Fundamental units: physics and metrology

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Abstract. The problem of fundamental units is discussed in the context of achievements of both theoretical physics and modern metrology. On one hand, due to fascinating accuracy of atomic clocks, the traditional macroscopic standards of metrology (second, metre, kilogram) are giving way to standards based on fundamental units of nature: velocity of light $c$ and quantum of action $h$. On the other hand, the poor precision of gravitational constant $G$, which is widely believed to define the “cube of theories” and the units of the future “theory of everything”, does not allow to use $G$ as a fundamental dimensional constant in metrology. The electromagnetic units in SI are actually based on concepts of prerelativistic classical electrodynamics such as ether, electric permittivity and magnetic permeability of vacuum. Concluding remarks are devoted to terminological confusion which accompanies the progress in basic physics and metrology.

1 Introduction

The problem of fundamental units has many facets, three of which seem to be most important: theoretical, experimental and technological. At present they are inseparable. Theory, the so called Standard Model, formulates basic physical laws and mathematical methods of their application. Theoretical laws were established and continue to be established and tested on the basis of ingenious experiments and astronomical observations of higher and higher accuracy for an expanding space of parameters.

Precision experiments and observations, in their turn, are unthinkable without modern high technologies, including lasers and computers. These technologies are indissolubly connected with metrology – creation, perfection and unification of standards of physical units, while metrology is widely using the results of such theories as quantum mechanics, relativity theory, electrodynamics, condensed matter theory etc. Thus the circle is closed.

The situation is additionally complicated by the fact that the Standard Model is not a complete theory. It has many unsolved problems. Perhaps the most burning is the problem of existence of fundamental scalar particles (higgses), responsible for the masses of all fundamental particles (from the heaviest one – $t$-quark to the lightest of the three neutrinos). We still do not understand the role of the three families of leptons and quarks. It would be too naive to think that their only justification is CP-violation. We still lack a successful theory unifying electroweak and strong interactions. The hypothesis of existence of moderately violated supersymmetry – symmetry between fundamental bosons and fermions...
is still not confirmed by experiments. Mathematical constructs of the type of superstrings and M-theory which in the beginning were considered as attempts to unify quantum gravity with electroweak and strong interaction, as time goes by, withdraw into a separate field of mathematics, whose practitioners do not promise any applications to the real physical world. The situation might become drastically different if manifestations of extra space dimensions are discovered; in particular if laws of gravity turned out to change at TeV scale.

Sections 2-6 are devoted to the history of units based on $c$, $\hbar$, $G$. Sections 7-9 deal with the units based on $c$, $\hbar$, $e$ and precision frequency measurements. Section 10 compares the Gaussian units and SI units. It is argued that while the latter are more convenient for practical purposes, the former allow to better understand the basic notions of modern physics. Therefore the use of both systems of units should be allowed in physics textbooks. Section 11 contains concluding remarks.

2 Fundamental parameters and units

The essence of theoretical physics is expressed by dimensionless equations for dimensionless quantities. However one cannot do experimental physics (and to teach physics) without dimensional quantities and dimensional units.

In what follows we shall refer to dimensionless fundamental constants such as $e^2/\hbar c$, or $m_e/m_p$ as fundamental parameters (here $e$ is the electron charge, $\hbar$ is the reduced quantum of action ($\hbar = \hbar/2\pi$) and of angular momentum, $c$ – velocity of light in vacuum, $m_e$ and $m_p$ are masses of electron and proton respectively).

In the absence of established terminology we shall refer to dimensional fundamental constants as fundamental units. Examples of units: $c$ (for velocity), $\hbar$ (for action and angular momentum). According to our definition $G$ is also a fundamental unit (though indirectly).

3 Planck units

When in 1899-1900 Planck discovered $\hbar$ [1], he used this discovery to introduce universal units, which at present are written in the form

$$ l_P = \hbar/m_P c, \quad t_P = \hbar/m_P c^2, \quad m_P = (\hbar c/G)^{1/2}, $$

(1)

where $G$ is Newton’s gravitational constant.

Planck derived his units by using dimensional order of magnitude relations:

$$ c = \frac{l_P}{t_P}, \quad \frac{Gm_P^2}{l_P^2} = m_P c^2, \quad \frac{Gm_P^2}{l_P^2}t_P = \hbar. $$

(2)

He was inspired by the idea that his units are universal (contrary to “handcrafted” earthbound ordinary units – meter, second, gram): they are the same at any far away corner of the universe.
Planck also considered as universal the Planck temperature \( T_P = \frac{m_P c^2}{k} \). But Boltzman’s \( k \) is not a universal unit, it is a conversion factor: \( k = 8.6 \cdot 10^{-5} \) eV/K (hint: \( \hbar \omega / kT \)).

### 4 \( c, h, G \) – units

From the point of view of the future “theory of everything” it is natural to use \( c, h, G \) as fundamental dimensional constants.

In 1928 Gamov, Ivanenko and Landau \footnote{2} considered the theory “of the world as whole” in terms of dimensional fundamental constant \( c, h, G \). In 1928 Landau was 20 years old, Gamov and Ivanenko – 24 (see Figs. 1, 2). They had written the paper “World Constants and Limiting Transitions” (see Fig. 3) as a humorous birthday present to their friend, a young lady. None of them ever referred this paper in their subsequent publications. But the ideas of the paper were fundamental. In 1936 Bronstein \footnote{3} worked at a theory in which all three constants are finite. It was one of the first papers on relativistic quantum gravity. In 1967 ideas of refs. \footnote{2,3} were presented in the form of a cube (see Figs. 4, 5) by Zelmanov \footnote{4}. Later on it was further developed by others \footnote{5,6}.

The vertices of this cube represent nonrelativistic mechanics (NM), nonrelativistic gravity (NG), nonrelativistic quantum mechanics (QM), special relativity (SR), quantum field theory (QFT), general relativity (GR) and finally relativistic quantum gravity (QGR) or theory of everything (TOE).

The cube, made of units, is “endowed” with dimensionless parameters like \( \alpha, \alpha_s, \) mixing angles, mass ratios, etc. Their values are expected to follow from TOE. Similar to the cube is “dimensional pyramid” (Kuchar \footnote{9}, Sanchez \footnote{10}) with 4 vertices and 4 planes, Fig. 4.

Note that Einstein tried to build a unified theory of electricity and gravity ("TOE") in the left-hand vertical plane of the cube, without Quantum Mechanics, without \( h \).

Planck units allow one to deal in the equations of TOE only with dimensionless functions of dimensionless variables and dimensionless parameters of the type \( \alpha = e^2/\hbar c \) or \( m_e/m_p \). Conceptually Planck units are excellent, but practically they have serious shortcomings, caused by \( G \), the same \( G \) which allows to bring gravitation and cosmology into the realm of quantum phenomena. Thus the source of strength at the same time turns out to be a source of weakness.

### 5 Planck units are impractical

The obvious shortcoming of Planck units is that they differ by many orders of magnitude from atomic units commonly used in physics. Their values are natural for the early universe and TOE, but not for mundane physics:

\[
l_P = 10^{-35} \text{ m} , \quad t_P = 10^{-43} \text{ s} , \quad E_P = m_P c^2 = 10^{19} \text{ GeV} .
\]

The energy which corresponds to the Planck mass is unattainable by accelerators even of the remotest future. (Note, however, that it is only a few orders
of magnitude larger than the grand unification scale of electroweak and strong interactions.) The Planck units of length and time are vanishingly small compared with atomic units. Of course the huge powers of ten are not frightful by themselves. As is well known, atomic units also differ by many orders of magnitude from SI units, which does not prevent atomic standards to be the base of modern metrology.

Much more essential is another shortcoming of Planck units, which stems from the fact that $G$ is known with rather poor accuracy (of order of $10^{-3}$, by five – four orders worse than those of $c$ and $\hbar$, and by 12 orders worse than the precision of atomic clocks). Thus it is impossible to use the Planck units as standards in modern precision physics and technology.

# 6 Units of Stoney

The use by Planck of $G$ as a basis for defining the unit of mass was caused by absence at the beginning of the 20th century of another natural, not “handcrafted”, candidate for the unit of mass. In that respect Planck’s universal units resemble the universal units suggested 30 years earlier by Irish physicist Stoney (1826 - 1910), secretary of Irish Royal Society (see Figs. 7,8). By studying electrolysis, he was the first who measured the value of elementary charge $e$ and introduced into physics the term “electron” for the carrier of this charge (in modern terminology it is ion). From $e, c, G$ Stoney [9] constructed in 1870 - 1880 universal units with dimensions of length, time and mass:

$$l_S = e\sqrt{G}/c^2, \quad t_S = e\sqrt{G}/c^3, \quad m_S = e/\sqrt{G},$$

which he derived from dimensional equations:

$$c = l_S/t_S, \quad e^2 = Gm_S^2, \quad e^2/l_S = m_Sc^2.$$

Let us note that units of Stoney are only by a factor $\sqrt{\alpha}$ smaller than those of Planck.

Stoney’s units look “tailored” for Einstein’s unified theory. Constants $e, c, G$ contain the gist of classical electrodynamics and gravity. There is no $\hbar$ in them. Comparison with $c, \hbar, G$ shows that $\hbar$ is brought into Stoney’s set of constants “through the back door of $\alpha$”. Therefore $e, c, G$ do not form a cube of theories with its limiting transitions considered by Gamov, Ivanenko and Landau [2].

# 7 Atomic clocks and $c$

During the 20th century the situation with standard of mass (time, length) has changed drastically. The fundamental identity of elementary particles and hence of atoms produced many candidates for standard of mass, known with much, much better precision than $G$. Thus, from the point of view of dimensions the necessity to use $G$ disappeared. However from the point of view of unifying physics the Planck units became even more attractive.
Let us now look at two other fundamental constants: \( c \) and \( h \).

Let us start from \( c \) and the frequencies of light and radio waves. In the second half of the 20th century physicists learned how to measure them in a digital way by counting the number of crests. This raised the accuracy of atomic (Cesium-133) clocks (first suggested by I. Rabi in 1945) to the level of 1 second in 300 years (NBS, 1955). (Now this has become 1 second in \( 20 \cdot 10^6 \) years: LPTF, NIST, PTB.) But even the first figure was sufficient for the introduction into SI of an atomic unit of a second (in 1967):

\[
1 \text{ s} = 9 192 631 770 \text{ periods of radiation in transition between levels of hyperfine splitting of the atomic ground state of Cs-133}.
\]

This, together with the independence of the velocity of light on its frequency, impelled Bay et al. \cite{10} to suggest, instead of unit of length (meter), to use as the basic unit the unit of velocity, namely the velocity of light \( c \). In 1983 the definition

\[
c = 299 792 458 \text{ m/s}
\]

was introduced in SI. The traditional standard of length gave way to the new standard based on the value of the velocity \( c \). This velocity is defined as a number without uncertainty. Further improvements of experiments which measured \( c \) would mean further improvement of the realization of the meter. An international report “Practical realization of the definition of the metre, including recommended radiations of other optical frequency standards” (2001) was published by T. Quinn in 2003 \cite{11}. (Note that both spellings “metre” and “meter” are used in the literature, the former in metrology, while the latter one in physics.)

Further progress in accuracy of atomic clocks is connected with passing from microwave to optical frequencies \cite{12,13}.

8 Towards kilogram based on \( h \)

Thus metrology made two momentous steps in the direction of fundamental physics: the place of macroscopic clocks and ruler (the famous rod at BIMP, in Sevre, near Paris) became occupied by the velocity of light and by atoms of Cs-133. There remains now only one macroscopic standard – the kilogram at Sevre. The prospect of expressing it through the quantum of action \( h \) is connected with precision measurements in atomic and condensed matter physics. There are many promising quantities which are good candidates for such measurements. I shall touch upon only one project which is connected with two outstanding discoveries in condensed matter physics: the Josephson effect \cite{14} (Nobel Prize 1973) and the von Klitzing effect \cite{15} (Nobel Prize 1985).

Josephson theoretically predicted the existence of a supercurrent and its remarkable properties. A supercurrent is a current of Cooper pairs tunneling through an insulator separating two superconductors. A supercurrent can exist without external voltage. An external voltage \( V \) creates an alternating supercurrent of frequency \( \nu \). The steps in \( V \) are given by the relation:

\[
V(n) = \nu n / K_J,
\]
where \( n \) is an integer, while the coefficient \( K_J \) is universal and is called the Josephson constant. It is reproduced in various experiments with unprecedented accuracy and is determined only by the ratio of fundamental constants:

\[
K_J = \frac{2e}{h} .
\]  

(6)

The effect, discovered by von Klitzing, is called the quantum Hall effect. This effect shows that there exists in Nature a universal electric resistance, one which can be expressed in terms of fundamental constants.

As is well known, the ordinary Hall effect occurs in a solid conductor (or semiconductor) with density of current \( j \) in a magnetic field \( H \) which produce an electric field \( E \) (with voltage \( V_H \)) orthogonal both to \( j \) and \( H \).

The quantum Hall effect was discovered in a two-dimensional electron system separating two parts of a silicon field transistor at very low temperature (< 4 K) and very strong magnetic field (~ 14 Tesla). It was established that the Hall resistance

\[
R_H = \frac{V_H}{I} ,
\]

(7)

where \( I \) is the total current, has quantum jumps:

\[
R_H(n) = \frac{R_K}{n} ,
\]

(8)

where \( n \) is an integer, while \( R_K \) is the von Klitzing constant:

\[
R_K = \frac{h}{e^2} .
\]

(9)

It is obvious that

\[
h = 4K_J^2 R_K^{-1} .
\]

(10)

This permits measurement of \( h \) using macroscopic apparatus. A special two-story-high watt balance compared electrical and mechanical forces:

\[
VI/v = mg ,
\]

(11)

where \( m \) is the measured mass of a body, \( g \) – local gravitational acceleration, \( V \) – the voltage in a coil moving with a vertical velocity \( v \) in a magnetic field, while \( I \) is the current in the same coil, this time fixed in the same magnetic field. By calibrating \( V \) and \( V/I \) through the Josephson and von Klitzing effects Williams et al. \[16\] succeeded in connecting \( h \) and the kilogram within uncertainty \( 8.7 \cdot 10^{-8} \).

It is hoped that in the not too distant future this accuracy might be improved by an order of magnitude, which would allow to use the watt balance for gauging the standards of mass and thus get rid of the Sevres kilogram and to define the value of \( h \). As a result the value of \( h \) would have no uncertainties in the same way as it occurred with \( c \). Thus fundamental units of nature \( c \) and \( h \) would become fundamental SI units of metrology.
9 Kilogram as frequency $\nu_K$

Another definition of the kilogram has been suggested \[17\] on the basis of equations

\begin{align*}
E &= h\nu, \\
E &= mc^2 ;
\end{align*}

(12)

(13)

"The kilogram is the mass of a body at rest whose equivalent energy equals the energy of collection of photons whose frequencies sum to $13.5639274 \times 10^{49}$ hertz".

This definition should be taken with a grain of salt. The combined use of equations (12) and (13) implies that a photon of frequency $\nu$ has mass $h\nu/c^2$. This implication persists in spite of the words "equivalent energy". The words "the mass of the body at rest" imply that mass is not Lorentz invariant, but depends on the velocity of a reference frame. It would be proper to replace equation (13) by

\begin{equation}
E_0 = mc^2 ,
\end{equation}

where $E_0$ is the rest energy (see e.g. ref. \[18\]). But then it would take some additional considerations in order to define the frequency $\nu_K$ corresponding to one kilogram. In particular massive atoms emitting and absorbing photons should be taken into account. From practical point of view the measurement of "frequencies sum" of order $10^{50}$ hertz is by eight orders of magnitude more difficult than that of the Planck frequency $\nu_P = 1/t_P$.

10 Electromagnetism and Relativity

Electromagnetism – the kinship of electricity and magnetism, discovered in 1820 by Oersted, rather soon became the foundation of Ampère’s electrodynamics. The development of the latter by Faraday and other outstanding physicists culminated in 1873 in the Treatise of Maxwell \[19\] who linked electric currents with electric and magnetic fields and with the properties of light. None of these great physicists knew the genuine nature of the phenomena. Maxwell considered vacuum filled with ether; the carriers of charges were unknown to him. The electromagnetic field was described by four vector quantities: electric field $E$, electric induction (or displacement) $D$, magnetic field $H$, and magnetic induction (or flux density) $B$.

On the basis of these notions practical units (such as volt, ampere, coulomb, joule) were introduced by International Electrical Congresses in 1880s. The electric permitivity $\varepsilon_0$ and magnetic permeability $\mu_0$ ascribed by Maxwell to the ether were accepted by the community of engineers and physicists: $D = \varepsilon_0 E$, $B = \mu_0 H$. In the middle of the 20th century these practical units became the basis of the Système International d’Unités (SI).

The end of the 19th and beginning of the 20th century were marked by great successes in understanding and applying classical electrodynamics. On practical
side it was the use of electric currents in industry, transport and radio communications. On theoretical side it was unification of electrodynamics, optics and mechanics in the framework of special relativity [20].

According to special relativity, the 4-radius vector is $x^i = (ct, \mathbf{r})$ ($i = 0, 1, 2, 3$), the 4-momentum vector is $p^i = (E/c, \mathbf{p})$, the 4-potential of electromagnetic field $A^i = (\varphi, \mathbf{A})$, the density of the 4-current $j^i = (\rho v, \mathbf{j})$, where $\mathbf{j} = \rho \mathbf{v}$, and $\rho = e \delta(\mathbf{r} - \mathbf{r}_a)$, $e$ is electric charge. (The current $j^i$ is consistent with the definitions of $p^i$ and $A^i$, due to an appropriate coefficient $c$ in front of $\rho$. The source of the field, the charge, is pointlike. Otherwise there appears a problem of the field inside the finite-size cloud of charge.) The upper index $i$ of a 4-vector indicates a contravariant 4-vector; a lower index $i$ indicates covariant 4-vector, its space components have minus sign. Raising or lowering of indices is done with the diagonal metric tensors $g^{ik}$ or $g_{ik}$ respectively.

The 3-vectors $\mathbf{E}$ and $\mathbf{H}$ are components of the 4-tensor of electromagnetic field

$$ F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} . \quad (15) $$

The tensors $F_{ik}$ and $F^{ik}$ can be represented by matrices:

$$ F_{ik} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E & 0 & -H_3 & H_2 \\ -E_2 & H_3 & 0 & -H_1 \\ -E_3 & -H_2 & H_1 & 0 \end{pmatrix} , \quad (16) $$

and

$$ F^{ik} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E & 0 & -H_3 & H_2 \\ E_2 & H_3 & 0 & -H_1 \\ E_3 & -H_2 & H_1 & 0 \end{pmatrix} , \quad (17) $$

respectively, or in a condensed form:

$$ F_{ik} = (\mathbf{E}, \mathbf{H}) , \quad (18) $$

$$ F^{ik} = (-\mathbf{E}, \mathbf{H}) . \quad (19) $$

This 4-tensor is obviously antisymmetric. From the definition of $F_{ik}$ it follows that dimensions of $\mathbf{E}$ and $\mathbf{H}$ are the same: $[\mathbf{E}] = [\mathbf{H}]$.

The field equations have the form in Gaussian units:

$$ \vec{F}_{ik} = 0 , \quad (20) $$

$$ \frac{\partial F^{ik}}{\partial x^k} = -\frac{4\pi}{c} j^i . \quad (21) $$

Here

$$ F_{ik} = \varepsilon^{iklm} F_{lm} , \quad (22) $$

where $\varepsilon^{iklm}$ is fully antisymmetric tensor ($\varepsilon^{0123} = +1$).
The equation describing the motion of charge in the electromagnetic field is given by
\[ \frac{dp}{dt} = eE + \frac{e}{c}[vH], \tag{23} \]
where
\[ v = \frac{pc^2}{E}. \tag{24} \]

Note that according to special relativity there is no ether, \( \varepsilon_0 = \mu_0 = 1 \), and the strength of magnetic field in vacuum \( H \) has the same dimension as that of \( E \), the identity of \( \varepsilon_0 = \mu_0 = 1 \) immediately follows from the fact that the same \( e \) determines the action of the charge on the field and of the field on the charge. (See expression for the action in ref. \[21\], eq. (27.6).) Thus, there is no need to consider \( B \) and \( D \) in the case of vacuum. In classical electrodynamics they appear only in the continuous media due to polarization of the latter \[22\].

In a number of classical monographs and textbooks on classical electrodynamics \( E \) and \( H \) are consistently used for description of electric and magnetic fields in vacuum with \( \varepsilon_0 = \mu_0 = 1 \) \([20], \[21], \[23], \[24], \[25]\). Their authors use Gaussian or Heaviside-Lorentz (with \( 1/4\pi \) in Coulomb law) units. Many other authors use \( B \) instead of \( H \), sometimes calling \( B \) magnetic field and sometimes – magnetic induction in vacuum \[24\]. Most of them use the SI units, according to which \( \varepsilon_0 = \mu_0 = 1 \) \([23], \[24], \[25]\). The classical electromagnetic fields in vacuum are described by four physical quantities \( D, H \) and \( E, B \), all four of them having different dimensions at variance with the spirit of special relativity.\(^1\) In that respect vacuum is similar to a material body. The SI units are very convenient for engineers, but not for theorists in particle physics.

In fact, theorists are not less responsible than metrologists for the gap between deductive basis of modern physics and mainly prerelativistic inductive basis of modern metrology. A good example is the 1935 article \[27\] by A. Sommerfeld and his book “Electrodynamics” based on lectures given in 1933-34 \[28\].

His argument against absolute system (that is based on units of time, length and mass) was the presence in it of fractional exponents (for instance from Coulomb law the dimension of charge is \( g^{1/2} \text{cm}^{3/2}s^{-1} \)). This argument was not very compelling in the 1930s and is even less so today. His argument against Gaussian or Heaviside-Lorentz system was based on inductive, prerelativistic view on electromagnetism. Though he was not quite happy\(^2\) with the new clumsy expression for fine structure constant \( \alpha \) introduced by him before the World War I, he kept insisting on MKSA units and against Gaussian units. His authority was not the least in the decision to legally enforce after World War II the SI as the obligatory system of units for all textbooks in physics.

\(^1\) Sometimes one can hear that the identity \( \varepsilon_0 = \mu_0 = 1 \) is similar to putting \( c = 1 \), when using \( c \) as a unit of velocity. However this similarity is superficial. In the framework of special relativity one can use any unit for velocity (for instance, m/s). But the dimensions and values of \( \varepsilon_0 \) and \( \mu_0 \) are fixed in SI.

\(^2\) “What is especially painful for me is that the fine structure constant is no more \( e^2/\hbar c \), but \( e^2/4\pi\varepsilon_0\hbar c^3 \)”.

Z. Phys. 36 (1935) 818.
Coming back to classical electrodynamics let us note that it is not a perfect theory: it has serious problems at short distances. To a large part these problems are solved by quantum electrodynamics (QED). Therefore the latter should be used as a foundation of a system of electromagnetic units. By the way, QED is used to extract the most accurate value of $\alpha$ from the precision measurements of the magnetic moment of electron.

In the framework of QED $\alpha$ is not a constant but a function of momentum transfer due to polarization of vacuum. Let us stress that this polarization has nothing to do with purely classical non-unit values of $\varepsilon_0$ and $\mu_0$.

11 Concluding remarks

Mutually fruitful “crossing” of fundamental physics and metrology gives numerous practical applications. One of them should be specially mentioned: the use of general relativity in global positioning system [29,30].

Remarkable achievements of metrology are not always accompanied by elaboration of adequate terminology. Here we will mention only a few of widely spread delusions.

The choice of $c$ as a unit of velocity leads many authors to the false conclusion that $c$ should be excluded from the set of fundamental units. They insist that $c = 1$, because $c$ in units of $c$ is equal 1. (The same refers to $h$ in units of $h$.) But number 1 is not a unit of measurement, because such units are always dimensional. Equations $c, h = 1$ are simply wide spread jargon. Some authors go even further by identifying space and time. (A detailed discussion can be found in ref. 31.)

The number of physical units is not limited. When solving a given problem the choice of units is determined by considerations of convenience. However from the point of view of “the world as a whole” $c, h$ and $G$ (or instead of $G$ some other quantity representing gravity) are definitely singled out as fundamental dimensional constants. Of course they must be accompanied by a number of dimensionless parameters. But the number of fundamental units could not be less than three [31].

The inclusion of candela into the set of base units (see Fig. 9) seems to be unconvincing from the point of view of physics. Of course, practically it is convenient to use it when discussing the brightness of light. But it does not look logical to put it on the same footing as units of length, time and mass.

As SI is imposed on the physics literature by governmental laws, the obligatory usage in textbooks of such notions as permittivity $\varepsilon_0$ and permeability $\mu_0$ of vacuum, makes it difficult to appreciate the beauty of the modern electrodynamics and field theory. It corresponds to the prerelativistic stage of physics.

This list can be extended, but it seems that the above remarks are sufficient for a serious discussion. The metrological institutes and SI are of great importance for science and technology. Therefore the metrological legal documents should be to a greater degree based on modern physical concepts. Especially
they should give more freedom to the usage of Gaussian and Heaviside-Lorentz systems of units in the textbooks.

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Fig. 1. Meeting in Kharkov, 1928, attended by Gamow, Ivanenko, and Landau.
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Fig. 1. Meeting in Kharkov, 1928, attended by Gamow, Ivanenko, and Landau

Fig. 2. Who is who in Fig. 1.

1. Dunaev 12. Kopp 23. Vereschagin
2. Heitler 13. Kotsarova 24. Slutsky
3. Arsenyeva 14. Kalfin 25. Gamov
4. Davydov 15. Efimovich 26. Shubnikov
5. Todorovich(?) 16. Ogievetsky 27. Landau
6. Frish 17. Grommer 28. Shtrumm
7. Bursian 18. Muskeliashvili 29. Frenkel
8. Ivanenko 19. Korsunsky 30. Rosenkevich
9. Obreimov 20. Gorwitz 31. Finkelshteyn
10. Fock 21. Ambartsumian 32. Jordan
11. Leipunsky 22. Mandel 33. Timoreva
World Constants and Limiting Transition

G. Gamow, D. Ivanenko, and L. Landau

In constructing a system of units in physics, there exist two basic methods for choosing units of any new quantity:

(i) One merely specifies an arbitrary standard of measure (this is the way in which one introduces the usual definitions of, say, gram or ohm).

(ii) By employing some law—we denote it by $A$—that relates the quantity in question to those that are known and which involves a numerical coefficient, one chooses a standard in such a way as to reduce this coefficient to unity (this is exemplified by the definition of a charge unit in terms of the Coulomb law).

Technical difficulties apart, one can always make use of either method of the above two. In the first case, we have a new arbitrary standard; that is, we increase the number of units forming the basis of the theory of dimensions. Moreover, the coefficient in the law $A$ then takes a specific numerical value that appears to be a new world constant.

In the second case, both the number of basic arbitrary standard and the number of world constants remain unchanged; for measuring the quantity in question, we only obtain a unit that is natural with respect to preceding ones. This unit will change in response to changes in basic standard. The character of this variation is studied within a dimensional analysis that introduces the concept of dimensions of a given physical quantity.

Constants of zero dimensions are independent of the choice of basis units and can therefore be treated as mathematical constants (numbers). One can hope that all these numerical constants can be obtained theoretically. Within a given system of dimensions, world constants from which one can compose a combination of zero dimension must therefore obey a mathematical relation, so that they are not independent.

From the aforesaid, it follows that we can always reduce the number of basic standard (number of dimensions) using one of the world constants for this and setting it to unity. Below, this process, which is equivalent to going over from the first definition to the second one, will be referred to as a reduction.

For a complete reduction (that is, a reduction to the number of standard that is equal to zero) to be possible, it is necessary that the number of independent constants not be less than the number of dimensions forming the basis of the system of units being considered. Obviously, the number of independent constants cannot be greater than the number of basic independent units in our system of dimensions.

For example, only the reduction to two units was possible in Newtonian mechanics, since, in the presence of three basic dimensions of $T, L,$ and $M$, there was only one law featuring a world constant, that is,

$$f = \frac{\alpha m v^2}{r^2}.$$ 

A second constant, which enables a reduction to one dimension is introduced by the special theory of relativity via the relation

$$x = it.$$ 

Finally, the last missing constant $h$ appears in the framework of quantum mechanics:

$$\phi = \frac{2 \pi W}{h}.$$ 

(this is the expression for the phase $\phi$ in terms of the action $W$).

Usually, we are dealing with the case where the number of constants known from experiments and not yet reduced to a smaller number by establishing mathematical relations is much greater than the adopted number of basic units. In this case, it is advisable to choose the most general constants for performing complete reduction.

The quartic system CGS\textsuperscript{1} is employed in modern experimental physics. In technologies, however, practical considerations dictate the use of a much greater number of standards (cm, g, s, $1^\circ$, $\text{ft}$, $\text{lb}$, ...); there, one adopts some CGS/$\Omega$ system.

Yet another example of choosing a basic system is provided by Planck's natural system of units ($c$, $\chi$, $h$, $k$).

§ 2. We have seen above that each constant is a representative of a physical law (theory), a world
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Fig. 5. Cube of theories.
Fundamental units: physics and metrology

Fig. 6. Dimensional pyramid
Fig. 7. Cover of the book dedicated to G.J. Stoney
LXI. On the Physical Unity of Nature. By G. J. Stony.

Vol. XL.—Fifth Series. JANUARY—JUNE 1881.

1. WHEN mathematicians apply the science of measurement to the investigation of Nature, they find it convenient to select such units of the several kinds of quantity with which they have to deal as will get rid of any coefficients in their equations which it is possible in this way to avoid. Every advance in our knowledge of Nature enables us to see more distinctly that it would contribute to our further progress if we could effect this simplification not only within reference to certain classes of phenomena, but throughout the whole domain of Nature.

* From the *Scientific Proceedings* of the Royal Dublin Society of February 10, 1881, being a paper which had been read before Section A of the British Association at the Belfast Meeting in 1874. Communicated by the Author.

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