Shear Modulus of Compacted Sandy Clay from Various Laboratory Methods

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Abstract. Nowadays, in geotechnical engineering, laboratory investigations are necessary in order to assess their engineering properties, like stiffness characteristics. Knowledge about soil shear modulus (G) in a strain range of 1.0⋅10⁻⁴÷1% is very important to solve soil response subjected to dynamic loading. For purpose of that paper, shear modulus (G) of compacted sandy clay was measured in Water Centre – Laboratory, at Warsaw University of Life Sciences – SGGW by means of three different laboratory techniques, i.e., resonant column (RC), bender element (BE) and torsional shear (TS). Two methods of travel time identification in BE testing were applied: start to start (STS) and peak to peak (PTP). A brief description of these experimental techniques is given, with a special attention to strain level and excitation frequency they relate to. The main objective of this paper is to compare static against dynamic test results and monotonic against cyclic for cohesive compacted soil. In the next step, the behaviour of compacted sandy clay was compared with the behaviour of natural soil. The results suggest that in the case of BE measurements soil stiffness tends to be overestimated compared with stiffness obtained by RC and/or TS tests. The RC results are in good agreement with TS test results. The results received from the analyzed techniques indicate significant change in shear modulus with frequency and strain. The comparison of the results from different tests should be done at similar frequencies and referred to the same strain level. It is also shown that natural material is characterized by a greater stiffness than a compacted one.

1. Introduction
Nowadays, in geotechnical engineering, laboratory and field investigations are necessary in order to classify geomaterials as well as to assess their engineering properties, like stiffness characteristics. Knowledge about soil shear modulus (G) in a strain range of 1.0⋅10⁻⁴÷1% is very important in seismic response analysis, for designing various geotechnical structures subjected to dynamic, liquefaction assessment, and many further soil dynamics problems. From this perspective, seismic wave-based techniques have lately gained a significant attention. However, it should be remembered that measuring parameter G either in field or in laboratory is a great challenge.
2. Literature review
Bender element (short version BE), resonant column (RC), cyclic simple shear (CSS) tests and others are the most common laboratory techniques usually employed for measuring dynamic material properties of soil [1,2]. Their popularity is connected with promptness of measurements, non-destructive and relatively low-cost of tests. Moreover, the same measurement can be performed in laboratory and in-situ [3].

Resonant column (RC) permits testing an axis-symmetric specimen under torsional and longitudinal excitation. Two parameters are obtained from this technique: the resonant frequency and the material damping coefficient. In the next order, wave velocity and attenuation are computed [4]. A complete description of test setup of conventional RC test has been reported in literature, e.g. [5].

Bender elements (BEs) are currently among the most popular techniques for measuring soil properties at the small strain range [2,6]. This technique involves the use of one piezoceramic element at each end of a soil specimen. BE at one end of the specimen (so called “transmitter”) is excited with a voltage signal. The signal generates a shear wave pulse which is next propagated through the specimen and finally received by BE located at the other end (“receiver”). Shear wave velocity (V_S) of the tested material is subsequently computed by evaluating the travel time of the signal and distance between two BEs [7].

Torsional shear (TS) tests are static, or quasi-static, monotonic or cyclic tests, during which an axially confined cylindrical specimen is sheared through rotating one of its ends. This method is based on the stress-strain relationship hysteresis loop. The loop is obtained by cycling a known torque at the top of the specimen and measuring the resulting displacement by means of a proximitor on the same specimen [8].

The main aim of the presented work is to discuss the measurements of shear modulus (G) of compacted sandy clay at low strains by three different laboratory methods, namely: resonant column (RC), bender element (BE) and torsional shear (TS). These techniques have played a key role in the experimental research on dynamic soil properties for the past decades, although each one has both advantages and disadvantages [9]. The authors examined the performance of RC, BE and TS tests by comparing shear modulus values of the same soil specimens. They wanted to verify as well if these values could be used interchangeably.

3. Materials and methods
3.1. Material and Specimen Preparation
Laboratory tests were conducted on cohesive soil, classified as sandy clay (saCl), in accordance with [10]. Information on the origin of the tested material can be found in [11]. The preparation of the specimen involved special attention. The specimen, having around 70 mm diameter and approx. 140 mm height, was prepared by using the moist tamping technique, with normal compaction, from 3 to 5 layers and 25 blows per layer. Each layer was under compacted to its successive layer. The specimen was directly prepared on the resonant column base using split mould.

3.2. Physical properties analysis
Before the stiffness measurements (RC, BE and TS tests) were performed, the major soil physical properties were determined, i.e., texture and structure, colour, permeability, consistency, pore space, etc. [12]. The optimum moisture content was conducted using the [13], in the Proctor mould with a volume of 2.2 dm^3. The authors used a standard compaction energy of 0.59 J/cm^3. Selected physical and mechanical parameters are given in Table 1.

| Table 1. Basic properties of tested material |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| **Description** | **Density of solid particles** | **Bulk density of soil** | **Dry density of soils particles** | **Moisture content** | **Void ratio** | **Plastic limit** | **Liquid limit** | **Plasticity index** | **Liquidity index** | **Maximum dry unit density** |
| **Unit** | g·cm\(^{-3}\) | g·cm\(^{-3}\) | g·cm\(^{-3}\) | % | - | % | % | % | - | g·cm\(^{-3}\) |
| **Value** | 2.64 | 2.08 | 1.84 | 12.82 | 0.43 | 10.3 | 18.9 | 8.6 | 0.29 | 2.08 |
3.3. Resonant column tests
Main laboratory tests were carried out in a fixed-free configuration of the resonant column apparatus (RCA), supplied by GDS company, Hampshire, U.K. The biggest advantage of the described RCA is that \(BE\) and \(TS\) tests can be simultaneously carried out on identical specimens, which can surely improve the reliability of the test results. The full description of the test equipment can be found in the other authors’ publications, e.g., [11, 14]. A schematic illustration of the modified resonant column apparatus is shown in figure 1.

Just to sum up briefly, in general, in \(RC\) test a torsional or flexural excitation is applied by means of an electromagnetic drive system. The drive system is composed of four electromagnets. During the torsional test, all magnets act in series which apply a net torque to the soil specimen. When flexural test, only two magnetic coils work in order to apply a net horizontal force at the top of the specimen [15]. To find the resonant frequency of the examined specimen associated with a given input voltage (maximum value of 1.0 V), the software allows running both broad and fine frequency sweep operation. The specimen can be vibrated up to a maximum frequency of 0.3 kHz [16]. Resonant frequency obtained for a given input voltage is accepted as the frequency corresponding to the maximum amplitude. For \(RC\) tests, resonant frequency and acceleration determine modulus and strain level.

Shear modulus \((G)\) is calculated from shear wave velocity following equation (1):

\[
G = \rho \cdot V_S^2 = \rho \cdot \left( \frac{2\pi f l}{\beta} \right)^2
\]

(1)

Where \(\rho\) is the mass density of the soil tested, \(V_S\) is shear wave velocity, \(f\) is the natural frequency of the specimen from \(RC\) test, \(l\) means specimen’s length, and \(\beta\) stands for the parameter depends on quotient of mass polar moment of inertia of soil specimen \((I)\) and mass polar moment of inertia of resonant column drive system \((I_0)\) [17].

RC tests comprised the application of isotropic confining pressures at two stages, namely 45 kPa and 90 kPa. At the specified effective confinement, the saturated specimen (i.e., the value of Skempton’s B parameter exceeded 0.95) was allowed to consolidate until the end of primary consolidation. Next, \(RC\) tests were performed. Dynamic properties were then evaluated under drained conditions.
In order to excite the electromagnetic field and induce a wave propagation through the specimen, the corresponding coil voltages values were placed. The magnetic field in the coils interacted with the magnets attached to the driving plate, that in turn conveyed a torsional oscillation to the top of the specimen [5]. The authors started with the coil voltage value of 0.01 V, increasing it every 0.01 V up to 0.1V, and increasing again every 0.1 V to a maximum of 1.0 V. Following this procedure, RC tests were repeated several times. Every time there was a progressive increase in the amplitude of the input voltage, which subsequently allowed to obtain shear modulus corresponding to increasing values of shear strain [18]. For each frequency, 100 cycles, which is referred to as the value of the cycle constant, were used to measure the response of the accelerometer [16]. By using the accelerometer, RC measurements benefit from the harmonic relationship between acceleration and displacement. This allowed the authors to achieve measurements at strain amplitudes near 10^{-4} %.

3.4. Bender element tests

Bender elements tests were carried out in the modified resonant column apparatus. The transmitter element is mounted at the pedestal of RC chamber, while the receiver element is inserted on the top of the specimen. Electric pulses are applied to the transmitter by the waveform generator. They produced continuous sine waves that travel through the soil before being recorded by the receiver on the other end [19].

Although the BE technique is simple, its application for the measurement of small-strain stiffness ($G_{max}$) may not be straightforward. It is caused by difficulties in estimating the exact travel time between the input and output signals. Determination of shear wave velocity is a key element for establishing $G_{max}$ in BE tests [2].

BE tests were performed after finishing RC tests, at the same level of mean effective stress. Following RC test at $p' = 45$ kPa, BE test was conducted. Then, pressure was increased up to $p' = 90$ kPa and the whole procedure was repeated, first RC test next BE test. Source bender element was excited by applying an excitation voltage from 1 V to 14 V.

Two-time domain methods were employed in order to estimate wave velocities, namely start to start, STS, (time difference between the starting and arrival points in the excitation and the response signals, respectively) and first peak to peak, PTP, (time difference between the peak of the transmitted signal and the first major peak of the received signal) techniques. Detailed description of these test techniques is contained in the other authors’ article [2].

The authors generated a sine wave in a range of period from 0.10 to 0.20 ms, with a step every 0.02ms. These periods correspond to the excitation frequencies between 4.5 and 10 kHz. The applied wave’s periods allowed reaching the $L/\lambda$ ratio ranging from 3 to 10. The near field effect was greatly reduced by a very high input voltage. Signal-to-noise ratio was kept by the authors above the level of 4 dB during BE tests, which also helped in improving the reliability of the results. The strain range was assumed not greater than 10^{-3}%%. In order to compute shear modulus ($G$) of compacted sandy clay from BE tests, the authors used equation (2) given by:

$$G = \rho \cdot V_s^2 = \rho \cdot \left( \frac{L_u}{\Delta t} \right)^2$$

, where $L_u$ is the wave’s travel distance (the distance between the transmitter and the receiver), and $\Delta t$ stands for the wave’s travel time [20].

3.5. Torsional shear tests

Torsional shear tests were performed as the last one on the same soil specimen, after RC and BE tests, just like previous research in the modified resonant column apparatus. The shear strain is determined by means of proximity transducer with a resolution of 0.1 μm. The transducer’s target is fixed to the drive system of RCA at a radial distance of about 30 mm, therefore they rotate with the specimen. The proximity transducer is blocked on the frame supporting the coils and monitors the overall rotational movement [9, 21].
TS test enables to evaluate dynamic properties of soils at frequencies usually between 0.0001-0.001 kHz and shear strain levels from 10^{-3} % to 1 %. The deviator stress is plotted against the axial strain and a hysteretic loop is generated. The phase lag between the applied stress and the resulting strain represents the ability of the specimen to store and dissipate energy in a finite period of time [22].

In the presented study, a certain number of loading cycles, namely 10, were applied to the specimen using sinusoidal variation of torque at a constant frequency equal to 1 Hz. Shear modulus was calculated based on equation (3) as:

$$ G = \frac{\tau}{\gamma} $$

where $\tau$ is the maximum shear stress, and $\gamma$ means the maximum shear strain, registered during the 10th excitation cycle hysteresis loop [8]. In order to receive the shear modulus degradation curve, the authors changed the amplitude of excitation from 0.02 V (for $p' = 45$ kPa) or 0.04 V (for $p' = 90$ kPa) to 1 V.

4. Results and discussion
4.1. Compacted sandy clay

Shear modulus of compacted sandy clay from three different laboratory methods (RC, BE and TS) is plotted versus the log of shear strain in figures 2a and 2b, depending on the applied mean effective stress, 45 kPa (figure 2a) or 90 kPa (figure 2b). For BE tests, the strain level was not precisely defined, but it was accepted to be equal to 1·10^{-4} %. The single $G$ value from start to start (STS) and peak to peak (PTP) technique was obtained using the special procedure of selecting shear wave velocity. Details of it can be found in the publication [23]. In the case of RC and TS tests, typical stiffness degradation curves were received. The presented strain’s range was achieved by a stepped approach with an applied voltage to the drive system, see e.g. [5]. Furthermore, with the use of broken lines the corresponded strain limits in these two laboratory methods are marked, i.e., 1.0·10^{-4} % ÷ 2.0·10^{-2} % for $p' = 45$ kPa and 1.0·10^{-4} % ÷ 1.3·10^{-2} % for $p' = 90$ kPa. It may be seen that the same value for the initial and the maximum shear modulus was obtained from cyclic dynamic resonant column tests. $G_{max}$ was gained for the smallest strain, 1.0·10^{-4} %, for both applied stresses. Regarding monotonic static torsional shear tests, the above-mentioned $G$ values also overlapped. $G_{max}$ was gained for 1.0·10^{-4} % as in the case of dynamic tests. The results from BE tests are not in a good agreement with the others. On the other hand, however, there is a relatively good agreement between the data obtained from torsional shear and resonant column tests, especially for higher stress ($p' = 90$ kPa). The average difference between the $G$ values from RC and TS method amounted to 6.3 MPa ($p' = 45$ kPa) and around 4 MPa ($p' = 90$ kPa), which is approx. 14 % and 5 % in favour of RC study. The data from dynamic tests are 1.2 and 1.1 times bigger than from static tests. It is possible to notice that the differences between the $G$ values of these two tests are decreasing with increasing strain. The curves from RC and TS tests deviate gradually as the strain level increases. The sudden drop of shear modulus at increased strain indicates that the soil specimen has reached the linear viscoelasticity limit, where stresses are approx. proportional to strains. The limit of linear viscoelastic response is an important parameter as it suggests when irreversible permanent deformation in the soil can be initiated. Concerning dynamic tests, the difference between shear modulus becomes very large as the strain approached about 1.0·10^{-1} %, i.e., from 79 % to 64 %.

In figure 3, the variation of shear modulus with excitation frequency for all three techniques under two levels of mean effective stress is shown. Analysing figure 3, it can be seen that with an increase of mean effective stress, $G$ increased as well, for all three methods. This trend is widely recognized by the worldwide researchers [24]. The frequency effect, observed in this figure, is evaluated at the same strain level, i.e., 1.0·10^{-4} %. The lowest frequency was used in TS tests ($f = 1$ Hz), then in RC tests (30 Hz < $f < 90$ Hz), and the largest in BE tests (4500 Hz < $f < 10000$ Hz). According to the literature [25], no significant variation in shear modulus of dry and saturated sands in torsional shear and resonant column tests at frequencies between 0.1 Hz and 100 Hz was reported. Similar conclusion can be drawn from the behaviour of the examined soil. Experiments on saturated compacted sandy clay at
comparable strain level using TS and RC tests show no great difference in dynamic properties with frequency: $\Delta G_{\text{max}}(45 \text{ kPa})=9 \text{ MPa}$, $\Delta G_{\text{max}}(90 \text{ kPa})=4.6 \text{ MPa}$. Though, the results from BE tests are significantly different from the others. It can be therefore stated that 100 Hz is the frequency limit. At frequencies smaller than this shear modulus does not exhibit frequency dependency, while at frequencies bigger already shows it. Moreover, this dependency has nonlinear character. As the best regression function, reflecting $G$ dependency on $f$, the power function was proposed (figure 3).

**Figure 2.** Shear modulus of compacted sandy clay from RC, BE and TS tests at $p'=45$ kPa (figure 2a) and $p'=90$ kPa (figure 2b) (broken lines show the same range of strain for RC and TS tests)

**Figure 3.** Comparison of shear modulus from RC, BE and TS tests as a function of excitation frequency at $p'=45$ and 90 kPa

Analysing all three figures (Figs. 2a, 2b, 3) and considering the two applied methods of travel time identification in BE testing, namely start to start and peak to peak, the $G$ values from STS method are bigger than those from PTP method, on average from 16 MPa to 3 MPa. Greater differences in the results were obtained for smaller stress ($p'=45$ kPa).

In Table 2, the summary of maximum shear modulus ($G_{\text{max}}$) obtained by RC, BE and TS tests is presented. Regardless of effective stress applied during experiments, the minimum small-strain stiffness was received from monotonic static torsional shear tests, whereas the maximum from dynamic bender element tests, using start to start method of interpretation.
The difference between $G_{\text{max}}$ from STS and $G_{\text{max}}$ from TS is of the order of 57 % (p’=45 kPa) and 46 % (p’=90 kPa). The difference between $G_{\text{max}}$ from STS and $G_{\text{max}}$ from PTP is thought of the order of 18 % (p’=45 kPa) and 2 % (p’=90 kPa).

The $G_{\text{max}}$-data from Table 2 confirms also the observation from the literature [5,21,26] that the higher the stress of the lateral confinement, the higher the value of shear modulus ($G$). The higher the confining stress leads to interlocking of the clay grains.

Table 2. Maximum shear modulus of compacted sandy clay from RC, BE and TS tests

| Effective stress p’ [kPa] | Maximum shear modulus Gmax [MPa] |
|-------------------------|----------------------------------|
|                         | Resonant Column | Torsional Shear | Bender Element |
| 45                      | 49.00           | 40.00           | 92.55          | 75.75          |
| 90                      | 76.70           | 72.13           | 134.53         | 131.80         |

In Figure 4, the ratio between $G_{\text{max}}$ values (Table 2) from all techniques studied in the article is shown. Based on this figure, the authors would like to compare all methods with each other. This ratio varies between 1.23 and 2.31 at p’=45 kPa and 1.065 and 1.87 at p’=90 kPa. The ratio closer to 1 means good determined results. Irrespective of p’, experimental data from RC and TS tests are well estimated. Again, it can be observed that these two methods are in a good agreement with each other. BE technique, in turn, allows to receive at least one and a half times overestimated results. This finding is in concordance with the previous studies, see e.g., [26].

4.2. Comparison with natural sandy clay

The dynamic properties of compacted sandy clay were next compared with the dynamic properties of natural sandy clay. High quality specimens were extracted from the depth approximately of 2.5m with a thin-walled aluminium tube sampler. The natural soil was obtained from the test site called “Fort Służew”, located in the south of Warsaw, capital of Poland, in the district Ursynów. This material was used for the comparison precisely because of its great similarity to the examined compacted soil. Its moisture content was equal to 12 %, density of solid particles amounted to 2.67 g·cm$^{-3}$, and bulk density to 2.11 g·cm$^{-3}$. All three previously described research techniques (RC, BE and TS) were used to investigate the $G$ parameter of these soils. For comparative tests, one sample subjected to the same mean effective stress as the compacted soil was selected.
As the first results, shear modulus versus shear strain amplitude is shown (Fig. 5a – at $p' = 45$ kPa, Fig. 5b – at $p' = 90$ kPa). It is seen that the behaviour of compacted material is similar to that of natural one under dynamic and cyclic loading. The following general observations coincide:

1/ shear modulus decreases with increasing shear strain;
2/ mean effective stress has a significant influence on shear modulus reduction curve;
3/ as effective stress increases modulus reduction curve shifts to a higher position;
4/ in the case of RC tests, shear modulus keeps at the highest value as long as the shear strain is less than one certain value due to the linearity of the curve in nature; the initial shear modulus is as well the maximum one;
5/ with an increase in strain amplitude beyond a threshold level, the curves from RC tests demonstrate an apparent nonlinearity in nature;
6/ in the case of TS tests, the initial $G$ value corresponding to the maximum one;
7/ shear modulus obtained by BE tests is decidedly bigger than the one from the other analyzed laboratory methods.

![Figure 5](image)

**Figure 5.** Shear modulus versus shear strain for natural and compacted sandy clay at $p' = 45$ kPa (figure 5a) and $p' = 90$ kPa (figure 5b)

In Table 3, the results of maximum shear modulus ($G_{\text{max}}$) of natural sandy clay from RC, BE and TS tests are presented. It should be noted that the measured $G_{\text{max}}$ values by means of monotonic static torsional shear tests were the lowest. However, in the case of the highest $G_{\text{max}}$ values, there is a difference compared to the compacted sandy clay. The highest shear modulus was received this time from peak to peak technique. The authors believe that this is due to lack of data from start to start tests.

| Effective stress $p'$ [kPa] | Maximum shear modulus $G_{\text{max}}$ [MPa] |
|---------------------------|---------------------------------------------|
|                           | Resonant Column | Torsional Shear | Bender Element |
|                           | Start-to-start | Peak-to-peak    |               |
| 45                        | 54.48          | 47.45           | 101.19         |
| 90                        | 90.32          | 78.82           | 150.68         |

$\Delta G_{\text{max}} = G_{\text{max nat}} - G_{\text{max com}}$

| Effective stress $p'$ [kPa] | $\Delta G_{\text{max}}$ [MPa] |
|---------------------------|-------------------------------|
| 45                        | 5.48                          |
| 90                        | 13.62                         |

$\Delta G_{\text{max}}$ = $G_{\text{max nat}}$ - $G_{\text{max com}}$
In the second part of Table 3, the differences between the $G_{\text{max}}$ values ($\Delta G_{\text{max}}$) measured for natural sandy clay ($G_{\text{max nat}}$) and compacted sandy clay ($G_{\text{max com}}$) are shown. This kind of comparison clearly demonstrates that natural material is characterized by a greater stiffness than compacted one. The differences between the $G_{\text{max}}$ results were an average of 13% in favor of natural sandy clay. This result can be explained by taking into account the physical properties of the two materials. Natural sandy clay soil was semi-stiff and medium cohesive. Compacted material had to be softer, less rigid, at most in hard plastic state. Usually compacted cohesive soils are in a plastic or hard plastic state, but a plastic one is preferred. Hence, there are the resulting $G$ differences. Obviously, their value depends on the applied laboratory techniques. The greatest discrepancies were achieved using $BE$ $PTP$ method, whereas the smallest from $TS$ method.

5. Conclusions
Dynamic properties represented by shear modulus for compacted sandy clay were measured using three different laboratory techniques. Resonant column ($RC$), bender element ($BE$) and torsional shear ($TS$) tests were performed in a small ($\gamma < 10^{-3}$%) and moderate ($\gamma < 10^{-1}$%) strain range at two mean effective stress (45 kPa and 90 kPa). Two methods to determine the travel time in $BE$ testing were selected, namely start to start ($STS$) and peak to peak ($PTP$). The experimental programme carried out enabled the comparison of static against dynamic test results and monotonic against cyclic test results for compacted and natural cohesive soil. On the basis of experimental results shown, it is possible to draw the following conclusions.

(a) $BE$ technique provides an overestimated results of shear modulus of examined soil. The $G$ values from $BE$ tests are from 1.55 to 2.31 times larger than those from other two laboratory methods.

(b) The values of $G$ between $RC$ and $TS$ tests correspond well and the $G_{\text{max}}$ results are in a good agreement with each other. Cyclic dynamic tests give slightly greater results than those from static monotonic tests. The difference between $G_{\text{max}}$ from $RC$ and $G_{\text{max}}$ from $TS$ is between 20% and 6%.

(c) Dynamic properties of cohesive soil studied in this paper are frequency and strain dependent. Shear modulus from $BE$ tests is a high-frequency shear modulus because it is reliably measured at frequencies from 4.5 kHz to 10 kHz, which falls in the high-frequency region. On the other hand, shear modulus from $RC$ and $TS$ tests is a low-frequency shear modulus, determined by the excitation frequencies of the Hz range. The observed values of $G$ obtained by $BE$ method are significantly larger, up to around 60%, than those from $RC$ and/or $TS$ tests at the same strain level.

(d) When comparing soil stiffness obtained by different laboratory methods, it is a great importance to make such a comparison at similar frequencies and at similar strain level.

(e) From comparison with natural sandy clay, it results that compacted soil exhibits similar behaviour under dynamic and cyclic loading to natural one. However, small-strain stiffness of compacted material is around 13% smaller than small-strain stiffness of natural soil. The authors believe that this is due to the difference in the state of these two materials. Nevertheless, this deserves further research.

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