About zone structure of a stack of a cholesteric liquid crystal and isotropic medium layers

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Abstract. The optical properties of a stack of metamaterial-based cholesteric liquid crystal (CLC) layers and isotropic medium layers are investigated. CLCs with two types of chiral nihility are defined. The peculiarities of the reflection spectra of this system are investigated and it is shown that the reflection spectra of the stacks of CLC layers of these two types differ from each other. The influence of: the CLC sublayer thicknesses; incidence angle; local dielectric (magnetic) anisotropy of the CLC layers; refraction indices and thicknesses of the isotropic media layers on the reflection spectra and other optical characteristics of the system is investigated.

1. Introduction

Material science has energetically been developing recently, and its part, concerning the optical materials, has been developing more vigorously. In particular, metamaterials are of great interest [1]. Investigations of photonic crystals (PCs) still are of great interest both for their wide application in science and techniques, and for developing of the modern technology of creating new media. They have a photonic band gap (PBG) in their transmittance spectrum that can be changed either by external fields, or by changes of the crystal internal structure [2]. The optical devices made on the base of PCs possess such properties, as: multifunctionality and tunability; compactness and low energetic losses; high reliability and good compatibility with other optical devices. Cholesteric liquid crystals (CLCs) are known as PCs with easily tunable parameters (their parameters can be tuned by external electric, magnetic and strong light fields, thermal gradients, or UV radiation, etc.). Then it was reported in some works [3, 4] about multiple PBGs of one dimensional structures containing CLC and isotropic layers. Recently the chiral nihility media exhibit great interest. The concept of chiral nihility in electromagnetism was introduced by Lakhtakia [5], as a medium in which both dielectric and magnetic permittivities are zero. Then Tretyakov et al applied the nihility concept to the isotropic chiral metamaterials [6].

In the present work, the concept of nihility is generalized for the structurally chiral media (such as CLCs) and, using it, the peculiarities of the stack formed of CLC layers with nihility and isotropic media layers are investigated.
2. Reflection Spectra. Spectra of the Polarization Characteristics

The problem was solved by the Ambartsumian’s layer addition modified method [4]. We analyze the spectra peculiarities of the multilayer structure that is a stack of CLC layers with nihility and an isotropic medium layers. As it is known (for instance, see [7]), the solutions of Maxwell’s equations (for the normal light incidence) for the CLC with the dielectric and magnetic tensors having the forms:

\[
\varepsilon(z) = \varepsilon_m \begin{pmatrix} 1 + \delta_1 \cos^2 az & \delta_1 \sin^2 az & 0 \\ \delta_1 \sin^2 az & 1 - \delta_1 \cos^2 az & 0 \\ 0 & 0 & 1 - \delta_1 \end{pmatrix}, \mu(z) = \mu_m \begin{pmatrix} 1 + \delta_2 \cos^2 az & \delta_2 \sin^2 az & 0 \\ \delta_2 \sin^2 az & 1 - \delta_2 \cos^2 az & 0 \\ 0 & 0 & 1 - \delta_2 \end{pmatrix},
\]  

are as follows:

\[
\mathbf{E}(z,t) = \sum_{j=1}^{4} [E_j^n \exp(ik_j^+z) + E_j^{-n} \exp(ik_j^-z)]\exp(-i\omega t),
\]  

where: \( \varepsilon_m = (\varepsilon_1 + \varepsilon_2) / 2, \mu_m = (\mu_1 + \mu_2) / 2, \delta_1 = (\varepsilon_1 - \varepsilon_2) / (\varepsilon_1 + \varepsilon_2), \delta_2 = (\mu_1 - \mu_2) / (\mu_1 + \mu_2); \varepsilon_1 \) and \( \varepsilon_2 \) are the principal values of the dielectric tensor; \( \mu_1 \) and \( \mu_2 \) are those of the magnetic tensor and \( a = 2\pi / p, \) and \( p \) is the helix pitch; \( \mathbf{n}_k = (x \pm iy) / \sqrt{2} \) are the circular polarization orts; \( \omega \) is the light frequency in vacuum; \( k_j^+ \) and \( k_j^- \) are the \( z \) components of the wave vectors \( (k_j^+ - k_j^- = 2\alpha) \), which are defined from the dispersion equation and have the following form:

\[
k_j^+ = a \pm K_{1,2}, \quad k_j^- = -a \pm K_{1,2}, \quad j=1,2,3,4,
\]  

where

\[
K_{1,2} = \sqrt{\frac{\omega^2}{c^2} \varepsilon_m \mu_m \pm \alpha^2 \pm a^2 \pm \frac{\left( \frac{\omega^2}{c^2} \varepsilon_1 \mu_2 - \varepsilon_2 \mu_1 \right)^2}{2} + 4\alpha^2 \frac{\omega^2}{c^2} \varepsilon_m \mu_m}.
\]

As it is well known, (see [6]) isotropic chiral nihility metamaterials are the media with the parameters, \( \varepsilon = 0, \mu = 0, \) and \( \rho \neq 0 \) (\( \rho \) is the chirality parameter). As the CLC is locally anisotropic, we define the CLC with chiral nihility as the CLC with \( \varepsilon_m = \mu_m = 0 \) and \( p \neq 0 \). There can be two types of helicoidal structures with such parameters, namely: 1. \( \frac{\varepsilon_1}{\mu_1} = \frac{\varepsilon_2}{\mu_2} < 0 \) (the first type chiral nihility); 2. \( \frac{\varepsilon_1}{\mu_1} = \frac{\varepsilon_2}{\mu_2} > 0 \) (the second type chiral nihility).

Both types differ from each other only by the phase of the dielectric and magnetic permittivities modulation; in the first case these phases coincide, and in the second case the phase difference is \( \pi / 2 \). For these conditions, \( \sqrt{\frac{\omega^2}{c^2} \varepsilon_1 \mu_2 - \varepsilon_2 \mu_1 \pm \alpha^2} = 0 \) and we have:

\[
K_{1,2} = \sqrt{\frac{\omega^2}{c^2} \varepsilon_1 \mu_2 + \varepsilon_2 \mu_1 \pm a^2}. \quad K_{1,2} > 0 \text{ and, therefore, } K_{1,2} \text{ are always real}.
\]

According to these, in the first case, the CLC layer is transparent, and in the second case, there appears a PBG of a new type,
namely, it comes out in the spectrum region, \( \lambda \leq p \sqrt{\varepsilon_1 \mu_2 + \varepsilon_2 \mu_1 / 2} \), where the CLC completely reflects the incidence light with any polarization.

First, let it be noticed once more that the optical property peculiarities of the stack composed of CLC and isotropic medium layers at low dielectric and magnetic anisotropies of CLC layers were discussed in [8]. Below we consider the simplest case if the isotropic layers refraction coefficient, \( n = 1 \), that is, we investigate the peculiarities of the equidistant layers band gap of the CLC with nilility. Then, we consider the case, \( n_0 = 1 \), that is, we assume that the stack is in vacuum. Besides, if it is not stipulated beforehand, we assume the first sublayer of the system isotropic. As our numerical calculations show, for the first type nilility stack, reflection is completely absent \( (R = 0) \) for any polarization of the incident light, as the case is for the single CLC layer. In figure 1 the reflection spectrum of the normally incident light for the stack with CLC layers with the second type nilility is presented. The incident light has left (the solid curve) and right (the dashed curve) circular polarization. The CLC layers helix is a right one. It is seen from the figure that in the shortwave region of the spectrum, \( \lambda \leq p \sqrt{\varepsilon_1 \mu_2 + \varepsilon_2 \mu_1 / 2} \), there is a PBG of the same type as in the case of the single CLC layer, but new PBGs are formed, too. Comparison of the reflection spectra with those for the single CLC layer shows:

1. In the case of the stack, polarization sensitivity emerges, despite the non-sensitivity of the isotropic layers and single CLC layers of first type nilility.

2. In contrast to the second type nilility single CLC layer, this system has multiple PBGs of both types, namely, the ones independent of the incident light polarization and the others selective in regard to the polarization.

In figure 2, the polarization plane rotation and polarization ellipticity spectra (the first row) and those for the azimuth and the ellipticity of the first eigen polarization (the second row) for the first type nilility CLC stack (the left column) and those for the second type nilility CLC stack (the right column) are presented for the normal light incident. In figure 3a, b, the incident light is polarized along the x axis. As it is seen from figure 3, in both cases the system possesses optical activity, and the transmitted light has elliptic polarization, and its ellipticity has been changed strongly with the change of \( \lambda \). The eigen polarizations (EPs) of the system are orthogonal elliptic polarizations for both types of nililities. In the first case the ellipticity of the EP monotonously increases (with respect to its module) if \( \lambda \) increases, and in the second case the EPs are orthogonal circular polarizations.
Figure 2. The polarization plane rotation (the dashed line) and polarization ellipticity (the solid line, the first row) and the spectra of the azimuth (the dashed line) and the ellipticity (the solid line) of the first eigen polarization (the second row) for the stack with CLC layers with the first type nilility (the left column) and the second type nilility (the right column). $s = 50$ (the left column), $s = 10$ (the right column). The other parameters are the same as in figure 1.

3. Isotropic Layers Thicknesses Influence.
Now we study the influence of the isotropic layers thicknesses on the band gap structure of the subject system. As it was mentioned above, in the case of the first nilility CLC stack, reflection is completely absent independent of the incident light polarization and of the thicknesses of the isotropic layers. In figure 3 the evolution of the reflection spectra for the second type nilility CLC layers stack, if the isotropic layers thickness, $d_1$, is changed. The incident light has left (a) and right (b) circular polarization. The bright regions correspond to stronger reflection. As it is seen from the figure, the short wavelength PBG is non-sensitive to the thickness changes of the isotropic layers. It is also seen from the figure, that, changing the isotropic layers thicknesses, one can change the number, the frequency width and frequency location of the PBG, consequently, one can tune the reflection/transmission of the subject system.

Figure 3. The reflection spectra evolution, if the isotropic layers thicknesses change. The incident light has left (the right column) and right (the left column) circular polarizations. The other parameters are the same as in figure 1.
4. CLC Layers Thicknesses Influence.

Now we pass to the investigation of the influence of the CLC layers thickness changes on the reflection spectra. We consider the case – as above – when the first sub-layer is isotropic. In figure 4, the reflection spectra evolution if the CLC layers thickness, \(d\), is changing, is presented for system with CLC layers with the second type nihility. As it is seen from the figure, the frequency locations and frequency widths of the PBGs are functions of the CLC layers thicknesses. The short wavelength PBG practically does not change if the layers thickness, \(d\), is changed. When the CLC layers thicknesses increase, the PBGs number and their frequency widths are changed. The PBGs frequency widths increase periodically; then they decrease, and all these take place in significantly large intervals, especially in the long-wave region. Also, it is seen from this figure, that the bright lines are practically vertical in the long-wave region for the right circularly polarized incident light. This means that the CLC layers thickness changes can either give rise PBGs with a very large frequency width, or lead to their vanishing.

5. Influence of the CLC local dielectric anisotropy.

The dielectric anisotropy, \(\Delta = (\varepsilon_1 - \varepsilon_2)/2\), is of the order of 0.5 or less, for the ordinary CLCs, but recently some artificial crystals (metamaterials) have been created having dielectric anisotropy varying within large intervals. It seems that there can be made such CLC-like helical periodic media on their base which possess huge local anisotropy. Such media with comparatively weaker anisotropy have been made long ago [9]. On the other hand, recently the interest in CLCs doped with nano-particles (or ferro-electric, or ferro-magnetic particles) is very great (see [10] and the citations in there). The presence of these nano-particles in the CLC structure leads to: significant increase of local (dielectric and magnetic) anisotropy; a significant change of the temperature of the phase transition from the isotropic phase to the liquid crystalline one; significant change of the frequency width and frequency location of the PBG; a change of the CLC elasticity coefficients; significant increase of tunability of CLCs, etc.

Figure 4. The reflection spectra evolution, if the CLC layers thicknesses change. The incident light has left (the left column) and right (the right column) circular polarizations. The other parameters are the same as in figure 1.

It follows from the above-said that the investigation of the optical properties of the stack of CLC layers with chiral nihility and isotropic medium layers, having different values of local dielectric and magnetic anisotropy, can have great interest. Presenting the principal values of the local dielectric and magnetic tensors of the CLC sub-layers in the form: \(\epsilon_{1,2} = \mp (\varepsilon_0 - x)\) and \(\mu_{1,2} = \pm (\mu_0 - x)\), we investigate the influence of \(x\) on the reflection spectra.
Figure 5. The evolution of the reflection spectra, when $x$ (characterizing the anisotropy) is changing. The incident light has right (the left column) and left (the right column) circular polarizations. The other parameters are the same as in figure 1.

In figure 5, the evolution of the reflection spectra if $x$ (characterizing the local dielectric and magnetic anisotropies) is changed are presented. As it is seen from the figure, no PBG is formed near the values, $x = \varepsilon_0 = \mu_0$ ($\varepsilon_0 = \mu_0 = 2.5$), that is, at $\varepsilon_1 = \varepsilon_2 = \mu_1 = \mu_2 = 0$, which is natural. In the case of the stack with CLC layers with chiral nihility of the second type, if $|\varepsilon_0 - x|$ (or $|\mu_0 - x|$) increases, the frequency width of the PBGs that are not selective to the incident light polarization is significantly increased and, for the larger values of this parameter, total reflection takes place practically in the whole subject wavelength range. This is natural too, because PBG is the above-said shortwave one with its longwave border shifted closer to the longwave region with the increase, $|\varepsilon_0 - x|$ (or $|\mu_0 - x|$).

4. Conclusion
Concluding, we have to note that we investigated the reflection spectra peculiarities of the stack of CLC layers with chiral nihility and isotropic medium layers. These investigations give much information about new possibilities of application of chiral PCs, in optics and photonics. Two types of CLCs with chiral nihilities were defined.

The main peculiarities of the reflection spectra of the usual single CLC layer are their polarization sensitivity (the reflection spectra are not identical, for the two orthogonal circular polarizations of the incident wave). In the case of the single CLC layer with the second type chiral nihility, a new type PBG appears which does not possess polarization sensitivity. In the stack case, this new type PBGs appears in any type of isotropic layers. Moreover, this system possesses multiple PBGs. This property of the subject system can find wide application, in particular, in display manufacturing.

It was shown that changing the system parameters (the CLC layers thicknesses, their dielectric anisotropy, the isotropic layer thicknesses, their refraction indices, etc.) one can change the PBGs' number, their frequency width and frequency distance (and in an essentially wide range) their character (as they are selective or non-selective with respect to the incident light polarization) their gyrotropic properties, etc.

Taking into account the possibility of tuning of local parameters by the changes of the external fields (electric, magnetic, mechanical, thermal, light, etc.), especially in the case if the system structure is soft, or the possibility of changing the internal structure of the system, the subject system has great perspectives, in the sense of its applications in photonics.
In particular, the subject system can find application as a tunable optical filter or mirror, or as a system allowing to obtain 100% polarized radiation from a non-polarized light (without any loss), or as an anti-reflecting systems, etc.

References
[1] Shalaev V M 2007 Nature Photonics 1 41
[2] Sakoda K 2001 Optical Properties of Photonic Crystals (Berlin: Springer)
[3] Ha N Y, Ohtsuka Y, et al 2008 Nature Mat. 7 43
[4] Gevorgyan A H 2012 Phys. Rev. E. 85 021704
[5] Lakhtakia A 2002 Int. J. of Infrared and Millimeter Waves 23 813
[6] Tretyakov S, Nefedov I, et al 2003 J. of Electromag. Waves and Appl. 17 695
[7] de Vries H 1951 Acta Crystallogr. 4 219
[8] Harutyunyan M Z, Gevorgyan A H and Matinyan G K. 2013 Opt. Spectrosc. 114 601.
[9] Robbie K, Brett M J and Lakhtakia A 1996 Nature 384 616.
[10] Jeng S-C, Hwang S-J, Hung Y-H and Chen S-C. 2010 Opt. Express. 18 22572.