Relativistic Zero-Energy Solitons at Faster Than Light Speeds in $1+1$ Dimensions

M. Mohammadi

Physics Department, Persian Gulf University, Bushehr 75169, Iran.

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Regardless of any discussion of causality, a classical relativistic system of fields is introduced in $1+1$ dimensions, which yields an energetically stable soliton with zero-energy and zero-momentum at the speeds faster than the speed of light. It can not move at the speeds less than the speed of light.
Keywords: faster than light; zero-energy; soliton; energetically stability; extended Klein-Gordon; solitary wave

1. INTRODUCTION

The usual classical concept of a particle is a stable localized energy density function which may be found at any arbitrary velocity. Hence, the theory of the relativistic classical fields with solitary wave solutions and solitons has been an attempt in this regard in the recent decades. Solitons are considered as stable solitary wave solutions whose energy density functions are localized\(^1\)\[^\[1\]–[4]\]. In many respect, they resemble the real stable particles. For example, they satisfy the same well-known relativistic energy-rest mass-momentum relations of the special relativity and would contract in the direction of motion according to the Lorentz contraction. There are many works on relativistic solitons and solitary wave solutions, among which one can mention the kink (anti-kink) solitons of the real nonlinear klein-Gordon (KG) systems \[^\[5\]–[30]\], the Q-ball solutions of the complex nonlinear KG systems \[^\[31\]–[46]\], the Skyrme model \[^\[1\] , [17], [48]\] of baryons, and 't Hooft Polyakov model \[^\[1\] , [4] , [49], [51]\] which yields monopole soliton solutions.

According to the common special relativity, it is quoted that the motion of any particle is restricted to be at speeds less than the speed of light. However, for first time, the hypothetical faster than light (FTL) particles, was proposed by Gerald Feinberg and called tachyons \[^\[52\]–[53]\] as the quanta of a special relativistic quantum field theory with imaginary mass. Considering the complex speeds is another possibility to build a theory with hypothetical particles at FTL speeds \[^\[54]\]. For the hypothetical FTL particles, the main implication is the violation of causality, which still remains an open problem. Furthermore, in 1935 a famous paper by Albert Einstein, Boris Podolsky, and Nathan Rosen \[^\[55\]\], which briefly known as the EPR paper, was published. They try to show that the quantum mechanics is not a complete theory and is incompatible with the soul of special relativity. In fact, they showed that the accuracy of the quantum mechanic depends on the transmission of information at FTL speeds. Einstein called this result "spooky action at a distance". However, the predictions of quantum mechanics were verified experimentally in tests in which the spin or polarization of entangled particles were measured at separate locations \[^\[56\] , [57]\]. Especially in 2013 a group of Chinese physicists experimentally showed that the information is transmitted at least 10000 times FTL speed for quantum entangled particles \[^\[56]\]. Quantum entanglement has

\(^1\) According to some well-known references such as \[^\[1\]\], the stability is just a necessary condition for a solitary wave solution to be a soliton; more precisely, a solitary wave solution is a soliton if it reappears without any distortion after collisions. In this paper, we only accept the stability condition for the definition of a soliton solution.
been demonstrated experimentally with photons \[58-63\], neutrinos \[64\], electrons \[65\], molecules \[66\]. Hence, some of the physicists believe that the FTL information exist but the FTL particles can not exist.

In the paradigm of the relativistic fields with solitons solutions, there have not been introduced a special system so far, which yields a solitary wave or soliton solution at FTL speeds. However, in this paper, we show that mathematically how a relativistic classical field theory leads to a zero-energy soliton solution at FTL speeds in 1+1 dimensions. Here, the speed of light is again a limiting speed for the motion of the zero-energy soliton solution, but in a different way, meaning that, the special soliton solution can not be at the speeds less than the speed of light. Any hypothetical particle like this special soliton solution since its energy and momentum are zero can be considered as an information transmitter, i.e. it can move from one real particle to another without any spending of energy at FTL speeds.

The special FTL soliton solution is obtained for a 1 + 1 relativistic field model which can be considered as a toy mathematical model to show that the theory of special relativity may not be inherently inconsistent with FTL particle-like solutions! In other words, we have no purpose in discussing the philosophical and physical implications (such as the causality problem) of the existence of the FTL particles. Our main aim is, as an example, to say that mathematically the theory of special relativity may not be incompatible with FTL particle-like solutions of the relativistic classical fields in principle. Moreover, in this regard, there are some works which can be notable for the interested reader \[67, 68\].

In this paper, the special Lagrangian density which is used to obtain a zero-energy FTL soliton solution is an extended KG Lagrangian density (see Refs. \[69-71\]). Briefly, for a set of scalar fields \(\phi_i (i = 1, 2, \cdots, N)\) the extended KG systems have Lagrangian densities which are not linear in the kinetic scalar terms \(S_{ij} = S_{ji} = \partial_\mu \phi_i \partial^\mu \phi_j\). In general, such Lagrangian densities can be called non-standard Lagrangian (NSL) densities too \[73-78\]. There are many works which are dealing with the extended KG systems among which one can mention the works of Riazi et al. \[79, 80\] and El-Nabulsi \[76, 77\]. There are many works which deal with such systems among which one can mention the works of Riazi et. al. \[79, 80\] and El-Nabulsi \[76, 77\]. Moreover, in cosmology the NSL are used for describing dark energy and dark matter \[40, 81-84\].

The organization of this paper is as follows: In Section 2 we will introduce a standard nonlinear KG system with a unstable FTL solitary wave solution. In Section 3 an extended KG will be introduced with an energetically stable zero-energy soliton solution at FTL speed. The last section is devoted to summary and conclusions.
2. AN UNSTABLE SOLITARY WAVE SOLUTION AT FTL SPEEDS

In the standard relativistic theory of the classical fields, it is common to first introduce a proper Lagrangian density and then try to find its solitary wave solutions. There is another approach where one can first consider a special proposed solitary wave solution and then try to find a proper Lagrangian density for it \[69\], \[70\]. A solitary wave solution is a special solution that has a localized energy density function. For example, based on the second approach, for a single real scalar field \(\varphi\), we can think about an unknown standard nonlinear KG Lagrangian density,

\[
\mathcal{L} = \partial^\mu \varphi \partial_\mu \varphi - U(\varphi) = \dot{\varphi} - \varphi' - U(\varphi),
\]

which is assumed to have a special solitary wave solution in the following form:

\[
\varphi_o = \exp(-x^2). \tag{2}
\]

Here \(U(\varphi)\) is called the field potential and should be determined in such a way that Eq. (2) to be a special solitary wave solution of the Lagrangian density (1). Note that, in Eq. (1) the dot (prime) indicates the time \((x)\) derivative, and for the sake of simplicity, throughout the paper, we assume the speed of light equal to one. In fact, Eq. (2) is considered to be a special solution of the dynamical equation belongs to the Lagrangian density (1):

\[
\Box \varphi = \frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial x^2} = -\frac{1}{2} \frac{dU}{d\varphi}. \tag{3}
\]

Hence, for the proposed static solution \(2\), the dynamical equation (3) is reduced to

\[
\frac{d^2 \varphi_o}{dx^2} = \frac{1}{2} \frac{dU(\varphi_o)}{d\varphi_o}. \tag{4}
\]

Moreover, from Eq. (2), one can invert \(x\) as function of \(\varphi_o\), i.e. \(x = \pm \sqrt{-\ln|\varphi_o|}\). Thus, if one inserts \(\varphi_o\) into (4), it is easy to check that the right field potential \(U\) is

\[
U(\varphi_o) = -4\varphi_o^2 \ln|\varphi_o|. \tag{5}
\]

Now, one can omit the index \(o\) and write the above result for \(\varphi\) in general. Note that, such a non-topological solitary wave solution \(2\) is essentially unstable and spontaneously blows up.

The most important advantage of the relativistic systems is that, if one can find a solution at rest, the moving version can be obtained easily just by applying a relativistic boost. In other words, one should replace \(x\) and \(t\) with \(\gamma(x-vt)\) and \(\gamma(t-vx)\) respectively, where \(\gamma = 1/\sqrt{1-v^2}\) and \(v\) is any arbitrary velocity. For example, the moving version of the special solution (2) is
\( \varphi_v = \exp(-\gamma^2(x - vt)^2) \). In general, for a system of the scalar fields \( \phi_i \) (\( i = 1, 2, \cdots, N \)), if one can find a special solution at rest: \( \phi_{io}(x, t) \) (\( i = 1, 2, \cdots, N \)), the moving version of that would be \( \phi_{iv}(x, t) = \phi_{io}(\gamma(x - vt), \gamma(t - vx)) \) (\( i = 1, 2, \cdots, N \)). Moreover, the same standard relativistic energy (\( E \))-rest mass (\( m_o \))-momentum (\( P \)) relations would be exist between the moving and non-moving versions of any relativistic special solitary wave solution in general, i.e. \( E_v = \gamma E_o \) and \( P = \gamma m_o v \).

Now, instead of the localized solution (2), let us to consider the Lagrangian density (1) with a non-localized solution in the following form:

\[ \varphi_o = \exp(x^2). \]  
\( \text{(6)} \)

Similar to the same approach which yields the appropriate field potential (5) for the requested special solution (2), here one can find another appropriate field potential for the non-localized solution (6) as well:

\[ U(\varphi) = 4\varphi^2 \ln(|\varphi|). \]  
\( \text{(7)} \)

The moving version of (6) would be

\[ \varphi_v = \exp(\gamma^2(x - vt)^2). \]  
\( \text{(8)} \)

This non-localized solution have no physical valency. But for the speeds larger than light, if we take the transformations \( x \rightarrow \gamma(x - vt) \) and \( t \rightarrow \gamma(t - vx) \) as a general rule, since \( v^2 > 1 \) and then \( \gamma = 1/(i\sqrt{v^2 - 1}) \) would be a pure imaginary number, thus the moving solution (8) turns to

\[ \varphi_v = \exp\left(-\frac{(x - vt)^2}{v^2 - 1}\right), \]  
\( \text{(9)} \)

which is now a real localized moving solution and can be interesting. Note that, if \( \varphi_v \) for the FTL speeds does not turn to a real function, we would not achieve our goal. In fact, we deliberately choose the proposed non-moving solution (8) as a function of \( x^2 \) for this goal. One can simply check that Eq. (9) is also a solution of the general dynamical equation (3) with the potential (5).

At present, we do not intend to argue about the physical implications of this outcome. In fact, we are going to show that the existence of a fully relativistic field system with an FTL speed solitary wave solution (9) is mathematically possible.

Numerically or theoretically, it is easy to show that the new FTL solitary wave solution (6) is essentially unstable. For example, based on a simple finite difference method for the PDE (3), one can simply simulate the motion of the special solitary solution (6) in Matlab. For a brief but
FIG. 1. The numerical simulation of the motion of an FTL solitary wave solution (9) of a real nonlinear KG system (1). The potential (7) in the range $0 < \varphi < 1$ is negative, then the system and its solutions are essentially unstable. Hence, the emergence of some spontaneous fluctuations is related to this inherent instability of the system (1).

remarkable time, it can be seen that a localized solitary wave solution at FTL speed can actually exist in our simulation program (see Fig. 1). After a while, the form of the FTL solitary wave solution (6) does not remain stationary and disrupts along the time.

In general, according to the Noether’s theorem, the energy density and momentum density belong to the Lagrangian density (1) would be

$$
\varepsilon(x, t) = \dot{\varphi}^2 + \varphi'^2 + U(\varphi), \quad \text{and} \quad p(x, t) = 2\dot{\varphi}\varphi',
$$

respectively. The integration of these functions above the whole space, for any arbitrary localized solution, yields the related total energy and momentum. Therefore, if one applies these integrations for the FTL solitary wave solution (6), they lead to

$$
E_v = \int_{-\infty}^{\infty} [\dot{\varphi}_v^2 + \varphi_v'^2 + U(\varphi_v)]dx = \sqrt{\frac{2\pi}{v^2} - 1}, \quad \text{and} \quad P_v = \int_{-\infty}^{\infty} [2\dot{\varphi}_v\varphi_v']dx = -v \sqrt{\frac{2\pi}{v^2} - 1}.
$$

(10)
The total energy $E_v$ is always positive, but the relation between the total momentum $P$ and velocity $v$ is in terms of the opposite sign, which physically is weird! Moreover, contrary to what we expect, higher speeds here does not lead to larger total energy. In fact, a moving solitary wave solution at the speed of light (infinity) has infinite (zero) energy!

3. AN EXTENDED KG SYSTEM WITH A ZERO-ENERGY SOLITON SOLUTION AT FTL SPEEDS

In line with [69, 70, 72], here we are going to introduce an extended KG system in $1 + 1$ dimensions, which leads to a single zero-energy soliton solution at FTL speeds. For this goal, three scalar fields $\varphi$, $\theta$ and $\psi$ are used to introduce a proper Lagrangian density. First, let us to introduce four independent relativistic functional scalars as follows:

$$S_1 = \partial_{\mu} \theta \partial^{\mu} \theta - 1,$$

$$S_2 = \partial_{\mu} \varphi \partial^{\mu} \varphi + 4\varphi^2 \ln |\varphi|,$$

$$S_3 = \partial_{\mu} \varphi \partial^{\mu} \varphi + 4\varphi \psi,$$

$$S_4 = \partial_{\mu} \psi \partial^{\mu} \psi + 4\psi^2 \ln |\varphi| + 8\psi^2 + 4\varphi \psi,$$

$$S_5 = \partial_{\mu} \psi \partial^{\mu} \psi + 4\varphi^2 \ln |\varphi| (\ln |\varphi| + 1)^2.$$

These scalars are build deliberately in such a way that four conditions $S_i = 0$ ($i = 2, 3, 4, 5$), as four independent PDEs, have a unique common solution (when it is at rest) in the following form:

$$\varphi_o = \pm \exp(x^2), \quad \psi_o = \pm x^2 \exp(x^2).$$

The moving version of this solution would be:

$$\varphi_v = \pm \exp(\gamma^2(x - vt)^2), \quad \psi_v = \pm \gamma^2(x - vt)^2 \exp(\gamma^2(x - vt)^2).$$

Although such functions for speeds less than the speed of light are non-localized and can not be considered as a particle-like solution, but for the FTL speeds $v > 1$, they turn to localized functions:

$$\varphi_v = \mp \frac{1}{\sqrt{v^2 - 1}} \exp \left( \frac{-1}{v^2 - 1}(x - vt)^2 \right), \quad \psi_v = \mp \frac{1}{\sqrt{v^2 - 1}} (x - vt)^2 \exp \left( \frac{-1}{v^2 - 1}(x - vt)^2 \right),$$

which together can be considered as a localized FTL solution for four independent conditions $S_i = 0$ ($i = 2, 3, 4, 5$).
Now, the proper Lagrangian density is

\[ \mathcal{L} = \sum_{i=1}^{5} K_i^3, \]  
(20)

where

\[ K_1 = h_1^2 S_1, \]  
(21)

\[ K_2 = h_2^2 S_1 + S_2, \]  
(22)

\[ K_3 = h_3^2 S_1 + S_3, \]  
(23)

\[ K_4 = h_4^2 S_1 + S_4, \]  
(24)

\[ K_5 = h_5^2 S_1 + S_5, \]  
(25)

and

\[ h_1 = \varphi, \]  
(26)

\[ h_2 = \varphi(2 \ln |\varphi| + 1) \]  
(27)

\[ h_3 = 2\varphi + \psi \]  
(28)

\[ h_4 = 2\varphi + 5\psi + 2\psi \ln |\varphi| \]  
(29)

\[ h_5 = \sqrt{2}\varphi(\ln |\varphi| + 1)^2. \]  
(30)

Note that, since \( K_i \)'s \((i = 1, 2, \ldots, 5)\) are introduced as five independent linear combinations of \( S_i \)'s \((i = 1, 2, \ldots, 5)\), thus five conditions \( S_i = 0 \) are equivalent to \( K_i = 0 \) \((i = 1, 2, \ldots, 5)\).

Using the Euler-Lagrange equations for the new Lagrangian density [20], one can obtain the related dynamical equations easily:

\[ \sum_{i=1}^{5} K_i \left[ 2(\partial_{\mu} K_i) \frac{\partial K_i}{\partial (\partial_{\mu} \theta)} + K_i \frac{\partial}{\partial (\partial_{\mu} \theta)} \right] = 0, \]  
(31)

\[ \sum_{i=1}^{5} K_i \left[ 2(\partial_{\mu} K_i) \frac{\partial K_i}{\partial (\partial_{\mu} \varphi)} + K_i \frac{\partial}{\partial (\partial_{\mu} \varphi)} \right] - K_i \frac{\partial K_i}{\partial \varphi} = 0, \]  
(32)

\[ \sum_{i=1}^{5} K_i \left[ 2(\partial_{\mu} K_i) \frac{\partial K_i}{\partial (\partial_{\mu} \psi)} + K_i \frac{\partial}{\partial (\partial_{\mu} \psi)} \right] - K_i \frac{\partial K_i}{\partial \psi} = 0. \]  
(33)

Moreover, the related energy density function would be

\[ \varepsilon(x,t) = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} + \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} \dot{\varphi} + \frac{\partial \mathcal{L}}{\partial \dot{\psi}} \dot{\psi} - \mathcal{L} = \sum_{i=1}^{5} K_i^2 [3C_i - K_i] = \sum_{i=1}^{5} \varepsilon_i, \]  
(34)
which is divided into five distinct parts, in which

\[ C_i = \frac{\partial K_i}{\partial \theta} \dot{\theta} + \frac{\partial K_i}{\partial \phi} \dot{\phi} + \frac{\partial K_i}{\partial \psi} \dot{\psi} = \begin{cases} 
2h_i^2 \dot{\phi}^2 & \text{i}=1, \\
2h_i^2 \dot{\phi}^2 + 2\dot{\phi}^2 & \text{i}=2, \\
2h_i^2 \dot{\theta}^2 + 2\dot{\phi}^2 & \text{i}=3, \\
2h_i^2 \dot{\theta}^2 + 2\dot{\psi}^2 & \text{i}=4, \\
2h_i^2 \dot{\theta}^2 + 2\dot{\psi}^2 & \text{i}=5.
\end{cases} \] (35)

After a straightforward calculation, one can obtain:

\[ \varepsilon_1 = K_1^2[5h_1^2 \dot{\theta}^2 + h_1^2 \theta^2 + \varphi^2] \geq 0, \] (36)
\[ \varepsilon_2 = K_2^2[5h_2^2 \dot{\theta}^2 + h_2^2 \theta^2 + 5\dot{\varphi}^2 + \varphi^2 + \varphi^2 + 4(\varphi \ln |\varphi|)^2] \geq 0, \] (37)
\[ \varepsilon_3 = K_3^2[5h_3^2 \dot{\theta}^2 + h_3^2 \theta^2 + 5\dot{\varphi}^2 + \varphi^2 + 4\varphi^2 + 4\dot{\psi}^2] \geq 0, \] (38)
\[ \varepsilon_4 = K_4^2[5h_4^2 \dot{\theta}^2 + h_4^2 \theta^2 + 5\dot{\psi}^2 + \psi^2 + 2\dot{\varphi}^2 + 4(\varphi + 2\psi + \psi \ln |\varphi|)^2] \geq 0, \] (39)
\[ \varepsilon_5 = K_5^2[5h_5^2 \dot{\theta}^2 + h_5^2 \theta^2 + 5\dot{\psi}^2 + \psi^2 + 2\varphi^2(1 + \ln^2 |\varphi|)(1 + \ln |\varphi|)^2] \geq 0. \] (40)

Since all bracket terms \([\cdots]\) in Eqs. (31)-(41) and (36)-(40) are multiplied by the scalar functionals \(K_i\) or \(K_i^2\) \((i = 1, 2, \cdots, 5)\), thus any set of functions \(\theta, \varphi\) and \(\psi\) for which \(K_i's= 0 (S_i's= 0)\) simultaneously, is a special zero-energy solution. As we mentioned before, for four conditions \(S_i's= 0 (i = 1, 2, 3, 4)\), there is a unique localized common solution (18). But, the condition \(S_1 = 0\), which is in no way related to other conditions \(S_i's= 0 (i = 2, 3, 4, 5)\), has infinite solutions such as \(\theta = \pm t, \theta = \frac{1}{\sqrt{3}}(2t - x), \theta = \frac{1}{\sqrt{12}}(4t + 2x)\), and so on. Hence, for a zero-energy localized solution, for which \(K_i's= 0 (i = 1, 2, \cdots, 5)\), the form of \(\varphi\) and \(\psi\) are unique (18), but for \(\theta\) there is not a unique form. In fact, in this model, the main scalar fields are \(\varphi\) and \(\psi\), whose responsibility are to create the concept of a particle-like solution (18), but \(\theta\) can be considered as a catalyzer field which is free to be in any arbitrary format, provided \(\partial_\mu \theta \partial^\mu \theta = 1\).

More precisely, the catalyzer field \(\theta\) is considered to build a system for which all terms in the energy density function to be positive definite (see Eqs. (36)-(40)). In other words, the catalyzer field \(\theta\) guarantees the energetically stability of the special solution (18), i.e. for any arbitrary variation above the background of that, the total energy always increases. In fact, since \(K_i's\) \((i = 1, 2, \cdots, 5)\) are five completely independent functional of three scalar fields \(\theta, \varphi\) and \(\psi\), it is not possible to be zero simultaneously except the special solution (18) along with one of the solutions of \(S_1 = 0\). Therefore, for any arbitrary (non-trivial) variation above the background of the special solution (18), i.e. \(\varphi = \varphi_v + \delta \varphi\) and \(\psi = \psi_v + \delta \psi\), at least one of the \(K_i's\) \((i = 2, 3, 4, 5)\)
changes and takes non-zero values. In other hand, since all terms in the energy density functional are positive define, then for any arbitrary variation, the energy density function changes and would be a non-zero positive function too. Thus, for any arbitrary variation, the total energy always increases. In other words, the special solution (18) has the minimum total energy among the other solutions, i.e. it is a soliton solution.

For the vacuum solution, i.e. \( \varphi = \psi = 0 \), according to Eqs. (36)-(40) since \( h_i ' s = 0 \) (i = 1, 2, \cdots, 5), thus all \( \varepsilon_i ' s = 0 \) (i = 1, 2, \cdots, 5). This means when \( \varphi = \psi = 0 \), it does not matter what is the form of the scalar field \( \theta \) at all. But, when \( \varphi \neq 0 \) and \( \psi \neq 0 \), to have a zero-energy solitary wave solution (18), \( \theta \) must satisfy this equation: \( \partial \mu \theta \partial^\mu \theta = 1 \) (i.e. \( S_1 = 0 \)). In sum, the role of the phase field \( \theta \) is like a road in all space on which the zero-energy particle (18) moves easily and is stable.

4. SUMMARY AND CONCLUSION

For the relativistic classical field systems in 1 + 1 dimensions, there is a general rule to obtain the form of a moving solution from its known non-moving version, that is \( x \to \gamma(x - vt) \) and \( t \to \gamma(t - vx) \) or the same Lorentz transformations, where \( \gamma = 1/\sqrt{1 - v^2} \). Based on this general rule, if one finds a solution at rest which it is a function of the even powers of \( x \) and \( t \), mathematically, it may turn to a localized solitary wave solution at FTL speeds. In fact, for the speeds larger than the speed of light, \( \gamma \) would be a pure imaginary number, but the even powers of \( \gamma \) would be a real number. Thus, for FTL speeds: \( x^2 \to \frac{1}{\sqrt{v^2 - 1}}(x - vt)^2 \) and \( t^2 \to \frac{1}{\sqrt{v^2 - 1}}(t - vx)^2 \), which remain real expressions. For example, a relativistic field system with a special non-localized solution \( \varphi = \exp(x^2) \) at rest, physically is not interesting at all, but for \( v > 1 \) it turns to a localized real function \( \varphi = \exp(\frac{1}{\sqrt{v^2 - 1}}(x - vt)^2) \), which physically can be interesting. Therefore, obtaining a special non-moving solution of a nonlinear field system may not be of physical importance, but it may turn to an interesting localized moving solution at FTL speeds.

In this paper, we introduced an extended nonlinear KG system (20) for three scalar fields \( \theta, \varphi \) and \( \psi \). It was shown that this system has a single non-localized zero-energy solution (17) at rest, but it turns into a single localized stable zero-energy solution at FTL speeds (19). All terms in the energy density function of this new extended system are positive definite and are zero just for the special solution (19). In other words, the special solution (19) has the minimum energy among the other solutions. In fact, the special solution (19) is an energetically stable solution for which any arbitrary variation in its internal structure leads to an increase in the total energy. In this model,
the main fields for building the special solution (19) are $\varphi$ and $\psi$. But, the role of the field $\theta$ is like a background in all space-time which guarantees the stability of the special solution (19). In other words, the field $\theta$ is free and would not be in a specific format in all space-time, nevertheless it has a crucial role in the stability of the special solution (19).

For the future works, the idea can be given whether there is a similar system with a single zero-energy FTL soliton solution in 3+1 dimensions? Or, is it possible to have a relativistic field model with a nonzero-energy FTL soliton solution? The complementary discussion of some implications such as casualty can be the context of the future works.

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