A Proposed Test of Charge Symmetry in $\Sigma$ Decay

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Abstract

The semi-leptonic decays of $\Sigma^{\pm}$ offer a vehicle for observing charge symmetry-breaking. The effect is expected to be about 6%, enhanced due to the replacement of two $u$ quarks by $d$ quarks. We propose that present experimental data be improved to search for this effect.
In the recent 1st International Symposium on *Symmetries in Subatomic Physics* Thomas\(^1\) discussed enhanced charge symmetry-breaking effects in the difference between the valence distribution of down quarks in the proton and up quarks in the neutron. The asymmetry is about 5-10 % for large values of Bjorken x, larger about than expected because there are two spectator u quarks in the proton and two spectator d quarks in the neutron.

There is a simpler process where the same enhancement occurs. This is a comparison of the semi-leptonic decay rates \(\Sigma^+ \rightarrow \Lambda^0 e^+ \nu_e\) to \(\Sigma^- \rightarrow \Lambda^0 e^- \bar{\nu}_e\). Since the \(\Sigma^+\) is \(uuu\) and the \(\Sigma^-\) is \(dds\), the rate comparison tests charge symmetry in a situation where two up quarks are replaced by two down ones.

As shown by Frampton and Tung\(^3\) and others\(^4\), the non-leptonic decay rate can be written as

\[
\Gamma = \left(\frac{G_F \sin \theta_c f_{\Sigma \Lambda}}{2^6 15 \pi^4 (1 + \delta)^3}\right) M_\Sigma^2 (3|f_1|^2 + 5|g_1|^2),
\]

where \(G_F\) is the Fermi coupling constant, \(\theta_c\) is the Cabibbo angle, and \(f_{\Sigma \Lambda}\) is an f-type SU(3) coefficient. Also we have defined

\[
M_- = M_\Sigma - M_\Lambda, \quad M_+ = M_\Sigma + M_\Lambda, \quad \delta = M_- / M_+,
\]

\(f_1\) is the matrix element of the weak vector current which we assume to be a dipole form factor \(f_1 = M_V^4 (q^2 + M_V^2)^{-2}\), with \(M_V \approx 0.84\) GeV, and \(q\) the 4-momentum transfer in the decay. In Eq. (1) \(g_1\) is the matrix element of the axial vector operator which we take as a number \(C_A\) times a form factor, \(g_1 = M_A^4 (q^2 + M_A^2)^{-2}\), where \(M_A \approx 1.05\) GeV. For \(\Sigma^\pm\) decay \(C_A\) is the matrix element of \(\bar{q} \gamma^\mu \gamma_5 \tau_\pm q\). If charge symmetry holds,

\[
|\Sigma^+ > = P_{cs} |\Sigma^- >,
\]

where the the charge symmetry operator\(^2\) \(P_{cs} = e^{i \pi T_2}\). This operator converts \(u\) quarks into \(d\) quarks and vice versa: \(P_{cs}|u> = -|d>\), \(P_{cs}|d> = |u>\). Charge symmetry leads to the result that \(C_A(\Sigma^+) = -C_A(\Sigma^-)\).
In this limit, we obtain

$$R \equiv \frac{\Gamma(\Sigma^-)}{\Gamma(\Sigma^+)} = \frac{[M_-(\Sigma^-)]^5}{[M_-(\Sigma^+)]^5} \frac{[1 + \delta(\Sigma^+)]^3}{[1 + \delta(\Sigma^-)]^3}$$

(4)

to an accuracy of better than 0.1%. The deviation from unity in Eq. 4 arises only from the difference in phase space factors caused by the mass difference between the $\Sigma^+$ and $\Sigma^-$. The difference of the form factors $f_1$ and $g_1$ for $\Sigma^+$ and $\Sigma^-$ decay is negligible. The use of Eq. (4) and known masses of $\Sigma^\pm$, $\Lambda^0$, give

$$R = 1.665 \pm 0.009.$$ 

(5)

At present, the decay rates are given by $\Gamma(\Sigma^+) = (2.5 \pm 0.6) \times 10^5 s^{-1}$, $\Gamma(\Sigma^-) = (3.87 \pm 0.18) \times 10^5 s^{-1}$. This gives

$$R^{exp} = 1.6 \pm 0.4.$$ 

(6)

The present branching ratio of $\Sigma^-$ is measured to about 5%, but that of $\Sigma^+$ only to 25%. These errors are much larger than typical charge symmetry breaking effects, which are typically of the order of 1-3%.

It is worthwhile to provide estimates of the size of the charge symmetry breaking effects of $C_A$. The simplest effect to consider is that of $|\Lambda^0 > -|\Sigma^0 >$ mixing. The physical $|\Lambda >$ and $|\Sigma^0 >$ are thought to be mixtures of pure isospin states

$$|\Lambda^0 > = |I = 0 > + \alpha |I = 1 >,$$

$$|\Sigma^0 > = -\alpha |I = 0 > + |I = 1 >,$$

(7)

with $\alpha \approx 0.013$ as estimated in Ref. 6. The quark-model origin of this effect is the charge symmetry breaking mass difference between up and down quarks which enters in the one-gluon exchange interaction. We estimate the matrix elements of the vector $\hat{V}$ and axial-vector $\hat{A}$ operators between the $|\Sigma^\pm >$ and the physical $\Lambda >$, using SU(6) wave
functions. The matrix element of $\hat{V}$ between the $|I = 0\rangle$ and $|I = 1\rangle$ pure isospin states vanishes, so the effects of mixing in Eq. (1) are of second order in $\alpha$ in the ratio $R$ and ignorable. A simple evaluation of the matrix element of $\hat{A}$ reveals that

$$<\Lambda|\hat{A}|\Sigma^\pm> = -(\pm \frac{2}{3}\sqrt{3} - \frac{4}{3}\alpha),$$

so the ratio of the square of the matrix elements is

$$(\frac{g^+}{g^-})^2 \approx 1 - \frac{8}{3}\sqrt{3}\alpha \approx 6\%.$$ (9)

This is a relatively large effect. Other charge symmetry breaking effects occur in the wave functions, but these are much smaller. The $\Sigma^+$ decay involves a $u$ in the $\Sigma^+$ changing into a $d$ in the $\Lambda$, while $\Sigma^-$ decay involves a $d$ in the $\Sigma^-$ changing into a $u$ in the $\Lambda$. In the simplest bag or constituent quark model the $u$ and $d$ wave functions of the $\Sigma$ and $\Lambda$ would be the same, but the quark mass difference would cause the $u$ wave function to differ from that of the $d$. However, that effect would give no charge asymmetry here because there is a single $u$ and a single $d$ wave function in both matrix elements. The charge asymmetry occurring via the effects of spatial wave function is therefore a subtle effect. In the MIT bag model\(^7\), a mass difference $\delta m$ between the up and down quarks is required to explain the mass difference between the neutron and proton. Using $\delta m = 4.3$ MeV\(^8\) leads to the result that $4$ MeV ($\equiv \Delta M$) of the $8.3$ MeV difference ($M_{\Sigma^+} > M_{\Sigma^-}$) must be supplied by electromagnetic and one gluon exchange effects. The use of the bag model stability equations gives $\Delta R/\tilde{R} = \frac{4\Delta M}{3M} \approx 1/900$, This is barely perceptible, and therefore very sensitive to a host of corrections. In any case, it is small and can therefore be neglected.

We also make an estimate using the non-relativistic quark model. The variational principle, with harmonic oscillator trial wave functions\(^9\), is used to simplify this first calculation. The result is that $\Delta R/\tilde{R} = \frac{3\Delta M}{4M}$ for oscillator confinement and $\Delta R/\tilde{R} =$
Here $\bar{R}$ and $\Delta R$ refer to the radius parameter of the harmonic oscillator wave functions. Computing the relevant overlaps gives a difference of 0.6 % (oscillator) or 1.2 % (linear) for the square of the matrix elements. There is extreme model dependence, but the effect is much smaller than the 6% expected from baryon mixing.

We would like to urge increased analysis of existing data or of new data to search for charge symmetry-breaking at the 1% level. Since this symmetry tests the effects of isospin mixing in the baryon wavefunction, it is of particular interest. No such asymmetry has yet been observed for a hadron.

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