Is Bohm’s quantum theory equivalent to standard quantum theory?

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Abstract. It is conventionally believed that Bohm’s quantum theory yields all the predictions of standard quantum theory. However, in this paper, we show that the transmission and reflection coefficients (i.e., probabilities) predicted by Bohm’s quantum theory can be different from those predicted by standard quantum theory for energy eigenstates of the step barrier.

1. Introduction

In 1952, David Bohm \cite{bohm1, bohm2} showed that quantum theory, besides predicting probabilities, could also provide a deterministic account of the actual unobserved motion of a particle. In Bohm’s quantum theory, the actual motion of a particle is governed by an equation that has the same form as Newton’s second law. But in addition to the classical force(s), the particle also experiences what Bohm called a quantum force that is determined by the amplitude of the quantum wave function.

It is conventionally believed \cite{bohm1, bell} that Bohm’s quantum theory yields all the predictions of standard quantum theory. According to Bohm \cite{bohm1} himself: “This suggested interpretation leads to precisely the same results for all physical processes as does the usual interpretation.” According to Bell \cite{bell}: “It [Bohm’s quantum theory] is experimentally equivalent to the usual version insofar as the latter is unambiguous.” However, in this paper, we show that the transmission and reflection coefficients (i.e., probabilities) predicted by Bohm’s quantum theory can be different from those predicted by standard quantum theory for energy eigenstates of the step barrier.

2. Reflection and transmission coefficients from Bohm’s quantum theory

Consider a particle in a one-dimensional potential barrier whose initial wavefunction is one of the energy eigenstates $\phi_E(x)$ of the potential barrier with energy $E$. To predict the particle trajectory using Bohm’s theory \cite{bohm1, bohm2}, the particle initial position has to be specified. Assuming that the source of the particle extends from $x = a$ to $x = b$ on the left of the barrier, the initial position $x_0$ of the particle is in the range $[a, b]$.

The normalized probability distribution for the particle initial position $x_0$ is given by

$$P_E(x_0) = \frac{|\phi_E(x_0)|^2}{\int_a^b |\phi_E(x_0)|^2 dx_0}.$$  \hfill (1)
The reflection and transmission coefficients are [4]-[6], respectively,

\[ R(E) = \int_a^b P_E(x_{0R})dx_{0R} \]  \hspace{1cm} (2)

\[ T(E) = \int_a^b P_E(x_{0T})dx_{0T} \]  \hspace{1cm} (3)

where the integrations are, respectively, over the set of initial positions of trajectories that are reflected (not transmitted) and transmitted through the barrier. The reflection coefficient and transmission coefficient above clearly sum to unity

\[ R(E) + T(E) = 1. \]  \hspace{1cm} (4)

3. Step barrier

Consider a one-dimensional step potential defined by

\[ V(x) = \begin{cases} 0 & x > 0 \\ V & x \leq 0 \end{cases}, \]  \hspace{1cm} (5)

where \( V \) is the height of the step. We will consider the cases \( E < V \) and \( E > V \) in turn.

3.1. \( E < V \)

For any energy eigenstate with energy \( E \) less than the step height \( V \), the particle, according to Bohm’s theory [7], always at rest, independent of its initial position \( x_0 \) on the left of the step barrier. Thus, Bohm’s quantum theory predicts \( R(E) = 1 \) and \( T(E) = 0 \).

According to standard quantum theory [8], \( R(E) = 1 \) and \( T(E) = 0 \).

In this case, the two theories yield the same predictions for the transmission and reflection coefficients.

3.2. \( E > V \)

For any energy eigenstate with energy \( E \) greater than the step height \( V \), the particle, according to Bohm’s theory [7], always move to the right, independent of its initial position \( x_0 \) on the left of the step barrier. Therefore, Bohm’s quantum theory predicts \( R(E) = 0 \) and \( T(E) = 1 \).

However, according to standard quantum theory [8]

\[ T(E) = \frac{4\sqrt{1 - \frac{V}{E}}}{\left[1 + \sqrt{1 - \frac{V}{E}}\right]^2}. \]  \hspace{1cm} (6)

The transmission coefficient given by equation (6), see figure 1, approaches 1 as \( V/E \) approaches 0, i.e., as \( E \) approaches infinity for fixed \( V \). For \( 0 < V/E < 1 \), the transmission coefficient from standard quantum theory is not equal to 1 (the value predicted by Bohm’s theory). The difference between the transmission coefficients from the two theories grows with increasing \( V/E \), the difference is close to 1 when \( V/E \) is close to 1.

4. Conclusion

Contrary to conventional belief, we have shown with the step barrier that the transmission and reflection coefficients predicted by Bohm’s quantum theory can be different from those predicted by standard quantum theory. Differences between the predictions of the two theories for the coefficients, if they exist for experimentally realizable barrier systems, allow another possibility of testing Bohm’s quantum theory, which has not been conclusively tested so far [9]-[13].
Figure 1. Transmission coefficients (from standard quantum theory —— and from Bohm’s quantum theory - - - -) versus \( V/E \) for the case \( E > V \).

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