Finite Element Model Modification of an Arch Bridge Based on Response Surface Method

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Abstract. Usually the structure of the initial finite element model can not reflect the actual situation of structure, the finite element model updating is for the initial finite element model of the structure, according to the static and dynamic response of the structure of the measured optimize the model parameters are adjusted, the revised model response is more close to the actual structural response, so as to get a more accurate finite element model. In this paper, the finite element model of the bridge is updated by using the second order incomplete polynomial response surface model. Then determine the sample data, use the least squares method to fit the undetermined coefficients of the second-order incomplete polynomial, and perform the accuracy test. Then the objective function based on frequency is constructed, and the updated optimal design parameters are obtained by quadratic sequence programming. Finally, using the optimal design parameters for modal analysis, it is found that the frequency error has decreased. It is proved that the correction by the response surface method (RSM) can improve the accuracy of the finite element model.

1. Introduction
This paper takes a through tied arch bridges with single support plane as the engineering background in Zhuanghe city, Dalian. The length of the bridge is 272.13m and the width of the bridge is 35m. ANSYS finite element analysis software [1] is applied to establish the finite element model of the arch bridge, which is mainly composed of main girder, arch rib, suspender and abutment. According to the nature of the structure of each part of the arch bridge, the established finite element model needs to adopt different elements to simulate each part, so as to make the finite element model reflect the actual situation of the structure as faithfully as possible. In the finite element model of arch bridge established in this paper, the arch ribs, main beams and bridges are simulated by BEAM188 element, the suspender is simulated by LINK10 element, and 67 MASS21 elements are distributed on the arch ribs to simulate the quality of the diaphragm. The finite element model of arch bridge has 279 elements and 428 nodes. The constraint at the connection between arch foot and main beam is considered as full constraint and full constraint at support. The finite element model is shown in Figure 1.
2. Basic Theory of RSM

The modification of structural finite element model is often reduced to an optimization problem. In the process of continuous iterative optimization, the finite element model is usually called many times to calculate. For large bridge structures, the finite element model often has a large number of elements. This method of iterative optimization by calling finite element model many times has a large amount of computation, and it is not easy to converge, so it is difficult to apply it to practical problems [2, 3]. RSM is a function approximation tool based on statistics and mathematics. The explicit approximate function relationship between the characteristic quantity of the structure and the design variable, that is, the response surface model, is established to replace the finite element model for iterative calculation. When the model is modified on the basis of the response surface model, the finite element model can be avoided every calculation, which greatly improves the calculation speed, effectively solves the contradiction between the large finite element model and the fast analysis, and the method is simple to use, and the universality and independence are also very strong. RSM was first proposed by Box and Wilson [4] in 1951. Myers [5] summarizes the theory, development and application of response surface method, which points out the way for the application of response surface method in engineering. Finally, the response surface method has been widely used in the finite element model modification of bridge structures [6, 7].

The mathematical expression of the response surface function is (1):

\[ y = f(x_1, x_2, ..., x_k) + \varepsilon \]  

\( f \) is the mapping relationship between input and output variables; \( x \) is the input variable; \( k \) is the number of parameters; \( \varepsilon \) is the statistical error.

3. Finite Element Design Method of RSM

3.1. Selection of Response Surface Function

The response surface function form has a great influence on the accuracy of model correction, so the appropriate function form should be selected according to the different structural characteristics. The following two points should be paid attention to in the selection: one is that the selected function form can express the complex relationship between the design variable and the characteristic variable; the other is that the selected function form should be as concise as possible in order to reduce the amount of calculation and the number of tests as much as possible. In the modification of bridge finite element model, the first-order polynomial and the second-order polynomial function model are the most widely used response surface function models [8], and the second-order Polynomials include second-order complete Polynomials and second-order incomplete Polynomials.

The undetermined coefficient of the response surface function of the first order polynomial is less and the amount of computation is small, but it can only fit the linear function well. Han Jianping [9] constructs the response model by using the second-order complete polynomial and the second-order
incomplete polynomial function respectively, and applies it to a bridge to verify it. The results show that the correction accuracy of the two methods is higher, but the computational complexity of the second-order incomplete polynomial response model is less. Therefore, in this paper, the second order incomplete polynomial function is selected for the regression of response surface function.

The mathematical expression of the second-order incomplete polynomial function is (2):

\[ f_i = \beta_0 + \beta_1 D_1 + \beta_2 E_1 + \beta_3 D_2 + \beta_4 E_2 + \beta_5 E_3 + \beta_6 E_4 + \beta_7 D_1^2 + \beta_8 E_1^2 + \beta_9 D_2^2 + \beta_{10} E_2^2 + \beta_{11} E_3^2 + \beta_{12} E_4^2 \]  

(2)

\( f_i \) is the response surface function of the \( i \) order frequency, \( i = 1,2,3, \ldots, 8 \); \( \beta_0, \beta_1, \beta_2, \ldots, \beta_{12} \) is the 12 undetermined coefficients of the response surface function, and \( D_1, E_1, D_2, E_2, E_3, E_4 \) (Elastic modulus \( E_1 \) and density \( D_1 \), elastic modulus \( E_2 \) and density \( D_2 \) of steel arch rib and suspender, elastic modulus \( E_3 \) of beam arch joint section, and elastic modulus \( E_4 \) of abutment) is the 6 parameters of the structure to be modified.

3.2. Finite Element Model Modification of Arch Bridge

The basic process of structural finite element model modification based on response surface method is as follows:

1. Determine the design parameters and state parameters of the structure, and determine the range of parameters;
2. Select sample points in the parameter design space through experimental design, and conduct finite element calculation to obtain the response of sample points;
3. Select the response surface function type, adopt the least square method [10] to fit the response surface model according to the sample data, and check the accuracy of the model;
4. Determine the objective function and constraints, replace the iterative optimization of the finite element model with the response surface model, and obtain the correction value of the characteristic parameters;
5. The modified value is substituted into the finite element model to obtain a more accurate finite element model after correction.

Based on the sample data obtained by uniform design, the undetermined coefficients of response surface functions of each order are obtained by regression fitting. The response surface function model of each order frequency can be obtained by bringing the determined undetermined coefficient into (2). The response surface model function of first order frequency is shown in figure 2. The response surface model function of the second order frequency is shown in figure 3.

(a) Surface of \( f_1 \) and parameter \( D_1, D_2 \)  
(b) Surface of \( f_1 \) and parameter \( D_1, E_2 \)
Figure 2. First Order Frequency Response Surface Model

(a) Surface of $f_1$ and parameter $D_1, D_2$
(b) Surface of $f_1$ and parameter $D_1, E_2$
(c) Surface of $f_1$ and parameter $D_1, E_3$
(d) Surface of $f_1$ and parameter $D_1, E_4$

Figure 3. Second Order Frequency Response Surface Model

(a) Surface of $f_2$ and parameter $D_1, D_2$
(b) Surface of $f_2$ and parameter $D_1, E_2$
(c) Surface of $f_2$ and parameter $D_1, E_3$
(d) Surface of $f_2$ and parameter $D_1, E_4$

3.3. Accuracy Test of Response Surface Models

For the fitted eighth order frequency response surface function, $R^2$ test and RMSE test are respectively used to check its accuracy. The calculation results are shown in table 1.
**Table 1.** Accuracy Test Value of Each Response Surface Model

| Response | $R^2$ | RMSE |
|----------|-------|------|
| $f_1$    | 0.9995| 0.0041|
| $f_2$    | 0.9998| 0.0016|
| $f_3$    | 0.9998| 0.0021|
| $f_4$    | 0.9981| 0.0072|
| $f_5$    | 0.9998| 0.0031|
| $f_6$    | 0.9997| 0.0035|
| $f_7$    | 0.9996| 0.0015|
| $f_8$    | 0.9538| 0.0391|

It can be seen from the above table that the $R^2$ of each response model is very close to 1, and the RMSE value is very close to 0, indicating that the difference between the calculated value of response surface function and the true value is very small. Therefore, in the parameter design space, the second-order incomplete polynomial response surface function can well fit the relationship between parameters and response, and the response surface model can replace the finite element model for model correction.

4. **Objective Function and Error Analysis**

4.1. **Objective Function Based on Frequency**

The square sum of the relative error is calculated as the objective function by using the eighth order frequency of the measured structure and the corresponding eighth order frequency calculated by the response surface. Based on MATLAB platform, the optimization iteration is carried out by sequential quadratic programming. MATLAB has a special optimization toolbox, which can directly call the fmincon function. The relative error of frequency is used as the objective function instead of absolute error, which mainly considers the different frequency of each order and avoids the error caused by the different weight. Constructed objective functions such as (3):

$$
y = \min \sum_{i=1}^{8} \left( \frac{f_{Ai} - f_{Ei}}{f_{Ei}} \right)^2
$$

where $f_{Ai}$ is the measured value of frequency, and $f_{Ei}$ is the calculated value of response surface of frequency, $i=1,2,3...8$. Bring the measured frequency value into (3) to get (4):

$$
y = \min \left[ \left( \frac{f_{A1} - 0.634}{0.634} \right)^2 + \left( \frac{f_{A2} - 0.873}{0.873} \right)^2 + \left( \frac{f_{A3} - 1.352}{1.352} \right)^2 + \left( \frac{f_{A4} - 2.846}{2.846} \right)^2 + \left( \frac{f_{A5} - 1.002}{1.002} \right)^2 + \left( \frac{f_{A6} - 1.983}{1.983} \right)^2 + \left( \frac{f_{A7} - 2.92}{2.92} \right)^2 + \left( \frac{f_{A8} - 3.863}{3.863} \right)^2 \right]
$$

Constraint condition:

- $2496.44 \leq D_1 \leq 3377.54$
- $2.94 \times 10^{10} \leq E_1 \leq 3.96 \times 10^{10}$
- $6672.4 \leq D_2 \leq 9027.6$
- $1.73 \times 10^{11} \leq E_2 \leq 2.37 \times 10^{11}$
- $2.92 \times 10^{10} \leq E_3 \leq 3.98 \times 10^{10}$
- $2.75 \times 10^{10} \leq E_4 \leq 3.75 \times 10^{10}$

4.2. **Comparison of modified results of finite element Model**

The finite element model is modified by using the optimal value of the design parameters, and the modal analysis of the modified model is carried out, and the corresponding eighth order frequency is obtained. Compare the initial design parameters with the optimal values of the design parameters obtained by the response surface method, as shown in Table 2.
Table 2. Comparison of Design Parameters before and after Correction

| Design Parameters | Initial Value | Correction Value | Error Value/% |
|-------------------|---------------|-----------------|--------------|
| $D_1$/kg·m$^3$    | 2938          | 3028            | 3.0642       |
| $E_1$/Pa          | $3.46 \times 10^{10}$ | $3.27 \times 10^{10}$ | -5.5071     |
| $D_2$/kg·m$^3$    | 7851          | 8057            | 2.6241       |
| $E_2$/Pa          | $2.06 \times 10^{11}$ | $1.96 \times 10^{11}$ | -4.877      |
| $E_3$/Pa          | $3.46 \times 10^{10}$ | $3.26 \times 10^{10}$ | -5.7970     |
| $E_4$/Pa          | $3.26 \times 10^{10}$ | $3.15 \times 10^{10}$ | -3.3845     |

The measured eighth order frequency is compared with the corresponding eighth order frequency calculated by the modified model. As shown in Table 3, the initial error between each order analysis frequency and the measured frequency of the modified finite element model is greatly reduced.

Table 3. Frequency Comparison before and after Correction

| Frequency               | Measured Value/Hz | Initial Value/Hz | Initial Error/% | Correction Value/Hz | Error Value/% |
|-------------------------|-------------------|------------------|-----------------|---------------------|--------------|
| First order vertical bending | 0.636             | 0.66917          | 5.3794          | 0.64155             | 1.0330       |
| Second order vertical bending | 0.875             | 0.87437          | 0.0411          | 0.87427             | 0.3202       |
| Third order vertical bending | 1.354             | 1.4037           | 3.7397          | 1.3797              | 1.9807       |
| Fifth order vertical bending | 2.848             | 2.522            | -11.4505        | 2.7047              | -4.9946      |
| First order torsion     | 1.002             | 1.0307           | 2.9569          | 1.0131              | 1.2187       |
| Second order torsion    | 1.985             | 2.0248           | 2.0513          | 2.022               | 1.9656       |
| Fourth order torsion    | 3.863             | 4.0188           | 4.0341          | 3.9522              | 2.3381       |

5. Summary
The sample data are determined by uniform design method, and the second order incomplete polynomial response surface function of each order frequency is obtained by least square method fitting, and the accuracy test is carried out by using $R^2$ test and RMSE test. The results show that the response surface function of each order frequency has high accuracy.

The design parameter correction value is brought into the finite element model and the modal analysis is carried out, the corresponding modal frequency parameters are obtained, the frequency parameters of the model are corrected by comparing the response surface method, the calculated frequency of the two modified models is found to be closer to the measured frequency value, And the corrected model accuracy is improved to a certain extent.

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References
[1] Liu Hao. ANSYS 15.0 finite element analysis from entry to mastery [M]. Beijing: Tsinghua University Press, 2016.
[2] Du Qing, Cai Meifeng, Zhang Xianmin. RC bridge dynamic finite element model updating [J]. Journal of High-way and Transportation Research and Development, 2006, 23(1): 60- 62.
[3] Ozaki I, Kimura F, Berz M.Higher- order sensitivity analysis of finite element method by
automatic differentiation [J]. Computational Mechanics, 1995, 16(4):223-234.

[4] Box G E, Wilson K B. On the experimental attainment optimum conditions [J]. Stat, 1951, 13(1): 1-45.

[5] Myers, Raymond H. Response surface methodology-current status and future directions [J]. Journal of Quality Technology, 1999, 31(1): 30-44.

[6] Guo Qintao, Zhang Lingmi, Fei Qingguo. Response surface method and its experimental design for deterministic computer simulation [J]. Acta Aeronautica et Astronautica Sinica, 2005, 27(1): 55-61.

[7] Chen Huabin. Response surface based finite element model updating of civil engineering structures[D]. Fuzhou: Fuzhou University, 2007.

[8] Sun Songsong. Research on modification of finite element model of sling-and-pull composite bridge [D]. Chengdu: SouthWest JiaoTong University, 2013.

[9] Han Jianping, Luo Yongpeng, Zheng Peijuan. Finite element model correction of rigid frame-continuous composite beam bridge based on response surface [J]. Engineering Mechanics, 2013, 30(12): 85-90.

[10] He Tao. Application research on correction method of bridge structure model based on dynamic and static load test [D]. Taiyuan University of Technology, 2015.