Quantum Weak Equivalence Principle and the Gravitational Casimir Effect in Superconductors

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Abstract

We will use Fisher information to properly analyze the quantum weak equivalence principle. We argue that gravitational waves will be partially reflected by superconductors. This will occur as the violation of the weak equivalence principle in Cooper pairs is larger than the surrounding ionic lattice. Such reflections of virtual gravitational waves by superconductors can produce a gravitational Casimir effect, which may be detected using currently available technology.

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General relativity is built upon the weak equivalence principle (WEP), which has been demonstrated to be valid up to $\delta \sim 10^{-15}$ in precision (where $\delta$ in the Eötvös parameter) [1]. The WEP states that due to the equivalence between the gravitational and inertial masses, the trajectory of a classical particle in a gravitational field does not depend on the mass of that particle. Now for a quantum particle, trajectories are replaced by quantum probability distributions, which in principle can violate the WEP. However, the violation of the WEP has not been observed in several quantum systems placed in constant gravitational fields [2–6]. As these quantum systems were studied using constant gravitational fields, it is still possible that the WEP can be violated in varying gravitational backgrounds, like gravitational waves. Thus, we will discuss the violation of the WEP by quantum particles in a gravitational wave (GW), and to do this we need to precisely define the quantum analog for the WEP.

The absence of information about the mass of the particle from its classical trajectory can be generalized to probability distributions to obtain a definition for quantum WEP. This is done using Fisher information, which measures the amount of information that an observable random variable provides about an unknown parameter. Now for a particle, the random variable would be its measured position $x$, and the unknown parameter would be its mass $m$. So, for a quantum particle with wave function $\psi(x,t)$, the Fisher information is given by [7, 8]

$$F_x(m) = \int d|x|\psi(x,t)|^2 \left[ \frac{\partial}{\partial m} \log |\psi(x,t)|^2 \right]^2 .$$  \hspace{1cm} (1)

One notes that it is possible to obtain the mass information from the position of a quantum particle. However, for the quantum WEP to hold, this Fisher information in a gravitational field $F_x(m)$ should be exactly equal to the Fisher information in the absence of such a gravitation field $F^\text{free}_x(m)$. In fact, it can be observed that for constant gravitational fields, $F_x(m) = F^\text{free}_x(m)$ [9], and so the WEP holds, as expected from previous observations [2–6].

However, we can demonstrate that new mass information can be obtained for a quantum particle in a GW. We start from the metric for a generally polarized linear plane GW

$$ds^2 = -c^2 dt^2 + dz^2 + (1-2v)dx^2 + (1+2v)dy^2 - 2udx dy ,$$  \hspace{1cm} (2)

where $u = u(t - z)$ and $v = v(t - z)$ are functions which describe a wave propagating in the $z$-direction. Now for a circularly polarized GW traveling along the $z$-direction, $v = f = f_0 \cos(kz - \omega t)$ and $u = if$. We can write the non-relativistic Hamiltonian for a quantum particle (using the Foldy-Wouthuysen transformation [10, 11] of the Dirac equation in curved
FIG. 1: The difference in the mass Fisher information between a free quantum particle and a quantum particle in a GW. Here WEP is violated as $|F_x(m) - F^\text{free}_x(m)| \neq 0$, and its value fluctuates in time. Units are chosen such that $\hbar, m, f_0, \omega$ are unity.

In spacetime) in the GW background as [12, 13]

$$H_{GW} = \frac{1}{2m}(\delta^{ij} + 2T^{ij})p_i p_j + m c^2,$$

$$T^j_i = \begin{pmatrix} v & -u & 0 \\ -u & -v & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (3)$$

The unitary transformation $U = \exp(-iH_{GW}t)$ gives the time evolution of a localized quantum particle (with a one dimensional wave packet $\psi(x, 0) = (2/\pi)^{1/4} e^{-(x-x_0)^2}$) in a GW background [9, 12]

$$\psi(x, t) = \left(\frac{2}{\pi}\right)^{1/4} \frac{e^{-(x-x_0)^2/b}}{\sqrt{b}}, \quad b \equiv 1 + \frac{2\hbar t}{m}(1 + f_0 \cos \omega t). \quad (4)$$

We can use this wave packet to calculate the Fisher information in GWs. In Fig. 1, we plot the absolute value of the difference between the mass Fisher information in a GW background $F_x(m)$ and mass Fisher information for free particle $F^\text{free}_x(m)$. We observe that even though $|F_x(m) - F^\text{free}_x(m)| \neq 0$ (breaking WEP), its value fluctuates over time. So, we can now define a parameter $\beta^A$ to measure the magnitude of violation of WEP for a quantum system $A$ in GWs (with Fisher information $F^A_x(m)$) as

$$\beta^A = \frac{\int |F^A_x(m) - F^\text{free}_x(m)|dt}{\int |F^\text{free}_x(m)|dt}. \quad (5)$$
Now for two quantum systems $A$ and $B$, if $\beta_A^A > \beta_B^B$, then the magnitude of violation of WEP is larger in $A$ than $B$.

![Diagram](image)

**FIG. 2:** Mechanism for spectral reflection of GWs from superconductors. (a) The negatively-charged CP deforms the positively-charged ionic lattice. (b) The GW accelerates the delocalized CP relative to the lattice. However, the positively-charged ionic lattice suppresses this acceleration, thereby partially reflecting the GW.

This violation of the WEP may be experimentally detected in superconductors because the delocalization of Cooper pairs (CPs) are expected to violate the WEP more than localised particles. This can be observed by considering the two-particle wave function of the CP

$$\psi_0(r^{(1)}, r^{(2)}) = \sum_k g_k e^{i k \cdot r^{(1)}} e^{-i k \cdot r^{(2)}}, \quad (6)$$

where $r^{(i)}$ is the coordinate of particle $i$. We can now use the unitary transformation operator $U = \exp(-i H_{GW}(\mathbf{p}^{(1)}, \mathbf{p}^{(2)}) t)$ to get the time-evolution of the CP in a GW background, where

$$H_{GW}(\mathbf{p}^{(1)}, \mathbf{p}^{(2)}) = \frac{1}{2m_{cp}} (\delta^{ij} + 2 T^{ij}) (p_i^{(1)} p_j^{(1)} + p_i^{(2)} p_j^{(2)}) + m_{cp} c^2 + V, \quad (7)$$

with $V$ as the interaction with the ionic lattice. As the CP are delocalized relative to the localized ionic lattice (shown in Fig. 2(a)), they exhibit a larger magnitude of WEP violation, $\beta_{\text{Cooper}} > \beta_{\text{ion}}$, because

$$\beta_{\text{Cooper}} = \frac{\int |F_x^{\text{Cooper}}(m_{cp}) - F_x^{\text{free}}(m_{cp})| dt}{\int |F_x^{\text{free}}(m_{cp})| dt} > \frac{\int |F_x^{\text{ion}}(m_{ion}) - F_x^{\text{free}}(m_{ion})| dt}{\int |F_x^{\text{free}}(m_{ion})| dt} = \beta_{\text{ion}}. \quad (8)$$
Thus, GWs will tend to accelerate the CPs relative to the ionic lattice due to the larger magnitude of WEP violation in CPs relative to the ionic lattice, as depicted in Fig. 2(b). As the CPs and ionic lattice are oppositely charged, they will resist any charge separation, thereby (partially) reflecting the GW, analogous to the way conductors repel electromagnetic waves. It was initially proposed that such a reflection of GWs occurs due to CPs not moving on the geodesics on which ions move [15–18], however, as the system is quantum mechanical, the reflection of GWs in superconductors actually occurs due to the difference in Fisher information for CPs and ions.

Now GWs with wavelengths of the same order as the CP coherence length would be required to test this WEP violation, but such GWs are difficult to generate in controlled experiments. However, we can utilize the full spectrum of the virtual GWs (formed from virtual gravitons in the vacuum) in a gravitational Casimir effect to test such violations of WEP [19, 20]. This can be explicitly demonstrated using the Einstein field equations linearized around a flat spacetime metric (which resemble the Maxwell equations with a magnetic monopole) [21–25],

\[
\begin{align*}
\nabla \cdot E &= \kappa \rho^{(E)}, \\
\nabla \cdot B &= \kappa \rho^{(M)}, \\
\n\nabla \times E &= -\frac{\partial B}{\partial t} - \kappa J^{(M)}, \\
\n\nabla \times B &= \frac{\partial E}{\partial t} + \kappa J^{(E)},
\end{align*}
\]

where \( \kappa \equiv \frac{8\pi G}{c^4} \), \( E_{ij} \equiv C_{0i0j}, B_{ij} \equiv \star C_{0i0j}, C_{\alpha\beta\mu\nu} \) is the Wely tensor, \( \star \) denotes Hodge dualization\(^1\), and \( \rho_i^{(E)} \equiv -J_{i00}, \rho_i^{(M)} \equiv -\star J_{i00}, J^{(E)}_{ij} \equiv J_{i0j}, J^{(M)}_{ij} \equiv \star J_{ij} \)\(^2\). Now for parallel superconducting plates separated by a distance \( b \) along the \( z \)-axis (with \( k_\parallel \equiv \sqrt{k_x^2 + k_y^2} \)), the gravitational Casimir energy can be obtained by summing over \((\omega_n^+, \omega_n^-)\), which are the relevant modes of virtual GWs for the system [20, 26],

\[
E_0(b) = \frac{\hbar}{4\pi} \int_0^\infty k_\parallel dk_\parallel \sum_n (\omega_n^+ + \omega_n^-) \sigma,
\]

where \( \sigma \) is the surface area of the plates. It may be noted that it diverges due to the summation over the infinite number of allowed modes, and we need to renormalize it by subtracting the Casimir energy at infinite separation [19]

\[
E_R(b) = \frac{E_0}{\sigma} - \lim_{b \to \infty} \frac{E_0}{\sigma}.
\]

\(^1\) Dualization is defined by \( \star C_{\mu\nu\rho\sigma} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} C_{\alpha\beta\rho\sigma}, \star J_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} J^{\alpha\beta} \), with \( \epsilon_{\mu\nu\alpha\beta} \) as the Levi-Civita tensor.

\(^2\) The matter current \( J_{\mu\nu} \equiv (\eta_{\mu[\rho} T_{\nu\sigma]}/3) - T_{\rho[\mu,\nu]} \) is obtained from stress perturbations \( T_{\mu\nu} (T \equiv T^{\alpha}_{\alpha}) \).
It is this finite renormalized gravitational Casimir energy that can be detected using superconducting parallel plates [27]. The strength of the gravitational Casimir force will depend on the difference between the magnitude of WEP violation experienced by the delocalized CP (β_{Copper}) and the localized ionic lattice (β_{ion}). Although the gravitational Casimir effect would be smaller than the electromagnetic Casimir effect, it can be measured from the additional force produced at the onset of superconductivity (which is experimentally easier than measuring absolute Casimir forces).

![Diagram](image)

**FIG. 3:** General schematic of experiments: a) A superconducting nanomembrane is fabricated above a superconducting substrate. b) Once cooled below the superconducting transition temperature $T_c$, one would observe the sudden change in distance between membrane and substrate due to the gravitational Casimir effect. c) One could measure this change in displacement using a number of currently available displacement probes, which can measure distances on the atomic scale.

A gravitational Casimir experiment with superconductors would be relatively simple, as shown in Fig 3. On a microchip, one can achieve remarkable parallelism [27], making it possible to fabricate a free-standing superconducting nanomembrane over a superconducting substrate [28]. The entire microchip can then be cooled below the superconducting transition temperature $T_c$, experiencing an additional gravitational Casimir force, as depicted in Fig 3(a, b). Any small additional forces at $T_c$ would lead to large displacements of these ultra-sensitive nanomembranes due to their high-aspect-ratio [28]. A displacement probe such as those used in scanning tunneling microscopy [29], or a photonic crystal probe [30], could measure displacements of this membrane with atomic precision, as depicted in Fig 3(c). The magnitude of WEP violation in CPs depends on the CP coherence length,
which in turn depends on the properties of superconductors \[31, 32\], so one may use various superconducting materials to measure the gravitational Casimir effect with current technology. As the WEP is a building block of general relativity, convincingly demonstrating its violation would signify a drastic departure from our current understanding of gravity at short distances.

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