The quantum simulation setup of $^{87}$Rb Bose-Einstein condensates and numerical analysis of disorder induced dynamic-equilibrium localization

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Abstract. We report on a rapid production of $^{87}$Rb Bose-Einstein condensates(BECs) in a tight confinement hybrid trap based on single-beam optical dipole trap(ODT) and magnetic quadrupole trap(MQT). By the help of preliminary evaporation cooling in MQT, more atoms can be transferred to ODT. Then the BEC phase transition will be achieved by forced evaporation in ODT. This setup has good optical access for loading multiple optical lattices and laser speckle to quantum simulation.

In theory, we numerically analyze the dynamic behavior of Bose-Einstein condensates in one-dimensional disordered potential before its completely losing spatial quantum coherence. We find that localization length can be remarkably affected by both the disorder statistics and the atom interaction. We also find that the phase of the condensates is broken into many small pieces while the system approaching localization, showing a counter-intuitive step-wise phase but not a thoroughly randomized phase. Although the condensates as a whole showing less flow and expansion, large currents occur where the phase changes abruptly. Thus we show explicitly that the localization of a finite size BEC is dynamic-equilibrium.

1. Introduction

Ultracold atoms, with their unprecedented controllability of interactions and other parameters, become the new powerful tools to simulate and study some problems of condensed matter physics, such as localization[1] and metal-insulator transition[2], super-fluidity et.al. In ultracold atoms system we can accurately control the interactions between atoms as well as other parameters, and utilize optical method to generate and control optical lattice. Based on these advantages we can directly and quantificationally research some complicated quantum systems, such as disorder. Here we use $^{87}$Rb BECs and laser speckle to simulate disorder system and research the dynamical properties of atoms in disordered potential.

Disorder is indeed an intrinsic property of all the real systems. Although sometimes the strength of disorder is much smaller than the chemical potential of a system, it still produces remarkably effects on the system. In this paper we introduce our experimental design for disorder system and some results of numerical simulation about Bose-Einstein condensates in one-dimensional disordered potentials(1D laser speckle). We found that the statistic characteristics
of the disorder potential can significantly affect the expansion of the condensate. Increasing the intensity and the autocorrelation length of speckles can both inhibit the expansion of the condensate. In addition, we calculated the phase distribution of the condensate when it becomes localized. We found the distribution of phase is in shape of disordered steps, and the abrupt changes of the phase always locates in between localized density peaks, corresponding strong atom currents.

2. Experimental Setup
Our $^{87}$Rb BEC system is based on a magnetical quadrupole trap and a red-detuned optical dipole trap[3]. We collect atoms in magneto-optical trap firstly and then transfer them to magnetic quadrupole trap and force RF evaporation preliminary. Finally the BEC is achieved in the single beam optical dipole trap by reduce its light intensity. This setup has good optical access for loading speckle on this system. The large numerical aperture is necessary to submicron size laser speckles.

We utilize a diffuser and a high N.A. lens to generate laser speckle. The amplitude(AMDP) and self-correlation length(SCL) of the speckle can be easily and precisely controlled. The AMDP is defined by the standard deviation of disordered potentials. Fig. 1 is the diagram of our experimental setup.

3. Numerical Simulation
We numerically analyze the dynamic behavior of Bose-Einstein condensate (BEC) in a one-dimensional disordered potential before its completely losing spatial quantum coherence. We use the 1D Gross-Pitaevskii equation (GPE) to describe a system with fixed average number of weakly interacting bosonic atoms, trapped in a harmonic potential. The form of 1D GPE[4] is

$$\hbar \frac{\partial \varphi}{\partial t} = \left[ -\hbar^2 \frac{\partial^2}{2m \partial z^2} \varphi(z) + V_{ho}(z) + V_{dis}(z) + U0 |\varphi|^2 \right] \varphi \tag{1}$$

where $V_{ho}(z)$ represent harmonic potential with a trapping frequency $\omega$, $V_{dis}(z)$ represents a disordered potential, and $U0 = g_{1D}N$. Here $N$ is atom number and $g_{1D}$ is the coupling constant, given by $g_{1D} = g/2\pi a_s^2 = 2a\hbar \omega_r$, where $a$ is s wave scatter length and $\omega_r$ is the transverse trapping frequency.
Using the imaginary time split-step Fourier method, we can obtain the initial wave function, which is the ground state wave function in a harmonic trap. Then we load the ground state into disordered potential and watch the evolution of the density and phase distribution of the wave function with time.

3.1. The effects of disorder statistics on atoms extension

![Figure 2.](image1)

**Figure 2.** The RMS size of the BEC wave function evolving in the speckle for several values of the amplitude after 1s evolved. The inset is the wave function of BEC after 1s evolved. The autocorrelation length of speckle is fixed at 1.5Unit.

![Figure 3.](image2)

**Figure 3.** The RMS size of the BEC wave function in speckle for several values of the autocorrelation length after 1s evolved. The inset is the wave function of BEC after 1s evolved. The amplitude of speckle is fixed at 0.05Er.

In Fig.2 and Fig.3, we plot the RMS size of the wave function as a function of the amplitude and autocorrelation length of disordered potential respectively. We find that increasing the intensity and the autocorrelation length of disordered potentials can both inhibit the expansion of the condensate.

3.2. The dynamic-equilibrium localization

we have discussed the effect of disordered potential on the probability density of wave function. But only the probability density can not indicate the whole property of wave function. We can not neglect the effect of disorder on the phase of wave function. The information of phase concerns the important physics property of BEC: Spatial coherence and Velocity of atoms. 

In harmonic trap the phase of BEC is continuous and consistent. The BEC has good coherence. But now the BEC is loaded into disordered potential and approach localization, we want to know how the disorder affect the phase and what is the presenting state of phase when the wave function approach localization. Here we analyzed the evolution of phase in disordered potential. As the Fig. 4a show, the phase becomes more and more disordered as the evolution time increasing. When the wave function approached localization the phase present some disordered step-wise shape. There are some abrupt changes of phase where the density of atoms occurred minimum value, Fig. 4b.

Through the distribution of phase we can calculate the particle current distribution of BEC. As the Fig. 4c shows, when the BEC approached localization though the whole wave function no longer expands, there are strong particle currents in the position of abruptly change of phase. So the localization is a dynamic equilibrium state.
4. Conclusion

we have simulated the evolution of a Bose-Einstein condensate in a 1D disorder potential, which can be generated by laser speckles. We found that the statistic characteristics of the disorder potential can significantly affect the expansion of the condensate. Increasing the intensity and the autocorrelation length of speckles can both inhibit the extension of the condensate. In addition we calculated the phase distribution of the condensate when it becomes localized. We found the distribution of phase is in shape of disordered steps, and the abrupt changes of the phase always locates in between localized density peaks, corresponding strong atom currents. These results illustrate that the localized state is not a static but dynamic equilibrium, and atom interchange between density peaks happens all the time.

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Figure 4. (a) The time evolution of the phase of BEC in speckle; (b) The corresponding relation between amplitude and phase of the localized wavefunction; (c) The corresponding relation between phase and particle current.