Equation of state for quark matter with strong magnetic field and hybrid stars

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Abstract. Our previous studies indicate that quark matter in the states of the lowest Landau level of a strong magnetic field must be described by a stiffer equation of state and a mass for the hybrid star that easily rises above $2\,M_\odot$. In contrast, the inclusion of higher Landau levels may permit a softer equation of state. Because this general scenario seems to be complex, as a first step, we look into introducing only the second Landau level to elicit trends. We find that to sustain a heavy star of mass $2\,M_\odot$, the equation of state must remain sufficiently stiff. We also discuss the mass-radius relation for these hybrid stars in the light of recent observations.

1. Introduction

Recently, many researchers have shifted their attention towards neutron stars, following the observation of the gravitational wave from the binary neutron star merger. From this gravitational event, one can constrain the tidal deformability of neutron stars, which in turn constrains their radius [1, 2, 3, 4]. Moreover, as $2M_\odot$ neutron stars have been observed [5, 6], the material composing it must be able to resist compression, and so the equation of state (EOS) has to be significantly “stiff”, in the high-density region. Neutron stars are endowed with many interesting physical features such as high density, strong magnetic field, cooling, and rapid orbital spin. Of these features, the strength of the magnetic field has been a focus of constant attention. Indeed, the behavior of high-density matter in strong magnetic fields has been an important theme in EOS studies. To date, the strong magnetic fields of compact stars have been investigated by many researchers (e.g., [7, 8, 9, 10, 11, 12, 13]).

The surface magnetic field for standard neutron stars is $\sim O(10^{12})$ G. Observationally, neutron stars that have surface magnetic fields of $\sim O(10^{15})$ G, much stronger than for typical stars, have been discovered. These strongly magnetized neutron stars are called magnetars. Because the magnetic fields inside neutron stars are generally stronger than at their surface, field strengths inside the stars may rise to $\sim O(10^{18})$ G [14, 15, 16, 17, 18]. For such values, one has to consider the practical effects of such magnetic fields, which become comparable to the hadronic energy scale. To investigate quark matter in such strong magnetic fields, we need to take into account the Landau levels of this matter-field system.

Two of the authors (H. S. and T. T.) investigated a quark matter in the states of the lowest Landau level (LLL) and showed the EOS describes a material that is significantly stiff and
resistant to compression [19, 20], and yields a mass for the hybrid star that is easily over $2M_{\odot}$. However, this previous work on the effect of the LLL had not considered $\beta$ equilibrium. In this context, the effect of higher Landau levels is of interest. As a preliminary study, we consider quark matter in states up to the second Landau level (2LL) and derive the EOS, properly taking $\beta$ equilibrium into account. We next discuss the properties of hybrid stars constructed with our EOS by integrating the Tolman–Oppenheimer–Volkoff (TOV) equation.

2. Formulation and numerical results
Assuming a strong uniform magnetic field $B$ along the quark core axis (the $z$-axis), the $n$th energy level of a charged particle in this field is in general given by

$$E_n^i = \sqrt{c^4 m_i^2 + c^2 p_z^2 + \hbar c |e_iB| [2n + 1 + \text{sgn}(e_iB)s]},$$

(1)

where $m_i$, $e_i$, and $s$ denote the mass, charge, and spin degree of freedom, respectively, of a particle species labeled by index $i = u, d, s, e$.

Similar to our previous study [19], we consider a hybrid star where a quark core exists and hadronic matter surrounds the quark core. For simplicity, we suppose the field contribution only in the LLL.

To take into account the effect of the lowest two Landau levels, we derive the critical magnetic field strength, $B_c^{(2)}$ for each particle [19] below which the 2LL is occupied. For the various species $i$, the values of $B_c^{(2)}$ with the $\beta$ equilibrium condition are found to be $B_c^{(u)} = 9.4 \times 10^{18} \times (n_b/n_0)^{2/3}$ G, $B_c^{(d)} = B_c^{(s)} = 14.8 \times 10^{18} \times (n_b/n_0)^{2/3}$ G, and $B_c^{(c)} = 4.5 \times 10^{18} \times (n_b/n_0)^{2/3}$ G. From these results, one observes that the $d$ and $s$ quarks first occupy the 2LL with decreasing magnetic field strength. In the following, we consider a configuration in which the $d$ and $s$ quarks are occupied up to the 2LL and the $u$ quarks and electrons remain only in the LLL.

As indicated in Fig. 1, the magnetized quark matter with Fermi energy $E_{\text{IF}}$ may exist in the state either with momentum $p_{\text{IF}}^{(1)}$ above the lowest energy level $E_0^{(1)}$ or with $p_{\text{IF}}^{(2)}$ above the first excited energy $E_0^{(2)}$, for which $E_{\text{IF}}$ is given by

$$E_{\text{IF}} = c p_{\text{IF}}^{(1)} = \left( c^2 p_{\text{IF}}^{(2)^2} + 2 \hbar c |e_iB| \right)^{1/2}.$$  

(3)

The energy density for this situation is found to be

$$\varepsilon_{2LL} = ac \left\{ \frac{3n_b}{2a} - \left( \frac{p_{\text{IF}}^{(1)}}{p_{\text{IF}}^{(1)} + 2} \sqrt{p_{\text{IF}}^{(1)^2} - \frac{2\hbar eB}{3c}} \right)^2 \right. 
+ ac \left( \frac{p_{\text{IF}}^{(1)^2}}{p_{\text{IF}}^{(1)^2} + 2p_{\text{IF}}^{(1)} + 2 \frac{4\hbar eB}{3c} \ln \left| \frac{p_{\text{IF}}^{(1)}}{p_{\text{IF}}^{(2)}} \right| - \frac{2\hbar eB}{3c} \ln \left( \frac{2\hbar eB}{3c} \right) \right) 
+ \frac{ac}{2} \left( \frac{2}{7} \left( \frac{p_{\text{IF}}^{(1)} - 2}{p_{\text{IF}}^{(1)^2} - \frac{2\hbar eB}{3c}} \right)^2 + \mathcal{B} \right), 
$$  

(4)
Figure 1. Energy levels for the configuration in which the $u$ quarks and electrons are in the LLL, whereas the $d$ and $s$ quarks are in the 2LL. Here $E_{dF}$, $E_{sF}$ ($= E_{dF}$), $E_{uF}$, and $E_{eF}$ denote the Fermi energies of the $d$, $s$, $u$ quarks, and electrons, respectively. Subscripts $\pm$ indicate the spin $s = \pm 1$.

Figure 2. (Left panel) EOS for different magnetic field strengths; (right panel) corresponding mass-radius relationship for a hybrid star. Labels “a” and “b” distinguish two different field configurations.

where $a \equiv eB/(2\pi^2h^2c)$.

In this study, we considered different types of density-dependent magnetic fields, rather than the standard one adopted in previous studies, to maintain the same configuration available for all densities, i.e.,

$$B(n_b) = C_B \times 10^{18} \times \left( \frac{n_b}{n_0} \right)^{2/3} \text{G},$$

where coefficient $C_B$ is an arbitrary dimensionless constant.

The contribution from the strong magnetic field generally depends on the magnetic field configuration inside the star, which usually leads to an anisotropic pressure. However, because the details of the magnetic field configuration are still uncertain, we simply consider two different field contributions, which are sometimes assumed in many studies. One assumes an isotropic field contribution of $(eB)^2/(8\pi\hbar c)$ added to the energy density and a contribution $(eB)^2/(24\pi\hbar c)$ added to the pressure [12, 21]. The other option adds a field contribution $(eB)^2/(8\pi\hbar c)$ directly to both the energy density and the pressure. Hereafter, we distinguish the two respective configurations with a subscript “a” and “b”.

In Fig. 2 (left panel), the quark EOS is drawn with a solid line for the “a” field contribution and with a dotted lines for the “b” field contribution, in which the LLL and the 2LL correspond to different field strengths. Specifically, they are $C_B = 14.83$ for the LLL and $C_B = 13.44$ for the 2LL, below which more complicated configurations, beyond the present ones, have to be considered. From this figure, one observes from the EOS that the quark core becomes
softer as the magnetic field strength decreases. As in our previous study, the EOS for hadronic matter is simply connected to the quark EOS at the transition point, which is determined as the intersection between the quark EOS and the hadronic EOS (Fig. 2). Here, for the EOS for hadronic matter, we specifically used “SLy4”, which is based on the Skyrme-type effective interaction [22, 23]. For reference, the EOS for hadronic matter is also shown (Fig. 2, thick-solid line).

Finally, we applied our EOS to the TOV equation. Plotting the mass–radius relation for hybrid stars described by the EOS (Fig. 2 (right panel), one sees that the mass and radius are very sensitive to the field strength, but less sensitive to the inclusion of field contributions. We note that the maximum mass of the hybrid star with an occupied 2LL is still above $2M_\odot$.

3. Summary and concluding remarks
In this study, we took into account the $\beta$ equilibrium condition and found that the differences between two configurations with and without the condition are very small. We derive the EOS of quark matter in the 2LL state and apply our EOS to the TOV equation. We found that the EOS with the 2LL is sufficiently stiff to sustain $2M_\odot$ neutron stars.

We have assumed that the d and s quarks are in states of the 2LL and the magnetic field is tuned to maintain this species configuration, but realistically the density dependence of the magnetic field in the inner core should be different from our choice. Specifically, each particle should be in the LLL state in some region and excited to the 2LL state in other regions. To consider this more realistic situation, we need to identify the magnetic field configuration inside the star. Once determined, we may then be able to take into account the effect of anisotropic pressure consistently rather than to suppose the simple approximation imposed in this study.

Moreover, having used a simple model for quark matter, it would be interesting to study other models such as the Nambu–Jona–Lasinio model or the color-superconducting model. Furthermore, if we considered mixed hadron–quark phases [24, 25] in a strong magnetic field, many interesting features could appear.

References
[1] Abbott B P et al. 2017 Phys. Rev. Lett. 119, 1601101
[2] Bauswein A et al. 2017 Astrophys. J. 850 L34
[3] Fattoyev F J et al. 2018 Phys. Rev. Lett. 120 172702.
[4] Most E R, Weih L R, Rezzolla L and Schaffner-Bielich J 2018 Phys. Rev. Lett. 120 261103
[5] Demorest P, Pennucci T, Ransom S, Roberts M and Hessels J 2010 Nature 467 1081
[6] Antoniadis J et al. 2013 Science 340 123232.
[7] Broderick A., Prakash M and Lattimer J M 2000 Astrophys. J. 537 351
[8] Felipe R G, Martinez A P, Rojas H and Orsaria M 2008 Phys. Rev. C 77 015807
[9] Huang X G, Huang M, Rischke D H and Sedrakian A 2018 Phys. Rev. D 81 045015
[10] Martinez A P, Felipe R G and Paret D M arXiv:1001.4038[astro-ph.HE]
[11] Casali R H, Castro L B and Menezes D 2014 Phys. Rev. C 89 065805
[12] Lopes L and Menezes D 2015 Journal of Cosmology and Astroparticle Physics 08 002 1508
[13] Mukhopadhayya S, Atta D and Basu D N 2017 Rom. Rep. Phys. 69 101
[14] Fushiki I, Gudmundsson E and Pethick C 1989 The Astrophysical Journal 342 958
[15] Lai D and Shapiro S L 1991 The Astrophysical Journal 383 745
[16] Cardall C Y, Prakash M and Lattimer J M 2001 The Astrophysical Journal 554 322
[17] Ferrer E J et al. 2010 Physical Review C 82 065802
[18] Makishima K et al. 2014 Phys. Rev. Lett. 112 171102
[19] Sotani H and Tsumaki T 2015 Mon. Not. R. Astron. Soc. 447 3155
[20] Sotani H and Tsumaki T 2017 Mon. Not. R. Astron. Soc. 467 1249
[21] Serot B D 1992 Rep. Prog. Phys. 55 1855
[22] Douchin F and Haensel P 2001 Astron. Astrophys. 380 151
[23] Haensel P and Potekhin A Y 2004 Astron. Astrophys. 428 191
[24] Endo T, Maruyama T, Chiba S and Tatsumi T 2006 Prog. Theor. Phys. 115 337
[25] Endo T 2011 Phys. Rev. C 83 068801