Role of the $\Delta$ isobar in the reaction $NN \to NN\pi$ near threshold

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Abstract

A model calculation for pion production in nucleon–nucleon collisions is presented. Direct production, pion rescattering and contributions from pair diagrams are taken into account. The amplitudes for the elementary processes are based on well-established microscopic models of nucleon–nucleon and pion–nucleon scattering. The $\Delta(1232)$ is included explicitly and is found to play an important role even at energies close to production threshold. A good overall agreement with existing data from the pion production threshold up to the Delta resonance region is achieved.

Over the last decade or so a wealth of rather accurate data on pion production in nucleon–nucleon ($NN$) collisions near threshold has become available. This concerns reaction channels with a two–body final state such as $pp \to d\pi^+$ [1–3], as well as channels with three–body final states such as $pp \to pp\pi^0$ [4–6] or $pp \to pn\pi^+$ [8,9]. The same period of time also witnessed lively activity on the theoretical side. In this case, however, most of the efforts concentrated on the study of one particular reaction, namely the process $pp \to pp\pi^0$. Clearly this development was initiated by the unexpected failure of the first model studies [10,11] to reproduce the corresponding data [4]. The underprediction of the $pp \to pp\pi^0$ total cross section by a factor of 5 turned out to be a challenging problem for theorists.

These first investigations followed very closely the approach of Koltun and Reitan [12]. In this model it is assumed that the pions are produced either directly from the nucleon (Fig. 1a) or via pion rescattering (Fig. 1b), where in the latter the $\pi N$ amplitude is approximated by the corresponding $\pi N$ s–wave scattering lengths. Subsequently two ”new” mechanisms have been suggested.
that, in principle, allow one to obtain quantitative agreement with the data on \( \pi^0 \) production close to threshold. In one of them the pion is produced via an intermediate virtual antinucleon–nucleon state in conjunction with the exchange of a heavy meson (\( \rho, \omega, \sigma, \ldots \)) (HME), as depicted in Fig. 1c. It was found in Refs. [13,14] that the contributions from \( \sigma \) and \( \omega \) exchange can enhance the production cross section significantly enough to reproduce the data. In the other mechanism the static \( \pi N \) interaction (i.e., the on–shell \( \pi N \) amplitude at threshold) used in the Koltun–Reitan model for the rescattering process is replaced by an off–shell \( \pi N \) amplitude. Physically, the latter is required anyway because the exchanged pion in the rescattering process is off–mass–shell. Since the isoscalar part of the \( \pi N \) amplitude, which is responsible for rescattering in the \( \pi^0 \) production, is practically zero on–shell and at threshold due to constraints of chiral symmetry, but can be fairly large once one goes off–shell, the use of an off–shell amplitude, given by a certain model, leads likewise to an appreciable enhancement of the production cross section [15–17].

Despite these apparent successes there is still a controversy over which of the two processes if any, is the physically correct one. The HME mechanism has been put into question in a recent paper by J. Adam et al. [18]. These authors work in the context of the relativistic Gross equation where contributions from intermediate antinucleon–nucleon states are generated automatically by the scattering equation and summed up to all orders. They see only moderate effects from these virtual intermediate states, resulting in contributions that are considerably smaller than the ones obtained in the perturbative treatment of Refs. [13,14]. Likewise, results based on the off–shell \( \pi N \) amplitude have come into dispute. Calculations done in lowest order chiral perturbation theory [19–21] give rise to a \( \pi N \) amplitude that differs in sign from the one employed in Refs. [15–17] when extrapolated off–shell. Accordingly, the amplitude from the rescattering diagram also changes sign and then interferes destructively with the contributions from the other production mechanisms. Consequently, agreement with the data in this case can be no longer achieved. For a thorough discussion of this topic see also Ref. [22]. Furthermore we want to call attention to a recent paper by Bernard et al. [23], where it is argued that heavy baryon chiral perturbation theory (on which the works [19–21] are based) can not be used in reactions with as large momentum transfers as they are typical for meson production in nucleon–nucleon collisions.

One possible way to learn more about the role of the individual production mechanisms has been suggested in Ref. [17,22]. It consists of a comparative study of all relevant pion production channels (\( pp \rightarrow pp\pi^0 \), \( pp \rightarrow pn\pi^+ \), \( pp \rightarrow d\pi^+ \) and \( pn \rightarrow pp\pi^- \)) in a consistent approach (i.e., with the same model and same parameters). Moreover, higher partial waves should be taken into account. Most of the aforementioned investigations consider only the lowest partial waves in the outgoing channel, which means that the \( NN \) system
is in an $S$–wave state (or in the deuteron bound state) and the pion is likewise in an $s$–wave relative to the nucleon pair. Such calculations permit only conclusions on the absolute magnitude of the production cross section near threshold. The inclusion of higher partial waves, in the $NN$ as well as the $\pi N$ sector, allows one to calculate predictions for differential cross sections and, in particular, spin–dependent observables. Therefore it is possible to examine whether the considered production mechanisms lead to the proper onset of higher partial waves, as suggested by the data. Results of the latter kind are particularly interesting because they reflect the spin–dependence of the production processes and therefore should be very useful in discriminating between different mechanisms.

In this letter we present a model calculation for the reactions $pp \to pp\pi^0$, $pp \to pn\pi^+$, $pp \to d\pi^+$, and $pn \to pp\pi^-$. It is an extension of our earlier study for $s$–wave pion production [16,17] to higher partial waves. Now all $NN$- and $\pi N$ partial waves up to orbital angular momenta $L = 2$, and all states with relative orbital angular momentum $l \leq 2$ between the $NN$ system and the pion are considered in the final state. Furthermore, the excitation of the $\Delta(1232)$ resonance is taken into account explicitly. Therefore we are able to calculate meaningful predictions for the different reaction channels of $NN \to NN\pi$ from their threshold up to the $\Delta$ resonance region.

The reaction $NN \to NN\pi$ is treated in a distorted wave born approximation, in the standard fashion. The actual calculations are carried out in momentum space. For the distortions in the initial and final $NN$ states we employ the model CCF of Ref. [24]. This potential has been derived from the full Bonn model [25] by means of the folded–diagram expansion. It is a coupled channel ($NN$, $N\Delta$, $\Delta\Delta$) model that treats the nucleon and the $\Delta$ degrees of freedom on equal footing. Thus, the $NN \leftrightarrow N\Delta$ $T$–matrices that enter in the evaluation of the pion production diagrams involving the $\Delta$ isobar (cf. Fig. 2) and the $NN$ $T$–matrices that are used for the diagrams in Fig. 1 are consistent solutions of the same (coupled–channel) Lippmann–Schwinger–like equation.

The $\pi N \to \pi N$ $T$–matrix needed for the rescattering process is taken from a microscopic meson–exchange model developed by the Jülich group [26]. This interaction model is based on the conventional (direct and crossed) pole diagrams involving the nucleon and $\Delta$ isobar as well as $t$–channel meson exchanges in the scalar ($\sigma$) and vector ($\rho$) channel derived from correlated $2\pi$–exchange. Note that in our model of the reaction $NN \to NN\pi$ contributions where the pions are produced directly from the nucleon or $\Delta$ (cf. Figs.1a and 2a–c) are taken into account explicitly. Therefore, the corresponding nucleon and $\Delta$ pole terms have to be taken out of the $\pi N$ $T$–matrix in order to avoid double counting.

For the $\pi NN$ and $\pi N\Delta$ coupling constants at the pion production vertices
we take the values $f_{N\pi}^2/4\pi = 0.0778$ [24] and $f_{N\Delta\pi}^2/4\pi = 0.26$ [27]. The form factors at these vertices are chosen to be soft (We use a monopole form with a cutoff mass $\Lambda_\pi = 900$ MeV) in line with recent QCD lattice calculations [28] and other information [29,30]. The width of the $\Delta$ isobar is taken into account by using a complex $\Delta$ energy in the propagator. Specifically, we employ a parameterization of the width given by Kloet and Tjon in Ref. [31] which is energy– as well as momentum–dependent. We wish to point out, however, that the width influences the observables only for energies $T_{\text{lab}} \geq 420$ MeV. For lower energies the results obtained with and without $\Delta$ width are practically the same. Note that for simplicity we have suppressed the three–body singularity that appears in the pion rescattering diagram for energies above threshold by fixing the corresponding three–body propagator to its threshold value. Thus, three particle unitarity is not fulfilled in our calculation. Earlier studies [32] have shown, however, that a large part of the imaginary part produced by the three particle cut of the pion exchange is cancelled by the imaginary part arising from the nucleon self energy contribution above pion threshold. Therefore we expect the effect of our approximation to be small, at least for energies close to threshold. Furthermore, a non–relativistic boost is applied for the $\pi N$–$T$–matrix. We expect this to be appropriate for the s–waves. The effect of this treatment for the higher partial waves in the $\pi N$–system needs further study. We come back to this point when we discuss our results.

Results for total cross sections in all experimentally accessible channels are shown in Fig. 3 as a function of $\eta$, the maximum momentum of the produced pions in units of the pion mass. Evidently the predictions of the model are in good overall agreement with the data over a wide energy range. We emphasize that the results for the channels $pp \to pn\pi^+$ and $pp \to d\pi^+$ do not involve any adjustable parameters and are, therefore, genuine predictions of our model. In the case of $pp \to pp\pi^0$, however, the basic model (including direct production plus rescattering) yields only about 60% of the measured cross section. (Corresponding results are indicated by the dash-dotted curve in Fig. 3.) Here we have added contributions from the HME mechanism, Fig. 1c, and fixed their ”strength” so that we can reproduce the data in the near-threshold region (cf. Ref. [17]). Specifically, we have included contributions due to $\omega$ exchange using the vertex parameters $g_{\omega NN}^2/4\pi = 10$ and $\Lambda_{\omega NN} = 1.5$ GeV (monopole form factor). We would like to emphasize, however, that we do not view our HME contribution as being due to a genuine process but rather as an effective parametrisation of short range mechanisms [33] not considered explicitly.

Note that, compared to our earlier work [16,17], now both time orderings of the rescattering diagram (Fig. 1b as well as 2d and e) are properly included (the one where the pion is emitted off one nucleon and interacts with the other before emission as well as the one where one nucleon emits two pions, one of which is absorbed on the other nucleon). This leads to a considerable enhance-
ment of the rescattering contribution so that now no additional contribution from the HME mechanism due to the $\sigma$ meson is needed. The effect of the HME contributions on the reaction channels $pp \to d\pi^+$ and $pp \to pn\pi^+$ is negligible [17,34], so that the corresponding results remain practically unchanged. Therefore we do not show them separately in Fig. 3.

Let us discuss the influence of the $\Delta$ resonance on the production cross sections close to threshold. Our model includes pion production from the $\Delta$ directly (Fig. 2 a–c) or via $\pi N$ rescattering (Fig. 2 d,e). The latter clearly gives contributions to $s$–wave pion production. In this context we wish to point out that now – unlike the purely nucleonic case – charge–exchange rescattering is possible even in the reaction $pp \to pp\pi^0$ via a $\Delta^{++}n$ intermediate state. However, it is less known that also direct pion production from the $\Delta$ gives a non-zero contribution at threshold. This can be easily seen from the standard reduction of the $\pi N\Delta$ vertex starting from the Lagrangian

$$L_{\pi N\Delta} = \frac{f_{\pi N\Delta}}{m_\pi} \bar{\psi} T^\mu \partial_\mu \Phi \psi + h.c. ,$$

where $\psi$, $\Phi$ and $\psi_\mu$ denote the nucleon, $\pi$ and $\Delta$ field operators, respectively, which leads to the following expression:

$$M_{fi} \propto [(\vec{S}' \cdot \vec{q} - \frac{\vec{S}' \cdot \vec{p}}{M_\Delta} (\omega_q - \frac{\vec{q} \cdot \vec{p}}{M_\Delta + E_p}))].$$

Here $\vec{p}$ ($E_p = \sqrt{M_\Delta^2 + \vec{p}^2}$) is the momentum (energy) of the incoming $\Delta$, $\vec{q}$ ($\omega_q = \sqrt{m_\pi^2 + q^2}$) the momentum (energy) of the produced pion and $\vec{S}$ the spin transition operator. Evidently, even for vanishing pion momentum $\vec{q}$, the term proportional to $\omega_q/M_\Delta$, which is the analog to the recoil term appearing in the $NN\pi$–vertex, survives.

Results for the production cross sections without inclusion of the $\Delta$ isobar are indicated in Fig. 3 by the dashed lines. These curves are obtained by setting the $\pi N\Delta$ coupling in the production operator to zero. Obviously, for the reaction $pp \to pp\pi^0$ the contributions involving the $\Delta$ lead to a decrease of the cross section in the near threshold region. This reduction (by about 20 %) is entirely due to the direct production mechanism; the contribution from rescattering off the $\Delta$ is negligibly small. At first this is very surprising, especially because – as was mentioned before – the $\Delta^{++}$ intermediate state allows also rescattering in the dominant $\pi N$ isovector channel. However, a detailed inspection of our results reveals that the rescattering contribution is only small because of a strong cancellation between the diagrams Fig. 2d and e. Individually their magnitudes are quite significant.
Also for the reaction $pp \to d\pi^+$ we find that the direct production is the dominant contribution among the ones involving the $\Delta$. It increases the production cross section in the threshold region by about 30%. Thus, its contribution is partly responsible for the observed overestimation of the $d\pi^+$ cross section close to threshold. Still we should stress that the contributions from the $\Delta$ that we get in our model are moderate as compared to the ones reported by Niskanen [34]. In his case the inclusion of the $\Delta$ leads to an increase of the $d\pi^+$ cross section close to threshold by almost a factor of 3. We believe that this difference is due a different treatment of the three particle propagator in the rescattering contribution involving the $\Delta$-resonance. Niskanen fixes the energy of the exchanged pion by putting it on–shell [35]. This choice maximizes the contribution of the $\Delta$ [36].

Analyzing powers for the reactions $pp \to d\pi^+$, $pp \to pn\pi^+$ and $pp \to pp\pi^0$ at some selected energies are shown in Fig. 4. Evidently the data for this polarization observable are nicely reproduced by our model. This means that the model predicts the correct onset of higher partial waves, especially of p–waves, and also the correct ratio of p–waves to s–waves. The dashed curve in Fig. 4 shows the results without contributions involving the $\Delta$ isobar. It is clear that the inclusion of the $\Delta$ isobar is essential for reproducing the data. It plays an important role even for energies very close to threshold ($\eta \leq 0.25$).

Double polarization observables, namely the spin–dependent total cross section $\Delta \sigma_T/\sigma_{\text{tot}} = -(A_{xx} + A_{yy})$ and the spin correlation coefficient $A_{xx} - A_{yy}$, have recently been measured at IUCF for energies $\eta \geq 0.56$ [37]. In Fig. 5 we present the predictions of our model for these observables. Obviously the description of these data is not that good. Especially in case of $A_{xx} - A_{yy}$ our model overestimates the data by a factor of about two. Since the result for $A_{xx} - A_{yy}$ without the $\Delta$ (dashed line) looks much better one could get the impression that our treatment of the $\Delta$-resonance is incorrect. However, this observable is also very sensitive to the non-resonant part of $\pi N$ p-wave rescattering. To illustrate this we show in Fig. 5 a calculation, where all contributions from $\pi N$ p-wave rescattering are switched off (dotted line). Also in this case the description of the $A_{xx} - A_{yy}$ data clearly improves whereas the results for $\Delta \sigma_T/\sigma_{\text{tot}}$ and $A_y$ remain practically unchanged. Thus, it seems that $A_{xx} - A_{yy}$ is a particularly interesting observable for learning more about the relation between the resonant and non-resonant contributions from $\pi N$ p-wave rescattering.

With regard to our model calculation we have already mentioned that we use an approximate description for the boost of the $\pi N$ T–matrix. It is possible that this approximation leads to an overestimation of the contributions from $\pi N$ p-wave rescattering. Further studies in this direction are required.

In summary, we have presented a model for pion production in nucleon–
nucleon collisions where the production operator is derived in a framework consistent with the interaction potentials that are used for generating the amplitudes in the elementary ($NN$ and $\pi N$) processes. The $NN$ interaction includes explicit coupling to the $N\Delta$ channel so that a consistent evaluation of pion production from $NN$ and $N\Delta$ states can be done. The $\pi N$ amplitude is taken from a meson-exchange model of $\pi N$ scattering developed by the Jülich group. A good overall description of cross section data for the reaction channels $pp \rightarrow pp\pi^0$, $pp \rightarrow pn\pi^+$, $pp \rightarrow d\pi^+$ and $pn \rightarrow pp\pi^-$ from the threshold up to the $\Delta$ resonance region is achieved. Quantitative agreement, not only with integrated cross sections but also with analyzing powers, is found over a wide energy range. Thereby the inclusion of $\pi N$ rescattering as well as of the $\Delta$ degree of freedom plays an important role. We also demonstrated that polarization observables are a powerful tool to investigate details of the dynamics of the process.

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Fig. 1. Pion production diagrams taken into account in our model – nucleonic contributions.

Fig. 2. Pion production diagrams taken into account in our model – contributions involving the $\Delta$-resonance.
Fig. 3. Total cross section for $NN \rightarrow NN\pi$ in the different charge channels. The dash-dotted line shows the result of the coherent sum of the direct production and the rescattering. For the solid line heavy-meson-exchange contributions are added as described in the text. The dashed line shows the result without contributions involving the $\Delta$ isobar.
Fig. 4. The analyzing power $A_y$. The solid line is the result of the full model, the dashed line is the result without the $\Delta$–isobar. Upper panel: $pp \to d\pi^+$ at $T_{Lab} = 290.7, 330$ and $425$ MeV, respectively. Experimental data are from Refs. [38,39]. Middle panel: $pp \to pn\pi^+$ at $T_{Lab} = 300, 320$ and $330$ MeV, respectively. Experimental data are from Refs. [9]. Lower panel: $pp \to pp\pi^0$ at $T_{Lab} = 310, 480$ and $530$ MeV, respectively. Experimental data are from Ref. [7].
Fig. 5. Predictions of our model for the spin–correlation functions of the reaction $pp \rightarrow pp\pi^0$. The solid line is the result of the full model. The dashed line is the result without the $\Delta$–isobar, whereas the dotted curve shows the results, when the contributions from non-resonant $\pi N$ p-wave rescattering are switched off. The data are from Ref. [37].