Improved analysis for the baryon masses to order $\Lambda_{QCD}/m_Q$
from QCD sum rules

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Abstract

We use the QCD sum rule approach to calculate the masses of the $\Lambda_Q$ and $\Sigma_Q$ baryons to the $\Lambda_{QCD}/m_Q$ order within the framework of heavy quark effective theory. We compare the direct approach and the covariant approach to this problem. Two forms of currents have been adopted in our calculation and their effects on the results are discussed. Numerical results obtained in both direct and covariant approaches are presented. The splitting between spin 1/2 and 3/2 doublets derived from our calculation is $\Sigma_Q^* \sim 0.35 \pm 0.03 \text{GeV}^2$ which is in good agreement with the experiment.

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I. INTRODUCTION

Important progresses in the theoretical description of hadrons containing a heavy quark have been achieved with the development of the Heavy Quark Effective Theory (HQET) [1–3]. Based on the spin-flavor symmetry of QCD, exactly valid in the infinite $m_Q$ limit, this framework provides a systematic expansion of heavy hadron spectra and both the strong and weak transition amplitudes in terms of the leading contribution, plus corrections decreasing as powers of $1/m_Q$. HQET has been applied successfully to learn about the properties of mesons and baryons made of both heavy and light quarks.

The effective Lagrangian of the HQET, up to order $1/m_Q$, can be written as

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot \overrightarrow{D} h_v + \frac{1}{2m_Q} \mathcal{K} + \frac{1}{2m_Q} \mathcal{S} + O(1/m_Q^2),$$

(1)

where $h_v(x)$ is the heavy quark field in effective theory. Apart from leading contribution, the Lagrangian density contains to $O(1/m_Q)$ accuracy two additional operators $\mathcal{K}$ and $\mathcal{S}$. $\mathcal{K} = \bar{h}_v (iD^\perp)^2 h_v$ is the non-relativistic kinetic energy operator and $\mathcal{S} = \frac{1}{2} \left( \frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right)^{3/\beta_0} \bar{h}_v \sigma_{\mu\nu} g_\sigma G^{\mu\nu} h_v$ is the chromo-magnetic interaction. Here $(D^\perp)^2 = D_\mu D^\mu - (v \cdot D)^2$, with $D_\mu = \partial_\mu - i g A_\mu$ the covariant derivative and $\beta_0 = 11 - \frac{2}{3} n_f$ is the first coefficient of the $\beta$ function.

The matrix elements of the operators $\mathcal{K}$ and $\mathcal{S}$ in (1) play a most significant role in many phenomenological applications such as the spectroscopy of heavy hadrons [4] and the description of inclusive decay rates [5]. For the ground-state $\Lambda_Q$ and $\Sigma_Q$ baryons, one defines two hadronic parameters, $\lambda_1$ and $\lambda_2$, as

$$\langle B(v) | \mathcal{K} | B(v) \rangle = \lambda_1,$$

$$\langle B(v) | \mathcal{S} | B(v) \rangle = d_M \lambda_2.$$

(2)

where $d_M$ is zero for $\Lambda_Q$ and $-\frac{1}{2}$, 1 for $\Sigma^*_Q$, $\Sigma_Q$ baryons, respectively. The constant $d_M$ characterizes the spin-orbit interaction of the heavy quark and the gluon field. Therefore, the mass of heavy baryon up to order $1/m_Q$ corrections can be written in a compact form

$$M = m_Q + \bar{\Lambda} - \frac{1}{2m_Q} (\lambda_1 + d_M \lambda_2),$$

(3)

where parameter $\bar{\Lambda}$ is the energy of the light degrees of freedom in the infinite mass limit. Thus the splitting of the spin $1/2$ and $3/2$ doublets is

$$\Sigma^*_Q - \Sigma^2_Q = \frac{3}{2} \lambda_2.$$

(4)

The hadronic parameters $\lambda_1$ and $\lambda_2$ are nonperturbative ones that should be either determined phenomenologically from experimental data or estimated in some nonperturbative approaches. A viable approach is the QCD sum rules [6] formulated in the framework.
of HQET \cite{7}. This method allows us to relate hadronic observables to QCD parameters via the operator product expansion (OPE) of the correlator. In the case of heavy mesons, those two matrix elements thus have been calculated to perfect first by Ball and Braun \cite{8} and latterly by Neubert \cite{9} taking a different approach. Masses of excited meson states had been calculated up to $1/m_Q$ order in \cite{10}. For the case of heavy baryons, there are several attempts to calculate the baryonic matrix elements of $\mathcal{K}$ and $\mathcal{S}$ using QCD sum rules, Colangelo et. al. have derived the value of $\lambda_1$ for $\Lambda_Q$ baryon \cite{11}. Furthermore, the baryonic parameters $\lambda_1$ and $\lambda_2$ for the ground state baryons had been calculated in \cite{12} by evaluating the two-point correlation functions. The mass parameters of the lowest lying excited heavy baryons had also been determined recently in \cite{13}. In the present work we shall calculate the baryonic parameters $\lambda_1$ and $\lambda_2$ for ground state $\Lambda_Q$ and $\Sigma_Q$ baryons using QCD sum rules in the HQET. Following Ball et. al. \cite{8} and Neubert’s \cite{9} work done for the meson, we adopt these two approaches, namely direct approach and covariant approach, to evaluate the three-point correlators and obtain the values of baryonic parameters. It is of interest to compare the two methods in the analysis.

The remainder of this paper is organized as follows. In Sec. II A we introduce the interpolating currents for baryons and briefly present the two-point sum rules. The direct Laplace sum rules analysis for the matrix elements is presented in Sec. II B. Another feasible approach (covariant approach) to this aim can be found in Sec. II C. Sec. II is devoted to numerical results and our conclusions. Some comments are also available in Sec. II.

II. DERIVATION OF THE SUM RULES FOR $\lambda_1$ AND $\lambda_2$

A. Heavy baryonic currents and two-point sum rules

The basic points in the application of QCD sum rules to problems involving heavy baryon are to choose a suitable interpolating current in terms of quark fields and to define the corresponding vacuum-to-baryon matrix element. As is well known, the form of interpolating currents for baryon with given spin and parity is not unique \cite{14–16}, the choice of which one is just a question of predisposition. The most generally used form of the heavy baryon current can be written as \cite{15}

$$j^\nu = \epsilon_{abc}(q_1^T a CT \tau q_2^b)\Gamma h_c^\nu,$$

in which $C$ is the charge conjugation matrix, $\tau$ is the flavor matrix which is antisymmetric for $\Lambda_Q$ baryon and symmetric for $\Sigma_Q^{(*)}$ baryon, $\Gamma$ and $\Gamma'$ are some gamma matrices, and $a$, $b$, $c$ denote the color indices. $\Gamma$ and $\Gamma'$ can be chosen co-variantly as
\[ \Gamma = \gamma_5 \, , \quad \Gamma' = 1 \, , \quad (6) \]

for \( \Lambda_Q \) baryon, and

\[ \Gamma = \gamma_\mu \, , \quad \Gamma' = (\gamma_\mu + v_\mu) \gamma_5 \, , \quad (7) \]

for \( \Sigma_Q \) baryon, and

\[ \Gamma = \gamma_\nu \, , \quad \Gamma' = -g_\mu\nu + \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3} (\gamma_\mu v_\nu - \gamma_\nu v_\mu) + \frac{2}{3} v_\nu v_\mu \, , \quad (8) \]

for \( \Sigma_Q^* \) baryon. Also the choice of \( \Gamma \) is not unique. We can insert a factor \( \phi \) before \( \Gamma \) defined by equations (6)-(8). The currents given by Eqs.(6)-(8) are denoted as \( j^v_1 \) and that with \( \phi \) insertion as \( j^v_2 \), which are two independent current representations.

The baryonic coupling constants in HQET are defined as follows

\[ \langle 0 | j^v | \Lambda(\nu) \rangle = F_\Lambda u, \]
\[ \langle 0 | j^v | \Sigma(\nu) \rangle = F_\Sigma u, \]
\[ \langle 0 | j^v | \Sigma^*(\nu) \rangle = \frac{1}{\sqrt{3}} F_{\Sigma^*} u^\alpha, \quad (9) \]

where \( u \) is the spinor and \( u^\alpha \) is the Rarita-Schwinger spinor in the HQET, respectively. The coupling constants \( F_\Sigma \) and \( F_{\Sigma^*} \) are equivalent since \( \Sigma_Q \) and \( \Sigma_Q^* \) belong to the doublet with the same spin-parity of the light degrees of freedom.

The QCD sum rule determination of these coupling constants can be done by analyzing the two-point function

\[ i \int dx e^{ikx} \langle 0 | T \{ j^v(x) \bar{j}^v(0) \} | 0 \rangle = \frac{1 + \phi}{2} Tr[\tau^\tau^+] \Pi(\omega), \quad (10) \]

where \( k \) is the residual momentum and \( \omega = 2v \cdot k \). It is straightforward to obtain the two-point sum rule:

\[ F_\Lambda^2 e^{-2\Lambda v/T} = \frac{3 T^6}{2^5 \pi^4} \delta_5(\omega_v/T) + \frac{T^2}{2^7 \pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \delta_1(\omega_v/T) + \frac{1}{6} \langle \bar{q}q \rangle^2, \]
\[ F_{\Sigma^*}^2 e^{-2\Lambda v^*/T} = \frac{9 T^6}{2^5 \pi^4} \delta_5(\omega_v/T) - \frac{T^2}{2^7 \pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \delta_1(\omega_v/T) + \frac{1}{2} \langle \bar{q}q \rangle^2. \quad (11) \]

The functions \( \delta_n(\omega_v/T) \) arise from the continuum subtraction and are given by

\[ \delta_n(x) = \frac{1}{n!} \int_0^x dt t^n e^{-t} = 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!}. \quad (12) \]

The second term of the last equation is assigned to the continuum mode, which can be much larger than the ground state contributions for the typical value of parameter \( T \) due to the high dimensions of the spectral densities.
B. The direct approach

In order to evaluate the matrix elements \( \lambda_1 \) and \( \lambda_2 \) we consider the three point correlation functions with \( K \) and \( S \) inserted directly between two interpolating currents at zero recoil as below

\[
\begin{align*}
  i^2 \int dx \int dy e^{ikx-ik'y} \langle 0 \mid T\{j^v(x)K(0)\bar{j}^v(y)\} \mid 0 \rangle &= \frac{1 + \hat{p}^2}{2} Tr[\tau \tau^+] T_K(\omega, \omega') \\
  i^2 \int dx \int dy e^{ikx-ik'y} \langle 0 \mid T\{j^v(x)S(0)\bar{j}^v(y)\} \mid 0 \rangle &= d_M \frac{1 + \hat{p}^2}{2} Tr[\tau \tau^+] T_S(\omega, \omega') \quad (13)
\end{align*}
\]

where the coefficients \( T_K(\omega, \omega') \) and \( T_S(\omega, \omega') \) are analytic functions in the “off-shell energies” \( \omega = 2v \cdot k \) and \( \omega' = 2v \cdot k' \) with discontinuities for positive values of these variables. Saturating the three-point functions with complete set of baryon states, one can isolate the part of interest, the contribution of the lowest-lying baryon states associated with the heavy-light currents, as one having poles in both the variables \( \omega \) and \( \omega' \) at the value \( \omega = \omega' = 2\bar{\Lambda} \):

\[
\begin{align*}
  T_K(\omega, \omega') &= 4 \frac{\lambda_1 F^2}{(2\Lambda - \omega)(2\Lambda - \omega')} + \cdots \\
  T_S(\omega, \omega') &= 4 \frac{\lambda_2 F^2}{(2\Lambda - \omega)(2\Lambda - \omega')} + \cdots \quad (14)
\end{align*}
\]

where the ellipses denote the contribution of higher resonances. In the theoretical calculation of the correlator it is convenient to choose the residual momenta \( k \) and \( k' \) parallel to the \( v \), such that \( k_\mu = \frac{\omega}{2} v_\mu \) and \( k'_\mu = \frac{\omega'}{2} v_\mu \).

The leading contribution to the matrix element of kinetic energy is of order 1, whereas to the chromo-magnetic interaction is of order \( \alpha_s \). Confining us to take into account these leading contributions of perturbation and the operators with dimension \( D \leq 6 \) in OPE, the relevant diagrams in our calculation are shown in Fig. 1. and Fig. 2. The calculation of the diagram (a) in Fig. 2. is the most tedious one. It can be computed using Feynman parameterization and the integral representation of the propagators, which is the standard technique [18,19]. The factorization approximation has been used to reduce the four-quark condensates to \( \langle \bar{q}q \rangle^2 \) in the calculation.

On theoretical side the correlators \( T_K(\omega, \omega') \) and \( T_S(\omega, \omega') \) can be casted into the form of integrals of the double spectral densities as

\[
\begin{align*}
  T_K(\omega, \omega') &= \int \int \frac{ds ds'}{s - \omega s' - \omega'} \rho_K(s, s') \\
  T_S(\omega, \omega') &= \int \int \frac{ds ds'}{s - \omega s' - \omega'} \rho_S(s, s') \quad (15)
\end{align*}
\]

where the double spectral density functions are
\[
\rho_{K}^{\Lambda,1}(s, s') = -\frac{3^3}{2^4 \pi^4} s^7 \delta(s - s') - \frac{7}{2^6 \pi^2} \frac{(\alpha_s G^2)}{3!} s^3 \delta(s - s'),
\]
\[
\rho_{K}^{\Sigma,1}(s, s') = -\frac{3^3}{2^4 \pi^4} s^7 \delta(s - s') - \frac{11}{2^6 \pi^2} \frac{(\alpha_s G^2)}{3!} s^3 \delta(s - s'),
\]
\[
\rho_{K}^{\Lambda,2}(s, s') = -\frac{3^2}{2^4 \pi^4} s^7 \delta(s - s') + \frac{1}{2^6 \pi^2} \frac{(\alpha_s G^2)}{3!} s^3 \delta(s - s'),
\]
\[
\rho_{K}^{\Sigma,2}(s, s') = -\frac{3^2}{\pi^4} s^7 \delta(s - s') - \frac{19}{2^6 \pi^2} \frac{(\alpha_s G^2)}{3!} s^3 \delta(s - s'),
\]
\[
\rho_{S}^{\bar{\Sigma}}(s, s) = \frac{\alpha_s}{2^3 \pi^5} [\Theta(s - s') \int_0^{s'} dx (s - x)(s' - x)x^3 + \Theta(s' - s) \int_0^s dx (s - x)(s' - x)x^3]
- \frac{1}{48 \pi^2} \frac{(\alpha_s G^2)}{3} s^3 \delta(s - s') + \frac{4\alpha_s}{3\pi} \langle \bar{q}q \rangle^2 [s\delta(s') + s'\delta(s)].
\]

The unitary normalization of flavor matrix \(T \tau[\tau \tau^+] = 1\) has been applied to get those densities. Here we use the numbers 1, 2 to denote the results corresponding to the different choice of currents \(j_1^v\) and \(j_2^v\). Those we do not discriminate with numeric superscripts means that with or without \(v\) insertion the results are identical. Following Refs. 20, 22, we then introduce new variables \(\omega_+ = \frac{1}{2}(\omega + \omega')\) and \(\omega_- = \omega - \omega'\), perform the integral over \(\omega_-\), and employ quark–hadron duality to equate the remaining integral over \(\omega_+\) up to a “continuum threshold” \(\omega_c\) to the Borel transform of the double-pole contribution in (14). Then following the standard procedure we resort to the Borel transformation \(B_{\tau}^\omega\), \(B_{\tau'}^\omega\) to suppress the contributions of the excited states. Considered the symmetries of the correlation functions it is natural to set the parameters \(\tau, \tau'\) to be the same and equal to \(2T\), where \(T\) is the Borel parameter of the two-point functions. We end up with the set of sum rules

\[
-4 \lambda_1^{\Lambda,1} F^2 e^{-2\lambda_1/T} = \frac{3^3}{(2 \pi)^4} \frac{T^8}{\pi^2} \delta_7(\omega_c/T) + \frac{7}{2^6 \pi^2} \frac{(\alpha_s G^2)}{3!} \delta_3(\omega_c/T),
\]
\[
-4 \lambda_1^{\Lambda,2} F^2 e^{-2\lambda_1/T} = \frac{3^2}{(2 \pi)^4} \frac{T^8}{\pi^2} \delta_7(\omega_c/T) - \frac{T^4}{2^6 \pi^2} \frac{(\alpha_s G^2)}{3!} \delta_3(\omega_c/T),
\]
\[
-4 \lambda_1^{\Sigma,1} F^2 e^{-2\lambda_1/T} = \frac{3^2}{(2 \pi)^4} \frac{T^8}{\pi^2} \delta_7(\omega_c/T) + \frac{11}{2^6 \pi^2} \frac{(\alpha_s G^2)}{3!} \delta_3(\omega_c/T),
\]
\[
-4 \lambda_1^{\Sigma,2} F^2 e^{-2\lambda_1/T} = \frac{3^2}{(2 \pi)^4} \frac{T^8}{\pi^2} \delta_7(\omega_c/T) + \frac{19}{2^6 \pi^2} \frac{(\alpha_s G^2)}{3!} \delta_3(\omega_c/T),
\]
\[
4 \lambda_2 F^2 e^{-2\lambda_2/T} = \frac{12}{(2 \pi)^4} \frac{T^8}{\pi^2} \delta_7(\omega_c/T) - \frac{T^4}{8 \pi^2} \frac{(\alpha_s G^2)}{3!} \delta_3(\omega_c/T) + \frac{32 T^2 \alpha_s}{3 \pi} \langle \bar{q}q \rangle^2 \delta_1(\omega_c/T).
\]

It is worth noting that the next-to-leading order \(\alpha_s\) corrections have not been included in the sum rule calculations. However, the baryonic parameter obtained from the QCD sum rules actually is a ratio of the three-point correlator to the two-point correlator results. While both of these correlators are subject to large perturbative QCD corrections, it is expected that their ratio is not much affected by these corrections because of cancellation. On the other hand, we have only calculated the diagonal sum rules by using the same type
interpolating current in the correlator. As to the non-diagonal sum rules, the only non-vanishing contributions in the OPE of correlator are terms with odd number of dimensions, thus the perturbative term gives no contribution. The resulting sum rules are dominated by the quark-gluon condensates. It is expected that the non-diagonal sum rules will give no more information than diagonal ones. This has been proved to be true in the analysis of Ref. [17].

C. The covariant approach

In the previous subsection we have completed the task of the determination of the matrix elements for both the operators of kinetic energy and chromo-magnetic interaction by direct calculation of three-point correlation functions. In fact, there exists a field-theory analog of the virial theorem [23,24] in consideration of the restrictions the equation of motion and the heavy quark symmetry imposing on baryons, which relates the kinetic energy and chromo-interaction to each other and ensure the intrinsic smallness of the kinetic energy explicitly. In this subsection we shall follow Neubert’s procedure [9] and take those restrictions into account to deduce a new result of the kinetic energy (the chromo-magnetic interaction is identical).

The main idea of that procedure is that the coefficients of the covariant decomposition of the bilinear matrix elements, the so-called invariant functions, can be related to the kinetic energy and chromo-interaction at the zero recoil. Following the discussion in [4,25] we have the general decomposition (see Appendix):

$$\langle \Lambda | \bar{h} \sigma_{\mu\nu} g_{s} G^{\mu\nu} h | \Lambda' \rangle = \phi_{1}(v', v)(v'_\mu v_\nu - v_\mu v'_\nu)\bar{u} \sigma^{\mu\nu} u',$$

(18)

for $\Lambda_Q$ baryon, in which $u$ is the spinor in HQET, and

$$\langle \Sigma | \bar{h} \sigma_{\mu\nu} g_{s} G^{\mu\nu} h | \Sigma' \rangle = \phi_{\alpha\beta}^{\mu\nu}(v', v)\bar{\Psi}^{\alpha} \sigma^{\mu\nu} \Psi^{\beta},$$

(19)

for $\Sigma_Q$ baryon, where $\phi_{\alpha\beta}^{\mu\nu}$ bears the decomposition

$$\phi_{\alpha\beta}^{\mu\nu} = \phi_{1}(g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) + \phi_{2}(g_{\mu\beta} v_\nu v'_{\alpha} - g_{\mu\alpha} v_\nu v'_{\beta} + g_{\nu\alpha} v_{\beta} v'_{\mu} - g_{\nu\beta} v_{\alpha} v'_{\mu})$$

$$+ \phi_{3}(g_{\mu\alpha} v_{\nu} v'_{\beta} - g_{\nu\alpha} v_{\mu} v'_{\beta} + g_{\nu\beta} v'_{\alpha} v'_{\mu} - g_{\beta\mu} v'_{\alpha} v'_{\nu})$$

$$+ \phi_{4}(v'_\mu v_\nu - v'_\nu v_\mu) g_{\alpha\beta} + \phi_{5}(v'_\mu v_\nu - v'_\nu v_\mu) v'_{\alpha} v_{\beta},$$

(20)

in which we use the covariant representation of the doublets $\Psi_{\mu} = u_{\mu} + \frac{1}{\sqrt{3}}(v_{\mu} + \gamma_{\mu})u$ with restrictions $\not{v} u = u$, $v_{\mu} u_{\mu} = 0$ and $\gamma_{\mu} u_{\mu} = 0$. The normalization of those coefficients at zero recoil is $\phi_{1}(1) = -\frac{1}{3} \lambda_{1}$ for $\Lambda_Q$ baryon and

$$\pm \lambda_{2} = 2 \phi_{1}(1),$$

$$\pm \lambda_{1} = \phi_{0}(1) = \phi_{1}(1) - 2(\phi_{2}(1) - \phi_{3}(1)) - 3\phi_{4}(1),$$

(21)
for $\Sigma_Q$ baryon. The foregoing minus sign corresponds to the $\Sigma_Q$ baryon and plus to $\Sigma_Q^*$ baryon.

Let us now derive the Laplace sum rules for the invariant functions $\phi_i(w)$. The analysis proceeds in complete analogy to that of the Isgur–Wise function. We shall only briefly sketch the general procedure and refer for details to Refs. [20, 21]. We consider, in the HQET, the three-point correlation function of the local operator appearing in (19) with two interpolating currents for the ground-state heavy baryons:

$$i^2 \int dx \, dy \, e^{ik \cdot x - i k' \cdot y} \langle 0 | T \{ j^{\mu}(x), \bar{h}_{\nu} i g_s \Gamma G^{\nu \rho} h_{\rho}(y), j^\nu(y) \} | 0 \rangle$$

$$= \Phi_{\alpha \beta}^{\mu \nu}(v', v, k') \Gamma_\alpha^{\nu'} \frac{1 + \gamma_5}{2} \Gamma_{\beta}^{\nu'}$$

(22)

where $k$ and $k'$ are the residual momenta. The Dirac structure of the correlation function, as shown in the second line, is a consequence of the Feynman rules of the HQET. $\Phi_{\alpha \beta}^{\mu \nu}$ obeys a decomposition analogous to (20), with coefficient functions $\Phi_i(\omega, \omega', w)$ that are analytic in the “residual energy” $\omega = 2v \cdot k$ and $\omega' = 2v' \cdot k'$, with discontinuities for positive values of these variables. These functions also depend on the velocity transfer $w = v \cdot v'$.

The lowest-lying states are the ground-state baryons $B(v)$ and $B'(v')$ associated with the heavy-light currents. They lead to a double pole located at $\omega = \omega' = 2\Lambda$. The residue of this double pole is proportional to the invariant functions $\phi_i(w)$. We find

$$\Phi_i^{\text{pole}}(\omega, \omega', w) = \frac{4 s_c \phi_i(w) F^2}{(\omega - 2\Lambda)(\omega' - 2\Lambda)},$$

(23)

where $s_c$ is the structure constant, 1 for $\Lambda_Q$, $-2\bar{\Lambda}/3$ for $\Sigma_Q$ and 1 for $\Sigma_Q^*$ baryons. In the deep Euclidean region the correlation function can be calculated perturbatively because of asymptotic freedom. Following the standard procedure, we write the theoretical expressions for $\Phi_i$ as double dispersion integrals and perform a Borel transformation in the variables $\omega$ and $\omega'$, then set the associated Borel parameters equal: $\tau = \tau' = 2T$. All goes like that in the direct approach, we introduce new variables $\omega_+ = \frac{1}{2}(\omega + \omega')$ and $\omega_- = \omega - \omega'$, perform the integral over $\omega_-$, and get the Laplace sum rules at zero recoil:

$$(8s_c F^2) \phi_1 e^{-2\Lambda \Sigma/T} = \frac{4}{\pi^4} \frac{\alpha_s}{\pi} T^8 \delta_7(\omega_c/T) - \frac{1}{24\pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \delta_3(\omega_c/T) + \frac{32\alpha_s}{9\pi} (\bar{q}q)^2 T^2 \delta_1(\omega_c/T),$$

$$(8s_c F^2) \phi_2 e^{-2\Lambda \Sigma/T} = 8s_c F^2 2(\phi_2 - \phi_3) e^{-2\Lambda \Sigma/T},$$

$$(8s_c F^2) \phi_4 e^{-2\Lambda \Sigma/T} = \frac{2}{\pi^4} \frac{\alpha_s}{\pi} T^8 \delta_7(\omega_c/T),$$

$$(8s_c F^2) \phi_5 e^{-2\Lambda \Sigma/T} = -\frac{1}{\pi^4} \frac{\alpha_s}{\pi} T^8 \delta_7(\omega_c/T)$$

(24)

for $\Sigma_Q$ baryon, and

$$(8F^2) \phi_1 e^{-2\bar{\Lambda}} = -\frac{2}{\pi^4} \frac{\alpha_s}{\pi} T^8 \delta_7(\omega_c/T),$$

(25)
for $\Lambda_Q$ baryon. After some simple algebra we find

$$-4\lambda_1 F^2 e^{-2\bar{\Lambda}_Q/T} = \frac{9}{\pi^4} \frac{\alpha_s}{\pi} T^8 \delta(\omega_c/T),$$

$$4\lambda_2 F^2 e^{-2\bar{\Lambda}_Q/T} = \frac{12}{\pi^4} \frac{\alpha_s}{\pi} T^8 \delta(\omega_c/T),$$

for $\Sigma_Q$ baryon, and

$$-4\lambda_1 F^2 e^{-2\bar{\Lambda}_Q/T} = \frac{32}{\pi^4} \frac{\alpha_s}{\pi} (\bar{q}q)^2 \delta(\omega_c/T),$$

$$4\lambda_2 F^2 e^{-2\bar{\Lambda}_Q/T} = \frac{8}{\pi^4} \frac{\alpha_s}{\pi} T^8 \delta(\omega_c/T),$$

for $\Lambda_Q$ baryon. The minus sign of the $\Lambda_Q$ baryon result may seem bizarre, in Sec. III we will return to dwell on this point.

### III. NUMERICAL RESULTS AND CONCLUSIONS

In order to get the numerical results, we divide our three-point sum rules by two-point functions to obtain $\lambda_1$ and $\lambda_2$ as functions of the continuum threshold $\omega_c$ and the Borel parameter $T$. This procedure can eliminate the systematic uncertainties and cancel the parameter $\bar{\Lambda}$. As for the condensates, we adopt the standard values

$$\langle \bar{q}q \rangle = -(0.23 \text{ GeV})^3,$$

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \text{ GeV}^4.$$  

From two-point sum rules one has known that there exist stable windows between $0.8 < T < 1.2 \text{ GeV}$ and $\omega_c = (2.2 - 2.7) \text{ GeV}$ for $\Lambda_Q$ baryon, and $0.7 < T < 1.1 \text{ GeV}$ and $\omega_c = (2.6 - 3.3) \text{ GeV}$ for $\Sigma_Q$ baryon. The stability window for three-point functions starts almost from values of the Borel parameter at $0.7 \text{ GeV}$ and stretches practically to $T \to \infty$. It is known that stability at lager Borel parameter could not give any valuable information, since in this region the sum rule is strongly effected by the continuum model. The usual criterium that both the higher-order power corrections and the continuum contribution should not be very large restricts the working region considerably. In the case of the three-point functions, the results are severely smeared by the continuum contributions for the high dimension of spectral densities and thus it is very difficult to ensure the contribution of the continuum mode is small. The working region of the three-point functions should be determined by the stable region of the two-point functions. So it does not necessarily coincide with the stable windows of the three-point functions [8]. Thus we find our working region for three-point functions is $T = 0.8 - 1.2 \text{ GeV}$ for $\Lambda_Q$ baryon and $T = 0.7 - 1.1 \text{ GeV}$ for $\Sigma_Q$ baryon. The results for $\lambda_1$ of the direct approach with two different choices of currents are shown in Fig. 3 for $\Lambda_Q$ baryon and Fig. 4 for $\Sigma_Q$ baryon. Results for kinetic energy of $\Lambda_Q$ and $\Sigma_Q$ baryons obtained by the covariant approach are presented in Fig. 5.
The forms of the chromo-magnetic interaction obtained by both approaches do not differ from each other, so we plot that unique curve in one figure, Fig. 6. For the $\Lambda_Q$ baryon we obtain the residual mass $\bar{\Lambda}_\Lambda = 0.8 \pm 0.1$ GeV and

$$-\lambda_1^1 = 0.4 \pm 0.1 \text{ GeV}^2,$$
$$-\lambda_2^1 = 0.5 \pm 0.1 \text{ GeV}^2,$$

(29)
in the direct approach, where the superscripts denote the different choices of currents, and

$$-\lambda_1 = -(0.08 \pm 0.02) \text{ GeV}^2;$$

(30)
in the covariant approach. For the $\Sigma_Q$ baryon the effective mass we obtained is $\bar{\Lambda}_\Sigma = 1.0 \pm 0.1$ GeV and

$$-\lambda_1^1 = 0.7 \pm 0.2 \text{ GeV}^2,$$
$$-\lambda_2^1 = 1.0 \pm 0.2 \text{ GeV}^2,$$

(31)
in the direct approach, where the superscripts also denote different currents, and

$$-\lambda_1 = 0.11 \pm 0.03 \text{ GeV}^2,$$

(32)
in covariant approach. For the chromo-magnetic interaction for $\Sigma$ baryon the results read

$$\lambda_2 = 0.23 \pm 0.02 \text{ GeV}^2.$$

(33)

Then we get the splitting of the spin 1/2 and spin 3/2 doublets is

$$\Sigma_Q^2 - \Sigma_Q^1 = \frac{3}{2} \lambda_2 = 0.35 \pm 0.03 \text{ GeV}^2.$$

(34)

All error quoted before is due to the variation of Borel parameter $T$ and the continuum threshold $\omega_c$. When it is scaled up to the bottom quark mass scale there will be a factor $\sim 0.8$ approximately due to the renormalization group improvement.

As for the effects on the correlation function of the different choices of the interpolating currents we may assert some facts and inspections. From the preceding numerical results it is clear that the interpolating currents with the $\bar{\phi}$ insertion give a considerable larger result to the kinetic energy than those without the insertion. Nevertheless, the two-point sum rules do not differ with the different currents [12,15]. From our calculation it is explicit that the sum rules associated with chromo-interaction insertion are identical. In our covariant calculation we find that the invariant functions do differ from each other generally, but interestingly they coincide at the zero recoil thus the chromo-interaction and kinetic energy do not take a different form. Naively, we can tell that the disparity of the two forms of the kinetic energy obtained in our direct calculation mainly comes from
the Lorenz structural differences of the two interpolating currents. It may be noted that
derivative operator acts differently on the currents with or without \( \phi \) insertion, thus with the
insertion it is easier for the continuum contamination to go into the correlation functions.
It is urgently needed to exclude the continuum contribution which smeared heavily the
results of both the direct and covariant approaches. It is this continuum contamination
that makes the prediction of the kinetic energy more intriguing. All previous theoretical
calculations with QCD sum rule approach or lattice calculation give various results and
can differ from each other by several times \([11,12,24,29]\). Current experimental data is
not enough to judge which one is right and what we can get is some restrictions on the
kinetic energy \([27]\) or a rough estimate of the kinetic energy extracted from experimental
data with some assumption \([28]\). As demonstrated in Refs. \([24,29]\) using a toy model of
harmonic oscillator, the main origin of the discrepancy between the direct and covariant
approach is the continuum smeared contribution. In the direct approach the first excited
contribution plays an important role. If we want to suppress this contribution, we go to
such a large Borel parameter that the power corrections blow up. For acceptable Borel
parameter, we get an over-estimated sum rule for the the kinetic energy. In the covariant
approach (via virial theorem) the situation is especially bad. The excited contribution
consists of two components – the diagonal transitions and off-diagonal ones – and each one
is large, but they have opposite relative signs. For highly excited states the sign-alternating
terms are smeared to zero after summation. However, the first two terms do not cancel
with each other and screen the ground state contribution. Thus a lower-estimated result
will be obtained. The minus sign before the kinetic energy of \( \Lambda_Q \) baryon in the covariant
approach may be seen as a manifestation of this assertion. Due to the unknown weight, we
cannot annihilate those contributions by weighted averaging just like that in the Quantum
Mechanics. But we may safely take the results of the direct and covariant approach as
lower-bound and higher-bound of the kinetic energy parameter, respectively. Then, follow
\([30]\), to take the mean value of the direct and covariant approach results as an rough
estimate. The result thus obtained is

\[
-\bar{\lambda}_1^1 \simeq 0.18 \pm 0.06 \text{ GeV}^2,
-\bar{\lambda}_2^2 \simeq 0.24 \pm 0.06 \text{ GeV}^2,
\]

for \( \Lambda_Q \) baryon and

\[
-\bar{\lambda}_1^1 \simeq 0.39 \pm 0.12 \text{ GeV}^2,
-\bar{\lambda}_2^2 \simeq 0.54 \pm 0.12 \text{ GeV}^2,
\]

for \( \Sigma_Q \) baryon. Taking all results obtained the mass of the ground state baryon is on hand.
From \( m_{\Lambda_c} \) and \( m_{\Lambda_b} \) \([31]\), we determine the heavy quark masses \( m_c \simeq 1.41 \pm 0.16 \text{ GeV} \)
and $m_b \simeq 4.77 \pm 0.12$ GeV. In the determination we have taken the average of the results obtained from two interpolating currents to be the physical pole masses of the heavy quarks because that the difference of the corresponding mass does not exceed the error bar. These values give the following results:

\begin{align*}
m_{\Sigma_c} &\simeq 2.47 \pm 0.20 \text{ GeV}, \\
m_{\Sigma_b^*} &\simeq 2.59 \pm 0.20 \text{ GeV}, \\
m_{\Sigma_b} &\simeq 5.79 \pm 0.13 \text{ GeV}, \\
m_{\Sigma_b^*} &\simeq 5.82 \pm 0.13 \text{ GeV},
\end{align*}

(37)

with interpolating current $j_1^v$ and

\begin{align*}
m_{\Sigma_c} &\simeq 2.52 \pm 0.20 \text{ GeV}, \\
m_{\Sigma_c^*} &\simeq 2.64 \pm 0.20 \text{ GeV}, \\
m_{\Sigma_b} &\simeq 5.80 \pm 0.13 \text{ GeV}, \\
m_{\Sigma_b^*} &\simeq 5.83 \pm 0.13 \text{ GeV},
\end{align*}

(38)

with interpolating current $j_2^v$. The spin average of the doublets are free of the chromo-interaction contribution and thus free of the uncertainties involved in the calculation of $\lambda_2$. Average over the doublets we have the quantity

\[
\frac{1}{3} (M_{\Sigma_Q} + 2M_{\Sigma_Q^*}) = m_Q + \bar{\Lambda}_\Sigma + \frac{1}{2m_Q} \bar{\lambda}_1
\]

which is more reliable. For the $c$ quark case, it is $2.55 \pm 0.20$ GeV with current $j_1^v$ and $2.60 \pm 0.20$ GeV with current $j_2^v$. For the $b$ quark case it is $5.81 \pm 0.13$ GeV with current $j_1^v$ and $5.83 \pm 0.13$ GeV with current $j_2^v$. Experimentally $M_{\Sigma_c} = 2453 \pm 0.2$ MeV [31]. There is experimental evidence for $\Sigma_c^*$ at $M_{\Sigma_c^*} = 2519 \pm 2$ MeV [32]. If we take this value for $\Sigma_c^*$, we have $\frac{1}{3} (M_{\Sigma_c} + 2M_{\Sigma_c^*}) = 2497 \pm 1.4$ MeV which is in reasonable agreement with the theoretical prediction. For lack of experimental data the corresponding quantity for the bottom quark will be checked in the future. If we take the preceding masses of the charmed $\Sigma$ baryons the splitting thus reduced is $0.33$ GeV$^2$ and our theoretical splitting is in considerable agreement with the experimental data.

As the kinetic energy of $\Lambda_Q$ baryon can be related to the spectrum via the kinetic energy of meson as

\[
(m_{\Lambda_b} - m_{\Lambda_c}) - (\overline{m}_B - \overline{m}_D) = [\lambda_1(\Lambda_b) - \lambda_1(B)] \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) + O(1/m_Q^2),
\]

(39)

\[\text{1}\] The relation between $\mu_\pi^2$ in Ref. [33] and $\lambda_1$ in this paper is $\mu_\pi^2 = -\lambda_1$
where $m_B = \frac{1}{4}(m_B + 3m_B^*$) and $m_D = \frac{1}{4}(m_D + 3m_D^*)$ denote the spin-averaged meson masses, the difference between the kinetic energy of $B$ meson and that of $\Lambda_b$ baryon can be extracted as

$$\lambda_1(\Lambda_b) - \lambda_1(B) = 0.01 \pm 0.02 \text{ GeV}^2,$$

which is consistent with the value obtained in Ref. [33]. Resorting to the recent experimental data for the mesonic kinetic energy parameter obtained in the inclusive semileptonic $B$ decays [34], $-\lambda_1 = 0.24 \pm 0.11 \text{ GeV}^2$, one can thus get the value of baryonic kinetic energy as

$$-\lambda_1(\Lambda_b) = 0.23 \pm 0.13 \text{ GeV}^2,$$

which is in reasonable agreement with our theoretical prediction given in (35).

For conclusions, we have calculated the $1/m_Q$ corrections to the heavy baryon masses from the QCD sum rules within the framework of the HQET. Two approaches have been adopted in the evaluation of the three-point correlators. Our final results read

$$M_{\Sigma Q} = m_Q + \bar{\Lambda}_\Sigma + \frac{1}{2m_Q}(0.16 \pm 0.12 \text{ GeV}^2),$$

$$M_{\Sigma^* Q} = m_Q + \bar{\Lambda}_\Sigma + \frac{1}{2m_Q}(0.51 \pm 0.12 \text{ GeV}^2),$$

for interpolating current without $\bar{\varphi}$ insertion and

$$M_{\Sigma Q} = m_Q + \bar{\Lambda}_\Sigma + \frac{1}{2m_Q}(0.31 \pm 0.12 \text{ GeV}^2),$$

$$M_{\Sigma^* Q} = m_Q + \bar{\Lambda}_\Sigma + \frac{1}{2m_Q}(0.66 \pm 0.12 \text{ GeV}^2),$$

for interpolating current with $\bar{\varphi}$ insertion. The $1/m_Q$ corrections are small. We have taken the mean value of the direct and covariant approach as the rough estimate of the kinetic energy parameter $\lambda_1$. Our theoretical predictions are in agreement with the recent experimental data. For a more precise treatment of the kinetic energy, more sophisticated technique to distinguish the smearing continuum contribution is in urgent necessity to be developed.

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In this appendix, we present the decomposition of bilinear matrix element. There exists the decomposition of $\Lambda_Q$ baryon, we present it here merely for completeness and convention.

First, let us consider the bilinear matrix element over $\Lambda_Q$ baryons

$$\langle \Lambda \mid \bar{h}_v (-i \slashed{D}_\mu) \Gamma^{\mu\nu} iD_\nu h_{\nu'} \mid \Lambda' \rangle = \psi_{\mu\nu}(v', v) \bar{u} \Gamma^{\mu\nu} u', \quad (A1)$$

the coefficients obey the symmetric relation $\psi_{\mu\nu}(v, v) = \psi^{\ast}_{\nu\mu}(v, v')$. It is convenient to write the coefficient $\psi$ into the sum of symmetric and anti-symmetric parts $\psi_{\mu\nu} = \frac{1}{2} \left[ \psi^A_{\mu\nu} + \psi^S_{\mu\nu} \right]$ which can be presented covariantly as

$$\psi^A_{\mu\nu} = \psi^A_1 (v_\mu v_\nu - v_\mu v'_\nu),$$

$$\psi^S_{\mu\nu} = \psi^S_1 g_{\mu\nu} + \psi^S_2 (v + v')_\mu (v + v')_\nu + \psi^S_3 (v - v')_\mu (v - v')_\nu, \quad (A2)$$

the HQET equation of motion implies that $v'_\nu \psi_{\mu\nu} = 0$ from which we can obtain the relations between those coefficients

$$\psi^S_1 + (1 + y) \psi^S_2 + (1 - y) \psi^S_3 + y \psi^A_1 = 0,$$

$$(1 + y) \psi^S_2 + (y - 1) \psi^S_3 - \psi^A_1 = 0, \quad (A3)$$

with

$$\bar{h} i \slashed{D}_\mu \Gamma h' + \bar{h} i D_\mu \Gamma h' = i \partial_\mu (\bar{h} \Gamma h') \quad (A4)$$

bear in mind we can get

$$\langle \Lambda \mid \bar{h}_v \Gamma^{\mu\nu} iD_\mu iD_\nu h_{\nu'} \mid \Lambda' \rangle = \psi_{\mu\nu}(v', v) \bar{u} \Gamma^{\mu\nu} u' + \bar{\Lambda}(v' - v)_\mu \xi_\nu \bar{u} \Gamma^{\mu\nu} u', \quad (A5)$$

using $x$ dependence of state in HQET $\mid B(x) \rangle = e^{-i \bar{\Lambda} x} \mid B(0) \rangle$. The $\xi_\nu$ are defined as

$$\langle \Lambda \mid \bar{h}_v \Gamma^{\nu'} iD_\nu h_{\nu'} \mid \Lambda' \rangle = \xi_{\nu'} \bar{u} \Gamma^{\nu'} u', \quad (A6)$$

Similarly, we can define the matrix elements for the operators of kinetic energy and chromo-interaction over baryon states with different velocities

$$\langle \Lambda \mid \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_{\nu'} \mid \Lambda' \rangle = \phi_1 (v_\mu v_\nu - v_\mu v'_\nu) \bar{u} \sigma^{\mu\nu} u', \quad (A7)$$

$$\langle \Lambda \mid \bar{h}_v (iD^\perp)^2 \Gamma h_{\nu'} \mid \Lambda' \rangle = \phi_0 \bar{u} \Gamma u', \quad (A8)$$

once such defined, the $\psi_i$ can be expressed via two $\phi_i$
we have the normalization of not dwell on. The covariant representation of the doublet is \( \Psi \)

Hence we will give the forms of decomposition and the final desired relation, others we will go almost the same. The only difference lies on the decomposition of the matrix element. The symmetric and antisymmetric decomposition of the coefficients like that in \( \Lambda \) matrix element is \( \psi \) in which the coefficients obey symmetric relation

The generalization can be made to the higher spin states such as \( \Sigma \)

\[
\begin{align*}
\psi^A &= \phi_1 - \bar{\Lambda}^2 \xi \frac{y}{y + 1}, \\
\psi^s &= \phi_0 + y \phi_1 + \bar{\Lambda}^2 \xi \frac{y}{y + 1}, \\
\psi^s &= \frac{(1 + 2y)\phi_1 + \phi_0}{2(y - 1)} - \frac{y}{2(y + 1)} \bar{\Lambda}^2 \xi, \\
\psi^s &= \frac{\psi^A - (y - 1)\psi^S}{1 + y},
\end{align*}
\]

the normalization of \( \phi_0, \phi_1 \) are \( \phi_0(1) = \lambda_1, \phi_1(1) = -\frac{1}{3} \phi_0(1) \), thus we get the desired result.

Generalization can be made to the higher spin states such as \( \Sigma_Q \) baryon. The procedure goes almost the same. The only difference lies on the decomposition of the matrix element. Hence we will give the forms of decomposition and the final desired relation, others we will not dwell on. The covariant representation of the doublet is \( \Psi_\mu = u_\mu + \frac{1}{\sqrt{3}}(v_\mu + \gamma_\mu)u \). The matrix element is

\[
\langle \Sigma | \bar{h}_\nu(-i\bar{D}_\mu)\Gamma^{\mu \nu}D_\nu h_{\nu'} | \Sigma' \rangle = \psi_{\mu \nu}^{\alpha \beta} (\nu, v) \bar{\Psi}_\alpha \Gamma^{\mu \nu} \Psi'_\beta,
\]

in which the coefficients obey symmetric relation \( \psi_{\mu \nu}^{\alpha \beta} (\nu, v') = \psi_{\mu \nu}^{\beta \alpha} (\nu, v) \). Adopt the same symmetric and antisymmetric decomposition of the coefficients like that in \( \Lambda_Q \) baryon case, we have

\[
\psi_{\mu \nu}^{\alpha \beta, A} = \psi^A_1 (g_{\mu \alpha}g_{\nu \beta} - g_{\mu \beta}g_{\nu \alpha}) + \psi^A_2 (g_{\mu \beta}v_\nu v_\alpha' - g_{\nu \beta}v_\mu v_\alpha' + g_{\nu \alpha}v_\beta v_\mu' - g_{\mu \alpha}v_\beta v_\mu') + \psi^A_3 (g_{\alpha \nu}v_\mu v_\beta + g_{\nu \beta}v_\alpha v_\mu' - g_{\nu \alpha}v_\beta v_\mu') + \psi^A_4 (v_\mu v_\nu - v_\nu v_\mu)g_{\alpha \beta}
\]

\[
+ \psi^A_5 (v_\mu' v_\nu' - v_\nu' v_\mu')g_{\alpha \beta},
\]

\[
\psi_{\mu \nu}^{\alpha \beta, S} = \psi^S_1 g_{\alpha \beta}g_{\mu \nu} + \psi^S_2 (g_{\mu \alpha}g_{\nu \beta} + g_{\mu \beta}g_{\nu \alpha}) + \psi^S_3 (g_{\mu \beta}v_\nu v_\alpha' - g_{\nu \beta}v_\mu v_\alpha' + g_{\nu \alpha}v_\beta v_\mu' - g_{\mu \alpha}v_\beta v_\mu') + \psi^S_4 (g_{\alpha \nu}v_\mu v_\beta + g_{\nu \beta}v_\alpha v_\mu' - g_{\nu \alpha}v_\beta v_\mu') + \psi^S_5 (v_\mu - v_\nu)g_{\alpha \beta}
\]

\[
+ \psi^S_6 (v_\mu' - v_\nu')g_{\alpha \beta} + \psi^S_7 (v_\mu + v_\nu)g_{\alpha \beta} + \psi^S_8 (v_\mu + v_\nu)g_{\alpha \beta} + \psi^S_9 (v_\mu + v_\nu)g_{\alpha \beta} + \psi^S_{10} g_{\alpha \beta},
\]

introduce other universal parameters in the leading order

\[
\langle \Sigma | \bar{h}_\nu \Gamma^{\nu}D_\nu h_{\nu'} | \Sigma' \rangle = \xi^{\alpha \beta}_\mu (v, v') \bar{\Psi}_\alpha \Gamma^{\mu \nu} \Psi'_\beta,
\]

\[
\langle \Sigma | \bar{h}_\nu \Gamma^{\nu}(-i\bar{D}_\nu)h_{\nu'} | \Sigma' \rangle = \xi^{\beta \alpha}_\nu (v, v') \bar{\Psi}_\alpha \Gamma^{\mu \nu} \Psi'_\beta,
\]

as usual, \( \xi^{\alpha \beta}_\mu (v, v') \) can be decomposed into the general form

\[
\xi^{\alpha \beta}_\mu (v, v') = \xi_1 (v + v')_\mu g_{\alpha \beta} + \xi_2 (v - v')_\mu g_{\alpha \beta} + \xi_3 (v + v')_\nu v_\beta v_\alpha' + \xi_4 (v - v')_\nu v_\beta v_\alpha' + \xi_5 v_\alpha' v_\beta g_{\nu \beta} + \xi_6 v_\beta g_{\alpha \nu}.
\]

the equation of motion implies that \( v_\nu^{\alpha \beta} = 0 \) and \( v_\nu^{\nu \mu \rho \alpha \beta} = 0 \) from which we can derive relations.
\[ w\psi_3^A - \psi_2^A + \psi_3^S + \psi_4^S = 0, \]
\[ w\psi_2^A - \psi_1^A - \psi_3^A + \psi_2^S + w\psi_3^S + \psi_4^S = 0, \]
\[ w\psi_4^S + \psi_1^S + (1 - w)\psi_6^S + (1 + w)\psi_8^S = 0, \]
\[ \psi_2^A + w\psi_5^A + \psi_3^A + (1 - w)\psi_5^S + (1 + w)\psi_7^S + \psi_9^S = 0, \]
\[ (1 + w)\psi_8^S - \psi_4^A - (1 - w)\psi_6^S = 0, \]
\[ \psi_4^S + (w - 1)\psi_5^S - \psi_3^A - \psi_5^A + (1 + w)\psi_7^S = 0, \]
\[(A14)\]

and

\[ (1 + w)\xi_1 + (1 - w)\xi_2 = 0, \]
\[ (1 + w)\xi_3 + (1 - w)\xi_4 + \xi_6 = 0, \]
\[(A15)\]

take the difference of the two terms in Eqs. (A12) and use (A4) we can reach

\[ \xi_1 = \frac{w - 1}{w + 1} \frac{c_1}{2} \bar{\Lambda}, \]
\[ \xi_3 = \frac{w - 1}{w + 1} \frac{c_2}{2} \bar{\Lambda} - \xi_6, \]
\[ \xi_2 = \frac{c_1}{2} \bar{\Lambda}, \]
\[ \xi_4 = \frac{c_2}{2} \bar{\Lambda}, \]
\[ \xi_5 = \xi_6. \]
\[(A16)\]

where \(c_1, c_2\) parameterize the matrix element

\[ \langle \Sigma | \bar{h}_\nu \Gamma h_{\nu'} | \Sigma' \rangle = (c_1 g_{\alpha\beta} + c_2 v_{\beta'} v_{\alpha'}) \bar{\Psi}^\alpha \Gamma \Psi'^\beta. \]
\[(A17)\]

The matrix elements for the kinetic energy and chromo-magnetic interaction are defined similar to those for the \(\Lambda_Q\) baryon

\[ \langle \Sigma | \bar{h}_\nu (iD^\perp)^2 \Gamma h_{\nu'} | \Sigma' \rangle = (\phi_0 g_{\alpha\beta} + \bar{\phi}_0 v_{\beta'} v_{\alpha'}) \bar{\Psi}^\alpha \Gamma \Psi'^\beta, \]
\[(A18)\]
\[ \langle \Sigma | \bar{h}_\nu \sigma_{\mu\nu} iG_{\mu\nu} h_{\nu'} | \Sigma' \rangle = \phi_{\mu\nu} \bar{\Psi}^\alpha \sigma^{\mu\nu} \Psi'^\beta, \]
\[(A19)\]

where \(\phi_{\alpha\beta}\) bear the same decomposition as \(\psi_{\alpha\beta}\) and they have simple relations between each other

\[ \phi_1 = \psi_1^A, \]
\[ \phi_2 = \psi_2^A - \xi_6 \bar{\Lambda}, \]
\[ \phi_3 = \psi_3^A - \xi_6 \bar{\Lambda}, \]
\[ \phi_4 = \psi_4^A - 2\xi_1\bar{A}, \]
\[ \phi_5 = \psi_5^A - 2\xi_3\bar{A}, \]
\[ \phi_0 = 2\psi_1^s + \psi_2^s + (1 - w)\psi_6^s + (1 + w)\psi_8^s + 2(1 - w)\xi_2\bar{A}, \]
\[ \bar{\phi}_0 = 2\psi_3^s + (1 - w)\psi_5^s + (1 + w)\psi_7^s + 2(1 - w)\xi_4\bar{A}, \]  \hspace{1cm} (A20)

the normalization condition is that \( \phi_1(1) = A\lambda_2, \) \( \phi_0(1) = B\lambda_1 \) where \( A = -1/2, 1/2 \) and 
\( B = 1, -1 \) for \( \Sigma_{Q, \bar{Q}}^\ast \) respectively. At zero recoil \( \lambda_1 \) can be expressed via \( \phi_0 \)
\[ \phi_0(1) = \phi_1(1) - 2[\phi_2(1) - \phi_3(1)] - 3\phi_4(1). \]  \hspace{1cm} (A21)
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Figure Captions

Fig. 1. Non-vanishing diagrams for the kinetic energy: (a) perturbative contribution, (b) to (e) gluon-condensate. The kinetic energy operator is denoted by a white square, the interpolating baryon currents by black circles. Heavy-quark propagators are drawn as double lines. Diagrams (b) to (e) are calculated in Fock-Schwinger gauge. The lower right vertices of those diagrams are set to the origin in coordinate space.

Fig. 2. Non-vanishing diagrams for the chromo-magnetic interaction: (a) perturbative contribution, (b) gluon-condensate, (c) quark-condensate. The chromo-magnetic interaction (velocity-changing current) operator is denoted by a white square, the interpolating baryon currents by black circles.

Fig. 3. Sum rules for $\Lambda_Q$ baryon: (a) for $j_1^v$, (b) for $j_2^v$. The dash-dotted, dashed and solid curves correspond to the threshold $\omega_c = 2.4, 2.6, 2.8$ GeV, respectively. The working region is $T = 0.8 - 1.2$ GeV.

Fig. 4. Sum rules of the kinetic energy for $\Sigma_Q$ baryon: (a) for $j_1^v$, (b) for $j_2^v$. The dash-dotted, dashed and solid curves correspond to the threshold $\omega_c = 2.9, 3.1, 3.3$ GeV, respectively. The working region is $T = 0.7 - 1.1$ GeV.

Fig. 5. Covariant sum rules of the kinetic energy: (a) for $\Lambda_Q$ baryon, (b) for $\Sigma_Q$ baryon. The dash-dotted, dashed and solid curves correspond to $\omega_c = 2.4, 2.6, 2.8$ GeV for $\Lambda_Q$ baryon, and $\omega_c = 2.9, 3.1, 3.3$ GeV for $\Sigma_Q$ baryon. The working region is $T = 0.8 - 1.2$ GeV for $\Lambda_Q$ baryon and $T = 0.7 - 1.1$ GeV for $\Sigma_Q$ baryon.

Fig. 6. Sum rules for the chromo-magnetic interaction. The dash-dotted, dashed and solid curves correspond to $\omega_c = 2.9, 3.1, 3.3$ GeV. The working region is $T = 0.7 - 1.1$ GeV.
Fig. 1.

Fig. 2.
Fig. 3.

Fig. 4.

Fig. 5.
Fig. 6.