Modeling of the atmospheric electric field local variations in the turbulent surface layer

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Abstract. An electrodynamic model of the surface layer is constructed in the approximation of the turbulent electrode effect in the atmosphere. The variation of the electric field that occurs due to changes in the degree of turbulent mixing during the day is calculated. The dependence of the spatial and temporal electric field characteristics on the values of the turbulent diffusion coefficient in the surface layer is studied. It is shown that the local changes in the degree of turbulent mixing lead to disturbances of the electric field comparable to the global unitary variation.

1. Introduction

Diurnal variations of the electric field have two components: global and local. The global component is usually a unitary variation of the ionosphere potential, which has a morning minimum (03-05 UT) and an evening maximum (19-20 UT) [1]. Local disturbances are determined by electrical processes in the surface layer associated with the action of the electrode effect near the earth's surface. Allocation of the electric field global variations against the background of local data variability requires accurate information about the physical causes of disturbances occurring near the earth's surface. According to experimental data [8] of atmospheric-electric observations, the daily course of the electric field is influenced by meteorological conditions that are local in nature, but they lead to periodic changes in the values of the turbulent mixing coefficient over time.

This work is a development of theoretical research presented in [2, 3] on electrodynamic processes in the atmosphere, in particular, on the mechanisms of generating diurnal variations of the electric field in a turbulent surface layer and its relationship with the global current. In [4-6], the process of occurrence of the electric field variations due to changes in the values of the total electric current at the upper boundary of the electrode layer under conditions of stable stratification in the surface layer was studied in the approximation of the turbulent electrode effect [7]. The dependence of the spatio-temporal characteristics of the electric field on the values of the turbulent diffusion coefficient, which was assumed to be constant, was proven.

The aim of this work is to model variations of the electric field near the earth's surface as a result of non-stationary turbulent transport in the surface layer.
2. Mathematical modeling and analysis of results

We set the task to find the dependence of the electric field distribution \((E)\) in the turbulent surface layer on the daily variation of the turbulent diffusion coefficient \((D)\), assuming the value of the current density at the upper boundary of the electrode layer constant. In this case, the total current equation \((j)\) in the surface layer can be written as:

\[
\frac{\partial E}{\partial t} - D(t) \frac{\partial^2 E}{\partial z^2} + 4\pi jE = 4\pi j
\]

where the right side is a constant, \(z\) is the height.

The law describing the daily variations of the turbulent diffusion coefficient is given as:

\[
D = D_0(2 - \cos(\omega t + \phi_0)), \quad \omega = \frac{2\pi}{T}, \quad T = 24h.
\]

The electrical conductivity of the air will be considered constant: \(\lambda = \text{const}\), the initial phase \((\phi_0)\) will be assumed to be zero.

The initial and boundary conditions for equation (1) will be considered as:

\[
E|_{t=0} = E_0e^{\frac{z}{L}}, \quad E|_{z=0} = E_0, \quad \frac{\partial E}{\partial z}|_{z=L} = 0.
\]

Here \(E_0\) is the value of the electric field at the earth’s surface, \(L=10\ m\).

We will search for the solution of the problem (1) – (2) using the Fourier method [9]. To do this, we will replace the variables \(E - E_0 = E_1(t, z)\). With this substitution, equation (1) retains its structure, but the boundary conditions become homogeneous:

\[
\frac{\partial E_1}{\partial t} - D_0(2 - \cos \omega t) \frac{\partial^2 E_1}{\partial z^2} + 4\pi \lambda E_1 = 4\pi (j - \lambda E_0),
\]

\[
E_1|_{t=0} = E_0(e^{\frac{z}{L}} - 1),
\]

\[
E_1|_{z=0} = 0, \quad \frac{\partial E_1}{\partial z}|_{z=L} = 0.
\]

As a result, we obtain an inhomogeneous initial-boundary value problem for a parabolic equation of type (3) with respect to an unknown function \(E_1(t, z)\) with inhomogeneous initial and homogeneous boundary conditions (4) and (5). The unknown function is represented as a series:

\[
E_1 = \sum_{n=1}^{\infty} T_n(t)\Phi_n(z),
\]

where the functions \(\{\Phi_n(z)\}\) are the solution of the Sturm-Liouville problem:

\[
-L_0 \frac{d^2\Phi}{dz^2} = \mu \Phi(z),
\]

with boundary conditions:

\[
\Phi(0) = \Phi(L) = 0.
\]

Eigenvalues and eigenfunctions of problem (7) have the form:

\[
\mu_n = \left(\frac{\pi(2n - 1)}{2L}\right)^2, \quad \Phi_n(z) = \sqrt{\frac{2}{L}} \sin \frac{\pi(2n - 1)}{2L} z
\]
To define functions \( \{T_n(t)\}_{n=1}^\infty \), you need to substitute the series (6) in equations (3), (4) (conditions (5) are fulfilled automatically). The correspondence of coefficients for a basic set \( \{\Phi_n(z)\}_{n=1}^\infty \) leads to the following Cauchy problem for defining functions \( T_n(t) \):

\[
T_n(t) + D(t) \mu_n T_n(t) = \alpha_n, \tag{10}
\]

with initial conditions:

\[
T_n(0) = \beta_n, \tag{11}
\]

where the functions \( \alpha_n \) and \( \beta_n \) are the coefficients of the expansion of the initial condition (4) and the right side of the equation (3), respectively, in the system of eigenfunctions \( \{\Phi_n(z)\}_{n=1}^\infty \):

\[
\alpha_n = \frac{8(j - \lambda E_0)}{2n - 1}, \quad \beta_n = -E_0 \sqrt{\frac{2}{L}} \left[ \frac{2Le^{-\frac{\pi(t(2n - 1))}{2L}}(2n - 1)}{\pi^2L^2(2n - 1)^2 + 4} \right].
\]

The Cauchy problem (10) – (11) for an ordinary differential equation is solved explicitly. Its solution can be found as the sum of the General solution \( \bar{T}_n(t) \), the corresponding homogeneous equation, and the partial solution of the inhomogeneous equation \( T_n(t) \).

Due to the fact that equation (10) is inhomogeneous with variable coefficients, the optimal solution method is the Lagrange method, which gives the following representation of the solution of problem (10) – (11):

\[
T_n(t) = \beta_n \left[ e^{-\frac{\pi(t(2n - 1))}{2L}} \left( 2r - \frac{1}{\alpha} \sin \alpha \right) \right] + \left[ e^{-\frac{\pi(t(2n - 1))}{2L}} \left( 2r - \frac{1}{\alpha} \sin \alpha \right) \right] \int e^{-\frac{\pi(t(2n - 1))}{2L}} \left( 2r - \frac{1}{\alpha} \sin \alpha \right) dt. \tag{12}
\]

The general representation for the electric field strength in the form of a Fourier series for the system of eigenfunctions of the problem (3) – (5) after the transition to the original unknown function will have the form:

\[
E(t, z) = E_0 + \sum_{n=1}^\infty \sqrt{\frac{2}{L}} \beta_n \left[ e^{-\frac{\pi(t(2n - 1))}{2L}} \left( 2r - \frac{1}{\alpha} \sin \alpha \right) \right] + \left[ e^{-\frac{\pi(t(2n - 1))}{2L}} \left( 2r - \frac{1}{\alpha} \sin \alpha \right) \right] \int e^{-\frac{\pi(t(2n - 1))}{2L}} \left( 2r - \frac{1}{\alpha} \sin \alpha \right) dt \sin \frac{\pi(t(2n - 1))}{2L}. \tag{13}
\]

It should be noted that the exponential integral included in expression (12) is not calculated in elementary functions, so an additional evaluation of the behavior of the specified integral is necessary to evaluate the behavior of the Fourier series describing the field strength distribution.

Thus, in contrast to the model that assumed the constancy of the turbulent diffusion coefficient and considered in [4-6], in this case it is not possible to obtain an expression for the intensity entirely by analytical methods.

As a rough estimate of the behavior of the series (13), we can note the negative exponents included in the time component of this series, which, combined with the periodic nature of the coordinate component of the series, allows us to talk about its convergence.

If we talk about a more subtle estimation, then by conducting estimates of the coefficients of the time component of the series (13), similar to those performed in [3, 5], we can show that the coefficients of higher orders are order values \( o(1/(2n-1)^2) \), which, in accordance with the known ratio \( \sum_{n=1}^\infty 1/(2n - 1)^2 = \pi^2 / 8 \), indicates that the upper limit of the coefficients of higher order is a value \( \pi^2 / 8 \).
Thus, the first members of the series play a determining role in the behavior of the function $E(z,t)$, and the remainder of the series will tend to zero. To evaluate the behavior of tension, it is sufficient to use only the first members of the series (13).

In addition to the estimates of the coefficients of the series made in [4, 6], the Matlab™ mathematical package implements a procedure for approximating and evaluating the integral coefficients of the series (13) as part of the construction of theoretical curves. The results of the corresponding numerical experiment are shown in figure 1, where a family of curves describing daily variations of the electric field at a height of 1 m from the surface at different amplitude values of the turbulent diffusion coefficient is constructed. The electric field strength at the earth's surface is equal to $E_0 = -100 \text{ V/m}$ and the thickness of the turbulent electrode layer is $L = 10 \text{ m}$.

![Figure 1](image.png)

**Figure 1.** Daily variations of the electric field under conditions of non-stationary turbulent mixing.

Analyzing the obtained curves of the daily course of the electric field, we can note the morning minimum of 20% in (03-05 UT) and the daily maximum of 30% in (14-16 UT), as well as the second maximum of the electric field 12% in (08-10 UT). The amplitude values of the electric field change with the change in the amplitude of the turbulent diffusion coefficient. There is also a temporary shift of the evening maximum to the area of 16 UT with an increase in the degree of turbulent mixing.

3. **Conclusion**

It is obvious that the observed variations of the electric field are associated with daily changes in the values of the turbulent diffusion coefficient, which is taken into account in the framework of the considered mathematical model. Disturbances in the electric field caused by local changes in the turbulent transport are comparable in value to the global unitary variation due to the potential of the ionosphere.

Therefore, it is necessary to clarify the role of turbulent transport in the surface layer. The fact that turbulent mixing in the atmosphere occurs anywhere on the earth's surface and at the same time has a well-defined daily course, due to the difference in night and day temperatures, reflects the global side of this meteorological process. Of course, the diurnal fluctuations of the electric field associated with turbulent diffusion occur in local time, whereas, for example, the global unitary variation of the electric field potential gradient is manifested simultaneously throughout the globe, regardless of the location of the observation point in the time zone. However, this periodic perturbation of the electric field against the background of other local factors should be taken into account when analyzing
observations of the atmospheric electric field, especially when highlighting variations of global and local origin.

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