Disorder-free localization in a simple $U(1)$ lattice gauge theory

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Localization due to the presence of disorder has proven crucial for our current understanding of relaxation in isolated quantum systems. The many-body localized phase constitutes a robust alternative to the thermalization of complex interacting systems, but recently the importance of disorder has been brought into question. Starting from translationally invariant (1+1)-dimensional quantum electrodynamics, we modify the dynamics of the gauge field and reveal a mechanism of disorder-free localization. We consider two different discretizations of the continuum model resulting in a free-fermion soluble model in one case and an interacting model in the other. We diagnose the localization in the far-from-equilibrium dynamics following a global quantum quench.

I. INTRODUCTION

Revealing the effect of disorder has been crucial for our understanding of how complex quantum systems can relax. It was found by Anderson [1] that the presence of disorder leads to a perfect insulator in non-interacting systems due to the exponential localization in space of single particle wavefunctions. Importantly, in interacting systems it is possible to have a robust many-body localized phase (MBL) of matter [2–6]. Unlike integrable or Anderson localized systems, MBL systems are stable to generic perturbation and are not protected by symmetry or local conservation laws. MBL provides an alternative to the more generic expectation that complex interacting systems should be ergodic and eventually thermalize [7–9].

More recently, the question has been posed whether disorder is a requirement for localization or whether non-ergodic behaviour is possible in translationally invariant systems. An alternative was demonstrated in a model related to a $\mathbb{Z}_2$ lattice gauge theory (LGT) [10], where the disorder-free localization mechanism emerged from the local gauge symmetry. This mechanism has since been utilised in a range of simple $\mathbb{Z}_2$ LGTs [11], the massless one-dimensional lattice Schwinger model of quantum electrodynamics (QED) [12], and a two-dimensional $U(1)$ quantum link model [13]. As well as the LGT approach, localization without disorder has also been achieved using a variation of Wannier-Stark localization [14, 15] and there is also evidence in some kinetically constrained models [16, 17].

Disorder-free localization in LGTs is remarkable as they are central in the study of interacting physics, from the description of the standard model of particle physics to strongly-correlated phases of many-body systems [18]. Even the simplest (1 + 1)-dimensional $U(1)$ Schwinger model of quantum electrodynamics (QED) [19] captures the Schwinger mechanism of particle-antiparticle pair production [20] and is a toy model of quark confinement [21]. In this paper our starting point is a modification of (1 + 1)-dimensional continuum QED, that differs only in the dynamics of the gauge field. The modification to the gauge field dynamics is such that there is a soluble discretization of the model that can be mapped onto free-fermions with an emergent disorder-free localization mechanism. This solubility allows us to identify the localized behaviour and perform large scale numerical simulations. In addition, we diagnose the localization using the persistence of local information under far-from-equilibrium dynamics after a global quantum quench [22, 23]. These quench protocols are relevant to experiments, such as those in cold atom optical lattice [24, 25], where they are routinely used to diagnose localization behaviour.

The lattice discretization of a relativistic continuum field theory is not unique and is complicated by the Nielsen-Ninomiya no-go theorem [26] and the fermion doubling problem [27, 28]. The former states that in any discretization, one of the following symmetries must be broken: hermiticity, locality, chiral symmetry or discrete translation invariance. The fermion doubling problem refers to non-physical degrees of freedom that result from the naive discretization. Out of the several possible approaches, we consider Staggered fermions and Wilson fermions [29]. Staggered fermions break the discrete translational symmetry whereas Wilson fermions do not respect chiral symmetry. Remarkably, we find that despite having the same continuum limit, the former results in a soluble lattice model mappable to free-fermions while the latter is manifestly interacting.

This paper is structure as follows. We begin by defining the continuum model in Sec. II, where we highlight the difference to standard (1+1)-dimensional QED. In Sec. III we provide the details of the Staggered fermion discretization of the model resulting in a one-dimensional lattice gauge theory. We then reveal the disorder-free mechanism for localization and demonstrate this behaviour using global quench protocols in Sec. IV. Following this, in Sec. V we consider the Wilson fermion approach to discretization. Finally, we close with a discussion of our results and an outlook.
II. THE CONTINUUM LIMIT

We begin by defining a (1+1)-dimensional continuum model with a local $U(1)$ gauge symmetry. Following the convention in high-energy physics, we label the time-dimension 0 and the space dimension 1 and work in the temporal gauge. The Hamiltonian reads

$$\hat{H} = -\int dx \left( i \bar{\Psi}(x) \gamma^1 \left[ \partial_1 + ig \hat{A}^1(x) \right] \Psi(x) \right) + \int dx \left( m \bar{\Psi}(x) \Psi(x) + B \left[ \partial_1 \hat{E}(x) \right]^2 \right),$$  

where $\partial_1$ is the partial derivative with respect to $x$, $\gamma^0$ and $\gamma^1$ are the gamma matrices in two spacetime dimensions, and $\Psi = \Psi^1 \gamma^0$. The Dirac field $\Psi(x)$ has two components. The parameters $m$ and $g$ represent the fermion mass and the coupling constant, respectively, and $B$ is a constant with dimensions $(\text{length})^2$.

The first term represents the coupling of the fermionic matter field with the vector gauge field $\hat{A}^1(x)$, and the second term is the fermionic mass term. The last term is the energy of the electric field $\hat{E}(x)$, which gives dynamics to the gauge field. Our Hamiltonian differs from QED in (1 + 1)-dimensions [27, 30] only in this last term where we have made the change $\int dx \hat{E}^2(x) \rightarrow \int dx \left[ \partial_1 \hat{E}(x) \right]^2$.

Note that we have chosen to work in the temporal gauge, where there is still a gauge freedom. The generators of the remaining gauge freedom are

$$\hat{G} = \partial_1 \hat{E} - g \bar{\Psi} \gamma^1 \Psi,$$  

which commute with the Hamiltonian.

Now, if we proceed to naively discretize the Dirac Hamiltonian on the lattice, we encounter the so-called doubling problem resulting in non-physical degrees of freedom, called fermion doublers [28]. We can avoid this problem with the use of Staggered [31] or Wilson [32, 33] fermions. Both discretizations give the desired continuum limit and result in lattice models with a local gauge invariance.

III. STAGGERED FERMIONS

We first consider the approach of Staggered fermions, see Ref. [31] for more details. Here we map the continuous Dirac fermion field onto spinless (single-component) fermion operators, $\hat{\Phi}_n$, where $n$ are site indices for a one-dimensional chain with lattice spacing $a$. The two spin components of the Dirac field are instead represented by a two-site unit cell with positive and negative mass terms on alternating sites. The corresponding discrete Hamiltonian is

$$\hat{H} = \frac{B}{a} \sum_n \left( \hat{E}_n - \hat{E}_{n-1} \right)^2 - \frac{ig}{2a} \sum_n \left( \hat{\Phi}_n^1 e^{-ig \hat{A}_n^1} \hat{\Phi}_{n+1} - \text{h.c.} \right) + m \sum_n (-1)^n \hat{\Phi}_n^1 \hat{\Phi}_n,$$

where the fermion operators satisfy the canonical anti-commutation relations $\{ \hat{\Phi}_n^1, \hat{\Phi}_m \} = \delta_{nm} \hat{1}$, and $\{ \hat{\Phi}_n, \hat{\Phi}_m \} = 0$. The first term represents the energy of the electric field $\hat{E}_n$. Notice the form $(\hat{E}_n - \hat{E}_{n-1})^2$ coming from the spatial derivative in Eq. (1) in contrast to the standard $\hat{E}_n^2$ appearing in the lattice Schwinger model. The second term is the coupling between the vector potential $\hat{A}_n^1$ and the single-component fermion fields $\hat{\Phi}_n$. Finally, we have the staggered mass term for the fermions. The lattice sites take values $n = 1, \ldots, N$ with $N$ the total number of sites.

In the Hamiltonian (3) we treat the electric field and vector potential as spin-1 systems with discrete spectra. In particular we take

$$\hat{E}_n = g \hat{L}_n, \quad -ag \hat{A}_n^1 = \hat{\theta}_n,$$

where the operators $\hat{\theta}_n$ and $\hat{L}_n$ are defined on the ‘link’ between the two lattice sites $n$ and $n + 1$, and have the commutation relations $[\hat{\theta}_n, \hat{L}_m] = -i \delta_{nm} \hat{1}$. In terms of these operators the $U(1)$ parallel transporters are $\hat{U}_n = e^{i \hat{\theta}_n}$. The operators $\hat{L}_n$ are generally quantized with spectrum, $\hat{L}_n = 0, \pm 1, \pm 2, \pm 3, \ldots$, but for our quench procedures discussed in the following we have chosen bounded link variables $\hat{L}_n = 0, \pm 1$ to define a quantum link model [34]. Our following discussion can be generalized to an arbitrary number of levels on the links.

In terms of these link operators the Hamiltonian becomes

$$\hat{H} = J_3 \sum_n \left( \hat{L}_n - \hat{L}_{n-1} \right)^2 - iJ_2 \sum_n \left( \hat{\Phi}_n^1 e^{i \hat{\theta}_n} \hat{\Phi}_{n+1} - \text{h.c.} \right) + J_1 \sum_n (-1)^n \hat{\Phi}_n^1 \hat{\Phi}_n,$$

where $J_3 = \frac{a^2 B}{n}$, $J_2 = \frac{1}{2a}$, and $J_1 = m$. The above Hamiltonian is similar to that of the lattice Schwinger model using the Kogut-Susskind formulation [35], with a change in the term related to the energy stored by the electric field.

The Hamiltonian of Eq. (5) is gauge invariant and commutes with the generators of $U(1)$ gauge symmetry, $\hat{G}_n$: $[\hat{G}_n, \hat{H}] = 0$. The local generators $\hat{G}_n$ are given by [36]

$$\hat{G}_n = \hat{L}_n - \hat{L}_{n-1} - \hat{\Phi}_n^1 \hat{\Phi}_n + \frac{1}{2} \left( 1 - (-1)^n \right) \hat{1}.$$  

These local conservation laws allow us to split the Hilbert space into superselection sectors labelled by the eigenvalues \( q_a \), i.e.,

\[
\hat{G}_n |\Psi(q_a)\rangle = q_n |\Psi(q_a)\rangle.
\] (7)

The standard Gauss’ Law requires these local charges to be zero \( \hat{G}_n |\text{physical}\rangle = 0 \), which corresponds to the subspace with gauge invariant states [35]. However, here we consider unconstrained gauge theories [11], and the states \( |\Psi(q_a)\rangle \) correspond to a distribution of local vacuum charges \( q_a \), the charge superselection sector of the model. Therefore, the Hamiltonian is gauge invariant, whereas the Hilbert space of interest is not. The existence of gauge invariance for the Hamiltonian is crucially related to the emergence of disorder in the system, as will be discussed below.

In order to reveal the free-fermion solubility of the model, our goal is to eliminate the gauge field from the Hamiltonian. We achieve this by a redefinition of the fermionic fields [37],

\[
\hat{\tilde{\Phi}}_n = e^{i \sum_{m=1}^{n-1} \hat{d}_m \hat{\Phi}_n},
\] (8)

which satisfy the same anti-commutation relations, and by substituting \( \hat{L}_n - \hat{L}_{n-1} = \hat{G}_n + \hat{\tilde{\Phi}}_n \hat{\tilde{\Phi}}_n - \frac{1}{2} (1 - (-1)^n) \hat{1} \) from Eq. (6). The Hamiltonian then becomes

\[
\hat{H} = J_3 \sum_n \left( \left[ (-1)^n - 1 \right] \hat{\tilde{\Phi}}_n \hat{\tilde{\Phi}}_n + [2 \hat{G}_n + 1] \hat{\tilde{\Phi}}_n \hat{\tilde{\Phi}}_n \right) - i J_2 \sum_n \left( \hat{\tilde{\Phi}}_n \hat{\tilde{\Phi}}_{n+1} - h.c. \right) + J_1 \sum_n (-1)^n \hat{\tilde{\Phi}}_n \hat{\tilde{\Phi}}_n.
\] (9)

Note, the above procedure of eliminating the gauge fields, is similar to that used in reference [36], where they proceeded with a Jordan-Wigner transformation, and eliminated the gauge fields by using the Gauss law, as well as a residual gauge transformation.

The first term corresponds to the emergent disorder with the effective on-site potential controlled by \( \hat{G}_n \), which are related to the conserved charges in our model. In a given charge sector these can be replaced by the their eigenvalues \( q_n \), leaving a quadratic free-fermion Hamiltonian for that sector. By changing the ratio \( J_3/J_2 \) we change the strength of the emergent disorder. The second term is the fermion hopping term and the last term is the staggered fermion mass. In the following we set the mass \( (J_1) \) to be zero for simplicity. In each charge sector, the Hamiltonian in Eq. (9) corresponds to a free-fermion model that is exactly solvable.

## IV. EMERGENT DISORDER

To study the localization behaviour in our model, which is manifestly translationally invariant, we consider a global quantum quench protocol. We start from an easily prepared initial state with an inhomogeneous fermion density and a macroscopic energy density. We then evolve with the Hamiltonian (9) and measure the density imbalance and spreading of correlations. We consider the initial state of the form [12]

\[
|\Psi(t = 0)\rangle = |\psi\rangle_g \otimes |\psi\rangle_f.
\] (10)

The fermionic part \( |\psi\rangle_f \) is a Slater determinant with fermions on the odd sites and the even sites are not occupied. The gauge fields, defined on the bonds, are set to be in an equal weight superposition of the eigenstates of \( \hat{L}_n \) with eigenvalues \( L_n = 0, \pm 1 \). We can write

\[
|\Psi(t = 0)\rangle = \left[ \otimes_n |\tilde{L}_n\rangle \right] \otimes |1010 \cdots\rangle_f,
\] (11)

with

\[
|\tilde{L}_n\rangle = \frac{1}{\sqrt{3}} \left( |(-1)^n + 0\rangle + |0\rangle_n + |1\rangle_n \right),
\] (12)

for \( n = 1, \ldots, N - 1 \). The initial state can be rewritten as

\[
|\Psi(t = 0)\rangle = \frac{1}{\sqrt{N_{sec}}} \sum_{q_n} \left[ \otimes_n |q_n\rangle \right] \otimes |1010 \cdots\rangle_f,
\] (13)

where \( N_{sec} \) is the total number of superselection sectors of charges, and the fermions are in the basis of \( \hat{\tilde{\Phi}} \) operators. The local charges \( q_n \) of each sector are given by
Eq. (6) and depend on the distribution of the fermions and the gauge fields. Specifically, each superselection sector corresponds to a particular distribution of the gauge fields with local eigenvalues $L_n = 0, \pm 1$. Schematically, the decomposition of the initial state is shown in Fig. 1. The time evolution of the state is then given by
\[ |\Psi(t)\rangle = \frac{1}{\sqrt{N_{\text{sec}}}} \sum_{\{q_a\}} e^{-itH_{\{q_a\}}} |\{q_a\}\rangle \otimes |\psi\rangle, \]
where in the Hamiltonians $\hat{H}_{\{q_a\}}$ the generators $\hat{G}_n$ are replaced by the local charges $q_n$ that correspond to a particular charge sector. We consider the dynamics of the density imbalance between odd and even sites,
\[ \Delta \rho(t) = \frac{1}{N} \sum_{j=1}^{N-1} |\langle \Psi(t) | (\hat{n}_j - \hat{n}_{j+1}) | \Psi(t) \rangle|, \]
and the connected density correlator
\[ \langle \Psi(t) | \hat{n}_j \hat{n}_k | \Psi(t) \rangle_c = \langle \Psi(t) | \hat{n}_j \hat{n}_k | \Psi(t) \rangle - \langle \Psi(t) | \hat{n}_j | \Psi(t) \rangle \langle \Psi(t) | \hat{n}_k | \Psi(t) \rangle, \]
where $N = N - 1$ with $N$ the number of sites and $\hat{n}_j = \hat{\Phi}_j^\dagger \hat{\Phi}_j$. These quantities allow us to diagnose the localization behaviour in our model signalled by the persistence of the initial density imbalance [23, 38, 39] and the lack of correlation spreading after the quench [22].

In order to compute these observables we note that the density imbalance can be written as
\[ \Delta \rho(t) = \frac{1}{NN_{\text{sec}}} \sum_j \sum_{\{q_a\}} |\langle \psi(t) | f (\hat{n}_j - \hat{n}_{j+1}) | \psi(t) \rangle_f |, \]
where $|\psi(t)\rangle_f = e^{-it\hat{H}_{\{q_a\}}}|\psi\rangle$ is the fermion state evolved under the Hamiltonian in a given charge sector. The density correlator can similarly be written as an average of free-fermion correlators. We calculated the average density imbalance numerically using free-fermion methods, as summarized in Ref. [22]. We average over a limited number of superselection sectors and find that the system is self-averaging and a small fraction of the total number of sectors is sufficient for converged results, see Fig. 4(inset).

In Fig. 2 we show the result of the density imbalance. Initially $\Delta \rho(t = 0) = 1$ but at long time we expect $\Delta \rho(t \to \infty) = 0$ for an ergodic system, which corresponds to a uniform fermion density. For our model we find that $\Delta \rho(t \to \infty) \neq 0$ for all non-zero values of $J_3/J_2$. This indicates that the system has long-time memory of the initial state and the remaining imbalance increases monotonically with $J_3/J_2$, as shown in Fig. 2. We compare this with the case with no effective disorder $J_3 = 0$, where the density imbalance decays to zero at long-times (red curve) in the thermodynamic limit. We note that the observed power-law decay is due to the integrability of the model.

Next we consider the spreading of the density correlations, defined in Eq. (16) and shown in Fig. 3. We show the free case $J_3 = 0$ in Fig. 3(a). The density correlations can be seen to spread ballistically until they reach the edge of the system. In comparison, for non-zero $J_3/J_2$ we find that the spreading of correlations is quickly halted and the correlations have a finite extent at long times, as shown in Figs. 3(b) and 3(c). This lack of spreading is also evidence of the localization behaviour in the model.

At long times the spatial profile of the density correlations decay exponentially, as shown in the inset of Fig. 4. This exponential decay is related to the localization of the single particle wavefunctions. In Ref. [10] it was found that this profile is given by $\exp(-L/2\lambda)$, where $\lambda$ is the single-particle localization length, which allows us to extract the localization length from these results. This localization length is shown in Fig. 4, which decreases with increasing effective disorder strength $J_3/J_2$.

V. WILSON FERMIONS

Having shown the localization behaviour in a simple LGT using staggered fermions we now considered an alternative discretization in terms of Wilson fermions [33]
\[ \hat{H}_W = \frac{B}{a} \sum_n \left( \tilde{E}_n - \tilde{E}_{n-1} \right)^2 \]
\[ + a \sum_n \left( \hat{\Psi}_n^\dagger \gamma^0 \left[ m + \frac{r}{a} \right] \hat{\Psi}_n \right) \]
\[ - \frac{1}{2} \sum_n \left( \hat{\Psi}_n^\dagger \gamma^0 \left[ i \gamma^1 + r \right] \hat{\Psi}_n \Psi_{n+1} + h.c. \right). \]
If we write the upper and lower component of the Dirac spinor $\hat{\Psi}_n$ as $\hat{\Psi}_{n,1}$ and $\hat{\Psi}_{n,2}$ respectively, the Hamiltonian of Eq. (18) can be written on the lattice as

$$\hat{H} = \frac{B}{a} \sum_n \left( \hat{E}_n - \hat{E}_{n-1} \right)^2 + \sum_n \left( m + \frac{1}{a} \right) \left( \hat{\Psi}_{n,1}^\dagger \hat{\Psi}_{n,2} + \hat{\Psi}_{n,2}^\dagger \hat{\Psi}_{n,1} \right) + \frac{1}{a} \sum_n \left( \hat{\Psi}_{n,1}^\dagger \hat{U}_n \hat{\Psi}_{n+1,2} + h.c. \right),$$

(21)

where we redefined the fermionic fields as $\hat{\Psi}_{n,i} \rightarrow (-1)^n \sqrt{a} \hat{\Psi}_{n,i}$ and set the Wilson parameter to $r = 1$. The result of the above field redefinition, is that the fermionic fields now are dimensionless. We followed the same approach as Ref. [40], writing the Hamiltonian in a convenient form. These operators satisfy the following commutation and anti-commutation relations

$$[\hat{\Psi}_{n,\alpha}, \hat{\Psi}_{m,\beta}^\dagger] = \delta_{\alpha\beta} \delta_{nm} \hat{1},$$

(22)

and $[\hat{E}_n, \hat{U}_m] = g \delta_{nm} \hat{U}_m$.

With the aim to obtain a free-fermionic soluble model as before, we follow the same steps as in the case of Staggered fermions studied in Sec. III. We introduce the operators $\hat{\theta}_n$ and $\hat{L}_n$ as defined in Eq. (4). Equation (21) then becomes

$$\hat{H} = \frac{B g^2}{a} \sum_n \left( \hat{L}_n - \hat{L}_{n-1} \right)^2 + \sum_n \left( m + \frac{1}{a} \right) \left( \hat{\Psi}_{n,1}^\dagger \hat{\Psi}_{n,2} + \hat{\Psi}_{n,2}^\dagger \hat{\Psi}_{n,1} \right) + \frac{1}{a} \sum_n \left( \hat{\Psi}_{n,1}^\dagger \hat{U}_n \hat{\Psi}_{n+1,2} + h.c. \right),$$

(23)

with $\hat{U}_n = e^{i \hat{\theta}_n}$, and the generators $\hat{G}_n$ take the form:

$$\hat{G}_n = g \hat{L}_n - g \hat{L}_{n-1} - g \left( \hat{\Psi}_{n,1}^\dagger \hat{\Psi}_{n,2} - h.c. \right).$$

(24)

The next step is to redefine the fermionic fields

$$\hat{\Psi}_{n,j} = e^{i \sum_{\ell=1}^{n-j} \hat{\theta}_\ell} \hat{\Psi}_{n,j}.$$

(25)
for $j = 1, 2$ and substitute Eq. (24) into the Hamiltonian to get
\[
\hat{H}_W = \frac{B}{a} \sum_n \left( \hat{c}_n - g \left[ 1 - \hat{\Psi}^\dagger_{n,1} \hat{\Psi}_{n,1} - \hat{\Psi}^\dagger_{n,2} \hat{\Psi}_{n,2} \right] \right)^2 \\
+ \left( m + \frac{1}{a} \right) \sum_n \left( \hat{\Psi}^\dagger_{n,1} \hat{\Psi}_{n,2} + \hat{\Psi}^\dagger_{n,2} \hat{\Psi}_{n,1} \right) \\
+ \frac{1}{a} \sum_n \left( \hat{\Psi}^\dagger_{n,1} \hat{\Psi}_{n+1,2} + h.c. \right). \tag{26}
\]
In contrast to the staggered fermions, the first term contains both a disordered potential and a fermionic interaction term, namely
\[
\hat{H}_{\text{int}} = \frac{B}{a} \sum_n 2g^2 \left( \hat{\Psi}^\dagger_{n,1} \hat{\Psi}_{n,1} \hat{\Psi}^\dagger_{n,2} \hat{\Psi}_{n,2} \right). \tag{27}
\]
Interactions lead to a model that is not free-fermion soluble and a numerical study of its properties is beyond the scope of this paper.

VI. DISCUSSION

In this paper we have studied a simple $U(1)$ lattice gauge theory derived from a variant of continuum QED in $(1+1)$ dimension. By changing only the dynamics of the gauge field we remove the effective long-range interactions that result in confinement, and instead reveal emergent disorder coming from the local symmetry of the model. We focused on the staggered fermion formulation of the lattice model where the model was free-fermion soluble and amenable to large scale numerical simulations, and when the disorder-free localization mechanism was most evident.

Surprisingly, the Wilson fermion formulation results in a model that is manifestly interacting. In this setting it is then a question whether the lattice model is many-body localized. In taking the continuum limit we need to pay careful attention to relevance of these interacting terms due to the scaling of the fermion fields. It remains an open yet interesting question whether, through a common continuum limit, we can deduce anything about this interacting model from the free-fermion staggered fermion model.

We have shown that in the disorder-free localization mechanism the emergent disorder potential depends on the distribution of the fermion and gauge field configuration of the initial state Eq. (6). For our choice of quench set-up the resulting disorder was only weakly correlated leading to full localization. An interesting topic for future research would be a systematic study of whether special choices of initial states can give rise to correlated disorder which in principle could result in mobility edges or delocalization even in one dimension.

Another extension of this work would be to consider different gauge fields, in particular non-abelian gauge theories, as well as studying higher-dimensional models. In these settings, we could ask if we can also find continuum gauge theories that have soluble discretizations and whether this disorder-free mechanism for localization extends more generally.

Finally, our simple modification of $(1+1)$-dimensional quantum electrodynamics allowed us to find a basic free fermion solution, which in turn could be of potential interest for the study of dynamical phenomena in the gauge invariant sector of the model like pair-production protocols.

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