Collapse of the many-worlds interpretation: Why Everett’s theory is typically wrong

Aurélien Drezet

1Univ. Grenoble Alpes, CNRS, Grenoble INP, Institut Neel, F-38000 Grenoble, France

Abstract

We analyze the objective meaning of probabilities in the context of the many-worlds interpretation of Everett. For this purpose we study in details the weak law of large numbers and the role of typicality and universally negligible probabilities (through the works of Cournot and Borel). We demonstrate that Everett’s theory doesn’t provide any clue for fixing a probability rule and therefore contradicts irrevocably empirical facts and Born’s law.

*Electronic address: aurelien.drezet@neel.cnrs.fr
I. INTRODUCTION AND MOTIVATION

Everett’s many-worlds interpretation [43], abbreviated as MWI in the following, is certainly one of the most known and discussed attempt for finding an alternative ontology of quantum mechanics. Unlike the standard Copenhagen or ‘collapse’ interpretation the MWI is based on a purely unitary ontology based on the Schrödinger equation and doesn’t require the unfamous, and vaguely defined, separation between observers and observed systems.

The motivations for developing the MWI were clearly announced by Everett [9, 36, 43]. On the one side, we want a complete theory describing the Universe as a whole and without an arbitrary ‘shifty split’ à la Heisenberg-von Neumann between different levels of reality. On the other side, we also want the theory to be able of recovering the many statistical features and successes of the orthodox/Copenhagen interpretation. Most importantly, we need to recover the usual Max Born probability law

\[ P^{(\text{Born})}_\alpha = |\Psi_\alpha|^2 = |\langle \alpha | \Psi \rangle|^2 \]  

for observing an outcome \( \alpha \) after a quantum measurement acting upon the state \( |\Psi\rangle = \sum_\alpha \Psi_\alpha |\alpha\rangle \).

Recovering Born’s rule and the statistical interpretation has been claimed by DeWitt to constitute the EWG (Everett-Wheeler-Graham) ‘meta-theorem’:

*The mathematical formalism of the quantum theory is capable of yielding its own interpretation.* [36, 37]

This ‘proof’ (and its descendants [2, 3, 33, 44, 47, 54, 59, 79, 80, 104, 112]) has been strongly criticized over the years and accused of circularity in particular because it relies on many subtleties concerning the application of the Bernoulli law of large-numbers, e.g., concerning very long sequences of repeated experiments [1, 7, 10, 11, 60, 64, 85, 99, 100].

More recently, the mere idea that a deterministic ontology based on Schrödinger’s unitary equation could lead to something like a justification of the probabilistic properties of our Universe has been criticized and named the ‘incoherence problem’ by philosophers [5, 74, 92]. Indeed, in the MWI all the alternatives of a quantum measurement acting upon \( |\Psi\rangle = \sum_\alpha \Psi_\alpha |\alpha\rangle \) are actual: None of them are collapsing and all of them are required to survive in order to preserve the unitary time-evolution of the whole universal wave-function. In this context, how could we with the MWI even hope to justify stochasticity, i.e., as observed
in the lab? Nevertheless, several authors have attempted to answer repeated objections made against the MWI and have tried to ‘precise’ the definition of probabilities used in this theory. For instance, it has often been claimed, in reply to criticisms, that probabilistic concepts used in the MWI are not worse (and perhaps not better) than they are in other interpretations of quantum mechanics or even in classical statistical mechanics [37, 43, 81, 117]. In parallel, Vaidman [76, 106–109], partly in response to Albert [5, 92], has developed suggestive narratives and a complete semantics for speaking about probability and ‘self-locating uncertainty’ as understood and perceived by observers in quantum branches of the universal wavefunction (see also [53, 102, 103]). One of the proposed solution advocated by McQueen and Vaidman [76, 106–109] is to assume an additional postulate for the MWI: the so-called Born-Vaidman rule [102]

**MWI probability postulate:** The probability of self-location in a world with a given set of outcomes is the absolute square of that world’s amplitude. [76]

As stressed by these authors ‘the postulate is not about some fundamental physical process, it is about the experience of the observer’. This is a subjective definition of probability that would probably not reject de Finetti himself [45]. Moreover, accepting a objective-Bayesianist reading of probability and Lewis’s ‘principal-principle’, it must also be assumed that the degree of belief or credence \( C_\alpha \), for the observer to be in one branch, is tied to an objective chance \( P_\alpha \) \(^1\). Following Vaidman, this objective property is just the ‘measure of existence’ \( P_\alpha = |\Psi_\alpha|^2 \), i.e., as given by Born’s rule. This supplementary rule is needed in order to constraint the statistics in the MWI to follow empirical evidences like Born’s law. Yet, this strategy constitutes an amendment to the original Everett goal and not every advocates of the MWI subscribe to it.

Furthermore, several new ideas based on decision-theoretic scenario à la Deutsch [35, 89, 90, 116, 117] or ‘envariance’ à la Zurek [8, 95, 120, 121] have been discussed as alternative ‘proofs’ for making sense of probability in the MWI, i.e., purportedly without incoherence and with the correct Born’s rule. These ideas have also been strongly criticized [5, 65, 74, 92] because they rely on some purely personalist and Bayesian definitions of probability that are, apriori, unrelated to experimental facts (i.e., relative frequencies). In this decision-theoretic

\(^1\) More precisely we write \( C_\alpha := C(\alpha|P_\alpha) = P_\alpha \). [71].
‘Oxfordian’ approach it is only proved that if we could define a probability function in quantum mechanics this function will satisfy Born’s rule. However, the physical explanation of the probability assumption (i.e., the qualitative or incoherence problem) is standing unjustified and apparently leads to circular deductions if we don’t add something to the MWI [1, 38].

In the present work, we are not going to further discuss the meaning of subjective probabilities in the decision-theoretic and Bayesian scenarios. Instead, we are going back to the old EWG theorem which is still we think at the core of the problem. More precisely, we believe that it is only by going back to the classical definitions of a probability as given in the work of Bernoulli, Laplace, Borel and many others that one can understand the objective and pragmatic role of probabilities as estimators of relative frequencies (even though if all these authors also emphasized the subjective role of probabilities). We will show that the mathematical and physical definitions of probabilities, and particularly the weak law of large numbers (WLLN), constitute actually fatal issues for the MWI. It is therefore, we think, very important to understand the consequences and the unavoidability of the problem. In turn, it will we think demonstrate that the MWI can just not be true in its basic form. It implies that one must either modify the ontology (like for instance in the many de Broglie-Bohm worlds strategy [26, 91, 105]), or add something to the current MWI as suggested by Vaidman (the present author has recently speculated a toy model of that kind [40] based on a unitary version of the many-minds interpretation [4]).

Of course, these are very strong claims and here we propose to justify this view with a very elementary scenario using a Bernouilli sequence of a simple quantum coin tossing (an elementary description adapted to the MWI is given in section II). During the discussion we will in section III study the history of the WLLN that is central to understand the MWI. Furthermore, we will discuss the notions of typicality and universally negligible probabilities that are needed to decipher the meaning of that WLLN. In particular, we will in section IV bring new elements coming from the deterministic de Broglie-Bohm theory or pilot-wave interpretation (PWI) [15], that is an alternative hidden-variables quantum theory, in order to clarify the concept of probability involved in the MWI and show that the full Everettian theory can just not survive in its present form.
II. QUANTUM BERNOULLI PROCESS

We start with a two-level quantum system or qubit described by the wave-function

\[ |\Psi_1\rangle = a|\spadesuit\rangle + b|\heartsuit\rangle \]  \hspace{1cm} (2)

with \( a, b \in \mathbb{C} \) and \( \langle \heartsuit | \spadesuit \rangle = 0, \| |\spadesuit\| = \| |\heartsuit\| = 1 \) (we also impose the normalization \( |a|^2 + |b|^2 = 1 \)).

Now, we suppose a quantum experiment involving a kind of Stern-Gerlach apparatus to separate the two components \( \spadesuit/\heartsuit \) of the system. After this splitting (that involves a form of external field acting on the qubit) we have not yet a quantum measurement. In the standard ‘collapse’ interpretation we require the separated wave-packets associated with the two components \( \spadesuit/\heartsuit \) to interact with amplifying devices or detectors bringing the information from the microscopic quantum world of potentiality to the macroscopic world where only one outcome has been registered.

However, in the MWI we preserve unitarity and we must include everything, i.e., even the observers, into the quantum realm. Indeed, the awesome idea of Everett \[9, 36, 43\] was to introduce observers as quantum mechanical devices participating to the experiment. As Everett wrote in his doctoral thesis:

\begin{quote}
As model for observers we can, if we wish, consider automatically functioning machines, possessing sensory apparatus and coupled to recording devices capable of registering past sensory data and machine configurations. [36, p. 64]
\end{quote}

Here, before the measurement (lets say at time \( t_0 \)) we write \( |\mathcal{M}_0, \text{Aurélien}_0\rangle_{t_0} \) the joint quantum state involving the detector \( \mathcal{M} \), the observer and his environment \(^2\). The interaction or measurement process that we consider has several important steps summarized on the

\(^2\) These sub-systems are strongly entangled, e.g., due to decoherence, and therefore our notations don’t even try to clearly identify and separate the Hilbert space belonging to the observer from the one associated with the apparatus or with the environment.
quantum state evolution

\[ |\Psi_1\rangle_{t_0} \otimes |\mathcal{M}_0, \text{Aurélien}_0\rangle_{t_0} \]

\[ \rightarrow (a|\spadesuit\rangle_{t_1} + b|\heartsuit\rangle_{t_1}) \otimes |\mathcal{M}_0, \text{Aurélien}_0\rangle_{t_1} \]  \hspace{1cm} (3)

\[ \rightarrow a|M_{\spadesuit}', \text{Aurélien}_{\spadesuit}\rangle_{t_2} + b|M_{\heartsuit}', \text{Aurélien}_{\heartsuit}\rangle_{t_2} \]  \hspace{1cm} (4)

\[ \rightarrow a|M_{0}, \text{Aurélien}_{\spadesuit}\rangle_{t_3} + b|M_{0}, \text{Aurélien}_{\heartsuit}\rangle_{t_3}, \]  \hspace{1cm} (5)

where we have added a time label to follow the chronology. At time \( t_1 \), i.e., Eq. 3, the splitting of the initial wave packet \( |\Psi_1\rangle_{t_0} \) already occurred: we obtain two disjoints and orthogonal wave packets \( |\spadesuit\rangle_{t_1} \) and \( |\heartsuit\rangle_{t_1} \) (i.e., \( \langle \heartsuit | \spadesuit \rangle_{t_1} = 0, ||\spadesuit||_{t_1} = ||\heartsuit||_{t_1} = 1 \)) moving in the direction of the detectors. At that time the detectors and observer are still factorized from the qubit state \(^3\).

At time \( t_2 \), i.e., Eq. 4, the detectors have detected and absorbed the qubit (the notation \( \mathcal{M}' \) means that the devices are excited). Rigorously, this involves two detectors: one being located along the trajectory of the \( \spadesuit \) wave packet and one along the path of the \( \heartsuit \) wave packet (i.e., we have here a kind of coincidence measurement like the one involved in Hanbury Brown and Twiss interferometers for single photons). We have \( \langle \mathcal{M}_{\heartsuit}', \text{Aurélien}_{\heartsuit} | \mathcal{M}_{\spadesuit}', \text{Aurélien}_{\spadesuit}\rangle_{t_2} = 0 \) and \( ||\mathcal{M}_{\spadesuit}', \text{Aurélien}_{\spadesuit}\rangle_{t_2}|| = 1, ||\mathcal{M}_{\heartsuit}', \text{Aurélien}_{\heartsuit}\rangle_{t_2}|| = 1 \) as required by unitarity.

At the final stage \( t_3 \), i.e., Eq. 5, the two detectors have relaxed to their ground states and the environment has kept an information about the process which for simplicity is included in the state of the observer. Once again, unitarity imposes that we get: \( \langle \mathcal{M}_0, \text{Aurélien}_{\heartsuit} | \mathcal{M}_0, \text{Aurélien}_{\spadesuit}\rangle_{t_2} = 0 \) and \( ||\mathcal{M}_0, \text{Aurélien}_{\spadesuit}\rangle_{t_2}|| = 1, ||\mathcal{M}_0, \text{Aurélien}_{\heartsuit}\rangle_{t_2}|| = 1 \). We finally obtain two orthogonal universes in which the two copies of the observer unaware of each other are living with their own memories of the experimental result.

We stress that the coefficients \( |a|^2 \) and \( |b|^2 \), that are in the orthodox interpretation identified with the Born probabilities for observing \( \spadesuit \) or \( \heartsuit \), i.e.,

\[ P_{\spadesuit} = |a|^2 \text{ or } P_{\heartsuit} = |b|^2, \]  \hspace{1cm} (6)

are here not yet identified with probabilities, but merely represent intensities or if you wish ‘measures of existence’ \(^{107, 108}\) for the two orthogonal branches.

\(^3\) Note that \( |\mathcal{M}_0, \text{Aurélien}_0\rangle_{t_1} \) is generally different from \( |\mathcal{M}_0, \text{Aurélien}_0\rangle_{t_0} \) but that unitarity imposes that the norm is preserved: \( |||\mathcal{M}_0, \text{Aurélien}_0\rangle_{t_1}|| = ||\mathcal{M}_0, \text{Aurélien}_0\rangle_{t_0}|| = 1. \)
The next step in our analysis is to consider a long sequence of $N$ trials of the same experiment (i.e., a Bernoulli process as sketched in Figure 1). We begin with a product state that reads $|\Psi_N\rangle = \bigotimes_{i=1}^N |\Psi_1^{(i)}\rangle := |\Psi_1^{(1)}\rangle \otimes ... \otimes |\Psi_1^{(N)}\rangle$ where, as in Eq. 2, we have $|\Psi_1^{(i)}\rangle = a|\spadesuit^{(i)}\rangle + b|\heartsuit^{(i)}\rangle$ ($i$ labeling the different Hilbert spaces associated with the different qubits). Each of the $N$ qubits are successively analyzed using our Stern-Gerlach apparatus and the sequence of results is carefully registered and memorized by the observer. In the end, we obtain a sum of $2^N$ orthogonal decohered branches or universes which reads

$$|\Psi_N\rangle_{t_0} \otimes |\mathcal{M}_0, \text{Aurélien}_0\rangle_{t_0} \rightarrow \sum_h a^{N_h \spadesuit} b^{N_h \heartsuit} |\mathcal{M}_0, \text{Aurélien}_h\rangle_{t_f}. \quad (7)$$

In Eq. 7 $h := [\alpha_1^h, \alpha_2^h, ..., \alpha_N^h]$ with $\alpha_i^h = \spadesuit$ (or $\heartsuit$) denotes one history in which the $i^{th}$ measurement led to the outcome $|\spadesuit^{(i)}\rangle$ (or $|\heartsuit^{(i)}\rangle$).

To each history $h \in \mathcal{H}$ (where $\mathcal{H} := \{\spadesuit, \heartsuit\}^N$ is the ensemble of all histories with cardinality $2^N$) is associated a quantum observer state $|\mathcal{M}_0, \text{Aurélien}_h\rangle_{t_f}$. These states constitute a complete set such that for each possible pair of histories $h, h'$ we have $\langle \mathcal{M}_0, \text{Aurélien}_{h'}, \mathcal{M}_0, \text{Aurélien}_h\rangle_{t_f} = \delta_{h,h'}$ (where $\delta_{h,h'}$ is a Kronecker symbol).

Moreover, the important quantity in Eq. 7 is the branch amplitude $a^{N_h \spadesuit} b^{N_h \heartsuit}$ which involves the number of times $N_h \spadesuit$ (respectively $N_h \heartsuit$) that a $\spadesuit$ (respectively $\heartsuit$) appeared in the sequence $h$ of length $N$. We have $N_h \spadesuit + N_h \heartsuit = N$, and by definition of $h$ we also have

![Schematic representation as a fractal tree of the quantum Bernoulli process associated with the quantum state $|\Psi_1\rangle = a|\spadesuit\rangle + b|\heartsuit\rangle$.](image)
\( N^h_\alpha = \sum_{i=1}^{i=N} \delta_{\alpha, \alpha_i} \) where \( \alpha = \♠ \) or \( \heartsuit \). It is convenient to introduce the relative frequency of occurrence \([36, 59]\) for the outcome \( \alpha \) in the history \( h \) as

\[
Q^h_\alpha := \frac{N^h_\alpha}{N} = \frac{1}{N} \sum_{i=1}^{i=N} \delta_{\alpha, \alpha_i}
\]

with clearly \( Q^h_\♠ + Q^h_\heartsuit = 1 \). All these concepts are required for the statistical analysis discussed in the next section.

### III. THE LAW OF LARGE NUMBERS: PROBABILITY AND TYPICALITY

In order to discuss the concept of probabilities we consider first the previous Bernoulli process from the point of view of the Copenhagen interpretation (or if we wish any collapse or reduction model of the quantum world \([48]\)).

More precisely, from the point of view of a super-observer localized on the classical side of the quantum-classical boundary and watching the full experiment described by Eq. \([7]\) we know that \( A_h = a^N_\♠ b^N_\heartsuit \) represents a ‘probability amplitude’ for the alternative \( h \). Therefore, applying Born’s rule and Eq. \([6]\) we obtain the probability \( P_h \) for the sequence \( h \) as

\[
P_h = |A_h|^2 = \mathcal{P}^N_\♠ \mathcal{P}^N_\heartsuit.
\]

The concept of probability that is used here is still vague and its relation to the experimental world will be precised in the following in connection with the works of Laplace and others.

First, observe that for such a sequence of Bernoulli trials we are not interested in \( P_h \) but instead in the sum

\[
\sum_{h \in \mathcal{H}_{N_\♠}} P_h := \mathcal{P}(N_\♠)
\]

in the subset \( \mathcal{H}_{N_\♠} \) of histories corresponding to a fixed value of \( N_\♠ \) (and therefore \( N_\heartsuit = N - N_\♠ \)). From pure combinatorics we immediately deduce the binomial probability for recording a number \( N_\♠ \) of \( \♠ \) in a sequence of \( N \) independent experiments:

\[
\mathcal{P}(N_\♠) = \frac{N!}{N_\♠! N_\heartsuit!} \mathcal{P}^N_\♠ \mathcal{P}^N_\heartsuit \simeq e^{-\frac{1}{2} \left( \frac{Q_\♠ - Q_\♠}{2\delta Q_\♠} \right)^2}
\]

where the second equality results from the Moivre-Laplace central-limit theorem valid if \( N \gg 1 \) (i.e., after a direct application of Stirling formula). Here, the probability \( \mathcal{P}_\♠ \) is
identified with the mean value $\langle Q_\bullet \rangle$ of the random variable $Q_\bullet = N_\bullet / N$ and similarly
$$\sqrt{\left( \frac{P_\bullet P_\bigcirc}{N} \right)}$$
with the fluctuation $\delta Q_\bullet = \sqrt{\langle (Q_\bullet - \langle Q_\bullet \rangle)^2 \rangle}$.

Most importantly for the present discussion, mathematics shows that for any real number $\sigma \in [0, \min(P_\bullet, P_\bigcirc)]$ the probability $P(|Q_\bullet - P_\bullet| \geq \sigma \delta Q_\bullet) = \sum_{|Q_\bullet - P_\bullet| \geq \sigma \delta Q_\bullet} P(N_\bullet)$ admits the upper bound:
$$P(|Q_\bullet - P_\bullet| \geq \sigma \delta Q_\bullet) \leq 2e^{-2N\sigma^2\delta Q_\bullet^2} = 2e^{-2\sigma^2 P_\bullet P_\bigcirc}$$
known as Hoeffding’s bound \(^4\). After writing $\varepsilon = \sigma \delta Q_\bullet$ Eq. \([13]\) is used to obtain a version of the (weak) law of large numbers (WLLN):
$$\lim_{N \to +\infty} P(|Q_\bullet - P_\bullet| \geq \varepsilon) = 0, \ \forall \varepsilon > 0 \quad (14)$$
showing that in the $N \to +\infty$ limit $Q_\bullet$ converges probabilistically to $P_\bullet$ \(^5\). This result, which was originally obtained by Jacob Bernoulli around 1692 is often considered as a ‘golden-theorem’ of the probability calculus.

Moreover, the WLLN is used to show that for ‘almost all’ alternatives $h \in H$ the frequency $Q_\bullet$ (respectively $Q_\bigcirc$) is a very good estimator of the probability $P_\bullet$ (respectively $P_\bigcirc$) when $N$ is very large. By almost we precisely mean that histories for which Born’s rule $Q_\bullet \simeq |a|^2$, $Q_\bigcirc \simeq |b|^2$ holds true are ‘typical’ in probability.

At that level of our discussion we must now try to introduce a definition of probability. We remind that the classical definition of a probability \([55, 101]\) given by Laplace is

\[ One \ has \ seen \ in \ the \ introduction \ that \ the \ probability \ of \ an \ event \ is \ the \ ratio \ of \ the \ number \ of \ cases \ that \ are \ favourable \ to \ it, \ to \ the \ number \ of \ possible \ cases, \]

\(^4\) More generally, in the continuous limit where $N \gg 1$ we have:
$$P(|Q_\bullet - P_\bullet| \leq \sigma \delta Q_\bullet) \simeq \frac{1}{\sqrt{2\pi}} \int_{-\sigma}^{\sigma} dx e^{-\frac{x^2}{2}} = \text{erf}(\frac{\sigma}{\sqrt{2}})$$
which leads to the famous 3-$\sigma$ rule corresponding to $\sigma = 3$ in Eq. \([12]\) i.e., $P(|Q_\bullet - P_\bullet| \leq 3\delta Q_\bullet) \simeq 99.73\%$.

\(^5\) Note that the WLLN is more often deduced from the Bienaymé–Chebyshev inequality $P(|Q_\bullet - P_\bullet| \geq \varepsilon) \leq \delta Q_\bullet^2 / \varepsilon^2$. Note also, that in the continuous limit Eq. \([11]\) implies that $P(aN \leq N_\bullet \leq bN) \simeq \int_{a}^{b} dQ \rho_{\delta Q_\bullet}(Q - P_\bullet)$ where the Gaussian probability distribution

\[ \rho_{\delta Q_\bullet}(X) = \frac{1}{\delta Q_\bullet \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{X}{\delta Q_\bullet} \right)^2} \]
asymptotically approaches the Dirac distribution $\delta(X)$ in the $N \to +\infty$ limit.
when there is nothing to make us believe that one case should occur rather than
any other, so that these cases are, for us, equally possible. [68, p. 181].

A similar definition was already given by de Moivre in 1718:

The probability of an event is greater or less, according to the number of chances
by which it may happen, compared with the number of all the chances, by which
it may either happen or fail. [78, p. 1].

There are several parts in the de Moivre-Laplace definition but let us here focus on this
counting or enumeration procedure for ‘chances’ i.e., physically distinct alternatives. Moreover,
contrarily to Laplace we don’t accept an ignorance-based justification for probabilities
in a deterministic Universe. To paraphrase Poincaré: information given by the probability
calculus will not stop to be true the day when these phenomena will be better known [83,
p. 3]. Therefore, we assume that the so-called ‘subjective probabilities’ are only estimations
of (objective) probabilities. The central question is of course which objective meaning can
have the word chance in a purely deterministic Universe if we don’t want to introduce some
new mysterious and occult fluid (see the remark in the footnote 8). We should try to answer
this question in the following. Moreover, accepting the ‘classical’ Moivre-Laplace definition
the probability \( P(A) \) for an outcome or event \( A \) is given by the ratio

\[
P(A) := \frac{M(A)}{M}
\]

between the number of elementary alternatives \( M(A) \) leading to the outcome \( A \) and the
total number of alternative \( M \) in the game.

Now, of course counting alternatives or chances is not without ambiguities. Take a coin
(classical or quantum): we have only two elementary alternatives so that apriori probabilities
for elementary outcomes ♠ and ♥ are necessarily \( P♠ = P♥ = \frac{1}{2} \). Moreover, in order
to extend or apply Laplace’s definition to the cases of unfair coins one must go beyond
simple ‘geo-metrical’ considerations. Consider again our coin with two alternatives ♠, ♥,
and now suppose that each alternative is actually degenerated and divided in many ‘physically
available’ sub-alternatives \( M♠ \) and \( M♥ \) respectively. The classical definition now applies
and allows for any non-negative (rational number) probabilities \( P♠/♥ = \frac{M♠/♥}{M♠+M♥} \) with the
normalization \( P♠ + P♥ = 1 \). Now, it seems that we actually introduced mysterious new
features in the formalism. What are actually those internal degrees of freedom with numbers
By introducing such elements we actually defined a new metric or weight on the configuration space \( \{♠, ♥\} \) with cardinality 2. An alternative way to see that is to consider that each elementary physical alternative ♠, or ♥ corresponds to a urn filled with \( M♠ \) and \( M♥ \) distinguishable balls or particles. When \( M → +∞ \) we have with this atomic model a way to picture and approximate any continuous non negative distributions of probability \( P♠ \) with \( P♠ + P♥ = 1 \). In others words, we have here progressively building the representation of a continuous fluid that is reminiscent of the definition made by Gibbs of a ‘fictitious’ probability fluid or ensemble 6. More precisely, Gibbs following Maxwell and Boltzmann wrote

\[
\text{We may imagine a great number of systems of the same nature, but differing in the configurations and velocities which they have at a given instant, and differing not merely infinitesimally, but it may be so as to embrace every conceivable combination of configuration and velocities.\[49\] p. vii}
\]

This conception of a ‘great number of systems’ is clearly reminiscent of Maxwell/Boltzmann gas theory in which huge numbers \( M ≃ 10^{23} \) of molecules or atoms are considered. Yet, for Gibbs this frequentist analogy is not the end of the story since the word ‘imagine’ emphasizes the mental 7 or at least not actual nature of this ensemble. For Gibbs this is just a physical model for building a quasi-continuous probability fluid. In the end, the Gibbs atomic model (like the mechanical analogies proposed by Maxwell for modeling the Aether) is not required anymore and can be eliminated: we have obtained the general notion of a non-negative ‘measure’, ‘weight’ or ‘metric’ \( μ(A) \) for a set of events \( A \), i.e., as it was later formally axiomatized by Borel, Lebesgue and Kolmogorov at the beginning of 20th century.

Following Laplace and thus Gibbs, the ‘confidence level’ probability \( \mathcal{P}(|Q♠−P♠| ≤ \sigmaδQ♠) \) must be interpreted as a fraction \( \frac{M_{\text{Typical}}}{2^N} \) of the number of typical histories over the total number of alternatives, i.e., here \( 2^N \). The WLLN states that when \( N \) is growing this ratio also grows and tends to the maximum value \( \frac{2^N}{2^N} = 1 \) asymptotically. With this property, and assuming a precisely defined probability measure on the whole history set

\[\text{6 Of course, the Gibbs ensembles were historically not built on a discrete space \{♠, ♥\} but on a continuous phase-space } Γ ∈ \mathbb{R}^{6r} \text{ where } r \text{ is an integer and } 6r \text{ the number of degrees of freedom of the ensemble (not to be confused with the cardinality } \text{card}(Γ) = (2^{8r})^{6r} = 2^{8r}).\]

\[\text{7 Schrödinger [93, p. 3] uses the expression ‘mental copies’ to denote a Gibbs ensemble.}\]
\( \mathcal{H} \), overwhelmingly all possible histories \( h \in \mathcal{H} \) ultimately belong to the typical history set \( \mathcal{H}_{\text{typical}} \sim \mathcal{H} \).

For all practical purposes this means that by watching a long Bernoulli sequence with \( N \gg 1 \) (for example \( N \sim 10^{10} \)) the frequency \( Q_\alpha \) deviates negligibly from the characteristic frequency, i.e., the probability \( P_\alpha \), for almost all eventualities and possibilities \( h \). Of course, this doesn’t mean that an atypical event is forbidden: There is nothing in the dynamics that prohibits an atypical event to happen (this would contradict the mere definition of a Bernoulli process).

Importantly, it is by applying this WLLN that we make sense of the usual frequentist definition of a probability:

\[
\lim_{N \to +\infty} Q^h_\alpha \equiv P_\alpha
\tag{17}
\]

where \( h \) belongs to the typical set \( \mathcal{H}_{\text{typical}} \) tending probabilistically to the whole set \( \mathcal{H} \) in the \( N \to +\infty \) limit. Once again, we stress \([19, 20, 32, 67]\) that this concept of ‘limit in probability’ based on typicality is different from the more usual point-wise limit \( \lim_{N \to +\infty} Q^h_\alpha = P_\alpha \). Here, the notion of limit in probability is conditioned on the existence of a typical set \( \mathcal{H}_{\text{typical}} \) itself defined probabilistically. This key point, often neglected, implies that the WLLN can not be used to define the probability as a frequency without further hypothesis or physical considerations on the notion of limit and sequences.

This has led to long standing debates concerning the physical relation between frequencies (i.e., statistics) and probabilities (i.e., possibilities). In particular, whereas the mathematical condition Eq. \( [17] \) recovering probabilities as relative frequencies in the limit \( N \to +\infty \) is apriori not questioned, it appears nevertheless unphysical since related to an infinitely long Bernoulli sequence which is aposteriori never encountered in nature. Moreover, for finite \( N \gg 1 \) the typicality reasoning leading to the WLLN is also often criticized for being circular, i.e. leading to an infinite regress fallacy. To quote David Wallace:

\textit{We cannot prove that in the long-run relative frequencies converge to probabilities. What we can prove, is that in the long-run relative frequencies converge to probabilities... Probably. }[118]

Furthermore, in the short-run the relation between relative frequencies and probabilities disappears and as a direct consequence the physical meaning of probabilities seems to vanish as well. In other words, probabilities can not simply, i.e., with certainty, be identified
with long-run frequencies without clearly understanding what is the physical meaning of typicality.

We stress that the intuitive idea to define probability as frequency, i.e., ‘frequentism’ has been defended to some extent by Ellis, Cournot and Venn [31, 42, 113] in the 19th century, and was later supported by von Mises [77] and Reichenbach [87] who considered infinite collectives. This interpretation of probability is often advocated by statisticians and physicists [101] to justify the existence of statistical ensembles (e.g., in statistical mechanics [88]). Moreover, frequentism has also been strongly criticized by those who, like Keynes [66], Ramsey [86] and de Finetti [45], prone a more subjectivist and personalist perspective on probability. Sometimes, mathematicians like Borel [23] accept prudently a more nuanced and ‘rationalist’ approach mixing subjectivism and frequentism (this view goes back at least to Laplace and is also named objective-Bayesianism [27, 62, 63]). Repeated objections done against frequentism include the ‘single-case’ and ‘reference class’ problems (for a recent and interesting review of these objections against pure frequentism see [56, 57]) that are both connected to the interpretation of a probability assigned to a singular event. Here, we dont subscribe to the frequentism advocated by von Mises and others. Better, as we will show, we assume an objective interpretation of probability based on the concept of typicality. In this view frequencies are still fundamental but unlike von Mises we dont require infinite collectives and an alternative axiomatics to define probability.

To further understand and decipher the central role played by typicality in an objective approach to probability it is important to realize how efficient are these probabilistic predictions obtained from the WLLN. To take an example, with $N = 10^{10}$ (which is already a large number for a quantum experimentalist) and $P_{\bullet} = \frac{1}{2}$ we see from Eq. 13 that the
probability for the frequentist Born rule $Q_\star \simeq \frac{1}{2} \pm \varepsilon$ to hold true experimentally with an uncertainty of at most $\varepsilon \simeq 4 \cdot 10^{-5}$, i.e., corresponding to $\sigma = 10$ in Eq. 13 is $\sim 1 - 10^{-23}$. Furthermore, for a ‘macroscopic’ sample involving $N \sim 10^{24}$ quantum coins the Born rule $Q_\star \simeq \frac{1}{2} \pm 5 \cdot 10^{-11}$ is obtained for $\sigma = 100$ with a gigantic probability (also named confidence level) $\sim 1 - 10^{-2174}$. But what is the physical meaning of an atypical event with probability like $10^{-20}$ or $10^{-2000}$?

Already in 1713 Bernoulli considered events with small probabilities as ‘morally impossible’ and those with high probabilities as ‘morally certain’. While we don’t here accept a subjective approach of probabilities the same issue was discussed more objectively at the end of the 18th century by d’Alembert and Buffon, and in 1843 by the philosopher and mathematician Cournot who wrote:

*The physically impossible event is therefore the one that has infinitely small probability, and only this remark gives substance-objective and phenomenal value-to the theory of mathematical probability.* [33, p. 78]

This ‘Cournot principle’, as it was later named, is sometimes considered as a corner stone of modern probability theory (for a review see [96]), although it belongs more to the applicative and empirical side of the theory than to the abstract measure-theoretic side associated with pure mathematics and enumeration of alternatives.

Borel, for example, considered that negligible probabilities associated with atypical events (and not only infinitesimal as believed by Cournot) are necessary in order to attribute a physical content to the WLLN and to define an ‘empirical law of randomness’ [21, p. 54]. In a short article ‘on universally negligeable probabilities’ published in 1930 [22], Borel wrote:

*The necessary conclusion is that the probabilities that can be expressed by a number smaller than $10^{-1000}$ are not only negligible in the common practice of*
life, but universally negligible, that is they must be treated as rigorously equal to zero in all questions concerning our Universe. 22

We stress that for Borel such negligible probabilities were both subjectively and physically negligible 23. The analysis of such atypical events are therefore representing a central element for interpreting both subjective and objective probabilities 10 and for giving a ‘practical and philosophical value to probabilities’ 23, 46.

Kolmogorov (following Markov, Chuprov and many others in Russia 97) also included this ‘Cournot principle’ as part of the physical axioms needed to connect the abstract probabilistic formalism to the actual world, and wrote:

if \( P(A) \) is very small, one can be practically certain that when conditions \( E \) are realized only once, the event \( A \) would not occur at all. 67, p. 4

Following all these mathematicians, we are tempted to say that great probabilities, i.e., ratios like \( \frac{M_{\text{typical}}}{2N} \sim 1 - 10^{-2174} \) are for all practical needs identified with negligible deviations from the absolute certainty. In other words, atypical events, i.e., maverick histories, are completely negligible in probability. Their atypicality should make them ‘physically’ or ‘factually’ impossible.

To be more precise on the justification and application of this Cournot’s principle and to understand the physical meaning of Borel’s ‘universally negligible probabilities’ we must ask ourself how can we measure an ‘un-probability’ like \( 10^{-1000} \)? In turn this must explain what we mean by ‘factually impossible’. If we can’t answer this question we are apparently going to enter the infinite regress loop that is (implicitly) alluded to in Wallace quote 118. However, we first note with Borel 24 that if during an experiment (\( E \)) (let say a Bernoulli sequence of \( N \) trials of a quantum coin tossing) an atypical event has a tiny probability \( P_{\text{atypical}} = 1/n \), with \( n \gg 1 \), then the probability to observe at least one time this atypical event in a super-Bernoulli sequence of \( n \) trials of the experiment \( E \) is \( 1 - (1 - 1/n)^n \approx 1 - 1/e \approx 63\% \). Such a probability is clearly not anymore negligible 11

---

10 As emphasized by Borel 20, 22, 23, we ‘certainly know’ as a rule of thumb that when a gambler is predicting that ‘for sure’ the next long-run \( N \sim 10^{10} \) of a Bernoulli game will produce an atypical sequence with \( Q_\star \) strongly deviating from \( P_\star \) (e.g., \( \varepsilon \sim 4 \cdot 10^{-5} \)) we can almost be certain that this gambler is lying; the certainty being here defined, at least, with a probability like \( 1 - 10^{-23} \).

11 Note that by pushing the number of trials to 100\( n \) we would obtain a probability \( 1 - (1 - 1/n)^{100n} \approx 1 - e^{-100} \approx 1 - 10^{-43} \), i.e, a quasi-certainty 22, 24, 25. We also stress that the notion of typicality and
and the event is expected to be observed (but without certainty of course since we are only considering probabilities, i.e., possibilities à la Laplace): This is the probabilistic meaning of the usual sentence ‘the atypical event as a chance of 1 over \( n \) to happen’.

At that stage we still have a WLLN statement: ‘a frequency is a probability with a high probability’. Therefore, in order to avoid the infinite regess loop, we must introduce an additional element. Moreover, with Borel [24, 25], we observe that very quickly, i.e., for \( \sigma \) large enough, the number \( n \) becomes extremely large and goes far beyond the possibilities physically allowed by the present Universe. To understand what are these limits suppose for example a cosmological system (‘our Universe’) with \( m \sim 10^{81} \) atoms and consider, as an estimation, that the number of trials physically possible for each atom is given by the ratio \( T_U/T_P \sim 10^{62} \) of the typical ‘age’ of our Universe \( T_U \) to the Planck time \( T_P \). We thus get a maximal total number of trials \( n_{\text{max}} \sim m \frac{T_U}{T_P} \sim 10^{143} \) allowed in our cosmological ‘Hubble volume’ (i.e., limited by causal considerations and the specific model used). This number should be compared with \( n \), or more precisely \( nN \simeq n \), the whole number of elementary trials in the super-Bernoulli process we want to realize. Therefore, if we consider \( P_{\text{atypical}} := P(|Q_{\bullet} - P_{\bullet}| \geq \sigma_{\text{max}} \delta Q_{\bullet}) = 1/n_{\text{max}} \sim 10^{-143} \), i.e., the probability for an atypical event in a Bernoulli sequence (\( E \)), we obtain from Eq. 13 (see Footnote 4) the largest possible value for \( \sigma_{\text{max}} \simeq 25.5 \) corresponding to \( \varepsilon_{\text{max}} = \sigma_{\text{max}} \delta Q_{\bullet} \simeq \frac{12}{\sqrt{N}} \). This represents the physical limits for the application of the WLLN in our Universe. Beyond this value, i.e., \( \sigma > \sigma_{\text{max}} \), we have \( P_e = 1/n < 10^{-143} \) which is universally negligible.

Most importantly for us, in discussing the experimental verification of the results predicted by the probability calculus Borel wrote:

\[ \text{negligible probabilities advocated by Borel was popularized in his book 'le hasard' [20] with the illustrative example of a group of monkeys using typewriters for randomly reproducing pages of famous books.} \]

\[ \text{We emphasize that the discussion presented here is limited to the Bernoulli process based on the binomial distribution. To take a known analogy, this corresponds to the problem of drawing a ball (N times) in a urn with replacement (the urn containing } M_{\bullet} \text{ and } M_{\heartsuit} \text{ balls respectively). Alternatively, if we consider drawing of a ball without replacement we have to use the hypergeometric distribution. In this case where the urn contains a finite number of balls } M = M_{\bullet} + M_{\heartsuit} \text{ the maximum number of extractions without replacement is naturally limited by } n_{\text{max}} := M, \text{ i.e., the sample size can not be larger than the whole population. We stress, that we believe the hypergeometric distribution to be a good starting point for a clean foundation of probability. In this approach the population } M_{\bullet}/M \text{ constitutes a definition of the probability } P_{\bullet} \text{ which can therefore be identified with the Laplace-Gibbs representation of the not-so fictitious probability fluid given by Eq. 16.} \]
the verification consists then to notice that the event whose probability is very low, that is negligible at the cosmic scale never happens. \[23\] p. 12]

This idea of an event whose probability is sufficiently small to never occur is central for understanding and justifying the application of the algorithmic probabilistic method to the physical world. It represents the ultimate (also named strong) interpretation of Cournot’s principle assumed by Borel and allows us to state that actual events are always typical. In other words, what is needed in order to break the infinite regress paradox is to eliminate atypical events from the actual world:

**Atypical events are never happening.**

This is the true message of typicality. Clearly, when we say that maverick alternatives are not happening we don’t say that they could not happen. Better, if atypical events are not happening this is because of a choice made by the Universe itself. In a deterministic Universe for example, assuming fixed evolution laws like Newtonian or Bohmian mechanics, the initial conditions on a distribution of, let say, particles over a space-like hyper surface \(\Sigma_0\) is all what is needed to fix the subsequent evolution of the system. This initial condition choice is not law-like (as the dynamical laws are) but better fact-like, i.e., contingent, since other choices were clearly possible and available in the configuration or phase space. In purely stochastic theories, like GRW \[48\], the ontology is about a cloud of events (e.g., a Bernoulli sequence) distributed over space-time and there is no law for the individuals only for the collective. Moreover, contrarily to the claims made by von Mises \[77\] this collective has not to be infinite. The WLLN associated with the strong Cournot principle leads to a statistical Born rule emerging at the collective level for \(N\) large but finite. In this approach atypical histories with low probabilities are simply not happening. In a certain sense, we could say that this is a conspiracy of nature albeit a typical one, i.e., a conspiracy that looks so natural to us that we don’t see it as extraordinary (at the difference of superdeterminism or retrocausality).

A misguided (but interesting) objection to this idea of an universally small probability regime (and therefore of atypicality) is that, after all, every branches \(h\) of the Bernoulli tree

---

\[13\] The different between a fact-like and law-like description is often arbitrary since many laws of nature that are once considered as fundamental are later accepted as emergent and are derived from more fundamental hypotheses (e.g., the thermodynamic relations are now derived from statistical mechanics).
have a small probability (e.g., $P_h = 1/2^N$ for the case $P_{\spadesuit/\heartsuit} = 1/2$). Therefore, if $N$ is large enough we have always $P_h \ll 10^{-143}$ that is universally negligible, and consequently the event $h$ should (according to a naive reading of Cournot principle) not occur. But since this is true for any $h$ none of the history $h$ should be allowed to occur and we have thus a paradox (this paradox was discussed by Borel in [23, p. 15]). The response to this paradox, is that when we speak of facts we are actually considering a Universe containing a machine or device programmed to find a specific event: for example the device could be programmed for finding the single history $h_1 := [\spadesuit, \spadesuit, \ldots, \spadesuit, \spadesuit] \in \mathcal{H}_{\text{atypical}}$ or alternatively the machine could be programmed for finding $h_2 := [\spadesuit, \heartsuit, \ldots, \spadesuit, \heartsuit] \in \mathcal{H}_{\text{typical}}$. The rule here, is that the only way for the machine to check if the outcome will actually occur is to try again and again (assuming the machine has no other information about the system), and the notion of atypicality advocated here says that this event with universally negligible probability will never happen in our Universe: this is a postulate. Now, to say that a machine or ‘observer’ programmed to find $h_1$ (or $h_2$) will not succeed is certainly different from telling that no machine at all will find $h_1$ (or $h_2$). The central difference is that in the former case the machine was programmed to find $h_1$ (or $h_2$) and in the latter not. Moreover, in the real world we, as devices, are not interested into such fine grained information but rather on the frequencies $Q_{\spadesuit/\heartsuit}$. It is with these observables $Q_{\spadesuit/\heartsuit}$ that probabilities $P(N_{\spadesuit})$ are defined and the WLLN is obtained (see Eqs. 11, 12). The notion of typicality and universally negligible probabilities used by Borel have therefore acquired a factual and physical meaning through the existence of typical observers and the non existence of atypical ones.

This reading of typicality looks probably extreme, or even odd, to many but we believe it is the only one which is devoid of contradiction and circularity. Indeed, typicality is a mere ‘geometrical’ statement about the size of a set $\mathcal{H}_{\text{typical}}$ relatively to $\mathcal{H}$ assuming a measure on these sets. Mathematically, it is just a characterization of a set. Yet, there is nothing in this definition which can force the system in the typical set to be actual or not (even if the typical set is overwhelmingly dominating the whole ensemble at a measure-theoretic level). Therefore, this dilemma requires a different axiom, as we suggested, in order to break the infinite regress paradox and to avoid to call typicality a pure tautology.

Therefore, the suggestion ‘atypical events are not happening’ is mathematically to reverse the order of the axioms in probability theory for all practical purposes. Here, we start with actual frequencies $Q_{\spadesuit}, Q_{\heartsuit}$ recorded during a Bernoulli sequence with $N$ independent
trials \(^{14}\) and we consider \(P\), \(P\) as estimators associated with a fictitious fluid satisfying the probability axiomatics (in particular the so called chain rule for independent trials that is fundamental for deriving the WLLN). The aim of these estimators is to reproduce accurately the recorded data \(Q\), \(Q\). And the best fit is, following the WLLN, obtained when \(Q\simeq P\), \(Q\simeq P\). Naturally, this inversion between the leading roles of frequencies and probabilities is only a practical rule of thumb, i.e., for an experimentalist interested into frequencies. Moreover, objective probabilities are fundamentally more important since they reveal an attractor for frequencies using the WLLN. Therefore, probabilities, like temperature or pressure, are objective properties of the whole system (or population) that are only probed and approached with samples of size \(N\). Importantly, our principle or method is valid for \(N \gg 1\) (i.e., in order to apply Stirling’s approximation needed for obtaining Eqs. \(^{11,13}\)). We don’t require the system to be actually or hypothetically infinite (unlike von Mises with his frequentist theory of probability \(^{77}\)). Additionally, the hypotheses leading to the WLLN are such that the equalities \(Q\simeq P\), \(Q\simeq P\) are not to be strict. The formalism allows for fluctuations so that for a recorded value \(Q\) we can define a confidence interval for \(P\). That is, for \(N \gg 1\) we have approximately \(^{15}\)

\[
|P - Q| \leq \sigma_{\text{max}} \frac{\sqrt{(Q(1-Q))}}{\sqrt{N}}
\]

(19)

with a confidence level \(\text{erf}(\frac{\sigma_{\text{max}}}{\sqrt{2}}) \simeq 1 - 10^{-143}\) identified with certainty.

Before to conclude this section we remind that in physics the importance of typicality was already stressed long ago by Boltzmann \(^{16,18}\) in his research for a clean foundation of statistical mechanics. For instance, replying to a well-known criticism made by Zermelo on the consistency of the probabilistic approach, Boltzmann wrote:

\(^{14}\) The notion of independence here means that we reproduce the same external conditions for the experiments and that the chain rule for independent probabilities apply. This is of course only a definition that is justified only by the physical consequences that we can get from it. Here we assume that nature satisfies this rule.

\(^{15}\) The confidence interval for \(P\) is obtained by inversion from the rigorous confidence interval for \(Q\), i.e, \(|P - Q| \leq \sigma_{\text{max}}Q\). The rigorous inversion leads to:

\[
|P - Q + \frac{\sigma_{\text{max}}^2}{2N}| \leq \sigma_{\text{max}} \frac{\sqrt{(Q(1-Q)) + \frac{\sigma_{\text{max}}^2}{4N}}}{\sqrt{N(1 + \frac{\sigma_{\text{max}}^2}{N)}}}
\]

(18)

that reduces to Eq.\(^{19}\) for \(N \gg 1\).
I assert on the contrary that by far the largest number of possible states are “Maxwellian” and that the number that deviate from the Maxwellian states is vanishingly small. [28, p. 395]

In the last decades, Boltzmann’s idea that discussions about typicality, i.e., about very large and very small probabilities, must play a central role for interpreting statistical mechanics and also quantum mechanics (e.g., in the de Broglie Bohm interpretation [41]) as evolved into a ‘typicality school’ 16, famously advocated by Lebowitz [70], Goldstein [52], and Penrose [82]. We emphasize that sometimes for advocates of the typicality school, typicality is considered as more fundamental than probability (i.e., too often assumed as a purely epistemic concept), and an opposition is made between Boltzmann and Gibbs. Here, we don’t subscribe to this radical view: typicality should not be thought as an alternative to probability, better typicality, i.e., through the strong Cournot principle, provides a practical and algorithmic way to use probabilistic deductions (i.e., it makes the calculus of probabilities safe of contradiction). Furthermore, for advocate of the typicality school, the typicality measure (e.g., the Liouville/Lebesgue measure in statistical mechanics or the equivariant $|\Psi|^2$ measure in Bohmian mechanics) is supposed to have a natural, preferred physical meaning justifying his unambiguous use. Once again, we disagree: the choice of a typicality (i.e., probability) measure is actually guided by experiments and should not be (only) grounded on mathematical symmetry or elegance (even if we agree that equivariance is a good characterization of the actual state of our Universe obeying Born’s rule).

IV. WHY THE MANY-WORLDS INTERPRETATION IS TYPICALLY DEAD

Assuming the previous interpretation of typicality and universally negligible probabilities we can go back to the MWI proposed by Everett [9, 36, 43]. As we explained, the MWI preserves the purely unitary evolution and attempts to give an ontic value to the quantum state $|\Psi\rangle_t$ driven at every time $t$ by Schrödinger’s equations $i\hbar\partial_t|\Psi\rangle_t = H|\Psi\rangle_t$. Following this interpretation, if we consider a quantum Bernoulli process, like the one described in section II, we must assume that all branches $h$ in the history tree $\sum_h a^{X_h} b^{Y_h} |M_0, \text{Aurélien}_h\rangle_{tf}$, i.e.,

16 For recent philosophical discussions about some controversial features of typicality and probability in the context of statistical, quantum mechanics, and cosmology see [6, 10, 61, 69, 73, 75, 111, 114, 115, 119].
associated with Eq. 7 and depicted in Fig. 1 must be considered on an equal footing. There is no collapse in the MWI, and, unlike in the Copenhagen interpretation, the wave-function $|\Psi_t\rangle$ can not just be used as a book-keeping for quantum alternatives before the actualization of only one history $h_0$.

In this context, it is not very difficult to see that the mere philosophy of the MWI directly conflicts with the notion of typicality discussed in section III. Indeed, if all branches should be preserved in the full unitary evolution $U_t$, then $U_t$ necessarily includes so-called maverick or atypical branches which, following our Cournot-Borel principle ‘atypical events are not happening’, must be forbidden to occur. It is here central to compare the MWI with ‘single-world’ interpretations (like the Copenhagen or Bohmian interpretation) which assume the different alternatives $h$ as virtually existing in a configuration or phase space. In the end, for these various single-world frameworks there is only one reality for the quantum system and only one $h$ is actualized (whatever ‘actual’ meant in these approaches). The fact that this actualization occurs, in agreement with Cournot’s principle, in the typical set $\mathcal{H}_{\text{typical}}$ satisfying Born’s rule is not contradictory and can safely be made an integral part of the ontology. That is clearly not the case in the MWI and maverick branches can not just be eliminated without changing the ontology (i.e., by modifying the dynamics or adding some new ideas as suggested in the introduction).

Interestingly enough, this issue has always been known, even though, perhaps not fully appreciated by all advocates of the MWI. One of the earliest discussion of this topic was probably done by DeWitt who, after reminding that ‘all the worlds are there, even those in which everything goes wrong and all the statistical laws break down’, wrote:

\begin{quote}
We can perhaps argue that in those branches in which the universe makes a habit of misbehaving on this way, life fails to evolve; so no intelligent automata are around to be amazed by it. 37
\end{quote}

In other words, it is claimed that the maverick branches are not existing because they violate some entropic and anthropic rules which would otherwise conflict with our own physical existence. This is somehow introduced by fiat as a contingent fact. Moreover, DeWitt even goes to suggest that

\begin{quote}
It is also possible that maverick worlds are simply absent from the grand superposition. 37
\end{quote}
DeWitt subsequently speculated about the possibility of a finite Universe containing only a finite number of branches [37]. Here we are not going to discuss nonlinear extensions of the MWI but these are serious possibilities to be considered.

Yet, the importance of maverick worlds or branches has been recently underlined by McQueen and Vaidman [76] in a response to Albert [5]. In complete agreement with Dewitt (at least on that issue) they indeed wrote:

*But now there are two responses: (i) once we start talking about worlds with different natural processes, we begin talking about worlds that may not support life and therefore measurement outcomes; and (ii) even if such worlds do support life, we find it very implausible that it would be possible to construct a reasonable theory of natural phenomena in such worlds.* [76]

The explanation of McQueen and Vaidman is motivated by the critical analysis made by Albert [5] concerning the absence (in the MWI) of empirical confirmations of the statistical Born’s rule \( Q_\alpha \sim P_\alpha \) for atypical histories \( h \in \mathcal{H}_{atypical} \). Albert wrote:

Why (for example) should it come as a surprise on a picture like this, to see what we would ordinarily consider a low-probability string of experimental results? Why should such a result cast any doubt on the truth of this theory (as it does, in fact, cast doubt on quantum mechanics)? [5, p. 162]

We completely agree with Albert, and we actually think that this is a fatal objection against the MWI. Interestingly, McQueen and Vaidman [76] try an answer by suggesting a difference

\[ A'_{h} = a^{N_{h}^\bullet} b^{N_{h}^\dagger} \text{Rect} \left( \frac{Q^\bullet - P^\bullet}{2\sigma_{\text{max}}^{\delta} Q^\bullet} \right) \] (20)

where \( y = \text{Rect}(x) \) is a normalized boxcar function such that \( y = 1 \) if \( |x| \leq \frac{1}{2} \) and \( y = 0 \) otherwise. All branches outside the confidence interval given by Eq. 19 are thus eliminated from the ‘grand superposition’. For a different but related idea see [39]. An other approach would be to assume the many de Broglie Bohm ontology [26, 94, 105] and assumes that there is only a finite number of world-lines associated with the population of Universes. This population is distributed according to Born’s rule and during a long Bernoulli process one could imagine that some atypical branches would be ultimately empty because the population of world-lines is not large enough to fill every branches (this is perhaps the meaning given by DeWitt in [37]).
between confirmation and deduction of a probability rule. They suggest that the probability rule must be deduced from the theory itself, i.e., the MWI, including all branches $h$, but that only the experimental records of observers belonging to the typical set $H_{\text{typical}}$ are physically significant for us as empirical confirmation (i.e., assuming we are typical observers).

The previous quote of McQueen and Vaidman concerning the difficulty or impossibility of confirmation for an observer belonging to $H_{\text{atypical}}$ is linked to this suggestion of DeWitt of prohibiting atypical worlds, i.e., by suggesting that life or sophisticated creatures able to records data are simply absent in those maverick branches. Furthermore, note that McQueen and Vaidman \[76\] assume that the situation is not better no worst that it is in other statistical and quantum interpretations. Replying to Albert they wrote:

Consequently, we can ask Albert’s question: Why should it come as a surprise, on a collapse theory, to see what we would ordinarily consider a low-probability string of experimental results? After all, such a string is logically consistent with a collapse theory. The answer is the same as in the MWI: there is no reason for why one should take such a string to refute collapse theories if a much broader set of data (including e.g. the reason for the sky’s colour) supports the Born rule. \[76\]

But assuming Cournot’s principle we believe that the previous analysis is inappropriate: it is definitely not a question of being ‘logically consistent’. According to strong-Cournot’s principle atypical strings don’t occur not because of a dynamical law but because of a contingent choice made on the initial conditions of the Universe (i.e., in a deterministic case) or because of a choice done in the whole tapestry of the Universe in a long-run sequence (i.e., in a stochastic case). This must be assumed as a fundamental property of the Universe and this even if logically speaking the laws of physics don’t prohibit maverick worlds. It is interesting to observe that the paradoxical conclusions obtained by McQueen and Vaidman are the same as could be made by somebody objecting against the self-consistency of the probability calculus, i.e., by ignoring the strong Cournot principle. This inappropriate objection states that the observation of an event with very low probability would not invalidate the principle of the probability calculus since this theory only deals with probabilities (that are by definition associated with the absence of certainty), whereas the non observation of such an event would not prove the theory either. In a sense the theory of probability would be
irrefutable or unfalsifiable and, as a consequence, non-scientific! This objection misses the point that one must necessarily include Cournot’s principle in any application of the whole probability method to the real physical world. This is actually equivalent to assuming that all observed events are typical, and therefore, that atypical or maverick events never occur. Once again, this strong Cournot principle makes perfectly sense in a single-world picture if the ontology can afford this constraint (e.g., by playing with the initial conditions like in Bohmian mechanics). This is not true in the MWI where unitarity requires all branches, even very atypical ones, to exist. Therefore, the analysis of DeWitt and McQueen-Vaidman are untanable and indefendable if we stick to the framework of the pure unitary MWI 18.

Going back to this difference between confirmation and derivation of Born’s rule advocated by McQueen and Vaidman, we already stressed in the introduction that many authors in the past have correctly emphasized the circularity and difficulties of all the purported proofs of Born’s postulate in the MWI [1, 7, 10, 11, 60, 64, 85, 99, 100]. Putnam [85], for instance, like Hemmo and Pitowsky [60], have questioned why a simple branch counting should not be favored instead of the $|\Psi|^2$-rule. More recently, Bricmont [27] has been very clear about it. Commenting about what will occur in a asymmetric situation where $P_\uparrow = 3/4$ he wrote:

\begin{quote}
But then the same use of the law of large numbers leads to the conclusion that, in the vast majority of worlds, the quantum predictions will not be observed, since our descendants will still see $N/2$ splittings where the cats end up alive and $N/2$ splittings where they end up dead, instead of the $(3/4, 1/4)$ frequencies. [27, p. 203]
\end{quote}

Despite many attempts made by advocates of the MWI over the years, we believe that this Bricmont objection about the simple branch-counting constitutes a valid objection against the standard MWI. Moreover, the situation is even worse than that: The question is not to know if the simple branch-counting could be a natural alternative to Born’s rule 19.

---

18 A recent article by Saunders reproduces the same error. He wrote ‘Denizens of anomalous branches, or of anomalous stretches of history in one-world theories, will be misled by the observed statistics of measurements. They will conclude that quantum mechanics (or at least the Born rule) is false. But they will simply be unlucky’ [91]. This inappropriate notion of ‘bad luck’ conflicting with Cournot’s principle is also advocated by Tappenden [103] in his criticism of Adlam work [1].

19 Note, that the motivation for the simple branch-counting procedure considered here should not be confused with a naive perspective that we could call the ‘god-eye perspective’ for which the observer metaphysically
Better, the question is to understand whether or not any probability rule can be chosen unambiguously in the current framework of the MWI. We believe that the answer is not, and that the MWI can not survive to this criticism without important amendments. In a remarkable paper Adlam [1] has been very lucid about that issue and wrote

Moreover, the Everett approach entails not only that mod-squared amplitudes cannot play this role but, but also that nothing else in the theory can play it either: since the theory does not single out any one sequence of outcomes, no entity defined within that theory can be responsible for our having seen the particular sequence of results that we have, and it follows that in an Everettian universe we would not be able to establish reference to the theoretical entities required by the Everettian theory. [1, p. 26]

Once again, we fully agree with this diagnostic: Both the Bricmont and Adlam analysis drive the final nails into the coffin of the old MWI. Moreover, here we want to play the role of the cononer and, based on our previous analysis of typicality, we actually provide the autopsy report of the MWI.

Remind first, Borel dictum ‘Atypical events are never happening’, and the consequence this message has on the pragmatic use of probabilities $P_\alpha$ as estimators for frequencies $Q_\alpha$. Remind also, that in quantum mechanics the Bernoulli process represents a kind of fractal tree sketched in Figure. [1] This tree is before all a topological structure and if we want to define a probability measure on this tree we must define a kind of fundamental metric for weighting the different branches: this what is named defining the theory by McQueen and Vaidman [76].

However, for any branches $h$ on the tree we can define a frequency $Q_h$ and by applying Eq. [19] for a number of trials $N \gg 1$ we can always estimate $P_\uparrow \sim Q_h$ by using the parameter $\sigma_{\text{max}}$ associated with universally negligible probabilities à la Cournot-Borel. And note again, that since the MWI is rigorously preserving unitarity, this is actually valid for every branches $h$, i.e., every sequences of outcomes. Therefore, once we assume the existence of all branches we have no other choice than to accept that the theory contains all the probability

located outside the Universe count the branches. Instead, here we are correctly discussing the inner perspective advocated by Everett where the observer is an integral part of the unitary evolution in the Bernoulli tree. Such an observer is not aware of the existence of the other branches.
laws, i.e., including the Born rule but also the simple branch-counting and anything else we want only constrained by the law \( P_\triangle + P_\triangledown = 1 \). In other words, in the MWI in order to fit \( Q_\triangle \), we can arbitrarily change the metric \( P_\triangle, P_\triangledown \) and subsequently the domain of typicality on the branches of the fractal Bernoulli-tree. And note once again, that this peculiar situation is specific of the MWI. In all accepted single-world quantum interpretations with collapse (like GRW) or not (like the PWI) there is no problem because we can postulate Born’s law as an external rule for univocally reproducing empirical statistics. As a matter of fact, the MWI, as it is built only from unitarity, can just not be true since it can not unambiguously impose Born’s rule.

The previous conclusion has its cost. If every frequencies are allowed in the MWI then maverick worlds that looks very atypical using Born’s rule have necessarily their place in the fractal Bernoulli-tree. Consider for example the case where \( a = b = 1/\sqrt{2} \) in Eq. 2, i.e., where Born’s rule apriori imposes \( P_\triangle = P_\triangledown = \frac{1}{2} \). The typicality reasoning based on the WLLN imposes thus \( Q_\triangle \approx \frac{1}{2} \) as it should be. However, atypical branches such as \( Q_\triangle \approx 0 \) or \( Q_\triangle \approx 1 \) can not be prohibited in the MWI. Theses worlds will be associated with their own regimes of typicality where \( P_\triangle \approx 0 \) or \( P_\triangle \approx 1 \). And this is the only way to make sense at the same time of the MWI and of Cournot’s principle. In other words, this is the only solution for building a theory that preserves unitarity together with the self-consistency of the probability calculus. Moreover, this theory, i.e., the bare MWI, clearly disagrees with experiments imposing Born’s rule: Consequently, the bare MWI must be rejected! 20

At this step of the analysis, many proponents of the MWI should probably getup on the stage to oppose this conclusion. After all, quantum mechanics has a preferred way to weight branches. Born’s rule \( P = |\psi|^2 \) is not arbitrary: it is imposed by quantum mechanics itself. Indeed (would oppose a typical many-worlder), we have several ways to justify and introduce Born’s rule: (i) We can obtain it from Schrödinger’s equation \( i\hbar \partial_t |\Psi\rangle_t = H |\Psi\rangle_t \) as a consequence of the local law of conservation \( -\partial_t |\psi|^2 = \nabla \cdot J_\psi \) where \( J_\psi \) is a current. This is obtained rigorously from Noether’s theorem under phase (gauge) invariance. (ii) We have Gleason’s and Everett’s reasoning motivating the choice of an additive measure

20 We stress that this rejection constitutes a way for solving the problem of the quantum suicid or ‘quantum russian roulette’ popularized by Tegmark [104] and relying on the existence of a extremely atypical branch where the player is always surviving and winning: This atypical branch is on the long run violating Cournot’s principle and thus Born’s rule... The impossibility of such a crazy situation is clearly demonstrative.
\( f(|a|^2) + f(|b|^2) = f(|a|^2 + |b|^2) \) \([13, 50]\). (iii) we have ‘equivariance’ that also motivates the choice \( \mathcal{P} = |\psi|^2 \) from current conservation (this is often used by Bohmians \([39, 51]\)).

(iv) We have also more Everettian-like derivations based on Deutsch-Wallace decision theory \([35, 89, 90, 116, 117]\) or ‘envariance’ \( \text{à la Zurek} \) \([8, 95, 120, 121]\). (v) Finally, we have Vaidman derivations \(21\) based on locality, symmetries and equiprobability \([76, 107, 109]\).

All these derivations, in addition to unitarity, require some ‘natural’ hypotheses that are motivated by empirical facts (like locality and equiprobabilities based on Keynes-Laplace’s indifference principle) or elegant symmetries (like equivariance and envariance).

However, despite naturalness or elegance these results conflicts with our previous conclusion concerning the MWI and typicality. It is not here our aim to review all the previous purported proofs of Born’s rule in order to find the source of the disagreement. Instead, we want to emphasize the role of a old but often neglected result \(22\) obtained by de Broglie \([29]\) and Bohm \([12]\) in the context of the pilot wave mechanics. Consider once again the local conservation law \(- \partial_t |\psi|^2 = \nabla \cdot J_{\psi} \) deduced from Schrödinger equation. In the pilot wave interpretation we introduce the velocity field \( v_\psi(x,t) = \frac{J_{\psi}(x,t)}{|\psi|^2} := \frac{dx(t)}{dt} \) (associated with the motion of the beables with coordinates \( x(t) \) in the configuration space) and the conservation law reads

\[- \partial_t |\psi|^2 = \nabla \cdot (v_\psi |\psi|^2). \tag{21}\]

Moreover, de Broglie and Bohm suggested that the most general probability distribution \( \rho(x,t) = f(x,t)|\psi|^2 \) carried by the same velocity field \( v_\psi(x,t) \) must also satisfy the conservation

\[- \partial_t \rho = \nabla \cdot (v_\psi \rho). \tag{22}\]

\(21\) We stress that for Vaidman there is no real derivation of the Born’s rule that do not assume additional axioms \([107, 109]\). Moreover, following our analysis of typicality and Cournot’s principle, Vaidman deductions motivated by locality can only work in those branches where Born’s rule hold. This is thus circular and inappropriate for ‘maverick branches’. The same is true for decision theoretic derivations \( \text{à la Deutsch Saunders Wallace} \) \([35, 90, 117]\) where Born’s ‘epistemic’ probability, i.e., defined as a subjective credence, is assigned to every branches including Maverick ones where the rule contradicts bare facts (that is in conflict with the Lewis principal principle).

\(22\) Over the years, this result has been nevertheless discussed by some advocates of the PWI (especially Bohm and Vigier \([13]\), Bohm and Hiley \([15]\), Valentini \([110, 111]\) including the present author \([39]\)). For a complete review of different probabilistic Bohmian interpretations see \([31]\).
comparing the two laws of conservation for $|\psi|^2$ and $\rho$ we immediately deduce the Lagrange derivative

$$(\partial_t + v_\psi \cdot \nabla)f(x,t) := \frac{d}{dt}f(x(t),t) = \frac{d}{dt}f_0 = 0$$

showing that the function $f$ is transported as a whole along a trajectory $x(t)$. This fundamental result is the counterpart of Liouville theorem’s $\frac{d}{dt}\eta(x(t),p(t),t) = 0$ in statistical mechanics where $\eta(x,p,t)$ is the probability density in the phase space. To enhance this analogy, the configuration space measure must be written $\delta\gamma(x,t) = |\psi(x,t)|^2dx$ in the configuration space, i.e., as the counterpart of the Lebesgue-Liouville measure $\delta\Gamma(q,p,t) = dqdp$ in the phase space [38]. In both cases, we have $\frac{d}{dt}\delta\Gamma_t = 0 = \frac{d}{dt}\delta\gamma_t$ meaning that the measure is preserved during the time evolution. Note that this result is very robust and can be obtained with all known Hamiltonians used in quantum mechanics. Furthermore, the theorem doesn’t require us to subscribe to the PWI: the velocity fluid can be seen as an hydrodynamic representation of the Schrödinger equation and is ontologically neutral. Importantly, the probability $dP = \rho dx$ to find the beables at point $x,t$ in the infinitessimal volume $dx$ is also written $dP = f d\gamma$ where $f$ plays the role of the genuine probability density with respect to the $\gamma$–measure. Like for the Liouville theorem the relation $\frac{d}{dt}f_0 = 0$ shows that once we define the density $f_0$ at a time $t_0$ we know its value at any time $t$ along the path. Therefore, if we postulate, like de Broglie did in 1928, $f_0 = 1 \forall x$ this will also be true at any time $t$, and the Born rule $\rho(x,t) = |\psi(x,t)|^2$ will be satisfied. Yet, we are not obliged to do so and consequently the condition $f = 1$, i.e., Born’s rule, is not mandatory.

It is at that step that one must add other principles to justify $f = 1$. This can be done in the PWI by postulating some distributions over the initial conditions and we can try, like Bohm and others [12, 13, 38, 110, 111], to justify the robustness of the condition $f \simeq 1$ as a kind of attractor under a chaotic dynamic. However, in the MWI such studies are not relevant since the coordinates $x(t)$ are not recognized as beables of the Everettian approach (unlike in the many de Broglie-Bohm worlds [26, 94, 105]). The problem is that there is no place in the theory for introducing an additional postulate for $f = 1$ without pervading the original goal of Everett. Therefore the diagnostic is irrefutable: in order to save the MWI one must modify or amend the theory (e.g., by adding a Born-Vaidman principle physically justified).

Going back to the quantum Bernoulli process studied in this article we will now detail
this diagnostic and show that the wave-function \( |\Psi_1\rangle = a|\uparrow\rangle + b|\downarrow\rangle \) allows us to discuss the role of the \( f\)-distribution in the MWI. To fix the idea consider that \( \uparrow / \downarrow \) are the two \( z\)-spin states of a single (non-relativistic) electron described by the wave-function

\[
|\Psi_1\rangle_t = \int d^3x [\psi_\uparrow(x,t)|\uparrow\rangle + \psi_\downarrow(x,t)|\downarrow\rangle]
\]  

(24)

with \( \psi_\uparrow, \psi_\downarrow \) the two components of the electron-spinor \( \phi \) and \( x \in \mathbb{R}^3 \) the spatial electron coordinates. Within the Bohm-Schiller-Tiomno theory \[14\], that expresses the PWI in the context of the Pauli equation for the non-relativistic electron with spin, we deduce the conservation law \(- \partial_t \rho_\psi = \nabla \cdot \mathbf{J}_\psi \) where \( \rho_\psi = \phi^\dagger \phi := |\psi_\uparrow|^2 + |\psi_\downarrow|^2 \) is the generalization of Born’s density of probability (i.e., the equivariant distribution) in presence of continuous configurations space variables \( x \) and discrete spin variables \( \uparrow / \downarrow \). Writing the current \( \mathbf{J}_\psi = \rho_\psi \mathbf{v}_\psi \) we obtain the hydrodynamic velocity \( \mathbf{v}_\psi(x,t) := \frac{dx(t)}{dt} \) of the de Broglie-Bohm fluid. For the present purpose the exact expression of the \( \mathbf{v}_\psi \) field is not relevant \[23\]. What is instead relevant is to understand that an initial localized single-electron wavepacket \( |\Psi_1\rangle_{t_0} \) defined at time \( t_0 \) (i.e., with \( \psi_\uparrow(x,t_0) = aF(x) \), \( \psi_\downarrow(x,t) = bF(x) \) and \( F(x) \) is a narrow wave function picked on \( x = 0 \) interacts with the magnetic field of a Stern-Gerlach apparatus and subsequently splits at time \( t_f \) into two secondary and non overlapping wavepackets

\[
\psi_\uparrow(x,t_f) = aF(x + d/2), \quad \psi_\downarrow(x,t_f) = bF(x - d/2) \quad \text{(i.e., assuming that the separation \( d \) between the two wavepackets is larger than their width)}.
\]

If \( F(x) \) is normalized the total \( \gamma \)-measure of the initial wavepacket defined as \( \gamma_{t_0} = \int d^3x \phi^\dagger(x,t_0)\phi(x,t_0) \) reads \( \gamma_{t_0} = (|a|^2 + |b|^2) \int d^3x |F(x)|^2 = |a|^2 + |b|^2 = 1 \). After the splitting the two wavepackets are associated with measures \( \gamma_{t_f}^{(\uparrow)} = |a|^2 \int d^3x |F(x + d/2)|^2 = |a|^2 \) and

\[
\gamma_{t_f}^{(\downarrow)} = |b|^2 \int d^3x |F(x - d/2)|^2 = |b|^2 \text{ respectively, and we have naturally conservation of the total measure, i.e.,} \quad \gamma_{t_0} = \gamma_{t_f}^{(\uparrow)} + \gamma_{t_f}^{(\downarrow)} = 1. \]

Moreover, from the existence of the \( \mathbf{v}_\psi \)-field we deduce that a fraction \( \gamma_{t_0}^{(\uparrow)} = |a|^2 \) (respectively \( \gamma_{t_0}^{(\downarrow)} = |b|^2 \)) of the initial \( \gamma \)-fluid is continuously evolving to transform into the domain \( \gamma_{t_f}^{(\uparrow)} = |a|^2 \) (respectively \( \gamma_{t_f}^{(\downarrow)} = |b|^2 \)) of the final \( \gamma \)-fluid.

Moreover, this fluid is for the moment just a mathematical measure and to transform it

\[23\] The general expression for \( \mathbf{v}_\psi \) is \( \mathbf{v}_\psi = \frac{|\nabla \cdot \mathbf{v}S_\psi/m + |\psi_\phi|^2 \nabla S_\psi/m - e/m \mathbf{A}}{|\psi_\uparrow|^2 + |\psi_\downarrow|^2} \) where \( e \) and \( m \) are the electric charge and mass of the electron respectively, and \( \mathbf{A}(x,t) \) is an external magnetic vector potential acting on the fluid \[14\]. The current of the Bohm-Schiller-Tiomno theory can be modified in order to agree with the non-relativistic limit of Dirac’s equation by adding to \( \mathbf{J}_\psi \) a spin-current term \( \mathbf{J}_{\psi}^{(S)} = \frac{\nabla \times (d^2 \sigma \phi)}{2m} \) \[15\].

29
into a probability fluid à la Laplace-Gibbs we must introduce the $f$-distribution. For the present purpose it is enough to use a coarse-grained description assuming that at time $t_f$ after the splitting $f$ is constant over the finite supports of the two wave packets $\psi_\spadesuit(x,t_f)$, $\psi_\heartsuit(x,t_f)$. More precisely, we introduce $f_\spadesuit$ and $f_\heartsuit$ such that

$$P_\spadesuit(t_f) = f_\spadesuit |a|^2, \quad P_\heartsuit(t_f) = f_\heartsuit |b|^2,$$

with $f_\spadesuit |a|^2 + f_\heartsuit |b|^2 = 1 \text{ (i.e., } f_\heartsuit = \frac{1-f_\spadesuit |a|^2}{1-|a|^2})$. Furthermore, applying the conservation rules we have at time $t_0$ $P_\spadesuit(t_0) = f_\spadesuit |a|^2$, and $P_\heartsuit(t_0) = f_\heartsuit |b|^2$.

Usually in the PWI we put $f_\spadesuit = f_\heartsuit = 1$ corresponding to the quantum equilibrium regime, i.e., Born’s rule, but this is not necessary. For instance, if we select $f_\spadesuit = \frac{|a|^2+|b|^2}{|a|^2} = |a|^{-2}$ and $f_\heartsuit = 0$ we have $P_\spadesuit(t_f) = 1$ and $P_\heartsuit(t_f) = 0$ corresponding to a highly non-equilibrated distribution. Following the WLLN and Cournot-Borel principle, this implies that on a long-run Bernoulli sequence we have typically $Q_\spadesuit \simeq 1$ and $Q_\heartsuit \simeq 0$, i.e., strongly conflicting with Born’s rule (i.e., $Q_\spadesuit \simeq |a|^2$, $Q_\heartsuit \simeq |b|^2$).

Now, in the PWI we can motivate or even justify Born’s rule $f_t = 1$ since this justification is ultimately connected to a postulate on the initial conditions of the Bohmian particles (or more generally beables) in a deterministic Universe. For a collapse model like GRW this is postulated as a stochastic law for the actual world. However, for the MWI this postulate is not possible and can not be motivated by any fundamental principle since everything in this theory is based on the pure unitary evolution $U_t$ and nothing more!

But actually this is even worse: Remember what we said previously about the absence of typicality in the MWI. There is no way in this interpretation to define univocally a Cournot-Borel condition for fixing $Q_\spadesuit/\heartsuit$. Since all branches are needed a case like $Q_\spadesuit \simeq 1$ and $Q_\heartsuit \simeq 0$ (that is strongly conflicting with Born’s rule) must be allowed. And now, we see that the Schrödinger theory or the Bohm-Schiller-Tiomno theory completely justifies the existence of probability distributions associated with a regime of non-equilibrium (e.g., $P_\spadesuit(t_f) = 1$ and $P_\heartsuit(t_f) = 0$). With our previous discussion about estimators and confidence intervals, it is always possible to find values $f_\spadesuit$ and $f_\heartsuit$ for fitting the empirical values $Q_\spadesuit$.

\footnote{Note, however that these probabilities $P_\spadesuit/\heartsuit(t_0)$ should not be falsely interpreted as telling that the particle has actually a spin component $\spadesuit/\heartsuit$ at time $t_0$. Instead, this is just a fluid conservation property, and the spin at time $t_0$ could be better defined using the Bohm-Schiller-Tiomno theory.}
and \( Q_{\heartsuit}^h \) in the branch \( h \) where we are located; even if that branch is a maverick one! \(^{25}\)

Therefore, all the purported attempts to prove Born’s rule within the current many-worlds framework are necessarily rooted in circular or misguided reasonings contradicting the mere spirit of the Everettian ontology. In other words, whereas for a ‘normal’ single-world theory (like the PWI or GRW) the probability function \( P_\alpha \) has a univocal and objective meaning, this can not be so in the MWI where only the frequencies \( Q_{\alpha}^h \) can be defined for the history \( h \) considered. Moreover, there is no rule for the convergence of \( Q_{\alpha}^h \) and all maverick and magical worlds are allowed in conflict with Cournot’s principle. For example, suppose one is repeating \( N_1 = 10^{10} \) times a quantum toss experiment and then again \( N_2 = 10^{10} \) more times the same experiment. We could find worlds where we have for the \( N_1 \) first trials \( h_1 = [\spadesuit, \spadesuit, \ldots] \) and subsequently \( h_2 = [\heartsuit, \heartsuit, \ldots] \) for the \( N_2 \) next trials. This looks completely odd and atypical but such worlds exist in the MWI. After the first sequence we could fit the frequency \( Q_{\spadesuit}^{h_1} = 1 \) with \( f_{\spadesuit} = |a|^{-2} \), \( f_{\heartsuit} = 0 \), and the second sequence \( Q_{\heartsuit}^{h_2} = 1 \) could be analyzed with \( f_{\heartsuit} = |b|^{-2} \) and \( f_{\spadesuit} = 0 \) (of course regrouping the two maverick sequences in a single one we get \( Q_{\spadesuit}^{h_1 \cup h_2} = 1/2 \) and thus \( f_{\spadesuit} = \frac{1}{2|a|^2} \), \( f_{\heartsuit} = \frac{1}{2|b|^2} \)). Following our application of the strong-Cournot principle that would support crazy laws of nature that are never really observed and would conflict with all known empirical quantum facts such as Born’s rule.

As a second example of crazy feature that can not be rejected in the MWI imagine an experiment with a beam-splitter, like before, where the observer sees \( Q_{\spadesuit}^h \simeq 1 \) even though \( |a| = |b| = 1/\sqrt{2} \). This corresponds, as we explained, to an atypical ‘maverick’ or ‘crazy’ statistics. If the number of trials \( N \) is very large the confidence level is so high that deviations are universally negligible in the Borel sense. Now, imagine that the observer after having obtained his/her results do a second experiments with a second beam-splitter after the first one for reuniting the two beams (the exit beams of the first beam-splitter are the input beams of the second one). This is a Mach-Zehnder interferometer and by tuning the phase we can force the beam to always exit in one door corresponding to the state \( |+\rangle \) : the other door (i.e., \( |-\rangle \)) being always empty. Now, that means that we have a really strange thing because with the two experiments (repeated \( N \gg 1 \) times on different qubits) the observer can conclude

\(^{25}\) The results obtained with the simple branch counting advocated by Putman and others \([27, 60, 85]\) correspond to \( f_{\spadesuit} = \frac{1}{2|a|^2} \), \( f_{\heartsuit} = \frac{1}{2|b|^2} \) which yields \( P_{\spadesuit} = P_{\heartsuit} = \frac{1}{2} \) selecting branches with frequencies \( Q_{\spadesuit} \simeq Q_{\heartsuit} \simeq \frac{1}{2} \) in the topological Bernoulli tree with \( 2^N \) branches illustrated in Fig 1.
that i) with the first experiment all the particles exit in the ♠ state: which implies that the beam ♣ is empty, ii) whereas in the second experiment the particles go always to the + state which is a signature of interference and therefore seems to imply that something was in both input branches ♠ and ♣ (in the PWI that would be the signature of an empty-wave). Again, this is a consequence of the strong non-equilibrium recorded in the ‘crazy’ observer branch. Finally, that means that observers in different branches of the Bernoulli tree can define their own domain of typicality with their own $f_t$ function. Therefore, in Everett’s relative state theory, i.e., the MWI, probabilities can not have a objective meaning: probabilities are relative and consequently badly defined (in strong conflict with Cournot’s principle). And don’t forget that Born’s rule, as we know it, is very robust empirically since the strong Cournot principle doesn’t allow significant deviations for $N$ large. But again, if we accept the usual MWI there is nothing that explains why we don’t see maverick worlds: This, we believe, irrevocably constitutes a dead-end for the MWI unless we are going to modify the theory as suggested by several authors.

V. CONCLUSION

To conclude this analysis it is important to go back to Everett himself. In his PhD thesis Everett provided a motivation for introducing a typicality reasoning. As he wrote:

*We wish to make quantitative statements about the relative frequencies of the different possible results of observation—which are recorded in the memory— for a typical observer state; but to accomplish this we must have a method for selecting a typical element from a superposition of orthogonal states.* [36, p. 70]

As we showed in our analysis of the typicality concept this is the point where he failed. As we showed in the usual probability calculus method the notion of typicality acquires a physical meaning through the strong Cournot principle advocated by Borel. Following this principle to apply the probability calculus we must define a regime of universally negligible probability or of atypicality. Without this concept it is impossible to transform a potentiality

---

26 An example of the same kind could be done with the which-path and double-slit experiments. A first experiment would reveal that all particles are going to slit 1, whereas a second experiment would reveal interference fringes characteristic of a wave involving the two slits 1 and 2.
into actuality and the probability calculus would remain unfalsifiable and useless. Many objections raised in the past against the relevance of probability for science were based on inappropriate applications of such a Cournot principle.

Moreover, in the MWI one of the subtle point often challenged is that all alternatives occur in decohered or orthogonal branches. Therefore, the concept of actuality seems to be in jeopardy and the standard Laplace notion of probability applied to alternatives has been criticized. This confusion has certainly inconsciously be used by many advocates of the MWI to justify a new and specific concept of probability used in the theory and to refute criticisms. However, Everett was relatively clear on that point. The introduction of the observer as an automaton with memory of only one of the numerous branches of the Bernoulli tree removes potential ambiguity and connects the standard probability method for alternatives (i.e., as used in single-world theories) to the apparently different situation occuring in the MWI. As he wrote:

*We have, then, a theory which is objectively causal and continuous, while at the same time subjectively probabilistic and discontinuous.* [9, p. 69]

In other words the branching tree of life of the observer memory provides the so called ‘illusion of probability’ advocated by Vaidman. However, as we showed this subjective perspective is not enough to emulate all aspects of the standard probability calculus. The illusion of probability is not enough to give us quantitative predictions for reproducing frequencies. Indeed, the success of the probability calculus relies on the unambiguous application of the WLLN itself relying on the strong Cournot principle. This methods shows that we can use unambiguously probabilities as estimators of frequencies for all practical purposes. The probabilistic convergence of frequencies \( Q_\alpha \) to \( P_\alpha \) is postulated to be so efficient that never we can see atypical maverick histories or branches in our real world. By ‘real’, we mean the empirical evidences provided by standard quantum mechanics imposing Born’s rule univocally. However, this is not possible in the bare MWI and no subjective/Bayesian notion of probability associated with the observer could ever changes that matter of fact. As we showed, with our example of a quantum coin tossing, the fractal Bernoulli tree with \( 2^N \) branches is coherent with infinitely many measures of probability and typicality \( P_\bullet \). According to the WLLN, each sectors of typicality selects a frequency \( Q_\bullet \) in the limit \( N \) large and therefore all maverick worlds have their own rationality and justification in the
MWI. It would be wrong to say, as many advocates of the MWI nevertheless do, that the situation with the MWI is not worse than in other interpretations of quantum mechanics, or like Everett, that ‘the situation is fully analogous to that of classical statistical mechanics’ [9, p. 125]. The situation could not actually be more different! In every single world models (or even in the many Bohm-de Broglie worlds ontology) reproducing quantum mechanics the application of Cournot’s principle and of the WLLN leads to a clean univocal definition of typicality. Like in classical statistical mechanics this is mandatory in order to apply the probability calculus method.

Furthermore, the measure of typicality Everett introduced is far from being univocal as we showed with our analysis of the de Broglie-Bohm theorem \( \frac{d}{dt} f_t = 0 \). As we analyzed the role of the \( f_t \) function often neglected provides the most general framework for discussing (continuously evolving) probability distributions in quantum mechanics. This refutes the claim of a natural and univocal measure of probability imposed by symmetries or anything else in the bare version of the MWI.

Finally, to close on a positive point we believe that this analysis provides clues and motivations for developing other ontologies based on the MWI. By adding a Born-Vaidman postulate it is already shown that the old MWI can be saved. However, we believe that this can only be done if we provide a ‘physical explanation’ for the additional principle. By physical explanation we don’t only mean that the principle should reproduce and justify all empirical evidences like Born’s rule and locality (that is the road followed currently by Vaidman [109]), but also that this principle should find it explanatory power in the common and accepted methods of theoretical physics. These accepted explanatory recipes tell that if the physical theory has to be deterministic then the new explanations should either comes from new evolution equations (may be nonlinear [30, 37, 58]), specific boundary or initial conditions (like in the PWI or the many de Broglie-Bohm worlds approach [26, 94, 105]), or perhaps by adding some structures (like observers) exploiting the pseudo-randomness associated with initial conditions in the deterministic Universe [40]. One of this strategy could perhaps provide an atypical path for making sense of Everett’s program.

[1] Adlam, E.: The problem of confirmation in the Everett interpretation. Studies in history and philosophy of modern physics 47, 21 (2014).
[2] Aguire, A. and Tegmark, M.: Born in an infinite universe: A cosmological interpretation of quantum mechanics. Phys. Rev. D 84, 105002 (2011).

[3] Aharonov, Y. and Reznik, B.: How macroscopic properties dictate microscopic probabilities. Phys. Rev. A 65, 052116 (2002).

[4] Albert, D. and Loewer, B.: Interpreting the many-worlds interpretation. Synthese 77, 195 (1988).

[5] Albert, Z. D.: After physics. Harvard University Press, Harvard (2015).

[6] Allori, V.: Some reflections on the statistical postulate: Typicality, probability and explanation between deterministic and indeterministic theories. In Statistical mechanics and scientific explanation, pp. 65-111. World Scientific, Singapore (2020).

[7] Ballentine, L. E.: Can the statistical postulate of quantum theory be derived? A critique of the many-universes interpretation. Found. Phys. 3, 229 (1973).

[8] Barnum, H.: No-signalling-based version of Zurek’s derivation of quantum probabilities: A note on ‘Environment-assisted invariance, entanglement, and probabilities in quantum physics’. arXiv:quant-ph/0312150v1 (2003).

[9] Barrett, J. A. and Byrne, P.: The Everett interpretation of quantum mechanics: Collected works 1955-1980 with commentary. Princeton University Press, Princeton (2012).

[10] Barrett, J. A.: Typical worlds. Studies in History and Philosophy of Science part B. Studies in History and Philosophy of Modern Physics 58, 31-40 (2017).

[11] Benioff, P.: A note on the Everett interpretation of quantum mechanics. Found. Phys. 8, 709 (1978).

[12] Bohm, D.: Proof that probability density approaches $|\psi|^2$ in causal interpretation of the quantum theory. Phys. Rev. 89, 458-466 (1953).

[13] Bohm, D., Vigier, J. P.: Model of the causal interpretation of quantum theory in terms of a fluid with irregular fluctuations. Phys. Rev. 96, 208-216 (1954).

[14] Bohm, D., Schiller, R., Tiomno, J.: A causal interpretation of the pauli equation (A). Nuovo Cim. 1, 48–66 (1955).

[15] Bohm., D and Hiley, B.J.: The undivided Universe. Routledge, London, (1993).

[16] Boltzmann, L.: Über die beziehung dem zweiten Haubtsatze der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung respektive den Sätzen über das Wärmegleichgewicht. Wiener Berichte 76, 373-435 (1877). English translation in \[98\].
[17] Boltzmann, L.: Vorselugen über gastheorie (Part I and II). Barth, Leipzig (1898). English translation by Brush, S. G.: Lectures on gas theory. University of California press, Berkeley (1964).

[18] Boltzmann, L.: Entgegnung auf die warmetheoretischen betrachtungen der Hrn. E. Zermelo. Ann. der Phys. 57, 773-784 (1896). English translation in [28, p. 392-402] as Reply to Zermelo’s remarks on the theory of heat.

[19] Borel, E.: Les probabilités denombrables et leurs applications arithmétiques, Rendiconti del Circolo Matematico di Palermo, 27 1055-1079 (1909).

[20] Borel, E.: Le Hasard. Librairie Félix Alcan, Paris (1914).

[21] Borel, E., Deltheil R.: Probabilités Erreurs. Armand Colin, Paris (1923).

[22] Borel, E.: Sur les probabilités Universellement Négligeables. C. R. Acad. Sci. (Paris) 190, 537-540 (1930).

[23] Borel, E.: Valeurs pratiques et philosophie des probabilités. Gauthiers-Villars, Paris (1939).

[24] Borel, E.: Probabilité et certitude. Dialectica 3, 24-27 (1949).

[25] Borel, E.: Probabilité et certitude. Presse Universitaire de France, Paris (1950).

[26] Boström, K. J.: Quantum mechanics as a deterministic theory of a continuum of worlds. Quantum Studies: Mathematics and Foundations 2, 315 (2014).

[27] Bricmont, J.: Making sense of quantum mechanics. Springer, Berlin (2016).

[28] Brusch, S.G.: The kinetic theory of gases: An anthology of classic papers with historical commentary; in History of modern physical sciences vol. 1. Imperial College Press, London (2003).

[29] De Broglie, L.: La mécanique ondulatoire. Mémorial des Sciences Physiques. Gauthier-Villars, Paris (1928).

[30] Buniy, R. V., Hsu, S. H. D., Zee, A.: Discreteness and the origin of probability in quantum mechanics. Phys. Lett. B 640, 219-223 (2006).

[31] Callender, C.: The emergence and interpretation of probability in Bohmian mechanics. Studies in History and Philosophy of Science part B.: Studies in History and Philosophy of Modern Physics 66, 351-370 (2007).

[32] Cantelli, F. P.: Considérations sur la convergence dans le calcul des probabilités, Annales de l’IHP, 5 3-50 (1935).

[33] Coleman, S.: Sidney Coleman Dirac lecture: ‘quantum mechanics in your face (1994)’.
[34] Cournot, A. A.: Exposition de la théorie des chances et des probabilités, in: Bru, B. (ed.). Hachette, Paris (1843).

[35] Deutsch, D.: Quantum theory of probability and decisions. Proc. R. Soc. A 455, 3129 (1999).

[36] DeWitt, B. S. and Graham, N.: The Many-Worlds interpretation of quantum mechanics. Princeton University Press, Princeton (1973).

[37] DeWitt, B. S.: Quantum mechanics and reality. Phys. Today 23, 30 (1970).

[38] Drezet, A.: Analysis of Everett’s quantum interpretation from the point of view of a Bohmian. Int. J. Quantum Found. 2, 67-88 (2016).

[39] Drezet, A.: How to justify Born’s rule using the pilot wave theory of de Broglie?. Ann. Found. Louis de Broglie 42, 103 (2017).

[40] Drezet, A.: Making sense of Born’s rule $p_\alpha = \|\psi_\alpha\|^2$ with the many-minds interpretation. To appear in Quantum Studies: Mathematics and Foundations (2021).

[41] Dürr, D., Goldstein, S., Zanghì, N.: Quantum equilibrium and the origin of absolute uncertainty. J. Stat. Phys. 67, 843 (1992).

[42] Ellis, L.: On the foundations of the theory of probabilities. John William Parker, London (1843).

[43] Everett, H. III.: ‘Relative State’ formulation of quantum mechanics. Rev. Mod. Phys. 29, 454 (1957).

[44] Farhi, E. and Goldstone, J.: How probability arises in quantum mechanics. Ann. Phys. 192, 368 (1989).

[45] De Finetti, B.: La prévision ses lois logiques, ses sources subjectives. Ann. Inst. Henri Poincaré 7, 1-68 (1397).

[46] Galavotti, M. C.: Pragmatism and the birth of subjective probability. European Journal of Pragmatism and american philosophy XI-1, doi.org/10.4000/ ejpap.1509 (2019).

[47] Geroch, R.: The Everett interpretation. Noûs 18, 617 (1984).

[48] Ghirardi, G. C., Rimini, A., Weber, T.: Unified dynamics for microscopic and macroscopic systems. Phys. Rev. D 27, 293-298 (1986).

[49] Gibbs, J. W.: Elementary principle in statistical mechanics: Developped with especial reference to the rational foundation of thermodynamics. Charles Scribner’s son, New York (1902).

[50] Gleason, A. M.: Measures on the closed subspaces of a Hilbert space. Indiana Univ. Math.
[51] Goldstein, S., Struyve, W.: On the Uniqueness of Quantum Equilibrium in Bohmian Mechanics. J. Stat. Phys. 128, 1197-1209 (2007).

[52] Goldstein, S.: Typicality and notions of probability in physics. In Ben-Menahem, Y. and Hemmo, M., Probability in physics, Chapt. 4, p. 59-71. Springer, Heidelberg (2012).

[53] Greaves, H.: Understanding Deutsch’s probability in a deterministic multiverse. Studies in History and Philosophy of Science part B.: Studies in History and Philosophy of Modern Physics 35, 423 (2004).

[54] Gutmann, S.: Usual classical probability to guarantee properties of infinite quantum sequences. Phys. Rev. A 52, 3560 (1995).

[55] Hacking, I.: The emergence of probability: A philosophical study of early ideas about probability induction and statistical inference. Cambridge University Press, London (1975).

[56] Hájek, A.: “Mises Redux”–Redux: fifteen arguments against finite frequentism. Erkenntnis 45, 209-227 (1997).

[57] Hájek, A.: Fifteen arguments against hypothetical frequentism. Erkenntnis 70, 211-235 (2009).

[58] Hanson, R.: When Worlds Collide: Quantum Probability from Observer Selection? Found. Phys. 33, 1129 (2003).

[59] Hartle, J. B.: Quantum mechanics of individual systems. Am. J. Phys. 36, 704 (1968).

[60] Hemmo, M. and Pitowsky, I.: Quantum probability and the many worlds. Studies in history and philosophy of modern physics 38, 333 (2007).

[61] Hubert, M.: Reviving frequentism. To appear in Synthese (2021).

[62] Jaynes, E. T.: Probability theory: The logic of science. accessible at the website http://bayes.wustl.edu/etj/prob.html (1999).

[63] Jeffreys, H.: Theory of Probability. Oxford University Press, London. (1961).

[64] Kent, A.: Against many-worlds interpretation. Int. J. Mod. Phys. A 5, 1745 (1990).

[65] Kent, A.: Does it makes sense to speak of self-locating uncertainty in the universal wave-function? Remarks on Sebens and Carroll. Found. Phys. 45, 211 (2015).

[66] Keynes, J. M.: Treatise on probability. Macmillan and Co., London (1921).

[67] Kolmogorov, A. N.: Foundations of the theory of probability. Chelsea Publishing, Chelsea (1950).
[68] Laplace, P. S.: Théorie analytique des probabilités. Courcier, Paris (1820). Originally published in 1812 and reproduced in Oeuvres complètes Vol. 7. Académie des Sciences, Paris (1878-1912). The English translation used here is by Hacking I. [55, p. 132].

[69] Lazarovici, D., Reichert, P.: Typicality, irreversibility and the status of macroscopic laws. Erkenntnis 80, 689-716 (2015).

[70] Lebowitz, J.: Boltzmann’s entropy and time’s arrow. Phys. Today 46, 32-38 (1993).

[71] Lewis, D. K.: A subjectivist’s guide to objective chance, in Jeffreys, R. C. (ed.) Studies in inductive logic and probability. Vol. 2 University of California press, Berkeley (1980).

[72] Martin, T.: La valeur objective du calcul des probabilités selon Cournot. Math. Inf. Sci. Hum. 1127, 5-14 (1994)

[73] Maudlin, T.: What could be objective about probabilities? Studies in History and Philosophy of Science part B.: Studies in History and Philosophy of Modern Physics 38, 275-291 (2007).

[74] Maudlin, T.: Philosophy of Physics: Quantum Theory. Princeton University Press, Princeton (2019).

[75] Maudlin, T.: The grammar of typicality. In Statistical mechanics and scientific explanation, pp. 231-255. World Scientific, Singapore (2020).

[76] McQueen, K. J. Vaidman, L.: In defence of the self-location uncertainty account of probability in the many-worlds interpretation. Studies in History and Philosophy of Science part B.: Studies in History and Philosophy of Modern Physics 66, 14-23(2019).

[77] Von Mises, R.: Probability, statistics and truth. MacMillian, New York (1957).

[78] De Moivre, A.: The doctrine of chances: or, A method of calculating the probabilities of events in play. Pearson, W., London (1718).

[79] Ochs, W.: When does a projective system of state operators have a projective limit?. J. Math. Phys. 20, 1842 (1979).

[80] Ohkuwa, Y.: Decoherence functional and probability interpretation. Phys. Rev. D 48, 1781 (1993).

[81] Papineau, D.: Many Minds are no worse than one. Brit. J. Phil. Sci. 47, 233 (1996).

[82] Penrose, R.: Cycles of times: An extraordinary new view of the Universe. The Bodley Head, London (2010).

[83] Poincaré, H.: Calcul des probabilités. Gauthier-Villars, Paris (1912).

[84] Popper, K.: Quantum theory and the schism in physics. Postcript to the Logic of scientific
discovery, vol. 3. Hutchinson and co., London (1982).

[85] Putnam,, H.: A philosopher looks at quantum mechanics (again). Brit. J. Phil. Sci 56, 615 (2005).

[86] Ramsey, F. P.: Philosophical papers, edited by Mellor, D. H. Cambridge University Press, Cambridge (1990).

[87] Reichenbach, H.: The theory of probability. University of California Press, Berkeley (1949).

[88] Reif, F.: Fundamentals of statistical and thermal physics. MacGraw-Hill, New York (1965).

[89] Saunders, S.: What is a probability?, in Quo Vadis quantum mechanics. edited by Eltizur, A. Dolev, S., and Kolenda N. Springer, Berlin (2005).

[90] Saunders, S. Wallace, D.: Branching and uncertainty. Brit. J. Phil. Sci. 59, 293 (2008).

[91] Saunders, S.: The Everett interpretation: Probability. arXiv:2103.03966 (2021).

[92] Saunders., S. Barrett, J. Kent, A. and Wallace, D.: Many Worlds?: Everett, quantum theory, and Reality. Oxford University Press, Oxford, (2010).

[93] Schrödinger, E.: Statistical thermodynamics. Cambridge University Press, Cambridge (1952).

[94] Sebens, C. T.: Quantum mechanics as classical physics. Philosophy of Science 82, 266 (2014).

[95] Sebens, C. T., and Carroll, S.: Self-locating uncertainty and the origin of probability in Everettian quantum mechanics. Brit. J. Phil. Sc. 69, 25 (2018).

[96] Shafer, G.: From Cournot’s principle to market efficiency, in: Touffut, J. P. (ed.), Augustin Cournot: Modelling economics, chapter 4. Edward Elgar Publishing, Cheltenham (2007).

[97] Shafer, G., Vovsk, V.: The source of Kolmogorov’s Grundbegriffe. Statistical Science 21, 70-98 (2006).

[98] Sharp, K., Matschinsky F.: Translation of Ludwig Boltzmann’s Paper “On the Relationship between the Second Fundamental Theorem of the Mechanical Theory of Heat and Probability Calculations Regarding the Conditions for Thermal Equilibrium” Sitzungberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissen Classe. Abt. II, LXXVI 1877, pp 373-435 (Wien. Ber. 1877, 76:373-435). Reprinted in Wiss. Abhandlungen, Vol. II, reprint 42, p. 164-223, Barth, Leipzig, 1909. Entropy 17, 1971-2009 (2015).

[99] Squires, E.: On an alleged ‘proof’ of the quantum probability law. Phys. Lett. A 145, 67 (1990).

[100] Stein, H.: The Everett interpretation: Many worlds or none?. Noûs 18, 635 (1984).
[101] Stigler, S. M.: The history of statistics: The measurement of uncertainty before 1900. The Bellnap Press of Harvard University Press, Cambridge (1986).

[102] Tappenden, P.: Evidence and uncertainty in Everett’s multiverse. Brit. J. Phil. Sci. 62, 99 (2010).

[103] Tappenden, P.: Everettian theory as a pure wave mechanics plus a no-collapse probability postulate. Synthese https://doi.org/10.1007/s11229-019-02467-4 (2019).

[104] Tegmark, M.: The interpretation of quantum mechanics: Many worlds or many words? Fortschr. Phys. 46, 855 (1999).

[105] Tipler, F.: Quantum nonlocality does not exist. PNAS 111, 11281 (2014).

[106] Vaidman, L.: On schizophrenic experiences of the neutron or why we should believe in the many-worlds interpretation of quantum mechanics. International Studies in the Philosophy of Science 12, 245 (1998).

[107] Vaidman, L.: Probability in the many-worlds interpretation of quantum mechanics, in Probability in physics, The frontiers collection, edited by Y. Ben Menuhem, M. Hemmo. Springer, Berlin (2012).

[108] Vaidman, L.: Quantum theory and determinism. Quantum Studies: Mathematics and Foundations 1, 5 (2014).

[109] Vaidman, L.: Derivations of the Born rule, in Quantum, probability, logic in physics, Jerusalem Studies in philosophy and history of science, edited by M. Hemmo, O. Shenker. Springer Nature, Switzerland (2020).

[110] Valentini, A.: On the pilot-wave theory of classical, quantum and subquantum physics. International School for advanced studies, Trieste (1992).

[111] Valentini, A.: Foundations of statistical mechanics and the status of the Born rule in the de Broglie-Bohm pilot-wave theory. In Statistical mechanics and scientific explanation, pp. 423-477. World Scientific, Singapore (2020).

[112] Van Wesep, R. A.: Many worlds and the appearance of probability in quantum mechanics. Ann. Phys. 321, 2438-2452 (2006).

[113] Venn, J.: The logic of chance, second edition. MacMillian, London (1876).

[114] Werndl, C.: Justifying typicality measures of Boltzmannian statistical mechanics and dynamical systems. Studies in History and Philosophy of Science part B.: Studies in History and Philosophy of Modern Physics 44, 470-479 (2013).
[115] Volchan, S. B.: Probability as typicality. Studies in History and Philosophy of Science part B.: Studies in History and Philosophy of Modern Physics 38, 801-814 (2007).

[116] Wallace, D.: Quantum probability from subjective likelihood. Studies in History and Philosophy of Science part B.: Studies in History and Philosophy of Modern Physics 38, 311 (2007).

[117] Wallace, D.: The emergent multiverse: quantum theory according to the Everett interpretation. Oxford University Press, Oxford (2012).

[118] Wallace, D.: The Probability puzzle and many-worlds interpretation of quantum mechanics. https://www.youtube.com/watch?v=8turL6Xnf9U (2015).

[119] Wilhelm, I.: Typical: A theory of typicality and typicality explanation. The British Journal for the Philosophy of Science. doi.org/10.1093/bjps/axz016 (2019).

[120] Zurek, W. H: Environment-assisted invariance, entanglement, and probabilities in quantum physics. Phys. Rev. Lett. 90, 120404 (2003).

[121] Zurek, W. H.: Probabilities from entanglement, Born’s rule $p_k = |\psi_k|^2$ from envariance. Phys. Rev. A 71, 052105 (2005).