Trajectory Dependent Structural Vibration and Its Effects on Surface Generation

Mengxiang Yang¹, Xinyong Mao¹*, Yili Peng¹, Bin Li¹,², Xuchu Jiang¹ and Liangjie Li¹

¹National NC System Engineering Research Centre, Huazhong University of Science and Technology, Wuhan 430074, PR China
²State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan 430074, PR China

*Correspondence E-mail: maoxyhust@hust.edu.cn, maoxyhust@163.com (X. Mao).

Abstract. The vibration of machine tool plays an important role in surface generation. However, the vibration characteristics is different for different machine tool structures and different machining path. This paper proposed a new method of structural sensitivity analysis to identify trajectory dependent structural vibration and its effect on surface generation in milling. The experimental and theoretical results verify that (i) Multimode low-frequency vibration of the sensitive structures (MLFVSS) directly determines the surface topography in milling; (ii) The sensitivity is different for different structures or the same structure at different modes; (iii) Structural vibration characteristics is related to the machining path, resulting in different influence of the structural vibration on surface generation at different position of the machining path. Finally, a surface topography prediction model was established based on the experimental results. This study is helpful to the optimization of machine tool structures and the selection of machining parameters.

1. Introduction
The surface quality of products reflects the machining accuracy, but also directly affects the products' functions. In recent years, researchers have explored a series of factors influencing the surface generation, including machining conditions [1], material swelling and recovery [2, 3], crystal orientation [4], the vibration of machine tool [5, 6], etc. There are two primary sources of affecting the surface generation caused by machine tool vibration: one is the high frequency vibration source, mainly refers to cutting tool vibration; the other is the low frequency vibration source, mainly refers to the structural vibration.

However, the above researches considered the high frequency vibration of the cutting tool, usually higher than 10000Hz. While the structural low-frequency vibration has not been taken into account.

On the other hand, the vibration characteristics is different for different structures which will cause different influences on surface generation. Chen, G., et al. carried out sensitivity analysis in the design of five-axis ultra-precision machine tool to find out the sensitive structures causing the machining error.

This paper proposed a new method to identify the trajectory dependent structural vibration and its effect on surface generation in milling based on structural sensitivity analysis.
2. Theory of sensitivity analysis

Stiffness is different for different structures due to the existence of dynamics joints. In this part, a theoretical derivation of modal mass distribution matrix (MMDM) is performed which can reflect the structural vibration energy. If the local amplitude is higher in the MMDM, it means that the corresponding components are the sensitive structures. The following is the theoretical derivation:

For general machine tool system, the modal mass of \( i \)th mode can be derived:

\[
m_i = \{ \varphi_i^T \} \begin{bmatrix} M \\ \vdots \\ 0 \end{bmatrix} \{ \varphi_i \} \tag{1}
\]

Then Eq. (1) can be replaced with:

\[
m_i = \begin{bmatrix} M_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & M_n \end{bmatrix} \begin{bmatrix} \varphi_{i1} \\ \vdots \\ \varphi_{in} \end{bmatrix} = \sum_{k=1}^{n} M_k \varphi_{ik} \tag{3}
\]

Then all modal mass values can form a matrix in spatial distribution called modal mass distribution vector (MMDV):

\[
\{ m'_i \} = \begin{bmatrix} M_1 \varphi_1^2 & M_2 \varphi_2^2 & \cdots & M_n \varphi_n^2 \end{bmatrix}^T = \begin{bmatrix} M \end{bmatrix} \{ \varphi_i \} \ast \{ \varphi_i \} \tag{4}
\]

Where \( \ast \) represents an overlapping multiplication operator. \( \{ m_i \} \) represents a vector consisting of a column elements of the modal mass matrix. All MMDV constitute the MMDM:

\[
\{ m' \} = \begin{bmatrix} \{ m'_1 \} & \{ m'_2 \} & \cdots & \{ m'_n \} \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \{ \varphi \} \ast \{ \varphi \} \tag{5}
\]

Where \( \{ \varphi \} \) is the scaled mode shape matrix under unit modal mass (UMM) principles.

As shown in Eq. (5), the modal mass expressions is similar to the kinetic expression. If the kinetic energy is mainly concentrated in some local DOFs, it can be deduced that the corresponding structure of those DOFs is the sensitive component.

3. Verification of the MLFVSS method

Before implementation, 0-400Hz was selected as the interest and analysis frequency band. In order to validate the MLFVSS method, the following steps need to be implemented.

Firstly, experimental modal analysis (EMA) was conducted through an impact testing to identify the natural frequencies of the whole machine tool structures between 0Hz and 400Hz.

Secondly, sensitivity analysis was implemented to identify the sensitive structures at these natural frequencies through the modal mass distribution matrix proposed in section 2. Thirdly, a trajectory-related milling experiment was carried out and the surface topography was measured to validate the trajectory dependent structural vibration and the MLFVSS method.

3.1. Impact testing setup

In this part, an impact testing was conducted on a milling machine tool (model number TC500). The experimental machine tool, impact point and measurement points as shown in Figure 1.
3.2. EMA and sensitivity analysis

In this part, the natural frequencies of the whole machine tool structures between 0-400Hz were estimated through EMA using the LMS PolyMAX algorithm. Then the sensitive structures were identified at each natural frequency based on MMDM. Table 1 shows the natural frequencies of the TC500 machine (0-400Hz).

| Mode | Frequency (Hz) |
|------|---------------|
| 1    | 151.6         |
| 2    | 225.9         |
| 3    | 289.5         |
| 4    | 315.7         |
| 5    | 330.9         |
| 6    | 381.6         |

Figure 1. Impact testing system

Figure 2. The modal mass distribution of the first six modes in table 1
Figure 2 is the modal mass distribution of the first six modes in Table 1 respectively, where the ‘Amplitude’ is the value of the modal mass matrix $m_i$, ‘n’ represents the corresponding DOFs of measuring points. In the experiment, 21 measurement points were selected and each point is composed of three DOFs (X, Y, Z). The No. 1 to No. 12 DOFs belong to the worktable, No. 13 to No. 30 DOFs belong to the column, No. 31 to No. 45 DOFs belong to the X-slide, No. 46 to No. 63 DOFs belong to the headstock. The sensitive structures of the TC500 machine are worktable and X-slide.

3.3. Verification of MLFVSS method

3.3.1. Trajectory-related milling experiment. Figure 3a shows the cutting experiment device and data acquisition system. In order to better reflect the influence of structural vibration on the surface topography, a curved trajectory and flank milling were adopted (as shown in Figure 3b). The spindle speed and feed rate were set to 2200rpm and 1500mm/min respectively. The transverse cutting depth was set to 0.1mm per cycle.

![Image](3a)

![Image](3b)

**Figure 3.** The photograph of milling experiment and work piece

3.3.2. Verification of trajectory dependent structural vibration. The time-varying cutting direction and cutting force at different positions will cause different performance of the structural sensitivity, resulting in different influence of the structure vibration on surface topography. Figure 4 and Figure 5 show the local and filtered surface topography map of the work piece at position 1 and position 2, respectively. From the comparison of these two pictures, it can be seen that there are some regular ‘small peaks’ in the surface topography of the work piece in Figure 5, separated by the green dashed line (as shown in Figure 5), but not in Figure 4. Through measurement, the width of the ‘small peaks’ in the surface topography is between 0.07mm-0.09mm. According to the feed rate ($F=1500\text{mm/min}$), the frequency reflected by the ‘small peaks’ is 277.8Hz to 357.1Hz. Therefore, a conclusion can be inferred that the ‘small peaks’ are caused by structure vibration, rather than the tool vibration (usually higher than 10000Hz).

![Image](4)

**Figure 4.** The filtered surface topography map of

![Image](5)

**Figure 5.** The filtered surface topography map
the work piece at position 1 of the work piece at position 2

Through the comparison of Figure 4 and Figure 5, it can be deduced that the effect of structural vibration on surface topography is different at different machining positions. Figure 6 shows the single path processing responses of work-piece in X and Y direction. From the comparison of work-piece responses at position 1 and position 2, the amplitude of the response signal at position 2 is higher and more complex than at position 1. The result indicate that the structure sensitivity of the machine tool is related to the machining trajectory, resulting in different performance of the structure vibration on surface topography at different machining position.

![Figure 6](image_url)

**Figure 6.** The single path processing responses of workpiece in X and Y direction

3.3.3. **Verification of MLFVSS method.** In order to validate the proposed method that the multimode low-frequency vibration of the sensitive structures (MLFVSS) has a significant impact on the surface topography. The FRFs signals of different structures of the TC500 machine needed to be compared. Figure 7 shows the comparison of FRFs of different structures. As shown in Figure 7, the sensitive structures (worktable, X-slide) have higher amplitude rather than non-sensitive components (headstock, column) at the natural frequencies in Table 1, indicated that the multimode low-frequency vibration of the sensitive structures (MLFVSS) directly determines the surface topography in actual processing.

![Figure 7](image_url)
Figure 7. The comparison of FRFs of different structures of TC500 machine.

4. Conclusions
In this paper, a new approach identifying the structure vibration on surface topography in milling machining was proposed combined with the method of structure sensitivity analysis. According to the trajectory-related milling experiments on TC500 machine, this new method was validated and a mathematical model to predict the products’ surface topography in milling machining was established.

The following is the conclusions and highlights:

(1) This new method combined structure sensitivity analysis with surface topography prediction.
(2) Multimode low-frequency vibration of the sensitive structures (MLFVSS) directly determines the surface topography in milling machining.
(3) The sensitivity is different for different structures at the same mode or the same structure at different modes.
(4) The structure sensitivity is related to the machining trajectory, resulting in different influence of the structure vibration on surface topography.
(5) The surface topography prediction model in milling was related to the feed rate and the natural frequencies of sensitive structures.

References
[1] Lee, W.B. and C.F. Cheung, A dynamic surface topography model for the prediction of nano-surface generation in ultra-precision machining. International Journal of Mechanical Sciences, 43(4): p. 961-991 (2001).
[2] Liu, K. and S.N. Melkote, Effect of plastic side flow on surface roughness in micro-turning process. International Journal of Machine Tools and Manufacture, 46(14): p. 1778-1785 (2006).
[3] To, S., et al., Influence of material swelling on surface roughness in diamond turning of single crystals. Materials Science and Technology, 17(1): 102-108 (2001).
[4] C.F. Cheung, S. To, W.B. Lee, Anisotropy of surface roughness in diamond turning of brittle single crystals, Materials and Manufacturing Processes, 17(2): 251–267 (2002).
[5] Zhang, S.J., et al., Dynamic characteristics of an aerostatic bearing spindle and its influence on surface topography in ultra-precision diamond turning. International Journal of Machine Tools and Manufacture, 62: p. 1-12 (2012).
[6] Kim DS, Chang IC and Kim SW. Microscopic topographical analysis of tool vibration effects on diamond turned optical surfaces. Precis Eng, 26(2): 168–174 (2006).