Theoretical study of small signal modulation behavior of Fabry-Perot germanium-on-silicon lasers

Ying Zhu1,2, Liming Wang1, Zhiqiang Li1, Ruitao Wen2 and Guangrui Xia3,*

1 Department of Materials Engineering, the University of British Columbia, Vancouver, BC V6T 1Z4 Canada
2 Department of Material Science and Engineering, Southern University of Science and Technology, Shenzhen 518055, People’s Republic of China
3 Crosslight Software Inc., Vancouver, BC V5M 2A4, Canada
* Author to whom any correspondence should be addressed.
E-mail: guangrui.xia@ubc.ca

Keywords: Germanium laser, optical modulation, silicon photonics

Abstract
This work investigated the modulation responses of Fabry–Perot Ge-on-Si lasers by modeling and simulations. The 3 dB bandwidth dependence on the structure parameters such as poly-Si cladding thickness, Ge cavity width and thickness, and minority carrier lifetime were studied. A 3 dB bandwidth of 33.94 GHz at a biasing current of 270.5 mA is predicted after Ge laser structure optimization with a defect limited carrier lifetime of 1 ns. The eye diagrams simulated show a stable eye-opening window of 33.94 GHz at a biasing current of 270.5 mA is predicted after Ge laser structure optimization with a defect limited carrier lifetime of 1 ns. The eye diagrams simulated show a stable eye-opening window at 20 Gb·s⁻¹ NRZ. The improvement to 10 ns minority carrier lifetime would reduce the threshold current to 6.85 mA, and increase the 3 dB bandwidth to 36.89 GHz.

1. Introduction

Optical communication utilizes light to carry and transport information, which possesses a large transmission bandwidth, fast communication speed and a low transmission loss compared to metal interconnects. In short-reach communication, optical communication has also been playing an increasingly significant role in applications such as backbone interconnect in data centers. For communication of mm and shorter distance, such as on-chip communication, silicon photonics has been considered as a powerful tool to address the slow-speed and high energy consumption of metal interconnects [1–5].

Although most silicon-integrated photonic components are mature, Si-compatible lasers have been sought for decades [6–10]. III-V semiconductor materials have been widely used as active laser gain mediums, and III-V semiconductor lasers integrated on Si substrate by heteroepitaxy [11, 12] or bonding techniques [6, 13, 14] have shown very good performance up to now. Even though the direct heteroepitaxial growth of III-V quantum dot (QD) lasers on Si substrates has been demonstrated to have great potentials [15–17], the high cost and unavoidable contamination problems impede the entrance of III-V semiconductors into the mainstream Si fabrication facility. In recent two decades, germanium (Ge), a group IV element semiconductor, has been demonstrated as a promising gain medium material [18–25]. Compared with III-V semiconductors, the fabrication of Ge is more compatible with the mainstream Si fabrications processes, which has great benefits in realizing Si-compatible lasers [26–29].

Ge is an indirect bandgap (0.664 eV) material. It only has a small energy difference (136 meV) between the direct valley (Γ) and indirect valley (L) [30]. Meanwhile, the direct bandgap of Ge is 0.8 eV, corresponding to a wavelength of 1550 nm, which exactly lies in the low loss window of Si dioxide. In 2007, Jifeng Liu et al have predicted that Ge can achieve an optical gain > 500 cm⁻¹ and a relatively high differential gain of 8.0 × 10⁻¹⁶ cm² under an injected carrier density of 2.0 to 4.0 × 10¹⁷ cm⁻³ by combining the strain-induced bandgap engineering and n-type doping above 4.0 × 10¹⁹ cm⁻³ [7, 18]. Based on that, an optically pumped edge-emitting Ge-on-Si laser was successfully demonstrated with a gain spectrum in 1590–1610 nm in 2010 [20]. Later in 2012 [23] and 2015 [31], electrically pumped Fabry–Perot Ge-on-Si lasers were realized. However, these early Ge-on-Si lasers had a high threshold current density of 280 kA·cm⁻² and a low wall-plug efficiency of 0.5 ~ 4% [23, 32].
In comparison, commercial III-V lasers have low threshold current densities of 0.01 – 1 kA·cm$^{-2}$ and high wall-plug efficiencies in the range from 50% to 65%. Some theoretical studies were available to calculate the performance potential of Ge lasers, including 0-dimensional (0D) analysis of Ge optical gains and their strain and doping dependences. 2D Ge-on-Si laser modeling and simulation studies from our group predicted that Ge-on-Si lasers could be greatly improved to reach a wall-plug efficiency of 43.8% and a threshold current of 4 mA (current density of 3 kA·cm$^{-2}$) by structure optimization, strain engineering and Ge material improvement. Except for these theoretical studies, there have been many experimental works of Ge lasers worldwide. The investigation of Ge lasers involved alloying Ge with Sn to become direct band [35], or strain engineering of Ge band structure using Ge nanowires or microbridges to amplify the tensile strain in Ge. They have demonstrated that using alloying method and special structures, compact group IV lasers can be realized with a low threshold value (an optical pumping threshold density of ~3.0 kW·cm$^{-2}$) and high efficiency (a differential quantum efficiency close to 100%) [36]. There are also some research efforts on Ge LEDs and photodetectors.

So far, Ge lasers research has been focused on the static properties, such as optical gains and losses, threshold current densities and efficiencies. It is important to investigate the direct modulation potentials of Ge lasers, and the methods to improve the bandwidth of Ge lasers with insights gained from III-V semiconductor lasers. For applications in optical links, direct modulated lasers (DMLs) are desired, as they can eliminate the needs for external modulators in photonic circuits. For DMLs, dynamic properties, such as small signal modulation behaviors, are crucial, which motivated this study.

This work is a 2D theoretic study of Ge-on-Si small signal modulation behaviors. We have investigated the small signal modulation responses of Ge-on-Si lasers, and how modulation bandwidth can be improved by structure optimization and material quality enhancement.

### 2. Ge laser modeling and calibration

The 2D Ge-on-Si laser modeling was conducted and calibrated in our group’s previous studies in [34, 44], where models of Ge energy band structure (with biaxial strain and doping) and material loss (including the bandgap narrowing effect and the energy separation effect) were implemented in LASTIPTM, a mainstream commercial 2D laser simulation software. The bandgap narrowing effect (BGN) model in Ge is adopted from literature and implemented in LASTIPTM, which is shown as follows:

$$E_g(N_D = 0) - E_g = E_{BGN} + \Delta_{BGN}N_D,$$

where $N_D$ is the dopant concentration, in the unit of cm$^{-3}$, $E_{BGN}$ is BGN turn-on offset energy reduction, and $\Delta_{BGN}$ is the BGN coupling parameter. The data has been yielded from literature [32, 45]: $E_{BGN} = 0.013$ eV and $\Delta_{BGN} = 10^{-21}$ eV/cm$^{-3}$.

Due to the lack of experimental data, the laser structure and light-current ($L-I$) curve in this work are mainly based on the MIT’s electrically pumped Ge-on-Si laser experiment [23]. The Fabry–Perot laser structure model is shown as figure 1. The thickness, doping, and strain parameters of each layer material are the same as those in the experiments in [23], as shown in table 1.

To reduce the computation time, 2 μm thick Si substrate is used, and a virtual contact is set at the bottom of Si substrate and at the top of the metal layers for biasing purpose. The active Ge region has a width of 1 μm, a length of 270 μm long and a thickness of 200 nm which is the average value of the 100 to 300 nm thickness from the experiment results owing to the process non-uniformity. The interface between the Si and Ge semiconductors is considered as an abrupt heterojunction in LASTIPTM and the physical model used for the current transport across the junction is the thermionic emission model.

The metal-semiconductor heterojunctions were aligned by electron affinity. The reflectivity values of the two facets are $R_1 = 23\%$ and $R_2 = 38\%$, which corresponds to a mirror loss $\alpha_m$ of 45 cm$^{-1}$ based on the MIT work [23, 24, 32]. The reflectivity of an ideal Ge facet is calculated to be 38% by considering the index contrast between Ge and air. However, according to their experiment report, a thin oxide layer was deposited on one facet for surface protection. The reflectivity of the Ge/Si dioxide facet is estimated to be 23%. The refractive index values of all materials are wavelength dependent for Si substrate, Ge, and poly-Si layers, respectively. These material parameters mainly come from literature [34, 44] and [32]. The Ge based laser device is index guided. The Auger coefficients are set as $C_{\text{ann}} = 3.0 \times 10^{-32}$ cm$^6$ s$^{-1}$ and $C_{\text{ppn}} = 7.0 \times 10^{-32}$ cm$^6$ s$^{-1}$ [18]. The defect limited lifetime ($\tau_{\text{def}}$) is an important material quality parameter and has been estimated conservatively to 1 ns for the epitaxial grown Ge films to calculate the defect recombination, based on the measurements in recent work [47]. The surface recombination would become significant when the pillar diameter or stripe width of the laser structure is in nm scale according to the Shockley-Read-Hall theory. The Ge width is 1 μm, making the surface recombination less important compared to the other two recombination paths mentioned above. As
relevant experiment data of Ge thin films is still lacking up to now, surface recombination was not considered in our simulations here.

As for the laser’s performance, the optical loss will be one of the most influenced factors. Generally, the optical loss is composed of two loss mechanisms: the internal loss $\alpha_i$ and the mirror loss $\alpha_m$. Here the internal loss $\alpha_i$ is assumed to be dominated by the free carrier absorption (FCA) [7]. In LASTIPTM, for a narrow wavelength range, the free carrier absorption (FCA) is described by

$$ \alpha_i = AN + BP, \quad (2) $$

where $A$ and $B$ are constants, in the unit of cm$^2$, $N$ and $P$ are the electron and hole density, respectively, in the unit of cm$^{-3}$. We have used first principle calculations of free carrier absorption results in the $n$-type Ge [32] and experiment measurements in the $p$-typed Ge [48] as a starting point and obtained the best fitting result to the L-I curve with the following free carrier absorption relationship:

$$ \alpha_i = 4.18 \times 10^{-19}N + 1.021 \times 10^{-17}P, \quad (3) $$

The free carrier absorption relationship in the doped Si substrate and poly-Si cladding layer were obtained from work [49, 50].

As experimental lasers were measured from one side of the laser, we used the laser optical power from the polished facet (R1 = 23%) as the optical output power to fit. Based on all these material parameters, the simulation model provided a current density $J_{th}$ of 297.3 kA·cm$^{-2}$ or threshold current $I_{th}$ of 802.7 mA at 15 °C with the transverse electric (TE) mode lasing at $\lambda = 1676$ nm, shown as figure 2(a). This fitted result was very closed to experimental results of 280 kA·cm$^{-2}$ and lasing wavelength of 1650 nm [23]. The spatial mode distribution of the TE mode is shown in figure 2(b) with a confinement factor of 0.43. Only one TE longitudinal mode is observed and lased from the Ge active region.

3. Relevant theories to calculate the frequency responses

After obtaining the best-fitting parameters with LASTIPTM, we were ready to calculate the small signal modulation properties. First, let’s review the relevant theories on that.
3.1. Frequency calculations
The relaxation resonance frequency is given by
\[ f_r = \frac{1}{2\pi} \sqrt{\frac{\nu_g}{q}} \Gamma \eta_i \frac{dg}{dN} (I_b - I_{th}) / V, \]  
(4)
where \( \nu_g \) is the group velocity, \( q \) is the elementary charge, \( \Gamma \) is the optical confinement factor, \( \eta_i \) is the internal efficiency, \( \frac{dg}{dN} \) is the differential gain, \( I_b \) is the biased current, \( I_{th} \) is the threshold current, and \( V \) is the volume of the active region. When the damping factor is small or negligible, the electrical 3 dB down frequency is given by
\[ f_{3\,dB} = \sqrt{1 + \frac{75}{4} f_r^2} \approx 1.55f_r, \]  
(5)
Based on the equation (4), for a given active material, there are mainly five strategies to enhance the modulation frequency: (1) to improve \( \Gamma \); (2) to minimize \( V \); (3) to maximize \( I_b - I_{th} \); (4) to enhance \( \eta_i \); and (5) to enhance \( \frac{dg}{dN} \).

Next, we will discuss the differential gain of strained Ge with n-doping.

3.2. Differential gain of strained Ge with doping
Optical gain, \( g \), is another important parameter for the active medium materials, which determines the capability of laser medium to increase the output power. The differential gain, \( \frac{dg}{dN} \), is a critical parameter in high-speed laser applications, because the relaxation resonance frequency of the laser depends on the square root of the differential gain \[ f_r = \frac{1}{2\pi} \sqrt{\frac{\nu_g}{q}} \Gamma \eta_i \frac{dg}{dN} (I_b - I_{th}) / V, \]  
(4)
in high-speed laser applications, because the relaxation resonance frequency of the laser depends on the square root of the differential gain. \( g \) is related to the carrier density, \( N \), by a simple two parameter logarithmic formula [51], shown in equation (6):
\[ g = g_{0N} \ln \frac{N}{N_{tr}} (g > 0), \]  
(6)
where \( g_{0N} \) is the gain coefficient, in the unit of cm\(^{-1}\), \( N_{tr} \) is the transparency carrier density, in the unit of cm\(^{-3}\). In order to obtain positive optical gain, the carrier density must be larger than the transparency carrier density \( (N > N_{tr}) \). Under this condition, the differential gain \( \frac{dg}{dN} \), which is of great importance in our simulation and directly influenced by the carrier density, \( N \) and gain coefficient, \( g_{0N} \), is given by differentiating equation (7):
\[ \frac{dg}{dN} = \frac{g_{0N}}{N}. \]  
(7)
Above threshold, the carrier density, \( N \), equals to the threshold carrier density, \( N_{th} \).

3.3. Gain and differential gain calculation by LASTIPTM
The optical gain behavior of 0.25% tensile strained n-doped Ge has been modeled and simulated by Jifeng Liu et al [18]. They have demonstrated that a significant net gain of about 400 cm\(^{-1}\) could be achieved in the 0.25% tensile strained n-doped Ge with an extrinsic electron density of 7.6 \times 10^{19} cm\(^{-3}\) and that a high differential gain of 8 \times 10^{-16} cm\(^{-2}\) could be obtained at a relatively low injected carrier density. It should be noted that the
The differential efficiency $\eta_d$ is defined as the product of internal efficiency $\eta_i$ and extraction efficiency $\eta_{ext}$, shown as equation (9). The extraction efficiency $\eta_{ext}$ is defined by equation (10), in which $\alpha_i$ is internal loss, the weighted average of the local loss.

$$P_{out} = \eta_i \frac{\alpha_m}{\alpha_i + \alpha_m} \frac{h\nu}{q} (I - I_b) = \eta_i \frac{h\nu}{q} (I - I_b),$$

$$\eta_d = \frac{\Delta P}{\Delta I} \frac{hc}{q\lambda} = \eta_i \frac{\alpha_m}{\alpha_i + \alpha_m} = \eta_i \eta_{ext}.$$
where \( \frac{\Delta P}{\Delta I} \) is the slope of the L-I curve, \( h \) is the Plank constant, \( c \) is the speed of light, \( q \) is the elementary charge, \( \lambda \) is the lasing wavelength, \( I \) is the biased current, \( I_{th} \) is the threshold current. After LASTIPTM calculated the L-I curves, \( I_{th} \) values were obtained from the plots, and the slopes \( \frac{\Delta P}{\Delta I} \) of the L-I curves in the above threshold region were calculated. Moreover, the internal loss \( \alpha_i \) was given by LASTIPTM. Bring all these known or calculated parameters back to the equation (9), and the internal efficiency could be calculated.

The threshold current \( I_{th} \) and threshold carrier density \( N_{th} \) are expressed in the following equations [51]:

\[
I_{th} = \frac{q d W L}{\eta_i} (R_{SRH}(N_{th}, P_{th}) + R_{rad}(N_{th}, P_{th}) + R_{Auger}(N_{th}, P_{th})) = \frac{q d W L N_i}{\eta_i} \tau_i, \tag{11}
\]

\[
N_{th} = N_{tr} + \frac{\alpha_i + \alpha_m}{\Gamma(d, W) G'}, \tag{12}
\]

where \( q \) is the elementary charge, \( d, W, L \) are the thickness, width, and length of the active region, respectively, \( \eta_i \) is the internal efficiency, \( R_{SRH} \) is the Shockley-Read-Hall non-radiative recombination rate generating at defects [53], \( R_{rad} \) is the spontaneous recombination rate, \( R_{Auger} \) is the nonradiative recombination rate due to the Auger recombination process, \( \tau_i \) is the carrier lifetime, \( N_{tr} \) is the transparency carrier density, \( \Gamma \) is the optical confinement factor which is influenced by the thickness \( d \) and width \( W \) of the active region, \( G' \) is the material gain, which equals to \( G/\Gamma \).

3.5. Model assumptions and limitations

In our modeling, we had some assumptions and conditions to make the calculation more reasonable and simpler. First, with the high pumping current, high heat generation is expected to cause electron migration and contact metal melting. In this work, these thermal effects have not been considered here, and the temperature \( T \) was maintained at 288 K and contact metal melting. In this work, these thermal effects have not been considered here and the temperature \( T \) was maintained at 288 K. Second, LASTIPTM is a 2D simulator, which ignores the phase matching condition and assumes that only a single longitudinal mode exists, and that lasing occurs at a wavelength with the peak modal gain [54]. In a real Fabry–Perot laser, lasing is at the wavelength where the cavity round-trip gain peaks. Third, the gain saturation effect is not included here, as we assumed that in our simulation such high photon density would not be encountered. Theoretically, gain saturation will reduce the differential gain and then affect the robustness of the laser. Fifthly, gain broadening model is not imposed on the gain of bulk material. The reason is that the level broadening coefficient is relatively small (1.2 meV) and the un-broadened gain spectrum does not have a sharp edge near the bandgap as in the quantum well case [54].

3.6. Small signal response simulated without laser structure optimization

The small-signal-modulation property of the laser structure (figure 1) was calculated by LASTIPTM under a biased current ranging from 756 to 999 mA, shown in figure 4. This current range was selected to cover a current window from below to above the threshold current of 802.7 mA. Under a simplified condition without considering the electron migration and metal heating effect of the metal contact, the 3 dB bandwidth was calculated to be about 6.2 GHz at the maximum simulated biased current of 999 mA. The bandwidth value is relatively small, and the biased current is very high for practical applications. Next, we investigated the impacting factors, such as the laser structure parameters and the minority carrier lifetime, to improve the modulation bandwidth.

4. Optimization of Fabry–Perot Ge-on-Si lasers

To investigate the small signal modulation responses of the Ge lasers and these influenced factors, we optimized the structure of the Ge laser. There are mainly three optimizing parameters, including the thickness of the poly-silicon cladding layer \( d_{polySi} \), the width of Ge cavity \( W_{Ge} \), and the thickness of the Ge cavity \( d_{Ge} \). The length of Ge cavity is maintained at 270 \( \mu \)m. Based on the previous simulations [44], this length gives low effect of threshold
currents. Ge is not a well-understood optical gain material, and many model parameters do not have widely agreed values or ranges. Therefore, during our simulation process, our goal is not to obtain the ultimate optimal point and the exact values for these properties, but rather to demonstrate that the performance of Ge-on-Si lasers could be improved greatly and how each factor influences the small-signal modulation responses.

After the calibration of our model parameters and analysis of the laser key performance equations, we started optimizing the Fabry–Perot Ge-on-Si lasers with LASTITP™. The starting point was the laser structure and parameters in figure 1 and table 1, where the active Ge region has a width of 1 μm, a length of 270 μm long and a thickness of 0.2 μm and the poly-Si thickness is 0.18 μm. For an ideal laser, small threshold current, high efficiency and large bandwidth are all desired properties, but they cannot be obtained at the same time. As we concentrate on studying the small signal modulation of lasers, the 3 dB bandwidth is chosen as the most important optimization criteria. In the simulation, the biased current is set at $I_{th} = 270.5$ mA through all the 3 dB bandwidth calculations to keep only one variable at one time and make the geometry improvement simpler to view. This biased current value is chosen to be larger than the threshold currents of all geometry simulated, and not higher than 10 times of $I_{th}$. With all these things settled down, the optimizations begin with the poly-Si thickness.

4.1. Poly-Si thickness $d_{\text{poly-Si}}$ dependence

The thickness of the poly-Si layer has a significant effect on the optical internal loss, which directly impacts the differential efficiency and threshold current as shown in equations (9) and (11). From figure 5(a), the internal loss decreases significantly with thicker poly-Si cladding and then becomes steady. The extraction and differential efficiency increase at first and finally reach a plateau. This is because the metal contact has a much higher optical absorption loss than poly-Si. As the poly-Si becomes thicker, the top metal contact is moved further away from the Ge active region and the internal loss $\alpha_c$ decreases [57]. Hence the internal efficiency increases a lot at first and then becomes steady. Meanwhile, the optical confinement factor also increases at the beginning due to the metal part being moved away and significantly reduction of its optical absorption. With the keeping increase of the poly-Si thickness, the optical absorption part in the poly-Si increases, which leads to a slight decrease of the optical confinement factor (figure 5(b)).

Since the internal loss decreases, the threshold current exhibits a similar decreasing behavior and becomes steady eventually, as shown in figure 5(c). Moreover, the threshold carrier density also decreases and then reaches a plateau since there is a smaller loss to compensate for. According to equation (7), the differential gain, $dg/dN$, displays an opposite trend to the threshold carrier density, that is, increases greatly first and then becomes steady (figure 5(d)). Consequently, with the increase of poly-Si thickness, the 3 dB bandwidth climbs to a maximum value of 27.08 GHz at the biased current of 270.5 mA at $d_{\text{poly-Si}} = 0.7$ μm and then slightly decreases due to the minor reduction of confinement factor (figure 5(d)). Therefore, 0.7 μm has been chosen as the optimized poly-Si cladding layer thickness as it shows the highest bandwidth value.

4.2. Ge width $W_{\text{Ge}}$ dependence

The width of Ge has a direct impact on the bandwidth through two parameters: (1) optical confinement factor, $\Gamma$, and (2) active region volume, $V$. The influence of the Ge width on $\alpha_c$, $\eta_B$, $\eta_c$, $\eta_i$, $I_{th}$, $V$, $dg/dN$ are shown in the figure 6. With the increase of the Ge width, the internal loss becomes smaller, and the differential and extraction efficiency rise a little and then plateau as in figure 6(a), because a wider Ge cavity results in a larger Ge...
active region and larger confinement factor in figure 6(b), and the optical mode will less extend into the lateral layers. Meanwhile, the internal efficiency shows very little decreasing tendency with wider Ge cavity for a narrower waveguide is beneficial for the uniform current injection.

It is obvious that the volume of the active region is in a monotonically linear increasing relationship with Ge width, shown in figure 6(c). Based on the equation (11) for threshold current, the threshold current is directly proportional to the width of Ge cavity and has an indirect relationship with internal loss by the threshold carrier density, \( N_{th} \). Under the combined action of Ge width and internal loss, the threshold current exhibits a minimum value of 32.73 mA at a width of 0.5 \( \mu \)m and then increases greatly with wider Ge width. The variation tendency of theoretical threshold carrier density can be inferred from equation (12). With the decrease of internal loss and increase of confinement factor, the \( N_{th} \) displays a decreasing tendency and then becomes steady. Hence, the variation trend of differential gain shows an increasing part at first and then plateaus.

Combined with all these competing factors and based on equations (4), (5), the 3 dB bandwidth climbs to a maximum value of 31.72 GHz at a width of 0.6 \( \mu \)m, and then reduces a lot due to the rapidly increasing active region volume. Although the 0.6 \( \mu \)m Ge width does not possess the lowest threshold current, it is a tradeoff to choose the optimization point between threshold current and 3 dB bandwidth. Since the 3 dB bandwidth is our final goal, the Ge width has been set at 0.6 \( \mu \)m to get the highest performance in modulation.

4.3. Ge thickness \( d_{Ge} \) dependence
As for the thickness of Ge active region, it has a similar dependence to the Ge width dependence, (1) optical confinement factor, \( \Gamma \), and (2) active region volume, \( V \). The influence of the Ge width on \( \alpha_i \), \( \eta_d \), \( \eta_{ext} \), \( \Gamma \), \( \eta_i \), \( I_{th} \), \( V \), \( dg/dN \) are shown in the figure 7. The internal loss exhibits a decreasing trend because the thicker Ge layer leads to less optical mode overlapping with lossy poly-Si cladding and metal contact layers. Thus, the differential and extraction efficiency rise at the beginning and then become steady (figure 7(a)). The internal efficiency shows minor decrease due to larger resistivity in Ge. The confinement factor increases dramatically with the thicker Ge layer which can provide better vertical confinement (figure 7(b)). Also, the volume of active region is directly proportional to the thickness of Ge layer (figure 7(c)). According to equation (11), the threshold current is influenced by Ge thickness, internal loss, and confinement factor. Under these competing actions, the threshold current is finally dominated by the thickness of Ge and monotonically increasing.
With the increase of Ge thickness, the lasing wavelength has a red shift, and the refractive index becomes smaller, which influences the material gain coefficient and differential gain. The differential gain peaks at a Ge thickness of 0.3 μm and then decreases to a plateau (Figure 7(d)). Based on all these variables, the 3 dB bandwidth shows a peak of 33.94 GHz at a thickness of 0.3 μm and then decreases greatly, which is dominated by the increase of the active region volume.

Above all, we have come to an optimization point of the Ge-on-Si lasers, which is $d_{\text{poly-Si}} = 0.7 \mu m$, $W_{\text{Ge}} = 0.6 \mu m$ and $d_{\text{Ge}} = 0.3 \mu m$, with a 3 dB bandwidth of 33.94 GHz at a biased current of 270.5 mA and a threshold current of 46.42 mA.

4.4. Defect-limited minority carrier lifetime dependence

In the previous optimization, the defect limited minority lifetime ($\tau_{n,p}$) was set as 1 ns for conservative estimation while the minority carrier lifetime strongly depends on the concentration of recombination centers and can be utilized to determine the Shockley-Read-Hall (SRH) recombination rate, $R_{\text{SRH}}$ [53].

$$R_{\text{SRH}} = \sigma_{n,p}^\delta n \tau_{n,p},$$

where $\sigma_{n,p}$ is the electron and hole capture cross sections of deep traps, $\nu_{n,p}$ is the thermal velocity of electrons and holes, $N_{\tau}$ is the trap (or defect) density, $\delta n$ is the excessive electron concentration. In LASTIPSM, the SRH recombination rate is given as [54]:

$$R_{n}^{\text{SRH}} = e_{n} \tau_{n} N_{\tau} (1 - f_{j}) - e_{n} n_{j} N_{\tau} f_{j},$$

$$R_{p}^{\text{SRH}} = e_{p} \tau_{p} N_{\tau} f_{j} - e_{p} p_{j} N_{\tau} (1 - f_{j}),$$

where $e_{n}$ and $e_{p}$ are the capture coefficients for electrons and holes, respectively, $n_{j}$ is the electron concentration when the electron quasi-Fermi level coincides with the energy level $E_{j}$ of the $j$th trap. A similar definition applies to $p_{j}$. $N_{\tau}$ is the density of the $j$th deep trap and $f_{j}$ is the occupancy of the $j$th deep trap level. The capture coefficient $\sigma_{n,p}$ for electrons and holes is related to the minority lifetime of the carrier by following equations [53]:

Figure 6. Ge width dependence in the range of 0.4 to 1.2 μm ($d_{\text{poly-Si}} = 0.7 \mu m$, $d_{\text{Ge}} = 0.2 \mu m$). (a) The internal loss, $\eta_{i}$, differential efficiency, $\eta_{d}$, and the extraction efficiency, $\eta_{ext}$. (b) Confinement factor, $\Gamma$, and internal efficiency, $\eta_{i}$. (c) Threshold current, $I_{th}$, and volume of the Ge active region, $V$. (d) The differential gain, $d_{\text{gain}}$, and 3 dB bandwidth at the biased current of 1002 A m⁻¹ or 270.5 mA. This biased current value was chosen to be larger than the threshold current of all geometry changing range, and not higher than 10 times $I_{th}$ to guarantee the lasers working properly.
where \( N_t \) is the trap density, \( \sigma_{n,p} \) is the electron and hole capture cross sections of deep traps, \( v_{th,n,p} \) is the thermal velocity of electrons and holes. Ge-on-Si layers with higher quality and longer minority lifetime can be realized by growing Ge on a GOI substrate or directly wafer bonding \([58]\) and chemical mechanical polishing (CMP) \([59]\). Researchers have reported minority lifetime of 3.12 ns and 5.3 ns of Ge layers using these strategies \([58, 59]\). Thin film delamination from bulk Ge wafers, like smart cut technology in Si, may also be able to provide higher quality Ge thin films.

The effect of minority lifetime and possible defect density range have been investigated based on the optimized geometrical structure and only changing \( N_t \) listed in table 2. From these results, we can tell that by enhancing the Ge materials quality, the laser performance can be improved. With longer minority carrier lifetime, the carriers can stay longer and recombine slower in the cavity, which means the injection carrier is less needed for the lasers and therefore the threshold current decreases. The threshold current can be reduced by 6.7 times by increasing the minority lifetime from 1 ns to 10 ns, and 15 times when increasing to 100 ns. The differential efficiency and differential gain show no variation with the change of minority lifetime, because this only influences the SRH recombination rate, \( R_{SRH} \), which affects the threshold current but does not change the internal loss or optical confinement factor. The modulation bandwidth depends on the square root of the

### Table 2. The laser performance with different minority lifetime.

| Minority lifetime (typical dislocation density) | Threshold Current (mA) | Differential efficiency (%) | Differential Gain (m²) | 3 dB bandwidth (GHz) |
|-----------------------------------------------|------------------------|-----------------------------|------------------------|----------------------|
| 1 ns \( (1 \times 10^7 \text{ cm}^{-2})^3 \) | 46.42                  | 17.6                        | \( 1.09 \times 10^{-20} \) | 33.94                |
| 10 ns \( (1 \times 10^5 \text{ cm}^{-2})^3 \) | 6.85                   | 17.6                        | \( 1.09 \times 10^{-20} \) | 36.89                |
| 100 ns \( (1 \times 10^5 \text{ cm}^{-2})^3 \) | 2.92                   | 17.6                        | \( 1.09 \times 10^{-20} \) | 37.01                |

\[ \frac{1}{\tau_{n,p}} = c_{n,p} N_t \]  \hspace{2cm} (16)

\[ c_{n,p} = \sigma_{n,p} v_{n,p} \]  \hspace{2cm} (17)
relative value of biased current to the threshold current value. Under this condition, the 3 dB bandwidth increases slightly with the improvement of minority lifetime.

After finding out the optimal point of 3 dB bandwidth, the ability of digital transmission of the optimized laser structure was demonstrated by using eye diagrams, which are useful in visualizing intersymbol interference between data bits and diagnosing communication link problems. Therefore, based on the above optimized laser structure, we also predicted the digital modulation property of our optimized laser device (\(d_{\text{poly-Si}} = 0.7 \ \mu\text{m}, W_{\text{Ge}} = 0.6 \ \mu\text{m}, d_{\text{Ge}} = 0.3 \ \mu\text{m}, \tau_{n,p} = 1 \ \text{ns}\)), shown in figure 8. The transmission bit rates varied from 10 to 40 Gb·s\(^{-1}\), which is in a back-to-back (BTB) configuration with an extinction ratio of 3.44 dB at a biased current of 270.5 mA. With a 10 and 20 Gb·s\(^{-1}\) non-return-to-zero (NRZ) signal, a clear eye opening of the optical signal is obtained and the amount of noise is small as seen in figures 8(a)–(b).

However, when the bit rate is raised up to 32 and 40 Gb·s\(^{-1}\), eye-opening window narrows a lot and becomes distorted, in which the overshoot of eye pattern reaches a higher level and the larger width of the eye corners indicates higher distortion of zero crossings and then more severe jitter effect, as shown in figures 8(c)–(d). Hence, according to the simulated eye diagrams for the optimized structure, the capability of digital transmission goes down with the increase of the bit rates. For our optimized laser structure, it was estimated that signals can be transmitted well at 20 Gb·s\(^{-1}\) in a BTB system with an extinction ratio of 3.44 dB at a biased current of 270.5 mA.

5. Conclusion

In this work, LASTIP\textsuperscript{TM} was used to model and simulate the small signal modulation responses of the Fabry–Perot Ge-on-Si laser diodes. The geometrical parameters, such as poly-Si cladding thickness, Ge cavity width and thickness were studied and optimized for better 3 dB bandwidth. A threshold current of 46.42 mA and a 3 dB bandwidth of 33.94 GHz at a biased current of 270.5 mA were predicted with an optimized laser structure, where \(d_{\text{poly-Si}} = 0.7 \ \mu\text{m}, W_{\text{Ge}} = 0.6 \ \mu\text{m} \) and \(d_{\text{Ge}} = 0.3 \ \mu\text{m}\) with 1 ns minority carrier lifetime. The eye diagrams simulated show a stable eye-opening window at 20 Gb·s\(^{-1}\) NRZ. The improvement to 10 ns minority carrier lifetime would reduce the threshold current to 6.85 mA and increase the 3 dB bandwidth to 36.89 GHz.
Better Ge material quality, strain, doping refinement, laser structure and cladding material optimization are all important methods to reduce the threshold current, enhance the wall-plug efficiency and increase the 3 dB bandwidth. Our work paved the way for further improvement of Ge lasers and shed light on the silicon integrated optoelectronic devices.

Acknowledgments

Dr Rodolfo Camacho-Aguilera at Luminous Computing is acknowledged for helpful discussions and proofreading. Southern University of Science and Technology (Shenzhen, China) is acknowledged for funding this work.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

ORCID iDs

Guangrui Xia @https://orcid.org/0000-0002-5290-4963

References

[1] Zhou Z, Yin B and Michel J 2015 On-chip silicon light sources for silicon photonics Light: Science & Applications 4 e358
[2] Doerr C R 2015 Silicon photonic integration in telecommunications Front. Phys. 3 37
[3] Wang J and Long Y 2018 On-chip silicon photonic signaling and processing: a review Sci. Bull. 63 1267–310
[4] Dong P, Chen Y-K, Duan G-H and Neilson D T 2014 Silicon photonic devices and integrated circuits Nanophotonics-Berlin 3 215–28
[5] Liu Y, Wang S, Wang J, Li X, Yu M and Cai Y 2022 Silicon photonic transceivers in the field of optical communication Nano Communication Networks 31 100379
[6] Fang A W, Park H, Cohen O, Jones R, Paniccia M J and Bowers J E 2006 Electrically pumped hybrid AlGaInAs-silicon evanescent laser Opt. Express 14 9203–10
[7] Liu J 2014 Monolithically integrated Ge-on-Si active photonics Photonics 1 162–97
[8] Chen S M, Tang M C, Wu J, Jiang Q, Dorogan V G, Benamara M, Mazur Y L, Salamo G J, Seeds A J and Liu H 2014 1.3 μm InAs/ GaAs quantum-dot laser monolithically grown on Si substrates operating over 100 °C Electron. Lett. 50 1467–8
[9] Szegel R et al 2019 Hybrid III–V Silicon technology for laser integration on a 200-mm fully CMOS-compatible silicon photonics platform IEEE J. Sel. Top. Quantum Electron. 25 1–10
[10] Yerci S, Li R and Dal Negro L 2010 Electroluminescence from Er-doped Si-rich silicon nitride light emitting diodes Appl. Phys. Lett. 97 081109
[11] Kataria H, Metaferia W, Junesand C, Chong Z, Julian N, Bowers J E and Lourdoudoss S 2014 Simple epitaxial lateral overgrowth process as a strategy for photonic integration on silicon IEEE J. Sel. Top. Quantum Electron. 20 350–6
[12] Groenert M E, Leitz C W, Pitera A J, Yang V, Lee H, Ram R J and Fitzgerald E A 2003 Monolithic integration of room-temperature cw GaAs/AlGaAs lasers on Si substrates via relaxed graded GeSi buffer layers J. Appl. Phys. 93 362–7
[13] Sun X, Zadok A, Shearn M J, Diest K A, Ghaffari A, Atwater H A, Scherer A and Yariv A 2009 Electrically pumped hybrid evanescent Si/InGaAs lasers Opt. Lett. 34 1145–7
[14] Keyvaninia S, Muneeb M, Stankovic S, Veldhoven P J V, Thourhout D V and Roelkens G 2013 Ultra-thin DVS-BCB adhesive bonding of III–V wafers, dies and multiple dies to a patterned silicon-on-insulator substrate Opat. Mater. Express 3 35–46
[15] Liu A Y, Zhang C, Norman J, Snyder A, Lubyhev D, Fastenau J M, Liu A W K, Gossard A C and Bowers J E 2014 High performance continuous wave 1.3 μm quantum dot lasers on silicon Appl. Phys. Lett. 104 820107
[16] Chen S et al 2014 Electrically pumped continuous-wave III–V quantum dot lasers on silicon Nat. Photonics 10 307–11
[17] Lee A, Jiang Q, Tang M, Seeds A and Liu H 2012 Continuous-wave InAs/GaAs quantum-dot laser diodes monolithically grown on Si substrate with low threshold current densities Opt. Lett. 34 22181–7
[18] Liu J, Sun X, Pan D, Wang X, Kimerling L C, Koch T L and Michel J 2007 Tensile-strained, n-type Ge as a gain medium for monolithic laser integration on Si Opt. Express 15 11272–7
[19] Sun X, Liu J, Kimerling L C and Michel J 2009 Room-temperature direct bandgap electroluminescence from Ge-on-Si light-emitting diodes Opt. Lett. 34 1198–200
[20] Liu J, Sun X, Camacho-Aguilera R, Kimerling L C and Michel J 2010 Ge-on-Si laser operating at room temperature Opt. Lett. 35 679–81
[21] Xiao Chen S, Lifeng L, Kimerling L C and Michel J 2010 Toward a germanium laser for integrated silicon photonics IEEE J. Sel. Top. Quantum Electron. 16 124–31
[22] Sanchez-Perez J R, Boztug C, Chen F, Sudradjat F F, Paskiewicz D M, Jacobson R B, Lagally M G and Paiella R 2011 Direct-bandgap light-emitting germanium in tensely strained nanomembranes Proc. Natl. Acad. Sci. U.S.A. 108 18893–8
[23] Camacho-Aguilera R E, Cai Y, Patel N, Bessette J T, Romagnoli M, Kimerling L C and Michel J 2012 An electronically pumped Ge laser Opt. Express 20 11316–20
[24] Yan C, Zhaohong H, Xiaoxin W, Camacho-Aguilera R E, Kimerling L C, Michel J and Lifeng L 2013 Analysis of threshold current behavior for bulk and quantum-well germanium laser structures IEEE J. Sel. Top. Quantum Electron. 19 1901009
[25] Fei ET et al 2013 Investigation of germanium-quantum-well light sources Opt. Express 23 22424–30
[26] Mashanovich G Z et al 2017 Germanium Mid-Infrared photonic devices J. Lightwave Technol. 35 624–30
[27] Reboud V et al 2017 Germanium based photonic components toward a full silicon/germanium photonics platform Prog. Cryst. Growth Charact. Mater. 63 1–24
[28] Armand Pillon F T et al 2019 Lasing in strained germanium microbridges *Nat. Commun.* **10** 2724

[29] Lin Y, Lee K H, Bao S, Guo X, Wang H, Michel J and Tan C S 2017 High-efficiency normal-incidence vertical p-i-n photodetectors on a germanium-on-insulator platform *Photonics Research* **5** 702–9

[30] Madelung O 1991 *Semiconductors Group IV Elements and III–V Compounds* (Verlag Berlin Heidelberg: Springer)

[31] Koerner R, Oehme M, Gollhofer M, Schmid M, Kostekci K, Bechler S, Widmann D, Kasper E and Schulze J 2015 Electrically pumped lasing from Ge Fabry–Perot resonators on Si *Opt. Express* **23** 14815–22

[32] Camacho-Aguilera R E 2013 *Ge-on-Si LASER for silicon photonics* Materials Science & Engineering Georgia Institute of Technology, Massachusetts Institute of Technology

[33] Jiang J and Sun J 2016 Theoretical analysis of optical gain in uniaxial tensile strained and n“+-doped Ge/GeSi quantum well *Opt. Express* **24** 14523–37

[34] Ke J, Chrostowski I. and Xia G 2017 Stress engineering with silicon nitride stressors for Ge-on-Si lasers *IEEE Photonics J.* **9** 1–15

[35] Chretien J et al 2019 GeSn lasers covering a wide wavelength range thanks to uniaxial tensile strain *Ac Photonics* **6** 2462–9

[36] Bao S et al 2017 Low-threshold optically pumped lasing in highly strained germanium nanowires *Nat. Commun.* **8** 1845

[37] Sukhdeo D S, Nam D, Kang J H, Brongersma M L and Saraswat K C 2014 Direct bandgap germanium-on-silicon inferred from 57% (100) uniaxial tensile strain [Invited] *Photonics Research* **2** A8

[38] Tani K, Okamura T, Oda K, Deura M and Ide T 2021 On-chip optical interconnection using integrated germanium light emitters and photodetectors *Opt. Express* **29** 28021–36

[39] Qin S, Sun J, Jiang Z, Zhang Y, Cheng M, Yu L, Wang K, Kai L, Shi H and Huang Q 2021 Monolithic integrated emitting-detecting configuration based on strained Ge microbridge *Nanophotonics-Berlin* **10** 2847–57

[40] Morton P A, Logan R A, Tanbun-Ek T, Sciortino P F, Sergent A M, Montgomery R K and Lee B T 1992 25 GHz bandwidth 1.55 μm GaInAsP p+-doped strained multiquantum-well lasers *Electron. Lett.* **28** 2156–7

[41] Kobayashi W, Ito T, Yamanaka T, Fujisawa T, Shibata Y, Kurosaki T, Kohtoku M, Tadokoro T and Sanjoh H 2013 50-Gb/s direct modulation of a 1.3-μm InGaAlAs-Based DBR laser with a ridge waveguide structure *IEEE J. Sel. Top. Quantum Electron.* **19** 1500908

[42] Golovnyshky S, Datsenko O I, Seravalli L, Trevisi G, Frigeri P, Babichuk I S, Golovynska I and Qu J 2018 Interband photocconductivity of metamorphic InAs/InGaAs quantum dots in the 1.3-1.55–μm window *Nanoscale Res. Lett.* **13** 103

[43] Yamaoka S et al 2020 Directly modulated membrane lasers with 108 GHz bandwidth on a high-thermal-conductivity silicon carbide substrate *Nat. Photonics* **15** 28–35

[44] Li X, Li Z, Li S, Chrostowski I. and Xia G 2016 Design considerations of biaxially tensile-strained germanium-on-silicon lasers *Semicond. Sci. Technol.* **31** 065015

[45] Camacho-Aguilera R, Han Z, Cai Y, Kimerling L C and Michel J 2013 Direct band gap narrowing in highly doped Ge *Appl. Phys. Lett.* **102** 152106

[46] Sue SM and Ng K K 2007 *Physics of Semiconductor Devices* (New York: Wiley)

[47] Newman R and Tyler W W 1957 Effect of impurities on free–hole infrared absorption in p-type germanium *Phys. Rev.* **105** 885–6

[48] Schroder D K, Thomas R N and Swartz J C 1978 Free carrier absorption in silicon *IEEE J. Solid-State Circuits* **13** 180–7

[49] Ogata O 2010 Free–carrier effects in polycrystalline silicon-on-insulator photonic devices *Microelectronic Engineering, Rochester Institute of Technology.

[50] Coldren L A, Corzine S W and Masanovic M L 2012 *Diode lasers and photonic integrated circuits* 2nd Edn, ed (Hoboken, New Jersey: John Wiley & Sons, Inc.)

[51] Shen C C et al 2019 Design, modeling, and fabrication of high-speed VCSEL with data rate up to 50 Gb/s *Nanoscale Res. Lett.* **14** 276

[52] Chuang S L 2009 *Physics of Photonic Devices* 2nd ed. (New Jersey: Wiley)

[53] Crosslight LASTIP. Available https://crosslight.com/products/lastip/ 2020

[54] Huang J and Casperson L W 1993 Gain and saturation in semiconductor lasers *Appl. Phys. Lett.* **63** 2847–50

[55] Loh T H, Nguyen H S, Murthy R, Yu M B, Loh W Y, Lo G Q, Balasubramanian N, Kwong D L, Wang J and Lee S J 2007 Selective epitaxial germanium on silicon-on-insulator high speed photodetectors using low-temperature ultrathin Si 0.8 Ge 0.2 buffer *Appl. Phys. Lett.* **91** 073503

[56] Cai Y 2014 Materials science and design for germanium monolithic light source on silicon *Department of Materials Science and Engineering, Massachusetts Institute of Technology.

[57] Geiger R, Frigerio J, Stuess M J, Chrastina D, Isella G, Spolenak R, Faist J and Sigg H 2014 Excess carrier lifetimes in Ge layers on Si *IEEE J. Solid-State Circuits* **49** 14815–22

[58] Nam D, Kang J H, Brongersma M L and Saraswat K C 2014 Observation of improved minority carrier lifetimes in high-quality Ge-on-insulator using time-resolved photoluminescence *Opt. Lett.* **39** 6205–8