Uplink User-Assisted Relaying Deployment in Cellular Networks

Hussain Elkotby, Student Member IEEE and Mai Vu, Senior Member IEEE

Abstract

We use stochastic geometry to analyze the performance of a partial decode-and-forward (PDF) relaying scheme applied in a user-assisted relaying setting where an active user relays data through another idle user in uplink cellular communication. We present the geometric model of a network deploying user-assisted relaying and propose two geometric cooperation policies for fast and slow fading channels. We analytically derive the cooperation probability for both policies which is further used in the analytical derivation of the moments of inter-cell interference power caused by system-wide deployment of this user-assisted PDF relaying. We then model the inter-cell interference power statistics using the Gamma distribution by matching the first two moments analytically derived. This cooperation and interference analysis provides the theoretical basis for quantitatively evaluating performance impacts of user-assisted relaying in cellular networks. We then numerically evaluate the average transmission rate performance and show that user-assisted relaying can significantly improve per-user transmission rate despite of increased inter-cell interference. This throughput gain is significant for active users near the cell edge and further increases with higher idle user density, supporting user-assisted relaying as a viable solution to crowded population areas.

Keywords: user-assisted relaying; partial decode-and-forward; stochastic geometry.

I. INTRODUCTION

Mobile operators driven by the increasing number of subscribers and continual customer demand for new and better services place pressing requirements on the underlying wireless technologies to provide high data rates and wide coverage. Next generation networks that promise higher data rates and multifold increase in system capacity include 3GPP Long Term Evolution—Advanced (LTE-A, 4G) and the emerging 5G systems. The fourth generation (4G) wireless systems were designed to fulfill the requirements of the International Mobile Telecommunications – Advanced (IMT-A). LTE as a practical 4G wireless system has been recently deployed in some countries and LTE-A is expected to be deployed soon around the globe [1].
It is well established that 4G networks have just reached the theoretical limit on the data rate with current technologies. These technologies are being complemented in the fifth generation (5G) wireless systems by designing and developing new radio concepts to accommodate higher data rates, larger network capacity, higher energy efficiency, and higher mobility necessary to meet the new and challenging requirements of new wireless applications. 5G wireless systems are expected to support peak data rate of 10 Gb/s for low mobility and 1 Gb/s for high mobility. These networks are expected to be standardized and deployed around and beyond 2020. Various promising technologies are proposed for 5G wireless communication systems such as massive MIMO, energy-efficient communications, Device-to-Device (D2D) communications, millimeter-wave (mmWave), and cognitive radio networks [1]–[3].

A. Background and Related Works

D2D and Relaying cooperative communications will play important roles in next generations wireless networks. D2D communications enable two proximity users to transmit signal directly without going through the base station; subsequently, 5G wireless systems are expected to relax the restrictions on the need to route all user data through the core network. D2D communications can increase network spectrum utilization and energy efficiency, reduce transmission delay, offload traffic for the base station, and alleviate congestion in the cellular core networks, which make it a promising technology for future wireless systems [3]–[5]. Relay-aided cooperative communication techniques represent another promising technology that improves performance in poor coverage areas by enabling ubiquitous coverage even for users in the most unfavorable channel conditions. The latest release of the LTE standard allows the deployment of fixed wireless relays to help cell-edge mobiles. Yet, other advanced cellular relaying modes are expected in 5G systems to improve the topology and robustness of a cellular network and decrease power consumption. These new technologies include mobile relaying, multi-hop relaying structures, and user-equipment based (user-assisted) relaying which will be enabled by the D2D communications technology [4]–[7].

Several modes of relay-aided communication have been studied in the literature, including fixed relay station, mobile relay station, and using other user equipment (UE) as relay nodes [8]–[10]. Most existing results are derived for the first two modes of fixed and mobile relay stations. For example, simulation is used to compare the difference between relaying network architectures with mobile or fixed relay stations and contrast their performance gains in [8].
Resource allocation for uplink OFDMA-based cooperative relay networks is studied in [9]. The third mode of relaying through other idle UEs (or user-assisted relaying) has only been studied through system simulations for decode-and-forward relaying in [10].

User-assisted relaying, nevertheless, provides more flexibility than fixed relaying in expanding the base station (BS) coverage into obscured areas especially where there is high density of idle UEs [7]. The density of idle UEs that are willing to participate in relaying transmission is expected to increase over time with the development of novel pricing models to tempt devices to participate in this type of communication [5]. The issue of battery power drainage of mobile UEs due to their cooperation in relaying other users data to the base station has also been examined through the emerging energy harvesting techniques [11]. In this paper, we analyze the performance of user-assisted relaying when deploying system-wide in a cellular network.

For cellular network analysis, stochastic geometry has been shown to be analytically tractable and capture some of the main performance trends. Stochastic geometry is used to develop a tractable model for downlink heterogeneous cellular networks which is comparable to the grid model and actual 4G deployment data in [12]. This model is further used in [13] to analyze downlink coordinated multipoint beamforming in which each user equipped with a single antenna can be served by either one or two base stations connected over backhaul links of infinite capacity. A user decides whether to connect to one or two base stations based on geometric policies taking into account its relative distances to the two closest base stations. In [29], Poisson spatial distribution is used to develop an analytic interference model for multi-cell multiple-input multiple-output cellular networks and derive its downlink average capacity. Stochastic geometry is also used in [15] to analyze the performance of decode-and-forward relaying techniques in uplink cellular networks under the specific setting of a fixed number of relays deployed at a fixed distance from the BS with equal angular separation in each cell.

B. Main Results and Contributions

In this paper, we study the performance of a partial decode-and-forward (PDF) user-assisted relaying scheme in uplink cellular networks. To the best of our knowledge, our work is the first that analyzes user-assisted relaying in a network-wide context. The main question under consideration is how network-wide deployment of user-assisted relaying affects the system performance? Since some idle users are now transmitting by relaying information of other users, the amount of interference generated to the network will increase. We use stochastic geometry
as a tool to model and analyze this interference as well as the cooperation policy which governs how to select the idle user to act as a relay. In this work, we consider the practical policy where each active UE selects the closest idle UE to relay its message to the destination, given that D2D communications will be enabled between UEs that are in proximity of each other. Other more complex policies such as selecting the idle user with strongest link instead of the closest one, or choosing the one resulting in the highest relayed data rate can also be considered in our proposed framework and are left as future work. As a base for analysis, we assume all nodes are equipped with a single antenna; generalization to the multiple antennas case is straightforward and can also be considered in a future work.

We provide the geometric basis for a rigorous analysis of performance metrics such as outage probability or throughput for the whole cellular system in contrast to the standalone analysis in [16]. The contributions and novelties of this paper are summarized as follows:

1) We propose a geometric model for user-assisted relaying in uplink cellular networks. The model assumes a reuse factor of one in the whole network. We consider scenarios where there are multiple idle users as potential candidates for helping an active user per resource block in each cell, indicated by the ratio between idle and active user densities.

2) We propose two practical cooperation policies: a pure geometric policy and a hybrid fading and geometric policy for fast fading and slow fading channels, respectively. Further, we analytically derive the cooperation probability for these policies and compare them with simulation results when the nodes know the channel state information perfectly, and show that these policies are approximate of each other.

3) We analytically formulate the out-of-cell interference power at both the destination base station and the relaying user within a cell of a given radius. This formulation takes into account the random locations of all users and base stations as well as both small and large scale channel fading. We then derive the Laplace transforms of these interference powers, which allow us to analytically compute any moments of the interference.

4) We use second moment matching to model the interference power as a Gamma distribution and numerically evaluate the fit of this model. Results show that the Gamma distribution provides a good fit within the range of regulated transmit power and for all range of user-base station distances. This result therefore provides a tractable analytic model for the out-of-cell interference power generated by network deployment of user-assisted relaying.
We use the developed analytic model of interference power to numerically evaluate system performance and provide a quantitative analysis of uplink user-assisted relaying using the average data rate as the metric. Results show that the rate gain is significant when the active user is located in the one-half or one-third ring near the cell edge. The average rate gain increases with higher idle user density and can be up to 50% when the idle users are six times denser than active users. The maximum gain can be as high as 200% when the idle user is ideally located about half way between the active user and the base station.

The remainder of this paper is organized as follows: Section II describes the system model and the considered relaying scheme. Section III describes the network geometric model. Section V provides the interference analysis and the derivation of cooperation probability. Then, Section VI shows the numerical results. Finally, Section VIII presents our conclusion.

II. SYSTEM DESCRIPTION

A. System Overview

In this section, we give an overview of the system under consideration and the network description. We consider a cellular system which consists of multiple cells, each cell has a single base station and each base station serves multiple users. Each of the users uses a distinct resource block, subsequently, no intra-cell interference is present. The only interference that affects each user is the out-of-cell interference due to frequency reuse in all other cells. We assume that each user is served by the single base station that is closest to that user. Within this system, we study the impact on performance of the cooperation technique in which a user can relay its message to the base station through the closest user that is in an idle state using the PDF relaying scheme described in Section II-C.

B. Relaying vs. Direct Transmission Channel Model

In this section we describe the channel model for the PDF relaying scheme proposed in [16] and show the modifications to account for the effects of interference in network deployment. We consider the half-duplex relay channel shown in Fig. 1 in which a source $S$ conditionally decides to exploit the help of a relay node $R$ to communicate a message to the destination $D$, or directly convey its message to the destination depending on the relative quality of the source-to-relay and source-to-destination links.
1) Physical Channel Model: In the relaying case, we assume flat fading over the two phase period. We model the received signal at the $i^{th}$ relay and destination, respectively, during the first phase as

$$Y_{r,i}^b = h_{sr}^{(i)} x_{s,i}^b + I_{r,i}^b + Z_{r,i}^b$$

$$Y_{d,i}^b = h_{sd}^{(i)} x_{s,i}^b + I_{d,i}^b + Z_{d,i}^b$$

where $b$ stands for broadcast transmission in which the source broadcasts to both the relay and destination. The signals $x_{s,i}^b$ and $x_{s,i}^m$ are the transmitted codewords from the source in the first and second phase; $x_{r,i}^m$ is the transmitted codeword from the relay in the second phase; $I_{r,i}^b$ and $I_{d,i}^b$ represents the interference received at the $i^{th}$ relay and destination during the first phase; $Z_{r,i}^b$ and $Z_{d,i}^b$ are i.i.d $\mathcal{CN}(0,\sigma^2)$ that represent the noise at the $i^{th}$ relay and destination; and $h_{sr}^{(i)} = e^{j\theta_1} |h_{sr}^{(i)}|$ and $h_{sd}^{(i)} = e^{j\theta_2} |h_{sd}^{(i)}|$ are the complex channel gains that captures both the small and large scale fading of the source-to-relay and source-to-destination channels with uniformly distributed phases $\theta_1$ and $\theta_2 \sim U[0, 2\pi]$, respectively.

Also, we can model the received signal at the $i^{th}$ destination during the second phase as

$$Y_{d,i}^m = h_{sd}^{(i)} x_{s,i}^m + h_{rd}^{(i)} x_{r,i}^m + I_{d,i}^m + Z_{d,i}^m$$

where $m$ denotes multiple access transmission in which both the source and the relay send information to the destination; $h_{rd}^{(i)} = e^{j\theta_r} |h_{rd}^{(i)}|$ captures the small and large scale fading of the relay-to-destination channel with phase shift $\theta_r$; $I_{d,i}^m$ represents the interference received at the $i^{th}$ destination during the second phase; and $Z_{d,i}^m \sim \mathcal{CN}(0,\sigma^2)$ represents the noise at the $i^{th}$ destination.

In the direct transmission case, we model the received signal at the $i^{th}$ destination as

$$Y_{d,i} = h_{sd}^{(i)} x_{s,i} + I_{d,i} + Z_{d,i}$$

where $I_{d,i}$ represents the average interference received at the $i^{th}$ destination during the transmission period which is equivalent to $I_{d,i}^b$, the interference received from source nodes only; and $Z_{d,i} \sim \mathcal{CN}(0,\sigma^2)$ represents the noise at the $i^{th}$ destination.

We note that the codeword $x_{s,i}^m = x_{s,i}^{m1} + x_{s,i}^{m2}$ represents the superposition of a common message codeword $x_{s,i}^{m1}$ and a private message codeword $x_{s,i}^{m2}$ which assume Gaussian distribution with
zero mean and variances defined as follows:

\[
\mathbb{E} \left[ |x_{s,i}^m|^2 \right] = \mathbb{E} \left[ |x_{s,i}^{m1}|^2 + |x_{s,i}^{m2}|^2 \right] = P_{s,i}^{m1} + P_{s,i}^{m2} = P_{s,i}^m,
\]

\[
\mathbb{E} \left[ |x_{r,i}^m|^2 \right] = P_{r,i}^m, \quad \mathbb{E} \left[ |x_{s,i}^b|^2 \right] = P_{s,i}^b, \quad \mathbb{E} \left[ |x_{s,i}|^2 \right] = P_{s,i}.
\] (5)

Also, the power allocation to each of the codewords has to satisfy the following constraints

\[
\alpha_1 P_{s,i}^b + \alpha_2 P_{s,i}^m = P_{s,i}, \quad \alpha_2 P_{r,i}^m = P_{r,i}.
\] (6)

where \( P_{s,i} \) and \( P_{r,i} \) represent the total power allocated to the source and relay nodes within a single transmission period. \( \alpha_1 \) and \( \alpha_2 = 1 - \alpha_1 \) represent the portions of the transmission time allocated to the first and second phases, respectively. Note that in our model, we do not use power control at each user but assume the worst-case scenario where each user is transmitting at the maximum allowable power. Thus our analysis result should represent a lower bound on the actual system performance, in the sense that the actual out-of-cell interference will likely be less than what we derive in our subsequent analysis.

For a given setting of nodes locations, we make use of the fact that interference at either the relay or destination is the sum of an infinite number of independent messages transmitted by an infinite number of sources that are distributed in the infinite 2-D plane and use the law of large numbers to approximate the interference as a complex Gaussian distribution. Also, since the transmitted codewords are realization of a complex Gaussian with zero mean, it is justified to set the mean of interference to zero. To fully characterize interference as a complex Gaussian distribution, we define the distributions as \( I_{d,i}^b \sim \mathcal{CN}(0, Q_{d,i}^b) \), \( I_{d,i}^m \sim \mathcal{CN}(0, Q_{d,i}^m) \), and \( I_{r,i}^b \sim \mathcal{CN}(0, Q_{r,i}) \) with the variances as detailed in Section V. The power of these interference terms which correspond to the variance of the Gaussian random variables are function of the node locations and hence vary with different network realizations.

2) Model Reformulation into Standard Form: The aforementioned model in Section II-B1 in case of relaying can be reformulated to capture the effects of interference into the channel fading
as follows

\[ \tilde{Y}_{r,i}^b = \tilde{h}_{sr}^{(i)} x_{s,i}^b + \tilde{Z}_{r,i}^b, \quad \tilde{Y}_{d,i}^b = \tilde{h}_{sd}^{(b,i)} x_{s,i}^b + \tilde{Z}_{d,i}^b, \]

\[ \tilde{Y}_{d,i}^m = \tilde{h}_{sd}^{(m,i)} x_{s,i}^m + \tilde{h}_{rd}^{(i)} x_{r,i}^m + \tilde{Z}_{d,i}^m, \]  

(7)

where the new channel fading terms can be defined as

\[ \tilde{h}_{sr}^{(i)} = \frac{h_{sr}^{(i)}}{\sqrt{\mathcal{Q}_{r,i} + \sigma^2}}, \quad \tilde{h}_{sd}^{(b,i)} = \frac{h_{sd}^{(b,i)}}{\sqrt{\mathcal{Q}_{d,i}^b + \sigma^2}}, \]

\[ \tilde{h}_{sd}^{(m,i)} = \frac{h_{sd}^{(m,i)}}{\sqrt{\mathcal{Q}_{d,i}^m + \sigma^2}}, \quad \tilde{h}_{rd}^{(i)} = \frac{h_{rd}^{(i)}}{\sqrt{\mathcal{Q}_{d,i}^m + \sigma^2}}, \]  

(8)

where \( \mathcal{Q}_{r,i}, \mathcal{Q}_{d,i}^b, \) and \( \mathcal{Q}_{d,i}^m \) are random variables that represent interference power respectively at the relay, the destination during 1\textsuperscript{st} phase and the destination during 2\textsuperscript{nd} phase, which depends on the realizations of the interfering nodes locations and are studied in details in Section \[V\]. The equivalent noise terms \( \tilde{Z}_{r,i}^b, \tilde{Z}_{d,i}^b, \) and \( \tilde{Z}_{d,i}^m \) are all i.i.d \( \mathcal{CN}(0,1) \).

On the other hand, in the direct transmission case, the received signal at the \( i \)\textsuperscript{th} destination during the whole transmission period can be remodeled as

\[ \tilde{Y}_{d,i} = \tilde{h}_{sd}^{(b,i)} x_{s,i} + \tilde{Z}_{d,i}, \]  

(9)

where the new channel fading is assumed to be the same as for the relaying case during the first phase resulting from active users (source nodes) only. Only active users are the source of interference during the first phase, in case of relaying, due to our assumption of perfect frame synchronization among all transmitting nodes in the network which is justified in Section \[III\]. Also, \( \tilde{Z}_{d,i} \sim \mathcal{CN}(0,1) \) is the normalized noise at the \( i \)\textsuperscript{th} destination.

C. Relaying Scheme Description

In this section, we describe the half-duplex partial decode-and-forward (PDF) relaying scheme presented in [16]. In this scheme, each transmission block is divided into 2 phases. In each block, the source \( S \) uses superposition coding and splits its information into a cooperative part, \( U_{s,i}^b \) sent in the 1\textsuperscript{st} phase and \( U_{s,i}^m \) sent in the 2\textsuperscript{nd} phase, and a private part \( V_{s,i}^m \) sent in the 2\textsuperscript{nd} phase. The common and private parts are transmitted at rates \( C_1 \) and \( C_2 \), respectively. The relay \( R \) decodes \( U_{s,i}^b \) in the 1\textsuperscript{st} phase and sends \( U_{r,i}^m = U_{s,i}^m \) in the 2\textsuperscript{nd} phase. At the end of the 2\textsuperscript{nd} phase, the destination \( D \) utilizes the received signals in both phases to decode both parts using joint maximum likelihood (ML) decoding rule.
Using Gaussian signaling, $S$ and $R$ constructs their transmit signals in the 1st and 2nd phase, respectively, as follows

$$x_{b,s,i}^b = \sqrt{P_{b,s,i}^U} x_{b,s,i}^b, \quad x_{b,r,i}^b = \sqrt{P_{m,r,i}^U} x_{b,r,i}^m,$$

(10)

$$x_{m,s,i}^m = \sqrt{P_{m,s,i}^U} x_{m,s,i}^m + \sqrt{P_{m,2,s,i}^U} x_{m,2,s,i}^m.$$

(11)

Using the ML decoding rule, we obtain the following achievable rate which ensures reliable decoding at $R$ and $D$:

$$R \leq \min(C_1 + C_2, C),$$

(12)

where

$$C_1 = \alpha_1 \log (1 + |\bar{h}_{sr}^{(i)}|^2 P_{s,i}^b), \quad C_2 = \alpha_2 \log (1 + |\bar{h}_{sd}^{(i)}|^2 P_{s,i}^m),$$

$$C = \alpha_1 \log (1 + |\bar{h}_{sd}^{(i)}|^2 P_{s,i}^b) + \alpha_2 \log (1 + |\bar{h}_{sd}^{(i)}|^2 P_{s,i}^m + (|\bar{h}_{sd}^{(i)}|^2 + |\bar{h}_{rd}^{(i)}|^2 P_{r,i}^m^2)^2).$$

(13)

The proposal in [16] is to adapt the scheme to the channel configuration to obtain two transmission cases: Direct transmission whenever $|\bar{h}_{sr}^{(k)}|^2 \leq |\bar{h}_{sd}^{(b,k)}|^2$ and PDF relaying otherwise. Note that knowledge about the phase offset between two transmitting nodes - the source and the relay - is assumed in this scheme in order to achieve coherent source-relay transmission, as usually done in the literature [17]–[19]. This assumption can be further justified in our model by noting that phase offset between the two transmitting nodes can be estimated at the destination base station and fed-back using a dedicated resource. Another more appealing technique is to benefit from the channel reciprocity to estimate the channels at the transmitting nodes and exploit the fact that D2D communications in future cellular networks are expected between two devices separated by short distances; therefore reliable communication links can be established over some dedicated resource blocks at low transmit powers in order to send the required transmit phase information. Even such phase information exchange allows the extensive reuse of the resources among different D2D connections within the same cell because of the proximity of such exchanges [20].

### III. NETWORK GEOMETRIC MODEL

#### A. Network Model Assumptions

In this section, we describe the geometric model of the uplink cellular network under study which is shown in Fig. 3. We assume that the users that will contend for the same resource block
causing interference on each other are distributed on a two-dimensional plane according to a homogeneous and stationary Poisson point process (p.p.p.) $\Phi_1$ with intensity $\lambda_1$. We also assume that $\Phi_1$ is independent of another p.p.p. $\Phi_2$ with intensity $\lambda_2$ that represents the distribution of another set of UEs that are in an idle state and can participate in relaying the messages transmitted by UEs in $\Phi_1$. Furthermore, under the assumption that each BS serves a single mobile in a given resource block, we follow the same approach in describing BSs distribution as proposed in [21] where each BS is uniformly distributed in the Voronoi cell of its served UE. In Fig. 3, we present an example layout of the suggested model for the uplink user-assisted relaying cellular network.

To model the interference as in Section III-B, we use the system and relay scheme description in Section II along with the frame synchronization assumption. The assumption of perfect frame synchronization can be justified by the fact that LTE-Advanced imposes very strict requirements on time synchronization since failure to comply with the synchronization requirements impacts the performance of the various features developed in the standard such as LTE-A Coordinated multi-Point (CoMP) and enhanced Inter-Cell Interference Coordination (e-ICIC) [3], [22], [23]. Carrier networks achieve the necessary precision and accuracy in synchronization based on a very precise and accurate primary reference which is mainly obtained by signals transmitted by GNSS satellite systems. The same synchronization requirements can be exploited in this proposed user-assisted relaying model.

It is worth noting that the perfect frame synchronization assumption can be relaxed to assume only transmission phase synchronization by incorporating another Bernoulli random variable as in [24]. In such synchronization, a user can be in phase 1 while another user can be in phase 2,
as long as these transmission phases are time-synchronized at the beginning.

B. Out-of-Cell Interference Model

Hence, we can express interference at the destination, during the first and second phase, and the relay, respectively, as follows

\[ I_{d,i}^b = \sum_{k \neq i} B_k h_{sd}^{(k,i)} x_{s,k} + (1 - B_k) h_{sd}^{(k,i)} x_{s,k}, \]

\[ I_{d,i}^m = \sum_{k \neq i} B_k \left( h_{sd}^{(k,i)} x_{s,k} + h_{rd}^{(k,i)} x_{r,k} \right) + (1 - B_k) h_{sd}^{(k,i)} x_{s,k}, \]  \( (14) \)

\[ I_{r,i}^b = \sum_{k \neq i} B_k h_{sr}^{(k,i)} x_{s,k} + (1 - B_k) h_{sr}^{(k,i)} x_{s,k}, \]  \( (15) \)

where the summation is over all the active users. Note that the interference at the destination and relay nodes during the first phase results from the active (source) nodes only in either cooperation or direct transmission mode, and the interference at the destination during the second phase results from both the active and relaying users if in cooperation mode or the active user if in direct transmission mode. Here, \( h_{sd}^{(k,i)} \) and \( h_{rd}^{(k,i)} \), respectively, are the channel fading from the \( k \)th active UE in \( \Phi_1 \) and the associated relaying UE in \( \Phi_2 \) to the BS associated with the \( i \)th active UE in \( \Phi_1 \). \( h_{sr}^{(k,i)} \) is the channel fading from the \( k \)th active UE in \( \Phi_1 \) to the relaying UE associated with the \( i \)th active UE in \( \Phi_1 \).

The Bernoulli random variable \( B_k \sim \text{Bern} (\rho_1) \) captures the transmission strategy of the \( k \)th UE in \( \Phi_1 \) with success probability \( \rho_1 \), where \( B_k = 1 \) is used to indicate the \( k \)th active UE decision to exploit the help of another idle UE and apply the relaying transmission strategy, and \( B_k = 0 \) indicates direct transmission. A Bernoulli random variable can represent the transmission strategy with a certain probability, \( \rho_1 \), because, as we show in Section IV, the developed cooperation policies will be independent for each active user. We derive the cooperation probability \( \rho_1 \) for the different policies later in that section.

The channel fading terms can be split into their small scale and path loss fading components as

\[ \left| h_{sd}^{(i)} \right|^2 = g_{sd}^{(i)} \left\| Z_i \right\|_2^{-\alpha}, \quad \left| h_{sr}^{(i)} \right|^2 = g_{sr}^{(i)} \left\| Y_i \right\|_2^{-\alpha}, \]

\[ \left| h_{rd}^{(i)} \right|^2 = g_{rd}^{(i)} D^{-\alpha}, \quad \left| h_{sd}^{(k,i)} \right|^2 = g_{sd}^{(k,i)} \left\| Z_k \right\|_2^{-\alpha}, \]  \( (16) \)

\[ \left| h_{sr}^{(k,i)} \right|^2 = g_{sr}^{(k,i)} \left\| Z_k - Y_i \right\|_2^{-\alpha}, \quad \left| h_{rd}^{(k,i)} \right|^2 = g_{rd}^{(k,i)} \left\| Z_k \right\|_2^{-\alpha}, \]  \( (17) \)
with
\[ D^2 = \|z_i\|^2 + \|y_i\|^2 - 2\|z_i\|\|y_i\| \cos \psi_0, \]  
where we use the law of cosines to obtain Eq. (18); \( z_k \) and \( y_i \) are vectors representing the 2-D locations of the UEs in \( \Phi_1 \) and \( \Phi_2 \), respectively; and \( \psi_0 \sim U[0:2\pi] \) is a uniform random variable that represents the angle between the two vectors \( z_i \) and \( y_i \) connecting the \( i^{th} \) UE to its base station and relaying node. \( g_{sd}^{(k,i)}, g_{rd}^{(k,i)}, \) and \( g_{sr}^{(k,i)} \), are all i.i.d. \( \sim \exp(1) \) and represent the small scale channel fading.

Note that in Eq. (17) we use the out-of-cell interference far field approximation and set the location of out-of-cell source interferer and its associated relay to be the same as also done in [13]. The results in [13] were shown to have an analytic performance similar to that of simulation without the far field approximation in the case of non-cooperating base station, but this similarity does not completely hold in case of full cooperation between neighboring base stations. The case is different in our model since, as discussed in Section IV next, we restrict the selection of each active UE for its relay to be the closest idle UE that is located within the coverage of their base station. Hence, the expected distance between the cooperating nodes is much less than that between neighboring base stations, justifying the far field approximation.

**IV. COOPERATION POLICIES AND PROBABILITY**

In this section, we identify three different cooperation policies: an ideal policy \( E_1 \), a pure geometric policy \( E_2 \) and a hybrid policy \( E_3 \) that defines whether a UE should select a relaying strategy or a direct transmission strategy. Then, we analytically develop closed form expressions for two of these policies.

**A. Cooperation Policies Definition**

We first introduce the cooperation policy, \( E_1 \), as an ideal policy that requires the decision making nodes, whether it is the base station only or both the active user and the closest idle user associated with it as a relay, to know the instantaneous channels between each other and between the active user and its serving base station. It also requires full knowledge of interference at the relay user and base station on decision making. Effectively, the ideal policy relies on knowledge of the instantaneous SINRs of the relay link and the direct link at the decision making node.
This policy can be defined according to the scheme described in [16] as

\[ E_1 = \left\{ \left| \tilde{h}_{kr}(k) \right|^2 \geq \left| \tilde{h}_{sd}(k) \right|^2 \right\} \]

\[ \simeq \left\{ g_{sr_2}r_2^{-\alpha} \geq g_{sd_2}r_2^{-\alpha} \right\} \quad (19) \]

where \( r_1 = \| z_i \|_2 \) and \( r_2 = \| y_i \|_2 \) denote the source-to-destination and source-to-relay distances, respectively, as shown in Fig. 2. This event identifies whether an idle UE will be associated as a relay for the \( k^{th} \) UE and participate in transmission, and hence cause interference to the network.

In an interference limited scenario, we can ignore the effect of the noise variance \( \sigma^2 \) and hence follows Eq. (20) from which we can conclude that when the interference powers at the relay and at the destination are approximately the same, the cooperation event depends mainly on the distances from the source to relay and to the destination. Taking this fact into account and averaging out the effects of small scale fading, we identify a pure geometric cooperation policy, \( E_2 \), as an approximate to the cooperation event in Eq. (20) which will simplify the interference power model analysis by assuming independence between the cooperation policy and interference, i.e., the interference terms \( I_{d,k}^b, I_{d,k}^{m}, \) and \( I_{r,k}^b \) will not affect the local cooperation decision of the \( k^{th} \) UE in \( \Phi_1 \).

The pure geometric policy \( E_2 \) is defined as

\[ E_2 = \left\{ r_1 \geq r_2, D \leq r_1 \right\} \quad (20) \]

Policy \( E_2 \) is more practical than policy \( E_1 \) in the sense that it does not require full knowledge of both the channel state information and the interference at the decision making node. Instead, it only requires the decision making nodes to know the distances from the active user to the nearest idle user and to the base station. It represents a practical decision making strategy for fast fading channels since it does not require any knowledge of the channel fading. The extra condition \( D \leq r_1 \) ensures the elimination of all the cases that will result in an infinite interference at the relay. This approximation, as stated earlier, is useful in simplifying the interference model analysis to follow in Section V.

To realize policy \( E_2 \), different network aided positioning techniques can be used by BSs to obtain information about UEs locations as surveyed in [25]. A location tracking algorithm based on the Kalman filter with velocity estimation and direction finder is proposed in [26].
This algorithm is used in [27] to develop an interference coordination algorithm in a D2D communications underlaying cellular networks based on the locations of both D2D users and cellular UEs.

The last policy, $E_3$, is introduced for slow fading channels where small scale fading parameters estimation and their feedback to the decision making node is feasible. We denote this policy as the hybrid fading and geometric policy and define it as

$$E_3 = \{ g_{sd}r_1^{-\alpha} \leq g_{sr}r_2^{-\alpha}, D \leq r_1 \}. \quad (22)$$

Note that this cooperation policy is still independent from the interference as in the pure geometric cooperation policy $E_2$.

### B. Cooperation Probabilities

Here we analytically derive the cooperation probabilities for the geometric policy $E_2$ and the hybrid policy $E_3$. For the ideal policy $E_1$, analytic evaluation of the cooperation probability is rather complicated because of the inter-dependency between the cooperation decision and consequential interference among different cells, hence we use numerical simulations instead.

We perform the analysis at a random BS assuming that it is associated with the $i^{th}$ UE. We assume that randomly picking this BS is equivalent to selecting a point uniformly distributed in the $\mathbb{R}^2$ plane as done in [21]. Under this assumption, the distribution of the distance $r_1$ between the $i^{th}$ UE and its associated BS can be shown to be Rayleigh distributed directly from the null probability of a two dimensional p.p.p. distribution.

Moreover, we can assume due to the stationarity of the Poisson point processes and the independence of $\Phi_2$ from BSs distribution that the location of the UE associated with the BS under study represents the origin (typical) point of $\Phi_2$. Then, each UE in $\Phi_1$ chooses the closest UE in $\Phi_2$ to assist it in relaying its message to the serving BS. Hence, similar to source-to-destination distance, the distribution of the source-to-relay distance $r_2$ between the $i^{th}$ UE and its associated relaying UE can be also shown to be Rayleigh distributed directly from the null probability of a two dimensional p.p.p..

Thus, we have

$$f_{r_1}(r_1) = 2\pi \lambda_1 r_1 e^{-\lambda_1 \pi r_1^2}, \quad f_{r_2}(r_2) = 2\pi \lambda_2 r_2 e^{-\lambda_2 \pi r_2^2}. \quad (23)$$

The probabilities of cooperation policies $E_2$ and $E_3$ in (21) and (22) denoted as $\rho_2$ and $\rho_3$, respectively, which can be approximations of the cooperation probability $\rho_1$ of $E_1$ in Eq. (20) are derived in closed form as in Theorem [1]
Theorem 1 (Cooperation probabilities). The probability of deploying user-assisted relaying for a randomly located active user within a cell can be evaluated as follows:

(i) For policy $E_2$

$$\rho_2 = \int_{-\pi/2}^{-\pi/3} \frac{2\lambda_2 \cos^2 \psi_0}{\pi(\lambda_1 + 4\lambda_2 \cos^2 \psi_0)} d\psi_0 + \int_{-\pi/3}^{\pi/2} \frac{2\lambda_2 \cos^2 \psi_0}{\pi(\lambda_1 + 4\lambda_2 \cos^2 \psi_0)} d\psi_0 + \frac{\lambda_2}{3(\lambda_1 + \lambda_2)}$$

(ii) For policy $E_3$

$$\rho_3 = \int_0^2 f_\beta(z) \left[ \int_{-\pi/2}^{\cos^{-1}(z/2)} \frac{2\lambda_2 \cos^2 \psi_0}{\pi(\lambda_1 + 4\lambda_2 \cos^2 \psi_0)} d\psi_0 + \int_{\cos^{-1}(z/2)}^{\pi/2} \frac{2\lambda_2 \cos^2 \psi_0}{\pi(\lambda_1 + 4\lambda_2 \cos^2 \psi_0)} d\psi_0 \right] dz + \int_2^\infty \int_{-\pi/2}^{\pi/2} f_\beta(z) \frac{2\lambda_2 \cos^2 \psi_0}{\pi(\lambda_1 + 4\lambda_2 \cos^2 \psi_0)} d\psi_0 dz,$$

where $\beta = \left(\frac{g_{sr}}{g_{sd}}\right)^{1/\alpha}$ and $f_\beta(z)$ is the probability density function (PDF) of $\beta$ defined as

$$f_\beta(z) = \frac{\alpha z^{\alpha-1}}{(1 + z^\alpha)^2}.$$ (26)

Proof: See Appendix A for details.

We use numerical integration to compare between these probabilities in Section VI-A. We can note by investigating equations (24) and (25) that they are both proportional to the users density ratio $\lambda_2/\lambda_1$ and that both probabilities $\rho_2$ and $\rho_3$ achieve their maximum when this ratio approaches infinity, i.e. as the density of the idle users increases. We can evaluate the maximum probability achieved by both cooperation policies $E_2$ and $E_3$ as

$$\rho_2^{\text{max}} = \lim_{\lambda_2 \to \infty} \rho_2(\lambda_1, \lambda_2) = 0.5, \quad \rho_3^{\text{max}} = \lim_{\lambda_2 \to \infty} \rho_3(\lambda_1, \lambda_2) = 0.5.$$ (27)

The maximum probabilities achieved of 0.5 is because of our restriction of the spatial cooperation domain to the idle UEs that are closer to the BS than the active user seeking cooperation. Effectively, we only consider potential relays approximately in a half circle centered at the active user and inside the cell under consideration.

V. Out-of-Cell Interference Analysis

User-assisted relaying actually increases the amount of out-of-cell interference in the network as some idle users are now transmitting when relaying information of active users. It is therefore necessary to understand this out-of-cell interference power, particularly its distribution, in order to assess the overall impact of user-assisted relaying on system performance. Given the stochastic geometry system model described in Section III with Poisson point process distributions, tools
from stochastic geometry can be used to analytically derive the moments of the interference power in the network. It is, however, difficult to describe the exact distribution of that interference power. To study the performance of user assisted relaying, we consider a cell with fixed radius, \( R_c = 1/2\sqrt{\lambda_1} \), that is proportional to the active users density, \( \lambda_1 \), and analyze the out-of-cell interference to this cell as a typical case for the network. Here we choose to model the interference power to the cell under study as a Gamma distribution by fitting the first two moments of the interference power analytically developed using stochastic geometry from the field of interferers outside that cell. The fit of this Gamma distribution model will be evaluated in the next section Section VI to access the impact of the model on the accuracy of system performance. Having an analytical interference model can significantly simplify system performance analysis by removing the need for time and labor intensive simulation. Further, such an analytical interference model also allows tractable performance analysis with detailed understanding of the impact of each parameter which may not be feasibly obtained by simulation.

A. Analytic Development of Moments of the Interference Power

We start by deriving interference power moments, for which, we first build up from the interference expressions in Eq. (14) to develop the interference power at the \( i^{th} \) destination BS during the first and second phase, respectively, as in Eq. (27) and Eq. (28). Similarly, interference power at the idle UE associated as a relay with the \( i^{th} \) active UE can be written as in Eq. (32). Here, \( \theta_{k2,i} \) and \( \theta_{kr,i} \) are realizations of independent and uniformly distributed random angle variables. We use the approximation described in [13] where interference is averaged over the reception angles and since \( \mathbb{E}_{\theta_{k2,i},\theta_{kr,i}} \left[ \cos (\theta_{k2,i} - \theta_{kr,i}) \right] = 0 \), we can rewrite the interference
power at the \(i^{th}\) destination BS during the second phase as in Eq. (29). This approximation implies even though the out-of-cell source-relay pair transmits coherently (i.e. beamform) to its own destination, the two signals go through different channels to the cell under consideration and appear independent of each other.

Next, we use the Laplace transform of interference power at the \(i^{th}\) destination BS during the first and second phase as derived in Appendix B to characterize the moments of interference as follows.

**Theorem 2** (Interference Power Laplace Transform). To characterize the moments of interference power, its Laplace transform is derived as follows:

(i) At the \(i^{th}\) destination BS during the first and second phase, respectively

\[
\mathcal{L}_{Q_{d,i}}(s) = \exp \left( -2\pi \lambda_1 \int_{R_c}^{\infty} \left( 1 - \mathcal{L}_{J_{d,i}}(s, r) \right) r dr \right),
\]

\[
\mathcal{L}_{Q_{m,i}}(s) = \exp \left( -2\pi \lambda_1 \int_{R_c}^{\infty} \left( 1 - \mathcal{L}_{J_{m,i}}(s, r) \right) r dr \right),
\]

where \(\mathcal{L}_{J_{d,i}}(s, \|z_k\|_2)\) and \(\mathcal{L}_{J_{m,i}}(s, \|z_k\|_2)\) are expressed as

\[
\mathcal{L}_{J_{d,i}}(s, \|z_k\|_2) = \rho_1 \mathcal{L}_G \left( s \|z_k\|_2^{-\alpha} P_{b,s,k} \right) + (1 - \rho_1) \mathcal{L}_G \left( s \|z_k\|_2^{-\alpha} P_{s,k} \right),
\]

\[
\mathcal{L}_{J_{m,i}}(s, \|z_k\|_2) = \rho_1 \mathcal{L}_G \left( s \|z_k\|_2^{-\alpha} P_{m,s,k} \right) \mathcal{L}_G \left( s \|z_k\|_2^{-\alpha} P_{r,k} \right) + (1 - \rho_1) \mathcal{L}_G \left( s \|z_k\|_2^{-\alpha} P_{s,k} \right).
\]

(ii) At the idle UE associated as a relay for the \(i^{th}\) active UE

\[
\mathcal{L}_{Q_{r,i}}(s) = \exp \left( -\lambda_1 \int_0^{2\pi} \int_{R_c}^{\infty} \left( 1 - \mathcal{L}_{J_{r,i}}(s, r, \theta) \right) r dr d\theta \right),
\]

where \(\mathcal{L}_{J_{r,i}}(s, \|z_k\|_2, \theta_k)\) is defined as

\[
\mathcal{L}_{J_{r,i}}(s, \|z_k\|_2, \theta_k) = \rho_1 \mathcal{L}_G \left( s \|z_k - y_i\|_2^{-\alpha} P_{b,s,k} \right) + (1 - \rho_1) \mathcal{L}_G \left( s \|z_k - y_i\|_2^{-\alpha} P_{s,k} \right),
\]

and the term \(\|z_k - y_i\|_2\) can be written in terms of \(\|z_k\|_2\), distance \(D\) defined in (18) and \(\theta_k\) using the law of cosines as

\[
\|z_k - y_i\|^2 = \|z_k\|^2 + D^2 - 2\|z_k\|D \cos \theta_k.
\]

**Proof:** See Appendix B for details.

**Lemma 1** (Interference Power Statistics). For network-wide deployment of user-assisted relaying, the out-of-cell interference generated at the destination BS and the relaying UE have the following statistics:
The first two moments, mean and variance, of interference power at the destination BS during the 1st and 2nd phase, respectively, are

\[
\mathbb{E}\left[Q_{d,i}^b\right] = \frac{2\pi \lambda_1 \zeta_1}{\alpha - 2} R_c^{2-\alpha}, \quad \mathbb{E}\left[Q_{d,i}^m\right] = \frac{2\pi \lambda_1 \zeta_3}{\alpha - 2} R_c^{2-\alpha},
\]

\[\text{var}\left[Q_{d,i}^b\right] = \frac{\pi \lambda_1 \zeta_2}{\alpha - 1} R_c^{2(1-\alpha)}, \quad \text{var}\left[Q_{d,i}^m\right] = \frac{\pi \lambda_1 \zeta_4}{\alpha - 1} R_c^{2(1-\alpha)}.\]

The first two moments, mean and variance, of interference power at the idle UE associated as a relay with the \(i\)th active UE are

\[
\mathbb{E}\left[Q_{r,i}\right] = \lambda_1 \zeta_1 \int_0^{2\pi} \int_R^\infty (r^2 + D^2 - 2rD\cos(\theta))^{-\frac{\alpha}{2}} r dr d\theta,
\]

\[\text{var}\left[Q_{r,i}\right] = \lambda_1 \zeta_2 \int_0^{2\pi} \int_R^\infty (r^2 + D^2 - 2rD\cos(\theta))^{-\alpha} r dr d\theta,
\]

where

\[
\zeta_1 = \rho_1 P_{s,k}^b + (1 - \rho_1) P_{s,k},
\]

\[
\zeta_2 = 2 \left[ \rho_1 (P_{s,k}^b)^2 + (1 - \rho_1) P_{s,k}^2 \right],
\]

\[
\zeta_3 = \rho_1 (P_{s,k}^m + P_{r,k}^m) + (1 - \rho_1) P_{s,k},
\]

\[
\zeta_4 = 2 \left[ \rho_1 (P_{s,k}^m + P_{r,k}^m)^2 + (1 - \rho_1) P_{s,k}^2 - \rho_1 P_{s,k} P_{r,k}^m \right].
\]

**Proof:** The proof is straightforward by using the results of the interference power Laplace transform in Theorem 2 and evaluating the following formulas:

(i) At the destination

\[
\mathbb{E}\left[Q_{d,i}^b\right] = -\frac{\partial L\left[Q_{d,i}^b(s)\right]}{\partial s}\bigg|_{s=0}, \quad \mathbb{E}\left[Q_{d,i}^m\right] = -\frac{\partial L\left[Q_{d,i}^m(s)\right]}{\partial s}\bigg|_{s=0},
\]

\[\text{var}\left[Q_{d,i}^b\right] = \frac{\partial^2 L\left[Q_{d,i}^b(s)\right]}{\partial s^2}\bigg|_{s=0} - (\mathbb{E}\left[Q_{d,i}^b\right])^2, \quad \text{var}\left[Q_{d,i}^m\right] = \frac{\partial^2 L\left[Q_{d,i}^m(s)\right]}{\partial s^2}\bigg|_{s=0} - (\mathbb{E}\left[Q_{d,i}^m\right])^2.
\]

(ii) At the relay

\[
\mathbb{E}\left[Q_{r}\right] = -\frac{\partial L\left[Q_{r}(s)\right]}{\partial s}\bigg|_{s=0}, \quad \text{var}\left[Q_{r}\right] = \frac{\partial^2 L\left[Q_{r}(s)\right]}{\partial s^2}\bigg|_{s=0} - (\mathbb{E}\left[Q_{r}\right])^2.
\]

Clearly from the above results for interference power statistics, the interference power is directly proportional to both the active users density, \(\lambda_1\), and the transmission power levels represented by \(\zeta_i, i \in [1 : 4]\) in Eq. (43), which agrees with intuition. Note also that the variance at the destination during the second phase is directly proportional to \(\zeta_4\) which includes the full...
correlation term $P_{s,k}^m P_{r,k}^m$ between the $k^{th}$ active user and its associated relay transmission, even though the active user uses only part of its power during the second phase to coherently transmit with its associated relay. The above results further show the effect of the path loss exponent, $\alpha$, and cell radius, $R_c$, on the interference power statistics. For the practical cases when $\alpha \geq 2$, the interference power statistics are inversely proportional to the cell radius and approaches zero at the destination BS as the cell radius increases.

### B. Modeling Interference Power Distribution

A parameterized probability distribution, which includes a wide variety of curve shapes, is useful in the representation of data when the underlying model is unknown or difficult to obtain in closed form. A parameterized probability distribution is usually characterized by its flexibility, generality, and simplicity. Although distributions are not necessarily determined by their moments, the moments often provide useful information and are widely used in practice. For example, a four-parameter probability distribution is introduced in [28], [29] which is used to fit a set of data and match up to its fourth order moment.

In [30], the two-parameter Gamma distribution is used in a study of the downlink performance in a fixed-size cell within a cellular network to match the first two moments of a given distribution representing either the product of the small-scale and lognormal fading or the out-of-cell co-tier and cross-tier interference power distributions. It is shown that the Gamma distribution is a good approximation for the interference when the point under study is closer to the cell center, but fails to represent the actual interference distribution whenever the point under study is exactly at the cell edge. We use the same approach here and match a Gamma distribution to the first two moments of the interference power terms derived earlier in Lemma 1. In Section VI-B, we study the validity of this interference model in our network while varying parameters such as the locations of the active user and the associated relaying user, and the maximum transmit power allowed in the network.

The Gamma distribution is specified by two parameters, a shape parameter $k$ and a scale parameter $\theta$. Given a Gamma random variable $\gamma[k, \theta]$, its probability density function is defined as $F_\gamma(q|k, \theta) = \frac{q^{k-1} e^{-q/\theta}}{\theta^k \Gamma(k)}$, where the Gamma function $\Gamma(t)$ is defined as $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$. 

The mean and variance of $\gamma[k, \theta]$ can be written in the following form \[31\]

$$
\mathbb{E}[\gamma] = k\theta, \quad \text{(46)}
$$

$$
\text{var}[\gamma] = k\theta^2. \quad \text{(47)}
$$

Given that we have analytically obtained the first two moments of the interference power terms of interest, we can easily estimate the shape and scale parameters of the Gamma distributed random variables $\gamma_d^b, \gamma_d^m, \text{ and } \gamma_r^b$ that fit interference power terms $Q_d^b, Q_d^m, \text{ and } Q_r^b$, respectively, using the moments estimation described below in Lemma 2 as introduced in \[30], \[32].

**Lemma 2** (Estimation of Gamma Distribution Parameters using Moments Matching). *Given a distribution $Q_i$ with mean $\mathbb{E}[Q_i]$ and variance $\text{var}[Q_i]$, the shape and scale parameters $k_i$ and $\theta_i$ of the Gamma distributed random variable $\gamma_i[k_i, \theta_i]$ can be estimated as*

$$
k_i = \frac{\left(\mathbb{E}[Q_i]\right)^2}{\text{var}[Q_i]}, \quad \theta_i = \frac{\text{var}[Q_i]}{\mathbb{E}[Q_i]}. \quad \text{(48)}
$$

**Proof:** The proof follows easily, given that we know $\mathbb{E}[Q_i]$ and $\text{var}[Q_i]$, by equating the first two moments of the actual distribution random variable, $Q_i$, to the moments of a Gamma distributed random variable, $\gamma_i[k_i, \theta_i]$, defined in (46) and (47). Then, using (46), we have $k_i = \mathbb{E}[Q_i]/\theta_i$ and substituting into (47), we get $\theta_i = \text{var}[Q_i]/\mathbb{E}[Q_i]$. Hence, $k_i = \mathbb{E}[Q_i]^2/\text{var}[Q_i]$. ■

### VI. MODEL EVALUATION AND DISCUSSION

In this section, we use our geometric network model to numerically verify the validity of the analytical results and models established in the previous two sections. We first verify the validity of using the Rayleigh distribution to model the source-to-relay distance. We then discuss and compare the different cooperation policies probabilities. Next, we evaluate and discuss the validity of using the Gamma distribution to model the interference power in the user-assisted relaying network. These validations confirm our analytical results and the fit of the Gamma distribution for out-of-cell interference power for the range of practical system parameters.

#### A. Geometric Model and Cooperation Probability

We present a sample layout of the network model for uplink user-assisted relaying in cellular system in Fig. [3] where we choose the idle users density to be twice that of the active users, $\lambda_2 = 2\lambda_1$. Both simulation and analytical results show that even for such a low density of idle
users, there is a good probability to exploit cooperation in the network to help cell edge users. In Fig. 4, we first validate our choice of the Rayleigh distribution to model the distance of the source-to-relay link by generating around $5 \times 10^6$ sample network layouts using the stochastic geometry model where $\lambda_2 = 2$ is chosen for the results shown. As Fig. 4 shows, the Rayleigh distribution matches perfectly the simulated data.

In Fig. 5, we use numerical simulation to evaluate the probability of the cooperation policy $E_3$ and compare it to the analytic closed-form probabilities obtained for both policies $E_2$ and $E_3$. In simulation, we used policy $E_3$ where perfect knowledge of the small scale fading channel is assumed but not the out-of-cell interference. Based on Fig. 5, we can see that both cooperation policies almost have the same probability and they match to a close extent the simulation results. These results show that even in the case of the least knowledge about the channel state information, policy $E_2$ can provide a similar cooperation result as other more complex policies requiring more information. The results suggest $E_2$ is suitable for practical deployment.

B. Interference Distribution Model Validation

We now validate the choice of Gamma distribution to model the interference power. In Fig. 6, we show two samples of interference power distribution at the destination and at the relay during the first phase. We also show the Gamma distribution fitting to the data numerically obtained through simulation and compare it to the analytically obtained interference power model in Lemma 1 and 2. Both the numerically fitted and the analytic distributions match perfectly. Further, they both can be considered good approximation to the actual interference power distribution for the different network parameters of practical interest as shown and discussed in Figs. 7 and 8.
Fig. 6: Sample fitting of interference power into Gamma distribution at the destination BS and the relay in the 1st phase, at transmit SNR = 15 dB

In Fig. 7, we study the effect of user transmit power level on the proposed analytic model for the interference power in two scenarios, when the active user is exactly half way between the destination BS and the cell edge, and when it is very close to the cell edge. Here we assume that the relaying node is co-located with the active user. The results in Fig. 7 show that the interference power analytic model causes the analytic performance to slightly diverge from simulation only at high transmit power. We note that the maximum transmission power defined in the LTE standard is 23 dBm, and for this range of practical power, the analytic results closely match that of simulation.

In Fig. 8, we study the effect of the locations of both the active user and its associated relay user on the interference power model. Since we assume a fixed-size cell, changing the locations of the active user and the relaying node does not affect the interference model at the destination. Hence, in this study we are mainly concerned with the interference model at the relaying node. Results in Fig. 8 show a close match for all locations of the relay user up to the cell edge.
We observe that there is only a single singular point when the relaying node is exactly at the cell edge at which the analytic interference power model fails to capture the actual simulation performance. This event, however, can be practically ignored given the low probability of having an idle node associated with an active user as a relay and located exactly on the cell edge. Fig. 8 further confirms only a slight difference in performance when comparing analytic results to simulation at a transmit power of 26 dBm as also observed in Fig. 7, while the analytic results match simulation perfectly for all other lower transmit power levels.

These validation results suggest that using the Gamma distribution to model the out-of-cell interference power is valid for all range of practical transmit powers and user distances. Next we will use this model to evaluate and analyze the performance gain of network-wide deployment of user-assisted relaying.

VII. PERFORMANCE ANALYSIS OF USER-ASSISTED RELAYING

In this section, we evaluate the performance of the PDF user-assisted relaying scheme when deploying in the network, taking into account cooperation decisions and out-of-cell interference. We use the transmission average rate as a performance metric. Consider an active UE located within a typical cell of radius \( R_c = 300 \) m, which is proportional to the active user density \( \lambda_1 = \frac{1}{(16 \times 150^2)} \) as discussed earlier in Section V. We discuss numerical results where we use \( P_{s,i} = P_{r,i}; \quad P_{s,i}^b = P_{s,i}^{m_1}; \) and \( P_{s,i}^{m_2} \) and \( P_{s,i}^{m_2} \) are allocated optimally to maximize the transmission rate of the active user. Note that here we perform numerical integrations to compute the average rates based on our developed analytic interference model and cooperation probabilities. We then
compare these numerical results with system simulation where we carry out detailed simulation of a multi-cell network as discussed in Section III. To make a fair comparison between our analytic models and simulation, we assume in case of simulation that all cells other than the cell of interest independently use the hybrid cooperation policy, $E_3$, whereas the cell under study uses the ideal cooperation policy, $E_1$. We use this simulation setting instead of the case of all cells using $E_1$ policy since the simulated setting gives more realistic performance and presents no inter-cell dependency between cooperation decision and interference, making the simulation computationally feasible.

A. Rate Gain versus Relay Location

In Fig. 9 we compare with simulation results the average rate numerically obtained using our interference power analytic models and the developed cooperation policies versus the ratio between source-to-relay and source-to-destination distances. These results show that performance of the system using the pure geometric cooperation policy, $E_2$, gets worse than simulation using the ideal cooperation policy, $E_1$, when the source-to-relay and source-to-destination distances ratio, $r_2/r_1$, is above $65\%$, i.e. when the relay is closer to the destination than the source. This difference is mainly due to the lack of the small scale fading information at the transmitting node in deciding whether to perform cooperation or not. An improvement to the performance is observed when we make use of the small scale fading knowledge as in the hybrid cooperation policy, $E_3$, which exhibits a close match with simulation results. Note that there is a slight difference between the analytic and simulation performance results in Fig. 9 at the transmit power $26$ dBm, which is mainly due to our exploit of the high transmission power as observed
and discussed in Fig. 7. At lower transmit power, this difference almost vanishes as also shown in Fig. 9.

In Fig. 10, we show the effect of cell size on performance using policy $E_3$. For an active user at a fixed distance from the destination BS, as we increase the cell radius, the effective interference at both the relaying node and the destination BS is reduced and hence the performance improves as expected from intuition. This result can also be interpreted in terms of reduced cell size and transmit power. As we decrease the cell size, the amount of transmit power required to reach a user at the cell edge is also decreased. Consequently, the out-of-cell interference is reduced, leading to similar performance. In other words, we can reduce both cell size and transmit power without affecting the performance of user-assisted relaying.

Both Fig. 9 and Fig. 10 show that it is usually more beneficial to have the relaying node closer to the active user than to the destination BS especially when we lack to the knowledge of the small scale fading of the channel. The maximum gain is achieved when the relay user is approximately midway between the active user and the BS.

B. Rate Gain Averaged over all Idle User Locations

Fig. 11 shows the rate gain averaged over all possible locations of the relaying user versus the distance from the active user to the destination BS and compare it to maximum gain of the ideal case where the active user always finds a relaying node at exactly 0.4 distance between the active user and the destination BS. We note that uplink user-assisted relaying poses significant gains for cell-edge users while can be irrelevant for active users too close to the destination BS. This result suggests user-assisted relaying is most applicable to the farthest 33% or 50% percentile of active users in the cell. For example, at six times more idle users than active users, uplink user-assisted relaying can achieve an average gain of up to 50%, and a maximum gain of up to 200% when the idle user is ideally located about halfway between the active user and the base station. The average gain results support the use of D2D communications among proximity users for helping to relay data of each other. Further, when the relay user is optimally placed, we get significantly higher gain which can be applicable in smaller cell settings.

Fig. 12 provides a quantitative evaluation of the average rate gain obtained for active users occupying the farthest 1/3 and 1/2 of the cell radius, averaged over all the possible locations of both the active and relay users. The gain increases with higher density of idle users, suggesting that user-assisted relaying is suitable for crowded metropolitan areas. These results show that
when applying our scheme to the users towards the cell edge, we can achieve higher percentage gain. Also, as the cell radius increases, user-assisted relaying becomes more beneficial to the cell edge users and brings higher rate gains. The average rate gain is almost 30% for the active users located on the one-half of cell radius towards the cell edge, and can increase to almost 50% when the cell size is doubled.

VIII. CONCLUSION

In this paper, we analyze system-wide performance impact of deploying user-assisted partial decode-and-forward relaying in a cellular network. Using a stochastic geometry model for user and base station locations, we analytically derive the probability of cooperation and interference power generated to the relayed user and destination base station. This cooperation and interference analysis provides a solid theoretical basis for evaluating system performance metric such as outage, coverage and throughput. Numerical results verify our analysis and show that user-assisted relaying can significantly improve the per-user transmission rate, despite increased out-of-cell interference. The throughput gain increases with higher idle user density and is more significant for active users closer to the cell edge, suggesting that user-assisted relaying is viable for crowded metropolitan areas to improve data rate of near-cell-edge users. Performance gain in terms of outage and coverage will be analyzed in a near future work.

APPENDIX A: PROOF OF THEOREM 1

In this section, we derive the probability of both the geometric cooperation policy, $E_2$, in (21) and the hybrid fading and geometric cooperation policy, respectively, which can be one of the representations of the cooperation probability $\rho_1$. First, we show the derivation of the policy $E_2$ probability $\rho_2$ detailed as follows

\[
\rho_2 = \mathbb{P}\{E_2\} = \mathbb{P}\{r_1 \geq r_2, r_1^2 + r_2^2 - 2r_1r_2\cos\psi_0 \leq r_1^2\}
\]

\[
= \mathbb{P}\{r_1 \geq r_2, r_2 \leq 2r_1\cos\psi_0\} = \int_{-\pi/3}^{\pi/3} \mathcal{E}_1 d\psi_0 + \int_{\pi/3}^{\pi/2} \mathcal{E}_1 d\psi_0 + \int_{-\pi/3}^{\pi/3} \mathcal{E}_2 d\psi_0,
\]

where

\[
\mathcal{E}_1 = 2\pi\lambda_1\lambda_2 \int_0^{\infty} \int_0^{2r_1\cos\psi_0} r_1r_2e^{-\pi(\lambda_1r_1^2 + \lambda_2r_2^2)}dr_2dr_1 = \frac{2\lambda_2\cos^2\psi_0}{\pi(\lambda_1 + 4\lambda_2\cos^2\psi_0)},
\]

\[
\mathcal{E}_2 = 2\pi\lambda_1\lambda_2 \int_0^{\infty} \int_0^{r_1} r_1r_2e^{-\pi(\lambda_1r_1^2 + \lambda_2r_2^2)}dr_2dr_1 = \frac{\lambda_2}{2\pi(\lambda_1 + \lambda_2)},
\]

substituting Eqs. (50) and (51) into Eq. (49), we obtain Eq. (24) in Theorem 1.
Now, we derive the probability \( \rho_3 \) of the hybrid fading and geometric policy \( E_3 \)

\[
\rho_3 = \mathbb{P}\{E_3\} = \mathbb{P}\{r_2 \leq \left( \frac{g_{xx}}{g_{sd}} \right)^{1/\alpha} r_1, r_1^2 + r_2^2 - 2r_1r_2 \cos \psi_0 \leq r_1^2 \} = \mathbb{P}\{r_2 \leq \beta r_1, r_2 \leq 2r_1 \cos \psi_0 \} = \int_{\mathbb{R}^2} f_\beta(z) \left[ \int_{-\pi/2}^{\pi/2} \mathcal{E}_1 d\psi_0 + \int_{-\cos^{-1}(z/2)}^{\cos^{-1}(z/2)} \mathcal{E}_1 d\psi_0 \right] dz = \int_{-\pi/2}^{\pi/2} \mathcal{E}_1 d\psi_0 dz
\]

where \( \beta = \left( \frac{g_{xx}}{g_{sd}} \right)^{1/\alpha} \) and \( f_\beta(z) \) is the probability density function (PDF) of \( \beta \).

To obtain the PDF of \( \beta \), we first derive the cumulative distribution function (CDF), \( F_\beta(z) \), as follows

\[
F_\beta(z) = \mathbb{P}\left\{ \left( \frac{x_1}{x_2} \right)^{1/\alpha} \leq z \right\} = \mathbb{P}\{ x_1 \leq z \alpha x_2 \} = \int_0^\infty \int_0^{z \alpha x_2} e^{-(x_1 + x_2)} dx_1 dx_2 = \int_0^\infty e^{-x_2} \left( 1 - e^{-z \alpha x_2} \right) dx_2 = 1 - \frac{1}{1 + z \alpha}, \quad z \in [0, \infty). \tag{53}
\]

The PDF \( f_\beta(z) \) is then obtained by differentiating \( F_\beta(z) \), as follows

\[
f_\beta(z) = \frac{dF_\beta(z)}{dz} = \frac{\alpha \cdot e^{-z \alpha}}{\left(1 + z \alpha \right)^2}, \quad z \in [0, \infty). \tag{54}
\]

**APPENDIX B: PROOF OF THEOREM**

In this section we derive the Laplace transform of the different interference power terms which is used to characterize the moments of interference. The developed moments are then used to develop the interference power distribution analytic model. We first develop the Laplace transform of the interference power at the destination during the 1st phase as follows

\[
\mathcal{L}_Q_{d,i}^b(s) = \mathbb{E}_{Q_{d,i}^b} e^{-sQ_{d,i}^b} = \mathbb{E}_{Q_{d,i}^b} \left[ \prod_{z_k \in \Phi 1 \setminus z_i} e^{-sB_k |h_{sd}^{(k,i)}|^2} p_{s,k}^{-s(1-B_k)|h_{sd}^{(k,i)}|^2} P_{s,k} \right]
\]

\[
= \mathbb{E}_{\Phi 1, g_{sd}^{(k,i)}} \left[ \prod_{z_k \in \Phi 1 \setminus z_i} e^{-sB_k \alpha |h_{sd}^{(k,i)}||z_k|^2} p_{s,k}^{-s(1-B_k)|z_k|^2} P_{s,k} \right]
\]

\[
= \mathbb{E}_{\Phi 1, g_{sd}^{(k,i)} \big|_{z_k \in \Phi 1 \setminus z_i}} \prod_{z_k \in \Phi 1 \setminus z_i} \rho_1 e^{-s g_{sd}^{(k,i)} |z_k|^2} + \left( 1 - \rho_1 \right) e^{-s |z_k|^2} P_{s,k} \bigg|_{z_k \in \Phi 1 \setminus z_i}
\]
where the last equality follows from the Laplace functional expression for p.p.p. using polar coordinates along with the fixed cell radius assumption; $G \sim \exp(1)$ and $\mathcal{L}_{J_{d,i}^b}(s, \|z_k\|_2)$ is expressed as seen in Eq. (34).

Now, we develop the Laplace transform of the interference power at the destination during the 2nd phase as follows

$$
\mathcal{L}_{Q_{d,i}^m}(s) = \mathbb{E}_{Q_{d,i}^m}[e^{-sQ_{d,i}^m}] = \mathbb{E}_{Q_{d,i}^m}\left[ \prod_{z_k \in \Phi_1 \setminus z_i} e^{-sB_k\left(h_{s,k}^{(k,i)}\right)^2 P_{s,k}^m + h_{r,d}^{(k,i)} P_{r,k}^m - s(1-B_k) h_{s,d}^{(k,i)} P_{s,k}} \right]
$$

$$
= \mathbb{E}_{\Phi_1}\left[ \prod_{z_k \in \Phi_1 \setminus z_i} \mathbb{E}_{G_{ad}^{(k,i)}}\left[ \rho_1 e^{-s\|z_k\|^\alpha_{G_{ad}^{(k,i)}} + g_{r,d}^{(k,i)} P_{r,k}} + (1 - \rho_1) e^{-s\|z_k\|^\alpha_{G_{ad}^{(k,i)}} P_{s,k}} \right] \right]
$$

$$
= \mathbb{E}_{\Phi_1}\left[ \prod_{z_k \in \Phi_1 \setminus z_i} \rho_1 \mathcal{L}_G(s \|z_k\|^\alpha_{P_{s,k}^m}) \mathcal{L}_G(s \|z_k\|^\alpha_{P_{r,k}^m}) + (1 - \rho_1) \mathcal{L}_G(s \|z_k\|^\alpha_{P_{s,k}}) \right]
$$

$$
= \mathbb{E}_{\Phi_1}\left[ \prod_{z_k \in \Phi_1 \setminus z_i} \mathcal{J}_{d,i}^m(s, \|z_k\|_2) \right] = \exp\left( -2\pi \lambda_1 \int_{R_c}^\infty \left( 1 - \mathcal{L}_{J_{d,i}^m}(s, r) \right) r dr \right),
$$

(56)

where the last equality also follows from the Laplace functional expression for p.p.p. using polar coordinates along with the fixed cell radius assumption; $\mathcal{L}_G(s) = 1/(1 + s)$ is the Laplace transform of an exponential random variable $G \sim \exp(1)$ and $\mathcal{L}_{\mathcal{J}_{d,i}^m}(s, \|z_k\|_2)$ is expressed as in Eq. (35).

Finally, the Laplace transform of the interference power at the relay during the 1st phase can be develop in a similar way to the case at the destination during the 1st phase.

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