The Wouthuysen Field Absorption Trough in Cosmic Strings Wakes

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Abstract. The baryon density enhancement in cosmic string wakes leads to a stronger coupling of the spin temperature to the gas kinetic temperature inside these string wakes than in the intergalactic medium (IGM). The Wouthuysen Field (WF) effect has the potential to enhance this coupling to such an extent that it may result in the strongest and cleanest cosmic string signature in the currently planned radio telescope projects. Here we consider this enhancement under the assumption that X-ray heating is not significant. We show that the size of this effect in a cosmic string wake leads to a brightness temperature at least two times more negative than in the surrounding IGM. If the Voytek \cite{1} et al. experiment confirms a WF absorption trough in the cosmic gas, then cosmic string wakes should appear clearly in redshift surveys of $z = 10$ to 30.
1 Introduction

Before the first luminous sources produced a large enough number of ultraviolet (UV) photons, the 21 cm spin temperature $T_S$ of the cosmic gas was determined by a competition between Compton scattering and collisions. Compton scattering couples $T_S$ to the CMB radiation temperature $T_\gamma$, whereas collisions couple $T_S$ to the much cooler kinetic temperature $T_K$ of the cosmic gas. In higher density regions such as string wakes, collisions will lower the spin temperature and lead to an enhancement in the 21 cm brightness temperature. This enhancement can be large enough to give a signal above noise for a string tension $G\mu \gtrsim 3 \times 10^{-8}$ [2].

Current constraints on the cosmic string tension from analyses of the angular power spectrum of CMB anisotropy maps yield $G\mu < 1.5 \times 10^{-7}$ using combined data from WMAP and SPT microwave experiments [3]. In previous work [2, 4] we considered the signal of a single cosmic string in a 21 cm redshift map as well as the angular power spectrum of a scaling solution of cosmic string wakes [5].

This work on 21 cm cosmic string signatures has ignored the effect of UV radiation. Hydrogen atoms can change hyperfine state through the absorption and re-emission of Lyman-$\alpha$ photons in what is known as the Wouthuysen-Field (WF) effect [6, 7]. Once enough UV photons are produced by the first galaxies, these transitions will again couple $T_S$ to $T_K$ leading to a more negative brightness temperature.

Galaxies may also produce X-rays which heat the cosmic gas, and eventually reionization begins. Since the details of the sources driving these events is uncertain, it is not known when the WF effect will occur. If it occurs before the IGM has been sufficiently heated, this will enhance the absorption signal in the brightness temperature. But if insufficient UV photons are produced, the cosmic gas may reach the radiation temperature before the spin temperature couples to it. It is an open question as to whether this does or does not occur and experiments such as [1] may soon give us an answer. Here will will assume X-ray heating is not significant since our concern is to compare the absorption signal, assuming it does exist, in the cosmic gas to that coming from a cosmic string wake.

Many works [9–16] have calculated the 21 cm brightness temperature in different scenarios for the redshift range $10 < z < 30$. Our purpose here is to show that the physics that leads to an absorption trough in the brightness temperature somewhere in this redshift range, will lead to an even larger effect in a cosmic string wake.
We begin by reviewing the 21 cm brightness temperature both in the IGM and in cosmic string wakes in section 2 and then approximating the possible size of the WF absorption trough. In order to calculate and compare the size of the absorption trough in a cosmic string wake versus the surrounding cosmic gas we need to model the production of UV photons from the first luminous sources. We do this in section 3 and use this to calculate the Lyman alpha coupling coefficient $x_\alpha$. This permits us to calculate the effect of these photons on the brightness temperature. In section 4 we further discuss the measurement of a wake’s brightness temperature. We present our results in section 5 and finally we discuss our conclusions in section 6.

2 The 21 cm brightness temperature of the IGM and string wakes

As explained in [8], the observation strategy for the 21 cm line is to measure the brightness temperature difference, $\delta T_b(\nu)$, a comparison of the temperature coming from the hydrogen cloud with the “clear view” of the 21 cm radiation from the CMB.

$$\delta T_b(\nu) = \frac{T_\gamma(\tau_\nu) - T_\gamma(0)}{1+z} \approx \frac{(T_S - T_\gamma(0))}{1+z} \tau_\nu . \quad (2.1)$$

$\tau_\nu$ is the optical depth and is given by:

$$\tau_\nu(s) = \frac{3hc^2 A_{10} x_{HI} n_H \Delta s \phi(s,\nu)}{32\pi\nu k_B} T_S \approx 2.6 \times 10^{-12} \text{ mK cm}^2 \text{s}^{-1} \frac{x_{HI} n_H \Delta s \phi(s,\nu)}{T_S} \quad (2.2)$$

where $A_{10} = 2.85 \times 10^{-15} \text{s}^{-1}$ is the spontaneous emission coefficient of the 21 cm transition, $x_{HI}$ is the neutral fraction of hydrogen, $n_H$ is the hydrogen number density, $\Delta s$ is the thickness of our hydrogen cloud, $\phi(s,\nu)$ is the 21 cm line profile, and $T_S$ is the spin temperature. Hence,

$$\delta T_b(z) \approx [2.6 \times 10^{-12} \text{ mK cm}^2 \text{s}^{-1}] \frac{1}{1+z} \left(1 - \frac{T_\gamma}{T_S}\right) x_{HI} n_H \Delta s \phi(s,\nu). \quad (2.3)$$

Up to this point the hydrogen cloud could be anything, the cosmic gas or a cosmic string wake. It is the combination $x_{HI} n_H \Delta s \phi(s,\nu)$ and $T_S$ that differ for each. For the cosmic gas the brightness temperature difference is [8]

$$\delta T_b(z) = [9 \text{ mK}] \frac{1 + \delta_b x_{HI}(1+z)^{1/2}}{1 + \frac{\partial v_{pec}/\partial r}{H(z)/(z+1)}} \left(1 - \frac{T_\gamma}{T_S}\right) \left(\frac{\Omega_b}{0.05}\right) \sqrt{\frac{0.3}{\Omega_m 0.7}} \cdot (2.4)$$

and for a cosmic string wake it is given by [2, 4, 5]

$$\delta T_{b\text{wake}}(z) = \frac{[9 \text{ mK}]}{\sin^2\theta} \frac{n_{HI}^{\text{wake}}}{n_{HI}^b} \frac{(1 + \delta_{b\text{wake}} x_{HI}^{\text{wake}}(1+z)^{1/2}}{1 + \frac{\partial v_{pec}/\partial r}{H(z)/(z+1)}} \left(1 - \frac{T_\gamma}{T_S}\right) \left(\frac{\Omega_b}{0.05}\right) \sqrt{\frac{0.3}{\Omega_m 0.7}} \cdot (2.5)$$

where $\theta$ is the angle of the 21 cm ray with respect to the vertical to the wake (see fig 2), $v_{pec}$ is the peculiar velocity, and $\Omega_b, \Omega_m$ are the baryon and matter fractions today. The $\sin^{-2}(\theta)$ factor comes from the line profile $\phi(s,\nu)$. It is derived in [2]. The factor does not lead to a divergence in a physical measurement of the brightness temperature since it cancels out for small $\theta$ when the resolution of the measurement is taken into account as we will further discuss in section 4.
Observing 21 cm radiation depends crucially on $T_S$. When $T_S$ is above $T_\gamma$ we have emission, when it is below $T_\gamma$ we have absorption. Interaction with CMB photons, spontaneous emission, collisions with hydrogen, electrons, protons, and scattering from UV photons will drive $T_S$ to either $T_\gamma$ or $T_K$. Since the timescales for these processes is much smaller than the Hubble time, the spin temperature is determined by equilibrium in terms of the collision and UV scattering coupling coefficients, $x_c$ and $x_\alpha$, as well as the kinetic and colour temperatures $T_K, T_C$:

\[
\left( 1 - \frac{T_\gamma}{T_S} \right) = \frac{x_c}{1 + x_c + x_\alpha} \left( 1 - \frac{T_\gamma}{T_K} \right) + \frac{x_\alpha}{1 + x_c + x_\alpha} \left( 1 - \frac{T_\gamma}{T_K} \right) \tag{2.6}
\]

The optical depth for Lyman alpha photons is given by the Gunn-Peterson optical depth $\tau_{GP} \approx 2 \times 10^5 x_{HI} (z + 1)^{1/2}$. Before reionization is significant ($x_{HI}$ not small), the large $\tau_{GP}$ value means that $T_C$ is driven to $T_K$ of the IGM. For the rest of this work we work with $x_{HI}$ close to 1 and we take $T_C \approx T_K$. Thus:

\[
\left( 1 - \frac{T_\gamma}{T_S} \right) = \frac{x_c + x_\alpha}{1 + x_c + x_\alpha} \left( 1 - \frac{T_\gamma}{T_K} \right) \tag{2.7}
\]

The collision coefficients $x_c = \frac{C_{10} T_\star}{A_{10} T_\gamma}$ for cosmic string wakes were discussed and calculated in [2, 4, 5]. ($C_{10}$ is the de-excitation rate per atom for collisions) We discuss the Lyman coupling coefficient $x_\alpha$ in section 3.

We can approximate the size of the Wouthuysen Field effect in the cosmic gas under the assumption that X-ray heating is negligible. Before the kinetic temperature of the cosmic gas is significantly heated and reionized, we can approximate $T_K \approx 0.02 K (1 + z)^2, x_{HI} \approx 1$. With $T_\gamma = 2.725 K (1 + z)$ we have:

\[
\delta T_b(z) \approx [9 mK](1 + z)^{1/2} \frac{x_c + x_\alpha}{1 + x_c + x_\alpha} \left( 1 - \frac{136}{1 + z} \right), \tag{2.8}
\]

In eq. 2.8 and for the rest of this paper, we ignore the peculiar velocities, baryon density fluctuations, and take $\Omega_b = 0.05, \Omega_m = 0.3, h = 0.7$.

If $x_c + x_\alpha \gg 1$ then $T_S \approx T_K$. At redshift $z \sim 30$ collisions are rare in the IGM except for higher density regions such as minihaloes. In the mean density regions such a condition will not be reached until the Wouthuysen-Field effect is saturated, i.e. $x_\alpha \gg 1$.

\[
\delta T_b(z) \approx [9 mK](1 + z)^{1/2} \left( 1 - \frac{136}{1 + z} \right), \tag{2.9}
\]

We see that if the WF effect is saturated before the cosmic gas is heated, the 21 cm line would show a strong absorption, with $\delta T_b < -170$ mK for $z < 30$. Once heating begins the kinetic temperature approaches the radiation temperature, this strong absorption disappears.

3 UV photons and the Ly$\alpha$ coupling

To calculate the brightness temperature absorption trough due to the Wouthuysen Field effect we first need the Lyman coupling $x_\alpha$ and to do that we need a model for the production of UV photons. The Lyman coupling coefficient can be written as [9–12] :

\[
x_\alpha = \frac{P_{10}(z) T_\star}{A_{10} T_\gamma(z)} = 1.805 \times 10^{11} \text{ cm}^2 \frac{S_{10} J_\alpha(z)}{z + 1} \tag{3.1}
\]
where $T_s = 0.06817$ K is the equivalent temperature of the energy splitting between the two hyperfine states, $A_{10} = 2.85 \times 10^{-15}$ s$^{-1}$ is the spontaneous emission Einstein coefficient, and $T_s(z) = 2.725$ K $(1 + z)$, is the photon temperature. $P_{10}(z)$ is the de-excitation rate per atom from the triplet to singlet hyperfine state: $P_{10}(z) = 0.020564$ cm$^2$s$^{-1}$ $S_n J_\alpha(z)$.

$S_n$ is a correction factor of order one that accounts for spectral distortions [11]. We use the approximation given in eq. 43 of ref. [8]

$$S_n = \exp \left[ -0.803 \left( \frac{T_K}{\text{Kelvin}} \right)^{2/3} \left( \frac{T_{\text{GP}}}{10^6} \right)^{1/3} \right]$$

(3.2)

where $T_{\text{GP}}$ is the Gunn-Peterson optical depth.

$J_\alpha(z)$ is the average Ly$\alpha$ flux in units of cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ sr$^{-1}$. It is given by [11, 12]

$$J_\alpha(z) = \sum_{n=2}^{n_{\text{max}}} J_\alpha^{(n)}(z)$$

(3.3)

where $J_\alpha^{(n)}(z)$ is the background from photons that originally redshift into the Ly$\alpha$ resonance, $\nu_n = (1 - n^{-2}) \nu_{LL}$, and cascade down to Ly$\alpha$.

$$J_\alpha^{(n)}(z) = \frac{(1 + z)^2}{4\pi} f_{\text{rec}}(n) \int_z^{z_n} dz' \frac{c}{H(z')} \epsilon(\nu_n, z')$$

(3.4)

$\nu_n' = \nu_n(1 + z')/(1 + z)$ is the frequency at redshift $z'$ that redshifts into that resonance at redshift $z$, and $z_n$ is the largest redshift from which a photon can redshift: $(1 + z_n)/(1 + z) = (1 - (n + 1)^{-2})/(1 - n^{-2})$. The recycle fraction $f_{\text{rec}}(n)$ is the fraction of Ly$\alpha$ photons that cascade through Ly$\alpha$: $f_{\text{rec}}(2) = 1, f_{\text{rec}}(3) = 0, f_{\text{rec}}(4) = 0.2609$, and monotonically increase thereafter levelling off to 0.359 for large $n$ [11, 12]. Following [10, 12] we truncate the infinite sum at $n_{\text{max}} = 23$ to exclude levels for which the horizon lies within the H II region of a typical galaxy.

The emissivity $\epsilon(\nu, z)$ gives the number of photons emitted at frequency $\nu$ and redshift $z$ per comoving volume, per proper time, per frequency.

$$\epsilon(\nu, z) = f_\nu n_b^0 \epsilon_b(\nu) \frac{d}{dt} f_{\text{coll}}(M_{\text{min}}, z(t)).$$

(3.5)

where $f_\nu$ is the efficiency that gas is converted to stars in haloes, $n_b^0 = \Omega_b \rho_{\text{crit}}^0/m_H$ is the mean baryon number density today, $\epsilon_b(\nu)$ is the number of photons produced at frequency $\nu$ per frequency per baryon in stars, and $f_{\text{coll}}(M_{\text{min}}, z)$ is the fraction of mass collapsed in haloes with mass $M > M_{\text{min}}$.

Following [13], we take $f_\nu = 0.1$ or 0.01 for Pop II or Pop III stars, respectively. In [10] the emissivity $\epsilon_b(\nu)$ is taken as a separate power law in frequency between every pair of consecutive levels of atomic hydrogen so that the total Pop II stars emit 9690 and Pop III stars emit 6520 photons per baryon. We can approximate $\epsilon_b(\nu)$ as a constant equal to 9690/$(\nu_{LL} - \nu_0)$ or 4800/$(\nu_{LL} - \nu_0)$ for either Pop II or Pop III, and find better than 30% or 6% agreement, respectively, with the power law frequency dependence.

To determine $f_{\text{coll}}(z)$ we use the halo mass function $f_{ST}$ of Sheth & Tormen [20] with the parameters given in [14]:

$$f_{\text{coll}}(m, z) = \int_{\Delta(z)}^\infty d(\ln \nu) f_{ST}(\nu)$$

(3.6)
We assume that the minimum mass $M_{\text{min}}$ is set by the virial temperature $T_{\text{vir}} \geq 10^4$ K, as in [11, 12], and we use the relationship between $M_{\text{min}}$ and $T_{\text{vir}}$ for a neutral gas given by:

$$
\frac{M_{\text{min}}}{M_\odot} = 1.05 \times 10^7 \left[ \frac{T_{\text{vir}}}{10^4 \text{K}} \right]^{3/2} \left( \frac{0.3}{\Omega_m} \right)^{1/2} \left( \frac{0.7}{h} \right) \quad (3.7)
$$

The time dependence in $f_{\text{coll}}$ occurs only through the redshift dependence of the linearized critical density $\delta_c(z) = \delta_c^0/D(z) \approx \delta_c^0(1+z)$, where $\delta_c^0 = 1.686$ and $D(z)$ is the linear growth factor. Thus

$$
\frac{d}{dt} f_{\text{coll}}(M_{\text{min}}, z(t)) = (1 + z) H(z) \frac{d}{dz} f_{\text{coll}}(m, z) \bigg|_{m=M_{\text{min}}} = H(z) f_{ST}(\frac{\delta_c(z)}{\sigma(M_{\text{min}})}) \quad (3.8)
$$

and

$$
J^{(n)}_\alpha(z) = \frac{c}{4\pi} f_* \bar{n}_b \bar{\epsilon}_b f_{\text{rec}}(n) \sigma(M_{\text{min}}) \frac{\sigma_0}{\delta_c^0} (1 + z)^2 \int \frac{\delta_c(z)/\sigma(M_{\text{min}})}{\delta_c(z)/\sigma(M_{\text{min}})} d\nu f_{ST}(\nu) \quad (3.9)
$$

We now have everything we need to calculate the Ly$\alpha$ coupling $x_\alpha$. We do this for photons produced by Population II and Population III stars and present our result in figure 1.

![Figure 1](image.png)

**Figure 1.** The Lyman scattering coefficients $x_\alpha$ when UV photons are produced by Pop II (blue) and Pop III (red) stars.

4 The wake’s measured brightness temperature

It would appear from the $(\sin \theta)^{-2}$ factor in eq. 2.5 that there is a singularity at $\theta = 0$ in the wakes brightness temperature. However if one considers the measured brightness temperature this is not so.

As shown in fig. 2, $\theta$ is the angle between the 21 cm ray reaching the observer and the normal to the wake. In a string wake only the planar directions expand in the Hubble flow, whereas the width grows by gravitational accretion, and hence any wake at a nonzero $\theta$ has a velocity gradient along the line of sight that depends on $\theta$. The relative velocity between the back and the front of the wake gives rise to a nonzero width of the 21 cm line and the line profile $\phi(\nu)$ is inversely proportional to this width. The brightness temperature, in turn,
Figure 2. A 21 cm light ray traverses a cosmic string wake of width \( w \).

is proportional to the line profile. As \( \theta \) goes to zero so does the line width, and hence the singularity in the line profile and brightness temperature. However any measurement of the 21 cm line involves a finite frequency resolution and so the measured brightness temperature shows no divergence.

For small \( \theta \), the frequency resolution of the measurement \( \Delta \nu_{\text{res}} \) will be greater than the frequency difference \( \delta \nu_{\text{wake}} \) from photons coming from the front and the back of the wake. Only at large angles will \( \Delta \nu_{\text{res}} \) be greater than \( \delta \nu_{\text{wake}} \).

In an experiment the wake’s measured brightness temperature is:

\[
[\delta T^w_b(z)]_{\text{measured}} = \int dz' W_z(z') \, \delta T^w_b(z') \quad (4.1)
\]

where \( W_z(z') \) is a window function, peaked at \( z \), that depends on the details of the experiment. We take \( W_z(z') \) it to be a top hat function of width \( \Delta z_{\text{res}} \) centred at \( z' \). The redshift resolution \( \Delta z_{\text{res}} \) is given by the frequency resolution of the measurement. For \( \Delta z_{\text{res}} \) is greater than the wake’s redshift thickness \( \Delta z_{\text{wake}} \), we have,

\[
[\delta T^w_b(z)]_{\text{measured}} = \frac{\Delta z_{\text{wake}}}{\Delta z_{\text{res}}} \delta T^w_b(z) + (1 - \frac{\Delta z_{\text{wake}}}{\Delta z_{\text{res}}}) \delta T^\text{IGM}_b(z) \quad \Delta z_{\text{res}} > \Delta z_{\text{wake}} \quad (4.2)
\]

The redshift ratio \( \Delta z_{\text{wake}}/\Delta z_{\text{res}} \) is equivalent to the frequency ratio \( \delta \nu_{\text{wake}}/\Delta \nu_{\text{res}} \) and hence we have

\[
[\delta T^w_b(z)]_{\text{measured}} = \frac{\delta \nu_{\text{wake}}}{\Delta \nu_{\text{res}}} \delta T^w_b(z) + (1 - \frac{\delta \nu_{\text{wake}}}{\Delta \nu_{\text{res}}}) \delta T^\text{IGM}_b(z) \quad \Delta \nu_{\text{res}} > \delta \nu_{\text{wake}} \quad (4.3)
\]

As shown in [2]

\[
\delta \nu_{\text{wake}} = \frac{H(z) \, w \, \sin^2 \theta}{c \, \cos \theta} \, \nu_2 \quad (4.4)
\]
where \( w \) is the wake’s width. \( \delta \nu_{\text{wake}} \) increases monotonically in \( \theta \) until \( \theta \) reaches the value \( \theta_1 \) such that \( \delta \nu_{\text{wake}}(\theta_1) = \Delta \nu_{\text{res}} \). Then for angles between \( \theta_1 \) and \( \pi/2 \), \( [\delta T_b^{\text{wake}}(z)]_{\text{measured}} = \delta T_b^{\text{wake}}(z) \). When this holds, we will get the strongest wake signal, since it will not be diluted by the cosmic gas as it is in eq. 4.3.

Let us see what frequency resolution we need to get a wide range of angles for which \( [\delta T_b^{\text{wake}}(z)]_{\text{measured}} = \delta T_b^{\text{wake}}(z) \). We can use eq. 4.4 to find the value of \( \sin^2(\theta_1) \) for which \( \delta \nu_{\text{wake}}(\theta_1) = \Delta \nu_{\text{res}} \):

\[
\sin^2(\theta_1) = B \sqrt{1 + \frac{B^2}{4} - \frac{B^2}{2}} \quad B = \frac{\Delta \nu_{\text{res}}}{\nu_{\text{21}}} \frac{c}{w H(z)} \quad (4.5)
\]

The wake width \( w \) is proportional to \( G\mu(z + 1)^{-1/2} H(z)^{-1} \) for shock heated wakes and to \( G\mu(z + 1)^{5/2} H(z)^{-1} \) for diffuse wakes [2]. For the small string tensions we are interested in \( (G\mu \lesssim 10^{-8}) \) the wakes tend to be diffuse, and so

\[
B = \frac{0.107 \, G\mu \, \Delta \nu_{\text{res}} \, (z_i + 1)^{1/2}}{(\nu_{\text{21}})^2 \, 1 \, \text{MHz} \, (z + 1)^{5/2}} \quad (4.6)
\]

If we take \( z = 10, z_i = 3000, (\nu_{\text{21}})^2 = 1/3, G\mu = 10^{-9}, \nu_{\text{res}} = 0.01 \, \text{MHz} \), we have that \( B = 0.44 \), and \( \theta_1 = 0.36 \) radians. For these parameters and the range of angles between 0.36 and \( \pi/2 \) radians we have that \( [\delta T_b^{\text{wake}}(z)]_{\text{measured}} = \delta T_b^{\text{wake}}(z) \). Decreasing \( G\mu \) allows us to take a coarser resolution since the diffuse wake widens with decreasing string tension. At larger redshift \( z \) the parameter \( B \) decreases and an even larger range of angles is possible. Thus we can evaluate our wake brightness temperature at a fiducial value of \( \pi/4 \) for comparison with the background IGM value.

## 5 The brightness temperature evolution with Ly\(\alpha \) photons

With this in hand we calculate the brightness temperature for the cosmic gas and for cosmic string wakes (diffuse and shock heated). A cosmic string segment laid down at time \( t_i \) (we are interested in \( t_i \geq t_{\text{eq}} \)) will generate a wake with physical dimensions:

\[
l_1(t_i) \times l_2(t_i) \times w(t_i) = t_i \quad c_1 \quad t_i \quad v_s \gamma_s \quad t_i \quad 4\pi G\mu \nu_s \gamma_s \quad (5.1)
\]

where \( c_1 \) is a constant of order one and \( v_s \gamma_s \) is the speed time the Lorentz gamma factor of the string. After being laid down, the lengths Hubble expand whereas the wake will grow by gravitational accretion. At a later time, parametrized by redshift \( z \), a shock heated wake will have grown to physical dimensions.

\[
l_1(z) \times l_2(z) \times w(z) = \left( \frac{3}{2} H(z) \sqrt{\frac{z_i + 1}{z + 1}} \right)^{-3} \left( c_1 \times v_s \gamma_s \times 4\pi G\mu \nu_s \gamma_s \frac{3}{10} \frac{z_i + 1}{z + 1} \right) \quad (5.2)
\]

where \( z_i \) is the redshift that corresponds to time \( t_i \). A diffuse wake will be wider by a factor discussed in eq. 3.2 of reference [2]. We take \( c_1 = 1 \) and \( (v_s \gamma_s)^2 = 1/3 \) and we restrict ourselves to the wakes laid down at matter radiation equality, \( z_i \sim 3000 \), since these will generically have the largest absorption brightness temperature [2, 4, 5]. We use eqs. 4.3, 4.5 for the brightness temperature with the spin temperature given by eq. 2.7.
The Wouthuysen Field effect couples $T_S$ to $T_K$ when $x_\alpha \approx 1$. For Population II stars we find (see figure 1) that $x_\alpha \approx 1$ at redshift $z \approx 18$, and for Population III stars this occurs at about $z \approx 13$. In figures 3 and 4 we plot the $G\mu$ dependence of the wake’s brightness temperature for these cases. Below $G\mu \lesssim 10^{-8}$, the brightness temperature absorption trough plateaus at a value of approximately $-240 \text{ mK}$ with Pop II stars and $-290 \text{ mK}$ with Pop III stars whereas the brightness temperature of the IGM at $z = 18$ and $z = 13$ corresponding to the Pop II and Pop III stars is $-120 \text{ mK}$ and $-140 \text{ mK}$, respectively. This plateau occurs because for a diffuse wake with small enough string tension, both the kinetic temperature and baryon density of the wake approach that of the cosmic gas [2]. The difference in brightness temperature is only due to the different line profiles arising from the different velocity gradients in a cosmic string wake versus the surrounding IGM.

**Figure 3.** The brightness temperatures (vertical axis) in degrees Kelvin with Pop II stars at a redshift of $z = 18$ as a function of the string tension $(G\mu)_6$ ($G\mu$ in units of $10^{-6}$).

**Figure 4.** The brightness temperatures (vertical axis) in degrees Kelvin with Pop III stars at a redshift of $z = 13$ as a function of the string tension $(G\mu)_6$ ($G\mu$ in units of $10^{-6}$).

Figures 3 and 4 also show that the strongest signal occurs for $G\mu \approx 8 \times 10^{-8}$ and $G\mu \approx 5 \times 10^{-8}$ for Pop II and Pop III stars respectively. These values of $G\mu$ are largely
determined by the shock heating condition $T_{\text{K wake}} \gtrsim 3 T_{\text{K CG}}$ which in turns gives a condition on the smallest $\mu$ at a given redshift for which shock heating will occur:

$$G\mu \gtrsim 1.6 \times 10^{-9}(z + 1)^{3/2}.$$  \hspace{1cm} (5.3)

When this condition is no longer met, our wakes becomes diffuse, with an increasing width but a decreasing overdensity. This occurs at $G\mu \approx 10^{-8}$ for redshifts between 13 and 18. As the string tension decreases even further the decrease in overdensity becomes more important than the increase in width and when $G\mu$ drops below $10^{-8}$ the brightness temperature plateaus, as we discussed in the previous paragraph.

Finally, figures 3 and 4 show a decrease in the absolute value of the brightness temperature as the string tension continues to increase above $8 \times 10^{-8}$ and $5 \times 10^{-8}$ for Pop II and Pop III stars respectively. This is because the kinetic temperature in the shock heated wake increases as $(G\mu)^2$ (see [2, 4]), and hence both the wake’s kinetic and spin temperature approach the temperature of the CMB.

In figures 5 and 6 we plot the brightness temperature of the cosmic gas and of a cosmic string wake with string tension $G\mu \lesssim 10^{-9}$ as a function of redshift. The absorption troughs rapidly become more significant at redshifts lower than those corresponding to an $x_\alpha = 1$, i.e. $z = 18$ or 13, for Pop II or Pop III starts respectively. For example for Pop II stars at $z = 16$, the IGM has a $\delta T_b(16) = -204$ mK, with the $\delta T_{\text{wake}}(16) = -410$ mK, a factor of two more negative. And for Pop III stars at $z = 11$, the corresponding numbers are -220 mK for the IGM, and -450 mK for the wake. Even at redshifts where $x_\alpha < 1$ there is a significant trough. For Pop II stars we have $\delta T_b(20) = -40$ mK, $\delta T_{\text{wake}}(20) = -80$ mK. For Pop III stars we have $\delta T_b(16) = -40$ mK, $\delta T_{\text{wake}}(16) = -80$ mK.

![Figure 5](image)

**Figure 5.** The brightness temperatures (vertical axis) in degrees Kelvin as a function of redshift $z$ (horizontal axis) where the UV photons are produced by Population II stars. The surrounding cosmic gas is in blue. A cosmic string wake with $G\mu = 10^{-9}$ is in red.

6 Discussion and conclusion

We have seen that in the absence of significant heating from X-rays, the Wouthuysen Field effect leads to a large negative brightness temperature on the order of hundreds of millikelvin for the IGM and at least twice that for a cosmic string wake, even for a very small string
Figure 6. The brightness temperatures (vertical axis) in degrees Kelvin as a function of redshift $z$ (horizontal axis) where the UV photons are produced by Population III stars. The surrounding cosmic gas is in blue. A cosmic string wake with $G\mu = 10^{-9}$ is in red.

tension. For small string tensions the wake temperature and the wake baryon density are not significantly different from that of the IGM, however they have decoupled from the Hubble flow and because of that the line profile of the 21 cm ray reaching the observer from a wake leads to a brighter brightness temperature. The enhancement in the brightness temperature relative to the cosmic gas is expressed through the $(\sin \theta)^{-2}$ factor in eq. 2.5.

The WF absorption trough is even greater in shocked cosmic string wakes. There the higher density regions make collisions more important than in the cosmic gas, and they are also hotter. However shocked wakes tend to occur for string tensions larger than $G\mu = 5 \times 10^{-8}$, which are already at the limit of being excluded.

We have not discussed how this effect compares to the extragalactic noise [21]. Experiments such as that in [1] must deal with this noise to measure the IGM’s brightness temperature, and once they do, then experiments with a finer resolution will be able to see cosmic string wake.

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