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Seventy years of tensegrities (and counting)

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Abstract We try to make a long way short by proceeding per exempla from Kenneth Snelson’s sculptures and Richard Buckminster Fuller’s coinage of the term tensegrity to modern tensegrity metamaterials. We document the passage from initial interest in tensegrity frameworks for their visual impact to today’s interest, driven by their peculiar structural performances. In the past seventy years, the early art pieces and roofing structural complexes have been followed by formalization of the principles governing the form-finding property of ‘pure’ tensegrity structures and by engineering hybridization leading to a host of diverse practical applications, such as variable-geometry civil engineering structures, on-earth and in-orbit deployable structures and robots, and finally to recent and promising studies on tensegrity metamaterials and small-scale tensegrity structures.

Keywords Tensegrity · Form finding · Adaptive structures · Tensegrity lattices · Tensegrity metamaterials

1 Introduction

In the late 1950s, a new concept—earlier suggested by certain unusual sculptures by Kenneth Snelson [1], who later developed it into spectacular decorative structures (Fig. 1)—was popularized by Snelson’s advisor, the American architect Richard Buckminster Fuller, who coined the term tensile integrity. In the same years, the French architect David George Emmerich had been studying similar structures [2].1 Quickly, the tensegrity (TS) concept became familiar to structural engineers and architects through a variety of applications. In 1978, a first step toward an engineering characterization was taken by Calladine [7] and later expanded by him and Pellegrino [8] and by Pellegrino [9,10]. Mathematical studies started in the 1980s by Roth and Whiteley [11] and by Connelly [12] were continued later by these authors and others.2

1 A controversy originated about who first proposed the tensegrity concept, if Fuller, Snelson, or Emmerich. For what it matters, their patents are dated, respectively, 1962, 1965, and 1964 [3–5]; more historical information is found in Gómez-Jáuregui [6].

2 In addition to [13–16], we refer the reader to Connelly’s webpage http://pi.math.cornell.edu/~connelly/ for a complete and updated list of papers. Other relevant results are discussed in [17].

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On informal inspection, tensegrities (TS’es) appear as pin-connected frameworks, whose elements are bars, which may carry tension or compression, and cables, which carry no compression; in stable placements, cables act as tendons, while bars are usually compressed and hence act as struts. On looking at Snelson’s, Buckminster Fuller’s and Emmerich’s realizations, we may compile the following list of defining properties:

1. cables form a connected set \((\text{tensile integrity})\);
2. no two bars are ever connected \((\text{floating compression})\);
3. frameworks may include \(\text{infinitesimal mechanisms}\), stabilized by a \(\text{self-stress state}\).

Over time, different definitions have been given. Certain authors regard both properties 1 and 2 as essential, others insist only on property 1. Snelson himself used the term \(\text{endoskeletal}\) for his structures since, as in the Baltimore installation in Fig. 1, bars are all internal to the envelope of the cable network, a fact that inspired Buckminster Fuller’s analogy of a balloon resisting the pseudo air pressure exerted by a bar complex in a regime Snelson termed \(\text{floating compression}\) \cite{19}; for Motro (2003) \cite{20} (see also \cite{21}), tensegrities are \(\text{free-standing systems}\) (i.e., systems with no external anchorages) satisfying property 1.

Mathematicians interested in tensegrities did not include any of the three properties above into their definitions. Their interest was scattered by their previous involvement in characterizing rigidity of conventional reticular structures, a notion suitable to be so extended as to apply to tensegrity-like systems. \(\text{Stability}\) of TS’es has been studied extensively within the framework of rigidity theory (see \cite{22} and the literature cited therein). In particular, these studies have been expedient in making precise a notion of central importance, to be introduced and discussed in Sect. 4, the notion of \(\text{form finding}\).

Before proceeding any further, we think it best to recall the limits in coverage and detail, and hence in size, we set for our present study: neither too large a selection of eye-catching pictures of TS sculptures and structures nor too detailed an account of tensegrity mathematics. Notwithstanding, we hope the reader will find our title justified by our choice of significant applications of the TS concept, which ranges from the early art pieces and structural complexes in Sect. 2 to the diverse and sophisticated TS-inspired structures in our final Sects. 7 and 8. In this connection, we should tell the reader that our long reference list has no pretenses to be exhaustive; a long essay, similar in spirit to ours, appeared in 2009 \cite{23}; we apologize for those references we have not included.

The leading intention of our historical account to document the progressive passage from an initial interest for TS frameworks mostly aroused by their visual impact to today’s interest, which is mostly driven by their structural performances.

No doubt, ‘pure’ TS systems—those qualifiable in Snelson parlance as “endoskeletal, free-standing, floating-compression systems”; those containing “islands of compression in a sea of tension”, in the imaginative language of Buckminster Fuller—are not, when regarded as civil engineering structures, as efficient as traditional structures. As an artist, Snelson did not care at all: the only load his creations were expected

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3 In \cite{18}, Skelton proposes to classify TS’es according to the maximum number of compression members connected in a node.

4 There are two main reasons for this: one is that, struts being not contiguous, loads are not transferred to foundations in a continuous manner; the other is that to prevent buckling instabilities is an especially taxing job \cite{24–26}.
to bear was their own weight, he used to say [19]. As an architect, Buckminster Fuller did care, e.g., the outer compression ring of his celebrated Aspension Dome with (see Sect. 2.2) makes it into an ‘impure’ but conveniently rigid and light roofing structure.

We call metatensegrities the civil engineering structures in Sect. 3, whose well-calibrated ‘impurity’ allows coupling of light weight and high performance with remarkable visual attraction. It was after formalization of certain TS distinguishing properties—in particular, of the form-finding property we discuss in Sect. 4—that more and more researchers began to see how to exploit TS principles to design innovative, not necessarily civil, structures. A variety of form-finding related theories and methods were developed; a necessarily condensed report of these is found in Sect. 5. New application-dependent technological problems had to be addressed and solved; moreover, growing awareness of limitations intrinsic to TS systems (see Sect. 6) had the positive outcome of directing attention to more and more new fields where recourse to TS-inspired structures was advantageous because of their deployability and variable geometry; see Sect. 7 for both terrestrial and spatial TS’es, as well as for robotic applications. In Sect. 8.1 we discuss studies which have clarified the peculiar adaptivity of TS structures and their nonlinear behavior both in statical and dynamical circumstances. In our opinion, the promising applications we review—tensegrity metamaterials in Sect. 8.2 and small-scale tensegrity structures in Sect. 8.3—are enough to let us believe that the counting of TS years is not at all over.

2 Early tensegrities

Buckminster Fuller was over three decades senior to Snelson. When the latter, one of his resident students at the Black Mountain College in North Carolina, presented his X-piece to him [1], Buckminster Fuller was more struck by the innovative structural implications than by the three-dimensional visual impact of that sculpture. Ever since, their work lines were to keep this basic difference in inspiration and scope. Consistently, we distinguish the early tensegrity frameworks we include in this section, whoever their creators, into tensegrity sculptures, decorative in nature, and tensegrity structures, as such primarily conceptual.

2.1 Tensegrity sculptures

We begin with the Skylon (Fig. 2a), a construction erected in London on the occasion of the 1951 Festival of Britain and dismantled after the closing of that event. This construction was designed by Hidalgo Moya, Philip Powell and Felix Samuely independently from Snelson’s and Buckminster Fuller’s first studies. It consisted of a vertical cigar-shaped strut, suspended by cables to three smaller ground-anchored struts. Because struts and cables were not assembled in a tensegrity-like way, the Skylon may be regarded as an antecedent rather than a precedent of Snelson’s towers, perhaps the most surprising and certainly the most popular works of that prolific and variously creative artist.

Snelson’s towers are built by juxtaposition along a vertical axis of smaller and smaller tensegrity prisms; the 1968 20-meter Needle Tower I shown in Fig. 2b is located in Washington, D.C., the 1969 30-meter Needle Tower II in Otterlo (Holland). Winds make Snelson’s towers sound as quite unusual musical instruments. Similarly, Buckminster Fuller’s mast, an early work, had bars and cables assembled in a non-tensegrity fashion (Fig. 2c). Just as cables suspend two X-shaped rigid objects in Snelson’s X-piece, in Buckminster Fuller’s mast a number of cable-edged tetrahedra, each of which has a 4-arm rigid body inside, are pair-wise suspended by cables, while other vertical cables running from bottom to top materialize the edges of a prism of square cross section.

In contrast to the mast, the Warnowturm in Fig. 2d was designed in fair observance of tensegrity principles, by Gerkan, Marg & Partner (Hamburg) in collaboration with Schlaich, Bergermann & Partner (Stuttgart). This 49-meter tower was built by MERO Structures Inc. in only ten days, near the Warnow river in Rostock (Germany), in the occasion of the 2003 International Gardening Fair.

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5 Neither of the two was aware of the proto-tensegrities proposed by the Latvian constructivist artist Kārlis Johansons in the very early 1920s https://en.wikipedia.org/wiki/Karlis_Johansons.
6 In fact, a tensegrity harp has been built; if interested to see it and listen to it when played, visit http://www.squid-labs.com/projects/tensegrity/index.html.
7 Observance is not strict, because two superposed tensegrity modules are interconnected at adjacent bars. Schlaich, Bergermann & Partner also designed a 4.6-meter tensegrity sphere conceived by Markus Heinsdorff and built in Wiesloch (Germany), in 2018.
Tensegrity towers are closely related to tensegrity arches: all it takes is to make the axis curvilinear. In 1959, Snelson, who surely could not miss the connection, built the Bead Arch in Fig. 3a. In 2001, in collaboration with our colleague Silvano Stucchi at the University of Rome Tor Vergata, we designed a strictly tensegrity arch inspired by Snelson’s Rainbow Arch (Fig. 3b); this arch, 50 meters in span (Fig. 3c), was meant to serve as an entry portal to our campus but was never built [27].

Just as the Skylon is reminiscent of Snelson’s needle towers, the arch in Fig. 3d is reminiscent of a tensegrity arch. It was designed by Janos Baracs in 1992 and built in Saint-Hyacinthe, Quebec (Canada); its span is 85 meters. A number of rigid rectangular elements are suspended from two tubular stringers, anchored to terminal concrete foundations and stabilized by guy ropes out from the vertical mid-plane of the arch. Baracs’ arch is one of the first examples of hybrid structure, whose rigidity may be assessed by applying the mathematical theory developed for tensegrity systems.8

We close this section with the tree-shaped tensegrity sculpture built in Canterbury (UK) to commemorate the 50th anniversary of the University of Kent and the 10th anniversary of its School of Architecture [28]. This project was inspired by studies described in [29,30]; the plan view in Fig. 4 suggests how the design exploits the concepts of concentric tensegrity balloons and nested endoskeletal systems [31].

2.2 Tensegrity structures

Buckminster Fuller’s spheres are realizations of his balloon analogy mentioned in the Introduction. On moving from a truncated regular polyhedron (such as the icosahedron depicted in Fig. 5a), he built the model structure in Fig. 5b, consisting of an external cable network, the balloon’s surface in the analogy, coupled with a Snelson’s endoskeleton of bars, standing for the gas pressure. On indulging to one of his frequent Pindaric flights, Buckminster Fuller dreamed of roofing all of a city by one of his spheres.9 In fact, as was shown in [26], a small dome obtained from a Buckminster Fuller’s tensegrity sphere weighs about three times more than a geodesic dome designed to bear the same loads.

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8 Not surprisingly, Baracs was a co-founder of Structural Topology, a journal devoted to such subjects as geometry and statical behavior of structures, issued from 1979 to 1997.

9 D. Ingber’s daring suggestion that the mechanical behavior of cells is reminiscent of that of tensegrity systems was no Pindaric flight: in 1998, it made it to the front cover of the January issue of Scientific American [32], but eventually failed to be substantiated by experimental evidence.
Fig. 3  a Snelson’s 1959 Bead Arch and b 2001 Rainbow Arch (image from http://kennethsnelson.net);  c the Tor Vergata Arch Project;  d the 1992 Baracs’ Arch in Saint-Hyacinthe, Quebec (Canada)

Fig. 4 The 2015 Kent Tensegritree (image from https://expedition.uk.com/projects/tensegritree-university-of-kent/). In the top view on the right, radial bars are depicted in blue, “circumferential” cables in red, and stabilizing cables in green
Another structural type devised by Buckminster Fuller was his A(scending su)spension Dome (Fig. 6), patented in 1964 [33]; its surprisingly simple, original concept may be deciphered with the help of Fig. 7. On starting from the idealized bicycle wheel depicted in Fig. 7(a, b), where radial cables connect an outer compression ring to an inner tension ring, imagine duplicating the inner ring with the insertion of vertical bars, as shown in (c, d), and then continuing the process as suggested by (e, f) until a cable dome with the desired number of smaller and smaller concentric layers is obtained (the layers of the model Aspension Dome in Fig. 6 are eight).

The Aspension Dome was to serve as an inspiring model for a number of very large roofing structures, beginning with the Olympic Fencing and Gymnastic Arena in Seoul (South Korea), realized by David Geiger in 1988 (Fig. 8).10 Geiger’s design differs from Buckminster Fuller’s because the radial cables running from the

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10 Other roof structures, some earlier than Geiger’s other essentially coeval, are in some ways reminiscent of the Aspension Dome. One such structure is the Spodek, a stadium designed by Waclaw Zalewski and built in Katowice (Poland) in between 1964
inner to the outer ring, instead of forming a triangular network, belong to vertical planes passing through the central symmetry axis of the whole structure. A triangulated layout of radial cables conforming to Buckminster Fuller’s original prescription is found in the very large (233.5 m in length, 186 m in width) oval-shaped aspension roof of the Georgia Dome (Fig. 9), designed by Matthys P. Levy and built by Waidlinger Inc. in Atlanta in 1992, to be dismantled in 2017. The complications induced by the lack of central symmetry of the Georgia Dome are similar to those encountered in designing and realizing the bi-lobed roof of the La Plata Stadium in Argentina (Fig. 9 (right)), a daring structure completed in 2011, topped by ‘juxtaposition and fusion’ of two circular rings, 85 m in diameter, whose centers are 48 m apart.
Spheres and domes were not the only geometric shapes to be interpreted tensegrity-wise. It appears that studies of ring-shaped TS'es were first performed by Burkhardt in 2003 [7]; small scale models have been built by Pars (www.tensegriteit.nl/index.html) and by Pena et al. in 2009 [34]. A torus-shaped TS is analyzed both by Peng et al. in 2006 [35] and by Yuan et al. in 2008 [36]; at the end of 2009, a tensegrity torus was assembled by the Swiss company Jakob Rope System.11

3 Metatensegrities

The design freedom allowed by judicious deviations from the defining principles listed in Sect. 1 permits one to couple lightness with high performance and high visual impact, as with Buckminster Fuller’s Aspension Dome and the roof structures mentioned in Sect. 2.2. We call metatensegrities those high-performance and eye-catching systems that are at times referred to as TS structures just because they feature cables and struts, although some of their parts are of a different nature, as for example when some or all cables are replaced by membranes (see [37,38] and the literature cited therein). A second example of metatensegrity is the Blur Building (Fig. 10), a construction designed by Diller & Scofidio and built offshore in the Neuchatel lake (Switzerland) in the occasion of the 2002 Expo. According to its designers, this building is ‘made of water’, in the form of an artificial fog obtained by (filtering and) nebulizing the water from the lake. In fact, the oval platform consists of octahedral steel moduli, each of which, justifying our classification, has a vertical strut suspended by cables at its center, while a network of cables connects the top and bottom ends of all struts.

Another metatensegrity is the TorVergata Footbridge we designed in 2003, again in collaboration with our colleague Silvano Stucchi (Fig. 11). Users of this 30-m-long structure were meant to access the area of TorVergata School of Engineering and have an unusual perception of a tensegrity structure while walking inside and through it. Our intention to adhere as much as possible to Snelson’s tensegrity notion led us to confront ourselves with advanced various design problems, which we solved by making the footbridge consist of interconnected tensegrity moduli and by guaranteeing the indispensable global rigidity by prestressed cables.

At variance with the TorVergata Footbridge, the 128-m-long bike-and-pedestrian Kurilpa Bridge in Fig. 12 has no more than an optical tensegrity touch. It has been designed by Cox Rayner Architects together with Arup Engineers and built around the end of 2009 in Brisbane, Queensland (Australia). Four irregularly oriented stays are found at each pile, other stays in pairs along the deck, with primary and secondary desk-supporting cables. The actual structural regime is akin to that of a cable-stayed bridge.

11 Visit www.jakob.com/ie/en/news/a-gate-to-the-sky-jakobr-tensegrity-torus.
Fig. 12 The 2009 Kurilpa bridge, Brisbane, Queensland, (Australia). (Image credit: Paul Guard at English Wikipedia, licensed under the CC BY-SA 3.0.)

Fig. 13 Reduced-scale model of the deployable tensegrity footbridge studied in [39–41] (image courtesy of Ian F.C. Smith, EPFL)

Fig. 14 The sinuous TS footbridge presented in [42] (image courtesy of Jonas Feron and Pierre Latteur, UCLouvain)
A 16-meter deployable footbridge concept was presented by Rhode-Barbarigos et al. [39]. The 1:4 reduced-scale model shown in Fig. 13 was realized and tested experimentally [40,41]; the two halves of the model deploy from the abutments and connect to each other at mid-span.

Latteur and coworkers [42] designed and optimized a TS footbridge obtained by juxtaposition of prismatic tensegrity modules (Fig. 14).

4 The form-finding property of tensegrity frameworks

4.1 A user-friendly tensegrity notion

That a tensegrity framework suggests its equilibrium form(s)—that it possesses an intrinsic form-finding (FF) property—becomes evident when we try and build a TS by hand. On elaborating a 1997 observation due to Oppenheim and Williams [43], let us suppose that we are given three bars and nine cables, all we need to assemble the simplest three-dimensional tensegrity structure shown in Fig. 15 (left). Once we realize all but one connections between elements, we notice that the partial assembly we obtained has no definite shape, no stiffness, and that there are many possible configurations with slack cables. But, the length of the last element is determined when we try to decrease, if it is a cable (to increase, if it is a bar), the distance between the two nodes that element is due to connect. In fact, as recalled by Calladine [7], in [44] Maxwell observed that the system acquires its shape when that distance takes a minimum (maximum) value. Moreover, if the two terminal nodes are forced to get closer (further apart), then the system develops a self-stress state with the last element in tension (compression).

Figure 16 illustrates the FF property for a two-element system. The length of the element on the left is fixed, the length of the right element is not; the central node can only be on the dashed arc shown in a. On shortening the right element, placement b is reached; further shortening induces a tensional self-stress state in both elements; when lengthened as in placement c, the right element is compressed, the left element is stretched.

With these elementary examples in mind, we may state the form-finding property as follows: given a $N$-elements tensegrity system, if the lengths of $(N-1)$ elements are fixed, then a stable equilibrium shape obtains when the last cable (bar) has minimal (maximal) length. For a fixed topology, i.e., once a collection of nodes connected by bars and cables has been chosen, a passing from a stable placement to another may be achieved by changing the lengths of two or more elements. We anticipate from Sect. 6 that it is precisely the form-finding property of tensegrity systems, with their related ability to change their shape, which suggests use

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12 A procedure of this type to achieve a continuum path of equilibrium shapes has been devised by Micheletti and Williams in 2006 [45], building on results by Williams [46].
4.2 A formal tensegrity notion and the associated formulation of the form-finding problem

A tensegrity framework may be identified in terms of three mathematical ingredients:

(i) its graph, an abstract object consisting of a set of vertices plus a set of edges connecting pairs of vertices;
(ii) an embedding of its graph, i.e., a map associating to each vertex a point, or node, in $n$-dimensional Euclidean space;
(iii) an edge-by-edge labeling of its graph, according to the physical character of its elements, e.g., bar labels for edges which can carry either tension or compression, cable labels for edges which can only carry tension.

In order to be of interest, a tensegrity framework identified as specified under (i)–(iii) should be susceptible of taking at least one configuration in which

(iv) the stress distribution is admissible, that is to say, stress is tensile or null in cables;
(v) the system is stable, i.e., the total potential energy attains a local minimum.

Figure 17 serves to illustrate in a simplest instance both the terminology used in stating properties (i)–(iii) and the feasibility conditions (iv) and (v). In particular, a basic FF problem may be stated as follows: given a graph and its labeling complying with both (i) and (iii) (Fig. 17c), specify as in (ii) an embedding such that conditions (iv) and (v) be satisfied. Granted this formulation of the problem, Figs. 17d and e show instances of, respectively, unstable and stable solutions.

As a matter of fact, for tensegrity systems stability is a word requiring at times further qualification, e.g., according to Connelly [12], one speaks of prestress stability when prestress imparts stiffness to an otherwise flexible tensegrity structure; of superstability whenever a tensegrity is stable independently of prestress, that is to say, ceteris paribus, it does not buckle whatever the prestress (see also [47–49]).

13 Strictly speaking, admissible labelings should comply with the floating–compression requirement.
5 Form-finding related theories and methods

The abstract formulation we just put down is not the only possible formulation of form-finding problems for tensegrity structures nor are such problems peculiar of this type of structures. After Pugh’s 1976 account [50], solutions to FF mathematical and design problems are available for many regular or symmetric shapes (prisms, towers, polyhedra, and others) [51–61]; moreover, several FF problems and the relative attack methods have been reviewed in Tibert and Pellegrino [62] and in Hernandez Juan and Mirats Tur [63].

Hereafter we proceed to list various approaches to tensegrity problems by mathematicians, architects, and structural engineers. Curiously and characteristically, these approaches span from Sakantamis and Popovic Larsen’s [64], proceeding by manual construction of physical models, to De Guzman and Orden’s [65] symbolic approach, supplied by an interesting theorem, which states that every TS can obtained from elementary units, referred to as atoms. Here are some other approaches.

- Genetic algorithms were employed by Paul et al. [66] and Xu and Luo [67]. The genetic algorithm given in Rieffel et al. [68] has the remarkable feature of leading to discovery of tensegrities with new underlying graphs.
- Li et al. [69] employed simple graph-theory tools to derive a TS from any vertex-truncated polyhedron.
- Connelly and Back [15] and Pandia Raj and Guest [58] applied group-representation theory to discover symmetric placements.
- Since 1994, techniques of nonlinear programming have been extensively used by Burkhardt, whose often-updated handbook appeared in [51].
- Motro [70] employed the dynamic-relaxation method introduced by Day [71] and then reliably applied to tensile structures (see, e.g., Barnes [72]) and to many other nonlinear problems. A refined dynamic-relaxation procedure has been employed in Zhang et al. [73].
- Pellegrino [74] formulated an equivalent constrained minimization problem.
- Liapi and Kim [75] proposed a parametric approach for the design of modular vaulted TS’es.
- Vassart and Motro [76] and Zhang and Ohsaki [77] employed the force-density method, first introduced by Linkwitz and Schek [78] (see also Schek [79]) for form-finding of tensile structures.
- Zhang and Ohsaki [80], Gomez Estrada et al. [81] and Tran and Lee [82] presented new numerical methods based on a force-density formulation.
- Williamson et al. [83] presented an algebraic approach specialized to floating-compression structures.
- Ebara and Kanno [84] devised a method to find the set of edges for a given placement of nodes.
- Micheletti [85] studied reciprocal diagrams, which can be used also to find new TS’es.
- Maceri et al. [86] presented an algebraic model for TS’es taking into account the unilateral behavior of cables.
- Nagase and Skelton [87] proposed a unified framework for finding minimal-mass TS’es.
- Pietroni et al. [88] devised a new framework for form-finding and designed free-form TS’es (Fig. 18).
- Latteur et al. [89] presented a design methodology for TS’es based on a stiffness-and-volume optimization algorithm.
- Kan et al. [90] presented a static and dynamic complementarity framework for the analysis of TS’es with slack cables.
- Aloui et al. [91] built on the work in [65] and introduced a general methodology to obtain complex spatial TS’es from elementary cells by cellular morphogenesis.

A list of form-finding related structural proposals follows.

- Raducanu and Motro patented a class of double-layer TS grids [92], described also in [93]. In these grids, the ‘islands of compression’ are constituted by series of bars arranged in a zigzag pattern (Fig. 19).
- Genovese [94], Moored and Bart-Smith [95], and Bel Hadj Ali [96] presented formulations for TS’es with clustered sliding cables.
- Gómez-Jáuregui et al. [97] proposed some new double-layer TS grids.
- Fraddosio et al. [98] studied different variants of tensegrity modules (Fig. 20) to assemble modular beams and double-layer grids; in [99], they proposed a new method to determine feasible self-stress states consistent with the unilateral behavior of cables.
6 Technological problems and intrinsic limitations

Certain technological issues and intrinsic structural limitations need to be successfully addressed in order to develop viable applications of tensegrity concepts.

Relevance and solution method of technological problems depend strongly on the application of current interest. Take two such issues, node design and actuation procedures of deployable structures, both during construction and in service: for civil engineering structures, nodes require a design and actuation calls for procedures (cf. Moored and Bart-Smith [95]) that are not at all the same as for the spatial structures considered in Sect. 7.1.2.

On turning to intrinsic limitations, TS-like structures have intrinsically low stiffness (see Wang [24]). This is mostly due to the fact that the overall stiffness of such structures is only furnished by cables, because bars
are not directly interconnected. Also, the edges-to-nodes ratio is lower than that of conventional frameworks. However, low stiffness, which makes any TS system inefficient to sustain relevant loads, is a lesser or no problem when such a system is due to bear small or no mechanical actions, as in the case of space applications.

Complexity is a second aspect which has limited practical realizations up to now: indeed, TS systems have complex geometry and complex mechanical behavior. As reported in Sect. 5, complex geometry has been dealt with by improving early form-finding methods and by developing new ones. However, the choice of formal and yet all-important conceptual TS ingredients (such as ingredients (i), (ii) and (iii) listed in Sect. 4.1) still remains a crucial step in determining success of a design solution.

Complex mechanical behavior is mainly due to nonlinearities, which are associated both with overall stiffening response in statics and with unilateral behavior of cables (cf. Maurin et al. [100]), let alone possible collisions and interferences between elements (cf. Le Saux et al. [101]). Complexity is also measured by the generally high number of variables involved in an optimization process such as, e.g., optimization of a whole path of equilibrium placements in the case of deployable structures.14

7 Tensegrity-inspired structures

The main reason why tensegrity concepts find application to deployable and variable-geometry structures is that TS structures have the FF property: by the use of actuator cables or bars, they can pass from one configuration to another through a continuous path of equilibrium placements. Moreover, due to the absence of hinges between bars, the mechanical behavior of floating–compression systems can be predicted with better accuracy than the behavior of conventional hinged systems.

7.1 Deployable structures

Studies for foldable/deployable TS’es pioneered by Furuya [105] and Hanaor [106] were continued by Oppenheim and Williams [43], Bouderbala and Motro [107], and Sultan and Skelton [108].

7.1.1 Terrestrial TS’es

The problem of foldable double-curvature grids with contiguous bars based on the pattern proposed by Raducanu and Motro (Fig. 19) was addressed in El Smaili et al. [109]. In [110], path planning and feedback control were used for the deployment of the TS footbridge shown in Fig. 13, while in [111], a novel type of deployable double-layer TS grid was designed and realized (Fig. 21). The design of a retractable aspension dome, which makes use of clustered cables, was presented in [112].

14 One of the first attempts to formulate an optimization problem for TS’s is that of Masic et al. [102]. In this work, the problem of finding a TS with optimal mass-to-stiffness ratio is solved employing a standard software package. The labeling of the underlying graph is not a floating–compression one and is not fixed a priori but is obtained as a result of the procedure. Ohsaki and Zhang [103] presented various approaches for designing TS, based on the force-density formulation and nonlinear programming. Rhode-Barbarigos et al. [104] showed that a better minimal-cost solution is found by a stochastic method than by a parametric analysis.
7.1.2 Spatial TS’es

Investigations on TS’es for space applications were performed in Tibert [113]; a deployable TS reflector for small satellites was described in [114], a deployable mast in [115]; various different deployable TS masts were analyzed in [116]. Crane et al. (2000) proposed a deployable antenna based on a tensegrity prism [117], which inspired a patent by Stern [118]. Russell and Tibert [119] modeled the deployment of a TS with inflatable tension elements, which need not be rigidified.

From 2009 to 2011, together with Tibert and Kayser Italia, a company based in Livorno (Italy), we took part in “Large aperture deployable systems”, a project of the European Space Agency aimed to concept designing and prototype testing of a deployable tensegrity ring (Fig. 22) to unfurl a mesh reflector in space [120,121]. The TS structure we designed was eventually patented in Europe [122] and the US [123]. The same kind of TS ring was studied further in [124].
7.2 Variable-geometry structures

In the literature, a number of studies about variable-geometry TS’es are found. Defossez [125] described the interesting bistability properties of some particular TS’es. Fest et al. [126] designed and realized a prestressed modular system, which can actively respond to external actions; the length of telescopic struts is adjusted using a stochastic search algorithm developed ad hoc. Schenk et al. [127] analyzed and designed a zero-stiffness TS prism, which can move along a continuous path of equilibrium symmetric placements, exhibiting a mechanism-like behavior. Arsenault [128] analyzed different variable-geometry TS structures. Fraternali et al. investigated the use of TS’es for ‘kinetic’ solar facades of smart buildings. The use of shape-memory-alloy cable actuators to obtain adaptive TS’es was considered in [129,130]; in particular, in [129] a shape-morphing three-stage tower with frequency tuning capabilities is realized and tested, while in [130], a matrix formulation is given for the simulation of TS’es with antagonistic actuation.

7.3 Robots

In the last few years, attention on robotic applications of TS’es has increased significantly. Following the work by Skelton et al. [18], Aldrich et al. [131] presented a study on a light and agile robot, in which the cable forces necessary to move along a prescribed path in minimum time are determined. Paul et al. [132] employed triangular and square TS prism to realize robots capable of moving on ground by exploiting evolutionary control algorithms. Fivat and Lipson realized a rolling robot based on the Expanded Octahedron, a classical TS, without publishing results (Fig. 23a, b). Rovira and Mirats Tur [133] demonstrated a robot with path-finding ability, again based on a TP. In Shai et al. [134], a robot obtained by stacking TP’s on top of each other was analyzed and built. Shibata and Hirai [135] used the expanded octahedron for rolling locomotion. Ushigome et al. [136] realized moving and interactive TS’s. TS robots were also considered by NASA as a viable solution for planetary exploration [137,138] (Fig. 23c). Unlike wheeled robots, a cable-actuated rolling locomotion allows these robots to move on rough terrain with no risk of tipping over.

8 Promising research lines

We expect research on foldable/deployable systems and robots to continue in the future. Likewise, the complex nonlinear behavior of TS’es may turn out to be a valuable, perhaps the most valuable, resource for innovative applications. In fact, a complex behavior—oftentimes a drawback for conventional structures at medium/large scales—may be successfully exploited at small scales. In this connection, we find it appropriate to quote a general guiding principle laid down by R.E. Skelton in [139], that is, design/optimization of a structure and its control system should be simultaneously pursued.

15 These facades, which are inspired by the spectacular solar screens of Al Bahar towers in Abu Dhabi, are termed kinetic because they are motor-driven so as to control the effects of solar radiation.
8.1 Adaptivity, nonlinear behavior

Certain tensegrity systems may ‘adapt’ because their rigidity or geometry is made to change. When exploited by way of slow processes, adaptivity opens up many promising areas of application, as exemplified by the deployable and variable-geometry structures listed in Sects. 7.1 and 7.2. But adaptation may also be exploited in a fast modality.

Crucial to a full understanding of the nonlinear static and dynamic behavior of TS-like complexes, whatever their scale, has been the study of one T3 prism first, then of multiple T3 complexes. Between 1997 and 2001, Oppenheim and Williams analyzed the nonlinear response of a T3, both in statics [140] and in dynamics [141]; they also suggested the possibility of using TS’es to realize adaptive structures [43]. Building on the developments in [140], Fraternali and coworkers [142] provided a detailed description of the nonlinear static response of a T3 prism: they found that it may stiffen or soften in response to applied loads depending on its aspect ratio, a behavior that was experimentally confirmed [143]. Moreover, it was shown in [49] that some other elastic TS’es feature two different bistable regimes, which can be explored either by changing their geometry (but not their connectivity) or by increasing their prestrain. Continuing this research line in [141], free and forced oscillations of T3 prisms, were studied in [144], and the steady-state nonlinear dynamics of one T3 prism was investigated by Michielsen et al. [145] by means of simulations and experiments.

In the last decade, TS dynamics has attracted more and more attention. In [146], Fraternali et al. have shown that a TS column consisting of T3 modules supports propagation of solitary waves, a feature exploited later to realize patented devices for wave generation and detection [147]. Prestress tunability of this type of solitary waves has been investigated in [148], where it was also shown that, for a T3 prism of suitable geometry, a compressive impulsive load may generate a rarefaction solitary wave. Other studies have demonstrated both the relevance of certain chirality characters to the impulsive dynamics of T3 chains [149] and the occurrence of compact compression waves (compactons) in two- and three-dimensional T3 lattices [150] (Fig. 24).

8.2 Tensegrity metamaterials

A number of recent advances have opened the way to development of new metamaterials with unprecedented mechanical properties, termed TS metamaterials. The nonlinear response of TS lattices consisting of elementary cells was investigated in [151,152]. Interestingly, it was shown in [153] that frequency band gaps of

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16 The T3 prism, depicted in Fig. 15, is the simplest three-dimensional TS.
‘monoatomic’ and ‘biatomic’ chains consisting of T3 prisms can be adjusted by fine-tuning of internal and external prestressing forces. In [154], \( n \)-dimensional assemblies of tensegrity units were analyzed at different prestress levels; various wave-propagation regimes were shown possible. In [155], it was found that the symmetries of the homogenized elasticity tensor of a tensegrity lattice depend on prestress. In [156], a systematic design framework was proposed for prestress-tunable tensegrity metamaterials, which allows also for the generation of floating–compression TS lattices. Prestress-controllable band-gap tunability of modular TS lattices was observed in experiments on polyamide specimens, additively manufactured by selective laser sintering [157].

8.3 Small-size tensegrity structures

Whatever the scale, fabrication of TS structures is challenging, due to the presence of tension-only members. We mention here three successful attempts at the centimeter scale: in [158], TS’es are realized by additive manufacturing of titanium-alloy bars, in combination with assembling and post-tensioning cables in ultrahigh molecular weight polyethylene; in [159], deployable TS’es were realized using shape-memory polymer struts and elastomeric cables; and a peculiar additive-manufacturing procedure to realize programmable soft tensegrity robots was proposed in [160].

Other studies concern tensegrity-like lattices, obtained by replacing by no-prestress bars all cables of a parent TS lattice. The interest of these tensegrity-like lattices consists in the fact that they inherit the nonlinear response of the parent TS lattice in absence of cable slackening. In [161] and [162], TS lattices with snap-through and bistable response were designed and additively manufactured, again at the centimeter scale; in this design, nodal connections between bars were realized by reducing bar diameter near nodes so as to obtain flexure hinges (Fig. 25a). Tensegrity-like lattices were fabricated also at the micrometer scale by multiphoton lithography. This was done in [163], where T3-based bistable lattices were subjected to cyclic compression tests (Fig. 25b), and in [164], where evidence is given that tensegrity-like lattices exhibit delocalized deformation at failure in compression.

9 Conclusions

We have striven for documenting in a concise but fairly complete way the as of today seventy-year-long history of the tensegrity concept and its manifold applications and hybridizations. In our perception, this story is far from being complete. To the contrary, the recent and current research lines we mentioned in the previous section have both special conceptual attraction and high applicative potential. Although we refrain from indulging into
predictions, it seems to us quite likely that the subject of the tensegrity concept and its applications will soon require updating.

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Declarations

Conflict of interest The authors have no competing interests to declare.

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Seventy years of tensegrities (and counting)

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