Lower Stagnation Point Flow of Convectively Heated Horizontal Circular Cylinder in Jeffrey Nanofluid with Suction/Injection

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ABSTRACT

Lower stagnation point flow of Jeffrey nanofluid from a horizontal circular cylinder is addressed under the influences of suction/injection, mixed convection and convective boundary conditions. Copper (Cu) is taken as the nanoparticles while Carboxymethyl cellulose (CMC) water is taken as the base fluid. The transformed boundary layer equations through the non-dimensional variables and non-similarity transformation variables are subsequently tackled by means of the Runge-Kutta Fehlberg method (RKF 45). The impact of dimensionless parameters such as the suction/injection, nanoparticles volume fraction and Deborah number are graphically presented and discussed in detail. The outcomes reveal that the velocity and temperature profiles are both augmented with rising values of nanoparticles volume fraction. Velocity profile escalates as suction/injection parameter rises but declines as Deborah number upsurgs. Temperature profile reduces when suction/injection parameter enlarges and augments when Deborah number increases.

Keywords:
Lower stagnation point; Jeffrey nanofluid; convective boundary conditions; suction/injection; mixed convection

1. Introduction

Fluid suction or injection through the bounding surface can substantially modify the flow field and also affect the surface heat transfer rate. According to Al-Sanea [1], enhancement in skin friction and heat transfer coefficients is perceived as a result of suction while injection acts contradictorily. Besides, fluid injection via a porous heated or cooled surface can improve the heating or cooling of the system and also assist the postponement of fluid transition from laminar flow [2].

Exploration of the non-Newtonian fluid models is a topic of ample research due to their diverse nature and industrial and engineering applications such as crystal growing, polymeric melt, dilute...
polymer solutions, cosmetic products, drilling muds, foods, glass blowing and coated sheets. Various constitutive relationships have been established in the previous works to exhibit the complex features of non-Newtonian fluids. Such establishment stems from the fact that the imperative characteristic of non-Newtonian fluids is incapable to be analyzed by the classical equations of Navier-Stokes, which is only usable to assess the Newtonian fluid features. A number of latest interesting studies concerning non-Newtonian fluids may be retrieved from the works of Arifin et al., [3], Zokri et al., [4], Mustafa et al., [5], Kumar et al., [6] and Ashraf et al., [7]. The model of non-Newtonian Jeffrey fluid has been proven quite efficacious for its capability in determining the viscoelasticity property of materials, namely the dual components of retardation and relaxation times. An example of this fluid model as mentioned by Hayat et al., [8] is dilute polymer solution.

Nevertheless, non-Newtonian fluids are well-known to unable fulfill the requirements of high intensity heat transfer because of their unsatisfactory thermal conductivity. The theory of suspending the nanoparticles such as metallic, non-metallic or polymeric nano-sized powders into the non-Newtonian fluid has been one of the contemporary innovative ideas in recent years to increase the thermal conductivity as well as enhance the heat transfer performance. These suspensions are called as nanofluid and are of size lesser than 100nm. Due to unique physical and chemical properties, nanofluid has been widely used in transportation industry, electronic application (micro-electromechanical systems and cooling of microchips), pharmaceutical processes and biomedical (nano-cryosurgery, nano-drug delivery, cryopreservation and cancer therapeutics). An experimental work carried out by Choi and Eastman [9] has revealed that the features of the base fluid had momentarily enhanced due to the dispersion of nanoparticles. This groundbreaking work has prompted the researchers to inspect the involvement of nanofluids in various conventional fluids, geometries and amalgamation of several effects. Pal and Mandal [10] analyzed the mutual impacts of microrotation and nanoparticle together with non-uniform heat source/sink, thermal radiation, suction and magnetic field. They examined four kinds of nanoparticles, for instance silver, alumina, copper and titania. Lu et al., [11] addressed the nonlinear thermal radiation effect in micropolar fluid suspended with nanoparticles and induced by a nonlinear vertical stretching sheet. The influences of magnetohydrodynamics, mixed convection and heat generation/absorption under the convectively heated boundary conditions were accounted. Also, the Ferric Oxide (Fe₃O₄) nanoparticles in the water-based micropolar nanofluid is investigated. Very recently, Kumam et al., [12] presented the applications of entropy generation for single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs) based on kerosene oil for Casson nanofluid flow from a rotating channel. The outcome of the study was tackled by means of the homotopic approach.

Interest in fluid flow passing through a horizontal circular cylinder with mixed convection has been revealed through a number of publications ever since the work of Merkin [13]. Extension of his problem was carried out by Aldoss et al., [14] and Aldos and Ali [15] who incorporated the impacts of MHD and suction and blowing, respectively. Then, Nazar et al., [16] and Nazar et al., [17] inspected the flow of micropolar fluid by taking into account of the constant wall temperature and constant heat flux cases, respectively, while the viscoelastic fluid flow with constant wall temperature was analyzed by Anwar et al., [18]. In the following year, the impact of temperature-dependent viscosity was explored by Ahmad et al., [19] while the Newtonian heating condition was assimilated by Salleh et al., [20] in a viscous fluid. By utilizing the nanofluid model proposed by Tiwari and Das, Nazar et al., [21] inspected three kinds of nanoparticles such as Cu, Al₂O₃ and TiO₂ and water-based fluid. Shortly after, Tham et al., [22] continued the study by examining the porous medium effect. The problem scrutinized by Anwar et al., [18] was prolonged by Kasim et al., [23] to the constant heat flux. Mohamed et al., [24] and Mohamed et al., [25] examined the viscous dissipation effect in the respective viscous and nanofluid model with constant wall temperature. Zokri et al., [26]
implemented the Buongiorno model to investigate the Jeffrey nanofluid model with viscous dissipation effect. Very recently, Mahat et al., [27] adopted the Tiwari and Das model to examine the copper and Carboxymethyl cellulose (CMC) water in viscoelastic nanofluid model.

Driven by the discussions pointed out above, it is clear that the Jeffrey nanofluid flow passing through horizontal circular cylinder has so far never been investigated using the Tiwari and Das model. Therefore, it is imperative to explore the impacts of mixed convection, suction/injection and convective boundary condition on lower stagnation point flow of Jeffrey nanofluid from a horizontal circular cylinder. Here, the Carboxymethyl cellulose (CMC) water represents based fluid while copper signifies nanoparticles. The closely related existing publications were from Zokri et al., [26] and Mahat et al., [27].

2. Mathematical Formulation

Suppose a Jeffrey fluid flow suspended with nanoparticles towards a horizontal circular cylinder in the presence of suction/injection and mixed convection is scrutinized. Figure 1 demonstrates the cylinder of radius \( a \), being heated to a convective boundary conditions with ambient temperature \( T_\infty \). The \( \bar{x} \) and \( \bar{y} \) coordinates of the cylinder surface are measured starting from the lower stagnation point \( \bar{x} = 0 \) and perpendicular to it, respectively. Then, the relevant equations governing the flow are:

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}
\]

\[
\rho_{nf} \left( \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = \rho_{nf} \bar{u}_e \frac{d\bar{u}_e}{d\bar{x}} + \mu_{nf} \left[ \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \lambda_1 \left( \frac{\partial^3 \bar{u}}{\partial \bar{x} \partial \bar{y}^2} + \frac{\partial^3 \bar{v}}{\partial \bar{x} \partial \bar{y}^2} - \frac{\partial \bar{u}}{\partial \bar{x}} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial \bar{u}}{\partial \bar{x}} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right) \right] +
\]

\[
g(\rho \beta_T)_{nf} (T - T_\infty) \sin \frac{\bar{x}}{a}, \tag{2}
\]

\[
\left( \rho C_p \right)_{nf} \left( \bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} \right) = k_{nf} \frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}
\]

![Fig. 1. Physical model of the coordinate system](image-url)
where the density of nanofluid, thermal expansion coefficient of nanofluid, heat capacity of nanofluid, effective viscosity of nanofluid and effective thermal conductivity of nanofluid are symbolized as $\rho_{nf}$, $(\rho \beta_T)_{nf}$, $(\rho C_p)_{nf}$, $\mu_{nf}$ and $k_{nf}$, respectively and defined as follows

$$
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s,
(\rho \beta_T)_{nf} = (1 - \phi)(\rho \beta_T)_f + \phi(\rho \beta_T)_s,
(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s
$$

$$
\mu_{nf} = \frac{\mu_f}{(1-\phi)\tau_s},
k_{nf} = k_f \left(\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}\right)
$$

The appropriate boundary conditions are

$$
\begin{align*}
\bar{u}(\bar{x}, 0) &= 0, \quad \bar{v}(\bar{x}, 0) = V_w, \quad -k_f \frac{\partial T}{\partial \bar{y}} = h_f (T_f - T) \quad \text{at} \quad \bar{y} = 0 \\
\bar{u}(\bar{x}, \infty) &\to \bar{u}_e, \quad \bar{v}(\bar{x}, \infty) \to 0, \quad T(\bar{x}, \infty) \to T_\infty \quad \text{as} \quad \bar{y} \to \infty
\end{align*}
$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ axes, respectively, $T$ is the fluid temperature, $\lambda$ is the ratio of relaxation to retardation times, $\lambda_1$ is the retardation time, $g$ is the gravity acceleration, $\phi$ is the nanoparticle volume fraction of nanofluid, $V_w$ is the uniform suction or injection velocity, $h_f$ is the heat transfer coefficient, $T_f$ is the hot fluid, $k_f$ is the thermal conductivity and $\bar{u}_e(x)$ is the external velocity, denoted as

$$
\bar{u}_e(x) = U_\infty \sin \left(\frac{\bar{x}}{a}\right)
$$

where $U_\infty$ is the free stream velocity. Table 1 presents the thermophysical properties of the base fluid and nanoparticles. The carboxymethyl cellulose (CMC) water is used as the non-Newtonian base fluid, as suggested by Lin et al. [28]. CMC-water exhibits shear thinning or pseudoplastic rheological behavior and has been experimentally proven to be one of the common types of time-independent non-Newtonian fluid [29].

| Table 1 | Thermophysical properties of base fluid and nanoparticles |
|---------|---------------------------------------------------------|
| Physical properties | $\rho (kg \ m^{-3})$ | $C_p (J kg^{-1} K^{-1})$ | $k (W \ m^{-1} K^{-1})$ | $\beta_T \times 10^5 (K^{-1})$ |
| Base fluid (CMC) | 997.1 | 4179 | 0.613 | 21 |
| Nanoparticle (Cu) | 8933 | 385 | 401 | 1.67 |

Now, the following non-dimensional variables are imposed to reduce the dimensional governing Eq. (1)-(3) to dimensionless form:

$$
x = \frac{\bar{x}}{a}, \quad y = Re^{\frac{1}{2}} \frac{\bar{y}}{a}, \quad u = \frac{\bar{u}}{U_\infty}, \quad v = \frac{\bar{v}}{U_\infty}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \bar{u}_e = \frac{\bar{u}_e}{U_\infty}
$$

Then, the dimensionless form of governing equations is

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
$$
\[ \frac{d\theta}{dx} + \frac{d\theta}{dy} = \frac{k_{nf}/k_f}{1-\phi+(\rho c_p)/(\rho c_p)} \frac{d^2\theta}{dy^2} \]

with the related boundary conditions

\[ u(x, 0) = 0, \quad v(x, 0) = \frac{V_w Re_x^{1/2}}{U_{\infty}}, \quad \frac{\partial \theta}{\partial y}(x, 0) = -Bi \left(1 - \theta(x, 0)\right) \text{ at } y = 0 \]

\[ u(x, \infty) \to u_\infty, \quad v(x, \infty) \to 0, \quad 0(x, \infty) \to 0 \text{ as } y \to \infty \]

where \( \lambda_2 = \frac{\lambda_1 u_{\infty}}{a} \), \( Pr = \left(\frac{c_p\mu}{k}\right) \), \( \gamma = \frac{Gr_x}{Re_x}, \quad Gr_x = \frac{a^2 f \left(T_f - T_{\infty}\right) a^3}{v_f^2} \) and \( Re_x = \frac{U_{\infty} a}{v_f} \) are the respective Deborah number, Prandtl number, mixed convection parameter, Grashof number and Reynolds number. Following Merkin [13], Eq. (7)-(9) are solved by seeking the succeeding non-similarity transformation variables

\[ \psi = xf(x, y), \quad \theta = \theta(x, y), \]

where \( \psi \) is the stream function, denoted as \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \) and \( \theta \) is the rescaled dimensionless temperature of fluid. Now, Eq. (7) is identically satisfied while Eq. (8) and Eq. (9) produce

\[ \frac{1}{(1-\phi)2.5(1+\lambda)} \left[ \frac{\partial^2 f}{\partial y^2} + \lambda_2 \left( \left(\frac{\partial^2 f}{\partial y^2}\right)^2 - \frac{\partial^4 f}{\partial y^4} \right) \right] + C_1 \left( f \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 \right) + C_2 \frac{\sin x}{x} \gamma \theta + C_1 \frac{\sin x \cos x}{x} = \]

\[ xc_1 \left( \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2} + \frac{\lambda_2}{c_1(1-\phi)2.5(1+\lambda)} \left( \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2} - \frac{\partial^2 f}{\partial y^2} \frac{\partial^2 f}{\partial x \partial y^2} \right) \right) \]

\[ \frac{1}{Pr} \frac{k_{nf}/k_f}{1-\phi+(\rho c_p)/(\rho c_p)} \frac{d^2\theta}{dy^2} + C_3 \frac{\partial \theta}{\partial y} = xc_3 \left( \frac{\partial f}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial \theta}{\partial y} \right), \]

with \( C_1, C_2 \) and \( C_3 \) are constants and be defined as

\[ C_1 = 1 - \phi + \phi \frac{\rho_s}{\rho_f}, \quad C_2 = 1 - \phi + \phi \frac{\rho \beta_f}{\rho \beta_f}, \quad C_3 = 1 - \phi + \phi \frac{\rho c_p}{\rho c_p} \]

and the boundary conditions Eq. (10) become

\[ f(x, 0) = f_w, \quad \frac{\partial f}{\partial y}(x, 0) = 0, \quad \frac{\partial \theta}{\partial y}(x, 0) = -Bi \left(1 - \theta(x, 0)\right) \text{ at } y = 0 \]

\[ \frac{\partial f}{\partial y}(x, \infty) \to \frac{\sin x}{x}, \quad \frac{\partial^2 \theta}{\partial y^2}(x, \infty) \to 0, \quad \theta(x, \infty) \to 0 \text{ as } y \to \infty \]

Eq. (12) and Eq. (13) give rise to the succeeding ordinary differential equations at lower stagnation region, \( x \approx 0 \)
\[
\frac{1}{(1-\phi)^2(1+\lambda)} \left[ \frac{\partial^2 f}{\partial y^2} + \lambda_2 \left( \left( \frac{\partial^2 f}{\partial y^2} \right)^2 - f \frac{\partial f}{\partial y} \right) \right] + C_1 \left( f \frac{\partial^2 f}{\partial y^2} - \left( \frac{\partial f}{\partial y} \right)^2 \right) + C_2 \gamma \theta + C_1 = 0, \tag{15}
\]

\[
\frac{1}{Pr} \frac{k_n f}{k_f} \frac{\partial^2 \theta}{\partial y^2} + C_3 f \frac{\partial \theta}{\partial y} = 0, \tag{16}
\]

with the boundary conditions

\[
f(0) = f_w, \quad \frac{\partial f}{\partial y}(0) = 0, \quad \frac{\partial \theta}{\partial y}(0) = -Bi(1 - \theta(0))
\]

\[
\frac{\partial f}{\partial y}(\infty) \rightarrow 1, \quad \frac{\partial^2 f}{\partial y^2}(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0
\]  

where \( f_w = -\frac{V_w R e}{U_\infty} \) is the suction/injection and \( Bi = -\frac{h_f a}{k_f R e} \) is the Biot number.

3. Results and Discussion

The outcomes of several physical parameters such as suction/injection parameter \( f_w \), nanoparticle volume fraction \( \phi \) and Deborah number \( \lambda_2 \) are examined graphically over the velocity and temperature profiles. A numerical method named Runge-Kutta Fehlberg method (RKF 45) encoded in Maple software is applied to obtain the solution for nonlinear ordinary differential Eq. (15) and Eq. (16) together with boundary conditions of Eq. (17). The numerical values are taken as follows: \( \lambda = \phi = \gamma = 0.1, \lambda_2 = f_w = 0.2, \text{Pr} = 6.2 \) and \( Bi = 0.5 \). Table 2 demonstrates the comparison between the results generated through this endeavor and existing works by Merkin [13], Nazar et al., [16], Rashad et al., [30] and Zokri et al., [26] for dissimilar values of \( \gamma \). The values are perceived to be in an outstanding agreement, hence validating the obtained graphical results as revealed later.

Importance of physical parameters \( f_w, \phi \) and \( \lambda_2 \) on velocity and temperature profiles is elucidated via Figure 2 to Figure 7. It is noticed from Figure 2 that the thickness of velocity boundary layer shrinkages with rising values of suction parameter, \( f_w > 0 \). This is due to the fact that, suction effect tends to remove the fluid from the system, thereby diminishes the thickness of momentum boundary layer. In contrast, injection effect, \( f_w < 0 \) permits the fluid to go into the system which subsequently thickens the velocity boundary layer thickness. Furthermore, as perceived in Figure 3, the thermal boundary layer thickness is intensified when \( f_w < 0 \), while it lessened when \( f_w > 0 \).

Figure 4 and Figure 5 present the effect of dissimilar \( \phi \) values on both velocity and temperature. With increasing \( \phi \) from 0 to 0.5, the velocity profile is seen to be accelerating owing to the augmentation of the energy transport. Besides, the increase of thermal boundary layer thickness is very much connected with the incremented thermal conductivity of the nanofluid. This increment is supplemented by larger values of thermal diffusivity that aid in reducing the temperature gradients and subsequently, increase the thickness of thermal boundary layer.
Table 2
Comparative values of $-\theta'(0)$ with preceding publications for dissimilar values of $\gamma$ when $\lambda = \phi = 0$, $Pr = 1$ and $\lambda_2 \to 0$ (very small)

| $\gamma$ | Merkin [13] | Nazar et al., [16] | Rashad et al., [30] | Zokri et al., [26] | Present |
|----------|-------------|--------------------|---------------------|------------------|---------|
| -1       | 0.5067      | 0.5080             | 0.5068              | 0.506679         | 0.506678|
| -0.5     | 0.5420      | 0.5430             | 0.5421              | 0.542072         | 0.542065|
| 0        | 0.5705      | 0.5710             | 0.5706              | 0.570484         | 0.570470|
| 0.5      | 0.5943      | 0.5949             | 0.5947              | 0.594546         | 0.594534|
| 0.88     | 0.6096      | 0.6112             | 0.6111              | 0.610775         | 0.610762|
| 0.89     | 0.6110      | 0.6116             | 0.6114              | 0.611182         | 0.611169|
| 1        | 0.6158      | 0.6160             | 0.6160              | 0.615601         | 0.615587|
| 2        | 0.6497      | 0.6518             | 0.6518              | 0.651507         | 0.651492|
| 5        | 0.7315      | 0.7320             | 0.7319              | 0.731529         | 0.731510|

Fig. 2. Variation of $f'(y)$ due to $f_w$

Fig. 3. Variation of $\theta(y)$ due to $f_w$

Fig. 4. Variation of $f''(y)$ due to $\phi$

Fig. 5. Variation of $\theta(y)$ due to $\phi$
4. Conclusions

A detailed theoretical study concentrating on the Jeffrey nanofluid flow over a horizontal circular cylinder near the lower stagnation point with suction/injection, mixed convection and convective boundary conditions has been deliberated. Copper (Cu) and Carboxymethyl cellulose solution (CMC) were selected to be the nanoparticles and base fluid, respectively. The resulting ordinary differential equations were subsequently tackled via the Runge-Kutta Fehlberg method (RKF 45). The benchmark of the solution is attained by way of comparison with limiting cases of existing publications. The results were perceived to be in an excellent consistency. Summarization of the present findings can be outlined as below

i. Velocity rises but temperature reduces as $f_w$ escalates.

ii. Both velocity and temperature are increased as $\phi$ intensifies.

iii. Velocity declines while temperature upsurges as $\lambda_2$ augments.

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