Mathematical modeling of active thermo-emission thermal protection system of aircraft in high speed air flow

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Abstract. A mathematical model of the process of nonstationary heat transfer between active thermo-emission thermal protection system (ATETPS) and convective gas flow is given. The effect of electron’s evaporation (emission) from the surface of the emitter on lowering its temperature is studied.

1. Introduction
It is important for hypersonic aircraft’s (HA) design to research the behavior of their thermal protection systems. The temperature of some parts of the body during flight can exceed 2500 – 3000 K [1]. There are many different thermal protection systems - passive, active and combined [1-3]. The potentials of using the thermionic method [4] for active thermal protection are considered. In this method the heating energy is converted directly to the electrical one. The evaporation of thermal electrons from the emitter by a decrease in the temperature of the latter [5] is accompanied. In the ATETPS a number of interrelated processes [5] take place: emission, electric, plasma, thermal, adsorption, etc. Experimental studies of thermionic installations are rather complex and expensive [6, 7]. Therefore much attention is paid to mathematical modeling of the processes taking place in them [5]. In this paper, the model of ATETPS is presented and studied. It is shown that the choice of protection parameters can significantly reduce the temperature of its structures.

2. Formulation of problem
Consider the multi-element construction of electrical generating elements (EGE), where each of them has its own zone of influence with a characteristic size $L_k$. The geometry of every EGE is a system of finite thickness’s truncated cones with small angle of taper. Figure 1 schematically shows active protection layers for a fixed axial angle with its own zone of influence and a characteristic transverse dimension. Further the problem of heat exchange within a typical EGE, by which we mean a composite region with a heat-insulated wall at $s = s_A$, $0 \leq n \leq L_2$ (except the collector $L_4 \leq n \leq L_5$) and at $s = s_k$, $L_k < n \leq L_8$ (except for the emitter $L_2 \leq n \leq L_3$) is investigated. The coordinate $n$ is directed from the outer surface of the body into the interior of the body’s shell (see figure 1). The layer 1 consists from molybdenum alloy. The layer 2 is emitter’s insulation from zirconium carbide (ZrC). The layer 3 includes an insulator (2), an emitter from W (3), and a current lead from W (3). The layer 4 consists of a current lead from molybdenum (Mo) (4), a collector from (Mo) (4) and an insulator (5). The layers 5 and 7 are the coolant’s capacities from aluminum oxide ($Al_2O_3$). The layer 6 consists from cooling...
air. The layer 8 is a receiver of electrical energy. The emitter 3 and the collector 4 constitute a thermionic element. \( d \) denotes the value of the inter-electrode gap (IEG). \( L_j, j = 1,\ldots,8 \) are distances from the outer surface of body. \( \delta_j, j = 1,\ldots,7 \) are the thicknesses of the layers 1..3 and 5..8 respectively in figure 1.

The study of the characteristics of an EGE is based on the current-voltage characteristics of an isothermal thermionic converter (TEC) [5-8], which in turn are integral characteristics of diverse processes in the IEG and on electrodes [5, 8] and are determined by the transfer of particles and energy in the plasma, ionization, adsorption and other processes. The mathematical formulation of the electrical model is taken from [7], but the general case is considered: the collector is not equipotential and the electrical conductivity of the electrodes and switching components depends on their temperature.

To find prototypes of the HA, which can justify the installation of ATETS, it is desirable to find the level of heat fluxes taken from the outer open shell of the emitter (layer 3 in figure 1) and the external surface of the collector (layer 4 in figure 1) by using electronic cooling and radiation processes. In addition, it is necessary to know the aerodynamic heating’s rates of the outer part of the refractory metal (layer 1 in figure 1). The heat fluxes for the outer open parts of layers 3 and 4 have the form [5, 7]:

\[
q_{E3} = -(q_1 + q_\epsilon + q_{C_\epsilon}), \quad q_{E4} = q_2 + q_\epsilon + q_{C_\epsilon},
\]

\[
q_1 = J(T_{2,4},T_{1,3},\Delta V)[\varphi_1(T_{1,3}/T_{C_\epsilon})/e + 2kT_{1,3}/e],
\]

\[
q_2 = J(T_{2,4},T_{1,3},\Delta V)[\varphi_2(T_{2,4}/T_{C_\epsilon})/e + 2kT_{1,3}/e],
\]

\[
q_\epsilon = \sigma_\epsilon(T_{1,3}^4 - T_{2,4}^4), \quad q_{C_\epsilon} = (\lambda_{C_\epsilon}/d)(T_{1,3}^4 - T_{2,4}^4),
\]

where \( k \) is the Boltzmann constant, \( e \) is the electron charge, \( \sigma \) is the Stefan-Boltzmann constant, \( \lambda_{C_\epsilon} \) is the coefficient of thermal conductivity of cesium vapor in the IEG, \( \varphi_j, j = 1, 2 \) is the work function of the emitter and of the collector materials, \( \epsilon_\epsilon \) is the emissivity of the emitter surface and the collector. Lower indices 1 and 2 in the left part of the formulas (2) and (3) correspond to the parameters of the emitter and the collector. Lower index \( C_\epsilon \) corresponds to cesium vapor. Point \( A \) is the conjugation boundary of the sphere-cone in figure 1. \( s_k \) is a terminal value of the coordinate \( s \). Lower index * corresponds to a characteristic quantity.
The problem of calculating the characteristics of heat transfer using natural coordinates reduces to solving the system of equations [2] with $s_A < s < s_k$

$$
c_{p_j}(T_{i,j}) \rho_j \frac{\partial T_{i,j}}{\partial t} = \frac{\partial}{\partial n} \left[ \lambda_j(T_{i,j}) \frac{\partial T_{i,j}}{\partial n} \right] + \frac{\partial}{\partial s} \left[ \lambda_j(T_{i,j}) \frac{\partial T_{i,j}}{\partial s} \right]
$$

$$+ \frac{\lambda_j(T_{i,j})}{r} \left( \frac{\partial T_{i,j}}{\partial s} \sin \theta - \frac{\partial T_{i,j}}{\partial n} \cos \theta \right), \quad j = 1, 2, \quad 0 < n < L_2, \quad s_A < s < s_k \tag{5}
$$

$$c_{p_2}(T_{1,3}) \rho_2 \frac{\partial T_{1,3}}{\partial t} = \frac{\partial}{\partial n} \left[ \lambda_2(T_{1,3}) \frac{\partial T_{1,3}}{\partial n} \right] + \frac{\partial}{\partial s} \left[ \lambda_2(T_{1,3}) \frac{\partial T_{1,3}}{\partial s} \right]
$$

$$+ \frac{\lambda_2(T_{1,3})}{r} \left( \frac{\partial T_{1,3}}{\partial s} \sin \theta - \frac{\partial T_{1,3}}{\partial n} \cos \theta \right), \quad L_2 < n < L_3, \quad s_A < s < s_1 \tag{6}
$$

$$c_{p_3}(T_{1,3}) \rho_3 \frac{\partial T_{1,3}}{\partial t} = \frac{\partial}{\partial n} \left[ \lambda_3(T_{1,3}) \frac{\partial T_{1,3}}{\partial n} \right] + \frac{\partial}{\partial s} \left[ \lambda_3(T_{1,3}) \frac{\partial T_{1,3}}{\partial s} \right]
$$

$$+ \frac{\lambda_3(T_{1,3})}{r} \left( \frac{\partial T_{1,3}}{\partial s} \sin \theta - \frac{\partial T_{1,3}}{\partial n} \cos \theta \right) + P_{1}^{v}, \quad L_2 < n < L_3, \quad s_1 < s < s_k \tag{7}
$$

$$c_{p_4}(T_{2,4}) \rho_4 \frac{\partial T_{2,4}}{\partial t} = \frac{\partial}{\partial n} \left[ \lambda_4(T_{2,4}) \frac{\partial T_{2,4}}{\partial n} \right] + \frac{\partial}{\partial s} \left[ \lambda_4(T_{2,4}) \frac{\partial T_{2,4}}{\partial s} \right]
$$

$$+ \frac{\lambda_4(T_{2,4})}{r} \left( \frac{\partial T_{2,4}}{\partial s} \sin \theta - \frac{\partial T_{2,4}}{\partial n} \cos \theta \right) + P_{2}^{v}, \quad L_5 < n < L_6, \quad s_A < s < s_2 \tag{8}
$$

$$c_{p_5}(T_{2,4}) \rho_5 \frac{\partial T_{2,4}}{\partial t} = \frac{\partial}{\partial n} \left[ \lambda_5(T_{2,4}) \frac{\partial T_{2,4}}{\partial n} \right] + \frac{\partial}{\partial s} \left[ \lambda_5(T_{2,4}) \frac{\partial T_{2,4}}{\partial s} \right]
$$

$$+ \frac{\lambda_5(T_{2,4})}{r} \left( \frac{\partial T_{2,4}}{\partial s} \sin \theta - \frac{\partial T_{2,4}}{\partial n} \cos \theta \right), \quad L_5 < n < L_6, \quad s_2 < s < s_k \tag{9}
$$

$$c_{p_6}(T_{2,4}) \rho_6 \frac{\partial T_{2,4}}{\partial t} = \frac{\partial}{\partial n} \left[ \lambda_6(T_{2,4}) \frac{\partial T_{2,4}}{\partial n} \right] + \frac{\partial}{\partial s} \left[ \lambda_6(T_{2,4}) \frac{\partial T_{2,4}}{\partial s} \right]
$$

$$+ \frac{\lambda_6(T_{2,4})}{r} \left( \frac{\partial T_{2,4}}{\partial s} \sin \theta - \frac{\partial T_{2,4}}{\partial n} \cos \theta \right), \quad L_6 < n < L_8, \quad s_4 < s < s_k, \quad j = 5, 6, 7 \tag{10}
$$

where $r = (R_N - n) \cos \theta + (s - s_A) \sin \theta, t$ is time, $c_{p_j}, \lambda_j, \rho_j, j = 1, \ldots, 7$ are the coefficients of specific heat, thermal conductivity and material’s densities of ATETPS layers, $P_{i}^{v}(s) = \xi_j I_{j}^{2}(s) / S_j^2$, $i = 1, 2$ – volumetric Joule heat dissipation of the emitter and collector, $I_{j}(s) = 2\pi R_i \int_{r_{i}}^{s} J(T_{2,4}, T_{3,3}, \Delta V) ds$, $I_{2}(s) = 2\pi R_i \int_{r_{2}}^{s} J(T_{2,4}, T_{3,3}, \Delta V) ds$ – the current flowing along the emitter and the collector was found by the method [7, 8], $\xi_j, j = 1, 2$ the electrical conductivity of the cathode and the anode is known as a function of temperature in [5], and $S_i = 2\pi \delta_i (R_i + \delta_i / 2), \quad S_j = 2\pi \delta_i (R_N - \delta_i / 2)$ – cross-sectional area of the emitter and collector [7], $R_i = R_N - L_3$ – internal radius of the emitter, $R_2 = R_N - L_4$ – external radius of the collector, $R_N$ – radius of spherical blunting, $\theta$ is the taper angle.

The system of equations (5) – (10) must be solved taking into account the following initial and boundary conditions. Initial conditions are as follows:
The heat transfer condition is set according to the Newton's law and heat transfer of the ZrC surface by the radiation
\[ \lambda_2 \left( \partial T_{1,3} / \partial n \right)_{n=L_3} = \Delta_1 \left( T_{1,3} \right)_{n=L_3} - T_{n-1}, \quad s_{A} \leq s \leq s_{L_3}. \]  
(13)

On the surface of the layer 3 (the emitter at \( n = L_3 \), \( s_{A} \leq s \leq s_{L_3} \)) a third kind boundary condition is set
\[ q_{L_3} = -\lambda_1,3 \left( \partial T_{1,3} / \partial n \right)_{n=L_3}, \quad s_1 < s < s_2. \]  
(14)

On the surface of the layer 3 (the emitter at \( n = L_3 \), \( s_2 \leq s \leq s_k \)) a third kind boundary condition is set and heat transfer of the W surface by the radiation
\[ -\lambda_3 \left( \partial T_{1,3} / \partial n \right)_{n=L_3} = \Delta_1 \left( T_{1,3} \right)_{n=L_3} - T_{n-1}, \quad s_2 \leq s \leq s_k. \]  
(15)

On the outer surface of layer 4 (the collector at \( n = L_4 \), \( s_{A} \leq s \leq s_{L_4} \)) heat transfer condition is set according to the Newton's law
\[ -\lambda_4 \left( \partial T_{2,4} / \partial n \right)_{n=L_4} = \Delta_2 \left( T_{2,4} \right)_{n=L_4} - T_{n-1}, \quad s_{A} \leq s < s_{L_4}. \]  
(16)

On the outer surface of the layer 4 (the collector at \( n = L_4 \), \( s_1 \leq s < s_2 \))
\[ q_{L_4} = -\lambda_4 \left( \partial T_{2,4} / \partial n \right)_{n=L_4}, \quad s_1 \leq s < s_2. \]  
(17)

On the outer surface of the layer 4 (the insulator \( \lambda_2 O_3 \) at \( n = L_4 \), \( s_2 \leq s \leq s_k \)) a third kind boundary condition is set
\[ -\lambda_5 \left( \partial T_{2,4} / \partial n \right)_{n=L_4} = \Delta_2 \left( T_{2,4} \right)_{n=L_4} - T_{n-1}, \quad s_2 \leq s \leq s_k. \]  
(18)

On the surface of the layer 7 (the substrate at \( n = L_7 \)) the heat transfer condition is set according to the Newton's law
\[ -\lambda_7 \left( \partial T_{2,7} / \partial n \right)_{n=L_7} = \Delta_7 \left( T_{2,7} \right)_{n=L_7} - T_0, \quad s_{A} \leq s \leq s_{L_7}. \]  
(19)

On the conjugation lines \( n = L_j, j = 1, 2 \) and \( n = L_i, i = 5, 6, 7 \) the ideal contact's conditions and the temperature equalities are set
\[ \lambda_i \left( \partial T_{i-1,1} / \partial n \right)_{n=L_{i-1}} = \lambda_{i+1} \left( \partial T_{i+1,1} / \partial n \right)_{n=L_{i+1}}, \quad i = 1, 2, 3, i_{i-1} \left( \partial T_{i-1,2} / \partial n \right)_{n=L_{i-1}} = \lambda_i \left( \partial T_{2,2} / \partial n \right)_{n=L_{i-1}}, \quad i = 5, 6, 7. \]  
(20)

On the left end \( s = s_{A} \) and on the right end \( s = s_k \) of layers 1-2, 5-7 the conditions of heat insulation are set
\[ (\partial T_{i,1} / \partial s)_{n=s_{A}} = 0, \quad (\partial T_{i,1} / \partial s)_{n=s_k} = 0, \quad i = 1, 2, \]  
(21)

On the left end \( s = s_{A} \) and on the right end \( s = s_k \) of the layer 4 heat transfer is set according to the Newton's law
\[ -\lambda_4 \left( \partial T_{2,4} / \partial s \right)_{n=s_{A}} = \Delta_4 \left( T_{2,4} \right)_{n=s_{A}} - T_{n-1}, \quad -\lambda_4 \left( \partial T_{2,4} / \partial s \right)_{n=s_k} = \Delta_4 \left( T_{2,4} \right)_{n=s_k} - T_{n-1}. \]  
(22)

At the right end \( s = s_k \) (the cathode) of the layer 3 a third kind boundary condition is set and heat transfer of the W surface by the radiation
\[ -\lambda_5 \left( \partial T_{1,3} / \partial s \right)_{n=s_{A}} = \Delta_5 \left( T_{1,3} \right)_{n=s_{A}} - T_{n-1}, \quad -\lambda_5 \left( \partial T_{1,3} / \partial s \right)_{n=s_k} = \Delta_5 \left( T_{1,3} \right)_{n=s_k} - T_{n-1}. \]  
(23)
$T_0$ is the initial temperature of the HA’s body, $\varepsilon_i$, $i = 1, 2$ – are the emissivity of the surface of W and ZrC, $T_{1w}, T_{2w}$ – the characteristic temperatures of the air near the end face of the emitter at $s = s_d$ and the collector at $s = s_{0d}$, and $\delta_1, \Delta_1, \Delta_2$ – the temperature conductivity coefficients of the different layers with the external environment.

The boundary value problem (5)–(23) was solved numerically using the locally one-dimensional splitting method [9]. An implicit, absolutely stable, monotonic difference scheme with a total error of $O(\tau + H_n^2 + H_n^3)$ was used, where $H_n$ is the maximum step along the space along the coordinate $n$ ($H_n = \max H_i, i = 1, \ldots, 7$), $H_s$ is a step along the space along the coordinate $s$, $\tau$ is a step on time.

Formulas [10] were used to specify the convective heat flux from turbulent boundary layer to the conical part of the HA’s body:

$$q_w = \alpha_e (h_{1w} - h_{2w}), \quad h_{2w} = T_{1w} c_1 + c_z T_{2w}^2 / 2, \quad \alpha_e = \frac{16.4 \nu_e^{1.25} \rho_e^{0.8}}{R_e \nu_e^2 (1 + h_{1w} / h_{10})^{2/3}} \times \frac{2.2 \bar{p}(u_e / v_m)}{\varsigma_0^{0.4} \bar{p}_{t2}^{0.2}}, \quad \bar{p} = P_e / P_{e0},$$

$$u_e / v_m = (1 - \bar{p}_e)^{0.5}, \quad \bar{p}_e = \cos \theta + (\bar{s} - \bar{s}_d) \sin \theta, \quad \varsigma = (\gamma_{ef} - 1 + 2/M_{\infty}^2)/(\gamma_{ef} + 1), \quad \gamma = (\gamma_{ef} - 1)/\gamma_{ef}.$$

The calculations of chemically equilibrium air flow around a cone with the half angle of taper $\theta = 5^\circ$ were carried out for the following conditions:

$H_{\infty} = 3.0 \times 10^7$ m, $h_{10} = 6.758 \times 10^6$ J/kg, $\nu_e = 4 \times 10^3$ m/s, $P_{e0} = 1.197 \times 10^3$ Pa, $\rho_{\infty} = 1.84 \times 10^{-2}$ kg/m$^3$, $g_\infty = 9.73 \ m/s^2$, $a_\infty = 257 \ m/s$, $T_\infty = 293 \ K$, $R_N = 0.1 \ m$, $M_\infty = 14$, $\gamma_{ef} = 1.21$, $\delta_i = 2 \times 10^{-3}$ m, $\delta_i = 1 \times 10^{-3}$ m, $i = 2, 3$, $\delta_1 = 10^{-3}$ m, $i = 4, 5, 6, 7$, $d = 2.5 \times 10^{-4}$ m, $s_k = 0.2084 \ m$, $s_d = 0.1484 \ m$.

$\nu_{\infty}$ is the speed of the air’s incident flow, $h$ is an static enthalpy, $M_\infty$ is the Mach number, $c_i, i = 1, 2$ are constants, $P$ is pressure, $H_{\infty}$ is flight altitude, $a_\infty$ is sound speed, $g_\infty$ is acceleration of gravity. The lower indices: $w$ corresponds to the parameters on the outer surface of HA’s body, $e$ and $e0$ – to the values at the outer edge of the boundary layer and at the stagnation point of the HA’s body, $\infty$ – to the values of the air’s incident flow, the line – to dimensionless parameters, $z$ – to the terminal moment of time, $ef$ – to effective value, $0$ – to initial value, $m$ – to maximum value.

3. The numerical solution results and their analysis

In figures 2 – 3 the temperature dependences for the surface of the HA’s body, for the emitter (solid curves) and for the collector (dashed curves) at presence of the EGE alongside coordinate $s$ are shown. Curves 1 – 4 in figures 2 – 3 correspond to next moments of time: 1–10, 2–20, 3–40, 4–t = $t_2$ (t$_2 = 60$ s corresponds to steady regime of body’s heating). Dashed curve in figure 2 denotes to regime without the EGE at moment of time $t_2 = 60$ s. The presence of the EGE decreased temperature for the surface of the HA’s body by 170 – 200 K. At presence the TEC the curves for temperatures of W surface $T_w$ and the emitter’s surface $T_{1w}$ are concave downward and curve for temperature for the collector’s surface $T_{2w}$ is concave upward. The decrease in the temperature of the outer shell in the region of the TEC is related to the electronic cooling of the emitter, which agrees qualitatively with the data of [5].

For practice, the value the temperature $T_{2w,t_2}$ of the internal wall of the structure for the collector $T_{2w,t_2}$ is of interest. The temperature of the inner wall of the substrate increased insignificantly at $s_*(s_k - s_d)/2$ and the time $t = t_2$ up to $T_{2w,t_2} = 311$ K.
4. Summary
A mathematical model of active thermionic thermal protection system for convective heating of a multilayer shell was developed. A decrease in the surface temperature of the shell and the temperature of the emitter surface as a result of the thermal emission of electrons from the emitter surface was detected. Qualitative agreement of calculation results with known data was obtained.

References
[1] Polezhaev Yu V and Yurevich F B 1976 Thermal Protection (Moscow: Energia) p 392
[2] Grishin A M, Golovanov A N, Zinchenko V I, Efimov K N and Yakimov A S 2011 Mathematical and Physical Modeling of Thermal Protection (Tomsk: Publishing House of Tomsk State University) p 358
[3] Zinchenko V I, Efimov K N and Yakimov A S 2011 Calculation of the characteristics of the conjugate heat-mass transfer in the spatial flow around a blunt body using a system of combined thermal protection High Temperature 49(1) 81–91
[4] Kernozhitsky V A, Kolychev A V and Makarenko A V 2014 Development of a technique for calculating the multielement thermionic thermal protection of hypersonic aircraft Proceedings of the MAI No 75
[5] Ushakov B A, Nikitin V D and Emelyanov I Ya 1974 Fundamentals of Thermionic Energy Conversion (Moscow: Atomizdat) p 288
[6] Sinyavsky V V 2000 Methods and Means of Experimental Research and Reactor Tests of Thermionic Power Generating Assemblies. (Moscow: Energoatomizdat) p 375
[7] Brovalsky Yu A, Rozhkova N M and Sinyavsky V V 1969 Generalized calculation of the current-voltage characteristics and temperature fields of thermionic converters based on the data of tests of an isothermal TEP Thermal Emission Energy Converters (Moscow: VNIIT) p 281
[8] Babushkin Yu V and Zimin V P 2006 Methods for calculating the current-voltage characteristics of thermionic power generating assemblies Izvestiya Tomsk Polytechnic University 309(2) 135–9
[9] Samarsky A A 1971 Introduction to the Theory of Difference Schemes (Moscow: Nauka) p 552
[10] Zemlyansky B A and Stepanov G I 1981 On the calculation of heat transfer in the spatial flow of thin blunted cones hypersonic airflow Izvestiya AN SSSR. Mechanika Zhidkosti i Gaza No 5 173–7