Statistical description of galaxy clusters in Finzi model of gravity

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Abstract
We exploit a new theory of gravity proposed by Finzi, which gives stronger interaction at large scales, to study the thermodynamic description of galaxy clusters. We employ a statistical model to deduce various thermodynamics equations of state. In addition, we analyze the behavior of clustering parameter in comparison to its standard (Newtonian) counterpart. The general distribution function and its behavior with varying strength of clustering parameter are also studied. The possibility of phase transition is investigated and it is observed that a phase transition is possible though hierarchically. We also analyze the model by comparing the results with data available through SDSS-III, and obtain the parameters involved.

1. Introduction

The distribution of matter on large scale is mainly described by the gravitational interaction. It is believed that the large scale structure of the Universe is a gravitationally amplified descendant of a faint noise field believed to be seeded by quantum fluctuations in the early universe. A linear theory for initial perturbations to the present observed matter distribution has been developed extensively [1]. The formation of first structures in the Universe took place at a red-shift of 10–30 in the dark matter halos of masses, \( M > 10^5 \sim 10^8 M_\odot \) [2]. This structure formation took place on the imprints of small matter density perturbations in the primordial matter density field. It has been verified by N-body simulations that the initial density perturbations have the potential to grow to the scale of present day observed structure [3].

It is well-known that the peculiar velocity of galaxies in a cluster doesn’t agree well with the total mass of the visible matter. The estimated mass was 200–400 times less than the mass required to prevent the rupture of the galaxies from the cluster [4]. This led to the concept of dark matter. There is a strong support to the dark matter hypothesis, but the identification of the particles that compose this matter is yet under discussion. One explanation to this could be a strong correlation between the dark matter and baryonic matter [5]. The other intriguing idea could be a relevance of modified Newtonian dynamics (MoND) or any new dynamics on galactic scales.

The formation and distribution of galaxy clusters are crucial to understand the evolution of the structure formation in the Universe. Saslaw and Hamilton developed a thermodynamic model for nonlinear regime of the clustering of galaxies in an expanding universe [6]. The theoretical predictions of this model involve the distribution of voids as well as galaxies corresponding to under-dense and over-dense regions, respectively, in the initial density field. The probability distribution function of \( N \) galaxies found in some volume \( V \) predicted by this theory is given by
\[ f(N) = e^{-N(1-b)} - N b \frac{N(1-b)}{N!} [N(1-b) + Nb]^{N-1}, \]

where \( \bar{N} \) is the ensemble-average of particles found in any volume \( V \) and \( b \) is clustering parameter.

It is well known that the force accounting for the flat rotation curve of galaxies and galaxy clusters requires stronger gravity at very larger spatial distances than produced by Newton’s law of gravity. Recently, there have been several attempts to account for this discrepancy through the modification to the general theory of relativity [7]. For instance, in \( f(R) \) group of theories, a function \( f(R) \) of Ricci scalar is used in place of Ricci scalar \( R \) to account for the enhanced gravitational interaction on large scales. The force field that governs the large scale structure formation also governs the distribution of mass at this scale. We can employ statistical methods to find the statistical distribution of matter at largest possible scales [8]. Recently there has been a lot of progress on the study of the effect of modified gravity laws on the clustering of galaxies using statistical mechanics [9–20]. The effect of the modifications incorporated has been anticipated in the strength of the clustering parameter \( b \). For instance, in [13], the modification to the clustering parameter as a function of correction factor has been studied.

In 1963, Finzi proposed a law of gravitation that gives a stronger interaction at relatively larger distances than predicted by Newton’s law of gravity [21]. This force form can explain the larger velocities of galaxies in clusters without involving the concept of dark matter. The potential energy function proposed by Finzi is given by

\[ \Phi(r) = G_a n^2 \left( \frac{1}{r^{3/2}} \right), \]

where \( G_a = -2k/\rho^{1/2}, k \) is a constant (equivalent to gravitational constant \( G \)) and \( \rho \) is a characteristic length beyond which the potential becomes significant. This potential reduces to the usual Newtonian potential at distances \( \rho \approx r \). The correlation and distribution function of galactic clusters has not been studied for Finzi approximation. Here, we try to bridge this gap and also make a comparison of the effect of this model to the already studied theories.

Our investigations are presented systematically as following. We construct the general partition function under Finzi potential for our system of galaxies in section 2. Here, in order to avoid divergence, we consider extended nature of galaxies. We derive the various thermodynamic potentials along with their behavior under different circumstances, e.g., Helmholtz free energy, pressure, internal energy, entropy and chemical potential in section 3. In section 4, we compare the clustering parameters with increasing radial distance corresponding to Newtonian and Finzi gravity. In section 5, we study the statistical distribution of galaxies in the new gravity law. The power-law behavior for the correlation function is presented in section 6. Finally we investigate the possibility of phase transition in section 7. We present the results and their importance in the last section.

2. The partition function

In this section, we develop the general partition function by taking into consideration a system of gravitationally interacting particles, with the interaction defined by Finzi gravity. We assume the clustering of galaxies in the expanding background to be in quasi-equilibrium state forming an ensemble of co-moving cells. We assume the system consists of an ensemble of cells of equal volume \( V \) with number density \( \bar{N} \). The form of the partition function for such a system of pairwise interacting particles having correlation energy \( \Phi \) and average temperature \( T \) is given by [8]

\[ Z_N(T, V) = \frac{1}{N!} \left( \frac{2\pi mT}{\lambda^2} \right)^{N/2} Q_N(T, V). \]

Here, \( \lambda \) is the normalization constant and the factor \( N! \) takes into account the distinguishability of the system particles. The Boltzmann’s constant, \( k_B \), is set equal to unity. Configuration part of the equation (3) can be written as

\[ Q_N(T, V) = \int \ldots \int \exp \left[ -T^{-1} \Phi(\tau_1, \tau_2, \ldots \tau_N) \right] d^{3N} r. \]

The gravitational potential energy function \( \Phi(\tau_1, \tau_2, \ldots \tau_N) \) is a function of the relative positions \( r = |\tau_i - \tau_j| \) and is summed over all the pairs of particles. In the system of gravitationally interacting bodies, the potential energy \( \Phi(\tau_1, \tau_2, \ldots \tau_N) \) is due to all pairs of particles present in the system, i.e.

\[ \Phi(\tau_1, \tau_2, \ldots \tau_N) = \sum_{1 \leq i < j \leq N} \Phi_{ij}(r). \]
With this simplification (5), equation (4) can now be written as

\[ Q_N(T, V) = \int \prod_{i<j=N}^{1} \exp \left(-T^{-1} \Phi_j(r)\right) d^{3N}r, \]

(6)

where \( \Phi_j \) is the two point interaction energy between the \( i^{th} \) and \( j^{th} \) particle. The configuration integral can be written in terms of two-point function \( f_{ij}(=e^{-\frac{a}{r}} - 1) \) defined as:

\[ Q_N(T, V) = \int (1 + f_{12})(1 + f_{13})(1 + f_{23})...(1 + f_{N-1,N})d^3r_1d^3r_2...d^3r_N. \]

(7)

Here, we note that the two-point function \( f_{ij} \) is non-zero only when there are interactions present. The Finzi potential (2) diverges for the point-particle nature of galaxies. Therefore, we need to express potential (2) by considering the extended nature of galaxies (galaxies with halos). This is done by introducing the softening parameter \( \epsilon \) (0.01 \( \leq \epsilon \leq 0.05 \)) in the potential as

\[ \Phi(r) = \frac{G_{\text{N}} m^2}{(r + \epsilon)^{1/2}}. \]

(8)

By substituting the two-point function corresponding to potential (8) in equation (7), the value of \( Q_N \) for various values of \( N \) can be easily calculated. For instance, for \( N = 1 \), we have

\[ Q_1(T, V) = V. \]

(9)

For \( N = 2 \) value, we evaluate the integral (equation (7)) by fixing the position of \( r_1 \) and integrating over all the other system particles. This simplifies the integral to

\[ Q_2(T, V) = V^2[1 + X\alpha], \]

where \( X = \frac{2G_{\text{N}} m^2}{\hbar^2c^2} \) and \( \alpha = \sqrt{1 + \frac{a}{r}[3 - 4(\epsilon/r) + 8(\epsilon/r)^2]}. \)

Proceeding in similar fashion, the values of \( Q_N \) for \( N = 3, 4, 5, 6, 7, \ldots, N \) can be easily calculated. For \( N = 3 \), we get

\[ Q_3(T, V) = V^3[1 + X\alpha]^2. \]

(10)

In general the above equation, equation (10), for \( N \) number of particles takes the form,

\[ Q_N(T, V) = V^N[1 + X\alpha]^{N-1}. \]

(11)

Finally, substituting equation (11) into equation (3), we obtain the general partition function for an interacting system of \( N \) particles (galaxies) as

\[ Z_N(T, V) = \frac{1}{N!}\left(\frac{2\pi m T}{\lambda^2}\right)^{\frac{3N}{2}} V^N[1 + X\alpha]^{N-1}. \]

(12)

equation (12) is the standard partition function (Canonical) of the system of \( N \) particles interacting through the modified gravity law (2). The correction to the partition function is inherent in the parameter \( \alpha \).

3. Thermodynamic equations of state of the system

The partition function equation (12) contains all the necessary information about the macroscopic variable (free energy, entropy, internal energy, pressure, chemical potential) of the system. The Helmholtz free energy for the system of galaxies can be deduced from the partition function via the relation \( F = -T \ln Z_N(T, V) \). For our system of galaxies the free energy takes the following form:

\[ F = NT \ln \left(\frac{N}{V} \frac{T}{\lambda^2}\right) - NT - \frac{3}{2} NT \ln \left(\frac{2\pi m T}{\lambda^2}\right) - NT \ln[1 + X\alpha], \]

\[ = NT \ln \left(\frac{N}{V} \frac{T}{\lambda^2}\right) - NT - \frac{3}{2} NT \ln \left(\frac{2\pi m T}{\lambda^2}\right) + NT \ln[1 - b_a]. \]

(13)

The behavior of free energy with respect to particle number is depicted in figure 1. Here, we approximated \( N - 1 \approx N \). In equation (13), the parameter \( b_a \) is defined as

\[ b_a = \frac{\alpha X}{1 + \alpha X}. \]

(14)

This is the modified clustering parameter which contains information of the strength of correlation that governs the time evolution of the Galaxy cluster and can take values between 0 and 1.

Once the free energy is known, other thermodynamic equations of state can be estimated easily. For example, we can calculate the entropy of the system of galaxies utilizing the relation, \( S = -\frac{\partial F}{\partial T}\bigg|_{V,N} \).

Substituting equation (13) in this relation the entropy of the system takes the following form:

\[ S = \frac{3}{2} NT \ln \left(\frac{2\pi m T}{\lambda^2}\right) + NT \ln[1 - b_a] \]

\[ + NT \ln \left(\frac{N}{V} \frac{T}{\lambda^2}\right). \]
Figure 1. Variation of the free energy of the system with particle number for various values of the $\alpha$ and unit value for the rest of parameters.

Figure 2. The variation of entropy $(S - S_0)$ with an increasing particle number in the system for various values of clustering parameter $b_n$.

Figure 3. The variation of internal energy function $U$ of the system with increasing number of particles for various values of correlation parameter $b_n$. 
The entropy varies with particle number as shown in figure 2. Specific entropy i.e., entropy per particle of the system corresponding to equation (13) takes the following form:

\[
S = N \ln \left( \frac{V}{N} T^{3/2} \right) + N \ln[1 + X \alpha] - 3N \frac{X \alpha}{1 + X \alpha} + S_0,
\]

where \( S_0 = \frac{5}{2} N + \frac{2}{3} N \ln\left( \frac{\tau m}{\lambda} \right) \). The entropy varies with particle number as shown in figure 2.

The total internal energy of the interacting system of galaxies can be calculated using the basic definition, \( U = F + TS \). Upon substituting the values for free energy \( F \) (13) and entropy \( S \) (14), the relation for internal energy of the system in terms of the new clustering parameter takes the form:

\[
U = \frac{3}{2} NT \left[ 1 - 2 \frac{X \alpha}{1 + X \alpha} \right]
\]

\[
= \frac{3}{2} NT \left[ 1 - 2b_n \right].
\]

Figure 3 shows the graphical visualization of the effect of modified clustering parameter \( b_n \) on the internal energy function of the system of galaxies interacting gravitationally.

The equation of the pressure caused by the particles in the system can be obtained utilizing the fundamental relations \( P = -\left( \frac{\partial F}{\partial N} \right)_{T,N} \). Using relation (13), the pressure of the system takes the form:

\[
P = -\frac{1}{V} \left[ F - \frac{V}{N} TS \right] = -\frac{1}{V} \left[ \left( \frac{V}{N} T^{3/2} \right) + N \ln[1 + X \alpha] - 3N \frac{X \alpha}{1 + X \alpha} + S_0 \right] = -\frac{1}{V} \left[ N \ln \left( \frac{V}{N} T^{3/2} \right) - N \ln(1 - b_n) - 3b_n + \frac{5}{2} + \frac{3}{2} \ln\left( \frac{\tau m}{\lambda} \right) \right].
\]
The behaviour of pressure is depicted in figure 4.

Finally, we deduce the relation for the chemical potential \( \mu \) of the system using the fundamental relation, 
\[
\mu = \left( \frac{\partial F}{\partial N} \right)_{T, V, \alpha},
\]

as
\[
\mu = T \left[ \ln \left( \frac{N}{V T^{3/2}} \right) + T \ln \left( 1 - \frac{X}{1 + X} \right) - \frac{3}{2} T \ln \left( \frac{2 \pi m}{\lambda^2} \right) \right].
\]

The graphical visualization of the variation in the chemical potential of the system with an increase in the particle number can be seen in figure 5.

4. A comparison of new and old clustering parameter

In this section the clustering parameter \( b_n \) developed in this work based on the Finzi model of gravity is compared with the standard parameter \( b \) defined in [8]. The correlation parameter gives the strength of correlation and will in turn decide the time-length of clustering. From the graph figure 6 it can be seen that the new clustering parameter is more strong than the standard one. This is due to the effect of the increased strength of the potential energy function at large distances as defined in the Finzi model.

5. General form of the distribution function

The general form of the distribution function \( f(N) \), which characterizes the Galaxy clustering, describes the distribution of voids as well as the number of galaxies in fixed volume cells distributed through out the system. Here we let the system particles cross the cell boundaries which in turn can change the particle number in each cell. Thus we derive the grand canonical partition function defined as
\[
Z_G(T, V, z) = \sum_{N=0}^{\infty} \exp \left( \frac{N \mu}{T} Z_N(T, V) \right).
\]

The probability distribution function of \( N \) particles contained in cells of fixed volume \( V \) in a grand canonical ensemble is given by
The factor \( z = \exp \frac{U}{T} \) is the fugacity of the system and it determines the activity within the system. From this basic equation (21), the general form of the distribution function of system can be determined easily. Utilizing the partition function (20) with chemical potential equation (12) the distribution function of the system takes the following form:

\[
F(N) = \sum_{i=0}^{N} \frac{\exp \left( \frac{Nu}{T} \right) \exp -\frac{U}{T}}{Z_0(T, V, z)} = \frac{\exp \left( \frac{Nu}{T} \right) Z_0(T, V)}{Z_0(T, V, z)}. \tag{21}
\]

The graphical representation of the two point correlation function can be visualized from the figure 8. We can see the strength of the correlation function decreases with increasing radius. This leads us to an important result about the power law for two-point correlation function.

### 6. Power law for two-point correlation function

Here we study the behavior of the two-point correlation function in the new model of gravity. We write the clustering parameter in the form (22)

\[
b_n = \frac{G \rho_n}{6T} \int \left[ \frac{1}{(r + \epsilon)^{3/2}} \right] \xi_2(\rho, r, T) dV,
\]

where \( \rho \) is the number density and \( \xi_2 \) is the two-point correlation function.

Differentiating with respect to \( V \), equation (23) yields

\[
\frac{\partial b_n}{\partial V} = \frac{G \rho_n}{6T} \frac{\partial}{\partial V} \int \left[ \frac{1}{(r + \epsilon)^{3/2}} \right] \xi_2(\rho, r, T) dV
\]

\[
+ \frac{G \rho}{6T} \left( - \frac{\partial \rho}{\partial V} \right) \int \left[ \frac{1}{(r + \epsilon)^{3/2}} \right] \xi_2(\rho, r, T) dV,
\]

where we have used \( \frac{\partial V}{\partial \rho} = - \frac{\rho}{V} \).

Using the relation \( \frac{\partial b_n}{\partial \rho} = \frac{b_n(1 - b_n)}{\rho} \) and equation (24), we obtain the following expression for the power law

\[
\xi_2(\rho) = \frac{9Tb_n^2}{2\pi G \rho \left( (r + \epsilon)^{3/2} \right)}.
\]

The graphical representation of the two point correlation function can be visualized from the figure 8. We can see the strength of the correlation function decreases with increasing radius. This leads us to an important result about...
the correlation of system points in different cells. The galaxies within a cell are more correlated than the galaxies in the adjacent cells. Thus the transfer of system points from cell to cell are less likely although not negligible.

7. Possibility of phase transition

If an interaction is introduced in a system characterized by Poisson distribution the system has a high chance of changing phase from less correlated to highly correlated. The possibility of phase transition in case of our system of galaxies interacting gravitationally can not be ignored. Here we will try to find out if the Finzi interaction can cause a phase transition in the system of gravitationally interacting point particles. The
result is important as the possible phase transition can break the homogeneity of the system and cause lumpiness in the structure. Among many indicators of phase transition, specific heat is an important candidate to track.

**Figure 10.** The theoretical (solid line) and calculated (dotted line) probability distribution of galaxy clusters in various red-shift and radius bins.
The specific heat (at constant volume) $C_V$ is defined as

$$C_V = \frac{1}{N} \left( \frac{\partial U}{\partial T} \right)_{N,V}.$$  \hspace{1cm} (26)
Using the relation for internal energy (17), the specific heat of the system takes the following form

\[ C_V = \frac{3}{2} \left[ \frac{1 + 6\alpha X - 4\alpha^2 X^2}{(1 + \alpha X)^2} \right] \]  (27)

As \( b_n \rightarrow 0, C_V \rightarrow 3/2 \), this corresponds to no interaction among the system particles. As \( b_n \rightarrow 1, C_V \rightarrow -3/2 \), which means the system is fully virialized. Between these two extreme values lie the maximum values of specific heat at some critical value of temperature. This extreme value of specific heat indicates a possible phase transition at \( T = T_C \).

\[ \frac{\partial C_V}{\partial T} \bigg|_{T=T_C} = 0. \]

This gives an expression for the critical temperature as

\[ T_C = \left[ \frac{N \alpha (GM^2)^3}{V} \right]^{1/3} \]  (28)

In terms of critical temperature the specific heat of the system, \( C_V \), given in (27) can be written as

\[ C_V = \frac{3}{2} \left[ 1 - \frac{2(1 - 4(T/T_C)^3)}{(1 + 2(T/T_C)^3)^2} \right] \]  (29)

At \( T = T_C, C_V = 5/2 \), a property of a diatomic gas. This is an indication of system symmetry breaking due to the formation of binaries. The phase transition is hierarchical and not spontaneous as observed in many other physical systems. The graphical behavior of the specific heat is visualized in figure 9.

8. Observational data

In this section we test our model with the data obtained through Sloan Digital Sky Survey III (SDSS-III) through its newest Data Release (DR12). SDSS-III contains additional sky coverage and better galaxy estimates than SDSS-I and SDSS-II. The data is present in catalog [2-3]. This catalog contains an information (RA, DEC, z, N etc) of almost 132,684 clusters. The catalog gives a parameter \( r_{200} \) which is the distance up to which mean density is \( \approx 200 \) and also the number of galaxy clusters in it i.e \( N_{200} \). Figures 10(a)–(i) shows the model fitted to the data. In figure 11 the sky distribution of galaxy clusters in various RA and DEC coordinates is presented from this catalog.

First we bin the data on the basis of radius (R) and Redshift (z). The bin size is chosen to be \( \Delta R \approx 0.35 \text{Mpc} \) and \( \Delta z \approx 20 \). We divide cells by physical boundaries i.e., \( R \). The probability distribution of each cluster is determined by substituting arbitrary values for the fitting parameter \( b_n \) and \( \rho \). The model is then fit through Application Programming Interface (API) Scipy.optimize.curve_fit of SciPy python Library. We obtained the optimized values for the parameters involved in equation(22), listed in table 1.

The value of the characteristic length, \( \rho \), is obtained through the relation,

\[ \sqrt{\rho} = \frac{1 - b_n}{b_n} \]  (30)

We have set all other parameters to unity in (30). The characteristic length \( \rho \) is measured in the same units as \( r \). For instance, the value \( \rho = 5 \) means for 1 unit of \( r \), the characteristic length is 5 units.

From the plots 10, we observe that the model fits very closely to data in redshift ranges \( 0.250 < z < 0.450 \) and radius ranges \( 0.40 < R < 0.45, 1.0 < R \leq 1.45 \), figures 10(b), (e). In bins \( 0.450 < z < 0.650; 0.75 < R < 1.10 \), figure 10(f) the model does not fit very closely to the data.

9. Conclusion

The velocities of galaxies in a cluster are much higher than depicted by the visible matter which brings in the concept of gravitating non-visible matter called dark matter. While the hunt for the exact description is underway, it is plausible to reconsider Newtonian dynamics and modify it to fit the observed data.

In this work, we have considered a gravitational potential proposed by Finzi and, by using statistical methods, we deduced various thermodynamic quantities along with correlation function. A graphical analysis of various thermodynamic equations of state was also done. A new clustering parameter was deduced based on this theory which shows a greater strength as compared to the standard one. We have also made a graphical comparison of the the clustering parameters for Finzi and Newtonian gravity. From the behavior of the clustering parameter, it is obvious that the Finzi model of gravity at larger scale is more significant. The power-
law of the correlation function for Finzi model is also discussed. We also studied possibility of phase transition within the system. The comparison of our results with the data was also studied and it could be seen that the model fits the data very closely in some Redshift $z$ and radius ranges $R$, while in some regions the fit is not too appropriate although there is agreement with the trend.

**Data availability statement**

All data that support the findings of this study are included within the article (and any supplementary files).

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