New absolute distance measurement technique with a self-mixing interferometer

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Abstract. A new compact self-aligned noncontact range finder is described. It uses the self-mixing effect inside a laser diode. Double modulation technique is proposed to improve the measurement accuracy. Wavelength modulation (WM) of the laser beam is obtained by modulating the injection current of the laser diode. Phase modulation (PM) of the laser beam is obtained by an electro-optic crystal (EOC) in the external cavity. Absolute distance of the external target is determined by Fourier analysis method. Experimental results show that an accuracy of ±0.3mm can be achieved for absolute distance ranging from 277mm to 477mm.

Keywords. self-mixing interference, absolute distance measurement, double modulation technique

1. Introduction

Laser ranging techniques are widely used to perform contactless measurement of the distance of a remote target for both industrial and scientific applications. Laser range finders can technically be put into three categories: time of flight, triangulation, and interferometric techniques. [1] Coherent techniques based on interferometric methods are generally based on Michelson interferometers which use a laser source with continuously tuneable emission wavelength. Wavelength sweeping generates an interferometric signal that carries information about the difference in length of the two interferometer arms. This method can exhibit a high accuracy, but at the expense of a complicated experimental set–up, that often requires a double interferometer. [2-3]

Self-mixing interference (SMI) system is simpler than the conventional interferometers because many optical elements like the beamsplitter, reference mirror, and the external photodetector are not required. Based on self-mixing interference, many smart and simple sensing systems have been developed. Self-mixing interferometer was used to measure absolute distance by counting the number of mode hops or measuring a beat signal frequency, achieving a resolution of few mm.[4-5] In this paper, we present a novel approach to absolute distance measurement based on the self-mixing effect. Double modulation technique is introduced to improve the absolute distance measurement resolution. Wavelength modulation (WM) of the laser beam is obtained by sinusoidal modulating the injection current of the laser diode. Phase modulation (PM) of the laser beam is obtained by an electro-optic crystal (EOC) in the external cavity. Absolute distance of the external target is determined by Fourier
analysis method. The principle of the measurements is described and experimental results are given in this paper.

2. Theoretical analysis

2.1. Self-mixing interference

![Figures 1(a) and 1(b)]

Figure 1. (a) A laser diode with an external cavity. (b) Equivalent model.

The basic theory of the self-mixing effect can be explained by two Fabry-Perot cavities shown in figure 1 (a) and its equivalent Fabry-Perot cavity is represented by figure 1(b). Here \( r_1 \) and \( r_2 \) are the amplitude reflectivity of LD facets, \( r_3 \) is the amplitude reflectivity of the external target, \( l \) is the length of the laser cavity and \( L \) is the length of the external cavity.

The laser beam emitted from a LD is reflected from the target, a portion of the laser output back into its laser cavity, and the self-mixing interference occurs. Neglecting the multiple reflections in the external cavity, the frequency of the equivalent cavity \( \nu_0 \) and the emitted optical power can be expressed as [6]

\[
\nu_0 = \frac{C}{2\pi l_t} \sin(2\pi \nu_0 t + \arctan \alpha) \tag{1}
\]
\[
l = I_0 [1 + mF(\phi)] = I_0 [1 + m\cos(4\pi\nu L/c)] \tag{2}
\]

Where \( \nu_0 \) and \( I_0 \) are the optical power and the optical frequency of LD without optical feedback, \( \alpha \) is the linewidth enhancement factor, and \( m \) is the undulation coefficient. The external feedback strength parameter \( C \) is defined by

\[
C = \frac{L}{l_t} \zeta \sqrt{1 + \alpha^2} \tag{3}
\]

The parameter \( \zeta \) is the coupling coefficient. \( l_t \) and \( l_t \) denote the time of flight within the external cavity and the laser cavity. For a weak feedback level (\( C<1 \)), the variation in the laser frequency caused by optical feedback is very small. The function \( F(\phi) \) is nearly sinusoidal and the signal amplitude increases for increasing level of the optical feedback. At higher feedback level (\( C=1 \)), the interferometric waveform exhibits a slight distortion. For moderate optical feedback (\( C>1 \)) the interferometric signal waveform becomes sawtooth-like and it exhibits hysteresis. In the following theoretical analysis, only the case when \( C<1 \) is discussed.

The self-mixing configuration has been demonstrated to be capable of performing interferometric measurements of the absolute distance between the laser source and the remote target. The conventional principle of absolute distance measurement by self-mixing is as follows. When the LD emission wavelength is modulated by an amount \( \Delta \lambda \), a self-mixing interferometric signal is generated, because the optical phase \( \Phi = 4\pi L/\lambda \) is correspondingly varied by the amount \( \Delta \Phi = -4\pi L \Delta \lambda/\lambda^2 \).

Practical implementation of this principle has been demonstrated by applying a triangular modulation to the LD injection current, that generates a known wavelength variation \( \Delta \lambda \). By counting the number \( N \) of fringes occurring in the self-mixing interferometric signal within one semi-period, the target distance \( L \) can be retrieved as \( L = \lambda^2 N/(2 \Delta \lambda) \). The resolution of the fringe counting method is limited by the fringe quantization error: \( \Delta L_{\text{error}} = \pm \lambda^2/(2 \Delta \lambda) \). This formula reveals that measurement accuracy can be improved by increasing the extent of the wavelength modulation. Unfortunately, continuous thermal wavelength tuning by injection current in Fabry-Perot LDs is typically limited to about 0.1nm
by longitudinal mode-hopping. Hence, when Fabry-Perot LDs are used as light sources for this application, the attainable resolution cannot be reduced below a few mm.

2.2. Principle of the target distance determination by double modulation technique

In order to determine the target distance $L$ with high resolution, double modulation technique is introduced. Wavelength modulation of the laser beam is obtained by sinusoidal modulating the injection current of the laser. The injection current of the laser diode consists of a dc bias component and a sinusoidal modulation signal $\text{asino}_{t}\theta$, where $a$ is the modulation depth of the injection current, $\omega_{m}$ is the modulation frequency and $\theta$ is the initial phase of modulation. As for the modulation depth $a$, one point must be taken into account is that the value of $a$ must be small enough so that the mode jump of the laser diode does not occur. The modulation of injection current results in both an intensity modulation and a wavelength modulation given by:

$$h(t) = h_0 + g \cdot a \cdot \sin(\omega_{m}t + \theta)$$

$$\lambda(t) = \lambda_0 + \Delta \lambda(t) = \lambda_0 + k \cdot a \cdot \sin(\omega_{m}t + \theta)$$

Where $h_0$ is the output intensity of the laser diode without modulation, $g$ is the factor of proportional relationship between the output intensity and the injection current, $\lambda_0$ is the wavelength of the laser without modulation and $k$ is the factor of proportional relationship between the wavelength and the injection current. The self-mixing interference signal can be expressed as:

$$S(t) = h(t)[1 + m \cos(\phi(t))]$$

Where

$$\phi(t) = \frac{4\pi L}{\lambda_0 + \Delta \lambda} \approx \frac{4\pi L}{\lambda_0} - \frac{\Delta \lambda(t)4\pi L}{\lambda_0^2} \approx \phi_0 - T \sin(\omega_{m}t + \theta)$$

$$T = 4\pi Lka / \lambda_0^2$$

From equation (8) we can see that the amplitude of the phase $\phi(t)$ (i.e. $T$) is proportional to the target distance $L$. In order to extract the value of $T$ from the interference signal, sinusoidal phase modulation of the laser beam is introduced by an EOC in the external cavity, which can be given by:

$$\psi(t) = b \sin(\omega_{m}t + \beta)$$

Where $b$ is phase modulation depth, $\omega_{m}$ is the modulation frequency which satisfies $\omega_{m} >> \omega_{m}$, $\beta$ is the initial phase of the modulation. Considering that the beam pass through the EOC twice in self-mixing interference, the double modulated interference signal can be written as:

$$S(t) = h(t) + h(t) \cdot m \cdot \cos(\phi(t) + 2b \sin(\omega_{m}t + \beta))$$

Expanding equation (10) in a Fourier series, harmonics at frequency $\omega_{m}$ and $2\omega_{m}$ have following expressions:

$$S(\omega_{m}, t) = 2mh(t)J_{1}(2b)\sin(\phi(t))\sin(\omega_{m}t + \beta)$$

$$= A_1(t)\sin(\omega_{m}t + \beta)$$

$$S(2\omega_{m}, t) = 2mh(t)J_{2}(2b)\cos(\phi(t))\cos(2\omega_{m}t + 2\beta)$$

$$= A_2(t)\cos(2\omega_{m}t + 2\beta)$$

Where $J_n(2b)$ is the $n$th-order Bessel function. The phase $\phi(t)$ can be extracted from the amplitude of the first harmonic $A_1(t)$ and second harmonic $A_2(t)$ using the subsequent relation:

$$\phi(t) = \arctan \left[ \frac{A_1(t) \cdot J_2(2b)}{A_1(t) \cdot J_1(2b)} \right]$$

From equation (13) we can see that when $J_1(2b)$ or $J_n(2b)$ is zero, the phase $\phi(t)$ can not be extracted correctly. As we all know, $J_1(2b)$ and $J_2(2b)$ are oscillating functions of parameter $b$, with a series of zero-crossings as shown in figure2. These values of $b$ should be avoided when choosing the modulation depth of EOC.
Fourier analysis method is proposed here to determine the values of \( A_1(t) \) and \( A_2(t) \). It mainly takes following steps: (1) taking the Fourier transform of the interference signal; (2) selecting the first harmonic \( I(f_m) \) and second harmonic components \( I(2f_m) \) of the Fourier spectra with two windows: \( f_m / 2 < f < 3f_m / 2 \) and \( 3f_m / 2 < f < 5f_m / 2 \); (3) taking the inverse Fourier transform of the filtered component, represented by \( I_{m1}(t) \) and \( I_{m2}(t) \) respectively; and (4) computing \( A_1(t) \) and \( A_2(t) \) using the subsequent relation:

\[
A_1(t) = \text{Im} \left[ I_{f_m}(t) / e^{i(\omega_m t + \phi)} \right] \tag{14}
\]
\[
A_2(t) = \text{Re} \left[ I_{2f_m}(t) / e^{i(2\omega_m t + 2\phi)} \right] \tag{15}
\]

As the modulation frequency is concerned, \( \omega_m >> \omega_a \) must be satisfied so that the occurrence of band overlapping in the frequency domain does not occur. The phase \( \phi(t) \) obtained using the Fourier analysis method is wrapped within the region of \( -\pi \) and \( \pi \). After unwrapping the phase, \( \phi(t) \) is extracted and its amplitude \( T \) can be determined. The target distance \( L \) can be obtained using the relationship in equation (8).

3. Experimental results

The experimental setup is shown in Fig.3. It consists of a LD package with a photodetector, an aspherical collimating lens, an EOC (New Focus 4002) and an object target. The central wavelength \( \lambda_0 \), maximum output power of the LD are 638 nm and 5mW respectively. The temperature of both the laser mount and the lens holder, which are embodied in one piece of aluminum alloy is stabilized at \( T < 0.01^\circ C \). And the driving current of LD is stabilized at \( \Delta i < 4\mu A \). The angle between the polarization
direction of the laser diode and the electro-optically active axis of EOC is 0°. The EOC can provide pure phase modulation with extremely low amplitude modulation. The interference signal monitored by the PD in the LD package is sent through a transimpedance amplifier and digitized with a 200-KHZ, 12-bit analog-to-digital board on a PC bus (National Instrument, NI 6024E).

In our experiments, the amplitude $a$ and frequency $f_a$ of the current modulation is 0.1mA and 20HZ respectively. The modulation amplitude $b$ and modulation frequency $f_m$ of EOC is 1.2rad and 2KHZ respectively. The first rising edge of TTL level from the signal generator which drives the EOC driver is introduced to the A/D card as the trigger of data acquisition. The initial phase of EOC modulation can be controlled at $\pi/2$. Figure4.(a) is detected double modulated self-mixing interference signal. Figure4.(b) is the Fourier spectra of the detected signal in which the first harmonic component centered at $f_m$ (2KHZ) and the second harmonic component centered at $2f_m$ (4KHZ). Following the demodulating steps we proposed in this paper, the phase variation of the laser beam corresponding to current modulation is calculated as in Figure4. (c). After unwrapping the phase, the phase variation $\phi(t)$ is reconstructed in Figure4. (d) and the amplitude of the phase variation $T$ can be obtained.

In experiments, the initial distance of the external target is $L=277$mm. The target is fixed on a stage which can provide a long distance smooth movement and the position $\Delta L$ can be digitally displayed with a resolution of 0.01mm. At each position, we measure the value of $\Delta T$ and get the measured distance $\Delta L_m$ using the relationship in Eq. (8). The distance measurement result for a range from $\Delta L=0$mm to $\Delta L=200$mm is shown in figure5. To assess the accuracy achievable with the proposed system, repeated measurements have been made for each distance and their statistical distribution is analyzed. The asterisks in figure5 represent the average values. In figure6, the black points represent the measurement error and the error bars represent the standard deviation $\sigma$. These experimental results are in good agreement with the theoretical analysis and an accuracy of $\pm0.3$mm can be obtained.
4. Conclusion
We have proposed and experimentally demonstrated a new technique for the measurement of the absolute distance of a remote target without contact. The technique is based on the self-mixing interferometric configuration, which makes use of a low cost laser diode and has a very simple optical set-up, only made by the laser, a focusing lens, EOC and the target. The proposed technique brings about an improvement in the measurement resolution with respect to the conventional self-mixing distance measurement method, which is based on fringe counting. A first prototype of the instrument has been built to confirm the validity of the proposed technique and the absolute distance have been measured with a resolution of ±0.3mm over the range 277mm – 477mm. In order to increase the measurable range to about a few meters, it is necessary to increase the laser output power and to reduce the laser FM noise.

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