Distortional Hardening Behavior and Strength Different Effect of Pure Titanium Grade 1 Sheets: Experimental Observation and Constitutive Modeling

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Abstract. This study presents a systematic methodology that used to identify the subsequent yield surfaces of a pure titanium grade 1 sheet at different levels of equivalent plastic work. Several experimental tests including uniaxial tensile tests (UT), hydraulic bulge test (BT), simple shear tests (SS), and uniaxial compressive tests (UC) have been conducted for samples prepared in different orientations to achieve a comprehensive experimental data of yielding behaviors observed in different forming modes. Under the condition of equivalent plastic work, the yielding behaviors are characterized and normalized to clarify the distortional hardening behavior and strength different effect for the tested material. It is seen that the yielding surface of the tested material distorts largely during plastic deformations and approaches to its final shape at an equivalent plastic work value of 40 MPa. Under the plane-stress assumption, experimental data obtained from on-axis tests (UT, BT, UC) are used to calibrate three constitutive models: Yld2k, CPB06, and CB04. Finite element analyses for a simple shear test have been performed in ABAQUS/explicit software to validate the developed material models. In conclusion, the CPB06 model provides the best prediction for plastic yielding behaviors of the tested material.

1. Introduction

Pure titanium Grade 1 sheet material is more and more utilized in industrial to make plate type heat-exchanger owing to its high heat-transfer rate, high formability during press forming, and excellent corrosion resistance. Recent efforts have been made to provide better understanding on material behaviors when they are subjected to press forming processes, which are believed to reduce the experimental cost for designing and manufacturing sheet metal products as well as plate heat exchangers [1].

Due to their hexagonal close packed structure, yielding responses of commercially pure Titanium (CP Ti) sheets subjected to tensile and compressive loads are different. The difference between yield stresses in tension and compression of sheet materials is conventionally called strength different (SD) effects. Additionally, the yielding surface of the tested material is found to be distorted under external loads. As consequences, the SD effects of CP Ti sheet are gradually changed during plastic deformation [2].
This study investigates the relation between the distortional hardening behavior and the SD effect of CP Ti sheets subjected plastic deformation. To this goal, a series of standard testing methods have been conducted to observe material responses in different forming modes. Based on these observations, various symmetric and asymmetric yield functions including Yld2k [3], CP04 [4], and CPB06 [5] have been applied to describe the subsequent yielding behaviors of the examined material. These models are then adopted in Abaqus/Explicit package to simulate several forming processes as validations.

2. Experimental procedures

The tested material in this study is pure titanium Grade 1 sheets with a thickness of 0.5 mm which were produced at POSCO, Korea with 600 mm in width and length. To characterize the in-plane anisotropy of the tested material, a series of uniaxial tensile (UT) tests was conducted in the rolling direction (RD), diagonal direction (DD), and transverse direction (TD). In addition, the SD effect was characterized by performing uniaxial compressive (UC) tests along the rolling and transverse directions. Fig. 1 shows the geometry of UT and UC specimens used in this study. All these tests were conducted at a deformation speed of 1 mm/min to achieve material’s responses under an assumption of quasi-static deformation.

Material characterization under balanced-biaxial tension mode was estimated through hydraulic bulge tests. These tests also provide information of stress-strain relationship over large strain ranges. Fig. 2 shows experimentally measured stress-strain curves obtained from these tests.

Hence, these curves are converted into an effective stress-strain curve under the theory of equivalent plastic work. To this goal, each data point is converted into a corresponding effective data point [6] as follows

\[
\sigma_e = \frac{\sigma_i}{k_i} \quad (1a)
\]

\[
\varepsilon_e = \frac{\varepsilon_i}{k_i} \quad (1b)
\]

In these equations, subscript \(i\) denotes the considering forming mode, i.e. RD-UT, TD-UT, RD-UC, TD-UC, etc; \(k_i\) denotes the corresponding convert factor. It is noted that the stress-strain curve obtained from the UT-RD test was used as the reference stress-strain curve to reproduce the hardening behavior of the tested material. Therefore, its corresponding convert factor always is the unit, as shown in Fig. 3.

It is seen from this figure that the variations of these convert factors according to the increment of the equivalent plastic work are significant and un-ignorable. That further result in the distortion of the yield surface, as reported in our previous work [7]. According to Fig. 3, these convert factors seem to be gradually decreased when plastic deformations are generated and finally approach to their saturated values in large-deformation stages.
3. Constitutive modeling

The hardening law is one of the essentials for simulating sheet metal forming processes with finite element method (FEM). To identify the hardening law of the tested material, the Kim-Tuan hardening model is adopted to fit with experimental stress-strain data obtained from UT in RD. This hardening model is able to provide good estimation for stress-strain relationship in large strain ranges [8-9].

In order to describe the elastoplastic behavior of the tested material, three constitutive material models are developed under an assumption of associated flow rule. These models include Yld2000-2d symmetric function (Yld2k) [3], Cazacu-Barlat 2004 asymmetric function (CB04) [4], and Cazacu-Plunkett-Barlat 2006 function (CPB06) [5]. Details on each model can be found in references. The formulation of Yld2000-2d model can be expressed in following equation:

\[
\text{Yld2k: } \bar{\sigma} = \left\{ \frac{1}{2} \left[ |X'_1 - X'_2|^m + |X''_1 - X''_2|^m + |X''_2 - X''_1|^m \right] \right\}^{1/m},
\]

In this equation, \(X'_{1,2}\) are principals of two tensor transformations: \(X' = L'.\sigma\) and \(X'' = L''.\sigma\); \(L'\) and \(L''\) are two transform operators, which include eight parameters i.e. \(\alpha_1 \sim \alpha_8\). Using a tensor transformation approach, Cazacut et al. [5] estimated the equivalent stress in the following formula:

\[
\text{CPB06: } \bar{\sigma} = A \left[ (|S_1| - kS_1)^a + (|S_2| - kS_2)^a + (|S_3| - kS_1)^a \right]^{1/a},
\]

where \(A\) is a constant used to match the calculated stress-strain curve in RD_UT with the referenced data; \(S_{1,2,3}\) are principals of the transformed tensor \(S = L.\sigma\); \(L\) is the transform operator including seven parameters i.e. \(c_1 \sim c_7\); \(k\) is the parameter describing the SD effect.

Using different approach, the equivalent stress in the CB04 model can be calculated as follows

\[
\text{CB04: } \bar{\sigma} = \tilde{J}_{2,3}^{3/2} - c\tilde{J}_3
\]

where \(\tilde{J}_{2,3}\) are anisotropic forms of stress invariants \(J_{2,3}\) which include ten parameters i.e. \(a_1 \sim a_{10}\); \(c\) is a parameters for describing the SD effect.
A Matlab code is developed to identify values of parameters of each model following a strategy presented in the work of Pham et al. [7]. In this approach, value of parameters in Eqs. (2-4) should be determined at several discrete levels of equivalent plastic work ($W_p$). Hence, their variations according to increment of $W_p$ reproduce the distortion of the calculated yield surface during plastic deformations.

**Figure 4.** Evolution of yield loci predicted by different models

**Figure 5.** Evolution of normalized yield stress.

Fig. 5 shows predictions of yield loci at several levels of $W_p$ based on different models. All of three models present good prediction for experimental data points. Due to its formula, Yld2k model provides symmetric yield loci, which are unable to capture the SD effect of the examined material. Unlike the Yld2k model, CPB06 and CB04 models predict plat loci in the biaxial-compressive regions. In addition, Fig. 5 compares the predictions of these models for the normalized yield stresses obtained from UT tests with experimental data. All of these models capture well the yield stresses in RD, DD, and TD. The predictions of Yld2k and CPB06 models are almost similar. However, the predictions of CB04 model are largely different from the others. Their predictions of yield loci and normalized yield stresses may affect on the simulated results with finite element method.

**4. Numerical verification**

This section provides numerical verification for the developed constitutive models. To this goal, simulations of a simple shear test in RD have been performed in Abaqus/Explicit package through VUMAT user’s subroutines. Fig. 6 shows an image of un-deformed mini-shear specimen [10] and full mesh on the numerical specimen. In this study, shell element (S4R) with 9 integration points through thickness was adopted to model the specimen.

During these simulations, loading forces according to the displacement of an initial gauge length ($L_0 = 8$ mm) were summarized and reported in Fig. 7. As seen in this figure, Yld2k and CB04 models seem to underestimate the measured loading forces. The observation is relevant to their predictions of yield loci shown in Fig. 4 since these models estimated lower yield stresses than the measured data of TD-UC and SS specimens.

Prediction of the CPB06 model shows a good agreement with the measured loading forces in the test. In this manner, the CPB06 model provides the best proper constitutive modeling for the tested material. However, simulations for other testing methods, for example, in-plane plane-strain tension or punch-stretching tests should be performed to validate its accuracy.
5. Conclusions
This study presents experimental observations of distortional hardening behavior and strength difference effect in a pure titanium grade 1 sheet by conducting a series of standard testing methods such as uniaxial tensile tests, uniaxial compressive tests, bulge tests, simple shear tests. Based on the observations, three different constitutive models have been developed to describe material’s responses during the tests. The developed material models are implemented in Abaqus/Explicit to simulate a simple shear test to verify their accuracy. It is found that the CPB06 model provides the best prediction for loading force during the test. Further studies are needed to validate the applications of the developed models in simulating real forming processes for the tested material.

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References
[1] Hama T, Kobuki A and Takuda H 2017 Int. J. Plast. 91 77-108
[2] Okulov I, Kuhn U, Romberg J, Soldatov I, Freudenberger J, Schultz L, Eschke A, Oertel C G, Skrotzki W, and Eckert J 2014 Mat. Design 62 14–20
[3] Barlat F, Brem J C, Yoon J W, Chung K, Dick R E, Lege D J, Pourboghrat F, Choi S H and Chu E 2003 Int. J. Plast. 19(9) 1297–1319
[4] Cazacu O and Barlat F 2004 Int. J. Plast. 20(11) 2017–45
[5] Cazacu O, Plunkett B and Barlat F 2006 Int. J. Plast. 22(7) 1171–94
[6] Kuwabara T, Mori T, Assano M, Hakoyama T and Barlat F 2017 Int. J. Plast. 97 164-186
[7] Pham Q T, Lee M G and Kim Y S 2019 Int. J. Mech. Sci. 10 90–102
[8] Pham Q T and Kim Y S 2016 Key Eng. Mat. 716 87–98
[9] Pham Q T, Lee M G and Kim Y S 2018 IOP Conf. Series: Journal of Physics: Conf. Series 1063 012024
[10] Abedini A, Butcher C and Worswick M J 2017 Experimental Mechanics 57 75-88