Digital simulation of non-Abelian anyons with 68 programmable superconducting qubits

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Non-Abelian anyons are exotic quasiparticle excitations hosted by certain topological phases of matter [1, 2]. They break the fermion-boson dichotomy and obey non-Abelian braiding statistics [3–5]: their interchanges yield unitary operations, rather than merely a phase factor, in a space spanned by topologically degenerate wavefunctions. They are the building blocks of topological quantum computing [1, 6–9]. However, experimental observation of non-Abelian anyons and their characterizing braiding statistics have remained elusive hitherto, in spite of various theoretical proposals [10–21]. Here, we report such an observation with up to 68 programmable superconducting qubits arranged on a two-dimensional lattice. By implementing the toric-code model with twists through quantum circuits, we demonstrate that twists exchange electric and magnetic charges and behave as a particular type of non-Abelian anyons—the Ising anyons. In particular, we show experimentally that these anyons follow the fusion rules and non-Abelian braiding statistics of the Ising type, and can indeed be used to encode topological logical qubits. Furthermore, we demonstrate how to implement both single- and two-qubit logical gates through applying a sequence of elementary Pauli gates on the underlying physical qubits and then measuring some nonlocal observables. Our results establish a versatile digital approach to realize and manipulate non-Abelian anyons, thus paving an alternative way to topological quantum computing.

Quantum theory classifies all fundamental particles in nature as either bosons or fermions [22]. For instance, photons are bosons and electrons are fermions. This dichotomy classification has profound implications and plays a crucial role in understanding a variety of physical phenomena, ranging from metal-insulator transitions [23] to superconductivity [24] and Bose-Einstein condensation [25]. However, in two dimensions it is possible that emergent particles (quasiparticles) would circumvent this dichotomy principle and obey anyonic statistics [26], where their exchange of positions would result in a generic phase factor that is neither 2π nor π (as for bosons or fermions), or even a unitary operation that shifts the system between different topologically degenerate states [1]. These quasiparticles are dubbed anyons [3].

While braiding Abelian anyons only leads to a phase factor, braiding non-Abelian anyons would lead to a unitary transformation [1–5]. This intriguing property gives rise to the notion of topological quantum computation [1, 6–9], where quantum information is encoded nonlocally and quantum computations are implemented by braiding and fusing non-Abelian anyons. The nonlocal encoding and the topological nature of braiding make topological quantum computation naturally immune to local errors, thus providing intrinsic fault tolerance at the level of hardware. However, despite numerous theoretical proposals for realizing non-Abelian anyons with a wide range of systems [10–21], including fractional quantum Hall states [4, 18], cold atoms [27], topological superconductors [17], and Majorana zero modes [16], the direct experimental observation of non-Abelian anyons and their braiding statistics still remains elusive so far [28–31]. Recent developments with quantum processors have demonstrated vast potential in simulating exotic phases of matter [32–36] and demonstrating topological quantum error-correcting codes [37–40], giving rise to unprecedented opportunities for realizing and manipulating non-Abelian anyons with these highly controllable systems.

In this paper, we report the observation of non-Abelian anyons with up to 68 programmable superconducting transmon qubits featuring median lifetimes of 109.8 µs, and median fidelities of simultaneous single- and two-qubit gates above 99.91% and 99.4%, respectively (Fig. 1). In particular, through efficient quantum circuits we successfully prepare the twisted toric-code ground states that are predicted to hosts non-Abelian anyons. We demonstrate that the twists exchanges electric and magnetic charges when an odd number of them are winded around, and their braiding and fusion rules mimic exactly as that of Ising-type anyons. We show that these twists can be explored to encode topological logical qubits and both single- and two-qubit logical gates can be implemented by applying a sequence of elementary Pauli gates on the physical superconducting qubits and then measuring the corresponding nonlocal observables. Based on this, we successfully prepare a logical two-qubit Bell state with a fidelity above 0.84. Our results provide smoking-gun evidence for the long-sought-after observation of non-Abelian anyons, thus marking a crucial step towards topological quantum computing.

**Framework and experimental setups**

We consider the toric-code model with qubits living on the
vertexes of a square lattice described by the following Hamiltonian [6, 41]:

$$H = -\sum_k A_k, \quad A_k = X_k Z_{k+i} Z_{k+i+j} X_{k+i+j}. \quad (1)$$

Here $k = (a, b)$ indexes the spins in the $a$-th row and $b$-th column and $i = (1, 0), j = (0, 1)$. $X$, $Y$, and $Z$ are Pauli operators. Noting that all the plaquette operators $A_k$ commute with each other, the ground state of this Hamiltonian can be simply described by the condition $\langle A_k \rangle = 1$ for all $k$. There are two types of quasiparticle excitations (corresponding to $\langle A_k \rangle = -1$), dubbed $e$ anyons (or “electric charges”) living at the dark plaquettes and $m$ anyons (or “magnetic charges”) living at the light plaquettes, respectively (see Fig. 1a). These are Abelian anyons [6, 41], and they can be created and moved by string operators, which are products of sequences of Pauli operators for simplicity and better illustration. c, Four Majorana operators are used to describe the four Ising anyons and the corresponding conceptual figure. The fusion and braiding of Ising anyons can be conveniently implemented by these operators [13]. d, A photograph of the superconducting quantum processor I with the chosen 68 qubits can be experimentally highlighted in cyan.

To obtain non-Abelian anyons, we consider introducing dislocations in the lattice [13, 42]. For instance, in Fig. 1a we deform the lattice so as to obtain two pairs of pentagonal plaquettes, and modify the Hamiltonian with new pentagonal plaquette operators accordingly. After the deformation, each pentagonal plaquette hosts a twist, which involves a lattice site that is shared by three (rather than four) neighboring plaquettes. As shown in Fig. 1c, due to the dislocation a string winding around a twist cannot close, and consequently an $e$ anyon winds around a twist will become an $m$ anyon or vice versa. Theoretically, it has been predicted that twists behave as non-Abelian Ising anyons when braided and fused [13]. Throughout this paper, we will use twist and Ising anyons interchangeably.

Our experiments are performed on two flip-chip superconducting quantum processors labelled as I and II [43]. Processor I (II) encapsulates $11 \times 11$ ($6 \times 6$) frequency-tunable transmon qubits arranged in a square lattice, with tunable couplers...
FIG. 2. The processor I and demonstration of the fusion rules. a, Layout of the processor and the measured stabilizer values of the deformed toric-code ground state. The purple dots represent the chosen qubits used in our experiment, whereas the grey (white) dots denote the functional but unused (non-functional) qubits. The black line connecting two adjacent qubits shows where a controlled-Z (CZ) gate can be implemented. The dark (light) grey plaquettes host \(e\) (\(m\)) anyons, and the brown blocks denote the natural defects explored in our experiment to implement twists. The stabilizer for the smallest squares reads \(A_k = X_k Z_{k+1} Z_{k+1} X_{k+1}\), and that for each defect is the product of all \(A_k\) it encloses. The three pairs of crosses connected by dashed lines denote the six twists used to demonstrate non-Abelian anyons. The integer number in each plaquette shows the corresponding measured stabilizer value (in percentage) of the prepared deformed toric-code state.

b, The sketch of the defects with eight twists. In the experiment, we create a pair of twists. The stabilizer for the smallest squares reads \(A_k = X_k Z_{k+1} Z_{k+1} X_{k+1}\), and that for each defect is the product of all \(A_k\) it encloses. The three pairs of crosses connected by dashed lines denote the six twists used to demonstrate non-Abelian anyons. The integer number in each plaquette shows the corresponding measured stabilizer value (in percentage) of the prepared deformed toric-code state.

c, Demonstration of the fundamental fusion rules. We initialize the system to be a ground state with the generalized charges being \(i\) for the chosen six twists marked in a (\(S_\sigma = i\), corresponding to six \(\sigma_+\) anyons, see Methods). We then create pairs of \(e\), \(m\), and \(\epsilon\) quasiparticles, and measure the stabilizer values corresponding to the defects, which specify the fusion results. The integer numbers in the defects show the measured stabilizer values (in percentage), respectively. In the lower left panel, the table indicates the Pauli operations used to generate the quasiparticles.

connecting adjacent qubits. Each qubit capacitively couples to its own readout resonator for qubit state measurement. We select 68 (20) qubits on processor I (II) to observe non-Abelian anyons and their associated braiding statistics. Through optimizing device fabrication and controlling process, we push the median lifetime of the qubits on processor I (II) to 109.8 \(\mu\)s (137.5 \(\mu\)s) and the median simultaneous single- and two-qubit gate fidelities greater than 99.91% and 99.4% (99.93% and 99.5%), respectively. The chosen 20 qubits on the processor II form a regular rectangular lattice and the desired twists are artificially created to some extent. Whereas, the 68 qubits on the processor I are more sporadically distributed due to limited capacities in both wirings of our dilution refrigerator and measurement electronics, and twists can be constructed by taking advantages of the imperfect geometry. In the main text, we mainly discuss the results obtained from the processor I (Fig. 1d), so as to stress that our approach for observing non-Abelian anyons bears the merit of generally applicable to quantum processors with unintended imperfections. The experiments carried out on the processor II follow a similar protocol, and we present the corresponding results in the extended data figures for comparison.

Topological charges and fusion rules

We first demonstrate the creation of both Abelian and non-Abelian anyons and the fundamental fusion rules. Based on the geometric structure for the chosen 68 qubits on processor I and the available connections between them, we use a quantum circuit with a circuit depth of 43 (containing 294 single-qubit rotations and 113 two-qubit controlled-Z gates, see Extended Data Fig. 6) to prepare the ground state of the
FIG. 3. Encoding of logical qubits with anyons and demonstration of logical gates. a, The encoding scheme described with fusion trees [1]. Here, for succinctness we only show part of the encoded logical basis $\ket{0}$ and $\ket{\bar{0}}$ (Supplementary Information I.B). b, On the left shows the braiding sequence corresponding to the process of $XHZH \ket{0} = \ket{\bar{0}}$. Time flows from up to down. The top middle panel sketch the defects and the three pairs of twists (denoted by $\sigma_i, i = 1, 2, 3, 4, 5, 6$). The black solid circle denotes the string operator to be measured as the logical $Z$ observable (corresponding to the Majorana correlator $-\sigma_1 \sigma_2$) for the first logical qubit. Other panels in the middle column outline the measured string operators after the application of the sequential logical gates in the encoding space. The right column shows the logical states at each stage and the corresponding measured Majorana correlators (shaded values with error bars), with the unshaded indices (0, ±1) indicating the corresponding ideal theoretical values. The two black dashed circles sketch the string operators applied to the physical qubits, which correspond to the braidings shown in b and implement the logical $Z$ and $\bar{X}$ gates applied on the first logical qubit. c, The braiding sequence corresponding to applying two logical controlled-$X$ (CX) gates consecutively on states $\ket{00}$ or $\ket{\bar{1}0}$. Time flows from left to right. We measure the corresponding Majorana correlators at each step and show their measured values in the lower panel.

corresponding stabilizer Hamiltonian (Methods and Supplementary Information I.C). In Fig. 2a, we sketch the geometric structure of the chosen 68 qubits and plot the individual stabilizer values measured after the ground state preparation. We achieve an average stabilizer value of 0.73±0.04, which is notable given the fact that certain stabilizers involve multi-qubit (up to 14 qubits) measurements. The deviation between the ideal theoretical prediction and experimental result is mainly attributed to limited gate fidelity and coherence time. All stabilizer values are positive, implying that there are no anyon excitations for the ground states. See also Extended Data Fig. 2 for experimental results from processor II, where an average stabilizer value of 0.946 ± 0.011 is obtained.

With twists, the system can host six generalized topological charges [13]: $1, e, m, \epsilon, \sigma_+, \text{ and } \sigma_-$. They obey the following fundamental fusion rules that can be verified using proper string operators

$$
\sigma_\pm \times e = \sigma_\pm \times m = \sigma_\mp, \quad \sigma_\pm \times \epsilon = \sigma_\pm, \\
\sigma_\pm \times \sigma_\mp = 1 + \epsilon, \quad \sigma_\pm \times \sigma_\mp = e + m.
$$

(2)

In our experiments, we demonstrate only some of the fusion rules for simplicity and concreteness. The remaining fusion rules either follow trivially or can be demonstrated in a similar way. After preparing the ground state of the Hamiltonian with twists, we create a pair of $e$ anyons and move one of them to wind around the twists located at the bottom of the $D_2$ defect (Fig. 2b), through applying corresponding string operators on the relevant qubits. We measure the four stabilizers (labeled by $A_{P_1}, A_{P_2}, A_{P_3}, \text{ and } A_{P_4}$, respectively) below $D_2$ at each step of this process and plot their values in the right panel of Fig. 2b. It is clear that, at the beginning, all measured stabilizer values are positive, indicating that there is no anyon excitation in the system. After the creation of a pair of $e$ anyons, $A_{P_1}$ becomes negative, which indicates that there is an excitation at plaquette $P_1$ (an $e$ anyon). This $e$ anyon is then moved around the first twist and becomes an $m$ anyon, which is confirmed in the experiment by the observation that $A_{P_2}$ becomes negative whereas $A_{P_1}$ changes back to be positive at step 2. The $m$ anyon is further moved around the second twist and becomes an $e$ anyon, as confirmed by the measured stabilizer values at step 3. At step 4, we further move the $e$ anyon around the third twist and it becomes an $m$ anyon again. This clearly shows that winding around an odd number of twists exchanges electric and magnetic charges. See also
Extended Data Fig. 3 for experimental results from processor II, where an \( \epsilon \) anyon winding around two twists will remain as an \( \epsilon \) anyon is demonstrated as well.

In Fig. 2c, we plot the measured results for \( A_{D_1}, A_{D_2}, \) and \( A_{D_3} \) (in the experiment, we have in fact measured all the stabilizers. Here, for better illustration we only plot three of them that are most relevant for the discussion), which determines the topological charges for the relevant twists marked as crosses in Fig. 2a and hence verifies the fusion rules. At the beginning, we prepare the system to the ground state of the stabilizer Hamiltonian, where all three measured stabilizer values are positive as shown in Fig. 2a. In other words, the system possesses six \( \sigma_+ \) anyons at the beginning. We then create a pair of \( \epsilon \) anyons and a pair of \( m \) anyons by applying the \( X \) gate on both qubits at sites \((9,3)\) and \((11,3)\) respectively, as depicted in the left upper panel of Fig. 2c. We measure the stabilizers \( A_{D_1}, A_{D_2}, \) and \( A_{D_3} \), and find \( \langle A_{D_1} \rangle = -0.265 \pm 0.015, \langle A_{D_2} \rangle = -0.629 \pm 0.013, \) and \( \langle A_{D_3} \rangle = -0.496 \pm 0.027, \) consistent with two \( \sigma_- \) anyons at the same positions where the original \( \sigma_+ \) anyons live. This verifies the fusion rules \( \sigma_+ \times \epsilon = \sigma_+ \times m = \sigma_- \).

We further create three pairs of \( m \) anyons and one pair of \( \epsilon \) anyons, with their locations depicted in the right lower panel of Fig. 2c. We find that \( \langle A_{D_1} \rangle \) and \( \langle A_{D_2} \rangle \) remain negative \((\langle A_{D_3} \rangle = -0.281 \pm 0.012 \) and \( \langle A_{D_3} \rangle = -0.620 \pm 0.005)\), but \( \langle A_{D_3} \rangle = 0.510 \pm 0.017 \) becomes positive. This demonstrates the fusion rules of \( \sigma_- \times m = \sigma_+ \) and \( \sigma_- \times \epsilon = \sigma_- \). The demonstrated rule \( \sigma_- \times \epsilon = \sigma_- \) implies that adding an \( \epsilon \) quasi-particle to \( \sigma_- \) will not change the total topological charge, which reflects the fact that twists can act as sources and sinks for \( \epsilon \) anyons. To study the braiding and fusion of two \( \sigma_+ \) anyons, we define Majorana operators as string operators winding around twists with the same end points, as illustrated in Fig. 1c (see Methods and Supplementary Information LB and LD). We measure the corresponding string operators and find that \( \langle ic_1c_2 \rangle \) can both be negative and positive as shown in Fig. 3a, which verifies the nontrivial fusion rule \( \sigma_+ \times \sigma_+ = 1 + \epsilon \) for \( \sigma_+ \) anyons. See also Extended Data Fig. 4 for experimental results from processor II, where more fusion rules are demonstrated in a similar fashion.

**Braidings and logical gates**

Braiding of non-Abelian anyons results in unitary operations in a space spanned by topologically degenerate states. This is the characteristic feature of non-Abelian anyons. As discussed above, we consider six \( \sigma_+ \) anyons hosted by the six twists marked in Fig. 2a. For simplicity and convenience, we omit the symbol “4" and label them by \( \sigma_i \) \((i = 1, \cdots, 6)\), as depicted in Fig. 3a and Fig. 3b. In general, one needs to move the twists to braid the \( \sigma \) anyons, which can be implemented by either adiabatically transforming the “geometry" of the Hamiltonian or through code deformations [44–46]. Here, we implement braidings of the \( \sigma \) anyons by exploring the corresponding Majorana operators (see Methods and Supplementary Information I-B).

In Fig. 3, we illustrate how frequently-used single- and two-qubit gates can be carried out on the encoded logical qubits. Given that the total topological charge is 1, the six \( \sigma \) anyons can be used to encode two logical qubits. In Fig. 3a, we illustrate the encoding scheme used in our experiment. We first show how single-qubit gates can be implemented for the first logical qubit, through braiding the first three \( \sigma \) anyons. To this end, in Fig. 3b we show the braiding sequence to implement the unitary circuit \( XHZH\bar{Z}\bar{H}\langle 0 \rangle \) and our experimental results on measuring the relevant Majorana correlations after the application of each gate. We prepare the system in the logical \( \langle 0 \rangle \) state. We measure the Majorana correlation \( \langle ic_1c_2 \rangle \), which corresponds to measuring the fusion charge of \( \sigma_1 \) and \( \sigma_2 \). We find \( \langle ic_1c_2 \rangle = 0.741 \pm 0.010 \), which is consistent with the fact that the system is in the logical \( \langle 0 \rangle \) state initially. A Hadamard gate on the first logical qubit can be applied by sequential braidings of \( \sigma_1 \) and \( \sigma_2 \), which corresponds to measuring the fusion charge of \( \sigma_1 \) and \( \sigma_2 \). We find \( \langle ic_1c_2 \rangle = 0.005 \pm 0.002 \), which agrees well with the theoretical prediction of \( \langle ic_1c_2 \rangle = 0 \). The logical \( \bar{Z} \) gate and \( \bar{X} \) gate can be carried out by applying the string operators \( -ic_2c_3 \) and \( -ic_1c_2 \) on the physical qubits, respectively (Methods and Supplementary Information I.E). See also Extended Data Fig. 5 for experimental results from processor II, where the corresponding Majorana operators are explicitly shown.

The two-qubit logical controlled-X gate (\( \bar{C}X \)) can be im-

**FIG. 4. Entangled logical state. a, The sketch of the three pairs of twists and the string operators corresponding to the logical \( \bar{Z}_1, \bar{Z}_2, X_1, \) and \( X_2 \) operations (see Supplementary Information I.E). The encoding scheme is the same as in Fig. 3a. b, Expectation values of \( XX, YY, \) and \( ZZ \) for the entangled state of anyon-encoded logical qubits. The overlap between the prepared state \( \rho_{\text{opt}} \) with the ideal Bell state \( |\Psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2} \), defined as \( F = \langle \Psi|\rho_{\text{opt}}|\Psi\rangle \equiv (1 + \langle \bar{X}\bar{X} \rangle - \langle \bar{Y}\bar{Y} \rangle + \langle \bar{Z}\bar{Z} \rangle)/4, \) is \( 0.845 \pm 0.002 \).**
plemented by the braiding operations shown in Fig. 3c, which in turn is equivalent to a change of the fusion basis under the encoding scheme shown in Fig. 3a. In order to demonstrate the action of $\overline{\text{CX}}$ gate, we prepare the system in either logical $|00\rangle$ or $|10\rangle$ state, and then apply sequentially two $\overline{\text{CX}}$ gates. We measure the corresponding Majorana correlations at each step. Our experimental results are shown in Fig. 3c, which agree with the corresponding theoretical values. For the state $|00\rangle$, applying the $\overline{\text{CX}}$ gate keeps it unchanged and thus the measured Majorana correlations all remain positive. In contrast, for the state $|10\rangle$, applying the $\overline{\text{CX}}$ gate evolves it to $|11\rangle$, resulting in a sign flip for the relevant Majorana correlations. The deviation of the measured results from the ideal theoretical predictions is due to experimental imperfections and the fact that the measurements of the Majorana correlations are multiqubit nonlocal measurements.

Entangled logical states
With the logical gates discussed above, we can prepare an entangled state for the logical qubits. To demonstrate this in experiments, we first prepare the system to be the logical $|00\rangle$ state in the same way as in Fig. 3c. We then apply the logical Hadamard and $\overline{\text{CX}}$ gates to evolve the state to the logical Bell state $|\Psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. In Fig. 4a, we sketch the string operators corresponding to the logical $\bar{X}_1$, $\bar{Z}_2$, $\bar{X}_1$, and $\bar{X}_2$ operations. We measure the expectation values of $\langle \bar{X}\bar{X} \rangle$, $\langle YY \rangle$, and $\langle ZZ \rangle$, and the results are shown in Fig. 4b. From this figure, we obtain that the fidelity between the experimentally prepared logical state and the ideal Bell state $|\Psi\rangle$ is $0.845 \pm 0.002$, which is larger than 0.5, indicating that the two logical qubits are entangled. We mention that, due to the particular encoding scheme used in our experiment, the procedure of preparing the Bell state in our experiment is in fact equivalent to a change of the fusion basis.

Conclusions
In summary, we have experimentally observed signatures of non-Abelian anyons and their associated non-Abelian braiding statistics with programmable superconducting quantum processors. We demonstrated that the twists exchange electric and magnetic charges and exactly mimic Ising anyons under braiding and fusion. We showed how to implement both single- and two-qubit logical gates through braiding and prepared an entangled state of two logical qubits encoded into six anyons. The digital approach explored in our experiment is highly flexible and generally applicable to realize and manipulate a wide range of non-Abelian anyons, including possibly Fibonacci anyons whose braiding give rise to a universal gate set for quantum computation.

Methods
Characterization of the model Hamiltonian
As we have introduced in the main text, the Hamiltonian of the surface code is a sum of stabilizers shown in Eq. (1). Any two of these stabilizers share an even number of common qubits attached to different Pauli operators. Thus, all the stabilizers commute with each other and can be simultaneously diagonalized. They restrain the ground state of the surface code to satisfy $A_k |G\rangle = |G\rangle$ for all $k$. Each restriction $A_k = 1$ will halve the number of free parameters to describe the ground state. For a system with $N$ qubits, the dimension of the corresponding Hilbert space is $2^N$. Its ground state can be uniquely determined by a complete set with $N$ independent stabilizers, up to an irrelevant global phase.

The deformation of the lattice may reduce the number of independent stabilizers and thus bring extra degeneracy to the ground states. For example, the deformation area of the lower left corner in Fig. 2a is the product of two stabilizers $A_k$, written as $A_{D_3} = A_{(9,1)} A_{(9,2)}$. As a result, the adding of deformation $D_3$ will reduce the number of independent stabilizers by one. Another way to define the stabilizer for a deformation is to attach Pauli-$Y$ operators to qubits on its edge, as shown in Extended Data Fig. 2. The lattice sites shared by three (rather than four) neighboring plaquettes are defined as twists, which must appear in pairs. The existence of one pair of twists reduces the number of independent stabilizers by one, resulting in double degeneracy of the ground states. This double degenerate ground states can be used to define the two possible fusion results of Ising anyons. Generally speaking, one pair of twists correspond to the double degeneracy of the ground states, and can be used to mimic the behavior of a pair of Ising anyons whose topological dimension is two.

On quantum processor I, the imperfect geometry for the chosen 68 qubits naturally introduces twists that mimic Ising anyons under braiding and fusion. As shown in Fig. 2a, we choose three pairs of twists and initialize their corresponding fusion results into vacuum as shown in Fig. 3a. The initialization method is introduced in the Supplementary Information LC. The braiding of Ising anyons and the logical quantum gates are performed on the ground-state space where the stabilizers in Fig. 2a are all positive. We mention that these are not a complete stabilizer set for 68 qubits and not all the degenerate ground states are employed.

String operators
In this work, observables and unitary operations are all described by string operators. For concreteness, we take the quantum processor II as an example to explain the quantum circuit implementation corresponding to the string operators. The chosen 20 qubits on Quantum processor II form a regular rectangular lattice (Extended Data Fig. 1), so all string operators can be displayed more succinctly and conveniently. The explicit definition rules for stabilizers on deformed lattice are exemplified in Extended Data Fig. 2. In addition, we also use the Zig-Zag index for simplicity and convenience: for $k = (a, b)$, we use $k' = (a - 1) \times \text{col} + b$ as the index, where \text{col} denotes the number of columns.

Now we introduce the constructions and properties of string operators. A string operator is an ordered sequence of Pauli operators defined on the vertices of the lattice, which is a Her-
mitian Pauli string. The corresponding Pauli operator is $X$ if the string operator is oblique to the right, and $Z$ if it is oblique to the left. The string operator may have cross, which should be attached with both Pauli $X$ and $Z$. In this work we attach Pauli $Y$ to the crosses of string operators to ensure that string operators are Hermitian.

One segment of the string operator can flip the sign of two adjacent stabilizers on its diagonal. The Pauli $X$ on position $k$ flips the sign of $A_{k-i}$ and $A_{k-j}$. It can create or annihilate excitations in pairs. Meanwhile, the Pauli $Z$ on position $k$ flips the sign of $A_{k-i-j}$ and $A_{k}$. The string operator flips the sign of stabilizers on its starting and ending positions, which is illustrated in Extended Data Fig. 1b and c. Particularly, a closed string operator does not influence the sign of all $A_{k}$. It commutes with the Hamiltonian and can be implemented on the ground state without resulting any excitations. The closed string operators have several important properties. A closed string operator is a Hermitian Pauli string commuting with all $A_{k}$. It is taken as a stabilizer to be measured and characterize different degenerate ground states. The string operator is also unitary, so it can be represented by a quantum circuit and applied to a physical state. As a result, string operators are used to describe both logical Pauli operators and logical observables in this work.

In a square lattice with no deformation, string operators create or move the charge $e$ and $m$ but cannot change their charge types. Closed string operators act trivially and generate a charge of vacuum. To demonstrate the non-Abelian statistics, we change the “geometry” of Hamiltonian on quantum processor II. The new stabilizers and experimental results of the ground state are shown in Extended Data Fig. 2. The deformation introduces four twists that exhibit non-Abelian statistics. When deformations exist, the string operator can start at $e$ and end at $m$ if it winds around an odd number of twists. This means that the charge $e$ is transformed to charge $m$. Whereas, the charge $e$ is preserved after surrounding an even number of twist. These statements also apply to charge $m$. The effect of string operators that cross different number of twists is illustrated in Extended Data Fig. 3.

**Charge of Ising anyons**

The charges of anyons are identified by the emergent global phase when braiding them. For example, whether there are an odd number of $e$ anyons can be detected by circling this area and returning to its original position with an $m$ anyon. The global phase of -1 indicates the existence of an odd number of $e$ anyons. An interferometric measurement can identify the phase, but it might be challenging experimentally. On the other hand, the string operators that drag anyons around are Hermitian for $e$, $m$, and $c$. Thus, we can directly measure the string operator to distinguish these charges from the vacuum.

The case of charge $c$ is a bit complicated since the twist can change the charge. An $e$ anyon cannot return to its original position after winding around a twist. One solution is to circle the twist with another twist. This requires geometric deforma-


tion of the lattice thus not practical in experiments. Another solution is to circle the twist twice to close the corresponding string operator. This scheme can be realized with a bunch of single qubit gates since the string operator is composed of Pauli operators, as shown in Fig. 1b and Extended Data Fig. 4a. Note that string operators circling an odd number of twists have eigenvalues of $\pm i$. The complex number emerges at the cross of $S_{e}$, where Pauli $Z$ and $X$ act on the same qubit successively, noting $ZX = -iY$. To ensure the string operators being Hermitian, we attach Pauli $Y$ to the crosses of string operators instead of $ZX$. Thus, the string operators to be measured are Hermitian Pauli sequences of $-iS_{c}$.

Like other string operators characterizing topological charges, those strings corresponding to the charge of Ising anyons can be adiabatically transformed to topologically equivalent ones. Specifically, they can be arbitrarily multiplied by the stabilizers in the Hamiltonian of Eq. (1). The resulted string operators characterize the charge of Ising anyons in different areas. In Extended Data Fig. 4a, we show the shortest string operators that can characterize the charge of four Ising anyons on the quantum processor II. The Pauli strings of these shortest Ising charge operators are same as the stabilizers on the corresponding areas, which is consistent with the cases of charges $e$, $m$, and $c$. Some fusion rules from Eq. (2) are verified on the quantum processor II. The corresponding experimental results are shown in Extended Data Fig. 4b.

**Majorana operators**

Anyons with non-Abelian statistics can be used to perform topological quantum computing [1]. One way to determine whether given anyons bear non-Abelian statistics or not is to study the dimensions of their fusion space. The anyons with definite fusion result are Abelian anyons. For example, the charges $e$, $m$, and $c$ considered in this work only have trivial fusion space, thus are Abelian. The braiding sequence of these anyons can be exchanged arbitrarily and cannot be used for quantum computing. However, if we consider a subset of the charges $\{1, \sigma_{+}, c\}$, their fusion rules from Eq. (2) are exactly the fusion rules of Ising anyons. One promising topological quantum computing scheme is to implement logical quantum gates by braiding the Ising anyons, and to access the measurements of logical qubits with the fusion results [1].

In this work, the fusion and braiding of Ising anyons are described by Majorana operators, which are in turn defined as string operators across different twists. Four Majorana operators on quantum processor II are shown in Extended Data Fig. 5a. All these Majorana operators start and end at the same positions, which is used to ensure that the product operator of any two of them is a closed string operator that commutes with the system Hamiltonian. Since two different Majorana operators start and end at the same positions, they have to cross an odd number of times. Given the definition of string operators, two Majorana operators always anti-commute with each other. Taking Majorana operators $c_{1}$ and $c_{2}$ shown in Extended Data Fig. 5a as an example, we have $c_{1} = X_{1}Z_{2}Z_{6}X_{8}$ and $c_{2} =$
The two flip-chip superconducting quantum processors I and II are designed to encapsulate, respectively, 11 × 11 and 6 × 6 frequency-tunable transmon qubits in square lattices, with tunable couplers between adjacent qubits for coupling strengths up to -25 MHz. Processor I (II) possesses 110 (36) functional qubits, and the typical values of the energy relaxation times of these qubits are summarized in Extended Data Fig. 7 (Ref. [43]). Due to limited capacities in both wirings of our dilution refrigerator and measurement electronics, we measured all the qubits on processor I during two rounds of cooldowns, and have selected 68 qubits for this experiment facing unexpected experimental realities such as broken wires resulted from thermal cycling. Detailed benchmarks of the 68 qubits on processor I and the 20 qubits on processor II covering key parameters related to qubit coherence, gate performance, and readout fidelity can be found in Supplementary Information Sec. II A.

The experimental circuit for preparing the ground state on the 68-qubit system is plotted in Extended Data Fig. 6, which is composed of 22 layers of single-qubit Clifford gates and 21 layers of two-qubit CZ gates. Each single-qubit Clifford gate is composed of a single-qubit Z rotation followed by a single-qubit XY rotation. We realize the single-qubit XY rotation with a 30-ns long (15-ns full-width half-maximum) DRAG pulse [47], and implement the Z rotation by a virtual-Z gate [48]. We realize the CZ gate by bringing $|11\rangle$ and $|02\rangle$ (or $|20\rangle$) for the subspace of the two neighbouring qubits in near resonance and turning on the coupling for a specific time of 30 ns. The median Pauli errors characterized by simultaneous cross entropy benchmarking (XEB) are $1.3 \times 10^{-3}$ ($9 \times 10^{-4}$) for the single-qubit gates and $6.7 \times 10^{-3}$ ($5.6 \times 10^{-3}$) for the two-qubit gates on Processor I (II).

Since the total topological charge is 1, fermions are expected to appear in pairs. As a result, post-selection upon an even number of fermions is legitimate to partially mitigate the error induced by gate imperfections and decoherence. In our experiment, $-i c_1 c_2$, $-ic_3 c_4$ and $-ic_5 c_6$ can be sampled simultaneously. With the expectation values of these operations we can distinguish the parity of the number of fermions. Specifically, only zero or two out of the three expectation values are allowed to be -1. Here we have performed post-selection, whenever possible, for the experimental results shown in Fig. 3 and 4.

Note: During the preparation of our manuscript, we became aware of the independent works in Ref. [49] and Ref. [50] that appeared recently on arXiv.

Data availability The data presented in the figures and that support the other findings of this study will be publically available upon its publication.

Code availability The data analysis and numerical simulation codes for this study will be publically available upon its publication.
Acknowledgement The device was fabricated at the Micro-Nano Fabrication Center of Zhejiang University. We acknowledge the support of the National Natural Science Foundation of China (Grants No. 11725419, 12075128, T2225008, 12174342, 12274368, 12274367, U20A2076, and 92065204), and the Zhejiang Province Key Research and Development Program (Grant No. 2020C01019). Z.-Z.S., W. L., W. J., and D.-L.D. are supported by Tsinghua University, and the Shanghai Qi Zhi Institute.

Author contributions S.X. and K.W. carried out the experiments under the supervision of C.S. and H.W.. H.L. fabricated the device supervised by H.W.. Z.-Z.S. designed the quantum circuits under the supervision of D.-L.D., W.L., W.J., L.Y., Z.-Z.S., and D.-L.D. conducted the theoretical analysis. All authors contributed to the experimental set-up, analysis of data, discussions of the results, and writing of the manuscript.

Competing interests All authors declare no competing interests.

Extended data

Extended Data Fig. 2. Experimental results of the ground state preparation on processor II. Each polygon surrounded by white edges represents a stabilizer, which can be expressed as the product of the Pauli operators shown in that polygon. The integer numbers in polygons show the measured stabilizer values (in percentage) of the prepared deformed toric-code state. The average stabilizer value is $0.946 \pm 0.011$.

Extended Data Fig. 3. Experimental demonstration of exchanging charges between $e$ and $m$ on processor II. a. The creation of an $e$ anyon with a string operator. b. The change of an $e$ anyon to an $m$ anyon after cycling one twist. c, d. The conservation of the $e$ anyon after cycling two twists generated either by different deformations or by the same deformation. We measure all stabilizers at each step, whose values (in percentage) are shown in their corresponding polygons.

[1] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Non-abelian anyons and topological quantum computation, Rev. Mod. Phys. 80, 1083 (2008).
[2] A. Stern, Non-abelian states of matter, Nature 464, 187 (2010).
[3] F. Wilczek, Fractional statistics and anyon superconductivity, Vol. 5 (World scientific, 1990).
[4] G. Moore and N. Read, Nonabelions in the fractional quantum hall effect, Nucl. Phys. B 360, 362 (1991).
[5] X. G. Wen, Non-abelian statistics in the fractional quantum hall states, Phys. Rev. Lett. 66, 802 (1991).
[6] A. Y. Kitaev, Fault-tolerant quantum computation by anyons, Ann. Phys. 303, 2 (2003).
[7] M. H. Freedman, M. Larsen, and Z. Wang, A modular functor which is universal for quantum computation, Comm. Math. Phys. 227, 605 (2002).
[8] S. D. Sarma, M. Freedman, and C. Nayak, Topological quantum computation, Phys. Today 59, 32 (2006).
[9] A. Stern and N. H. Lindner, Topological quantum computation-
Extended Data Fig. 4.  **Demonstration of fusion rules on processor II.**  **a,** The string operators $-iS_{\sigma}$ that specify the generalized charges of the four Ising anyons. A conceptual graph demonstrating the spatial relationship of the twists and the corresponding string operators is shown at the bottom. **b,** Experimental results to verify some fusion rules from Eq. (2). We start from the ground state with four $\sigma_+$ anyons (blue trapezoids shown in the upper left panel). Next, we verify the fusion rules between $\sigma_+$ and $\epsilon, m, \epsilon$ by creating the corresponding quasiparticles in the deformations respectively. The lower left panel shows that by fusing an $\sigma_+$ anyon with an $m$ anyon we can obtain an $\sigma_-$ particle; The upper middle panel shows that by fusing an $\sigma_+$ anyon with either an $\epsilon$ or an $m$ anyon we can always obtain a $\sigma_-$ particle; The lower middle panel shows that by fusing an $\sigma_+$ anyon with an $\epsilon$ anyon we can obtain a $\sigma_-$ particle. Lastly, we use the $\sigma_-$ anyons (red trapezoids) generated at the previous step to verify the fusion rules between $\sigma_-$ and $\epsilon, m, \epsilon$ quasiparticles, respectively. The integer numbers (in percentage) in the trapezoids show the measured stabilizer values.

from basic concepts to first experiments, Science 339, 1179 (2013).
[10] A. Kitaev, Anyons in an exactly solved model and beyond, Ann. Phys. 321, 2 (2006).
[11] M. Barkeshli, C.-M. Jian, and X.-L. Qi, Twist defects and projective non-abelian braiding statistics, Phys. Rev. B 87, 045130 (2013).
[12] J. C. Y. Teo, A. Roy, and X. Chen, Unconventional fusion and braiding of topological defects in a lattice model, Phys. Rev. B 90, 115118 (2014).
[13] H. Bombin, Topological order with a twist: Ising anyons from an abelian model, Phys. Rev. Lett. 105, 030403 (2010).
[14] H. Zheng, A. Dua, and L. Jiang, Demonstrating non-abelian statistics of majorana fermions using twist defects, Phys. Rev. B 92, 245139 (2015).
[15] B. J. Brown, K. Laubscher, M. S. Kesselring, and J. R. Wootton, Poking holes and cutting corners to achieve Clifford gates with the surface code, Phys. Rev. X 7, 021029 (2017).
[16] J. Alicea, Y. Oreg, G. Refael, F. von Oppen, and M. P. Fisher, Non-abelian statistics and topological quantum information processing in 1d wire networks, Nat. Phys. 7, 412 (2011).
[17] D. A. Ivanov, Non-abelian statistics of half-quantum vortices in $p$-wave superconductors, Phys. Rev. Lett. 86, 268 (2001).
[18] P. Bonderson, A. Kitaev, and K. Shtengel, Detecting non-abelian statistics in the $\nu = 5/2$ fractional quantum hall state, Phys. Rev. Lett. 96, 016803 (2006).
[19] D. J. Clarke, J. Alicea, and K. Shtengel, Exotic non-abelian anyons from conventional fractional quantum hall states, Nat. Commun. 4, 1348 (2013).
[20] N. Tantivasadakarn, R. Verresen, and A. Vishwanath, The shortest route to non-abelian topological order on a quantum processor, arXiv:2209.03964 (2022).
[21] Y.-J. Liu, K. Shtengel, A. Smith, and F. Pollmann, Methods for simulating string-net states and anyons on a digital quantum computer, PRX Quantum 3, 040315 (2022).
[22] D. J. Griffiths and D. F. Schroeter, Introduction to quantum mechanics (Cambridge university press, 2018).
[23] N. F. MOTT, Metal-insulator transition, Rev. Mod. Phys. 40, 677 (1968).
[24] M. Tinkham, Introduction to superconductivity (Corporation, 2004).
[25] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Theory of bose-einstein condensation in trapped gases, Rev. Mod. Phys. 71, 463 (1999).
[26] F. Wilczek, Quantum mechanics of fractional-spin particles, Phys. Rev. Lett. 49, 957 (1982).
[27] D.-L. Deng, S.-T. Wang, K. Sun, and L.-M. Duan, Proposal for observing non-abelian statistics of majorana-shockley fermions in an optical lattice, Phys. Rev. B 91, 094513 (2015).
[28] M. Banerjee, M. Heiblum, Y. Umansky, D. E. Feldman, Y. Oreg, and A. Stern, Observation of half-integer thermal hall conductance, Nature 559, 205 (2018).
[29] Y. Kasahara, T. Ohnishi, Y. Mizukami, O. Tanaka, S. Ma, K. Sugii, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, et al., Majorana quantization and half-integer thermal quantum hall effect in a kitaev spin liquid, Nature 559, 227 (2018).
Extended Data Fig. 5. The Majorana operators and demonstration of logical gates on processor II. a, Four Majorana operators corresponding to the four Ising anyons. b, The demonstration of $H:\bar{Z}\bar{H}|\bar{0}\rangle = \bar{X}|\bar{0}\rangle$ in the logical space, which is realized by braiding the Ising anyons with the four Majorana operators. Here, the encoding of the logical qubit is $|\bar{0}\rangle = (|\sigma_1\times\sigma_2\rangle_1,|\sigma_3\times\sigma_4\rangle_1)$. The experimentally measured Majorana correlations (shaded values with error bars) and the corresponding ideal theoretical values (unshaded integers) are shown in the bottom of each panel at each step, which correspond to the expectation values of the logical $\bar{Z}$ observable.

[30] M. Dolev, M. Heiblum, V. Umansky, A. Stern, and D. Mahalu, Observation of a quarter of an electron charge at the $v=\frac{1}{2}$ quantum hall state, Nature 452, 829 (2008).

[31] H. Bartolomei, M. Kumar, R. Bisognin, A. Marguerite, J.-M. Berroir, E. Bouquillon, B. Placais, A. Cavanna, Q. Dong, U. Gemmer, et al., Fractional statistics in anyon collisions, Science 368, 173 (2020).

[32] K. SATzinger, Y.-J. Liu, A. Smith, C. Knapp, M. Newman, C. Jones, Z. Chen, C. Quintana, X. Mi, A. Dunsworth, et al., Realizing topologically ordered states on a quantum processor, Science 374, 1237 (2021).

[33] P. T. Dumitrescu, J. G. Bohnet, J. P. Gaebler, A. Hankin, D. Hayes, A. Kumar, B. Neyenhuis, R. Vasseur, and A. C. Potter, Dynamical topological phase realized in a trapped-ion quantum simulator, Nature 607, 463 (2022).

[34] A. Kyprianidis, F. Machado, W. Morong, P. Becker, K. S. Collins, D. V. Else, L. Feng, P. W. Hess, C. Nayak, G. Pagano, et al., Observation of a prethermal discrete time crystal, Science 372, 1192 (2021).

[35] X. Mi, M. Ippoliti, C. Quintana, A. Greene, Z. Chen, J. Gross, F. Arute, K. Arya, J. Atalaya, R. Babbush, et al., Time-crystalline eigenstate order on a quantum processor, Nature 601, 531 (2022).

[36] X. Zhang, W. Jiang, J. Deng, K. Wang, J. Chen, P. Zhang, W. Ren, H. Dong, S. Xu, Y. Gao, et al., Digital quantum simulation of Floquet symmetry-protected topological phases, Nature 607, 468 (2022).

[37] R. Acharya, I. Aleiner, R. Allen, T. I. Andersen, M. Ansmann, F. Arute, K. Arya, A. Asfaw, J. Atalaya, R. Babbush, et al., Suppressing quantum errors by scaling a surface code logical qubit, arXiv:2207.06431 (2022).
[48] D. C. McKay, C. J. Wood, S. Sheldon, J. M. Chow, and J. M. Gambetta, Efficient z gates for quantum computing, Phys. Rev. A 96, 022330 (2017).
[49] T. I. Andersen, Y. D. Lensky, K. Kechedzhi, I. Drozdov, A. Bengtsson, S. Hong, A. Morvan, X. Mi, A. Opremcak, R. Acharya, et al., Observation of non-Abelian exchange statistics on a superconducting processor, arXiv:2210.10255 (2022).
[50] M. Kalinowski, N. Maskara, and M. D. Lukin, Non-Abelian Floquet Spin Liquids in a Digital Rydberg Simulator, arXiv:2211.00017 (2022).
[51] S. Bravyi, I. Kim, A. Kliesch, and R. Koenig, Adaptive constant-depth circuits for manipulating non-abelian anyons, arXiv:2205.01933 (2022).
[52] S. Bravyi, Universal quantum computation with the $\nu = 5/2$ fractional quantum hall state, Phys. Rev. A 73, 042313 (2006).
[53] L. S. Georgiev, Towards a universal set of topologically protected gates for quantum computation with Pfaffian qubits, Nucl. Phys. B 789, 552 (2008).
[54] J. Preskill, Quantum computing in the nisq era and beyond, Quantum 2, 79 (2018).
[55] S. Bravyi, M. B. Hastings, and F. Verstraete, Lieb-Robinson Bounds and the Generation of Correlations and Topological Quantum Order, Phys. Rev. Lett. 97, 050401 (2006).
[56] C. Song, K. Xu, H. Li, Y.-R. Zhang, X. Zhang, W. Liu, Q. Guo, Z. Wang, W. Ren, J. Hao, et al., Generation of multicomponent atomic Schrödinger cat states of up to 20 qubits, Science 365, 574 (2019).
[57] M. A. Rol, L. Ciorciaro, F. K. Malinowski, B. M. Tarasinski, R. E. Sagastizabal, C. C. Bultink, Y. Salathe, N. Haandbaek, J. Sedivy, and L. DiCarlo, Time-domain characterization and correction of on-chip distortion of control pulses in a quantum processor, Appl. Phys. Lett. 116, 054001 (2020).
[58] J. Kelly, R. Barends, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, A. G. Fowler, L.-C. Hoi, E. Jeffrey, et al., Optimal quantum control using randomized benchmarking, Phys. Rev. Lett. 112, 240504 (2014).
[59] M. Gario and A. Micheli, Pysmt: a solver-agnostic library for fast prototyping of smt-based algorithms, in SMT Workshop 2015 (2015).
[60] W. Ren, W. Li, S. Xu, K. Wang, W. Jiang, F. Jin, X. Zhu, J. Chen, Z. Song, P. Zhang, et al., Experimental quantum adversarial learning with programmable superconducting qubits (2022), arXiv:2211.05803.
[61] A. Kissinger and J. van de Wetering, PyZX: Large Scale Automated Diagrammatic Reasoning, in Proceedings 16th International Conference on Quantum Physics and Logic, Chapman University, Orange, CA, USA., 10-14 June 2019, Electronic Proceedings in Theoretical Computer Science, Vol. 318, edited by B. Coecke and M. Leifer (Open Publishing Association, 2020) pp. 229–241.
[62] M. Treinish, Qiskit/qiskit: Qiskit 0.39.2 (2022).
Extended Data Fig. 6. *Experimental circuit for preparing the ground state with the 68-qubit system.* The whole circuit contains 43 layers (labeled from 0 to 42 at the top of the figure), with each layer containing only single- or two-qubit gates that are applied simultaneously. Every single-qubit gate (red dots) is a Clifford gate realized by combining XY and Z rotations experimentally. The additional echo gates (yellow dots) are implemented by Y gates. There are in total 294 single-qubit gates (including 66 echo gates) and 113 two-qubit CZ gates.
Extended Data Fig. 7. **Typical $T_1$ values of the 110 functional qubits on the processor I.** a, Distribution of $T_1$. We obtained the heatmap during two rounds of cooldowns due to the limitation of wirings in the dilution refrigerator and measurement electronics. $T_1$ values may vary with frequency, drift over time and incur a sudden change at a different cooldown. b, Integrated histogram of $T_1$. Dashed line indicates the median value, from which we obtain that $T_1$ has a median value of 100.3 $\mu$s.
Supplementary Information: Digital simulation of non-Abelian anyons with 68 programmable superconducting qubits

CONTENTS

References .................................................. 9

I. Theory ....................................................... 15
   A. Non-Abelian anyons .................................. 15
   B. Logical qubits, gates, and measurements ......... 16
   C. Quantum circuit for the initialization ............. 18
   D. Quantum circuit for braiding ....................... 20
   E. Quantum circuit for processor I ............... 21

II. Experimental Details ................................. 22
   A. Device information .................................. 22
   B. Calibration procedure .............................. 22
      1. Single-qubit calibration .......................... 22
      2. Multiple-qubit calibration .................... 23
   C. Experiment circuit .................................. 23

I. THEORY

A. Non-Abelian anyons

It is well recognized that bosonic and fermionic statistics are the only two allowed quantum statistics in three-dimensional (3D) space where we live in. Since there is only one topologically distinct way to exchange two identical particles in 3D space. In such a scenario, the many-body wave functions of identical particles host only two possible symmetries under exchange: the symmetric wave functions for bosons, and the anti-symmetric wave functions for fermions. However, such a boson-fermion dichotomy can be broken in 2D space, where the swap of two identical particles may give rise to a topological non-trivial path, and thus the exotic anyon statistics would emerge.

From the perspective of group representation theory in mathematics, the bosonic and fermionic statistics are described by the two one-dimensional irreducible representations of the permutation group, whereas the anyonic statistics are described by the braid group. According to the representation of braid group, anyons can be divided into two categories, Abelian and non-Abelian anyons. Abelian anyons usually correspond to the one-dimensional irreducible representations of braid group, i.e., all of the braid group elements commute. Whereas non-Abelian anyons usually correspond to the higher dimensional irreducible representations of the braid group, and the braid elements multiplication is non-commutative.

In physics, the study of Abelian anyons dates back more than forty years ago. For demonstration, let us suppose that we have $N$ identical particles with the wave function $|\Psi(r_1, r_2, \cdots r_N)\rangle$, where $r_i$’s denote the positions of identical particles in real space. Exchanging the position of arbitrary two particles could give rise to an overall phase

$$|\Psi(r_1, r_2, \cdots r_N)\rangle \rightarrow e^{i\theta}|\Psi(r_1, r_2, \cdots r_N)\rangle. \quad (S1)$$

The special cases $\theta = 0$ or $\pi$ correspond to the bosonic or fermionic statistics. While for other values of the statistical angle $\theta \in [0, 2\pi)$ except 0 or $\pi$, the corresponding identical particles are called Abelian anyons.

The Abelian anyons are expected to appear in a variety of two-dimensional quantum systems. For example, in the $\nu = 1/3$ fractional quantum Hall system, there appear localized excitations (e.g. quasiparticles or quasiholes) carrying quantized magnetic fluxes and fractional charges. Such localized excitations obey Abelian anyonic statistics, since exchanging the corresponding two composite particles would give rise to a non-trivial phase owing to their mutual Aharonov-Bohm effect. Thus, the localized excitations can be regarded as Abelian anyons. Besides that, the Abelian anyonic excitations are also predicted in the lattice model, like the Kitaev $\mathbb{Z}_2$ toric code model with ground state degeneracy depending on the topological structure of the two-dimensional system. In such $\mathbb{Z}_2$ toric model, the pairs of Abelian anyons $e(m)$ can be excited by applying the Pauli-$Z(X)$ operator on the local lattice site. By applying the string operator on the excited states, we can move the corresponding anyons on the lattice. Encircling the $e$-anyon around the $m$-anyon results in a $e^{i\pi}$ phase factor to the corresponding state, rather than...
the phase factor 1 for bosons or fermions. This behaves the Abelian anyonic statistics. However, braiding Abelian anyons in
the subspace spanned by the degenerate ground states cannot be utilized for quantum computation, since the braiding operation
would only induce an overall phase factor to the corresponding state.

Different from the exchange of Abelian anyons, exchanging non-Abelian anyons can give rise to a unitary operation on
the degenerate ground state space. For example, suppose we have a set of degenerate ground states \( \{|\Phi_i(r_1, r_2, \cdots r_N)\rangle\} \) of
a topological quantum system with \( N \) identical non-Abelian anyonic excitations. Then by exchanging the positions of two
non-Abelian anyons, one obtains

\[
|\Phi_i(r_1, r_2, \cdots r_N)\rangle \rightarrow U_{ij}|\Phi_j(r_1, r_2, \cdots r_N)\rangle,
\]

where \( U \) denotes the unitary operation. The study of non-Abelian anyons has attracted extensive interest in both theory and
experiments, owing to not only their exotic physics but also their potential applications in topological quantum computation
against local noises. The non-Abelian anyons are predicted to appear in \( \nu = \frac{2}{3} \) fractional quantum Hall system, quantum double
models of non-Abelian finite group, and so on. One of the simplest non-Abelian anyon models is the Ising anyon model, which
is described by the \( SU(2) \) topological quantum field theory. The Ising anyon model includes three anyons, the vacuum \( 1 \), the
Ising anyon \( \sigma \), and the Majorana fermion \( \epsilon \). They obey the following fusion rules

\[
\sigma \times 1 = 1 \times \sigma = \sigma,
\epsilon \times 1 = 1 \times \epsilon = \epsilon,
\sigma \times \epsilon = \epsilon \times \sigma = \sigma,
\epsilon \times \epsilon = 1,
\sigma \times \sigma = 1 + \epsilon.
\]

For a system with \( 2N \) Ising anyons fusing to the vacuum, the total Hilbert space spanned by the fusion bases is \( 2^{N-1} \). Thus
the quantum dimension of the Ising anyon is \( d = \sqrt{2} \). Applying the braiding operations on the Ising anyons, one can thus
obtain the \( 2^{N-1} \) - dimensional irreducible representation of the \( 2N \)-strand braid group \( B_{2N} \). From the perspective of quantum
computation, the fusion bases of \( 2N \) Ising anyons span the full computational space, and the elements of the braid group serve
as the quantum logical gates. Thus, the universality of the topological quantum computation model based on braiding operations
would depend on the universality of the braid group in expressing the \( SU(2^{N-1}) \) group.

One candidate to realize the Ising anyon statistics is based on the Majorana zero modes (MZM). In recent years, extensive
experimental efforts have been devoted to demonstrating the presence of the MZM in quantum materials. However, it remains a
challenging task to confirm the existence of MZM in a solid-state system, as well as braiding such MZMs.

As mentioned above, the non-Abelian anyonic excitations can also emerge in the quantum double model of non-Abelian finite
group \( G \), where the dimension of each local lattice site is equivalent to the group size \(|G|\). The simplest quantum double lattice
model of the non-Abelian finite group is \( D(S_3) \), where \( S_3 \) denotes the 3-strand symmetric group. The eight anyonic excitations
correspond to the eight irreducible representations of the Hopf algebra \( D(S_3) \). In a recent work [51], the authors show that the
ground states and anyon excitations of the model \( D(S_3) \) can be realized by adaptive circuits with constant-depth, assisted by
those geometrically local unitary gates and mid-circuit measurements.

### B. Logical qubits, gates, and measurements

The logical qubits are defined in the fusion space of Ising anyons. Two different fusion channels of Ising anyons into the
vacuum or fermion can be used to represent the \( |0\rangle \) and \( |1\rangle \) state, respectively [52]. Since the quantum dimension of a pair
of Ising anyons is two, two pairs of Ising anyons have enough dimensions to define a logical qubit and three pairs of Ising anyons
can define two logical qubits. In this work, we define the case of the first two Ising anyons fusing to the vacuum as the logical
\( |0\rangle \) state, which is denoted as \( |0\rangle \). The case of the first four Ising anyons fusing to the vacuum is defined as \( |0\rangle \) state of the
second logical qubit. There are several notations for the logical states in the main text to facilitate writing and vividly illustrate
the braidings. We show a complete set of the logical bases represented by these notations in Fig. S1.

It has been introduced in the main text that we adopt two methods to implement logical quantum gates. The first one is to
braid the Majorana operators. For a given encoding scheme and quantum gate, calculating the corresponding braiding sequence
is complicated but eminently solvable [1, 53]. The effect of braiding Majorana operators has been introduced in the main text.
In Fig. S2, we show all the explicit braiding sequences and their effects on Majorana operators for the logical gates \( \bar{Z}, \bar{X}, \bar{H} \),
and \( \bar{C} \).

The braiding can change the indexes and phase factors of Majorana operators, and accordingly change the string operators
to be measured. For example, the measurement of \( \bar{Z} \) on the first logical qubit is always the measurement of string operators
Extended Data Fig. S1. Dirac notations, fusion trees, and braiding sequences of logical qubit states. a, $|0\rangle$ represents logical $|0\rangle$ state. This state is indicated by that the fusion result of the first two Ising anyons is the vacuum, as shown in the Dirac notation and fusion tree notation. The braiding sequence on the right shows that these two pairs of Ising anyons are both generated from the vacuum, which is consistent with their fusion results. b, Logical state $|1\rangle$. The braiding sequence of $|1\rangle = X|0\rangle$ is also shown. c, Logical state $|0\rangle$. The first logical qubit being logical state $|0\rangle$ is indicated by that the first two Ising anyons fuse to the vacuum. And the second logical qubit being $|0\rangle$ is indicated by that the first four Ising anyons fuse to the vacuum. The second logical qubit can also be represented with the fusion results of the last two Ising anyons, due to the conservation of total topological charge. d, Logical state $|\bar{0}\rangle$. e, Logical state $|\bar{1}\rangle$. f, Logical state $|\bar{1}\rangle$.

Extended Data Fig. S2. The braiding of Majorana operators corresponding to all logical operators in this work. a, $\bar{Z} = B_{12}^2$. Braiding the first two Ising anyons twice gives $c_1 \mapsto -c_1$, $c_2 \mapsto -c_2$, $c_3 \mapsto c_4$, and $c_4 \mapsto c_4$, corresponding to logical $\bar{Z}$ gate. b, $\bar{H} = B_{12}B_{23}B_{12}$. c, $X = B_{23}^2$. d, $\bar{CX} = B_{12}B_{34}B_{45}B_{56}^{-1}B_{45}^{-1}B_{56}^{-1}$.

defined by the product of the first two Majorana operators. But which two are the first two Majorana operators will change, and so will their phase factors. In the context of using quantum circuits to realize logical operators, those logical operators that do not change the positions of Ising anyons are much easier to realize. We can apply the logical operators $X$ and $\bar{Z}$ by their corresponding string operators introduced in the main text. The measurement results of $\bar{X}$ and $\bar{Z}$ are obtained by measuring these string operators. While the quantum circuit implementation of $\bar{H}$ where Ising anyons do not all return to their original positions requires a sequence of multi-qubits gates [49]. The scheme to implement this kind of logical operators by changing the indexes and phase factors of corresponding Majorana operators might be more practical to be used for large scale topological quantum computing with noisy intermediate-scale quantum (NISQ) [54] devices.
C. Quantum circuit for the initialization

Here we introduce the method to prepare the ground state of Hamiltonian $H = -\sum_k A_k$, where $A_k = X_k Z_{k+1} Z_{k+3} X_{k+1+3}$, as well as the initialization of the logical states in the fusion space. Initially, the system is in the physical state $|0\rangle^\otimes N$, where $\langle A_k \rangle = 0$ for all $k$. We aim to prepare the ground states of the Hamiltonian, such that all of the local operators $A_k$ share the same eigenvalue of 1. An intuitive initialization method is to perform non-destructive measurements on all $A_k$ and annihilate the excitations with string operators. This scheme can create the ground state with only one layer of measurements [20]. However, performing non-destructive measurements and feed-forward string operators is expensive under the existing experimental conditions.

The non-destructive measurements of $A_k$ can be achieved by the Hadamard test that requires an ancillary qubit. The procedure is to apply the unitary $A_k$ on quantum state $|\psi\rangle$ of the lattice system conditioned on the ancillary state of $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Then by applying a Hadamard gate on the ancillary qubit, we obtain

$$\frac{1}{2} [ |0\rangle \langle I + A_k | \psi\rangle + |1\rangle \langle I - A_k | \psi\rangle ].$$

(S4)

Simple calculations show that the measurement result on the ancillary qubit equals the expected value $\langle \psi | A_k | \psi\rangle$. The projection of $|\psi\rangle$ to the state with $A_k = 1$ can be realized by projecting the ancillary qubit to $|0\rangle$. According to Eq. (S4), projecting the ancillary qubit to $|0\rangle$ is to project the quantum state $|\psi\rangle$ to $\frac{1}{\sqrt{2}} ( |I + A_k \rangle | \psi\rangle )$, whose eigenvalue corresponding to $A_k$ is +1.

The term $\frac{1}{\sqrt{2}} ( I + A_k )$ is not unitary and cannot be directly implemented by quantum circuit. However, we can initialize a representative qubit for each $A_k$ to create the superposition [21]. Let us use the example of $A_1 = X_1 Z_2 X_3 Z_4$ with the representative qubit of $Q_1$ to show how to implement $\frac{1}{\sqrt{2}} ( I + A_1 )$ in detail. Starting from $|0000\rangle$, we firstly initialize the representative qubit to $|+\rangle = \frac{1}{\sqrt{2}} ( |0\rangle + |1\rangle )$, which is stabilized by $X_1$. Then we apply the operators of $A_1$ except $X_1$ controlled by the representative qubit. The result state is

$$|0\rangle |000\rangle + X_1 A_1 |1\rangle |000\rangle = \frac{1}{\sqrt{2}} ( I + A_1 ) |0000\rangle.$$

(S5)

Here $X_1 A_1$ is the product of all single-qubit operators in $A_1$ except $X_1$ (note that $X_1 X_1 = I$). The resulted state of Eq. (S5) is the target ground state corresponding to $A_1 = 1$. This process is shown in Fig. S3a. The key point to understand this process is that the representative qubit does not change under the action of $A_1$. Thus, we can use it to implement $\frac{1}{\sqrt{2}} ( I + A_1 )$, which is superposition of operators. For a surface code with no deformation, every qubit (except those on the edge) is shared by four square plaquettes. Thus, stabilizers corresponding to every plaquette can be assigned to an isolated representative qubit.

Since an entangled qubit cannot be stabilized by a single Pauli operator, the representative qubit should not be entangled to the rest qubits until it has controlled the corresponding $A_k$. Because all $A_k$ commute with each other, the operation $\frac{1}{\sqrt{2}} ( I + A_k )$ will preserve stabilizers that are already well-prepared when the expected value of $A_k$ is zero. For a state satisfying $\langle \phi | A_j | \phi\rangle = 1$ ($j \neq k$), we have

$$\langle \phi | \frac{1}{\sqrt{2}} ( I + A_k ) A_j \frac{1}{\sqrt{2}} ( I + A_k ) | \phi\rangle = \langle \phi | ( I + A_k ) A_j | \phi\rangle = 1 + \langle \phi | A_k | \phi\rangle = 1.$$

(S6)

To keep the representative qubit not entangled with other qubits until all operations controlled by it are performed, we can activate the representative qubits layer by layer. The number of layers in the initialization circuit is proportional to the diameter of the rectangular system. This scaling is essentially optimal for the topologically ordered system [55]. We show a sketch of initializing the ground state of the surface code through the activating sequence of representative qubits in Fig. S3b.

There is a practical method to approximately halve the number of two qubits gates when initializing the ground state of surface code. Note that some stabilizers have no “conflict”, which means they do not share common qubits attached by different Pauli operators. In other words, the “conflict” is where two different string operators cross but regardless of their tangent point. These stabilizers with no “conflict” can be prepared simultaneously. For example, we only need to implement two Hadamard gates on the qubits attached to $X$ to obtain the ground state of an independent stabilizer $A$. This process is shown in Fig. S3a. Since half of the plaquettes can be prepared with this method, it reduces the workload of representative qubits by half, and so do the number of layers. This optimized initialization method is shown in Fig. S3b.

The existence of twists seems to reduce the number of terms in the system Hamiltonian, and reduce the difficulty of initialization. However, this is not true if we consider them as Ising anyons generated in pairs from the vacuum or fermion. A pair of twists will reduce one term in the system Hamiltonian. This degree of freedom is used to characterize the fusion result of a pair.
Extended Data Fig. S3. The method to prepare the ground state of the surface code. **a**, The method to prepare the ground state of one plaquette, where the corresponding stabilizer is \( A_1 = X_1 Z_2 X_3 Z_4 \). The first step is to initialize the representative qubit to a superposition state. The second step is to implement the rest part of \( A_1 \) on qubit 2, 3, and 4 controlled by the representative qubit. These controlled operations are CZ, CX, and CZ that can be sequentially implemented. The final state is the eigenstate of \( A_1 \) with the corresponding eigenvalue equal +1. **b**, The method to prepare the ground state of the surface code. Starting with the state of \(|0\rangle \otimes N\), we first initialize the plaquette at the bottom, then the second layer above it, and so on. The representative qubits can function well without interfering with each other under the order shown in this figure, where circuit depth is linearly related to the diameter. The colors of representative qubits are only for clarity of layering.

\[
\begin{align*}
A_1 &= X_1 Z_2 X_3 Z_4 \\
|0000\rangle &\quad \langle A_1 \rangle = 0 \\
H_1 &= \frac{1}{\sqrt{2}}(Z_1 + X_1) \\
\langle X_1 \rangle &= 1 \\
H_3 &\quad |+000\rangle \\
A_1 &= 1 + \langle A_1 |+000\rangle
\end{align*}
\]

Extended Data Fig. S4. The optimized method to prepare the ground state of the surface code. **a**, The method to prepare the ground state of one independent plaquette, where the corresponding stabilizer is \( A_1 = X_1 Z_2 X_3 Z_4 \). This process can be described as \( H_1 H_3 |0000\rangle = |+0 + 0\rangle \) and \( \langle +0 + 0 | A_1 |+0 + 0\rangle = 1 \). **b**, The optimized method to prepare the ground state of the surface code. Under this scheme, half of the plaquettes can be prepared to the corresponding ground states by one layer of Hadamard gates. Thus, the number of representative qubits and the corresponding two qubits gates is halved.

\[
\begin{align*}
A_1 &= X_1 Z_2 X_3 Z_4 \\
|000\rangle &\quad \langle A_1 \rangle = 0 \\
H_1 &\quad |+00\rangle \\
H_3 &\quad |+0 + 0\rangle \\
A_1 &= 1 + \langle A_1 |+0 + 0\rangle
\end{align*}
\]

The method of shown in Fig. S4a is efficient but requires that the string operators to be initialized have no “conflict”. Now we...
show the method to define string operators that uniquely determine the fusion results of twist and have no “conflicts”. As shown in Fig. S5b, three string operators uniquely determine the fusion results of three pairs of twists. These results are recorded as $f_b$, $f_g$, and $f_y$ for the blue, green, and yellow string operators, respectively. String operators corresponding to these fusion results have eight “conflicts” and cannot be initialized with the method of S4a. This obstacle can be circumvented if we turn to initialize the string operators corresponding to $f_b$, $f_b \times f_g$, $f_b \times f_g \times f_y$, as shown in Fig. S5c. These string operators have no “conflict” and can uniquely determine the fusion results of these three pairs of twists.

Extended Data Fig. S5. The string operators corresponding to the fermion charge. a, Topological equivalent string operators on the 20 qubits quantum processor II. The blue strings represent the fermion charge of the upper two twists and the green ones represent the fermion charge of the lower two. The string operators on the right are the shortest ones containing the corresponding pairs of twists, which are the string operators used in the initialization. b, The string operators that uniquely determine the fusion results of three pairs of twists. The eigenvalue of these string operators being +1 indicates the vacuum and being −1 indicates ϵ. The desired fusion results corresponding to these string operators are recorded as $f_b$, $f_g$, and $f_y$ for the blue, green, and yellow string operators, respectively. These string operators have eight “conflicts” marked by red arrows, which make the method in Fig. S4a incapable to initialize them. c, Three “conflict”-free string operators uniquely determining the fusion results of three pairs of twists. The blue, green, and yellow string operators correspond to $f_b$, $f_b \times f_g$, $f_b \times f_g \times f_y$, respectively.

D. Quantum circuit for braiding

The logical gates defined in the fusion space of Ising anyons are achieved by braiding them. We use two different ways to realize this braiding. The first way is to use the string operators that evolve the corresponding Majorana operators. For one logical qubit encoded by four Ising anyons, the logical $Z$ corresponds to braid the first two Ising anyons twice, and can be realized by the string operator $U_Z = -ic_1c_2$. All braiding sequences that do not change the positions of Ising anyons can be implemented in this way. Logical $X$ and logical $Z$ suit this condition and are realized by corresponding string operators in our experiment. Meanwhile, the braiding sequences that change the positions of Ising anyons are more complicated. A simple example is the logical $S$ gate which corresponds to the braiding $B_{12}$. The unitary to realize it is

$$U_S = \sqrt{-i}c_1c_2 = \sqrt{-ie^{\frac{\pi}{4}}c_1 c_2} = e^{-\frac{\pi}{4}}e^{\frac{\pi}{4}c_1 c_2} = e^{-\frac{\pi}{4}}\frac{1}{\sqrt{2}}(I + c_1 c_2).$$

This is a non-local unitary and is hard to implement on large scale quantum processors. Thus, we directly change the indexes and phase factors of Majorana operators (i.e., change of the fusion basis, which corresponds to a F-move in the language of topological quantum computing), which is shown in Fig. 3, Fig. 4, and Extended Fig. 5 in the main text.

An alternative way to move Ising anyons is directly moving the corresponding twists since Ising anyons are defined on them. This process is to adiabatically transform the “geometry” of the Hamiltonian [13]. In other words, some stabilizers in the Hamiltonian are changed. A simple way to achieve this is to measure the new stabilizer $S_{\text{new}}$. However, this non-destructive measurement method is difficult to experimentally realize. The initialization methods shown in Fig. S3 and Fig. S4 do not apply to the present situation because of two differences. The first one is that there is no free qubit that is not entangled with the system. Thus, the scheme of representative qubits fails. The second difference is that there will be one old stabilizer $S_{\text{old}}$ becomes invalid once the new stabilizer $S_{\text{new}}$ is activated. Since this process can be realized by implementing non-destructive measurement, $\{S_{\text{new}}, S_{\text{old}}\} = 0$. Since both these string operators are Hermitian Pauli strings, their product multiplied by $i$ is Hermitian, written as:

$$[iS_{\text{new}}S_{\text{old}}]^\dagger = -iS_{\text{old}}^\dagger S_{\text{new}} = -iS_{\text{old}}S_{\text{new}} = iS_{\text{new}}S_{\text{old}}.$$

This is an important property since it means the operator

$$\frac{1}{\sqrt{2}}(I + S_{\text{new}}S_{\text{old}}) = \frac{1}{\sqrt{2}}[I + i(\text{new}_{S_{\text{old}}})] = e^{\frac{\pi}{4}i(-iS_{\text{new}}S_{\text{old}})} = e^{\frac{\pi}{8}}S_{\text{new}}S_{\text{old}}$$

(S9)
Extended Data Fig. S6. String operators and their attached Pauli matrices corresponding to $-ic_1c_2$, $-ic_3c_4$, $-ic_5c_6$, $-ic_1c_4$, $-ic_4c_5$, and $-ic_3c_6$. The subscripts of Majorana operators give the indexes of Ising anyons being surrounded. For example, $-ic_1c_2$ represents a string operator surrounding $\sigma_1$ and $\sigma_2$. It specifies the fusion result of these two Ising anyons. The complex phase factor is introduced to keep these operators both Hermitian and unitary. These two properties make them severing as quantum circuits and measurements performed on the physical qubit. Their implementations and corresponding logical operators are shown in Fig. 3 and Fig. 4 of the main text.

is a unitary operator. Note that $S_{\text{old}}$ is a stabilizer of the state $|\psi\rangle$ before moving the twist. We can obtain

$$\frac{1}{\sqrt{2}} (I + S_{\text{new}}S_{\text{old}}) |\psi\rangle = \frac{1}{\sqrt{2}} (I + S_{\text{new}}) |\psi\rangle .$$

(S10)

Now we have the unitary operator $e^{\pi S_{\text{new}}S_{\text{old}}}$ to achieve same effect of $\frac{1}{\sqrt{2}} (I + S_{\text{new}})$ on the state $|\psi\rangle$ before moving the twist. Based on what we have discussed around Eq. (S4), the unitary operator $e^{\pi S_{\text{new}}S_{\text{old}}}$ is equivalent to projecting $|\psi\rangle$ to the eigenstate of $S_{\text{new}}$ with the corresponding eigenvalue equaling $+1$. In this way, a unitary operation to slowly change the “geometry” of the Hamiltonian is provided.

E. Quantum circuit for processor I

In this subsection, we show the quantum circuits corresponding to the string operators that characterize the fusion results of two Ising anyons in Fig. S6. These string operators are both Hermitian and unitary, which are used as observables to be measured and quantum circuits to be implemented in Fig. 3 and Fig. 4 of the main text. Detailed introductions of these circuits are in the caption of Fig. S6.
II. EXPERIMENTAL DETAILS

A. Device information

We observe the non-Abelian exchange statistics on two different quantum processors, referred to as version I (main text Fig. 1) and version II [43], both of which were fabricated using the flip-chip recipe as described elsewhere [36]. Processor I (II) hosts an array of $11 \times 11$ ($6 \times 6$) frequency-tunable transmon qubits with tunable couplers between adjacent qubits. On both processors, the maximum resonance frequencies of the qubit and coupler are around 4.8 GHz and 9.0 GHz, respectively. The effective coupling strength between two neighboring qubits can be dynamically tuned up to $-25$ MHz. Each qubit capacitively couples to its own readout resonator, designed at the frequency around 6.5 GHz, for qubit state measurement. In our experiments, 68 (20) qubits out of the 110 (36) functional qubits on processor I (II) are used, whose relaxation times and Hahn echo dephasing times measured at idle frequencies are shown in Fig. S7, with median values of $T_1 = 109.8$ ($137.5$) µs and $T_2 = 17.9$ ($16.4$) µs, respectively. The cumulative distribution of the readout fidelities are also shown, which are used to mitigate measurement errors [56]. In addition, we also plot the distribution of single-qubit parameters such as idle frequencies and Pauli errors of single-qubit gate on processor I for the 68 qubits.

B. Calibration procedure

Quantum operations on our superconducting quantum processors are physically realized by applying on each qubit/coupler analog signals with continuous control parameters such as amplitudes and phases. A calibration procedure is a collection of experiments that is used to learn and optimize these control parameters and enables us to gain full control of the processor. A systematic, automated and scalable calibration procedure is essential for achieving high-fidelity qubit operations across the whole device. We separate our calibration procedure into two sessions: a single-qubit calibration session that is used to bring up all the qubits/couplers individually from scratch and collect basic device and control parameters, and a multiple-qubit calibration session that is used to calibrate the processor on a system level and achieve high-fidelity simultaneous single- and two-qubit gates. Below we illustrate a couple of key steps during the bootstrapping calibration procedure.

1. Single-qubit calibration

In the single-qubit calibration session, we first bring up each qubit individually. We isolate each qubit from the others by applying on it a flux bias to set its transition frequency to around 4.5 GHz while keeping the other qubits/couplers at their sweet points. Then we bring up the qubit following the procedure as summarized below:

- Perform qubit spectroscopy and power Rabi oscillations to optimize the parameters (drive frequency and power) of the π pulse that excites the qubit to $|1\rangle$ state.
- Perform qubit spectroscopy as a function of bias to find the relation between the qubit transition frequency and bias.
- Calibrate the timing between the qubit microwave pulse and flux bias pulse.
- Measure the response of the qubit to a detuning pulse to eliminate the impact of pulse distortions [57, 58].
- Measure the spectrum of $T_1$, which is later used in allocating the qubit energy level in the multiple-qubit calibration session.

After bringing up all the qubits, we move on to bring up all the couplers. We isolate each coupler from the others by applying on it a flux bias to set its transition frequency to around 5.6 GHz while keeping the other couplers at their sweet points. At the same time, we set one of its neighboring qubits to around 4.5 GHz, which is used to readout the coupler state, and another to below 4 GHz, while keeping other qubits at their sweet points. Then we bring up the coupler following the procedure as summarized below:

- Calibrate the timing between the microwave pulse of the readout qubit and flux bias pulse of the coupler by applying a square flux bias pulse on the coupler to dispersively detune the qubit.
- Measure the swapping dynamics of the readout qubit and the coupler by applying a flux bias pulse on the coupler with a fixed time of 5 ns and vary its amplitude. Determine the amplitude with which a complete state transfer between the coupler and readout qubit is realized.
• Perform coupler spectroscopy and power Rabi oscillations to optimize the parameters (drive frequency and power) of the \( \pi \) pulse that excites the coupler to \( |1\rangle \) state. The microwave pulse is applied via the microwave line of the readout qubit.

• Measure the response of the coupler to a detuning pulse to eliminate the impact of pulse distortions.

Now that we have tuned up all the qubits and couplers individually, we move on to do calibrations that require a qubit-qubit or qubit-coupler pair at a time:

• Construct a directed acyclic graph (DAG) according to the topological structure of the device with the breadth-first search (BFS) algorithm (Fig. S9 a), and fine tune the timing between the flux bias pulses of each qubit-coupler pair from the root to every branches in the DAG. The pulse sequences and typical data are shown in Fig. S9 b. After this step, all the control pulses are synchronized.

• Characterize the flux bias crosstalk matrix among all the qubits and couplers, which is used to actively cancel out the flux bias crosstalk effect [56].

2. Multiple-qubit calibration

In the multiple-qubit calibration session, we allocate all the qubits and couplers to proper frequencies with the knowledge obtained from the previous session, based on which we can achieve high-fidelity single- and two-qubit gates. The procedure is summarized as below:

• Allocate the energy levels of all the qubits by solving a constraint optimization problem with a Satisfiability Modulo Theory (SMT) solver [59]. With the constraints we avoid the impact of various factors such as TLS defects, stray couplings between qubits, carrier frequency leakage, and pulse distortions. We then allocate the readout energy levels of all the qubits in a similar way.

• Characterize the microwave crosstalk coefficients for qubit pairs that are allocated within 60 MHz, which are used to actively cancel out the microwave crosstalk effect [60].

• Allocate the idle frequencies of all the couplers to minimize the unwanted qubit-qubit coupling.

• Fine tune parameters (qubit resonant frequency, drive power, DRAG coefficient) to realize high-fidelity simultaneous single-qubit \( \pi \) and \( \pi/2 \) pulses.

• Fine tune parameters (qubit bias, coupler bias, interaction frequency) to realize high-fidelity individual two-qubit CZ gates. We achieve a CZ gate with a fidelity as high as 0.9987 during this procedure (Fig. S10).

• Benchmark the performance of the CZ gates when executed simultaneously for each two-qubit gate layer in the experimental circuit. Adjust the interaction frequencies when necessary.

C. Experiment circuit

The circuit for preparing the ground state in the 68-qubit system is plotted in Extended Data Fig. 6 (main text), which contains 22 layers of single-qubit gates and 21 layers of two-qubit CZ gates. In our experiment, arbitrary single-qubit gates can be realized by combining XY and Z rotations. During the application of XY pulses, we use active microwave cancellation technique at each qubit’s frequency to mitigate the microwave crosstalk effects. The two-qubit CZ gate is realized by dynamically steering the resonant frequency of the coupler along a well-designed trajectory, so that the effective ZZ-type coupling strength can be turned on for a specific amount of time, accumulating a conditional \( \pi \) phase angle while minimizing the leakage. CZ gate parameters are optimized for each two-qubit CZ gate layer. To characterize the performance of the quantum gates used in this experiment, we perform simultaneous cross entropy benchmarking (XEB). The median Pauli errors are \( 1.3 \times 10^{-3} \) (9 \( \times 10^{-4} \)) for the single-qubit gates and \( 6.7 \times 10^{-3} \) (5.6 \( \times 10^{-3} \)) for the two-qubit gates on Processor I (II), with the experimental data shown in Fig. S7 j and Fig. S8.

After constructing the original circuit for generating the ground state with the method depicted in 1C, we use ZX-calculus [61] and Qiskit [62] to reduce the depth of the circuit and recompile it with single- and two-qubit gates selected from the gateset \{RX, RY, RZ, CZ\}, where RX, RY and RZ denote single-qubit rotations around the x-, y-, and z-axis in the Bloch sphere,
and CZ denotes the two-qubit CZ gate. Then we align all gates right while avoiding performing single- and two-qubit gates simultaneously. As a result, the optimized circuit consists of a staggered arrangement of single- and two-qubit gate layers. During the execution of the circuit, we apply CPMG sequences to minimize the impact of dephasing, which is realized by applying two Y gates (rotation around the y-axis by $\pi$ angle) on qubits whose successive idle layers span more than six single-qubit gate layers.
Extended Data Fig. S7. **Heatmaps of various single-qubit parameters for processor I and corresponding integrated histograms for processors I and II.**

- **a.** Qubit idle frequency.
- **b.** Qubit relaxation time measured at the idle frequency.
- **c.** Qubit dephasing time measured using Hahn echo sequence.
- **d.** Pauli error of the single-qubit gate.
- **e.** Readout fidelity of the qubit $|0\rangle$ state.
- **f.** Readout fidelity of the qubit $|1\rangle$ state.
- **g.** Corresponding integrated histograms for processor I (blue) and II (orange). Dashed lines indicate the median values.

Heatmaps show parameter values for different qubit states and processors, with integrated histograms displaying the distribution of these parameters.
Extended Data Fig. S8. **Integrated histograms of the two-qubit CZ gate Pauli errors on processors I and II with the sample sizes of 70 and 20, respectively.** Dashed lines indicate the median values.

Extended Data Fig. S9. **Synchronization of the flux bias pulses of qubit-coupler pairs.**

**a.** The directed acyclic graph (DAG) of the 68-qubit system constructed with the breadth-first search (BFS) algorithm, which is used to align the timing among all the flux bias pulses across the device. Each circle denotes a qubit indexed by the numbers inside. The calibration is performed with the order indicated by the direction of the arrows. b, Experimental pulse sequences for calibrating the timing between the flux bias pulses of a qubit-coupler pair. After exciting the qubit to $|1\rangle$ with a $\pi$ pulse and waiting for a fixed delay $t_c$, we apply a flux bias pulse with a length of around 5 ns on the coupler, which can transfer the photon from the qubit to the coupler when the qubit stays at the idle frequency. At the same time we apply a flux bias pulse with the same length on the qubit and adjust its beginning time $t_q$. c, The experimentally measured $|0\rangle$-state probability (dots) of the qubit vs. the time offset $\Delta t (= t_q - t_c)$. When the two flux bias pulses are synchronized, the qubit will stay at $|1\rangle$ state, resulting in a dip on the $P_0$ curve. We fit the location of dip to adjust the timing between the two pulses.
Extended Data Fig. S10. **Best individual CZ gate.**

**a**, Interleaved RB data with a sample size of \( n = 30 \).

**b**, Simultaneous XEB data to characterize the single-qubit gates.

**c**, XEB data to characterize the same CZ gate as in **a**, where each cycle contains two single-qubit gates in parallel and a CZ gate. The CZ Pauli errors extracted from RB and XEB are 0.173% and 0.160%, respectively.