On the effectiveness of local vortex identification criteria in the vortex representation of wall-bounded turbulence

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Compressing complex flows into a tangle of vortex filaments is the basic implication of the classical vortex-representation notion. This work focuses on the effectiveness of the local identification criteria in the vortex representation of wall-bounded turbulence. Basically, five local identification criteria regarding vortex strength and three criteria for vortex axis are considered. Instead of separately evaluating the two classes of criteria, the current work defines vortex vectors by arbitrarily combining the vortex strength and vortex axis expressed by various criteria, and attempts to figure out the most effective one regarding the vortex representation. The effectiveness of these vortex vectors is evaluated based on two aspects: first, the alignment of the vortex axis and vortex iso-surface should be well established, which benefits the simplification of the vortex filaments; second, vortices could be viewed as the “gene code” of turbulent flows, which means reconstructing the velocity fields based on them should be effective. For the first aspect, the differential geometry method is employed to describe the vortex isosurface-axis alignment property quantitatively. For the second aspect, the Biot-Savart law is employed to accomplish the vortex-to-velocity reconstruction. Results of this work provide some reference for the applications of vortex identification criteria in wall-bounded turbulence.

Wall-bounded turbulence, Vortex identification, Biot-Savart law

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1. Introduction

Extracting vortex structures from available velocity fields is a routine procedure when analyzing the physics of complex flows [1-4]. Although a widely-accepted mathematical definition for vortex is not available yet, the concept typically refers to the rotating motion of a multitude of material particles around a common center [5]. This notion of vortex brings two aspects of convenience for the corresponding analysis work. First, as an intensely rotational motion, a strong vortex retains coherence for comparatively larger spatial separations and longer temporal intervals, which implies a significant contribution to the transfer of mass and momentum. Second, the vortex cores or the vortex skeletons typically presents a tube-like shape and thus can be simplified as a curve with variable radius and strength, which facilitates the vortex-based modelling works [6]. As Perry and Chong [7] stated, “these vortex skeletons are the ‘genetic code’ of the flow field, since this requires very little specification and the Biot-Savart law can be used to generate the velocity field.”

Plenty of research works focus on the vortex identification criteria, and these works have been comprehensively reviewed by Chakraborty et al. [8], Kolár [9] and Epps [10]. Chakraborty et al. [8] classified the identification criteria into two types: the local approaches based on velocity gradient tensor (∇u) and the non-local ones. In this work, only the
local identification criteria are discussed, which includes the
discriminant $J$ [11], the second invariant $Q$ [12], the imag-
inary part $\lambda_{ci}$ of the complex eigenvalues of $\nabla u$ [13], and
the second largest eigenvalue $\lambda_2$ for pressure Hessian ($\nabla \nabla p$) [14]. These criteria are mostly employed as scalar indica-
tors for the local vortex strength, and the vortex cores can be
represented by the iso-surfaces of these criteria for certain
prescribed thresholds.

Kolář [9] argued that the vortex identification criterion
should recognize the vortex axis as well as the vortex strength
and he developed a triple decomposition technique for $\nabla u$
in order to meet this requirement. In fact, as reminded by
Cucitore et al. [15], some of the preceding scalar criteria also
imply privileged directions in their definitions, which are cor-
related with the vortex axis. The $\lambda_1$ criterion recognizes the
eigendirection $\Lambda_p$ associate with the minimum eigenvalue of
pressure Hessian as the vortex axis [14]. The $\lambda_{ci}$ criterion
involves two privileged directions: the real eigenvector $\Lambda_c$
of $\nabla u$, which indicates the compressing or stretching direc-
tion of the local flow, and the direction perpendicular to the
complex eigenvectors $\Lambda_{cit} \pm i\Lambda_{ci}$, which refers to the swirl-
plane-normal direction (i.e., $\Lambda_{cit} \times \Lambda_{ci}$) [13]. These indicating
vectors for vortex axis have also been employed in the investi-
gations of wall-bounded turbulence. Pirozzoli et al. [16]
compared $\Lambda_t$, $\Lambda_{ct} \times \Lambda_{ci}$, and $\Lambda_p$ inside the identified vortices,
and found they are visually equivalent for recognized vor-
tices except for the region very close to the wall. Gao et al.
[17] employed $\Lambda_t$ to identify the vortex axis and refined the
vortex strength, density and orientation based on planar partic-
le image velocimetry (PIV) data and DNS data. Wang et al.
[18] extended the work of Gao et al. [17] for higher Reynolds
numbers based on the volumetric PIV data and analyzed the
role of $\Lambda_t$ in controlling the evolution of the vorticity.

Recently, Liu et al. [19] and Tian et al. [20] proposed a
new identification criterion involving both the vortex strength
and the vortex axis, which triggered widespread interest in
the physical interpretation and application of the new method
[21-24]. Basically, the new method considers the rotation
state of the local material line segment and recognizes a
direction without rotation as the vortex axis. The vortex
strength is defined as twice the minimum angular speed in the
plane normal to the vortex axis, which was named “Rortex”
($R$) by Liu et al. [19]. Coincidentally, the definition of Rortex
is consistent with several prevenient research works. While
the magnitude of Rortex is equivalent to the residual vorticity
in the triple decomposition of Kolář [9] for two-dimensional
(2D) situations [25], the orientation of Rortex was demon-
strated to be $\Lambda_p$ [24]. Tian et al. [26] revisited the definition
of Rortex by addressing the physical meaning of local fluid
rotation.

Although these identification criteria share the same pur-
pose of extracting vortex cores, quantitatively comparing and
ranking their effectiveness is impossible unless the specific
applying situation is confined. While Jeong and Hussain
[14] tried to show the superiority of the $\lambda_2$ criterion, Cuci-
tore et al. [15] argued that any of the well-known criteria
has counter examples where unsatisfying identification re-
sults are obtained. Chakraborty et al. [8] demonstrated that
these identification criteria (not including the newly proposed
Rortex) are compatible with one another for typical turbulent
flows once using the equivalent thresholds. Chen et al. [27]
compared various criteria for planar velocity fields in wall-
bounded turbulence, and investigated equivalent thresholds
in order to facilitate quantitative comparison of the results
from different criteria. Gao and Liu [24] showed the advan-
tage of Rortex in removing the influence of addictive shear
flows. Zhan et al. [28] compared the $Q$ criterion and Rortex
in the application of an in-stream structure and concluded
Rortex is more suitable in their analysis. Kolář and Šístek
[25] investigated the stretching response of Rortex and other
criteria and found that both Rortex and $\lambda_3$ are stretching-
insensitive schemes, and they allow an arbitrary axial strain.

The current work focuses on the application of these vortex
identification criteria in the investigations of wall-bounded
turbulence. In wall turbulence community, vortices were usu-
ally viewed as the building blocks of turbulence. In the theo-
retical work of Perry and Chong [29], A-shape or II-shape
vortex tubes were regarded as the representative candidates
of attached eddies, and they formulated the profiles for vari-
ous statistics based on the randomly piling up of these vor-
tex tubes. Adrian [30] highlighted the roles of hairpin vor-
tex structures observed by particle image velocimetry and re-
fined them as a useful paradigm to explain many flow fea-
tures. The shared spirit of these investigations is to repre-
sent complex turbulent flows by simplified vortex structures,
which is the so-called vortex-presentation principle. Bas-
ically, the practice of this principle imposes two requiremen-
t on the vortex identification criteria: first, the recognized vor-
tices should be simple and easy for description, based on the
modelling considerations; second, the original turbulent
flows can be explained as the induced motions of vortices,
in the sense that the velocity fields can be effectively recon-
structed based on the vortices.

In the current work, the identification criteria will be quan-
titatively evaluated based on the requirements of the vortex
representation. Instead of separately assessing the criteria for
vortex strength (VS, including $J$, $Q$, $\lambda_2$, $\lambda_{ci}$, $R$) and the ones
for the vortex axis (VA, including $\Lambda_t$, $\Lambda_{ct} \times \Lambda_{ci}$, $\Lambda_p$), the cri-
teria for VS and VA are combined into $5 \times 3$ different types
of vortex vectors before the quantitative assessment works. It
is expected that evaluating the vortex vectors might be more
convenient than separately evaluating the VS and VA criteria.

Firstly, the alignment of the vortex iso-surface and vortex axis is quantitatively evaluated. For an ideal vortex tube, the vortex axis should be tangent to the local iso-surface and align with the centerline direction of the tube. The alignment benefits the simplifying and modelling works for vortices. Zhou [31] demonstrated the alignment of Λ, and the iso-surface of λ by tracking Λ vectors inside a hairpin vortex identified by λc. Liu et al. [19] found that Λ vector is tangent to the iso-surface of R and they defined the vortex vector as the combination of R and Λ. To judge different versions of the vortex vectors, quantitative indicators reflecting the alignment degree should be proposed.

Secondly, the vortex fields expressed by the identification criteria will be employed to reconstruct the original velocity field. From the perspective of vortex representation, the original velocity field should be reconstructed with high accuracy. Thus, the correlation coefficients between the original DNS velocity fields and reconstructed ones are used as another evaluating indicator. In this work, the vortex-to-velocity (V2V) reconstruction will be implemented by the Biot-Savart law [32].

The remaining parts of this work are arranged as follows. In Sect. 2, the DNS data employed in this work and a collection of the definitions for vortex identification criteria are introduced. Particularly, the threshold equivalence is focused on in order to facilitate the following comparing and assessment works. Subsequently, in Sect. 3, the alignment of the identified vortex iso-surface and vortex axis is evaluated. In Sect. 4, the Biot-Savart law is employed to reconstruct the velocity fields based on the vortex fields, and the corresponding reconstruction accuracy is compared. Lastly, the conclusion is provided in Sect. 5.

2. DNS data and the vortex identification criteria

2.1 DNS data

The data employed in this investigation come from an open-access direct numerical simulation (DNS) database for incompressible turbulent boundary layers (Fluid Dynamics Group of Universidad Politecnica de Madrid, available at https://torroja.dmt.upm.es/). Details for the DNS code and the validation works can be found in the papers of Borrell et al. [33] and Sillero et al. [34]. The DNS data have been used in several research works, including Marusic et al. [35], Wang et al. [36], Wang et al. [37], and Wang et al. [18]. The whole DNS data correspond to a spatially evolving turbulent boundary layer, containing 15361, 4096 and 535 collocation points for the streamwise, spanwise and wall-normal directions, respectively. In this work, only the data segment for Re ≈ 1200 is truncated from the whole DNS data. The truncated DNS segment has a dimension of 8δ × 13.4δ × 0.3δ (δ is the averaged boundary thickness), which corresponds to 1412 × 4096 × 96 calculation nodes. The wall-normal range considered is 0-0.3δ, covering the whole buffer layer, the whole logarithmic layer, and part of the wake layer. The separation between adjacent collocation points along the wall-normal direction is not uniform, which is determined by the resolution of the spatial discretization scheme and the local Kolmogorov scale [33]. The streamwise spacing and the spanwise spacing are 6.80 and 3.93 wall-units, respectively.

In the following discussion, let the three coordinate axes of x, y, z be aligned with the streamwise, spanwise and wall-normal direction. u, v, w stand for the three fluctuating velocity components, respectively. A superscript of "+" indicates the quantity is normalized based on the local wall unit or the friction velocity. The three-component fluctuating velocity field and its gradient tensor are denoted as u and ∇u, respectively.

Distinguishing between pure shearing motions and the actual swirling motion of a vortex is an essential requirement for the vortex-identification criteria. That means these criteria should be robust to the mean shear of wall turbulence. However, Pirozzoli et al. [16] found that these criteria for vortex axis (such as Λi and Λp) sometimes fail in the presence of intense mean shear. In order to alleviate such deficiency, they determined the geometrical properties of vortices by artificially subtracting out the mean shear, as was also done by Robinson [38]. In this work, we followed them and calculated all the criteria based on the fluctuation velocity fields. It should be pointed out that the current work is mainly limited to the geometry properties of vortices. For the dynamic aspects, the vortices could not be decoupled from the background shear layer. In fact, the mean shear layer plays an important role in governing the generation and evolution of vortices. Therefore, the original velocity fields should be considered when analyzing the dynamic aspects of vortices in wall turbulence.

2.2 Identification criteria for vortex strength and equivalent thresholds

To make it convenient for the following comparison, it is necessary to collect the existing vortex identification criteria and discuss their relations. This subsection will focus on the identification criteria regarding the vortex strength (namely the VS criteria) and discuss how to choose the equivalent threshold.

Definitions of the five identification criteria regarding the vortex strength are collected into Table 1. These definitions
could be found in the review article of Chakraborty et al. [8] (for the top four criteria) and Wang et al. [39] (for the $R$ criterion). It is seen that all the criteria are directly related to the matrix of $\mathbf{V}\mathbf{u}$. Physical interpretations for these criteria were also introduced in the literatures mentioned above, which would not be detailed herein. Instead, we simply discuss their inherent relation based on the correlated formulas. As listed in the fourth column of this table, $\Delta$ and $Q$ can be determined by the complex eigenvalue ($\lambda_{ci}$) of $\mathbf{V}\mathbf{u}$ [8]. A similar relation for $\lambda_2$ is not available unless $\mathbf{V}\mathbf{u}$ is a normal tensor. The explicit expression for $R$ is provided by Wang et al. [39], which relates $R$ to $\lambda_{ci}$ and the vorticity component along $\lambda_r$ (i.e., $\omega_k$).

The effective ranges for these criteria are shown in the third column of the table, which corresponds to the lowest requirements for identifying vortices. As indicated by the formulas in the fourth columns, the effective ranges for different criteria are not equivalent. While $Q > 0$ yields $\lambda_{ci} > 0$ and $|\lambda_{ci}|/\lambda_{ci} < \sqrt{3}/3$, $\lambda_2 < 0$ demands $\lambda_{ci} > 0$ and $|\lambda_{ci}|/\lambda_{ci} < 1$. The ratio $\lambda_{cr}/\lambda_{ci}$ is the inverse spiraling compactness, which measures the local orbital compactness in a vortex [8]. The effective ranges for the other three criteria (i.e., $\Delta > 0$, $\lambda_{ci} > 0$ and $R > 0$) are mathematically equivalent. They do not involve the requirement of spiraling compactness and thus would isolate more regions for the recognized vortices.

Of particular note is that these criteria have different dimensions. Specifically, $\Delta$ has the dimension of $\nu$ (denotes the vorticity magnitude); $Q$ or $\lambda_2$ has the dimension of $\omega^2$; $\lambda_{ci}$ or $R$ has the dimension of $\omega$. In order to make these criteria comparable and also to facilitate the discussion regarding equivalent thresholds, we need to regularize these criteria. The regularization process only considers the effective ranges of the identification criteria (see the second column of Table 1). The values outside this range are directly set as zeros. We wiped out the values of the criteria in non-effective regions (such as $Q < 0$) in order to make these criteria more suitable for the definition of vortex strength. For the regions not recognized as vortices, the vortex strength should be defined as zeros. However, $\Delta$, $Q$ and $\lambda_2$ return non-zero values for the regions which are not recognized as vortices. For the other two criteria, $\lambda_{ci}$ and $R$, the values outside the vortex regions are all zeros by their definition. Thus no wiping process is necessary for these two criteria. In the regularization process, the criteria are first processed by certain power operations and then normalized by the corresponding root mean squares (RMSs) at a fixed wall-normal position. The latter is a popular processing technique used to relieve the influence of wall-normal variation of the vortex strength [40]. Explicit
formulas for the regularization process are shown in the last column of Table 1. As a convention, a symbol with a hat implies that it is the non-dimensional form obtained by the regularization process.

To show the effectiveness of the regularization process, the correlation degrees among these criteria before and after the process were analyzed by using the overall correlation coefficients. For example, the overall correlation coefficient between \( \hat{A} \) and \( Q \) is defined as:

\[
c(\hat{A}, Q) = \frac{\int_{\Omega} \hat{A} Q d\Omega}{\sqrt{\int_{\Omega} \hat{A}^2 d\Omega \int_{\Omega} Q^2 d\Omega}}.
\]

where \( \Omega \) represents the spatial domain of the considered DNS segment. The statistically averaged correlation coefficients between any two criteria from Table 1 are displayed in Fig. 1 by a matrix of three-dimensional bars. It shows that the regularization process significantly increases the correlation coefficients of these criteria. Notably, the correlation coefficients between \( \hat{A} \) and the other criteria are improved by 0.37-0.74. Another criterion deserving more attention after the regularization process is \( \lambda_{ci} \) because the correlation coefficients between \( \lambda_{ci} \) and all the other criteria are larger than 0.9. The peculiarity of \( \lambda_{ci} \) has also been noticed in the fourth column of Table 1, where all the other criteria are linked to \( \lambda_{ci} \) in the explicit formulas. These results validate that these five criteria could be highly correlated to each other after the regularization process, which facilitates the following works regarding how to determine the equivalent thresholds.

Choosing the threshold is always the most challenging part in the application of the identification criteria. Usually, investigators chose different volume fractions for different research purposes. In the research work of del Álamo et al. [40], the fraction of volume occupied by the considered vortexes in the channels are 2%-3%, while Tanahashi et al. [41] counted in all the vortexes recognized by \( Q > 0 \), occupying a volume fraction of 39% in isotropic turbulence. For the visualization purpose, a suitable threshold should lead to well-recognized vortex structures without tangling too much to influence the isolation of individual structures. Jiménez [42] suggested that the threshold could be determined based on the “percolation” transition of isolated structures [43], which typically takes place for an isolated volume fraction of 5%-10%. In this work, a volume fraction \( (Vf) \) of 6% for recognized vortex structures is prescribed. Considering the DNS data have non-uniform grid spacings in the wall-normal direction, the volume fractions are calculated based on the following steps. First, in the wall-parallel plane, the DNS nodes satisfying the condition (such as \( Q > \) threshold) are counted. The resulting number is divided by the total node number of the whole plane, which gives the in-plane volume fraction of vortices at this height (denoted as \( V_{f, \text{in-plane}}(z) \)). Second, the integral of \( V_{f, \text{in-plane}}(z) \) from 0 to 0.3δ were calculated. The integral number was further normalized by 0.3δ to obtain the averaged volume fraction in the 3D domain considered. Besides the standard case of \( V_f = 6\% \), the influence of variable threshold would also be considered in the following discussions.

The resulting thresholds for these criteria are collected into Table 2. As we can see, these thresholds take comparable values, which is owing to the preceding regularization process. A quantitative comparison for the five criteria is given in Fig. 2, where the in-plane volume factions for identified vortexes are plotted against the wall-normal positions. It shows that with the increase of wall-normal positions, the volume faction first dramatically increases to the maximum peak and then slowly reduces to a flat plateau of about 6%, which is equivalent to the prescribed value of the volume fraction. According to the Refs. [44-46], the number density of vortexes reaches a peak at \( z^+ = 30-50 \), which is roughly consistent

![Figure 1](image-url)
with Fig. 2. All the criteria give approximately equal volume fractions for any given wall-normal positions, and the consistency is notably better for the logarithmic region.

Figure 3 shows the vortex structures identified by the five criteria using the thresholds listed in Table 2. It shows that all the results present consistent tube-like vortex structures tangling with one another, which is a typical pattern for turbulent flows. The typical quasi-streamwise vortices and distorted hairpin-like vortices populate in the near-wall region and the logarithmic region (i.e., $z^+ > 80$), respectively. Comparing the results from different criteria, we can recognize that the thickness and length of the same tubular vortex from different subplots are visually equivalent. To quantitatively compare the vortex structures recognized by various criteria, we calculated the overlap volume ratio following the work of Chakraborty et al. [8]. Specifically, the overlap volume of the vortexes recognized by $\lambda_{ci}$ and the other criteria was divided by the volume of the vortexes recognized by $\lambda_{ci}$. Herein, the vortexes recognized by $\lambda_{ci}$ are regarded as the references because $\lambda_{ci}$ is highly correlated with all the other criteria, as shown in Fig. 1. The results of overlap volume ratios are shown in Fig. 3f. It shows that the least overlap volume ratio occurs for $R$ and $\lambda_{ci}$, which is 78.2%. Besides, all the other overlap volume ratios are larger than 83%. It indicates that these five criteria are reasonably consistent by using the current threshold. The results of Figs. 2 and 3 validate the equivalence of the thresholds for different criteria in the sense of isolating consistent structures with equal volume fractions. This is a prerequisite for fairly comparing these results in the following discussions.

| Table 2 | The thresholds of various criteria (the non-dimensional form) |
|---------|-------------------------------------------------------------|
| Threshold | $\Delta$ | $Q$ | $\lambda_2$ | $\lambda_{ci}$ | $R$ |
| 2.63 | 2.45 | 2.44 | 2.55 | 2.41 |

2.3 Identification criteria for the vortex axis

In this subsection, we will first illustrate the physical implications of the three VA criteria from a new perspective. And then we will discuss the possible definitions for the vortex vector by combining the VS and VA criteria, which lays the foundation for the following comparing and evaluating works.

As introduced in Sect. 1, three vectors associated with the $\lambda_2$ or $\lambda_{ci}$ criterion are candidate indicators of VA, which are the real eigenvector of $\nabla u (A_c)$, the normal vector of the plane spanned by the real part and imaginary part of the complex eigenvector ($A_c \times A_{ci}$), and the eigenvector corresponding to the minimum eigenvalue of $\nabla \nabla p (A_p)$. Unlike the VS criteria, the relationships between these VA criteria have not been sufficiently discussed in the published literatures. Herein, we will illustrate the physical meaning and the relationships of these criteria in an intuitive way.

Consider the relative velocity and pressure regarding one reference point, which is recognized as vortex by both $\lambda_{ci}$ and $\lambda_2$. $\nabla u$ contains all the information of the relative velocity. Specifically, for any relative position vector $\delta r$, the corresponding relative velocity is $\delta u = \nabla u \cdot \delta r$ based on the definition of $\delta u$. The relative pressure $\delta p$ could be determined by a similar formula but with a higher-order truncation, which yields $\delta p = \nabla p \cdot \delta r + \delta r \cdot \nabla \nabla p \cdot \delta r/2$. The term $\nabla p \cdot \delta r$ corresponds to a linear distribution for pressure in space. For the vortex core, $\nabla p$ is supposed to be along the vortex axis direction, which indicates that it has a weak influence on the pressure distribution in the plane perpendicular to the vortex axis. Thus the term $\nabla p \cdot \delta r$ is neglected, which leads to $\delta p = \delta r \cdot \nabla \nabla p \cdot \delta r/2$. According to Jeong and Hussain [14], $\nabla \nabla p = -\rho (\Omega^2 + S^2)$ ($\rho$ is the density of the fluid) once the unsteady term and viscous terms are neglected. Thus, we can see both the relative velocity and pressure could be determined by $\nabla u$.

Now take an example case of $\nabla u$:

$$\nabla u = \begin{pmatrix} 1 & -3 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & -2 \end{pmatrix}$$

(2)

where the trace of $\nabla u$ equals zero indicating the situation of incompressible flow. The zero elements in the above matrix do not delimit the scope of the following discussion, considering any $3 \times 3$ matrix could be transformed into this form by only a coordinate rotation [20].

Figure 4 shows the relative velocity and pressure on the surface of an infinitely small sphere centered at the reference point, with the sphere radius normalized as a unit. Note that only the tangential velocity components are displayed.
as vectors for clarity. The sphere surface shows two polar centers with zero tangential velocity which are located at the intersection points of the sphere surface and the third coordinate axis (z). The polar centers indicate the non-rotation axis (Λr) according to Tian et al. [26]. In the side view (Fig. 4c), the velocity vectors represent a converging trend at the place of the black line. This converging line forms a close circle on the sphere, which marks the swirling plane with zero off-plane velocities. Mathematically, the corresponding normal direction of the swirling plane could be indicated by Λcr × Λci. The distribution of relative pressure shows two minimum points on the sphere surface at the neighboring regions of the polar centers. The minimum points correspond to the eigendirection of \( \nabla p \) associated with the smallest eigenvalue, which is the definition of Λp. Lastly, it should be pointed out that all of the three criteria correspond to two possible directions by adjusting the signs in their definitions. Following Gao et al. [17] and Tian et al. [20], only the direction with a smaller deviation angle with the local vorticity is used as the vortex axis direction. This convention is adopted in all the remaining parts of this article.

It seems that each one of the three criteria recognizes a privileged direction from a specific perspective, and they all present some sense of rationality as the criteria of the vortex axis. Theoretically, the three recognized axes do not collapse unless \( \nabla u \) is a normal tensor, i.e., \( \partial w/\partial x = \partial w/\partial y = 0 \) in this example. Liu et al. [19] and Tian et al. [20] emphasized the physical interpretation of Λr, and they defined the vortex vector by combining the direction of Λr and the strength of R. In a loose sense, the vortex vector could be defined as any other combination of the foregoing criteria for the vortex strength and the vortex axis. This consideration necessitates revisiting the definition of the vortex vector by trialling the other possible combinations. A collection of all possible combinations of the criteria for the vortex strength and the vortex axis is displayed in Table 3. Notably, the definition of Liu et al. [19] is equivalent to V1-5 in this table. A comprehensive comparison of these variants of vortex vectors will help to
settle down the controversy on the most reasonable definition, which is meaningful for the foundation and application of the vortex identification criteria. The comparing and evaluating works will be introduced in Sects. 3 and 4.

3. The alignment of the vortex iso-surface and vortex axis

For typical tube-like vortices, the vortex iso-surface and the vortex axis usually presents good alignment, which means that the vortex axis is tangential to the skeletons of the vortex tubes. The alignment property provides a possible route to simplify the complex vortex structures. For example, the vortices could be characterized by the 3D curves of skeletons, varying radii of vortex cores and a Gaussian distribution of vortex vectors tangential to the vortex skeletons, which is in line with the vortex-skeleton presentation of Perry and Chong [29]. In the previous investigations, the alignment property was demonstrated mostly by intuitive observations on the instantaneous vortex structures, such as the work of Gao and Liu [24]. In contrast, we will quantitatively evaluate the alignment property of various versions of vortex vector in Table 3.

As suggested by Wang et al. [32], the alignment property could be described by using the differential geometry theory. To introduce the method, we show the geometry of a typical vortex tube in Fig. 5 and focus on a small surface element on the tube as marked by the red colour. The normal unit vector of $S$ pointing to the outside is denoted as $\mathbf{n}$. According to the theory of differential geometry, two orthogonal directions can be determined on $S$, which correspond to the directions with the largest and the smallest curvatures, denoted as $\kappa_1$ and $\kappa_2$, respectively. For a typical tube-like geometry, such as for a cylinder-shape one, it can be inferred that the two principal directions should be along the axis direction and along the azimuthal direction of the cross-section, as shown by $l_1$ and $l_2$ in the plot. Since $l_1$ bends towards the negative direction of $\mathbf{n}$, the corresponding curvature ($C_2$) is negative. The curve $l_2$ bends harder than the curve $l_1$, which means $|C_2| > |C_1|$, or $C_2 < C_1$ if the sign issue is considered. Thus, the vortex axis direction should be along the principal direction with the largest curvature ($C_1$), which is named as the first principle direction ($\kappa_1$) in this work.

As an example, Fig. 6 shows the iso-surfaces of $\lambda_2$ for the prescribed threshold. $\mathbf{A}_1$ and $\mathbf{A}_2$ are also displayed as red arrows and line segments, respectively. Firstly, we can see that the vectors of $\mathbf{A}_1$ are tangential to the iso-surface, namely $\mathbf{A}_1 \cdot \mathbf{n} \approx 0$. This property is reminiscent of the vorticity-surface field (VSF) suggested by Wang et al. [32] when visualizing vortex structures. VSF is defined as a scalar field with the gradient direction perpendicular to the local vorticity. By this definition, $\lambda_2$ could be approximately regarded as the “vortex-surface field” once $\mathbf{A}_1$ is viewed as the local vortex orientation. The more interesting aspect is that the directions indicated by $\mathbf{A}_1$ and $\kappa_1$ agree well for most cases, which is consistent with the foregoing analysis regarding the ideal vortex tubes. Particularly, longer and thinner vortex tubes represent better alignment for the two directions while the vortex blobs and the ends of vortex tubes see comparatively worse alignment.

To quantitatively describe the alignment of the vortex axis and the vortex iso-surface, the pointwise correlation coefficients between $\mathbf{A}_1$ and $\kappa_1$ are calculated based on

$$\rho(\mathbf{A}_1, \kappa_1) = \frac{\mathbf{A}_1 \cdot \kappa_1}{|\mathbf{A}_1||\kappa_1|},$$

(3)

Table 3 A collection of various definitions for vortex vector

| Vortex axis ($\mathbf{A}$) | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ |
|--------------------------|------------|------------|------------|------------|------------|
| $\mathbf{A}_1$          | V1-1       | V1-2       | V1-3       | V1-4       | V1-5       |
| $\mathbf{A}_2 \times \mathbf{A}_3$ | V2-1 | V2-2 | V2-3 | V2-4 | V2-5 |
| $\mathbf{A}_4$          | V3-1       | V3-2       | V3-3       | V3-4       | V3-5       |

Figure 5 A sketch for one vortex tube and the principal directions of local curvatures.

Figure 6 An example vortex field showing the alignment of $\mathbf{A}_1$ (shown by red vectors) and the first principal direction of the local curvatures $\kappa_1$ (displayed by black lines). The vortex iso-surfaces identified by $\lambda_2$ are displayed as references with contours showing the local wall-normal positions.
respectively. Considering that both \( \mathbf{n} \) and \( \mathbf{k}_1 \) might change directions by adjusting signs in the definition, only the magnitude for \( \rho(\Lambda_c, \mathbf{k}_1) \) is considered, i.e., \( |\rho(\Lambda_c, \mathbf{k}_1)| \). A good alignment of the vortex axis and the vortex iso-surface promises a \( |\rho(\Lambda_c, \mathbf{k}_1)| \) close to one.

Generally, for each point of the DNS data, there exists an vortex iso-surface crossing this point. Thus, the curvatures of local iso-surfaces can be calculated on every grid points of the DNS data. The calculation for principal directions of local curvatures is not detailed in this article. The reader can also refer to the article of Wang et al. [32], who provided a detailed introduction in the Appendix. All the grid points recognized as vortices by the prescribed threshold (see Table 2) are counted in a statistical average of \( |\rho(\Lambda_c, \mathbf{k}_1)| \). The results are denoted as \( \langle |\rho(\Lambda_c, \mathbf{k}_1)| \rangle \), with a bracket indicating the statistically averaging operation. Similarly, the calculation and statistical process could be performed on other definitions of the vortex axis \( (\Lambda_{ci} \times \Lambda_{ci} \text{ and } \Lambda_p) \), and the details are not repeated herein. To make it convenient for the following discussion, the results for these correlation coefficients are collectively denoted as \( \langle |\rho(\Lambda, \mathbf{k}_1)| \rangle \).

Figure 7 displays an ensemble of the average correlation coefficients based on the various definitions for the vortex vector (see Table 3), varying with the wall-normal position. The \( \Delta \) criterion perform the worst in the alignment of the vortex iso-surface and axis, with comparatively larger \( \langle |\rho(\Lambda, \mathbf{n})| \rangle \), and smaller \( \langle |\rho(\Lambda, \mathbf{k}_1)| \rangle \). The results of \( Q, \lambda_2, \lambda_{ci} \) are very close for all the cases. The newly-proposed criterion \( R \) does not perform very well in this examination. Besides comparing the three criteria regarding VA, a horizontal comparison shows that \( \Lambda_c \) criterion leads to the best alignment, whose superiority is more evident for the near-wall region.

In this region, \( \Lambda_c \) criterion brings a maximum peak for the correlation coefficients between VA and \( \mathbf{k}_1 \), while the results of \( \Lambda_{ci} \times \Lambda_{ci} \) and \( \Lambda_p \) maintain at a low level. In fact, most of the vortices in the buffer layer are quasi-streamwise vortices, which take simple tube-like shapes compared to the irregular ones in the logarithmic region. The bad performance of \( \Lambda_{ci} \times \Lambda_{ci} \) and \( \Lambda_p \) in this region might be caused by the intense shear layer embedded in this region. It implies that the orientations indicated by the two criteria are more susceptible to the influence of shear flow.

Figure 7 shows a low correlation coefficient between \( \mathbf{k}_1 \) and different vortex axis results at \( z^+ < 20 \), which could be explained by two reasons. First, in the region very close to the wall, the shear layers surrounding the low-speed streaks are very strong, which might cause the failure of the identification criteria for vortex axis [16]. Second, the foregoing alignment property is established for the flank surface of the vortex tubes. The end parts of the vortex tubes are very complex regarding the local curvatures, which would make the alignment property invalid. In wall turbulence, large numbers of vortex tubes are rooted in the near-wall region and then extend outward at certain inclined angles. Therefore, the near-wall region contains a larger proportion of the root (end) parts of these vortex tubes, which also results in the misalignment trend shown in Fig. 7.

To analyze the influence of the threshold, the thresholds for these criteria were adjusted, which makes the volume fractions of recognized vortices vary from 1% to 30%. The corre-

![Figure 7](image-url)
sponding results of \( \langle \rho(\mathbf{A}, \mathbf{k}_1) \rangle \) are derived and shown as functions of \( V_f \) in Fig. 8. It shows that \( \langle \rho(\mathbf{A}, \mathbf{k}_1) \rangle \) always drops with the increase of \( V_f \) of the considered vortices. This is expectable because stronger vortices occupying smaller volume usually present better tube-like features, which correspond to better alignment performance. \( \langle \rho(\mathbf{A}, \mathbf{k}_1) \rangle \) maintains above 0.75 for the best vortex definition (V1-4). Throughout the threshold range considered, the rank of the performances of these criteria remains unchanged. Specifically, for the five strength criteria, \( Q, \lambda_2, \lambda_3 \) outperform \( R \) and \( \Lambda \); and among the three VA criteria, \( \Lambda_1 \) performs the best.

4. The accuracy of the vortex-to-velocity reconstruction

From the perspective of the effective vortex representation, the vortex fields should retain the essential kinematic information of the complex flows. As the quotation cited in the first paragraph of this article, vortices can be viewed as the “gene code” of turbulent flows. Thus, it is reasonable to anticipate that the velocity fields can be reconstructed based on the vortex fields with high accuracy. The vortex-to-velocity reconstruction is much like a decoding process, which infers the turbulent features based on the vortex field. It should be pointed out that this vortex-based velocity reconstruction is essentially different from the vorticity-based velocity reconstruction since vortex is not equal to vorticity by the definitions shown in Table 3. More discussions about the motivations and applications of the V2V reconstruction are reported by Wang et al. [32]. In the present work, the V2V reconstruction provides an essential tool to examine and evaluate the effectiveness of these vortex definitions.

The Biot-Savart law provides a useful tool to reconstruct velocity field based on the corresponding vorticity field. As reported by literatures, the Biot-Savart law was also employed for the purpose of the vortex-to-velocity reconstruction. For example, Pirozzoli et al. [47] employed the Biot-Savart law to reconstruct velocity fields based on the vortex-tube fields defined by \( 2 \lambda_i \). In the attached eddy model, the Biot-Savart law was used to recover the velocity fields induced by an arrangement of vortex tubes. Recently, Wang et al. [32] proposed a data-driven method to reconstruct the velocity field based on the vortex field, which was named as field-based stochastic estimation (FLSE). FLSE performs better than the Biot-Savart law in the near-wall region. However, the implementation of FLSE is expensive, and it was also demonstrated that the two methods perform equally well in the logarithmic region. Therefore, we would adopt the Biot-Savart law to reconstruct the velocity fields from the vortex fields defined in Table 3.

The Biot-Savart law regarding the vortex-to-velocity reconstruction takes the following form [32]:

\[
\mathbf{u}(r) = \frac{\mu}{4\pi} \iiint_{V} \frac{\mathbf{A}(r') \times (r - r')}{|r - r'|^3} \, d\Omega,
\]

where \( r' \) is the running position vector \((x', y', z')\) for the volume integration. \( \mathbf{A}(r') \) is the vortex field, which could be defined by any one in Table 3. \( \mathbf{u}(r) \) is the reconstructed velocity field with \( r = (x, y, z) \). \( \mu \) is an artificial multiplier added to
make the resulting velocity field comparable to the real velocity fields in magnitude. In this work, the parameter was determined by minimizing the reconstruction error [32].

To facilitate the implementation, Eq. (4) could be reformed as:

\[ u_i(r) = \sum_{j} \Omega_{ij} \Lambda_j(r')d\Omega = P_{ij} \Lambda_j, \quad (5) \]

where both \( i \) and \( j \) are indices varying from one to three. The repeated index \( j \) indicates a summation for all the realizations of \( j = 1, 2, 3 \). * stands for the 3D convolutional operation and

\[ P_{ij} = \frac{\mu}{4\pi(x^2 + y^2 + z^2)^2 + \epsilon} \begin{pmatrix} 0 & -y & -z \\ -z & 0 & x \\ y & -x & 0 \end{pmatrix}. \quad (6) \]

Herein, \( \epsilon = 10^{-7} \), which is added to avoid the singularity at the origin. \( P_{ij} \) is discretized based on the same grid spacings as the original DNS data. It was believed that the inducing effect of vortices could be neglected at a place far away from the origin. Thus, the streamwise and spanwise ranges for \( P_{ij} \) were truncated as \( \pm 0.5\delta \) and \( \pm 0.25\delta \). This aspect ratio gains support from the statistical results for attached structures [40]. Figure 9 provides the contour map of the reconstructed velocity field based on the vortex field defined by V1-4 in Table 3, compared with the original velocity field. It shows that the reconstructed velocity field is remarkably consistent with the original one. The results validate the implementation scheme of the Biot-Savart law, which consolidates the foundations of the following work.

To quantitatively describe the reconstruction accuracy, the overall correlation coefficient between the original velocity field and the reconstructed one is defined as:

\[ c(u_{DNS}, u_{rec}) = \sqrt{(c(u_{DNS}, u_{rec})^2 + c(v_{DNS}, v_{rec})^2 + c(w_{DNS}, w_{rec})^2)/3}, \quad (7) \]

where \( c(u_{DNS}, u_{rec}), c(v_{DNS}, v_{rec}), c(w_{DNS}, w_{rec}) \) stand for the overall correlation coefficients for three velocity components field as defined by the following equation:

\[ c(u_{DNS}, u_{rec}) = \frac{\int_{\Omega} u_{DNS} u_{rec} d\Omega}{\sqrt{\int_{\Omega} u_{DNS}^2 d\Omega \int_{\Omega} u_{rec}^2 d\Omega}}. \quad (8) \]

Figure 10 shows the averaged correlation coefficient \( (c(u_{DNS}, u_{rec})) \) between the original velocity fields and the reconstructed ones based on different versions of vortex vector fields (as listed in Table 3), as a function of the wall-normal position. For each type of vortex vector, two reconstruction cases are considered. In the first case, the vortex fields defined by Table 3 directly participate in the V2V reconstruction, which means that the contributions of all the non-zero data points of the criteria are counted in. In the second case, only intense vortices whose strength is larger than the threshold (see Table 2) are considered in the reconstruction, which involves a threshold-filtering process.

The corresponding results for the two cases are shown in the first and second rows of Fig. 10, respectively, which is revealing in several aspects. Firstly, in each subplot, it shows that the correlation coefficients dramatically increase with \( z^+ \) before reaching the plateau in the logarithmic region, and then decrease with \( z^+ \). Two factors could explain the variation trend of the correlation coefficients. On the one hand, the Biot-Savart law has weak performance at the near-wall region due to the viscosity effect [32]. On the other hand, the considered DNS data segment was limited below \( z = 0.3\delta \), which indicates that the vortices beyond \( z = 0.3\delta \) are not counted in the reconstruction. Therefore, the correlation coefficients become smaller nearby \( z = 0.3\delta \).

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Figure 9  a-c Original DNS velocity fields and d-f the reconstructed velocity fields based on the vortex vector field defined by V1-4. The section is extracted from \( z^+ = 119.4 \), and the three columns correspond to \( u^+ \), \( v^+ \) and \( w^+ \), respectively.
Secondly, for the non-threshold case (the first row in Fig. 10), the \( \Delta \) criterion leads to the highest reconstruction accuracy, closely followed by the \( \lambda_{ci} \) criterion. The performances of the other three criteria lag far behind, compared to the former two criteria. The superiority of the \( \Delta \) criterion and \( \lambda_{ci} \) criterion is attributed as the comparatively looser requirement in recognizing vortices, which can be indicated by the formulas in the fourth column of Table 1. In other words, the \( Q \) and \( \lambda_2 \) criteria only recognized part of the vortices which have been recognized by the \( \Delta \) or \( \lambda_{ci} \) criterion. The \( R \) criterion is an exception for this explanation since \( R > 0 \) are equivalent to \( \Delta > 0 \) or \( \lambda_{ci} > 0 \). By definition, the \( R \) criterion is determined by the minimum angular speed of material lines in the vortex-axis-normal plane. By neglecting the influence of the maximum angular speed, the \( R \) criterion has a good performance in removing the influence of the shear flow [24]. However, the local maximum angular speed is very important information for inferring the flow field, and missing the information causes larger biases in the reconstructed results.

As for the case for the threshold-filtering reconstruction (the second row in Fig. 10), the results are somewhat different. The reconstruction accuracy is obviously lower for this case, which is expected since the threshold filtering keeps only 6% of the total volume with non-zero values. In this case, the \( \Delta \) criterion performs the worst while \( Q \) and \( \lambda_2 \) criteria achieve the best results. The performance of the \( \lambda_{ci} \) is also satisfying because its results are very close to the best ones. Thus, it seems that the \( \lambda_{ci} \) criterion is a stable choice for the vortex identification with promising performances for both weak and strong vortices. Another advantage of the \( \lambda_{ci} \) criterion is that it has the same dimension as vorticity, which means \( \lambda_{ci} \) could be directly viewed as the vortex strength, without the additional regularization process as introduced in Sect. 2. On the other hand, a horizontal comparison among the results of various VA criteria in Fig. 10 indicates that the \( \Lambda_\text{c} \) and \( \Lambda_\text{p} \) criterion bring approximately equivalent reconstruction accuracy, while the \( \Lambda_\text{c} \times \Lambda_\text{ci} \) criterion causes comparatively poor reconstruction accuracy.

The influence of the threshold on the reconstruction accuracy is illustrated by Fig. 11, where the correlation coefficients are displayed as functions of the volume fractions corresponding to various thresholds. It shows that the correlation coefficients continuously increase with the increase of the volume fractions of vortices counted in the reconstruction. When \( V_T > 0.2 \), the correlation coefficients retain above 0.8 for all the cases. For the \( V_T \) range considered in Fig. 11 (\( V_T < 0.3 \)), the performances of \( \Delta \) criterion and \( R \) criterion are not as good as the other three criteria. A horizontal comparison shows that both \( \Lambda_\text{c} \) and \( \Lambda_\text{p} \) criterion perform better than \( \Lambda_\text{c} \times \Lambda_\text{ci} \) criterion. While the \( \Lambda_\text{c} \) criterion performs the best for smaller \( V_T \), the \( \Lambda_\text{p} \) criterion has a slight advantage at larger \( V_T \), consistent with the results in Fig. 10. These results, together with the results in Sect. 3 validate that the \( \Lambda_\text{c} \) criterion is the best candidate for the definition of the vortex axis among the three criteria considered, which supports the application of \( \Lambda_\text{c} \) in wall-bounded turbulence [17, 18].
5. Conclusions

This work investigated the effectiveness of various vortex identification criteria regarding the vortex representation of wall-bounded turbulence, which involves two essential aspects: the alignment of the vortex axis and the vortex iso-surfaces, and the accuracy for the V2V reconstruction process. Notably, five criteria for the vortex strength including $\Delta$, $Q$, $\lambda_2$, $\lambda_{ci}$, $R$ and three criteria for the vortex axis including $\Lambda_r$, $\Lambda_{ci} \times \Lambda_{ci}$, and $\Lambda_p$ were considered in this investigation.

We first discussed the relationship between these identification criteria. To facilitate the following comparison, we regularized the criteria for the vortex strength to get their non-dimensional forms. The thresholds were prescribed so that the isolated structures took a volume fraction of 6%. It showed that the recognized structures from various criteria share comparatively large overlap volumes and the volume fractions as functions of wall-normal positions collapse well, which was essential for making a fair comparison. The criteria for the vortex axis were interpreted in an intuitive way, which indicated that each of the criteria recognized a privileged direction. The definition of vortex vector was revisited by combining various criteria for the vortex strength and the vortex axis.

The vortex tube with a simple geometry promises a good alignment of the vortex axis and the vortex iso-surface, which facilitates the compressed representation of vortex fields. The alignment degree was quantitatively evaluated by using the pointwise correlation coefficients between the vortex axis and the first principal direction of local curvatures. It was shown that the vortex axis tended to be tangential to the local vortex iso-surface and aligned with the first principal direction of local curvatures. The statistical results showed that while $Q$, $\lambda_2$, $\lambda_{ci}$ are better criteria for the vortex strength, $\Lambda_r$ performed the best in the three criteria for vortex axis regarding the good alignment of the vortex axis and the vortex iso-surface.

Evaluating the performance of the vortex representation necessitates a V2V reconstruction. The V2V reconstruction was implemented by the Biot-Savart law. Promising reconstruction results were found by comparing the reconstructed velocity fields to the original DNS velocity fields. Quantitative comparison on the reconstruction accuracy based on various definitions of vortices validated that the $\lambda_{ci}$ criterion is a good choice for the vortex strength since its performance demonstrated promising results for both the non-threshold case and the threshold case. As for the criteria of the vortex axis, $\Lambda_r$ and $\Lambda_p$ criterion performed equivalently well while the $\Lambda_{ci} \times \Lambda_{ci}$ criterion caused poor accuracy.

To collect all the results, we can conclude that the combination of $\hat{Q}$, $\hat{\lambda}_2$, or $\hat{\lambda}_{ci}$ for the vortex strength and the $\Lambda_r$ criterion for the vortex axis performs the best regarding the vortex representation of wall-bounded turbulence. Particularly, the $\lambda_{ci}$ criterion is recommended for the vortex strength because it has the same dimension with the vorticity, thus needs no regularization process. The evaluating principles employed...
in the current work also provide some clues for the assessment works of vortex identification criteria in the application of other flows.

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旋涡识别准则在壁湍流涡表征方面的有效性比较

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摘要   将复杂、多尺度的流动表示为丝状旋涡结构的诱导作用是经典旋涡表征观点的基本内容。本文主要研究不同旋涡识别准则在壁湍流旋涡表征方面的有效性。本文考察五个关于旋涡强度的识别准则以及三个关于涡轴方向的识别准则。通过任意组合这两类旋涡识别准则定义不同版本的旋涡矢量，然后从两个方面对这些旋涡矢量的有效性进行了评估。首先，涡轴方向与旋涡等值面伸展的方向应具有一致性，这是丝状旋涡结构的特征；其次，旋涡是湍流的“基因密码”，这意味着基于它们重建速度场是有效的。对于第一个方面，采用微分几何方法定量地描述涡轴与旋涡等值面的关联性。对于第二个方面，采用毕奥-萨伐尔定律来实现旋涡到速度场的重建。本文的研究结果对壁湍流旋涡识别和表征方面的工作具有一定的参考意义。