The scalable quantum computation based on quantum dot systems

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We propose a scheme for realizing the scalable quantum computation based on nonidentical quantum dots trapped in a single-mode waveguide. In this system, the quantum dots simultaneously interact with a large detuned waveguide and classical light fields. During the process, neither the waveguide mode nor the quantum dots are excited, while the sub-system composed of any two quantum dots can acquire phases conditional upon the states of these two quantum dots and the certain detunings between the waveguide mode and corresponding external light fields. Therefore, it can be used to realize selective quantum phase gates, graph states, N-qubit controlled phase π gates, and cluster states.

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I. INTRODUCTION

Semiconductor quantum dots (QDs) embedded in a photonic crystal (PC) cavity provides a promising system to investigate cavity quantum electrodynamics and quantum information processing (QIP) in the solid state [1]. In the past decade, it has attracted considerable experimental and theoretical attention. Both the weak and strong couplings have been achieved in experiment [2–7]. However, the practical and useful QIP requires a large number of qubits, and it is difficult to achieve so many spatial separation QDs in a PC cavity experimentally [8]. On the other hand, experiments have proved that the above system also can be used to harvest single photons by coupling a single QD to an enhanced cavity mode[7]. Nevertheless, since generated single photons must be coupled out of the cavity, the overall efficiency of this kind of single-photon source isn’t high enough. In order to overcome this challenge, Hughes et al. presented several theory proposals based on PC waveguides [9, 10]. These schemes show single QDs also can coupled a PC waveguides efficiently. And it has been proved in experiment by Lund-Hansen et al. [11].

Very recently, Feng et al. proposed a scheme to realize a quantum computation with atoms in decoherence-free subspace by using a dispersive atom-cavity interaction driven by strong classical laser fields [12]. But their proposal is based on identical qubits, and each qubit is driven with four laser fields. Motivated by these works, we present a scheme for realizing the scalable quantum computation based on nonidentical QDs trapped in a single-mode PC waveguide. In this scheme, any two QDs can acquire different phases conditional upon their different states and corresponding detunings between the waveguide mode and external light fields. And selective gate operations for any two QDs can be acquire in this way. For this reason, this scheme also can be employed to achieve graph states, N-qubit controlled phase π gate (NCZ gate), and cluster states with different number of gate operations. During the gate operation, neither the QDs nor the waveguide is excited. Comparing with Ref. [12], the logical gate is extended to nonidentical qubits, and the number of laser fields is decreased. In addition, this scheme is the first scheme to realize the scalable quantum computation with spatially separated and nonidentical QDs.

The organization of this paper is as follows. In Sec II, we introduce the theoretical model and effective Hamiltonian. In Sec III, we present how to realize the selective quantum phase gate between any two QDs. In Sec IV, we give the operations to achieve the graph states, NCZ gate, and cluster state. In Sec V, we show the simulation and realizability of the above gate operations and entangled states. The conclusion is given in Sec VI.

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II. THEORETICAL MODEL AND EFFECTIVE HAMILTONIAN

Let us consider that $N$ charged and spatially separated GaAs/AlGaAs QDs are trapped in a single-mode waveguide. Each dot has two lower states $|g\rangle = |\uparrow\rangle$, $|f\rangle = |\downarrow\rangle$ and a higher state $|e\rangle = |\uparrow\downarrow\rangle\rangle$, here ($|\uparrow\rangle$, $|\downarrow\rangle$) and ($|\uparrow\rangle\rangle$, $|\downarrow\rangle\rangle$) denote the spin up and spin down for electron and hole, respectively. The transitions $|g\rangle \leftrightarrow |e\rangle$ and $|f\rangle \leftrightarrow |e\rangle$ are correspondingly coupled to the vertical polarization and horizontal polarization lights $|f\rangle$ and $|e\rangle$ can coupled to the waveguide mode and classical laser fields. In this situation, the Hamiltonian describing the interaction between QDs and fields can be written as:

$$\hat{H}_I = \sum_{j=1}^{N} (g_j a e^{i\Delta_j^C t} + \frac{\Omega_j}{2} e^{i\Delta_j t} + \frac{\Omega_j^f}{2} e^{-i\Delta'_j t})\sigma_j^+ + H.c.,$$

(1)

where $g_j$ is the coupling constant between QD $j$ and the waveguide mode with the detuning $\Delta_j^C$, $a$ is the annihilation operator for the waveguide mode, $\Omega_j$ and $\Omega_j^f$ are the Rabi frequencies driven by the laser fields with the detunings $\Delta_j$ and $\Delta'_j$, respectively, and $\sigma_j^+ = |e\rangle\langle j|g\rangle$ (see FIG. 1).

![FIG. 1: (Color online) Each of QDs is driven with two classical fields and one quantum field.](image)

In order to derive the effective Hamiltonian of the system, we will use the method proposed in Refs. [12][15][16] under the following conditions: (1) $|\Omega_j| = |\Omega_j^f|$; (2) $\Delta_j = \Delta'_j$; (3) the large detuning condition: $|\Delta_j|, |\Delta'_j| >> |g_j|, |\Omega_j|, |\Omega_j^f|$; (4) $\delta_j = \Delta_j^C - \Delta_j$. The first condition together with the second condition can cancel the Stark shifts and related terms completely. Under the large detuning condition, the probability for QDs absorbing photons from the light fields or being excited is negligible. The last condition ensures that $\delta_j$ is only related with detuning between the waveguide field and light fields. In this situation, if the QDs are initial in the ground states, the excited states will not be populated and can be adiabatically eliminated. Thus we can obtain the effective Hamiltonian:

$$\hat{H}_{eff-1} = -\sum_{j=1}^{N} (\frac{|g_j|^2}{2\Delta_j^C} a^+ a + \lambda_j a^+ e^{i\delta_j t} + \lambda_j^* a^+ e^{-i\delta_j t})|g_j\rangle\langle g_j|,$$

(2)

where $\lambda_j = \frac{\Omega_j g_j}{4}(\frac{1}{\Delta_j} + \frac{1}{\Delta_j^C})$. The first term describes a Stark shift caused by the waveguide mode, the last two terms shows the indirect coupling between laser fields and waveguide field, which is caused by the virtually excited QDs.

Under the condition $\delta_j >> |g_j|^2/\Delta_j^C, |\lambda_j|$, the waveguide mode cannot exchange energy with the classical fields, the photon in the waveguide is only virtually excited, and any two QDs can interfere with each other. So the effective Hamiltonian takes the form:

$$\hat{H}_{eff-2} = -\sum_{j=1}^{N} \frac{|g_j|^2}{2\Delta_j^C} a^+ a + \eta_j \lambda_j \sigma_j^+ \sigma_j^+ + 2 \sum_{j=1, k=1, j \neq k}^{N} \eta_{jk} \sigma_j^+ \sigma_k^+ \sigma_k^+ \cos(\delta_{jk} t),$$

(3)

where $\delta_{jk} = \delta_j - \delta_k$ and $\eta_{jk} = \frac{|\lambda_j \lambda_k|}{2}(\frac{1}{\delta_j} + \frac{1}{\delta_k})$. With the initial state for the waveguide mode being in the vacuum state assumed, the effective Hamiltonian reduces to...
\[
\hat{H}_{e,f} = \begin{cases}
\sum_{j=1}^{N} \eta_{jj} \sigma_{j}^{-} \sigma_{j}^{+} + 2 \sum_{j=1, k=1 \neq j}^{N} \eta_{jk} \sigma_{j}^{-} \sigma_{k}^{+} \sigma_{j}^{-} \sigma_{k}^{+}, & \delta_j = \delta_k \\
\sum_{j=1}^{N} \eta_{jj} \sigma_{j}^{-} \sigma_{j}^{+} + 2 \sum_{j=1, k=1 \neq j}^{N} \eta_{jk} \sigma_{j}^{-} \sigma_{k}^{+} \sigma_{j}^{-} \sigma_{k}^{+} \cos(\delta_{jk} t) & \delta_j \neq \delta_k
\end{cases}
\]

This equation can be understood as follows. Under the condition of \(\delta_j = \delta_k\), with the laser field acting, QDs will take place the Stark shifts and acquire the virtual excitation, and this virtual excitation will induce the coupling between the vacuum waveguide mode and classical fields. As the Stark shifts are nonlinear in the number of any two QDs in the state \(|g\rangle\), the subsystem composed by arbitrary two QDs can acquire a phase conditional upon the number of these two QDs in the state \(|g\rangle\). On the contrary, in the case of \(\delta_j \neq \delta_k\), there might be not additional phase for the two QDs in the state \(|g\rangle\). As a result, this system can be employed to construct the selective controlled phase gate.

**III. THE SELECTIVE QUANTUM CONTROLLED PHASE**

Now, we take QDs \(m\) and \(n\) as an example to discuss how to construct the selective quantum controlled phase gates with arbitrary two QDs. In order to do so, states \(|f\rangle\) and \(|g\rangle\) are used to store the quantum information at first. In the case of \(\delta_m = \delta_n\), by using the appropriate light fields for QDs \(m\) and \(n\), we can get \(\delta_m = \delta_n = n \delta_0\), \(\lambda_m = \lambda_n = \sqrt{n} \lambda_0\). Then, the effective Hamiltonian \(\hat{H}_{es}\) takes the form of

\[
\hat{H}_{es} = \epsilon \left( \sum_{j=m,n} \sigma_{j}^{-} \sigma_{j}^{+} + 2 \sigma_{m}^{-} \sigma_{n}^{+} \sigma_{m}^{-} \sigma_{n}^{+} \right),
\]

here, \(\epsilon = \frac{|\lambda_0|^2}{2\delta_0}\). After that, according to the Hamiltonian \(\hat{H}_{es}\) for the same detuning, the evolutions of the logical states are:

\[
\begin{align*}
|ff\rangle_{m,n} & \rightarrow |ff\rangle_{m,n}, \\
|fg\rangle_{m,n} & \rightarrow \exp(-i\epsilon t)|fg\rangle_{m,n}, \\
|gf\rangle_{m,n} & \rightarrow \exp(-i\epsilon t)|gf\rangle_{m,n}, \\
|gg\rangle_{m,n} & \rightarrow \exp(-i4\epsilon t)|gg\rangle_{m,n}.
\end{align*}
\]

With the application of the single-qubit operation \(|g\rangle_j \rightarrow \exp(i\epsilon t)|g\rangle_j\), the above equation can be rewritten as

\[
\begin{align*}
|ff\rangle_{m,n} & \rightarrow |ff\rangle_{m,n}, \\
|fg\rangle_{m,n} & \rightarrow |fg\rangle_{m,n}, \\
|gf\rangle_{m,n} & \rightarrow |gf\rangle_{m,n}, \\
|gg\rangle_{m,n} & \rightarrow \exp(-2i\epsilon t)|gg\rangle_{m,n}.
\end{align*}
\]

This transformation for QDs \(m\) and \(n\) corresponds to the quantum phase gate operation, in which if and only if both controlling and controlled qubits are in the states \(|g\rangle\), there will be an additional phase in the system. During the operation, none of QDs and waveguide modes is excited. It is worth to point out that, although QDs are nonidentical, \(\delta_j = \Delta_j - \Delta_j^C\) is a tunable constant, which is decided by the frequency of detuning between the laser field and waveguide mode. Therefore, this system can construct the controlled phase gate with different QDs.

On the other hand, in the case of \(\delta_m \neq \delta_n\), with the choice of appropriate light fields for QDs \(m\) and \(n\), we can get \(\delta_j = j \delta_0\), \(\lambda_j = \sqrt{j} \lambda_0\), \(\eta_{jj} = \epsilon\), and \(\delta_{mn} = (m-n) \delta_0\). So the effective Hamiltonian \(\hat{H}_{ed}\) takes the form:

\[
\hat{H}_{ed} = \sum_{j=m,n} \epsilon \sigma_{j}^{-} \sigma_{j}^{+} + 2 \eta_{mn} \sigma_{m}^{-} \sigma_{n}^{+} \sigma_{m}^{-} \sigma_{n}^{+} \cos(\delta_{mn} t).
\]

And the time evolutions of four logical states for the two QDs \(m\) and \(n\), under the Hamiltonian \(\hat{H}_{ed}\) for the different detunings, are given by:

\[
\begin{align*}
|ff\rangle_{m,n} & \rightarrow |ff\rangle_{m,n}, \\
|fg\rangle_{m,n} & \rightarrow \exp(-i\epsilon t)|fg\rangle_{m,n}, \\
|gf\rangle_{m,n} & \rightarrow \exp(-i\epsilon t)|gf\rangle_{m,n}, \\
|gg\rangle_{m,n} & \rightarrow \exp(-i2(\epsilon t + \frac{\eta_{mn}}{\delta_{mn}} \sin(\delta_{mn} t)))|gg\rangle_{m,n}.
\end{align*}
\]
After the performance of the single-qubit operation \(|g\rangle_j \rightarrow \exp(i\epsilon t)|g\rangle_j\), there is

\[
\begin{align*}
|ff\rangle_{m,n} & \rightarrow |ff\rangle_{m,n}, \\
|fg\rangle_{m,n} & \rightarrow |fg\rangle_{m,n}, \\
|gf\rangle_{m,n} & \rightarrow |gf\rangle_{m,n}, \\
|gg\rangle_{m,n} & \rightarrow \exp(-i\frac{\theta_{mn}}{\delta_{mn}}\sin((m-n)\delta_0 t))|gg\rangle_{m,n}.
\end{align*}
\]

(10)

It means, in the situation of \(\delta_0 t = k\pi\) for \(k = 1, 2, 3, \ldots\), there will be no the controlled phase gate operation for the QDs \(m\) and \(n\).

As a result, with the choice of \(2\epsilon t = \pi\), this system can realize the several selective controlled phase \(\pi\) gate (SCZ gate) operations for the different QDs with different detuning at the same time.

**IV. GRAPH STATES, NCZ GATE, AND CLUSTER STATES**

Here, we show how to acquire graph states, NCZ gate, and cluster states in the system. At first, we will review the definition of an \(N\)-qubit graph state in brief. In a system of \(N\) qubits, if each qubit is in the state \(|+\rangle = (|g\rangle + |f\rangle)/\sqrt{2}\), and to all pairs \(\{m, n\}\) of qubits joined by a controlled phase \(\pi\) gate (\(CZ_{m,n}\) gate), these \(N\) qubits are in the graph state \([17]\), which can be expressed as

\[|G\rangle = \otimes_{m,n\in N} CZ_{m,n}(\otimes_{j\in N}|+\rangle_j).\]

(11)

We will prepare this state as follows. Assume \(N + 1\) QDs are in the initial state \(|\Psi\rangle_{N+1} = \otimes_{j\in N+1}|+\rangle_j\), they are trapped in a single-mode waveguide, and simultaneously driven by the appropriate laser fields. The transition \(|g\rangle \leftrightarrow |e\rangle\) is initial far off resonant with the fields, and all the detunings between the single-mode waveguide and laser fields are the same. If the waveguide mode is initial in the vacuum state, the state evolution for any two QDs is governed by Eq.(7). Waiting for a controlled phase \(\pi\) gate operation time, a graph state for \(N + 1\) QDs can be created. After the above graph state is generated, with the implement of controlled phase \(\pi\) gate operations for \(N\) of these \(N + 1\) QDs again, a NCZ gate can be constructed. The process of generating the graph state and NCZ gate can refer the figures in Ref. \([18]\). In addition, as the selective controlled phase \(\pi\) gate can be realized in several different groups at the same time, with the choice of \(\delta_J = J\delta_0\) and \(\lambda_J = \sqrt{J}\lambda_0\) for group \(J\), several different graph states or NCZ gates can be achieved simultaneously.

As it is well known, cluster states can be acquired by the local unitary operation from graph states. If each dot is initial in the state \(|+\rangle\) and encoded with \(ABABABAB\ldots\), a 1D cluster state can be realized by two steps (see FIG.2-(a)). First step: Apply the SCZ gate operations for A and B; second step: Apply the SCZ gate operations for B and A. After that, a 1D cluster state is achieved. The operations for constructing the 2D cluster states are listed as follows (see FIG.2-(b)). i) Name dots with ABCDABCDABCD\ldots; ii) With the application of SCZ gate operations for A and B, C and D, successively, several 1D cluster states are generated ; iii) After applying the SCZ gate operations for A and C, B and D, respectively, a 2D cluster state is created.

![FIG. 2: (Color online) The operations for generate cluster states](image)
V. SIMULATIONS OF DECOHERENCE

In the following, let us discuss the realizability of the experiment. According to the above discussion, the influences of spontaneous emission from the excited states and the waveguide decay can be ignored. As a matter of fact, under the condition of the largely detuned couplings the excited state is rarely populated, so the influence of the spontaneous emission can be neglected, and the main decoherence effect in our scheme is due to waveguide decay. Then, the master equation can be given as follows:

\[ \dot{\rho} = -i[H_I, \rho] + \frac{\gamma}{2} (2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a) \]  

(12)

where \( \gamma = 1/\tau_w \) is the waveguide decay rate, and \( \tau_w \) is the decay time of the waveguide mode. And the fidelity of the entangled states and gate operations can be expressed as \( F = Tr(\rho \rho' ) \) with \( \rho(\rho' ) \) being the density operator of the system in the case with (without) waveguide decay. According to experimentally achievable parameters in the system of QDs embedded in a single-mode waveguide [11, 19, 20], the coupling of QDs and waveguide is about 0.1 meV, the decay time for waveguide is \( \tau_w \sim 1 \text{ ns} \). With the choices of the coupling constants and detunings as FIG.3, which apparently satisfy the approximation conditions mentioned above, we can get \( \lambda_j = 0.0025 \text{ meV} \). The performance of the any two QDs A and B controlled phase \( \pi \) gate (CZ\(_{A,B}\) gate) operations versus the waveguide decay time \( \tau_0 \), and the fidelities of the entangled states and NCZ gate operations versus the number of qubits are given in FIG. 4 and FIG. 5.

FIG. 3: (Color online) \( \Omega_j \) versus \( g_j \) and \( \Delta_j \)

FIG. 4: (Color online) (a) Numerical simulation of the fidelity of the any two qubits CZ\(_{A,B}\) gates versus the waveguide decay time \( \tau_0 \), with the parameters \( g_A = 0.10 \text{ meV}, g_B = 0.08 \text{ meV}, \Omega_A = 10 \text{ meV}, \Omega_B = 13.75 \text{ meV} \). The detunings of blue line are given by \( \Delta_A = 200.00 \text{ meV}, \Delta_B = 220.00 \text{ meV}, \) and the detunings of green line are given by \( \Delta_A = 200.09 \text{ meV}, \Delta_B = 220.09 \text{ meV}, \) respectively. \( \tau_0 \) is the decay time. (b) the fidelity of the graph states and NCZ gates versus the number of QDs with the decay time of \( \tau_w \). And the Fidelities for graph states and NCZ gates are the blue line and green line, respectively.

FIG.4 (a) presents, with the increase of \( \tau_w/\tau_0 \), the fidelity for the two-qubit quantum controlled phase \( \pi \) gate is decreasing. It means that the waveguide decay affects the fidelity of the gate operation largely [12]. And the fidelity is 0.9877 for \( \tau_0 = \tau_w = 1 \text{ ns} \). Moreover, in this case the gate operation time is about 50 ns, comparing with effective decay time of waveguide \( 1.5 \times 10^4 \text{ ns} \left( \approx \tau_w/(|\lambda|^2) \right) \), it is possible to perform hundreds two-qubit controlled phase \( \pi \) gates within the effective decay time. FIG. 4 and FIG. 5 show, with increasing the number of QDs, the fidelities for the graph state, NCZ gate operation and cluster state decreases. It is due to that, with the number of QDs increasing,
the probability of waveguide mode in the excited state increases. Moreover, FIG. 5 also presents the relationships $F_{M \times N} = F_{N \times M}$ (for $M \times N$ QDs) and $F_{1 \times 12} > F_{2 \times 6} > F_{3 \times 4}$. The reason for these is, the operation time for $M \times N$ and $N \times M$ is the same, while the operation time increases from $1 \times 12$ to $3 \times 4$. For the same reason, with the same number of QDs, the fidelity for 1D cluster state is higher than the one for 2D cluster state, which can be seen from FIG. 5.

![FIG. 5: (Color online) Numerical simulation of the fidelity of the cluster states versus the number of $M \times N$ QDs with the decay time of $\tau_{w}$](image)

VI. CONCLUSION

In summary, we have shown that in a single-mode PC waveguide, $N$ nonidentical and spatially separated QDs can be used to realize the scalable quantum computation with the application of the classical light fields. During the process, neither the waveguide mode nor the QDs are excited. The distinct advantages of the proposed scheme are as follows: firstly, this system is scalable and controllable; secondly, there is no waveguide photon population involved and the QDs are almost in their ground states; thirdly, as the QDs are non-identical, it is more practical. Therefore, we could use this scheme to construct a kind of scalable and controllable solid-state optical logical devices. In addition, this method opens up a prospect to realize a scalable quantum computation in QD system.

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