Multi-attribute Decision Method Based on Normal Random Variable in Economic Management Risk Control

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Abstract

This article proposes a distribution function based on normal distribution to express the distribution of criterion values in the interval. This method considers that the economic evaluation target of investment projects is multi-attribute. The economic management risk control environment is divided into information environment, time and space environment, and subject environment. The paper gives the corresponding multi-attribute economic evaluation method for the six possible combinations of project risk environments under the possibility of mutual comparison between interval numbers under the normal distribution. In the end, an example is used to illustrate the practicability and simplicity of the decision-making method.

Keywords: Multiple attribute decision making, standard random variables, economic management, risk control, time-series decision making, group decision making

AMS 2010 codes: 60G50

1 Introduction

The traditional economic evaluation theory of investment projects believes that the subject of investment decision-making is an ‘economic man.’ The broker’s investment behavior is ‘completely rational’, and he pursues the goal of maximization. When he makes a ‘maximum’ selection decision, all measures or programmes are ‘known’ or ‘given’.

Multi-attribute decision-making (MADM) problems generally exist in various fields such as economy, education, management and the military. Therefore, MADM is essential in decision-making theory and method research [1]. MADM is a limited option selection problem related to multiple attributes. Due to the complexity and uncertainty of MADM problems, it is difficult for decision-makers to determine the exact value of the attribute value in practice. But the decision-maker can give the measurement or evaluation result of the attribute value.
value in the form of the random variable. The random variable whose attribute value obeys or approximately obeys the normal distribution is the most common form. For example, products, customer or market demand, etc., obey normal distribution. Therefore, to find out the ways and means to solve the MADM problem with standard random variables is of great significance.

At present, there are not many analysis methods for MADM problems, with the attribute value being a standard random variable. Some scholars use the Monte-Carlo method to calculate the possibility to analyse the evaluation value of random variables and then compare and rank the plans. According to the $3\sigma$ principle, some scholars transform the MADM problem of standard random variables into a comparison of interval numbers’ advantages to realize the comparison and sorting of the schemes. Some scholars have put forward the possibility of mutual comparison between interval numbers under the normal distribution. However, they still adopt a MADM method based on interval numbers [2]. Therefore, these methods do not reflect the characteristics of MADM problems with standard random variables. This article will point out the crux of the problem and propose a new statistical method for MADM with standard random variables. This method uses the knowledge of probability and statistics to compare standard random variables that reflects the characteristics of normal random variables. The paper gives the corresponding multi-attribute economic evaluation method for the six possible combinations of project risk environments under the possibility of mutual comparison between interval numbers under the normal distribution.

2 Traditional economic management risk control evaluation method

Evaluation methods improved based on traditional investment project evaluation methods, such as the multi-objective analytic hierarchy process (AHP) method, data envelopment analysis (DEA), accurate options method, etc. Although there are some improvements, these methods have not changed [3]. Though they are fundamentally correct, there are shortcomings in traditional project economic evaluation. Specifically, these methods include the following aspects:

2.1 Completely rational assumption

Traditional economic evaluation theory assumes that the subject of investment decision-making has both ‘economic’ characteristics and ‘rationality’ in the course of action. Such people know all aspects of their environment. Even if this knowledge is not complete, at least it is pretty rich. Furthermore, such people have strong calculation skills, and they rely on these skills to determine which programme can reach the highest point on their preference scale [4]. However, this utterly rational assumption is difficult to realise in real life, and the rationality of the evaluation subject has limitations: first, the environment is complicated. In the form of impersonal exchange, people are faced with a complex and uncertain world. And the more transactions, the greater the uncertainty and the less complete the information. The second is that people’s ability to calculate and understand the environment is limited. Therefore, in this case, the project’s economic evaluation results will not be entirely rational.

2.2 Incomplete information definition

According to the degree of completeness available and the possibility of corresponding decision failure, traditional project economic evaluation theory divides information into three states: deterministic information, risk information, and uncertainty information [5]. From the above definition of information, it can be seen that traditional project economic evaluation ignores information that cannot be reached or obtained by human rationality.
2.3 Blurred information processing distortion

Traditional project economic evaluation methods tend to quantify the absolute number of fuzzy information in processing fuzzy information. The realisation of fuzzy linguistic values will make the information distorted or even lost. In addition, the evaluation subject may not be sure of a particular attribute value of the item and can only give an approximate range of the attribute value. Traditional economic evaluation theory rarely mentions the processing of this kind of fuzzy information.

2.4 Ignore the evaluation of social and environmental goals

In the traditional investment project evaluation, the good or bad of a project mainly determines the choice of the project from the level of economic income. As a result, it often leads to insufficient understanding of the role of investment projects, emphasising economic benefits and ignoring social and ecological environmental effects [6]. In this way, some projects, even though they are with good social and ecological environmental impacts, have been rejected because of poor economic returns.

3 A multi-attribute economic evaluation of investment projects

The traditional economic evaluation theory of investment projects still has shortcomings such as entirely rational assumptions, incomplete information definitions, distortion of fuzzy information processing, neglect of social and environmental evaluation, static evaluation of prices and arbitrary individual decision-making. The economic evaluation of investment projects, in reality, should first be multi-objective. This includes not only economic benefit goals but also social and ecological benefits. Secondly, the project evaluation and decision-making information environment should be multi-dimensional. This includes not only deterministic information but also risky and completely uncertain information. At the same time, the fuzzy processing of information by this theory should be reliable, objective and reasonable. Once again, project evaluation decision-making time and space should also be dynamic. One should pay attention to the benefits of the project at the current point in time and consider the future growth potential of the project and past growth experience. Finally, the subject of project evaluation and decision-making is not entirely rational and needs to draw on the knowledge of the group for the final evaluation of the project.

MADM theory is also called limited-plan multi-objective decision-making theory. It is a kind of multi-criteria decision theory. A multi-criteria decision-making problem with a limited number of options is called MADM. A multi-criteria decision-making problem with an unlimited number of options is called multi-objective decision-making [7]. MADM is joint in engineering, social and economic systems. Therefore, it is an important content of research on decision-making theory and methods. The main problem it wants to solve is the selection of the scheme or the ordering of schemes. So far, the MADM theory has been successfully applied in various fields and has become a new research hotspot for experts at home and abroad. MADM theory can solve problems including deterministic decision-making, fuzzy decision-making, group decision-making and dynamic decision-making.

4 Proposing the problem of multiple decision attributes

Considering a MADM problem with standard random variables, we set \( A = \{A_1, A_2, \cdots, A_m\} (m \geq 2) \) as the set of alternatives. \( A_i \) represents the \( i \) decision plan. \( C = \{C_1, C_2, \cdots, C_n\} (n \geq 2) \) is the attribute set, \( C_j \) represents the \( j \) attribute, and \( C_1, C_2, \cdots, C_n \) is independent of each other. The attribute weight vector is \( \vec{\omega} = (\omega_1, \omega_2, \cdots, \omega_n)^T \), where \( \omega_j \) is the weight or importance of an attribute \( C_j \) and satisfies it \( \sum_{j=1}^{n} \omega_j = 1 \), \( \omega_j \geq 0 \), \( j = 1, 2, \cdots, n \).

Let \( X = [X_{ij}]_{m \times n} \) be the decision matrix. Where \( X_{ij} \) represents the attribute value of the scheme \( A_i \) cor-
responding to the attribute \( C_j \). This paper considers that \( X_{ij} \) is a standard random variable \( X_{ij} \sim N(\mu_{ij}, \sigma_{ij}^2) \). The problem to be solved is how to select the best solution or sort the solutions based on the decision-making knowledge matrix \( X \) and the attribute weight vector \( \omega \). Assume that \( X \) has been standardised [8]. Then the comprehensive evaluation value \( x_i \) of the scheme \( A_i \) at this time is

\[
x_i = \sum_{j=1}^{n} \omega_j x_{ij}, \quad i = 1, 2, \cdots, m
\]

(1)

According to the nature of the normal distribution, \( X_i \sim N(\mu_i, \sigma_i^2) \):

\[
\mu_i = \sum_{j=2}^{n} \omega_j \mu_{ij}, \quad \sigma_i^2 = \sum_{j=2}^{n} \omega_j^2 \sigma_{ij}^2, \quad i = 1, 2, \cdots, m
\]

(2)

The MADM problem of standard random variables mainly uses the concept of the statistical confidence interval and the 3\( \sigma \) criterion of the normal distribution to transform standard random variables into interval numbers \([\mu - 3\sigma, \mu + 3\sigma]\). Then the method transforms the random variable into a certain interval number and then uses the interval number MADM theory and method to solve this problem. However, this processing method does not fully reflect the characteristics of random variables, especially standard random variables [9]. Therefore, further research is needed.

5 Statistical methods for MADM with standard random variables

Solving the MADM problem with standard random variables mainly involves two steps: one is comparing the total evaluation value of the MADM programmes with standard random variables, and the other is the ordering of the programmes.

5.1 Comparison of standard random variables

The comparison of the size of standard random variables has probabilistic significance and can therefore be defined as:

Definition 1: We set \( \xi \sim N(\mu_A, \sigma_A^2) \) and \( \eta \sim N(\mu_B, \sigma_B^2) \) and they are independent of each other, then define the comparison result of the size of standard random variables \( \xi \) and \( \eta \) as

\[
p(\xi > \eta) = \Phi \left( \frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}} \right)
\]

(3)

\[
p(\eta > \xi) = \Phi \left( \frac{\mu_B - \mu_A}{\sqrt{\sigma_A^2 + \sigma_B^2}} \right)
\]

(4)

Where \( \Phi(\cdot) \) is the cumulative distribution function of the standard normal distribution:

\[
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \varphi(t)dt, \quad \varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}
\]

(5)

Obviously, according to the nature of the normal distribution:

\[
P\{\xi > \eta\} = \Phi \left( \frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}} \right) = 1 - \Phi \left( \frac{\mu_B - \mu_A}{\sqrt{\sigma_A^2 + \sigma_B^2}} \right) = 1 - P\{\eta > \xi\}
\]

(6)

Namely: \( P\{\xi > \eta\} + P\{\eta > \xi\} = 1 \). It is not difficult to see from the above Definition 1:
(1) If \( \mu_A > \mu_B \), there must be:

\[
P\{\xi > \eta\} = \Phi\left(\frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}}\right) > \frac{1}{2}
\] (7)

The mean of the normal distribution plays a leading role in the comparison of standard random variables. The following proves that the comparison probability of the size of the standard random variable given by Equation (7) is moderately transitive.

Proposition: Let \( X_i \sim N(\mu_i, \sigma_i^2) \), \( X_j \sim N(\mu_j, \sigma_j^2) \), \( X_k \sim N(\mu_k, \sigma_k^2) \) be three known pairwise independent normal distributions. If \( q_{ij} = P(X_i > X_j) \geq 1/2, q_{jk} = P(X_j > X_k) \geq 1/2 \), there must be \( q_{ik} = P(X_i > X_k) \geq 1/2 \) (if it is true, it is called moderate transitivity).

Proof: Because

\[
q_{ij} = \Phi\left(\frac{\mu_i - \mu_j}{\sqrt{\sigma_i^2 + \sigma_j^2}}\right)
\] (8)

\[
q_{jk} = \Phi\left(\frac{\mu_j - \mu_k}{\sqrt{\sigma_j^2 + \sigma_k^2}}\right)
\]

\[
q_{ik} = \Phi\left(\frac{\mu_i - \mu_k}{\sqrt{\sigma_i^2 + \sigma_k^2}}\right)
\]

\( \Phi(\cdot) \) is the cumulative distribution function of the standard normal distribution. Introduce the mark: \( \phi_{ij} = \sqrt{\sigma_i^2 + \sigma_j^2} \), and there is \( \phi_{ij} = \phi_{ji} \). Because: \( \mu_i - \mu_k = (\mu_i - \mu_j) + (\mu_j - \mu_k) \), the following relational expression can be obtained:

\[
\phi_{ij}\Phi^{-1}(q_{ij}) = \mu_i - \mu_j
\] (9)

\[
\phi_{jk}\Phi^{-1}(q_{jk}) = \mu_j - \mu_k
\]

\[
\phi_{ik}\Phi^{-1}(q_{ik}) = \mu_i - \mu_k
\]

And then there are relations:

\[
\phi_{ik}\Phi^{-1}(q_{ik}) = \phi_{ij}\Phi^{-1}(q_{ij}) + \phi_{jk}\Phi^{-1}(q_{jk})
\] (10)

Reuse:

\[
\phi_{ik} = \phi_{ij}, \phi_{ik}\Phi^{-1}(q_{ki}) = \mu_k - \mu_i = -\phi_{ik}\Phi^{-1}(q_{ik}) = -\phi_{ik}\Phi^{-1}(q_{ik})
\] (11)

It is easy to see:

\[
\phi_{ik}\Phi^{-1}(q_{ki}) + \phi_{ij}\Phi^{-1}(q_{ij}) + \phi_{jk}\Phi^{-1}(q_{jk}) = 0
\] (12)

If the conditions are met:

\[
q_{ij} \geq 1/2
\] (13)

You can launch:

\[
q_{ij} = \Phi\left(\frac{\mu_i - \mu_j}{\sqrt{\sigma_i^2 + \sigma_j^2}}\right) \geq 1/2 \Rightarrow \mu_i - \mu_j \geq 0
\] (14)

\[
q_{jk} = \Phi\left(\frac{\mu_j - \mu_k}{\sqrt{\sigma_j^2 + \sigma_k^2}}\right) \geq 1/2 \Rightarrow \mu_j - \mu_k \geq 0
\] (15)
Thus:

$$\mu_i \geq \mu_j \geq \mu_k \Rightarrow \mu_i \geq \mu_k$$  \hspace{1cm} (16)

Thus

$$q_{jk} = \Phi \left( \frac{\mu_i - \mu_k}{\sqrt{\sigma_j^2 + \sigma_k^2}} \right) \geq 1/2$$  \hspace{1cm} (17)

That is, the size of standard random variables is relatively transitive, which is moderate transitivity.

(2) If $\mu_A = \mu_B$, there must be:

$$P\{\xi > \eta\} = \Phi(0) = 1/2, \quad P\{\xi > \eta\} = P\{\eta > \xi\} = 1/2 \hspace{1cm} (18)$$

It can be seen that this definition cannot distinguish when the mean is equal. To this end, it is necessary to compare the variance of the normal distribution further. In the case of equal means, the smaller the variance, the better the solution. From this, the basic principle of the ordering of MADM schemes with standard random variables can be expressed as follows: First, the schemes are sorted according to the mean value of the total evaluation value of each scheme. The larger the mean, the better the scheme. Then, when the mean values are equal, compare the variances. Again, the smaller the variance, the better the solution. When the variances are also equal, it is determined that the two schemes are in the same order.

5.2 The sorting method of the scheme

This paper proposes the ordering steps of MADM schemes with standard random variables as follows:

Step 1: We set the attribute weight vector as $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$.

Step 2: Calculate the total attribute value of each scheme:

$$G_i = \sum_{j=1}^{n} \omega_j r_{ij}$$  \hspace{1cm} (19)

Since the attribute value $r_{ij}$ obeys a normal distribution, the attribute weight $\omega_1, \omega_2, \cdots, \omega_n$ is an actual number. Therefore, the total attribute value $G_i$ is also a random variable that obeys a normal distribution and is denoted as $G_i = N(\mu_i, \sigma_i^2)$, $i = 1, 2, \cdots, m$.

Step 3: (1) If $\mu_i > \mu_j$, then $G_i \succ G_j$. (2) If $\mu_i = \mu_j$ and $\sigma_i^2 < \sigma_j^2$, then $G_i \succ G_j$. (3) If $\mu_i = \mu_j$ and $\sigma_i^2 > \sigma_j^2$, then $G_j \succ G_i$. (4) If $\mu_i = \mu_j$ and $\sigma_i^2 = \sigma_j^2$, then $G_i = G_j$.

Step 4: According to Step 3, sort all $m$ MADM schemes with standard random variables.

6 Project economic evaluation model based on MADM method

6.1 System environment portfolio of investment projects

The choice of multi-attribute economic evaluation methods for investment projects depends on the environment in which the investment project system is located (the combination of project information environment, time and space environment, and subject environment).

1. Complete project information and no need to investigate dynamic benefits Project information are complete means that the project meets the following conditions simultaneously: First, the project attribute index value can be determined [10]. Quantitative indicators of the project can calculate the quantitative
value, and qualitative indicators can be given based on a certain level of superiority and inferiority. They can give a clear quantitative score based on this. Second, the attribute weight value of an investment project can be determined. The attribute weights of investment projects can be determined in advance or determined by the subjective weighting method or objective weighting method.

2. The project information is complete but dynamic benefits need to be examined. The project information, including project, attribute values and attribute weight values, is complete from the current perspective. But looking at a particular benefit index or comprehensive benefit index of the candidate project from the reference period, there is still a certain degree of fluctuation [11]. Therefore, we need to evaluate the project’s growth potential and future benefits based on the project investment income during the reference period.

3. Project information is incomplete, and there is no need to investigate dynamic benefits. The project information is incomplete and contains the following two meanings: First, the attribute index value of the investment project is uncertain. Due to the limited project data, the quantitative indicators of investment projects cannot give a definite quantitative value but can only give a specific range of variation [12]. There may be different forms of fuzzy number scale for the processing of these fuzzy languages. Evaluation subjects with different preferences will also give different forms and ranges of fuzzy quantification. Second, the attribute weight value of investment projects is uncertain. The attribute weight of an investment project may be a fuzzy number that varies within a specific range.

6.2 Selection of economic evaluation methods for investment projects

We can choose the corresponding multi-attribute economic evaluation method according to the different combinations of investment project environments. Then, when the investment project information is complete, and there is no need to examine the dynamic benefits of the project, according to the corresponding relationship between the investment project information environment and the investment project subject environment. At this time, we adopt the accurate individual number evaluation method to obtain good investment projects.

When the investment project information is complete, and the dynamic benefits of the investment project need to be examined, the economic evaluation of the investment project starts from two dimensions [13]. One is the individual accurate number evaluation, which evaluates the comprehensive benefits of the project at the current point in time. The second is time series evaluation, which evaluates the total benefit and benefits increment during the inspection period.

When the investment project information is not complete, we do not need to examine the dynamic benefits of the project. The group fuzzy economic evaluation method can be carried out by first carrying out the individual fuzzy evaluation. Then integrate the individual fuzzy evaluation results to achieve.

When the investment project information is incomplete, and the dynamic benefits of the project need to be examined, we need to adopt the group fuzzy economic evaluation method. This needs to eliminate as much as possible the influence of individual decision-making preferences while adopting time-series economic evaluation methods to evaluate the dynamic benefits of the project over time.

6.3 Process of economic evaluation method for multi-attribute projects

According to the multi-attribute economic evaluation flowchart shown in Figure 1, we first judge whether the project information is confirmed or not. Then, in a deterministic environment, a multi-attribute economic evaluation is carried out at the individual level. Individual fuzzy evaluation is carried out in an uncertain environment first, and then the group fuzzy evaluation ranking result is obtained by weighted integration.
6.4 Case analysis

Suppose there are four schemes, and the evaluation indicators involved in each scheme have three attributes. Then the decision matrix we give is

\[ R = (r_{ij})_{4 \times 3} = \begin{bmatrix}
0.49, 0.023 & 0.52, 0.005 & 0.73, 0.003 \\
0.45, 0.047 & 0.61, 0.010 & 0.69, 0.090 \\
0.66, 0.053 & 0.68, 0.020 & 0.50, 0.033 \\
0.64, 0.055 & 0.70, 0.065 & 0.70, 0.010
\end{bmatrix} \tag{20}\]

Among them, \( r_{ij} = \{\mu_{ij}, \sigma_{ij}^2\} \). If the weight vector of these three attributes is

\[ \omega = (0.258, 0.373, 0.369)^T \tag{21}\]

Then it is not difficult to calculate the comprehensive evaluation value of these four options

\[ G_1 \sim N(0.5898, 0.0026), \ G_2 \sim N(0.5982, 0.0168) \tag{22}\]
\[ G_3 \sim N(0.6084, 0.0108), \ G_4 \sim N(0.6845, 0.0141) \tag{23}\]

According to the scheme ranking method given in Section “The sorting method of the scheme”, \( A_4 \succ A_3 \succ A_2 \succ A_1 \). Scheme 4 is the optimal scheme. The ranking results are the same as the actual results, but the method is more superficial.

7 Summary

The economic evaluation theory of investment projects under the perspective of MADM researched and discussed in this article is conducive to the ultimate realisation of a multi-attribute synergy of the investment
project’s goals of its value, economic benefits, social benefits and ecological benefits. On the other hand, the economic evaluation of investment projects can comprehensively consider the environment of multiple information attributes (deterministic and uncertain information), multiple temporal and spatial attributes (static and dynamic evaluation), and multiple subject attributes (individual evaluation and group evaluation). As a result, project investment decision-making and management are more standardised, scientific and reasonable. This paper describes and proves the moderate transitivity of standard random variables. This gives a comparative method of risk management and control schemes for multi-attribute decision economic management with standard random variables. The calculation example shows the conciseness and practicality of the method proposed in this paper.

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