Proxy-SU(3): A symmetry for heavy nuclei

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Abstract.

The SU(3) symmetry realized by J. P. Elliott in the sd nuclear shell is destroyed in heavier shells by the strong spin-orbit interaction. On the other hand, the SU(3) symmetry has been used for the description of heavy nuclei in terms of bosons in the framework of the Interacting Boson Approximation, as well as in terms of fermions using the pseudo-SU(3) approximation. A new fermionic approximation, called the proxy-SU(3), has been recently introduced and applied to the even rare earths. We show that the applicability of proxy-SU(3) can be extended to even nuclei in the 28-50 proton shell, to even superheavy elements, as well as to odd-odd and odd rare earths. Parameter free predictions for the β and γ deformation parameters are presented and compared to alternative theoretical predictions and to existing data.

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1 Introduction

A new algebraic approach to heavy deformed nuclei, based on fermionic symmetries and called the proxy-SU(3) scheme, has been introduced recently \cite{1,2}. Its basic assumptions and microscopic justification have been discussed in Ref. \cite{1}.
and are further considered in the present Workshop in Ref. [3]. A first success of
the proxy-SU(3) scheme is the explanation of the prolate over oblate dominance
in deformed nuclei, which has been considered in Refs. [2, 4] and is further dis-
cussed in the present Workshop in Ref. [5]. The border of the prolate to oblate
transition is also determined [2, 5]. In addition, parameter-free predictions for
the deformation parameters $\beta$ and $\gamma$ for even rare earths have been predicted in
Ref. [2] and successfully compared to Relativistic Mean Field predictions [6]
and to existing data [7].

In the present work we obtain parameter-free predictions for the deformation
parameters $\beta$ and $\gamma$ for nuclei in the 28-50 proton shell, as well as for even
superheavy elements and we compare them to alternative theoretical predictions
and to existing data. Furthermore, we apply the proxy-SU(3) scheme in odd-odd
rare earths and odd rare earths and compare the results to existing theoretical
predictions.

2 Numerical results

2.1 Even nuclei in the 28-50 proton shell

This shell is of particular current interest, because of the presence of the $Z = 40$
subshell closure and the appearance of shape coexistence [8, 9] around it.

![Figure 1. Proxy SU(3) predictions for $Z = 32-46$ for $\gamma$. See Section 2.1 for further discussion.](image)

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Parameter-free proxy-SU(3) predictions for the $\gamma$ deformation parameter are shown in Fig. 1 and are compared to D1S Gogny calculations [10] in Fig. 2, while in Fig. 3 the parameter free proxy-SU(3) predictions for the $\beta$ deformation parameter are shown and compared to theoretical predictions by Relativistic Mean Field with the NL3 parametrization [6], D1S Gogny interaction [10], and the FRDM (2012) mass tables [11], as well as to available data [7].

In general, good agreement is observed for the $\beta$ deformation parameter between the proxy-SU(3) predictions and the predictions of alternative theories and the data. Deviations are stronger in the Sr, Zr, and Mo isotopes ($Z = 38 - 42$), which lie on or close to the $Z = 40$ subshell closure, which is not taken into account in the proxy-SU(3) scheme in any specific way.

In the case of the $\gamma$ deformation parameter, proxy-SU(3) shows a tendency to values near 30 degrees, indicating triaxial shapes, in the region around $N = 74$, while it climbs at even higher values, approaching 60 degrees (indicating oblate shapes) in the Ru and Pd isotopes in this region. Further study of these results is needed, taking into account that nuclei near the end of the neutron shell are not very well deformed, as indicated, for example, by their $R_{4/2} = E(4^+_1)/E(2^+_1)$ ratios [12].

2.2 Odd-odd and even-odd rare earths

Proxy-SU(3) results have been obtained for odd-odd and even-odd rare earths with $Z$ in the $sdg$ proxy-SU(3) shell (which is an approximation of the 50-82 shell) and neutrons in the $pfh$ proxy-SU(3) shell (which is an approximation of the 82-126 shell).

Parameter-free proxy-SU(3) predictions for the $\beta$ and $\gamma$ deformation parameters for odd-odd nuclei are shown in Figs. 4 and 5, compared to predictions for $\beta$ reported in the mass table FRDM(2012) [11], where they have been calculated within the finite-range droplet macroscopic model and the folded-Yukawa single-particle microscopic model. Good agreement is observed in general, with the largest deviations appearing for Au ($Z = 79$), i.e., near the end of the 50-82 proton shell.

A similar set of figures for odd-$N$ rare earths appears in Figs. 6 and 7. Again, the higher deviations appear near the end of the 50-82 proton shell, at the Pt ($Z = 78$) isotopes.

2.3 Even superheavy elements

Parameter independent proxy-SU(3) results have been obtained for superheavy elements (SHE) with $Z$ in the $pfh$ proxy-SU(3) shell (which is an approximation of the 82-126 shell) and neutrons in the $sdgi$ proxy-SU(3) shell (which is an approximation of the 126-184 shell), as well as in the $pfhj$ proxy-SU(3) shell.
(which is an approximation of the 184-258 shell). For the illustrative and pedagogical purposes of this work, we take the relevant shells for the actinides and super heavy nuclei as $Z = 82-126$, $N = 126-184$, and $N = 184-258$, although the upper bounds are by no means certain and microscopic calculations give many varying scenarios. Results are shown for $100 \leq Z \leq 114$. In order to have results from alternative calculations to compare our results with, we confine ourselves to $128 \leq N \leq 220$.

We compare our results to predictions contained in the following sources.

Extended results for $10 \leq Z \leq 110$ and $N \leq 200$ with the D1S Gogny interaction are given in [10] for the mean ground state $\beta$ deformation, as well as for the mean ground state $\gamma$ deformation.

Extended results for the proton deformation $\beta_p$ and the neutron deformation $\beta_n$ with covariant density functional theory for $96 \leq Z \leq 130$ and $N$ from the proton drip line up to $N = 196$ are given in Ref. [13] for the functionals PC-PK1 and DD-PC1.

Extended results for the deformation $\beta$ within a microscopic-macroscopic method (MMM) for $98 \leq Z \leq 126$ and $134 \leq N \leq 192$ are given in Ref. [14].

Extended results for the deformation $\beta$ up to $A = 339$ are reported in the mass table FRDM(2012) [11], calculated within the finite-range droplet macroscopic model and the folded-Yukawa single-particle microscopic model.

The results are summarized in Figs. 8 and 9. Overall good agreement is observed between the parameter-free proxy-SU(3) predictions and the alternative calculations.

3 Conclusion

In the present work, the applicability of the proxy-SU(3) scheme is tested, through parameter independent predictions for the deformation parameters $\beta$ and $\gamma$ for even nuclei in the 28-50 proton shell and even superheavy elements, as well as in odd-odd nuclei and odd-$N$ rare earths. In general, good agreement is obtained with results of alternative calculations, as well as with existing data. Some deviations seen, as for example around the $Z = 40$ subshell closure, which is playing a central role in shape coexistence in the relevant region, call for further investigations.

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Figure 2. Proxy SU(3) predictions for $Z = 32-46$ for $\gamma$, compared with D1S-Gogny calculations (D1S-Gogny) [10]. See Section 2.1 for further discussion.
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Figure 3. Proxy SU(3) predictions for $Z = 32-46$ for $\beta$, compared with results by relativistic mean field theory (RMF) [6], the D1S-Gogny interaction (D1S-Gogny) [10], and the mass table FRDM(2012) [11], as well as to the data (exp.) [7]. See Section 2.1 for further discussion.
Figure 4. Proxy SU(3) predictions for $\beta$ for odd-odd rare earths with $Z = 53$-67, compared with results reported in the mass table FRDM(2012) [11]. See Section 2.2 for further discussion.
Figure 5. Proxy SU(3) predictions for $\beta$ for odd-odd rare earths with $Z = 69-79$, compared with results reported in the mass table FRDM(2012) [11]. In the two bottom panels, the proxy-SU(3) predictions for $\gamma$ are reported for $Z = 53-79$. See Section 2.2 for further discussion.
Figure 6. Proxy SU(3) predictions for $\beta$ for even-odd rare earths with $Z = 54-68$, compared with results reported in the mass table FRDM(2012) [11]. See Section 2.2 for further discussion.
Figure 7. Proxy SU(3) predictions for $\beta$ for even-odd rare earths with $Z = 70-78$, compared with results reported in the mass table FRDM(2012) \cite{11}. In the two bottom panels, the proxy-SU(3) predictions for $\gamma$ are reported for $Z = 54-78$. See Section 2.2 for further discussion.
Figure 8. Proxy SU(3) predictions for $Z=100-114$ for $\beta$, compared with covariant density functional theory with the DD-PC1 functional (DD-PC1) [13] (in which case different values for protons (DD-PC1$_p$) and neutrons (DD-PC1$_n$) are reported), the microscopic-macroscopic method (MMM) [14], the D1S-Gogny interaction (D1S-Gogny) [10], and the mass table FRDM(2012) [11]. See section 2.3 for further discussion.
Figure 9. Proxy SU(3) predictions for $Z = 100$-$114$ for $\gamma$, compared with results by the D1S-Gogny interaction (D1S-Gogny) [10]. See section 2.3 for further discussion.