DECENTRALIZED ATTRIBUTION OF GENERATIVE MODELS

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ABSTRACT

There have been growing concerns regarding the fabrication of contents through generative models. This paper investigates the feasibility of decentralized attribution of such models. Given a set of generative models learned from the same dataset, attributability is achieved when a public verification service exists to correctly identify the source models for generated content. Attribution allows tracing of machine-generated content back to its source model, thus facilitating IP-protection and content regulation. Existing attribution methods are non-scalable with respect to the number of models and lack theoretical bounds on attributability. This paper studies decentralized attribution, where provable attributability can be achieved by only requiring each model to be distinguishable from the authentic data. Our major contributions are the derivation of the sufficient conditions for decentralized attribution and the design of keys following these conditions. Specifically, we show that decentralized attribution can be achieved when keys are (1) orthogonal to each other, and (2) belonging to a subspace determined by the data distribution. This result is validated on MNIST and CelebA. Lastly, we use these datasets to examine the trade-off between generation quality and robust attributability against adversarial post-processes.

1 INTRODUCTION

Recent advances in generative models (Goodfellow et al., 2014) have enabled the creation of synthetic contents that are indistinguishable even by naked eyes (Pathak et al., 2016; Zhu et al., 2017; Zhang et al., 2017; Karras et al., 2017; Wang et al., 2018; Brock et al., 2018; Miyato et al., 2018; Choi et al., 2018; Karras et al., 2019a; b; Choi et al., 2019). Such successes raised serious concerns (Kelly, 2019; BRELAND, 2019) regarding adversarial applications of generative models, e.g., for the fabrication of user-profiles (Satter, 2019), articles (Keskar et al., 2019), images (Korshunova et al., 2017), audios (Kumar et al., 2019), and videos (Wang et al., 2018a; Fried et al., 2019). Necessary measures have been called for the filtering, analysis, tracking, and prevention of malicious applications of generative models before they create catastrophic sociotechnical damages.

Existing studies primarily focused on the detection of machine-generated contents. Marra et al. (2019) showed empirical evidence that generative adversarial networks (GANs) may come with data-specific fingerprints in the form of averaged residual over the generated distribution, yet suggested that generative models trained on similar datasets may not be uniquely distinguishable through fingerprints. Yu et al. (2018) showed on the other hand that it is empirically feasible to attribute a finite and fixed set of GAN models derived from the same dataset, i.e., correctly classifying model outputs by their associated GANs. While encouraging, their study did not prove that attribution can be achieved when the model set continues to grow (e.g., when GAN models are distributed to end-users in the form of mobile apps). In fact, Wang et al. (2019b) showed that detectors trained on one generative model are transferable to other models trained on the same dataset, indicating that individually trained detectors may perform incorrect attribution, e.g., by attributing images from one model belonging to user A to another model belonging to user B. It should be highlighted

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that most of the existing detection mechanisms are centralized, i.e., the detection relies on a registry that collects all models and/or model outputs and empirically looks for collection-wise features that facilitate detection. This fundamentally limits the scalability of detection tools in real-world scenarios where an ever growing number of models are being developed even for the same dataset.

**Problem formulation and protocol** We are thus motivated to investigate the feasibility of a decentralized approach to ensuring the correct attribution of generative models. Specifically, we assume that for a given dataset $D$, the registry only distributes keys, $\Phi := \{\phi_1, \phi_2, \ldots\}$, to users of generative models without collecting information from the users’ models. Each key is held privately by a user, whose key-dependent model is denoted by $G_\phi(\cdot; \theta): \mathbb{R}^{dz} \to \mathbb{R}^{dx}$ where $z$ and $x$ are the latent and output variables, respectively, and $d_z$ and $d_x$ the corresponding dimensionalities. $\theta$ denotes the model parameters. When necessary, we will suppress $\theta$ and $\phi$ to reduce the notational burden. The distribution of each key is accompanied by that of a public verification service, which tells whether a query belongs to $G_\phi$ (labeled as $1$) or not (labeled as $-1$). We call the underlying binary classifier a verifier and denote it by $f_\phi: \mathbb{R}^{dz} \to \{-1, 1\}$. In this paper, we model the verifier as linear classifier: $f_\phi(x) = \text{sign}(\phi^T x)$. On multiple GAN benchmarking sets, we show the linear verifier is effective.

**Motivating example** A company develops a GAN model for image post-processing. A third-party organization (the registry) assigns keys to the company, who is then required to embed a watermark to the GAN models according to the keys for image post-processing. A third-party organization (the registry) assigns keys to the company, who is then required to embed a watermark to the GAN models according to the keys for users to download. With the keys, the registry can trace the GAN generated images back to the user-end models.

The following quantities are central to our investigation: the distinguishability of $G_\phi$ is defined as

$$D(G_\phi) := \frac{1}{2} \mathbb{E}_{x \sim P_{G_\phi}, x_0 \sim P_D} \left[ \mathbb{I}(f_\phi(x) = 1) + \mathbb{I}(f_\phi(x_0) = -1) \right],$$

where $P_D$ is the authentic data distribution, and $P_{G_\phi}$ the model distribution induced by $G_\phi$.

The attributability of a collection of generative models $\mathcal{G} := \{G_1, \ldots, G_N\}$ is defined as:

$$A(\mathcal{G}) := \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}_{x \sim G_i} \mathbb{I}(\phi_i^T x < 0, \forall j \neq i, \phi_i^T x > 0).$$

Distinguishability of $G$ (attributability of $\mathcal{G}$) is achieved when $D(G) = 1$ ($A(\mathcal{G}) = 1$). Lastly, We denote by $G(\cdot; \theta_0)$ (or shortened as $G_0$) the root model sent to all users along the key, and assume $P_{G_0} = P_D$. We will measure the (lack of) generation quality of $G_\phi$ by the FID score [Heusel et al., 2017] and the $l_2$ norm of the mean output perturbation:

$$\Delta x(\phi) = \mathbb{E}_{z \sim P_z} \left[ G_\phi(z; \theta) - G(z; \theta_0) \right],$$

where $P_z$ is the latent distribution.

This paper addresses the following critical question: What are the rules for designing keys according to the proposed protocol, so that the resultant generative models can achieve distinguishability individually and attributability collectively, while maintaining their generation quality?

**Contributions** We claim the following contributions:

1. We develop sufficient conditions for distinguishability and attributability, to connect these metrics with the geometry of the data distribution, the angles between keys, and the generation quality.

2. The sufficient conditions yield simple design rules for the keys: keys should be (1) data compliant, i.e., $\phi^T x < 0$ for $x \sim P_D$, and (2) orthogonal to each other.
3. We validate the design rules and study the capacity of keys using DCGAN (Radford et al., 2015) on MNIST (LeCun & Cortes, 2010) and CelebA (Liu et al., 2015).

4. We empirically test tradeoffs between generation quality and robust attributability under post-processes including random image blurring, cropping, noising, JPEG conversion, and a combination of all, and show that robust attributability can be achieved, with limited and hard-to-perceive loss of generation quality.

2 SUFFICIENT CONDITIONS FOR ATTRIBUTABILITY

Distinguishability through watermarking We consider data-compliant keys $\phi \in \mathbb{R}^d$, such that $f_\phi(x) = -1$ for all $x \sim P_D$, i.e., the authentic data are correctly attributed as not belonging to any of the generators. All keys are constrained by $||\phi|| = 1$ for identifiability, where $|| \cdot ||$ is the $l_2$ norm. A key-dependent generative model $G_\phi$ achieves distinguishable output distribution $P_{G_\phi}$ by adding a uniform and bounded perturbation $\Delta x$ to the output of the root model $G_0$. We can solve the following problem with respect to $\Delta x$ to achieve distinguishability:

$$\min_{||\Delta x|| \leq \varepsilon} \mathbb{E}_{x \sim P_D} \left[ \max \{ \phi^T (x + \Delta x), 0 \} \right],$$

(4)

where $\varepsilon > 0$. We have the following proposition (proof in Appendix A):

**Proposition 1.** Let $d_{\max} (\phi) := \max_{x \sim P_D} |\phi^T x|$. If $\varepsilon \geq d_{\max} (\phi)$, then $\Delta x^* = d_{\max} (\phi)\phi$ solves Eq. (4), and $f_\phi (x + \Delta x^*) > 0$, $\forall x \sim P_D$.

Proposition 1 shows that to achieve distinguishability, there is a data geometry-dependent threshold on generation quality ($\varepsilon$).

Watermarking user-end models The perturbation $\Delta x^*$ can potentially be reverse engineered and removed when generative models are white-box to end users. Therefore, we propose to instead retrain the user-end models $G_\phi$ on the perturbed dataset $D_{\gamma, \phi} := \{ x + \gamma \phi \mid x \sim G_0 \}$ with some $\gamma > 0$. Different from Proposition 1, we will show in Theorem 1 that $\gamma$ needs to be larger than $d_{\max} (\phi)$ in order to guarantee distinguishability of $G_\phi$. To explain, note that $G_\phi$ is trained to match $(z, G(z; \theta_0) + \phi)$ in a supervised manner. Therefore, we introduce the following model:

$$G_\phi (z; \theta) = G(z; \theta_0) + \gamma \phi + \varepsilon,$$

(5)

where the random error $\varepsilon \sim N (\mu, \Sigma)$. Sec. 3 provides estimates of $\mu$ and $\Sigma$ using sampled cs from DCGANs on MNIST and CelebA.

We have the following sufficient condition for model distinguishability (proof in Appendix B):

**Theorem 1.** Let $d_{\max} (\phi) = \max_{x \in \mathbb{D}} |\phi^T x|$, $\sigma^2 (\phi) = \phi^T \Sigma \phi$, $\delta \in [0, 1]$, and $\phi$ be a data-compliant key. $D(G_\phi) \geq 1 - \delta/2$ if

$$\gamma \geq \sigma (\phi) \sqrt{\log \left( \frac{1}{\delta^2} \right) + d_{\max} (\phi) - \phi^T \mu.}$$

(6)

Remarks The computation of $\sigma (\phi)$ requires $G_\phi$, which in turn requires $\gamma$. Therefore, an iterative search will be needed to determine $\gamma$ that is small enough to limit the loss of generation quality, yet large enough for distinguishability (see Alg. 1).

**Attributability** We can now derive the sufficient conditions for attributability of the generative models from a set of $N$ keys (proof in Appendix C):

**Theorem 2.** Let $d_{\min} = \min_{x \in \mathbb{D}} |\phi^T x|$, $d_{\max} = \max_{x \in \mathbb{D}} |\phi^T x|$, $\sigma^2 (\phi) = \phi^T \Sigma \phi$, $\delta \in [0, 1]$, $A(G) \geq 1 - N \delta$ if $D(G) \geq 1 - \delta$ for all $G_\phi \in \mathcal{G}$ and for any pair of data-compliant keys $\phi$ and $\phi'$:

$$\phi^T \phi' \leq -1 + \frac{d_{\max} (\phi') + d_{\min} (\phi') - 2 \phi^T \mu}{\sigma (\phi') \sqrt{\log \left( \frac{1}{\delta^2} \right) + d_{\max} (\phi') - \phi'^T \mu}}.$$

(7)

Remarks When $\sigma (\phi')$ is negligible for all $\phi'$ and $\mu = 0$, RHS of Eq. (7) is approximately $d_{\max} (\phi') / d_{\max} (\phi') > 0$, in which case $\phi^T \phi' \leq 0$ is sufficient for attributability. In Section 3 we empirically show that this is the case for MNIST and CelebA.
Figure 2: (a) Validation of Theorem 1. RHS=LHS on the red line. (b) Validation of Theorem 2. All RHS of Eq. (7) are positive. (c) For 100 keys, all trained keys are orthogonal. $D(G_{\phi_i})$ and $A(\{G_{\phi_j}\}_{j=1}^{i})$ are close to 1.

3 EXPERIMENTS AND ANALYSIS

The theorems suggest that data-compliant and orthogonal keys guarantee an attributability lower bound. In this section we validate the theorems through experiments.

Key generation We generate keys by iteratively solving the following convex problem starting from an empty key set $\Phi$:

$$\phi_i = \arg \min_{\phi} E_{x \sim P_D, G_0} \left[ \max\{1 + \phi^T x, 0\} \right] + \sum_{j=1}^{i-1} \max\{\phi_j^T \phi, 0\}. \tag{8}$$

The orthogonality penalty is omitted for the first key. The resultant keys will be normalized before inserted into the next problems. We note that $P_D$ and $P_{G_0}$ do not perfectly match in practice, and therefore expectations are taken over both distributions for the data compliance requirement. $G_0$s are trained using the standard DCGAN architecture on MNIST and CelebA using ADAM with learning rate 0.001, $\beta_1 = 0.9$, $\beta_2 = 0.99$. The training details are in Appendix E.

User-end generative models The training of $G_{\phi}$ follows Alg. 1 where we fine-tune $\gamma$ to balance generation quality and distinguishability. For each $\gamma$, we collect a perturbed dataset $D_{\gamma,\phi} := \{(z, G(z; \theta_0) + \gamma \phi)\}$ with $z \sim P_z$, and solve the following training problem:

$$\min_{\theta} E_{(z,x) \sim D_{\gamma,\phi}} \left[ ||G_{\phi}(z; \theta) - x||^2 \right], \tag{9}$$

starting with $\theta = \theta_0$. If the resultant model does not meet the distinguishability requirement (due to discrepancy between $D_{\gamma,\phi}$ and $G_{\phi}$), we extend the perturbation by $\gamma = \alpha \gamma$. In experiments, we use a standard normal distribution for $P_z$, $\delta = 10^{-2}$, and $\alpha = 1.1$.

Validation of Theorem 1 We first validate the sufficient condition for distinguishability. Fig. 2(a) compares the LHS and RHS values of Eq. (6) for 100 keys, along with the empirical distinguishability of the corresponding $G_{\phi}$s. The mean distinguishability and the standard deviation are $1 - 10^{-4}$ and $3 \times 10^{-4}$, respectively. Calculation of the RHS requires an estimation of $\mu$ and $\Sigma$. To do this, we sample

$$\epsilon(z) = G_{\phi}(z; \theta) - G(z; \theta_0) - \gamma \phi \tag{10}$$
using 5000 samples $z \sim P_z$ with $G_φ$ and $γ$ output from Alg. 1. $Σ$ and $μ$ are then estimated by the sampled $cs$ for each $φ$. The Fig. 3 demonstrates the estimates of $Σ$. We note that $μs$ have elements close to zero for the test datasets. Results in Fig. 2(a) show that the sufficient condition (Eq. (6)) is satisfied for all $G_φs$ through the training specified in Alg. 1. Lastly, we notice that the LHS values for MNIST are farther away from the equality line than those for CelebA. This is due to the fact that perturbations for samples from MNIST are more likely to exceed the bounds for pixel values due to their binarized values in $D$. Clamping of these invalid pixel values reduces the effective perturbation length. Therefore to achieve distinguishability, Alg. 1 seeks $γ$'s larger than needed. This issue is less observed in CelebA, in which case data points in $D$ rarely have values close to boundaries.

Algorithm 1: Training of $G_φ$

| input | $φ, G_0$ |
|-------|----------|
| output: | $G_φ, γ$ |
| 1 | set $γ = d_{max}(φ)$ |
| 2 | collect $D_{γ,φ}$ |
| 3 | train $G_φ$ by solving Eq. (9) using $D_{γ,φ}$ |
| 4 | compute empirical $D(G_φ)$ |
| 5 | if $D(G_φ) < 1 - δ$ then |
| 6 | set $γ = αγ$ |
| 7 | goto step 2 |
| s | end |

Figure 3: Sample covariance matrix of errors

Validation of Theorem 2 Recall that from Theorem 2, we derive the simple design rule that keys should be orthogonal, based on the observation that the RHS of Eq. (7) is almost always positive. Here we empirically test this assumption, as we plot the RHS values derived from 100 keys in Fig. 2(b), where the values are all positive for MNIST and close to zero for CelebA. The result suggests that orthogonality of keys is a good heuristic for attributability. Note that the condition is sufficient thus could be more stringent than necessary. We conjecture that this is the reason why attributability is empirically achieved with orthogonal keys. Lastly, while it is feasible to further reduce the lower bound on the angles between keys, e.g., some acute angles will still facilitate attribution in MNIST, doing so would require the derivation of new keys to rely on knowledge about existing user-end models (to compute the RHS of Eq. (7)).

Empirical results on distinguishability, attributability, and generation quality Table 1 reports the metrics of interest empirically measured on 20 user-end models for each dataset. Specifically, we achieve high attributability while all models are trained separately only for distinguishability. As a comparison, we demonstrate cases where keys are 45 deg apart ($φ^T_δ φ = 0.71$). Note that due to the perturbations, all $G_φs$ have worse generation quality than $G_0$ (i.e., higher FIDs than those of the root models).

Capacity of keys For real-world applications, we hope to maintain attributability for a large set of keys. Our study so far suggests that the capacity of keys is constrained by the data compliance and orthogonality requirements. While sequentially computing keys is convex and affordable (Eq. 3), finding the maximum number of feasible keys is a problem about optimal sphere packing on a manifold (Fig. 4), which is an open challenge (Cohn et al. 2017; Cohn 2016). Here we use MNIST on DCGAN to empirically explore the capacity of keys. Results on 100 keys are reported in Fig. 2(c) using the following metrics with respect to an increasing number of keys $n$: average orthogonality $o_n = \frac{\sum_{j=1}^{n-1} |φ_j φ_n|}{n - 1}$ ($o_1 = 0$), perturbation length $γ_n$, distinguishability $D(G_φ_n)$, and attributability $A\{\{G_φ\}_{j=1}^n\}$, for $n = 1, ..., 100$. We show that attributability is almost always 1 while all models are trained only for distinguishability.

Robust training We now consider the scenario where outputs of the generative models are post-processed (by adversaries) before being attributed. When the post-processes are known, we can take counter measures through robust training, which intuitively, will lead to additional loss of generation quality. To assess this tradeoff between robustness and generation quality, we train $G_φ$ against post-

Figure 4: Capacity of keys as a sphere packing problem: The feasible space (arc) is determined by the data compliance and generation quality constraints, and the size of spheres by the minimal angle between keys.
lack of theoretical guarantee in this scenario, we resort to the following robust training problem:

\[ \max_{\theta} \mathbb{E}_{z \sim P_z, T \in P_T} \left[ \max \left\{ 1 - \mathcal{J}(G_0(z; \theta_1)), 0 \right\} \right] + C \| \nabla_x \| \quad \text{subject to } G_0(z) - G_0(z; \theta_1) \|_2^2, \]

where \( C = 10 \) is the hyper-parameter for a penalty on the generation quality. We consider five types of post-processes: blurring, cropping, noise, JPEG compression and the combination of these four. Examples of the post-processed images are shown in Fig. 5. Blurring uses Gaussian kernel widths uniformly drawn from \( \frac{1}{8} \{1, 3, 5, 7, 9\} \). Cropping crops images with uniformly drawn ratios between 80% and 100%, and scales the cropped images back to the original size using bilinear interpolation. Noise adds white noise with standard deviation uniformly drawn from [0, 0.3]. JPEG applies JPEG compression. Combination performs each attack with a 50% chance in the order of Blurring, Cropping, Noise and JPEG. We use implementations for differentiable blurring (Riba et al. (2020)) and JPEG (Zhu et al. (2018)). For robust training against each post-process, we apply the post-process to mini-batches with 50% probability.

Table 1: Empirical average of distinguishability \( \mathbb{E}_{x \sim P_x} |D(x) - D(G_0(x; \theta_0))| \) and FID scores, and attributability \( \mathbb{E}_{x \sim P_x} \| \nabla_x \| \) from 20 generative models for each dataset. Std in parenthesis (all close to zero for distinguishability). FID \( D \) is the FID for \( D_0 \).

| GANs     | Angle       | Dataset  | \( \bar{G} \) | \( A(G) \) | \( ||\Delta x|| \) | \( \mathbb{E}_{x \sim P_x} |D(x) - D(G_0(x; \theta_0))| \) |
|----------|-------------|----------|---------------|------------|-------------------|-----------------------------|
| DCGAN    | Orthogonal  | MNIST    | 0.98          | 5.05       | 5.05 (0.09)       | 4.98 (0.15)                 | 5.36 (0.12) |
| DCGAN    | 45 degree   | MNIST    | 0.99          | 5.39       | 5.39 (0.14)       | -                           | 5.42 (0.13) |
| DCGAN    | Orthogonal  | CelebA   | 0.99          | 5.63       | 5.63 (0.11)       | 33.95 (0.13)                | 53.91 (2.20) |
| DCGAN    | 45 degree   | CelebA   | 0.99          | 5.78       | 5.78 (0.21)       | -                           | 53.92 (2.56) |

Table 2: Distinction before (Bfr) and after (Aft) robust training, attributability Bfr and Aft, \( ||\Delta x|| \) after robust training, and FID after robust training. Std in parenthesis. Higer is better for distinguishability and attributability. Lower is better for \( ||\Delta x|| \) and FID.

| GANs     | Dataset  | Blurring | Cropping | Noise | JPEG | Combination |
|----------|----------|----------|----------|-------|------|-------------|
| DCGAN    | MNIST    | 0.49     | 0.96     | 0.52  | 0.99 | 0.85        |
| DCGAN    | CelebA   | 0.99     | 0.25     | 0.53  | 0.99 | 0.88        |
| DCGAN    | MNIST    | 0.02     | 0.97     | 0.03  | 0.99 | 0.77        |
| DCGAN    | CelebA   | 0.00     | 0.99     | 0.00  | 0.99 | 0.89        |
| DCGAN    | MNIST    | 15.96    | 9.67     | 9.17  | 0.65 | 5.93        |
| DCGAN    | CelebA   | 11.83(0.65) | 9.30(0.31) | 4.75(0.17) | 6.01(0.29) | 13.69(0.59) |
| DCGAN    | MNIST    | 41.11(20.43) | 21.58(2.44) | 5.79(0.19) | 6.50(1.70) | 68.16(24.67) |
| DCGAN    | CelebA   | 73.62(6.70) | 98.86(9.51) | 59.51(1.60) | 60.35(2.57) | 87.29(9.29) |

Table 3: 1st-4th rows: distinguishability before (Bfr) and after (Aft) robust training, attributability Bfr and Aft, \( ||\Delta x|| \) after robust training, and FID after robust training. Std in parenthesis. Higer is better for distinguishability and attributability. Lower is better for \( ||\Delta x|| \) and FID.
mean is taken over the latent space. For MNIST, these eigenvectors overlap with the digits; for CelebA, they are structured color patterns. On the other hand, the smallest eigenvectors represent directions rarely covered by the sensitivity vectors, thus resembling random noise. Based on this finding, we test the hypothesis that keys more aligned with the eigenspace of the small eigenvalues will have smaller $d_{\text{max}}$. We compute that the correlations between $d_{\text{max}}$ and $\phi^T M \phi$: the pearson correlations are 0.33 and 0.53 for MNIST and CelebA, respectively. Lastly, we compare outputs of generative models using the largest and the smallest eigenvectors of $M$ as the keys in Fig. 7.

4 RELATED WORK

Detection and attribution of model-generated contents This paper focused on the attribution of contents from generative models rather than the detection of hand-crafted manipulations (Agarwal & Farid (2017); Popescu & Farid (2005); O’Brien & Farid (2012); Rao & Ni (2016); Huh et al. (2018)). In this context, it has been shown that generative models based on convolutional neural network leave fingerprints in their outputs (Odena et al. (2016)), allowing them to be detected (Zhang et al. (2019b); Marra et al. (2019); Wang et al. (2019a)). However, generative models trained on similar datasets have similar fingerprints (Marra et al. (2019)). Thus fingerprints cannot be used for attribution. Recent studies show that attribution is achievable through a centralized manner, where multiclass classifiers are trained on all models (Yu et al. (2018); Albright et al. (2019)). However, such classifiers are not proven to exist, and computing them is not practical when the number of
models grows arbitrarily. Skripniuk et al. (2020) studied decentralized attribution. Instead of using linear classifiers for attribution, they train a watermark encoder-decoder network that embeds (and reads) watermarks into (and from) the content, and compare the decoded watermark with user-specific ones. They do not provide sufficient conditions of the watermarks for distinguishability and attributability. Their empirical attributability is measured on a set of six generative models for CelebA. In comparison, we do so on 100 DCGANs on MNIST and CelebA.

**IP protection of digital contents** Watermarks have conventionally been used for IP protection of digital contents Tirkel et al. (1993); Van Schyndel et al. (1994); Bi et al. (2007); Hsieh et al. (2001); Pereira & Pun (2000); Zhu et al. (2018); Zhang et al. (2019a). In this context, watermarks are often added to contents as a post-process, and therefore can be removed when the models are white-box to users. Another approach towards content IP protection is through blockchain Hasan & Salah (2019). However, this approach requires meta data to be transferred along with the contents, which may not be realistic in adversarial settings. E.g., one can simply take a picture of a synthetic image to remove any meta data attached to the image file.

**IP protection of machine learning models** Aside from IP protection of contents, mechanisms for protecting IP of discriminative and generative models have also been studied Uchida et al. (2017); Nagai et al. (2018); Le Merrer et al. (2019); Adi et al. (2018); Zhang et al. (2018); Fan et al. (2019); Szyller et al. (2019); Zhang et al. (2020). Model watermarking is usually done by adding watermarks into model weights Uchida et al. (2017); Nagai et al. (2018), by embedding unique input-output mapping into the model Le Merrer et al. (2019); Adi et al. (2018); Zhang et al. (2018), or by introducing a passport mechanism so that model accuracy drops if the right passport is not inserted Fan et al. (2019). In the wake of model extraction attacks, where the attacker can reverse engineer the model from its output, existing solutions resort to watermarking of outputs Szyller et al. (2019); Zhang et al. (2020). While closely related, existing work on model IP protection focused on the distinguishability of individual models, rather than the attributability of a model set.

5 CONCLUSION

This paper investigated the feasibility of decentralized attribution for generative models. We put forward a protocol where a registry generates keys to watermark user-end models, for which the outputs can be correctly attributed by the registry. Our investigation led to simple design rules for the design of the keys to achieve correct attribution while only requiring each user-end model to be distinguishable from the authentic dataset. With concerns about adversarial post-processes, we empirically show that robust attribution can be achieved using the developed design rules, with additional loss of generation quality.
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A Proof of Proposition 1

**Proposition 1.** Let $d_{\text{max}}(\phi) := \max_{x \sim P_D} |\phi^T x|$. If $\varepsilon \geq 1 + d_{\text{max}}(\phi)$, then $\Delta x = (1 + d_{\text{max}}(\phi)) \phi$ solves Eq. (4), and $f_\phi(x + \Delta x) > 0 \forall x \sim G_0$.

**Proof.** Let $\phi$ be a data-compliant key and let $x$ be sampled from $P_D$. First, from the KKT conditions for Eq. (4) we can show that the solution $\Delta x^*$ is proportional to $\phi$:

$$\Delta x^* = \phi / \mu^*,$$

where $\mu^* \geq 0$ is the Lagrange multiplier. To minimize the objective, we seek $\mu$ such that

$$1 - (x + \Delta x)^T \phi = 1 - x^T \phi - 1/\mu^* \leq 0,$$

for all $x$. Since $x^T \phi < 0$ (data compliance), this requires $1/\mu^* = 1 + d_{\text{max}}(\phi)$. Therefore, when $\varepsilon \geq 1 + d_{\text{max}}(\phi)$, $\Delta x^* = (1 + d_{\text{max}}(\phi)) \phi$ solves Eq. (4). And $f_\phi(x + \Delta x^*) = \phi^T (x + (1 + d_{\text{max}}(\phi)) \phi) = \phi^T x + 1 + d_{\text{max}}(\phi) > 0$.

□

B Proof of Theorem 1

**Theorem 1.** Let $d_{\text{max}}(\phi) = \max_{x \in D} |\phi^T x|$, $\sigma^2(\phi) = \phi^T \Sigma \phi$, $\delta_d \in [0, 1]$, and $\phi$ be a data-compliant key. $D(G_\phi) \geq 1 - \delta_d / 2$ if

$$\gamma \geq \sigma(\phi) \sqrt{\log \left( \frac{1}{\delta_d^2} \right)} + d_{\text{max}}(\phi) - \phi^T \mu.$$

**Proof.** We first note that due to data compliance of keys, $E_{x \sim P_D}[\mathbb{I}(\phi^T x < 0)] = 1$. Therefore $D(G_\phi) \geq 1 - \delta_d / 2$ iff $E_{x \sim P_{G_\phi}}[\mathbb{I}(\phi^T x > 0)] \geq 1 - \delta_d$, i.e., $Pr(\phi^T x > 0) \geq 1 - \delta_d$ for $x \sim P_{G_\phi}$.

We now seek a lower bound for $Pr(\phi^T x > 0)$. To do so, let $x$ and $x_0$ be sampled from $P_{G_\phi}$ and $P_{G_0}$, respectively. Then we have

$$\phi^T x = \phi^T (x_0 + \gamma \phi + \epsilon) = \phi^T x_0 + \gamma + \phi^T \epsilon,$$

and

$$Pr(\phi^T x > 0) = Pr \left( \phi^T \epsilon > -\phi^T x_0 - \gamma \right).$$

Since $d_{\text{max}}(\phi) \geq -\phi^T x_0$, we have

$$Pr(\phi^T x > 0) \geq Pr \left( \phi^T \epsilon > d_{\text{max}}(\phi) - \gamma \right) = Pr \left( \phi^T (\epsilon - \mu) \leq \gamma - d_{\text{max}}(\phi) + \phi^T \mu \right).$$

The latter sign switching in equation [17] is granted by the symmetry of the distribution of $\phi^T (\epsilon - \mu)$, since $\phi^T (\epsilon - \mu) \sim \mathcal{N}(0, \phi^T \Sigma \phi)$. A sufficient condition for $Pr(\phi^T x > 0) \geq 1 - \delta_d$ is then

$$Pr \left( \phi^T (\epsilon - \mu) \leq \gamma - d_{\text{max}}(\phi) + \phi^T \mu \right) \geq 1 - \delta_d.$$

Recall the following tail bound of $x \sim \mathcal{N}(0, \sigma^2)$ for $y \geq 0$:

$$Pr(x \leq \sigma y) \geq 1 - \exp(-y^2/2).$$

Compare equation [19] with equation [18] the sufficient condition becomes

$$\gamma \geq \sigma(\phi) \sqrt{\log \left( \frac{1}{\delta_d^2} \right)} + d_{\text{max}}(\phi) - \phi^T \mu.$$

□
C  Proof of Theorem 2

**Theorem 2.** Let \( d_{\text{min}} = \min_{x \in \mathcal{D}} |\phi^T x|, d_{\text{max}} = \max_{x \in \mathcal{D}} |\phi^T x|, \sigma^2(\phi) = ||\phi||^2_2, \) \( \delta_a \in [0, 1], \)

\[ A(\mathcal{G}) \geq 1 - N \delta \text{ if } D(\mathcal{G}) \geq 1 - \delta \text{ for all } G_\phi \in \mathcal{G} \text{ and for any pair of data-compliant keys } \phi \text{ and } \phi': \]

\[ \phi^T \phi' \leq -1 + \frac{d_{\text{max}}(\phi') + d_{\text{min}}(\phi') - 2\phi^T \mu}{\sigma(\phi') \sqrt{\log \left( \frac{1}{\delta^2} \right)} + d_{\text{max}}(\phi') - \phi^T \mu}. \quad (21) \]

**Proof.** Let \( \phi \) and \( \phi' \) be any of two keys with \( \phi^T \phi' \leq 0 \), and \( x \) and \( x_0 \) be sampled from \( P_{G_{\phi}} \) and \( P_{G_{\phi'}}, \) respectively. We first derive the sufficient conditions for \( \Pr(\phi^T x < 0) \geq 1 - \delta_a \). Since \( x = x_0 + \gamma \phi + \epsilon \) for \( x \in G_\phi \), we have

\[ \phi^T x = \phi^T (x_0 + \gamma \phi + \epsilon) = \phi^T x_0 + \gamma \phi^T \phi' + \phi^T \epsilon. \]

Then

\[ \Pr(\phi^T x < 0) = \Pr(\phi^T \epsilon < -\phi^T x_0 - \gamma \phi^T \phi') \]

\[ \geq \Pr(\phi^T (\epsilon - \mu) < d_{\text{min}}(\phi') - \gamma \phi^T \phi' - \phi^T \mu), \]

where \( d_{\text{min}}(\phi') := \min_{x \in \mathcal{D}} |\phi^T x| \) and \( \phi^T (\epsilon - \mu) \sim N(0, \sigma^2(\phi')). \) Using the same tail bound of normal distribution and Theorem 1, we have \( \Pr(\phi^T x < 0) \geq 1 - \delta \) if

\[ -\gamma \phi^T \phi' \geq \sigma(\phi') \sqrt{\log \left( \frac{1}{\delta^2} \right)} - d_{\text{min}}(\phi') + \phi^T \mu \]

\[ \Rightarrow \phi^T \phi' \leq -1 + \frac{d_{\text{max}}(\phi') + d_{\text{min}}(\phi') - 2\phi^T \mu}{\sigma(\phi') \sqrt{\log \left( \frac{1}{\delta^2} \right)} + d_{\text{max}}(\phi') - \phi^T \mu}. \quad (24) \]

Note that \( \Pr(A = 1, B = 1) = 1 - \Pr(A = 0) - \Pr(B = 0) + \Pr(A = 0, B = 0) \geq 1 - \Pr(A = 0) - \Pr(B = 0) \) for binary random variables \( A \) and \( B \). With this, it is straight forward to show that when \( \Pr(\phi^T x < 0) \geq 1 - \delta \) for all \( \phi' \neq \phi \), and \( \Pr(\phi^T x > 0) \geq 1 - \delta \) for all \( \phi \), then \( \Pr(\phi^T x > 0, \phi^T x < 0 \forall \phi' \neq \phi) \geq 1 - N \delta \) and \( A(\mathcal{G}) \geq 1 - N \delta \).

\[ \square \]

D  Examples of GANs

In the main draft, we show examples from DCGAN on CelebA. Here, we illustrate DCGAN on MNIST. For Fig. 8 (a) 1st-2rd row: authentic data, samples from the non-robust generator (b-f) 1st-2rd rows: worst-case post-process, samples from robust training against the specific post-processes (before the post-processes). 3rd row for all: numerical differences between 2nd row of (a) and 2nd row of each case. Thus, the differences show the effect of robust training on attribution.

E  Training Details

E.1  Parameters

We adopt Adam optimizer for gradient descent. We attach other parameters in Table 5.

E.2  Training Time

For experimental validations, we use V:100 Tesla GPUs. Exact number of GPUs are reported in Table 5.
Table 3: Hyper-parameters for solving Eq.(Key generation) (top) and Eq.(Generative models) (btm). Equations are in Implementation.

| GANs   | Dataset | Batch Size | Learning Rate | $\beta_1$ | $\beta_2$ | Epochs |
|--------|---------|------------|---------------|-----------|-----------|--------|
| DCGAN  | MNIST   | 16         | 0.001         | 0.9       | 0.99      | 10     |
| DCGAN  | CelebA  | 64         | 0.001         | 0.9       | 0.99      | 2      |

F  ABLATION STUDY

We report the table of ablation study of robust training. We compute how $C$ affects distinguishability, attributability, $||\Delta x||$, and FID scores in Table 5 and Table 6. $C$ improves $||\Delta x||$ and FID for every generators (Table 6). However, $C$ is in inverse proportional to the robustness (Table 5). We can observe that, as $C$ increases, robustness decreases but generation quality increases. In the main document, we used twenty keys and generators. But, in this section, we computed five keys and generators for each of $C$. 
Table 4: Training time (in minute) of one key (Eq.(Key generation)) and one generator (Eq.(Generative models)). DCGAN\textsubscript{MNIST}: DCGAN for MNIST, DCGAN\textsubscript{CelebA}: DCGAN for CelebA. Equations are in Implementation.

| GANs     | GPUs | Key | Naive | Blurring | Cropping | Noise | JPEG | Combination |
|----------|------|-----|-------|----------|----------|-------|------|-------------|
| DCGAN\textsubscript{MNIST} | 1    | 1.77| 14    | 4.12     | 3.96     | 4.19  | 5.71 | 5.12        |
| DCGAN\textsubscript{CelebA} | 1    | 5.31| 15    | 10.33    | 9.56     | 10.35 | 10.25| 10.76       |

Table 5: Distinguishability (top), attributability (btm) before (Bfr) and after (Aft) robust training. DCGAN\textsubscript{MNIST}: DCGAN for MNIST, DCGAN\textsubscript{CelebA}: DCGAN for CelebA.

| GANs     | C   | Blurring | Cropping | Noise | JPEG | Combination |
|----------|-----|----------|----------|-------|------|-------------|
| -        | -   | Bfr      | Aft      | Bfr   | Aft  | Bfr  | Aft  | Bfr | Aft  | Bfr  | Aft  | Bfr  | Aft  | Bfr  | Aft  | Bfr  | Aft  | Bfr  | Aft  | Bfr  | Aft  | Bfr  | Aft  | Bfr  | Aft  | Bfr  | Aft  | Bfr  | Aft  |
| DCGAN\textsubscript{MNIST} | 10  | 0.49     | 0.97     | 0.51  | 0.99 | 0.84  | 0.99 | 0.53 | 0.99 | 0.50 | 0.63 |
| DCGAN\textsubscript{MNIST} | 100 | 0.49     | 0.61     | 0.51  | 0.98 | 0.76  | 0.98 | 0.53 | 0.99 | 0.50 | 0.52 |
| DCGAN\textsubscript{MNIST} | 1K  | 0.49     | 0.50     | 0.51  | 0.81 | 0.69  | 0.91 | 0.53 | 0.97 | 0.50 | 0.51 |
| DCGAN\textsubscript{CelebA} | 10  | 0.49     | 0.99     | 0.49  | 0.99 | 0.96  | 0.99 | 0.50 | 0.99 | 0.49 | 0.85 |
| DCGAN\textsubscript{CelebA} | 100 | 0.50     | 0.96     | 0.49  | 0.99 | 0.92  | 0.93 | 0.50 | 0.99 | 0.49 | 0.61 |
| DCGAN\textsubscript{CelebA} | 1K  | 0.50     | 0.62     | 0.49  | 0.97 | 0.88  | 0.91 | 0.50 | 0.99 | 0.49 | 0.51 |

Table 6: $||\Delta x||$ (top) and FID score (btm). Standard deviations in parenthesis. DCGAN\textsubscript{MNIST}: DCGAN for MNIST, DCGAN\textsubscript{CelebA}: DCGAN for CelebA, Combi.: Combination attack. Lower is better.

| GANs     | C   | Baseline | Blurring | Cropping | Noise | JPEG | Combi. |
|----------|-----|----------|----------|----------|-------|------|--------|
| -        | -   |          |          |          |       |      |        |
| DCGAN\textsubscript{MNIST} | 10  | 5.05     | 15.96    | 9.17    | 5.93  | 6.48 | 17.08  |
| DCGAN\textsubscript{MNIST} | 100 | 4.09     | 12.95    | 7.62    | 4.57  | 4.70 | 12.70  |
| DCGAN\textsubscript{MNIST} | 1K  | 3.88     | 7.17     | 7.43    | 4.22  | 5.12 | 7.56   |
| DCGAN\textsubscript{CelebA} | 10  | 5.63     | 11.83    | 9.30    | 4.75  | 6.01 | 13.69  |
| DCGAN\textsubscript{CelebA} | 100 | 3.08     | 10.00    | 7.80    | 3.20  | 4.26 | 11.65  |
| DCGAN\textsubscript{CelebA} | 1K  | 2.55     | 7.68     | 7.13    | 2.65  | 3.39 | 9.23   |