Research on improving ERT reconstruction precision based on combined algorithm

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Abstract. Among the traditional image reconstruction algorithm for ERT (Electrical Resistance Tomography) system, the Landweber algorithm is the most commonly used iterative algorithm, with moderate computation and good reconstruction quality. However, because "soft field" error is usually ignored in reconstruction, there is still much room for improvement in the quality of reconstructed images. Aiming this problem, a combined algorithm is proposed. The Landweber reconstruction results were taken as the initial population position of the particle swarm optimization (PSO). Through the random forest regression model, the "soft field" error prior condition is obtained, and used in the construction of the PSO objective function to eliminate the influence of ignoring the "soft field" error. The simulation experiment results show that the proposed algorithm effectively improves the accuracy of the reconstructed image.

1. Introduction
Electrical Resistance Tomography is regarded as a promising process tomography technology with advantages of non-invasive, rapid, low cost and non-radioactivity. Whether ERT technology can be successfully applied largely depends on the accuracy and speed of imaging algorithm [1]. In order to improve the accuracy of ERT reconstruction, various image reconstruction algorithms are constantly proposed. Li et al. proposed an improved Conjugate Gradient (CG) algorithm to overcome the low convergence rate of the CG method [2]. Yan et al. proposed an image reconstruction method based on improved sensitivity matrix, they enhanced the sensitivity values in the central region, and reduced the ill-conditioned degree of sensitivity matrix [3].

High quality ERT image reconstruction is always a problem worth exploring. Among many commonly used image reconstruction algorithms, the most representative ones are LBP and Landweber algorithm. LBP calculation speed is fast but reconstruction quality is poor. The Landweber is iterative algorithm and can provide better reconstruction results. However, in the image reconstructed by Landweber algorithm, the boundary between target and background target is still relatively fuzzy, and the image quality still has a large room to improve [4].

In order to further improve the reconstruction quality of ERT system, an algorithm combining Landweber algorithm and PSO algorithm (Landweber-PSO) is proposed in this paper. The fitness function considering "soft field" error is established, and PSO algorithm is used to further improve the reconstruction results of Landweber algorithm. The effectiveness of the proposed method is verified by simulation experiments.
2. The forward problem model of ERT and "soft field" effect

Image reconstruction is generally realized by using the forward problem model based on the sensitivity matrix. The forward problem model of ERT based on the sensitivity matrix is usually shown in Equation (1).

\[ \lambda = S G \]  

\( S \) is the normalized sensitivity matrix; \( G \) is the normalized conductivity, namely the gray value vector; \( \lambda \) is the normalized boundary voltage vector.

The sensitive field of ERT is not uniformly distributed. The sensitivity of the central region of the sensor is far lower than that of the edge region. At the same time, the sensitivity is also affected by the conductivity distribution of the measured medium, which is usually called the "soft field" effect [5].

ERT image reconstruction uses the sensor to measure the boundary voltage and adopts the corresponding reconstruction algorithm to carry out the image reconstruction. In essence, it is an inverse problem solving process. The inverse problem of ERT is usually solved using the sensitivity matrix of empty tube, that is, the sensitivity matrix is obtained in the case of no target medium in the measured area. When there is target medium in the measured area, the sensitivity matrix is different from that of the empty tube due to "soft field". Therefore, high quality image reconstruction is difficult to achieve because of ignoring "soft field" error. If "soft field" error is considered and reduced as the optimization goal, the reconstruction accuracy can be further improved.

3. A combined algorithm based on Landweber and PSO

The basic idea of the combined algorithm proposed in this paper can be stated as follows. The normalized conductivity reconstructed by Landweber algorithm was taken as the initial position of PSO individuals, and the fitness function considering "soft field " error was constructed. The prior condition of the "soft field " error is obtained by using the random forest regression prediction algorithm, and the optimal solution of the fitness function was obtained by using the PSO algorithm.

3.1 The Landweber iterative algorithm

The essence of the Landweber algorithm is to obtain a generalized inverse matrix of sensitivity matrix to approximate inverse matrix of sensitivity matrix [6]. The Landweber iterative algorithm is shown in Equation (2):

\[ G_{k+1} = G_k + \frac{2}{\gamma_{\text{max}}} S^T (\lambda - SG_k) \]

\[ G_0 = S^T \lambda \]

\( G_k \) is the normalized conductivity vector of the iteration in step \( k \); \( G_0 \) is the initial gray value of image calculated by LBP algorithm; \( S \) is the sensitivity matrix; \( \lambda \) is the normalized boundary voltage vector; \( \gamma_{\text{max}} \) is the maximum eigenvalue of \( S^T S \).

3.2 The particle warm optimization algorithm

In this paper, two combined algorithms of Landweber and PSO algorithm are proposed. One is the fusion of Landweber with the standard particle swarm called Landweber-SPSO. The other is the fusion of Landweber with double particle swarm called Landweber-DPSO. Both of the PSO algorithms adopt the decayed weight mechanism \( \exp(-\alpha_k) \). Where, \( \alpha_k \) is the decayed constant, and after repeated verification, its value is 1. The purpose of adopting this mechanism is to reasonably control the particle velocity updating process and balance the local and global optimization capabilities.

3.3 The standard particle swarm optimization algorithm

In the \( D \) dimensional search space, \( n \) particles form a population. Each particle contains a \( D \)
dimensional position vector $\mathbf{X}_i$ and a velocity vector $\mathbf{V}_i$. The position vector $\mathbf{X}_i$ represents a feasible solution to the optimization problem. The velocity vector $\mathbf{V}_i$ reflects the updating process of position, which affects the convergence speed of the algorithm. In the standard PSO algorithm, particle update status is shown in Equation (3):

$$
\mathbf{V}(t+1) = \exp(-\alpha)\mathbf{V}(t) + c_1r_1[p_{best}(t) - \mathbf{x}(t)] + c_2r_2[p_g(t) - \mathbf{x}(t)]
$$

$$
\mathbf{X}(t+1) = \mathbf{X}(t) + \mathbf{V}(t+1)
$$

(3)

It is difficult to find the optimal solution of image reconstruction by standard PSO algorithm. Because, in order to achieve a high precision in the reconstructed image, the particle position must have a high dimension, and multi-dimension is often trapped into local optimal due to the loss of particle population diversity.

### 3.4 Double particle swarm optimization algorithm based on Lotka-Volterra model

In order to balance the contradiction between particle diversity and convergence speed, Lotka-Volterra model, a new cooperative-competitive scheme is introduced [7]. Set the size of particle swarm $N_A$ and $N_B$ is $A$ and $B$, $\mathbf{V}^A_i$ and $\mathbf{X}^A_i$ ($\mathbf{V}^B_j$ and $\mathbf{X}^B_j$) are the velocity and position of the $i$th (or $j$th) particle in particle swarm $A$ (or $B$), respectively. The particle position is the conductivity distribution, namely the solution of image reconstruction. The corresponding acceleration factor of the two particle swarms have the same value, $C_1 = 2.15$, $C_2 = C_3 = 1.03$. $r_1$, $r_2$, and $r_3$ are random numbers between [0,1]. $p^A_i$ ($p^B_j$) is the individual historical optimal position of the particle swarm $A$ (or $B$); $p^A_g$ and $p^B_g$ are the historical optimal positions respectively; $p_g$ is the historical optimal position of two particle swarms, that is, the current optimal solution of image reconstruction. Its calculation method is shown in Equation (4):

$$
p_g = \min\left(p^A_g(t), p^B_g(t)\right)
$$

(4)

The update status of the velocity and position of the two-particle swarm is shown in Equation (5):

$$
\mathbf{V}^A_i(t+1) = \exp(-\alpha)\mathbf{V}^A_i(t) + C_1r^A_1[p^A_i(t) - \mathbf{x}^A_i(t)] + C_2r^A_2[p^A_g(t) - \mathbf{x}^A_i(t)]
$$

$$
= C_3r^A_3[p^A_g(t) + p^A_g(t) - 2\mathbf{x}^A_i(t)]
$$

$$
\mathbf{X}^A_i(t+1) = \mathbf{X}^A_i(t) + \mathbf{V}^A_i(t)
$$

$$
\mathbf{V}^B_j(t+1) = \exp(-\alpha)\mathbf{V}^B_j(t) + C_1r^B_1[p^B_j(t) - \mathbf{x}^B_j(t)] + C_2r^B_2[p^B_g(t) - \mathbf{x}^B_j(t)]
$$

$$
= C_3r^B_3[p^B_g(t) + p^B_g(t) - 2\mathbf{x}^B_j(t)]
$$

$$
\mathbf{X}^B_j(t+1) = \mathbf{X}^B_j(t) + \mathbf{V}^B_j(t)
$$

(5)

The Lotka-Volterra model can fully consider all kinds of relationships among groups and avoid the problem that the standard particle swarm cannot be taken into account. In this process, each particle is not only attracted by the individual optimal of its own particle, but also affected by the historical optimal of another subgroup and the global optimal of two subgroups, thus increasing the diversity of particles and effectively avoiding premature convergence. It gives consideration to both local and global search capabilities.

### 3.5 Optimization of objective function and prediction of prior conditions

To construct the optimization objective function, $\Delta \lambda$ in Equation (6) is defined as the difference between the voltage measured value $\lambda$ and the voltage value corresponding to the true distributed conductivity $\mathbf{g}_m$, namely the actual voltage deviation, which reflects the error caused by the "soft
field” effect. In Equation (6), \( \Delta \lambda \) is defined as the difference between the voltage measured value \( \lambda \) and the voltage value corresponding to the reconstructed conductivity \( g_r \), namely the reconstructed voltage deviation.

\[
\Delta \lambda_1 = \lambda - S \cdot g_m \\
\Delta \lambda_2 = \lambda - S \cdot g_k
\]  

(6)

Usually, the intelligent algorithm takes \( \| \Delta \lambda_1 \| \) minimum as the optimal objective function for ERT reconstruction, but such objective function cannot correct the "soft field" error. For this reason, this paper proposed an objective function to consider "soft field" error, that is, the minimum difference between \( \| \Delta \lambda_1 \| \) and \( \| \Delta \lambda_2 \| \) is defined as the objective function, as shown in Equation (7). This objective function can make the reconstructed conductivity \( g_k \) closer to the true distributed conductivity \( g_m \).

\[
f = \min \{ \| \Delta \lambda_1 \| - \| \Delta \lambda_2 \| \}
\]  

(7)

Since different distribution have different prior condition \( \| \Delta \lambda_i \| \), this paper adopts random forest regression prediction algorithm to obtain prior condition \( \| \Delta \lambda \| \).

In the training stage, the random forest uses bootstrap sampling to collect several different sub training data sets from the input training data set to train multiple different decision trees in turn; in the prediction stage, the random forest averages the prediction results of multiple decision trees to get the final result.

In this paper, four typical prototypes were generated as the sample data set, where 8 samples for each prototype, and a total of 32 groups of sample data. The data set included the conductivity distribution of each prototype and its voltage deviation vector. The input of random forest is \( \{(g_{m1}, \Delta \lambda_{i1}),(g_{m2}, \Delta \lambda_{i2}),\ldots,(g_{mi}, \Delta \lambda_{ni})\} \), each of tree decision-making produces a prediction value, and the final prediction value \( \| \Delta \lambda \| \)is the average of these prediction results [8]. Since the real distribution in the actual measurement process is unknown, in this paper, the reconstructed result of Landweber is used as the \( g_m \). By inputting \( g_m \) to the pre-trained prediction model, \( \| \Delta \lambda \| \) can be finally predicted.

4. Steps of combined algorithm
The combined algorithm proposed in this paper can be divided into two stage.

Stage one, random forest training. In this stage, 32 groups of conductivity distribution and voltage deviation are input to construct the sample data set. Using the resampling technique, the new training sample sets containing 20 samples are generated and used to train the decision tree prediction model.

Stage two, image reconstruction. Following steps are included in this stage.

(1) Initial reconstruction. An initial reconstruction results is obtained by the Landweber algorithm, the number of iteration is set as 200.

(2) Prior condition prediction. The reconstruction result of the Landweber algorithm is regarded as true distributed conductivity and assigned to \( g_m \), then the prior condition \( \| \Delta \lambda \| \) is obtained by the decision tree prediction model.

(3) Initialize particle swarm parameters. Population number, iteration times, etc., and the reconstruction results of the Landweber algorithm is assigned to the initial positions of all particles.

(4) The velocity and position of each particle are updated according to equations (3) ~ (5);

(5) The fitness value of each particle is calculated, the individual optimal particle and the global optimal particle are updated:
Determine whether the maximum number of iterations has been reached. If so, the algorithm will end; otherwise, go back step (5) and continue searching until output result.

5 Analysis of simulation reconstruction results
In order to verify the effectiveness of the algorithm, the 16 electrode ERT system is used for simulation experiment. The five typical distributions were reconstructed with LBP, Landweber, Landweber-SPSO and Landweber-DPSO algorithms. During reconstruction, the pipeline section was divided into 48×48 grids, of which 1,804 were located in the effective imaging area. The number of PSO algorithms iteration steps is 100. Figure 1 shows the reconstructed images of the prototype and the results of four algorithms. The prototypes (a)–(d) are similar to the sample data set, but the radius size and center position of the target are different from the sample data, while the prototype (e) is not in the data set sample. The reconstruction errors of the four algorithms are shown in Table 1.

It can be seen from Table 1 that the LBP cannot correctly reconstruct the central position of the measured target when the targets are close to each other. Compared with LBP, the reconstruction quality of Landweber algorithm is improved obviously, but there are still fuzzy boundaries. Compared with the Landweber algorithm, the target shape reconstructed by the Landweber-SPSO algorithm is closer to the prototype, but the background area has subtle scattered spots. Compared with the other three algorithms, landweber-DPSO algorithm can make the reconstructed image closer to the prototype, and the reconstruction quality is significantly improved.

| Prototype | LBP     | Landweber | Landweber-SPSO | Landweber-DPSO |
|-----------|---------|-----------|----------------|----------------|
| (a)       | CC 0.5459 | 0.8730    | 0.8743         | 0.7982         |
|           | SIE 21.05 | 2.0167    | 29.0667        | 0.5500         |
| (b)       | CC 0.4991 | 0.8945    | 0.8967         | 0.8094         |
The data in Table 2 can objectively reflect the difference between the reconstruction effects of each algorithm. The correlation coefficient (CC) of Landweber, Landweber-SPSO and Landweber-DPSO is obviously superior to LBP, and landweber-DPSO has the smallest spatial image error (SIE), which is significantly smaller than the other three algorithms. However, the SIE of Landweber-SPSO is relatively large, which is due to the low gray scatter artifact in the background.

6 Conclusions

In order to improve the accuracy of ERT image reconstruction, a combined image reconstruction algorithm based on the fusion of Landweber and PSO algorithm was proposed. The "soft field" error prior condition is obtained by the random forest regression model, and used in the construction of the PSO objective function to eliminate the influence of ignoring the "soft field" error. The simulation experiment results show that the proposed algorithm effectively improves the accuracy of the reconstructed image.

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