Student error analysis in global meta-level algebraic thinking on treffinger learning assisted by scaffolding

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Abstract. This study aims to: (1) find out the quality of Treffinger learning assisted by scaffolding; (2) find out whether students’ abilities in global meta-level algebraic thinking in Treffinger learning assisted by scaffolding reached minimal completeness criteria score; (3) describe the types and causes of student errors in global meta-level algebraic thinking; (4) describe the types of scaffolding that could minimize student errors in global meta-level algebraic thinking. This study used a mixed-method. The results showed that: (1) the quality of Treffinger learning assisted by scaffolding was very good; (2) students’ abilities in global meta-level algebraic thinking on Treffinger learning assisted by scaffolding reached minimal completeness criteria score; (3) upper group students tended to make comprehension errors, middle group students tended to make comprehension and process skills errors, lower group students tended to make comprehension, process skills, and encoding error; (4) the type of scaffolding was given to students include the distribution of student worksheets which providing assistance according to the group’s level (environmental provisions), explaining, reviewing, and restructuring.

1. Introduction
Mathematics as the subjects given to school can be applied in various aspects of life both from the simplest to the most complex. Therefore, mathematics has a very important function in everyday life. Teaching mathematics at lower levels and middle classes such as in junior high school helps to guide students thinking more so when it comes to handling concrete to abstract ideas through the practice of solving daily math problems or others subject related to daily life [1]. However, for some students mathematics is not a pleasant subject, indeed a lot of students assume mathematics as a difficult subject. One of the factors that cause the low students’ ability to understand mathematics is because they are accustomed to learning mathematical concepts and formulas by memorizing without understanding their purpose, content, and usefulness. This causes students tend to have difficulty in understanding concepts and materials in mathematics. The difficulty of students to learn mathematics is strengthened by data from the results of the national examinations in the academic year of 2017/2018 which has decreased compared to the previous two years, namely in 2017/2018 an average score of the national examinations is 43.34, whereas in the academic year of 2015/2016 and 2016/2017 the average score is 50.24 and 50.31.

Based on Decree of The Indonesian Minister of Education and National Number 22 Year of 2006, one of the scopes of mathematics that must be mastered by junior high school students in curriculum 13 so that objectives of learning mathematics can be achieved is algebra. Algebra is an important subject for students’ educational and economic futures, however student interest in mathematics declines over
adolescence [2]. The algebraic understanding of junior high school or Islamic junior high school students in Semarang Regency can said to be still low. This is reinforced by data from Puspendik in 2018 especially in Junior High School 1 Ungaran. Based on these data, the percentage of students’ mastery of State Junior High School 1 Ungaran on the national mathematics exam for the academic year of 2017/2018 is presented in Table 1 below.

#### Table 1. Mastery of Junior High School/Islamic Junior High School National Examination Material for Academic Year of 2017/2018

| Material Coverage                | School | District | Prov | Nat |
|---------------------------------|--------|----------|------|-----|
| Number                          | 88.02  | 53.30    | 48.20| 44.99|
| Algebra                         | 78.47  | 41.43    | 41.47| 41.88|
| Geometry and Measurement        | 79.33  | 43.02    | 43.07| 41.40|
| Statistics and Opportunities    | 87.16  | 53.45    | 52.66| 45.71|

Based on Table 1 above, it can be seen that the percentage of algebraic mastery at the school, district, and provincial levels is lower compared to the scopes of other mathematics. Mastery of algebraic concepts requires abilities that must be mastered by students that are thinking algebraic. According to [3], in school mathematics, algebra refers to use of symbols to state and manipulate generalizations within the context of numbers. [4] stated that algebra is an important core subject area at the middle school level and according to [5], algebraic thinking is at the heart of teaching and learning algebra at the elementary and middle school levels. According [6] categorizes algebraic thinking into three forming abilities namely generational abilities, transformational abilities, and global meta-level abilities.

Based on the results of preliminary tests conducted by a researcher at State Junior High School 1 Ungaran, it was found that students’ abilities of global meta-level algebraic thinking in problem-solving were still low. The reason for the low students’ abilities to global meta-level algebraic thinking in solving problems can be determined by analysis of global meta-level algebraic thinking errors. According to [7] by analysis of global meta-level algebraic thinking errors can be used as information to improve the learning process and the errors are not repeated.

One method that can be used to analyze student errors is the Newman Error Analysis. According to [8], Newman error analysis is a method that analyzes student errors when completing the problem description. Newman classifies errors made by students into five types, namely (1) reading error, (2) comprehension error, (3) transformation error, (4) process skills error, and (5) encoding error.

A very important aspect of the teacher in the learning process is that it can guide students in minimizing errors made by students. One effort that can be done to improve student understanding so as not to repeat the same mistake is to provide scaffolding. The scaffolding support required may be different for each individual, and may depend on several factors such as motivation, prior knowledge, psycho-physiological needs, interest, and context. [9], while according to [10] scaffolding is support or assistance provided in the early stages of learning and then the assistance will be removed when students can learn independently. [10] suggests three levels of scaffolding, namely (1) environmental provisions, (2) explaining, reviewing, restructuring, and (3) developing conceptual thinking.

Algebra learning in schools, in general, is only in the form of delivery of information and does not involve many students to be able to build their understanding. This causes the low students’ abilities to algebraic thinking. So that learning is needed to minimize student errors in solving algebra problems. Of the many existing learning models, one of the learning models that can be used is Treffinger learning. The Treffinger learning model will be very suitable to be used to train students so that they can improve their learning outcomes [11]. According [12], In this model, it also focuses on the involvement of cognitive and affective skills at each level of this model.

Based on the description above, purpose of this study is: (1) find out quality of Treffinger learning assisted by scaffolding on the ability of students to think algebraic in global meta-level; (2) whether
students’ abilities in global meta-level algebraic thinking in Treffinger learning assisted by scaffolding reached minimum completeness criteria; (3) describe the types and causes of student errors in global meta-level algebraic thinking; (4) describe the type of scaffolding that could minimize student errors in global meta-level algebraic thinking.

2. Method
This study used mixed-method with the concurrent embedded design which consists of research in qualitative descriptive. This study was conducted at State Junior High School 1 Ungaran where the population in this study was eighth-grade students of SMP Negeri 1 Ungaran in the academic year of 2018/2019. By using a simple random sampling technique obtained VIII B as the study class that given the Treffinger learning model assisted by scaffolding, while class VIII C as instrument trials in abilities of global meta-level algebraic thinking test. The selection of research subjects in this study used a purposive sampling technique that was selected based on group level and the results of students test in global meta-level algebraic thinking. The research subjects consisted of 9 students, namely 3 students from the upper group, 3 students from the middle group, and 3 students from the lower group.

Data collection methods for qualitative data are test, observation, and interviews, while quantitative data use test methods. The meta global -level algebraic thinking ability test used in this study consisted of 4 essay questions. This test is used to determine the level of students’ abilities on meta global -level algebraic thinking, the types of errors made by students, and the completeness of student learning after using Treffinger learning assisted by scaffolding. Before the test is given, first the test questions are tested on instrument trial class to find out the validity, reliability, difficulty level, and discrimination power. Interviews were conducted with semi-structured interviews. The purpose of the interview is to describe the location and type of student errors in algebraic thinking and the causes of errors of each research subject. After the data is obtained, then it is analyzed qualitatively. Consists of data reduction, data display, and verification. Quantitative data analysis includes the normality test using the Shapiro-Wilk test, testing the completeness for the average with the t statistic test and the proportion test with z statistic.

3. Results and Discussion

3.1. The quality of Treffinger learning assisted by scaffolding
In this study, the quality of Treffinger learning assisted by scaffolding is measured by paying attention to the three stages of learning based on the opinion of [13], namely: (1) planning of the learning process consisting of syllabus, lesson plans, and written test items, (2) implementation of the learning process consisting from the observation sheet of teacher activities and student activities, and (3) assessment of learning outcomes consisting of quizzes. Learning is said to be of quality if the plan and implementation aspects of the learning process meet the minimum criteria both and the aspects of the assessment of learning outcomes show more than 75% of students reach minimum completeness criteria. The results of Treffinger learning assisted by scaffolding quality for each aspect are presented in the following table.

| Learning Step          | Component                        | Score    | Criteria  |
|------------------------|----------------------------------|----------|-----------|
| Learning Process Plan  | Syllabus                         | 92.5%    | Very Good |
|                        | Lesson Plan                      | 94%      | Very Good |
|                        | Written Test Items               | 94.3%    | Very Good |
| Learning Process       | Teacher Activity Observation Sheets | 82.5%    | Very Good |
| Implementation         | Students Activity Observation Sheets | 84.7%    | Very Good |
| Learning Process Result| Quiz                            | 87.3%    | Very Good |
| Average                |                                  | 89.2%    | Very Good |
Based on the table, it can be seen that at the planning stage of the learning process has been carried out very well and the components at the planning stage are feasible to be used in Treffinger learning assisted by scaffolding. From the results of the acquisition value of teacher and student activities shows that both teacher and students have implemented Treffinger learning assisted by scaffolding very well. At the evaluation stage, the learning outcomes have been carried out very well and the acquisition of the quiz shows that the students’ grades have reached the minimum completeness criteria.

Based on the reason above, it can be concluded that the quality of mathematics learning with the Treffinger learning assisted by scaffolding on the students’ abilities in global meta-level algebraic thinking is very good.

### 3.2. Global meta-level algebraic thinking ability test results

After learning for three meetings, an evaluation was conducted to measure students' global meta-level algebraic thinking skills. The test questions consist of 4 items with 50 minutes of processing time. The test was implemented by 32 students. After conducting an evaluation, students obtained a global meta-level algebraic thinking ability test score with an average of 80.59; the standard deviation of 5.0365; the highest value is 90; and the lowest value is 67.

The test result data is then analyzed with the normality test and the learning completeness test. The test results of students' global meta-level algebraic thinking skills are said to be complete if the average value of the global meta-level algebraic thinking ability test of students in the research class has reached the minimum completeness criteria if at least 75% of students in the research class scored ≥ 77. Based on the normality test using the Shapiro-Wilk test, it was found that the global meta-level algebraic thinking ability data were normally distributed. From the data obtained $t_{count} = 9.825$ and with $\alpha = 5\%$ obtained $t_{table} = 1.70$ so $t_{count} > t_{table}$ it can be concluded that the average value of the algebraic thinking ability test of the global meta-level of students > 77. The results of the calculation of the proportion of the right side obtained $z_{count} = 1.8845$ and with $\alpha = 5\%$ we get $z_{table} = 1.64$. Because $z_{count} > z_{table}$, it can be concluded that the percentage of students in the research class who obtained ≥ 77 has reached more than 75%. From the t and z test it was concluded that the average test score of students' ability to think global meta-level algebra had reached the the minimum completeness criteria in accordance with predetermined

Treffinger Learning Outcomes apparently can bring students to reach the minimum completeness criteria, this is because at the Treffinger learning stage students are actively invited. At the understanding of the challenge stage, students can discover the concept of the surface area and volume formulas of cubes and beams in worksheets by discussing with a group of friends and can understand the purpose of the problem given. In the generating ideas stage, students can find the right solutions and solve the problems given by discussion. At the stage of preparing for action, students can independently discuss the solution of more complex problems with a group of friends then some groups will write the results of their group discussions on the board and present them to the class and discuss with other groups. Besides, at the stage of preparing for action, students can independently solve more complex problems in quiz activities.

Treffinger learning can be supported by applying scaffolding because it makes students more active in learning activities and more independent in solving problems. The form of scaffolding provided in the form of Worksheet (environmental provisions) in which contains assistance by the level of ability to think algebraic levels of global meta, and in the form of teacher interaction with students through explaining, reviewing, and restructuring. The provision of scaffolding is carried out in stages until later the assistance is eliminated so that students can solve problems independently.

### 3.3. Types and causes of student error

After obtaining the results of grouping students' algebraic thinking skills from the quiz results, each group was selected 3 research subjects so that in this study there were 9 students. The nine subject selected in the study are presented in Table 3 below.
Table 3. Study Subject

| Algebraic Thinking Ability in Global-Meta Level | High | Medium | Low |
|-----------------------------------------------|------|--------|-----|
| $X > 89.53$                                    |      | $33.79 \leq X$ | $X < 33.79$ |
| $\leq 89.53$                                  |      |        |     |
| S-01 (B-03)                                   | S-04 (B-12) | S-07 (B-07) |
| S-02 (B-06)                                   | S-05 (B-17) | S-08 (B-22) |
| S-03 (B-11)                                   | S-06 (B-24) | S-09 (B-25) |

Nine selected research subjects were then interviewed. The following is an analysis of the types and causes of errors made by students for each group level.

3.3.1. Student Errors in High Groups Analysis. The fact is that the S-01 made a mistake at the comprehension stage, which is not quite right in writing what is asked in the problem. This is because S-01 is not careful in reading the information provided in the problem so S-01 makes a mistake in writing what is asked of the problem.

Based on the results of data analysis, high group students have high global meta-level algebraic thinking skills. Each subject made a mistake at the comprehension stage. Errors of comprehension are indicated by subjects who are unable to write information by problem. This error includes errors in writing what is asked of the by the problem. Errors made by high group subjects are caused by the subject being not careful in writing the information that is known and what is asked from the problem is not because of the subject's lack of understanding in solving the problem.

3.3.2. Student Errors in Medium Groups Analysis. The fact that the S-06 made a mistake at the stage of comprehension and process skills. S-06’s error at the comprehension stage is not complete in writing what is asked in the problem, while at the process skill stage is wrong in performing multiplication calculations. The error made by S-06 is caused by S-06 being inaccurate in reading the information provided in the problem and inaccurate in doing the calculation process.

Based on the results of data analysis, group students currently can to think algebraically at a global level. Each subject made a mistake at the stage of comprehension and process skills. Errors of comprehension are indicated by subjects who are unable to write information by problem. These errors include errors in writing what is known and what is asked of the problem by the information provided. The comprehension error made by the medium group subject is being caused because it is incomplete in identifying the information that is known and asked from the problem. Process skill errors are indicated by subjects who are not able to do the right completion process after determining the completion strategy. These errors include errors in calculating operations and errors in determining the next completion step. The process skill error made by the medium group subject is being caused because the subject is not careful in doing the calculation process and the subject is unable to solve the problem so the original subject determines the next completion step.

These results are in line with [14] which states that medium group students are likely to make mistakes at only two stages, namely transformation and process skills. The mistakes made by medium group students are wrong in forming mathematical models of problem problems and wrong in the process of mathematical calculation to find solutions that cause errors in determining the solution of an equation. This is due to the inaccuracy of students in implementing problems into mathematical modeling and student inaccuracies in the calculation process.

3.3.3. Student Errors in Low Groups Analysis. The fact that the S-09 made a mistake at the stage of comprehension, process skills, and encoding. S-09’s error at the comprehension stage is not quite right in writing what is asked in the problem, at the process skill stage is wrong in doing the calculation
process, while at the encoding stage because S-09 does not write the conclusion answers. Mistakes made by S-09 are caused by S-09 being inaccurate in reading information given to the questions, not understanding the material of the questions, and not careful in doing the calculation process.

Based on the results of data analysis, low group students have low global meta-level algebraic thinking skills. Each subject made a mistake in the stages of comprehension, process skills, and encoding. Errors of comprehension are indicated by subjects who are not quite right in writing information according to the problem. These errors include errors in writing what is known and asked according to the information provided from the problem. The comprehension error made by low-group subjects is caused by being out of sync in writing what is known and what is asked of the problem with the information provided in the problem. Process skill errors are indicated by subjects who are not able to do the right completion process after determining the completion strategy. These errors include errors in the calculation process, errors in determining the next completion step, and not completing the calculation process. Process skill errors made by low-group subjects are caused by the subject being not careful in doing the calculation process, the subject is unable to complete the calculation process, and the subject lacks mastery of the material so that he cannot do the problem correctly. Encoding errors are indicated by the subject of the carelessness of the subject in writing the concluding sentence. These errors include errors in writing the unit used in the final answer, mistakes in writing the concluding sentence by what is asked, and not writing the conclusion of the final answer. The encoding error made by low-group subjects was caused by the subject being inaccurate in the calculation process so that it caused an error in writing the final answer, the subject was not careful in writing the units used for the final answer, and the subject could not complete the calculation process and ran out of time in the process of working on the problem so do not write the conclusion of the final answer.

This result is in line with research conducted by [15] which states that underclass students tend to make mistakes at all stages. Errors made by students include errors in writing information given to the problems, errors in the process of calculation and problem-solving, and errors in writing the conclusions of the final answers. The cause of errors made by students in the lower group is due to the inaccuracy of students in writing what is known and asked on the problem and the lack of students’ understanding of the material provided so that students cannot solve the problem with the right steps.

3.4. **Scaffolding forms given to minimize student error**

3.4.1. **Form of Scaffolding in High Group.** The form of scaffolding that can be given to minimize the mistakes of high-level students who tend to make comprehension errors is through the provision of worksheets in which there is assistance in the form of key answers to the final questions and no assistance is given in the form of steps in the process of work (environmental provisions). Not only that but scaffolding is also given during the group discussion process by asking students to correct the results of the work (reviewing). It aims to minimize student errors caused by inaccuracies in students writing information about the questions.

3.4.2. **Form of Scaffolding in Medium Group.** The form of scaffolding that can be given to minimize the mistakes of medium-group students who tend to make errors of comprehension and process skills is through the provision of worksheets in which there is assistance in the form of completion steps (environmental provisions). Not only that, scaffolding is also given during the group discussion process by asking students to correct the results of the work they are doing whether they have completed all the stages of completion or not and to re-correct the results of the work whether it is in accordance with the questions of the questions or not (reviewing). It aims to minimize student errors caused by the inaccuracy of students in writing information from questions and inaccuracies of students in carrying out the calculation process.

3.4.3. **Form of Scaffolding in Low Group.** The form of scaffolding that can be given to minimize errors of low-group students who tend to make mistakes comprehension, process skills, and encoding
is the provision of worksheets in which there is assistance in the form of filling one of the information from the problem (environmental provisions). Not only that, scaffolding is also given during the group discussion process, namely by providing an explanation of the purpose of the problem if the students do not understand (explaining), rebuilding their understanding of the purpose of the problem by giving simpler examples then the example is associated with the questions worked by students (restructuring), and ask students to review their work (reviewing).

4. Conclusions
Based on the results and discussion it can be concluded: (1) the quality of Treffinger's learning aided by scaffolding on the ability to think algebraic levels of global meta is very good; (2) students' ability to think global meta-level algebra in scaffolding-assisted Treffinger learning has reached the minimum completeness criteria; (3) high group students tend to make mistakes at the comprehension stage, the cause is due to the lack of accuracy of students in checking the suitability of information and questions in the questions with what they write on the answer sheet. Students in medium groups are inclined to make mistakes at the stage of comprehension and process skills, the cause is because it is incomplete in identifying the information that is known and asked of the questions and the lack of student accuracy in the process of working on the questions. Low group students tend to make mistakes at the comprehension, process skill, and encoding stages, the cause is at the comprehension stage because it is out of sync in writing what is known and asked of the questions, at the process skill stage the students lack accuracy in doing the calculation process, the lack of student understanding related problem solving, and lack of mastering the material so that students, while at the encoding stage because students are wrong in doing the process; (4) the form of scaffolding given to high group students is by giving worksheet (environmental provisions) in which there are answer key questions and asking students to recheck their work (reviewing). The form of scaffolding that is given to medium group students is by providing worksheet (environmental provisions) in order, asking students to recheck their work (reviewing), and conducting question and answer to guide students to get the correct solution (restructuring). The form of scaffolding provided for low-group students is to provide worksheet (environmental provisions) which contains help with work steps, provide an explanation to students about the purpose of the problem (explaining), provide guidance to rebuild understanding by providing other examples that simpler then linking it with problems that students have done (restructuring), and asking students to double check the results of their work (reviewing).

Suggestions that can be given (1) teachers should give questions more often in the form of essays that require interpretation so that students are accustomed to understanding the information provided on the questions so as to minimize errors at the stage of comprehension and transformation; (3) it should ensure that students have mastered the prerequisite material properly so as to minimize errors at the process skill stage; (4) students should get used to solving problems from writing information given to the problems to the final conclusions so as to minimize errors at the comprehension and encoding stages.

References
[1] Pasani C F 2018 Int J Eng Res Technol 11 3 451
[2] Walkington C, & M Bernacki 2015 J. Math. Behav. 40 171
[3] Kızıltoprak A and Köse N Y 2017 Int Electron J Elem Educ 10 1 131
[4] Zhang Z 2018 Int Educ Stud 11 2 106
[5] Magiera M T, den Kieboom L A and Moyer J C 2013 Educ Stud Math 84 1 93
[6] Kieran C 2014 Math Educ 8 1 139
[7] Agustina I R, Mulyono & Asikin M 2016 Unnes J Math Educ 5 2 92
[8] Jha S K 2012 Int J Comput Appl Eng Sci II 1 17
[9] Baalsrud Hauge J M, Stanescu I A, Arnab S, Moreno Ger P, Lim T, Serrano-Laguna A, et al 2015 Int J Serious Games 2 1 30
[10] Anghileri J 2006 J Math Teach Educ 9 1 33
[11] Sumiara W T and Subroto 2019 Int. J. Educ. Res. Rev. Special Issue, 74
[12] Nizham H, Suhendra S, P. BA 2017 *Int J Sci Appl Sci Conf Ser* 2 1 130
[13] Suryosubroto, B 2009 *Proses Belajar Mengajar di Sekolah Edisi Revisi* (Jakarta: Rineka Cipta)
[14] Agoestanto A, Isnarto I, Sukestiyarno Y L, Rochmad R and Lestari M D 2018 *Int J Inst.* 12 1 1431
[15] Fitriani H N, Turmudi T, Prabawanto S 2018 *Int Conf Math Sci Educ* 3:791