Collisionless Shocks – Magnetic Field Generation and Particle Acceleration

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Abstract

We present numerical results from plasma particle simulations of collisionless shocks and ultra-relativistic counter-streaming plasmas. We demonstrate how the field-particle interactions lead to particle acceleration behind the shock-front. The post-shock particle energy spectra are found to be segmented power laws. Specifically, we observe power law indices consistent with $p \approx -2.2$ for high gamma particles, and $p \approx 0.5$ for low gamma particles. The break is found at a gamma higher by $\sim 0.5 M_i/m_e$ than the gamma of the up-stream plasma relative to the shock. Further, we find that ultra relativistic counter-streaming plasmas create large scale magnetic fields and that the generated field propagate at $v \sim c$. The magnetic field generation is due to a Weibel-like two-stream instability. These results may help explain the origin of the magnetic fields and accelerated electrons responsible for afterglow synchrotron radiation from gamma ray bursts.

1 Introduction

One requirement in validating MHD models of a plasma is that collisions are able to maintain the distribution function close to a Maxwellian. However, in the context of the external shock model of gamma ray burst after-glow [1] the mean free path for collisions is much larger than the depth of the fireball shell. Thus, collisions are so rare that the distribution functions departs radically from Maxwellsians. In the reference frame of the shock one observes two counter-streaming plasmas. In the absence of collisions this gives rise to a deep interpenetration of two streaming near-delta-function populations. Such a situation clearly requires a kinetic treatment of the plasma.

The existence of a strong magnetic field in the expanding fireball shell is required in order to explain the observed synchrotron radiation. Medvedev & Loeb [2] showed through a linear kinetic treatment how a two-stream magnetic instability (a generalization of the Weibel instability [3] [4]) can generate a strong magnetic field ($\epsilon_B \sim 10^{-5}$ to $10^{-1}$ of equipartition energy density) in the collisionless shock front (see
also discussion in [5]). We note in passing that this instability is well-known in other plasma physics disciplines (e.g. laser-plasma interactions [6] [7]).

A power-law distributed population of relativistic electrons is also required to produce the observed synchrotron radiation spectra. All together, we therefore identify the following questions as being essential in the discussion of collisionless shocks and GRB afterglows: 1) Which mechanism is responsible for the particle acceleration in collisionless plasma shocks – and what implications can be inferred from the answer to this question? 2) Can one possibly avoid \textit{ab initio} assumptions of a magnetic field in GRB after-glow, and instead come up with a picture where the magnetic field is selfconsistently generated, using known and defensible physics?

## 2 Simulation Tool

Simulations were performed using a self-consistent time-averaged implicit fully 3D electromagnetic particle-in-cell code compliant with special relativity. The PIC-code was originally written by Dr. Michael Hesse for simulating reconnection topologies [8], and has been redeveloped and enhanced by the present authors to obey special relativity and to be fully $O(\Delta t^2) + O(\Delta x^2)$.

The code solves the Maxwell equations for the electromagnetic field

$$
\partial_t \mathbf{E} = c^2 (\partial_x \times \mathbf{B} - \mu_0 \mathbf{J}), \quad \partial_t \mathbf{B} = -\partial_x \times \mathbf{E}, \quad \partial_x \cdot \mathbf{B} = 0, \quad \partial_x \cdot \mathbf{E} = \rho_c/\epsilon_0 .
$$

Particle velocities and positions are defined continuously throughout phase space. Fields and field source terms are defined on a fully 3D Yee lattice [9]. The sources in Maxwell’s equations are formed by weighted averaging of particle data to the field grid, using quadratic spline weighted interpolation (Triangular Shaped Cloud scheme).

The particle-to-mesh (source) and mesh-to-particle (force) interpolations have identical weighting schemes, in order to obey momentum conservation and eliminate particle self-forces through coupling to the grid [10]. The ion to electron mass ratio is typically chosen in the range $m_{0i}/m_{0e} \sim 8 - 25$. The particle equations of motion are integrated by solving, for all particles,

$$
\partial_t (\gamma (v_i) \mathbf{v}_i) = q_i m_{0i}^{-1} (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}), \quad \partial_t \mathbf{r}_i = \mathbf{v}_i .
$$

A typical grid size is $\{n_x, n_y, n_z\} = \{50, 50, 400\}$, with about 25 particles/cell or $\sim 10^7$ particles. Typical dimensions $L_\perp \sim 4\delta_i \sim 16\delta_e$ and $L_z \sim 25\delta_i \sim 100\delta_e$, chosen so as to resolve a significant number of ion skin-depths $\delta_e$ – vital to our conclusions.
3 Magnetic Field Generation

A small anisotropic perturbation to an isotropic probability density distribution for a plasma gives rise to a plasma instability that generates a magnetic field (with a relativistic generalization by [4]). A similar two-stream situation is described in detail by Medvedev and Loeb [2]: two counter-streaming plasmas penetrate each other with a relative velocity and relativistic bulk gamma factor $\Gamma_{sh}$. In the presence of an infinitesimal magnetic seed field, electrons and ions deflect into separate channels. This creates currents that amplify the existing magnetic configuration, producing an instability and magnetic field amplification.

From the requirement that the total plasma momentum should be conserved, and since

$$P_{tot} = \sum_i p_i(r_i) + \int_V \mathbf{S}(r) \, dr,$$

the (electro)magnetic field produced by the two-stream may acquire part of the momentum lost by the entire two-stream population in the shock; this opens the possibility that magnetic field structures created in the shock migrate downstream of the shock and thus carry away some of the momentum impinging on the shock. Here, $\mathbf{S}$ denotes the Poynting flux in a volume $V$, and $p_i$ are the particle momenta in $V$.

Our experiments show that this does indeed happen; the magnetic field structures created in the shock are also carried along downstream. Since the impinging beam of particles looses energy only gradually downstream of their first encounter with the denser plasma, there is ample energy present for sustaining and amplifying the field further.

We find that, as the instability develops downstream, the perpendicular size of the structures increase systematically with distance from the shock. The growth mechanism is somewhat similar to the merging of smaller cells into larger ones below the surface of a strongly stratified convection zone in a star. Smaller cells are advected by larger cells, and their constituent structures merge into those of the larger cells.

Figure 1 shows the resulting patterns, in a slice at 75% of the covered downstream region. Note that, even though the patterns started out on scales similar to the skin depths, they grow to cover essentially the whole box width in our model.


4 Particle Acceleration

If the correlation length of the electro-magnetic field is smaller than typical particle gyro radii the field is able to work as an efficient scatter-mechanism for the particles. Coupling of the two streams through fluctuating $B$- and $E$-fields then provide a mechanism for heating the quiescent population (in either reference frame). Also, it will act as a channel to transfer and equalize the kinetic energy between ions and electrons in each of the two beams. Hence, electrons will be accelerated strongly ($\gamma_e \sim \frac{1}{2} \gamma_{sh} m_0 i / m_0 e$). Of course, some of the kinetic energy must go to the production of electromagnetic turbulence, to provide the scatter mechanism in the first place.

There is thus the intriguing possibility that a chain of energy transfers between various energy pools may provide a selfconsistent picture where both magnetic field generation and particle acceleration is explained:

Initially the energy resides predominantly in the bulk kinetic energy of the heavier protons (and ions). The fluctuating magnetic field created by the two-stream instability helps scatter some of the incoming protons, and also carries away some of the bulk momentum. The scattered protons create a fluctuating electric field, which tends to equilibrate the energy between protons and electrons, thus accelerating the electrons. This mechanism is qualitatively different from Fermi acceleration in that acceleration is provided $in situ$ in the down-stream plasma, rather than by scattering of particles back and forth across the shock.

![Probability Distribution Functions](image)

Figure 2: Probability Distribution Functions as produced by the collisionless shock in one of our experiments.

The Probability Distribution Functions illustrated in Fig. 2 shows that this chain of events actually takes place, and that the mechanism is capable of producing a population of accelerated electrons.
5 Conclusions

A central point to be made here is that magnetic field generation and particle acceleration may well be generic and unavoidable in collisionless relativistic shocks. If so, the ratios $\epsilon_B$ and $\epsilon_e$ are both results of the same process, and cannot be regarded as independent, free parameters.

Qualitatively, the electro-magnetic field acts as the ‘catalyst’ that allows the desired ‘reaction’ to proceed; the bulk kinetic energy in the up-stream plasma needs to be converted to randomized particle kinetic energy while maintaining the bulk impulse, for the conventional shock picture to be applicable. A priori, it is unclear how this happens in a collisionless shock. The scenario proposed here, inspired by the results of the simulations, offers a likely mechanism for the thermalization of the bulk kinetic energy, which automatically also provides a natural explanation for the presence of a strong magnetic field and a power law population of accelerated electrons.

One may hope that further studies along these lines will provide quantitative predictions for the fractions of the bulk kinetic energy that go into magnetic field energy and energy of accelerated electrons, $\epsilon_B$ and $\epsilon_e$, respectively; these are the parameters than are needed / assumed in conventional GRB afterglow shock modeling. However, what is ultimately observed is the resulting synchrotron-like radiation spectrum. Thus, rather than first trying to abstract scalar parameters $\epsilon_B$ and $\epsilon_e$ from the simulations (where these ratios may be expected to actually depend on the distance from the shock) it may be a better approach to compute synthetic radiation spectra directly from the models, and to use scaling laws to predict what would be observed from the corresponding, real afterglow shocks.

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Discussion

Berger: Is it possible to explain the wide range of p values which is inferred to be about 1.4 - 2.8 from afterglow observation?

Nordlund: That remains to be seen; we have so far carried out only a few experiments.

Brandenburg: Very near the shock front, the ion and electron densities were very nearly stagnant in the lateral direction; only further away did the structures wobble. What is it that holds electron and ion concentrations so nearly fixed near the shock?

Nordlund: The feature itself. This IS the first encounter of the beam with any resistance, which creates a standing perturbation.

Rossi: Do your result indicate that we should change the theory that we use to fit afterglow? If yes, how?

Nordlund: We should certainly not assume constant $\epsilon_B$ and $\epsilon_e$ to hold in the entire shocked plasma, but rather develop an understanding of their evolution and mutual dependence.