Influence of thermonuclear effects on the collapse of supermassive stars

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Abstract.

We present results of general relativistic simulations of collapsing supermassive stars using the two-dimensional general relativistic numerical code Nada, which solves the Einstein equations written in the BSSN formalism and the general relativistic hydrodynamic equations with high resolution shock capturing schemes. These numerical simulations use a tabulated equation of state which includes effects of radiation and gas pressure, and those associated with the electron-positron pairs. We also take into account the effect of thermonuclear energy released by hydrogen and helium burning. We find that objects with mass \( \approx 5 \times 10^5 M_\odot \) and initial metallicity greater than \( Z_{CNO} \approx 0.004 \) do explode if non-rotating, while the threshold metallicity for an explosion is reduced to \( Z_{CNO} \approx 0.002 \) for objects uniformly rotating.

1. Introduction

The observation of luminous quasars detected at redshifts higher than 6 in the Sloan Digital Sky Survey (SDSS) implies that SMBHs with masses \( \sim 10^9 M_\odot \), which are believed to be the engines of such powerful quasars, were formed within the first billion years after the Big Bang (see [1] for a recent review). However, it is still an open question how SMBH seeds are formed and grow to reach such high masses in such a short amount of time [2].

A number of different routes based on stellar dynamical processes, hydrodynamical processes or a combination of both have been suggested. One of the theoretical scenarios proposes that if sufficient primordial gas in massive halos is unable to cool below \( T_{vir} \approx 10^4 K \), it may lead to the formation of a supermassive star (SMS), with mass \( \geq 5 \times 10^4 M_\odot \), which would eventually collapse to form a SMBH. This route assumes that fragmentation and cooling is suppressed by the presence of sufficiently strong UV radiation that prevents the formation of molecular hydrogen [3]. If isentropic SMSs [4, 5] form, it is expected that their quasi-stationary evolution of cooling and contraction will drive the stars to the onset of a general relativistic gravitational instability that would cause their gravitational collapse, and possibly also leads to the formation of a SMBH [6].

Fuller et al. [7] performed simulations of non-rotating SMSs in the mass range \( 10^5-10^6 M_\odot \) with post-Newtonian corrections, an equation of state including \( e^\pm \)-pairs (which reduce the adiabatic index during collapse), and nuclear reactions describing hydrogen burning by the CNO cycles and the break-out from the \( \beta \)-limited hot CNO (HCNO) cycle. They found that SMSs with masses \( M \geq 10^5 M_\odot \) and initial metallicities \( Z_{CNO} \lesssim 0.005 \) do not explode, whereas SMSs with initial metallicities \( Z_{CNO} \gtrsim 0.005 \) do explode.
General relativistic simulations of collapsing supermassive stars [8, 9, 10, 11] neglecting the effects of nuclear burning (justified in the case of SMSs with masses exceeding $10^6 M_\odot$) indicate that about 90% of the initial mass would end up in the SMBH with a spin parameter $J/M^2 \approx 0.75$.

It is still an open question whether hydrogen burning by the $\beta$-limited HCNO cycle and its break-out via the $^{15}$O$(\alpha, \gamma)^{19}$Ne reaction (rp-process) can halt the gravitational collapse of rotating SMSs with non-zero initial metallicities and generate enough thermal energy to lead to an explosion. To address this issue, we perform a series of general relativistic hydrodynamic simulations with a microphysical tabulated equation of state accounting for contributions from radiation, $e^\pm$-pairs, and baryonic matter. We also take into account the net thermonuclear energy released by the nuclear reactions involved in hydrogen and helium burning. The numerical simulations have been carried out with the Nada code [12].

2. Basic equations

We follow the so-called BSSN formulation of the Einstein equations [13, 14]. BSSN makes use of a conformal 3-metric, $\tilde{\gamma}_{ij} = e^{-4\phi} \gamma_{ij}$ and the conformal-traceless extrinsic curvature $\tilde{A}_{ij} = e^{-4\phi}(K_{ij} - \gamma_{ij} K/3)$, with the conformal factor $\phi$ satisfying $e^{4\phi} = \gamma^{1/3} \equiv \det(\gamma_{ij})^{1/3}$. In addition to evolution equations for $\tilde{\gamma}_{ij}$ and $\tilde{A}_{ij}$, there are evolution equations for the conformal factor $\phi$ (or $\chi \equiv e^{-4\phi}$), the trace of the extrinsic curvature $K$, and the “conformal connection functions” $\Gamma^i$ [14]. The lapse $\alpha$ and the shift vector $\beta^i$ are computed using the so-called “moving puncture gauge”.

The GRHD equations are the local conservation laws of momentum and energy, encoded in the stress-energy tensor $T^{\mu\nu}$, and of the matter density, $J^\mu$ (the continuity equation), i.e., $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu J^\mu = 0$, where $\nabla_\mu$ stands for the 4-dimensional covariant derivative. Following [15] the GRHD equations are written in flux-conservative form in cylindrical coordinates.

In order to avoid the complexity and stiffness from the solution of a nuclear reaction resulting network coupled to the hydrodynamic evolution, we follow an approximate method to take into account the net effect of the nuclear reactions of interest on the dynamics of the collapsing SMSs. We compute the nuclear energy release by hydrogen burning (through pp-chain, cold and HCNO cycles, and its break-out via the $^{15}$O$(\alpha, \gamma)^{19}$Ne reaction) and helium burning (through 3-\alpha reaction) as a function of rest-mass density, temperature and mass fractions of hydrogen ($H$), helium ($He$) and CNO metallicity ($Z_{CNO}$). This nuclear energy generation rate is added as a source term in the right-hand-side of the energy evolution equation.

3. Numerical approach and initial data

The evolution equations are integrated by the method of lines, for which we use a strong stability-preserving (SSP) Runge-Kutta algorithm of fourth-order. Since it is not possible to follow the gravitational collapse of a SMS from the early stages to the phase of black hole formation with a uniform Cartesian grid (computationally too demanding), we adopt a “regridding” procedure [10].

All initial models are chosen such that they are gravitationally unstable, i.e. their central rest-mass density is slightly larger than the critical central density required for the onset of the collapse. A list of the six different SMSs we have considered is given in Table 1. Models S1.a, S1.b and S1.c represent spherically symmetric SMSs with a gravitational mass of $M = 5 \times 10^5 M_\odot$, whereas R1.a, R1.b and R1.c are uniformly rotating initial models with the same gravitational mass. The rotating initial models are computed with the Lorene code [16]. We introduce a perturbation to trigger the gravitational collapse by reducing the pressure by about 2%. In order to determine the threshold metallicity required to halt the collapse and produce an explosion we carried out several numerical simulations for each initial model with different values of the initial metallicity as listed in Table 1.
Table 1. Main properties of the initial unstable models studied and their fate. From left to right the columns show: model name, gravitational mass, initial central rest-mass density, ratio of rotational to gravitational energy $\frac{T}{|W|}$, initial central temperature, metallicity, and the outcome of the evolution.

| Model  | $M_{grav}$ [$M_\odot$] | $\rho_c$ [g/cm$^3$] | $\frac{T}{|W|}$ | Temp [K] | Initial metallicity $Z_{CNO}$ | Fate |
|--------|------------------------|----------------------|-----------------|----------|-----------------------------|------|
| S1.a   | $5.0 \times 10^5$      | $2.42 \times 10^{-2}$ | 0               | $5.8 \times 10^4$ | $2.0 \times 10^{-3}$  | BH   |
| S1.b   | $5.0 \times 10^5$      | $2.42 \times 10^{-2}$ | 0               | $5.8 \times 10^7$ | $4.0 \times 10^{-3}$  | Explosion |
| S1.c   | $5.0 \times 10^5$      | $2.42 \times 10^{-2}$ | 0               | $5.8 \times 10^7$ | $5.0 \times 10^{-3}$  | Explosion |
| R1.a   | $5.0 \times 10^5$      | $4.0 \times 10^{-1}$  | 0.0088          | $1.3 \times 10^8$ | $5.0 \times 10^{-4}$  | BH   |
| R1.b   | $5.0 \times 10^5$      | $4.0 \times 10^{-1}$  | 0.0088          | $1.3 \times 10^8$ | $8.0 \times 10^{-4}$  | Explosion |
| R1.c   | $5.0 \times 10^5$      | $4.0 \times 10^{-1}$  | 0.0088          | $1.3 \times 10^8$ | $2.0 \times 10^{-3}$  | Explosion |

4. Results
First we consider a gravitationally unstable spherically symmetric SMS with a gravitational mass of $M = 5 \times 10^5 M_\odot$ (S1.a, S1.b and S1.c), which corresponds to a model extensively discussed in [7], and therefore allows for a comparison with the results presented here. In [7] it was found that an unstable non-rotating SMSs with $M = 5 \times 10^5 M_\odot$ and initial metallicity $Z_{CNO} = 2 \times 10^{-3}$ collapses to a BH, while a model with an initial metallicity $Z_{CNO} = 5 \times 10^{-3}$ explodes due to the nuclear energy released by the HCNO cycle. They also found that the central density and temperature at the thermal bounce (moment where collapse is reversed to explosion) are $\rho_{c,b} = 3.16$ g/cm$^3$ and $T_{c,b} = 2.6 \times 10^8$ K, respectively.

The left panels in Figure 1 show the increase in the central rest-mass density (upper panel) and temperature (lower panel) for model S1.a ($Z_{CNO} = 2 \times 10^{-3}$) with dashed lines until the point where an apparent horizon is found, indicating the formation of a BH. The solid lines represent the central rest-mass density and temperature evolution for model S1.c ($Z_{CNO} = 5 \times 10^{-3}$). They show that a thermal bounce occurs (at approximately $t \sim 9 \times 10^5$ s) entirely due to the HCNO cycle. The rest-mass density at bounce is $\rho_{c,b} = 2.9$ g/cm$^3$ and the temperature $T_{c,b} = 2.56 \times 10^8$ K, thus up to a few percent in agreement with [7]. Furthermore, we also find that model S1.b with metallicity $Z_{CNO} = 4 \times 10^{-3}$ explodes.

The evolutionary tracks for the central density and temperature of the rotating models R1.a, R1.b and R1.c are also shown in Figure 1. We refer with a dashed line to model R1.a with an initial metallicity $Z_{CNO} = 5 \times 10^{-4}$, which collapses to a BH, and with a solid line to model R1.c with $Z_{CNO} = 2 \times 10^{-3}$, which explodes due to the energy released by the HCNO cycle. Model R1.b with a lower metallicity of $Z_{CNO} = 8 \times 10^{-4}$ is a marginal case that explodes when the central temperature is high enough to trigger burning by the rp-process. As a result of the kinetic energy stored in the rotation of models R1.b and R1.c, the critical metallicity needed to lead to an explosion decreases relative to the non-rotating case. We observe that rotating models with initial metallicities up to $Z_{CNO} = 5 \times 10^{-4}$ do not explode even via the rp-process, which dominates at temperatures higher than $T \approx 5 \times 10^8$ K and increases the hydrogen burning rate by $200 \sim 300$ times relative to that of the HCNO cycle. We note that the time scale for the collapse and bounce phase is reduced because rotating models are more compact and have higher initial central density and temperature than spherical ones at the onset of the gravitational instability.

The right panel in Figure 1 shows the total nuclear energy generation rate in erg/s for the exploding models as a function of time during the collapse and subsequent bounce. The major contribution to the nuclear energy generation is due to hydrogen burning by the HCNO cycle. As expected the maximum nuclear energy needed to produce an explosion is lower in the rotating models.
Figure 1. Time evolution of the central rest-mass density for all initial models (upper panel), and of the central temperature (lower panel). Horizontal dotted lines mark the temperature range where nuclear energy in primarily released by the HCNO cycle. The right panel shows the total nuclear energy generation rate [\text{erg/s}] for the exploding models around the time of bounce.

5. Summary
We present results of general relativistic simulations of collapsing supermassive stars using the two-dimensional general relativistic numerical code Nada. We employed a tabulated equation of state which includes effects of radiation, $e^\pm$-pairs, and gas pressure. We took into account the effects of thermonuclear energy release by hydrogen and helium burning, and we determined the critical initial metallicity at which the collapse to a black hole is prevented by a thermonuclear-powered explosion of the rotating and non-rotating SMSs with mass $M = 5 \times 10^5 M_\odot$.

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