PAPR Reduction Techniques in Generalized Inverse Discrete Fourier Transform Non-orthogonal Frequency Division Multiplexing System

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1. INTRODUCTION

The idea of 5G is to fulfil the demands of more network capacity by 2020. 5G needs to have higher carrier frequencies with bigger bandwidths to support high data rates [1]. Therefore, multicarrier systems are capable technologies that have been discovered give higher bandwidth efficiency in Physical (PHY) layer[2]. Orthogonal Frequency Division Multiplexing (OFDM) is the most recognized multicarrier system that has become the leading technology for high speed wireless communication[1], [3]. The fundamental idea of OFDM is to split the bit streams into many of substreams. Each of these substreams will modulate a different carrier with low data rate. This process can reduce a lot of Intersymbol Interference (ISI) with a drawback of high intercarrier interference (ICI) [4], [6]. Therefore, it is needed a proper design of channel coding and equalization to overcome the problem.

In order to achieve 5G standard, exploitation of orthogonality technique in multicarrier can be a promising technology [1]-[2], [4]. The advantages of non-orthogonal systems is better bandwidth efficiency [4], [6] as the ICI can fulfil the unused subcarrier spacing to benefit the required bandwidth[2]. The modulation can be categorised as filtered [7] and Discrete Fourier Transform (DFT) based. One of modulation that using square root of Nyquist filter as carrier was first introduced by Saltzberg [8]. Filter Bank Multicarrier (FBMC) modulation techniques were introduced even before the introduction of the OFDM [9]. It is an alternative and can be a replacement to OFDM in order to fulfil transmitting data packets
at a very high rate [8]. This technique was developed to prevent both ISI and ICI, such as dispersion in both time and frequency. The fundamental of FBMC is a set of parallel data symbols were transmitted through a bank of modulated filters. The use of this filter is more efficient in spectrum utilization where it minimized interference across the subcarriers. Another filter based technique is, Generalized Frequency Division Multiplexing (GFDM) that was studied by Fettweis et al. is based on flexible pulse shaping [11]. This technique might be an encouraging technology for positioning in cognitive radio networks and for 5G wireless communication [11], [7]. The technique allows the flexible pulse shaping of the spectrum for efficient multiuser scheduling, where each single subcarrier can be modulated individually. However, the performance analysis of GFDM is longer 6.25 times compares to OFDM due to larger FFT sized used. GFDM offer lower PAPR than OFDM, but the performance may be reduced if PAPR technique is applied.

As filter bank based have become more popular this days, DFT based modulation also now rapidly evolving. Spectrally Efficient Frequency Division Multiplexing (SEFDM) was an interesting technique investigated by M. Rodrigues and I. Darwazeh where it save the spectral by multiplexed first the subcarriers to create new symbol and relocated it at nearer spacing [3], [6]. The main idea of this technique is to abandon the orthogonality and synchronisation of the usual practice of OFDM [3]. Besides SEFDM, Overlapped Frequency Division Multiplexing (OvFDM) which was introduced by J. Wang et al has shown that the bandwidth can be larger than the frequency coherence bandwidth although the subcarriers were overlapped together [12]. Next promising multicarrier technique that was studied by N. Ahmad et al [13] is GIDFT n-OFDM which is more bandwidth efficient compare to OFDM system. This approached allow more data to be transmitted with fewer samples which can utilize the bandwidth more efficiently. The GIDFT n-OFDM system used self-cancellation minimum mean squared error (SC-MMSE) turbo equalizer to detect the transmitted signal which shows better error rate performance even at higher ICI occurrence.

Even though non-orthogonal multicarriers gives promising result in higher data transmission, high Peak to Average Power Ratio (PAPR) still an issue that need to be considered. An analysis conducted by S. S. K. C. Bulusu et al [14] had introduced a PAPR reduction technique which is Dispersive Selective Mapping (DSLM) to decrease the PAPR value in FBMC. It was found that, the method produced a very near to performance of OFDM with conventional SLM technique. The drawback of the DSLM is that the PAPR shows no reduction if it is calculated in the current symbol period duration as most of the symbol energy lies in the two successive symbol periods. Whereas in GFDM, Z. Sharifian et al [7] carried out an investigation Polynomial-based Companding Technique (PCT). For SEFDM, I. Isam and I. Darwazeh [15] has conducted several PAPR reduction techniques such as Clipping, Selective Mapping (SLM), Partial Transmit Transform (PTS) and a new technique, Sliding Window (SLW) in the SEFDM system. After numerical simulations, it showed that SLW had shown a great amount of PAPR reduction power without side effects.

Whereas, to date, in GIDFT n-OFDM system, PAPR analysis has not been formally study yet. Despite the important of PAPR reduction in multicarrier, there has been no research in implementing any PAPR techniques in this GIDFT n-OFDM. In this paper, the GIDFT n-OFDM system model and PAPR will be described in section 2. In section 3, the proposed PAPR reduction techniques will be discussed. For section 4 simulation result and discussion will be presented and the conclusion in section 5.

2. SYSTEM MODEL

2.1. Generalized Inverse Discrete Fourier Transform Non-Orthogonal Frequency Division Multiplexing (GIDFT n-OFDM)

This section explaining the promising technique of GIDFT n-OFDM system. GIDFT n-OFDM can transmit large data information by exploiting the orthogonality with huge overlapping among those subcarriers [13]. The sample is smaller or less than the subcarrier and matrix size of this technique transformed to rectangular size or ‘fat’ rather than square size such as in OFDM. Figure 1 shows the block diagram of basic GIDFT n-OFDM system.

![Figure 1. Block diagram of basic GIDFT n-OFDM transmitter](image)

The n-OFDM signal is expressed as:
Where $N_p$ is total number of subcarrier, $s_{n,m}$ is the $n^{th}$ subcarrier in the $m^{th}$ $n$-OFDM symbol, while $j = \sqrt{-1}$, whereas $\Delta f$ is the guard frequency of the subcarriers, and $T$ is the $n$-OFDM symbol duration. In order to have non-orthogonal signal, the $\eta$ must be less than one so that $\Delta f$ is narrower compared to OFDM. Therefore the bandwidth will be used efficiently as shown in (2) as $\eta$ will determines the bandwidth efficiency factor and given as (3) with $N_c$ is the tootal number of sampling.

$$\Delta f = \frac{1}{\eta T}$$
(2)

$$\eta = \frac{N_c}{N_p}$$
(3)

The discrete time GIDFT n-OFDM is expresses as

$$X[k] = \frac{1}{\sqrt{N_c}} \sum_{n=0}^{N_p-1} s_n \exp \left( j \frac{2 \pi kn}{N_p} \right)$$
(4)

where $X[k]$ is the $k^{th}$ time sample of the first symbol of $x(t)$ in (1) with $k = [0,1,2, ..., N_c-1]$ and $\frac{1}{\sqrt{N_c}}$ is the normalization factor. For more simplification, the received signal can be expressed as

$$y = G_{N_c} S + v,$$
(5)

where $y = [y_0, y_1, ..., y_{N_c-1}]^T$ is the received signal vector, $S = [s_0, s_1, ..., s_{N_p-1}]^T$ is the input signal vector, $v = [v_0, v_1, ..., v_{N_c-1}]^T$ is zero-mean complex White Gaussian noise with variance $\sigma_v^2$.

2.2. Peak to Averarege Power Ratio (PAPR)

Peak to Average Power Ratio (PAPR) issue must be taken care seriously as it will lead to huge fluctuation in multicarrier modulation system of OFDM[16]. PAPR occurred from the cumulative peaks that were produce when multiple sinusoids are added together to form the multicarrier[17]. When all the higher peak of the multicarrier meet each other at the same time, the high PAPR will occurred[18]. The PAPR of the transmitted OFDM signal can be defined as:

$$PAPR = \frac{\max |x[n]|^2}{E[|x[n]|^2]},$$
(6)

The PAPR in dB is defined as,

$$PAPR(x[n])_{dB} = 10 \log_{10}(T(x[n]))$$
(7)

3. PROPOSED PAPR REDUCTION TECHNIQUES IN GIDFT N-OFDM SYSTEM

In this section, three types of PAPR reduction techniques that were proposed will be described. The techniques used were previously used in OFDM were applied to GIDFT n-OFDM system. It is to investigate the efficiency of the techniques in this new system and to analyse the performance of the PAPR.

3.1. Clipping

Clipping is the simplest and easy technique that has been done to reduce PAPR [19]. The basic knowledge of clipping is where it will clip the amplitude level to a certain value that has been set. It is
determine as Clipping Level (CL) [18] or can be called cut-off value which decides the exceeding peaks to be clipped [17]. This clipping process can be defined as:

\[
C(x[n]) = \begin{cases} 
  x[n], & |x[n]| \leq A \\
  A e^{j \angle x[n]}, & |x[n]| > A 
\end{cases}
\]  

(8)

In this paper, clipping level is set as \( A \) and it is a positive real number and \( \angle x[n] \) is the angle of \( x[n] \). Then, \( C(x[n]) \) is GIDFT n-OFDM clipped signal. Despite of the simplest technique, clipping technique tend to produce high bit error rate (BER) degradation as it will clipped most of the important signal data [20].

### 3.2. Selective Mapping.

Selective Mapping is widely used in multicarrier system to reduce the PAPR [15], [19]. This technique reduced the PAPR without required more power and suffering from data rate loss. Initial idea of SLM is to construct alternative transmit sequences from the same data source and the sequence of the transmitted data vectors will be multiplied with the data vectors by some random phase \( p^u = [p_0^{(m)}, p_1^{(m)}, \ldots, p_{N-1}^{(m)}]^T \) where \( p_i^{(m)} = e^{j \theta_i} \) or \( (\pm 1, \pm j) \). When the multiplication is done, in order to transform the signal frequency domain to time domain, the multiplied data will go through Inverse Fast Fourier Transform (IFFT). The result will generate the data block of GIDFT n-OFDM system that has different time domain signals and this process will change the PAPR properties. After the IFFT process, the lowest PAPR among the independent data blocks and the candidate with the lowest PAPR will be compared and to be selected for transmission as \( x'(p) \). Figure 2 shows the block diagram of selective mapping technique.

![Block diagram of GIDFT n-OFDM for SLM technique](image)

**Figure 2. Block diagram of GIDFT n-OFDM for SLM technique**

### 3.3. Partial Transmit Sequence (PTS)

In Partial Transmit Sequence (PTS), the input data block of \( X \) is divided into a \( N \) of disjoint symbol subsequence or sub-blocks [18], [21]. This can be represented by the vector \( X^{(n)} = \{ X_0^{(n)}, X_1^{(n)}, \ldots, X_{N-1}^{(n)} \} \), therefore:

\[
X = \sum_{n=0}^{N-1} X^{(n)}
\]

(9)

Then, the subblocks \( X^{(n)} \) were transformed into \( N \) time-domain partial transmits sequences through the IFFT process. These partial sequences are independently rotated by phase factors \( b_n = e^{j \theta_n}, \theta_n \in \{ \frac{2\pi k}{N} | k=0,1,\ldots,N-1 \} \). Next, the multiplied result signal subsequences are summed as,

\[
x' = \sum_{n=0}^{N-1} b_n X^{(n)}
\]

(10)
Assuming that there are $W=4$ phase angles to be allowed. Before transmitting the signal, side information will be added. After that, the PAPR is calculated for each resulting sequence and then the lowest PAPR of the signal sequence was selected and transmitted as $x'(b)$. Figure 3 shows the block diagram of the PTS technique.

![Figure 3. Block diagram of GIDFT n-OFDM base PTS technique](image)

3. SIMULATION AND RESULTS ANALYSIS

This section presents the simulation results of GIDFT n-OFDM system with three different types of PAPR reduction using MATLAB simulation. Table 1 summarized the simulation parameters for PAPR analysis in GIDFT n-OFDM system.

| Parameters                  | Type/Value |
|-----------------------------|------------|
| Modulation                  | BPSK       |
| Number of samples ($N_x$)   | 32         |
| Number of blocks            | 8192       |
| Number of subcarriers ($N_b$) | $N_c/\alpha$ |
| Overlapping factor ($\alpha$) | 0.2, 0.4, 0.6, 0.8 |
| Types of PAPR               | Clipping, SLM, PTS |

In this simulation, we are using 32 number of samples, $N_x$ where we varied number of subcarriers, $N_b$ based on the overlapping factor, $\alpha$. This allowed more information to be transmitted in the system with the same number of samples $N_x$. Clipping, SLM and PTS of PAPR reduction technique is applied in this GIDFT-n-OFDM system with variety of $\alpha$ to investigate the relationship of PAPR reduction with the overlapping factor.

From Figure 4, it shows the result of GIDFT n-OFDM without PAPR and with implementation of three types of PAPR reduction techniques. It shows that at $10^{-3}$, Clipping has the worst performance compared to the original GIDFT n-OFDM performance as the overlapping factor increases. Even though it is a simple technique, it cannot be use in this system. Besides that, PTS also shows a promising performance which 6 dB PAPR reduction. SLM technique shows improvement result compared to others which is 8 dB PAPR reductions. After that, a simulation with many choices of overlapping factor $\alpha$ was done in GIDFT n-OFDM with SLM technique. In Figure 5, as expected, it showed the result of SLM decreased as the overlapping factors $\alpha$ increased. The overlapping factors correlated with significant PAPR reduction performances. It also shows that $\alpha=0.2$ has given the most reduction PAPR values with reduction of 9 dB. This happened because, at $\alpha=0.2$, the overlapping factor of the frequency is 80% where the number of subcarriers $N_b$ are 160 and subcarrier, $N_c$ is fixed at 32. Hence, the bigger the data input, the overlapping will increase and the frequency separation $\Delta f$ is decreased.
4. CONCLUSION

In this paper, the Peak to Average Power Ratio (PAPR) in GIDFT n-OFDM system was investigated and analysed. Simulations of GIDFT n-OFDM system without any PAPR reduction as well as with three PAPR reduction techniques were perform to improve the PAPR values. The results of the lowest PAPR values for all simulation were recorded and analysed using CCDF. Simulation results show, SLM has given the best PAPR value of compared to other PAPR reduction techniques that has been perform in this paper. Besides that, we can conclude that the level of PAPR reduction varies with the level of frequency overlapping indicated as $\alpha$. As the overlapping frequency increase, the PAPR reduction also decrease in great values. In addition, even though the overlapping percentage is increase, only PTS and SLM still manage to reduce the PAPR despite of large input data. Therefore, this shows that, GIDFT n-OFDM system efficiency can be improved by using SLM and PTS techniques. In the future work, the simulation and analysis will be extended at the receiver part in order to investigate the Bit Error Rate (BER) of the whole GIDFT n-OFDM system with reduction technique of PAPR.

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