Magnetothermal Transport of Oriented Graphite at Low Temperatures

Konstantin Ulrich and Pablo Esquinazi

Abteilung Supraleitung und Magnetismus, Universität Leipzig, Linnéstrasse 5, D-04103 Leipzig, Germany

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1. INTRODUCTION

One of the authors (P.E.) of this contribution met Siegfried Hunklinger for the first time in a warm day of August in the year 1983 at the Institut für Angewandte Physik II located near the Philosophenweg in the beautiful city of Heidelberg. Part of the work done in the two and a half years of postdoc stay was guided by him and Georg Weiss. That research work done twenty years ago was mainly based on the interaction between tunneling systems (TS) and conduction electrons in amorphous metals. At that time an anomalous behavior in the acoustic properties of amorphous superconductors below the superconducting critical temperature was found, indicating that the interaction of TS and conduction electrons was not as simply as the Korringa-based models predicted. After all these years, the main observations and this subject in general remain still without clear answers. However, we know that the applied strain energy and the interaction between TS play an important role and should be taken into account. The work initiated and guided by Siegfried Hunklinger was stimulating and ambitious. For P.E. it was a pleasure to work and collaborate with him. We are glad to have here...
K. Ulrich and P. Esquinazi

the possibility to honor his scientific trajectory with a contribution on a relatively new topic in solid state physics.

An unaware reader may be surprised to see a paper reporting on the magnetothermal transport of graphite. In fact, graphite is one of the most studied materials in physics and chemistry and the literature contains a substantial number of measurements and theoretical work on the electrical and thermal transport properties of this semi-metal. In contrast to the common believe, however, several transport properties of graphite are not understood and some theoretical assumptions done in the past seem now less plausible. The number of open questions regarding the transport properties of graphite is significant. From the experimental side this is partially due to the fact that the quality of the samples we can measure today is much better than that obtained in the past. From the theoretical side, we know now that the physics of an electron system in two dimensions cannot be simply derived from the Fermi liquid theory and an unusual and still controversial behavior for the transport properties is predicted and partially observed experimentally. For ideal, two-dimensional graphite the situation becomes even more interesting because of the predictions that at the $K$-points of the Brillouin zone, the electrons should behave as relativistic, massless Dirac-Fermions (DF) with a linear dispersion relation, similar to, e.g., the quasiparticles (QP) at the gap nodes in the high-temperature superconductors (HTS).

The subject of this contribution is the thermal conductivity and its magnetic field dependence of highly oriented pyrolytic graphite (HOPG) samples. This property and its field dependence was indeed study in oriented samples of less quality in the past by Ayache in his thesis and by Woollam, but those experimental curves were apparently never published. The behavior of the thermal conductivity at the quantum limit (applied magnetic field normal to the graphene planes $B > 1$ T) where signs of the quantum Hall effect are observed, is of interest as well as its behavior at low fields ($B \sim 0.1$ T) where a field-driven metal-insulator-like transition (MIT) is measured in the electrical resistivity. We will show below that the thermal conductivity is a useful and important transport property to study the behavior of the QP in graphite. Our main results reveal that $\kappa(B)$ shows an anomalous behavior at the MIT, which resembles that expected from the magnetic catalysis (MC) theory and that the amplitude of the quantum oscillations in $\kappa(B > 1$ T) decreases with decreasing temperature following a $T^n$ law with $n = 3.0 \ldots 3.5$, in clear contrast to the electrical Hall effect behavior.
Magnetothermal Transport of Graphite

2. PREVIOUS EXPERIMENTAL WORK AND THEORETICAL BACKGROUND

In a recent paper\textsuperscript{16} we have reported on the magnetothermal conductivity of a HOPG sample at temperatures $5 \leq T \leq 20$ K for fields $(0 \leq B \leq 9)$ T parallel to the $c$–axis. With the measured longitudinal electrical resistivity we showed that the Wiedemann-Franz law is not able to explain the large oscillations in $\kappa(B)$ observed in the high-field regime ($B > 1$ T) where a quantum-Hall-like behavior is measured. The temperature dependence of the oscillation amplitude $\Delta k$ between the minimum (at $B \simeq 3.7$ T) and maximum (at $B \simeq 5.5$ T) showed a rough maximum at $\sim 7$ K, i.e. near the limit of the measuring range. Because the phonon-electron interaction does not appear to provide an answer for the large oscillations in $\kappa(B)$,\textsuperscript{16} lower temperature measurements are necessary to get more hints on their origin.

The scientific interest on graphite has been recently renewed by transport measurements on HOPG,\textsuperscript{17,12,18,19} which shows a similar MIT as in two-dimensional (2D) electron (hole) systems (which takes place either varying carrier concentration or applying a magnetic field $B$).\textsuperscript{20} The MIT in graphite is remarkable: the in-plane electrical resistivity can increase a factor 10 after the application of a magnetic field of $\sim 0.2$ T perpendicular to the graphene planes and the temperature dependence changes from metallic-like ($d\rho(T)/dT > 0$) to semiconducting-like ($d\rho(T)/dT < 0$) at a “critical” field $B_c \sim 0.04\ldots 0.1$ T.

Theoretical analysis\textsuperscript{14,15} suggests that the MIT in graphite is the condensed-matter realization of the magnetic catalysis (MC) phenomenon\textsuperscript{21} known in relativistic theories of $(2 + 1)$ - dimensional DF. According to this theory,\textsuperscript{14,15} the magnetic field opens an insulating gap in the spectrum of DF associated with an electron-hole (e-h) pairing, below a transition temperature $T_{c\varepsilon}(B)$. In qualitative agreement with theory, experiments show a $T_{c\varepsilon}(B)$ that increases with field from a field of the order of 50 mT.\textsuperscript{22}

The influence of the MC on the electronic contribution to the thermal conductivity for a DF system has been calculated in Ref. 13. The MC was suggested to be the reason for the kink behavior found in the magnetic field dependence of the thermal conductivity of some high-temperature superconducting samples.\textsuperscript{23,24} Experimentally, however, this kink behavior is not systematically observed,\textsuperscript{25–27} probably due to a in-plane anisotropy of the cuprates\textsuperscript{28} added to the fact that QP-vortices interaction plays a main role in the field dependence of $\kappa$ in the superconducting state. It seems that HTS materials are not the best candidates to study the behavior of DF as a function of field. The MC theory predicts that above a “critical” field $B_c(T)$, $\kappa(B)$ shows a plateau-like behavior being larger than the conductiv-
K. Ulrich and P. Esquinazi

ity one would obtain from, e.g., the WF law. The reason for this effective increase of $\kappa(B > B_c)$ in the theory is due to the decrease of the efficiency of the relevant scattering processes for the thermal transport of the excitonic pairs.\cite{13} According to the authors\cite{13} the appearance of a kink in $\kappa(B)$ is model independent, being only determined by the critical behavior of the induced dynamical mass near the phase transition. We may qualitatively expect that the effect of the MC should be seen when the applied field $B \gtrsim B_c$, the field needed to generate the dynamical mass at a temperature $T$, as described in Ref. 13. In a previous experiment\cite{16} we did not find clear evidence for a kink in $\kappa(T \gtrsim 5\text{K}, B < 1\text{T})$ in contrast to what we are reporting here. The results suggest that the kink is sample dependent and that the intrinsic disorder may play a role. We remark that for the relativistic spectrum of 2D-QP in graphite the violation of the WF law is expected simply because the total electrical current differs from the total heat current for interband excitations that create quasiholes.\cite{29}

3. EXPERIMENTAL DETAILS

We have studied two HOPG samples: UC-sample from Union Carbide (rocking curve FWHM $\simeq 0.26^\circ$), AC-sample from Advanced Ceramics (rocking curve FWHM $\simeq 0.4^\circ$). The UC-sample is a different piece from the same batch of the HOPG sample reported in Ref. 16. The samples typical length and width were $\sim 1\ldots \sim 2\text{mm}$ and thickness $\sim 0.1\text{mm}$.

The thermal conductivity measurements at $T \geq 5\text{K}$ of the UC-sample were done in a 4 K cryostat equipped with a 9 T superconducting solenoid. For the measurement of the temperature gradient (between 100 mK and 200 mK in $\sim 1\text{ mm}$ length) we used a previously field- and temperature-calibrated type E thermocouples with a dc-picovoltmeter.\cite{30} The thermocouple ends were positioned both at the same surface of the sample. A detailed calibration of the thermocouple as a function of field and temperature was performed because the thermopower of our thermocouple is specially sensitive on the magnetic field below 10 K with a non-simple dependence.\cite{30,31} The experimental arrangement was recently used to study the longitudinal and Hall thermal conductivities of high-temperature superconducting crystals.\cite{32} Our system enables us to measure $\kappa(B)$ with a relative resolution better than 0.1% above 5 K. The thermal stability was better than 1 mK in the measured magnetic field and temperature range. The absolute error in the thermal conductivity was estimated to be $\leq 30\%$.

The lower temperature measurements were done in a dilution cryostat ($0.2\text{ K} \leq T \leq 4\text{ K}$) equipped with a 8 T superconducting solenoid. For these measurements and due to the necessity of using small pieces of RuO$_2$
4. EXPERIMENTAL RESULTS AND DISCUSSION

4.1. Deviations from the WF law at magnetic fields $B \sim 0.1$ T

Figure 1 shows the temperature dependence of the thermal conductivity of the AC-sample measured in zero field and under 8 T. In the same figure we show the corresponding data for two UC-samples at $B = 0$. The measured temperature dependence as well as the absolute value agree with earlier measurements done in HOPG samples (dashed line). We assume that the
K. Ulrich and P. Esquinazi

thermal transport of graphite is given by two contributions:

\[ \kappa = \kappa_p(T) + \kappa_e(T, B), \]

where \( \kappa_p \) is due to the phonons and \( \kappa_e \) due to charged carriers (conduction electrons and/or holes). The field dependence of the thermal conductivity is given only by the electronic part \( \kappa_e(T, B) \), which can be estimated with the WF relation. Experimental evidence shows that the field dependence of \( \kappa \) decreases with temperature being negligible at 100 K \((\kappa(B) - \kappa(0))/\kappa(0) < 0.1\% \) at \( B = 9 \) T), indicating that the phonon contribution does not change with field. The universal WF-relation relates the electrical resistivity \( \rho(T, B) \) with the thermal conductivity due to electrons by

\[ \frac{\kappa_e \rho}{T} = L_0, \]

through the universal constant \( L_0 = 2.45 \times 10^{-8} \) WΩK\(^{-2} \). The relation (2) holds strictly for elastic or quasielastic electron scattering and therefore the range of validity is usually set, either at low enough temperatures where the resistivity is temperature independent (impurity scattering dominates), or at high enough temperatures where the electron-phonon scattering is large. For the samples measured in this work, the temperature dependence of the electrical resistivity indicates a saturation below 10 K (curves for similar samples can be seen in Refs. 19,22) and therefore at \( T \leq 10 \) K we expect to be roughly in the validity range of the WF-law. We stress that within the Fermi liquid theory the field dependence of \( \kappa_e \) should be given by the WF law. Deviations from the expected field dependence obtained from the measured electrical resistivity should be taken as a hint for a non-Fermi liquid behavior, whatever their origin.

Indeed, the estimates of the electronic part from Eq. (2) using the measured resistivity are in general in agreement with experimental results at zero applied field. For example, from Fig. 1 we estimate that at the crossover between the \( T^3 \)- and \( T \)-dependence of \( \kappa \) at \( T \sim 1.5 \) K and zero field, \( \kappa_e \sim \kappa_p \sim 0.5 \) W/Km. Taking the measured resistivity for this sample \( \rho(2 \text{K},0) \sim 0.14 \) µΩm from (2) we obtain \( \kappa_e \sim 0.25 \) W/Km. Taking into account the 30% uncertainty in the absolute value in both \( \kappa \) and \( \rho \) we obtain an apparent agreement between the WF law and the experimental result. On the other hand, the experimental error does not allow us to rule out the contribution of other QP with a different effective Lorentz number at \( B = 0 \).

With magnetic field, however, the apparent agreement clearly breaks down. As expected from the increase of the electrical resistance with field, the crossover from \( T^3 \) to \( T \) in the temperature dependence of \( \kappa \) is shifted to lower temperatures, as has been observed experimentally in earlier work.\(^{37} \)
Magnetothermal Transport of Graphite

Fig. 2. Normalized thermal conductivity as a function of applied field at different constant temperatures of the UC-sample measured in this work. The continuous lines (1) to (3) are calculated using the WF law, see Eq.(2), and the measured in-plane electrical resistivity at 8.0 K, 6.5 K and 5.5 K, respectively.

From our results, see Fig. 1, this crossover is observed at $T \sim 0.3$ K at $B = 8$ T and therefore we can estimate $\kappa_e(0.3 \text{K}, 8 \text{T}) \sim 10^{-3}$ W/Km. The estimate from Eq. (2) provides however $\kappa_e \sim 10^{-4}$ W/Km using the measured resistivity, i.e. much smaller than the estimated value from the measurements. The $T^3$ dependence found at $B = 8$ T and $T > 0.3$ K appears to be due to phonon transport limited by grain boundaries, as a simple estimate indicates. Using the specific heat from literature $C \sim 0.0325 \times 10^{-3} T^3$ J/mol K (page 183 in Ref. 3) and an average sound velocity $v \sim 2 \times 10^4$ m/s (page 89 in Ref. 4) we obtain a phonon mean free path $l \sim 3 \mu$m, which is of the order of the measured crystallite size for similar samples.3,36 Nevertheless and due to the failure of the WF law we cannot rule out that an enhanced QP contribution still exists at large fields. This contribution may influence only weakly the $T^3$ dependence due to phonons.

Let us discuss now the field dependence and its comparison with the WF law in detail. Figure 2 shows the field dependence of the UC-sample at three different constant temperatures. The observed behavior agrees well with that published recently.16 However, the measurements for this sample show a clear kink at a field of $B \sim 0.1$ T, which is near the field at which the
MIT occurs.\textsuperscript{22} The continuous lines (1) to (3) in Fig. 2 are obtained from

\[
\frac{\kappa(B)}{\kappa(0)} = \frac{\kappa_e(B) - \kappa_e(0)}{\kappa(0)} + 1,
\]

assuming that the phonon conductivity does not depend on magnetic field and calculating $\kappa_e$ from Eq.(2). Certainly, due to the uncertainty in the absolute values of $\kappa$ and $\rho$ the calculated curve can be shifted up or down within experimental error. The WF curves show in Fig. 2 were calculated using $\kappa(T,0)$ values that best match the field dependence of $\kappa$ at $B < 0.1$ T; the deviation obtained with other values of $\kappa(T,0)$ are discussed below. From Fig. 2 we recognize a "kink" in $\kappa(B)$ at a field $B \sim 0.1$ T. This represents a clear deviation from the WF law and it occurs near the MIT, in qualitative agreement with theoretical predictions based on the MC phenomenon.\textsuperscript{13}

We note that $B \sim 0.1$ T is slightly larger than the "critical" field $\sim 0.07$ T obtained from scaling arguments using electrical resistivity data.\textsuperscript{38} Measurements of the electrical resistivity as a function of temperature at fields $0 < B < B_c$ indicate a minimum at a temperature $T_{\text{min}}(B) \propto \sqrt{B - B_c}$ (in qualitative agreement with theory\textsuperscript{14,15}). From these data the "critical" field at $T \lesssim 10$ K for the UC-sample is a factor of two to three smaller\textsuperscript{22} than the field at which $\kappa(B)$ shows the kink. The difference may be related to different response of the electrical resistivity and thermal conductivity to the induced phases and the intrinsic disorder, which may have an influence in the formation and behavior of the "insulating" phase. The sample disorder potential may affect the formation of the bound states. Therefore, we would expect that at lower temperatures, when the thermally activated behavior of the carriers decreases, the field necessary to induce the bound state in the electronic system increases. This is perhaps the origin for the increase of the field at the kink measured in the AC sample below 2 K, see inset in Fig. 4(a). Note, however, that above $\sim 4$ K the field at the kink shifts slightly to larger values the higher the temperature, see Fig. 2, in agreement with the general behavior found experimentally.\textsuperscript{22} Due to the relatively small QP contribution at $T > 10$ K, the experimental resolution is not enough to resolve with certainty the evolution of the kink position at higher temperatures.

The fact that other piece of the same UC sample shows such a clear kink behavior might be related to the sample quality and/or smaller dimensions compared to that one reported in Ref. 16. A less disordered sample may enhance the sensitive of $\kappa$ to the MIT, as the results obtained for the AC-sample indicate (see below). It may also be that the position of the thermocouple used to measure the temperature gradient has some influence; in this work both ends of the thermocouple pair are attached at the same HOPG surface in contrast to the arrangement used in Ref. 16. Experiments
Fig. 3. Normalized thermal conductivity as a function of applied field at different constant temperatures for the AC sample.

with different samples having different dimensions are necessary to clarify this point.

The results for the AC sample are shown in Fig. 3. This sample is intrinsically more disordered than the UC sample as the FWHM of the rocking curve indicates. A comparison with the WF law indicates that the deviation starts or is at largest at the MIT. Figure 4(a) shows the same data at 0.6 K as in Fig. 3. In this figure we show also two curves calculated with Eqs. (2) and (3) using two values of $\kappa(0.6K,0)$ with a difference of $\sim 4\%$, which is within experimental error. In one case we choose an absolute value for $\kappa(0.6K,0)$ in such a way that the WF law matches the lower field data (continuous line in Fig. 4(a), as in Fig. 2). In the other case the theoretical curve based on the WF law matches the experimental data at very high fields (dashed line in Fig. 4(a)). From the difference between the experimental and theoretical curves, see Fig. 4(b), we conclude that either the deviation starts at a field near the MIT or is at largest at this field $B_c(T)$. Whatever definition we use for the critical field, the low-temperature data below 2 K indicate that $B_c(T)$ increases decreasing temperature, see inset in Fig. 4.

4.2. Oscillations due to Landau quantization at high fields

(a) Quasiparticles contribution at high fields: In what follows we discuss the oscillations due to Landau quantization of the electronic levels, observed
Fig. 4. (a) Normalized thermal conductivity as a function of field at $T = 0.6$ K. The dashed line is obtained from Eq.(3) choosing a value of $\kappa(0)$ that matches the measured thermal conductivity at $B = 8$ T. The continuous line was calculated in a similar way but $\kappa(0)$ was chosen to fit the low field range. The difference between the two values of $\kappa(0)$ is $\sim 4\%$ and therefore within experimental error. (b) Relative difference ($\kappa_r = \kappa(B)/\kappa(0)$) between the measured and calculated values from (a). ($\circ$) represents the values obtained from the curve that matches the data at 8 T, and ($\blacksquare$) those obtained from the other curve (continuous line in (a)). Inset in (a) shows the temperature dependence of the field at which the deviation from the WF law starts (or of the field at the minimum, ($\circ$) in (b)) for the AC sample ($\blacksquare$). The point ($\circ$) is obtained for the UC sample, see Fig. 2.
Magnetothermal Transport of Graphite

in $\kappa(B)$ at the quantum limit ($B > 1$ T) and their possible origins. We have recently shown that these oscillations are correlated with the QHE features observed in HOPG. A simple estimate indicates that the amplitude of the oscillations cannot be explained using the WF law and the measured resistivity. The reason is that at $B > 1$ T the resistivity increases by a factor of 100, therefore and according to WF the electronic contribution to the thermal transport should be much less than the phonon contribution in the temperature range of our measurements. However, as we showed above, the WF law does neither reproduce the field dependence of $\kappa$ at or above the MIT nor the apparent large electronic contribution to $\kappa$ observed at high fields and at the lowest temperatures. Therefore, we may speculate that the QP in graphite may still have a direct contribution to the thermal transport at the quantum limit. We note that the deviation from the WF law can be partially explained arguing that the measured longitudinal resistivity at large fields is not the real resistivity due to the influence of a Hall voltage at high enough fields as has been recently suggested for inhomogeneous semiconductors. The quasi-linear and non-saturating magnetoresistance of highly oriented graphite would speak for such a Hall contribution.

As a characterization of the oscillations we have defined in Ref. 16 the oscillation amplitude $\Delta \kappa = \kappa(B \simeq 3.7 \text{ T}) - \kappa(B \simeq 5.5 \text{ T})$ and showed that $\Delta \kappa$ has an apparent maximum around 7 K. Figure 5 shows those data with new points obtained for the other piece of the same sample. The oscillation amplitude was measured in the AC sample down to 0.2 K. The data indicate that the oscillation amplitude vanishes at low temperatures following roughly a $T^3$ or a $T^{3.5}$ law below $\sim 5$ K, see Fig. 5. Because the QHE features are clearly measured below 1 K, this $T$-dependence suggests that the oscillations might be related to phonon-mediated transitions, i.e. either the oscillations are due to a change in the phonon mean free path through their interaction with the QP or the QP contribution to the transport, being not as negligible as the WF law predicts, is influenced by the scattering of QP with phonons. Another interesting behavior that is not expected from the WF law is the steadily increase (leaving the oscillations of $\kappa$ by side) of the mean value of $\kappa(B > 1\text{ T}, T \sim 5\text{ K})$. This increase with field is reproducible in all measured samples and is at largest at the temperature where the oscillation amplitude is at maximum, see Fig. 5. Taking into account the experimental and theoretical research of the anomalous properties of graphite, the still unclear effects of the linear dispersion relation for the QP including Landau quantization and phonon-QP interaction in the theory for thermal transport and the unclear origin of the linear magnetoresistance, at the present stage we are not able to provide a conclusive answer on the QP contribution to the thermal transport at the quantum limit.
Fig. 5. The thermal conductivity difference between the minimum (at $B \simeq 3.7$ T) and maximum (at $B \simeq 5.5$ T) (see Fig. 2) as a function of temperature for (■) the AC sample, (●) the UC sample studied in this work and (○) the UC sample from Ref. 16. The continuous and dashed lines show a $T^3$ and $T^{3.5}$ temperature dependence, respectively.

(b) Phonon-electron interaction: Due to the correlation of the oscillations with the Hall effect, it is clear that the QP play a role. As in Antimony,$^{40}$ we may argue that phonon-electron interaction is the main scattering mechanism for phonons that leads to the oscillations in the phonon contribution of $\kappa$, through an effect of the quantization on the phonon-electron scattering, due to, e.g. the field dependence of the density of states (or carrier density) of the QP. However, the relatively low density of QP ($10^{-4}$ to $10^{-5}$ carriers per atom) indicates that this scattering mechanism in graphite should be less important than in usual metals. The inelasticity parameter $\eta = v/\lambda \omega_c$ (here $v$ is the sound velocity, $\lambda$ the magnetic length and $\omega_c$ the cyclotron frequency), that provides an estimate of the efficiency of the phonon-electron scattering, is $\sim 0.01$ at 4 T for graphite, apparently too small to indicate a strong phonon-electron interaction. From the other electronic side, the thermally activated $T$—dependence of the longitudinal electrical resistivity obtained for graphite at low magnetic fields, also observed in several 2D electron systems,$^{41}$ and the $T$—dependence for $\kappa$ do not speak for a large electron-phonon interaction.

It may be possible that at certain magnetic fields the phonon-electron scattering is significantly enhanced when the separation energy between Landau levels is of the order of the characteristic acoustic-phonon energy.$^{42}$ This is expected to occur at a temperature $T_\lambda \sim \hbar \omega = \hbar v q$ when the phonon
wave vector is of the order $1/\lambda$, the inverse of the magnetic length. Putting numbers one obtains that $T_\lambda \sim 2$ K at $B = 3.5$ T, surprisingly near the maximum temperature of the oscillation amplitude. However, energy conservation requires that the phonon energy matches the separation between Landau levels. At $B = 5$ T, $\hbar \omega_c \sim 100$ K taking the effective mass of electrons $m^* \sim 0.05 m_0$, and therefore much larger than $T_\lambda$, making the one-phonon electron transition inefficient. Multi-phonon processes may, however, provide the necessary efficiency for that transition to occur. One may also argue that the low effective mass of the carriers used above to estimate $\hbar \omega_c$ is not the appropriate one, since in this field range electron-hole excitonic pairs may act as the main carriers.

Other possibility for a resonance phonon-electron scattering is a substantial decrease of the energy between Landau levels due to the intersection of two Landau levels belonging to different size-quantization subbands (due to electrons and holes, for example). There is however no clear experimental evidence for such a crossover. Some details of the field dependence of the Hall effect depend on sample quality and its internal disorder, and a rather complicated algorithm is used to obtain details of the band structure from these data. Very recently, clear quantized plateaus were obtained for the Hall conductivity in two different geometries in high quality and small HOPG samples. If this behavior represents that of ideal graphite, then some characteristics of the carriers and the electronic band structure picture of graphite reported in the past should be revised.

When discussing the efficiency of the phonon-electron scattering in graphite we should take into account an effective Debye temperature $\theta^*$. This effective temperature provides the limiting value for the change of the scattered phonon wave vector $q$. Because of the low density of carriers $2k_F \ll q_D$ the effective Debye temperature for the phonon-electron interaction is estimated as $\theta^* \sim 2k_F v_f \hbar / k_B \sim 9$ K using the 2D value for $2k_F$ obtained from Hall effect measurements on the same sample. This value should be compared with the (anisotropic) Debye temperature for graphite with a value $\theta > 500$ K. This effective Debye temperature $\theta^*$ is near the temperature at which we observe a maximum in the oscillation amplitude, see Fig. 5.

5. Summary

In summary, we have shown that the magnetic field dependence of the thermal conductivity in highly ordered graphite shows a kink anomaly at or near the “critical” field of the metal-insulator transition. Above this field a clear plateau is measured. The observation of this kink depends on sample; experimental evidence suggests that the larger the disorder the
less clear is the kink, but more systematic work is necessary. For the two measured samples we have shown that the WF law does not account for the field dependence of $\kappa$ showing a systematic deviation at the metal-insulator transition. The overall behavior appears to be accounted for by the magnetic catalysis model, the opening of an excitonic gap and the increase of the effective mass of the carriers due to the electron-hole pairing. We have shown also that the WF law does not account for the oscillations in $\kappa$ due to Landau quantization at the quantum limit. Measurements to 0.2 K show that the oscillation amplitude increases with temperature below $\sim$ 5 K with a power law $T^n$ with $n = 3.0 \ldots 3.5$ suggesting that phonons play a role.

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Magnetothermal Transport of Graphite

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