REGULARIZATIONS, ANOMALIES AND FERMION NUMBER NON-CONSERVATION IN CHIRAL GAUGE THEORIES

Sinya Aoki

Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan

ABSTRACT

We study how fermion number conservation fails in fermion number preserving regularization schemes. We show that the fermion number have to be carried by the gauge field configurations with non-zero winding number in this scheme and this fermion number is not conserved in the presence of instantons. We also consider other types of regularization scheme which have different global symmetries. In particular, we point out that the fermion number is conserved in the lattice chiral gauge theories with the Wilson-Yukawa coupling.
I. Introduction

It is well-known[1] that the sum of the baryon number \((B)\) and the lepton number \((L)\) is not conserved in the standard model due to an anomaly and \(SU(2)\) instantons. Recently it is pointed out[2], however, that the lattice formulation of chiral gauge theories with the so-called Wilson-Yukawa coupling[3-6] can not produce this anomaly, hence \(B\) and \(L\) are conserved. This fact is rather surprising since this means that the structure of the anomaly could depend on the way how one regularizes the theory. In this paper we investigate the relation among the regularizations, anomalies and the fermion number non-conservation.

Before considering the relation, we first mention subtleties of regularizations for chiral gauge theories. We consider a chiral gauge theory without Higgs fields, whose partition function is given by

\[
Z = \int D\psi D\overline{\psi} Dw_\mu \exp[S_0(\psi, \overline{\psi}, W_\mu) + S_{\text{reg}}(\psi, \overline{\psi}, W_\mu)] \tag{1.1}
\]

where \(\psi\) are fermion fields, \(W_\mu\) are gauge fields, \(S_0\) is the classical part of the action, and \(S_{\text{reg}}\) is the regulator part of the action. The problem in the regularization of chiral gauge theories is that \(S_{\text{reg}}\) is not invariant under the gauge transformation

\[
\begin{align*}
\psi^h &= (h_L P_L + h_R P_R) \psi \\
\overline{\psi}^h &= \overline{\psi} (h_L^\dagger P_R + h_R^\dagger P_L) \\
W_{\mu}^h &= h W_{\mu} h^\dagger - i(\partial_\mu h) h^\dagger
\end{align*}
\tag{1.2}
\]

where \(h\) is a gauge transformation function which satisfies \(h^\dagger h = 1\), \(h_X = D_X(h)\) for \(X = L\ or R\), and \(D_X\) is some unitary representation of the gauge group for the fermions.

Using the identity

\[
\int Dg \exp[S_{\text{GF}}(\psi^g, \overline{\psi}^g, W_{\mu}^g)] = 1 \tag{1.3}
\]

where \(S_{\text{GF}}\) is a gauge-fixing functional including the Faddeev-Popov determinant\(^1\), we obtain

\[
Z = \int Dg D\psi D\overline{\psi} Dw_\mu \exp[S_0(\psi, \overline{\psi}, W_\mu) + S_{\text{reg}}(\psi^g, \overline{\psi}^g, W_{\mu}^g) + S_{\text{GF}}(\psi, \overline{\psi}, W_\mu)]. \tag{1.4}
\]

\(^1\) We can take \(S_{\text{GF}} = 0\) for the lattice regularization.
Since $S_{reg}$ is not gauge invariant, the group-valued field $g$ appears in $S_{reg}$, so that the gauge volume $\int \mathcal{D}g$ can not be factored out. There are two approaches for treating the scalar field $g$. In the first approach we try to decouple $g$ field from the renormalized theory by adding local gauge non-invariant counter terms. This procedure gives a renormalizable and gauge (or BRST) invariant renormalized perturbation theory for anomaly free theories. However, the gauge invariance (1.2) is lost at the regularized level. We call this approach as the gauge non-invariant scheme. In the other approach $g$ field is considered to be the Nambu-Goldstone part of the Higgs field, so that the regularized theory describes a chiral gauge theory with the Higgs field. Although the gauge symmetry (1.2) is lost, the regularized action is invariant under another local transformation:

\[
\begin{align*}
\psi^h &= (h_L P_L + h_R P_R) \psi \\
\bar{\psi}^h &= \bar{\psi}(h_L^\dagger P_R + h_R^\dagger P_L) \\
W_h^\mu &= h W_\mu h^\dagger - i(\partial_\mu h) h^\dagger \\
g^h &= gh^\dagger
\end{align*}
\]

(1.5)

which is identical to the gauge transformation of the gauge-Higgs-fermion system. However, the theory is not manifestly renormalizable due to the non-linearity of $g$. We call this approach as the gauge invariant scheme. The lattice formulation of chiral gauge theories with the Wilson-Yukawa coupling belongs to this scheme.

If we consider theories without Yukawa couplings, the calculation of the fermion determinant for background gauge fields is identical in both schemes if we take the unitary gauge such that $g = 1$. In order to interpret the results we impose the gauge invariance (1.2) for the renormalized theory in the gauge non-invariant regularization scheme, while the gauge symmetry (1.5), not (1.2), is relevant in the gauge invariant regularization scheme and (1.5) is automatically satisfied in this scheme.

II. Anomalies of Noether Currents

In this section we give a formula by which we can easily obtain the divergence of the vector and axial-vector Noether currents as well as their variation under (1.2).
We consider the theory defined by the action:

\[ S = \int dx \sum_{\mu=1}^{4} \bar{\psi}(x) \gamma^{\mu} [\partial_{\mu} + iW_{\mu}^{L}(x)P_{L} + iW_{\mu}^{R}(x)P_{R}] \psi(x) \]  

(2.1)

where \( \psi, \bar{\psi} \) are fermion fields, \( P_{L,R} = \frac{1 \pm \gamma^{5}}{2} \), and \( W_{\mu}^{L,R} \) are general background chiral gauge fields. Introducing some regularization the effective action can be defined;

\[ S_{\text{eff}}(W^{L},W^{R}) = \text{Tr} \ln[\gamma^{\mu}(\partial_{\mu} + iW_{\mu}^{L}(x)P_{L} + iW_{\mu}^{R}(x)P_{R})]. \]  

(2.2)

We denote the parity-odd part of \( S_{\text{eff}} \) as \( \Gamma \). The gauge anomalies can be calculated through

\[ \delta \Gamma(W^{L},W^{R}) \equiv \Gamma(W^{L} + \delta W^{L},W^{R} + \delta W^{R}) - \Gamma(W^{L},W^{R}) \]  

(2.3)

where

\[
\begin{aligned}
\delta W^{L}(x)_{\mu} &= -\partial_{\mu} \theta^{L}(x) + i[\theta^{L}(x),W_{\mu}^{L}(x)] \\
\delta W^{R}(x)_{\mu} &= -\partial_{\mu} \theta^{R}(x) + i[\theta^{R}(x),W_{\mu}^{R}(x)]
\end{aligned}
\]  

(2.4)

and \( \theta^{L,R} \) are infinitesimal gauge transformations of (1.2) so that \( O((\theta^{L,R})^{2}) \) terms are neglected in the definition of \( \delta \Gamma \).

In order to calculate divergences of vector and axial-vector currents we replace \( W^{L,R} \) with

\[
\begin{aligned}
\tilde{W}^{L}_{\mu}(x) &= W^{L}_{\mu}(x) + V_{\mu}(x) + A_{\mu}(x) \\
\tilde{W}^{R}_{\mu}(x) &= W^{R}_{\mu}(x) + V_{\mu}(x) - A_{\mu}(x)
\end{aligned}
\]  

(2.5)

where \( V_{\mu} \ (A_{\mu}) \) is an external U(1) vector( axial-vector) field. They transform as

\[ \delta V_{\mu} = -\partial_{\mu} \theta^{V}(x) \quad \text{and} \quad \delta A_{\mu} = -\partial_{\mu} \theta^{A}(x) \]  

(2.6)

under infinitesimal gauge transformations \( \theta^{V,A} \). Since the U(1) vector and axial-vector Noether currents are defined by

\[
\begin{aligned}
J^{\mu}_{V}(x) &= \frac{\partial S}{\partial V(x)} = i\bar{\psi} \gamma^{\mu} \psi \\
J^{\mu}_{A}(x) &= \frac{\partial S}{\partial A(x)} = i\bar{\psi} \gamma^{\mu} \gamma^{5} \psi
\end{aligned}
\]  

(2.7),
the expansion of $\delta \Gamma$ gives the following relation:

$$
\delta \Gamma(\bar{W}^L, \bar{W}^R) = \delta \Gamma(W^L, W^R) + \int dx \left[ \theta^V(x) \partial_\mu \langle J^\mu_V(x) \rangle + \theta^A(x) \partial_\mu \langle J^\mu_A(x) \rangle \right] \\
+ \int dx \left[ V_\mu(x) \delta \langle J^\mu_V(x) \rangle + A_\mu(x) \delta \langle J^\mu_A(x) \rangle \right] \\
+ O(V^2, A^2, VA)
$$

(2.8)

where

$$
\langle J^\mu_V(x) \rangle = \frac{\partial \Gamma}{\partial V_\mu(x)}(W^L, W^R) \\
\langle J^\mu_A(x) \rangle = \frac{\partial \Gamma}{\partial A_\mu(x)}(W^L, W^R) \\
\delta \langle J^\mu_V(x) \rangle = \delta \frac{\partial \Gamma}{\partial V_\mu(x)}(W^L, W^R) = \frac{\partial (\delta \Gamma)}{\partial V_\mu(x)}(W^L, W^R) \\
\delta \langle J^\mu_A(x) \rangle = \delta \frac{\partial \Gamma}{\partial A_\mu(x)}(W^L, W^R) = \frac{\partial (\delta \Gamma)}{\partial A_\mu(x)}(W^L, W^R)
$$

Here $\langle O \rangle$ denotes the vacuum expectation value of $O$ in the presence of the background gauge fields $W^L$ and $W^R$. From (2.8) we can easily obtain the divergences of U(1) vector and axial-vector currents $\partial_\mu \langle J^\mu_{V,A}(x) \rangle$ as well as their variation $\delta \langle J^\mu_{V,A}(x) \rangle$ under (1.2).

Finally it is noted that $\Gamma$ term can be divided into 2 terms:

$$
\Gamma = \Gamma_{pure} + \Gamma_{local}
$$

(2.10)

where $\Gamma_{pure}$ has the non-local form of $W^L$ and $W^R$ while $\Gamma_{local}$ only contains the local terms. $\delta \Gamma_{pure}(W^L, W^R)$ is the usual (consistent) gauge anomaly and is regularization-independent.

III. Results of Dirac Type regularizations

The dimensional regularization[8] for the Dirac fermions is defined by

$$
S_0 + S_{reg} = \int d^D x \sum_{\mu=1}^D \bar{\psi}(x) \gamma^\mu (\partial_\mu + iW^L_\mu P_L + iW^R_\mu P_R) \psi(x)
$$

(3.1)

where $D = 4 - 2\varepsilon$, the background chiral gauge fields $W^L,R_\mu$ are defined only in 4 dimensions and we use the ’t Hooft-Veltman definition[8] of $\gamma^5$ satisfying $\gamma^5 \gamma^\mu = -\gamma^\mu \gamma^5$ for $\mu = 1 \sim 4$.
and \( \gamma^5 \gamma^\nu = \gamma^\nu \gamma^5 \) for \( \nu = 5 \sim d \). It is noted that because of this \( \gamma^5 \) property the left-handed part of \( \psi \) couples to the right-handed part of \( \bar{\psi} \) through the term \( \sum_{\mu=5}^D \bar{\psi} \gamma^\mu \partial_\mu \psi \). Therefore we have to assign which left-handed (Weyl) fermion couples to which right-handed (Weyl) fermions in order to form one Dirac fermion and, for the one generation standard model, we have to introduce a right-handed neutrino at the regularized level. We call this type of regularization the Dirac type.

The lattice version of the Dirac type regularization is given by

\[
S_{\text{lat}}(\nu) = \frac{r}{4a} \sum_{x,\mu} \bar{\psi}(x) \{U_{\mu}^L(x) + U_{\mu}^R(x)\} \psi(x + a\mu) - \{U_{-\mu}^L(x) + U_{-\mu}^R(x)\} \psi(x - a\mu)
\]

(3.2)

where \( a \) is the lattice spacing, \( U_{\mu}^{L,R} = \exp[i\mu W_{\mu}^{L,R}] \), and \( r \) in \( S_{\text{reg}} \) is the Wilson-Yukawa coupling. We need \( U^L + U^R \) in \( S_{\text{reg}} \) in order to use the formula for the U(1) Noether currents in Sect. II.

For the dimensional regularization we obtain

\[
\delta \Gamma_{\text{dim}}^{(L,R)} = \int dx \frac{i \mu \nu \alpha \beta}{24 \pi^2} \text{tr} \left[ \partial_\mu (W^L_\nu \partial_\alpha W^L_\beta + \frac{i}{2} W^L_\nu W^L_\alpha W^L_\beta) \right. \\
- \left. \partial_\nu (W^R_\mu \partial_\alpha W^R_\beta + \frac{i}{2} W^R_\nu W^R_\alpha W^R_\beta) \right] (x)
\]

(3.3)

and for the lattice regularization

\[
\delta \Gamma_{\text{lat}}^{(L,R)} = i \mu \nu \alpha \beta \int dx \text{tr} \left[ (I_1 + I_2) W^L_\nu \partial_\alpha W^L_\beta + i (I_1 - I_2) W^L_\nu W^L_\alpha W^L_\beta \right] \\
- \theta^R \partial_\mu \left( (I_1 + I_2) W^R_\nu \partial_\alpha W^R_\beta + i (I_1 - I_2) W^R_\nu W^R_\alpha W^R_\beta \right) (x)
\]

(3.5)

\[
\Gamma_{\text{lat}}^{(L,R)} = i \mu \nu \alpha \beta \int dx \text{tr} \left[ \{2I_1/3 \partial_\mu W^L_\nu + \partial_\mu W^R_\nu\} + i I_3 (W^L_\mu W^L_\nu + W^R_\mu W^R_\nu) \right] \\
\times (W^L_\alpha W^R_\beta - W^R_\alpha W^L_\beta) (x).
\]

(3.6)
Since
\[ I_1 = \int_{-\pi}^{\pi} d^4p \frac{4r MS_\mu^2 - M^2 C_\mu}{F^3} C_\nu C_\alpha C_\beta = \frac{1}{32\pi^2}, \] (3.7a)
\[ I_2 = \int_{-\pi}^{\pi} d^4p \frac{4r M^3 S_\mu^2 - M^4 C_\mu}{F^4} C_\nu C_\alpha C_\beta = \frac{1}{96\pi^2}, \] (3.7b)
\[ I_3 = \int_{-\pi}^{\pi} d^4p \frac{4r MS_\mu^2 - M^2 C_\mu}{2F^4} S^2 C_\nu C_\alpha C_\beta = \frac{1}{96\pi^2}, \] (3.7c),
where \( S_\mu = \sin p_\mu, C_\mu = \cos p_\mu, S^2 = \sum_\mu S_\mu^2, M = r \sum_\mu (1 - C_\mu) \) and \( F = S^2 + M^2 \), we find that
\[ \delta \Gamma^{(\text{dim})}_{\text{pure}} = \delta \Gamma^{(\text{lat})}_{\text{pure}} \quad \text{and} \quad \delta \Gamma^{(\text{dim})}_{\text{local}} = \delta \Gamma^{(\text{lat})}_{\text{local}} \] (3.8)

From the expression above for \( \Gamma \) and the formula in the previous section it is easy to see

(1) The U(1) axial-vector Noether current has an anomaly:
\[ \langle \partial_\mu J^\mu_A \rangle = i \frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \text{tr} \partial_\mu \left[ W^L_\nu \{ 2 \partial_\alpha W^L_\beta + i W^L_\alpha W^L_\beta + i W^R_\alpha W^R_\beta \} + W^R_\nu \{ 2 \partial_\alpha W^R_\beta + i W^R_\alpha W^R_\beta + i W^L_\alpha W^L_\beta \} + 2W^L_\nu \partial_\alpha W^R_\beta \right]. \] (3.9)

For the QCD case \( W^L = W^R = W \) this agrees with the well-known result[9]:
\[ \langle \partial_\mu J^\mu_A \rangle = i \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2} \text{tr} G_{\mu\nu} G_{\alpha\beta} \] (3.10)
where \( G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + i[W_\alpha, W_\beta] \). It is noted that the local term \( \Gamma_{\text{local}} \) is necessary to obtain the gauge invariant result (3.10) with the correct normalization \( 1/(16\pi^2) \).

(2) The axial-vector Noether current is not invariant under (1.2):
\[ \delta \langle J^\mu_A \rangle = i \frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \text{tr} \left[ \partial_\nu (\theta^L - \theta^R) \{ \partial_\alpha (W^L_\beta - W^R_\beta) + i W^L_\alpha W^L_\beta - i W^R_\alpha W^R_\beta \} + (\theta^L - \theta^R) \{ i[W^R_\alpha, \partial_\nu W^L_\beta] - i[W^L_\alpha, \partial_\nu W^R_\beta] \} + (W^R_\nu W^R_\alpha W^R_\beta + W^R_\nu W^L_\alpha W^L_\beta - W^L_\nu W^R_\alpha W^R_\beta - W^L_\nu W^L_\alpha W^R_\beta) \right]. \] (3.11)

It is clear that the current is invariant under (1.2) only for vector gauge theories such as QCD \( (\theta^L = \theta^R) \).

(3) Since the regularized action is manifestly invariant under the U(1) vector transformation, the U(1) vector Noether current is conserved:
\[ \langle \partial_\mu J^\mu_V \rangle = 0. \] (3.12)
The fermion number $Q_F = \int d^3 x J^0_V$ defined from the vector Noether current is conserved. This is the problem recently pointed out [2] for the lattice action. However this vector Noether current also is not invariant under (1.2):

$$\delta \langle J^\mu_V \rangle = i \epsilon^{\mu\nu\alpha\beta} \frac{8\pi^2}{3} \text{tr} \partial_\nu \left[ (\theta^L - \theta^R) (\partial_\alpha (W^L_\beta + W^R_\beta) + i[W^L_\alpha, W^R_\beta]) \right].$$

(3.13)

Therefore, in the gauge non-invariant regularization scheme, $Q_F$ does not correspond to the observed fermion number. The following modified non-Noether currents:

$$\tilde{J}^\mu_V = J^\mu_V + K^\mu_V$$

$$\tilde{J}^\mu_A = J^\mu_A + K^\mu_A$$

are gauge invariant under (1.2) [10]:

$$\delta \langle \tilde{J}^\mu_V \rangle = \delta \langle \tilde{J}^\mu_A \rangle = 0,$$

(3.15)

where

$$K^\mu_V = i \epsilon^{\mu\nu\alpha\beta} \frac{8\pi^2}{3} \text{tr} \left[ W^L_\nu \{ \partial_\alpha (W^L_\beta + W^R_\beta) + \frac{i2}{3} [W^L_\alpha, W^L_\beta] \} ight.$$  

$$\left. - W^R_\nu \{ \partial_\alpha (W^L_\beta + W^R_\beta) + \frac{i2}{3} [W^R_\alpha, W^R_\beta] \} \right],$$

$$K^\mu_A = i \epsilon^{\mu\nu\alpha\beta} \frac{24\pi^2}{3} \text{tr} (W^L_\nu - W^R_\nu) \left[ \partial_\alpha (W^L_\beta - W^R_\beta) + iW^L_\alpha W^L_\beta - iW^R_\alpha W^R_\beta \right].$$

(3.16)

These modified currents give the desired anomalies:

$$\partial_\mu \langle \tilde{J}^\mu_V \rangle = i \epsilon^{\mu\nu\alpha\beta} \frac{32\pi^2}{3} \text{tr} \left[ G^{L}_{\mu\nu} G^{L}_{\alpha\beta} - G^{R}_{\mu\nu} G^{R}_{\alpha\beta} \right]$$

(3.18a)

$$\partial_\mu \langle \tilde{J}^\mu_A \rangle = i \epsilon^{\mu\nu\alpha\beta} \frac{32\pi^2}{3} \text{tr} \left[ G^{L}_{\mu\nu} G^{L}_{\alpha\beta} + G^{R}_{\mu\nu} G^{R}_{\alpha\beta} \right]$$

(3.18b)

where $G^{L}_{\mu\nu} = \partial_\mu W^L_\nu - \partial_\nu W^L_\mu + i[W^L_\alpha, W^L_\beta]$ and $G^{R}_{\mu\nu} = \partial_\mu W^R_\nu - \partial_\nu W^R_\mu + i[W^R_\alpha, W^R_\beta]$. The anomalies appear in the gauge-invariant non-Noether currents, not in the Noether currents.

The gauge invariant fermion number is given by

$$\tilde{Q}_F = Q_F + Q_B = \int d^3 x \overline{\psi} \gamma^0 \psi + \int d^3 x K^0_V.$$

Here $Q_B$ is equal to the winding number of the gauge field, which is not conserved in the presence of instantons. In the Dirac type regularization scheme the fermion number carried
by the gauge field is changed by the instantons while the fermion number carried by the fermion field is always conserved. Since the instantons induce no fermionic zero-modes which violate fermion number in this regularization scheme, no fermion-number violating vertex[1] is generated from the fermion determinant. If we can calculate the transition rate among the states with different $Q_B$ by some non-perturbative method, it directly gives the rate of fermion number non-conservation without referring to fermionic matrix elements.

Now we consider the case of the gauge invariant Dirac-type regularization scheme. In this scheme, the relevant symmetry is (1.5) and the conserved vector Noether current is invariant under (1.5). Therefore, the fermion number can not be violated in this formulation as pointed out in ref.[2]. Since the currents are always gauge invariant due to the presence of scalar fields $g_L$ and $g_R$, the gauge invariant fermion number is conserved in the lattice chiral gauge theories with the Wilson Yukawa coupling.

IV. New Regularization for Weyl Fermions

The Dirac-type regularization used in the previous sections can not deal with the one-generation standard model without right-handed neutrinos. (It can deal with the one-generation standard model without right-handed neutrinos. See ref.[11].) In this section, we propose a new regularization scheme in order to deal with a single left-handed (right-handed) fermion, even at the regularized level. We call this regularization scheme the Weyl type. We combine the Weyl-type regularization with the dimensional regularization and the lattice regularization, though we can combine it with any other regularizations such as the Pauli-Villars regularization.

The action for one left-handed fermion is

$$S_0 + S_{reg} = \int d^D x \sum_{\mu=1}^{4} \overline{\psi}^L \gamma^\mu_L (\partial^\mu + igW^L_\mu) \psi^L + \frac{1}{2} \sum_{\mu=5}^{d} (\psi^L C^L \gamma^\mu_L \partial^\mu \psi^L - \overline{\psi}^L \gamma^\mu_L \partial^\mu C^L \psi^L) \quad (4.1)$$
for the dimensional regularization[11] and
\[ S_{\text{reg}} = -\frac{r}{4} \sum_{x,\mu} \left[ \psi^L(x) C^L \{ \psi^L(x + a\mu) + \psi^L(x - a\mu) - 2\psi^L(x) \} \right. \\
- \left. \overline{\psi}^L(x) C^L \{ \overline{\psi}^L(x + a\mu) + \overline{\psi}^L(x - a\mu) - 2\overline{\psi}^L(x) \} \right] \\
\] (4.2)
for the lattice regularization[12]. Here \( \psi^L = P_L \psi, \gamma^\mu_L = \gamma^\mu P_L, \) \( C^L = C P_L, \) and \( C \) is the charge conjugation matrix which satisfies \( C \gamma^\mu C^{-1} = -(\gamma^\mu)^T. \) It is easy to see that the fermion number as well as the gauge symmetry is violated by the Majorana type terms in \( S_{\text{reg}} \).

We obtain
\[ \delta \Gamma_{\text{pure}}(W_L) = \int dx \frac{i\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \text{tr} \left[ \partial^L \partial_\mu \left( W^L_\nu \partial_\alpha W^L_\beta + i\frac{1}{2} W^L_\nu W^L_\alpha W^L_\beta \right) \right] \] (4.3)
and \( \Gamma_{\text{local}}(W^L) = 0 \) for both regularizations. It is noted again that the parity-even terms are neglected here. From the above expression, we find that the U(1) left-handed Noether current has non-zero divergence;
\[ \partial_\mu \langle J^\mu_L \rangle = i\frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \text{tr} \partial_\mu (W^L_\nu \partial_\alpha W^L_\beta + i\frac{1}{2} W^L_\nu W^L_\alpha W^L_\beta), \] (4.4)
and the current is not gauge invariant;
\[ \delta \langle J^\mu_L \rangle = i\frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \text{tr} \partial_\mu \theta^L (2\partial_\alpha W^L_\beta + i\frac{1}{2} W^L_\alpha W^L_\beta). \] (4.5)
It is obvious that there is no mixing term between left- and right- gauge fields and the right-handed Noether current becomes
\[ \partial_\mu \langle J^\mu_R \rangle = -i\frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \text{tr} \partial_\mu (W^R_\nu \partial_\alpha W^R_\beta + i\frac{1}{2} W^R_\nu W^R_\alpha W^R_\beta). \] (4.6)
It is noted that if we use this regularization for QCD, the divergence of the U(1) axial-vector Noether current, \( J^L_\mu - J^R_\mu, \) does not agree with the desired result, eq.(3.10). If we use this type of regularization for the one-generation standard model without right-handed neutrinos, we obtain the anomaly for the baryon number and the lepton number, as expected. However the result is not invariant under (1.2) and, therefore, the left-handed fermion number, \( \int d^3 x J^0_L, \) is not an observable in the gauge non-invariant scheme.
The gauge invariant left-handed non-Noether current can be defined as

\[ \tilde{J}^\mu_L = J^\mu_L + K^\mu_L \]  

(4.7)

where

\[ K^\mu_L = i \frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \text{tr} W^L_\nu (2\partial_\alpha W^L_\beta + i \frac{3}{2} W^L_\alpha W^L_\beta). \]  

(4.8)

This modified current has the desired anomaly;

\[ \partial_\mu \langle \tilde{J}^\mu_L \rangle = i \frac{\epsilon^{\mu\nu\alpha\beta}}{32\pi^2} \text{tr} G^L_{\mu\nu} G^L_{\alpha\beta}. \]  

(4.9)

Again the left-handed fermion number can be carried by the gauge field as well as the fermion fields and both bosonic and fermionic left-handed fermion numbers are violated by the instantons. Finally we obtain

\[ \partial_\mu \langle \tilde{J}^\mu_L \rangle = \partial_\mu \langle \tilde{J}^\mu_L + \tilde{J}^\mu_R \rangle = i \frac{\epsilon^{\mu\nu\alpha\beta}}{32\pi^2} \text{tr} \left[ G^L_{\mu\nu} G^L_{\alpha\beta} - G^R_{\mu\nu} G^R_{\alpha\beta} \right] \]  

(4.10a)

\[ \partial_\mu \langle \tilde{J}^\mu_A \rangle = \partial_\mu \langle \tilde{J}^\mu_L - \tilde{J}^\mu_R \rangle = i \frac{\epsilon^{\mu\nu\alpha\beta}}{32\pi^2} \text{tr} \left[ G^L_{\mu\nu} G^L_{\alpha\beta} + G^R_{\mu\nu} G^R_{\alpha\beta} \right] \]  

(4.10b)

These anomalies in the Weyl type regularization are identical to those in the Dirac type regularization, eqs.(3.18a-b).

From the result of sect. III. and IV., it is concluded that anomalies of the gauge invariant non-Noether currents do not depend on types of regularizations, Dirac type or Weyl type. However so far we do not have regularization schemes which have gauge-invariant vector and axial-vector Noether currents.

V. Discussions

In this paper we have investigated the relation among regularizations, anomalies and the fermion number non-conservation in general chiral gauge theories. The gauge invariance (1.2) play a crucial role in giving the unique anomaly of the vector current, (3.18a) or (4.10a), which does not depend on global symmetries of regularization schemes, and in inducing the fermion number non-conservation. Fields which carry the fermion number,
however, depend on global symmetries of regularization schemes. In particular we show that bosonic fields can have non-zero fermion number non-perturbatively. The structure of fermionic zero modes in the presence of instantons also depends on global symmetries of regularization schemes. In the Dirac type regularization scheme there is no fermionic zero mode which violates fermion number.

The fermion number non-conservation can not occur in lattice chiral gauge theories with the Wilson-Yukawa coupling since the relevant gauge symmetry is different from (1.2). Even if the fermion number is explicitly broken by the (gauge invariant) Majorana Wilson-Yukawa coupling, this explicit breaking does not lead to the correct anomaly (4.10a). (This is also true for the proposal of ref. [18].) So far there is no satisfactory gauge invariant lattice formulation which gives the correct anomaly of the vector current.

Finally we consider the lattice formulation by the Rome group[19] where a dummy gauge singlet fermion \( \chi \) is introduced to construct the Wilson term. This formulation can deal with the left-handed (right-handed) fermion only and is classified into the gauge non-invariant scheme. The action for the regulator becomes,

\[
S_{\text{reg}} = \frac{1}{2a} \sum_{x, \mu} \overline{\chi}^R(x) \gamma^\mu \left[ \chi^R(x + a\mu) - \chi^R(x - a\mu) \right] - \frac{r}{2a} \sum_{x, \mu} \left[ \chi^R(x) \{ \psi^L(x + a\mu) + \chi^R(x - a\mu) - 2\chi^R(x) \} \right] + \psi^L(x - a\mu) - 2\psi^L(x) + \overline{\psi}^L(x) \{ \chi^R(x + a\mu) + \chi^R(x - a\mu) - 2\chi^R(x) \} \right]
\]

for the left-handed fermion \( \psi^L \). The left-handed fermion number for the physical field \( \psi^L \) is violated by the Wilson term in \( S_{\text{reg}} \). This formulation is equivalent to the Dirac type regularization for a Dirac field \( (\psi^L, \chi^R) \) with the gauge fields \( (W^L_\mu, 0) \). Thus, we obtain

\[
\delta \Gamma_{\text{pure}}(W^L) = \int dx \, i \frac{\epsilon^{\mu \nu \alpha \beta}}{24\pi^2} \text{tr} \left[ \theta^L \partial_\mu (W^L_\nu \partial_\alpha W^L_\beta + \frac{i}{2} W^L_\nu W^L_\alpha W^L_\beta) \right] (x)
\]

which is identical to the result of the Weyl type regularization, (4.3). Therefore we obtain the same results for the anomalies (4.4) and (4.5): The U(1) left-handed Noether current for \( \psi^L \) has non-zero divergence and the current is non-invariant under (1.2). The invariant left-handed non-Noether current has the desired anomaly (4.9) and the fermion number is carried by both the fermion field and the gauge field.
ACKNOWLEDGMENTS

We would like to thank Drs. H. Kawai, K. Fujikawa, J. Shigemitsu, R. Shrock and A. Ukawa for useful discussion and Drs. H. Murayama and T. Yanagida for useful comment on the formula in sect.II.

References

[1] G. ’t Hooft, *Phys. Rev. Lett.* **37**(1976)110; *Phys. Rev. D**14**(1976)3432.
[2] T. Banks, *Phys. Lett.* **B272**(1991)75.
[3] P.D. Swift, *Phys. Lett.* **B145**(1984)256.
[4] J. Smit, *Acta Phys. Polon.* **B17**(1986)531; *Nucl. Phys. B*(Proc. Suppl.)**4**(1988)451.
[5] S. Aoki, *Nucl. Phys. B*(Proc. Suppl.)**4**(1988)479; *Phys. Rev. Lett.* **60**(1988)2109;

  *Phys. Rev. D**38**(1988)618; *Nucl. Phys. B*(Proc. Suppl.)**9**(1989)584.
[6] K. Funakubo and T. Kashiwa, *Phys. Rev. Lett.* **60**(1988)2133.
[7] S. Aoki, *Phys. Lett.* **B247**(1990)357; *Phys. Rev. D**42**(1990)2806.
[8] G. ’t Hooft and M. Veltman, *Nucl. Phys. B**44**(1972)189.
[9] S.L. Adler, *Phys. Rev.* **177**(1969)2426;

  J.S. Bell and R. Jackiw, *Nuovo Cimento* **60A**(1969)47.
[10] M.J. Dugan and A.V. Manohar, *Phys. Lett.* **B265**(1991)137.
[11] S. Aoki, ”Chiral Symmetry, fermion masses and global anomalies”, UTHEP-224, talk given at *Hot Summer Daze Workshop*, BNL, New York, USA, 1991.
[12] S. Aoki, in Proceedings of 1991 International Symposium on Lattice Field Theory, UTHEP-230, Nucl. Phys. B (Proc. Suppl.) in press.
[13] S. Aoki, I-H. Lee, and S.-S. Xue, *BNL Report 42494* (Feb. 1989); *Phys. Lett. B**229**(1989)79;

  I-H. Lee, *Nucl. Phys. B*(Proc. Suppl.)**17**(1990)457.
[14] S. Aoki, I-H. Lee, J. Shigemitsu, and R.E. Shrock, *Phys. Lett. B**243**(1990)403.
[15] S. Aoki, I-H. Lee, and R.E. Shrock, *Nucl. Phys. B**355**(1991)383;
S. Aoki, in *Strong Coupling Gauge Theories and Beyond*, eds. T. Muta and K. Yamawaki (World Scientific, 1991) p. 349;

*Nucl. Phys. B*(Proc. Suppl.)20(1991)589.

[16] W. Bock, A.K. De, K. Jansen, J. Jersak, T. Neuhaus, and J. Smit, *Phys. Lett.* B232(1989)436; *Nucl. Phys. B344*(1990)207; W. Bock and A.K. De, *Phys. Lett.* B245(1990)207; A.K. De, *Nucl. Phys. B*(Proc. Suppl.)20(1991)572; W. Bock, A.K. De, C. Frick, K. Jansen, and T. Trappenberg, *HLRZ-91-21*.

[17] M. Golterman and D. Petcher, *Phys. Lett.* B247(1990)370.

[18] E. Eichten and J. Preskill, *Nucl. Phys. B145*(1986)179.

[19] A. Borrelli, L.Maiani, G. Rossi, R. Sisto and M. Testa, *Phys. Lett.* B221(1989)360; *Nucl. Phys. B333*(1990)335;

Y. Kikukawa, *Mod. Phys. Lett.* A7(1992)871.