A model of a transition neutral pion formfactor measured in annihilation and scattering channels

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We consider an alternative explanation of newly found growth of neutral pion transition form factor with virtuality of one of photon. It is based on Sudakov suppression of quark-photon vertex. Some applications to scattering and annihilation channels are considered including the relevant experiments with lepton-proton scattering.

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I. INTRODUCTION

A lot of attention was paid to the problem of describing the transition form factor of neutral pion [1–3]. It is the behavior was obtained in process \(e^+e^-\rightarrow e^+e^-\pi^0\). The kinematics, when one of photon is almost real and the other is highly virtual was considered [4, 5]. The result was presented by authors of the experiment as a some nondecreasing function of the module of square of momentum of a virtual photon. Such type of behavior is in clear contradiction with the predictions of factorization theorem applying to this process (see [68] and references therein).

Below we consider another reason to explain such type of behavior, using the well known expression of a virtual photon-quark vertex (so called Sudakov form factor \([9]\)) which is entered in the triangle Feynman diagram, describing the conversion of two photon to the neutral pseudoscalar meson. It is the motivation of this paper.

Both channels of pseudoscalar mesons production in elastic electron-positron collisions - the scattering and annihilation ones are considered. The second one \(e^+e^-\rightarrow\pi_0l^+l^-\) can be the subject of experimental investigation.

II. SCATTERING CHANNEL

In the scattering type of experiments

\[ e^+(p_+) + e^-(p_-) \rightarrow e^+(p'_+)e^-(p'_-)^0(q_{\pi}) , \]

\( (p_+^2 = p'_+^2 = m_{\pi}^2, \quad q_0^2 = M^2) \) neutral pion is created by two photons with momenta \( q = p_+ - p'_+ \) and \( q_1 = p_- - p'_- \) that involving into lepton interaction as it is presented on Fig. [1]. Due to Weizsäcker-Williams (WW) kinematics of this process (the scattered electron assumed to move close to beam direction) one of photons is almost real \(|q^2| \ll M^2\) and other is off mass shell \(Q^2 = -q^2 \gg M^2\).

Matrix element has a form (see for details in the Appendix[A])

\[ M = \frac{2s(4\pi\alpha)^2}{q_0^2 q_1^2} |q \times q_1| V(Q^2) N_+ N_-, \]

\[ V(Q^2) = \frac{M_\pi^2}{2\pi^2 F_\pi Q^2} F \left( \frac{Q^2}{M_\pi^2} \right), \] \[ |N_\pm|^2 = \frac{1}{s^2} Tr[p_-p_+p_-p_+] = 2, \]

where \( s = (p_+ + p_-)^2 \gg Q^2 \) and \( V(Q^2) \) is the transition form-factor of pion, \( F_\pi = 93 \text{ MeV} \) is the decay constant of pion, \( M_\pi = M_u = M_d = 280 \text{ MeV} \) is the quark mass, \( q, q_1 \), are transversal to the beam axes (\( z \)) direction components of photon momenta. The quantity \( F(Q^2/M_\pi^2) \) has a form:

\[ F(Q^2/M_\pi^2) = -\int \frac{d^4k}{i\pi^2} \times \]

\[ \times \frac{Q^2 V S(Q^2; p_1^2, p_2^2)}{(k^2 - M_\pi^2 + i0)(p_1^2 - M_\pi^2 + i0)(p_2^2 - M_\pi^2 + i0)}. \]
where $p_1 = k + q_\pi$ and $p_2 = k + q_\pi$ and Sudakov vertex function $V_S$ [10, 11] is:

$$V_S(Q^2, p_1^2, p_2^2) = \exp \left( -\frac{\alpha_s C_F}{2\pi} \ln \frac{Q^2}{|p_1^2|} \ln \frac{Q^2}{|p_2^2|} \right), \quad (4)$$

where $Q^2 > |p_1^2| > M_\pi^2$ and $C_F = (N^2 - 1) / (2N) = 4/3$. We use here the Goldberger-Treiman relation on the quark level $F_\pi = M_\pi / g_{q\bar{q}\pi} = 93$ MeV.

The cross section of the process [1] has a form:

$$d\sigma = \frac{1}{8s} |M|^2 d\Gamma_3 \quad (5)$$

The phase volume of the final state $d\Gamma_3$ can be expressed through the Sudakov parametrization of the photon’s momenta which turns out to be convenient:

$$q_1 = \alpha_1 \tilde{p}_+ + \beta_1 \tilde{p}_- + q_{1\perp},$$
$$q = \alpha \tilde{p}_+ + \beta \tilde{p}_- + q, \quad (6)$$
$$a_\perp p_\pm = 0, \quad q_{1\perp}^2 = -q_{1\perp}^2,$$

and we imply the 4-vectors $\tilde{p}_\pm$ to be light-like, $2\tilde{p}_+ \tilde{p}_- = s$. Therefore (details in Appendix A):

$$d\Gamma_3 = (2\pi)^{-5} \left( p_+ + p_+ - p'_+ - p'_- - q_\pi \right) \times \frac{d^2 p'_+ d^2 p'_- q_\pi^2 q_\pi}{2E_+^2 E_-'^2 E_\pi^2} = (2\pi)^{-5} \frac{d\beta_1}{4s} \frac{1}{\beta_1 (1 - \beta_1)} d^2 q_1 d^2 q,$$

$$Q^2 + M^2 < \beta_1 < 1. \quad (7)$$

Using the expression for the square of momentum of "almost" real photon

$$q_1^2 = -\frac{1}{1 - \beta_1} \left[ q_{1\perp}^2 + m_\pi^2 \beta_1^2 \right], \quad q_{1\perp}^2 \ll Q^2, \quad (8)$$

and performing the integration on the parameters of scattered electron ($\beta_1, q_1$), moving close to $z$ axis we obtain for the cross section:

$$\frac{d\sigma}{dQ^2} = \alpha^4 \frac{4Q^2}{4Q^2} V(Q^2) J(Q^2),$$
$$J(Q^2) = \frac{1}{2} L_s^2 + L_e (L_e - 1) - (L_e + 1), \quad (9)$$

where

$$L_s = \ln \frac{s}{Q^2 + M^2}, \quad L_e = \ln \frac{Q^2}{m_\pi^2}. \quad (10)$$

There are several approaches to infer the value $V(Q^2)$, which is named as pion transition formfactor.

One of them is based on QCD collinear factorization theorem [2]

$$V^{BL}(Q^2) = \frac{2F_\pi}{3Q^2} \int_0^1 dx x Q^2 \phi_\pi(x, s). \quad (11)$$

and in the papers [6, 8] different forms of pion wave function $\phi_\pi(x, s)$ was used. Another possible mechanisms of the effect was given in [7, 12].

Also in the paper [13, 14] was pointed that pion form factor in the frames constituent quark model has the double logarithmic asymptotic at large momentum transfer.

Another one which use the approach of Nambu-Jona-Lasinio model [15, 16] gives:

$$V^{NJL}(Q^2) = \frac{2F_\pi}{3Q^2}. \quad (12)$$

The approach used here is based on the Sudakov form of the vertex function which describe interaction of a photon with large four momentum square $|q^2|$ with two quarks of an anomalous three angles quark diagram describing the conversion of two photons to the neutral pion.

The three angle quark-loop diagram itself at large $Q^2$ has the double-logarithm asymptotic [14] and insertion of Sudakov form of the vertex function which includes also QCD-inspired corrections gives the asymptotic in $\ln (Q^2 / m^2)$ behavior.

The final expression is (details are in Appendix A):

$$V(Q^2) = A \frac{M^2}{2\pi F_\pi \alpha_s C_F} \Phi(z_B), \quad (13)$$

where

$$\Phi(z) = \int_0^1 dx \left( 1 - e^{-z_B x (1 - x)} \right),$$

$$z_B = \frac{C_F \alpha_s}{2\pi} \ln^2 \frac{Q^2}{BM^2},$$

where $A, B$ can be considered as a positive fitting parameters of order of unity. We find it through Babar data fitting. Function $Q^2 V(Q^2)$ is presented in Fig. [3] where the experimental data also presented.
III. ANNIHILATION CHANNEL

Let us consider now the annihilation channel depicted on Fig. 3

\[ e^+ (p_+) + e^- (p_-) \rightarrow \gamma^* (q) \rightarrow \pi^0 (q) l^+ (q_+) l^- (q_-), \]

(14)

where \( l = e, \mu \) and \( p_+^2 = 0, q_+^2 = m_f^2, q_-^2 = M_f^2, s = q^2 = (p_+ + p_-)^2, \) \( s_1 = q_1^2 = (q_+ + q_-)^2. \) We put the matrix element of this process in the form:

\[ M = \frac{(4\pi\alpha)^2}{q_1^2 q^-} J^\mu J^{\nu(l)} V(s) \epsilon_{\mu\nu\alpha\beta} q^\alpha q^\beta q_1^\beta, \]

\[ J^\mu = \bar{u} (p_+) \gamma_\mu u (p_-), \]

\[ J^{\nu(l)} = \bar{v} (q_+) \gamma_\nu u_\nu (q_-), \]

(15)

where quantity \( V(s) \) describes conversion of two off mass shell photons to the neutral pion (pion transition form-factor) is defined in (2).

Let perform the phase volume of the final state

\[ d\xi_3 = (2\pi)^{-5} \delta^4 (p_+ + p_- - q_+ - q_- - q_3) \times \]

\[ \times d^3 q_+ d^3 q_- q_3^0 q_3 \]

\[ \frac{2E_+ 2E_- 2E_3}{2E_3}, \]

(16)

to the form

\[ d\Gamma_3 = (2\pi)^{-5} \prod_{q_i} d^2 q_i^2 \frac{\Lambda_{1/2} (s, q_i^2, M_f^2)}{2s}, \]

(17)

where \( \Lambda (a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc) \) and

\[ d\Gamma_{q_i} = \int d^3 q_+ d^3 q_- \delta^4 (q_1 - q_+ - q_-). \]

(18)

Performing the integration on the invariant mass square of the lepton pair (we use the approximation \( s_1 \ll s \) and \( s_1 < M_f^2 \)) the relevant cross section have a form (details in Appendix A):

\[ \sigma^{e\gamma \rightarrow \pi l^- l^+} = \frac{\alpha^4 V(s)^2}{6} \left( 1 - \frac{M_f^2}{s} \right)^3 \left[ \ln \frac{s}{m_f^2} - \frac{5}{3} \right]. \]

(19)

IV. CONCLUSION

From our point of view the QCD corrections connected with the vertex of interaction of the highly virtual photon with quarks is essential and pion can be considered as a point particle. So at rather large values of \( Q^2 \) the details of pion wave function becomes irrelevant.

In literature presents the alternative explanations (see the end of Section III) of the experimental data BaBar [5].

On the Figure 3 we represent a numerical estimation fit of the BaBar data. We obtain the qualitative logarithmic-logarithmical growth (see Eq. (17)) of the transitional form-factor (13). On the plot we put the best fitting of BaBar data with two adjustable parameters \( A \) and \( B. \)

The similar phenomena can take place as well for the case of scalar mesons production.

We remind as well the possibility to measure the transition pion form factor in electro-proton scattering \( ep \rightarrow e\pi_0 p. \) The relevant cross section will be

\[ \frac{d\sigma^{e\gamma \rightarrow e\pi_0 p}}{dQ^2} = \left( \frac{\alpha g_{pqq} g_{pNN}}{8\pi (Q^2 + M_f^2)} \right)^2 V^2(Q^2) \]

\[ \times \left[ F_1^2(Q^2) + \frac{Q^2}{4M_f^2} F_2^2(Q^2) \right] J(Q^2), \]

(20)

where \( F_1, F_2 \) are Dirac and Pauli proton form factors. Here instead of virtual photon the virtual vector meson takes place; \( g_{pqq}, g_{pNN} \) are the \( \rho \) meson couplings with quarks and nucleons correspondingly. In this case a problem with background (\( ep \rightarrow e\Delta^+ \rightarrow e\pi^0 p \)) must be overcomed.
We consider Sudakov form factor for time-like transfer momentum. With ordinary particles in the loop we must take into account the imaginary part of relevant amplitude. For quarks inside a loop imaginary part is absent.

Taking into account the non-leading terms in expression of Sudakov exponent results in modification of quark mass and a general shift of normalization:

\[ F(q^2, p_1^2, p_2^2) \rightarrow AF(q^2, p_1^2, p_2^2), \]

\[ Q^2 \alpha \beta > BM_Q^2, \]

with \( A \sim B \sim 1 \) can be considered as a fitting parameters \( A, B > 0 \).

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Appendix A: Details of calculation

Transformation of the phase volume of 2-gamma creation process \( 2 \rightarrow 3 \) consist in introduction of two transferred vectors \( q_1, q \):

\[
d\Gamma_3 = (2\pi)^{-5}d^4q_1d^4q_2d^4q_3d^4p_+d^4p_- \times \\
\quad \delta (p_+ - q_1 - q_2) \delta (p_+ - q - p_+) \times \\
\quad \delta (q + q_1 - q_2) \delta \left((q_1 - p_+)^2 - m_q^2\right) \times \\
\quad \delta \left((q - p_+)^2 - m_q^2\right) \delta \left((q_1 + q)^2 - M_1^2\right),
\]

and using the Sudakov parametrization for the scattering channel \( (1) \) we put it in form:

\[
d\Gamma_3 = (2\pi)^{-5}d^4q_1d^4q_2d^4q_3d^4p_+d^4p_- \times \\
\quad \delta (s_1 \beta - q_1^2 - s_3 \beta - m_q^2) \times \\
\quad \delta (s_{13} \beta - q_1^2 - s_{13} - m_q^2) \times \\
\quad \delta \left(s_2 \beta - (q + q_1)^2 - M_1^2\right),
\]

Performing the integrations over \( \alpha_1, \alpha, \beta \), we obtain the result given above (see Eq. 7).

Expression for scalar loop integral with 3 denominators and Sudakov vertex inserted has a form:

\[
F\left(\frac{Q^2}{M_q^2}\right) = \int \frac{d^4k}{i\pi^2} Q^2 V_S(Q^2, p_1^2, p_2^2),
\]

\[
(k) = k^2 - M_q^2 + i0; \\
(1) = (k + q_1)^2 - M_q^2 + i0; \\
(2) = (k + q_2)^2 - M_q^2 + i0.
\]

where \( V_S(Q^2, p_1^2, p_2^2) \) was defined in [4].

To perform the integration we use Sudakov parametrization of the loop momentum:

\[ k = \alpha n_1 + \beta n_2 + k_\perp, \quad (A2) \]

with \( n_{1,2} \) are light-like 4 vectors \( n_1^2 = 0 \), transversal to \( k_\perp \), \( n_1 k_\perp = 0 \), built from the 4-vectors \( q_\pm, q_1 \) such that \( 2n_1n_2 = Q^2 \). In such a parameterization

\[ d^4k = \frac{Q^2}{2} d\alpha d\beta d^2k_\perp. \quad (A3) \]

Expressing the denominators of quark Green functions:

\[(k) = Q^2\alpha \beta - \vec{k}^2 - M_q^2 + i0; \quad (1) \approx Q^2\alpha; \quad (2) \approx Q^2\beta, \]

and performing the integration over \( k_\perp^2 = -\vec{k}^2 \) as (we imply that principal part does no contribute)

\[
\int \frac{\pi d\vec{k}}{(k)} = -i\pi^2 \int d\vec{k}^2 \delta(-\vec{k}^2 + Q^2\alpha \beta - M_q^2) = \\
- i\pi^2 \theta(Q^2\alpha \beta - M_q^2), \quad (A4)
\]

we obtain

\[
F\left(\frac{Q^2}{M_q^2}\right) = \frac{1}{2} \int_0^1 \frac{d\alpha}{M_q^2/Q^2} \int_0^1 \frac{d\beta}{M_q^2/Q^2} \theta(Q^2\alpha \beta - M_q^2) \times \\
\exp\left(-\frac{\alpha_s C_F}{2\pi} \ln \frac{1}{\alpha} \ln \frac{1}{\beta}\right). \quad (A5)
\]

Performing one integration we obtain:

\[
F\left(\frac{Q^2}{M_q^2}\right) = \frac{\pi}{\alpha_s C_F} \Phi(z), \quad (A6)
\]

where

\[
\Phi(z) = \int_0^1 \frac{dx}{x} \left(1 - e^{-zx(1-x)}\right), \\
z = \frac{\alpha_s C_F L^2}{2\pi}, \quad L = \ln \frac{Q^2}{M_q^2}.
\]

For large values of \( Q^2 \) we obtain

\[
F\left(\frac{Q^2}{M_q^2}\right) = \frac{\pi}{\alpha_s C_F} \left(\ln(z) + c\right), \quad (A7)
\]

where

\[
c = \int_0^1 \frac{dx}{x} (1 - e^{-x}) - \int_1^\infty \frac{dx}{xe^x} \approx 0.57. \quad (A8)
\]

In reality the quantity \( L \sim 5 \) for light consistent quarks \( u, d \) which are present in the neutral pion and \( L \sim 1 - 2 \) for s-quark in \( \pi' \) meson.
When considering the integration on the pair phase volume in annihilation channel we use the relation (consequence of gauge invariance):

\[ \sum_{\text{pol}} \int d\Gamma q_1 J^{(\mu(1))}(J^{(\nu(1))})^* = -\frac{1}{3} \left( g_{\mu\nu} - \frac{q_{1\mu} q_{1\nu}}{q_1^2} \right) (q_1^2 + 2m_1^2) \pi \beta_-, \]  

(A9)

\[ \beta_- = \sqrt{1 - \frac{4m_1^2}{q_1^2}}. \]

The differential cross section is

\[ d\sigma_{e\bar{e}\to \pi^0 l\bar{l}} = \frac{\alpha^4 M^4}{24\pi^3 F_s^2 s^{3/2}} \left( s, q_1^2, M^2 \right) \frac{dx}{x} \times \left( 1 + \frac{1}{2x} \right) \sqrt{1 - \frac{1}{x}}, \]  

(A10)

where \( x = q_1^2 / (4m_1^2) > 1. \)

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