Divergent energy strings in $AdS_5 \times S^5$ with three angular momenta

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Abstract

In this paper, novel solutions for strings with three angular momenta in $AdS_5 \times S^5$ geometry are presented; the divergent energy limit and the corresponding conserved charges, as well as dispersion relation are also determined. Interpretations of these configurations as either a giant magnon (GM) or a spiky string (SS) are discussed.

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I. INTRODUCTION

Divergent energy solutions have an important role in classical string theory. Such strings can be interpreted in terms of the gauge theory, but even in cases where an exact interpretation is lacking, there are enough characteristic features in these solutions to characterize them in a class of their own. The most well-understood cases concern solutions known as giant magnons [1], and when one string of this kind has one angular momentum in the $\mathbb{R} \times S^2$ subspace of $AdS_5 \times S^5$, it obeys the dispersion relation

$$E - J = 2T \left| \sin \frac{P}{2} \right| ,$$

(1)

where $E$, $J$ and $T$ are the energy, the angular momentum, and the tension of the string, respectively. $p$ is identified with the angle between the extremes of the giant magnon and $E, J \rightarrow \infty$. In another case, a solution known as spiky string [2, 3] also describes a divergent energy string with one angular momentum and it too obeys the dispersion relation

$$E - T \Delta \phi = 2T \left( \pi - \theta \right) ,$$

(2)

where $T$ is the string tension, $\Delta \phi \rightarrow \infty$ is the deficit angle and $\theta$ is the coordinate where the string peaks.

Variations in dispersion relations can be found in strings rotating in various dimensions and in different geometries. For example, giant magnons are found in $AdS_5 \times S^5$ with two [2, 4, 8] and three [2, 4] angular momenta. In the Lunin-Maldacena background, GM’s are found with one [9, 10] and two [9–11] angular momenta; and in $AdS_4 \times \mathbb{CP}^3$, with one [8, 12, 14], two [8, 13, 15, 19] and three [20] angular momenta.

In the case of spiky strings different examples have already been found. In $AdS_5 \times S^5$ geometry, spiky strings have been found with one [21], two [9] and three [4] angular momenta; and in $AdS_4 \times \mathbb{CP}^3$ with one [13], two [16] and three [20] angular momenta.

The more angular momenta are added to the string, the more the dispersion relation changes. If we have two angular momenta, the dispersion relation [22, 23]

$$E - J_1 = \sqrt{J_2^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{P}{2}}$$

(3)

describes a giant magnon string in $AdS_5 \times S^5$ where $E$ and $J_1$ are divergent and $J_2$ is finite. If we allow more divergent quantities, the complexity increases, and we find the following divergent dispersion relation [24]

$$\sqrt{E^2 - J_\phi^2} - J_\psi = T \sin \Delta \psi$$

(4)

found in giant magnons in $AdS_4 \times \mathbb{CP}^3$ [13, 20], and in the case of many divergent angles the spiky string-like dispersion relation can be even more complex [20]. The interpretation of these solutions in the case of gauge theory is unknown, but if the gauge/gravity correspondence is correct, these solutions also have a dual description.

In a previous study [20], we addressed several multi-divergent solutions for strings in the $AdS_4 \times \mathbb{CP}^3$ background with three angular momenta. These solutions are important in order to answer the question of the existence of giant magnons and spiky strings with three angular momenta in the aforementioned background. In the case of $AdS_5 \times S^5$, the references cited above include giant magnons and spiky strings with various angular momenta, but not with various divergent angular momenta and deficit angles, and the purpose of this article is to fill in this void in current literature. In some sense we generalize the former results.

The contents of this paper are organized as follows: in the second section, multi-divergent solutions are constructed in their full generality, while in the third and fourth sections, we take the particular cases where divergent energy is found and interpret them in terms of giant magnons and spiky strings. The fourth section presents our conclusions.

II. THE GENERAL SOLUTION

We start with the complete $AdS_5 \times S^5$ metric

$$ds^2 = R^2 \left( - \cosh^2 \rho dt^2 + d\phi^2 + \sinh^2 \rho d\Omega_3^2 \right) + R^2 \left( d\theta^2 + \sin^2 \theta d\psi^2 + \sin^2 \theta \cos^2 \psi d\phi_1^2 + \cos^2 \theta d\phi_2^2 + \sin^2 \theta \sin^2 \psi d\phi_3^2 \right) .$$

(5)
The first term of expression (5), in brackets, corresponds to the metric of a five dimensional anti-de Sitter space whose elements are time, \( t \), a radial coordinate, \( \rho \), and a three dimensional sphere. The second term in brackets is a metric of a five dimensional sphere, whose coordinates are parameterized as \( \theta, \psi \in [0, \pi/2] \) and \( \phi_i = (1, 2, 3) \in [0, 2\pi] \). We are seeking solutions at the center of the anti-de Sitter space, which means that \( \rho = 0 \) and the string is confined in a \( \mathbb{R} \times S^5 \) subspace. The motion of the string we are interested in has four degrees of freedom, as the coordinate \( \theta \) is kept constant. Of course, if we chose \( \psi \) as the constant, the results would be exactly the same. Thus, we chose the following ansatz for the varying coordinates

\[
 t = \kappa \tau, \quad \theta \in (0, \pi/2), \quad \psi = \psi(y), \quad \phi_i = (1, 2, 3) = \omega_i \tau + f_i(y)
\]

where \( y = \alpha \sigma + \beta \tau \), and \( \alpha, \beta, \kappa, \) and \( \omega_i \) are constants, and \( \sigma \) and \( \tau \) parameterize the dynamics of the world sheet of the string. The dynamics of a string with tension \( T = \frac{\sqrt{A}}{2\pi} \) can be described by Polyakov action

\[
 S = \frac{T}{2} \int d\sigma \, dt \, \gamma^{ab} g_{\mu\nu} \partial_\mu X^a \partial_\nu X^b,
\]

and the Virasoro constraints

\[
 g_{\mu\nu} \partial_\tau X^a \partial_\tau X^b = 0 \quad \text{and} \quad g_{\mu\nu} \left( \partial_\tau X^a \partial_\sigma X^b + \partial_\sigma X^a \partial_\tau X^b \right) = 0,
\]

where \( \gamma^{ab} = (-1, -1, 1) \) is the world sheet metric. Using (6) and the equations of motion for \( \phi_i \), we obtain

\[
 \partial_y f_i = \frac{1}{\alpha^2 - \beta^2} \left( \frac{A_i}{g_i} + \beta \omega_i \right)
\]

where \( A_i \) is the integration constant, and \( g_i \) the metric tensor component corresponding to \( \phi_i \). Using (9) and the Virasoro constraints (8) we get

\[
 \psi_y^2 + w_1^2 \cos^2 \psi + w_2^2 + w_3^2 \sin^2 \psi + \frac{a_1^2}{\cos^2 \psi} + a_2^2 + \frac{a_3^2}{\sin^2 \psi} - k^2 = 0
\]

such that \( \psi_y = \partial_y \psi \) and the equation constants come from a redefinition of the older ones, namely

\[
 a_1 = \frac{1}{\alpha^2 - \beta^2} \frac{A_1}{\sin^2 \theta}, \quad a_2 = \frac{1}{\alpha^2 - \beta^2} \frac{A_2}{\cos^2 \theta}, \quad a_3 = \frac{1}{\alpha^2 - \beta^2} \frac{A_3}{\sin^2 \theta},
\]

\[
 w_1 = \sin^2 \theta \frac{\alpha \omega_1}{\alpha^2 - \beta^2}, \quad w_2 = \cos^2 \theta \frac{\alpha \omega_2}{\alpha^2 - \beta^2}, \quad w_3 = \sin^2 \theta \frac{\alpha \omega_3}{\alpha^2 - \beta^2}
\]

and

\[
 k^2 = \frac{\alpha^2 + \beta^2}{(\alpha^2 - \beta^2)^2} \kappa^2.
\]

With the change of variables \( X = \cos 2\psi \) so that (11) becomes

\[
 \frac{1}{16} \lambda_y^2 = \frac{w_1^2 - w_3^2}{8} \lambda^3 + \left( \frac{w_1^2 + w_3^2}{8} - \frac{m}{4} \right) \lambda^2 + \\
 + \left( \frac{w_2^2 - w_1^2}{8} + \frac{a_1^2 - a_3^2}{2} \right) X - \frac{w_1^2 + w_3^2}{8} - \frac{a_1^2 + a_3^2}{2} + \frac{m}{4},
\]

and \( m = k^2 - w_2^2 - a_2^2 \). Equation (11) shows that the polynomial degree drops from three to two, if \( w_1^2 = w_3^2 \). Thus, we study the two different cases separately.

A. \( w_1^2 \neq w_3^2 \)

To obtain divergent energy, we choose

\[
 a_i = 1, 3 = \frac{1}{4} (m - w_1^2).
\]
The energy is real if \( w_3^2 > w_1^2 \), and so finally we get
\[
d y = \frac{1}{\sqrt{2 (w_3^2 - w_1^2)}} \frac{d \mathcal{X}}{\mathcal{X} \sqrt{\mathcal{X}_0 - \mathcal{X}}},
\tag{13}
\]
where \( \mathcal{X}_0 \in (0, 1) \) is given by
\[
\mathcal{X}_0 = \frac{w_1^2 + w_3^2 - 2m}{w_3^2 - w_1^2}. \tag{14}
\]
Expression (13) can be used to calculate the conserved charges
\[
E = T \int \frac{2\pi}{0} d \sigma \hat{t}, \quad \text{and} \quad J_i = T \int \frac{2\pi}{0} d \sigma g_i \hat{\varphi}_i. \tag{15}
\]
Using \( d \sigma = \alpha dy \) and \( \mathcal{X} \in [0, \mathcal{X}_0] \), the energy is divergent and the momenta and the deficit angles are
\[
J_i \left( \sin^2 \theta \right) = \left( \beta a_i + \frac{\alpha w_i}{2} \right) E \kappa \pm \sqrt{2 \left( w_3^2 - w_1^2 \right)} I_0
\]
\[
\Delta \phi_i \left( \sin^2 \theta \right) = \left( \alpha a_i + \beta w_i \right) E \kappa T + \frac{2 a_i}{\sqrt{2 \left( w_3^2 - w_1^2 \right)}} I_i
\tag{16}
\]
\[
J_2 = \frac{J_2}{\cos^2 \theta} = \left( \beta a_2 + \alpha w_2 \right) E \kappa \quad \text{and} \quad \Delta \phi_2 = \left( \alpha a_2 + \beta w_2 \right) E \kappa T,
\]
where
\[
I_0 = \int_{\mathcal{X}_0}^{\mathcal{X}_0} \frac{d \mathcal{X}}{\sqrt{\mathcal{X}_0 - \mathcal{X}}} = 2 \sqrt{\mathcal{X}_0}
\]
\[
I_1 = \int_{\mathcal{X}_0}^{\mathcal{X}_0} \frac{1}{\mathcal{X} - 1} \frac{d \mathcal{X}}{\sqrt{\mathcal{X}_0 - \mathcal{X}}} = - \frac{2}{\sqrt{1 - \mathcal{X}_0}} \arctan \sqrt{\frac{\mathcal{X}_0}{1 - \mathcal{X}_0}}
\tag{17}
\]
\[
I_3 = \int_{\mathcal{X}_0}^{\mathcal{X}_0} \frac{1}{\mathcal{X} + 1} \frac{d \mathcal{X}}{\sqrt{\mathcal{X}_0 - \mathcal{X}}} = \frac{2}{\sqrt{1 + \mathcal{X}_0}} \arctanh \sqrt{\frac{\mathcal{X}_0}{1 + \mathcal{X}_0}}
\]
Expressions (16) and (17) summarize all the information about the conserved charges of the system. We point out that if \( \mathcal{X}_0 \geq 1 \), all the charges are divergent without a finite term, so this case does not generate either giant magnons or spiky strings.

**B. \( w_1^2 = w_3^2 \)**

In this case, we use \( \cos 2 \psi = \gamma \), \( w_1^2 = w_3^2 = w \) and so (11) changes to
\[
\frac{1}{8} \gamma_0^2 = \frac{n}{2} \gamma^2 + (a_1^2 - a_2^2) \gamma + \frac{n}{2} - a_2^2 - a_3^2,
\tag{18}
\]
so that \( n = m - w^2 \). Divergent energy is obtained as long as \( n = n \pm (a_1 \pm a_3)^2 \), and in this case (18) spans
\[
d y = \frac{1}{2 \sqrt{n}} \frac{d \gamma}{\gamma_0 - \gamma}, \tag{19}
\]
so that \( \gamma_0 \in (0, 1] \), because if \( \gamma_0 > 1 \) there is no divergence in the integral. Thus, either
\[
\gamma_0 = \frac{a_1 - a_3}{a_1 + a_3} \quad \text{if} \quad n = n_+,
\quad \text{otherwise} \quad \gamma_0 = \frac{a_1 + a_3}{a_1 - a_3}. \tag{20}
\]
The conserved charges to this string are

\[ \mathcal{J}_{i=1,3} = \left( \beta a_i + \alpha w_1 \frac{1 \pm \sqrt{\gamma}}{2} \right) E \frac{1}{\kappa} \pm T w_3 \gamma_0 \]

\[ \Delta \phi_i = \left( \beta w + \frac{2 a_i}{1 \pm \sqrt{\gamma}} \right) E \frac{1}{\kappa T} + \frac{2 a_i}{1 \pm \sqrt{\gamma}} \ln(1 \pm \sqrt{\gamma}). \]  

(21)

\( \mathcal{J}_2 \) and \( \Delta \phi_2 \) have equal expressions as (16).

C. Common features

We classify a solution according to the dispersion relations it generates, and we claim that both cases have giant magnons and spiky strings described by similar dispersion relations. What is meant by similar is that the relation among the divergent quantities is totally equal, with only the finite term being different, as it is generated by the particular constraints among the constants of the problem. We justify our claim by establishing the following equivalence relations:

\[ (J_1 + J_3)_{w_1^2 \neq w_3^2} \sim (J_1 - J_3)_{w_1^2 = w_3^2} \]  

(22)

\[ (J_2)_{w_1^2 \neq w_3^2} \sim (J_1 + J_2 + J_3)_{w_1^2 = w_3^2}. \]  

(23)

Expression (22) relates quantities formed by a finite and a divergent term, and (23) relates two intrinsically divergent quantities. Analogous considerations can be carried out with regards to the deficit angles. So, the relations that can be built among these quantities are the same in both the equivalence classes. Of course, the finite term will have different expression, but with the same physical meaning. Also, the cases are physically equivalent, and in the following sections, we study different dispersion relations by considering only the most general situation, where \( w_1^2 \neq w_3^2 \).

III. GIANT MAGNON SOLUTIONS

Here we need finite deficit angles and divergent energy and momenta, and we obtain these by choosing values for the constants in (16). The most obvious possibility, \( \alpha a_i = -\beta w_i \), forces \( w_1^2 = w_3^2 = 0 \), which is not acceptable. So, we pick a more general condition

\[ \alpha (a_1 + a_3) = -\beta (w_1 + w_3), \quad \alpha a_2 = -\beta w_2 \quad \text{and} \quad \alpha a_{1,3} \neq -\beta w_{1,3}, \]  

(24)

so that, for \( J = J_1 + J_3 \) and \( \Delta \phi = \Delta \phi_1 + \Delta \phi_3 \)

\[ J = \alpha (\omega_1 + \omega_3) \left( \frac{1}{2} - \frac{\beta^2}{\alpha^2} \right) E \frac{1}{\kappa} + T (\omega_1 - \omega_3) I_0, \]

\[ J_2 = \omega_2 \left( 1 - \frac{\beta^2}{\alpha^2} \right) \frac{E}{\kappa} \]

\[ \Delta \phi = 2 (a_1 I_1 + a_3 I_3) \]

\[ \Delta \phi_2 = 0. \]  

(25)

A corresponding set of momenta and divergent angles has already been found in \( AdS_4 \times \mathbb{CP}^3 \) [16, 20] with a particular dispersion relation akin to (1). Hence (25) describes a solution that is a giant magnon of this kind. The only thing to do is to choose the correct constraints among the parameters of the problem. The compatibility between the difference of the Virasoro constraints and a relation that comes from the dispersion relation results in a constraint which permits us to determine \( a_1 \) as

\[ a_1 = \frac{w_1 + w_3}{w_3 - w_1} \frac{\beta}{\alpha} \left[ \frac{1}{2} - \frac{\beta^2}{\alpha^2} \right] (w_1 + w_3) - w_3 \]  

(26)

In addition, as \( X_0 \in (0, 1) \), we have

\[ w_2^2 \left( 1 - \frac{\beta^2}{\alpha^2} \right) + w_1^2 < k^2. \]  

(27)
The deficit angle has a constraint which allows it to be related to the finite term in the dispersion relation as follows:

\[
\sin \Delta \phi = (w_1 - w_3) I_0. \tag{28}
\]

Relations (26), (24), and (28) allow us to eliminate the integration constants and so define a complex constraint among the parameters of the problem, which we not include here because it does not contribute anything with regards to the physics of the problem. All the conditions above allow us to write the dispersion relation of the giant magnon as

\[
\sqrt{E^2 - J^2} - J = T \sin \Delta \phi. \tag{29}
\]

The above solution (29), which has a known giant magnon dispersion relation [16, 20, 24], also has the presumed feature that the limit \(J \to 0\) recovers the usual giant magnon dispersion relation. Compared to the former solution of a giant magnon with three angular momenta [2], we stress that all three angular momenta of (29) are divergent, while the former solution has one divergent angular momentum and two finite angular momenta. On the other hand, both have the same limit when two angular momenta are taken to zero and only one divergent angular momentum remains. Thus, the above result and [2] are different generalizations of the usual giant magnon with one divergent angular momentum. Neither of these generalizations is understood on the gauge side.

We can also obtain a curious giant magnon solution simply by choosing different constants in the \(\phi_2\) direction, such that \(J_2 = 0\), and \(\Delta \phi_2\) is divergent. The dispersion relation in this case is given by

\[
\sqrt{E^2 - (T \Delta \phi_2)^2} - J = T \sin \Delta \phi. \tag{30}
\]

A corresponding case, where a deficit angle and a momenta play interchanged roles has already been discovered in a spiky string by [16]. The effect of simply changing the momentum to the deficit angle in the same direction does not have an interpretation, but (30) certainly describes a GM because the deficit angle and the subtracting momentum belong to the same coordinate, and the limit \(\Delta \phi_2 \to 0\) recovers the GM dispersion relation.

### IV. SPIKY STRING SOLUTIONS

This case is built from divergent deficit angles and finite momenta, so that the analysis of the giant magnon case is valid here, only changing \(J\) by \(T \Delta \phi\) in the appropriate places. As for the giant magnon case, we cannot make the angular momenta finite in an independent manner, because this implies \(w^2_{i=1,3} = 0\). Thus we impose

\[
2\beta(a_1 + a_3) = -\alpha(w_1 + w_3), \quad \beta a_2 = -\alpha w_2 \quad \text{so that} \quad \alpha a_{1,3} \neq -\beta w_{1,3}, \tag{31}
\]

to obtain, with the very notation of the preceding section, that

\[
J = \sqrt{2 \frac{w_3 + w_1}{w_3 - w_1}} I_0, \quad J_2 = 0, \tag{32}
\]

\[
\Delta \phi = \left(1 - \frac{\alpha^2}{2 \beta^2} \right) \beta (w_1 + w_3) \frac{E}{\kappa T} + \sqrt{\frac{2}{w_3^2 - w_1^2}} (a_1 I_1 + a_3 I_3) \tag{33}
\]

\[
\Delta \phi_2 = \left(1 - \frac{\alpha^2}{2 \beta^2} \right) \beta w_2 \frac{E}{\kappa T}. \tag{34}
\]

As \(\lambda_0 \in (0, 1)\), we also write that

\[
w_2^2 \left(1 + \frac{\alpha^2}{\beta^2} \right) + w_1^2 < k^2. \tag{35}
\]

And if we choose

\[
a_1 = \frac{w_1 + w_3}{w_3 - w_1} \frac{\alpha}{\beta} \left[ \left( \frac{1 - \alpha^2/\beta^2}{1 - \alpha^2/\beta^2} \right) \left( w_1 + w_3 \right) - \frac{w_3}{2} \right], \tag{36}
\]

we write a dispersion relation to a spiky string

\[
\sqrt{E^2 - (T \Delta \phi_2)^2} - T \Delta \phi = 2T \left( \frac{\pi}{2} - \psi_0 \right), \tag{37}
\]

\[\text{VI.}\]
so that
\[
\psi_0 = \frac{\pi}{2} - \sqrt{\frac{2}{w_3^2 - w_1^2}} (a_1 I_1 + a_3 I_3)
\]
\[
\cos 2\psi_0 = X_0.
\]

This is a very interesting result: it is totally analogous to the giant magnon case, and it is a novel dispersion relation. A similar case can be obtained from [20] by imposing suitable constraints, although this was not done by the authors. As in the preceding case, the limit \(\Delta \phi_2 \to 0\) generates the usual spiky string dispersion relation, and the unusual form of (37) comes from the fact that there are more geometric degrees of freedom to the motion of the string, and this greater geometric freedom requires a less parametric freedom, thus complicating the constraints (38) involving the constants of the model. When comparing (37), which has two finite angular momenta and three divergent deficit angles, to the spiky string found in [16], which has one divergent angular momentum and one divergent deficit angle, we see that both the results have a usual spiky string as a limit when only one deficit angle remains as a divergent quantity. Thus both cases have identical limits, even though the strings are in different geometries. Accordingly, we infer that these solutions probably have the same physical interpretation.

As for the giant magnon, we can have a different dispersion relation to the spiky string. When we choose appropriate values for the constants, we get
\[
\sqrt{E^2 - J_2^2} - T \Delta \phi = 2T\left(\frac{\pi}{2} - \psi\right).
\]

A similar situation appears is evident in AdS\(_4\) × CP\(_3\) geometry, as studied by [16]. As previously stated with regards to the GM case, we do not have an interpretation for the interchange of momenta and the deficit angle, but it is certainly a spiky string solution because it behaves thus in the \(J_2 \to 0\) limit.

To round off this section, we will mention that a dispersion relation where both a giant magnon and a spiky string appears coupled, as the case in [20], does not seem to be possible in any choice of constants for the strings described in this article. The reason for this is that in order to construct a dispersion relation akin to that found in the reference would require quantities in \(\phi_2\) which are not intrinsically divergent, and \(\mathcal{J}_2\) and \(\Delta \phi_2\) in [16] do not have a finite term to fulfill this feature.

V. CONCLUDING REMARKS

In this article we have presented new giant magnon and spiky string solutions. These solutions are different from the usual giant magnon and spiky string cases because they have divergences in various dimensions. Naturally, this feature changes the dispersion relation of the conserved charges, but the interpretation of these objects in the gauge side of the duality remains, in principle, the same. A point which supports this hypothesis is the satisfactory behavior of the solutions, which recover lower dimensional GM and SS in the appropriate limits. This assumption also relies on the AdS/CFT conjecture, and although it has not yet been proven, the dispersion relations found here must be found in the dual gauge according to the correspondence hypothesis. We expect future studies in the gauge side of the correspondence to confirm this prediction.

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