On the parameter sensitivity in structural parameter identification using Eigensystem Realization Algorithm for a MW-size wind turbine

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Abstract. Accurate estimation of structural parameters such as the natural frequencies, modal shapes and damping ratios is important for both numerical modelling and structural health monitoring, and ambient vibration analysis is usually used for the estimation. Although the Eigensystem Realization Algorithm (ERA) is widely used, the results are known to depend on the selection of the calculation parameters. In this study, first the sensitivity of the calculation parameters of the NExT-ERA method for the full-scale measurement data of a 2.4MW wind turbine is studied to provide reference for practical application of the method. Small initial model order resulted in unstable estimation of the damping ratios, while higher values gave robust results. The optimum value for the initial model order was 400 to 600, which was much larger than the recommendation in the literature as four times the model order. The optimal values for the correlation length parameter are 3500 to 4000 for the 1st modes, and 100 to 350 for the 2nd modes, which corresponds to the length of the free decay process seen in the cross-correlation function. Higher number of points used for FFT gave lower damping ratios for the 1st mode in both fore-aft and side-side directions. The effect of the number of the FFT points is smaller in 2nd mode, which may be due to difference of the length of data required to capture the characteristics of the system. The structural parameters estimated with NExT-ERA method agreed well with the results of the previously conducted forced vibration test except for the fore-aft 1st mode damping ratio, of which value was especially small compared to other modes.

1. Introduction
Information of structural parameters such as the natural frequencies, modal shapes and damping ratios is important not only for the numerical modelling but also for the health monitoring of a structure. Although excitation tests with known input force give more accurate results, ambient vibration analysis is often used for the identification of the structural parameters for large structures due to the limitations of the source of the generation of controlled excitation force. The Eigensystem Realization Algorithm (ERA) is one of the widely used methods to extract structural parameters from the ambient vibration signals. The results of these methods, however, are considerably affected by the selection of the calculation parameters such as the model order and the size of the Hankel matrix. Theories have been developed for the selection of the model order. However, for other parameters such as the size of the Hankel matrix, theoretical criteria are not fully developed. Although several practical guidelines have been proposed for these parameters, they are not generally applicable for all measured data that

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contains various amount of measurement noise as the optimal values of the calculation parameters are affected by the ratio of the measurement noise to the excited vibration. This makes the information of the sensitivity of the calculation parameters for the measurement data from similar structures highly useful in the practical application.

Previous efforts have been made in the identification of the structural parameters of wind turbines using ambient vibration analysis. James et al. (1992) applied Natural Excitation Technique (NExT) to a vertical axis wind turbine and Powell (2011) adopted ERA to the structural identification of a 900kW and a 1.5MW horizontal axis wind turbines. Stochastic Subspace Identification (SSI) has been applied to the operating offshore wind turbine with Jacket and tri-pod foundation (Haeckell and Rolfs (2013)) and suction bucket foundation (Dong et al. (2018)), as well as the onshore wind turbine (Oliveira et al. (2018)). However, little focus was made in the literature on the sensitivity of the calculation parameters for wind turbines.

In this study, Eigensystem Realization Algorithm method is applied to the ambient vibration of a 2.4MW wind turbine of which structural parameters are known from previously conducted forced excitation tests. Sensitivity of the initial model order and the correlation length parameter to the estimated modal damping ratios were studied for each level of wind speed for the 1st and 2nd mode in fore-aft and side-side directions. Finally by comparing the estimated results using optimized calculation parameter to forced excitation tests, accuracy of the method in application to MW-size wind turbine is discussed.

2. NExT-ERA

2.1. Outline of NExT-ERA

A brief theoretical background of ERA is given in this section. The equation of motion is shown in Eq. (1) where \( M \) is the mass matrix, \( C \) is the damping matrix, \( K \) is the stiffness matrix, \( u(t) \) is the displacement vector, and \( f(t) \) is the external force vector. Using Eq. (2) the equation of motion is rewritten into the state-space equation shown in Eq. (3).

\[
M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = Bf(t) \\
x(t) = \begin{bmatrix} u(t) \\ \dot{u}(t) \end{bmatrix} \\
\dot{x}(t) = A_c x(t) + B_c f(t), \quad A_c = \begin{bmatrix} 0 & I_- M^{-1}K \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ -M^{-1}B \end{bmatrix}
\]

The observed output of the system \( y(t) \) that can be expressed with linear combination of \( x(t) \) is shown in Eq. (4). In this study \( y(t) \) is the acceleration data, in which case \( C_c = [-M^{-1}K \quad -M^{-1}C] \). When the system is in free-vibration condition, \( f(t) \) becomes zero in Eq. (3), and using Eq. (4), \( y(t) \) at time step \( k \) can be obtained asymptotically as shown in Eq.(5).

\[
y(t) = C_c x(t) \\
y(k) = C_c A_c^k x(1)
\]

Using Eq. (5), the Hankel matrix is formed as shown in Eq. (6). Here, the number of rows of the Hankel matrix, \( \alpha \), is called the initial model order, and the number of columns of the Hankel matrix, \( \beta \), is called the correlation length parameter.

\[
H(k-1) = \begin{bmatrix} y(k) & \cdots & y(k+\beta) \\ \vdots & \ddots & \vdots \\ y(k+\alpha) & \cdots & y(k+\alpha+\beta) \end{bmatrix}
\]

Singular value decomposition is then applied to \( H(0) \) as shown in Eq. (7). Here, under ideal conditions for a system with the number of modes of \( g \), \( S \) obtained with Eq. (7) can be expressed as Eq. (8), where \( S_g \) is a \( g \) by \( g \) matrix. The number of modes is given with the model order \( g \). Finally \( A_c \) is calculated using Eq. (9).

\[
H(0) = USV^T \\
S = \begin{bmatrix} S_g & 0 \\ 0 & 0 \end{bmatrix}
\]

2
\[ A_e = S^{-1/2}U^T H(1)V S^{-1/2} \]  

In ERA, the impulse response of the system needs to be estimated to form the Hankel matrix in Eq.(6). In this study, this estimation is conducted with Natural Excitation Technique (NExT)\(^4\). The method utilize that the impulse response are expressed as Eq.(10), and that the cross-correlation function of the ambient vibration responses between two locations are expressed as Eq. (11). Here \( x_{rk} \) is the response at location \( r \) caused by the impulse input force at location \( k \), \( \Phi_i^r \) is the \( i \)-th modal shape at location \( r \), \( \lambda_i \) is the \( i \)-th eigenvalue. \( R_{rs} \) is the cross-correlation function of the system response between location \( r \) and \( s \) under random input force, and \( \alpha_{ki} \) is the scale factor of the cross-correlation function. Comparison of the two equations shows that the cross-correlation function have the form of decaying sinusoids, and therefore the same characteristics with the impulse response function.

\[
x_{rk}(t) = \sum_{i=1}^{2N} \Phi_i^r \Phi_i^k e^{\lambda_it} \\
R_{rs}(T) = \sum_{k=1}^{N} \sum_{i=1}^{2N} \sum_{j=1}^{2N} \Phi_i^r \Phi_i^s \alpha_{ki} \int_{-\infty}^{\infty} e^{(\lambda_i+\lambda_j)t} d\tau \tag{10} \\
\tag{11}
\]

\[ \]  

2.2. Parameters in NExT-ERA

In the ERA method, the model order, the initial model order and the correlation length parameter are the calculation parameters known to affect the estimated results. The model order is usually determined by the stabilization diagram and is usually recommended to use two times the number of expected modes\(^3\). Considering that the number of modes is small for wind turbine in parked condition and that the estimated frequency was stable for a range of model orders as is shown below, sensitivity of the model order is not discussed in this study. For the initial model order, which is the number of rows of the Hankel matrix, the literature recommends to use four times the number of expected modes\(^3\). For the correlation length parameter, which is the number of columns of the Hankel matrix, it is recommended to include as much information from the cross-correlation function as possible. For NExT method, the calculation of the cross-correlation function is often performed using cross-spectral density, where the number of points used in the FFT (Fast Fourier Transformation) is also a parameter that affects the results.

3. Outline of measured ambient vibration data

3.1. The target wind turbine

The measured wind turbine is a pitch-regulated MHI 2.4 MW wind turbine located at 3.1 km offshore Choshi, Japan. The wind turbine and the steel tower are supported with a gravity foundation up to 10.83 m. The rotor diameter is 96 m and the total masses are 5.9E+4 kg for the rotor and 8.1E+6 kg for the tower. Four sets of accelerometers are installed at 74.7 m (Ch1), 58.9 m (Ch2), 36.5 m (Ch3), and 20.5 m (Ch4) height of the tower. Ambient vibration analysis was performed using the acceleration measured at these locations. Outline of the wind turbine and the location of the accelerometers are shown in Figure 1. Measurement campaign has been carried out since 2012 for tower motions and loads. The sampling frequency is 50 Hz and each data length is 10 minutes.

An active mass damper (AMD) is installed at 55 m height of the tower, and was used as the excitation force in previously conducted forced excitation tests\(^6\) to obtain the modal frequencies and damping ratios of the parked wind turbine. Two types of excitation tests were carried out. One is the sinusoidal vibration test, where the tower was excited for a range of frequencies and the structural parameters were obtained by fitting the measured acceleration to the theoretical equation. Another is the free decay test, where the active mass damper is ordered a sudden stop after the steady state of feathering condition. Natural frequencies and damping ratios of 1\(^{st}\) and 2\(^{nd}\) modes obtained in the excitation test are shown in Table 1.
3.2. Ambient vibration data selection and analysis

In order to eliminate other sources of uncertainties to the estimation of the NExT-ERA method than the calculation parameters, parked wind turbines are targeted in this study. Measured acceleration data was selected for those with the blade pitch angles at feathering condition, and with the nacelle yaw angle quiescent for the recorded 10 minutes. A total number of 157 data sets were selected from January 9th, 10th, and 21st, 2017. All data was converted into fore-aft and side-side directions using the nacelle yaw angle. In order to study the effect of the wind speed on the parameter sensitivity, selected data sets were divided by into five bins; 7 m/s bin including data from 6 to 8 m/s, 9 m/s bin including data from 8 to 10 m/s, 11 m/s bin including data from 10 to 12 m/s, 13 m/s bin including data from 12 to 14 m/s, and 15 m/s bin including data from 15 to 17 m/s. 15 data sets were randomly selected from each bin to study the parameter sensitivities, and finally all data sets were used for the comparison with the forced excitation tests.

For the NExT method, Ch 1 is chosen for the reference channel. Examples of the cross-spectral density of each channel with the reference channel are shown in Figure 2 along with the stabilization
The figure shows that mainly 1<sup>st</sup> mode and 2<sup>nd</sup> mode are excited for the present wind turbine in the parked condition. In higher wind speed, blade flap-wise modes are also excited. Estimated natural frequencies were stable for the model orders of 3 to 14 for the 1<sup>st</sup> mode and of 3 to 6 for the 2<sup>nd</sup> mode. In this study, the model order of 6 is used for all cases. For the estimation of the cross-spectral density, Welch’s method is used with Hann’s window. The number of points for the FFT is the same with the window length. Overlapping is not used.

Considering that the differences in both natural frequency and the modal amplitude of 1<sup>st</sup> mode and 2<sup>nd</sup> mode are large as shown in Figure 2, band-pass filters are applied to all data for more accuracy in the extraction of the free-decay characteristics. The width of the filter was 0.2 to 2.5 Hz for the 1<sup>st</sup> mode estimation and 2.5 to 4 Hz for the 2<sup>nd</sup> mode estimation. The modal assurance criterion (MAC) values<sup>3</sup> are used as initial criteria, and estimations with MAC values larger than 0.95 is selected.

The examples of the cross-correlation functions of the accelerations filtered for the 1<sup>st</sup> mode are shown in Figure 3. It is seen that the characteristics of the 1<sup>st</sup> mode free decay is well captured with the NExT method. For low amplitude modes such as the 7 m/s bin in fore-aft direction and all cases in side-side direction, more effects of noise can be seen. The same trend can be observed also for the cross-correlation function of accelerations filtered for the 2<sup>nd</sup> mode shown in Figure 4.

![Cross-correlation function](image1)

**Figure 3.** Examples of the cross-correlation function of 1<sup>st</sup> mode accelerations at each height with the reference channel for (a) fore-aft 7 m/s bin, (b) fore-aft 11 m/s bin, (c) fore-aft 15 m/s bin, (d) side-side 7 m/s bin, (e) side-side 11 m/s bin, and (f) side-side 15 m/s bin

![Cross-correlation function](image2)

**Figure 4.** Examples of the cross-correlation function of 2<sup>nd</sup> mode accelerations at each height with the reference channel for (a) fore-aft 7 m/s bin, (b) fore-aft 11 m/s bin, (c) fore-aft 15 m/s bin, (d) side-side 7 m/s bin, (e) side-side 11 m/s bin, and (f) side-side 15 m/s bin
4. Results

4.1. Sensitivity of initial model order

ERA is performed with a range of values of the initial model order to study its sensitivity to the estimated damping ratios. The results of the 1st mode damping ratios is shown in Figure 5 for three wind speed bins; 7 m/s bin, 11 m/s bin, and 15 m/s bin. Here, the number of points for FFT is 8192 and the correlation length parameter was set at 800. It is seen from the figure that when the initial model order is small the estimation of the damping ratios is unstable for both fore-aft and side-side directions. For the fore-aft direction, the estimations become stable with the increase of the initial model order. The result of the side-sider direction in the high wind speed bin shows the same trend with fore-aft direction, while higher initial model orders resulted in larger deviation between each data set for the low and middle wind speed bins. This may due to the large signal to noise ratio in these cases. Generally, the initial model order of 400 to 600 resulted in stable and converged results for the 1st mode. This is about several hundred times of the expected number of modes in the time-series, which is much larger than the recommendation in the literature as four times the model order.

![Figure 5](attachment:image.png)

Figure 5. Sensitivity of initial model order to the 1st mode damping ratio for (a) fore-aft 7 m/s bin, (b) fore-aft 11 m/s bin, (c) fore-aft 15 m/s bin, (d) side-side 7 m/s bin, (e) side-side 11 m/s bin, and (f) side-side 15 m/s bin

![Figure 6](attachment:image.png)

Figure 6. Sensitivity of initial model order to the 2nd mode damping ratio for (a) fore-aft 7 m/s bin, (b) fore-aft 11 m/s bin, (c) fore-aft 15 m/s bin, (d) side-side 7 m/s bin, (e) side-side 11 m/s bin, and (f) side-side 15 m/s bin
Sensitivity of the initial model order to the estimated 2\textsuperscript{nd} mode damping ratios is shown in Figure 6 for 7 m/s, 11 m/s, and 15 m/s wind speed bins. Here, 8192 points were used in the FFT, and the correlation length parameter was set as 200 for all cases. It is seen from the figure that for both fore-aft and side-side direction, the deviation of the damping ratio between each data set is larger in lower wind speeds and smaller in higher wind speeds. Small initial model order resulted in unstable estimations for all cases, while they converged well with the increase in the initial model order. The initial order of 350 to 400 showed robust results for all cases.

4.2. Sensitivity of correlation length parameter

Sensitivity of the correlation length parameter to the estimated 1\textsuperscript{st} mode damping ratios is shown in Figure 7 for 7 m/s, 11 m/s, and 15 m/s wind speed bins. The number of FFT of 8192 and the initial model order of 400 are used. It is seen from the figure that for both fore-aft and side-side directions, the estimated damping ratios become stable with the increase in the correlation length. The correlation length parameter of 3500 to 4000 gives robust estimation to all cases. These values correspond to 70 sec to 80 seconds in the cross-correlation function, which is the length that covers the total free-decay process as shown in Figure 3. These results are consistent with the recommendations in literature\(^1\) to include as much information from the cross-correlation function as possible. Meanwhile, it is also seen from Figure 7 that the standard deviations of the obtained damping ratios between each data set remains similar for all correlation length parameters. This indicates that the deviation of the damping ratios between each data set is not affected by the choice of the parameter.

Results of the study of the sensitivity of the correlation length parameter to the estimated 2\textsuperscript{nd} mode damping ratios are shown in Figure 8 for 7 m/s, 11 m/s, and 15 m/s wind speed bins. It is seen from the figure that for the 7 m/s wind speed bin, the estimated damping ratios are unstable and variations among each data set remains large regardless of the choice of the parameters. For the 11 m/s and 15 m/s wind speed bin, the results converge with the increase in the correlation length parameters, and the values of 200 to 250 gave converged results. This corresponds to 4 to 5 sec in the cross-correlation function, which contains most information of the whole free-decay process as shown in Figure 4. As well as the 1\textsuperscript{st} mode, the standard deviations were found to remain similar regardless of the selection of the correlation length parameters for both fore-aft and side-side directions.

![Figure 7](image)

Figure 7. Sensitivity of correlation length parameter to the 1\textsuperscript{st} mode damping ratio for
(a) fore-aft 7 m/s bin, (b) fore-aft 11 m/s bin, (c) fore-aft 15 m/s bin,
(d) side-side 7 m/s bin, (e) side-side 11 m/s bin, and (f) side-side 15 m/s bin.
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4.3. Comparison with the forced excitation test

The results of the NExT-ERA method with the optimum calculation parameters found in the previous sections are compared to those obtained from the forced excitation test shown in Table 1. As the number of points used in the FFT for NExT also affects the estimation, the ambient vibration analysis is performed with two cases with numbers of FFT points of 4098 and 8192. Comparison of the results of the NExT-ERA and the forced excitation test for the 1st mode frequencies and damping ratios are shown in Figure 9. Figure 9 (a) and (c) show that the difference of the estimation and the forced vibration test is 1 to 2% for 1st mode natural frequencies in both fore-aft and side-side directions. Dependency of the natural frequencies on the wind speed is limited. Larger number of points in FFT resulted in smaller natural frequencies in the fore-aft direction. Figure 9 (b) and (d) show that for the 1st mode damping ratios, the results from the ambient vibration analysis largely overestimated those from the forced excitation test in fore-aft directions, while the two results showed relatively good agreement in the side-side direction. This can be due to the small value of the damping ratio in fore-aft direction, where the amount of change of the amplitude in the free-decay process is small and therefore estimations become more sensitive to the measurement noises. For both directions, larger number of points in FFT gave lower results for damping ratios.

Comparison of the results of the NExT-ERA and the forced excitation test for the 2nd mode frequencies and damping ratios are shown in Figure 10. Figure 10 (a) and (c) show that the estimated natural frequencies in both fore-aft and side-side directions decrease with the increase of the wind speed and gradually approach to the results from the forced excitation test. In high wind speeds, the estimated frequencies agreed well with the experiment. Figure 10 (b) shows that the estimated 2nd mode damping ratio agreed well with the experiment, although large deviation among each time-series is observed in lower wind speeds. It is seen from Figure 10 (d) that the estimated 2nd mode damping ratio roughly matched the experiment in side-side direction with relatively large deviations. The effect of the choice of the number of points in FFT is smaller for the estimation of 2nd mode compared to the 1st mode, which may be due to difference of the length of data required to capture the characteristics of the system. The NExT-ERA method gave good results for 1st mode and 2nd mode natural frequencies, 1st mode damping ratios in side-side direction, and 2nd mode damping ratios in fore-aft direction. For the purpose of the estimation of the structural parameters, the method is applicable for structural structural modes with certain amount of damping ratios. However, as it is observed that some deviations between each time-series for estimated damping ratios still remains even with the optimal selection of the calculation parameters, additional effort such as the statistical processing of the estimated results is required for the purpose that requires higher accuracy such as the structural health monitoring.
In this study the sensitivities of the calculation parameters in NExT-ERA method are studied for a 2.4MW wind turbine. Following conclusions are drawn.

a) Small initial model order resulted in unstable estimation of damping ratios, while higher order gave robust results. The optimum value for the initial model order was 400 to 600, which was much larger than the recommendation in the literature as four times the model order.

b) The optimal values for the correlation length parameter are 3500 to 4000 for 1st modes, and 100 to 350 for 2nd modes, which corresponds to the length of the free decay process seen in the cross-correlation function. The effect of the selection of the correlation length parameter on the deviations between each data set was found limited.
c) Higher number of points used for FFT gave lower damping ratios of 1st mode for both fore-aft and side-side directions. The effect of the parameter is smaller in 2nd mode, which may be due to difference of the length of data required to capture the characteristics of the system.

d) The structural parameters estimated with NExT-ERA method agreed well with the results of the forced vibration test except for the fore-aft 1st mode damping ratio, of which value was especially small compared to other modes.

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