On the Higgs-Confinement Complementarity

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Abstract

It has been noticed long ago that in Higgs models with ‘complete symmetry breaking’ one can move from the confinement to the Higgs regime without crossing a phase boundary, a fact sometimes called referred to as ‘complementarity’. In a recent paper some doubt was raised about the correctness of the mathematics underlying this fact and it was claimed that the supposed ‘flaw’ would resolve the ‘paradox’ seen in this complementarity. Here we briefly revisit the facts both from a mathematical and a physical point of view and point out that (a) there is no paradox and (b) there is no flaw in the mathematical reasoning.

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1 Introduction

After K. Wilson [1] created lattice theory and thereby found a well-defined non-perturbative formulation of gauge theories (albeit with an ultraviolet cutoff) that did not require gauge fixing, it became possible to address questions not answerable in perturbation theory. The first and most important such question, the question of confinement of quarks, was answered by Wilson himself (with later mathematical refinement in [2]) for strong enough bare coupling.

Very soon the interest turned to the Higgs mechanism, which in perturbation theory with gauge fixing is conventionally described as spontaneous symmetry breaking (SSB). In [2] it was shown using a cluster expansion that a Yang-Mills-Higgs model having a property conventionally denoted as ‘complete breakdown of symmetry’ has a mass gap at arbitrarily weak gauge coupling, provided the Higgs potential is sufficiently strong. This result was analyzed in [3] and physically interpreted as the absence of a phase boundary between ‘confinement’ and ‘Higgs’ regions, a property that became known as ‘complementarity’.

Lately is has been been claimed by Grady [4] that the proof of the relevant theorem in [2] contains a flaw, that the theorem is invalid and thereby a ‘paradox’ is solved. Here we will first remind the reader why there is no paradox; to set the record straight we then briefly revisit the arguments entering the proof and show why there is no flaw.

2 There is no paradox

G. ’t Hooft [5] in 1979 explained confinement-Higgs complementarity in physical terms. He showed that the Higgs mechanism may just as well be interpreted as confinement: bare Yang-Mills quanta and bare Higgs quanta may be combined into gauge invariant, permanently bound compounds forming the gauge invariant physical ‘W’ and Higgs bosons, just as quarks and gluons are bound into hadrons. This is gratifying, because from Elitzur’s theorem [6] we know that gauge invariance cannot be broken spontaneously if no gauge fixing term is present. This fact is in some sense obvious, because the local invariance means that there is no preference for aligning even nearby degrees of freedom (i. e. there is not even short range order).

A little later Fröhlich, Morchio, and Strocchi [7] set out to formulate perturbation theory for such models in terms of gauge invariant composite fields without the usual assumption of SSB via a nonvanishing expectation value of the Higgs field. The procedure outlined there is not necessarily useful for computational purposes, but it is important to recognize that it is possible in principle. Fröhlich et al also showed in that paper that SSB is absent even in the presence of certain

2
gauge fixes.

Another important, unfortunately much too little recognized result is due to Kennedy and King [8]. They considered abelian lattice gauge theories in 4 dimensions with a covariant gauge fixing term, as employed usually in continuum perturbation theory. There finding was: SSB occurs only in the Landau gauge, i.e. the gauge with minimal infrared fluctuations. In all other covariant gauges infrared fluctuations destroy SSB, much like infrared fluctuations destroy long range ordering in 2D ferromagnets (a fact known as the Mermin-Wagner theorem [9]).

Provocately one might say that SSB in Higgs models, if it occurs at all, is a gauge fixing artefact.

3 There is no flaw in the proof of analyticity

Here want to briefly review the structure of the proof of analyticity given in [2], in order to clear up some misconceptions. It proceeds by a convergent cluster expansion in the form developed by Glimm, Jaffe and Spencer [10] in the context of Constructive Quantum Field Theory. This expansion expresses expectation values first as a sum of finite volume quantities which are clearly analytic in the parameters; then it is shown that the expansion converges uniformly in the volume in a certain complex domain of parameters. This by itself is, however, not sufficient to prove analyticity of the infinite volume limit, as correctly remarked by Grady (it is also not sufficient to establish the existence of that limit). Grady goes on to suggest that the limits of the individual terms might be nonanalytic.

But in [2] it is remarked that existence of the infinite volume limit and its analyticity follow by standard applications of the cluster expansion, for which the authors refer to [10]. Obviously the authors of [2] did not want to repeat the arguments given in [10], as they were considered ‘standard’ in their community. So let me repeat the structure of the argument for analyticity of the infinite volume limit:

(1) The cluster expansion converges uniformly (in the volume) in a certain complex domain in the parameters of the model (the conditions necessary for lattice gauge theories are given in OS).

(2) From this follows uniform (in the volume) exponential clustering for pairs of observables.

(3) This clustering in turn implies convergence to the thermodynamic limit because boundary effects decouple exponentially.

(4) Vitali’s convergence theorem for holomorphic functions then implies analyticity of the limit in the domain specified before.
It should also be said that there are other, possibly more transparent ways of arriving at the results; in particular so-called polymer expansions described in [11] and later in the textbook [12] produce a somewhat different series; the main advantage of this approach is the fact that every term in the series reaches its thermodynamic limit already at a certain finite volume, so analyticity of each term is obvious, because it is equal to a finite volume quantity.

4 Conclusions

On the lattice there is, at least in the case of ‘complete symmetry breakdown’, no phase boundary completely separating the ‘Higgs’ and ‘Confinement’ regimes (just as in the liquid-gas case); the old proofs are valid. This is, however, not a paradox; the phenomenological explanation provided by 't Hooft makes the fact intuitively plausible.

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