Perspectives of implementing QED cascade production with the next generation of laser facilities

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Abstract. QED cascade in an intense laser field can start developing if the electron proper acceleration expressed in Compton units $\chi$ becomes of the order of unity. We derive a general formula expressing the quantum dynamical parameter $\chi(t)$ of an initially slow electron in an arbitrary electromagnetic field, which is valid for a time range $t \ll 1/\omega$, where $\omega$ is the field carrier frequency. Using this formula, we identify a special field configuration of colliding focused laser beams, which provides cascade production at intensity of the order of $10^{23}$ W/cm$^2$. Such intensity is believed to become attainable with the new generation of laser facilities, particularly the ELI Beamlines, in the coming years.

1. Introduction

Pair creation from vacuum in strong electromagnetic field is being discussed in the literature for almost 90 years, see e.g. [1–13]. But only nowadays, due to a remarkable progress of laser technology, the possibility to observe such kind of effects has been appeared. There are two conceivable possibilities to probe them experimentally. The first option might be causing direct pair production from vacuum [1–7]. However, calculations in [7] had demonstrated that spontaneous pair creation from vacuum would become observable only for the field intensity above about $10^{26}$ W/cm$^2$. Hence, taking into account that the maximum laser intensity attainable nowadays is only $2 \cdot 10^{22}$ W/cm$^2$ [14], we arrive at conclusion that it would be probably extremely hard to implement such an option in the nearest future.

The second option available would be observation of QED cascades [8–13]. In this paper we consider a cascade seeded by a charged particle that was initially at rest. Laser field accelerates such a particle causing emission of hard photons, that in turn can create electron-positron pairs, and so on. Such an avalanche-like process would then last until the generated particles finally exit the focal area of the laser field. Numerical simulations performed in [8,10] for some particular models of laser pulses have indicated that the threshold intensity required for cascade development would be still above $10^{24}$ W/cm$^2$.

A new new generation of laser facilities, particularly the ELI Beamlines [16] in Czech Republic and VULCAN10 in UK [17] have been recently announced, that would provide intensities of the order $10^{23}$ W/cm$^2$. Since it still remains less than the threshold intensity for arising QED
cascades as calculated in [8, 10], there arises a question if this threshold could be decreased somehow at least by an order of magnitude.

The probabilities for pair creation and hard photon emission processes [3]

\[ W_{e,\gamma}(\chi_{\gamma,e} \gg 1) \sim \frac{\alpha m^2 e^4}{\hbar \varepsilon_{e,\gamma}} \chi_{\gamma,e}^{2/3}, \quad W_e(\chi_{\gamma} < 1) \sim e^{-\frac{8}{3\chi_{\gamma}}} \]  

(1)

where \( \varepsilon_{e,\gamma} \) is the energy of radiating electron or pair creating photon and \( \alpha \) is the fine structure constant, are defined by the quantum dynamical parameter

\[ \chi = \frac{e\hbar}{m^3c^4} \sqrt{-\left(F_{\mu\nu}p^\nu\right)^2}. \]  

(2)

Here \( F_{\mu\nu} \) is the electromagnetic field tensor, \( m \) and \( e \) are the electron mass and charge, \( p^\nu \) is the 4-momentum. Since the probability of the pair creation process is exponentially suppressed for \( \chi_{\gamma} < 1 \) and since \( \chi_{\gamma} \) for emitted photon is always less than \( \chi_e \) for an emitting particle, the problem is to find such a field configuration that provides the fastest growth of \( \chi \) during the period of ‘acceleration’\(^1\). Then by numerical calculation we can find the minimum laser field intensity needed for arising of a QED cascade in such an optimal field configuration.

2. Growth of \( \chi(t) \) at the ‘acceleration’ stage (general case).

Strong electromagnetic field is usually obtained by using focused laser pulses. But a standing wave like the field of two counterpropagating pulses is more efficient for cascade production than a single focused pulse of the same total power [8]. Therefore we consider an arbitrary focused field with the only restriction that the magnetic field vanishes at the center of the focus (the origin \( r = 0 \)).

For the time range \( t \ll 1/\omega \), where \( \omega \) is the field carrier frequency, the electric and magnetic fields can be both expanded into the Taylor series

\[ E(x) = E_0 + E_k(0)x^k, \quad B(x) = B_k(0)x^k \]  

(3)

with respect to time and coordinates \( x^k = \{ct, r\} \). Besides, in what follows we always assume that the laser field is thus strong that the particle becomes ultrarelativistic after acceleration, \( eEt/mc \gg 1 \), where \( t \) is the acceleration time, but at the same time still remains much weaker than the QED critical field, \( E \ll E_{cr} = m^2c^3/e\hbar = 1.3 \cdot 10^{16} \text{V/cm} \).

Let the electric field at the focus be initially directed along the 3-axis. Then for \( t \gg m/eE_0 \) \(^2\) the solution of equations of motion accurate to the zeroth order in \( \omega t \) reads

\[ p^{(0)} = eE_0 t, \quad \varepsilon^{(0)} = eE_0 t, \quad x = y = 0, \quad z = t. \]  

(4)

where \( p \) and \( \varepsilon \) are the momentum and the energy of a particle.

Using the first order expressions for the fields in \( \omega t \), \( E(x) = E_0 + E't, \quad B(x) = B't, \) where

\[ E' = \frac{\partial E}{\partial t}(0) + \frac{\partial E}{\partial z}(0), \quad B' = \frac{\partial B}{\partial t}(0) + \frac{\partial B}{\partial z}(0) \]  

(5)

we can now solve the first order equation of motion

\[ \frac{d\mathbf{p}^{(1)}}{dt} = e \left( \mathbf{E}' + \frac{[\mathbf{E}_0 \times \mathbf{B}']}{E_0} \right) t, \]  

(6)

\(^1\) Herein and after we drop the induces thus denoting e.g. \( \chi_e \) as just \( \chi \).

\(^2\) We assume below that \( \hbar = c = 1 \).
thus obtaining the formula for \( \chi(t) \)

\[
\chi(t) = \frac{e^2 E_0 t^2}{2m^3} \sqrt{(E'_x - B'_y)^2 + (E'_y + B'_x)^2}.
\] (7)

The result (7) is in agreement with the special cases of rotating purely electric field and standing plane wave obtained previously in Refs. [9, 13]. Hence, to optimize the field configuration for cascade production we must increase the coefficient at \( t^2 \) in the formula (7) as much as possible.

3. Case of two colliding focused beams

Now consider a realistic model of the focused laser beam suggested in Ref. [6]

\[
A = \frac{\sqrt{P} f(\xi) e^{-i\omega t}}{\omega^3 |\epsilon|} \left[ \hat{n} \times \nabla_\perp \right] \left( [\epsilon \times \hat{n}] \cdot \nabla_\perp \right) \sin(\omega R) \frac{\sin(\omega R)}{R},
\] (8)

which is an exact solution of the Maxwell equations. Here, \( R = (r_\parallel - ib) \sqrt{1 + \frac{r_\parallel^2}{(r_\parallel - ib)^2}} \), \( \hat{n} \) is a unit vector directed along the focal axis of the beam, \( r_\parallel \) and \( r_\perp \) are the components of the radius vector parallel and perpendicular to \( \hat{n} \), respectively, \( \epsilon \) is the polarization vector, \( P \) is the beam power, \( b \) is the Rayleigh length, \( \xi = b \omega t \) and

\[
f(\xi) = \frac{8\sqrt{\pi} \xi^{5/2} e^\xi}{\sqrt{e^{4\xi}(4\xi^2 - 6\xi + 3) - 4\xi^2 - 6\xi - 3}}.
\] (9)

More details about the model can be found in Appendix A below.

Using (7) for a pair of such colliding beams propagating along and opposite the \( x \)-axis, see the left panel of Fig. 1, we obtain

\[
\chi(t) = \frac{e^2 P \omega^3 |c_1 c_2|}{m^3 c_1^2 + c_2^2} g(\xi) t^2,
\] (10)
where the polarization vectors of both beams have been chosen as \( \mathbf{e} = e_1 \hat{y} - ic_2 \hat{z} \), \( P \) is their total power, and

\[
g(\xi) = \frac{128e^{2\xi}(\xi \cosh \xi - \sinh \xi)}{\xi^3 (e^{4\xi^2} - 6\xi^2 + 3) - 4\xi^2 - 6\xi - 3} \times \\
(3 (\xi^2 + 4) \sinh \xi + \xi (\xi^2 - 12) \cosh \xi). \tag{11}
\]

Note that dependence of \( \chi \) on both the degree of focusing and polarization have been separated. Therefore from Eq. (10) it follows that the optimal polarization for cascade production, which provides the most rapid growth of \( \chi(t) \) is the circular one, \( c_1 = c_2 \).

4. Case of \( n \) colliding focused beams

A method for reducing the total threshold power for spontaneous pair creation from vacuum via the multibeam technology was suggested in Ref. [7], where it was clearly demonstrated how the key parameter of that problem, the magnitude of the electric field strength at the focus, could be increased considerably due to interference by using several pairs of colliding laser beams instead of single one of the same power. In our problem we can not follow this way straightforwardly, because it was expedient in Ref. [7] to use identical linearly polarized beams, but such a field would never produce cascades in the focus.

Nevertheless, let us consider an even number \( n \) of beams propagating in the \( xy \) plane at angles

\[
\varphi_k = 2\pi (k - 1)/n, \quad k = 1 \ldots n
\]

(12)
to the \( x \) axis, see the right panel of Fig. 1, and find their optimal polarization for cascade production. If we choose polarization vectors as \( \mathbf{e}_k = e_x^k \hat{x} + e_y^k \hat{y} - ie_z^k \hat{z} \), then using (7) we obtain

\[
\chi_n(t) = \frac{e^2 \omega^3 P g(\xi) F}{m^3} \xi^2, \tag{13}
\]

where the factor

\[
F = \frac{e_z \sqrt{e_x^2 + e_y^2}}{n} \sum_{k=1}^{n} \left[ (e_x^k)^2 + (e_y^k)^2 + (e_z^k)^2 \right], \quad e = \sum_{k=1}^{n} e^k \tag{14}
\]
determines the dependence of \( \chi \) on polarization and the number of beams.

Due to the symmetry of the problem the polarization vector of the total electric field can lie in any plane that is parallel to \( z \)-axis. Let it be the \( yz \)-plane. Since for each beam the electric field is transverse to the direction of propagation \( \hat{n} \), we can rewrite the projections of polarization vectors to the \( xy \)-plane as \( e_x^k = h_k \sin \varphi_k, \ e_y^k = h_k \cos \varphi_k \). Then

\[
F = \frac{e_x e_y}{\sum_{k=1}^{n} (h_k^2 + (e_z^k)^2)}, \quad e_y = \sum_{k=1}^{n} h_k \cos \varphi_k. \tag{15}
\]

Besides, let us define four \( n \)-dimensional vectors \( \mathbf{Z} = \{ e^k_x \}, \ \mathbf{H} = \{ h^k \}, \ \mathbf{\alpha} = \{ \cos \varphi_k \} \) and \( \mathbf{\beta} \), such that all the components of \( \mathbf{\beta} \) equal unity. Then the expression (15) takes the form

\[
F = \frac{(\mathbf{\beta} \mathbf{Z}) \langle \mathbf{\alpha} \mathbf{H} \rangle}{\sqrt{||\mathbf{Z}||^2 + ||\mathbf{H}||^2}} \tag{16}
\]
Table 1. Power and intensity QED cascade thresholds for configurations with \( n \) colliding beams \((\omega = 1\, \text{eV}, \xi = 8)\).

| \( n \) | \( P, \text{PW} \) | \( I, 10^{23}\text{W/cm}^2 \) |
|---|---|---|
| 2 | 23 | 17 |
| 4 | 14 | 10 |
| 6 | 10 | 7.3 |
| 8 | 7.9 | 5.6 |

The maximization of the expression (15) thus follows from the chain of inequalities

\[
F \leq \frac{1}{2} \frac{(\beta Z) (\alpha H)}{|Z| |H|} \leq \frac{|\alpha| |\beta|}{2} = \frac{n}{2\sqrt{2}},
\]

i.e. the growth rate of the parameter \( \chi \) is proportional to the number of beams \( n \). Hence \( F \) achieves its maximum value when \( Z \parallel \beta, H \parallel \alpha \) and \( |Z| = |H| \). Consequently,

\[
e^1_z = e^2_z = \ldots = e^n_z, \quad e_z = ne^1_z,
\]

\[
h_k = h_1 \cos \varphi_k, \quad e_y = h_1 \sum_{k=1}^n \cos^2 \varphi_k = \frac{n}{2} h_1
\]

and \( e^1_z = \frac{h_1}{\sqrt{2}} \). Therefore

\[
e_z : e_y = \sqrt{2} : 1,
\]

i.e. the optimal polarization of the total field is elliptical with the semi-major to semi-minor axis ratio \( \sqrt{2} : 1 \). The polarization vectors of the beams in this optimal case are

\[
\epsilon_k = \{ \sqrt{2} \sin \varphi_k \cos \varphi_k, \sqrt{2} \cos^2 \varphi_k, -i \}.
\]

Note, that Eq. (20) and its derivation are valid if \( n > 2 \), while for \( n = 2 \) the optimal polarization is circular, as was already mentioned after Eq. (11).

5. Numerical results

The actual threshold power required for cascade production can be found by numerical simulations. Since such a threshold is never sharp, let us define it by demanding a single pair creation on average by each seeding electron during the passage of laser pulses through the focus. As a model of a focused laser beam we still use the exact solution (8) for the Maxwell equations. The wavelength is taken to be \( \lambda = 1.24\, \mu\text{m} \), which corresponds to the laser photon energy \( 1\, \text{eV} \). Finiteness of pulse duration was introduced artificially by limiting the simulation time. The simulation time is chosen to be 5 laser periods as is typical for the modern laser facilities. The focusing parameter is chosen as \( \xi = \frac{b_0}{\omega} = 8 \). In this case the FWHM focal spot diameter equals to the wavelength, as at the HERCULES facility [14] (see also Appendix).

For numerical simulation we use a code implementing the Monte-Carlo (MC) algorithm, in which one assumes that \( e^- \) and \( e^+ \) are moving classically between the acts of the emission events, which are determined in the manner similar to Refs. [10, 12]. The seed particles are represented by a bunch of \( 10^3 \) electrons placed initially at rest at the center of the focal region. There exist bounds for a number of focused beams that can be allocated concurrently in a plane avoiding intersections [7,20]. In our case \( \xi = 8 \) this number can not exceed 8 (see Appendix
Figure A1. Distribution of the electric field strength (in units of $E_{cr}$) in the $xy$-plane for a single beam propagating along the $x$-axis at $t = 0$, $z = 0$ for $\epsilon = \{0, 1, -i\}$, $\xi = 8$, $\omega = 1\text{eV}$ and $P = 1\text{PW}$.

A). The resulting thresholds obtained by numerical simulation of QED cascades in the cases of $n = 2, 4, 6$, and 8 beams with optimal polarization (21) colliding in the $xy$-plane are presented in Table 1. One can observe from the given data that the required laser power and intensity for cascade production decreases significantly as the number of beams increases.

6. Conclusion

We have shown that the QED cascades can arise in laser-target interaction at laser power below 10 PW and intensity below $10^{24}\text{W/cm}^2$. This is realizable by a multibeam setup with several pairs of colliding laser beams. In order to determine how to arrange different beams, we have derived a general analytical expression for the dynamical quantum parameter $\chi(t)$ on the ‘acceleration’ stage, which controls the hard photon emission and pair photoproduction. Then, by specifying the case of several colliding beams, we have identified the optimal polarization of the beams with the most rapid growth of $\chi(t)$ in the focus. For such an optimal configuration with the fixed total laser power, $\chi$ is proportional to the number of beams. As an illustration, we have computed the QED cascade thresholds for the particular case of 8 colliding beams of optimal polarization. According to our simulations, the threshold total power is $P_{th} = 7.9\text{PW}$ and the threshold intensity is $I_{th} = 5.6 \cdot 10^{23}\text{W/cm}^2$. Such parameters can be very likely achieved with the next generation of laser facilities [16, 17] in the nearest future.

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Appendix A. The focused field model

Following [6], consider a vector potential

$$A(r, t) = \int \frac{d\mathbf{k}}{(2\pi)^3} a(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \quad \mathbf{k} \cdot a(\mathbf{k}) = 0,$$

(A.1)

which provides an exact solution of Maxwell equations $\ddot{\mathbf{A}} - \Delta \mathbf{A} = 0$ with the transverse gauge $\nabla \cdot \mathbf{A} = 0$. If we choose

$$a(\mathbf{k}) = (\hat{n} \times \mathbf{k}_\perp)(C \cdot \mathbf{k}_\perp)e^{i(k_x - \omega)t}\delta(k^2 - \omega^2),$$

(A.2)
where \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \) are the components of the wave vector \( \mathbf{k} \) parallel and orthogonal to the direction \( \hat{n} \) of the wave propagation, \( \mathbf{C} = c[\mathbf{e} \times \hat{n}] \) with some constant \( c \) and evaluate the integral (A.1) (see [6] for details), then we arrive at

\[
A = \frac{e^{-\omega(\mathbf{k}_1 \cdot \mathbf{b} + \mathbf{k}_2 \cdot \mathbf{a})}}{(2\pi)^2} (\hat{n} \times \nabla_\perp)(\mathbf{C} \cdot \nabla_\perp) \frac{\sin(\omega R)}{R},
\]

(A.3)

that coincides with the representation (8) up to a constant factor.

In order to calculate the beam power, consider a beam propagating along the \( x \)-axis (\( \hat{n} = \hat{x} \)) and assume that \( \mathbf{e} = c_1 \hat{y} - ic_2 \hat{z} \). By singling out the outgoing part of the wave, i.e. the part of (A.1) proportional to \( e^{i\omega R} \), in the far region \( (r = \sqrt{x^2 + y^2 + z^2} \to \infty) \) we obtain

\[
E = c^3 \frac{\sin^2 \theta e^{i\omega(bR + \mathbf{c} \cdot \hat{n})}}{8\pi^2} (c_1 \sin \phi + ic_2 \cos \phi) \times \{0, \sin \phi, - \cos \phi\},
\]

(A.4)

\[
B = c^3 \frac{\sin^2 \theta e^{i\omega(\mathbf{c} \cdot \hat{n})}}{8\pi^2} (c_1 \sin \phi + ic_2 \cos \phi) \times \{-\sin \theta, \cos \theta \cos \phi, \cos \theta \sin \phi\},
\]

(A.5)

where \( \phi = \arctan \frac{y}{x} \) and \( \theta = \arctan \frac{z}{\sqrt{x^2 + y^2}} \). By noting that \( R \to r \left(1 - \frac{ib\xi}{r^2}\right) \) for \( r \to \infty \), we can compute the beam power

\[
P = \frac{1}{8\pi} \int \text{Re}[\mathbf{E} \times \mathbf{B}^*]d\mathbf{S},
\]

(A.6)

where \( \xi = b\omega \). This proves the choice of the normalization factor \( g(\xi) \) in (8).

As it was mentioned above, we demand as tight focusing that the circle of the wave length diameter in the focal plane contains a half of the beam power (practical realization of such degree of focusing was reported in Ref. [14]). Calculating the integral for the power over the circle area of the diameter \( \lambda = 1.24 \cdot 10^{-4} \) cm in the plane \( z = 0 \) numerically, we obtain that this condition is satisfied if \( \xi = 8 \).

Distribution of the electric field magnitude in the \( xy \)-plane is displayed in Fig. A1. One can see that it has the distinctive shape of the focused beam. At far distance from the focal region the field is contained inside the cone with the aperture depending on the focusing parameter \( \xi \). It can be shown, that in our case \( \xi = 8 \), this aperture \( \sim \pi/4 \), therefore the number of beams that can be allocated in the plane without intersections can not exceed 8.

Finally, to clarify polarization of the beam in our model, consider the electric field at the focus

\[
\mathbf{E}(t, 0) = \frac{e^{-i\omega t}c\sqrt{P(\xi \cosh \xi - \sinh \xi)}}{\xi^3|\epsilon|} \{0, ic_1, c_2\}.
\]

(A.7)

This expression proves that the field is polarized elliptically.

References

[1] Sauter F 1931 Z. Phys. 69 742
Sauter F 1932 Z. Phys. 73 547
[2] Schwinger J 1951 Phys. Rev. 82 664
[3] Nikishov A I and Ritus V I 1964 Sov. Phys. JETP 19 529
Nikishov A I and Ritus V I 1967 Sov. Phys. JETP 25 1135
Baier V N and Katkov V M 1967 Phys. Lett. 25 A 492
[4] Nikishov A I 1970 Sov. Phys. JETP 30 660
[5] Narochny N B, Bulanov S S, Mur V D and Popov V S 2004 Phys. Lett. A 330 1
Bulanov S S, Narochny N B, Mur V D and Popov V S 2006 JETP 102 9
[6] Fedotov A M 2009 Laser Physics 19 214
[7] Bulanov S S, Mur V D, Narozhny N B, Nees J and Popov V S 2010 Phys. Rev. Lett. 104 220404
[8] Bell A R and Kirk J G 2008 Phys. Rev. Lett. 101, 200403
   Kirk J G, Bell A R and Arka I 2009 Plasma Phys. Control. Fusion 51 085008
[9] Fedotov A M, Narozhny N B, Mourou G and Korn G 2010 Phys. Rev. Lett. 105 080402
[10] Elkina N V et al 2011 Phys. Rev. STAB 14 054401
[11] Nerush E N et al 2011 Phys. Rev. Lett. 106 035001
[12] Duclous R, Kirk J G and Bell A R 2011 Plasma Phys. Control. Fusion 53 015009
[13] Bashmakov V F et al 2014 Phys. Plasmas 21 013105
[14] Bahk S W et al 2004 Opt. Letters 29 2837
   Yanovsky V et al 2008 Opt. Express 16 2109
[15] Strickland D, Mourou G 1985 Opt. Communications 55 447
[16] http://www.extreme-light-infrastructure.eu
[17] http://www.stfc.ac.uk/clf/New+Initiatives/ The+Vulcan+10+Petawatt+Project/14684.aspx
[18] Narozhny N B and Fofanov M S 2000 JETP 90 753
   Narozhny N B and Fofanov M S 2000 Phys. Lett. A 295 87
[19] Landau L D and Lifshitz E M 1988 Theoretical Physics: The Classical Theory of Fields, vol. 2 (Course of Theoretical Physics Series) (London: Pergamon Press)
[20] Gonoskov I, Aiello A, Heugel S and Leuchs G 2012 Phys. Rev. A 86 053836