Electromagnetically-induced transparency and light storing of a pair of pulses

A. Raczyński¹ *, J. Zaremba¹ and S. Zielińska-Kaniasty²

¹Instytut Fizyki, Uniwersytet Mikołaja Kopernika, ul.Grudziądzka 5, 87-100 Toruń, Poland,
²Instytut Matematyki i Fizyki, Akademia Techniczno-Rolnicza, Al. Prof. S. Kaliskiego 7, 85-796 Bydgoszcz, Poland.

Abstract

Electromagnetically-induced transparency and light storing are studied in the case of a medium of atoms in a double Λ configuration, both in terms of dark- and bright-state polatitons and atomic susceptibility. It is proven that the medium can be made transparent simultaneously for two pulses following their self-adjusting so that a condition for an adiabatic evolution has become fulfilled. Analytic formulas are given for the shapes and phases of the transmitted/stored pulses. The level of transparency can be regulated by adjusting the heights and phases of the control fields.

PACS numbers: 42.50.Gy, 03.67.-a

*email: raczyn@phys.uni.torun.pl
I. INTRODUCTION

It is well known that an atomic medium irradiated by a control laser field may become transparent for a signal field which in the absence of the first field would be almost immediately absorbed. This phenomenon, known as an electromagnetically-induced transparency (EIT) [1,2], has recently been used to drastically change the velocity of a light pulse or even to stop or store it: by changing the control field in time it is possible to make the medium opaque at the moment at which the signal pulse is inside [3–5]. The pulse is then transformed into an atomic coherent excitation which is rather robust against relaxation and after quite a long time in the atomic scale it is possible to switch the control field on again and to release the trapped signal. Such processes have been observed experimentally and explained theoretically both in the language of so-called polaritons, being collective atom+field excitations, and in terms of atomic susceptibility [6,7]. An elementary atomic systems for which such processes are possible is an atom with three active (resonantly coupled) states in the \( \Lambda \) configuration.

Adding a fourth active state and a second control field i.e. extending the atomic system to a double \( \Lambda \) configuration allows one to consider new nonlinear, resonantly enhanced optical processes [8]. In particular it is possible to simultaneously propagate two optical pulses of different frequencies through a medium in the conditions of EIT. In our earlier paper [9] we have pointed out that one can stop one pulse and release a pulse of a different frequency or two different pulses. We have also mentioned that making the medium transparent simultaneously for two pulses is possible provided that the fields are in some special relation, which allows for an adiabatic evolution of the system. If on the other hand the initial conditions do not satisfy this relation the pulses are partially absorbed, which is connected with nonadiabatic phenomena. In the present work we give a detailed quantitative analysis of the conditions of a joint medium transparency for two pulses. A discussion in terms of dark- and bright-state polaritons, presented in the next section, includes simple analytic formulas which allow one to predict the amplitudes, phases and time evolution of the transmitted
and restored signals and/or the value and space distribution of the atomic coherence due to
the trapped pulses. Section III shows how the non-adiabaticity of the evolution is reflected
on the atomic susceptibility. Section IV contains a comparison of the predictions of the
polariton approach with the results of complete numerical solutions of the Bloch-Maxwell
equations.

II. POLARITONS

We consider a medium composed of atoms in a double Λ configuration presented in Fig. 1.
Two driving fields $\epsilon_{2,4}(t)$, induce transparency for two weak signal fields $\epsilon_{1,3}$. The evolution
of the atomic density matrix $\sigma$ and the propagation of the signal pulses are described by
the a set of Bloch-Maxwell equations, while propagation effects for the driving fields are
neglected.

In the rotating-wave and slowly-varying-envelope approximations, in the resonance con-
ditions without relaxations for the transitions shown in Fig. 1, we are left in the first order
with respect to the signal fields with the equations

\[
\begin{align*}
\left(\frac{\partial}{\partial t} + \epsilon \frac{\partial}{\partial z}\right) R_1 &= S_1, \\
\left(\frac{\partial}{\partial t} + \epsilon \frac{\partial}{\partial z}\right) R_3 &= S_3, \\
\frac{\partial}{\partial t} S_1 &= -\kappa_1^2 (R_1 + U_2 \sigma_{bc}), \\
\frac{\partial}{\partial t} S_3 &= -\kappa_3^2 (R_3 + U_4 \sigma_{bc}), \\
\frac{\partial}{\partial t} \sigma_{bc} &= U_2^* S_1 + U_4^* S_3.
\end{align*}
\]

(1)

In the above formulas $\kappa_{1,3}^2 = \frac{|d_{1,3}|^2 \omega_{1,3} N}{4 \epsilon_0 \hbar}$, $\omega_j$ is the frequency of the field $j$, $N$ is the density of
atoms, $\epsilon_0$ is the vacuum electric permittivity, $d_j$ are the transition matrix elements: $d_1 = d_{ab}$,
$d_2 = d_{ac}$, $d_3 = d_{db}$, $d_4 = d_{dc}$, $R_j = \frac{\epsilon_j d_j^*}{2 \hbar \kappa}$, $U_{2,4} = \frac{\epsilon_j d_j^*}{2 \hbar \kappa_{1,3}}$, $S_1 = -i \kappa_1 \sigma_{ba}$, $S_3 = -i \kappa_3 \sigma_{bd}$ and
$\sigma = \sigma(z, t)$ is the atomic density matrix after transforming-off the rapidly oscillating terms.

The adiabatic approximation would consist in setting the time derivatives of $S_{1,3}$ equal to
zero, which would mean an evolution during which the atomic upper states are not populated
at all. However, one can see from the third and fourth equation in the set (1) that this is possible if the four laser fields remain in a certain proportion: $\frac{R_1}{U_2} = \frac{R_3}{U_4}$. This reflects the fact that during an adiabatic evolution each atom makes a continuous transition from the initial state $b$ to the dark state, which is a superposition of the two lower states $b$ and $c$ with the coefficients determined by the instantaneous values of the intensities of the signal and control fields. However, in a double $\Lambda$ system this combination is a dark state simultaneously for the two $\Lambda$’s only in the special situation.

After eliminating the variables $S_{1,3}$ the above equations can be rewritten in a new set of variables - the so-called polaritons

$$R_1 = \exp(i \arg(U_2))(\cos \theta \cos \phi \Psi + \sin \theta \cos \phi \Phi + \sin \phi X),$$

$$R_3 = \exp(i \arg(U_4))(\cos \theta \sin \phi \Psi + \sin \theta \sin \phi \Phi - \cos \phi X),$$

$$\sigma_{bc} = - \sin \theta \Psi + \cos \theta \Phi,$$

where $\sin \theta = (|U_2|^2 + |U_4|^2 + 1)^{-1/2}$ and $\tan \phi = \frac{|U_4|}{|U_2|}$.

The evolution equations for the polaritons in the case of time-independent driving fields read

$$\frac{\partial}{\partial t} \Psi + c \cos^2 \theta \frac{\partial}{\partial z} \Psi + c \sin \theta \cos \theta \frac{\partial}{\partial z} \Phi = 0,$$

$$- \sin^2 \theta \frac{\partial^2}{\cos \theta \partial t^2} \Psi + \sin \theta \frac{\partial^2}{\partial t^2} \Phi = - \frac{1}{\sin \theta} \left( \kappa_1^2 \cos^2 \phi + \kappa_3^2 \sin^2 \phi \right) \Phi - \sin \phi \cos \phi (\kappa_1^2 - \kappa_3^2) X,$$

$$\frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \right) X = - \frac{\sin \phi \cos \phi}{\sin \theta} (\kappa_1^2 - \kappa_3^2) \Phi - (\kappa_1^2 \sin^2 \phi + \kappa_3^2 \cos^2 \phi) X.$$

In the case of a single $\Lambda$ system (i.e. for $X = 0$) an adiabatic evolution meant that the dark state polariton traveled with the velocity $c \cos^2 \theta$ keeping its shape, with the bright state polariton $\Phi = 0$. The only necessary condition of adiabaticity was that the pulses 1 and 2 should be smooth enough. For a double $\Lambda$ system, again assuming smoothness of all the pulses, the new element is that the bright state polariton $X$ is in general different from zero at the beginning of propagation: the decomposition of the incoming signal fields is given by Eqs (2) (with $\Phi = 0$)
\[ R_1^0 = \exp[i \arg(U_2) \theta_0 \cos \phi_0 \Psi + \sin \phi_0 X], \]
\[ R_3^0 = \exp[i \arg(U_4) \theta_0 \sin \phi_0 \Psi - \cos \phi_0 X). \]

Then during the evolution \( \Psi \) keeps its shape while \( X \) is damped. Thus the conditions of adiabaticity are gradually created and the three dynamical variables tend to the following values

\[ R_1 \rightarrow \exp[i \arg(U_2) \cos \theta \cos \phi \cos \theta_0 (\cos \phi_0 \exp[-i \arg(U_2) \theta_0 \cos \phi_0 \Psi + \sin \phi_0 X] R_1^0 + \sin \phi_0 \exp[-i \arg(U_4) \theta_0 \sin \phi_0 \Psi - \cos \phi_0 X] R_3^0) \]
\[ R_3 \rightarrow \exp[i \arg(U_4) \cos \theta \sin \phi \cos \theta_0 (\cos \phi_0 \exp[-i \arg(U_2) \theta_0 \cos \phi_0 \Psi + \sin \phi_0 X] R_1^0 + \sin \phi_0 \exp[-i \arg(U_4) \theta_0 \sin \phi_0 \Psi - \cos \phi_0 X] R_3^0), \]
\[ \sigma_{bc} \rightarrow -\frac{\sin \theta}{\cos \theta_0} (\cos \phi_0 \exp[-i \arg(U_2) \theta_0 \cos \phi_0 \Psi + \sin \phi_0 X] R_1^0 + \sin \phi_0 \exp[-i \arg(U_4) \theta_0 \sin \phi_0 \Psi - \cos \phi_0 X] R_3^0). \]

The argument of the functions \( R_{1,3}^0 \) in the r.h.s of Eqs (5) has to take into account the shift by \( \int_0^t c \cos^2 \theta(\tau) d\tau \) during the time \( t \). In particular if we assume that at \( t = 0 \) the edge of the pulse reaches the sample, when calculating the position of the maximum we have to take into account that the maximum moves first with the velocity \( c \) until it reaches the sample and later travels with the velocity \( c \cos^2 \theta \).

The above formulas allow one to predict the shape and the numerical parameters of the transmitted pulses in typical EIT, when the control fields are kept constant, as well as in the process of light storing, when the control pulses are slowly varying. Note in particular that a single initial signal in the presence of two control fields leads to an appearance of the other signal pulse. In Section IV we will compare the predictions based on these formulas with the results of the numerical solutions of the Bloch-Maxwell equations.

### III. ATOMIC SUSCEPTIBILITY

A complementary picture of the above evolution is obtained in terms of the susceptibility. If we rewrite the first three equations of Eq. (1) in the original variables admitting the detuning and relaxations terms

\[ i \dot{\sigma}_{ba} = \frac{1}{2\hbar} \epsilon_1 d_1^* + \Omega_2 \sigma_{bc} + \Delta_1 \sigma_{ba} \]
\[ i\sigma_{bd} = \frac{1}{2\hbar} \varepsilon_3 d_3^* + \Omega_4 \sigma_{bc} - \Delta_3 \sigma_{bd} \]  
\[ i\sigma_{bc} = \Omega_2 \sigma_{ba} + \Omega_4 \sigma_{bd} - \delta \sigma_{bc}, \]

where \( \Omega_{2,4} = \frac{\varepsilon_2 d_2^*}{2\hbar}, \) \( h\Delta_1 = E_b - E_a - h\omega_1 - \frac{i}{2} \Gamma_1, \) \( h\Delta_3 = E_d - E_b - h\omega_3 - \frac{i}{2} \Gamma_3 \) and \( h\delta = E_b + h\omega_1 - E_c - h\omega_2 - i\gamma \) are the detunings including the relaxation rates for the coherences \( ab, \) \( db \) and \( bc. \)

If we now pass to the frequency domain, assuming that \( \omega_1 - \omega_2 = \omega_3 - \omega_4, \) we can calculate the elements of the density density matrix \( \sigma \) and express the components of the polarization in terms of the signal fields

\[ N d_1 \sigma_{ba}(\omega) = 2\pi \varepsilon_0 [\chi_{11}(\omega) \varepsilon_1(\omega) + \chi_{13}(\omega) \varepsilon_3(\omega)] \]
\[ N d_3 \sigma_{bd}(\omega) = 2\pi \varepsilon_0 [\chi_{31}(\omega) \varepsilon_1(\omega) + \chi_{33}(\omega) \varepsilon_3(\omega)], \]

where

\[ \chi_{11}(\omega) = \frac{N |d_1|^2}{4\pi \hbar \varepsilon_0} \frac{i(\Delta_1 - \omega)(\delta - \omega) - |\Omega_4|^2}{(\Delta_3 - \omega)(\Omega_4)^2 - (\Delta_3 - \omega)|\Omega_2|^2}, \]
\[ \chi_{13}(\omega) = \frac{N d_1^* d_3}{4\pi \hbar \varepsilon_0} \frac{e^{-i\Omega_4^*}}{\Omega_4^* \Omega_4} \frac{i(\Delta_1 - \omega)(\delta - \omega) - |\Omega_4|^2}{(\Delta_3 - \omega)(\Omega_4)^2 - (\Delta_3 - \omega)|\Omega_2|^2}, \]
\[ \chi_{31}(\omega) = \frac{N d_3^* d_1}{4\pi \hbar \varepsilon_0} \frac{e^{i\Omega_4^*}}{\Omega_4^* \Omega_4} \frac{i(\Delta_1 - \omega)(\delta - \omega) - |\Omega_4|^2}{(\Delta_3 - \omega)(\Omega_4)^2 - (\Delta_3 - \omega)|\Omega_2|^2}, \]
\[ \chi_{33}(\omega) = \frac{N |d_3|^2}{4\pi \hbar \varepsilon_0} \frac{i(\Delta_1 - \omega)(\delta - \omega) - |\Omega_2|^2}{(\Delta_3 - \omega)(\Omega_4)^2 - (\Delta_3 - \omega)|\Omega_2|^2}. \]

It is important to notice that under the assumptions of the polariton analysis, i.e. in the resonance conditions and neglecting the relaxations, the expressions for the susceptibility \( \chi \) become singular at \( \omega = 0, \) e.g.,

\[ \chi_{11}(\omega) = \frac{N |d_1|^2}{4\pi \hbar \varepsilon_0} \frac{\omega^2 - |\Omega_4|^2}{\omega^3 - \omega(|\Omega_2|^2 + |\Omega_4|^2)}, \]
\[ \chi_{13}(\omega) = \frac{N d_1^* d_3}{4\pi \hbar \varepsilon_0} \frac{\omega^3 - \omega(|\Omega_2|^2 + |\Omega_4|^2)}{\Omega_2^* \Omega_4}, \]

with analogous expressions for \( \chi_{31} \) and \( \chi_{33}. \) One cannot thus speak of a transparency window and the pulses’ propagation occurs with a significant distortion. Fulfilling the adiabaticity condition \( \frac{\hbar v_1}{\Gamma_1} = \frac{\hbar v_2}{\Gamma_2} \) (cf. the third and fourth of Eqs (1)) means that the singularities in Eqs
(7) cancel out and the induced transparency inside a transparency window of a finite size is possible. In that case after substituting $\epsilon_3 = \epsilon_1 \frac{d_1 \Omega_4}{\alpha_3 \Omega_2}$ we obtain

$$Nd_1 \sigma_{ba} = 2\pi \epsilon_0 \chi_{11}(\omega) \epsilon_1(\omega),$$

(10)

with

$$\chi_{11}(\omega) = \frac{N|d_1|^2}{4\pi \hbar \epsilon_0} \frac{-\omega}{\omega^2 - (|\Omega_2|^2 + |\Omega_4|^2)},$$

(11)

which is the expression as for a single $\Lambda$ system, with the only difference that the denominator is corrected by the $|\Omega_4|^2$ term, which means that the transparency window is widened compared with the case of a single $\Lambda$ system.

### IV. NUMERICAL ILLUSTRATION

A numerical illustration of the above results is presented below. We have performed calculations for the atomic model with $E_a = -0.10$ a.u., $E_b = -0.20$ a.u., $E_c = -0.18$ a.u., $E_d = -0.05$ a.u., the width of the upper levels $a$ and $d$ connected with the spontaneous transitions to the lower levels $b$ and $c$ were taken $\Gamma_{a,d}^{b,c} = 2.4 \times 10^{-9}$ a.u., from which the dipole moments have been calculated; the relaxation of the lower-states’ coherence has been neglected. The medium density has been taken $N = 3 \times 10^{-13}$ a.u. ($2 \times 10^{12}$ cm$^{-3}$). The length of the sample was $10^7$ a.u (0.5 mm). The initial length of the signal pulses was $10^{11}$ a.u. (2.4 $\mu$s) and their amplitudes, modeled by a sine-square, were of order of $10^{-10}$ a.u., which correspond to the power density of $3.5 \times 10^{-4}$ Wcm$^{-2}$. The amplitudes of the control pulses were larger by about an order of magnitude.

Because there are essentially two reasons of nonadiabaticity of the evolution, namely the discontinuity of pulses or the fields’ failing to satisfy the proportion following Eq. (1), we first check the role of the former effect. In Fig. 2 we show the shape of the initially rectangular pulse in a single $\Lambda$ system in a control field with a time-independent amplitude. The evolution is initially nonadiabatic. However, the signal propagating in the medium is
gradually smoothed, which corresponds to an absorption of the polariton $\Phi$. An analogous calculation with a smooth (sine square) pulse yields to a very good approximation a conservation of the pulse shape during the propagation, which means an absence of the polariton $\Phi$ from the very beginning. Because in our further investigations on double $\Lambda$ systems we use sine square pulses, it follows that we may indeed neglect the polariton $\Phi$, as mentioned in Section II.

In Fig. 3 we show the transmitted pulses 1 and 3 compared with their initial shapes. The horizontal lines show the values calculated from Eq. (5). The predictions of the heights of the pulses are excellent. We have checked that Eq. (5) predicts properly also the phases of the transmitted signal pulses.

The effective transparency for the two pulses can be regulated by choosing the heights and phases of the control fields. In Fig. 4 we present the transmitted pulses 1 and 3 for two different phase relations. By a suitable choice of the phase of any of the control fields one can regulate the heights of the signal pulses.

If we switch the control fields off the medium becomes opaque for the two signal pulses, which are then "stopped" in the form of the single atomic coherence $\sigma_{bc}$, the form of which is given by the third of Eqs (5). Of course the absolute value of this coherence depends on the place inside the sample but the phase relations are more general. In Fig. 5 we show the argument of the complex coherence as a function of the phase shift of the field 3; the simultaneous adiabatic switch-off of the two control fields has been modeled by a hyperbolic tangent. One can see that the predictions of the last of Eqs (5) are also excellent.

In Fig. 6 we show the space distribution of the coherence due to the trapped pulse. The length of the sample was $3.5 \times 10^8$ a.u. and the control pulses were switched off again simultaneously as a hyperbolic tangent, but somewhat later than before so that the whole pulse could be trapped inside the medium without any additional effects of the boundary of the sample. The distribution can be modeled with a good accuracy by a sine square, i.e. the common shape of the incoming pulses. No parameters have been fitted: the amplitude has been obtained from the last of Eqs (5), the width is the original width multiplied by the
compression factor $\cos^2 \theta_0$ (the pulse has been compressed when entering the sample but did not change its width any more during the storing stage) and the position of the maximum has been calculated as explained above following Eqs (5).

V. CONCLUSIONS

We have discussed a simultaneous propagation of two signal pulses in the medium of four-level atoms in the double $\Lambda$ configuration. We have shown quantitatively that it is practically possible to divide the incoming pulses into a dark- and bright-state polaritons, of which the dark one survives during the evolution. This is connected with the pulses’ self-adjusting during the initial stage of the propagation, so that the condition of adiabaticity becomes fulfilled. A knowledge of the shape and position of the dark-state polariton allows one to predict the characteristics of the transmitted pulses or, in case the pulses were stored, the value and space distribution of the induced atomic coherence. Those characteristics can be dynamically controlled by changing the parameters of the control fields, e.g. their phase difference. We have also demonstrated that an adiabatic character of the pulses’ evolution is reflected in a cancellation of singularities of the atomic susceptibility.
ACKNOWLEDGMENTS

This work is a part of a program of the National Laboratory of AMO Physics in Toruń, Poland.
REFERENCES

[1] S. E. Harris, Phys. Today **507**, 36 (1997).

[2] M. O. Scully and M. S. Zubairy, *Quantum Optics*, (Cambridge University Press, 1997).

[3] C. Liu, Z. Dutton, C. H. Behroozi and L. V. Hau, Nature **409**, 490 (2001).

[4] D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth and M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).

[5] A. B. Matsko, O. Kocharovskaya, Y. Rostovtsev, G. R. Welch, A. S. Zibrov, and M. O. Scully, Adv. in At. Mol. Opt. Phys. **46**, 191 (2001).

[6] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000).

[7] O. Kocharovskaya, Y. Rostovtsev and M. O. Scully, Phys. Rev. Lett. **86**, 628 (2001).

[8] M. D. Lukin, P. R. Hemmer and M. O. Scully, Adv. At. Mol. Opt. Phys. **42**, 347 (2000).

[9] A. Raczyński and J. Zaremba, Opt.Commun. **209**, 149 (2002).
FIGURES

FIG. 1. Level and coupling schemes; the indices 1 and 3 refer to signal fields and 2 and 4 - to control fields.

FIG. 2. The shape of the propagating initially rectangular pulse as a function of the local time $t' = t - \frac{z}{c}$, in the case of a single Λ system for a control field of a time-independent amplitude $\epsilon_2 = 1.2 \times 10^{-9}$ a.u.: at the beginning of the sample (1), at the depths: $3 \times 10^6$ a.u. (2), $1.5 \times 10^7$ a.u. (3), $3 \times 10^7$ a.u. (4). Note the process of smoothing the pulse.

FIG. 3. The shapes of the transmitted pulses 1 (curve 1) and 3 (curve 3), compared with their initial shapes (curves 10 and 30, respectively, for the control fields $\epsilon_2 = 1.2 \times 10^{-9}$ a.u. and $\epsilon_4 = 1.8 \times 10^{-9}$ a.u. The horizontal lines show the predicted heights of the transmitted pulses. Note also the slowdown of the transmitted pulses.

FIG. 4. The shapes of the transmitted pulses 1 (curves 1a and 1b and 3 (curves 3a and 3b) for the data as in Fig. 3 but with additional phase shifts $\phi_j$ of the pulse $j$: $\phi_2 = \frac{\pi}{2}$, $\phi_3 = \frac{2\pi}{3}$, $\phi_4 = \frac{7\pi}{6}$ (case a) and $\phi_4 = \frac{\pi}{6}$ (case b).

FIG. 5. The phase shift of the coherence $\sigma_{bc}$ due to the trapped pulses as a function of the phase shift of the field 3. The crosses are the results of the numerical computations and the solid line is obtained from the last of Eqs (5). The control fields of the maximum values as in Fig. 3 were switched off simultaneously as a hyperbolic tangent.

FIG. 6. The spatial distribution of the coherence $\sigma_{bc}$ due to the trapped pulse. The crosses are the results of the numerical computations and the solid line is a sine square, of which the amplitude has been calculated from the last of Eqs (5), the position of the maximum is given by the formula following Eqs (5) and the width is equal to the width of the incoming pulses divided by $\cos^2 \theta_0$. The control fields were of the same shape as in Fig. 5 but were switched off somewhat later.
field amplitude ($10^{-11}$ a.u.)

t' ($10^{10}$ a.u.)
\[ \text{arg}(\sigma_c) \text{ (rad)} \]

\[ \text{arg}(\varepsilon_3(0)) \text{ (rad)} \]
