Modelling and improvement of the design of hinged centralizer for casing

I Shatskyi¹*, I Vytvytskyi², M Senyushkovych² and A Velychkovych³

¹Ivano-Frankivsk Branch of Pidstryhach-Institute for Applied Problems in Mechanics and Mathematics, NAS of Ukraine, Department of modelling of damping systems, Mykytynetska 3, Ivano-Frankivsk, Ukraine
²Ivano-Frankivsk National Technical University of Oil and Gas, Department of Drilling, Karpatska 15; Ivano-Frankivsk, Ukraine
³Ivano-Frankivsk National Technical University of Oil and Gas, Department of Construction, Karpatska 15; Ivano-Frankivsk, Ukraine

E-mail: ipshatsky@gmail.com

Abstract. We have researched an important problem for the oil-and-gas field, which is the problem of centralizing of cases in wellbores with a complex profile. Usually this problem is solved by equipping the case in problematic areas with special centralizing devices. Centralizers have mutually competing requirements: low stiffness in order not to slow down the descent of the case into wellbore and high stiffness in order to provide proper clearance gap for high-quality cementing of the annular space. The aim of the paper is to solve this problem by means of the construction of a hinged-rod centralizer, additionally equipped with a thrust ring. To model the arch-shaped link of the centralizer, the classical theory of the flat rods is used. On the basis of analytical solution of contact problems with unilateral connections, we investigated the stress-strain state of the rod in two cases: before and after the interaction with the axial obstruction. We have determined the non-linear dependencies between the contact force and the mutual approach of the case and the wall of the wellbore, and also the expressions of the acceptable contact loadings that provide structural integrity of the construction.

1. Introduction
Centering of casing strings is one of the key factors that ensure the quality of well cementing. Millions of centralizers are annually produced and used in the world, but problems associated with inadequate casing of the wellbores continue to emerge. The deviation of the casing depends on the trajectory and the size of the well, the size of the casing, the location and properties of the centralizers, and other factors. One of the tasks the operators and service companies must fulfill is choosing the correct type of centralizer. Despite the wide range of engineering solutions aimed at increasing the reliability of wells casing [1–4], the problem of effective centering of the casing in the complex configuration of the well axis remains relevant for the oil and gas industry. This problem is particularly important due to the increasing volumes of directional drilling.

Different types of centering devices are used to center the casing in an open wellbore. In order to assess the efficiency of the centering devices of different types, it is recommended to calculate their elastically rigid characteristics according to different schemes of attaching them on the casing, and as a consequence to prevent their over-consumption or insufficient quantities in the interval of centering.
The most commonly used in the practice of drilling vertical and directional wells are typical designs of elastic rod "bow" type centralizers [5, 6].

The main operational characteristic of centering devices is considered their ability to concentrically locate the casing in the well in any position of its axis. When choosing the centralizer, two mutually competing demands are put together at the same time: low stiffness, in order not to slow down the descent of the string in the well, and high stiffness to provide a proper gap for the qualitative cementing of the annular space [7–10]. The paper aims at solving this problem at the expense of the construction of a hinged centralizer, additionally equipped with a thrust ring.

2. The design of the centralizer with axial thrust

To solve the described problem, the authors propose to use a centralizer design with variable stiffness (figure 1). Such a centralizer has a hinged fixing of the lower ends of the elastic plates 1 with folding collars 2 and free attachment of the upper ends of the elastic plates 1, fixed with a flexible and stretchable ring 4, placed in a loop with a rounded shoulder. From the side of free attachment of the ends of the elastic links at a certain distance \( \Delta \), the centering system is equipped with an additional thrust 7.

![Figure 1. Hinged-rod centralizer with an axial thrust: 1 – elastic plate, 2 – folding collars, 3 – segmental slots, 4 – elastic ring, 5 – stop ring, 6 – casing, 7 – stiff axial thrust.](image)

3. Results of mathematical modeling and analysis

General problems of interaction of columns with the borehole were considered, in particular, in the papers [11–14]. Various approaches to the engineering simulation of contact phenomena in shell and rod structures using classical theories of rods and shells are proposed in publications [15–22]. The authors of the paper [23] have initiated the procedure for formulation of analytical bilateral assessment of the rigidity and strength of the hinged centralizers of the casing. The numerical-analytical [24] studies touch upon the analysis of the mechanical properties of the centralizers and the interaction of the column-equipped well with the well wall.

For simulation, we used the classical linear theory of hollow arch-shaped rods. We consider the tensile stiffness of the run-around thread to be small. This means that each link of the centralizer works quasi autonomously and one can only consider the deformation of a single link loaded by the contact force (figure 2).
Let $f$ be the lifting arm of the centralizer’s link, $2l$ is the length of the rod projection on the axis of abscissa, $2\alpha$ is the small slope of the arc and $R$ is the radius of curvature. The rod is considered flat, so

\[
\left(\frac{f}{l}\right)^2 << 1, \quad \frac{1}{R} \approx \frac{2f}{l^2}, \quad \alpha \approx \frac{2f}{l}.
\]

The key equations of the model are as follows:

- equilibrium equation

\[
\frac{d^2N}{dx^2} = 0, \quad \frac{dQ}{dx} + \frac{N}{R} = -P\delta(x), \quad \frac{dM}{dx} - Q = 0, \quad x \in (-l, l);
\]  

- physical relationships

\[
\frac{du}{dx} + \frac{w}{R} = 0, \quad M = EJ \frac{d\vartheta}{dx}, \quad EJ \frac{d^2w}{dx^2}, \quad x \in (-l, l).
\]  

Here $P$ is the contact force, $N$ is axial force, $Q$ is transverse force, $M$ is bending moment; $EJ$ is stiffness of the rod to the bend; $u$ is tangential displacement; $\vartheta$ is the angle of rotation, $w$ is transverse displacement of the rod; $x$ is the coordinate; $\delta(x)$ is Dirac function.

**Figure 2.** Scheme of the contact of the centralizer’s link with the well wall.

The boundary conditions for fastening the rod are as follows:

- on the left end ($x = -l$):

\[
M(-l) = 0, \quad u_x(-l) = u(-l) + w(-l)\alpha = 0, \quad u_y(-l) = w(-l) + u(-l)\alpha = 0;
\]  

- on the right end ($x = l$):

\[
M(l) = 0, \quad u_x(l) = w(l) - u(l)\alpha = 0; \quad N_x(l) = N(l) - Q(l)\alpha = 0, \quad u_y(l) = u(l) + w(l)\alpha \leq \Delta,
\]  

or

\[
u_x(l) = u(l) + w(l)\alpha = \Delta, \quad N_y(l) = N(l) - Q(l)\alpha < 0.
\]  

Thus, (1) – (5) is the boundary problem for the system of differential equations of the 6th order with boundary conditions in the form of inequalities corresponding to one-sided joints.
The solution to this problem \( \mathbf{z}(x) = (N, Q, M, \mathcal{G}, w, u)^T \) is found in the form:

\[
\mathbf{z}(x) = \begin{cases}
P \mathbf{z}^{(1)}(x), & P > P_i; \\
P \mathbf{z}^{(1)}(x) + (P - P_i) \mathbf{z}^{(2)}(x), & P < P_i;
\end{cases}
\]

\[
= P \mathbf{z}^{(1)}(x) + (P - P_i)H(P - P_i) \mathbf{z}^{(2)}(x) - \mathbf{z}^{(1)}(x).
\]  

(6)

Here \( H(x) \) is the Heaviside function, and \( P_i = \frac{12 f EJ}{5 l^4} \).

In addition, \( \mathbf{z}^{(i)}(x) = (N^{(i)}, Q^{(i)}, M^{(i)}, \mathcal{G}^{(i)}, w^{(i)}, u^{(i)})^T \) are the solutions of two auxiliary problems: 

- \( i = 1 \) – the problem of the action of a single concentrated force \( P = 1 \) on an arch with a hinged moving fixing (condition (5) should be replaced by \( N_i(l) = 0 \));
- \( i = 2 \) – the problem of the action of a single concentrated force on an arch with a hinged immovable fixing (instead of condition (5) it is necessary to take the condition \( u_i(l) = 0 \)).

Using the results of [23], we decode the components of vectors \( \mathbf{z}^{(1)}(x) \) and \( \mathbf{z}^{(2)}(x) \):

\[
N^{(1)}(x) = -1 \cdot \frac{f}{l}, \quad Q^{(1)}(x) = 1 \cdot \left( 1 - \frac{1}{2} H(x) \right), \quad M^{(1)}(x) = 1 \cdot \left( \frac{1}{2} - \frac{x}{11} H(x) \right),
\]

\[
\mathcal{G}^{(1)}(x) = \frac{1 \cdot l^2}{EJ} \left( -\frac{1}{4} + \frac{1}{4} \frac{(x + l)^2}{2l^2} - \frac{x^2}{2l^2} H(x) \right), \quad w^{(1)}(x) = \frac{1 \cdot l^3}{EJ} \left( -\frac{1}{4} + \frac{1}{4} \frac{(x + l)^2}{2l^2} - \frac{x^2}{2l^2} H(x) \right),
\]

\[
u^{(1)}(x) = \frac{1 \cdot l^2 f}{EJ} \left( \frac{1}{4} - \frac{1}{4} \frac{(x + l)^2}{2l^2} - \frac{x^2}{4l^2} + \frac{x^4}{4l^2} H(x) \right);
\]

\[
N^{(2)}(x) = -1 \cdot \frac{25 l}{64 f}, \quad Q^{(2)}(x) = 1 \cdot \left( 1 - \frac{9}{32} \frac{x + l}{1l} - H(x) \right),
\]

\[
M^{(2)}(x) = 1 \cdot \left( 1 - \frac{9}{32} \frac{x + l}{1l} + \frac{25}{32} \frac{(x + l)^2}{2l^2} - \frac{x}{1l} H(x) \right),
\]

\[
\mathcal{G}^{(2)}(x) = \frac{1 \cdot l^2}{EJ} \left( \frac{1}{96} - \frac{9}{32} \frac{x + l}{2l} \frac{(x + l)^2}{2l^2} + \frac{25}{32} \frac{(x + l)^2}{3l^2} - \frac{x^2}{2l^2} H(x) \right),
\]

\[
w^{(2)}(x) = \frac{1 \cdot l^3}{EJ} \left( \frac{1}{96} - \frac{9}{32} \frac{x + l}{3l^2} + \frac{25}{32} \frac{(x + l)^2}{4l^2} - \frac{x^3}{3l^2} H(x) \right),
\]

\[
u^{(2)}(x) = \frac{1 \cdot 2 l^2 f}{EJ} \left( -\frac{1}{96} \frac{(x + l)^2}{2l^2} + \frac{9}{32} \frac{(x + l)^2}{4l^2} - \frac{25}{32} \frac{(x + l)^4}{5l^2} + \frac{x^4}{4l^2} H(x) \right).
\]

(7)

(8)

We evaluate the stiffness of the structure by analyzing the radial approach of the string with the well wall \( \Delta_y = -w(0) \) and the axial movement of the right end of the rod \( \Delta_z = u_i(l) = u(l) + w(l) \alpha \). From the expressions (6) – (8) we have:

\[
\Delta_y = \frac{P}{C^{(1)}} + (P - P_i)H(P - P_i) \left( \frac{1}{C^{(2)}} - \frac{1}{C^{(1)}} \right).
\]
\[ \Delta_y = \frac{P}{C_y(2)} \left[ \left( P - P_e \right) H \left( P - P_e \right) \left( \frac{1}{C_y(2)} - \frac{1}{C_y(1)} \right) \right], \]  

where \( P_e = C_x^{(1)} \Delta_x, \quad C_x^{(1)} = \frac{12 f E J}{l^3}, \quad C_x^{(2)} = \infty, \quad C_y^{(1)} = \frac{6 E J}{l^3}, \quad C_y^{(2)} = 256 \frac{E J}{l^3}. \)

We evaluate the bearing capacity of the construction according to the classical theory of the greatest normal stresses [25]:

\[ \text{max } \sigma_{eq}(x) = \text{max} \left\{ \frac{N(x)}{F} \pm \frac{M(x)}{W} \right\} \leq \sigma, \]  

where \( \sigma \) is permissible stress for rod material and \( F, W \) are area and moment of resistance of the cross-section.

**Figure 3.** Loading diagram of centralizer with thrust.

**Figure 4.** The greatest equivalent stresses.

In particular, we have identified the most equivalent stress required to assess the strength of the centralizer. By the formulas (6)–(8), (10):

\[ \text{max } \sigma_{eq}(x) = \sigma_{eq}(0) = P \left( \frac{f}{l} + \frac{1}{2} \frac{F l}{W} \right) + \left( P - P_e \right) H \left( P - P_e \right) \left( \frac{1}{C_y(2)} - \frac{1}{C_y(1)} \right) \left( \frac{l - f l}{l - F l} \right). \]  

The obtained results (9) and (11) are illustrated in figures 3 and 4. For calculations we considered: \( f / l = 0.1 \) and \( F l / W = 25 \). The kinks of the lines in the plots are due to changing boundary conditions (5).

**4. Conclusion**

A new design of a rod centralizer of a casing column is offered. The key features of this design are as follows: the possibility of autonomous operation of each of the links of the centralizer, united through a non-rigid ring; the presence of an additional axial thrust, which allows one to operate the device at high contact loads.

The boundary-value problem for the system of differential equations of equilibrium of an arc-shaped rod with boundary conditions in the form of inequalities is formulated and solved. The analytical assessment of the stiffness and strength of the centralizer with rigid axial thrust is performed.
The described in the paper design of the hinged centralizer with axial thrust has essentially nonlinear characteristics, which contribute to the efficient centering of the casing. At slightly deviated areas of the well with small clamping forces, the centralizer behaves like a soft one and does not impede the descent of the string; in areas with a large curvature the centralizer becomes stiff due to the thrust. In both cases, the radial clearance gap required for high-quality cementing is provided.

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