Hadronic decays of $B_c$ mesons with flavor $SU(3)_F$ symmetry

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Abstract

We study implications of a recent observation of non-leptonic $B_c^+ \rightarrow D^0K^+$ decay and a bound on $B_c^+ \rightarrow D^0\pi^+$ transition on CP-violating asymmetries in $B_c$ decays. In the U-spin symmetry limit, we derive a relation between the CP-asymmetries in the $B_c^+ \rightarrow D^0K^+$ and $B_c^+ \rightarrow D^0\pi^+$ channels and the corresponding branching ratios. We also derive several relations between non-leptonic $B_c$ decays into the final states with $D$ mesons in the flavor $SU(3)_F$ limit. We point out that a combined study of $SU(3)_F$ amplitudes in these decays can be used to constrain the angle $\gamma$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

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I. INTRODUCTION

The $B_c$ meson contains a $\bar{b}$ and a $c$ quark, making it a open-flavored meson with two heavy quarks. Just like its heavy-light cousins $B_{d,s}^0$ and $B^{\pm}$, it decays via weak interactions in the Standard Model (SM). However, unlike those states, decays of the $B_c^+$ meson involve weak decays of either heavy quark ($\bar{b}$ or $c$). Moreover, since tree-level decays of the charm quark involve transitions between first and second generation quarks, the CKM matrix elements that come into play ($V_{cs}$ or $V_{cd}$) are large. In contrast, tree-level decays of the bottom quark involve transitions from the third generation to the second (or the first) generation and the associated CKM matrix elements ($V_{ub}$ or $V_{cb}$) are suppressed by one (or more) powers of the Wolfenstein parameter ($\lambda \sim 0.2$) [1]. Therefore, tree-level weak decays of the $B_c^+$ meson are dominated by the $c \to s$ transition. However, unlike in decays of the charm quark where penguin amplitudes are suppressed by the small Wilson coefficients of the corresponding operators, in bottom quark decays, the Wilson coefficients of the penguin operators are quite large. These facts make $B_c$ mesons interesting laboratories for simultaneous studies of $b$ and $c$ quark decays.

Recently, the LHCb collaboration observed clear evidence for a $B_c$ decay that proceeds through the decay of the $b$ quark. Using data with integrated luminosity of 3.0 $fb^{-1}$ and center-of-mass energies of 7 and 8 TeV, the LHCb collaboration observed the decay $B_c^+ \to D^0 K^+$ at 5.1 $\sigma$ significance [2]. The same search also found no evidence for the U-spin related decay $B_c^+ \to D^0 \pi^+$. This is a rather interesting result, as the color-allowed tree-level amplitude in $B_c^+ \to D^0 \pi^+$ is larger than that for $B_c^+ \to D^0 K^+$ by a factor of $|V_{ud}/V_{us}| \sim 5$. These observations prompted the collaboration to conclude that the decays are dominated by weak-annihilation and penguin diagrams rather than the color-favored tree diagrams.

The LHCb observation of the hadronic $B_c^+$ decay has opened a gateway to further studies of $B_c^+$ decays, as future measurements in other decay channels can be expected [3]. In this letter we study various implications of the LHCb observation of $B_c$ mesons via the decays of a $b$ quark, particularly for the observation of CP-violating asymmetries in $B_c$ decays. Using a U-spin symmetry relation between the decay amplitudes of $B_c^+ \to D^0 \pi^+$ and $B_c^+ \to D^0 K^+$ we derive a model-independent relation between CP-violating asymmetries in these channels. We then generalize our considerations to discuss relationships between
different $B_c^+$ nonleptonic decays under flavor $SU(3)_F$ symmetry. Compared to similar studies in heavy-light mesons $B_q$, flavor $SU(3)_F$ relations are simpler for $B_c$ mesons, owing to the fact that $B_c$ state is an $SU(3)_F$ singlet. Our studies complement earlier predictions for branching ratios and CP asymmetries for various $B_c^+$ decays using perturbative QCD [4] and other techniques [5–7].

This letter is organized as follows. In Section II we show that U-spin symmetry leads to a convenient relationship between branching ratios and CP asymmetries in several $B_c^+$ meson decays. Section III includes a discussion of more general relationships between decays from a flavor-$SU(3)_F$ symmetry perspective. We conclude in Section IV.

II. U-SPIN SYMMETRY RELATIONS IN $B_c^+$ DECAYS

Decays of the $B_c^+$ meson with $\Delta b = 1$ and $\Delta s = 0(1)$ proceed through the quark-level transitions $\bar{b} \rightarrow \bar{q}d(\bar{s})q$, where $q = u, c$ at tree-level. Additionally, $B_c^+$ decays are mediated by transition operators that represent gluonic-penguin operators $(\bar{b} \rightarrow \bar{d}(\bar{s})\sum_q q\bar{q},$ with $q = u, d, s, c)$ and electro-weak penguin operators $(\bar{b} \rightarrow \bar{d}(\bar{s})\sum_q e_q q\bar{q},$ with $q = u, d, s, c)$. All these operators generate decay amplitudes that are symmetric under the interchange of $d$ and $s$ quarks (or U-spin symmetry) due to the fact that $u, c, b,$ and $(\bar{d}d + \bar{s}s)$ are all singlets under U-spin. Thus, there are $\Delta s = 0(1)$ $B_c^+$ decay pairs that are related by U-spin symmetry, implying amplitude-level relationships between pairs of decays obtained through the exchange $s \leftrightarrow d$, such as $B_c^+ \rightarrow D^0\pi^+(K^+)$ and $B_c^+ \rightarrow D^+K^0(D_s^+\bar{K}^0)$. It is interesting that those U-spin relations between the observed branching ratios also imply relations between CP-violating asymmetries on those channels.

Let us define CP-asymmetries in the conventional way,

$$A_{CP}(B_c \rightarrow f) = \frac{\Gamma(B_c^+ \rightarrow f) - \Gamma(B_c^- \rightarrow \bar{f})}{\Gamma(B_c^+ \rightarrow f) + \Gamma(B_c^- \rightarrow f)},$$

(1)

where $\Gamma(B_c \rightarrow f)$ is a partial width for $B_c$ transition to the final state $f$. Note that experimentally reported branching ratios $\mathcal{B}(B_c \rightarrow f)$ are usually averaged over the CP-conjugated states,

$$\mathcal{B}(B_c \rightarrow f) = \frac{1}{2\Gamma}[\Gamma(B_c^+ \rightarrow f) + \Gamma(B_c^- \rightarrow \bar{f})],$$

(2)

where $1/\Gamma = \tau = (0.507 \pm 0.009)$ ps is a total lifetime of a $B_c$ state [3].
The decays $B_c^+ \rightarrow D^0\pi^+$ and $B_c^+ \rightarrow D^0K^+$ are related by the $U$-spin symmetry. Indeed, we can write the transitions amplitudes for those decays

$$A(B_c^+ \rightarrow D^0\pi^+) = V_{cb}^* V_{cd} A_d^c + V_{ub}^* V_{ud} A_d^u,$$

$$\overline{A}(B_c^- \rightarrow D^0\pi^-) = V_{cb} V_{cd}^* A_d^c + V_{ub} V_{ud}^* A_d^u,$$

$$A(B_c^+ \rightarrow D^0K^+) = V_{cb}^* V_{cs} A_s^c + V_{ub} V_{us} A_s^u,$$

$$\overline{A}(B_c^- \rightarrow D^0K^-) = V_{cb} V_{cs}^* A_s^c + V_{ub} V_{us}^* A_s^u,$$

where CP-conjugate amplitudes were obtained by changing the sign of the CP-violating phases in the Cabibbo-Kobayashi-Maskawa matrix elements. Note that we already used unitarity of the CKM matrix to eliminate top-quark-related combination $V_{td}^* V_{tq}$ for $q = s, d$. This implies that hadronic matrix elements contain the corresponding penguin contributions. One can then construct the following differences in squared amplitudes,

$$|A(B_c^+ \rightarrow D^0\pi^+)|^2 - |\overline{A}(B_c^- \rightarrow D^0\pi^-)|^2 = 4 \text{ Im}[V_{cb}^* V_{cd} V_{ub} V_{ud}] \text{ Im}[A_d^c A_d^u],$$

$$|A(B_c^+ \rightarrow D^0K^+)|^2 - |\overline{A}(B_c^- \rightarrow D^0K^-)|^2 = 4 \text{ Im}[V_{cb}^* V_{cs} V_{ub} V_{us}] \text{ Im}[A_s^c A_s^u].$$

U-spin implies relationships between the amplitudes: $A_d^{c,u} = A_s^{c,u}$. Further, unitarity of the CKM matrix leads to the following relationship

$$\text{Im}[V_{cb}^* V_{cd} V_{ub} V_{ud}] = -\text{Im}[V_{cb}^* V_{cs} V_{ub} V_{us}].$$

Using the above relationships we can show that

$$|A(B_c^+ \rightarrow D^0\pi^+)|^2 - |\overline{A}(B_c^- \rightarrow D^0\pi^-)|^2 = - |A(B_c^+ \rightarrow D^0K^+)|^2 + |\overline{A}(B_c^- \rightarrow D^0K^-)|^2.$$

Now converting the amplitudes to partial widths and using Eqs. (1) and (2) we obtain

$$\frac{A_{\text{CP}}(B_c^+ \rightarrow D^0\pi^+)}{A_{\text{CP}}(B_c^+ \rightarrow D^0K^+)} = - \frac{p_{\pi^+}^* B(B_c^+ \rightarrow D^0\pi^+)}{p_{K^+}^* B(B_c^+ \rightarrow D^0K^+)},$$

where $p_{M}^*$ represents the magnitude of the three-momentum of the daughter particles in the rest frame of the decaying $B_c^+$ meson for the decay process $B_c^+ \rightarrow D^0M$. Note that while we have established this relation for the U-spin related pair of decays $B_c^+ \rightarrow D^0\pi^+(K^+)$, the same relation also exists for other U-spin related pairs such as $B_c^+ \rightarrow D^+K^0(D_s^0\overline{K}),$

The LHCb collaboration recently observed the process $B_c^+ \rightarrow D^0\pi^+$ with a significance of 5.1 standard deviations [2]. However, since the absolute production rate for $B_c^+$ at LHCb
is unknown, the measured observable includes a normalizing factor $f_c/f_u$ that compares the production rate of $B_c^+$ to that of $B_u^+$ using the decay constants of the two mesons,

$$R_{D^0 K} = \frac{f_c}{f_u} \times B(B_c^+ \rightarrow D^0 K^+) = \left(9.3^{+2.8}_{-2.5} \pm 0.6\right) \times 10^{-7}.$$  \hspace{1cm} (8)

At the same time LHCb did not observe any events in the $D^0\pi^+$ channel thereby putting an upper bounds on the corresponding branching ratio,

$$R_{D^0\pi} = \frac{f_c}{f_u} \times B(B_c^+ \rightarrow D^0\pi^+) < 3.9 \times 10^{-7} \text{ at 95 \% c.l.}.$$ \hspace{1cm} (9)

The above limit is a direct consequence of no $B_c^+ \rightarrow D^0\pi^+$ signal events being seen at the LHCb. Combining the above limits and using $p_{\pi^+}^* / p_{K^+}^* = 1.008$ we find the following bound on the ratio of CP-violating asymmetries,

$$\left| \frac{A_{CP}(B_c^+ \rightarrow D^0\pi^+)}{A_{CP}(B_c^+ \rightarrow D^0K^+)} \right| \gtrsim 2.4,$$ \hspace{1cm} (10)

which is independent of the unknown ratios of the production rates $f_c/f_u$. Note that the signs of CP-violating asymmetries for $B_c^+ \rightarrow D^0\pi^+$ and $B_c^+ \rightarrow D^+K^0$ are opposite of each other. Note also that earlier predictions of CP-violating asymmetries \cite{4,5} explicitly violate our model-independent bound Eq. (10).

### III. FLAVOR SU(3)$_F$ ANALYSIS OF B$_c^+$ DECAYS

The U-spin relations derived in the previous section can be generalized by using full flavor $SU(3)_F$. Here we shall concentrate on the relations among different two-body decays of the $B_c$ meson to the final state containing an open-flavor heavy and a light pseudoscalar meson $M = \pi, \eta, K$, such as $B_c \rightarrow DM$ and $B_c \rightarrow BM$.

We use standard $SU(3)_F$ representations of the initial and final state mesons. The initial $B_c$ meson transforms as a singlet under $SU(3)_F$, while $B_q$ and $D$ mesons containing light quarks ($u, d, s$) form triplets and can be represented as row vectors,

$$B_i = (B^-, B^0_d, B^0_s), \quad D_i = (D^0, D^+, D_s).$$ \hspace{1cm} (11)

The octet of pseudoscalar mesons $M$ formed by the light quarks ($u, d, s$) can be represented by a $3 \times 3$ matrix $M^j_i$ where the upper index represents the rows (quarks) and the lower
index represents the columns (antiquarks). In this notation, the matrix $M_{ij}$ can be expressed as,

$$M = \begin{pmatrix}
\pi^0 + \frac{\eta_8}{\sqrt{2}} & \pi^+ & K^+ \\
\pi^- & -\pi^0 + \frac{\eta_8}{\sqrt{2}} & K^0 \\
K^- & K^0 & -\sqrt{\frac{2}{3}}\eta_8
\end{pmatrix}.$$  \hspace{1cm} (12)

For simplicity we ignore $\eta - \eta'$ mixing and assume $\eta = \eta_8$ from now on. In order to write $SU(3)_F$ relations for the $B_c$ decay matrix elements we need to specify representations of the effective Hamiltonians.

I. $B_c$ decays via $b$-quark decay

The low energy effective Hamiltonian governing $\Delta b = 1, \Delta c = 0$ weak decays of the $\bar{b}$ quark are well known \[8, 9\],

$$\mathcal{H} = \frac{4G_F}{\sqrt{2}} \sum_{q=d,s} [V_{ub}^* V_{uq} [C_1 (\bar{b}_L \gamma^\mu u_L) (\bar{q}_L \gamma_\mu q_L) + C_2 (\bar{b}_L \gamma^\mu u_L) (\bar{q}_L \gamma_\mu u_L)] - V_{tb}^* V_{tq} \sum_{i=3}^{10} C_i Q_i^{(q)}].$$ \hspace{1cm} (13)

The terms corresponding to the coefficients $C_{1,2}$ are generally referred to as the “tree” part, while the rest of the Hamiltonian involving the coefficients $C_i (i = 3 - 10)$ are collectively known as the “penguin” part. There are four “gluonic” penguin operators ($i = 3 - 6$) and four “electro-weak” penguin operators ($i = 7 - 10$). The precise form of these penguin operators is \[8\],

$$O_3^{(q)} = \sum_{q''=u,d,s} (\bar{b}_L \gamma^\mu q_L) (\bar{q''}_L \gamma_\mu q''_L), \quad O_4^{(q)} = \sum_{q''=u,d,s} (\bar{b}_L \gamma^\mu q''_L) (\bar{q}_L \gamma_\mu q_L),$$

$$O_5^{(q)} = \sum_{q''=u,d,s} (\bar{b}_L \gamma^\mu q_L) (\bar{q''}_R \gamma_\mu q''_R), \quad O_6^{(q)} = \sum_{q''=u,d,s} (\bar{b}_L \gamma^\mu q''_L) (\bar{q}_R \gamma_\mu q_R),$$

$$O_7^{(q)} = \frac{3}{2} \sum_{q''=u,d,s} (\bar{b}_L \gamma^\mu q_L) e_{q''} (\bar{q''}_R \gamma_\mu q''_R), \quad O_8^{(q)} = \frac{3}{2} \sum_{q''=u,d,s} (\bar{b}_L \gamma^\mu q''_L) e_{q'} (\bar{q}_R \gamma_\mu q_R),$$

$$O_9^{(q)} = \frac{3}{2} \sum_{q''=u,d,s} (\bar{b}_L \gamma^\mu q_L) e_{q'} (\bar{q''}_L \gamma_\mu q''_L), \quad O_{10}^{(q)} = \frac{3}{2} \sum_{q''=u,d,s} (\bar{b}_L \gamma^\mu q''_L) e_{q''} (\bar{q}_L \gamma_\mu q_L).$$ \hspace{1cm} (14)

In the above operators $q_{L(R)}$ represents left-handed (right-handed) quark fields and $e_q$ represents the electric charge of the quark $q$. 
Written in the form of Eq. (14), the operators $O_i$ mix under flavor $SU(3)_F$ transformations. A standard approach\(^1\) (pioneered in [10] for kaon decays) is to decompose the operators according to different representations of $SU(3)_F$ [11].

The light quarks ($u, d, s$) transform as a triplet under flavor $SU(3)_F$. The tree part of the Hamiltonian proportional to $V_{ub}^* V_{uj}$ is made up of four-quark operators of the form $(bq_1)(q_2q_3)$ where $q_i$ represents a light quark. These operators transform as a $3 \times 3 \times 3 \equiv 15 + 3 + 3$ of $SU(3)_F$.

The part of the Hamiltonian proportional to $V_{cb}^* V_{cq}$ gets contributions from both trees and penguins. Ignoring contributions from the electroweak penguin operators $O^{(q)}_{7-10}$, this part transforms as a triplet of $SU(3)_F$.

In what follows, we use the notations introduced by Savage and Wise [11] to carry out group-theoretic calculations. Let us first consider $\Delta s = 0$ transitions. The triplet Hamiltonian with quantum numbers of $(bc)(\bar{c}d)$ can be represented as $H_i = (0,1,0)$. The Hamiltonian with the quantum numbers of $(bu)(\bar{u}d)$ is obtained by considering its $SU(3)_F$ decomposition. Here $H(\bar{3})^i \equiv (0,1,0)$ is a three-component vector, while $H(\bar{15})_k^i$ and $H(6)_k^i$ are traceless three-index tensors that are symmetric ($\bar{15}$) and antisymmetric (6) on their upper indices [11]. The non-zero elements of these tensors are [11]

$$
H(\bar{15})_{12} = H(\bar{15})_{21} = 3, \quad H(\bar{15})_{22} = -2, \quad H(\bar{15})_{31} = H(\bar{15})_{23} = -1, \\
H(6)_{12} = -H(6)_{21} = 1, \quad H(6)_{32} = -H(6)_{23} = -1.
$$

The effective Hamiltonian then takes the following form,

$$
H_{\text{eff}} = V_{cb}^* V_{cd} \, H^{\bar{c}d} + V_{ub}^* V_{ud} \, H^{\bar{u}d}, \quad \text{where}
$$

$$
H^{\bar{c}d} = \alpha B_c H^i M_i^j D_j, \\
H^{\bar{u}d} = A_{(3)} B_c H(\bar{3})^i M_i^j D_j + A_{(6)} B_c H(6)_k^i M_k^j D_j + A_{(\bar{15})} B_c H(\bar{15})_k^i M_k^j D_j.
$$

The coefficients $\alpha$, and $A_{(r)}$ respectively represent the reduced matrix elements from the corresponding group-theoretic operators. The amplitude for every $B_c^+ \to MD$ transition can then be expressed as $\langle DM | H_{\text{eff}} | B_c^+ \rangle$. We list the results for the corresponding rates in the upper part of Table [1].

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\(^1\) Another approach to decomposing decay amplitudes in terms of $SU(3)_F$ matrix elements can be found in [12].
The non-zero elements of these tensors are 

Table I: List of $B_c^+ \rightarrow MD$ decay rates for $\Delta s = 0,1$ processes

| Process                  | Rate                                                                 |
|--------------------------|-----------------------------------------------------------------------|
| $B_c^+ \rightarrow D^+\pi^0$ | $\frac{1}{2} |V_{cb}^* V_{cd} \alpha + V_{ub}^* V_{ud} (A_3 - A_6 - 5A_{\overline{3}})|^2$ |
| $B_c^+ \rightarrow D^0\pi^+$    | $|V_{cb}^* V_{cd} \alpha + V_{ub}^* V_{ud} (A_3 - A_6 + 3A_{15})|^2$       |
| $B_c^+ \rightarrow D_s^+K$     | $|V_{cb}^* V_{cd} \alpha + V_{ub}^* V_{ud} (A_3 + A_6 - A_{15})|^2$       |
| $B_c^+ \rightarrow D^+\eta$    | $\frac{1}{2} |V_{cb}^* V_{cd} \alpha + V_{ub}^* V_{ud} (A_3 + A_6 + 3A_{15})|^2$       |
| $B_c^+ \rightarrow D^+K^0$     | $|V_{cb}^* V_{cs} \alpha + V_{ub}^* V_{us} (A_3 + A_6 - A_{15})|^2$       |
| $B_c^+ \rightarrow D^0K^+$     | $|V_{cb}^* V_{cs} \alpha + V_{ub}^* V_{us} (A_3 - A_6 + 3A_{15})|^2$       |
| $B_c^+ \rightarrow D_s^+\pi^0$ | $2 |V_{ub}^* V_{us} (A_6 + 2A_{\overline{3}})|^2$                         |
| $B_c^+ \rightarrow D_s^+\eta$ | $\frac{2}{3} |V_{cb}^* V_{cs} \alpha + V_{ub}^* V_{us} (A_3 - 3A_{15})|^2$       |

The consideration of $\Delta s = 1$ transitions arising from weak-interaction Hamiltonians with the quantum numbers of $(\bar{b}c)(\bar{c}s)$ and $(\bar{b}u)(\bar{u}s)$ is quite similar. The triplet Hamiltonian with quantum numbers of $(\bar{b}c)(\bar{c}s)$ is $H_i = (0,0,1)$ [11]. The Hamiltonian with the quantum numbers of $(\bar{b}u)(\bar{u}s)$ is once again decomposed into irreducible representations of $SU(3)_F$. Here, $H(\overline{3})^i \equiv (0,0,1)$ is a three-component vector, while $H(\overline{15})^i_k$ and $H(6)^i_k$ are traceless three-index tensors that are symmetric $(\overline{15})$ and antisymmetric $(6)$ on their upper indices. The non-zero elements of these tensors are

$$H(\overline{15})_1^{13} = H(\overline{15})_1^{31} = 3, \quad H(\overline{15})_3^{33} = -2, \quad H(\overline{15})_2^{23} = H(\overline{15})_2^{32} = -1,$$

$$H(6)_1^{13} = -H(6)_1^{31} = 1, \quad H(6)_2^{23} = -H(6)_2^{32} = -1. \quad (18)$$

As before, the effective Hamiltonian takes the form,

$$H_{\text{eff}} = V_{cb}^* V_{cs} H^{\overline{3}s} + V_{ub}^* V_{us} H^{6s} , \quad \text{with}$$

$$H^{\overline{3}s} = \alpha B_c H^i M^j D_j ,$$

$$H^{6s} = A_{(\overline{3})} B_c H(\overline{3})^i_j M^j D_j + A_{(6)} B_c H(6)^{ij}_k M^k_i D_j + A_{(\overline{15})} B_c H(\overline{15})^{ij}_k M^k_i D_j .$$

We list the results for the rates of $\Delta s = 1$, $B_c^+ \rightarrow DM$ decays in the bottom part of Table

8
From the results given in Table I we can once again see the relations between the ratios of CP-violating asymmetries and the corresponding branching ratios,

\[ \frac{A_{CP}(B_c^+ \rightarrow D^0 \pi^+)}{A_{CP}(B_c^+ \rightarrow D^0 K^+)} = -\frac{p_{\pi^+}^*}{p_{K^+}^*} \frac{B(B_c^+ \rightarrow D^0 K^+)}{B(B_c^+ \rightarrow D^0 \pi^+)} , \]

\[ \frac{A_{CP}(B_c^+ \rightarrow D_s \overline{K}^0)}{A_{CP}(B_c^+ \rightarrow D^+ K^0)} = -\frac{p_{D_s}^*}{p_{D^+}^*} \frac{B(B_c^+ \rightarrow D_s \overline{K}^0)}{B(B_c^+ \rightarrow D^+ K^0)} . \] (20)

Table I lists eight decay processes, the branching ratio and direct CP asymmetry for each of which can in principle be measured. Thus there are 16 observable quantities that can provide information about these decays. However, flavor SU(3)F symmetry introduces 2 relationships between these observables, so that not all of them are independent. If we fix one overall CP-even phase, then there are a total of 7 hadronic parameters that can fully characterize the 4 complex SU(3)F matrix elements represented by \( \alpha \), and \( A_{(r)} \). Clearly, enough information will be available from the branching ratios and CP asymmetries of the 8 decay processes to determine the 7 hadronic parameters using a phenomenological fit. However, most of these decay rates and CP asymmetries have not yet been measured. Future measurements of these observables will make it possible to study the hadronic parameters in further detail.

In addition to the seven hadronic parameters mentioned above, there is one CP-odd phase (the CKM angle \( \gamma \), the phase of the CKM matrix element \( V_{ub} \)), which can be included as an unknown parameter in a fit. Such a fit can provide a way of obtaining information about the parameter \( \gamma \), independent of those commonly used to study it. This further emphasizes the need to measure the observables mentioned above.

II. \( B_c \) decays via \( c \)-quark decay

Similar SU(3)F relations can be obtained for the two-body \( B_c^+ \) decays to a \( B \) meson and a light pseudoscalar meson. Unlike the decays considered in the previous sub-section, these decays are \( \Delta b = 0, \Delta c = 1 \) transitions. The Hamiltonian governing quark-level transitions in which the \( c \) quark decays, is similar to that governing the decay of the \( b \) quark. However, in this the penguin operators are dynamically much more suppressed, because the heaviest quark that can run in the penguin loop is now a \( b \) quark. Thus, the largest contributions come simply from the tree-level operators corresponding to \( O_1 \), and \( O_2 \).
Table II: List of $B_c^+ \to MB$ decay rates for CF, SCS, and DCS processes

| Process                  | Rate                                                                 |
|--------------------------|----------------------------------------------------------------------|
| $B_c^+ \to B_s^0\pi^+$   | $|V_{cs}V_{ud}(2A_{15}^c - A_{15}^a)|^2$                           |
| $B_c^+ \to B^+ K^0$      | $|V_{cs}V_{ud}(2A_{15}^c + A_{15}^a)|^2$                           |
| $B_c^+ \to B_s^0 K^+$    | $|V_{cs}V_{us}(A_{15}^c + A_{15}^a)|^2$                            |
| $B_c^+ \to B^+ \eta$     | $\frac{1}{6} |V_{cs}V_{us}(3A_{15}^c - A_{15}^a)|^2$                    |
| $B_c^+ \to B_d^0\pi^+$   | $|V_{cd}V_{ud}(A_{15}^c + A_{15}^a)|^2$                            |
| $B_c^+ \to B^+\pi^0$     | $\frac{1}{2} |V_{cd}V_{ud}(3A_{15}^c - A_{15}^a)|^2$                    |
| $B_c^+ \to B_d^0 K^+$    | $|V_{cd}V_{us}(2A_{15}^c - A_{15}^a)|^2$                            |
| $B_c^+ \to B^+ K^0$      | $|V_{cd}V_{us}(2A_{15}^c + A_{15}^a)|^2$                            |

Tree-level decay amplitudes where the $c$ quark decays can be classified into three categories based on the CKM matrix elements that are involved. Cabibbo favored (CF) amplitudes are proportional to the combination $V_{cs}^*V_{ud}$. Singly Cabibbo suppressed (SCS) amplitudes are proportional to $V_{cd}^*V_{ud}$ or $V_{cs}^*V_{us}$. Doubly Cabibbo suppressed (DCS) amplitudes are proportional to the combination $V_{cd}^*V_{us}$. Compared to the CF amplitudes, the SCS (DCS) amplitudes are suppressed by one (two) power(s) of the Wolfenstein parameter $\lambda$. The hadronic part of the decay amplitudes can be expressed in terms of group-theoretic amplitudes $A_{(r)}^c$ where the superscript $c$ denotes that these amplitudes represent decays of the $c$ quark. The amplitudes for these processes are given in Table II.

We find that the amplitudes listed in Table II satisfy the following relations,

$$
\left| \frac{A(B_c^+ \to B_s^0\pi^+)}{V_{cs}^*V_{ud}} + \frac{A(B_c^+ \to B^+\overline{K}^0)}{V_{cs}^*V_{ud}} \right| = \frac{A(B_c^+ \to B_d^0 K^+) + A(B_c^+ \to B^+ K^0)}{V_{cs}^*V_{us}} = \frac{A(B_c^+ \to B_d^0 K^+) + \sqrt{6}A(B_c^+ \to B^+ \eta)}{V_{cs}^*V_{us}} = \left| \frac{A(B_c^+ \to B_d^0\pi^+)}{V_{cd}^*V_{ud}} + \sqrt{2}A(B_c^+ \to B^+\pi^0) \right|. \tag{21}
$$

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The LHCb collaboration has observed the decay process $B_c^+ \to B_s^0 \pi^+$ [13]. However, the other decays have not yet been measured.

IV. CONCLUSIONS

We considered implications of a recent observation of non-leptonic $B_c^+ \to D^0 K^+$ decay and a bound on $B_c^+ \to D^0 \pi^+$ transition on CP-violating asymmetries in $B_c$ decays. We derived two model-independent relations between the CP-asymmetries in the $B_c^+ \to D^0 K^+$ and $B_c^+ \to D^0 \pi^+$ and $B_c^+ \to D_s \bar{K}^0$ and $B_c^+ \to D^+ K^0$ channels and the corresponding branching ratios. We also derived several relations between non-leptonic $B_c$ decays into the final states with $D$ and $B$ mesons in the flavor $SU(3)_F$ limit.

While we concentrated on the final states with the pseudoscalar mesons, the same relations also hold for the pseudoscalar-vector final states. In particular, the results listed in Tables I and II hold exactly for the $D^* M$ and $B^* M$ final states with trivial substitutions $D(B) \to D^*(B^*)$ and $\alpha \to \beta$ and $A_{(r)}^{(c)} \to B_{(r)}^{(c)}$, where $\beta$ and $B_{(r)}^{(c)}$ are reduced matrix elements for the effective Hamiltonians describing $B_c \to MD^*(B^*)$ transitions. Only upper bounds for such decay rates are currently available [3].

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