Empirical Likelihood for Contextual Bandits

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Abstract

We apply empirical likelihood techniques to contextual bandit policy value estimation, confidence intervals, and learning. We propose a tighter estimator for off-policy evaluation with improved statistical performance over previous proposals. Coupled with this estimator is a confidence interval which also improves over previous proposals. We then harness these to improve learning from contextual bandit data. Each of these is empirically evaluated to show good performance against strong baselines in finite sample regimes.

1 Introduction

Contextual Bandits [ACFS02, LZ07] are now in widespread practical use ([LCLS10, CABJ17, PGC+14]). Key to their value is the ability to do counterfactual estimation [HT52] of the value of any policy enabling sound train/test regimes similar to supervised learning. A limiting factor on the lower (data) scale of application is the variance of counterfactual estimation. How can we find the tightest-possible confidence interval on counterfactual estimates? And since tight confidence intervals are deeply dependent on the form of their estimate, how can we find a tight estimate? And given what we discover, how can we leverage this for improved learning algorithms?

We discover good answers to these questions through the application of empirical likelihood [Owe01]. Applying this first to estimation, we construct a simply-specified estimator in section 2.1 and convert this into a computationally tractable solution via duality resulting in a low bias/low variance estimator for the value of a policy which is particularly relevant in regimes where the number of samples $n$ is of the same order as the smallest inverse probability $1/p$ of an action.

Next we elaborate a computationally tractable asymptotically exact confidence interval in section 2.2. Typically confidence intervals are either small but fail to guarantee prescribed coverage, or guarantee prescribed coverage but are too wide to be useful. Our interval is both small and (despite having only an asymptotic guarantee) empirically honors prescribed coverage.

Turning to learning in section 2.3, we utilize our confidence interval to construct a robust counterfactual learning objective with which we experiment with empirically in section 3.

1.1 Contributions

The estimator, confidence interval, and learning objective presented here are all new. Of these, the estimator and learning objective are useful improvements, while the confidence interval is a large improvement over previous approaches as shown in figure 1.

1.2 Related Work

The empirical likelihood framework [Owe01] forms the framework for our approach. It is a non-parametric maximum likelihood approach that treats the sample as a realization from a multinomial

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Figure 1: A comparison of confidence intervals on contextual bandit data. The MLE confidence interval is dramatically tighter than an approach based on a binomial confidence interval while avoiding chronic undercoverage as per the asymptotic Gaussian confidence interval. Note that in some regimes, the asymptotic Gaussian CI both undercovers and has greater average width. This is possible as the MLE CI has a different functional form than a multiplier on the Gaussian CI. On the left, shaded area represents 90% of the empirical distribution indicating the MLE CI width varies less over realizations. On the right, shaded area represents 4 times the standard error of the mean indicating coverage differences are everywhere statistically significant.

distribution with an infinite number of categories. Surprisingly, empirical likelihood results in both efficient algorithms and efficient estimators with guarantees similar to those of parametric maximum likelihood with a well specified model.

There are many previous estimators for contextual bandits. The simplest estimator for contextual bandits is the "Inverse Propensity Score" (IPS) approach [HT52] which is unbiased, but suffers from high variance. The Self-Normalized IPS (SNIPS) [SJ15b] estimate is a simple modification which is biased but has superior mean squared error. An orthogonal way to reduce variance is to incorporate a reward estimator. This can be done via doubly robust (DR) estimation [RR95, DLL11] which is unbiased even when the reward estimator is biased and has lower variance when the reward estimator is good. The SWITCH estimator [WAD17] provides a method for switching between a double robust estimator and direct application of a reward estimator to optimize mean square error. The estimator presented here is a natural alternative to IPS and SNIPS, can be seamlessly combined with DR or SWITCH (replacing their IPS part), and provides lower mean squared error. We briefly discuss how to incorporate a reward predictor.

There is less work on confidence intervals for contextual bandits. A simple baseline approach for on-policy confidence intervals randomly rounds the rewards to \{0, 1\} and applies a Binomial confidence interval. For off-policy evaluation this approach can still be used by randomly rounding weights to the largest weight value or 0 yielding an even looser confidence interval. The confidence interval we create here is much tighter. A simple asymptotically motivated approach fits the observations to a normal distribution and uses the confidence interval for the normal, an approach previously applied to contextual bandits [LCKG15]. This results in a too-tight interval, e.g., when all observed rewards are zero resulting in zero empirical variance. The MLE confidence intervals are also asymptotically motivated but we use a “missing mass” argument to empirically mitigate undercoverage.

There are many contextual bandit learning algorithms including theoretical [ACFS02, LZ07], reduction oriented [DLL11], optimization-based [SJ15a], and Bayesian style [MKLL12] algorithms. A recent paper about empirical contextual bandit learning [BAL18] informs our experiments.

Ideas from empirical likelihood have previously been applied to robust supervised learning [DCN16]. Our combination of confidence intervals with learning is a contextual bandit analogue to robust supervised learning. Regularizing counterfactual learning via lower-bound optimization has been previously considered, e.g., based upon empirical Bernstein bounds [SJ15a] or divergence-based trust regions grounded in lower bounds from conservative policy iteration [SLA+15, KL02].
2 Empirical Likelihood Applications to Contextual Bandits

We consider the standard contextual bandit problem, with contexts \( x \in \mathcal{X} \), a finite set of actions \( A \), and bounded real rewards \( r \in A \rightarrow [r_{\min}, r_{\max}] \). The environment generates i.i.d. context-reward pairs \( (x, r) \sim D \) and reveals \( x \) to the policy, the policy samples \( a \in A \) from a context-conditional distribution \( \pi : X \rightarrow \mathcal{P}(A) \) and observes reward \( r(a) \). We denote the all ones vector as \( \mathbf{1} \) and the indicator function as \( \mathbb{I} \).

2.1 Off-Policy Evaluation

We assume a dataset \( \{(x_n, a_n, r(a)_n)\}_{n \in N} \), generated from a fixed historical policy \( h \), with which we want to estimate the value of another fixed policy \( \pi \). The value of \( \pi \) is

\[
V(\pi) = \mathbb{E}_{(x,r) \sim D} \left[ r(a) \right] = \mathbb{E}_{(x,r) \sim D} \left[ \frac{\pi(a|x)}{h(a|x)} r(a) \right],
\]

where \( \pi(a|x) = \mathbb{E}_{a' \sim \pi(x)}[\mathbb{I}_{a=a'}] \) and analogously for \( h(a|x) \). Define \( w = \frac{\pi(a|x)}{h(a|x)} \), and assume the joint distribution of \( w \) and \( r \) has (possibly infinite) discrete support. Then we can represent the joint distribution of \( w \) and \( r \) for data generated from \( h \) as a matrix \( Q \) via

\[
Q_{w,r} \triangleq \mathbb{E}_{(x,r) \sim D} \left[ \mathbb{I}_{w=\pi(a|x)} \mathbb{I}_{r_n=r} \right], \quad V(\pi) = \sum_{w,r} w Q_{w,r} r = \mathbf{w}^\top Q \mathbf{r}.
\]

We assume \( w \) is bounded between \([w_{\min}, w_{\max}]\); given \( h \) and \( \pi \) these limits are easily determined in practice. For simplicity, we assume \( 0 \leq w_{\min} < 1 \) and \( w_{\max} > 1 \). This precludes \( w_{\min} = w_{\max} = 1 \), which is the (degenerate) case of on-policy evaluation.

To estimate \( V \) we first estimate \( Q \) and then use \( V(\pi) = \mathbf{w}^\top \hat{Q} \mathbf{r} \). To estimate \( Q \) we solve the following empirical maximum likelihood optimization:

\[
\max_{Q \succeq 0} \sum_n \log(Q_{w_n,r_n}), \quad \text{subject to} \quad \mathbf{w}^\top Q \mathbf{1} = 1, \quad \mathbf{1}^\top Q \mathbf{1} = 1.
\]

We defer the motivation for this procedure to section 2.2. The dual variable for each constraint is shown in parentheses. The constraints (1) and (2) respectively normalize the counterfactual and factual distribution. Theorem 1 characterizes the solution.

**Theorem 1.** The solution to equation (1) satisfies for every observed \((w, r)\) pair

\[
\hat{Q}_{w,r} = \sum_n \frac{\mathbf{1}_{w=w_n, r=r_n}}{\beta^*(w_n - 1) + N},
\]

where \( \beta^* \) is the solution to the dual problem

\[
\sup_{\beta} \sum_n \log (\beta(w_n - 1) + N) \quad \text{subject to} \quad \forall w : \beta(w + 1) + N \geq 0.
\]

Moreover, if \( w_{\min} \) or \( w_{\max} \) are not observed the solution to (1) puts mass on these according to the solution of the non-negative linear feasibility program

\[
w_{\min} \hat{q}_{\min} + w_{\max} \hat{q}_{\max} = 1 - \sum_n \frac{w_n}{\beta^*(w_n - 1) + N}, \quad \hat{q}_{\min} + \hat{q}_{\max} = 1 - \sum_n \frac{1}{\beta^*(w_n - 1) + N},
\]

where \( \hat{q}_{\min} \geq 0 \) and \( \hat{q}_{\max} \geq 0 \) are associated with \( w_{\min} \) and \( w_{\max} \) respectively. This additional mass can be distributed arbitrarily over \( r \in [r_{\min}, r_{\max}] \), implying the value estimate is an interval.

**Proof.** See appendix A.1
When empirical likelihood estimators are subject to additional constraints they can place mass on unobserved portions of the sample\(^{3}\). In our case the additional mass is due to the \(\beta\) constraint. Note once both extreme values \(\hat{w}_{\text{min}}\) and \(\hat{w}_{\text{max}}\) have been observed, all mass is placed upon the sample. Until then, it might be possible to increase the likelihood of the observed data while satisfying the constraint by shifting some mass to an unobserved extreme value. The dual is a one dimensional convex problem which we solve to accuracy \(\epsilon\) in \(O(N \log(\frac{N}{\epsilon}))\) time via bisection in \((\frac{N}{1-\hat{w}_{\text{max}}}, \frac{N}{1-\hat{w}_{\text{min}}})\). Given the dual solution, the additional mass can be found via primal feasibility, as indicated in appendix \(A.2\).

The resulting value estimate is

\[
\hat{V}(\text{MLE}) = \rho + \sum_{n} \frac{w_{n}(r_{n} - \rho)}{\beta^{*}(w_{n} - 1) + N},
\]

where \(\rho\) is arbitrary in \([r_{\text{min}}, r_{\text{max}}]\) and only affects the value estimate if there is mass placed outside the sample. Note \(\hat{q}_{\text{min}}\) and \(\hat{q}_{\text{max}}\) need not be explicitly computed.

Comparing the MLE with the standard IPS [HT52] and SNIPS [SJ15b] estimates in the same notation,

\[
\hat{V}(\text{IPS}) = \sum_{n} \frac{w_{n}r_{n}}{N}, \quad \hat{V}(\text{SNIPS}) = \sum_{n} \frac{w_{n}r_{n}}{\sum_{n} w_{n}}.
\]

and assuming \(\hat{q}_{\text{max}} = \hat{q}_{\text{min}} = 0\), reveals that IPS corresponds to \(\beta^{*} = 0\). This implies the \(\beta\) constraint is not active at the optimum, i.e., IPS is the MLE when the sum of the importance weights of the realization equals the number of examples. In that case SNIPS is also the MLE.

**Incorporating a reward predictor** The MLE estimator is analogous to the IPS estimator, which can be augmented with a reward predictor via the doubly-robust estimator [DLLL11]. Analogues to the doubly-robust predictor exist in the empirical likelihood literature [LYLL16]. The simplest approach is to center the rewards prior to applying maximum likelihood, and then add back the expected shift. Given reward predictor \(\tilde{r} : \mathcal{X} \times \mathcal{A} \rightarrow [r_{\text{min}}, r_{\text{max}}]\), we construct data for the MLE

\[
(w_{n}, \tilde{r}_{n}) \leftarrow \left(\frac{\pi(a_{n}|x_{n})}{h(a_{n}|x_{n})}, r_{n} - \tilde{r}(x_{n}, a_{n})\right),
\]

apply the MLE on this data (with modified \(\tilde{r}_{\text{min}}\) and \(\tilde{r}_{\text{max}}\)), and then adjust the result via

\[
\hat{V}(\text{rpmle}) = \hat{V}(\text{mle}) + \sum_{n} \sum_{a} \pi(a_{n}|x_{n})\tilde{r}(x_{n}, a_{n}).
\]

### 2.2 Confidence Intervals

An advantage of the MLE is that it comes with an asymptotically exact coverage interval defined by a likelihood level set. Let

\[
\log R_{\text{w}} = \sup_{Q} \left\{ \sum_{n} \log(NQw_{n}, r_{n}) \mid \vec{w}^{\top}Q\overline{1} = 1, \overline{1}^{\top}Q\overline{1} = 1 \right\}
\]

\[
\log R_{\text{w}}(v) = \sup_{Q} \left\{ \sum_{n} \log(NQw_{n}, r_{n}) \mid \vec{w}^{\top}Q\overline{1} = 1, \overline{1}^{\top}Q\overline{1} = 1, \vec{w}^{\top}Q\vec{r} = v \right\}
\]

with \(Q^{(\text{prof})}\) the maximizer of \(\log R_{\text{w}}(V(\pi))\) and \(Q^{(\text{mle})}\) the maximizer of \(\log R_{\text{w}}\). Then we can apply Theorem 3.5 of [Owe01] which states that \(-2(\log R_{\text{w}}(V(\pi)) - \log R_{\text{w}}) \rightarrow \chi_{1}^{2}\) in distribution as \(n \rightarrow \infty\). Letting \(\chi_{1}^{2}(1-\alpha)\) be the \(1 - \alpha\) quantile of the \(\chi^{2}\)-squared distribution with one degree of freedom, we have that for all \(\alpha\)

\[
\lim_{n \rightarrow \infty} \Pr \left( \sum_{n} \log Q^{(\text{mle})}w_{n}, r_{n} - \sum_{n} \log Q^{(\text{prof})}w_{n}, r_{n} \leq \frac{1}{2} \chi_{1}^{2}(1-\alpha) \right) = 1 - \alpha.
\]

\(^{3}\) The factor \(N\) can be refined to be the size of a histogram of \((w, r)\) pairs. On a laptop, a C++ implementation finds the MLE of a histogram with \(N = 10^8\) in a second (to single precision).
To get the lower bound of the confidence interval for $V(\pi)$ we just search for the minimum possible $\hat{V}(\pi)$ that still respects the prescribed coverage $1 - \alpha$. This leads to the optimization problem

$$\min_{Q \geq 0} \quad u^T Q \hat{r},$$

subject to

$$u^T Q \hat{I} = 1, \quad (\beta)$$
$$\bar{I}^T Q \hat{I} = 1, \quad (\gamma)$$
$$\Delta + \sum_n \log Q_{w_n,r_n} \geq \sum_n \log Q_{w_n,r_n}^{mle}, \quad (\kappa)$$

where $\Delta$ should asymptotically be $\frac{1}{2} \chi^{2,1-\alpha}_{(1)}$. Considerations from the proof in [Owe01] suggest that setting $\Delta$ to be half the $1 - \alpha$ quantile $F_{1, N-1}^{1-\alpha}$ of the corresponding $F$-distribution leads to better coverage for small samples. Theorem 2 characterizes the solution.

**Theorem 2.** The solution to equation (5) satisfies for every observed $(w, r)$ pair

$$\hat{Q}_{w,r} = \kappa^* \sum_n \frac{1_{w=w_n, r=r_n}}{\gamma^* + \beta^* w_n + w_n r_n},$$

where $(\kappa^*, \beta^*, \gamma^*)$ is the solution to the dual problem

$$\sup_{\kappa \geq 0, \beta, \gamma} \quad \sum_n \left( -\kappa \log \kappa + \kappa \left( -\frac{\Delta}{N} + 1 + \log \frac{\gamma + \beta w_n + w_n r_n}{\beta^{mle}(w_n - 1 + N) - \beta} \right) \right) \quad (7)$$

subject to

$$\forall w, r : \gamma + \beta w + w r \geq 0,$$

where $\beta^{mle}$ is the optimal dual variable for the MLE. Moreover the solution can put mass on the unobserved extreme values $(w_{\text{min}}, r_{\text{min}})$ and $(w_{\text{max}}, r_{\text{min}})$. This mass can be computed by the solution of the linear feasibility program

$$w_{\text{min}} \hat{q}_{\text{min}} + w_{\text{max}} \hat{q}_{\text{max}} = 1 - \kappa^* \sum_n \frac{w_n}{\gamma^* + \beta^* w_n + w_n r_n},$$

$$\hat{q}_{\text{min}} + \hat{q}_{\text{max}} = 1 - \kappa^* \sum_n \frac{1}{\gamma^* + \beta^* w_n + w_n r_n},$$

$$\hat{q}_{\text{min}} \geq 0, \hat{q}_{\text{max}} \geq 0,$$

where $\hat{q}_{\text{min}}$ and $\hat{q}_{\text{max}}$ correspond to $(w_{\text{min}}, r_{\text{min}})$ and $(w_{\text{max}}, r_{\text{min}})$ respectively.

**Proof.** See appendix A.3

Given the optimal dual variables from Theorem 2 the lower bound is

$$V^{(lb)}(\pi) = r_{\text{min}} + \kappa^* \sum_n \frac{w_n (r_n - r_{\text{min}})}{\gamma^* + \beta^* w_n + w_n r_n}.$$  

Alternatively, the lower bound is equal to the value of the dual objective at the optimum. For the upper bound, an analogous result to Theorem 2 is obtained by negating $r$ everywhere and placing additional mass at $r_{\text{max}}$ instead of $r_{\text{min}}$.

### 2.3 Learning From Logged Bandit Feedback

In this setting the goal is to learn a policy $\pi$ based upon a dataset $\{(x_n, a_n, h(a_n|x_n), r(a_n))\}_{n \in \mathbb{N}}$ generated from a fixed historical policy $h$, i.e., without interactive experimental control over the system generating the data. One strategy is to leverage a counterfactual estimator to reduce policy learning to optimization [LCKG15], suggesting the use of the MLE estimator in the objective. We can go one step further and use the lower bound of the MLE confidence interval as the objective that policy $\pi$ should maximize. This is similar to recent work which employs regularized learning, e.g., based upon empirical Bernstein bounds [SJ15a] or divergence-based trust regions grounded in lower bounds from conservative policy iteration [SLA+15, KL02].
Figure 2: Mean squared error of MLE and other estimators on synthetic data. Asymptotics are similar while MLE dominates in the small sample regime. Line width is 4 times the standard error of the population mean.

| MLE vs. Exploration | Wins | Ties | Losses |
|---------------------|------|------|--------|
| IPS                 | 10:2 | 0:2  | 0:1    |
| SNIPS               | 10:2 | 0:2  | 0:1    |

Table 1: Off-policy evaluation results where $\epsilon = 0.25$ is $\epsilon$-greedy exploration, bags=10 is bootstrap exploration with 10 replicas, and cover=10 is online cover [AHK14] with 10 policies.

For optimizing the lower bound, it’s useful to partially optimize over $\kappa$ analytically yielding the simpler dual

$$\sup_{\beta,\gamma} N \exp \left( -\frac{\Delta}{N} + \frac{1}{N} \sum_n \log \frac{\gamma + \beta w_n + w_n r_n}{\beta^{(\text{mle})}(w_n - 1) + N} \right) - \gamma - \beta, \quad (8)$$

subject to

$$\forall w, r : \gamma + \beta w + w r \geq 0,$$

$$\kappa^* = \exp \left( -\frac{\Delta}{N} + \frac{1}{N} \sum_n \log \frac{\gamma + \beta w_n + w_n r_n}{\beta^{(\text{mle})}(w_n - 1) + N} \right).$$

This is a two dimensional convex problem and can be solved to accuracy $\epsilon$ in $O(N \log \frac{N}{\epsilon})$ via the ellipsoid method. In practice we use Newton’s method.

Suppose $\pi$ is parameterized by $\theta$. For each $\theta$, $\pi$ induces a set of importance weights $w_n(\theta)$ and solving (8) gives optimal values $(\kappa^*(\theta), \beta^*(\theta), \gamma^*(\theta))$. Reward lower bound maximization becomes:

$$\sup_{\theta} r_{\min} + \kappa^*(\theta) \sum_n \frac{w_n(\theta)(r_n - r_{\min})}{\beta^*(\theta) w_n(\theta) + w_n(\theta)r_n}, \quad (9)$$

subject to

$$w_n(\theta) = \frac{\pi(a_n|x_n; \theta)}{h(a_n|x_n)}.$$

We can view lower bound optimization as a game between two players: one controlling the distribution $Q$ via the dual variables $(\kappa, \beta, \gamma)$ and one controlling the policy $\pi$. Theorem 2 shows how to implement the first player given any policy $\pi$. The second player can be implemented by a learning algorithm that searches for the policy with the best reward under $Q$. Optimizing the MLE policy value estimate is analogous but leveraging the dual equation (3).

3 Experiments

Replication instructions and scripts are available at [http://github.com/pmineiro/elfcb](http://github.com/pmineiro/elfcb).

Off Policy Evaluation, Synthetic Data We begin with a synthetic example to build intuition. First, an environment is sampled. For all environments, the historical logging policy is $\epsilon$-greedy with possible importance weights $(0, 2, 1000)$. We choose $\pi$ to induce the maximum entropy distribution over importance weights consistent with $E[w^2] = 100$. Rewards are binary with the conditional distribution of reward varying per environment draw such that the value of $\pi$ is uniformly distributed on $[0, 1]$. Once an environment is drawn a set of examples is sampled from that environment, and the squared error of the value estimate is computed.

Figure 2 shows the mean squared error (MSE) over 10,000 environment samples for various estimators. The best constant predictor of $1/2$ ("Constant") has a MSE of $1/12$, as expected. ClippedDR is
A doubly robust estimator with the best constant predictor of $1/2$ clipped to the range $[0, 1]$. In other words, we use $\min(1, \max(0, \frac{1}{2} + \sum_n \frac{w_n}{N} (r_n - 1/2)))$ which is strictly superior to vanilla DR for MSE. SNIPS is the self-normalized estimator IPS estimator. For MLE, we use a reward of $\frac{1}{2}$ for mass placed outside the realization. When a small number of large importance weight events is expected in a realization, both ClippedDR and SNIPS suffer due to their poor handling of the $E[w] = 1$ constraint. Asymptotically all estimators are similar.

**Off Policy Evaluation, Realistic Data** We employ an experimental protocol inspired by the operations of the Decision Service [ABC+16], an industrial contextual bandit platform. Details are in appendix [B.1]. Succinctly, we use 40 classification datasets from OpenML [VvRBT13]; apply a supervised-to-bandit transform [DLL11]; and limit the datasets to 10,000 examples to study the small sample regime. Each dataset is randomly split $20\% / 60\% / 20\%$ into Initialize/Learn/Evaluate subsets with “Initialize” used to learn a historical policy $h$ which is then applied to other datasets. “Learn” is used to create off-policy data drawn from the historical policy $h$ for learning an updated policy $\pi$ and “Evaluate” is used to evaluate the updated policy $\pi$ with off-policy data drawn from the historical policy $h$. Learning is done via Vowpal Wabbit [LLS07] using various exploration strategies implemented therein, with default parameters and $\pi$ initialized to $h$.

We compare the mean square error of MLE, IPS, and SNIPS using the true value of $\pi$ on the evaluation set (available because the underlying dataset is fully observed and the action distribution of $\pi$ is known). For each dataset we evaluate multiple times, with different actions chosen by the historical policy $h$. Table 1 shows the results of a paired $t$-test with 60 trials per dataset and 95% confidence level: “tie” indicates null result, and “win” or “loss” indicates significantly better or worse. IPS is clearly dominated, The MLE is overall superior. Additional results are presented in Table 4 of appendix [B.1].

**Confidence Intervals, Synthetic Data** We use the same synthetic $\epsilon$-greedy data as described above. Figure 1 shows the mean width and empirical coverage over 10,000 environment samples for various confidence intervals at 95% nominal coverage. Binomial CI is the Clopper Pearson confidence interval on the random variable $\frac{w}{w_{\max}} R$. This is an is excessively wide confidence interval. Asymptotic Gaussian is the standard z-score confidence interval around the empirical mean and standard deviation motivated by the central limit theorem. Intervals are narrow but typically violate nominal coverage. The MLE interval is narrow and obeys nominal coverage throughout the entire range despite only having asymptotic guarantees.

Once again there is a qualitative change when the sample size is comparable to the largest importance weight. The Binomial CI interval only begins to make progress at this point. Meanwhile, the asymptotic Gaussian interval widens as large importance weight events increase empirical variance. Appendix [B.2] contains two additional figures. The first demonstrates empirically that the MLE CI width does not depend upon the cardinality of the support. This is an unintuitive but well-known result from the empirical likelihood literature. The second demonstrates that the MLE CI width increases or decreases as the variance of the importance-weighted random variable increases or decreases, unlike the Binomial CI width which essentially assumes worst-case variance.

**Confidence Intervals, Realistic Data** We use the same datasets mentioned above, but produce a 95% confidence interval for off-policy evaluation rather than the maximum likelihood estimate. With 40 datasets and 60 evaluations per dataset we have 2400 confidence intervals from which we compute the coverage and the ratio of the width of the interval to the MLE. As expected from simulation, the Binomial Confidence Interval (Binom) overcovers and has wider intervals. MLE widths are comparable to asymptotic Gaussian (AG) on this data, but AG undercovers. A 95% binomial confidence interval on the coverage of AG is [90.0%, 92.3%], indicating sufficient data to conclude undercoverage.

**Learning From Logged Bandit Feedback** We use the same 40 datasets as above, but with a $20\%/20\%/60\%$ Initialize/Learn/Evaluate split. For optimizing the policy parameters and the distrib-
## Exploration CI LB MLE

| Exploration          | Wins | Ties | Losses | Wins | Ties | Losses |
|----------------------|------|------|--------|------|------|--------|
| $\epsilon = 0.05$  greedy | 16   | 18   | 6      | 11   | 26   | 3      |
| $\epsilon = 0.1$  greedy     | 16   | 19   | 5      | 13   | 24   | 3      |
| $\epsilon = 0.25$ greedy     | 15   | 22   | 3      | 3    | 34   | 3      |
| bagging, 10 bags     | 21   | 18   | 1      | 11   | 28   | 1      |
| bagging, 32 bags     | 4    | 26   | 10     | 7    | 31   | 2      |
| cover, 10 policies   | 18   | 21   | 1      | 6    | 30   | 4      |
| cover, 32 policies   | 9    | 29   | 2      | 6    | 34   | 0      |

Table 3: Learning From Logged Bandit Feedback. “CI LB” uses the lower bound dual problem of equation (8), while “MLE” uses the estimation dual problem of equation (3).

We compare the true value of $\pi$ on the evaluation set resulting from learning with the different objectives. For each dataset we learn multiple times, with different actions chosen by the historical policy $h$. Table 3 shows the results of a paired t-test with 60 trials per dataset and 95% confidence level: “tie” indicates null result, and “win” or “loss” indicates significantly better or worse evaluation value for the CI lower bound. Using the CI lower bound overall yields superior results. Using the MLE estimate also provides some lift but is less effective than using the CI lower bound.

## 4 Conclusions and Future Work

Empirical likelihood techniques are particularly useful for contextual bandits since they effectively incorporate the normalization constraint for both historical and policy distributions, removing a source of slack which is particularly relevant in the regime where the number of samples is smaller than the largest importance weight of unbiased estimation. This slack removal sharpens estimators and learning algorithms while greatly improving the quality of confidence intervals.

Several avenues of future work are possible including more challenging counterfactual scenarios, e.g., combinatorial contextual bandits [SKA+17] and off-policy actor-critic methods [ESM+18]. Another open question is whether empirical likelihood can be used to create effective contextual bandit exploration strategies. For learning from logged bandit feedback, the simple alternating optimization strategy employed here can presumably be improved upon. Finally, we have assumed a fixed historical policy $h$ in our derivation, corresponding to the iid assumption in empirical likelihood. In practice the historical policy can vary over time. All the quantities presented herein can be computed using logged propensities, but it is unclear what theoretical guarantees are available in this case.

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A Off-Policy Evaluation

A.1 Proof of Theorem 1

Theorem 1. The solution to equation (1) satisfies for every observed (w, r) pair

\[ \hat{Q}_{w,r} = \sum_n \frac{1_{w=w_n, r=r_n}}{\beta^*(w_n - 1) + N}, \]  

(2)

where \( \beta^* \) is the solution to the dual problem

\[ \sup_{\beta} \sum_n \log (\beta(w_n - 1) + N) \quad \text{subject to} \quad \forall w : \beta(w - 1) + N \geq 0. \]  

(3)

Moreover, if \( w_{\min} \) or \( w_{\max} \) are not observed the solution to (1) puts mass on these according to the solution of the non-negative linear feasibility program

\[ w_{\min} \hat{q}_{\min} + w_{\max} \hat{q}_{\max} = 1 - \sum_n \frac{w_n}{\beta^*(w_n - 1) + N}, \quad \hat{q}_{\min} + \hat{q}_{\max} = 1 - \sum_n \frac{1}{\beta^*(w_n - 1) + N}, \]

where \( \hat{q}_{\min} \geq 0 \) and \( \hat{q}_{\max} \geq 0 \) are associated with \( w_{\min} \) and \( w_{\max} \) respectively. This additional mass can be distributed arbitrarily over \( r \in [r_{\min}, r_{\max}] \), implying the value estimate is an interval.

Mass on the Realization

Starting from equation (1) we construct the Lagrangian:

\[ L(Q, \beta, \gamma) = \beta(\bar{a}^T \bar{Q} \bar{I} - 1) + \gamma(\bar{I} \bar{Q} \bar{I} - 1) + \sum_n -\log(Q_{w_n, r_n}). \]

The Lagrange dual function is

\[ g(\beta, \gamma) = \inf_{Q \geq 0} L(Q, \beta, \gamma) = \inf_{Q \geq 0} \beta(\bar{a}^T \bar{Q} \bar{I} - 1) + \gamma(\bar{I}^T \bar{Q} \bar{I} - 1) + \sum_n -\log(Q_{w_n, r_n}) \]

\[ = -\beta - \gamma + \inf_{Q \geq 0} \sum_{(w,r)} (\beta w + \gamma)Q_{w,r} - \sum_n 1_{w=w_n, r=r_n} \log(Q_{w,r}) \]

This is a separable optimization and each term can be optimized separately. Observe that for \( c \geq 0 \) and \( y \geq 0 \)

\[ \inf_{q \geq 0} yq - c \log(q) = c - c \log(c) + c \log(y), \]  

(10)

with the infimum attained at \( q^* = c/y \) (and unbounded if \( y < 0 \)). This, together with later simplifications establishes the form of \( Q \). Using \( c_{w,r} = \sum_n 1_{w=w_n, r=r_n} \) and (10) leads to

\[ g(\beta, \gamma) = \begin{cases} 
-\beta - \gamma + N - \sum_{(w,r)} c_{w,r} \log(c_{w,r}) + \sum_n \log(w_n \beta + \gamma) & \text{if } \forall w : \beta w + \gamma \geq 0 \\
-\infty & \text{otherwise}
\end{cases} \]

The dual for equation (1) follows directly from this and strong duality. Ignoring constants yields

\[ \sup_{\beta, \gamma} -\beta - \gamma + \sum_n \log(w_n \beta + \gamma) \quad \text{subject to} \quad \forall w : \beta w + \gamma \geq 0. \]

\( \gamma \) can be eliminated by summing the KKT stationarity conditions. For this we introduce dual variables \( \phi \geq 0 \) corresponding to \( Q \geq 0 \), and leverage complementary slackness and primal feasibility:

\[ \sum_n \frac{1_{w=w_n, r=r_n}}{Q_{w,r}} = \phi_{w,r} + w \beta + \gamma \]  

(KKT stationarity),

\[ \Rightarrow \sum_{w,r} Q_{w,r} \sum_n \frac{1_{w=w_n, r=r_n}}{Q_{w,r}} = \sum_{w,r} Q_{w,r} \phi_{w,r} + \beta \bar{a}^T \bar{Q} \bar{I} + \gamma \bar{I}^T \bar{Q} \bar{I}, \]

\[ N = \sum_{w,r} Q_{w,r} \phi_{w,r} + \beta \bar{a}^T \bar{Q} \bar{I} + \gamma \bar{I}^T \bar{Q} \bar{I} \]

\[ = 0 + \beta \bar{a}^T \bar{Q} \bar{I} + \gamma \bar{I}^T \bar{Q} \bar{I} \]  

(complementary slackness)

\[ = \beta + \gamma. \]  

(primal feasibility)
Substitution results in:

$$\sup_\beta -N + \sum_n \log (w_n \beta + (N - \beta)) \quad \text{subject to} \quad \forall w : \beta w_n + N - \beta \geq 0.$$ 

which ignoring constants gives

$$\sup_\beta \sum_n \log ((w_n - 1)\beta + N) \quad \text{subject to} \quad \forall w : \beta (w_n - 1) + N \geq 0,$$

as per equation (3). Equation (2) follows from the KKT stationarity conditions.

**Additional Mass**  For an unobserved \((w, r)\) pair with \(Q_{w, r} > 0\) we have

\[
\begin{align*}
0 &= \phi_{w, r} + w\beta + \gamma \quad \text{(KKT stationarity)} \\
 &= w\beta + \gamma \quad \text{(complementary slackness)} \\
 &= (w - 1)\beta + N \quad \text{(dual variable relationship)}
\end{align*}
\]

which due to the inequality constraints can only occur for a single value of \(w\), either the smallest value \(w_{\min}\) if \(\beta > 0\) or the largest value \(w_{\max}\) if \(\beta < 0\); unless \(\beta = 0\) in which case \(1^T Q1 = 1\) and there is no missing mass.

If \((w, r)\) is observed than

\[
\begin{align*}
0 &= \phi_{w, r} + w\beta + \gamma \quad \text{(KKT stationarity)} \\
 &= \phi_{w, r} + (w - 1)\beta + N \quad \text{(dual variable relationship)}
\end{align*}
\]

therefore additional mass can only be assigned to an unobserved importance weight. The distribution over \(r\) for this \(w\) is not determined, resulting in an interval corresponding to extreme values of \(r\). □

**A.2 Primal Recovery**

Given the dual optimum \(\beta^*\) of equation (3) we can determine the mass assigned to unobserved \(w\) via primal feasibility. Introducing \(q_{\min}\) and \(q_{\max}\) to represent the mass at \(w_{\min}\) and \(w_{\max}\) respectively, we have

\[
\max_{q_{\min} \geq 0, q_{\max} \geq 0} 1 \quad \text{(11)}
\]

subject to

\[
\begin{align*}
w_{\min} q_{\min} + w_{\max} q_{\max} &= 1 - \sum_n \frac{w_n}{\beta^*(w_n - 1) + N}, \\
q_{\min} + q_{\max} &= 1 - \sum_n \frac{1}{\beta^*(w_n - 1) + N}.
\end{align*}
\]

Because the dual optimum is determined to finite precision, in practice (11) can be infeasible. Therefore we actually solve the non-negative least squares problem

\[
\min_{q_{\min} \geq 0, q_{\max} \geq 0} \left\| \begin{pmatrix} 1 & 1 \\ w_{\min} & w_{\max} \end{pmatrix} \begin{pmatrix} q_{\min} \\ q_{\max} \end{pmatrix} - \begin{pmatrix} 1 \\ 1 - \sum_n \frac{1}{\beta^*(w_n - 1) + N} \end{pmatrix} \right\|^2,
\]

which is equivalent when (11) is feasible but otherwise is more robust.

**A.3 Proof of Theorem 2**

**Theorem 2.** The solution to equation (5) satisfies for every observed \((w, r)\) pair

\[
\hat{Q}_{w, r} = \kappa^* \sum_n \frac{\mathbb{1}_{w_n = w, r_n = r_n}}{\gamma^* + \beta^* w_n + w_n r_n}, \quad \text{(6)}
\]
where \((\kappa^*, \beta^*, \gamma^*)\) is the solution to the dual problem

\[
\sup_{\kappa \geq 0, \beta, \gamma} \sum_n \left( -\kappa \log \kappa + \kappa \left( -\frac{\Delta}{N} + 1 + \log \frac{\gamma + \beta w_n + w_n r_n}{\beta \text{mle}(w_n - 1) + N} - \frac{\gamma - \beta}{N} \right) \right)
\]

subject to \(\forall w, r : \gamma + \beta w + wr \geq 0\),

where \(\beta \text{mle}\) is the optimal dual variable for the MLE. Moreover the solution can put mass on the unobserved extreme values \((w_{\text{min}}, r_{\text{min}})\) and \((w_{\text{max}}, r_{\text{min}})\). This mass can be computed by the solution of the linear feasibility program

\[
\begin{align*}
w_{\text{min}} q_{\text{min}} + w_{\text{max}} q_{\text{max}} &= 1 - \kappa^* \sum_n w_n \\
q_{\text{min}} + q_{\text{max}} &= 1 - \kappa^* \sum_n \frac{1}{\gamma + \beta w_n + w_n r_n} \\
q_{\text{min}} \geq 0, q_{\text{max}} \geq 0,
\end{align*}
\]

where \(q_{\text{min}}\) and \(q_{\text{max}}\) correspond to \((w_{\text{min}}, r_{\text{min}})\) and \((w_{\text{max}}, r_{\text{min}})\) respectively.

**Mass on the Realization** The Lagrangian for equation (5) is

\[
L(\beta, \gamma, \kappa, Q) = w^T Q r + \kappa \left( -\Delta - \sum_n \log Q_{w_n, r_n} + \sum_n \log Q_{w_n, r_n}^{\text{mle}} \right) + \gamma \left( \bar{1}^T Q \bar{1} - 1 \right) + \beta \left( \bar{w}^T Q \bar{1} - 1 \right)
\]

\[
= \kappa \left( -\Delta + \sum_n \log Q_{w_n, r_n}^{\text{mle}} \right) - \gamma - \beta
\]

\[
+ \sum_{w, r} \left( -\kappa \left( \sum_n 1_{w=w_n, r=r_n} \right) \log (Q_{w, r}) + (\gamma + \beta w + wr) Q_{w, r} \right),
\]

implying dual boundedness (primal feasibility) requires \(\forall w, r : \gamma + \beta w + wr \geq 0\). Setting the derivative w.r.t. \(Q_{w, r}\) to 0 gives us

\[
Q_{w, r} = \frac{\kappa \sum_n 1_{w=w_n, r=r_n}}{\gamma + \beta w + wr}
\]

Substituting back in, we get:

\[
\inf_{Q \geq 0} L(\beta, \gamma, \kappa, Q) = L(\beta, \gamma, \kappa, Q) \bigg|_{Q_{w, r} = \frac{\kappa \sum_n 1_{w=w_n, r=r_n}}{\gamma + \beta w + wr}}
\]

\[
= \kappa \left( -\Delta + N - N \log \kappa \right) - \gamma - \beta + \kappa \sum_n \left( \log Q_{w_n, r_n}^{\text{mle}} - \log \frac{\sum_n 1_{w=w_n, r=r_n}}{\gamma + \beta w_n + w_n r_n} \right)
\]

\[
= N \left( -\kappa \log \kappa - \frac{\gamma + \beta}{N} \right) + \kappa \left( -\frac{\Delta}{N} + 1 + \frac{1}{N} \sum_n \log \frac{\gamma + \beta w_n + w_n r_n}{\beta \text{mle}(w_n - 1) + N} \right),
\]

resulting in equation (7).

**Additional Mass** If the realization is empty, a solution with the smallest possible lower bound can be constructed by placing mass solely on the 2 extreme values of \((w_{\text{min}}, r_{\text{min}})\) and \((w_{\text{max}}, r_{\text{min}})\). Therefore assume the realization is not empty.

Introducing dual variables \(\phi \geq 0\) corresponding to \(Q \geq 0\), for an unobserved \((w, r)\) pair with \(Q_{w, r} > 0\) we have

\[
0 = \gamma + \beta w + wr + \phi_{w, r} \quad \text{(KKT stationarity)}
\]

\[
= \gamma + \beta w + wr. \quad \text{(complementary slackness)}
\]

This condition can only exist at extreme points because \(\gamma + \beta w + wr\) is linear in \(w\) and \(r\) and \(\gamma + \beta w + wr \geq 0\) implies that the only points with equality can be on the boundary of the allowed set of \(w\) and \(r\). When \(w > 0\), only \(r_{\text{min}}\) is eligible, whereas for \(w = 0\) all values of \(r\) are equivalent for the objective; there only considering \(r_{\text{min}}\) is sufficient.
MLE vs. Exploration Wins Ties Losses

| MLE vs. Exploration | Wins | Ties | Losses |
|---------------------|------|------|--------|
| IPS                 |      |      |        |
| $\epsilon = 0.05$-greedy | 26   | 11   | 3      |
| $\epsilon = 0.1$-greedy | 27   | 10   | 3      |
| $\epsilon = 0.25$ | 28   | 9    | 3      |
| bags=10            | 13   | 19   | 8      |
| bags=32            | 22   | 10   | 8      |
| cover=10           | 16   | 16   | 9      |
| cover=32           | 11   | 13   | 16     |

| SNIPS               |      |      |        |
| $\epsilon = 0.05$-greedy | 5    | 34   | 1      |
| $\epsilon = 0.1$-greedy | 7    | 33   | 0      |
| $\epsilon = 0.25$ | 2    | 37   | 1      |
| bags=10            | 7    | 30   | 3      |
| bags=32            | 4    | 34   | 2      |
| cover=10           | 7    | 33   | 0      |
| cover=32           | 6    | 29   | 5      |

Table 4: Additional off-policy evaluation results.

**Primal Recovery.** Given the dual optimum $(\beta^*, \gamma^*, \kappa^*)$ of equation (7) we can determine the mass assigned to unobserved $(w, r)$ via primal feasibility by solving a linear program. Again, due to finite precision, non-negative least squares is recommended in practice.

**B Experiments.**

Our experimental design is inspired by the operational cycle of the Decision Service [ABC+16], in which an initial policy is deployed to a production endpoint which makes (randomized) decisions and collects rewards; the resulting data is used to produce a new policy initialized at the previous policy and trained via learning from logged bandit feedback; and then the new policy is optionally deployed if off-policy evaluation on additional collected data compares favorably with the initial policy. Consequently, each dataset is split into Initialize, Learn, and Evaluate sets. The Initialize set is used to produce a plausible initial policy; we use on-policy learning to achieve this. The Learn set corresponds to the off-policy step used to produce a new policy. The Evaluate set corresponds to the gated deployment step.

Replication instructions and scripts are available at [http://github.com/pmiveiro/elfcb](http://github.com/pmiveiro/elfcb).

**B.1 Off-Policy Evaluation, Realistic Data.**

We use the following 40 datasets from OpenML [VvRBT13] identified by their OpenML dataset id: 1216, 1217, 1218, 1233, 1235, 1236, 1237, 1238, 1241, 1242, 1412, 1413, 1441, 1442, 1443, 1444, 1449, 1451, 1453, 1454, 1455, 1457, 1459, 1460, 1464, 1467, 1470, 1471, 1472, 1473, 1475, 1481, 1482, 1483, 1486, 1487, 1488, 1489, 1496, 1498. For each dataset we convert to Vowpal Wabbit format, shuffle the dataset, and utilize up to the first 10,000 examples as data. We utilize a 20%/60%/20% Initialize/Learn/Evaluate split sequentially by line number. Note the shuffle and split is done only once per dataset. We create a historical policy $h$ using on-policy learning on the Initialize dataset, and then learn a new policy $\pi$ on the Learn dataset using off-policy learning with data drawn from $h$. These Initialize and Learn steps are done only once per dataset. Only the off-policy evaluation step is done multiple times per dataset, and the random variations are due to the different actions selected by $h$ over the Evaluate set. For each evaluation, we compute the squared error of the different predictors, i.e., the squared difference between the off-policy value estimate and the true value of $\pi$. Note the true value of $\pi$ can be computed (and is independent of the choices of $h$ on the evaluation set) because the underlying datasets are fully observed. Using the squared error as the random variable, we apply a paired $t$-test between MLE and the other predictors to determine win, loss, or tie for each dataset. We use default settings for Vowpal Wabbit except for the choice of exploration strategy.
B.2 Confidence Intervals, Synthetic Data

Figure 3 demonstrates additional interesting properties of the MLE CI.

First, by holding the number of examples fixed but drawing examples from the maximum entropy distribution satisfying different $E[w^2]$, we can change the statistical difficulty of the problem. Larger $E[w^2]$ implies (slightly) more frequent use of the largest importance weight and (more pronounced) less frequent use of the smallest non-zero importance weight. Essentially the policy whose value is being estimated is “more off-policy” when $E[w^2]$ increases, and the MLE CI width is larger.

Second, by adding small magnitude noise to a dataset we can create a family of datasets that are nearly equivalent in all moments but have any desired cardinality. Under these conditions the MLE CI width does not degrade, indicating no fundamental dependence upon the cardinality of the support.

B.3 Learning from Logged Bandit Feedback

We first utilize the same 40 datasets as above, but with a 20%/20%/60% Initialize/Learn/Evaluate split. The Initialize step is done once per dataset, then the Learn and Evaluate steps are done multiple times per dataset. Note the Evaluate step here is using the true value of $\pi$, i.e., is deterministic and independent of $h$ given $\pi$. Using the evaluation score as the random variable, we apply a paired $t$-test between MLE and the other predictors to determine win, loss, or tie for each dataset. We use Vowpal Wabbit in IPS learning mode with default settings, and do 4 passes over the data. At the beginning of each pass, we optimize the dual variables holding the policy fixed, then use the resulting dual variables during the learning pass to compute importance weights.