Entropy Production at RHIC

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For central heavy ion collisions at the RHIC energy, the entropy per unit rapidity $dS/dy$ at freeze-out is extracted with minimal model dependence from available experimental measurements of particle yields, spectra, and source sizes estimated from two-particle interferometry. The extracted entropy rapidity density is consistent with lattice gauge theory results for a thermalized quark-gluon plasma with an energy density estimated from transverse energy production at RHIC.

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Lattice quantum chromodynamics (QCD) calculations indicate that at zero net baryon density at a critical temperature of $T_c \sim 170$ MeV and energy density $\epsilon_c \sim 1$ GeV/fm$^3$ a color deconfined and a chirally restored quark-gluon plasma (QGP) phase is formed. These energy densities are attained in relativistic heavy ion collisions, where it is believed that a dense system of quarks and gluons is created which undergoes rapid collective expansion before the partons hadronize and eventually decouple.

The ongoing experimental program at RHIC is devoted to the search for the signals of QGP. Unfortunately, measurements are confined to final-state particles which are mostly hadrons. One interesting quantity that may provide valuable insight into the state of the matter in the early stages of the collision is the net entropy which is roughly conserved between initial thermalization and freeze-out. After freeze-out, when particles are free-streaming, the entropy is fixed. Entropy would also be conserved during the expansion stage if the mean free paths were very short. Viscous effects, shock waves, and the decoupling process can only increase the entropy. Thus, the entropy from the final state provides an upper bound for the entropy of the initial state.

Compared to a pion gas at the same energy density, the QGP is a high-entropy state as can be understood by considering the equation,

$$\Delta S = \frac{Q}{T}, \quad (1)$$

where $\Delta S$ is the change in entropy resulting from the addition of an amount of heat $Q$ into a fixed volume at temperature $T$. Lattice calculations show that the transition is nearly a first order with a distinct peak in the specific heat near $T_c$. In this region, the temperature stays relatively fixed while the energy density changes substantially. Thus, the existence of a latent heat keeps the temperature lower than a pion gas at the same energy density, which results in a higher entropy as can be seen from Eq. (1). At high energy densities, the temperature of a QGP remains lower than that of the pion gas due to the disparity in the number of degrees of freedom.

Asymptotically, the energy density of a QGP approaches that of a massless gas of quarks and hadrons,

$$\epsilon = \left(N_B + \frac{7}{8}N_F\right)\frac{\pi^2}{30}T^4, \quad (2)$$

where $N_B$ and $N_F$ are the number of Bosonic and Fermionic degrees of freedom. Whereas a pion gas has 3 bosonic degrees of freedom, a QGP has 47.5 effective degrees of freedom if one considers gluons and the three light quark flavors. Thus, relative to a pion gas, the QGP has a lower temperature and higher entropy. If the partonic degrees of freedom were not fully liberated at RHIC, one would thus expect a lower entropy for the same energy density. Furthermore, if one were to consider a full hadron gas, incorporating all the resonances from the particle data book at energy densities far above the point where the hadronic models are justified, one would expect to have a state of matter with even more entropy than a QGP for temperatures near or above 250 MeV.

The entropy density per unity rapidity $dS/dy$ can be calculated from the final phase space density, $f(p,r,t_f)$, evaluated at the freeze-out time $t_f$. Spectra provide information only about the final momenta, $p$. The spatial information in $f$ must be determined by correlations which can provide the characteristic dimensions of $f(p,r,t)$ for given values of $p \parallel \perp$. In fact, it has been suggested that the softer equation of state associated with a first order phase transition to QGP with a large latent heat results in long lived and/or large source sizes. The fact that the QGP models tend to accurately reproduce spectra while over-predicting source sizes suggests that the entropy at RHIC might fall below the expectations for a quark-gluon plasma.

In this letter we extract $dS/dy$ for hadrons at freeze-out from yields and spectra combined with the source dimensions in central Au+Au collisions at the RHIC energy of $\sqrt{s} = 130$ A GeV. By comparing this estimated net hadron $dS/dy$ with the entropy at the early stage in the parton medium, we address several key issues concerning...
the initial stage of the collisions and the equilibration of partonic degrees of freedom.

In the Landau quasi-particle approximation the entropy is given by

\[ S = (2J + 1) \int \frac{d^3 r d^3 p}{(2\pi)^3} \left[ - f \ln f + (1 \pm f) \ln(1 \pm f) \right] \]

\[ \approx (2J + 1) \int \frac{d^3 r d^3 p}{(2\pi)^3} \left[ - f \ln f + f + f^2/2 \right], \quad (3) \]

where the \( \pm \) sign corresponds to bosons/fermions with total spin \( J \). In the classical limit \( f \ll 1 \), the \( f^2 \) and higher orders of \( f \) can be neglected. The classical limit is sufficient for all particles except for pions. Even for pions, the \( f^2 \) term contributes only on the 3% level. Assuming a three-dimensional Gaussian form for the density profile in coordinate space, the single-particle phase space density at any time \( t \) at or after freeze-out [14] is

\[ f(r, p, t) = f_{\text{max}}(p) \exp \left( \frac{-x^2}{2R_x^2} + \frac{y^2}{2R_y^2} + \frac{z^2}{2R_z^2} \right), \quad (4) \]

where \( R_x, R_y \) and \( R_z \) are implicit functions of \( p \). Since spectra are obtained from integrating the phase space density, \( f_{\text{max}} \), and therefore \( f(p, r, t) \), can be determined from the radii and spectra.

\[ \frac{dN}{dp} = (2J + 1) \int \frac{d^3 r}{(2\pi)^3} f(r, p), \]

\[ f_{\text{max}}(p) = \frac{(2\pi)^{3/2}}{(2J + 1)} \frac{dN}{dp} \frac{1}{R^3}, \quad (5) \]

where \( R^3 \) is the product of the three radii. Chemical equilibration has not been invoked as the expressions do not involve temperature or chemical potentials. Instead the phase space densities and therefore the entropies will be functions of the spectra and the source sizes which can be inferred from experiment.

In Hanbury-Brown Twiss (HBT) correlation measurements for identical particles, if the single-particle emission source is characterized by a Gaussian as in Eq. (4), the Gaussian widths are each inversely proportional to the source dimensions in the canonically conjugate spatial variables [14]. One then obtains the two-particle correlation function in terms of the radii,

\[ C(K, q) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} \]

\[ = 1 + \exp \left( -q_0^2 R_0^2 - q_1^2 R_1^2 - q_2^2 R_2^2 \right), \quad (6) \]

where \( K = (p_1 + p_2)/2 \) and \( q = p_1 - p_2 \). The momentum difference \( q \) has been decomposed into “out-side-long” components defined by the beam direction \( q_0 \), parallel to the total momentum of the pair in the transverse plane \( q_0 \), and the direction orthogonal to the above two direction \( q_{\perp} \). Data are generally analyzed in the longitudinal co-moving system (LCMS) frame in which the longitudinal component of the pair momentum vanishes.

The Gaussian widths of Eq. (6) in the LCMS frame are related to the radii in the two-particle rest frame via a contraction factor \( \gamma = E_T/m \). Phase space densities depend on the product of the three radii. In the rest frame of the pair, this product is referred to as \( R^3_{\text{inv}} = \gamma R_x R_y R_z \). Substituting the HBT radii in Eq. (14), the entropy rapidity density from Eq. (3) in the classical limit is then

\[ \frac{dS}{dy} = \int \frac{d^2 p_T}{p} \frac{dN}{d^3 p} \left[ \frac{5}{2} \ln \frac{F(p)}{1} \exp \left( \frac{1}{2} \ln F(p) \right) \right], \]

\[ F(p) = \frac{1}{m(2J + 1)} \frac{dN}{d^3 p} \frac{1}{R^3_{\text{inv}}}. \quad (7) \]

Thus, \( dS/dy \) of the final state can be estimated from the Gaussian source-size parameters and spectra.

The two-pion correlation function has been measured by STAR [16] and PHENIX [17] for \( \approx 11\% \) central Au+Au collisions at \( \sqrt{s} = 130 \) A GeV at midrapidity. In spite of about 70% increase in pion multiplicity from SPS to the RHIC energy the extracted radius, \( R_{\text{inv}} \approx 6.3 \) fm at RHIC, was very similar to what was measured at the SPS [13]. Moreover, the prediction of \( R_0 \) larger than \( R_s \) as a QGP signature was not observed. Instead \( R_0/R_s \approx 1 \) was measured over a broad range of transverse momenta \( K_T = (p_{T1} + p_{T2})/2 \). The smaller values of \( R_l \) and \( R_o \), as compared to hydrodynamic models, may be suggestive of a rather high final state temperature.

In our estimate of \( \pi^- \) entropy we use the scaling relation \( R_l = A_l/\sqrt{m_T} \) where \( m_T = \sqrt{K_T^2 + m^2} \) with the fitted value \( A_l = 3.32 \pm 0.03 \) fm GeV\(^{1/2} \) [17]. While for the s-o direction we employ the blast-wave inspired parameterization, \( R_s = R_o = R_{\text{geom}}(1 + \beta^2 m_T/T) \), where \( R_{\text{geom}} = 6.7 \pm 0.2 \) fm, \( \beta = 0.69 \) and the freeze-out temperature is \( T = 125 \) MeV [17]. The measured corresponding single particle \( \pi^- \) spectra at RHIC [20] (after correcting for electromagnetic and weak decays) is fitted to a Bose-Einstein distribution \( E dN/d^3 p = A'/[\exp(m_T/T_{\text{eff}}) - 1] \) where \( A' = 739 \) GeV\(^{-2} \) and effective (slope) temperature \( T_{\text{eff}} = 197 \) MeV. The entropy per unit rapidity estimated from Eq. (7) is \( dS/dy|_{\pm} = 680 \), and is shown in Table I.

Pions resulting from electromagnetic decays, i.e., those from \( \eta \) and \( \eta' \) decays, should be subtracted from the spectra. From a thermal model with a chemical equilibrium temperature of 170 MeV, we estimate that 12% of the pions come from such decays and correct the spectra accordingly. We add in the entropy of the \( \eta \) and \( \eta' \) by assuming that the \( \eta \) contributes the same amount of entropy as a single species of kaon which has a similar mass, and that the \( \eta' \) contributes half the amount of entropy as the average of protons and anti-protons. The factor of 1/2 is applied to account for the lower spin degeneracy of the \( \eta' \). The entropies of the \( \eta \) and \( \eta' \) are displayed in Table I.
TABLE I: The entropy content per unit rapidity, $dS/dy$, for hadrons produced at midrapidity in $\sim 11\%$ most central Au+Au collisions at the RHIC energy of $\sqrt{s} = 130A$ GeV.

|   | $\pi$ | $\eta$ (y') | $K^-(K^+)$ | $\bar{p}$ (p) | $\Lambda^{+}(\Lambda^{+})$ | $\Sigma^{+}(\Sigma^{+})$ | $\Sigma^{-}(\Sigma^{-})$ | $\Xi^-(\Xi^+)$ |
|---|---|---|---|---|---|---|---|---|
| $d$ | 680 | 256 (69) | 256 (286) | 118 (159) | 81 (111) | 31 (42) | 23 (27) | 5 (5) |

For a dilute gas in chemical equilibrium, it was demonstrated that the entropy per nucleon carried by the nucleons is related to the ratio of the deuteron to the proton yield,

$$S = 5/2 - \ln R_{dp}/3\sqrt{2}.$$  

(8)

If, within a given phase space cell, the ratio of a species $a$ to the product of two species $b$ and $c$, which can undergo a reaction $a + b \leftrightarrow c$, is determined by the ratio of available phase space for bound and unbound states, one can determine the Gaussian source parameters from coalescence arguments.

$$R_{inv}^3 = \pi^{3/2} \frac{(2J_a + 1)}{(2J_b + 1)} \frac{N_a dN_a}{m_a d^3p_a} \frac{N_b dN_b}{m_b d^3p_b} \frac{N_c dN_c}{m_c d^3p_c}.$$  

(9)

This expression is based on the assumption that the relative populations within a given phase space cell are determined by the ratios of the available phase space for bound and unbound states; whereas Eq. (8) requires the assumption that all phase space cells are equilibrated according to a single chemical potential. Once $R_{inv}$ is obtained as a function of the momentum, one can extract the entropy from Eq. (7).

Assuming that $\bar{p}$ and $\pi$ possess identical phase space densities, which in turn is identical to that for $d$ at the same relative velocity, one can determine a source size for $\bar{p}$ using measured anti-proton and anti-deuteron spectra. Figure 1 shows the $R_{inv}$ for antiprotons (open triangles) estimated (using Eq. (9)) from the measured spectra of $d$ and weak-decay-corrected $\bar{p}$s at three $K_T$ bins for the $\sim 18\%$ most central Au+Au collisions at $\sqrt{s} = 130A$ GeV. As expected, the radii for the massive $\bar{p}$ exhibits a smaller decrease with $K_T$ compared to that of pions. The $\bar{p}$ transverse momentum spectra for this centrality from the STAR [23] and the PHENIX [21] data are corrected for the weak hyperon decay and matched in the overlap region. We fit this spectra in the range $0.25 < p_T < 3.2$ GeV/c with a Gaussian distribution $EdN/d^3p = dN/dy \exp(-p_T^2/2\sigma^2)/(2\pi\sigma^2)$ resulting in a yield of $dN/dy = 11.4$ and width $\sigma = 872$ MeV. Given the limited range and poor statistics for anti-deuterons, we choose a constant average value, $R_{inv} = 5.34$ fm, independent of $K_T$. Applying Eq. (7), the entropy carried by the anti-protons is then $dS/dy|_{\bar{p}} = 118$.

The procedure to estimate the $\bar{p}$ source radii can be identical applied to extract the kaon radii using the $\phi$ meson, $\phi \leftrightarrow K^+K^-$, in place of the deuteron. The coalescence arguments used in Eq. (8) ignore the binding energy of the composite particle, which is justified if the binding energies are much less than the breakup temperatures. This is an excellent assumption for the deuteron and a reasonable assumption for the $\phi$ which is a resonance with a binding energy of $B = -30$ MeV. Although we neglect the binding energy in this analysis, one could adjust the expression for $R_{inv}^3$ by a factor of $e^{B/T}$ to account for the binding energy.

![Figure 1](attachment:image.png)

**FIG. 1**: Top panel: The $K_T$ dependence of HBT radii for $\pi^-$ (open circles) and $\bar{p}$ (open triangles) estimated from invariant yields at midrapidity for central Au+Au collisions at $\sqrt{s} = 130A$ GeV. The measured $\pi^-$ radii (solid circles) from PHENIX [20] are also shown. Bottom panel: The average freeze-out phase space density as a function of $K_T$ for the hadrons.
with that of the preliminary three-dimensional measurements \cite{27} at the highest RHIC energy of $\sqrt{s} = 200$ A GeV. We note that the $m_T$ dependence of the HBT radii $R_{\text{inv}} = A/\sqrt{m_T}$ for these hadrons at RHIC are not consistent with one scaling coefficient as observed at the SPS energy \cite{28}; the fitted values here are $A = 3.81, 4.45, 5.32$ fm GeV$^{1/2}$ for $\pi^-, K^-, \bar{p}$. The measured single particle $m_T$ spectra for $K^-$ and $K^+$ \cite{26} are fitted with an exponential function $E dN/dp = dN/dy \exp(-m_T - m_{K^\mp})/T)/[2\pi T (m_T + T)]$ that results in a rapidity density of $dN/dy = 38.2(42.1)$ for $K^-(K^+)$ using a slope parameter of $T = 280$ MeV. Inserting the spectra along with the the parameterization of $R_{\text{inv}}$ into Eq. (4) yields $dS/dy|_{K^-} = 256$ and $dS/dy|_{K^+} = 286$. We assume that the neutral (anti)kaons carry the same entropy as the corresponding charged (anti)kaons.

The measured \cite{24} spectra for $\bar{\Lambda}$ for the 11% most central collisions at midrapidity has feed-down from the electromagnetic decay of the $\Xi^-$ and the weak decays of the $\Xi^+\Xi^0$, and $\Xi^0$. The feed-down from the charged $\Xi$ and $\Omega$ states is corrected by using the exponential parametrization of the the measured spectra for $\Xi^-$ and $\Omega^-$ of Ref. \cite{29}. The corrected $\bar{\Lambda}$ spectra is then fitted with an exponential function resulting in $dN/dy = 7.4$ and slope parameter $T = 390$ MeV. The entropy rapidity density for the antihyperons estimated by assuming that they have the same source radii as the antiprotons are presented in Table I. To estimate $dS/dy$ for $\Sigma^+$, we note that the particle density in the statistical model in the Boltzmann approximation is $N/V = gm^2 T K_2 (m/T) \exp(\mu/T)/(2\pi^2)$, where the notations are as usual. Assuming isospin symmetry $\Sigma^+ \approx \Xi^0 \sim \Sigma^-$, and same temperature $T_{\Xi} \approx T_{\Sigma} = 173$ MeV, the thermodynamical model gives the ratio $dN/dy = 1.47$ and thus $\Sigma^+ = (\bar{\Lambda} + \Xi^0)/2.94$. The $dS/dy$ extracted under this assumption for $\Sigma^+$ is listed in Table I.

To extract the entropy density for baryons, we employ the measured ratio for the yields $\bar{\pi}/p = 0.72, \bar{\Lambda}/\Lambda = 0.71, \Xi^+/\Xi^- = 0.83, \Omega^+/\Omega^- = 0.95$ \cite{30}, and assume that all (anti)baryons have the same source radii as antiprotons. Furthermore, the ratios are found to be independent over a large range a $p_T$ suggesting that the transverse momentum distribution and thus the slope temperatures of the baryons and their antibaryons can be taken to be identical. The $dS/dy$ for the baryons thus estimated are shown in Table I.

The values of $R_{\text{inv}}$ shown in Fig. 1 (top panel) may be used in conjunction with the corresponding spectra of the hadrons to calculate at a given $p_T$ the average phase space density at freeze-out $(f(p_T)) = F/2\sqrt{2}$, where $F$ is given by Eq. (4). This is displayed in Fig. 1 (bottom panel) where the pion $(f)$ at RHIC is remarkably higher than the values extracted at SPS \cite{18,31} and AGS \cite{32} energies.

Under the assumption of isospin symmetry the total entropy rapidity density for the hadron-gas at freeze-out for central collisions at RHIC is 4451. Despite the large uncertainty in source sizes, this number is surprisingly robust since the radii only enter logarithmically. A 30% change of $R_{\text{inv}}$ changes the entropy by 0.26 units per particle. For baryons, which have approximately 10 units of entropy per particle, the overall entropy would change by only a few percent. Another source of error comes from estimating the contribution of the $\eta$ and $\eta'$. However, since an $\eta$ carries two to three times more entropy than a pion, and since the $\eta$ decays into two or three pions, the net entropy is rather insensitive to the fraction of pions assigned to $\eta$s. The same would be true if one were to explicitly include the entropy of the $\phi$ meson while correspondingly reducing the kaon yields. Since the yields are not measured to better than 10% accuracy, the systematic error in estimating the final-state entropy is estimated $\sim 10%$.

We now estimate the entropy per rapidity density generated in the QCD partonic matter in the early stage at RHIC. Assuming boost-invariant longitudinal expansion, the energy density averaged over transverse coordinate is

$$\epsilon_{\text{Bj}}(\tau) = \frac{dE_T}{dy} \frac{1}{\pi R^2}.$$ (10)

Using $dE_T/dy \approx 542$ GeV \cite{33} and $\pi R^2 \approx 120$ fm$^2$ for the $\sim 11%$ most central Au+Au collisions, one obtains $\epsilon(\tau) = 4.5$ GeV/fm$^3$ at a proper time $\tau = 1$ fm/c. Present data from lattice QCD simulation at $\mu_B = 0$ shows a rapid increase of $\epsilon/T^4$ by about an order of magnitude within a narrow temperature range $\Delta T < 0.1 T_c \sim 16$ MeV near the critical temperature $T_c$. This may indicate a first order transition, or nearly first-order transition. For three degenerate quark flavors, $n_f = 3$, with a mass $m/T = 0.4$, transition to a plasma phase occurs at $T_c = 154$ MeV at a critical energy density $\epsilon/T^4 \approx 6$. Assuming the initial matter at RHIC is already thermalized at 1 fm/c, lattice data indicates that $\epsilon(\tau) = 4.5$ GeV/fm$^3$ would correspond to an (average) initial temperature of $T \approx 290$ MeV and pressure density $p \approx 3T^4 = 1.1$ GeV/fm$^3$. The resulting entropy density for the 3 flavor system is $s = (\epsilon + p)/T = 24.3$ fm$^{-3}$, and the average entropy rapidity density at $\tau = 1$ fm/c is

$$\left.\frac{dS}{dy}\right|_{\text{QGP}, n_f=3} = s \approx 2922.$$ (11)

The Bjorken estimate of the energy density only provides a lower bound since it ignores the energy of the local longitudinal (along the beam axis) motion and neglects the longitudinal work caused by the expansion of the plasma. The inclusion of local longitudinal motion would increase the estimate by a factor of 4/3 if the emitted particles were massless and if there was no transverse collective flow. Accounting for transverse flow and
masses should result into the increase being in the 10-15% range. For massless particles undergoing a purely one-dimensional Bjorken expansion, the energy density would fall as \( \tau^{-4/3} \), and \( dE_T/dy \) would change by a factor of \( \tau^{-1/3} \). This would be a factor of 2 for times between 1 and 8 fm/c. The loss of \( E_T \) from longitudinal work is proportional to the \( \pi R^2 \int Pd\tau \) in hydrodynamic models, or \( \pi R^2 T_\perp d\tau \) if the stress tensor is non-isotropic, i.e., viscous effects are included. The effects of viscosity have been studied within the context of cascade models. Whereas, one-dimensional hydrodynamic models might show that \( dE_T/dy \) is higher than that at breakup, the inclusion of viscosity reduces that increase to the 50% range \[33, 36\]. After including the effects of longitudinal work and longitudinal motion, estimates of the initial average energy density might be in the range of 7 ± 2 GeV/fm\(^3\), depending on the equation of state and the viscosity.

Of course, the initial entropy is less than the final state entropy. One expects the entropy to increase during the reaction by the order of 10% from viscous effects or shocks \[37\]. If one were to lower the final-state estimate of \( dS/dy \) by 10% it would come close to the value of \( dS/dy \) predicted by the lattice gauge calculation. A lack of complete thermalization of the initial state would also point to a lack of entropy at early times. If the initial energy was stored in classical gluon fields \[38\], the entropy would be negligible. An underpopulation of quark degrees of freedom would also lower the entropy. Since flavor and color charges cannot sample the entire volume, finite-size effects can also reduce the initial entropy, but only by a few percent \[39, 40\]. Since the amount of entropy produced during the expansion is unknown, it is impossible to rule out that the initial state might have been of lower entropy.

Thus, our estimate of the final-state entropy, \( dS/dy = 4451 \), does not seem inconsistent with expectations from lattice calculations from analyzing Fig. 2. It therefore seems puzzling that hydrodynamic calculations based on lattice-inspired equations of state matched spectra and failed to fit HBT sizes \[8, 10, 11, 12\]. An overestimate of a factor of two in \( R_{inv}^3 \) would suggest an overestimate of the entropy of the order of \( \ln(2) \) per pion, or \( \sim 400 \) units per rapidity. However, it appears from the data that other degrees of freedom, especially the baryonic degrees of freedom, compensate for the lack of entropy in the pions. Although the pions comprise two thirds of the final-state particles, they provide less than half the final-state entropy. The shift of entropy from pions to the (anti)baryon sector was discussed by Rapp and Shuryak \[41\] as a natural consequence of an increased phase space density of pions. This follows from the reaction \( p\pi \leftrightarrow N\pi \). Since pion number population is difficult to maintain, the overpopulation of pions was expected in an isentropic expansion \[42\]. An additional factor in increasing the overpopulation of pions would be a lowering of hadronic masses associated with chiral symmetry restoration \[43\]. A lowering of hadron masses would lead to an increase in the population of heavier hadrons, many of which would decay back to multiple pions after the normal QCD vacuum is restored \[44\].

It should be stressed that the small pionic sources extracted from HBT remain a puzzle. Since the overall entropy of the final state is consistent with the lattice gauge theory, the possibility remains open that a quark-gluon plasma is indeed created at RHIC but that the dynamics of the expansion and decoupling somehow differ from current hydrodynamic descriptions \[45, 46\].

To illustrate the sensitivity of the entropy to the number of degrees of freedom, Fig. 2 also displays \( dS/dy \) for a pion gas. Since the pion gas has only three effective degrees of freedom, it has a much higher temperature for a given energy density than does the lattice equation of state. From Eq. 4 it therefore has a lower entropy when compared at the same energy density. At high temperatures, a hadron gas would effectively incorporate more degrees of freedom, and for energy densities near or above 10 GeV/fm\(^3\) would have more effective degrees of freedom than a plasma due to the extremely large number of baryonic resonances with masses between 1.0 and 2.0 GeV. However, since the volume of a baryon is in the neighborhood of 2 fm\(^3\), energy densities above 0.5 GeV/fm\(^3\) preclude a serious description in terms of a non-interacting hadron gas.

In conclusion, we have calculated the final-state entropy from 130A GeV Au+Au collisions at RHIC to be \( dS/dy = 4451 \). We estimate that this number, which is based solely on measured spectra and extracted source-size radii of the principal final-state constituents, to have
a systematic uncertainty of 10%. This number is not out of line with expectations from lattice gauge theory. Improved experimental measurements of spectra and source size might reduce the systematic uncertainties to the 5% level, but it is the uncertainty in the reaction dynamics between the initial and final stages of the collision that precludes our making a more definitive conclusion about the initial state entropy. Although our number represents an important upper bound on the initial state entropy, the amount of entropy generated throughout the reaction and the initial energy density remain objects of conjecture. These issues will be settled largely by other observables. For instance, elliptic flow measurements are sensitive to the viscosity of the initial stages [47] and HBT measurements provide important three-dimensional constraints about the dynamics.

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