Solving the single track train scheduling problem via Deep Reinforcement Learning

Valerio Agasucci\textsuperscript{a,b}, Giorgio Grani\textsuperscript{a,c}, Leonardo Lamorgese\textsuperscript{b}

\textsuperscript{a}DIAG, Sapienza University of Rome, Rome, Italy
\textsuperscript{b}OPTRAIL, Rome, Italy
\textsuperscript{c}SINTEF, Oslo, Norway

Abstract

Every day, railways experience small inconveniences, both on the network and the fleet side, affecting the stability of rail traffic. When a disruption occurs, delays propagate through the network, resulting in demand mismatching and, in the long run, demand loss. When a critical situation arises, human dispatchers distributed over the line have the duty to make their best to minimize the impact of the disruptions. Unfortunately, human operators have a short depth of perception of how what happens in distant areas of the network may affect their control zone. In recent years, decision science has focused on developing methods to solve the problem automatically, to improve the capabilities of human operators. In this paper, machine learning-based methods are investigated when dealing with the train dispatching problem. In particular, two different Deep Q-Learning methods are proposed. Numerical results show the supremacy of these techniques concerning the classical linear Q-Learning based on matrices.

Keywords: Scheduling; Reinforcement Learning; Rail Optimization

1. Introduction

Railway systems rely on a complex scheme of routes and scheduled stops. A rail company organizes its fleet to accommodate expected demands, maximizing revenue and coverage, so that the service is provided to customers as far as possible. From a practical point of view, companies have to make decisions for two different time horizons: offline and online. Offline decisions deal with the problem of routing trains in advance, so that the basic path for each train is decided and in normal conditions, these are the one that will be followed. Decisions in this sense are made sporadically in a year, typically once every three to six months. The planned routes and schedules are usually hand-engineered according to regulation, safety measures, and demand requirements.

As said, planned routes are the ones preferred in normal conditions, but this rarely happens since disruptions occur daily in the network. A broken train, a not working switch, delays in the preparation of the train, and many more real-life problems may affect the overall network. Sometimes the delay introduced is small and the planned schedule can still be used, but on other occasions, online re-routing and re-scheduling have to be applied. In literature, this online decision making is called the Train Dispatching problem (TD), a real-time variant of the Train Timetabling problem (known to be NP-hard \textsuperscript{3}). Indeed, the very little time admitted
for computation (often less than a few seconds) makes this problem very challenging to solve in practice. The task of dispatching trains is in the hands of human operators, the so-called dispatchers, that, to this day, are generally provided with little or no decision support in the process. This makes it difficult for dispatchers to go beyond their local view of the network and take into account the knock-on effects of their decisions. The Train Dispatching problem has spurred much research interest over the years, leading to a vast amount of literature, in particular in the Optimization community. Different models and solutions approaches have been proposed over the years, primarily based on integer programming, graph theory, and ad-hoc heuristics. Refer to [11] for a deeper insight on the Train Dispatching problem, and to the many surveys on the topic for an overview of these approaches (e.g. [2, 5, 8]).

Recent developments in both Machine Learning and Optimization have led to the definition of new learning-based paradigms to solve hard problems. Our focus is on Reinforcement Learning, for which a lot of interesting results have been achieved so far. In particular, in the well known AlphaGo algorithm (see [15]) the authors tackle the complex game of Go, developing an outstanding framework able to overcome the best human player. Several approaches have followed AlphaGo, like AlphaZero [13, 14] and MuZero [12], increasing every time the degree of generalization possible. Recently in [10], this approach has been extended to combinatorial problems with a more general range of possible applications. To achieve their results, the authors combined Deep Q-Learning with graph convolutional neural networks, which are a specialized class of models for graph-like structures. A very similar approach has been followed in [7].

In this paper, TD is tackled by means of Deep Q-Learning on a single line. More specifically, two approaches have been investigated: decentralized and centralized. In the former, each train can be seen as an independent agent with the ability to see only a part of the network, namely some blocks ahead and so beyond. This approach can be easily generalized, but of course, it lacks the depth of prediction useful to understand network dynamics. The former method takes as input the entire line and learns to deal with delay propagation. In the second case, the generalization capability is more limited, even if computational tests show a relatively short time for learning. The approaches are compared to their linear counterpart: the matrix Q-learning approach proposed in [9].

The paper is organized as follows: in Section 2 basic concepts of train dispatching as a problem, and reinforcement learning are introduced formally. In Section 3 the two algorithms are discussed. Specifically, in sub-section 3.1 the decentralized approach is tackled, whereas in sub-section 3.2 the centralized approach is presented. Finally in Section 4 a numerical analysis is conducted to prove the quality of deep reinforcement learning for the train dispatching problem.

2. Preliminaries

Two basic ingredients characterize this paper: the train dispatching problem (TD) and (deep) reinforcement learning. In the following, a short presentation of both is presented, with the aim of being introductory and not comprehensive.

2.1. Train dispatching problem

The train dispatching problem refers to the rearrangement of trains on a network after a disruption has modified the nominal functioning of the system. The objective is to introduce as little modifications as possible, matching the programmed timetable with the lowest delay. The fundamental elements of TD are trains, the railway network, and routes.
Trains are of curse the means of transportation in this system. On a real network, different typologies of trains can operate at the same time. By the way, from the point of view of the dispatcher, the relevant attributes of a train are its priority and its length. Priority refers to the fact that certain trains may prevail when a delay solution has to be taken, i.e. cargo trains have usually less priority concerning a passenger one. The length usually takes discrete values, coherently with the regulation and market conditions.

A railway network is composed of a set of tracks and stations. Tracks are physical connections between two stations, and they are separated into sections so that two trains with the same direction can occupy a track simultaneously if a sufficient number of free sections separates them. In principle, tracks can host any number of vehicles according to the physical capacity, but in practice, there exists a fixed distance between two trains for safety reasons.

Stations are represented by a set of track segments. A track segment is an atomic element in a railway system, with different functionalities according to its function. In particular, it is possible to distinguish between interlocking routes and stopping points. Interlocking routes occur between stopping points, where a stopping point models an operational point in the station. On a less granular level, stations can eventually have platforms or simply be crossovers. Crossovers contain pairs of switches, where a switch is a mechanical installation enabling trains to be guided from one track to the other. Each station contains at least two switches, one entering and the other exiting. A switch can be occupied by one train at a time. Platforms, on the other hand, imply the possibility to board people or load goods. Each platform can be occupied by one train at the same time, so no train can enter a platform until the last train leaves it by a certain safety distance.

When entering a track, a check is made on the time elapsed since the previous train has entered. If the time measured is under a certain threshold, safety rules impose to wait. This threshold is called line headway time.

Finally, a route is the expected path of a train with programmed times. The path is specified by a starting station, a set of boarding points, and the arrival station.

2.2. Reinforcement learning

In a few words, reinforcement learning (RL) is an approximated version of dynamic programming, as stated in [1]. This paradigm learns, in the sense that the approximated function built takes into account statistical information, obtainable from previous iterations of the same algorithm or external data. The term deep reinforcement learning (DeepRL) usually refers to RL where a deep neural network is used to build the approximation.

Please look at [16] and [1] for a comprehensive dissertation on RL.

RL is usually explained through agent and environment. The agent is the part of the algorithm that learns and takes decisions. To do so, it has to analyze the surrounding environment. Once it takes a decision, the environment reacts to this decision and the agent can perceive what effect its action has produced. To understand if the action taken has been successful or not, the agent may receive a reward associated with the action and the new environment observed.

In the decentralized approach in Section 3.1, the agent is the train and the environment the observable line, whereas in Section 3.2 the agent is a line coordinator, deciding for each train, and the environment is the entire line.

For the purposes of this paper, the reinforcement procedure will move towards a discrete-time step $t$. In the RL vocabulary, the state represents the formal representations of the environment at a time step $t$, and it is formalized by the vector $s_t \in S_t$, where $S_t \subseteq \mathbb{R}^n$ is the set of all possible states at time $t$. The action taken by the agent is $a_t \in A_t$, where $A_t \subseteq \mathbb{R}^n$ is the set of all possible actions at time $t$. Once the agent observes $s_t$ and accomplishes $a_t$, the environment reacts by producing the new state $s_{t+1} \in S_t$ and a reward $r_{t+1} \in R$, where $R \subseteq \mathbb{R}$ is the set of
all possible rewards. In this case, \( R \) is mono-dimensional, but in general, it could be a vector, according to the problem examined. In figure 1, a simplified flowchart of RL.

Q-learning is a branch of RL that uses an action-value function (usually referred to as q-function) to identify the action to take. More formally, the q-function \( Q(s_t, a_t) \) represents the reward that the system is expected to achieve. At each step \( t \), the new action is chosen according to

\[
   a_t = \arg \max_{a \in A_t} Q(s_t, a)
\]

In algorithm 1, \( L(\hat{y}, y) \) is the loss function used to perform the training. \( \epsilon \in (0, 1) \) is the probability to pick up a random action, \( T \) is the final state of the process (i.e. when it is not possible to move anymore), \( \gamma \in (0, 1] \) is the future discount, a hyper-parameter reflecting the fact that future rewards may be less important. To make the estimation more consistent, usually, a replay mechanism is used, so that the agent interacts with the system for a few virtual steps before learning. This is suitable in situations where the environment can be efficiently manipulated or simulated so that the computational cost is affected marginally.

Algorithm 1: Q-learning algorithm

\begin{itemize}
  \item \textbf{Input:} \( \mathcal{P} \) a set of instances, \( \mathcal{D} = \emptyset \) the memory, a loss function \( L(\hat{y}, y), \epsilon \in (0, 1), \gamma \in (0, 1] \), \#episodes, \#moves
  \item \textbf{Output:} the trained predictor \( Q(\cdot, \cdot) \)
  \item for \( k = 1, \ldots, \#\text{episodes} \) do
    \item Sample \( P \in \mathcal{P} \)
    \item Initialize episode \( t = 0, s_0 = 0 \)
    \item for \( t = 0, \ldots, \#\text{moves} \) do
      \item Select an action \( a_t = \begin{cases} \text{Unif}\{A_t\}, & \text{with probability } \epsilon \\ \arg \max_{a \in A_t} Q(s_t, a), & \text{with probability } 1 - \epsilon \end{cases} \)
      \item Observe \( s_{t+1}, r_{t+1} = \text{ENVIRONMENT}(s_t, a_t) \)
      \item Store \( (s_t, a_t, y_{t+1}) \) in the memory \( \mathcal{D} \), where \( y_{t+1} = \begin{cases} r_{t+1}, & \text{if } t + 1 = T \\ r_{t+1} + \gamma \arg \max_{a \in A_t} Q(s_{t+1}, a), & \text{oth.} \end{cases} \)
      \item Sample batch \( (x, y) \subseteq \mathcal{D} \)
      \item Learn by making one step of stochastic gradient descent w.r.t. the loss \( L(Q(x), y) \)
        \item if \( t + 1 = T \) then \break
        \item end
    \item end
  \item end
end
\end{itemize}
3. Algorithms

In this section, basic ideas regarding states, actions, and rewards to solve TD are presented. Two approaches have been applied: decentralized and centralized. The difference between them relies mainly on the topology of the state, and therefore in the capability of the approximating q-function to capture the right policy. The decentralized approach returns the worst solutions, but it is faster to train and easier to generalize, whereas the centralized achieves better solutions but is tied to the structure of the network.

When describing the state, both of the approaches use the concept of a resource. A resource is a block on the line that can be occupied by one or more trains. As studied in [4, 17], enriching the information available to the agent affects the space of policies to be learned. For this reason, six features have been adopted for each resource, specifically:

1. status, which is a discrete value chosen among stopping point, track, blocked, and failure.
2. number of trains
3. train priority (if there is more than one train, the train with the highest priority is reported)
4. direction, which is a discrete value chosen among: follower, crossing, or empty
5. length check, a Boolean identifying if the train length is less than or equal to the resource size
6. number of parallel resources w.r.t. to the current one

For what that concerns the learning process, both approaches share the same model for the q-function, which is represented by a feed-forward deep neural network (FNN) with two fully connected hidden layers of 60 neurons each, and a third layer mapping into the space of the actions. The output of the third layer is then combined with a mask, disabling infeasible moves. Figure 2 shows the structure of the network.

As loss, both of the approaches use the mean squared error. Given an input $X$ and label $Y$ the loss $L$ is given by

$$L(X, Y) = E (||Q(X) - Y||_2^2)$$

Single train railway networks are characterized by the alternation of one station and one track, but sometimes it happens that between two stations there are two single tracks or more. In this situation, if two crossing trains occupy the two of these tracks at the same time a deadlock arises. In few words, given a generic train $T_0$ positioned in $R_0$ that wishes to occupy the resource $R_1$, then the method evaluates the position of all visible trains $T_i$ and resources $R_i$, for $i = 1, \ldots, K$, converging to $R_1$. For each train $T_i$, the flow of resources $F_i$ from $R_i$ to $R_1$ is then computed. Finally, if there exists at least one $i \in \{1, \ldots, K\}$ such that $F_i$ does not contain parallel resources, then $T_0$ must wait to avoid deadlock.

The majority of models for TD relies on several approximations to make the problem more readable and easy to handle when doing mathematics. Using RL allows us to integrate inside the ENVIRONMENT a lot of these hidden aspects, introducing a new level of complexity with a relatively small effort. One of the most important is the train length. Generally, models tend to

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1 It has to wait until another train cross it.
2 The system can not be recovered.
approximate a train with a point, but this is not the case in real life where a train may occupy one or more resources at the same time.

Another important step is to introduce safety rules like the line headway time, the safety distance inside the same track, or the role of switches, all three already discussed in section 2.

3.1. Decentralized approach
In dispatching problems, the dispatcher has to take a sequence of decisions to reach the minimum weighted delay. The main operation is to decide whether the train can access a specific resource or it has to stop. This decision is taken at a control point, whereas otherwise, the train takes the next available resource automatically. In the decentralized approach, the DeepRL algorithm acts as if the train was able to take decisions by itself. To this aim, the train cannot access the whole network, but only a limited number of resources ahead and behind. This mimic to some extent what is implemented by human dispatchers, which focus only on the local area of the line surrounding the control point. For each visible resource, the six features discussed before are considered.

Given the state, the action to be taken is one of the following:

- halt
- go to the best resource
- go to a reachable resource in order to avoid failure

Of course the train can always be halted, while the other two possibilities depend on the state.

In this approach, the most delicate part is in the definition of the reward. The objective is to minimize the weighted delay, but with a local visibility there is no way to associate the delay produced with the overall network objective. For this reason, the reward is assigned at the
end of each episode, and then the data collected is stored. In this fashion, the reward for each state-action pair generated assumes a penalty value if the episode ends up in a deadlock, a minor penalty if the weighted delay is greater than 1.25 times the minimum weighted delay found so far, and a prize if the weighted delay is less than or equal to 1.25 times the minimum weighted delay found so far.

The last aspect to be discussed is the memory management. Since no prior knowledge has been used, the composition of the sample is critical to drive a smooth learning process. Therefore, the memory has been divided into three data-sets: best, normal and deadlock. Action-state-reward items are stored according to the level of reward if and only if the action mask in the FNN allows more than one action, so to strengthen the learning process only on critical moves. Deadlock and normal data-sets are never deleted, while the best memory is updated every time a new best weighted delay is found.

3.2. Centralized approach

In the centralized, the DeepRL algorithm takes advantage of knowing the state of the overall network at each step. It can be thought about the agent as a line coordinator, deciding critical issues at control points, predicting the expected effect on the network. The difference w.r.t. the decentralized case is that now the model is strictly linked to the topology of the network, and generalization is less obvious. On the other hand, the learning process is affected by a larger dimension of the input space, resulting in longer computations.

In particular, the state has now the dimension of all the resources of the network, and for each resource, it is specified priority, direction and length of the train on it.

The action space now takes into account not only the singular move (as in the decentralized case), but also a network choice selecting the train to which apply the singular move.

More interestingly, the reward is the total amount of (negative) delay produced by the action until a new event occurs. If the algorithm ends up in a deadlock, then a penalty is introduced. If the episode ends up in a feasible solution, then the final reward is proportional to the cumulative weighted delay, so that the prize is directly linked to the real objective.

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4. Numerical results

The two algorithms have been compared to the linear Q-learning algorithm proposed in [9]. This paper aims to show that DeepRL is superior to linear Q-learning. In particular, the quality of the solutions in terms of final delay is the main driver used to discriminate the quality of an algorithm. For what that concerns time, all the procedures need less than one second to complete an episode, and therefore they are all suited for online applications.

All the tests were performed on an Intel i7 processor, with no use of GPUs. The rail network is single-track and composed of 140 linear blocks, with 42 parallel resources.

The experiments have been conducted to study the following properties:

- exp.1 ability to solve unseen instances generated from the same distribution
- exp.2 capacity to generalize when more and longer trains are present
4.1 First experiment

Since the structure of the network is fixed, instances of the TD problem differentiate the one from the other according to several parameters like number of trains, position, direction, priority and length.

The range for each parameter has been chosen according to real-world instances available from our commercial partners, and more specifically:

- number of trains $N$ is chosen randomly between 4 and 10 with the following probability distribution: $P(N = 4) = 0.1$, $P(N = 5) = 0.2$, $P(N = 6) = 0.2$, $P(N = 7) = 0.2$, $P(N = 8) = 0.15$, $P(N = 9) = 0.1$, $P(N = 10) = 0.05$

- position is chosen using a uniform distribution on the available resources

- direction is chosen uniformly

- priority $A$ is chosen randomly between 1 and 5 with the following probability distribution $P(A = 1) = 0.05$, $P(A = 2) = 0.15$, $P(A = 3) = 0.23$, $P(A = 4) = 0.27$, $P(A = 5) = 0.3$

- length is chosen uniformly in the set

\[ \{4000, 4500, 5000, 5500, 6000, 6500\} \]

Additionally, a penalty factor is multiplied to the delay to express the importance related to different priorities. Given a priority $A \in \{1, 2, 3, 4, 5\}$, the coefficient $\omega_A$ is such that $\omega_1 = 20$, $\omega_2 = 10$, $\omega_3 = 5$, $\omega_4 = 2$ and $\omega_5 = 1$.

The models are trained on 100 randomly generated instances, with the fixed rail network, and tested on 100 unseen instances with the same specifications. The training phase has been performed using 10,000 episodes for each algorithm. An episode is stopped either when all trains are at their last resource or after 2 hours of future dispatching have been produced.

Figure 3 reports the performance profiles for the three algorithms concerning the value of the delay obtained on 100 instances of the same distribution of the training set. Performance profiles have been used similarly as proposed in [6]. Given a set of solvers $S$ and a set of problems $P$, the performance profile takes as input a ratio between the performance of a solver $s \in S$ on problem $p \in P$ and the best performance obtained by any solver in $S$ on the same problem. Consider the cumulative function $\rho_s(\tau) = |\{p \in P : r_{p,s} \leq \tau\}|/|P|$ where $t_{p,s}$ is the delay and $r_{p,s} = t_{p,s}/\min\{t_{p,s'} : s' \in S\}$. The performance profile is the plot of the functions $\rho_s(\tau)$ for $s \in S$. In any case, keep in mind that the higher the curve the better the component. Here, the components are algorithms and a victory is achieved by comparing delays. The graph shows the supremacy of deep architectures with respect to the simple linear one, which is reasonable due to the known complexity of the problem. Unexpectedly, the centralized model seems to perform worst than the decentralized one. The motivation is that both centralized and decentralized share the same network architecture, but the space of decisions and complexity of the centralized approach is much larger so that the neural network is too simple to achieve the desired results in the training time provided by the experiment.

| Delay       | Linear Q-learning | Decentralized | Centralized |
|-------------|-------------------|---------------|-------------|
| minimum     | 886               | 0             | 0           |
| average     | 42399.04          | 9914.86       | 13810.75    |
| maximum     | 174262            | 30578         | 50181.01    |
| std dev     | 35422.18          | 7287.825      | 11122.6     |
| # deadlocks | 4                 | 5             | 10          |

Table 1: Basic statistics on 100 test instances from the same distribution of the training.
Table 1 resumes some basic statistics regarding the performances over the test instances. It is clear that the average delay is smaller on average, with smaller variance, so that DeepRL captures inner non-linearities more efficiently. On percentage, the centralized approach returns 30% higher results than decentralized, whereas the linear Q-learning is five times higher. Deadlocks are comparable among the approaches.

4.2. Second experiment

As a second experiment, the capacity of the three algorithms to generalize to longer trains with different frequencies is analyzed. In particular, for the test set the new parameters adopted are:

- Number of trains $N$ is chosen randomly between 4 and 12 with the following probability distribution: $P(N = 4) = 0.05, P(N = 5) = 0.15, P(N = 6) = 0.15, P(N = 7) = 0.15, P(N = 8) = 0.15, P(N = 9) = 0.1, P(N = 10) = 0.1, P(N = 11) = 0.1, P(N = 12) = 0.05$

- length is chosen uniformly in the set $\{4000, 4500, 5000, 5500, 6000, 6500, 7000, 7500, 8000\}$, to be intended in feet

These two parameters have been chosen because they reflect a great portion of the total complexity in a TD problem.

In this case, the DeepRL models are trained for 20 000 episodes, whereas the linear Q-learning for 50 000.

The approaches were trained on 200 instances from the distribution described in the first experiment and tested on 200 instances with the specifics specified above. Table 2 resumes some basic statistics regarding the performances over these test instances. The decentralized approach has 50% lower delays than the centralized on average, and simple linear Q-learning has roughly 4.5 worst performances. Here, the critical point refers to the number of deadlocks. Linear Q-Learning finds the minimum number of deadlock because, in many test instances, trains are halted before they can reach a potential deadlock, translating into incredibly high delays. Interestingly, the centralized algorithm fails one-third of the time, and this is due to the inability to learn as fast as the decentralized. Figure 4 shows the performance profiles in this case.

Figure 3: Performance profiles for the three algorithms compared together based on the value of the delay obtained on 100 instances of the same distribution of the training set.
Table 2: Basic statistics on 200 test instances from the same distribution of the training.

|          | Linear Q-learning | Decentralized | Centralized |
|----------|-------------------|---------------|-------------|
| minimum  | 50573.49          | 13966.1       | 26807.19    |
| average  | 52318.1           | 14992.37      | 20817.36    |
| maximum  | 256883.1          | 86309.01      | 96922       |
| std dev  | 52318.19          | 14092.37      | 20817.36    |
| # deadlocks | 18              | 26            | 36          |

Figure 4: Performance profiles for the three algorithms compared together based on the value of the delay obtained on 100 with higher number of trains (10 to 15) than the training set.

5. Conclusions

In this study, deep Q-learning has been compared to linear Q-learning for the train dispatching problem. Two architectures were proposed: decentralized and centralized. The former considers a train as an agent with a limited perception of the rail network. The latter observes the entire network and operates on all the trains simultaneously. Numerical analysis shows the supremacy of the deep approach with respect to the linear case. The generalization to larger problems shows room for improvement, both in terms of search strategy and complexity of the network used. In any case, when the instance is generated by the same distribution of the training set, the algorithms can deal efficiently with the problem providing effective solutions in a very short time, being perfectly suitable for online applications.

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References

[1] Dimitri P Bertsekas. *Reinforcement learning and optimal control*. Athena Scientific Belmont, MA, 2019.

[2] V. Cacchiani, D. Huisman, M. Kidd, L. Kroon, P. Toth, L. Veelenturf, and J. Wagenaar. An overview of recovery models and algorithms for real-time railway rescheduling. *Transportation Research Part B*, 63:15–37, 2014.
[3] Alberto Caprara, Matteo Fischetti, and Paolo Toth. Modeling and solving the train timetabling problem. *Operations Research*, 50(5):851–861, 2002.

[4] Yichen Chen, Lihong Li, and Mengdi Wang. Scalable bilinear π learning using state and action features. *arXiv preprint arXiv:1804.10328*, 2018.

[5] Francesco Corman and Lingyun Meng. A review of online dynamic models and algorithms for railway traffic management. *IEEE Transactions on Intelligent Transportation Systems*, 16(3):1274–1284, 2015.

[6] E. Dolan and J. Moré. Benchmarking optimization software with performance profiles. *Mathematical Programming*, 91:201–213, 2002.

[7] Iddo Drori, Anant Kharkar, William R Sickinger, Brandon Kates, Qiang Ma, Suwen Ge, Eden Dolev, Brenda Dietrich, David P Williamson, and Madeleine Udell. Learning to solve combinatorial optimization problems on real-world graphs in linear time. *arXiv preprint arXiv:2006.03750*, 2020.

[8] W. Fang, S. Yang, and X. Yao. A survey on problem models and solution approaches to rescheduling in railway networks. *IEEE Transactions on Intelligent Transportation Systems*, 16(6):2997–3016, Dec 2015.

[9] Harshad Khadilkar. A scalable reinforcement learning algorithm for scheduling railway lines. *IEEE Transactions on Intelligent Transportation Systems*, 20(2):727–736, 2018.

[10] Elias Khalil, Hanjun Dai, Yuyu Zhang, Bistra Dilkina, and Le Song. Learning combinatorial optimization algorithms over graphs. In *Advances in Neural Information Processing Systems*, pages 6348–6358, 2017.

[11] Leonardo Lamorgese, Carlo Mannino, Dario Pacciarelli, and Johanna Törnquist Krassmann. Train dispatching. In *Handbook of Optimization in the Railway Industry*, pages 265–283. Springer, 2018.

[12] Julian Schrittwieser, Ioannis Antonoglou, Thomas Hubert, Karen Simonyan, Laurent Sifre, Simon Schmitt, Arthur Guez, Edward Lockhart, Denis Hassabis, Thore Graepel, et al. Mastering atari, go, chess and shogi by planning with a learned model. *arXiv preprint arXiv:1911.08265*, 2019.

[13] David Silver, Thomas Hubert, Julian Schrittwieser, Ioannis Antonoglou, Matthew Lai, Arthur Guez, Marc Lanctot, Laurent Sifre, Dharshan Kumaran, Thore Graepel, et al. Mastering chess and shogi by self-play with a general reinforcement learning algorithm. *arXiv preprint arXiv:1712.01815*, 2017.

[14] David Silver, Thomas Hubert, Julian Schrittwieser, Ioannis Antonoglou, Matthew Lai, Arthur Guez, Marc Lanctot, Laurent Sifre, Dharshan Kumaran, Thore Graepel, et al. A general reinforcement learning algorithm that masters chess, shogi, and go through self-play. *Science*, 362(6419):1140–1144, 2018.

[15] David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, et al. Mastering the game of go without human knowledge. *nature*, 550(7676):354–359, 2017.

[16] Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*. MIT press, 2018.
[17] Lin F Yang and Mengdi Wang. Sample-optimal parametric q-learning using linearly additive features. *arXiv preprint arXiv:1902.04779*, 2019.