What can we learn about the dynamics of transported spins by measuring shot noise in spin–orbit-coupled nanostructures?

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Abstract
We review recent studies of the shot noise of spin-polarized charge currents and pure spin currents in multiterminal semiconductor nanostructures, while focusing on the effects brought by the intrinsic Rashba spin–orbit (SO) coupling and/or extrinsic SO scattering off impurities in two-dimensional electron gas (2DEG) based devices. By generalizing the scattering theory of quantum shot noise to include the full spin-density matrix of electrons injected from a spin-filtering electrode, we show how decoherence and dephasing in the course of spin precession can lead to the substantial enhancement of the Fano factor (noise-to-current ratio) of spin-polarized charge currents. These processes are suppressed by decreasing the width of the diffusive Rashba wire, so that purely electrical measurement of the shot noise in a ferromagnet–SO-coupled-diffusive-wire–paramagnet setup can quantify the degree of quantum coherence of transported spin through a remarkable one-to-one correspondence between the purity of the spin state and the Fano factor. In four-terminal SO-coupled nanostructures, injection of unpolarized charge current through the longitudinal leads is responsible not only for the pure spin Hall current in the transverse leads, but also for nonequilibrium random time-dependent current fluctuations. The analysis of the shot noise of transverse pure spin Hall current and zero charge current, or transverse spin current and non-zero charge Hall current, driven by unpolarized or spin-polarized injected longitudinal charge current, respectively, reveals a unique experimental tool to differentiate between the intrinsic Rashba and extrinsic SO mechanisms underlying the spin Hall effect in 2DEG devices. When the intrinsic mechanisms responsible for spin precession start to dominate the spin Hall effect, they also enhance the shot noise of transverse spin and charge transport in multiterminal geometries. Finally, we discuss the shot noise of transverse spin and zero charge currents in the quantum-interference-driven spin Hall effect in ballistic four-terminal Aharonov–Casher rings realized using high-mobility 2DEG with the Rashba SO coupling. The modulation of the Rashba coupling by the gate electrode imprints the oscillatory signature of constructive and destructive spin interference around the ring on both the spin and charge shot noise, which differ from the corresponding oscillations of the spin Hall conductance, thereby revealing quantum-interference-driven temporal correlations between spin-resolved charge currents of opposite spins.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Over the past two decades, the exploration of the shot noise accompanying charge currents in mesoscopic conductors has become one of the major tools for gathering information about microscopic mechanisms of transport and temporal correlations between charge carriers which cannot be extracted from traditional measurements of time-averaged quantities [1–3]. Such nonequilibrium time-dependent fluctuations arise due to the discreetness of the electrical charge, persist down to...
zero temperature and require stochasticity induced by either quantum-mechanical [4] backscattering of charge carriers (as in mesoscopic and nanoscopic devices) or by the random injection process (as in the textbook example of a Schottky vacuum tube where the cathode emits electrons randomly and independently).

A theoretical description of the shot noise is typically formulated in terms of current fluctuations in a conductor with a non-fluctuating bias voltage applied between the contacts [1]. For zero bias voltage, \( V = 0 \), or in macroscopic systems where electrons thermalize in a short time to remain near equilibrium even under finite \( V \), one observes thermal noise. This equilibrium noise vanishes at \( T = 0 \) and, being directly related to the conductance through the fluctuation–dissipation theorem, does not give any new information [1, 3]. The situation changes when the time it takes for an electron to equilibrate becomes comparable to the time of flight through the conductor, which can be achieved by reducing the size of the system or by lowering the temperature. In this limit, nonequilibrium effects become essential, and the relevant energy scale for the noise is set by the bias voltage \( eV \) rather than the temperature \( k_B T \) [5]. At low frequencies, the nonequilibrium current noise is dominated by time-dependent conductance fluctuations (arising from the random motion of impurities), termed ‘1/f noise’ because of the characteristic frequency dependence of their spectral density which is a quadratic function of the mean (i.e., time-averaged) current \( I \). At higher frequencies, two principal signatures signifying the shot noise emerge—noise spectral density linearly depends on current while being frequency independent.

Macroscopic metallic conductors typically exhibit thermal noise, but no shot noise—in wires of length \( L \) longer than the temperature-dependent inelastic electron–phonon scattering length \( L_{\text{e-ph}} \), the shot noise power is expected to be reduced by a factor \((L_{\text{e-ph}}/L)^p (p > 0)\) [5–7] when electron–phonon interaction is able to efficiently drain extra energy from the electron subsystem to bring it closer to local thermal equilibrium [8]. While this leads to a priori assumption of vanishing nonequilibrium noise [1–3] in macroscopic metallic samples, which is typically confirmed experimentally [8], finite shot noise can be encountered in specific devices that are much longer than \( L_{\text{e-ph}} \) [6, 9]. In contrast, inelastic electron–electron scattering, which persists to much lower temperatures than electron–phonon scattering, does not suppress shot noise, but slightly enhances the noise power [1, 3, 5, 10]. The low sensitivity of the shot noise to electron–electron scattering is due to its inability to drain the external-electric-field-supplied energy from the electron subsystem, so that shot noise may be considered as a direct result of such deviation from equilibrium [5].

The zero-frequency shot noise spectral density, \( S = 2FeI \), of conventional unpolarized charge current in two-terminal non-interacting conductors reaches the maximum value \( F = 1 \) (the Poissonian limit) when transport is determined by uncorrelated stochastic processes. This is the situation encountered in modern tunnel junctions (or vacuum tubes explored in the early 1900s) where the mean occupation of a state is so small that the Pauli principle is inoperative. On the other hand, correlations among electrons reduce the noise \( F < 1 \), where the dominant source of correlations is the Pauli principle preventing double occupancy of an electronic state. While Coulomb repulsion is another source of correlations, in a metal it is strongly screened and ineffective. Nevertheless, electron–electron interactions in specific setups (e.g., involving various regimes under the Coulomb blockade condition [10], as reviewed in [2, 11]) can lead to the experimental observation of the super-Poissonian \( F > 1 \) shot noise [10].

For some of the basic types of two-terminal nanostructures, the Fano factor characterizing the transport of non-interacting quasiparticles assumes universal values [1] that are independent of the details of the system (such as impurity distribution, band structure and shape of the conductor): \( F = 1/2 \) for a symmetric double barrier, \( F = 1/3 \) for a diffusive wire, \( F = 1/2 \) for a dirty interface, \( F = 1/4 \) for a symmetric ballstic chaotic cavity and \( F = 0 \) for a ballstic conductor (e.g., quantum point contact in the plateau regime of its quantized conductance). These sub-Poissonian results have been confirmed experimentally [12, 13] and derived theoretically by various approaches [1, 14]—they are considered to be semiclassical in nature (in the sense that they can be reproduced via approaches based on the Boltzmann equation with Langevin random forces [5]). The notion of noise can be generalized to multiterminal conductors [15] where temporal correlations calculated between currents in different terminals are always negative due to electrons obeying the Fermi statistics [1].

Interestingly enough, the mature field of the shot noise of non-interacting particles has been revived very recently by the studies of ballistic transport through evanescent modes in two-terminal graphene nanoribbons, where the scattering theory predicts \( F = 1/3 \) [16]. This surprising result, which is incidentally [16] the same as the Fano factor for transport through diffusive semiconductor or metallic wires, has been confirmed experimentally in large aspect ratio (width \( \gg \) length) graphene samples [17]. However, it is not universally applicable to all graphene nanoribbons with different types of edges or contacts with metallic electrodes [17, 18]. It is also a genuinely quantum-mechanical feature since it requires classically forbidden evanescent wavefunctions that decay exponentially from metallic electrodes into the graphene sample.

An example of underlying physics revealed by the Fano factor of the shot noise, such as \( F = 1/3 \) for diffusive wires [19] or \( F = 1/2 \) for dirty interfaces [20, 21], is the interplay of randomness in quantum-mechanical impurity scattering and the Pauli blocking imposed by the Fermi statistics of transported quasiparticles. In both of these cases, the Fano factor confirms bimodal distributions [1, 3, 20, 21] of the transmission eigenvalues of the device. Similarly, in chaotic ballistic quantum dots stochasticity is introduced by electron scattering at its irregularly shaped boundaries. Nevertheless, the Fano factor reaches \( F = 1/4 \) [3, 4, 22] only in the fully developed quantum regime where the electron dwell time \( \tau_{\text{dwell}} \gg \tau_{\text{F}} \) is greater than the Ehrenfest time \( \tau_{\text{F}} \) which is roughly equal to the time it takes for the chaotic classical
dynamics to stretch an initially narrow wave packet, of the size of the Fermi wavelength, to some relevant classical length scale \( [4, 22] \). The shot noise of chaotic quantum dots is reduced below \( F = 1/4 \) in the classical-to-quantum crossover regime \( \tau_{\text{dwell}} < \tau_E \), where it depends sensitively on the degree of chaoticity thereby allowing one to extract its Lyapunov exponent \([22]\).

1.1. Recent trends in theoretical studies of spin-dependent shot noise

In contrast to the wealth of information acquired on the shot noise in spin degenerate transport (only briefly touched above, reviewed in \([3]\), and extensively covered in \([1]\) and its ‘update’ \([2]\)), it is only recently that the study of spin-dependent and spin-resolved ferromagnet-normal-metal systems has been initiated for two-terminal \([23–32]\) and multiterminal structures \([33–36]\). In these devices, ferromagnetic sources (for simplicity, often assumed to be half-metallic ferromagnets \([27–30]\)) inject spin-polarized charge current into a paramagnetic central region where spin-dependent interactions affect spin dynamics in the course of transport of electrons to which spins are attached.

For example, Mishchenko \([27]\) analyzed shot noise in diffusive spin valves for parallel and antiparallel magnetizations of their ferromagnetic electrodes, finding setups with significant increase of the Fano factor (when compared to conventional diffusive wires with \( F = 1/3 \)) caused by generic spin-flip scattering. Lamacraft \([28]\) calculated the effect of spin–orbit (SO) coupling, magnetic impurities and precession in an external magnetic field on the noise in the experimentally relevant cases of diffusive wires and lateral semiconductor dots, finding possible dramatic enhancements\(^1\) of the Fano factor. Through the specific values of the Fano factor enhancement, different types of spin-flip mechanisms leave distinctive signatures on the shot noise of injected spin-polarized charge currents, thereby making it possible to extract the spin-relaxation times associated with different microscopic mechanisms from electrical noise measurements in open mesoscopic systems \([28]\). In \([29]\), Nagaev and Glazman studied finite frequency shot noise in spin valves which originates from random spin flips due to SO-dependent scattering and magnetic impurities. Although the latter mechanism does not contribute to the average current, and its effect on the noise is smaller than that of SO scattering, it can be distinguished by a unique low-frequency noise dispersion that results from impurity–spin reorientations. By studying the shot noise of spin-polarized current injected into a diffusive ferromagnetic wire attached to two half-metallic ferromagnetic electrodes via tunnel contacts, Hatami and Zareyan \([30]\) demonstrated how the enhanced shot noise (most conspicuous when the electrodes are perfectly polarized in opposite directions) can probe the intrinsic density of states and the extrinsic impurity scattering contributions to the current polarization of the wire.

Similar studies for the multiterminal spin-valve-type devices by Zareyan and Belzig \([37]\) also encountered enhanced (when compared to the values in identical devices but with paramagnetic electrodes) shot noise and cross-correlations measured between currents in two different ferromagnetic terminals. The enhancement depends on the relative orientation of the magnetization of electrodes, the degree of spin polarization of the terminals and the strength of the spin-flip scattering in the normal central region. This makes it possible to determine the spin-flip scattering rate by changing the polarization of the ferromagnetic terminals \([37]\).

These results emphasize how the enhanced Fano factor of the shot noise of spin-polarized electrons is capable of unveiling additional mechanisms of current fluctuations, as well as temporal correlations between carriers of opposite spin, which are not visible in the conventional noise of unpolarized currents. This is due to the fact that any spin flip converts a spin– subsystem particle into a spin– subsystem particle, where the two subsystems differ when spin degeneracy is lifted. The non-conservation of the number of particles in each subsystem as the origin of an additional source of current fluctuations is analogous to a more familiar example of fluctuations of electromagnetic radiation in random optical media due to the non-conservation of the number of photons \([38]\). Microscopically, spin flips are either instantaneous events generated by the collision of electrons with magnetic impurities and SO-dependent scattering off static disorder \([29]\), or continuous spin precession \([28, 31, 36]\) during electron-free propagation in magnetic fields imposed externally or effectively generated by the intrinsic SO couplings \([39, 40]\) associated with an electronic band structure. Unlike the external magnetic field, the ‘internal’ magnetic field \( B_{\text{int}}(p) \) corresponding to the intrinsic SO couplings is momentum dependent, does not break the time-reversal invariance \([41]\), and it is capable of spin splitting the energy bands.

In \([31]\), we addressed two key problems for the shot noise in two-terminal nanostructures with intrinsic SO coupling illustrated by the device setup in figure 1: what is the connection between the Fano factor and the degree of quantum coherence \( |P_{\text{out}}| \) of transported spins? How does the shot noise depend on the Bloch polarization vector \( P_{\text{in}} \) of injected spins and its direction with respect to \( B_{\text{int}}(p) \)? As illustrated by figure 1, the spin-polarization vector of the detected current \( P_{\text{out}}(42) \) is rotated by coherent precession, and can be shrunk \( 0 \leq |P_{\text{out}}| < 1 \) by the D’yakonov–Perel’ (DP) spin dephasing \([40, 43–45]\) due to random changes in \( B_{\text{int}}(p) \) after the electron scatters off impurities or boundaries (note that these collisions themselves do not involve spin flip). We review in section 3 necessary extensions \([31]\) of the scattering approach to shot noise to handle the information about the spin coherence of injected states encoded by \( P_{\text{in}} \), and then discuss in section 4

\(^1\) Note that throughout the paper the ‘enhancement’ of noise-to-current ratio is measured with respect to the reference value determined by transport processes in the absence of spin-dependent interactions. For example, in the case of two-terminal diffusive wires, the reference value of the Fano factor is the standard \( F = 1/3 \) and the ‘enhancement’ of spin-dependent shot noise is considered to be any value, \( F > 1/3 \).
Figure 1. Generic two-terminal low-dimensional semiconductor nanostructure for the study of spin transport and spin decoherence. Fully spin-polarized current (comprised of pure states |Ps> = 1) is injected from the left lead 1 and detected in the paramagnetic right lead 2 which accepts both spin species. The central 2DEG region contains the Rashba SO coupling due to structural inversion asymmetry and can be in the ballistic or in the diffusive transport regime. The source and the drain electrodes are modeled as ideal (with no spin or charge interactions) multichannel semi-infinite leads. The detected current will have its spin-polarization vector rotated by coherent spin precession in the 2DEG region, where the effective magnetic field $B_{eff}(p)$ is along the y-axis, as well as shrunk |Ps> < 1 due to processes (such as scattering off static impurities or interfaces in the presence of SO coupling) which lead to the loss of spin coherence.

Section 2.2 introduces the topic of the shot noise of pure spin and zero charge currents generated by the mesoscopic spin Hall effect (SHE) [50, 52, 53] in multiterminal nanostructures with SO couplings. We discuss its computation for simply-connected 2DEGs attached to four electrodes in section 5 and physical insights brought by this recently initiated type of analysis [36]. In section 6, we apply the same type of analysis to multiply-connected (i.e., ring-shaped) devices attached to four electrodes where noise can probe spin-dependent quantum interference effects on transport.

Among the studies of the spin-dependent shot noise, a subject that has evolved into a vast subfield [11, 23, 24, 32, 35, 54, 55] on its own (and outside of the scope of this paper) deals with current fluctuations and temporal correlations in nanoscale conducting islands (such as quantum dots, carbon nanotubes and magnetic molecules) attached to ferromagnetic electrodes [11, 56]. In such devices, an interplay between Coulomb blockade and spin accumulation takes place with shot noise offering tools to probe various aspects of its phenomenology that cannot be extracted from mean current [11, 32]. Another set of topics which involves spin-dependent shot noise, and is better suited for a separate review [57] in the context of spin qubits for quantum computing, is the shot noise probing [3, 33, 58] of two-electron spin-entangled states.

1.2. Experimental studies of spin-dependent shot noise

Despite increasing theoretical activity on the spin-dependent shot noise in recent years, only few experiments have been performed, mostly focusing on the shot noise in magnetic tunnel junctions (MTJ). While high magnetoresistance of MTJs makes them well suited for fine magnetic field sensors, their low-frequency operation is limited by the presence of a relatively large 1/f noise [59]. Although the shot noise is not the most important among noise sources [59] as the key limiting factor [60] for MTJ applications, it is a sensitive tool to probe the properties of different types of insulating barriers [21] responsible for tunneling.

For example, the measurement of the Fano factor of spin-dependent shot noise in MTJs with a MgO insulating barrier can be employed to test the quality (i.e., the presence of imperfections) of epitaxially grown crystalline MTJs—obtaining Poissonian limit $F = 1$ signals pure spin-dependent direct tunneling and validates high structural quality of the MgO barrier [61]. In [62], Guerrero et al measured the sub-Poissonian Fano factor $F < 1$ in Al$_2$O$_3$ MTJs whose value was dependent on the alignment of the ferromagnetic electrodes for certain barrier conditions. This was attributed ($F \simeq 1$ for the Cr-doped and $F < 1$ for the Cr-free insulating barrier) to sequential tunneling via impurity levels inside the tunnel barrier. On the other hand, Garzon et al [63] measured super-Poissonian shot noise in small-area MTJs whose Fano factor $F > 1$ depends on the magnetization state of the ferromagnetic electrodes. Although intertwined spin and charge blockade facilitated by localized states within the barrier could account for these measurements, the search for super-Poissonian shot noise in MTJs and its theoretical explanation is still in its infancy.
2. Overview of recent analyses of the SO coupling effects on the shot noise

2.1. Shot noise in Rashba SO-coupled systems

The crucial role played by the SO interactions in all-electrical control of spin in semiconductor nanostructures [40, 64] has also provoked recent studies of their effects on the shot noise. For example, the Rashba SO coupling [39] induced by the structural inversion asymmetry of the semiconductor heterostructure hosting the 2DEG can be tuned by a gate electrode covering the device [65, 66]. This is envisaged as a key ingredient of the ‘second generation’ spintronic devices [64], such as semiconductor-based spin transistors that manipulate propagating coherent spin states [40].

In a pioneering work on the shot noise in Rashba SO-coupled systems, Egues et al [33, 58] unveiled how the Rashba SO coupling present in a localized region of one of the incoming leads of a clean four-terminal beam splitter can modulate the Fano factor of the shot noise of injected spin-polarized electrons. This suggests a direct way to measure the incoming leads of a clean four-terminal beam splitter can

2.2. Shot noise in mesoscopic spin Hall systems with the intrinsic Rashba SO coupling

The recently discovered spin Hall effect [74] in paramagnetic semiconductor [75–77] and metallic [78–81] systems holds great promise to revolutionize electrical generation, control and detection of nonequilibrium spin populations in the envisioned ‘second-generation’ spintronic devices [64]. The SHE actually denotes a collection [74] of phenomena manifesting as transverse (with respect to injected unpolarized charge current) separation of spin-up and spin-down states, which then comprise either a pure spin current or accumulate at the lateral sample boundaries. Its Onsager reciprocal phenomenon—the inverse SHE [82, 83] where longitudinal pure spin current generates transverse charge current or voltage between the lateral boundaries—offers one of the most efficient schemes to detect elusive pure spin currents [50] by converting them into electrical signal [78–81]. The basic phenomenology of both the direct and the inverse SHE, as manifested in multiterminal nanostructures, is illustrated in figure 2.

While SHE does not require an external magnetic field, it essentially relies on the SO coupling effects in solids. In addition, its magnitude can depend on the type of microscopic SO interaction, impurities, charge density, geometry and dimensionality. Such a variety of SHE manifestations poses immense challenge for attempts at a unified theoretical description of spin transport in the presence of relativistic effects, which has not been resolved by early hopes [50, 84, 85] that the auxiliary spin current operator $j_z$ (for $S_z$-spins
transported along the y-axis) and spin conductivity \( \sigma_{\text{SH}} = \langle \hat{j}_z \rangle / E \) (as the linear response to the longitudinal electric field \( E \)) of infinite homogeneous systems could be elevated to universally applicable and experimentally relevant quantities.

Thus, the key task emerging for theoretical analysis is to provide guidance for increasing and controlling the spin accumulation in confined geometries [53, 86–88] (observed SHE in semiconductors is presently rather small [75, 76]) or outflowing spin currents [52, 53] driven by them. In this respect, understanding of the intrinsic [84, 85, 89] (due to the SO-induced spin-split band structure) or extrinsic [82, 90] (due to SO-dependent scattering off impurities) origin of the SHE has been one of the central topics in interpreting experiments [89] and developing SHE-based spintronic devices [64]. For example, the intrinsic SO couplings are predicted [89] to yield much larger SHE response [77], which, moreover, can be controlled electrically by the gate electrodes covering low-dimensional semiconductor devices [52, 91]. The extrinsic ones are fixed and the corresponding much smaller SHE is hardly controllable (except through charge density and mobility [64]).

However, the measurements of standard quantities associated with transverse spin and charge transport are often unable to resolve the intrinsic versus extrinsic controversy [50, 89] or probe the crossover between these limiting regimes [90]. This long-standing issue is well known from the studies of the anomalous Hall effect (AHE) [92] in ferromagnetic materials (SHE can be viewed as the zero magnetization limit of AHE). For example, the frequent analysis of the AHE experimental data—fitting of the Hall resistivity versus longitudinal zero-field resistivity by a power law—is typically insufficient [93] to clearly differentiate a variety of mechanisms [90, 92] driven by SO coupling effects.

Here lessons from mesoscopic quantum physics might shed new light: as discussed in section 1, much more information about the transport of non-interacting or interacting quasiparticles is contained in time-dependent nonequilibrium current (or voltage) fluctuations [1] than in traditional time-averaged quantities such as conductances and conductivities. Furthermore, recent theoretical and experimental studies have suggested that shot noise in systems with spin-dependent interactions provides a sensitive probe to differentiate between magnetic impurities, spin-flip scattering and continuous spin precession effects, as overviewed in sections 1.1 and 1.2.

Although seminal arguments [85] for the intrinsic SHE in infinite homogeneous 2DEGs in the clean limit have predicted ‘universal’ SHE conductivity \( \sigma_{\text{SH}} = e/8\pi \), \( a \) posteriori analysis has found that for SO couplings linear in momentum (such as the Rashba one), any scattering that leads to a stationary electric current via deceleration of electrons by impurities or phonons will result in exact cancellation of the bulk spin Hall current in the dc case [74, 94, 95]. Such cancellation can be avoided by moving into the ac domain with frequencies exceeding the inverse spin relaxation time [95] or by making sufficiently small and clean structures to support ballistic transport (mean-free path greater than the system size) across the device. The latter case gives rise to the so-called mesoscopic SHE [50, 52] which, unlike ‘universal’ SHE [85] in an infinite 2DEG where the electric-field-driven acceleration of electron momenta and associated precession of spins [90] plays a crucial role, was introduced [52, 53, 86] in ballistic finite-size systems attached to multiple current and voltage probes with electric field being absent in the SO-coupled central region [52, 96–99].

In two-terminal SO-coupled ballistic wires, mesoscopic SHE is characterized by spin accumulation of opposite sign along opposite lateral edges [86–88]. In four-terminal and other multiterminal [96] nanostructures, its description [52] in terms of the total charge currents \( I_\alpha = I_{\alpha \uparrow} + I_{\alpha \downarrow} \) and conserved total spin currents \( I_{\alpha \uparrow} = I_{\alpha \uparrow}^S - I_{\alpha \downarrow}^S \) outflowing through spin and charge interaction-free electrodes (ensuring terminal spin currents that do not change at different cross sections of the leads [53]) is particularly suited for spin-dependent shot noise analysis. The SHE in four-terminal systems is quantified by the spin conductance (for labeling of the total currents and voltages in the terminals, see figure 2):

\[
G_{\text{SH}}^\alpha = I_{\alpha \uparrow}^S / (V_1 - V_2).
\]

Unlike in three-dimensional semiconductor [75] and metallic devices [78–81], which are always disordered and where extrinsic contribution to the SHE is therefore present or dominant, ballistic conditions for the mesoscopic SHE can be achieved in low-dimensional semiconductor systems. For example, the very recent experiment on nanoscale H-shaped structures built on high-mobility HgTe/HgCdTe quantum wells has reported for the first time the detection of the mesoscopic SHE via non-local and purely electrical measurements [77].

The spin Hall conductance\(^2\) in clean four-terminal 2DEG devices is shown in figure 3(a), and in disordered ones in figure 3(b). In the general cases [76, 90], where both the extrinsic and intrinsic SO-interaction effects are present, the intrinsically driven contribution to spin Hall current in finite-size devices starts to dominate [100] when the ratio of characteristic energy scales [50] for the disorder and SO coupling effects satisfies \( \Delta_{\text{SO}}/\tau \hbar \gtrsim 10^{-1} \) (\( \Delta_{\text{SO}} \) is the SO-induced spin splitting of quasiparticle energies [39, 40], and \( \hbar/\tau \) is the disorder-induced broadening of energy levels due to the transport scattering time \( \tau \)).

However, spin current is not a directly measurable quantity and has to be converted into other quantities (such as spin accumulation, voltage or charge current) to be measured by conventional techniques. Following [36], section 5 discusses how the information stored in the shot noise of transverse spin Hall current, as well as the noise of associated transverse charge transport, can provide a new tool to separate different types of SO interactions driving the SHE. We draw inspiration for this approach from the following recent intriguing theoretical findings: (i) the intrinsic aspects of AHE have been related to (transverse) voltage shot noise by Timm et al [101]; (ii) Hatami and Zareyan [30] argued that shot

\(^2\) Note that the spin conductance has a natural unit \( e^2/4\pi = (\hbar/2e)(e^2/\hbar) \) taking into account that spin current carries angular momenta \( h/2 \) instead of charge \( e \). Nevertheless, to simplify the noise analysis we use the same units for both the spin and the charge current, so that the unit of \( G_{\text{SH}}^\alpha = e^2/\hbar \).
noise of spin-polarized current injected into a ferromagnetic diffusive wire can probe aspects of its AHE; (iii) Erlingsson and Loss [102] pointed out that the measurement of charge currents and their auto- and cross-correlation shot noise on a multiterminal bridge could be used to obtain the spin Hall conductance solely in terms of these purely electrical quantities (independently of the underlying microscopic SO mechanism); (iv) as discussed in [31] and reviewed in section 4, the shot noise of spin-polarized charge current offers a sensitive electrical probe of spin precession and spin dephasing in two-terminal nanostructures, where spin precession represents a crucial aspect [90] of SHEs driven by intrinsic SO couplings.

2.3. Shot noise of the quantum-interference-driven spin Hall effect in four-terminal Aharonov–Casher rings with the intrinsic Rashba SO coupling

The superpositions of quantum states and thereby induced quantum interference effects are one of the most fundamental aspects of quantum mechanics. The interference experiments are difficult to perform with electrons in solids which are typically coupled to a large decohering environment [103, 104]. Nevertheless, the advent of mesoscopic structures, which are smaller that the phase coherence length \( L_\phi \approx 157a \) (along which spin precesses by an angle \( \pi \)). The disorder strength sets the transport properties (independently of the underlying microscopic SO mechanism); (ii) as discussed in [31] and reviewed in section 4, the shot noise of spin-polarized charge current offers a sensitive electrical probe of spin precession and spin dephasing in two-terminal nanostructures, where spin precession represents a crucial aspect [90] of SHEs driven by intrinsic SO couplings.

The superpositions of quantum states are sensitive to the acquired AB topological phases [112] in the external magnetic field. The pursuit of fundamental spin interference effects, as well as spin transistors with unpolarized (unlike Datta–Das spin-FET [40]) charge currents [113], has also generated considerable interest to demonstrate the Aharonov–Casher (AC) effect [112] via transport experiments in SO-coupled semiconductor nanostructures. The electromagnetic duality (i.e., charge and...
spin, as well as electric and magnetic fields, interchanged) entails the AC effect, originally discussed in terms of a neutral magnetic dipole moment going around a charged line to acquire the AC phase [112, 114]. Very recent vigorous experimental activity [114, 115] has been focused on detecting the AC phase (which in one-dimensional rings is the sum of the SO Berry phase and spin dynamical phase [116, 117]) difference for opposite spin states traveling clockwise and counterclockwise around the two-terminal ring with the Rashba SO coupling. This results in oscillatory behavior of the ring conductance, more complicated than in the case of AB rings, as a function of the SO interaction strength [105, 116–118].

Furthermore, the quantum-interference-driven SHE (QIDSHE) was predicted in [91] to occur in a four-terminal ballistic mesoscopic ring with homogeneous Rashba SO coupling within the ring-shaped central region. The recent studies [119, 120] have extended the possibility of such unusual SHE (which cannot be captured by semiclassical analysis to which standard SHE can be reduced [92]) to multiterminal rings with different types of SO couplings. This AC-ring-type nanostructure generates pure spin current in the transverse electrodes as a response to unpolarized charge current injected through the longitudinal leads (labeled as 1 and 2 in the inset of figure 4). The transverse spin Hall current can be modulated between zero and a large value (when compared to small extrinsically generated spin currents in dirty 2DEGs [76]) by changing the voltage on the gate electrode covering the ring [65, 66, 115], as demonstrated by figure 4. This gives an unambiguous experimental signature that is particularly visible in single-channel devices [91, 105]. In section 6, we investigate if the shot noise of transverse spin Hall and zero charge current can provide additional insights into interference and dephasing effects in general multichannel AC ring devices, or if it can offer an alternative measuring scheme to confirm QIDSHE electrically.

![Figure 4. The spin Hall conductance \( G_{SHE} = \frac{I_S}{V_1 - V_2} \) of a one-dimensional Aharonov–Casher ring attached to four single-channel leads as the function of the dimensionless Rashba SO coupling \( Q_R \) that can be tuned by the gate electrode covering the ring. (Adapted from [91].) ](image-url)

3. The scattering approach to spin-resolved shot noise: inclusion of the spin-density matrix of injected electrons

At low temperatures, where small enough conductors become phase coherent and the Pauli blocking renders regular injection and collection of charge carriers from the bulk electrodes, the scattering theory of quantum transport provides the celebrated formula [1, 3, 14] for the shot noise power in terms of the transmission eigenvalues \( T_n \):

\[
S = \frac{4e^3V}{\hbar} \sum_{n=1}^{M} T_n(1 - T_n). \tag{2}
\]

Here \( V \) is the linear response time-independent bias voltage. The physical interpretation of equation (2) is quite transparent—in the basis of eigenchannels, which diagonalize \( t^t \), where \( t \) is the transmission matrix of a two-terminal device, a mesoscopic structure can be viewed as a parallel circuit of \( M \) (= number of transverse propagating orbital wavefunctions in the leads) independent one-dimensional conductors, each characterized by the transmission probability \( T_n \). To get the shot noise through disordered systems, equation (2) has to be averaged [1] over a proper distribution \([21]\) of \( T_n \). However, this standard route becomes inapplicable for spin-polarized injection where one has to take into account the spin-density matrix of injected electrons [42] and, therefore, perform the calculations in the natural basis [1] composed of spin-polarized conducting channels of the electrodes.

We recall [41] that the density matrix \( \hat{\rho} = \hat{\rho}^\dagger \); \( \text{Tr} \hat{\rho} = 1 \) of spin-\( \frac{1}{2} \) particles is a 2 × 2 matrix (\( 1 \) denotes the 2 × 2 unit matrix):

\[
\hat{\rho} = \begin{pmatrix} \rho_{11}^\dagger & \rho_{12}^\dagger \\ \rho_{21} & \rho_{22} \end{pmatrix} = \frac{1}{2} (1 + \mathbf{P} \cdot \hat{\sigma}),
\]

which is determined solely by the three real numbers comprising the spin-polarization vector \( \mathbf{P} = \text{Tr}[\hat{\rho} \hat{\sigma}] \). The magnitude \( |\mathbf{P}| \) quantifies the degree of quantum coherence or purity of a quantum state. Thus, \( \hat{\rho} \) and the corresponding \( \mathbf{P} \) provide the most general quantum-mechanical description of a spin-\( \frac{1}{2} \) system, accounting for both pure (i.e., fully coherent) \( \hat{\rho}^2 = \hat{\rho} \Leftrightarrow |\mathbf{P}| = 1 \) and mixed \( \hat{\rho}^2 \neq \hat{\rho} \Leftrightarrow 0 \leq |\mathbf{P}| < 1 \) states [41].

Below we discuss a generalization of the scattering-matrix-based formulae for the shot noise which allow one to compute spin-resolved noise as the building block of charge and spin current noise computation, while including both the ‘direction’ of injected spins and the degree of their quantum coherence. That is, all of our analytical formulae contain the spin-polarization vector \( \mathbf{P}_{\text{in}} \) which determines the density matrix \( \hat{\rho}_{\text{in}} = (1 + \mathbf{P}_{\text{in}} \cdot \hat{\sigma})/2 \) of injected spins. Moreover, in section 4.1 we argue that the value of the Fano factor in the right electrode is directly connected to the degree of quantum coherence \( |\mathbf{P}_{\text{out}}| \) of outgoing spins, as extracted from the recently developed [42] scattering approach to their spin-density matrix, \( \hat{\rho}_{\text{out}} = (1 + \mathbf{P}_{\text{out}} \cdot \hat{\sigma})/2 \).

The analysis of the spin-dependent shot noise requires us to evaluate temporal correlations between spin-resolved
charge currents $I^\uparrow_\alpha$ and $I^\downarrow_\alpha$ due to the flow of spin-up and spin-down electrons through the terminals of a nanostructure [47]:

$$S_{\alpha\beta}^\sigma\sigma'(t - t') = \frac{1}{2}(\delta I^\sigma_\alpha(t)\delta I^\sigma'_{\beta}(t') + \delta I^{\sigma'}_{\beta}(t')\delta I^\sigma_\alpha(t)).$$

(4)

Here $\hat{I}^\sigma_\alpha(t)$ is the quantum-mechanical operator of the spin-resolved charge current carrying spin-$\sigma$ ($\sigma = \uparrow, \downarrow$) electrons in lead $\alpha$. The current-fluctuation operator at time $t$ in lead $\alpha$ is

$$\hat{I}^\sigma_\alpha(t) = \hat{I}^\sigma_\alpha(t) - \langle \hat{I}^\sigma_\alpha(t) \rangle.$$

(5)

We use $(\cdots)$ to denote both quantum-mechanical and statistical averaging over the states in the macroscopic reservoirs to which a mesoscopic conductor is attached via semi-infinite interaction-free leads [10]. The spin-resolved noise power between terminals $\alpha$ and $\beta$ is (conventionally defined [10] as twice) the Fourier transform of equation (4),

$$S_{\alpha\beta}^\sigma\sigma'(\omega) = 2 \int d(t - t') \exp[-i\omega(t - t')] S_{\alpha\beta}^\sigma\sigma'(t - t').$$

(6)

The total noise power of charge current,

$$I_\alpha = I^\uparrow_\alpha + I^\downarrow_\alpha,$$

(7)

is given by

$$S_{\alpha\beta}^{\text{charge}}(\omega) = S_{\alpha\beta}^{\uparrow\uparrow}(\omega) + S_{\alpha\beta}^{\uparrow\downarrow}(\omega) + S_{\alpha\beta}^{\downarrow\uparrow}(\omega) + S_{\alpha\beta}^{\downarrow\downarrow}(\omega),$$

(8)

while the total noise power of spin current,

$$I_\alpha = I^\uparrow_\alpha - I^\downarrow_\alpha,$$

(9)

is obtained from the spin-resolved noise powers as

$$S_{\alpha\beta}^{\text{spin}}(\omega) = S_{\alpha\beta}^{\uparrow\uparrow}(\omega) + S_{\alpha\beta}^{\downarrow\downarrow}(\omega) - S_{\alpha\beta}^{\uparrow\downarrow}(\omega) - S_{\alpha\beta}^{\downarrow\uparrow}(\omega).$$

(10)

Selecting the same electrode $\alpha = \beta$ yields the auto-correlation noise, while for different electrodes $\alpha \neq \beta$ we get the cross-correlation noise.

In the scattering theory of quantum transport, the operator of spin-resolved charge current carrying spin-$\sigma$ electrons through terminal $\alpha$ is expressed as

$$\hat{I}^\sigma_\alpha(t) = \frac{e}{\hbar} \sum_{n=1}^M \int dE dE' d^{(E - E')} |M| \hat{a}_{\alpha \uparrow}(E) |\hat{a}_{\alpha \downarrow}(E')^\dagger - \hat{b}_{\alpha \uparrow}(E) |\hat{b}_{\alpha \downarrow}(E')^\dagger |.$$ (11)

The operators $\hat{a}_{\alpha \sigma}(E)$ and $\hat{a}_{\alpha \sigma}^\dagger(E)$ create (annihilate) incoming electrons in lead $\alpha$ which have energy $E$, spin-$\sigma$, and the orbital part of their wavefunction (i.e., ‘conducting channel’) is the transverse propagating mode $|n\rangle$ [11]. The corresponding operators $\hat{b}_{\alpha \sigma}(E)$ and $\hat{b}_{\alpha \sigma}^\dagger(E)$ act on the outgoing states. Inserting $\hat{I}^\sigma_\alpha(t)$ in equation (4), and taking its Fourier transform, leads to the following formula for the spin-resolved noise power spectrum:

$$S_{\alpha\beta}^\sigma\sigma'(\omega) = \frac{e^2}{\hbar} \sum_{E, E'} \operatorname{Tr} \left[ A_{\alpha \beta}^{\sigma\sigma'}(\alpha, \sigma, E, E + \hbar\omega) \times A_{\alpha \beta}^{\sigma\sigma'}(\beta, \sigma', E + \hbar\omega, E) \right] |f^\sigma(E)| |1 - f^\sigma(E + \hbar\omega)|$$

$$+ f^\sigma(E + \hbar\omega) \left[ 1 - f^\sigma(E) \right].$$

(12)

Here $f^\sigma(E)$ is the Fermi function of spin-$\rho$ electrons ($\rho = \uparrow, \downarrow$), kept at temperature $T$ and having spin-dependent chemical potential $\mu^\sigma_\rho$ in lead $\gamma$. The Büttiker’s current matrix [1] $A_{\alpha \beta}^{\sigma\sigma'}(\alpha, \sigma, E, E')$, whose elements are

$$[A_{\alpha \beta}^{\sigma\sigma'}(\alpha, \sigma, E, E')]_{\alpha\beta} = \sum_{n\alpha} \delta_{\alpha n} \delta_{\beta n} \delta_{\rho\rho'} \delta^\sigma\sigma'.$$

(13)

is now generalized to include explicitly spin degrees of freedom through the spin-resolved scattering matrix connecting operators $\hat{a}_{\alpha \sigma}(E)$ and $\hat{b}_{\alpha \sigma}(E)$ via

$$\hat{b}_{\alpha\sigma}(E) = \sum_{\beta \mu} [a_{\alpha \sigma}^\mu(E)]_{\alpha \beta} [a_{\beta \sigma}^\mu(E')]_{\beta \mu}.$$ (14)

In the zero-temperature limit the thermal (Johnson-Nyquist) contribution to the noise vanishes and the Fermi function becomes a step function $f^\sigma(E) = 0(E - \mu^\sigma_\rho)$. 3.1. Two-terminal spin-resolved shot noise

The evaluation of equation (12) for zero temperature and zero frequency, $S_{\alpha\beta}^{\sigma\sigma'}(\omega) \equiv S_{\alpha\beta}^{\sigma\sigma'}(\omega = 0, T = 0)$, in the right lead $\alpha = 2 = \beta$ of a two-terminal mesoscopic device yields the scattering theory formulae for the shot noise arising in the course of propagation of spin-polarized current through a central region with arbitary spin-dependent interactions:

$$S_{zz}^{\uparrow\uparrow} = \frac{4e^2}{\hbar} \left[ \left| \operatorname{Tr} \left( \hat{t}_{z1}^\uparrow \hat{t}_{z2}^\uparrow \right) \right| eV + \left| \operatorname{Tr} \left( \hat{t}_{z1}^\downarrow \hat{t}_{z2}^\downarrow \right) \right| eV - \left| \operatorname{Tr} \left( \hat{t}_{z1}^\uparrow \hat{t}_{z2}^\downarrow \right) \right| eV - \left| \operatorname{Tr} \left( \hat{t}_{z1}^\downarrow \hat{t}_{z2}^\uparrow \right) \right| eV \right]. \tag{15a}$$

$$S_{zz}^{\uparrow\downarrow} = \frac{4e^2}{\hbar} \left[ \left| \operatorname{Tr} \left( \hat{t}_{z1}^\uparrow \hat{t}_{z2}^\downarrow \right) \right| eV + \left| \operatorname{Tr} \left( \hat{t}_{z1}^\downarrow \hat{t}_{z2}^\uparrow \right) \right| eV - \left| \operatorname{Tr} \left( \hat{t}_{z1}^\uparrow \hat{t}_{z2}^\uparrow \right) \right| eV - \left| \operatorname{Tr} \left( \hat{t}_{z1}^\downarrow \hat{t}_{z2}^\downarrow \right) \right| eV \right], \tag{15b}$$

$$S_{zz}^{\downarrow\uparrow} = \frac{4e^2}{\hbar} \left[ \left| \operatorname{Tr} \left( \hat{t}_{z1}^\downarrow \hat{t}_{z2}^\uparrow \right) \right| eV + \left| \operatorname{Tr} \left( \hat{t}_{z1}^\uparrow \hat{t}_{z2}^\downarrow \right) \right| eV + \left| \operatorname{Tr} \left( \hat{t}_{z1}^\uparrow \hat{t}_{z2}^\uparrow \right) \right| eV - \left| \operatorname{Tr} \left( \hat{t}_{z1}^\uparrow \hat{t}_{z2}^\downarrow \right) \right| eV \right], \tag{15c}$$

$$S_{zz}^{\downarrow\downarrow} = \frac{4e^2}{\hbar} \left[ \left| \operatorname{Tr} \left( \hat{t}_{z1}^\downarrow \hat{t}_{z2}^\downarrow \right) \right| eV + \left| \operatorname{Tr} \left( \hat{t}_{z1}^\uparrow \hat{t}_{z2}^\uparrow \right) \right| eV + \left| \operatorname{Tr} \left( \hat{t}_{z1}^\uparrow \hat{t}_{z2}^\downarrow \right) \right| eV - \left| \operatorname{Tr} \left( \hat{t}_{z1}^\downarrow \hat{t}_{z2}^\uparrow \right) \right| eV \right]. \tag{15d}$$

Here the elements of the transmission matrix $\hat{t}_{z\alpha}^{\sigma\sigma'}$, which is a block of the full scattering matrix [11], determine the probability $| \hat{t}_{z\alpha}^{\sigma\sigma'} |^2$ for the spin-$\sigma'$ electron incident in lead 1 in the orbital conducting channel $|m\rangle$ to be transmitted to lead 2 as a spin-$\sigma$ electron in channel $|n\rangle$. The direction of
the spin-polarization vector of injected electrons selects the spin-quantization axis for \(\uparrow, \downarrow\). Its magnitude quantifies the degree of spin polarization, which is introduced in equation (12) via the spin-dependent electrochemical potentials in the injecting (left) lead:

\[
\mu_\uparrow = E_F + eV,
\]

\[
\mu_\downarrow = E_F + eV \frac{1 - |P_m|}{1 + |P_m|},
\]

In the collecting (right) lead the electrochemical potentials for both spin species are the same \(\mu_\uparrow = \mu_\downarrow = E_F\), where \(E_F\) is the Fermi energy. For instance, injection of fully spin-\(\uparrow\) polarized current \(|P_m| = 1\) from the left lead (e.g., made of half-metallic ferromagnet) means that there is no voltage drop for spin-\(\downarrow\) electrons \(\mu_\downarrow = \mu_\downarrow = E_F\), so that they do not contribute to transport.

Equations (15a)–(15d), together with the expressions for mean spin-resolved currents collected in the right paramagnetic lead. The spin-resolved two-terminal conductances

\[
I_\uparrow^\uparrow = \langle \hat{I}_\uparrow^\uparrow(t) \rangle = \left( G_{\uparrow\uparrow}^{\uparrow\uparrow} \frac{1 - |P_m|}{1 + |P_m|} \right) V,
\]

\[
I_\uparrow^\downarrow = \langle \hat{I}_\uparrow^\downarrow(t) \rangle = \left( G_{\uparrow\downarrow}^{\uparrow\downarrow} \frac{1 - |P_m|}{1 + |P_m|} \right) V,
\]

define the Fano factors for parallel and antiparallel spin-valve setups:

\[
F_{\uparrow\rightarrow\uparrow} = \frac{S_{\uparrow\uparrow}^{\uparrow\uparrow}(|P_m| = 1)}{2eI_\uparrow^\uparrow(|P_m| = 1)},
\]

\[
F_{\uparrow\rightarrow\downarrow} = \frac{S_{\uparrow\downarrow}^{\uparrow\downarrow}(|P_m| = 1)}{2eI_\uparrow^\downarrow(|P_m| = 1)},
\]

These equations also yield the Fano factor for a ferromagnetSO-coupled-wireparamagnet configuration

\[
F_{\uparrow\rightarrow\uparrow\downarrow} = \frac{S_{\uparrow\downarrow}^{\uparrow\downarrow}(|P_m| = 1)}{2eI_\uparrow^\downarrow(|P_m| = 1)},
\]

where \(I_\uparrow = I_\uparrow^\uparrow + I_\uparrow^\downarrow\) is the sum of both spin-resolved currents collected in the right paramagnetic lead. The spin-resolved two-terminal conductances

\[
G_{\sigma\sigma}^{\uparrow\downarrow} = \frac{\hbar e^2}{\pi} \sum_{\alpha,\beta=1}^M |\langle \sigma||\mathbf{e}_{\alpha\beta}||\sigma\rangle|_m|^2
\]

are given by the usual Landauer formula.

### 3.2. Four-terminal spin-resolved shot noise

The evaluation of \(S_{\alpha\beta\sigma\sigma}^{\sigma\sigma} \equiv S_{\alpha\beta\sigma\sigma}^{\sigma\sigma}(\omega = 0, T = 0)\) at zero temperature and zero frequency in the top lead \(\alpha = \beta = 3\) of the four-terminal bridge in figure 2, typically employed in the analysis of the mesoscopic SHE [52, 53, 97, 100], yields explicit expressions for \(S_{33}^{\uparrow\uparrow}\), \(S_{33}^{\downarrow\downarrow}\) and \(S_{33}^{\uparrow\downarrow}\) noise power. They are too lengthy to be written down explicitly here due to numerous terms arising from the effect of other leads on the shot noise in selected lead 3. We note that using the unitarity of the scattering matrix, \(S_{33}^{\uparrow\uparrow}\) can be expressed solely in terms of the transmission matrix \(T_{\alpha\beta}^{\sigma\sigma}\). The spin-quantization axis for \(\uparrow\) and \(\downarrow\) spin states is assumed to be the \(z\)-axis, so that all spin currents and noises in lead 3 describe the SHE response of 2DEG [53].

Since in two-terminal devices the spin dynamics affecting the shot noise is most pronounced when injected current is spin polarized [27, 30, 31], we also evaluate in section 5 the noise correlators for setups where spin-polarized charge current is injected through lead 1 thereby driving the transverse charge Hall current [121] through leads 3 and 4. In this case, the magnitude \(|P_m|\) of the spin-polarization vector enters into equation (12) via the spin-dependent electrochemical potentials in the injecting lead 1, \(\mu_\uparrow = E_F + eV\) and \(\mu_\downarrow = E_F + eV (1 - |P_m|)/(|1 + |P_m|)\), in complete analogy with the two-terminal spin-resolved noise formulae in section 3.1. Such a setup [121] is closely related to the inverse SHE, where \(\mu_\uparrow = E_F + eV = \mu_\downarrow\) and \(\mu_\downarrow = E_F = \mu_\downarrow\) describe the injection of two counter-propagating fully spin-polarized charge currents of opposite \(P\) and, therefore, no net longitudinal charge current [83].

### 3.3. Effective SO Hamiltonian and nonequilibrium Green functions for computing the spin-resolved noise

The consequences of equation (12) can be explored by analytical means, such as the waveformfunction matching [58] (for one or two channel leads attached to ballistic structures [58]) or random matrix theory applicable to ‘black-box’ disordered and chaotic ballistic structures [28, 99] which are smaller than the spin precession length \(L_{SO}\). However, to take into account concurrent microscopic modeling [90] of the impurity scattering, SO effects (skew scattering and side jump [90, 92]) in the electric field of an impurity, and fast spin precession induced by strong intrinsic SO coupling effects [45], it is more advantageous to employ the nonequilibrium Green function (NEGF) [122] technique via numerically exact real \(\otimes\) spin space [52, 53] computation. The NEGF formalism can take as an input the microscopic Hamiltonian of both weakly \((L \ll L_{SO})\) and strongly \((L \geq L_{SO})\) SO-coupled nanostructures of arbitrary shape and disorder attached to many multichannel electrodes.

In general, the central 2DEG region (such as those employed in recent SHE experiments [76]) can be modeled by the effective mass Hamiltonian which takes into account intrinsic and extrinsic SO coupling effects, as well as the impurity \(V_{\text{imp}}(x, y)\) and confining \(V_{\text{conf}}(y)\) potentials:

\[
\hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m^*} + V_{\text{imp}}(x, y) + V_{\text{conf}}(y)
\]

\[
+ \frac{\lambda}{\hbar} \left( \hat{p}_x \hat{\sigma}_z - \hat{p}_z \hat{\sigma}_x \right) \times (\hat{\sigma} \times \hat{p}) \cdot \nabla V_{\text{imp}}(x, y).
\]

Here the fourth term is the intrinsic Rashba SO coupling [39] due to the structural inversion asymmetry of the quantum well, \(\langle \hat{\sigma}_z \hat{\sigma}_x \hat{\sigma}_y \rangle\) denotes the vector of the Pauli matrices, and \(\hat{\rho} = (\hat{p}_x, \hat{p}_y)\) is the momentum operator in 2D space. The Rashba coupling is responsible for the spin splitting, \(\Delta_{SO} = 2\alpha k_F\), of quasiparticle energies at the Fermi level \((h k_F\) is the Fermi momentum). The fifth term is a relativistic correction to the Pauli equation for a spin-\(\frac{1}{2}\) particle where the minuscule value of \(\lambda\) in vacuum can be renormalized
enormously by the band structure effects due to strong crystal potential (leading to, e.g., $\lambda / \hbar = 5.3 \AA^2$ for GaAs [39]).

The effective momentum-dependent magnetic field $B_{\text{int}}(p)$ of the Rashba SO coupling lies in the plane of a 2DEG, thereby forcing the injected $z$-polarized spins to precess. This process is characterized by the spin precession length $L_{SO}$, along which injected out-of-plane polarized spins precess by an angle $\pi$. The $L_{SO}$ scale plays a crucial role in the mesoscopic SHE [52]. It also plays the role of DP spin dephasing length in weakly disordered bulk SO-coupled systems [40, 43–45, 123]. This scale is inversely proportional to the Rashba coupling strength,

$$L_{SO} = \frac{\pi \hbar^2}{2m^* |\alpha_R|},$$

and can be extracted from the measurements of spin dephasing in both ballistic and diffusive systems [45].

For NEGF computation, we represent the general 2DEG Hamiltonian (23) in the local orbital basis [100]:

$$\hat{H}_{TB} = \sum_{m,\alpha} \epsilon_m^e \hat{c}_{m\alpha}^\dagger \hat{c}_{m\alpha} + \sum_{\langle mm' \rangle} \sum_{\alpha \sigma} \tilde{\epsilon}_{m\alpha} \hat{c}_{m\alpha}^\dagger \hat{c}_{m'\alpha\sigma} \hat{c}_{m'\sigma} - i\hat{\sigma}_{SO} \sum_{m,\alpha\beta} \sum_{ij} \sum_{\gamma} \epsilon_{ij;\gamma} \langle \hat{c}_{m+\gamma} \hat{c}_{m-\gamma} \rangle - \epsilon_{m+\gamma} \epsilon_{m-\gamma} \hat{c}_{m\alpha}^\dagger \hat{c}_{m\alpha} + \hat{c}_{m\alpha}^\dagger \hat{c}_{m\alpha}$$

$$\times \mathbf{e}_{\alpha\beta\gamma} \cdot \mathbf{e}_{\alpha\gamma\beta} \mathbf{e}_{\alpha\beta\gamma}.$$  \hspace{1cm} (25)

The first term accounts for isotropic short-range spin-independent static impurity potential, where $\epsilon_m \in [-W_{\text{ins}}/2, W_{\text{ins}}/2]$ is a uniform random variable. The second term is the tight-binding representation of the Rashba SO coupling whose nearest-neighbor ($\langle mm' \rangle$) hopping is a non-trivial $2 \times 2$ Hermitian matrix $t_{m\alpha m'\sigma}$ in the spin space [53]:

$$t_{m\alpha m'\sigma} = -t_0 I_3 - i t_{SO} \hat{\sigma}_y (\mathbf{m} = \mathbf{m'} + \mathbf{e}_x)$$

$$-i t_0 I_3 + i t_{SO} \hat{\sigma}_y (\mathbf{m} = \mathbf{m'} + \mathbf{e}_x).$$

(26)

Here $I_3$ is the unit $2 \times 2$ matrix in the spin space, and $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors along the $x$- and $y$-axes, respectively. The strength of the SO coupling is measured by the parameter $t_{SO} = \alpha_R / 2a$ ($a$ is the lattice spacing), and the spin splitting of the band structure is expressed as $\Delta_{SO} = 4at_{SO}\alpha_R$ in terms of $t_{SO}$. A direct correspondence between the continuous effective mass Hamiltonian (23) and its lattice version (25) is established by selecting the Fermi energy of the injected effective mass electrons to be close to the bottom of the band where tight-binding dispersion reduces to the parabolic one, and by using $t_0 = \hbar^2 / (2m^* a^2)$ for the orbital hopping which yields the effective mass $m^*$ in the continuum limit [53]. The labels in the third term, which involves both nearest-neighbor and next-nearest-neighbor hopping, are the dimensionless extrinsic SO scattering strength $\lambda_{SO} = \lambda / (4a^2)$, $\epsilon_{ij;\gamma}$ stands for the Levi-Civita totally antisymmetric tensor with $i, j$ denoting the in-plane coordinate axes, and $\nu, \gamma$ are the dummy indices taking values $\pm 1$.

The central NEGF quantity for the computation of the transmission coefficients is the retarded Green function of the scattering region,

$$G' = [E - H_{\text{open}}]^{-1},$$

(27)

associated with the matrix representation of the Hamiltonian $H_{\text{open}} = H_{TB} + \sum_{\sigma,\mu} \Sigma_{\sigma,\mu}$ of an open system ($H_{TB}$ is the matrix representation of equation (25)). Here non-Hermitian retarded self-energy matrices $\Sigma_{\sigma,\mu}$ introduced by the interaction with the leads determine escape rates of spin-$\sigma$ electrons into the electrodes. The block $G_{\sigma,\mu}^{\sigma',\mu'}$ of the retarded Green function matrix, consisting of those matrix elements which connect the layer of the sample attached to lead $\beta$ to the layer of the sample attached to lead $\alpha$, yields the spin-resolved transmission matrix:

$$t_{\sigma,\mu}^{\sigma',\mu'} = 2\sqrt{-\text{Im} \Sigma_{\sigma,\mu}^{\sigma',\mu'}} G_{\sigma,\mu}^{\sigma',\mu'} \sqrt{-\text{Im} \Sigma_{\mu,\sigma}^{\sigma',\mu'}}.$$  \hspace{1cm} (28)

For simplicity, we assume that $\hat{\Sigma}_r^{\sigma,\mu} = \hat{\Sigma}_r^{\sigma,\mu}$, which experimentally corresponds to identical conditions for injection of both spin species.

By replacing transmission matrices with equation (28) in the spin-resolved noise expressions in equations (15a)–(15d), or the corresponding expressions in four-terminal structures, we arrive at NEGF formulae that can easily be evaluated in terms of $G'$ and $\Sigma_{\sigma,\mu}$ matrices [31, 36]. The equivalent route to such NEGF expressions for the noise would be to start from the spin-resolved current expression in terms of NEGF, rather than the scattering formula equation (11), and then work through a lengthy derivation similar to that provided in [32, 122].

4. Shot noise in two-terminal diffusive Rashba SO-coupled wires

In this section, the spin-dependent shot noise formalism introduced in section 3 is applied to diffusive SO-coupled quantum wires of different widths where recent experiments [124] demonstrate how transverse confinement affects the degree of transported spin coherence [42]. The quantum wires are realized using 2DEG (in the $x$-$y$-plane so that the unit vector $\mathbf{e}_y$ is orthogonal to it) with a tunable Rashba SO coupling, as described by the effective mass Hamiltonian (23) and its lattice version (25) (we assume that extrinsic SO scattering effects are negligible, $\lambda_{SO} = 0$). The internal magnetic field $B_{\text{int}}(p) = -(2a / g\mu_B)(p \cdot \mathbf{e}_x)$ of the Rashba SO coupling is nearly parallel to the transverse $y$-axis in the case of quantum wires [125]. Therefore, the injected $z$-axis polarized spins are precessing within the wires, while the $y$-axis polarized spins are in the eigenstates of the corresponding Zeeman term and do not precess. This leads to a difference in the shot noise when changing the spin-polarization vector of the injected current in the 'polarizer–analyzer' scheme in the top and middle panels of figures 5(a) and (b).

Moreover, in both cases and within the asymptotic limit $L \gg L_{SO}$ (L is the wire length) we find that the Fano factor of the shot noise increases above the universal value $F = 1 / 3$ (characterizing diffusive wires with zero SO coupling, $L_{SO} \to \infty$) for all three measurement geometries in figure 5:

(i) spin valves with parallel magnetization of the electrodes where $\uparrow$-electrons are injected from the left lead and $\uparrow$-electrons are collected in the right lead—a situation described by the Fano factor $F_{\uparrow \to \uparrow};$
(ii) spin valves with antiparallel magnetization of the electrodes where \(\uparrow\)-electrons are injected through a perfect Ohmic contact and \(\downarrow\)-electrons are collected, as described by the Fano factor \(F_{\uparrow\rightarrow\downarrow}\); 
(iii) a setup with only one spin-selective electrode where \(\uparrow\)-electrons are injected and both \(\uparrow\)- and \(\downarrow\)-electrons are collected in the normal drain electrode, as described by the Fano factor \(F_{\uparrow\rightarrow\downarrow}\).

The spin precession length (24) defined by the clean Rashba Hamiltonian can be rewritten as

\[
L_{SO} = \frac{a_\pi t_0}{2SO},
\]

in terms of the parameters of the corresponding lattice Rashba Hamiltonian (25). For very small SO coupling and, therefore, large \(L_{SO} \rightarrow \infty\), the Fano factors \(F_{\uparrow\rightarrow\downarrow}\) and \(F_{\uparrow\rightarrow\downarrow}\) start from the universal value \(F = 1/3\) characterizing the diffusive unpolarized transport, and then increase toward their asymptotic values, \(F_{\uparrow\rightarrow\downarrow}(L \gg L_{SO}) \approx F_{\uparrow\rightarrow\downarrow}(L \gg L_{SO}) \approx 0.7\) and \(F_{\uparrow\rightarrow\downarrow}(L \gg L_{SO}) \approx 0.55\). Such enhancement of the spin-dependent shot noise is due to spin decoherence and dephasing processes [42] in SO-coupled structures that are responsible for the reduction [103, 104] of the off-diagonal elements of the spin-density matrix \(\hat{\rho}_{\text{out}}\) of detected current. Note that in these setups, the density matrix \(\hat{\rho}_{\text{in}} = \hat{\rho}_{\text{out}}\) describes pure injected spin states comprising fully spin-polarized current in the lead.

However, these asymptotic Fano factor values are lowered in narrow wires where transverse confinement slows down the DP spin relaxation in the picture of semiclassical spin diffusion [43, 44], or reduces the size of the ‘environment’ composed of orbital conducting channels (i.e., smaller ‘environment’ means smaller number of channels) to which the spin can entangle or which provide ‘ensemble dephasing’ [104] in a fully quantum transport picture [42] employed to obtain \(\hat{\rho}_{\text{out}}\) versus the wire width \(W\) (at the fixed length \(L\) and the Rashba SO coupling strength) in figure 5(d). The ability of geometrical confinement effects to increase spin coherence in narrow wires [42–44, 123] has been confirmed in a very recent optical spin detection experiment [124]. Their utilization could be essential for the realization of all-electrical semiconductor spintronic devices [40, 64] where spin is envisaged to be manipulated via SO couplings while avoiding their detrimental dephasing effects [42].

The shot noise in the antiparallel configuration reaches the full Poissonian value \(F_{\uparrow\rightarrow\downarrow}(L \ll L_{SO}) \approx 1\) in the limit of small SO coupling since the probability that the spin state which has huge overlap with \(|\uparrow\rangle\) can enter into the right electrode whose spin-\(\uparrow\) states are empty is vanishingly small. This leads to a tunneling-type [1, 27] shot noise where electrons propagate independently without being correlated by their Fermi statistics. In the asymptotic limit \(L \gg L_{SO}\), injected spins loose their memory on a very short length scale, so that \(F_{\uparrow\rightarrow\downarrow}(L \gg L_{SO})\) acquires the same asymptotic value as \(F_{\uparrow\rightarrow\downarrow}(L \gg L_{SO})\).

Since the present spintronic experiments are usually conducted by injecting partially spin-polarized charge currents \(|\hat{\rho}_{\text{in}}| < 1\), we employ our general formulæ (15a)–(15d) to obtain the Fano factor:

\[
F_{\uparrow\rightarrow\downarrow} = \frac{\sum_{i} I_{i}^2 |\hat{\rho}_{\text{in}}(0, 0, P)|^4}{2e I^2 |\hat{\rho}_{\text{in}}(0, 0, P)|^2},
\]

This represents a generalization of \(F_{\uparrow\rightarrow\downarrow}\) to characterize the shot noise in an experimental setup where partially

---

**Figure 5.** Panels (a) and (b) show the Fano factor versus the spin precession length \(L_{SO}\) for different two-terminal setups (figure 1) where 100% spin-\(\uparrow\) polarized charge current is injected from the source electrode (e.g., a half-metallic ferromagnet) into a diffusive Rashba SO-coupled wire and spin-resolved charge currents \(I^2\) (top), \(I^2\) (middle) or both \(I^2 + I^2\) (bottom), are collected in the drain electrode. Panel (c) shows the corresponding decay of the degree of quantum coherence of transported spin, as quantified by the magnitude \(|\hat{\rho}_{\text{out}}|\) of the spin-polarization vector of detected charge current. The injected current in the left lead is composed of fully coherent pure spin state elements of the spin-density matrix \(\hat{\rho}_{\text{in}}\), that our wires are in the diffusive transport regime for selected disorder strength. (Adapted from [43].)
polarized (along the z-axis) electrons are injected from the left lead while both spin species are collected in the right lead. Figure 6 suggests that predictions for the excess shot noise $F_{\text{excess}}(0,0\to\uparrow\downarrow) > 1/3$ should be observable even for small polarization of injected current $J_{\text{inj}} = 0.1$. Note also that for conventional unpolarized current $J_{\text{inj}} = 0$, one does not observe any noise enhancement in two-terminal device geometry. In fact, the Fano factor $F_{\text{excess}}(0,0\to\uparrow\downarrow)$ in Figure 6 decreases with increasing strength of the Rashba SO coupling due to weak antilocalization correction [14] that reduces $S_{\text{Z}}$ for $J_{\text{inj}} = (0, 0, 0)$ and increases $I_{\text{Z}}$ for $J_{\text{inj}} = (0, 0, 0)$ in equation (30), as discussed in more detail in section 4.2 and in Figure 7.

4.1. The Fano factor as a quantifier of transported spin coherence

To understand the evolution of quantum coherence of transported spin, we use the fully quantum transport formalism of [42] which treats both the spin dynamics and orbital propagation of electrons to which the spins are attached phase coherently. This allows us to obtain the spin-density matrix of charge current in the right lead in terms of the same spin–resolved transmission matrix $T_{\text{out}}$ which determines the shot noise power $S_{\text{Z}}^\text{out}$. Note that the traditional description of DP spin dephasing treats charge propagation semiclassically while the dynamics of spin attached to charges is described via quantum evolution of the spin-density matrix [40, 43–45].

Here we summarize principal steps, put forth by Nikolić and Souma in [42] (applied or extended in numerous recent studies of electron [126] and hole [127] transport in low-dimensional systems with SO couplings and the magnetic field affecting their spins), which make it possible to define the spin-density matrix of an ensemble of phase-coherently transported spins comprising the detected current in the right lead within the framework of the scattering approach [14] to quantum transport. Suppose that a spin-$\uparrow$ polarized electron is injected from the left lead through a conducting channel $|\text{in} \rangle \equiv |n\rangle \otimes |\uparrow\rangle$. Then, a pure state emerging in the right lead after the electron has traversed the sample is described by a linear combination of the outgoing channels,

$$|\text{out} \rangle = \sum_{m} [t_{m1}^{|\uparrow\rangle \otimes |\sigma\rangle}].$$

Such a non-separable state [41] encodes entanglement of spin and the ‘environment’ composed of orbital conducting channels $|n\rangle$. Any entanglement to the environment is a source of spin decoherence [103, 104]. That is, the spin-density matrix obtained by tracing the full density matrix $|\text{out} \rangle |\text{out} \rangle$ of the pure state $|\text{out} \rangle$ over the orbital transverse propagating modes $|n\rangle$ in the right lead,

$$\rho_{\text{out}}^{\uparrow\rightarrow\text{out}} = \frac{1}{Z} \text{Tr}_{\text{orbital}} |\text{out} \rangle \langle \text{out} | = \frac{1}{Z} \sum_{m=1}^{M} |\text{out} \rangle \langle \text{out} |, \tag{32}$$

will have, in general, the polarization vector magnitude $|P_{\text{out}}| < 1$ reduced below one when transport takes place through multichannel wires [42]. Here $Z$ is the normalization factor ensuring that $\text{Tr}_{\text{spin}} \rho_{\text{out}}^{\uparrow\rightarrow\text{out}} = 1$.

Further decrease of the observable degree of quantum coherence encoded in the off-diagonal elements [103, 104] of the spin-density matrix is generated by spin dephasing [42] due to averaging over all orbital incoming channels:

$$\rho_{\text{out}}^{\uparrow} = \sum_{m} \rho_{\text{out}}^{m\uparrow\rightarrow\text{out}}. \tag{33}$$

Note that this type of dephasing is equivalent to ‘fake decoherence’ or ‘ensemble dephasing’ discussed through examples in recent monographs on quantum decoherence [103, 104]. The spin dephasing, whose meaning is defined precisely through equation (33), can be effective in reducing the off-diagonal elements of $\rho_{\text{out}}^{\uparrow}$ even if every electron in the right lead continues to be in the orbital conducting channel through which it was originally injected, so that $|\text{out} \rangle$ state...
Figure 7. Zero-frequency spin-resolved shot noise power $S^\sigma \sigma'$ (panels (a) and (d)) and spin-resolved conductances $G^\sigma \sigma'$ (panels (b) and (e)), which define different Fano factors in figure 5, for current detected in the right lead after the injection of spin-polarized (along the $z$-axis) charge current from the left lead into the diffusive wire with the Rashba SO coupling of strength $L/L_{SO}$. The quantum wire is wide in panels (a)–(c) and narrow in panels (d)–(f). The inset in panel (b) shows weak antilocalization enhanced detected current in the right lead $I_2 = I^\uparrow_2 + I^\downarrow_2$ of a ferromagnet|SO-coupled-wire|paramagnet setup. The spin-resolved shot noise for unpolarized current injection is shown in panels (c) and (f), whose sums give limiting curves (for $|P_{in}| = 0$) on the surface plots in figure 6. (Adapted from [31].)

Emerges in the right lead as a separable quantum state and ‘true decoherence’ [103] is absent. This procedure finally leads to the spin-density matrix associated with the detected charge current in the right lead [42]:

\[
\hat{\rho}_{\text{out}} = \frac{e^2}{h} \frac{1}{G_{21}^+ + G_{21}^-} \sum_{n,m=1}^M \left[ \frac{[t_{21}^{+}]_{nm}^* [t_{21}^+]_{nm} [t_{21}^-]^* [t_{21}^-]_{nm}}{[t_{21}^+]_{nm}^* [t_{21}^-]_{nm}^*} \right] \right] = \frac{1}{2} (\mathbf{1} + P_{\text{out}} \cdot \hat{\sigma}). \tag{34}
\]

From it, one can also extract the experimentally measurable spin-polarization vector $\mathbf{P}_{\text{out}}$.

Figure 5(e) shows that in narrow wires the quantum coherence of transported spin quantified by $|P_{\text{out}}|$ remains close to one for $L \lesssim L_{SO}$. In wires of fixed length, the suppression of spin decoherence in figure 5(d) is governed by the wire width $W$ and the spin precession length $L_{SO}$, which are also invoked as characteristic length scales to explain recent experiments [124]. Figure 5(e), where the Fano factor value is directly related to $|P_{\text{out}}|$, demonstrates an exciting possibility for a novel experimental tool to quantify purity of transported spin state via electrical means where the measurement of the Fano factor $F_{\uparrow \rightarrow \uparrow \downarrow}$ does not require demanding [40] spin selective detection in the right lead. The preservation of spin coherence also allows for spin-interference signatures to become visible in the shot noise in figure 5(a) as oscillations of the Fano factor between $F_{\sigma \rightarrow \sigma} = 1/3$ and $F_{\sigma \rightarrow \sigma} = 1$ along the $L_{SO}$ spatial scale.

4.2. Discussion

The phenomenological model of [28], characterized by the spin-relaxation length $L_{S}$ (which in the bulk SO-coupled systems with weak disorder is identical $L_{S}$ and $L_{SO}$), finds $F_{\uparrow \rightarrow \uparrow \downarrow} (L \gg L_{S}) \simeq 0.55$ governed by the parameters of microscopic Rashba Hamiltonian where
further modification of $F_{\uparrow \rightarrow \uparrow \downarrow}(L \gg L_S) < 0.55$ can be induced by geometrical confinement effects acting against spin decoherence and dephasing.

As regards the spin-valve setups, the semiclassical Boltzmann–Langevin approach [1] applied to spin-dependent shot noise in [27] predicts Fano factors $F_{\uparrow \rightarrow \uparrow}(L \gg L_S) = F_{\downarrow \rightarrow \downarrow}(L \gg L_S) = 1/3$ for arbitrary microscopic spin-relaxation processes within the normal region, while we find $F_{\uparrow \rightarrow \uparrow}(L \gg L_{SO}) = F_{\downarrow \rightarrow \downarrow}(L \gg L_{SO}) \approx 0.7$ for a specific case of wide Rashba SO-coupled wires. Furthermore, oscillatory behavior of the Fano factor versus specific case of wide Rashba SO-coupled wires. Furthermore, oscillatory behavior of the Fano factor versus specific case of wide Rashba SO-coupled wires.

To elucidate the source of these apparent discrepancies, we provide in figure 7 a detailed picture of auto- and cross-correlations between spin-resolved charge currents. S$_{\sigma \sigma}$ are fully spin-polarized current was completely dephased imposed on its relation to the system size), rather than by the full spin-density matrix.

5. Spin and charge shot noise in mesoscopic spin Hall systems

One of the principal outcomes of the analysis of spin-dependent shot noise for two-terminal nanostructures with the Rashba SO coupling in section 4 is understanding of how spin precession and spin decoherence can increase the Fano factor of the shot noise (above its value in the absence of SO coupling) for injected current that is spin polarized. The analysis of the same effects in multiterminal devices is more complicated [15] due to non-local effects where other leads contribute to the noise in a selected lead. Therefore, straightforward conclusions about the absence of the shot noise enhancement in the case of unpolarized current injection in two-terminal devices, found in figures 6 and 7, cannot be extended to four-terminal devices that serve as the generators of the mesoscopic SHE when unpolarized charge current is injected into them. Instead, we proceed in this section to analyze shot noise of transverse spin Hall transport for both unpolarized and spin-polarized longitudinal charge current injection.

5.1. Multiterminal spin Hall and charge current shot noise in ballistic 2DEG nanostructures

In this section and related figure 8 we assume ballistic transport ($V_{in}(x, y) = 0$ or $\epsilon_m = 0$) through 2DEG with non-zero $L_{SO}$ due to the Rashba coupling. We recall that in the two-terminal ballistic structure the stream of electrons (injected from noiseless electrodes) is completely correlated by the Pauli principle in the absence of impurity backscattering, so that the corresponding shot noise vanishes, $S = 0$ (except at the subband edges where new conducting channels open up) [1]. However, in four-terminal structures in figure 8 transmission is not perfect because of the presence of the transverse leads (even if they do not draw current [15]), so that non-zero noise appears in the absence of SO coupling. While large Rashba coupling would introduce backscattering [52, 97] at the interface between the electrodes with no SO coupling and the sample, we find this effect not to be the crucial one for noise discussion below since similar results are obtained for the bridge in figure 2(a) where leads 1 and 2 have the same Rashba SO coupling as in the central 2DEG sample.

The magnitude of pure spin currents flowing out of the mesoscopic SHE device through ideal (with no SO coupling) electrodes is governed by the spin precession length $L_{SO}$ in equations (24) and (29). This mesoscopic length scale (e.g., $L_{SO} \sim 100$ nm in typical 2DEG experiments [65, 66]) has been identified through intuitive physical arguments [74] as an important parameter for spin distributions—for example, in clean systems the spin response to an inhomogeneous field diverges at the wave vector $q = 2/L_{SO}$. In fact, the mesoscopic SHE analysis predicts [52] via numerically exact calculations that the optimal device size for achieving large spin polarizations and spin currents is indeed $L \simeq L_{SO}$, as demonstrated by figure 3(a). This is further confirmed by an alternative analysis of the SHE response in disordered finite-size 2DEGs in [128]. Therefore, we employ the 2DEG sample of the size $L_{SO} \times L_{SO}$ to study the dependence of the shot noise on the Fermi energy (i.e., charge density). We also assume that 2DEG is smaller than the inelastic scattering length $L_{\text{SO-ph}}$ because in larger samples electron–phonon scattering would average out the ‘mesoscopic’ values [6] of the shot noise to zero [8], as discussed in section 1.

The most conspicuous feature of the spin-resolved shot noise in figure 8 is the emergence of highly non-trivial temporal correlations between spin-resolved currents encoded by $S_{\sigma \sigma}(\tau) = S_{\sigma \sigma}(\tau + \tau') < 0$ (more pronounced for polarized $P_{in} = (0, 0, 1)$ injection). This stems from spin flips in the form of continuous spin precession of the $z$-axis oriented spins in the effective...
momentum-dependent magnetic field $B_{\text{int}}(p)$ of the Rashba SO coupling. Such cross-correlations can be manipulated by changing the Fermi energy in the case of polarized injection ($|P_{\text{in}}|=1$) or Rashba coupling in the case of unpolarized longitudinal current ($|P_{\text{in}}|=0$), thereby imprinting signatures of the intrinsic SO coupling on experimentally measurable charge current noise $S_{\text{charge}}$.

Another feature specific to mesoscopic manifestations of the SHE, which is also exhibited by the SHE conductance $G_{\text{int}} = I_3^2/(V_1 - V_2)$ [52, 97], is the appearance of sharp peaks in figure 8(a) in the vicinity of subband edges. At these energies new conducting channels in the leads become available for transport (top panel of figure 8(a)). Although this multiterminal noise property of ballistic conductors persists even in the absence of SO coupling, additional features of this type can arise at the energies of bound states in the cross device geometry whose mixing with propagating states via SO coupling introduces resonances in the transmission [121].

We emphasize [52, 100] that achieving pure ($I_3 = I_4 = 0$) spin Hall current $I_3^3 = -I_4^3$, akin to SHE in infinite systems [85, 89], requires us to apply [52, 102] voltages $\mu_3 = \mu_4 = 0$ to transverse leads of the clean bridge biased with $I_1 = eV/2$ and $\mu_2 = -eV/2$. Despite zero charge current $I_3 = 0$ in this case, we find non-zero fluctuations around the zero average value (found also in some other pure spin current induction setups [48]), as quantified by $S_{\text{charge}}^{\text{charge}}(|P_{\text{in}}|=0)$ in figure 8. The noise power increases in the same setup, at fixed $E_F$ and with fast spin dynamics in samples $L/L_{SO} \gtrsim 2$, by switching from unpolarized to polarized injection of longitudinal current $I_1$ responsible for non-zero transverse charge Hall current [121].

5.2. Multiterminal spin Hall and charge current shot noise in diffusive 2DEG nanostructures

To bring a multiterminal SHE bridge into the diffusive transport regime, we introduce disorder into the 2DEG through on-site potential $\varepsilon_m \in [-W_{\text{dis}}/2, W_{\text{dis}}/2]$ in Hamiltonian (25) and tune its strength $W_{\text{dis}} = 1.1\mu_0$ (mean free path $\ell \approx 29\mu$) to ensure that the shot noise in lead 1 attains the universal value $F_{11} = S_{11}^{\text{charge}}/2eI_1 = 1/3$ characterizing diffusion in multiterminal devices [15]. In the absence of the SO coupling, the noise in the other three leads does not display any universal features ($F_{11} = 1/3$ is expected to be independent of the impurity distribution, band structure and the shape of the conductor [15]) because of nonlocal effects. That is, other leads contribute to the noise in the electrode $\alpha \neq 1$, thereby making possible arbitrarily large values of $F_{11}$ [15].

In the presence of disorder, one can expect both extrinsic and intrinsic contributions to $I_3^3$. Their importance (as in the case of experimentally explored SHE systems based on 2DEGs [76]) is governed [100] by the ratio of the characteristic energy or length scales, $\Delta_{SO}/\ell = \Delta_{SO}/L_{SO}$. For simplicity, we analyze separately 2DEGs with dominant intrinsic $\alpha_R \neq 0, \lambda = 0$ in figures 9(a) and (b)) and extrinsic ($\alpha_R = 0, \lambda/\ell = 5.3\mu$) regimes of the SHE. The most important insight brought about by figure 9 is the substantial difference between the shot noise in the intrinsic (figures 9(a) and (b)) and extrinsic (figure 9(c)) regimes, where the former exhibits non-zero cross-correlations $S_{11}^{\text{charge}} = S_{33}^{\text{charge}} < 0$ akin to its ballistic counterpart in figure 8, but smaller. The shot noise of the extrinsic SHE device in figure 9(c) has no temporal correlations of this type for the $z$-axis spins, while exhibiting orders of magnitude smaller cross-correlation noise, $S_{33}^{\text{charge}}$ and $S_{33}^{\text{charge}}$ for the $x$- or $y$-spins, respectively, which carry no spin...
current $I_{3}^{x} = I_{3}^{y} = 0$ when $\lambda \neq 0$. We also find that hypothetical (i.e., experimentally not accessible) increase of $\lambda$ would give orders of magnitude smaller noise change (in fact, decrease) compared to significant spin $S_{33}^{\text{spin}}(|P_{m}| = 0)$ or charge $S_{33}^{\text{charge}}(|P_{m}| = 1)$ shot noise enhancement with increasing of the intrinsic $\alpha_{R}$ that can be experimentally controlled [65, 66],

Note that due to $I_{1} = 0$ in the SHE setup ($|P_{m}| = 0$), we plot raw noise values in figures 8 and 9, rather than normalizing them to $2eI_{1}$ or $2eI_{3}^{x}$ to get conventionally defined Fano factors. A useful Fano factor can actually be defined if we normalize noise to the injected current in lead 1:

$$F_{33}^{\text{charge}} = \frac{S_{33}^{\text{charge}}}{2eI_{1}}, \quad F_{33}^{\text{spin}} = \frac{S_{33}^{\text{spin}}}{2eI_{1}}.$$  \hspace{1cm} (35)

We plot such Fano factors in figure 10 for zero charge current $I_{1}$ when injected current $I_{1}$ is unpolarized ($|P_{m}| = 0$), as well as for non-zero charge current $I_{1}$ at partially ($|P_{m}| = 0.1$ and $|P_{m}| = 0.5$) or fully polarized ($|P_{m}| = 1$) current is injected through lead 1. Their dependence on the ratio of characteristic energy or length scales demonstrates that around $\Delta_{SO}t_{\perp} / h = \pi t / L_{SO} \sim 10^{-1}$, for which the intrinsic Rashba SO coupling becomes dominant SHE mechanisms in disordered 2DEGs in figure 3(b), both the spin and the charge Fano factors start to increase above their reference values set in the limit of zero SO coupling ($L_{SO} \to \infty$). Thus, figure 10, as one of our principal results in section 5, suggests that by measuring the enhanced charge Fano factor in transverse lead 3 at given polarization of the injected current one could confirm in an unambiguous fashion the dominance of the intrinsic SO mechanisms in the induction of (much more difficult to measure) spin Hall current in the same lead.

The Fano factor reference value in figure 10, defined at negligible SO coupling strength $\pi t / L_{SO} \to 0$, depends on $|P_{m}|$ even in this limit. At first sight, this might seem surprising since this feature is absent in figure 6 for two-terminal devices. In fact, this is a consequence of the four-terminal device geometry where noise power in a given lead depends on contributions from all other leads. For example, one can view the four-terminal device as an eight-terminal one with fully polarized electrodes, so that the injection of spin-polarized current through lead 1 corresponds to one of these eight leads being totally (for $|P_{m}| = 1$) or partially blocked (for $0 < |P_{m}| < 1$). This decreases both the noise power in lead 3 (as shown in figure 9) and the current $I_{1}$ normalizing the noise to define the Fano factor.

6. Quantum interference effects on the shot noise of spin Hall and charge currents in four-terminal Aharonov–Casher rings

The stationary states of the system one-dimensional (1D) ring + two 1D leads can be found exactly by matching the wavefunctions in the leads to the eigenstates of the clean ($V_{\text{dis}} = 0$ and $\lambda = 0$) ring Hamiltonian (23), and then computing the charge conductance from the Landauer transmission formula [116]. However, attaching two extra leads in the transverse direction, as well the finite width of the ring and/or the presence of disorder within the ring region, requires us to switch from wavefunctions to the NEGF formalism discussed in section 3.3 in order to compute numerically exact transmission matrices $t_{a\sigma}$ connecting the four leads $\alpha, \beta = 1, \ldots, 4$. The computation of the retarded Green function matrix $G'$ in equation (27) can be done efficiently using the lattice-type Hamiltonian akin to equation (25) (assuming $\Delta_{SO} = 0$), which was introduced in [105] as a set of $M$ concentric chains composed of $N$ lattice sites spaced at a distance $a$. Besides the usual energy scales introduced by such Hamiltonian—the orbital hopping $t_{0}$ and the Rashba hopping $t_{SO}$ between nearest-neighbor sites discussed in section 3.3—it is advantageous to employ a dimensionless parameter, $Q_{R} \equiv (t_{SO}/t_{0})N/\pi$, to...
measured the strength of the SO coupling within the ring region [105, 117].

The charge conductance of a two-terminal 1D AC ring [105, 116, 117] becomes zero at specific values of $Q_R^{\text{min}}$ for which destructive spin interference of opposite spins traveling in opposite directions around the ring takes place. For example, in a simplified treatment [117] $G = \frac{e^2}{h}[1 - \cos \left(\frac{\Phi_{\text{AC}}}{2}\right)]$ (the complete analytical solution is given in [116]), where $\Phi_{\text{AC}} = \pi \left(1 + \sigma \sqrt{Q_R^2 + 1}\right)$ is the AC phase acquired by a spin-↑ or spin-↓ quantum state, so that charge conductance minima $G(Q_R^{\text{min}}) = 0$ are at $Q_R^{\text{min}} \approx \sqrt{n^2 - 1}$ ($n = 2, 4, 6, \ldots$). However, adding two transverse leads onto the same 1D ring lifts the minima of the longitudinal conductance to $G_{\text{L}}(Q_R^{\text{min}}) = I_2/(V_1 - V_2) \approx \varepsilon^2/h$ due to the contribution from incoherent (indirect) paths, $1 \rightarrow 3 \rightarrow 2$ and $1 \rightarrow 4 \rightarrow 2$, which do not exhibit destructive spin interference effects that characterize coherent (direct) paths from terminal 1 to 2 (see the inset in figure 4 for labeling of the terminals). In these cases, the electron goes into the macroscopic reservoirs, where the phase of its wavefunction is lost, before reaching these cases, the electron goes into the macroscopic reservoirs, before reaching the second longitudinal electrode. Nevertheless, $G_{\text{L}}$ vanishes at $Q_R^{\text{min}}$, while the amplitude of its quasiperiodic oscillations gradually decreases at large $Q_R$ because of the reflection at the ring–lead interface, as shown in figure 4.

This type of four-terminal spin interferometer and its two-terminal counterpart [105], based on tunable Rashba SO coupling, share the same limitations with other types of interferometers discussed in section 2.3. The visibility of quantum interference effects encoded into the quasi-periodic oscillations of $G_{\text{SH}}(Q_R)$ in figure 4 is reduced by ‘dephasing’ when accumulated AC phases are averaged over many Feynman paths through 2D rings with $M > 1$ (see figure 12(a)).

Here we examine, if the shot noise of pure spin Hall current $I^S_{\text{CH}}$ and zero charge current $I_3$ contains any additional insights, beyond the spin Hall conductance $G_{\text{SH}}$ in figure 4, about the spin interference effects in four-terminal AC rings. The result for spin $S_{33}^{\text{spin}}$ ($|P_{\text{in}}^S| = 0$) and charge $S_{33}^{\text{charge}}$ ($|P_{\text{in}}^C| = 0$) noise power as the function of the Rashba coupling strength $Q_R$ is plotted in figure 11. In contrast to naive expectations [2] about the shot noise in interferometers simply retracing the oscillations of $G_{\text{SH}}$ in figure 4, we see a more complicated pattern for the total spin noise, as well as maxima of the total charge noise (for the zero time-averaged value of the charge current $I_3 = 0$) at around zeros of $G_{\text{SH}}$. Furthermore, the spin interference effects are inducing a highly non-trivial oscillatory pattern in the cross-correlation noise $S_{33}^{\text{spin}}$, as seen in figure 7, which contains only for finite values of $Q_R$.

As discussed in section 2.3, the key issue for experimental [106] realization of solid-state interferometers is the confinement of electrons to single channel transport to avoid averaging of the phase of their wavefunction when many channels contribute to measured transport properties [107]. In the case of QIDSHE, Souma and Nikolić [91] examined the effect of 2D transport within the ring and electron injection through 1D leads (assuming that, e.g., point contact has been introduced between the lead and the ring), finding that oscillations of $G_{\text{SH}}$ due to spin interference effects are still
clearly visible. However, a more realistic device amenable to nanofabrication [115] is ballistic 2D ring attached to few-channel leads where electrons can be injected through one or more conducting channels of lead 1 by tuning their Fermi energy. The spin Hall conductance of such a device is shown in figure 12, where we still find non-zero QIDSHE, but with greatly distorted oscillations of $G_{zH}$ even when electrons are injected through a single transverse propagating mode of lead 1. Nevertheless, the Fano factor of transverse zero charge current, defined as $S_{\text{charge}}^{33}/2eI_1$, where $I_1$ is used for normalization taking into account that $I_3 = 0$, displays much more regular oscillations with maxima appearing at similar values as in the case of the noise in strictly 1D structures of figure 11. Analogously to figure 11, we show the origin of these Fano factors in terms of the spin-resolved shot noise contributions to it plotted in figure 13.

7. Concluding remarks

The number of theoretical studies on spin-dependent shot noise has grown at an accelerated pace in recent years, carried by a wave of interest in spintronics and spin-based quantum computing, as well as by fundamental interest to unravel new tools for probing spin dynamics and electron-electron interactions in nanostructures. In particular, akin to earlier studies of the shot noise in spin-degenerate mesoscopic devices, the results on spin-dependent shot noise have divulged how random time-dependent current fluctuations encode the signatures of interactions of transported spin with magnetic impurities, SO couplings, and other internal and external magnetic fields. These unique signatures are not visible when measuring the time-averaged currents and conductances.

In contrast to theoretical endeavors, large-scale experimental effort on spin-dependent shot noise in semiconductor spintronic devices is still lacking. The recent spin-dependent shot noise measurements have mostly been focused on magnetic tunnel junctions. While shot noise does not impose the most important limiting factor (when compared to debilitating $1/f$ noise) for MTJ applications, it does offer a sensitive tool to probe microscopic features of their imperfect insulating barriers or Coulomb interaction effects in spin-polarized tunneling.

Here we focused on reviewing, as well as extending, recent results [31, 36] on the effect of the Rashba SO coupling on: (i) the shot noise of spin-polarized current injected into two-terminal diffusive quasi-1DEG-based nanowires, (ii) the shot noise of pure spin and charge currents generated by the mesoscopic SHE in four-terminal ballistic and diffusive 2DEG nanostructures and (iii) the shot noise associated with quantum-interference-driven SHE in four-terminal Aharonov–Casher rings realized using 2DEG.

Figure 11. The ballistic spin-resolved shot noise in the transverse electrode 3 of a four-terminal 1D Aharonov–Casher ring (top and middle panels), as well as the total shot noise of transverse pure spin Hall current $I^S_3$ and zero charge current $I_3 = 0$ (bottom panel), driven by unpolarized $|P_{in}| = 0$ injected charge current $I_1$. The spin conductance of the QIDSHE as the function of the Rashba SO coupling within the 1D ring is shown in figure 4 for the same device parameters ($E_F$ is the Fermi energy of injected electrons through single $M = 1$ channel leads attached to 1D ring discretized using $N = 100$ sites).

Figure 12. (a) The spin Hall conductance of a four-terminal 2D Aharonov–Casher ring attached to multichannel ($M = 3$) electrodes as the function of the dimensionless Rashba coupling $Q_R$ within the ring. (b) Fano factor characterizing the shot noise of zero charge current $I_3 = 0$, normalized to current in lead 1. The Fermi energy $E_F$ is tuned to inject electrons from lead 1 through a single ($E_F = -2.7t_0$, solid line), two ($E_F = -1.7t_0$, thin solid line) and three ($E_F = -0.17t_0$, dotted line) conducting channels. The ring is modeled [105] using three coupled concentric circles, each discretized with $N = 100$ sites.
To study these problems requires us to extend [31] the conventional scattering theory formulae for spin-degenerate noise [1] to spin-resolved shot noise, while taking as an input the degree of quantum coherence of injected spins $|\mathbf{P}_m|$, and the direction of the spin-polarization vector $\mathbf{P}_m$ with respect to relevant internal and external magnetic fields within the sample. The application of this formalism to two-terminal multichannel diffusive quantum wires with the Rashba SO coupling shows how decoherence and dephasing of spin dynamics are essential to observe enhancement of charge shot noise in spin-polarized transport. That is, in narrow wires, where loss of spin coherence is suppressed and $|\mathbf{P}_{\text{out}}|$ decays much slower than in the bulk systems, the increase of the Fano factor (above $F = 1/3$ of spin-degenerate diffusive transport $|\mathbf{P}_m|$) in the strong SO coupling regime ($L >> L_{\text{SO}}$ inducing fast spin dynamics within the sample) is reduced when compared to wide wires. This occurs despite the fact that partially coherent spin state continues to ‘flip’, but through (partially coherent [42]) spin precession $0 < |\mathbf{P}_{\text{out}}| < 1$. To obtain the Fano factor of charge currents comprised of partially coherent spins requires us to treat both charge propagation and spin dynamics quantum mechanically, as suggested by the spin-resolved shot noises and conductances in figure 7 (which cannot be reproduced by semiclassical approaches to spin-dependent shot noise where spin dynamics is captured only through the phenomenological spin-flip diffusion length [27]).

A remarkable one-to-one correspondence between the values of $F_{f-1}$ and the degree of quantum coherence $|\mathbf{P}_{\text{out}}|$ predicted in figure 5(e) offers an exciting possibility of measuring the coherence properties of transported spin as a magnetic degree of freedom in a purely charge transport experiment on open SO-coupled systems. This offers an all-electrical alternative to usually employed optical tools to probe transport of spin coherence in semiconductors [124].

While the enhancement of the shot noise due to fluctuations involving spin flips (i.e., continuous spin precession) in two-terminal Rashba SO-coupled devices is absent when the injected current is unpolarized, this conclusion cannot be trivially extended to multiterminal devices due to non-local effects where other leads contribute to the noise in a selected lead. In fact, in the multiterminal SO-coupled device exhibiting mesoscopic SHE, where intrinsic SO mechanisms relying on precessing spins can dominate over the extrinsic ones, we find a possibility for a significant enhancement of the shot noise of transverse spin and charge transport even when the current injected through longitudinal leads is unpolarized. This is related to the fact that extrinsic SO scattering off impurities in 2D has no measurable effect on the shot noise. Therefore, experiments observing shot noise enhancement in the transverse electrodes upon changing the voltage of the gate electrode [65, 66] covering 2DEG could unambiguously resolve the dominance of the intrinsic contribution to the spin Hall or the charge Hall effect (and related inverse SHE) in multiterminal nanostructures. The central result of this novel approach to long-standing ‘intrinsic versus extrinsic controversy’ [50, 74, 92] surrounding experimental tests of the origin of SHE (and related AHE) is shown in figure 10 — the spin and charge current Fano factors in the transverse electrode starts to increase in the same region of intrinsic SO coupling strength in which intrinsic mechanisms begin to dominate [100] SHE manifestations in figure 3(b). The specific Fano factor values that can be measured electrically are set by the polarization of injected current. Thus, by detecting the increase of the Fano factor of transverse charge current while increasing the Rashba SO coupling via the gate voltage will confirm that intrinsic mechanisms (i.e., spin precession associated with them) dominate the induction of the SHE [76] in the same device.

Finally, in four-terminal Aharonov–Casher rings we find that both the spin and charge shot noise of the spin Hall transport in the transverse electrodes oscillate as the strength of the Rashba SO coupling is modified by the gate electrode covering the ring to tune constructive and destructive spin interference effects. However, the pattern of these oscillations is much more different from the oscillations of time-average quantities, such as the spin Hall conductance.

Figure 13. The ballistic spin-resolved shot noise in the transverse electrode 3 of a four-terminal 2D Aharonov-Casher ring (top and middle panels), as well as the total shot noise of transverse pure spin Hall current $I^S_3$ and zero charge current $I_1 = 0$ (bottom panel), driven by unpolarized $|\mathbf{P}_m| = 0$ injected charge current $I$. The spin conductance of the QIDSHE and the Fano factor of zero transverse charge current as the function of the Rashba SO coupling within the 2D ring is shown in figure 12 for the same device parameters ($M = 3, N = 100$) and the same number of open channels (one in (a), two in (b) and three in (c)) used to inject electrons from lead 1 by tuning their Fermi energy $E_F$. 
or longitudinal charge conductance. This is related to a complicated (when compared to the same quantities studies in simply connected SO-coupled nanostructures in section 4.2) oscillatory pattern of the spin-resolved shot noise measuring temporal correlations between currents of spin-up and spin-down electrons. Despite the net transverse charge current being identically zero, there is still a non-zero oscillatory charge shot noise due to the opposite flow of spin-up and spin-down electrons in the course of pure spin Hall current induction. This noise reaches maxima at around zeros of the spin Hall conductance.

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