Is It Possible to See the Breaking Point of General Relativity near the Galactic Center Black Hole? Consideration of Scalaron and Higher-dimensional Gravity

P. C. Lalremruati and Sanjeev Kalita

Department of Physics, Gauhati University, Guwahati, 781014, India

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Abstract

The Galactic center black hole is a putative laboratory to test general relativity (GR) and constrain its alternatives. $f(R)$ scalaron gravity is an interesting alternative to GR and has tremendous prospects for astrophysics and fundamental physics near the black hole. In this work, we search for breaking points of GR through estimation of pericenter shift of stellar orbits with semimajor axis $a = (45–1000)$ au. The black hole spin is taken as the maximum $\chi = 0.99$, and orbital eccentricity is taken as $e = 0.9$. We work with theoretical scalaron field amplitude and coupling, predicted by Kalita, and also consider the constraints reported by Hees et al. The scalaron mass is taken in the range $(10^{-22}–10^{-17})$ eV. It is found that GR suppresses scalaron gravity at all orbital radii for the theoretical values of scalaron field coupling predicted by Kalita. Breaking point arises only for higher scalaron coupling resulting from the Hees et al. observations within a few tens of au to $a = 1000$ au. We also estimate the pericenter shift with a power-law potential $V(r) \sim 1/r^2$ arising in five-dimensional gravity and obtain allowed ranges of the five-dimensional Planck mass through existing bounds on the parameterized post-Newtonian parameters coming from the orbits of S-2, S-38, and S-55. The breaking point for GR arises for a five-dimensional Planck mass of about $10^4$ GeV. Constraint on this parameter, expected from the astrometric capabilities of existing and upcoming large telescopes, is also presented.

1. Introduction

General relativity (GR) has been successfully tested in the scale of the solar system (Will 2001) and binary pulsars (Hulse & Taylor 1974; Taylor & Weisberg 1982; Stairs et al. 1998). Detection of gravitational waves predicted by GR and emission from coalescences of binary black holes is also an indication of the correctness of the theory (Abbott et al. 2016a, 2016b, 2017). Tests in the scale of galaxies through lensing observation have also been initiated, and GR has been found to be satisfactory on these scales (Collett et al. 2018). Local position invariance (LPI) inbuilt in the theory has been validated through monitoring of stellar orbits near the Galactic center (GC) black hole Sgr A* (Amorim et al. 2019). The strong principle of equivalence has been tested through the observation of triple-compact star systems (Archibald et al. 2018). Two recent reports near Sgr A* have successfully tested GR in a relatively strong field (large curvature) environment. They are (a) gravitational redshift of light from the star S-2 near the supermassive black hole (Gravity Collaboration et al. 2018; Do et al. 2019) and (b) detection of Schwarzschild precession (first-order post-Newtonian effects) of the orbit of S-2 around Sgr A* (Gravity Collaboration et al. 2020). The GC black hole has now been realized as a potential laboratory to test gravitational theories. This has been made possible through the advent of adaptive optics (AO)-facilitated near-IR (NIR) observing facilities realized in Very Large Telescope (VLT)/GRAVITY and the Keck Observatory.

Despite its successes, GR has not been independently tested in the regime of cosmology without the assumption of the Lambda cold dark matter (CDM) model (Wojtak et al. 2011). The physics of its ingredients are not yet clear. The theory is still unable to explain the problems of dark matter and dark energy in a cosmological setting (Zakharov 2018). Moreover, GR naturally undergoes quantum correction either near black holes or in the very early universe. Modifications to GR are not yet strongly tested astronomically. Over the last decades, several extended theories of gravity have been proposed, such as modified Newtonian dynamics (MOND; Milgrom 1983), scalar–tensor theories (Brans &Dicke 1961; Moffat 2005, 2006), conformal gravity (Behnke et al. 2002; Babichev et al. 2009), Yukawa-like gravity theory (Fischbach & Talmadge 1992; Cardone & Capozziello 2011), and massive gravity theories (Rubakov &Tinyakov 2008; Babichev et al. 2009; de Rham et al. 2011; Gong 2013). Alternative approaches to Newtonian gravity in weak field limit of fourth-order gravity theory have been proposed (Zakharov et al. 2006, 2007; Frigerio Martins &Salucci 2007; Nuccita et al. 2007; Capozziello et al. 2009; Iorio 2010; Borka et al. 2012, 2013; Zakharov et al. 2014). These theories have been put into tests through observations of different scales ranging from solar systems to clusters of galaxies (Capozziello & Fang 2002; Capozziello et al. 2003, 2014; Carroll et al. 2004).

Of all these alternatives to GR, the most studied one is the $f(R)$ gravity which modifies the Einstein–Hilbert action through a general function of the Ricci scalar. $f(R)$ theory was first proposed by Buchdahl in 1970 (Buchdahl 1970) to study open universes oscillating between nonsingular states. Starobinsky (1980) used the $f(R)$ theory to reproduce the accelerated expansion of the early universe (geometric inflation). $f(R)$ gravity has been found to be capable of explaining some anomalies with the dark matter sector of the universe. The theory has the ability to explain the structure formation in the universe without incorporating dark matter (Capozziello et al. 2014). Capozziello et al. (2007) reported that the observed flatness of the rotation curves of spiral galaxies may indeed be a breakdown of GR rather than the evidence of dark matter halos. To verify the hypothesis they considered a sample of 15
low surface brightness galaxies with combined H\textsc{i} and H\textalpha{} measurements of the rotation curves extending far beyond the optical edge of the disk and assumed f(R) theory without dark matter. The possibility of flat rotation curves of spiral galaxies using power-law f(R) theory (f(R) \sim R^n) without the assumption of dark matter was also investigated by Frigerio Martins & Salucci (2007). The CDM paradigm is successful in explaining the large-scale structure of the universe. But it faces persistent challenges on the scales of galaxies, and the discrepancy has been treated with the help of modification to the theory of gravity such as MOND, modified gravity (MG), and f(R) gravity (de Martino et al. 2020; Salucci et al. 2021). Although these theories are not yet strong competitors of the dark component, their testability has been realized.

Origin of f(R) gravity through curvature correction to quantum vacuum fluctuation in black hole spacetime and astrophysical implications near the GC black hole have been extensively studied by Kalita (2018, 2020). The framework of f(R) gravity naturally leads to Yukawa-type gravitational potential in weak field limits (Zakharov 2017; Kalita 2018). From the consideration of geometrically corrected quantum vacuum fluctuation, Kalita (2018) deduced the potential V(r) = -\alpha(GM)\exp(-\psi r) r, where, \alpha \sim \psi^{-1} is the coupling strength with \psi = dR/dR being the scalar field known as a scalaron—a new scalar fifth force, M, is the central mass (in this case the black hole), and M_\psi is the mass of the scalaron field—the quanta of the scalar fifth force. Several effects near the GC black hole were discussed with scalaron mass in the range 10^{-22}\text{ to } 10^{-17} \text{ eV}. Amorim et al. (2019) reported the effect of scalar field mass M_\psi = (10^{-22} \text{ to } 10^{-15} \text{ eV}) on stellar orbits similar to that of S-2. The background scalaron field amplitude \psi_{\text{max}} which is related to the coupling strength \alpha as \alpha = 1/3\chi_{\psi} (Kalita 2018) was calculated in Kalita (2020) from the consideration of UV and IR cut off of the geometrically corrected quantum vacuum fluctuations near the black hole. The value of the coupling \alpha \approx 2.7 \times 10^{-3} corresponding to the scalaron amplitude \psi_{\text{max}} \approx 1217 reported in Kalita (2020) is found to be remarkably consistent with the constraint derived from recent detection of Schwarzschild precession of S-2 (Gravity Collaboration et al. 2020). This strengthens further consideration of scalaron gravity in the region near the GC black hole.

The orbit of S-2 has also been realized as a potential probe for constraining graviton mass in massive gravity theories (Zakharov et al. 2018). It is expected that after the graviton mass bound of 1.2 \times 10^{-22} \text{ eV} given by the LIGO-Virgo collaboration (Abbott et al. 2016b), the future observations of bright stars’ orbits near the GC black hole will bring down the upper limit of graviton mass to about 10^{-23} \text{ eV} (Zakharov et al. 2018). Scalarons predicted by Kalita (2020) are massive compared to gravitons. Therefore, the GC black hole is a convenient laboratory for studying interesting modifications to GR. Deviation from GR in the form of scalaron gravity, which may be subject to further astronomical tests, is an interesting avenue for tests of gravity near the GC black hole. For instance, f(R) gravity with f(R) \sim R^n (power-law gravity) can cause retrograde precession of stellar orbits near the GC black hole and may, therefore, be degenerate with the effect of extended mass distribution near the GC black hole (Rubilar & Eckart 2001; Borka et al. 2012). There are extensive studies on the possibility of testing MG near the GC black hole. Studies of the motion of S-2 in the framework of f(R) gravity and other modified theories of gravity have been utilized to see a small departure from GR near the black hole (De Laurentis et al. 2018a; De Martino et al. 2021; Monica et al. 2022). It is also believed that the pericenter shift due to binary systems (two neutrons or pulsars) near the GC has the capability to give a strong constraint on Yukawa gravity (De Laurentis et al. 2018b). Kalita (2021) discussed the conditions on the black hole spin for which scalaron gravity-induced precession shift of stellar orbits may touch the order of other relativistic effects beyond the Schwarzschild effect. Therefore, consideration of departure from GR is an essential study that is going to accompany future observations near the GC black hole.

In addition to the Yukawa potential given by the scalaron field, we find interesting power-law potentials in higher-dimensional gravitational theories such as the Lovelock gravity (Dadhich et al. 2013b). One of the subsets of Lovelock gravity is the five-dimensional Kaluza–Klein (henceforth KK) theory. These effects were not studied earlier in the context of the GC black hole. By adopting scalaron gravity and higher-dimensional theories, we ask the question: is it possible to see the breaking point of GR near the GC black hole through upcoming observations? We also examine whether the precession of stellar orbits affected by higher-dimensional gravity is close to the effect of f(R) scalarons studied earlier by Kalita (2020). Implications of this comparative study are presented. Although we have not seen any deviation from GR in recent tests near the Sgr A*, experimental philosophy does not prohibit us to look beyond the horizon of current measurements and to find possible corrections to the theory.

In this work, we take the pericenter shift of the stellar orbits as a probe. The eccentricity of the orbits is chosen as e = 0.9 (see Lalremruati & Kalita 2021 for reasons for high eccentricity). The black hole spin is randomly chosen as \chi \approx 0.99 to maximize the relativistic precession. The semimajor axis is chosen in such a way that the lower bound is set by the timescales of gravitational wave emission and stellar ages whereas the upper bound is set as 1000 au (corresponding to the semimajor axis of the S-2 star). The idea is that estimation of pericenter shift requires stellar orbits with semimajor axis in such a way that the timescale of gravitational waves emitted by the extreme mass ratio inspirals (S stars + GC black hole) become comparable or longer than the age of these stars. Thus semimajor axis must have a minimum bound. While estimating these pericenter shifts in f(R) gravity, the semimajor axes were unconstrained toward the bottom in the earlier work by Kalita (2018, 2020). In the present work, we consider the lower bound given by gravitational wave emission constraint. For scalaron gravity, we adopt scalaron coupling and mass derived in Kalita (2020). For KK five-dimensional theory we consider five-dimensional Planck masses lying above the supersymmetric scale (1000 GeV). We have also presented an estimation of this parameter on the basis of existing bounds on MG reported by Gravity Collaboration et al. (2020). The possibility of constraining the parameter through existing and upcoming ground-based astrometric facilities is also highlighted.

The paper is organized as follows. Section 2 contains precession of stellar orbits in scalaron gravity and relativistic (GR) contribution including spin-induced effects. In Section 3, precession in higher-dimensional gravity theory is presented. The MG parameter appearing as the five-dimensional Planck
mass has been estimated. Constraint on this parameter to be placed by existing and upcoming large telescopes is also presented in this section. Section 4 presents results and discussion. Section 5 summarizes.

2. Pericenter Shift in Scalaron Gravity

In this section, we estimate the precession of stellar orbits in presence of Yukawa potential given by the scalaron field of $f(R)$ gravity studied earlier by Kalita (2020). Relativistic contributions to precession (Schwarzschild precession + spin/frame-dragging effect + quadrupole-moment effect) are also estimated. The upper bound for the semimajor axis of the considered stellar orbits is set to 1000 au (S-2 semimajor axis) and the lower bound for the semimajor axis is generated by considering the timescale of gravitational wave emission expressed by Gualandris & Merritt (2009) as

$$t_{GW} = \frac{5c^5}{256f(e)G^3\mu(M + M_{BH})^2},$$

where

$$f(e) = (1 - e^2)^{-\gamma/2}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$

(2)

and

$$\mu = M M_{BH} (M + M_{BH})^{-1}. \tag{3}$$

$M$ is the mass of the typical S star considered here to be $\sim 10 M_\odot$. $M_{BH}$ accounts for the mass of the Galactic center black hole which is recently estimated as 4.25 x 10$^6 M_\odot$ (Gravity Collaboration et al. 2020). Genzel et al. (2010) have reported the age of the S stars in the nuclear cluster near Sgr A$^*$ to be within (6–400) Myr. Thus, these stars must survive at least for 6 Myr before their orbits rapidly evolve through gravitational radiation. With all the above considerations, the lower bound on the semimajor axis of the star for eccentricity, $e = 0.9$ is found to be 45 au.

The precession rate induced for a Yukawa-like fifth force is given as (Iorio 2012)

$$\Omega_{\text{MG}} = \frac{\alpha}{e}\exp\left(-\frac{\alpha}{\lambda}\right)I_1(ae/\lambda)\sqrt{1 - e^2}, \tag{4}$$

where $\alpha$ is the coupling strength in the fifth force and for the present case it is equal to $1/3\psi_a$, $e$ is the eccentricity of the Keplerian orbit and $I_1(x)$ is the modified Bessel function of the first kind. The superscript MG stands for modified gravity. The term $\lambda$ is simply the inverse of the scalaron mass, $M_{\psi}$. The function $I_1(x)$ takes the form $I_1(x) = 1 + \frac{x^2}{2} + \frac{x^4}{16} + \frac{x^6}{64} + \ldots$ if the argument $x < 1$ (here $x = ae/\lambda$).

Depending upon the value of $a$, $e$, and $\lambda$, the function $I_1(x)$ varies (for this work, the value of $I_1(x)$ is obtained from the python environment).

$n = 2\pi/P$ (P being the period in the orbit) is the mean Keplerian motion with $P = 2\pi\sqrt{a^3/GM}$ being the orbital period. However, in this work, we consider only the pericenter shift angle of the orbits and shall be ignoring any necessary contributions made by the orbital period of the stars. Thus pericenter shift for Yukawa fifth force becomes

$$\Delta \psi_a = \frac{2\pi}{3\psi_a}\left(M_{\psi}a/\lambda\right)\exp(-M_{\psi}a)I_1(M_{\psi}ae)\sqrt{1 - e^2}. \tag{5}$$

In addition to scalaron induced pericenter shift, we also estimate general relativistic pericenter shift arising from Schwarzschild precession, spin/frame-dragging, and quadrupo-lamoment up to third post-Newtonian (3PN) correction. This is expressed as (see Tucker & Will 2019)

$$\Delta \psi_{\text{GR}} = \frac{6\pi GM_{BH}}{c^2a(1 - e^2)} + \frac{3\pi G^2 M_{BH}^3(18 + e^2)}{2c^4a(1 - e^2)^2}$$

$$+ \frac{45\pi GM_{BH}^3}{2c^6a^3(1 - e^2)^3} + \frac{8\pi G^3/2 M_{BH}^2}{c^8a^5(1 - e^2)^3/2}$$

(6)

$$\times \left(1 + \frac{2\lambda^2GM_{BH}}{c^2a(1 - e^2)}\right).$$

The pericenter shift arising from both scalaron gravity and general relativity are compared for the semimajor axes bounded within (45–1000) au. Figures 1–4 shows the comparison of the pericenter shift for a scalaron mass, $M_{\psi} = (10^{-22} - 10^{-11})$ eV by assuming a theoretical scalaron field amplitude, $\psi_0 = 1217.05$ obtained by Kalita (2020) and the field amplitudes derived from the Yukawa coupling constants reported by Hees et al. (2017) by employing observational data of stellar orbits near Sgr A$^*$. These field amplitudes are $\psi_0 = 33.33, 1.01, and 0.33$ (estimated by Hees et al. 2017 from observational data collected from stellar orbits near Sgr A$^*$). The reason behind choosing the abovementioned range of scalaron mass is the following. These scalarons are massive compared to gravitons (when recent constraints coming from gravitational radiation are taken into account; Abbott et al. 2016b). A new category of ultralight scalar dark matter with $10^{-19}$ eV reported in Bar et al. (2019) falls in this range. Kodama & Yoshino (2012) found that scalar fields with masses equal to or below $10^{-17}$ eV can form bound states near supermassive black holes of the type of Sgr A$^*$. Prospects of scalar particles for observations near the GC black hole have appeared in the literature. Using standard results of orbital perturbation theory, recent observations of the orbit of S-2 are expected to be sensitive to a scalar field with mass $10^{-20}$ eV $\leq m_\psi \leq 10^{-18}$ eV (Gravity Collaboration 2019). De Martino et al. (2017) discussed the light axionic dark matter of mass $10^{-22}$ eV in the context of galactic core formation. The possibility of constraining black hole hair produced by ultralight scalar fields through the study of shadows near Kerr black holes such as the one in M87 has been discussed by Cunha et al. (2019). They considered scalar mass around $10^{-20}$ eV and discussed the potential of the Event Horizon Telescope (EHT) for constraining such scalar particles. Scalar particles with mass (10$^{-18}$–10$^{-20}$) eV (axionic dark matter) were considered by Luu et al. (2020) to explain galaxy core masses and the presence of nuclear star clusters.

3. Pericenter Shift in Higher-dimensional Gravitational Theory

Black holes in higher-dimensional spacetime are interesting fields of study for theories beyond GR (see the review by Emparan & Reall 2008). Tangherlini (1963) first generalized the Schwarzschild solution to extra dimensions ($d > 4$) and Myers & Perry (1986) extended the Kerr solution.
Carter & Neupane (2005) reported stability and black hole thermodynamics of higher-dimensional Kerr-anti-de Sitter black holes. Particle orbits such as red–blueshift photons in Myers-Perry black holes and the Penrose process have been studied by Sharif (2016) in the context of a supermassive black hole in the GC. The possibility of imaging higher-dimensional black hole shadows was discussed recently by Hertog et al. (2019). Here we are concerned with power-law gravitational potential that appears in some novel higher-dimensional theories. We estimate the pericenter shift of stellar orbits near the GC black hole with such potential.

Power-law potential often occurs in modification to Newtonian gravitational potential. For example, in first-order post-Newtonian approximation to general relativity potential proportional to $r^{-3}$ occurs leading to perihelion shift. Uniform dark energy is modeled by a potential proportional to $r$ (cosmological constant). In this section, we consider the Newtonian version of gravitational potential arising from a generalized higher-dimensional modification to general relativity. We consider Lovelock gravity which is the most generalized metric theory in higher dimensions, $d > 4$. It is expressed by the gravitational Lagrangian

$$L = -2\lambda + \sum_{m=1}^{\infty} \frac{C_m}{2m} \epsilon_i^{cd} b_c^{d_1} \cdots a_{ab} R_1^d b^1 \cdots R_2^{d_2} \cdots R_\text{abcd}. \tag{7}$$

Here $\lambda$ is the cosmological constant, $\delta$ is the antisymmetric determinant (see Konoplya & Zhidenko 2020), and $R_{abcd}$ is the Riemann tensor. Lovelock gravity contains GR for $m = 1$, Gauss–Bonnet (quadratic, low-energy approximation of string theory) gravity for $m = 2$, and so on. In Lovelock gravity, the potential varies as $V(r) \sim 1/r^{\alpha}$, where $\alpha = (d - 2m - 1)/m$ (Dadhich et al. 2013b). For $d = 3m + 1$, Lovelock gravity becomes identical to Einstein’s gravity in four dimensions and the potential goes as $V(r) \sim 1/r$. Lovelock gravity exhibits two interesting features: (a) bound orbits exist for $2m + 1 < d < 4m + 1$ (Dadhich et al. 2013a), and (b) black holes with $\alpha \geq 1$ or $d \geq 3m + 1$ are stable against scalar, vector and tensor perturbations (Takahashi & Soda 2009; Gannouji & Dadhich 2014). We, therefore, consider an attractive power-law potential $V(r) = -\beta/r^\alpha$ which corresponds to $d = 4m + 1$.

The analytical form of the pericenter shift for noncircular orbits with power-law potential was worked out by Adkins & McDonnell (2007); the general formula for pericenter shift is given by Equation (30) of these authors. For power-law potential of the type $V(r) = a_{n-1}/r^{n+1}$, the pericenter shift is expressed as (see Equation (38) of Adkins & McDonnell 2007)

$$\theta_{\text{prec}} = \frac{-\pi a_{n-1}}{GMm} \chi_{\alpha}(e) \frac{L^n}{L^\alpha}. \tag{8}$$

Here $L = a(1-e^2)$, $a$ being the semimajor axis and $e$ being the orbital eccentricity and

$$\chi_{\alpha}(e) = n(n + 1) F_{1/2} \left( 1 - \frac{n}{2}; 1 - \frac{n}{2}; 2; e^2 \right). \tag{9}$$
2F1 is the hypergeometric function. In the present case \( n = 1 \) and \( a^{-(n+1)} = -\beta \):

\[
\theta_{\text{prec}} = \frac{\pi \beta}{GMm a(1 - a^2)}.
\]

(10)

The parameter \( \beta \) is evaluated as follows. In the Newtonian version of general relativity (four-dimensional Einstein gravity), the gravitational potential energy is expressed as

\[
V(r) = \frac{G_{(4)}Mm}{r^3} \equiv \left(\frac{Mm}{M_{\text{Pl}(4)}}\right)\left(\frac{1}{r}\right),
\]

(11)

where \( M_{\text{Pl}(4)} = G_{(4)}^{-1/2} \) is the four-dimensional standard Planck mass related to the four-dimensional gravitational constant. The generally accepted value of this mass is \( M_{\text{Pl}(4)} \approx 10^{19} \) GeV. This idea resounds in modern versions of higher-dimensional gravitational theories. In a \( d \)-dimensional gravitational theory, the gravitational potential can be expressed as

\[
V(r) = \frac{G_{(d)}Mm}{r^{d-3}} \equiv \left(\frac{Mm}{M_{\text{Pl}(d)}}\right)\left(\frac{1}{r^{(d-4)/2}}\right).
\]

(12)

Here, \( M_{\text{Pl}(d)} \) is the \( d \)-dimensional Planck mass. Clearly, a five-dimensional theory gives \( 1/r^2 \) variation of the potential. Extradimensional corrections to general relativity occur at higher energy scales which probe new gravitational physics below a compactification scale \( R_c \). In this case, the \( d \)-dimensional Newtonian potential can be expressed as

\[
V(r) = \frac{G_{(d)}Mm}{r^{d-3}} \equiv \left(\frac{Mm}{M_{\text{Pl}(d)}}\right)\left(\frac{1}{r^{(d-4)/2}}\right).
\]

(13)

The relation between \( d \)-dimensional and four-dimensional Planck masses is obtained from Equations (11) and (12) as

\[
M_{\text{Pl}(d)} = \frac{M_{\text{Pl}(4)}}{(M_{\text{Pl}(d)}R_c)^{(d-4)/2}}.
\]

(14)

For the present case of interest \( d = 5 \), leading to the relation between five- and four-dimensional Planck masses:

\[
M_{\text{Pl}(d)} = \frac{M_{\text{Pl}(4)}}{(M_{\text{Pl}(5)}R_c)^{(d-4)/2}}.
\]

(15)

Intriguing theoretical possibilities arise if \( M_{\text{Pl}(5)} \ll M_{\text{Pl}(4)} \) which demands very large \( M_{\text{Pl}(5)}R_c \). We resort to a few educated
guesses, $M_{Pl(5)} = 1.9 \times 10^3, 10^4, 10^5, 10^6$ GeV so that the energy scale of new gravitational physics lies at or above the supersymmetry breaking scale ($\sim 1000$ GeV). The scale $10^3$ GeV (1 TeV) is also of particular interest in the five-dimensional KK theory. It is expected that the lightest KK particle with a mass of about $1.9 \times 10^3$ GeV can generate the abundance of dark matter ($\Omega_{DM} \sim 0.3$) in the universe (Servant & Tait 2002). Cheng et al. (2002) proposed the lightest KK particles as candidates of CDM. This is an interesting avenue for cosmology and experimental searches of dark matter particles. From Equation (13), we can write the inverse square potential for $r \leq R_s$ as

$$V(r) = \frac{G_{55}Mm}{r^2} \equiv \left( \frac{Mm}{M_{Pl(5)}^3} \right) \left( \frac{1}{r^2} \right),$$

which gives $\beta = Mm/M_{Pl(5)}^3$. Equation (10) then gives the pericenter shift as

$$\theta^{HD}_{\text{prec}} = \frac{M_{Pl(4)}^2}{M_{Pl(5)}^4} \frac{\chi_1(e)}{a(1 - e^2)}.$$  

The superscript “HD” stands for higher dimension. If $M_{Pl(5)} = cM_{Pl(4)}$, then $c = 1.9 \times 10^{-16}, 10^{-15}, 10^{-14}, 10^{-13}$ satisfy the three values of $M_{Pl(5)}$, Equation (17) can be written as

$$\theta^{HD}_{\text{prec}} = \frac{1}{c^4M_{Pl(4)}^4} \frac{\chi_1(e)}{a(1 - e^2)}.$$  

Variation of pericenter shift of stellar orbits due to the power-law potential is shown with that contributed by standard GR in Figure 5. We also estimate the allowed range of the parameter $M_{Pl(5)}$ based on constraints of the parametrized post-Newtonian (PPN) parameters ($\gamma, \beta$) estimated by Gravity Collaboration et al. (2020) through orbital precession of S-2 and those given by Gainutdinov (2020) through orbital measurements of S-2, S-38, and S-55. A forecast on this parameter has also been presented on the basis of astrometric uncertainties of the Keck telescope, GRAVITY interferometer in the VLT, and the upcoming Thirty Meter Telescope (TMT).

In order to extract the new parameter $M_{Pl(5)}$ through the constraints on ($\gamma, \beta$) we proceed as follows. In the PPN formulation of general relativity, the pericenter shift for MG theories is expressed as (Will 2014)

$$\theta^{\text{prec}}_{\text{PPN}} = \left( 2 + 2\gamma - \beta \right) \frac{\pi R_s}{a(1 - e^2)}.$$  

Here ($\gamma, \beta$) are the PPN parameters ((1,1) for general relativity) and $R_s$ is the Schwarzschild radius of the central mass which is the GC black hole. Comparing this form with the expression of

![Figure 3. Comparison of pericenter shift due to MG and that of relativistic precession up to 3PN for a scalar field amplitude $\psi_o = 1.01$ and a scalaron mass range, $M_{\psi} = (10^{-22} - 10^{-17}$) eV.](image-url)
pericenter shift in higher-dimensional gravity theory (equation (18)), we get

$$2 + 2\gamma - \beta = \frac{\chi_1(e) M_{\psi(4)}}{M_{P(5)} R_s^2}.$$  \hspace{1cm} (20)

In the mass-energy unit, $1/R_s \approx 10^{-25} \text{ GeV}$. We use current constraints on the two PPN parameters to place bounds on $M_{P(5)}$. The constraints $\gamma = 1.18 \pm 0.34$, $\beta = 1.05 \pm 0.11$ have been derived from pericenter shift of S-2 Gravity Collaboration et al. (2020). Through the measurements on the orbits of S-2, S-38, and S-55, Gainutdinov (2020) has derived the constraints $\gamma = 0.81_{-0.60}^{+0.46}$, $\beta = 0.97_{-0.42}^{+0.37}$. These bounds are not competitive with those derived from solar system measurements. They are weak constraints when compared with the constraints coming from the Shapiro time delay measured by the Cassini spacecraft and the VLBI measurements of light deflection using quasars (Will 2014). But these bounds have been obtained for the first time in the new environment of the black hole. The statistical errors may still hide some information on MG. The allowed ranges of $M_{P(5)}$ for the constraints of Gravity Collaboration et al. (2020) and Gainutdinov (2020) are displayed in Figure 7.

Existing and upcoming astrometric facilities such as the Keck telescope, GRAVITY in the VLT, and the upcoming TMT carry sufficient potential to put constraints on the MG parameter ($M_{P(5)}$) through the measurement of pericenter shift. Here, we present a forecast on this parameter on the basis of astrometric uncertainties ($\sigma$) of Keck, GRAVITY, and TMT which are 0.16 mas, 0.30 mas, and 0.015 mas respectively.

With the assumption that future astrometric observations will be able to detect GR-induced pericenter shift with accuracy $\delta = 2\sigma$ (2 standard deviations so that the angles are measurable up to 95% confidence level; see Zakharov 2018) the bound on the MG parameter $M_{P(5)}$ can be obtained through the condition

$$\frac{M_{P(4)}^2}{M_{P(5)}^3} \chi_1(e) a(1 - e^2) \leq \delta.$$  \hspace{1cm} (21)

Equality in the above condition gives the lower bound on $M_{P(5)}$. For $a = 45$, 100, and 1000 au, these bounds on the parameter are shown in Table 1.

### 4. Results and Discussion

Pericenter shift due to scalaron gravity and general relativity is shown in Figures 1–4. The general relativistic effect has been estimated by considering effects up to 3PN and including spin-induced effects with maximum black hole spin $\chi = 0.99$. The trends of the pericenter shift for different scalaron field amplitudes (inverse of scalaron coupling) are discussed below.

For field amplitude $\psi_0 = 1217.05 (\alpha \approx 2.73 \times 10^{-4} )$, very low mass scalarons, $M_\psi = (10^{-22} - 10^{-20})$ eV show rising trend of pericenter shift up to the orbit of S-2, $a \approx 1000$ au. Within this mass range, the pericenter shift rises from $10^{-4}$ arcsec to about 10 arcsec. The rising trend bends downward for $M_\psi = (10^{-19} - 10^{-17})$ eV. For both low mass and high mass scalarons, the effect of scalaron gravity is well suppressed by

![Image of Figure 4: Comparison of pericenter shift due to MG and that of relativistic precession up to 3PN for a scalar field amplitude $\psi_0 = 0.33$ and a scalaron mass range, $M_\psi = (10^{-22} - 10^{-17})$ eV.](image-url)
GR within $a = (45–1000)$ au, with the suppression being heavy for most massive scalarons.

For field amplitude $\psi_o = 33.33(a \approx 0.01)$ the rising trend of pericenter shift, remaining well below GR, is observed for scalarons with $M_P = (10^{-22}–10^{-21})$ eV. But, for $M_P = (10^{-20}–10^{-19})$ eV we find intersections between GR and scalaron gravity where pericenter shift is predominantly due to scalarons. These breaking points drift toward smaller orbital radii (below the orbit of S-2) for $M_P = 10^{-19}$ eV. For more massive scalarons, $M_P = (10^{-18}–10^{-17})$ eV the pericenter shift is again suppressed by GR and the breaking point disappears for the entire range of orbital size. It remains hidden for much smaller orbits ($a \ll 45$ au).

For field amplitude $\psi_o = 1.01(\alpha \approx 0.33)$ rising trend of scalaron induced pericenter shift which is suppressed by GR is observed only for the lowest mass scalaron, $M_P = 10^{-21}–10^{-20}$ eV. For scalarons with $M_P = (10^{-21}–10^{-18})$, we find breaking points of GR. As scalaron mass increases from $10^{-21}$ eV to $10^{-19}$ eV, these breaking points drift from the neighborhood of S-2 like orbits to orbital radii well below that of S-2. As one increases scalaron mass above $10^{-19}$ eV, MG-induced precession dominates only for very compact orbits, unlike the effect of scalarons with masses, $M_P < 10^{-17}$ eV. For larger orbital radii GR again suppresses scalarons, with the breaking point disappearing within $a = (45–1000)$ au for $M_P = 10^{-17}$ eV. The above pattern of variation is retained for scalaron field amplitude $\psi_o = 0.33(3.33 \approx 1.01)$.

Effect of power-law gravity $V(r) \sim r^{-2}$ induced by KK theory having five-dimensional Planck masses in the range $M_{P(5)} = (10^3–10^6)$ GeV is also studied. It is seen that the effect of power-law gravity with $M_{P(5)} = (10^3–10^5)$ GeV lies above that of GR for orbits within S-2. However, for $M_{P(5)} \geq 10^5$ GeV power-law gravity gets suppressed by GR through several orders of magnitude. Interestingly, maxima of pericenter shift in the power-law gravity with larger five-dimensional Planck masses may become similar to the size realized for GR and $f(R)$ gravity for some scalarons. Kalita (2021) reported scalaron masses within $M_P = (10^{-22}–10^{-17})$ eV and black hole spin in the range $0 < \chi \leq 1$ for which scalaron induced pericenter shift becomes

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**Table 1**

| Astrometric uncertainties(σ) | $M_{P(5)}$ (GeV) |
|-------------------------------|-----------------|
| For semimajor axis, $a = 45$ au |
| $\sigma_{Kcock} = 0.16$ mas | $7.29 \times 10^6$ |
| $\sigma_{GRAVITY} = 0.03$ mas | $1.27 \times 10^7$ |
| $\sigma_{TMT} = 0.015$ mas | $1.6 \times 10^7$ |
| For semimajor axis, $a = 100$ au |
| $\sigma_{Kcock} = 0.16$ mas | $5.59 \times 10^6$ |
| $\sigma_{GRAVITY} = 0.03$ mas | $9.7 \times 10^6$ |
| $\sigma_{TMT} = 0.015$ mas | $1.23 \times 10^7$ |
| For semimajor axis, $a = 1000$ au |
| $\sigma_{Kcock} = 0.16$ mas | $2.59 \times 10^6$ |
| $\sigma_{GRAVITY} = 0.03$ mas | $4.53 \times 10^6$ |
| $\sigma_{TMT} = 0.015$ mas | $5.7 \times 10^6$ |

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**Figure 5.** Pericenter shift due to higher gravitational power law and Schwarzschild precession for semimajor axis, $a = (45–1000)$ au. $M_{P(5)} = 1.9 \times 10^3$ corresponds to the Kaluza–Klein mass.
comparable to relativistic effects (spin-induced effects). Since there are cases for which pericenter shift in five-dimensional KK gravity becomes comparable to that observed in scalarons and GR, the results obtained here are of potential use for better understanding five-dimensional gravity independently through monitoring stellar orbits lying below S-2. The allowed range of the five-dimensional KK mass has been obtained through available bounds on the PPN parameters ($\gamma$, $\beta$) coming from the study of the orbits of S-2, S-38, and S-55. These ranges are displayed in Figure 7. It has been found that given the present bounds on ($\gamma$, $\beta$), $M_{Pl(5)} \approx 10^4$ GeV is allowed. It is seen from Figure 6 that higher-dimensional gravity with $M_{Pl(5)} \approx 10^4$ GeV causes pericenter shift to stay above general relativistic shift within $a = (45–1000)$ au. This is, however, a crude estimation arising from less precise data. But it is important as it indicates the possibility of extracting five-dimensional Planck mass lying not very far away from the supersymmetry breaking scale.

Relatively tight constraint on $M_{Pl(5)}$ appears when one employs astrometric capabilities of Keck, GRAVITY in the VLT, and TMT. For $a = 45, 100, \text{ and } 1000$ au, the lower bounds on $M_{Pl(5)}$ are obtained as $(10^6–10^7)$ GeV. This is
obtained for astrometric uncertainties 0.16 mas, 0.03 mas, and 0.015 mas of Keck, GRAVITY, and TMT respectively. This is expected to be a much tighter constraint relative to the allowed ranges of $M_{\psi(5)}$ obtained from relatively poor constraint ($\gamma$, $\beta$). Estimation of $M_{\psi(5)}$ from the above two considerations indicates that it is possible to constrain five-dimensional gravity with KK masses lying not very far away from the supersymmetric breaking scale (1000 GeV).

5. Summary

For higher scalaron coupling ($\alpha = 0.33$, 1.01), scalaron masses in the range, $M_o = (10^{-21} - 10^{-19})$ eV yield breaking points of general relativity. Scalarons overwhelm GR-induced precession for orbital radii within a few tens of au to about $a = 1000$ au. Massive scalarons with $M_o = (10^{-18} - 10^{-17})$ eV dominate GR only at smaller orbital radii, much below $a = 1000$ au. For the highest mass, scalarons are completely suppressed by GR at all scales within $a = (45 - 1000)$ au. For moderate coupling ($\alpha \approx 10^{-2}$) breaking points of GR appear near the orbit of S-2 for $M_o = (10^{-20} - 10^{-19})$ eV. Other scalarons are suppressed by GR within $a = (45 - 1000)$ au.

For extremely low coupling ($\alpha \approx 10^{-4}$) expected from the theoretical prediction of Kalita (2020), scalaron effects are well suppressed with no breaking point of GR within the entire interesting range of semimajor axes considered here. The scalaron induced precession of stellar orbits is smaller than that of GR by 2 to a few tens of orders of magnitude (see Figure 1), for orbital size in the range, 100 au $\leq a < 1000$ au. As these scales are within the reach of existing and upcoming large telescopes, a lower scalaron coupling will reveal no breaking point of GR in observations of pericenter shift below the orbit of S-2. The coupling $\alpha \approx 10^{-4}$ corresponding to the scalaron field $\psi_o \approx 1217$ is consistent with recent constraint (upper bound) on Yukawa coupling given by the detection of Schwarzschild precession of S-2 (Gravity Collaboration et al. 2020). Therefore, the upper bound on the coupling arising from precession measurements of S-2 increases the possibility of ruling out higher scalaron coupling at least within the orbit of S-2.

Power-law gravity-induced by the five-dimensional KK theory shows a noticeable effect in the pericenter shift of S-2 depending on the five-dimensional KK mass scale $M_{\psi(5)}$. This is realized to be a naïve opportunity to severely constrain KK dark matter particles which are otherwise impressively speculated in cosmological problems. We look forward to possible constraints on $f(R)$ scalaron gravity and higher-dimensional gravity in the upcoming observations of the pericenter shift of compact stellar orbits near the GC black hole.

A breaking point may arise in five-dimensional gravity theory for the interesting scale, $M_{\psi(5)} \approx 10^5$ GeV allowed by existing bounds on the PPN parameters near the black hole. This is an interesting clue for fundamental physics. Much tighter constraints on this parameter are expected from future measurements on the PPN parameters. Running and upcoming astrometric facilities such as Keck, GRAVITY, and TMT are capable of constraining five-dimensional gravity with five-dimensional Planck mass lying not too distant from the supersymmetry breaking scale. These constraints will also help cosmology in narrowing down the list of particle dark matter candidates expected from new physics.

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