A CRITERION FOR PHOTOIONIZATION OF PREGALACTIC CLOUDS EXPOSED TO DIFFUSE ULTRAVIOLET BACKGROUND RADIATION

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ABSTRACT

To elucidate the permeation of cosmic ultraviolet (UV) background radiation into a pregalactic cloud and the subsequent ionization, the frequency-dependent radiative transfer equation is solved, coupled with the ionization process, for a spherical top-hat cloud composed of pure hydrogen. The calculations properly involve scattering processes of ionizing photons that originate from radiative recombination. As a result, it is shown that the self-shielding, although it is often disregarded in cosmological hydrodynamic simulations, could start to emerge shortly after the maximum expansion stages of density fluctuations. Quantitatively, the self-shielding is prominent above a critical number density of hydrogen, which is given by $n_{\text{crit}} = 1.4 \times 10^{-2} \text{ cm}^{-3} \left( M/10^8 \text{ M}_\odot \right)^{-1/3} I_{21}^{1/2}$ for $10^4$ K gas, where $M$ is the cloud mass and the UV background intensity is assumed to be $I = 10^{-21} I_{21} (v/v_L)^{-1}$ ergs cm$^{-2}$ s$^{-1}$ sr$^{-1}$ Hz$^{-1}$, with $v_L$ being the Lyman limit frequency. The weak dependence of $n_{\text{crit}}$ upon the mass is worth noting. The corresponding critical optical depth ($\tau_{\text{crit}}$) turns out to be independent of either $M$ or $I_{21}$, which is $\tau_{\text{crit}} = 2.4$ for $10^4$ K gas. The present analysis reveals that the Strömgren approximation leads to overestimation of the photoionization effects. Also, the self-shielded neutral core is no longer sharply separated from surrounding ionized regions; a low but noticeable degree of ionization is caused by high-energy photons even in the self-shielded core. The present results may be substantial when one considers the biasing by photoionization against low-mass galaxy formation.

Subject headings: cosmology: theory — diffuse radiation — galaxies: formation — radiative transfer

1. INTRODUCTION

Recently, a recalcitrant problem on galaxy formation has been pointed out: that low-mass galaxies are overproduced as compared with observations in the context of the hierarchical bottom-up theory of galaxy formation (e.g., White & Frenk 1991; Kauffmann, White, & Guiderdoni 1993; Cole et al. 1994). Hence, some process that inhibits the formation of low-mass galaxies is required. Photoionization has been considered as one such mechanism (Dekel & Rees 1987; Babul & Rees 1992; Efstathiou 1992; Chiba & Nath 1994; Thoul & Weinberg 1996; Quinn, Katz, & Efstathiou 1996). In photoionized media, the cooling efficiency is dramatically reduced in the temperature range of $10^4$ K $< T < 10^5$ K. Also, a bulk of energy could be carried into a cloud, so that the enhanced thermal pressure could suppress the gravitational collapse of a subgalactic cloud with the virial temperature lower than several times $10^4$ K, i.e., $M \lesssim 10^5 \text{ M}_\odot$ in gas mass (Umemura & Ikeuchi 1984, 1985; Ikeuchi 1986; Rees 1986; Bond, Szalay, & Silk 1988; Steinmetz 1995; Thoul & Weinberg 1996). In cosmological hydrodynamic simulations, an optically thin medium against ionizing photons has been mostly assumed so far (Umemura & Ikeuchi 1984, 1985; Thoul & Weinberg 1996; Quinn et al. 1996). In semianalytic approaches, naive analytic corrections for opacity effects have been made, based on the optical depth criterion (Efstathiou 1992) or the Strömgren approximation (Chiba & Nath 1994). In the case of interstellar clouds, it is claimed that the Strömgren approximation could be misleading in regard to the real effect of the penetration of diffuse UV (Flannery, Roberge, & Rybicki 1980; Maloney 1993), and therefore the radiative transfer equation should be solved properly. However, as far as we know, no study has hitherto been made on determining the ionization structure inside a pregalactic cloud by solving a radiative transfer equation for diffuse UV photons. Hence, the effects of photoionization on the evolution of a pregalactic cloud have not been assessed satisfactorily. In this paper, we solve the radiative transfer equation for diffuse UV radiation coupled with an ionization process to elucidate the self-shielding of pregalactic clouds from UV background radiation and provide a practical criterion for the self-shielding.

2. RADIATIVE TRANSFER WITH AN IONIZATION PROCESS

We assume for simplicity a spherical top-hat (uniform) density distribution of a cloud and place 100 radial meshes for solving radiation transfer. Also, the cloud is assumed to be composed of pure hydrogen so that we could readily compare the results of the frequency-dependent radiative transfer with an analytic estimate, although we should keep in mind that helium of cosmic abundance could alter the ionization degree maximally by the order of 10% (Osterbrock 1989; Nakamoto, Sasa, & Umemura 1998).

The intensity of UV background radiation at high redshifts is inferred from the so-called proximity effect of Lyman-$\alpha$ absorption lines in QSO spectra (Bajtlik, Duncan, & Ostriker 1988; Giallongo et al. 1996). The observations require the diffuse UV radiation to be at a level of $I_{\nu,0} = 10^{-21} \pm 0.5$ ergs cm$^{-2}$ s$^{-1}$ sr$^{-1}$ Hz$^{-1}$ at the hydrogen Lyman edge at $z = 1.7–4.1$. In this paper, we assume the specific intensity of the UV background as $I_{\nu,0} = 10^{-21} I_{21} (v/v_L)^{n}$ ergs cm$^{-2}$ s$^{-1}$ sr$^{-1}$ Hz$^{-1}$, where we set $n = -1$ and vary $I_{21}$ in the range $0.1 < I_{21} < 2$. As for the ionization process, we presume the ionization balance, because the ionization or recombination timescale for clouds with density of interest is much shorter than the...
The equation of the ionization balance is

\[ \Gamma_{\chi H I} + \Gamma^c \chi_{HI} (1 - \chi_{HI}) n = \alpha_c(T)(1 - \chi_{HI})^2 n, \]  

where \( \chi_{HI} \) is the fraction of neutral hydrogen, \( \Gamma^c \) is the photoionization rate, \( \Gamma^s \) is the collisional ionization rate, \( n \) is the hydrogen number density, and \( \alpha_c(T) \) is the total recombination coefficient to all bound levels of hydrogen. \( \Gamma^s \) is given by

\[ \Gamma^s = \int_{0}^{\infty} \frac{d\Omega}{\omega} \int_{0}^{4\pi} \frac{d\Omega}{\omega} \frac{I(\tau)}{\hbar \nu} a_\nu, \]  

with the photoionization cross section being \( a_\nu = 6.3 \times 10^{-18} (\nu_y/\nu)^3 \) cm\(^2\), where the local UV intensity \( I(\tau) \) at radius \( \tau \) is determined by solving a transfer equation. The value \( \Gamma^s \) is given by \( \Gamma^s = 1.2 \times 10^{-3} T_4^{1/2} e^{-15.8/T_4} \) cm\(^3\) s\(^{-1}\), with \( T_4 \equiv T/10^4 \) K. We assume \( T_4 = 1 \) except where other values are specified. The value \( \alpha_c(T) \) is well fitted by \( \alpha_c(T) = 2.1 \times 10^{-13} T_4^{1/2} \phi(16/T_4) \) cm\(^3\) s\(^{-1}\), where \( \phi(y) = 0.5(1.7 + \ln y + 1/6y) \) for \( y \geq 0.5 \) or \( y(-0.3 + 1.2 \ln y) + y^2(0.5 - \ln y) \) for \( y < 0.5 \) (Sherman 1979).

Photoionization and recombination processes can be regarded as extinction and emission, respectively, with respect to ionizing photons. Thus, the radiative transfer equation for ionizing photons is described as

\[ \frac{dI_\nu}{ds} = -\chi_\nu I_\nu + \eta_\nu, \]  

where \( \chi_\nu \) is the extinction coefficient (\( \chi_\nu = a_\nu n_{HI} \)) and \( \eta_\nu \) is the emissivity. If a free electron recomines directly to the ground state of hydrogen, the emitted photon has enough energy to cause further photoionization. But, if an electron is captured to an excited state of hydrogen, the emitted photon does not have enough energy to ionize hydrogen, because the kinetic energy of a free electron is of order 0.1 ryd for \( \sim 10^4 \) K gas. Thus, the former process is regarded as scatterings that provide the emissivity, while the latter process is pure absorption. The effective scattering albedo is given by \( \omega = (\alpha_c(T) - \alpha_p(T))/\alpha_c(T) \), which is 0.4 at \( 10^4 \) K (e.g., Osterbrock 1989), where \( \alpha_p(T) \) is the recombination coefficient to all excited levels of hydrogen. Hence, we set \( \eta_\nu = 0 \) for \( \nu > \nu_L \) and \( \eta_\nu = h\alpha_{HI} n_e n_p/4\pi\delta v \) for \( \nu = \nu_L \), where \( \delta v = \kappa T_h / \hbar \) and \( n_p \) is the proton number density.

Taking into account the frequency dependence of the emergent UV intensity, it is convenient to divide the photoionization rate into two parts: \( \Gamma^s = \Gamma^s_{\nu \nu} + \Gamma^s_{\nu \nu_L} \). As for Lyman limit photons, the transfer equation is the integro-differential equation, which includes a source term due to the scattering processes of recombination photons. Thus, the equation is numerically solved by including the iterative procedure. Without invoking an on-the-spot approximation, which could be misleading for large mean free path photons, we solve the equation by means of an impact-parameter method of high accuracy, which allows us to treat diffuse photons correctly (Stone, Mihalas, & Norman 1992). We deal with 156 impact parameters for light rays. Also, in order to converge the intensity, we employ the lambda-iteration method (e.g., Mihalas & Mihalas 1984, p. 366).

In the range of \( \nu > \nu_L \), since photoionization is regarded as pure absorption, the UV intensity is obtained by \( I_\nu = I_{\nu,0} e^{-\tau_\nu} \) alone, where \( I_{\nu,0} \) is the boundary intensity and \( \tau_\nu \) is the ionization optical depth. So, \( \Gamma^s_{\nu \nu_L} \) is determined by

\[ \Gamma^s_{\nu \nu_L} = \int_{0}^{\infty} \frac{d\Omega}{\omega} \int_{0}^{4\pi} \frac{d\Omega}{\omega} \frac{I_{\nu,0} e^{-\tau_\nu}}{\hbar \nu} a_\nu \frac{(\nu_L/\nu)^3}{\nu L}. \]  

Since the optical depth is \( \tau_\nu = \tau_{\nu L}(\nu_L/\nu)^{2} n_{HI} \) with Lyman limit optical depth \( \tau_{\nu L} \) and the assumed boundary intensity is \( I_{\nu,0} = I_{\nu L}(\nu_L/\nu)^{2} \), the above equation can be analytically integrated using the incomplete gamma function \( \gamma \) if \( \chi_{HI} \) is given:

\[ \Gamma^s_{\nu \nu_L} = \int_{0}^{\infty} \frac{d\Omega}{\omega} \frac{a_\nu I_{\nu L}}{\hbar} \frac{\gamma(4/3, \tau_{\nu L})}{3(\nu L)^{3}}. \]  

With this integration, the overall procedure to solve the transfer equation is as follows:

1. Initially, give the cloud mass \( M \) and the radius \( R \) (therefore the density), and derive \( \chi_{HI} \) by assuming an optically thin medium. Specify the boundary UV intensity by \( I_{21} \).
2. Solve the transfer equation at \( \nu = \nu_L \) for a given \( \chi_{HI} \), and calculate \( \Gamma^s_{\nu \nu_L} \). Obtain \( \Gamma^s_{\nu \nu_L} \) analytically by equation (5). Then determine the total photoionization rate \( \Gamma^s \).
3. Solve for the ionization equilibrium using \( \Gamma^s \), obtained above, and thereby redefine \( \chi_{HI} \).
4. Continue steps 2 and 3 until \( \chi_{HI} \) converges at a level of relative error of \( 10^{-6} \). (Typically 100 iterations are performed.)

Note that the analytical integration (eq. [5]) with respect to frequencies enables us to reduce the computational cost dramatically on solving such a frequency-dependent transfer equation, including scatterings. The validity of this method is confirmed by solving the transfer equation exactly, using a number of meshes for frequencies.

3. NUMERICAL RESULTS

We consider the cloud mass range \( M = 10^{5} - 10^{9} M_\odot \) and vary the radius in the range of \( R = 0.1 - 25 \) kpc. Figure 1 shows the growth of self-shielded regions when a cloud of \( 10^{8} M_\odot \) is contracting, embedded in the UV background of \( I_{21} = 1 \). The self-shielding is prominent when \( R < 4 \) kpc. It is noted that the distributions of the H i fraction are a gradual function of radii even in the self-shielded stage, where we cannot recognize a clear boundary between the neutral core and the ionized envelope; also a low but noticeable ionization is left in the self-shielded regions. Such distributions seem to be realized by the high-frequency photons far above the Lyman edge, which could penetrate into deeper regions because of the smaller cross section for ionization. To ensure this conjecture, we solve the transfer equation solely for the Lyman edge photons to obtain \( I_{\nu L}(\tau) \) and tentatively set the form of UV radiation spectrum to be \( I_{\nu L}(\tau) = I_{\nu L}(\tau)(\nu_L/\nu)^{2} \) at any radius (which implies that photons of higher frequencies are absorbed with the same ionization cross section as that at the Lyman edge). We can see an outstanding difference between the two cases, as shown in Figure 1. In the tentative case, we can see a steep inward increase of neutral fraction and a very sharp transition from ionized regions to neutral regions.

Figure 2 shows the neutral hydrogen fraction at the center as a function of cloud size. We see that the central \( \chi_{HI} \) varies abruptly at a certain critical size. Here we define the critical radius (\( R_{crit} \)) at which the H i fraction at the center...
drops just below 0.1. In Figure 3, we plot the critical radii obtained from all the numerical results as a function of UV background intensity. Different symbols represent different masses of gas clouds. All the results can be remarkably well fitted by a simple formula, which is a function of the cloud mass and the UV background intensity:

\[ R_{\text{crit}} = 4.10 \text{kpc} \left( \frac{M}{10^8 \text{M}_\odot} \right)^{2/5} I_{21}^{-1/5}. \]  

Equivalently, the corresponding critical number density of the cloud is

\[ n_{\text{crit}} = 1.40 \times 10^{-2} \text{ cm}^{-3} \left( \frac{M}{10^8 \text{M}_\odot} \right)^{-1/5} I_{21}^{1/5}. \]  

It is worth noting that the critical density is quite weakly dependent upon \( M \). When the cloud is highly ionized and optically thin, the neutral fraction should be \( \chi_{\text{H}_1,0} = 0.15n_{\text{crit}} \), if collisional ionization is neglected. Then, we define the critical optical depth \( \tau_{\text{crit}} \) at the Lyman edge as a measure in such a way that the self-shielding becomes effective: \( \tau_{\text{crit}} \equiv n_{\text{crit}} \chi_{\text{H}_1,0} a_{\text{H}_1} R_{\text{crit}} = 2.4 \), which turns out to be independent not only of \( M \), but also of \( I_{21} \). Hence, such a simple criterion of \( \tau = 1 \), as adopted by Efstathiou (1992), is found to be heuristically practical for assessing the self-shielding for \( \sim 10^4 \text{K} \) clouds.

For clouds of different temperature, we have found that \( R_{\text{crit}} \) is scaled by \( a_{\text{H}_1}(T)^{1/5} \), and therefore \( n_{\text{crit}} \propto a_{\text{H}_1}(T)^{-3/5} \). Resultantly, \( \tau_{\text{crit}} \) is scaled by \( a_{\text{H}_1}(T)/a_{\text{H}_1}[1 - (1 - \omega)^{-1}] \). Both \( a_{\text{H}_1}(T) \) and \( a_{\text{H}_1}(T) \) are decreasing functions of temperature, but \( \omega \) increases with temperature, so that \( \tau_{\text{crit}} \) is larger for higher temperature. For instance, \( \tau_{\text{crit}} = 2.6 \) for \( 3 \times 10^4 \text{K} \). In an extreme case of infinite temperature (although unrealistic), the complete scattering \( (\omega = 1) \) leads to \( \tau_{\text{crit}} = \infty \). In other words, clouds with any optical depth can be ionized because of photon diffusion.

4. COMPARISON WITH ANALYTIC ESTIMATES

Here, we try to analytically estimate the critical radius based upon the Strömgren approximation. (A similar estimate is found in Chiba & Nath 1994.) By equating the number per unit time of ionizing photons that enter from the surface to the number per unit time of photons that are absorbed in the cloud, we have \( R_{\text{H}_1} = \left[ R^3 - 3\pi R^2 I_{\text{L},0}/h n^2(1 - \chi_{\text{H}_1})^2 \xi_{\text{H}_1}(T) \right]^{1/3} \), where the ionized regions are assumed to be sharply separated from the neutral core of radius \( R_{\text{H}_1} \). Then, the critical radius can be estimated by setting \( R_{\text{H}_1} = 0 \); resultantly, we find \( R_{\text{crit}} = 3.5 \text{kpc}(M/10^8 \text{M}_\odot)^{2/5} I_{21}^{-1/5} \), and the corresponding critical optical depth is \( \tau_{\text{crit}} = 5.3 \). Hence, the dependence of equation (6) upon the cloud mass and the UV intensity can
be fundamentally understood by this argument. But, from a quantitative point of view, this approximation obviously leads to considerable overestimation of photoionization effects as recognized by the critical optical depth. The overestimation comes from the assumption that all photons that enter into the cloud always cause ionization. In fact, some photons that have especially low incident angles do escape from the gas cloud without causing ionization. The diffusion process of ionizing photons tends to enhance this effect. Furthermore, a sharply edged neutral core, which is the basic assumption in the Strömgren approximation, is no longer realistic, as shown above.

5. DISCUSSION

The present results seem of great significance, considering the biasing by photoionization against the formation of low-mass galaxies. It is shown in previous analyses that if a cloud is assumed to be optically thin, the photoionization suppresses the collapse of clouds with \( M \leq 10^9 \, M_\odot \) (Umemura & Ikeuchi 1984) or circular velocities smaller than 30 km s\(^{-1}\) (Thoul & Weinberg 1996). In the present analysis, the permeation of UV radiation is characterized by a different criterion, and the critical density has turned out to be almost independent of the mass. Hence, the evolution of subgalactic clouds would not be determined solely by the cloud mass or the circular velocity. The maximum expansion radius of a top-hat density fluctuation is given by

\[
R_{\text{max}} = 10.7 \, \text{kpc} \left( \frac{M}{10^9 \, M_\odot} \right)^{1/3} \left[ \frac{10^2 (1 + z_{\text{max}})}{1 + z} \right] h_{50}^{-2/3}
\]

in an Einstein–de Sitter universe, where \( z_{\text{max}} \) is the maximum expansion epoch, \( h_{50} \) is the present Hubble constant in units of 50 km s\(^{-1}\) Mpc\(^{-1}\), and the baryon density parameter is assumed to be 0.05. Comparing \( R_{\text{max}} \) with equation (6) and taking into account a possibility that the UV intensity might be significantly lower at \( z > 5 \), we can speculate that the self-shielding could be quite effective shortly after the maximum expansion stages. Thus, in order to assess the effects of photoionization properly, we should consider the frequency-dependent radiative transfer of diffuse UV photons.

The present results are also relevant to the formation of primordial hydrogen molecules, which provide key physics for the formation of the first-generation objects (Tegmark et al. 1997). UV radiation will naively suppress the formation of hydrogen molecules and, thereby, the cooling (Haiman, Rees, & Loeb 1997). However, when the reionized gas is self-shielded in the course of evolution, H\(^-\) or H\(^2\)\(^+\) ions could form efficiently because of the residual ionization, which may lead to the effective production of hydrogen molecules (e.g., Kang & Shapiro 1992). Since the formation of primordial hydrogen molecules is a nonequilibrium process, the permeation of even a small portion of UV background photons may play an important role for subsequent molecule formation and thereby the evolution of neutral core.

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