Systematic Investigations of the Free Fermionic Heterotic String Gauge Group Statistics: Layer 1 Results

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Using software under development at Baylor University, we explicitly construct all layer 1 gauge, weakly coupled free fermionic heterotic string models up to order 22 in four large space-time dimensions. The gauge models consist primarily of gauge content making a systematic construction process efficient. We present an overview of the model building procedure, redundancies in the process, methods used to reduce such redundancies and statistics regarding the occurrence of various combinations of gauge group factors and GUT groups. Statistics for both $\mathcal{N} = 4$ and $\mathcal{N} = 0$ models are presented.

I. INTRODUCTION

Recent work puts the number of possible string derived models on the order of $10^{500}$ [1, 2]. Consequently, any efforts to explore this landscape of string vacua require the use of high-performance computing and a choice of construction method. Each construction method has access to different, overlapping regimes of the landscape; here we will focus on the weakly coupled free fermionic heterotic string (WCFFHS) construction formalism [3–5]. The WCFFHS formalism has produced some of the most phenomenologically viable models to date [6–41] and is ideal for computer construction. Random examinations of the landscape, using this formalism, have been performed in the past [42, 43]; however, due to the many-to-one nature of this construction a random survey of the input parameters has many endemic problems that are non-trivial to address [44]. One way to deal with these problems is to systematically survey the valid input parameters.

Two software frameworks, currently under development at Baylor University, are being designed and used specifically for the purpose of performing such systematic surveys of the WCFFHS landscape. One such framework, the Gauge Framework, focuses on systematically building gauge models in four large space-time dimensions. A detailed explanation of what is meant by a “gauge model” is provided in section II. These surveys serve multiple purposes including aiding in attempts at understanding and reducing the redundancies inherent to the construction method. Furthermore we can use the results of these searches to guide slower, more detailed surveys.

The results presented here are intended more as a proof-of-concept than as an exhaustive analysis of the data. The framework is a work in progress but currently generates approximately 400 models per second. Future iterations are projected to exceed 1000 models per second and will have the capability of generating models in any number of large spacetime dimensions, of any order and at any layer.

A detailed account of the WCFFHS formalism is provided here from the perspective of gauge model building. We include discussion of the input space and the systematic generation of the inputs. To discuss redundancy we need to first define what it means for a model to be “unique”; this definition is provided in section II C. As mentioned, the many-to-one nature of this formalism remains a difficult problem to overcome. We discuss several steps taken that have reduced redundancy to a level that admits systematic surveys of gauge models in section II D. We conclude...
with statistics from a layer 1 survey of WCFFHS models from order 2 through order 22. These statistics divide nicely into two categories, correlations between the input space and the model space, and the statistics of various properties of the low energy effective field theories themselves independent of input. We look at the number of models generated at each order, section III A, as well as the number of group factors of each rank in the unique models built, section III B, and the combinations of group factors and GUT groups, section III C, that occur in both SUSY and non-SUSY models. Many of the results are presented in a truncated form for brevity. The full range of statistics and complete data sets will be made available at http://homepages.baylor.edu/eucos/svp.

II. GAUGE MODEL BUILDING

The Gauge Framework focuses on the construction of gauge models. Further discussion requires a more concrete definition of what a “gauge model” is.

Definition (Gauge Model). A model is a **gauge model** if it can be built from a set of basis vectors in which every basis vector beyond the all-periodic and SUSY basis vectors is bosonic, that is of the form \((\vec{0}^{10} \parallel \vec{\alpha})\), within the free fermionic construction [3–5, 45].

These models are in many ways some of the most simple models that one can build. We can think of them as the basis from which more complex models can be built. This makes them interesting as a starting point for systematic surveys. We can use what we learn about these models to guide further searches. Here we review the WCFFHS construction method with gauge models in mind.

Within the free fermionic framework two inputs are required, the set of basis vectors, \(A\), and the GSO projection coefficient matrix, \(k\). In order to systematically build these models we need to systematically build the input set \(\{A, k\}\) ensuring that all of the modular invariance constraints are met.

A. Basis Vectors

In 4 large spacetime dimensions, the basis vector set is defined as

\[
A = \left\{ \tilde{\alpha}_i \mid \tilde{\alpha}_i \in \mathbb{Q}_{32} \cap (-1, 1)^{32} \right\},
\]

where \(i = 1, 2, \ldots, l\). For our purposes we will always take \(l \geq 3\) and will refer to \(L = l - 2\) as the layer. Each of these basis vectors represents the boundary conditions of complex worldsheet fermion degrees of freedom. We will be taking \(\alpha_j^i\) with \(j = 1, \ldots, 10\) to represent the boundary conditions of the left-moving supersymmetric string and \(\alpha_j^i\) with \(j = 11, \ldots, 22\) to represent the right-moving bosonic string boundary conditions. The order of each basis vector, \(N_i\), is the smallest integer such that

\[
N_i \alpha_j^i = 0 \pmod{2}.
\]

(2.2)

Of course, the choices of these basis vectors are constrained by modular invariance in such a way that

\[
N_i \alpha_j^i = \begin{cases} 
0 \pmod{8} & \text{if } N_i \text{ even} \\
0 \pmod{4} & \text{if } N_i \text{ odd}
\end{cases}
\]

(2.3)

and

\[
N_{ij} \tilde{\alpha}_i \cdot \tilde{\alpha}_j = 0 \pmod{4},
\]

(2.4)

where \(N_{ij} \equiv \text{LCM}(N_i, N_j)\).

Since we are dealing with \(L = 1\), we have three basis vectors, two of which will always be the same for every basis vector set we generate:

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1 We do not allow for chiral Ising models here. So all left- or right-moving real fermions can be paired to form left- or right-moving complex fermions.
• The first basis vector, denoted $1$, is the all periodic boundary conditions: $(\bar{1}^{10} \parallel \bar{1}^{22})$.

• The second basis vector is the SUSY generator, $S$, which is $(1 (100)^3 \parallel \bar{0}^{22})$.

Generating these basis vectors can introduce excessive redundancy hindering systematic search algorithms. However, an algorithm for efficiently generating only the modular invariant basis vectors was developed and implemented [46]. Further discussion of the redundancies of this process is found in section II D.

B. GSO Projection

Once a modular invariant set of basis vectors has been generated, the states can be built. To do so we begin by generating all sectors as linear combinations of the basis vectors with $m^j_1 \in \mathbb{N}$ and $m^j_1 < N_j$, namely

$$\tilde{V}^j = \sum_i m^j_i \alpha_i. \quad (2.5)$$

The sectors describe how the worldsheet fermions, $f_j$, transform around non-contractible loops on the worldsheet

$$f_j \rightarrow -e^{i\pi V^j} f_j. \quad (2.6)$$

To each of these sectors we apply fermion number operators, $\tilde{F}^i$,

$$\tilde{Q}^i = \frac{1}{2} \tilde{V}^i + \tilde{F}^i \quad (2.7)$$

to build the charges. We can then express the masses of the states in terms of the charges as

$$\alpha' m_{left}^2 = \frac{1}{2} \left( \tilde{Q}^i_{left} \right)^2 - \frac{1}{2} \quad (2.8a)$$

$$\alpha' m_{right}^2 = \frac{1}{2} \left( \tilde{Q}^i_{right} \right)^2 - 1 \quad (2.8b)$$

Because we are working at the string scale and only interested in the low-energy effective theory, these states must be massless. Thus,

$$\left( \tilde{Q}^i_{left} \right)^2 = 1 \quad (2.9a)$$

$$\left( \tilde{Q}^i_{right} \right)^2 = 2 \quad (2.9b)$$

However, once the states are constructed we must ensure that they are, in fact, physical. This requires the application of a GSO projection, and hence the specification of GSO projection matrix, $k$. This matrix is, in our case, $(L + 2) \times (L + 2)$ and is constrained by modular invariance:

$$k_{ij} + k_{ji} = \frac{1}{2} \alpha_i \cdot \alpha_j \quad (2.10a)$$

and

$$k_{ii} + k_{i0} = \frac{1}{4} \alpha_i \cdot \alpha_i - s_i \quad (2.10b)$$

with

$$N_j k_{ij} = 0 \quad (2.11)$$

It is clear from Equation 2.10 that, in general, we have $\frac{1}{2} (L + 1)(L + 2) + 1$ degrees of freedom in our choice of $k$. However, one of the degrees of freedom, our choice of the $k_{i0}$ element, has no effect on the model generated so we fix
it to 1. This reduces us back to \(\frac{1}{2}(L + 1)(L + 2)\), meaning we can specify the lower-triangle of our GSO projection matrix. There is however, a caveat; not every choice of the lower-triangle yields a modular invariant matrix. In layer 1 models specifically, there are eight possible lower-triangles each of which specifies a GSO projection matrix. However, when the additional basis vector is odd, only two are modular invariant. We can represent this freedom in \(k\) as

\[
\begin{pmatrix}
1 & S & \tilde{\alpha} \\
1 & 0 & 0 \\
S & 2 & 0 \\
\tilde{\alpha} & 2 & 0
\end{pmatrix}
\]

where each entry represents the number of choices for that element of \(k\). Now, when we consider an \(\tilde{\alpha}\) of odd order, and using Equation 2.11, we see that the only choices of \(k_{20}\) and \(k_{21}\) that admit \(k_{22}\) elements of \(2\mathbb{Z}\) values can be made. It is easy to see that, regardless of the choice of \(k_{21}\), \(k_{12} = k_{21}\), because the forms of \(S\) and \(\tilde{\alpha}\) presupposed. Consequently, because \(k_{21} \in \{0, 1\}\) only \(k_{21} = 0\) is admitted. This takes the number of admissible \(k\) choices to four. We can apply a similar analysis to the \(k_{20}\) choice. We know that \(k_{20} \in \{0, 1\}\), and that this choice determines both \(k_{02}\) and \(k_{22}\). It has been shown that \(k_{02}\) and \(k_{22}\) are either both even or both odd [47]. It is easy to see that switching between choices of the \(k_{20}\) flips the parity of \(k_{i2}\). So, this again halves the number of accessible GSO projection matrices to two.

We then simply apply the GSO projection,

\[
\tilde{\alpha}_i \cdot \tilde{Q}^j = \sum_{l=1}^{L+2} m_i^l k_{il} + s_i \pmod{2}
\]

with \(s_i\) being the space-time component of \(\tilde{\alpha}_i\). Now we are in a position to consider the space-time supersymmetry of these models.

From a model building perspective, the simplest way to determine the number of supersymmetries in a model is to count the number of gravitinos in the model. Gravitinos have a specific form,

\[
\begin{pmatrix}
\frac{1}{2} \\
\frac{1}{2} \\
\frac{1}{2} \\
\frac{1}{2}
\end{pmatrix}
\]

when considering \(SU(2)^6\) worldsheet SUSY. In our models, these states can only arise from the SUSY sector generated by \(S\). Half of these eight gravitinos have positive inner products with the SUSY generator, so when the GSO projection is carried out, exactly half of the gravitinos are ejected from the model; the half that is kept is determined by the \(k_{22}\) element of \(k\). Now, because the additional basis vector, \(\tilde{\alpha}\) has a zero inner product with the charges, the gravitino states are only kept if \(k_{21}\) zero. Thus, in these models, for a given set of basis vectors, if \(k_{21} = 0\) for any \(i > 2\), SUSY is preserved at \(\mathcal{N} = 4\) and is broken to \(\mathcal{N} = 0\) otherwise.

We saw previously that if the \(\tilde{\alpha}\) is of odd order, \(k_{2i}\) is necessarily 0, thus at layer 1 there are no odd order, \(\mathcal{N} = 0\) models.

### C. Uniqueness

When considering uniqueness of models, both gauge and matter content should be considered. The nice thing about gauge models is that, when the model is supersymmetric, models with the same gauge group will always have the same matter spectrum and are thus identical. This, however, is not true for non-SUSY models so, in general, to consider uniqueness we must investigate the matter content of these models. Fortunately, the exact particle spectrum of these models is not of interest here. We are only concerned with the gauge content and whether the model is
supersymmetric. From that, we can use additional software to prepend left-movers to our basis vectors and build models using these gauge models as a starting point. When there is no left-right pairing, the new models will either keep or break the gauge group of their base gauge models. So, for our purposes we will define uniqueness as follows:

**Definition (Uniqueness).** A model is considered unique if no other model has been previously generated with both the same gauge group and number of space-time supersymmetries.

By this we mean that as we generate models, if we build one that has a combination of gauge groups and number of space-time supersymmetries that has not yet been created we retain it. However, any model after that with the same combination of gauge states and SUSY will be discarded. This has an impact on the statistics of the non-SUSY models which will be discussed in more detail in subsequent sections.

We can easily determine the maximum number of unique models that can be built by considering that only simply-laced gauge groups can appear and there are no rank cuts [46], thus the total rank must be 22 in $D = 4$. Determining all of the combinations of simply-laced gauge groups whose rank sums to 22 and doubling that, for SUSY and non-SUSY, gives us at most 48952 unique gauge models. Of course, it is unlikely that all of these combinations can exist, especially at $L = 1$. In fact, if we perform the same analysis on $D = 10$ we find that 5714 models could occur, but it is well known that only 9 models are realized by the $D = 10$ heterotic landscape [48], even when considering full matter content. However, because the $D = 4$ landscape is much more complex, we should expect a higher occurrence of unique models than at $D = 10$. These calculations have been performed for $D = 4$ through $D = 10$ and are provided in Table I.

| $D$ | # of Models |
|-----|-------------|
| 10  | 5714        |
| 9   | 4140        |
| 8   | 11988       |
| 7   | 8576        |
| 6   | 24508       |
| 5   | 17341       |
| 4   | 48952       |

**D. Redundancies**

The free fermionic construction formalism has the inherent problem of redundancy; the mapping from input space to output space is many-to-one. This property is what condemns random surveys and remains a problem for systematic searches. Reducing these redundancies will bring systematic surveys within current technological limits. Both the basis vectors and the GSO projection coefficients present redundancies that can be accounted for and removed in many cases.

The systematic generation of basis vectors admits redundancy in at least two ways, permutations and charge conjugacy. Permutations of the elements of a basis vector leave the mapping invariant as long as the same permutation is applied to each of the basis vectors in the set, i.e.

\[
\begin{pmatrix}
\vec{1}^{10} & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & \vec{1}^{18} \\
\vec{1}^{10} & 1 & 1 & 0 & 0 & 1 & 1 & \vec{1}^{18}
\end{pmatrix}
\cong
\begin{pmatrix}
\vec{1}^{10} & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & \vec{1}^{18} \\
\vec{1}^{10} & 1 & 1 & 1 & 1 & 0 & 1 & \vec{1}^{18}
\end{pmatrix}
\]  

Here the third column of the right-movers was switched with the sixth. These two sets will generate the same output, given that the same GSO projection matrix is chosen. A scheme for removing these permutation redundancies was developed in [46]. Additionally, we can always flip the signs of the charges as long as that change does not remove modular invariance, i.e.

\[
\begin{pmatrix}
\vec{1}^{10} & 1 & 1 & 1 & 0 & 0 & 0 & 0 & \vec{1}^{18} \\
\vec{1}^{10} & \frac{2}{3} & \frac{2}{3} & -\frac{3}{2} & 0 & 0 & 0 & 0 & \vec{1}^{18}
\end{pmatrix}
\cong
\begin{pmatrix}
\vec{1}^{10} & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & \vec{1}^{18} \\
\vec{1}^{10} & \frac{2}{3} & \frac{2}{3} & \frac{3}{2} & 0 & 0 & 0 & 0 & \vec{1}^{18}
\end{pmatrix}
\]
This is referred to as charge conjugacy and does not change the gauge group. These two redundancies are not sufficient to completely remove the many-to-one nature of the mapping, and studies are currently under way to find more sources of basis vector redundancy.

Systematically generating the GSO projection matrices also introduces two redundancies. The first redundancy is in our choice of which 4 gravitinos to remove via the S GSO projection. The choice does not affect the number of supersymmetries nor what gauge group the model possesses and is specified by our choice of \( k_{10} \). By our definition of uniqueness, simply changing the \( k_{10} \) value will leave the model the same. Recall that \( k_{10} \in \{0, 1\} \), so we can choose to only build the \( k \)'s with \( k_{10} = 0 \). This means we now only have 4 choices of \( k \) for layer 1, even order basis vector sets and 1 for odd order sets. The second redundancy is in our choice of \( k_{20} \) for layer 1, even order sets. Models built with either choice of \( k_{20} \) are identical by our definition. Why this is the case has not been shown analytically, but has been confirmed up to order 22. This reduces our choices down to 2 and 1 for layer 1, even and odd basis vector sets, respectively.

The result of accounting for these redundancies is a significant improvement in the volume of models that must be built. Table II depicts the effects of these redundancies. One thing to note is that each of these affects even and odd orders to differing extents. However, if we account for all of them the result is that the number of models that must be built (not the number of unique models) at orders \( 2N \) and \( 2N + 1 \) are of the same order.

### TABLE II: Number of \( L = 1 \) Models

For each order we list the most models possible and number of models after the permutation, charge conjugacy and GSO projection redundancies are accounted for. These are not necessarily distinct models, in fact the majority are still redundant.

| \( N \) | Initial Models | Permutation | Charge Conjugacy | GSO Projection |
|-------|----------------|-------------|------------------|----------------|
| 2     | \( 8.39 \times 10^6 \) | 20          | 20               | 10             |
| 3     | \( 3.14 \times 10^{10} \) | 47          | 7                | 7              |
| 4     | \( 3.76 \times 10^{13} \) | 640         | 152              | 76             |
| 5     | \( 2.38 \times 10^{15} \) | 873         | 55               | 55             |
| 6     | \( 2.63 \times 10^{17} \) | 8292        | 772              | 386            |
| 7     | \( 3.91 \times 10^{18} \) | 9352        | 328              | 328            |
| 8     | \( 1.48 \times 10^{20} \) | 71724       | 3748             | 1874           |
| 9     | \( 9.85 \times 10^{20} \) | 70759       | 1679             | 1679           |
| 10    | \( 2.00 \times 10^{22} \) | 463948      | 16172            | 8086           |
| 11    | \( 8.14 \times 10^{22} \) | 413948      | 7339             | 7339           |
| 12    | \( 1.10 \times 10^{24} \) | 2434404     | 62704            | 31352          |
| 13    | \( 3.21 \times 10^{24} \) | 2007773     | 28979            | 28979          |
| 14    | \( 3.28 \times 10^{25} \) | 10756336    | 223020           | 111510         |
| 15    | \( 7.49 \times 10^{25} \) | 8378335     | 104453           | 104453         |
| 16    | \( 6.19 \times 10^{26} \) | 41719604    | 730020           | 365010         |

### III. STATISTICS

Traditionally, the collection of string derived, low energy effective field theories (LEEFTs) is referred to as the landscape. However, because we are interested less in the field theories and more in the mapping from the WCFFHS input space to this landscape, we can consider only those LEEFTs that are mapped to by a particular input sub-space; namely the layer 1, order 2 through 22 gauge input space. For our purposes it is sufficient to refer to the LEEFTs that these inputs map to as the “layer 1 landscape.” We can then look at the relationships between the input and output spaces as well as the mapping between them.

Using the WCFFHS formalism, we constructed all unique, layer 1 gauge models from order 2 through 22. This amounted to 68 SUSY and 502 non-SUSY models and required a total of 31, 863, 121 models to be built. Of all of the group combinations found, 50 had both SUSY and non-SUSY realizations. In this section we review the statistics for these 570 models as well as how we may use these results to improve further surveys and what LEEFTs are accessible from these types of inputs.
A. Model Generation and Redundancy

Here we look at relationships between model generation, the basis vector order and redundancy. Strictly speaking, these relationships have no physical meaning; however, they are important when creating algorithms for the systematic generation of WCFFHS models, particularly for studies into how redundancies manifest themselves in the gauge input space.

In our model building process, all models of a particular order are generated before progressing to the next. This allows us to ask how the number of unique models generated is dependent on the order. There is a subtlety to these questions in that any model can be, in general, generated at other orders. However, because we have imposed an ordering on the build process this inherently gives preference to lower orders. This has the advantage of improving the efficiency of the build process and does not affect statistics beyond the physically meaningless question of “at what order was this model generated?”

We know from section II B that there are no odd-order $N = 0$ models. This immediately suggests that there is a difference in the way SUSY and non-SUSY models are generated at each order. This difference can be seen in Figure 1 where the number of unique models generated is plotted with respect to order for both SUSY and non-SUSY data sets. We see that a statistical majority of SUSY models are generated at low order, from order 2 through 12, while a statistically significant number of non-SUSY models are generated through 22.

FIG. 1: Number of New Models at Each Order - Order 6 generates the most unique SUSY models at 18, and order 12 generates the most non-SUSY with 96. Note that the non-SUSY curve only has data for even orders because no odd order non-SUSY models exist.

One may also be interested in how higher orders subsume lower orders. That is, because higher orders admit a significantly higher number of potential models, one might suspect that higher orders may well contain all of the models generated at lower orders. To verify this we look at the number of unique models generated at each order as well as the number of models that were generated at lower orders but are absent from higher orders, Figure 2.

FIG. 2: Number of Additional and Absent Models at Each Order - At each order we look at the number of models generated in addition to the models previously created as well as the number of models that are absent at that order. Note that no $N = 0$ models are generated at odd orders so, for brevity, those orders are not plotted.
This information can be used to more efficiently generate models. For example, even order SUSY models completely subsume lower orders. This means that we could simply build SUSY order 22 and we would get everything below it. That reduces the number of SUSY models that must be built from approximately $1.82 \times 10^7$ to $8.4 \times 10^6$, roughly in half. This is not quite as nice for non-SUSY models in that we would have to build orders 16 through 22. This amounts to 98.89% of the total number of models. Only SUSY would benefit from this approach. Unfortunately, there is no known way to predict which orders subsume lower orders.

B. Group Distribution Statistics

We now focus on specific properties of the LEEFTs, in particular how group factors of each rank manifest themselves across the layer 1 landscape. We begin by considering the number of models with a group factor of a particular rank, $M_n$, for each, SUSY and non-SUSY, data set. Figure 3.

**FIG. 3: Number of Models with Factors of Each Rank** - For each rank and class of gauge group, the number of models with at least one factor of that type is plotted. The label on each bar is the total number of models with at least one group of that rank. The plots for the SUSY and Non-SUSY models are provided for comparison.

$SU_2$ is highly prevalent in both datasets because it is relatively simple to generate. It amounts to a single, disjoint charge in our gauge states and consequently occurs often when groups are broken.

For the $N = 0$ models we see that $M_n > M_{n+2}$ for all classes of group factor, $SU_N$, $SO_N$ and $E_N$. However, this trend only occurs for $SU_N$ of odd rank up to $n = 11$. Additionally, we can see $M_{2n-1} > M_{2n}$ up to $n = 10$ for non-SUSY models.
This does not speak to how the factors are distributed amongst the models. Of the non-SUSY models, 314 have at least one factor of $SU_2$, but generally we can expect more than one for a particular model, approximately 1.74 on average. The average number of factors of each rank, $M_n$, is plotted in Figure 4.

FIG. 4: Average Number of Factors of Non-Abelian Groups - For each rank, the average number of factors for each class of groups is plotted for each set of statistics, (a) Non-SUSY Models and (b) SUSY Models.

C. Group Combinations

In this subsection we present statistics for occurrence of specific group factors in various combinations across the layer 1 landscape. As well as combinations of two group factors, we look at combinations of specific compound factors in conjunction with single and other compound factors. Such compound factors include $E_6 \otimes E_6$, $G_{PS} \equiv SU_4 \otimes SU_2 \otimes SU_2$ (Pati-Salam), $G_{LRS} \equiv SU_3 \otimes SU_2 \otimes SU_2$ (Left-Right Symmetric), $G_{SM} \equiv SU_3 \otimes SU_2 \otimes U_1$ (Standard Model), and $G_{RSM} \equiv SU_3 \otimes SU_2$ (Reduced Standard Model). We also include $F_{SU_5} \equiv SU_5 \otimes U_1$, though, because we are not considering matter content, we can only say that the model has the $F_{SU_5}$ gauge group; it may not actually be $F_{SU_5}$.

Recall that the modular invariance constraints and redundancies lead to two GSO projection matrices for even order models and only one for odd order models. In either case, the GSO projection that admits $N = 4$ SUSY is consistent, as discussed in section II B. The SUSY landscape exhibits 68 unique models. From these, the percentage of models exhibiting each combination of group factors at least once is calculated as a straight percentage of the 68 models. These statistics are provided in Table III.

| N = 68  | $U_1$ | $SU_2$ | $SU_3$ | $SU_5$ | $SU_{N>5}$ | $SO_8$ | $SO_{10}$ | $SO_{N>10}$ | $E_N$ | $E_6 \otimes E_6$ | $F_{SU_5}$ | $G_{PS}$ | $G_{LRS}$ | $G_{RSM}$ | $G_{SM}$ |
|--------|------|--------|--------|--------|-----------|--------|--------|------------|------|--------------|----------|--------|--------|--------|--------|
| $U_1$  | 29.41| 11.76  | 4.41   | 4.41   | 11.76     | 5.88   | 36.76  | 4.41       | 7.35 | 11.76       | 5.88     | 1.47   | 5.88   | 2.94   | 0      |
| $SU_2$ | –    | 29.41  | 0      | 7.35   | 0         | 32.35  | 11.76  | 7.35       | 25.00 | 10.29       | 0        | 1.47   | 0      | 1.47   | 0      |
| $SU_3$ | –    | –      | 2.94   | 0      | 1.47      | 2.94   | 0      | 0          | 0     | 0           | 1.47     | 0      | 0      | 0      | 0      |
| $SU_5$ | –    | –      | –      | –      | 5.88      | 0      | 11.76  | 1.47       | 4.41 | 1.47        | 2.94     | 1.47   | 0      | 2.94   | 0      |
| $SU_{N>5}$| – | –      | –      | –      | –         | 4.41   | 2.94   | 0          | 0     | 0           | 0        | 4.41   | 0      | 0      | 0      |
| $SO_8$ | –    | –      | –      | –      | –         | –      | –      | –          | –     | 33.82       | 7.35     | 12.24  | 16.18  | 8.82   | 1.47   |
| $SO_{10}$| – | –      | –      | –      | –         | –      | –      | –          | –     | 32.35       | 7.35     | 0      | 0      | 0      | 0      |
| $SO_{N>10}$| – | –      | –      | –      | –         | –      | –      | –          | –     | –           | –        | 17.65  | 13.24  | 0      | 0      |
| $E_N$  | –    | –      | –      | –      | –         | –      | –      | –          | –     | 11.76       | 1.47     | 0      | 1.47   | 0      | 0      |
| $E_6 \otimes E_6$ | – | –      | –      | –      | –         | –      | –      | –          | –     | 0           | 0        | 0      | 0      | 0      | 0      |

TABLE III: $N = 4$ Gauge Group Combinations (All Models) - The percentage of all unique $N = 4$ models with each combination of gauge groups is tabulated. For example, 11.76% of the 68 unique SUSY models have the combination $SU_4 \otimes U_1$ at least once.
We included the $G_{LRS}$, $G_{RSM}$ and $G_{SM}$ entries for completeness. While they are identically zero for $N = 4$ models, this is not true for $N = 0$ models, thus we include them for future consistency.

$SU_3$ never occurs in tandem with $SU_2$. This means there is no Standard Model gauge group in the SUSY layer 1 landscape, as defined here. Pati-Salam and $FSU_5$ occur in an equal number across the SUSY landscape but never in the same model. The only compound factor that occurs more than once in any model is $FSU_5$ and it does so 75% of the time, though this only amounts to 3 models in total.

Turning our attentions to $N = 0$ models, we can perform the same statistical analysis we did above. This time, however, we note that there are 502 unique non-SUSY models.

### TABLE IV: $N = 0$ Gauge Group Combinations (All Models)

| $N = 502$ | $U_1$ | $SU_2$ | $SU_3$ | $SU_4$ | $SU_5$ | $SU_{N>5}$ | $SO_8$ | $SO_{10}$ | $SO_{N>10}$ | $E_N$ | $E_6 \otimes E_6$ | $FSU_5$ | $G_{PS}$ | $G_{LRS}$ | $G_{RSM}$ | $G_{SM}$ |
|-----------|-------|--------|--------|--------|--------|------------|--------|-----------|-------------|-------|-------------|---------|--------|--------|--------|-------|
| $U_1$     | 75.70 | 49.80  | 25.30  | 40.64  | 20.92  | 63.75      | 11.75  | 13.35     | 10.36       | 10.36 | 1.39        | 20.92  | 15.54  | 8.76   | 14.74  | 14.74 |
| $SU_2$    | -     | 42.63  | 14.74  | 24.50  | 12.35  | 39.24      | 12.55  | 8.17      | 12.75       | 9.96  | 1.00        | 12.35  | 9.36   | 4.78   | 8.76   | 8.76  |
| $SU_3$    | -     | -      | 13.94  | 11.95  | 10.76  | 15.74      | 1.00   | 1.20      | 0           | 0.60  | 0           | 10.76  | 3.78   | 6.57   | 8.96   | 8.96  |
| $SU_4$    | -     | -      | -      | 19.12  | 9.56   | 28.09      | 6.37   | 5.98      | 4.18        | 3.98  | 0.60        | 9.56   | 8.37   | 3.78   | 7.17   | 7.17  |
| $SU_5$    | -     | -      | -      | -      | 8.76   | 12.35      | 0.80   | 1.00      | 0           | 0.60  | 0           | 8.76   | 2.59   | 4.38   | 7.17   | 7.17  |
| $SU_{N>5}$| -     | -      | -      | -      | -      | -          | 31.27  | 9.76      | 10.16       | 9.56  | 0.40        | 12.35  | 9.36   | 3.00   | 7.17   | 7.17  |
| $SO_8$    | -     | -      | -      | -      | -      | -          | 4.58   | 1.79      | 4.98        | 3.78  | 0           | 0.80   | 2.39   | 0      | 0      | 0     |
| $SO_{10}$ | -     | -      | -      | -      | -      | -          | -      | 2.59      | 1.20        | 2.19  | 0.40        | 1.00   | 2.39   | 0      | 0      | 0     |
| $SO_{N>10}$| -     | -      | -      | -      | -      | -          | -      | 5.98      | 5.38        | 0.20  | 0           | 0.40   | 0      | 0      | 0      | 0     |
| $E_N$     | -     | -      | -      | -      | -      | -          | -      | -         | 3.78        | 0.20  | 0           | 0.60   | 1.59   | 0      | 0      | 0     |
| $E_6 \otimes E_6$ | -     | -      | -      | -      | -      | -          | -      | -         | -           | -    | 0           | 0.20   | 0.20   | 0      | 0      | 0     |
| $FSU_5$   | -     | -      | -      | -      | -      | -          | -      | -         | -           | -    | 8.76        | 2.59   | 4.38   | 7.17   | 7.17   | 7.17  |
| $G_{PS}$  | -     | -      | -      | -      | -      | -          | -      | -         | -           | -    | 2.99        | 0.80   | 1.99   | 1.99   | -      | -     |
| $G_{LRS}$ | -     | -      | -      | -      | -      | -          | -      | -         | -           | -    | 1.99        | 3.78   | 3.78   | -      | 6.57   | 6.57  |
| $G_{RSM}$ | -     | -      | -      | -      | -      | -          | -      | -         | -           | -    | 6.57        | -      | -      | 6.57   | -      | -     |
| $G_{SM}$  | -     | -      | -      | -      | -      | -          | -      | -         | -           | -    | -           | -      | -      | -      | 6.57   | -     |
| Total     | 83.67 | 62.55  | 25.30  | 41.83  | 20.92  | 66.53      | 19.32  | 13.94     | 21.51       | 15.54 | 1.39        | 20.92  | 16.33  | 8.76   | 14.74  | 14.74 |

It is interesting to note that the occurrence of $N = 0$ group combinations is not simply an extension of the $N = 4$. That is, groups that are less common in $N = 4$ models are not necessarily less common in $N = 0$. We also find that $SU_3 \otimes SU_2$ combinations never occur with $SO_N$ nor $E_N$.

### IV. CONCLUSION

We have shown the computational feasibility of systematically generating models of this type by creating all possible layer 1 gauge models through order 22. While these are by no means phenomenologically realistic models, some show properties that suggest possible avenues of investigations. Furthermore, several redundancies in the construction process have been demonstrated and will be explored further with future work. Additionally, the development process has provided tools that can be used for higher layer surveys as well. One such survey, examining the contents of the Layer 2 landscape through order 22, is currently underway. In future work, we hope to extend these models by
prepending left-movers to determine how well these surveys can be used to guide more exhaustive and detailed surveys excluding left-right pairing.

[1] R. Bousso and J. Polchinski, JHEP 06, 006 (2000), hep-th/0004134.
[2] S. Ashok and M. R. Douglas, JHEP 01, 060 (2004), hep-th/0307049.
[3] I. Antoniadis, C. P. Bachas, and C. Kounnas, Nucl. Phys. B289, 87 (1987).
[4] I. Antoniadis and C. Bachas, Nucl. Phys. B298, 586 (1988).
[5] H. Kawai, D. C. Lewellen, and S. H. H. Tye, Nucl. Phys. B288, 1 (1987).
[6] G. B. Cleaver, A. E. Faraggi, D. V. Nanopoulos, and J. W. Walker, Nucl. Phys. B593, 471 (2001), hep-ph/9910230.
[7] J. L. Lopez, D. V. Nanopoulos, and K.-j. Yuan, Nucl. Phys. B399, 654 (1993), hep-th/9203025.
[8] A. E. Faraggi, D. V. Nanopoulos, and K.-j. Yuan, Nucl. Phys. B335, 347 (1990).
[9] A. E. Faraggi, Nucl. Phys. B387, 239 (1992), hep-th/9208024.
[10] I. Antoniadis, G. K. Leontaris, and J. Rizos, Phys. Lett. B245, 161 (1990).
[11] G. K. Leontaris and J. Rizos, Nucl. Phys. B554, 3 (1999), hep-th/9901098.
[12] A. E. Faraggi, Phys. Lett. B278, 131 (1992).
[13] A. E. Faraggi, Nucl. Phys. B403, 101 (1993), hep-th/9208023.
[14] A. E. Faraggi, Nucl. Phys. B407, 57 (1993), hep-ph/9210256.
[15] A. E. Faraggi, Phys. Lett. B274, 47 (1992).
[16] A. E. Faraggi, Phys. Rev. D47, 5021 (1993).
[17] A. E. Faraggi, Phys. Lett. B377, 43 (1996), hep-ph/9506388.
[18] A. E. Faraggi, Nucl. Phys. B487, 55 (1997), hep-ph/9601332.
[19] G. B. Cleaver, Nucl. Phys. Proc. Suppl. 62, 161 (1998), hep-th/9708023.
[20] G. B. Cleaver and A. E. Faraggi, Int. J. Mod. Phys. A14, 2335 (1999), hep-ph/9711339.
[21] G. Cleaver, M. Cvetic, J. R. Espinosa, L. L. Everett, and P. Langacker, Nucl. Phys. B525, 3 (1998), hep-th/9711178.
[22] G. Cleaver, M. Cvetic, J. R. Espinosa, L. L. Everett, and P. Langacker, Nucl. Phys. B545, 47 (1999), hep-ph/9805133.
[23] G. Cleaver et al., Phys. Rev. D59, 055005 (1999), hep-ph/9807479.
[24] G. Cleaver et al., Phys. Rev. D59, 115003 (1999), hep-ph/9811355.
[25] G. B. Cleaver (1998), hep-ph/9812262.
[26] G. B. Cleaver, A. E. Faraggi, and D. V. Nanopoulos, Phys. Lett. B455, 135 (1999), hep-ph/9811427.
[27] G. B. Cleaver, A. E. Faraggi, and D. V. Nanopoulos, Int. J. Mod. Phys. A16, 425 (2001), hep-ph/9904301.
[28] G. B. Cleaver, A. E. Faraggi, D. V. Nanopoulos, and J. W. Walker, Nucl. Phys. B593, 471 (2001), hep-ph/9910230.
[29] G. B. Cleaver (1999), hep-ph/9901203.
[30] G. B. Cleaver, A. E. Faraggi, D. V. Nanopoulos, and J. W. Walker, Mod. Phys. Lett. A15, 1191 (2000), hep-ph/0002060.
[31] G. B. Cleaver, A. E. Faraggi, and C. Savage, Phys. Rev. D63, 066001 (2001), hep-ph/0006331.
[32] G. B. Cleaver, A. E. Faraggi, D. V. Nanopoulos, and J. W. Walker, Nucl. Phys. B620, 259 (2002), hep-ph/0104091.
[33] G. B. Cleaver, D. J. Clements, and A. E. Faraggi, Phys. Rev. D65, 106003 (2002), hep-ph/0106060.
[34] G. B. Cleaver, A. E. Faraggi, and S. Nooij, Nucl. Phys. B672, 64 (2003), hep-ph/0301037.
[35] G. B. Cleaver (2002), hep-ph/0210093.
[36] G. Cleaver et al., Phys. Rev. D67, 026009 (2003), hep-ph/0209050.
[37] J. Perkins et al. (2003), hep-ph/0110555.
[38] J. Perkins et al., Phys. Rev. D75, 026007 (2007), hep-ph/0510141.
[39] G. B. Cleaver, A. E. Faraggi, E. Manno, and C. Timirgaziu, Phys. Rev. D78, 046009 (2008), 0802.0470.
[40] J. Greenwald, D. Moore, K. Pechan, T. Renner, T. Ali, and G. Cleaver (2009), 0912.5207.
[41] G. Cleaver et al. (2011), 1105.0447.
[42] K. R. Dienes, Phys. Rev. D73, 106010 (2006), hep-th/0602286.
[43] K. R. Dienes, M. Lennek, D. Senechal, and V. Wasnik, Phys. Rev. D75, 126005 (2007), 0704.1320.
[44] K. R. Dienes and M. Lennek, Phys. Rev. D75, 026008 (2007), hep-th/0610319.
[45] H. Kawai, D. C. Lewellen, J. A. Schwartz, and S. H. H. Tye, Nucl. Phys. B299, 431 (1988).
[46] M. Robinson, G. Cleaver, and M. B. Hunziker, Mod. Phys. Lett. A24, 2703 (2009), 0809.5094.
[47] T. Renner, J. Greenwald, D. Moore, and G. Cleaver (2011), 1107.3138.
[48] H. Kawai, D. C. Lewellen, and S. H. H. Tye, Phys. Rev. D34, 3794 (1986).