Consistent BCS+RQRPA formalism with application to the
double beta decay

A. Bobyk, Wieslaw A. Kamiński, P. Zaręba

Department of Theoretical Physics, Maria Curie-Skłodowska University,
Radziszewskiego 10, 20–031 Lublin, Poland

(March 31, 2022)

A new consistent analysis of the renormalized proton–neutron quasiparticle random phase approximation based on the simultaneous recalculation of the one–body density matrix and the pairing tensor has been used to study the double beta decay. We demonstrated that inclusion of the quasiparticle correlations at the BCS level reduces the ground state correlations in the particle–particle channel of the proton–neutron interaction. We also simplified the RQRPA equations significantly obtaining a low–dimensioned set of linear equations for the quasiparticle densities. The formalism was applied to the double beta decay in $^{76}$Ge.

The proton–neutron quasiparticle random phase approximation (pn–QRPA) has been considered the most powerful method for the beta and double beta transition calculations of nuclear systems which are far away from the closed shells. A remarkable success was achieved especially by the QRPA approach in revealing the suppression mechanism of the neutrino accompanied double beta decay, a long–standing problem of the theoretical treatment of this process. Further development of the approach went beyond many shortcomings and refined calculations of nuclear matrix elements involved in the double beta decay. Among others, the following problems were set and solved: particle number non-conservation, role of the proton–neutron pairing, violation of the Pauli exclusion principle, higher–order corrections to the ordinary QRPA, treatment of tran-
itions to final excited states \cite{16,17,18,20}, extension of the definition of phonon operators by means of the–so–called scattering terms \cite{21}, etc.

Most of such improvements disregarded, however, the main source of the formalism instability connected with violation of the Pauli exclusion principle by using the commutation relations for the QRPA phonon operators. To overcome this shortcoming of the pn–QRPA framework the renormalization technique was proposed \cite{12} and extended to include the proton–neutron pairing \cite{13}. The main goal of the method called in literature the renormalized QRPA (RQRPA) is to use a self–iteration of the QRPA equation to take into account the additional one–quasiparticle scattering terms in the commutation relations. But this procedure results in a non–vanishing quasiparticle content of the ground states and followed by some inconsistency between RQRPA and the BCS approach since the ground state approximated by the BSC state is chosen to be the quasiparticle vacuum. To minimize the influence of such a discrepancy one needs to reformulate the BCS equations in a way proposed in \cite{23}. Combining both RQRPA and so modified BCS one obtains the self–consistent BCS+RQRPA approach (SRQRPA) which we study in more detail in this paper.

In the QRPA (either ordinary or renormalized) approach one assumes the harmonicity of the nuclear motion and starts with the excited–state creation phonon operators of the form \cite{22,24}:

\[
Q_{m,\pi M}^{\dagger} = \sum_{pn} \left[ X_{(pn),\pi M}^{m} A_{(pn),\pi M}^{\dagger} - Y_{(pn),\pi M}^{m} \right],
\]

where \(X_{(pn),\pi}^{m}\) and \(Y_{(pn),\pi}^{m}\) are the forward–going and backward–going variational amplitudes, respectively. The quantities \(A_{(pn),\pi M}^{\dagger} \equiv [a_{p,\pi}^{\dagger} a_{n,\pi}^{\dagger}]_{\pi M}\) are the angular–momentum coupled two–quasiparticle creation operators. Since they do not fulfil the bosonic commutation relations exactly in the quasiboson approximation (QBA), that is used to derive the usual QRPA equations, the Pauli principle is violated. To avoid this serious drawback in the improved version of the theory one introduces the renormalized operators \cite{19}:

\[
A_{(pn),\pi M}^{\dagger} \equiv D_{pn}^{-1/2} A_{(pn),\pi M}^{\dagger},
\]

along with the renormalized amplitudes.
\( X_{(p_m),J^\pi}^m \equiv D_{p_m}^{1/2} X_{(p_m),J^\pi}^m, \)

\( Y_{(p_m),J^\pi}^m \equiv D_{p_m}^{1/2} Y_{(p_m),J^\pi}^m, \)

where the \( D_{p_m} \) matrix is defined by the expectation value in the RPA ground–state of the commutator:

\[ D_{p_m} \equiv \langle 0 \left| [A_{(p_m),J^\pi,M}, A_{(p_m),J^\pi,M}^\dagger] \right| 0 \rangle = (1 - n_p - n_n) \]

(5)

together with the quasiparticle densities \( (j_a \equiv \sqrt{2j_a + 1}) \):

\[ n_p \equiv \overline{j_p}^{-1} \langle 0 \left| [a_p^\dagger \bar{a}_p]_{00} \right| 0 \rangle, \]

(6)

\[ n_n \equiv \overline{j_n}^{-1} \langle 0 \left| [a_n^\dagger \bar{a}_n]_{00} \right| 0 \rangle. \]

(7)

The above equation has been derived using the exact fermionic commutation relations and thus it goes beyond the ordinary quasiboson approximation. One can prove easily that now the following relation holds:

\[ \langle 0 \left| [A_{(p_m),J^\pi,M}, A_{(p_n'),J^\pi,M'}^\dagger] \right| 0 \rangle = \delta_{p p'} \delta_{n n'} \delta_{j J} \delta_{j' J'} \delta_{\pi \pi'} \delta_{M M'}, \]

(8)

i.e. the renormalized operators behave as bosons, at least in the sense of the ground–state expectation value of their commutator. The phonon operator now reads:

\[ Q_{J^\pi,M}^m = \sum_{p_m} \left[ X_{(p_m),J^\pi,M}^m A_{(p_m),J^\pi,M}^\dagger - Y_{(p_m),J^\pi,M}^m A_{(p_m),J^\pi,M} \right], \]

(11)

and using e.g. the equation of motion (EOM) method \[22\] one gets the RQRPA equations of the usual form:

\[ \begin{pmatrix} A & B \\ B & A \end{pmatrix}_{J^\pi} \begin{pmatrix} X^m \\ Y^m \end{pmatrix}_{J^\pi} = \Omega_{J^\pi}^m \begin{pmatrix} X^m \\ -Y^m \end{pmatrix}_{J^\pi}, \]

(9)

with the new renormalized RPA matrices \( A \) and \( B \) defined in \[12\]. Here \( \Omega_{J^\pi}^m \equiv E_{J^\pi}^m - E_0 \) is the RPA excitation energy with respect to the ground–state.

Now there appears the question of the calculation of the \( D_{p_m} \) matrix entering the expressions for the RPA matrices \( A \) and \( B \). Using the mapping \[12\]:

\[ }
\[ [a^+_p a_p]_{00} \mapsto j_p^{-1} \sum_{J^* M n} A^+_p (J^*_M J^* M) A_p (J^*_M J^* M), \]  
\[ [a^+_n a_n]_{00} \mapsto j_n^{-1} \sum_{J^* M p} A^+_n (J^*_M J^* M) A_n (J^*_M J^* M), \]

and inverting (11) one derives the following equations for the quasiparticle densities:

\[ n_p = j_p^{-2} \sum_{J^* n m} D_{pn} |\mathcal{Y}^m_{(pn), J^*}|^2, \]  
\[ n_n = j_n^{-2} \sum_{J^* m p} D_{pn} |\mathcal{Y}^m_{(pn), J^*}|^2. \]

Inserting (14) into (13) one gets the following system of linear equations for \( n_p \) and \( n_n \):

\[ \mathcal{Y}'_p n_p + \sum_n \mathcal{Y}_{pn} n_n = \mathcal{Y}'_p, \]
\[ \mathcal{Y}'_n n_n + \sum_p \mathcal{Y}_{pn} n_p = \mathcal{Y}'_n, \]  

or, in the matrix form:

\[ \begin{pmatrix} \text{diag}(\mathcal{Y}'_p) & \mathcal{Y} \\ \mathcal{Y}^T & \text{diag}(\mathcal{Y}'_n) \end{pmatrix} \begin{pmatrix} n_p \\ n_n \end{pmatrix} = \begin{pmatrix} \mathcal{Y}'_p \\ \mathcal{Y}'_n \end{pmatrix}, \]  

where

\[ \mathcal{Y}_{pn} \equiv j^2 \sum_{J^*_m} |\mathcal{Y}^m_{(pn), J^*}|^2, \]
\[ \mathcal{Y}_p \equiv \sum_n \mathcal{Y}_{pn}, \quad \mathcal{Y}'_p \equiv j^2_p + \mathcal{Y}_p, \]
\[ \mathcal{Y}_n \equiv \sum_p \mathcal{Y}_{pn}, \quad \mathcal{Y}'_n \equiv j^2_n + \mathcal{Y}_n. \]  

It is worth mentioning, that the dimension of our linear problem is only \( 2n \times 2n \), where \( n \) is the dimension of the single-particle basis. This is of much advantage, since (14) has to be solved many times as one should iterate between (13) and (14) until convergence is achieved.

The other way round, inserting (13) into (11), as it has been done by several authors, e.g. [12,13], one obtains the equation:

\[ D_{pn} = 1 - j_p^{-2} \sum_{n'n} D_{pn'} \mathcal{Y}_{pn'} - j_n^{-2} \sum_{p'n} D_{p'n} \mathcal{Y}_{p'n}, \]  

\[ (16) \]
that, on the contrary to the claim expressed in [12], can be transformed into the $n^2 \times n^2$ linear system:

$$\sum_{p'n'} W_{pn,p'n'} D_{p'n'} = U,$$

(17)

where

$$W_{pn,p'n'} \equiv \delta_{pp'} \delta_{nn'} + \delta_{pp'} \hat{J}_p^{-2} \gamma_{pn'} + \delta_{nn'} \hat{J}_n^{-2} \gamma_{p'n'},$$

(18)

and $U$ is the vector of 1’s. In practice however, it takes much less time to solve this system using the standard linear algebra procedures than following iteration methods, as in [12,13]. But further reduction of the complexity of the problem to the form of (14) allows us to take all possible multipolarities into account for the calculation of the $D_{pn}$ renormalization factors. Although some of them are less important and neglected in [12,13], when one leaves only a few, the J–coupling scheme breaks, since the basis becomes incomplete and the validity of the mapping (10)–(11) is questionable. As can be seen further the results are the evidence for it.

With a non–vanishing quasiparticle content of the ground state one arrives at the inconsistency between RQRPA and BCS, since in the latter assumes the ground state to be the quasiparticle vacuum. One thus needs to reformulate the BCS equations [23], namely by recalculating the density matrix $\rho$ and the pairing tensor $\kappa$. With the standard Bogoliubov–Valatin transformation [24] they read now:

$$\rho_a \equiv \left\langle 0 \left| c_{a \alpha}^\dagger c_{a \alpha} \right| 0 \right\rangle = v_a^2 + (u_a^2 - v_a^2)n_a,$$

(19)

$$\kappa_a \equiv \left\langle 0 \left| \tilde{c}_{a \alpha} c_{a \alpha} \right| 0 \right\rangle = u_a v_a (1 - 2n_a),$$

(20)

and depend on the quasiparticle densities $n_a$, where $a$ runs over proton or neutron indices. The $u$ and $v$ coefficients are obtained by minimizing the ground–state energy, that by virtue of the Wick’s theorem is expressed as [24,25]:

$$\left\langle 0 \left| \hat{H} \right| 0 \right\rangle = \sum_a j_a \tilde{c}_a \rho_a + \frac{1}{4} \sum_{ab} j_a j_b \left\langle (aa)^{T=1} | V | (bb)^{T=1} \right\rangle \kappa_a \kappa_b$$

(21)
with the particle–number constraint:

\[ N_0 = \langle 0 \mid \hat{N} \mid 0 \rangle = \sum_a \hat{n}_a \rho_a. \]  (22)

In the above, \( \varepsilon_a \) are the single–particle energies and \( \langle (aa)^{T_T=0\uparrow} \mid V \mid (bb)^{T_T=0\uparrow} \rangle \) are the matrix elements of the two–body interaction.

To solve the SRQRPA equations we start with the ordinary BCS equations, putting \( n_p = n_n = 0 \), than to proceed with the corresponding RQRPA problem (inner iteration), that gives us new quasiparticle densities and loop with them back to BCS until the convergence is achieved (outer iteration). We arrived thus at the doubly–iterative problem and the question of efficient and accurate getting through all the calculation steps becomes very important. We then stress again, that without showing that the problem of calculating the \( D \)–matrix can be reduced to the linear system (14) of acceptable size the realization of this task would be hardly possible.

The ground–state to ground–state \( 2\nu\beta\beta \) Gamow–Teller matrix elements are expressed as follows:

\[
M^{2\nu}_{\text{GT}} = \sum_{mm'} \langle 0^+_{\text{gs}}(A, Z + 2) \mid \hat{\sigma}_+ \mid 1^+_{m'} \rangle \langle 1^+_{m'} \mid \hat{\sigma}_+ \mid 0^+_{\text{gs}}(A, Z) \rangle
\]

\[
\frac{1}{2} \left[ \Omega^m_{1+} + \Omega^{m'}_{1+} + Q^- (A, Z + 1) - Q^- (A, Z) \right],
\]  (23)

where the charge–changing transition densities are:

\[
\langle 0^+_{\text{gs}} \mid \hat{\sigma}_+ \mid 1^+_{m'} \rangle = \sum_{pn} \langle p \mid \hat{\sigma} \mid n \rangle \left( v'_{p} u'_{n} \chi_{(pn)1^+}^{m'} + u'_{p} v'_{n} \gamma_{(pn)1^+}^{m'} \right) \sqrt{D_{pn}},
\]  (24)

\[
\langle 1^+_m \mid \hat{\sigma}_+ \mid 0^+_{\text{gs}} \rangle = \sum_{pn} \langle p \mid \hat{\sigma} \mid n \rangle \left( u_{p} v_{n} \chi_{(pn)1^+}^{m} + v_{p} u_{n} \gamma_{(pn)1^+}^{m} \right) \sqrt{D_{pn}}
\]  (25)

and the overlap of intermediate excited states is assumed to be expressed as:

\[
\langle 1^+_{m'} \mid 1^+_m \rangle = \sum_{pn} \left( \chi_{(pn)1^+}^{m'} \chi_{(pn)1^+}^{m} - \gamma_{(pn)1^+}^{m'} \gamma_{(pn)1^+}^{m} \right).
\]  (26)

In the above the non–primed (primed) quantities result from the SRQRPA calculations based on the initial (final) ground–state.

To illustrate the differences between the QRPA, the RQRPA and the SRQRPA and to show much better stability of the SRQRPA solutions, the results of our calculations as a
function of the particle–particle \((g_{pp})\) and particle–hole \((g_{ph})\) factors, renormalizing the bare two–body NN interaction are plotted in Figs. 1–3. This commonly used renormalization is necessary due to the nucleus finite size (the bare NN matrix elements are calculated for the infinite nuclear matter) and due to the limited dimension of the single–particle basis. In our calculations we used the two–body matrix elements calculated from the Bonn–B nucleon–nucleon one boson exchange potential. The single–particle energies are calculated from the Coulomb–corrected Woods–Saxon potential with the Bertsch parametrization. We used several sets of single–particle levels to see how the choice of the basis influences the results. We find weak dependence of the RQRPA and the SQRPA results on the dimension of the single–particle basis. On the other hand, the QRPA shows no stability on the chosen basis. The conclusion is that the most suitable basis for the calculations of \(2\nu\beta\beta\) decay in \(^{76}\)Ge consists of 16 levels with \(^{16}\)O as a core. Therefore we used this basis in all the further studies described below. To compare with the experiment we have adopted the experimental half–life of \(T_{1/2}^{2\nu} = (1.42 \pm 0.03 \pm 0.13) \times 10^{21}\) yr from the latest measurement by the Heidelberg–Moscow \(\beta\beta\) cooperation \[26\].

In Fig. 1 the calculated double Gamow–Teller matrix elements \(M_{GT}^{2\nu}\) as a function of \(g_{pp}\) for two different \(g_{ph}\) values and for three different QRPA approaches are plotted. We would like to stress that in these calculations in the self–consistent iterations of the RQRPA and the SRQRPA all the intermediate multipolarities were present. The comparison between the QRPA, the RQRPA and the SRQRPA results in the physically acceptable region of the \(g_{pp}\) parameter \(0.8 \leq g_{pp} \leq 1.2\) shows two main features of the renormalized QRPA. First, the inclusion of the ground–state correlations beyond QRPA does not only improve the agreement between theoretical calculations and experimental data but also causes the stabilization of the dependence of \(M_{GT}^{2\nu}\) as a function of \(g_{pp}\). Second, the iteration procedure for quasiparticle densities which causes treating of RQRPA and BCS on the same footing stabilizes the results even further.

Some authors claim that only the limited set of these multipolarities plays a role in the evaluation of the double Gamow–Teller matrix elements \[12,13\]. In Fig. 2 there are shown
the results of calculations of $M_{GT}^{2\nu}$ as a function of $g_{pp}$ for a different number of multipolarities for the RQRPA and the SRQRPA, respectively. The basis is the same as in Fig. 1, but the value of $g_{ph}$ parameter is fixed to 1.0. It can be seen why the inclusion of all multipolarities is essential to obtain the reliable predictions of the RQRPA and the SRQRPA calculations. The solid line in Fig. 2 represents the QRPA calculations, the dot–dashed line the calculations with only $1^+$ multipolarity, the dotted line with multipolarities up to $2^+$, the dashed line up to $3^-$, the long–dashed up to $5^-$ and the thick solid line all considered multipolarities up to $11^+$. The inclusion of higher multipolarities causes the shift of the collapse of the RQRPA and the SRQRPA beyond the value of $g_{pp} = 1.0$. The additional advantage of the SRQRPA solutions is that the calculated matrix elements are less dependent on the $g_{pp}$ parameter.

In Fig. 3 the effect of including more multipolarities $J^\pi$ in the RQRPA and the SQRPA calculations of $M_{GT}^{2\nu}$ for fixed $g_{ph} = 1.0$ is shown. The filled symbols represent the calculations for $g_{pp} = 0.8$ and open symbols for $g_{pp} = 1.0$. One can see the saturation effect for higher multipolarities. The explanation of this behaviour is that the higher multipolarities $J^\pi$ are less collective. Their contribution to the ground–state correlations is much smaller than the lower ones. An interesting feature is the virtual independence of the matrix element on the number of multipolarities in SRQRPA around $g_{pp} = 1.0$. It is reflected in Fig. 2, where one can see that RQRPA ”diverges” when going with $g_{pp}$ from 0 to 1, while SRQRPA ”converges”. In our opinion, this is a clear evidence that the self–consistency between BCS and RPA, i.e. between the ground–state and excited–state properties is being restored in SRQRPA.

Finally, we would like to address the question of the Ikeda sum–rule violation. It is well known, that in the usual QRPA the Ikeda sum rule is conserved exactly if all the spin–orbit partners of the single–particle orbitals are present in the basis, i.e.

$$S_- - S_+ = \sum_m \left| \langle 0^+_{gs} | \sigma \tau_+ | 1^+_m \rangle \right|^2 - \sum_m \left| \langle 0^+_{gs} | \sigma \tau_- | 1^+_m \rangle \right|^2 = 3(N - Z).$$

(27)

The violation is marginal even if one or two of these partners are omitted. But this is not the case in RQRPA, where the Ikeda sum rule is violated up to 20% and is similar, up to
25% in the SRQRPA. There is some difference between two nuclei under consideration, i.e. germanium, where the situation gets worse when the ground–state correlations (SRQRPA) are taken into account, and selenium, where the results are slightly improved. One can conclude, that there is more to this than the ground–state correlations to restore the Ikeda sum rule. As already pointed out [28], the scattering terms present in the $\beta$–decay operators can be responsible for this effect [29]. They give no contribution to QRPA, because they are of the quasiparticle–quasihole character, but they should be taken into account when the ground–state quasiparticle densities are not assumed to be zero, like in the RQRPA or SRQRPA. It is necessary to extend the form of the QRPA phonon operator (1) by including new excitation modes, the so–called $B$–modes [21].

In conclusion, we have developed a new method of the nuclear matrix–element calculations for the neutrino accompanied double beta decay to the ground state in a frame of the consistent BCS+RQRPA approach. Using the $2\nu\beta\beta$–decay in germanium $^{76}$Ge as an example we demonstrated that the inclusion of the ground–state correlations beyond QRPA causes the stabilization of the dependence of the Gamow–Teller nuclear matrix element, but also weakens their influence on their magnitude because of the additional change of the quasiparticle densities during the iteration procedure with the modified BCS solution.

Unlike orthodox QRPA which requires fine tuning of the $g_{pp}$ parameter describing the particle–particle interaction–strength as the matrix element collapses near the physical strength, RQRPA and BCS+RQRPA give stable matrix elements over the whole range of physical strength and thus the latter approaches allow for more predictive power than the old method.

Owing to the development of the method to calculate the $D_{pn}$ normalization factors we could take all possible multipolarities into account. Then we were able to avoid possible J–scheme breaking because, by neglecting some of the multipolarities, the basis becomes incomplete and the validity of mappings (10)–(11) is questionable.

This work has been supported in parts by the State Committee for Scientific Researches (Poland) and in the frame of PECO project, Contract No. ERBCIPDCT 940603.
[1] P. Vogel, and M. R. Zirnbauer, Phys. Rev. Lett. 57, 3148 (1986).
[2] O. Civitarese, A. Faessler, T. Tomoda, Phys. Lett. B 194, 11 (1987).
[3] T. Tomoda, and A. Faessler, Phys. Lett. B 199, 475 (1987).
[4] J. Engel, P. Vogel, and R. Zirnbauer, Phys. Rev. C 37, 731 (1988).
[5] K. Muto, and H. V. Klapdor, Phys. Lett. B 201, 420 (1988).
[6] J. Suhonen, T. Taigel, and A. Faessler, Nucl. Phys. A486, 91 (1988).
[7] K. Muto, E. Bender, and H. V. Klapdor, Z. Phys. A334, 187 (1989).
[8] G. Pantis, A. Faessler, W. A. Kamiński, and J. D. Vergados, J. Phys. G 18, 605 (1992).
[9] O. Civitarese, A. Faessler, J. Suhonen, and X. R. Wu, J. Phys. G 17, 943 (1991).
[10] F. Krmpotić, A. Mariano, T. T. S. Kuo, and K. Nakayama, Phys. Lett. B 319, 393 (1993).
[11] M. K. Cheoun, A. Bobyk, A. Faessler, F. Šimkovic, and G. Teneva, Nucl. Phys. A561, 74 (1993); Nucl. Phys. A564, 329 (1993).
[12] J. Toivanen and J. Suhonen, Phys. Rev. Lett. 75, 410 (1995).
[13] J. Schwieger, F. Šimkovic, and Amand Faessler, Nucl. Phys. A600, 179 (1996).
[14] A. A. Raduta, A. Faessler, S. Stoica, and W. A. Kamiński, Phys. Lett. B 208, 7 (1991); A. A. Raduta, A. Faessler, and S. Stoica, Nucl. Phys. A 534, 149 (1991).
[15] S. Stoica, and W. A. Kamiński, Phys. Rev. C 47, 867 (1993); Nuovo Cimento A 106, 723 (1993).
[16] J. Suhonen, Nucl. Phys. A563, 205 (1993).
[17] A. Bobyk, and W. A. Kamiński, J. Phys. G 21, 229 (1995).
[18] J. Toivanen, and J. Suhonen, Phys. Rev. C 55, 2314 (1997).

[19] D. Karadjov, V. V. Voronov and F. Catara, Phys. Lett. B 306, 197 (1993); F. Catara, N. Dinh Dang and M. Sambataro, Nucl. Phys. A579, 1 (1994).

[20] J. Schwieger, F. Šimkovic, Amand Faessler, and Wiesław A. Kamiński, J. Phys. G 23, 1647 (1997); Phys. Rev. C 57, 1738 (1998).

[21] A. A. Raduta, C. M. Raduta, A. Faessler and W. A. Kamiński, Nucl. Phys. A (in press).

[22] D. J. Rowe, Rev. Mod. Phys. 40, 153 (1968); Phys. Rev. 175, 1283 (1968).

[23] R. V. Jolos and W. Rybarska–Nawrocka, Z. Phys. A296, 73 (1980); D. Karadjov, V. V. Voronov and F. Catara, J. Phys. G 20, 1431 (1994).

[24] M. Baranger, Phys. Rev. 120, 957 (1960).

[25] P. Ring and P. Schuck, The Nuclear Many–Body Problem (Springer, New York, 1980).

[26] A. Balysh et al., Phys. Lett. B 322, 176 (1994).

[27] F. Krmpotić, A. Mariano, E. J. V. de Passo, A. F. R. de Toledo Piza, and T. T. S. Kuo, Fizika B5, 93 (1996); Nucl. Phys. A612, 223 (1997).

[28] K. Ikeda, T. Udagawa, and H. Yamaura, Prog. Theor. Phys. 33, 22 (1965).

[29] F. Krmpotić, E. J. V. de Passo, D. S. Delion, J. Dukelsky and P. Schuck, nucl-th/9805038.
FIG. 1. Dependence of the double Gamow–Teller matrix element $M^{2\nu}_{\nu GT}$ on the renormalization factor in the particle–particle channel $g_{pp}$. Two different cases for $g_{pp} = 0.8$ and $1.0$ and for three different QRPA approaches are shown. Magnitude of the experimental estimate is marked by two parallel dotted lines $^{26}$.

FIG. 2. Influence of the different number of the added multipolarities on $M^{2\nu}_{\nu GT}$ for two types of the renormalization: RQRPA and SRQRPA. Full range of the strength factor $g_{pp}$ is scanned for fixed $g_{ph} = 1.0$. SRQRPA shows less dependence on $g_{pp}$ parameter than the RQRPA approach. The experimental values $^{26}$ are between two parallel dotted lines.

FIG. 3. Contributions of the individual multipolarities ($1^+, 0^+, ..., 11^+$) to the double Gamow–Teller matrix element within two different RQRPA approaches. Calculations were made for $g_{ph} = 1.0$ and for two different values of the renormalization constant in the particle–particle channel: $g_{pp} = 0.8, 1.0$. For details see the text.
$M_{GT}^{2}$ [MeV$^{-1}$] vs $g_{pp}$

- $g_{ph}=1.0$ QRPA
- $g_{ph}=0.8$ QRPA
- $g_{ph}=1.0$ RQRPA
- $g_{ph}=0.8$ RQRPA
- $g_{ph}=0.8$ SQRPA
- $g_{ph}=1.0$ SRQRPA

**Experiment**
added multipolarities

$M_{GT}^{2v}$ [MeV$^{-1}$]

experiment

$g_{pp} = 1.0$ RQRPA
$g_{pp} = 1.0$ SRQRPA
$g_{pp} = 0.8$ SRQRPA
$g_{pp} = 0.8$ RQRPA

added multipolarities

1$^+$ 0$^+$ 1$^+$ 2$^+$ 3$^+$ 4$^+$ 5$^+$ 6$^+$ 7$^+$ 8$^+$ 9$^+$ 10$^+$ 11$^+$