Quintessence and tachyon dark energy in interaction with dark matter: observational constraints and model selection

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Abstract

We derive two field theory models of interacting dark energy, one in which dark energy is associated with the quintessence and another in which it is associated with the tachyon. In both, instead of choosing arbitrarily the potential of scalar fields, these are specified implicitly by imposing that the dark energy fields must behave as the new agegraphic dark energy. The resulting models are compared with the Pantheon supernovae sample, CMB distance information from Planck 2015 data, baryonic acoustic oscillations (BAO) and Hubble parameter data. For comparison, the non-interacting case and the $\Lambda CDM$ model also are considered. By use of the $AIC$ and $BIC$ criteria, we have obtained strong evidence in favor of the two interacting models, and the coupling constants are nonvanishing at more than $3\sigma$ confidence level.
I. INTRODUCTION

Twenty years ago, two groups have discovered independently that the universe is in a period of accelerated expansion \[1\] \[2\]. In order to obtain such an acceleration in the expansion rate in the framework of the General Relativity, it is necessary that the universe be dominated by a component with negative pressure. Such component was called of dark energy, and the first candidate considered for the dark energy was the cosmological constant. In fact, the model of universe based on a cosmological constant and cold dark matter, the $\Lambda CDM$, has been capable of reproduce all observational data until now. However, there are two well known problems with the cosmological constant: the fine tuning and the coincidence problems. In order to solve such problems, many alternatives to the cosmological constant have been proposed. As many of these models are capable of reproduce the observational data with the same quality as the $\Lambda CDM$, such models cannot be discarded and, in fact, some of these models have the same number of free parameters as the $\Lambda CDM$ - one of them we will see in this work. A possibility is that the dark energy is constituted by a physical field. If this is the case, it is more natural to assume that dark energy interacts with dark matter, as fields generally interact, unless such an interaction be prevented by some specific symmetry. An advantage of an interaction between dark energy and dark matter is that both components will evolve in a coupled fashion, and if dark energy decays into dark matter, this will at least alleviate the coincidence problem. Many papers considering an interaction between dark energy and dark matter have been published, and some evidence of the interaction has been found, see e.g. \[3\] \[4\] \[5\] \[6\] \[7\] \[8\]. For a more complete list of references about evidences of the interaction, as well as for a discussion of theoretical aspects and cosmological implications, see \[9\]. However, in the most of these papers the interaction term in the equations of the model is derived phenomenologically. Much smaller is the number of papers where the interaction term is derived from a field theory. Examples of the works on this direction are \[10\] \[4\] \[11\] \[12\]. In this paper, we will follow this path.

It is very common to choose scalar fields as candidates to dark energy, as the canonical scalar field, called quintessence, or the tachyon field. They naturally arise in particle physics and string theory. For reviews about the use of scalar fields as dark energy see, e.g. \[13\] \[14\]. The quintessence has the equation of state parameter, $\omega_q \equiv \frac{P_q}{\rho_q}$, between $-1$ and $+1$. Quintessence models were investigated, e.g. in \[10\] \[15\] \[16\] \[17\] \[18\] \[19\] \[20\]. The
The tachyon field has the equation of state parameter $0 \leq \omega_t \leq -1$. The tachyon lagrangian was derived from brane developments in string theory. Tachyon as dark energy was studied, e.g., in references [21] to [26]. A natural question which arises is to choose the potential $V(\varphi)$ of the scalar field. Common choices are power law or exponential potentials. However, these choices are in fact arbitrary. It would be interesting to choose the potential by some physical criterion. Efforts on this direction were made in references [11] and [12]. More specifically, in those papers, two field theory models of dark energy interacting with dark matter had been constructed. In both the models, dark energy had been associated with a massive Dirac field, interacting via Yukawa coupling with a tachyon scalar field in one model and with a quintessence field in the other. However, instead of choosing a particular form for the potential $V(\varphi)$ of the scalar fields, this had been implicitly fixed by imposing that the energy density of dark energy must match the energy density of the holographic dark energy. In this model, the energy density of dark energy is given by $\rho_{de} = \frac{3M_{Pl}^2 c^3}{L^2}$, where $M_{Pl} = \frac{1}{\sqrt{8\pi G}}$ is the reduced Planck mass, $c$ is a free parameter and $L$ is an infrared cut-off. In references [35] to [36] was demonstrated that if one choose $L$ as the event horizon of the universe, the model reproduce the present period of accelerated expansion. Holographic dark energy models have been extensively studied in the literature, for a review and a list of references, see [37]. It was demonstrated in references [38] and [33] to [34] that there are correspondences between quintessence and tachyon and holographic dark energy, in the noninteracting cases. In references [11] and [12] the scalar fields were interacting, and in that cases the combination with the holographic dark energy in fact resulted in two new models of interacting dark energy.

However, there is a consistency problem, concerning causality, which would be pointed in the holographic dark energy model: this depends on the event horizon of the universe, and this in turn only exists if the period of accelerated expansion is forever. In reference [39] was proposed another model of dark energy, on which again $\rho_{de} = \frac{3M_{Pl}^2 n^2}{L^2}$, but $L$ being now the conformal time, $\eta(t) \equiv \int_0^t \frac{dt'}{a(t')} \ (n$ is again a free parameter of order unity). This model has not the consistency problem mentioned, and possesses another advantage: because the initial value of relative density of dark energy is not a free parameter, this model has one less parameter than the holographic dark energy, possessing, in the noninteracting case, the same number of free parameters as the $\Lambda CDM$. In this work, we will construct two field theory models of interacting dark energy, one in which the dark energy is associated with
the quintessence, and another in which the dark energy is the tachyon. However, instead of choosing the potentials $V(\varphi)$, we will specify these implicitly, by imposing that the energy density of the scalar fields, $\rho_\varphi$, must match the energy density of the new agegraphic dark energy, $\rho_{\text{de}} = \frac{3M_{\text{Pl}}^2 n^2}{\eta^2}$. This was the same reasoning used in [11] and [12] to construct the two models analysed in that papers, but there holographic dark energy was in place of the new agegraphic dark energy. Therefore, now the models possess different dynamical properties, as the new agegraphic dark energy model behaves itself different from the holographic dark energy, as already discussed in [39]. Moreover, the models have not the causality problem, and possesses one less parameter than before.

In this work we use the Natural Units system, in which $\hbar = c = k_B = 1$.

II. INTERACTING NEW AGEGRAPHIC DARK ENERGY

In [40] [41] [42], it was argued that a distance $t$ in Minkowski space cannot be measured with accuracy better than

$$\delta t = \lambda t_p^{2/3} t^{1/3},$$

(1)

where $\lambda$ is a dimensionless constant of order unity, $t_p$ is the reduced Planck time, given by $t_p = \frac{1}{M_{\text{Pl}}}$, being $M_{\text{Pl}}$ the reduced Planck mass. Because the time-energy uncertainty relation, this uncertainty on length measures implies that a region of size $\delta t^3$ possesses an energy content

$$E_{\delta t^3} \sim t^{-1}.$$  

(2)

Therefore, there is an energy density associated with the quantum fluctuations of the space-time, given by

$$\rho_\varphi \sim \frac{E_{\delta t^3}}{\delta t^3} \sim \frac{1}{t_p^2 t^2} \sim \frac{M_{\text{Pl}}^2}{t^2}.$$  

(3)

In [45], this energy density was associated with dark energy. More precisely, the energy density of dark energy would be given by

$$\rho_{\text{DE}} = \frac{3n^2 M_{\text{Pl}}^2}{T^2},$$

(4)

where the time $t$ was identified with the age of the universe $T$ and $n$ is a dimensionless parameter of order unity. The resultant model of dark energy was denominated agegraphic
dark energy. However, this model has a subtlety \([45]\), and in \([39]\) it was proposed that the age of the universe \(T\) be replaced by the conformal time \(\eta\), that is,

\[
\rho_{DE} = \frac{3n^2M^2_{\text{Pl}}}{\eta^2},
\]

where

\[
\eta(t) \equiv \int_0^t \frac{dt'}{a(t')}
\]

is the conformal time. The model of dark energy resulting was denominated new agegraphic dark energy model.

It is interesting to notice that, from a different argumentation it was obtained in \([35]\) \([36]\) the holographic dark energy model, whose expression for the energy density of dark energy is very similar to (5), namely

\[
\rho_{DE} = \frac{3c^2M^2_{\text{Pl}}}{R^2_h},
\]

where \(R_h\) is the event horizon, given by

\[
R_h = a(t) \int_t^\infty \frac{dt'}{a(t')}.
\]

The reasoning used to construct both the models, although different, has in common the point that in both is considered that for very small scales quantum effects of gravity must be considered. Therefore, although we don’t have yet a quantum gravity theory, the similarity of the expressions (5) and (7) perhaps suggests that we correctly incorporated some universal property of quantum gravity.

For a universe composed by dark energy and dark matter in interaction, and baryonic matter and radiation, the conservation equations are

\[
\dot{\rho}_{DE} + 3H\rho_{DE}(\omega_{DE} + 1) = Q,
\]

\[
\dot{\rho}_{DM} + 3H\rho_{DM} = -Q,
\]

\[
\dot{\rho}_b + 3H\rho_b = 0
\]

and

\[
\dot{\rho}_r + 4H\rho_r = 0,
\]

where the dot represents derivative with respect to time, and \(Q\) is the interaction term. The Friedmann equation for a flat universe reads
\[ H^2 = \frac{1}{3M_{Pl}^2} \left[ \rho_{DE} + \rho_{DM} + \rho_b + \rho_r \right] . \]  

(13)

Using eqs. (9), (10), (11), (12) and (13), it is possible to rewrite (9) as

\[ \dot{\Omega}_{DE} = 3H\Omega_{DE} \left[ -(1 - \Omega_{DE})\omega_{DE} + \frac{\Omega_r}{3} \right] + \frac{Q}{3M_{Pl}^2 H^2} \]  

(14)

On the other hand, the energy density of the new agegraphic dark energy is given by (5), which can also be written as

\[ \Omega_{DE} = \frac{n^2}{H^2 \eta^2} \]  

(15)

Deriving (9) with respect to time, and using (6) and (15), we have

\[ \dot{\rho}_{DE} = -H\rho_{DE}\frac{2\sqrt{\Omega_{DE}}}{na} \]  

(16)

Inserting (16) in (9) we obtain

\[ \omega_{DE} = -1 + 2\frac{\sqrt{\Omega_{DE}}}{3na} + \frac{Q}{3H\rho_{DE}} \]  

(17)

The interaction term \( Q \) is specified by the interacting dark energy model under consideration. In this work, we will construct two field theory models of interacting dark energy.

### III. THE MODELS

We consider the general action

\[ S = \int d^4x \sqrt{-g} \left\{ -\frac{M_{Pl}^2}{2} R + \mathcal{L}_\varphi (x) + \frac{i}{2} \left[ \bar{\Psi} \gamma^\mu \nabla_\mu \Psi - \bar{\Psi} \gamma^\mu \gamma^\nu \gamma^\rho \nabla_\mu \gamma^\nu \nabla_\rho \Psi \right] - (M - \beta \varphi) \bar{\Psi} \Psi + \sum_j \mathcal{L}_j (x) \right\} \]  

(18)

where \( M_{Pl} \equiv (8\pi G)^{-1/2} \) is the reduced Planck mass, \( R \) is the curvature scalar, \( \mathcal{L}_\varphi (x) \) is, unless of the coupling term, the lagrangian density for the scalar field, which we will identify with dark energy, \( \Psi \) is a massive fermionic field, which we will identify with dark matter, \( \beta \) is the dimensionless coupling constant and \( \sum_j \mathcal{L}_j (x) \) contains the lagrangian densities for the remaining fields. Notice that, in this work, we will only consider an interaction of dark energy with dark matter. If there was a coupling between the scalar field and baryonic matter, the corresponding coupling constant \( \beta_b \) should satisfy the solar system constraint

\[ \beta_b \lesssim 10^{-2} . \]  

(19)
We assume $\beta_b \equiv 0$, which trivially satisfy the constraint (19).

We consider two kinds of scalar fields: the canonical scalar field, or quintessence field, for which

$$\mathcal{L}_\varphi(x) = \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - V(\varphi), \quad (20)$$

and the tachyon scalar field, for which

$$\mathcal{L}_\varphi(x) = -\sqrt{1-\alpha \partial^\mu \varphi \partial_\mu \varphi} V(\varphi), \quad (21)$$

where $\alpha$ is a constant with dimension $MeV^{-4}$. Notice that in both cases, we assume a Yukawa coupling with the dark matter field $\Psi$.

**A. Quintessence field**

For the quintessence field, $\mathcal{L}_\varphi(x)$ in the action (18) is given by (20). From a variational principle, we obtain

$$i\gamma^\mu \nabla_\mu \Psi - M^*\Psi = 0, \quad (22)$$

$$i(\nabla_\mu \bar{\Psi})\gamma^\mu + M^*\bar{\Psi} = 0, \quad (23)$$

where $M^* \equiv M - \beta \varphi$, and

$$\nabla_\mu \partial^\mu \varphi + \frac{dV(\varphi)}{d\varphi} = \beta \bar{\Psi} \Psi. \quad (24)$$

Eqs. (22) and (23) are, respectively, the covariant Dirac equation and its adjoint, in the case of a nonvanishing interaction between the Dirac field and the scalar field $\varphi$. For homogeneous fields and adopting the flat Friedmann-Robertson-Walker (FRW) metric, $g_{\mu\nu} = \text{diag}(1, -a^2(t), -a^2(t), -a^2(t))$, where $a^2(t)$ is the scale factor, eqs. (22) and (23) lead to

$$\frac{d(a^3 \bar{\Psi} \Psi)}{dt} = 0$$

which is equivalent to

$$\bar{\Psi} \Psi = \bar{\Psi}_i \Psi_i \left(\frac{a_i}{a}\right)^3 \quad (25)$$

where the subscript ”$i$” denotes some initial time, and (24) reduces to

$$\ddot{\varphi} + 3H \dot{\varphi} + \frac{dV(\varphi)}{d\varphi} = \beta \bar{\Psi} \Psi, \quad (26)$$
where $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter.

From the energy-momentum tensor, we get

$$\rho_\varphi = \frac{1}{2} \dot{\varphi}^2 + V(\varphi) , \quad (27)$$

$$P_\varphi = \frac{1}{2} \dot{\varphi}^2 - V(\varphi) , \quad (28)$$

$$\rho_\Psi = M^* \bar{\Psi} \Psi , \quad (29)$$

$$P_\Psi = 0 . \quad (29)$$

From (27) and (28) we have $\omega_\varphi \equiv \frac{P_\varphi}{\rho_\varphi} = \frac{\frac{1}{2} \dot{\varphi}^2 - V(\varphi)}{\frac{1}{2} \dot{\varphi}^2 + V(\varphi)}$. Differentiating (27) and (29) with respect to time and using (25) and (26), we obtain

$$\dot{\rho}_\varphi + 3H \rho_\varphi (\omega_\varphi + 1) = \beta \dot{\varphi} \bar{\Psi}_i \Psi_i \left( \frac{a_i}{a} \right)^3 \quad (30)$$

and

$$\dot{\rho}_\Psi + 3H \rho_\Psi = -\beta \dot{\varphi} \bar{\Psi}_i \Psi_i \left( \frac{a_i}{a} \right)^3 . \quad (31)$$

Comparing (9) and (10) with (30) (31), we see that

$$Q = \beta \dot{\varphi} \bar{\Psi}_i \Psi_i \left( \frac{a_i}{a} \right)^3 . \quad (32)$$

Remembering that $\rho_{\Psi i} = 3 M^2 \Pi_i H^2 \Omega_{\Psi i}$ and using (29), we have

$$\bar{\Psi}_i \Psi_i = \frac{3 M^2 \Pi_i H^2 \Omega_{\Psi i}}{M - \beta \dot{\varphi}_i} , \quad (33)$$

where $\Omega_{\Psi i}$ is the initial relative energy density of the dark matter, $H_i$ is the initial value of the Hubble parameter, and $\varphi_i$ is the initial value of the quintessence field. From (27) and (28) we have

$$\dot{\varphi} = \text{sign}[\dot{\varphi}] \sqrt{3} M \Pi H \sqrt{\Omega_\varphi (1 + \omega_\varphi)} . \quad (34)$$

Substituting (33) and (34) in (32), we have

$$Q = \text{sign}[\dot{\varphi}] \delta M \Pi H^2 \sqrt{3 \Omega_{\Psi i}} \sqrt{\Omega_\varphi (1 + \omega_\varphi)} \left( \frac{a_i}{a} \right)^3 , \quad (35)$$

where we have defined the effective coupling constant

$$\delta \equiv \frac{\beta}{M - \beta \dot{\varphi}_i} . \quad (36)$$
Notice that \( \text{sign}[\phi] \) is in fact arbitrary, as it can be changed by redefinitions of the quintessence field, \( \varphi \to -\varphi \), and of the coupling constant \( \beta \to -\beta \). Substituting (35) in (14) we have

\[
\frac{d\Omega_\varphi}{dz} = 3 \Omega_\varphi \left\{ (1 - \Omega_\varphi) \omega_\varphi - \frac{\Omega_r}{3} - \sqrt{\frac{2}{3}} \gamma_q \sqrt{1 + \omega_\varphi} \right\},
\]

(37)

where

\[
\gamma_q(z) \equiv \frac{\delta M_{\text{Pl}}}{\sqrt{2}} \left( \frac{H_i}{H} \right)^2 \frac{\Omega_{\Psi_i}}{\Omega_\varphi} \left( \frac{1 + z}{1 + z_i} \right)^3.
\]

(38)

Notice that we rewrite the evolution equation for \( \Omega_\varphi \) in terms of redshift \( z \).

Inserting (35) in (17), we have

\[
\omega_\varphi = -1 + 2 \sqrt{\Omega_\varphi} (1 + z) + \sqrt{\frac{2}{3}} \gamma_q \sqrt{1 + \omega_\varphi}.
\]

(39)

Solving for \( \omega_\varphi \), we obtain

\[
\omega_\varphi(z) = -1 + 2 \sqrt{\Omega_\varphi} (1 + z) + \frac{\gamma_q}{3} \left[ \gamma_q + \sqrt{\frac{\gamma_q^2 + \frac{4}{3} \Omega_\varphi}{n}} (1 + z) \right].
\]

(40)

In an entirely analogue manner done to (9), one can rewrite (10), (11) and (12) as

\[
\frac{d\Omega_\Psi}{dz} = -3 \Omega_\Psi \left[ \Omega_\varphi \omega_\varphi + \frac{\Omega_r}{3} - \sqrt{\frac{2}{3}} \gamma_q \Omega_\varphi \sqrt{1 + \omega_\varphi} \right],
\]

(41)

\[
\frac{d\Omega_b}{dz} = -3 \Omega_b \left[ \frac{2 \Omega_\varphi}{1 + z} + \frac{\Omega_r}{3} \right]
\]

and

(42)

\[
\frac{d\Omega_r}{dz} = -3 \Omega_r \left[ \frac{\Omega_\varphi \omega_\varphi + \frac{\Omega_r}{3} - \frac{1}{3}}{1 + z} \right].
\]

(43)

Evidently, from (37), (41), (42) and (43), only three are independent, as for a flat universe, \( \Omega_\varphi + \Omega_\Psi + \Omega_b + \Omega_r = 1 \).

From (12) we have

\[
\rho_r = \rho_{ri} \left( \frac{1 + z}{1 + z_i} \right)^4.
\]

(44)

So

\[
\rho_{ri} = \rho_{r0} \left( \frac{1 + z_i}{1 + z_0} \right)^4,
\]

(45)

where the subscript "0" denotes the quantities today. \( \rho_{r0} = (1 + 0.2271 N_{\text{eff}}) \rho_{\gamma 0} \), where \( N_{\text{eff}} = 3.04 \) is the effective number of relativistic degrees of freedom, and \( \rho_{\gamma 0} \frac{\pi^2}{30} T_{\text{CMB}}^4 \) is the
energy density of photons, \(T_{CMB} = 2.725K\) is the CMB temperature today. Remembering that \(\rho_{ri} = 3M_{Pl}^2H_i^2\Omega_{ri}\) and from \([45]\), we have

\[
H_i = \lambda \frac{(1+z)^2}{\sqrt{\Omega_{ri}}},
\]  

(46)

where

\[
\lambda \equiv \pi \frac{3}{3M_{Pl}} \sqrt{\frac{1 + 0.2271N_{eff}T_{CMB}^2}{5}}.
\]  

(47)

On the other hand, from \([44]\) it is possible to write the Hubble parameter as

\[
H(z) = H_i \sqrt{\frac{\Omega_{ri}}{\Omega_r}} \left(\frac{1+z}{1+z_i}\right)^2
\]  

(48)

or, using \([46]\),

\[
H(z) = \lambda \frac{(1+z)^2}{\sqrt{\Omega_r}}.
\]  

(49)

According to \([39]\), the initial value \(\Omega_{\phi i}\) is not a free parameter, but, in the radiation era, it is related with the parameter \(n\) as

\[
\Omega_{\phi i} = \frac{n^2}{(1+z_i)^2},
\]  

(50)

where \(z_i\) is some redshift for which the universe was in the radiation era.

So, the evolution with redshift \(z\) of all quantities of the model are determined by three of the equations \([37], [41], [42], [43]\), with \(\omega_{\phi}\) given by \([40]\), \(\gamma_q(z)\) and \(H(z)\) given by \([38]\) and \([49]\) respectively. The free parameters of the model are \(\delta, n, \Omega_{\Psi_i}\) and \(\Omega_{b_i}\). (The initial condition \(\phi_i\) is in fact arbitrary, as only the effective coupling constant \(\delta\) is constrained by the observational data.) It is interesting to notice that for the noninteracting case, \(\delta = 0\), the model has three free parameters, \(n, \Omega_{\Psi_i}\) and \(\Omega_{b_i}\), the same number of free parameters as the \(\Lambda CDM\), for which the free parameters are \(\Omega_{\Lambda i}, \Omega_{\Psi_i}\) and \(\Omega_{b_i}\).

The relation \([34]\) can be rewritten in terms of redshift as

\[
\frac{d\phi}{dz} = -\sqrt{3M_{Pl}^2\sqrt{\Omega_{\phi}}(z)(1+\omega_{\phi}(z))} \frac{1 + \omega_{\phi}(z)}{1 + z}.
\]  

(51)

From \([27]\) and \([34]\) we have

\[
V(z) = 3M_{Pl}^2H^2\Omega_{\phi}(z)(1-\omega_{\phi}(z)).
\]  

(52)

From \([52]\) and \([51]\) it is possible to compute \(V(\phi)\). Hereafter, we denote the \textit{Interacting Quintessence New Agegraphic Dark Energy Model} simply as IQNADE.
B. Tachyon field

In the case of dark energy modeled as the tachyon scalar field, \( \mathcal{L}_\varphi(x) \) in the action (18) is given by (21). From a variational principle, we obtain

\[
i\gamma^\mu \nabla_\mu \Psi - M^* \Psi = 0 , \tag{53}
\]

\[
i(\nabla_\mu \bar{\Psi}) \gamma^\mu + M^* \bar{\Psi} = 0 , \tag{54}
\]

where \( M^* \equiv M - \beta \varphi \), and

\[
\nabla_\mu \partial^\mu \varphi + \alpha \frac{\partial \varphi (\nabla_\mu \partial_\nu \varphi) \partial^\nu \varphi + 1}{1 - \alpha \partial_\mu \varphi \partial_\mu \varphi} + \frac{1}{\alpha} \frac{d \ln V(\varphi)}{d \varphi} = \frac{\beta \bar{\Psi} \Psi}{\alpha V(\varphi)} \sqrt{1 - \alpha \partial_\mu \varphi \partial_\mu \varphi} . \tag{55}
\]

Eqs. (53) and (54) are the interacting covariant Dirac equation and its adjoint, respectively, i.e., (53) and (54) are almost the same as eqs. (22) and (23), the only difference is that the scalar field \( \varphi \) in \( M^* \) now is the tachyon field. For homogeneous fields and adopting the flat FRW metric, (55) reduces to

\[
\ddot{\varphi} = -(1 - \alpha \dot{\varphi}^2) \left[ \frac{1}{\alpha} \frac{d \ln V(\varphi)}{d \varphi} + 3H \dot{\varphi} - \frac{\beta \bar{\Psi} \Psi}{\alpha V(\varphi)} \sqrt{1 - \alpha \dot{\varphi}^2} \right] , \tag{56}
\]

whereas for the fermions, the equations of motion will reduce to eq. (25), as already obtained above:

\[
\bar{\Psi} \Psi = \bar{\Psi} i \Psi i \left( \frac{a_i}{a} \right)^3 . \tag{8}
\]

From the energy-momentum tensor, we get

\[
\rho_\varphi = \frac{V(\varphi)}{\sqrt{1 - \alpha \dot{\varphi}^2}} , \tag{57}
\]

\[
P_\varphi = -V(\varphi) \sqrt{1 - \alpha \dot{\varphi}^2} , \tag{58}
\]

\[
\rho_\Psi = M^* \bar{\Psi} \Psi ,
\]

\[
P_\Psi = 0 .
\]

From (57) and (58) we have

\[
\omega_\varphi \equiv \frac{P_\varphi}{\rho_\varphi} = \alpha \dot{\varphi}^2 - 1 . \tag{59}
\]

Differentiating (57) and (58) with respect to time and using (56) and (25), we get

\[
\dot{\rho}_\varphi + 3H \rho_\varphi (\omega_\varphi + 1) = \beta \dot{\varphi} \bar{\Psi} i \Psi i \left( \frac{a_i}{a} \right)^3 \tag{60}
\]
\[ \dot{\rho}_{\Psi} + 3H\rho_{\Psi} = -\beta\dot{\phi}\bar{\Psi}_i\Psi_i \left( \frac{a_i}{a} \right)^3, \]  

(61)

where the dot represents derivative with respect to time.

Notice that the interaction term is of the same form as before,

\[ Q = \beta\dot{\phi}\bar{\Psi}_i\Psi_i \left( \frac{a_i}{a} \right)^3. \]  

(62)

However, the scalar field now is the tachyon, its behaviour been determined by (56). Defining \( \phi \equiv \sqrt{\alpha}\varphi \), from (59), we have

\[ \dot{\phi} = \text{sign}[\phi]\sqrt{1 + \omega_{\phi}}. \]  

(63)

As before, we have

\[ \bar{\Psi}_i\Psi_i = \frac{3M_{Pl}^2H_i^2\Omega_{\Psi_i}}{M - \frac{\beta}{\sqrt{\alpha}}\phi_i}. \]  

(64)

Substituting (63) and (64) in (62), we have

\[ Q = \text{sign}[\dot{\phi}]\delta3M_{Pl}^2H_i^2\Omega_{\Psi_i}\sqrt{1 + \omega_{\phi}} \left( \frac{a_i}{a} \right)^3, \]  

(65)

where

\[ \delta \equiv \frac{\beta}{1 - \frac{\beta}{M\sqrt{\alpha}}\phi_i}. \]  

(66)

As before, \( \text{sign}[\dot{\phi}] \) is in fact arbitrary, as it can be changed by redefinitions of the tachyon field, \( \phi \rightarrow -\phi \), and of the coupling constant \( \beta \rightarrow -\beta \). Substituting (65) in (14) we have

\[ \frac{d\Omega_{\phi}}{dz} = \frac{3\Omega_{\phi}}{1 + z} \left\{ (1 - \Omega_{\phi})\omega_{\phi} - \frac{\Omega_r}{3} - \sqrt{\frac{2}{3}\gamma_t\sqrt{1 + \omega_{\phi}}} \right\}, \]  

(67)

where

\[ \gamma_t(z) = \frac{1}{\sqrt{6}}\frac{\delta H_i^2\Omega_{\Psi_i}}{H^3\Omega_{\phi}} \left( \frac{1 + z}{1 + z_i} \right)^3. \]  

(68)

In an analogue manner as done for quintessence, we obtain

\[ \omega_{\phi}(z) = -1 + \frac{2\sqrt{\Omega_{\phi}}}{3n}(1 + z) + \frac{\gamma_t}{3} \left[ \gamma_t + \sqrt{\frac{4\Omega_{\phi}}{n}(1 + z)} \right]. \]  

(69)

As before, the Friedmann equation reads

\[ H(z) = \lambda \frac{(1 + z)^2}{\sqrt{\Omega_r}}, \]  

(70)
where
\[ \lambda \equiv \frac{\pi}{3M_{Pl}} \sqrt{\frac{1 + 0.2271N_{eff}T_{CMB}^2}{5}}. \]  
(71)

Again, the initial value \( \Omega_{\phi i} \) is not a free parameter, but is determinated by \( n \) as
\[ \Omega_{\phi i} = \frac{n^2}{(1 + z_i)^2}. \]  
(72)

So the interacting tachyonic agegraphic dark energy model possesses four free parameters: \( \delta \), \( n \), \( \Omega_{\Psi i} \) and \( \Omega_{bi} \), which must be determined from comparison of the model with observational data. Again, \( \phi_i \) is arbitrary, as only the effective coupling constant \( \delta \) is constrained by the data.

We can obtain the evolution of \( \phi \) with redshift as
\[ \frac{d\phi}{dz} = -\frac{\sqrt{1 + \omega_\phi(z)}}{H(z)(1 + z)}. \]  
(73)

From (57), the potential can be written as
\[ V(z) = 3M_{Pl}^2H^2\Omega_{\phi}(z)\sqrt{-\omega_\phi(z)}. \]  
(74)

From (74) and (51), it is possible to compute \( V(\phi) \) for the tachyon field. Hereafter, we will refer to Interacting Tachyonic New Agegraphic Dark Energy Model as ITNADE.

It is interesting to notice that both interacting models, IQNADE and ITNADE, in the noninteracting case, \( \delta = 0 \), will be reduced to the NADE model. In other words, we will obtain reconstructions of NADE from the quintessence and tachyon fields, as already obtained in [47] [48] and [49].

IV. CONSTRAINTS FROM OBSERVATIONAL DATA

We include four sets of observational data: the 1048 SNIa data from the Pantheon sample [50], 9 BAO data as compiled, for instance, in [51] [52], measurements of the Hubble parameter in 31 different redshifts, as compiled, for instance, in [53], and the CMB distance priors from Planck 2015 data [54].

We compare our theoretical predictions for the distance modulus at redshift \( z \), \( \mu(z) \), with the 1048 observational values of \( \mu \) of the Pantheon sample [50]. The theoretical distance modulus is defined as
\[ \mu(z) = 5\log_{10} \left[ c(1 + z) \int_0^z \frac{dz'}{H(z')} \right] + 15. \]  
(75)
We compute the quantity

\[ \chi_{SN}^2 = \sum_{ij} \left( \mu_{th}^i - \mu_{data}^i \right) C_{ij}^{-1} \left( \mu_{th}^j - \mu_{data}^j \right), \]  

(76)

where \( \mu^{th} \) are the predicted model values calculated using (75), and \( \mu^{data} \) are the observational values of the Pantheon sample. \( C_{ij}^{-1} \) is the inverse of the covariance matrix for the Pantheon sample.

The Planck distance priors summarizes the information of temperature power spectrum of CMB. These includes the ”shift parameter” \( R \), the ”acoustic scale” \( l_A \) and the physical energy density of baryonic matter today, \( \Omega_{b0} h^2 \). These quantities are very weakly model dependent [55] [56]. \( R \) and \( l_A \) are given by

\[ R = \sqrt{\Omega_{m0} H_0 r (z_*)} \]

and

\[ l_A = \frac{\pi}{r_s (z_*)}, \]

where \( r (z) \) is the comoving distance to redshift of last scattering \( z_* \), \( r_s (z_*) \) is the comoving sound horizon at \( z_* \), \( \Omega_{m0} = \Omega_{dm0} + \Omega_{b0} \), the total energy density of matter today (dark matter plus baryonic matter) and \( H_0 \) is the Hubble parameter today. For a flat universe, \( r (z_*) \) and \( r_s (z_*) \) are given by

\[ r (z) = \int_{0}^{z} \frac{dz}{H(z)}, \]  

(77)

and

\[ r_s (z) = \int_{z}^{\infty} \frac{dz}{H(z) \sqrt{3 \left( 1 + \frac{\bar{R}_b}{1 + z} \right)}}, \]  

(78)

where \( \bar{R}_b/(1 + z) = 3 \Omega_b / (4 \Omega_{\gamma}) \). For the redshift of decoupling \( z_* \) we use the fitting function proposed by Hu and Sugiyama [57]:

\[ z_* = 1048 \left[ 1 + 0.00124 \left( \Omega_{b0} h^2 \right)^{-0.738} \right] \left[ 1 + g_1 \left( \Omega_{m0} h^2 \right)^{g_2} \right], \]

where

\[ g_1 = \frac{0.0783 \left( \Omega_{b0} h^2 \right)^{-0.238}}{1 + 39.5 \left( \Omega_{b0} h^2 \right)^{0.763}}, \]

and

\[ g_2 = \frac{0.560}{1 + 21.1 \left( \Omega_{b0} h^2 \right)^{1.81}}, \]

Table 1 shows the Planck distance information [54] used in this work.
Table 1: Planck distance information from Planck 2015 data.

|   |   |
|---|---|
| $R$ | 1.7448 |
| $l_A$ | 301.460 |
| $\Omega_{b0}h^2$ | 0.02240 |

The inverse of the covariance matrix associated with these data is given below

\[
C^{-1}_{ij}^{(PLANCK)} = \begin{pmatrix}
84362.33 & -1314.56 & 1650925.67 \\
-1314.56 & 157.90 & 6186.87 \\
1650925.67 & 6186.87 & 74320938.55
\end{pmatrix}
\]

Thus we add to $\chi^2_{\text{tot}}$ the term

\[
\chi^2_{CMB} = \sum_{ij} (x_i^{\text{th}} - x_i^{\text{data}}) C^{-1}_{ij}^{(PLANCK)} (x_j^{\text{th}} - x_j^{\text{data}}),
\]

where $x = (l_A, R, \Omega_{b0}h^2)$ is the parameter vector.

Baryonic Acoustic Oscillations (BAO) are described in terms of the cosmological distances

\[
D_V(z) = c \left[ \frac{z}{H(z)} \left( \int_0^z \frac{dz'}{H(z')} \right)^2 \right]^{1/3}, \quad (79)
\]

\[
D_A = \frac{c}{1 + z} \int_0^z \frac{dz'}{H(z')}, \quad (80)
\]

\[
D_H = \frac{c}{H}. \quad (81)
\]

The observational values of BAO which we use in this work are given in terms of the quotients of (79), (80) and (81) with $r_d$, the comoving sound horizon at $z_d$, the redshift of the drag epoch. The theoretical value of $r_d$ is calculated using (78) with $z = z_d$, where $z_d$ is calculated using the fitting function proposed by Eisenstein and Hu [58]:

\[
z_d = 1291 \frac{(\Omega_{m0}h^2)^{0.251}}{1 + 0.659 (\Omega_{m0}h^2)^{0.828}} \left[ 1 + b_1 (\Omega_{b0}h^2)^{b_2} \right],
\]

\[
b_1 = 0.313 (\Omega_{m0}h^2)^{-0.419} \left[ 1 + 0.607 (\Omega_{m0}h^2)^{0.674} \right],
\]

\[
b_2 = 0.238 (\Omega_{m0}h^2)^{0.223}.
\]

The observational values of BAO used here are given in tables 2 and 3 below, and were compiled in [52].
Table 2: Isotropic BAO scale measurements.

| z   | \(d_{iso}^i\) | Reference |
|-----|----------------|-----------|
| 0.106 | 2.98 ± 0.13    | [59]      |
| 0.15  | 4.47 ± 0.17    | [60]      |
| 1.52  | 26.1 ± 1.1     | [61]      |

Table 3: Anisotropic BAO scale measurements.

| z   | \(d_{aniso}^i\) | Reference |
|-----|-----------------|-----------|
| 0.38 | 7.42            | [62]      |
| 0.38 | 24.97           | [62]      |
| 0.51 | 8.85            | [62]      |
| 0.51 | 22.31           | [62]      |
| 0.61 | 9.69            | [62]      |
| 0.61 | 20.49           | [62]      |

The \(\chi^2_{BAO}\) is given by

\[
\chi^2_{BAO} = \chi^2_{iso} + \chi^2_{aniso}
\]

with

\[
\chi^2_{iso} = \sum_i \left( \frac{d_{iso}^i - d_{iso}^{i(th)}}{\sigma_i} \right)^2
\]

and

\[
\chi^2_{aniso} = \sum_{ij} \left( d_{aniso}^i - d_{aniso}^{i(th)} \right) C_{ij}^{-1(BAO)} \left( d_{aniso}^j - d_{aniso}^{j(th)} \right),
\]

where \(C_{ij}^{-1(BAO)}\) is the inverse of the covariance matrix for the anisotropic BAO [51], given by
The Hubble parameter $H$ in 31 redshifts. These data are compiled, e.g., in [53], and are listed in table 4.

Table 4: The $H(z)$ data. The values of $H$ are in $\frac{km}{sMpc}$.

| $z$  | $H(1\sigma)$ Ref. | $z$  | $H(1\sigma)$ Ref. |
|------|-------------------|------|-------------------|
| 0.07 | 69.0(19.6) [63]   | 0.473 | 80.9(9.0) [66]    |
| 0.09 | 69.0(12.0) [64]   | 0.48  | 97.0(62.0) [68]   |
| 0.12 | 69.0(12.0) [63]   | 0.5929| 104.0(13.0) [65]  |
| 0.17 | 83.0(8.0) [64]    | 0.6797| 92.0(8.0) [65]    |
| 0.1791| 75.0(4.0) [65]   | 0.7812| 105.0(12.0) [65]  |
| 0.1993| 75.0(5.0) [65]   | 0.8754| 125.0(17.0) [65]  |
| 0.2  | 72.9(29.6) [63]   | 0.88  | 90.0(40.0) [68]   |
| 0.27 | 77.0(14.0) [64]   | 0.9   | 117.0(23.0) [64]  |
| 0.28 | 88.8(36.6) [63]   | 1.037 | 154.0(20.0) [65]  |
| 0.3519| 83.0(14.0) [65]  | 1.3   | 168.0(17.0) [64]  |
| 0.3802| 83.0(13.5) [66]  | 1.363 | 160.0(33.0) [69]  |
| 0.4  | 95.0(17.0) [64]   | 1.43  | 177.0(18.0) [64]  |
| 0.4004| 77.0(10.2) [66]  | 1.53  | 140.0(14.0) [64]  |
| 0.4247| 87.1(11.2) [66]  | 1.75  | 202.0(40.0) [64]  |
| 0.4497| 92.8(12.9) [66]  | 1.965 | 186.0(50.4) [69]  |
| 0.47 | 89.0(50.0) [67]   |       |                   |

So we add to $\chi^2_{tot}$ the term

$$\chi^2_H = \sum_{i=1}^{31} \left( \frac{H(z_i) - H_{i(obs)}}{\sigma_H} \right)^2 .$$
Using the expression \( \chi^2_{tot} = \chi^2_{SN} + \chi^2_{CMB} + \chi^2_{BAO} + \chi^2_{H} \), the likelihood function is given by

\[
L(\delta, n, \Omega_{\Psi i}, \Omega_{bi}) \propto \exp\left[-\frac{\chi^2_{tot}(\delta, n, \Omega_{\Psi i}, \Omega_{bi})}{2}\right].
\]

So, by minimizing \( \chi^2_{tot} \) (what is obviously equivalent to maximize the likelihood function \( L \)), we obtain the best fit values for the parameters of the ITNADE and the IQNADE. For comparison, we also obtain the best fit values for the noninteracting case (NADE) and for the \( \Lambda CDM \) model. In the next section we show and discuss the results obtained.

V. RESULTS

Table 5 below shows the individual best fits for all models considered in this work. We integrate the equations of all models since the redshift \( z_i = 3 \times 10^{5} \) - to increase \( z_i \) in some orders of magnitude didn’t affect the results. Also are shown the \( \chi^2_{\text{min}} \), \( AIC \) (Akaike Information Criteria), \( BIC \) (Bayesian Information Criteria) and \( \Delta AIC \) and \( \Delta BIC \).

Table 5: Values of model parameters of the ITNADE, IQNADE, NADE and \( \Lambda CDM \) from SNeIa, BAO, CMB and \( H \). \( \delta \) is dimensionless: for ITNADE \( \delta \) is in fact \( \delta H_{0} \), where \( H_{0} = 2.133h \times 10^{-39}MeV \) and \( h = 0.7 \), and for IQNADE, \( \delta \) is in fact \( \delta M_{Pl} \), where \( M_{Pl} = 2.436 \times 10^{21}MeV \) is the reduced Planck mass. \( \Delta AIC = AIC_{\text{model}} - AIC_{\Lambda CDM} \) and \( \Delta BIC = BIC_{\text{model}} - BIC_{\Lambda CDM} \).

|       | ITNADE          | IQNADE          | NADE            | \( \Lambda CDM \)          |
|-------|-----------------|-----------------|-----------------|-----------------------------|
| \( n/\Omega_{\Lambda i} \) | 2.445^{+0.033}_{-0.033} | 2.453^{+0.038}_{-0.038} | 2.405^{+0.014}_{-0.014} | (7.850^{+0.064}_{-0.064}) \times 10^{-19} |
| \( \Omega_{\Psi i} \) | (8.80^{+0.18}_{-0.18}) \times 10^{-3} | (8.70^{+0.19}_{-0.19}) \times 10^{-3} | (9.180^{+0.051}_{-0.051}) \times 10^{-3} | (1.0525^{+0.0048}_{-0.0048}) \times 10^{-2} |
| \( \Omega_{bi} \) | (1.770^{+0.025}_{-0.025}) \times 10^{-3} | (1.775^{+0.025}_{-0.025}) \times 10^{-3} | (1.8200^{+0.0085}_{-0.0085}) \times 10^{-3} | (1.6960^{+0.0085}_{-0.0085}) \times 10^{-3} |
| \( \delta \) | -0.110^{+0.023}_{-0.023}+0.023+0.046+0.066 \times -0.023+0.046+0.066 | -0.065^{+0.014}_{-0.014}+0.028+0.042 \times -0.014+0.028+0.042 | - | - |
| \( \chi^2_{\text{min}} \) | 1117.65 | 1118.67 | 1144.95 | 1130.24 |
| \( AIC \) | 1125.65 | 1126.67 | 1150.95 | 1136.24 |
| \( BIC \) | 1145.63 | 1146.65 | 1165.93 | 1151.22 |
| \( \Delta AIC \) | -10.59 | -9.57 | 14.71 | - |
| \( \Delta BIC \) | -5.59 | -4.57 | 14.71 | - |
Notice that the noninteracting model (NADE) has the same number of parameters of the $\Lambda CDM$ ($n$, $\Omega_{\psi i}$ and $\Omega_{bi}$ for the NADE and $\Omega_{\Lambda i}$, $\Omega_{\Psi i}$ and $\Omega_{bi}$ for the $\Lambda CDM$). For a large number of degrees of freedom, the distribution of $\chi^2$ is gaussian, with mean equal to the number of degrees of freedom, $\chi^2 = \nu = 1091 - 3 = 1088$ in this case, and standard deviation $\sigma = \sqrt{2\nu} = 46.7$. If we define the criterion that values of $\chi^2_{\text{min}}$ within an interval of $2\sigma$ around the best value $\chi^2 = \nu$ are acceptable, or in other words, if we define the criterion that fits whose $\chi^2_{\text{min}}$ are within the interval $994.7 < \chi^2 < 1181.3$ are acceptable, then by this $\chi^2$ criterium, we can say that the NADE model fits well the present set of observational data (in fact, for the NADE $\chi^2_{\text{min}} = 1144.95 \simeq \nu + 1.2\sigma$, and for the $\Lambda CDM$ $\chi^2_{\text{min}} \simeq 1130.24 = \nu + 0.9\sigma$). For more details about the $\chi^2$ criterion see, e. g. [70].

We can see by the values of $\chi^2_{\text{min}}$ showed in the table 1, that the two interacting models fits the data better than the $\Lambda CDM$. But this improvement on the fit is sufficient to justify the introduction of one more free parameter (the coupling constant $\delta$) in the NADE model? This question can be answered using, for example, the $AIC$ [71] and $BIC$ [72] criteria. We can use the $AIC$ and $BIC$ criteria to answer if a given model is preferred by the data or not, or in other words, if the data furnishes sufficient evidence in favor of a given model. Obviously, what we want to know in this work is if there exists evidence in favor of an interaction between dark energy and dark matter.

The $AIC$ is basically a frequentist criterion, and for a large set of data and Gaussian errors, it is given by

$$AIC = -2 \ln L_{\text{max}} + 2p ,$$

(85)

where $p$ is the number of free parameters of the model. If we want to know if there exists evidence in favor of a given model, say $model 1$, in relation to another model, $model 2$, we need to compute $\Delta AIC = AIC_{model 1} - AIC_{model 2}$. If $4 < \Delta AIC < 7$ there is evidence in favor of the $model 2$, that is, the model with minor $AIC$ value. If $\Delta AIC > 10$ such an evidence is strong. For detailed discussions about $AIC$ and $BIC$ criteria see, e. g., [73] and [74].

The $BIC$ follows from a Gaussian approximation to the Bayesian evidence in the limit of large sample size [75]:

$$BIC = -2 \ln L_{\text{max}} + p \ln N ,$$

(86)

where $p$ is the number of free parameters and $N$ is the number of data points. In the same
manner as for AIC, if $2 < \Delta BIC < 6$, there is positive evidence in favor of the model with minor BIC value. Again, if $\Delta BIC > 10$, such an evidence is strong.

From table 5, we see that the AIC and BIC criteria furnishes strong evidence against the noninteracting case (NADE), in relation to the $\Lambda CDM$ model. Such a conclusion has already been obtained in [76] and [77], from different data sets. However, for both interacting models, ITNADE and IQNADE, AIC criterium furnishes strong evidence in favor of the interacting models, whereas BIC criterium furnishes moderate evidence. Therefore, considering both the criteria, in the present work we have obtained strong evidence in favor of both the interacting models. These result, combined with the fact that for both ITNADE and IQNADE the coupling constant is nonvanishing at more than $3\sigma$ confidence level, give us significative evidence of an interaction between dark energy and dark matter. Furthermore, the sign of the coupling is compatible with dark energy decaying into dark matter, alleviating the coincidence problem.

Figures 1 and 2 show the marginalized probability distributions of the (dimensionless) coupling constant $\delta$ and $n$, whereas figures 3 and 4 show the two parameter confidence regions of $1\sigma$, $2\sigma$ and $3\sigma$ for the ITNADE and the IQNADE models.

It is interesting to notice that there is a little degeneracy between the coupling constant $\delta$ and $n$, so that in both interacting models $n$ is bigger than from the noninteracting case. The differences, however, are less than $1\sigma$.

![Marginalized probability distributions](image)

**FIG. 1:** Marginalized probability distributions of $\frac{\delta}{H_0}$ and $n$ for the ITNADE model.
FIG. 2: Marginalized probability distributions of $\delta M_{Pl}$ and $n$ for the IQNADE model.

FIG. 3: Confidence regions of 1$\sigma$, 2$\sigma$ and 3$\sigma$ for two parameters for the ITNADE model.

FIG. 4: Confidence regions of 1$\sigma$, 2$\sigma$ and 3$\sigma$ for two parameters for the IQNADE model.
In summary, we have derived two field theory models of interacting dark energy and have made the comparison of these models with recent observational data. We have also made the comparison of the noninteracting and $\Lambda$CDM models with the data. From the application of the AIC and BIC model selection criteria, we have obtained strong evidence in favor of the two interacting models. Moreover, the coupling constants of the two models are nonvanishing at more than $3\sigma$ confidence level. Therefore, we have obtained significative evidence of an interaction in the dark sector of the universe. This conclusion go in the same direction of other works in recent years, e.g. [3] [4] [5] [6] [7] [8].

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The Pantheon SNeIa data and its covariance matrix can be obtained at https://archive.stsci.edu/prepds/ps1cosmo/scolnic_datatable.html.

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