The $OSp(32|1)$ versus $OSp(8|2)$ supersymmetric
M-brane action from self-dual $(2,2)$ strings

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Abstract

Taking the $(2,2)$ strings as a starting point, we discuss the equivalent integrable field theories and analyze their symmetry structure in $2+2$ dimensions from the viewpoint of string/membrane unification. Requiring the ‘Lorentz’ invariance and supersymmetry in the $(2,2)$ string target space leads to an extension of the $(2,2)$ string theory to a theory of $2+2$ dimensional supermembranes ($M$-branes) propagating in a higher dimensional target space. The origin of the hidden target space dimensions of the M-brane is related to the maximally extended supersymmetry implied by the ‘Lorentz’ covariance and dimensional reasons. The Kähler-Chern-Simons-type action describing the self-dual gravity in $2+2$ dimensions is proposed. Its maximal supersymmetric extension (of the Green-Schwarz-type) naturally leads to the $2+10$ (or higher) dimensions for the M-brane target space. The proposed $OSp(32|1)$ supersymmetric action gives the pre-geometrical description of M-branes, which may be useful for a fundamental formulation of F&M theory.

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Introduction. Since a discovery of string dualities, much evidence was collected for the idea that ‘different’ string theories can be understood as particular limits of a unique underlying theory whose basic formulation is yet to be found. The fundamental theory does not seem to be a theory of strings but it describes fields, strings and membranes in a democratic way. A candidate for the unified theory was also proposed under the name of M-theory [1, 2] or its refined F-theory formulation [3], which can be reduced to all known 10-dimensional superstrings and 11-dimensional supergravity as well. There should be also room for strings with extended world-sheet supersymmetry in the unified theory. The anticipated relation between the $N = (2,1)$ heterotic strings and M-theory in some particular low-dimensional backgrounds was recently used [4] to propose the definition of the underlying M-theory as a theory of $2 + 2$ dimensional membranes (called M-branes [5]) embedded in higher dimensions. The origin of M-branes should therefore be understood from the basic properties of $N=2$ strings. It is the purpose of this Letter to argue that the hidden membrane (both world-volume and target space) dimensions are in fact required by natural symmetries which are broken in the known $N=2$ string formulations. By the natural symmetries I mean ‘Lorentz’ invariance and supersymmetry which should be made explicit and linearly realized. That symmetries uniquely determine the dynamics of M-branes.

The basic idea for describing M-branes naturally arises from the known world-sheet/target space duality of $N=2$ strings. In the early days of $N=2$ string theory, when only two-dimensional target spaces were considered, Green [6] suggested to use the $N=2$ string world-sheet as the target space, which implies a duality between the world-sheet moduli and their target space counterparts. The four-dimensional nature of the $(2,2)$ string target space as a (hyper) Kähler manifold was understood later by Ooguri and Vafa [7], who suggested to associate with the $N=2$ string world-sheet (Riemann surface) a four-dimensional symplectic space – the so-called ‘cotangent bundle of the Riemann surface’. The latter has an equal number of moduli to be associated with non-trivial deformations of a complex structure and of a Kähler class. The duality (in fact, triality) symmetries then appear between the world-sheet moduli, the target space complex structure moduli, and the target space Kähler-class moduli. I am going to use the world-sheet/target space duality of $N=2$ strings as the (first) working principle of string/membrane unification, namely, as a route for constructing the self-dual theory of M-branes out of the target space field theory of $(2,2)$ strings, along the lines of ref. [4]. However, unlike the way of reasoning in ref. [4], which puts forward the $(2,1)$ heterotic strings, I consider closed and open $(2,2)$ strings as a starting point. The critical $(2,2)$ strings naturally live in $2 + 2$ dimensions, which are crucial for self-duality, whereas the $(2,0)$ or $(2,1)$ heterotic strings require the $1 + 1$
or 1 + 2 dimensional target space \[7\]. The duality principle is however not enough to deliver the M-brane action, since it does not say enough about the symmetries of the M-brane. Hence, I postulate the second working principle by requiring all the natural symmetries to explicitly appear in the target space action. Despite its innocent content, the ‘Lorentz’ invariance in 2 + 2 dimensions appears to be non-trivial for N=2 strings. By gauging the ‘Lorentz’ group \(SO(2,2)\), I formulate a Kähler-Chern-Simons-type gauge-invariant action in five dimensions, whose dynamics describes the self-dual gravity in four dimensions. It gives the relevant part of the M-brane action, according to the (first) duality principle above. The rest of the M-brane action is fixed by requiring the maximal supersymmetry (the second working principle) in the M-brane target space, whose dimension is \(2 + 10\), or it can be even higher. The supersymmetric action is proposed to describe the M-branes, which may be the fundamental constituents of the putative F&M theory.

2 Summary of (2,2) strings. The N=2 strings are strings with two world-sheet (local) supersymmetries. The gauge-invariant \(N = 2\) string world-sheet actions in the NSR-type formulation are given by couplings of a two-dimensional N=2 supergravity to a complex N=2 scalar matter \[10\], and they possess global \(U(1,1) \times \mathbb{Z}_2\) target space symmetry. A covariant gauge-fixing introduces conformal ghosts \((b, c)\), complex superconformal ghosts \((\beta^{\pm}, \gamma^{\mp}) = (\partial \xi^{\pm} e^{-\phi^{\mp}}, \eta^{\mp} e^{\phi^{\mp}})\), and real abelian ghosts \((\tilde{b}, \tilde{c})\), as usual. The chiral N=2 (superconformal) current algebra comprises a stress-tensor \(T(z)\), two supercurrents \(G^{\pm}(z)\), and an abelian current \(J(z)\). The critical closed and open (2,2) strings live in four dimensions with a signature \(2 + 2\). The current algebras of the N=2 heterotic strings have the additional abelian null current. It is needed for a nilpotency of the BRST charge, and implies a reduction of the N=2 string target spacetime dynamics down to \(1 + 2\) or \(1 + 1\) dimensions \[\text{or}\].

The BRST cohomology and on-shell amplitudes of N=2 strings were investigated by several groups \[7, 11, 12, 13\]. There exists only a single massless physical state in the open or closed (2,2) string spectrum. This particle can be identified with the Yang scalar of self-dual Yang-Mills theory for open strings, or the Kähler scalar of self-dual supergravity for closed strings, while infinitely many massive string modes are all unphysical. The (2,2) strings thus lack ‘space-time’ supersymmetry. Though twisting the N=2 superconformal algebra yields some additional twisted physical states which would-be the target space ‘fermions’, they actually decouple. It is consistent with another observation that the ‘space-time fermionic’ vertex operators constructed in

\[3\] See refs. \[8, 9\] for a review.

\[4\] The signature is dictated by the (2,2) world-sheet supersymmetry. The euclidean signature is excluded by trivial kinematics for massless particles.
ref. [13] anticommute modulo picture-changing, instead of producing ‘space-time’ translations required by the ‘space-time’ supersymmetry.

An n-point function of closed N=2 strings is given by a topological sum indexed by the genus g and the instanton number (Chern class) c, with each term in the sum being an integral over metric, N=2 fermionic and Maxwell moduli. The Maxwell moduli parameterize the space of flat connections or harmonic 1-forms $H$ on the n-punctured world-sheet $\Sigma$, and they enter the gauge-fixed action as $\int_{\Sigma} H \wedge \ast J$. Making a shift $H \rightarrow H + h$ changes the action as

$$\int_{\Sigma} h \wedge \ast J = \sum_{i=1}^{g} \left( \int_{a_i} h \oint_{b_i} \ast J - \int_{b_i} h \oint_{a_i} \ast J \right) + \sum_{l=1}^{n} \oint_{c_l} h \oint_{p_l} \ast J ,$$

where a canonical homology basis $(a_i, b_j)$ on $\Sigma$, the contours $c_l$ encircling punctures $p_l$, and a reference point $p_0$ have been introduced. Therefore, the shift gives rise to twists $SFO(\theta) \equiv \exp \{2\pi i \theta \int \ast J\}$ around the homology cycles as well as around the punctures, with $\theta \in [0, 1]$. This phenomenon is known as spectral flow. A twist around a puncture at $z$ can be absorbed into a redefined (twisted) vertex operator $[13]

$$V(z) \rightarrow V^{(\theta)}(z) = \exp \left\{2\pi i \theta \int_{z_0}^{z} \ast J\right\} V(z) .$$

(1)

The spectral flow operator $SFO$ is BRST-closed but only its zero mode is not BRST-exact. Hence, the position of $SFO$ in an amplitude is irrelevant, and all the n-point functions are invariant,

$$\langle V_1^{(\theta_1)} \cdots V_n^{(\theta_n)} \rangle = \langle V_1 \cdots V_n \rangle ,$$

(2)

as long as the total twist vanishes, $\sum_i \theta_i = 0$. The bosonized spectral flow operator reads

$$SFO(\theta) = e^{-2\pi i \theta \phi(z_0)} \exp \{2\pi i \theta \phi(z)\} ,$$

(3)

where the $U(1)$ current has been bosonized as $\ast J = d\phi$. The two factors in eq. (3) are separately neutral under the local $U(1)$, but carry opposite charges under the global $U(1)$ symmetry. Eq. (3) relates the spectral flow to Maxwell instantons on the world-sheet. Indeed, choosing $\theta = 1$ yields an instanton-creation operator, $ICO \equiv \lambda SFO(\theta = 1)$, which changes the world-sheet instanton number $c$ by one. Amplitudes with different instanton backgrounds are therefore related as

$$\langle V_1 \cdots V_n \rangle_c = \langle V_1 \cdots V_n (ICO)^c \rangle_{c=0} = \lambda^c \langle V_1^{(\theta_1)} \cdots V_n^{(\theta_n)} \rangle_{c=0} ,$$

(4)

with a total twist of $\sum_i \theta_i = c$. Hiding the reference point ambiguity by declaring the Maxwell coupling constant to be $\lambda = \exp \{2\pi i \phi(z_0)\}$ implies that both $ICO$ and $\lambda$
have a *non-vanishing* charge with respect to the $U(1)$ subgroup of the actual global symmetry group $U(1, 1) \subset SO(2, 2)$ [13].

The only non-vanishing N=2 string scattering amplitudes are 3-point trees (and, maybe, 3-point loops as well), while all the other tree and loop amplitudes vanish due to kinematical reasons. As a result, a (2,2) string theory appears to be equivalent to an *integrable* field theory. In particular, the open (2,2) string amplitudes are reproduced by either the *Yang* non-linear sigma-model action [14] or the *Leznov-Parkes* cubic action [15], each following from a field integration of the self-dual Yang-Mills (SDYM) equations in a particular gauge, and related to each other by a duality transformation. As far as the closed (2,2) strings in the zero-instanton sector are concerned, the equivalent non-covariant field theory action is known as the *Plebański* action [16] for the self-dual gravity (SDG). The world-sheet instanton effects lead to a *deformation* of self-duality: the Ricci-tensor does not vanish, while the integrability implies the self-dual *Weyl* tensor instead.

The natural (global) ‘Lorentz’ symmetry of a flat 2 + 2 dimensional ‘space-time’ is $SO(2, 2) \cong SU(1, 1) \otimes SU(1, 1)'$. The NSR-type N=2 string actions used to calculate the amplitudes have only a part of it, namely, $SU(1, 1)$, so is the symmetry of the N=2 string amplitudes. The full ‘Lorentz’ symmetry $SO(2, 2)$ can be formally restored in the *twistor* space, which adds the (harmonic) space $SU(1, 1)'/U(1)$ of all complex structures in 2 + 2 dimensions [11]. The ladder generators of the second $SU(1, 1)'$ factor can be explicitly constructed as follows [11]:

$$J_+ = \int \xi^+ \eta^+ (1 + \tilde{c} \tilde{b}) ICO^{-1} , \quad J_- = \int \xi^- \eta^- (1 - \tilde{c} \tilde{b}) ICO .$$

(5)

Closing the underlying N=2 superconformal algebra to be appended by the additional currents $J_\pm$ results in the so-called ‘small’ *twisted* N=4 superconformal algebra. This remarkable property allows one to treat the N=2 string theory as an N=4 *topological* field theory [11, 17]. The embeddings of the N=2 algebra into the N=4 algebra are just parameterized by twistors: a choice of a complex structure selects a $U(1, 1)$ subgroup of the ‘Lorentz’ group, while world-sheet Maxwell instantons rotate that complex structure.

The (real) coupling constant $g$ of the (2,2) string interaction and the Maxwell coupling constant (phase) $\lambda$ can be naturally unified into a single complex coordinate parameterizing the moduli space of complex structures. The complex N=2 string coupling can also be interpreted as the vacuum expectation value of a *complex dilaton*

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[11] Similar results are valid for open (2,2) strings too.
field. Hence, the N=2 string dilaton is not inert under the full ‘Lorentz’ transformations! The dilaton thus takes its values in SU(1,1′)/U(1)′, and it can therefore be represented by an anti-self-dual (closed) two-form ω satisfying a nilpotency condition ω ∧ ω = 0 (see sects. 4 and 5 also).

3 Adding supersymmetry in 2 + 2 dimensions. Because of the isomorphisms SU(1,1) ≡ SL(2,R) and SO(2,2) ≡ SL(2,R) ⊙ SL(2,R)′, it is natural to represent the 2 + 2 ‘space-time’ coordinates as xα,α′, where α = (+,−) and α′ = (+′,−′) refer to SL(2) and SL(2)′, respectively. The N-extended supersymmetrization of self-duality amounts to extending the SL(2) factor to OSP(N|2), while keeping the SL(2)′ one to be intact. One has \( \delta^{AB} = (\delta^{ab}, C^{\alpha\beta}) \), where \( \delta^{ab} \) is the SO(N) metric and \( C^{\alpha\beta} \) is the (part of) charge conjugation matrix, \( A = (a, \alpha) \). In superspace \( Z = (x^{\alpha,\alpha'}, \theta^{A}) \), the N-extended (gauged) self-dual supergravity (SDSG) is defined by the constraints on the spinorial covariant derivatives, \( \nabla_{A\alpha} = E_{A\alpha} M^\mu \partial_M + \frac{i}{2} \Omega_{A\alpha} BC M^{CB} \), as \[ \{ \nabla^{a\alpha}, \nabla^{b\beta} \} = C^{\alpha\beta} M^{ab} + \delta^{ab} M^{\alpha\beta} , \] (6a)

\[ \{ \nabla^{a\alpha}, \nabla_{b\beta} \} = \delta^{a} C^{\alpha\beta} \nabla_{b\beta} , \quad [ \nabla^{a\alpha}, \nabla_{b\beta} ] = \delta^{a\beta} \delta^{ab} \nabla_{b\beta} , \] (6b)

where \( M^{AB} = (M^{ab}, M^{\alpha\beta}, \nabla^{a\alpha}) \) are the generators of OSP(N|2). Eqs. (6) have the OSP(N|2) ⊙ SL(2)′ (local ⊗ global) symmetry, and they can be ‘solved’ in the light-cone gauge in terms of a SDSG pre-potential. It is well-known that, as far as the SDG is concerned, one has

\[ \frac{R_{a_1 a_2 a_3 a_4}}{a_1 a_2 a_3 a_4} \sim \partial_{a_1 \alpha'} \partial_{a_2 \alpha'} q_{a_3 a_4}^{-\alpha \alpha'} \sim \partial_{a_1 \alpha'} \partial_{a_2 \alpha'} \partial_{a_3 \alpha'} \partial_{a_4 \alpha'} V_{\alpha \alpha'} \] (7)

where the prepotential \( V_{\alpha \alpha'} \) has a single component representing the helicity (+2).

Eq. (7) can be generalized in superspace, \( R_{A_1 A_2 A_3 A_4}(Z) \sim \partial_{A_1 +} \cdots \partial_{A_4 +} V_{=\alpha \alpha'}(Z) \), where \( V_{=\alpha \alpha'} \) is a SDSG pre-potential of dimension (−1). The free field equation for the SDSG pre-potential, \( \partial_A \partial_B V_{=\alpha \alpha'}(Z) = 0 \), can be solved for all \( \theta^{a\alpha'} \) dependence. It reduces \( V_{=\alpha \alpha'}(Z) \) to a self-dual superfield \( V_{=\alpha \alpha'}(x^{\alpha,\alpha'}, \theta^{a\alpha'}) \), which merely depends on a half of \( \theta^{a\alpha'} \)'s. Of course, it breaks the ‘Lorentz’ symmetry. As a result, the SDSG constraints in the light-cone gauge can be reduced to a single equation for the pre-potential, which is obtained from the N-extended super-Plebański action \[ S_{SDSG} = \int d^{2+2}x d^N \theta \left[ \frac{1}{2} V_{=\alpha \alpha'} \Box V_{=\alpha \alpha'} + \frac{1}{6} V_{=\alpha \alpha'} (\partial^{\alpha \alpha'} \partial_A V_{=\alpha \alpha'}) \delta^{BA} (\partial_B V_{=\alpha \alpha'}) \right] \] (8)

As was noticed by Siegel [18], the action (8) implies the maximal supersymmetry! Indeed, dimensional analysis immediately yields \( N = 8 \), and the same follows from counting the total GL(1)′ charge of the action (8), where GL(1)′ is the unbroken part.
of the ‘Lorentz’ factor $SL(2)'$. Similarly, the $N$-extended super-Leznov-Parkes action for the self-dual supersymmetric Yang-Mills (SDSYM) theory implies $N = 4$ [18]:

$$S_{SDSYM} = \int d^{2+2}x d^4\theta \left[ \frac{1}{2} V_{\pm} \square V_{\pm} + \frac{i}{3} V_{\pm} \left( \partial^a + V_{\pm} \right) (\partial_{a+} V_{\pm}) \right] .$$

(9)

The SDSG and SDSYM theories in eqs. (8) and (9) are similar to the non-self-dual supersymmetric gauge theories in the light-cone gauge [19].

The natural appearance of the maximal $N = 8$ supersymmetry and the (gauged) $SO(8)$ internal symmetry in the supersymmetrized $(2,2)$ string effective action in $2 + 2$ dimensions is very remarkable, since that effective action is supposed to be a (dual) part of an M-brane action. We may now proceed in the usual way known in supergravity, and ‘explain’ the maximally extended local supersymmetry as a simple local supersymmetry in higher dimensions. For example, one may use the embedding

$$SO(2, 2) \otimes SO(8) \subset SO(2, 10) ,$$

(10)

which implies going up to $2 + 10$ dimensions. Indeed, the $2 + 10$ dimensions are the nearest ones in which Majorana-Weyl spinors and self-dual tensors also appear, like in $2 + 2$ dimensions. It should be noticed that twelve dimensions for string theory were originally motivated in a very different way, namely, by a desire to explain the S-duality of type IIB string in ten dimensions as the T-duality of a 12-dimensional F-theory dimensionally reduced on a two-torus. The type IIB string is then supposed to arise upon double dimensional reduction from the F-theory.

There is, however, a problem with that naive approach. One has to double the on-shell number (8) of the anticommuting coordinates in a covariant M-brane action while maintaining the number of their degrees of freedom. One then gets $2 \times 16 = 32$ off-shell components, which is just needed for a single Majorana-Weyl spinor in $2 + 10$ dimensions. As is well known in superstring theory, it is the $\kappa$-symmetry of the Green-Schwarz superstring action that makes the doubling to be possible, while the Green-Schwarz action itself can be understood as the particular Wess-Zumino-Novikov-Witten (WZNW) model with superspace as the target supermanifold [20]. Therefore, one should look for a Green-Schwarz-type reformulation of self-duality in $2 + 2$ dimensions, and then maximally supersymmetrize the target space, instead of (or, maybe, in addition to) the world-volume (or NSR-type) supersymmetrization.

4. Kähler-Chern-Simons actions for SDYM and SDG. A more geometrical (dual) description of the SDYM theory is provided by the five-dimensional hyper Kähler-Chern-Simons action [21]:

$$S_{hKCS} = -\frac{1}{4\pi} \int_Y \text{tr} \left( \tilde{A} \wedge \tilde{dA} + \frac{2}{3} \tilde{A} \wedge \tilde{A} \wedge \tilde{A} \right) \wedge \omega^i e_i ,$$

(11)
where $Y = M_4 \otimes R$, with $M_4$ being the 2 + 2 dimensional hyper Kähler world-volume and $R$ being the auxiliary dimension called extra ‘time’ $t$. Here $\tilde{A}$ is the Lie algebra valued 1-form on $Y$, $\omega^i$ is the hyper Kähler structure on $M_4$, and $e_\mu = (1, e_i)$ is a basis of quaternions. Since $\omega^i$ are closed, the action (11) is invariant under the gauge transformations $\tilde{A}_h = h\tilde{A}h^{-1} - dhh^{-1}$ which should be trivial on the boundary $\partial Y$. I assume that the boundary conditions for the gauge field $\tilde{A}$ are chosen in such a way that no boundary terms appear in the equations of motion. It is convenient to decompose both the gauge field and the exterior derivative into the ‘time’ and ‘rest’ components, $\tilde{A} = A_t + A$ and $\tilde{d} = dt \frac{\partial}{\partial t} + d$. One finds that $A_t$ and $\omega^i$ appear in eq. (11) as Lagrange multipliers, which implement the self-duality equations

$$F \wedge \omega^i = 0, \quad i = 1, 2, 3,$$

where the YM field strength $F = dA + A \wedge A$ has been introduced. Varying with respect to $A$ implies (in the gauge $A_t = 0$) that the A-field is $t$-independent. In the gauge $A_t = 0$, the gauge symmetry is represented by the $t$-independent gauge transformations. Therefore, the action (11) describes on-shell the SDYM in 2 + 2 dimensions. Eq. (12) for $i = 1, 2$ can be solved in complex coordinates ($z^a, \bar{z}^{\bar{a}}$) on $M_4$ as $A_a = (U)^{-1}\partial_a U$ and $A_{\bar{a}} = -\partial_{\bar{a}} U^\dagger(U^{-1})^{-1}$, where $U$ is locally defined. In terms of the gauge-invariant potential $J = UU^\dagger$, the remaining eq. (12) at $i = 3$ is just the Yang equation.

$$\omega \wedge \bar{\partial} (J^{-1}\partial J) = 0.$$

Eq. (13) can be obtained from the Donaldson-Nair-Schiff (DNS) action [21]

$$S_{DNS}[J; \omega] = -\frac{1}{4\pi} \int_{M_4} \omega \wedge \text{tr}(J^{-1}\partial J \wedge J^{-1}\bar{\partial} J) + \frac{i}{12\pi} \int_{M_4 \times [0,1]} \omega \wedge \text{tr}(J^{-1}dJ)^3. \quad (14)$$

The action similar to eq. (11) can also be constructed for SDG. Let us simply replace the YM Chern-Simons form by the ‘Lorentz’ Chern-Simons form $C_{3L}$,

$$C_{3L} = \text{tr} \left( \tilde{\Omega} \wedge \tilde{d}\tilde{\Omega} + \frac{2}{3} \tilde{\Omega} \wedge \tilde{\Omega} \wedge \tilde{\Omega} \right) = \text{tr} \left( \tilde{\Omega} \wedge \tilde{R} - \frac{1}{3} \tilde{\Omega} \wedge \tilde{\Omega} \wedge \tilde{\Omega} \right), \quad (15)$$

where $\tilde{R} = d\tilde{\Omega} + \tilde{\Omega} \wedge \tilde{\Omega}$, and the 1-form $\tilde{\Omega}$ takes values in the Lie algebra of $SO(2,2)$. The SDG action for a hyper Kähler manifold $M_4$ equipped with the anti-self-dual two-form $\omega$ is given by

$$S_{SDG}[\tilde{\Omega}; \omega] = -\frac{1}{4\pi} \int_Y C_{3L} \wedge \omega. \quad (16)$$

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6 There exist the Kähler (1,1) form $\omega$ and a closed (2,0) form $\omega^+$ on any hyper Kähler manifold $M_4$. The hyper Kähler structure is defined by $\omega^1 = \text{Re}\omega^+, \omega^2 = \text{Im}\omega^+$ and $\omega^3 = \omega$.

7 If one first solves eq. (12) for $i = 2, 3$, the remaining equation for $i = 1$ follows from the dual Leznov-Parkes action (sect. 2).
The self-duality condition \( R \wedge \omega = 0 \) appears from varying eq. (16) with respect to \( \Omega_t \) (in the gauge \( \Omega_t = 0 \)). One can check that the vanishing variation of the action (16) with respect to a Kähler potential to be associated with \( \omega \) is consistent with the self-dual geometry, and the physics associated with eq. (16) is four-dimensional indeed. The anti-self-dual two-form \( \omega \) is interpreted as the (2, 2) string dilaton field (sect. 2), rather than the world-volume gravity.

5. Higher dimensions versus extended supersymmetry. The maximally supersymmetric SDSG and SDSYM actions (8) and (9) have manifest \( OSp(8|2) \) or \( OSp(4|2) \) supersymmetry, respectively. In fact, they possess an even larger superconformal symmetry \( SL(8|4) \) or \( SL(4|4) \), respectively \([8]\), which may be the fundamental worldvolume symmetries of (closed or open) M-branes. Indeed, the conformal extension of \( SO(2, 2) \) is given by \( SO(3, 3) \cong SL(4) \), whereas its \( N \)-supersymmetric extension is just \( SL(N|4) \).

Since the internal symmetry of the supergroup \( SL(4|4) \) is also \( SL(4) \cong SO(3, 3) \), combining it with the ‘space-time’ conformal group \( SO(3, 3) \) implies ‘hidden’ twelve dimensions in yet another way: \( SO(3, 3) \otimes SO(3, 3) \subset SO(6, 6) \). The 6+6 dimensions is the only alternative to 2+10 dimensions where Majorana-Weyl spinors also exist. I do not consider this possibility.

The gauge actions (11) and (16) for SDYM and SDG, or the equivalent DNS action (14), can be naturally supersymmetrized à la Green-Schwarz. The simple supersymmetry with one spinor generator (minimal grading) in the maximal dimensions (twelve) amounts to the simple superalgebra \( OSp(32|1) \). The choice of \( OSp(32|1) \) is unique since it simultaneously represents the minimal supersymmetric extension of (i) the (self-dual) ‘Lorentz’ algebra in 2+10 dimensions, (ii) de Sitter algebra in 1+10 dimensions and (iii) the conformal algebra in 1+9 dimensions \([22]\). A supersymmetry part of \( OSp(32|1) \) reads (cf. eq. (6a)):

\[
\{Q_\alpha, Q_\beta\} = \gamma^\mu_{\alpha\beta} M_{\mu\nu} + \gamma^\mu_{\alpha\beta\mu_1\cdots\mu_6} Z^{\nu}_{\mu_1\cdots\mu_6},
\]

where \( Q_\alpha \) is a 32-component Majorana-Weyl spinor, the Dirac \( \gamma \)-matrices are chirally projected, \( M_{\mu\nu} \) are 66 ‘Lorentz’ generators, \( \mathbb{6} \) and 462 generators \( Z^{\nu}_{\mu_1\cdots\mu_6} \) comprise a self-dual six-form (all in 2+10 dimensions). The pre-geometrical action I propose for M-branes is given by

\[
S_M[\tilde{\Omega}; \omega] = -\frac{1}{4\pi} \int_Y \text{str} \left( \tilde{\Omega} \wedge \tilde{d}\tilde{\Omega} + \frac{2}{3} \tilde{\Omega} \wedge \tilde{\Omega} \wedge \tilde{\Omega} \right) \wedge \omega,
\]

\(^8\)At this point my approach differs from that of Bars \([23]\).
where $\omega$ is an anti-self-dual two-form (N=2 string dilaton !) in the world-volume, and $\tilde{\Omega}$ is the $OSp(32|1)$ Lie superalgebra valued 1-form gauge potential. The action (18) can be further (doubly) supersymmetrized with respect to the world-volume, as in sect. 3. Currently, it is unclear to me whether it should be done or not.

The action in eq. (18) is called pre-geometrical because of the apparent absence of the translation generators (momenta) in the gauged superalgebra $OSp(32|1)$. However, the momenta can be easily recovered after a Wigner-Inönü-type contraction of $OSp(32|1)$ to lower dimensions. For instance, the 66 Lorentz generators $M_{\mu\nu}$ are decomposed into 55 Lorentz generators and 11 translations in eleven dimensions. The additional generators $Z_{\mu_1 \cdots \mu_6}^+$ can be interpreted either as the off-shell charges that do not transform the physical states $\{22\}$, or as the active charges which are related to boundaries of extended objects (6-branes) $\{23\}$. The most degenerate contraction of $OSp(32|1)$ yields the flat target space in $66 + 462 = 528$ (!) dimensions (cf. ref. $\{23\}$).

6. Conclusion. My arguments in this Letter support the idea $\{4\}$ that the fundamental framework for describing the secret F(or M, S, ...) theory is provided by the 2 + 2 dimensional supermembranes (M-branes) living in $2 + 10$ dimensions. The integrability (or self-duality) of M-branes naturally substitutes and generalizes the conformal symmetry of the string world-sheet. The basic assumptions were merely the N=2 string (world-sheet/target space) duality, and the manifest 'Lorentz' invariance and supersymmetry in ‘space-time’. The hidden superconformal symmetries of M-branes are to be responsible for their full integrability and the absence of loop divergences in $2 + 2$ world-volume dimensions despite of the fact that the DNS action is non-linear and, hence, is formally non-renormalizable in four dimensions. It fact, the DNS action is known to be one-loop finite, at least $\{24, 25\}$. Its maximally supersymmetric extension may have no divergences at all, presumably because of having a chiral current symmetry algebra similar to that in the two-dimensional supersymmetric WZNW models $\{24, 25\}$. Unlike the N=2 strings having severe infrared divergences in loops $\{26\}$, no such problems are expected for M-branes due to the higher world-volume dimension. The theory of M-branes should therefore exist as a quantum theory, in which strings would appear as asymptotic states of M-branes.

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9A super-Poincaré algebra does not exist in 2 + 10 dimensions.
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