A historical Introduction to the LPM shower

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Abstract. The matter before and after the birth of the LPM showers is outlined. The most important tasks for the LPM shower studies are thoroughgoing examinations on the validity of the methods around calculations of the cascade showers.

1. Introduction

The LPM effect is a kind of density effect, which brings about the decrease of the bremsstrahlung due to multiple scattering at higher energies. Landau-Pomeranchuk treat this problem in semi-classical manner [1] and afterwards, Migdal formulates it in quantum electrodynamical manner [2].

Now, it is named as the LPM effect [3, 4]. Nowadays, it is well known that the LPM effect makes influence on the cosmic ray phenomena greatly at extremely high energies and the cascade showers characterized by the LPM effect are called as the LPM showers.

2. The history of the LPM showers

2.1. The pioneering work in the LPM showers by A.A. Pomansky

In 1968, Pomansky represented a paper entitled as Consideration of the cascade curve at super high energies [5]. This is the first application of the LPM effect to the electromagnetic cascade showers which is shown in figure 1. In his paper, he did not use the physical term of the LPM effect, only cited Migdal’s name in his paper, because at that time the physical term of the LPM effect did not exist.

Unfortunately, his results were not correct [6]. However, his work should be highly appreciated due to its pioneering nature in the history of the LPM showers. We quote a part of the first page of his paper in figure 1.

2.2. The first result of the correct LPM showers and their subsequent developments

In 1974, Misaki et al. calculated cascade curves where the cross sections obtained by Migdal were utilized by an exact Monte Carlo method. Their cascade showers were clearly different from the usual cascade showers under Bethe-Heitler’s cross section [7]. Then, they named this effect as L-P-M effect, because it made possible to calculate cascade showers in quantitative manner only by the advent of Migdal’s cross section. The figure 2 is reproduced from original one.

When their paper was presented at an international conference, where Khristiansen casted strong doubt around the reliability of their results. His strong criticism toward their results seemed to be quite reasonable at that time, because Kamata and Nishimura made the following comment in their famous paper on three-dimensional cascade theory [8].
“Recently, the limit of application of the complete screening cross sections at very high energies has been critically discussed by Landau-Pomeranchuk. They pointed out that, at very high energies, the path length of an electron which is the effective in the collision processes become so long that the interference effects of the adjacent atoms should be taken into account and that the cross section should decreases in the high energy regions. Even if his cross section should fall off at very high energies, the average energy of shower particles would soon decrease by order of magnitude by the rapid multiplication of particles. Thus these effects, if exist any, should concern only the beginning stage of shower development and the overall behavior should suffer little change.” [8]

In order to overcome Khristiansen’s bitter remark, and, furthermore, to exclude the conjecture by Kamata-Nishimura, they had to verify the validity of their calculation as rigorously as possible. The validity of the Monte Carlo method utilized was verified in the following ways.

i. First of all, they calculated cascade showers within the framework of the one-dimensional analytical theory under Approximation A [7]. The physical quantities, such as, total track lengths of shower particles and cascade curves for shower particles were obtained by the analytical theory, the accuracies of which are clarified [7].

ii. The total track lengths for shower particles and the cascade curves by the Monte Carlo method are compared with the corresponding ones by the analytical theory [9]. Both quantities obtained by mathematically different methods showed an excellent agreement. This fact confirms that the validity of their Monte Carlo method in one-dimensional was established.

iii. They extended their method to three-dimensional case. For the purpose, they built the effect of the multiple scattering due to shower electrons into their Monte Carlo procedure in one dimensional case.
Figure 2. Transition curves for a photon-initiated shower with energy $E_0 = 3 \times 10^{14}$ eV in lead. Reproduced from Proc. Inter. Cosmic Ray Symp. on Hugh. Ener. Phys. 148(1974), Tokyo

Figure 3. Transition curves of mean number of electrons in lead. Open circles show the results with Bethe-Heitler cross-sections ($E_0/E_{\text{min}}=10^3$) and closed circles, triangles and crosses show the results including the LPM effect initiated by photons with energies $E_0=10^{11}, 10^{13}, 10^{15}$ eV and $E_m=10^8, 10^{10}, 10^{12}$ eV, respectively.

Figure 4. Mean square lateral spread of electrons in lead. Marks are the same as figure 3. $\langle r^2 \rangle$ is normalized by multiplying by the factor $(E_m/E_s)^2$, where $E_m$ is the minimum energy of observation and $E_s$ is the scattering energy $E_s=21.2$ MeV. The solid curve shows analytical results under approximation A.

Figure 5. Mean square angular spread of electrons in lead. Marks are the same as figure 3. $\langle \theta^2 \rangle$ is normalized by multiplying by the factor $(E_m/E_s)^2$, where $E_m$ is the minimum energy of observation and $E_s$ is the scattering energy $E_s=21.2$ MeV. The solid curve shows analytical results under approximation A.
The validity of their three-dimensional procedure in the Monte Carlo method was finally checked by the comparison of the mean square lateral and angular spreads of shower particles by the Monte Carlo method with corresponding ones by the analytical procedure [9]. The corresponding quantities by the analytical theory had been proved to be right mathematically. They confirmed that the agreement between them were excellent (See, figures 3 to 5). As the results of it, they confirmed the validity of their Monte Carlo procedure in three dimensional cases. Figure 3 to 5 are reproduced from E.Konishi et al., Il Nuovo Cimento, 44A, (1978)509-503. The agreements between the Monte Carlo results and analytical ones in these figures under Approximation A guaranteed finally the validity of their three-dimensional Monte Carlo procedure.

iv. As the validity of the general structures of the three-dimensional Monte Carlo procedure in BH showers had been verified, the applications of this procedure to the LPM showers were automatically guaranteed, because the difference between two procedures was merely the difference in the numerical values between respective cross sections.

2.3. The final denial toward Khristiansen’s comment and the partially -denial towards Kamata-Nishimura’s conjecture

The title of the paper in which figures 3 to 5 are included is named as, “Three-Dimensional Cascade Showers in Lead taking account of the Landau-Pomeranchuk-Migdal effect”. Namely, the physical term of the LPM effect appeared firstly in a peer-reviewed journal [4].

It is clear from these figures that LPM effect influence really on the development of the cascade showers. Consequently, the criticism by Khristiansen should be finally excluded. However, it should be noticed that the conjecture by Kamata-Nishimura cited above are not completely excluded, because the minimum energies of shower particles in figures 3 to 5 are rather high where the LPM effect is still effective. In order to exclude their conjecture finally, they need to develop a new technique by which it makes possible to pursue shower particles down to lower energies where the LPM effect is completely neglected. Because, it was impossible to utilize the exact Monte Carlo method for the purpose due to huge time consuming at that time and even now. Thus, a new technique was forced to be developed for the purpose [10] and its details was given in [11].

In figure 6, the validity of the matrix method was verified through the agreements among cascade curves obtained by the methodologically independent means. This figure is reproduced from A.Misaki, Physical Rev.D49 (1989)3086—3096 by the matrix method [12]. The validity of this method was verified by other two independent methods, namely, the analytical and Monte Carlo methods on which are methodologically independent, as shown in figure 6. Based on the confirmation of the validity of the matrix method, the transition curves for shower particles both the cases without LPM effect (figure 7) and with LPM effect (figure8) are calculated.

In these numerical calculations, the effect of the ionization loss is included (in the sense of “under Approximation B”). It is clear from the comparison between figure 7 and figure 8 that the developments of the LPM showers are completely different from corresponding ones of the BH showers. Here, shower particle are pursued down to almost zero energies where LPM effect is completely neglected. Thus, it is clear from the figures that the conjecture by Kamata-Nishimura should be finally excluded.

In table 1, the physical quantities which characterize the behaviors of cascade showers are compared between the LPM showers and BH showers. Namely, Nmax, the maximum number for cascade showers, Tmax, the depth where the cascade showers attain at the maximum, and their FWHM, are compared between LPM showers and BH showers. It is clear from the table that the LPM showers are quite different from the BH showers.

The comparison of cascade curves (average pictures) without the LPM effect with the corresponding ones with LPM effect is made in figure 7 and 8. It is clear that the LPM showers are quite different from the BH showers as shown in figures 7 and 8, and table1, which are reproduced from A. Misaki, Fortschritte der Physik, 38 (1990)413-446.
**Figure 6.** Comparison between the present results and the results obtained by Konishi et al. [4]. The comparison is made for LPM showers without ionization loss in lead, keeping $E_0/E_{th}$ fixed and values of $E_0=10^{11}$, $10^{13}$, $10^{15}$ eV. The crosses, closed circles and open circles denote $E_0=10^{15}$, $10^{13}$, $10^{11}$ eV respectively. From A.Misaki, Phys.Rev.D,40, (1989)3086-3096

**Figure 7.** Transition curves of electron numbers in water under Approximation B initiated by a photon of $10^{20}$ eV for various threshold energies. The letter attached to each curve denotes the threshold energies: $a=10^3$ eV, $b=10^5$ eV, $c=10^7$ eV, $d=10^8$ eV, $e=10^9$ eV. From: A.Misaki, Fortschr. Phys. 38 (1990) 413-446

**Figure 8.** Transition curves of electron numbers in water in the presence of the LPM effect including ionization loss. The primary energy is $10^{20}$ eV and the threshold energies, $a$ to $e$ are the same as in figure 7. From: A.Misaki, Fortschr. Phys. 88 (1990) 413-446

**Figure 9.** Number $N$ of electrons in an electron-initiated shower of primary energy $E_0=10^{17}$ eV in lead as a function of depth in the absorber. The three curves refer to three different values of cut off energy $E_{m}=10^6$, $10^{12}$, $10^{14}$ eV. From: Konishi et al, Phys.G: Nucl.Part.Phys. 17(1991)719-732
Table 1. Numerical values for electron number at shower maximum, $N_{\text{max}}$, depth of shower maximum, $T_{\text{max}}$, and full width half maximum, FWHM, in both BH and LPM showers including ionization loss in water. In each row, numerical values on upper row are due to the LPM shower, while the corresponding ones on lower row are due to the BH shower. Reproduced from: A. Misaki, Fortschr. Phys. 38 (1990) 413-446.

| $E_0$/eV | $10^{15}$  | $10^{16}$  | $10^{17}$  | $10^{18}$  | $10^{19}$  | $10^{20}$  | $10^{21}$  |
|---------|------------|------------|------------|------------|------------|------------|------------|
| $N_{\text{max}}$ | $1.04 \times 10^6$ | $8.31 \times 10^6$ | $4.58 \times 10^7$ | $1.73 \times 10^8$ | $5.82 \times 10^8$ | $1.91 \times 10^9$ | $6.16 \times 10^9$ |
| $T_{\text{max}}$ | $107 \times 10^6$ | $100 \times 10^7$ | $9.48 \times 10^8$ | $9.04 \times 10^8$ | $8.63 \times 10^9$ | $8.26 \times 10^{10}$ | $7.97 \times 10^{11}$ |
| FWHM | 18 (r.l.) | 24 (r.l.) | 38 (r.l.) | 78 (r.l.) | 200 (r.l.) | 569 (r.l.) | 1696 (r.l.) |
|        | 17 (r.l.) | 19 (r.l.) | 22 (r.l.) | 24 (r.l.) | 26 (r.l.) | 29 (r.l.) | 31 (r.l.) |
|        | 13 (r.l.) | 16 (r.l.) | 28 (r.l.) | 75 (r.l.) | 223 (r.l.) | 676 (r.l.) | 2049 (r.l.) |
|        | 12 (r.l.) | 13 (r.l.) | 14 (r.l.) | 15 (r.l.) | 15 (r.l.) | 16 (r.l.) | 17 (r.l.) |

2.4. New aspect in the structure of the cascade showers

In 1991, E. Konishi et al. [13] found the quite new phenomena in cascade showers which are called as multi-peak structure now. In figure 9, one example of an individual shower with multiple peaks is shown. In the figure, there are three different cascade curves with different cut off energies for the same showers with the primary energy of $10^{17}$ eV in lead.

It should be noticed that smoothed cascade curves for different cut off energies in figure 9 are the resultants of the average of the individual cascade shower with multi-peak structures for different cut off energies. The existence of similarities in the cascade curves for different cut off energies in figure 9 suggests us that the essential energy configuration among shower particles in the LPM shower at extremely high energies are accidentally determined in higher energy particles. This is the origin of variety of the diversities among the LPM showers. Figure 9 is reproduced from E. Konishi et al, J. Phys. G: Nucl. Part. Phys. 17 (1991) 719-932.

3. Conclusion

The diversity of the LPM showers at extremely high energies is a quite new phenomenon which never appears in the usual BH cascade showers. The characteristic of the multi-peak structure at extremely high energies among the LPM showers will be clarified in another paper in this workshop, which play decisively important role in the study for extremely high energy electron neutrino astrophysics.

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