SymmetricTextures in $SO(10)$ and LMA Solution for Solar Neutrinos

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Abstract

We analyze a model based on SUSY $SO(10)$ combined with $SU(2)$ family symmetry and symmetric mass matrices constructed by the authors recently. Previously, only the parameter space for the LOW and vacuum oscillation (VO) solutions was investigated. We indicate in this note the parameter space which leads to large mixing angle (LMA) solution to the solar neutrino problem with a slightly modified effective neutrino mass matrix. The symmetric mass textures arising from the left-right symmetry breaking and the $SU(2)$ symmetry breaking give rise to very good predictions for the quark and lepton masses and mixing angles. The prediction of our model for the $|U_{e\nu_3}|$ element in the Maki-Nakagawa-Sakata (MNS) matrix is close to the sensitivity of current experiments; thus the validity of our model can be tested in the near future. We also investigate the correlation between the $|U_{e\nu_3}|$ element and $\tan^2 \theta_\odot$ in a general two-zero neutrino mass texture.

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The recently reported measurements from KamLAND reactor experiment \cite{1} confirmed the large mixing angle (LMA) solution to be the unique oscillation solution to the solar neutrino problem at 4.7 $\sigma$ level \cite{2,3,4}. The global analysis including Solar + KamLAND + CHOOZ indicate the following allowed region at 3$\sigma$ \cite{2},

\begin{align*}
5.1 \times 10^{-5} < \Delta m^2_{21} < 9.7 \times 10^{-5}eV^2 \\
0.29 \leq \tan^2 \theta_{12} \leq 0.86 \\
(0.70 \leq \sin^2 2\theta_{12} \leq 0.994)
\end{align*}

The allowed regions at 3$\sigma$ level based on a global fit including SK + Solar + CHOOZ for the atmospheric parameters and the CHOOZ angle are \cite{5}

\begin{align*}
1.4 \times 10^{-3} < \Delta m^2_{32} < 6.0 \times 10^{-3}eV^2 \\
0.4 \leq \tan^2 \theta_{23} \leq 3.0 \\
(0.82 < \sin^2 2\theta_{23}) \\
\sin^2 \theta_{13} < 0.06
\end{align*}

There have been a few SO(10) models constructed aiming to accommodate the observed neutrino masses and mixing angles (see for example, \cite{6,7,8}). By far, the LMA solution is the most difficult to obtain. Most of the models in the literature assume the mass matrices “lopsided”. In our model based on SUSY SO(10) $\times$ SU(2) \cite{6,7} (referred to “CM” herein), we consider symmetric mass matrices which are resulting from the left-right symmetric breaking of SO(10) and the breaking of family symmetry SU(2). Previously, we studied the parameter space for the LOW and VO solutions to the solar neutrino problem in our model. In view of the KamLAND result, we re-analyze our model and find the parameter space for the LMA solution.

The details of our model based on SO(10) $\times$ SU(2)$_F$ are contained in CM. The following is an outline of its salient features. In order to specify the superpotential uniquely, we invoke $Z_2 \times Z_2 \times Z_2$ discrete symmetry. The matter fields are

$$
\psi_a \sim (16, 2)^{++} \quad (a = 1, 2), \quad \psi_3 \sim (16, 1)^{+++}
$$

where $a = 1, 2$ and the subscripts refer to family indices; the superscripts $+/-$ refer to $(Z_2)^3$ charges. The Higgs fields which break SO(10) and give rise to mass matrices upon acquiring
The superpotential of our model is

\[ W = W_{Dirac} + W_{\nu RR} \]

\[ W_{Dirac} = \psi_3 \psi_3 T_1 + \frac{1}{M} \psi_3 \psi_a \left( T_2 \phi_{(1)} + T_3 \phi_{(2)} \right) \]
\[ + \frac{1}{M} \psi_a \psi_b \left( T_4 + \mathcal{C} \right) S_{(2)} + \frac{1}{M} \psi_a \psi_b T_5 S_{(1)} \]

\[ W_{\nu RR} = \psi_3 \psi_3 \mathcal{C}_1 + \frac{1}{M} \psi_3 \psi_a \Phi \mathcal{C}_2 + \frac{1}{M} \psi_a \psi_b \Sigma \mathcal{C}_2 \]

The mass matrices then can be read from the superpotential to be

\[ M_{u,\nu LR} = \begin{pmatrix}
0 & 0 & \langle 10^\pm_2 \rangle \epsilon' \\
0 & \langle 10^+_4 \rangle \epsilon & \langle 10^+_3 \rangle \epsilon \\
\langle 10^+_2 \rangle \epsilon' & \langle 10^+_3 \rangle \epsilon & \langle 10^+_1 \rangle \\
\end{pmatrix} \]
\[
\begin{pmatrix}
0 & 0 & r_2 e' \\
0 & r_4 e & \epsilon \\
r_2 e' & \epsilon & 1
\end{pmatrix}
M_U
\]

(14)

\[
M_{d,e} = \begin{pmatrix}
0 & \langle 10_5^- \rangle e' & 0 \\
\langle 10_5^- \rangle e' & (1, -3) \langle 126^- \rangle \epsilon & 0 \\
0 & 0 & \langle 10_1^+ \rangle 
\end{pmatrix}
= \begin{pmatrix}
0 & \epsilon' & 0 \\
\epsilon' & (1, -3) p e & 0 \\
0 & 0 & 1
\end{pmatrix}
M_D
\]

(15)

where \( M_U \equiv \langle 10_1^+ \rangle \), \( M_D \equiv \langle 10_1^- \rangle \), \( r_2 \equiv \langle 10_2^+ \rangle / \langle 10_1^+ \rangle \), \( r_4 \equiv \langle 10_4^+ \rangle / \langle 10_1^+ \rangle \) and \( p \equiv \langle 126^- \rangle / \langle 10_1^- \rangle \). The right-handed neutrino mass matrix is

\[
M_{\nu_{RR}} = \begin{pmatrix}
0 & 0 & \langle 126_2^0 \rangle \delta_1 \\
0 & \langle 126_2^0 \rangle \delta_2 & \langle 126_2^0 \rangle \delta_3 \\
\langle 126_2^0 \rangle \delta_1 & \langle 126_2^0 \rangle \delta_2 & \langle 126_1^0 \rangle \delta_3
\end{pmatrix}
= \begin{pmatrix}
0 & 0 & \delta_1 \\
0 & \delta_2 & \delta_3 \\
\delta_1 & \delta_3 & 1
\end{pmatrix}
M_R
\]

(16)

with \( M_R \equiv \langle 126_1^0 \rangle \). Here the superscripts \(+/-/0\) refer to the sign of the hypercharge. It is to be noted that there is a factor of \(-3\) difference between the \((22)\) elements of mass matrices \( M_d \) and \( M_e \). This is due to the CG coefficients associated with \( 126 \); as a consequence, we obtain the phenomenologically viable Georgi-Jarlskog relation. We then parameterize the Yukawa matrices as follows, after removing all the non-physical phases by rephasing various matter fields:

\[
Y_{u,\nu_{LR}} = \begin{pmatrix}
0 & 0 & a \\
0 & be^{i\theta} & c \\
a & c & 1
\end{pmatrix}
d
\]

(17)

\[
Y_{d,e} = \begin{pmatrix}
0 & ee^{-i\xi} & 0 \\
0 & e^{i\xi} & (1, -3) f \\
0 & 0 & 1
\end{pmatrix}
h
\]

(18)
This is one of the five sets of symmetric texture combinations (labeled set (v)) proposed by Ramond, Roberts and Ross [10].

We use the following inputs at $M_Z = 91.187 \text{ GeV}$ [11, 12]:

\begin{align*}
m_u &= 2.32 \text{ MeV} (2.33^{+0.42}_{-0.45}) \\
m_c &= 677 \text{ MeV} (677^{+56}_{-61}) \\
m_t &= 182 \text{ GeV} (181^{+13}_{-13}) \\
m_e &= 0.485 \text{ MeV} (0.486847) \\
m_\mu &= 103 \text{ MeV} (102.75) \\
m_\tau &= 1.744 \text{ GeV} (1.7467) \\
|V_{us}| &= 0.222 (0.219 - 0.224) \\
|V_{ub}| &= 0.0039 (0.002 - 0.005) \\
|V_{cb}| &= 0.036 (0.036 - 0.046)
\end{align*}

where the values extrapolated from experimental data are given inside the parentheses.

These values correspond to the following set of input parameters at the GUT scale, $M_{\text{GUT}} = 1.03 \times 10^{16} \text{ GeV}$:

\begin{align*}
a &= 0.00246, \\
b &= 3.50 \times 10^{-3} \\
c &= 0.0320, \\
d &= 0.650 \\
\theta &= 0.110 \\
e &= 4.03 \times 10^{-3}, \\
f &= 0.0195 \\
h &= 0.0686, \\
\xi &= -0.720 \\
g_1 &= g_2 = g_3 = 0.746
\end{align*}

the one-loop renormalization group equations for the MSSM spectrum with three right-handed neutrinos are solved numerically down to the effective right-handed neutrino mass scale, $M_R$. At $M_R$, the seesaw mechanism is implemented. With the constraints $|m_{\nu_3}| \gg |m_{\nu_2}|, |m_{\nu_1}|$ and maximal mixing in the atmospheric sector, the up-type mass texture leads us to choose the following effective neutrino mass matrix

\begin{equation}
M_{\nu_{LL}} = \begin{pmatrix}
0 & 0 & t \\
0 & 1 & 1 + t^{3/2} \\
t & 1 + t^{3/2} & 1
\end{pmatrix} \frac{d^2 v_u^2}{M_R} 
\end{equation}
and from the seesaw formula we obtain

\[
\begin{align*}
\delta_1 &= \frac{a^2}{c^2 t + a^2 (2t^{1/2} + t^2) + 2a (1 - c (1 + t^{3/2}))} \\
\delta_2 &= \frac{b^2 e^{2i\theta}}{c^2 t + a^2 (2t^{1/2} + t^2) + 2a (1 - c (1 + t^{3/2}))} \\
\delta_3 &= \frac{-a (be^{i\theta} (1 + t^{3/2}) - c) + bct e^{i\theta}}{c^2 t + a^2 (2t^{1/2} + t^2) + 2a (1 - c (1 + t^{3/2}))}
\end{align*}
\]

(21)

We then solve the two-loop RGE’s for the MSSM spectrum down to the SUSY breaking scale, taken to be \(m_t(m_t) = 176.4 \text{ GeV}\), and then the SM RGE’s from \(m_t(m_t)\) to the weak scale, \(M_Z\). We assume that \(\tan \beta \equiv v_u/v_d = 10\), with \(v_u^2 + v_d^2 = (246/\sqrt{2} \text{ GeV})^2\). At the weak scale \(M_Z\), the predictions for \(\alpha_i \equiv g_i^2/4\pi\) are

\[
\begin{align*}
\alpha_1 &= 0.01663, \quad \alpha_2 = 0.03374, \quad \alpha_3 = 0.1242
\end{align*}
\]

These values compare very well with the values extrapolated to \(M_Z\) from the experimental data, \((\alpha_1, \alpha_2, \alpha_3) = (0.01696, 0.03371, 0.1214 \pm 0.0031)\). The predictions at the weak scale \(M_Z\) for the charged fermion masses, CKM matrix elements and strengths of CP violation, are summarized in Table I of Ref. [7]. Using the mass square difference in the atmospheric sector \(\Delta m_{\text{atm}}^2 = 2.78 \times 10^{-3} \text{ eV}^2\) and the mass square difference for the LMA solution \(\Delta m_{\odot}^2 = 7.25 \times 10^{-5} \text{ eV}^2\) as input parameters, we determine \(t = 0.35\) and \(M_R = 5.94 \times 10^{12} \text{ GeV}\), and correspondingly \((\delta_1, \delta_2, \delta_3) = (0.00119, 0.000841 e^{i (0.220)}, 0.0211 e^{-i (0.029)})\). We obtain the following predictions in the neutrino sector: The three mass eigenvalues are give by

\[
(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = (0.00363, 0.00926, 0.0535) \text{ eV}
\]

(22)

The prediction for the MNS matrix is

\[
|U_{MNS}| = \begin{pmatrix}
0.787 & 0.599 & 0.149 \\
0.508 & 0.496 & 0.705 \\
0.350 & 0.629 & 0.694
\end{pmatrix}
\]

(23)

which translates into the mixing angles in the atmospheric, solar and reactor sectors,

\[
\begin{align*}
\sin^2 2\theta_{\text{atm}} &= \frac{4|U_{\mu\nu_3}|^2|U_{\tau\nu_3}|^2}{(1 - |U_{e\nu_3}|^2)^2} = 1 \\
\tan^2 \theta_{\text{atm}} &= \frac{|U_{\mu\nu_3}|^2}{|U_{\tau\nu_3}|^2} = 1.03
\end{align*}
\]
\[ \sin^2 2\theta_{\odot} = \frac{4|U_{e\nu_1}|^2|U_{e\nu_2}|^2}{1 - |U_{e\nu_3}|^2} = 0.93 \]

\[ \tan^2 \theta_{\odot} \equiv \frac{|U_{e\nu_2}|^2}{|U_{e\nu_1}|^2} = 0.58 \]

\[ \sin^2 \theta_{13} = \frac{|U_{e\nu_3}|^2}{|U_{e\nu_1}|^2} = 0.022 \]

To our precision, the atmospheric mixing angle is maximal, while the solar angle is within the allowed region at 1\( \sigma \) level (0.37 \( \leq \tan^2 \theta_{\odot} \leq 0.60 \) \[8\])). We comment that \( M_{\nu LL} \) given in Eq. 20 is a special case of a two-zero texture

\[
\begin{pmatrix}
0 & 0 & * \\
0 & * & * \\
* & * & *
\end{pmatrix}
\]

(25)

first proposed in \[6\] in which the elements in (23) block are taken to have equal strengths to accommodate near bi-maximal mixing. Here we consider a slightly different case

\[
\begin{pmatrix}
0 & 0 & t \\
0 & 1 & 1 + t^n \\
t & 1 + t^n & 1
\end{pmatrix}
\]

(26)

This modification is needed in order to accommodate a large, but non-maximal solar angle in the so-called “light side” region (0 < \( \theta < \pi/4 \)) \[13\]. We find that it is possible to obtain the LMA solution at 3\( \sigma \) level with \( n \) ranging from 1 to 2. To obtain the LMA solution within the allowed region at 1\( \sigma \) level, we have considered above \( n = 3/2 \). The correlation between \( |U_{e\nu_3}|^2 \) and \( \tan^2 \theta_{\odot} \) for different values of \( n \) is plotted in Fig. 1. The prediction of our model for the strengths of CP violation in the lepton sector are

\[ J^l_{CP} \equiv Im\{U_{11}U_{12}^*U_{21}^*U_{22}\} = -0.00690 \]

\[ (\alpha_{31}, \alpha_{21}) = (0.490, -2.29) \]

(27)

Using the predictions for the neutrino masses, mixing angles and the two Majorana phases, \( \alpha_{31} \) and \( \alpha_{21} \), the matrix element for the neutrinoless double \( \beta \) decay can be calculated and is given by \( |< m >| = 2.22 \times 10^{-3} \) eV. Masses of the heavy right-handed neutrinos are \( (M_1, M_2, M_3) = (1.72 \times 10^7, 2.44 \times 10^9, 5.94 \times 10^{12}) GeV \). As in the case of LOW and VO solutions in our model \[7\], the amount of baryogenesis due to the decay of heavy right-handed neutrinos is too small to account for the observed amount. Thus another mechanism
FIG. 1: Correlation between $|U_{e3}|^2$ and $\tan^2 \theta_\odot$ for different values of $n$. The value of $\Delta m_{\text{atm}}^2$ is $2.8 \times 10^{-3}eV^2$. The dotted line corresponds to the upper bound $\Delta m_{\odot}^2 = 10^{-4}eV^2$; the dotted-long-dashed line corresponds to the best fit value $\Delta m_{\odot}^2 = 7.3 \times 10^{-5}eV^2$; the dotted-short-dashed line corresponds to the lower bound $\Delta m_{\odot}^2 = 5 \times 10^{-5}eV^2$. So a generic viable prediction of the texture given in Eq. 26 is in the region bounded by the dotted line and the dotted-short-dashed line.

for baryogenesis is needed in our model. The prediction for the $\sin^2 \theta_{13}$ value is 0.022, in agreement with the current bound 0.06. Because our prediction for $\sin^2 \theta_{13}$ is very close to the present sensitivity of the experiment, the validity of our model can be tested in the foreseeable future.
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