INFLUENCE OF TENSION STIFFENING AND CRACKED SHEAR MODULUS MODELS ON NON-LINEAR ANALYSIS OF HIGH STRENGTH FIBROUS REINFORCED CONCRETE SLABS

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ABSTRACT

In the present study, new models are suggested and proposed for cracked shear modulus and tension stiffening to study their effect on the response of the slab. These models are used in the nonlinear analysis of High Strength Steel Fiber Reinforced Concrete (HSSFRC) slabs. The suggested models have multiple shapes depending on the curvature factor, these models are compared with the well-known formulas used in previous studies and great agreements are achieved. The Serendipity “eight-node” element type has been adopted for representing the concrete and layered approach is used to simulate the concrete elements and a smeared layer approach is used to represent the steel reinforcement. The concrete compression behavior is modeled using strain hardening plasticity method, the first two stress invariants of the yield condition is used. For finite element analysis, a computer program coded in Fortran 90 is developed and used for performing nonlinear analysis on the slab. In order to check the validity of the current models, many actual results for testing slabs “in the laboratory” are compared with the results from the present study and a great agreement is achieved. All studied slabs were simply supported from four sides and loaded with concentrated load at the middle of the slab, but slab S5 is simply supported by two opposite parallel sides with line load parallel to the supports at the middle of the span of the slab. For the curvature factors \( B_t, B_g \) it is found that the values \( B_t = 0.005 - 0.5, B_g = 0.001 - 0.05 \) give the best simulation for the slab. The effect of tension stiffening model is more than the effect of cracked shear modulus model and there is an interaction between tension stiffening and cracked shear modulus models.

KEYWORDS: Cracked shear modulus; Finite element; High strength concrete; New constitutive relationships; Tension stiffening; Reinforced concrete slabs; Steel fiber

1. GENERAL INTRODUCTION

The slabs are the first structural members subjected to the loads and they transfer these loads to the columns and beams, therefore it is important to study the slab’s behavior. Using of high strength concrete has developed and increased to reduce the cross sections of the members and that let to reduce the weight of the building and get a larger space of the areas, therefore using high strength concrete has a structurally and economically benefits [1]. The brittleness of high strength concrete is the main disadvantage of using it and this issue can be solved by adding steel fiber to the concrete to ensure high ductility during post crack stage in tension, and post peak stage in compression [2]. So, the benefits of adding fibers to the concrete are to improve its tensile strength, ductility, durability, toughness flexural strength [3]. It is very important to study the influence of the important relations on the structural response of

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the slabs such as tension stiffening and cracked shear modulus models. To reach this goal, many shapes of these models are suggested and the results of current study are compared with actual results from experimental studies in references [4-7].

2. MODELLING OF MATERIALS

The main scope of any theoretical study is predicting the response and strength of the member subjected to increasing load until the failure of the member. To achieve this goal, it is very important to study the material properties of the studied member which is HSSFRC, these properties should be studied and modeled correctly depending on experimental results.

2.1 Uniaxial tensile strength of HSSFRC

In the present study, the behavior of HSSFRC in tension can be divided into two stages, which are peak stage and post-peak stage as shown in Figure 1.

\[ \sigma_i = f_t \sin \left( \frac{\pi}{2} \varepsilon_i \right) \]  \hspace{1cm} (1)

2.1.1 Pre-peak stage

To simulate this stage, the following suggested equation is used and a great agreement with experimental values in reference [9] is achieved with index of determination equal to 99%.

\[ \sigma_i = f_t \sin \left( \frac{\pi}{2} \varepsilon_i \right) \]  \hspace{1cm} (1)

2.1.2 Post-peak stage

The path BCD represents the post-peak stage in uniaxial tensile behavior as shown in Figure 1. The tensile behavior after cracking of concrete depends on the pullout action of fibers. The shear strength between the steel fibers and the concrete \( \tau_u \) can be obtained from the following equation which is used in reference [1]:

\[ \tau_u = 0.85 f_{cf}^{0.6} + 2.4 F^{0.6} \]  \hspace{1cm} (2)

So, the average tensile stress \( f_u \) is calculated from the following equation:

\[ f_u = 0.41 \tau_u F \]  \hspace{1cm} (3)

A nonlinear stress strain behavior is assumed in this study to represent the post-peak stage as shown in Figure 2 (path CD). The proposed nonlinear equation has a curvature factor \( B_t \) can affect the nonlinearity of the equation when this value increased the nonlinearity of the equation will decrease and approach to the linear equation as shown in Figure 2.

\[ \sigma_i = f_u \frac{B_t}{B_t + X} \]  \hspace{1cm} (4)

\[ X = \frac{\varepsilon_i - \varepsilon_{ef}}{\varepsilon_m - \varepsilon_{ef}} \]  \hspace{1cm} (5)

Where \( (B_t) \) represent the curvature factor of the equation as shown in Figure 2.

The following suggested equation is adopted to get the value of \( \varepsilon_{ef} \), and this equation shows great agreement with experimental test result in reference [9] with 95.5% index of determination.

\[ \varepsilon_{ef} = \frac{11F + 3f_{cf}}{10^5} \]  \hspace{1cm} (6)
the maximum tensile strength $f_{tf}$ can be obtained from equation (7). A great agreement with 72.84% index of determination is achieved with experimental results in references [10-12].

$$f_{tf} = 0.3F_{f',f} + 2.8F_f + f_t$$  \(7\)

In some studies, the value of $f'_t$ is not measured, so the value of $f_{tf}$ can be calculated depending on the compressive strength of the concrete using the following suggested equation.

$$f_{tf} = 2.94F_f + 0.08f_{tf}$$  \(8\)

Acceptable agreement is achieved with experimental results in references [12, 13] with 65.62% index of determination.

$$G_F = \int_0^\delta \sigma(\delta)d\delta = \int_{\varepsilon_{tm}}^{\varepsilon_{tm}} G_F \sigma_{\varepsilon_{tm}} d\varepsilon$$  \(9\)

$$G_F = h_c(u(\ln(\varepsilon_m - \varepsilon_{tf} + B_t \varepsilon_m - B_t \varepsilon_{tf} - B_t \varepsilon_{tf}) - ln(B_t \varepsilon_m - B_t \varepsilon_{tf}) - ln(B_t \varepsilon_m - B_t \varepsilon_{tf}) - f_u B_t \varepsilon_{tf} + f_u B_t^2 \varepsilon_{tf} - f_u B_t^2 \varepsilon_{tf} - f_u B_t^2 \varepsilon_{tf})) d\varepsilon$$  \(10\)

This equation is achieved by completing the integration. After substituting the values of $h_c$, $f_u$, $G_F$, $\varepsilon_{tf}$, $B_t$ in equation (11) and solving the equation, the value of $\varepsilon_m$ can be calculated.

$$h_c = \sqrt{dA}$$  \(12\)

Where $G_F$ is fracture energy of high strength steel fiber concrete.

$dA$ is the area represented by Gauss point.

$h_c$ is the characteristic length of the concrete.

Fig. (2): The effect of curvature factor on the shape of stress strain curve in tension for HSSFRC in post peak stage.

It is important to compare the suggested model for tension stiffening with the models suggested by another researchers. Fig. (3 shows the comparison between present model (with $B_t = 0.5$) and the models in references [1, 14] and a good agreement is achieved.
2.2 Cracked Shear modulus of the concrete

For shear modulus, a nonlinear equation is proposed to simulate the degradation of the cracked shear modulus. In some studies, it is suggested that the reduction of the cracked shear modulus is following a linear model and some of them suggested a nonlinear model. For the present study the same formula for tension stiffening is used for modelling the cracked shear modulus reduction but the curvature factor here called $B_g$. The suggested equation is compared with the previous studies and a great agreement is achieved as shown in Fig. (4).

The cracked shear modulus will be reduced gradually until it will reach the value of zero at the maximum tensile strain ($\varepsilon_m$) and this value will be obtained from equation (11). Therefore, there is an interaction between tension stiffening and cracked shear modulus models.

$$\tilde{G} = A G B_g B_g^* X \frac{B_g}{B_g + X}$$

$$X = \frac{\varepsilon - \varepsilon_{if}}{\varepsilon_m - \varepsilon_{if}}$$

Where:

A is the Ratio ($\tilde{G}/G$) at the beginning of the crack which is equal to 0.4 according to reference [15].

$\tilde{G}$, G is the shear modulus after and before crack respectively.

$B_g$, $\nu$ are the Curvature factor “shown in Fig. (4)” and Poisson's ratio respectively.

2.3 Steel reinforcement modeling

The steel reinforcement is assumed as homogeneous material which mean that it has the same strength in compression and tension. The representation of the steel is done by using the smeared layer inside the concrete layers.

3. FORMULATION OF THE FINITE ELEMENT

The eight-node “isoparametric plate element (Ahmed element)” with three degrees of freedom at each node is adopted in this study. The formulation of this element is mentioned in reference [16]. For steel reinforcement, a smeared layer method is adopted. Perfect bond (no slip between steel reinforcement and concrete) is assumed between the reinforcement and concrete.
4. EXPERIMENTAL AND NUMERICAL RESULTS

4.1 Description of the tested slabs

To complete the finite element analysis of the slabs and study the effect of cracked shear modulus and tension stiffening models. Many experimental tests of HSSFRC slabs in references [4-7] are selected to compare them with the results of this study. The characteristics of the selected slabs are listed in Table 1. All studied slabs were simply supported from four sides and loaded with concentrated load at the middle of the slab, but slab S5 is simply supported by two opposite parallel sides with line load parallel to the supports at the middle of the span of the slab.

![Graph showing comparison between suggested model and previous models for cracked shear modulus of HSSFRC](image)

**Fig. (4):** Comparison between the suggested model in this study and the models used in previous references for cracked shear modulus of HSSFRC.

**Table (1):** the slabs properties

| Reference   | Slab number | Concrete | Steel |
|-------------|-------------|----------|-------|
|             | [4]         | [5]      | [6]   | [7]   |
| Slab number | S16         | S42      | S3    | S5    |
| V (%)       | 1.2         | 0.5      | 0.75  | 0.75  |
| l/df        | 37          | 133      | 50    | 50    |
| fcd (MPa)   | 59          | 50.8     | 55    | 71.15 |
| Esf (GPa)   | 38.91       | 36.13    | 37.56 | 42.73 |
| ft (MPa)    | 6.8         | 4.77     | 5.5   | 6.79  |
| εcu (x10^-3)| 6.09        | 7.667    | 5.625 | 5.625 |

**Concrete**
- fcd (MPa)
- Esf (GPa)
- ft (MPa)
- εcu (x10^-3)

**Steel**
- fy (MPa)
- ρy, ρx (%)
- Dimensions of the slabs (m)
- Thickness of the slab (mm)

| Reference   | Slab number | Concrete | Steel |
|-------------|-------------|----------|-------|
|             | S16         | S42      | S3    | S5    |
| V (%)       | 1.2         | 0.5      | 0.75  | 0.75  |
| l/df        | 37          | 133      | 50    | 50    |
| fcd (MPa)   | 59          | 50.8     | 55    | 71.15 |
| Esf (GPa)   | 38.91       | 36.13    | 37.56 | 42.73 |
| ft (MPa)    | 6.8         | 4.77     | 5.5   | 6.79  |
| εcu (x10^-3)| 6.09        | 7.667    | 5.625 | 5.625 |

- f = 0.2, Es = 200000 MPa.
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4.2 Modelling the finite element

A computer program coded in Fortran 90 was implemented for nonlinear analysis of HSSFRC slabs using finite element method. All the proposed models in this study are used in this program. Because the slabs are symmetric, one quarter of the slabs was modeled using 16 elements. Reduced integration rule (2 x 2 Gauss points) is used.

4.3 Analysis of the results

In the present study, many shapes of cracked shear modulus and tension stiffening models are used and the values of $B_t$ are (0.005, 0.05, 0.5 and 1.0) and the values of $B_g$ are (0.005, 0.05, 0.1 and 0.5) as shown in Fig. 2 and these values cover large possible shapes of these relations. The load deflection curves for this study using these models of cracked shear modulus and tension stiffening are compared with the actual results from experimental tests as shown in the following figures.

**Fig. (5):** Comparison between load deflection curves from this study with experimental load deflection curve for slab S16.

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Fig. (6): Comparison between load deflection curves from this study with experimental load deflection curve for slab S42.

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Fig. (7): Comparison between load deflection curves from this study with experimental load deflection curve for slab S3.

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From figures (5-8) it is found that there is good agreement in comparison between the numerical and experimental load deflection curves. From these figures, it is clear that the shape of the relation of the cracked shear modulus and tension stiffening models has an important effect on the response of the concrete slabs. The factors ($B_g$ and $B_t$) is responsible for the curvature of the curves, so by reducing these factors the curves are converted to very sharp curvature and the stiffness of the slabs will reduced. The effect of the tension-stiffening model is more than the effect of cracked shear modulus on the slab, and that is because the slab is subjected to bending stresses more than shear stresses.

From the previous figures, it is found that there is an interaction between the tension

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stiffening and the cracked shear modulus models. By changing the value of $B_t$ the value of $\varepsilon_m$ will change too to keep a constant value of $G_F$ (which is the area under the stress strain curve in tension) as shown in Fig. (2), and by changing the value of $\varepsilon_m$ the shape of the reduced shear modulus relation will change too.

The crack pattern of the slabs in the present study for (S42 and S5) at the failure load showed great agreement with the actual crack patterns as shown in Fig. (9) and Fig. (10).

In S42, the location and direction of the cracks is very similar to the actual crack pattern and the cracks direction is distributed in radial mode. In the corners, the direction of cracks is about 45° as assumed in the yield line analysis. For slab S5 the direction of the cracks in the middle of the slab are parallel to the supports as expected.

The ultimate load is one of the most important property of the slab, so the values of the ultimate load for the all proposed models used in this study are showed in Table 2.
Table (2): Failure load of slabs from this study and experimental test

| Slab identification | $B_t$  | $B_p$ | $P_u$ test | $P_u$ | $P_u$ test | Slab identification | $B_t$  | $B_p$ | $P_u$ test | $P_u$ | $P_u$ test |
|---------------------|-------|-------|------------|------|------------|---------------------|-------|-------|------------|------|------------|
|                     |       |       | (kN)       | exp. | / $P_u$ |       |       |       | (kN)       | exp. | / $P_u$ |
| S16                 | 0.005 | 1     | 198        | 176.8| 1.1199    | S3                  | 0.005 | 1     | 187      | 220  | 0.85       |
|                     | 0.005 | 0.5   | 198        | 176.8| 1.1199    |                     | 0.005 | 0.5   | 187      | 220  | 0.85       |
|                     | 0.05  | 1     | 202        | 176.8| 1.1425    |                     | 0.05  | 1     | 195.25   | 220  | 0.8875    |
|                     | 0.05  | 0.5   | 198        | 176.8| 1.1312    |                     | 0.05  | 0.5   | 195.25   | 220  | 0.8875    |
|                     | 0.05  | 0.005 | 190        | 176.8| 1.0747    |                     | 0.05  | 0.005 | 187      | 220  | 0.85       |
|                     | 0.1   | 1     | 202        | 176.8| 1.1425    |                     | 0.1   | 1     | 198      | 220  | 0.9        |
|                     | 0.1   | 0.5   | 202        | 176.8| 1.1425    |                     | 0.1   | 0.5   | 195.25   | 220  | 0.8875    |
|                     | 0.1   | 0.005 | 190        | 176.8| 1.0747    |                     | 0.1   | 0.005 | 187      | 220  | 0.85       |
|                     | 0.5   | 1     | 204        | 176.8| 1.1538    |                     | 0.5   | 1     | 203.5    | 220  | 0.925     |
|                     | 0.5   | 0.5   | 202        | 176.8| 1.1425    |                     | 0.5   | 0.5   | 200.75   | 220  | 0.9125    |
|                     | 0.5   | 0.05  | 200        | 176.8| 1.1312    |                     | 0.5   | 0.05  | 195.25   | 220  | 0.8875    |
|                     | 0.5   | 0.005 | 190        | 176.8| 1.0747    |                     | 0.5   | 0.005 | 192.5    | 220  | 0.85       |
| S42                 | 0.005 | 1     | 141.75     | 135.5| 1.0461    | S5                  | 0.005 | 1     | 127.5    | 126  | 1.0119    |
|                     | 0.005 | 0.5   | 143.5      | 135.5| 1.0590    |                     | 0.005 | 0.5   | 127.5    | 126  | 1.0119    |
|                     | 0.005 | 0.05  | 141.75     | 135.5| 1.0461    |                     | 0.005 | 0.05  | 127.5    | 126  | 1.0119    |
|                     | 0.005 | 0.005 | 138.25     | 135.5| 1.0203    |                     | 0.005 | 0.005 | 126      | 126  | 1.0000    |
|                     | 0.05  | 1     | 145.25     | 135.5| 1.0719    |                     | 0.05  | 1     | 130.5    | 126  | 1.0357    |
|                     | 0.05  | 0.5   | 145.25     | 135.5| 1.0719    |                     | 0.05  | 0.5   | 129      | 126  | 1.0238    |
|                     | 0.05  | 0.05  | 145.25     | 135.5| 1.0719    |                     | 0.05  | 0.05  | 130.5    | 126  | 1.0357    |
|                     | 0.05  | 0.005 | 143.5      | 135.5| 1.0590    |                     | 0.05  | 0.005 | 129      | 126  | 1.0238    |
|                     | 0.1   | 1     | 147        | 135.5| 1.0848    |                     | 0.1   | 1     | 130.5    | 126  | 1.0357    |
|                     | 0.1   | 0.5   | 147        | 135.5| 1.0848    |                     | 0.1   | 0.5   | 132      | 126  | 1.0476    |
|                     | 0.1   | 0.005 | 143.5      | 135.5| 1.0590    |                     | 0.1   | 0.005 | 132      | 126  | 1.0476    |
|                     | 0.5   | 1     | 148.75     | 135.5| 1.0977    |                     | 0.5   | 1     | 132      | 126  | 1.0476    |
|                     | 0.5   | 0.5   | 148.75     | 135.5| 1.0977    |                     | 0.5   | 0.5   | 132      | 126  | 1.0476    |
|                     | 0.5   | 0.05  | 147        | 135.5| 1.0848    |                     | 0.5   | 0.05  | 132      | 126  | 1.0476    |
|                     | 0.5   | 0.005 | 145.25     | 135.5| 1.0719    |                     | 0.5   | 0.005 | 132      | 126  | 1.0476    |
5. THE CONCLUSIONS

The eight-node plate element with 3 DoF at each node showed great results and agreement between the experimental results and numerical results, and approved to be valid and useful for nonlinear analysis of high strength steel fiber reinforced concrete slabs. Reduced integration (2x2 gauss points) is used and no shear locking and spurious modes detected. The smeared layer is adopted to simulate the steel reinforcement. This method approved to be used in the nonlinear finite element analysis.

For the ultimate failure load, excellent agreement is achieved in comparison between the results of this study and the actual experimental results for all slabs as listed in Table 2. For slabs (S3 and S5), the results of the ultimate load of this study is better than the results obtained from ANSYS program. The cracked shear modulus and tension stiffening models (values of $B_g$ and $B_t$) have an important effect on the response of the HSSFRC slabs. As the values of $B_g$ and $B_t$ reduced the slab stiffness reduced. For the curvature factors ($B_g$, $B_t$) the best values for them are ($B_g$ =0.005-0.5, $B_t$ =0.001-0.05). The effect of tension stiffening model is more than the effect of cracked shear modulus model. There is an interaction between tension stiffening and cracked shear modulus models, and the value of $G_F$ is the controlling factor of that interaction, so the value of the fracture energy is a very important factor and its model should be taken carefully. The proposed constitutive models are succeeded to simulate the HSSFRC slabs, and the load deflection curves and crack patterns obtained from this study showed great agreement with the actual experimental results.

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