Problem of the complete measurement for $CP$—violating parameters in neutral $B$—meson decays

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Abstract

Phenomenological $CP$-violating parameters in decays of neutral $B$-mesons are discussed. Special attention is given to the degree of their measurability. We emphasize important role of the sign of $\Delta m_B$ and suggest how it could be determined experimentally.

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1 Introduction

Two opinions may be considered now as generally accepted (see, e.g., reviews [1, 2]):

1. The origin of $CP$–violation will not be established till its manifestations are known only for neutral kaons.

2. The most promising test-ground for detailed studies of $CP$–violation is given by decays of neutral $B$–mesons.

As a result, much work was devoted to discussion of $B$–meson decay modes favourable for $CP$–violation searches and to experimental manifestations of possible sources of the violation (see, e.g., references in [1, 2]). A more straightforward problem, degree of measurability of phenomenological parameters describing $CP$–violation in $B$–meson decays, has not been considered. One possible reason could be a close similarity of neutral $B$–mesons to neutral kaons. But heavier masses of the third quark generation produce various differences, sometimes rather essential, in the meson decay properties. Therefore, in the present paper we reanalyse basic $CP$–violating parameters for $B$–mesons. Special attention is given to the question how one could achieve their complete measurement.

The presentation goes as follows. In Section 2 we discuss various parameters that are familiar to express $CP$–violation. Of special interest is degree of their rephasing (in)dependence. Section 3 considers how the parameters appear in experimental lifetime distributions. The role of width and mass differences for measurability of phenomenological parameters is emphasized. This consideration is continued in Section 4 where we also suggest how one could measure the sign of $\Delta m_B$ providing the basis for the complete measurement of all $CP$–violating parameters. Summary of our results and their short discussion are given in the last Section.

2 $CP$–violating parameters

As is well-known the time evolution of neutral $B$–mesons is determined by two states

$$B_\pm = \frac{1}{\sqrt{2(1+|\varepsilon_B|^2)}} \left[(1+\varepsilon_B)B^0 \pm (1-\varepsilon_B)\overline{B}^0\right].$$ (1)

If we use the phase convention

$$\overline{B}^0 = (CP)B^0,$$ (2)

the exact $CP$–conservation would lead to $\varepsilon_B = 0$ and the states $B_\pm$ having the definite $CP$–parity equal $\pm 1$. Generally, they are eigen-states of the effective (non-Hermitian) Hamiltonian. But possibility of rephasing $\overline{B}^0$ (with appearance of a phase-factor in relation (2) but without changing $B_+$ and $B_-$) means that $\varepsilon_B$ itself can not be measurable.
Only \(\left|\frac{1+\varepsilon_B}{1-\varepsilon_B}\right|\) is rephasing-invariant and admits measurement. The value

\[
\delta_B = \frac{|1 + \varepsilon_B|^2 - |1 - \varepsilon_B|^2}{|1 + \varepsilon_B|^2 + |1 - \varepsilon_B|^2} = \frac{2\text{Re}\varepsilon_B}{1 + |\varepsilon_B|^2}
\]

may be considered as the measure of \(CP\)-violation in \(B\bar{B}\) mixing.

Similar quantity for neutral kaons, \(\delta_K\), describes the charge asymmetry in semileptonic decays of \(K_L\) (or in decays of pure \(K_S\) if it could be separated). The same would be true for \(B\)-mesons as well. But the real separation of \(B_L\) and \(B_S\) is hardly possible because of expected smallness of \((\Gamma_S - \Gamma_L)_B\). Therefore, the problem arises how to find ways of physical identification for the states \(B_{\pm}\) in the absence of \(CP\)-conservation.

Mixing of \(B^0\) and \(\bar{B}^0\) is mainly determined by the very heavy intermediate state \(t\bar{t}\). The corresponding transition amplitude contains quarks from two generations only. Since \(CP\)-violation (by the standard CKM-mechanism) requires participation of all three quark generations, the value of \(\delta_B\) has an additional kinematic suppression by the factor \(m_c^2/m_t^2\) (more detailed discussion see, e.g., in [3]). Thus, contrary to neutral kaons, there are no hopes to find experimentally nonvanishing \(\delta_B\) in near future.

More promising are studies of decays

\[
B^0(\bar{B}^0) \rightarrow f
\]

with final states \(f\) having definite \(CP\)-parity [3,4]. As the measure of \(CP\)-violation for a particular decay one may use deviation of the parameter

\[
\lambda_B^{(f)} = \frac{1 - \varepsilon_B}{1 + \varepsilon_B} \cdot \frac{\langle f|\bar{B}^0\rangle}{\langle f|B^0\rangle}
\]

from the \(CP\)-parity value of the state \(f\). Any \(\lambda_B^{(f)}\) is rephasing-invariant and, hence, its complete measurement (i.e., of both the absolute value and phase) should be possible.

Now we can restrict ourselves to considering such independent states where final state interaction produces only elastic rescattering (really we mean states that diagonalize \(S\)-matrix of strong interactions; analogues in neutral kaon decays are, e.g., two-pion states with definite isospin values). Any other final state can be expanded as series over the independent ones.

Assumption of \(CPT\)-invariance leads to the conclusion that the ratio of amplitudes in expression (5) for an independent final state is a phase factor. So we have

\[
|\lambda_B^{(f)}| = \left|\frac{1 - \varepsilon_B}{1 + \varepsilon_B}\right|
\]

for every independent state \(f\).

Thus, the parameter \(\delta_B\) generated by the \(CP\)-violation in mixing appears to be universal and determines deviation of any \(|\lambda_B^{(f)}|\) from unity. Only one additional \(CP\)-violating
parameter, arg($\lambda_B^{(f)}$), may arise for each particular independent final state $f$ in the neutral $B$—decays. They are just the parameters that phenomenologically correspond to direct $CP$—violation in the particular decay mode.

Traditional $CP$—violating parameters $\eta$, similar to ones used for kaon decays, are simply related to $\lambda$’s. For $CP$—even states $f+$:

$$\eta_B^{(f+)} = \frac{1 - \lambda_B^{(f+)}}{1 + \lambda_B^{(f+)}};$$  \hspace{1cm} (7)

for $CP$—odd state $f-$:

$$\eta_B^{(f-)} = \frac{1 + \lambda_B^{(f-)}}{1 - \lambda_B^{(f-)}}.$$  \hspace{1cm} (8)

Comparing eqs.(5),(7),(8) shows that one is always able to find an appropriate phase convention for $B$ and $\bar{B}$ which changes $\varepsilon_B$ so to equate

$$\varepsilon_B = \eta_B^{(f)}$$  \hspace{1cm} (9)

for any chosen independent state $f$. Independently of any phase convention we have

$$\delta_B = \frac{2\text{Re} \eta_B^{(f)}}{1 + |\eta_B^{(f)}|^2}.$$  \hspace{1cm} (10)

Note that all the above relations, including eq.(10), are true for neutral kaons as well. This leads to new experimental predictions based on the $CPT$—invariance. E.g., $CP$—violating parameters in decays $K^0(\overline{K}^0) \to 2\pi$ and $K^0(\overline{K}^0) \to 3\pi$ should satisfy equality

$$\frac{\text{Re} \eta_K^{(2\pi)}}{1 + |\eta_K^{(2\pi)}|^2} = \frac{\text{Re} \eta_K^{(3\pi)}}{1 + |\eta_K^{(3\pi)}|^2} = \frac{1}{2} \delta_K$$  \hspace{1cm} (11)

(we assume here both $2\pi$ and $3\pi$ states to have a definite isotopic structure). Thus, measurement of the corresponding $|\eta|^2$ immediately leads to determination of arg $\eta$ (double-valued, up to the sign of Im$\eta$). Eq.(11) should be applicable also to the decay $K^0(\overline{K}^0) \to \pi^+\pi^-\gamma$ where $CP$—violation has been observed experimentally [5]. It does really work within available precision.

Specific feature of neutral $B$—mesons, having no analogues for neutral kaons, is the presence of decays

$$B^0(\overline{B}^0) \to fK^0(\overline{K}^0),$$  \hspace{1cm} (12)

with $f$, again, being definite $CP$—parity states. They are induced by the quark decay $b \to c\bar{s}s$. The most popular final state of such a kind is $J/\psi K^0(\overline{K}^0)$. Unique property of decays (12) is the coherence of neutral $B$ and neutral $K$ evolutions [3].
Decays (12) generate new set of parameters:

\[
\lambda^{(f)}_{BK} = \frac{1 - \varepsilon_B}{1 + \varepsilon_B} \cdot \frac{1 + \varepsilon_K}{1 - \varepsilon_K} \cdot \frac{\langle fK^0|B^0 \rangle}{\langle fK^0|B^0 \rangle} \tag{13}
\]

similar to (5). They are invariant under rephasing of both \( B \) and \( K \) mesons and should also be completely measurable. If the final system may be considered as independent (in the above sense) then the ratio of amplitudes in eq.(13) is again a phase factor and deviation of \(|\lambda^{(f)}_{BK}|\) from unity becomes universal. But it is influenced, differently from \(|\lambda^{(f)}_B|\), by the \( CP \)–violation in both \( B \)– and \( K \)–mixing.

3 Experimental manifestation and measurability

To suggest ways for the complete measurement of the parameters \( \lambda \) we should first consider how they reveal themselves experimentally. The problem of measurement for \( \delta_B \) looks quite clear and we will not discuss it here anymore. Situation is not so clear for parameters \( \lambda \).

Standard calculations for the decay (4) in the case of the initially pure \( B^0 \)–meson lead to the time distribution

\[
W^{(f)}_{B^0}(t) \sim \left| \frac{1 + \lambda^{(f)}_B}{2} \right|^2 \exp(-\Gamma_+ t) + \left| \frac{1 - \lambda^{(f)}_B}{2} \right|^2 \exp(-\Gamma_- t) + \exp\left(-\frac{\Gamma_+ + \Gamma_-}{2} t\right) \left(1 - |\lambda^{(f)}_B|^2\right) \cos \Delta m_B t - \Im \lambda^{(f)}_B \cdot \sin \Delta m_B t \right).
\]

Here \( \Delta m_B = m_+ - m_\mp; m_+, \Gamma_+ \) and \( m_-, \Gamma_- \) are the mass and width of the corresponding state \( B_+ \) or \( B_- \). To obtain the distribution for the initially pure \( B^0 \)–meson one should change \( \lambda \rightarrow 1/\lambda \). Eq.(14) has the same structure as, e.g., distribution of decays \( K^0(t) \rightarrow \pi \pi \). The first two terms are contributions of states \( B_\pm \), the last two terms describe their interference.

Distribution (14) contains contributions of \( |\lambda^{(f)}_B|^2 \), \( \Re \lambda^{(f)}_B \) and \( \Im \lambda^{(f)}_B \) multiplied by different functions of time. So, at first sight the three quantities can all be easily extracted if the distribution is experimentally measured with sufficient accuracy.

However \( \Re \lambda^{(f)}_B \) does not appear explicitly in distribution (14) if \( \Gamma_+ \) and \( \Gamma_- \) coincide. Thus, a very small expected difference of \( \Gamma_+ \) and \( \Gamma_- \), contrary to neutral kaons, may prevent direct measurement of \( \Re \lambda^{(f)}_B \). The situation for \( \Im \lambda^{(f)}_B \) is not so simple as well. In eq.(14) it is multiplied by \( \sin \Delta m_B t \), which sign is still unknown since only \( \Delta m_B \) has been measured.

Therefore, distribution (14) suggests straightforward measurement for \( |\lambda^{(f)}_B| \) and \( |\Im \lambda^{(f)}_B| \). The sign of \( \Im \lambda^{(f)}_B \) can be measured only in respect to the sign of \( \Delta m_B \).

Surely, even if \( \Re \lambda^{(f)}_B \) can not be directly measured one is able to calculate \( |\Re \lambda^{(f)}_B| \) from \( |\lambda^{(f)}_B| \) and \( |\Im \lambda^{(f)}_B| \). After that the only unknown pieces of information on \( \lambda^{(f)}_B \) are the signs of \( \Re \lambda^{(f)}_B \) and \( \Im \lambda^{(f)}_B \). Let us discuss them in more details.
Note, first of all, that definition (1) can not be used for unambiguous determination of the states $B_{\pm}$. Indeed, rephasing may even interchange the two expressions. So we need some physical identification for the states. For neutral kaons it was easily done due to large difference of lifetimes of two neutral kaon states (i.e., of $K_L$ and $K_S$). But separation of two states by their lifetimes does not show by itself which of the states $K_L$ and $K_L$ (or $B_S$ and $B_L$) corresponds to, e.g., $K_+$ (or $B_+$ respectively) in the sense of eq.(1). For this purpose one should accurately study particular decays (for the kaon case they are pion decays).

To identify the states $B_{\pm}$ let us first introduce amplitudes $a^{(f)}_{\pm}$ for decays $B_{\pm} \rightarrow f$. Then we may rewrite eq.(5) as

$$\lambda_B^{(f)} = \frac{a^{(f)}_+ - a^{(f)}_-}{a^{(f)}_+ + a^{(f)}_-},$$

and

$$\text{Re} \lambda_B^{(f)} = \frac{|a^{(f)}_+|^2 - |a^{(f)}_-|^2}{|a^{(f)}_+ + a^{(f)}_-|^2}; \quad \text{Im} \lambda_B^{(f)} = 2 \frac{\text{Im}(a^{(f)}_+ a^{(f)*}_-)}{|a^{(f)}_+ + a^{(f)}_-|^2}. \quad (16)$$

Consider, for definiteness, a $CP$–even final state $f^+$. If $CP$ were conserved it would be natural to define $B_{\pm}$ as the states with $CP$–parity $\pm 1$. Then $a^{(f^+)}_+ = 0$ and $\lambda^{(f^{+})}_B = +1$ (for $CP$–odd states $f^-$ we would have $a^{(f^-)}_+ = 0$ and $\lambda^{(f^-)}_B = -1$).

In the $CP$–violation case one can not use the $CP$–parity to identify the states $B_{\pm}$. But assuming smallness of the violation we can define $B_+(B_-)$ as being approximately $CP$–even ($CP$–odd). It means, by definition, that

$$|a^{(f^+)}_+| > |a^{(f^+)}_-|, \quad |a^{(f^-)}_+| > |a^{(f^-)}_-|.$$ \quad (17)

Surely, such a case of approximate $CP$–conservation leads to the same sign(Re $\lambda_B^{(f)}$) as in the exact $CP$–conservation case.

Now, without any preliminary assumptions, we can choose one particular final state $f$ (with a definite $CP$–parity) and identify states $B_{\pm}$ by the corresponding inequality (17). In other words, we ascribe the $CP$–parity of the particular final state $f$ to the approximate $CP$–parity for that of two states $B_{\pm}$ which has larger partial width for the decay to $f$.

If the $CP$–violation is small indeed then inequalities (17) for all other $f$’s are satisfied as well. However, if the $CP$–violation is really intrinsically large then after fixing the states $B_{\pm}$ and $B_-$ some of inequalities (17) might degenerate to equalities or even reverse their signs (in other words, various decay channels might ascribe different values of approximate $CP$–parity to the same particular state of the pair $B_{\pm}$). The latter case can be tested experimentally by comparing signs of various $\text{Re} \lambda_B^{(f)}$ determined from time dependencies (14) for various final states. It is possible only if the experiment is exact enough to discriminate $\Gamma_+$ and $\Gamma_-$. Surely, such an experiment would also allow one to identify two neutral $B$–meson states by their lifetimes as $B_L$ and $B_S$, just similar to neutral kaons.

In the absence of such possibility we assume that all inequalities (17) are correct simultaneously and, thus, sign($\text{Re} \lambda_B^{(f)}$) is known for any state $f$ being the same as if $CP$ were
conserved. After that to make $\lambda^{(f)}_B$ completely measured one needs to find $\text{sign}(\text{Im} \lambda^{(f)}_B)$ as well.

Note, first of all, that this sign may be definite only for a definite choice of the states $B_\pm$. Indeed, according to eq.(16) their interchange reverses $\text{sign}(\text{Im} \lambda^{(f)}_B)$. But even if we identified the states in one way or another we can not fix the sign by some convention similar to that suggested above for the sign of $\text{Re} \lambda^{(f)}_B$. The reason is that in the limit of $CP$-conservation $\text{Re} \lambda^{(f)}_B$ tends to the definite finite limit, while $\text{Im} \lambda^{(f)}_B$ vanishes. As a result, the sign of $\text{Re} \lambda^{(f)}_B$ is ”kinematic” at not very large $CP$—violation, while the sign of $\text{Im} \lambda^{(f)}_B$ is ”dynamic” at any degree of the violation.

Thus, we see that the complete measurement of parameters $\lambda_B$ for decays (4) requires to determine $\text{sign}(\text{Im} \lambda_B)$ from experiment, i.e. from the corresponding distribution (14). However, such distributions can only relate signs of $\text{Im} \lambda^{(f)}_B$ and $\Delta m_B$, but cannot measure them separately. Therefore, the complete measurement of parameters of direct $CP$—violation is possible only if one knows $\text{sign}(\Delta m_B)$.

The situation is the same for neutral kaons where $\text{sign} (\text{arg} \eta_K)$ can be measured only in respect to $\text{sign}(\Delta m_K)$. Essential difference between kaons and $B$—mesons is much longer lifetime of kaons (even for $K_S$) which gave possibility to measure $\text{sign}(\Delta m_K)$ in complicated regeneration experiments. Similar experiments for $B$—mesons are impossible, and experiments on decays (4) or flavor-tagged decays (including semileptonic ones) are insensitive to the absolute sign of $\Delta m_B$ (just as corresponding decays of neutral kaons).

Thus, neutral $B$—meson decays (4) are able to demonstrate manifestations of direct $CP$—violation. But they can provide the complete measurement for the corresponding $CP$—violating parameters only if $\text{sign}(\Delta m_B)$ is known from some different experiments.

4 Measurability of the sign of $\Delta m_B$

In the preceding Section we discussed only parameters $\lambda_B$ for decays (4). Parameters $\lambda_{BK}$ for decays (12) studied in [6] have similar properties. In particular, signs of various $\text{Re} \lambda_{BK}$ may be used for identifying states $B_\pm$ and testing intrinsic smallness of $CP$—violation by inequalities similar to (17). On the other side, signs of $\text{Im} \lambda_{BK}$ can not be fixed by any convention for the choice of states and should be determined from experiment. For more detailed discussion on these parameters see [6].

Time distributions in decays (12) are more complicated than distributions (14) in decays (4). They were also studied in [8]. Here we will not describe them in detail but summarize two essential points:

- The neutral kaon produced in the decay (12) can be observed only after its own decay by the decay products. As a result, coherence of $B^0(\bar{B}^0)$ and $K^0(\bar{K}^0)$ evolutions relates the primary decay (of neutral $B$) and the secondary one (of neutral $K$) to
each other. Distribution in primary $t_1$ and secondary $t_2$ lifetimes appears, generally, non-factorizable and depends on the secondary decay mode.

E.g., distribution in $t_1$ at $t_2 \to 0$ for kaon semileptonic decays has the same form as for direct semileptonic decays of neutral $B$–mesons (though with different normalization). Two-pion kaon decay in the same limit $t_2 \to 0$ produces $t_1$–dependence of the form (14) with substitution

$$\lambda_B \to \lambda_{BK} \lambda_K^{\pi\pi}, \quad \lambda_K^{\pi\pi} = \frac{1 - \eta^{\pi\pi}_K}{1 + \eta^{\pi\pi}_K}.$$  \hfill (18)

The opposite extreme case $t_2 \to \infty$ restores factorization since only $K_L$ survives in the limit. The corresponding $t_1$–distribution, independent of kaon decay modes, is given by eq.(14) with

$$\lambda_B \to -\lambda_{BK}.$$  \hfill (19)

• What is most interesting for purposes of the present paper, interference of $K_L$ and $K_S$ in the intermediate region of $t_2$ together with interference of $B_+ and B_-$ produces time distributions sensitive to the sign of $\Delta m_B$ relative to known signs of kaon parameters. This sensitivity survives even after integration over $t_1$.

Therefore, decays (12) allow one not only to search for $CP$–violation but also to determine experimentally $\text{sign}(\Delta m_B)$ and, thus, provide a necessary basis for the complete measurement of all parameters of the direct $CP$–violation in neutral $B$–meson decays. It can be done in various ways. For example, one can fix both $t_1$ and $t_2$ lifetimes and investigate their correlations in observed time distributions. Alternatively, one may not select definite $t_1$ and study only time distribution of secondary kaon decays integrated over $t_1$. Corresponding general expressions for both approaches are given in [6].

As an illustration let us consider here the sequence of decays

$$B^0(\bar{B}^0) \to J/\psi K^0(\bar{K}^0), \quad J/\psi \to \ell^+\ell^-, \quad K^0(\bar{K}^0) \to \pi^+\pi^-,$$  \hfill (20)

which has clear experimental manifestation. Using experimental branching ratios for $B^0 \to J/\psi K^0$ [7], $J/\psi \to e^+e^-$, $\mu^+\mu^-$ [8], $K_S \to \pi^+\pi^-$ [8] and the factor 1/2 for $K^0 \to K_S$ we find the effective branching ratio for events (20) to be equal

$$(Br)_{\pi^+\pi^-}^{\text{eff}} \approx 0.47 \cdot 10^{-4}.$$  \hfill (21)

According to [3], the initial pure $B^0$–state produces the secondary $\pi^+\pi^-$ yield integrated over $t_1$ with the secondary decay time distribution

$$W_B^{\pi^+\pi^-}(t_2) \sim D \cdot \exp(-\Gamma_S t_2) + |\eta|^2 E \exp(-\Gamma_L t_2)$$  
$$+ 2\text{Re} \{\eta \cdot F \exp(-i\Delta m_K t_2)\} \exp \left(-\frac{\Gamma_L + \Gamma_S}{2} t_2\right),$$  \hfill (22)
where
\[
D = \frac{1}{1 - y_B^2} \left( \frac{1 + |\lambda|^2}{2} - y_B \text{Re} \lambda \right) + \frac{1}{1 + x_B^2} \left( \frac{1 - |\lambda|^2}{2} - x_B \text{Im} \lambda \right),
\]
\[
E = D(-\lambda),
\]
\[
F = \frac{1}{1 - y_B^2} \left( \frac{1 + |\lambda|^2}{2} + iy_B \text{Im} \lambda \right) + \frac{1}{1 + x_B^2} \left( \frac{1 + |\lambda|^2}{2} - ix_B \text{Re} \lambda \right).
\] (23)

The notations used here are:
\[
y_B = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-}, \quad x_B = 2 \frac{\Delta m_B}{\Gamma_+ + \Gamma_-} = 2 \frac{m_- - m_+}{\Gamma_+ + \Gamma_-},
\] (24)
\[
\lambda \equiv \lambda_{BK}^{J/\psi}, \quad \eta \equiv \eta_{K}^{\pi^+\pi^-}.
\]

Surely, \( \Gamma_L \) and \( \Gamma_S \) in eq.(22) are widths of neutral kaons.

Distribution (22) and its coefficients (23) illustrate similarity and difference between properties of decays (4) and (12). Similar to distribution (14), coefficients \( D \) and \( E \) are sensitive to the relative signs of \( \Delta \Gamma_B = \Gamma_+ - \Gamma_- \) and \( \text{Re} \lambda_{BK} \), of \( \Delta m_B \) and \( \text{Im} \lambda_{BK} \). But the coefficient \( F \) has another structure. It may be obtained from \( D \) by interchange of \( y_B \) and \( ix_B \). As a result, this coefficient and the corresponding part of distribution (22), differently from distribution (14), are sensitive to the relative signs of \( \Delta \Gamma_B \) and \( \text{Im} \lambda_{BK} \), of \( \Delta m_B \) and \( \text{Re} \lambda_{BK} \). Therefore, if we identify the states \( B_{\pm} \) by fixing the sign of \( \text{Re} \lambda_{BK} \) we can measure three other signs.

Hence, each particular decay (12), in difference with decays (4), can provide by itself the complete measurement of the corresponding parameter \( \lambda_{BK} \). Of more universal interest is that any decay (12) can be used for measuring \( \text{sign}(\Delta m_B) \), thus providing necessary information for the complete measurement of any parameter \( \lambda_{B} \) as well.

For such purposes we may neglect here \( CP\)–violation in the primary decay (20) and use \( \lambda_{BK}^{J/\psi} = -1 \). We also neglect, for simplicity, the small quantity \( |y_B| \). Then
\[
D = E = 1, \quad F = \cos \alpha_B \cdot e^{i\alpha_B}, \quad \tan \alpha_B = x_B.
\] (25)

Available data [8] give
\[
|x_B| = 0.71 \pm 0.06, \quad |\alpha_B| = (35 \pm 2)^\circ, \quad \cos \alpha_B = 0.815 \pm 0.023.
\] (26)

The distribution (22) may be rewritten now as
\[
W_{B^{\pi^+\pi^-}}(t_2) \sim \exp(-\Gamma_st_2) + |\eta|^2 \exp(-\Gamma_L t_2)
\]
\[
+ 2 |\eta| \cdot \cos \alpha_B \cdot \cos(\alpha_B + \varphi - \Delta m_K t_2) \cdot \exp\left(-\frac{\Gamma_S + \Gamma_L}{2} t_2\right),
\] (27)

where [8]
\[
\varphi = \arg \eta = \varphi_{+-} = (44.3 \pm 0.8)^\circ.
\]
The value of $|\alpha_B|$ is large and comparable to $\varphi$. Thus, two possible signs of $\alpha_B$ (i.e., of $x_B$) produce very different phases of oscillation in the third term of distribution (27). However, because of the small factor $|\eta| \approx 2 \cdot 10^{-3}$, their discrimination requires high experimental statistics.

Therefore, $B-$factories look inappropriate to determine the sign of $\Delta m_B$ from events (20). More promising might be LHC. The detector LHC-B dedicated for $B-$physics at LHC \[\text{is expected to accumulate 55000 events (20) per year (10}^7 \text{ seconds) at restricted luminosity } 1.5 \cdot 10^{32} \text{cm}^{-2}\text{s}^{-1}. \text{We have used Monte Carlo simulation based on PYTHIA to estimate their statistical meaning for the above task (note that modifications of the original PYTHIA were necessary to account for the coupled coherence of } B- \text{ and } K-\text{evolutions; more details will be published elsewhere). Our results show that reliable determination of sign}(\Delta m_B) \text{ requires at least an order more events (20), i.e. about } 10^6 \text{ events. This could be achieved if LHC-B were modified so to work with higher LHC luminosity.}

Another way is to use different sequence of decays

$$B^0(\bar{B}^0) \rightarrow J/\psi K^0(\bar{K}^0), \quad J/\psi \rightarrow \ell^+\ell^-, \quad K^0(\bar{K}^0) \rightarrow \pi^\pm \ell^\mp \nu \quad (28)$$

with the effective branching ratio (compare to (21))

$$(Br)^{\ell\ell}_{eff} \approx 0.45 \cdot 10^{-4}. \quad (29)$$

Its secondary decay time distribution, again integrated over $t_1$ and with $\lambda_{B_K}^{J/\psi} = -1$, has the form (compare to (27))

$$W^\pm_B(t_2) \sim \exp(-\Gamma_S t_2) + \exp(-\Gamma_L t_2) \quad (30)$$

$$\quad \pm 2 \cos \alpha_B \cdot \cos(\alpha_B - \Delta m_K t_2) \cdot \exp\left(-\frac{\Gamma_S + \Gamma_L}{2} t_2 \right).$$

Here $W^\pm_B$ refers to the secondary lepton $\ell^\pm$ in the kaon decay. Note that all the distributions (22), (27) and (30) are written for the initially pure $B^0-$state. For the initial $\bar{B}^0-$state one should change the sign of the interference term in (27) and (30) and, additionally, substitute $\lambda \rightarrow 1/\lambda$ in (22).

At $\alpha_B = 0$ the expression (30) coincides with distributions of kaon semileptonic decays. In difference with (27), it does not contain small factor $|\eta|$. Nevertheless, some smallness appears here as well since only a small part of decays, in the $t_2$ interval of order $\tau_S$, demonstrates oscillations while their main part, with characteristic time $\tau_L \gg \tau_S$, does not. Experiments on kaon semileptonic decays show [10] that oscillations are observable only up to $t_2 \lesssim 10^{-9}s$. Comparing to $\tau_L \approx 5 \cdot 10^{-8}s$ we see that not more than 1/50 of all events (28) may be used to extract the oscillating term. This number noticeably exceeds, however, the smallness parameter $|\eta| \approx 2 \cdot 10^{-3}$ for events (20).

Therefore, one may hope that determination of the sign of $\Delta m_B$ will be really achieved at LHC-B or some other facilities by studying time distribution of events (28). More reliable estimation of necessary statistics requires detailed investigation of trigger efficiencies for such events in a particular detector.
5 Summary and conclusions

Here we briefly summarize results of the above considerations.

There are several kinds of $CP$-violating parameters in decays of neutral $B$-mesons. One of them, $\delta_B$ (see Eq.(3)), is universal and related to $B\bar{B}$ mixing. However it can be measured only if the experiment is sensitive enough to discriminate $\Gamma_L$ and $\Gamma_S$ for $B$-mesons. But even in such a case, the conventional CKM-mechanism of $CP$-violation strongly suppresses $\delta_B$ and makes it hardly measurable.

Another set of parameters corresponds to direct $CP$-violation in $B$-meson decays (4) to final states having definite $CP$-parity. It can be identified with phases of quantities $\lambda_B$ (see Eq.(5)) for the decays with pure elastic final-state rescattering (one of neutral kaon analogues is the kaon decay to $2\pi$ with the definite isospin).

One more set of direct $CP$-violating parameters having no analogues in neutral kaon decays is generated by decays (12) of neutral $B$-meson to neutral kaon accompanied by a definite $CP$-parity system (e.g., $J/\psi$). It can be identified with phases of various $\lambda_{BK}$ (see Eq.(13)).

Both values and signs for the direct $CP$-violating phases are physically meaningful and worth to measure. For example, in kaon decays only one sign of $\text{arg}\eta$ leads to agreement of experimental data with the superweak model of $CP$-violation [11]. Moreover, the CKM-mechanism with 3 quark generations should unambiguously relate the signs of $CP$-violating parameters for neutral $B$-meson and neutral kaon decays (they are all expressible through only one $CP$-violating parameter of the CKM-matrix). However we demonstrate that any decay (4) by itself can not provide measurement of the sign of the corresponding $CP$-violating phase. To achieve the complete measurement of $CP$-violating parameters one should separately find the sign of $\Delta m_B$.

Therefore, $\text{sign}(\Delta m_B)$ appears to be a universal element which knowledge is very important for studies of $CP$-violation in neutral $B$-meson decays. We suggest how one can measure the sign of $\Delta m_B$. This goal may be achieved by extracting the secondary kaon decay oscillations in the decay sequences (20) or (28). Monte Carlo simulations show that experimental statistics will be insufficient in the near future for events (20) with two-pion kaon decays. The situation looks more promising for events (28) with semileptonic kaon decays. Corresponding measurements could be done at LHC-B or some other facilities.

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