THE EVOLVING ADAPTIVE NEURAL NETWORK FOR DATA PROCESSING WITH MISSING OBSERVATIONS

The problem of computational intelligence systems synthesis in on-line mode, capable for processing stochastic signals with missing observations in the data is considered. An adaptive approach based on using of orthogonal polynomials is developed.

Keywords: neural network, orthogonal polynomials, Chebyshev polynomials, incomplete data with missing observations.

INTRODUCTION

Artificial neural networks, neuro-fuzzy systems, hybrid systems of computational intelligence currently are widespread for solving data processing problems, Data Mining, prediction, identification and control of nonlinear stochastic and chaotic objects and systems [1–5].

The most attractive properties of these systems are their universal approximation properties and learning ability, usually understood as a possibility of tuning the parameters by optimizing some quality criterion (objective function, learning criterion). In a wider sense, it is possible to configure the system architecture too. Currently, there exist number of approaches, but the most widely used is so-called constructive approach in which the system of computational intelligence, starting with the simplest architecture, gradually increasing its complexity and at the same time tuning their parameters to achieve the desired quality of the solution. This approach has formed a new direction in computational intelligence, known as evolving systems [6, 7]. At the same time, most of these systems process information in a batch mode, which makes them difficult to use in cases where the data for processing are fed in on-line mode in the form of time series.

In many practical applications involving the processing of sequences of real data, there exists situation when some observations of the controlled time series, for whatever reasons, are missing (lost). It is understood that for the normal operation of the neural network or a hybrid system, these missing data must somehow be restored. The problem of restoring the missing values has received sufficient attention [8–10], in this case as the most effective are neural networks [11–14]. However, the known approaches for restoration of missing values in time series are effective only in cases when the whole data set is given a priori, the amount of missing values is known, and the time series has a fixed number of observations. It is natural that in problems where data are fed for processing in real time, the number of missing values is unknown beforehand and the sequence has nonstationary character, the known approaches are ineffective.

So, the problem of synthesis of computational intelligence systems in on-line mode, capable for processing stochastic signals with missing observation in the data is interesting and useful.

SEQUENTIAL RESTORATION OF MISSING OBSERVATIONS IN THE TIME SERIES

The proposed approach is based on the use of classical orthogonal polynomials [15] and first of all Chebyshev polynomials (T-systems) [16, 17], that have a number of good properties in terms of the approximation problem using quadratic criterion [16].

Generally Chebyshev polynomials can be compactly represented using trigonometric functions

\[ T_l(x) = \cos(l \arccos x), \quad x \in [-1, 1], \quad l = 0, 1, 2, ..., \]  

or using recurrent relation

\[ T_l(x) = 2xT_{l-1}(x) - T_{l-2}(x). \]  

In some situations it is easier to solve the problem in the interval \( x \in [0, 1] \), which can be used with biased Chebyshev polynomials:

\[ T_l^B(x) = T_l(2x - 1), \]  

\[ T_l^{B'}(x) = (4x - 2)T_{l-1}(x) - T_{l-2}(x). \]  

It is also easy to write biased polynomials for any interval \( x \in [x_{\text{min}}, x_{\text{max}}] \).

To approximate arbitrary function of a scalar argument \( y = f(x) \) given on a set of nodes \( x_1, x_2, ..., x_N \); \( y_1, y_2, ..., y_N \), let us transform the original data using simple relations

\[ \tilde{x}_k = 2 \frac{x_k - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} - 1; \]

\[ \tilde{y}_k = 2 \frac{y_k - y_{\text{min}}}{y_{\text{max}} - y_{\text{min}}} - 1, \quad k = 1, 2, ..., N \]
for the polynomials (1), (2) and
\[ \bar{x}_k = \frac{x_k - x_{\min}}{x_{\max} - x_{\min}}; \quad \hat{y}_k = \frac{y_k - y_{\min}}{y_{\max} - y_{\min}}, \quad k = 1, 2, ..., N \]
for (3), (4) and write approximating T-system as
\[ T^N_h(\bar{x}) = \sum_{l=0}^{h} w_l T_l(\bar{x}), \]
whose parameters \( w_l \) are determined by the simple relation
\[ w_l = \sum_{k=1}^{N} \frac{\hat{y}_k T_l(\hat{x}_k)}{\sum_{k=1}^{N} T^2_l(\hat{x}_k)}, \]
with conditions of orthogonality.

It is easy to see that these expressions are standard estimates of least squares. The values \( T_l(\hat{x}_k) \) are calculated using the recurrent relations
\[ T_0(\hat{x}_k) = \hat{x}_k - b_1, \]
\[ T_l(\hat{x}_k) = (\hat{x}_k - b_l)T_{l-1}(\hat{x}_k) - a_lT_{l-2}(\hat{x}_k), \]
where index \( N \) in \( T_{IN} \) means that the corresponding polynomials are orthogonal on the interval \( k \in [1, N] \).

After this it is easy to restore the missing observations in the form
\[ \hat{x}_k = \sum_{l=0}^{h} w_l T_l(\hat{x}_k) \quad \forall k \in X_G. \]

Combining the available \( (\hat{x}_k \in X_P) \) and restored \( (\hat{x}_k \in X_G) \) values of the time series, it’s possible to form a sequence containing all \( N \) values, referred \( \hat{x}_1, \hat{x}_2, ..., \hat{x}_N \).

Next, let’s introduce the parameter vector \( w = (w_0, w_1, ..., w_h)^T \) and T-system of orthogonal polynomials \( T_N(k) = (T_{0N}(k), T_{1N}(k), ..., T_{hN}(k))^T \) and rewrite (5) in vector form
\[ T^N_h(k) = w^T T_N(k), \]
where vector \( w \) is calculated by the standard method of least squares
\[ w(N) = \left( \sum_{k \in X_P} T_N(k)T_N^T(k) \right)^{-1} \sum_{k \in X_P} T_N(k)\hat{x}_k = \]
\[ = P(N) \sum_{k \in X_P} T_N(k)\hat{x}_k. \]

After that we can finally write
\[ \hat{x}_k = w^T (N)T_N(k) \forall k \in X_G, \]
\[ \hat{x}_k = \hat{x}_k \forall k \in X_P. \]
Using relations (6) and (7) processing is realized in batch mode for a fixed number of points \(N\). If the data are fed to the processing sequentially we have to organize on-line data processing. In [17] for this purpose it was proposed to use recurrent least-squares method in the form

\[
\begin{align*}
T_{\alpha}(\hat{k}) & = \frac{1}{\sqrt{s}}, \\
T_{\beta}(\hat{k}) & = \frac{3}{s(s^2 - 1)}(2\hat{k} - s - 1), \\
T_{\delta}(\hat{k}) & = (a_{\delta}k + b_{\delta})T_{\gamma(\hat{k})} - c_{\gamma}T_{\gamma(\hat{k}-1)}(\hat{k}), l = 2, 3, ..., h, \\
da_{l} & = \frac{s + 1}{l} \sqrt{\frac{d_{l}(d_{l} - 2)}{g_{l}}}, \\
b_{l} & = s + 1 \frac{d_{l}(d_{l} - 2)}{l} \frac{1}{2} a_{l}, \\
c_{l} & = \frac{l - 1}{l} \left( \frac{d_{l}(d_{l} + g_{l} - 2)}{g_{l}} \right), \\
d_{l} & = 2l + 1, \\
g_{l} & = s^2 - l^2. \\
\end{align*}
\]

Then the estimate of type (6) with the fixed structure of the polynomials \(T_{\alpha}(N)\) can be written as

\[
w_{\gamma}(N) = \left( \sum_{\hat{k} = k \in \mathcal{X}_{P}}^{s - 1} T_{\gamma}(\hat{k})T_{\gamma}^{T}(\hat{k}) \right)^{-1} \sum_{\hat{k} = k \in \mathcal{X}_{P}}^{s - 1} T_{\gamma}(\hat{k})\hat{x}_{\gamma} = P_{\gamma}(N) \sum_{\hat{k} = k \in \mathcal{X}_{P}}^{s - 1} T_{\gamma}(\hat{k})T_{\gamma}^{T}(\hat{k})
\]

and estimates of missing observations

\[
\hat{\hat{x}}_{k} = w_{\gamma}^{T}(N)T_{\gamma}(\hat{k}) \forall k \in \mathcal{X}_{G}.
\]

Implementation of the described approach is very convenient with the apparatus of artificial neural networks using ortho-synapse [18], shown in Fig. 1 and trained by the algorithm (10). Ortho-synapse on the structure coincides with the nonlinear synapse of neo-fuzzy neuron [19, 20], but instead of the membership functions it contains the orthogonal activation functions \(T_{\gamma}\), making the learning process easier and quicker. It is also important that due to using of a sliding window, these activation functions are not changed in the learning process. The value of the sliding window is selected from empirical considerations \(s \geq h + 1\). Thus if approximating sequence is nonstationary, the value should not significantly exceed the number of parameter estimates.

**ARCHITECTURE OF ORTHOGONAL NEURAL NETWORK**

Using orthogonal polynomials as activation functions leads to the creation of a whole group of orthogonal neural networks [21–29] possessing good approximating properties and high speed learning of synaptic weights. In [30–33] growing orthogonal networks based on the ortho-synapses and ortho-neurons have been proposed [18]. These networks are characterized by simplicity of learning of synaptic weights and architecture.

Fig. 2 shows the architecture of orthogonal neural network for data processing with the lost observations that implements a nonlinear mapping

\[
\hat{y}_{\gamma} = \hat{f}(\hat{x}_{\gamma}) = \sum_{i = 1}^{n} \sum_{l = 0}^{h} w_{l\gamma} T_{l\gamma}(\hat{x}_{\gamma}),
\]

where \(k = 1, 2, ..., \) – current discrete time, \(w_{l\gamma}\) – tuned synaptic weights, \(T_{l\gamma}(\bullet)\) – \(l\)-th orthogonal activation function of the type (1) or (3) for the input signal \(\hat{x}_{\gamma}\), \(i = 1, 2, ..., n\).

At the zero (receptive) layer of the network series of observations \(\hat{x}_{\gamma}\) containing a priori unknown number of missing values are fed. Note also that the external training signal \(\hat{y}_{\gamma}\) also can contain gaps.
The same function for the reference training signal \( k_y \) performs ortho synapse \( M \). As a result, in on-line mode the time series formed us \( \hat{k} \), \( k \) \( x \). The output layer is formed by ortho-neuron that coincides with the architecture of neo-fuzzy neuron, but with orthogonal activation functions instead of the usual membership functions. Error \( \hat{k} \), \( k \) \( y \) is used by learning algorithm to tune both the weights and architecture.

The present architecture contains \( 2n + 1 \) ortho-synapses \( M \) of output layer is chosen rather arbitrarily, so if it is found that the synthesized neural network does not provide the required quality of the information processing, the number of these functions can be increased (or decreased, if necessary) in on-line mode directly in the learning process. Thus network evolutionary properties by adapting the structure, due to orthogonality with activation functions for the calculated correction synaptic weights made simply.

Suppose that at the instant \( N = N^* \) it was decided that each ortho-synapse \( O - S_i \) of output layer need to add an activation function \( h \) so that to the network should be introduced \( n \) additional synaptic weights \( w_{h+1} \).

The estimate \( w(N) \) (14) should be corrected in this situation as follows:

\[
\hat{y}_k = w^T T(\hat{s}_k) = \sum_{i=1}^{N} w_i^T T_i(\hat{s}_k). \tag{12}
\]

To tune the vector of synaptic weights \( w \) in a sequential mode it’s possible to use a recurrent least squares method (8) processing non-stationary signals is using adaptive procedures that have tracking properties, such as the same method of least squares on the sliding window, which in this situation, can be written as:

\[
\begin{align*}
\hat{w}(N) &= w(N-1) + \frac{P(N-1)(y^* - w^T(N-1)T(\hat{s}_N))T(\hat{s}_N)}{1 + T^T(\hat{s}_N)P(N-1)T(\hat{s}_N)}, \\
\hat{P}(N) &= P(N-1) - \frac{P(N-1)T(\hat{s}_N)T^T(\hat{s}_N)P(N-1)}{1 + T^T(\hat{s}_N)P(N-1)T(\hat{s}_N)}, \\
P(N) &= \hat{P}(N) + M \left( \frac{\hat{P}(N)T(\hat{s}_{N-1})T^T(\hat{s}_{N-1})P(N)}{1 - T^T(\hat{s}_{N-1})P(N)T(\hat{s}_{N-1})} \right), \\
w(N) &= \hat{w}(N) - \frac{\hat{P}(N)(y^* - w^T(N)T(\hat{s}_{N-1}))T(\hat{s}_{N-1})}{1 - T^T(\hat{s}_{N-1})P(N)T(\hat{s}_{N-1})},
\end{align*}
\]

or

\[
w(N) = \left( \sum_{k=N-s+1}^{N} T(\hat{s}_k)T^T(\hat{s}_k) \right)^{-1} \sum_{k=N-s+1}^{N} T(\hat{s}_k)y_k^* = P_s(N) \sum_{k=N-s+1}^{N} T(\hat{s}_k)y_k^*, \tag{14}
\]

thus, due to the diagonality of matrix \( P_s(N) \) calculation of estimation (14) even at large \( n \) and \( h \) does not cause difficulty.
where \(((h+1)n \times n)\)-matrix \(R_{l,h+1}(\tilde{x}_k)\) formed by elements

\[
\sum_{k=N-x+1}^{N} T_{h,i}(\tilde{x}_k) T_{h+1,j}(\tilde{x}_k), \quad l = 0, 1, \ldots, h; \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, n; \quad (n \times n)\)-matrix \(R_{h+1,h+1}(\tilde{x}_k)\) – formed by elements

\[
\sum_{k=N-x+1}^{N} T_{h+1,i}(\tilde{x}_k) T_{h+1,j}(\tilde{x}_k); \quad (n \times 1)\) – the

column \(\eta_{h+1,i}(\tilde{x}_k)\) – formed by elements

\[
\sum_{k=N-x+1}^{N} T_{h+1,i}(\tilde{x}_k) y_{k}^{*},
\]

Applying the Frobenius formula, estimate (15) can be rewritten as

\[
w^{*}(N) = \left( \sum_{k=N-x+1}^{N} T(\tilde{x}_k) y_{k}^{*} \right) \left( \sum_{k=N-x+1}^{N} T(\tilde{x}_k) y_{k}^{*} \right)^{-1} \times \left( \sum_{k=N-x+1}^{N} T(\tilde{x}_k) y_{k}^{*} \right) \left( \sum_{k=N-x+1}^{N} T(\tilde{x}_k) y_{k}^{*} \right)^{-1}
\]

where

\[
H^{-1}(\tilde{x}_k) = R_{h+1,h+1}(\tilde{x}_k) - R_{h+1,h+1}(\tilde{x}_k) P_{s}(N) R_{h+1,h+1}(\tilde{x}_k).
\]

It’s obvious that architectural adaptation is more difficult than synaptic. The possibility of its implementation in online mode, by using of orthogonal activation functions, can achieve the required approximating properties in the learning process.

**CONCLUSION**

The evolving neural network that due to using orthogonal activation functions tunes the synaptic weights and structure in the learning process is proposed. Another important feature of the proposed network is connected with the possibility to process in on-line mode information, spoiled by missing values in the data. The neural network under consideration is characterized by high speed and can to process distorted nonlinear nonstationary stochastic and chaotic signals in real time.

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Стаття надійшла до редакції 16.10.2013.
Ключевые слова: нейронная сеть, ортогональные полиномы, полиномы Чебышева, неполные данные с потерными наблюдениями.

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