Probing quark gluon plasma properties by heavy flavours

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The Fokker Planck (FP) equation has been solved to study the interaction of non-equilibrated heavy quarks with the Quark Gluon Plasma (QGP) expected to be formed in heavy ion collisions at RHIC energies. The solutions of the FP equation have been convoluted with the relevant fragmentation functions to obtain the D and B meson spectra. The results are compared with experimental data measured by STAR collaboration. It is found that the present experimental data can not distinguish between the $p_T$ spectra obtained from the equilibrium and non-equilibrium charm distributions. Data at lower $p_T$ may play a crucial role in making the distinction between the two. The nuclear suppression factor, $R_{AA}$ for non-photonic single electron spectra resulting from the semileptonic decays of hadrons containing heavy flavours have been evaluated using the present formalism.

I. INTRODUCTION

The nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) energies are aimed at creating a phase of matter where the properties of the matter is governed by quarks and gluons [1]. Such a phase of matter, composed of mainly light quarks and gluons - is called quark gluon plasma (QGP). The study of the bulk properties of QGP is a field of high contemporary interest and the heavy flavours namely, charm and bottom quarks play a crucial role in such studies, because they are produced in the early stage of the collisions, they are not part of the bulk properties of the system and their thermalization time scale is larger than the light quarks and gluons and hence can retain the interaction history more effectively.

The successes of the relativistic hydrodynamical model [2, 3] in describing the host of experimental results from RHIC [4] indicate that the thermalization might have taken place in the system of quarks and gluons formed after the nuclear collisions. The strong final state interaction of high energy partons with the QGP i.e. the observed jet quenching [5, 6] and the large elliptic flow ($v_2$) [7, 8] in Au+Au collisions at RHIC indicate the possibility of fast equilibration. On the one hand the experimental data indicate early thermalization time $\sim 0.6$ fm/c [9] on the other hand the pQCD based calculations give a thermalization time $\sim 2.5$ fm/c [10] (see also [11]). The gap between these two time scales suggests that the non-perturbative effects play a crucial role in achieving thermalization. It has also been pointed out that the instabilities [12, 13, 14, 15] may derive the system towards faster equilibrium.

The two pertinent issues regarding the equilibration which will be addressed here are (i) do the the heavy quarks achieve equilibrium and (ii) in case they achieve equilibration, can the equilibrium be maintained during expansion of the system. The second issue will be addressed first.

We make a rather strong assumptions that the heavy quarks produced initially is in thermal equilibrium and check whether it can maintain the equilibrium during the entire evolution processes by comparing their scattering rates with the expansion rate of the matter. This issue will be addressed with different equation of states (EoS) which affects the expansion rate. In case the heavy quarks are unable to maintain the equilibrium then the analysis of the transverse momentum of the mesons carrying heavy flavours can not be done by using thermal phase space distribution. The analysis will require non-equilibrium statistical mechanical treatment. We solve Fokker-Planck (FP) equation [16, 17, 18, 19, 20, 21, 22, 23] to address this issue as discussed below.

The perturbative QCD (pQCD) calculations indicate that the heavy quark thermalization time, $\tau_i^Q$ is larger [18] than the light quarks and gluons thermalization scale $\tau_i$. Gluons may thermalize before up and down quarks [18, 24], in the present work we assume that the QGP is formed at time $\tau_i$. Therefore, the interaction of the non-equilibrated heavy quarks ($Q$) with the equilibrated QGP for the time interval $\tau_i < \tau < \tau_i^Q$ can be treated within the ambit of the FP equation i.e. the heavy quark can be thought of executing Brownian motion in the heat bath of QGP during the said interval of time. The solution of the FP equation can be used to study $p_T$
spectra of heavy mesons in the spirit of blast wave method.

In the next section we address the issues of thermalization in a rapidly expanding system. The results indicate that the heavy quark can not maintain the equilibrium at RHIC and LHC energies during the entire evolution history of the QGP. This demands the treatment of the problem within the framework of non-equilibrium statistical mechanics, which is discussed in section III. Section IV is devoted to summary and conclusions.

II. THERMALIZATION IN AN EXPANDING SYSTEM

We consider a thermally equilibrated partonic system of quarks, anti-quarks and gluons produced in relativistic heavy ion collisions. We would like to study whether the system can maintain thermal equilibrium when it evolves in space and time. Relativistic hydrodynamics (with boost invariance along longitudinal direction and cylindrical symmetry) have been used to describe the space-time evolution. For this purpose, the scattering time scale ($\tau_{\text{scatt}}$) of the partons are compared with the expansion time scale ($\tau_{\text{exp}}$). For maintenance of thermal equilibrium the following criteria should be satisfied:

$$\tau_{\text{exp}} \geq \alpha \tau_{\text{scatt}} \quad (1)$$

where $\alpha \sim O(1)$ is a constant. The criteria given in Eq. 2 is reverse to the one used to study the freeze-out of various species of particles during the evolution of the early universe [23] (similar condition is used in [26] in heavy ion collisions also).

The $\tau_{\text{scatt}}$ is determined for each partons by the expression

$$\tau_{\text{scatt}}^i = \frac{1}{\sum \sigma_{ij} v_{ij} n_j} \quad (2)$$

where $\sigma_{ij}$ is the total cross section for particles $i$ and $j$, $v_{ij}$ is the relative velocity between the particles $i$ and $j$ and $n_j$ is the density of the particle type $j$.

To calculate the scattering time we use the following processes $gg \rightarrow gg$, $gg \rightarrow q\bar{q}$, $q(\bar{q})q \rightarrow q(\bar{q})g$, $qq \rightarrow q\bar{q}$, $q\bar{q} \rightarrow q\bar{q}$ for light flavours and gluons [24]. Here $q$ stands for light quarks and $g$ denotes gluons. For evaluating $\tau_{\text{scatt}}$ for heavy quarks ($Q$) the pQCD processes are taken from [28]. The infrared divergence appearing in case of massless particle exchange in the $t$-channel has been shielded by Debye mass.

The expansion time scale can be defined as:

$$\tau_{\text{exp}}^{-1} = \frac{1}{\epsilon(\tau, r)} \frac{d\epsilon(\tau, r)}{d\tau} \quad (3)$$

where $\epsilon(\tau, r)$ is the energy density, $\tau$ and $r$ are proper time and the radial co-ordinate respectively. $\epsilon(\tau, r)$ is calculated by solving the hydrodynamical equation:

$$\partial_{\mu}T^{\mu\nu} = 0 \quad (4)$$

with the assumption of boost invariance along longitudinal direction [29] and cylindrical symmetry of the system [30]. In Eq. (4), $T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} - g^{\mu\nu}P$ is the energy momentum tensor, $P$ is the pressure, $u^{\mu}$ denotes four velocity and $g^{\mu\nu}$ stands for metric tensor. We consider a net baryon free QGP here, therefore the baryonic chemical potential ($\mu_B$) is zero.

The expansion rates for RHIC and LHC energies have been calculated using the initial conditions, $T_i = 400$ MeV, $\tau_i = 0.2$ fm for RHIC which gives $dN/dy \sim 1100$ [4] and $T_i = 700$ MeV, $\tau_i = 0.08$ fm for LHC giving $dN/dy = 2100$ [31]. The initial radial velocity has been taken as zero for both the cases. Two sets (SET-I and SET-II) of equation of state (EoS) have been used to study the sensitivity of the results on EoS.

SET-I: In a first order phase transition scenario - we use the bag model EOS for the QGP phase and for the hadronic phase all the resonances with mass $\leq 2.5$ GeV have been considered [32]. and SET-II: The EOS is taken from lattice QCD calculations performed by the MILC collaboration [33].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Expansion time vs scattering time (calculated with pQCD process) in RHIC energy. Here the expansion time scale has been calculated at $r = 1$ fm.}
\end{figure}

In Fig 1 the scattering time scale is contrasted with the expansion time scale for two types of EoS mentioned above. For the sake of comparison the expansion rate for the extreme case of free streaming is also displayed. The scattering rates are evaluated.
FIG. 2: Same as Fig. 1 with the pQCD cross section enhanced by a factor 2

with pQCD cross sections. The condition for equilibration in Eq. 1 indicates that the gluons remains close to equilibrium, however the charm and bottom (not shown in the figure) quarks remain out of equilibrium during the entire evolution history.

However, as mentioned in the introduction the analysis of the experimental data within the ambit of relativistic hydrodynamics suggest that the matter formed in Au+Au collisions at RHIC achieve thermalization. One possible reasons for the thermalization to occur is that the partons interact strongly after their formation in the heavy ion collisions. It is argued in [34] that the onset of thermalization in the system formed in heavy ion collisions at relativistic energies can not be achieved without non-perturbative effects. It has also been shown in [35] that a large enhancement of the pQCD cross section is required for the reproduction of experimental data on elliptic flow at RHIC energies. Therefore, the pQCD cross sections used to derive the results shown in Fig. 1 should include non-perturbative effects. To implement this we enhance the pQCD cross sections by a factor of 2. The resulting scattering time is compared with the expansion time in Fig. 2. It is observed that the gluons are kept in equilibrium throughout the evolution, light quarks are closer to the equilibrium as compared to the heavy flavours.

In Figs. 3-4 the results for LHC are displayed for the two time scales mentioned above for pQCD and enhanced cross sections. The expansion becomes faster at LHC than RHIC because of the higher internal pressure. As a consequence, it is interesting to note that the thermalization scenario at LHC does not differ drastically from RHIC.

FIG. 3: Same as Fig. 1 for LHC energy.

FIG. 4: Same as Fig. 2 for LHC energy.

III. NON-EQUILIBRIUM PROCESS

It is argued in [19] that the relaxation time for heavy quarks is larger than the corresponding quantities to light partons by a factor of \( M/T \), where \( M \) is the mass of the heavy flavour and \( T \) is the temperature. In the present work we have also seen that the heavy flavours do not maintain equilibration throughout the evolution scenario, but the gluons are close to equilibrium. Therefore, we treat this problem as an interaction between equilibrium and non-equilibrium degrees of freedom and FP equation provides an appropriate framework for such studies.
The Boltzmann transport equation describing a non-equilibrium statistical system reads:

\[
\frac{\partial f}{\partial t} + \frac{p}{E} \frac{\partial f}{\partial x} + F \frac{\partial f}{\partial p} = \frac{\partial}{\partial t} \frac{\partial f}{\partial t} \tag{5}
\]

The assumption of uniformity in the plasma and absence of any external force leads to

\[
\frac{\partial f}{\partial t} = \frac{\partial f}{\partial t} \tag{6}
\]

The collision term on the right hand side of the above equation can be approximated as (see \[17, 36\] for details):

\[
\left[ \frac{\partial f}{\partial t} \right]_{col} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_i} [B_{ij}(p)f] \right] \tag{7}
\]

where we have defined the kernels

\[
A_i = \int d^3 p \omega(p, k) k_i
\]

\[
B_{ij} = \int d^3 p \omega(p, k) k_i k_j. \tag{8}
\]

and the function \(\omega(p, k)\) is given by

\[
\omega(p, k) = g_j \int \frac{d^3 q}{(2\pi)^3} f_j(q) v_{ij} \sigma_{p,q\rightarrow p-k,q+k} \tag{9}
\]

where \(f_j\) is the phase space distribution for the particle \(j\), \(v_{ij}\) is the relative velocity between the two collision partners, \(\sigma\) denotes the cross section and \(g_j\) is the statistical degeneracy. The co-efficients in the first two terms of the expansion in Eq. 7 are comparable in magnitude because the averaging of \(k_i\) involves greater cancellation than the averaging of the quadratic term \(k_i k_j\). The higher power of \(k_i\)'s are smaller \[37\].

With these approximations the Boltzmann equation reduces to a non-linear integro-differential equation known as Landau kinetic equation:

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_i} [B_{ij}(p)f] \right] \tag{10}
\]

The nonlinearity is caused due to the appearance of \(f\) in \(A_i\) and \(B_{ij}\) through \(\omega(p, k)\). It arises from the simple fact that we are studying a collision process which involves two particles - it should, therefore, depend on the states of the two participating particles in the collision process and hence on the product of the two. Considerable simplicity may be achieved by replacing the distribution functions of the collision partners of the test particle by their equilibrium Fermi-Dirac or Bose-Einstein distributions (depending on the statistical nature) in the expressions of \(A_i\) and \(B_{ij}\). Then Eq. 10 reduces to a linear partial differential equation - usually referred to as the Fokker-Planck equation \[38\] describing the motion of a particle which is out of thermal equilibrium with the particles in a thermal bath. The quantities \(A_i\) and \(B_{ij}\) are related to the usual drag and diffusion coefficients and we denote them by \(\gamma_i\) and \(D_{ij}\) respectively (\textit{i.e.} these quantities can be obtained from the expressions for \(A_i\) and \(B_{ij}\) by replacing the distribution functions by their thermal counterparts):

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ \gamma_i(p)f + \frac{\partial}{\partial p_i} [D_{ij}(p)f] \right] \tag{11}
\]

We evaluate the value of the \(\gamma_i\) and \(D_{ij}\) for the reaction: \(\text{gQ} \rightarrow \text{gQ}\) and \(\text{qQ} \rightarrow \text{qQ}\) for both zero and non-zero quark chemical potential \((\mu = \mu_B/3)\). In Fig. 5 we depict the variation of the drag coefficients as a function of the transverse momentum of the charm and bottom quarks at a temperature, \(T = 200\) MeV. The momentum dependence is weak. For non-zero quark chemical potential the value of the drag increases, however, the nature of the variations remain same. In Fig. 6 the temperature variation of the drag co-efficient is plotted for both zero and non-zero quark chemical potential. Qualitatively, the inverse of the drag co-efficient gives the magnitude of the relaxation time. Therefore, the present results indicate that a system with fixed temperature achieves equilibrium faster for non-zero \(\mu\). In Fig. 7 the diffusion coefficients are plotted as a function of \(p_T\) for \(T = 200\) MeV. The diffusion co-efficient for non-zero \(\mu\) is larger as compared to the case of vanishing \(\mu\). The same quantity is displayed in Fig. 8 as a function of temperature. In the present work we confine to \(\mu = 0\). Recently the heavy quark momentum diffusion co-efficient has been computed \[39\] at next to leading order within the the ambit of hard thermal loop approximations. For \(T \sim 400\) MeV our momentum averaged pQCD value of the diffusion co-efficient is comparable to the value obtained in \[39\] in the leading order approximation for the same set of inputs (\textit{e.g.} strong coupling constant, number of flavours etc).

The inverse of the drag co-efficients gives an estimate of the thermalization time scale. Results obtained in the present work indicate that the heavy quarks are unlikely to attain thermalization at RHIC and LHC energies \[40\].

The total amount of energy dissipated by a parton depends on the path length it traverses through the plasma. Each parton traverse different path length which depends on the geometry of the system and on the point where its is produced. The probability that a parton is created at a point \((r, \phi)\) in the plasma depends on the number of binary collisions at that
point which can be taken as [21]:

\[
P(r, \phi) = \frac{2}{\pi R^2} (1 - \frac{r^2}{R^2}) \delta(R - r)
\]  

(12)

where \( R \) is the nuclear radius. A parton created at \((r, \phi)\) in the transverse plane propagate a distance \( L = \sqrt{R^2 - r^2 \sin^2 \phi - r \cos \phi} \) in the medium. In the present work we adopt the following averaging procedure for the transport coefficients. For the drag coefficient (\( \gamma \)):

\[
\Gamma = \int r dr d\phi P(r, \phi) \int_{L/v}^{L/v} d\tau \gamma(\tau)
\]  

(13)

where \( v \) is the velocity of the propagating partons. The quantity \( \Gamma \) appears in the solution of the FP equation as the input for the expression involving diffusion coefficients to take into account the geometry of the system.

Using the drag and diffusion co-efficients as inputs we solve the to FP equation with the following parametrization of the initial momentum distribution of the charm quarks generated in p-p collisions at \( \sqrt{s} = 200 \) GeV:

\[
\frac{d^2N_c}{dp_T^2} = C \frac{(p_T + A)^2}{(1 + \frac{p_T}{B})^\alpha}
\]  

(14)

where \( A = 0.5 \) GeV, \( B = 6.6 \) GeV, \( \alpha = 21 \) and \( C = 0.845 \) GeV\(^{-4}\). We do not elaborate here on the procedure of solving the FP equation as this has been discussed in detail in ref. [20]. The corresponding initial distributions for bottom quarks can be obtained from the results obtained in Ref. [41] for pp collisions at \( \sqrt{s} = 200 \) GeV. Obtaining the solution of the FP equation for the heavy (charm and bottom) quarks we convolute it with the fragmentation functions of the heavy quarks to obtain the \( p_T \) distribution of the \( D \) and \( B \) mesons. The following three sets of fragmentation functions have been used to check the sensitivity of the results:

1. SET-I [42]

\[
f(z) \propto \frac{1}{z^{1+r_Q bm_Q}} (1 - z)^a exp(- \frac{bm_Q^2}{z})
\]  

(15)

where \( m_Q \) is the mass of the charm quark, \( r_Q = 1 \), \( a = 5 \), \( b = 1 \) and \( m_T^2 = m_Q^2 + p_T^2 \). It has been explicitly checked that the \( R_{AA} \) is not very sensitive to the values of \( a \) and \( b \).

2. SET-II [43]

\[
f(z) \propto z^\alpha (1 - z)
\]  

(16)

where \( \alpha = -1 \) for charm quark and it is 9 for bottom

3. SET-III [44]

\[
f(z) \propto \frac{1}{z^{2 - \frac{\epsilon_e}{1 - z}}}
\]  

(17)

for charm quark \( \epsilon_e = 0.05 \) and for bottom quark \( \epsilon_b = (m_c/m_b)^2 \epsilon_c. \)

Recently, the \( p_T \) spectra of \( D \) mesons has been measured by the STAR collaboration [13] in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV. The \( p_T \) spectrum of hadrons can be written as [16]

\[
\frac{dN}{d^2p_Tdy} = \frac{g}{(2\pi)^3} \int r dr d\phi (m_T \cosh(\eta - y) dr - p_T \cos(\eta dr)) f(u^\gamma p_u)
\]  

(18)

\( \eta \) is the space time rapidity, \( p^\mu \) is the four momentum and \( u^\gamma = \gamma(1, \beta) \) is the hydrodynamic four velocity,
$u^\mu p_\mu$ is the energy of the hadrons in the co-moving frame of the plasma and $f(u^\mu p_\mu)$ is the momentum space distribution. In the spirit of the blast wave method we can write Eq. (18) as

$$\frac{dN}{d^2 p_T dy} = \frac{g}{(2\pi)^3} \int \tau r d\phi d\eta m_T \cosh(\eta - y) f(u^\mu p_\mu) dr$$  \hspace{1cm} (19)$$

Taking the surface velocity profile as:

$$\beta(r) = \beta_s \left( \frac{r}{R} \right)^n$$  \hspace{1cm} (20)$$

and choosing $n=1$, the $p_T$ spectra of $D$ mesons is evaluated.

Before comparing the data with non-equilibrium momentum distribution we analyze the data within the ambit of the blast wave method assuming an equilibrium distribution for the $D$-meson. The values of the blast wave parameters i.e. the radial flow velocity at the surface, $\beta_s$ and the freeze-out temperature, $T_F$ are 0.6 and 0.170 GeV respectively. The data is reproduced well (Fig. 9). The value of $T_F$ is close to $T_c$, which indicate that the $D$-mesons (even if the charm is in equilibrium in the partonic phase) can not maintain it in the hadronic phase. This is reasonable because of the low interaction cross sections of the $D$ mesons with other hadrons. Next we replace the equilibrium distribution in Eq. (19) by the solution of FP equation appropriately boosted by the radial velocity. The results are displayed in Fig. 9. The data is reproduced well for all the three sets of fragmentation functions mentioned before. The value of the freeze-out temperature is 170 MeV and the flow velocity at the surface is 0.45, 0.35 and 0.4 for SET-I, SET-II and SET-III fragmentation functions respectively. The values of $\beta_s$ is lower here than the equilibrium case for all the fragmentation function. It is interesting to note that at low $p_T (\leq 0.5 \text{ GeV})$ domain the results for equilibrium distribution substantially differ from the non-equilibrium distribution for all the three sets of fragmentation functions. Therefore, measurements of the heavy meson spectra at low $p_T$ domain will be very useful to distinguish between the equilibrium and the non-equilibrium scenarios. The two scenarios also give different kind of variation at large $p_T$. The $p_T$ integrated quantity, i.e. the $D$ meson multiplicity may also be useful to understand the difference between the equilibrium and non-equilibrium scenarios.
FIG. 10: Nuclear suppression factor, $R_{AA}$ as function of $p_T$.

FIG. 11: Same as Fig. 10 with enhancement of the cross section by a factor of 2.

IV. NON-PHOTONIC SINGLE ELECTRON FROM HEAVY FLAVOURS.

The STAR [48] and the PHENIX [49] collaborations have measured the non-photonic single electron inclusive $p_T$ spectra recently both for Au+Au and p+p collisions at $\sqrt{s_{NN}} = 200$ GeV. The quantity

$$R_{AA}(p_T) = \frac{N_{Au+Au}}{N_{coll} \times \frac{dN_{D}}{E_T dE_T}}$$

related to the rest frame spectrum for the decay $D(B) \rightarrow X e\nu$ by the following relation [51]

$$\frac{1}{\Gamma} \frac{dT}{dE_e} = \omega g(E_e).$$

GeV indicating the interaction of the plasma particles with the charm and bottom quarks from which electrons are originated through the process: $c(b)$ (hadronization) $\rightarrow D(B)(\text{decay}) \rightarrow e + X$. The loss of energy of high $p_T$ heavy quarks propagating through the medium created in Au+Au collisions causes a depletion of high $p_T$ electrons.

To evaluate $R_{AA}$ theoretically, the solution of the FP equation for the charm and bottom quarks should be convoluted by the fragmentation functions to obtain the $p_T$ distribution of the $D$ and $B$ mesons which subsequently decay through the processes: $D \rightarrow X e\nu$ and $B \rightarrow X e\nu$. Similar formalism has been used in [50] to study the evolution of light quark momentum distributions. The resulting electron spectra from the decays of $D$ and $B$ mesons can be obtained as follows [51, 52, 53]:

$$\frac{dN^e}{p_T dP_T} = \int dQ_T \frac{dN^D}{Q_T dQ_T} F(p_T, Q_T)$$

where

$$F(p_T, Q_T) = \omega \int \frac{d \langle p_T \cdot Q_T \rangle}{2 \langle p_T \cdot Q_T \rangle} g(p_T, Q_T/M)$$

where $M$ is the mass of the heavy mesons ($D$ or $B$), $\omega = 96(1 - 8m^2 + 8m^6 - m^8 - 12m^4lnm^2)^{-1}M^{-6}$ ($m = m_X/M$) and $g(E_e)$ is given by

$$g(E_e) = \frac{E_e^2(M^2 - M^2 - 2MeE_e)^2}{(M - 2E_e)}$$
We evaluate the electron spectra from the decays of heavy mesons originating from the fragmentation of the heavy quarks propagating through the QGP medium formed in heavy ion collisions. Similarly the electron spectrum from the p-p collisions can be obtained from the charm and bottom quark distribution which goes as initial conditions to the solution of FP equation. The ratio of these two quantities gives the nuclear suppression $R_{AA}$. For a static system the temperature dependence of the drag and diffusion co-efficients of the heavy quarks enter via the thermal distributions of light quarks and gluons through which it is propagating. However, in the present scenario the variation of temperature with time is governed by the equation of state or velocity of sound of the thermalized system undergoing hydrodynamic expansion. In such a situation the quantities like $\Gamma$ (Eq. 13) and hence $R_{AA}$ becomes sensitive to velocity of sound in the medium.

The results for $R_{AA}$ is displayed in Fig. 11. The theoretical results are obtained for fragmentation function of SET-I [12]. The velocity of sound for the QGP phase is taken as $c_s = 1/\sqrt{4}$ corresponds to the equation of state, $p = \epsilon/4$. The results failed to describe the data in this case. Next we generate $R_{AA}$ by changing the value of $c_s$ to $1/\sqrt{5}$ and keeping all the other parameters fixed. The resulting spectra describes the data reasonably well. Lower value of $c_s$ makes the expansion of the plasma slower enabling the propagating heavy quarks to spend more time to interact in the medium and hence lose more energy before exiting from the plasma which results in less particle production at high $p_T$. Further lowering of $c_s$ gives further suppressions.

However, as mentioned earlier the non-perturbative effects are important for the interaction of the heavy quarks with the plasma. Therefore, we enhance the cross section by a factor of 2 and find that the experimental results can also be explained by taking an EoS $P = \epsilon/4$ (Fig. 11) and keeping other quantities like fragmentation functions etc unchanged. The ideal gas EoS $P = \epsilon/3$ can not reproduce the data even if the cross section is enhanced by a factor of 2. With $c_s = 1/4$ and enhanced cross section (by a factor 2) the data can also be described with for fragmentation functions of set-II and set-III Fig. 12. Several mechanisms like inclusions of non-perturbative contributions from the quasi-hadronic bound state [54], 3-body scattering effects [53] and employment of running coupling constants and realistic Debye mass [57] have been proposed to improve the description of the experimental data. It is demonstrated here that the EoS of the medium and the non-perturbative effects play a crucial role in determining the nuclear suppression factor.

V. SUMMARY AND CONCLUSIONS

The transverse momentum spectra of the $D$ and $B$ mesons have been studied within the ambit of Fokker-Planck equation where the charm and bottom quarks are executing Brownian motion in the heat bath of light quarks and gluons. We have evaluated the drag and diffusion co-efficients both for zero and non-zero quark chemical potential. Results for non-zero baryonic chemical potential will be very useful for studying physics at low energy RHIC run [56]. The results are compared with experimental data measured by STAR collaboration. It is found that the present experimental data can not distinguish between the $p_T$ spectra obtained from the equilibrium and non-equilibrium charm distributions. Data at lower $p_T$ may play a crucial role in making the distinction between the two. Since the results for equilibrium and non-equilibrium scenarios differ, the $p_T$ integrated quantity i.e. the $D$ meson multiplicity may also be very useful to put constraints on the model. The nuclear suppression factor for the measured non-photonic single electron spectra resulting from the semileptonic decays of hadrons containing heavy flavours have been evaluated using the present formalism. The experimental data on nuclear suppression factor of the non-photonic electrons can be reproduced within this formalism if the expansion of the bulk matter is governed by a equation of state $p = \epsilon/4$ and the partonic cross sections are taken as 2 $\times \sigma_{pQCD}$. Three different kinds of fragmentation functions for the charm and bottom quarks hadronizing to $D$ and $B$ mesons respectively have been used and found that $c_s \sim 1/\sqrt{4}$ can describe the data reasonably well. The data can not be reproduced with $c_s \sim 1/\sqrt{3}$ even after enhancing the cross section by a factor of 2. The loss of energy by the heavy quarks due to radiative process may be suppressed due to dead cone effects. In the present work the radiative loss is neglected. The FP equation needs to be modified to include the radiative loss [58, 59, 60, 61, 62] (see [63] for a review), work in this direction is in progress [56].

The calculations may be improved by making the space time evolution picture more rigorous as follow. In the absence of any external force the evolution of the heavy quark phase space distribution is governed by the equation:

$$\left(\frac{\partial}{\partial t} + v_p \cdot \nabla_r\right) f(p, r, t) = C[f(p, r, t)]$$

(26)

As mentioned before the FP equation can be obtained from the above equation by linearizing the collision term, $C[f(p, r, t)]$. To take into account the energy loss of the heavy quarks in the thermal bath
the ideal hydrodynamic equation needs to be modified as:

\[ \partial_t T^{\mu\nu} = J^{\nu} \]  

(27)

containing a source term \( J^{\nu} \) corresponding to the energy-momentum deposited in the thermal system along the trajectory of the heavy quark, which may be taken as \( J^{\nu} \sim dp^{\nu}/dT \) (\( p^{\mu} \) is the four-momentum vector). Eq. (27) should be solved for \( T(r,t) \) with appropriate equation of state which can be used to obtain the surface of hadronization by setting

\[ T(r_c, t_c) = T_c. \]

Subsequently the solution of Eq. (27) \( f(r_c, t_c) \) for the heavy quark on this surface should be convoluted with the fragmentation function to obtain the \( B \) and \( D \) distribution.

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