Schwinger effect at modern laser facilities

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Abstract. The theoretical and experimental investigation of physical processes in strong fields of different nature (electromagnetic, gravitational, etc.) is one of the important directions of modern physics. Particular interest is devoted to the area of extremely strong fields, in which qualitatively new effects become important due to the restructuring of the physical vacuum which accompanies the creation of matter from the vacuum at modern laser facilities. Such kind of time-dependent strong field vacuum effects can be appropriately described within a kinetic theory approach as an effective instrument of theoretical investigations. A short review of recent achievements in the direction of the dynamical Schwinger effect is given in this contribution.

1. Introduction: The physical vacuum in strong, time-dependent fields

The physical vacuum (PV) is understood as the state of a certain region of space, which is characterized by the absence of observable particles and physical fields. In the ordinary sense this is emptiness, a perfect vacuum. Nevertheless, the PV represents a specific material medium. In order to describe its simplest properties, it is necessary to clarify some usual definitions used above in the description of the PV.

Observable particles are understood as localizable material objects characterized by a set of charges (electric, color, ...), momentum \( p \), mass \( m \), energy \( \epsilon \), etc. It is important that for energy, mass and momentum of a free observable particle the Einstein relation holds

\[
\epsilon = c \sqrt{m^2 c^2 + p^2}.
\]

When this relationship is fulfilled one says that the energy and momentum lie on the mass shell in the four-dimensional space of \( \epsilon \) and \( p \).

Under the absence of physical fields in the PV one understands the vanishing of the mean values of their field strength, which manifests itself in maintaining the state of rest for a test particle under conservation of the corresponding charge and mass.

The PV is one of the fundamental and at the same time complex manifestations of Nature. However, in contrast to the ether of the XIX century, which largely remained a tentative concept, the PV provided answers to the thoughtful questions of the physicists. Direct experimental evidence for
the existence of the PV represent such tiny physical effects as the Lamb shift of the energy levels of
the hydrogen atom and hydrogen-like atoms, the anomalous magnetic moment of the electron, the
Casimir effect (in the simplest case the mutual attraction of uncharged conducting plates).

There are, however, groups of vacuum effects which until now have not found a direct
experimental confirmation despite long standing and reliable theoretical predictions. Primarily this
concerns the effect of vacuum particle creation under the influence of strong external constant electric
fields, predicted in their classical works by F. Sauter (1931) [1], W. Heisenberg and H. Euler (1936)
[2] as well as J. Schwinger (1951) who based this effect on a strong foundation in the framework of
quantum electrodynamics [3]. Not entirely correct, this effect is often called Schwinger effect (SE).
For brevity, we shall use this abbreviation.

\[ E \]

**Figure 1.** The left panel shows the structure of the energy states of fermions with mass
\( m \), in the PV of the classical Dirac theory. The upper region represents allowed energy
levels of free states. The lower region the Dirac sea of negative energy states. These
regions are separated by an energy gap of width \( 2mc^2 \). The influence of an external
electric field is shown on the right panel: the energy gap is converted to an energy
barrier accessible for tunneling into the observable region.

The principle of the effect is the following. Under the influence of a static electric field of strength
\( E_o \) a particle with charge \( e \) a potential energy of \( eE_o x \) emerges. This gives rise to a tilt of the energy
gap separating the states with positive and negative energies in the Dirac picture of the PV (figure 1,
\( e < 0 \)). This is a consequence of the change in the relationship between the energy and momentum (1)
under the influence of the field,

\[
(e - |E_o x|)^2 = c^2 \left( m^2 c^2 + p^2(x) \right)
\]

The result is that the electron from the Dirac sea is separated from the region of observable values
of the energy by a potential barrier of triangular shape (thick line on the right panel of figure 1), in
the inner region of which the momentum \( p(x) \) becomes imaginary (forbidden region for classical
particle).

From the point of view of quantum mechanics, a particle incident on a potential barrier of arbitrary
shape, can penetrate through it with a finite probability (Gamow’s tunnel effect [4])

\[
w \sim \exp \left\{ -\frac{2}{\hbar} \int_{x_2}^{x_1} dx p(x) \right\}
\]
Here we omitted the pre-exponential factor which is usually of the order of unity. Two turning points \( x_1 = \left( \frac{c - mc^2}{\hbar E_0} \right) \), \( x_2 = \left( \frac{c + mc^2}{\hbar E_0} \right) \) are found from the condition of vanishing particle momentum \( p(x) = 0 \). Substituting these values in the formula (3), we obtain

\[
w \sim \exp \left\{ -\frac{4m^2c^3}{\hbar E_0} \int_0^1 ds (1-s^2) \right\} = \exp \left\{ -\frac{\pi m^2c^3}{\hbar E_0} \right\}, \tag{4}
\]

which is in good agreement with the exact solution found by Schwinger for the probability of the formation of electron-positron pairs (EPP) per unit volume per unit time, i.e. for the pair production rate of EPP [3]

\[
w = \frac{ce^2E_0^2}{4\pi\hbar} \exp \left\{ \frac{\pi m^2c^3}{\hbar E_0} \right\} = \frac{ce^2E_0^2}{4\pi\hbar} \exp \left\{ -\frac{\pi E_c}{E_0} \right\}, \tag{5}
\]

where \( E_0 \) - the constant external field strength, and \( E_c = \frac{m^2c^3}{\hbar e} \) - the critical field strength. The result (5) is unique in the sense that it can only be obtained using nonperturbative methods, i.e. without the involvement of the standard perturbation theory in the hyperfine structure constant \( \alpha = e^2 / 4\pi\hbar c \approx 1/137 \), with which most of the results of quantum electrodynamics (QED) are obtained. Indeed, the expansion of the exponent in equation (5) for small strengths of the external electric field, i.e. in orders of the ratio \( E_0 / E_c \ll 1 \), is meaningless.

According to formulas (4) and (5) the effect of vacuum EPP creation should be noticeable if the electric field becomes comparable to the Schwinger critical field strength \( E_c = 1.323 \times 10^{16} \) V/cm, at which the breakdown of the electromagnetic vacuum occurs. Such gigantic constant fields are unlikely to be achievable even in the long term. For comparison, we give the value of the electric field at the Bohr orbit of the hydrogen atom: \( E_n = 10^9 \) V/cm.

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**Figure 2.** History and growth prospects of the energy density in the focal spot of laser systems (from the work of Mourou et al. [5]).

High hopes for achieving near-critical fields are associated with the rapid development of laser technologies and the increasing power of the generated laser radiation. The idea of using laser light to observe the SE was expressed by F. V. Bunkin and I. I. Tugov already in 1970 [6]. So far, these expectations were not realized. However, existing projects offer hope that the Schwinger limit, estimated by the radiation intensity \( I_c = \frac{eE_c^2}{4\pi} \), will be achieved in the near future. Figure 2 shows the growth curve of the optical laser radiation intensity and the expectations for this parameter. Here
the most promising optical laser systems are HiPER (Rutherford Laboratory, England), ELI (Extreme Light Infrastructure), XCELS (Russia) and the X-ray Free Electron Laser (XFEL).

The generalization of equation (5) to the case of time-dependent fields is not obvious. This circumstance caused the development of approaches based on different versions of the kinetic theory. It is this direction in which in recent years the majority of results were obtained.

The simplest is the approach proposed by the authors (D.B.B. and S.A.S.) in [7]. Similar to the result by J. Schwinger (5), the kinetic equation (KE) obtained in this work is based on non-perturbative methods of QED and sets from this point of view the standard. It is formulated for a linearly polarized electric field $E(t)$, which depends on time in an arbitrary way, but is spatially homogeneous, i.e. it does not depend on the point of observation. Such conditions are satisfied approximately in the focal spot of two counter propagating laser beams (see figure 3) with dimensions of the order of the laser wavelength.

![Figure 3](image-url)

**Figure 3.** Scheme of a laser collider, providing a standing wave in the focal spot which approximately realizes the situation of a homogeneous, time-dependent and strong electric field (“pump” field, modifying the PV). A probe beam interferometer is sketched which shall measure the refraction index and/or birefringence of the PV region modified by EPP creation due to the “pump” field.

Because of the importance of this KE in the following, we give it in the integro- differential form for the distribution function $f(p,t)$ of the electrons. If the initial state prior to the application of the external field was a vacuum with zero electric charge, by virtue of charge conservation in the following an equal number of electrons and positrons will be created which move in opposite directions, and the positron component will be described by the distribution function $f(-\bar{p},t)$. The KE describing the vacuum creation of electrons and positrons (we will talk about the birth of an electron-positron pair plasma - EPP) has the form

$$\dot{f}(\bar{p},t) = \frac{1}{2} \lambda(\bar{p},t) \int d\bar{p}' \lambda(\bar{p}',t) \left[ 1 - 2f(\bar{p},t) \cos \theta(k,\bar{p},t)^{2} \right],$$  

(6)

where the vacuum transition amplitude is (here we use units in which $\hbar = c = 1$)
\[ \lambda(\vec{p},t) = \frac{eE(t)}{E(\vec{p},t)} \] (7)

\[ \varepsilon_\perp = \sqrt{m^2 + p^2_\perp} \] is the transverse energy, which depends on the components of the momentum \( p_\perp \) orthogonal to the direction of the external field \( \vec{E}(t) \). Finally,

\[ e(\vec{p},t) = \sqrt{\varepsilon_\perp^2 + \left(p_\parallel - eA(t)\right)^2} \] (8)

is the energy of quasiparticle excitations (electrons or positrons) in the external field, \( p_\parallel \) - component of the momentum vector parallel to the direction of the external field \( \vec{E}(t) = -\dot{A}(t) \), where \( A(t) \) - vector potential of the electromagnetic field. The dot over a symbol denotes here and on the left hand side of the KE (6) the time derivative. Thus the KE (6) describes the strongly nonequilibrium evolution of the EPP by the distribution function \( f(\vec{p},t) \). The KE (6) is non-Markovian, i.e. taking into account the whole evolution of the EPP, starting from the moment \( t_0 \) when the external field starts: \( t_0 \leq t \leq t' \).

The nontriviality of the KE (6) is largely due to the two-scale evolution of the EPP. The large time scale is determined by the period of the characteristic changes of the external field \( T = 2\pi/\omega, \) where \( \omega \) is some characteristic frequency. The small time scale corresponds to the Compton time \( \tau_c = 2\pi/m_c \) comparable with the period of vacuum oscillations in the presence of the field (8). In the absence of an external field, this corresponds to the vacuum fluctuations (Zitterbewegung) with the frequency (1). Usually these scales vary greatly, \( T \gg \tau_c \) or \( 2\pi/T \ll e(\vec{p},t) \). The interaction of the time dependent external field with vacuum oscillations leads to complex parametric phenomena accompanying vacuum creation of EPP. These effects can be analyzed by means of numerical and approximate solutions of the KE (6).

Currently, there is a fairly complete picture of the dependence of the structure of the nonequilibrium distribution function on the laser radiation parameters: amplitude, frequency spectrum and laser pulse shape (Section 3). Restricting ourselves to the main characteristics of laser light, the amplitude of the field strength \( E_0 \) and the characteristic wavelength \( \lambda \), the interest is in the following region of parameters:

\[ 0 \leq E_0 \leq E_c , \quad \lambda_c \leq \lambda < \lambda_0 , \] (9)

where \( \lambda_c = 2\pi/m \) is the Compton wavelength and \( \lambda_0 \sim 1\mu m \) is an upper limit determined by restrictions in the numerical solution of the KE.

However, the distribution function is not accessible to direct experimental observation due to the extreme conditions under which the EPP is born. Observable may be some secondary effects that are available for registration at a sufficient distance from the focal spot. Among these effects are:

- photons radiation from annihilation events in the EPP;
- birefringence of the EPP.

One can expect some other observable manifestations of the EPP created from the PV. Here, the corresponding results have usually only preliminary character. They will be shortly explained in Section 4. To understand the nature of the difficulties that arise here, we first consider the mechanisms of vacuum creation of EPP.

2. Vacuum pair creation process in strong, time-dependent fields
Let us first consider the long-wavelength limit \( \lambda \rightarrow \infty \) in the sector (9), which corresponds to the constant field case with the field strength \( E_0 \). The effect of vacuum particle creation was qualitatively predicted initially based on the Dirac picture of the PV (figure 1), which in the presence of a constant
field leads to the tunneling interpretation of the effect. This so-called Schwinger limit (5) is well supported both numerically and analytically by solutions of the KE (6).

In the transition to non-stationary electric fields it is useful representation of the external field as an infinite reservoir of photons with characteristic frequencies determined by the spectral decomposition of the field. Constant field corresponds to the reservoir of photons with an infinitely large wavelength. We now increase the frequency of the external field up to extremely high frequencies $\nu \geq 2m/e$. Then the energy of one photon of this kind would be sufficient to ensure that there was a breakdown of the energy gap, generating an electron-positron pair. In this speculative limiting case acts the one-photon mechanism [8]. Figure 4 (left) illustrates the effect of such a sort of vacuum photoelectric effect, the photon knocks an electron-positron pair off the PV.

In the transition to lower frequencies (e.g., in the optical or X-ray range) for the breakdown of the energy gap it is necessary that a large number of "soft" photons must act coherently. This corresponds to the multiphoton mechanism of EPP creation. In this case, each of the photons from the external photon reservoir excites the PV to some virtual level (this process is shown on the right panel of Figure 4). From this point of view, the conventional tunneling mechanism can be interpreted as a joint action of an infinite number of coherent photons with frequency tending to zero. It is clear just these multiphoton processes are responsible for the excitation of the EPP in real laser fields.

It is also important to note that during the action of the external field not real commonly observed electrons and positrons are born from the PV, but their quasiparticle analogues having the same charge and spin as the real particles, but with a set of dynamic variables including the characteristics of the external field: a quasimomentum $\vec{p} = \vec{p} - eA(t)$ and a quasieigenenergy $\epsilon(p) = \sqrt{m^2 + p^2}$. Here, there is no restriction to linear polarization. Unlike real particles the quasiparticle excitations do not lie on the mass shell (1). It is also important that for the description of processes involving quasiparticles the usual methods such QED are not applicable, such as the S-matrix formalism, which considers the evolution of the system from an infinitely distant past to an infinitely remote future.

Thus, during the laser pulse a quasiparticle EPP is created from the vacuum. After the pulse the electron-positron quasiparticle excitations become real particles and go to the mass shell (1). In this way a residual EPP forms (see figure 5). Both of these stages of evolution give their specific contributions to the formation of secondary observable effects.

**Figure 4.** The single-photon (left) and multiphoton (right) mechanisms of the EPP excitation.
Figure 5. Evolution of the PV under the influence by a pulse of a time dependent electric field.

3. Landscape of EPP distributions

The full information about the EPP in the quasiparticle as well as in the real final state is contained in the distribution function (DF) \( f(\rho, \tau) \). Based on the KE (6), below will be considered the reaction of the vacuum on the action of a monochromatic laser pulse with a frequency \( \nu(2\pi\nu = \omega) \) and with the Gaussian envelope

\[
E(t) = E_0 \cos(\omega t + \varphi) e^{-t^2/2\tau^2},
\]

where \( \tau \) determines the half-width of the pulse. Typically, \( 1/\tau << \nu \). Increasing \( \tau \), one can model a monochromatic field [9].

The dependence of the DF in the final state on \( \rho \) is of particular interest because it determines the spectral characteristics of the experimentally observable real electrons and positrons created by the field. Here appear two characteristic features.

Firstly, an increase of the wavelength in relation to \( \lambda_c \), corresponding to a reduction of the circular frequency \( \omega \) in (10) leads to a proportional stretch of the spectrum of produced particles along the direction of the field.

Such a significant complication of the DF is the result of the interference of the contributions of several successive field pulses. Since during the action of each subsequent pulse the quasiparticle density exceeds by many times the residual density accumulated during the preceding stages, the DF saves the memory about the entire history of its evolution.

Increasing the pulse duration by increasing the parameter \( \tau \) along with the complication of DF form leads to a substantial increase in its maximum value and the total number of produced particles. The total amount of produced particles (per unit volume of the field region) is directly proportional to the pulse duration.

The character of the evolution of DF during the action of the field pulse and its dependence on \( \rho \) is usually associated with the adiabaticity parameter \( \gamma = E_0 \omega / E_0 m \). In figure 6 a general picture of the behavior of this parameter in dependence on the wavelength and the amplitude of the applied field is given [10] (another variant of such kind of landscape is presented in the work [11]).
The main area is occupied by the "calm valley", where there is no accumulation effect for which the production of EPP would increase with time (i.e. with increasing pulse duration). This accumulation effect of EPP appears in two areas, limiting the "calm valley" on the left and the top. This is a narrow band in the region of extremely short wavelengths $\lambda \lesssim 10^{-7}$ nm and the region of very strong fields, starting with $E_0 \geq 0.1E_c$. The long-wavelength limit in the accumulation region for strong fields corresponds to the Schwinger production rate of EPP.

4. Experimental verification of the dynamical Schwinger effect at modern laser facilities

Summarizing, we can say that on the basis of numerical and approximate solutions of the KE (6) one can trace the evolution of the DF in sufficient detail during the entire period of the action of the field pulse up to the stage of the freezeout of the EPP. Of course, the limits of applicability of the KE (6) are determined by the model of a linearly polarized spatially uniform external field. These model assumptions can be dropped, however, at the price of a drastic complication of the basic KE. But this does not alter the qualitative picture of the phenomena.

Against this background, it becomes highly actual to perform correct estimations of secondary observable effects such as radiation of annihilation photons from the region of the EPP creation, birefringence etc.

Such a choice of suggested experiments is based on the conditions for the creation of EPP in the focal spot of colliding laser beams, forming there a standing electromagnetic wave (figure 3). The latter is necessary to satisfy the condition of existence of the effect obtained by J. Schwinger. It is clear that in such circumstances the instruments for the detection of EPP generated from the vacuum should be located at sufficient distance from the focal spot.

The above mentioned effects satisfy this requirement. In the case of the annihilation effect there is a finite probability for the annihilation of EPP generated from the vacuum resulting in one or two photons per pulse (figure 7), which can be detected away from the focal spot. Note that single-photon channel being forbidden in the standard QED becomes open in strong fields (i.e. in the quasiparticle stage of evolution EPP).

The idea of the experiment which is based on the effect of birefringence is also simple (see the same figure 3): a test laser beam is split into two, one of which is directed through the region of generation of EPP and afterwards merged again with the other in order to observe their interference
pattern. Then, during the term of the main pulse, due to the interaction of one of the rays with the generated EPP its spectral composition is modified which results in changes of the interference pattern.

![Figure 7](image)

**Figure 7.** Diagrams of two annihilation channels of EPP produced in the focal spot of two laser beams.

Currently, reliable theoretical estimates of these effects are absent even though quite detailed quantitative descriptions of the effect of EPP creation from the vacuum exist. Preliminary estimates indicate that both of these effects can be detected when the electric field intensity at the level $E_0 \geq 0.1$ will be reached [12]. So in this direction there is still sufficiently elaborated work to be done.

On the other hand, there are doubts as to the possibility of achieving in most advanced laboratory conditions electric fields comparable to the Schwinger limit. The reason is seen in the fact that the Schwinger effect assumes idealized conditions when there is no surrounding matter. In real experiments in a vacuum chamber, covering the area of the focus of the laser beams, there are always trace amounts of gases and vapors which are ionized under the influence of fields which are still very far from the Schwinger limit. The free electrons appearing as a result are accelerated and generate an avalanche of further creation of charged particles. This charged plasma starts screening the electric field and will prevent achieving Schwinger-like field strengths. In this picture, however, also not everything is clear. For example, in the presence of a superstrong field, any electron resulting from ionization, immediately becomes a quasi-particle, indistinguishable from those generated from the vacuum, and the avalanche itself becomes a quasiparticle process.

5. Summary

Despite all contradictions and incompleteness of the current state of the problem, there is no doubt that the vacuum particle creation in strong fields, starting with the creation of matter in the early stages of the Universe, is the basis of a number of phenomena in the laboratory of Nature in all its diverse appearances in atomic and molecular physics, condensed matter physics, astrophysics, physics of ultrarelativistic heavy-ion collisions etc. Thus the key mystery remains unresolved: how proceeds the creation of matter, and what is the basis of the Universe?

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