Derivation of Maxwell-like equations from the quaternionic Dirac’s equation

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Abstract – Expanding the ordinary Dirac’s equation, \( \frac{1}{i\hbar} \frac{\partial \psi}{\partial t} + \vec{\alpha} \cdot \vec{\nabla} \psi + imc^2 \bar{\psi} = 0 \), in quaternionic form has yielded Maxwell-like field equations. As in the Maxwell’s formulation, the particle fields are represented by a scalar, \( \psi_0 \) and a vector \( \vec{\psi} \). The analogy with Maxwell’s equations requires that \( \psi_0 = -c\beta \vec{\alpha} \cdot \vec{\psi} \), \( \vec{E}_D = c^2 \vec{\alpha} \times \vec{\psi} \), and \( \vec{B}_D = \vec{\alpha} \psi_0 + c\beta \vec{\psi} \). An alternative solution suggests that monopole-like behaviour accompanies Dirac’s field. In this formulation a field-like representation of Dirac’s particle is derived. It is shown that when the vector field of the particle, \( \vec{\psi} \), is normal to the vector \( \vec{\alpha} \), Dirac’s field represents a medium with maximal conductivity. The energy flux (Poynting vector) of the Dirac’s fields is found to flow in opposite direction to the particle’s motion. An equivalent symmetrised Maxwell’s equations are introduced. A longitudinal (scalar) wave traveling at speed of light is found to accompany magnetic charges flow. This wave is not affected by presence of electric charges and currents.

Introduction. – Maxwell has unified the laws of electricity and magnetism in a consistent way into a set of four equations [1]. The sources of the electric and magnetic fields are the charges and currents (moving charges). De Broglie had postulated that a moving particle exhibits a wave-like nature. This wave is concomitant with the particle as it moves on in a form of wave packet centered around the particle. On the other hand, the electric and magnetic fields associated with the moving charges and currents spread away in space and have oscillatory behavior. Biot-Savart showed that the electric and magnetic fields due to a charged particle moving at constant speed are stationary and centered around the particle. In quantum mechanics, Dirac and Schrodinger equations govern the motion of a particle (e.g., and electron) owing to its mass (matter) nature irrespective of its charge. However, a moving charged particle can interact with the electromagnetic field existing in

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space. The proper formation is described by electrodynamics merging quantum mechanics with Maxwell’s theory. The question that normally arise is that how do the two waves interact, the matter waves (due to mass) and the field waves (due to charge). Moreover, one can also think of another wave nature, that is governed by some equation, pertaining to another physical property of the particle, like spin, instead of incorporating it in the former formulations. In a recent paper we have derived the ordinary Dirac equation (in an unfamiliar form), Klein-Gordon, and Schrodinger equations from a quaternionic form of Dirac’s equation [2]. Earlier we have made an analogy between hydrodynamics and electrodynamics and showed that hydrodynamics laws can be written in A Maxwell-like fields equations. Following a same line of reasoning, we have presented an analogy between matter (de Broglie) waves and the electromagnetic waves [3]. Moreover, a quaternionic Maxwell’s equation is found to allow for scalar (longitudinal) waves beside the ordinary electromagnetic transverse waves [4]. The use of quaternions in formulating physical laws is found to be very rich [5, 6].

In this work we extend our formulation to investigate another form of quaternionic Dirac’s equation. We aim in this work to describe the Dirac particle (matter nature) by assigning a field character rather than a matter-wave character. We then compare the equations governing these fields with the corresponding Maxwell’s equations. This approach will enhance deepening the notion of the duality nature exhibited by micro-particles. However, a complete wave-particle duality is recently investigated [7]. The new quaternionic Dirac’s equation formulation is found to be very promising in bringing the above ideas into reality. The resulting set of equations emerging from the quaternionic Dirac’s equation are analogous to Maxwell’s electromagnetic field equations.

We introduce the classical electromagnetism in section 1, and the quaternionic Dirac’s equation in section 2. We then defined the analogous electric and magnetic matter fields, and compared these with that of the Biot-Savart fields arising from a charge moving at constant velocity. In section 3 we provide the energy and momentum densities of the two analogous fields, and showed that they are described by the same set of equations. We have found that while the electromagnetic fields can dissipate energy in some cases, the matter fields spread without energy loss. The direction of the matter field flux density is opposite to the particle’s motion. A symmetrised Dirac’s field equations are obtained by relaxing one of our conditions on the Dirac’s fields. This latter condition allows for longitudinal wave nature for the Dirac’s field and permits monopoles to exist. This is longitudinal (scalar) wave, \( \Lambda \), has an energy flux \( -\Lambda \vec{B} \) and energy density \( \frac{\Lambda}{\varepsilon_0} \Lambda^2 \).

Maxwell’s fields equations. – The force on a charged particle \( q \) in the presence of electric \( \vec{E} \), and magnetic field \( \vec{B} \), is given by Lorentz force as

\[
\vec{F}_L = q(\vec{E} + \vec{v} \times \vec{B}) \,.
\]  

(1)
According to the Newton’s second law of motion the particle (charge) acceleration \( \ddot{a} \) is given by \( \vec{F}_L = m\ddot{a} \). However, the electromagnetic fields are determined by Maxwell’s equations \[1\],

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \]

\[ \nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}, \]

and

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}, \]

\[ \nabla \cdot \vec{B} = 0. \]

Here \( \vec{J} \) and \( \rho \) represent the source of the electromagnetic fields. In Maxwell’s formulation the matter field hasn’t been considered, as this was not yet known at that time.

We would like now to employ quaternions to rewrite Dirac’s equation, and to compare the resulting equations with Maxwell’s equations. This is because the electron charge nature is expressed by Maxwell’s equations, while its wave nature is described by Dirac. To this aim, we should express Dirac particles by fields rather than wavefunctions (spinors) in the ordinary situation. This is because we know that a photon besides its wavefunction nature it is represented by a field. This field embodies all of the photon its properties rather than its wavefunction nature as in Dirac and Schrodinger representations. Moreover, the electron and the photon do interact owing to their electromagnetic nature.

The quaternionic Dirac’s equation. – The ordinary Dirac’s equation of a spin-1/2 particle with rest mass \( m \) is expressed as \[8\]

\[ p^\mu \gamma_\mu \psi = mc \psi, \]

where \( \gamma^\mu \) are expressed in terms of Pauli matrices, \( c \) is the speed of light, and \( \psi \) are the spinors representing the Dirac’s wavefunction. In quaternionic form eq.(6) reduces to

\[ \hat{P} \hat{\gamma} \hat{\psi} = mc \hat{\psi}, \]

where

\[ \hat{P} = \left( \frac{i}{c} E, \vec{p} \right), \quad \hat{\gamma} = \left( \frac{i}{c} \beta, \vec{\alpha} \right), \quad \hat{\psi} = \left( \frac{i}{c} \psi_0, \vec{\psi} \right). \]

Here \( \psi_0 \) and \( \vec{\psi} \) will be later related to the Dirac’s particle fields. Let us now employ the quaternion multiplication rule for two quaternions, \( \hat{A} = (a_0, \vec{a}) \) and \( \hat{B} = (b_0, \vec{b}) \), viz. \[4\]

\[ \hat{A} \hat{B} = (a_0 b_0 - \vec{a} \cdot \vec{b}, a_0 \vec{b} + \vec{a} b_0 + \vec{a} \times \vec{b}). \]

Now apply eqs.(8) and (9) in eq.(7) and then equate the real and imaginary parts to each other, to obtain

\[ \nabla \cdot (c^2 \vec{a} \times \vec{\psi}) = \frac{mc^2}{\hbar} \psi_0, \]
\[ \vec{\nabla} \times (c^2 \alpha \times \vec{\psi}) = -\frac{\partial}{\partial t} (\vec{\alpha} \psi_0 + c\beta \vec{\psi}) + \vec{\nabla} (c^2 \vec{\alpha} \cdot \vec{\psi} + c\beta \psi_0), \]  
(11)

\[ \vec{\nabla} \times (\vec{\alpha} \psi_0 + c\beta \vec{\psi}) - \frac{1}{c^2} \frac{\partial}{\partial t} (c^2 \vec{\alpha} \times \vec{\psi}) = \frac{mc^2}{\hbar} \vec{\psi}, \]  
(12)

and

\[ -\vec{\nabla} \cdot (c \vec{\psi} + \beta \vec{\alpha} \psi_0) + \frac{\partial}{\partial t} \left( \frac{\psi_0}{c} + \beta \vec{\alpha} \cdot \vec{\psi} \right) = 0. \]  
(13)

To reproduce a set of equations similar to Maxwell’s equations we make the following choices:

\[ \vec{E}_D = c^2 \vec{\alpha} \times \vec{\psi}, \]  
(14)

\[ \vec{B}_D = \vec{\alpha} \psi_0 + c\beta \vec{\psi}, \]  
(15)

\[ \vec{J}_D = \frac{mc^2}{\mu_0 \hbar} \vec{\psi}, \]  
(16)

\[ \rho_D = \frac{m}{\mu_0 \hbar} \psi_0, \]  
(17)

and

\[ \psi_0 = -c\beta \vec{\alpha} \cdot \vec{\psi}. \]  
(18)

Hence, eqs.(10) - (13) become

\[ \vec{\nabla} \cdot (c^2 \vec{\alpha} \times \vec{\psi}) = \frac{mc^2}{\hbar} \psi_0, \]  
(19)

\[ \vec{\nabla} \times (c^2 \vec{\alpha} \times \vec{\psi}) = -\frac{\partial}{\partial t} (\vec{\alpha} \psi_0 + c\beta \vec{\psi}), \]  
(20)

\[ \vec{\nabla} \times (\vec{\alpha} \psi_0 + c\beta \vec{\psi}) = \frac{mc^2}{\hbar} \vec{\psi} + \frac{1}{c^2} \frac{\partial}{\partial t} (c^2 \vec{\alpha} \times \vec{\psi}), \]  
(21)

and

\[ \vec{\nabla} \cdot (\vec{\alpha} \psi_0 + c\beta \vec{\psi}) = 0. \]  
(22)

Applying eqs.(14) - (18) in eqs.(19) - (22) yields the Dirac-Maxwell’s-like equations

\[ \vec{\nabla} \cdot \vec{E}_D = \frac{\rho_D}{\varepsilon_0}, \]  
(23)

\[ \vec{\nabla} \times \vec{E}_D = -\frac{\partial \vec{B}_D}{\partial t}, \]  
(24)

\[ \vec{\nabla} \times \vec{B}_D = \mu_0 \vec{J}_D + \frac{1}{c^2} \frac{\partial \vec{E}_D}{\partial t}, \]  
(25)

and

\[ \vec{\nabla} \cdot \vec{B}_D = 0. \]  
(26)

It is remarkable to see that eqs.(16) and (17) can be expressed as

\[ \rho_D = \frac{1}{2} \sigma_m \psi_0, \quad \vec{J}_D = \frac{1}{2} \sigma_m c^2 \vec{\psi}, \quad \sigma_m = \frac{2m}{\mu_0 \hbar}. \]  
(27)
Here $\sigma_m$ represents the maximal conductivity that any system can attain as recently hypothesized [3,10]. Hence, eqs.(14) and (27) yield $|\vec{E}_D| = c^2|\vec{a}|$ and $|\vec{J}_D| = \sigma_mc^2|\vec{\psi}|$, where $|\vec{a}| = 1$. Thus, the Ohm’s law, $\vec{J} = \sigma \vec{E}$ is satisfied for a medium with maximal conductivity where $\vec{a}$ is normal to $\vec{\psi}$. This shows that eq.(24) is applicable to a conducting medium that has a maximal conductivity, i.e., $\sigma = \sigma_m$. Such systems (media) are recently shown to exist in white dwarfs and neutron stars [9]. Hence, the Dirac’s field is equivalent to Maxwell fields propagating in a medium with maximal conductivity. In Dirac’s theory $\vec{a}$ is related to the spin of the particle [8].

Since the electric and magnetic fields have wave character, the Dirac’s fields will have a wave character too. Note that as apparent from eqs.(23) - (26) together with eqs.(16) and (17) that the fields of a massless Dirac’s particle propagate with speed of light.

The system of equations, eqs.(10)-(13) or eqs.(19)-(22), satisfies the continuity equation, i.e.,

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0.$$  \hspace{1cm} (28)

This is evident if we take the divergence of eq.(21) and use eq.(19) to obtain

$$\nabla \cdot \vec{\psi} + \frac{\partial \psi_0}{\partial t \ c^2} = 0.$$  \hspace{1cm} (29)

This is in agreement with the definition in eqs.(16) and (17). Note that in electromagnetism the electric and magnetic fields in the rest frame of a charge moving with constant velocity satisfy [1]

$$\vec{v} \cdot \vec{E} = \vec{v} \cdot \vec{B} = 0, \quad \vec{E} = -\vec{v} \times \vec{B}, \quad \vec{B} = \frac{\vec{v} \times \vec{E}}{c^2}. \hspace{1cm} (30)$$

Equation (30) describes the fields of a point charge moving at constant velocity. They are usually valid for $v << c$. These fields are associated with the particle and aren’t radiated away. These are often called Biot-Savart law. We obtain from eqs.(15) and (16) the following equations

$$c\vec{a} \cdot \vec{E}_D = c\vec{a} \cdot \vec{B}_D = 0, \quad \vec{E}_D = -\beta c\vec{a} \times \vec{B}_D, \quad \vec{B}_D = \frac{\beta c\vec{a} \times \vec{E}_D}{c^2}. \hspace{1cm} (31)$$

Note that in the Dirac’s theory, $\vec{v} = c\vec{a}$, but here we have $\vec{v} = c\beta \vec{a}$. By applying this in eq.(31) and comparing the resulting equations with eq.(30) we disclose that the Dirac’s fields are consistent with Maxwell’s fields. Moreover, we reveal that, since $\beta$ has eigenvalues $\pm$, the magnetic and electric fields of a moving charged particle spread always in two directions (to the left and the right of the particle motion) irrespective of the particle’s velocity direction. It is interesting to see that the appearance of $\beta$ in eq.(31) implies that the fields exist in both directions irrespective of the particle velocity directions. That means, $\vec{B}_D = \pm \frac{\vec{v} \times \vec{E}_D}{c^2}$ and $\vec{E}_D = \pm \vec{v} \times \vec{B}_D$.

The Lorentz force on the particle vanishes, as evident in substituting eqs.(14) and (15) in (1). This is so because the particle is moving at constant velocity.
The Dirac’s fields energy. – Owing to the analogy existing between Dirac’s fields and Maxwell’s field, we can now find the energy and momentum densities of these fields. The energy equation in Maxwell’s theory is defined by \[ \frac{\partial u}{\partial t} + \nabla \cdot \vec{S} = -\vec{J} \cdot \vec{E}, \quad u = \frac{\varepsilon_0}{2} E^2 + \frac{B^2}{2 \mu_0}, \quad \vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}. \] (32)

For Dirac’s fields, the dissipation energy term, \( \vec{J}_D \cdot \vec{E}_D = \frac{mc^4}{m \hbar} \vec{\psi} \cdot (\vec{\alpha} \times \vec{\psi}) = 0, \) vanishes. Consequently the Dirac’s fields don’t lose energy when spread out. Hence, the energy conservation equation for Dirac’s fields is given by \[ \frac{\partial u_D}{\partial t} + \nabla \cdot \vec{S}_D = 0, \] (33)

where \( u_D \) and \( \vec{S}_D \) are the corresponding energy and momentum densities of the Dirac’s fields. Using eqs.(14) and (15) together with eqs.(32) and (18), one finds \[ \vec{S}_D = \frac{c^2 \psi_0}{\mu_0} \vec{\psi}, \quad u_D = \frac{c^2}{\mu_0} \psi^2. \] (34)

This implies that the energy spreads out along the vector field \( \vec{\psi} \). Since the magnetic field of a point charge is not distributed uniformly, the Dirac’s fields follow a similar pattern too.

We now notice that eqs.(16) and (17) suggest that \[ \vec{J}_D = \rho_D \vec{v}_D, \quad \vec{v}_D = \frac{c^2}{\psi_0} \vec{\psi}. \] (35)

This clearly shows that the Dirac’s vector field of the particle ushers in the velocity direction. Moreover, eq.(34) and (35) using eq.(18) reveal that \[ \vec{S}_D = -u_D \vec{v}_D. \] (36)

Hence, the energy flux outflows in the opposite direction of the particle motion. This behaviour agrees with our earlier findings for matter waves in general [3,11].

New solution. – Let us now assume that eq.(18) is not satisfied, and instead let \[ \Lambda_D = c\beta \psi_0 + c^2 \vec{\alpha} \cdot \vec{\psi}. \] (37)

Substituting eq.(37) in eqs.(10) - (13) yields

\[ \nabla \cdot \vec{E}_D = \frac{\rho_D}{\varepsilon_0}, \] (38)

\[ \nabla \times \vec{E}_D = -\frac{\partial \vec{B}_D}{\partial t} + \nabla \Lambda_D, \] (39)

\[ \nabla \times \vec{B}_D = \mu_0 \vec{J}_D + \frac{1}{c^2} \frac{\partial \vec{E}_D}{\partial t}, \] (40)
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and

$$\vec{\nabla} \cdot \vec{B}_D = \frac{1}{c^2} \frac{\partial \Lambda_D}{\partial t}. \quad (41)$$

Now take the divergence of eq.(39) and use eq.(41) to obtain

$$\frac{1}{c^2} \frac{\partial^2 \Lambda_D}{\partial t^2} - \nabla^2 \Lambda_D = 0. \quad (42)$$

This shows that $\Lambda_D$, which has a dimension of electric field, satisfies the wave equation. It is interesting to note that the existence of $\Lambda_D$ in eqs.(39) and (41) neither disturbs the continuity equation nor the wave nature of the electromagnetic field. We may associate here $\Lambda_D$ with a longitudinal wave (electroscalar) that is concomitant with Dirac’s fields \[3, 4\].

Equation (41) shows that the magnetic Dirac’s field is not solenoidal as in Maxwell’s theory. This reveals that Dirac’s monopole can exist, while Maxwell’s one doesn’t. We call the system of equations, eqs.(38) - (41), the symmetrised Dirac’s field equations. This inspires us to consider a similar set of Maxwell’s equations. The existence of $\Lambda_D$ in the field equations above, eqs.(38) - (41), will have many physical implications. Now taking the dot product with respect to $dS$ of eq.(39), and integrate eq.(41) with respect to the volume element $dV$ employing the Stokes’s and divergence theorems, and eq.(42), we arrive at the electromotive force

$$\int \vec{E}_D \cdot d\vec{l} = \int \left( \nabla^2 \Lambda_D - \frac{1}{c^2} \frac{\partial^2 \Lambda_D}{\partial t^2} \right) dV = 0. \quad$$

Considering a complex version of Maxwell’s equations, we have recently shown that one can obtain a system of equations, viz., \[4, 12\]

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \frac{\partial S}{\partial t}, \quad (43)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (44)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \vec{\nabla} S, \quad (45)$$

and

$$\vec{\nabla} \cdot \vec{B} = 0. \quad (46)$$

Here $S$ is analogous to $\Lambda_D$. The symmetrised Maxwell’s equations exhibiting monopole features are \[12, 13\]

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0}, \quad (47)$$

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t} = \vec{J}_m, \quad (48)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_e + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}, \quad (49)$$

and

$$\vec{\nabla} \cdot \vec{B} = \rho_m, \quad (50)$$
where $\rho_e$ and $\rho_m$ are the electric and magnetic charge densities, and $\vec{J}_e$ and $\vec{J}_m$ are the electric and magnetic current densities, respectively. Now comparing the system of equations, eqs.(47) - (50) with eqs.(38) - (41) reveals that
\[
\vec{\nabla} \Lambda_D = -\vec{J}_m, \quad \frac{1}{c^2} \frac{\partial \Lambda_D}{\partial t} = \rho_m.
\] (51)
Now take the divergence of the first equation in eq.(51) and the partial derivative of the second equation, and use eq.(42) to obtain
\[
\vec{\nabla} \cdot \vec{J}_m + \frac{\partial \rho_m}{\partial t} = 0.
\] (52)
Moreover, eq.(51) discloses that
\[
\vec{\nabla} \times \vec{J}_m = 0, \quad \vec{\nabla} \rho_m + \frac{1}{c^2} \frac{\partial \vec{J}_m}{\partial t} = 0.
\] (53)
It is interesting that the magnetic current and charge densities in eqs.(52) and (53) satisfy the system of generalized continuity equation, we have recently introduced [14]. Equation (52) states that the magnetic charge is conserved. Hence, the system of equations, Eqs.(38) - (41) and eqs.(47) - (50) are equivalent.

**Energy conservation.** Let us now consider the system of equations, (38) - (41) instead of (47) - (50) and discuss the evolution of monopoles. We then would like now to see whether the scalar wave $\Lambda$ governed by eq.(42) carries energy and momentum. To do this multiply (dot) eq.(39) by $\vec{B}$ and eq.(40) by $\vec{E}$, and subtract the two resulting equation to obtain
\[
\frac{\partial u'}{\partial t} + \vec{\nabla} \cdot \vec{S}' = -\vec{J} \cdot \vec{E},
\] (54)
where
\[
u' = \frac{B^2}{2\mu_0} + \frac{\varepsilon_0}{2} E^2 + \frac{\varepsilon_0}{2} \Lambda^2, \quad \vec{S}' = \frac{\vec{E} \times \vec{B} - \Lambda \vec{B}}{\mu_0}.
\] (55)
It is really very interesting that the new scalar wave, $\Lambda$, is a physical wave. Thus, the new system of Maxwell’s equations, (38) - (41), could be experimentally tested, and a possible new scalar wave accompanying the ordinary electromagnetic wave can be observed. The energy flux of this new wave points opposite to the direction of the magnetic field, and its energy density is proportional to $\Lambda^2$. In standard electromagnetic theory, monopole can be introduced in Maxwell’s equations and we get a symmetrical set of equations. However, in the present formulation, monopoles are inherited and emerge naturally, in addition they give rise to scalar waves. Thus, existence of monopoles can be deduced differently. We can say that this scalar wave accompanied magnetic charges flow.

Let us now consider a pure magnetic wave arising from magnetic charges only. In this case, $\vec{E} = 0$. Hence, the energy equation of this wave is
\[
\frac{\partial}{\partial t} \left( \frac{B^2}{2\mu_0} + \frac{\varepsilon_0}{2} \Lambda^2 \right) + \vec{\nabla} \cdot (-\Lambda \vec{B}) = 0.
\] (56)
We have already encountered such an equation when we deal with longitudinal wave arising from several considerations \cite{3,11}. It is also similar to eq.(33). It is evident that such wave doesn’t lose energy as it propagates in a conducting medium. It travels at speed of light. It is thus the best wave that we should use to send information. Hence, a new era of communication is imminent. Therefore, if a magnet is somehow disintegrated (separated) into two monopoles (north/south) a huge energy will be released. This is analogous to disintegration of a nucleus into its constituents which releases an enormous amount of energy. The question is how much energy is released in disintegrating a magnet? It could be even more greater than nuclear energy! Then a safe and clean source of energy is around the corner. Should we say that our universe is created out of disintegration of a supermagnet (magnetic big-bang)? This process of disintegration may create huge energy (binding energy) and magnetic charges moving in opposite directions. Moving magnetic charges produce magnetic currents that induce electric charges (electric field). Thus, the origin of electric charge may be understood.

If we connect $\Lambda$ to vacuum contribution and the fact that this contribution decays with time, so when $\Lambda$ becomes significantly small, then eq.(56) reads

\[
\frac{\partial}{\partial t} \frac{B^2}{2\mu_0} = 0. \tag{57}
\]

Hence, at that instance the magnetic field will reach a steady state, freezing out at that value, and becomes some kind of a background (fossil) permeating the whole space. The presence of magnetic monopoles is therefore relied on existence of such magnetic wave.

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