Enhanced weathering to capture atmospheric carbon dioxide: Modeling of a trickle-bed reactor

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Abstract
Enhanced weathering (EW) of alkaline minerals can potentially capture CO2 from the atmosphere at gigaton scale, but the reactor design presents great challenges. We model EW with fresh water in a counter-current trickle flow packed bed batch of 1–10 mm calcite particles. Weathering kinetics are integrated with the mass transfer of CO2 incorporating transfer enhancement by chemical reaction. To avoid flooding, flow rates must be reduced as the particles shrink due to EW. The capture rate is mainly limited by slow transfer of CO2 from gas to liquid although slow dissolution of calcite can also play a role in certain circumstances. A bed height of at least 7–8 m is required to provide sufficient residence time. The results highlight the need to improve capture rate and reduce energy and water consumption, possibly through enriching the feed with CO2 and further chemical acceleration of the mass transfer.

KEYWORDS
carbon dioxide removal, enhanced weathering, enhancement factor, mathematic modeling, trickle bed reactor

1 | INTRODUCTION

Although the rate at which greenhouse gases are accumulating in the atmosphere could be reduced by cutting emissions, elevated levels of CO2 will remain for centuries.1 Climate models show that peak CO2-induced global warming is mainly determined by cumulative CO2 emissions, rather than the emission pathway.2 As a consequence, it may well become necessary to remove large amounts of CO2 from the atmosphere—a process known as carbon dioxide removal (CDR). The IPCC has warned that “All pathways that limit global warming to 1.5°C with limited or no overshoot project the use of CDR on the order of 100–1000 Gt CO2 over the 21st century.”3,4

Among the potential routes to CDR, enhanced weathering (EW) based on the exposure of crushed alkaline minerals to the atmosphere is a promising option. In chemical terms, EW is similar to the process of liming fields to reduce their acidity, as practiced by farmers for centuries.5 Currently, the weathering of rock by carbon dioxide and water, a natural process, absorbs about 1.1 Gt CO2 per year from the atmosphere, mainly stored as bicarbonate in the ocean.6 Many types of carbonate and silicate mineral show CO2 sequestration ability.7–11 An advantage of the EW approach to CDR is that the CO2 is captured in the form of carbonate or bicarbonate ions, removing the need for CO2 compression and underground storage. A potential disadvantage is that, in cases where the bicarbonate ion is a product, chemistry occurring in the natural environment can reverse the reaction, releasing CO2 again to the atmosphere. A further challenge is the potential scale of operations and the need to handle many tons of crushed rock, and also very large amounts of water, for every ton of CO2 sequestered. Moreover, it is known that rates of natural weathering are relatively low, and it is not certain that these can be enhanced to an extent sufficient to make EW useful in countering climate change in this century. For the implementation of EW, suggestions include spreading crushed minerals on the soil8,9 or in coastal environments,10 and heap leaching.11 Exploring the potential of a more “engineered”
approach, the motivation for the modeling study presented here is to quantify capture rates and water and energy requirements for an EW process taking place in a chemical reactor. We chose calcite as a model mineral, because its weathering kinetics have been relatively well-studied. By modeling the behavior of a known chemical system in equipment with well-known characteristics we provide a benchmark for process requirements and illustrate the technical issues that might arise when attempting EW at a large scale, in a novel setting.

The dissolution of calcite in the CO₂-water system was first ascribed to three simultaneous reactions occurring at the solid–liquid interface by Plummer et al.¹² They also drew attention to the importance of pH. Buhmann and Dreybrodt¹³ developed a model for the water film sandwiched between a plane calcite surface and atmosphere. They found that reaction at the calcite surface was the rate-limiting process when the fluid was in turbulent flow, but in laminar flow the rate-limiting process strongly depended on the ratio of the volume of the solution and the surface area of calcite. Their later work¹⁴ confirmed that the conversion of CO₂ into HCO₃⁻ was the rate-limiting process when the liquid film on the calcite surface was very thin. The studies of rate-limiting processes provided valuable insights to the mechanisms of weathering of calcite. However, most previous work was limited to batch reactors operated with a CO₂ pre-saturated solution.¹²⁻¹⁴ The mass transport of CO₂ from atmosphere into liquid was rarely integrated with the kinetics of calcite weathering.

The reactor type studied in this work is the packed bed column, which is very widely used in the process industries for carrying out both reactions and separations in multi-phase systems. In this work, we consider the use of packed bed columns for EW, where the reacting mineral particles are contacted with air and water and provide interfacial area for gas–liquid mass transfer. An important feature of the process is that the calcite particles change size during dissolution, which alters the bed hydrodynamics as the reaction proceeds. We also consider the possible advantages of using air enriched with carbon dioxide, and the benefits of increasing the rates of reaction (e.g., by catalysis) to accelerate the mass transfer. The resulting model provides useful guidance on the design and operation of a trickle flow packed bed for CO₂ sequestration through EW.

2 | MODEL DEVELOPMENT

2.1 | Main assumptions

In the mathematical model we integrate the transport of CO₂ from gas to liquid with the kinetics of calcite dissolution and liquid-phase reactions in a trickle flow packed bed which is illustrated in Figure 1. The following are the main assumptions used:

- Solid particles are loaded into the reactor at the start of the operation, with no further replenishment.
- The flow arrangement in the reactor is counter-current. Water enters at the top, air at the bottom.
- A one-dimensional model is developed to describe the spatial variation of concentrations through the height of the cylindrical packed bed. No radial gradients of concentration or velocity are considered.
- The velocities of gas and liquid are assumed not to vary along the bed height.
- An average particle-shrinking rate is applied to all calcite particles, independent of the location inside the packed bed. The particles retain their shape (assumed spherical) during dissolution.
- The ability of the chemical reactions to accelerate the rate of carbon dioxide absorption is taken into account by the use of enhancement factors.

2.2 | Mass conservation and boundary conditions

For CO₂ in the gas phase, diffusive and dispersive mass transport is negligible compared to convection (Peclet number ≫ 1) and is therefore omitted from consideration. An unsteady state mass balance for CO₂(g) over a volume element of the bed yields for the gas phase

$$\frac{\partial}{\partial t} \varepsilon_G \rho_{CO_2(g)} + u_G \nabla \cdot \rho_{CO_2(g)} = -r_{G-L}$$

where $\varepsilon_G$ is gas holdup, defined as the volume fraction of the bed occupied by gas phase and $r_{G-L}$ (mol m⁻³ s⁻¹) is the source term, representing transfer of CO₂ from gas to liquid. $u_G$ (m s⁻¹) is the gas superficial velocity.

An unsteady state mass balance for aqueous species $i$ over a volume element of the bed is given by:

$$\varepsilon_L \frac{\partial}{\partial t} c_i - \varepsilon_L D_{L,i} \nabla^2 c_i + u_L \nabla c_i = r_{i}$$

where $\varepsilon_L$ is liquid holdup, defined as the volume fraction of the bed occupied by liquid phase, $u_L$ (m s⁻¹) is the superficial liquid velocity, $c_i$
(mol m\(^{-3}\)) is the concentration of species \(i\) in the liquid phase, \(D_{ij}\) (m\(^2\) s\(^{-1}\)) is the hydrodynamic dispersion coefficient of species \(i\) in liquid, and \(r_i\) (mol m\(^{-3}\) s\(^{-1}\)) is the source term of species \(i\). The aqueous species include \(CO_2(aq)\) and \(Ca^{2+}\), and we define the total carbon ions (TCI), to be the sum of HCO\(_3^-\) and CO\(_2^2-\):

\[
c_{TCI} = c_{HCO_3^-} + c_{CO_2^2-}
\]

(3)

The boundary condition of Equation (1) is:

\[
r_{TCI} = n\left[k_{2CO_2}C_{HCO_3^-} - k_{2CO_2}C_{CO_2^2-} \cdot CO_2(aq) + k_{61}C_{CO_2(aq)}\right] - k_{2CO_2}C_{CA}^2 - c_{HCO_3^+} + k_{71} + k_{72}C_{Ca^2+} \cdot c_{HCO_3^-} \cdot c_{OH^-}
\]

(15)

where \(a_w\) (m\(^{-1}\)) is the wetted area of solids per unit volume of the bed.

The boundary condition of Equation (2) is the Danckwerts' boundary conditions:

\[
\text{at liquid inlet: } x = 0, c_{CO_2(aq)} = c_{CO_2(aq),\text{inlet}}
\]

(4)

The boundary conditions of Equation (2) are the Danckwerts' boundary conditions:

\[
\text{at liquid inlet: } x = H_{bed}, u_i c_{i,\text{inlet}} = u_i c_i - D_{iJ} \nabla c_i.
\]

\[
\text{at liquid outlet: } x = 0, \nabla c_i
\]

(5)

The subscript \(i\) represents the aqueous species, for example, \(CO_2(aq), Ca^{2+}\) and TCI.

2.3 | Reaction kinetics

The reactions involved in weathering of calcite are as follows:

\[
CO_2(aq) + OH^- \rightleftharpoons k_{11} HCO_3^-
\]

(6)

\[
HCO_3^- + OH^- \rightleftharpoons k_{12} CO_3^{2-} + H_2O
\]

(7)

\[
H^+ + OH^- \rightleftharpoons k_{13} H_2O
\]

(8)

\[
CO_2(aq) + H_2O \rightleftharpoons k_{14} HCO_3^- + H^+
\]

(9)

\[
CaCO_3(s) + H^+ \rightleftharpoons k_{15} Ca^{2+} + HCO_3^-
\]

(10)

\[
CaCO_3(s) + CO_2(aq) + H_2O \rightleftharpoons k_{16} Ca^{2+} + 2HCO_3^-
\]

(11)

\[
CaCO_3(s) + H_2O \rightleftharpoons k_{17} Ca^{2+} + HCO_3^- + OH^-
\]

(12)

The rates of reaction and equilibria in the aqueous phase are modeled, with steps (7) and (8) assumed to be in equilibrium, as follows:

\[
r_{CO_2(aq)} = r_{CO_2} = n\left[k_{2CO_2}C_{HCO_3^-} - k_{2CO_2}C_{CO_2^2-} \cdot CO_2(aq) + k_{61}C_{CO_2(aq)}\right] - k_{2CO_2}C_{CA}^2 - c_{HCO_3^+} + k_{71} + k_{72}C_{Ca^2+} \cdot c_{HCO_3^-} \cdot c_{OH^-}
\]

(13)

\[
c_{CO_2^2-} = K_{2CHCO_3^+} \cdot c_{OH^-}
\]

(14)

\[
r_{TCI} = n\left[k_{2CO_2}C_{HCO_3^-} - k_{2CO_2}C_{CO_2^2-} \cdot CO_2(aq) + k_{61}C_{CO_2(aq)}\right] - k_{2CO_2}C_{CA}^2 - c_{HCO_3^+} + k_{71} + k_{72}C_{Ca^2+} \cdot c_{HCO_3^-} \cdot c_{OH^-}
\]

(15)

where \(a_w\) (m\(^{-1}\)) is the wetted area of solids per unit volume of the bed.

The source term of TCI is:

\[
r_{TCI} = n\left[k_{2CO_2}C_{HCO_3^-} - k_{2CO_2}C_{CO_2^2-} \cdot CO_2(aq) + k_{61}C_{CO_2(aq)}\right] + a_w\left[k_{61}C_{CO_2(aq)}\right] + 2\left[k_{62}C_{Ca^2+} \cdot c_{HCO_3^-} - k_{63}C_{CO_2(aq)}\right] + k_{71} + k_{72}C_{Ca^2+} \cdot c_{HCO_3^-} \cdot c_{OH^-}
\]

(17)

The values and sources of rate and equilibrium constants are listed in Table 1.

The composition of the inlet liquid is calculated assuming saturation with CO\(_2\) at atmospheric conditions, by solving the following algebraic equations:

\[
c_{CO_2(aq),\text{inlet}} - \frac{p_{CO_2,\text{atmos}}}{k_H} = 0
\]

(18)

\[
c_{HCO_3^-} - \frac{K_{4CO_2(aq),\text{inlet}}}{c_{OH^+},\text{inlet}} = 0
\]

(19)

\[
c_{CO_2^2-} - \frac{K_{2CHCO_3^+},\text{inlet}}{c_{OH^+},\text{inlet}} = 0
\]

(20)

\[
c_{OH^+},\text{inlet} + c_{HCO_3^-},\text{inlet} + 2c_{CO_2^2-},\text{inlet} - c_{H^+},\text{inlet} = 0
\]

(22)

where \(p_{CO_2,\text{atmos}}\) (Pa) is the partial pressure of CO\(_2\) in the atmosphere and \(k_H\) (Pa m\(^3\) mol\(^{-1}\)) is Henry's constant for CO\(_2\) in water, which can be expressed, with \(\rho_L\) in kg m\(^{-3}\), as a function of temperature:

\[
k_H = \frac{1}{\rho_L} \exp(-6789.04/T - 11.4519\log T - 0.010454T + 94.4914)
\]

(23)

The values of parameters defining the base-case condition are given in Table 2.

2.4 | Mass transfer, interfacial areas and phase fractions

In Equation (1), \(r_{G-L}\) (mol m\(^2\) s\(^{-1}\)) is the source term for the mass transfer of CO\(_2\) from atmosphere into the aqueous phase, which may be expressed as:

\[
r_{G-L} = a_{G-L}K_{dl}\left(c_{CO_2(aq)} - c_{CO_2(aq)}\right)
\]

(24)
The base-case operating condition is as follows:

| Parameter                          | Value                        |
|------------------------------------|------------------------------|
| Operating temperature and pressure | 20°C, 1 atm                  |
| Diameter of reactor column         | 1 m                          |
| Gaseous CO₂ concentration at gas   | 305 ppm                      |
| Concentrations at liquid inlet:    |                              |
| CO₂ (aq)                           | 1.646 × 10⁻² mol m⁻³         |
| HCO₃⁻                              | 2.556 × 10⁻³ mol m⁻³         |
| CO₃²⁻                              | 4.180 × 10⁻⁸ mol m⁻³         |
| H⁺                                 | 2.559 × 10⁻³ mol m⁻³         |
| OH⁻                                | 2.652 × 10⁻⁶ mol m⁻³         |
| Ca²⁺                               | 0                            |

where \( a_{G>L} \) (m⁻¹) is the gas-liquid interfacial area per unit volume of the bed and \( K_{OL} \) (m/s) is the overall liquid phase mass-transfer coefficient. The saturated concentration of CO₂ in the liquid phase, \( c_{CO₂|L} \) (mol m⁻³), can be calculated by Henry's law:

\[
c_{CO₂|L} = c_{CO₂|G}RT/k_H
\]  

(25)

The expression used to determine the interfacial area for mass transfer depends on the pattern of interactions between the three phases. For a trickle flow bed, a commonly used expression based on Onda's work is24,25:

\[
a_{G>L} = a_t \left( 1 - \exp \left( -1.45 \left( \frac{\sigma_C}{\sigma_l} \right)^{0.75} Re_L^{0.01} Fr_L^{0.05} We_L^{0.2} \right) \right)
\]  

(26)

where \( \sigma_C \) (N m⁻¹) is the critical surface tension of the packing material (2.17 × 10⁻² N m⁻¹ at 20°C) and \( \sigma_l \) (N m⁻¹) is the surface tension of the liquid (7.28 × 10⁻² N m⁻¹ at 20°C).26

TABLE 1  Rate constants and equilibrium constants

| Parameter | Value       | Unit         | Reference |
|-----------|-------------|--------------|-----------|
| \( k_1 \) | \( k_2/k_W \) | m³ mol⁻¹     | Calculated |
| \( k_2 \) | \( 10^{(15689.9T - 2.5866 - 6.737 \times 10^{-3}T)} \) | m³ mol⁻¹ | 16, 18 |
| \( k_3 \) | \( 1/K \) | m⁶ mol⁻²     | Calculated |
| \( k_4 \) | \( \rho_W exp^{-12092.1/\rho_W - 36.7816nT + 235.482} \) | mol m⁻³ | 19 |
| \( k_5 \) | \( K_{C/K}\) | mol⁻³        | Calculated |
| \( k_6 \) | \( K_{C/K}\) | mol⁻³        | Calculated |
| \( k_7 \) | \( K_{C/K}\) | mol⁻³        | Calculated |
| \( k_8 \) | \( 10^{(13.870 - 3059/\rho_W - 0.04035\rho_W)} \) | mol² L⁻² | 20 |
| \( k_{11} \) | \( 10^{(13.635 - 2695/\rho_W)} \) | L mol⁻¹ s⁻¹ | 22 |
| \( k_{21} \) | \( 6.0 \times 10^6 \) | m⁴ mol⁻¹ s⁻¹ | 21 |
| \( k_{31} \) | \( 1.4 \times 10^8 \) | m³ mol⁻¹ s⁻¹ | 16 |
| \( k_{41} \) | \( 10^{(21.65 - 110.541/\rho_W - 17266.4/\rho_W)} \) | s⁻¹ | 23 |
| \( k_{51} \) | \( 10^{(0.198 - 44/\rho_W)} \) | cm s⁻¹ | 12 |
| \( k_{61} \) | \( 10^{(2.84 - 2177/\rho_W)} \) | cm s⁻¹ | 12 |
| \( k_{71} \) | \( 10^{(-5.86 - 317/\rho_W)} \) | mol cm⁻² s⁻¹ | 12 |

In the above equations, \( a_t \) (m⁻¹) is the total specific particle surface area of the packed bed, a function of particle diameter \( d_p \) (m) and bed porosity \( \varepsilon_M \):

\[
a_t = 6(1 - \varepsilon_M)/d_p
\]  

(28)

For a random packing, bed porosity is calculated by28

\[
\varepsilon_M = 0.390 + \frac{1.740}{(d_p/d_e + 1.14)^2}
\]  

(29)

where \( d_e \) (m) is the diameter of the reactor.

For a trickle flow bed operated below the loading point, \( \varepsilon_L \) is given by29,30

\[
\varepsilon_L = \left( \frac{12 Fr_L}{Re_L} \right)^{1/3}
\]  

(30)

The longitudinal dispersion coefficient is calculated from the following empirical correlation31:

\[
\frac{1}{Pe_L} = \frac{Pe_m}{5} \left( 1 - p \right)^2 + \frac{Pe_m^2}{25} p(1-p)^3 \left( \exp \left( -\frac{5}{p(1-p) Pe_m} \right) - 1 \right) + \frac{1}{Pe_m}
\]  

(31)
with
\[ p = \frac{0.48}{Sc^{0.115}} + \left( \frac{1}{2} - \frac{0.48}{Sc^{0.115}} \right) \exp \left( -\frac{755c}{Pe_m} \right) \]  
(32)

where \( Pe_L \) and \( Pe_m \) are the Peclet number based on longitudinal dispersion coefficient and molecular diffusivity, respectively. \( Sc \) is the Schmidt number, and \( r \) is the tortuosity of the packed bed.

\[ \rho_L = \frac{\overline{u}_L d_p}{D_L}, \quad Pe_L = \frac{\overline{u}_L d_p}{\nu_L}, \quad Sc = \frac{\mu_L}{\rho_L D_m} \]  
(33)

where \( \overline{u}_L \) (m/s) is the mean interstitial liquid velocity, \( D_L \) and \( D_m \) (m²/s) are the longitudinal dispersion and molecular diffusion coefficients, respectively. The mean interstitial velocity is found from the liquid holdup, based on \( r_L = \overline{u}_L / \overline{u}_L \). The tortuosity of a bed of randomly packed spherical particles can be estimated from\(^ {22} \)

\[ r = 1.23 \left( \frac{1 - \epsilon_m}{\epsilon_m} \right)^{4/3} \]  
(34)

\( K_{OL} \) (m/s) for the transfer of carbon dioxide is calculated from the following expression which takes the enhancement factor, \( E \), into account:\(^ {23} \)

\[ \frac{1}{K_{OL}} = \frac{1}{E K_L} + \frac{RT}{k \epsilon} \]  
(35)

where \( k_L \) and \( k_G \) (m/s) are the mass-transfer coefficients on the liquid and gas side, respectively. These are calculated from Onda’s correlations:\(^ {25} \)

\[ k_G = 5.23 \left( \frac{\rho_G u_G}{\rho_{GC}} \right)^{0.7} \left( \frac{\rho_G}{\rho_{GC}} \right)^{1/3} (d_G)^{-2} \]  
(36)

\[ k_L = 0.0051 \left( \frac{\rho_L u_L}{\rho_{GC} - \rho_L} \right)^{2/3} \left( \frac{\rho_L}{\rho_{GC}} \right)^{3} (d_G)^{0.4} \]  
(37)

where \( D_{GC,m} \) and \( D_{G,m} \) (m²/s) are the molecular diffusivity of CO₂ in gas and liquid, respectively. The dimensionless numbers used in this work include \( Re \) (Reynolds number), \( Fr \) (Froude number), \( We \) (Weber number), \( Mo \) (Morton number) and \( Ga \) (Galileo number). These are:

\[ R_e = \frac{u_L \rho_L \sigma_L}{\alpha_L}, \quad F_r = \frac{u_G^2 d_G}{g}, \quad W_e = \frac{u_G^2 \rho_G}{\rho_L \sigma_L}, \quad M_o = \frac{\rho_L u_L^4}{\rho_G \sigma_L^3}, \quad G_a = \frac{g u_G^2}{\rho_L \sigma_L^3} \]  
(38)

where \( \rho_L \) (Pa s) and \( \rho_L \) (kg m⁻³) are the viscosity and density of liquid, respectively. \( g \) (m/s²) is the acceleration due to gravity. Onda’s correlations including Equations (26), (36) and (37) are valid for \( 0.04 < R_e_L < 500, \quad 1.2 \times 10^{-7} < W_e < 0.27 \) and \( 2.5 \times 10^{-9} < F_r_L < 1.8 \times 10^{-2} \).

### 2.5 Enhancement factor

There are two chemical reactions, Equations (6) and (9), which, if occurring in the liquid film to any appreciable extent, will accelerate the rate of absorption of CO₂ above what would be expected from a purely physical absorption with the same concentration driving force. The factor by which the liquid film transfer coefficient is then increased is the enhancement factor, \( E \), as shown in Equation (35). The calculation scheme for \( E \) used in this work is that developed by Hogendoorn et al\(^ {23} \) for reversible reactions of finite rate in chemically loaded solutions. It starts with estimation of the asymptotic or maximum attainable enhancement factor of a reversible (pseudo-)first-order reaction, \( E^\infty \), as given by

\[ E^\infty = 1 + \frac{D_{HCO3:L} (c_{HCO3:L} - c_{HCO3:L}^\infty)}{D_{CO2:aq:L} (c_{CO2:aq:L} - c_{CO2:aq:L}^\infty)} \]  
(39)

where \( D_i \) (m²/s) and \( c_i \) (mol m⁻³) are the molecular diffusivity and concentration of species \( i \), the subscript \( L \) and \( G \) specifying gas–liquid interface and bulk liquid, respectively. To calculate \( E^\infty \) from Equation (39) requires the interfacial compositions of only four species, namely \( CO2aq, HCO3^−, H^+, \) and \( OH^- \), since there is negligible conversion to carbonate. To find these, three equilibria are applied at the interface—Henry’s law Equation (18), the CO₂ hydrolysis equilibrium Equation (9) and the ionic product of water Equation (8). The necessary fourth equation is provided by conservation of mass since the net generation rate of \( HCO3^- \) is equal to the generation rate of \( H^+ \) plus the consumption rate of \( OH^- \). Equating the rates of transfer between interface and bulk liquid we obtain:

\[ D_{HCO3:L} (c_{HCO3:L} - c_{HCO3:L}^\infty) = D_{H^+:L} (c_{H^+:L} - c_{H^+:L}^\infty) - D_{OH^-:L} (c_{OH^-:L} - c_{OH^-:L}^\infty) \]  
(40)

Solving the three equilibria at the interface together with Equation (40), the particular interfacial concentrations \( c_{CO2:aq:L}, c_{HCO3:L}, c_{H^+:L} \) and \( c_{OH^-:L} \) can be found and \( E^\infty \) can then be calculated from Equation (39). As shown by Hogendoorn et al\(^ {33} \) this value of \( E^\infty \) enables us to use DeCoursey’s expression originally derived for an irreversible reaction, for our cases of reversible reactions with a chemically loaded solution. The apparent enhancement factor for reaction \( n \) is calculated by DeCoursey’s expression which is\(^ {34} \):

\[ E_n = \frac{-H_{a,n}^2}{2L(\epsilon^\infty - 1)} \left[ \frac{H_{a,n}^2}{4(\epsilon^\infty - 1)^2} + \frac{E^\infty H_{a,n}^2}{(\epsilon^\infty - 1)} + 1 \right] \]  
(41)

where \( H_{a,n} \) is the Hatta number for reaction \( n \). For the reactions represented by Equations (6) and (9), the Hatta number is respectively expressed as\(^ {33,34} \):

\[ H_{a1} = \sqrt{\frac{k_{11} D_{CO2:aq} COH^-:L}{k_L}} \]  
(42)

\[ H_{a2} = \sqrt{\frac{k_{12} D_{CO2:aq} COH^-:L}{k_L}} \]  
(43)
The overall enhancement factor of parallel reactions is calculated by the method of Gaspar and Fosbøl:

\[ E = 1 + \sum (E_n - 1) \]  

(44)

### 2.6 Determination of the applicable range of superficial velocities

Since the particles shrink during reaction and the liquid holdup increases as the particles get smaller, the EW process could potentially lead to flooding of the bed. Our chosen operating policy to counter this is to operate continuously at the same “distance” from the flooding limits (i.e., at a constant fraction of flood), which requires the operating gas and liquid velocities to be adjusted as a function of particle size. According to the correlations of Billet and Schultes, the superficial gas velocity at the flooding point is given by:

\[ u_{G,FL} = \sqrt{\frac{g}{\rho_L}} \left( \frac{\epsilon_M - \epsilon_{L,FL}}{\rho_G - \rho_L} \right) \]  

(45)

where \( \psi_{FL} \) is the resistance coefficient and \( \epsilon_{L,FL} \) is the liquid holdup, both at the flooding point. Note that \( u_{G,FL} \) is a function of the specific area in the bed, which is correlated with the size of the particles. \( \psi_{FL} \) and \( \epsilon_{L,FL} \) can be found from:

\[ \psi_{FL} = \frac{1}{C_{FL}} F_N \left( \frac{\mu_L}{\mu_G} \right)^{0.2} \]  

(46)

where \( C_{FL} \) and \( n_{FL} \) are the constants for specific packing and \( F_N \) is the flow parameter, a ratio defined by:

\[ F_N = \frac{u_L}{u_G} \sqrt{\frac{\rho_G}{\rho_L}} \]  

(47)

When \( F_N \) is smaller than 0.4, the liquid trickles downwards over the internals as a dispersed phase, while if \( F_N \) is greater than 0.4, liquid in the void spaces tends to join up and flow down as a continuous phase. In our work, the flow parameter is kept below 0.4. In this case, as suggested by Billet and Schultes, the value of the constant \( C_{FL} \) was taken as 2.132, using the values for a wide range of column packings tabulated by Billet and Schultes as a guide (60 values, mean \( C_{FL} \) = 2.006, SD = 0.291).

The liquid holdup at the flooding point is determined by:

\[ \epsilon_{L,FL}^2 (3\epsilon_{L,FL} - \epsilon_M) = \frac{6 \epsilon_M R e_{L,FL}}{G d_l} \]  

(48)

Once the liquid holdup at the flooding point is known, the corresponding superficial liquid velocity at the flooding point, \( u_{L,FL} \), can be calculated using the following equation:

\[ u_{L,FL} = \frac{g}{3 \rho_L} \left( \frac{\epsilon_M - \epsilon_{L,FL}}{\epsilon_M} \right) \left( 1 - \frac{3 (\epsilon_M - \epsilon_{L,FL})}{\epsilon_M} \right) \]  

(49)

As pointed out by Billet and Schultes, an iteration is needed to calculate \( u_{G,FL} \) and \( u_{L,FL} \) from these correlations. The iteration starts with \( F_N \) (chosen for process reasons) as shown in Figure 2. Briefly, we first find \( u_{L,FL} \) using an initial guessed value \( u_{L,FL}^0 \) which determines a corresponding value of \( u_{G,FL} \) according to Equation (47). We can then calculate \( u_{L,FL} \) from Equation (48) and a new value for \( u_{L,FL} \) from Equation (49). The procedure is repeated, replacing \( u_{L,FL}^0 \) with the new value of \( u_{L,FL} \) until the match is sufficiently close, when \( u_{L,FL} \) and \( u_{G,FL} \) will have been determined, for the given \( F_N \). The superficial liquid and gas velocities \( u_L \) and \( u_G \) applied in the reactor simulation should be a fraction of those at the flooding point. This fraction is termed the operating parameter, \( OP \), in our work.

The superficial gas and liquid velocities at the flooding point of the trickle flow packed bed reactor at the selected \( F_N \) and particle diameters, are shown in Figure 3. When operating at a constant flow parameter \( F_N \) as the particles become smaller both the gas and liquid velocities must be reduced proportionately to retain the same fraction of flood (i.e., same \( OP \)).

To save computational time, we have regressed the superficial velocity of liquid at the flooding point as a function of the flow parameter and particle size. The corresponding gas velocity is then also found:

\[ u_{G,FL} = d u_{L,FL} \]  

(50)

\[ u_{L,FL} = a (d_p)^2 + b (d_p) + c \]  

(51)

The values of parameters \( a \), \( b \), \( c \), \( d \) and the R-squared are shown in Table 3. The superficial velocities of gas and liquid at flooding point as a function of particle diameter can be found in Figure S1. Once the velocities at flooding are known for a particular flow parameter, it is straightforward to determine the velocities at a particular \( OP \):

\begin{itemize}
  \item Choose a \( F_N \)
  \item Give \( u_{L,FL}^0 \) an initial value
  \item Calculate \( u_{L,FL} \) and \( u_{G,FL} \) at a given \( F_N \)
  \item \( u_{L,FL} - u_{L,FL}^0 \) \begin{align*}
    \text{No} \quad \text{Yes}
  \end{align*}
  \item A pair of \( u_{L,FL} \) and \( u_{G,FL} \) at a given \( F_N \) is determined
\end{itemize}

\[ \text{FIGURE 2} \quad \text{Iteration process to calculate superficial velocities of liquid and gas} \]
\[ u_L = \frac{OP_{u_L}}{v_f}; u_G = \frac{OP_{u_G}}{v_f} \] (52)

2.7 Calcite particle shrinkage by dissolution

The total mass loss rate (kg s\(^{-1}\)) of the solid is calculated by

\[ \frac{dm}{dt} = M_p r_{Ca^{2+}} V_{bed} \] (53)

where \( m \) (kg) is the total solid mass and \( V_{bed} \) (m\(^3\)) is the total volume of the packed bed. \( M_p \) (kg mol\(^{-1}\)) is the molecular weight of material dissolving out of the solid phase and \( r_{Ca^{2+}} \) (mol m\(^{-3}\) s\(^{-1}\)) is the reaction rate computed by Equation (16) an average value for the bed since the particle shrinking rate is assumed to be location-independent. The density of the solids, \( \rho_s \) (kg m\(^{-3}\)) is 2,710 kg m\(^{-3}\) for calcite, so the rate of loss of solids volume may be found from Equation (53). The bulk density of the bed is \( \rho_s (1-\varepsilon_M) \) and the porosity is given by Equation (29), so the rate of change of bed volume and thus bed height at any point of the operation may also be found. The rate of change of particle diameter, \( r_{dp} \) (m s\(^{-1}\)) and bed height, \( r_{H_{bed}} \) (m s\(^{-1}\)) are then given by

\[ r_{dp} = \frac{1}{3} \frac{M_p r_{Ca^{2+}}}{\rho_s (1-\varepsilon_M)} d_p \] (54)

\[ r_{H_{bed}} = \frac{M_p r_{Ca^{2+}}}{\rho_s (1-\varepsilon_M)} \left( \frac{d_p}{d_0} \right)^3 H_{bed}/H_{0_{bed}} \] (55)

where \( d_p^0 \) (m) and \( H_{0_{bed}} \) (m) are the initial particle diameter and bed height, respectively.

Neglecting the insignificant change in porosity with particle diameter for large equipment, the particle diameter \( d_p \) is related to its initial value \( d_0 \) by.

\[ d_p = \left( \frac{d_0}{C16/C17} \right)^{3} \] (56)

To ensure that the same mass of calcite is dissolved from a given initial mass of solids when simulating batches with different initial particle sizes, the ratio of the terminal and initial particle diameter must be identical.

2.8 Energy and water consumption

We calculate energy and water consumption in this work considering only the operation of the reactor; other demands (e.g., the energy cost for rock grinding) are outside the scope of this assessment. More specifically, the energy required by the EW process as calculated here is for maintaining gas and liquid flow. Gas flowing through the bed must overcome the bed pressure drop which needs a blower or compressor. The energy input to this device, the work of compression, is

\[ W_G = \frac{1}{\eta} \int_{t_0}^{t_1} \left( \rho_G \gamma_{in} \gamma_{f-1} \left( \frac{P_{out}}{P_{in}} \right)^{(1-\beta)} - 1 \right) \] (57)

where \( \eta \) is compressor efficiency, assumed to be 0.8, \( \gamma \) is the ratio of the specific heats at constant pressure and constant volume, \( Q_{in}^G \) (m\(^3\) s\(^{-1}\)) is the inlet volumetric flow rate of gas, \( P_{in} \) (Pa) is the pressure of

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**FIGURE 3** Superficial gas and liquid velocities at flooding with different particle diameters [Color figure can be viewed at wileyonlinelibrary.com]

**TABLE 3** Parameters \( a, b, c, d \) and \( R^2 \) for regressed expression of gas and liquid velocities (m s\(^{-1}\)) at flooding in the packed bed

| \( F_n \) | \( a \) | \( b \) | \( c \) | \( d \) | \( R^2 \) Equation (50) | \( R^2 \) Equation (51) |
|---|---|---|---|---|---|---|
| 0.10 | -7.664 | 0.20965 | 2.4126 \( \times \) 10\(^{-4}\) | 292.996 | .9999 | 1.0000 |
| 0.15 | -11.702 | 0.30647 | 2.7032 \( \times \) 10\(^{-4}\) | 194.827 | .9997 | 1.0000 |
| 0.20 | -13.962 | 0.37630 | 3.4978 \( \times \) 10\(^{-4}\) | 146.501 | .9978 | 1.0000 |
| 0.25 | -17.418 | 0.46182 | 3.7826 \( \times \) 10\(^{-4}\) | 117.020 | .9977 | 1.0000 |
| 0.30 | -22.147 | 0.52315 | 4.2776 \( \times \) 10\(^{-4}\) | 97.603 | .9979 | 1.0000 |
| 0.35 | -22.129 | 0.59853 | 4.5637 \( \times \) 10\(^{-4}\) | 83.643 | .9979 | 1.0000 |
| 0.40 | -24.205 | 0.66346 | 4.7883 \( \times \) 10\(^{-4}\) | 73.254 | .9981 | 1.0000 |
TABLE 4 Default values and range of studied parameters

| Parameter (unit) | Default value | Range          |
|------------------|---------------|---------------|
| Packed bed height (m) | 10            | 1–60          |
| Initial particle diameter (mm) | 10            | 1.25, 2.5, 5, 10 |
| Enhancement factor | Calculated by Equation (44) | 1.0–5.0      |
| CO₂(g) concentration | 410 ppm       | 410 ppm to 20% |

FIGURE 4 Composition within the packed bed with full-kinetics and semi-kinetics models in base-case operating condition. $F_g = 0.4$, $OP = 30\%$. Feed gas concentrations: 410 ppm and 5% CO₂. $H_{bed}$ decreases from 10 to 7.3 m, $d_p$ decreases from 10 to 9 mm. (a) CO₂(aq), (b) OH⁻, (c) HCO₃⁻, (d) Ca²⁺, (e) CO₂(g), (f) pH [Color figure can be viewed at wileyonlinelibrary.com]
the feed stream to be compressed, generally close to atmospheric, \( p_{\text{out}} \) (Pa) is the pressure at the bottom of the bed, which is \( p_0 + \Delta p \).

The bed pressure drop is given by\(^{30}\)

\[
\frac{\Delta p}{H_{\text{bed}}} = \frac{\Delta p_0 \psi_L}{H_{\text{bed}} \psi_0 (\epsilon_M - \epsilon_L)^3}
\]

where \( \Delta p_0 / H_{\text{bed}} \) (Pa m\(^{-1}\)) is the pressure drop per unit height of the dry bed. Details can be found in Billet and Schultes\(^{30}\); \( \psi_0 \) and \( \psi_L \) are the resistance coefficients of dry bed and wet bed respectively.

Water must be pumped to the top of the packed section, and the energy required, taking into account the decrease in bed height due to particle shrinkage, is

\[
W_L = \frac{1}{\eta} \int_0^{t_0} (\rho_L Q_L g H_{\text{bed}}) dt
\]

where \( Q_L \) (m\(^3\) s\(^{-1}\)) is the volumetric flow rate of liquid, \( H_{\text{bed}} \) is the height of the bed. The efficiency \( \eta \) is again taken as 0.8.

The total energy consumption, summing that for gas compression and liquid pumping, is

\[
W = W_G + W_L
\]

The water consumption is calculated by the following equation:

\[
V_L = \int_0^{t_0} u_L S_R dt
\]

where \( S_R \) (m\(^2\)) is the cross-sectional area of the column.

## 2.9 | Meshing and numerical solution methods

Commercial software COMSOL Multiphysics 5.4 was employed to implement the fully coupled governing equations based on the finite element method (FEM) using a personal desktop with i7 processor and 32 GB RAM. In the COMSOL model, an adaptive mesh was mapped over the computational domain with finer resolution near the...
liquid inlet and coarser resolution toward the liquid outlet. The maximum element size is 0.1 m, the maximum element growth rate is 1.1, and the number of elements is 500 with an element ratio of 0.01 and a growth formula of arithmetic sequence. The MUMPS time-dependent solver of the software was adopted with default parameter settings, and the tolerance was set to be physics-controlled with a relative tolerance of 0.01.

3 | RESULTS AND DISCUSSION

The energy consumption, water consumption and CO2 capture rate were studied as a function of flow parameter ($F_{lv}$), OP, initial packed bed height ($H_{bed}$) and initial particle diameter ($d_0$). Some calculations were also performed for a feed stream enriched with CO2. To simulate the effect of a hypothetically accelerated liquid phase reaction, the enhancement factor ($E$) was also varied. Default values and range of varied input parameters are shown in Table 4. To avoid excessive computation time and without affecting the prediction of prominent trends, all reported simulations were carried out for partial (not full) dissolution of the particles, with the extent stated for each set of studies.

3.1 | Spatial distribution of species

Figure 4a-f show the variation of composition in the bed for two cases, one with air as the feed gas (bed exposure time 35,900 hr), and the other using air enriched with CO2 to 5 mol% (bed exposure time 3,300 hr). Figure 4 shows the compositions when the particle diameter reached 90% of its initial value ($d_p = 9$ mm).

Water entering the bed is pre-saturated with CO2 at atmospheric conditions (to give a concentration $0.01646$ mol m$^{-3}$, see Table 2). Although the incoming water contains no calcium, at the top of the bed Ca$^{2+}$ is present due to longitudinal dispersion in the liquid phase. Immediately below the liquid inlet, the chemical environment in the

![Figure 6](https://wileyonlinelibrary.com)
bed causes a sharp drop in the CO$\text{$_2$}$(aq) and OH$^-$ concentrations and a rise in bicarbonate concentration. Further down the bed, weathering of the solid particles causes concentrations of Ca$^{2+}$ and bicarbonate to rise steadily in the trickling liquid, and CO$\text{$_2$}$(aq) concentration also rises after its initial drop. The gas phase concentration of CO$_2$ decreases as this component is absorbed on rising through the bed, though the fractional removal is relatively modest here—$18\%$ for the air at $410$ ppm CO$_2$, and $1.64\%$ for the air enriched to $5\%$ CO$_2$. In the latter case we can see that the lower half of the bed is rather inactive, there being a pinch at the bottom of the bed, where the liquid phase has become saturated with CO$_2$; evidently there is insufficient water flow to absorb much of the CO$_2$ being offered to the reactor. A greater value of the flow parameter is required, but $F_{lv} = 0.4$, the value used in Figure 4, is the maximum at which trickling flow can be maintained.

For the case shown in Figure 4 the change in CO$_2$ concentration is almost linear with height in both the liquid and gas phases over the lower 7 m of bed height. In this region the log mean concentration driving force for mass transfer can then be calculated, and from that
TABLE 5 Summary results of trickle flow bed for CO₂ capture through enhanced weathering (EW) in base-case operating condition

| Parameter                        | Values for the least energy consumption (Case A) | Values for the least water consumption (Case B) | Values for the highest CO₂ capture rate (Case C) |
|----------------------------------|-------------------------------------------------|------------------------------------------------|-------------------------------------------------|
| Flow parameter FLv               | 0.4                                             | 0.1                                             | 0.4                                             |
| Operating parameter OP           | 10%                                             | 10%                                             | 60%                                             |
| Particle diameter range (mm)     | 2.5–2.0                                         | 1.25–1.0                                        | 10.0–8.0                                        |
| Total energy consumption (MJ/kg CO₂ captured) | 7.26                                             | 33.15                                           | 35.47                                           |
| Energy for water pumping Ww (% of total) | 67.73                                           | 12.00                                           | 34.52                                           |
| Energy for air blowing WC (% of total) | 32.27                                           | 88.00                                           | 65.48                                           |
| Water consumption Vl (m²/kg CO₂ captured) | 53.75                                           | 43.11                                           | 134.5                                           |
| Time required t (hr)             | 2.743 × 10⁵                                     | 8.865 × 10⁵                                     | 4.01 × 10⁴                                      |
| Average calcite dissolution rate (kg/m² surface area/~hr) | 2.36 × 10⁻⁶                                     | 3.65 × 10⁻⁷                                     | 6.45 × 10⁻⁵                                     |
| Average CO₂ capture rate (kg hr⁻¹) | 9.73 × 10⁻³                                     | 3.07 × 10⁻³                                     | 5.64 × 10⁻²                                     |
| Average mass-transfer coefficient KCL (m s⁻¹) | 1.71 × 10⁻⁵                                     | 8.36 × 10⁻⁶                                     | 8.35 × 10⁻⁵                                     |
| Average superficial gas velocity uG (m s⁻¹) | 1.25 × 10⁻²                                     | 1.30 × 10⁻²                                     | 0.186                                           |
| Average superficial liquid velocity uL (m s⁻¹) | 1.71 × 10⁻⁴                                     | 4.43 × 10⁻⁵                                     | 2.54 × 10⁻³                                     |
| Average packing specific area aL (m²/m³) | 1830.0                                          | 3660.0                                          | 457.4                                           |
| Average gas–liquid interfacial area aGL (m² m⁻³) | 70.67                                           | 65.33                                           | 78.54                                           |
| Average interfacial area of the wetted area αw (m² m⁻³) | 1146.7                                          | 1923.3                                          | 375.49                                          |

Note: The average values are based on particle diameter from its initial size to the terminal size. Packed bed height: 10 m, initial atmospheric CO₂ concentration: 410 ppm. The ranges of studied parameters: (a) initial particle diameter: 1.25–10 mm; (b) flow parameter: 0.1–0.4; (c) operating parameter: 10–60%.

the height of a transfer unit (HTU). Commercial equipment for mass transfer in trickle flow generally provides an HTU of 1 m or less, though higher values may be expected when there is significant liquid phase resistance, as in this case: Here the calculated value of HTU, HTU based on the overall gas phase driving force, is 32 m. This is significantly poorer than most commercial absorbers because the absorbing component is not very soluble in the solvent, there is little chemical enhancement of transfer rate and the gas–liquid interfacial area is comparatively small.

In the “full-kinetics” case, all the participating chemical reactions (6)–(12) are dynamically modeled (see the addition rate expressions in the Supporting Information). In the “semi-kinetics” case the fast proton transfer reactions (7) and (8) are taken to be at equilibrium. Figure 4 shows that the calculated concentrations of various species with the semi-kinetics model are effectively identical to those with the full-kinetics model. Assuming equilibrium in reactions (7) and (8) reduces the complexity of the model and saves considerable computational time without significant loss of accuracy.

3.2 Effect of flow parameter (FLv) and OP

Figure 5a–d show four key performance indicators for trickle bed simulations in which the particle size was allowed to shrink from 10 to 8 mm, so that the bed decreased in height from 10 to 5.12 m. All the results are mean values averaged over the simulated period. A range of flow parameters and OP were chosen, to illustrate the effect of changes in velocities.

The total CO₂ capture capacity, defined as kg of CO₂ captured when 1 kg of calcite is dissolved, is shown in Figure 5a. According to the overall stoichiometry of the calcite reaction (CaCO₃ + CO₂ + H₂O = Ca²⁺ + 2HCO₃⁻), 0.44 kg (or 10 mol) CO₂ would be captured for each kg (or every 10 mol) of calcite dissolved. We see that in all cases the capture capacity is close to this theoretical maximum. There is some loss in effectiveness of the use of dissolved calcite, more evident at higher flow parameters and higher OP. The calculated source terms of calcite dissolution, as shown in Figure 5a, confirm that step (12) is the dominant process of calcite dissolution, with the liquor is still some way from saturation with the incoming air (in Figure 4a, xC₂O₃(FLv) = 410 ppm: CO₂[(aq)] = 0.0074 mol m⁻³ while saturation is 0.01646 mol m⁻³). A faster CO₂ capture rate is achieved when larger FLv and OP are used because parameters such as mass-transfer coefficients and interfacial areas increase. But as calcite particles...
The CO₂ capture capacity, Figure 6a, initially increases markedly as taller beds are used, but the rate of increase declines as the capture capacity approaches the theoretical maximum of 0.44 kg CO₂ per kg of calcite dissolved. The capacity is lower at larger \( F_{lv} \) and \( OP \), but the effect of these parameters is relatively minor, as previously seen in Figure 5a (note the different y-axis scales between Figures 5a and 6a).

In our simulations which were each run at constant \( F_{lv} \) and constant \( OP \), the liquid and gas superficial velocities are determined by particle size (Equations 50, 51 and 52). For various given initial bed heights, the initial and terminal particle sizes are fixed, so the volumetric flow rates of liquid and gas through beds of different height are the same. But the total amount of calcite dissolved out of a taller bed is greater, leading to a higher concentration of Ca²⁺ at liquid outlet, as shown in Figure S5. For example, as the bed height increases from 1.0 to 10 m, the Ca²⁺ concentration at the outlet increases from 0.118 to 0.215 mol/m³. The increase in Ca²⁺ would lead to a larger CO₂ capture capacity at equilibrium because the final product of weathering is calcium bicarbonate. However, although plenty of calcite is dissolved in reaction step (12), the generated OH⁻ must be consumed by reaction step (6), which requires CO₂ to arrive by mass transfer from the gas phase. When the rate of mass transfer is insufficient, some of the generated Ca²⁺ and OH⁻ is flushed out of the bed without reacting with aqueous CO₂. This leads to the lower CO₂ capture capacity of shorter beds, which have less gas–liquid interfacial area for mass transfer.

The variation of CO₂ capture capacity with initial bed height is consistent with the increase in CO₂ capture rate shown in Figure 6b. As more bed height is added, more CO₂ capture occurs, and the effectiveness of extra bed height only decreases slowly, though of course there is an energy penalty, as shown in Figure 6d.

Figure 6c shows the effect of bed height on water consumption. With heights less than 7–8 m very large volumes of water are required, because the beds are short and there is too little CO₂ captured to utilize the dissolved calcite effectively. Figure 6c mirrors Figure 6a. As more bed height is added, the saturation level of the liquid increases, and the water consumption approaches the theoretical minimum, \( \sim 42.1 \text{ m}^3/\text{kg} \), when the effluent is at equilibrium with air at 410 ppm CO₂.

The energy consumption for both liquid pumping and gas blowing increase almost linearly with bed height, as shown in Figure 6d, which is a warning that the increase in CO₂ capture rate and reduced consumption of water in taller beds come at an energy cost. These results suggest that when the initial bed height is set at 7 m, the ratio of CO₂ capture capacity and energy consumption is at a maximum, which is perhaps an optimum. In this case, the water consumption is \( \sim 3 \) times the theoretical minimum and the CO₂ capture rate is around one third that calculated with a bed height of 60 m, but other trade-offs are also possible.

### 3.3 Effect of bed height

The CO₂ capture capacity, Figure 6a, initially increases markedly as taller beds are used, but the rate of increase declines as the capture capacity approaches the theoretical maximum of 0.44 kg CO₂ per kg of calcite dissolved. The capacity is lower at larger \( F_{lv} \) and \( OP \), but the effect of these parameters is relatively minor, as previously seen in Figure 5a (note the different y-axis scales between Figures 5a and 6a).

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### 3.4 Effect of particle size

When investigating the effect of particle size on the reactor performance, to obtain the same amount of calcite dissolution, the ratio of the particle diameters at the start and end of a run must be the same. We investigated four size ranges of the particle diameter, namely...
dissolution from 10 to 8 mm, 5 to 4 mm, 2.5 to 2 mm and 1.25 to 1 mm at base-case operating condition with the default value of other parameters.

As shown in Figure 7a, the particle size has little effect on CO2 capture capacity, though the capacity is somewhat higher with smaller particles due to the combination of potentially conflicting factors (e.g., increase in mass transfer area and decrease in the mass-transfer coefficient, cf. Figure 8). At these conditions the CO2 capture capacity is close to the maximum value, 0.44 kg kg\(^{-1}\). The increase in CO2 capture rate seen in Figure 7b for the larger particle sizes occurs because the system operates in those cases at a higher velocity which increases mass-transfer coefficients, as shown in Figure 8. The absolute values of \(K_{OL}\) in Figure 8 are rather low compared with what would normally be expected in an industrial absorption process—in these packed beds of smallish spherical particles, there is little opportunity for the turbulence and convection that enhance transfer at the gas–liquid interface at higher gas velocities with high open-area packing. In fact, the values of \(K_{OL}\) here would be even lower but for the (modest) enhancement of mass transfer rates caused by reaction steps (6) and (9) which particularly help the beds of smaller particles—see Figure 8. The smaller particles, with their greater surface area, offer the advantage of lower water consumption for the same quantity of calcite dissolution, as shown in Figure 7c. However, with their lower operating velocities (and thus longer run times) and higher gas phase pressure drop (see Figure S6), the smaller particles can have a significant energy penalty, as demonstrated in Figure 7d.

3.5 Results of optimizing strategies

Table 5 summarizes simulation results where the flow parameter \(F_{lv}\), operating parameter \(OP\) and particle size were varied according to three different optimizing strategies, seeking (a) least energy consumption, (b) least water consumption, and (c) highest CO2 capture rate. The least energy consumption, 7.26 MJ/kg CO2, is found at an intermediate particle size with the largest \(F_{lv}\) and smallest \(OP\), and a water consumption only slightly (25%) more than the minimum value. The other two optima incur an energy consumption 4–5 times greater. The highest CO2 capture rate gives much the lowest batch time, with the largest particles. This limited search for optimal operating
conditions illustrates the kind of trade-offs that can be made, and Table 5 shows typical values for some of the key variables in a trickle flow reactor. A wider search for optimal operations should include other types of reactor such as slurry columns, but this is outside the scope of the present work.

The height of an overall gas phase mass transfer unit \((HTU_{OG})\) can be calculated for the average values given in Table 5, with \(HTU_{OG} = \frac{u_G k_H}{(K_{OL}/C_1 RT/c_1 a_G - L)}\). The values for cases A, B and C are 11, 25 and 29 m respectively. These rather large values again reflect the poor solubility of CO\(_2\) (large \(k_H\)), and the low gas–liquid interfacial area.

The simulations of these three optimizing strategies can also be used to analyze the relative importance of mass transfer and chemical reactions as the rate-controlling mechanisms. Among the chemical reactions, Equations (6), (9) and (11) are the routes by which dissolved CO\(_2\) (that is, CO\(_2\)\(_{aq}\)) is removed from the liquid phase. At the pH of the solutions in this weathering system (with CO\(_2\) supplied at 410 ppm), Equation (6) makes negligible contribution to the removal of CO\(_2\)\(_{aq}\). Considering only forward reactions for the sake of a simplified analysis, the rate of chemical reaction of dissolved CO\(_2\) in unit volume of bed is thus approximately given by

\[
\frac{r_{CO_2(aq)}}{C_3 CO_2(aq)} = \frac{k_{41} c_{CO_2(aq)} a_w}{C_3 CO_2(aq)}
\]

The maximum rate of chemical reaction is then

\[
\frac{r_{CO_2(aq)}}{C_3 CO_2(aq)} = \frac{k_{61} c_{CO_2(aq)} a_w}{C_3 CO_2(aq)}
\]

And the maximum rate of mass transfer, from Equation (24) is

\[
r_{G - L} = a_{G - L} K_{OL} c_{CO_2(aq)}
\]

We can expect the rate of EW to be limited by the gas–liquid mass transfer when \(r_{G - L} < r_{CO_2(aq)}\). That is, the mass transfer is limiting when

\[
\frac{a_{G - L} K_{OL}}{k_{41} a_w + k_{61} a_w} < 1
\]

Calculated from the data in Table 5, the values of the left-hand-side expression for Cases A, B and C are 0.041, 0.011 and 0.59, respectively. These values imply that the rate of EW in Cases A and
B is primarily controlled by the rate of gas–liquid mass transfer. In Case C, both the weathering reactions and the mass transfer have similar rates, so both play a role. The criterion (65) demonstrates the importance of chemical rates, but also mass-transfer coefficients and the characterizing parameters \( \alpha_C - L \), \( \varepsilon_L \) and \( \alpha_w \). Cases like A and B would obviously benefit more from enhancement of gas to liquid mass transfer, than cases like C. The approach of this analysis is general and could be applied to other weathering systems with known kinetics.

### 3.6 Accelerating mass transfer by chemical reaction

For an absorption process like the one discussed here in which liquid phase mass transfer is a limiting factor, the acceleration of rates offered by chemical reaction in the liquid may prove crucial to viability. Reactions naturally occurring in the aqueous calcite system offer only a small enhancement (Figure 9) whereas many commercial absorptions make use of catalysis or additives which have a much larger effect. This may be difficult to accomplish in practice in EW, but we have examined theoretically the potential benefits in our model. We disabled the routines which calculated the enhancement factor (Section 2.5) and chose instead values of \( E \) in the range of 1.0–5.0. As shown in Figure 10a,b, an increase in enhancement factor improves the capture capacity and capture rate as if the bed had been made taller. However, this has occurred without any increase in the bed height, so there is a reduction in energy consumption, which is more pronounced for the shorter bed (Figure 10d). There is also a useful reduction in water consumption (Figure 10c). It is observable that the rate of improvement declines as \( E \) increases, and for enhancement factors greater than 3.0, there is little further benefit to energy or water consumption especially with the smaller bed height. Further simulation analysis (results not shown here) has confirmed that under these conditions the role of calcite dissolution in rate controlling becomes more significant.

### 3.7 Weathering with CO2-enriched air

The effect of initial CO2 concentration, in the range of 410 ppm to 20%, on the reactor performance is shown in Figure 11 a–d. The CO2 capture capacity and capture rate are both greater at higher feed CO2 concentration as the rate of mass transfer of CO2 from gas into liquid is enhanced by a higher CO2 partial pressure. We see that increasing the feed gas CO2 concentration from 410 ppm to even 0.5% reduces the energy and water consumption by 80%, and raising it again to 5% causes a further halving (Figure 11c,d). These results confirm the suggestion of Keleman et al.\(^{28}\) that reaction times would be significantly reduced when utilizing air enriched to a few percent in carbon dioxide to sparge through reacting alkaline rock.

## 4 CONCLUSIONS

We conclude that a computerized advection-reaction-dispersion model can be developed for the EW of calcite with CO2 and water in a trickle bed reactor with air through-flow. Performance indicators including CO2 capture capacity and rate as well as water and energy requirements could be calculated from the model, and are shown to be influenced by factors such as bed height, particle size and the velocities of gas and liquid flows. In particular, the model shows that a bed height of at least 7–8 m is needed to enable sufficient weathering and thus capture of CO2 with 1–10 mm particles. The simulation is possible because the dissolution and reaction kinetics of this mineralogical system have been well researched by many previous workers, and data fitted with theory and correlations as needed in the modeling. Given the importance of EW as a CDR technology, similar investigation of other minerals—dissolution and reaction kinetics—is urgently recommended. The model also requires mass transfer and hydrodynamic parameters for packed beds of particles, and again we have been able to use well-known results of previous workers, though extrapolation is needed to estimate properties of beds of rock particles. The hydrodynamic and mass transfer behavior of such mineral beds also deserves much more attention, if, as has been suggested, we are to apply EW at the scale required to capture gigatons per year of carbon dioxide.

An important feature of the model is its ability to calculate the chemical enhancement of mass transfer through application of the results of Hogendoorn et al.\(^{33}\) DeCoursey\(^{34}\) and Gaspar and Fosbol\(^{35}\). The relatively simple estimation method described here means that the computer model can be used to screen the performance of EW reactors with a reduced need for experimentation—though of course verification of this is needed. The computer model is able to deal with shrinkage of the bed which results from dissolution of the solids, and this is another topic that would benefit from further experimental research. The model shows that even a small enrichment of the CO2 concentration of the inlet air, perhaps to as little as 0.5%, would make a significant saving in energy and water requirements of EW. Research to develop such schemes might be fruitful.

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### AUTHOR CONTRIBUTIONS

Lei Xing: Conceptualization; data curation; formal analysis; investigation; methodology; writing-original draft. Richard Darton: Formal analysis; methodology; project administration; supervision; writing-review and editing. Aidong Yang: Conceptualization; formal analysis; funding acquisition; methodology; project administration; supervision; writing-review and editing.

### NOTATION

\( S \) | area, \( m^2 \)
---|---
\( \bar{u} \) | average interstitial velocity, \( m\,s^{-1} \)
\( c \) | concentration, \( mol\,m^{-3} \)
\(d\) diameter, m

\(D\) diffusion coefficient, \(\text{m}^2\ \text{s}^{-1}\)

\(E\) enhancement factor

\(K\) equilibrium constant

\(F_{iv}\) flow parameter

\(Fr\) Froude number

\(Ga\) Galileo number

\(Ha\) Hatta number

\(HTU\) height of transfer unit

\(k_H\) Henry's constant, \(\text{Pa.m}^3\ \text{mol}^{-1}\)

\(h\) holdup

\(m\) mass, kg

\(M\) molecular weight, \(\text{kg.mol}^{-1}\)

\(Mo\) Morton number

\(n\) number

\(OP\) operating parameter, fraction of flood

\(C_p\) packing constant

\(Pe\) Peclet number

\(p\) pressure, \(\text{Pa}\)

\(k\) rate constant/mass-transfer coefficient

\(r\) reaction source/sink, \(\text{mol.m}^{-3}\ \text{s}^{-1}\)

\(Re\) Reynolds number

\(Sc\) Schmidt number

\(a\) specific/interfacial area, \(\text{m}^{-1}\)

\(u\) superficial velocity, \(\text{m.s}^{-1}\)

\(T\) temperature, \(K\)

\(t\) time, \(\text{s}\)

\(V\) volume, \(\text{m}^3\)

\(Q\) volumetric flow rate, \(\text{m}^3\ \text{s}^{-1}\)

\(We\) Weber number

\(W\) work/energy, \(\text{J}\)

\(\rho\) density, \(\text{kg.m}^{-3}\)

\(\eta\) efficiency

\(\varepsilon\) porosity

\(\gamma\) ratio of the specific heat

\(\psi\) resistance coefficient

\(\sigma\) surface tension, \(\text{N/m}\)

\(\tau\) tortuosity

\(\mu\) viscosity, \(\text{Pa.s}\)

Superscript

\(\ast\) saturated

\(0\) initial

\(\infty\) maximum

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SUPPORTING INFORMATION
Additional supporting information may be found online in the Supporting Information section at the end of this article.

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