Probability Distribution of the Ratio of Consecutive Level Spacings in Interacting Particle Systems

N. D. Chavda\textsuperscript{a} and V. K. B. Kota\textsuperscript{b}
\textsuperscript{a}Applied Physics Department, Faculty of Technology and Engineering, M. S. University of Baroda, Vadodara 390 001, India
\textsuperscript{b}Physical Research Laboratory, Ahmedabad 380 009, India
(Dated: May 28, 2013)

We study the probability distribution of the ratio of consecutive level spacings for embedded one plus two-body random matrix ensembles with and without spin degree of freedom and for both fermion and boson systems. The agreement between the numerical results and the recently derived analytic form for the distribution and other related quantities is found to be close. This establishes conclusively that local level fluctuations generated by embedded ensembles follow the results of classical Gaussian ensembles.

I. INTRODUCTION

Large number of investigations in the last two decades have established that Embedded Gaussian orthogonal ensembles of one plus two-body interactions [EGOE(1+2)] operating in many particle spaces [1−3], apply in a generic way to isolated finite interacting many-particle quantum systems such as nuclei, atoms, quantum dots, small metallic grains, interacting spin systems modeling quantum computing core and so on. For sufficiently strong interaction, EGOEs exhibit average-fluctuation separation in eigenvalues with the smoothed eigenvalue density being a corrected Gaussian and the local fluctuations are of GOE type [2, 4, 5].

The classical Gaussian Orthogonal Ensemble (GOE), Gaussian Unitary Ensemble (GUE) and Gaussian Symplectic ensemble (GSE) are ensembles of multi-body (m-body for m-particle systems) interactions. Therefore they are unrealistic for understanding quantum many-body chaos as interactions for most finite quantum systems are largely one- plus two-body in character (one-body part corresponds to a mean-field). The concept of Embedded Ensembles (EE) precisely takes care of the GOE version of EE. For m spin-less fermions occupying say N single particle (sp) states, with the Hamiltonian (H) matrix in two-particle spaces represented by GOE and then constructing the many-particle H matrix, with the m-particle basis states being direct products of sp states, gives the Embedded Gaussian Orthogonal Ensemble of two-body interactions [EGOE(2)] in m-particle spaces. Similarly, for interacting spin-less boson systems, the Embedded Gaussian Orthogonal Ensemble of two-body interactions can be defined and to distinguish these from those of fermion systems, they are denoted by BEGOE(2) [6]. Addition of the mean-field, one-body part, gives EGOE(1+2) and BEGOE(1+2) for fermion and boson systems respectively. EGOE(1+2) can be defined for fermions or bosons with spin degree of freedom and also with many other symmetries [2, 7]. It is useful to note that EGOE(1+2)\textsuperscript{s} [BEGOE(1+2)\textsuperscript{s}] reduce to EGOE(2)\textsuperscript{s} [BEGOE(2)\textsuperscript{s}] in the strong interaction limit.

Normally one uses the Nearest Neighbor Spacing Distribution (NNSD) \(P(S)dS\) for establishing GOE nature of level fluctuations. It is well known that if the system is in integrable domain, corresponding to the regular behavior of the system, the form of the NNSD is close to the Poisson distribution \([P(S) = \exp(-S)]\). However, if the system is chaotic, NNSD is described by the Wigner surmise which is essentially the GOE result \([P(S) = (\pi/2)S\exp(-\pi S^2/4)]\). In constructing NNSD for a given set of energy levels (or eigenvalues), the spectra have to be unfolded to remove the variation in the density of eigenvalues [4, 8]. For the spectra generated by a random matrix ensemble, the unfolding of the spectrum can be done in two different ways. One of them being spectral unfolding, i.e. spectrum of each individual member of the ensemble is unfolded separately and then ensemble averaged NNSD is constructed. A second method is ensemble unfolding and here a single unfolding function is used for all the members. For EGOE, it is well known that spectral unfolding gives GOE fluctuations while there are deviations from GOE when ensemble unfolding is used [4, 9]. Because of this, in the past there was serious confusion with regard to the usefulness of EE; see the discussion on page 424 of [4] and also [10]. Another minor issue is, although eigenvalue density for EGOEs is close to Gaussian form, many groups use polynomial of a high degree for unfolding purposes; see for example [11]. Therefore, to conclusively establish that EGOEs generate GOE fluctuations, it is important to use fluctuation measures that are independent of the unfolding function and unfolding procedure. The purpose of the present letter is to report the results of the analysis of level fluctuations in EGOEs using a measure introduced recently that is independent of unfolding the spectrum.

Oganesyan and Huse [12] in 2007 considered the distribution of the ratio of consecutive level spacings of the energy levels which requires no unfolding as it is independent of the form of the density of the energy levels. This distribution allows a more transparent comparison with experimental results than the traditional level spacing distribution. This measure was used to quantify the
distance from integrability on finite size lattices [13, 14], and also to investigate numerically many-body localization [12, 15–17]. More importantly, recently Atas et. al [18], derived expressions for the probability distribution of the ratio of two consecutive level spacings for the classical GOE, GUE and GSE ensembles of random matrices. These expressions, called Wigner-like surmises, are shown to be very accurate when compared to numerical calculations in the large matrix size limit. Also, results from a quantum many-body lattice model and from zeros of the Riemann zeta function are shown to be in excellent agreement with the analytical formulas derived. Going beyond these, in the present letter we show that the probability distribution for ratio of two consecutive level spacings and related averages for many different EGOEs are very close to the GOE formulas. This then confirms conclusively that EGOE level fluctuations follow GOE. Now, we will give a preview.

Analytical results for GOE for the probability distribution of the ratio of consecutive level spacings and related averages are briefly discussed in Section II. The five different EGOEs used in the present analysis are described in Section III. The numerical EGOE (and BEGOE) results of the probability distribution for ratio of consecutive spacings and related averages are presented in Section IV. Finally, Section V gives concluding remarks.

II. PROBABILITY DISTRIBUTION OF THE RATIO OF CONSECUTIVE LEVEL SPACINGS

Let us consider an ordered set of eigenvalues (energy levels) $E_n$, where $n = 1, 2, ..., d$. The Nearest-Neighbor Spacings is given by $s_n = E_{n+1} - E_n$. Then, the ratio of two consecutive level spacings is $r_n = s_{n+1}/s_n$. The probability distribution for consecutive level spacings is denoted by $P(r)dr$. If the system is in integrable domain, then NNSD is Poisson and $P(r)$ is [denoted by $P_P(r)$],

$$P_P(r) = \frac{1}{(1+r)^2}.$$  \hspace{1cm} (1)

Similarly, for GOE, derived using $3 \times 3$ real symmetric matrices, the $P(r)$ is given by Wigner-like surmise [18],

$$P_W(r) = \frac{27}{8} \frac{r + r^2}{(1 + r + r^2)^{5/2}}.$$  \hspace{1cm} (2)

It is also suggested in [18] that the difference $\delta P(r) = P(r) - P_W(r)$ between numerics and the surmise (2) can be approximated by the following expression,

$$\delta P(r) = \frac{C}{(1+r)^2} \left( \left( r + \frac{1}{r} \right)^{-1} - 2 \frac{\pi - 2}{4 - \pi} \left( r + \frac{1}{r} \right)^{-2} \right),$$  \hspace{1cm} (3)

where the parameter $C$ is to be obtained by fitting the expression $P(r) = P_W(r) + \delta P(r)$. In addition to $r_n$, Oganesyan and Huse [12] considered the distribution of the ratios $\tilde{r}_n$ defined by

$$\tilde{r}_n = \frac{\min(s_n, s_{n-1})}{\max(s_n, s_{n-1})} = \min(r_n, 1/r_n)$$  \hspace{1cm} (4)

As pointed out in [18], it is possible to write down $P(\tilde{r})$ given $P(r)$. In practice, it is also useful to consider $\langle \tilde{r} \rangle$, the average value of $r$. For GOE we have $\langle r \rangle = 1.75$ and for Poisson it is $\infty$. However, $\langle \tilde{r} \rangle = 0.536$ for GOE and 0.386 for Poisson. We will use $P(r)$, $\langle r \rangle$ and $\langle \tilde{r} \rangle$ in the analysis of energy levels presented in Section IV.

III. EMBEDDED ENSEMBLES FOR FERMION AND BOSON SYSTEMS WITH AND WITHOUT SPIN DEGREE OF FREEDOM

In the present study five different EGOEs are employed. For spin-less fermion and boson systems with a mean-field and two-body interactions we have EGOE(1+2) and BEGOE(1+2) respectively [1, 19] and the Hamiltonian $H = h(1) + \lambda(V(2))$. Here, $h(1)$ is the mean-field one-body part defined by single particle (sp) energies $\epsilon_i$ and $\{V(2)\}$ represents EGOE(2) or BEGOE(2), i.e. $V(2)$ matrix in two-particle spaces is represented by GOE with matrix elements variance unity. The parameter $\lambda$ is the strength of the two-body interaction in units of average spacing $\Delta$ of the single sp levels.

Note that $\{ - - \}$ denotes ensemble with matrix elements variance unity in two-particle spaces. Going beyond spin-less systems, we have considered three embedded ensembles with spin degree of freedom. For fermions with spin $s = 1/2$ degree of freedom, we have EGOE(1+2)-s [20]. Here, the interaction $V(2)$ will have two parts as the two particle spin $s = 0$ and 1 giving $H = h(1) + \lambda_0\{V^{s=0}(2)\} + \lambda_1\{V^{s=1}(2)\}$. Similarly, for two species boson systems it is possible to consider bosons with a fictitious ($F$) spin 1/2 degree of freedom.

Then, we have BEGOE(1+2)-F [21]. Again here also, the interaction will have two parts as the two particle $F$-spin $f = 0, 1$. Therefore for BEGOE(1+2)-F, $H = h(1) + \alpha\{V^{f=0}(2)\} + \lambda_1\{V^{f=1}(2)\}$. Note that here and also for EGOE(1+2)-s, the sp levels will be doubly degenerate. Finally, we have also considered bosons spin one degree of freedom, i.e. BEGOE(1+2)-S1 [22]. Here, the interaction will have three parts as the two particle spin $s = 0, 1$ and 2 and therefore $H = h(1) + \lambda_0\{V^{s=0}(2)\} + \lambda_1\{V^{s=1}(2)\} + \lambda_2\{V^{s=2}(2)\}$ with sp levels defining $h(1)$ triply degenerate. In all the five ensembles, without loss of generality, we choose the average spacing between the sp levels to be unity so that all $\lambda$I's are unit-less.

The following choices are made for constructing the five EGOEs in many particle spaces:

1. EGOE(1+2) for $m = 6$ fermions in $N = 12$ sp states with $H$ matrix of dimension 924. The sp energies are chosen as $\epsilon_i = i + 1/2$, $i = 1, 2, ..., 12$ and the interaction strength $\lambda = 0.1$; see Ref. [1] for details.
2. EGOE(1+2)-s for \( m = 6 \) fermions occupying \( \Omega = 8 \) sp levels (each doubly degenerate) with total spin \( S = 0 \) and \( S = 1 \) giving the \( H \) matrices of dimensions 1176 and 1541 respectively. The sp energies are chosen as \( \epsilon_i = i + 1/i, i = 1, 2, \ldots, 8 \) and the interaction strength \( \lambda = \lambda_0 = \lambda_1 = 0.1 \); see Ref. [20, 23] for details.

3. BEGOE(1+2) for \( m = 10 \) bosons in \( N = 5 \) sp states with \( H \) matrix of dimension 1001. The sp energies are chosen as \( \epsilon_i = 1 + 1/i, i = 1, 2, \ldots, 5 \) and the interaction strength \( \lambda = 0.03 \); see Ref. [19, 24] for details.

4. BEGOE(1+2)-F for \( m = 10 \) bosons occupying \( \Omega = 4 \) sp levels (each doubly degenerate) with total \( F \)-spin \( F = 2 \) and \( F = F_{\text{max}} = 5 \) giving the \( H \) matrices of dimensions 750 and 286. The sp energies are chosen as \( \epsilon_i = i + 1/i, i = 1, 2, 3, 4 \) and the interaction strength \( \lambda = \lambda_0 = \lambda_1 = 0.05 \); see Ref. [21] for details.

5. BEGOE(1+2)-S1 for \( m = 8 \) bosons occupying \( \Omega = 4 \) sp levels (each triply degenerate) with total spin \( S = 4 \) giving the \( H \) matrix of dimension 1841. The sp energies are chosen as \( \epsilon_i = i + 1/i, i = 1, 2, \ldots, 4 \) and the interaction strength \( \lambda = \lambda_0 = \lambda_1 = \lambda_2 = 0.2 \); see Ref. [22] for details.

All the ensembles considered in the analysis have 500 members. Let us add that the \( \lambda \) values in the ensemble calculations are chosen such that the system is in Gaussian domain, i.e. the state density and local density of states will be close to Gaussian in form and level and strength fluctuations (when an appropriate unfolding function is used) exhibit GOE fluctuations. Now we will turn to the numerical results.

IV. NUMERICAL RESULTS FOR EXAMPLES FROM EGOE(1+2), BEGOE(1+2), EGOE(1+2)-s, BEGOE(1+2)-F AND BEGOE(1+2)-S1

Following the definitions given in Section II, we have constructed the distribution of the ratio of consecutive level spacings \( P(r) \), using the middle 80\% part of the spectrum, for the EEs described in Section III. The results are shown in the Figs. 1–5 (upper panels). In each calculation, averaging over the 500 members of the ensemble, histogram for \( P(r) \) is constructed using a bin size of 0.1. The agreement between the numerical embedded ensemble results and the surmise given by Eq. (2) is very good for all the examples. Results are also shown for small value of \( r \) \( (r \leq 0.5) \) for the EGOE(1+2) and BEGOE(1+2) examples; see the inset plots in Figs. 1 and 3 respectively. They show that the agreements are indeed very close for \( r \) over all the range. We have also calculated the difference between numerical results and the surmise for all the examples considered in the study. These results are shown in the lower panels of the Figs. 1–5. The smooth red curves (shown in lower panels) are obtained by fitting Eq. (3) with one parameter \( C \). The values of \( C \) are given in Table I. It is seen from Figs. 1–5 that the \( \delta P(r) \) given by Eq. (3) fits quite well the numerical results. In addition, we have also calculated the averages \( \langle r \rangle \) and \( \langle \tilde{r} \rangle \) and the results are given in Table I. Again the calculated values are seen to be close to GOE values.

In the past it is demonstrated that for EGOE(1+2) and BEGOE(1+2) ensembles, as the strength \( \lambda \) of the two-body interaction increases, generically there is Poisson to GOE transition in level fluctuations [1, 19, 21, 23]. In order to examine this in terms of \( P(r) \), we have calculated \( \tilde{r} \) for spin-less fermion EGOE(1+2) and spin-less boson BEGOE(1+2) ensembles as a function of the interaction strength \( \lambda \). Figure 6 shows the results. It is clearly seen that as \( \lambda \) increases from \( \lambda = 0 \), the value of \( \tilde{r} \) changes from close to Poisson value to finally the GOE value. Similar results are obtained using lattice models in [16, 17]. In order to quantify the results in Fig. 6, it is necessary to derive a formula for \( P(r) \) in a random matrix model that generates Poisson to GOE transition; see ahead for further discussion.

With all the good agreements seen in Figs. 1–5 for EGOE(1+2)s with sufficiently large value for the interaction strength and including middle 80\% of the levels in the analysis, the questions that one may ask are: (i) will there be deviations if we use all the levels; (ii) do we get good agreements if we use EE without mean-field; (iii) will the \( P_W(r) \) given by Eq. (2) fit well the results if we use only the lowest 10 or 20 levels. Briefly, in order to answer (i) and (ii) we have carried out EGOE(2) and BEGOE(2) calculations using all the levels for \( (m = 6, N = 12) \) and \( (m = 5, N = 10) \) systems respectively. The results in Fig. 7 and Table 1 clearly show that the agreement with \( P_W(r) \) given by Eq. (2) is good even when we include all the levels in the analysis. Question (iii) above is important as levels close to the ground state are in general expected to show departures from GOE. In fact, Flores et al [25] showed, using eigenvalues from large nuclear shell model calculations, that the so-called semi-Poisson distribution gives a better fit to the NNSD when only low-lying levels are used along with spectral unfolding. To investigate deviations from GOE if any present in low-lying levels generated by EEs, we have constructed \( P(r) \) considering lowest 10 levels and 20 levels for EGOE(2), BEGOE(2), EGOE(1+2)-s and BEGOE(1+2)-F examples. Results are shown in Fig. 8. There are clearly deviations from GOE for \( r \lesssim 0.5 \). In order to quantify the departures from GOE for low-lying levels, it is necessary to extend the results in [18] to Poisson to GOE interpolating region. Using Eq. (7) of [18], we suggest the following form for Poisson to GOE interpolation for \( P(r) \).

\[
P_{P-\text{GOE}}(r : \beta) = \frac{1}{Z_\beta} \frac{(r + r^2)^\beta}{[1 + (2 - \beta)r + r^2]^{\frac{1}{2}}},
\]

Then, \( \beta = 0 \) gives Poisson and \( \beta = 1 \) GOE correctly.
TABLE I. Values of the constant \( C \) and the averages \( \langle \tilde{r} \rangle \) and \( \langle r \rangle \) for various EE examples used in the present study. Given are also the values for Poisson, GOE, EGOE(2) and BEGOE(2).

| Example            | \( C \)          | \( \langle \tilde{r} \rangle \) | \( \langle r \rangle \) |
|--------------------|------------------|-------------------------------|--------------------------|
| EGOE(1+2)          | 0.2198 ± 0.0109  | 0.5304                        | 1.7727                   |
| EGOE(1+2)-s        | \( S = 0 \)      | 0.1909 ± 0.0119               | 0.5313                   | 1.7700                   |
|                    | \( S = 1 \)      | 0.1528 ± 0.0074               | 0.5318                   | 1.7661                   |
| BEGOE(1+2)         | 0.3264 ± 0.0083  | 0.5279                        | 1.7960                   |
| BEGOE(1+2)-F       | \( F = 5 \)      | 0.2926 ± 0.0159               | 0.5283                   | 1.7962                   |
|                    | \( F = 2 \)      | 0.2940 ± 0.0120               | 0.5286                   | 1.7885                   |
| BEGOE(1+2)-S1      | \( S = 4 \)      | 0.2303 ± 0.0082               | 0.5303                   | 1.7718                   |
| EGOE(2)            | all levels       | 0.2153 ± 0.0088               | 0.5303                   | 1.7823                   |
| BEGOE(2)           | all Levels       | 0.2788 ± 0.0093               | 0.5286                   | 1.9340                   |
| GOE                |                  | 0.2334                        | 0.5359                   | 1.75                      |
| Poisson            |                  | 0.3863                        | \( \infty \)             |

The condition \( \int_0^\infty \langle \tilde{r} \rangle P(r)dr = 1 \) gives \( Z_\beta \). Eq. (5) fits well the results in Fig. 8 and some examples are shown in Fig. 9. This will be explored in more detail elsewhere.

V. CONCLUSIONS

Embedded ensembles (EGOE), appropriate for isolated finite quantum many-particle systems, generate eigenvalue density close to Gaussian and this is quite different from the GOE semi-circle density. In the past it was shown [4, 10] that only with proper spectral unfolding, EGOEs exhibit GOE level fluctuations. Therefore, nature of level fluctuations in EGOEs is still not fully understood. Addressing this issue, in this Letter we have demonstrated, with examples from EGOE(1+2), EGOE(1+2)-s, BEGOE(1+2), BEGOE(1+2)-F and BEGOE(1+2)-S1, that the probability distribution \( P(r) \) of ratio of consecutive level spacings for embedded random matrix ensembles follow GOE for strong enough interaction. The Wigner-like surmise for \( P(r) \) is found to agree very well with the numerical calculations. Also, the difference between the surmise and the exact calculations is small and can be fitted by a one-parameter polynomial formula with very good accuracy. As \( P(r) \) is independent of unfolding, the results in Section IV conclusively establish that level fluctuations in EGOE(1+2)s follow GOE for strong enough interaction. Let us add that we have also verified that \( P_W(r) \) is close to the \( P(r) \) from a nuclear shell model example as shown in Fig. 10. Note that the results from shell model example can be considered as the results of a typical member of EGOE(1+2)-JT [1, 4] and this ensemble is usually called TBRE in literature [26]. Finally, in future it is useful to derive a formula for \( P(r) \) for Poisson to GOE transition (see [27] for a discussion of this for NNSD) and also for pseudo integrable systems (see [28] for NNSD for these systems). These and Eq. (5) should be useful in quantifying departures from GOE in low-lying levels generated by EEs.
FIG. 2. Probability distribution of the ratio of consecutive level spacings \( P(r) \) vs. \( r \) and \( \delta P(r) \) vs. \( r \) for a 500 member EGOE(1+2)-s ensemble. Results are shown for spins \( S = 0 \) and 1. See Fig. 1 and text for details.

FIG. 4. Probability distribution of the ratio of consecutive level spacings \( P(r) \) vs. \( r \) and \( \delta P(r) \) vs. \( r \) for a 500 member EGOE(1+2)-F ensemble. Results are shown for \( F \)-spins values \( F = 5 \) and 2. See Fig. 1 and text for details.

FIG. 3. Probability distribution of the ratio of consecutive level spacings \( P(r) \) vs. \( r \) and \( \delta P(r) \) vs. \( r \) for a 500 member BEGOE(1+2) ensemble. See Fig. 1 and text for details.

FIG. 5. Probability distribution of the ratio of consecutive level spacings \( P(r) \) vs. \( r \) and \( \delta P(r) \) vs. \( r \) for a 500 member BEGOE(1+2)-S1 ensemble. See Fig. 1 and text for details.
given by Eq. (2).

FIG. 6. Ensemble averaged value of \( \tilde{r}_m \) (denoted as \( \langle \tilde{r} \rangle \)) defined by Eq. (4) as a function of two-body interaction strength \( \lambda \), calculated using 500 member ensembles for EGOE(1+2) ensemble with \((m, N) = (6, 14)\) and BEGOE(1+2) ensemble with \((m, N) = (10, 5)\). The horizontal dotted-lines represent Poisson estimate (bottom reference line) and Wigner estimate (top reference line). In the calculations sp energies are drawn from the center of a GOE. As the systems considered are not strictly Poisson for \( \lambda = 0 \), it is seen from the figure that for very small \( \lambda \) value, the \( \langle \tilde{r} \rangle \) value is larger than the Poisson value with the difference more for BEGOE(1+2) as \( N \) is much smaller for this example.

FIG. 7. Results for \( P(r) \) vs \( r \) for EGOE(2) and BEGOE(2) examples including all the levels in the analysis. Inset figures show the results for \( r \leq 0.5 \). The red smooth curve is \( P_W(r) \) given by Eq. (2).

FIG. 8. Probability distribution of the ratio of consecutive level spacings \( P(r) \) vs. \( r \) for the lowest 10 and 20 levels using for EGOE(2), BEGOE(2), EGOE(1+2)-s and BEGOE(1+2)-F ensembles. For comparison, results from \( P_P(S) \) (red dash curve) and \( P_W(S) \) (red smooth curve) are shown. See text for details.

FIG. 9. Histograms represent \( P(r) \) vs. \( r \) results for the lowest 10 levels obtained for EGOE(1+2)-s, BEGOE(1+2)-F EGOE(2) and BEGOE(2) examples. The red smoothed curves are due to fitting histograms with Eq. (5) and values of parameter \( \beta \) are given in the figure. We have also analyzed the lowest 20 levels for the same examples and the \( \beta \) values obtained are : 0.8399[EGOE(1+2)-s], 0.7895[BEGOE(1+2)-F], 0.8795[EGOE(2)] and 0.6205[BEGOE(2)].
FIG. 10. Probability distribution of the ratio of consecutive level spacings $P(r)$ vs. $r$ for Nuclear shell model example: $^{24}\text{Mg}$ with 8 nucleons in the $(2s1d)$ shell with angular momentum $J = 2$ and isospin $T = 0$. The matrix dimension is 1206 and all levels are used in the analysis. See Ref.[5] for further details. Inset figure shows $\delta P(r)$ vs. $r$. Given in the figure are also the calculated values of $\langle \tilde{r} \rangle$ and $\langle r \rangle$. See Fig. 1 and text for details.

ACKNOWLEDGMENTS

We thank V. Potbhare for many useful discussions and for his interest in the present work. One of the authors (NDC) acknowledges support from UGC (New Delhi) grant No: F.40-425/2011(SR).

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