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High-frequency simulation of acoustic lenses based on Fresnel zone plate

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Abstract. In this work, we develop a Fortran program to simulate the propagation of high-frequency plane acoustic wave through Fresnel zone plate (FZP) in two dimensions. The simulation was carried out at the frequency of 200 kHz with the wave speed of 1,500 m/s. The thickness of FZP was also varied to determine its effect on the focusing of FZP. Numerical results show that FZP could successfully focus the acoustic wave. The results also matched well with the simulation result of a previously published work of underwater acoustics. An efficiency comparison between FZP and a scattering-type acoustic lens focusing a wave at the frequency of 2140 Hz and wave speed of 330 m/s was also reported.

1. Introduction
A Fresnel zone plate (FZP) is a thin lens that focuses wave fields based on the concept of Fresnel zones and diffraction. It has been applied to both electromagnetic waves [1–4] and acoustic waves [5]. This study demonstrated that sound waves could be focused to a desired position. Using an FZP acoustic lens. The optimize design parameters for fabricating a lens [6] used a hybrid genetic-greedy algorithm constrained to a linear structure. The experimental results were compared to the simulation results for ultrasonic waves in air[7], verifying the simulation method. The simulation for ultrasonic underwater, used FZP to focusing[8]. The results showed that a designed FZP effected low energy loss. Moreover, the authors did not disclose the computer codes that they used in their studies. Therefore, we had to develop a computer program in Fortran programs to evaluate our design.

Materials and Method.
In this work we develop a serial Fortran computer program for performing numerical simulation of acoustic wave propagation through FZP in two dimensions. We also study of the relationship of the focal point and the resolution with the thickness of Fresnel zone plate. Finally, we compare FZP acoustic lens with the scattering-type acoustic lens of Sanchis et al[9].

2. Fresnel zone plate
A Fresnel zone plate is a periodic of symmetric rings patterns, which alternate between opaque zone and transparent zone as shown in Figure 1. Waves hitting the opaque zone of zone plate will reflect and transmitting the transparent zone will diffract and interfere at a focal point.
Figure 1. Fresnel zone plate with opaque (black) and transparent (white) zones where \( r_n \) is the radius of the \( n^{th} \) zone.

In previous work, we shown how to derive the formula of the radius of the \( n^{th} \) zone for focusing plane waves. According to Figure 2, the path difference between paths SOP and SQP is

\[
\left( \sqrt{z_1^2 + r_n^2} + \sqrt{z_2^2 + r_n^2} \right) - (z_1 + z_2) = \frac{n\lambda}{2}, \quad n = 1, 2, \ldots
\]

(1)

Where \( \lambda \) is the wavelength.

A plane wave can be thought of as a spherical wave travel from a point source located very far from the zone place, i.e., \( z_1 \to \infty \).

Then the first term in Equation 1 becomes \( \sqrt{z_1^2 + r_n^2} \approx z_1 \) and, consequently, Equation 1 can be rewritten as

\[
\left( z_1 + \sqrt{z_2^2 + r_n^2} \right) - (z_1 + z_2) = \frac{n\lambda}{2}, \quad n = 1, 2, \ldots
\]

(2)

Rearranging Equation 2 yields

\[
r_n^2 = n\lambda z_2 + \left( \frac{n\lambda}{2} \right)^2
\]

(3)

3. **Numerical simulation of FZP acoustic lenses**

We present numerical methods for the simulation of wave propagation acting on FZP by solving the two-dimensional acoustic wave equation in this section.

\[
\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} - \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = s(x, y, t),
\]

(4)

where \( u \) and \( s \) are the pressure and source fields, respectively, and \( c \) is the phase velocity of acoustic wave.

3.1 **Finite difference method**

The second-order derivatives in equation 4 are approximated by the second-order finite difference approximation. Therefore, the wave field \( u(x,y,t) \) approximating all second derivatives in the wave equation yields the discrete wave equation

\[
\frac{1}{c_i^2} D_t^2 u_{i,j}^n - (D_t^2 u_{i,j}^n + D_t^2 u_{i,j}^n) = s_{i,j}^n
\]

(5)

The discrete wave equation can be rearranged as an explicit time-marching scheme

\[
u_{i,j}^{n+1} = (2 - 4C^2)u_{i,j}^n + C^2\delta u - u_{i,j}^{n+1} + (C_i^2\Delta t) s_{i,j}^n,
\]

(6)

where \( \delta u = (u_{i+1,j}^n + u_{i-1,j}^n + u_{i,j+1}^n + u_{i,j-1}^n) \) and \( C = c_{i,j} \Delta t / h \).
3.2 Boundary conditions

In this work, we use two types of boundary conditions: perfectly matched layer and Neumann boundary condition.

3.2.1 Perfectly matched layer. We simulate an acoustic wave propagation in an unbounded domain using the perfectly matched layer (PML) method proposed by Berenger to absorb wave energy at the boundary of the computational domain. To absorb waves propagating in the x direction using PML, a complex coordinate stretching

\[ x \rightarrow x + i \beta(x) / \omega, \tag{7} \]

is used where \( \beta(x) \) is a positive real-valued attenuation function, \( i = \sqrt{-1} \), and \( \omega \) is the angular frequency. Consequently, the spatial derivative \( \partial / \partial x \).

3.2.2 Neumann boundary condition. The zero-flux boundary condition \( \frac{\partial u}{\partial n} = 0 \), is used to propagate a plane wave along a boundary as shown in Figure 4.

![Figure 3](image-url) Simulation of a spherical wave propagation in 2D (a) without PML and (b) with PML.

![Figure 4](image-url) Simulation of a plane wave propagation along a zero-flux Neumann boundary condition.

4. Numerical results

In this section, we present the numerical results of simulating acoustic wave propagation in 2D domain through two types of Fresnel zone plate. In all simulations, the source frequency is 200 kHz, and the wave speed is 1,500 m/s. a desired focal position at \( z = 10 \) cm and \( x = 0 \) cm. Then we use Fortran programs to simulate this. Due to the significant effect of the thickness of FZP for plane wave, we set it to 3 different values: 4, 8, and 16 mm. Figure 4-6. show the simulation results with these 3 thickness, respectively. The results for the 4-mm lens were the following: the focal point was at \( z = 9.8 \)
cm. and \( z = 0 \) cm. and a peak to null = 16 mm. Those for the 8-mm lens were the following: the focal point was at \( z = 9.6 \) cm. and \( x = 0 \) cm. and a peak to null = 17 mm, while those for the 16-mm lens were as follows: the focal point was at \( z = 9.0 \) cm. and \( x = 0 \) cm. and a peak to null = 17 cm. The conclusion is that the thicker the lenses the farther the resultant focal point is displaced from the desired focal point. In other words, the thinner the lenses the better the quality of the focusing mechanism. In addition, our results agree closely to those reported, demonstrating that our developed simulation programs is reliable.

Figure 5. Numerical results of FZP for plane waves for high frequency and the thickness of FZP is 4 mm.

Figure 6. Numerical results of FZP for plane waves for high frequency and the thickness of FZP is 8 mm.
Figure 7. Numerical results of FZP for plane waves for high frequency and the thickness of FZP is 16 mm

Figure 8. Numerical results of FZP for plane waves for frequency 2140 HZ

5. Conclusion
The presented formulas for Fresnel zone plates for plane waves were validated by numerical simulation based on the finite-difference solution of a PML formulation of a two-dimensional acoustic wave equation. The simulations performed by using our own developed Fortran programs showed that the acoustic waves were accurately focused at the designed locations in both cases. The thickness of the FZP was shortened, and it was found that the narrower the thickness the more effective the focusing mechanism. The simulation results were found to agree closely to those in a published paper, indicating that our developed simulation programs were a reliable. Compared to the scattering method, the FZP was found to be more effective at focusing acoustic waves.
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