Multipath Stealth Communication with Jammers

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Abstract—We consider the problem of stealth communication over a multipath network in the presence of an active adversary. The multipath network consists of multiple parallel noiseless links, and the adversary is able to eavesdrop and jam a subset of links. We consider two types of jamming — erasure jamming and overwrite jamming. We require the communication to be both stealthy and reliable, i.e., the adversary should be unable to detect whether or not meaningful communication is taking place, while the legitimate receiver should reconstruct any potential messages from the transmitter with high probability simultaneously. We provide inner bounds on the robust stealth capacities under both adversarial erasure and adversarial overwrite jamming.

I. INTRODUCTION

Suppose an activist (Alice) occasionally wishes to communicate with a news agency, say BBC (Bob), and can use several social media accounts she has to do so. However, the government James is eavesdropping on some of these accounts (Alice and Bob do not know which ones), and is able to jam (i.e., erase or corrupt) information on these. The goal is to ensure that (i) the activist Alice can communicate with the BBC Bob even if the government James attempts to disrupt communication, and (ii) Alice’s communication should be stealthy — any communication posted on the social media that James observes should be explainable as “innocent behaviour”.

The classical information-theoretic security problem aims to hide the content of communication. However, in certain scenarios the mere fact that communication is taking place should also be hidden. Stealth communication, first studied in [1] for Discrete Memoryless Channels (DMCs), requires that the transmitter Alice should be able to reliably communicate with the legitimate receiver Bob, and simultaneously ensure the communication is undetectable by a malicious adversary James. The work [2] generalizes the communication medium from classical DMCs to networks, and particularly studies stealth communication over a noiseless multipath network wherein James is able to eavesdrop on a subset of links.

This paper builds upon the model examined in [2]. Our setup is as follows. Suppose Alice and Bob communicate over a multipath network, which consists of $C$ parallel noiseless links. Unlike [2] wherein James is only able to eavesdrop on a subset of links passively, this work considers the situation in which James also has the ability to jam the same subset of links to disturb any potential communication (even if he cannot detect the existence of communication). When Alice does not wish to communicate with Bob, her transmissions on the $C$ links are sampled according to an innocent distribution (known a priori to Bob and James). When she is communicating with Bob, her transmissions are chosen from a codebook (also known a priori to Bob and James). In both scenarios, James is able to control (eavesdrop on/jam) at most $Z$ out of $C$ links (where $1 < Z < C/2$), but which subset of links is controlled is not known to Alice and Bob. Note that using “conventional” error-correcting codes does not suffice — the correlations introduced across links by such codes may reveal to James that Alice is indeed actively communicating.

James first estimates whether or not Alice is transmitting by observing the transmission patterns on the links he controls. The stealth is measured via a hypothesis-testing metric — the communication is deemed to be stealthy if regardless of James’ estimator, his probability of false alarm plus his probability of missed detection always approaches one asymptotically. Afterwards, on the basis of his observations, James tries to adversarially jam the links he controls. We consider two types of jamming — erasure jamming and overwrite jamming. Erasure jamming means that James can erase everything on the links he controls, while overwrite jamming allows him to replace the original transmission with his carefully designed transmission patterns.

A. Comparison with Related Work

Stealth communication is closely related to the well-studied covert communication problem. Prior work has successfully investigated the fundamental limits of covert communication under different settings, including AWGN channels [3], DMCs [4], [5], Binary Symmetric Channels (BSCs) [6], etc. The major difference lies in the assumptions on the innocent distribution (when no communication happens) — covert communication requires that, under innocent transmission, the channel inputs must be the “zero symbols”, while stealth communication allows the inputs to follow a non-zero innocent distribution. As a consequence, the throughput with guarantees on both stealth and reliability, in this work and also in [1], [2], scale linearly in the blocklength (rather than being restricted by the square-root law in usual covert communication setups). Another, somewhat technical difference, is that in our setup, 1 it is impossible to communicate stealthily and reliably when $G_1 = C/2$, since James can always “symmetrize” — send a fake message pretending to be Alice (and using her codebook) on the (at least) half of the links he controls.

1The square-root law states that one can only transmit $O(\sqrt{n})$ bits covertly and reliably over $n$ channel uses.
the channel from Alice to James is not known a priori to Alice and Bob because of James’ flexibility in choosing which subset of $Z$ links to sit on, as opposed to a fixed channel from Alice to James in most relevant work (other than [2], [7]).

Instead of the broadly studied random noise channels, the work [8] (on covert communication) shifts the focus to the adversarial noise channels, i.e., the channel between Alice and Bob can be maliciously jammed by James, and the coding scheme there should be resilient to every possible (including even the worst) jamming strategy induced by James. This work inherits the eavesdrop-and-jam framework studied in [8]. In both scenarios, the jammer may cleverly design its jamming strategy based on his observations to disturb any potential communication. Without the stealth/covertness constraint, the eavesdrop-and-jam framework has been investigated in myopic adversarial channels [9], correlated jamming channels [10], and multipath networks [11]. Furthermore, we point out that the functionalities of the jammer in this work is fundamentally different from [12], wherein the jammer is present to help Alice and Bob by sending “artificial noise” to the eavesdropper (similar to the cooperative jamming [13] for security problem).

Reliable communication (without the stealth constraint) over a multipath network in the presence of a jammer has been well-studied in the past. The work [14] shows that as long as $Z < C/2$, Alice and Bob can fully utilize the rest of links to communicate, regardless of the types of jamming (either erasure or overwrite). Robustness against erasure jamming is relatively straightforward while robustness against overwrite jamming requires non-trivial coding schemes (such as pairwise hashing [14]). Similar results are obtained in this work while also taking stealth into account.

B. Our Contributions and High-level Intuition

This work builds upon [2] by considering an active malicious adversary, who can maliciously disturb the transmission. The best rate one can hope for is in general smaller than in [2], since the links being controlled do not carry information anymore (under erasure jamming), or may even carry misleading information (under overwrite jamming).

Firstly, we provide an inner bound on the robust stealth capacity under erasure jamming. The channel between Alice and James can be viewed as an aggregation of all the links controlled by James, while the channel between Alice and Bob can be viewed as an aggregation of the complement of these links (since James erases everything on the links he controls). The stealth constraint imposes a lower bound on the rate (as a consequence of the channel resolvability [4]), while the reliability constraint imposes an upper bound. Moreover, as is standard in wiretap secrecy problems, creating an artificial noisy channel at the encoder (or equivalently, adding an auxiliary random variable) may hurt James more than Bob, and in turn lead to a higher throughput.

Coding against an overwrite jammer is significantly more non-trivial since all possible jamming strategies should be considered. In this work we prove that there still exists a robust coding scheme allowing communicating stealthily and reliably. The crux of our proof is to take advantage of James’ uncertainty about the message/codeword conditioned on his observations. This is inspired by the novel ideas in [9] for reliable communication over myopic adversarial channels.

From a stealth perspective, the major challenge in this work is to design communication schemes that introduce redundancy across the $C$ links (so as to enable resilience to James’ jamming) without allowing the resulting correlation across links to reveal to James that Alice is actually communicating.

While the focus of this work is on robustness to active jamming, it has not escaped our attention that composing our schemes with well-known techniques in the information-theoretic literature allows us to get schemes that are secure against both information leakage and active jamming attacks in this stealth communication setting. A full characterization of this communication setting with trifold objectives is a source of ongoing investigation.

II. MODEL

Random variables and their realizations are respectively denoted by uppercase letters and lowercase letters, e.g., $X$ and $x$. Sets are denoted by calligraphic letters, e.g., $X$. Vectors of length-$n$ are denoted by boldface letters, e.g., $X$ and $x$. If the single-letter distribution on $X$ is $P_X$, then the corresponding $n$-letter product distribution $\prod_{t=1}^{T} P_X$ is denoted by $P_X^n$.

The multipath network consists of $C$ parallel links $L_1, L_2, \ldots, L_C$, each link $L_i$ carries a symbol from the alphabet $X_i$ per time instant. The alphabet for all the links taken together is denoted by $X = \prod_{i=1}^{C} X_i$. Alice’s transmission status is denoted by $T \in \{0, 1\}$ — $T = 0$ if Alice is innocent, whereas $T = 1$ if Alice is active. The message $M$ is either $0$ (if Alice is innocent) or uniformly distributed over $\{1, 2, \ldots, N\}$ (if Alice is active). Note that no prior distribution is assigned to $T$ and only Alice knows $T$ and $M$ a priori. Let $n$ be the blocklength (number of time instants). The length-$n$ vector transmitted on the $j$-th link is denoted by $x_j$, and the collection of vectors on $C$ links is denoted by $X = [x_1^T \ x_2^T \ \ldots \ x_C]^T$. Note that $x$ can also be viewed as a length-$n$ vector over $X$. The system diagram is illustrated in Figure 1.

**Innocent distribution:** When Alice is innocent ($T = 0$), at each time instant $t$ ($1 \leq t \leq n$), an innocent transmission pattern on the $C$ links is sampled according to the time-independent innocent distribution $P_X^{inn} \in \mathcal{P}(X)$, where $\mathcal{P}(X)$ denotes the set of all distributions on $X$. For any subset $J \subseteq \{L_1, L_2, \ldots, L_C\}$, the marginal innocent distribution is
denoted by $P_{X_J|c}$. Over $n$ time instants, the corresponding $n$-letter innocent distribution (resp. $n$-letter marginal innocent distribution) is a product distribution with the form $P_{X_J}^n = \prod_{t=1}^n P_{X_J}^t$ (resp. $P_{X^n,J|c} = \prod_{t=1}^n P_{X^n,J|c}$).

**Encoder:** Alice’s encoder $\Psi(\cdot, \cdot)$ takes the transmission status $T$ and the message $M$ as input, and outputs a length-$n$ vector $X$. If $T = 1$ and message $m$ is transmitted, the encoder $\Psi(1, m)$ outputs the corresponding length-$n$ codeword $X(m)$. The codeword $C$ is the collection of all codewords $X(m)$, $\forall m \in \{1, 2, \ldots, N\}$, and the rate is defined as $R \triangleq (\log N)/n$. If $T = 0$ (hence $M = 0$), the encoder $\Psi(0, 0)$ outputs an innocent vector $X$ according to the innocent distribution. We assume that the codeword $C$ is public, i.e., it is known to all parties, including the jammer.

**Active distribution:** The active distribution, averaged over all the codewords $X(m)$ in the codeword $C$, is denoted by $P_X$. Similarly, for any subset $J \subseteq \{L_1, L_2, \ldots, L_C\}$, the marginal active distribution is denoted by $P_{X_J}$.

**James’ estimation and jamming:** James is able to control any subset of links of size at most $Z$, and let $\mathcal{J}$ be the class of all possible subsets of size at most $Z$. James selects a specific subset $J \in \mathcal{J}$, which is unknown to both Alice and Bob. On the basis of his observation $X_J$ and his knowledge about the codeword $C$, James estimates Alice’s transmission status $T$, and also non-causally jams the subset $J$ to prevent reliable communication irrespective of his estimation.

**Estimation:** James’ estimator $\Phi(\cdot)$ outputs a single bit $\hat{T} = \Phi(X_J)$ to estimate Alice’s transmission status $T$. The stealth is measured by the hypothesis-testing metric. Let $\alpha(\Phi) = \Pr_{X}(\hat{T} = 1|T = 0)$ and $\beta(\Phi) = \Pr_{YM,X}(\hat{T} = 0|T = 1)$ respectively be the probability of false alarm and the probability of missed detection of an estimator $\Phi$. Stealth communication requires that regardless of which estimator $\Phi$ is chosen, $\alpha(\Phi) + \beta(\Phi)$ should approach one asymptotically. A classical result on hypothesis testing [15] shows that the optimal estimator $\Phi^*$ satisfies $\alpha(\Phi^*) + \beta(\Phi^*) = 1 - \mathcal{V}(P_{X_J}, P_{X_J|^c})$, where $\mathcal{V}(P_{X_J}, P_{X_J|^c}) = \frac{1}{2} \sum_{x_J} |P_{X_J}(x_J) - P_{X_J}(x_J^c)|$ is the variational distance between the marginal active distribution and the marginal innocent distribution. Hence we say the communication is stealthy if $\lim_{n \to \infty} \mathcal{V}(P_{X_J}, P_{X_J|^c}) = 0$.

**Jamming:** James can also jam the subset $J$ that he controls. Under erasure jamming, the transmission $X_J$ (on the subset $J$) is completely replaced by the erasure symbols ‘⊥’, while under overwrite jamming, $X_J$ is replaced by a carefully designed vector $\tilde{Y}_J$. In particular, James is able to choose the jamming vector $\tilde{Y}_J$ stochastically according to any conditional distribution $P_{X_J|c}$, since he knows $X_J$ and the codewbook.

**Decoder:** Bob receives $Y$ through the multopath network.

1. Under erasure jamming, $Y_J = X_{J^c}$ on the subset $J^c$ (where $J^c$ denotes the complement of set $J$), and $Y_J$ equals the erased symbols ‘⊥’ on the subset $J$.
2. Under overwrite jamming, $Y_J = X_{J^c}$ on the subset $J^c$, and $Y_J$ is arbitrarily chosen by James.

Note that if James ignores the knowledge of $X_J$, a naive estimator $\hat{\Phi}$ (which always outputs $T = 0$ or $T = 1$) can also guarantee $\alpha(\hat{\Phi}) + \beta(\hat{\Phi}) = 1$. Therefore, the definition for stealth communication implies that James’ optimal estimator $\Phi^*$ cannot be much better than the naive estimator $\hat{\Phi}$. Note that Bob can easily figure out the subset $J$ under erasure jamming due to the appearance of ‘⊥’, while it is not the case under overwrite jamming. Bob reconstructs the message $M$ by applying his decoding function $\Gamma(\cdot)$ to his observation. The probabilities of error under erasure and overwrite jamming are respectively defined as

$$P_e(\Psi, \Gamma) \triangleq \max_{J \in \mathcal{J}} \sum_{t \in \{0, 1\}} \Pr(\hat{M} \neq M|T = t),$$

$$P_e(\Psi, \Gamma) \triangleq \max_{J \in \mathcal{J}} \sum_{t \in \{0, 1\}} \Pr(\hat{M} \neq M|T = t).$$

**Achievable rate:** A rate $R$ is said to be achievable under erasure jamming (resp. achievable under overwrite jamming) if there exists an infinite sequence of codes $(\Psi_n, \Gamma_n)$ such that each code in the sequence has rate at least $R$, and ensures $\lim_{n \to \infty} \mathcal{V}(P_{X^n,J}, P_{X^n,J|^c}) = 0$ (resp. $\lim_{n \to \infty} P_e(\Psi_n, \Gamma_n) = 0$).

### III. MAIN RESULTS

To facilitate the statement of our results, we first define an optimization problem $(A)$, which includes an auxiliary random variable $U$, for a fixed innocent distribution $P_{X^n}$ and a non-negative integer $Z < C/2$ as follows:

$$(A) \ \sup_{P_{X|U}, P_{X^n|U}} \ \min_{J \in \mathcal{J}} I(U; X_J)$$

subject to

$$P_{X^n,J} = \sum_u P_{U} \cdot P_{X^n,J|U}, \ \forall J \in \mathcal{J},$$

$$\max_{J \in \mathcal{J}} I(U; X_J) < \min_{J \in \mathcal{J}} I(U; X_{J^c}).$$

The optimal value of $(A)$ is denoted by $K(P_{X^n}, Z)$. Consider another optimization

$$(B) \ \sup_{P_X} \ \min_{J \in \mathcal{J}} H(P_{X^n,J})$$

subject to

$$P_{X^n,J} = P_{X^n,J}, \ \forall J \in \mathcal{J},$$

$$\max_{J \in \mathcal{J}} H(P_{X^n,J}) < \min_{J \in \mathcal{J}} H(P_{X^n,J}),$$

and let the optimal value be $K(P_{X^n}, Z)$. It is worth noting that $K(P_{X^n}, Z)$ is always bounded from above by $K(P_{X^n}, Z)$, since $(A)$ is equivalent to $(B)$ by restricting $U = X$. As is usual in wiretap secrecy problems, Theorem 1 below shows that a higher rate $K(P_{X^n}, Z) - \epsilon$ is achieved by introducing an auxiliary variable $U$.

**Theorem 1 (Erasure jamming).** For any $P_{X^n}$ and non-negative integer $Z < C/2$, the rate $R = K(P_{X^n}, Z) - \epsilon$ is achievable under erasure jamming for any small $\epsilon > 0$.

**Remark 1 (Cardinality Bound).** Bounding the cardinality of the auxiliary variable $U$ is possible. Following standard cardinality bound arguments (c.f. [16]), given any feasible $(U, X)$ in $(A)$, there always exists a feasible $(U^*, X)$ with $|U^*| \leq |X| + 2|Z| - 1$ that yields the same objective value.

Compared with erasure jamming, dealing with overwrite jamming is much more challenging due to the fact that James, knowing Alice’s codewbook, may attempt to “spoof” Alice’s transmissions. Bob’s decoder should be robust to any
jamming strategy $P_{X|U,X,J,c}$, including the one that maximizes his probability of decoding error. However, our next result shows that stealth communication is still possible.

**Theorem 2 (overwrite jamming).** For any $P_{X|U,X,J,c}$ and non-negative integer $Z < C/2$, the rate $R = \mathcal{K}(P_{X|U,X,J,c}, Z) - \epsilon$ is achievable under overwrite jamming for any small $\epsilon > 0$.

**IV. PROOF SKETCHES OF THEOREMS 1 & 2**

**A. Erasure jamming (Theorem 1)**

The optimal distributions in optimization (A) are denoted by $P_U$ and $P_{X|U}$.

**Encoder:** Let $R = \min_{J \subseteq I} I(U; X_J) - \epsilon$ (for any small $\epsilon > 0$). For each message $m \in \{1, 2, \ldots, N\}$, where $N = 2^{nR}$, the intermediate codeword $u(m)$ is generated according to the $n$-letter distribution $P_U$. To transmit $m$, Alice chooses $u(m)$ and stochastically maps $u(m)$ to $x(m)$ with probability $P_{X|U}(x(m)|u(m))$. The length-$n$ codeword $x(m)$ is transmitted over the multipath network.

**Decoder:** Bob first determines the subset $J$ (controlled by James) based on the erasure symbol ‘1’. And then applies typicality decoding based on $y_J$. Note that $y_J = x_J$ since the subset $J'$ is not controlled by James. He decodes to $x(J)$ according to the $n$-letter distribution $P_{X|U}(x(m)|u(m))$. The length-$n$ codeword $x(m)$ is indistinguishable from the marginal innocent distribution $P_{X|U}^m(m)$. Note that

$$P_{X_J}(x_J) = \sum_{m=1}^{N} \frac{1}{N} P_{X_J}(x_J|u(m)),$$  \hspace{1cm} (5)

$$P_{X_J}^m(x_J) = \sum_{u} P_U(u) P_{X_J|U}(x_J|u),$$  \hspace{1cm} (6)

Equation (6) follows from the constraint in (1), which ensures that the stochastic process $\sum_u P_U \cdot P_{X_J|U}$ simulates the encoder $\Psi$ identical to the marginal innocent distribution $P_{X|U}^m$. The constraint (2) ensures the size of the codebook to be large enough so that with high probability (w.h.p.) the active distribution $P_{X_J}$ is sufficiently close to $\sum_u P_U \cdot P_{X_J|U}$ — it turns out that $R > I(U; X_J)$ is sufficient, as noticed in [4], from a channel resolvability perspective. To prove it, we first denote the typical set of $X_J$ by $A_{X_J}$, and the joint typical set (resp. joint type class) of $U$ with respect to a typical $x_J$ by $A_{U|x_J}$ (resp. $T_{U|x_J}$). Recall that proving stealth is equivalent to bounding the variational distance $\mathcal{V}(P_{X_J}^m, P_{X_J}) = \frac{1}{2} \sum_{x_J} |P_{X_J}^m(x_J) - P_{X_J}(x_J)|$. For any typical $x_J$, we have

$$|P_{X_J}^m(x_J) - P_{X_J}(x_J)| \leq \sum_{u \in A_{U|x_J}} P(u) P(x_J|u) - \sum_{m: u(m) \in A_{U|x_J}} \frac{1}{N} P(x_J|u(m)) \leq \sum_{T_{U|x_J}} \sum_{u \in T_{U|x_J}} P(u) P(x_J|u) - \sum_{m: u(m) \in T_{U|x_J}} \frac{P(x_J|u(m))}{N},$$

where the approximation (a) is obtained by discarding negligible atypical events, (b) is achieved by dividing the typical set $A_{U|x_J}$ into typical type classes $T_{U|x_J}$, and (c) follows since $P(x_J|u)$ is identical for all $u \in T_{U|x_J}$. Note that

$$P(U \in T_{U|x_J}) = \sum_{T_{U|x_J}} P(U \in T_{U|x_J}) = \frac{|m: u(m) \in T_{U|x_J}|}{N}.$$

For any $\epsilon > 0$, we have marginal distributions that look innocent. Therefore, Alice is constrained to using a stealth codebook, and hence any set of $Z$ links must have marginal distributions that look innocent. The ability to overwrite $X_J$ together with the partial knowledge about $X_{J'}$, may make it possible for James to fool Bob.

Nonetheless, as shown in Theorem 2, it is still possible for Alice and Bob to communicate at a positive rate, and we sketch the proof as follows. Let $P_Y$ be the optimal distribution in (B).

**Encoder:** Let $R = \min_{I \subseteq J} I(U; X_I) - \epsilon$ (for any small $\epsilon > 0$). For each message $m$, the codeword $x(m)$ is generated according to the $n$-letter distribution $P_{X_J}$. Alice encodes $m$ to $x(m)$, and transmits $x(m)$ over the multipath network. The codebook $C \triangleq \{x(m)\}_{m=1}^N$.

**Decoder:** Since Bob does not know the set $J$ controlled by James a priori, he attempts to decode based on every possible choice of $J \in J$ and applies an erasure-like decoding on its corresponding decoding set $J'$. For a specific $J$, Bob outputs a message $m$ to his list $L$ if there is a unique $m$ such that

$$P(x_J|u(m)) \geq \frac{1}{N} P(U \in T_{U|x_J}) - \frac{|m: u(m) \in T_{U|x_J}|}{N}.$$
The proof for stealth is similar to that in Section IV-A, hence we focus on reliability only. Suppose Alice is active (T = 1) and the subset J is controlled by James. When Bob decodes according to the “correct” decoding set \( J^c = J^j \), the transmitted message \( m \in \mathcal{L} \) w.h.p., since \( J^c \) is noiseless and the rate \( R < H(P_X) \). If Bob decodes according to any other \( J^c \) (\( J \neq J^j \)), we argue that w.h.p., no other message \( m' \neq m \) falls into \( \mathcal{L} \). We now consider the worst case wherein \( J \subseteq J^c \) (the decoding set \( J^c \) contains all the links controlled by James). The set of “good” links in \( J^c \) is denoted by \( \mathcal{G} \triangleq J^c \setminus J \). James is able to replace \( x_j \) with \( y_j \) according to any distribution \( P_{X_j|X,C} \), and the probability of error with respect to set \( J^c \) and \( P_{X_j|X,C} \) is given as

\[
\sum_{m=1}^{N} \frac{1}{N} \left| \sum_{y_j \neq x_j} \sum_{x_j \in \mathcal{X}_j} P(y_j|x_j,(m),C) \cdot \mathbb{I} \{ (x_G(m), y_j) \in \mathcal{C} \} \right|
\]

where \( \mathbb{I} \{ (x_G(m), y_j) \in \mathcal{C} \} = 1 \) if there exists a message \( m' \neq m \) such that the sub-codewords of \( m' \) on \( \mathcal{G} \) and \( J \) equals \( x_G(m) \) and \( y_j \) respectively. Note that in (10) we only consider \( y_j \neq x_j \), since when \( y_j = x_j \), Bob’s decoder outputs the true message \( m \) to the list \( \mathcal{L} \) and no error occurs. By considering typical events only and gathering all messages with the same sub-codeword on \( J^c \) together, we approximate (10) by

\[
\sum_{x_j \in \mathcal{X}_j} \sum_{m : x_j(m) = x_j} \frac{1}{N} \sum_{y_j \neq x_j} P(y_j|x_j,C) \cdot \mathbb{I} \{ (x_G(m), y_j) \in \mathcal{C} \}
\]

\[
= \frac{1}{N} \sum_{x_j \in \mathcal{X}_j} \sum_{m : x_j(m) = x_j} P(y_j|x_j,C) \cdot \mathbb{I} \{ m : x_j(m) = x_j \cap (x_G(m), y_j) \in \mathcal{C} \}
\]

Finally, a union bound over all \( J \in \mathcal{J} \) shows that w.h.p., there does not exist a fake message \( m' \neq m \) falling into \( \mathcal{L} \), which in turn implies the list \( \mathcal{L} \) contains a unique message \( m \).

When Alice is innocent (\( T = 0 \)), a similar proof technique shows that \( \mathcal{L} \) is empty with high probability.

**Remark 2.** It would be interesting to see if it is possible to modify the proof technique above to show that the rate \( K(P_X, Z) - \epsilon \) is also achievable. The main challenge is to deal with the complicated joint typicality relationship among \( (u, y_j, x_j) \), since we introduce an auxiliary variable \( u \) and use typicality decoding. We believe that the following strategy likely works and conjecture the following achievability.

**Conjecture 1.** For any \( P_X \) and non-negative integer \( Z < C/2 \), the rate \( R = K(P_X, Z) - \epsilon \) is also achievable under overwire jamming for any small \( \epsilon > 0 \).

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