Longitudinal Fluctuations in Partonic and Hadronic Initial State

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(Dated: August 26, 2011)

Collective flow in collisions between Lead nuclei at LHC are influenced by random initial state fluctuations, especially for odd harmonics. Here we extend fluctuation studies to longitudinal fluctuations, which may have significant effect on the rapidity distribution of odd harmonics. Furthermore center of mass rapidity fluctuations are measurable, but not yet analysed. Here in the PACIAE parton and hadron molecular dynamics model we make an analysis of initial state fluctuations. As previous analyses discussed mainly the effects of fluctuations on eccentricity and the elliptic flow we pay particular attention to the fluctuations of the Center of Mass rapidity of the system, which is conservatively estimated in our model as \( \Delta_{\text{CM}} = 0.1 \), by neglecting all pre-equilibrium emission effects that are increasing the \( \Delta_{\text{CM}} \) fluctuations.

PACS numbers: 12.38.Mh, 25.75.-q, 25.75.Nq, 51.20.+d

I. INTRODUCTION

Global collective observables are becoming the most essential observables in ultra-relativistic heavy ion reactions\textsuperscript{1}. When we want to extract precise knowledge from experiments, both on the Equation of State (EoS) and the transport properties of matter \textsuperscript{2,3} we have to invoke a most realistic description with fully 3+1 dimensional dynamical evolution at all stages of the reaction, including the initial state. This most adequate description of all stages can only be achieved by the multi-module, hybrid models. (See e.g. refs. \textsuperscript{4,5}.)

The initial state, where we have very little direct experimental information, is of paramount importance in the theoretical description. This leads to a wide variety of initial state models, which behave differently. Theoretical models and experimental results indicate that the initial state fluctuations are essential in understanding the data, although in the global continuum (fluid dynamical or field theoretical) models these fluctuation effects may inherently not be present and even may not survive to the hadronic final state. Nevertheless, we need to analyze the behavior of these initial state models from the point of view of fluctuations. (See e.g. refs. \textsuperscript{6}.)

However, one has to take into account that the Center of Mass (CM) rapidity is not exactly the same for all events because of random fluctuations in the initial state caused by the difference of participant nucleon numbers from projectile and target. This leads to considerable fluctuations at large impact parameters, where the flow asymmetry is the strongest, but the number of participant nucleons is the smallest.

Just as all initial state fluctuations, we have two sources of CM-rapidity fluctuations: First, the number of nucleons are randomly located in the configuration space and due to their fluctuating location, the number of participants from the target and projectile nucleus must not be the same event-by-event, even in the symmetric, A+A, collisions. Second, those nucleons, which are in the geometrical participant zone, may actually not collide with any single nucleon from the opposite nucleus, consequently these will not become participants. Some recent results on the subject concerning the \( v_2 \) and \( v_3 \) fluctuations are discussed in refs. \textsuperscript{6,7}.

Up to now less attention is paid to the fluctuations in the beam direction. The expected momentum and/or rapidity fluctuations in this direction may be bigger due to the large beam momentum in recent experiments. In case of CM-rapidity fluctuations there is an additional problem: It is not obvious how tightly bound system is the initial state. The number of participant nucleons may not come from the projectile and the target nuclei equally, there can be one or a few more nucleons from one side. The momentum carried by the extra nucleons, may be shared (i) by all participants equally in a tightly bound system (a single large confined QGP bag, may be considered as such a system) or (ii) by a loosely connected cloud of nucleons (where the extra nucleons have little direct effect on the participant matter). In the later case, although the total momentum is conserved, the internal energy of the participant matter is increased considerably by the energy of the extra nucleons but the momentum of the participant matter is not correlated with the momenta of the extra nucleons. So, the collective rapidity change is much less.

It is important to mention that the phase transitions and the consequent fluctuations both in and out of QGP may enhance the collective behavior of the system. However, it is rather difficult to estimate the consequences of such transitions and fluctuations to the CM-rapidity fluctuations. From the point of view of initial state fluctuations we have to arrive at system, which is close to local equilibrium, thus, at high energies the transition to QGP has to happen earlier than the formation of the initial state. Thus, it is important to study the CM-rapidity fluctuation as an observable on its own to...
learn about energy deposition, and also due to its strong effect on flow observables. (See e.g. ref. [10]).

In this work, after some simple considerations, we present an analysis of these fluctuations in the PACIAE model, where the major sources of fluctuations are taken into account.

II. ANALYTICAL ESTIMATES FOR THE CM-RAPIDITY FLUCTUATIONS

As mentioned above, the initial state fluctuation is stemming from the participant nucleon number \((N_a + N_b = N_{part})\) fluctuation. Here \(N_a\) and \(N_b\) are the numbers of participant nucleons from the projectile and target nuclei, respectively. The participant matter forms then the initial state system. In the following examples we present three situations where different fraction of the beam energy is contributing to the total transverse mass of the locally equilibrated participant matter.

Let us first estimate the effect of fluctuations of the participant matter for a impact parameter of \(b\) of the locally equilibrated participant matter.

In the initial state model based on expanding flux tubes or streaks [11] used in fluid dynamical calculations [10, 12], the initial state system is tightly bound and unexcited system. We assume that one extra nucleon from the projectile nucleus will be absorbed into the participant matter for an impact parameter of \(b\) of the locally equilibrated participant matter.

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In the above considerations show that the question of initial state fluctuations is a rather complex and model dependent question. After all, the collectivity or looseness of the initial state must be estimated experimentally. The CM-rapidity fluctuations may be quite substantial. In this case a large fraction of beam energy should be carried away through other channels, like pre-equilibrium emission.

For the initial state in hadronic transport models the momentum of extra nucleons are hardly influencing the momenta of the other participant nucleons. The extra nucleons are not stopped in this picture, the transverse mass \(M_t^{CM}\) in the above expression includes large prethermal momenta, but \(M_t^{CM}\) can still be proportional to \(m_t \cdot \sinh(y_0)\). In such a model the CM-rapidity fluctuation will be significantly smaller. For example, in the above \(b = 0.7b_{max}\) Pb+Pb reaction at \((1.38+1.38)\) A-TeV if we assume 65 + \(\delta N\), (where \(\delta N = 1\)) participant nucleons and full equilibration, so that \(2/3\) of the beam kinetic energy is converted into the transverse mass of the participant matter, and \(M_t^{CM}\) can be approximated as \(M_t^{CM} = N_{part}(m_t + \epsilon_0 \cdot 2/3)\) where \(\epsilon_0 = 1.38\text{ TeV per nucleon in the Lab/CM frame. Then the CM-rapidity fluctuation can be approximated as}\n
\[
\Delta y_{CM} \approx \arcsinh \left[ \frac{\delta N m_t}{N_{part}(m_t + 1 GeV) \sinh(y_0)} \right] = 3.1.
\]

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\[
\Delta y_{CM} \approx \arcsinh \left[ \frac{\delta N m_t}{N_{part}(m_t + 2\epsilon_0/3) \sinh(y_0)} \right] = 0.025.
\]
2. The participant nucleons are distributed randomly inside the overlap zone, starting from nucleons inside the corresponding nuclear sphere having an isotropic Woods-Saxon distribution. Nucleons are given beam momentum, and a particle-list of initial nucleons is constructed.

3. An A+A (A+B) collision is decomposed into nucleon-nucleon (NN) collision pairs and every one with a collision time calculated by assuming that the nucleons propagate along straight line trajectories and interact with the NN inelastic (total) cross sections. Then the initial NN collision-list is constructed by these NN collision pairs. The PACIAE model assumes that if a NN collision happens both colliding nucleons become participants, and eventual occupations of final particle states are disregarded. These approximations would decrease the longitudinal fluctuations and angular asymmetries [13].

4. A NN collision pair with the earliest collision time is selected from the collision list, and the final state of the collision is obtained by the PYTHIA model with string fragmentation switched-off. Afterwards the diquarks (anti-diquarks) are broken randomly into quark pairs (anti-quark pairs), and one obtains a configuration of quarks, anti-quarks, and gluons, beside a few hadronic remnants for a NN collision. Although gluons are treated as point like particles, this treatment is not accurate, as gluons are mediating the interaction among the color charges and they have significant role in the formation and hadronization of QGP. In these transitions, the energy of gluons is connected to the masses of the hadrons and to the energy of the emitted high energy photons. We neglect photons in the initial state and so we neglect pre-equilibrium photon emission also. The detailed treatment of the gluons, hadron-parton transition and pre-equilibrium emission would increase the fluctuations. To include these effects would be overly complicated and not realized in models similar to PACIAE. Thus, instead we chose to neglect the gluon contribution to $y_{CM}$. In the present highly approximate treatment, where gluons are treated as point like classical particles, the inclusion of the gluons would reduce $y_{CM}$ fluctuations contrary to the physical expectations.

5. Each of the particles (nucleons) travels along straight line trajectories between two consecutive NN collisions. After the collision the particle list and collision time list are updated, the last step and this process are repeated until the NN collision list becomes empty (the NN collision pairs are exhausted).

The hadron and parton cascade model, PACIAE, includes the most important geometrical effect of the fluctuation of center of mass momentum in heavy ion collisions, as the positions of the initial nucleons are random following the original Woods-Saxon profiles of the projectile and target nuclei. Then in the overlap region nucleons may collide with each other according to NN cross section, and those which do not, will become spectators. This construction provides the participant nucleons, their positions and momenta, as well as the number of spectators from the projectile and the target separately. All other effects, which would influence the $y_{CM}$ fluctuations are neglected. In this way the model gives a lower limit for the fluctuations of the initial state CM rapidity.

From the point of view of global collective flow phenomena, we would have to consider an initial system of particles in local thermal equilibrium. This system does not contain non-thermalized, pre-equilibrium emitted particles, jets, high energy direct gammas, etc. In the present estimate we neglect all these effects, as the quantitative theoretical estimate of all these effects is exceedingly difficult, and even the definition of which particles could be considered belonging to the collective initial state is not settled. These channels take away considerable energy and momentum from the collective initial state, so the center of mass rapidity of the collective initial state will be bigger than the "lower limit" estimate provided by the model PACIAE.

### A. Particle number asymmetries in PACIAE model

We first estimate the probability distribution of the participant nucleons suffered at least one nucleon-nucleon collision. Let us have $N_{a}$ participant nucleons from the projectile and $N_{b}$ from the target. When $N_{a} = N_{b}$ the participant matter is symmetric, so the CM momentum and the CM-rapidity vanish.

At a given impact parameter we have a possibility for symmetric fluctuations when $N_{a} = N_{b}$ change by equal number of nucleons. This will not affect the Center of Mass. If we have an asymmetry, $\delta N = N_{a} - N_{b}$, this...
leads to a change of the CM-rapidity.

Taking into account the effect of overlap geometry and of the nucleon-nucleon cross section, the PACIAE model \[13\], estimates the $\delta N$ distribution from $N_{\text{part}}$ fluctuations as presented in Figures 1 and 2.

In our model calculations the centrality bins are defined in terms of the geometrical cross section, $b_{\text{max}}^2 = (2R_A)^2\pi$, and for example a centrality bin of 60-70% corresponds to an impact parameter range $[b_i, b_j]$, such that $(b_i^2) / (b_{\text{max}}^2) = 0.6$ and $(b_j^2) / (b_{\text{max}}^2) = 0.7$.

As shown in Figure 1 for the central RHIC collisions $|\delta N| / <N_{\text{part}}> \approx 1.5\%$, while for peripheral collisions it is 5%. In peripheral collisions the longitudinally moving uppermost and lowest layers have relatively more particles than in central collisions, and so the random fluctuations have include relatively more particles, although the absolute number of particle asymmetry is less.

In Figure 2 the same results for LHC collisions are 1.4% for central and 6% for peripheral collisions. Thus, the relative number fluctuation for central collisions decreased slightly due to the difference in the number of participants, while for peripheral collisions the small increase is primarily caused by the difference in the centrality bin. The small difference indicates that the relative number fluctuation in peripheral collisions is less sensitive to centrality bin selection than the absolute numbers.

At higher energy the cross sections are bigger, so both the number of realized primary-primary collisions and primary-secondary collisions are bigger. This results in an increase in the participant number in the same overlap domain. This leads to the observed fact that while the absolute numbers are increasing the relative number fluctuations show a smaller increase.

**B. Rapidity fluctuations in PACIAE model**

Let us make a simple estimate: what is the resulting CM-rapidity fluctuation. The extra nucleons, $\delta N$, carry a longitudinal momentum of $\delta p_z = \delta N m_N \sinh(y_0)$. The total momentum of the symmetric part, $\langle Na + Nb - |\delta N| \rangle$, of the participant matter vanishes. We assume a fix impact parameter, $b$ and neglect mass number fluctuations of the symmetric part of participant matter. Then we can assume the mass number of the symmetric part to be $<N_{\text{part}}> - |\delta N| >$. If we assume further that all of the reaction energy is absorbed in the participant matter and $<N_{\text{part}}>> |\delta N|$ then we get

$$\Delta y_{CM}(\delta N) \approx \text{artanh} \left[ \frac{\delta N}{<N_{\text{part}}> \tanh(y_0)} \right].$$

Thus, the CM-rapidity distribution becomes a series of delta functions according to the $\delta N$-distribution. If we allow for the fluctuation of the symmetric mass number for a range of impact parameters or a range of multiplicities, or we allow other channels mentioned above leaking energy from the initial state the peaks of the CM-rapidity distribution will be smoothed out.

Figure 3 shows this delta function structure in the resulting partonic initial state generated by PACIAE model for 1.38+1.38 A·TeV 0-5% central Pb+Pb collisions.

![FIG. 3: Initial state CM-rapidity fluctuation in 1.38+1.38 A·TeV 0-5% central Pb+Pb collisions in PACIAE model.](image)

The sharp peak structure indicates that all other channels (pre-equilibrium emissions, etc.) are neglected in our estimate, so the source of rapidity fluctuations is the momentum of those extra nucleons, which are not matched in originating from the projectile and the target.
C. CM-fluctuations of different matter components

In the partonic initial state generated by the PACIAE model a large part of reaction energy is invested into gluons. The gluons are treated as classical point like particles just as the quarks and anti-quarks. If these gluons were regarded as a distinct gluon field, then this gluon field might keep the partonic initial state system more bound and uniform. Then the remaining part (quarks and anti-quarks) of the partonic initial state fluctuates stronger.

Gluons have an important role in developing collective flow still in the QGP phase (indicated by the constituent quark number scaling observed at RHIC). This collective flow at high energies may lead to a collective rotation [5, 10] where a significant part of the collision energy remains in longitudinal flow, and so it does not contribute to the transverse mass of the system. This would lead to a form of collective energy from the gluons, which leads to increased $y_{CM}$ fluctuations because this energy reduces the transverse mass of the system. Such, collective effects are not included in PACIAE, as gluons are treated as classical point like particles.

The initial state fluctuations of the energetic partonic matter may be important because the developments of these components may not be identical, especially at the final FO and hadronization stages of the reaction. The gluon fields may contribute to forming the final rest masses of the hadrons, and they may contribute different amount of thermal and collective kinetic energy to different hadrons [10]. All effects mentioned above would increase the center of mass rapidity fluctuation of the initial state, but these are not included in the PACIAE model we used.

Figure 4 gives CM-rapidity fluctuation of the quarks and anti-quarks in the partonic initial state calculated for 1.38+1.38 A·TeV, 0-5% central and 60-70% peripheral Pb+Pb collisions by PACIAE model. The fact that, the massive gluon field may carry energy and momentum, makes it possible to incorporate part of the fluctuations. This enables the model to achieve around a few times larger CM-rapidity fluctuations than without a flexibly moving massive gluon field as one can see in comparing Fig. 3 with Fig. 4. Figure 5 gives the fluctuation of the CM-longitudinal momentum per participant nucleon of the quarks and anti-quarks in the partonic initial state, i.e. $p_z$ fluctuation.

In the PACIAE model calculations above, nearly 57.6% of the total collision energy is shared by the quarks and anti-quarks and 42.4% by the gluons in the 60-70% centrality Pb+Pb collisions. These values are 57.9% and 42.1% for quarks and anti-quarks and gluons, respectively, in the 0-5% central Pb+Pb collisions. So, how}

![Figure 4: The CM-rapidity fluctuation of quarks and anti-quarks in the initial state calculated for 1.38+1.38 A·TeV, 0-5% central and 60-70% peripheral Pb+Pb collisions by PACIAE model.](image)

**FIG. 4:** The CM-rapidity fluctuation of quarks and anti-quarks in the initial state calculated for 1.38+1.38 A·TeV, 0-5% central and 60-70% peripheral Pb+Pb collisions by PACIAE model.

| Figure | Description |
|--------|-------------|
| 4      | CM-rapidity fluctuation of quarks and anti-quarks in Pb+Pb collisions |

![Figure 5: The fluctuation of the CM-longitudinal momentum per participant nucleon of the quarks and anti-quarks in partonic initial state, i.e. $p_z$ fluctuation calculated for 1.38+1.38 A·TeV 0-5% central and 60-70% peripheral Pb+Pb collisions by the PACIAE model.](image)

**FIG. 5:** The fluctuation of the CM-longitudinal momentum per participant nucleon of the quarks and anti-quarks in partonic initial state, i.e. $p_z$ fluctuation calculated for 1.38+1.38 A·TeV 0-5% central and 60-70% peripheral Pb+Pb collisions by the PACIAE model.

| Figure | Description |
|--------|-------------|
| 5      | Fluctuation of CM-longitudinal momentum |

IV. CONCLUSIONS

Initial state fluctuations were analyzed in the PACIAE model, with particular attention to the CM-rapidity fluctuations. It was found that in central collisions the longitudinal asymmetry, arising from different number of projectile and target participants, in longitudinal momentum is around 1.5% only, while for peripheral reactions it can reach $\pm 5 - 6\%$ (see Figs. 1 and 2). In central collisions the CM-rapidity fluctuations arise from this longitudinal asymmetry is not large in the PACIAE model as indicated by Figure 3.

We can see in Fig. 4 that the arising CM-rapidity fluctuation is around $\pm 0.03$ units for central collisions and around $\pm 0.1$ units in peripheral ones. These are about
5–10 times smaller than the assumptions used in ref. [10], and this would result in less reduction of the original $v_1(y)$ calculations. On the other hand, the PACIAE estimates can be considered as conservative lower limits of the $y_{CM}$ fluctuations, so the measured $y_{CM}$ fluctuations may exceed these values. In the present formulation of PACIAE, with point like gluons, the gluon contribution would decrease the CM-rapidity fluctuations (cf. Fig. 3).

In the PACIAE partonic initial state study above, we do not include the pre-equilibrium emission, the collective effects as e.g. rotation, and the formation of excited intermediate states. These could lead to the increase of CM-rapidity fluctuations. The developing collective flow may increase and decrease fluctuations, depending on the quantitative details of the developing flow pattern. The structure of the collective flow will be detected at the end of the reaction, but this pattern develops from the initial state in the QGP phase where the gluon component is essential. The collective flow has both transverse and longitudinal components. The pre-collision initial state has exclusively longitudinal collective motion. At the time point of strongest stopping, this longitudinal flow energy is reduced to about 30% of the initial value, while in average at the end of the reaction the longitudinal and transverse energy has about 50–50% share[17]. Soft EoS (like QGP) and collective rotation may increase the share of longitudinal flow energy. The increased longitudinal energy (especially from rotation) and the projectile/target participant asymmetry may in itself contribute to direct increased longitudinal fluctuation.

The share of longitudinal and transverse flow energies also influence the transverse mass of the system, which indirectly contributes to longitudinal fluctuations. The transverse part of the flow energy increases the transverse mass, while the longitudinal part reduces it. Larger transverse mass reduces the $y_{CM}$ fluctuations. We know that with increasing beam energy the collective flow became more energetic and it is the most dominant phenomenon at LHC energies. This arises from the initial energy and momentum distribution, including the gluon components, as these are necessary for the development of the large collective flow processes.

The PACIAE model with point like gluons has less ability to incorporate these collective flow effects, and about two-thirds of the available energy will contribute to the transverse mass, while no direct longitudinal flow fluctuation will develop from the initial state asymmetries. Thus, PACIAE with point like gluons underestimates the $y_{CM}$ fluctuations.

The initial state longitudinal fluctuations are essential for the analysis of the directed flow, as these fluctuations have significant effect on the measurable $v_1$-flow[10]. The present situation regarding the directed flow is rather complex as at RHIC and LHC energies that, the observed collective $v_1$ flow is rather weak, $|v_1| \leq 0.001$ at $\eta = 0.8$, so the $v_1$-flow from the initial state fluctuations may exceed the global collective $v_1$ flow. Thus, the evaluation of $v_1(p_t)$ at low momenta and low rapidities is a complex problem, where the two processes are interacting[18]. The event-by-event longitudinal fluctuations may be important in the assessment and separation of the global directed flow and the directed flow arising from the initial state random fluctuations.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grants No. 11075217, 11047142, 10975062 and the 111 project of the foreign expert bureau of China. L.P. Csernai thanks for the kind hospitality of the Institute of Particle Physics of the Huazhong Normal University, where part of this work was done.
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