Bandwidth widening in nonlinear electromagnetic vibrational generator by combined effect of bistability and stretching

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Abstract. This work reports the novel concept of frequency response widening of vibrational energy harvesters exploiting the combined effects of bistable and monostable nonlinearities in a single device. The bistability is introduced into the system by repulsive arrangement of magnets, while monostable nonlinear force through the stretching is incorporated by large deformation of two-end-fixed cantilevers. The simulation results show wider bandwidth in the bistability and stretching combined configuration compared to the bistable only or the monostable stretching only configuration.

1. Introduction
The continual reduction in power requirement of wireless sensor nodes (WSNs) and the possibility of generating the required power from ambient energy sources have made the widespread deployment of self-powered autonomous WSNs feasible in recent times [1]. In this context, the conversion of ambient vibrations into usable electrical power has become a thriving research topic over the last few years due to the abundance of vibration sources in the modern urban environment [2]. However, most of the reported vibrational energy harvesters are based on linear resonant oscillators capable of producing usable energy only within a very narrow range of vibration frequency near the resonance [3-5]. The real vibration sources, on the other hand, often produce vibration that are either random in nature, or the frequency of vibration vary considerably over time [6]. An attractive solution to this problem is the intentional incorporation of nonlinearity into the system dynamics to widen the output response. Some of the earliest reported nonlinear energy harvesting systems employed bistable mechanisms [7-9] and they were proven to be superior in terms of harvested power and frequency bandwidth in realistic multi-frequency stochastic vibration environments. Nonlinear monostable devices based on cantilever stretching effects were also studied [10, 11] and these configurations also demonstrated bandwidth widening effects. The present work aims to combine the effects of bistability and stretching based nonlinearity into a single novel device configuration in order to augment the overall performance of nonlinear VEH systems.

2. Design of the energy harvester
The device, consisting of FR4 device structure, NdFeB magnets, mild steel keepers and copper wire-wound coils is shown in figure 1. The device is based on an oscillator carved out of 0.15 mm thick FR4 sheets using computer controlled laser processing tools. The movable proof mass, consisting of four NdFeB magnets (8mm × 4mm × 2mm) and two mild steel plates (8mm × 8mm × 1.6mm) are
bonded on either sides of a slot in the FR4 device structure using epoxy adhesive. The four magnets are arranged in oppositely polarized orientation in such a way that high flux density prevails in the slot between the magnet assemblies [3, 4]. The mild steel keepers are used on both sides of the four-pole magnet arrangement to concentrate the magnetic flux lines through the coil, as depicted in figure 1(b).

Figure 1: (a) Vibrational energy harvesting device structure. (b) Cross-section of the four-pole magnet configuration. Red arrows denote the polarity of the magnets.

When the mass vibrates in the vertical direction, the pair of centrally fixed and two-ends-guided cantilevers supporting the mass gets stretched in addition to their bending. This stretching introduces a nonlinear restoring force proportional to the third power of the proof mass displacement, into the system. Two more pairs of repulsively oriented magnets are arranged on both sides of the proof-mass. This magnetic repulsive force introduces bistable nonlinearity into the system which can be controlled by adjusting the gap between the repulsive magnets.

3. Electromechanical model and numerical analysis

The theory of operation of the device can be described as a spring-mass-damper system in conjunction with a nonlinear spring force, due to stretching of the spring arm. Another nonlinear force term is added due to the bistable potential well introduced by the magnetic repulsive force on the FR4 based oscillator.

Figure 2: (a) Vertical deflection of the device under external force. (b) Applied force vs vertical deflection.
The equivalent mass, linear spring coefficient and total damping of the system is denoted by $m$, $k$, and $D$ respectively. A periodic force $F$ due to external vibration is applied in vertical ($Z$) direction. The resulting vertical displacement of the equivalent mass $m$ from the initial equilibrium position is denoted by $z(t)$ and the distance between the repulsively arranged pair of magnets become $r$. Since the cantilever arms supporting the equivalent mass are fixed at the middle and guided (i.e. allowed to move only in the vertical direction) on both ends, beams would undergo stretching as well as bending when vibrating with amplitude larger than the thickness of the beams [11]. The potential energy due to bending and stretching of the supporting beams is given by the Duffing potential function as,

$$ U(z) = \frac{1}{2} k z^2 + \frac{1}{4} k_n z^4 $$

(1)

where $k_n$ is the nonlinear spring coefficient due to stretching. The values of the linear and nonlinear spring constants are determined from stationary simulation using COMSOL Multiphysics [Figure 2].

The magnetic potential energy [12] due to the force of repulsion between the magnets is then given by

$$ V_m(z) = -\frac{\mu_0}{4\pi r^3} \left[ 3 m_1 m_2 \left( \frac{d}{r} \right)^2 - m_1 m_2 \right] + \frac{\mu_0 m_1 m_2}{4\pi} \frac{\left( 2d^2 - z^2 \right)}{\left( z^2 + d^2 \right)^{5/2}} = C \frac{\left( 2d^2 - z^2 \right)}{\left( z^2 + d^2 \right)^{5/2}} $$

(2)

where $\mu_0$ is the permeability of air and $m_1$, $m_2$ are the magnetic dipole moments due to the repulsively oriented magnets, and $C$ is a constant pre-factor.

The total (mechanical and magnetic) potential energy and restoring force of the system is then given by,

$$ V(z) = \frac{1}{2} k z^2 + \frac{1}{4} k_n z^4 + C \frac{\left( 2d^2 - z^2 \right)}{\left( z^2 + d^2 \right)^{5/2}} $$

(3)

and,

![Figure 3: Variation of potential energy function and restoring force with gap distance ($d$) between repulsively positioned magnets without stretching and including stretching.](image-url)
The variation of potential energy \( V(z) \) and restoring force \( F_{\text{Rest}}(z) \) with displacement \( z \) for the cases of the bistable VEH excluding stretching and including stretching at different gap distances \( d \) between the magnets are shown in figure 3. As \( d \) is gradually decreased from higher values, the potential energy profile and restoring force transform into nonlinear bistable from linear response. Subsequently, with inclusion of the stretching effects, the potential wells become narrower due to the hardening nonlinearity brought into effect by stretching of the cantilever spring arms. Figure 3 show that without stretching, the bistable restoring force follows almost a linear path while away from the equilibrium position. Then, as stretching effects are included, \( F_{\text{Rest}}(z) \) follows a nonlinear cubic force path when deflected away from the equilibrium position. The dynamical equation of the system can be represented as follows,

\[
m\ddot{z} + k_z + k_n z^3 + C \frac{3z(z^2 - 4d^2)}{(z^2 + d^2)^{3/2}} + D \dot{z} = F \cos(\omega t)
\]  

Total damping coefficient \( D \) is a combination of mechanical damping \( D_P \) and electromagnetic damping \( D_{\text{EM}} \). The load power harvested across a load resistance \( R_{\text{Load}} \) is given by,

\[
P_{\text{Load}} = \frac{R_{\text{Load}}}{R_{\text{Coil}} + R_{\text{Load}}} \frac{1}{T} \int_{t}^{T} D_{\text{EM}}(\dot{z})^2 \, dt
\]  

where \( R_{\text{Coil}} \) is the resistance of the copper coil. The analytical model was numerically simulated for different parameters and excitation conditions using MATLAB.

4. Simulation results and discussions
The numerical model of the proposed energy harvester was simulated under 1g (10 m/s\(^2\)) and 0.5g (5 m/s\(^2\)) acceleration in a frequency range from 10 Hz to 140 Hz. At 1g acceleration, when the device is linear, i.e. no effect due to stretching or bistability is included, the average power spectrum produces a sharp peak at the resonance frequency (58.7 Hz) and the peak power is 2.9 mW [figure 4(a)].

Figure 4: Power frequency response at (a) 1g acceleration, (b) 0.5g acceleration
As the repulsively oriented magnets are brought closer by reducing $d$, the system become bistable, and hysteresis occur in the frequency response. The oscillator jumps from one potential well to the other under the applied vibration (1g) producing peak power of 2.87 mW at 43.7 Hz frequency and the half-power bandwidth in upward frequency sweep is 6.62 Hz. When the system is made nonlinear based on stretching effects of the supporting beams and bistability is not included, peak power frequency shifts towards right (95.4 Hz), and the bandwidth becomes 12 Hz. However, combining both the effects of bistability and stretching, the bandwidth becomes even wider at 15 Hz, and the peak power frequency is shifted to 85.37 Hz, generating almost the same power as in all the other cases. The frequency response shows nonlinear spring hardening behaviour in all of these situations. Fig. 4(b) shows the power-frequency responses at 0.5g acceleration. When the system is made bistable in this case, the oscillator is confined within one potential well, producing spring softening behaviour in frequency response. The bandwidth in stretching only and bistable only cases are 6.7 Hz and 4.7 Hz respectively, with a peak power of about 0.7 mW. As both the effects of bistability and stretching are combined, the peak power comes down to 0.29 mW, while the bandwidth during downward frequency sweep becomes significantly wider at 31.5 Hz. Hence, in comparison to VEH devices employing only bistable or only stretching based nonlinearity, the bandwidth can be expanded significantly in a device incorporating the effects of bistability and stretching simultaneously. However, the effect of stretching on the frequency response become prominent in the bistable configuration when the oscillator is confined within a single potential well and deflected further away from the initial equilibrium, facilitating stretching dependent nonlinear force.

5. Conclusions
In this work the design and simulation of an electromagnetic vibration energy harvester, exploiting both bistability and stretching based nonlinearity was described. The expressions of total potential energy function and restoring force in the numerical model shows that while the effect of bistability is dominant for small displacements, i.e. near the equilibrium position, stretching based nonlinearity dominates when the displacement is larger. The simulation results show that simultaneous incorporation of bistability and stretching effects are able to widen the bandwidth significantly. However, if the oscillator is confined within one of the potential wells, the output power is lowered. Based on these results, suitable modifications and optimization of the device structure could achieve even wider frequency response.

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