Scrubbing Supergravity Models through Neutrino Telescopes

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ABSTRACT

Galactic halo neutralinos (\(\chi\)) captured by the Sun or Earth produce high-energy neutrinos as end-products of various annihilation modes. These neutrinos can travel from the Sun or Earth cores to the neighborhood of underground detectors (“neutrino telescopes”) where they can interact and produce upwardly-moving muons. We compute these muon fluxes in the context of the minimal \(SU(5)\) supergravity model, and the no-scale and dilaton \(SU(5) \times U(1)\) supergravity models. At present, with the Kamiokande 90\% C.L. upper limits on the flux, only a small fraction of the parameter space of the \(SU(5) \times U(1)\) models is accessible for \(m_\chi \sim m_{Fe}\), which in turn implies constraints for the lightest chargino mass around 100 GeV for a range of \(\tan \beta\) values. We also delineate the regions of parameter space that would be accessible with the improvements of experimental sensitivity expected in the near future at Gran Sasso, Super-Kamiokande, and other facilities such as DUMAND and AMANDA, currently under construction. We conclude that if neutralinos are present in the halo, then this technique can be used to eventually explore more than half of the allowed parameter space of these specific models, and more generally of a large class of supergravity models, in many ways surpassing the reach of traditional collider experiments.
1 Introduction

Even though there appear to be several indications that low-energy supersymmetry is indeed a symmetry of Nature [4], no more converts should be expected until an actual sparticle is observed experimentally or unequivocal indirect experimental evidence becomes compelling. For either discovery mode to be efficient, one must provide as accurate experimental predictions as possible. In supersymmetric models where the number of parameters is large, as in the minimal supersymmetric standard model (MSSM), easily falsifiable experimental predictions are hard to obtain since the various parameters usually allow a myriad of possibilities. One basic step forward is to study specific supersymmetric models wherein the assumptions made are well motivated by physics at very high energies, such as grand unification, supergravity, and superstrings. In this way one can hope to test general theoretical frameworks experimentally, as opposed to just ruling out small regions of a many-dimensional parameter space. This line of thought has been in existence for almost as long as supersymmetric phenomenology has. However, it has only been since the firm establishment of the standard model at LEP that real candidates for the theory beyond the standard model have emerged from a morass of pre-LEP challengers.

The aim of this paper is to study a very promising indirect experimental procedure to discover supersymmetry, in the context of a class of $SU(5)$ and $SU(5) \times U(1)$ supergravity models, via interactions of upwardly-moving muons in underground detectors. As we will show, this method of experimental exploration is quite competitive with other indirect probes which have been recently investigated in the context of this same class of models, namely supersymmetric contributions to the one-loop electroweak LEP observables [2], to the rare radiative $b \rightarrow s\gamma$ decay [3], and to the anomalous magnetic moment of the muon [4].

The basic idea is that the neutralinos $\chi$ (lightest linear combination of the superpartners of the photon, $Z$-boson, and neutral Higgs bosons), which are weakly interacting massive particles (WIMPs), are assumed to make up the dark matter in the galactic halo, and can be gravitationally captured by the Sun or Earth [5, 6], after losing a substantial amount of energy through elastic collisions with nuclei. The neutralinos captured in the Sun or Earth cores annihilate into all possible ordinary particles, and the cascade decays of these particles as well as their interactions with the solar or terrestrial media produce high-energy neutrinos as one of several end-products. These neutrinos can then travel from the Sun or Earth cores to the vicinity of underground detectors, and interact with the rock underneath producing detectable upwardly-moving muons [4]. Such detectors are rightfully called “neutrino telescopes”, and the possibility of indirectly detecting various WIMP candidates has been considered in the past by several authors [7]. More recent analyses can be found in Refs. [8, 9, 10, 11, 12, 13, 14].

1In this paper, we do not consider the less promising “contained events” in which neutrino interactions take place within the detector, since the event rate for such events is proportional to $E_\nu$, as opposed to $E_\nu^2$ for the upwardly-moving muon events.
Making use of the techniques in the literature [8, 10, 11], our current work focuses on the predictions for the upwardly-moving muon event rates (fluxes) in two distinct supersymmetric models, namely, the minimal $SU(5)$ supergravity model and the string-inspired $SU(5) \times U(1)$ supergravity model. In these models, because of the underlying structure of supergravity, the low-energy couplings and masses are completely determined via renormalization group equations (RGEs) in terms of only five parameters: the three universal soft-supersymmetry-breaking parameters ($m_{1/2}, m_0, A$), the top-quark mass ($m_t$), and the ratio of Higgs vacuum expectation values ($\tan \beta$). The restriction to just five parameters is possible because of the radiative electroweak symmetry breaking mechanism, which allows to determine some further parameters, like the magnitude of the Higgs mixing term $\mu$ (but not its sign). This mechanism imposes further constraints on the model parameters (e.g., it requires $\tan \beta > 1$) and involves the value of $m_t$ in a fundamental way [15]. These features make it possible to incorporate all currently available experimental constraints, both direct and indirect, as well as some theoretical consistency conditions into a complete determination of the allowed parameter space of these models. We calculate the upwardly-moving muon fluxes induced by the neutrinos from the Sun and Earth in the still-allowed parameter space of these models, and compare them with the currently most stringent 90\% C.L. experimental upper bounds, obtained at Kamiokande, for neutrinos from the Sun [16] and Earth [12] respectively, i.e.

\[ \Gamma_{\text{Sun}} < 6.6 \times 10^{-14} \text{cm}^{-2} \text{s}^{-1} = 2.08 \times 10^{-2} \text{m}^{-2} \text{yr}^{-1}, \]
\[ \Gamma_{\text{Earth}} < 4.0 \times 10^{-14} \text{cm}^{-2} \text{s}^{-1} = 1.26 \times 10^{-2} \text{m}^{-2} \text{yr}^{-1}. \]  

Aiming at the next generation of underground experimental facilities, such as MACRO and other detectors at the Gran Sasso Laboratory [17], Super-Kamiokande [18], DUMAND, and AMANDA [19], where improvements in sensitivity by a factor of 2–100 are expected, we also delineate the region of the parameter space of these models that would become accessible with an improvement of experimental sensitivity by modest factors of two and twelve.

This paper is organized as follows. In Sec. 2 we describe the minimal $SU(5)$ and $SU(5) \times U(1)$ supergravity models. In Sec. 3 we outline the calculation of the capture rates at the Sun and Earth, while in Sec. 4 we outline the calculation of the corresponding detection rates. In Sec. 5 we present the results of our computations for the two supergravity models described in Sec. 2. Finally, we conclude in Sec. 6 with some comments.

2 The Supergravity Models

We work in the context of two supergravity models based on the gauge groups $SU(5)$ [20] and $SU(5) \times U(1)$ (“flipped $SU(5)$”) [21]. In these two models, spontaneous supersymmetry breaking takes place in the “hidden” sector, which manifests itself in the “observable” sector Lagrangian as a set of soft-supersymmetry-breaking terms
that are universal at the respective unification scales of the models. The low-energy particle content of these two models is the same as that of the MSSM. However, mainly because of the different gauge group structure, these two models have rather different low-energy phenomenologies, which we now briefly describe in turn.

In the minimal \( SU(5) \) supergravity model, the unification of the standard model gauge couplings takes place at a scale \( M_U \sim 10^{16} \text{ GeV} \) \(^2\), as a combined result of the \( SU(5) \) gauge group and the minimal matter content. This property of the model has been shown to hold even after the most general low-energy and high-energy threshold corrections have been incorporated, as well as the two-loop expressions for the RGEs \(^2\). The unified symmetry implies the existence of additional heavy particles with masses of order \( M_U \). In particular, the Higgs sector includes at least the \( 24 \) representation to break \( SU(5) \) down to \( SU(3) \times SU(2) \times U(1) \), as well as the \( 5, \bar{5} \) superfields, whose scalar doublet components \( (H_2) \) are responsible for the electroweak gauge symmetry breaking and should be kept light, while the scalar triplet components \( (H_3) \) must be heavy to avoid fast proton decay through dimension-six operators. It also follows that the Yukawa couplings of the bottom-quark and tau-lepton must be unified at \( M_U \), which when evolved to low energies entails a constraint on the relation between the ratio of Higgs vacuum expectation values \( (\tan \beta) \) and the top-quark mass \( (m_t) \). Perhaps the most distinguishing feature in the minimal \( SU(5) \) model has to do with the genuinely supersymmetric proton decay via dimension-five operators, mediated by the exchange of superpartners of the heavy Higgs triplets \( (\tilde{H}_3) \). Even for sufficiently large values of \( M_{\tilde{H}_3} \), it is necessary to tune the sparticle spectrum so that this type of proton decay remains at an acceptable level \(^3\). This usually requires light charginos and neutralinos, while squarks and sleptons should be heavy. We study a generic supersymmetry breaking scenario characterized by the universal parameters \( (m_{1/2}, m_0, A) \). It turns out that, because of the correlation between the relic density of the lightest neutralino and the soft supersymmetry breaking patterns \(^4\), the cosmological constraint together with the proton decay constraint dramatically reduce the parameter space of the minimal \( SU(5) \) model, and what remains are essentially points corresponding to the resonances in the neutralino pair annihilation cross section where \( m_{\chi} \sim \frac{1}{2} M_Z, \frac{1}{2} m_h \) \(^4\). In addition, the ratio \( \xi_0 = m_0/m_{1/2} \) must be significantly larger than unity, and \( \tan \beta \) must not be too large \( (\tan \beta \lesssim 3 - 5) \) \(^3\) \(^4\). The ensuing simple relations among the different sparticle masses in this model can be summarized as:

\[
\begin{align*}
\text{Higgs} : & \quad 60 \text{ GeV} < m_h < 125 \text{ GeV}, \\
\chi^0_1, \chi^+_1 : & \quad 2m_{\chi^0_1} \sim m_{\chi^+_1} \sim 0.3 m_{\tilde{g}}, \\
\text{Squarks} : & \quad m_{\tilde{q}} \approx \frac{1}{3} m_{\tilde{g}} \sqrt{6 + \xi_0^2}, \\
\text{Sleptons} : & \quad m_{\tilde{e}_R} \approx \frac{1}{3} m_{\tilde{g}} \sqrt{0.15 + \xi_0^2}, \quad m_{\tilde{e}_L} \approx \frac{1}{3} m_{\tilde{g}} \sqrt{0.5 + \xi_0^2},
\end{align*}
\]

where \( \xi_0 \gtrsim 3 - 6 \) depending on the value of the triplet higgsino mass used \( (M_{\tilde{H}_3} < (3 - 10) M_U) \). The actual five-dimensional parameter space to be explored in this paper for the minimal \( SU(5) \) model has been obtained in Ref. \(^4\).

In the string-inspired \( SU(5) \times U(1) \) supergravity model, the preferred unification
scale is $M_U \sim 10^{18}$ GeV, as expected in the context of string theory \[27\], which has been realized by the effects of extra vector-like matter fields at some intermediate scales \[28\]. Contrary to the minimal $SU(5)$ model, the Higgs bosons needed to break the $SU(5) \times U(1)$ symmetry belong to the $10$ and $\overline{10}$ representations, which can be easily accommodated by the simplest string models (with Kac-Moody level $k = 1$), while the adjoint representation $(24)$ may only appear in more complicated constructions. Because of the $SU(5) \times U(1)$ gauge group structure, the doublet-triplet splitting of the pentaplet Higgs superfields occurs in a very simple way, and the dimension-five proton decay operators are automatically suppressed. In the case of the $SU(5) \times U(1)$ supergravity model, we study two string-inspired supersymmetry breaking scenarios: (i) the no-scale model \[29\], where $m_0 = A = 0$ \[30\], and (ii) the dilaton model \[31\], where $m_0 = \sqrt{3} m_{1/2}, A = -m_{1/2}$ \[32\]. Therefore, the $SU(5) \times U(1)$ model (either scenario) is quite predictive since it depends on only three parameters: $m_t, \tan \beta, m_{1/2}$. Also, in these two scenarios the cosmological constraint is automatically satisfied \[24\].

The mass relations in this model are:

$$SU(5) \times U(1) : \begin{cases} \text{Higgs} : & 60 \text{ GeV} < m_h < 125 \text{ GeV}, \\ \chi^0_{1,2}, \chi^{\pm}_1 : & 2m_{\chi^0_1} \approx m_{\chi^0_2} \approx m_{\chi^+_1} \approx 0.28m_\tilde{g}, \\ \text{Squarks} : & m_{\tilde{q}} \approx m_\tilde{g}, \\ \text{Sleptons} : & m_{\tilde{e}_R} \approx 0.18(0.33)m_\tilde{g}, \quad m_{\tilde{e}_L} \approx 0.30(0.41)m_\tilde{g}, \end{cases}$$

(4)

for the no-scale (dilaton) case. Note that the squark and slepton mass relations in Eq. (3) do not reduce to those in Eq. (4), for values of $\xi_0 = 0$ (no-scale) and $\xi_0 = 1/\sqrt{3}$ (dilaton), because of slight changes in the running of the scalar masses down to low energies from the different starting value of $M_U$. The allowed three-dimensional parameter space of this model has been determined in Refs. \[24, 31\] for the no-scale and dilaton cases respectively.

We now comment on some other features about these two models which are particularly relevant to our current work. First, for $m_\tilde{g} \lesssim 1$ TeV the lightest neutralino cannot be a pure higgsino \[33\]. In fact, we have found that for almost all the points in the allowed parameter spaces of both models, the lightest neutralino is a “mixed” state with substantial gaugino and higgsino components. In the MSSM, such “mixed” neutralinos normally cannot even account for the halo dark matter \[34\], because the efficient annihilation via Higgs boson exchange renders the relic density rather small ($\Omega_\chi h^2_0 < 0.05$). This situation is improved in the two supergravity models, as a result of the one-loop radiative corrections to the masses of Higgs bosons which we have included in our analysis. Since the one-loop corrected Higgs masses are normally larger than the tree-level values, the overall effect is an enhancement of the relic density for “mixed” neutralinos. Therefore, the lightest neutralinos in these two models, although mostly “mixed” states, are still good candidates for the major component of the galactic halo. Such neutralinos are mainly captured by the Sun and Earth through their coherent (spin-independent) scattering off nuclei due to the exchange of Higgs bosons. Furthermore, in these two models the neutralinos typically have masses in the 20–150 GeV range, so their capture by the Earth is expected to be
enhanced when the neutralino mass closely matches that of the abundant elements in the Earth’s core (Fe) and mantle (Si and Mg), while the same effect is irrelevant for the capture by the Sun [8]. We will discuss the implications of these features in Sec. 5.

3 The Capture Rate

In order to calculate the expected rate of neutrino production due to neutralino annihilation, it is necessary to first evaluate the rates at which the neutralinos are captured in the Sun and Earth. Following the early work of Press and Spergel [3], the capture of WIMPs by a massive body was studied extensively by Gould [6]. In this paper, we make use of Gould’s formula, and follow a procedure similar to that of Refs. [10, 11] in calculating the capture rate.

From Eq. (A10) of the second paper in Ref. [4], the capture rate of a neutralino of mass $m\chi$ by the Sun or Earth can be written as

$$C = \left(\frac{2}{3\pi}\right)^{\frac{1}{2}} M_B \rho_\chi \bar{v}_\chi \sqrt{\sum_i f_i \sigma_i X_i},$$

(5)

where $M_B$ is the mass of the Sun or Earth, $\rho_\chi$ and $\bar{v}_\chi$ are the local neutralino density and rms velocity in the halo respectively, $\sigma_i$ is the elastic scattering cross-section of the neutralino with the nucleus of element $i$ with mass $m_i$, $f_i$ is the mass fraction of element $i$, and $X_i$ is a kinematic factor which accounts for several important effects: (1) the motion of the Sun or Earth relative to Galactic center; (2) the suppression due to the mismatching of $m_\chi$ and $m_i$; (3) the loss of coherence in the interaction due to the finite size of the nucleus (see Ref. [6] for details).

In the summation in Eq. (5), we only include the ten most abundant elements for the Sun or Earth respectively, and use the mass fraction $f_i$ of these elements as listed in Table A.1 of Ref. [10]. We choose $\bar{v}_\chi = 300 \text{ km sec}^{-1}$, a value within the allowed range of the characteristic velocity of halo dark matter particles. To take into account the effect of the actual neutralino relic density, we follow the conservative approach of Ref. [10] for the local neutralino density $\rho_\chi$: (a) $\rho_\chi = \rho_h = 0.3 \text{ GeV}/\text{cm}^3$, if $\Omega_\chi h_0^2 > 0.05$; while (b) $\rho_\chi = (\Omega_\chi h_0^2/0.05)\rho_h$, if $\Omega_\chi h_0^2 \lesssim 0.05$. As for $\sigma_i$, the dominant contribution is the coherent interaction due to the exchange of two CP-even Higgs bosons $h$ and $H$ and squarks, and we use the expressions (A10) and (A11) of Ref. [11] to compute the spin-independent cross section for all the elements included. In addition, for capture by the Sun, we also evaluate the spin-dependent cross section due to both $Z$-boson exchange and squark exchange for the scattering from hydrogen according to Eq. (A5) (EMC model case) of Ref. [11]. It should be noted that in all these expressions the squarks were assumed to be degenerate. In the two supergravity models that we consider here this need not be the case, although for

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2We have corrected a sign error for the $H\chi\chi$ coupling in (A10) of Ref. [11].
most of the parameter space this is a fairly good approximation. Hence, we simply use the average squark mass $m_{\tilde{q}}$ in this part of the calculation.

The kinematic factor $X_i$ in Eq. (5), can be most accurately evaluated once the detailed knowledge of the mass density profile as well as the local escape velocity profile are specified for all the elements. In practice, this can be done by performing a numerical integration with the physical inputs provided by standard solar model or some sort of Earth model. Instead of performing such an involved calculation, we approximate the integral for each element by the value of the integrand obtained with the average effective gravitational “potential energy” $\phi_i$ times the integral volume. The values of $\phi_i$ are taken from Table A.1 of Ref. [10].

4 The Detection Rate

We next describe the procedure employed by us to calculate the detection rate of upwardly-moving muons, resulting from the particle production and interaction subsequent to the capture and annihilation of neutralinos in the two supergravity models. The annihilation process normally reaches equilibrium with the capture process on a time scale much shorter than the age of the Sun or Earth. We assume this is the case, so that the neutralino annihilation rate equals half of the capture rate. The detection rate for neutrino-induced upwardly-moving muon events is then given by

$$\Gamma = \frac{C}{8\pi R^2} \sum_{i,F} D_i B_F \int \left( \frac{dN}{dE_\nu} \right)_{iF} E_{\nu}^2 dE_{\nu}. \quad (6)$$

In Eq. (6), $D_i$ is a constant, $R$ is the distance between the detector and the Sun or the center of the Earth, and $(dN/dE_\nu)_{iF}$ is the differential energy spectrum of neutrino type $i$ as it emerges at the surface of the Sun or Earth due to the annihilation of neutralinos in the core of the Sun or Earth into final state $F$ with a branching ratio $B_F$. It should be noted that in Eq. (3) that $i$ is summed over muon neutrinos and anti-neutrinos, and that $F$ is summed over final states that contribute to the high-energy neutrinos. The only relevant fermion pair final states are $\tau\bar{\tau}$, $c\bar{c}$, $b\bar{b}$, and (for the $SU(5) \times U(1)$ model) $t\bar{t}$ when $m_\chi > m_t$. The lighter fermions do not produce high-energy neutrinos since they are stopped by the solar or terrestrial media before they can decay [8].

The branching ratio $B_F$ can be easily calculated as the relative magnitude of the thermal-averaged product of annihilation cross section into final state $F$ ($\sigma_F$) with the Møller velocity $v_M$. Since the core temperatures of the Sun and Earth are very low compared with the neutralino mass ($T_{\text{Sun}} \sim 1.34 \times 10^{-6}$ GeV; $T_{\text{Earth}} \sim 4.31 \times 10^{-10}$ GeV), only the s-wave contributions are relevant, hence it is enough here to use the usual thermal average expansion up to zero-order of $T/m_\chi$ ($v_M \rightarrow 0$ limit), i.e.

$$B_F = \frac{\langle \sigma_F v_M \rangle}{\langle \sigma_{tot} v_M \rangle} = \frac{a_F}{a_{tot}}. \quad (7)$$
In Eq. (7), all kinematically allowed final states contribute to \( a_{\text{tot}} \). Besides all the fermion pair final states, we have also included boson pair final states \( WW, ZZ \) and \( hA \) in our calculation of \( B_F \). Due to the parameter space constraints, these channels are not open for the minimal \( SU(5) \) model. But \( WW \) and \( ZZ \) channels are generally open in the \( SU(5) \times U(1) \) model, and the \( hA \) channel also opens up for large values of \( \tan \beta \) in the dilaton case. We should also remark that the annihilation channel into lightest CP-even higgs pair \( hh \) is always allowed kinematically in some portion of the parameter space for both supergravity models we consider, but since its \( s \)-wave contribution vanishes, we do not include it in the calculation of \( B_F \). However, this channel is taken into account in the calculation of the neutralino relic density, which does affect the capture rate through the scaling of local density \( \rho_\chi \) when \( \Omega_\chi h_0^2 < 0.05 \) (see Sec. 3). In addition, we have kept all the nonvanishing interference terms in the evaluation of \( a_{\text{tot}} \) and \( a_F \).

The calculation of the neutrino differential energy spectrum is somewhat involved, since it requires a reasonably accurate tracking of the cascade of the particles which result from neutralino annihilation into each of the final state \( F \). This involves the decay and hadronization of the various annihilation products and their interactions with the media of the Sun or the Earth’s cores. In addition, at high energies, neutrinos interact with and are absorbed by solar matter, a fact that affects the spectrum. In Ref. [8], Ritz and Seckel rendered this calculation tractable by their adaptation of the Lund Monte Carlo for this purpose. Subsequently, analytic approximations to the Monte Carlo procedure outlined in their paper were refined and employed by Kamionkowski to calculate the neutrino energy spectra from neutralino annihilation for the MSSM in Ref. [11]. The procedure involved is described below for completeness.

Since the probability for producing an underground muon that traverses the detector is proportional to the square of the neutrino energy, the primary quantity of interest is the second moment \( \langle N z^2 \rangle m_\chi^2 \), defined as

\[
\langle N z^2 \rangle_{iF} \equiv \frac{1}{m_\chi^2} \int \left( \frac{dN}{dE_\nu} \right)_{iF} E_\nu^2 dE_\nu.
\]  (8)

Once the second moments are obtained, the detection rate for the neutrino-induced upwardly-moving muon events \( \Gamma \) may be conveniently written as

\[
\Gamma = \kappa_B \left( \frac{C}{\text{sec}^{-1}} \right) \left( \frac{m_\chi}{\text{GeV}} \right)^2 \sum_i a_i b_i \sum_F B_F \langle N z^2 \rangle_{iF} \text{ m}^{-2} \text{yr}^{-1},
\]  (9)

where \( \kappa_B = 1.27 \times 10^{-29} \) (7.11 \( \times 10^{-21} \)) for neutrinos from the Sun (Earth), the scattering coefficients \( a_i = 6.8 \) (3.1) for neutrinos (anti-neutrinos), while the muon range constants \( b_i = 0.51 \) (0.67) for neutrinos (anti-neutrinos).

The approximations of Refs. [8, 11] consist in obtaining expressions for the second moment without detailed knowledge of the functional form of the differential energy spectra. We now list these approximate expressions for \( \langle N z^2 \rangle_{iF} \) as follows:
(i) **Fermion pair final states** \((τ\bar{τ}, c\bar{c}, b\bar{b}, t\bar{t})\). The simplest case is that of fermions injected into the core of the Earth, since all interactions are negligible. We use

\[
\langle Nz^2 \rangle = \frac{1}{3} \langle N \rangle \langle y^2 \rangle \left[ \langle z_f^2 \rangle - \frac{m_f^2}{4E_{in}^2} \right].
\]

Here \(\langle N \rangle\) and \(\langle y^2 \rangle\) are the rest frame yield and second moments, while \(\langle z_f^2 \rangle\) is the second moment of the fragmentation function, obtained from Table 2 and Table 3 respectively in [8]. \(m_f\) and \(E_{in}\) are the mass and energy of the injected fermion.

Fermions injected into the core of the Sun undergo interactions and decay before final state neutrinos emerge at the surface. These are analytically approximated reliably for fermion injection energies relevant to our situation by [11]

\[
\langle Nz^2 \rangle = \left[ ax_0e^{x_0} \int_{x_0}^{\infty} \frac{e^{-x}}{x} dx \right]^2,
\]

where \(x_0 = 155/E_{in} (275/E_{in})\) and \(a = 0.056 (0.052)\) for neutrinos (anti-neutrinos) from \(c\) quarks, while \(x_0 = 185/E_{in} (275/E_{in})\) and \(a = 0.086 (0.082)\) for neutrinos (anti-neutrinos) from \(b\) quarks. For the top quark we have considered three different masses: 130, 150 and 170 GeV in the \(SU(5) \times U(1)\) model. In this case the approximations are less reliable [11], but better at energies below the 100 GeV scale than above it. The appropriate values here are \(a = 0.18 (0.14)\) and \(x_0 = 110/E_{in} (380/E_{in})\) for neutrinos (anti-neutrinos). In the above (and in what follows), \(E_{in}\) and other energies are taken in GeV when obtaining numerical values. The above expressions include the hadronization of \(b\) and \(c\) and \(t\) quarks in the solar medium. The \(τ\) lepton decays almost instantly, and does not hadronize, so must be treated differently. The second moment for the \(τ\) is approximated by

\[
\langle Nz^2 \rangle = ae^{-E_{in}/E_a}
\]

where \(a = 0.0204 (0.0223)\) and \(E_a = 476 (599)\) for neutrino (anti-neutrino) production.

(ii) **Gauge boson pair final states** \((WW, ZZ)\). The production of \(ν_μ\) and \(\bar{ν}_μ\) by \(W, Z\) boson pair and Higgs boson pair is related to that by fermions, since the bosons primarily decay into fermions, hence the subsequent cascade and hadronization remains the same. For the annihilation final state \(WW\), the \(W\) boson with velocity \(β\) and energy \(E\) will decay into \(μν_μ\) with a fractional width \(Γ(W \rightarrow μ\bar{ν}_μ)\). The second moment for this annihilation final state for the neutrinos from the Sun is [11]

\[
\langle Nz^2 \rangle = \frac{Γ(W \rightarrow μ\bar{ν}_μ) 2 + 2E_{in}τ_i (1 + α_i) + E_{in}^2τ_i^2α_i(1 + α_i)}{βE^3 \alpha_iτ_i^2(α_i^2 - 1)(1 + E_{in}τ_i)^{1+α_i}} \bigg|_{E_{in}=E(1-β)/2} \bigg|_{E_{in}=E(1+β)/2}.
\]

Here \(τ_i = 1.01 \times 10^{-3}\) GeV\(^{-1}\) (3.8 \times 10^{-4}\) GeV\(^{-1}\), \(α_i = 5.1 (9.0)\) for neutrinos (anti-neutrinos). For the neutrinos from the Earth, the second moments is approximated by a simpler expression

\[
\langle Nz^2 \rangle = Γ(W \rightarrow μ\bar{ν}_μ)(3 + β^2)/12
\]
due to the absence of interactions. The expressions for the \( ZZ \) final state can be similarly obtained \([11]\).

(iii) Higgs boson pair final state \((hA)\). For the annihilation final state \(hA\), each Higgs boson \(S = h, A\) with velocity \(\beta_S\) and energy \(E_S\) will decay into fermion pair \(ff\) with a branching ratio \(BR(S \rightarrow ff)\). Assuming such decays are isotropic, the second moment of this final state is

\[
\langle N_z^2 \rangle = \sum_{S=h,A} \sum_f \frac{2BR(S \rightarrow ff)}{\beta_S E_S} \int_{E_S(1-\beta_S)/2}^{E_S(1+\beta_S)/2} \langle N_z^2 \rangle_f dE_f. \tag{15}
\]

Here \(E_f\) is the fermion energy, and \(\langle N_z^2 \rangle_f\) is the second moment for the fermion pair \(ff\) \((\tau\bar{\tau}, c\bar{c}, b\bar{b}, t\bar{t})\) as given in Eqs. \((10)-(12)\).

5 Results and Discussion

For each point in the parameter spaces of the two models described in Sec. 2, namely the minimal \(SU(5)\) and the \(SU(5) \times U(1)\) supergravity models (both the no-scale and dilaton cases), we have determined the relic abundance of neutralinos and then computed the capture rate in the Sun and Earth (as described in Sec. 3) and the resulting upwardly-moving muon detection rate (as described in Sec. 4). In Figs. 1 and 2, the predicted capture and detection rates in the minimal \(SU(5)\) supergravity model are shown, based on the assumption that the mass of the triplet higgsino, which mediates dimension-five proton decay, obeys \(M_{\tilde{H}_3} < 3M_U\). We have redone the calculation relaxing this assumption to \(M_{\tilde{H}_3} < 10M_U\), in which case the results for the muon fluxes remain qualitatively the same, except that the parameter space is opened up somewhat. The dashed lines in Fig. 2 represent the current Kamiokande 90% C.L. upper limits \((1)\) and \((2)\). Similarly, the predictions of the \(SU(5) \times U(1)\) supergravity model are presented in Figs. 3 and 4 for the no-scale scenario, and in Figs. 5 and 6 for the dilaton scenario, again along with the Kamiokande upper limits (dashed lines). In Figs. 3–6, we have taken the representative value of \(m_t = 150\) GeV. Similar results are obtained for other values of \(m_t\).

Several comments on these figures are in order. First, the kinematic enhancement of the capture rate by the Earth manifests itself in all figures as the big peaks near the Fe mass \((m_{Fe} = 52.0\) GeV), as well as the smaller peaks around Si mass \((m_{Si} = 26.2\) GeV). Second, there is a severe depletion of the rates near \(m_\chi = \frac{1}{2}M_Z\) in Figs. 3–6, which is due to the decrease in the neutralino relic density. In the case of Earth capture, this effect is largely compensated by the enhancement near the Fe mass. As mentioned in Sec. 3, in our procedure, the relic density affects the local neutralino density \(\rho_\chi\) only if \(\Omega_\chi h_0^2 < 0.05\), while in the minimal \(SU(5)\) model this almost never happens, therefore, the effect of the Z-pole is not very evident in Figs. 1 and 2. Also, in Figs. 3–6, the various dotted curves correspond to different values of \(\tan \beta\), starting from the bottom curve with \(\tan \beta = 2\), and increasing in steps of two. These curves clearly show that the capture and detection rates increase with increasing \(\tan \beta\), since
the dominant piece of the coherent neutralino-nucleon scattering cross section via the exchange of the lightest Higgs boson \( h \) is proportional to \((1+\tan^2\beta)\). The capture rate decreases with increasing \( m_\chi \), since the scattering cross section falls off as \( m_h^{-4} \) and \( m_h \) increases with increasing \( m_\chi \). It is expected that the detection rate in general also decreases for large value of \( m_\chi \), since it is proportional to the capture rate. However, the opening of new annihilation channels, such as the \( WW \), \( ZZ \) and \( hA \) channels in the \( SU(5) \times U(1) \) model, could have two compensating effects on the detection rate: (a) the presence of a new channel to produce high-energy neutrinos which leads to an enhancement of the detection rate; and (b) the decrease of the branching ratios for the fermion pair channels, which makes the neutrino yield from \( \tau\bar{\tau} \), \( c\bar{c} \) and \( b\bar{b} \) smaller and hence reduces the detection rate. Therefore, these new annihilation channels could either increase or decrease the detection rate, depending which of these two effects wins over. We found that, for small values of \( \tan\beta \) and \( \mu < 0 \), the \( WW \) channel can become dominant if open, basically because in this case the neutralino contains a rather large neutral wino component. This explains the distortion of the detection rate curves in the \( \mu < 0 \) half of Figs. 4 and 6. The effect of the \( ZZ \) channel turns out to be negligible in the \( SU(5) \times U(1) \) model, since neutralinos with \( m_\chi > M_Z \) have very small higgsino components. The same argument applies to the \( hA \) channel which sometimes opens up in the dilaton case for rather large values of \( \tan\beta \).

In the dilaton case, for large values of \( \tan\beta \) the CP-odd Higgs boson \( A \) can be rather light, and the presence of the \( A \)-pole when \( m_\chi \sim \frac{1}{2}m_A \) makes the relic density very small. \( \Omega_\chi h_0^2 \) as a function of \( m_\chi \), is first lower than 0.05, it increases with \( m_\chi \), and eventually reaches values above 0.05, when neutralinos move away from the \( A \)-pole. Thus, the capture and detection rates also show this behavior, which can be seen as the few “anomalous” lines in Figs. 3 and 4. For lower values of \( \tan\beta \), \( \Omega_\chi h_0^2 < 0.05 \), and there is no such effect. In the minimal \( SU(5) \) model, since the allowed points include different supersymmetry breaking scenarios and several values of \( m_t \) and \( \tan\beta \), all these features are blurred. Nonetheless, in the same range of \( m_\chi \) and for same values of \( \tan\beta \) and \( m_t \), we have found the results of these two models comparable, with the rates in the \( SU(5) \times U(1) \) model slightly smaller due to the smaller relic density.

It is clear that at present the experimental constraints from the “neutrino telescopes” on the parameter space of the two supergravity models are quite weak. In fact, only the Kamiokande upper bound from the Earth can be used to exclude regions of the parameter space with \( m_\chi \approx m_{\text{Fe}} \) for both models, in particular for the \( SU(5) \times U(1) \) model, due to the enhancement effect discussed above. However, it is our belief that the results presented in this paper will be quite useful in the future, when improved sensitivity in underground muon detection rates become available. An improvement in experimental sensitivity by a factor of two should be easily possible when MACRO \([17]\) goes into operation, while a ten-fold improvement is envisaged when Super-Kamiokande \([18]\) announces its results sometime by the end of the decade. More dramatic improvements in the sensitivity (by a factor of 20–100) may be expected from DUMAND and AMANDA \([19]\), currently under construction. In addition, as recently argued \([35]\), we think that perhaps the full parameter space
of a large class of supergravity models, including the two specific ones considered here, may only be convincingly probed by a detector with an effective area of 1 km². It is interesting to note that, with a sensitivity improvement by a factor of 100, a large portion of the $\mu < 0$ half parameter space of the minimal SU(5) model can be probed. Unfortunately, the remaining portion, with fluxes below $\sim 10^{-4}$, can hardly be explored by underground experiments in the foreseeable future. For the $SU(5) \times U(1)$ supergravity models, in Figs. 7 and 8 we have plotted the allowed points in the $(m_{\chi^\pm_1}, \tan \beta)$ space. These points are those obtained originally in Refs. [29, 31], such that the neutrino telescope constraint is also satisfied; no other constraints have been imposed. It can be seen here that the small voids of points for $m_{\chi^\pm_1} \approx 100$ GeV and a variety of values of $\tan \beta$ are excluded by the constraint from the “neutrino telescopes”. In these figures we have marked by crosses the points in the $(m_{\chi^\pm_1}, \tan \beta)$ plane that MACRO should be able to probe (assuming an increase in the sensitivity by a factor of two) for the no-scale and dilaton scenarios respectively. Finally, Figs. 8 and 10 show that, using this indirect technique, Super-Kamiokande can cover nearly half of the parameter space of the $SU(5) \times U(1)$ model, assuming that an improvement by a factor of about twelve can be achieved. As expected, the constraints from future “neutrino telescopes” will be strictest for large values of $\tan \beta$.

To keep the above results in perspective, we now discuss some effects that have not been taken into account in our analysis. As we have shown, for the two supergravity models, the capture of the neutralinos by the Earth is more important than by the Sun. In the Earth case, we have considered only the primary direct capture of the neutralinos from the galactic halo, in which a neutralino is trapped by the Earth’s gravitational field only when its velocity falls below the escape velocity at a point inside the Earth, as a consequence of its interaction with the nuclei around. However, there exists yet another mechanism by which neutralinos can be captured by the Earth, namely, the secondary indirect capture, first studied by Gould [6]. In this mechanism, a neutralino first loses only enough energy to be bound in a solar orbit (“orbit capture”), but not enough to be directly captured by the Earth. The orbit-captured neutralinos further weakly interact with the nuclei in the Earth, and a fraction of them subsequently can be indirectly captured into the Earth’s core. It was found by Gould that, for Dirac neutrinos of mass 10–80 GeV, direct and indirect capture by the Earth are of the same order of magnitude, and the kinematic enhancement of the total capture rate becomes enlarged and broadened [4]. We believe that this result should also apply to the case of neutralino capture, which means that the Earth capture rate that we have computed in this work may have been significantly underestimated. Although to our knowledge the indirect capture was also not considered in recent works that dealt with Earth capture [11, 12, 13, 14], we feel that this important mechanism should be taken into account in future analyses of this type. In fact, from Figs. 7 and 8, we see that even before MACRO, interesting constraints may already be extracted from the current Kamiokande upper limits, had we included the indirect capture in our calculation.

Recently, after the calculations in this paper had been completed, we received
Refs. [36, 37] in which some other issues of particle physics relevant to our work have been addressed. Given the fact that the neutralino in the two supergravity models we study is mostly a “mixed” state, in particular for $m_\chi \sim m_{Fe}$, according to Fig. 7 of Ref. [36], we expect that the results of Ref. [36], when they become appreciable, could lead to a shift of our detection rates in both directions by about 10% or less. We have not redone our calculations of the capture rate using the cross section of Ref. [37]. The inclusion of the new neutralino-gluon scatterings [37], when they become appreciable, would increase the chance that neutralinos can be captured by both the Sun and the Earth, barring the normally small interference effects which sometimes could render the total cross section of Ref. [37] smaller than what it would be without these new scatterings [38]. Therefore, the effect of the results of Ref. [37] on the detection rate, could somehow enhance or compensate that of Ref. [36], depending whether the latter is an increase or decrease of the detection rate. However, even as a conservative estimate, we do not expect that the combined effect of Refs. [36, 37] would alter our results quantitatively by more than 20%, a change not large enough to affect our conclusions qualitatively.

We have also looked into the possibility that the high energy neutrino flux may be altered by MSW oscillations [39] in the Sun. This issue has been addressed in a recent paper [40]. From their results we conclude that for the neutrino energy range relevant to this work, $\nu_\mu$ to $\nu_e$ oscillations are negligible. The $\nu_\mu$ flux may also be altered by $\nu_\mu$ to $\nu_\tau$ vacuum oscillations during passage from the Sun to the Earth. This effect will be significant only if the $\nu_\mu$-$\nu_\tau$ mixing angle is large. Large mixing is disfavored by the general GUT based see-saw arguments [41]. An analysis based on these considerations coupled with phenomenological arguments in the context of the $SU(5) \times U(1)$ model [42] supports a value for $\sin^2 2\theta \approx 10^{-4}$, which is much too small to affect the flux values predicted here.

6 Conclusions

We have explored the possibility of detecting supersymmetry indirectly through the measurement of upwardly-moving muons in underground detectors or “neutrino telescopes”. These muons originate from high-energy neutrino interactions in the rock below the detector, and these neutrinos result from annihilation of neutralinos in the Earth or Sun cores. The latter would have been captured by these heavenly bodies if the neutralinos constitute an important part of the galactic halo—an important assumption which should not be overlooked.

The present day experimental upper bounds on the muon flux are only weakly constraining. In fact, this is mostly because the large possible fluxes for light neutralinos (see for example Fig. 1) for small $m_\chi$ have already been ruled out by the LEP lower bounds on the neutralino mass. Nonetheless, there is a region of parameter space with $m_\chi \approx 100$ GeV (corresponding to $m_\chi \approx m_{Fe}$) and a range of values of $\tan \beta$, which is excluded at the 90% C.L. However, expected increases in experimental sensitivity in the next few years should turn this technique into a very efficient way
of probing the parameter space of the specific supergravity models considered here, even surpassing the reach of traditional direct detection collider experiments.

We should remark that even though our explicit computations apply only to the $SU(5)$ and $SU(5) \times U(1)$ models, the correlations among sparticle and Higgs boson masses, which play such a fundamental role in the quantitative results, are common to a large class of supergravity models with radiative electroweak symmetry breaking [15]. In this respect, the specific models considered here include values of $\xi_0 = m_0/m_{1/2}$ which are small ($\xi_0 = 0$, no-scale), moderate ($\xi_0 = 1/\sqrt{3}$, dilaton), and large ($\xi_0 \gg 1$, minimal $SU(5)$), and therefore span the whole allowed range. Thus, our results: (i) represent a significant sampling of what would be obtained using an arbitrary selection of parameters in a generic supergravity model, and (ii) will allow to select the correct supergravity model when experimental data start to restrict the parameter space.

Acknowledgments

This work has been supported in part by DOE grant DE-FG05-91-ER-40633. The work of R.G. and K.Y. has been supported by a World-Laboratory Fellowship. The work of J.L. has been supported by an SSC Fellowship.
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Figure Captions

Figure 1: The neutralino capture rate for the Sun and Earth as a function of the neutralino mass in the minimal $SU(5)$ supergravity model.

Figure 2: The upwardly-moving muon flux in underground detectors originating from neutralino annihilation in the Sun and Earth, as a function of the neutralino mass in the minimal $SU(5)$ supergravity model. The dashed lines represent the present Kamiokande 90% C.L. experimental upper limits.

Figure 3: The neutralino capture rate for the Sun and Earth as a function of the neutralino mass in the no-scale $SU(5) \times U(1)$ supergravity model. The representative value of $m_t = 150$ GeV has been used. Note the depletion of neutralinos in the halo near the $Z$-resonance, and the enhancement in the Earth capture rate near the iron nucleus mass (52.0 GeV).

Figure 4: The upwardly-moving muon flux in underground detectors originating from neutralino annihilation in the Sun and Earth, as a function of the neutralino mass in the no-scale $SU(5) \times U(1)$ supergravity model. The representative value of $m_t = 150$ GeV has been used. The dashed lines represent the present Kamiokande 90% C.L. experimental upper limits.

Figure 5: Same as Fig. 3 but for the dilaton $SU(5) \times U(1)$ supergravity model.

Figure 6: Same as Fig. 4 but for the dilaton $SU(5) \times U(1)$ supergravity model.

Figure 7: The allowed parameter space of the no-scale $SU(5) \times U(1)$ supergravity model (in the $(m_{\chi^\pm}, \tan \beta)$ plane) after the present “neutrino telescopes” (NT) constraint has been applied. Two values of $m_t$ (130, 150 GeV) have been chosen. The crosses denote those points which could be probed with an increase in sensitivity by a factor of two.

Figure 8: Same as Fig. 7 but the crosses now denote points which could be probed with an increase in sensitivity by a factor of 12.

Figure 9: Same as Fig. 7 but for the dilaton $SU(5) \times U(1)$ supergravity model.

Figure 10: Same as Fig. 8 but for the dilaton $SU(5) \times U(1)$ supergravity model.