Distributed Coded Modulation Schemes for Multiple Access Relay Channels

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Abstract—In this paper, we investigate network nest coded modulation schemes for multiple access relay channels. The performance of the distributed systems which are based on distributed convolutional codes with network coded modulation is presented. An analytical upper bound on bit error probability performance for the studied distributed systems with Maximum Likelihood Sequence Detection (MLSD) is derived. The constructed bounds for the investigated systems are shown to be asymptotically tight for increasing channel Signal-to-Noise Ratio (SNR) values.

I. INTRODUCTION

Distributed coding, as a special channel coding strategy developed for cooperative communication networks [1], [2], attracted large attentions recently. The distributed code construction concept has been applied on conventional channel coding to form such as distributed turbo codes [3], distributed low-density parity-check (LDPC) codes [4] and distributed rateless codes [5]–[17]. These published results show that the proposed schemes can improve the transmission reliability over point-to-point wireless communication channels.

The distributed coding schemes discussed above are developed for small-scale unicast relay networks, in which the messages are sent from a single source to a single destination through single/multi-hop relays. In this work, we consider a scenario that multiple source nodes communicate with multiple destination nodes via a Relay Node (RN). A classical way to pass such kind of information is through routing, where the relay nodes simply store and forward the received packets to the destination.

In [18], a network coding (NC) approach is proposed to replace routing. In NC, the relay nodes are allowed to encode the packets received from multiple source nodes. The combined information is subsequently sent to the destination. It has been shown in [19]–[27] that compared with traditional routing, NC can enhance the network capacity and throughput.

For the existing research in distributed network-channel codes (DNCC) design, many open questions in the design and implementation of distributed codes still have not been addressed. In this work, we develop the design of distributed codes for uplink transmissions in cellular systems and analyze the code performance based on the framework of uplink cellular systems.

The main contributions of this work are the propose of a distributed physical layer network coded system, the derivation of the analytical upper bound on the error probability.

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The outline of the paper is given below. The system model is described in Section II. The decoding algorithm is depicted in Section III. Performance analysis in terms of the upper bound on the error probability is given in Section IV. Code search results are given in Section V. In Section VI, numerical and simulation results are presented. Finally, conclusions are drawn in Section VII.

II. SYSTEM MODEL

We consider a wireless network with one relay node and a set of source and destination nodes. The source nodes send data packets to the destination nodes via the relay node. Let us denote by $\mathcal{S}$ the set of source nodes and by $\mathcal{T}$ the set of destination nodes. A connection between a source node $S$ and a destination node $T$ is established through the relay node $R$, where $S \in \mathcal{S}$ and $T \in \mathcal{T}$. The system can be modeled with a directed hypergraph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N}$ is the set of nodes, $\mathcal{N} = \mathcal{S} \cup \mathcal{T} \cup R$ and $\mathcal{E}$ is a set of hyperedges. A hyperedge $(n, \mathcal{D})$ consists of the directed edges between a start node $n$ and a set of end nodes $\mathcal{D}$, where $n \in \mathcal{N}$, $\mathcal{D} \subset \mathcal{N}$ and $\mathcal{D}$ is non-empty. A hyperedge $(n, \mathcal{D})$ represents broadcasting links between node $n$ and the nodes in $\mathcal{D}$.

An example of this scenario is illustrated in Fig. 1, in which four source nodes transmit the information packets to four destination nodes via the relay node. In this case, $\mathcal{S} = \{A, C, D, F\}$ and $\mathcal{T} = \{B, C, E, F\}$. Denote by $\mathbf{I}_A$, $\mathbf{I}_C$, $\mathbf{I}_D$, $\mathbf{I}_F$ the packets originated from the source node $A$, $C$, $D$, $F$, respectively. We assume that all the source nodes have packets to transmit and all the packets have equal length of $\kappa$ bits. In the concept of network coding, the packets received by the relay node will be linearly combined over a finite field $GF(2)$ and then broadcast to all the destination nodes. With the assumption that all the destination nodes have the side information from the neighboring source nodes, the destination nodes can successfully decode the packets which are intended to them. For example, if the destination node $B$ knows the packets transmitted from source nodes $C$, $A$, $F$, then it can successfully decode the packet $\mathbf{I}_D$. In practical systems, however, the complete side information from the source nodes may not be available. For example, node $B$ may only have side information from the neighbor nodes $A$ and $C$, in this case the node $B$ cannot successfully decode the packet $\mathbf{I}_D$.

In this work, we consider the system that the information blocks, e.g., $\mathbf{I}_A$, are encoded at the source node $A$ before its transmission to the relay node. The received codewords from all the source nodes are linearly combined using the XOR operation at the relay node and then broadcast to all the
destination nodes. Assume error free transmission from the source nodes to the relay node. Then the codeword generated at the relay node is

$$C = I_A G_A \oplus I_C G_C \oplus I_D G_D \oplus I_F G_F,$$

(1)

where $G_A$, $G_C$, $G_D$, $G_F$ are the generator matrices for the corresponding source nodes with a code rate of $\kappa/\eta N_T$, where $N_T$ is the cardinality of the set of destination nodes $\mathcal{T}$. Each destination node decodes the received codeword with its own side information. Let $\Theta_\omega$ be the side information for the destination node $\omega$. At the relay, the received codeword is combined with the side information,

$$\tilde{C}_\omega = \tilde{C} \oplus \Theta_\omega.$$

(2)

$\tilde{C}_\omega$ denotes the received mixed codewords which are unknown to the node $\omega$. It represents a corrupted version of the codeword with a code rate $(N_T - |\Theta_\omega|)\kappa/\eta N_T$, where $|\Theta_\omega|$ is the length of the set of destination nodes which are known by node $\omega$. The receiver at node $\omega$ can then decode $\tilde{C}_\omega$ with the corresponding code rate. Therefore, a single codeword $C$ can be interpreted differently by different destination nodes. This is also the basic idea of nested codes. For more detailed description of nested codes, please refer to [28, 29].

For a special case of the above mentioned system, we consider a wireless cellular uplink transmission system, in which a number of MTs send data packets to a BS via a RN. We assume that there are direct links between the source MTs and the RN and between the RN and the BS, but no direct links between the BS and the MTs. The transmission can be separated into two phases. In the first phase, the source MTs first encode the information packets then broadcast to the RN. In the second phase, the RN decodes the received codewords from the source MTs, then the codewords will be re-encoded, modulated and broadcasted to the BS.

The system model is illustrated in Fig. 2, in which a number of $S$ source MTs transmit information packets to a BS via a RN. The $i$th source MT uses the encoder $C_i$ with a code rate of $1/n_i$. We assume that all the source nodes have packets to transmit and all the packets have equal length of $\kappa$ bits. In the concept of network coding, the packets received by the relay node will be linearly combined over a finite field $\text{GF}(2)$ and then broadcast to all the destination nodes.

In this work, the transmitted information of each source node is encoded with different linearly independent generators and then forwarded to the relay node. Since the different transmitted information will be nested at the relay node with XOR operation, the rate of nested packets will be easily higher than one if we encode source information with a high rate channel code. For example, if we choose a code with rate $R = k/n$ to encode different source information separately and XOR them, the nested rate will be $R_{\text{nested}} = S \cdot k/n$ ($S$ denotes the number of source nodes), which easily achieves a much higher rate than one. However, such a high rate can only be decoded by perfect prior knowledge at the destination nodes.

Alternatively, select a low rate $R/S = k/(nS)$ code to encode information at different source node, and then perform XOR operation at the relay node, mathematically,

$$c_{\text{nested}} = i_1 G_1 \oplus i_2 G_2 \oplus \cdots \oplus i_S G_S,$$

(3)

$$= [i_1, i_2, \cdots, i_S] [G_1, G_2, \cdots, G_S]^T,$$

(4)

where $G_1, G_2, \cdots, G_S$ are mutually linearly independent generators corresponding to rate $k/(nS)$ codes. $\oplus$ denotes XOR operation. At the relay, the output codewords $c_{\text{nested}}$ are then fed into a Network Encoder (NE) and modulated with a memoryless modulator (MM). By properly choosing the NE and the MM, we can form a class of digital coded modulation scheme. To simplify the study, here we borrow the idea of coded digital phase modulation scheme, such as CPFSK [31], in which the NE is a Recursive Systematic Convolutional (RSC) encoder. It was shown in [32] that a CPFSK scheme can be decomposed into a concatenation of a RSC and a MM. CPFSK has the advantage of creating a differentially encoded waveform stream and is further attractive due to its good spectral properties, which has been widely used in many communication systems, e.g. digital video broadcasting (DVB) [33], satellite communication networks [34–35], etc. The generated waveform from the digital phase modulator is transmitted over the channel. The channel is assumed to be the AWGN channel.
III. Decoding with Nested Coded Modulation at the Destination Node

The received passband signal at the destination node can be expressed as, \( r(t, \mu) = s(t, \mu) + n(t) \), where \( n(t) \) is a zero mean Gaussian random process. \( s(t, \mu) \) is the transmitted signal from the relay with binary information symbols \( \mu \in \{0, 1\} \), which can be written as

\[
s(t, \mu) = \text{Re} \left\{ s_b(t, \mu) e^{j(2\pi f_1 t + \phi_0)} \right\}
\]

where \( s_b(t, \mu) = \sqrt{2E_c} e^{jW(t, \mu)} \) is the complex baseband equivalent signal \( \mathcal{R} \). \( f_1 = f_c - h/2T \) is a shift of the carrier frequency \( f_c \), \( \phi_0 \) is the initial phase of the carrier and \( \mu \) is the data symbol sequence. \( E_s \) and \( T \) are the symbol energy and the symbol interval duration, respectively.

The tilted information carrying phase \( ^\perp \) during symbol interval \( n(t = \tau + nT) \) is given by \( (Q, \{\} \) where \( \mathcal{R} \{\} \) is the modulo \( x \) operator and \( W(\tau) \) is given by (5), which represents the data-independent terms. The phase response \( q(\cdot) \) is found from the frequency pulse \( g(t) \) according to \( q(t) = \int_{-\infty}^{\infty} g(\tau) d\tau \). \( L \) is the length of the frequency response. For CPFSK, \( L = 1 \) and the frequency pulse used in this work is Rectangular (REC) pulse defined as:

\[
\text{REC} \quad g(t) = \frac{1}{2T}, \quad 0 \leq t \leq T
\]

The equivalent complex baseband continuous-time signal can be written as \( r_b(t, \mu) = s_b(t, \mu) + n_b(t) \),

\[
r_b(t, \mu) = s_b(t, \mu) + n_b(t), \quad (6)
\]

where \( n_b(t) \) is a complex baseband representation of the additive white Gaussian random process having zero mean double-sided power spectral density \( 2N_0 \) \( \mathcal{R} \).

In each symbol interval, a set of possible CPM sequences consists of \( P \cdot M^L \) various complex valued signals. Thus, a bank of \( P \cdot M^L \) complex valued filters, matched to those signals and sampled once every symbol interval produces a sufficient statistics, since they form a basis for signal space \( \mathcal{R} \). A component of the vector \( r_n \) representing the sampled outputs at symbol interval \( n \), can be calculated by

\[
r_{i,n} = \int_{(n-1)T}^{nT} r(t, \mu) e^{jW(t, \mu_i)} dt, \quad (7)
\]

where \( i \in \{1, 2, \ldots, P \cdot M^L\} \) and \( \mu_i \) is the \( i \)-th hypothesis sequence offered by the receiver. All the \( P \cdot M^L \) various complex signals must be generated by using various \( \mu \) s and the index \( i \) on the right hand side of (7) is intended to reflect this.

Sufficient statistics can be produced in many other ways, corresponding to choosing other basis functions for the signal space. Changing from one to another is done by a linear transformation. Given the above choice, signal space basis may not be orthogonal. There can be linear deterministic dependencies among the components of \( r_n \). Hence \( r_n \) has a non-diagonal covariance matrix \( \mathbf{A} \).

We denote by \( \chi_n \) the expectation of \( r_n \). Because of the Gaussian channel, \( r_n \) is a Gaussian random vector. The joint Probability Density Function (PDF), conditioned on the CPE output vector \( \nu_n \), is

\[
p(r_n | \nu_n) \propto \exp \left\{ - (r_n - \chi_n)^H \mathbf{A}^{-1} (r_n - \chi_n) \right\} \quad (8)
\]

where \( (\cdot)^H \) denotes Hermitian transpose. A more detailed description of CPM can be found in \( \mathcal{R} \).

Based on (8), we can use maximum a posterior (MAP) algorithm to decode the received CPFSK signal as described in \( \mathcal{R} \). Please note that since the generators at the source nodes and the CPFSK modulator formed a super-trellis at the destination nodes, we can employ the MAP algorithm over the super-trellis to decode the transmitted signal for each source node.

At different destination nodes, after the demodulation and decoding process, we get output signal \( \hat{c} = c_{\text{nested-os}} + e \), where \( e \) is the binary error pattern. At the destination node \( d_i \),

\[
\hat{c} = \bigoplus_{l \notin \kappa_d} i_l G_l \bigoplus_{l \in \kappa_d} i_l G_l + e, \quad (9)
\]

where \( \kappa_d \) denotes the indices of the information prior known to the destination node \( d_i \). Part \( \bigoplus_{l \notin \kappa_d} i_l G_l \) represents the known information to \( d_i \), which can be canceled by XOR operation. Then,

\[
\hat{d}_i = \hat{c} \bigoplus_{l \notin \kappa_d} i_l G_l = \bigoplus_{l \notin \kappa_d} i_l G_l + e. \quad (10)
\]

The remained part of \( \bigoplus_{l \notin \kappa_d} i_l G_l + e \) indicates the combined unknown nested packets to the destination node \( d_i \). To separate the desired information, we need to employ the feature of nested codes. Because of the nested approach in Eq. (3), where \( \left[ G_1, G_2, \ldots, G_{N_d} \right] \) can be regarded as a new “stacking” generator matrices, it is possible to interpret \( i_l \) \( l \notin \kappa_d \) from \( \bigoplus_{l \notin \kappa_d} i_l G_l + e \) with corresponding “stacking” generator matrices.

The LLR of the \( i \)-th bit in \( c_n \) can be computed as Eq. (11). There is no information lost through the above cancelation operation, because it only changes the sign of the LLR. Then we get the calculated LLR \( L_{\text{c}} \), which is the estimated soft information of unknown packets to receiver \( d \). Based on the linearly independent feature of nested codes, we can separate all the desired information at different destination nodes. Particularly, regarding \( \left[ G_1, G_2, \ldots, G_{\kappa_d} \right] \) from Eq. (3) as a new “stacking” generator matrix, we can multiple interpret \( i_l \) \( l \notin \kappa_d \) from \( L_{\text{c}} \).

IV. Analytical Bounds on the Bit Error Probability for the Multiple Access Relay Channels

In this work, we will develop analytical upper bounds on bit error probability for the investigated network coded system under MLSD. The different convolutional encoders at different source nodes and the NE at the RN constitute a super-trellis encoder. At the destination nodes, we can develop a ML decoder for the super-trellis encoder. Let \( P_b \) be the bit error probability for the links from all source nodes to the relay
\[
\psi(\tau + nT, \mu) = R_{2\pi} \left\{ 2\pi h R_P \left\{ \sum_{i=0}^{\eta-1} \mu_i \right\} + 4\pi h \sum_{i=0}^{L-1} \mu_{i}q(\tau + iT) + W(\tau) \right\}, \quad 0 \leq \tau < T \tag{6}
\]

\[
W(\tau) = \pi h(M - 1)\frac{\tau}{T} - 2\pi h(M - 1) \sum_{i=0}^{L-1} q(\tau + iT) + \pi h(M - 1)(L - 1), \quad 0 \leq \tau < T \tag{5}
\]

Let \( P_b = 1 - (1 - P_b^u)(1 - P_b^{d}) \). \( \tag{12} \)

Let \( N_s \) be the cardinality of the set of source nodes \( \mathcal{S} \). We assume that at each source node, the source information is firstly convolutional encoded, then modulated with BPSK and transmitted over an AWGN channel. We further assume that the sequences from different source node are transmitted over orthogonal channels, so that they do not interfere with each other. Under the above assumptions, \( P_b^u \) can be expressed as

\[
P_b^u = 1 - \frac{N_s}{1} \prod_{i=1}^{N_s} (1 - P_b^{u,\sigma}(i)), \tag{13}
\]

where \( P_b^{u,\sigma}(i) \) is the bit error probability for the link from the \( i \)th source node to the relay node. Suppose the code rate at the \( i \)th source node is \( r_i \), and the bit energy is \( E_b^i \), then \( P_b^{u,\sigma}(i) = Q(\sqrt{r_i E_b^i/N_0}) \). Thus, \( P_b^u \) can be expressed in a closed form.

Now let us look at \( P_b^{d} \). Exact expression of \( P_b^{d} \) is hard to find, however, we can give an upper bound on \( P_b^{d} \) under MLSD. The state \( \sigma_j \) of the super-trellis encoder at discrete time \( j \) is defined as \((\sigma^{ec}_j, \sigma^{ne}_j)\), where \( \sigma^{ec}_j \) and \( \sigma^{ne}_j \) denote the state of the Joint Distributed Convolutional Encoder (JDCE) and the state of RSC at discrete time \( j \), respectively. For a JDCE having \( m \) memory elements, and a CPFSK scheme, the total number of states is \( 2^{m+1} \). The state transition \( \sigma_j \rightarrow \sigma_{j+1} \) is determined by the input of the source nodes. Associated with this transition is also the input symbol \( \mu \in \{0, 1\} \) of the NE, i.e. the RSC encoder, and the mean vector which is obtained by letting the transmitted waveform pass through a bank of complex filters which are matched to the transmitted signals \( \text{[8]} \).

In this work, we assume all the source nodes have the same transmit power. Let \( E_b \) be the information bit energy

and \( N_0/2 \) be the double sided power spectral density of the additive white Gaussian noise. The bit error probability for a memoryless information source sequence of the distributed network coded modulation system will follow the following theorem.

**Theorem 1:** Under the MLSD and the assumption that the source block is infinitely long, the bit error probability for a distributed convolutional encoded CPFSK system with a discrete memoryless uniform digital source sequence, can be upper bounded by Eq. (14), where \( d_{\text{min}} \) is the minimum NSED and \( \eta, \epsilon, \zeta \) are dummy variables and \( r \) is the code rate of the joint trellis encoder of all the source nodes. \( r = p/s \) as described in Section II. The average transfer function is given by Eq. (15), where \( W_{j, s_1, \ldots, s_r} \) is the number of error events that start at time \( j \) from state \( s_k \), and have NSED \( d^2 \), length \( t \) and total number of symbol errors caused by the error event given by \( \tau \). The \( Q \) function is defined as

\[
Q(x) = \left( \frac{\sqrt{2\pi}}{2} \right)^{-1} \int_x^{\infty} e^{-z^2/2} dz. \quad \Box
\]

The transfer function \( F(j, \kappa, \sigma, \epsilon, \zeta) \) can be obtained by using a product state diagram \( \text{[49, 50]} \). A product state at time \( j \) is defined as \((\sigma_j, \hat{\sigma}_j)\), where \( \sigma_j \) is a state of the super-trellis encoder of the distributed system and \( \hat{\sigma}_j \) represents a state of the decoder. The transition \((\sigma_j, \hat{\sigma}_j) \rightarrow (\sigma_{j+1}, \hat{\sigma}_{j+1})\) is labeled with

\[
\sum_{\Delta \tau} \sum_{\Delta d^2} b(\Delta \tau, \Delta d^2) \eta \epsilon^\Delta \tau \zeta^\Delta d^2, \tag{16}
\]

where \( \Delta \tau \) and \( \Delta d^2 \) are the number of the symbol errors and NSED, respectively, \( b(\Delta \tau, \Delta d^2) \) denotes the number of paths having NSED \( \Delta d^2 \) and symbol errors \( \Delta \tau \) for this state transition.

For the distributed convolutional coded CPFSK system, the bit error rate only depends on the output of the distributed convolutional encoder. In other words, it is independent of the pair state of the RSC within the CPFSK modulator \((\sigma^{ne}_j, \hat{\sigma}^{ne}_j)\). Furthermore, the NSED \( d^2 \) only depends on the difference of the CPFSK states \((\sigma^{ne}_j - \hat{\sigma}^{ne}_j)\) \( \text{[38]} \). Therefore, the product state can be reduced. The reduced product state can be
The product states can be divided into initial states, transfer states, and end states. The end states represent the transitions from transfer states to transfer states in one step. The transfer function can be written as \((\sigma_j^{cc}, \tilde{\sigma}_j^{cc}, \omega_j)\), where \(\omega_j = R_P \{\sigma_j^{ne} - \tilde{\sigma}_j^{ne}\} = R_P \{\sum_{n=0}^{j-L} \gamma_n\}\) is the difference phase state. The total number of product states for the distributed convolutional coded CPFSK systems is \(2^{2m+1}\).

The product states can be divided into initial states, transfer states, and end states. A product state is an initial state if an error event can start from it. A product state is an end state if an error event can end in it. The conditions for initial states and end states are \(\sigma_j^{cc} = \tilde{\sigma}_j^{cc}\) and \(\omega_j = 0\). Other states are referred to as transfer states.

Let \(A_{k,j}\) represent the state transitions from an initial state \(s_k\) to transfer states in one step at time \(j\). Let us denote by \(B_j\) the transitions from transfer states to end states, and by \(C_j\) the transitions from transfer states to transfer states in one step at time \(j\). Let \(D_{k,j}\) represent the transitions from an initial state \(s_k\) to end states in one step. The transfer function can be calculated by \(F(j, k, \eta, \epsilon, \zeta) = 1 \cdot (B_j(I - C_j)^{-1}A_{k,j} + D_{k,j})\) (17)

where 1 is an all one vector and \(I\) represents the identity matrix.

Eq. (14) can be further expressed as

\[
P^d_b \leq \sum d^j \cdot W_d \cdot Q(\sqrt{\frac{d^j E_{br}}{N_0}}),
\]

(18)

where \(W_d = 2^{m-j} \sum_{j=1}^{2m} \sum_{\kappa=1}^{2m-j} \sum_{\ell} \sum_\tau W_{j,s_k,\ell,\tau,d} \cdot 2^{j+1}\).

It can be seen from (18) that the minimum NSED \(d^j_{min}\) and \(W_{d_{min}}\) (the number of error events with \(d^j_{min}\)) dominate the asymptotical symbol error rate of the system.

Based on (12), (13) and (14), we can get the overall bit error probability for the investigated distributed network coded modulation system via eq. (19).

V. CODE SEARCH

To achieve the network coded modulation scheme with nested codes, the code design is assumed to satisfy the following criterion:

1) the generators assigned to different nodes should be mutually linearly independent.

2) the rate of “stacked” generator matrix should be less than 1.

3) the selected code should not be a catastrophic convolutional code.

We construct several good codes based on the modified FAST algorithm in [61], which are presented in Table I.

| Rate | Memory number | Generator matrices | \(d_{free}\) |
|------|---------------|---------------------|------------|
| 2/3  | 2             | \(\begin{bmatrix} 6 & 5 & 1 \\ 7 & 2 & 5 \end{bmatrix}_8\) | 5          |
| 2/4  | 2             | \(\begin{bmatrix} 3 & 7 & 1 & 6 \\ 4 & 7 & 6 & 3 \end{bmatrix}_8\) | 8          |
| 3/4  | 2             | \(\begin{bmatrix} 5 & 4 & 3 & 2 \\ 4 & 6 & 5 & 5 \end{bmatrix}_6\) | 6          |
| 4/6  | 2             | \(\begin{bmatrix} 5 & 6 & 5 & 6 & 7 & 4 \\ 7 & 0 & 7 & 6 & 6 & 2 \\ 4 & 5 & 2 & 6 & 6 & 5 \end{bmatrix}_8\) | 8          |

Then, we can choose different rows of one generator matrix as different linearly independent generators. In this work, we choose a rate 2/3 code from Table I. Table II illustrates the code performance of the selected 2/3 code. Then, one of the possible strategies to assign the generators to different nodes can be

\[
G_1 = [6 \ 5 \ 1],
\]

(20)

\[
G_2 = [7 \ 2 \ 5],
\]

(21)

VI. SIMULATION RESULTS FOR DISTRIBUTED NETWORK CODED SYSTEMS

Computer simulations have been performed for the distributed physical layer network coded and digital phase modulated systems over Rayleigh fading and AWGN channels. We consider two mobile stations communicate with BS via a single relay node. The two mobile stations are encoded with the nested codes, the code design is assumed to satisfy the following criterion:

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| 3/4  | 2             | \(\begin{bmatrix} 5 & 4 & 3 & 2 \\ 4 & 6 & 5 & 5 \end{bmatrix}_6\) | 6          |
| 4/6  | 2             | \(\begin{bmatrix} 5 & 6 & 5 & 6 & 7 & 4 \\ 7 & 0 & 7 & 6 & 6 & 2 \\ 4 & 5 & 2 & 6 & 6 & 5 \end{bmatrix}_8\) | 8          |
$P_b = 1 - (1 - P^u_b)(1 - P^d_b) = 1 - (1 - Q(\sqrt{rE_b/N_0})^{N_s})(1 - P^d_b) \leq 1 - (1 - (1 - Q(\sqrt{rE_b/N_0})^{N_s})(1 - Q(\sqrt{2d_{min}E_b/2N_0}))) \exp \left(\frac{d_{min}E_b}{2N_0}\right) \frac{\partial F(\eta, \epsilon, \zeta)}{\partial \epsilon} \bigg|_{\eta=1/2, \epsilon=1, \zeta=\epsilon^{-1}} \leq 1 - (1 - Q(\sqrt{rE_b/N_0})^{N_s}) \left(1 - \sum_{d^2} W_d \cdot Q(\sqrt{d^2 E_b/2N_0})\right)$ (19)

Table II

| Rate | Generator matrices | $d_{free}$ | $\Delta_{i}\phi_{j}(d_{free}+1)$ | $\Delta_{i}\phi_{j}(d_{free}+1)$ |
|------|-------------------|------------|-----------------------------|-----------------------------|
| 1/3  | (6 5 1)\(_a\)     | 5          | 1                           | 1                           |
|      | (7 2 5)\(_a\)     | 6          | 2                           | 2                           |
| 2/3  | (6 5 1)\(_a\)     | 5          | 3                           | 3                           |

![Simulation vs Analytical bound](image)

Figure 3. Analytical upper bounds and one simulation result for the investigated systems.

VII. SUMMARY

In this work, we proposed a distributed network coded modulation scheme for multiple access channel in a cellular system. We considered a system in which a number of source MTs transmit information data packets to a BS via a relay node.

The decoded codewords from the MTs at the relay were then fed into a network coded modulator to perform network encoding and digital phase coded modulation. Analytical bound on BER performance for the proposed distributed network coded modulation systems is derived. The bounds are shown asymptotically tight. These bounds can be served as the guideline for the design of the investigated distributed network coded modulation systems.

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