Innovative Models for Crop Planning Problem to Improve Production Efficiency in Agricultural Management under Uncertainty

Takeshi Itoh
Graduate School of Management and Economics Tohoku University
27-1 Kawauchi Aoba Sendai 980-8576 Japan

Abstract: Several mathematical optimization models for agricultural management have been proposed and developed. Crop planning model is one of these models that maximizes the farmers’ incomes by finding an optimum assignment of crops in their farmlands. Although the crop planning model is formulated as a linear programming problem and considers cultivation areas (sizes) for objective crops, it does not indicate the assignment location of crops. In addition, the problem deals with the farmer income in a certain season. In many actual cases, if we continue to cultivate a crop at the same farmland for some seasons, the harvest will change with the season because of replant failures, i.e., the optimum assignment is not applicable for all seasons. Therefore, we have to formulate more useful models that consider multiple seasons. In this paper, we propose a mathematical model that discusses crop assignment in a farmland and a multi-period crop rotation programming problem with replant failure effects to develop an innovative crop planning model. Furthermore, we try to describe the harvest transition using a network and discuss methods that yield an optimum crop rotation sequence using network programming and quality-control techniques.

Key Words: crop planning problem, knapsack problem, branch-and-bound method, fuzzy number, fuzzy order, network programming

1. Introduction

The Japanese food self-sufficiency ratio has remained at a low level for decades. Although we need to discuss what ratio is suitable for Japan, the value undoubtedly does not show an optimistic status. We can identify the various reasons why the ratio is at such a low level, e.g., decrease in farm numbers and cultivatable farmlands and small farmland size, among others. In any case, productivity drive in agriculture is a critical factor, and it requires a policy to improve the food self-sufficiency ratio to raise the quantity of production per unit area by effectively utilizing farmlands. In other words, we can achieve a favorable ratio by considering improvement in management efficiency in each farm.

Crop planning problem exists as a major mathematical optimization model for agricultural management [6]. The model features the same concept as that of the financial portfolio optimization and aims to determine the crop areas that can maximize the farmer profit under some constraints. Because the original problem is formulated as a linear programming model, analyzing its structure and finding an optimal solution are easy. However, such an optimal solution does not show the location but only the areas (size) for the corresponding crops. Indeed, if soil condition is uniform, we can obtain a similar harvest per unit area, irrespective of location, and thus we do not have to consider the cropping locations. However, in actual practice, we should consider the locations because replant failures influence the harvest.

Moreover, we should consider some multi-period models because the influence is sustained for a long time. Although the concept of crop rotation can become an alternative, crop rotation is programmed on the basis of the degree of influence of the pre-planted crops on the harvest of post-planted crops in a farmland [1],[5]. Because such estimation depends on the intuition or experience of experts in most cases, an amateur will have difficulty in estimating the influence and planning crop rotations. Further, the influence of a replant failure may span for several years. Therefore, we have to consider crop rotations with a several-year cycle; thus, the number of combinations accordingly increases.

In this paper, we propose some crop allocation models in which we attempt to determine proper cropping locations for a single-period case and a multi-period crop rotation programming problem to maximize farmer profit. We describe the original crop planning problem in Section 2. In Section 3, we formulate the crop allocation problem by introducing reduction ratios. Although we assume in this section that the reduction ratios are constant, these assumed values are very rare in real life. Therefore, in Section 4, we fuzzify the allocation model by defining the ratios as fuzzy numbers. In Section 5, we show a crop rotation model that is valuable in utilizing the intuition and experience of an expert as one of the network programming problems.

2. Crop Planning Problem

The original crop planning problem is expressed as follows:

Maximize \( \sum_{i=1}^{m} c_i x_i \)
subject to \[ \sum_{j=1}^{m} w_i x_{ij} \leq W \]
\[ \sum_{i=1}^{m} x_{ij} \leq H \]
\[ x_{ij} \geq 0 \quad (i = 1, 2, \ldots, m) \]

where \( x_{ij} \), \( c_j \) and \( w_i \) are the cultivation area (decision variable), profit coefficient, and required agricultural work time per unit area for a crop \( i \), respectively. The number of object crops is \( m \), \( W \) is the total available workforce, and \( H \) is the total available area for cultivation. The first and second constraints are called “labor constraint” and “land constraint”, respectively.

Because the above problem is a linear programming model, we can easily obtain the optimal solutions.

3. Crop Allocation Problem

3.1 Problem Formulation

We assume that an object farmland is rectangular and can be divided into \( n \) square cells, as shown in Figure 3.1.

![Division of a farmland](image)

The size of these cells corresponds to a minimum available area for actual cultivation. Now, we can grow any of the \( m \) crops at each square cell. If crop \( i \) is grown at cell \( j \) then the decision variable \( x_{ij} \) should be set to one; otherwise, it is set to zero. Although in the crop planning problem the profit coefficients are constant irrespective of where we plant the corresponding crops, \( c_j \) is supposed to be reduced by \( a_{ij} \) because of replant failures.

From the above assumptions, we formulate the crop allocation problem as follows:

Maximize \[ \sum_{i=1}^{m} \sum_{j=1}^{n} (c_i a_{ij}) x_{ij} \]
subject to \[ \sum_{j=1}^{n} w_i x_{ij} \leq W \]
\[ \sum_{i=1}^{m} x_{ij} \leq 1 \quad (j = 1, 2, \ldots, n) \]
\[ x_{ij} = 0 \text{ or } 1 \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \]

The second constraint is considered because we can grow only one crop in a cell. Although originally, the following inequality should be included as a land constraint in this problem, it is implied by the second constraint, and we do not have to consider it.

We can recognize the above problem as a special type of the 0-1 knapsack problem [8]. Therefore, mathematically solving this problem is generally difficult. However, we can actually obtain an optimal solution using some integer programming methods, e.g., the branch-and-bound method.

In composing the search tree, we have to consider a subproblem at each node and the corresponding linear relaxation problem where the constraints “\( x_{ij} = 0 \text{ or } 1 \)” in the original problem are replaced with “\( 0 \leq x_{ij} \leq 1 \)” in order to describe whether imposing boundary on latter branches are proper.

Whereas linear programming problems can be easily solved, the larger the number of nodes and branches is, the more the computational efforts increase. Therefore, we must avoid wasteful searching of latter branches using the following conditions:

- If an optimal solution of a relaxation problem is composed of integers, i.e., zero or one, no optimal solution exists for the latter branches.
- The objective functions of the subproblems are non-increasing in terms of the depth direction.
- If \( x_{ij} = 1 \) for some \( i = i_t \), then \( x_{ij} = 0(i = 1, 2, \ldots, m; i \neq i_t) \) from \( \sum_{i=1}^{m} x_{ij} \leq 1 \).

3.2 Numerical Example

We consider a farmland and divide it into four cells, as shown in Figure 3.2, and assume that the parameters are given as follows:

\[ m = 3, n = 4, W = 9 \]
\[ (c_1, c_2, c_3) = (5, 3, 4) \]
\[ (w_1, w_2, w_3) = (6, 4, 2) \]
\[ \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix} = \begin{pmatrix} 0.7 & 0.8 & 0.5 & 0.2 \\ 0.5 & 0.9 & 1 & 0.4 \\ 1 & 0.1 & 0.4 & 0.2 \end{pmatrix} \]

The second constraint is considered because we can grow only one crop in a cell. Although originally, the following inequality should be included as a land constraint in this problem, it is implied by the second constraint, and we do not have to consider it.

Then, our problem is described as follows:

Maximize \[ 3.5x_{11} + 4x_{12} + 2.5x_{13} + x_{14} + 1.5x_{21} + 2.7x_{22} + 3x_{23} + 1.2x_{24} + 4x_{31} + 0.4x_{32} + 1.6x_{33} + 0.8x_{34} \]
subject to  

\[
\begin{align*}
6(x_{11} + x_{12} + x_{13} + x_{14}) \\
+4(x_{21} + x_{22} + x_{23} + x_{24}) \\
+2(x_{31} + x_{32} + x_{33} + x_{34}) & \leq 9 \\
\end{align*}
\]

Now, checking a binary tree based on the depth-first search where the edges connecting the root correspond to \(x_{11} = 0, 1\) and the latter edges are composed on the order of \(x_{31}, x_{31}, x_{12}, x_{22}, \ldots, x_{34}\), for the above problem, we can obtain the optimal solution as follows:

\[
\begin{bmatrix}
x_{11} & x_{12} & x_{13} & x_{14} \\
x_{21} & x_{22} & x_{23} & x_{24} \\
x_{31} & x_{32} & x_{33} & x_{34}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0
\end{bmatrix}
\]

immediately after addressing the sub-problem for \(x_{11} = 1\). As a result, we can determine that cultivating crop 3 at cells 1 and 3, crop 2 at cell 2, and no crop at cell 4 is proper in this situation.

4. Fuzzy Crop Allocation Problem

In this section, we assume that \(a_{ij}\) in the previous section is replaced with an \(L\) fuzzy number \(\tilde{A}_{ij} = (m_{ij}, a_{ij})_{L}\), which is defined as follows [9]:

**Definition 1 (L fuzzy number)**

We consider the following shape function \(L:\)

1. \(L(t) = L(-t) = 0\) for \(t > 0\)
2. \(L(0) = 1\)
3. \(L(t) = 1\) for \(t > 0\)
4. \(L(t)\) is non-increasing for \(t > 0\)
5. There exists \(i\) such that \(i = \inf \{t > 0 | L(t) = 0\}\).

We define \(\tilde{A}\) as an \(L\) fuzzy number whose membership function is \(L(t - m)/\alpha\), where \(m \geq 0\) is called “center” (which is the most possible value) and “width”, respectively. This \(L\) fuzzy number is simply denoted as \(\tilde{A} = (m, \alpha)_{L}\).

Our problem has been formulated as a fuzzy crop allocation problem, i.e.,

\[
\begin{align*}
\text{Maximize} & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\tilde{A}_{ij} \\
\text{subject to} & \quad \sum_{j=1}^{n} w_{ij}x_{ij} \leq W \\
& \quad \sum_{i=1}^{m} x_{ij} \leq 1 \quad (j = 1, 2, \ldots, n) \\
& \quad x_{ij} = 0 \text{ or } 1 \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)
\end{align*}
\]

where \(\tilde{A}_{ij} = (m_{ij}, a_{ij})\) for all \(i, j\).

From the extension principle, the objective function is equal to the following:

\[
\left( \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}m_{ij}x_{ij} \right)_{L} + \left( \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \right)_{L}
\]

Because this is an \(L\) fuzzy number and simply maximizing it is impossible, we discuss its maximization by introducing the fuzzy order [2].

**Definition 2 (fuzzy order)**

For two \(L\) fuzzy numbers \(\tilde{A} = (m_{1}, a_{1})_{L}, \tilde{B} = (m_{2}, a_{2})_{L}\), \(\tilde{A} \preceq \tilde{B}\) if and only if there exists \(c\) such that

\[
\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x) \quad (x < c) \\
\mu_{\tilde{A}}(x) \geq \mu_{\tilde{B}}(x) \quad (x > c)
\]

As shown in [2], this definition is equivalent to the following:

\[
\tilde{A} \preceq \tilde{B} \iff \tilde{A} \preceq \tilde{B} \iff a_{1} - a_{2} \preceq m_{2} - m_{1}
\]

This order is a partial order; therefore, we seek the maximal ones.

**Definition 3**

\(\tilde{A}^{*}\) is maximal for fuzzy numbers \(\tilde{A}_{1}, \tilde{A}_{2}, \ldots, \tilde{A}_{n}\) if there exists no other \(\tilde{A}\), satisfying \(\tilde{A}^{*} \preceq \tilde{A}\).

For the order relationship, we can easily imply the following theorem from the above equivalence relationship.

**Theorem 1**

The following fuzzy numbers are maximal for \(\tilde{A}_{i} = (m_{i}, a_{i})_{L}\) (\(i = 1, 2, \ldots, n\));

\[
\tilde{A}_{POS} = (m_{i}, a_{i})_{L} \text{ for a maximizer } i \text{ of } m_{i} \\
\tilde{A}_{OPT} = (m_{i}, a_{i})_{L} \text{ for a maximizer } i \text{ of } (m_{i} + \tilde{a}_{i}) \\
\tilde{A}_{PES} = (m_{i}, a_{i})_{L} \text{ for a maximizer } i \text{ of } (m_{i} - \tilde{a}_{i})
\]

where we can regard \(\tilde{A}_{POS}, \tilde{A}_{OPT}\) and \(\tilde{A}_{PES}\) as the most possible, optimistic, and pessimistic cases, respectively.

From Theorem 1, we can reformulate the problem as follows:

\[
\text{Maximize} \quad Z_{POS} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}m_{ij}x_{ij}
\]

subject to

\[
\sum_{j=1}^{n} w_{ij}x_{ij} \leq W \\
\sum_{i=1}^{m} x_{ij} \leq 1 \quad (j = 1, 2, \ldots, n) \\
x_{ij} = 0 \text{ or } 1 \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)
\]

where

\[
Z_{POS} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}m_{ij}x_{ij}
\]

Of course, whereas an optimal solution of the above problem yields a maximal allocation in the most possible case, we should replace \(Z_{POS}\) with

\[
Z_{OPT} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}m_{ij}x_{ij} + \tilde{a}_{ij}
\]

or

\[
Z_{PES} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}m_{ij}x_{ij} - \tilde{a}_{ij}
\]

if we assume the most optimistic or pessimistic case.

The above problems have the same structure as the crop allocation problem (non-fuzzy version), i.e., a special type of the 0-1 knapsack problem. Therefore, we can solve the problems using the branch-and-bound method and find the maximal allocations for the fuzzy crop allocation problem.
5. Multi-Period Crop Rotation Programming Problem with Replant Failure

5.1 Problem Formulation

We consider the problem as follows:

\[ \text{P} \text{ := "A farmer wants to maximize his total profit during } p \text{ years or seasons, taking into account replant failures. This is a type of multi-period management. What crop rotation should be adopted?"} \]

where \( p \) is a positive integer. We also assume the following items for the problem discussion:

(A1) The cultivation area that the farmer can use is finite and constant, and its soil condition is uniform.

(A2) The farmer simultaneously grows uniformly a single-species crop in the objective cultivation area.

(A3) The number of crop alternatives that the farmer can grow is finite, i.e., 1, and the alternatives do not change for \( p \) years (seasons).

(A4) The planting and harvesting seasons of the objective crops are known.

(A5) For each crop, the period from planting to harvesting is within one year.

(A6) Each crop is uniquely classified into one of the \( n \) plant classes.

(A7) Only a repeated cultivation of crops belonging to a class causes a replant failure.

(A8) The period \( \Delta C(i) \) for a whole year starts in January and increases from one to \( n \) every month.

(A9) The farming resources for cultivation, i.e., water, agricultural, labor power, and so on, are adequately supplied.

\[ C_i(i = 1, 2, \ldots, l) \text{ and } f_i(i = 1, 2, \ldots, n) \text{ denote the profit in a season for crop } i \text{ without any replant failure and the integer parameter that indicates the level of replant failure for each plant class, respectively. Because the effect of a replant failure disappears from the soil sometime according to (A8), we assume that the remaining period (months) for the effect is known and define } k_i \text{ by adding one to this period. When the cultivation of a crop in class } i \text{ is completed, the corresponding } f_i \text{ is set to one and increases from one to } k_i \text{ every month. } f_i \text{ is reset to one at the next cultivation of another crop in the class } i \text{ and does not increase over } k_i. \text{ The soil condition } F_j \text{ comprising } f_s \text{ at the beginning of the } j \text{th year is as follows:} \]

\[ F_j = (f_1, f_2, \ldots, f_p) (j = 1, 2, \ldots, p) \]

While discussing multi-period income, we count the feasible crop rotation sequences \( R_i \) for a whole year starting from January. In this procedure, we have to check and memorize the first and the last months of each \( R_i \) and whether \( R_i \) is completed in December. If \( R_i \) is not completed by the end of December, we call it an extended crop rotation. Although the number of permutations for the crops is \( l! \), the branches of the enumeration tree are bounded to some degrees because of positional constraints for the planting and harvest seasons of the crops.

If \( R_i \) includes crop \( j \), the \( j \text{th element of } l \text{ dimensional vector } r_j(i = 1, 2, \ldots, m) \text{ is set to } C_j; \text{ otherwise, it is set to zero.} \]

Moreover, each element of \( n \text{ dimensional vector } \Delta C(i) \) denotes the cultivation period (months) of the corresponding crop in crop rotation \( R_i \), and it is zero in case \( R_i \) does not include the crop. Similarly, each element of \( n \text{ dimensional vector } \Delta Q(i) \) denotes the value obtained by adding one to the postharvest months until the year-end of the corresponding crop in crop rotation \( R_i \), and it is zero in case \( R_i \) does not include the crop. If \( R_i \) is an extended crop rotation, the element of \( \Delta Q(i) \) that corresponds to the last crop in \( R_i \) must be one.

Now, we define \( \Delta U(i) \) as follows: If some elements of \( \Delta C(i) + \Delta Q(i) \) are zero, then the corresponding elements of \( \Delta U(i) \) are 12, and the other elements of \( \Delta U(i) \) are equal to the corresponding elements of \( \Delta Q(i) \).

For assumptions (A8) and (A9), replant failures influence the harvests, and the profit for crop \( j \) in class \( i \) is supposed to be

\[ C_j \times \frac{f_i}{k_i} (i = 1, 2, \ldots, n; j = 1, 2, \ldots, l), \]

where \( f_i \) is the current parameter for class \( i \) including crop \( j \). In other words, when we execute crop rotation \( R_i \) after \( R_s \), the profit for \( R_i \) is computed as the product of \( r_j \) and \( a_s \), which is an \( l \text{ dimensional vector that denotes the effects of replant failures and whose } j \text{th element is equal to the current } f_j/k_i \) based on class \( i \) that includes crop \( j \).

5.2 Solution Procedure

We introduce the concept of network programming to obtain the solutions of problem \( P \). We consider a \( p \)-layered system network where alternative crop rotation sequences for each year are assigned to nodes in the corresponding layer. Therefore, for a layer, in case the crop rotation of the preceding layer in question is an extended crop rotation, we must note the relative position between the end of the extended crop rotation in the preceding layer and the beginning of the current rotation in the layer. Further, we introduce super source node \( S \) just before the first layer and super sink node \( T \) just after the \( p \)th layer (see Figure 5.2) [4].

We assign profits to the corresponding arcs as their distances according to the start and end points of the arcs after computing \( a_i \) in the following. We note that all arcs that connect to the super sink node have the same distance, which we can treat as zero for convenience.

**Step 1** For all \( i, j \), if \( R_i \) includes a crop in class \( i \), then set \( F_j = \Delta U(s) \) for the \( i \text{th element and } F_j = F_j - 1 + \Delta U(s) \) for the other elements.

**Step 2** Compute \( F_j' = F_j + F(t) \).

**Step 3** Derive \( a_i \) according to elements \( f_s \) of current \( F_j' \).

By finding the longest path from the super source node to the super sink node for the objective network, we can obtain
an optimum solution for problem P, i.e., an optimum crop rotation sequence. The path is the very critical path on the arrow diagram in the quality control [3],[7].

Fig. 3 Example of Network

6. Discussion and Conclusion

We have proposed the crop allocation problem as a type of integer programming problem and its fuzzified version to support agricultural management. We have also considered recommendable approaches based on the branch-and-bound method to find optimal solutions for both problems.

However, we admit enclave cropping, which means that a crop is grown at discontinuous areas, as a result of our models. Because the enclave cropping in our models has no constraint, the structures of our models are very simple. At the same time, they cannot exclude unrealistic situations from the perspective of workability. We will attempt to revise our models to reflect more practical situations.

Because we define the optimality based on a partial order in our fuzzy model, we cannot help but accept non-dominated solutions. Therefore, we think of a new model that uses a total order and defines the very optimality as further research. In our models, we deal with profit coefficients as constant values by applying algorithms in the network programming or some techniques for the finding the path will not be essential. We can use various methods in the network programming or some techniques for the critical path in the quality control.

Whereas we proposed the single-period and the multi-period models, these models were independently discussed. Preferably, they should be combined if we desire a more efficient management. We will consider the complex model in our further research.

Acknowledgments

This work was supported by JSPS KAKENHI [Grant-in-Aid for Challenging Exploratory Research] Grant Number 24651174.

References

[1] Bohrerova, Z., Stralkova, R., Podesova, J., Bohrer, G. and Pokorny, E., The Relationship between Redox Potential and Nitritification under Different Sequences of Crop Rotations, Soil & Tillage Research, 77 (1), 2004, pp. 25-33.

[2] Furukawa, N., A Parametric Total Order on Fuzzy Numbers and Fuzzy Shortest Route Problem. Optimization, 30, 1994, pp. 367-377.

[3] Hu, T. C., Combinatorial Algorithms, Addison-Wesley Publishing Company, 1982.

[4] Itoh, T. and Ishii, I., Fuzzy Sharing Problem by Possibility Measure, European Journal of Operational Research, 102 (3), 1997, pp. 648-656.

[5] Krupinsky, J.M., Tanaka, D.L., Merrill, S.D., Liebig, M.A. and Hanson, J.D., Crop Sequence Effects of 10 Crops in the Northern Great Plains, Agricultural Systems, 88 (2-3), 2006, pp. 227-254.

[6] Nanseki, T., Stochastic Programming, Gendai-Sugakusha, Kyoto, 1995 (in Japanese).

[7] Nayatani, Y., New Seven Tools in QC, Japan Standards Association, 1987 (in Japanese).

[8] Winston, W. L., Operations Research: Applications and Algorithms, Wadsworth Publishing Company, California, 1994.

[9] Zimmermann, H. L., Fuzzy Set Theory: and Its Applications, Kluwer Academic Publishers, Massachusetts, 1991.

Takeshi Itoh

is Professor at Graduate School of Management and Economics, Tohoku University. He holds B. A. and M. E., and received D. E. degree in Graduate School of Engineering, Osaka University. His main areas of interests are fuzzy mathematical programming, stochastic programming, and scheduling.