Cosmological models with one extra dimension.

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Abstract

We consider cosmological models in which a homogeneous isotropic universe is embedded as a 3+1 dimensional surface into a 4+1 dimensional manifold. The size of the extra dimension depends on time. It is small compared to the size of the universe only if the energy of gravitational self-interaction of the universe through the compact extra dimension dominates over all other kinds of energy. The self-interaction energy gives the main contribution into the Friedmann equation, which governs the dynamics of the scale factor of the universe.

The possibility that space-time has more than three spatial dimensions arose for the first time in the Kaluza-Klein approach to unification of gravity with electromagnetism \cite{1}. This idea is implemented now in different models of quantum field theory and string theory. It is natural to ask whether the presence of additional space-time dimensions could be tested experimentally. Of course, if the size $L$ of the additional compact dimensions is small compared to all experimentally attainable scales (for example, of order of the

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Planck length $L \sim L_{pl}$) it would be extremely difficult to develop an experiment in which this size could be determined directly. Recently N.Arkani-Hamed et al. [2] proposed a framework for solving the hierarchy problem in which the space-time manifold contains "large" extra dimensions with the size $L \gg L_{pl}$. Within this framework the conventional $(3 + 1)$ dimensional space-time $M_{3+1}$ is embedded as a surface into a higher dimensional manifold $M_{(3+n)+1}$. All the matter fields are confined to live on the surface $M_{3+1}$ while the gravitational field can propagate in $M_{(3+n)+1}$. According to [2], the conventional four-dimensional gravitational constant $G_4$ is a derived quantity, it is related to the fundamental $4+n$ dimensional gravitational constant $G_{4+n}$ through the size $L$ of the extra dimensions

$$G_4 \sim \frac{G_{4+n}}{L^n}$$

Therefore, although the four dimensional Planck mass $M_{pl} = \sqrt{\hbar c/G_4}$ is of order of $10^{19}$ GeV, the fundamental $4+n$ dimensional Planck mass $\tilde{M}_{pl} = (\hbar^{1+n}c^{1-n}/G_{4+n})^{1/(2+n)}$ could be of order of TeV. This opens the possibility to detect the presence of extra dimensions experimentally in collider experiments [3].

Apart from the direct consequences for particle physics, the presence of extra dimensions would have a set of testable predictions in cosmology. If we want to check whether the possibility that space-time has compact extra dimensions is allowed from the cosmological point of view, we need first to find the solutions of Einstein equations which describe a homogeneous isotropic universe in this framework. In this Letter we concentrate our attention on cosmological models with one extra dimension. Although the case of just one extra dimension is not favored in the original argumentation of [2], it deserves attention due to its simplicity. Besides, if the number of large extra dimensions if bigger than one, it is not necessary that all of them have equal size $L_1, ..., L_n = L$. If the sizes are arranged hierarchically, we could have $L_1 \gg L_i, i > 1$ and in this case the problem with just one extra dimension could serve as a first approximation to the complete problem.

Cosmological models with extra dimensions can differ significantly from four-dimensional ones. When the universe surface $M_{3+1}$ is embedded into a higher dimensional manifold $M_{(3+n)+1}$ it can self-interact through the $(3+n)$ dimensional volume. This self-interaction could affect somehow on the...
internal geometry of the surface $M_{3+1}$, that is the dynamics of the scale factor of the universe. Besides, this interaction becomes stronger if the size of the extra dimensions decreases. Thus, even if the size of the extra dimensions is very small, a cosmology with extra dimensions could be very different from the standard four-dimensional one. There were recently some attempts to develop cosmological models with one extra dimension \[4, 5\]. In these models the effect of self-interaction of the universe through the additional compact dimension could not be traced, because the authors introduce a “hidden brane” apart from the “visible” one into $\mathcal{M}_{4+1}$. Besides, the Ansatz used for the $(4+1)$ dimensional metric implies that the size of the extra dimension is kept fixed, rather than determined from Einstein equations. The authors of \[4, 5\] point out some problems with the realization of the standard scenario of primordial nucleosynthesis in their cosmological models with one extra dimension.

In what follows we find solutions of the $(4+1)$ dimensional Einstein equations with just one brane $M_{3+1}$ which could gravitationally self-interact when it is embedded into $\mathcal{M}_{4+1}$. The size of the extra dimension is defined dynamically from Einstein equations. The condition that it must be much smaller than the size of the (visible part of the) universe implies that the energy of self-interaction of the universe through the additional dimension dominates over all other kinds of energy in the universe.

Let us suppose that a homogeneous isotropic $(3+1)$ dimensional universe with metric

$$dl^2 = dr^2 - R^2(t) \left( d\chi^2 + f(\chi) \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right)$$

$$f(\chi) = \begin{cases} 
\sin^2 \chi & \text{for a closed universe} \\
\chi^2 & \text{for a flat universe} \\
\sinh^2 \chi & \text{for an open universe}
\end{cases}$$

is embedded into a $(4+1)$ dimensional manifold $M_{3+1} \subset \mathcal{M}_{4+1}$. If the $(4+1)$ dimensional metric in the coordinates $(t, r, \chi, \theta, \phi)$ has the form

$$ds^2 = e^{\nu(t,r)} dt^2 - e^{\lambda(t,r)} dr^2 - r^2 \left( d\chi^2 + f(\chi) \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right)$$

then the equation of embedding is

$$r = R(t)$$
for some function $R(t)$.

The functions $\lambda(t, r)$, $\nu(t, r)$ which define the metric (4) must be found from Einstein equations in $\mathcal{M}_{4+1}$. Suppose that all the matter is concentrated on the surface $M_{3+1}$, and the stress energy tensor has the form

$$\mathcal{T}_\mu^\nu = T_\mu^\nu \delta(r - R(t)). \quad (6)$$

Its projection on the surface $M_{3+1}$ is

$$T^j_i = \text{diag}(\rho, -p, -p, -p) \quad (7)$$

(indices $i, j$ run through the coordinates on $M_{3+1}$ while indices $\mu, \nu$ run over the coordinates in $\mathcal{M}_{3+1}$). Outside the surface $M_{3+1}$, the metric (4) is a solution of the vacuum Einstein equations in $(4 + 1)$ dimensions. As it is shown in the Appendix, the functions $\lambda, \nu$ are given by

$$e^\nu = e^{-\lambda} = k - \frac{\mu}{r^2}, \quad (8)$$

where $k = +1, 0, -1$ for a closed, flat and open universe, respectively. $\mu$ is a free parameter. The behavior (8) of the $(4 + 1)$ dimensional metric is easy to understand. For example, in the case of a closed universe one immediately recognizes in (8) the generalization of the Schwarzschild solution to the case of $(4 + 1)$ space-time dimensions [6]. Indeed, the metric (4) is spherically symmetric in this case and (8) describes a $(4 + 1)$ dimensional spherically-symmetric black hole. The gravitational mass $M$ of the black hole is related to the parameter $\mu$ as

$$\mu = \frac{8G_5}{9\pi} M, \quad (9)$$

where $G_5$ is the gravitational constant in $(4 + 1)$ dimensions. The horizon of the black hole is situated at

$$r_g = \sqrt{\mu}. \quad (10)$$

The Einstein equations on the surface $M_{3+1} \subset \mathcal{M}_{4+1}$ can be written in the form [7]

$$[K^j_i] - \delta^j_i [K^l_l] = 8\pi G_5 T^j_i$$

$$T^j_i = 0$$

$$K^j_i T^i_j = 0, \quad (11)$$
where $K_{ij}$ is the extrinsic curvature tensor of $M_{3+1}$. Here the semicolon denotes the covariant derivative with respect to the induced metric (2) and the square brackets $[A]$ denote the jump of a function $A(t,r,\chi,\theta,\phi)$ across the surface.

For the surface (5) we have $K^2_2 = K^3_3 = K^4_4$ and the stress-energy tensor (7) also has a simple form. Thus, the system (11) can be transformed into a more simple system of equations

$$[K^2_2] = \frac{8}{3}\pi G_5 \rho$$

$$\dot{\rho} + 3\frac{\dot{R}}{R}(\rho + p) = 0,$$  \hspace{1cm} (12)

where dot denotes differentiation with respect to “cosmological” time $d/d\tau$ (3). The $K_{22}$ component of the extrinsic curvature tensor for the surface (4) is

$$K_{22} = -\frac{\sigma}{R} \sqrt{\dot{R}^2 + e^{-\lambda}} = -\frac{\sigma}{R} \sqrt{\dot{R}^2 + k - \frac{\mu}{R^2}} \hspace{1cm} (13)$$

where $\sigma = +1$ if the transversal spatial coordinate grows in the direction of the surface normal and $\sigma = -1$ if it decreases in this direction (see [4, 8] for a stricter definition). Equations (12), (13) govern the dynamics of the scale factor of a universe which is embedded as a surface in a $(4+1)$ dimensional space-time.

Up to now we have discussed only the local structure of the solutions of Einstein equations and payed no attention to their global properties. Now our task will be to find solutions with a compact extra dimension. Let us first consider the case of a closed universe. In this case the metric outside $M_{3+1}$ is the Schwarzschild metric. Therefore, $M_{4+1}$ is a part of the $(4+1)$ dimensional Schwarzschild manifold. The global structure of the Schwarzschild space-time in $(4+1)$ dimensions is essentially the same as in $(3+1)$ dimensions. A conformal diagram for this space-time is presented on Fig. 1. We have two asymptotically flat regions with spatial infinities $i_0$ and two space-like singularities (lines $i_-i_-$ and $i_+i_+$) in the absolute past and the absolute future. The surface $M_{3+1}$ is represented by a line which starts at the past singularity, then expands to a maximal radius and then contracts to the future singularity.

A typical space-like section of this space-time has wormhole geometry: the coordinate $r$ changes from $\infty$ at the left $i_0$ to a minimal value $r_{min}$ and
then back to $\infty$ at the right $i_0$ along a space-like section. But $r$ parameterizes the “extra” dimension of “our” space-time $M_{3+1}$. Thus, if we want this extra dimension to be compact, we cannot allow $r$ to change unboundly. In order to “compactify” the extra dimension let us implement the following procedure.

First, we introduce another surface $M'_{3+1}$ on the other side of the wormhole, placed symmetrically to the surface $M_{3+1}$ with respect to the mouth of the wormhole (see Fig. 1). Next, we cut the Schwarzschild manifold along the surfaces $M_{3+1}$ and $M'_{3+1}$ so that only the region $V$ of Fig. 1 is left after this step. The induced metrics on $M_{3+1}$ and $M'_{3+1}$ are the same and we can identify $M_{3+1} \equiv M'_{3+1}$. In the obtained $(4+1)$ dimensional space-time $M_{4+1}$ the coordinate $r$ changes along a spatial section from the maximal value $r_{\text{max}} = R(\tau)$ at $M_{3+1}$ to $r = r_{\text{min}}$ and then back to the maximum. Thus, the extra dimension is compact.

The same procedure can be implemented in the case of open and flat universes. The global structure of the $(4+1)$ dimensional space-time with the metric (4), (8) in the cases $k = 0, -1$ and $\mu > 0$ is shown on Fig. 2. There exists only the past singularity $i_0i_0$. For these values of the parameters the coordinate $r$ is time-like and the coordinate $t$ parameterizes the

Figure 1: The structure of $(4+1)$ dimensional space-time with a closed universe.
“extra” dimension. The space-like section also has wormhole geometry, because $t$ can change from $-\infty$ to $+\infty$ along a space-like section. Again, we can place the two identical surfaces $M_{3+1}$ and $M'_{3+1}$ at the different sides of the wormhole, cut the asymptotic regions and glue the surfaces together in order to get a compact extra dimension. The procedure is shown on Fig. 4.

![Figure 2: The structure of (4+1) dimensional space-time with a flat or open universe.](image)

The dynamics of the surface $M_{3+1}$ in $\mathcal{M}_{4+1}$ is governed by the set of equations (12). The space-time metric in the volume $V$ is specified by the parameter $\mu$ which in the case of a closed universe is related to the mass $M$ (9) of the black hole. In order to write down the final form of equations (12) we need also to determine the quantity $\sigma$ in (13). From the definition of $\sigma$ we see that $\sigma = +1$ at the surface $M_{3+1}$ and $\sigma = -1$ at $M'_{3+1}$. The jump of the $K_{22}$ component of the extrinsic curvature on the surface $M_{3+1} = M'_{3+1} \subset \mathcal{M}_{4+1}$ is thus

$$\left[ K_{22}^2 \right] = \frac{2}{R} \sqrt{\dot{R}^2 + k - \frac{\mu}{R^2}} = \frac{8}{3} \pi G \rho.$$  \hspace{1cm} (14)

Squaring the last equation we get an equation which is analogous to the conventional Friedmann equation of the $(3 + 1)$ dimensional cosmology

$$\frac{\dot{R}^2}{R^2} + k \frac{\rho}{R^2} = \frac{16 \pi^2 G^2}{9} \rho^2 + \frac{\mu}{R^4}.$$ \hspace{1cm} (15)
The essential difference of (13) from the usual Friedmann equation is that
the energy density $\rho$ enters (15) quadratically. There is an additional con-
tribution into the r.h.s. of the Friedmann equation in our model due to the
self-interaction of the brane universe $M_{3+1}$ through the compact extra di-
mension. This contribution is similar to the contribution which would come
from radiation (with equation of state $p = \rho/3$) with energy density

$$\rho_s = \frac{3\mu}{8\pi G_4 R^4}$$  \hspace{1cm} (16)

($G_4$ is the four-dimensional gravitational constant).

The essential requirement imposed on any cosmological model with extra
dimensions is that the size $L$ of extra dimensions must be much smaller than
the parameter $R(\tau)$

$$L \ll R$$  \hspace{1cm} (17)

(in the case of a closed universe $R(\tau)$ is the size of the universe, while in
the case of flat universe we must modify this requirement and speak about
the size of the visible part of the universe). Let us estimate the values of
the parameters of our model, when (17) is satisfied. In a matter dominated
closed universe we have $p = 0, k = 1$. In this case

$$\rho = \frac{3}{4\pi} \frac{m}{R^3},$$  \hspace{1cm} (18)

where $m$ is the total mass of the universe. Taking $\dot{R} = 0$ in (15) we find that
the maximal expansion of the universe is

$$R_{max}^2 = \frac{1}{2} \left( \mu + \sqrt{\mu^2 + 4G_4^2 m^2} \right).$$  \hspace{1cm} (19)

The size of the extra dimension at the moment of maximal expansion can
be defined as the distance between the points $A$ and $B$ of Fig. 1 along the
spatial section $t = 0$ of the Schwarzschild space-time

$$L_{max} = \int_A^B \frac{dr}{\sqrt{1 - \mu/r^2}} = 2\sqrt{R_{max}^2 - \mu}.$$  \hspace{1cm} (20)

We see that the size of the extra dimension is small compared to the size of
the universe if

$$R_{max} \approx r_g = \sqrt{\mu},$$  \hspace{1cm} (21)
i.e. if the maximal scale factor is close to the gravitational radius of the 
(4 + 1) dimensional black hole. From (19) we find that this is true when

\[ M \gg m \tag{22} \]

that is the gravitational mass (1) of the universe must be much greater than its bare mass (18).

But when (22) is true, the second term in the r.h.s. of Friedmann equation (15) dominates over the first one. Neglecting the term proportional to \( \rho^2 \) in zero approximation we get the following law of expansion

\[ R_l(\tau) = \sqrt{2\sqrt{\mu} \tau - \tau^2}. \tag{23} \]

We have added the subscript \( l \) to \( R(\tau) \) because the dependence (23) is exactly the dependence of the radius on proper time along the line \( l \) (the mouth of the wormhole) of Fig. [1]. In zero approximation the trajectories of both \( M_{3+1} \) and \( M'_{3+1} \) follow the line \( l \), and therefore the size of the extra dimension is equal to zero.

The size of the extra dimension is not constant in our model. When it is small, we can implement the following procedure in order to define \( L(\tau) \). Let us introduce a locally Minkowsky reference frame which is comoving to the brane \( M_{3+1} \) at some moment of time \( \tau_0 \). In this frame we can use the usual nonrelativistic definition of spatial distances (and therefore of the “size” of the extra dimension) in the vicinity of the brane. Since both \( M_{3+1} \) and \( M'_{3+1} \) move close to the line \( l \) of Fig. [1], we can use a locally Minkowsky frame comoving to \( l \) instead of the frame comoving to \( M_{3+1} \).

In order to find a comoving frame at a given point \( \tau_0 \) of \( l \) we first introduce the analog of the Kruskal coordinate system in the white hole region of the (4 + 1) dimensional Schwarzschild space-time

\[
\begin{align*}
U &= -e^{-u/\sqrt{\mu}}, \\
V &= -e^{v/\sqrt{\mu}},
\end{align*}
\tag{24}
\]

where

\[
\begin{align*}
u &= t - r^* \\
v &= t + r^*
\end{align*}
\tag{25}
\]

\[
r^* = r + \frac{\sqrt{\mu}}{2} \ln \frac{\sqrt{\mu} - r}{\sqrt{\mu} + r}.
\]
In these coordinates the metric (4) takes the form
\[ ds^2 = -\frac{\mu(r + \sqrt{\mu})^2}{r^2 e^{2r/\sqrt{\mu}}}dUdV - r^2 \left( d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2) \right). \] (26)

The line \( l \) is defined by the equation \( U - V = 0 \). The coordinate frame
\[
\begin{cases}
T = e^{-R_l(\tau_0)/\sqrt{\mu}}\frac{\sqrt{\mu}(R_l(\tau_0) + \sqrt{\mu})}{2R_l(\tau_0)}(U + V) \\
X = e^{-R_l(\tau_0)/\sqrt{\mu}}\frac{\sqrt{\mu}(R_l(\tau_0) + \sqrt{\mu})}{2R_l(\tau_0)}(U - V)
\end{cases}
\] (27)
is a comoving locally Minkowsky frame at the moment of time \( \tau_0 \). If the trajectory of the surface \( M_{3+1} \) in this locally Minkowsky frame is \((T(\tau), X(\tau))\), the size of the extra dimension is
\[ L(\tau_0) = 2X(\tau_0). \] (28)

The coordinate \( \tilde{X} = (U - V)/2 \) changes along the brane trajectory according to the equation
\[ \frac{d\tilde{X}}{d\tau} = \frac{1}{2\sqrt{\mu}} \left( (U(\tau) - V(\tau))\frac{dr^*}{d\tau} - (U(\tau) + V(\tau))\frac{dt}{d\tau} \right) \] (29)
(see (24), (25)). From (15) we find that in first approximation in the small parameter \( m/M \)
\[ \frac{d\tilde{X}}{d\tau} + \frac{R_l}{2\sqrt{\mu}(\mu - R_l^2)^{1/2}}\tilde{X} = \frac{G_5me^{R_l/\sqrt{\mu}}}{\sqrt{\mu}(\sqrt{\mu} + R_l)^{3/2}(\sqrt{\mu} - R_l)^{1/2}}, \] (30)
where \( R_l \) is defined in (23). The last equation can be integrated
\[ \tilde{X}(R_l) = \frac{G_5m(\sqrt{\mu} - \sqrt{\mu - R_l^2})e^{R_l/\sqrt{\mu}}}{\mu(\sqrt{\mu} + R_l)} \] (31)
and from (27), (28) we find that the size \( L \) of the extra dimension changes as
\[ L = \frac{2G_5m(\sqrt{\mu} - \sqrt{\mu - R_l^2})}{R_l\sqrt{\mu}} \] (32)
It is equal to zero at the moment of the big bang $\tau = 0$, then grows up to the maximal value (20) at the moment of maximal expansion and then contracts back to zero at the moment of the big crunch.

According to (1) the variation of the size of the extra dimension leads to a variation of the effective 4-dimensional gravitational constant in time

$$G_4 \sim \frac{R_t \sqrt{\mu}}{m(\sqrt{\mu} - \sqrt{\mu - R_t^2})}$$

It decreases when the universe expands.

The analogous analysis can be done in the cases of flat and open universes. In zero approximation (when the size of the extra dimension is equal to zero) the surface $M_{3+1} = M'_{3+1}$ follows the line $l : \{ t = 0 \}$ of Fig. 4. The scale factor changes with time as

$$R_t = \begin{cases} \sqrt{2\sqrt{\mu} \tau}, & k = 0 \\ \sqrt{2\sqrt{\mu} \tau + \tau^2}, & k = -1. \end{cases}$$

When the parameter $m/M$ is not equal to zero, but the energy of self-interaction of the universe through the compact extra dimension still dominates over all other kinds of energy located at the surface $M_{3+1}$, the size of the extra dimension depends on the scale factor of the universe as

$$L(R_t) = \frac{2G_5 m}{\sqrt{\mu}} \begin{cases} R_t/(2\sqrt{\mu}), & k = 0 \\ (\sqrt{\mu + R_t^2} - \sqrt{\mu})/R_t, & k = -1. \end{cases}$$

it grows with time. But, in the case of an open universe it reaches asymptotically the finite value

$$L_f = \frac{2G_5 m}{\sqrt{\mu}},$$

while in the case of a flat universe it grows unboundly.

To summarize, the gravitational self-interaction of the brane universe through the additional compact dimensions must be a general feature of higher-dimensional cosmological models. This strong self-interaction could explain why the size of the extra dimensions is very small compared to the size of the (visible part of the) universe. The “energy density” (16) of this self-interaction gives an essential contribution to the Friedmann equation (15) which governs the dynamics of the scale factor of the universe.
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2 Appendix.

In this Appendix we find solutions of the vacuum Einstein equations in (4+1) dimensions. If we take the space-time metric in the form (4) the components of the Ricci tensor are

\[ R_{00} = \frac{1}{4} (\dot{\lambda} \dot{\nu} - \dot{\lambda}^2 - 2\ddot{\lambda}) + \frac{1}{4} (\nu'^2 - \nu' \lambda' + 2\nu'') e^{\nu - \lambda} + \frac{3\nu'}{2r} e^{\nu - \lambda} \]
\[ R_{11} = \frac{1}{4} (\dot{\lambda}' \dot{\nu}' - \dot{\nu}'^2 - 2\nu'') + \frac{1}{4} (\ddot{\lambda} - \ddot{\nu} + 2\ddot{\lambda}) e^{\lambda - \nu} + \frac{3\lambda'}{2r} \]
\[ R_{01} = \frac{3\dot{\lambda}}{2r}; \quad R_{22} = \frac{1}{2} r e^{-\lambda} (\lambda' - \nu') - 2e^{-\lambda} + 2k \]
\[ R_{33} = f R_{22}; \quad R_{44} = f \sin^2 \theta R_{22} \]

where \( k \) is defined right after equation (8). The relevant components of the Einstein equations which define the functions \( \lambda, \nu \) in (4) are

\[ \left( \frac{\lambda'}{2r} - \frac{1}{r^2} \right) e^{-\lambda} + \frac{k}{r^2} = T^0_0 \] \hspace{1cm} (37)
\[ -\left( \frac{\nu'}{2r} + \frac{1}{r^2} \right) e^{-\lambda} + \frac{k}{r^2} = T^1_1. \] \hspace{1cm} (38)

Taking \( T^\beta_\alpha = 0 \) one finds that for the vacuum solutions of Einstein equations

\[ e^\nu = e^{-\lambda} = k - \frac{\mu}{r^2}. \] \hspace{1cm} (39)

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