Simulation of Lamb wave interactions with defects in a thin plate

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Abstract. The understanding of Lamb wave propagation and its interaction with defects are crucial part of SHM technology. Being of high relevance due to the costs of experimental equipment, the dynamic finite element analysis is performed in this study. FEA analysis was implemented to simulate propagation of Lamb waves in healthy and defective aluminium plate. The wavefield visualisation shows a high degree of difference between these two plates, which shows the interaction with defects. This study is useful for a good understanding of the Lamb wave interaction with defect before applying this method for a real application.

1. Introduction

Structural health monitoring (SHM) has gained more attention among researcher due to its ability to allow a continuous supervision of the structural integrity of any operational systems. Among the various developed techniques, Lamb wave has shown capability to be utilised as one of the SHM approaches. The principle advantage of this method is performing the inspection over long distances from a single probe position [1]. They are very sensitive to any changes of the structural properties even at a tiny scale or underneath structural surfaces and able to detect any types and sizes of defects [2, 3]. Moreover, they feature prominent merits like low attenuation, low energy consumption and convenience in actuation or acquisition. These characteristics make Lamb wave as one of the promising diagnostic tool. However, there are several challenges when utilising Lamb wave method in the SHM system. Multimodal and dispersive nature makes the Lamb wave interpretation difficult [4]. Multimodal property refers to the existence of at least two modes that coexist at any given frequency, which leads to multiple wave packets in the acquired signal [5]. On top of that, other wave characteristics such as wave dispersion, which is associated with the velocity of each mode which, varies with respect to its frequency have added complexity to the analysis of the signal [6]. This complexity may lead to the wrong diagnosis and false alarm in the inspection. Thus, the understanding of Lamb waves propagation plays an important role in the successful application of Lamb wave for damage detection. This paper presents the visualisation simulation of the Lamb wave responses for an intact and damaged isotropic plate. This visualisation will help the beginner in the Lamb wave based SHM filed to fully understand the propagation of the Lamb wave which contain various wave modes. The modes were clearly appeared and their interaction with the defects was successfully simulated.

Visualisation of the wave propagation generated from the simulation is used to study in details about the effect of defects to the Lamb wave propagation. The changes of the wave contour when interacting with the defects may be used as an indicator for the existence of damage. The extracted features can be used as the defects signature for the SHM diagnostic system.
2. Theory of Lamb wave

Lamb wave is ultrasonic guided wave, which is induced by free surfaces of plate, shell and tube, and propagates along the directions of length and width. They can be classified as symmetric and anti-symmetric modes, which exist simultaneously and propagate independently of each other [7]. The symmetric modes are symmetric with respect to the mid-plane of the plate and are noted $S_i$, where $i$ is the order or number of the mode. The anti-symmetric modes are anti-symmetric with respect to the mid-plane of the plate and are noted $A_i$. Fundamental symmetrical, $S_0$ and anti-symmetrical, $A_0$ mode is the lowest order of the mode and followed by $A_1$, $A_2$, $A_3$, … $A_\infty$, respectively. Symmetric mode having symmetric deformation about the mid-plane and the particles inside the plate primarily have in-plane motions. The displacements in $x$ direction are larger than the displacements in $y$ direction. Particles at the top and at the bottom experiencing the same direction of displacements in the $x$ component and opposite direction in the $y$ component as illustrated in Figure 1(a).

For the $A$ mode, the deformation of the plate is anti-symmetric and the particles inside the plate have out-of-plane motions. The displacements in the $y$ direction are larger than the $x$ direction. The displacements for the $y$ direction are in the same direction and the $x$ component in the opposite directions for particles at the top and bottom [8] as shown in Figure 1 (b).

![Figure 1. Propagation of the (a) symmetric mode and (b) anti-symmetric mode](image)

The number of existing modes of a plate structure depends on the product of excitation frequency and thickness $(f d)$. An excitation frequency must be decided properly before performing any analysis regarding Lamb wave propagation in order to minimize the number of generated modes. For signals simplification, it is a common practice to apply low excitation frequencies, so that only fundamental modes ($S_0$ and $A_0$) exist. The reference for the appropriate excitation frequency can be referred to theoretical dispersion curve and it was generated using equation below [9]

$$\begin{align*}
\frac{\tan qh}{\tan ph} & = \frac{4k^2pq}{(q^2-k^2)^2} = 0 \\
\frac{\tan qh}{\tan ph} & = \left(\frac{q^2-k^2}{4k^2pq}\right) = 0
\end{align*}$$

(1)

(2)

Where the parameters $p$ and $q$ are defined as follows:

$$p^2 = \frac{\omega^2}{c_L^2} - k^2, \quad q^2 = \frac{\omega^2}{c_T^2} - k^2, \quad h = \frac{d}{2} \quad \text{and} \quad k = \frac{\omega}{c_p},$$

where $\omega$ is circular wave frequency, $k$ is wavenumber, $c_p$ is phase velocity, $c_L$ and $c_T$ are the propagation velocities of bulk waves. From equation 1 and 2, the phase velocity, $c_p$ dispersion curves can be plotted. The other useful plot is group velocity dispersion curves which can be derived from the phase velocity curves using equation (3). The $c_p$ is propagating velocity of the wave with a single frequency and $c_s$ is the propagation velocity of the wave packet with adjacent multi frequencies.
\[
\frac{d}{dk} (k c_p) = c_p + k \frac{d c_p}{dk}
\]

(3)

Referring to Equation (1) and (2), different material with different plate thickness will produce different dispersion curves. Figure 2 depicts the dispersion curve for aluminium plate with thickness of 2 mm. This theoretical curve is generated using PACshare Dispersion Curves software. It can be seen that the number of modes increases with the excitation frequency. Thus, infinite number of symmetric and anti-symmetric modes can exist. Referring to Figure 2, excitation frequency more than 800 kHz can generate more than two modes leading to the complexity of the recorded signals. For this study, the excitation frequency of 200 kHz is chosen for the Lamb wave actuation, which is actuated only two modes that is \( S_0 \) and \( A_0 \). The characteristics of these two modes has been identified for a better understanding of their interactions with defects. The existing feature signatures influenced by the defects were extracted for further studies.

![Figure 2](image)

**Figure 2.** Theoretical dispersion curve of group velocity against frequency for aluminium plate with thickness of 2 mm

3. Finite element modelling

A 90 cm x 90 cm x 0.2 cm aluminium plate with stress-free boundary condition was considered in this study. The properties of the aluminium are: density = 2580 kg/m\(^3\), Young’s Modulus = 72.7 GPa and Poisson’s ratio = 0.33. A commercial ABAQUS finite element software package was used to perform the dynamic simulation. A three dimensional FE method using eight-node brick solid elements was used to model the geometry and a square element type was applied for meshing generation. Figure 3 shows the healthy and damaged plate model used in this study. The damage slot was introduced with the size of 0.2 cm x 2 cm as presented in Figure 3 (c).
Figure 3. Plate model for (a) healthy plate, (b) damaged plate and (c) close-up for damage slot

3.1. Defect plate simulation
Dynamic simulation of the healthy plate was performed in order to obtain the baseline signals that will be used as the reference for the comparison analysis for damaged plate. The interaction of the Lamb wave with the defects was extracted by using this baseline signals. Referring to the wavefield visualisation in Figure 4(a), two main wave packets were appeared for the healthy plate. Theoretical dispersion curve in Figure 2 was used to identify the types of mode of that wave packets. At the given excitation frequency (200 kHz), there is only two modes exist, $S_0$ and $A_0$. Based on that projection, it was assumed the first wave packet is the $S_0$ and the second wave packet is the $A_0$. This assumption is due to $S_0$ mode is moving faster than $A_0$ mode based on dispersion curve in Figure 2. Healthy plate shows a smooth contour of the waves propagation as presented in Figure 4 (a).
Figure 4. Visualisation of the wave propagation for (a) intact plate, (b) damaged plate and (c) close-up of the disturbed propagated wave for damaged plate.

A through damaged slot was introduced at distance of 15 cm from the wave exciter, as presented in Figure 3(b) and (c). The same simulation set up was applied for damaged plate simulation. The obtained wavefield visualisation shows a high degree of difference compared with the healthy plate. The $S_0$ mode seems to attenuate when interacting with defect, leaving $A_0$ mode to dominantly propagate in the plate. Thus, only one mode appeared in the visualisation as presented in Figure 4(b). Close-up figure shows that $A_0$ mode contour experiencing disturbances when interacting with defect. Distorted contour was obviously noticeable which is happened before and after the defects. They are labelled as disturbed direct wave and disturbed transmitted wave in Figure 4(c). This sensitivity characteristic of Lamb wave with the changes of structural properties is useful in the SHM application.

4. Conclusions
Lamb wave was attracted by many researchers due to their advantages. However due to multimode and dispersive nature, well understanding of their behaviour is crucial for acquiring accurate results for SHM system. Simulation work was performed in this present work for a better understanding of its interaction with the defects. A smooth propagation of $A_0$ and $S_0$ modes was observed in the healthy plate. On the other hand, only $A_0$ mode was appeared in the damaged plate but in distorted condition compared with healthy plate. $S_0$ mode is assumed to attenuate when interacting with defects. Existence of defects in the structure alters the behaviours of the wave. It can be concluded that Lamb wave is one of the promising tool for the diagnostic analysis due to their sensitivity to changes of the structural properties.

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