Various experiments have shown superluminal group and signal velocities recently. Experiments were essentials carried out with microwave tunnelling [1], with frustrated total internal reflection [2], and with gain-assisted anomalous dispersion [3]. According to textbooks a superluminal signal velocity violates Einstein causality implying that cause and effect can be changed and time machines known from science fiction could be constructed. This naive analysis, however, assumes a signal to be a point in the time dimension neglecting its finite duration. A signal is not presented by a point nor by its front, but by its total length. On the other hand a signal energy is finite thus its frequency band is limited, the latter is a fundamental physical property in consequence of field quantization with quantum $h\nu$. All superluminal experiments have been carried out with rather narrow frequency bands. The narrow band width is a condition sine qua non to avoid pulse reshaping of the signal due to the dispersion relation of the tunnelling barrier or of the excited gas, respectively [4]. In consequence of the narrow frequency band width the time duration of the signal is long so that causality is preserved. However, superluminal signal velocity shortens the otherwise luminal time span between cause and effect.

Can a signal travel faster than light? If this happens, would it really violate the principle of causality stating that cause precedes effect [5, 6]? The latter statement has been widely assumed as a matter of fact. It has been shown according to the theory of special relativity that a signal velocity faster than light allows to change the past. The line of arguments how to manipulate the past in this case is illustrated in Fig. [6, 7]. There are two frames of reference displayed. In the first one at the time $t = 0$ lottery numbers are presented, whereas at $t = -10$ ps the counters were closed. Mary (A) sends the lottery numbers to her friend Susan (B) with a signal velocity twice the velocity of light. Susan
moving in the second inertial system at a relative speed of 0.75c, sends the data back at an even faster speed of 4c, which arrives in the first system at $t = -50$ ps, thus in time to deliver the correct lottery numbers before the counters close at $t = -10$ ps.

Figure 1: Coordinates of two observers A (0,0) and B with O(x,t) and O'(x',t') moving with a relative velocity of 0.75c. The distance L between A and B is 0.1 m. A has available a signal velocity $v = 2c$ and B $v' = 4c$. Taking into consideration a finite signal duration, the lottery fraud is impossible as shown in Fig. 3. (The numbers in the example are chosen according to [6].)

The time shift of a point on the time coordinate into the past is given by the relation [6]:

$$t_A = \frac{-L}{c} \left( \frac{c}{N} - \frac{c}{N'} + \frac{v}{NN'} \right) \frac{1}{c - \frac{v}{NN'}} \frac{1}{c - \frac{v}{N}},$$

where $L$ is the transmission length of the signal, $v$ is the relative velocity of the two inertial systems A and B, and Nc, N’c are the signal velocities in A and B, respectively. N and N’ are numbers assumed to be $> 1$.

This is an example often encountered in the literature supposed to show that a superluminal signal velocity results in negative times and allows to manipulate the past. We show now that this simple model is not correct.

First we are going to recall the basic properties of a signal. Microwave pulses [1, 8] and quite recently light pulses [3] of frequency $\nu$ and bandwidth $\Delta \nu$ have been shown to travel at a velocity much faster than light. The pulses in the two experiments correspond to signals used nowadays in telephone as well as in inter-computer communication. Frequency band limitation of signals, the basis of the sampling theorem [9], is a backbone of digital communication technology discussed in detail in the literature (e.g. in Encyclopedia Britannica), but scarcely addressed to in textbooks of physics. The
finite energy content of a signal actually implies the frequency band limitation \[4\]. This fundamental physical property is in consequence of the energy of any frequency components of a signal to be \(n h \nu\) where \(n\) is a whole number, \(h\) the Planck constant, and \(\nu\) the frequency.

A pulse represents an amplitude modulated (AM) signal on a carrier frequency. The carrier frequency is in charge of the receivers address and the half-width of the pulse represents the number of digits, i.e. the information. In the case of modern fiber optics the relative frequency band width is \(10^{-3}\) approximately, in the superluminal microwave experiments the band width was \(10^{-1}\) and in the optical experiment mentioned above it was less than \(10^{-9}\). Due to the narrow frequency bands there was no significant pulse reshaping neither in the microwave tunnelling experiment nor in the gain-assisted light propagation experiment. The superluminal signals are shown together with the luminal reference signals in Fig. 4.

![Figure 2: Display of the superluminal gain–assisted optical pulses (left [3]) and tunnelled microwave (right [8]). The pulses are normalized and compared with the air born or the wave-guided signals. The measured velocities have been –310c and 4.7c, respectively.](image)

Thus in both experiments the signal travelled at a superluminal velocity, e.g. with 4.7c [8] or with –310c [3], respectively. Nevertheless, the principle of causality has not been violated in both experiments.

In the example with the lottery data the signal was assumed to be a point on the time coordinate. However, a signal has a finite duration as the pulse sketched along the time coordinate in Fig. 3. (In the experiments in question 7.5\(\mu\)s and 5 ns, see Fig. 2) Any information like a word has a finite extension on the time coordinate. In the two cited superluminal experiments the superluminal time shift compared with the pulse length is about 30% in the microwave experiment with the velocity 4.7c and about 1% in the light experiment with the velocity –310c. Due to the signal’s finite duration of 200 ps the information is obtained only at positive times under the assumptions as illustrated in
In contrast to Fig. 1, the pulse has a finite duration of 200 ps. This data is used for a clear demonstration of the effect. (In both experiments, the pulse length is extremely long compared with measured time shift in consequence of the superluminal signal velocity as shown in Fig. 2.)

Fig. 3. The same holds a fortiori for the two discussed experiments. The finite duration of a signal is the reason that a superluminal velocity does not violate the principle of causality. On the other hand, a shorter signal corresponds to a broader frequency band. In consequence of the dispersion relation of either a tunnelling barrier or of an excited atomic gas with an extremely narrow frequency regime of anomalous dispersion strong pulse reshaping would occur. Summing up, the principle of causality has not been violated by the experiments with superluminal signal velocities, but amazing the time span between cause and effect has been reduced compared with luminal propagation.

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