Explosive black hole fission and fusion in large extra dimensions

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ABSTRACT: Black holes are the densest form of energy, and in the presence of compact dimensions black objects may take one of several forms including the black-hole and the black-string, the simplest relevant background being $\mathbb{R}^{3+1} \times S^1$. Recent understanding of the first order nature of the transition indicate a powerful “hysteresis” curve, where black objects may undergo fusion or fission during a tachyonic decay with Planck power and duration of the order of the size of the compact dimension $L$. Such explosions which scale with $L$ could be test signatures for large extra dimensions in either astronomical observations or accelerators.
1. Introduction

Black holes are by definition the densest form of energy. Black holes which are small enough to be on the human scale (∼ 1m) attract special fascination. For example a black hole of radius 9mm has the following striking properties: it is long lived, has the same mass as the earth, attracts matter 1m away with an acceleration of $4 \cdot 10^{13} \, g$, has Hawking temperature $0.02 \, K$ and emission power of $9 \cdot 10^{-18} \, W$. Any process involving such objects would involve energies and forces which are extremely high on a human scale.

On the other hand, extra dimensions are theoretically motivated by Kaluza-Klein theory and String Theory, and have recently attracted much attention in phenomenology. Various extra dimensions scenarios were suggested but for the purpose of this note it will be enough to consider 4 extended dimensions together with a fifth periodically identified coordinate $z$, $z \sim z + L$, and it is enough to know that “large extra dimensions” much larger than the Planck length cannot be ruled out experimentally today, for $L \lesssim h/1 \, \text{TeV}$ in ordinary circle compactification\(^1\) or even as high as $L \lesssim 1 \mu m$ for theories where gauge fields are confined to a brane localized in the extra dimension and only gravity propagates in the extra dimension.

In the presence of a compact dimension massive solutions of General Relativity may take one of several forms including the black-hole and the black-string. Comparison of the entropies of the two solutions reveals that for low masses (Schwarzschild radius smaller than $L$) the black hole is preferred, while at large masses the black string is stable, and the black hole phase is expected to disappear following the merger of the “north” and “south” poles. This leads one to suspect that a phase transition occurs. Indeed some phenomenological consequences of this transition were discussed in [19, 20, 21, 22, 23]. The purpose of this note is to stress the phenomenological consequences of a recent qualitative understanding of the transition [5, 10] (based on [1, 2, 3, 4]) and give estimates of the energy and power

\(^{1}\)Notation - while $c$ is set to 1, both $\hbar$ and $G_4$ (the 4d Newton’s gravitational constant) will be shown explicitly in order to distinguish classical from quantum processes. The (4d) Planck length is defined by $l_{p4}^2 \sim \hbar G_4$, and the Planck mass is $m_{p4} \sim \hbar/l_{p4}$. 
emitted during transition. See also [11, 13, 2, 14, 5, 16, 7, 18] for a non-representative list of recent papers on black holes and and large extra dimensions/ brane worlds/ accelerator prospects.

This note is organized as follows: we start with a review of the suggested phase diagram of [10] in section 2, then the fusion and fission processes are described in section 3 together with explicit numerical estimates and we conclude with applications and open numerical questions in section 4.

2. Phase diagram

Let us set-up the problem. Consider a static (no angular momentum) black object in an $\mathbb{R}^{3+1} \times S^1$ gravitational background, namely extended 3 + 1 space-time with a periodic fifth dimension which will be denoted by the $z$ coordinate. The system is characterized by 3 dimensionful constants: $L$ the size of the extra dimension $z \sim z + L$, $M$ the (4d) mass of the system measured at infinity of $\mathbb{R}^3$, and $G_5$ the 5d Newton constant. From these a single dimensionless parameter can be constructed

$$\mu = \frac{(G_5 M)}{L^2}, \quad (2.1)$$

while the 4d effective Newton constant is given by $G_4 = G_5/L$.

The isometries of these solutions are $SO(3) \times U(1)$, where the $SO(3)$ comes from the spherical symmetry in $\mathbb{R}^3$ and the $U(1)$ comes from time independence. The most general metric with these isometries is

$$ds^2 = -e^{2A}dt^2 + ds^2_{(r,z)} + e^{2C}d\Omega^2, \quad (2.2)$$

which is a general metric in the $(r, z)$ plane together with two functions on the plane $A = A(r, z), C = C(r, z)$. The horizon is a line determined by $e^{2A} = 0$ and as usual $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. In 4d one can choose a coordinate system that is especially adapted to cylindrical symmetry, but this works best only in 4d.

At least two phases of solutions can be distinguished - a black string with an $S^2 \times S^1$ horizon topology and a 5d black hole with an $S^3$ horizon. The uniform black string (figure 1) is given by the 4d Schwarzschild metric with $z$ added as a spectator coordinate

$$ds^2 = -(1 - r_4/r) dt^2 + (1 - r_4/r)^{-1} dr^2 + dz^2 + r^2 d\Omega^2, \quad (2.3)$$

where the Schwarzschild radius is $r_4 = 2G_4 M$.

The small 5d black hole (see figure 2) ($r_5 = ((8/3\pi)G_5 M)^{1/2} \ll L$) can be approximated by a combination of two solutions. Denoting the distance from the black hole by $\rho$, for $\rho \ll r_5$ it can be approximated by the 5d Schwarzschild BH

$$ds^2 = -(1 - r_5^2/\rho^2) dt^2 + (1 - r_5^2/\rho^2)^{-1} d\rho^2 + \rho^2 d\Omega^2_{S^3}, \quad (2.4)$$

\footnote{Taken from [10].}

\footnote{In the Lorentzian solutions it is the non-compact version of $U(1)$.}
where $d\Omega^2_{S^3} = d\chi^2 + \sin^2 \chi d\Omega^2$. For $\rho \gg r_5$ the Newtonian approximation is valid, and the potential is proportional to the Green function

$$V(w) = \frac{1}{4r}[\coth(w/2) + \coth(\bar{w}/2)].$$ (2.5)

where $w := 2\pi (r + iz)/L$. Indeed close to the source point $w \to 0$, the potential has the expected 5d behavior $V \sim (1/4r)((2/w) + (2/\bar{w})] = L/(2\pi(r^2 + z^2))$, while for $\text{Re}(w) \to \infty$, the 4d behavior is restored $\coth(w) = 1 + 2\exp(-2w)$, $V \sim (1/2r) + O(\exp(-r))$. Presumably a full solution can be built perturbatively in $r_5/L$ from these two approximations.

Note that following the equipotential surfaces of (2.5) one encounters already a primitive version of a topology change - close to the source the surface has an $S^3$ topology, then there is one singular surface with a conic singularity (a cone over $S^2$) after which the topology of the surfaces changes to $S^2 \times S^1$.

Comparing the areas of the two solutions one finds that for the string the area is

$$A_{\text{string}} = 4\pi r_4^2 L \sim \mu^2,$$ (2.6)

while for the small BH

$$A_{BH} \simeq 2\pi^2 r_5^3 \simeq \mu^{3/2}.$$ (2.7)

Hence, for small $\mu$ the black-hole is preferred, while for larger $\mu$ it is not clear if the black-hole phase exists, but even if it does then if its area would grow as in (2.7) it would be dominated by the black-string. Therefore a phase transition is expected from the outset.

We now turn to describe the phase diagram suggested in [10] - figure 3 (some refinements are in order near the merger point and will be discussed below). The vertical axis is the parameter $\mu$, while the horizontal axis is an order parameter$^4$ of non-uniformity $\lambda$. For small $\lambda$ it is defined by $\lambda := (1 - r_{\text{min}}/\bar{r})$ where $r_{\text{min}}$, $\bar{r}$ are the minimum and average respectively of $r = r(z)$, so that $\lambda = 0$ exactly when the solution is a uniform string. For large $\lambda$ no precise definition is offered, but consider it to continue to be a measure of non-uniformity even in the black-hole phase.

The diagram contains two stable phases, the (uniform) black-string and the black hole. The black string is stable for $\mu > \mu_1$ (the numerical value of $\mu_1$ is unknown) where a first

$^4$Even though normally a phase diagram does not include order parameters.
order transition occurs to an equal entropy black hole. Since the first order phase transition requires tunneling, a big black string will be long-lived and one may consider lowering $\mu$ further until one gets to the Gregory-Laflamme point \cite{1,2} $\mu_{GL} \simeq 0.070$ (or $(L/r_4) \simeq 7.2$) where a certain metric mode becomes marginally tachyonic signaling the breaking of the translational symmetry along the $z$ axis. The end-point of this decay is somewhat unclear and controversial – while from the current point of view and also according to \cite{1,2} the end-point is the black-hole, it was also argued recently that the end-point must be a stable non-uniform string \cite{3}. At any rate, these distinctions do not change the main result of this note, although they are important for the details of the processes.

Figure 3: A suggested phase diagram, which will be refined further in the area of the merger transition. $\mu$ is the dimensionless parameter, and $\lambda$ is an order parameter (measure of non-uniformity). Stable (unstable) phases are denoted by solid (dashed) lines, while the dotted lines denote transitions - a first order transition at $\mu_1$ and a tachyonic decay from the two other points.
The unstable non-uniform string emanating from the GL point was determined to be unstable by the analysis of Gubser and by some first results of a numeric study. This determination is crucial because it shows the transition to be first order and explosive, rather than, say second order and mild. However, the unstable phase never realizes physically.

The black-hole phase is stable for $\mu < \mu_1$ and long-lived until the onset of a perturbative instability, which presumably takes one to the uniform black-string phase. Now it is time to explain the nature of the due refinement in figure 3. Actually the black hole must turn perturbatively unstable already at $\mu_2 < \mu_{\text{merger}}$. This is because the cone over $S^2 \times S^2$ which is a central player in this transition and locally models the merger and topology change is unstable 5, and accordingly changes are expected in the diagram in the vicinity of the merger point.

3. Fission (evaporation) and fusion (accretion)

The terms “fission” and “fusion” refer to the following process: we are interested in two kinds of transitions - black-hole (BH) $\leftrightarrow$ black-string (see figure 4). In “black-string $\rightarrow$ BH”, the string fissions into a BH, while in the “BH $\rightarrow$ black-string” the BH fuses itself into a string.

The system was seen to include a typical first order transition. In particular it traces a sort of “hysteresis” curve, with two possible tachyonic decays. These decays are bound to release energy in the form of gravitational waves, with characteristic energy and time determined by their classical nature. Since at transition the system has only one energy scale (classically) $M \simeq L/G_4 \simeq L^2/G_5$ the released energy must be proportional to it $\Delta M = \eta M$, with some efficiency coefficient $\eta < 1$ where $\eta$ may differ for the two processes BH $\rightarrow$ black-string or black-string $\rightarrow$ BH, and it can be expected to equal a few percent by analogy with black hole collision simulations (see and references therein). For relevant $L$’s this can be restated as

$$\Delta M = 1.21 \cdot 10^{38} \eta \frac{\mu}{1 \mu m} = 2.89 \cdot 10^{28} \text{ Ton TNT } \eta \frac{L}{\mu} \frac{L}{1 \mu m}$$

5Curiously this instability disappears for $d > 10$ spacetime dimensions.
\[
= 2.40 \cdot 10^{25} J \frac{\eta}{\mu} \frac{L}{h1 \text{ TeV}^{-1}} = 5.71 \cdot 10^{15} \text{ Ton TNT} \frac{\eta}{\mu} \frac{L}{h1 \text{ TeV}^{-1}}
\]

where 1 Ton TNT := 4.2 GJoule, and for the black string \( \mu = \mu_{GL} = 0.070 \) while for the black hole \( \mu \sim 1 > \mu_{GL} \).

The time scale for decay is again determined by the single classical time scale present

\[
\tau \sim L,
\]

and so the power of emitted radiation is

\[
P \sim \eta \frac{1}{G_4}
\]

It is interesting whether the emitted energy is originally produced mostly in the form of gravitational waves, creating other secondary particles, or whether other particles are significantly produced.

Let us see how the fission of the black-string into a black-hole fits with the Hawking evaporation process (see figure 5).

For \( \mu > \mu_{GL} \) the emitted power is

\[
P = \sigma A T^4 \simeq \frac{h}{(G_4 M)^2} \simeq \frac{h}{r_4^2} = 2.2 \cdot 10^{16} W \left( \frac{h1 \text{ TeV}^{-1}}{r_4} \right)^2
\]

where \( \sigma = \pi^2 k^4/(60 h^3 c^2) = 5.67 \cdot 10^{-8} M m^{-2} K^{-4} \) is the Stefan-Boltzmann constant. Notice that this is a quantum process suppressed by a power of \( h \). When \( \mu_{GL} \) is reached the black string destabilizes and collapses ("fission") emitting at Planck powers (3.3). As a 5d black hole it continues to evaporate though with a different rate \( \sim h/r_5^2 \sim h/(G_5 M) \). In the last stages of the evaporation for \( M \sim m_{p4} \) the power gets close to Planck power again but only over Planck periods, so the total emitted energy is much smaller. Thus the phase transition stands out as the most exothermic part of the evaporation process.

Going in the opposite direction, a black hole may be "charged" up to \( \mu_2 \), say by an accretion disk, where it becomes unstable and collapses to a string ("fusion"). The energy and power parameters of this explosion are very similar to the parameters for evaporation (3.1, 3.2, 3.3). The ensuing evolution after the explosion will depend on the balance of evaporation versus accretion.

Figure 5: A typical evaporation process. The power is depicted as a function of time on a log-log scale, and the kink between 4d and 5d behaviour is over-accented. At transition an explosion occurs with Planck power and duration of order \( L \).
4. Discussion

It would be interesting to apply these explosions (or their absence) as a test or signature for extra dimensions. As we saw, any Hawking evaporation process necessarily goes through a fission explosion, the explosion being bigger the bigger $L$ is. Fusion could also occur naturally for a growing small black hole fed by an accretion disk. In relation to astronomical observations one could estimate the chances for small black holes to exist and explode within a time and space window such that they would be noticable. One could also consider implications for scenarios where black holes would be produced by accelerators.

One cannot avoid considering bomb construction using either fission or fusion ignoring the problems of creating and handling small black holes. A black hole could be prepared close to its critical size ready to fuse and then activated by adding matter and reaching “critical mass”. Timing the fission process is more difficult, since the time for decay is determined by the Hawking radiation which cannot be hurried artificially, but perhaps it is possible to balance the Hawking radiation with incoming radiation until activation. However, these bombs have a big disadvantage, since if one is in possession of small black holes, one could collide them with very similar explosions occurring both in emitted energy and time scale (being a classical process), and since igniting the phase transition seems to be the more complicated process it would probably be disfavoured, and luckily not used for destructive ends.

I would like to conclude by mentioning the basic dimensionless constants of the problem which remain to be determined numerically: $\eta_{1,2}$ the real-time efficiency for radiating energy for either fission or fusion, $\mu_1$ the location of the first order transition and $\mu_2$ the location of the onset of the tachyon.

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