Matrix Elements and Parton Showers in Hadronic Interactions

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Abstract: A method is suggested to combine tree level QCD matrix for the production of multi jet final states and the parton shower in hadronic interactions. The method follows closely an algorithm developed recently for the case of $e^+e^-$ annihilations [1].

Keywords: QCD, Jets, Deep Inelastic Scattering, Hadronic Colliders.
1. Introduction: ME vs. PS

The production of multi jet final states is one of the searching grounds for new physics at current and future collider experiments. Therefore their Monte Carlo simulation, both for signal and background processes is of crucial importance for the success of the experiments. Currently, two alternative ways to fill the phase space for particle emission can be formulated:

1. One might use exact matrix elements (ME) at some given perturbative order in the coupling constant(s), say $\alpha_s$. In most cases, these MEs are available at the tree level, i.e. at the lowest perturbative order for the production of a specific final state. Then, the final state particles are usually identified with jets and their phase space is cut accordingly. In some cases higher perturbative orders are known as well, and blur this simple correspondence. However, in this paper we will focus on tree level MEs, therefore we can stick to the simple identification of partons with jets. The virtue of the MEs is that they are exact and take all interference effects at the perturbative order into account. The downsize is that more and more diagrams have to be considered with rising numbers of outgoing particles populating a multidimensional phase space. Clearly, our calculational abilities do not live up to this situation. Furthermore, for the transition of the partons into hadrons a phenomenological fragmentation model has to be invoked with parameters, that are a priori free and depend on the fragmentation scale. Hence, applying a fragmentation scheme immediately at the hard scale...
of the ME necessitates a re-tuning of its parameters for each c.m. energy in order to gain reliable results on the hadron level. This is an impossible task and limits the applicability of using pure MEs considerably.

2. Alternatively one might use the parton shower (PS). By expanding around the soft and collinear limits of parton emission the radiation pattern factorises into individual parton branchings, each giving rise to potentially large soft and collinear logarithms. Via multiple emission according to the individual branching probabilities the PS then resums all the large logarithms. In fact, the coherent, angular ordered PS is correct up to Next–to Leading Logarithmic (NLL) accuracy, i.e. it correctly resums all terms of the form $\alpha_s^n (L^{2n} + L^{2n-1})$, where $L$ denotes a large logarithm of the form $\ln s/Q_0^2$ with $s$ the hard scale of the process and $Q_0^2$ the soft parton resolution scale. Obviously, the PS is able to connect both the hard scale and the fragmentation scale. Hence, using the PS allows us to tune the fragmentation parameters at one scale, say at LEP1 energies, where huge statistics were amassed, and to use the same parameters at all other scales. However, due to its factorised structure the PS misses important interference effects that may play a significant role for detailed analyses.

To gain some insight into the effect of these interference effects, let us compare the emission of a gluon in $e^+e^- \rightarrow q\bar{q}$. Pictorially we can write:

$$\frac{d\sigma_{\text{ME}}}{dx_1dx_2} \sim \text{interference terms} + \frac{d\sigma_{\text{PS}}}{dx_1dx_2} \sim$$

![Pictorial representation of contributions to single gluon emission in $e^+e^- \rightarrow q\bar{q}g$ as given by the ME and the PS. As indicated, the PS does not take into account the interference contributions present in the ME. This leads to the ratio $ME/PS$ as depicted in the contour plot. At the soft and collinear boundaries of the phase space in the $x_1-x_2$ plane (where $x_i$ corresponds to the energy fraction of the quark and antiquark) the PS correctly reproduces the ME, whereas in the region of hard gluon emission the PS omits the (destructive) interference contribution.](image)

**Figure 1:** Pictorial representation of contributions to single gluon emission in $e^+e^- \rightarrow q\bar{q}g$ as given by the ME and the PS. As indicated, the PS does not take into account the interference contributions present in the ME. This leads to the ratio $ME/PS$ as depicted in the contour plot. At the soft and collinear boundaries of the phase space in the $x_1-x_2$ plane (where $x_i$ corresponds to the energy fraction of the quark and antiquark) the PS correctly reproduces the ME, whereas in the region of hard gluon emission the PS omits the (destructive) interference contribution.

From the figure above, it can be seen clearly, that in the ratio $d\sigma_{\text{ME}}/d\sigma_{\text{PS}}$ the omitted interference terms have a large impact in the region where the gluon is hard and emitted at a large angle. Obviously it would be of great interest to combine both ME and PS to take full advantage of their respective virtues. A number of attempts in this direction were made in [2]-[11], some of them also incorporate NLO corrections for specific processes. For the latter, a recent paper [12] provides a general method to construct process specific
algorithms to combine MEs at NLO with the PS. This method, however, is limited by the number of processes that are calculated at NLO and that the user wants to implement.

In contrast, in the framework of this paper I would like to restrict the discussion on the merging of MEs at tree level with the PS. This allows for a process independent method that can be applied immediately. In fact, what I propose in this paper is the extension of a previous paper [1], addressing the merging of arbitrary MEs for jet production at tree level and the PS in $e^+e^-$ annihilations, to processes with hadronic initial states.

The basic idea of this method is to divide the phase space for parton radiation in two disjoint regions, one region of jet production that is filled with help of MEs and a complementary region of jet production, that is filled with the PS. The combined ME+PS then should correctly reproduce all large logarithms up to NLL accuracy and the subleading terms present in the ME. Consider as an example the process $e^+e^- \rightarrow$ jets. The respective orders in $\alpha_s$ and $L$ are given in the figure to the right. For the case of four jet production, the method incorporates all the contributions labelled with a 4, i.e. the full tree level ME, plus all NLL contributions of higher perturbative order, represented by the green blobs to the right of the vertical four jet line.

The challenge in such a procedure is to avoid double counting. In our example of four jet production this translates into ensuring that the green blobs labelled by a 4 are taken into account only once. This can be achieved by applying weights on the tree level ME such that the NLL terms of higher perturbative order in $\alpha_s$ are correctly treated and by a veto on the production of extra jets in the PS. To see how this works, let us stick first to the example of jet production in $e^+e^-$ annihilations before going to more general cases.

2. ME+PS in $e^+e^-$ annihilations

2.1 Jet rates at NLL accuracy

I’d like to begin the discussion of the combination procedure with a brief review of the PS. There, individual branchings, i.e. parton emissions, are governed by the Sudakov form
factor,
\[
\Delta(T, t) = \exp \left\{ - \int_t^T \frac{dt'}{t'} \int_{z_+ (t')}^{z_- (t')} dz \frac{\alpha[s] [p_\perp (z, t')]}{2\pi} P_{a\rightarrow bc} (z) \right\},
\]  
(2.1)
yielding a probability for no branch of parton \(a\) into partons \(b\) and \(c\) between scales \(T\) and \(t\). In Eq.(2.1) the scales \(T\) and \(t\) might denote virtual masses like for instance in PYTHIA [13] or scaled opening angles like in HERWIG, [14]. \(z\) denotes the energy fraction of the parton after branching, the limits on the \(z\)–integration depend on the meaning of the scale parameters as does the particular form of the transverse momentum \(p_\perp\) in the argument of \(\alpha_s\), and the splitting function \(P_{a\rightarrow bc}\) depends on the parton types involved. In the following we will consider only angular ordered, coherent PS [15]. Then the scales \(T\) and \(t\) play the role of scaled opening angles, i.e. the branching scale of parton \(a\), \(t_a\) reads
\[
t_a = E_a^2 (1 - \cos \theta_{bc})
\]  
(2.2)
and the \(z\)–limits are given by
\[
\sqrt{t_0/t_a} < z < 1 - \sqrt{t_0/t_a}
\]  
(2.3)
where \(t_0\) is connected to a minimal branching angle.

However, up to NLL accuracy we can then perform the \(z\)–integrals in Eq.(2.1) yielding the NLL–Sudakov form factor
\[
\Delta_{\text{NLL}}^{q,g} (Q, q) = \exp \left[ - \int_q^Q dq' \Gamma_{q,g} (q', Q) \right]
\]  
(2.4)
with the integrated splitting functions
\[
\Gamma_q (q, Q) = \frac{2C_F}{\pi} \frac{\alpha_s (q)}{q} \left( \log \frac{Q}{q} - \frac{3}{4} \right),
\]
\[
\Gamma_g (q, Q) = \frac{2C_A}{\pi} \frac{\alpha_s (q)}{q} \left( \log \frac{Q}{q} - \frac{11}{12} \right).
\]  
(2.5)
In Eqs.(2.4) and (2.5) I have explicitly denoted the parton type – quark \((q)\) or gluon \((g)\) – and the scales \(Q\) and \(q\) are now directly related to transverse momenta. For further details I would like to refer to [16].

Now we are in the position to give expressions for \(n\) jet rates in \(e^+e^-\) annihilations in the Durham– or \(k_\perp\) scheme [17, 18]. In this scheme two partons belong to different jets, if
\[
y_{ij} = \frac{2\min \{E_i^2, E_j^2\} (1 - \cos \theta_{ij})}{s} > y_{\text{jet}},
\]  
(2.6)
where the parameter \(y_{\text{jet}}\) regulates the "hardness" of the jets and \(s = E_{\text{cm}}^2\) is the c.m. energy squared of the \(e^+e^-\) pair. Defining the jet resolution scale
\[
Q_{\text{jet}} = \sqrt{y_{\text{jet}} E_{\text{cm}}}
\]  
(2.7)
we have

\[ R_{2}^{NLL} = [\Delta_{q}^{NLL}(E_{\text{cm}}, Q_{\text{jet}})]^{2} \]

\[ R_{2}^{NLL} = 2 [\Delta_{q}^{NLL}(E_{\text{cm}}, Q_{\text{jet}})] \]

\[ \times \int_{Q_{\text{jet}}}^{E_{\text{cm}}} dq \frac{\Delta_{q}^{NLL}(E_{\text{cm}}, Q_{\text{jet}})}{\Delta_{q}^{NLL}(q, Q_{\text{jet}})} \Gamma_{q}(q, E_{\text{cm}}) \Delta_{q}^{NLL}(q, Q_{\text{jet}}) \Delta_{g}^{NLL}(q, Q_{\text{jet}}) \]

\[ = 2 [\Delta_{q}^{NLL}(E_{\text{cm}}, Q_{\text{jet}})]^{2} \int_{Q_{\text{jet}}}^{E_{\text{cm}}} dq \Gamma_{q}(q, E_{\text{cm}}) \Delta_{q}^{NLL}(q, Q_{\text{jet}}) \]

(2.8)

The interpretation is quite simple: Since on the parton level jet rates are mere probabilities, the two jet rate is just the combined probability that neither the quark nor the anti quark have emitted another parton at scales above the jet resolution scale. In turn the two jet rate is a combination of two possible "histories". In either history, one quark does not emit a parton resolvable at \( Q_{\text{jet}} \), whereas the other one first propagates down to an intermediate scale \( q \) without having radiated a parton resolvable at \( Q_{\text{jet}} \). Then, at \( q \) it decays into a quark and a gluon and both decay products experience an evolution down to the jet resolution scale without branching any further. That way, the rates for higher jet configurations can be constructed as well, for details see [18]. However, let us note that omitting the integral over \( q \) differential jet rates at NLL can be constructed, i.e. expressions like \( dR_{3}/dq \).

### 2.2 Differential jet rates and the ME weight

This leads us directly to the construction of the weights on the ME. Having chosen the final state momenta \( p_{i} \) according to the ME with \( \alpha_{s} \) taken at the jet resolution scale the weight is construct along the following lines:

1. Construct the correct "PS history":
   - Merge the two partons \( i \) and \( j \) with the smallest \( y_{ij} \). Combine only "allowed" pairs, like for instance \{quark, gluon\} or \{quark, anti-quark\}. The momentum of the new parton is the sum of the momenta of \( i \) and \( j \).
   - Repeat the step above until only a \( q\bar{q} \) pair remains.

2. For each parton line of type \( p \) between \( q_{\text{in}} \) and \( q_{\text{out}} \) apply a weight

\[ \frac{\Delta_{p}^{NLL}(q_{\text{in}}, Q_{\text{jet}})}{\Delta_{p}^{NLL}(q_{\text{out}}, Q_{\text{jet}})} \]

(2.9)

where \( q_{\text{out}} \) might be \( Q_{\text{jet}} \) for outgoing partons.

3. For each QCD node apply a correction factor

\[ \frac{\alpha_{s}(q_{\text{node}})}{\alpha_{s}(Q_{\text{jet}})} \]

(2.10)

4. Accept or reject the kinematical configuration according to the combined weight.
As an example let us consider three jet production as in Fig.2.2. In this case the correction weight reads

\[ \mathcal{W} = \Delta_q(Q_{\text{jet}}, Q) \frac{\Delta_q(Q_{\text{jet}}, Q)}{\Delta_g(Q_{\text{jet}}, Q)} \frac{\alpha_s(q)}{\alpha_s(Q_{\text{jet}})} \]

yielding the correct jet rate \( R_3 \) after taking into account configurations where the gluon gets clustered with the other quark line and after inserting the appropriate integrated splitting function and integration over all scales \( q \).

In this way, all the integrated splitting functions of the differential jet rates at NLL accuracy have been replaced by the correct ME. This leads to an improved description of the region \( y_{qg}, y_{qg} > y_{\text{jet}} \), i.e. the region of hard jet emission where interference effects matter more and more. Clearly the combined weight of this procedure and the ME takes into account the full ME for jet production at tree-level plus all the higher order leading and next-to leading logarithmic contributions.

From the considerations above we can formulate the re-weighting procedure as a cookbook recipe as follows:

1. Expand the PS weight up to the perturbative order in \( \alpha_s \) for jet production at tree level. In fact this boils down to a product of integrated splitting functions.

2. Replace this leading term of the result with the full ME squared.

Let me note in passing that this is exactly the approach utilised by Pythia [13] and Herwig [14] but only up to the first order in \( \alpha_s \). In both programmes, either the first or the hardest emission off both the quark and the anti-quark line is re-weighted with the ME \(^1\), thus suitably replacing the splitting function. The re-weighting procedure makes use of the fact, that \( d\sigma_{\text{ME}}(q\bar{q}g) < d\sigma_{\text{PS}}(q\bar{q}g) \) in the full phase space of gluon emission. This relation is not valid for higher parton configurations any more and consequently the re-weighting procedure has to be modified. The most obvious way to keep the re-weighting applicable is to enhance the value of \( \alpha_s \) in the Sudakov form factor, \( \alpha_s \rightarrow k \cdot \alpha_s \) with \( k > 1 \) a suitable constant factor. However, due to a potentially large number of rejected emissions, this modification might lead to tremendously inefficient parton showers. For more details on the rejection procedure in various processes see [5]-[7].

### 2.3 Vetoing the PS

Having corrected the ME on differential jet rates exact up to NLL accuracy, we are now left with the task of performing the PS. The question then naturally arises, at which scale to start the PS. Naively one might try to start the parton shower at the jet resolution scale, i.e. at \( Q_{\text{jet}} \). However, this is wrong. A naive counter argument is that in such a case, there

\(^1\)the weight is given by \( d\sigma_{\text{ME}}(q\bar{q}g)/d\sigma_{\text{PS}}(qqg) \)
would be a radiation dip at scales just below $Q_{\text{jet}}$. For a more formal argument, consider two jet events. Their rate is

$$R_{\text{NLL}}^{2}(Q_{\text{jet}}) = \left[ \Delta_{q}^{\text{NLL}}(E_{cm}, Q_{\text{jet}}) \right]^{2},$$

(2.11)

see Eq.(2.8). The probability for a two jet event at an even smaller resolution scale $Q_{0} < Q_{\text{jet}}$ should read

$$R_{\text{NLL}}^{2}(Q_{0}) = \left[ \Delta_{q}^{\text{NLL}}(E_{cm}, Q_{0}) \right]^{2}.$$

(2.12)

If we start the PS at the jet resolution scale, however, the probability for two jet events at scale $Q_{0}$ would read

$$\left[ \Delta_{q}^{\text{NLL}}(E_{cm}, Q_{\text{jet}}) \Delta_{q}^{\text{NLL}}(Q_{\text{jet}}, Q_{0}) \right]^{2},$$

(2.13)

which is clearly wrong. The correct answer is to start the shower at the hard scale $E_{cm}$ and veto all emissions with $q > Q_{\text{jet}}$. Identifying vetoed emissions with crosses in the figure on the right we then find for one quark line

$$\frac{\Delta_{q}(E_{cm}, Q_{0}) \Delta_{q}(E_{cm}, Q_{\text{jet}})}{\Delta_{q}(E_{cm}, Q_{\text{jet}})} = \frac{\Delta_{q}(E_{cm}, Q_{0}) \Delta_{q}(E_{cm}, Q_{\text{jet}})}{\Delta_{q}(E_{cm}, Q_{\text{jet}})}$$

(2.14)

exactly as demanded. Similar reasoning holds true for more jets, again I want to refer to [1]. This leads to the following algorithm for the PS:

- The PS evolution for any outgoing parton starts at the nodal scale $q > Q_{\text{jet}}$ where the parton was produced.
- In any branch at some nodal value $q$, the softer of the two outgoing partons is produced at $q$ whereas the harder parton is produced at some larger scale. This is of specific interest for $g \to gg$ nodes.

Taken everything together it can be shown that in the combination of re-weighting the MEs and vetoing the PS correct NLL jet rates are reproduced at all resolution scales above the fragmentation scale. In particular, the dependence on $Q_{\text{jet}}$ cancels at NLL accuracy. Furthermore, colour configurations are chosen in a gauge independent way.

3. ME+PS for hadron collisions

For the construction of a combination procedure with similar virtues we employ the same strategy of dividing the phase space for parton emission into a region of jet production
and a region of jet evolution. Again, this division is achieved by means of a $k_\perp$-measure [19, 20], and again MEs together with a suitable weight are responsible for the production of jets whereas the PS together with a veto on the emergence of extra jets takes care of the jet evolution down to the fragmentation scale.

### 3.1 Some brief reminders

Before I go into more detail I’d like to review briefly the PS in the initial state. For definiteness let us constrain ourselves on the case of processes with hadrons as initial states, since the extension to processes involving both hadrons and leptons in the initial state (like for instance Deep–Inelastic Scattering, DIS) is straightforward. First, some process $h_1 h_2 \rightarrow X$ and its configuration in phase space is generated according to

$$
\sigma_{h_1 h_2 \rightarrow X} = \sum_{p_1, p_2} \int dx_1 dx_2 d\hat{\Phi} f_{p_1}^{h_1}(x_1, Q^2) f_{p_2}^{h_2}(x_2, Q^2) \frac{d\sigma_{p_1 p_2 \rightarrow X}}{d\Phi},
$$

where $x_i$ are the energy fractions the partons $p_i$ carry. Furthermore, $d\sigma/d\hat{\Phi}$ is the partial differential cross section of the partonic subprocess w.r.t. the differential phase space element $\hat{\Phi}$ of the outgoing particles. The scale $Q^2$ entering the parton distribution functions $f_p$ (PDF) depends on the phase space point under consideration. In Drell-Yan processes, for instance, this scale is given by the invariant mass of the lepton pair, whereas in QCD-type subprocesses this scale is given by the $p_\perp^2$ of the outgoing partons.

Starting from this scale the PS in the initial scale now proceeds backwards down to some infrared scale $t_0$. This backward evolution is due to the fact that in difference to the final state PS here the parton configurations both at the hard and the soft scales are already known. The idea behind the backward evolution is to include the PDFs in order to generate only physically meaningful parton ensembles during the PS evolution.

In this framework, the probability for a parton $b$ to be evolved backwards from $(t_2, x_2)$ to $(t_1, x_2)$ with no branching resolvable at the scale $t_0$ reads

$$
\Pi(t_1, t_2; x_2) = \frac{f_b(x_2, t_1) \Delta_b(t_2, t_0)}{f_b(x_2, t_2) \Delta_b(t_1, t_0)}.
$$

The inclusion of the PDFs actually ensures the correct physical behaviour at low and high values of $x_2$. Having chosen a scale $t_1$, the corresponding value $z = x_2/x_1$ of the splitting $a \rightarrow bc$ has to be selected according to

$$
\frac{\alpha_s(p_\perp(z, t_1))}{2\pi} \frac{x_2/z f_a(x_2/z, t_1)}{x_2 f_b(x_2, t_1)} P_{a \rightarrow bc}(z).
$$

After each branching, the system is boosted into the c.m. frame of the two outermost incoming partons.

Let us continue the discussion now with the outline of the algorithm I’d like to propose before I start constructing the weights on the MEs.
3.2 Proposed algorithm

The algorithm I propose goes as follows:

1. Cluster initial and final state particles backwards according to the longitudinal invariant $k_\perp$ scheme \cite{19, 20} until a $2 \rightarrow 2$ process remains. This clustering is achieved step by step in the c.m. system of the incoming partons. In this scheme the initial and final state partons are included in the following fashion:

   - If the two particles considered are both outgoing, their measure $y_{ij}$ is given by
     \[
     y_{ij} = \frac{2 \min \{ E_i^2, E_j^2 \} (1 - \cos \theta_{ij})}{\hat{s}} \rightarrow \frac{\min \{ p_{\perp, i}, p_{\perp, j} \} \left[ (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2 \right]}{\hat{s}},
     \]
     see Eq. (2.6). Here, $\hat{s}$ is the invariant mass squared of the outgoing particles, $p_{\perp}$ are their transverse momenta w.r.t. the beam axis and $\eta$ and $\phi$ are their pseudorapidities and azimuthal angles.

   - If two outgoing, i.e. final state, particles are clustered, the resulting particle again is a final state particle with $p = p_i + p_j$.

   - If one of the two particles, say $j$ is one of the two incoming partons then
     \[
     y_{ij} = \frac{2 E_i^2 (1 - \cos \theta_{ij})}{\hat{s}} \rightarrow \frac{p_{\perp, j}^2}{\hat{s}}.
     \] (3.4)

   If an incoming and an outgoing particle are clustered, the new particle is incoming, and its momentum is $p = p_j - p_i$. Note that in such a case a boost is in order to the c.m. frame of the new pair of two incoming particles.

In this scheme, the outgoing and incoming particles are clustered “towards” the hard $2 \rightarrow 2$ process, i.e. clustering two particles leads to another particle resolved at a higher scale.

Note that in case we consider DIS–like processes the measures are given by the energies and the cosines, whereas in case of purely hadronic initial states the measures are given in terms of transverse momenta.

2. Find the hardest $k_\perp^2$ in the “core” $2 \rightarrow 2$–subprocess. Examples for this hardest $k_\perp^2$ are:

   - $\hat{s} = M_{ll}^2$ in Drell-Yan type $q\bar{q} \rightarrow l\bar{l}$ subprocesses.
   - $\frac{2s^2l^2}{s^2 + l^2 + u^2}$ in QCD subprocesses.

3. Apply a weight constructed by comparing histories. Some examples will be given below.

4. Start the initial and final state parton showers from the $2 \rightarrow 2$–subprocess with corresponding starting conditions, i.e. every leg starts its evolution at the scale where it was produced.
5. Veto on jet emissions in the PS.

3.3 Constructing the ME weight

As an example let us consider Drell-Yan processes. Similar to the case of $e^+e^-$ annihilations we first construct a PS weight, that consists of ratios of Sudakov form factors and splitting functions. Taking $t_{\text{jet}}$ as the jet resolution scale and $t$ as the hard scale $t = M_{\ell\ell}^2$ this weight reads for the configuration in the example above

$$W_{PS} = \frac{q(x_1/z_1, t_1)}{z_1 q(x_1, t)} \frac{\Delta_q(t, t_{\text{jet}})}{\Delta_q(t_1, t_{\text{jet}})} \cdot \alpha_s(t_1) P_{q\rightarrow gg}(z_1) \times \frac{q(x_1/z_1 z_2, t_2)}{z_2 q(x_1/z_1, t_1)} \frac{\Delta_q(t_1, t_{\text{jet}})}{\Delta_q(t_2, t_{\text{jet}})} \cdot \alpha_s(t_2) P_{q\rightarrow gg}(z_2) \times \frac{q(x_1/z_1 z_2, t_{\text{jet}})}{q(x_1/z_1, t_{\text{jet}})} \frac{\Delta_q(t_{\text{jet}}, t_{\text{jet}})}{\Delta_q(t_{\text{jet}}, t_{\text{jet}})} \cdot \alpha_s(t_2) P_{q\rightarrow gg}(z_2).$$

(3.5)

We have to combine this with the PDFs contained in the expression for the cross section for pure Drell-Yan processes with $Q^2 = t$,

$$d\sigma_{DY} = \int dx_1 dx_2 q(x_1, Q^2) \bar{q}(x_2, Q^2) d\hat{\sigma}_{q\bar{q}\rightarrow \ell\ell}(\hat{s}, \alpha_s(Q^2)),$$

see Eq. (3.1) and compare the result with a similar expression for the production of two additional gluons at a lower scale,

$$d\sigma_{DY+gg} = \int dx_1' dx_2 q(x_1' = x_1/z_1 z_2, t_{\text{jet}}) \bar{q}(x_2, t_{\text{jet}}) d\hat{\sigma}_{q\bar{q}\rightarrow \ell g g}(\hat{s}/z_1 z_2, \alpha_s(t_{\text{jet}})).$$

We can easily read off the correction weight on the matrix element, namely

$$W_{\text{corr.}} = \Delta_q(t, t_{\text{jet}}) \cdot \frac{\alpha_s(t_1)}{\alpha_s(t_{\text{jet}})} \cdot \frac{\alpha_s(t_2)}{\alpha_s(t_{\text{jet}})} \cdot \alpha_s(t_{\text{jet}}).$$

(3.6)

Note that the missing factor $z_1 z_2$ in the denominator as well as the missing PDF in the numerator is compensated by the differential cross section for the "Drell-yan + 2 gluon"-process at the jet resolution scale.
Of course, things become more complicated, if some flavour changing branchings like for instance gluon splitting occurs between the jet resolution and the hard scale. For the example above the PS weight would read

$$ W_{PS} = \frac{g(x_1/z_1,t_1)}{z_1g(x_1,t)} \frac{\Delta g(t,t_{jet})}{\Delta g(t_1,t_{jet})} \cdot \alpha_s(t_1)P_{g \rightarrow qq}(z_1) $$

$$ \times \frac{g(x_1/z_1z_2,t_2)}{z_2g(x_1/z_1,t_1)} \frac{\Delta g(t_1,t_{jet})}{\Delta g(t_2,t_{jet})} \cdot \alpha_s(t_2)P_{g \rightarrow gg}(z_2) $$

resulting in the following correction weight

$$ W_{corr.} = \Delta g(t_1,t_{jet}) \cdot \frac{\Delta q(t,t_{jet})}{\Delta q(t_1,t_{jet})} \cdot \frac{\alpha_s(t_1)}{\alpha_s(t_{jet})} \cdot \frac{\alpha_s(t_2)}{\alpha_s(t_{jet})}. $$

We read off that the correction factor is a combination of Sudakov form factors depending only on the nodes of emissions and the flavours of the incoming lines. This is in fact quite similar to what we already knew from pure final state considerations.

### 3.4 Vetoed showers

Finally, let us turn to the PS. For the final state particles, the PS evolution is done in the same fashion as in the case of $e^+e^-$ annihilations. For the two initial state lines the evolution from the harder – spacelike – scales down to lower scales proceeds as follows:

1. Evolve to lower scales with help of the Sudakov form factors.
2. Veto on all emissions with $t > t_{jet}$.
3. For the first allowed branching with $t_{all.} \leq t_{jet}$ the PDF part of the weight reads:

$$ W_{PDF} = \frac{f(\tilde{x}/\tilde{z},t_{all})}{\tilde{z}f(\tilde{x},t_{jet})}. $$

In the two examples above, $\tilde{x} = x_1/z_1z_2$.

### 4. Summary and outlook

In this paper I have suggested a method to combine MEs and PS for hadronic interactions. This method is an extension of the already known one for $e^+e^-$ annihilations [1] that has been implemented and successfully tested vs. experimental data in the new event generator AMEGIC++/APACIC++ [21, 22].

In both cases the method correctly takes into account the full perturbative order in $\alpha_s$ of the tree–level ME for the corresponding multi jet production. Additionally, all contributions up to NLL accuracy are reproduced in the jet rates via a re-weighting procedure. Suitable starting conditions and a veto applied on the PS avoid double counting.

Nevertheless, I would like to stress that the extension of the method I proposed still waits for the proof of correctness and for the implementation into an event generator. This is work in progress.
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