Superresonance Phenomenon from Acoustic Black Holes in Neo-Newtonian Cosmology

Ines. G. Salako$^a$ *, Abdul Jawad$^b$ †

$^a$Institut de Mathématiques et de Sciences Physiques (IMSP) 01 BP 613 Porto-Novo, Bénin.

$^b$Department of Mathematics, COMSATS Institute of Information Technology, Lahore-54000, Pakistan.

Abstract

We explore the possibility of the acoustic analogue of a super-radiance like phenomenon, i.e., the amplification of a sound wave by reflection from the ergo-region of a rotating acoustic black hole in the fluid draining bathtub model in the presence of the pressure be amplified or reduced in agreement with the value of the parameter $\gamma = (1 + k \ n \ \rho_0^{n-1})$. We remark that the interval of frequencies depend upon the neo-newtonian parameter $\gamma$ ($\bar{\Omega}_H = \frac{2}{1 + \gamma} \ \Omega_H$) and they can be wider or narrower. As a consequence, the tuning of the neo-newtonian parameter $\gamma = (1 + k \ n \ \rho_0^{n-1})$ changes the rate of loss of the acoustic black hole mass.

1 Introduction

The certain aspects as well as problems of black hole (BH) physics was firstly introduced by Unruh in the theory of supersonic acoustic flows [1, 2]. He has also investigated the Hawking radiation and some other phenomena for

*inessalako@gmail.com
†jawadabi81@yahoo.com; abduljawad@ciitlahore.edu.pk
understanding quantum gravity. Hawking radiation is an important quantum effect of black hole (BH) physics. Hawking (in 1974) has tried to develop quantum gravity and pointed out that classically a BH does not radiate. However, He argued that it emits thermal radiation at a temperature proportional to the horizon surface gravity when we consider the quantum effects.

Upto now, many acoustic BHs have been widely discussed in the literature \cite{3-5} and also possesses several fundamental properties of BHs of general relativity. There exists a large number of fluid systems which have been analyzed on many analog models of acoustic BHs. These are water \cite{6}, optical fiber \cite{7}, electromagnetic waveguide \cite{8}, gravity wave \cite{9} and slow light \cite{10}. Some other models have also been developed for introducing the acoustic BH geometry in the laboratory which are atomic Bose-Einstein condensates \cite{11,12}, superfluid helium II \cite{13} and one-dimensional Fermi degenerate non-interacting gas \cite{14}.

The analogous systems employ a classical as well as Newtonian treatment generally, and also some quantum systems are considered. Also, a BH is a relativistic gravitational phenomenon which requires the inertial and gravitational effects of the pressure for a reasonable description of the system. However, relativistic pressure effects can be incorporated in a Newtonian framework in some cases approximatively. This is called neo-Newtonian cosmology and it is modification of the usual Newtonian cosmology by comprising the enough pressure into the dynamics.

In the present work, our main goals is firstly to realize the process of drawing the acoustic BH for the neo-Newtonian hydrodynamics, on the other hand, to analyze the impact of neo-Newtonian parameter on the phenomenon of super-radiance especially on the frequencies of the waves. The paper is organized as follows. In Sec. 2 we will review the development process of the acoustic BH after recalling the basic concepts of the neo-Newtonian cosmology \cite{15}. In Sec. 3 we will develop superresonance phenomenon and in Sec. 4 the numerical results. In Sec. 5 we make our final conclusions.
2 Newtonian Hydrodynamics in an Expanding Background: Cosmology

In this section, we present the neo-Newtonian hydrodynamics applied to cosmology. However, we consider the standard case of Newtonian equations firstly. In the presence of invisible perfect fluid, the basic equations of Newtonian hydrodynamics are

$$\dot{\rho} + \nabla \cdot (\rho \vec{v}) = 0,$$

$$\rho \frac{d\vec{v}}{dt} \equiv \rho [\dot{\vec{v}} + (\vec{v} \cdot \nabla) \vec{v}] = -\nabla p,$$

where $\rho$, $p$ and $\vec{v}$ are fluid density, corresponding pressure and the velocity field, respectively. The dot shows the differentiation with respect to cosmic time $t$. The above system of equations becomes suitable to study cosmology adopting the velocity field $\vec{v} = H(t) \vec{r}$ (Hubble’s law) where $H(t) = \frac{\dot{a}(t)}{a(t)}$, being $a(t)$ the scale factor. It is worth noting the trivial solution for the continuity Eq. $\rho(a) = \rho_0 / a^3$, where the today’s scale factor $a_0 = 1$ gives the today’s density of the fluid $\rho_0$.

Gravitational interaction is coupled into Euler’s equation (2) as

$$\dot{\vec{v}} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{\nabla p}{\rho} - \nabla \Psi,$$

where the gravitational potential $\Psi$ obeys the Poisson equation

$$\nabla^2 \Psi = 4\pi G \rho.$$

Eqs. (1) and (3) represents the fluid picture of cosmic medium which is gravitationally self-interacting through Poisson Eq. (4). In the framework of Newtonian cosmology, the Friedmann equations are given as follows

$$\frac{\dot{a}^2}{a^2} + \frac{(-2E)}{a^2} = \frac{8\pi G}{3} \rho \quad \text{and} \quad \dot{H} + H^2 = -\frac{4\pi G}{3} \rho,$$

where $E$ appears as integration constant associated to the energy of system. Moreover, the pressure does not correspond to homogeneous and isotropic background. With the inclusion of Newtonian cosmology, we can not model a radiation dominated as well as dark energy dominated epoch. This approach is restricted to a description of the Einstein-de Sitter universe.
2.1 Including Pressure: The neo-Newtonian Cosmology

The neo-Newtonian equations has been developed by McCrea [16] and Harrison [17] which ensures the effects of pressure as well the simplicity of Newtonian physics. Later, a crucial study related to the perturbative behavior of the neo-Newtonian equations has helped in developing the final expression for the fluid equations in this approach [18] (see also [19]) which are given by

\[ \partial_t \rho + \nabla \cdot [(\rho + p) \vec{v}] = 0, \]  

\[ \dot{\vec{v}} + (\vec{v} \cdot \nabla) \vec{v} = -\nabla \Psi - \frac{\nabla p}{\rho + p}, \]  

\[ \nabla^2 \Psi = 4\pi G \left( \rho + 3p \right). \]  

Combining Eqs. (6), (7) and (8), we can obtain the following equations

\[ \frac{\dot{a}^2}{a^2} + \frac{(-2E)}{a^2} = \frac{8\pi G}{3} \rho, \]  

\[ \dot{H} + H^2 = -\frac{4\pi G}{3} (\rho + 3p). \]  

These equations are exactly correspond to the relativistic Friedmann equations. The main idea behind the neo-Newtonian formalism relies on the following substitutions: Firstly, it is necessary to redefine the concept of inertial and passive gravitational mass density. With the redefinition

\[ \rho_i \rightarrow \rho + p, \]  

we rewrite the continuity and the Euler equation.

The second step is the interpretation of the active gravitational mass density i.e., the density that source the gravitational field. Hence the following redefinition

\[ \rho_g \rightarrow \rho + 3p, \]  

which is related to the trace of the energy-momentum tensor, will become the source of the Poisson equation. The generalization of this result in the presence of pressure has been evaluated in [16]. Moreover, this approach has been modified [17] which leads to neo-Newtonian cosmology.

When \( p = 0 \) we find the Newtonian equations. The interesting feature of above equations is that the inertial mass present in the Newtonian equation
can be replaced by $\rho + \frac{P}{\rho}$. From cosmological structure formation point of view, it is noted that equation (6) does not provide correct growth of matter density perturbations in the scenario of homogeneous, isotropic and expanding background [20, 21, 22]. Actually, the correct growth is obtained if equation (6) is modified as

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) + p \nabla \cdot \vec{v} = 0.$$  

This equation shows correspondence with effective metric of usual Newtonian scenario when applied to fluid configuration considered. It represents that one can see, how the neo-Newtonian formulation is sensible to the the specific symmetries and hypothesis of the problem in another context of the cosmological one. This fact may indicate that the construction of a Newtonian counterpart of a relativistic problem may vary from problem to problem, deserving in the general case a deeper analysis.

2.2 Acoustic Black Holes in neo-Newtonian cosmology

The wave equation from the perturbation equations (13), (7) and (8) is given by [15]

$$- \partial_t \left\{ c_s^2 \rho_0 \left[ \partial_t \phi_1 + \left( \frac{1}{2} + \frac{\gamma}{2} \right) \vec{v}_0 \cdot \nabla \phi_1 \right] \right\} + \nabla \cdot \left\{ - c_s^2 \rho_0 \vec{v}_0 \left[ \left( \frac{1}{2} + \frac{\gamma}{2} \right) \partial_t \phi_1 \right. \right. \\
+ \left. \left. \gamma \vec{v}_0 \cdot \nabla \phi_1 \right] + \rho_0 \nabla \phi_1 \right\} = 0.$$  

(14)

which we can rewrite as

$$\partial_{\mu} (f^{\mu \nu} \partial_\nu \phi_1) = 0$$  

(15)

where we get

$$f^{\mu \nu}(t, \vec{x}) = \frac{\rho_0}{c_s^2} \left( \begin{array}{ccc}
-1 & -1+\frac{\gamma}{2} v^x & -1+\frac{\gamma}{2} v^y \\
-1+\frac{\gamma}{2} v^x & c_s^2 -(v^x)^2 & -\gamma v^x v^y \\
-1+\frac{\gamma}{2} v^y & -\gamma v^x v^y & c_s^2 -(v^y)^2
\end{array} \right).$$  

(16)

assuming a (2+1) dimensional spacetime.

By using the Klein-Gordon equation [3] for a massless scalar field

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu \nu} \partial_\nu \phi_1) = 0,$$  

(17)
we obtain the effective (acoustic) metric related to this equation (in cylindrical coordinates)

\[
\begin{align*}
\rho_0^2 \sqrt{c_s^2 + (v_r^2 + v_\phi^2)(\gamma - \frac{1}{2})^2} & \left[ -c_s^2 dt^2 + (dr - \gamma v_r \, dt)^2 (rd\phi + \gamma v_\phi \, dt) 
\right. \\
+ & \left. d\phi^2 \right]
\end{align*}
\]

(18)

where \[15\]

\[
\gamma = (1 + k n \rho_0^{n-1})
\]

(19)

### 2.3 Ergo-region and event horizon

Considering again a position-independent density, we can find again the same velocity field and potential from the continuity equation as follows \[15\], i.e.,

\[
\vec{v} = \frac{A}{r} \hat{r} + \frac{B}{r} \hat{\theta}
\]

(20)

and

\[
\phi(r, \theta) = -A \ln r - B \theta.
\]

(21)

Therefore, considering (20) and (21) and

\[
dt = dt' + a_1 dr, \quad d\phi = d\phi' + a_2 dr,
\]

(22)

with

\[
a_1 = \frac{A r (1 + \gamma)}{2(A^2 \gamma - c_s^2 r^2)}; \quad a_2 = \frac{A B \gamma}{r(A^2 \gamma - c_s^2 r^2)},
\]

the acoustic metric (13)

\[
\begin{align*}
\rho_0^2 \beta_1 & \left\{ -\left[ 1 - \frac{\gamma (A^2 + B^2)}{c_s^2 r^2} \right] c_s^2 dt^2 + \frac{1 + \beta_2}{1 - \frac{\gamma A^2}{r^2 c_s^2}} dr^2 - 2B \beta_3 \, d\phi \, dt 
\right. \\
+ & \left. [r^2 + \beta_4] \, d\theta^2 + d\ell^2 \right\}.
\end{align*}
\]

(23)

with can be recast in a Kerr form as

\[
\beta_1 = \sqrt{\frac{1}{1 + \frac{\gamma (A^2 + B^2)}{c_s^2 r^2} (\gamma - \frac{1}{2})^2}}, \quad \beta_4 = \left(\frac{A(\gamma - 1)}{4c_s}\right)^2
\]
\[ \beta_2 = \gamma \left( \frac{A^2 + B^2}{c_s^2 r^2} \right) \left( \frac{\gamma - 1}{2} \right)^2 \quad \beta_3 = \frac{1 + \gamma}{2} \quad (24) \]

which gives the radius of ergo-region and event horizon, i.e.,

\[ r_e = \frac{\sqrt{\gamma}}{c_s} \sqrt{A^2 + B^2}, \quad r_H = \frac{\sqrt{\gamma}}{c_s} |A|. \quad (25) \]

3 Superradiance phenomenon

Now, we introduce the metric (23) in the relation Klein Gordon given by

\[ \sqrt{-g} \partial_{\mu} \left( \sqrt{-g} g^{\mu\nu} \partial_{\nu} \phi_1 \right) = 0 \quad (26) \]

Bearing in mind the stationarity and axisymmetry spherical metric (23), we can separate the potential as follows:

\[ \phi_1(t, r, \theta) = R(r) e^{i (w t - m \theta)} \quad (27) \]

where \( m \) is a real constant (azimuthal number) and \( w \) the rotational frequency of the acoustic BH. Considering (23), (26), (27), The radial function \( R(r) \) satisfies the linear second order differential equation

\[ R''(r) + P_1(r) R'(r) + P_2(r) R(r) = 0 \quad (28) \]

where

\[ P_1(r) = \frac{\Gamma_1}{\Gamma_2} \quad (29) \]

\[ \Gamma_1 = \left\{ c_s^2 r^2 [8c_s^2 r^2 + B^2(-1 + \gamma)^2] + 3A^4 \gamma(-1 + \gamma)^2 + A^2[3B^2(-1 + \gamma)^2 \times \gamma + c_s^2 r^2(1 + \gamma(6 + \gamma))] \right\} \quad (30) \]

\[ \Gamma_2 = 2r[4c_s^2 r^2 + A^2(-1 + \gamma)^2 + B^2(-1 + \gamma)^2(c_s^2 r^2 - A^2 \gamma)^2 \quad (31) \]

and

\[ P_2(r) = \frac{\Gamma_3}{\Gamma_4} \quad (32) \]
where
\[ \Gamma_3 = r^2 \left[ 4c_s^2 + A^2(-1 + \gamma)^2 + B^2(-1 + \gamma)^2 \right] \left[ -4c_s^4 r^2 m^2 + A^2 r^2 w^2 \right] (1 + \gamma)^2 + 4c_s^2 [(A^2 + B^2)m^2 \gamma - i B m r w(1 + \gamma) + r^4 w^2], \]
and
\[ \Gamma_4 = 4 \left[ 4c_s^2 r^2 + A^2(-1 + \gamma)^2 + B^2(-1 + \gamma)^2 \right] \left( c_s^2 r^2 - A^2 \gamma \right)^2. \] (33)

Thus, the problem has reduced to a one dimensional Schrödinger problem. Two further analytical simplifications can be made \(^{(28)}\): first, we introduce the tortoise coordinate \( r^* \) through the equation
\[ r^* = \left( 1 + \frac{(A^2 + B^2)}{4 c_s^2} (\gamma - 1)^2 \right) \left( r + \frac{|A| \sqrt{\gamma}}{2 c_s} \log \left| \frac{r c_s - A \sqrt{\gamma}}{r c_s + A \sqrt{\gamma}} \right| \right). \] (34)

Note that the tortoise coordinate spans the entire real line as opposed to \( r \) which spans only the half-line; the horizon \( r = \frac{\sqrt{\gamma} |A|}{c_s} \) maps to \( r^* \to -\infty \), while \( r \to \infty \) corresponds to \( r^* \to +\infty \). Next, introducing a new radial function \( Z(r) G(r^*) = R(r) \). The equation \(^{(28)}\) becomes:
\[ \frac{d^2 G(r^*)}{dr^*^2} + Q(r) G(r^*) = 0 \] (35)

with
\[ Q(r) = \frac{1}{\Delta^2} \left( \frac{1}{Z(r)} \frac{d^2 Z(r)}{dr^2} + P_1 \frac{1}{Z(r)} \frac{dZ(r)}{dr} + P_2 \right) \]
\[ = \frac{1}{16} \left( \frac{1}{(1 + \gamma)^2 (c_s^2 + \gamma)^2} \right)^2 \left\{ \frac{(144 c_s^4 r^2)}{(4 c_s^2 r^2 + A^2(-1 + \gamma)^2 + B^2(-1 + \gamma)^2)^2} \right\} \]
\[ + \frac{5}{r^2} \left( \frac{8 c_s^2}{(4 c_s^2 r^2 + A^2(-1 + \gamma)^2 + B^2(-1 + \gamma)^2)} \right)^2 \left\{ (2(8 c_s^2 r^2 + A^2(-1 + \gamma)^2 + B^2(-1 + \gamma)^2) \right\}^2 \]
\[ \times \gamma + A^2(3 B^2(-1 + \gamma)^2) + c_s^2 r^2(1 + \gamma)(6 + \gamma)) \} \right\} + \{ 4 r^2 (A^2 + A^2 \}
\[ \times (\gamma + A^2(3 B^2(-1 + \gamma)^2) + c_s^2 r^2(1 + \gamma)(6 + \gamma)) \} \right\} + \{ 4 r^2 (A^2 + A^2 \}
\[ \times (\gamma + A^2(3 B^2(-1 + \gamma)^2) + c_s^2 r^2(1 + \gamma)(6 + \gamma)) \} \right\} + \{ 4 r^2 (A^2 + A^2 \}
\[ \times (\gamma + A^2(3 B^2(-1 + \gamma)^2) + c_s^2 r^2(1 + \gamma)(6 + \gamma)) \} \right\} + \{ 4 r^2 (A^2 + A^2 \}
\[ \times (\gamma + A^2(3 B^2(-1 + \gamma)^2) + c_s^2 r^2(1 + \gamma)(6 + \gamma)) \} \right\} + \{ 4 r^2 (A^2 + A^2 \}
\[ \times (\gamma + A^2(3 B^2(-1 + \gamma)^2) + c_s^2 r^2(1 + \gamma)(6 + \gamma)) \} \right\} + \{ 4 r^2 (A^2 + A^2 \}
\[ \times (\gamma + A^2(3 B^2(-1 + \gamma)^2) + c_s^2 r^2(1 + \gamma)(6 + \gamma)) \} \right\} + \{ 4 r^2 (A^2 + A^2 \}
\[ \times (\gamma + A^2(3 B^2(-1 + \gamma)^2) + c_s^2 r^2(1 + \gamma)(6 + \gamma)) \} \right\} + \{ 4 r^2 (A^2 + A^2 \}\]
We remark that the main advantage of the new radial equation (35) over (28) is the absence in the former of a first derivative of the radial function. We analyze differential equation (35) in two distinct radial regions near the horizon, i.e. $r^* \to -\infty$ and at asymptopia, i.e. $r^* \to +\infty$. In the asymptotic region, Eq. (35) can be written approximately as,

$$\frac{d^2 G(r^*)}{dr^*^2} + \sqrt{\frac{w^2}{c_s^2 + \frac{1}{4}(A^2 + B^2)(1 + \gamma)^2}} G(r^*) = 0$$

of which one solution is

$$G(r^*) = \exp\left[i r^* \sqrt{\frac{w^2}{c_s^2 + \frac{1}{4}(A^2 + B^2)(1 + \gamma)^2}}\right] + R \exp\left[-i r^* \sqrt{\frac{w^2}{c_s^2 + \frac{1}{4}(A^2 + B^2)(1 + \gamma)^2}}\right].$$

where $R$ is the reflection coefficient. In equation (37), the first term is the incident wave and the second term is the reflected wave. The Wronskian of solution (37) is:

$$W(+\infty) = -2i \left\{ \frac{c_s^2}{w^2} \right\} T^2 (1 - |R|^2).$$

Considering the second field ($r^* \to -\infty$), the equation (35) becomes:

$$\frac{d^2 G(r^*)}{dr^*^2} + \left\{ A^2 (4c_s^2 r^2 + A^2)(-1 + \gamma) \right\} G(r^*) = 0.$$  

The physical solution of equation (39) is the following.

$$G(r^*) = T \exp\left[-i r^* \left\{ A^2 (4c_s^2 r^2 + A^2)(-1 + \gamma) \right\} \right]/\left\{ A^2 (4c_s^2 r^2 + A^2)(-1 + \gamma) \right\}.$$  

$$W(-\infty) = \left\{ -2i \left\{ 4(-2B c_s^2 m + A^2(1 + \gamma) w^2) \right\} T^2 \right\} /\left\{ A^2$$
\( \times \left( 4c_s^2 r^2 + A^2(-1 + \gamma)^2 + B^2(-1 + \gamma)^2)(A^2(1 + \gamma)^2 + B^2(-1 + \gamma)^2) \right) \). \tag{41}

Since both equations are actually limiting approximations of the differential Eq. \((35)\), which, as we have mentioned, has a constant Wronskian, it follows that (\[23\], \[24\], \[25\], \[26\])

\[ W(-\infty) = W(+\infty), \tag{42} \]

so that, from \((38)\) and \((41)\), we obtain the relation

\[ |\mathcal{R}|^2 = 1 - \frac{\left[ 1 + \gamma - \frac{2B m c_s^2}{w A^2} \right]}{\sqrt{(A^2(1 + \gamma)^2 + B^2(-1 + \gamma)^2)}} T^2. \tag{43} \]

Superradiance occurs when the norm of the reflected wave is greater than the norm of the incident wave, that is, when the reflection coefficient is greater than unity \([27, 28]\). Hence, we can observe in eq. \((43)\) that, for frequencies in the range

\[ 0 < w < m \bar{\Omega}_H \tag{44} \]

with

\[ \bar{\Omega}_H = \frac{2}{1 + \gamma} \Omega_H, \quad \Omega_H = \frac{B c_s^2}{A^2}, \tag{45} \]

the reflection coefficient has a magnitude larger than unity whose imply the amplification relation of the ingoing sound wave near horizon. With this condition we can extract the energy of the system \([26]\). Here \(m\) is the azimuthal mode number and \(\bar{\Omega}_H\) is the angular velocity of the usual Kerr-like acoustic BH, the angular velocity of the usual Kerr-like acoustic BH depends of the pressure \((n, k)\). Thus, we show that the presence of the the neo-newtonian parameter \(\gamma\) modifies the quantity of removed energy of the acoustic BH and that is either possible to accentuate or attenuate the amplification of the removed energy of the acoustic. The effect of superresonance can be eliminated when \(\gamma = \left(\frac{2m \Omega_H}{\omega} - 1\right)\).

### 4 Numerical results

To see the effect of pressure on the phenomenon, we have research to express and plot the potential energy, which leads to rewrite \(Q(r)\) from the equation
We can see from (11) when the \( \gamma \) parameter becomes larger, the potential energy increases, which allows us to say that the presence of pressure ascentue the phenomenon of superradiance.
Figure 1: The graphs illustrating the evolution of the potential $V(r)$ for each value of $\gamma$. The function is plotted for $A = B = c_s = m = 1$.

5 Conclusion

In this paper we shown that the presence of the pressure modify the quantity of removed energy of the acoustic BH and that, it is possible to accentuate or to attenuate the amplification of the removed energy of the acoustic BH and still exists the possibility to cancel the superradiance effect i.e the reflection coefficient is equal to unity, when $\gamma = \left(\frac{2m\Omega_H}{\omega} - 1\right)$ where $\Omega_H$ is the angular velocity of the acoustic black hole. Furthermore, the interval of frequencies can be wider or narrower depending on the neo-newtonian parameter $\gamma$. As a consequence, the tuning of the neo-newtonian parameter $\gamma$ changes the rate of loss of mass of the acoustic BH.

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