Research and Application of Multiobjective Robust $H_2 / H_\infty$ Fuzzy Control Algorithms for Double Reheat Steam System

Miao Liu, Gengda Li* and Baowei Chen

Smart Power Generation Technology Research Center, Guodian New Energy Technology Research Institute Co., Ltd, Beijing, China

*Corresponding author e-mail: gengda.li@chnenergy.com.cn

Abstract. In order to ensure the tracking and anti-interference performance of the second reheat steam system of the ultra supercritical unit, a multi-objective robust $H_2 / H_\infty$ fuzzy tracking control scheme is proposed. T-S model is used to describe the reheat steam system, and the problems of state observation and control error are considered in the design of control strategy. The control design problem is transformed into a multi-objective constraint problem. The design process and robustness of the method are illustrated by a simulation example.

1. Introduction

Thermal power generation is still the main power source in China in the medium and long term, so it is of great significance to carry out the construction of economic, clean, safe and efficient intelligent thermal power generation.[1] At the same time of improving efficiency, the operation of ultra supercritical secondary unit also brings many challenging technical problems [2].

Compared with the primary reheating, the thermal efficiency of the secondary reheating unit is 2% ~ 3% higher, which can reduce the coal consumption of power supply by 8 ~ 10g / kW · H. Compared with the conventional unit, the operation parameters of the ultra supercritical reheate unit are higher, the materials of components are different greatly, and it has a more complex auxiliary system, and its operation mode is also different from the conventional unit. The second reheat unit realizes the second reheat process of working medium in the thermal cycle. Figure 1 shows the Rankine cycle of secondary reheat unit.

![Figure 1. T–S diagram of Rankine cycle.](image-url)
Generally speaking, steam temperature is one of the important monitoring parameters of large thermal power units. The quality of steam temperature control affects the efficient operation of the unit[3]. In recent years, many control technologies have been developed for nonlinear systems, such as fuzzy model predictive control, adaptive control, state observation control and so on [4, 5, 6]. In the past two decades, people have become more and more interested in fuzzy control for nonlinear systems[7], especially the control technology based on T-S fuzzy model[8]. This fuzzy control technology has been applied to the control design of reheat steam system. In addition, the secondary reheat steam temperature is easily affected by disturbances, such as unit load changes. $H_\infty$ control is one of the most effective techniques to reduce the influence of external interference[9]. When the augmentation system is undisturbed, mixed $H_2/H_\infty$ tracking can reduce control performance index[10]. However, in real time, some objects have model uncertainty, and many state variables are unmeasurable [11, 12].

In this paper, the T-S fuzzy model is used to model the non-linear secondary reheat steam temperature system. A state observer is added to the system, and the tracking error is taken as the input state matrix of the state variable. A fuzzy robust tracking control based on the full state observer is proposed. The method of combining linear matrix inequality (LMI) technology with MOEA[13] is used to solve the simultaneous optimization of MOP in multi-objective tracking control of fuzzy systems. Finally, the design process and simulation results of the non-linear superheated steam temperature system under external disturbances are illustrated by simulation examples.

2. The expression of fuzzy model and fuzzy control

The nonlinear fuzzy model of the reheat temperature system of a secondary reheating unit can be described as:

If $z_i(t)$ is $F_{i_1}, \cdots, z_i(t)$ is $F_{i_2}$, Then

$$
\dot{x}(t) = A_i x(t) + B_i u(t) + E_\omega(t) \\
y(t) = C_i x(t)
$$

(1)

where, $F_i$ are fuzzy set, $z(t)$ are premise variable. $A \in R^{n \times n}$, $B \in R^{n \times l}$ and $C \in R^{l \times n}$ are known real matrix, $i \in Z = \{1, 2, \cdots, N\}$, $N$ is the if-then rule number. Therefore, a non-linear system can be written as an equivalent system as follows:

$$
\dot{x}(t) = A(\mu(t)) x(t) + B(\mu(t)) u(t) + E_\omega(t) \\
y(t) = C(\mu(t)) x(t)
$$

(2)

where, $\mu((t))$ is a membership function. The reference model is:

$$
\dot{x}_r(t) = A_r(\mu(t)) x_r(t) + r(t)
$$

(3)

System state observer $\hat{x}(t)$ and $\hat{y}(t)$. The error:

$$
\dot{e}_x(t) = \hat{x}(t) - \bar{x}(t)
$$

(4)

Tracking error:

$$
e_x(t) = \hat{x}(t) - x_r(t)
$$

(5)

$$
E_y = \int_0^t e_y(t) dt
$$

(6)

Derivatives of formulas (5) and (6) are obtained respectively $\dot{e}_x(t)$ and $E_y$. Then the augmented system:

$$
\dot{\bar{x}}(t) = \bar{A}_b(\mu(t)) \bar{x}(t) + \bar{B}(\mu(t)) u(t) + \bar{E}(\mu) \bar{\omega}(t)
$$

(7)
where,\[
\bar{A}_y(\mu) = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
L(\mu)C(\mu) & A(\mu) & 0 & A(\mu) + A_i(\mu) & 0 \\
0 & 0 & A(\mu) & 0 & 0 \\
0 & 0 & 0 & A_i(\mu) & 0 \\
0 & 0 & 0 & 0 & C(\mu) \\
0 & 0 & 0 & 0 & C_i(\mu)
\end{bmatrix}, \Phi(\mu) = A(\mu) - L(\mu)C(\mu).
\]

For the above augmented system, the control law is:
\[
u(t) = K(\mu)\bar{x}(t)
\]
(8)
where, \[K(\mu) = \begin{bmatrix} K(\mu) & 0 & 0 & \sum_{j=1}^{N} \mu_j(z(t))K_j \end{bmatrix}.\]

A closed-loop augmented system is obtained by introducing control law (7) into system (8).
\[
\ddot{x}(t) = \bar{A}(\mu)x(t) + \bar{E}\ddot{\omega}(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \mu_i\mu_j [\bar{A}_i x(t) + \bar{E}\ddot{\omega}(t)]
\]
(9)
where, \[\bar{A}_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\
L_i C_i & A_i + B_i K_j & 0 & A_i + A_i i & B_i K_j \\
0 & B_i K_j & A_i & 0 & B_i K_j \\
0 & 0 & 0 & A_i i & 0 \\
0 & 0 & 0 & 0 & C_i \\
0 & 0 & 0 & 0 & C_i i
\end{bmatrix}.
\]

Next, for the augmented system (10), considering the control performance of \(H_\infty\):
\[
J_\infty(u) = \int_0^\infty e_i^T(t)Q_e(t)dt / \int_0^\infty \ddot{\omega}_i^T(t)\ddot{\omega}(t)dt = \int_0^\infty \ddot{x}_i^T(t)\bar{Q}\ddot{x}(t)dt / \int_0^\infty \ddot{\omega}_i^T(t)\ddot{\omega}(t)dt
\]
(10)
where, known weight matrix \(Q \in R^{n_\infty} \) of \(e_i(t)\). Equation (10) shows the influence of external interference on tracking error.
When disturbance \(\ddot{\omega}(t)\) does not exist:
\[
J_2(u) = \int_0^\infty [e_i^T(t)Qe_i(t) + u_i^T(t)Ru(t)]dt = \int_0^\infty \ddot{x}_i^T(t)\bar{Q}\ddot{x}(t) + u_i^T(t)Ru(t)]dt
\]
(11)
where, \(Q \in R^{n_\infty} \) and \(R \in R^{m_\infty}\) are weight matrices of \(e_i(t)\) and \(u(t)\), respectively. In formula (10) and formula (11), \(\bar{Q}\) is expressed as
\[
\bar{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\
0 & Q & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \in R^{4n_\infty x 4n_\infty}
\]

The controller design is to minimizes the \(H_\infty\) tracking performance (10) and \(H_2\) tracking performance (11).
\[
\min_{u(t)}(\alpha \quad \beta)
\]
(12)
\[\text{s.t.} \quad J_2(u) \leq \alpha \quad J_\infty(u) \leq \beta\]

3. State observation \(H_2 / H_\infty\) fuzzy tracking controller
Considering \(\ddot{x}(0) \neq 0\), the \(H_\infty\) performance is as follows:
\[
\int_0^\infty \ddot{x}_i^T(t)\bar{Q}\ddot{x}(t)dt \leq \ddot{x}(0)P\ddot{x}(0) + \beta \int_0^\infty \ddot{\omega}_i^T(t)\ddot{\omega}(t)dt
\]
(13)
Next we give theorem 1.

Theorem 1 Considering a fuzzy system (9), if there exists a constant matrix \( P = \text{diag}\{P_1, P_2, P_3\} > 0 \), parameter \( \alpha > 0 \) and parameter \( \beta > 0 \), so that the following multi-objective optimization problems can be solved, then the \( H_\infty \) performance index satisfies, and the \( H_2 \) performance index satisfies.

\[
\begin{align*}
(\alpha_\text{a}, \beta_\text{b}) &= \min_{w_{11}, w_{22}, \gamma_j} (\alpha, \beta) \\
\text{s.t.} & \\
\left[
\begin{array}{cccc}
\Phi_j & \Delta_j & \Gamma_j & \mathbf{B}_j \mathbf{Y}_j \\
0 & \Sigma_j & \mathbf{B}_j \mathbf{Y}_j^T & 0 \\
* & * & T_j & \mathbf{W}_{33} \mathbf{C}_{11}^T \\
* & * & * & 0 \\
* & * & * & -R_j^{-1} \\
* & * & * & -Q_j^{-1}
\end{array}
\right] & \leq 0
\end{align*}
\]

\[
\begin{align*}
-\alpha I & \quad \mathbf{x}_j^T(0) \\
-\mathbf{W}_{11} & \quad 0 \\
* & -\mathbf{W}_{22} \\
* & * & -\mathbf{W}_{33} \\
* & * & * & -\mathbf{W}_{44}
\end{align*}
\]

\leq 0

where, \( \Delta_j = \mathbf{Y}_j^T \mathbf{B}_j^T - \mathbf{Y}_j, \Gamma_j = \hat{\mathbf{A}} \mathbf{W}_{33} - \mathbf{A}_j \mathbf{W}_{33} + \mathbf{Y}_j^T, \Sigma_j = \mathbf{W}_{22} \mathbf{A}_j^T + \mathbf{A}_j \mathbf{W}_{22}, T_j = \mathbf{A}_j \mathbf{W}_{33} + \mathbf{W}_{33} \mathbf{A}_j^T, \Phi_j = \mathbf{W}_1 \hat{\mathbf{A}}^T + \hat{\mathbf{A}} \mathbf{W}_1 + \mathbf{B}_j \mathbf{Y}_j + \mathbf{Y}_j^T \mathbf{B}_j \mathbf{Y}_j^T, \mathbf{W}_1 = P_{11}^{-1}, \mathbf{W}_2 = P_{22}^{-1}, \mathbf{Y}_j = \mathbf{K}_j \mathbf{W}_1, j \in \mathbf{Z} = \{1, 2, \ldots, N\}, \mathbf{K}_j = \mathbf{Y}_j \mathbf{W}_1^{-1}, \mathbf{Y}_j = \mathbf{K}_j \mathbf{W}_2, \mathbf{Y}_j = \mathbf{L} \mathbf{W}_2; * \text{ is the symmetric part of a matrix.}

Proof for \( H_2 \) tracking performance, when perturbation \( \tilde{\mathbf{w}}(t) = 0 \), due to \( \mathbf{x}_j^T(t)\mathbf{P}_\infty(t) \geq 0 \), we can get

\[
J_2(u) \leq \mathbf{x}_j^T(0)\mathbf{P}_\infty(0) + \int_0^T \{\mathbf{x}_j^T(t)\mathbf{Q}_\infty(t) + u_j(t)\mathbf{R}_\infty(t)\mathbf{x}_j(t)\mathbf{P}_\infty(t) + \mathbf{x}_j^T(t)\mathbf{P}_\infty(t)\mathbf{P}_\infty(t)\}dt
\]

The formula (8) and the formula (9) are brought into the formula (18), and the following formula is obtained.

\[
J_2(u) \leq \mathbf{x}_j^T(0)\mathbf{P}_\infty(0) + \int_0^T \{\mathbf{x}_j^T(t)\mathbf{Q}_\infty(t) + \sum_{\mu_j} \sum_{\mu_j} \mathbf{x}_j^T(t)[\mathbf{A}_j^T P + \mathbf{P} \mathbf{A}_j + \mathbf{K}_j^T \mathbf{R} \mathbf{K}_j] \mathbf{x}(t)\}dt
\]

Obviously, if

\[
\mathbf{A}_j^T P + \mathbf{P} \mathbf{A}_j + \mathbf{K}_j^T \mathbf{R} \mathbf{K}_j + \mathbf{Q} \leq 0
\]

and \( \mathbf{x}_j^T(0)\mathbf{P}_\infty(0) \leq \alpha \)

are established, then we can get \( J_2 \leq \alpha \).
First, on both sides of inequality (21), we multiply $\text{diag}(P_{11}^{-1}, P_{22}^{-1}, P_{33}^{-1}, P_{44}^{-1})$ by left and right, and then LMI (15) can be obtained by applying Schur complement theorem. LMI (16) can be obtained by applying Schur complement theorem to inequality (17).

The analysis method of $H_2$ tracking performance index is similar, because $\tilde{x}^T(t')P\tilde{x}(t') \geq 0$ and $\tilde{x}^T(0)P\tilde{x}(0) = 0$, we can get:

$$
\int_0^T \tilde{x}^T(t)Q\tilde{x}(t)dt \leq \int_0^T \{ \sum_{i=1}^N \sum_{j=1}^N \mu_i \mu_j [\tilde{x}^T(t)A_{ij}^TP\tilde{x}(t)+\tilde{x}^T(t)PA_{ij}\tilde{x}(t)] + \tilde{x}^T(t)\tilde{x}(t)P\tilde{x}(t)] \} dt 
$$

(22)

Formula (2) can be arranged as

$$
\int_0^T \tilde{x}^T(t)Q\tilde{x}(t)dt \leq \int_0^T \{ \sum_{i=1}^N \sum_{j=1}^N \mu_i \mu_j [\tilde{x}^T(t)A_{ij}^TP\tilde{x}(t)+\tilde{x}^T(t)PA_{ij}\tilde{x}(t)+\tilde{x}^T(t)\tilde{x}(t)\tilde{x}(t)\tilde{x}(t)] \} dt 
$$

(23)

If the following inequalities hold true

$$
\tilde{Q}^T(t)P\tilde{Q}(t)+\tilde{P}\tilde{Q}(t)+\tilde{P}\tilde{Q}(t)\tilde{Q}(t) \leq 0
$$

(24)

We can obtain $\beta$. By using Schur complement theorem, linear matrix inequalities (17) can be obtained by using both left and right multiplication matrices $\text{diag}(P_{11}^{-1}, P_{22}^{-1}, P_{33}^{-1}, P_{44}^{-1})$ on both sides of formula (23).

Proof completed.

Combined with multi-objective problem solving method[13,14], we can obtain (25).

$$
\min_{\alpha, \beta} \left( \alpha, \beta \right) 
$$

s.t.

$$
\Psi_{ij} < 0 \quad i \in Z
$$

(26)

$$
\frac{\Psi_{ij}}{\Psi_{ij}} + \frac{\Psi_{ij}}{\Psi_{ij}} < 0 \quad i \neq j \in Z
$$

(27)

where,

$$
\Psi_{ij} = \begin{bmatrix}
\Phi_{ij} & \Delta_{ij} & \Gamma_{ij} & B_{Y_{ij}} & Y_{ij}^T & 0 \\
\Sigma_i & B_{Y_{ij}} & Y_{ij}^T & W_{22} & 0 & 0 \\
* & T_i & W_{33}C_{ri}^T & 0 & 0 & 0 \\
* & * & Y_{ij}^T & 0 & 0 & 0 \\
* & * & * & -P^{-1} & 0 & 0 \\
* & * & * & * & -Q^{-1} & 0 
\end{bmatrix}
$$

(28)

And

$$
\Delta_{ij} = Y_{ij}^T B_{ij}^T - Y_{ij}, \quad \Delta_{ij} = A_{i}W_{33} - A_{n}W_{33} + Y_{ij}^T, \quad \Sigma_i = W_{22}A_{i}^T + A_{i}W_{22}, \quad T_i = A_{n}W_{33} + W_{33}A_{i}^T,
$$

$$
\Phi_{ij} = W_{11}A_{ij}^T + A_{i}W_{11} + B_{Y_{ij}} + Y_{ij}^T B_{i}^T, \quad W_{11} = P_{11}^{-1}, \quad W_{22} = P_{22}^{-1}, \quad Y_{ij} = K_{i}W_{11}, \quad K_{j} = Y_{ij}W_{11}, \quad Y_{ij} = K_{j}W_{22}
$$

and

4. Simulation

Figure 2 is the fuzzy tracking control structure of the double-in, double-out reheat steam temperature system with state observer.
In reference [14], the state space models of reheat steam temperature for 750 MW and 1000 MW secondary reheating units are given. The reheat steam temperature is regulated by flue gas regulating baffle, flue gas recirculation baffle and reheater water spray desuperheater valve in order to achieve the regulation of reheat steam temperature.

Reference state space model for two channels:

\[
A_r = \begin{bmatrix}
-0.025 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -0.025 & 0
\end{bmatrix}, \quad B_r = \begin{bmatrix}
0.125 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0.125
\end{bmatrix}, \quad C_r = \begin{bmatrix}
0.2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.2
\end{bmatrix}.
\]

The T-S model:

Rule 1: if \( f = 750 \text{MW} \), then \( \dot{x} = A_{75}x + B_{75}u + E\omega \)

Rule 2: if \( f = 1000 \text{MW} \), then \( \dot{x} = A_{100}x + B_{100}u + E\omega \)

where, \( x \) is the system state, \( \omega \) is a system disturbance, \( f \) is the unit load, \( E = I \) is the disturbance coefficient. Triangle membership function is used to simplify the design.

To solve the optimization problem (28), the optimal parameters of multi-objective fuzzy \( H_2 / H_\infty \) tracking control coordination controller for reheat steam temperature system were obtained. As a comparative experiment, we choose the control strategy (CS) 1: \( u_k = K e_k \) in the reference [14], and the CS2: \( (x I y)_k + Ke_k(t) + K e(t) \) in this paper. Next, we select the controller parameters under 750MW. In the case of mismatch model with anti-integrator saturator: when \( t = 1s - 30000s \), the expected value \( r_1 = 1 \); when \( t = 30001s - 80000s \), the expected value \( r_1 = 2 \); when \( t = 1s - 20000s \), the expected value \( r_2 = 1 \); when \( t = 20001s - 80000s \), the expected value \( r_2 = 1.5 \). When \( t = 50000s \), add 10% disturbance to the output \( y_1 \); and \( t = 70000s \), add -10% perturbation to output \( y_2 \). Add anti-integral saturation link, set controller range, assuming \( U_{\text{max}} = [2, 20] \) and \( U_{\text{min}} = [-2, -20] \). When the model is mismatched, observe the output response of CS1 and CS2 and the output curve of the controller.
Figure 3. Model mismatch

It can be seen from Figure 3 (a) that when the model is mismatched, the adjustment time and rise time of CS1 and CS2 increased. The overshoot of CS1 response curve increases compared with the overshoot of CS1 response curve under the determined model, and there is difference control, but the overshoot of CS2 is basic. At the same time, when the model is mismatched, the adjustment time of CS2 response output is faster than that of CS1 response output when the rise time of CS2 response output is basically the same as that of CS1 response output. CS2 also shows strong robustness.

Next, the variable load experiment is carried out. The load instructions are shown in Figure 4. When $t = 0s - 10000s$, load $f = 750MW$; when $t = 10001s$, the load is increased at $1MW/s$ speed until the load is balanced after $f = 1000MW$. Assuming $U_{max} = [2 15]$ and $U_{min} = [-2 -15]$. From Figure 5, it can be seen that CS1 and CS2 can guarantee the stability of control system when model mismatch occurs in the system tracking stage ($t = 0s - 10000s$). However, the output quality of CS1 is poor, while CS2 with observer basically maintains the system. Output quality, overshoot basically unchanged, and adjustment time slightly increased. In this stage, the output fluctuation of the controller of CS2 is significantly less than that of CS1. In the load-raising stage (after $t = 10001s$), the output quality of CS1 and CS2 is similar, but as far as the output quality of the controller is concerned, the output quality of the controller of CS2 is better than that of the controller of CS1.

Figure 4. Load command
5. Conclusion

In this paper, the multi-objective robust $H_2 / H_{\infty}$ fuzzy tracking control for the non-linear dynamic secondary reheat steam temperature system of supercritical units is studied. Firstly, using T-S model to describe the reheat steam temperature system of supercritical reheat unit. Secondly, a multi-objective robust fuzzy tracking controller is designed. The LMI method is used to transform the controller design problem into a MOP problem with LMI constraints. The design of multi-objective fuzzy tracking control is accomplished simultaneously by combining LMI-MOEA search method. Finally, we propose a multi-objective robust $H_2 / H_{\infty}$ fuzzy tracking controller is illustrated by a simulation example of the secondary reheat temperature system in a USC power plant. The comparative experiments also show that the robust $H_2 / H_{\infty}$ fuzzy tracking CS2 with state observer has more advantages than CS1 in practical engineering.

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