The role of energy-momentum conservation in emission of Cherenkov gluons

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Abstract

The famous formula for the emission angle of Cherenkov radiation should be modified when applied to hadronic reactions because of recoil effects. They impose the upper limit on the energy of the gluon emitted at a given angle. Also, it leads to essential corrections to the nuclear refractive index value as determined from the angular position of Cherenkov rings.

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1 Introduction.

The famous formula for the emission angle of Cherenkov radiation [1]

\[ \cos \theta = \frac{1}{\beta n} \]  

has been successfully used for Cherenkov photons. However, it was pointed out already long ago [2] that energy-momentum conservation

\[ E = E_r + \omega, \quad p = p_r + k \]  

asks for a somewhat modified relation between the cone angle \( \theta \) and the refractive index of a medium \( n \). Here, \( E, p \) and \( E_r, p_r \) are energies and momenta of the primary and recoiled emitters (electrons) while \( \omega, k \) correspond to the emitted quantum (photon).

Such a relation was first derived in [2]. The only new element compared to usual energy-momentum conservation laws in vacuum is the relation between

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the momentum of the emitted quantum \( k \) and its energy \( \omega \) in a medium with the refractive index \( n \):

\[
k = \omega n. \tag{3}
\]

Let us stress here that this relation is a consequence of the collective response of the medium to the quanta passing through it. This is described macroscopically by the refractive index \( n \). Its quantitative microscopic interpretation is still missing. The whole process is considered in the rest system of the infinite medium.

The solution of energy-momentum conservation conditions for relativistic \((\beta \approx 1)\) electrons with this requirement leads to the following formula \([2]\):

\[
\cos \theta = \frac{1}{n} \left( 1 + \frac{\omega}{2E} (n^2 - 1) \right). \tag{4}
\]

The second term in the brackets is positive for \( n > 1 \) and diminishes the cone angle \( \theta \). It is small for low energies of photons \( \omega \ll E \) and/or for the refractive index \( n \) close to 1. It would become important for energetic photons and large refractive indices.

In realistic situations with usual Cherenkov effect, the correction term appears to be inessential for photons because their energies are much less than the energy of electrons-emitters and the refractive indices of most ordinary media are rather close to 1. Therefore Eq. (1) can be used for Cherenkov photons with good accuracy. At the same time this correction happened to be important when considering the anomalous Doppler effect \([3]\).

2 Cherenkov gluons.

The situation changes if one considers the emission of Cherenkov gluons (first proposed in \([4, 5]\)) whose energies can be comparable by an order of magnitude to the energy of a parton-emitter producing a jet. Moreover, the partons-emitters are color charged and can be refracted in the nuclear medium. We’ll discuss this in Section 3, and first consider just the refraction of emitted gluons.

In contrast to Cherenkov photons propagating in ordinary media, the nuclear refractive index \( n \) is unknown for Cherenkov gluons. Theoretical attempts to estimate it are related either to its connection with the real parts
of hadronic forward scattering amplitudes using further assumption on applicability of their common features to gluons as well [4, 5] or to calculations of the polarization operator in simplified models [6] (more discussion can be found in the review papers [7, 8]). They depend strongly on the unknown density of scattering centers in hadronic media. Therefore, it becomes the main goal to find \( n \) by measuring the cone angle in high energy nucleus-nucleus collisions. From Eq. (4) applied now to gluons the nuclear refractive index \( n \) can be expressed as a function of the angle \( \theta \):

\[
\frac{E}{\omega} \left( \cos \theta - \left( 1 - \frac{\omega}{E} \right)^2 - \sin^2 \theta \right)^{1/2}.
\] (5)

Considering real values of \( n \) one gets from (5) the upper limit on the allowed range of gluon energies

\[
\omega \leq E(1 - \sin \theta).
\] (6)

This region becomes narrower at large cone angles. For gluons with larger energies \( \omega \) the refractive index acquires the imaginary part. Therefore the damping increases.

The relation (5) is very simple for rather soft gluons satisfying the condition \( \omega \ll 0.5E \cos^2 \theta \):

\[
n \approx \frac{1}{\cos \theta} \left( 1 + \frac{\omega}{2E} \tan^2 \theta \right).
\] (7)

One concludes that the traditional estimates of \( n \) from the measured angle \( \theta \) using only the first term of (7) are lower than its actual value. The second term can be rather important at large cone angles \( \theta \) even at small gluon energies \( \omega \ll E \).

Let us relate these formulas to experimental findings. The cone structure of hadron emission around the away-side jets has been observed in several experiments on nucleus-nucleus collisions at RHIC [9, 10, 11, 12, 13, 14]. The cone angle values are quite large so that the second term in Eq. (7) is important. They vary between 60° and 70°. This undefinedness is related to different methods of rings presentation and to their finite widths. Surely, this range will become shorter with newly coming data. We present our estimates for two values of \( \cos \theta \) equal to 0.5 (60°) and 0.342 (70°) to demonstrate how important are the corrections obtained above.
At $\theta = 60^\circ$ one gets $\omega < 0.134E$, $n = 2(1 + 1.5\omega/E) < 2.4$. If the total energy of the away-side jet is $E = 5$ GeV, it implies $\omega < 0.67$ GeV, i.e. only states in the low-mass wing of $\rho$-resonance can be created.

At $\theta = 70^\circ$ one gets $\omega < 0.06E$, $n = 2.9(1 + 3.8\omega/E) < 3.6$. For $E = 5$ GeV we get $\omega < 0.3$ GeV, i.e. in this case only very low energy tail of $\rho$-resonance can play a role so that one would expect low intensity for such a process. The value $n = 3$ used in [7, 8] lies in between the above limits.

### 3 Discussion.

In electrodynamics, the charged current (electrons) differs strongly from neutral emitted radiation (photons). Therefore, the medium impact on these components in Cherenkov effect is different. The refractive index is introduced for photons while the electron motion is considered as almost undisturbed. It is supported by smallness of energy losses for Cherenkov radiation. That is why the relation (3) was used for photons only. In general, the refractive index depends on energy but it is constant with high precision for visible light.

In chromodynamics, the partons-emitters (quarks or gluons) are colored as well as the emitted gluons. They differ only by their energies. In principle, all three of them can be refracted by the nuclear medium. If there is no dispersion (energy dependence) of the nuclear refractive index, the nuclear Cherenkov effect is impossible. In above treatment, it was implicitly assumed that the nuclear refractive index is constant and exceeds 1 for comparatively soft emitted gluons while for emitters with high energies it is inessential, i.e. close to 1. Then Eq. (4) is valid. This assumption can be justified only relying on possible analogy of partonic properties to hadronic reactions used earlier (see reviews in [7, 8]).

It is well known for all studied hadron collisions that the real part of the forward scattering amplitude depends on energy. It is positive in the low-mass wings of the Breit-Wigner resonance regions, then it is negative at higher energies and becomes again positive at extremely high energies but the latest region is not considered right now. Since excess of $n$ over 1 is proportional to this real part, one can argue [7, 8] that Cherenkov gluons can excite the collective modes of the nuclear medium within the resonance region. Thus the energy behaviour of the nuclear refractive index $n$ plays
an important role for such emission\textsuperscript{2}. Since energies of partons-emitters are higher than those of Cherenkov gluon, one can neglect their refraction because $n$ is close to 1 (and below it) in this region above resonances. Then the formula (4) is justified for Cherenkov gluons if applied to results of trigger experiments at RHIC.

What concerns the non-trigger experiments with extremely high energy partons at LHC (in the third region described above), the refractive indices of all three participants can be taken into account. It leads to the simple generalization of above formulas so that Eq. (5) is replaced by

$$n = \frac{E}{\omega} n_0 \left( \cos \theta - \frac{n_r^2}{n_0^2} (1 - \omega/E)^2 - \sin^2 \theta \right)^{1/2},$$

where $n_0$ and $n_r$ are the refractive indices of the initial and recoiled partons, correspondingly.

In particular, if the energies of the initial and recoiled partons are approximately the same then their refractive indices are almost equal ($n_0 \approx n_r$), and one should use Eq. (7) with $n$ replaced by the ratio $n/n_0$. At high energies the refractive indices decrease with energy increasing (see [7, 8]). Thus $n_0$ is much closer to 1 than the refractive index $n$ of the gluon emitted with lower (but still very high) energy. The above formulas work quite well in this region also.

To conclude, we have shown that energy-momentum conservation laws play a crucial role in determining main characteristics of emission of Cherenkov gluons. They should be carefully taken into account in analysis and interpretation of experimental data about the ring-like structure recently observed in high energy nucleus-nucleus collisions. Such analysis must reveal the values of the nuclear refractive index and their energy dependence. Herefrom one would conclude about the state of the matter formed in high energy nucleus-nucleus collisions.

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\textsuperscript{2}Its energy dependence can be easily taken into account as it was done, e.g., in [15] but above we used constant $n$ treating it as an effective average value in the resonance region.
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