Random Decision Stumps for Kernel Learning and Efficient SVM

Gemma Roig * Xavier Boix * Luc Van Gool

Computer Vision Lab, ETH Zurich, Switzerland
{boxavier,gemmar,vangool}@vision.ee.ethz.ch
* Both first authors contributed equally.

Abstract. We propose to learn the kernel of an SVM as the weighted sum of a large number of simple, randomized binary stumps. Each stump takes one of the extracted features as input. This leads to an efficient and very fast SVM, while also alleviating the task of kernel selection. We demonstrate the capabilities of our kernel on 6 standard vision benchmarks, in which we combine several common image descriptors, namely histograms (Flowers17 and Daimler), attribute-like descriptors (UCI, OSR, and a-VOC08), and Sparse Quantization (ImageNet). Results show that our kernel learning adapts well to these different feature types, achieving the performance of kernels specifically tuned for each, and with an evaluation cost similar to that of efficient SVM methods.

1 Introduction

The success of Support Vector Machines (SVMs), e.g. in object recognition, stems from their well-studied optimization and their use of kernels to solve non-linear classification problems. Designing the right kernel in combination with appropriate image descriptors is crucial. Their joint design leads to a chicken-and-egg problem in that the right kernel depends on the image descriptors, while the image descriptors are designed for familiar kernels.

Multiple Kernel Learning (MKL) [1] eases kernel selection by automatically learning it as a combination of given base kernels. Although MKL has been successful in various vision tasks (e.g. [2,3]), it might lead to complex and inefficient kernels. Recently, Bazavan et al. [4] introduced an approach to MKL that avoids the explicit computation of the kernel. It efficiently approximates the non-linear mapping of the hand-selected kernels [5,6,7], thus delivering impressive speed-ups.

We propose another way around kernel learning that also allows for efficient SVMs. Instead of combining fixed base kernels, we investigate the use of random binary mappings (BMs). We coin our approach Multiple Binary Kernel Learning (MBKL). Given that other methods based on binary decisions such as Random Forests [8] and Boosting decision stumps [9] have not performed equally well on image classification benchmarks as kernel SVMs, it is all the more important that we will show MBKL does. Not only does MBKL alleviate the task of selecting the right kernel, but the resulting kernel is very efficient to compute and can scale to large datasets.

At the end of the paper, we report on MBKL results for 6 computer vision benchmarks, in which we combine several common image descriptors. These descriptors
are histogram-based (Flowers17 [10] and Daimler [11]), attribute-based (OSR [12], a-PASCAL VOC08 detection [13], and UCI [14]), and Sparse Quantization [15] (ImageNet [16]). We demonstrate for the first time that a classifier based on BMs can achieve performances comparable to those of the hand-selected kernels for each specific descriptor. Moreover, it is as fast as the fastest kernel approximations, but without the need of interactively selecting the kernel.

2 Efficient SVM and Kernel Learning

In this section, we revisit the SVM literature, with special emphasis on efficient and scalable kernel learning for object recognition.

Efficient SVM. We use \((w, b)\) to denote the parameters of the SVM model, and \(\phi(x)\) for the non-linear mapping to a higher-dimensional space. The classification score for a feature vector \(x\) then is \(w^T \phi(x) + b\). The SVM aims at minimizing the hinge loss.

For the SVM implementation, one typically applies the kernel trick: with Lagrange multipliers \(\{\alpha_i\}\) the classification score becomes

\[
    w^T \phi(x) + b = \sum_i \alpha_i y_i \left( \phi(x)^T \phi(x_i) \right) + b = \sum_i \alpha_i y_i K(x, x_i) + b
\]

where \(K(x, x_i) : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}\). The optimal multipliers \(\alpha_i\) tend to be sparse and select relatively few ‘support vectors’ from the many training samples. The kernel trick bypasses the computation of the non-linear mapping by directly computing the inner products \(K(x, x_i)\).

The strength of SVMs is that they yield max-margin classifiers. At test time, the computational cost is the number of support vectors times the cost of computing the kernel. The problem is that the latter may be quite expensive. Also, during training, the complexity of computing the kernel matrix grows quadratically with the number of training images, which renders it intractable for large datasets.

Several authors have tried to speed up kernel-based classification. Ideas include limiting the number of support vectors [17,18] or creating low-rank approximations of the kernel matrix [19]. These methods are effective, but do not scale well to large datasets because they require the kernel distances to the training set. Rahimi and Recht introduced Random Fourier Features [6], thereby circumventing the approximation of the explicit feature map, \(\phi(x)\). Such techniques have been explored further for kernels used with common image descriptors, such as \(\chi^2\) and RB-\(\chi^2\) kernels [5,7] or the intersection kernel [20,21]. Other approaches use kernel PCA to linearize the image descriptors [22] or sparse feature embeddings [23]. Recently, Wu [24] introduced the power mean kernel, which generalizes the intersection and \(\chi^2\) kernels, among others, and achieves a remarkably efficient, scalable SVM optimization.

These methods approximate specific families of kernels. Our aim is to learn a fast kernel, which eases kernel selection, rather than approximating predefined kernels.
Multiple Kernel Learning (MKL). MKL aims at jointly learning the SVM and a linear combination of given base kernels. The hope is that such committee of base kernels is a more powerful kernel. Denoting the base kernels as \( \hat{K} \), the final kernel \( K \) takes the form

\[
K(x, x_i) = \sum_k \theta_k \hat{K}_k(x, x_i),
\]

where the weights \( \theta_k \in \mathbb{R}^+ \) can be discriminatively learned. There have also been some approaches to find non-linear combinations of kernels, e.g. \[25,26\], that we do not further consider here.

In recent years, many advances have been made to improve the efficiency of MKL, and various optimization techniques have been introduced, e.g. semi-definite programming \[27\], SMO \[1,28\], semi-infinite linear programming \[29\] and gradient-based methods \[30,26\]. Yet, scalability to large datasets remains an issue, as these methods explicitly compute the base kernel matrices. Therefore, Bazavan et al. \[4\] exploit Random Fourier Features, which approximate the non-linear mapping of the kernel, and allow to scale to large datasets.

Our approach is related to the latter in that it also aims at efficient and scalable kernel learning. Yet, MBKL’s base kernels are not hand-selected. Instead of approximating a distance coming with a pre-selected kernel, we explore the use of random BMs to learn a distance for classification. Indeed, in large-scale image retrieval, there is an increasing body of evidence that suggests that BMs are effective to evaluate distances, e.g. \[31,32,33\].

In the next section, we introduce our formulation for kernel learning built from BMs. This, in turn, will yield a kernel very efficient to learn and to evaluate (Section 3). Moreover, the kernel will adapt to most image descriptors, since it is learned from the input data (Section 5).

3 Multiple Binary Kernel Learning

In this section, we introduce a kernel that is a linear combination of binary kernels, defined from a set of simple decision stumps.

MKL with Binary Base Kernels. BMs have been used to speed-up distance computations in large-scale image retrieval, e.g. \[31,32,33\]. In these methods, the input feature is transformed into a binary vector that preserves the locality of the original feature space. In the context of classification, we can further enforce that the kernel in an SVM separates the image classes well.

We adopt the MKL formulation (see eq. (2)) as starting point of our kernel, since it aims at jointly learning the classifier and the kernel distance, yet incorporate BMs and restrict the base kernels to only take on binary values. The binary base kernels are defined as:

\[
\hat{K}_k(x, x_i) = I[\sigma_k(x) = \sigma_k(x_i)],
\]
where $I[\cdot]$ is the indicator function, which returns 1 if the input is true and 0 otherwise, and $\sigma_k(x)$ is a BM of the input feature, $\sigma_k(x): \mathbb{R}^n \rightarrow \{0, 1\}$. Each base kernel is built upon one single BM. The BMs need not be linear and can be adapted to each problem if desired. In the sequel, we explore different possibilities, but our kernel is not restricted to any of them. In all cases, $\sigma_k(x)$ divides the feature space into two sets and the indicator function returns whether the two input samples fall in the same part of the feature space or not.

The final kernel, $K$, is a linear combination of the binary kernels: $K(x, x_i) = \sum_k \theta_k I[\sigma_k(x) = \sigma_k(x_i)]$. Note that $K$ is not restricted to be binary, though the base kernels are. In the supplementary material we show that such ‘Multiple Binary Kernel’ (MBK) is a valid Mercer kernel. An appropriate choice of the $\sigma_k(x)$ will be important to arrive at good classifications. For instance, in a two class problem, the better the set of $\sigma_k(x)$ separate the two classes, the better the kernel might be.

### Explicit Non-linear Mapping

We analyze the form of the non-linear mapping of MBKL. In the Supplementary Material, we derive the non-linear mapping $\phi(x)$ that induces the MBKL kernel, and it is

$$[\sqrt{\theta_1} \sigma_1(x), \sqrt{\theta_1} \bar{\sigma}_1(x), \sqrt{\theta_2} \sigma_2(x), \sqrt{\theta_2} \bar{\sigma}_2(x), \ldots]^T,$$

(4)

where $\bar{\sigma}_k(x)$ is $\sigma_k(x)$ plus the not operation, and the SVM parameters are

$$w^T = [\sqrt{\theta_1} c_1, \sqrt{\theta_1} c_0, \sqrt{\theta_2} c_1, \sqrt{\theta_2} c_0, \ldots],$$

(5)

where $c_1, c_0 \in \mathbb{R}$ are two learned constants, which correspond to the underlying parameters of the classifier. We can see that this mapping recovers the MBKL kernel in the form $\phi(x)^T \phi(x_i)$. Thus, to evaluate MBKL at test time, we do not need to evaluate the kernel because we have access to the non-linear mapping $\phi$.

### Benefits of kernel learning

MBKL generalizes a SVM with BMs as input features. This can be easily seen by fixing $\theta = 1$ in eq. (4) and (5). But learning $\theta$ rather than fixing it to 1 has several advantages. Recall that the kernel distance does not depend on the image class we are evaluating. A BM with $\theta_k$ equal to 0 does not contribute to the final kernel distance, and hence, can be discarded for all image classes. This is crucial to arrive at a competitive computational complexity. Moreover, MBKL aims at learning a kernel distance adapted to the image descriptors, that can be used for tasks other than classification.

MBKL is not a particular instance of any of the kernels in the literature. Rather the opposite may be true, since most kernels can be approximated with a collection of BMs [31].

### 4 BMs as Random Decision Stumps

We found that defining the $\sigma_k(x)$ as simple random decision stumps achieves excellent results with the image descriptors commonly used in the literature. Decision stumps select a component in a feature vector and threshold it. We randomly select a component
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Fig. 1. BMs for Kernel Learning. On Flowers 17 training set: (a) Comparison between \(\chi^2\) distance and a MBKL distance with \(\theta = 1\) (MKBL kernel is normalized with the amount of BMs). Each point represents the distance between two images in the training set (we indicate in orange color that the \(\chi^2\) distance is lower than 0.2). (b) MBKL kernel with \(\theta = 1\), and (c) MBKL kernel with the learned \(\theta\). For (b) and (c) images are sorted with their class label, this is why some semantic clusters can be seen around the diagonal.

\(i \in \mathbb{N}\) of the input feature vector, using a uniform probability distribution between 1 and the feature length. Then, the BM is calculated applying a threshold, \(\sigma_k(x) = I[x_i > t]\), where \(t \in \mathbb{R}\) is the threshold value. Again, this threshold is generated from a uniform probability distribution, here over the interval of values observed during training for component \(i\). Note that we generate \(\sigma_k(x)\) randomly, without using labeled data. In contrast, the supervised learning of the kernel and the SVM will use labeled data to appropriately combine the BMs (Section 5).

We may need thousands of random BMs to arrive at the desired level of performance. Since the decision stumps have cost \(O(1)\), the computational complexity of evaluating MBKL at test time grows linearly with the number of BMs. In the experiments we show that this is of the same order of magnitude as the feature length, or one order higher. This allows to achieve a competitive computational cost compared to other methods, as we report in the experimental section.

Intuitively, random decision stumps may seem to quantize the image descriptor too crudely. That might then affect the structure of the feature space and deteriorate performance. Yet, in classification, decision stumps are known to allow for good generalization [34,35,36]. As an illustration, Fig. 1a compares the \(\chi^2\) distance and MBKL with decision stumps. We use the experimental setup of Flowers17 (see Section 6), for which \(\chi^2\) is the best performing kernel, but the other kernel distances and datasets in the paper yield the same conclusions. MBKL uses 30,000 random decision stumps and for the time being we simply put \(\theta = 1\), i.e. all \(\theta_k = 1\). We can see that the distances produced by both methods are highly correlated. The decision stumps do change the structure of the feature space, but keep it largely intact. Since the kernel distance is parametrized through \(\theta\), that modulates the contribution of each BM, MBKL can further adjust the kernel distance to the SVM objective. Fig. 1b shows the final MBKL kernel, and Fig. 1c the MBKL \(\theta\)-adjusted kernel, as learned in Section 5. Observe that the high kernel values between images of different classes in the non-learned kernel, are smoothed out in the learned kernel.

Also, note that using decision stumps with MBKL differs substantially from boosting decision stumps. Apart from the differences in the loss function, boosting optimizes
Algorithm 1: Multiple Binary Kernel Learning

Input: \((x_i, y_i), \forall i\)
Output: \(\sigma(x), \theta, w\)

\(\{\sigma_k\} = \text{Generate Random Tests};\)

Step 0: \(\{c_1^k, c_0^k\} = \text{Initial Guess}\) \((\{\sigma_k(x)\}, y)\);  
Step 1: \(\theta = \text{SVM}_{l_1}\) \((\{c_1^k, c_0^k\}, y)\);  
\(\sigma = \text{Select}\) \(\{\theta_k\} > 0\);  
Step 2: \(\{c_1^k, c_0^k\} = \text{SVM}(\phi(x), y)\);  

the parameters of the BMs individually, using labeled data, and progressively adds them  
to the final classifier. MBKL generates all the BMs all at once, with random parameters  
and without using labeled data.

5 Efficient Two-stage Learning

In this section, we introduce the formulation for learning the kernel and the classifier  
parameters, once the random BM have already been generated. MBKL pursues minimizing the SVM objective. Rather than jointly optimizing the kernel and the classifier – which may be not feasible in practice for thousands of binary kernels – we decompose the learning in two stages to make it tractable. All stages optimize the same SVM objective, but either \(\theta\) or the classifier parameters are kept fixed. Firstly, we fix the classifier parameters to an initial guess, and we learn the kernel, \(\theta\). Secondly, the classifier is trained with the learned kernel. We could extend this algorithm to iteratively re-learn \(\theta\) and the classifier parameters, but this would obviously raise the computational cost while we did not observe any increase in performance. Also, note that we do not re-sample new BMs after discarding some.

Next we describe the two steps of the learning in more detail. Before the actual optimization starts, we have to initialize the classifier, which we describe as the prior. Step 0 then learns the kernel parameters, after which Step 2 learns the actual classifier. We show that the most complex optimizations can be solved with off-the-shelf SVM solvers in the primal form. We summarize all steps of the learning in Algorithm 1.

**Step 0: Efficient Initial Guess of the Classifier**. Recall that each binary kernel has two parameters associated: \(c_1^k, c_0^k\) (eq. [4], [5]). In order to efficiently get an initial guess of these parameters, we learn each pair of parameters individually, without taking into account the other kernels. The downside is that this form of learning is rather myopic, blind as it is to the information coming from the other kernels. However, this is alleviated by the global learning of the kernel weights \(\theta\) and the SVM classifier in Steps 1 and 2 of the algorithm.

For the initial guess of \((c_1^k, c_0^k)\) we also use the SVM objective, but we formulate it for each kernel individually. The classifier and the non-linear mapping of a single kernel becomes

\[\phi_k(x)^T = \sqrt{\theta_k(\sigma_k(x), \bar{\sigma}_k(x))}, \quad w_k^T = \sqrt{\theta_k(c_1^k, c_0^k)}, \quad (6)\]
and we place them in an SVM objective function. We can ignore the dependence on \( \sqrt{\sigma_k} \) because it only scales the classification score, and can be compensated by \( w_k \).

Thus,

\[
\phi_k(x)^T = (\sigma_k(x), \bar{\sigma}_k(x)), \quad w_k^T = (c_k^1, c_k^0).
\] (7)

Interestingly, because when \( \sigma_k(x) \) is 1 then \( \bar{\sigma}_k(x) = 0 \), and v.v., \( w_k^T \phi_k(x) \) can only take two values, i.e., either \([1, 0]^T = c_k^1 \) or \([0, 1]^T = c_k^0 \). As a consequence, we can show that when we optimize \( w_k^T \) with a linear SVM with input features \((\sigma_k(x), \bar{\sigma}_k(x))\), then, \( w_k^T = (a, -a) \), where \( a \in \mathbb{R} \) (see Supplementary Material).

This shows that the SVM classifier for one binary kernel only requires learning a single parameter, \( a \). Further, introducing this result into \( w_k^T \phi_k(x) \) yields

\[
w_k^T \phi_k(x) = (a \mathbf{I}[\sigma_k(x) = 1] - a \mathbf{I}[\sigma_k(x) = 0]).
\] (8)

If we let \( a = \text{sign}(a) |a| \), and discard \( |a| \) because it is only a scale factor that can be later absorbed by \( \theta_k \) (if \( a \neq 0 \)), we obtain that \([c_k^1, c_k^0]\) is determined by \( \text{sign}(a) \) when \( a \neq 0 \). Thus,

\[
w_k^T \phi_k(x) = \text{sign}(a) \mathbf{I}[\sigma_k(x) = 1] - \text{sign}(a) \mathbf{I}[\sigma_k(x) = 0],
\] (9)

when \( a \neq 0 \). Using the proportion of samples that responded \( \sigma_k(x) = 1 \), we can determine the sign of \( a \), and when \( a = 0 \). Let \( t_p \) be the number of samples of the class we are learning the classifier (positive sample), and \( t_n \) the number of samples for the rest of classes (negative). Let \( p \) and \( n \) be how many samples of each class have test value \( \sigma_k(x) = 1 \). Table 1 shows in which cases \( a = 0 \) or, otherwise \( \text{sign}(a) \). In case \( a = 0 \) we discard it, since in Eq. (8) \( a = 0 \) can not contribute in any way to the final classification score. These rules can be deduced by fulfilling the max-margin of the SVM objective.

For of multi-class problem, we do the one-vs-rest strategy, and initialize the parameters \([c_k^1, c_k^0]\) independently for each classifier.

**Step 1: Learning the Kernel Parameters.** We fix \([c_k^1, c_k^0]\) using the initial guess previously learned and we learn the \( \theta \) that minimizes the SVM loss. In order to write the SVM loss directly depending on \( \theta \), we reorder \( w_k^T \phi_k(x) \). From eq. (4) and (5), we obtain \( w_k^T \phi(x) = \theta^T s(x) \), where \( s(x) = [s_1(x), \ldots, s_k(x), \ldots] \), and \( s_k(x) = c_k^1 \sigma_k(x) + c_k^0 \bar{\sigma}_k(x) \). Note that \( s_k(x) \) is known because it only depends on the already guessed \( c_k^1, c_k^0 \), and \( \sigma_k(x) \). With some algebra, the SVM objective that we are pursuing becomes

\[
\min_{\theta, \xi} \| \theta \|_1 + C \| \xi \|_1, \quad \text{s.t.} \forall i : y_i \theta^T s(x) + b \geq 1 - \xi_i.
\] (10)
Observe that the regularizer becomes \( \| \mathbf{w} \|_2^2 = \| \mathbf{\theta} \|_1 \) since \( \{ c_k^1, c_k^0 \} \) is either 1 or \(-1\), and can be discarded because of the square. Thus, we learn \( \mathbf{\theta} \) with a 2-class linear \( \ell_1 \)-SVM where the input features are \( s(\mathbf{x}) \). Since the off-the-shelf SVM optimizers do not constrain \( \theta_k \geq 0 \), it might happen that for some kernels this is not fulfilled. In that case, we directly set the \( \theta_k \) that are not positive to 0. Note that this is necessary to yield a valid Mercer kernel.

For a multi-class problem, the kernel parameters, i.e. \( \{ \sigma_k(\mathbf{x}) \} \) and \( \mathbf{\theta} \), are the same for all classes, whereas there is a specific set \( \{ (c_k^0, c_k^1) \} \) for each class, denoted as \( \{ (c_{k,y}^0, c_{k,y}^1) \} \). We follow the same learning strategy, i.e. we optimize \( c_{k,y}^0, c_{k,y}^1 \) according to Table 1, but using the multi-class heuristic of one-vs-all. Let \( s_y(\mathbf{x}_i) \) be the responses corresponding to class \( y \). Because \( \mathbf{\theta} \) is the same for all classes (the kernel does not change with the class we are evaluating), the SVM in eq. (10) is still a two-class SVM in which we take as positive samples the \( s_y(\mathbf{x}_i) \) evaluated for the true class, i.e. when \( y = y_i \), and the others as negative. This may yield a large negative training set, but we found that in practice it suffices to use a reduced subset of examples. In practice, we generate the subset of negative samples by randomly extracting examples whose object class is different from the target class, and taking into account that the amount of examples per object class is balanced. The number of samples for each dataset is detailed in the experiments section.

**Step 2: Learning the Classifier.** Finally, we use the learned kernel to train a standard SVM classifier, thus replacing the initial guess of the classifier. We discard the BMs with \( \theta_k = 0 \) before learning the SVM, because they do not contribute to the final kernel. From eq. (4) and (5), we can deduce that optimizing the SVM objective with \( \{ c_k^1, c_k^0 \} \) as the only remaining parameters can be done with an SVM in the primal form with \( \phi(\mathbf{x}) = [\theta_1 \sigma_1(\mathbf{x}), \theta_1 \bar{\sigma}_1(\mathbf{x}), \theta_2 \sigma_2(\mathbf{x}), \theta_2 \bar{\sigma}_2(\mathbf{x}), \ldots] \) and \( \mathbf{w}^T = [c_1^1, c_0^1, c_2^1, c_0^2, \ldots] \).

**Computational Cost and Scalability.** We can see by analyzing all the steps of the algorithm that it scales linearly to the number of training data. Step 0 only requires evaluating \( \sigma_k(\mathbf{x}) \) for the training set, and use the simple rules in Table 1. Besides, since the \( \{ c_k^1, c_k^0 \} \) are learned independently, this can be parallelized. Step 1 and 2 are optimized with SVMs in the primal form. Note that for most practical cases, the computational cost of Step 1 may be the bottleneck of the algorithm. The feature length of Step 1 is equal to the initial number of BMs, which is larger than the number of BM with \( \theta_k \neq 0 \), used in Step 2.

Moreover, all steps of the learning algorithm also scale linearly to the number of classes. Note that Step 0 and 2 can be solved with the one-vs-all strategy, and Step 1 is always a two-class SVM.

### 6 Experiments

In this section we report the experimental results on 6 benchmarks, in order to evaluate MBKL in the context of a variety of vision tasks and image descriptors. After introducing the most relevant implementation details, we discuss the results.
Table 2. Summary of the Datasets. The number of images for training and testing are reported for 1 split.

| Dataset          | Daimler | Liver | Sonar | Flowers17 | OSR   | a-VOC08 | ImgNet |
|------------------|---------|-------|-------|-----------|-------|---------|--------|
| # Classes        | 2       | 2     | 2     | 17        | 8     | 20      | 909    |
| # Im. Train      | 19,600  | 276   | 166   | 1,020     | 240   | 6,340   | 1e6    |
| # Im. Test       | 340     | 2,448 | 69    | 32        | 9,800 | 6,355   | 5e4    |
| Descr.           | HoG     | Attr  | Attr  | BoWs      | HoG+Attr | HoG+Attr | S.Quant.|
| Feat. Len.       | 558     | 6     | 60    | 31e3      | 518   | 9,752   | 21e3   |

6.1 Experimental Setup

All the experiments are run on 4 CPUs Intel i7@3.06GHz. We chose the $C$ parameter of the SVM among $\{0.01, 0.1, 1, 10, 100, 1000\}$ by cross-validation on the training set, and we fix it for computing the times. We use liblinear [37] library when using linear SVM, and libsvm [38] library when using a kernel.

Table 2 summarizes the datasets used, as well as their characteristics and the features used. In case the features are attributes, we normalize them with a logistic function to lie in the interval $[0, 1]$. We use this normalization for all methods. For each dataset, the standard evaluation procedures described in the literature are used. Further details are provided in the Supplementary Material. We evaluate MBKL’s efficiency for all the datasets except UCI, for which the computational cost is very little for all methods.

- **Daimler** [11] (Pedestrian detection). This is a two-class benchmark, consisting of 5 disjoint sets, each of them containing 4,800 pedestrian samples and 5,000 non-pedestrian examples. We use 3 splits for training and 2 for testing. Testing is done on the two other sets separately, yielding a total of 6 testing results. The HoG descriptor is used.

- **UCI** [14] (Object Recognition). We report results on two-class problems, namely Liver and Sonar, using 5 cross-validations. We use the attribute-based descriptors that are provided, which are of length 6 and 60, respectively.

- **Flowers17** [10] (Image classification). It consists of 17 different kinds of flowers with 80 images per class, divided in 3 splits. We describe the images using the features provided by [39]: SIFT, opponent SIFT, WSIFT and color attention (CA), building a Bag-of-Words histogram, computed using spatial pyramids.

- **OSR** [12] (Scene recognition). It contains 2,688 images from 8 categories, of which 240 are used for training and the rest for testing. We use the 512-dimensional GIST descriptor and the 6 relative attributes provided by the authors of [12].

- **a-PASCAL VOC08** [13] (Object detection). It consists of 12,000 images of objects divided in train and validation sets. The objects were cropped from the original images of VOC08. There are 20 different categories, with 150 to 1,000 examples per class, except for people with 5,000. The features provided with the dataset are local texture, HOG and color descriptors. For each image also 64 attributes are given. In [13] it is reported that those attributes were obtained by asking users in Mechanical Turk for semantic attributes for each object class in the dataset. They can be used to improve classification accuracy. We use both the features and the attributes.
Fig. 2. Analysis of MBKL in Flowers17. (a) Accuracy, (b) training time, (c) testing time, and (d) amount of selected BMs, when varying the amount of initially randomly generated BMs.

| ImageNet [16]| We create a new dataset taking a subset of 1,065,687 images. This subset contains images of 909 different classes that do not overlap with the synset. We randomly split this subset into a set of 50,000 images for testing and the rest for training, maintaining the proportion of images per class. For evaluation, we report the average classification accuracy across all classes. We use the Nested Sparse Quantization descriptors, provided by [15], using their setup. We use 1024 codebook entries with max-pooling in spatial pyramids of 21 regions (1 × 1, 2 × 2 and 4 × 4). As for many state-of-the-art (s-o-a) descriptors for image classification, better accuracy is achieved with a linear SVM than with kernel SVM. Additionally, we test a second descriptor in ImageNet. We use the same setup as [15] to create a standard Bag-of-Words by replacing the max-pooling by average pooling. This descriptor performs better with kernel SVM, but it achieves lower performance than max-pooling in a linear SVM.

6.2 Analysis of MBKL

We investigate the impact of the different MBKL parameters. Results are given for Flowers17. We conducted the same analysis over the rest of the tested datasets (except ImageNet for computational reasons), and we could extract similar conclusions for all of them. To conduct our analysis we use the following baselines:

- MBKL with binarized $\chi^2$ projections (MBKL proj.): To test other BMs in MBKL, we replace the decision stumps by random projections. We generate the random projections...
using the explicit feature map for the approximate $\chi^2$ by [7]. To obtain the BMs, we threshold them using decision stumps. We use the $\chi^2$ random projections because they achieve high performance on Flowers17, and the computational cost is of the same order as any other random projection.

- **MBKL with $\theta = 1$**: In order to analyze the impact of $\theta$, we do not learn it and directly set it to 1. This is equivalent to using BMs as the input features to a linear SVM.

- **BM as input features for $\ell_1$-SVM**: We use the BMs as the input features to SVM with a $\ell_1$ regularizer, which allows for discarding more BMs. Note that in contrast to MBKL, the selection of BMs is different for each class.

**Impact of the BMs.** In Fig. 2 for MBKL and the different baselines, we report the accuracy, training time, testing time, and the final amount of BMs with $\theta_k \neq 0$, when varying the amount of initial BMs. Comparing random projections and decision stumps, we observe that both obtain similar performance. Also, note that each random projection has a computational complexity in the order of the feature length, $O(n)$, and decision stumps of $O(1)$. This is noticeable at test time, and not in training, since the learning algorithm is much more expensive than computing the BMs.

We can observe that when increasing the number of BMs, the accuracy saturates and does not degrade. MBKL does not suffer from over-fitting when including a large amount of BMs. We believe that this is because the BMs are generated without labeled data, and then are used in a kernel SVM that is properly regularized.

**Impact of $\theta$.** In Fig. 2 we also show the results of fixing $\theta = 1$. This yields a performance close to that of MBKL, because the SVM parameters compensate for the lack of learning $\theta$. Note that fixing $\theta = 1$ lowers the training time since the kernel need not be learned. Yet, learning the kernel is justified because it allows to discard BMs for all classes together (recall that the kernel does not vary depending on the image class), which yields a faster testing time. The number of BMs diminishes after learning the kernel. We can see that this is also the case for the BMs in an $\ell_1$ framework, which is efficient to evaluate, but degrades the performance.

Interestingly, observe that the original feature length is 31,300, and when using an initial amount of 5,000 BMs, the performance is already very competitive. Also, note that the number of BMs with $\theta_k \neq 0$ saturates at around 10,000 (Fig. 2). This is because there are redundant feature descriptors, or non-informative feature pooling regions, and MBKL learns that they are not relevant for the classification. We only observed this drastic reduction of the feature length on Flowers17, where we use multiple descriptors.

**Computational Cost of Kernel Learning.** Table 3 shows the impact of the parameters on the computational cost of the learning algorithm. We also report for all datasets the MBKL parameters that we use in the rest of the experiments. Recall that the parameters of Step 1 are the initial number of BMs and the number of training samples we use to learn the two-class SVM. We set these parameters to strike a good balance between accuracy and efficiency. The proportion of negative vs. positive training samples is set
Table 3. Learning Parameters of MBKL. We report the amount of BMs randomly generated (# Initial BMs), the number of BMs selected (# $\theta_k \neq 0$), the proportion of Positive vs Negative training samples (Neg/Pos), the amount of samples (Samples), and the training times for one split.

| Datasets | Daimler | Flowers17 | OSR | a-VOC08 | ImgNet | ImgNet |
|----------|---------|-----------|-----|---------|--------|--------|
| Descr.   | HoG     | BoW       | HoG+Attr | HoG+Attr | BoW   | S.Quant. |
| Descr. Length | 558 | 31c3 | 518 | 9,752 | 21c3 | 21c3 |
| # Initial BMs | 1e4 | 3e4 | 3e4 | 1e5 | 2e5 | 21c3 |
| # $\theta_k \neq 0$ | 3,832 | 9,740 | 2,906 | 6e4 | 9e4 | 18,341 |
| Step 1: Neg/Pos | 7 | 2 | 2 | 2 | 2 |
| Step 1: Samples | 19,600 | 11,220 | 1,920 | 19,020 | 5e4 | 5e4 |
| Step 1 Time | 24s | 56s | 5s | 480s | 5e3s | 2c3s |
| Total Time | 5e3s | 60s | 5.5s | 690s | 2e5s | 5e4s |

6.3 Comparison to state-of-the-art

We compare MBKL with other learning methods based on binary decisions, and also with the state-of-the-art (s-o-a) efficient SVM methods. In all cases we use the code provided by the authors.

Methods based on binary decision: We compare to Random Forest (RF) [8] using 100 trees of depth 50, except for UCI where we use 50 trees of depth 10. We also compare to the AdaBoost implementation of [9], using 500 iterations. These parameters were the best found.

Predefined kernels: We use $\chi^2$, RB-$\chi^2$ kernels, and Intersection kernel (IK) [20]. For RB-$\chi^2$ we set the hyper-parameter of the kernel to the mean of the data.

Fast kernel approximations: We use some of the state-of-the-art methods:

- Approx. $\chi^2$ by Vedaldi and Zisserman [7]: we use an expansion of 3 times the feature length, which is reported in [7] to work best. We also use an expansion of 9 times, which gives a feature length similar to MBKL (we indicate this with: x3).
- Approx. Intersection Kernel (IK) by Maji et al. [20]: following the suggestion by the authors, we use 100 bins for the quantization. We did not observe any significative change in the accuracy when further increasing the number of bins for the quantization.
- Power mean SVM (PmSVM) by Wu [24]: We use the $\chi^2$ approximation and the intersection approximation of [24], with the default parameters. The features are scaled following the author’s recommendation.
We also analyzed the use of BMs for kernel approximation by Raginsky and Lazebnik [31] (using the available code). This approach combines BMs and the random projections by Rahimi and Recht [6]. In contrast to MBKL, that method learns the kernel distance to preserve the locality of the original descriptor space. We use the resulting kernel in a linear SVM, and we found that it performs poorly (we do not report it in the tables). Note that this method was designed to preserve the locality, which is a useful criterion for image retrieval but may be less so for image classification. Moreover, it is based on the random projections of [6] that approximate the RBF kernel, which might not be adequate for the image descriptors we use.

We do not report the accuracy performance of the MKL method for large-scale data by Bazavan et al. [4] (for which the code is not available). This is because [4] uses the approximations of the predefined kernels that we already report, and the accuracy very probably is comparable to those approximations with the parameters set by cross-validation.

**Performance accuracy.** The results are reported in Table 4. We can observe that MBKL is the only method that for each benchmark achieves an accuracy similar to the best performing method for that benchmark. Note that the kernels and their approximations do not perform equally well for all descriptor types. Their performance may degrade when the descriptors are attribute-based features, and also, when descriptors are already linearly separable, such as s-o-a descriptors in large scale datasets. In most cases, the approximations to a predefined kernel perform similarly to the actual kernel. We observed that the performance of PmSVM is lower when the feature length is small. We can conclude MBKL outperforms the other methods based on binary decisions, including Random Forest and Boosting.

For all tested datasets the accuracy of MBKL is comparable to the s-o-a, which is normally only achieved by using different methods for different datasets. We even outperform the s-o-a for UCI [40], as well as for the Daimler [20] benchmark since we...
found better parameters for the HoG features. For ImageNet, MBKL outperforms \cite{15} using the same descriptors, achieving a good compromise between accuracy and efficiency (computing the descriptors for the whole dataset in less than 24h using 4 CPUs).

**Test Time.** Fig. 3 compares the testing time of MBKL to that of the efficient SVM methods. We report the testing times relative to the test time of MBKL, as well as the accuracy relative to MBKL. MBKL achieves very competitive levels of efficiency. MBKL’s computation speed depends on the number of BMs with $\theta_k \neq 0$. For Flowers17, MBKL is faster than linear SVM because there are fewer final BMs than original feature components. Note also that if the final length of the feature map is the same for MBKL and approximate $\chi^2$, MBKL can be faster because decision stumps are faster to calculate than the projections to approximate $\chi^2$. PmSVM achieves better accuracy and speed than MBKL in two cases, but note that for the rest of the cases this is opposite. Note that in UCI datasets, which are not Fig. 3 but in Table 4, PmSVM performs poorly compared to MBKL, since the descriptors are attribute-based.

**Training Time.** When learning MBKL, any of the optimizations can be done with an off-the-shelf linear SVM. Thus, the computational complexity depends mainly on the optimizer. We use liblinear, but we could use any other more efficient optimizer. Comparing the different SVM optimizers is out of the scope of the paper. We do not compare the training time of MBKL to that of the methods that use predefined kernels, because these methods do not have the computational overhead of learning the kernel.

**7 Conclusions**

This paper introduced a new kernel that is learned by combining a large amount of simple, randomized BMs. We derived the form of the non-linear mapping of the kernel that allows similar levels of efficiency to be reached as the fast kernel SVM approximations. Experiments show that our learned kernel can adapt to most common image descriptors, achieving a performance comparable to that of kernels specifically selected for each image descriptor. We expect that the generalization capabilities of our kernel can be exploited to design new, unexplored, image descriptors.
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