Generation of Cosmic Magnetic Fields at Recombination

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ABSTRACT

It is shown that the standard cosmological model predicts ab initio generation of large-scale but very small-amplitude cosmic magnetic fields at the epoch of recombination of the primeval plasma. Matter velocities dominated by coherent flows on a scale \( L \approx 50 h^{-1}(1 + z)^{-1} \) Mpc lead to a dipole of radiation flux in the frame of the moving matter. Thomson scattering of the radiation differentially accelerates the electrons and ions, creating large-scale coherent electric currents and magnetic fields. This process is analyzed using magnetohydrodynamic equations which include a modification of Ohm’s law describing the effect of Thomson drag on the electrons. The field strength is estimated to be \( B \approx 10^{-20} \) G.

1. Introduction

The behavior of cosmic matter and radiation at \( z \approx 1100 – 1300 \), the recombination era, is now broadly understood from the concordance of direct observations (e.g. de Bernardis et al 2000, Lange et al. 2000, Hanany et al. 2000) and detailed theoretical models (e.g. Bond et al. 1996, 1999, Hu et al. 1996, Lawrence et al. 1999) of anisotropy in the cosmic background radiation. The amplitude and shape of the anisotropy spectrum at Legendre multipoles \( l \leq 400 \) (and especially the first acoustic peak at \( l \approx 200 \)) confirm the main physical ingredients of the model: baryonic matter, thermal blackbody radiation, collisionless dark matter, and large-scale primordial adiabatic perturbations. This note analyzes the generation of large-scale coherent electric and magnetic fields in this system. Although the fields are critical for dynamically coupling the ions and electrons and apply forces comparable to the Thomson drag of radiation on the matter, the stresses of the residual magnetic fields are dynamically negligible.

The process described here generates the fields from scratch, without the need for an exotic early-universe seed field: it acts as a “battery” rather than a “dynamo”. The fields are generated by currents created by differential radiation pressure on the electrons and ions as plasma moves under the influence of gravity; the electrons experience a much stronger force from the radiation than the ions do, tending to create an electron-ion drift and hence an electric current. However, a large-scale electric field arises to cancel the Thomson current, induced by the formation of a

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large-scale magnetic field of comparable magnitude. This effect makes little practical difference in models of the recombination era, which assume perfect coupling between the electrons and rest of the plasma; it is interesting however that the coupling actually depends on large-scale electrical and magnetic fields, and therefore on the validity of Maxwell’s equations (including zero photon mass) over scales of about the horizon size (100 kpc) at recombination. The battery analyzed here resembles models of batteries in other astrophysical environments (e.g. Zeldovich et al. 1983).

2. Currents from Thomson Drag

Before decoupling, baryons and photons are tightly coupled; the radiation pressure provides a large restoring force so baryon perturbations are oscillating acoustic waves with sound speed \( c_S \) not much below \( c/\sqrt{3} \). Oscillations on scales with favorable phases for maximizing the density perturbation at decoupling lead to “acoustic peaks” in the angular spectrum of background anisotropy. After decoupling, the baryons no longer feel the pressure of the radiation and their own pressure is negligible, so they simply flow into the dark matter potentials. We focus here on the transition epoch around last scattering, when the photon path length is larger than the scales under consideration (so the photons are no longer tied to the baryons and oscillations have ceased), but when the radiation density and ionization are high enough that the photon drag on matter is still significant.

The generation of fields is controlled by the amplitude and spatial coherence scale of radiation dipoles in the matter frame, which (for small optical depth) depend mainly on the structure of the velocity flow caused by linear perturbations. The typical rms dark matter velocities corresponding to density perturbations on scale \( L \) are \( v_L \approx LH(\delta \rho/\rho)_L \), where \( H \) is the Hubble rate and \( (\delta \rho/\rho)_L \) the rms density contrast of the dark matter. In terms of the fluctuation power spectrum \( P(k) \), the rms peculiar velocities \( v_L \propto k^{3/2}v_k \propto k^{1/2}P(k)^{1/2} \), in standard CDM are maximized at the familiar scale determined by the comoving horizon size, \( ct_{eq}(1 + z_{eq}) = 50h^{-1}\text{Mpc}(\Omega_M h/0.2)^{-1} \), at matter-radiation equality \( (1 + z_{eq} = 4780[\Omega_M h/0.2]) \), and fall off (as \( v_L \propto L^{-1} \) and \( L \) on larger and smaller scales. (This is the same pattern on the same comoving scales as linear large-scale flows today, but with velocities smaller by a factor \( \approx (1 + z)^{-1/2} \).)

The dark matter velocity is about \( v/c \approx \delta T/T \approx 10^{-5} \) or \( v \approx 3\text{km s}^{-1} \) as each scale enters the horizon, and in the matter-dominated era grows thereafter as \( (1 + z)^{-1/2} \). (The acoustic velocity of the baryons, \( v \approx c_S(\delta \rho/\rho) \), depends on the phase of the acoustic oscillations and so has a more complicated dependence on scale.) For the present discussion we adopt the simplified picture that once the photon path length exceeds \( L_{eq} \), the background radiation as viewed from the moving frame of the baryons has a dipole anisotropy coherent over scales \( L \approx ct_{eq}(1 + z_{eq})/(1 + z) \), and we adopt \( v = v_{10} \times 10\text{km s}^{-1} \) as a typical value for baryon velocities on this scale.

The radiation dipole produces a drag on the residual electrons by Thomson scattering. This is by far the most important dynamical interaction of the radiation background with the matter, and
remains so even after the fractional ionization becomes small. An electron moving with velocity \( \vec{v} \) relative to frame in which the dipole vanishes experiences a drag force (e.g. Peebles 1993, Peacock 1999)

\[
\vec{F}_{\text{Thomson}} = -\frac{4}{3} \sigma_T a T_\gamma^4 \vec{v}/c
\]

where \( \sigma_T \) is the Thomson cross section and \( T_\gamma = 2728 z_{1000} K \) is the radiation temperature. The acceleration of the electron, if there are no other forces, is then

\[
\vec{a}_{\text{Thomson}} = \frac{\vec{F}_{\text{Thomson}}}{m_e} = -1.4 \times 10^{-2} \text{ cm s}^{-2} z_{1000}^4 \vec{v}_{10}. \tag{2}
\]

We can ignore the corresponding direct acceleration of the ions by radiation; the scattering is suppressed by two powers of mass, and the acceleration by one more.

If there are no macroscopically organized electromagnetic fields, the main other force experienced by an electron is friction on the ions, dominated by long-range, small angle electron-proton scatterings out to the Debye length. (Because of the long range of the Coulomb force, this remains true even if the ionization is low). The momentum transfer between electrons and ions of number density \( n_e \) is characterised by a rate (Spitzer 1962, Shu 1992)

\[
\nu_c \approx 3 \times 10^{-3} s^{-1} n_e T_{3000}^{-3/2}
\]

(This corresponds to an electrical resistivity \( \eta \approx c^2 n_e \nu_c/4\pi n_e e^2 \approx 0.6 \times 10^{13} \ln \Lambda T^{-3/2} \text{ cm}^2 \text{ s}^{-1} \), with \( T \) in K and \( T_{3000} = T/3000 \), where in this case the Spitzer factor \( \ln \Lambda \approx 20 \).) Thus the electron gas, in the absence of a macroscopic electromagnetic field, would develop a velocity relative to the ions

\[
\vec{v}_{ie} \approx \vec{a}/\nu_c = -4 \text{ cm s}^{-1} z_{1000}^4 \vec{v}_{10} n_e^{-1} T_{3000}^{3/2}, \tag{4}
\]

in the process of transferring the radiation drag momentum impulse to the rest of the plasma. In other words, this is the velocity an electron acquires before its accumulated drift momentum is randomized by scattering. (Note that the drift of charged particles relative to neutrals is much larger than this, but has no effect on this argument and will be neglected.) This relative velocity can develop without an accumulation of net charge, but corresponds to an electrical current coherent over the large scale of the matter flow.\(^2\) The numerical value seems like a small velocity, but the corresponding current density \( \vec{J} = -en_e \vec{v}_{ie} \) is actually very large. The magnetic field on scale \( L \) estimated from Ampere's law, \( \vec{\nabla} \times \vec{B} = (4\pi/c)\vec{J} \), has an amplitude

\[
B \approx 4\pi c^{-1} n_e v_{ie} L, \tag{5}
\]

which for typical numbers at recombination yields a field with \( B \geq 10^4 \text{ G} \) ! This clearly violates the assumption we have made of zero fields. It does however indicate that cosmic recombination with primordial perturbations inevitably generates \textit{ab initio} large-scale coherent fields.

\(^2\)Note that because of the Thomson drag, the velocity field is not an irrotational potential flow, so no symmetry prevents current loops from forming. Proper treatment of return currents however requires consideration of retarded fields and transients, which are omitted here.
3. Magnetohydrodynamics with Thomson drag

The field does not actually reach such a large strength; instead, the electric field increases (mostly via induction) until the transfer of (photon-drag-induced) momentum from the electrons to the ions and neutrals then occurs via the field rather than electron-ion friction. This reduces the electron-ion differential velocity, limiting the current. The situation is best described with equations of classical magnetohydrodynamics (Spitzer 1962, Jackson 1975, Shu 1992), but with the addition of Thomson drag on the electrons.

The main new effect is a modification of Ohm’s law. We adopt a “laboratory frame” in which the mean radiation dipole vanishes, and describe a fluid moving with velocity $\vec{v}$ in which we neglect ion-neutral drift. The medium responds with the same conductivity $\sigma = c^2 / 4\pi\eta$ whether electron accelerations arise from electric fields or from Thomson drag. Therefore in the fluid frame (denoted by primes), Ohm’s law for the current density now includes a drag term,

$$\vec{J}' = \sigma [\vec{E}' + (m_e/e)\vec{a}_{\text{Thomson}}],$$

Transforming to the lab frame,

$$\frac{\vec{J}}{\sigma} = \vec{E} + \frac{1}{c}(\vec{v} \times \vec{B}) - \frac{\vec{v}}{c \sigma T a T^4 e}.$$

As we have seen, the MHD approximation applies that $\sigma$ is very large; the fields adjust themselves such that the terms on the right side nearly cancel (i.e., $\vec{J}/\sigma \rightarrow 0$). (The new Thomson term however breaks the usual MHD phenomenon of “field freezing”, even though the conductivity is very high.) Therefore the electric field approximately balances the Thomson drag everywhere,

$$\vec{E} \approx -\frac{\vec{v}}{c \sigma T a T^4 e},$$

and has the same macroscopically organized structure and dynamical importance—indeed over time the electric field does the same work on the electrons as the Thomson drag. (However that work is almost cancelled by work done by the ions on the same field— which is the main dynamical coupling between ions and electrons.)

The electric field grows quickly (due to the “displacement current”) until it is sufficient to cancel the Thomson drag. The electric field in turn grows a magnetic field by Faraday induction,

$$\nabla \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = 0.$$

However, this corresponds to a very slow growth of the magnetic field; even by the end of the process the magnetic field is only about as strong as the electric field was all along, eq. (8), corresponding to a magnetic field of only about $10^{-20} \nu_{10} \text{G}$, independent of scale (for a given $\nu$). The coherence scale of the fields is comparable to the scale of the velocity flows, $L \approx 50h^{-1}(1+z)^{-1}\text{Mpc}$. 


The magnetic stress is too small to affect the flow of scattering matter or the pattern of cosmic anisotropy. The electron density and the radiation density both decrease rapidly after recombination, and the battery shuts down when the matter loses its purchase on the radiation. The Thomson drag time on the plasma as a whole exceeds the Hubble time when the ionization falls below \( n_e/n \leq 10^{-2} z_{1000}^{-5/2} \), which happens for standard ionization history at \( z \approx 900 \). However, the ionization remains high enough \( (n_e/n \geq 10^{-3.5}) \) to keep the fields frozen to the plasma. The fields passively follow the still nearly-uniform expanding medium, preserving the coherent \( \approx 50 h^{-1} \) Mpc-scale comoving pattern of fields as the field strength redshifts like \( B \propto (1 + z)^2 \). Linear perturbations in the baryon density grow in the usual way, responding mainly to dark matter gravity rather than magnetic stresses.

The magnetic fields never become dynamically important, although in principle they might form the seeds of dynamo fields. In essence the process acts like other astrophysical batteries, albeit on a much larger scale. As the field remains frozen to the matter, the magnetic field in a system of baryon density \( n \) at some later time has

\[
B \approx 10^{-22} G (n/\text{cm}^{-3})^{2/3}.
\]

More likely, the observed large scale galactic fields (see Kronberg 1994, Beck et al. 1996 and Zweibel and Heiles 1997 for reviews), fully-developed microgauss galactic fields at high redshift (Wolfe et al. 1992), large-scale fields \( \geq 0.2 \mu G \) between clusters in the intergalactic medium (Kim et al. 1989), and lower limits \( \geq 0.1 \sim 0.4 \mu G \) in the intracluster medium of galaxy clusters (Kim et al. 1991, Rephaeli et al. 1994, Sreekumar et al. 1996) arise as an ejection of strongly dynamo-amplified fields from compact systems such as AGNs, and have nothing to do with recombination.

An earlier version of this paper reached essentially opposite conclusions about the field strength—having wrongly guessed that the amplitude reached was limited by equipartition rather than the induction-limited growth rate. I am grateful for a strong contingent of alert readers for finding this classical error: A. Loeb, M. Rees, D. Spergel, and especially J. Goodman. I am grateful for critical comments by K. Jedamzik and D. Scott, and for useful conversations with S. Phinney, J. Wadsley and J. Dalcanton. This work was supported at the University of Washington by the NSF.

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