PrivMin: Differentially Private MinHash for Jaccard Similarity Computation

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Abstract

In many industrial applications of big data, the Jaccard Similarity Computation has been widely used to measure the distance between two profiles or sets respectively owned by two users. Yet, one semi-honest user with unpredictable knowledge may also deduce the private or sensitive information (e.g., the existence of a single element in the original sets) of the other user via the shared similarity.

In this paper, we aim at solving the privacy issues in Jaccard similarity computation with strict differential privacy guarantees. To achieve this, we first define the Conditional $\epsilon$-DPSO, a relaxed differential privacy definition regarding set operations, and prove that the MinHash-based Jaccard Similarity Computation (MH-JSC) satisfies this definition. Then for achieving strict differential privacy in MH-JSC, we propose the PrivMin algorithm, which consists of two private operations: 1) the Private MinHash Value Generation that works by introducing the Exponential noise to the generation of MinHash signature. 2) the Randomized MinHashing Steps Selection that works by adopting Randomized Response technique to privately select several steps within the MinHashing phase that are deployed with the Exponential mechanism. Experiments on real datasets demonstrate that the proposed PrivMin algorithm can successfully retain the utility of the computed similarity while preserving privacy.

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1 Introduction

With the widespread of real-world big data applications such as recommendation systems and social network, similarity computation has become one essential process as it measures the distances between different user profiles. Utilizing the similarity between users or items, the service providers can further carry out data analytic tasks such as clustering, classification or recommendation. Among the varieties of similarity measures, Jaccard Similarity is a popular one that has been widely used to compare the similarity of two given sets. More specifically, for two sets $S_A$ and $S_B$, their Jaccard similarity is defined as the ratio between the size of their intersection and the size of their union.

However, because of the adoption of Jaccard similarity in real-world applications, one increasing concern is the potential privacy leakage. Let us consider one example scenario as described below.

**Example 1** In cloud services, the similarity computation may be available for users who want to know the “semantic distance” between their data.

Assume a cloud platform which provides such a service of Jaccard similarity computation between two sets $S_A$ and $S_B$ privately owned by users $U_A$ and $U_B$, respectively. Each set contains a fixed number of textual tags that reflect the user’s reading preferences, such as $S_A = \{\text{History, Politics, Science, Law, Travel}\}$ and $S_B = \{\text{History, Science, Travel, Cookbooks, Fiction}\}$. The cloud service may estimate the Jaccard similarity as $\frac{|S_A \cap S_B|}{|S_A \cup S_B|} = 0.429$, and make it available for users $U_A$ and $U_B$.

Remarkably, based on the above shared similarity and the prior knowledge that the value of $|S_A \cup S_B|$ should fall into the range of 5 to 10, $U_B$ can easily work out the values of $|S_A \cap S_B|$ and $|S_A \cup S_B|$, which are 3 and 7 respectively. Moreover, if $U_B$ further knows in advance that $U_A$ does not like the book genres such as cookbooks and fiction, he would basically make sure that his three tags in common with $U_A$ are $\{\text{History, Science, Travel}\}$. In addition, when we take into account the fact that there are two collusive users $U_B$ and $U_C$ who are interested in $U_A$’s private information, it would not take these collusive users much background knowledge to achieve their purpose. $U_B$ would also achieve the attack goal easily through the similarity with $U_A$ via different carefully constructed sets.

As shown in the above example, users with background knowledge can induce other users’ private information with high probability by observing their shared similarity. Hence, how to preserve the privacy in the Jaccard similarity computation is an emerging issue that needs to be addressed.

In the past decade, Differential Privacy has emerged as a solid privacy model with a provable privacy guarantee, regardless of the adversary’s background knowledge. Recently, some researches have focused on the privacy issue in similarity computation by incorporating the differential privacy mechanism. Alaggan et al. [1] proposed several secure protocols to compute differentially private values of Scalar Product and Cosine similarity. Their follow-up paper [2] proposed a differentially private method for randomizing the intermediate outputs instead of adding noise to the final Cosine similarity outputs. Wong et al. [22] first proposed a secure protocol for a specific Jaccard similarity computation for the binary data. However, those tailored Jaccard similarity computations cannot be generalized to other situations. To the best of our knowledge, there is limited researches that have addressed the privacy concerns in the general Jaccard similarity computation while maintaining the acceptable utility and efficiency.
As the advances in Hashing techniques, such as the MinHash and SimHash, the current research barriers can be tackled in a natural way. The MinHash technique [6] was proposed to efficiently approximate the value of Jaccard similarity instead of the precise one, so it can significantly improve the computation efficiency when a large collection of data involved [7, 8]. For convenience, we refer to this processing workflow as MinHash-based Jaccard Similarity Computation (MH-JSC).

In this paper, we will present an intuition that the MH-JSC is internally connected with a relaxed differential privacy, because its expected error $\theta$ can be regarded as noise. This intuition opens the opportunity to design a differentially private Jaccard similarity computation algorithm, which protects the certainty of presence/absence of any element in the original profile. However, there are still two main challenges when designing the differentially private Jaccard similarity computation algorithm:

- The first challenge is how to measure the randomness within the MH-JSC for further analyzing its relationship with the differential privacy.
- The second challenge lies on how to leverage the minimum hash value computation process within the MinHashing phase for achieving strict differential privacy in MH-JSC while maintaining an acceptable utility.

For the first challenge, we investigate the relationship between the MH-JSC and the differential privacy via a relaxed differential privacy definition, Conditional $\epsilon$-DPSO. Based on this, we intend to design a differentially private Jaccard similarity computation algorithm via the Exponential mechanism, which leverages the minimum hash value computation process within the MinHashing phase. As the introduced Exponential noise will distort the utility in a large extent, the second challenge can be solved by introducing the Randomized Response technique to privately select some MinHashing steps for adopting the Exponential mechanism.

Based on these, we finally present the PrivMin algorithm to achieve the differentially private Jaccard similarity computation, and the contributions in this paper can be summarized as follows:

- Firstly, through the relaxed differential privacy, Conditional $\epsilon$-DPSO, we investigate the randomness within the MH-JSC and provide relevant privacy analysis in detail.
- Secondly, we design a practical differential private Jaccard similarity computation algorithm, PrivMin, which maintains an acceptable utility. Theoretical analysis and extensive experiments are provided to verify the improved performance.

The rest of this paper is organised as follows. We present the preliminaries and related works in Section 2 and provide the problem statement in Section 3. In Section 4 we define the Conditional $\epsilon$-DPSO to depict a relax situation when considering differential privacy for set operations, followed by theoretical privacy analysis of the MH-JSC under this definition. In Section 5 we describe the PrivMin algorithm for achieving the differentially private MinHash-based Jaccard similarity computation. The theoretical privacy analysis and utility analysis of the algorithm are proposed in Section 6. Section 7 presents experimental results, and conclusions are given in Section 8.

2 Preliminaries and Related Works

This section reviews four fundamental concepts: Jaccard Similarity, MinHash, Differential Privacy and Randomized Response, and then briefly surveys the related works in Differentially Private Similarity Computation.
Table 1 lists the relevant notations used in this paper.

Table 1: Notations

| Symbol      | Description                                                                                   |
|-------------|----------------------------------------------------------------------------------------------|
| MH-JSC      | abbreviation of MinHash-based Jaccard similarity computation                                  |
| $S$         | user’s private profile $S = \{s_1, s_2, ..., s_N\}$                                           |
| $J(S_A, S_B)$ | original Jaccard similarity of $S_A$ and $S_B$                                                |
| $\sigma$   | a conventional notation to represent the value of $J(S_A, S_B)$                               |
| $J_{mh}(S_A, S_B)$ | original MinHash-based Jaccard similarity of $S_A$ and $S_B$                                |
| $\theta$   | expected error in $J_{mh}(S_A, S_B)$ compared with $J(S_A, S_B)$                            |
| $\Delta J_{mh}$ | perturbed MinHash-based Jaccard similarity of $S_A$ and $S_B$                              |
| $K$         | number of hash functions                                                                      |
| $h_k(S)$   | hash values set of profile $S$ when given a hash function $h_k(\cdot)$                       |
| $\min\{h_k(S)\}$ | the minimum hash value in $h_k(S)$                                      |
| $h_{(K)}(S)$ | original MinHash signature vector                                                             |
| $h_{(K)}(S)$ | perturbed MinHash signature vector                                                            |
| $V$        | original flip vector                                                                          |
| $P_r$      | bit flipping probability in original flip vector generation                                 |
| $V'$       | perturbed flip vector                                                                         |
| $P_t$      | bit flipping probability in perturbed flip vector generation                                 |
| $\epsilon$ | overall privacy budget                                                                       |

2.1 Preliminaries

**Definition 1 (Jaccard Similarity).** Assume $S_A$ and $S_B$ are two sets owned by user $A$ and user $B$ respectively. Their Jaccard similarity is defined as

$$J(S_A, S_B) = \frac{|S_A \cap S_B|}{|S_A \cup S_B|}. \quad (1)$$

2.1.1 MinHash

The MinHash was initially proposed in [6, 7] for quickly estimating the similarity between two textual documents which have been respectively expressed as sets $S_A$ and $S_B$. The basic intuition for the MinHash technique is the replacement of the original sets $S_A$ and $S_B$ by their relevant MinHash Signatures $h_{(K)}(S_A)$ and $h_{(K)}(S_B)$ when computing the Jaccard similarity. For the convenience of the following descriptions, we refer to the above similarity estimating process as *MinHash-based Jaccard Similarity Computation (MH-JSC)*.

In practice, the MH-JSC between textual documents usually involves three main phases: the *Shingling* phase to formulate the textual documents into set representations, the *MinHashing* phase to generate the relevant MinHash signatures, followed by the approximate computation phase.

Specifically, in the *Shingling* phase, the document is firstly segmented into $N$ parts (shingles) and represented as the set $S = \{s_1, s_2, ..., s_N\}$; in the process of the MinHashing phase, $K$ hash functions $h_k$ with $k \in [1, K]$ are orderly applied to $S$ and generate $h_k(S) = \{h_k(s_1), h_k(s_2), ..., h_k(s_N)\}$, and then the minimum hash value $\min\{h_k(S)\}$ is selected as the $k$-th element of the MinHash Signature $h_{(K)}(S)$, as shown in Fig 1.
a) In each step $k$, first calculate the hash values of $S = \{s_1, ..., s_n\}$ under the given hash function $h_k()$.

```
step 1: { $h_1(s_1)$, $h_1(s_2)$, ..., $h_1(s_n)$ } --→ $\min h_1(S)$
... { ... ... ... } --→ ...
step K: { $h_K(s_1)$, $h_K(s_2)$, ..., $h_K(s_N)$ } --→ $\min h_K(S)$
```

b) Then select the minimum hash value in $h_k(S) = \{h_k(s_1), ..., h_k(s_N)\}$.

Figure 1: MinHashing Phase

Given the MinHash signatures $h_{(K)}(S_A)$ and $h_{(K)}(S_B)$ for two textual documents $S_A$ and $S_B$, an unbiased estimate of the Jaccard similarity between $S_A$ and $S_B$ is formulated as

$$J_{mh}(S_A, S_B) = \frac{|h_{(K)}(S_A) \cap h_{(K)}(S_B)|}{K},$$

with an expected error $\theta = O\left(\frac{1}{\sqrt{K}}\right)$.

For the convenience of description, if we adopt the notation $\sigma$ to represent the value of $J(S_A, S_B)$, the probability for $J_{mh}(S_A, S_B)$ to fall into the range $[\sigma - \theta, \sigma + \theta]$ can be calculated via the following equation [6]:

$$p(K, \sigma, \theta) = \sum_{K(\sigma - \theta) \leq t \leq K(\sigma + \theta)} \binom{K}{t} (\sigma)^t (1 - \sigma)^{K-t}.$$  

(3)

### 2.1.2 Differential Privacy

Differential privacy is based on the principle that the output of a computation should not allow inference about any element’s presence or absence from the computation’s input. Hence in the context of Jaccard similarity computation, the present or absent status of the elements within input data is expected to be protected under the rigorous differential privacy definition which is described below.

**Definition 2** ($\epsilon$-Differential Privacy [11]). A randomized algorithm $M$ gives $\epsilon$-differential privacy if for all neighbour sets $S$ and $S'$ differing on at most one element, and all $O \subseteq \text{Range}(M)$, we have

$$\Pr[M(S) \in O] \leq e^\epsilon \cdot \Pr[M(S') \in O].$$

Algorithm $M$ is associated with the sensitivity, which measures the maximum change on the result of query function $f$ when one element from the set $S$ changes [10].

**Definition 3** (Sensitivity). For any function $f : S \rightarrow \mathbb{R}^d$, and for all $S$, $S'$ differing in at most one element, the sensitivity of $f$ is $\Delta f = \max_{S,S'} \|f(S) - f(S')\|_1$.

To satisfy the definition of differential privacy, two basic mechanisms are usually utilized: the Laplace mechanism and the Exponential mechanism. And the Laplace mechanism is suitable for numeric output and relies on the strategy of adding the Laplacian noise $\text{Laplace}(\cdot)$ to the query result [12]. It is formally defined as the following:

**Definition 4** (Laplace Mechanism). Given a function $f : S \rightarrow \mathbb{R}^d$, the mechanism,

$$M(S) = f(S) + (Y_1, ..., Y_d),$$

where $Y_1, ..., Y_d \sim \text{Laplace}(\frac{\Delta f}{\epsilon})$.  

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where $Y_i$ are i.i.d random variables drawn from $\text{Laplace}(\frac{\Delta f}{e})$.

The Exponential mechanism focuses on queries with non-numeric output [17]. It pairs with an application dependent Score Function $q(S, \psi)$, which represents how good an output scheme $\psi$ is for query $q$. The Exponential mechanism is formally defined as

**Definition 5 (Exponential Mechanism).** An Exponential mechanism $M$ is $\epsilon$-differential privacy if

$$M(S) = \{\text{return } \psi \text{ with the probability } \propto \exp\left(\frac{q(S, \psi)}{2\Delta q}\right)\}.$$

To guarantee the overall privacy when it comes to a sequence of differentially private operations, we have the following composition properties [16].

**Theorem 1 (Sequential Composition).** Given $n$ independent randomized algorithms $A_1, A_2, \ldots, A_n$ where $A_i(1 \leq i \leq n)$ satisfies $\epsilon_i$-differential privacy, a sequence of $A_i$ over the dataset $S$ satisfies $\epsilon$-differential privacy, where $\epsilon = \sum_{i=1}^{n} \epsilon_i$.

**Theorem 2 (Parallel Composition).** Given $n$ independent randomized algorithms $A_1, A_2, \ldots, A_n$ where $A_i(1 \leq i \leq n)$ satisfies $\epsilon_i$-differential privacy, a sequence of $A_i$ over a set of disjoint datasets $S_i$ satisfies $\max(\epsilon_i)$-differential privacy.

### 2.1.3 Randomized Response

Randomized Response is a commonly used survey technique in statistics [20]. When a respondent is asked a sensitive question for which the answer can be either yes or no, he has the opportunity to answer the question with plausible deniability. To do so, the respondent flips a biased coin before answering the question. If the coin turns head with a probability $p$, he gives his true answer; otherwise, he reports the opposite of the true answer. It has pointed out that Randomized Response can be regarded as a specific randomized algorithm that satisfies the $\epsilon$-differential privacy, if the coin flipping probability $p$ of the algorithm has the following relationship with the privacy budget $\epsilon$ [13, 4]:

$$p = \frac{e^\epsilon}{1 + e^\epsilon}.$$  \hfill (4)

### 2.2 Related Works

#### 2.2.1 Differentially Private Similarity Computation

For the applications such as recommender system [3, 22], several works have been proposed to address the potential privacy issues in two-party profiles computation [1, 5], in user profiles collection [19, 18] and in the profile related data releasing [2, 23, 24] by the third party.

Most of these works were focused on the distributed environments in which the involved users are semi-honest while the third party (if it existed) is assumed as semi-trusted or even untrusted. Therefore, users profile must be perturbed or encrypted before being sent to other users or the third party for further processing such as similarity computation, an essential component in collaborative filtering. In addition, sometimes the released similarity also needs to be perturbed.

For achieving the differentially private similarity computation by output perturbation, the line of research was pioneered by Alaggan et al. [1], who introduced the Laplace mechanism into the **Scalar Product** and Cosine similarity computation. In particular, their proposed secure protocols were partially based on **Homomorphic Encryption** and worked by adding the Laplacian noise to the
similarity. Following a similar strategy, Wong et al. [22] presented a secure protocol for a specific Jaccard similarity computation of binary data.

For profile perturbation, Alaggan et al. [2] considered the scenario of profile release and proposed the BLIP mechanism in which the Bloom filter of user profile would be distorted by Randomized Response before being released to the public. The Scalar Product and Cosine similarity were considered in this work. Barthe et al. [3] proposed a two-party protocol for computing Hamming distance between bit-vectors via Homomorphic Encryption and the Laplace mechanism. Boutet et al. [5] designed an obfuscation protocol and a randomized dissemination protocol for two-party Jaccard similarity computation. Besides, existing works focused also on the perturbation of user profiles for dataset release and for multi-level privacy needs instead of specific similarity computation needs. Zhu et al. [23, 24] considered the privacy issues in releasing and sharing of tagging datasets in tagging recommender systems and presented the private tagging release algorithm PriTop based on the topic generation model, on the Laplace mechanism and on the Exponential mechanism. Shen et al. [19, 18] aimed to achieve multi-level privacy control in user profile perturbation and proposed the DP-MultiUPP and EpicRec frameworks based on the Laplace mechanism and optimization techniques.

Table 2 gives the comparison among the existing works for differentially private similarity computation. The main details of our proposed PrivMin algorithm are also listed in the table, and the differences between our work and the existing works will be discussed in the next section.

### Table 2: Differentially Private Similarity Computation Comparison

| Related Work       | Third Party Setting | Similarity Type               | Perturbation Approach | Involved Method                              |
|--------------------|---------------------|--------------------------------|-----------------------|----------------------------------------------|
| Alaggan et al.     | None or Semi-trusted| Scalar Product, Cosine Similarity| Output Perturbation   | Homomorphic Encryption, Laplace Mechanism    |
| Wong et al. [22]   | Semi-trusted        | Jaccard Similarity             | Output Perturbation   | Homomorphic Encryption, Laplace Mechanism    |
| Barthe et al. [3]  | None                | Hamming Distance               | Profile Perturbation  | Homomorphic Encryption, Laplace Mechanism    |
| Alaggan et al. [2] | None                | Scalar Product, Cosine Similarity| Profile Perturbation  | Bloom Filter, Randomized Response            |
| Boutet et al. [5]  | None                | Jaccard Similarity             | Profile Perturbation  | Compact profile construction, Randomized Response |
| PrivMin            | Trusted             | Jaccard Similarity             | Profile Perturbation  | MinHash Signature, Exponential Mechanism, Randomized Response |

#### 2.2.2 Discussion

Based on the above analysis, the differences between our work and the existing works can be concluded in three aspects:

- Firstly, compared to the distributed setting of the existing works, our work is focused on the centralized setting. Moreover, this work mainly assumes that the third party is trusted while the existing works generally assumed a semi-trusted third party or no third party at all. The main reason of such assumptions in our work is that in many real-world applications of recommendation and plagiarism detection, the service providers always have access to users profiles and then use their storage capacity and computing ability to provide users with varieties of services. Even so, the proposed PrivMin algorithm can also be extended for the
untrusted third party scenario, since it has the ability to release perturbed users profiles before entering the similarity computation phase.

- Secondly, for the research of differentially private Jaccard similarity computation, the existing work such as [22] was partially focused on applying the Laplace mechanism to the original Jaccard similarity computation equation, which cannot maintain a high utility of the released similarity. Besides, due to its assumption of binary input data, the current work failed to meet the privacy needs in the application scenario as shown in Example [1]. Moreover, few attention has been devoted to the relationship between MinHash-based Jaccard similarity computation and differential privacy, which is the basic rationale and main contribution of our work.

- Thirdly, in order to adopt the Randomized Response technique, the existing works (e.g., [2, 5]) were focused on directly distorting user profiles which are represented as binary expressions. However, if relying on some specific value computation process such as MinHashing, we find that the combination of Randomized Response technique with Exponential mechanism could provide the possibility to design a differentially private algorithm with acceptable utility. To the best of our knowledge, this is the first attempt to incorporate the Exponential mechanism with the Randomized Response in the context of differentially private Jaccard similarity computation.

2.3 Summary

For differentially private similarity computation, the existing works have established two perturbation strategies to address relevant issues, and provided referential ideas and solutions for differentially private algorithm design. However, currently there has been limited research attention in the MinHash-based Jaccard Similarity Computation (MH-JSC) to design the differentially private Jaccard similarity computation algorithm. Based on the observation that the internal randomness within MH-JSC is related with a relaxed differential privacy, this paper aims to address the following specific research issues:

- How to measure the randomness within MH-JSC?
- How to achieve strict differential privacy in MH-JSC?

3 Problem Statement

This section first introduces the system and threat model considered in this work, and then clearly presents the differentially private Jaccard similarity computation problem, along with its challenges.

3.1 System and Threat Model

Since in many real-life scenarios users are expected to provide their true data to the cloud platform in order to access add-on services such as accurate recommendation, we assume that the cloud platform will not collude with any user and is trusted. Meanwhile, the platform users are supposed to be semi-honest, namely they are willing to provide their own data to the cloud but also curious about other users’ sensitive information.

Fig. 2 shows the system and treat model.
3.2 Problem Definition

In this work, we are addressing the problem of **differentially private Jaccard similarity computation**, which can be described as follows.

Assume two users $U_A$ and $U_B$, each of them respectively maintains the profile $S_A$ and $S_B$ on the trusted cloud platform. Given their profiles $S_A$ and $S_B$ and the privacy budget $\epsilon$, the cloud platform is expected to calculate the Jaccard similarity $J(S_A, S_B)$ and shares a perturbed version with these two users. On one hand, the overall similarity computation mechanism should satisfy $\epsilon$-differential privacy such that no semi-honest users can infer the present or absent status of the elements in other user profiles based on the shared similarity. On the other hand, the shared perturbed similarity should also maintain acceptable utility for further data analysis or value-adding services.

3.3 Research Issues and Challenges

In this paper, we aim to solve the differentially private Jaccard similarity computation problem by leveraging the MinHash and MH-JSC. However, directly introducing differential privacy into the MH-JSC brings up two major challenges.

**How to measure the randomness within MH-JSC?** As introduced in Section 2, the MinHash-based Jaccard Similarity Computation can estimate the Jaccard Similarity with an expected error $\theta$. It seems that this error can be regarded as a kind of internal randomized noise, which makes it possible for MH-JSC to achieve differential privacy. If the above hypothesis is proved right, it is not necessary to add any extra external noise to MH-JSC since the internal noise could be enough.

In Section 4, we will define $\epsilon$-DPSO and conditional $\epsilon$-DPSO, to analyze the relationship between the randomness within the MH-JSC and differential privacy.

**How to achieve strict differential privacy in MH-JSC?** Based on the randomness analysis in Section 4, we will show that MH-JSC only satisfies a relaxation of strict differential privacy, the conditional $\epsilon$-DPSO. For achieving strict differential privacy, although the Laplace mechanism can be applied to perturbed the original MinHash-based Jaccard similarity, the final utility of similarity will be distorted in a large extent. In Section 5, we will adopt the Profile Perturbation approach and propose two private operations to constitute the PrivMin algorithm, along with the relevant privacy analysis in Section 6. This algorithm also exploits the minimum hash
value computation process within the MinHashing phase. In the meanwhile, the Exponential mechanism and Randomized Response will be carefully adopted for maintaining an acceptable utility.

4 Randomness Analysis within MH-JSC

In this section, in order to study the relationship between the internal randomness within the MH-JSC and the differential privacy, we first provide a relaxed definition of differential privacy ($\epsilon$-DPSO) for set operations, and then prove that the MH-JSC satisfies the $\epsilon$-DPSO.

4.1 Differentially Private Set Operations

In the definition of differential privacy the neighbouring datasets are $S$ and $S'$ which differ in one element, while the algorithm $M$ is randomized with its non-deterministic output $M(S)$ which belongs to $\text{Range}(M)$. In what follows, we relax this definition for set operations.

For a data set pair $\{S_A, S_B\}$ that consists of two data sets $S_A$ and $S_B$, its neighboring data set pair $\{S_A', S_B\}$ or $\{S_A, S_B'\}$, where $S_A$ differs in one element with $S_A'$, and $S_B$ differs in one element with $S_B'$. The randomized algorithm $\tilde{M}$ is a set operation process with a nondeterministic output in the range $\text{Range}(\tilde{M})$. Based on the above setting, the Differentially Private Set Operations ($\epsilon$-DPSO) is formally defined as

**Definition 6 ($\epsilon$-DPSO).** A randomized set operation algorithm $\tilde{M}$ gives $\epsilon$-differential privacy if for all neighbouring data set pairs $\{S_A, S_B\}$ and $\{S_A', S_B\}$ differing on at most one element, and all $O \subseteq \text{Range}(\tilde{M})$, we have

$$\Pr[\tilde{M}(\{S_A, S_B\}) \in O] \leq e^\epsilon \cdot \Pr[\tilde{M}(\{S_A', S_B\}) \in O].$$

Next, for a randomized set operation algorithm $\tilde{M}$, we observe that although all its possible outputs belong to $O$, there maybe exist a narrower outputs set $O_\sigma$ that includes the most possible outputs of the algorithm. For example, in the MH-JSC, the probability for its output $J_{\text{mh}}(S_A, S_B)$ to be in the range $[\sigma - \theta, \sigma + \theta]$ could be relatively high if given appropriate parameters, as shown in Eq. 3. Therefore, as a condition, if we only focus on the most possible outputs $O_\sigma$ instead of all possible outputs $O$ of a randomized set operation algorithm $\tilde{M}$, the Conditional $\epsilon$-DPSO can be further defined as

**Definition 7 (Conditional $\epsilon$-DPSO).** A randomized set operation algorithm $\tilde{M}$ gives conditional $\epsilon$-differential privacy if for all neighbouring data set pairs $\{S_A, S_B\}$ and $\{S_A', S_B\}$ differing on at most one element, and for the most possible outputs $O_\sigma \subseteq \text{Range}(\tilde{M})$,

$$\Pr[\tilde{M}(\{S_A, S_B\}) \in O_\sigma] \leq e^\epsilon \cdot \Pr[\tilde{M}(\{S_A', S_B\}) \in O_\sigma].$$

4.2 Privacy Analysis of MH-JSC

Here, we will show that the MH-JSC satisfies the conditional $\epsilon$-DPSO:

**Theorem 3.** The MH-JSC satisfies the conditional $\epsilon$-DPSO.
Proof. Assume that $\sigma$ and $\sigma'$ are the value of $J(S_A, S_B)$ and $J(S_A, S_B)'$, respectively, and all sets $S_A$, $S_A'$, $S_B$ and $S_B'$ have the same size, that is, $|S_A| = |S_A'| = |S_B| = |S_B'| = n$, according to equation 1, we have

$$J(S_A, S_B) = \frac{|S_A \cap S_B|}{|S_A \cup S_B|} = \sigma$$

and

$$J(S_A', S_B) = \frac{|S_A' \cap S_B|}{|S_A' \cup S_B|} = \sigma'$$
or

$$J(S_A, S_B') = \frac{|S_A \cap S_B'|}{|S_A \cup S_B'|} = \sigma'$$

Then we can have $|\sigma - \sigma'| \leq \frac{1}{n}$, because the maximum change of the numerator between $\sigma$ and $\sigma'$ is 1 and the minimum of the denominator between $\sigma$ and $\sigma'$ is $n$.

According to Eq. 3 and above conclusion, we have

$$\frac{\Pr[J_{mh}(S_A, S_B) \in O_{\sigma}]}{\Pr[J_{mh}(S_A, S_B) \in O_{\sigma}]} = \frac{\Pr[J_{mh}(S_A, S_B) \in [\sigma - \theta, \sigma + \theta]]}{\Pr[J_{mh}(S_A, S_B') \in [\sigma - \theta, \sigma + \theta]]}$$

$$\leq \frac{\Pr[J_{mh}(S_A, S_B', S_B') \in [\sigma' - (\theta + \frac{1}{n}), \sigma' + (\theta + \frac{1}{n})]]}{\Pr[J_{mh}(S_A, S_B) \in [\sigma - \theta, \sigma + \theta]]}$$

$$= \frac{p(K, \sigma, \theta)}{p(K, \sigma', \theta + \frac{1}{n})}$$

$$= \frac{\sum_{K(\sigma - \theta) \leq t \leq K(\sigma + \theta)} \binom{K}{t} (\sigma)^t (1 - \sigma)^{K-t}}{\sum_{K(\sigma' - \theta + \frac{1}{n}) \leq t' \leq K(\sigma' + \theta + \frac{1}{n})} \binom{K}{t'} (\sigma')^{t'} (1 - \sigma')^{K-t'}}$$

$$= e^\epsilon.$$

Based on above formula derivations, we can conclude that if we use the $O_{\sigma}$ instead of $O$ to represent the most possible outputs of $M$, the computation process of MinHash-based Jaccard Similarity satisfies the conditional $\epsilon$-DPSO with $\epsilon = \ln\left(\frac{p(K, \sigma, \theta)}{p(K, \sigma', \theta + \frac{1}{n})}\right)$.

Since the above privacy property is based on the observation of a particular subset of the output space of MH-JSC, the MH-JSC still cannot achieve the strict differential privacy in which the privacy property should be maintained across all the output space. That is to say, the randomness within the MH-JSC can only lead to a limited indistinguishability of its outputs, and external noise is still required for MH-JSC to achieve the $\epsilon$-differential privacy, as shown in Section 5.

5 Private Jaccard Similarity Computation

In this section, we propose a Private MinHash-based Jaccard Similarity Computation (PrivMin) algorithm to achieve the strict differential privacy in MH-JSC.
5.1 Algorithm Overview

The PrivMin algorithm aims to release the MinHash-based Jaccard similarity between any two cloud users by the Profile Perturbation approach, which ensures that each user’s private information can be protected from the passive attack similar to the one in Example 1. That is, based on observation of the released similarity, potential adversaries cannot re-identify the elements in the original user profiles. The rationale for PrivMin algorithm is shown in Fig. 3.

![Figure 3: Rationale for PrivMin Algorithm](image)

We examine the minimum hash value computation process within the MinHashing phase, and add the Exponential noise to the original MinHash signatures through leveraging the Randomized Response strategy. From the perturbed MinHash signatures, the adversary cannot infer the sensitive information within the users’ input profiles. Specifically, we conceptualize the PrivMin algorithm into two private operations:

**Private MinHash Value Generation** Based on the Exponential mechanism, this operation privately selects the minimum hash value in each step within the MinHashing phase. By default this operation will be executed in all the \( K \) steps and then the perturbed MinHash signature will be generated.

**Randomized MinHashing Steps Selection** In the generation of perturbed MinHash signature, this operation privately shrinks the number \( K \) into \( m \) by the Randomized Response technique, so that the total added Exponential noise is tightly controlled.

Details for the **Private MinHash Value Generation** is presented in Section 5.2, followed by the **Randomized MinHashing Steps Selection** in Section 5.3.

5.2 Private MinHash Value Generation

In this operation, we attempt to add the Exponential noise through the minimum hash value computation process within the MinHashing phase, and then generate the perturbed MinHash signatures \( h_{(K)}(S_A) \) and \( h_{(K)}(S_B) \), for the profiles \( S_A \) and \( S_B \). The intuition behind this operation is that we aim to add just enough noise by leveraging the internal noise in MH-JSC. In this way, by using the perturbed MinHash signatures, the final similarity would also be a noisy version from which the semi-honest users cannot successfully launch a passive attack.

More specifically, as proved in Section 4, the MH-JSC satisfies the Conditional \( \epsilon \)-DPSO because of its internal randomness. Herein, we first show that the MinHashing phase produces such randomness and it only satisfies the differential privacy in certain situations.

**Lemma 1.** The MinHashing phase only satisfies the \( \epsilon \)-differential privacy at certain situations in which the element difference between \( S \) and \( S' \) has an impact on the equality of their minimum hash value.
This is because when all the steps within the MinHashing phase adopt the Exponential MinHashing phase and makes full use of the internal noise, its utility remains far from acceptable.

Although the above operation relies on the minimum hash value computation process within the MinHashing phase and makes full use of the internal noise, its utility remains far from acceptable. This is because when all the steps within the MinHashing phase adopt the Exponential mechanism,
Algorithm 1 Private MinHash Value Generation

**Input:** Profile $S = \{s_1, s_2, ..., s_N\}$, $K$ hash functions, overall privacy budget $\epsilon$.

**Output:** Perturbed MinHash signature $\tilde{h}_{(K)}(S)$.

1. Initialize a null vector $\tilde{h}_{(K)}(S)$;
2. for $k \leftarrow 1...K$ do
   3. for $n \leftarrow 1...N$ do
      4. Compute the hash value $h_k(s_n)$;
      5. end for
   6. Construct the hash values set $h_k(S) = \{h_k(s_1), h_k(s_2), ..., h_k(s_N)\}$;
   7. Select the minimum hash value $\min\{h_k(S)\}$ with probability proportional to $\exp^{(\epsilon q(h_k(S), \psi))_{2K/\Delta \psi}}$;
   8. Append to $\tilde{h}_{(K)}(S)$;
9. end for
10. return $\tilde{h}_{(K)}(S)$

The cumulative noise would seriously distort the accuracy of outputs. Fortunately, we find that if we use the Randomized Response technique to select steps for adopting the Exponential mechanism, the combined algorithm will successfully achieve both in rigorous differential privacy and in acceptable utility. More specifically, the Randomized MinHashing Steps Selection operation consists of two main steps which are described as the following.

1. The operation randomly selects several steps out of all the $K$ steps within the MinHashing phase with probability $P_t$. For the convenience of recording this result, we maintain an Original Flip Vector $\vec{V}$ in which the binary value in the $k^{th}$ ($1 \leq k \leq K$) place represents whether the $k^{th}$ step is initially implemented with the Exponential mechanism. As the above generated vector should be prevented from potential attack and should not be directly used in the Private MinHash Value Generation operation, a Perturbed Flip Vector $\vec{V}'$ will be generated in the next step.

2. Aiming to satisfy the differential privacy in this step, we generate the Perturbed Flip Vector $\vec{V}'$ with Randomized Response technique as described in Section 2.1.3 and similar to the Permanent Randomized Response proposed in [13]. The Perturbed Flip Vector $\vec{V}'$ will indicate which steps within the MinHashing phase will finally be implemented with the Exponential mechanism. Specifically, given an Original Flip Vector $\vec{V}$, for the value $V_k$ in each bit $k \in [0, K]$ of $\vec{V}$, this step generates a perturbed binary value $V'_k$ which equals to:

$$V'_k = \begin{cases} 
1, & \text{with probability } 1/2P_t, \\
0, & \text{with probability } 1/2P_t, \\
V_k, & \text{with probability } 1 - P_t,
\end{cases}$$

where $P_t$ is the threshold probability to flip the original binary value. In this way, the Perturbed Flip Vector $\vec{V}'$ will be generated. It is noted that we directly set the $P_t = P_r$ in our experiments as a default setting. An intuition description of this procedure is shown in Fig 4.

By incorporating the Private MinHash Value Generation with the Randomized MinHashing Steps Selection, the pseudocode of the PrivMin algorithm is given in Algorithm 2. It starts with
Figure 4: Generation of Perturbed Flip Vector

a profile $S$ of a user, the overall privacy budget $\epsilon$ and $K$ hash functions. **Firstly,** the cloud platform initializes a null vector $\tilde{h}_{(K)}(S)$ (Line 1) and divides the overall privacy budget $\epsilon$ into two equal parts, $\epsilon_1$ and $\epsilon_2$ (Line 2-3). The former one is used to calculate the value of the probability threshold $P_t$ used in Randomized MinHashing Steps Selection while the latter will be assigned to the Private MinHashing Signature Generation. **Secondly,** the Perturbed Flip Vector generation would be triggered as described above (Line 4-5). **Thirdly,** according to the generated $\vec{V'}$, the Exponential mechanism would be deployed on the marked steps within the MinHashing phase (Line 6-15). And for the steps which are not marked in $\vec{V'}$, compute their original outputs (Line 16-22). **Finally,** the computed minimum hash values of each step would be appended to Perturbed MinHash Signature $\tilde{h}_{(K)}(S)$. The generated Perturbed MinHash Signature can be further used to compute the MinHash-based Jaccard similarity.

6 Algorithm Analysis

The proposed PrivMin algorithm aims to achieve the differential privacy while maintaining an acceptable utility. In this section, we will prove that the algorithm satisfies $\epsilon$-differential privacy and then provide the utility analysis.

6.1 Privacy Analysis

Based on the Sequential Composition and the Parallel Composition as in Theorem 1 and 2, we have the following theorem on the privacy guarantee of the proposed algorithm.

Theorem 4. The PrivMin algorithm satisfies $\epsilon$-differential privacy.

**Proof.** Two independent private operations of the PrivMin algorithm can respectively satisfy relevant level of differential privacy as follows:

- Based on the proofs in [13], since we adopt a similar randomized response approach as the Permanent Randomized Response in [13], we can conclude that the Randomized MinHashing Steps Selection operation satisfies $\epsilon_1$-differential privacy where $\epsilon_1 = K\ln(\frac{P_t}{1-P_t})$.

- As the Private MinHash Value Generation operation adopts the Exponential mechanism successively in the privately selected steps within the MinHashing phase, this operation satisfies $\epsilon_2$-differential privacy since the $m$ selected steps respectively achieve $\frac{\epsilon_2}{m}$-differential privacy.
Algorithm 2 PrivMin algorithm

Input: Profile $S = \{s_1, s_2, ..., s_N\}$, $K$ hash functions, overall privacy budget $\epsilon$.

Output: Perturbed MinHash signature $h_{(K)}(S)$.

1: Initialize a row vector $\tilde{h}_{(K)}(S)$;
2: $\epsilon_1 \leftarrow \epsilon/2$, $P_t \leftarrow \frac{e^{\epsilon_1/K}}{1 + e^{\epsilon_1/K}}$;
3: $\epsilon_2 \leftarrow \epsilon/2$;
4: Construct the Original Flip Vector $\vec{V}$ by randomly choosing its $K$ elements $r_1, r_2, ..., r_K$ from $\{0, 1\}$ with probability $P_r$;
5: Construct the Perturbed Flip Vector $\vec{V}'$ by implementing Randomized Response technique described in Section 5.3;
6: $m \leftarrow$ Compute the numbers of the elements in $\vec{V}'$ that is equal to 0;
7: $\epsilon' \leftarrow \epsilon_2/m$;
8: for $k \leftarrow 1...K$ do
9: if $V_k' = 0$ then
10: for $n \leftarrow 1...N$ do
11: Compute the hash value $h_k(s_n)$;
12: end for
13: Construct the hash values set $h_k(S) = \{h_k(s_1), h_k(s_2), ..., h_k(s_N)\}$;
14: Select the minimum hash value $\min\{h_k(S)\}$ with probability proportional to $\exp(\frac{\epsilon'(h_k(S), \psi)}{2\Delta_q})$;
15: Append to $\tilde{h}_{(K)}(S)$;
16: else
17: for $n \leftarrow 1...N$ do
18: Compute the hash value $h_k(s_n)$;
19: end for
20: Construct the hash values set $h_k(S) = \{h_k(s_1), h_k(s_2), ..., h_k(s_N)\}$;
21: Select the minimum value $\min\{h_k(S)\}$;
22: Append to $h_{(K)}(S)$;
23: end if
24: end for
25: return $\tilde{h}_{(K)}(S)$
Consequently, according to the *Sequential Composition*, we can conclude that the PrivMin algorithm satisfies $\epsilon$-differential privacy where $\epsilon = \epsilon_1 + \epsilon_2$.

### 6.2 Utility Analysis

Here we adopt $(\alpha, \delta)$-usefulness to measure the Semantic Loss in each step of the Private MinHash Value Generation operation.

**Theorem 5.** For all $\delta > 0$, with probability at least $1 - \delta$, the SLoss of the MinHash signatures in the Private MinHash Value Generation operation is less than $\alpha$. When

$$1 - \frac{3}{2}P_t + P_t^2 \leq \delta \alpha,$$

where $P_t = \frac{e^{\epsilon_1/K}}{1 + e^{\epsilon_1/K}}$, and the Private MinHash Value Generation operation is satisfied with $(\alpha, \delta)$-useful.

**Proof.** According to Markov’s inequality, we have

$$\Pr(\text{SLoss} > \alpha) \leq \frac{E(\text{SLoss})}{\alpha} \tag{6}$$

For each minimum hash value $\min\{h_k(S)\}$ in $\hat{h}_{(K)}(S)$, the probability of “unchange” in the randomized private selection is proportional to

$$P_r \cdot \left(\frac{1}{2}P_t + 1 - P_t\right) + \left(1 - P_r\right) \cdot \frac{1}{2}P_t = P_r - P_rP_t + \frac{1}{2}P_t$$

$$= \frac{3}{2}P_t - P_t^2.$$

Therefore, we have

$$E(\text{SLoss}) = \sum_{\min\{h_k(S)\} \in \hat{h}_{(K)}(S)} d(\min\{h_k(S)\}, \hat{\min\{h_k(S)\}}) (1 - \frac{3}{2}P_t + P_t^2).$$

According to Eq. (6), the evaluation of the SLoss is

$$\Pr(\text{SLoss} > \alpha) \leq \frac{\sum_{\min\{h_k(S)\} \in \hat{h}_{(K)}(S)} d(\min\{h_k(S)\}, \hat{\min\{h_k(S)\}}) (1 - \frac{3}{2}P_t + P_t^2)}{\max d \cdot |h_{(K)}(S)| \cdot \alpha}.$$ 

When we take the maximal $d(\min\{h_k(S)\}, \hat{\min\{h_k(S)\}}) = K$, it can be simplified as

$$\Pr(\text{SLoss} > \alpha) \geq 1 - \frac{1 - \frac{3}{2}P_t + P_t^2}{\alpha} \tag{7}.$$ 

Let

$$1 - \frac{1 - \frac{3}{2}P_t + P_t^2}{\alpha} \geq 1 - \delta,$$

thus

$$1 - \frac{3}{2}P_t + P_t^2 \leq \delta \alpha \tag{8},$$

where $P_t = \frac{e^{\epsilon_1/K}}{1 + e^{\epsilon_1/K}}$.

The proof shows that the Semantic Loss of the Private MinHash Value Generation operation mainly depends on the privacy budget $\epsilon_1$ and the hash function number $K$. 

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7 Experiment and Analysis

In this section, we conduct experiments to examine the performance of the proposed PrivMin algorithm by answering the following questions:

**How does the PrivMin algorithm preserve the utility?** The PrivMin algorithm aims to release Jaccard similarities with acceptable utility. In Section 7.2, we will investigate its performance in terms of $F_1$ Score and Mean Squared Error (MSE) on the released similarities, and compare it with the Baseline algorithm and MH-JSC.

**How will the main parameters impact on the performance of it?** The PrivMin algorithm has two parameters $\epsilon$ and $K$: $\epsilon$ controls the privacy level of algorithms; and $K$ determines the total number of Hash functions which are used in MinHashing phase. In Section 7.2.1 and 7.2.2, we will investigate and analyze their impacts on the involved three algorithms.

7.1 Experiment Setting

7.1.1 Datasets and Configuration

We evaluate the compared algorithms on four real textual datasets:

- **Alpine Dale**: The Alpine Dale dataset\(^1\) was retrieved from the course website of “text technologies for data science” by the University of Edinburgh, and includes 10000 news stories for plagiarism detection. In the following experiments, we will use a subset with 1000 records.

- **BBC Sport**: This dataset was derived from Insight Project Resources\(^2\). It contains 737 documents from the BBC Sport website corresponding to sports news articles in five topical areas from 2004 – 2005.

- **Opinosis\[^{21,14}\]**: This dataset from Paraphrase Grouped Corpora\[^{3}\] is a subset of the Opinosis corpus\[^{4}\]. It contains 669 sentences which were manually grouped according to their meaning.

- **MSRP\[^{21,9}\]**: This dataset from Paraphrase Grouped Corpora is a subset of the the Microsoft Research Paraphrase corpus\[^{5}\]. It contains 859 sentences which was automatically grouped according to its original manually annotated meaning.

The involved three algorithms are implemented in Python 2.7 based on the code by Chris McCormick\[^{6}\]. All the experiments are conducted on an Intel Core i5-3210M 2.50GHz PC with 6GB memory. In each experiment, every algorithm is executed 10 times, and its average score is reported.

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\(^1\)http://www.inf.ed.ac.uk/teaching/courses/tts/assessed/assessment3.html
\(^2\)http://mlg.ucd.ie/datasets/bbc.html
\(^3\)http://white.ucc.asn.au/resources/paraphrase˙grouped˙corpora/
\(^4\)http://kavita-ganesan.com/opinosis-opinion-dataset
\(^5\)http://research.microsoft.com/en-us/downloads/607d14d9-20cd-47e3-85bc-a2f65cd28042/
\(^6\)http://mccormickml.com/2015/06/12/minhash-tutorial-with-python-code/
7.1.2 Experiment Parameters

We consider two parameters $\epsilon$ and $K$ since the performance of algorithms could be affected by them:

the privacy budget $\epsilon$  Although the tradeoff between the privacy budget $\epsilon$ and the utility under the naive Laplace mechanism and the Exponential mechanism is known, we also expect to discover the situation in which the Randomized Response cooperates with the Exponential mechanism.

the number of Hash functions $K$  Although it is clear that a smaller number of hash functions may lead to worse accuracy in similarity, we expect that the PrivMin algorithm to perform well when $K$ is relatively small.

In our experiments, we will vary above two parameters to study their impacts on the involved algorithms, in terms to the metrics as mentioned in Section 7.1.3.

7.1.3 Utility Metrics

We adopt the $F_1$ score and Mean Squared Error (MSE) to measure the utility performance among the proposed PrivMin algorithm, the Baseline algorithm and the MH-JSC.

$F_1$ Score  The $F_1$ Score is the harmonic mean of Precision $P$ and Recall $R$, which can measure the algorithm outputs’s accuracy compared with the given ground truth. A higher $F_1$ Score means a better accuracy. Herein, the accurate Jaccard similarity of given two profiles is set as the ground truth. We aim to investigate the statistical differences between the released perturbed Jaccard similarity and the accurate one in several tests. The $F_1$ Score can be calculated as

$$F_1 = \frac{2 \times P \times R}{P + R},$$

(9)

$$P = \frac{TP}{TP + FP},$$

(10)

$$R = \frac{TP}{TP + FN},$$

(11)

where $TP$ is true positive, $FP$ is false positive, $TN$ is true negative, $FN$ is false negative. Table 3 shows the details of setting for these four variables. According to the specific characteristics of the textual records within four datasets, we empirically set the related thresholds as 0.5, 0.4, 0.5 and 0.3.

Mean Squared Error (MSE)  The Mean Squared Error (MSE) is a measure of the quality of an estimator by calculating the error between the estimator’s predicted value and its accurate value. A lower MSE means a better accuracy. The MSE in the following experiments can be calculated as

$$MSE = \frac{1}{n} \sum_{t=1}^{n} (predicted_t - accurate_t)^2,$$

(12)

where the $predicted_t$ and $accurate_t$ are corresponding to the perturbed Jaccard similarity and accurate Jaccard similarity, respectively. We aim to investigate the numerical errors between the released perturbed Jaccard similarity and the accurate one in several tests.
Table 3: Settings of TP, FP, TN and FN

|   | Description                                                                 |
|---|-----------------------------------------------------------------------------|
| TP | the number of test in which both the perturbed Jaccard similarity and the accurate Jaccard similarity are above a given threshold. |
| FP | the number of test in which only the perturbed Jaccard similarity is above a given threshold and the accurate Jaccard similarity is not. |
| TN | the number of test in which both the perturbed Jaccard similarity and the accurate Jaccard similarity are below a given threshold. |
| FN | the number of test in which only the perturbed Jaccard similarity is below a given threshold and the accurate Jaccard similarity is not. |

7.1.4 Compared Algorithms

We consider a Baseline algorithm and the MH-JSC as the competitors of the PrivMin algorithm.

Baseline The Baseline algorithm is based on the Output Perturbation approach which introduces differential privacy by directly adding Laplacian noise to the output similarity \( J_{mh}(S_A, S_B) \) of MH-JSC:

\[
\tilde{J}_{mh}(S_A, S_B) = J_{mh}(S_A, S_B) + \text{Laplace}(\frac{\Delta J_{mh}}{\epsilon}).
\]

The added Laplacian noise is calibrated to the sensitivity of MH-JSC as the following:

\[
\Delta J_{mh} = \max_{S_B, S_{B'} \text{neighbours}} \|J_{mh}(S_A, S_B) - J_{mh}(S_A, S_{B'})\|
\]
\[
= \max_{S_B, S_{B'} \text{neighbours}} \|h(K)(S_A) \cap h(K)(S_B) - |h(K)(S_B) \cap h(K)(S_A)|\|
\]
\[
\leq \max_{S_A, S_B} \left\| h(K)(S_A) \cap h(K)(S_B) - |h(K)(S_A) \cap h(K)(S_B)| \pm 1 \right\|
\]
\[
= \frac{1}{K}.
\]

Finally, the Baseline algorithm will release the perturbed similarity \( \tilde{J}_{mh}(S_A, S_B) \). to the users \( U_A \) and \( U_B \). Since the Baseline algorithm intuitively adds coarse-grained noise to achieve differential privacy, we expect that it will underperform the PrivMin algorithm in most cases.

MH-JSC The MinHash-based Jaccard Similarity Computation (MH-JSC) can be regarded as a comparative algorithm which maintains an empirical utility upper bound. Since MH-JSC does not add any external noise, we expected that it will outperform both the PrivMin algorithm and the Baseline algorithm in most cases, and it’s performance will also be much closer to that of the PrivMin algorithm.

7.2 The Performance of PrivMin

7.2.1 Impact of Privacy Budget

Firstly, we fix \( K = 5, 10, 15, 20, 25 \) and report the utility measures of different algorithms when varying \( \epsilon \) from 0.1 to 1.0. Fig. 5 shows the F1 score over four datasets with the change of \( \epsilon \). We
observe that the PrivMin algorithm has higher F1 Scores than the Baseline algorithm when given smaller K and ε on all datasets. Specifically in Fig. 5D, when K = 5 and ε = 0.2, PrivMin achieves a F1 Score of 0.3060 while Baseline achieves only 0.0006, with an improvements by 50900%. When ε = 0.5, PrivMin achieves a F1 Score of 0.6881 and outperforms the Baseline by 49050%. The improvements by PrivMin can also be observed in other subfigures in Fig. 5. For the Baseline algorithm, the larger ε, the higher F1 scores. However, we observe that the PrivMin algorithm is not clearly affected by changing the privacy budget.

For the Mean Squared Error (MSE), Table 4 shows that the PrivMin algorithm generally outperforms the Baseline algorithm with the changing privacy budget. And in some conditions, it also maintains a better utility compared with the MH-JSC.

Table 4: Comparison of the MSE on Different Algorithms

| Parameters | K      | 5   | 15  | 25  |
|------------|--------|-----|-----|-----|
|            | ε      | 0.01| 0.1 | 1   | 0.01| 0.1 | 1   |
| Alpine Dale| Baseline | 801.6931 | 8.003136 | 0.080065 | 88.86992 | 0.080652 | 0.008917 | 32.03135 | 0.320059 | 0.003222 |
|            | PrivMin | **0.000055** | 0.000060 | 0.000063 | **0.000024** | 0.000026 | 0.000026 | **0.000016** | 0.000017 | 0.000016 |
|            | MH-JSC  | 0.000117 |             |             |         | 0.000031 |             | 0.000022 |
| BBC Sport  | Baseline | 798.8903 | 8.030287 | 0.080235 | 88.95124 | 0.89008 | 0.00896 | 31.94539 | 0.320657 | 0.003230 |
|            | PrivMin | **0.000090** | **0.000088** | 0.000096 | **0.000060** | 0.000062 |         | **0.000046** | **0.000044** | 0.000056 |
|            | MH-JSC  | **0.000156** |             |             | **0.000031** |             |             | **0.000023** |
| Opinosis   | Baseline | 800.8486 | 7.991925 | 0.080309 | 88.57464 | 0.888713 | 0.008927 | 32.03530 | 0.319903 | 0.003221 |
|            | PrivMin | 0.000100 | **0.000099** | 0.000106 | 0.000048 | 0.000053 | **0.000042** | 0.000037 | **0.000035** | **0.000035** |
|            | MH-JSC  | 0.000120 |             |             | 0.000040 |             |             | 0.000025 |
| MSRP       | Baseline | 798.3747 | 7.996413 | 0.080602 | 89.04559 | 0.887925 | 0.008916 | 31.98979 | 0.320710 | 0.003216 |
|            | PrivMin | 0.000135 | **0.000114** | **0.000108** | **0.000099** | **0.000124** | 0.000101 | 0.000084 | 0.000089 | **0.000073** |
|            | MH-JSC  | 0.000095 |             |             | 0.000031 |             |             | 0.000018 |

7.2.2 Impact of Hash Function Number

For the F1 Score, we fix ε = 0.1, 0.5, 1.0 and vary the size K of a single MinHash signature, to study its impact on the utility of each algorithm. The results are shown in Fig. 6. As expected, for the Baseline algorithm its utility measures increase when K increases. We also observe that the PrivMin algorithm is not clearly affected by the changing K.

For the Mean Squared Error (MSE), Table 4 shows that the PrivMin algorithm generally outperforms the Baseline algorithm with the changing K. And in some conditions, it also maintains a better utility compared with the MH-JSC.

7.2.3 Summary and Recommendations

Remarkably, although the Baseline algorithm can achieve ε-differential privacy by adopting the Laplace mechanism, it can hardly maintain an acceptable F1 Score unless both ε and K are large (e.g., ε ≥ 1.0 and K ≥ 20, or ε ≥ 0.8 and K ≥ 25 in experimental results shown in Fig. 6 and Fig. 5). This is because that in the design of the Baseline algorithm, the MH-JSC is directly considered as a “black box”, and the noise adding is applied on this box’s output without exploiting the minimum hash value computation process inside the box. In contrast, the intuition behind the PrivMin
Figure 5: Varying $\epsilon$: F1
Figure 6: Varying $K$: F1
algorithm is that introducing the Exponential noise to the minimum hash value computation process of MinHashing phase in a privately randomized way, which helps the proposed algorithm achieve both $\epsilon$-differential privacy and acceptable utility. Based on the above empirical results in terms of utility metrics, when using the proposed PrivMin algorithm, we recommend that $\epsilon \leq 1.0$ and $K \leq 20$ or, $\epsilon \leq 0.8$ and $K \leq 25$.

8 Conclusions

Jaccard Similarity Computation is an essential process which has been widely used in many real-world applications such as recommendation and plagiarism detection. However, its potential privacy leakage is an emerging issue that needs to be addressed. Current research pay little attention on the MinHash-based Jaccard Similarity Computation (MH-JSC) for designing a differentially private algorithm. This paper studies the MH-JSC under the relaxed and the strict differential privacy with the following contributions:

- We first provide a relaxed definition of $\epsilon$–DPSO that extends the differential privacy into set operations. It is found that the MH-JSC satisfies the Conditional $\epsilon$ – DPSO naturally. Relevant theorem and detailed proof of privacy analysis are provided in Section 4.

- Based on the above analysis, we then proposed the PrivMin algorithm in Section 5 to achieve the differential privacy. The proposed algorithm consists of two private operations, the Private MinHashing Value Generation that applies the naive Exponential mechanism for the MinHashing phase, and the Randomized MinHashing Steps Selection which takes the advantages of the Randomized Response technique.

These contributions constitute a practical solution to the differentially private Jaccard similarity computation with less utility loss. Our theoretical and experimental analysis in Section 6 and 7 show that the proposed PrivMin algorithm could reserve acceptable utility.
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