Selective Harmonic Elimination of a Multilevel Voltage Source Inverter using Whale Optimization Algorithm

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ABSTRACT

In this paper, the whale optimization algorithm is proposed for harmonics elimination in a cascaded multilevel inverter. In selective harmonic elimination pulse width modulation, the selected low-order harmonics are eliminated by solving nonlinear equations, while the fundamental of output waveform is adjusted to a desired value. In this paper, whale optimization algorithm is applied to a 7-level cascaded H-bridge inverter to solve the equations. Also, it was validated by experimental results, since this algorithm has an ability to search in entire solution space, the probability of catching a global best solution is very high. This method has higher accuracy and probability of convergence than the genetic algorithm. The optimization and comparison of whale optimization algorithm and genetic algorithm have been done in MATLAB software. A 1 kW prototype of this converter is built and the results are presented. The effectiveness and the theoretical analysis of this method are verified through both simulation and experimental results.

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1. INTRODUCTION

Multilevel voltage source inverters (MVSI) have been widely applied in medium voltage (MV) and high power industry applications such as AC power supply, motor drives, HVDC, interconnection of distributed generation (DG) units to a grid and wind energy systems [1-3]. MVSI has several advantages compared to the traditional two-level inverter such as low harmonic contents, low switching stresses and losses, low electromagnetic interface (EMI), and high system efficiency. Owing to have low total harmonic distortion (THD), MVSI can follow the target signal more accurately. Moreover, the smaller harmonic filters are required and the insulation requirements are reduced [4]. For improving MVSI performance and quality of the output waveforms, different methods have been proposed such as sinusoidal pulse width modulation (SPWM) [5], optimal minimization of THD [6, 7], space-vector modulation (SVM) [8], and selective harmonic elimination pulse width modulation (SHEPWM) [9, 10].

SHEPWM has been paid a great attention for its ability to eliminate the specific low-order harmonic contents, and to leave triple harmonics uncontrolled to facilitate the advantages of the three phase system. Several works have been done to solve the nonlinear equations related to SHEPWM with different optimization methods such as genetic algorithm [11, 12], particle swarm optimization (PSO) [13], ant colony systems [14]. In these works, the objective function is defined and minimized. GA is widely used which is simpler and more applicable.

One of the best methods for solving optimization problems is whale optimization algorithm (WOA) which is proposed by several researchers [15-18]. Whale optimization algorithm is inspired by the bubble-net hunting strategy of the humpback whale. The trajectory path of the bubble-net attack mechanism of WOA is based on the shrinking circling mechanism on the spiral track. As a result, the probability of catching the global best solution is very high as it covers total space. The motion of the whale in the WOA is divided in two parts:
A linear direction (in shrinking part) with 50% probability and circled spiral direction by other 50% probability. Selection between these two parts is done or decided by a random number.

Some valuable research works are published related to the presented work in the current. WOA is implemented to decide optimum switching angles for three-phase Voltage Source Inverter of eliminating some high order harmonics while providing the required voltage [19]. Moreover, authors show that WOA gives faster and more accurate results in terms of decreasing THD than PSO algorithm in SHE applications. Unfortunately, no experimental results is reported in this paper to approve the claim. The selective harmonic elimination of an eleven level inverter using WOA was investigated by Dash et al. [16] and the results are satisfying the proposed method. However, the obtained results are not compared with the results from using another optimization method. A modified WOA is presented for harmonic elimination of a 11-level three-phase VSI [17]. The results showed a superior performance compared to the simple WOA results. An original asymmetrical half-cascaded multilevel inverter structure that its half-cascaded part has operated as zero-positive step creator, and then, H-bridge part provides the perfect staircase quasi-sinusoidal voltage [18]. The novel VSI has been optimized using WOA.

In this study, WOA is utilized to solve the nonlinear equations to eliminate the low-order harmonics while adjusting the fundamental harmonic to a desired value. The optimization results with WOA including the probability of catching a global minimum and optimum value of objective functions in three-phase and single-phase Cascaded Half-Bridge (CHB) inverters are compared with the results of GA. The results show the superiority of WOA to GA.

The features of the presented research work can be listed as:
1) Investigating a 7-level VSI structure,
2) Implementing selective harmonic elimination using WOA,
3) Comparing the WOA results with the GA results using Cumulative Distribution Function (CDF),
4) Investigating the WOA results for single-phase and three-phase MVSI structures,
5) Obtaining satisfying results for harmonic elimination using WOA in the proposed structures,
6) Implementing the optimum inverter to obtain the experimental results for evaluation purpose.

The main goal of presenting all the aforementioned features in the current study is to find an appropriate approach to reduce the size of the output filter for the MVSI especially in ultrasonic converter application.

The rest of this paper is organized as follows. The whale optimization algorithm is described at section II. Section III contains the problem formulation for multilevel inverters, the optimization results, and comparison of WOA with GA. Section V reports the experimental results of WOA algorithm for single-phase and three-phase 7-level cascaded H-bridge inverters.

2. METHOD

2.1. Whale Optimization Analysis

One of the nature-inspired meta-heuristic algorithm, which can be utilized in optimization problems, is whale optimization algorithm (WOA) that mimics the hunting behavior of humpback whales. The foraging behavior of these whales is called bubbled net feeding method. In this foraging method, the whale creates distinctive bubbles along a “9” shaped path around the prey and swims up toward the surface to capture it as illustrated in Figure 1. The process of hunting consists of three steps, 1) encircling, 2) bubble-net attacking, and 3) searching of a prey. In the following section, the mathematical model is briefly discussed.

2.1.1. Encircling of a Prey

The humpback whale can recognize the location of a prey and encircle it. After defining the best search agent, the whale tries to move toward this best agent. The mathematical description of this step is as follows [18]:

\[ \bar{D} = |\bar{C}, \bar{X}(i) - \bar{X}(i)| \]  
\[ \bar{X}(i + 1) = [\bar{X}(i) - A, \bar{D}] \]  

![Figure 1. Flowchart of whale optimization algorithm for optimally selecting (20)](image)
where \( i \) is the iteration’s number, \( \vec{A} \) and \( \vec{C} \) are coefficient vectors, \( \vec{X} \) is the position vector, \( \vec{X}^* \) is the position of the best agent, \( || \) is the absolute value of the vector, and \( \cdot \) is an element by element multiplication. The vectors \( \vec{A} \) and \( \vec{C} \) are obtained as follows [18]:
\[
\vec{A} = 2\vec{a}\vec{r} - \vec{a} \tag{3}
\]
\[
\vec{C} = 2\vec{r} \tag{4}
\]
where \( \vec{a} \) is linearly decreased from 2 to 0 over the iteration and \( \vec{r} \) is a random vector in \([0, 1]\).

2. 1. 2. Bubble-net Attack Method (Exploitation Phase) The whale swims around the prey within a shrinking circle and along a “9” shaped path simultaneously with 50% percent probability. The mathematical model for this attacking behavior is as follows [18]:
\[
\vec{X}(i+1) = \begin{cases} \vec{X}^*(i) - \vec{A}\vec{B} & \text{if } p < 0.5 \\ \vec{X}^*(i) - \vec{A}\vec{D} & \text{if } p > 0.5 \end{cases} \tag{5}
\]
where \( \vec{D} \) indicates the distance between the whale to the prey (best agent obtained so far), \( \vec{b} \) is a constant value, and \( l \) is a random number in \([-1, 1]\).

2. 1. 3. Search for Prey (Exploration Phase) According to the random coefficient vector \( \vec{A} \), WOA forces to search on a global level. When \( ||\vec{A}|| > 1 \), humpback whale starts foraging in the entire region. This step mathematically described as follows [18]:
\[
\vec{B} = |\vec{C} - \vec{X}_{\text{rand}}(i)| \tag{6}
\]
\[
\vec{X}(i+1) = \vec{X}_{\text{rand}}(i) - \vec{A}\vec{B} \tag{7}
\]
where \( \vec{X}_{\text{rand}} \) is a random position vector (a random whale) chosen from the current population. The WOA starts with a set of random solution. In every iteration, the search agents update their positions based on a randomly chosen search agent or the best solution so far.

2. 2. Harmonic Elimination using a WOA

2. 2. 1. Problem Formulation In this paper, without loss of generality, single-phase and three-phase 7-level CHB inverters are chosen as a case study which consists of three series connected H-bridge cells (shown in Figure 2) in each phase. Three different voltage levels (\( -V_s, 0, \) and \( V_s \)) can be generated by each cell. In order to implement SHEPWM, three switching angles (\( \alpha_1, \alpha_2, \) and \( \alpha_3 \)) are utilized as illustrated in Figure 3. The frequency of each cell is equal to the fundamental frequency (\( f_i \)). By these definitions, the output phase voltage of the 7-level CHB inverter (\( V_s \)) can be expressed using Fourier Transform as follows [17]:
\[
V_s = \sum_{n=1}^{50} (A_n \cos(n\omega t) + B_n \sin(n\omega t)) \tag{8}
\]
where \( \omega \) is the output angular frequency, and \( A_n \) and \( B_n \) are the coefficients of Fourier series. As it is shown in Figure 3, it can be concluded that the even harmonics are absent owing to the symmetry of the waveforms. Consequently, the coefficient \( A_n \) is zero. Therefore, Equation (8) declines to [18]:
\[
V_s = \sum_{n=1}^{50} B_n \sin(n\omega t) \tag{9}
\]
The coefficient \( B_n \) can be calculated as Equation (9) [16]:
\[
B_n = \left(\frac{4E}{\pi n}\right) \sum_{k=1}^{n-1} \alpha_k \cos(n\alpha_k), \ n = \text{odd} \tag{10}
\]
where \( E \) is the DC bus voltage of each cell, and \( \alpha_k \) is the switching angle of \( k^\text{th} \) cell. The modulation index is defined to be a representative value of the fundamental harmonic \( V_{1} \) [17]:
\[
m = \frac{\pi V_s}{12E}, \quad (0 \leq m \leq 1) \tag{11}
\]
In this study, the switching angles are found such that low-order harmonics are eliminated and the magnitude of the fundamental harmonic reaches to the desired value. In a single-phase CHB, the low-order harmonics are third and fifth harmonics. Therefore, a general objective function \( F_{SP}(\alpha) \) of the optimization problem is defined as follows:
\[
F_{SP} = \left| 100 \frac{B_{3m}}{m} \right|^4 + \left| \frac{1550}{3} \frac{B_{5m}}{V_s} \right|^2 + \left| \frac{50}{3} \frac{B_{5m}}{V_s} \right|^2 \tag{12}
\]
In three-phase CHB, the triple harmonics are not existed in line voltages. In this case, the objective function can be written to eliminate the fifth and seventh harmonics as below:

\[ F_{TP} = \left[ 100 \frac{B_k - m}{m} \right]^4 + \frac{1}{5} \left[ 50 \frac{B_k}{n_i} \right]^2 + \frac{1}{7} \left[ \frac{50 B_k}{n_i} \right]^2 \]  \hspace{1cm} (13)

The objective function \( F_{SP} \) and \( F_{TP} \) are subjected to the following constraint:

\[ 0 \leq \alpha_k \leq \frac{\pi}{2}, \quad k = 1, 2, 3 \]  \hspace{1cm} (14)

According to Equations (12) and (13), for any variation of fundamental component form desired value lower than \( \%1 \), the first term gets a negligible value. Although, if its variation is more than \( \%1 \), the first term of these equations fines it by a power of 4. Other terms neglect harmonics under \( 2 \% \) of the fundamental. If any harmonic gets value more than this limit, the objective function fines it by power of 2. Finally, by weighting each harmonic by inverse of its order, elimination of low-order harmonics gets higher importance. The more detailed mathematical model for harmonic elimination using WOA is presented in previous published research works [15-18].

3. RESULTS

3.1. Solving SHEPWM Equations  As mentioned in the previous section, the whales search for the prey with random initial positions. Then they move toward the best or random search agent in the search area. The WOA is considered as a global optimizer because it has the exploring and exploiting abilities. In addition, this algorithm defines a search space in the nearby of the best solution which allows other search agents to exploit the current best solution inside this domain. To solve the optimization problem discussed in the previous section, WOA is written in MATLAB software. The size of population is 100. For each run the number of iteration is 100.

Figure 3 shows the optimum objective functions for different values of \( m \) by step of 0.01 applied in 7-level single-phase and three-phase CHB inverters. The objective functions are defined in a way that the variation of fundamental harmonic from desired value is lower than \( 1\% \) and two low-order harmonics are lower than \( 2\% \) of the fundamental. As a result, if the objective function has a low value, the result is satisfactory and acceptable. As it is illustrated in Figure 3, for a range of [0.3, 0.85], the optimum objective functions have small values. Therefore, WOA can successfully find the optimum switching angles. For other range of \( m \), the SHEPWM does not have perfect results.

Optimum switching angles are illustrated in Figure 4. As the desired fundamental harmonic increases (\( m \) increases), switching angles are shifted to the origin so that regulates the fundamental harmonic to desired value. For low value of modulation index, all switching angles have a value close to 90 degree. In addition, as it is illustrated in Figure 4, for all set of switching angles the constraint of optimization problem is satisfied. Therefore, it can be concluded that WOA is a reliable algorithm for solving SHE equations.

Low-order harmonics and THD level are calculated by Equation (10) and the results are depicted in Figure 5. It is expected that fundamental harmonic regulates completely due to using a penalty for first harmonic. Other harmonics and THD level have a negligible value when objective function is closed to zero. Although, WOA is not able to eliminate harmonics in a low value of \( m \).

3.2. Comparison  As mentioned in the previous section, WOA is considered as a global optimizer. In order to show this ability, results of WOA algorithm are compared with the results of GA for solving SHEPWM equations. Figure 6(a) shows the optimum values of \( F_{SP} \) obtained by these algorithms. It can be noted that, in a range of [0.55, 0.7], WOA algorithm has suitable results. For three-phase applications, Figure 6(b) shows that WOA finds better solutions for a range of [0.4, 0.85]. As a matter of fact, it is pointed out that WOA can find the global minimum. In these cases, GA finds the local minimum. In order to compare the probability of finding global minimum using WOA and GA, the cumulative distribution function (CDF) is defined as follows [19]:

\[ CDF(x) = P(X < x) \]  \hspace{1cm} (15)

Figure 4. Switching angles versus modulation index \( m \) (a) for a single-phase structure (b) for a three-phase structure

Figure 5. Fundamental, two low-order harmonics and THD versus \( m \) (a) single-phase CHB (b) three-phase CHB.
Figure 6. Comparison between the answers of WOA and GA versus m: (a) to minimize $F_{SP}$, (b) to minimize $F_{TP}$.

For a given probability distribution, the probability of having a value less than or equal to $x$ for a random variable $X$ is called cumulative distribution function (CDF). The CDF curve shows the probability of reaching to a value lower or equal to a specified level of objective function. Figure 7 illustrates CDF curve of WOA and GA to solve Equations (12) and (13). It is clear that CDF of WOA is above of CDF of GA. Therefore, WOA is more likely to converge comparing to GA. As a case in point, CDF ($10^{-17}$) of WOA is 14% and CDF ($10^{-7}$) of GA is 2% for a single-phase CHB, and CDF ($10^{-7}$) of WOA is 48% and CDF ($10^{-7}$) of GA is 22% for a three-phase CHB.

3.3 Experimental Results

A 1kW 7-level CHB inverter is constructed in laboratory as illustrated in Figure 8. It consists of three H-bridge inverters that are connected in series. The main parameters of the systems are shown in Table 1. It is assumed that each cell has a constant DC voltage 25V. The output voltage frequency is 50Hz. Isolated-gate bipolar transistor (IGBT) switches have been employed as power switches in this structure. In this prototype, the SHEPWM modulation is implemented over a Spartan-3 field-programmable gate array (FPGA). The switching angles which are obtained offline by WOA, are loaded in a FPGA processor as a lookup table. Therefore, for each modulation index m, the processor finds the optimum switching angles from this lookup table. For isolation of the power circuit from the control board, optocoupler 6N137 have been used. At the end, the isolated signals are reached to the power switches by IGBT driver HCPL-3120 to provide the amount of charges required to turn on IGBT switches.

To validate the results of WOA, two operation points are implemented in single-phase and three-phase CHB inverters when the modulation index are $m=0.6$ and $m=0.8$. The switching angles for each case are tabulated in Table 2. Figures 9 and 10 show the output phase voltage of CHB inverter and frequency spectrum in the single-phase and three-phase CHB inverters. It can be noted that, the third and fifth harmonics in single-phase CHB, and fifth and seventh harmonics in three-phase CHB, are completely eliminated that confirms the results of WOA. Figure 11 shows the phase voltage of single-phase CHB when a step change in the modulation index is applied.

Table 1. Parameters of the 7-level CHB inverter

| Parameters                  | Sym. | Value                           |
|-----------------------------|------|---------------------------------|
| Number of cell per phase    | $N$  | 3                               |
| DC voltage of cell          | $E$  | 25 [V]                          |
| Switching frequency         | $f_{sw}$ | 50 kHz                       |
| IGBT switches               | $S$  | IKW40N120H3                     |

Table 2. Switching angles for $m = 0.8$

| 7-level CHB | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ |
|-------------|-------------|-------------|-------------|
| Single phase| 19.998297   | 20.183009   | 58.349913   |
| Three phase | 11.504323   | 28.713562   | 57.104075   |

Figure 8. The constructed 7-level CHB inverter: A- CHB inverter B- FPGA processor and interface C- DC voltage sources D-auxiliary voltage sources
5. CONCLUSION

In this paper, the whale optimization algorithm has been used to solve nonlinear equations of SHEPWM for multilevel inverters. It is shown that WOA has an ability to find a global solution for two objective functions. Comparison of the optimization results of WOA and the results of GA proves that this method has higher accuracy and probability of convergence than GA and shows the superiority of WOA to GA. A 1 kW prototype of 7-level CHB inverter is built and the results are presented. The low-order harmonics are eliminated completely and the first harmonic is adjusted to the specific value.

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Persian Abstract
چکیده
در این مقاله الگوریتم بهینه سازی هارمونیک در یک اینورتر پل H به کار می‌رود. الگوریتم بهینه سازی هارمونیک انتخابی در حل معادلات غیر خطی هارمونیک و انتخاب کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در این مقاله به کار رفته در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی کارایی مقدار دلخواه ترکیب شده است. الگوریتم بهینه سازی هارمونیک در حل معادلات غیر خطی به بهینه‌سازی кар

اریکشی و تجزیه و تحلیل نظری این روش از طریق نتایج شبیه‌سازی و تجربی تایید می‌گردد.