in section 4. Then, the Stokes operator \( S_2 \approx \sqrt{\phi}/Z \cdot y \), where \( \phi \) is the photon flux. \( S_2 \) can be measured with polarization homodyning techniques as depicted in figure 2. The light beam is sent through a halfwave plate and a polarizing beam splitter (PBS). The signals from the detectors situated in the output ports of the PBS are subtracted and the difference signal is analysed at the Larmor frequency \( \Omega \) with a lock-in amplifier, since we are interested in detecting a signal from the spins in the rotating frame. Additionally the measurement outcome can be weighted with suitable mode functions \( f(t) \) and after suitable normalization we are thus able to measure the light observables \( y_{c,-} \) or \( y_{c,+} \) defined in equation (3).

5.1. Atomic state reconstruction

Now the variances of the collective atomic operators \( P_{\epsilon}^c = 1/\sqrt{2}(P_{\epsilon}^c + P_{\epsilon}^\Pi) \) and \( P_{s}^c = 1/\sqrt{2}(X_{s}^c - X_{s}^\Pi) \) can be found from measurements on the transmitted light. From the variances of the outgoing operators \( y_{c,-} \), using the input–output relations given in equation (4), it follows that
\[ \text{var}(P_{\epsilon}^c) = \frac{1}{\kappa^2} \left( \text{var}(y_{c,-}) - \sigma_\kappa^2 \left( 1 - \frac{\kappa^2}{Z^2} \right) \right), \]
\[ \text{var}(P_{s}^c) = \frac{1}{\kappa^2} \left( \text{var}(y_{s,-}) - \sigma_\kappa^2 \left( 1 - \frac{\kappa^2}{Z^2} \right) \right), \] (6)

where \( \sigma_\kappa^2 \) is the shot noise of light, assuming that the incoming light is in a coherent state. The coupling constant is defined as \( \kappa = Z \sqrt{1 - e^{-\gamma_2 T}} \). The normalized EPR variance of atomic noise is \( \xi \equiv \text{var}(P_{\epsilon}^c) + \text{var}(P_{s}^c) = \text{var}(P_{\epsilon}^c + P_{\epsilon}^\Pi)/2 + \text{var}(X_{s}^c - X_{s}^\Pi)/2 \).

5.2. Including decay and detection efficiency

To include the decay of the atomic spin due to spontaneous emission, magnetic field instabilities, etc into the input–output equations (equation (4)), we assume a decay with the rate \( \gamma_{\text{extra}} \) towards the CSS. Since the interaction times which are used to perform the read out are short, this is an adequate approximation. The atomic part of the input–output equations becomes
\[ P_{\epsilon\epsilon} = P_{\epsilon\epsilon}^0 \cdot e^{-\gamma T} + \frac{1}{Z^2} e^{-\kappa^2/2} + \varepsilon \sqrt{1} - e^{-\gamma_2 T} \cdot F_{\epsilon\epsilon}, \]
\[ X_{\epsilon\epsilon} = X_{\epsilon\epsilon}^0 \cdot e^{-\gamma T} + \kappa q_{\epsilon\epsilon} \varepsilon \sqrt{1} - e^{-\gamma_2 T} \cdot F_{\epsilon\epsilon}, \] (7)

with the two-cell noise operators \( F_{\epsilon\epsilon} = 1/NF \int t^0 e^{-\gamma(T-t)} F(t) dt \) and \( F_{\epsilon\epsilon}^0 = \frac{1}{2} \) and \( \varepsilon^2 = \gamma_{\text{extra}}/\gamma' \). The relevant light mode is exponentially growing with the total decay rate \( \gamma = 1/T_2 = \gamma_s + \gamma_{\text{extra}} \). The coupling constant is reduced due to the decay and defined as \( \kappa = Z \sqrt{1 - e^{-\gamma^2 T}} \).

Also the equations for the light are adjusted accordingly:
\[ y_{c,-} = \varepsilon^2 y_{c,-} + \varepsilon \sqrt{1} - e^{-\gamma_2 T} \cdot F_{\epsilon\epsilon}, \]
\[ q_{c,-} = \varepsilon^2 q_{c,-} + \varepsilon \sqrt{1} - e^{-\gamma_2 T} \cdot F_{\epsilon\epsilon}, \]
\[ -\kappa Z^2 \sqrt{1} - e^{-\gamma_2 T} \cdot F_{\epsilon\epsilon}, \]
\[ -\kappa Z^2 \sqrt{1} - e^{-\gamma_2 T} \cdot F_{\epsilon\epsilon}, \] (8)
The nonorthogonal exponentially growing and falling light modes are now mixed due to the decay. The reconstruction equation (6) must then be adjusted accordingly:

\[
\text{var}(P_{\text{c}}^m) = \frac{1}{\kappa^2} \left( \text{var}(y_{\text{out}}) - U^2 \cdot \sigma_{\text{in}}^2 - V^2(F_t^2) \right),
\]

\[
\text{var}(X_{\text{c}}^m) = \frac{1}{\kappa^2} \left( \text{var}(y_{\text{out}}) - U^2 \cdot \sigma_{\text{in}}^2 - V^2(F_t^2) \right),
\]

with the corrected, reduced coupling constant and \(U^2(\kappa^2, T_2)\) and \(V^2(\kappa^2, T_2)\) which can be calculated directly from equation (8)\textsuperscript{10}. To extract the atomic noise from the light noise measurements, it is therefore only necessary to know the coupling constant \(\kappa\) for which a measurement procedure is explained below and the decay time \(T_2\) which can also be easily measured.

Additionally, the detection efficiency \(\eta = 0.84(4)\) which arises from light losses and imperfect detection can be included by assuming a beam-splitter with transmission \(\eta\).

### 5.3. Measurement of the coupling strength

The most important experimental parameter for the reconstruction is the coupling constant \(\kappa\). It would be possible to determine \(\kappa^2\), by performing noise measurements on known atomic states, e.g. the CSS or the thermal atomic state. However, imperfect state preparation or additional noise sources can spoil such measurements. The approach we implement here is therefore based on measurements of mean values as opposed to noise measurements. The modus operandi is to transfer a coherent light state with a known displacement to the atoms and then read out the atomic state [58, 33].

First, a pulse is sent through two oppositely oriented atomic samples with a displacement in \(q_{\text{c}}\), so in \(S_1\). Following equation (4) and assuming that the atoms possess no initial displacement, this leaves the atomic sample with a mean value in the X-quadratures:

\[
X_{\text{c}} = \kappa (X_{\text{c}}^\text{1st}).
\]

To be able to read out those atomic mean values via a measurement on \(y_{\text{c}}\), and thus gain information on \(\kappa\), we apply a \(\pi/2\)-pulse to the atomic spin rotating \(X\) into \(P\). This can be done by adding a magnetic field in the \(\hat{x}\)-direction, so that the spins rotate a little faster or slower in between the pulses and \(J_{z,1} \rightarrow J_{z,2}\) and \(J_{z,3} \rightarrow J_{z,4}\), leading to \(X_{\text{c}} \rightarrow P_{\text{c}}\). Then, we send a second light pulse for the readout. The outcome of the light measurement of the second pulse reveals

\[
\langle y_{\text{c}}\rangle_{\text{out}} = \kappa (\langle y_{\text{c}}\rangle_{\text{in}}) = \kappa^2 (\langle y_{\text{c}}\rangle_{\text{in}}). \tag{11}
\]

The coupling strength can be calculated as \(\kappa^2 = \langle y_{\text{out}}^2 \rangle / \langle y_{\text{c}}^2 \rangle\).

The displaced coherent light states are produced with the help of an electro optical modulator (EOM) [59]. The strongly polarized beam is sent through an EOM whose optical axis

\textsuperscript{10}Note that the exponentially growing and falling modes are not orthogonal. Taking this into account: \(U^2 = e^x + e^{-2T_f}(1 - e^x)\), \(V^2 = 2e^{-2T_f}f_T(T_f)\) and \(\gamma = e^{-2T_f}f_T(T_f)\)Chcch(T_f) \(\approx 0\), \(e^x\) can be easily extracted from \(\kappa^2\), \(e^x = (1 - \kappa^2)/(1 - e^{-2T_f}/2^2)\).

is slightly tilted compared to the input polarization. Then, a dc voltage and a small modulation at 322 kHz can be used to rotate a small portion of the large polarization component in the \(\hat{y}\)-direction into the \(\hat{x}\)-polarization mode. The value of the dc voltage determines the phase of the modulation in phase space, so the position in the \(S_3-S_4\) plane. The strength and phase of the RF modulation determine the size of the displacement of the cosine and sine modes.

For the measurement of \(\langle y_{\text{c}}\rangle\), a \(\pi/2\)-plate is inserted in the detection path to switch to an \(S_3\) measurement\textsuperscript{11}.

In figure 5, measurements of \(\kappa^2\) are shown for different numbers of atoms \(N\) with a fixed probe power of 5 mW and a probe duration of 1 ms. \(N\) is varied by changing the temperature of the atomic ensemble. It can be monitored by sending a weak linearly polarized probe beam in the direction of the macroscopic orientation. Due to the Faraday effect, the light polarization is rotated by \(\theta\) \(J_1\) and for the CSS \(J_1 \approx 4 \cdot N\). The orientation of the atomic ensembles can be tested via magneto optical resonance spectroscopy [60]. Orientations of 0.997(3) are reached regularly in this experimental setup. Figure 5(c) displays how the measured decay rate \(\gamma = 1/T_2\) can be decomposed in \(\gamma_c\) and \(\gamma_{\text{extra}}\). Clearly \(\gamma_c \propto \gamma \cdot \theta_f\).

### 5.4. PN measurement

When the coupling constant is known, equation (9) can be used to reconstruct the collective atomic operator noise from measurements of noise on \(y_{\text{c}}\) of the outgoing light. In figure 5(d), measurements of the atomic noise in units of projection noise (PN) are shown for different \(N\). There is a small additional noise component, probably arising from technical noise, which scales with the number of atoms. Only a small range of \(\theta_f\) and thus \(N\) is shown. For higher \(N\), additional classical noise of unknown origin is measured, disqualifying higher atomic densities as a working point for quantum noise-limited measurements in this setup.

To find the time evolution of the atomic noise, while probe light is present (as the curves shown in figure 3(a)), \(y_{\text{c},t}\) is evaluated in the time interval \([t, t + \Delta t]\), where \(\Delta t\) is the evaluation time with \(\kappa (\theta_f)\) to find the atomic variances at time \(t\). The corresponding pulse sequence is shown on the left of figure 6.

### 5.5. Measurement of the conditional atomic noise variance

The theoretical background for the generation of the steady state of atoms with the noise reduced by combining dissipation with the measurement is described in section 4. The experimental procedure for the generation of a state with reduced conditional noise variance is described in the following. We wish to squeeze the atomic variances at time \(t\) by measuring the outgoing light operators \(y_c\) and \(y_s\) for the preceding time. The time sequence is illustrated on the right of figure 6. The probe pulse is divided into sections, where measurements in the first time interval \([0, \tau]\) are used

\textsuperscript{11}To test if the measurement is set up correctly, one creates a fixed signal coming from the atoms in one atomic ensemble e.g. created by a magnetic RF pulse. Following equation (4), the atomic signal should be suppressed by \(1/Z^2\) in an \(S_3\) - compared to an \(S_2\) - measurement.
for the conditional noise reduction of the atomic operators at time \( t \), and the consecutive time slice \([t, t + t_{\text{probe}}]\) is used for the reconstruction of the atomic state as described above. Here, the quantity of interest is the conditionally reduced atomic variance at time \( t \): \( \text{var}(P_{\text{out}}^{\text{con}}) = \text{var}(P_{\text{out}} - \alpha y_{\text{probe}}) = \text{var}\left(\frac{1}{\sqrt{2N}} (J_{\gamma} t \pm J_{\chi} t) - \alpha \frac{1}{\sqrt{2N}} y_{\text{probe}}\right) \) (compare equation (5)). The superscript ‘probe’ refers to the first time slice from 0 to \( t \). To achieve an optimal noise reduction, \( \alpha \) and the temporal mode function of the probe section are optimized. The optimized mode function is exponentially rising with a rate that is typically faster than \( 1/T_2 \). This means that only the last bit of the long preceding pulse is used for the conditional noise reduction. This behaviour is rooted in the additional decoherence mechanisms. The atomic state acquires a noise component piling up with rate \( \gamma_{\text{extra}} \). The measurements closest in time to the atomic state which one wishes to squeeze should be weighted most.

The actual measurement of \( \text{var}(P_{\text{out}}^{\text{con}}) \) is done by evaluating \( y_{\text{out}}^{\text{probe}} \) in the second time slice \([t, t + t_{\text{probe}}]\). The same reconstruction mechanism as for the unconditional atomic state reconstruction is used; therefore, it is the second time slice evaluated with an exponentially falling mode function with \( \gamma = \gamma_s + \gamma_{\text{extra}} \). Again \( y_{\text{out}} \) is utilized to establish the value of \( \text{var}(P_{\text{out}}) \) at time \( t \). Accordingly, \( \text{var}(P_{\text{out}}^{\text{con}}) = \text{var}(y_{\text{out}}^{\text{con}} - \alpha^2 y_{\text{probe}}^{\text{con}}) \) can be used to find \( \text{var}(P_{\text{out}}^{\text{con}}) \) with the same reconstruction procedure. The conditional variance can be extracted from the two-time correlation functions \( \langle y_{\text{out}}^{\text{con}} y_{\text{out}}^\ast \rangle, \sigma_{\text{cond}}^{2,\text{out}} = \text{var}(y_{\text{out}}^{\text{con}}) \) by optimizing \( \alpha^2 \); this is assuming that there are no correlations between the light operators at different points in time \( \langle y(t)y(t') \rangle = \delta(t-t') \).

### 6. Conclusions

The method discussed in this paper allows one to deterministically generate entangled states between two atomic ensembles over a macroscopic distance. The desired quantum state is thereby stabilized by the dissipative mechanism, which renders the created entanglement robust and long-lived. Purely dissipative entanglement generation does not require postselection nor conditioning.

For two-level systems, the dissipative mechanism explained in section 3 can be used to produce steady state entanglement, i.e. entanglement which is available permanently. This is even possible in the presence of imperfections and noise sources. The use of atoms with a

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**Figure 5.** (a) The pulse sequence is shown. The probe light is turned on and off smoothly to avoid noise contributions at 322 kHz. (b) \( \chi^2 \) for a varied number of atoms and a light power \( P = 5 \text{ mW} \) and (c) the corresponding decay constants \( \gamma_{\text{extra}} \) and \( \gamma_s \) are shown. (d) The scaling of the normalized atomic noise over \( N \).

**Figure 6.** Pulse sequences for long probe durations. Left: a pulse sequence with a long probe pulse is depicted, displaying the evaluation time interval \([t, t_{\text{probe}}]\) used to find the atomic noise at time \( t \). Right: we show how a measurement prior to \( t \) can be used to conditionally reduce the atomic noise. For measurements like this typically, a fast exponentially growing mode function is used.
The mechanism considered here is applicable in any system which can be coupled to two photonic sideband modes accordingly through a tunable quadratic interaction, for example, for the generation of entanglement between two mechanical oscillators. Even though the limited coherence times of quantum systems impose typically severe restrictions on the life times of quantum superposition states, dissipative methods for quantum state engineering allow one to produce event-ready quantum states for applications in quantum information science. Exploring and exploiting all advantages of dissipative approaches will require both devising new protocols which are capable of generating and processing steady states and finding realistic and practical ways of implementing the required coupling of physical systems to a bath.

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Appendix A. Derivation of the master equation

The interaction of atoms and light illustrated in figure 1 can be described by the effective ground state Hamiltonian

$$H = H_A + H_L + H_{\text{int}},$$

where excited states have been eliminated using the fact that the detuning $|\Delta|$ is large compared to the Doppler width $\delta_{\text{Doppler}}$ and atomic decay rates $\Gamma_{\text{atomic}}$. Here and in the following, $\Gamma_{\text{atomic}}$ denotes the largest effective rate for atomic ground states including single particles as well as collective rates (see below). $H_A = \Omega (J_{s,l} - J_{s,l})$ accounts for the Zeeman splitting of the atoms in the external magnetic field and $H_L = \int dk \omega_k a_k^\dagger a_k$ is the free Hamiltonian of the light field, where $a_k$ is the annihilation operator of a photon with wave vector $k$ and frequency $\omega_k$. In a rotating frame, the interaction Hamiltonian is given by

$$H_{\text{int}} = \int d\omega_k \sum_{\Lambda} g(\omega_k) \left( \mu \sum_{i=1}^N \sigma_{I,i}^- e^{i \omega_k r_i} - v \sum_{j=1}^N \sigma_{II,j}^- e^{i \omega_k r_j} \right) a_k^\dagger + \int d\omega_k \sum_{\Lambda} g(\omega_k) \left( \mu \sum_{j=1}^N \sigma_{II,j}^+ e^{i \omega_k r_j} - v \sum_{i=1}^N \sigma_{I,i}^+ e^{i \omega_k r_i} \right) a_k^+ + \text{h.c.},$$

(A.1)

where $\Lambda_k$ specifies the the two orthogonal polarizations of the light mode with wave vector $k$. The first and second integral cover narrow bandwidths $\Delta \omega_{\text{int}}$ and $\Delta \omega_{\text{ph}}$ centred around the lower and upper sideband, respectively. The atomic operator $\sigma_{I,I,i} = |\uparrow\rangle_{I,I,i}\langle\downarrow|$ refers to a particle in ensemble I/II at position $r_i$, $\Delta k = k_L - k$ and $k_L$ is the wavevector of the applied classical field. $g(\omega_k)\mu$ and $g(\omega_k)v$ denote the effective coupling strengths of the passive (beamsplitter-like) part of the interaction and the active (squeezing) component of the Hamiltonian, respectively. The laser field covers a very narrow bandwidth around the central frequency $\omega_k$ and is sufficiently off-resonant such that the interaction is well within the dispersive regime and absorption effects can be neglected.

Starting from the Hamiltonian (A.1), a master equation of Lindblad form can be derived for the reduced atomic density matrix $\rho(t)$. To this end, the Born–Markov approximation is used, which is well justified for optical frequencies. Since the Larmor splitting exceeds well atomic decay rates $\Omega \gg \Gamma_{\text{atomic}}$, the two sideband modes can be treated as independent baths. The effect of atomic motion gives rise to noise terms and can be included in the master equation in the form of averaged coefficients, where the average in space corresponds to an average in space. This is legitimate in the fast motion limit, where the time scale set by the average velocity of the atoms $v$ is fast compared to the time scale of the radiative decay $\Gamma_{\text{atomic}} \ll 1$. In this case, the emission of light can be described independently of the evolution of atomic positions.

Using the definitions

$$A = \frac{1}{\sqrt{N}} \sum_{i=1}^N \sigma_{I,i}^- - v \frac{1}{\sqrt{N}} \sum_{j=1}^N \sigma_{II,j}^-,$$

$$B = \mu \frac{1}{\sqrt{N}} \sum_{i=1}^N \sigma_{I,i}^+ - \frac{1}{\sqrt{N}} \sum_{j=1}^N \sigma_{II,j}^+,$$

the resulting master equation can be cast in the form

$$\frac{d}{dt} \rho(t) = \frac{1}{2} d\Gamma A \rho(t) A^\dagger + \frac{1}{2} d\Gamma B \rho(t) B^\dagger$$

$$+ \frac{1}{2} \Gamma \mu^2 \sum_{i=1}^N (\sigma_{I,i}^- \rho(t) \sigma_{I,i}^+ + \sigma_{II,i}^- \rho(t) \sigma_{II,i}^+),$$

$$+ \frac{1}{2} \Gamma \nu^2 \sum_{i=1}^N (\sigma_{I,i}^+ \rho(t) \sigma_{I,i}^- + \sigma_{II,i}^+ \rho(t) \sigma_{II,i}^-),$$

$$+ \cdots,$$

(A.2)

where $\Gamma$ is the single particle decay rate and a shorthand notation was used. Master equations of Lindblad form
\[ \frac{d}{dt}\rho(t) = \frac{1}{2} \{ [A(t)A(t)]^\dagger - A(t)A(t)] + h.c. \}\] with decay rate \( \gamma \) and jump operator \( A \) is abbreviated by the expression \( \frac{d}{dt}\rho(t) = \frac{1}{2} \{ [A(t)A(t)]^\dagger + \cdots \} \). \( d \) denotes the resonant optical depth of one atomic ensemble. Note that the entangling terms in the first line are enhanced by a factor \( d \), such that for sufficiently optically thick samples, the additional noise terms reflecting thermal motion are small compared to the desired contributions.

Next, additional cooling and heating processes, as well as dephasing, are included. The full master equation is given by
\[ \frac{d}{dt}\rho(t) = \frac{1}{2} \{ [A(t)A(t)]^\dagger - A(t)A(t)] + h.c. \} + \frac{1}{2} \{ [B(t)B(t)]^\dagger - B(t)B(t)] + h.c. \} \]
\[ + \frac{1}{2} \{ [C(t)C(t)]^\dagger - C(t)C(t)] + h.c. \} + \frac{1}{2} \{ [D(t)D(t)]^\dagger - D(t)D(t)] + h.c. \} \]
with \( \sigma_{t} \) the longitudinal spin and \( \sigma_{l} \) the transverse spin. The first two terms are obtained by applying the decorrelation approximation of equation (A.2) to all \( \langle [A(t)A(t)]^\dagger - A(t)A(t) \rangle \) to \( \langle [B(t)B(t)]^\dagger - B(t)B(t) \rangle \) for the first (second) ensemble. The three last terms represent single particle processes. Hence, they do not feature a collective enhancement factor as the entangling terms in the first line. As shown in [3], equation (A.3) includes all terms that need to be taken into account. Collective dephasing processes, as well as collective contributions due to pump and repump fields can be neglected. Similarly, the distance \( R \) between the two ensembles does not play a role in \( k_{l} \rightarrow R/L \) and \( k_{l}L \gg 1 \), where \( L \) is the spatial extent of an atomic ensemble. For the experimental setting under consideration, this condition is fulfilled.

**Appendix B. Calculation of entanglement**

Below it is shown how the entanglement measured by the quantity \( \xi = \Sigma_{J} / \{2([J_{z}])\} \), where \( \Sigma_{J} = \text{var}(J_{z} - J_{z}) + \text{var}(J_{x} - J_{x}) \), can be determined in the limit \( N \gg 1 \), assuming that the number of atoms in the two-level system \( N_{2} \) depends on time. For clarity, operators referring to the two-level model are labelled with subscript ‘2’. The time derivative of the variance \( \Sigma_{J} \) is calculated using equation (A.3). By applying the decorrelation approximation \( [J_{z}]/(J_{z})_{2} \approx (J_{z}/(J_{z}))_{2} \) for mean values of products of transverse and longitudinal spins, one obtains
\[ \frac{d}{dt}\Sigma_{J}(t) = -\tilde{\Gamma} + d(t)P_{z}(t)\Sigma_{J}(t) \]
\[ + N_{2}(t) \{ \tilde{\Gamma} + d(t)P_{z}(t)^{2}(\mu - \nu)^{2} \} \]
with \( d(t) = \text{d}N_{2}(t)/N, \tilde{\Gamma} = \Gamma_{\text{cool}} + \Gamma_{\text{heat}} + \Gamma_{\text{deph}} \) and \( P_{z}(t) = 2(J_{z}(t))/\{N_{2}(t)\} \). For \( t \rightarrow \infty \) and \( N_{2} = N \),
\[ \Sigma_{J}(\infty) = N \frac{\tilde{\Gamma} + d(t)P_{z}(\mu - \nu)^{2}}{\Gamma_{\text{cool}} + d(t)P_{z}(\mu - \nu)^{2}} \]
where $\Gamma_{\text{out}}^d$ is the rate at which atoms leave the two-level subsystem due to radiative transitions caused by the driving field. $\Gamma_{\text{repump}}$ is the rate at which the applied repump fields transfer atoms back. Transitions within the two-level subsystem occur at the rates

$$\Gamma_{3,4} = \mu^2 \Gamma + \Gamma_{\text{pump}} + \Gamma_{\text{col}},$$

$$\Gamma_{4,3} = \nu^2 \Gamma + \Gamma_{\text{col}}.$$  

$\mu^2 \Gamma$ and $\Gamma_{\text{pump}}$ are the driving field and pump field-induced cooling rates, respectively. The heating rate caused by the driving field is given by $\nu^2 \Gamma$. Note that the application of pump fields leads to an increased dephasing rate $\Gamma$. In contrast, repump fields do not have an effect on $\Gamma$ [3]. We estimate the effect of these fields on the dephasing rate by adding $2\Gamma_{\text{pump}}$ to $\Gamma$.

Figure 3(b) shows the predicted time evolution of entanglement for $d = 55,100,150$. The parameters take the values used to fit the experimental data measured in the absence of additional fields (compare figure 2, panels a and b in [1]), $\Gamma = 0.002 \text{ ms}^{-1}$, $\tilde{\Gamma} = 0.193 \text{ ms}^{-1}$, $\Gamma_{\text{col}} = 0.002 \text{ ms}^{-1}$ and $Z = 2.5$. The presence of both pump and repump fields is included as described above with $\Gamma_{\text{pump}} = \Gamma_{\text{repump}} = 0.160 \text{ ms}^{-1}$.

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