Lane-Free Crossing of CAVs through Signal-Free Intersections as a Minimum-Time Optimal Control Problem

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Abstract—Unlike traditional cars, connected and autonomous vehicles (CAVs) can cross intersections in a lane-free and signal-free order to increase the temporal-spatial capacity of intersections. This paper presents an optimal strategy to centrally control the CAVs entering an intersection to minimise the worst crossing time of the vehicles, or equivalently maximise the throughput of the CAVs entering an intersection to minimise the worst crossing regardless of the number of CAVs.

Index Terms—throughput; path planning; connected and autonomous vehicles; signal free intersection; dual problem

I. INTRODUCTION

Traffic lights schedule the intersections inefficiently due to the limitations of human drivers, and this makes the intersections one of the major roots of congestion in urban traffic networks. Intersections also account for a large portion of all accidents (e.g. 47% in the United State in 2010 [1]). CAVs will be able to execute more complex manoeuvres than human drivers to cross the intersections faster and safer, and hence to reduce energy consumption and increase traffic throughput [2]. However, these potential improvements require addressing three different challenges: collision avoidance, finding the minimum-time optimal solution instead of only checking the feasibility, and real-time implementation.

First, previous studies proposed three approaches to ensure collision avoidance among CAVs:

i) Reserving the whole intersection for one of the CAVs at a time; The authors in [3, 4] designed an algorithm based on solving an OCP where each CAV reserves the intersection for a period of time to avoid collisions. A similar work is presented by Tallapragada et al. in [5], where CAVs are split into clusters and each cluster reserves the whole of the intersection for a period of time. The authors in [6, 7] introduced a scheduling method where CAVs are placed into a virtual lane based on their distance to the centre of the intersection and their risk of collision. Then, crossing time of the intersection is scheduled between CAVs in the virtual lane.

ii) Reserving a finite number of specific points (called conflict points) instead of the whole of the intersection; Mirheli et al. [8, 9] designed 16 conflict points for a four-leg intersection. Each leg of the intersection includes exclusive left turn and straight lanes. The proposed algorithm enforces CAVs to reserve the approaching conflict point(s) prior to their arrival. Moreover, [10] presents a more recent study of the conflict zone reservation approach, where CAVs are capable of travelling to any direction.

iii) Utilising the whole space of intersections freely, a.k.a. lane-free; Generally speaking, reservation-based collision avoidance approaches require CAVs following predefined paths and not fully exploiting the intersection area. These types of collision avoidance approaches are not efficient in terms of reducing travelling time and energy consumption. Prior studies developed lane-free algorithms based on OCP in [11, 12]. To avoid collisions, the Euclidean distance between any pair of CAVs are constrained to be greater than a safe margin. This formulation of the collision avoidance constraints is non-convex [13], and hence any optimisation problem including them are difficult to solve. Li et al. in [12] divided the non-convex problem of intersection crossing into two stages to make it tractable. At stage one, CAVs inform the central controller with their intention and then make a standard formulation which is computed online. At stage two, the controller searches an offline constructed lookup table for the intended crossing scenario and finds the control inputs of each CAV. The authors suggested to solve offline an individual optimal control problem for any possible crossing scenario to construct the lookup table of the control commands. However, the resulting offline problems are still non-convex and solving them for all possible scenarios of 24 CAVs take around 358 years [12]. Alternatively, Li et al. in [11] fixed the crossing time to a constant value and convert the minimum-time optimal control problem to a feasibility problem to solve online.

Second, the algorithms that only find a feasible (collision-free) solution to the problem of the intersection crossing do not fully exploit the CAVs’ advantages to minimise the crossing time. In other words, minimising the crossing time is not part of their objectives. The studies carried out in [11, 5] focus on the passenger comfort and addressed the challenge by minimising fluctuation of the vehicles’ acceleration. Other
TABLE I: Summary of the challenges of the intersection crossing problem and the corresponding techniques for the challenges

| Challenge                                                                 | Techniques to address                                                                 | Implemented in               |
|--------------------------------------------------------------------------|--------------------------------------------------------------------------------------|------------------------------|
| Collision avoidance                                                      | Reservation of the whole Intersection                                              | [5, 6, 3, 4, 7]              |
|                                                                           | Reservation of conflict points                                                       | [8, 10, 9]                  |
|                                                                           | Lane-free                                                                            | [12, 11]                    |
| Finding the minimum-time optimal solution instead of only checking the feasibility | Minimisation of fluctuation of the vehicles’ acceleration                          | [11, 5]                     |
|                                                                           | Minimisation of deviation from the speed limit                                       | [14, 15, 16, 17]            |
|                                                                           | Minimisation of the crossing time                                                    | [12]                        |
| Real-Time implementation                                                 | Centralised strategies with fully-observable data                                   | [12, 11, 15, 16]            |
|                                                                           | Decentralised strategies with fully connected CAVs                                   | [18, 19]                    |
|                                                                           | Decentralised strategies with partially connected CAVs                               | [20, 7]                     |

Researchers in [14, 15, 16, 17] optimised the motion of CAVs to move on the predefined paths with as close speed as possible to the speed limit of the intersection. Both of these approaches formulate the intersection crossing as a simpler problem to solve rather than a minimum-time optimal problem. However, the results show that these methods find the feasible solutions and only marginally improve the crossing time as compared to the traditional signalised intersections. The authors in [12] formulate the intersection crossing problem of CAVs as a minimum-time optimal problem to minimise the crossing time without any restrictions on the crossing paths (except the road boundaries). However, their algorithm is not time-effective for real-time applications.

Finally, it is always challenging to implement optimal control strategies in real-time. CAVs are intelligent agents communicating to each other and to the infrastructure to share the information such as location, speed, and intentions. The optimal strategies for crossing intersections, therefore, must operate on a network of cars (i.e., networked controller), considering the shared information to control multiple CAVs which are seeking conflicting objectives (like minimising their individual crossing times). A centralised topology with a fully available information of all the CAVs or different decentralised topologies, where the CAVs are fully or partially connected, can be used to calculate the optimal crossing strategy of all the CAVs. The centralised controllers receive information of all vehicles, compute trajectories, and send back the calculated trajectory of each individual CAV. There is no path planning at the CAV level and vehicles only follow the provided trajectories. Li et al. [12] proposed a centralised, but computationally expensive, optimal controller for multiple CAVs crossing a lane-free intersection. The centralised algorithms in [14, 16] split the problem into two stages, finding the crossing order and calculating the control inputs to follow the attained crossing orders, to make it computationally tractable.

In the decentralised strategies, on the other hand, each CAV computes its own trajectory by solving or approximating the solution of an optimal control problem to achieve a level of both the local and global objectives. The authors in [19, 18] formulated a decentralised OCP controller of the CAVs crossing intersections where each CAV has access to the shared information of all the others. Although decentralised algorithms find sub-optimal solutions, it is shown to be less computationally expensive as compared to the centralised counterparts [19, 18].

However, the CAVs which are crossing an intersection cannot be practically fully connected to each other at all the times. This means that at any instance of time, each CAV only communicates with a subset of the others, i.e., partially connected. Bian et al. [20] proposed a framework where the CAVs travelling on the same lane can communicate to each other, but they estimate the states of the other not-connected vehicles. Reference [7] proposes a partially connected distributed algorithm based on the concept of virtual platooning. CAVs, first, form a virtual platoon and then optimise their arriving time to the intersection to avoid collision. This is a decentralised reservation-based algorithm that allows only one CAV at a time within the intersection. Generally speaking, unlike the centralised controllers which are capable of finding the global optimum solution, decentralised controllers can only find sub-optimal strategies [21]. Table I provides a summary of challenges and the corresponding techniques.

In summary, majority of the literature propose reservation-based algorithms which calculate the feasible collision-free crossing. To the best of the authors’ knowledge, there is a limited number of literature dealing with lane-free minimum-time crossing of intersections. The lane-free minimum-time crossing is a non-convex problem and the current state-of-the-art approaches are computationally expensive. This paper addresses these gaps by the following novel contributions:

- Formulation of the lane-free crossing of CAVs through signal-free intersections as a minimum-time OCP;
- Smoothing and convexification of the constraints that avoid collisions of CAVs with each other and with road boundaries. The constraints are replaced with the dual optimisation problem of their relaxed necessary conditions;
- Minimisation of the crossing time of multiple CAVs passing through intersections. It is shown that the minimum crossing time is fixed to a constant value for a junction regardless of the number of CAVs till reaching the maximum temporal-spatial capacity of the intersection.

The remainder of this paper is organised as follows: Section II describes the system of multiple CAVs crossing an intersection and presents the notations used throughout the paper; Section III formulates the lane-free crossing of CAVs through a signal-free intersection as a minimum-time optimal control
problem; Section IV provides numerical results obtained from simulation along with discussions, and Section V concludes the outcomes.

II. SYSTEM DESCRIPTION

A. Lane-Free and Signal-Free Intersections

Fig. 1 illustrates an example layout of a lane-free and signal-free intersection. The intersection includes four approaches, each of them has a separate incoming and outgoing lane. In a lane-free intersection, vehicles can freely change their lanes in favour of faster crossing through the intersection. For instance, Fig. 1 shows three CAVs which are moving from their initial points, depicted with the most solid colour, towards their intended destinations which are with the most transparent colour. The green CAV overtakes the black CAV by using the opposite lane. It is worth noting that the intersection does not have traffic lights because the CAVs can directly communicate their states and intentions. In this study, there is a coordinator that receives all the information from the CAVs and centrally control the vehicles to efficiently and safely cross the intersection. This fully-connected and centralised topology provides the optimal solution as the benchmark for testing any decentralised strategies.

The black and green CAVs also show the collision avoidance. In this regard, the expression $-b_i^T \lambda_{ij} - b_i^T \lambda_{ji}$ is the dual representation of the distance between a pair of CAVs and $S_{ij}^\circ$ is the separating hyperplane placed between them. Similarly, the red CAV and the highlighted road boundary show the road boundary avoidance. $-b_i^T \lambda_{ir} - b_i^T \lambda_{ri}$ is the dual representation of distance between a CAV and a road boundary and $S_{ir}^\circ$ is the separating hyperplane between them. These equations are further explained in section III.

B. Vehicle Kinematics

This study represents the lateral behaviour of CAVs with the bicycle model [23]. The bicycle model consists of two degree-of-freedom (DoF) which are sideslip angle $\beta_i$ and yaw rate $r_i$, as in Fig. 2. The model also includes an additional DoF for the longitudinal velocity $V_i$. The equations of these DoFs along with other three to model the ground-fixed location, construct a set of non-linear differential equations to represent CAV as follows:

$$\begin{align*}
\frac{d}{dt} \begin{bmatrix}
  r_i \\
  \beta_i \\
  V_i \\
  y_i \\
  \theta_i \\
  x_i
\end{bmatrix}(t) &= \begin{bmatrix}
  \frac{N_f}{m} \cdot \beta_i(t) + \frac{N_r}{m} \cdot r_i(t) \\
  \left(\frac{V_i^2}{m \cdot V_i(t)} - 1\right) \cdot r_i(t) + \frac{Y_f}{m \cdot V_i(t)} \cdot \beta_i(t) \\
  V_i(t) \cdot \cos \theta_i(t) \\
  V_i(t) \cdot \sin \theta_i(t) \\
  r_i(t)
\end{bmatrix} + \begin{bmatrix}
  0 \\
  0 \\
  1 \\
  0 \\
  0 \\
  0
\end{bmatrix} \begin{bmatrix}
  a_i \\
  \delta_i
\end{bmatrix}(t), t \in [t_0, t_f],
\end{align*}$$

where $x = [r_i, \beta_i, V_i, x_i, y_i, \theta_i]^T$ and $u = [a_i, \delta_i]^T$ are, respectively, the system states and control inputs of CAV, $z_i = [x_i, y_i, \theta_i]^T$ refers to the pose of CAV in non-inertial reference system, $a_i(t)$ and $\delta_i(t)$ are, respectively, the acceleration (in $m^2$) and steering angle (in rad) of the vehicle. The constants $m$ and $I_z$ denote mass (kg) and moment of inertia (kg.m$^2$) of the vehicle. $t_0$ and $t_f$ (s) represent the starting and final time of crossing the intersection. The vehicle parameters $N_r$, $N_\beta$, $N_\delta$, $Y_r$, $Y_\beta$ and $Y_\delta$ are calculated as follows [23]:

$$\begin{align*}
N_r &= \frac{1}{V_i(t)} \cdot (l_f^2 \cdot C_F + l_r^2 \cdot C_R), \\
N_\beta &= -l_f \cdot C_F, \\
N_\delta &= -l_r \cdot C_R, \\
Y_r &= \frac{1}{V_i(t)} \cdot (l_f \cdot C_F - l_r \cdot C_R), \\
Y_\beta &= C_F + C_R, \\
Y_\delta &= -C_F.
\end{align*}$$

where $C_F$ and $C_R$ are, respectively, the cornering stiffness of the front and rear tyres. $l_f$ and $l_r$ are distance of the front and rear axis from center of gravity of the vehicle.

To ensure CAVs drive within their dynamic limitations, the following constraints are enforced for each CAV:

$$\begin{align*}
V &\leq V_i(t) \leq \bar{V}, \\
\bar{a} &\leq a_i(t) \leq \underline{a}, \\
\bar{\beta} &\leq |\beta_i(t)| \leq \underline{\beta}, \\
\bar{r} &\leq |r_i(t)| \leq \underline{r}, \\
\bar{\delta} &\leq |\delta_i(t)| \leq \underline{\delta}.
\end{align*}$$

where $\bar{\cdot}$ and $\underline{\cdot}$ are, respectively, the upper and lower boundaries.
Based on these representations, there is no collision between CAV\(_i\) and CAV\(_j\) iff \(\beta_i(z_i; t) \cap \beta_j(z_j; t) = \emptyset, \forall t \in [t_0, t_f]\). Similarly, CAVs do not collide road boundaries when the intersection of their sets is always empty, i.e. \(\beta_i(z_i; t) \cap O_r = \emptyset, \forall t \in [t_0, t_f]\).

### III. Problem Formulation

This section formulates simultaneous crossing of multiple CAVs through a lane-free and signal-free intersection as an optimal control problem. The formulated OCP minimises the crossing time of the intersection whilst avoiding collisions of each CAV with others and with road boundaries. Rest of the sections provide collision avoidance constraints, initial and terminal conditions and the objective function before presenting the overall OCP formulation.

#### A. Constraints to Avoid Collisions Between CAVs

To avoid collisions between any CAV\(_i\) and CAV\(_j\) \(\forall i \neq j \in \{1..N\}\), their polytopic sets should not intersect, i.e. \(\beta_i \cap \beta_j = \emptyset\) where \(\beta_i = \{X \in \mathbb{R}^2 | A_i X \leq b_i\}\) and \(\beta_j = \{Y \in \mathbb{R}^2 | A_j Y \leq b_j\}\). However, these are non-convex and non-differentiable conditions and enforcing them as constraints in an OCP will make the problem non-convex and difficult solve. To preserve differentiability and continuity, \(\beta_i \cap \beta_j = \emptyset\) is replaced by the following necessary condition which has negligible effect on the optimality of the solution for small values of \(d_{min}\) [23]:

\[
dist(\beta_i, \beta_j) = \min_{X,Y} \{||X-Y\|_2 \mid A_i X \leq b_i, A_j Y \leq b_j\} \geq d_{min}; \quad \forall i \neq j \in \{1..N\}. \quad (6)
\]

where \(d_{min}\) is a minimum safety distance between any pair of CAVs.

It is known that the problem of finding the minimum distance between a polytope \(\beta_i\) and another given polytope \(\beta_j\) is convex [24]. Therefore the substituted necessary condition, \(\text{dist}(\beta_i, \beta_j) \geq d_{min}\), is convex and since \(\beta_i\) is not an empty set, the strong duality holds [23]. This means that the solution of the primal problem \(\text{dist}(\beta_i, \beta_j) \geq d_{min}\) is the same as the one of its dual problem which is as follows:

\[
dist(\beta_i, \beta_j) = \max_{\lambda_{ij}} -b_i^T \lambda_{ij} - b_j^T \lambda_{ji} \quad \text{s.t.} \quad A_i^T \lambda_{ij} + s_{ij} = 0, A_j^T \lambda_{ji} - s_{ij} = 0, \quad ||s_{ij}||_2 \leq 1, -\lambda_{ij} \leq 0, -\lambda_{ji} \leq 0; \quad \forall i \neq j \in \{1..N\}. \quad (7)
\]

where \(\lambda_{ij}\), \(\lambda_{ji}\), and \(s_{ij}\) are the dual variables and \(A_i\) and \(b_i\) are as in [5] (the deviation of dual problem [7] from primal problem [6] is shown in [23]).

Combining (7) with (6), the objective function of (7) subject to its constraints must be greater than or equal to \(d_{min}\) in order to avoid collisions. However, (7) can be substituted by \(\exists \lambda_{ij} \geq 0, \lambda_{ji} \geq 0, s_{ij} : -b_i^T \lambda_{ij} - b_j^T \lambda_{ji} \geq d_{min}, A_i^T \lambda_{ij} + s_{ij} = 0, A_j^T \lambda_{ji} - s_{ij} = 0, ||s_{ij}||_2 \leq 1\) because the existence of a feasible solution \(\lambda_{ij,feas}, \lambda_{ji,feas}\), and \(s_{ij,feas}\) where \(-b_i^T \lambda_{ij,feas} - b_j^T \lambda_{ji,feas} \geq d_{min}\) is a sufficient condition to...
ensure $dist(\beta_i, \beta_j) \geq d_{\text{min}}$, i.e. to avoid collisions \(^{[23]}\). As seen in Fig. \(1\) $S_{ij}$ is a separating hyperplane between CAV\(_i\) and CAV\(_j\), and $S_{ij} = S_{ji}$.

### B. Constraints to Avoid Collisions with Road Boundaries

Each CAV must also avoid all the road boundaries, i.e. $\beta_i \cap O_r = \emptyset$ where $\beta_i = \{X \in \mathbb{R}^2| A_i X \leq b_i\}$ and $O_r = \{Y \in \mathbb{R}^2| A_r Y \leq b_r\}$. Similar to section III-A collision avoidance between CAVs, $\beta_i \cap O_r = \emptyset$ is replaced by the following necessary condition:

$$\text{dist}(\beta_i, O_r) = \min_{X,Y} \{\|X-Y\|_2 | A_i X \leq b_i, A_r Y \leq b_r\} \geq d_{\text{min}}; \forall r \in \{1..N_r\}. \tag{8}$$

where $d_{\text{min}}$ is the minimum safety distance between CAVs and road boundaries.

The dual problem of (8) is then substituted with the sufficient condition $\exists \lambda_{ir} \geq 0, \lambda_{ri} \geq 0, s_{ir} : -b_i^T \lambda_{ir} - b_r^T \lambda_{ri} \geq d_{\text{min}}, A_i^T \lambda_{ir} + s_{ir} = 0, A_r^T \lambda_{ri} - s_{ir} = 0, \|s_{ir}\|_2 \leq 1$ where $\lambda_{ir}, \lambda_{ri}$, and $s_{ir}$ are the dual variables. $S_{ir}$ is the separating hyperplane between CAVs and road boundaries (see Fig. \(1\)).

### C. Objective Function

CAVs are expected to reach their terminal pose as fast as possible. Therefore, this paper proposes the objective function \(^{[9]}\) that minimises the error between the current and final pose along with the overall time that all CAVs cross the intersection:

$$J(z_i(\cdot), \ldots, z_N(\cdot)) = \int_{t_0}^{t_f} \sum_{i=1}^{N} \left[ (z_i(t) - z_i(t_f))^T Q(z_i(t) - z_i(t_f)) \right] dt + \sum_{i=1}^{N} z_i(t_f) R z_i(t_f) + P(t_f - t_0). \tag{9}$$

where $Q$ and $R$ are gain matrices related to the CAVs’ pose and $P$ is the gain for the crossing time. The gains are selected based on try and error to best normalise the cost function. The expression $z_i(t_f) R z_i(t_f)$ enforces CAV\(_i\) to arrive at an exact final pose and expression $P(t_f - t_0)$ penalises the total crossing time of all CAVs.

### D. Optimal Control Problem

Lane-free crossing of multiple CAVs through a signal-free intersection is formulated as the following optimal control problem:

$$\{a_i(\cdot), \delta_i(\cdot)\}^* = \arg \min_{t_f, a_i(\cdot), \delta_i(\cdot)} J(z_1(\cdot), \ldots, z_N(\cdot)) = \{a_i(\cdot), \delta_i(\cdot)\}, \tag{10a}$$

s.t. \(11a\), \(11b\), \(11c\), \(11d\), \(11e\), \(11f\), \(11g\), \(11h\), \(11i\), \(11j\), \(11k\),

where $A_i$ and $b_i$ are functions of each CAV’s pose $z_i(t)$, and present CAV\(_i\) polytope at each time step $t_f$. Problem \(11\) is solved at time $t_0$ for $N$ CAVs until the terminal time $t_f$. The solution to this problem is optimal trajectories of the control signals $a_i(\cdot)$ and $\delta_i(\cdot)$ of each CAV\(_i\) for each $t \in [t_0, t_f]$, as well as a terminal time $t_f$. CAV\(_i\) follow their calculated trajectories to arrive final destination at the terminal time $t_f$.

The initial pose $z_i$, i.e. initial position and heading angle, and initial speed $V_i$ of all CAV\(_i\) $\forall i \in \{1..N\}$ within the control zone are known. The remaining of the states and the initial inputs to the CAVs are also assumed as zero. These initial conditions at $t = t_0$ are feasible solutions of the OCP. The final pose of CAVs, $z_i(t_f)$ is chosen randomly that indicates the intended destination of each CAV.

### IV. Numerical Simulation and Discussion

This paper employs CasADi \(^{[26]}\) and IPOPT \(^{[27]}\) to solve the formulated nonlinear OCP \(11\). CasADi directly discretises the continuous-time OCP \(11\), using a collocation method, to construct an equivalent large nonlinear programming (NLP) \(^{[28]}\). The resulting NLP is then solved using interior-point method (IPM) \(^{[27]}\). The paper also improves computation time by linking IPOPT to Intel® oneAPI Math Kernel Library (oneMKL, \(https://software.intel.com\)), which includes high-performance implementation of the MA27 linear solver.
This section verifies the developed control strategy with two test scenarios and different number of CAVs. All the results are calculated with Matlab running on a Mackbook 2.5 GHz quad-core intel core i7 and 16 GB memory. Table III provides the chosen values for important parameters of the algorithm. In order to assist with understanding, a detailed video of the two test scenarios is also provided on https://youtu.be/L_aFgKt38U

A. Test Scenario One: Benchmark comparison

The first test scenario compares the effectiveness of the proposed algorithm in terms of crossing time and average speed against two benchmarks methods. The first benchmark is the reservation-based approach in [10] which is a bi-level controller where an upper-level controller determines a unique arrival time to the conflict points (to reserve the point), as well as the final crossing time of each CAV. The lower-level control, on the other hand, calculates the control inputs, i.e. acceleration/deceleration, of each CAV to comply with the provided timings by the upper-level controller.

The second benchmark is the lane-free method proposed by Li et al. [11]. This method calculates the control inputs for a fixed final time, however, there is no reservation of conflict points and the CAVs can freely use all the space of the junction, as long as there is no collisions.

Table III summarises the crossing time and average speed of the CAVs controlled by the proposed strategies in this paper, in [10] and in [11]. The scenario consists of testing performance of the algorithms for different number of CAVs between 2 and 12. All tests consists of right-turn maneuver by at least one CAV. The starting and terminal positions, allowable range of speed and acceleration of CAVs are same for all the cases.

The results in Table III show that the proposed strategy improves crossing time of CAVs by, respectively, up to 65% (and an average improvement of 52% for different number of CAVs), and 54% as compared to the reservation-based approach in [10] and lane-free method in [11]. The average speed of CAVs is also increased to around 16 (m/s). The reservation-based methods restrain the motion of CAVs to orderly pass the critical points which resulted in a higher crossing time and lower average speed values than the proposed strategy. Furthermore, the lane-free method in [11] fixes the crossing time to make the calculation tractable, however, it lowers average speeds and therefore increases crossing time.

It is worth noting that the crossing time and average speed of CAVs are almost constant with the proposed algorithm regardless the number of CAVs. This is an interesting outcome that is showing, unlike the traditional signalised junctions, the optimal crossing time (or throughput) of junctions with CAVs is limited by the layout of the junction rather than the number of passing CAVs. The proposed strategy finds the optimal crossing time (or throughput) of a junction which can be used as the benchmark to analyse sensitivity of different control strategies with respect to the number of crossing CAVs.

Overall, Table III demonstrates that the presented lane-free control strategy strongly outperforms the reservation-based method in [10] and lane-free method in [11] in terms of both crossing time and average speed of CAVs.

B. Test Scenario Two: A more complex scenario

The second test evaluates performance of the proposed strategy for a complex scenario that involves 21 CAVs and
allows any direction of travel (right, left, and straight). In this scenario, CAVs are given a random initial and final pose to cross the intersection while avoiding collision with other CAVs and with road boundaries.

Table IV compares the crossing time and average speed for different number of CAVs which are controlled by the proposed strategy when passing through a junction. As seen in Table IV, the crossing time is the same as the one for the test scenario 1 and again does not change regardless of the number of CAVs. This determines that the crossing time is not sensitive to the type of scenario and number of CAVs and the proposed lane-free control strategy enables CAVs to employ the most of the temporal-spatial capacity of the intersection. In other words, 4.56 s seems to be the global optimal crossing time of this particular junction.

Fig. 3 and 4a illustrate, respectively, the calculated motion trajectories and speed profiles of all the CAVs in the second test scenario. Fig. 3 displays that CAVs freely move and use opposite lanes if they are free while avoiding road boundaries. Fig. 4a shows that the longitudinal speed of CAVs changes linearly with the maximum admissible acceleration/deceleration, i.e. $\pm 3 \, (m/s^2)$, which indicates a bang-bang strategy. Fig. 4b displays the optimal steering angle of 21 CAVs in test scenario two. Please check out a provided video on https://youtu.be/L_aFGkKT38U for more details.

C. Computational Time

The computational time of test scenario two is presented in Fig. 5. The computation time of each number of CAV is the average of 10 times of running the scenario. The standard deviations of all the tests are less than a negligible value of 0.5% and are not shown. Fig. 5 shows that the computational complexity of the proposed algorithm is of the order of $O(\epsilon^{0.13n})$ in terms of number of CAVs $n$. The future work of this study is to reduce the computational time to the order of $O(\log(n))$ by decentralising the proposed strategy.

V. Conclusion

This study formulates the lane-free crossing of CAVs through a signal-free intersection as an optimal control problem that minimises crossing time while avoiding obstacles. The proposed formulation substitutes the non-differentiable and non-convex constraints that avoid collisions of CAVs with obstacles and road boundaries, with the dual problem of a necessary condition of the constraints. The resulting smooth OCP is then solved by CasADi to optimally control CAVs to safely pass through a junction within the minimum possible time. It is shown that the proposed strategy is capable of significantly reducing crossing time as compared to the state-of-the-art reservation-based or other lane-free strategies.
layout of intersection and is independent to the number or manoeuvres of the crossing CAVs. Therefore, the proposed algorithm provides the benchmark to evaluate the performance of other strategies in terms of their effects on traffic throughput.

Computational complexity of the algorithm is currently of the order of $O(\alpha^{0.13+\epsilon})$, where $n$ is the number of CAVs passing through the intersection, that has to be reduced to the order of $O(\log(n))$ for real-time implementation. This can be fulfilled by decentralising calculations amongst the CAVs which will be added as a future work.

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