Global conjecturing process in pattern generalization problem

Sutarto¹, Toto Nusantara², Subanji³, Intan Dwi Hastuti³, Dafik⁴
¹Mathematics Education Departement, IKIP Mataram, Mataram, Indonesia
²Department of Mathematics, UM Malang, Malang, Indonesia
³Mathematics Education Departement, Universitas Muhammadiyah Mataram, Mataram, Indonesia
⁴Mathematics Education Departement, University of Jember, Jember, Indonesia

Email: sutarto@ikipmataram.ac.id

Abstract. The aim of this global conjecturing process based on the theory of APOS. The subjects used in study were 15 of 8th grade students of Junior High School. The data were collected using Pattern Generalization Problem (PGP) and interviews. After students had already completed PGP; moreover, they were interviewed using students work-based to understand the conjecturing process. These interviews were video taped. The result of study reveals that the global conjecturing process occurs at the phase of action in which subjects build a conjecture by observing and counting the number of squares completely without distinguishing between black or white squares, finally at the phase of process, the object and scheme were perfectly performed.

1. Introduction
One of the standard assessments used in this study begins from preschool until secondary school level [19] is making and examining the mathematic conjecture. Furthermore, it is explained that making conjecture is important work because it functions as the basic to develop and increase new perception for further study. Making and examining conjecture is a step in mathematical study [12], reasoning [21], and mathematical thinking [16].

Conjecture is the logical statement where its truth is not yet certain [3, 8, 16, 21]. Along with this idea, conjecture is a statement concerning all the possible cases, based on the empirical facts, but with the doubtful elements [4]. Based on these arguments, it can be argued that conjecture is the statement based on the reasoning process where its truth is not yet certain.

Conjecture and problem solving are linked activity. Conjecture and problem solving are the important parts and interrelated in the mathematic activity [4, 2] Moreover, it is said that the problem solving involves the finding whiler conjecture is the main road for the finding [19]. In problem solving, conjecture helps the problem solver to find the solution for the problems faced. As a result, conjecture does not just show up, but there is a process and the process is the conjecturing process.

The conjecturing process is the process of constructing the conjecture [4,16]. Assumes that the conjecturing process is the mental activity expressed in problem solving based on the knowledge which has been owned and the trust is necessary to be proven [8]. Based on the argument, it can be concluded that the conjecturing process is the mental activity in constructing conjecture based on the possessed knowledge. The mental activity is the process in the mind which can be seen ins the students’ behavior in problem solving [11, 30, 31].

There will be the conjecturing process if the students face any problems. In the conjecturing process, the students construct the conjecture based on their knowledge. There is no validation in the conjecture built by the students. It depends on the students’ involment. If there is a validation for the conjecture they built, the conjecture is considered as the correct one. Only after the conjecture is
validated then one can consider it as correct or incorrect. If the conjecture does not have incorrect value the process of conjecturing will be processed again until the it has the correct value. The conjecture with the correct value is the solution for the problems faced by the students.

In relation to conjecturing process, the familiar one used is mathematic problem solving as it is the conjecturing type of empirical induction from a finite number of discrete cases [4]. This type of conjecturing process consists of seven steps namely observing the case, organizing the case, finding and predicting the pattern, formulating the conjecture, validating the conjecture, generalizing the conjecture, and validating the generalization. The conjecturing process of this type—by inducting from a finite number of discrete cases—is mostly found in the problem related to the numbers, where the pattern under observation is consistent. In the problem solving involving the numbers with consistent pattern, the seven conjecturing processes do not always take place; there are many factors affecting process such as the type of task or the students’ characteristics involved [3].

The pattern described as the regularity which can be predicted above commonly involves the numerical, spatial or logical relations [17]. Many mathematicians state that the mathematics is a ‘science about pattern’ [22, 25]. They highlight the pattern existence in all mathematic fields [25]. In particular the pattern is considered by some researchers as the strategy applied in algebraic field because the pattern is the basic measure to construct the generalization which is the mathematic essence [29].

Generalization of pattern is an important aspect in school mathematical activities [5, 18, 26, 29]. Along with this idea, the generalization must be the core of the school mathematical activities [10]. The generalization of pattern itself is the activity making the pattern common rule based on some special examples. The common rule obtained is the conjecture, and the generalization is the specific type of conjecture obtained from common reasoning [28].

The significant contribution to the conjecture or conjecturing has been studied by researchers who consider the conjecture as the intuition of expression [8]. It shows the important of conjecturing atmosphere [15]. Research has been carried out on the production of conjectures within a dynamic geometry environment [9] by analyzing how the students verify the conjecture and how the teachers’ trust related to this process [1]. Researchers developed an open classical analogy in geometry construction [13] by designing mathematic conjecturing activities to foster thinking and constructing actively [14]. Then, they analyze various familiar types and steps of conjecturing process in problem solving [4]. Among the studies, it is not revealed yet how the students—through conjecturing process—generalizing the pattern of problem solving.

The conjecturing process described above correlates to some theories [3, 4, 20, 21, 23]. These theories are the basic of conjecturing process of empirical induction from a finite number of discrete cases. They consist of four inductive reasoning processes in problem solving, namely (1) observing specific cases, (2) formulating the conjecture based on previous case, (3) generalization, and (4) conjecture verification with specific new cases [20]. Reid uses the inductive reasoning process in the context of empirical induction from a finite number of discrete cases by the steps: (1) observing specific cases, (2) observing the pattern, (3) formulating the conjecture for common cases (with doubtfulness), (4) generalization, and (5) using generalization to prove [21]. There are seven steps in describing the inductive reasoning process, namely (1) observing cases, (2) organizing cases, (3) searching for and predicting patterns, (4) formulating a conjecture, (5) validating the conjecture, (6) generalizing the conjecture, (7) justifying the generalization [3].

There are seven steps in describing the inductive reasoning process drive from Canada’s, as such a types of conjecturing process namely empirical induction from finite number of discrete cases. The conjecturing process in this study is empirical induction from a finite number of discrete cases. The explanation of those seven steps take place in conjecturing process [24]. Based on its explanation and indicator adapted from the students. the conjecturing process in generalization of pattern problem solving is grouped into two types, i.e. global conjecturing, and local conjecturing. The global conjecturing is the mental activity in constructing the conjecture by observing the problems intact, and The local conjecturing is the mental activity of constructing the conjecture by observing the problems
separately. While, The global conjecturing process is often done by the students in generalization of pattern problem solving. Thus, this study will describe the global conjecturing process based on APOS theory.

The mental activity in conjecture analyzes using APOS theory, because APOS theory is a theory that can be used as the analytical tool to describe one’s developmental scheme in a mathematic topic--as it is the totality of the related knowledge (aware or unaware) to the topic [6]. This theory is based on the hypothesis of mathematic knowledge that may use to solve the situation as the mathematical problem by constructing the action, process and object as well as regulating the scheme to comprehend the situation and solve the problem [7]. This theory is called APOS and use to describe an action at the interiorization as the Process. The Process is encapsulated in an object. Then, it is related to other knowledge in a schema. A schema can also be encapsulated as an object. Problem of Research: How is the global conjecturing process in the solving of pattern generalization problem based on APOS theory?

2. Methodology of Research

2.1. Subjects
The subjects in this study are 15 students of class VIII derived from 9 students of VIII State Junior High School 1 Malang, and 6 students of State Junior High School 3 Malang.

2.2. Instrument
There are two types of instruments use. The main instrument is the researchers themselves who act as planners, data collectors, data analysts, interpreters, and reporters of research results. The auxiliary instrument used in this study is a Pattern Generalization Problem (PGP) and interviews. The problem given aims to obtain a description of the process of conjecturing of the students, while the interview used was unstructured interview. The PGP is presented in Figure 1.

![Figure 1. The Pattern Generalization Problem (PGP)](image)

2.3. Data Analysis
This study is a qualitative research with descriptive exploratory approach. At the data analysis step, the activities conducted by researchers were (1) transcribing the data obtained from interviews, (2) data condensation, including explaining, choosing principal matters, focusing on important things, removing the unnecessary ones, and organizing raw data obtained from the field (3) encoding the data from PGP answer sheet and interviews refer based on indicators of local conjecturing process are
presented in Table 1, (4) describing the global conjecturing process in the solving of pattern generalization problem based on APOS theory, and (5) conclusion.

3. Results of Research

Based on the analysis results of the answer sheets and the interview results, it is obtained the data on the global conjecturing process conducted by the students in the generalization of pattern problem solving based on the APOS theory. After getting bored for the subject taking process, it is obtained 6 subjects who conduct the global conjecturing process, 5 subjects who conduct the contrast conjecturing and 3 subjects who conduct the local conjecturing and generalizing symbolic. Out of 6 subjects, it will be described two subjects making the global conjecturing process of generalization of pattern problem-solving which is the S1 subject and S2 subject. The data presented is obtained by the procedures (1) the subjects complete the PGP, and (2) after the subjects complete the PGP, they are interviewed to explore about the global conjecturing process which has been conducted. The data presentation and analysis of the global conjecturing processes in the generalization of pattern problem solving is as the following.

3.1. S1 subject data presentation

In generalizing the S1 patterns, it has realized that 1st figure, 2nd figure, and 3rd figure form a pattern. To find a common formula on the number of square at the n th figure, S1 observes and counts the number of square regardless the black square and white ones at the 1st figure, 2nd figure, and 3rd figure. Here are the interview quotation and the S1 work results in completing the following PGP.

Figure 2. S1 subject work result

S104 : this is the different of the figure, Sir. This is the first, 7 and the second one is 11, and the third one, the number is 15 (while pointing to the square figure). The different is 4, so the following figure is plus 4, plus 4, plus 4.

Based on the number of square in the 1st figure, 2nd figure, and 3rd figure, S1 organizes the cases by ordering the number row pattern. Then, S1 finds and predicts the pattern by seeing the different between the 2 nd figure and the 2 nd figure, the 3 rd figure and the 2 nd figure and thinking how the following figure is plus 4, plus 4, plus 4. This is confirmed by the S104 interview quotation and the students’ work results in completing the following PGP.

Figure 3. S1 subject work result

To formulate the conjecture, S1 subject sees the addition of 1 st figure into the 2 nd figure is 2, and the 2 nd figure into the 3 rd figure is also 4, by seeing this addition, S1 formulates the n th formula conjecture in setting the number of square at the figure is \( n + 4 \). After that, S1 validates the conjecture by seeing the appropriateness with the number of square at the 4 th figure and 3 rd figure, then saying that the n th formula is incorrect. The following is the S1 interview quotation.
S108: this adds by 4, then adds by 4, then adds by 4. But, thinking it continuously, it may be incorrect. It is why, when 4 adds by 4, it is 8, then when n is 3, it adds by 4 the result is only 7. So it is incorrect.

After realizing that the conjecture formulated is incorrect, S1 tries new strategy to formulate the conjecture namely by finding the initial number before it is plus 4 because the pattern always adds by 4. S1 finds the initial number by looking for the different of the number of the square at the 1st figure, 2nd figure and 3rd figure by 4. The subtraction results consecutively are 3, 7, 11. S1 realizes that the initial number searched is not yet correct because its initial number is still different. This is shown by the interview quotation and the S1 work results as the following.

S111: this is initially still different (3, 7, 11) meaning that it is incorrect (while pointing out the work result)

\[ \begin{array}{c}
7 - 4 = n \\
11 - 4 = n \\
15 - 4 = n \\
\end{array} \]

Figure 4. S1 subject work result

S1 subject then uses the new strategy which is to look for the initial numbers before adding by 4 times n. S1 writes down the initial number symbols with x, for the 2nd figure \( x + (4 \times 2) = 11 \), then \( x = 3 \), for the 3rd figure \( x + (4 \times 3) = 15 \) then \( x = 3 \) so the initial number before plus 4 times n is 3. After finding the initial figure, S1 formulates the conjecture namely the common formula which is \( 3 + (4 \times n) \) and validates the conjecture based on the number of squares which are already known. After validating S1 generalizes the conjecture as to believe that the common formula is \( 3 + (4 \times n) \). It is also shown from the quotation interview and S1 work result as the following.

S111: I look for the initial number before it is plus 4 times n. The second figure is the same to \( x + (4 \times 2) = 11 \) so \( x = 3 \). Then the 3rd figure is similar to \( x + (4 \times 3) = 15 \) so \( x = 3 \). So, the initial number before it is plus 4 times n is 3.

P13: Okay, then are you sure by the common formula you obtain?
S113: yes, Sir I am...

\[ \begin{array}{c}
6_2 = x + (4 \times 2) \\
11 = x + (4 \times 2) \\
15 = x + (4 \times 3) \\
\end{array} \]

Figure 5. S1 subject work result

S1 justifying the generalization with the aim to convince others that the conjecture obtained is correct with a particular example. S1 describes how to obtain the formula, and shows an example of the formula suitability with a number for a square at the 1st, 2nd, 3rd figures and calculate the number of square at the 4th figure like what has done at the validation step which \( n = 3 + (4 \times 4) = 19 \) and 19 is
also obtained from the 3rd figure plus 4 is 19, from this example S1 justifying the resulting generalizations. This is shown by the interview quotation as the following.

\[ P_{19} : \text{Okay, then how do you explain that the resulting formula is correct?} \]

\[ S_{19} : \text{I will explain how I get the formula and show the example for the 1st, 2nd, 3rd and 4th figures. For example, for the 4th figure, } n = 3 + (4 \times 4) = 19. \text{ 19 is also obtained from the 3rd fig., } 15 + 4 = 19. \text{ (while pointing put the work).} \]

From the data described based on the conjecturing process steps, it can be described the S1 subject thinking structure analyzed based on the APOS step. The S1 conjecturing process in the generalization of pattern problem solving begins with the action steps namely observing case, and organize the case, then S1 internalizes the action into prose by finding and predicting the pattern. Once internalized the action into the process, S1 encapsulates the process into the object by formulating the conjecture and validating the conjecture. At the following scheme step, S1 generalizes the conjecture and justifying the resulting conjecture. S1 thinking structure is presented in Figure 6.

![Figure 6. S1 subject thinking structure](image)

**Notes:**

- **a**: The problem proposed is to find the common formula to set the number of square at n<sup>th</sup> figure
- **b**: Counting and observing the number of square at the 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> figures.
- **c**: Counting the number of square at the 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> figures
- **d**: Writing down the row pattern of 7, 11, 15
- **e**: Counting the square different at the 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> figures and thinking of the following object
- **f**: Stating the row different is 4
- **g**: The addition is 4 so n = n + 4
- **h**: Finding the initial number before being

**Activity sequence:**

- **l**: The n<sup>th</sup> formula is 3 + (4 \times n)
- **m**: Believing in that The n<sup>th</sup> formula is 4n + 3
- **n**: Validating the n<sup>th</sup> formula by pointing out at the example at the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> figures, supposed the 4<sup>th</sup> figure n = 3 + (4 \times 4) = 19. 19 is also obtained from the 3<sup>rd</sup> figure, 15 + 4 = 19.
- **o**: Done
3.2 $S_2$ subject data presentation

In generalizing the patterns, $S_2$ subject has been aware that the 1st figure, 2nd figure and 3rd figure form a pattern. To find a common formula of the number of square at the nth figure, $S_2$ observes and counts the number of square regardless the black square and white one at 1st figure, 2nd figure and 3rd Figure. Here is the interview quotation of $S_2$.

\[ P\ 04 : \text{what do you think first when reading this problem?} \]
\[ S_2 04 : \text{at first I look at the figure, then from this figure, I look at another figure continuously, then it compare both (while pointing out at the PGP)} \]
\[ P\ 05 : \text{Then you compare, what does it mean?} \]
\[ S_2 05 : \ldots\text{When comparing both, I find if in each figure there is 4 addition, four square addition. I still can not see the white and black. I don’t see it. Then at first, I think of this continuously, the pattern is always like this(while pointing out at the PGP)} \]

Based on the number of square at the that the 1st figure, 2nd figure and 3rd figure, $S_2$ subject organizes cases by signing up to number one with the 1st figure, number two with the 2nd figure, number three with the 3rd figure, and so on. Furthermore, $S_2$ locates and predicts the patterns by comparing the number of squares at the 1st figure, 2nd figure and 3rd figure and finds that the number of additional figure is always 4 and thinks of that the pattern always continues. This is confirmed by $S_2$ 05 interview quotation and the student’s work results in completing the following generalization of pattern problem solving.

To formulate a conjecture $S_2$ tries to determine suitable n based on the figure sequence. For example, 7 squares and 11 squares, this means that it has to plus 4, if n then n + 4 it can not be. So it must determine a suitable n based on the figure sequence. After $S_2$ tries to enter 1st figure (one) into the formula because one is also n, by trying one by one starting from \((1 \times 1) + 6\), \((1 \times 2) + 5\), and \((1 \times 4) + 3\). Then, $S_2$ formulates a common formula of conjecture to determine the number of square at the nth figure = \((n \times 4) + 3\) and validates the conjecture based on the number of squares at the 4th figure and 5th Figure. After validating, $S_1$ generalizes the conjecture as to believe that the common formula is \((n \times 4) + 3\). It is also shown from the interview quotation and the students’ work in completing the following generalization of pattern problem solving.

\[ S_2 08 : \text{so, I try the first one, supposed } 1 \times 1, 1 \times 1 \text{ must be added with what number to be 7, eee.. then it is plus 6, but if it is supposed } 2 \times 1 + 6 \text{ then the results is not 11, so I keep trying the} \]

![Figure 7. S2 subject work result](image)
closest ine which is the most appropriate answer for this square ( while pointing out at the square figure at PGP). I keep trying 

\[(1 \times 2) + 5 = 7\]

is correct, then this one 

\[(2 \times 2) + 5\]

the result is 9 and not 11, so I keep trying four

\[(1 \times 4) + 3 = 7\]

I try if it is 

\[(2 \times 4) + 3 = 11\]

I try again the 

\[(3 \times 4) + 3 = 15\]

because I am still doubt I try this one

\[(4 \times 4) + 3 = 19\]

So that is my thinking pattern.

![Formula](image)

**Figure 8.** $S_2$ subject work result

$S_2$ subject justifying the generalization with the aim of convincing others that the resulting conjecture is correct with a particular example. $S_2$ points out, from this example, $S_2$ justifying the generalization results in. This is shown by the following interview quotation.

**P 14**: Okay, then how do you explain to others that the resulting formula is correct?

**S_2 14**: I will show the results at the 1st, 2nd, 3rd figures and so on. This is the proof ( while pointing out the work result). I have tried it continuously, then it is correct. So I believe in the formula.

From the data described based on the conjecturing process steps, it describes $S_2$ subject thinking structure which is analyzed based on the APOS step. $S_2$ Conjecturing process in the generalization of pattern problem solving begins by observing case the action step, and organizing the case, then $S_2$ internalizes the action into prose by finding and predicting the pattern. Once internalized into the action, $S_2$ encapsulates the action into the by formulating the conjecture and validating the conjecture. Then at the following schema step, $S_2$ Generalizing the conjecture and justifying the resulting conjecture. $S_2$ thinking structure is presented in Figure 9.

![Diagram](image)

**Figure 9.** $S_2$ subject thinking structure

**Notes:**

|   |   |
|---|---|
| a : | The problem proposed is to find the common formula to set the number of square at $n^{th}$ figure |
| b : | Counting and observing the number of square at the 1st, 2nd, and 3rd figures. |
| c : | Counting the number of square at the 1st, 2nd, |
| m : | The $n^{th}$ formula is $(n \times 4) + 3$ |
| n : | Believing in that The $n^{th}$ formula is $(n \times 4) + 3$ |
| o : | Validating the $n^{th}$ formula by specific case |
3.3 Global Conjecturing Process Schema of S1 subject and S2 subject in Generalization of Pattern Problem Solving based on APOS

In generalizing the patterns, S1 and S2 have been aware that the 1st figure, 2nd figure and 3rd figure form a pattern. To find a common formula of the number of square at the nth Figure At this action step, S1 and S2 observe and count the number of square regardless the black square and white one, at the 1st figure, 2nd figure and 3rd Figure Based on the number of square at the 1st figure, 2nd figure and 3rd figure, S1 organizes cases by sorting the number row patterns and S2 registers to relate number one with the 1st figure, number 2 with 2nd figure, number 3 with the 3rd figure, and so on. Then, at the process step, S1 and S2 are searching for and predicting the pattern by looking at the difference between the 2nd figure and the 1st figure, 3rd figure and the 2nd figure and think that the following figure increases by 4.

The object step, to formulate conjecture, S1 sees the 1st figure addition to the 2nd figure is 4, and the 2nd figure to the 3rd figure is also 4, by looking at the addition, S1 looks for the initial number before adding by 4 times n. For the 2nd figure \( x + (4 \times 2) = 11 \), then \( x = 3 \), for the 3rd figure \( x + (4 \times 3) = 15 \) then \( x = 3 \) so the initial number before added 4 times n is 3. S2 Subject tries to determine the suitable n based on the figure sequence, then S2 tries to enter the 1st (one) figure into the formula because one is n, by trying one by one starting from (1 \( \times 1 \) + 6), (1 \( \times 2 \) + 5), and (1 \( \times 4 \) + 3). The conjecture produced by S1 and S2 is \( 3 + (4 \times n) \) and validate the conjecture based on specific examples obtained at the action or process step.

The scheme step, S1 and S2 subjects generalize the conjecture to believe that the conjecture resulted is correct after validating the conjecture in the previous steps. In justifying the generalization with the aim of convincing others that the resulting conjecture is correct, S1 and S2 use specific examples obtained at the action step or process step. Justifying the generalizations made by the subject
S1 and S2 is the same as what has done at the validation step namely using the specific examples. The Global conjecturing process scheme of S1 subject and S2 subject in the solving of pattern generalization problem is presented in Figure 10.

![Figure 10](image)

Figure 10. The schema of global conjecturing process

4 Discussion

This section will discuss the research findings related to global conjecturing process to solve problem of pattern generalization. In generalizing the pattern, S1 and S2 on the "action" step have realized that the 1st figure, 2nd figure, and 3rd figure form a pattern. To find a common formula number of square in the nth, S1 and S2, it observed by counting number of square regardless the black and white squares on the 1st figure, 2nd figure, and 3rd figures. Based on the number of square at the 1st figure, 2nd figure, and 3rd figure, S1 organizes the case by sorting the row pattern of 7, 11, 15 and so on while S2 makes a list or a table to relate number one with the 1st figures, number 2 with the 2nd figure, number 3 with the 3rd figure. This shows that at the "action" step, S1 subject and S2 subject observe and organizes the cases regardless the black and white squares, therefore the conjecturing process conducted by the subjects is referred to as the global conjecturing process. Observing cases and organizing cases regardless the black and white squares are based on the Gestalt laws in observation, called similarity Law which is a person tends to perceive the same holistic stimulus [27].

This "process" step, the subjects internalize the action to find and predict the pattern by investigating the distinguish between number of square at the 2nd, 1st, 3rd, and 2nd figures. and think that the following figure has the same pattern, namely obtaining the increased 4. In formulating the conjecture step, S1 conducts the encapsulation to generate the object which is to see the 1st figure
addition to the 2nd figure is 4, and the 2nd figure to 3rd figure is also 4. S1 seeks the initial number before adding 4 times n. for the 2nd figure, \(-2x + (4 \times 2) = 11\) so \(x = 3\), for the 3rd figure \(-3x + (4 \times 3) = 15\) so \(x = 3\) so the initial number before being added 4 times n is 3. After finding the initial number, S1 formulate the common formula of the conjecture namely \(3 + (4 \times n)\) and validating the conjecture based on the number of square known. S2 tries to det the appropriate n based on the figure sequence, after that S2 tries to enter the 1st figure into the formula because one is also n, by trying one by trying one by one starting from \((1 \times 1) + 6\), \((1 \times 2) + 5\), and \((1 \times 4) + 3\) then S2 formulates the common formula of the conjecture to set the number of square at the \(n^{th}\) figure \(n = (n \times 4) + 3\) and validate the conjecture based on the number of squares on the 4th figure and 5th figure. The way done by S1 is looking for the initial number using the x symbol and S2 seeks the appropriate based on the figure sequence. Both ways are different but meaningful for itself to find the common formula of the conjecture, it describes the knowledge possessed. This is consistent to what expressed by[28]that the mathematical symbol is a tool for coding and describing the knowledge as well as communicating the mathematical knowledge. At this process step and object step, the subjects conduct it perfectly.

At this scheme step, S1 subject and S2 subject generalize the conjecture to believe that the resulting conjecture is correct. In justifying the generalization with the aim of convincing others that the resulting conjecture is correct, S1 and S2 use the specific examples obtained at the action step and object step, S1 justifying the generalization with the aim of convincing others that the resulting conjecture is correct with the specific example. S2 counts the number of square at the 4th figure namely \(n = 3 + (4 \times 4) = 19\) and 19 is also obtained from 3rd figure plus 4 is 19, from this example, S1 validates the resulting generalization. S2 validates the generalization with aim of convincing others that the resulting conjecture is correct with specific example obtained at the object step by pointing out \((1 \times 4) + 3 = 7\), \((2 \times 4) + 3 = 11\), \((3 \times 4) + 3 = 15\), and \((4 \times 4) + 3 = 19\). In justifying the generalizations, S1 subject and S2 subject conduct their own way, this is based on the [3] that the students do not just simply use the notation or symbols but also their presentation and give a reason mathematically, make conclusions and generalizations in their way. At this scheme step, it is also conducted perfectly.

5 Conclusions

Based on the findings in the global conjecturing process conducted by the students in the generalization of pattern problem solving, it increases the conjecturing process theories [4] about the type of empirical induction from a finite number of discrete cases which has seven steps and not study the students’ thinking process in constructing the conjecture generalization. The results show that the global conjecturing process occurs at the step of action in which subjects build a conjecture by observing and counting the number of squares complete, at the step of process, the object and scheme were perfectly performed.

Acknowledgements

This research is a grant from the Direktorat Riset dan Pengabdian Kepada Masyarakat (DRPM) Ristekdikti. The author would like to express sincere appreciation for all the support provided

References

[1] Bergqvist T 2005 How students verify conjectures: teachers’ expectations. Journal of Mathematics Teacher Education 8 171–191
[2] Sutarto and Hastuti I D 2015 Conjecturing Dalam Pemecahan Masalah Generalisasi Pola. Jurnal Ilmiah Mandala Education 1 2 172-178
[3] Cañadas M C and Castro E 2005 A proposal of categorisation for analysing inductive reasoning In M. Bosch (Ed.), Proceedings of the CERME 4 International Conference 401-408
[4] Cañadas M C, Deulofeu J, Figueiras L, Reid D A and Yevdokimov O 2007 The conjecturing process: Perspectives in theory and implications in practice Journal of Teaching and Learning, 1 55–72
[5] Dindyal J 2007 High school students’ use of patterns and generalisations In J. Watson & K. Beswick (Eds), Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia 236-245

[6] Dubinsky E 2001 Using a theory of learning in college TaLUM 12 10 – 15

[7] Dubinsky E and Mc Donald M 2011 APOS: A constructivist theory of learning in undergraduate mathematics education research In D. Holton et al. (Eds), The teaching and learning of mathematics at university level: An ICMI Study 273-280

[8] Fischbein E 2002 Intuition in science and mathematics An Educational Approach NewYork: Kluwer Academic Publisher.

[9] Furinghetti F and Paola D 2003 To produce conjectures and to prove them within a dynamic geometry environment: A case study In N. A. Pateman, B. J. Doherty, & J. Zilliox (Eds), Proceedings of the Twenty-Seventh Annual Conference of the International Group for the Psychology of Mathematics Education 397-404

[10] Küchemann D 2010 Using patterns generically to see structure Pedagogies: An International Journal 5 3 233 –250

[11] Hastuti I D, Nusantara T and Susanto H 2016 Constructive metacognitive activity shift in mathematical problem solving Educational Research and Reviews 11 8 656

[12] Lakatos I 2015 Proofs and refutations: The logic of mathematical discovery Cambridge university press

[13] Lee K H and Sriraman B 2010 Conjecturing via reconceived classical analogy Educational Studies in Mathematics 76 2 123–140

[14] Lin F L 2006 Designing mathematics conjecturing activities to foster thinking and constructing actively. APEC-TSUKUBA International Conference, Tsukuba, Japan

[15] Mason J 2002 Generalization and algebra: Exploiting Children’s Powers In L. Haggerty (Ed.), Aspects of Teaching Secondary Mathematics: Perspectives on Practice 105-120

[16] Mason J, Burton L and Stacey K 2010 Thinking mathematically second edition, England: Pearson Education Limited

[17] Mulligan J and Mitchelmore M 2009 Awareness of pattern and structure in early mathematical development Mathematics Education Research Journal 21 2 33-49

[18] Mullingan J T, Mitchelmore MC, English LD and Robertson G 2011 Implementing a Pattern and Structure Mathematics Awareness Program (PASMAP) In Kindegarden In L. Sparrow, B. Kissane, & C. Hurst (Eds.) Shaping the Future of Mathematics Education. Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia 797–804

[19] Nasional Council of Teacher of Mathematics 2000 Principles and standards for school mathematics Reston, VA: NCTM

[20] Pólya G 1967 Le découverte des mathématiques Paris: DUNOD

[21] Reid D 2002 Conjectures and refutations in grade 5 mathematics Journal of Research in Mathematics Education 33 1 5-29

[22] Resnik M D 2005 Mathematics as a science of mathematics Oxford: University Press

[23] Sutarto Toto N and Subanji S 2015 Indicator of conjecturing process in a problem solving of the pattern generalization In Proceeding International Conference on Educational Research and Development (ICERD), UNESA Surabaya 32-45

[24] Sutarto T N and Subanji S 2016 Local conjecturing process in the solving of pattern generalization problem Educational Research and Reviews 11 8 732

[25] Tikekar V G 2009 Deceptive patterns in mathematics International Journal of Mathematical Science Education 2 1 13-21

[26] Vogel R 2005 Patterns: A fundamental idea of mathematical thinking and learning ZDM 37 5 445-449

[27] King D B and Wertheimer M 2009 Max Wertheimer & Gestalt Theory New Brunswik, New Jersey: Transaction Publisher
[28] Schwartz J L, Yerushalmy M and Wilson B The geometric supposer: what is it a case of? Hillsdale, NJ Lawrence Erlbaum Associates
[29] Zazkis R and Liljedahl P 2002 Generalization of patterns: the tension between algebraic thinking and algebraic notation Educational Studies in Mathematics 49 379–402
[30] Hastuti I D 2016 Pergeseran aktivitas metakognitif siswa dalam pemecahan masalah matematika Disertasi tidak diterbitkan Malang: PPs UM
[31] Sutarto T N and Subanji S 2016 Local conjecturing process in the solving of pattern generalization problem Educational Research and Reviews 11 8 732