Analysis of 2-user ZF coordinated with user-wise MRTC diversity

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Abstract: Multi-user MIMO and spatial diversity are attractive techniques to improve the link capacity. In this paper, multi-user zero-forcing (ZF) coordinated with user-wise maximal-ratio transmitting and combining (MRTC) diversity is studied. A closed-form expression for the received signal-to-noise ratios (SNRs) achievable with 2-user ZF coordinated with user-wise MRTC diversity is derived for the given MIMO channel condition. It is shown from the derived SNR expression that the uplink capacity is higher than the downlink capacity and that the uplink capacity is not necessarily the same among users while the downlink SNR is always identical for users. This is confirmed by numerical evaluation assuming a Rayleigh fading environment.

Keywords: multi-user MIMO, zero-forcing, diversity, coordinated transmitter and receiver

Classification: Wireless Communication Technologies

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1 Introduction

Multi-user multi-input multi-output (MIMO) [1] has been recognized as an indispensable technique for efficiently utilizing the limited bandwidth. Meanwhile, spatial diversity [2] still remains an indispensable countermeasure against multipath fading. A simple way of dealing with inter-user and inter-antenna interference is to employ the zero-forcing (ZF) precoding at the base station (BS) transmitter. It is required that the number of BS antennas is larger than the total number of user antennas. To relax this requirement, the authors in [3, 4] presented a framework for the multi-user coordinated transmit-receive beamforming by extending the idea of joint transmitter and receiver diversity scheme of [5]. If the BS knows each user’s beamforming weight vector, it can create a set of virtual single-antenna channels and can design the ZF precoding matrix. Then, the requirement can be relaxed to that the total number of BS antennas is no smaller than the number of users irrespective of the total number of user antennas.

Inspired by the above ZF based multi-user coordinated transmit-receive beamforming, assuming time division duplex (TDD), we apply the maximal-ratio transmitting and combining (MRTC) diversity [2, 6, 7] to the user transceiver and study downlink and uplink multi-user ZF coordinated with user-wise MRTC diversity. The optimal multi-user coordinated transmit-receive beamforming requires computationally expensive iterative algorithms to determine the beamforming at BS and user sides [3, 4, 5]. On the other hand, the multi-user ZF coordinated with user-wise MRTC diversity does not require iterative algorithm and thus, simple.

In this paper, we assume a 2-user case and derive closed-form expressions for the downlink and uplink signal-to-noise ratios (SNRs) for the given multi-user MIMO channel. Using the derived SNR expressions, downlink and uplink capacities in a Rayleigh fading environment is numerically evaluated. Then, the impacts of the number of BS antennas and that of user antennas on the link capacity are investigated.

In this paper, the superscripts $T$, $H$, and $^*$ represent the transpose, Hermitian transpose, and complex conjugate operations, respectively. $E[\cdot]$ and $\|\cdot\|$ represent the ensemble average operation and the Frobenius norm.

2 Downlink transmission

BS and each user are assumed to be equipped with $M$ antennas and $N$ antennas, respectively. The channel between the $m$ ($=0$–$M-1$)th BS antenna and $n$ ($=0$–$N-1$)th antenna of the $u$ ($=0,1$)th user is represented as $h_{um}^{(n)}$ with $E\left[|h_{um}^{(n)}|^2\right] = 1$. The full MIMO channel of size $2N \times M$ can be represented as
\( \mathbf{H} = \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_1 \end{bmatrix} \), where \( \mathbf{H}_u = \begin{bmatrix} h_{u,m}^{(r)} \\ h_{u,m}^{(i)} \end{bmatrix} \) is the \( N \times M \) MIMO channel associated with the \( u \) (\( = 0, 1 \))th user.

### 2.1 Downlink transmission model

The downlink transmission model is illustrated in Fig. 1. The transmit signal vector \( \mathbf{x} = [x_0 \, \cdots \, x_{M-1}]^T \) at BS and the received signal vector \( \mathbf{r} = [r_0, r_1]^T \) at 2 users are expressed as

\[
\begin{align*}
\mathbf{x} &= \alpha \mathbf{W} \sqrt{2P} \mathbf{d} \\
\mathbf{r} &= \mathbf{H}_{\text{virtual}} \mathbf{x} + \mathbf{\eta},
\end{align*}
\]

where

\[
\mathbf{H}_{\text{virtual}} = \begin{bmatrix} \mathbf{h}_0 \\ \mathbf{h}_1 \end{bmatrix}
\]

represents the virtual MIMO channel between \( M \)-antenna BS and 2 users with \( \mathbf{h}_u = [h_{u,0} \, \cdots \, h_{u,M-1}] \) and \( \mathbf{W} = [w_{m,u}]_{m=0:M-1, u=0:1} \) represents the ZF precoding matrix of size \( M \times 2 \). In Eq. (1), \( \mathbf{d} = [d_0, d_1]^T \) with \( E[\mathbf{d}_u^* \mathbf{d}_{u'}] = \delta_{uu'} \) (Kronecker delta), \( \mathbf{P} = \text{diag}(P_0, P_1) \), \( \alpha \), and \( \mathbf{\eta} = [\eta_0, \eta_1]^T \) represent the data symbol vector, the user power allocation vector, the power normalization factor to keep the sum of user transmit powers intact after the precoding, and the noise vector, respectively.

Letting \( \mathbf{\beta}_u = \left[ \beta_u^{(0)} \, \cdots \, \beta_u^{(N-1)} \right] \) be the \( u \)th user’s MRTT weight vector, \( \mathbf{h}_u \) constituting \( \mathbf{H}_{\text{virtual}} \) in Eq. (2) is given as

\[
\mathbf{h}_u = \mathbf{\beta}_u \mathbf{H}_u \quad \text{s.t.} \quad \|\mathbf{\beta}_u\|^2 = 1,
\]

where \( \mathbf{\beta}_u \) is determined so that the \( u \)th user’s virtual channel gain \( |h_{uu}| \) is maximized. Since \( \|\mathbf{\beta}_u\|^2 = 1 \), \( \eta_0 \) and \( \eta_1 \) of \( \mathbf{\eta} \) in Eq. (1) are characterized as independent zero-mean complex Gaussian variables having the same variance of \( 2N_0/T_s \), where \( N_0 \)
and $T_s$ are the noise power spectrum density and the data symbol length in time, respectively. It is assumed that BS and each user have the perfect knowledge of $H_{\text{virtual}}$ and $H_u$, respectively.

The $n$th element $\beta^{(n)}_u$ of $\beta_u$ is given as [2]

$$\beta^{(n)}_u = \left( h^{(n)}_{uu} \right)^* \left( \sum_{n=0}^{N-1} |h^{(n)}_{uu}|^2 \right)^{-1/2} \text{ for } u = 0, 1. \quad (4)$$

It is worth noting that the MIMO channel matrix size to determine the ZF precoding matrix is reduced from $2N \times M$ to $2 \times M$ irrespective of $N$.

### 2.2 Obtaining $W$ and $\alpha$

The ZF precoding matrix $W$ is expressed as $W = H_{\text{virtual}}^H \left( H_{\text{virtual}} H_{\text{virtual}}^H \right)^{-1}$. Since $H_{\text{virtual}} H_{\text{virtual}}^H$ is a $2 \times 2$ matrix, obtaining its inverse matrix is rather simple. After some manipulation, we obtain $W$ and $\alpha$ as follows.

$$W = H_{\text{virtual}}^H \begin{bmatrix} \|h_1\|_2^2 - \langle h_0, h_1^H \rangle & - \langle h_0, h_1^H \rangle^* \|h_0\|_2^2 \\ \|h_0\|_2^2 - \langle h_0, h_1^H \rangle^2 \end{bmatrix} \|h_0\|_2^2$$

$$\alpha = \sqrt{(P_0 + P_1) \frac{\|h_0\|_2^2 \|h_1\|_2^2 - \|h_0 h_1^H\|_2^2}{P_0 \|h_1\|_2^2 + P_1 \|h_0\|_2^2}}. \quad (5)$$

### 2.3 Obtaining the received signal-to-noise ratio (SNR)

Since $H_{\text{virtual}} W = I$, the received signal vector $r$ becomes

$$r = \alpha \sqrt{2P_d} + \eta$$

$$= \sqrt{(P_0 + P_1) \frac{\|h_0\|_2^2 \|h_1\|_2^2 - \|h_0 h_1^H\|^2}{P_0 \|h_1\|_2^2 + P_1 \|h_0\|_2^2}} \sqrt{2P_d} + \eta, \quad (7)$$

from which the downlink SNR $\gamma_{1u}$ can be obtained as

$$\gamma_{1u} = \Gamma_u \frac{\Gamma_0 + \Gamma_1}{\Gamma_0 \|h_1\|_2^2 + \Gamma_1 \|h_0\|_2^2} \left( \frac{\|h_0\|_2^2 \|h_1\|_2^2 - \|h_0 h_1^H\|_2^2}{\|h_0\|_2^2 \|h_1\|_2^2 - \|h_0 h_1^H\|_2^2} \right), \quad (8)$$

where $\Gamma_u = P_u \cdot T_s / N_0$ represents the $u$th user’s transmit symbol energy-to-noise power spectrum density ratio.

### 3 Uplink transmission

Assuming TDD transmission mode, the channel reciprocity between the uplink and downlink can be exploited. The transmit power associated with the $u$th user is assumed to be the same $P_u$ for the downlink and uplink transmissions although they can be set different. Furthermore, the uplink transmit power control is not considered, i.e., the transmit power is set to the predetermined value $P_u$ irrespective of the MIMO channel condition. The downlink MRTC weight vector $\beta_u$ whose
nth weight is given by Eq. (4) is used for the uplink transmission. Accordingly, the uplink virtual MIMO channel can be expressed as $H_{\text{virtual}}^T$ and the uplink ZF postcoding matrix can be given as $W^T$. Consequently, the received signal after ZF postcoding can be expressed as

$$r = W^T \left( H_{\text{virtual}}^T \sqrt{2P_d} + \xi \right) = \sqrt{2P_d} + W^T \xi,$$

(9)

where $\xi = [\xi_0, \xi_1]^T$ is the noise vector representing the noises received on 2 BS antennas. $\xi_0$ and $\xi_1$ are assumed to be independent zero-mean complex Gaussian variables having the variance of $2N_0/T_s$ as assumed for the downlink reception.

It can be understood from Eq. (9) that, unlike from the downlink, the received SNR is different between 2 users because of the effect of noise after ZF postcoding. After some manipulation, we obtain the uplink SNR $\gamma_{1u}$ as follows.

$$\begin{align*}
\gamma_{10} &= \Gamma_0 \frac{||h_0||^2_2 ||h_1||^2_2 - |h_0 h_1^H|^2}{||h_1||^2_2} \\
\gamma_{11} &= \Gamma_1 \frac{||h_0||^2_2 ||h_1||^2_2 - |h_0 h_1^H|^2}{||h_0||^2_2}
\end{align*}$$

(10)

### 4 Discussion

For simplicity, we assume $M = 2$ and $P_0 = P_1 = P$ ($\Gamma_0 = \Gamma_1 = \Gamma$). From Eqs. (8) and (10), after some manipulation, we obtain the following very simple expressions for the downlink and uplink SNRs:

$$\begin{align*}
\gamma_1 &= \gamma_{10} = \gamma_{11} \\
&= 2\Gamma \frac{|h_{00} h_{11} - h_{01} h_{10}|^2}{|h_{00}|^2 + |h_{01}|^2 + |h_{10}|^2 + |h_{11}|^2} \quad \text{for downlink} \\
\gamma_{10} &= \Gamma \frac{|h_{00} h_{11} - h_{01} h_{10}|^2}{|h_{10}|^2 + |h_{11}|^2} \\
\gamma_{11} &= \Gamma \frac{|h_{00} h_{11} - h_{01} h_{10}|^2}{|h_{00}|^2 + |h_{01}|^2} \quad \text{for uplink.}
\end{align*}$$

(11a, 11b)

The link capacity $C$ [bps/Hz] can be computed using $C = \log_2(1 + \gamma)$. It can be understood from Eq. (11) that the downlink capacity is identical between 2 users, while the uplink capacity is not necessarily the same when $\{h_{um}; u \text{ and } m = 0, 1\}$ are given. From Eq. (11), we also obtain the following relationship between the downlink and uplink SNRs (its derivation procedure is omitted for brevity).

$$\min(\gamma_{10}, \gamma_{11}) \leq \gamma_1 \leq \max(\gamma_{10}, \gamma_{11}).$$

(12)

It can be understood from Eq. (12) that in a fading environment, the probability of the uplink capacity falling below a certain small value is higher than the downlink, while the probability of the uplink capacity exceeding a certain large value is higher than the downlink.

An important feature of the multi-user ZF coordinated with user-wise MRTC diversity is twofold: each user can determine its MRTC diversity weight vector based
on its own MIMO channel only and BS can determine the ZF precoding/postcoding matrix based on the virtual MIMO channel without the knowledge of the number of user antennas. This can be achieved by exchange of orthogonal pilots between BS and users (note that BS and users need to share the pilot information in advance). Firstly, BS transmits orthogonal pilots from its $M$ antennas and each user constructs its own MIMO channel using the pilots received on its $N$ antennas to determine the MRTC diversity weight vector. Next, each user transmits back the orthogonal pilot precoded by its MRTC diversity weight vector to BS and BS constructs the virtual MIMO channel matrix using the received pilots to determine the ZF precoding/postcoding matrix.

5 Numerical evaluation
The downlink and uplink capacities in uncorrelated Rayleigh faded MIMO channels are evaluated assuming $P_0 = P_1 = P$ ($\Gamma_0 = \Gamma_1 = \Gamma$), i.e., the transmit power is set to the same for downlink and uplink transmissions. The pathloss and shadowing loss are not considered. The multi-user ZF coordinated with user-wise MRTC diversity allows $U \cdot N \geq M$ as far as $U \leq M$ is satisfied. The cumulative distribution function (CDF) of the user capacity $C_{\text{user}}$ when $\Gamma = 20$ dB obtained by the numerical evaluation is plotted in Fig. 2(a) with $M$ as a parameter when $N = 2$ and in Fig. 2(b) with $N$ as a parameter when $M = 2$. It can be seen from Fig. 2 that as either $M$ or $N$ increases, the link capacity improves because of increasing diversity gain. Also seen is that the capacity at CDF = 90% is higher for the uplink than for the downlink. It is seen when $M = 4$ and 8 (Fig. 2(a)) or $N = 4$ and 8 (Fig. 2(b)) that the capacity at CDF = 1% is slightly higher for the downlink than for the uplink. These results are consistent with the observations made in Sect. 4.

6 Conclusions
In this paper, multi-user ZF coordinated with user-wise MRTC diversity was studied. Its important feature is twofold: each user can determine its MRTC diversity
weight vector based on its own MIMO channel only and BS can determine the ZF matrix based on the virtual MIMO channel without the knowledge of the number of user antennas. The closed-form expression for the received SNR achievable with 2-user ZF coordinated with user-wise MRTC diversity was derived for uplink and downlink transmissions. It was confirmed by the numerical evaluation that the spatial diversity is effective to improve the link capacity and that the higher link capacity is achieved for the uplink than the downlink in a Rayleigh fading environment.

In a broadband communication suffering from frequency-selective fading, frequency-domain transmission techniques, e.g. OFDM and single carrier with minimum mean square error based frequency-domain equalization (SC-FDE), are adopted. The multi-user ZF coordinated with user-wise MRTC diversity can be applied directly to each subcarrier of OFDM and an additional frequency-diversity gain can be obtained. However, SC-FDE cannot completely eliminate the inter-symbol interference caused by frequency-selective fading, and hence, a joint optimization of user-wise diversity, FDE, and multi-user multiplexing methods is necessary. This is left as a future study issue.

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