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Peak-easing strategies for urban subway operations in the context of COVID-19 epidemic

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ABSTRACT
Subways play an important role in public transportation to and from work. In the traditional working system, the commuting time is often arranged at fixed time nodes, which directly leads to the gathering of “morning peak” and “evening peak” in the subway. Under the COVID-19 pandemic, this congestion is exacerbating the spread of the novel coronavirus. Several countries have resorted to the strategy of stopping production to curb the risk of the spread of the epidemic seriously affecting citizens’ living needs and hindering economic operation. Therefore, orderly resumption of work and production without increasing the risk of the spread of the epidemic has become an urgent problem to be solved. To this end, we propose a mixed integer programming model that takes into account both the number of travelers and the efficiency of epidemic prevention and control. Under the condition that the working hours remain the same, it can adjust the working days and commuting time flexibly to realize orderly off-peak travel of the workers who return to work. Through independent design of travel time and reasonable control of the number of passengers, the model relaxes the limitation of the number of subway commuters and reduces the probability of cross-travel between different companies. We also take the data of Beijing subway operation and apply it to the solution of our model as an example. The example analysis results show that our model can realize the optimal travel scheme design of returning to work at the same time node and avoiding the risk of cross infection among enterprises under different epidemic prevention and control levels.

1. Introduction

1.1. Background and research motivation

The COVID-19 pandemic broke out in early 2020 and has since completely disrupted normal life around the world. By first week of November 2021, the pandemic has caused untold devastation with more than 224 million people already infected with the virus, with a death toll of more than five million people. Due to the fast infection spread, high infectivity and mutations to many variants, countries around the world have resorted to declaring states of emergency, and have taken unprecedented actions like evacuating...
overseas citizens and sealing off borders. In the context of the current state of complete globalization, these defensive countermeasures have inevitably led to the interruption of industrial supply chains. If production throughout the world is not resumed quickly, the supply of critical medical equipment cannot be guaranteed effectively, while also causing immense shortages of essential consumer goods (Govindan et al., 2020, Ivanov, 2020). However, hasty resumption of production and other economic activities would further intensify the epidemic prolonging the epidemic cycle. Countries all over the world, therefore, are struggling with the problem of how to balance and coordinate the resumption of work and production on one hand and the prevention and control of the epidemic on the other.

There is an obvious positive correlation between the infection speed of COVID-19 and the crowd size (Chen et al., 2020). Countries and cities with a higher population density are emerging as main contagious areas. If these countries and cities want to resume work, production and school, urban public transport will inevitably lead to a peak in passenger flow, which will accelerate the epidemic spread (Seong et al., 2021). In a city like Beijing, for example, most of the workers go to work by public transport, especially the subway. Therefore, the prevention of work-related transmission is very important (Lan et al., 2020). To cope with this problem, many public transportation systems have adopted various unconventional anti-epidemic measures (Zhou et al., 2020). For example, some systems are implementing unprecedented management methods where passengers are required to wear masks and gloves and take separate seats in the carriages at a distance from each other. The entire subway stations are sterilized several times a day and passenger information is accurately traced using QR codes (Jiang and Li, 2020). In China, Beijing subway stations, for example, have adopted the procedure where passengers need to make reservations before entering the stations (Beijing daily, 2020). To ensure a safe environment, Shanghai subway stations have taken measures at 17 subway stations to split the crowds effectively where passengers are divided into smaller groups and their body temperature is taken before embarking. These measures have produced great results for effective epidemic prevention and control, however, they can only be implemented if the number of people on public transport is controlled so that the epidemic prevention and control measures can be maximized. Therefore, it is more critical to eliminate the travel peak of subway commuters while ensuring the basic resumption of work and production, and effectively control the number and timing of passenger flow, and thus balance the operations of the traffic network. In the nearly two years since the outbreak began, the continuous prevention and control of COVID-19 has become the new normal for all countries. Under the condition of ensuring orderly resumption of work and production of enterprises, off peak travel will also be very important.

1.2. Research questions and major findings

In brief, considering the trade-off between the spread of COVID-19 and the necessity of restoring disrupted supply chain operation, this paper focuses on the following issues. As mentioned earlier, the characteristics of COVID-19 transmission are directly related to population concentration. Large-scale resumption of work and production will inevitably lead to crowds gathering in urban public transportation systems and increase the risk of epidemic transmission. But non-resumption of work and production will cause continued disruption of the supply chain and will also hamper the handling of the epidemic situation. The question is: how to take measures to reconcile this contradiction.

Flexible work schedules have been adopted by a large number of enterprises even under normal conditions. However, excessive flexibility in working hours may cause more returning workers to cross their commuting paths, making it very difficult to track those who cross. Here the question is: how to control the commuting within a relatively fixed range of time, and at the same time, achieve the smoothing of the peak travel in the subway by distributing the rush hour. This is an important step to reduce the congestion level so that transmission of the pandemic is curtailed. To achieve this, can we still use traditional optimization methods, or an innovative method needs to be developed?

To find answers to the above questions, we reviewed the relevant state-of-the-art research results. It was found that there were few research findings on COVID-19 prevention and control, and on the topics of resumption of work and production in the field of public transportation. To resume production and supply chain operation during the epidemic, the above-mentioned problem of crowded public transportation must be solved. To address this, we propose to achieve the peak-easing of the subway. Peak-easing will reduce the peak crowd to a lower level and therefore will reduce the risk of cross-infection. This will enable the enterprises to open their production facilities. We have developed a mixed integer programming model for this peak-easing problem. The objective of this model is to reflect the ratio of employees returning to work, and find a flexible work schedule arrangement in a fixed period. This method is also feasible in reality. In China, many cities like Chengdu and Shenzhen have adopted a flexible working schedule to avoid COVID-19 transmission.\footnote{Evidence can be accessed at: https://m.gmw.cn/baijia/2020-02/02/1300913014.html; accessed date: Nov. 25, 2021; https://epaper.fsonline.com.cn/fsrb/html/2020-02/11/content_29735_160617.htm; accessed date: Nov. 25, 2021.} Not only that, working days are not limited from Monday to Friday. Working on Saturday and Sunday can also be considered.\footnote{For example, in long national holidays, such as Tomb Sweeping Day, Dragon Boat Festival, and Mid-Autumn festival (3-day holiday), and the National Day (7-day holiday), firms often change the conventional working schedule and let employees work on weekend days.} At the same time, the model fully considers the effective reduction of the number of cross-travel, the stability of the resumption of work period, the guarantee of the number of employees for resumption of work, and the promotion and application of non-epidemic period. By solving this model, we can tackle the problems of daily commute to work and inter-period commute to work with different peaks. In order to verify the effectiveness of the model and the degree of optimization, real data from Beijing Subway...
systems in China are utilized and analyzed in Section 3. The results show that the expansion of the traditional work cycle and the flexible working system of fixed time period achieves better results, can effectively reduce the problem of rush hour peaks, and can guide enterprises, government and transportation departments to adjust the resumption of work and production, and thereby adopt epidemic prevention strategies at the micro and macro levels.

1.3. Contribution and paper structure

Our findings enable us to make the following fourfold contributions. First, different from other studies in the context of COVID-19 infection prevention strategy for public transportation (Amankwah-Amoah, 2020; Trovao, 2020), we consider the resumption of supply chain operations and enterprise production even under the epidemic situation. While many countries have adopted a travel ban strategy (Choi, 2020), our study provides a reference for lifting this restriction policy. Second, as a derivative of the peak-easing strategy of the public transportation system (Nuzzolo et al., 2012; Spiess and Florian, 1989), our model can effectively reduce the emergence of congestion in the public transportation system, and is not only suitable for the daily peak-easing problem, but is also suitable for setting the optimal shift strategy with strict passenger flow restriction during the epidemic period. Thirdly, we change the daily commuting modes of “working from Monday through Friday, resting on weekends” and the “9 am to 5 pm working hours”. This avoids the rush hours during mornings and evenings of workdays, thereby reducing the risks of epidemic spread. The model introduced in this paper overcomes the limitation of the traditional equilibrium traffic allocation problem which only considers a single time point or takes one day as the optimization unit (Daganzo, 2013; Jia et al., 2016; Luo, 2020). To guarantee the traceability of metro commuters under control, our model considering flexible working schedule is arranged in the fixed period. It has become a new peak model that takes the information tracking of passengers into consideration. Finally, this paper also overcomes the shortcomings of the traditional epidemic prevention model which lacks the support of actual quantitative data. This study provides detailed data support for enterprises and subway operation departments, and has the potential of minimizing the costs of epidemic prevention for enterprises and the country as a whole.

The equilibrium assignment model proposed in this paper is a mixed integer programming model. The traditional equilibrium assignment model, such as the Wardrop equilibrium method, is an integral model (Zhang et al., 2011) that requires a large number of partial derivatives calculations. This makes it difficult to generate multiple points assignments and long-term assignments (Babazadeh et al., 2020). Our model has the advantages in solution scale and speed. In addition, considering that enterprises usually return to work in weeks, we set up the problem of return to work on a weekly basis in fixed period and continuous period with peak-easing. We set up a cross period and peak-shifting reworking model with a single station on a weekly basis. The model is designed to be adaptable also to the multiple sites case.

This paper is organized as follows. Section 2 briefly reviews the related literature. Section 3 introduces the present situation of Beijing subway operation and the existing problems under COVID-19. In Section 4, we introduce the mixed integer programming model to establish the urban subway evacuation model under the epidemic situation. Section 5 shows a specific example analysis of the Beijing subway operation and the existing problems under COVID-19. In Section 4, we introduce the mixed integer programming model to establish the urban subway evacuation model under the epidemic situation. Section 5 shows a specific example analysis of the urban subway evacuation model under the epidemic situation. Section 6 shows a specific example analysis of the urban subway evacuation model under the epidemic situation.

2. Literature review

In logistics and transportation literature, optimization of urban public transportation literature has been rather well-researched (Szymański et al., 2018; He et al., 2019; Li et al., 2019). For example, a popular topic in traffic operations over the past decade is about the influence of an epidemic. Chatterjee et al. (2016) studied the dynamic process of epidemic outbreaks in the public transport network in 6 cities in India, and developed an anti-epidemic model based on the epidemic outbreak nodes of the stations, to control the spread of the epidemic. Their study is one of the earliest concerning the impact of public transport mode on epidemic spread. Sanna and Hsieh (2017) introduced the RAMPP (Risk Associated with Metro Passengers Presence) model to quantify the proximity and passenger flow at stations during the outbreak of dengue in Gao Xiong city, Taiwan. They found that a gathering of large passenger flows would increase the spread of dengue, which confirmed the impact of public transportation system on the spread of the disease. Ren et al. (2019) studied the urban villages (UV) in Guangzhou, China, and found that the dengue fever would spread among UV through the public transportation system. The above results validated the reality of the spread of epidemics in urban public transportation and identified the role public transportation plays in the spread. However, these studies have not given reasonable prevention suggestions that combine epidemic spread and the role of public transportation in work resumption. Among the most recent studies, Park (2020) paid attention to the public transportation system under the impact of COVID-19 outbreak, and identified the impact of the age of the passengers and the degree of crowdedness in the stations on the choice of public transportation. By analyzing the use pattern of the subway system, he has formulated corresponding countermeasures applicable to different situations. This study posits that passengers’ recognition of risks and the changes in social distancing requirements are the key elements that are currently affecting the spread trajectory of COVID-19. However, this study ignores the time-varying pattern of congestion in public transport. In practice, however, as long as the flow of passengers can be controlled at a specific time, the problem of transmission can be mitigated.

The data used in this paper are the actual operational data of Beijing subway in September 2019. Muren contributes to obtaining the data during his postdoctoral work in the State Key Laboratory of Rail Transportation, Beijing Jiaotong University.
The model proposed in this paper achieves this and its resultant finding is one of the contributions of our paper.

Management of public transportation network operations is a very important topic (Xia et al., 2020). There are different research questions, including the problem of congestion and disruption of public transportation. In existing literature, regarding the question of congestion (Wang and Zacharias, 2020), Muñoz et al. (2018) used a platform gate design to ease passenger flow, guide passengers and optimize subway line operations. Tsai et al. (2020) solved the problem of passenger crowdedness in public transportation vehicles, using a simulated annealing algorithm to find the suitable number of neurons in a deep neural network. They predicted the number of bus passengers, to optimize the transportation network. Similarly, Xu et al. (2016) used data envelopment analysis (DEA) and genetic algorithm (GA) to solve the congestion problem of service equipment under uncertain requirements, and conceptually expanded the service capacity of stations. Using the subway map as a planning tool, Guo et al. (2017) found that factors such as replanning the length of lines on the subway map based on the degree of congestion could significantly influence the route choice of passengers. The disruption of urban public transportation network will lead to serious productivity loss. The above research mainly focuses on the general public traffic congestion, solving the problems of urban traffic capacity allocation and passenger flow facilitation. However, in the context of COVID-19, the key is to avoid passenger clustering. The research contribution of our paper is to suggest ways to effectively prevent public traffic congestion by controlling the number of passengers and their commuting time, and thus reduce the resulting spread of the epidemic. Under the COVID-19 epidemic, disruption of the city’s public transport network cannot be avoided, which may result in a significant loss of productivity. The design of the prevention strategy in response to interruption interference has been the concern of several researchers (Cicerone et al., 2009; Chen et al., 2011; Noland and Hanson, 2015; Fartaj et al., 2019). Meyer and Belobaba (1982) are the first to qualitatively study the process of emergency planning related to the rail transit system. Jin et al. (2016) introduced the bus bridge-connection services to deal with the degradation of the urban rail transit network, based on the travel needs of commuters under traffic interruption conditions. In general, people are not allowed to congregate in public transportation systems under COVID-19. Some countries have strict rules that prohibit citizens from leaving their homes because of the disruption of public transport systems (Choi, 2020). To ease this restriction, our research designs a mixed integer programming model to resolve the conflict between the resumption of work and production and the ban on public transport under the epidemic situation by expanding the time constraints of commuters’ fixed work schedule, and to optimize the commuting peak. Different from the above two research directions, we have not only overcome the problem of travel prohibition under the epidemic situation, but also achieved the management of public transport congestion. It also provides guidelines for government and transportation departments as they attempt to mitigate the spread of the epidemic without disrupting public transportation operations.

Since the outbreak of the COVID-19 epidemic, new research topics on public transportation have emerged. By analyzing the “bring-service-near-your-home” business operations mode in Hong Kong, Choi (2020) reveals the value of urban logistics for enterprises to resume work and production under COVID-19 from an innovative perspective. From the perspective of hygiene and health, Yu (2020) put forward technical requirements for operations management, personnel demands, and sanitary protection of passenger transportation sites and transportation modes such as airplanes, trains, subways, buses, coaches, taxis, and ships, to reduce the impact of COVID-19 epidemic on the transportation industry and the health of people. Horve et al. (2019) proposed that social distance and duration of stay inside buildings are the key factors for the spread of new coronavirus, revealing the importance of keeping effective social distance in public transportation facilities and planning reasonable duration inside buildings. However, the above research on hygiene and health all considers the impact of COVID-19 characteristics on public transport travel, but do not recommend specific strategies to guide public transport operations. Shen (2020) proposed that urban rail transit companies should be forward-looking during the epidemic period. They should first initiate epidemic prevention and control measures inside company operations, and later adjust and supplement those measures according to specific situations vis-a-vis epidemic development. Zheng et al. (2020) used an improved Tabu search algorithm to control the load capacity in the determination of passenger flow, providing a simulation reference for the resumption of work and production in enterprises. Though the above two research articles provide us with the design ideas of public transport operation plans under COVID-19, they did not combine the transportation modes to further control the travel time of passenger flow and the number of passengers at the start and end points of the routes. In this paper, we bridge this gap in the existing research by innovatively expanding the internal mechanism of flexible working schedules using a mathematical model solved by a proposed algorithm. We suggest ways to balance the trade-offs between work and production resumption, which would help improve the way of keeping the pandemic under control at a time when people need to use public transportation to get back to work.
3. Beijing subway operation status and problem analysis

In this section, we present the data describing the passenger flow and the passenger travel characteristics of all passengers of the Beijing Subway system. The Beijing subway is the longest and the fastest-growing subway system in China. This subway network has formed the basis, and its operating characteristics basically represent the current status, of the development of the subway system as a whole in China. The data used in our study is the sampling data and the survey data of the Beijing subway system. It does reflect the actual operation status of the Beijing subway.

3.1. Passenger flow distribution

(1) Time distribution.

Fig. 1 shows the passenger flow in and out of the Beijing subway stations during 5 working days at 15 min intervals. Through statistical analysis, it is found that the number of passengers entering and leaving the stations showed an expected bimodal distribution. The number of passengers increases significantly during 6:30–10:00 in the mornings and 17:00–20:00 in the evenings. The number of passengers in and out the stations at other times during the day is relatively stable at a lower volume, averaging around 50,000 people.

(2) Space distribution.

In the subway network, the passenger flow of each subway line varies greatly due to the different station settings, route coverage, and facilities along the lines, and is unevenly distributed in the network. The passenger flow is mainly concentrated on certain lines or some specific stations, as shown in Fig. 2 and Fig. 4. The statistical analysis of the passenger flow of each line shows that the lines in the central area of the city carry many more passengers, especially the lines connecting the suburbs and inner ring roads of Beijing.

3.2. Passengers’ travel characteristics

Passenger flow is the basis for the allocation of subway transport capacity, and the study of the passengers’ travel pattern is a prerequisite for improving subway service level and reducing operating costs. The average travel distance of subway passengers in Beijing is 15.95 km. The travel distances of passengers are highly concentrated within the range of 10–15 km, with an average of 12.48 km. The travel distance of 48.22% of passengers is more than 15 km, and only 2.29% of passengers travel more than 40 km (Zhang, 2018). The passenger flow entering the subway stations is the most direct indicator reflecting the intensity of passenger flow at subway stations and lines.

Fig. 3 shows the statistics of the number of passengers entering Beijing subway stations at 15 min intervals. On workdays from Monday to Friday, the passenger flow entering the stations shows double peaks. Passengers traveling during 6:30 to 10:00 account for 36.74% of the total passenger flow of the day. The duration of the evening peak hours is longer, which is from 16:00 to 19:30, accounting for 31.01% of the total passenger flow of the day. These figures indicate that the residents largely depend on the subway for their daily commute. On weekends, there is no obvious bimodal distribution. The passenger flow distribution curve is relatively smooth, and the passenger flow is smaller, indicating that most subway passengers are commuters, and their demand for subway transportation is, therefore, less on weekends.

Some relevant data analyses of the time distribution of passengers entering the subway stations on workdays from Monday through Friday for each line reveals the following characteristics: (1) The distribution of passenger flow of each line in the subway network is seen to be consistent over all the lines. (2) For each station, the passenger flows entering the stations all exhibit morning-peak and evening-peak distributions. (3) The morning passenger flow is greater than the evening passenger flow. (4) The evening peak lasts longer than the morning peak. These findings provide basic data reference for improving operations efficiency by taking measures such as setting different train intervals at different times, and increasing runs during peak hours in the mornings and evenings, and so on. This data is also used to set the upper limit of the flow level for different lines.

![Fig. 2. Statistics of passenger flow in and out of each subway line during workdays (Unit: person).](image-url)
4. Urban subway dispersion model under COVID-19 epidemic

Peak-Easing strategies for urban subway systems play an important role in the optimization of subway operations (Chen et al., 2018), especially in the context of COVID-19 (Chen et al., 2020). However, in the traditional equilibrium traffic allocation models, scholars pay more attention to the equilibrium distribution problem in a single time node or in a single day (Daganzo, 2013; Jia et al., 2016; Luo, 2020). There is no relevant literature on the equilibrium distribution problem considered across a weekly time unit. In fact, employees of many companies tend to have a work schedule on a weekly basis, working daily from Monday through Friday and having Saturday and Sunday off. In the period of severe epidemic, including the two weekend days in the work schedule would have a great research value. Therefore, off-peak travel and making full use of the weekend travel time have become the best options and form the basis of the recommendations of this paper.

We propose a new mixed integer programming model, through an in-depth study of the theory of the balanced flow assignment problem, combined with the concept of linearization of appropriate indicators (Muren et al., 2020), to make the flow in each line as re as possible. The optimal solution of the models can be obtained by using appropriate software. We propose a possible solution method using the real-life subway commuting data, that was presented in Section 3, and generate insights for a new proposed solutions method that can effectively increase the total number of travelers without over-crowding which may cause rapid spread of the epidemic. Our paper focuses on the impact of subway operations only and not on the impact of travelers’ behaviors using other modes. This assumption is justified by the fact that subway commuting captures most of the commuters in the city of Beijing.

4.1. Accurate peak-easing model for reworking in fixed period

Statistical analysis (see Figs. 1 and 3 above) shows that most passenger commuting happens on workdays, while the number of passengers traveling on weekends is relatively stable. In terms of travel time, most passenger commuting happens in the morning when people are going to work, and in the evening when they are returning home. The greater the passenger density, the more likely it is for the epidemic to spread. One of the effective methods of epidemic control is to minimize the gathering of people (Reluga, 2010). In view of this observation, one of the key proposals in this paper is to implement peak-staggering commuting and thereby avoiding the gathering of passengers in morning and evening peak hours. Telecommuting can reduce the travel needs of employees to a certain extent. However, for those whose commute is required and cannot be avoided, it is necessary to advocate, to the extent possible, that enterprises adopt newer work methods such as requiring only a proportion of all employees to come to the work sites, and separating the work hours of different batches of employees, among other things. These measures can reduce the number of people traveling to work on a daily basis and thereby reduce the risk of epidemic spread both during the commuting process and the working hours.

Currently, most enterprises follow a work method of five consecutive workdays during the weekdays and two consecutive rest days during weekends. However, in order to alleviate the problem of crowd gathering in the morning and evening rush hours of workdays, we propose two innovative approaches. One is to adjust the working days in the week. The other is to adjust the working hours during the day. We will still keep a 5-consecutive-day working week and keep the number of hours worked in a day unchanged. For example, an enterprise could allow its employees to start the five consecutive workdays starting from any day in a week rather than all on Mondays, and then take two days off. This meets the work requirements of the enterprise, but at the same time minimizes the number of commuters on a given day. In this way, the number of employees going to work sites everyday by subway would be greatly reduced, and the number of passengers at each station would be evenly scheduled.

According to our analysis of Beijing subway, commuters’ commuting in the morning and evening is the main cause of congestion, so commuters traveling around nearby stations (both same line and different lines) are not taken into account. Moreover, after the peak-easing arrangement for a single station, although there is overlap between multiple stations and multiple lines, it will not cause congestion concentrated in fixed periods.

Model assumptions:

(1) There are $m$ enterprises, each planning to carry out $s$ periods of peak-easing allocation during rush hours.
(2) The weekly commuting time of each enterprise is relatively fixed, and the commuting time can be designed according to the needs of epidemic prevention and control.

(3) When the commuting time is relatively fixed, the arrival and departure of employees approximately follow the normal distribution, so that there will be no over-concentration of employees in a single time period.

(4) The work resumption rate of employees near the subway stations does not exceed \( p_j \) (\( j = 1, 2, \ldots, m \)), and it will be dynamically adjusted according to the epidemic level. For example, when there is no epidemic situation \( p_j = 1 \) and when the epidemic situation is most serious, \( p_j = 0 \).

(5) The ratio of employees who take subway to work does not exceed \( q_j \) (\( j = 1, 2, \ldots, m \)).

The specific values of the above two variables can be calculated according to the actual status of urban subway operation and the epidemic risk level.

(6) Subway departure has a certain regularity, and its speed and dwell time between any two stations are relatively stable.

(7) The travel pattern of commuters is relatively stable, and the number of multiple round trips is minimal.

Next, we introduce the notation used in our model and the model parameters in Table 1.

The optimization model to achieve this is shown as Model 1.

(Model 1)

\[
\begin{align*}
\min & \quad -1^s \sum_{i=1}^{7} \sum_{j=1}^{m} \sum_{k=1}^{7} y_{ijk} + \sum_{i=1}^{s} \sum_{k=1}^{7} (y_{ijk} + y_{jk}) \\
\text{s.t.} & \quad \sum_{j=1}^{m} y_{ijk} \leq R, \forall i = 1, 2, 3, 4, 5, 6, 7, \forall k = 1, 2, \ldots, s \\
& \quad \sum_{i=1}^{7} x_{ij} = 1, \forall j = 1, 2, \ldots, m \tag{1.2} \\
& \quad Y_{jmin} \leq \sum_{k=1}^{7} y_{jk} \leq \min \{ Y_{jopt}, Y_{jmax} \}, \forall i = 1, 2, 3, 4, 5, 6, 7, \forall j = 1, 2, \ldots, m \tag{1.3} \\
& \quad \sum_{i=1}^{7} x_{ij} \geq Y_{jmin}, \forall j = 1, 2, \ldots, m \tag{1.4} \\
& \quad \sum_{k=1}^{7} y_{jk} \geq Y_{jmax} \left( Y_{jmax} - Y_{jmin} \right)(1 - x_{ij}), \forall i = 1, 2, 3, 4, 5, 6, 7, \forall j = 1, 2, \ldots, m \tag{1.5}
\end{align*}
\]

**Table 1**

| Notations and Parameters: |
|---------------------------|
| **Decision Variables**    | Description |
| \( x_{ij} \)             | Whether the \( j \)-th enterprise will arrange rest for two consecutive days on \( i \)-th day |
| \( y_{ijk} \)             | Number of employees of the \( j \)-th enterprise returning to work in the \( k \)-th period of the \( i \)-th day |
| \( z_{ik} \)              | Whether the \( j \)-th enterprise resumes work in the \( k \)-th period of the \( i \)-th day |
| **Parameters**            | Description |
| \( p_j \)                 | The work resumption rate of employees |
| \( q_j \)                 | The ratio of employees who take subway to work |
| \( w_j \)                 | Total number of employees in the \( j \)-th enterprise |
| \( Y_{jopt} \)            | Number of employees the \( j \)-th enterprise expects to go to work by subway, \( j = 1, 2, \ldots, m \), can be reported by each enterprise according to their own actual situation |
| \( Y_{jmax} \)            | Upper limit of the number of employees who take subway to work each period and it is related to \( w_j, q_j, p_j \) |
| \( Y_{jmin} \)            | Minimum number of employees of the \( j \)-th enterprise who take subway to work every day, \( j = 1, 2, \ldots, m \), can be determined by government or management department according to the epidemic level |
| \( S \)                   | The number of staggered peak periods (the rush hours are divided into equal sections \( s \)). |
\[
\sum_{k=1}^{s} y_{i,j,k} \leq Y_{\text{min}} + (Y_{\text{max}} - Y_{\text{min}})(1 - x_{y}), \forall i = 1, 2, 3, 4, 5, 6, \forall j = 1, 2, \ldots, m \tag{1.6}
\]

\[
\sum_{k=1}^{s} y_{i,k} \leq Y_{\text{min}} + (Y_{\text{max}} - Y_{\text{min}})(1 - x_{y}), \forall j = 1, 2, \ldots, m \tag{1.7}
\]

\[
\sum_{i=1}^{m} z_{i,k} = 1, \forall i = 1, 2, 3, 4, 5, 6, 7, \forall j = 1, 2, \ldots, m \tag{1.8}
\]

\[
z_{i,k} = z_{i,k} = z_{k} = z_{k} = z_{i,k} = z_{i,k}, \forall j = 1, 2, \ldots, m, \forall k = 1, 2, \ldots, s \tag{1.9}
\]

\[
y_{i,k} \leq Y_{\text{max}}, \forall i = 1, 2, 3, 4, 5, 6, 7, \forall j = 1, 2, \ldots, m, \forall k = 1, 2, \ldots, s \tag{1.10}
\]

\[
x_{y}, z_{i,k} = 0 \text{ or } 1, y_{i,k} \text{ are integers}, i = 1, 2, 3, 4, 5, 6, 7, \forall j = 1, 2, \ldots, m, \forall k = 1, 2, \ldots, s \tag{1.11}
\]

In Model 1, the first item \(-1^{*} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{s} y_{i,j,k} \cdot \text{of the objective function (Eq. (1.0)) represents the maximization of the total number of people of each enterprise returning to work in every week’s rush hours. The second item \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{s} \left( y_{i,j,k} + y_{i,j,k} \right) \text{divided by } 2^{*} R \text{ is to ensure that the maximum value of } \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{s} \left( y_{i,j,k} + y_{i,j,k} \right) \text{ does not exceed } 2^{*} R, \text{ so as to ensure that the number of people returning to work on weekends } \left( \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{s} y_{i,j,k} \right) \text{ will be further optimized after } \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{s} y_{i,j,k} \text{ obtains the optimal value. This representation avoids solving the model twice, and does not need to take a specific parameter value, which is more general. The value of } s \text{ is closely related to the congestion duration and the upper limit of the number of commuters.} \text{ Assuming that the arrival time of workers returning to work is evenly distributed, the relatively loose peak-shifting time is more convenient for workers returning to work, but it is easy to form local peak flow in relatively large resumption period (} s \text{ is smaller), leading to the failure of peak-easing. On the contrary, the shorter the peak shifting time is selected (} s \text{ is larger), which is beneficial to peak-easing, but the travel time of workers who resume work is more strictly constrained, resulting in employees’ dissatisfaction. In real life, if the planned number of passengers at a station and the number of passengers that the subway can accommodate in a unit time are known, the specific value of } s \text{ can be derived according to the peak shifting time.}

Constraint (1.1) means that the number of people returning to work in each period of one day is less than or equal to the maximum number of people returning to work.

Constraint (1.2) ensures that each enterprise should choose a certain day in a week to have two consecutive rest days.

Constraint (1.3) means that the number of people returning to work every day of each enterprise is more than the specified lower limit, that is, less than or equal to the smaller of the expected number of resumption of work provided by the enterprise and the maximum number of resumption of work provided by the government or management department.

Constraint (1.4) ensures that the total number of people returning to work in one week of each enterprise is more than the specified lower limit. This restriction is to guarantee that employees of each enterprise can resume work by subway fairly. If there is no such restriction, some enterprises may not be able to resume work by subway, thus causing dissatisfaction among employees.

Constraints of (1.5), (1.6) and (1.7) represent the upper limit of the number of people returning to work for two consecutive days of rest for each enterprise. When \( x_{y} = 1 \), the number of workers returning to work for two consecutive days is less than or equal to \( Y_{\text{min}} \). When \( x_{y} = 0 \), the constraint is going to be less than or equal to \( Y_{\text{max}} \).

Constraint (1.8) means that each enterprise should select a time period to resume work every day.

Constraint (1.9) restricts the return to work time of each enterprise to be the same every week. The main purpose of this hypothesis is to ensure the uniformity of the time when employees return to work (In fact, employees have other social tasks, such as picking up and dropping off the kids, so a consistent commute is important. Unfixed start times can easily lead to a disruption in employees’ body clocks and an increase in lateness), and also effectively reduce the risk of cross-infection and the difficulty of tracking.

Constraint (1.10) means that only in a certain period of time on a certain day when an enterprise chooses to resume work (i.e., \( z_{i,k} = 1 \)), a certain number of personnel may choose to go to work (i.e., \( y_{i,k} > 0 \)).

The last constraint (1.11) is the variable constraint.

If the enterprise does not plan to have a midweek break (i.e., two consecutive days off in a period between Monday and Friday) and can only arrange a rest on Saturday and Sunday, the following constraints need to be further added to Model 1:

\[
x_{y} = 1, \forall j = 1, 2, \ldots, m \tag{1.12}
\]

Although \( y_{i,k} \) is an integer in Model 1, it is assumed that \( y_{i,k} \) is a real number, to make the solution speed faster. Since each variable is an integer, even when \( y_{i,k} \) is assumed to be a real number, the value of \( y_{i,k} \) in the optimal solution obtained is found to be always an integer.
integer.

The adjustment of the resumption time of enterprises may bring some inconveniences to commuters. In particular, many
commuters would be reluctant to accept earlier work time. In view of this, we suggest that these commuters who go to work too early
should be given appropriate travel subsidies or incentives. The fairness of commuting time of employees in various enterprises can also
be guaranteed through a kind of clockwise rotation system of working hours.

4.2. Accurate peak-easing model for reworking in continuous periods

In Model 1, all enterprises are required to resume work at the same time. This will inevitably lead to a decline in the total number of
people returning to work. If all enterprises are allowed to resume work in a continuous period, constraints (1.9), (1.10) and (1.11) and
variable $s_k$ are only needed to be removed from Model 1. However, if all enterprises are allowed to resume work at different times, it
will inevitably increase the risk of epidemic spread and management. To this end, we have considered a compromise: allow companies
to resume work for two consecutive periods. At this point, Model 1 can be modified into Model 2 where work is allowed to be resumed
in one of two consecutive periods.

(Model 2)

\begin{equation}
\min (\sum_{s=1}^{7} \sum_{j=1}^{m} \sum_{i=1}^{4} y_{ijk} + \sum_{s=1}^{7} \sum_{j=1}^{m} (y_{ijk} + y_{ijk}^t)) / 2^s x^s R
\end{equation}

s.t. Constraints (1.1) to (1.9).

\begin{align}
y_{ijk} & \leq Y_{max} z_{ij}, \forall j = 1, 2, 3, 4, 5, 6, 7, \forall j = 1, 2, \ldots, m \\
 y_{ijk} + y_{ijk+1} & \leq Y_{max} (z_{ij} + z_{ij+1}), \forall j = 1, 2, 3, 4, 5, 6, 7, \forall j = 1, 2, \ldots, m, \forall k = 1, 2, \ldots, s \quad (1.13) \\
x_{ij} = 0 or 1, y_{ijk} \geq 0 are integers, i = 1, 2, 3, 4, 5, 6, 7, \forall j = 1, 2, \ldots, m, \forall k = 1, 2, \ldots, s \quad (1.14)
\end{align}

Constraints (1.1) to (1.9) have the same meanings as those set in Model 1. The constraint (1.13) in Model 2 represents that the
commuters’ number of the j-th enterprise returning to work in the first period on i-th day cannot exceed the maximum number of the j-
th enterprise returning to work. Constraint (1.14) means that in the case of resumption of work in a continuous period, the daily limit of
resumption of work of each enterprise is subject to two constraints: whether the previous period resumes work or whether the current
stage resumes work, indicating that each enterprise is allowed to resume work in two consecutive periods. Of course, we can also
specify that several specific enterprises are allowed to resume work for two consecutive periods. In this case, we only need to change
the value of $j$ in the constraint (1.14) to the designated enterprises.

4.3. Discussion

For passengers, there are two dangerous epidemic spread time periods during the subway riding process. The first time period is on
entering the subway station and waiting for the train. During this time period, the passengers are mostly the ones entering the station
from nearby places and the transfer passengers. For newly entered passengers, decreasing the entering rate through passenger flow
control methods can greatly reduce the risks of epidemic spread. For transfer passengers, all lines should be scheduled synergistically to
shorten the stay time of transfer passengers at stations as much as possible. The second time period is the train riding period. Dense
passenger crowds can vastly increase the risks of epidemic spread. Considering the above two time periods, and under the premise of
not excessively increasing the operating costs, effective control of human flow and peak-easing travel can reduce the risk of epidemic
transmission.

As seen above, in this paper, the peak-easing is achieved by the peak-staggering method of the off-duty time node. After
the realization of the off-duty peak-staggering, the on-duty peak-staggering can also be realized by considering the relatively fixed stay
time of commuters in the subway. In addition, considering the relative stability of commuting passengers’ travel patterns and the
stability of subway running rules, the problem of peak-staggering of departing station after work and entering station for work can be
solved relatively easily.

As a decision-maker, it is natural to worry about whether the implementation of off-peak work in a single point would cause
congestion in other nodes. According to the relevant data analysis diagram of Beijing subway (Fig. 1 and Fig. 3), it can be seen that the
number of subway trips in weekends is relatively stable at a much lower level, while there are obvious travel peaks in the mornings
and evenings from Monday through Friday. Therefore, commuting for work can be considered as the most important cause of subway
congestion. Under the basic assumption that the departure rule of subway is relatively stable, the off-peak commuting at a single point
achieves the distribution of the excessive peak-time travel demand of a station to other time nodes.

While reducing the probability of subway congestion, the method of easing of the single high peak does not create new travel
demand, so its significance is obvious. In addition, under the assumption that the travel demand of commuters is relatively fixed, the
travel crossover probability between two stations is low, so the multi-point off-peak work can be implemented independent of each other.

In Model 1, according to different epidemic levels, reasonable assignment of weekly working staff was realized in different en-
terprises. On the premise of satisfying the minimum travel demand of each enterprise, the model attempts to meet the expected weekly
travel demand of enterprises while at the same time, making reasonable use of Saturdays and Sundays to further achieve the peak-easing target of the working week. On the basis of Model 1, Model 2 further realizes the peak-easing of the number of passengers at different times of the day through reasonable assignment of working time nodes. Through this model, we achieve the optimal peak-easing of the number of working people in a week and the peak-easing of the number of working people at each time node every day. A point to note is that the traditional flat peak model fails to achieve the above goals. In addition, we also note that Model 1 can be independently used for daily subway flat peak problem even under non-epidemic conditions by adding a new constraint, and can also be used for reasonable peak-easing of queuing problems under uneven customer arrival conditions.

Through Model 2, we achieved a further increase in the number of people returning to work. This, in turn, increases the scope of application of the peak-easing reworking model even further. At the same time, in both Model 1 and Model 2, we ensured that all enterprises could resume work at the same or adjacent periods every day, which is not only conducive to epidemic prevention and control and tracking, but will also eliminate the requirement of flexible commuting time for employees when enterprises implemented flexible work schedule.

5. Example analysis

In Section 4, we introduced our peak-easing model that will take into account the subway riders’ travel patterns and will smoothen out the peak demands in order to reduce the gathering of a large number of people in the stations and the subway cars which, in turn, will reduce the risks of propagation of COVID-19. In this section, we take the data for a specific Beijing subway station as an example and apply that to our model to carry out the calculation and analysis. Results from this analysis will test the efficiency of the peak-easing commuting scheme proposed in Model 1 and Model 2. At the same time, the efficiency comparison of multiple schemes with the number of cross trips as the measure of scheme superiority will be further analyzed.

5.1. Example setting

We present Fig. 4 that shows the analysis results of the degree of crowdedness of each station in the Beijing subway system within a regular time period. It is apparent from the figure that there are significant differences in the degree of crowdedness of each station. This gives an insight that different management measures should be adopted for different stations. Also, the work resumption rates of different stations should be set individually. In any case, the measure of jointly using multiple transportation facilities can also be considered.

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We screen stations with larger passenger flow. It is assumed that the maximum capacities per unit time of the target stations under different protection levels are determined. As an example, we selected Station No. 197 (Xi’erqi station, a shared station of Line 13 and the Changping Line). The maximum passenger capacity per hour of this station is known. It can be obtained by calculating the capacity,
departure interval and past data of subway. Through this analysis, it is found to be 5000.

The requirements of major enterprises (enterprises with large number of employees) were collected near each station on commuter subway lines, and we calculate the total number of employees of each enterprise going to work every day, based on the maximum capacity specified for each station. In this example, the total number of employees of the major enterprises near this station collected through data mining is 55663. Relevant data of returning to work by subway of 20 major companies near the Xi’erqi station are shown in Table 2. The total number of enterprises comes from the actual data of real enterprises around Xi’erqi Station. We assume that the lower limit of the number of people returning to work is set at 5% of the total number of people returning to work, and the upper limit is set at 50%. The setting of these ratios, however, can be adjusted according to the actual situation. In Table 1, we assume that the per week minimum work resumption rate by subway of each enterprise is 30%, and the expected work resumption rate by subway of all enterprises of each day is 33%. If there are no peak-easing strategies, the work resumption rate of each day would be 5000/55663 = 8.98%. This rate is significantly lower than each day work resumption rate. If the enterprise is allowed to have 3 h of peak-easing period, then the daily return to work rate is (5000*3)/55663 = 26.9%, which still cannot meet the minimum return to work rate of 30% when taking subway. If the employees also go to work on weekends and have a three-hour peak-easing time, then the work resumption rate by subway of (5000*3)/55663*(7/5) = 37.7% can be reached. This rate is greater than the expected work resumption rate by subway of all enterprises of each day and the minimum work per week resumption rate by subway.

5.2. Basic model analysis

Considering that if the peak-easing period is set as 1 h, there will still be travel peak at local time nodes. Therefore, the peak-easing period is set as half an hour, so that the upper limit of the number of passengers in each period becomes 5000/2 = 2500. In addition, in order to meet the needs of employees of various enterprises to commute by subway as much as possible, it is calculated that 6 peak-easing periods (s = 6) should be designed within one day, which can meet the needs of enterprises to resume work.

We use Model 1 to calculate and obtain data of the number of employees working in each enterprise from Monday through Sunday. Optimization results as shown in Table 3 are obtained by using our proposed algorithms.

It can be seen from the results in Table 3 that the final number of people returning to work with a probability of 88976/(18365*5 + 2689*2) = 91.5% that meets the expected number of enterprises to return to work, and full load has been realized from Monday to Friday. However, there is still a certain number of people returning to work on Saturday and Sunday, which does not achieve 100% of our optimization goals. For this reason, we further extend Model 2 to obtain the optimal work schedule when the first four enterprises are allowed to resume work in two continuous periods. The results are shown in Table 4.

It can be seen from Table 3 that we have 100% probability to meet the expected number of workers of all enterprises around the subway station. While there are a large number of workers returning to work from Monday to Friday, the number of workers returning to work on Saturday and Sunday has been reduced to a minimum. This further demonstrates the value of Model 2.

| Enterprises | Total number of employees | Minimum number of employees returning to work | Maximum number of employees returning to work | Expected number of employees returning to work | Minimum number of employees returning to work per week |
|-------------|---------------------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|------------------------------------------------|
| E1          | 11,600                    | 580                                         | 5800                                        | 3000                                        | 13,500                                        |
| E2          | 7457                      | 373                                         | 3729                                        | 2500                                        | 11,250                                        |
| E3          | 6792                      | 340                                         | 3396                                        | 2300                                        | 10,350                                        |
| E4          | 6500                      | 325                                         | 3250                                        | 2200                                        | 9900                                          |
| E5          | 6244                      | 312                                         | 3122                                        | 2100                                        | 9450                                          |
| E6          | 3863                      | 193                                         | 1932                                        | 1400                                        | 6300                                          |
| E7          | 3751                      | 188                                         | 1876                                        | 1300                                        | 5850                                          |
| E8          | 2736                      | 137                                         | 1368                                        | 1000                                        | 4500                                          |
| E9          | 2135                      | 107                                         | 1068                                        | 800                                          | 3600                                          |
| E10         | 1532                      | 77                                          | 766                                         | 600                                          | 2700                                          |
| E11         | 1130                      | 57                                          | 565                                         | 400                                          | 1800                                          |
| E12         | 752                       | 0                                           | 376                                         | 300                                          | 1500                                          |
| E13         | 327                       | 0                                           | 164                                         | 150                                          | 750                                           |
| E14         | 258                       | 0                                           | 129                                         | 90                                           | 450                                           |
| E15         | 251                       | 0                                           | 126                                         | 80                                           | 400                                           |
| E16         | 129                       | 0                                           | 65                                          | 50                                           | 250                                           |
| E17         | 82                        | 0                                           | 41                                          | 40                                           | 200                                           |
| E18         | 55                        | 0                                           | 28                                          | 25                                           | 125                                           |
| E19         | 41                        | 0                                           | 21                                          | 20                                           | 100                                           |
| E20         | 28                        | 0                                           | 14                                          | 10                                           | 50                                            |
| Total       | 55,663                    | 2689                                        | 27,836                                      | 18,365                                      | 83,025                                        |

Note: The data of enterprises collected in Table 2 may have certain discrepancies. Meanwhile, it is stipulated that the number of employees going to work must not exceed 50% for each enterprise in each week; the minimum number of employees going to work is greater than or equal to 4.5 times the expected number; and the expected number is usually provided by the enterprise.
5.3. Further analysis of models

In this section, we conduct analysis to show the rationale of our formulation of the peak easing problem. We do this by tightening or relaxing selected constraints in Model 1 and Model 2, solve the resulting Model and then analyze the results to contrast with our original results. In this way, we can clearly show the advantages of the proposed model and the superiority of the conclusions obtained from the two models.

5.3.1. Analysis of the case of not arranging work resumption on Saturday or Sunday

The setting of Model 1 is based on the fact that the enterprise can return to work on any day of the week, but can only return to work at a fixed time of the day. In this section, we will demonstrate that our proposal of working on any day of the week-including Saturdays

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| Table 3 | Optimal results of the number of employees going to work in each enterprise from Monday through Sunday based on Model 1. |
|---------|-------------------------------------------------------------------------------------------------|
| Peak-easing period | Enterprises | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday | Total |
| 1 | E1 | 2500 | 2500 | 2500 | 2500 | 2500 | 580 | 580 | 13,660 |
| 2 | E2 | 2500 | 2500 | 2500 | 2500 | 2500 | 373 | 373 | 13,246 |
| 3 | E3 | 2235 | 2285 | 2235 | 2300 | 2200 | 340 | 340 | 11,935 |
| 4 | E4 | 2123 | 2123 | 2200 | 1900 | 1600 | 325 | 325 | 10,596 |
| 5 | E5 | 1250 | 2100 | 1710 | 1993 | 1983 | 312 | 312 | 9660 |
| 6 | E6 | 698 | 963 | 1213 | 188 | 188 | 1300 | 1300 | 5850 |
| 7 | E7 | 698 | 963 | 1213 | 188 | 188 | 1300 | 1300 | 5850 |
| 8 | E8 | 800 | 293 | 693 | 107 | 107 | 800 | 800 | 3600 |
| 9 | E9 | 77 | 77 | 300 | 600 | 600 | 600 | 600 | 2854 |
| 10 | E10 | 400 | 57 | 57 | 400 | 400 | 400 | 400 | 2114 |
| 11 | E11 | 300 | 300 | 0 | 0 | 300 | 300 | 300 | 1500 |
| 12 | E12 | 0 | 0 | 150 | 150 | 150 | 150 | 150 | 750 |
| 13 | E13 | 90 | 90 | 90 | 0 | 0 | 90 | 90 | 450 |
| 14 | E14 | 80 | 80 | 0 | 0 | 80 | 80 | 80 | 400 |
| 15 | E15 | 50 | 0 | 0 | 50 | 50 | 50 | 50 | 250 |
| 16 | E16 | 40 | 40 | 40 | 0 | 0 | 40 | 40 | 200 |
| 17 | E17 | 25 | 25 | 25 | 0 | 0 | 25 | 25 | 125 |
| 18 | E18 | 20 | 20 | 0 | 0 | 20 | 20 | 20 | 100 |
| 19 | E19 | 10 | 10 | 0 | 0 | 10 | 10 | 10 | 50 |
| Total | 15,000 | 15,000 | 15,000 | 15,000 | 15,000 | 6988 | 6988 | 88,976 |

| Table 4 | Optimal results of the number of employees going to work in each enterprise from Monday through Sunday based on Model 2. |
|---------|-------------------------------------------------------------------------------------------------|
| Peak-easing period | Enterprises | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday | Total |
| 2 | E1 | 812 | 180 | 180 | 2300 | 2307 | 1507 | 2200 | 9486 |
| 3 | E1 | 2188 | 400 | 400 | 700 | 693 | 1493 | 800 | 6674 |
| 1 | E2 | 812 | 180 | 180 | 180 | 180 | 1507 | 2200 | 5239 |
| 2 | E2 | 1688 | 2320 | 2320 | 193 | 193 | 993 | 340 | 807 |
| 5 | E3 | 1455 | 592 | 592 | 903 | 903 | 0 | 340 | 4785 |
| 6 | E3 | 845 | 1708 | 1708 | 1397 | 1397 | 340 | 0 | 7395 |
| 3 | E4 | 312 | 2100 | 2100 | 1800 | 1800 | 325 | 325 | 8762 |
| 4 | E4 | 1888 | 100 | 100 | 400 | 400 | 0 | 0 | 2888 |
| 5 | E5 | 312 | 2100 | 2100 | 2100 | 2100 | 312 | 11,124 |
| 1 | E6 | 1400 | 1400 | 1400 | 1400 | 193 | 193 | 7386 |
| 5 | E7 | 188 | 1300 | 1300 | 1300 | 1300 | 188 | 6876 |
| 6 | E8 | 1000 | 137 | 137 | 1000 | 1000 | 1000 | 1000 | 5274 |
| 1 | E9 | 107 | 800 | 800 | 800 | 800 | 107 | 107 | 4214 |
| 6 | E10 | 600 | 600 | 600 | 77 | 77 | 600 | 600 | 3154 |
| 5 | E11 | 400 | 400 | 400 | 57 | 57 | 400 | 400 | 2114 |
| 4 | E12 | 300 | 300 | 300 | 0 | 0 | 300 | 300 | 1500 |
| 5 | E13 | 150 | 150 | 150 | 150 | 0 | 150 | 150 | 750 |
| 5 | E14 | 90 | 0 | 0 | 90 | 90 | 90 | 90 | 450 |
| 1 | E15 | 180 | 80 | 80 | 180 | 180 | 0 | 0 | 400 |
| 5 | E16 | 50 | 50 | 50 | 0 | 0 | 50 | 50 | 250 |
| 1 | E17 | 40 | 40 | 40 | 0 | 0 | 40 | 40 | 200 |
| 6 | E18 | 25 | 25 | 25 | 0 | 0 | 25 | 25 | 125 |
| 6 | E19 | 20 | 20 | 0 | 0 | 20 | 20 | 20 | 100 |
| 6 | E20 | 10 | 10 | 0 | 0 | 10 | 10 | 10 | 50 |
| Total | 14,772 | 14,992 | 14,992 | 14,992 | 14,992 | 13,028 | 9435 | 97,203 |
and Sundays-is a sound one. For this analysis, we tighten the conditions in Model 1 for enterprises to return to work on any day of the week. Such restrictions restore the normal traffic and commuting without epidemic situation.

We modify constraint 1.2 in Model 1 to 1.12, and Model 1 will be transformed into the case of not returning to work on Saturday and Sunday. If we still use the data in Table 2 to solve, the model has no feasible solution, because the model cannot meet the total return to work demand of enterprises taking the subway in a week. Therefore, to find a solution, we assume that $Y_{j(min)}$ is four times of $Y_{opt}$. Now, the subway resumption results can be calculated, as shown in Table 5.

It can be seen from Table 4 that although we have maximized the number of people returning to work from Monday to Friday, we have not made full use of the weekend time to return to work. Therefore, compared with Table 3, the number of people returning to work is less by 8598, and with a probability of $80378/(18365*5 + 2689*2)$ = 82.7%. This demonstrates the value of our proposal of working on Saturdays and Sundays. So, in order to ensure the stability of national income during the epidemic, it is necessary to make full use of Saturdays and Sundays, which can effectively avoid the complete disruption of the supply chain (for example, the shutdown caused by the insufficient supply of raw materials in fast food restaurants in the United States and other places).

5.3.2. Analysis of the case of no minimum limit of the number of commuters on Saturdays and Sundays

In this subsection, we demonstrate the use of the term $\sum_{j=1}^{n} \sum_{k=1}^{m} (y_{jk} - y_{jk})$ in the objective function of Model 1. This term was included to minimize the number of people returning to work on Saturdays and Sundays. In the analysis here, we remove the term from the object function of Model 1. The specific calculation results are shown in Table 6. It can be found that the number of people returning to work by subway reflected in Table 3 and Table 6 is 88976, but the number of people returning to work by subway on Saturdays and Sundays in Table 6 is significantly higher than that in Table 3 by 6045. Such commuting arrangements do not expand the number of people returning to work and will disrupt the fixed work cycle of some enterprise employees. Therefore, in model 1, we limit the number of people returning to work on Saturdays and Sundays, and keep the original work cycle of most enterprise employees as much as possible.

5.3.3. Analysis of the situation in which enterprises do not resume work at the same period every day

In this section, we relaxed the constraint that enterprises in Model 1 return to work at the same period every day. The purpose of this is to verify whether enterprises can be allowed to freely set commuting times in case of COVID-19 epidemic. Therefore, we removed the relevant constraint 1.9 (given below again) in model 1.

$z_{jk} = z_{jk} = z_{jk} = z_{jk} = z_{jk}, j \neq k, j = 1, 2, ..., m, \forall k = 1, 2, ..., s$

Table 7 shows the situation that the enterprise chooses to return to work at any time in a day. It can be found that the total number of people returning to work by subway is 97203, which has been significantly improved. However, considering the uncertainty of time when employees take the subway to return to work every day, it brings many inconveniences, such as disrupting the biological clock and being unable to shuttle children to and from school. More seriously, due to the uncertainty of travel time, the commuting arrangements of employees in various enterprises have more intersections, which will not only aggravate the spread of the epidemic, but also bring trouble to the tracking of the epidemic. In fact, because COVID-19 has the characteristics of asymptomatic infection, the infection may occur in the early stage of the carrier’s diagnosis, that is, it is transmitted during taking the subway. Therefore, it is necessary to avoid cross travel as much as possible. Meanwhile, in the analysis in the next section, we not only analyze the peak-easing

| Table 5 |
|------------------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Peak-easing period    | Enterprises | Monday 1   | Tuesday 1  | Wednesday 1 | Thursday 1 | Friday 1   | Saturday 1 | Sunday 1    |
|------------------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 6                      | E1         | 2200       | 2200       | 2200       | 2200       | 2200       | 580        | 580        |
| 3                      | E2         | 1903       | 1903       | 1903       | 1903       | 1903       | 373        | 373        |
| 5                      | E3         | 2051       | 2051       | 2051       | 2051       | 2051       | 340        | 340        |
| 2                      | E4         | 2185       | 2185       | 2185       | 2185       | 2185       | 325        | 325        |
| 4                      | E5         | 1555       | 1555       | 1555       | 1555       | 1555       | 312        | 312        |
| 1                      | E6         | 1043       | 1043       | 1043       | 1043       | 1043       | 193        | 193        |
| 5                      | E7         | 1010       | 1010       | 1010       | 1010       | 1010       | 188        | 188        |
| 4                      | E8         | 945        | 945        | 945        | 945        | 945        | 137        | 137        |
| 3                      | E9         | 597        | 597        | 597        | 597        | 597        | 107        | 107        |
| 5                      | E10        | 449        | 449        | 449        | 449        | 449        | 77         | 77         |
| 2                      | E11        | 297        | 297        | 297        | 297        | 297        | 57         | 57         |
| 6                      | E12        | 300        | 300        | 300        | 300        | 300        | 0          | 0          |
| 1                      | E13        | 150        | 150        | 150        | 150        | 150        | 0          | 0          |
| 2                      | E14        | 90         | 90         | 90         | 90         | 90         | 0          | 0          |
| 2                      | E15        | 80         | 80         | 80         | 80         | 80         | 0          | 0          |
| 2                      | E16        | 50         | 50         | 50         | 50         | 50         | 0          | 0          |
| 2                      | E17        | 40         | 40         | 40         | 40         | 40         | 0          | 0          |
| 2                      | E18        | 25         | 25         | 25         | 25         | 25         | 0          | 0          |
| 2                      | E19        | 20         | 20         | 20         | 20         | 20         | 0          | 0          |
| 2                      | E20        | 10         | 10         | 10         | 10         | 10         | 0          | 0          |
| Total                  |            | 15,000     | 15,000     | 15,000     | 15,000     | 15,000     | 2689       | 2689       |
|                         |            |            |            |            |            |            |            | 80,378     |
5.4. Consider the cross infection of subway during the peak-easing and resumption of work

In order to analyze the cross travel of enterprises during the peak-easing period, we conduct an in-depth analysis in this section on the above subway resumption schemes. First, we define the number of cross travels under different peak-easing and resumption schemes. Table 6 and Table 7 show the optimal results of the number of employees going to work in each enterprise from Monday through Sunday and not consider the minimum number of people returning to work based on Model 1 and Model 2, but also pay attention to the cross contact between return to work enterprises during the subway commute period, so as to reduce the probability of cross infection and further prevent and control the epidemic situation while ensuring to meet the return to work needs of enterprises. We, therefore, propose to keep the constraint that the employees return to work at the same period every day.

### Table 6

Optimal results of the number of employees going to work in each enterprise from Monday through Sunday and not consider the minimum number of people returning to work based on Model 1.

| Peak-easing period | Enterprises | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday | Total |
|-------------------|-------------|--------|---------|-----------|----------|--------|----------|--------|-------|
| 3                 | E1          | 2500   | 2500    | 2500      | 2500     | 580    | 580      | 13,660 |
| 4                 | E2          | 373    | 2500    | 2500      | 2500     | 373    | 13,246   |
| 1                 | E4          | 40     | 340     | 1860      | 1900     | 1810   | 2293     | 10,836 |
| 5                 | E5          | 1690   | 2073    | 2073      | 1984     | 312    | 312      | 10,134 |
| 6                 | E6          | 1312   | 1312    | 1063      | 1063     | 1400   | 193      | 6536   |
| 6                 | E7          | 188    | 188     | 1300      | 1300     | 874    | 1300     | 6450   |
| 6                 | E8          | 1000   | 1000    | 137       | 137      | 226    | 1000     | 4500   |
| 5                 | E9          | 800    | 107     | 186       | 800      | 800    | 800      | 3600   |
| 4                 | E10         | 600    | 600     | 600       | 600      | 77     | 77       | 3154   |
| 1                 | E11         | 250    | 250     | 57        | 386      | 400    | 1800     | 50     |
| 5                 | E12         | 0      | 300     | 300       | 300      | 300    | 1500     | 100    |
| 1                 | E13         | 0      | 0       | 150       | 150      | 150    | 750      | 50     |
| 4                 | E14         | 90     | 90      | 0         | 90       | 90     | 450      | 50     |
| 1                 | E15         | 0      | 0       | 80        | 80       | 80     | 400      | 50     |
| 1                 | E16         | 50     | 50      | 0         | 50       | 50     | 250      | 50     |
| 4                 | E17         | 40     | 40      | 40        | 0        | 40     | 200      | 50     |
| 1                 | E18         | 0      | 0       | 25        | 25       | 25     | 125      | 50     |
| 5                 | E19         | 0      | 20      | 20        | 20       | 20     | 100      | 50     |
| 5                 | E20         | 10     | 0       | 0         | 10       | 10     | 50       | 50     |
| **Total**         |             | 11,443 | 13,570  | 15,000    | 15,000   | 13,942 | 10,545   | 9476   | 88,976|

### Table 7

Optimal results of the number of employees going to work in each enterprise from Monday through Sunday based on Model 2 when $s = 4$, $R = 3750$.

| Peak-easing period | Enterprises | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday | Total |
|-------------------|-------------|--------|---------|-----------|----------|--------|----------|--------|-------|
| 1                 | E1          | 617    | 2917    | 0         | 1750     | 2233   | 580      | 0      | 8097  |
| 2                 | E1          | 2383   | 83      | 3000      | 1250     | 767    | 0        | 580    | 8063  |
| 3                 | E2          | 1367   | 1367    | 200       | 200      | 683    | 207      | 2500   | 6524  |
| 3                 | E2          | 1133   | 1133    | 173       | 173      | 1817   | 2293     | 0      | 6722  |
| 1                 | E3          | 2300   | 0       | 1750      | 0        | 0      | 340      | 0      | 4390  |
| 2                 | E3          | 0      | 2300    | 550       | 2300     | 2300   | 0        | 340    | 7790  |
| 3                 | E4          | 132    | 132     | 2184      | 2184     | 316    | 0        | 1405   | 6353  |
| 4                 | E4          | 2068   | 16      | 16        | 9        | 325    | 795      | 5297   |
| 4                 | E5          | 312    | 312     | 2100      | 2100     | 2100   | 2100     | 11,124 |
| 1                 | E6          | 193    | 193     | 1400      | 1400     | 1400   | 1400     | 7386   |
| 3                 | E7          | 1300   | 1300    | 188       | 188      | 1300   | 1300     | 1300   | 6876  |
| 3                 | E8          | 1000   | 1000    | 1000      | 1000     | 137    | 137      | 1000   | 5274  |
| 4                 | E9          | 800    | 800     | 800       | 800      | 107    | 107      | 107    | 4214  |
| 1                 | E10         | 600    | 600     | 600       | 600      | 77     | 77       | 600    | 3154  |
| 4                 | E11         | 400    | 400     | 400       | 400      | 57     | 57       | 57     | 2114  |
| 4                 | E12         | 0      | 0       | 300       | 300      | 300    | 300      | 1500   |
| 3                 | E13         | 150    | 150     | 150       | 150      | 0      | 0        | 750    |
| 4                 | E14         | 90     | 90      | 0         | 90       | 90     | 450      | 50     |
| 4                 | E15         | 80     | 80      | 80        | 80       | 0      | 0        | 80     | 400   |
| 4                 | E16         | 0      | 0       | 50        | 50       | 50     | 50       | 50     |
| 1                 | E17         | 40     | 40      | 0         | 40       | 40     | 40       | 200    |
| 3                 | E18         | 25     | 25      | 25        | 25       | 0      | 0        | 25     | 125   |
| 3                 | E19         | 0      | 0       | 20        | 20       | 20     | 20       | 20     | 100   |
| 3                 | E20         | 10     | 10      | 10        | 10       | 10     | 10       | 10     | 50    |
| **Total**         |             | 15,000 | 15,000  | 14,996    | 14,996   | 14,999 | 9423     | 12,789 | 97,203|
schemes:

(1) When the enterprise allows an independent commuting period, the cross number is defined as 0;
(2) When multiple enterprises allow employees to return to work by subway in the same period and the employees that cross travel by subway every day of the week remain unchanged, the cross number is defined as the maximum number of people who return to work by subway in a week;
(3) When employees are allowed to travel across periods and the number of employees taking subway cross travel every day of the week remains unchanged, the number of cross travels is defined as the sum of the maximum number of people taking subway to return to work in two adjacent periods;
(4) When employees are allowed to take the subway to resume work in different periods within a week, the number of people crossing each period is defined as the sum of the number of people in this period minus the number of independent travel enterprises, and then minus the number of people on the day when the subway travel employees are exactly the same on a certain two days.

According to the above definition, we draw the subway cross travel figures under different resumption schemes (see Fig. 5). The number at the top left of the figures represents the total cross number of people who take the subway to return to work in different periods; the number at the bottom right represents the total number of people returning to work on different days of the week; the width of the box in the middle represents the number of people who return to work by subway, and the number in the box represents the enterprise number who return to work by subway.

5.4.1. Resumption scheme without cross travel between enterprises

By analyzing the travel schemes in Sections 5.2 and 5.3, we can find that there is no cross travel among enterprises in some travel schemes, which greatly reduces the risk of epidemic spread. Below, we will focus on analyzing this kind of situation through intuitive figures.

(1) Analysis of travel scheme under fixed daily return period and return to work on Saturday and Sunday.

Combined with the optimal peak-easing resumption scheme in Table 3, the travel effect diagram of the subway resumption scheme shown in Fig. 5 is drawn by using MATLAB software. It can be seen from the Table 3 and Fig. 5 that E1 and E2 have independent travel time respectively. E4, E10 and E12 travel in the third period; E5, E9, E11, E17 and E20 travel in the fourth period; E6, E7 and E8 travel in the fifth period; E13, E14, E15, E16, E18 and E19 travel during the sixth period. The travel time of the employees of these enterprises is unified every day and the off-peak travel of different time periods realizes the independence of commuters. It can be seen that there is no cross commuting in the scheme, which greatly reduces the cross infection of COVID-19. This scheme can be adopted when the epidemic risk level is severe and, at the same time, the demand for commuting by subway is strong.

(2) Analysis of travel scheme under fixed daily return period and no return to work on Saturday and Sunday.

Combined with the optimal peak-easing travel scheme in Table 4 and MATLAB software, the travel effect diagram of the subway resumption scheme shown in Fig. 6 is drawn. It can be seen from the Table 4 and Fig. 6 that employees traveling at the same time can achieve mutual independence, so it is more conducive to the control of the epidemic. When it is not allowed to return to work on weekends, we maximize the number of people returning to work from Monday to Friday, but the total number of workers returning to work is small. This scheme can be adopted when the epidemic situation is severe and the demand for returning to work by subway is not strong.

![Fig. 5. Peak-easing commute effect corresponding to the situation in Table 3.](image-url)
(3) Analysis of travel scheme under fixed daily return period with no minimum number of return to work on Saturdays and Sundays.

Compared with the results of returning to work by subway in Table 3, the total number of returning to work in Table 6 is the same, but we have not minimized the number of returning to work on weekends in Table 6, so it is necessary to introduce the term \( \sum_{m=1}^{M} \sum_{s=1}^{S} (y_{6m} + y_{7m}) \) in the objective function.

Combined with the optimal subway resumption scheme in Table 5 and MATLAB software, the travel effect diagram of the subway resumption scheme shown in Fig. 7 is drawn. It can be seen from Table 6 and Fig. 7 that we have achieved the same total number of people returning to work by subway as Table 3, but the number of people returning to work on weekends is higher by 8045, which will cause more dissatisfaction of people returning to work by subway.

5.4.2. Resumption scheme with cross travel between enterprises

(1) Analysis of travel scheme across two consecutive periods with fixed daily return period and return to work on Saturday and Sunday.

Combined with the optimal peak-easing travel scheme in Table 4 and MATLAB software, the travel effect diagram of the subway resumption scheme shown in Fig. 8 is drawn. The wide red line in Fig. 8 indicates that there are cross trips of employees in the front and back periods. For example, the first period and the second period, E2 has personnel travel. As can be seen from Table 4 and Fig. 8, enterprise E1 can choose to travel in consecutive periods 2 and 3; E2 can choose to travel in consecutive periods 1 and 2; E3 can choose to travel in consecutive periods 5 and 6; and E4 can choose to travel in consecutive periods 3 and 4. Other enterprises all have unique travel periods. The cross-travel behaviors among enterprises are significantly more than the results in Table 3. Therefore, this scheme can be adopted when the epidemic risk level is relatively low and the demand for subway resumption is strong.

(2) Analysis on the scheme of returning to work by subway when the number of travel periods changes.

Combined with the optimal subway resumption scheme in Table 7 and MATLAB software, the travel effect diagram of the subway resumption scheme shown in Fig. 9 is drawn. It can be seen from the Table 7 and Fig. 9 that we have fulfilled the requirements for the resumption of work of all enterprises and minimized the number of people returning to work on weekends. As the return to work time on the subway becomes less, the travel time of passengers is relatively loose, which increases the risk of epidemic spread. This scheme can be adopted when the epidemic situation has been significantly alleviated, the demand for subway resumption is relatively strong and the travel constraints are relatively loose.

(3) Analysis on the effect of allowing employees to take subway to return to work across periods.

Combined with the discussion in section 5.2.4 and relevant Matlab algorithms, the analysis diagram of the effect of allowing employees to take the subway to return to work across time periods is drawn as shown in Fig. 10. It can be seen from Fig. 10 that although employees are allowed to return to work across time periods, the number of people returning to work has increased significantly, and the number of cross trips between enterprises has increased significantly. At the same time, considering the inconvenience of irregular return time of each enterprise every day, it is not recommended to adopt this scheme.

5.5. Overall evaluation of multiple schemes for subway peak-easing and resumption

In order to compare the overall evaluation results of different subway resumption schemes, the sub items and comprehensive rating
results of each scheme are given in Table 8. Decision makers can choose the better scheme according to the epidemic risk level and the immediate needs of returning to work.

In Section 5, we first solved our model as formulated, applying it on the real life subway data. Then we experimented with various modified schemes to show their applicability or otherwise. Finally, we produced Table 8 as a comprehensive evaluation of those schemes.

6. Conclusions

6.1. Concluding remarks and managerial implications

The prevailing epidemic situation has compelled many countries to draw up new requirements for enterprises to resume work and production. Urban public transport is a necessary link for enterprises to resume work and production. In this paper, we discuss a subway commuting optimization strategy for controlling the flow of passengers to support enterprises as their employees resume work and production without causing further spread of the pandemic. The precise model proposed in this paper effectively realizes the peak-easing commuting in a single fixed period and a continuous fixed period under the condition of the expected number of workers, thus reducing the probability of rapid spread of the epidemic and the difficulty of tracking the epidemic.

Two new commuting models of peak-easing are proposed in this paper. In the first model, the expected number of workers...
returning to work and the fairness of travel are satisfied as far as possible under the assumption that enterprises return to work at the same period every day, and ensuring that the number of passengers does not exceed the maximum number of passengers that can be accommodated by the subway under different epidemic levels. The second model allows enterprises to resume work in two consecutive periods.

Table 8
Evaluation form of multiple schemes for peak-easing travel.

| Scheme | Rating of total number of workers returning to work | Rating of Epidemic prevention and control | Rating of travel satisfaction | Overall rating results |
|--------|----------------------------------------------------|-----------------------------------------|------------------------------|------------------------|
| Table 2 scheme | ★★★ | ★★★★★ | ★★★★★ | ★★★★ | ★★★★★★★ |
| Table 3 scheme | ★★★★★ | ★ | ★ | ★★★★ | ★★★★★★★ |
| Table 4 scheme | ★ | ★★★ | ★★★ | ★★★ | ★★★★★★★ |
| Table 5 scheme | ★★★ | ★★★ | ★ | ★★★ | ★★★★★★★ |
| Table 6 scheme | ★★★★ | ★ | ★ | ★★★ | ★★★★★★★ |
| Fig. 9 scheme | ★★★★ | ★ | ★ | ★ | ★★★★★★★ |

Note: in Table 8, ★ represents the quality of subway resumption effect, more ★ means better resumption effect.

Fig. 9. Peak-easing commute effect corresponding to the situation in Table 7.

Fig. 10. Peak-easing commute effect of allowing employees to take subway to return to work across periods.

Note: in Table 8, ★ represents the quality of subway resumption effect, more ★ means better resumption effect.
periods, which widens the limit of fixed periods, so as to make more flexible schedule on the basis of guaranteeing regular periods. The model breaks through the limitation of the traditional equilibrium traffic distribution problem, which takes a single time node or a single day time section as the optimization target. After the development of the model and its solution methods, we verified the model, using the actual data from Beijing subways. The result showed the proposed method to be effective, and the resultant schedule to have effectively and efficiently staggered the peak gathering of commuters. In addition, we also considered the number of cross-trips under off-peak commuting as one of the factors to be considered in the evaluation of the return to work scheme, so as to weigh the advantages and disadvantages of multiple schemes. Further, the congestion problem during the rush hours of subway operation is also solved, effectively blocking the transmission of COVID-19 in the subway. This will ensure the normal and safe resumption of work and production of enterprises under the epidemic. Additionally, our research puts forward suggestions for the resumption of work and production of enterprises and subway construction in the general transportation sector. Some interesting findings, insights and managerial implications are summarized below.

At the micro level, the model verifies the value of a flexible working system in the fixed period for enterprises to resume work and production under the background of the prevailing epidemic situation. This finding has important implications for enterprise operations management. In reality, home-working mode has become a new strategy for enterprises under the influence of COVID-19. The simultaneous implementation of flexible working system and home-working mode can guide enterprises to choose the appropriate number of employees to return to work. This will not only alleviate the pressure on public transportation systems and prevent epidemic spread, but also solve the stringent requirements of companies for resuming production. In addition, the parameters in the model include the traditional problems of off peak working and travel ban.

At a macro level, as the government is the major entity in charge of the epidemic prevention work, our model can provide suggestions for the government and transportation departments to effectively balance the workings of public transport systems and the planned resumption of work and production.

6.2. Generalization

The methods proposed in this paper have an important reference value for the epidemic prevention problem of urban subway stations and are also suitable for the epidemic prevention of other transportation modes such as buses, airlines and trains. Moreover, the proposed methods can also be used as major means for the daily subway passenger flow control and peak-staggering. Our basic model, Model 1, is highly flexible in the sense that, by changing, deleting or adding constraints, various real life scenarios can be represented. For example, when the work resumption rate of enterprises is 1, then Model 1 is the model for daily subway passenger flow control and peak-easing. If we choose not to work on Saturdays or Sundays, we add a new set of constraints to Model 1. To allow two consecutive periods to resume work, again we change the related constraints. If the second term in the objective function of Model 1 is removed, it is transformed into the model without considering the minimization of the number of subway workers returning to work on weekends. If constraint condition 1.9 is removed, it is transformed into a model allowing enterprises to resume work by subway across time periods.

Therefore, by modifying the objective function or constraint conditions of the model proposed in this paper, complex travel problems with multiple factors can be represented and solved. At the same time, this model is also suitable for problems such as complex assignment of common resources or common places.

6.3. Limitations and future studies

To the best of our knowledge, our study is the first to analyze the strategies of subway operations to alleviate peak hours congestion and optimize public transportation under the COVID-19 epidemic situation. Nevertheless, our model has several limitations that could be addressed in future research. First, the model does not consider the psychological impact of COVID-19 on employees using public transportation. In order to avoid gatherings, many people have reduced public travel and instead have used private cars or shared bikes and shared electric vehicles. Secondly, some interesting research topics will be to further explore the complex relationships between the resumption of work and production, and the peak-easing of transport operations based on factors like the normal population density. It should be noted here that some enterprises also adopt the work-at-home mode. Utilization of public transportation by these employees (should they be considered for returning to work when enterprises return to normalcy), the residents’ income levels, the urban economic development index, and the capacity of various urban transportation facilities are also complicating factors that could be analyzed. Finally, further research is needed to solve the problem of large-scale peak-easing travel.

CRediT authorship contribution statement

Muren: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization, Funding acquisition. Shiyuan Zhang: Conceptualization, Investigation, Writing – original draft, Writing – review & editing. Lianlian Hua: Conceptualization, Investigation, Resources, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition. Bo Yu: Conceptualization, Methodology, Software, Data curation, Writing – original draft, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to
influence the work reported in this paper.

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