The Three Faces of $\Omega_m$: Testing Gravity with Low and High Redshift SN Ia Surveys

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ABSTRACT
Peculiar velocities of galaxies hosting Type Ia supernovae generate a significant systematic effect in deriving the dark energy equation of state $w$, at level of a few percent. Here we illustrate how the peculiar velocity effect in SN Ia data can be turned from a “systematic” into a probe of cosmological parameters. We assume a flat $\Lambda$-Cold Dark Matter model ($w = -1$) and use low and high redshift SN Ia data to derive simultaneously three distinct estimates of the matter density $\Omega_m$ which appear in the problem: from the geometry, from the dynamics and from the shape of the matter power spectrum. We find that each of the three $\Omega_m$’s agree with the canonical value $\Omega_m = 0.25$ to within 1σ, for reasonably assumed fluctuation amplitude and Hubble parameter. This is consistent with the standard cosmological scenario for both the geometry and the growth of structure. For fixed $\Omega_m = 0.25$ for all three $\Omega_m$’s, we constrain $\gamma = 0.72 \pm 0.21$ in the growth factor $\Omega_m(z)$, so we cannot currently distinguish between standard Einstein gravity and predictions from some modified gravity models. Future surveys of thousands of SN Ia, or inclusion of peculiar velocity data, could significantly improve the above tests.

Key words: large-scale structure of universe – cosmological parameters – surveys – galaxies: kinematics and dynamics

1 INTRODUCTION
The observed present acceleration of the universe was first confirmed a decade ago by two separate groups using Type Ia supernovae (SN Ia, Perlmutter et al. 1999; Riess et al. 1998). SN Ia are one of a number of probes needed to obtain tighter constraints on dark energy equation of state, including any possible time evolution. This will require surveys of thousands of supernovae out to high redshifts to accurately measure their luminosity distances from which parameters describing the dark energy can be inferred. To achieve the desired constraints on dark energy, in particular a few percent constraint on the equation of state parameter $w$, the supernovae will have to be accurately calibrated. It is therefore vital that this calibration is done accurately, and it is the low redshift supernovae which are vital to achieve this, for details see Aldering et al. (2002). At low redshift the supernovae distances have little or no dependence on the cosmological parameters such as $\Omega_m$, $\Omega_{\Lambda}$ and the dark energy equation of state $w$. They do however put a tight constraint on a combination of what is essentially the calibrated magnitude zeropoint $(M)$ and the Hubble constant $H_0$, whereas

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to this have been discussed recently in the literature. In Neill, Hudson, & Conley (2007) different flow models based on the IRAS PSCz survey (Branchini et al. 1994) were used to "correct" the luminosity distances by the known peculiar velocities before fitting them for the cosmological parameters of interest. They find the potential systematic error in $w$ caused by ignoring peculiar velocities is of the order of 4 percent, i.e. quite significant.

Radburn-Smith, Lucey, & Hudson (2004) compared peculiar velocities from 98 local supernovae with the gravity field predicted from IRAS. In Haugbølle et al. (2007) an angular expansion of the radial velocity field was used to probe the local dipole and quadrupole of the velocity field at three different distances. They found that the dipole is consistent with galaxy surveys (e.g. Erdöghu et al. 2006) at the same Hubble flow depths.

The third and somewhat different method is utilised by Hui & Greene (2006); Cooray & Caldwell (2006); Gordon, Land & Slosar (2007) who take a covariance matrix approach. From the fluctuation in the luminosity distance induced by the peculiar motions, (see Hui & Greene 2006; Pyne & Birkinshaw 2004; 1996; Sasaki 1987, for derivations), a covariance matrix for the resulting errors in the luminosity distance (or similarly the apparent magnitude) can be calculated. The covariance matrix depends on cosmological parameters which describe the growth and distribution of structure. In addition to the peculiar velocity effect this is due to gravitational lensing effect, which is important for redshifts larger than 1, and we shall ignore it in this Letter.

Cooray & Caldwell (2006) found that peculiar velocities of the low redshift supernovae may prevent measurement of $w$ to better than 10 percent, and diminish the resolution of the time derivative of $w$ projected for planned surveys. Gordon, Land & Slosar (2007) used the covariance matrix approach on current data, showing the changing constraints on $\sigma_8$, $\Omega_m$ and $w$ depending on the exact redshift range of the SN Ia sample and whether the full covariance was included or not. They also apply the analysis to forecasting constraints for future surveys.

Here we unify the analysis of SN Ia data to study simultaneously fits for the expansion of the universe and the growth of structure. There is plenty of discussion on the possibility that the accelerated expansion of the universe is caused by a modification of general relativity on large scales (e.g. Durrer & Maartens 2008; Huterer & Linder 2007, and references therein). By measuring the growth of structure, which directly effects the observed peculiar velocity field, information is gained to differentiate between the two scenarios.

The rest of the Letter is organised as follows. In Section 2 we describe the SN Ia sample used in this Letter. In Section 3 we describe the theory underlying SN Ia analysis in cosmology and the effect of the velocity field.

2 DATA

We analyse both nearby supernovae ($z \leq 0.12$) from Jha, Riess, & Kirshner (2007) and high redshift supernovae ($z \leq 0.176$) from a sample compiled by Davis et al. (2007) which includes data from Riess et al. (2007) and Wood-Vasey et al. (2007). Davis et al. (2007) combined the data from the two samples by normalising to the low redshift supernovae they had in common. Following Jha, Riess, & Kirshner (2007) 9 supernovae are excluded from the low redshift set, those that are unsuitable due to bad lightcurve fits. This includes supernovae with their first observation more than 20 days after maximum light, those that are hosted in galaxies with excessive extinction ($A_V > 2.0$ mag) and one outlier (SN1999e), which appears to have an extremely large peculiar velocity. This leaves 124 supernovae from the Jha, Riess, & Kirshner (2007) data set in the redshift range $z \in [0.0023, 0.12]$, and median redshift $\bar{z} = 0.017$. The overlapping SN Ia in the two data sets were used to estimate a small normalising offset to the magnitudes from the Davis et al. (2007) data set (the extra magnitude error is negligibly small). The same procedure was used by Davis et al. (2007) in normalising the two high redshift data sets. After eliminating duplicated SN Ia, our combined data set has 271 SNe with $z \in [0.0023, 1.76]$, and $\bar{z} = 0.29$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.pdf}
\caption{Assuming a flat universe with a constant equation of state $w$, these are 1 and 2 $\sigma$ likelihood contours showing the constraints in the $\Omega_m$-$w$ plane for the Riess et al 2007 supernova data. The red/dark contours are when using the full gold sample (182 SN Ia), the blue/light contours are when using the gold sample but removing all supernovae with redshifts less than 0.1 (146 SN Ia).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.pdf}
\caption{The 1 and 2 $\sigma$ contours on $\Omega_m^{dyn}$ and the parameter $\gamma$ from using all 271 low and high redshift SNe. Values of other parameters include $n_s = 1$ and $h = 0.7$, $\sigma_8 = 300\,\text{km}\,\text{s}^{-1}$, $\sigma_m = 0.1$ and the other two $\Omega_m$'s=0.25.}
\end{figure}
3 METHODOLOGY

We describe here how we utilise the SN Ia dataset described in Section 2 to estimate cosmological parameters by including the peculiar velocity covariance.

3.1 Covariance matrix approach

The luminosity distance $d_L$ is defined as

$$F = \frac{L}{4\pi d_L^2}$$

where $F$ is the observed flux of the supernova and $L$ is its intrinsic luminosity. The apparent magnitude $m$ of a supernova at redshift $z$ depends on the luminosity distance as follows

$$m(z) = 5 \log_{10} D_L(z) - 5 \log_{10}(H_0) + M + 25$$

where $M$ is the magnitude zero-point, and $D_L$ is defined without the Hubble constant as $D_L = H_0 d_L$ in $\text{km s}^{-1}$. The equation above ignores the additional terms which involve applying dust corrections, K corrections etc. For a flat universe containing a matter component and a dark energy component with a constant equation of state, the luminosity distance can be written as

$$D_L(z) = c (1 + z) \int_0^z \frac{dz'}{E(z')}$$

$$E(z') = (\Omega_m (1 + z')^3 + (1 - \Omega_m) (1 + z')^{-3(1+w)})^{\frac{1}{2}}$$

If the universe was truly homogeneous and isotropic (FRW) this would be the end of the story, the observed $D_L$ would be described accurately by Eq. 3. However peculiar velocities have the effect of perturbing the luminosity distance

$$\delta D_L = \frac{\nu_z}{c} \left( 1 - \frac{c(1+z)^2}{H(z) d_L(z)} \right)$$

where $\nu_z$ is the radial peculiar velocity of the supernova and $H(z)$ is the Hubble parameter. See Hui & Green (2006); Bonvin, Durrer, & Gasparini (2006); Pyne & Birkhoff (2004); Sasaki (1987) for a derivation. We emphasize that in the right hand side of the above equation $d_L$ is for an unperturbed FRW universe, derived at a perturbed redshift $z$.

Therefore the covariance of the perturbation in $D_L$, $\delta D_L / D_L$ for a pair $i, j$ is given by

$$C_{ij} = \frac{\langle \nu_i \nu_j \rangle}{c^2} \left( 1 - \frac{c(1+z)^2}{H(z) d_L(z)} \right)_i \left( 1 - \frac{c(1+z)^2}{H(z) d_L(z)} \right)_j$$

and

$$\langle \nu_i \nu_j \rangle = \xi_{ij} \cos \theta_i \cos \theta_j \langle \Psi_i \rangle (r) + \sin \theta_i \sin \theta_j \langle \Psi_{ij} \rangle (r)$$

is the linear theory radial peculiar velocity correlation function, Gorski (1988) and Groth, Juszkiewicz, & Ostriker (1989). The angles in Eq. 4 are defined by $\cos \theta_X = \mathbf{r} \cdot \mathbf{r}$ and the diagonal elements $\xi_i$ are given by Eq. 5 below. The $\langle \Psi_i \rangle (r)$ and $\langle \Psi_{ij} \rangle (r)$ can be calculated from the matter power spectrum using linear theory

$$\langle \Psi_{ij} \rangle (r) = D'(z_i) D'(z_j) \int \frac{P(k)}{2\pi^2} B_{ij} (kr) dk$$

where $P(k)$ is the matter power spectrum, $B_{ij} = \delta_{ij} - j_0(x)$ and $j_0$, $j_1$ are the first and second derivative of the zeroth order spherical Bessel functions respectively and $D'(z)$ is the derivative of the growth function at redshift $z$, $D'(z)$ is a function of the Hubble parameter and $\Omega_m$. See Section 3.3 for further discussion. The auto correlation is given by

$$\xi_i = \frac{1}{3} D'^2(z_i) \int \frac{P(k)}{2\pi^2} dk.$$

We can therefore calculate $C_{ij}$ for a pair of supernovae at $z_i$ and $z_j$ respectively given a set of cosmological parameters.

In the above equations the growth factor is calculated exactly numerically. More insight to the dependence on $\Omega_m$ is given by the commonly used approximation for the growth factor $f = d \ln \delta / d \ln a \approx \Omega_m (z)^\gamma$, where $\gamma \approx 0.6$ (Peebles 1980), with little dependence on the cosmological constant (Lahav et al. 1991), and a slight dependence on $w$ (Wang & Steinhardt 1998). Recent refined calculations predict $\gamma = 0.55$ for the concordance model, and $\gamma = 0.69$ (Linder & Cahn 2007) for a particular modified gravity model, DGP braneworld gravity (Dvali, Gabadadze, & Porrati 2000), though this is just an example of many possible modified gravity models. Below we shall constrain $\gamma$ from the SN Ia data.
From the equations in Section 3.1 it can easily be seen that $\sigma^2_i$ is a function of $\Omega_m$ through

$$\sigma^2 = \frac{\ln(10)^2}{5} \left( \sigma_m^2 + \mu^{err}_i \right)^2 \left( 1 - \frac{c(1+z)^2}{H(z)d_L(z)} \right) \frac{\sigma^2_z}{c^2} (12)$$

where $\sigma_m$ is often set to 300 km s$^{-1}$ and is included to account for nonlinear contributions to $\xi(z)$ (which is derived only in linear theory, Silberman et al. [2001]), and the velocity of the SN within the host galaxy. Here $\sigma_m$ is the intrinsic magnitude scatter and $\mu^{err}$ is the error from the light curve fitting.

### 3.2 Likelihood analysis

To find the set of cosmological parameters $\Theta_{\text{max}} = [\theta_1, \ldots, \theta_N]$ that best fit the data we find the set that maximise the likelihood function. Assuming that the data and the observational errors are Gaussian random fields the likelihood function can be written as

$$L = \frac{1}{\sqrt{(2\pi)^N|\Sigma|}} \exp \left( -\frac{1}{2} \sum_{i,j} D_i \left( \Sigma^{-1} \right)_{ij} D_j \right). \tag{10}$$

where $D_i$ is defined as $D_i = (D_L^{obs} - D_L(z))/D_L(z)$ and $\Sigma$ is the covariance matrix including the observational noise. Following [Gordon, Land & Slosar [2007]] we write this as

$$\Sigma_{ij} = C_{ij}^L + \sigma_i^2 \delta_{ij} \tag{11}$$

where $\sigma_i$ is the standard uncorrelated error given by

$$\sigma_i^2 = \frac{\ln(10)^2}{5} \left( \sigma_m^2 + \mu^{err}_i \right)^2 \left( 1 - \frac{c(1+z)^2}{H(z)d_L(z)} \right) \frac{\sigma^2_z}{c^2} (12)$$

where $\sigma_m$ is often set to 300 km s$^{-1}$ and is included to account for nonlinear contributions to $\xi(z)$ (which is derived only in linear theory, Silberman et al. [2001]), and the velocity of the SN within the host galaxy. Here $\sigma_m$ is the intrinsic magnitude scatter and $\mu^{err}$ is the error from the light curve fitting.

### 4 RESULTS

In the following analysis we assume a flat ΛCDM universe with a dark energy equation of state $w = -1$. To gain an insight for the effect of varying the other cosmological parameters, namely $H_0$, $n_s$, and the “nuisance” parameters $\sigma_m$ and $\sigma_v$ we do not marginalise over them but present the results at some choice values for these parameters. The effect of marginalising over $\sigma_v$ and $\sigma_m$ degrades the error on $\Omega_m^{\text{geom}}$ by about 10 percent, the error on $\Omega_m^{\text{dyn}}$ changes negligibly and the error on $\Omega_m^{\text{ps}}$ degrades by about 30 percent. The larger degradation of the error on $\Omega_m^{\text{ps}}$ is because of its strong degeneracy with $\sigma_n$. Other parameters are fixed as follows; $\Omega_b = 0.04$, $\Omega_{\Lambda} = 0$ and $\sigma_8 = 0.8$. For clarity most contours have only the 1σ confidence level, and where appropriate the assumed values of other parameters are stated.

Table 1 shows a set of results for each “face” of $\Omega_m$ under different parameter combinations. This table shows that the different $\Omega_m$'s are consistent with the canonical value of 0.25 to within 1σ. One can see the degeneracy direction of the errors $(\sigma_v, \sigma_n)$ and $\Omega_m$, discussed below. See table caption for an explanation of the columns.

| $\Omega_m^{\text{geom}}$ | $\Omega_m^{\text{dyn}}$ | $\Omega_m^{\text{ps}}$ |
|-------------------------|-------------------------|-------------------------|
| A                       | B                       | C                       |
| 0.25 ± 0.01             | 0.25 ± 0.01             | 0.25 ± 0.01             |
| 0.24 ± 0.01             | 0.10 ± 0.10             | 0.48 ± 0.52             |

4.3 The Three Faces of $\Omega_m$

From the equations in Section 3.1 it can easily be seen that $C_{ij}^L$ is a function of $\Omega_m$ through

(i) $\Omega_m^{\text{geom}}$, the geometry of the universe, from $H(z)$ and also $d_L(z)$ in Eq. 5. For a flat universe $\Omega_m^{\text{geom}} = 1 - \Omega_{\Lambda}$, with no dependence on other cosmological parameters. $\Omega_m^{\text{geom}}$ is most strongly constrained however by the high redshift SNe through Eqs. 6 and 7.

(ii) $\Omega_m^{\text{dyn}}$: the growth of structure in the universe, from $D_L(z)$ in Eqs. 8 and 9 or the equivalent growth factor parametrization $f = \Omega_m^{\text{dyn}}(z)^2$. We note a strong degeneracy through the product $\sigma_n \Omega_m(z)$ where

$$\Omega_m(z) = \frac{\Omega_m(z = 0)(1+z)^3}{\Omega_m(z = 0)(1+z)^3 + 1 - \Omega_m(z = 0)} \tag{13}$$

for a flat universe, our results can be scaled accordingly.

(iii) $\Omega_m^{\text{ps}}$: the matter power spectrum $P(k)$ in Eqs. 8 and 9. It is well known that the shape of the power spectrum depend on the product $\Gamma = \Omega_m^{\text{ps}} h$, with some degeneracy with e.g. the spectral index $n_s$, $\sigma_n$ and baryon and neutrino mass densities $\Omega_b$ and $\Omega_\nu$. Please note that Eq.s 8 and 9 contain all of the low redshift $\Omega_m$ terms. If the ΛCDM model of the universe is correct then when varying each of these “faces” of $\Omega_m$ separately the results should be consistent with each other. If not this suggests that the ΛCDM model is inconsistent and the data may favour a model which changes the theory of general relativity on large scales or other dark energy models.
Finally Figure 4 shows the effect of \( \sigma_v \) on all the contour pairs. It has the largest effect on \( \Omega_{\text{dyn}} \) and very little effect on \( \Omega_{\text{geom}} \). This is because the diagonal elements of \( C_{ij}^{\text{obs}} \) (see Eq. 4) are approximately proportional to \( (\Omega_{\text{dyn}}) \sigma_v \) and to \( \sigma_{ij} \). Therefore the best-fit \( \Omega_{\text{dyn}} \) decreases as \( \sigma_v \) increases.

5 CONCLUSIONS

We have presented in this paper a unified approach for probing both the expansion of the universe and the growth of structure with SN Ia data and to test the consistency of the ACDM model. We utilised the SN Ia data to derive three distinct estimates of the matter density \( \Omega_m \) which appear in the problem: from the geometry, from the dynamics and from the growth of galaxy peculiar velocity data (using \( D_{\text{L}} - \sigma \) and Tully-Fisher distance indicators) will also provide improvement on the \( \Omega_m^{\text{ps}} \) and \( \Omega_m^{\text{dyn}} \) constraints. Our approach can also be generalised for a range of other cosmological parameters and exotic models of dark energy and gravity.

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