Chaotic characteristics of microwave radio field strength over Nigeria

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Abstract. This paper accesses the dynamics of microwave radio field strength over selected stations in Nigeria by computing radio refractivity and its field strength on the basis of sets of measured atmospheric parameters. Hence, radio refractivity and field strength is computed from meteorological variables across seven locations ((Akure (7.0°N, 4.6°E), Anyigba (9.1°N, 7.4°E), Jos (10.7°N, 9.0°E), Minna (9.6°N, 6.4°E), Nsukka (6.9°N, 7.4°E), Port-Harcourt (4.7°N, 7.0°E), Yola (9.3°N, 12.3°E)) in Nigeria for two-years (January 2011 – December 2012). The results in this paper depends on Tropospheric Data Acquisition Network (TRODAN) data compiled and administered by the Centre for Atmospheric Research, National Space Research and Development Agency, Anyigba, Kogi State, Nigeria. The degree of chaoticity across the study locations, varying from rainy season to dry season have been quantified. Analysis of the computed radio field strength time series is based on nonlinear techniques which include the Phase Space Reconstruction (PPR), Average Mutual Information (AMI), False Nearest Neighbor (FNN), Lyapunov exponent (LE) and Tsallis entropy (TE).

1. Introduction
The communication system is a system that describes an exchange of information between two stations, transmitter and receiver and this information goes through a channel. Radio wave propagation is the transfer of electromagnetic energy at radio frequencies from a transmitter to a receiver [1]. Radio field strength in radio frequency telecommunications is the value of the obtained electromagnetic field which will stimulate a reference antenna and thereby induce a voltage at a certain frequency to provide an input signal to a radio receiver from the transmitting antenna [2]. Communication systems are affected by refraction which is the bending of electromagnetic wave due to the variation of refractive index in the atmosphere [3].

Radio wave signal propagating in the troposphere is affected by processes which include the variations of meteorological parameters such as temperature, pressure and relative humidity [4]. The atmosphere which is a channel for the propagation of radio signal tends to have so much influence on the quality of the radio signal, amplitude and phase [5].

Identification of chaotic behaviour shows how chaotic systems may be separated from stochastic ones and, at the same time, provides estimates of the degrees of complexity of the fundamental chaotic system. Chaos theory is well suited to study the unpredictability and complex dynamics of atmospheric parameters in the atmosphere [6,7].
2. Theory

2.1 Radio Field Strength

The atmospheric radio refractive index, n in relation to radio refractivity, N can be computed by the following formulas \[8,9\]:

\[
n = 1 + N \times 10^{-6}
\]

\[
N = 77.6 \frac{P}{T} + 72 \frac{e}{T} + 3.75 \times 10^5 \frac{e}{T^2}
\]  

where \(T\) = Atmospheric temperature (K) and \(P\) = Atmospheric pressure (hpa) \(e\) = Water vapour pressure (hpa) computed using:

\[
e = \frac{He_s}{100}
\]  

where \(H\) is the relative humidity (%) and \(e_s\) is the saturation vapour pressure (hPa) obtained from:

\[
e_s = 6.1121 \exp\left[\frac{17.502t}{t+240.97}\right] \frac{100}{100}
\]  

where \(t\) = temperature (°C).

Surface refractivity relates exceedingly with radio field strength, especially at the VHF bands. In the frequency range 30–300 MHz, a factor of 0.2 dB changes in field strength may be implemented for every unit change in \(N\) \[10,11\]. Thus, an assessment of field strength variability (FSV) in a given station is explored from daily ranges of \(N\) from the relation:

\[
FSV = \left(N_{(MAX)} - N_{(MIN)}\right) \times 0.2dB
\]

2.2. Time series and phase space reconstruction

To reconstruct the phase space of the time series \(x(t_0), x(t_1),..., x(t_i),..., x(t_n)\), we extend to a phase type of \(m\) dimensions phase space with time delay \(\tau\) [12].

\[
x = \begin{bmatrix}
  x(t_0) & x(t_1) & x(t_i) & x(t_n + m(m - 1)\tau) \\
  x(t_0 + \tau) & x(t_1 + \tau) & x(t_i + \tau) & x(t_n + m(m - 2)\tau) \\
  \vdots & \vdots & \vdots & \vdots \\
  x(t_0 + m(m - 1)\tau) & x(t_1 + m(m - 1)\tau) & x(t_i + m(m - 1)\tau) & x(t_n)
\end{bmatrix}
\]

A phase point of phase space comprises of every row in equation (6). Where variables \(x(t_0), x(t_0 + \tau),..., x(t_0 + m(m - 1)\tau)\) denote the values of variable \(x\) at times \(t_0, t_0 + \tau, t + 2\tau,\ldots, t_0 + m(m - 1)\tau\) with the embedding delay \(\tau\) and embedding dimension \(m\). The correct choice of proper time delay \(\tau\) and embedding dimension \(m\) can be done by using the AMI and the FNN methods respectively.
2.3. Average Mutual Information (AMI) Method

The mutual information can be computed using the following expression [13]:

\[
I(x(t), x(t+\tau)) = \sum_{i,j} P_{ij}(\tau) \log \left( \frac{P_{ij}(\tau)}{P_i P_j} \right)
\]  

(7)

where \(\tau\) is the time delay, \(P_i\) is the probability that \(x(t)\) is in bin \(i\) of the histogram constructed from the data points in \(x\), and \(P_{ij}(\tau)\) is the probability that \(x(t)\) is in bin \(i\) and \(x(t+\tau)\) is in bin \(j\).

2.4. False Nearest Neighbour (FNN) Method

The false nearest neighbour can be computed using the formula [14]. The D-dimensional phase space with \(r\)th nearest neighbour of a coordinate vector \((t)\) is considered and represented by \(y(r)(t)\), then the square of the Euclidean distance \((R_D)\) between time series \(y(t)\) and the \(rth\) nearest neighbour is given as [15]:

\[
\left[ \frac{R_D^{2+r}(t,r) - R_D^2(t,r)}{R_D^2(t,r)} \right]^{1/2} = \frac{|x(t+D\tau) - x^{(r)}(t+D\tau)|}{R_D^2(t,r)} > R_{tol}
\]  

(8)

where \(t\) and \(\tau\) are the times corresponding to the neighbour and the reference point, respectively. \(R_d\) denotes the distance in phase space with embedding dimension \(d\), and \(R_{tol}\) is the tolerance threshold. The embedding dimension needs to be sufficiently large to remove false crossings and ensure the presence of true neighbours for every trajectory in the reconstructed phase space [16].

2.5. Lyapunov exponents

Lyapunov exponent is one of the most common indicator of chaos in a system. The average rate of divergence of trajectories representing a time series is given as [17,18]:

\[
\lambda_1 = \lim_{n \to \infty} \frac{1}{t} \ln \frac{\Delta x(t)}{x(0)} = \lim_{n \to \infty} \frac{1}{t} \sum_{i=1}^{t} \ln \left( \frac{\Delta x(t_i)}{\Delta x(t_{i-1})} \right)
\]  

(9)

where \(n\) represents a small space of evolution in phase space, \(\Delta x\) represents the expansion of trajectories in \(n\) and \(t\) represents time of evolution in phase space [19].

2.6. Tsallis entropy

The TE in term of index \(q\) can then be expressed as [20]:

\[
S_q = k \frac{1}{q-1} \left( 1 - \sum_{i=1}^{w} p_i^q \right)
\]  

(10)
where W is the total number of probabilities, k is Boltzmann’s constant, p is the probability associated with the microscopic configuration.

3. **Methodology**

The research locations are in Akure (Ondo state), Anyigba (Kogi state), Jos (Plateau state), Minna (Minna State), Nsukka (Enugu State), Port Harcourt (Rivers state) and Yola (Adamawa state). Figure 1 indicates the map of Nigeria showing the study locations. The locations were chosen as a representation of the climatic regions in Nigeria. The data used in this research work were obtained from the archives of Tropospheric Data Acquisition Network (TRODAN) domiciled with the Centre for Atmospheric Research, National Space Research and Development Agency, Anyigba, Kogi State, Nigeria. The data for the seven weather stations have five minutes integration time from January 1, 2011 to December 31, 2012. The data recorded cover 24 hours each day from 00:00 hours to 23:00 hours local time at 5 minutes interval involving the dry and rainy season.

![Map of Nigeria showing the study locations](image_url)

**Figure 1.** Map of Nigeria showing the study locations

4. **Result and discussion**

Figure 2 shows the typical time series with different peaks for one of the seven selected stations, which reflect the information of dynamical systems. The time series plot shows the possible nonlinear nature of the system, which brings the necessity of carrying out a nonlinear assessment to discover the nonlinearity and also show the complexity of the system. The tendency of non-linearity which was initially observed in the time series plots may have been due to sporadic variations of the atmospheric parameters due to the effect of different atmospheric and climatic conditions.
Figure 2. Typical time series plots for the radio field for period 2011 – 2012

A graph of a typical mutual information against time delay is presented in Figure 3 which shows that the mutual information shows a minimum at different points, the choice of delay time $\tau = 15$ allowing one to gather accurate information about the system being represented with the aid of the set of measured data.

Figure 3. Mutual information as a function of time delay

Figures 4 shows that a significant portion of the false nearest neighbours. The delay time $\tau = 15$ and embedded dimension $m = 3$ for false nearest neighbour drops below the lowest value of $\tau$ which is considered to be true for 2011 and 2012 data by repeating the similar analysis for all the time series of different stations picking time delay $\tau = 15$ and embedded dimension $m = 4$. 
Figure 4. The false nearest neighbours as a function of embedding dimension

The phase space trajectories were constructed using estimated delay $\tau = 15$ and embedding dimension, $m = 3$. Common embedding parameters were chosen in order not to under-embed one system than the other. From the phase space plot, it was observed that there is a tendency of chaoticity in the system represented by the time series; as the time series of a non-chaotic system does not exhibit localized waveform but spread across the phase space. The clustering of the phase space around a local point and different peaks from the time series waveforms shows the tendency of nonlinearity in the field strength time series.

The Lyapunov Exponent shows that the time series is chaotic due to its positive values of the Lyapunov Exponent of the seven stations. All the data set for all the locations show similar variation. The values of LE in Table 1 have been observed within the phase space trajectory with $\tau = 15$ and $m = 3$ for all the seasons in each station. It was observed that the Lyapunov exponent was high in Yola and low in Akure and Port-Harcourt. This shows that the time series results for Lyapunov exponent computed from the entire data set for all the seven stations show that the Lyapunov exponents are chaotic as it has positive values. The positive trends by the Lyapunov exponents shows the presence of chaoticity for all the data sets considered [12].

The Tsallis entropy values as seen in Table 1 show similar patterns. It was observed that the Tsallis entropy was high in Akure and Port-Harcourt and low in Jos and Yola. It can be detected that the results of the chaoticity and dynamical complexity show degrees of correlation between the two major parameters computed from the radio field strength. The complexity during the rainy season is more compared to the dry season.
Table 1. Variation of Field Strength Variability (FSV), Lyapunov Exponent (LE) and Tsallis Entropy (TE) for years 2011-2012 at Akure, Anyigba, Jos, Minna, Nsukka, Port-Harcourt and Yola

| STATIONS     | FSV      | LE        | TE        |
|--------------|----------|-----------|-----------|
| AKURE        | 6.057176 | 2.3942    | 8.8293    |
| ANYIGBA      | 6.310499 | 2.6014    | 4.4723    |
| JOS          | 6.375974 | 2.5916    | 3.0131    |
| MINNA        | 6.514243 | 2.6356    | 3.6778    |
| NSUKKA       | 5.223463 | 2.6852    | 5.3869    |
| PORTHARcourt| 4.990072 | 2.6141    | 10.6252   |
| YOLA         | 6.152913 | 2.3154    | 2.9796    |

5. Conclusion

The degree of chaoticity across the study locations have been quantified, varying from rainy season to dry season. The constructed phase space trajectories, shows that phase space cluster for the dry seasons is usually more loosely packed and more closely packed during the rainy season. The magnitude of the Lyapunov exponent, it can be observed that there is low dimension chaos in the time series which infers the impossibility of the long term prediction. The behaviour of Lyapunov exponent corresponds with Tsallis entropy which confirms the high degree of complexity during the rainy season for the two years.

The results reveal similar pattern for Akure, Anyigba, Jos, Minna, Nsukka, Port-Harcourt and Yola having same trend for both quantifiers. The short term prediction of disorderliness during the rainy and dry seasons for the two years were observed and recorded. The higher the Field strength the higher the chaoticity which was observed during the rainy season compared to the dry season for the seven selected locations. Therefore, in other to avoid attenuation of radio signals along the radio communication links, it is necessary to examine the effect of meteorological parameters and the hydrometeors on the troposphere.

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