A0 condensate in QCD

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Abstract

A survey devoted to A0-condensate in gauge theories at high temperature is presented. Both the theoretical foundations of the spontaneously generated condensate and known methods of its calculation are discussed. As most important consequence the SU(N) global symmetry breakdown is investigated in details. Influence of A0 on matter fields is studied in different aspects. Some new results concerning this subject are reported as well.

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1 Introduction and Motivation

In the last few years an interest to problems of high-temperature QCD has considerably increased. Among commonly discussed topics on the deconfining phase transition and the chiral symmetry restoration, new possible phenomena - generation of the gauge classical field (the so-called $A_0$-condensate) and spontaneous breaking of the global gauge symmetry caused by $< A_0 >$ - have become of great importance. A number of new essential results obtained in various approaches as well as interesting ideas and hopes connected with the non-zero vacuum value $< A_0 > \neq 0$ impelled us to undertake an attempt to summarize here the most essential achievements and unsolved questions in this area of gauge theories. To be more transparent, we are going to formulate a general point of view both on physical nature and mathematical aspects of these phenomena representing them in different methods of calculations. We would like to discuss as well the most exciting problems and possible ways of their solving. Naturally, we are aware that it is a sufficiently difficult task to present a general survey concerning a particular question in a so rapidly developing area as gauge theories at finite temperature. Moreover, it cannot be excluded that by the moment of appearing our paper in the journal some questions discussed here will be solved. The situation gets complicated by the fact that as far as we know there is no (mathematically) strict proof at the present time that $A_0$-condensate must fall at high temperature. Nevertheless, we believe that calculation of $< A_0 >$ by different methods and, on the other hand, derivation of the most significant consequences of such a condensate on multi-particle systems make our attempt quite justified. Besides, the appearance of $A_0$-condensate and the breakdown of global gauge symmetry can undoubtedly lead to significant improvement of our conception both of the high temperature behaviour of the strongly interacting matter and of the physics of the gauge theories on the whole and surely have a connection to other problems currently under investigation (as, for instance, infrared problem, behaviour of quarks at non-zero baryonic number, etc.). All these questions will be considered in the paper.

Let us begin with a comprehensive consideration of some known facts obtained from the studies of QCD. The Hamiltonian can be formally written as the sum of the chromoelectric and the chromomagnetic terms and has the following form in lattice version of the theory

$$H = \frac{g^2}{2a}E^2 + \frac{1}{2ag^2} UUU^+U^+ = H_E + H_B.$$  \hspace{1cm} (1)

At finite temperature the behaviour of chromoelectric fields has been well studied both in the perturbative (in $g^2$) and especially in the non-perturbative regions. As is generally known, in the strong coupling approximation the main contribution to the partition function results from the chromoelectric part in eq. (1), because the chromomagnetic term being proportional to $g^{-2}$ can be treated perturbatively in $\frac{1}{g}$. At high temperature because of periodic boundary conditions the gauge field configurations called Polyakov loops develop a non-vanishing expectation value, which breaks $Z(N)_{gl}$ symmetry of the initial QCD-action and leads to the deconfinement. As the Polyakov
loops transform non-trivially under $Z(N)_{gl}$ rotations, their non-zero expectation value could mean screening of $Z(N)$-charges (or static quarks) at $T > T^D_c$ (where $T^D_c$ is the critical temperature of the deconfinement phase transition). If this is the case, one may claim that there exists a physical quantity which characterizes the phenomenon of screening. This quantity is called the Debye mass and is defined in the continuum as the zero momentum limit of the zero-zero component of the vacuum polarization tensor,

$$m^2_D(T) = -\Pi_{00}(\vec{k} \to 0, k_0 = 0).$$

This definition gives a gauge invariant value only in the lowest non-trivial order of the weak-coupling expansion for $SU(N_c)$ gauge theory with $N_f$ massless fermions

$$-\Pi_{00} = \left(\frac{N_c}{3} + \frac{N_f}{6}\right)g^2(T) + O(g^3).$$

Calculation of Eq.(2) even in the two-loop approximation reveals infrared divergences and gauge dependence leading to the conclusion that such a calculation cannot be trusted as high-order corrections are not calculable systematically. The starting point of the lattice studies is the connected correlation function

$$\Gamma(R, T) = \langle W(0)W(R) \rangle - \langle W(0) \rangle^2$$

of the two Polyakov loops

$$W(\vec{x}) = Sp \prod_{t=1}^{N_c} U_0(\vec{x}, t)$$

separated by a spatial distance $\vec{R}$. The colour averaged quark-antiquark potential $V(R)$ defined by this correlation

$$V(R) = -\ln \Gamma(R, T)$$

has been found to be of the screening form

$$V(R) \approx const \frac{R}{R} \exp(-m_D R)$$

In such a way we obtain an other definition of the Debye mass. In the weak coupling approximation the definitions (2) and (7) have to coincide.

Results of lower accuracy have been obtained from studies of the chromomagnetic fields behaviour at high temperature. First of all, two essential problems being important for understanding the finite-temperature physics of non-abelian fields on the whole are to be emphasized: infrared problem and area law for spatial Wilson loop. The infrared problem appears at attempts to study perturbatively the physics of the high temperature region: The static chromomagnetic sector develops infrared divergences which make the perturbative expansion invalid starting from $g^6$ order. Leading infrared divergences are those of the three-dimensional Yang-Mills theory. It is widely believed that some physical characteristic should exist leading to screening of the chromomagnetic forces and to curing of the divergences. It is usually called the
chromomagnetic mass though there are several definitions of this quantity in the literature.

1) By analogy with the definition of the Debye mass (4) the same limit of the space components of the \( \Pi_{\mu\nu}(k_0, \vec{k}) \) could be considered:

\[
m^2_{\mu}(T) = -\Pi_{ii}(\vec{k} \to 0, k_0 = 0)
\] (8)

However, this limit was found to be equal to zero in the one-loop approximation \[1\]. Two-loop calculations performed in the axial gauge have given finite result (but different in various schemes) \[6\]. Besides, specific difficulties of the axial gauge (coming from its singular character) bring an ambiguity in obtained results and, what is more important, lead to qualitatively different behaviour of \( \Pi_{ii} \) in comparison with relativistic gauges. The most general two-loop calculations of \( \Pi_{ii} \) in an arbitrary relativistic gauge were done in \[7\]

\[
-\Pi^{(2)}_{ii}(\vec{k}, k_0 = 0) = \frac{\pi g^4 N_c^2}{8(2\pi)^3 \beta^2} \left[ \frac{\alpha^3}{3} + \frac{4\alpha^2}{3} + \frac{13\alpha}{3} - 2 \right] \ln \frac{\vec{k}^2}{\mu^2}
\] (9)

where \( \alpha \) is the gauge fixing parameter (\( \alpha = 1 \) corresponds to the Feynman gauge). There is the infrared singularity in the last equation when \( \vec{k}^2 \to 0 \). One can see as well that the sign of the right-hand side of (9) can be changed by varying \( \alpha \). Hence one must conclude, expression (9) cannot define a physical mass. A detailed discussion of such a situation may be found in \[6\]. Consequently, at the present time we do not possess any regular methods for calculation of the chromomagnetic mass (defined in the sketched way). That is why other parameters should also be discussed.

2) By analogy with the definition of the Debye mass (4)-(7) the chromomagnetic mass can be determined from the corresponding correlation functions. In this case it has to be the spacelike Wilson loop in the adjoint representation. If \( m_\mu \neq 0 \), ”heavy” gluon current sources have to be screened. As far as we know such a calculation has not been performed yet.

3) If gluons acquire a magnetic mass, the magnetic fields of test charges will be screened. It means we can measure a monopole-antimonopole potential with the charges in the center of the gauge group. Corresponding Monte-Carlo simulations, although on the small lattices, were presented in \[8\]. The ”magnetic” potential really is of the screening form and the chromomagnetic mass determined from this potential is in a good agreement with the formula \( m_\mu = B g^2(T) T \), where \( g^2(T) \) is the coupling constant renormalized at the momentum scale \( T \). The computations were performed both for the Wilson pure \( SU(2) \) gluodynamics and for the Mack-Petkova modified \( SU(2) \)-model which does not contain dynamical monopoles.

It seems to be a logical conclusion that all of these three definitions determine the same physical quantity though we do not know a strict proof of this statement.

Second important (and strict) result which should be understood and obviously concerns the chromomagnetic fields was obtained by Borgs \[9\] (see also \[10\]). Its gist is the following: The spacelike Wilson loop obeys the area law at arbitrary high
temperature. In principle, this result does not look very strange. The chromomagnetic sector in Eq.(1) at finite $T$ does not generate (unlike the chromoelectric part) any configuration transforming non-trivially under $Z(N)_{gl}$-rotations. Therefore it seems natural that there is no screening effect of the corresponding sources (currents) being in the fundamental representation in the vacuum. Two points we would like to emphasize here are the following:

1) area law explains the impossibility to apply the standard weak coupling expansion at least to all modes of the chromomagnetic sector because the area law can never be achieved in this expansion.

2) as is known from the duality relations [11], [12], the area law for the Wilson loop implies the perimeter law for the ’t Hooft disorder parameter. In its line this has to signify a screening potential for a monopole-antimonopole pair.

Thus we can see that despite the fact that there is no strict proof that the chromomagnetic mass should exist we have to take into account that some kind of screening of the chromomagnetic forces does exist. We consider that the result by Borgs and the duality relations surely lead to this conclusion.

The crucial question coming from these facts concerns the nature of this screening. Undoubtedly, a mechanism of such a screening implies the existence of the magnetic mass in one of the meanings above.

Two possible mechanisms supplying gauge fields with mass and leading to the screening of chromomagnetic forces are available and discussed now in the literature: dielectric structure of the QCD vacuum and the spontaneous breaking of global gauge symmetry in the background $< A_0 >$.

1) Dielectric vacuum.

The attempts to formulate a confinement model in QCD framework led in the second half of the seventies to the so-called dielectric theory. In this model the gauge action is accompanied by additional terms corresponding to the auxiliary field $\rho$. If we calculate Gauss’ law from the action of this kind we shall formally find the relation with dielectric field as in the electrodynamics of the dielectrics. The definition of the pure gauge model is the following [13]

$$Z = \int \prod_{x,\mu} D(\Phi) \exp(-S(\Phi))$$

where

$$S(\Phi) = \sum_x \sum_{\mu,\nu} \lambda_{\mu\nu} F_{\mu\nu} F_{\mu\nu}^* + \frac{m^2}{2} \sum_x \Phi_\mu(x) \Phi_\mu^*(x) + \sum_x V[\rho(x)]$$

$$\Phi_\mu = \rho_\mu U_\mu, U_\mu \in SU(N), 0 \leq \rho < \infty$$

$$D(\Phi) = \rho^3 d\rho D\mu(U),$$

$D\mu(U)$ is the invariant group measure. The dielectric field is introduced via the auxiliary field $\rho$. $V(\rho)$ is a gauge invariant potential for this field. $\rho$ is a scalar field in the colour space for $SU(2)$ and a tensor field for $SU(N > 2)$. One of the most essential
features of such theories is a possibility to introduce a gauge-invariant mass for gauge fields $\Phi_\mu(x)$ (see the second term in (11)).

It has been recently shown that the lattice effective action for the infrared dangerous static modes appears at the dimensional reduction to be a little complicated version of the described dielectric theory [14]. Physical mass of the dielectric field is

$$m_d = T(\lambda^0 - 2\lambda_e \cos^2(a_\beta g < A_0 >))$$

(14)

where $\lambda^0 = 2$(flat function of $T, g$), $\lambda_e = \frac{a_\beta}{g^2 a_\beta}$. Undoubtedly, this mass leads to the screening of chromomagnetic forces and therefore of all gluon sources being in the adjoint representation [14]. It means in a sense the solution of the infrared problem, truly, perturbative expansion is invalid in any case since the dielectric field appears only at the non-perturbative level and vanishes if we try to apply the perturbative expansion.

2) $A_0$-condensate.

The second possibility is the spontaneous breaking of the $SU(N)$ global symmetry at high temperature accompanied by the generation of the chromomagnetic mass. Just this phenomenon will be in the focus of our interest here. The only known mechanism providing such a breakdown at high temperature is the generation of $A_0$-condensate which is a constant part of the temporal gauge field component $A_0 = const$ [13]. The nature of this parameter is connected with special properties of non-abelian gauge fields at finite temperature which display themselves in different ways depending on the particular method of calculations. We adduce below a comprehensive description of these properties. In general, the possibility of $< A_0 > \neq 0$ originates from the compactification of the imaginary time direction at $T \neq 0$. So, it is necessary to combine periodicity in the imaginary time $A_\mu(0) = A_\mu(\beta)$, $A'_\mu(0) = A'_\mu(\beta)$ and the gauge transformations of the fields $A'_\mu = U A_\mu U^+ + \frac{i}{g} U \partial_\mu U^+$ where $U$ is the gauge transformation operator. To make this transformation agree with the periodicity, the operator $U = \exp(i g A_0 \beta)$ (where $A_0 = A_0^a t^a$) must commute with generators $t^a$. Hence, $A_0$ should belong to the center of the gauge group: $A_0 = \frac{2\pi n}{gN}$, $n \in \mathbb{Z}_N$, $n = 0, 1, ..., N - 1$. Actually, the vacuum value of $A_0$ has to be calculated from a full effective action with quantum fluctuations included and if it will be found that $< A_0 >$ differs from $\frac{2\pi n}{gN}$, the spontaneous breaking of the gauge symmetry will be determined. In what follows, speaking about $A_0$-condensate we will assume it as a non-zero vacuum value of zero gauge field component obtained at a minimum position of the full effective action. The presence of the classical gauge field in vacuum may give a necessary missing parameter for solving the infrared problem. Moreover, this condensate effects various processes at high temperatures. In particular, it may lead to the spontaneous breaking of the baryon charge symmetry, etc.

As a matter of fact, the two screening mechanisms could be somehow connected. So it has been discussed in [14] that the mass of the dielectric field in the reduced theory can be proportional to $< A_0 >$. We shall consider this possibility in the corresponding part of the survey.
The described general idea can get different realization depending on applied calculation methods. A general picture of the effects of the $A_0$-condensate will appear when the results obtained in various particular approaches will be gathered together. The goal of the present review is to describe in a systematic way the results obtained in three approaches to $A_0$-condensate calculations. We discuss the method of effective Lagrangians for $A_0$-field, the Hamiltonian description of the $SU(N)$-symmetry breaking in the background $A_0$ both on the lattice and in the continuum theory, and the loop expansion of the effective action in QCD. In all these approaches the gauge field condensate has been determined in the framework of the appropriate approximation schemes. Anyway, at the present time there is no common opinion about realization of this phenomenon in the nature because of a number of problems concerning the accuracy, gauge invariance and precision of the calculations. So, in what follows we are going to adduce the comprehensive analysis of these problems and compare the results obtained.

Our Survey is organized as follows.

First we remind in brief the most essential features of gauge theories at finite temperature and introduce our notations (chapter 2). The general status of $A_0$-condensate is presented in chapter 3. In chapter 4 we go through the calculation of the condensate in the Hamiltonian formulation on the lattice. The gauge independence of the condensate in this approach is discussed as well. In chapter 5 the loop expansion for $\langle A_0 \rangle$ calculation is examined. The central point of this examination is the proof of gauge independence of the condensate by means of the Nielsen identities. Chapter 6 is devoted to elaborating of the effective Lagrangian method for the expectation value of $A_0$. In chapter 7 we compute the condensate in the theory with dynamical quarks. We consider both lattice and loop approaches and compare their results. Some consequences of the non-zero $\langle A_0 \rangle$ are reviewed in chapter 8. Here we are going to give some new results related to the $A_0$-condensate phenomenon. So we give a sketch of the calculations of the heavy quark potential in the background $\langle A_0 \rangle$. The infrared problem is reexamined at $\langle A_0 \rangle \neq 0$ in reduced lattice theory at high temperature. We give a proof that the adjoint spatial Wilson loop obeys perimeter law, which means the screening of chromomagnetic forces. Further, we discuss the spontaneous breaking of the baryon charge symmetry and some closely related problems. The general conditions for appearing of the Chern-Simons action in the background $\langle A_0 \rangle$ at high temperature are presented as well. Brief Summary and Discussion can be found in chapter 9.

2 Gauge theories at finite temperature. General outlook

In order to make our survey as self-contained as possible and for convenience of readers we would like to make a sketch of $SU(N)$ gauge theories at $T \neq 0$. To begin with, we remind the general formulation of the finite temperature theories both in the continuum
and on the lattice and introduce our notations. In the continuum the QCD action has
the form:

\[ S = \int[d^3x] \int_0^\beta dt L_{QCD}, \]

\[ L_{QCD} = \frac{1}{2g^2} Tr F_{\mu\nu} F^{\mu\nu} + \sum_{f=1}^{N_f} \nabla^f [D_n \gamma_n - m_f + (D_0 + i\mu) \gamma_0] \Psi^f, \]  

(15)

where

\[ F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + ig[A_{\mu}, A_{\nu}], \]

\[ D_\mu = \partial_\mu - igA_\mu, \quad A_\mu = A_\mu^a t^a, \quad (16) \]

\( \mu \) is the baryonic chemical potential.

On the lattice we consider the Wilson action:

\[ S = S^G + S^f \equiv \]

\[ \frac{2N_c}{g^2} \sum_P [1 - \frac{1}{2N_c} Tr U(\partial P)] + C.C. + \]

\[ \sum_{x,n=-d}^{d} \sum_f \nabla_{x}^f \Gamma_{x,x+n} U_n(x) \Psi^f_{x+n} + \sum_x m_f \nabla_{x}^f \Gamma \Psi^f_x \]  

(17)

where \( U_n(x) \in SU(N) \) and \( Tr U(\partial P) \) is the plaquette character in the fundamental
representation. The form of the matrices \( \Gamma_{x,x+n} \) and \( \Gamma \) depends on a sort of lattice
fermions.

It is well-known that the problem of quantization of gauge theories depends essentially on the boundary conditions imposed on the gauge fields. The compact topology
leads to another physical picture of the gauge model than the Euclidean topology
accompanied by the corresponding boundary conditions. QCD at zero temperature
is studied in the Euclidean space \( G = R^d \). A transition to the cylinder topology is
achieved by imposing the periodic boundary conditions on the gauge fields along the
imaginary time direction with period \( \beta = 1/T \) which plays the role of inverse temper-

ature.

Thus, the finite temperature theories are defined on the space \( G = R^{d-1} \otimes S^1 \). The
periodic boundary conditions generate the new observable known as the Polyakov loop
[16]:

\[ W = P \exp (ig \int_0^\beta dt A_0(x,t)) \]  

(18)

(or see (5) for the lattice definition).

The finite temperature formalism is used for studying the thermodynamical features
of QCD and its different phases. So, let us discuss in brief these features and, first of
all, the phase structure obtained from lattice calculations.

Nowadays two phase transitions are usually considered to be relevant to QCD at
finite temperature and/or baryon density. One is the deconfinement phase transition,
i.e. transition from the confining phase of the hadronic matter to the quark-gluon
plasma phase. The other is the chiral symmetry restoration phase transition. As is generally believed, the physical picture obtained from lattice studies could be the following. The pure gluonic QCD action has the global $Z(N)$ symmetry, which leads to the confinement of color quark charges in the low temperature phase. In the high temperature phase $Z(N)$ symmetry is spontaneously broken and quarks are screened by some specific configurations of the gluon fields. This transition is called the deconfinement phase transition and it is best studied in pure gluodynamics. Yaffe and Svetitsky [17] have proved the relation between the free energy of the infinitely heavy quark $F_q$ and the expectation value of the trace of the Polyakov loop, \( < TrW > = \exp(-\beta F_q) \). This relation and the behaviour of $TrW$ under $Z(N)_{gl}$ transformations

\[
trW \rightarrow ztrW
\]

were used as the basic ones to study the deconfinement phase transition in the Yang-Mills theory [18]. Difference of the expectation value of the $TrW$ from zero is the signal of the spontaneous breaking of $Z(N)_{gl}$ symmetry and also of the deconfining of the static colour charges transforming non-trivially under $Z(N)$:

\[
\langle \{N^{-1}trW_x\} \rangle = \begin{cases} 
0, & T < T^D_c, \text{ confinement phase}, \\
Z * f(T), & T > T^D_c, \ Z \in Z(N)_{gl}, \ f(T) \leq 1, \text{ deconf. phase}.
\end{cases}
\]

In the full QCD with dynamical quarks $Z(N)$ global symmetry is evidently broken from the beginning and no other appropriate order parameters for this transition are known. On the other hand, in the presence of the dynamical fermions the first order deconfining transition in the pure $SU(3)$ gauge theory is getting weaker as the quark mass decreases. This weakening is going on up to the quark mass of order $T$. However, for smaller masses the transition has been found to become stronger with the quark mass decreasing. This transition can be characterized by the chiral order parameter $\langle \sigma \rangle$ and is called the chiral phase transition. Numerical results show that the deconfinement and the restoration of the chiral symmetry, being distinctive by character at the first sight, are in fact indistinguishable in the region of intermediate quark masses.

Before proceeding further to discuss the gluon field condensation at high temperature let us make a short summary on the perturbative QCD vacuum at $T \neq 0$ (see [15], [19], [20] for a detailed review of the finite temperature properties of QCD) The perturbative vacuum is supposed to have no structures or condensates. Its main property is the asymptotic freedom ($g(T) \rightarrow 0$ in the limit $T \rightarrow \infty$ [15]). At $T \neq 0$ the momentum space is naturally divided in two parts: 1) $|k| \gg gT$ where the perturbative methods and results are to be reliable owing to asymptotic freedom; 2) $|k| \ll gT$, this is a truly infrared region. Just for these momenta one runs into infrared divergencies in the higher orders of perturbation theory, gauge dependence and other problems which signal that the perturbative vacuum is not adequate to the nature. So, as is expected and has been discussed in the introduction, some new macroscopic parameter (like the Debye mass) should be dynamically generated. As a candidate, the gluon magnetic mass, calculated by perturbative and non-perturbative methods [21], [3] has been discussed.
At the same time another possibility has also originated from the analysis by perturbative methods. Here we have in mind the dimension reduction process at high temperature \cite{13}, \cite{22}, \cite{23}, \cite{24}. The basic reason is as follows: in the gluon free propagator \( \frac{1}{k^2 + (2\pi n T)^2} \) of imaginary-time perturbation theory, the term \( 2\pi n T \) acts like a "mass" in the three-dimensional theory. According to the Appelquist-Carazzone theorem \cite{22} all nonstatic modes \( n \neq 0 \) decouple in the limit \( T \to \infty \), leaving the static (= three-dimensional) sector as the effective theory. This idea can be realized in different ways and, actually, a number of problems of calculation of the effective Lagrangian appeared \cite{24}. But in general it gives a possibility to search for a non-trivial vacuum at finite temperature. In the present paper the problem of the investigation of the gauge field vacuum via the described mechanism will be one of the discussed topics both in continuum QCD and beyond perturbative horizon on the lattice.

3 General status of \( A_0 \)-condensate

In this chapter we would like to give general reasons for condensation of the electrostatic potential and derive the corresponding mathematical foundation. We formulate a mathematical task of calculation of \( \langle A_0 \rangle \) and give an outlook on the previous investigations.

Firstly, we consider the lattice gauge theory without periodic boundary conditions (QCD at \( T = 0 \)) with the partition function of the form:

\[
Z = \int D\mu(U)D\bar{\Psi}D\Psi \exp(-S^G - S^F).
\]  

(21)

The gluonic \( S^G \) and the fermionic \( S^F \) parts of the action are expressed in eq. (17). The gauge field integration is performed over \( SU(N) \)-invariant group measure \( D\mu(U) \). Due to this fact the results of the calculations are gauge independent. For example,

\[
\langle B \rangle = \langle B(U) \rangle_{U_0 = 1 \ A_0 = 0},
\]

if \( B(U) \) is a gauge invariant function \cite{12}. This means that there is a gauge transformation which allows us to fix gauge \( U_0 = 1 \) or\( A_0 = 0 \). We can easily see that any constant \( \langle A_0 \rangle \) may be singled out by an appropriate gauge transformation.

The partition function at finite temperatures is calculated within the following periodic boundary conditions:

\[
U_\mu(x, \tau) = U_\mu(x, \tau + N_\beta),
\]

\[
\Psi(x, \tau) = -\Psi(x, \tau + N_\beta),
\]

\[
\bar{\Psi}(x, \tau) = -\bar{\Psi}(x, \tau + N_\beta),
\]

(22)

where \( N_\beta \) is a number of the lattice sites in the time direction. These boundary conditions are incompatible with the gauge \( A_0 = 0 \). We may fix a static diagonal gauge
only. Let \( U_0(x, \tau) = V_x \), where \( V_x \) is a diagonal time-independent \( SU(N) \) matrix. After performing the gauge transformations

\[
U_n(x, \tau) \longrightarrow (V_x)^\tau U_n(x, \tau)(V_{x+n})^{-\tau} \\
U_n^+(x, \tau) \longrightarrow (V_{x+n})^\tau U_n^+(x, \tau)(V_x)^{-\tau},
\]

\( n = 1, \ldots, d \)

\[
\Psi(x, \tau) \longrightarrow (V_x)^\tau \Psi(x, \tau) \\
\bar{\Psi}(x, \tau) \longrightarrow \bar{\Psi}(x, \tau)(V_x)^{-\tau}
\]  

(23)

(which lead to the gauge \( A_0 = 0 \) at zero temperature) we can remove the matrices \( V(x) \) in the action from all links but the last one where all of them are grouped gathering in the Polyakov loops \( W_x \). On the lattice in the static gauge the Polyakov loop becomes

\[
W_x = P \prod_{\tau=1}^{N_\beta} U_0(x, \tau) = \exp(i\beta g A_0(x)).
\]  

(24)

In such a way we obtain the new terms in the chromoelectric and in the fermionic parts of the action

\[
S^G(W) = \frac{2}{g^2} \sum_{x,n} \text{Re} \, Tr \, W_x U_n(N_\beta - 1, x) \cdot W_{x+n}^+ U_n^+(1, x),
\]

\[
S^F(W) = \sum_{x, \pm 0} \bar{\Psi}_x(N_\beta - 1) \Gamma_{x, \pm 0} \cdot W_x(\pm 0) \Psi_x(1),
\]

(25)

\[
W_x(\pm 0) = \begin{cases} W_x \\
W_x^+. \end{cases}
\]

These terms are absent in the theory without periodic boundary conditions and describe the interactions between gauge fields, fermionic fields and the Polyakov loops. Owing just to these new terms the \( SU(N) \) gauge theory has the non-trivial phase structure described above. Moreover, we can deduce from the last equations a possibility of the spontaneous breaking of the \( SU(N) \) global gauge symmetry caused by condensation of the chromoelectric potential. The symmetry group of the pure gauge theory is the direct product of the group of the local gauge symmetry, the group of the global gauge symmetry and its center subgroup: \( SU(N)_{\text{loc}} \times SU(N)_{\text{gl}} \times Z(N)_{\text{gl}} \). The last one acts only on the Polyakov loops. The fermionic part of the action (25) violates the global center symmetry explicitly, the symmetry group becomes \( SU(N)_{\text{loc}} \times SU(N)_{\text{gl}} \) and the expectation value of the \( Tr W_x \) is not the appropriate order parameter at all (25). Nevertheless the eigenvalues of the Polyakov loop

\[
W_x^I = \exp(i \varphi_I(x))
\]  

(26)

can be used for studying the behaviour of the system under \( SU(3)_{\text{gl}} \) symmetry because the Polyakov loop transforms under \( SU(3) \) rotations like the matter fields in the adjoint representation [13]

\[
W_x \rightarrow U W_x U^+, \quad U \in SU(N)
\]

(27)
The eigenvalues of the Polyakov loop can be chosen, for example, in the $SU(3)$ theory as
\[
(\beta g)^{-1} \varphi_1 = A_0^3 + \frac{1}{\sqrt{3}} A_0^8,
\]
\[
(\beta g)^{-1} \varphi_2 = -A_0^3 + \frac{1}{\sqrt{3}} A_0^8,
\]
\[
\sum_{l=3}^N \varphi_l = 0
\]
Therefore the expectation value of $W_l^x$ may be different from zero only if the spontaneous breaking of the $SU(3)_{gl}$ symmetry is possible. The constant field $A_0$ can be cancelled in the action only when $W(\langle A_0 \rangle)$ belongs to $Z(N)$, as one may easily see from eq. (25) because in this case the matrices $W$ commute with $U_n$. In any other case the constant $A_0$ will be present in the action and the global $SU(3)$-symmetry is broken up to its Cartan subgroup
\[
W(\langle A_0 \rangle) \ni Z(N) \rightarrow SU(N) \rightarrow [U(1)]^{N-1}
\]
It is clear now that such a mechanism can work both in the pure gluodynamics and in the theory with dynamical quarks unlike the mechanism of the spontaneous breaking of the $Z(N)_{gl}$ symmetry.

Thus, we have two possibilities to quantize the gauge theory in the space $G = R^{d-1} \otimes S^1$:

1) The theory in $G = R^{d-1} \otimes S^1$ must always possess the same symmetry of the vacuum and the same Lagrangian as the theory in the Euclidean space $G = R^d$. So, for instance, any constant $A_0$ should be singled out by the corresponding gauge transformation from eq.(23). This constraint leads to the restriction $W_x^l = \exp(\frac{2\pi i}{N} q_l(x)), q = 0, 1, ..., N - 1$ and for the quantum theory we obtain
\[
Z = \sum_{q_l(x)} \int_{U_n(r) = U_n(r+\beta)} D\mu(U) D\bar{\Psi} D\Psi \exp(-S(\exp(\frac{2\pi i}{N} q_l(x)), U, \bar{\Psi}, \Psi)).
\]

2) The global symmetries of the vacuum are determined by dynamics of the gauge system itself. Then, in eq.(24) we have $\sum_{q_l(x)} \rightarrow \int d\mu(W_x)$, where $d\mu$ is the invariant $SU(N)$ measure.

In the former case there is the only possibility of the spontaneous breaking of the global $Z(N)$ symmetry whereas in the latter one the spontaneous breaking of both $Z(N)$ and $SU(N)$ global symmetry can occur. It is difficult to give a preference to any of these formulations from the theoretical point of view. The second formulation commonly seems to be more relevant. In what follows just this formulation will be explored here.

Thus, from the picture formulated above we come to a task of calculating $\langle A_0 \rangle$ which can be formulated in the following way: The configurations $A_n = 0$ ($n = 1, ..., d$), $A_0 = \text{const}$ are the solutions of the Yang-Mills equations with the additional condition that
the classical action equals zero. At the classical level, however, the breakdown of the symmetry is absent and the system is in the symmetry invariant minimum. Can the quantum fluctuations generating the effective potential for \( A_0 \) lead to spontaneous breaking of the global symmetry and produce the minima of the effective action with a non-trivial value of \( \langle \beta g A_0 \rangle \neq \frac{2\pi}{N} \)? On the lattice the situation is the same: for the configurations \( U_n = 1, W_l = \text{const} \), the classical lattice action \( S^G \) is equal to zero and therefore we can ask the same question.

Actually, the investigation of the \( A_0 \)-condensate started ten years ago and \( \langle A_0 \rangle \neq 0 \) was determined in various approaches. Nevertheless, at the moment we have neither a strict proof that the condensate has to appear nor a common opinion about its generation. This is mainly due to the mathematical difficulties which have been encountered in the used approaches. For example, in the loop expansion method \( \langle A_0 \rangle \neq 0 \) is derived from the two loop effective action \( W(A_0, \xi) \) \[26\],\[27\],\[28\],\[29\],\[30\],\[31\],\[32\],\[33\],\[34\],\[35\]. So, to verify the result the three-loop contribution should be evaluated. This very complicated task has not been solved yet. Other approaches discussed in the literature \[36\],\[37\],\[38\],\[39\],\[40\] also contain either some uncertainties or unsolved problems.

Recently, the possibility of \( A_0 \) condensation has been called in question from the point of view of its gauge invariance \[29\]. As was found in the background \( R_\xi \) gauge, both the effective action and its minimum value appear to be dependent on the gauge fixing parameter \( \xi \) \[27\],\[31\],\[30\]. Hence, a doubt about gauge invariance of the phenomenon has arisen. To resolve this doubt in the perturbation theory several methods for gauge invariant calculations have been conjectured \[30\],\[33\],\[41\]. But in all of them the result - no real condensation at two loop level - has been stated. On the other hand, the gauge invariant results of the analytical lattice investigations \[30\] unambiguously show that condensate does appear. The same conclusion has been supported via the Nielsen identities method \[42\] at the two loop level (in order \( g^2 \) in coupling constant). Another approach to the problem of gauge invariance of the condensate was recently proposed in \[40\] where the authors built a partition function for the eigenvalues of the Polyakov loop in the Hamiltonian formulation of the continuum theory. The result \( A_0 = 0 \) was obtained. We shall discuss this result later on remarking that this conclusion is in obvious contradiction with apparently gauge invariant lattice calculations \[10\],\[30\].

The situation needs to be clarified in any way because \( A_0 \)-condensate, if it does realize in the nature at high temperature, would be a very essential element of the self-consistent finite temperature gauge theory. It seems to us that the only way to comprehend the situation is to fix the strong results of various approaches and to find out both their common point and sources of discrepancies. At the same time it would be very desirable to find crucial phenomena connected to \( \langle A_0 \rangle \neq 0 \) which can be detected in future experiments on the heavy-ion collisions. These are main two purposes of the present paper. We believe that being gathered together, the results of various approaches can help to elucidate a lot of difficulties and outline the prospects for future investigations.
4 Hamiltonian formulation of the gauge theories and $A_0$-condensate on the lattice

The first approach we would like to analyze here is the Hamiltonian formulation of the gauge theories. It is very desirable to have an apparently gauge invariant approach to defining and calculating $< A_0 >$-condensate. Possessing this essential property, the Hamiltonian approach allows us to be convinced that $A_0$ condensation will be a gauge invariant phenomenon. On the other hand, we want to have a strict proof that $< A_0 > \neq 0$ in the framework of the reliable approximation scheme. Fortunately, such an approximation does exist. It is the strong coupling expansion in the lattice theories. In what follows we consider, at first, the strong coupling region of the Hamiltonian lattice formulation. Then, the continuum version of the Hamiltonian approach will be discussed.

The detailed description of the lattice Hamiltonian formulation can be found in the series of the papers [43], [44] and in the review [45]. The Hamiltonian approach on the lattice was developed for the first time in [46]. In the construction of the Hamiltonian partition function we follow our own method [47] adducing a proof of its equivalent to earlier methods. The Hamiltonian of the lattice gluodynamics in the strong coupling approximation includes only the chromoelectric part

$$H = \sum_{\text{links}} \left( \frac{g^2}{2a} \right) E^2(l)$$

where $E(l) = i\partial/\partial(A_l)$ - are the chromoelectric field operators. In this approach the chromomagnetic term can be treated perturbatively at $g^2 \rightarrow \infty$. Calculation of the partition function

$$Z = \tilde{S}P \exp(-\beta H)$$

is connected with summing over local gauge-invariant states. This is reflected in sign $\tilde{S}P$ in (31). The corresponding physical Hilbert space is determined by Gauss’ law. To satisfy the latter the usual method is to introduce the necessary $\delta$-function. In order to solve this task we use the more general method connected with the projection operator technique. It should be emphasized at once that the Hamiltonian formulation allows to obtain an effective action for the eigenvalues of the Polyakov loop in the gauge $A_0 = 0$. In what follows we are going to demonstrate that carrying out the projection onto local gauge invariant states and summing over the eigenvalues of the diagonal operators of the gauge group are equivalent to integration over spatial gauge fields in the Euclidean version of the lattice theory without fixing any gauge. It is a rather important point which was missed, for instance, in Ref. [11].

Our starting point is the partition function (31) where we implant a projection operator in each lattice site

$$Z = S \exp(-\beta \sum_{\text{links}} \left( \frac{g^2}{2a} \right) E^2(l))P, P = \prod_x P_x^0$$

14
where the operator $P^r$ is defined as

$$P^r = \frac{h_r}{|g|} \sum_{j=1}^{[g]} \Omega_j^{*r}(g)U_j(g). \quad (33)$$

To make the next mathematical construction more transparent we have put down here $P^r$ for a discrete group, $h_r$ is the dimension of the representation $r$, $|g|$ is the rang of the group and $\Omega^r$ is the character of the irreducible representation $r$. The matrices $U_j(g)$ are located on the links. In eq. (33) we must consider all representations which can be combined in such a way to form singlets in each lattice site. Eq. (33) can be rewritten to the form

$$P^r = \frac{h_r}{|g|} \sum_{k=1}^{n} \Omega_k^{*r} \sum_{j=1}^{g_k} U_{k,j} = \frac{h_r}{|g|} \sum_{k=1}^{n} \Omega_k^{*r} C(k). \quad (34)$$

The second identity is the definition of the class operator $C(k)$ which contains $g_k$ elements. $n$ is the number of linearly independent class operators of the representation group $G$ [48]. There exists a representation for $C(k)$

$$C(k) = \frac{g_k}{|G|} \sum_{j=1}^{[G]} U_j U_k U_j^{-1}$$

from which it is easy to deduce the most important property of the class operator: it is invariant under transformations out of group of representation $G$ or in other words

$$[C(k), U(g)] = 0. \quad (36)$$

A generalization of (34) and (35) on the Lie group is not very complex and is founded on using the so-called “unitary Weyl’s trick”. As is known, any unitary representation of the Lie group may be presented in the form [49]

$$U(\Phi) = V(v)\xi(\phi)V^{-1}(v) \quad (37)$$

by the appropriate choice of the unitary coordinate system. In these terms the invariant measure is

$$d\Phi = dv d\mu(\phi), \quad (38)$$

which allows to integrate over $\nu$-variables. After that we have for $P^r$

$$P^r = h_r \int d\Phi \Omega^{*r}(\Phi) U(\Phi) = h_r \int d\mu(\phi) \Omega^{*r} \int dv U(\phi, \nu). \quad (39)$$

For class operator we obtain in the case of the Lie group

$$C(\phi) = \int dv U(\phi, \nu). \quad (40)$$

It is obvious that $C(\phi)$ possesses the property of the invariance analogous with (36). Just this property is the most important one in the selection of the local gauge-invariant
states. Let $C_l$ be the eigenvalues of the Hamiltonian (30) and $U^\mu_l(x)$ be the eigenfunctions of $E^2$. They are taken as variables on the lattice links, $x$ is the lattice site and $\mu$ is the unit vector. Then, the partition function (32) is rewritten in the form

$$Z = \sum_l K_l \exp(-\gamma C_l),$$

(41)

where $\gamma = \frac{3a^2}{2\alpha_x}$ . The operator $K_l$ selects gauge-invariant states among all eigenstates of the Hamiltonian and has the form

$$K_l = \int d\mu(U) \prod_{x,\nu} U^+_{x,\nu} P \prod_{x,\nu} U^l_{x,\nu}.$$

(42)

From above formulae it follows that

$$P \prod_{x,\nu} U^l_{x,\nu} = \int d\mu(\phi_x) \prod_{x,\nu} \Omega(\phi_x) U^l_{x,\nu} \Omega^+(\phi_{x+\nu}).$$

(43)

The operator $P$ satisfies the relation $PP = P$ and is nothing but the projection operator. How it is working may be seen from the last three equations. Next, we would like to describe how to connect this technique with standard projection and with Gauss’ law.

Let us choose the transformations $\Omega(\phi_x)$ in the form

$$\Omega(\phi_x)U_{x,\nu} = \exp(iE^a_{x,\nu}\phi^a_x)U_{x,\nu},$$

$$U_{x,\nu}\Omega^+(\phi_{x+\nu}) = U_{x,\nu}\exp(-iE^a_{x+\nu,\nu}\phi^a_x),$$

(44)

where the generators $E^a_{x,\nu}$ are the chromoelectric field tension operators defined on the lattice links. Then, grouping $E^a_{x,\nu}$ at the same $\phi^a_x$ we obtain that

$$P = \prod_x P^0_x, \quad P^0_x = \int d\mu(\phi_x) \exp(-iQ^a_x\phi^a_x)$$

(45)

where

$$Q^a_x = \sum_{\nu} E^a_{x,\nu}$$

(46)

are the colour charge operators and the generators of the gauge transformations in the lattice sites. $P^0_x$ in Eq. (45) is the projection operator onto states with zero charge and is a particular choice of the operator (39).

The connection of Gauss’ law $\delta(\Delta E)$ with a projection operator technique is not a trivial task as the calculations are quite complicated because the operators $Q^a$ do not commute with each other and, consequently, it is impossible to find eigenfunctions for all $Q^a$ at the same time. Applying ”Weyl’s trick” again we present $\delta(Q^a)$ in the form

$$\delta(Q^a) = \int d^{(N^2-N)}\nu d^{(N-1)}\phi \tilde{V}(\nu) \exp[i \sum \phi^a Q^a] \tilde{V}^+(\nu).$$
In the last equation the matrices $\tilde{V}(\nu)$ diagonalize $\exp[i \sum_a^{N-1} \phi^a Q^a]$ in the sense that in the final expression only $Q^a$ belonging to Cartan subgroup enter. After integration over $\nu$ one finds

$$\delta(Q^a) = \int d\mu(\phi) \exp[i \sum_a^{N-1} \phi^a Q^a] = P^0.$$  \hfill (47)

The last step we need to do is to introduce a summing over eigenvalues of the $Q^a$ of the Cartan subgroup. Just eigenfunctions of these operators and eigenfunctions of the Hamiltonian produce a full set of the local gauge invariant states in such a treatment.

The operator $P^r$ at $r \neq 0$ projects onto the invariant states in the presence of the background probe charges. To prove it we have to consider the $\delta$-function $\delta(Q^a - q^a)$ where $q^a$ are the probe charges and the generators of the $r$-th irreducible representation. Summing over all charges belonging to $r$-representation we easily find

$$\sum_{q^a \in r} \delta(Q^a - q^a) = P^r = \int d\mu(\phi) \Omega^r(\phi) \exp[i \sum_a^{N-1} \phi^a Q^a].$$ \hfill (48)

Now let us consider the Wilson lattice action in the same strong coupling approximation (restricting ourselves to chromoelectric part of the full action in (17) at finite temperature). It is known that after integration over space gauge fields and taking a limit $a \beta \to 0$ in the time direction we come to the same expression for the partition function as it appears in the Hamiltonian formulation. Since in the last case we do not presuppose any gauge and start from an apparently gauge invariant formulation let us demonstrate it once more. To integrate out the space gauge field configurations we follow the standard procedure and expand the Wilson action into series over the characters of the irreducible representations of $SU(N)$-gauge group. Then, performing the invariant integration over $dU_n(x)$ we come to the following expression

$$Z_G = \int \prod_{x,t} d\mu[U_0(x,t)] \prod_{x,n} \left[ \sum_r C_r^{N_\beta}(\beta_\sigma)(Sp \prod_{t=1}^{N_\beta} U_0^r(x,t))(Sp \prod_{t=1}^{N_\beta} U_0^{+,r}(x+n,t)) \right]$$ \hfill (49)

where $\beta_\sigma = \frac{2N g^2 a_\beta}{\beta}$. We can now choose Weyl’s representation (37) for the Polyakov loop $W^r = \prod_{t=1}^{N_\beta} U_0^r(x,t)$ rewriting the invariant measure in the corresponding form. After cancellation of the non-diagonal parts of the Polyakov loops we have

$$Z_G = \int \prod_{x,t} d\mu(\alpha_{x,t}) \prod_{x,n} \sum_r C_r^{N_\beta}(\beta_\sigma)(Sp W^r(\alpha_x))(Sp W^{+,r}(\alpha_{x+n}))$$ \hfill (50)

where $W_x = \exp[i \sum_{t=1}^{N_\beta} \alpha^a_x(t) T^a]$, $T^a$ are diagonal generators of $SU(N)$. It is enough to integrate in the last equation only over one chosen $\alpha(t')$. Because of the invariant integration the result will be independent of $\alpha(t \neq t')$. Thus, we obtain

$$Z_G = \int \prod_{x} d\mu(\alpha_x) \prod_{x,n} \sum_r C_r^{N_\beta}(\beta_\sigma)\Omega^r(\alpha_x)\Omega^{+,r}(\alpha_{x+n}).$$ \hfill (51)
We consider now the quark contribution to the partition function. To get it in the Hamiltonian formulation we have to pick up the operator $P^r$ when $r \neq 0$ and take into account both the states without charges and the contribution of the antiparticles when summing over $q^a$ is performed in (18). In such a way one finds the quark contribution to be
\[ \prod_x (1 + \text{Re} \Omega_x^r). \] (52)
The same contribution can be obtained in the Euclidean formulation where we do not fix any gauge. In this case we consider the interaction of matter fields with $U_0(x, t)$ and explore the Kogut-Susskind massless fermions [15]. Calculating the partition function
\[ Z_q = \int \prod_x d\Psi_x d\Psi_x \exp(-\Psi_x D_0 \Psi_y), \] (53)
where
\[ D_0 = \frac{1}{2}(U_0(x, t)\delta_{y,x+0} - U_0^+(x, t)\delta_{y,x-0}) \]
and $U_0$ belongs to the fundamental representation of $SU(N)$, we get
\[ Z_q = \prod_x (1 + \text{Re} \Omega_x^r). \] (54)
This contribution is exactly the same as in the Hamiltonian formulation (52) if we choose $r \in$ fundamental representation of $SU(N)$. To do the last step in our proof we notice that at $a_\beta \to 0, N_\beta \to \infty$ we have
\[ C_{r}^{N_\beta} = (\int d\mu \exp(\beta_a \Omega_\mu))^N_\beta \approx \exp(-\gamma C_2(r)) \]
up to an irrelevant constant, $\gamma = \frac{\beta g^2}{2a_g}$. Here, $C_2(r)$ is the quadratic Casimir operator which is, on other hand, the eigenvalue of the lattice Hamiltonian in the strong coupling approximation (30). Thus we can see the full equivalence between the lattice Hamiltonian formulation and lattice in the Euclidean space.

Two points we would like to point out once more are: 1) the integration over space gauge field plays in the Euclidean space a role of the projection onto local gauge invariant states; 2) just (and only) this integration or gauge transformation carried out at projection (14) produces the interactions of the eigenvalues of the Polyakov loops.

Certainly, all described above is quite known in the context of the lattice gauge theories. Reminding this mathematical picture we wanted to make the readers sure of two facts: 1) the lattice Hamiltonian formulation is quite reliable approach for calculation of the effective action for the eigenvalues of the Polyakov loop; 2) relying on apparent gauge invariance of the lattice approximation on the whole we can claim that if $<A_0> \neq 0$ appears it will be a gauge-invariant phenomenon. This question is a rather important one and it is, basically, the central point of the loop calculations presented in the next chapter.

Now we are able to consider the problem of constructing of the effective action for the eigenvalues of the Polyakov loop. At first we consider pure gluodynamics. The
quark contribution is calculated in the chapter 7. The first calculations of the condensate on the lattice were presented in [16] where the authors carried out the corresponding Monte-Carlo computations for $SU(3)$ gluodynamics. Although the computations were done on the small lattice the unambiguous result was obtained: $< A_0 > \neq 0$ in the deconfined phase and the condensate falls at the deconfinement phase transition. It signifies that $Z(N)$ and $SU(N)$ global symmetries are broken at the same critical temperature. Lastly, the similar result comes from [50] where a simulation of the pure gauge theory fixing the Landau gauge has been carried out. They have argued that $A_0$ develops a nonvanishing expectation value above the deconfinement transition temperature and have found the breaking of the colour charge conjugation symmetry.

Unfortunately, we do not know any other attempts to compute $< A_0 >$ by means of the MC-simulations. Here, we are going to explore some known analytical methods of statistical mechanics and spin systems. The following material is founded on the papers [36], [51], [52], [53] where all omitted (especially technical) details can be found.

Our starting point is the $SU(N)$ partition function (41), (42), with the projection operator (43). We define the effective action in the standard way as [54], [40]

$$S_{\text{eff}}(< \beta g A_0 > = \phi) = -\ln Z_\phi(\beta, g)$$  \hspace{1cm} (55)

where

$$Z_\phi = S p K_0^0 \exp(-\beta H)$$  \hspace{1cm} (56)

$$K_\phi^0 = \int \prod x, \nu dU_\nu(x)U_\nu(x) \int \prod x d\mu(\phi_x) \prod \Omega(\phi_x)U_{x,\nu}^{+,\dagger} \Omega^+(x) \delta(N\phi - \sum x \phi_x),$$  \hspace{1cm} (57)

and $N$ is the number of the lattice sites in the spatial directions. In these terms the operator (14) becomes after substitution $\phi_x \rightarrow \phi + \alpha_x$

$$P = \prod x P_x^0, \quad P_x^0 = \int d\mu(\phi_x) \exp(-iQ^a_x(\phi + \alpha_x)^a).$$  \hspace{1cm} (58)

Integration in (56) is performed over compact measure with the constraint

$$\sum x \alpha_x = 0.$$  \hspace{1cm} (59)

From the last two equations we can deduce that the operator (58) projects onto local gauge-invariant states with non-zero global colour charge. In a sense it clarifies the physical nature of $A_0$-condensate: it is an imaginary chemical potential for the colour charge out of $SU(N)$ group. The simplest case when $< A_0 > \neq 0$ may appear is the one of $SU(2)/Z(2)$ gauge group (since elements of center are absent in such a theory $< A_0 > = 0$ in the unbroken phase). Let us consider in brief this theory as a simple but non-trivial example which demonstrates apparently how the condensate appears in obvious manner. The initial action (51) for $SU(2)/Z(2)$ theory includes the adjoint
representation for gauge field matrix $U_n(x)$. It means that we must use the sum in (56) only over representations which transform trivially under $Z(N)$ rotations. Performing the general steps sketched above and summing over all irreducible representations we obtain $Z_\phi$ in the form
\begin{equation}
Z_\phi^{SU(2)/Z(2)} = \int_0^\pi d\phi_x \prod_{x} \Theta_2(e^{-\gamma}, \phi_x - \phi_{x+n}) - \Theta_2(e^{-\gamma}, \phi_x + \phi_{x+n}) \right)
\end{equation}
where $\Theta_2$ is the Jacobi function. Making use the obvious approximation $\phi_x \approx \phi + \delta \alpha_x$ and neglecting all fluctuations $\delta \alpha_x$ (as the fluctuations are small they can not influence the final result because for the gaussian free fields the constraint $< \delta \alpha_x > = 0$ is automatically fulfilled; of course, it may be not obvious and one should argue this point by calculating the corrections; we shall mention this point below for more realistic SU(2) group) one finds the effective action to be
\begin{equation}
-S_{eff}(\phi) = d \ln \left[ 1 - \frac{\Theta_2(e^{-\gamma}, 2\phi)}{f(\gamma)} \right] + (1 - d) \ln \sin^2 \phi.
\end{equation}
We omitted an irrelevant constant and $f(\gamma) = \sum_{l=0,1,..} \exp(-\gamma l(l + 1))$. Analyzing this expression we conclude that there exists such $\gamma_c$ that at $\gamma > \gamma_c$, $\phi \neq 0$ in the point of minimum of the effective action (61). This conclusion is quite understandable without the approximation we have used here. The Jacobi function in (60) which carries the sign "+" does not include the constant part of $\phi_x$. This constant part enters in the second Jacobi function with sign "−". As $\phi \neq 0$ lowers the Jacobi function it signifies that when $\phi \neq 0$, $Z_\phi$ increases and, thus, leads to deeper minimum of the effective action than $\phi = 0$. Unfortunately, such (almost "strict") proof can be done only in this simple example. The more realistic case of SU(2) gauge group has been investigated in [36]. As this case will be presented below and in the more general context we restrict ourselves to some phrases here. We used the same approximation as described above and calculated the corrections coming from Gaussian integration over $\delta \alpha_x$. The obtained $S_{eff}$ was investigated numerically. Our conclusions are similar to the previous case.

Lastly, we are studying QCD (SU(3) gauge group) in details by two independent methods of calculations in order to be convinced in the results. First, we suppose that all fluctuations are suppressed by coupling constant. Recalling that SU(3) representations are labelled by two independent indices $l = (l_1, l_2)$ and eigenvalues of $E^2$ are the quadratic Casimir operator eigenvalues $C_2(l) = \frac{1}{3}(l_1^2 + l_2^2 - l_1 l_2) - 1$ we can perform a summation in the partition function (56) over all irreducible representations. In this way we can calculate the single-site free energy in the form convenient for the following analysis
\begin{equation}
-S_{eff,G}^{SU(3)} = \ln [F(\phi)] = \ln \left[ \frac{\Xi^d(\phi)}{\mu^{d-1}(\phi)} \right]
\end{equation}
where for readability we have introduced the notations
\begin{equation}
\mu(\phi) = \sin^2 \left( \frac{\phi_1 - \phi_2}{2} \right) \sin^2 \left( \frac{2\phi_1 + \phi_2}{2} \right) \sin^2 \left( \frac{2\phi_2 + \phi_1}{2} \right)
\end{equation}
and

$$\Xi = \exp(\gamma) \sum_{\alpha=1}^{3} \left[ -\frac{1}{3} \Theta_3(\gamma/3, 0) \Theta_3(\gamma, 2\Phi_\alpha) - \frac{2}{3} \Theta_3(\gamma/3, \Phi_\alpha) \Theta_3(\gamma, \Phi_\alpha) + \frac{1}{3} \Theta_3(\gamma/3, \Phi_\alpha) \sum_{\beta > \sigma} \Theta_3(\gamma/3, \Phi_\beta - \Phi_\sigma) \right] + [\Theta_3 \to \Theta_2] \quad (64)$$

up to an irrelevant constant. Here $\Theta_i(\gamma, \phi)$ are the Jacobi functions and $\Phi_\alpha = \phi_\beta - \phi_\sigma$ at $\sum_\alpha \Phi_\alpha = 0$ ($\phi_\alpha$ has been defined after eq.(27)). Thus $<\phi>$ can be found from the local minima of $S_{effG}$. The $S_{effG}$ behaviour has been analyzed numerically at $d = 3$ and in the interval of $\gamma = [1, 2]$ with high precision. At larger $\gamma > \gamma_c$ (small temperature) the minima of $\ln[F(\phi)]$ are located inside of the every triangle plotted on Fig.1 at the following values: $<\phi_1> = 2\pi k$, $<\phi_2> = 2\pi (k + \frac{1}{3})$, $<\phi_3> = 2\pi (k - \frac{1}{3})$ and so on (six combinations are possible). Obviously, that this distribution is invariant under the $Z(3)$ transformations and we conclude that the system is in the confined phase where $<TrW> = 0$ and $<\phi> = 0$. With $\gamma$ decreasing (temperature increasing) at $\gamma = 1.75$ three secondary minima of the function $\ln[F(\phi)]$ develop and get deeper. At the same time the initial minimum of $\ln[F(\phi)]$ gets also deeper but it develops slower and at $\gamma = 1.62$ it disappears at all. But still earlier at $\gamma_c^G = 1.73$ the secondary minima become degenerate with the initial one and the system could undergo the phase transition presumably of the first order. In these newly developed minima $<\phi> \neq 0$ which signals the global gauge symmetry breakdown and forming temperature dependent condensate. It is clear that all these phenomena emerge at the deconfinement phase transition since above its critical temperature $<TrW> \neq 0$ in any minimum.

The second approach to evaluate the condensate for the $SU(3)$ gluodynamics has been presented in [51]. To understand the picture in more details authors of [51] have considered the first non-trivial term in sum over characters and reparametrized the conditions $\delta(N\phi^a - \sum_x \phi^a_x)$ in terms of the Polyakov loop. For calculation of the effective action Bogolyubov’s method of quasiaveraging has been utilized. Numerical studying of the effective action up to $\lambda^{12}$ (where $\lambda$ is an effective coupling constant which is small in the strong coupling region) shows that the condensate appears in the deconfinement phase and moreover the picture described above is approximately reproduced in the framework of Bogolyubov’s method and of redefinition of those quantities over which we have to minimize the effective action. In a sense, this method is close to those developed in [30], [41]. The difference is that we used the Polyakov loop to express the effective action (though in a special parametrization) from the very beginning. Our final result is directly opposite to the one obtained in [30], [41]. It might imply that Belyaev’s method is not self-consistent (a comprehensive analysis of this method we give in the next chapter).

Generally speaking, the situation could turn out to be more complicated than described above. At high temperature the Polyakov loop develops a non-vanishing expectation value. In quantum theory, the Polyakov loop, being the function of the random variable $A_0$, is an other random variable. It could mean that to obtain the real distribution of contributions to the free energy coming from the Polyakov loop and from
A_0\text{-condensate we need to construct a general effective action for both these quantities and to find minima of such an effective action. In other case it cannot be excluded that either some part of the condensate may be "transferred" into the Polyakov loop or vice versa. To verify this idea we carried out the corresponding calculation for SU(2) gauge group. Our preliminary results are the following [53]. Implanti
last point was missed in the discussed paper). It means the following: Choosing the same representation for $SU(N)$ characters we can formally cancel the determinant. But on the other hand, it means that we cancel some part of the projection operator. The sum over irreducible $SU(N)$ representations which survives after the cancellation can never be presented as the old projection operator (it can be easily demonstrated mathematically). The inconsistency of Ref. [40] is the usage of the same projection operator after the cancellation. Strictly speaking, this is incorrect. Namely this gives a possibility to rewrite the partition function in such a form that the constant part of $A_0$ will be only at imaginary unit. Thus, there are two possibilities: either one cancels the determinant but after that we must build a new projection operator (if it can exist) or we can work with the usual projection operator but the determinant is not canceled in this case. In chapter 6 we shall give a confirmation what we sketched above of. The same determinant appears not only in the Hamiltonian version but also in the static gauge for $A_0$ as the Fadeev-Popov determinant. There is no doubt that cancellation takes place and besides not only at one-loop level. But nevertheless the condensate falls at high temperature. It may signify only one fact: after the cancellation we have no old simple representation for the projection operator.

Let us consider in the static gauge the part of the Yang-Mills action quadratic in spatial fields. Basically, the term at linear power in $A_0$ is nothing but Gauss’ law. Integration over $A_0$ gives as a result the projection operator onto the states where Gauss’ law is fulfilled. Let us now make an integration over spatial fields. Since we have limited ourselves to quadratic part, this integration can be performed exactly. Resulting determinant appears to be the Vandermonde determinant but in $(-1/2)$ power as usual. In such a way we come to the obvious cancellation. We verified all these results in finite-difference lattice formulation in the same static gauge [56]. Then, in order to come to the partition function [40] we must look for such an expression for the projection operator where $A_0$ enters again at Gauss’ law in the action. Only then we will be able to separate the constant part of $A_0$ in the exponent. It is impossible, which of course can be easily seen from the formulae of this chapter.

Probably, a more appropriate partition function in the continuum theory just for $A_0$ gauge field has been conjectured in [57] where the authors have introduced an invariant integration over $A_0$ and postulated an effective action for this variable. This theory is able to describe confinement in QCD. We think that theory can exhibit spontaneous symmetry breaking by means of $A_0$ field condensation at high temperature [58].

5 $<A_0>$ in loop expansion

First, a possibility of $A_0$-condensation had been discussed in the standard loop expansion approach to the calculation of the effective action [15], [28], [26]. Just in this method the most essential questions such as gauge invariance of the condensate, the higher loop contributions, the thermodynamics of the $Z(N)$-phases have been examined. In this chapter we are going to go in a systematic way through the results of the
calculations and to discuss them. Our actual calculations will be done for QCD in the limit of high temperatures when the coupling constant is small and it is possible to expand in this parameter. All calculations are carried out in the background relativistic gauge $R_{\xi_{ext}}$ which incorporates the results of all relativistic gauges.

We begin with calculation of the two-loop effective action $W^{(2)}(A_0, \xi)$ of the gluonic external field due to the pure gluon contribution \[23, 28, 27, 31, 32\] (the quark contribution will be analyzed in chapter 7). The QCD Lagrangian in the relativistic background gauge reads:

$$L = \frac{1}{4} (G_{\mu\nu}^a)^2 + \frac{1}{2\xi} (D_{\mu} B_{\nu})^2 + \bar{\chi} D_{\mu} D_{\mu} \chi + \bar{\Psi} (\gamma_{\mu} \partial_{\mu} + im) \Psi + ig \bar{\Psi} \gamma_{\mu} (A_{\mu}^c + Q_{\mu}^c) (t^c)_{ab} \Psi^b,$$

(65)

$$G_{\mu\nu}^a = (D_{\mu} B_{\nu})^a Q_{\nu}^b - (D_{\nu} B_{\mu})^a Q_{\mu}^b - g f^{abc} Q_{\mu}^c Q_{\nu}^b,$$

$$Q_{\mu}^c = \delta_{\mu0}(\delta^{c2} A_0^2 + \delta^{cb} A_0^b),$$

where $(t^c)^a_b$ are the $SU(3)$ generators, $Q_{\mu}^a$ is the quantized field, $f^{abc}$ are the structure constants and $\bar{\chi}, \chi$ are the ghost fields. In what follows it will be convenient to introduce the "charged basis" of the gluonic fields:

$$\pi_{\mu}^\pm = \frac{1}{\sqrt{2}} (A_{\mu}^1 \pm iA_{\mu}^2) , \quad \pi_{\mu}^0 = A_{\mu}^3 , \quad K_{\mu}^\pm = \frac{1}{\sqrt{2}} (A_{\mu}^4 \pm iA_{\mu}^5) ,$$

$$\bar{K}_{\mu}^\pm = \frac{1}{\sqrt{2}} (A_{\mu}^6 \pm iA_{\mu}^7) , \quad \eta_{\mu} = A_{\mu}^8.$$

(66)

In this basis the background fields $A_{\mu}^3$ and $A_{\mu}^8$ enter into the momentum space Lagrangian as constant shifts of the zero momentum components. More details about the "charged basis" \[66\] and the Feynman rules are given in \[26\].

To determine the vacuum value of $A_0$ one should calculate the effective action $W(A_0, \xi)$ and find its minimum point $\langle A_0 \rangle_{min}$ via the minimization procedure. If this value occurs to be different from $\frac{2\pi n}{3g}$, it means that spontaneous breaking of the gauge symmetry happens.

The effective action $W(A_0, \xi)$ is given as a functional integral over periodic gauge and ghost fields and antiperiodic fermion fields \[23\]

$$\exp[-W(A_0)VT] = N \int \mathcal{D}Q \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{D}\bar{x} e^{- \int_0^\beta d^3x \left[ \mathcal{L} + J_0^a \varphi^a + \frac{g^2}{2} \sum_{i=1}^3 [B_2^2(a_i) + 2B_2(0)B_2(a_i)] + B_2(a_1)B_2(a_2) \right]},$$

(67)

where $\mathcal{L}$ is the Lagrangian \[34\], $N$ is a $T$-independent normalization factor, $V$ is space volume and $J_0^a$ is an external source. The effective action due to gluons up to two-loop order in the background gauges has been calculated in Ref. \[28, 24, 27, 31, 32, 34, 33, 33, 55\]. Since the result has crucial significance for what follows we have calculated it once more. Our result for $W(A_0, \xi)$ is the following

$$W(A_0, \xi) \beta^4 = \frac{4\pi^2}{3} (-\frac{1}{30} + \sum_{i=1}^3 B_4(a_i)) + \frac{g^2}{2} \sum_{i=1}^3 [B_2^2(a_i) + 2B_2(0)B_2(a_i)] + B_2(a_1)B_2(a_2).$$

24
\[
\begin{align*}
+ & B_2(a_2)B_2(a_3) + B_2(a_3)B_2(a_1) + \frac{1 - \xi}{3} g^2 [B_3(a_1)] [2B_1(a_1) + B_1(a_2) - B_1(a_3)] \\
+ & B_3(a_2)[2B_1(a_2) + B_1(a_1) + B_1(a_3)] + B_3(a_3)[2B_1(a_3) + B_1(a_2) - B_1(a_1)]
\end{align*}
\] (68)

where the notations have been introduced
\[
\begin{align*}
& x = \frac{g\beta A_0^3}{\pi}, \quad y = \frac{g\beta A_0^8}{\pi}, \\
& a_1 = \frac{x}{2}, \quad a_2 = \frac{1}{4}(x + \sqrt{3}y), \quad a_3 = \frac{1}{4}(-x + \sqrt{3}y),
\end{align*}
\] (69)

This expression differs from that of ref. [27] in the \(\xi\)-dependent part by the additional factor \(3/2\). Besides, all signs in the squared brackets are "plus" in [27]. Our result (68) coincides (up to the sign definition) with the result of ref.[33]. As is seen from (68), the gluon contribution depends on the gauge fixing parameter \(\xi\). In QCD at high temperature the coupling constant is small. So, one can calculate the minimum point of \(W(A_0, \xi)\) and the value of the functional in the minimum by an expansion in \(g^2\). Up to the second order we obtain:
\[
\beta^4 W_{gl} = -\frac{8}{45\pi^2} + \frac{1}{6} g^2 - \frac{g^4}{32\pi^2} (3 - \xi)^2 x_{min} = \frac{g^2}{4\pi^2} (3 - \xi), \quad y_{min} = 0
\] (70)

Here, the values of \(x_{min}\) and \(y_{min}\) have been found for the intervals \(0 \leq a_1, a_2 \leq 1, -1 \leq a_3 \leq 0\). Five other minima in the \((x, y)\)-plane can be found by means of consequent rotations of the coordinate system by the angle \(\frac{\pi}{3}\). From (70) it follows that the presence of the condensate lowers the action. So, the spontaneous generation of the condensate takes place. It is very essential that the depth of all minima is the same. Hence it follows that although the condensate breaks both gauge and \(Z(3)\) global symmetries, the generated phases preserve the rotation symmetry as in the case of \(Z(N)\)-phases. The description of the general structure will be done below after evaluating the quark contribution. These results are in the full accordance with those from the previous chapter. Nevertheless it is to be emphasized that in the loop expansion treatment there is no phase boundary whereas in the strong coupling lattice approach there is a phase transition to the state with \(< A_0 > \neq 0\).

As is also seen from (70), the vacuum value \(x_{min}\) as well as the minimum value of the functional \(W(x_{min})\) turn out to be dependent on the gauge fixing parameter \(\xi\). This is the \(\xi\)-dependence problem discussed by many authors [30], [27], [31], [32], [33], [35], [41]. In general, in gauge theories there are two ways of dealing with the problem of gauge invariance: 1) to use the explicitly gauge invariant formulation from the very beginning; 2) to apply the Green function analysis and extract the gauge invariant results for the observables via the Nielsen (the Ward type) identities. Besides, some hybrid method can be developed as well. As far as the problem of the \(A_0\)-condensate is concerned, methods of all kind have been applied and different conclusions have been extracted. In the previous chapter the explicitly gauge invariant approach to \(A_0\) was presented. In that scheme \(< A_0 > \neq 0\) is obviously a gauge invariant phenomenon. Here, we are going to explore the Nilson identity to demonstrate gauge invariance of the condensate.

25
One of the powerful methods of dealing with the $\xi$-dependence problem is to apply the Nielsen identities method which first was used in the investigations of spontaneous gauge symmetry breaking by radiative corrections [42], [58], [59]. As is well known, in models of the Coleman-Weinberg type both the effective potential $V(\Phi, \xi)$ of the scalar field $\Phi$ and its minimum point $\Phi_{\text{min}}$ are $\xi$-dependent. A very elegant resolution of this problem had been found by Nielsen [42] who proved the gauge invariance of this dynamical phenomenon. Namely, he formulated the Ward type identity describing an analytical dependence of $V(\Phi, \xi)$ on $\xi$. So any variation of the potential can be compensated by the corresponding variation in $\delta \Phi$ (along some characteristic line in the $(\Phi, \xi)$-plane). More details about these results can be found in [42], [58], [59]. In the papers [31], [32], [35] this method has been applied to the problem of $\xi$-dependence of the gluon condensate. First of all this gives a possibility to check the correctness of the calculations. Secondly, due to generality of the approach the relation to other methods of calculations can be established. Nielsen’s identities of a general form have been recently derived by Kobes, Kunstatter and Rebhan [60]. For the effective action they describe a variation of $W(\Phi)$ due to a variation of the gauge fixing term $F^\alpha(\Phi)$ and in the condensed De Witt notations are given by the expression:

$$\delta W(\bar{\Phi}) = W,^j \delta \Xi^j(\bar{\Phi}).$$  \hfill (71)

Here $\Phi^i$ is a gauge field, superscript “$i$” includes all discrete and continuous variables, $\bar{\Phi}^i$ denotes a vacuum value of the field, comma after $W$ means the variation derivative with respect to corresponding fields and the contraction means integration over continuous and summation over discrete variables. The variation $\delta \Xi^i$ describes changing of the mean field value due to the special gauge transformations

$$\delta \Phi^i = D^i,^\alpha(\Phi) \delta \Omega^\alpha$$  \hfill (72)

with the parameter

$$\delta \Omega^\alpha = -\Delta^\alpha,^\beta(\Phi) \delta F^\beta(\Phi).$$  \hfill (73)

This parameter is chosen to cancel the change $\delta S$ of the classical action

$$S_{g.f.}(\Phi) = S(\Phi) + \frac{1}{2} \eta_{\alpha,\beta} F^\alpha(\Phi) F^\beta(\Phi)$$  \hfill (74)

due to the variation of the gauge fixing term, $\delta F^\alpha(\Phi)$, and the ”metric” $\eta_{\alpha,\beta} \to \eta_{\alpha,\beta} + \delta \eta_{\alpha,\beta}$:

$$\delta' F^\beta = \delta F^\beta + \frac{1}{2} \eta^{\rho,\beta} \eta_{\alpha,\rho} F^\alpha$$  \hfill (75)

In Eqs.\hspace{1em} (72-75) $D^i,^\alpha(\Phi)$ are gauge transformation generators, $\Delta^\alpha,^\beta(\Phi)$ is the ghost propagator in the external field $\Phi$, $S(\Phi)$ is the classical action of the gauge fields. In quantum theory $\delta \Xi^i$ has to be calculated from the equation [50]

$$\delta \chi^i = - < D^i,^\alpha(\Phi) \Delta^\alpha,^\beta(\Phi) \delta' F^\beta(\Phi) >$$  \hfill (76)
where

\[ < O(\Phi) > = e^{-W(\Phi)} \int D\Phi O(\Phi) det[F^\alpha_i(\Phi)D^i_\beta] \exp(-S_{g.f.} - W_j(\bar{\Phi})(\Phi - \bar{\Phi}^j)) \]  

(77)

and we substituted the current \( J_j = -W_j \). In the background field gauge the gauge fixing functions depend on an additional external field \( \bar{\Phi}^i \). So the expression (76) should be replaced by

\[ \delta' \chi^i = (C^i_j)^{-1} \delta \chi^j \]  

(78)

where the function

\[ C^i_j(\bar{\Phi}) = \delta^i_j - < D^i_\alpha(\bar{\Phi})\Delta^\alpha_\beta(\Phi) \frac{\delta F^\beta(\bar{\Phi}, \Phi)}{\delta \Phi^j} > + \frac{1}{2} \eta^\alpha_\beta \frac{\delta \eta^\alpha_\gamma}{\delta \Phi^j} F^\gamma(\Phi, \bar{\Phi}) >_{\Phi = \bar{\Phi}} \]  

(79)

describes the dependence of the function \( F^\alpha \) on \( \bar{\Phi} \). The additional external field must be set equal to the vacuum value \( \Phi^i \) at the last step of the calculations. Besides, the latter has to be calculated from the effective action \( W(\bar{\Phi}, \Phi) \) with \( \Phi \) and \( \bar{\Phi} \) taken to be different. \( \delta^i_j \) is the Kroneker delta. In QCD one must use the total gauge field \( A^a_\mu = Q^a_\mu + \delta_{\mu 0}(\delta^{a3}A^3_\mu + \delta^{a8}A^8_\mu) \) as the field \( \Phi^i \). One must substitute \( D^i_\alpha = D^i_\mu(A + Q) \) as generators and \( \eta^\alpha_\beta = \frac{1}{2} \delta^{ab} \) as the metric tensor. Variations of \( \xi \) can be realized by the variations of the metric

\[ \delta \eta^\alpha_\beta = \delta' F^\alpha = -\frac{1}{2}(D^B_\mu(A)Q^a_\mu)\frac{\delta \xi}{\xi}. \]  

(80)

In the papers [31], [32] the relations (78), (79) have been adopted to the case of the background field gauge. To do it the external field \( \Phi = \bar{A} \) appearing in the gauge fixing function \( F^\alpha(\bar{\Phi}, \Phi) = (D^B_\mu(\bar{A})Q^a_\mu)^a \) should be initially assumed to be different from the actual background field \( B^a \equiv A^a = const. \) The former must be identified with the latter one at the end of the calculations. So the field \( \bar{A} \) in the covariant derivative \( D^B_\mu(\bar{A}) \) can be written as \( \bar{A}^\mu = (B + q)^\mu_a \) where \( q^a_\mu \) are the deviations from the solution \( B^a_\mu \). Taking \( q^a_\mu \) to be small one may calculate propagators, vertices, etc as series in \( q^a_\mu \) and keep only the linear terms. Then the functions \( \delta \chi^i(\bar{\Phi}) \) and \( C^i_j(\bar{\Phi}) \) are calculated by differentiating with respect to \( q^a_\mu \). On this way the identity (71) can be written as:

\[ \delta W(B) = W_q(B, q) |_{q=0} [C^a_\beta(B, q)]^{-1}_{q=0} \delta \chi^\beta(B, q) |_{q=0} \]  

(81)

where

\[ \delta \chi^\beta(B) = -\frac{1}{2} \frac{\delta \xi}{\xi} < D^B_d(B + Q)\Delta^d_\epsilon(B + q)(D^B_\mu(B + q)Q^\epsilon_\mu)_{q=0} >, \]  

(82)

\[ C^a_\beta = \delta^a_\beta + g f^{abc}\delta_{\mu 0}\delta^b_\beta < D^a_\epsilon(B + Q)\Delta^\epsilon_d(B + q)Q^\epsilon_\mu >_{q=0}. \]

The average values in (76), (82) should be calculated in perturbation theory considering quantum fluctuations \( Q^a_\mu \) and deviations \( q^a_\mu \) to be small. The internal index “\( a \)” in eq.
(81) takes now the values \( a = 3, 8 \) in accordance with the structure of the background field.

As is seen from eq. (68), the dependence of the effective action on \( \xi \) appears in the order \( g^2 \). So, in the lowest order both the left-hand side (LHS) and the right-hand side (RHS) of eq.(81) should be of the order \( g^2 \), as well. The covariant derivative \( D^\mu_a(\mathcal{A} + Q) \) contains the quantum field as a product \( gQ^\mu_a \). So, in the one-loop approximation the variation \( \delta \chi^{(1)} \) is of the order \( g \). The one-loop effective action \( W^{(1)}(B) \) has the zero order and its derivative with respect to \( B \) has the first order in coupling constant. Hence it follows that in the lowest order one must use in the identity (8 1) the one-loop functions \( \delta \chi^{(1)} \) and \( C^{(1)\alpha} \) to be equal to unity.

To calculate \( \delta \chi^a \) (82) it is necessary to take into account the explicit form of generators and the fact that only the third and eighth components of external field are to be non-zero. Thus, in the one-loop approximation the expression ( 82) reads

\[
\delta \chi^{(1) a} = -\frac{1}{2} g \frac{\delta \xi}{\xi} f^{abc} \Delta^d(x-y) \tilde{D}^d_{\nu} (B) G_{\nu 0}^c (x-y). \tag{83}
\]

The necessary structure constants in the basis (66) are

\[
f^{3\pi^+ \pi^-} = i, \quad f^{3k^+ k^-} = -f^{3k^+ k^-} = \frac{i}{2}, \quad f^{8k^+ k^-} = f^{8k^+ k^-} = \frac{i\sqrt{3}}{2} \tag{84}
\]

and the background covariant derivatives may be written as follows

\[
(D^B Q^a) = \tilde{D}^a_{\mu} \Pi^b_{\nu} \tag{85}
\]

where \( \Pi^b_{\nu} \) is the column

\[
\Pi^b_{\nu} = (\pi^+, \pi^-, \pi^0, k^+, k^-, \bar{k}^+, \bar{k}^-, \eta)^T \tag{86}
\]

and values of the variables \( a, b \) now are: \( a, b = \pi^+, \pi^-, ..., \eta \). Diagonal elements of \( \tilde{D}^{ab}_{\mu} \) are the following:

\[
diag \tilde{D}^{ab}_{\mu} = (\tilde{D}^{a_1}_{\mu}, \tilde{D}^{-a_1}_{\mu}, \partial_{\mu}, \tilde{D}^{a_2}_{\mu}, \tilde{D}^{-a_2}_{\mu}, \tilde{D}^{a_3}_{\mu}, \tilde{D}^{-a_3}_{\mu}, \partial_{\mu}) \tag{87}
\]

where \( a_i \) stand for the background fields (69) describing the contributions of the isospins \( I, V \) and \( U \) spin subgroups of the SU(3) group, respectively. Using the explicit forms of the gluon and ghost field propagators

\[
C_{\mu \nu}^{ab}(x-y) = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} e^{-ip(x-y)} \left( \delta_{\mu \nu} \frac{\delta_{ab}}{(p^\mu)^2} + (\xi - 1) \frac{p^\mu p^\nu}{(p^\mu)^4} \right) \tag{88}
\]

\[
\delta^{ab}(x-y) = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} e^{-ip(x-y)} \frac{\delta^{ab}}{(p^\mu)^2} \tag{89}
\]

28
where
\[ p_\mu^c = [p^0 + a^c, \vec{p}], \quad p^0 = \frac{2\pi n}{\beta}, n = 0, \pm 1, \ldots \] (90)
and
\[ a^{\pi\pm} = \pm g B_0^3, \quad a^{k\pm} = \pm \frac{g}{2} (B_3^3 + \sqrt{3} B_8^8), \quad a^{\pi} = \pm \frac{g}{2} (-B_0^3 + \sqrt{3} B_0^3), \quad a^0 = \alpha = 0 \] (91)
and substituting (85)-(89) in eq.(83) one obtains:
\[ \delta \chi^3 = \frac{g}{4\pi \beta} [B_1(a_1) + \frac{1}{2} B_1(a_2) - \frac{1}{2} B_1(a_3)] \delta \xi, \]
\[ \delta \chi^8 = \frac{g \sqrt{3}}{4\pi \beta} \left[ \frac{2}{3} B_3(a_2) + B_1(a_3) \right] \delta \xi. \] (92)
The derivatives of the one-loop parts of \( W(A_0, \xi) \) in eq.(68) with respect to \( B_3^3 = \lambda_0^3 \) and \( B_8^8 = \lambda_0^8 \) equal the expressions:
\[ \frac{\partial W_{gl}^{(1)}}{\partial B_3^3} = \frac{4g\pi}{3\beta^3} [2B_3(a_1) + B_3(a_2) - B_3(a_3)], \]
\[ \frac{\partial W_{gl}^{(1)}}{\partial B_8^8} = \frac{4g\pi}{3\beta^3} \sqrt{3} [B_3(a_2) + B_3(a_3)]. \] (93)
By summing up the corresponding products of expressions (92), (93) one obtains the gluon contribution to the RHS of eq.(81). The obtained expressions coincide up to the sign with the derivative of (68) with respect to \( \xi \). Thus, the Nielsen identity
\[ \frac{dW_{gl}}{d\xi} = \frac{\partial W_{gl}^{(1)}}{\partial \xi} + \frac{\partial W_{gl}^{(1)}}{\partial B_3^3} C_3^{(1)} + \frac{\partial W_{gl}^{(1)}}{\partial B_8^8} C_8^{(1)} = 0 \] (94)
is satisfied up to the two-loop order. In eq.(94) we have denoted \( C_{3,8}^{(1)} = \frac{\delta \chi_3, \delta \chi_8}{\delta \xi} \). The identity (94) is just the characteristic equation. So, from the equation it follows that along characteristic lines in the \((A_0, \xi)\)-plane the effective action is not changing. In particular, this is the case for its minimum value (70). In accordance with general theory of Nielsen’s identity approach this means that the gluon condensation at finite temperature is a gauge invariant phenomenon. The fact that identity (94) holds means as well that loop expansion of \( W(A_0, \xi) \) is the self-contained procedure and no other \( \xi \)-dependent diagrams should be included in the order \( g^2 \). Only \( \xi \)-independent terms may be added, in principle, to eq.(68). From Refs. [42], [58] it also follows that along an orbit which passes through the point \((\xi, x_{min} \neq 0)\) not only the minimum value of the effective action \( W(\xi, (A_0)_{min}) \) but all other observables (particle masses, \( S \)-matrix elements, etc) are to be constant as well. This property selects the unique orbit among other ones.

As it was mentioned before, \( \xi \) dependence of the effective action (68) has called in question a possibility of the gluon field condensation because physical phenomena
must be gauge independent. So, in a number of papers some gauge invariant methods of calculations have been proposed. Historically the first of them has been used by Belyaev ([30]) who introduced a special reparametrization of the background field $A_0$ in terms of the Polyakov loop ([18]). Applying this procedure to $SU(2)$ gluodynamics he came to the conclusion that there is no real condensation at the two-loop level. The same result has been derived by this method in $SU(N)$ gluodynamics in Ref.[41]. Thus, it turns out that two methods of calculations give opposite conclusions about $A_0$-condensate (at two-loop level). Let us try to find the origin of the discrepancy.

First of all let us briefly describe Belyaev’s method and consider $SU(2)$ case for simplicity ($I$-spin subgroup of the $SU(3)$ group in (68)). The main idea is to define the background field $x$ in (68) in terms of the Polyakov loop $<\text{Tr}W>$ with quantum fluctuations included. This ”classical” or ”measured” value, $x_{cl}$, can be calculated via $<\text{Tr}W>$ and in the one-loop approximation the relation between $x$ and $x_{cl}$ has the form

$$x = x_{cl} + g^2 f(x_{cl}) + o(g^2) \quad (95)$$

where $f(x_{cl})$ is a function which has to be found. After the redefinition the effective action reads

$$W(x) = W(x_{cl}) = W^{(1)}(x_{cl}) + g^2 \left[ \frac{d}{dx_{cl}} W^{(1)}(x_{cl}) f(x_{cl}) + W^{(2)}(x_{cl}) \right] + o(g^2). \quad (96)$$

It has been expected that at least the minimum position in $W(x_{cl})$ is gauge invariant. After calculating the Polyakov loop in the one-loop approximation the following relation has been obtained:

$$x = x_{cl} + \frac{g^2}{4\pi^2} B_1(x_{cl}/2)(\xi - \xi_0) \quad (97)$$

where $\xi_0$ is an arbitrary fixed number and $x = x_{cl}$ when $\xi = \xi_0$. It is the gauge where the renormalization of the gluon propagator is absent. Substituting (97) into the effective potential and taking into account the explicit form of the Bernoulli polynomials, the final result for the two-loop effective action has been found

$$\beta^4 W(x_{cl}) = \pi^2 \left[ -\frac{1}{15} + \frac{1}{12} x_{cl}^2 (x_{cl} - 2)^2 \right] + g^2 \left[ \frac{1}{24} - \frac{5}{96} x_{cl}^2 (x_{cl} - 2)^2 \right] + o(g^2). \quad (98)$$

This functional has a minimum at $x_{cl} = 0$, which means the absence of the condensate at two-loop level. Now, we analyze this procedure in more details. Actually, it interpolates between Green’s function methods (described above) and completely gauge invariant methods like the lattice formulation discussed in the previous chapter. Really, in the Refs. [30], [41] as a first step the Green functions are used to calculate the effective action. As a second one the reparametrization of the background field in terms of $<\text{Tr}W>$ is performed. The latter may occur to be not consistent. The point is that in the loop expansion method, as a rule, the background field $A_0$ is chosen to be a solution of classical field equations and is to be considered as a fixed parameter through all calculations. Its vacuum value must be determined via the minimization procedure. On the other hand, if one expresses $A_0$ in terms of colourless parameter $<W>$ (but,
in principle, other parametrizations are possible) with quantum corrections to be taken into account and substitutes it as an argument in the effective action then the variation of $W(A_0)$ with respect to $<W>$ may not correspond to the determination of vacuum value $<A_0>$ considered as the dominant classical configuration in the initial partition function. It seems to us that it would be more natural to express $W(A_0)$ in terms of $<W>$ from the very beginning and apply a loop expansion to this functional. This scheme was described in the previous chapter and we shown that the condensate does appear in such a treatment.

After these remarks it will be very instructive to describe Belyaev’s result in terms of variables on characteristics. The basic point here is that the relation (97) and "observable" background fields coincide identically with the characteristic in the $(x, \xi)$-plane which passes through an arbitrary point $(x_0 = x_{cl}, \xi_0)$. In order to find the value of $W(x, \xi)$ on the characteristic one must substitute eq.(97) in eq.(68) (for the $I$-spin subgroup only in $SU(2)$ case) and then expand in powers of $g^2$ to order $g^2$. After that one obtains the equation

$$
\beta^4 W_{\text{charact}}(x_0, \xi) = \frac{2\pi^2}{3} [B_4(0) + 2B_4(x_0/2)] + \frac{g^2}{2} [B_2^2(x_0/2) + 2B_2(0)B_2(x_0/2)]
+ \frac{2}{3} g^2 (1 - \xi_0)B_3(x_0/2)B_1(x_0/2) \quad (99)
$$

As has been expected, on the orbit the effective action is independent on the parameter $\xi$ but it is a constant depending on $x_0, \xi_0$. The minimum position and minimum value of $W_{\text{charact}}$ are

$$
(x_0(\xi_0))_{\text{min}} = \frac{g^2}{8\pi^2} (3 - \xi_0), \quad (100)
$$

$$
\beta^4 W_{\text{charact}}((x_0)_{\text{min}}, \xi_0) = -\frac{\pi^2}{15} + \frac{g^2}{12} - \frac{g^4}{192\pi^2} (3 - \xi_0)^2. \quad (101)
$$

Taking into account that $(x_0)_{\text{min}}$ can be identified with $x_{cl}$ one comes to the conclusion that $(x_0)_{\text{min}}$ in eq.(101) is the gauge invariant "measured" value of the gluon condensate. This is the exact meaning of the $\xi$-independence of $W_{\text{charact}}(x_0, \xi)$. Moreover, in this way the idea to express $W(A_0, \xi)$ in terms of the Polyakov loop can be realized in the Nielsen identity approach. As we remarked before, this possibility appears due to the fact that characteristics in the $(x, \xi)$-plane and the relation of $x_{cl}$ and $x$ are given by the same equation (97). If one puts $\xi_0 = 3$ in eqs. (99), (101), the effective action (98) and other results of Refs. [30], [41] immediately follow. Hence it is possible to conclude that Belyaev’s method corresponds to the choice of the orbit which passes through the point $x_{cl} = x_0 = 0, \xi_0 = 3$. This is a special gauge where $x_{cl} \neq 0$ might be determined in the three-loop approximation. All other gauges signal $< A_0 > \neq 0$ at two-loop level.

In Ref. [33], [41] the gauge-invariant thermodynamical potential $\Omega(A_0)$ has been calculated step by step in perturbation theory. In this method the minimum position of the next order of the potential $\Omega^{(n)}(A_0)$ has to be calculated via the effective action
$W^{(n-1)}(A_0, \xi)$ of the previous order. In this way the result $< A_0 >= 0$ in two-loop approximation has also been obtained. But at the same time the non-trivial minimum position for the three-loop thermodynamical potential $\Omega^{(3)}(A_0)$ was found. Hence, a good chance for determination of the gluon condensation at this level has appeared in the gauge-invariant method of calculation. Thus, we come to the idea that a final determination of $A_0$-condensate needs a calculation of higher loop contributions. This is not a surprise because $< A_0 >\neq 0$ is the two-loop effect and to prove it the first quantum corrections should be calculated. This very complicated mathematical task has not been solved yet. At the present time in the literature only some partial results of the role of the higher loops have been reported. For completeness we shall describe them here.

Generally speaking, when the Green function method is used all the observables and, in particular, the higher loop contributions should be calculated with Nielsen’s identities taken into account. In calculation of $W(A_0, \xi)$ it may occur that some diagrams of special kind give $\xi$-dependent contributions. Then the only way to check the correctness of the approximation scheme is to apply the Nielsen identities. From this standpoint it is very essential, as we have been convinced before, that the loop expansion is a consistent approximation scheme and no other $\xi$-dependent diagrams should be added to $W^{(2)}(A_0, \xi)$ in order $g^2$. Only $\xi$-independent contributions can be included. Keeping these arguments in mind, let us discuss the results of Refs. [29], [55] where the ring diagrams (plasmon diagrams) $W_D(A_0)$ have been calculated in the Feynman gauge ($\xi = 1$). As the main result it has been found that when one considers the sum of $W_D$ and $W^{(2)}$ the solution $< A_0 >= 0$ in the order $g^2$ follows. However, from the above remarks it is clear that the conclusion contradicts to what Nielsen’s identities tell us. To understand the situation in details we have calculated the contribution of the ring diagrams $W_D(A_0, \xi)$ in the $R_\xi^{ext}$ background gauge. It was found that this contribution is $\xi$-dependent as well as $W^{(2)}(A_0, \xi)$. Moreover, if we consider the sum $W^{(2)}(A_0, \xi) + W_D(A_0, \xi)$ the result $< A_0 >= 0$ follows again, as in [29]. At the same time, when one substitutes $W_D(A_0, \xi)$ in the identity (94), it does not fulfilled. So, the contribution $W_D(A_0, \xi)$ is inconsistent. The resolution of this contradiction lies in the way of calculation of $W_D(A_0, \xi)$ in [29], [55].

As is known [3], in QCD and QED (with $A_0 = 0$) the sum of the ring diagrams describes the contribution of infrared divergencies to the effective action and results in the non-analytic term of order $g^3$. In this case the infrared limit is calculated as follows: $k_n=0, |\vec{k}| \to 0$ [3]. The same definition is used in [29] in the case of $A_0 \neq 0$. But actually this may not be the case and another definition must be considered. In [61] the following one has been introduced: $k_0 = k_4 + g\beta A_0 = 0, |\vec{k}| \to 0$. Besides, the additional contribution resulting from the transversal part of the polarization tensor has also been calculated [11]. After that the obtained corrections appear to be gauge independent and in the limit $A_0 \to 0$ reproduce the result for the $A_0 = 0$ case. So, just this non-local term should be included to the effective action $W(A_0, \xi)$. With this contribution we have no $A_0$-condensate elimination.

Other essential result for understanding the properties of the higher loop effects
has been reported in [33] where, in particular, the position of the three-loop thermodynamical potential $\Omega^{(3)}(A_0)$ was determined which is both non-trivial and $\xi$-dependent. Hence, the hope to determine $\langle A_0 \rangle \neq 0$ at the three-loop level has obtained a real confirmation. Anyway, the calculation of the three-loop contribution to the effective action remains of great importance for the final solution of the discussed problem.

6 Effective Lagrangian approach to $A_0$ condensation

It is well-known that gauge fields are invariant under global $Z(N)$-transformations. But at finite temperature the Polyakov loop transforms under $Z(N)_{gl}$ subgroup as matter fields in the fundamental representation (19). This implies that eigenvalues of the Polyakov loop or $A_0$ have the charges of centre of $SU(N)$ like quarks. Besides, the Polyakov loop transforms under $SU(N)_{gl}$ group like the matter fields in the adjoint representation (27). It is a motivation to consider $A_0$ as the Higgs field. Basically, the action $S^G$ (25) has the Higgs form for $\phi_x$ defined in eq. (26). These fields are living in the lattice sites, they have non-trivial self-interaction and can provide a minimum far away from zero. So the action $S^G$ is more likely to be the ”standard” $SU(2)$ Higgs lattice action for the $\phi$-fields. The evidences that at high temperature $A_0$ indeed behaves as the Higgs field in the continuum theory and its Lagrangian has the Higgs form have been given in [37, 38, 39].

In the continuum theory the minimum of the pure gauge action is reached for the semiclassical values of the potentials $A_0^a = \text{const.}$ and $A_i^a = 0$. If we admit a semiclassical value of $A_0^a \neq 0$ we immediately find the massive chromomagnetic gluons in the effective theory. To obtain the action for the vacuum corresponding to $A_0^a \neq 0$ we integrate out these massive modes following the philosophy of Appelquist-Carrazzone decoupling theorem [22]. As a result we obtain an effective Higgs potential for quantum fluctuations of field $A_0$ treated as the Higgs field. This Higgs potential is periodic as a function of chromoelectric gauge field with period $2\pi g_\beta$. Due to this periodicity the new vacuum of the effective system possesses the periodic symmetry

$$A_0^a \rightarrow A_0^a = A_0^a + \frac{2\pi}{g_\beta}$$

together with the parity symmetry

$$A_0^a \rightarrow A_0^a = -A_0^a.$$

For $A_0$ within the interval of periodicity the effective $SU(2)$ three-dimensional theory has been built and it has been shown that the global gauge symmetry is broken due to formation of non-zero expectation value of $A_0$. On the contrary, in the case where all the semiclassical potentials vanish the corresponding effective system is completely different. We find there massless gluons interacting with Higgs field in a state with unbroken symmetry characterized by the effective Higgs potential with global minimum
for $A_0 = 0$ displaying only parity symmetry. Transition to the system described earlier must be accompanied by a symmetry breaking process.

Let us now give a sketch of the calculation and show how the global gauge symmetry breaking happens. The Fourier expansion with respect to the compactified dimension enables us to reduce the four dimensional theory to the effective three dimensional field model. The Fourier field components acquire frequency dependent masses and therefore it looks reasonable to apply for nonzero frequency modes the Appelquist-Carazzone decoupling theorem\cite{22} to calculate an effective theory for static field modes. This process generally known as dimensional reduction was described by Appelquist and Pisarski\cite{23} as a possible calculation scheme for high temperature behaviour of field theories. Following this approach, an effective static model in three dimensions was proposed, reasonably describing the full four dimensional theory in the distance range $RT \approx 1$ at sufficiently high temperature in the deconfinement phase\cite{38}. The idea of calculation consists in a reduction of a number of degrees of freedom in the continual integral definition of the partition function and/or the thermal mean values of the observables. The Fourier expansion of the fields with respect to the compactified direction was applied which enables us to integrate out the nonstatic modes of the fields. The frequency dependent “mass” term prevents the procedure from infrared divergencies. The ultraviolet divergencies are regularized by zeta function regularization method. This procedure results in a nonlocal functional determinant. Expanding the determinant we obtain finally the effective action for static modes in three dimensions. This expansion is obtained within the background field method for the chromoelectric field in the static gauge

$$A^3_0(x) = A^B_0 + \varepsilon(x) \tag{102}$$

where $\varepsilon(x)$ represents the quantum field fluctuations around the constant field value $A_0$. In the effective theory the chromoelectric field is identified with the Higgs field in the adjoint representation\cite{16}. The effective action depends explicitly on the static field value $A_0$ through the effective Higgs potential. The mean values of operators calculated by this method must be complemented by the information about the value of $A_0$ in the minimum of the effective potential. Depending on the number of degrees of freedom chosen we can calculate the different effective systems. To decide which effective system for fixed finite temperature corresponds to the full system is the question of comparison the minima of the effective potentials corresponding to the different effective actions derived from the action of the full system. We suppose that in our approach the partition function is not changed by the mathematical operations in the process of the calculation of the different effective systems. Therefore the system with lower minimum of the effective potential is the most probable effective system corresponding to the full theory at fixed temperature.

Starting from $SU(2)$ gluodynamics we have used the action (15) expanded up to the second order in the chromomagnetic fields. Following the calculation described above we come to the effective potential for $A^B_0$ of the form:

$$U_{\text{eff}}(X) = -\beta^{-4} \left( \frac{6\pi^2}{90} - \frac{4\pi^2}{3} [X^4 - 2X^3 + X^2] \right) - \frac{1}{V\beta} \ln(\text{Det}^{-\frac{1}{2}}(\text{S}_{\text{cor}}(\chi > \chi_{\text{min}}, X))) \tag{103}$$
where

\[
S_{\text{cor}}(k, X) = \frac{(4\pi l)^2}{2} + g_r^2 \left\{ 2(X^2 - X + \frac{1}{6}) - \frac{11}{3} l^2 [(1 - \ln(4\pi) - \Psi(X) - \Psi(1 - X)] + \frac{l^3 \pi^3}{\sin^2(\pi X)} \\
+ \sum_{-\infty}^{\infty} \left\{ - \left( \frac{2l^4}{|n + X|^2} + 2l^2 \right) \frac{l}{l} \arctan \frac{l}{|n + X|} \\
+ |n + X| + \frac{5}{3} \frac{l^2}{|n + X|} \right\}\right\}
\]  

(104)

Here we have used the new field variable \( X = g_{\beta A_0}/(2\pi)^{-1} \), \( g_r \) is the temperature dependent running coupling constant, and \( 4\pi l = \beta k \) is the dimensionless momentum.

The functional determinant in (103) is calculated by integration over variable \( \chi \) by means of the \( \zeta \)-function regularization scheme. Analyzing the last equation we find the following picture. For larger coupling constant (low temperature) the extreme of the \( U_{\text{eff}}(X) \) is achieved at \( X = \frac{1}{2} \). This value corresponds to the confinement phase where the expectation value of the Polyakov loop is equal to zero. When coupling constant is decreasing the minimum value of the potential is moving from the point \( X = \frac{1}{2} \) and there will appear two symmetric minima which signal that \( SU(2) \) global gauge symmetry is broken as in these minima \( X \neq n \) where \( n \) is any integer. This means that in this phase the condensate falls, in full accordance with our previous investigations. In the same time we found that the Polyakov loop in this phase differs from zero. In this sense this calculations are very similar to those presented in chapter 4 where the \( \langle A_0 \rangle \neq 0 \) generation is accompanied by deconfinement phase transition.

Our approach differs from that of Weiss one [28] by taking into account the quantum fluctuations around \( A_{B0} \) gauge potential.

It is interesting to compare the calculations presented above with the similar ones done by Oleszczuk in [38] where it has been shown that \( \langle A_0 \rangle \neq 0 \) does not appear at one-loop level. We find the differences in treating both the chromoelectric part of the initial action and massive static modes of the chromomagnetic potentials. In our approach the massive static modes of the chromomagnetic potentials were included in the effective potential whereas in [38] these modes were missed. As the result the Oleszczuk vacuum state contains two states with mass term proportional to \( (A_{B0}^2) \). It implies that the effective potential "wants to correct" the situation and final expression for the \( U_{\text{eff}} \) does not contain the third order of the background field and possesses the minimum for \( A_{B0}^2 = 0 \). In this minimum the masses proportional to \( (A_{B0}^2) \) vanish and the former massive modes become massless and, therefore, they are a part of the vacuum.

Nevertheless, if we build the semi-classical expansion around the gas of chromomagnetic monopoles [38] the result \( \langle A_0 \rangle \neq 0 \) can be recovered.

As we promised in chapter 4, we are now discussing the problem of the Vandermonde determinant cancellation. In the static gauge this determinant appears as the
Fadeev-Popov determinant and can be calculated directly without the ghost formalism. In [38] it has been shown that the Fadeev-Popov determinant is indeed cancelled by contributions coming from the integration over transversal chromomagnetic potentials. It happens, as was discussed in [40], even beyond one-loop level. However, it does not lead to the conclusion that $A_0 = 0$ in the vacuum [40]. In fact the determinant is cancelled from the effective potential even when $A_0^B \neq 0$ [38]. This fact has very simple and beautiful explanation. As we pointed out earlier the integration over chromomagnetic gauge field in the present approach is equivalent to projection onto gauge-invariant states. It means that if the cancellation takes place when $A_0^B = 0$ the same must happen also for $A_0^B \neq 0$ if the latter is a gauge-invariant phenomenon. From this fact we can deduce that the present calculations performed in the static gauge lead to the gauge-invariant results since the mentioned cancellation does take place.

7 $A_0$-condensate in theory with dynamical quarks

As it follows from the title, this chapter will be devoted to studying the influence of the dynamical quarks on the condensate. We may say that quarks do not change a general picture in essential way and both lattice strong coupling approximation with massive quarks [36], [51] and loop expansion with massless quarks [31], [34] have shown that condensate does not disappear. Nevertheless there are some disagreements between these two approaches which should be clarified. Let us begin with calculation of the quark contribution on the lattice.

Two approaches can be developed to solve this task. In [51] the hopping parameter expansion was utilized to include the effects of massive quarks in Euclidean version of the lattice theory. In [36] the quark contribution was obtained in Hamiltonian formulation by means of Banks-Ukawa method [25] and calculation of first not-trivial term of high temperature expansion of the fermionic determinant. Since the results have been obtained to be essentially the same, we limit ourselves here to elaborate on the second approach as it looks as being reliable in more broader range of parameters.

Lattice Hamiltonian for Kogut-Susskind fermions is of form

$$S_{K-S} = \frac{1}{2} \sum_{x,n=-d}^d \eta_n(x) \bar{\Psi}(x) U_n(x) \Psi(x + n) + m_q a \sum_x \bar{\Psi}(x) \Psi(x),$$

$$\eta_{-n} = -\eta_n, \quad U_{-n}(x) = U^+_n(x - n)$$

(105)

where $\eta_n(x) = (-1)^{x_1 + x_2 + \ldots + x_{n-1}}$. We remind that we should now use the projection operator (15) where

$$q^a = \bar{\Psi}(x) \lambda^a \Psi(x).$$

(106)

The integration over the quark part of the Hamiltonian yields the factor $Z_q$ and for the massless quarks (for the sake of simplicity let us take it for a short while) the result turns out to read

$$Z_q = Z^0_q \det(I + \frac{\delta_{ab}}{q^a_{\sigma} a_{\sigma}} D^{ab}_{xy})$$

(107)
where we have noted
\[ q^a_x = 1 + \exp(i\phi^a_x) \]  
(108)
and
\[ D_{xy} = \frac{1}{2} \sum_n (U_{xy}\delta_{y,x+n} - U^+_{xy}\delta_{y,x-n}). \]  
(109)

The diagonal part of the fermionic determinant \( Z^0_q \) resulting from the projection operator is explicitly given as \[ 25 \]
\[ Z^0_q = \prod_x [1 + \text{Re}\Omega(\phi_x)]. \]  
(110)

For non-diagonal part of the determinant the high-temperature expansion can be constructed. The necessary details can be found in \[ \text{[62]} \]. After calculation we come to the following quark contribution restricting ourself to the first non-trivial term in high temperature expansion
\[ -S^Q_{\text{eff}} = \ln Z^0_q + \ln \left( \prod_{x,n} dU_n(x) \exp\left[ -\frac{\beta^2}{8a^2} Tr \sum_{x,n} \frac{1}{q} U^1_q \right] \right). \]  
(111)

The dots represent here contributions of the next even terms of logarithmic function expansion. All these terms are negative therefore we can regard the maximal values of \( Z^0_q \) to correspond to the minimal values of the quark effective potential. It is clear that maximum of \( Z^0_q \) is achieved when \( <\phi> = 0 \). This result is very essential as it shows the gluon condensate generation is caused by gluonic kinetic energy only but not the quark contributions. This point is to be in disagreement with the loop expansion method where condensate appears either from gluon or from quark sector (see discussion below).

Further we choose to work with \( SU(3) \) gauge theory. As in the pure gluodynamics, there are six solutions here, however, due to the explicitly broken \( Z(N) \) symmetry they are not equivalent. Now the basic maximum is developing at \( <\phi_1> = <\phi_2> = 0 \). It is clear that evaluating the effect of massive quarks a la Banks-Ukawa \[ 25 \] i.e. making the substitution
\[ q^a_x \rightarrow \exp(\beta m_q) + \exp(i\phi^a_x) \]  
(112)
we come to the same conclusion: condensate does not appear at any temperature.

Performing now the invariant integration in (111) we have up to an irrelevant constant
\[ -S^Q_{\text{eff}} = \ln[1 + \text{Re}\Omega(\phi)] - \frac{d\beta^2}{16a^2} \left[ \frac{\text{Im}\Omega(\phi)}{1 + \text{Re}\Omega(\phi)} \right]^2 \]  
(113)
and then combining (112) and (113) with substitution (112) we have analyzed the effective potential \( S_{\text{eff,G}} + S^Q_{\text{eff}} \) at different values of \( \gamma \) and \( m_q \) and found the following picture. At larger \( \gamma \) there is one minimum where condensate is absent. With temperature increasing (\( \gamma \) decreasing) a phase transition takes place at \( \gamma^c_q \) which is somewhat larger than for pure gluodynamics. Below \( \gamma^c_q \) the effective action develops two minima where \( <\phi_1> \neq 0, <\phi_2> \neq 0 \). It implies that besides spontaneous breaking of the
global gauge symmetry the condensate generation may lead to spontaneous breaking of the colour charge symmetry. This conclusion is in some contradiction to the loop expansion method as well. Such a breaking means that in the corresponding minima the baryonic number is generated. We elaborate this result in details in the next chapter. However, at very high temperature when quark mass is vanishing we have found that the deepest minimum of the effective action appears to be $C$-symmetric again with $<\phi_1>\neq 0$, $<\phi_2>=0$. In such a way the disagreement with loop calculations can be avoided. Hence, our more essential conclusion is that $A_0$-condensate falls at high temperature in full QCD. $<A_0>$ is non-vanishing because of fluctuations of the gluonic kinetic energy and does not present in the quark sector of QCD.

Now let us present the results of loop calculations in the continuum QCD with dynamical quarks. All our notations follow chapter 5 where we discussed the loop approach in gluodynamics. The one-loop contribution of massless quarks to the $W(A_0)$ has been calculated in [15]. The two-loop quark functional was obtained in Refs. [34], [35]. The Feynman rules and necessary integrals and sums are listed in Appendix A of Ref. [34]. Thus, up to two-loop order we have

$$W_q(A_0) = -\frac{4\pi^2}{3} \sum_{i=1}^{3} \{B_4(c_i)\}$$

$$-\frac{1}{2} g^2 \{ B_2(a_1) [B_2(c_1) + B_2(c_2)] + B_2(a_2) [B_2(c_1) + B_2(c_3)]$$

$$+ B_2(a_3) [B_2(c_2) + B_2(c_3)] - \frac{1}{3} \sum_{i=1}^{3} [B_2^2(c_i) - 2B_2(0)B_2(c_i)]$$

$$- B_2(c_1)B_2(c_2) - B_2(c_2)B_2(c_3) - B_2(c_3)B_2(c_1) \}$$

$$+ \frac{(\xi - 1)}{3} g^2 \{ B_1(a_1) [B_3(c_1) - B_3(c_2)]$$

$$+ B_1(a_2) [B_3(c_1) - B_3(c_3)] + B_1(a_3) [B_3(c_2) - B_3(c_3)] \} \right) \), \tag{114}$$

where $x, y$ and $a_i$ are defined in [39] and

$$c_1 = \frac{1}{4} (x + \frac{1}{\sqrt{3}} y + 2), \quad c_2 = \frac{1}{4} (-x + \frac{1}{\sqrt{3}} y + 2),$$

$$c_3 = -\frac{1}{2\sqrt{3}} y + \frac{1}{2} \tag{115}$$

where $N_f$ is a number of quark flavors, $B_i(x)$ is the Bernoulli polynomial of the order $i$ defined modulo 1. Calculating the minimum position and minimum value of this effective action we obtain

$$\beta^4 W_q = -\frac{\pi^2}{60} \frac{7}{2} N_f + g^2 \frac{5}{72} N_f - g^4 N_f \frac{(3 - \xi)^2}{192\pi^2},$$

$$x_{\min} = g^2 \frac{3 - \xi}{4\pi^2}, \quad y_{\min} = 0. \tag{116}$$
Here, the minimum values were calculated for intervals

\[ 0 \leq a_1 \leq 1, \ 0 \leq a_2 \leq 1, \ -1 \leq a_3 \leq 0, \]
\[ 0 \leq c_1 \leq 1, \ 0 \leq c_2 \leq 1, \ 0 \leq c_3 \leq 1 \] (117)

and, as in the pure gluodynamics, five other minima in \((x, y)\)-plane can be found by means of consequent rotations by the angle \(\pi/3\). It is interesting to notice that the value of the condensate is the same both in the gluon and in the quark contributions. Moreover, both fermions and bosons act to lower the action of \(A_0\) field. This is rather unusual property that the condensate is produced in both sectors of the theory separately. As a total result, we have in the two loop approximation

\[
\beta^4(W_q + W_{gl})_{min} = -\pi^2\left(\frac{8}{45} + \frac{7}{60}N_f\right) + g^2\left(\frac{1}{6} + \frac{5}{72}N_f\right) - g^4\left(\frac{3 - \xi}{32\pi^2}\right)\left(1 + \frac{N_f}{6}\right) \] (118)

It is worth mentioning that \(y_{min} = 0\) in the deepest minimum (what means - in the vacuum). Hence, it follows that \(A_0\)-condensate does not effect a baryon charge at high temperature. Now let us elaborate the vacuum structure of gluon condensate which results from (70) and (118). The quarks change the symmetry of the vacuum. As is well known, the Bernoulli polynomials are defined modulo 1. Hence, the effective action possesses the symmetry in the \((x, y)\)-plane. We display this symmetry in the Fig.1. Any dot in the plane can be translated along dashed lines. Vacuum structure consists of the hexagonal and triangular elements, which cannot be transformed one to another by translations. At the translations the hexagonal elements pass to themselves. So do the triangular ones.

The global minima of the effective potential \(W_q + W_{gl}\) are marked by dots. Besides, in each triangular area there are six local minima which are not depicted. The local minima appear only if the quark and gluon contributions are included together. They disappear when one considers the gluon contribution only. The local minima depths decrease and the global ones increase while the number of quark flavors \(N_f\) becomes bigger.

At some \(N_f\) the local minima disappear completely. To understand the effect of quark mass we also have calculated the contribution (113) with \(m_q \neq 0\). Unfortunately, this calculation is incomplete because of complexity of integrals and the result has been obtained for small \(m_q\) only. It qualitatively looks as follows. The difference between the local and global minima decreases with the quark masses increasing. The local minima are equal to global ones and we obtain the same result as in the gluodynamics. Thus, mainly the light quarks bring up to an appearance of local minima which can be identified with metastable phases of the quark-gluon matter. So, one can assume that at some intermediate temperature the phase transition from the global minimum to the local one takes place. This transition may lead to \(y_{min} \neq 0\) in a vacuum. So, it can be expected that the chiral phase transition is accompanied by the baryon number generation and by spontaneous breakdown of the charge symmetry.

Now let us check the identity (94) for the quark contribution. Differentiating the
one-loop part of $W_q$ with respect to $B^3 = \overline{A}^3_0$ and $B^8 = \overline{A}^8_0$ we have

$$\frac{\partial W_q^{(1)}}{\partial B^3_0} = -\frac{4\pi}{3\beta^3} g[B_3(c_1) - B_3(c_2)],$$

$$\frac{\partial W_q^{(1)}}{\partial B^8_0} = -\frac{4\pi}{3\sqrt{3}\beta^3} g[B_3(c_1) + B_3(c_2) - 2B_3(c_3)].$$ (119)

Then by summing up the products of eq. (92), (119) the RHS of eq. (81) can be calculated. The obtained expressions coincide up to the sign with derivative of (115) with respect to $\xi$. Thus the Nielsen identity (94) is fulfilled for quarks as in the case of gluons. Taking into account that non-vanishing $A_0$-condensate lowers the actions $W_q, W_{gl}$ and $W_q + W_{gl}$ and the above result we see that both basic properties required by the Nielsen identities approach are held. In accordance with general theory we come to the conclusion that the gluon condensation at finite temperature is a gauge invariant phenomenon with quarks included in consideration.

Thus, both lattice and loop expansion considerations led us to the similar conclusion about $A_0$-condensate in the full QCD though the effect of the quark contribution to the effective action has been found out to be essentially different. This discrepancy does not have an explanation at the moment and should be clarified in the future investigations. The reason can be the following. In fact, the high temperature expansion used on the lattice to calculate a quark contribution is well reliable only at large $m_q$ whereas the loop calculations have been performed at vanishing quark masses. So it is very desirable to carry out both calculations at intermediate values of quark masses. Just this point can be the origin of another disagreement concerning a possibility of the baryon number generation which will be discussed in the next chapter.

8 $A_0$-condensate in hot gauge theories. Some consequences

What would be the consequences of such a condensate? We shall present some of them and give a brief review of the most important results and related problems. We begin our discussion with the problem which was actually one of that difficulties of hot non-abelian gauge theories which has impelled to develop the approach to these theories on the basis of the possibility of the global gauge symmetry spontaneous breaking. This is infrared problem which have been already discussed in different aspects. Now, the question is: what may we call a solution of the infrared problem? It has been shown in the first papers by Linde that appearing of infrared cut-off even of the order $T$ (the condensate appears to be just of this order) cannot save the perturbative expansion (in this case all terms of the perturbative expansion starting from the order $g^6$ are proportional just to this order and therefore all of them give equal in coupling constant contributions). From the mathematical point of view the answer is quiet clear - we need to construct a method of calculation which would give finite results for expectation...
values of physical observables in the continuum. One of such methods we discussed
in chapters 2 and 6. This is the reduction of gauge theories at high temperatures to
the effective three-dimensional models. It is possible to show that in the course of
the reduction the static modes of the chromomagnetic potentials acquire mass due to
formation of the $A_0$-condensate $[3]$, $[39]$. Thus, in this method of calculations a hope
to avoid the infrared divergences by means of the condensate does appear. We think
that the most essential point in proofing that $A_0$-condensate can cure the theory of
the divergences is the proof that just this condensate lead to screening all sources in
the adjoint representation at high temperatures. The existence of such a screening
would mean, in our opinion, the solution of the infrared problem. We remind here that
we should consider spatial correlators to calculate this screening (see our discussion in
the introduction). Now, we are going to show, omitting all technical details, how this
screening can appear in the reduction of lattice gauge theories $[62]$, $[14]$.

We present a method much analogous in its idea to the recent development of perturba
tive expansion resumming $[63]$ conjectured to cure the infrared divergences of
finite temperature QCD. Actually, the reduction looks like isolation of static contribu-
tion in the action after Fourier transforming gauge fields $A_\mu(x)$ with further calculation
of multi-loop corrections over massive non-static modes. This procedure has been al-
ready performed both in the continuum theory for the Yang-Mills action (see previous
chapter) and on the lattice for the Wilson action $[64]$. In the latter the reduction
beyond the perturbative horizon $[65]$, $[66]$ can be accomplished as the gauge matrix
$U = \exp(gaA)$ expansion is not necessary.

We develop proper high-temperature expansion and the corresponding reduction
dealing with the Fourier transformation of compact gauge matrices $U_\mu(x)$ rather than
gauge fields $A_\mu(x)$ calculating then a static contribution resulting from such an ex-
pansion. Two essential points distinguishing this approach from the preceding ones
$[23]$, $[39]$, $[64]$ appear to be the following. Firstly, the static sector generates compact
dielectric field leading to an effective dielectric theory. Secondly, since in the present
case Fourier transformation is not simple linear substitution the Jacobian of this trans-
formation is non-trivial and moreover it generates a mass of dielectric field (which is, in
fact, the mass of static modes). As to the non-static modes they are massive as before
$[22]$ with the mass proportional to $(nT)$ (in continuum limit). Therefore, in this way
we could construct quite reliable perturbative expansion for massive modes. All calcu-
lations of the resulting effective action can be found in $[62]$, $[14]$ where both corrections
to the main static contribution and quark contribution have been computed. Here, we
write down the effective action for static modes restricting to the pure gluodynamics.

\begin{align}
Z &= \int \prod_x d\mu(\alpha_x) \prod_{x,n} \exp(\beta' \cos \varphi_x \cos \varphi_{x+n} - 2\lambda_c \cos \alpha_x \cos \alpha_{x+n})Z_D, \\
Z_D &= \int \prod_l \left[ \rho_l^3 d\rho_l d\mu(U_l) \right] \exp S_D
\end{align}

$\beta' = 4(\lambda_c/\lambda^0)^N$, $l = (x, n)$ is a link. This effective action we call induced dielectric
action. In mentioned approximations we have

\[ S_D = S_E^{\text{stat}} + S_m^{\text{stat}} + S_0^{\text{mass}}, \]  

where we have laid

\[ S_m^{\text{stat}} = \lambda_m N_t S p \sum_p \rho(\partial \mathbf{\Pi}) U(\partial \mathbf{\Pi}), \]  

\[ S_0^{\text{mass}} = -\frac{N_t \lambda^0}{2} \sum_{x,n} S p(\Phi_n(x)\Phi_n^+(x)), \]  

\[ S_E^{\text{stat}} = N_t \lambda_e \sum_{x,n} S p(\Phi_n(x)V_x^{\Phi_n}(x)V_{x+n}^{\Phi_n}). \]  

\( V_x \) is zeroth component of gauge field matrices in the static gauge and \( \lambda_m = \frac{2 a_g}{g^2 a_s} \), \( \lambda_e = \frac{2 a_g}{g^2 a_s} \). In chosen approximation \( \lambda^0 \approx 2 \). The representation \( \Phi_n(x) = \rho_n(x)U_n(x) \) was using in the course of the reduction.

Comparing formulae (121)-(125) with (10)-(13) from Introduction one can easily deduce that (121) is nothing but some kind of LDGT. There exist two differences from formulation [13] described in the introduction to be emphasized. First one is that the theory (121) is a compact since the dielectric field obeys the relation \( 0 \leq \rho \leq 1 \) unlike (12). Another significant difference is the presence of the interaction term \( S_E^{\text{stat}} \) which describes interaction between gauge field \( U_n(x) \), dielectric field \( \rho_n(x) \) and Higgs field \( A_0 \).

The mass of the static mode can be calculated from a ”naive” effective potential for dielectric field. Its classical form can be easily obtained if we neglect the fluctuations of the gauge field \( U_n(x) \) and take the expectation value \( < A_0 > \) in \( S_H^D \). The result has been put down in eq.(14). It is clear from its form that in the continuum limit the mass is proportional to \( < A_0 > \) as we need to expand \( \cos(a_{\beta}gA_0) \) when \( a_{\beta} \) tends zero (temperature independent constant part should be avoided from the mass after adding zero temperature contribution to the free energy). Two facts following from the present consideration should be stressed. Firstly, as was conjectured in Introduction, two screening mechanisms - dielectric one and that caused by global gauge symmetry breaking - can be indeed related as \( A_0 \)-condensate supplies the dielectric field with gauge-invariant mass. Secondly, we have proved that the mass of the static mode appears not only on the level of the standard reduction but beyond the perturbative horizon if we take into account all powers of static modes. Now, there is no difficulty to prove that spatial adjoint Wilson loop obeys perimeter law at any temperature [14]. Moreover, it is possible to prove that the fundamental Wilson loop displays the area law behaviour with string tension to be proportional to the mass of the dielectric field [14] in full accordance with results of [9], [10]. All of that, according to duality relations, mean the screening of the gluon currents and confinement of the ”spatial” sources being in the fundamental representation. Thus, we can see that \( A_0 \)-condensate does lead to the screening of the chromomagnetic forces at high temperature. (Of course, our proof is not strict and more exact calculation of the adjoint Wilson loop in the effective theory with the corrections from non-static modes included is very desirable.)
The next interesting problem we are going to discuss here is evaluating the heavy quark potential in background $< A_0 >$. As is known, heavy quark potential at finite temperature is calculated in different phases by using the correlator of the Polyakov loops. We find the linearly rising potential in the confinement phase and the Debye screened potential in the deconfinement phase where $< A_0 > \neq 0$.

The heavy quark potential $V_{qq}$ in the finite temperature theory is connected with the Polyakov loop correlator as

$$e^{-\beta V_{qq}(x_1,x_2)} = \langle L(x_1), L(x_2) \rangle$$

In the static gauge we have for the Polyakov loop the simple expression ($SU(2)$ gauge group)

$$L(x) = \cos\left(\frac{1}{2}\beta g(A_0 + \epsilon(x))\right),$$

i.e. Polyakov loop depends on the chromoelectric field only. This justifies us to use for the calculation of the correlator the same method as we have used for the calculation of the effective potential in the chapter 6. The effective potential was obtained from the relation for the partition function expressed in terms of the effective action:

$$Z(\beta) = \int [D\epsilon] e^{S_{\text{eff}}}$$.}

For the effective action, applying the method described in chapter 6, we have found the relation:

$$S_{\text{eff}} = \frac{V}{\beta^3} \left[ \frac{3\pi^2}{45} - \frac{4\pi^2}{3} (X^4 - 2X^3 + X^2) \right] - 8 \int d^3l \, \tilde{\epsilon}(l) S_{\text{cor}}^{\text{eff}}(k, X) \tilde{\epsilon}(-l).$$

In the last expression $\tilde{\epsilon}(l)$ is the Fourier transform of the function $\beta \epsilon(x)$, $S_{\text{cor}}^{\text{eff}}(k, X)$ has been put down in (105) and all other notations follow chapter 6.

By the same method one finds for the correlator of two Polyakov loops at the distance $R$ the equation:

$$\langle L(0), L(R) \rangle = \frac{1}{2} e^{-\frac{g^2}{\beta} S^{-1}(0,0)} \left[ e^{\frac{g^2}{\beta} S^{-1}(0,R)} + \cos(2\pi X) e^{-\frac{g^2}{\beta} S^{-1}(0,R)} \right].$$

$S^{-1}$ means the inverse matrix element with continuum indices given by the relation:

$$S^{-1}(x,y) = \int \frac{d^3(\beta k)}{(2\pi)^3} \frac{1}{S_{\text{cor}}^{\text{eff}}(k, X)} e^{-ik(x-y)}$$

The direct application of last formula is not possible on the level of the approximations defined in[38], because $S_{\text{cor}}^{\text{eff}}$ is negative for small $k = |k|$ in the neighborhood of $X = 1/2$. This is not consistent with the definition of physically acceptable action in the functional integral. To remove this inconsistency in a natural way we carefully review the terms abandoned in the previous method of the calculation[38] with the hope...
to find some contributions making $S_{\text{cor}}^{\text{eff}}$ positive. The calculation is in progress now and we would like to stress its significance by pointing out the important consequences.

Here, we simply modify $S_{\text{cor}}^{\text{eff}}$ to the positive function $S_{\text{cor}}^{\text{mod}}$ by addition of a small positive constant. We rewrite Eq. (131) in the form:

$$S^{-1}(0, R) = \frac{1}{i(2\pi)^2 \rho} \left\{ \int_{-\infty}^{0} \frac{l \, dl}{S_{\text{cor}}^{\text{mod}}(-l, X)} e^{il\rho} + \int_{0}^{\infty} \frac{l \, dl}{S_{\text{cor}}^{\text{mod}}(l, X)} e^{il\rho} \right\}$$

(132)

where $\rho = TR$.

The singular structure of the integrand in the complex $l$ plane is defined by the singular structure of the $S_{\text{cor}}^{\text{mod}}$. The function $S_{\text{cor}}^{\text{mod}}(l, X)$ possesses two cuts, $(-i\infty, -iX)$, $(iX, i\infty)$ in the complex $l$ plane. Because the integrands in Eq.(132) have no singularities out of real and imaginary axes, moreover,

$$\lim_{|l| \to \infty} l \left[ S_{\text{cor}}^{\text{mod}}(l, X) \right]^{-1} = 0,$$

(133)

we change the range of the integration by closing the integration contour. Then we can rewrite Eq.(132) after substitution $l = i\tau$ in the form:

$$S^{-1}(0, R) = \frac{2}{(2\pi)^2 \rho} \int_{0}^{\infty} d\tau e^{-\tau \rho} \left| \frac{\tau}{S_{\text{cor}}^{\text{mod}}(i\tau, X)} \right|^2.$$

(134)

Applying the mean value theorem, the integral of Eq.(134) is estimated as:

$$S^{-1}(0, R) = C(T) \frac{e^{-\mu TR}}{TR},$$

(135)

where $C(T)$ is a calculable quantity. The quantity $\mu$ is a positive constant from the range of integration. For our purposes here it is sufficient to know that $\mu > 0$. The analytical evaluation of this quantity shows its dependence on the chromoelectric mass.

Eq.(134) is valid for $R \neq 0$. The matrix element $S^{-1}(0, 0)$ is infinite and represents the quark self energy in the heavy quark potential given in Eq.(130). In what follows we shall consider only the interaction energy of the quarks by subtracting $S^{-1}(0, 0)$ from all relations. We would like to point out the importance of the Eq.(135). All qualitative features of the following discussion are based on the particular approximate form of the right-hand side of Eq.(135) and are independent on details of Eq.(105).

Investigating the last equations we find the following picture.

For value of $X = \frac{1}{2}$ (which corresponds to the vanishing Polyakov loop value $\langle L \rangle = 0$) the heavy quark potential calculated from the Eqs.(126),(130),(133) possesses the desired $R$ dependence because for $R \to \infty$ the leading term is linear:

$$\frac{1}{T} \{V_{\text{interact}}(R) - V_{\text{self energy}}\} = -\mu RT + \text{const} = \ln RT + O(R^{-2}).$$

(136)

In the deconfinement phase characterized by $\langle L \rangle \neq 0$ the effective potential possesses two minima for $X \neq 1/2$ symmetric regarding to $X = 1/2$. The heavy quark potential in this case is usually calculated by the formula:

$$-\frac{1}{T} V_{qq}(R) = \ln \frac{\langle L(0), L(R) \rangle}{\langle (L(0)) \rangle^2}.$$

(137)
Inserting Eqs. (130, 135) into the last equation we find for $R \to \infty$ the result:

$$\frac{1}{T} V_{qq}(R) = tg^2(\pi X) \frac{g^2}{8} C(T) \frac{e^{-\mu T R}}{T R} + O(R^{-2}).$$  \hspace{1cm} (138)$$

In the leading term of the last expression we recognize the Debye screening of the colour charge potential in accordance with the lattice results[67] as well as analytical calculations[68]. Thus we can see that appearance of $X \neq n (< A_0 > \neq 0)$ leads to the Debye screened phase at high temperatures.

One of the most exciting consequences of $A_0$-condensate is the spontaneous breaking of the colour charge symmetry and, consequently, a generation of the baryonic number. If this phenomenon does realize in nature it would be a good possibility both for verifying the general theory described in this survey and for searching the new state of the strong interacting matter. It should be mentioned at once that the baryonic number is generated not at all possible values of the condensate but only when the situation described in the end of chapters 4, 5 is realized. In $SU(2)$ gauge theory such a generation is not possible at all because the appearance of the condensate leads to the symmetric under charge conjugation effective action. In $SU(3)$ theory the baryonic number is generated when both $< A_3 >$ and $< A_8 >$ differ from zero. The main statement is: in the presence of a non-vanishing Euclidean $\langle A_0 \rangle$ field the generation of the nonzero baryonic number is possible. This statement was proved in two cases.

1) We studied the Hamiltonian formulation of the lattice QCD in strong coupling limit [39], where the partition function is of form (the Kogut-Susskind fermions were used) [56], (57), (107):

$$Z = \int \prod_x D\mu(A_0) \prod_{x,n} \exp(-\gamma C_2(l)) \Omega_l(A_0(x)) \Omega^*_l(A_0(x + n)) * \int \prod_x D\mu(x) \prod_{x,n} \exp(i\varphi^\alpha(x)) + (T a_\sigma)^{-1} \sum_{n = d} \eta_n(x) U_n(x) \delta_{x,y,n}$$ \hspace{1cm} (139)

where $\gamma = g^2(2T a_\sigma)^{-1}$, $\Omega_l$ is character of the $l$-th irreducible representation and the high-temperature expansion in the Wilson loops was applied to calculate the fermionic determinant. For each colour quantum number the corresponding charge density generated by $\langle A_0 \rangle$ is given by:

$$\rho_\alpha = \rho[(A_0)_{\alpha,\alpha}] = \lim_{\mu \to 0} (N)^{-1} \partial \ln Z(g \varphi^\alpha \longrightarrow g \varphi^\alpha + i\mu^\alpha).$$  \hspace{1cm} (140)$$

The chemical potential is introduced into the fermionic determinant. The physical baryonic density is then obtained as:

$$Q = \sum_\alpha \rho[(A_0)_{\alpha,\alpha}].$$  \hspace{1cm} (141)$$

where $\alpha$ is the colour index. After integrating over gauge fields we get the result:

$$\rho_\alpha = \frac{Im W^\alpha(A_0)}{1 + Re W(A_0)} + (16 T^2 d)^{-1} \sum_n \langle \Delta^\alpha \rangle + O(T^{-4}), \Delta^\alpha \sim Im W^\alpha.$$ \hspace{1cm} (142)
From the last equation we can see that baryonic density is proportional to the imaginary part of the Polyakov loop. It is easy to conclude that all the higher orders of the $\beta$-expansion are also proportional to the imaginary part of the fundamental characters. We point out now that $\langle \text{Im}W \rangle$ will be different from zero if $\langle A^3_0 \rangle$ and $\langle A^8_0 \rangle$ are non-vanishing.

2). The similar result was obtained in weak coupling regime of the continuum QCD for free quarks being in the constant background field $A_0$ [81]. In that case the baryonic number is expressed through different representations of the imaginary part of the Polyakov loop:

$$Q = i \sum_{\alpha} T V^{-1} \frac{\partial \ln Z}{\partial (A_0)_{\alpha,\alpha}} = i \frac{2m^2 T}{\pi^2} \sum_{n=1}^{\infty} \sum_{\alpha} (-1)^n \frac{K_2(nm\beta)}{n} \sin(n\beta A^\alpha_0).$$

(143)

Thus we can suppose that in the general case as well a non-vanishing imaginary part of the Polyakov loop yields a non-zero baryonic density even when baryonic chemical potential equals zero. It is very important property of $\langle A_0 \rangle$-condensate which has to lead to observable effects in the relativistic heavy ion collisions.

Now, we would like to draw an attention to the following problem concerning the charge broken states. The free energy of $SU(3)$ gauge theory has three global minima at the high temperatures. On Fig.2 we have drawn them schematically for theory with dynamical quarks. In the minimum ”1” $\langle \text{Tr} \text{Im}W \rangle = 0$ and, therefore, $Q = 0$. In other two minima the charge symmetry is broken, $\langle \text{Tr} \text{Im}W \rangle \neq 0$ and, therefore, $Q \neq 0$. But we do not know in fact whether the C-symmetry broken states are stable or metastable and, consequently, short-lived. There are some results concerning this problem. One loop calculations ($\langle A_0 \rangle = 0$) with massless quarks have shown that the states where $\langle \text{Tr} \text{Im}W \rangle = 0$ are deeper [72]. More precise 2-loop calculations with massless quarks (and with $\langle A_0 \rangle \neq 0$) described earlier lead to the same conclusions. The lattice strong coupling evaluations have confirmed the such inference but calculations with massive quarks [82] predict the existence of some values of parameters $\gamma = \frac{\beta g^2}{2a}$, $\beta$, $m\beta$ (considered as independent) preferring the minima ”2”, ”3” to be deeper than minimum ”1” at $\gamma \rightarrow 0$. If it is the case we obtain the following scenario.

In the chiral symmetric phase at high temperature ($m_q = 0$) the main true minimum of the free energy will be always real and other two will be only metastable. If in the deconfinement phase the quarks have a nonzero mass, the charge broken minima become deeper and the baryonic number is nonzero. But at the chiral phase transition the system will be back the real minimum again. Basically, in this case the disappearance of the baryonic number and the transition of the system to the state where $\langle \text{Im} \text{Tr}W \rangle = 0$ ($\langle A^3_0 \rangle \neq 0$, $\langle A^8_0 \rangle = 0$) is just the chiral phase transition. To verify this picture we can look for the similar phenomena in $Z(3)$ spin system because most of features of the $Z(3)$ spin system in d-dimension are similar to the $SU(3)$ ones at finite temperature[17]. We consider $Z(3)$ theory with external either positive or negative
magnetic field, whose partition function is of the form:

\[
Z = \sum_{q_x = 0}^{N-1} \exp\{\lambda \sum_{x,n} Re(e^{2\pi i q_x \cdot e^{2\pi i q_{x+n}}} \cdot e^{2\pi i q_{x+n}}) + \eta \sum_x \cos(\frac{2\pi}{N} q_x)\}
\]  

(144)

It is well known that: 1) if \(\eta > 0\), the minimum is real, \(\langle Im \ W \rangle = 0\) and the phase transition of the second order is possible at sufficiently small \(\eta\); 2) if \(\eta < 0\), the minima are complex ones, \(\langle Im \ W \rangle \neq 0\), the phase transition of the Ising-type takes place to the state with broken symmetry \(q \rightarrow -q\).

What should be concluded from these points? At least at the hopping-parameter expansion of fermionic determinant the leading contribution including the Polyakov loops has positive sign which definitely leads us to the case \(\eta > 0\). Of course, it is enough dangerous to do conclusions comparing QCD with \(Z(3)\) theory, nevertheless, it seems to us that the real minimum will be deeper at any temperature in \(SU(3)\) theory as well and, hence, the states with nonzero baryonic number are, in fact, metastable. In principle, we have not to except other possibility as well. The situation can be much more complicated. For example, the discussion in \[81\] shows that in QCD slightly above the deconfinement phase transition the domains of different ”vacua” (with different background \(A_0\)) may exist. The scenario described above cannot be excluded, too, at least, until we obtain the connection (or some dependence) between \(\langle A_0 \rangle\) condensate and quark masses.

Another approach to resolve this problem has been conjectured in \[82\]: full QCD cannot be described by a grand canonical ensemble with respect to triality or quark number. The vacuum of QCD with dynamical fermions has triality zero and therefore degenerate \(Z_3\) phases and ordered-ordered phase transitions like pure gluonic QCD. This means that full QCD does not lead to metastable phases existing up to arbitrary high temperatures.

Certainly, it is very desirable to have more precise both analytical and Monte-Carlo calculations concerning the metastable phases with massive and massless quarks to clarify this very exciting problem.

The last question we would like to elaborate here is an inducing of the Chern-Simons action in finite temperature gauge theories. We adduce the sufficient conditions of such a generation in the background \(A_0\) field.

The Chern-Simons theories are very popular now owing to their unusual and attractive properties. The two areas are usually discussed to be relevant to the Chern-Simons action: the superconductivity and 4-dimensional field theory at finite temperature. There are some well-known examples when the Chern-Simons action is induced by fermionic determinant at high temperatures in four-dimensional theory. Redlich \[73\] has studied the model of two left-hand double fermions connected with \(SU(2)\) gauge field and imaginary chemical potential. In \[74\] QCD-like theory with axial \(U(1)\) charges was studied. In both cases the fermionic determinant generates at high temperature the Chern-Simons action for the static modes of gauge field. The coefficient in the front of the Chern-Simons term is proportional to the introduced chemical potential. In connection with it the following question is very interesting: is it possible to induce
the Chern-Simons term into finite-temperature QCD and what is the mechanism of it? At the first sight this question may seem a bit strange since there are some problems which forbid appearance of the Chern-Simons term in the QCD-action:

1) the Chern-Simons term is defined for 3-dimensional theories;
2) $S_{c-s}$ violates the $CP$-symmetry.

This part of our survey is founded on the article [75] where all these problems are discussed in details. In brief, the $S_{c-s}$ is induced for static gauge modes which are described by appropriate 3-dimensional theory. $CP$-symmetry is broken via certain regularization procedure on the lattice. This breakdown vanishes identically in the naive continuum limit. The Chern-Simons action is of the form

$$S_{c-s} = 8\pi^2 \kappa \int d^3x \, I(A),$$

$$I(A) = \frac{1}{8\pi^2} \varepsilon^{nmk} Sp(A_n \partial_m A_k + \frac{2}{3} A_n A_m A_k).$$ (145)

Partition function has the standard form

$$Z = \int [DA_\mu] \exp\{-\int_M [dx] L_G - \Gamma_{eff}(A)\}$$ (146)

where $\Gamma_{eff}(A)$ is the result of integration over quark fields. We looked for action (145) for smooth potentials of the static gauge modes in $\Gamma_{eff}$ because we do not know how it is possible to put down the action (145) on the lattice in an arbitrary case. We used the lattice regularization for the effective action and studied both the Wilson and the Kogut-Susskind actions for fermions with different boundary conditions for gauge fields. Both the perturbative expansion and the expansion in the Wilson loops of the determinant were applied to calculate $\Gamma_{eff}(A)$. The summary of our results concerning $A_0$-condensate and the Chern-Simons action is as follows: (for detailed calculations see [75]).

1. In perturbative expansion a coefficient at the Chern-Simons term is expressed through free fermion propagator $G$,

$$\kappa = \frac{1}{6} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d^3p}{(2\pi)^3} \varepsilon_{nmk} Sp[(G^{-1} \partial_n G)(G^{-1} \partial_m G)(G^{-1} \partial_k G)].$$ (147)

Analyzing this expression one can conclude that there is not any perturbative relation between $A_0$-condensate and the Chern-Simons action (though, exists a possibility of $\kappa \neq 0$ for some values of the Wilson parameter $R$; see below). Nevertheless, a nonperturbative connection does exist.

2. Either for QCD on the torus due to certain periodic boundary conditions for gauge fields

$$M = (S^1)^d : A_\mu(x + l_\nu) = \Omega_\nu A_\mu(x) \Omega_\nu^{-1} + \Omega_\nu \partial_\mu \Omega_\nu^{-1};$$

$$\Omega_\mu(x + l_\nu)\Omega_\nu(x) = \Omega_\nu(x + l_\nu)\Omega_\mu(x)$$ (148)

or if we take into account nonperturbative configurations out of center of $SU(N)$, the fermionic determinant will generate the $\theta$-term $F_{\mu\nu} F_{\rho\lambda} \varepsilon_{\mu\nu\rho\lambda}$. We suppose that
the vortex configurations of gauge field, existing on the lattice, can provide the same minimum of the effective action as fields $A_\mu = 0$ \[13\]:

$$Sp \ Z(\partial p)V(\partial p) = 1, \ Z(\partial p) \in Z(N),$$

$$V(\partial p) = V_n(x)V_m(x + n)V^*_n(x + n)V^*_m(x),$$

$$V_m(x) \in SU(N)/Z(N).$$

The $\theta$-term is reduced to the Chern-Simons action at high temperatures and when $A_0$ condensate falls. The coefficient at the action is $\kappa \approx \alpha n \beta \langle A_0 \rangle$ for the theory on the torus and $\kappa \approx \lambda \beta \langle A_0 \rangle F_{nm}(\Theta)$ for the second case. Here, $n$ is the winding number of matrices $\Omega$ and $\Theta$ is a vortex potential.

3. In theory with the $SU(3)$ gauge group and nonperturbative dielectric vacuum $S_{c-s}$ is generated when expectation value of the imaginary part of the Polyakov loop differs from zero. Nontrivial dielectric vacuum appears in an effective three-dimensional theory with the group having nontrivial center \[62\], \[14\] (see also our discussion above). In the last both cases the configurations from the center of $SU(N)$ are necessary for generation of $S_{c-s}$. So it is very plausible that such a mechanism is impossible in the standard continuum QCD where the variables from the center of $SU(N)$ are absent. Besides, there is another necessary condition of the appearing of $S_{c-s}$ here. In any of these cases QCD has to belong to definite class of universality. It means the following. Choosing the lattice fermionic action we had to solve the problem of doubling the fermion degrees of freedom. We should use either the Wilson action \[77\]

$$S_W = \frac{1}{2} a^3 \sum_{x,\mu} [\overline{\Psi}(x)U_\mu(x)(R - \gamma_\mu)\Psi(x + a_\mu) + \overline{\Psi}(x)U^*_\mu(x - a_\mu)(R + \gamma_\mu)\Psi(x - a_\mu)]$$

$$+ a^4 m \sum_x \overline{\Psi}(x)\Psi(x) - da^3 \sum_x \overline{\Psi}(x)R\Psi(x)$$

or the Kogut-Susskind action \[45\]

$$S_{K-S} = \frac{1}{2} \sum_{x,n=-d}^d \eta_n(x)\overline{\Psi}(x)U_n(x)\Psi(x + n) + ma \sum_x \overline{\Psi}(x)\Psi(x),$$

$$\eta_n = -\eta_{-n}, \ U_{-n}(x) = U^*_n(x - n)$$

which allow us to avoid undesirable degrees of freedom. The most general form of the Wilson parameter $R$ is \[12\], \[44\], \[78\]:

$$R = s \exp(i \theta \gamma_5)T, \ 0 < s \leq 1, \ 0 \leq \theta \leq \pi, T^2 = 1,$$

where $T$ is some matrix from the fermionic space. One of the $\eta$-symbols is \[79\]:

$$\eta_n(x) = (-1)^{x_1 + x_2 + \cdots + x_{n-1}} \exp(i \pi [k(x + n) - k(x)])$$

where $k$ is any integer. At the level of free quantum theory we cannot choose a unique form of these quantities. More than that, it is just valid at any choice of $R$ and $\eta$. 

49
- naive continuum limit is \((\theta, s, T, k)\)-independent and chiral symmetry is restored;
- fermionic propagator and lattice Feynman’s rules coincide in the continuum limit with standard Feynman’s rules;
- the property of the positivity is fulfilled.

Nevertheless, the theories with different \(R\) can belong to different classes of universality since they can define different quantum continuum theories. Examples of the such nonuniversality can be found in [80]. In the only case if \(\eta\)-symbols or parameter \(R\) satisfy some conditions, for example,

\[
\theta \neq 0, T = \gamma_0 \\
\eta_n(x)\eta_m(x+n)\eta_k(x+n+m)\eta_l(x+n+m+k) = \varepsilon_{nmkl} \\
n \neq m \neq k \neq l
\]

the Chern-Simons action will be generated in QCD. Fortunately this property does not depend on lattice regularization [80]. On the other hand it is very difficult task to prove that the \(S_{c-s}\) is unique nonuniversal contribution to effective action in finite-temperature QCD though it seems very plausible (see, for instance, [80]). The parameters \(R\) and \(\eta\) as in (154) violate the CP-symmetry. As we could see above, this violation disappears in the continuum theory both in the naive limit and in the free quantum theory. The appearance of \(S_{c-s}\) is the only remnant of this violation which survives in the continuum quantum limit. There is only one theoretical reason for the choice of \(R\) or \(\eta\), the property of the positivity [78]. But this property is satisfied for given \(R\) and \(\eta\). So we may formulate the question: is it possible that QCD belongs to the class of universality in which the Chern-Simons action exists? The answer is left unclear so far. Certainly, the presence of \(S_{c-s}\) has to lead to interesting phenomena at high temperatures because \(S_{c-s}\) can change a statistics of a matter fields connected with it. Thus, appearance of the Chern-Simons term in the QCD action at high temperature is approved by the following complex of the circumstances:

a) appearance of \(A_0\)-condensate;

b) existing of nonperturbative vacuum which is formed by vortex potentials from the center of the gauge group;

c) problem of universality.

To finish, we would like to notice that appearing of \(S_{c-s}\) could lead to solving of infrared problem as well, since the Chern-Simons action generates magnetic mass just for the infrared dangerous static modes.

9 Discussion and Summary. Unsolved problems

In the present survey we have considered the mechanism of the spontaneous breaking of the global gauge symmetry caused by the condensation of the gluon field at high temperatures. We have discussed in details the most important approximations for calculation of \(A_0\) condensate both in the continuum theory and within lattice gauge
models. We understand that it is almost impossible to give a full review of all questions related to $A_0$ condensation because breaking $SU(N)$ symmetry is certainly strong phenomenon which has to reflect itself on many aspects of high temperature QCD.

As an essence of the above analysis we would like to stress once more that in all considered approaches the gluon field condensation at $T \neq 0$ has been determined and proved to be a gauge invariant phenomenon. The nice and important fact that in all approaches we have discussed here the gauge invariance of the condensate is manifested by the different methods makes us sure that the condensation may indeed be realized in the nature. At the same time, a number of discrepancies in results obtained by different methods (mainly as the role of quarks is concerned) have been found. These points need to be investigated separately in order to have more reliable results.

It is natural that in the every method of calculation there are own problems which should be solved in the future. Generally speaking, they can be divided in three categories dependently on their importance and theoretical and practical significance. The main problem now is to derive a conclusion on $A_0$ condensation with higher loop contributions to be included. This is a complicated mathematical task which should be investigated on the base of Nielsen’s identities in order to obtain a correct gauge invariant result. Then, if $<A_0> \neq 0$ will be determined finally, the problem of constructing the consistent theory with $<A_0> \neq 0$ should be solved. This point should be investigated in details because just here a number of theoretical problems remain unsolved yet. Most essential of them is the construction of partition function and calculation of various observables with $<A_0> \neq 0$ to be taken into account. As we have seen, there are six minima with the same depth in the $(x, y)$-plane and $Z(3)$ symmetry is broken. This situation, but in the case when $Z(N)$ is preserved, has been discussed in the paper [81]. It was shown that an extension of standard description of thermal equilibrium is needed when discrete degenerate phases are to be included. However, how does it work in the case of spontaneous breaking of $SU(N)$ has not been investigated yet. In this line the investigations of infrared problem of gauge fields at $T \neq 0$, the gluon magnetic mass, asymptotic behaviour of running coupling constant are of great importance, too. Having these questions answered one obtains a consistent theory with $<A_0> \neq 0$. After that a phenomenology of finite temperature QCD will be more transparent. And only this third of mentioned points may give a possibility of a deriving of a final conclusion on gauge field condensate. Monte-Carlo simulations and analytical evaluations on the lattice making use some correlation inequalities could help in this problem as well.

At last, the very interesting and exciting problem is to find a dependence of $A_0$-condensate on quark mass in order to clarify the possible connection between $<A_0>$ and the order parameter of the chiral phase transition. One may hope that such a connection really exists since the $A_0$-condensate can drastically change the structure of the quark propagator. On this way we may also hope to find a solution of the problem of the baryonic number generation.

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