Fast and accurate pseudoinverse with sparse matrix reordering and incremental approach

Jinhong Jung
Jeonbuk National University

Lee Sael
Ajou University
Outline

- Introduction
- Proposed Method
- Experiment
- Conclusion
Research Question

Q. How can we compute the pseudoinverse of a sparse feature matrix efficiently and accurately?

- For solving machine learning problems

- Pseudoinverse is a generalized inverse for all types of matrices
  - Plays a crucial role in obtaining best-fit solutions to the linear systems
  - Various applications in machine learning domain
Problem Definition

- **Pseudoinverse of a sparse matrix**
  - Moore-Penrose Inverse via low-rank SVD
  
- **Inputs**
  - A **sparse** matrix \( \mathbf{A} \in \mathbb{R}^{m \times n} \) and a target rank \( r \)

- **Output**
  - MP Inverse \( \mathbf{A}^\dagger \approx \mathbf{V}_{n \times r} \mathbf{\Sigma}_{r \times r} \mathbf{U}_{r \times m}^\top \)
    - \( \mathbf{A} \) is decomposed into \( \mathbf{U} \mathbf{\Sigma} \mathbf{V}^\top \) via SVD
    - If \( r \) is the full rank, the equality holds.
    - Otherwise, it is a best approximate \( \mathbf{A}^\dagger \) for rank \( r \)

- **Target Application:** Multi-label Linear Regression
Limitations

- Previous methods have high costs for computing Pseudoinverse
  - Especially for relatively large rank $r$
    - Needed for high accuracy
  - SVD: $O(mn^2)$ & Randomized-SVD: $O(mr^2)$
  - Krylov sub-space method & frPCA for spares matrices
    - Effective for very low rank $r$

- C. How to efficiently compute it without loss of accuracy?
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Proposed Method

- **FastPI** (Fast PseudoInverse)
  - Novel, fast, and accurate method for approximate pseudoinverse for spare matrices

- **Ideas**
  - **Idea 1)** Many feature matrices are highly sparse and skewed
    - Can be reordered such that their non-zeros are concentrated
  - **Idea 2)** The SVD of a large and sparse block diagonal sub-matrix is easy-to-compute
  - **Idea 3)** The final SVD is efficiently obtained by an incremental update method
Sparse Feature Matrices

- Sparse feature matrices are considered as bipartite networks
  - Instance nodes to feature nodes

```
| Instances | Features |
|-----------|---------|
| Feature matrix (sparse) |
```

![Diagram of bipartite network]
Sparse Feature Matrices

- Degree distributions of bipartite networks from real-world feature matrices are highly skewed!

![Graphs showing skewed distributions](image-url)
Sparse Matrix Reordering

- Sparse rectangular matrix can be reordered as follows:
  - See the paper for details
  - The non-zero entries are concentrated

(a) The original matrix  (b) After thirty iterations  (c) After eighty iterations  (d) After 115 iterations  (e) After the final (119) iteration
SVD on Reordered Matrix

How can we compute SVD of the reordered matrix while exploiting the sparsity?

- **Step 1.** Compute the SVD of $A_{11}$
- **Step 2.** Incrementally update it for $A_{21}$
- **Step 3.** Incrementally update it for $\begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix}$
SVD on Reordered Matrix

- SVD of the large sparse rectangular diagonal submatrix $A_{11}$
  - Easy-to-compute by computing SVD of each rectangular block & the results are also sparse!
  - Reason that we can accelerate the speed!

```
\begin{align*}
& A_{11} \\
\rightarrow & \quad \text{Low-rank SVD} \\
\end{align*}
```
Experimental Setting

Experimental Questions

- Q1. Reconstruction error
- Q2. Accuracy of multi-label linear regression
- Q3. Efficiency

Datasets: 4 real-world feature matrices

- Amazon, RCV, Erulex, Bibtex

Methods

- FastPI (proposed), RandPI, KrylovPI, frPCA
Reconstruction Error & Accuracy

- FastPI produces similar reconstruction error & accuracy to other methods
- Can compute Pseudoinverse without loss of accuracy

(a) Amazon  (b) RCV

Reconstruction Error  Accuracy of Multi-label Linear Regression
Efficiency

- **FastPI** is faster than other methods
  - Especially for relatively large rank!
  - However, they are similar when the rank is small ⇒ Need to improve this as future work

![Graphs showing running time for Amazon, RCV, Eurlex, and Bibtex datasets](Image)

**Running Time**

*Journal*  
*ACML 2020*
Conclusion

- **FastPI** (Fast Pseudoinverse)
  - **Idea 1)** Many feature matrices are sparse and skewed
  - **Idea 2)** The SVD of a sparse block diagonal matrix is easy-to-compute
  - **Idea 3)** The final SVD is efficiently obtained by an incremental update method

- **Experimental Results**
  - **FastPI** computes the approximate pseudoinverse of a sparse matrix **without loss of accuracy**
  - **FastPI** is **faster** than other competitors for relatively large rank
Thank you!

Jinhong Jung
Jeonbuk National University

Lee Sael
Ajou University