Analyzing of a 2-D Magnet Array with Hexagon Magnet based on superposition

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Abstract. In this paper, a 2D Halbach array with the hexagon permanent magnets is introduced. The expression of magnetization intensity is derived by superposition principle and Fourier series. The expression of magnetic flux density is derived by magnetic scalar potential and method of separation of variables. The assumption of the relative permeability of magnet equals 1.0 is needless because of the compact structure. The accuracy of the expression is verified by comparing with the finite element model. The expression can be directly used in real time control by selecting the proper iteration steps for the coefficients.

1. Introduction

According to the characteristics of Halbach array[1], the magnetic field is significantly enhanced on one side and significantly weakened on the other side. The magnetic field of the Halbach array has good sinusoidal distribution. As for 2D Halbach array, it can be used in the magnetically levitated planar motor that can realize non-contact and non-abrasion plane motion [2]. Comparing with the conventional planar motion mechanism comprised by two linear motors, there is no supporting guide, and the motor with simple structure can realize high speed and acceleration motion. It has extensive application perspectives in high precision lithographic machines and so on [3, 4 and 5]. The different shapes except rectangular of magnets are applied to the 2D Halbach array in order to obtain higher magnetic flux density, such as trapezoid, triangle, cone, hexagon and so on [6,7 and 8]. The disadvantage is that the expressions of magnetic flux density deriving by analytical method are complicated. It is difficult to obtain the real time control mode. In this paper, take the 2D permanent magnet array using hexagon magnets for example [9]. The quasi-analytical expression of magnetic flux density is derived by using the superposition principle. So the real time control mode can be easily obtained by the quasi-analytical expression. The method can be applied to analysis of the permanent magnet array with the non-rectangular magnets.

2. Analysis of magnet flux density

Figure 1 shows the permanent magnet array with a top view and a cut-view. In figure 1, the arrow denotes the magnetization direction of the magnets from S-pole to N-pole. S means the direction towards the paper and N means out of paper. The hexagon magnets are in horizontal directions. \( \tau \) is the pole pitch, \( \tau_m \) is the length of the side of the magnets which are magnetized in z-direction, \( \tau_d \) is the length of the side of the hexagon magnets. The permanent magnets can be modelled as an infinitely large magnet array with Fourier series by neglecting the end-effects [10]. The residual magnetization vector can be expressed as
\[ \vec{M} = M_x \hat{x} + M_y \hat{y} + M_z \hat{z} \]  

(1)

The component of \( M_x \) can be seen as the distribution of hexagon magnets because the magnetization direction. In this paper, the hexagon magnet can be seen as a superposition of an infinite number of small rectangular segments. The derivation of \( M_x \) of the hexagon magnets is based on the superposition principle. Figure 2 shows the \( M_x \) projection distribution of one rectangle segment in the hexagon magnets, including the x-direction and y-direction.

**Figure 1.** The permanent magnet array (a) Top view. (b) Cut-view.

**Figure 2.** \( M_x \) projection distribution of one segment. (a) Magnet array. (b) x-direction. (c) y-direction.

The projection distribution is derived by using Fourier series. Then the expression of \( M_{ij} \) is

\[
M_{ij} = MX_{ij} \cdot MX_{ij} \cdot M = -M \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} a_j(k) b_j(l) \cos(k \omega x) \sin(l \omega y)
\]

(2)

where \( M = \frac{B_r}{\mu_0}, \omega = \frac{\pi}{\tau}, B_r \) is the remanent magnetization of the permanent magnet, \( \mu_0 \) is the vacuum permeability, \( j \) represents the sequence number of the segments, \( k \) and \( l \) are the harmonic numbers for the x- and y-direction, respectively, \( a_j(k) \) and \( b_j(l) \) are projection distribution coefficients,

\[
a_j(k) = \frac{8}{k \pi} \sin \frac{k \pi}{2} \sin \frac{k \omega (r_m + (2j-1)d)}{2} \sin \frac{k \omega d}{2}
\]

(3)

\[
b_j(l) = \frac{4}{l \pi} \sin \frac{l \pi}{2} \sin \frac{l \omega (r_m + (2j-1)d)}{2}
\]

(4)

If the hexagon magnet is divided into \( n \) segments, the residual magnetization \( M_x \) can be expressed

\[
M_x = \sum_{j=1}^{n} M_{ij} = -\sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \left( \sum_{j=1}^{n} a_j(k) b_j(l) \right) \cos(k \omega x) \sin(l \omega y)
\]

(5)

The other component of \( \vec{M} \) can be obtained by using the same approach. The function of \( M_y \) can be established by interchanging the variables \( x \) and \( y \) in the function of \( M_x \). The expression of the residual magnetization vector of this array is
where,

\[ c(k) = \frac{4}{k\pi} \sin \frac{k\pi}{2} \sin \frac{k\omega r_m}{2} \]  

(7)

\[ R_z = \sum_{j=1}^{n} a_j(k) b_j(l) \]  

(8)

\[ R_z = \sum_{j=1}^{n} a_j(l) b_j(k) \]  

(9)

\[ R_z = c(k)c(l) \]  

(10)

In order to derive the magnetic flux density distribution of the magnet array, the 3D space is divided into three regions. The cross section is shown in figure 3. Regions 1 and 3 are in air. The permanent magnets in region 2 are located in between \( m_t \) and \( m_b \). There is no air in the permanent magnet array because of the compact structure. So the assumption of air in the magnet array is not existed. The scalar magnetic potential equation is used to this problem because there is no current.

**Fig. 3.** Space divided into three regions.

The magnetic field equations and boundary conditions are listed in Table 1 and Table 2, respectively.

**Table 1.** Magnetic field equations.

| Region1 | Region2 | Region3 |
|---------|---------|---------|
| \( H_1 = -\nabla \Psi_1 \) | \( H_2 = -\nabla \Psi_2 \) | \( \vec{H}_1 = -\nabla \Psi_1 \) |
| \( \nabla \times \vec{H}_1 = 0 \) | \( \nabla \times \vec{H}_2 = 0 \) | \( \nabla \times \vec{H}_3 = 0 \) |
| \( \vec{B}_1 = \mu_0 \vec{H}_1 \) | \( \vec{B}_2 = \mu_0 \mu_t \vec{H}_2 + \mu_0 \vec{M} \) | \( \vec{B}_3 = \mu_0 \vec{H}_3 \) |
| \( \nabla \cdot \vec{B}_1 = 0 \) | \( \nabla \cdot \vec{B}_2 = 0 \) | \( \nabla \cdot \vec{B}_3 = 0 \) |
| \( \nabla^2 \Psi_1 = 0 \) | \( \nabla^2 \Psi_2 = \nabla \cdot \vec{M} / \mu_t \) | \( \nabla^2 \Psi_3 = 0 \) |

**Table 2.** Boundary conditions.

| \( z = \infty \) | \( z = m_t \) | \( z = m_b \) | \( z = -\infty \) |
|-----------------|-----------------|-----------------|-----------------|
| \( \Psi_1 = 0 \) | \( H_{1z} = H_{2z} \) | \( H_{3z} = H_{3z} \) | \( \Psi_3 = 0 \) |
| \( H_{1y} = H_{2y} \) | \( H_{2y} = H_{3y} \) | \( H_{2y} = H_{3y} \) | \( \Psi_3 = 0 \) |
| \( B_{1z} = B_{2z} \) | \( B_{2z} = B_{3z} \) | \( B_{2z} = B_{3z} \) | \( \Psi_3 = 0 \) |
In the table above, $\vec{B}$ is the magnetic flux density distribution, $\vec{H}$ is the magnetic field strength, $\Psi_1$ is the scalar potential, $\mu_r$ is the relative permeability of the magnets, $m_m, m_b$ is the height of the magnet.

The method of separation of variables is used to solve the Laplace equations. A solution of the scalar potential is substituted.

$$\Psi = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} Z(z) \sin(k\omega x) \sin(l\omega y)$$  \hspace{1cm} (11)

In regions 1 and 3, by solving the function the expression is

$$Z(z) = K_1 \exp(-\lambda z) + K_3 \exp(\lambda z)$$  \hspace{1cm} (12)

$$\lambda = \omega (k^2 + l^2)^{1/2}$$  \hspace{1cm} (13)

where $K_1$ and $K_3$ are constants. Because of the boundary conditions (zero scalar potential for $z = \pm\infty$)

$$\Psi_1 = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} K_1 \exp(-\lambda z) \sin(k\omega x) \sin(l\omega y)$$  \hspace{1cm} (14)

$$\Psi_3 = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} K_3 \exp(\lambda z) \sin(k\omega x) \sin(l\omega y)$$  \hspace{1cm} (15)

In region 2, a nonhomogeneous differential equation is obtained. The solution is

$$Z_2(z) = K_2 \exp(-\lambda z) + T_z \exp(\lambda z) - C$$  \hspace{1cm} (16)

$$C = \frac{\omega B_r}{\lambda^2 \mu \mu_0} \left(kR_s + lR_s\right)$$  \hspace{1cm} (17)

where $K_2$ and $T_z$ are constants.

The above constants can be calculated with the boundary conditions. The expression for the magnetic flux density distribution in region 3 is

$$\vec{B}_3 = -\mu_0 \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} K_3 \exp(\lambda z) \begin{bmatrix} k \omega \cos(k\omega x) \sin(l\omega y) \\ l \omega \sin(k\omega x) \cos(l\omega y) \\ \lambda \sin(k\omega x) \sin(l\omega y) \end{bmatrix}$$  \hspace{1cm} (18)

where

$$K_3 = -B_r \frac{\exp(-\lambda m_b)(\exp(-\lambda m_1) - \exp(-\lambda m_b))(d_1 kR_s + d_2 lR_s + d_3 (k^2 + l^2)^{1/2} R_s)}{\mu_0 \omega (k^2 + l^2)(\mu_1 + 1)^2 \exp(-2\lambda m_b) - (\mu_r - 1)^2 \exp(-2\lambda m_1)}$$  \hspace{1cm} (19)

$$d_1 = \mu_1 (\exp(-\lambda m_1) - \exp(-\lambda m_b)) + (\exp(-\lambda m_1) + \exp(-\lambda m_b))$$  \hspace{1cm} (20)

$$d_2 = (\exp(-\lambda m_1) - \exp(-\lambda m_b)) + \mu_1 (\exp(-\lambda m_1) + \exp(-\lambda m_b))$$  \hspace{1cm} (21)

The magnetic flux density at 4 mm below the magnet array is shown in figure 4. $t_m=17$mm, $B_r=1.24$T, $n=4$, $t=25$mm, the thickness of magnet array is 7mm. The magnetic flux density calculated by 3-D finite element model is shown in figure 4(a). The error between the quasi-analytical model and finite element model is shown in figure 4(b). The maximum difference value between the quasi-analytical model and finite element model is less than 0.015 T, which is 2.64% of the peak value of the finite element model.
Figure 4. Magnetic flux density, (a) Calculated by the finite element model, (b) Error between the quasi-analytical model and the finite element model.

3. Conclusions

The 2D Halbach array with compact structure by using the hexagon permanent magnets is introduced. The quasi-analytical expression of magnetic flux density is derived by the superposition principle. The accuracy is verified by comparing with finite element model. The real time control mode can be easily obtained by the quasi-analytical expression. The method can be applied in the design of permanent magnet array with the non-rectangular magnets of planar motors.

4. References

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