Decentralized robust active disturbance rejection control of modular robot manipulators: an experimental investigation with emotional pHRI

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This work is supported by the National Natural Science Foundation of China (Grant nos. 61773075, 62173047 and 61703055), the Scientific Technological Development Plan Project in Jilin Province of China (Grant no. 20200801056GH).

ABSTRACT This paper presents an active disturbance rejection control (ADRC) method for modular robot manipulators (MRMs) based on extended state observer (ESO), which solves the problem of trajectory tracking when modular robot manipulators facing the emotional physical human-robot interaction (pHRI). The dynamic model of MRMs is formulated via joint torque feedback (JTF) technique that is deployed for each joint module to design the model compensation controller. ESO is used to estimate the interconnected dynamic coupling (IDC) term and the interference term caused by emotional pHRI. An uncertainty decomposition-based robust control is developed to compensate the friction term. The terminal sliding mode control (TSMC) algorithm is introduced to the controller to provide faster convergence and higher precision control effect. Based on the Lyapunov theory, the tracking error is proved to be ultimately uniformly bounded (UUB). Finally, experiments demonstrate advantages of the proposed method.

INDEX TERMS Modular robot manipulators, Extended state observer, Terminal sliding mode, Emotional pHRI, Active disturbance rejection control.

I. INTRODUCTION

Modular robot manipulators (MRMs) have drawn widely attentions in robotics community since they possess better structural adaptability and flexibility than conventional robot manipulators. MRM consists standardized robotic modules, which consists of actuators, speed reducers, sensors and communication units. MRMs are always employed in dangerous and complex environments, such as space exploration, hazard survey and furthermore physical human-robot interaction (pHRI). Nowadays, pHRI has become a research hotspot in the field of robotics. Emotional pHRI can ensure that when humans produce different kinds of emotions, the robots can continue to complete predefined tasks according to the established requirements. Hence, appropriate control systems are required to guarantee the robustness and precision of MRMs in contact with pHRI.

Besides the properties of modularity and pHRI etc., for achieving high-precision robot control, some scholars develop control issues of manipulator under conditions of random delay [1], input dead zone [2], input saturation [3], visual servo [4], optimization verification [5], and micro-positioning [6], etc. Biglarbegian [7] gave an interval type-2 fuzzy controller for trajectory tracking of desired motion. Kasprzak [8] proposed motion planning for multcorporeal MRM based on hierarchical search algorithms. Xu [9] gave an adaptive sliding mode control to achieve interaction control. Pham [10] proposed a robust control method. All aforementioned methods used the centralized control scheme. For practicality, the centralized control strategy is not suitable for the MRMs system because of its modularity and high complexity. Decentralized control is more suitable for MRMs system compared with centralized control strategy [11]-[12]. The advantages of decentralized control are to simplify complexity of the MRM system and effectively improve the running speed [13]-[15].

In order to address the problems in enhancing interaction...
stability and robustness, lots of researches focused on investigating active disturbance rejection control (ADRC) method for nonlinear system [16]-[17]. ADRC algorithm includes tracking differentiator, extended state observer (ESO) and nonlinear feedback, and it has been applied in many fields. In the field of robotics, ADRC algorithm has been applied in the control system of manipulator [18], such as Stewart platform [19] and calibration-free robotic eye-hand coordination [20]. Among them, ESO is a more effective and mature algorithm in ADRC. Huang et al. [21] studied a centralized controller based on ESO compensation, and carried out control simulation for a four-jointed finger. Ren et al. [22] proposed a model predictive control method with friction compensation based on the ESO for the trajectory tracking control problem of a omnidirectional mobile robot. However, the existing ESO-based ADRC of robot control method does not considering the pHRI. Indeed, the control torques of each robotic joint may increase significantly along with the instantaneous deviations of the feedback of joint position, velocity and torque measurements when in contact with external circumstance. In order to improve the convergence speed and the noise tolerance of dynamic systems, the sliding mode control (SMC) system [23] is introduced. Linear SMC is an insightful method for robot control, and it has been widely applied for its simple algorithm, fast response, and strong robustness [24]-[27]. However, the linear SMC cannot guarantee that the system error will converge to zero within a finite time. In order to provide faster convergence speed and higher control accuracy to compensate the disturbance term caused by the uncertain external environment, the terminal sliding mode control (TSMC) is designed to reach an insightful method for robot control, and it has been widely applied for its simple algorithm, fast response, and strong robustness [24]-[27]. However, the linear SMC cannot guarantee that the system error will converge to zero within a finite time. In order to provide faster convergence speed and higher control accuracy to compensate the disturbance term caused by the uncertain external environment, the terminal sliding mode control (TSMC) is designed to reach zero within a finite time. In order to provide faster convergence speed and higher control accuracy to compensate the disturbance term caused by the uncertain external environment, the terminal sliding mode control (TSMC) is designed to reach zero within a finite time.

Assumption 1: There is no loss of torque transmission at the location of reducer.

Based on MRMs with n modules via interconnected sub-systems, the ith subsystem of MRMs is [34]:

\[
I_{mi} \dot{q}_i + f_i(q_i, \dot{q}_i) + I_{mi} \sum_{j=1}^{i-1} T_{mi} z_{mj} \dot{q}_j + \frac{\tau_{fi}}{\gamma_i} = \tau_i, \\
I_{mi} \sum_{j=1}^{i-1} T_{mi} z_{mj} \dot{q}_j + I_{mi} \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} T_{mi} (z_{mk} \times z_{mj}) \dot{q}_k \dot{q}_j + d_i(q_i) = \tau_i,
\]

where \(d_i(q_i)\) denotes the disturbance because of uncertain environments. 

The major contributions are summarized:

1. To the best of author’s knowledge, it is first time to solve decentralized robust ADRC problem of MRMs with pHRI.

2. Because of different kinds of emotion, we propose a control algorithm to meet the trajectory tracking problem of people in different emotions, and the tracking performance of MRM system can be improved by experimental verification.

II. DYNAMIC MODEL FORMULATION

A. DYNAMIC MODEL FORMULATION

Since MRMs have many mechanical modules, we formulate the MRM subsystem dynamic model (see Fig. 1). Similarly in [33], we assume following conditions.

\[
I_{mi} \dot{q}_i + f_i(q_i, \dot{q}_i) + I_{mi} \sum_{j=1}^{i-1} T_{mi} z_{mj} \dot{q}_j + \frac{\tau_{fi}}{\gamma_i} = \tau_i,
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\]

FIGURE 1. Schematic of joint module.
etc. When in anger or sadness emotion, humans will beat or collide with the robotic manipulator [35]-[37].

According to the reference in [38], \( f_i(q_i, \dot{q}_i) \) denotes:

\[
f_i(q_i, \dot{q}_i) = b_f \dot{q}_i + \left( f_{ci} + f_{si} e^{-f_{ri} \dot{q}_i^2} \right) sgn(q_i) + f_{qi}(q_i, \dot{q}_i),
\]

where \( f_{ci} \) is Coulomb friction, \( f_{si} \) is static friction, \( f_{ri} \) means Striebeck effect, \( b_f \dot{q}_i \) is viscous friction, \( f_{qi}(q_i, \dot{q}_i) \) denotes other friction errors.

We consider \( f_{si}, f_{ri} \) are approximate to actual values, and \( f_{si} e^{-f_{ri} \dot{q}_i^2} \) can be linearized as \( \hat{f}_{si}, \hat{f}_{ri} \). Hence, we have

\[
f_{si} e^{-f_{ri} \dot{q}_i^2} = (\hat{f}_{si} + \hat{f}_{ri}) e^{-f_{ri} \dot{q}_i^2} - \hat{q}_i^2 \hat{f}_{ri} e^{-f_{ri} \dot{q}_i^2} \\
\approx \hat{f}_{si} e^{-f_{ri} \dot{q}_i^2} + \hat{f}_{ri} e^{-f_{ri} \dot{q}_i^2} - \hat{q}_i^2 \hat{f}_{ri} e^{-f_{ri} \dot{q}_i^2}.
\]

(4)

Substituting (4) into (3), we have:

\[
f_i(q_i, \dot{q}_i) \approx \hat{b}_f \dot{q}_i + \left( \hat{f}_{ci} + \hat{f}_{ri} e^{-f_{ri} \dot{q}_i^2} \right) sgn(q_i) + f_{qi}(q_i, \dot{q}_i) + Y(q_i) \hat{F}_i,
\]

where \( \hat{F}_i = [\hat{b}_f - \hat{b}_f \hat{f}_{ci} - \hat{f}_{si} \hat{f}_{ri} - \hat{f}_{ri}^T] \) indicates friction parametric uncertainty, \( \hat{b}_f, \hat{f}_{ci}, \hat{f}_{ri}, \) and \( \hat{f}_{ri} \) denote estimated values of friction parameters, the vector is \( Y(q_i) \) regarded as

\[
Y(q_i) = \left[ q_i, sgn(q_i), e^{(f_{ri} \dot{q}_i^2)} sgn(q_i), f_{qi} e^{(f_{ri} \dot{q}_i^2)} sgn(q_i) \right].
\]

(5)

Rewrite the terms \( I_{mi} \sum_{j=1}^{i-1} z_{mi} \dot{z}_{qj} \dot{q}_j \) and \( I_{mi} \sum_{j=1}^{i-1} \dot{z}_{mj} \dot{q}_j \) for facilitating as follows:

\[
I_{mi} \sum_{j=1}^{i-1} z_{mi} \dot{z}_{qj} \dot{q}_j = I_{mi} \sum_{j=1}^{i-1} D_j \dot{q}_j
\]

\[
= \sum_{j=1}^{i-1} \left[ I_{mi} \dot{D}_j \right] \left[ I_{mj} \dot{q}_j \right]^T = \sum_{j=1}^{i-1} I_{ij} U_j
\]

(6)

\[
I_{mi} \sum_{j=2}^{i-1} \sum_{k=1}^{i-1} z_{jk} \dot{z}_{qj} \dot{q}_j = I_{mi} \sum_{j=2}^{i-1} \Theta_{jk} \dot{q}_j \dot{q}_k
\]

\[
= \sum_{j=2}^{i-1} \sum_{k=1}^{i-1} \left[ I_{mi} \Theta_{jk} \right] \left[ I_{mj} \dot{q}_j \dot{q}_k \right]^T
\]

\[
= \sum_{j=2}^{i-1} \sum_{k=1}^{i-1} \Theta_{jk} \dot{q}_j \dot{q}_k
\]

(7)

We obtain relationships of \( D_j = z_{mi} \dot{z}_{qj} \) and \( \dot{D}_j = D_j - \dot{D}_j \) in (7) and (8), where \( D_j \) is dot product of \( z_{mi} \) and \( \dot{z}_{qj} \) and \( \dot{D}_j \) is alignment error. We also have \( \Theta_{jk} = \Theta_{jk} (z_{mi} \dot{z}_{qj}) \), \( \dot{\Theta}_{jk} = \dot{\Theta}_{jk} (\dot{z}_{qj}) \), \( \Theta_{jk} \dot{z}_{qj} \) is alignment error. \( U_j \) and \( \dot{U}_j \) are model uncertainties.

**Property 1:** \( d_{is}(q_i) \) is caused by the emotional pHRi. The term \( d_{is}(q_i) \) is bounded, and up-bound is \( |d_{is}(q_i)| \leq \rho_{di}. \)

**Property 2:** For (3) and approximation (4), \( b_{fi}, f_{si}, f_{qi}, f_{ri} \) and their estimations are bounded, \( \hat{F}_i \) is also bounded, \( f_{qi}(q_i, \dot{q}_i) \) is bounded as \( |f_{qi}(q_i, \dot{q}_i)| \leq \rho_{fqi} \), where \( \rho_{fqi} \) is bounded for any position \( q_i, \dot{q}_i \).

\[\text{B. SYSTEM STATE SPACE DESCRIPTION}\]

Rewriting (1), we have

\[
\dot{\tilde{q}}_i = -B_i \left( \begin{array}{c}
\hat{f}_{ci} + \hat{f}_{si} e^{-f_{ri} \dot{q}_i^2} \\
y(q_i) \tilde{F}_i + d_i(q_i) + \frac{T_f}{\gamma_i} 4b_f \dot{q}_i - \tau_i
\end{array} \right) + \sum_{j=2}^{i-1} \sum_{k=1}^{i-1} V_{kj} + \sum_{j=1}^{i-1} U_j^i
\]

(9)

where \( B_i = (I_{mi} \gamma_i)^{-1} \). Define \( x_i = [x_{i1} \ x_{i2}]^T \). The state space equation is

\[
\begin{aligned}
\dot{x}_{i1} & = x_{i2} \\
\dot{x}_{i2} & = \phi_i(x_i) + h_i(x_i) + \theta_i(x_i) + B_i \tau_i
\end{aligned}
\]

(10)

where \( \phi_i(x_i) = -B_i (\hat{b}_f x_{i2} + (\hat{f}_{ci} + \hat{f}_{ri} e^{-f_{ri} \dot{q}_i^2}) sgn(x_{i2}) + \frac{\dot{\tau}_i}{\gamma_i}) \) represents precise modeling and measurable part, \( h_i(x) = -B_i (\sum_{j=1}^{i-1} U_j + \sum_{j=2}^{i-1} \sum_{k=1}^{i-1} V_{kj}) + d_i(x) \) represents IDC term and external disturbance, \( \theta_i(x_i) = -B_i f_{qi}(x_{i1}, x_{i2}) \) represents the uncertain part of the friction modeling.

Next, a decentralized robust ADRC in contact with emotional pHRi is proposed to ensure tracking error UUB.

**III. DECENTRALIZED ROBUST ADRC BASED ON TSMC A. DESIGN OF ROBUST SLIDING MODE CONTROLLER**

According to (9), we define the actuator output torque:

\[
\tau_i = (T_f/j_i) + u_i,
\]

(11)

where \( u_i \) is control input of \( i \)th joint. Define parameters:

\[
e_{i1} = q_i - q_{id}, r_i = \dot{e}_{i1} + \lambda_i e_{i1}, a_i = \ddot{q}_{id} - 2\lambda_i \dot{e}_{i1} - \lambda_i^2 e_{i1},
\]

(12)

where \( e_{i1} \) is the position tracking error, \( r_i \) is the filter error term, \( \lambda_i \) is the auxiliary error function, \( \lambda_i \) is arbitrary positive constant, \( q_{id} \) is the expected trajectory. Then \( u_i \) is defined as:

\[
u_i = u_{ic1} + u_{ir} + u_{in3},
\]

(13)

where \( u_{ic1} = I_{mi} \gamma_i e_{i1} + \hat{b}_f \dot{q}_i + (\hat{f}_{ci} + \hat{f}_{ri} e^{-f_{ri} \dot{q}_i^2}) sgn(q_i) \) compensates for the moment of inertia. \( u_{ic} \) is a robust controller to deal with friction modeling. \( u_{in3} \) is active disturbance rejection controller, which compensates IDC and disturbance torque item \( d_i(q_i) \).

Therefore, the decentralized robust ADRC problem has transformed obtaining \( u_{ic1}, u_{ir} \) and \( u_{in3} \), and in this way, to realize compensation of model uncertainty as well as emotional pHRi.

Next, we design a decentralized robust controller to compensate the model uncertainties.

First, we decompose \( \tilde{F}_i \) into:

\[
\tilde{F}_i = \tilde{F}_{ic} + \tilde{F}_{iv},
\]

(14)

where \( \tilde{F}_{ic} \) is unknown constant vector, \( \tilde{F}_{iv} \) is limited by:

\[
|\tilde{F}_{iv}| < \rho_{in}.
\]

(15)
Based on decentralized controller design, we choose:

\[ u_{ir} = u_{p}^i + Y(q_i)(u_{pc}^i + u_{pv}^i), \]  

(16)

where \( u_{p}^i \) is compensate \( f_{qi}(q_i, \dot{q}_i) \), \( u_{pc}^i \) and \( u_{pv}^i \) compensate \( \tilde{F}_{ic} \) and \( \tilde{F}_{iv} \). The control of the \( i \)-th joint \( u_{pc}^i \), \( u_{pv}^i \) and \( u_{u}^i \) are defined as:

\[
\begin{align*}
\dot{u}_{pc}^i &= -k \int_0^t Y(q_i)^T r_i dt, \\
\dot{u}_{pv}^i &= \begin{cases} 
-\rho_f \frac{\zeta_p}{\omega_p} | \dot{e}_{pi}^i | > \varepsilon_p, \\
-\rho_p \frac{\zeta_p}{\omega_p} | \dot{e}_{pi}^i | \leq \varepsilon_p, 
\end{cases}
\end{align*}
\]

(17)

(18)

(19)

where \( \zeta_p = Y(q_i)^T r_i, \varepsilon_p, \varepsilon_p^i \) are positive control parameters.

After that, to improve the convergence rate, the TSMC algorithm is introduced in this paper, firstly, the position tracking errors of \( e_{i1} \) and \( e_{i2} \) are defined as follows:

\[
\begin{align*}
e_{i1} &= q_i - q_{id} \\
e_{i2} &= \dot{q}_i - \alpha_i
\end{align*}
\]

(20)

where \( \alpha_i \) denotes virtual control, and as shown follow:

\[
\alpha_i = \dot{q}_{id} - \eta_k e_{i1},
\]

(21)

where \( \eta_k \) is a positive integer. The time derivative of \( e_{i1}, e_{i2} \) is taken:

\[
\begin{align*}
\dot{e}_{i1} &= e_{i2} - \eta_k e_{i1} \\
\dot{e}_{i2} &= \theta_i(x_i) + h_i(x) - (I_m \tilde{\gamma}_i)^{-1} \tau_i - \dot{\alpha}_i
\end{align*}
\]

(22)

Select the following Lyapunov functions for the \( i \)-th subsystem:

\[ V_{i1} = \frac{1}{2} e_{i2}^2. \]

(23)

The derivation of the above formula can be obtained:

\[ \dot{V}_{i1} = e_{i1} e_{i2} - \eta_k e_{i1}^2. \]

(24)

For ensuring stability of SMC, the state can follow desired state within a specified finite time, and tracking error converges asymptotically to 0.

Secondly, define TSMC:

\[ s_i = e_{i2} + \beta_i e_{i1}^{2p - 1}, \]

(25)

where \( \beta_i \) is arbitrary positive number, \( p_i \) and \( q_i \) are any positive odd numbers, and \( p_i > q_i > 0 \). By (22) and (23), we can obtain the derivative of \( s_i \) with respect to time:

\[
\begin{align*}
\dot{s}_i &= \dot{e}_{i2} + \beta_i \frac{q_i}{p_i} e_{i1}^{2p_i - 1} \dot{e}_{i1} \\
&= \theta_i(x_i) + h_i(x) - (I_m \tilde{\gamma}_i)^{-1} \tau_i - \dot{\alpha}_i \\
&\quad + (I_m \tilde{\gamma}_i)^{-1} \tau_i + \beta_i \frac{q_i}{p_i} e_{i1}^{2p_i - 1} \dot{e}_{i1}.
\end{align*}
\]

(26)

The decentralized sliding mode control law is:

\[
\begin{align*}
u_{ir1} &= \dot{\alpha}_i - \beta_i \frac{q_i}{p_i} e_{i1}^{2p_i - 1} \dot{e}_{i1} - k_i s_i - e_{i1} + \frac{\tau_f}{\gamma_i} \left( I_m \tilde{\gamma}_i \right)^{-1} \\
&= \dot{\alpha}_i + Y(q_i)(u_{pc}^i + u_{pv}^i) + \frac{\tau_f}{\gamma_i} \left( I_m \tilde{\gamma}_i \right)^{-1} e_{i1} - k_i s_i - e_{i1}.
\end{align*}
\]

(27)

In the end, the robust sliding mode controller is:

\[
\begin{align*}
u_{ir} &= u_{ir1} + u_{ir2} \\
&= \dot{\alpha}_i + Y(q_i)(u_{pc}^i + u_{pv}^i) + \frac{\tau_f}{\gamma_i} \left( I_m \tilde{\gamma}_i \right)^{-1} e_{i1} - k_i s_i - e_{i1} \\
&\quad + \frac{\tau_f}{\gamma_i} \left( I_m \tilde{\gamma}_i \right)^{-1} e_{i1} - k_i s_i - e_{i1}.
\end{align*}
\]

(28)

B. DESIGN OF ACTIVE DISTURBANCE REJECTION CONTROLLER

In this part, we introduce the ADRC based on the ESO. Based on the state space (10), the extended state is introduced to estimate IDC term and interference items which caused by contact with emotional pHRI.

Define the following terms for the active disturbance rejection controller design:

\[ z_2 = \dot{\bar{e}}_{i1} + k_1 e_{i1} = \dot{q}_i - k_1 q_{id} - x_{2eq}, \]

(29)

where \( k_1 \) is a positive feedback gain. ESO deals with modeling uncertainty in feedback linearization control [39]. First, (10) is now described as:

\[
\begin{align*}
\dot{x}_{i1} &= x_{i2} \\
\dot{x}_{i2} &= \phi_i(x_i) + \theta_i(x_i) + B_i u_{in3}, \\
\dot{x}_{i3} &= h_i(x)
\end{align*}
\]

(30)

From (30), a linear ESO is defined as:

\[
\begin{align*}
\dot{\hat{x}}_{i1} &= \hat{x}_{i2} - 3\omega_0 (\hat{x}_{i1} - x_{i1}) \\
\dot{\hat{x}}_{i2} &= \phi_i(x_i) + B_i u_{in3} + \hat{x}_{i3} + 3\omega_0^2 (\hat{x}_{i1} - x_{i1}), \\
\dot{\hat{x}}_{i3} &= -\omega_0^3 (\hat{x}_{i1} - x_{i1})
\end{align*}
\]

(31)

where \( \hat{x}_i = [\hat{x}_{i1}, \hat{x}_{i2}, \hat{x}_{i3}]^T \) is the state estimation and \( \omega_0 > 0 \) means the bandwidth of ESO. According to [40], we have

\[ \lambda_0(s) = (s + \omega_0)^3. \]

(32)

Denote \( \tilde{x}_{ij} = x_{ij} - \hat{x}_{ij}, j = 1, 2, 3 \) as estimation error. From (31), (32), the estimation error is:

\[
\begin{align*}
\dot{\tilde{x}}_{i1} &= \tilde{x}_{i2} - 3\omega_0 \tilde{x}_{i1} \\
\dot{\tilde{x}}_{i2} &= \tilde{x}_{i3} - 3\omega_0^2 \tilde{x}_{i1} \\
\dot{\tilde{x}}_{i3} &= h_i(x) - \omega_0^3 \tilde{x}_{i1}
\end{align*}
\]

(33)

Define \( e_{ij} = \tilde{x}_{ij}/\omega_0^j, j = 1, 2, 3 \), then, (33) can be rewritten as:

\[
\begin{align*}
\dot{e}_{ij} &= \omega_0 A_i e_{ij} + M_i \frac{h_i(x)}{\omega_0^j}, \\
\end{align*}
\]

(34)

where \( A_i \) is Hurwitz inferred in (33) and \( M = [0, 0, 1]^T \).

Lemma 1: Assuming \( h_i(x) \) is bounded, estimated states are bounded and exists \( \sigma_{ij} > 0, T_i > 0 \) such that \( |\tilde{x}_{ij}| \leq \sigma_{ij} \), \( \sigma_{ij} = O(\frac{1}{\omega_0^j}), j = 1, 2, 3, \forall t \geq T_1 \) for any positive integer \( c \).
So that we can obtain ESO-based control law:

\[ u_{in3} = (\dot{x}_{2eq} - \phi_i(x_i) - k_2z_2 - \dot{x}_{i3})/B_i. \] (35)

Combined with (11), (13), (28), and (35), the decentralized controller is:

\[ \tau_i = \frac{\tau_{fi}}{\gamma_i} + u_{ic1} + u_{ir} + u_{in3} = u_i - \hat{I}_mi_\gamma_i a_i + \dot{\hat{b}}_i x_i + \frac{\tau_{fi}}{\gamma_i} + (\dot{x}_{2eq} - \phi_i(x_i) - k_2z_2 - \dot{x}_{i3})/B_i + Y(x_{i2})u^{i}_{pc} + u^{i}_{pv} \]

\[ + \left( -\alpha_i + \beta_i q_i e_{i1} a_{i1} - \beta_{i1} a_{i1} e_{i1} + k_1s_i + e_i \right)/B_i \]

\[ + \left( \dot{f}_ci + \dot{f}_si e(\dot{f}_ci) \right) \text{sgn}(x_{i2}). \] (36)

**Remark 2:** With the combination of the sliding mode control and active disturbance rejection control, the estimated disturbance and chattering can be observed, and then adjust the switch gain to reduce chattering effect generated by the system.

**C. STABILITY PROOF**

**Theorem 1:** Consider \( n \) modules MRMs system with (1), and the model uncertainty is defined by (7) and (8), as well as the disturbance term caused by the emotional pHRI in (2). The tracking error can be guaranteed UUB under control law proposed by (36).

**Proof:** Select Lyapunov candidate function:

\[ V_{r2} = \frac{1}{2} \sum_{i=1}^{n} s_{i}^2. \] (37)

Then, the derivative of (37):

\[ \dot{V}_{r2} = \sum_{i=1}^{n} s_{i} \dot{s}_{i}. \] (38)

Therefore, the derivative of Lyapunov function is shown:

\[ \dot{V}_{i} = \sum_{i=1}^{n} \left( \dot{V}_{i1} + \dot{V}_{i2} \right) = \sum_{i=1}^{n} (e_{i1}e_{i2} - \eta_i e_{i1}^2 + s_i \dot{s}_i) \]

\[ = \sum_{i=1}^{n} \left( \dot{x}_i (\theta_i(x_i) + \ddot{\hat{b}}_i(x) - (I_{mi}\gamma_i)^{-1} \tau_{si} + (I_{mi}\gamma_i)^{-1} \tau_i) \right) \]

\[ = \sum_{i=1}^{n} \left( e_{i1}e_{i2} - \eta_i e_{i1}^2 + s_i \left( \dot{\theta}_i(x_i) + \ddot{\hat{b}}_i(x) - k_is_i \right) \right). \] (39)

\[ \dot{V}_{i} \leq \sum_{i=1}^{n} \left( -k_is_i^2 - \beta_i q_i e_{i1}/s_{i1} + s_i \left( \dot{\theta}_i(x_i) + \ddot{\hat{b}}_i(x) - k_is_i \right) \right) \leq 0. \] (40)

Since \( \dot{V}_{i} \leq 0 \) is negative semidefinite, which implies \( s_i \) is bounded. Denote \( V_p = \sum_{i=1}^{n} (k_is_i^2 + \beta_i q_i e_{i1}/s_{i1} + \eta_ie_{i1}^2) \) is integrated from 0 to \( t \), which is:

\[ \int_{0}^{t} V_p d\tau \leq - \int_{0}^{t} \dot{V}_p d\tau = V(0) - V(t). \] (41)

Since \( V(0) \) is bounded, \( V(t) \) is non-increasing with time, so that:

\[ \lim_{t \to \infty} \int_{0}^{t} V_p d\tau < \infty. \] (42)

According to Barbalat’s lemma, \( V_p(t) \to 0 \) is known when \( t \to \infty \) is used. It means \( e_{i1} \to 0, s_i(t) \to 0 \). and, if \( e_{i1} = 0, s_i(t) = 0 \). The tracking error of the MRMs system will asymptotically be 0, in the end the theorem is completed.

**IV. EXPERIMENTS**

**A. EXPERIMENTAL SETUP**

The experimental platform of this experiment is 2-DOF MRM (See Fig. 2) which produced by Quanser Company. The platform is composed of QPIDe data acquisition board, a linear power amplifier (LPA), two sets of joint modules and

![Experimental platform](image-url)
each of them contains a DC motor, a joint torque sensor, a speed reducer and an absolute encoder. The joint torque sensor measures the joint torque, the absolute encoder measures the position of the connecting rod end, the incremental encoder measures the motor end displacement in the joint and sends all the measured data back to the host computer through the QPIDe data acquisition board. The speed reducers are coupled with the DC motor.

We consider two kinds of emotional pHRI which include delighting and sadness (See Fig. 3). For the first situation, the player feels happy thus shaking hands with the end-effector of the manipulator. In the second scene, the player is dissatisfied and tap the manipulator at 30s and 90s. For the third situation, people beat the manipulator to demonstrate the motion, and after a while at 60s, people shake hands with manipulator initiatively to demonstrate the apology. The robotic joints are followed with the trajectories of

\[
q_{1d} = \frac{3}{10} \sin\left(\frac{\pi}{20}\right)t + \frac{\pi}{18} \sin\left(\frac{\pi}{10}\right)t, \\
q_{2d} = \frac{1}{5} \sin\left(\frac{\pi}{10}\right)t + \frac{\pi}{9} \sin\left(\frac{\pi}{20}\right)t,
\]

for the first situation and

\[
q_{1d} = -\frac{1}{10} \sin\left(\frac{\pi}{15}\right)t - \frac{\pi}{12} \sin\left(\frac{\pi}{30}\right)t, \\
q_{2d} = -\frac{1}{5} \sin\left(\frac{\pi}{10}\right)t - \frac{\pi}{6} \sin\left(\frac{\pi}{20}\right)t,
\]

for the second scene and the third situation is

\[
q_{1d} = \frac{2}{5} \sin\left(\frac{\pi}{25}\right)t + \frac{\pi}{8} \sin\left(\frac{\pi}{15}\right)t, \\
q_{2d} = \frac{4}{5} \sin\left(\frac{\pi}{20}\right)t - \frac{\pi}{10} \sin\left(\frac{\pi}{30}\right)t.
\]

The model parameters and control parameters of different emotion are given in Table 1. The physics parameters of various mood are shown in Table 2.

B. EXPERIMENTAL RESULTS

For comparing the advantages between existing [41]-[43] and proposed method, two different control scheme are taken into account.

(1) Position tracking performance

Figs. 4-7 are trajectory tracking curves with delighted emotion, respectively. From these figures, both methods are effective. Figs. 8-11 are trajectory tracking and error curves with sadness and fusion emotion, respectively. We can obtain the negative emotion appear, the position trajectory can also follow the desired trajectory.

(2) Velocity tracking performance

Figs. 12 and 13 are velocity error tracking curves under existing and proposed control methods with delighted emotion. The existing and proposed method of up-bound velocity error are less than 3e-3rad/s, 1e-3rad/s. That is because the
TABLE 1. Parameter setting.

| Parameter Type | Parameter Name | Value       | Parameter Name | Value       | Parameter Name | Value       |
|----------------|----------------|-------------|----------------|-------------|----------------|-------------|
| Model parameter | $b_{f_{i}}$   | 12:mNm/rad  | $f_{c_{i}}$    | 30:mNm      | $f_{s_{i}}$    | 40:mNm      |
|                | $f_{r_{i}}$   | $20s^{2}/rad^2$ | $I_{m_{i}}$   | 120:gcm$^2$ | $\gamma_{i}$  | 0.9         |
| Control parameter | $p_{d_{i}}$ | 5           | $p_{d_{c_{i}}}$ | 2.5:mNm     | $p_{f_{q_{i}}}$ | 1           |
| Condition One  | $\alpha_{i}$ | 0.8         | $\beta_{i}$   | 1.2         | $\eta_{i}$    | 5           |
| Control parameter | $p_{i}$     | 5           | $q_{i}$        | 5           | $\epsilon_{i}$ | 1.2         |
| Condition Two  | $\alpha_{i}$ | 0.5         | $\beta_{i}$   | 0.9         | $\eta_{i}$    | 2           |
| Control parameter | $p_{i}$     | 7           | $q_{i}$        | 5           | $\epsilon_{i}$ | 1.1         |
| Condition Three | $\alpha_{i}$| 0.66        | $\beta_{i}$   | 1.1         | $\eta_{i}$    | 4           |
|                | $p_{i}$      | 9           | $q_{i}$        | 7           | $\epsilon_{i}$ | 0.8         |

TABLE 2. Physics parameter setting of different emotions.

| Physics parameter | Delighting | Sadness | Fusion |
|-------------------|------------|---------|--------|
| Up-bound of disturbance torque | 0.3:Nm | 2.2:Nm | 1.3:Nm |
| Amplitude of trajectory | 0.5:rad | 0.3:rad | 1.2:rad |
| Frequency of trajectory | 40:s | 60:s | 55:s |

FIGURE 6. Position tracking curve via proposed method under situation one. (a) Joint One (b) Joint Two.

FIGURE 7. Position tracking curve via proposed method under situation two. (a) Joint One (b) Joint Two.

existing method has not considered ADRC to compensate the IDC effects and emotional pHRI. Fig. 14 is velocity error tracking curves under proposed control method with sadness emotion. When emotional pHRI happens, the error values decrease to normal ranges within short time, which may attribute to active disturbance rejection control. Fig. 15 is velocity error tracking curves under proposed control method with fusion emotion. We can get the similar results with delighted and sadness emotion, that is attributed to the effective decentralized robust ADRC method.

(3) Control torque

Figs. 16, 17 are control torque tracking curves via situation one of existing and proposed method. In Fig. 16, control torque curves are with serious chattering effect. Better curves are in Fig. 17, which profits from the proposed control methods. Figs. 18 and 19 are control torques under situation two and three. In the figures, we can see curves may multiply increase when emotional pHRI occurred and lead to the robotic system out of control. The proposed active disturbance rejection control realizes the optimization of tracking errors.
For the experimental results one can get the proposed ADRC method can guarantee stability as well as accuracy.

**V. CONCLUSION**

A decentralized ADRC scheme for MRM in contact with emotional disturbance is proposed. Based on JTF technique, we obtain the dynamic model of MRM. When MRM systems facing external environment, we transform the control problem with emotional pHRI into active disturbance rejection. Based on the strong estimation ability of ESO, we design ADRC, and it is used to estimate IDC term and the interference term caused by emotional pHRI. TSMC is used to guarantee high precision control as well as fast convergence. According to Lyapunov method, trajectory tracking error is proved to be UUB. Experiments are performed to confirm effectiveness.

In our future work, there will be a more detailed and completed analysis of what kind of interaction behavior human will do for different emotions.

**REFERENCES**

[1] Li Z, Xia Y, Sun F (2014) Adaptive fuzzy control for multilateral cooperative teleoperation of multiple robotic manipulators under random network-induced delays. IEEE Trans. Fuzzy Syst. 22(2): 437–450.

[2] He W, Dong Y, Sun C (2016) Adaptive neural impedance control of a robotic manipulator with input saturation. IEEE Trans. Syst. Man Cybern. Syst. 46(3): 334–344.

[3] He W, David A, Yin Z, Sun C (2016) Neural network control of a robotic manipulator with input deadzone and output constraint. IEEE Trans. Syst. Man Cybern. Syst. 46(6): 759–770.

[4] Sadeghzadeh M, Calvert D, Abdullah H (2015) Self-learning visual servoing of robot manipulator using explanation-based fuzzy neural networks and Q-learning. J. Intell. Robot. Syst. 78(1): 83–104.

[5] Ding B, Li X, Li Y (2021) FEA-based optimization and experimental
FIGURE 13. Velocity error curve via proposed method under situation one. (a) Joint One (b) Joint Two.

FIGURE 14. Velocity error curve via proposed method under situation two. (a) Joint One (b) Joint Two.

FIGURE 15. Velocity error curve via proposed method under situation three. (a) Joint One (b) Joint Two.

FIGURE 16. Control torque curve via existing method under situation one. (a) Joint One (b) Joint Two.
FIGURE 17. Control torque curve via proposed method under situation one. (a) Joint One (b) Joint Two.

FIGURE 18. Control torque curve via proposed method under situation two. (a) Joint One (b) Joint Two.

FIGURE 19. Control torque curve via proposed method under situation three. (a) Joint One (b) Joint Two.

[26] Jung S (2018) Improvement of tracking control of a sliding mode controller for robot manipulators by a neural network. Int. J. Control Autom. Syst. 16(2): 937–943.
[27] Wang Z, Su Y, Zhang L (2017) A new nonsingular terminal sliding mode control for rigid spacecraft attitude tracking. ASME. J. Dyn. Sys. Meas. Control. 140(5): 051006.
[28] Zhang F (2017) High-speed nonsingular terminal switched sliding mode control of robot manipulators. IEEE/CAA J. Autom. Sin. 4(4): 775–781.
[29] Ma Z, Sun G (2017) Dual terminal sliding mode control design for rigid robotic manipulator. J. Franklin Inst. 355(18): 9127–9149.
[30] Yang Y (2018) A time-specified nonsingular terminal sliding mode control approach for trajectory tracking of robotic airships. Nonlinear Dyn. 92(3): 1359–1367.
[31] Albu-Schaffer A, Ott C, Hirzinger G (2007) A unified passivity-based control framework for position, torque, and impedance control of flexible joint robots. Int. J. Robot. Res. 26(1): 23–39.
[32] Liu G, Abdul S, Goldenberg A (2008) Distributed control of modular and reconfigurable robot with torque sensing. Robotics. 26(1): 75–84.
[33] Zhang F (2017) High-speed nonsingular terminal switched sliding mode control of robot manipulators. IEEE/CAA J. Autom. Sin. 4(4): 775–781.
[34] Dong B, An T, Zhou F, Liu K, Li Y (2019) Decentralized robust zero-sum neuro-optimal control for modular robot manipulators in contact with uncertain environments: theory and experimental verification. Nonlinear Dyn. 97(1): 503–524.
[35] Chen L, Zhou M, Wu M, She J, Liu Z, Dong F (2018) Three-layer weighted fuzzy support vector regression for emotional intention understanding in human-robot interaction. IEEE T. Fuzzy Syst. 26(5): 2524–2538.
[36] Mohyeddini C, Pauli R, Bauer S (2009) The role of emotion in bridging the intention behaviour gap: the case of sports participation. Psychol. Sport Exerc. 10(2): 226–234.
[37] Wang P, Liu J, Hou F, Chen D, Xia Z, Guo S, (2021) Organization and understanding of a tactile information dataset TacAct for physical human-robot interaction. 2021 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS): 7328–7333.
[38] Liu G, Goldenberg A, Zhang Y (2004) Precise slow motion control of a direct-drive robot arm with velocity estimation and friction compensation. Mechatronics. 14(7): 821–834.
[39] Dong B, An T, Zhu X, Li Y, Liu K (2021) Zero-sum game-based neuro-optimal control of modular robot manipulators with uncertain disturbance using critic only policy iteration. Neurocomputing. 450: 183–196.
[40] Zheng Q, Gao L, Gao Z (2007) Stability analysis of active disturbance rejection control for nonlinear time-varying plants with unknown dynamics. Proc. IEEE Conf. Decision Control: 3501–3506.
[41] Dong B, An T, Zhou F, Yu W (2019) Model-free optimal decentralized sliding mode control for modular and reconfigurable robots based on adaptive dynamic programming. Adv. Mech. Eng. 11(12): 1687814019896923.
[42] Zhu X, Ma B, Dong B, Liu K, Li Y (2020) Adaptive dynamic programming-based sliding mode optimal position-force control for reconfigurable manipulators with uncertain disturbance. In: 2020 Chinese Control and Decision Conference; 421–427.

[43] Dong B, An T, Zhou F, Liu K, Yu W, Li Y (2019) Actor-critic-identifier structure-based decentralized neuro-optimal control of modular robot manipulators with environmental collisions. IEEE Access. 7: 96148-96165.

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