Superfluidity of Λ hyperons in neutron stars

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Abstract

We study the $^1S_0$ superfluidity of Λ hyperons in neutron star matter and neutron stars. We use the relativistic mean field (RMF) theory to calculate the properties of neutron star matter. In the RMF approach, the meson-hyperon couplings are constrained by reasonable hyperon potentials that include the updated information from recent developments in hypernuclear physics. To examine the $^1S_0$ pairing gap of Λ hyperons, we employ several ΛΛ interactions based on the Nijmegen models and used in double-Λ hypernuclei studies. It is found that the maximal pairing gap obtained is a few tenths of a MeV. The magnitude and the density region of the pairing gap are dependent on the ΛΛ interaction and the treatment of neutron star matter. We calculate neutron star properties and find that whether the $^1S_0$ superfluidity of Λ hyperons exists in the core of neutron stars mainly depends on the ΛΛ interaction used.

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I. INTRODUCTION

Neutron stars are natural laboratories for studying the physics of dense matter. They are associated with some of the most exotic environments in the universe. Our knowledge of neutron star interiors is still uncertain. The central density of neutron stars can be extremely high, and many possibilities for such dense matter have been suggested \[1,3\]. For densities below twice normal nuclear matter density \((\rho_0 \sim 0.15 \text{ fm}^{-3})\), the matter consists of only nucleons and leptons. When the density is higher than \(2\rho_0\), the equation of state (EOS) and composition of matter are much less certain. The presence of hyperons in neutron stars has been studied by many authors \[4,8\]. \(K^-\) condensation in dense matter was suggested by Kaplan and Nelson \[9\] and has been extensively discussed in many works \[2,4,10\]. It has been suggested that the quark matter may exist in the core of massive neutron stars, and the hadron-quark phase transition can proceed through a mixed phase of hadronic and quark matter \[1,2,6,11\]. If deconfined quark matter does exist inside stars, it is likely to be in a color superconducting phase \[1,6\], and various color superconducting phases have been intensively investigated in recent years \[12–14\].

Baryon pairing is believed to play an important role in the evolution of neutron stars \[2,15,16\]. The presence of neutron superfluidity in the crust and the inner part of neutron stars can be considered well established \[15\]. The neutron fluid in the crust probably forms a \(^1S_0\) superfluid. With increasing density, the \(^1S_0\) interaction turns repulsive, and the neutrons in the outer core mainly form a \(^3P_2\) superfluid. On the other hand, one can expect \(^1S_0\) proton pairing in the outer core because the small proton fraction brings about a low proton density in this region. In the inner core of neutron stars, hyperons may appear through the weak interaction because of the fast rise of the baryon chemical potentials with density. It is widely accepted that hyperons appear around \(2\rho_0\). The presence of hyperons tends to soften the EOS at high density and lower the maximum mass of neutron stars \[4,8\] as well as increase the neutron star cooling rate \[16,17\]. In general, the first hyperon to appear is \(\Lambda\), which is the lightest one with an attractive potential in nuclear matter. The potential of \(\Sigma\) hyperons is now considered to be repulsive; therefore \(\Sigma^-\) appears at a higher density than \(\Lambda\) in neutron star matter \[18,19\]. The \(^1S_0\) superfluidity of \(\Lambda\) hyperons is suggested to occur in the same way as that of neutrons arising from the attractive \(\Lambda\Lambda\) interaction in the \(^1S_0\) channel \[20,24\]. It is known that hyperon pairing can significantly affect the thermal
evolution of neutron stars by suppressing neutrino emission from the hyperon direct Urca process [24–26]. Young neutron stars cool primarily by neutrino emission from the interior. As discussed in Refs. [27–29], the neutrino emissivity in superfluid matter is exponentially suppressed when the temperature $T$ is much lower than the superfluid critical temperature $T_c$. On the other hand, superfluidity initiates a specific neutrino emission from the Cooper pair breaking and formation process, which is forbidden in nonsuperfluid matter. This process is exponentially suppressed when $T \ll T_c$, and it is much less efficient than the direct Urca process [29, 30]. Hence the presence of baryon superfluidity can drastically suppress the neutrino emission, which may play a key role in neutron star cooling. We are mainly interested in the possibility of $^1S_0$ superfluidity of $\Lambda$ hyperons in neutron stars. So far, the $^1S_0$ pairing gap of $\Lambda$ hyperons is still uncertain because it can be significantly influenced by both the properties of matter and the $\Lambda\Lambda$ interaction. More studies are needed to determine these uncertain factors using available information from recent developments in hypernuclear physics.

In this article, we focus on the $^1S_0$ superfluidity of $\Lambda$ hyperons in neutron star matter, which is composed of a chemically equilibrated and charge-neutral mixture of nucleons, hyperons, and leptons. To calculate the pairing gap, we need to specify how to treat the neutron star matter and the $\Lambda\Lambda$ interaction. In this article, we use the relativistic mean field (RMF) theory to calculate the properties of neutron star matter. The RMF theory has been successfully and widely used for the description of nuclear matter and finite nuclei [31–35]. It has also been applied to providing the EOS of dense matter for use in supernovae and neutron stars [36]. In the RMF approach, baryons interact through the exchange of scalar and vector mesons. The meson-nucleon coupling constants are generally determined by fitting to some nuclear matter properties or ground-state properties of finite nuclei. To examine the influence of the RMF parameters, we employ two successful parameter sets, TM1 [37] and NL3 [38], which have been widely used in many studies of nuclear physics [11, 35–39]. As for the meson-hyperon couplings, there are large uncertainties because of limited experimental data in hypernuclear physics. Generally, one can use the coupling constants derived from the quark model or the values constrained by reasonable hyperon potentials. The meson-hyperon couplings play an important role in determining the properties of neutron star matter [8, 40]. We use the values constrained by reasonable hyperon potentials that include the updated information from recent developments in hypernuclear physics. We take into account the
two additional hidden-strangeness mesons, $\sigma^*$ and $\phi$, which were originally introduced to obtain the strong attractive $\Lambda\Lambda$ interaction deduced from the earlier measurement [41]. A recent observation of the double-$\Lambda$ hypernucleus $^6\Lambda\Lambda\text{He}$, called the Nagara event [42], has had a significant impact on strangeness nuclear physics. The Nagara event provides unambiguous identification of $^6\Lambda\Lambda\text{He}$ production with a precise $\Lambda\Lambda$ binding energy value $B_{\Lambda\Lambda} = 7.25 \pm 0.19^{+0.18}_{-0.11} \text{ MeV}$, which suggests that the effective $\Lambda\Lambda$ interaction should be considerably weaker ($\Delta B_{\Lambda\Lambda} \approx 1 \text{ MeV}$) than that deduced from the earlier measurement ($\Delta B_{\Lambda\Lambda} \approx 4$–5 MeV). The weak hyperon-hyperon ($YY$) interaction suggested by the Nagara event has been used to reinvestigate the properties of multistrange systems, and it has been found that the change of $YY$ interactions affects the properties of strange hadronic matter dramatically [11, 43–45]. We would like to examine whether the $^1S_0$ superfluidity of $\Lambda$ hyperons exists in neutron star matter, and how large the pairing gap can be if it does, by considering recent developments in hypernuclear physics.

The aim of this article is to investigate the possibility of forming a $\Lambda$ superfluid in neutron stars. It has been suggested that hyperon superfluidity could significantly suppress the neutrino emission in the core of a neutron star and play a key role in neutron star cooling [24, 25]. Over the last decade, there has been some discussion in the literature about hyperon pairing in dense matter [20–24, 46]. In the work of Balberg and Barnea [20], the $^1S_0$ superfluidity of $\Lambda$ hyperons has been studied by using an effective $\Lambda\Lambda$ interaction based on a $G$ matrix calculation and an approximation of nonrelativistic effective mass obtained from single-particle energies. Their calculation predicts a gap energy of a few tenths of a MeV for $\Lambda$ Fermi momenta up to about 1.3 fm$^{-1}$. In Refs. [21, 22], Takatsuka and Tamagaki studied $\Lambda$ superfluidity using two types of bare $\Lambda\Lambda$ interactions based on the one-boson-exchange (OBE) model and two types of hyperon core models. They found that $\Lambda$ superfluidity could exist in a density region between $2\rho_0$ and $(2.6–4.6)\rho_0$, depending on the pairing interaction and the hyperon core model. A study of $\Lambda\Lambda$ pairing in a pure neutron background has been presented by Tanigawa et al. [23] using the relativistic Hartree-Bogoliubov model, where the $\Lambda\Lambda$ pairing gap was found to decrease with increasing background density and decreasing $\Lambda\Lambda$ attraction. Both $\Lambda$ and $\Sigma^-$ superfluidities in neutron star matter have been investigated by Takatsuka et al. [24] using three pairing interactions based on the OBE model and several nonrelativistic EOS with different incompressibilities. It was found that both $\Lambda$ and $\Sigma^-$ are superfluid as soon as they begin to appear at around $4\rho_0$, although the pairing gap and the
density region depend on the pairing interaction and the EOS of neutron star matter. The
effect of the Nagara event on $\Lambda$ superfluidity has been discussed in Refs. [23, 24], where the
weak attractive $\Lambda\Lambda$ interaction suggested by the Nagara event leads to very small pairing
gap in dense neutron matter [23] or the disappearance of $\Lambda$ superfluidity in neutron star
matter [24]. All these studies indicate that the $\Lambda\Lambda$ pairing gap in dense matter depends
both on the $\Lambda\Lambda$ interaction and on the EOS of matter. The key role of $\Lambda$ superfluidity in
neutron star cooling motivates us to investigate the possibility of forming a $\Lambda$ superfluid in
neutron stars by carefully considering the pairing interaction and the description of neutron
star matter with the updated information from recent developments in hypernuclear physics.

This article is arranged as follows. In Sec. II, we briefly describe the RMF theory for
the calculation of neutron star matter properties. In Sec. III, we discuss the $\Lambda\Lambda$ interaction
used in the gap equation. We present the numerical results in Sec. IV. Section V is devoted
to a summary.

II. RELATIVISTIC MEAN FIELD THEORY

We use the RMF theory to describe the neutron star matter, which is composed of a
chemically equilibrated and charge-neutral mixture of nucleons, hyperons, and leptons. In
the RMF approach, baryons interact through the exchange of scalar and vector mesons. The
baryons considered in this work are nucleons ($p$ and $n$) and hyperons ($\Lambda$, $\Sigma$, and $\Xi$). The
exchanged mesons include isoscalar scalar and vector mesons ($\sigma$ and $\omega$), an isovector vector
meson ($\rho$), and two additional hidden-strangeness mesons ($\sigma^*$ and $\phi$). The total Lagrangian
density of neutron star matter takes the form

$$
\mathcal{L}_{\text{RMF}} = \sum_B \bar{\psi}_B \left[i\gamma_\mu \partial^\mu - m_B - g_{\sigma B} \sigma - g_{\sigma^* B} \sigma^* - g_{\omega B} \gamma_\mu \omega^\mu - g_{\phi B} \gamma_\mu \phi^\mu \right] \psi_B + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} g_2 \sigma^3 - \frac{1}{3} g_3 \sigma^4 - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_{\mu\nu} \omega^{\mu\nu} - \frac{1}{2} m_\rho^2 \rho_{\mu\nu} \rho^{\mu\nu}
+ \frac{1}{4} c_3 (\omega_{\mu\nu})^2 - \frac{1}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} S_{\mu\nu} S^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_{\mu\nu} \phi^{\mu\nu} + \sum_l \bar{\psi}_l \left[i\gamma_\mu \partial^\mu - m_l \right] \psi_l,
$$

(1)
where $\psi_B$ and $\psi_l$ are the baryon and lepton fields, respectively. The index $B$ runs over the baryon octet ($p$, $n$, $\Lambda$, $\Sigma^+$, $\Sigma^0$, $\Sigma^-$, $\Xi^0$, $\Xi^-$), and the sum on $l$ is over electrons and muons ($e^-$ and $\mu^-$). The field tensors of the vector mesons, $\omega$, $\rho$, and $\phi$, are denoted by $W_{\mu\nu}$, $R_{i\mu\nu}$, and $S_{\mu\nu}$, respectively. In the RMF approach, the meson fields are treated as classical fields, and the field operators are replaced by their expectation values. The meson field equations in uniform matter have the following form:

\begin{align}
    m_\sigma^2 \sigma + g_2 \sigma^2 + g_3 \sigma^3 &= -\sum_B \frac{g_{\sigma B}}{\pi^2} \int_0^{k_B^F} \frac{m_B^*}{\sqrt{k^2 + m_B^*}} k^2 dk, \\
    m_\omega^2 \omega + c_3 \omega^3 &= \sum_B \frac{g_{\omega B} (k_F^B)^3}{3\pi^2}, \\
    m_\rho^2 \rho &= \sum_B \frac{g_{\rho B} T_{3B} (k_F^B)^3}{3\pi^2}, \\
    m_\sigma^* \sigma^* &= -\sum_B \frac{g_{\sigma^* B}}{\pi^2} \int_0^{k_B^F} \frac{m_B^*}{\sqrt{k^2 + m_B^*}} k^2 dk, \\
    m_\phi^2 \phi &= \sum_B \frac{g_{\phi B} (k_F^B)^3}{3\pi^2},
\end{align}

where $\sigma = \langle \sigma \rangle$, $\omega = \langle \omega^0 \rangle$, $\rho = \langle \rho_3 \rangle$, $\sigma^* = \langle \sigma^* \rangle$, and $\phi = \langle \phi^0 \rangle$ are the nonvanishing expectation values of meson fields in uniform matter; $m_B^* = m_B + g_{\sigma B} \sigma + g_{\sigma^* B} \sigma^*$ is the effective mass of the baryon species $B$, and $k_F^B$ is the corresponding Fermi momentum.

The meson-baryon coupling constants play an important role in determining the properties of neutron star matter. To examine the influence of the RMF parameters, we employ two successful parameter sets, TM1 [37] and NL3 [38], in the present calculation. These parameters have been determined by fitting to some ground-state properties of finite nuclei, and they can provide a good description of nuclear matter and finite nuclei, including unstable nuclei. With the TM1 (NL3) parameter set, the nuclear matter saturation density is 0.145 fm$^{-3}$ (0.148 fm$^{-3}$), the energy per nucleon is $-16.3$ MeV ($-16.3$ MeV), the symmetry energy is 36.9 MeV (37.4 MeV), and the incompressibility is 281 MeV (272 MeV) [37, 38]. As for the meson-hyperon couplings, we take the naive quark model values for the vector
coupling constants;

\[
\frac{1}{3} g_{\omega N} = \frac{1}{2} g_{\omega \Lambda} = \frac{1}{2} g_{\omega \Sigma} = g_{\omega \Xi},
\]

\[
g_{\rho N} = \frac{1}{2} g_{\rho \Sigma} = g_{\rho \Xi}, \quad g_{\rho \Lambda} = 0,
\]

\[
2g_{\phi \Lambda} = 2g_{\phi \Sigma} = g_{\phi \Xi} = -\frac{2\sqrt{2}}{3} g_{\omega N}, \quad g_{\phi N} = 0.
\] (7)

The scalar coupling constants are chosen to give reasonable hyperon potentials. We denote the potential depth of the hyperon species \(i\) in the matter of the baryon species \(j\) by \(U_{i}^{(j)}\). It is estimated from the experimental data of single-\(\Lambda\) hypernuclei that the potential depth of a \(\Lambda\) in saturated nuclear matter should be around \(U_{\Lambda}^{(N)} \simeq -30\) MeV \[47\]. For \(\Sigma\) hyperons, the analysis of \(\Sigma\) atomic experimental data suggests that \(\Sigma\)-nucleus potentials have a repulsion inside the nuclear surface and an attraction outside the nucleus with a sizable absorption. In recent theoretical works, the \(\Sigma\) potential in saturated nuclear matter is considered to be repulsive with a strength of about 30 MeV \[18, 47\]. Some recent developments in hypernuclear physics suggest that \(\Xi\) hyperons in saturated nuclear matter have an attraction of around 15 MeV \[18, 48\]. In this article, we adopt \(U_{\Lambda}^{(N)} = -30\) MeV, \(U_{\Sigma}^{(N)} = +30\) MeV, and \(U_{\Xi}^{(N)} = -15\) MeV to determine the scalar coupling constants. We obtain, for the TM1 (NL3) parameter set, \(g_{\sigma \Lambda} = 6.228\ (6.323)\), \(g_{\sigma \Sigma} = 4.472\ (4.709)\), and \(g_{\sigma \Xi} = 3.114\ (3.161)\), respectively. The hyperon couplings to the hidden-strangeness meson \(\sigma^*\) are restricted by the relation \(U_{\Xi}^{(\Xi)} \approx U_{\Lambda}^{(\Xi)} \approx 2U_{\Lambda}^{(\Lambda)} \approx 2U_{\Lambda}^{(\Xi)}\) obtained in Ref. \[49\]. The weak YY interaction implied by the Nagara event suggests \(U_{\Lambda}^{(\Lambda)} \simeq -5\) MeV, and hence we obtain \(g_{\sigma^* \Lambda} = 5.499\ (5.678)\) and \(g_{\sigma^* \Xi} = 11.655\ (11.899)\) for the TM1 (NL3) parameter set. We assume \(g_{\sigma^* \Sigma} = g_{\sigma^* \Lambda}\) and take \(m_{\sigma^*} = 980\) MeV and \(m_{\phi} = 1020\) MeV in this article.

For neutron star matter consisting of a neutral mixture of baryons and leptons, the \(\beta\)-equilibrium conditions without trapped neutrinos are given by

\[
\mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e,
\]

\[
\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n,
\]

\[
\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e,
\]

\[
\mu_\mu = \mu_e,
\]

where \(\mu_i\) is the chemical potential of species \(i\). At zero temperature, the chemical potentials
of baryons and leptons are given by
\[ \mu_B = \sqrt{k_B^2 + m_B^2 + g_\omega B \omega + g_\phi B \phi + g_{\rho B} \tau_3 B \rho}, \quad (12) \]
\[ \mu_l = \sqrt{k_l^2 + m_l^2 + g_\omega l \omega + g_\phi l \phi + g_{\rho l} \tau_3 l \rho}, \quad (13) \]
respectively. The electric charge neutrality condition is expressed by
\[ \rho_p + \rho_{\Sigma^+} = \rho_e + \rho_{\mu} + \rho_{\Sigma^-} + \rho_{\Xi^-}, \quad (14) \]
where \( \rho_i = \left( k_F^i \right)^3 / (3\pi^2) \) is the number density of species \( i \). We solve the coupled Eqs. (2)–(6), (8)–(11), and (14) self-consistently at a given baryon density \( \rho_B \). Then we can calculate the EOS and the composition of neutron star matter as well as the effective mass and the Fermi momentum of \( \Lambda \) hyperons, which are crucial in the study of \( \Lambda \Lambda \) pairing.

### III. GAP EQUATION AND \( \Lambda \Lambda \) INTERACTION

We study the \( ^1S_0 \) superfluidity of \( \Lambda \) hyperons in neutron star matter. The crucial quantity in determining the onset of superfluidity is the energy gap function \( \Delta(k) \), which can be obtained by solving the gap equation
\[ \Delta(k) = -\frac{1}{4\pi^2} \int k'^2 dk' \frac{V(k, k') \Delta(k')}{\sqrt{[E(k') - E(k^2)]^2 + \Delta^2(k')}} , \quad (15) \]
where \( E(k) \) is the single-particle energy of \( \Lambda \) with momentum \( k \). For \( \Lambda \) hyperons in neutron star matter, the single-particle energy in the RMF approach is given by
\[ E(k) = \sqrt{k^2 + m_{\Lambda}^2 + g_{\omega \Lambda} \omega + g_{\phi \Lambda} \phi} . \]
The effective mass \( m_{\Lambda}^* \) and the Fermi momentum \( k_{F\Lambda} \) are computed self-consistently at a given baryon density \( \rho_B \) within the RMF approach.

For the \( \Lambda \Lambda \) pairing interaction in the \( ^1S_0 \) channel, the potential matrix element can be written as
\[ V(k, k') = \langle k | V_{\Lambda\Lambda} (^1S_0) | k' \rangle = 4\pi \int r^2 dr \ j_0(kr) V_{\Lambda\Lambda}(r) j_0(k'r) , \quad (16) \]
where \( j_0(kr) = \sin(kr)/(kr) \) is the spherical Bessel function of order zero and \( V_{\Lambda\Lambda}(r) \) is the \( ^1S_0 \) \( \Lambda\Lambda \) interaction potential in coordinate space. Because of large uncertainties in the
ΛΛ interaction, we adopt several ΛΛ potentials. Most of them are based on the Nijmegen models and are used in double-Λ hypernuclei studies, which are of the three-range Gaussian form

\[ V_{\Lambda\Lambda}(r) = \sum_{i=1}^{3} v_i \exp\left(-r^2/\beta_i^2\right). \]  

(17)

The short-range term provides for a strong soft-core repulsion, whereas the medium-range and long-range terms provide for attraction. The parameters \( v_i \) and \( \beta_i \) are taken from Refs. [50–53], and we list them in Table I. The ND1 potential was given in Ref. [50] as an effective soft-core interaction fitted to the Nijmegen model D (ND) hard-core interaction. Another simulation of the ND interaction, called ND2 in this article, was obtained in Ref. [51]. The ESC00, NSC97b, NSC97e, and NSC97f potentials given in Ref. [51] were obtained by changing the strength of the medium-range attractive component of the three-range Gaussian potential such that they could reproduce the scattering length and the effective range as close to values by the corresponding Nijmegen models. It is well known that the Nagara event provides unambiguous identification of \( ^6\Lambda\Lambda\)He production with a precise ΛΛ binding energy value \( B_{\Lambda\Lambda} \) and has had a significant impact on strangeness nuclear physics. The NFs and NSC97s potentials given in Refs. [52, 53] were obtained by adjusting parameters to reproduce the experimental value of \( B_{\Lambda\Lambda}(^6\Lambda\Lambda\)He) from the Nagara event. We have also chosen an Urbana-type potential that has been successfully used to explain the experimental values of hypernuclei. The Urbana potential could be found in Ref. [54].

We plot in Fig. I all ΛΛ potentials considered in the present work. The strongest ΛΛ interaction is the ESC00 potential, whereas the weakest ΛΛ interaction is the NSC97f potential. We note that the NFs, NSC97s, and Urbana potentials simulate the experimental value of \( B_{\Lambda\Lambda}(^6\Lambda\Lambda\)He) from the Nagara event.

IV. RESULTS AND DISCUSSION

In this section, we investigate the \( ^1S_0 \) superfluidity of Λ hyperons in neutron star matter and neutron stars. We employ the RMF model with the parameter sets TM1 and NL3 to calculate the properties of neutron star matter, which is known to provide excellent descriptions of the ground states of finite nuclei, including unstable nuclei. The meson-hyperon couplings play an important role in determining the properties of neutron star matter. We use the values constrained by reasonable hyperon potentials that include the
updated information from recent developments in hypernuclear physics. As for the $\Lambda\Lambda$ pairing interaction used in the gap equation, we adopt several $\Lambda\Lambda$ potentials that have been used in double-$\Lambda$ hypernuclei studies. Some simulate the experimental value of $B_{\Lambda\Lambda}({}^6\Lambda\Lambda\text{He})$ from the Nagara event. With the effective mass and the Fermi momentum of $\Lambda$ hyperons obtained in the RMF approach, the gap equation [Eq. (15)] is solved numerically.

In Fig. 2, we show the resulting $^1S_0$ pairing gap of $\Lambda$ hyperons at the Fermi surface, $\Delta_F$, as a function of the baryon density, $\rho_B$, in neutron star matter. The results of TM1 and NL3 are plotted in Fig. 2 (top) and Fig. 2 (bottom), respectively. In the case of TM1 (NL3), the threshold density of $\Lambda$ is around $0.31 \text{ fm}^{-3}$ ($0.28 \text{ fm}^{-3}$), and $\Lambda$ hyperons form a $^1S_0$ superfluid as soon as they appear in neutron star matter. With increasing baryon density, $\Delta_F$ increases first, reaching a maximum value at $\rho_B \sim 0.34 \text{ fm}^{-3}$ ($\rho_B \sim 0.30 \text{ fm}^{-3}$), then decreases and finally vanishes at $\rho_B < 0.46 \text{ fm}^{-3}$ ($\rho_B < 0.38 \text{ fm}^{-3}$) for the case of TM1 (NL3) with the ESC00 potential. It is found that the maximal pairing gap is about 0.8 MeV with the ESC00 potential in the TM1 case. This is because the ESC00 potential has the strongest attraction among the $\Lambda\Lambda$ interactions used here. The pairing gaps with the ND1 and ND2 potentials are of the order of $0.1-0.2$ MeV, as shown in Fig. 2. In addition, we find that the pairing gaps are of the order of $10^{-4}$ MeV (TM1) or absent (NL3) with the NSC97e, NFs, NSC97s, and Urbana potentials. The $\Lambda$ pairing does not appear for the NSC97b and NSC97f potentials. We present in Table II the maximal pairing gap at the Fermi surface ($\Delta_F^{\text{max}}$) and the corresponding baryon density ($\rho_B$), effective $\Lambda$ mass ($m_\Lambda^*$), and Fermi momentum ($k_F^*$) using these potentials with the TM1 and NL3 parameter sets.

The $\Lambda$ pairing gap $\Delta_F$ depends not only on the $\Lambda\Lambda$ interaction but also on the properties of $\Lambda$ hyperons in neutron star matter. In Fig. 3, we show the particle fraction, $Y_i = \rho_i/\rho_B$, as a function of the baryon density, $\rho_B$, using the RMF model with the TM1 (Fig. 3 top) and NL3 (Fig. 3 bottom) parameter sets. It is seen that $\Lambda$ hyperons appear around $0.31 \text{ fm}^{-3}$ (TM1) or $0.28 \text{ fm}^{-3}$ (NL3) and then increase rapidly with increasing density. We note that hyperon threshold densities, fractions, and effective masses are dependent on the RMF parameters used. This dependence has an effect on the resulting pairing gap, as shown in Fig. 2. Our results with the ND1 potential can be compared with those in Table III of Ref. [24], where the same $\Lambda\Lambda$ interaction (called the ND-Soft potential) was used. The difference is in the treatment of neutron star matter, for which they use a nonrelativistic $G$ matrix-based effective interaction approach, whereas we use the RMF approach. In our case
of TM1 (NL3), the maximal pairing gap at the Fermi surface is 0.17 MeV (0.12 MeV), as given in Table II, where $\rho_B = 0.344 \text{ fm}^{-3}$ ($\rho_B = 0.303 \text{ fm}^{-3}$), $Y_\Lambda = 0.039$ ($Y_\Lambda = 0.044$), and $m_\Lambda^* = 743 \text{ MeV}$ ($m_\Lambda^* = 706 \text{ MeV}$). Takatsuka et al. [24] obtained the maximal pairing gap of 0.34 MeV at $\rho_B = 4.5\rho_0$ for the TNI6u EOS. The larger pairing gap at higher $\rho_B$ given in Ref. [24] is because of their smaller $Y_\Lambda$ and larger $m_\Lambda^*$. As discussed in Refs. [20, 23, 24], the pairing gap is very sensitive to the effective mass. Generally, a smaller effective mass leads to a higher single-particle energy and then yields a smaller pairing gap. In Fig. 4, we show the effective mass of $\Lambda$ hyperons, $m_\Lambda^*$, as a function of the baryon density, $\rho_B$, using the RMF model with the TM1 (solid line) and NL3 (dashed line) parameter sets. It is shown that $m_\Lambda^*$ decreases with increasing $\rho_B$. When $\Lambda$ hyperons appear around $0.31 \text{ fm}^{-3}$ (TM1) or $0.28 \text{ fm}^{-3}$ (NL3), the effective mass of $\Lambda$ hyperons is about 762 MeV or 727 MeV. It is found that the effective masses of $\Lambda$ hyperons in the NL3 case are smaller than those of TM1, which leads to a smaller pairing gap, as shown in Fig. 2. We note that the effective mass of $\Lambda$ hyperons is mainly determined by the coupling constants $g_{\sigma\Lambda}$ and $g_{\sigma^*\Lambda}$. Here we use the values constrained by reasonable hyperon potentials, which are suggested by the experimental data of single-$\Lambda$ hypernuclei and by the Nagara event.

To examine whether the $^1S_0$ superfluidity of $\Lambda$ hyperons exists in neutron stars, we solve the Tolman-Oppenheimer-Volkoff (TOV) equation with the EOS of the RMF theory over a wide density range. For the nonuniform matter at low density, which exists in the inner and outer crusts of neutron stars, we adopt a relativistic EOS based on the RMF theory with a local density approximation [7, 36]. The nonuniform matter is modeled to be composed of a lattice of spherical nuclei immersed in an electron gas with or without free neutrons dripping out of nuclei. The low-density EOS is matched to the EOS of uniform matter at the density where they have equal pressure. The neutron star properties are mainly determined by the EOS at high density. Using the EOS described in Sec. II, we calculate the neutron star properties and find that the maximum mass of neutron stars is about 1.70 $M_\odot$ (2.06 $M_\odot$) with the TM1 (NL3) parameter set. According to the compilation of measured neutron star masses [3, 55], some massive neutron stars were reported to be observed recently. However, the uncertainties in these mass measurements are rather large, and the mass of PSR J0751+1807 was corrected from $(2.1 \pm 0.2) M_\odot$ to $(1.26 \pm 0.14) M_\odot$ [56]. We note that the EOS used here could not be ruled out by current observations. In Figs. 5 and 6, we show the central baryon density as a function of the neutron star mass. We find that whether the $^1S_0$
superfluidity of Λ hyperons exists in the core of neutron stars depends on the ΛΛ interaction used. With weaker ΛΛ interactions, such as NSC97b and NSC97f, the Λ superfluidity does not appear inside neutron stars. For the NSC97e, NFs, NSC97s, and Urbana interactions, although we obtain the pairing gaps of the order of $10^{-4}$ MeV in the TM1 case, it is unlikely that Λ superfluidity can exist in observed neutron stars because of its low superfluid critical temperature $T_c \simeq 0.57\Delta_F/\kappa_B \sim 10^6$ K [24, 29]. With stronger ΛΛ interactions, such as ESC00, ND1, and ND2, the $^1S_0$ superfluidity of Λ hyperons may exist in massive neutron stars, as shown in Figs. 5 and 6. In the case of TM1 (NL3) with the ESC00 potential, Λ hyperons do not appear in neutron stars with $M < 1.37 M_\odot$ ($M < 1.50 M_\odot$). For neutron stars with $1.37 M_\odot < M < 1.63 M_\odot$ ($1.50 M_\odot < M < 1.82 M_\odot$), Λ hyperons in the core of neutron stars form a $^1S_0$ superfluid. However, when $M > 1.63 M_\odot$ ($M > 1.82 M_\odot$), not only superfluid Λ but also normal (nonsuperfluid) Λ can exist in the core of neutron stars because the central baryon density exceeds the upper limit of the range where Λ superfluidity exists. The presence of nonsuperfluid Λ hyperons in the core of massive stars would lead to a more rapid cooling than the case with only superfluid Λ hyperons. The mass region, where only superfluid Λ hyperons exist in the core of neutron stars, is shaded in Figs. 5 and 6. It is shown that the region with the ESC00 potential is the widest among all cases in these figures. This is because the ESC00 potential has the strongest attraction, and its pairing gap covers the widest density range, as shown in Fig. 2. We note that this region depends both on the ΛΛ interaction and on the EOS of neutron star matter.

V. SUMMARY

We have studied the $^1S_0$ superfluidity of Λ hyperons in neutron star matter and neutron stars. In this article, we employ the RMF model with the parameter sets TM1 and NL3 to calculate the properties of neutron star matter, which is composed of a chemically equilibrated and charge-neutral mixture of nucleons, hyperons, and leptons. The RMF theory has been successfully and widely used for the description of nuclear matter and finite nuclei, including unstable nuclei. In the RMF approach, baryons interact through the exchange of scalar and vector mesons. The baryons considered in this article are nucleons ($p$ and $n$) and hyperons ($\Lambda$, $\Sigma$, and $\Xi$). The exchanged mesons include isoscalar scalar and vector mesons ($\sigma$ and $\omega$), an isovector vector meson ($\rho$), and two additional hidden-strangeness
mesons ($\sigma^*$ and $\phi$). It is well known that the meson-hyperon couplings play an important role in determining the properties of neutron star matter. We have used the couplings constrained by reasonable hyperon potentials that include the updated information from recent developments in hypernuclear physics. To examine the $^1S_0$ pairing of $\Lambda$ hyperons, we have adopted several $\Lambda\Lambda$ potentials. Most are based on the Nijmegen models and have been used in double-$\Lambda$ hypernuclei studies. NFs, NSC97s, and Urbana potentials have simulated the experimental value of $B_{\Lambda\Lambda}(^6\Lambda\Lambda\text{He})$ from the Nagara event.

We have calculated the $^1S_0$ pairing gap of $\Lambda$ hyperons at the Fermi surface, $\Delta_F$, using the $\Lambda\Lambda$ potentials adopted in this article. It is found that $\Delta_F$ depends both on the $\Lambda\Lambda$ interaction and on the treatment of neutron star matter. The maximal $\Delta_F$ obtained in the present calculation is about 0.8 MeV with the ESC00 potential in the TM1 case. This is because the ESC00 potential has the strongest attraction among the $\Lambda\Lambda$ interactions used in this article. The ND1 and ND2 potentials yield somewhat smaller $\Delta_F$ of the order of 0.1–0.2 MeV. For the NSC97e, NFs, NSC97s, and Urbana potentials, the values of $\Delta_F$ are of the order of $10^{-4}$ MeV (TM1) or absent (NL3). The $\Lambda$ pairing does not appear for the NSC97b and NSC97f potentials. The difference in these results reflects the dependence of $\Delta_F$ on the $\Lambda\Lambda$ interaction. On the other hand, the magnitude and the threshold density of $\Delta_F$ are also dependent on properties of neutron star matter, especially on the effective mass and particle fraction of $\Lambda$ hyperons. In the case of TM1 (NL3) with the ESC00 potential, the threshold density of $\Delta_F$ is around 0.31 fm$^{-3}$ (0.28 fm$^{-3}$), reaches a maximum value at $\rho_B \sim 0.34$ fm$^{-3}$ ($\rho_B \sim 0.30$ fm$^{-3}$), and finally vanishes at $\rho_B < 0.46$ fm$^{-3}$ ($\rho_B < 0.38$ fm$^{-3}$). By solving the TOV equation, we have calculated neutron star properties and found that whether the $^1S_0$ superfluidity of $\Lambda$ hyperons exists in the core of neutron stars mainly depends on the $\Lambda\Lambda$ interaction used. With stronger $\Lambda\Lambda$ interactions, such as ESC00, ND1, and ND2, the $\Lambda$ superfluidity may exist in massive neutron stars. It is unlikely that $\Lambda$ superfluidity can exist in neutron stars with the NFs, NSC97s, and Urbana interactions, which have simulated the experimental value of $B_{\Lambda\Lambda}(^6\Lambda\Lambda\text{He})$ from the Nagara event.

In this article, we have considered the updated information from recent developments in hypernuclear physics and used the weak attractive $\Lambda\Lambda$ interactions suggested by the Nagara event. However, there are still large uncertainties in the hyperon-hyperon interaction and the EOS of neutron star matter. A more precise study of the $\Lambda$ pairing in neutron stars requires further development in hypernuclear physics.
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TABLE I: Parameters of $^1S_0$ AA interaction defined in Eq. (17), taken from Refs. [50–53]. The size parameters are the same for all cases, which are $\beta_1 = 1.342$ fm, $\beta_2 = 0.777$ fm, and $\beta_3 = 0.350$ fm. The strength parameters are in MeV.

|       | $v_1$  | $v_2$  | $v_3$  |
|-------|--------|--------|--------|
| ND1   | -21.92 | -283.5 | 4745   |
| ND2   | -21.49 | -379.1 | 9324   |
| ESC00 | -21.49 | -456.6 | 9324   |
| NSC97b| -21.49 | -182.1 | 9324   |
| NSC97e| -21.49 | -207.1 | 9324   |
| NSC97f| -21.49 | -177.1 | 9324   |
| NFs   | -10.96 | -141.8 | 2137   |
| NSC97s| -21.49 | -250.1 | 9324   |
TABLE II: Maximal pairing gap at the Fermi surface $\Delta_{F}^{\text{max}}$ obtained with several $\Lambda\Lambda$ potentials; $\rho_B$ is the total baryon density of neutron star matter where $\Delta_{F}^{\text{max}}$ is obtained, and $k_{F}^{\Lambda}$ and $m_{\Lambda}^{*}$ are the corresponding Fermi momentum and effective mass of $\Lambda$ hyperons, respectively.

|       | TM1                  | NL3                  |
|-------|----------------------|----------------------|
|       | $\rho_B$ (fm$^{-3}$) | $k_{F}^{\Lambda}$ (fm$^{-1}$) | $m_{\Lambda}^{*}$ (MeV) | $\Delta_{F}^{\text{max}}$ (MeV) | $\rho_B$ (fm$^{-3}$) | $k_{F}^{\Lambda}$ (fm$^{-1}$) | $m_{\Lambda}^{*}$ (MeV) | $\Delta_{F}^{\text{max}}$ (MeV) |
| ND1   | 0.344                | 0.738                | 743  | 0.17  | 0.303                | 0.731                | 706  | 0.12  |
| ND2   | 0.339                | 0.681                | 747  | 0.10  | 0.298                | 0.664                | 711  | 0.06  |
| ESC00 | 0.349                | 0.789                | 740  | 0.81  | 0.305                | 0.762                | 704  | 0.62  |
| NSC97b| -                    | -                    | -    | -     | -                    | -                    | -    | -     |
| NSC97e| 0.329                | 0.548                | 753  | $1.2 \times 10^{-4}$ | -                    | -                    | -    | -     |
| NSC97f| -                    | -                    | -    | -     | -                    | -                    | -    | -     |
| NFs   | 0.329                | 0.548                | 753  | $5.4 \times 10^{-4}$ | -                    | -                    | -    | -     |
| NSC97s| 0.329                | 0.548                | 753  | $4.0 \times 10^{-4}$ | -                    | -                    | -    | -     |
| Urbana| 0.329                | 0.548                | 753  | $5.5 \times 10^{-4}$ | -                    | -                    | -    | -     |

FIG. 1: (Color online) $^1S_0$ $\Lambda\Lambda$ interaction potentials used in this article.
FIG. 2: (Color online) $^1S_0$ pairing gap of Λ hyperons at the Fermi surface $\Delta_F$ as a function of baryon density $\rho_B$ in neutron star matter with the ND1, ND2, and ESC00 potentials: (top) TM1 and (bottom) NL3.

FIG. 3: (Color online) Particle fraction $Y_i = \rho_i / \rho_B$ as a function of baryon density $\rho_B$: (top) TM1 and (bottom) NL3.
FIG. 4: (Color online) Effective mass of Λ hyperons $m_\Lambda^*$ as a function of baryon density $\rho_B$. 
FIG. 5: Central baryon density $\rho_c$ as a function of neutron star mass $M$ in the TM1 case. The region where only superfluid $\Lambda$ hyperons exist in the core of neutron stars is shaded.
FIG. 6: Same as Fig. 5, but for the NL3 case.