The shortest time and/or the shortest path strategies in a CA FF pedestrian dynamics model

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Abstract—This paper deals with a mathematical model of a pedestrian movement. A stochastic cellular automata (CA) approach is used here. The Floor Field (FF) model is a basis model. FF models imply that virtual people follow the shortest path strategy. But people are followed by a strategy of the shortest time as well. This paper is focused on how to mathematically formalize and implement to a model these features of the pedestrian movement. Some results of a simulation are presented.

Keywords—Cellular automata; pedestrian dynamics; transition probabilities; artificial intelligence

I. INTRODUCTION

A stochastic cellular automata (CA) model of pedestrian flow is considered here. Our model takes inspiration from stochastic floor field (FF) CA model [2] that provides pedestrians with a map which “shows” the shortest distance from current position to a target. In this paper we focus on mathematical formalizing and implementation to the model such behavioral aspects of decision making process as: while moving people follow at least two strategies — the shortest path and the shortest time. Strategies may vary, cooperate and compete depending on current position.

This is a next attempt to extend basis FF model towards a behavioral aspect making more flexible/realistic decision making process and improve simulation of individual and collective dynamics of people flow.

II. STATEMENT OF THE PROBLEM

The space (plane) is known and sampled into cells 40cm × 40cm which can either be empty or occupied by one pedestrian (particle) only [2]. Cells may be occupied by walls and other nonmovable obstacles. So space is presented by 2 matrices:

\[ f_{ij} = \begin{cases} 1, & \text{cell } (i, j) \text{ is occupied by a pedestrian;} \\ 0, & \text{cell } (i, j) \text{ is empty,} \end{cases} \]

\[ w_{ij} = \begin{cases} 1, & \text{cell } (i, j) \text{ is occupied by an obstacle;} \\ 0, & \text{cell } (i, j) \text{ is empty.} \end{cases} \]

A Static Floor Field (SFF) \( S \) is used in the model. Field \( S \) coincides with the sampled space. A value of each \( S_{i,j} \) saves the shortest distance from cell \((i, j)\) to a nearest exit; i.e., \( S \) increases radially from exit cells where \( S_{i,j} \) are zero. It doesn’t evolve with time and isn’t changed by the presence of the particles. One can consider \( S \) as a map that pedestrians use to move to the nearest exit.

Starting people positions are known. A target point for each pedestrian is the nearest exit. Each particle can move to one of four its next-neighbor cells or to stay in present cell (the von Neumann neighborhood) at each discrete time step \( t \rightarrow t + 1 \). Generally speaking, a direction for each particle at each time step is random and determined in accordance with a transition probabilities distribution (and transition rules).

So a main problem is to determine “right” transition probabilities (and transition rules).

III. SOLUTION

A. Update rules

A typical scheme for stochastic CA models is used here. There is step of some preliminary calculations. Then at each time step transition probabilities are calculated, and direction is chosen. If there are more then one candidates to one cell a conflict resolution procedure is applied, and then a simultaneous transition of all particles is made.

Figure 1. Target cells for a pedestrian in the next time step [2].
In our case the preliminary step includes calculations of SFF $S$. Each cell $S_{i,j}$ saves shortest discreet distance to the nearest exit. The unit of such distance is a number of steps. To calculate the field $S$ (and only here) we admit diagonal transitions and consider that a vertical and horizontal movement to the nearest cell has a length of 1; a length of a diagonal movement to the nearest cell is $\sqrt{2}$. (It’s clear that movement through a corner of walls or collums is forbidden and around movement is admitted in such cases only.) It is made a discreet distance more close to continuous one.

Probabilities to move from cell $(i, j)$ to each of four the nearest cells are calculated in the following way:

$$
\begin{align*}
    p_{i-1,j} &= \frac{\hat{p}_{i-1,j}}{\text{Norm}_{i,j}},
p_{i,j+1} &= \frac{\hat{p}_{i,j+1}}{\text{Norm}_{i,j}},
p_{i+1,j} &= \frac{\hat{p}_{i+1,j}}{\text{Norm}_{i,j}},
p_{i,j-1} &= \frac{\hat{p}_{i,j-1}}{\text{Norm}_{i,j}},
\end{align*}
$$

where $\text{Norm}_{i,j} = \hat{p}_{i-1,j} + \hat{p}_{i,j+1} + \hat{p}_{i+1,j} + \hat{p}_{i,j-1}$.

Moreover

$$
\begin{align*}
p_{i-1,j} &= 0, & p_{i,j+1} &= 0, & p_{i+1,j} &= 0, & p_{i,j-1} &= 0 \quad (2)
\end{align*}
$$

only if

$$
\begin{align*}
w_{i-1,j} &= 1, & w_{i,j+1} &= 1, & w_{i+1,j} &= 1, & w_{i,j-1} &= 1
\end{align*}
$$

(3) correspondingly.

A probability to stay at present cell isn’t calculated directly. But decision rules are organized in a way that such opportunity may be realized, and a people patience is reproduced by this means.

Decisions rules are the following [3]:

1) If $\text{Norm}_{i,j} = 0$ then motion is forbidden, otherwise a target cell $(l, m)^*$ is chosen randomly using the transition probabilities.

2) a) If $\text{Norm}_{i,j} \neq 0$ and $(1 - f_{l,m}) = 1$ then a target cell $(l, m)^*$ is fixed.

b) If $\text{Norm}_{i,j} \neq 0$ and $(1 - f_{l,m}) = 0$ then the cell $(l, m)^*$ is not available for moving and a “people patience” can be realized. To do it probabilities of the cell $(l, m)^*$ and all other occupied the nearest neighbors are given to an opportunity not to leave the present position. A target cell is randomly chosen again among empty neighbors and the present position.

3) Whenever two or more pedestrians have the same target cell, the movement of all involved pedestrians is denied with the probability $\mu$, i.e. all pedestrians remain at their places [2]. One of the candidates moves to the desired cell with the probability $1 - \mu$. A pedestrian that is allowed to move is chosen randomly.

4) Pedestrians that are allowed to move perform their motion to the target cell.

5) Pedestrians that stand in exit cells are removed from the room.

These rules are applied to all particles at the same time; i.e., parallel update is used.

B. How to calculate probability?

Mostly in this paper we focus on transition probabilities. In normal situations people choose their route carefully (see [1] and reference therein). Pedestrians keep a certain distance from other people and obstacles. The more hurried a pedestrian is and the more tight crowd is the more smaller this distance is. While moving people follow at least two strategies — the shortest path and the shortest time.

In FF models people move to the nearest exit, and their wish to move there doesn’t depend on a current distance to the exit. From the probability view point this means that for each particle among all the nearest neighbor cells a neighbor with the smallest $S$ should have the largest probability. So a main driving force for each pedestrian is to minimize SFF $S$ at each time step. But in this case only a strategy of the shortest path is mainly realized, and a slight regard to an avoidance of congestions is supposed. This is not realistic for people movement.

An idea to improve a dynamics in a FF model is to introduce an environment analyzer in a probability formula. It should decrease an influence of a short path strategy and increase the possibility to move to a direction with favorable conditions for a moving. This will provide some kind of “trade off” between two main strategies.

In this paper we introduce a revised idea of the environment analyzer [3] and make an attempt to mathematically formalize a complex decision making process that people do choosing their path — while moving their strategies may vary: cooperate, coincide and compete depending on a current position and an environment; i.e., depending on a place and time.

At first let us present a probability formula and later we are discussing it in details. For example, the transition probability to move from a cell $(i, j)$ to the up neighbor is:

$$
\hat{p}_{i-1,j} = A^{SFF}_{i-1,j} A^{people}_{i-1,j} A^{wall}_{i-1,j} (1 - w_{i-1,j}).
$$

Here

- $A^{SFF}_{i-1,j} = \exp (k_S \triangle S_{i-1,j})$ — the main driven force:
  - 1) $\triangle S_{i-1,j} = S_{i,j} - S_{i-1,j}$;
  - 2) $k_S \geq 0$ — a sensitivity parameter (model parameter) that can be interpreted as the knowledge of the shortest way to the destination point, or as a wish to move to the destination point. $k_S = 0$ means that pedestrians don’t use information from the SFF $S$ and move randomly. The higher $k_S$ is the more directed is movement of pedestrians.

As far as, SFF depict direct distance from each cell to the nearest exit then $\triangle S_{i-1,j} > 0$ if cell $(i - 1, j)$ is
closer to exit than current the cell \((i, j)\), \(\triangle S_{i-1,j} < 0\) if the current cell is closer. And \(\triangle S_{i-1,j} = 0\) if cells 
\((i, j)\) and \((i - 1, j)\) are equidistant to the exit.

In contrast to other authors that deal with the FF model 
\(\text{(e.g.,} [2], [4], [5], [7])\) and use pure values of the field 
\(S\) in the probability formula we propose to use only 
\(\triangle S_{i-1,j}\). From a mathematical view it is the same 
but computationally this trick has a great advantage.

Values of SFF may be too high (it depends on a size of 
the space), and \(\exp(k_S S_{i-1,j})\) is uncomputable. This 
is a significant restriction of that models. At the same 
time \(0 \leq \triangle S_{i-1,j} \leq 1\), and problem of computing 
\(A_{SFF}^{i-1,j}\) is absent;

- \(A_{i-1,j}^{people} = \exp(-k_p D_{i-1,j}(r_{i-1,j}^*))\) — a factor that 
takes into account a people density in the direction:

1) \(r_{i-1,j}^*\) — a distance to the nearest obstacle in 
this direction \((r_{i-1,j}^* \leq r)\);
2) \(r > 0\) — a “visibility” radius (a model parameter) 
is a maximal distance (number of cells) at which 
the pedestrian can look through to collect information 
about the density and possible obstacles 
(but not pedestrians);
3) density \(0 \leq D_{i-1,j}(r_{i-1,j}^*) \leq 1\), if all \(r_{i-1,j}^*\) cells 
are empty in this direction then \(D_{i-1,j}(r_{i-1,j}^*) = 0\), if all \(r_{i-1,j}^*\) cells are occupied by people in this 
direction then \(D_{i-1,j}(r_{i-1,j}^*) = 1\). We estimate 
density by using idea of the kernel Rosenblat-
Parzen’s density estimate, and

\[
D_{i-1,j}(r_{i-1,j}^*) = \frac{\sum_{m=1}^{r_{i-1,j}^*} \Phi\left(\frac{r_{i-1,j}^*}{r_{i-1,j}}\right) f_i-m,j}{r_{i-1,j}^*},
\]

were

\[
\Phi(z) = \begin{cases} 
0.335 - 0.067(z^2) & \text{if } |z| \leq \sqrt{5}; \\
0 & \text{if } |z| > \sqrt{5}; 
\end{cases}
\]

\[
C(r_{i-1,j}^*) = \frac{r_{i-1,j}^*+1}{\sqrt{5}},
\]

4) \(k_p \geq k_S\) — a people sensitivity parameter (a 
model parameter) determines an influence of the 
people density. The higher \(k_p\) is the more 
pronounced is the strategy of the shortest path.

- \(A_{i-1,j}^{wall} = \exp(-k_W (1 - \frac{r_{i-1,j}^*}{r})) \tilde{1}(\triangle S_{i-1,j} - \max \triangle S_{i,j})\) — a factor that takes into account walls and obstacles:

1) \(k_W \geq k_S\) — a wall sensitivity parameter (a model parameter) determines an influence of walls and obstacles;
2) \(\max \triangle S_{i,j} = \max\{\triangle S_{i-1,j}, \triangle S_{i,j-1}, \triangle S_{i,j+1}, \triangle S_{i,j-1}\}\),

\[
\tilde{1}(\phi) = \begin{cases} 
0, & \phi < 0; \\
1 & \text{otherwise.}
\end{cases}
\]

An idea of the function \(\tilde{1}(\triangle S_{i-1,j} - \max \triangle S_{i,j})\) 
goes from a fact that people avoid obstacles only 
moving towards a destination point. But if people 
take detours (that means not minimizing the SFF) 
approaching to obstacles is not avoiding.

- NOTE that only walls and obstacles turn the probability to “zero”.

Probabilities to move from cell \((i, j)\) to each of four 
neighbors are:

\[
\tilde{p}_{i-1,j} = \exp[k_S \triangle S_{i-1,j} - k_p D_{i-1,j}(r_{i-1,j}^*) - k_W (1 - \frac{r_{i-1,j}^*}{r}) \tilde{1}(\triangle S_{i-1,j} - \max \triangle S_{i,j})] (1 - w_{i-1,j});
\]

\[
\tilde{p}_{i,j+1} = \exp[k_S \triangle S_{i,j+1} - k_p D_{i,j+1}(r_{i,j+1}^*) - k_W (1 - \frac{r_{i,j+1}^*}{r}) \tilde{1}(\triangle S_{i,j+1} - \max \triangle S_{i,j})] (1 - w_{i,j+1});
\]

\[
\tilde{p}_{i,j-1} = \exp[k_S \triangle S_{i,j-1} - k_p D_{i,j-1}(r_{i,j-1}^*) - k_W (1 - \frac{r_{i,j-1}^*}{r}) \tilde{1}(\triangle S_{i,j-1} - \max \triangle S_{i,j})] (1 - w_{i,j-1});
\]

In \((6)-(9)\) a product \(A^{people} A^{wall}\) is the environment 
analyzer that deals with people and walls. Parameters \(k_p\) and 
\(k_W\) allow to tune sensitivity of the model to the people 
density and the approaching to obstacles correspondingly. 
And as far as \(0 \leq \triangle S \leq 1\), \(0 \leq D(r^*) \leq 1\) and 
\(0 \leq 1 - \frac{r}{r^*} \leq 1\) both parameters shouldn’t be less then \(k_S\). The term \(A^{wall}\) is only to avoid obstacles ahead, we 
will not discuss it here and let \(k_W = k_S\).

The following the shortest path strategy means to take 
detours around high density regions if it is possible. The 
term \(A^{people}\) works as a reduction of the main driving force 
(that provides the shortest path strategy), and probability of 
detours becomes higher. The higher \(k_p \geq k_S\) is the more 
pronounced the shortest time strategy is. Note that the low 
person density makes influence of \(A^{people}\) small, and the probability of the shortest path strategy increases for the 
particle.

### IV. Simulations

Here we present some simulation results to demonstrate 
that our idea works. We use one space and compare 2 sets 
of parameters. Size of space is 14.8m × 13.2m (37 cells 
× 33 cells) with one exit (2.0m). Recall that the space is
a) Field $S$. b) Initial positions.

Figure 2.

$t = 25$  
$t = 65$  
$t = 135$  
$t = 165$  
$t = 180$  
$t = 225$

Figure 3. Evacuation for 300 people, $k_S = k_W = 4$, $r = 10$, $k_P = 6$.

the probability of the shortest path strategy depending on density's value.

V. Conclusion

Figures 3-4 show a great difference in the flow dynamics that obtained by following only one movement strategy and by “keeping in mind” both strategies at a time. The case of $k_P = 18$, i.e., when both strategies of the shortest path and the shortest time are well pronounced, gives a more realistic shape of flow. A model dynamics proper needs a careful investigation and it is go on. A necessity of the $k_P$ spatial adaptation is already clear.

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