Results from pQCD for A+A collisions at RHIC & LHC energies

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Abstract. This talk will discuss how to compute initial quantities in heavy ion collisions at RHIC (200 AGeV) and at LHC (5500 AGeV) using perturbative QCD (pQCD) by including the next-to-leading order (NLO) corrections and a dynamical determination of the dominant physical scale. The initial numbers are converted into final ones by assuming kinetic thermalization and adiabatic expansion.

The whole heavy ion physics community has entered into exciting and intense period as the first results from RHIC are expected to appear, and the era of LHC seems not more very distant. However, despite the solid status of QCD as the theory of strongly interacting matter, many uncertainties remain in the predictions for the outcome of the current experiments and various methods have been applied. Further experimental data will (hopefully) single out the best candidates for the correct approach.

As the collision energy is increased from that of the SPS, larger intrinsic scales are generated and the applicability of perturbative QCD (pQCD) becomes possible. The initial particle production is expected to be dominated by minijets, i.e. partons with $p_T \sim 1...2$GeV $\gg \Lambda_{QCD}$ [1]. By assuming independent multiple semi-hard parton-parton collisions the average energy carried by minijets with $p_T \geq p_0$ at the rapidity interval $\Delta Y$ in a central $b = 0$ AA collision is given in leading-order (LO) by [2]

$$E_{AA}^T = T_{AA}(0) \sigma \langle E_T \rangle$$

$$= T_{AA}(0) \sum_{q,q,g} \int_{p_0,\Delta Y} dp_1 dy_1 dp_1 dy_2 dx_1 f_{i/p}(x_1, Q^2) dx_2 f_{j/p}(x_2, Q^2) \frac{d\sigma_{ij\rightarrow jk}}{dt} p_T.$$ (1)

where $T_{AA}(b)$ is the standard nuclear overlap function and $p_0$ is the smallest transverse momentum scale to be considered. Collinear factorization is assumed to

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hold and any effects beyond it are neglected. This perturbative minijet approach suffers from two major sources of uncertainties:

(i) The next-to-leading order (NLO) corrections, which have not been known prior to [3,4], but rather have been simulated by ad hoc $K$-factors.

(ii) The determination of which value of $p_0$ to use at, say, RHIC or LHC energies. Should one stick to some constant universal value of $p_0$ or will this parameter possess some nontrivial $\sqrt{s}$- and $A$-dependence?

In the following we will provide answers to both of these questions and combine them to obtain numerical estimates of average transverse energies and charged particle multiplicities at RHIC and LHC energies.

As is evident from formula (1), to deal with the first uncertainty, the relevant quantity to compute in NLO pQCD is the $\sigma \langle E_T \rangle$, the first moment of the perturbative $E_T$ distribution in a pp-collision. The infrared safe NLO computation of this quantity has been presented in [4], where the computation was formulated via the subtraction algorithm of S. Ellis, Z. Kunszt and D. Soper [5].

The $E_T$ in central rapidity region is defined to be the total $p_T$ entering this region and originating from hard subprocesses in which at least an amount of $2p_0$ of transverse momentum is released. The numerical results are shown in fig. 1(a).

![Figure 1](attachment:figure1.png)

**Figure 1.** (a) The NLO $\sigma \langle E_T \rangle$. The LO' stands for the leading order result evaluated with 2-loop $\alpha_s$ and NLO parton distributions, whereas the LO stands for the leading order result evaluated with 1-loop $\alpha_s$ and LO parton distributions. The rapidity interval is chosen to be the central unit, and the parton distributions used are those of GRV-94-set [6]. (b) The average number of QCD-quanta produced with $p_T \geq p_0$ and $|y| \leq 0.5$ as a function of $p_0$. The saturation scale $p_{sat}$ is determined by the points of intersection with the dashed curve ($p_0^2 R_A^2$) labelled “saturation”.

To analyze the implications of these numbers, let us define two $K$-factors: $K = (\text{full NLO})/\text{LO}$ and $K' = (\text{full NLO})/\text{LO}'$, the first of these measuring the deviation of the NLO results from the consistent LO calculation and the latter measuring rather the relative difference between two subsequent terms of the perturbation series. The magnitude of these $K$-factors is due to a new kinematical region which manifests itself only at NLO. If one rejects this new domain, the resulting $K$-factors
would be a factor of two smaller than those obtained from fig. 1(a), but then a significant amount of perturbatively calculable $E_T$ would be neglected.

For a detailed description of the calculation and issues such as the scale choices, see ref. [4] and references therein. For the purposes to be considered here, the sufficient observation is that we now have control over the magnitude of NLO corrections which are stable relative to LO results even at few GeV scales, thus signalling the applicability of pQCD in this domain.

Turning to the uncertainty (ii), then, it is clear that as $p_0$ is decreased the cross section, as well as the uncertainty, grows. At certain value of saturation, $p_0 = p_{\text{sat}}$ the system becomes very dense and new physics enters [7]. For large nuclei and large collision energies this may happen already in the perturbative domain, which we concluded on the basis of the NLO analysis to include also the few GeV region. Then the corresponding values for the number of particles as well as for the amount of $E_T$ are easily produced via the perturbative computation at $p_0 = p_{\text{sat}}$, which effectively accounts for the contributions of all scales, since the partons with $p_T \gg p_{\text{sat}}$ are rare and those with $p_T \ll p_{\text{sat}}$, although numerous, contribute negligibly to total $E_T$.

Various ways to determine the actual magnitude of $p_{\text{sat}}$ can be conjectured. A simple geometric criterion has been presented in [8]. This is based on the idea that if one assigns an effective area $\pi/p_0^2$ to each gluon produced, then at certain value of $p_0$ the total area of $N_{AA}(p_0, \sqrt{s}\Delta Y)$ gluons produced will exceed the effective transverse area $\pi R_A^2$ of the nucleus. Therefore one can iterate the equation $N_{AA} = p_{\text{sat}}^2 R_A^2$ to determine $p_{\text{sat}}$ for given $A$ and $\sqrt{s}$, see fig.1(b). On the basis of the NLO analysis, we take here $K = 2$ to account for the NLO corrections and also implement nuclear shadowing via the EKS98 parametrization [9].

All the initial quantities then computed can well be fitted by a scaling law of a type $C A^b (\sqrt{s})^b$. In particular one finds that

\begin{equation}
\begin{align*}
    p_{\text{sat}}/\text{GeV} &= 0.208 A^{0.128} (\sqrt{s})^{0.191} \\
    \epsilon_i/(\text{GeV}/\text{fm}^3) &= 0.103 A^{0.504} (\sqrt{s})^{0.786} \\
    n_i \cdot \text{fm}^3 &= 0.370 A^{0.383} (\sqrt{s})^{0.574}
\end{align*}
\end{equation}

where the particle and energy densities are evaluated at $p_{\text{sat}}$, and the whole production process is then considered to take place at $\tau_0 = 1/p_{\text{sat}}$.

These, however, are just the initial numbers at 0.2 (0.1) fm/c at RHIC (LHC), and the major problem is how to get from these to the ones at later instants and to finally arrive at experimentally visible quantities. Let us therefore assume, not completely without reason (see [8]), that the system is initially thermalized in the sense that it possesses a correct ratio of energy per particle as far as the dominant gluonic particle content is considered, and expands conserving the total entropy $S \approx 3.6 N_i$. As the final particles consist dominantly of pions, for which $S \approx 4N_f$, we find that $N_f = 0.9 N_i$. As the initial volume is $V_i = \pi R_A^2 \Delta Y/p_{\text{sat}}$, formulae (2) give

\begin{equation}
N_f = 1.245 A^{0.92} (\sqrt{s})^{0.383}.
\end{equation}
After the conference the very first measurements by PHOBOS collaboration have been announced [10]. According to them, the charged particle multiplicity at midrapidity is $dN/d\eta = 408 \pm 12 \pm 30$ at 56 AGeV and $555 \pm 12 \pm 35$ at 130 AGeV. The framework described here gives $N_{ch} = 2/3N_f = 370$ and 530 per unit $\eta$ respectively, when taking into account that number of participants was reported to be 330 for $\sqrt{s} = 56$ AGeV and 343 for $\sqrt{s} = 130$ AGeV, and that $dN/dy = 1.15dN/d\eta$.

The final $E_T$ in this scenario is obtained by means of hydrodynamics [8,11] as

$$E_T = N_f \times \left[ 0.39 + 0.061 \ln(N_f/A) \right].$$

Numerical values then per unit $\eta$ are $E_T = 260$ GeV for $\sqrt{s} = 56$ AGeV and $E_T = 390$ GeV for $\sqrt{s} = 130$ AGeV using the multiplicites computed above and the quoted participant numbers. Measurements of these final transverse energies are awaited to appear soon. These measurements will then allow us to draw conclusions on the issues such as the true degree of thermalization in the system.

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REFERENCES

1. Blaizot J.P., and Mueller A.H., Nucl. Phys. B289 847 (1987);
   Kajantie K., Landshoff P.V. and Lindfors J., Phys. Rev. Lett. 59 2527 (1987).
2. Eskola K.J., Kajantie K. and Lindfors J., Nucl. Phys. B323 37 (1989).
3. Leonidov A. and Ostrovsky D., Eur. Phys. J. C11 495 (1999).
4. Eskola K.J. and Tuominen K., hep-ph/0002008.
5. Ellis S.D., Kunszt Z. and Soper D.E., Phys. Rev. D40 2188 (1989);
   Ellis S.D., Kunszt Z. and Soper D.E., Phys. Rev. Lett. 64 2121 (1990);
   Kunszt Z. and Soper D.E., Phys. Rev D46 192 (1992).
6. M. Glück, E. Reya and A. Vogt, Z.Phys. C67 433 (1995).
7. Gribov L.V., Levin E.M. and Ryskin M.G., Phys. Rept. 100 1 (1983);
   Mueller A.H. and Qiu J., Nucl. Phys. B268 427 (1986).
8. Eskola K.J., Kajantie K., Ruuskanen P.V. and Tuominen K., Nucl. Phys. B570 379 (2000). hep-ph/9909456.
9. Eskola K.J., Kolhinen V.J. and Ruuskanen P.V., Nucl. Phys. B535 351 (1998). hep-ph/9802350;
   Eskola K.J., Kolhinen V.J. and Salgado C.A., Eur. Phys. J. C9 61 (1999). hep-ph/9807297.
10. PHOBOS collaboration, Back B.B. et al., hep-ex/0007036.
11. Kataja M., Ruuskanen P.V., McLerran L. and von Gersdorff H., Phys. Rev. D34 2755 (1986).