Logarithmically rising $hh$ and $\gamma h$ cross-sections: some features of model versus experimental data.

E. Martynov

Bogoliubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine, 03143, Kiev-143, Metrologicheskaja 14b, Ukraine
e-mail: martynov@bitp.kiev.ua

A Regge model of the Pomeron with intercept equal to one, leading to rising cross-sections is considered. Analysis of the experimental data on hadron and photon induced interactions is performed within the model. It is shown that the available pure hadronic data as well as data on DIS are compatible with a high energy logarithmic behaviour of cross-sections and do not require a Pomeron intercept above one. The very important rôle of the preasymptotic contributions is especially emphasized.

In this talk I would like to discuss the importance of preasymptotic terms when the available data are analyzed and described in the framework of Regge approach. These terms strongly influence the conclusions about properties of the asymptotic term, the Pomeron. I shall explicit in asymptotic term, the Pomeron. I shall explicitly discussing the soft Pomeron model, not violating the Froissart-Martin bound, in which the contribution of secondary Reggeons, in particular $f$ Reggeon, can not be neglected, even at the highest accelerator energies. The model describes the hadron elastic scattering as well as the proton structure function.

The elastic scattering amplitude in Regge approach is given by the sum of Pomeron and secondary Reggeons

$$ A(s, t) = P(s, t) + R(s, t) $$

$$ = P(s, t) + f(s, t) + [\pm 2 \pm 3 \pm 4, ...] $$

where the signs of terms in square brackets depend on the process under consideration.

I concentrate here firstly on two aspects: the exchange degeneracy of various Reggeons and the phenomenological separation between the Pomeron and the $f$ Reggeon.

- **Exchange degeneracy.** The hypothesis is based on the fact that $f, \omega, \rho, a_2$ trajectories seem to coincide on a Chew-Frautschi plot.

  The linear parameterization

  $$ \alpha_{e-d}(m^2) = \alpha_{e-d}(0) + \alpha'_{e-d}m^2 $$

  of the single exchange-degenerate trajectory, fitted to the data, gives

  $$ \alpha_{e-d}(0) = 0.449, \quad \alpha'_{e-d} = 0.901 \text{ GeV}^{-2} $$

  and an extremely high $\chi^2$, namely $\chi^2/dof \approx 118$.

  Fitting in isolation each of the trajectories, we obtain the result shown in Fig. 1. Numerically we have

  $$ \alpha_f(0) = 0.697 \pm 0.041, \quad \alpha'_f = (0.801 \pm 0.002) \text{ GeV}^{-2}, \quad \chi^2/dof = 6.01, $$

  $$ \alpha_\omega(0) = 0.436, \quad \alpha'_\omega = 0.923 \text{ GeV}^{-2}, $$

  $$ \alpha_\rho(0) = 0.478 \pm 0.001, \quad \alpha'_\rho = (0.880 \pm 0.002) \text{ GeV}^{-2}, \quad \chi^2/dof = 3.31, $$

  $$ \alpha_{a_2}(0) = 0.512 \pm 0.041, \quad \alpha'_{a_2} = (0.857 \pm 0.023) \text{ GeV}^{-2}, \quad \chi^2/dof = 0.42. $$

  The bad $\chi^2$ for the $f$ trajectory is due to its evident nonlinearity (see below).

  Thus the first conclusion is the following: the available data on mesons lying on the $f, \omega, \rho$ and $a_2$ Regge trajectories contradict the exchange degeneracy assumption.

  It should be noted that, in accordance with the conclusion derived in [23], the hypothesis of exchange degenerate trajectories is not supported also by the forward scattering data.

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1 Because of the very restricted size of the talk I give only the references to our papers where further references can be found.
Certainly, trajectories must rise slower than first power of $t$ at $t \to \infty$. A more realistic $f$ trajectory chosen for example in the form

$$\alpha_f(t) = \alpha_f(0) + \beta_1(\sqrt{4m^2 - 4m^2 - t}) + \beta_2(\sqrt{12 - 1 - t})$$

leads to $0.77 < \alpha_f(0) < 0.87$.

Thus, we obtain a very interesting and important phenomenological consequence of the non-linearity: a higher intercept of the $f$ trajectory. Coming to the next conclusion:

**the intercept of the $f$ trajectory is $\geq 0.7$, but most probably, the lower bound is larger than this value.**

- **Separation of Pomeron and $f$ Reggeon.**
  Generally, there is an evident correlation between the intercept of the $f$ Reggeon and the model for the Pomeron. This is due to the fact that in all known processes Pomeron and $f$ Reggeon contribute additively. As a rule, a higher $f$ intercept is associated with a slower growth with energy due to the Pomeron contribution. In Fig. 3 we illustrate this observation and show how $\alpha_f(0)$ is correlated with a power of $\ell ns$ in the behaviour of the total cross-section, if the forward $pp(\bar{p}p)$ scattering amplitudes are parametrized in the form

$$A^{(hh)}(s, 0) = i \left[ C_1 + C_2 \ell n^\gamma(-is/s_0) \right]$$

$$+ f(s, 0) \pm \omega(s, 0),$$

with the standard form of secondary Reggeons contribution.

**Fig.3** Intercept of the $f$ trajectory correlated with the power $\gamma$ of $\ell ns$ in $\sigma_{tot}(s)$. Intervals for intercept of some trajectory parameterizations are shown.

Taking into account the restriction $\alpha_f(0) > 0.7$, revealed from spectroscopy data, one can conclude that $\gamma < 2$ or that total cross-section rises slower than $\ell n^2 s$.

**Fig.1** Chew-Frautschi plot for $f$, $\omega$, $\rho$ and $a_2$ Regge trajectories taken separately assuming linearity (the figure below is an enlargement for small masses).

- **$f$ trajectory.** The nonlinearity of the $f$ trajectory is illustrated unambiguously in Fig. 2, where the above mentioned linear fit is shown together with the parabola passing exactly through the three known resonances.

**Fig.2** Real part of the $f$ trajectory versus the squared mass of the resonance, $m^2$. Solid line is the parabola passed through the known resonances. Dashed straight line is the result of a linear fit.

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On the other hand it is known from a comparison of various Pomeron models that the "best" description (or $\chi^2$) is obtained if the Pomeron contribution to the total cross-section has the form $\sigma_P = C_1 + C_2 \ln(s/s_0)$. The difference in $\chi^2$ is quite small (~ a few percents) if $\sqrt{s} \geq \sqrt{s_{min}} = 9 \text{ GeV}$. An additional advantage of the logarithmic model is its stability (of the parameters as well as the $\chi^2$) when $\sqrt{s_{min}}$ is decreasing down to 5 GeV.

It is interesting to note two points concerning the logarithmic model:

1. Intercept of the $f$ Reggeon takes approximately its maximal value of Fig. 3, $\alpha_f(0) \approx 0.8$.

2. If the well known Supercritical Pomeron $P(s,0) = iC(-i\frac{s}{s_0})^\Delta$ is generalized for two components $i[C_1 + C_2(-i\frac{s}{s_0})^\Delta]$ (provided the Regge trajectories are non exchange degenerate) then a fit to the total cross-sections leads to a very small value of $\Delta(\approx 0.001)$. Hence, one can approximate $\tilde{C}_1 + \tilde{C}_2(-i\frac{s}{s_0})^\Delta \approx C_1 + C_2 \ln(-i\frac{s}{s_0})$,

$$C_1 = \tilde{C}_1 + \tilde{C}_2, \quad C_2 = \Delta \tilde{C}_2$$

with all parameters ($C_1, C_2$ as well as couplings and intercepts of the secondary Reggeons) coinciding with those obtained in the logarithmic Pomeron model.

This observation is valid not only for $pp$ and $\bar{p}p$ cross-sections but also for all hadronic, $\gamma p$ and $\gamma \gamma$ cross-sections.

It is necessary to emphasize that in this model the preasymptotic $f$ term gives quite a large part of the total cross-section even at the Tevatron energy, $\sigma_f(\sqrt{s} = 1.8\text{ TeV}) \approx 3 \text{ mb}$. It cannot be neglected at the available energies.

The third conclusion follows: the available data on the total cross-sections of hadron and photon induced processes are better described in the model yielding a moderate (logarithmic) rise of the cross-sections.

- **Dipole Pomeron model and elastic scattering.** I present here, without any details and references for an history of the subject, that can be found in [3], only the final results concerning the differential cross-sections of elastic $pp$ and $\bar{p}p$ interactions in the so-called Modified Additive Quark Model (MAQM).

In the MAQM, we have assumed that the Pomeron can be coupled not only with a single quark but also with a pair of quarks, giving rise to a small (~ 10%) but important correction. Also, some counting rules for the Reggeons contribution based on their quark contents have been suggested.

The above mentioned logarithmic Pomeron model can be named as "Dipole Pomeron" (DP) model because in the complex momentum plane ($j$-plane) a double pole at $j = \alpha_P(t)$ with $\alpha_P(0) = 1$ is the dominating contribution in the amplitude and leads asymptotically at $s \to \infty$ to $\sigma_{tot} \propto \ell n(s)$.

Applying the MAQM and DP (adding at $t \neq 0$ an Odderon contribution), we obtain a quite good description (with $\chi^2/\text{dof} \approx 2.38$) of the total cross-sections for $pp$, $\bar{p}p$, $\pi^+p$, $\gamma p$ and $\gamma \gamma$ and of the differential cross-sections of $pp$ and $\bar{p}p$ elastic scattering in a wide kinematical region ($\sqrt{s} \geq 5 \text{ GeV}$ for $t = 0$ [5] and $\sqrt{s} \geq 19 \text{ GeV}$ for $0 < |t| \leq 14 \text{ GeV}^2$).

Thus:

not only the forward scattering data but also the elastic scattering at small and large $|t|$ can be described with a high quality in the Dipole Pomeron model with the intercept $\alpha_P(0)$ equal to one.

- **Dipole Pomeron model and deep inelastic scattering.** Another example demonstrating the importance of preasymptotic terms, when the properties of Pomeron are derived from the experimental data, is the Dipole Pomeron model for the forward $\gamma^*p$ amplitude.

Defining the Dipole Pomeron model for DIS, we start from the expression connecting the transverse cross-section of $\gamma^*p$ interaction to the proton structure function $F_2$ and the optical theorem for forward scattering amplitude

$$\sigma_{\gamma^*p}(W, Q^2) = 3mA(W^2, Q^2; t = 0)$$

$$= \frac{4\pi^2\alpha}{Q^2(1-x)}(1 + 4m_p^2x^2/Q^2)F_2(x, Q^2),$$

where $\sigma_{\gamma^*p} = 0$ is assumed. The forward scattering at $W^2 = Q^2(1/x - 1) + m_p^2$ being far from the threshold $W_{ih} = m_p$ is dominated by the
Pomeron and the $f$ Reggeon\footnote{\textsuperscript{2}We ignored an $a_2$ Reggeon considering the $f$ term as an "effective" one at $W \geq 3$ GeV.}\footnote{\textsuperscript{3}An attempt to extract such a local slope as a function of $x$ and $Q^2$ is given in \cite{8}.}

\begin{equation}
A(W^2, Q^2; t = 0) = P + f, \quad P = P_1 + P_2,
\end{equation}

\begin{equation}
P_1 = iG_1(Q^2) \ln \left(-\frac{iW^2}{m^2_p}\right)(1 - x)^{B_1},
\end{equation}

\begin{equation}
P_2 = iG_2(Q^2)(1 - x)^{B_2},
\end{equation}

\begin{equation}
f = iG_f(Q^2) \left(-\frac{iW^2}{m^2_p}\right)^{\alpha_f(0) - 1}(1 - x)^{B_f}.
\end{equation}

It evidently follows from the experimental data that $Q^2 \sigma^{p\gamma} p(W,Q^2)$ decreases with $Q^2$, at least at high $Q^2$. We choose

\begin{equation}
G_i(Q^2) = \frac{g_i}{(1 + Q^2/Q_i^2)^{D_i}},
\end{equation}

expecting $D_i > 1$ at high $Q^2$. As it follows from the fit, $D_i$ and $B_i$ should be functions of $Q^2$. The details of the parameterization of the real functions $D_i(Q^2), B_i(Q^2)$ are given in \cite{3}.

A fit to the 1389 experimental points in the region $W \geq 3$ GeV$^2$, $0 \leq x \leq 0.85$, $Q^2 \geq 0$ was performed and a quite good description of data was obtained : $\chi^2/\text{dof} = 1.07$.

Again, as for the pure hadron case, the preasymptotic contributions in $\sigma^{p\gamma} p$, the constant (negative) component of the Pomeron term as well as the $f$ term are very important in the whole considered kinematical region.

To illustrate this statement, let us consider the "experimental" data on the "effective" Pomeron intercept and its dependence on $Q^2$. This quantity, $\alpha_{eff} = 1 + \Delta_{eff}$ (or $1 + \lambda_{eff}$) is extracted from the data on $F_2$ in accordance with the parametrization $F_2(x,Q^2) = C(1/x)^{\lambda_{eff}}$. Strictly speaking a more correct definition of an effective intercept (or $x$-slope of the structure function) is as follows

\begin{equation}
F_2(x, Q^2) = G(Q^2)(1/x)^{\Delta_{eff}(x,Q^2)},
\end{equation}

\begin{equation}
\Delta_{eff}(x,Q^2) = \partial F_2/\partial \ln(1/x).
\end{equation}

$\Delta_{eff}(x,Q^2)$ coincides with $\lambda_{eff}$ only if it does not depend on $x$. For an accurate comparison of a model with experiment we need the data on the local $x$-slope\footnote{To illustrate this statement, let us consider the "experimental" data on the "effective" Pomeron intercept and its dependence on $Q^2$. This quantity, $\alpha_{eff} = 1 + \Delta_{eff}$ (or $1 + \lambda_{eff}$) is extracted from the data on $F_2$ in accordance with the parametrization $F_2(x,Q^2) = C(1/x)^{\lambda_{eff}}$. Strictly speaking a more correct definition of an effective intercept (or $x$-slope of the structure function) is as follows}

\begin{equation}
\Delta_{eff}(x,Q^2)
\end{equation}

intervals of $x$. The investigated Dipole Pomeron has an intercept exactly equal to one. Nevertheless, due to interferences Pomeron-$f$ Reggeon, the model is able to describe well the data on averaged $x$-slope as it is shown in Fig. 4.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{The effective intercept calculated in the Dipole Pomeron model \cite{3} (triangles and solid line) compared with the experimental data.}
\end{figure}

We have shown that a careful account of the preasymptotic contributions allows to describe the available data within the model that does not violate the asymptotic bounds on the cross-sections.

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