Abstract

We analyze the production of triple pseudoscalar Higgs bosons in the decay channel of $Z \to AAA$ for light pseudoscalar bosons when the corresponding scalar boson is too heavy to be produced by $Z$ decay. Analytic results are obtained both at the tree level and at the one–loop level. The branching fraction can be as large as $10^{-5}$ which should be detectable at LEP.
There is essentially no stringent and model–independent limit on the mass of a pseudoscalar Higgs boson, generically denoted by $A$ in this paper. Such a pseudoscalar boson always exists in the extended Higgs sector beyond the Standard Model (SM). An identical pair of pseudoscalar bosons cannot be produced in pair in the $Z$ decay, as it is forbidden by Bose statistics. The potential limit on the mass of a pseudoscalar Higgs boson comes from LEP experiments. However, in all the analyses\cite{1,2,3}, the pseudoscalar bosons are assumed to be produced by the decay of a physical scalar boson $h$. For the case when the scalar boson is heavy (such as $m_h > m_Z$), no limit on $m_A$ has been extracted yet.

If the scalar partner $h$ is heavy enough, the mode $Z \to hA$ will not be allowed by kinematics. Nevertheless, the channel $Z \to AAZ^* \to AA\ell^+\ell^-$ is allowed if $A$ is light enough. However, its branching fraction was shown \cite{4} to be typically about $10^{-8}$, too small to be detectable for LEP. A pseudoscalar Higgs boson lighter than a $b$ quark can be ruled out by $b \to sA$\cite{5}, however the conclusion will be very much model dependent. (Therefore, it is still worthwhile to make a direct search at LEP even if the pseudoscalar mass is in this light range.) In any case, for a pseudoscalar boson whose mass is heavier than the $b$ quark and whose companion scalar boson is too heavy for the decay $Z \to hA$, the current model independent bound on its mass is very weak.

In this note, we look into another potential discovery channel $Z \to AAA$ for the pseudoscalar boson which may be detectable among the rare $Z$ decays. The channel is particularly interesting when the lightest scalar is heavier than the lightest pseudoscalar boson which can also be an axion. Experimentally, the $AAA$ final states was searched for by LEP detectors \cite{1} assuming that two $A$’s are decay product of a physical scalar boson $h$. A lot of that analysis can probably be borrowed immediately to the case when $h$ is off-shell.

In the popular Minimal Supersymmetric Standard Model (MSSM), at tree level the Higgs masses obey relations\cite{5} $m_h < m_Z < m_H$, $m_h < m_A < m_H$ and $m_{H^\pm} > m_W$, where $m_h$ is defined to be the lighter one of the two scalar bosons. These relationships are modified when one loop corrections, due to top quark, are taken into account. $m_h$ in this case no longer has to be lighter than $m_Z$. However, it is still constrained to be lighter than about 140 GeV (for $\tan \beta > 1$)\cite{5}. With the radiative corrections, it is also possible\cite{3} that $m_A < \frac{1}{2}m_h$. In this sense, our analysis is also very much relevant to the
MSSM in addition to the more general models.

The one–loop amplitude for $Z \to AAA$ via the virtual top–quark was roughly estimated by Li\cite{Li}. Here we study in details both the tree-level process due to a virtual scalar Higgs boson, and the one–loop process due to the top–quark loop.

In Fig. 1, we illustrate one of the Feynman diagrams that the triple pseudoscalar decay mode $Z \to AAA$ occurs through the gauge vertex $Z \to Ah^*$, followed by $h^* \to AA$\cite{Gribov}. Phenomenologically, one can describe the interaction among the scalar and the pseudoscalar Higgs bosons by an effective Lagrangian,

$$\mathcal{L} = \lambda \langle V \rangle hAA.$$  

The coefficient $\lambda \langle V \rangle$ is related to the vacuum expectation value of the Higgs field, the quartic bosonic couplings and also some mixing angles. Its value is about the scale of the electroweak interaction.

The amplitude for $Z(p_Z, \varepsilon_Z) \to A(p_1) + A(p_2) + A(p_3)$ can be written in term of the form factors as,

$$M = |M| = 4\pi\alpha\lambda^2(V)^2 \left[ \frac{2xy - 4(1 - z - a)}{(1 - x + a - h)(1 - y + a - h)} + \frac{z^2 - 4a}{(1 - z + a - h)^2} \right] + 2 \text{ other permutations of } x, y, z.$$  

We simplified our picture by assuming that the contribution of the lightest scalar Higgs boson dominates. The analysis is parameterized model independently such that it would be straight forward to adapt our study to a specific model.

From Eq. (2), we obtain the spin-summed amplitude squared as follows,

$$\sum |M|^2 = 4\pi\alpha\lambda^2(V)^2 \left[ \frac{2xy - 4(1 - z - a)}{(1 - x + a - h)(1 - y + a - h)} + \frac{z^2 - 4a}{(1 - z + a - h)^2} \right] + 2 \text{ other permutations of } x, y, z.$$  

Here $a = M_A^2/M_Z^2$, $h = M_h^2/M_Z^2$ and $x = 2p_1 \cdot p_Z/M_Z^2$, $y = 2p_2 \cdot p_Z/M_Z^2$, $z = 2p_3 \cdot p_Z/M_Z^2$, with $x + y + z = 2$. The allowed $x$ and $y$ ranges are

$$2\sqrt{a} \leq x \leq 1 - 3a,$$

$$1 - \frac{1}{2}(x + d) \leq y \leq 1 - \frac{1}{2}(x - d),$$

with $d = (x^2 - 4a)^{\frac{1}{2}}[1 - 4a/(1 - x + a)]^{\frac{1}{2}}$. 

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The partial width for the channel $Z \to AAA$ is

$$d\Gamma(Z \to AAA) = \frac{M_Z}{256\pi^3} \left( \frac{1}{3} \sum |M|^2 \right) \frac{dx dy}{3!}.$$  \hspace{1cm} (7)

In Fig. 2, we demonstrate the branching fractions of the process $Z \to AAA$ for different scenarios, (a) $M_h = 90$ GeV (dashed), (b) $M_h = 100$ GeV (solid), and (c) $M_h = 110$ GeV (long dashed), for the case $\lambda\langle V \rangle = 100$ GeV. We find that the size of the branching fraction can be as large as $10^{-5}$ for lighter $h$.

Next, we look at the induced amplitude at the one-loop level. This is potentially significant when $M_h$ is very large so that the tree-level amplitude is not important. It is also interesting because it only depends on the coupling of the pseudoscalar boson with the top quark, and independent of the details of the Higgs self-couplings. We shall parameterize the Yukawa coupling of the top quark to the light pseudoscalar boson ($A$) in the following model independent form,

$$\mathcal{L}_{\text{Yukawa}} = g_t \bar{t}_i \gamma^5 t A.$$  \hspace{1cm} (8)

The coupling between the top quark and the $Z$ gauge boson is given in the Standard Model,

$$g'_Z = \frac{e}{4 \sin \theta_W \cos \theta_W}.$$  

For the top–induced amplitude of the process $Z(p_Z) \to A(p_1)A(p_2)A(p_3)$, there are six Feynman diagrams. Under charge conjugation, they pair up into three sets,

$$\mathcal{M} = 3 \frac{g_t^3 g_Z^2 m_t}{96\pi^2} \int \int \int 3! d\alpha d\beta d\gamma \left( \frac{N_{123}^\nu}{(\mu_{123}^2)^2} + \frac{N_{132}^\nu}{(\mu_{132}^2)^2} + \frac{N_{213}^\nu}{(\mu_{213}^2)^2} \right) \cdot (\varepsilon_Z)_\nu,$$  \hspace{1cm} (9)

$$\hat{p}_{123} = \gamma p_1 - \delta p_3 - \frac{1}{2}(\alpha + \gamma \beta - \delta)p_Z,$$  \hspace{1cm} (10)

$$\mu_{123}^2 = p_{123}^2 - \frac{1}{4} M_Z^2 - (\gamma + \delta) m_A^2 + m_t^2 + (\delta p_3 + \gamma p_1) \cdot p_Z,$$  \hspace{1cm} (11)

$$N_{123}^\nu = p_{123}^e (M_Z^2 - 4 m_t^2 - 8 \mu_{123}^2 + 4 \rho_{123}^2) + 4[(p_3 - p_1)^\nu p_Z - 2 p_1^\nu p_3 + 2 p_1^\nu p_1] \cdot \hat{p}_{123}.$$  \hspace{1cm} (12)

The Feynman parameters satisfy $\alpha + \beta + \gamma + \delta = 1$ and $0 \leq \alpha, \beta, \gamma, \delta \leq 1$. In Fig. 3, we carefully show the choice of the momentum flow and the corresponding Feynman parameters. Our results can be easily produced following such convention. A color factor 3 has been explicitly included in Eq. (9). It is straightforward to generalize expressions (10–12) to other cases 132 and 213 by permutations. The charge conjugated diagrams
give equal contributions as one can easily check that $N_{123} = N_{321}$ etc. To arrange this
one-loop amplitude in a similar form as (3), we introduce the form factors $F^t(p_i, p_j)$ in
parallel with $F^h$,

$$F^t(p_1, p_3) = 3 \frac{2 g_t^3 g_Z m_t}{96 \pi^2} \int \int \int 3! d\alpha d\beta d\gamma \left[ (4 p_Z + 8 p_1) \cdot \hat{p}_{132} + (4 p_Z + 8 p_3) \cdot \hat{p}_{312} \right.$$ 

$$+ \frac{M_Z^2 - 4 m_t^2 - 8 \mu^2_{123} + 4 \hat{p}_{123}^2}{(\mu^2_{123})^2} \right]$$

(13)

The above formulas are ready for numerical integrations. However, for the purpose of
illustration we only extract the leading contribution in the large $m_t$ limit even though the
correction can be of order of $M_Z^2/m_t^2 \approx 0.25$. We also remove irrelevant constant terms
from $F^t$. Such constant terms cannot contribute to the overall amplitude for a physical
polarization $\varepsilon$ of the $Z$ boson. We obtain,

$$F^t(p_1, p_3) \approx 3 \frac{g_t^3 g_Z (p_1 \cdot p_3)}{8 \pi^2 m_t^3} .$$

(14)

Unfortunately, such a top-loop induced amplitude is so small that it produces, by itself, a
negligible branching fraction for $Z \rightarrow AAA$ below $10^{-10}$, even we assume a SM coupling
$g_t = (\sqrt{2} G_F)^{1/2} m_t$. This is much smaller than the previous rough estimate[7] by many
orders of magnitude. More likely, the signal of $Z \rightarrow AAA$ comes from the Higgs mediated
process.

Since one requires the scalar boson $h$ to be light enough (such as 90 GeV) in order to
get a large branching ratio, one may also consider the alternative production of $e^+e^- \rightarrow
Z^* \rightarrow hA \rightarrow AAA$. This possibility is already covered in some of the Higgs search
analysis[1, 2].

As the accumulated events of $Z \rightarrow hadrons$ among the four LEP groups have reached
$10^7$, A branching ratio of $10^{-5}$ is potentially detectable. The main difficulty seems to be
finding a clear signal with high efficiency for such events. If the pseudoscalar boson is
heavier than $2 m_b \approx 10$ GeV, then presumably it will decay dominantly into six $b$ quarks.
In MSSM[8], for bosons decay into $b\bar{b}$ about 90% of the time and about $6 - 8\%$ into $\tau^+\tau^-$. For the case $10$ GeV $\approx 2 m_b > m_A > 2 m_\tau \approx 3.5$ GeV, the $A$ boson can decay
dominantly into six $\tau$ leptons or six charm quarks. The two modes are competitive
with each other. One can search for $\tau^+\tau^-$ plus four jets or $\tau^+\tau^-\tau^+\tau^-$ plus two jets
or $\tau^+\tau^-\tau^+\tau^-\tau^+\tau^-$ in increasing detection efficiencies. The answer will depend on the
relative fraction between $\tau^+\tau^-$ and $c\bar{c}$ final states.
For the channel $Z \rightarrow AAA \rightarrow b\bar{b}b\bar{b}b\bar{b}$, the clear signal can be a number of $b$-tagged jets. Similar signal was searched before in the previous Higgs search analysis \cite{1} for lighter on-shell scalar boson $h$ using the same AAA final state. It was concluded\cite{1} that the current limit of this branching ratio is at about $10^{-4}$ level. With more recent data, this limit may be improved by a factor of 3 or more with improved statistics. To improve this further, one probably has to increase the efficiency in the identification of six jets from the three $A$ bosons and the efficiency in b-tagging. Typically a prize (of about $20 - 30\%$) has to be paid to impose tight cut to reject 3, 4 or 5 jets events. The efficiency will be higher for lighter pseudoscalar. In addition, one has to pay a prize for b-tagging. The current b-tagging efficiency of LEP detectors is roughly about $20\%$ per jet. Even if one tags only 3 out of 6 jets, the prize is already quite severe ($20 \times (0.2)^3 = 16\%$). These two effects combine to give $3 - 5\%$ efficiency of identifying $Z \rightarrow AAA$. (OPAL\cite{1} quoted $6 - 11\%$, but with rather high background).

In a general multi-doublet extension of Standard Model, it is possible that the pseudoscalar decays into $\tau$ leptons or $b$ quarks with similar branching fraction, in that case, the best modes to discover pseudoscalar boson may be $b\bar{b}\tau^+\tau^-\tau^+\tau^-$ or $b\bar{b}b\tau^+\tau^-$ final states.\cite{9} As far as we know, these modes have not been seriously searched for by LEP yet.

Our analysis indicates that there is a good chance that one can detect signal of pseudoscalar boson in $Z$ decay with a branching ratio of about $10^{-5}$. It is certainly far from trivial; however, the reward is that the pseudoscalar boson may be already there in the data waiting to be uncovered.

Acknowledgment

We thank Ling–Fong Li for useful discussion and encouragement for the completion of this work. D.C. also wish to thank Willis Lin, Augustine Chen and especially Yuan-Han Chang for illuminating discussions on LEP data analysis. This work was supported in part by the United States Department of Energy under Grant Number DE-FG02-84ER40173 and by a grant from National Science Council of Republic of China.
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Figure Captions

1. One of the tree–level Feynman diagrams for the process $Z \rightarrow AAA$ via the scalar $h$ Higgs boson. Two other diagrams are obtained by permuting the momenta.

2. Predicted branching fractions of the tree–level process $Z \rightarrow AAA$ for different scenarios, (a) $M_h = 90$ GeV (dashed), (b) $M_h = 100$ GeV (solid), and (c) $M_h = 110$ GeV (dot-dashed). We have set $\langle V \rangle = 100$ GeV.

3. One of the one–loop Feynman diagrams for the process $Z \rightarrow AAA$ via the virtual top–quark. The choice of momentum flow and the Feynman parameters are clearly labeled. Two other diagrams are obtained by permuting the momenta.

Fig. 1 One of the tree–level Feynman diagrams for the process $Z \rightarrow AAA$ via the scalar $h$ Higgs boson. Two other diagrams are obtained by permuting the momenta.
Fig. 3 One of the one-loop Feynman diagrams for the process $Z \to AAA$ via the virtual top–quark. The choice of momentum flow and the Feynman parameters are clearly labeled. Other diagrams are obtained by permuting the momenta.
Branching fraction of $Z \rightarrow \text{AAA}$

$\lambda(V) = 100 \text{ GeV}$

$M_A = 110 \text{ GeV}$

$M_A = 100 \text{ GeV}$

$M_A = 90 \text{ GeV}$