Branes and instantons intersecting at angles

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Abstract: We study in detail the system of D6 branes and euclidean D2-brane instantons intersecting at angles in type IIA string theory. We find that in the absence of orientifolds the system does not contribute to the low energy superpotential, in agreement with expectations based on effective field theory arguments. We also comment on the implications of our results for dual string theory pictures.
1. Introduction

In the last few years there has been growing interest in non-perturbative effects coming from euclidean D-brane instantons [1], mainly due to the realization [2, 3, 4] that they could be used for solving some longstanding difficulties in string model building coming from perturbative symmetries. We refer the reader to [5] for a survey of the recent developments.

Of central importance in the study of instanton effects is the issue of fermionic zero modes. They are Grassmann variables, and integration over their measure makes the contribution of the instanton to a given observable vanish unless they are properly saturated by explicit insertions. This severely constrains the quantities a given instanton will contribute to.

In this paper we will be mainly interested in contributions to the low energy effective superpotential $W$ of a $\mathcal{N} = 1$ theory. Our focus will be on four dimensional $\mathcal{N} = 1$ theories realized as Calabi-Yau compactifications of type II string theory in the presence of D$(3+p)$ branes, and for the most part we will be dealing with type IIA in the presence of D6 branes. We want to understand instanton effects coming from a single euclidean D2 brane wrapping some special Lagrangian cycle in the internal manifold, and localized in the four Minkowski directions.
Fermionic zero modes of the instanton come from open strings going from the instanton to itself, or to background D6 branes. Because of their charge under the gauge $U(1)$ of the D6 branes, zero modes of the second kind are commonly called *charged*, while those of the first kind are called *neutral*. The biggest constraint in model building with instantons comes from the neutral zero modes, they are the ones we will be trying to understand in this paper.

The issue with neutral zero modes in this class of compactifications is the following: despite the fact that the background preserves four supercharges, away from the background branes and orientifolds (and in the absence of flux, see below) there are eight conserved supercharges, since locally we are dealing with a Calabi-Yau compactification of type II. Our D-brane instantons are 1/2 BPS objects, so they break four of the eight supersymmetries. The broken supersymmetries will be nonlinearly realized as goldstinos in the worldvolume of the instanton. We will denote these zero modes $\theta^\alpha$ and $\bar{\tau}^\dot{\alpha}$. The two $\theta^\alpha$ goldstinos are expected for a superpotential: one gets the coupling in the Lagrangian from the term going with $\theta^2$ in the expansion of the superpotential superfield. In string theory this is realized as a disk diagram with explicit insertions of two $\theta^\alpha$ modes (see [2], for example). On the other hand, the $\bar{\tau}^\dot{\alpha}$ should be removed from the spectrum if we want to generate a superpotential.

There are various known mechanisms for saturating or removing the $\bar{\tau}^\dot{\alpha}$ modes. The simplest one is to put one or more space-filling branes on top of the instanton. In this case, reviewed in section 2, the instanton is interpreted as a gauge instanton in the theory of the space-filling brane.\(^1\) The case with a single space-filling brane is slightly special, since it does not admit a gauge theory interpretation, but the lifting of $\bar{\tau}^\dot{\alpha}$ modes still takes place \([7, 8, 9, 10, 11]\). When there are orientifolds in the system, instantons wrapping cycles invariant under the orientifold action and such that the gauge symmetry on their worldvolume is $O(1)$ (the so-called $O(1)$ instantons) are also interesting. In this case the orientifold projects out the undesired $\bar{\tau}^\dot{\alpha}$ modes \([12, 13, 14, 15]\) and superpotential contributions can be generated. Along similar lines, multi-instanton configurations which can recombine into a $O(1)$ instanton can saturate all zero modes and thus contribute to the superpotential \([16, 8, 17, 18]\). The last possibility is to consider the instanton in the presence of background flux \([19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 10, 15, 23, 30, 31]\). In most known scenarios the fluxes one has to introduce in order to lift $\bar{\tau}^\dot{\alpha}$ modes necessarily break supersymmetry \([10, 15, 29, 30, 31]\), a fact that can be understood most easily from the 4d effective field theory point of view \([18, 31]\).

The focus of this paper is a careful analysis of a system composed of a D6 brane and a rigid D-brane instanton intersecting at angles in the internal space, in the absence of orientifolds or fluxes.\(^2\) This system is of particular interest in the study of D-brane instanton effects in particle physics models. Our key result is that in this case the $\bar{\tau}^\dot{\alpha}$ modes are not lifted, and thus no superpotential is generated by the D-brane instantons.

\(^1\)The usual gauge effects coming from instantons are reproduced in this way. As an example, \([3]\) gives a concrete realization of the ADS superpotential using D-brane instantons.

\(^2\)A related system has been recently studied in \([32, 33]\).
This negative result has important implications for the particle physics model building, since it excludes such ubiquitous configurations from generating superpotential terms. In the following we will elaborate in detail on the origin of this rather strong result.

The main tool we will use in our analysis is a microscopic CFT calculation, although we will also discuss some general effective field theory arguments forbidding the generation of the superpotential. This is the main reason why we have chosen to work in type IIA: close to the point(s) where the instanton and the brane intersect one has a good CFT description of the system, and the fact that the \( \bar{\tau} \) modes are not lifted can be shown convincingly. The type IIA picture is also related by duality to backgrounds in type IIB, and we comment on implications of our result for such systems.

In our CFT analysis we will restrict to the oscillator modes that become massless when the D6 gets aligned with the instanton. More precisely, in terms of its dependence on the angles \( \theta_i \) parameterizing the local rotation between the instanton and the brane, the mass of any oscillator mode is of the form (in string units):

\[
m = c + \sum_{i=1}^{3} a_i \theta_i
\]

with \( c \) and \( a_i \) being some constants depending on the state. We will restrict to modes with \( c = 0 \). Modes with \( c \neq 0 \) are massive and genuinely stringy, and we believe that they will not affect the results of our analysis. The main reason for believing this is that similar modes are present in the gauge instanton case, where they do not modify the analysis in any substantial way.

We will also restrict to the vicinity of a single intersection, which can be modelled by a system of branes at angles in flat space. The general configuration of interest in model building will have D6 branes wrapping special Lagrangian manifolds, possibly intersecting at more than one point. Nevertheless, the result of the analysis still applies simply because \( \bar{\tau} \) is not saturated in any intersection, and thus cannot be saturated globally.

A holomorphy argument, and the importance of tachyons

We will be mostly interested in the case where the brane and the instanton share some supersymmetries. Nevertheless, the fact that there are some common supersymmetries is only true at very special points in complex structure moduli space, and in a generic point in complex structure moduli space the brane and the instanton will be misaligned, preserving no common supersymmetries. By holomorphy of the generated low energy superpotential, this implies \[8, 18\] that no superpotential can be generated anywhere in complex structure moduli space, including the points where the brane and the instanton share some supersymmetries.\[3\]

One worry with this argument is that away from the supersymmetric point in moduli space, the instanton might dissolve into the brane, restoring supersymmetry. As discussed

\[3\]For related considerations, we also refer the reader to the recent work \[14\], where the global behavior of instanton contributions to the superpotential has been linked to the topological string.
microscopically in [16] and applied for global considerations in [8, 18] something similar happens in the case where we have a two-instanton process where the two instantons misalign: the system will still give us a superpotential contribution if the two misaligned instantons can recombine into a single BPS instanton with the proper structure of zero modes. In the context of the system discussed in this paper, such a recombination process would be signalled by the appearance of a tachyonic open string between the instanton and the brane, coming from the bosonic modes we will denote $\omega_\alpha$ in the following.

It is easy to argue that there is no such tachyon; the argument goes as follows. We will see in section 3 that the bosonic modes have a positive mass in the supersymmetric case (see also [2]). Generic infinitesimal deformations of the angles make the system non-supersymmetric, while keeping the mass of $\omega_\alpha$ bigger than zero. In order for the misaligned instanton to contribute, it must have a tachyon at each non-supersymmetric point, so we conclude that recombination is not possible, and the instanton does not contribute.

Remarkably, there are systems where there is a tachyon between the brane and the instanton, and the misaligned instanton just dissolves into the brane, restoring supersymmetry. As an illustration of this effect, in section 5 we discuss the non-commutative gauge instanton.

The microscopic analysis

The previous macroscopic argument is powerful and general, but it does not show what goes wrong microscopically. Let us discuss here one such puzzle arising from the microscopic point of view.

Misaligned with respect to the brane or not, there are always 4 neutral fermionic zero modes in the instanton worldvolume due to the fact that it is a 1/2 BPS object of the Calabi-Yau background (we are assuming that there is no orientifold around). In the case of the gauge instanton, two of the four zero modes correspond to supersymmetries broken by both the instanton and the brane. This partial supersymmetry breaking by the background brane appears as an effective coupling in the instanton worldvolume lifting these extra “pseudo-goldstinos” (we review all this more carefully in section 2).

In the case where the instanton and the brane are intersecting and misaligned there are no shared supersymmetries, so one might expect four genuine goldstinos not lifted by any interactions. Thus, as mentioned above, the results of [18] are expected to apply. Nevertheless, it is not hard to see that there are couplings in the instanton action similar to those that lift the “pseudo-goldstinos” in the gauge instanton case (we will show this using CFT techniques in section 3), only this time $\tau_\alpha$ couples to massive modes. One might still wonder what is the effect of these couplings.

More precisely, in (3.41) we will find the following action for the modes going from the instanton to the brane:

$$
S^{E2-D6} = m_{\overline{\tau}_A} \overline{\tau}_A + m_{\mu A} \mu_A + m_{\mu B} \mu_B + m_\omega \omega + \overline{\tau} (\overline{\omega} \mu^0 + \overline{\mu}^0 \omega) + i \overline{D} : \omega \sigma \overline{\omega} 
$$

where $\overline{\tau}_\alpha$, $\mu_A$, $\mu'_A$, $\overline{\tau}_A$, and $\overline{\tau}_A$ are the Grassmann variables we have to saturate in the measure, $\overline{\omega}^A$ is the lightest bosonic modes from the instanton to the brane, and $D$ is the
D-term auxiliary field. Note the presence of the primed modes $\mu', \overline{\mu}'$, which are not present in the gauge instanton case. They are required in order to give masses to $\mu$ and $\overline{\mu}$. As we will describe in section 3.2, this action is compatible with the supersymmetries preserved by the system, and in fact its form is highly constrained by these same supersymmetries. It is also easy to see (we show this in detail in section 3.3) that with such interactions we cannot saturate all Grassmann integrations: if we bring down the second coupling in order to saturate $\overline{\mu}'$, then we also saturate $\mu$, and we cannot use the first term to saturate $\overline{\mu}$. We believe that our CFT analysis in section 3 settles conclusively this and related issues arising from the microscopic point of view.

This paper is organized as follows. In section 2 we review some background material on the embedding of gauge instantons in string theory that might be useful for comparison with the intersecting brane case. Section 3 contains the core of this work, a CFT analysis of the brane-instanton system. Section 4 contains some comments on systems related by duality. Finally, section 5 contains a short discussion of the non-commutative instanton as an illustration of a system where recombination is allowed.

2. A short review of gauge instantons in string theory

Before starting the analysis of the somewhat exotic intersecting brane and instanton system in section 3, let us review here some important features of the gauge instanton case. As we will see, there are some crucial differences between both cases, but also some important parallels that make it worth to keep the gauge case in mind. Here we will just discuss the main points of interest to us, we refer the reader to [35, 36] for much more detailed discussions.

In many ways the nicest system where we can discuss gauge instantons in string theory is $\mathcal{N} = 4$ $SU(N)$ SYM, which can be engineered simply by putting $N$ coincident D3 branes in flat space. The gauge instantons of this theory appear as $D(-1)$ branes dissolved on the worldvolume of the D3. The moduli space of instantons in field theory appears as the Higgs branch of the moduli space of vacua of the theory living on the worldvolume of the $D(-1)$ brane [37]. Although it is possible to work directly in the $0 + 0$ dimensional worldvolume theory of the $D(-1)$ instanton, and in fact for the intersecting case we will be forced to work in this way, for the $\mathcal{N} = 4$ theory it is simpler to work in the D5-D9 system instead, and at the end dimensionally reduce the results along the 6 common directions. This makes the effect of the supersymmetry clearer, and the interpretation of the fields in the $D(-1)$ worldvolume more transparent. Since they do not play a role in the intersecting case, we will set the fields living on the D9 to zero.

The instanton worldvolume theory

Let us describe the worldvolume theory on the D5. As a matter of convention, we will denote the spinorial indices in the 6d theory by $A, B, \ldots = 1 \ldots 4$, and the vector indices by $m, n, \ldots = 1 \ldots 6$. The D5 preserves 16 supersymmetries of the background, in the usual 6d notation we have $\mathcal{N} = (1, 1)$. The background D9 will break down this further
to $\mathcal{N} = (0, 1)$, so we will express the multiplets in this language. On the D5 there is a vector multiplet $(\chi_m, \bar{\tau}^A, D_i)$. Here $\chi_m$ is the gauge connection on the D5, $D_i$ is a triplet of auxiliary D-terms, and $\bar{\tau}^A$ is the left half of the reduction of the 10d Majorana-Weyl gaugino, which decomposes under $SO(6) \times SO(4) \cong SU(4) \times SU(2)_{R} \times SU(2)_{L}$ as:

$$16 \longrightarrow (4, 2, 1) + (4, 1, 2).$$

(2.1)

The right half $\theta^A$ of the decomposition appears in an adjoint hypermultiplet with four real bosonic components $x_\mu \sim x_{a\dot{a}}$. These two multiplets of $\mathcal{N} = (0, 1)$ join into the vector multiplet of the $\mathcal{N} = (1, 1)$ symmetry on the D5.

The strings going from the D5 to the D9 break down $\mathcal{N} = (1, 1)$ to $\mathcal{N} = (0, 1)$. They give a single hypermultiplet in the fundamental, with two complex bosonic components $\omega_{\dot{a}}$ and one Weyl fermion $\mu^A$.

There is a Lagrangian for the theory living on the D5, it can be written as the sum of three terms:

$$S_{\text{inst}} = \int d^6 \xi \text{Tr} \left( S_{\text{gauge}} + S_{\text{matter}}^{(1,1)} + S_{\text{matter}}^{(0,1)} \right)$$

(2.2)

with $\xi$ the coordinates in the D5. The first term encodes the gauge dynamics:

$$S_{\text{gauge}} = \frac{1}{2} \dot{\rho}^2 - i \bar{\tau} \bar{\psi} \tau + \frac{1}{2} \bar{D}^2$$

(2.3)

where we are using the reality condition on $\bar{\tau}$, and $\bar{D}$ is the covariant derivative contracted with the 6d chiral gamma matrices $\bar{\Sigma}$: $\bar{\psi} = \bar{\Sigma}_m D_m$. (In these expressions we will sometimes omit some obvious index contractions for clarity, so $\bar{\tau}_A \bar{\Psi}_{AB} \bar{T}^{B\dot{a}}$, etc.)

The other two terms encode the dynamics of the hypermultiplets. For the hypermultiplet $(x, \theta)$ in the adjoint of the D5 gauge group we have:

$$S_{\text{matter}}^{(1,1)} = D_m x_{a\dot{a}} D^m x^{a\dot{a}} - i \theta \bar{\psi} \psi - if(\theta^a, x_{a\dot{a}}) \bar{\tau}^A + i \bar{D} \cdot \text{tr}(x \bar{\sigma} x)$$

(2.4)

with $\bar{\sigma}$ the Pauli sigma matrices. Similarly for the hypermultiplet in the fundamental:

$$S_{\text{matter}}^{(0,1)} = D_m \omega^{\dot{a}} D^m \omega_{\dot{a}} - i \mu \bar{\psi} \chi - i (\alpha^A \omega_{\dot{a}} + \bar{\omega}_{\dot{a}} \mu^A) \bar{\tau}_A + i \bar{D} \cdot \omega_{\dot{a}} \bar{\sigma}^{\dot{a} \beta} \bar{\omega}_{\beta}$$

(2.5)

The theory on the D$(-1)$ can be easily obtained by dimensionally reducing (2.2), simply by discarding the derivatives. We will also reduce to the case of a single D3 and a single D$(-1)$, so all the commutators vanish. The fields now become zero modes of the instanton: $\chi_m$ parameterizes the position of the D(-1) on the internal $\mathbf{R}^6$ (i.e., it parameterizes motion away from the D3), $x_\mu$ is the position of the instanton in the $\mathbf{R}^4$ parallel to the D3, $\omega$ parameterizes the Higgs branch of the instanton, where it gets dissolved into the D3 and admits a classical interpretation as a gauge instanton, and $\bar{\tau}$ and $\theta$ are the Goldstinos of the supersymmetries broken by the instanton. The resulting action (restoring some constant factors we have disregarded in the analysis above) is given by [35]:

$$S_{\text{inst}} = -2\pi i \tau + \frac{1}{g_6^2} \bar{D}^2 + \chi^2 W_0 - \frac{2i}{\sqrt{8}} \mu^A \mu^B \Sigma^m_{AB} \chi_m + i \bar{D} \cdot \bar{W} + i \bar{\tau}_A \mu^A \omega_{\dot{a}} + \mu^A \bar{\omega}_{\dot{a}}$$

(2.6)

$\bar{\tau}^1 = \Sigma_\theta \bar{\tau}$, with $\Sigma$ the gamma matrix for anti-chiral 6d Weyl spinors.
where we have introduced
\[ g_0^2 = 4\pi(4\pi^2\alpha')^{-2}g_s \quad ; \quad \tau = C_0 - \frac{i}{g_s} \]  
with \( g_s \) the string coupling, \( C_0 \) the RR 0-form of type IIB, and
\[ W_0 = \ddot{\omega} \quad ; \quad W^c = \ddot{\omega}\bar{\sigma}^{\mu\nu}\bar{\eta}^c_{\mu\nu} \]
with \( \bar{\eta} \) the anti-self-dual 't Hooft symbol mapping anti-self-dual tensors to the adjoint of \( SU(2) \) (see Appendix A of [36] for explicit expressions, which we will not need here). We also have the relation:
\[ W_0^2 = \sum_c (W^c)^2 \]  

Notice that in the field theory limit of an instanton on its Higgs branch\(^5\) the second term in (2.6) disappears (since \( g_0 \to \infty \)) and \( \vec{D} \) becomes a Lagrange multiplier, which implements the bosonic ADHM constraints \([38]\). We can use the last term in (2.6) in order to saturate \( \tau \), the resulting insertion in the path integral implements the so-called fermionic ADHM constraints \( (\bar{\mu}^A\omega_{\tilde{a}} + \mu^A\tilde{\omega}_{\tilde{a}}) = 0 \). In section 2.1 we will be interested in the behavior of the small instanton limit, so we will keep the second term in (2.6) around.

### Supersymmetry transformations

The supersymmetry transformations of the zero modes under the unbroken supersymmetries \( \xi_{\tilde{a}A} \) can be found either by a CFT computation \([39, 35]\), similar to the one we will do in section 3, or by field theory considerations \([36]\). The result is:
\[ \delta x_{\tilde{a}A} = i\xi_{\tilde{a}A}\theta^A_{\dot{\alpha}} \quad ; \quad \delta \theta^A_{\dot{\alpha}} = 0 \]  
\[ \delta \chi_m = i\Sigma_m^{AB}\xi_{\tilde{a}A}\tau_B \quad ; \quad \delta \tau_A = \vec{D} \cdot \bar{\sigma}_{\dot{\alpha}} \]  
\[ \delta \omega_{\tilde{a}} = i\xi_{\tilde{a}A}\mu^A \quad ; \quad \delta \mu^A = 0 \]
where we have left out some terms that will not be relevant for our discussion (the full expressions can be found in section X.3.1 of [36]).

#### 2.1 Backreaction and the Coulomb branch of \( N = 4 \) instantons\(^6\)

As we discussed in the introduction, lifting the neutral zero modes of stringy instantons is somewhat involved in general type II compactifications. The main reason is that the compactification locally preserves 8 supersymmetries, and the instanton breaks 4 of these, so it necessarily has 4 goldstinos.

It is interesting to consider what happens when the backreaction of the background branes is taken into account. Typically, this analysis is complicated by the absence of

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\(^5\)This limit is somewhat subtle. We need to take \( \alpha' \to 0 \), keeping the string coupling small so that the D3 theory becomes weakly coupled SYM. We also need to restrict to vevs for \( \omega \) such that we stay away from the small instanton singularity.

\(^6\)We will not use the results of this section in the rest of the paper, so it can be skipped in a first reading. We include it here as an interesting side remark.
explicit expressions for the backreacted backgrounds, and the difficulty of applying CFT techniques to general curved backgrounds. In this section we would like to argue that the D3-D(−1) system in its Coulomb branch (defined by \( \chi_m \neq 0 \)) provides a particularly simple toy example in which the backreaction of the D3 lifts all the \( \mathfrak{T}_A^\alpha \) modes.

In the Coulomb branch of the instanton the modes going from the D3 to the D(−1) (the hypermultiplet in the fundamental, and its stringy excitations) acquire a mass growing with \( \chi_m \). Our strategy will be to integrate these modes out explicitly, finding an effective instanton action on the Coulomb branch depending just on \( \chi_m \). Schematically (we make this more precise below):

\[
e^{-S_{\text{inst}}(\chi_m)} \sim \int d[\omega, \mu, \ldots] e^{-S_{\text{inst}}(\chi_m, \omega, \mu, \ldots)} \tag{2.13}
\]

where the measure includes all the massive modes. We will find out that the integral on the right side saturates the integral over \( \mathfrak{T} \) for any \( \chi_m \). In terms of string diagrams, we want to compute the cylinder diagram with one boundary on the D3 and the other on the D(−1), with various insertions of \( \mathfrak{T} \) on the D(−1) boundary. This is a complicated problem, and we are just interested here in the qualitative behavior of zero mode lifting, so let us restrict ourselves to two particularly simple limits in which the cylinder contribution can be easily obtained.

The first limit we want to consider is \( \chi_m \) much smaller than the string scale. In this limit we can restrict to the open string modes of lowest mass \( \omega \) and \( \mu \), and just plug (2.6) into (2.13). It will be convenient to integrate out explicitly the D-terms first. If we do this we get:

\[
S_{\text{inst}} = -2\pi i \tau + \frac{g_0^2}{4} (W^c)^2 + \chi^2 W_0 - \frac{2i}{\sqrt{8}} \mu^A \mu^B \Sigma_{AB}^m \chi_m + i\mathfrak{T}_A^\alpha (\bar{\mu}^A \omega_\alpha + \mu^A \bar{\omega}_\alpha) . \tag{2.14}
\]

Furthermore, in doing this integration we pick up a prefactor of \( g_0^3 \) in the measure of integration of zero modes coming from the \( 1/g_0^2 \) coefficient of \( \mathcal{D}^2 \) in (2.4). The integral to perform is now:

\[
g_0^3 \int d[\omega, \bar{\omega}, \mu, \bar{\mu}] \exp \left[ 2\pi i \tau - \frac{g_0^2}{4} (W^c)^2 - \chi^2 W_0 + \frac{2i}{\sqrt{8}} \mu^A \mu^B \Sigma_{AB}^m \chi_m - i\mathfrak{T}_A^\alpha (\bar{\mu}^A \omega_\alpha + \mu^A \bar{\omega}_\alpha) \right] . \tag{2.15}
\]

The first term in the exponential does not depend on the massive modes, so we will ignore it in the following. We want to focus on the terms that saturate as many \( \mathfrak{T} \) modes as possible, so let us saturate all \( \mu, \bar{\mu} \) integrations by bringing down the last term eight times. We are left with:

\[
(\mathfrak{T})^8 \int d[\omega, \bar{\omega}] W_0^4 \exp \left[ -\frac{g_0^2}{4} (W^c)^2 - \chi^2 W_0 \right] . \tag{2.16}
\]

\[\text{7Nevertheless, there has been some recent progress in the analysis of instantons in the presence of fluxes from the point of CFT} \, [40, 30, 29].
\[\text{8A similar computation, with a different motivation, was performed in} \, [35]. \text{We will follow this reference in doing our computation.}\]
Here $\langle \overline{\tau} \rangle^8$ is shorthand for a coupling that saturates all $\tau$ modes, with the normalization:

$$\int d^8\overline{\tau} \langle \overline{\tau} \rangle^8 = 1 .$$  \hspace{1cm} (2.17)$$

We are left with the integral over $\omega, \bar{\omega}$. At this point it is very convenient to change variables from $\omega$ to $W^c$:

$$\int d[\omega, \bar{\omega}] = 2\pi \int \frac{dW_1 dW_2 dW_3}{W_0} .$$  \hspace{1cm} (2.18)$$

Since our integral (2.16) depends just on the radial coordinate $W_0$, we can perform the angular part of the integration, and we end up with:

$$e^{-S_{\text{inst}}(\chi_m)} = 8\pi^2 g_0^3 \langle \overline{\tau} \rangle^8 \int_0^\infty dW_0 W_0^5 \exp \left[ -\frac{g_0^2}{4} (W_0)^2 - \chi^2 W_0 \right]$$

$$= 8\pi^2 g_0^{-3} \langle \overline{\tau} \rangle^8 \int_0^\infty dw w^5 \exp \left[ -\frac{1}{4} (w)^2 - w \chi^2 \sqrt{g_0} \right]$$

$$= 8\pi^2 g_0^3 \langle \overline{\tau} \rangle^8 \left( 32t^4 + 144t^3 + 64 - (32t^5 + 160t^3 + 120t)\sqrt{\pi}e^t \text{erfc}(t) \right)$$  \hspace{1cm} (2.19)$$

where we have introduced the adimensional variables $w \equiv g_0 W_0$ and $t \equiv \chi^2 / \sqrt{g_0}$, and erfc($t$) denotes the complementary error function [11]. While this is perhaps not a particularly transparent expression, it does show what we want, namely that there is a lifting of the $\tau$ modes in the Coulomb branch of the instanton. This lifting is maximized at the origin of the Coulomb branch, and decreases fast as $\chi^2$ increases.

Let us now consider the opposite limit, where $\chi_m$ is much larger than the string scale. Here the cylinder can be computed in the closed string channel, with a closed string mode propagator connecting a disk with the boundary on the D($-1$) and a disk with the boundary on the D3. In this regime we can replace the D3 by its backreaction, and just compute the disk tadpole for the D($-1$) with a closed string mode inserted on it. The D3 sources the RR 4-form $C_4$ and the metric $G_{mn}$, so we need the couplings of a disk with boundary on the D($-1$), an insertion of $C_4$ or $G_{mn}$ on the bulk of the disk, and various insertions of $\overline{\tau}$ on the boundary. This computation, while easier than the whole cylinder diagram, is still quite challenging. We will not present the final result, but let us point out that the requisite couplings are present. It was argued in [42] that there is a coupling between $G_{mn}$ and four goldstinos of the instanton:

$$S_{\text{inst}} \sim \ldots + \partial_m \partial_n G_{\rho\sigma}(\overline{\Theta} \Gamma^{m\rho\nu} \Theta)(\bar{\Theta} \Gamma^{n\sigma\nu} \Theta)$$  \hspace{1cm} (2.20)$$

where $\Theta$ is the Majorana-Weyl spinor encoding the neutral fermionic zero modes of the D($-1$), and $\Gamma^{\mu\nu}$ is the antisymmetrized product of three $\Gamma$ matrices. The greek indices take values in the 10 euclidean dimensions. The coupling to $C_4$ can be obtained T-dualizing the coupling of a D0 brane to $C_5$ obtained in [43]. This gives the following coupling:

$$S_{\text{inst}} \sim \ldots + \partial_m \partial_n G_{\mu\rho\sigma}^{(4)}(\overline{\Theta} \Gamma^{m\mu\nu} \Theta)(\bar{\Theta} \Gamma^{n\rho\sigma} \Theta)$$  \hspace{1cm} (2.21)$$
Extension to theories with less supersymmetry

One very interesting aspect of the previous $\mathcal{N} = 4$ analysis is that it points towards a hitherto unnoticed mechanism for lifting neutral zero modes in $\mathcal{N} = 1$ cases. As mentioned in the beginning of this section, doing the analysis explicitly is difficult, but we can easily give a plausibility argument based on the $\mathcal{N} = 4$ case just discussed.

The previous discussion holds without big modifications in any theory where stringy instantons have both a Coulomb and Higgs branch, connected through the small instanton limit. One such theory is $\mathcal{N} = 2$ $SU(N)$ SYM, which we can engineer in string theory in the following way. Take a local geometry given by $\mathbb{R}^6$ times a (resolved) $A_1$ degeneration of K3:

$$x^2 + y^2 + w^2 = 0,$$

and wrap $N$ D5-branes on the resolved $\mathbb{S}^2$ times some $\mathbb{R}^4 \subset \mathbb{R}^6$. Similarly to the $\mathcal{N} = 4$ case, gauge instantons in this theory have a Coulomb branch, parameterizing the $\mathbb{R}^2$ where the D5 is pointlike, and represented on the field theory of the instanton by an adjoint hypermultiplet. As in the $\mathcal{N} = 4$ case, the whole Coulomb branch of the instanton will contribute to the theory on the D5.

Let us now deform the previous space by fibering the $A_1$ singularity over the $\mathbb{R}^2$ plane, which we parameterize by $z$:

$$x^2 + y^2 + w^2 = f(z)^2$$

This geometry breaks the supersymmetry down to $\mathcal{N} = 1$, and in particular lifts the Coulomb branch to a set of isolated points, the set of solutions to $f(z) = 0$. Close to each of these points, for sufficiently generic $f(z)$, we have a resolved conifold singularity. Branes wrapped on the $\mathbb{S}^2$ of these resolved conifolds are BPS, and in particular we will generically have isolated instantons not on top of any brane or orientifold. We can make $f(z)$ as small as we want, so we expect that the basic mechanism that lifted zero modes in the $\mathcal{N} = 4$ case still applies, and a superpotential is generated.

The utility of this mechanism for stabilizing K"ahler moduli is limited by the fact that the instanton is homologous to the gauge brane, so there will be a contribution to the potential only if there is gaugino condensation already (and our mechanism is rather subleading in this case). Nevertheless we find it of some theoretical interest, and worthy of further study.

3. Conformal field theory analysis

In this chapter we perform a detailed analysis of the instanton modes and their interactions using conformal field theory techniques. For concreteness we perform this analysis in type IIA, where the relevant instantons are euclidean D2 branes wrapping a three-cycle $\Sigma$ away from the orientifold plane in the internal manifold (and thus point-like in four-dimensional space-time). Generically, such an isolated $U(1)$ instanton exhibits four bosonic zero modes $x^\mu$ corresponding to the breakdown of four-dimensional Poincaré invariance and Goldstino
modes $\theta_A^i$ and $\tau_A^i$ associated with the supersymmetries broken by the instanton.\(^9\) So far this is just as for the gauge instanton case discussed in section 2. Remember that there we also had a background space-filling brane wrapping $\Sigma$. In this section we want to study in detail what happens when the background D6 brane is wrapping a cycle $\Sigma'$ homologically distinct to $\Sigma$. Generically there will be intersections between the instanton and the brane, and thus there will be some charged open string modes localized in there. These modes will be the focus of our analysis.

We start in section 3.1 by analyzing the instanton spectrum and determining the supersymmetry transformations in the case when the D-instanton D-brane system preserves some common supersymmetry. This analysis relies heavily on CFT techniques. At the end of section 3.1 we summarize our results. In section 3.2 we determine the effective instanton action by calculating various string amplitudes and discuss its supersymmetric nature. Finally in section 3.3 we discuss the saturation of the instanton zero modes by computing the path integral.

### 3.1 Description of the D-brane-instanton modes

The different massive and massless charged instanton modes appear as excitations of the NS- and R-vacuum respectively. Schematically they take the form

\[\sum_{k \in \mathbb{Z}} (\alpha^I_k - \theta_I)^N_I (\psi^I_k)^M_I |0\rangle_{NS}, \quad \sum_{k \in \mathbb{Z}} (\alpha^I_k - \theta_I)^N_I (\psi^I_k)^M_I |0\rangle_{R},\]  

(3.1)

where $\theta_I$ is the intersection angle between the D-brane and the instanton in the $I$-th dimension\(^10\) and $\alpha^I_k$ and $\psi^I_k$ denote the bosonic and fermionic creation operators. Note that the fermionic creation operator $\psi^I_k$ has Fermi statistics, and thus $M^I_k$ can only take the values 0 or 1, while $N^I_k$ can be any non-negative integer.

The mass of these states is given by:

**NS-sector:**  

\[M^2 \sim \varepsilon_0^{NS} + \sum_k \left( N^I_k (k - \theta_I) + M^I_k (k + \frac{1}{2} - \theta_I) \right)\]  

(3.2)

**R-sector:**  

\[M^2 \sim \varepsilon_0^{R} + \sum_k \left( N^I_k (k - \theta_I) + M^I_k (k - \theta_I) \right)\]  

(3.3)

where $\varepsilon_0^{NS,R}$ denotes the zero point energy in the NS or R sectors, and crucially depends on the intersection angles of the D6-brane and the E2-instanton. Note that due to the Dirichlet-Neumann boundary conditions in space-time\(^11\) the zero point energy for the NS sector is shifted by $1/2$ compared to the D6-D6 brane system. This implies that there are no tachyonic modes between the D-brane and the instanton and for non-trivial angles the

\(^9\)This is the minimal content of neutral zero modes. If $\Sigma$ is not rigid, as in the $\mathcal{N} = 4$ gauge instanton, then we also have moduli for the broken translation (super)symmetries in the internal space.

\(^10\)We take $\theta_I$ to be defined in the range $[-1, 1)$.

\(^11\)That implies that the intersection angles in space-time are $\theta_4 = \theta_5 = \frac{1}{2}$ in equation (3.1). From now on we ignore any excitations in space-time which are genuinely stringy in the sense that there mass is non-zero even if the instanton and D-brane wrap the same cycle.
bosonic modes are always massive. The absence of tachyons in this setup rules out the possibility of recombination of the instanton and D-brane.

In this work we will focus only on instanton modes whose mass takes the form

$$M^2 \sim \sum_{I=1}^{3} a^I \theta_I .$$

(3.4)

thus we ignore states which are genuinely stringy in the sense that even in the limit $\theta_I \to 0$ they are massive. That restricts the bosonic and fermion number $N^I_k$ and $M^I_k$. For the NS sector no fermionic excitations are allowed and only $N^I_k$ can be non-vanishing. Thus the only states in the NS-sector are the vacuum and a tower built up from an arbitrary number of excitations $\alpha^{I}_{-\theta_I}$.

$$\sum_I \left(\alpha^{I}_{-\theta_I}\right)^{N^I_0} |0\rangle^{NS} .$$

(3.5)

For the R-sector, the restriction to modes with masses of the type (3.4) implies that $N^I_k = M^I_k = 0$ for $k > 0$. Thus the generic state in R-sector has the form

$$\sum_I \left(\alpha^{I}_{-\theta_I}\right)^{N^I_0} \left(\psi^{I}_{-\theta_I}\right)^{M^I_0} |0\rangle^{R} .$$

(3.6)

In both (3.5) and (3.6) we find an infinite tower of states with similar properties, differing only on their mass. In the following we will restrict our analysis to the lowest states in the tower. Higher levels in the tower work in the same way.

The states (3.1) are subject to the GSO-projection which puts an additional constraint on the total number of fermionic creation operators in the R-sector. Depending on the intersection angles the number has to be odd or even. For the CPT conjugated sector the GSO-projection projects onto states whose total fermionic creation operator number is even if it was odd for the original sector and vice versa.

There is a one to one correspondence between the states and the respective vertex operators which has been analyzed for different intersection angles in [45] for an intersecting D6 brane setup\textsuperscript{12}. The difference to the E2-D6 setup we are investigating are the Neumann-Dirichlet (ND) boundary conditions in space-time between the instanton and the D-brane. For the vertex operators this implies the presence of twist fields in space-time associated with ND boundary conditions. Moreover, in the R-sector only states whose $U(1)$-wordsheet charge is $Q_{WS} = -\frac{1}{2} \mod 2$ are allowed, thus only chiral modes survive the GSO-projection [2, 3, 4, 52]. The latter is related to the fact that the total fermion number in the R-sector is constrained due to the GSO-projection.

In the following we analyze the instanton mode structure for a concrete setup, we determine the instanton spectrum their corresponding vertex operators as well as the SUSY transformations, in case the D-instanton D-brane system preserves some common supersymmetry. Later in subsection 3.2 we derive the interaction terms involving charged and

\textsuperscript{12}For string amplitude calculations involving states arising from intersecting D6-branes see for instance [46, 47, 48, 49, 50, 51, 45] and references therein.
neutral instanton modes. The latter are crucial for the saturation of fermionic instanton modes in the path integral and thus important for answering the question of whether a generic rigid $U(1)$ instanton gives contributions to the superpotential.

Without loss of generality we choose the sign of the angles to be:

$$\begin{align*}
\theta_1 > 0 & \quad \theta_2 > 0 & \quad \theta_3 < 0 , \\
\end{align*}$$

(3.7)

where we require the supersymmetry condition

$$\begin{align*}
\theta_1 + \theta_2 + \theta_3 = 0 .
\end{align*}$$

(3.8)

The preserved supercharge is $Q^0_\alpha$ while all the others $Q^A_\alpha$ with $A = 1, 2, 3$ are broken unless some angles are trivial as we will see later. Let us display the form of the supercharges explicitly since we will make use of them extensively in the following

$$\begin{align*}
Q^A_\alpha = S_\alpha S^A e^{-\varphi/2}
\end{align*}$$

(3.9)

where $S_\alpha$ is the spin field in space-time with negative chirality and $S^A$ the spin fields in the internal six dimension with negative chirality. The latter can be bosonized and then take the form

$$\begin{align*}
S^0 = \prod_{I=1}^3 e^{-\frac{i}{2}H_I} & \quad S^A = e^{\frac{i}{2}H_A} \prod_{I \neq A \neq 0} e^{-\frac{i}{2}H_I} .
\end{align*}$$

(3.10)

We will start with the states which arise from the $E2-a$ sector, where $a$ denotes the D6-brane. These states transform in the $(E2, \pi)$ representation under the gauge groups living on the instanton and D-brane (that is, they are bifundamentals). We will analyze first the states arising from the NS sector. If we require the mass to take the form (3.4) and ignore any bosonic excitation the only state is the vacuum

$$\begin{align*}
|0\rangle_{NS}^{E2a}
\end{align*}$$

(3.11)

whose vertex operator takes the form

$$\begin{align*}
V_{\omega_\alpha}^{-1}(z) = \omega_\alpha S^\alpha \Sigma e^{i\theta_1 H_1} \sigma_{\theta_1} e^{i\theta_2 H_2} \sigma_{\theta_2} e^{i\theta_3 H_3} \sigma_{1+\theta_3} e^{-\varphi} ,
\end{align*}$$

(3.12)

Here $\Sigma$ denotes the bosonic twist field which ensures Dirichlet-Neumann boundary conditions in space-time and $\sigma_{\theta_I}$ and $i^{\theta_I}H_I$ are the bosonic and fermionic twist fields ensuring the boundary conditions in the $I$-th complex internal dimension. The mass of $|0\rangle_{NS}^{E2a}$ is simply given by the zero-point energy in (3.2) which for this specific configuration is $\frac{1}{2}(\theta_1 + \theta_2 - \theta_3)$.

For the R-sector one observes four different states surviving the GSO projection. Note that with the conventions in (3.7), the total fermionic operator number in the $E2-a$ sector has to be even. Again we are ignoring any additional bosonic excitations. These four states are given by:

$$\begin{align*}
|0\rangle^{R}_{E2a} & \quad |\psi_{-\theta_1} \psi_{-\theta_2}|0\rangle^{R}_{E2a} & \quad |\psi_{-\theta_1} \psi_{\theta_3}|0\rangle^{R}_{E2a} & \quad |\psi_{-\theta_2} \psi_{\theta_3}|0\rangle^{R}_{E2a} .
\end{align*}$$

The masses and vertex operators (in the $-\frac{1}{2}$ ghost picture) for these states are displayed in the following expressions:
\begin{itemize}
  \item $\mu_0 = \psi_{-\theta_1}\psi_{-\theta_2}|0 \rangle_{E_{2a}}^R$, $M^2 = \theta_1 + \theta_2$
  \[ V_{\mu_0}^{-\frac{1}{2}}(z) = \mu_0 \sum e^{i(\theta_1 + \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 + \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 + \frac{1}{2})H_3} \sigma_{1+\theta_3} e^{-\varphi/2} \] (3.13)
  \item $\mu_1 = \psi_{-\theta_1}\psi_{\theta_3}|0 \rangle_{E_{2a}}^R$, $M^2 = \theta_1 - \theta_3$
  \[ V_{\mu_1}^{-\frac{1}{2}}(z) = \mu_1 \sum e^{i(\theta_1 + \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 - \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 - \frac{1}{2})H_3} \sigma_{1+\theta_3} e^{-\varphi/2} \]
  \item $\mu_2 = \psi_{-\theta_2}\psi_{\theta_3}|0 \rangle_{E_{2a}}^R$, $M^2 = \theta_2 - \theta_3$
  \[ V_{\mu_2}^{-\frac{1}{2}}(z) = \mu_2 \sum e^{i(\theta_1 - \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 + \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 - \frac{1}{2})H_3} \sigma_{1+\theta_3} e^{-\varphi/2} \]
  \item $\mu_3 = |0 \rangle_{E_{2a}}^R$, $M^2 = 0$
  \[ V_{\mu_3}^{-\frac{1}{2}}(z) = \mu_3 \sum e^{i(\theta_1 - \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 - \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 + \frac{1}{2})H_3} \sigma_{1+\theta_3} e^{-\varphi/2} \]
\end{itemize}

In the existing literature the massless charged mode $\mu_3$ is often denoted by $\lambda$.

In the sequel we will apply the four supercharges (3.13) to the fermionic fields. We will derive under which circumstances the supercharges are preserved and the transformation behavior of the fermionic fields under the respective supercharge. In order to do this we need to know the explicit form of the OPE’s of various conformal fields, they are given by the following expressions:

\[ e^{k \varphi(z)} e^{l \varphi(w)} \sim (z - w)^{-k l} e^{(k+l)\varphi(z)} \]
\[ S^\alpha(z) S^{\beta}(w) \sim e^{(z-w)^{-\frac{1}{2}}} \]
\[ e^{i k H_1(z)} e^{i H_3(w)} \sim (z-w)^{-k l} e^{i (k+l) H_1(z)} \] (3.14)

With this information is it simple to obtain the supersymmetry transformations of the conformal fields, they are computed as follows:

\[ \left[ \xi_0^\dagger Q_\alpha^0, V_{\mu_0}^{-\frac{1}{2}} \right] = \xi_0^\dagger \cdot \mu_0 \int dw e^{-\varphi/2(w)} e^{-\varphi/2(z)} S^\alpha(w) \Sigma(z) e^{-\frac{1}{2}H_1(w)} e^{i(\theta_1 + \frac{1}{2})H_1(z)} e^{-\frac{1}{2}H_2(w)} e^{i(\theta_2 + \frac{1}{2})H_2(z)} e^{-\frac{1}{2}H_3(w)} e^{i(\theta_3 + \frac{1}{2})H_3(z)} \sigma_{\theta_1}(z) \sigma_{\theta_2}(z) \sigma_{1+\theta_3}(z) \]
\[ = \xi_0^\dagger \cdot \mu_0 \int dw e^{-\varphi(z)} S^\alpha \Sigma e^{-\varphi} e^{i \theta_1 H_1} \sigma_{\theta_1} e^{i \theta_2 H_2} \sigma_{\theta_2} e^{i \theta_3 H_3} \sigma_{1+\theta_3} \frac{(z-w)^{1+\frac{1}{2}(\theta_1+\theta_2+\theta_3)}}{\xi_0^\dagger Q_\alpha^0, V_{\mu_0}^{-\frac{1}{2}}} \]

Thus for $\theta_1 + \theta_2 + \theta_3 = 0$ we recover the SUSY transformation

\[ \left[ \xi_0^\dagger Q_\alpha^0, V_{\mu_0}^{-\frac{1}{2}} \right] = \delta_{\xi_0^\dagger} V_{\omega_{\alpha}}^{-1} \] (3.15)

Similarly we obtain for the other fermionic states

\[ -\theta_1 + \theta_2 + \theta_3 = 0 : \left[ \xi_1^\dagger Q_\alpha^1, V_{\mu_1}^{-\frac{1}{2}} \right] = \delta_{\xi_1^\dagger} V_{\omega_{\alpha}}^{-1} \]
\[ \theta_1 - \theta_2 + \theta_3 = 0 : \left[ \xi_2^\dagger Q_\alpha^2, V_{\mu_2}^{-\frac{1}{2}} \right] = \delta_{\xi_2^\dagger} V_{\omega_{\alpha}}^{-1} \] (3.16)
\[ \theta_1 + \theta_2 - \theta_3 = 0 : \left[ \xi_3^\dagger Q_\alpha^3, V_{\mu_3}^{-\frac{1}{2}} \right] = \delta_{\xi_3^\dagger} V_{\omega_{\alpha}}^{-1} \]
Applying the same supercharge again we will get additional fermionic states. As we will see momentarily the presence of these states is required to define a proper mass term. Again we will be explicit for $Q^0_\alpha$ and state the result for the other supercharges,

$$[\xi^0_0 Q^0_\alpha (w), \partial Z^{-1} (z)] = \xi^0_0 \partial w \sum e^{i(\theta_1 - \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 - \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 - \frac{1}{2})H_3} \sigma_{1+\theta_3} (z - w)^{1+\frac{1}{2}(\theta_1+\theta_2+\theta_3)}/2 \ .$$

Here we applied the OPE’s given in (3.14) and we recovered, as expected, that $Q^0_\alpha$ is preserved for $\theta_1 + \theta_2 + \theta_3 = 0$. The SUSY transformation behavior is

$$[\xi^0_0 Q^0_\alpha, \partial Z^{-1}] = \delta_{\xi^0_0} V^{-\frac{1}{2}}_{\mu_0} \ .$$

(3.17)

The vertex operator of the associated state $\mu_0'$ in the $-\frac{3}{2}$ ghost picture takes the form

$$V^{-\frac{3}{2}}_{\mu_0'} (z) = \mu_0' e^{-3\varphi/2} \sum e^{i(\theta_1 - \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 - \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 - \frac{1}{2})H_3} \sigma_{1+\theta_3} \ .$$

The conformal dimension of $V^{-\frac{3}{2}}_{\mu_0'}$ is $1 - \theta_3$ and thus the mass is expected to be $M^2 = -\theta_3$. Let us apply the picture changing operator to get the vertex operator in the canonical $-\frac{1}{2}$ ghost picture. After applying the picture changing procedure

$$\lim_{w \rightarrow z} O_{PCO}(w) V^{-\frac{3}{2}}_{\mu_0} (z) = V^{-\frac{1}{2}}_{\mu_0'} (z) \ ,$$

(3.18)

we obtain the vertex operator in the canonical $-\frac{1}{2}$-ghost picture, which allows us to identify the corresponding state. The picture changing operator is given by

$$O_{PCO}(z) = e^{\varphi(z)} T_F (z) \ ,$$

(3.19)

where $T_F$ is

$$T_F (z) = \psi^\mu (z) \partial X^\mu (z) + e^{iH_1 (z)} \partial \bar{Z}^I + e^{-iH_1 (z)} \partial \bar{Z}^I \ ,$$

(3.20)

where $\partial \bar{Z}^I$ and $e^{iH_1 (z)}$ are the complexified bosonic and fermionic fields in the $I$-th internal dimensions, whereas $\partial X^\mu$ and $\psi^\mu$ denote the bosonic and fermionic fields in space-time. Using the OPE’s displayed in (3.14) and below [53,13]

$$\begin{align*}
\partial Z (z) \sigma_\theta (w) & \sim (z - w)^{\theta-1} \bar{\tau}_\theta (z) \\
\partial \bar{Z} (z) \sigma_\theta (w) & \sim (z - w)^{\theta} \bar{\tau}_\theta (z) \\
\partial Z (z) \sigma_{1-\theta} (w) & \sim (z - w)^{\theta} \bar{\tau}_{1-\theta} (z) \\
\partial \bar{Z} (z) \sigma_{1-\theta} (w) & \sim (z - w)^{\theta-1} \bar{\tau}_{1-\theta} (z)
\end{align*}$$

(3.21)

the vertex operator for $\mu_0'$ in the $-\frac{1}{2}$ ghost picture takes the form

$$V^{-\frac{1}{2}}_{\mu_0'} (z) = \mu_0' e^{i(\theta_1 - \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 - \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 + \frac{1}{2})H_3} \tau_{1+\theta_3} \bar{\tau}_{1-\theta} \ .$$

We read off that the corresponding state is given by $\mu_0' = \alpha_{\theta_3} |0\rangle_{E2a}$, which indeed has the right mass $M^2 = -\theta_3$. Analogously we obtain the other three states $\mu_1'$, $\mu_2'$ and $\mu_3'$

13The OPE’s with space-time fields do not play any crucial role here.
\[ \mu_1' = \alpha_{-\theta_1} |0\rangle_{E2a} \quad M^2 = \theta_2 \]
\[ V_{\mu_1'}^{-\frac{1}{2}}(z) = \mu_1' \sum e^{i(\theta_1 - \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 - \frac{1}{2})H_2} \tau_{\theta_2} e^{i(\theta_3 + \frac{1}{2})H_3} \sigma_{1+\theta_3} e^{-\phi/2} \]

\[ \mu_2' = \alpha_{-\theta_1} |0\rangle_{E2a} \quad M^2 = \theta_1 \]
\[ V_{\mu_2'}^{-\frac{1}{2}}(z) = \mu_2' \sum e^{i(\theta_1 - \frac{1}{2})H_1} \tau_{\theta_1} e^{i(\theta_2 - \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 + \frac{1}{2})H_3} \sigma_{1+\theta_3} e^{-\phi/2} \]

\[ \mu_3' = \frac{1}{3} \left[ \psi_{-\theta_2} \psi_3 \alpha_{-\theta_1} + \psi_{-\theta_3} \psi_1 \alpha_{-\theta_2} + \psi_{-\theta_3} \psi_{-\theta_2} \alpha_{\theta_3} \right] |0\rangle_{E2a} \quad M^2 = \theta_1 + \theta_2 - \theta_3 \]
\[ V_{\mu_3'}^{-\frac{1}{2}}(z) = \mu_3' e^{-\phi/2} \sum \left[ e^{i(\theta_1 - \frac{1}{2})H_1} \tau_{\theta_1} e^{i(\theta_2 + \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 - \frac{1}{2})H_3} \sigma_{1+\theta_3} + e^{i(\theta_1 + \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 - \frac{1}{2})H_2} \tau_{\theta_2} e^{i(\theta_3 - \frac{1}{2})H_3} \sigma_{1+\theta_3} + e^{i(\theta_1 + \frac{1}{2})H_1} \sigma_{\theta_1} e^{i(\theta_2 + \frac{1}{2})H_2} \sigma_{\theta_2} e^{i(\theta_3 + \frac{1}{2})H_3} \tau_{1+\theta_3} \right]. \]

Moreover, analogously to before one can show that \( \mu_A' \) is related to \( \omega \) if the supercharge \( \overline{Q}_A^A \) is preserved

\[ [\xi_1^A \overline{Q}_A^A, V_{\omega}^{-1}] = \delta_{\xi_1} V_{\mu_1'}^{-\frac{1}{2}} \]
\[ [\xi_2^A \overline{Q}_A^A, V_{\omega}^{-1}] = \delta_{\xi_2} V_{\mu_2'}^{-\frac{1}{2}} \quad (3.22) \]
\[ [\xi_3^A \overline{Q}_A^A, V_{\omega}^{-1}] = \delta_{\xi_3} V_{\mu_3'}^{-\frac{1}{2}}. \]

From (3.15), (3.16), (3.17) and (3.22) we can read off the SUSY transformations of the instanton modes \( \mu_A, \omega \) and \( \mu_A' \)

\[ \delta_{\xi} \mu_A' = i \xi_A^A \omega^\alpha \quad \delta_{\xi} \omega^\alpha = i \xi_A^A \mu_A. \quad (3.23) \]

Let us turn to the CPT-conjugated sector \( a-E2 \). Again ignoring any bosonic excitations there is only one state in the NS sector which is simply given by the vacuum \( |0\rangle_{aE2} \).

Its mass is \( M^2 = \frac{1}{2}(\theta_1 + \theta_2 - \theta_3) \) and the vertex operator takes the form

\[ V_{\omega}^{-1}(z) = \overline{\omega} \sum e^{-i\theta_1 H_1} \sigma_{1-\theta_1} e^{-i\theta_2 H_2} \sigma_{1-\theta_2} e^{-i\theta_3 H_3} \sigma_{-\theta_3} e^{-\phi}. \quad (3.24) \]

Note that \( \omega \) and \( \overline{\omega} \) have the same mass and one can show the presence of the mass term

\[ M_{\omega, \omega} = \overline{\omega} \] \quad (3.25)

by computing

\[ \langle V_{\omega}^{-1}(z_1) \overline{V_{\omega}^{-1}(z_2)} \rangle. \quad (3.26) \]

For the R-sector the GSO-projection projects onto states with odd total fermion number. Thus neglecting bosonic excitations the R-sector exhibits four different states

\[ \psi_{-\theta_1} |0\rangle_{aE2} \quad \psi_{-\theta_2} |0\rangle_{aE2} \quad \psi_1 |0\rangle_{aE2} \quad \psi_{-\theta_1} \psi_{-\theta_2} \psi_1 |0\rangle_{aE2}. \quad (3.27) \]

Below we display the corresponding vertex operators in the \(-\frac{1}{2}\)-ghost picture.
\[ \overline{\eta}_0 = \psi_{\theta_3} |0\rangle^R_{aE2} \quad M^2 = -\theta_3 \]

\[ V_{\overline{\eta}_0}^{-\frac{1}{2}}(z) = \overline{\eta}_0 \sum e^{-i(\theta_1 - \frac{1}{2})H_1} \sigma_{1-\theta_1} e^{-i(\theta_2 - \frac{1}{2})H_2} \sigma_{1-\theta_2} e^{-i(\theta_3 - \frac{1}{2})H_3} \sigma_{-\theta_3} e^{-\varphi/2} \tag{3.28} \]

\[ \overline{\eta}_1 = \psi_{-\theta_2} |0\rangle^R_{aE2} \quad M^2 = \theta_2 \]

\[ V_{\overline{\eta}_1}^{-\frac{1}{2}}(z) = \overline{\eta}_1 \sum e^{-i(\theta_1 - \frac{1}{2})H_1} \sigma_{1-\theta_1} e^{-i(\theta_2 + \frac{1}{2})H_2} \sigma_{1-\theta_2} e^{-i(\theta_3 + \frac{1}{2})H_3} \sigma_{-\theta_3} e^{-\varphi/2} \]

\[ \overline{\eta}_2 = \psi_{-\theta_1} |0\rangle^R_{aE2} \quad M^2 = \theta_1 \]

\[ V_{\overline{\eta}_2}^{-\frac{1}{2}}(z) = \overline{\eta}_2 \sum e^{-i(\theta_1 + \frac{1}{2})H_1} \sigma_{1-\theta_1} e^{-i(\theta_2 + \frac{1}{2})H_2} \sigma_{1-\theta_2} e^{-i(\theta_3 + \frac{1}{2})H_3} \sigma_{-\theta_3} e^{-\varphi/2} \]

\[ \overline{\eta}_3 = \psi_{-\theta_1} \psi_{-\theta_2} \psi_{\theta_3} |0\rangle^R_{aE2} \quad M^2 = \theta_1 + \theta_2 - \theta_3 \]

\[ V_{\overline{\eta}_3}^{-\frac{1}{2}}(z) = \overline{\eta}_3 \sum e^{-i(\theta_1 + \frac{1}{2})H_1} \sigma_{1-\theta_1} e^{-i(\theta_2 + \frac{1}{2})H_2} \sigma_{1-\theta_2} e^{-i(\theta_3 - \frac{1}{2})H_3} \sigma_{-\theta_3} e^{-\varphi/2} \]

In the case where all supersymmetry is broken the mass of \( \bar{\eta}_A \) is different from the mass of \( \eta_A \), and thus the mass terms for the fermionic modes cannot be of the form \( \mu_A \bar{\eta}_A \) but rather are given by

\[ \mu_0^0 \bar{\eta}_0 \quad \mu_1^1 \bar{\eta}_1 \quad \mu_2^2 \bar{\eta}_2 \quad \mu_3^3 \bar{\eta}_3 . \tag{3.29} \]

Again this can be verified by computing the string disc amplitude

\[ \left\langle V_{\overline{\eta}_A}^{-\frac{4}{3}}(z_1) V_{\overline{\eta}_A}^{-\frac{4}{3}}(z_2) \right\rangle . \tag{3.30} \]

In order to ensure ghost charge -2 of the whole amplitude we need the vertex operator of \( \mu_A' \) in the \( -\frac{3}{2} \) ghost picture which ensures the conservation of \( U(1) \) world sheet charge.

Similarly there should be three \( \bar{\eta}' \) states which give together with \( \mu_0, \mu_1 \) and \( \mu_2 \) the respective mass terms. Indeed acting with the supercharges \( \overline{\sigma}_A' \) with \( A = 0, 1, 2 \) on the bosonic state \( \overline{\eta}_{\alpha} \), analogously to the procedure performed for the sector \( E2-a \), we obtain the states

\[ \overline{\eta}_0 = \frac{1}{2} (\psi_{-\theta_1} \alpha_{-\theta_2} + \psi_{-\theta_2} \alpha_{-\theta_1}) |0\rangle^R_{aE2} \quad M^2 = \theta_1 + \theta_2 \]

\[ V_{\overline{\eta}_0}^{-\frac{1}{2}}(z) = \overline{\eta}_0 \sum e^{-i(\theta_1 - \frac{1}{2})H_1} \sigma_{-\theta_1} e^{-\varphi/2} \times \left[ e^{-i(\theta_1 - \frac{1}{2})H_1} \tau_{1-\theta_1} e^{-i(\theta_2 + \frac{1}{2})H_2} \sigma_{1-\theta_2} + e^{-i(\theta_1 + \frac{1}{2})H_1} \sigma_{1-\theta_1} e^{-i(\theta_2 - \frac{1}{2})H_2} \tau_{1-\theta_2} \right] \]

\[ \overline{\eta}_1 = \psi_{\theta_1} \alpha_{-\theta_1} |0\rangle^R_{aE2} \quad M^2 = \theta_1 - \theta_3 \]

\[ V_{\overline{\eta}_1}^{-\frac{1}{2}}(z) = \overline{\eta}_1 \sum e^{-i(\theta_1 + \frac{1}{2})H_1} \tau_{1-\theta_1} e^{-i(\theta_2 + \frac{1}{2})H_2} \sigma_{1-\theta_2} e^{-i(\theta_3 - \frac{1}{2})H_3} \sigma_{-\theta_3} e^{-\varphi/2} \]
$\overline{p}_2 = \psi_{\theta_3} \alpha_{\theta_2} |0\rangle_{aE2} \quad M^2 = \theta_2 - \theta_3$

$$V_{\overline{p}_2}^{-\frac{1}{2}}(z) = \overline{p}_2 \sum e^{-i(\theta_1 - \frac{1}{2})H_1} \sigma_{1-\theta_1} e^{-i(\theta_2 - \frac{1}{2})H_2} \tau_{1-\theta_2} e^{-i(\theta_3 - \frac{1}{2})H_3} \sigma_{-\theta_3} e^{-\phi/2}$$

Again one can show by computing the two point amplitude

$$\langle V_{-\frac{1}{2}}^{-1}(z_1) V_{-\frac{1}{2}}^{-2}(z_2) \rangle$$

that the mass terms

$$\mu_0 \overline{p}_0 \quad \mu_1 \overline{p}_1' \quad \mu_2 \overline{p}_2$$

are indeed present. Note that there is no state $\overline{p}_3$. This is expected since $\mu_3$ is massless, and hence no mass term of the form $\mu_3 \overline{p}_3$ should appear in the instanton action$^{14}$.

Analogously to the $E2-a$ sector, in case the D-instanton and the D-brane preserve some common supersymmetry one can show by applying the supercharges $\overline{Q}_{\bar{\alpha}}^A$ onto the vertex operators of $\overline{p}_A$, $\overline{\omega}$ and $\overline{p}_A$ that the instanton states are related to each other via the following transformations

$$\delta_{\xi} \mu_A = i \overline{\xi}_{\alpha} \omega^{\bar{\alpha}} \quad \delta_{\xi} \mu_A' = i \overline{\xi}_{\alpha} \mu_A \quad \delta_{\xi} \overline{\omega}_{\bar{\alpha}} = i \overline{\xi}_{\alpha} \overline{p}_A \quad \delta_{\xi} \overline{\omega}_{\bar{\alpha}} = i \overline{\xi}_{\alpha} \overline{p}_A$$

**Brief summary on the spectrum and SUSY transformation**

The D-instanton D-brane configuration considered above exhibits the intersection pattern

$$\theta_1 > 0 \quad \theta_2 > 0 \quad \theta_3 < 0$$

We determined for such a configuration all states whose mass squared is at most of linear order in the intersecting angles $\theta_I$. In table we present all states and their corresponding mass.

To each state corresponds a vertex operator and we have explicitly shown the existence of the mass terms$^{15}$

$$S_{E2}^{mass} = m_A \mu_A \overline{p}_A + m_\omega \omega \overline{\omega} + m_{\overline{p}_A} \overline{p}_B \mu_B$$

This agrees with the result displayed in table. Note in particular that in the case where all SUSY is broken by the E2-D6 system the masses of $\mu_A$ and $\overline{p}_A$ are different, and thus a mass term $\mu_A \overline{p}_A$ is never allowed.

As shown above if the D-instanton D-brane system preserves the supercharge $\overline{Q}_{\bar{\alpha}}^A$ the charged instanton modes are related to each other in the following way

$$\delta_{\xi} \mu_A = i \overline{\xi}_{\alpha} \omega^{\bar{\alpha}} \quad \delta_{\xi} \omega_{\bar{\alpha}} = i \overline{\xi}_{\alpha} \mu_A \quad \delta_{\xi} \overline{p}_A = i \overline{\xi}_{\alpha} \mu_A \quad \delta_{\xi} \mu_A' = i \overline{\xi}_{\alpha} \overline{p}_A \quad \delta_{\xi} \overline{\omega}_{\bar{\alpha}} = i \overline{\xi}_{\alpha} \overline{p}_A$$

If $\mu_A$ or $\overline{p}_A$ is massless we expect the absence of $\overline{p}_A$ or $\overline{p}_A$, respectively. This is analogous to the D6-D6 brane configuration, where we have in the massive case a hypermultiplet

$^{14}$Recall that $\mu_3$ is often referred to as $\lambda$ mode in the existing literature.

$^{15}$Here $A$ runs from 0 to 3, while $B$ runs only from 0 to 2.
\begin{tabular}{|c|c|c|c|c|}
\hline
E2-a & CFT state & a-E2 & CFT state & mass$^2$ \\
\hline
$\mu_0$ & $\psi^-_{-\theta_1} \psi^-_{-\theta_2} |0\rangle_{E_{2\alpha}}^R$ & $\mathcal{E}_0$ & $\frac{1}{2} (\psi^-_{-\theta_1} \alpha^-_{-\theta_2} + \psi^-_{-\theta_2} \alpha^-_{-\theta_1}) |0\rangle_{aE_2}^R$ & $\theta_1 + \theta_2$ \\
$\mu_1$ & $\psi^-_{-\theta_1} \psi_{\theta_2} |0\rangle_{E_{2\alpha}}^R$ & $\mathcal{E}_1$ & $\psi_{\theta_2} \alpha^-_{-\theta_1} |0\rangle_{aE_2}^R$ & $\theta_1 - \theta_2$ \\
$\mu_2$ & $\psi^-_{-\theta_2} \psi_{\theta_2} |0\rangle_{E_{2\alpha}}^R$ & $\mathcal{E}_2$ & $\psi_{\theta_2} \alpha^-_{-\theta_2} |0\rangle_{aE_2}^R$ & $\theta_2 - \theta_3$ \\
$\mu_3$ & $|0\rangle_{E_{2\alpha}}^R$ & None & None & $0$ \\
$\mu'_0$ & $\alpha_{\theta} |0\rangle_{E_{2\alpha}}^R$ & $\mathcal{E}_0$ & $\psi_{\theta_1} |0\rangle_{aE_2}^R$ & $-\theta_3$ \\
$\mu'_1$ & $\alpha \cdot \theta |0\rangle_{E_{2\alpha}}^R$ & $\mathcal{E}_1$ & $\psi_{-\theta_2} |0\rangle_{aE_2}^R$ & $\theta_2$ \\
$\mu'_2$ & $\alpha_- \theta |0\rangle_{E_{2\alpha}}^R$ & $\mathcal{E}_2$ & $\psi_{-\theta_1} |0\rangle_{aE_2}^R$ & $\theta_1$ \\
$\mu'_3$ & $\frac{1}{2} \epsilon_{ijk} \psi_{-\theta_1} \psi_{-\theta_2} \alpha_{-\theta_1} |0\rangle_{E_{2\alpha}}^R$ & $\mathcal{E}_3$ & $\psi_{-\theta_1} \psi_{-\theta_2} \psi_{\theta_1} |0\rangle_{aE_2}^R$ & $\theta_1 + \theta_2 - \theta_3$ \\
$\omega^\alpha$ & $|0\rangle_{E_{2\alpha}}^{NS}$ & $\mathcal{E}_4$ & $|0\rangle_{aE_2}^{NS}$ & $\frac{1}{2} (\theta_1 + \theta_2 - \theta_3)$ \\
\hline
\end{tabular}

Table 1: Instanton D-brane spectrum. We have paired modes by mass. Notice that, despite what the notation might suggest, $\mu$ modes can only pair up with $\mathcal{E}$ modes, and $\mu'$ modes can only pair up with $\mathcal{E}$ modes. Notice also the chiral spectrum at the massless level, encoded in the fact that $\mu_3$ has no mass partner.

(consisting of two chiral multiplets) while in the massless case we only have a single chiral multiplet. In case the D-instanton and D-brane wrap the same cycle all supercharges $\mathcal{Q}_A^{\alpha}$ are preserved and all $\mu_A$ and $\mathcal{E}_A$ are massless. Then the SUSY transformations reduce to the second and fourth equations of (3.36) which are exactly the usual SUSY transformations appearing in the ADHM case \[33, 34]\.

3.2 The instanton effective action

Above we saw that for D-branes and instantons intersecting at non-trivial angles we get the expected mass terms for the respective fields. Apart from these mass terms we have additional interaction terms. Let us assume for simplicity that the instanton wraps a rigid supersymmetric cycle on the Calabi-Yau threefold of $SU(3)$ holonomy. In this case the only neutral instanton zero modes are the four universal bosonic zero modes $x^\mu$ and four fermionic zero modes $\theta^\alpha \equiv \theta_0^\alpha$ and $\mathcal{E}_\alpha \equiv \mathcal{E}_0^\alpha$, the rest of the $\mathcal{E}_A, \theta_A$ modes being lifted by the holonomy of the background.

In this case we would like to argue (as was also done in \[34\]) that the ADHM-like coupling

$$\mathcal{E}_A (\omega^\alpha \mu_0 + \mathcal{E}_0 \omega^\alpha)$$

survives even for non-trivial intersections. Indeed it is easy to show that the $U(1)$ world sheet charge is conserved using the vertex operators given in (3.12), (3.13), (3.24), (3.28) and the vertex operator for $\mathcal{E}$

$$V_{\mathcal{E}}^{-\frac{1}{2}} (z) = \mathcal{E}_\alpha S^\alpha e^{-\frac{i}{2} H_1} e^{-\frac{i}{2} H_2} e^{-\frac{i}{2} H_3} e^{-\varphi/2}.$$  (3.38)

\[16\]Note that for the gauge instanton setup, which corresponds to all three intersection angles $\theta_i$ being 0, the vacuum is defined differently. In that case the $\mu'_A$ and $\mathcal{E}'_A$ modes are projected out and do not appear in the instanton mode spectrum.
Note that due to the absence of the $\Psi^A (A \neq 0)$ the other fermionic modes $\mu_A$ and $\bar{\mu}_A$ do not couple to the bosonic modes $\varphi$ and $\omega$.

One might wonder if $\tau$ also couples in a similar fashion to the $\mu_0'$ and $\bar{\mu}_0'$ modes. The corresponding amplitude vanishes since it violates $U(1)$ world-sheet charge, and thus the corresponding coupling in the instanton effective action is absent. In fact, apart from the mass terms in (3.29) and (3.32), there is no additional coupling involving $\mu_0'$ and $\bar{\mu}_0'$.

There is also the coupling of the bosonic fields $\omega$ and $\bar{\omega}$ to the auxiliary field $D^{\mu \nu} = \bar{D} \sigma_{\mu \nu}$

\[ i \bar{D} \bar{W} \]  

where $W^c$ is displayed in (2.8). The vertex operator in the 0-ghost picture of the auxiliary field $D^c$ is given by (for details see [39])

\[ V_0^D = \frac{1}{2} \bar{D} \sigma_{\mu \nu} \psi^{\mu} \psi^{\nu} \]  

and again it is easy to check that the $U(1)$ world sheet charge is preserved in this amplitude.

Summarizing, we have the following $E2$–$D6$ interaction terms:

\[ S^{E2-D6} = m_{\omega} \varphi^A \mu_A' + m_{\mu_0} \mu_B \bar{\mu}_B' + m_{\omega} \varphi \bar{\omega} + \tau \varphi \mu_0' + \bar{\tau} \bar{\omega} + i \bar{D} \cdot \varphi \bar{\sigma} \varphi \]  

As expected this action preserves the supercharge $Q^0_\alpha$. This can be easily verified using the transformation behavior for the charged and neutral instanton displayed below (see sections 2 and 3.1):

\[ \delta_{\xi} \mu_0' = i \xi^0_\alpha \varphi^{\dot{\alpha}} \]  
\[ \delta_{\xi} \omega^{\dot{\alpha}} = i \xi^0_\alpha \omega_0 \]  
\[ \delta_{\xi} \bar{\tau} = \xi^0_\alpha \bar{\sigma} \bar{D} \]  

and similarly for the conjugated modes:

\[ \delta_{\xi} \bar{\mu}_0' = i \xi^0_\dot{\alpha} \bar{\varphi}^{\alpha} \]  
\[ \delta_{\xi} \bar{\omega}_0 = i \xi^0_\dot{\alpha} \bar{\omega}_{\dot{\alpha}} \]  
\[ \delta_{\xi} \bar{\tau} = \xi^0_\dot{\alpha} \bar{\sigma} \bar{D} \]

For all other fields in (3.41) the transformation behavior is trivial. Note that the presence of the mass term for the bosons $m_{\omega} \varphi \bar{\omega}$ requires the appearance of the mass terms $m_{\mu_0} \mu_0 \bar{\mu}_0'$ and $m_{\bar{\mu}_0} \bar{\mu}_0' \mu_0$ with $m_{\omega} = m_{\mu_0} = m_{\bar{\mu}_0}$ to ensure invariance under the supercharge $Q^0_\alpha$. Supersymmetry also explains the lack of couplings between $\tau$ and the primed modes $\mu_0'$, $\bar{\mu}_0'$.

From (3.41) and (3.42) we have that in order to preserve supersymmetry the cubic coupling to $\tau$ must be of the schematic form:

\[ \tau \delta_{\xi} (\omega \bar{\omega}) \]  

and in particular $\tau$ only couples to modes that can be obtained from supersymmetry variations of $\bar{\omega}$ and $\omega$, which excludes couplings to $\mu_0'$ or $\bar{\mu}_0'$.

### 3.3 Saturation of the fermionic modes

With the explicit action (3.41) for instanton modes at hand it is a simple matter to argue that no superpotential is generated. In order to show this, we will perform the relevant
part of the path integral calculation explicitly, and argue that it is not possible to saturate all the integrals over fermionic modes of the instanton simultaneously.

The path integral takes the form (here we focus only on the terms potentially relevant for the saturation of the $\bar{\tau}$ modes):

$$
\int \prod_{A=0}^{3} d^2 \omega d^2 \bar{\omega} d\mu'_A d\mu_A \prod_{B=0}^{2} d\bar{\mu}_B d\bar{\mu}_B d\bar{\mu}_3 e^{-S_{E^2-D^6}^{E^2-D^6}}
$$

(3.45)

After performing the integration over the $\bar{\tau}$ modes and $\mu^0, \bar{\mu}^0$ we are left with

$$
\int \prod_{A=1}^{3} d^2 \omega d^2 \bar{\omega} d\mu'_A d\mu_A \prod_{B=1}^{2} d\bar{\mu}_B d\bar{\mu}_B d\bar{\mu}_3 d\mu'_0 d\mu_0 e^{-(m_{\mu_A} + m_{\phi} \mu^0) \bar{\tau}^A \bar{\mu}^A} e^{-m_{\mu_0} \bar{\mu}^0 \mu^0 \bar{\mu}^0}
$$

(3.46)

where we omit the term $\bar{D} \cdot \omega \bar{\sigma} \bar{\omega}$ which is irrelevant for the analysis. Next we use the mass terms for saturating the remaining fermionic charged instanton modes:

$$
\int \prod_{A=1}^{3} d^2 \omega d^2 \bar{\omega} d\mu'_A d\mu_A d\mu'_0 d\mu_0 e^{-(m_{\mu_0} + m_{\mu_0} \mu^0) \bar{\tau}^A \bar{\mu}^A} e^{-m_{\mu_0} \bar{\mu}^0 \mu^0 \bar{\mu}^0}
$$

(3.47)

Note that we cannot saturate the $\mu'_0$ and $\bar{\mu}'_0$ modes since we used already $\mu_0$ and $\bar{\mu}_0$ to saturate the universal $\bar{\tau}$ modes. Since there is no other way to soak up these modes the whole path integral vanishes. Thus we conclude that a generic rigid $U(1)$ instanton does not contribute to the superpotential, even when intersects a background D-brane.

Let us comment on the saturation of massive modes appearing at intersections of D-branes and instantons invariant under some orientifold projection. As mentioned in the introduction, introducing an orientifold is one of the conventional mechanisms for getting rid of $\bar{\tau}$ modes. In this case, due to the absence of the $\bar{\tau}$ modes we can saturate all fermionic modes via the mass terms apart from the massless $\mu_3$. As mentioned earlier this mode is the massless charged mode often called $\lambda$. It can couple to open string matter fields located between two D6-branes and thus induces superpotential contributions involving these open string matter fields.

Finally let us briefly compare this setup with the gauge instanton case discussed in section 2, for which a superpotential contribution can be generated. The crucial difference between these two configurations is the massiveness of the involved instanton modes in case of a non-trivial intersection. We showed that the naive expectation that the $\mu_A$ and $\bar{\mu}_A$ pair up to create a mass term fails, due to the fact that they have different masses in the regime in which all supersymmetry is broken. In order to have a proper mass term the presence of additional fermionic modes $\mu'_A$ and $\bar{\mu}'_A$ is required, whose presence we confirmed in section 3.1 by applying preserved and broken supersymmetry to the bosonic states $\omega$ and $\bar{\omega}$. While the undesired $\bar{\tau}$ modes can be saturated via the coupling (3.37), in the path integral there is no coupling to $\mu'$ and $\bar{\mu}'$ not involving $\mu$ and $\bar{\mu}$, respectively. Thus the modes $\mu'$ and $\bar{\mu}'$ cannot be saturated and the path integral vanishes.
4. Some related systems

In the previous discussion we focused on the system of branes intersecting at angles in type IIA string theory. Using some well-known duality relations of string theory it is simple to carry over the lesson from the intersecting case to other corners of the string theory moduli space. In particular, we will (briefly) discuss the implications for type IIB string theory. Before going into this, let us discuss a somewhat subtle point involving the stability of instantons as we move in moduli space.

Duality and holomorphy of instanton superpotentials

The issue is the following: generically mirror symmetry will map smooth geometric compactifications into dual conformal field theories with no simple geometrical description \([54, 55, 56]\). In order to obtain a geometric description of the dual we need to move in the dual Kähler moduli space into a region admitting a large volume description.

One might worry that upon doing this, since the spectrum of BPS instantons will generically change discontinuously as we move in moduli space, the results we obtained for intersecting branes are no longer relevant for the regions of interest. In other words, there exists the possibility that the non-perturbative superpotential is not a smooth function in Kähler moduli space, and we cannot extrapolate our vanishing results.

The resolution of this issue comes, as in the vanishing argument given in the introduction, from holomorphy of the resulting low-energy superpotential. The non-perturbative superpotential due to instantons is a holomorphic function in Kähler moduli space, and we can reliably extend the result we obtained in the type IIA description to the geometric regime of the dual, even if the microscopic description might change discontinuously. By analyticity this also means that, since the vanishing of the non-perturbative superpotential holds for an open subset of the type IIA complex structure moduli space, it must hold everywhere in the Kähler moduli space of the dual.

With these remarks in place, let us proceed to discuss the extension of our results to type IIB.

4.1 Branes at singularities in Type IIB

The first dual picture we want to discuss is that of branes at singularities in type IIB \([57, 58, 59]\). For definiteness, and in order to make contact with some existing literature, let us take the case of (fractional) D3 branes located at the \(\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2\) orbifold. This system was studied using CFT techniques in \([14]\). Of particular interest to us is the discussion in section 3.3 of that paper, where it was argued that instantons wrapping nodes not wrapped by fractional D3 branes (the dual of instantons wrapping cycles not homologous to the cycle of the branes) do not give contributions to the superpotential, agreeing with our results.

In \([14]\) only the massless modes from the instanton to the brane were considered, and one might wonder what is the dual of the tower of massive modes we found before. The answer is that the spacing of the modes in the tower, which in the intersecting brane case we took to be small (proportional to the small angles), is at the quiver point of the order of the string scale. This will become clearer in next section, when we blow up the singularity.
We will recover a tower of states with arbitrarily small mass separations, given by the Kaluza-Klein scale of the blown-up exceptional cycle. This scale is dual to the angle in the type IIA picture, and it becomes of the string scale at the quiver point.

We can generalize this discussion easily to branes located at arbitrary toric singularities (abelian orbifolds are particular examples of toric singularities). The technology to deal with these cases is well developed by now, in the form of dimer models [60, 61, 62, 63] (see also [64, 65] for reviews). Although there are no available CFT techniques for studying arbitrary toric singularities, all of them descend from orbifold singularities upon performing partial resolutions.\(^{17}\) These resolutions appear in the theory on the brane as Higgsings, which cannot generate a non-perturbative potential for stringy D-brane instantons if there was none in the original orbifold theory (and Higgsings cannot map gauge instantons into stringy instantons).

The duality between D3 branes at toric singularities (and the associated dimer models) and intersecting D6 branes in the mirror of the toric CY is well understood by now [63], and we refer the curious reader to that paper for details of the duality map.

4.2 Magnetized branes in Type IIB/F-theory

Let us now resolve the singularity by moving away from the quiver point in Kähler moduli space into the large volume regime. This can be done most systematically for singularities which can be resolved by blowing up a four cycle; one important example of this class are the complex cones over the del Pezzo surfaces. For instance, blowing up the \(\mathbb{C}^{3}/\mathbb{Z}_{3}\) orbifold gives the complex cone over \(dP_{0}\).

When this is the case, we have a rather powerful large volume description of the physics in terms of exceptional collections \([67, 68, 69, 70, 71]\) living on the blown up four-cycle. Physically we can interpret these exceptional collections as a nice basis of fractional D3 branes, which at large volume are interpreted as D7 (or \(D\bar{7}\)) branes with fluxes on their worldvolume. We can then map the fractional branes discussed in the previous section to elements in the exceptional collection by using the \(\Psi\) map proposed in [72].\(^{18}\)

Using these ideas we can follow the duality between the intersecting branes and the mirror large volume branes in detail. The Kaluza-Klein tower of states (going from the instanton to the brane) in the large volume picture is dual to the tower of states spaced by the brane intersection angle analyzed in section 3, and the whole discussion done there applies with little change to the system at hand.

Thus, this Type IIB mirror description of the system studied in section 3 does not seem to give a non-perturbative superpotential; it would be interesting to address these aspects within a fully F-theoretical context.

5. Non-commutative instantons

In the introduction we discussed an argument based on the considerations of [8, 18] why the

\(^{17}\)See for example [66] for efficient techniques to perform the resolutions.

\(^{18}\)For completeness, let us mention that the full physical interpretation of the large volume objects one obtains from the fractional branes is slightly more complicated: they are objects of the derived category of coherent sheaves \(D^{b}(X)\), where \(X\) is the Calabi-Yau cone, see for example [73] for a detailed discussion.
system with instantons and branes at angles should not contribute to the superpotential. The key observation was that the instanton can misalign from the brane without recombining, so it cannot contribute to the superpotential when misaligned, and by holomorphy of the superpotential on the closed string moduli that create the misalignment, it should not contribute to the superpotential when aligned either.

We wish to emphasize in this section the importance of the condition that the instanton should not be able to recombine with the brane. As verified explicitly in section 3, the spectrum of bosonic strings going from the instanton to the brane at an angle in the internal space has positive nonvanishing mass in the misaligned regime, so no recombination is possible. In this section we would like to discuss the non-commutative instanton in string theory as a simple example in which the instanton does dissolve into the brane when misaligned, due to the appearance of tachyons.

Let us consider a single D\((p+3)\) brane extended in the four Minkowski directions and wrapping a supersymmetric \(p\)-cycle \(\Sigma_p\) in the internal space, such that at low energies the theory on the worldvolume of the brane reduces to \(\mathcal{N} = 1\) SYM with gauge group \(U(1)\). We are interested in non-perturbative corrections to the superpotential coming from euclidean D\((p-1)\) branes wrapping \(\Sigma_p\). As discussed in [7, 8, 9, 10], even though in \(U(1)\) gauge theories there are no instanton effects, in string theory the D-brane instanton wrapping the same node as the brane does generate a low energy superpotential:

\[
W_{\text{eff}} = e^{-S_{\text{inst}}} = e^{-\frac{1}{g_s}V\{\Sigma_p\}} \tag{5.1}
\]

with \(V\{\Sigma_p\}\) the complexified volume of \(\Sigma_p\). It is possible to misalign the brane and the instanton by switching on a field strength on the Minkowski part of the D\((p+3)\) brane, or equivalently by switching on a \(B\)-field.

There is a well known dual description of such a system: non-commutative gauge theory [74]. We will see here that the result one obtains from non-commutative gauge theory nicely matches what one requires from holomorphy, and in particular a tachyon appears when the instanton becomes misaligned with the D-brane.

For completeness, we include here a telegraphic review of the main points we will need. We refer the reader to [75, 76] for some nice set of lectures on non-commutative gauge theories, and to [74] for the original embedding of non-commutative field theories in string theory that we will be using.\(^{19}\)

In a non-commutative gauge theory the space-time coordinates satisfy the algebra:

\[
[x_\mu, x_\nu] = i\theta_{\mu\nu} \tag{5.2}
\]

with \(\theta_{\mu\nu}\) some antisymmetric tensor. It will prove convenient to split \(\theta\) into its self-dual and anti-self-dual parts:

\[
\theta_{\mu\nu} = \xi^i\eta_{\mu\nu}^i + \xi^i\eta_{\nu\mu}^i \tag{5.3}
\]

\(^{19}\)There were interesting previous embeddings of non-commutative gauge theory in string theory, see for example [71, 72, 73].
with η and ıη the self-dual and anti-self-dual ’t Hooft symbols respectively (see appendix A of [36] for explicit expressions). The index i runs from 1 to 3.

The way we engineer such a deformation of the gauge theory in our stringy setup is simple: we just need to include a B-field in the four Minkowski directions [79, 74]. The non-commutativity parameter will then be given by θ = B⁻¹.

Let us discuss the supersymmetries preserved by such a background. We pick the convention that the D(p − 1) instanton preserves the same supersymmetry as a self-dual field strength on the D(p + 3) brane. By the same token, if we want to misalign the instanton with respect to the brane, we should switch on the anti-self-dual part of the field strength. In terms of θ, we are thus interested in switching on ξ⁻.

It is a well known fact that switching on ξ⁺ does not modify the moduli space of instantons for a U(1) theory (see [74], for example). In particular, it does not remove the small instanton singularity corresponding to the stringy D(p − 1) instanton. In contrast, switching on the anti-self-dual term ξ⁻ has a drastic effect on the moduli space: it removes the small instanton singularity.

Let us briefly recall how this modification of the moduli space comes about. The introduction of θ modifies the ADHM construction of instantons by the introduction of a Fayet-Iliopoulos term in the bosonic ADHM constraints:

\[ \bar{\pi} \sigma^i \omega = \xi^i \]  

(5.4)

Since the radius ρ of the instanton is given by ρ = \|ω\|, this implies that the instanton can no longer have zero size, and it necessarily dissolves into the brane, as advertised.

It is not hard to see the required tachyon explicitly, either. Let us write

\[ \omega = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \]  

(5.5)

and choose \( \xi^i_\_ = (0, 0, \xi) \). From (5.4), we have:

\[ |\phi_1|^2 - |\phi_2|^2 = \xi \]  

(5.6)

Equivalently, recalling that (5.4) is actually a D-term in the instanton action, we have:

\[ S_{inst} = \ldots + \left( |\phi_1|^2 - |\phi_2|^2 - \xi \right)^2 \]  

(5.7)

Note how for any nonzero ξ one of \( \phi_1, \phi_2 \) gets a negative mass squared, becoming the required tachyon. The condensation of the tachyon restores supersymmetry, and allows the (diluted) instanton to contribute.

6. Conclusions

In this paper we have discussed in some detail the system of branes and instantons intersecting at angles, with the conclusion that they cannot contribute to the superpotential. We gave both macroscopic arguments based on holomorphy (in the introduction), and microscopic arguments using a local CFT analysis (section 3). In section 4 we discussed some
important implications of our result for the dual type IIB picture, and in section 3 we briefly discussed how the presence of tachyons allows for non-perturbative contributions even for instantons that can be misaligned, which we illustrated in the interesting case of non-commutative instantons.

We believe our result to be robust, and widely applicable for any system where the instanton has a supersymmetry phase independent of the brane (following the arguments of [18]). We would like to emphasize at this point that the key issue is really the existence or not of tachyons at every point of moduli space where the instanton gets misaligned with respect to the background branes. In general this is a condition that can be easily checked. Modes with positive mass have no effect on the saturation of $\tau$, as we showed in detail in section 3. Microscopically we could also see this from the requirement of supersymmetry of the instanton effective action, which disallowed massive modes from saturating massless modes.

Possibly the most conspicuous omission in the analysis above is that of the massive modes of string scale. That is, modes with masses of the form:

$$m = c + \sum_{i=1}^{3} a_i \theta_i$$  \hspace{1cm} (6.1)

with $c \neq 0$. If such an effect were present, it would be in fact the leading contribution to the otherwise vanishing couplings in the superpotential. We do not expect such modes to alter the conclusion for the following reasons. As mentioned in the introduction, a similar structure of higher massive modes is present in the gauge instanton case, where it is known that they do not change the vanishing results one obtains by looking at the massless sector only. Another argument follows from the world-sheet charge conservation considerations in section 3: in order to construct a mass term we need two fermionic modes, but by world-sheet charge conservation arguments only one of them can couple to $\tau$.

Furthermore, one might also entertain the idea that $g_s$ effects might play a role in the lifting of $\tau$ zero modes in the intersecting instanton case. One strong argument against this possibility is the holomorphy argument which still holds in the low energy supergravity: if one can misalign the instanton with the brane no superpotential should be generated.

Another interesting avenue of research concerns higher F-terms [80, 81]. In general, when $\tau$ modes are not saturated BPS instantons contribute to higher fermionic F-terms [16], and here intersecting brane instantons can have an important effect. In particular, they will give interesting deformations of the moduli space of the string compactification.

We hope that this work helps to clarify part of the global structure of non-perturbative effects in string theory compactifications, a fascinating topic.

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