The Theta Term in QCD Sum Rules and the Electric Dipole Moment of the Vector Meson

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Abstract

We demonstrate that the QCD sum rule method can be successfully applied to the calculation of CP-odd electromagnetic observables induced by a vacuum \( \theta \)–angle. We implement the approach in calculating the electric dipole moment of the rho meson to \(~ 30\%\) precision, and find that the result can also be explicitly related to the vacuum topological susceptibility.

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I. INTRODUCTION

In this paper we demonstrate the feasibility of QCD sum rule calculations \[1\] for CP-odd electromagnetic observables induced by the QCD vacuum angle \(\theta\). This parameter labels different super-selection sectors for the QCD vacuum, and enters in front of the additional term in the QCD Lagrangian

\[
\mathcal{L} = \theta \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}
\]

which violates P and CP symmetries. As it is a total derivative, \(G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}\) can induce physical observables only through non-perturbative effects.

Experimental tests of CP symmetry suggest that the \(\theta\)–parameter is small, and among different CP-violating observables, the electric dipole moment (EDM) of the neutron is one of the most sensitive to the value of \(\theta\) \[2\]. The calculation of the neutron EDM induced by the theta term is a long standing problem. According to Ref. \[3\], an estimate of \(d_n(\theta)\) can be obtained within chiral perturbation theory. The result,

\[
d_n = -e\theta \frac{m_u m_d}{f^2(m_u + m_d)} \left( \frac{0.9}{4\pi^2} \ln(\Lambda/m_\pi) + c \right)
\]

is seemingly justified near the chiral limit where the logarithmic term becomes large and may dominate over other possible contributions parametrized in this formula by the constant \(c\). This constant is not calculable within this formalism and in principle can be more important numerically than the logarithmic piece away from the chiral limit. In fact it is also worth noting that in the limit \(m_u, m_d \to 0\), the logarithm is still finite, and stabilized, for example, by the electromagnetic mass difference between the proton and neutron. In any case, an inability to determine the size of corrections to the logarithm means that one is unable to estimate the uncertainty of this prediction.

If the logarithm is cut off at \(\Lambda \sim m_\rho\) and non-logarithmic terms are ignored, one can derive the following bound on the value of \(\theta\) using current experimental results on the EDM of the neutron \[4\]:

\[
\bar{\theta} < 3 \cdot 10^{-10}.
\]

Confronted with a naive expectation \(\theta \sim 1\), the experimental evidence for a small if not zero value for \(\theta\) constitutes a serious fine tuning problem, usually referred to as the strong CP problem. The most popular solution for the strong CP problem is to allow the dynamical adjustment of \(\theta\) to zero through the axion mechanism \[5\].

There are two main motivations for improving the calculation of the EDM of the neutron induced by the theta term. The first refers to theories where the axion mechanism is absent and the \(\theta\)–parameter is zero at tree level as a result of exact P or CP symmetries \[6,7\]. At a certain mass scale these symmetries are spontaneously broken and a nonzero \(\theta\) is induced through radiative corrections. At low energies, a radiatively induced theta term is the main source for the EDM of the neutron as other, higher dimensional, operators are negligibly small. As \(\theta\) itself can be reliably calculated when the model is specified, the main uncertainty in predicting the EDM comes from the calculation of \(d_n(\theta)\).
The second, and perhaps the dominant, incentive to refine the calculation of \( d_n(\theta) \) is due to efforts to limit CP-violating phases in supersymmetric theories in general, and in the Minimal Supersymmetric Standard Model (MSSM) in particular. Substantial CP-violating SUSY phases contribute significantly to \( \theta \) and therefore these models apparently require the existence of the axion mechanism. However, this does not mean that the \( \theta \)-parameter is identically zero. While removing \( \theta \sim 1 \), the axion vacuum will adjust itself to the minimum dictated by the presence of higher dimensional CP-violating operators which generate terms in the axionic potential linear in \( \theta \). This induced \( \theta \)-parameter is then given by:

\[
\theta_{\text{induced}} = -\frac{K_1}{|K|}, \quad \text{where} \quad K_1 = i \left\{ \int dx e^{iqx} \langle 0 | T \left( \frac{\alpha_s}{8\pi} G \tilde{G}(x), O_{CP}(0) \right) | 0 \rangle \right\}_{q=0},
\]

where \( O_{CP}(0) \) can be any CP-violating operator with \( \text{dim}>4 \) composed from quark and gluon fields, while

\[
K = i \left\{ \int dx e^{iqx} \langle 0 | T \left( \frac{\alpha_s}{8\pi} G \tilde{G}(x), \frac{\alpha_s}{8\pi} G \tilde{G}(0) \right) | 0 \rangle \right\}_{q=0}
\]

is the topological susceptibility correlator. In the case of the MSSM, the most important operators of this kind are colour electric dipole moments of light quarks \( \bar{q}_i g t^a G^a_{\mu\nu} \sigma_{\mu\nu} \gamma_5 q \), and three-gluon CP-violating operators. The topological susceptibility correlator \( K \) was calculated in [8,9] and the value of \( \theta \), generated by color EDMs can be found in a similar way [10]. Numerically, the contribution to the neutron EDM, induced by \( \theta_{\text{eff}} \) is of the same order as direct contributions mediated by these operators and by the EDMs of quarks. Therefore, the complete calculation of \( d_n \) as a function of the SUSY CP-violating phases must include a \( d_n(\theta) \) contribution and a computation of this value, beyond the naive logarithmic estimate (2), is needed.

Within the currently available techniques for the study of hadronic physics, it seems that the only chance to improve analytically on the estimate (2) is by use of the QCD sum rule method [1]. Given its success in predicting various hadronic properties, including the electromagnetic form factors of baryons [11,12], it appears highly suitable for the calculation of observables depending on \( \theta \). In the sum rule approach, physical properties of the hadronic resonances can be expressed through a combination of perturbative and nonperturbative contributions, the latter parametrized in terms of vacuum quark-gluon condensates. In the case of CP-odd observables induced by \( \theta \), the purely perturbative piece is absent and the result must be reducible to a set of the vacuum condensates taken in the electromagnetic and “topological” background. The expansion to first order in \( \theta \) will result in the appearance of correlators which have a structure similar to \( K \) and \( K_1 \) in Eq. (4). These correlators can then be calculated via the use of current algebra, in a similar manner to those considered in [3,4]. In this approach the \( \theta \)-dependence will arise naturally, with the correct quark mass dependence, and the relation to the U(1) problem will be explicit. This relation is manifest in the vanishing of any \( G \tilde{G} \)-induced observable in the limit when the mass of the U(1) “Goldstone boson” is set equal to the mass of pion.

Obviously, the calculation of the EDM of the neutron induced by the theta term will be a substantial task, although it appears that the main problem may be technical rather than conceptual – the calculation of \( d_n(\theta) \) needs to be at linear order in the quark mass as compared to the calculation of the anomalous magnetic moment which may be performed in
the limit \( m_{u,d} = 0 \). There are, however, additional subtleties relating to the correct choice for the nucleon current in the presence of non-zero \( \theta \).

Keeping in mind the importance of a sum rule calculation for the EDM of the neutron, we would like to test the applicability of this method by calculating the EDM(\( \theta \)) for a simpler system. The perfect candidate for this would be \( \rho \) meson which couples to the isovector vector current and whose properties have been predicted within QCD sum rules with impressive accuracy \([1]\). Thus in this paper we propose to study the feasibility of sum rule calculations for \( \theta \)-dependent electromagnetic observables in this mesonic system. We begin, in Section 2, with a tree-level analysis of the vector current correlator in a background with a nonzero electromagnetic field, and a theta term. We obtain the Wilson OPE coefficients for all \( \theta \)-dependent contributions up to neglected operators whose momentum dependence is \( O(1/q^6) \). This result is also briefly contrasted with the analogous expression for the tensor structure leading to the magnetic dipole moment. In Section 3, we turn to the phenomenological side of the Borel sum rule, and in Section 4 study the various contributions to the sum rule in some detail, obtaining a stable relation at the level of \( O(1/q^4) \) terms, and a reasonably precise extraction of \( d_\rho \). Section 5 contains further discussion, including comments on a consistent procedure for the definition of current operators away from the chiral limit in a background with nonzero \( \theta \).

II. CALCULATION OF THE WILSON OPE COEFFICIENTS

Since it is a spin 1 particle, the \( \rho \)-meson can possess on-shell two CP-odd electromagnetic form factors, the electric dipole and magnetic quadrupole moments. We shall concentrate here on the EDM of \( \rho^{+(-)} \) as the CP-violating form factors of \( \rho^0 \), induced by the theta term should vanish. This is a general consequence of C-symmetry, which is respected by the theta term.

Before commencing the calculation, it is useful to have a rough estimate of the EDM of \( \rho \) induced by theta. It is clear that the correct answer should have the “built-in” feature of vanishing when \( m_u \) or \( m_d \) are sent to zero. Thus we should expect a result of the form,

\[
d_\rho \sim \theta e \frac{m_u m_d}{m_\rho \Lambda(m_u + m_d)},
\]

where \( \Lambda \) is some scale at which the reduced quark mass is effectively normalized, presumably between \( f_\pi \) and \( m_\rho \). We note in passing that one could also use the approach of \([3]\) to obtain the contribution to \( d_\rho \) due to the chiral logarithm.

In order to calculate the \( \rho^+ \) EDM within the sum rule approach, we need to consider the correlator of currents with \( \rho^+ \) quantum numbers, in a background with nonzero \( \theta \) and an electromagnetic field \( F_{\mu\nu} \),

\[
\Pi_{\mu\nu}(Q^2) = i \int d^4x e^{iq\cdot x} \langle 0 | T \{ j^+_{\mu}(x) j^-_{\nu}(0) \} | 0 \rangle_{\theta,F},
\]

where we denote \( Q^2 = -q^2 \), with \( q \) the current momentum.

To simplify the presentation, we shall consider the example of \( \rho^+ \) in the \( m_u = m_d \) limit, for which the current reduces to \( j^+_{\mu} = \bar{u} \gamma_{\mu} d \). Since we always work to linear order in the quark mass, it is straightforward to resurrect the full dependence when required below, and
we shall always write the full mass dependence explicitly, with the implicit understanding that we set \( m_u = m_d \). With this current structure, the correlator reduces to the form

\[
\Pi^+(Q^2) = i \int d^4x e^{iqx} \langle 0 | \Pi \gamma_\mu S^d(x,0) \gamma_\nu u(0) + \bar{d}(0) \gamma_\mu S^u(0, x) \gamma_\nu d(x) | 0 \rangle_{\theta,F}
\]

\[
\equiv \pi^u_{\mu\nu} + \pi^d_{\mu\nu},
\]

(7)

where we have contracted two of the quark lines leading to the presence of the \( d \) and \( u \) quark propagators, \( S^u(x,0) \) and \( S^d(x,0) \), respectively.

We concentrate now on the contribution \( \pi^u_{\mu\nu} \), and note that at tree level the linear dependence on the background field \( F_{\mu\nu} \) can arise either through a vacuum condensate, or from a vertex with the propagator. These contributions are depicted in Fig. 1, and correspond to an expansion of the quark propagator to linear order in the background field. If we assume a constant field \( \partial_\rho F_{\mu\nu} = 0 \), the gauge potential may be written in covariant form \( A_\mu(x) = -\frac{1}{2} F_{\mu\nu}(0) x^\nu \), while similarly if we work in a fixed point gauge \([13]\), the gluon gauge potential may also be represented as \( A^a_\mu t^a(x) = -\frac{1}{2} G^a_{\mu\nu}(0) t^a x^\nu \). The expansion of the massless propagator, conveniently written in momentum space, then takes the form \([14]\)

\[
S(q) = \int d^4x e^{iqx} S(x,0) = \frac{1}{q^2} + \frac{q_\alpha}{(q^2)^2} \bar{F}_{\alpha\beta} \gamma_\beta \gamma_5 + \frac{q_\alpha}{(q^2)^2} g \bar{G}_{\alpha\beta} \gamma_\beta \gamma_5 + \cdots,
\]

(8)

where \( G_{\mu\nu} = C^a_{\mu\nu} t^a \), and we have introduced the dual field strengths \( \bar{F}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\gamma\delta} F^{\gamma\delta} \), and \( \bar{G}_{\alpha\beta} \). Since we are concerned only with the leading linear dependence on the quark mass, the mass structure of the propagator is very simple, and while not shown explicitly here, this structure is easily resurrected when required. The particular contributions we shall need will be given below.

While the expansion of the propagator \([8]\) apparently exhausts all possibilities for obtaining a linear dependence on the background field, it is important to also consider an expansion of the quark wavefunctions. The first order correction in the covariant Taylor expansion will be sufficient here, and is given by

\[
u(x) = u(0) + x_\alpha D_\alpha u(0) + \cdots,
\]

(9)

where \( D_\alpha = \partial_\alpha - ie_u A_\alpha(x) \) is the covariant derivative in the background field\([9]\).

Thus, if we substitute the expansions for the quark wavefunction and propagator into \( \pi^u_{\mu\nu} \), we find a sum of six terms,

\[
\pi^u_{\mu\nu} = i \left( \pi^u_1 + \pi^u_2 + \pi^u_3 + \pi^u_{\partial_1} + \pi^u_{\partial_2} + \pi^u_{\partial_3} \right),
\]

(10)

which may be conveniently represented in momentum space as follows: The first three terms, \( \pi^u_{1,2,3} \), given by,
FIGURES

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figures.png}
\caption{Contributions to the correlator at leading order in $F_{\mu\nu}$.}
\end{figure}

\[ \pi^u_1 = \langle 0 | \bar{u}(0) \gamma_\mu \frac{m_d}{(q^2)^2} \gamma_\nu u(0) | 0 \rangle_{\theta,F} \]  
\[ \pi^u_2 = g \langle 0 | \bar{u}(0) \gamma_\mu \frac{m_d}{2(q^2)^2} (G\sigma) \gamma_\nu u(0) | 0 \rangle_{\theta,F} \]  
\[ \pi^u_3 = e_d \langle 0 | \bar{u}(0) \gamma_\mu \frac{m_d}{2(q^2)^2} (F\sigma) \gamma_\nu u(0) | 0 \rangle_\theta \]

represent the contributions at leading order in the quark wavefunction, while $\pi^{u}_{\partial 1,2,3}$ correspond to the first order corrections,

\[ \pi^u_{\partial 1} = -i \langle 0 | \bar{u}(0) \overset{\leftarrow}{D}_\alpha \gamma_\mu \left( \frac{g_\alpha \beta}{q^2} - 2 \frac{q_\alpha q_\beta}{(q^2)^3} \right) \gamma_\beta \gamma_\nu u(0) | 0 \rangle_{\theta,F} \]
\[ \pi^u_{\partial 2} = -ig \langle 0 | \bar{u}(0) \overset{\leftarrow}{D}_\alpha \gamma_\mu \left( \frac{g_\alpha \beta}{(q^2)^2} - 4 \frac{q_\alpha q_\beta}{(q^2)^3} \right) \tilde{G}_{\beta\gamma}(0) \gamma_\gamma \gamma_5 \gamma_\nu u(0) | 0 \rangle_{\theta,F} \]
\[ \pi^u_{\partial 3} = -ie_d \langle 0 | \bar{u}(0) \overset{\leftarrow}{D}_\alpha \gamma_\mu \left( \frac{g_\alpha \beta}{(q^2)^2} - 4 \frac{q_\alpha q_\beta}{(q^2)^3} \right) \tilde{F}_{\beta\gamma}(0) \gamma_\gamma \gamma_5 \gamma_\nu u(0) | 0 \rangle_F. \]

Note that in evaluation of $\pi^u_2$ and $\pi^u_{\partial 2}$ we may turn off the background electromagnetic field.

We shall now consider each of these contributions in turn, although as a first step its helpful to re-express the quark wavefunction corrections $\pi^u_{\partial 1,2,3}$ in terms of the leading order condensates via use of the equations of motion. It proves convenient to first analyze the derivative term $\pi^u_{\partial 1}$. Writing this result in spinor notation, so as to factorise the $\gamma$-matrix structure, the matrix element we need to consider has the form

\[ \langle 0 | \bar{u} \overset{\leftarrow}{D}_\alpha \gamma_\mu \bar{u} | 0 \rangle = C_1 (\gamma_\beta)_{ba} \langle 0 | \bar{u} \gamma_\lambda \gamma_5 u | 0 \rangle \]

\[ + C_2 (\gamma_\beta \gamma_5)_{ba} \langle 0 | \bar{u} D_{\alpha \beta \gamma 5} u | 0 \rangle, \]

where $C_1$ and $C_2$ are two constants to be determined and $[\alpha, \beta]$ is used to denote anti-symmetrisation of indices (symmetrisation of indices will later be denoted by $\{\alpha, \beta\}$).

\[ ^2 \text{In principle one could also explicitly include colour indices. However, the trace over these indices will always be trivial in the examples to be considered here, so this dependence will be suppressed.} \]
Making use of the following identities,

\[ \{ \mathcal{D}, \sigma_{\alpha\beta} \} = -2\epsilon^{\alpha\beta\mu\lambda} \gamma^\lambda \gamma_5 D_\mu \]  
\[ [\mathcal{D}, \sigma_{\alpha\beta}] \gamma_5 = 2iD_{[\alpha} \gamma_{\beta]} \gamma_5, \]  
integrating by parts, and using the Dirac equation \( \mathcal{D} u = -im_u u \), we can reduce the natural Lorentz decomposition to a more recognizable form,

\[ \langle 0 | \overline{\pi}_a D_\alpha u_b | 0 \rangle = im_u C_1 (\gamma_\beta)_{ba} \langle 0 | \overline{\pi} \sigma_{\alpha\beta} u | 0 \rangle 
- m_u C_2 (\gamma_\beta \gamma_5)_{ba} \langle 0 | \overline{\pi} \sigma_{\alpha\beta} \gamma_5 u | 0 \rangle. \]  

(20)

Two equations for the constants \( C_2 \) and \( C_2 \) may be obtained by, in one case, contracting with \( (\sigma_{\mu\nu} \gamma_\alpha)_{ab} \), and in another, via multiplication by \( \gamma_\beta \) and then anti-symmetrising in \( \alpha, \delta \). One obtains \( C_1 = 0 \) and \( C_2 = -1/8 \), and thus we are left with only one structure in the decomposition.

On Fourier transforming to momentum space, and performing the straightforward \( \gamma \)–matrix algebra we find

\[ \pi^u_{\theta 1} = \frac{i}{q^2} \int \frac{d^d q}{q^2} \langle 0 | \overline{\pi} \sigma_{\mu\nu} u | 0 \rangle \theta_F - \frac{m_u}{4q^2} \epsilon_{\mu
u\gamma\delta} \langle 0 | \overline{\pi} \sigma_{\alpha\beta} \gamma_5 u | 0 \rangle \theta_F. \]  

(21)

We can now compare the first term here with \( \pi^u \) given in (14). Since the \( \theta \)–dependent CP-odd contribution corresponds to considering only the term antisymmetric in \( \mu \) and \( \nu \), the relation \( \gamma_\mu \gamma_\nu = -i\sigma_{\mu\nu} + \text{sym} \) implies that \( \pi^u \) precisely cancels the first term above. Thus we have,

\[ \pi^u_{\theta 1} + \pi^u_{\theta 2} = -m_u \frac{g_\alpha q_\gamma}{(q^2)^2} \epsilon_{\mu
u\gamma\delta} \langle 0 | \overline{\pi} \sigma_{\alpha\beta} \gamma_5 u | 0 \rangle \theta_F. \]  

(22)

Turning next to \( \pi^u_{\theta 2} \) (17), a little \( \gamma \)–matrix algebra shows that the matrix element may be rewritten in the form

\[ \pi^u_{\theta 2} = \left( \frac{g_{\alpha\rho}}{(q^2)^2} - \frac{4g_\alpha q_\rho}{(q^2)^3} \right) \epsilon_{\mu\sigma\nu\lambda} \tilde{F}_{\rho\sigma} \langle 0 | \overline{\pi} \gamma_\lambda u | 0 \rangle \theta_F. \]  

(23)

It can be shown that the condensate in this expression is in fact proportional only to \( g_\alpha \lambda u \langle 0 | \overline{\pi} u | 0 \rangle \theta \) and therefore does not contain any CP-violating piece.

The final wavefunction correction to consider is \( \pi^u_{\theta 3} \) (16) which may be handled in a similar manner to \( \pi^u_{\theta 2} \), via extracting the appropriate projections onto vacuum condensates, or alternatively by direct calculation. We shall follow the former approach here, and write down

\[ \langle 0 | \overline{\pi} (\tilde{G}_{\beta\gamma}^a) D_\alpha u_q | 0 \rangle = C_1 (t^a)_{\gamma} \langle 0 | \overline{\pi} (\tilde{G}_{\beta\gamma}^a) \mathcal{D} u | 0 \rangle. \]  

(24)

Note that another apparently valid Lorentz structure of the form \( \langle 0 | \overline{\pi} (\tilde{G}_{\beta\gamma}^u) \gamma_5 \mathcal{D} u | 0 \rangle \) vanishes on the equations of motion. Contracting with \( (\gamma_\alpha)_{pq} (t^a)_{ji} \), and recalling that \( t^a t^a = 4/3 \), one finds that \( C_1 = 3/64 \). The resulting expression for \( \pi^u_{\theta 3} \) is,

\[ \pi^u_{\theta 3} = -\frac{m_u g}{4} \left( \frac{g_{\alpha\beta}}{(q^2)^2} - \frac{4g_\alpha q_\beta}{(q^2)^3} \right) \epsilon_{\mu\nu\alpha\gamma} \langle 0 | \overline{\pi} \tilde{G}_{\beta\gamma} u | 0 \rangle. \]  

(25)
The next problem to address is that of extracting the leading $\theta$–dependence of these matrix elements, and we follow standard practice (see e.g. [9]) in making use of the anomalous Ward identity. To illustrate the procedure, consider the condensate $m_u \langle 0 | \pi \Gamma u | 0 \rangle$, with a generic Lorentz structure denoted by $\Gamma$. In the $\theta$–vacuum, to leading order, we have

$$m_u \langle 0 | \pi \Gamma u | 0 \rangle_{\theta} = m_u \int d^4 y \langle 0 | T \{ (\pi \Gamma u(0), i\theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}(y) ) \} | 0 \rangle. \quad (26)$$

We now make use of the anomalous Ward identity for axial currents [9] restricted to 2 flavours. A useful calculational simplification follows if we take as the anomaly relation a linear combination of the singlet equations for the $u$ and $d$ quarks. In particular, we use

$$\partial_{\mu} j_{\mu 5} = 2 m_\ast (\bar{u} \gamma_5 u + \bar{d} \gamma_5 d) + \frac{\alpha_s}{4\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}, \quad (27)$$

where

$$j_{\mu 5} = \frac{m_\ast}{m_u} \bar{u} \gamma_\mu \gamma_5 u + \frac{m_\ast}{m_d} \bar{d} \gamma_\mu \gamma_5 d \quad (28)$$

is the anomalous current, and we have introduced the reduced mass,

$$m_\ast \equiv \frac{m_u m_d}{m_u + m_d}. \quad (29)$$

Substituting the anomaly relation (27) for $G\tilde{G}$ into the correlator (26), we recall that the only contribution from $\partial_{\mu} j_{\mu 5}$ is a contact term $\propto \delta(y_0)$ due to the presence of the $T$–product. Through the use of the equal time commutator, we find that this leads to a local contribution (independent of $y$), and consequently we have

$$m \langle 0 | \pi \Gamma u | 0 \rangle_{\theta} = i\theta m_\ast \langle 0 | \pi \Gamma \gamma_5 u | 0 \rangle + i\theta \int d^4 y \langle 0 | T \{ m_\ast (\bar{u} \gamma_5 u(y) + \bar{d} \gamma_5 d(y)), m_u \bar{u} \sigma_{\mu\nu} u(0) \} | 0 \rangle. \quad (30)$$

The nonlocal contribution to this correlator, the second term above, is $O(m^2)$ in light quark masses. Nonetheless, this term would cancel the local contribution were there an intermediate state with mass squared of $O(m)$ – for example the Goldstone boson in the singlet channel. The crucial point, as stressed in [9], is that due to the $U(1)$ problem the lightest intermediate state $\eta$ has $m_\eta \gg m_\pi$ and thus the second term can be neglected at leading order in $m$. Thus for each of the contributions above, the leading dependence on $\theta$ is determined via the following relation,

$$m \langle 0 | \pi \Gamma u | 0 \rangle_{\theta} = i\theta m_\ast \langle 0 | \pi \Gamma \gamma_5 u | 0 \rangle. \quad (31)$$

We could of course have obtained this result in a simple manner by using the anomaly to rotate away the $G\tilde{G}$ term in the action. This induces a complex quark mass $m \rightarrow m + i\theta m_\ast \gamma_5$, and leads directly to the leading $\theta$–dependence (31) above. However, despite being somewhat more involved, the procedure we have followed is advantageous in that it makes quite explicit the role of the anomaly and, in particular, the conditions under which the higher order nonlocal terms may be neglected. We shall return to the issue of chiral rotations at the level of the action in Section 5.
The final effect to consider is that of the background field, $F_{\mu\nu}$. For the term $\pi_3^u$ (13), the leading $F$-dependence is already explicit, and may be extracted via introduction of spinor indices. For the other terms, we follow Ioffe and Smilga (11) and introduce “condensate susceptibilities”, $\chi$, $\kappa$, and $\xi$, defined as follows (11):

\[
\langle 0 | \sigma_{\mu\nu} q | 0 \rangle_F = e_q \chi F_{\mu\nu} \langle 0 | q | 0 \rangle_F \\
g \langle \overline{q} \gamma^\alpha (G_{\mu\nu}^a t^a) q | 0 \rangle_F = e_q \kappa F_{\mu\nu} \langle \overline{q} q | 0 \rangle_F \\
g \epsilon_{\mu\nu\lambda\sigma} \langle 0 | \gamma^\lambda (G_{\mu\nu}^a t^a) q | 0 \rangle_F = i e_q \xi F_{\mu\nu} \langle \overline{q} q | 0 \rangle_F ,
\]

where $q = u$ or $d$.

Using the relations (31, 32), and performing the Fourier transformation to momentum space, we can now gather all the results from $\pi_{1}^u$–$\pi_{2}^u$ (10) and combine them with the analogous results for $\pi_{1}^d$ in (7), to obtain

\[
\Pi^+_{\mu\nu} = m_s \theta(e_u - e_d) \langle 0 | q | 0 \rangle_F \left[ \frac{\tilde{F}_{\mu\nu}}{q^2} \left( -\chi - \frac{1}{q^2} \left( 1 + \kappa - \frac{1}{4} \xi \right) \right) \right] \\
- \left( \chi - \frac{\xi}{q^2} \right) \frac{g \epsilon_{\mu\nu\lambda\sigma} \langle \overline{q} \gamma^\lambda (G_{\mu\nu}^a t^a) q | 0 \rangle_F}{(q^2)^2} .
\]

This expression exhibits the two tensor structures one would have expected to appear on general grounds. However, only the first term contributes to the EDM, as one may check by choosing a rest frame for the current momentum, since the second tensor structure vanishes on shell.

Therefore, if we retain only the contribution which survives on-shell, we have as our final expression for the theoretical side of sum rule,

\[
\Pi^+_{\mu\nu} = m_s \theta(e_u - e_d) \langle 0 | q | 0 \rangle_F \left[ \frac{\tilde{F}_{\mu\nu}}{q^2} \left( -\chi - \frac{1}{q^2} \left( 1 + \kappa - \frac{1}{4} \xi \right) \right) \right] .
\]

As a short digression, it is instructive to contrast this result with the analogous expression one would obtain for the structure $F_{\mu\nu}$ which leads to an extraction of the magnetic dipole moment $\mu_{\rho}$. The crucial difference is that in this case a nonzero contribution survives in the chiral limit. Specifically, a perturbative 1-loop diagram in the background field leads to a contribution of the form,

\[
\Pi^+_{\mu\nu} = -\frac{1}{8\pi^2} (e_u - e_d) F_{\mu\nu} \ln \frac{\Lambda^2}{-q^2} + \cdots .
\]

Subleading power corrections have been ignored here. However, it turns out that such contributions generically have a form similar to those in (34) with coefficients $m_s \theta \to m_q$, and therefore vanish in the chiral limit $m_q \to 0$. One may also check that the first subleading terms of $O(m_q^0)$ actually vanish identically, and therefore the perturbative piece serves as the dominant contribution to $\mu_{\rho}$. An interesting corollary is that, while certain power corrections are closely related as above, there would appear to be no simple proportionality relation between the electric and magnetic dipole moments.
III. MESONIC SPECTRAL FUNCTION AND CONSTRUCTION OF THE SUM RULE

In order to extract a numerical value for the $\rho^+$ EDM from the OPE, we assume as usual that $\Pi^+$ satisfies a dispersion relation (ignoring subtractions) of the form,

$$\Pi(q^2) = \frac{1}{\pi} \int_0^\infty d\sigma \frac{Im \Pi(\sigma)}{(\sigma - q^2)},$$

which we then saturate with physical mesonic states ($\rho^+$, and excited states with the same quantum numbers which we denote collectively as $\rho'$). To suppress the contribution of excited states, we apply a Borel transform to $\Pi^+$, which we define, following [14,16], as

$$B\Pi^+ \equiv \lim_{s,n \to \infty, s/n = M^2} \frac{s^n}{(n - 1)!} \left( -\frac{d}{ds} \right)^n \Pi^+(s) = \frac{1}{\pi M^2} \int_0^\infty d\sigma e^{-\sigma/M^2} Im \Pi^+(\sigma),$$

where $s = -q^2$.

The phenomenological side of the sum rule may be parametrised by considering the form-factor Lagrangian which encodes the effective CP violating vertices (see Fig. 2). This has the form $\mathcal{L} = \sum f_n S(q) O_n S(q)$, where $f_n$ is the form factor, $S(q)$ is the on-shell propagator for $\rho^+$ or one of its excited states, and $O_n$ is the operator corresponding to the induced vertex.

As mentioned above, there are a priori two such operators which need to be considered at lowest order, $F_{\mu\nu} q^2$ and $q_\alpha q_\mu F_{\nu\alpha}$. As noted above, the second structure vanishes on-shell and thus does not enter the form-factor Lagrangian. Another on-shell T-odd form factor, the magnetic quadrupole moment, would appear only at the next order in momentum transfer, i.e. in front of the structure proportional to $\partial_\lambda F_{\mu\nu}$ [15]. Since we work only to linear order in photon momentum, the magnetic quadrupole moment cannot be recovered from the OPE form (34), and so we omit it on the phenomenological side as well.

![FIG. 2. Mesonic contributions to the current correlator in an external electromagnetic field. Possible excited states with the $\rho^+$ quantum numbers are denoted generically by $\rho'$.](image)

Consequently, for comparison with the OPE, we have on the phenomenological side in momentum space,

$$\Pi_{\mu\nu}^{(phen)} = 2 f(q^2) F_{\mu\nu} + \cdots,$$

where, since we work outside the dispersion relation we may add polynomials in $q^2$ to ensure transversality in the chiral limit, and optimum behaviour for large $q^2$, without affecting the physical spectral function $Im \Pi$. We then find that the function $f(q^2)$ takes the form,
\[ f(q^2) = \frac{f_1 \lambda^2}{(q^2 - m_{\rho}^2)^2} + \sum_n \frac{f_n}{(q^2 - m_{\rho}^2)(q^2 - m_n^2)} + \sum_{n,m} \frac{f_{nm}}{(q^2 - m_{n}^2)(q^2 - m_{m}^2)}. \]  

(39)

In this expression, \( \lambda \) is the dimension 2 coupling defined in terms of the transition amplitude for the vector current to go into the \( \rho^+ \) state, \( \langle 0 | j_\mu | \rho^+ \rangle = \lambda V_\mu \), where \( V_\mu \) is an appropriate vector, while \( f_1 \) is associated with the \( \rho^+ \) EDM, and \( f_n \) and \( f_{nm} \) correspond respectively to transitions between \( \rho^+ \) and excited states, and between the excited states themselves.

After performing a Borel transform on \( f \), we obtain

\[ f(M^2) = \frac{\lambda^2 f_1}{M^4} e^{-m_{\rho}^2/M^2} + \frac{1}{M^2} \sum_n \frac{f_n}{m_n^2 - m_{\rho}^2} e^{-m_n^2/M^2} + \sum_{n,m} \frac{f_{nm}}{(m_n^2 - m_{\rho}^2)(m_m^2 - m_{\rho}^2)} e^{-(m_n^2 + m_m^2)/M^2}. \]  

(40)

Since the gap from \( m_{\rho} \sim 0.77\text{GeV} \), to the first excited state \( m_{\rho'} \sim 1.7\text{GeV} \) is large, we shall, as in [18], ignore the continuum contribution as it is exponentially suppressed. Thus we may write,

\[ f(M^2) = \left( \frac{\lambda^2 f_1}{M^4} + \frac{A}{M^2} \right) e^{-m_{\rho}^2/M^2}, \]  

(41)

where \( A \) is an effective constant of dimension 2.

We are now in a position to write down the sum rule for the coefficient of \( \tilde{F}_{\mu \nu} \). From the Borel transform of (34), and also (38,41), we have

\[ \lambda^2 f_1 + AM^2 = \frac{1}{2} m_s \theta(e_u - e_d) M^4 e^{-m_{\rho}^2/M^2} \langle 0 | \bar{q} q | 0 \rangle \left( \frac{\lambda}{M^2} - \frac{1}{M^4} \left( 1 + \kappa - \frac{\xi}{4} \right) + \cdots \right). \]  

(42)

This is our final result for the CP-odd sum rule, and will be investigated numerically in the next section.

**IV. NUMERICAL ANALYSIS**

The coupling \( \lambda \) present in (42) may be obtained from the well known mass sum rule in the CP even sector. In this case, there is no need to consider a background electromagnetic field, and the sum rule takes the form (see e.g. [10]),

\[ \frac{\lambda^2}{m_{\rho}^2 M^2} e^{-m_{\rho}^2/M^2} = -\frac{1}{4\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) e^{-s_0/M^2} + \frac{1}{4\pi^2} \left( 1 + \frac{\alpha_s}{\pi} + \frac{\pi^2}{3M^4} \left( \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle + 24 \langle 0 | m_{\rho} \bar{q} q | 0 \rangle \right) \right). \]  

(43)

\(^3\)An alternative derivation of the Borel transform in this context, using double dispersion relations in order to parametrise \( \text{Im} \Pi \), was presented in [11].
Since there is no background field, the leading term is a single pole contribution, and we include a continuum term shifted to the right-hand side starting from the $\rho'$ threshold at $s_0 \sim 1.7$GeV. Note also that $\lambda$, as defined above, is related to the dimensionless coupling $g_\rho$ associated with the width of the resonance, via $\lambda = m_\rho^2/g_\rho$, so that $g_\rho$ is dimensionless.

The physical EDM parameter $d_\rho$ may be obtained by normalising the form factor $f_1$, introduced above, by the $\rho'$ mass. Furthermore, it will be convenient in what follows to define an additional parameter $\tilde{d}$ via the relation,

$$d_\rho = \frac{f_1}{m_\rho} \equiv \tilde{d} \frac{m_\rho}{m_\rho} \theta(e_u - e_d).$$

We shall now study the sum rule (42), making use of (43) to remove the $\lambda$–dependence. Its helpful to consider the various contributions to (42) in turn. At the most naive level, we can ignore the $O(1/M^4)$ corrections in (42), and also the continuum in (43). Taking ratios one finds,

$$\tilde{d}_1 \sim \frac{2\pi^2}{m_\rho^2} \frac{\chi\langle 0|\bar{q}q|0\rangle}{(1 + \alpha_s/\pi + \pi^2\langle \mathcal{O}_4 \rangle/M^4)}.$$  

where we have defined $\langle \mathcal{O}_4 \rangle \equiv \langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle + 24\langle 0|m_q\bar{q}q|0\rangle$ for convenience. For numerical calculation we make use of the following parameter values: For the quark condensate, we have

$$\langle 0|\bar{q}q|0\rangle = -(0.225 \text{ GeV})^3,$$  

while for the condensate susceptibilities, we have the values calculated in [17] and [18],

$$\chi = -5.7 \pm 0.6 \text{ GeV}^{-2} \text{ [17]}$$

$$\kappa = -0.34 \pm 0.1 \text{ [18]}$$

$$\xi = -0.74 \pm 0.2 \text{ [18]}$$

Note that $\chi$, which enters at $O(1/M^2)$, since it is dimensionful, is numerically significantly larger than $\kappa$ and $\xi$. With these parameters, the result for $\tilde{d}_1$ is shown in Fig. 3, where we have also used the 1-loop running coupling $\alpha_s(M)$ with two flavours, normalised to 0.34 at $M_\tau$. Note that the stability at large $M^2$ is an artefact of the cancelation of the leading $M$ dependence in (43). One should expect this relation to have reasonable accuracy only in the range $M^2 \sim m_\rho^2$.

To observe the effect of the $O(1/M^4)$ corrections, we now need to address the issue of the unknown constant $A$ in (42). Relative to $f_1$, there is a suppression factor of $M^2/(s_0 - m_\rho^2)$ associated with $A$, which near $M^2 = m_\rho^2$ is $\sim 0.25$. Although this is to be summed over all the excited states, we shall use this as justification to treat $A$ perturbatively, and solve for it in terms of $f_1$ using (42), but ignoring $O(1/M^4)$ terms. Its convenient to do this via first pre-multiplying (42) by $M^2$ and then differentiating by $1/M^2$. One obtains the relation

$$A \sim f_1 \lambda^2 \left( \frac{1}{m_\rho^2} - \frac{1}{M^2} \right) + \cdots,$$  

$$\lambda = m_\rho^2/g_\rho.$$
FIG. 3. The $\rho^+$ EDM parameter $\tilde{d}$ as a function of $M^2$ according to various components of the sum rules (45) and (51).

which vanishes when $M^2 = m^2_\rho$. Substituting this back into (42), we can isolate $\tilde{d}$ by taking the ratio with the quantity obtained by pre-multiplying (43) by $e^{s_0/M^2}$, and then differentiating by $1/M^2$,

$$\tilde{d} = 2\pi^2 \frac{\langle 0|qq|0 \rangle (s_0 + M^2 - m^2_\rho)}{(s_0(1 + \alpha_s/\pi) + \pi^2(s_0 + 2M^2)\langle O_4 \rangle/M^4)} \left( \frac{\chi}{M^2} - \frac{1}{M^4} \left( 1 + \kappa - \frac{\xi}{4} \right) + \cdots \right).$$

(51)

To study the various contributions to (51), we first set all the $O(1/M^4)$ corrections zero, and the result, denoted $\tilde{d}_2$, is shown in Fig. 3. We see the expected $1/M^2$ behaviour so that there is no stability region, although the relation is, as one would expect, very close to $\tilde{d}_1$ (43) near $M^2 = m^2_\rho$.

The leading correction at $O(1/M^4)$ may be isolated by setting $\kappa = \xi = 0$ in (51). The result, $\tilde{d}_3$, is shown in Fig. 3. The presence of the $1/M^4$ term induces a transition region in the $M^2$ dependence. Note that the numerical similarity with $\tilde{d}_2$ over the relevant range of the Borel parameter $M^2$ is in part due to the compensating effect of the $O(1/M^4)$ correction in the denominator of (51).

Finally, we can obtain an estimate of the corrections associated with $\kappa$ and $\xi$, by plotting the full expression in (51), which is displayed in Fig. 3 as $\tilde{d}_4$. We see that including these corrections have little effect on the behaviour or stability of the sum rule. This is encouraging, as the precise numerical values for $\kappa$ and $\xi$ are uncertain to a larger degree than that of $\chi$. Extracting a numerical estimate for $\tilde{d}$, and an approximate error, from the stability region $M^2 \sim 0.3 - 0.8\text{GeV}^2$ in Fig. 3, we find the result

$$d_\rho = (2.6 \pm 0.8)e\theta \frac{m_*}{(1\text{GeV})^2},$$

(52)

for the EDM of $\rho^+$, where $e = e_u - e_d$ is the positron charge. As is clear from Fig. 3, the dominant contribution arises from the term proportional to the susceptibility $\chi$, and thus the result is essentially linearly dependent on the value (and error) for this parameter.
It is interesting to note that comparison with the naive estimate (5) implies a value for the effective scale $\Lambda$ of order 1 GeV. This is very close to a similarly defined scale which would effectively appear in the chiral logarithm estimates for $d_n$, Eq. (2). In this sense, we can conclude that our result is in the expected range. One final point to note is that if we return to Eq. (51), which is written in terms of the condensates, and re-express the final answer in slightly different units, we can directly relate the EDM to the vacuum topological susceptibility correlator $K$ (4) as calculated in [8,9]:

$$d_\rho = 2.2 \times 10^{-3} e\theta \frac{m_\pi^2 f_\pi^2}{(100\text{MeV})^5} \frac{m_u m_d}{(m_u + m_d)^2} = 2.2 \times 10^{-3} e\theta \frac{K}{(100\text{MeV})^5}. \quad (53)$$

Note that we have used a normalisation (100MeV) which is adapted to the small size of $m_*$, and it is this which accounts for the small overall coefficient. This factor is essentially hidden in the result presented in (52).

V. DISCUSSION

Throughout the calculation we have intentionally kept $m_u = m_d$, knowing that the correct mass dependence is $m_*$. It is easy to see, however, that if $m_u \neq m_d$ the calculation does not automatically restore $m_*$ since, for example, the up-quark bilinear combination in Eq. (13) comes with a coefficient proportional to $m_d$. In the final result it would induce a contribution which would not vanish in the limit $m_u \to 0$. This means that if $m_u \neq m_d$ there should be additional contributions which would combine with the rest to form an overall $m_*$-dependent result.

At the same time, we recall that one can use a chiral transformation in the QCD Lagrangian and rotate the $\theta$-parameter to stand in front of the quark singlet combination $m_*(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)$. It is clear that in this situation the $\theta$-dependence for any physical observable will arise together with the correct mass dependence and will disappear at $m_u = 0$. The answer to this “puzzle” lies in the chirally non-invariant form of the quark current which we associate with $\rho^\pm$. Using this form of the current, additional contributions must arise which are associated with vector–axial-vector current mixing. In other words, the purely vectorial current is not diagonal due to the chiral anomaly, and one needs to consider all the $\langle j_V j_V \rangle$, $\langle j_V j_A \rangle$, and $\langle j_A j_A \rangle$ correlators to obtain a well-defined projection onto $\rho$. However, a more elegant approach would appear to involve a direct diagonalisation of the current. To see how this might be achieved, let us write down two forms of the QCD Lagrangian with an external vector source coupled to isovector quark current:

$$L_1 = \cdots - m_u \bar{u}u - m_d \bar{d}d - \theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} + V_\mu \bar{u}\gamma_\mu u + V_\mu^* \bar{d}\gamma_\mu d \quad (54)$$

$$L_2 = \cdots - m_u \bar{u}u - m_d \bar{d}d - \theta m_*(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d) + V_\mu \bar{u}\gamma_\mu u + V_\mu^* \bar{d}\gamma_\mu d \quad (55)$$

where the ellipses stand for the standard kinetic terms for the gauge and quark fields. In the absence of the external current $L_1$ and $L_2$ are equivalent (we consider $\theta$ to be small

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4We thank A. Vainshtein for discussions on this point.
The presence of the external current in the form written in the Eqs. (54) and (55) makes \( L_1 \) and \( L_2 \) explicitly inequivalent. The same chiral rotation transforms \( L_1 \) to \( L_2 \) plus an extra term

\[
L_1 \rightarrow L_2 + i \frac{m_d - m_u}{m_u + m_d} (V_\mu \bar{u} \gamma_\mu \gamma_5 d - V^*_\mu \bar{d} \gamma_\mu \gamma_5 u).
\]

(56)

In the limit of \( m_u = 0 \), this extra term contains \( \theta \) explicitly which will then enter in the physical amplitudes bilinear in \( V \). Thus we need to bear in mind that in the presence of \( \theta \) the choice of the current for further use in QCD sum rules is not “automatic”, if one wishes to avoid mixing with other contributions. In the case of \( \rho^+ \) with \( m_u \neq m_d \), the \( \bar{d} \gamma_\mu u \)-current should be used only in the basis where \( \theta \) is completely rotated to the quark mass term. In the basis where \( \theta \) enters in front of \( GG \), the current includes additional axial-vector pieces which restore the correct quark mass dependence in the final answer. As an additional check, one can calculate the next order \( \sim \theta^2 \)-corrections to CP-even observables and observe the dependence of the result on the choice of the current.

Of course, the calculation of the electric dipole moment of \( \rho \) does not have direct experimental implication. Our main motivation for calculating this quantity was to test the possibility of applying the QCD sum rule approach to the problem of EDM(\( \theta \)). We would like to mention here that the idea to consider the EDM of the \( \rho \) meson resulting from the EDM of quarks was used in [19].

Returning to the problem of \( d_n(\theta) \), it seems clear that there are a number of additional difficulties which may be encountered. One of them refers to the correct choice of the nucleon current at \( \theta \neq 0 \), as this choice can be ambiguous even in the normal CP-conserving case [20]. Another difficulty is related to the necessity for a simultaneous treatment of the mass operator and the electromagnetic form factors. This is because in the presence of \( \theta \) the mass operator develops an imaginary part which can influence the answer for \( d_n(\theta) \). Nevertheless, this calculation appears feasible and work in this direction is currently in progress. Only then can one have a reliable means to interpret \( d_n \) directly in terms of the high-energy parameters (CP-violating phases in the soft-breaking sector and masses of superpartners in the case of the MSSM).

In conclusion, we have demonstrated that QCD sum rules can be used for the calculation of CP-odd electromagnetic form factors induced by the theta term. The result for the EDM of the vector meson, calculated in this way, is stable and numerically dominated by the vacuum magnetic susceptibility. The set of correlators which appear in the OPE part of the QCD sum rule were calculated via the use of the anomaly equation, in a similar manner to the calculation of topological susceptibility. In this way, a direct relation to the U(1) problem becomes apparent as the total result vanishes in the limit \( m_\eta \rightarrow m_\pi \).

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