Evidence for a lost population of close-in exoplanets

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ABSTRACT

We investigate the evaporation history of known transiting exoplanets in order to consider the origin of observed correlations between mass, surface gravity and orbital period. We show that the survival of the known planets at their current separations is consistent with a simple model of evaporation, but that many of the same planets would not have survived closer to their host stars. These putative closer-in systems represent a lost population that could account for the observed correlations. We conclude that the relation underlying the correlations noted by Mazeh et al. (2005) and Southworth et al. (2007) is most likely a linear cut-off in the $M^2/R^3$ vs $a^{-2}$ plane, and we show that the distribution of exoplanets in this plane is in close agreement with the evaporation model.

Key words: (Stars:) Planetary systems – X-Rays: stars

1 INTRODUCTION

When Mayor & Queloz (1995) discovered the first planet around a main sequence star, its small orbital separation came as a surprise to the astronomical community, who had expected to find planets similar to those in our solar system. This motivated work into how and where these ‘hot Jupiters’ form and how they might migrate to their current separations (see for example Lin et al. 1996; Boss 1996). Many close-in planets are now known and intriguingly some planet properties seem to be correlated. Mazeh et al. (2005) suggest that planet mass is correlated with orbital period, while Southworth et al. (2007) find that surface gravity may be correlated with orbital period. These correlations have been re-plotted by various authors as more planets have been discovered (for example Hansen & Barman 2007; Pollacco et al. 2008). Both distributions are reproduced in our Figure 1 using the sample of transiting planets from this work (see Section 2.1).

It has been suggested that these correlations might be a result of the migration mechanism (e.g. Torres et al. 2008 and references therein) or that they are a result of evaporation of close-in planets, such as that reported to be ongoing in HD209458b by Vidal-Madjar et al. (2003) (see also Ben-Jaffel 2007; Vidal-Madjar et al. 2008). Lecavelier Des Etangs (2007) developed a simple model of exoplanet evaporation driven by X-ray/EUV radiation and considered the current evaporation rates of known planets. He found that known planets will not be losing significant mass at today’s irradiation rates. Penz et al. (2008) pointed out that X-ray/EUV irradiation will be much stronger around young stars and showed that, in principle, the mass distribution of close-in exoplanets could be significantly modified by evaporation.

In this paper we use a simple model of evaporation to investigate whether the trends in planet properties identified by Mazeh et al. (2005) and Southworth et al. (2007) can be accounted for by past evaporation. We find good agreement between the distribution of known planets and the cut-off predicted by the evaporation model, and we take this as evidence that a population of close-in exoplanets has been lost to evaporation.

2 METHOD

2.1 Selection of exoplanet sample

We select only transiting planets for our study because we require the planetary radius in order to consider surface gravity and evaporation rates. We selected planets from The Exoplanets Encyclopedia\textsuperscript{1} on 15 June 2008. Key parameters were selected from the encyclopedia and, where necessary, original discovery papers. These parameters allow the calculation of the energy incident upon the planet per unit time, and the total potential energy of the planet. Stellar quantities are shown in Table 1 and planetary properties in Table 2 for the sample of planets used in this work.

We used as many of the known transiting systems as

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2.2 Evaporation model

It has been shown by Lammer et al. (2003) that Jeans escape of material from hot Jupiters cannot drive significant mass loss. Instead they suggest that energy-limited escape of particles due to absorption of EUV/X-ray photons is the most-likely driver of mass loss in these systems.

In order to estimate the evaporation rates of exoplanets we adopt the model of Lecavelier Des Etangs (2007), who used the ratio of the gravitational potential of a planet (Sect. 4.3 of Lecavelier Des Etangs 2007), which suggests may underestimate mass loss by up to 40% when a planet is close to filling its Roche lobe. Further, we neglect any evolution of the planet radius, assuming the current value for each planet. Since unperturbed gaseous planets contract slowly with time, this will tend to underestimate mass loss rates.

2.3 Estimation of X-ray/EUV irradiation

X-ray/EUV emission from stars is believed to be driven by a dynamo action that decreases with time as the star spins down due to magnetic braking (Skumanich 1972). X-ray/EUV emission is seen to increase with stellar rotation rate, up to a maximum of $L_x/L_{bol} \approx 10^{-3}$, where it saturates (Vilhu 1984; Vilhu & Walter 1987). We expect therefore all known exoplanets to have suffered a period of high and approximately constant X-ray/EUV irradiation in the past, when their host stars were young and rapidly rotating. This saturated phase is likely to dominate their irradiation history.

As a result the saturated phase of X-ray emission depends on spectral type, but is not well known. For F type stars a saturation time of 0.1 Gyr is used, taken from Flaccomio et al. (2003) who used optical and X-ray telescope data of five star-forming regions of varying age ($\rho$ Ophiuchi, the Orion Nebula Cluster, NGC 2264, Chamaeleon I, and $\eta$ Chamaeleontis) and two young clusters (the Pleiades and NGC 2516). This is their upper limit for the saturation period of stars with the mass of F stars.

For G type stars we estimate a saturation time of 0.2 Gyr using the results of Güdel et al. (1997), who used the ASCA and ROSAT X-ray satellites to probe the coronae of a sample of nine solar-like G stars, with ages between 70 Myr and 9 Gyr.

The saturation timescale for K type stars is estimated from the observation (e.g. Wheatley 1998) that K stars are saturated in the Pleiades (age of 0.1 Gyr, Basri et al. 1996) but not in the Hyades (age of ~0.6 Gyr, Lastennet et al. 2000). We choose a value of 0.35 Gyr, but adjusting this...
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Table 1. Stellar parameters used in this work. Stellar mass, radius and spectral type are taken from The Exoplanet Encyclopedia unless otherwise noted, and bolometric magnitudes from Allen (2000). \( \rho_* \) is calculated using the standard formula from the data listed. \( L_{\text{sat}}^{\text{x}} \) is calculated by converting the bolometric magnitude into a bolometric luminosity, and using the relations from Pizzolato et al. (2003) as described in Section 2.3. \( \tau_{\text{sat}} \) is estimated as described in Section 2.3.

| Star     | Mass \((M_\odot)\) | Radius \((R_\odot)\) | \( \rho_* \) \(g \text{ cm}^{-3} \) | Spec.Type | Bol.Mag. \((\text{mag})\) | \( \log L_{\text{sat}}^{\text{x}} \) \((\text{ergs s}^{-1})\) | \( \tau_{\text{sat}} \) \((\text{Gyr})\) | ref       |
|----------|-------------------|----------------------|-------------------------------|----------|--------------------------|--------------------------|--------------------------|-----------|
| HAT-P-1  | 1.12              | 1.15                 | 1.04                          | GOV      | 4.2                      | 29.9                     | 0.2                      | a         |
| HAT-P-2  | 1.298             | 1.41                 | 0.65                          | F8       | 3.8                      | 30.06                    | 0.1                      | a         |
| HAT-P-3  | 0.936             | 0.82                 | 2.36                          | K        | 5.6                      | 30.14                    | 0.35                     | a         |
| HAT-P-4  | 1.26              | 1.59                 | 0.44                          | F7V      | 3.5                      | 30.18                    | 0.1                      | a         |
| HAT-P-5  | 1.16              | 1.17                 | 1.03                          | G1V      | 4.5                      | 29.78                    | 0.1                      | a         |
| HAT-P-6  | 1.29              | 1.46                 | 0.59                          | F1V      | 3.2                      | 30.3                     | 0.1                      | a         |
| HD 17156 | 1.2               | 1.47                 | 0.53                          | K0       | 4.2                      | 29.9                     | 0.2                      | a         |
| HD 189733| 0.8               | 0.75                 | 2.64                          | K1-K2    | 5.6                      | 30.04                    | 0.35                     | a         |
| HD 209458| 1.01              | 1.12                 | 1.01                          | GOV      | 4.2                      | 30.6                     | 0.2                      | a         |
| OGLE-TR-113 | 0.78           | 0.77                 | 2.41                          | K        | 5.6                      | 30.14                    | 0.35                     | a         |
| OGLE-TR-132| 1.26           | 1.34                 | 0.74                          | F5       | 3.5                      | 30.18                    | 0.1                      | a         |
| OGLE-TR-56| 1.17              | 1.32                 | 0.72                          | G0       | 4.2                      | 29.9                     | 0.2                      | a         |
| TrES-1   | 0.87              | 0.82                 | 2.23                          | KOV      | 5.6                      | 30.04                    | 0.35                     | a         |
| TrES-2   | 0.98              | 1.00                 | 1.38                          | GOV      | 4.2                      | 30.7                     | 0.2                      | a         |
| TrES-3   | 0.924             | 0.81                 | 2.43                          | G6V      | 5                        | 30.38                    | 0.2                      | a,b       |
| TrES-4   | 1.384             | 1.81                 | 0.33                          | F8V      | 3.8                      | 30.06                    | 0.1                      | a         |
| WASP-1   | 1.24              | 1.38                 | 0.66                          | F7V      | 3.65                     | 30.12                    | 0.1                      | a,c       |
| WASP-2   | 0.84              | 0.83                 | 2.04                          | K1V      | 5.7                      | 30.00                    | 0.35                     | a,c       |
| WASP-3   | 1.24              | 1.31                 | 0.78                          | F7V      | 3.65                     | 30.12                    | 0.1                      | a,c       |
| WASP-4   | 0.9               | 1.15                 | 0.84                          | G7V      | 5                        | 30.38                    | 0.2                      | a,c       |
| WASP-5   | 0.972             | 1.03                 | 1.27                          | G4V      | 4.8                      | 30.46                    | 0.2                      | a,c       |
| WASP-6   | 0.85              | 0.9                  | 1.65                          | G8       | 5.1                      | 30.24                    | 0.2                      | a,c       |
| WASP-7   | 1.28              | 1.24                 | 0.96                          | F5V      | 3.5                      | 30.18                    | 0.1                      | a,c       |
| WASP-8   | 0.87              | 0.91                 | 1.63                          | G6       | 5                        | 30.28                    | 0.2                      | a,c       |
| WASP-9   | 1.1               | 1.05                 | 1.34                          | G0       | 4.2                      | 30.6                     | 0.2                      | a,c       |
| WASP-10  | 0.71              | 0.78                 | 2.09                          | K5       | 6.7                      | 29.7                     | 0.35                     | a,c       |
| WASP-11  | 0.74              | 0.74                 | 2.58                          | K3       | 6.1                      | 29.94                    | 0.35                     | a,c       |
| WASP-12  | 1.1               | 1.05                 | 1.34                          | G0       | 4.2                      | 30.6                     | 0.2                      | a,d       |
| WASP-13  | 1.3               | 1.3                  | 0.84                          | F9       | 4                        | 29.98                    | 0.1                      | a,c       |
| WASP-14  | 1.319             | 1.3                  | 0.85                          | F5V      | 3.5                      | 30.18                    | 0.1                      | a,c       |
| WASP-15  | 1.4               | 1.4                  | 0.72                          | F5       | 3.5                      | 30.18                    | 0.1                      | a,c       |
| XO-1     | 1.00              | 0.93                 | 1.77                          | G1V      | 4.3                      | 30.56                    | 0.2                      | a         |
| XO-2     | 0.98              | 0.96                 | 1.54                          | KOV      | 5.6                      | 30.14                    | 0.35                     | a         |
| XO-3     | 1.213             | 1.38                 | 0.66                          | F5V      | 3.5                      | 30.18                    | 0.1                      | a         |
| XO-4     | 1.32              | 1.55                 | 0.5                           | F5V      | 3.5                      | 30.18                    | 0.1                      | a         |
| XO-5     | 1.00              | 1.11                 | 1.03                          | G8V      | 5.1                      | 30.24                    | 0.2                      | a         |

References:  

\(^{a}\) http://www.exoplanet.eu - retrieved 15/JUN/08.  
\(^{b}\) O’Donovan et al. (2007).  
\(^{c}\) Cameron et al. (2008)  
\(^{d}\) Hebb et al. (2008).

value to the extremes does not affect our conclusions (see Section 3.3).

For each of the exoplanets in our sample, we have calculated the total incident energy over the planet surface for the complete saturation interval of its host star. The stellar parameters required are listed in Table 1 and planetary properties in Table 2. The saturation duration is based only on the spectral type of the star. The X-ray saturation level used can be found by using the saturation level for stars of a given mass (in Table 3 of Pizzolato et al. 2003). Four stars in this work are slightly more massive than those studied by Pizzolato et al. (2003), and a saturated value of \( L_{\text{sat}}^{\text{x}} / L_{\text{bol}} = 10^{-3.9} \) is used, which is the average of the highest mass group in Pizzolato’s work.

A difference between our analysis and the population study of Penz et al. (2008) is that Penz et al. included a range of X-ray behaviour for stars of each type, whereas we assume average properties, since we do not know the individual histories of the objects in our study. Our estimates of X-ray irradiation would be biased if the X-ray emission of stars were correlated with the presence of planets. Indeed, Kashyap et al. (2008) find evidence for excess X-ray emission from stars with known planets. We choose not to modify our estimates by this factor because we are interested in the duration and strength of the saturated phase of X-ray emission, whereas the measurements of Kasyap et al. apply to the weaker X-ray emission of much older stars. Nevertheless, if the results of Kashyap et al. (2008) were found to apply to stars of all ages, then we may have underestimated the X-ray irradiation of our sample by around half an order of magnitude.
Table 2. Planetary parameters used in this work, includes calculated figures for \(\langle a \rangle\), \(a_{\text{roche}}\), PE and \(\langle \frac{dE}{dt} \rangle\).

Figures for \(a\), \(R_p\) and \(M_p\) are taken from The Exoplanet Encyclopedia\(^a\) unless otherwise specified. Densities may be compared with that of Jupiter, 1.33 g cm\(^{-3}\) (Allen 2000).

| Planet     | \(< a >\) (au) | \(a_{\text{roche}}\) (au) | \(M_p\) (M\(_J\)) | \(R_p\) (R\(_J\)) | \(\rho_p\) (g cm\(^{-3}\)) | \(\log(-PE)\) (ergs) | \(\log\left(\frac{\langle dE \rangle}{\langle r_{\text{sat}} \rangle}\right)\) | \(\log\left(\frac{\langle dE \rangle}{\langle r_{\text{roche}} \rangle}\right)\) | Ref. |
|------------|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----|
| HAT-P-1b   | 0.055          | 0.018           | 0.53           | 1.203          | 0.38           | 42.77          | 42.33          | 43.29          | a   |
| HAT-P-2b   | 0.077          | 0.006           | 8.64           | 0.952          | 12.42          | 45.29          | 41.1           | 43.21          | a   |
| HAT-P-3b   | 0.039          | 0.012           | 0.599          | 0.89           | 1.05           | 43             | 42.44          | 43.45          | a   |
| HAT-P-4b   | 0.045          | 0.018           | 0.68           | 1.27           | 0.41           | 42.96          | 42.75          | 43.51          | a   |
| HAT-P-5b   | 0.041          | 0.015           | 1.06           | 1.26           | 0.66           | 43.35          | 42.62          | 43.47          | a   |
| HAT-P-6b   | 0.052          | 0.017           | 1.057          | 1.33           | 0.56           | 43.32          | 42.67          | 43.65          | a   |
| HD 17156b  | 0.184          | 0.01            | 3.08           | 1.15           | 2.51           | 44.31          | 40.65          | 43.01          | a   |
| HD 189733b | 0.031          | 0.012           | 1.15           | 1.154          | 0.93           | 43.46          | 42.26          | 43.09          | a   |
| HD 209458b | 0.045          | 0.018           | 0.69           | 1.32           | 0.37           | 42.96          | 42.79          | 43.6           | a   |
| OGLE-TR-113b | 0.023       | 0.011           | 1.32           | 1.09           | 1.26           | 43.6           | 42.48          | 43.13          | a   |
| OGLE-TR-132b | 0.023       | 0.014           | 1.14           | 1.18           | 0.86           | 43.44          | 42.21          | 42.86          | a   |

References:

\(^a\)http://www.exoplanet.eu - retrieved 15/JUN/08. \(^b\)O’Donovan et al. (2007). \(^c\)Cameron et al.

3 RESULTS

3.1 Past evaporation of known systems

Figure 2 compares the potential energy of planets in our sample with the energy incident during the saturated X-ray emission phase of their parent star. As can be clearly seen, all of the systems considered in this work are able to survive the energy absorbed during the saturation period of their parent star. Of course, each of these systems has already weathered one saturation period, so Fig. 2 really tests whether these systems could survive a second saturation period. Since they all could survive another saturation period, it seems likely that they were only minimally affected by the first period. However, systems close to the dashed line may have lost around half of their mass during the saturation period they have already weathered.

3.2 Evaporation at the Roche limit

We know that all the planets in our sample have survived the saturation phase of their star. However most of these systems could have existed closer in, since most are well outside their Roche limit. In fact, there is a clear deficit of systems close to their Roche limits, as shown in Fig. 3, despite selection effects biasing surveys towards detection of close-in systems. The Roche limit sets the minimum orbital separation for a planet, within which tidal forces become too strong and pull the planet apart.

The Roche limit for a fluid body (gas giant planets are assumed to be best represented by a fluid model rather than a rigid body), as given by Aggarwal & Oberbeck (1974), is

\[ a_{\text{roche}} = 2.44a \log \left( \frac{\rho_p}{\rho_*} \right) \]
3.3 Effect on planet property distributions

Mazeh et al. (2005) and Southworth et al. (2007) have suggested that the apparent correlations found in mass-period and surface gravity-period plots for the known planetary systems may be due to evaporation processes. We have shown that if planets like the known sample existed closer to their parent stars they would not have survived the saturated phase of stellar X-ray/EUV emission, and hence would appear as a void in these plots.

If X-ray/EUV irradiation is important, then the underlying cut-off must be linear in the $M_p^2/R_p^3$ versus $a^{-2}$ plane, as the ratio of potential energy to irradiating energy per unit time defines the planet’s characteristic lifetime. A plot of this form is shown in Figure 5. A sparsely populated region is seen in the lower left, similar to those in Figure 4. For a given class of star it is possible to plot a power law on this diagram, representing a destruction limit, below which planets would either have evaporated completely or left planetary cores that are not currently detectable in transit surveys. The functional form of this destruction limit is

$$\frac{M_p^2}{R_p^3} = \frac{4 L_x^{sat} \tau_{sat}}{3 G a^2}$$

where $a_{\text{roche}}$ is the Roche limit, $a$ is the semi major axis, and $\rho_p$ and $\rho_*$ are the density of the planet and star respectively.

Re-plotting Figure 2 but moving the planets in to their Roche limit results in Figure 4. In this case $\sim 50\%$ of known systems would have absorbed enough energy to move their entire mass out to infinity. Therefore closer-orbiting analogs of the known planets would have suffered catastrophic evaporation. The loss of such systems may account for the cut-off seen in Figure 4 since they would be situated to the left of their current positions, thereby filling the sparsely populated areas of these diagrams. Planets that would have survived at their Roche limit tend to have either high densities and surface gravities, or already exist close to their Roche limits.

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planet hosting stars (Sect. 2.3) are all likely to result in underestimates of evaporation rates.

We note that uncertainties in saturation timescales alone cannot move the destruction limits up to the known planets. This would require timescales of 0.8, 0.6 and 3 Gyr for F, G and K stars respectively, which are unreasonably high values compared with existing X-ray observations (Sect. 2.3).

4 CONCLUSIONS

We have shown that a simple model of exoplanet evaporation during the saturated phase of stellar activity is consistent with the observed population of transiting planets. The model also suggests that these same planets would not, in general, have survived at their Roche limit. We conclude that there is likely to have been a population of close-in planets that have been lost to evaporation. If this is the case there is no need for migration models to account for the observed lack of planets close to their Roche limits. We note that uncertainties in saturation timescales alone cannot move the destruction limits up to the known planets. This would require timescales of 0.8, 0.6 and 3 Gyr for F, G and K stars respectively, which are unreasonably high values compared with existing X-ray observations (Sect. 2.3).

This lost population can account for the correlations between mass-period and surface gravity-period suggested by Mazeh et al. (2005) and Southworth et al. (2007). Further to this we suggest that a cut-off in the $M_p^2/R_p^3$ versus $a^{-2}$ plane is the underlying relation, and we show that evaporation in the saturated phase defines destruction limits that are a good match to the position of the lower edge of the observed distribution.

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