On the density dependent hadron field theory at finite temperature
and its thermodynamical consistency

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Abstract

In this work we study in a formal way the density dependent hadron field theory at finite temperature for nuclear matter. The thermodynamical potential and related quantities, as energy density and pressure are derived in two different ways. We first obtain the thermodynamical potential from the grand partition function, where the Hamiltonian depends on the density operator and is truncated at first order. We then reobtain the thermodynamical potential by calculating explicitly the energy density in a Thomas-Fermi approximation and considering the entropy of a fermi gas. The distribution functions for particles and antiparticles are the output of the minimization of the thermodynamical potential. It is shown that in the mean field theory the thermodynamical consistency is achieved. The connection with effective chiral lagrangians with Brown-Rho scaling is discussed.

PACS number(s): 21.65.+f, 21.30.-x, 25.70.-z

1 Introduction

Recently effective field theories have been often used in order to describe hadron properties since the solutions for the fundamental theory of strong interactions, the Quantum Chromo Dynamics (QCD), are still an unreachable problem at the hadron physics energy scale. The study of nuclear matter and finite nuclei properties at finite temperature has become an important problem since a large variety of data, where matter is being tested at
extreme conditions of density, pressure and non-zero temperature, are becoming available in the modern experimental facilities which are already operational.

The motivation for the formulation of Density Dependent Hadron Field Theory (DDHFT)\cite{1,2} may be linked with the relativistic Dirac-Brueckner Hartree-Fock (DBHF) theory\cite{3}. Recently, one approach that has been used\cite{4,5} is to consider the DBHF calculations for nuclear matter only as a guide for a suitable parametrization of density dependence of the meson-baryon coupling operators. In fact, the parameters are adjusted in order to describe the nuclear matter and some finite nuclei properties, as it is done in the usual Walecka model parametrizations\cite{6}. In the present work we discuss how to treat the nuclear matter in the DDHFT at finite temperature and show that, in the mean field approximation (MFA), the thermodynamical consistency is achieved. This is in line with previous calculations done on the same subject, where thermodynamical consistency was assumed\cite{7}. We also show that the mean field approximation is equivalent to the Thomas-Fermi approximation (TFA). We follow two different methods in obtaining our results. We first start from the calculation of the grand partition function in terms of the density dependent Hamiltonian, which is truncated at first order. Once the thermodynamical potential is found, all other thermodynamical quantities can be obtained from it. In the second approach, we start by writing the thermodynamical potential in terms of the energy density calculated in the Thomas-Fermi approximation and the entropy of a fermi gas. The distribution functions for particles and antiparticles are the result of the minimization of the thermodynamical potential.

Furthermore, there is a class of models, including those based on effective chiral lagrangians endowed with the Brown-Rho scaling, to which our formalism and our conclusions can be easily extended. This point is discussed later in this paper.

The paper is organized as follows: in section 2 the DDHFT is presented. In section 3 a mean field approximation is performed and some termodynamic quantities are calculated. In section 4 the Thomas-Fermi approximation is used and the results are compared with the ones obtained in section 3. Finally, in the last section the conclusions are drawn.
2 Density Dependent Hadron Field Theory Formalism

We start from the density dependent hadron field theory Lagrangian density of the well known Walecka-type models [1, 2, 6]:

\[
\mathcal{L} = \bar{\psi} \left[ \gamma_\mu \left( i \partial_\mu - \Gamma_\omega(\hat{\rho}) \omega_\mu - \frac{\Gamma_\phi(\hat{\rho})}{2} \vec{\rho} \cdot \vec{\rho}_\mu - e \left( 1 + \tau_3 \right) A_\mu \right) - (M - \Gamma_\phi(\hat{\rho}) \phi) \right] \psi \\
+ \frac{1}{2} \left( \partial_\mu \phi \partial_\nu \phi - m_\phi^2 \phi^2 \right) - \frac{1}{4} \omega_\mu \omega_\nu + \frac{1}{2} m_\omega^2 \omega_\mu \omega_\mu \\
- \frac{1}{4} \vec{\rho}_\mu \cdot \vec{\rho}_\nu + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}_\nu - \frac{1}{4} F_\mu \nu F_\mu \nu ,
\]

where \( \phi, \omega_\mu, \vec{\rho}_\mu \) and \( A_\mu \) are the scalar-isoscalar, vector-isoscalar and vector-isovector meson fields and the photon field respectively, \( \omega_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), and \( \vec{\rho}_{\mu \nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu - \Gamma_\rho \left( \vec{\rho}_\mu \times \vec{\rho}_\nu \right) \). \( M \) is the nucleon mass, \( m_\phi, m_\omega, m_\rho \) are the masses of the mesons and \( \tau_3 = -1 (\tau_3 = 1) \) for neutrons (protons). The nucleon-meson density dependent coupling operators \( \Gamma_\phi(\hat{\rho}), \Gamma_\omega(\hat{\rho}), \Gamma_\rho(\hat{\rho}) \) are taken to be dependent on a Lorentz scalar functional, \( \hat{\rho} (\bar{\psi}, \psi) \). In what follows we assume the vector density dependence description [1], i.e., \( \hat{\rho}^2 = j^\mu j_\mu \), where \( j_\mu = \bar{\psi} \gamma_\mu \psi \). Actually this has been the functional dependence most often used in recent applications [4, 5, 8]. Notice that other possibilities for the functional dependencies of the coupling operators may be considered as well, for example, \( \Gamma_\phi \) has been taken to be a function of \( \hat{\rho} \bar{\psi} \psi \) in [1].

In this work our emphasis is to consider hadronic models in the context of the Walecka model. However, our formalism can be applied to a larger class of systems, i.e., any Lagrangian density of the type \( \mathcal{L} = \mathcal{L}_F + \mathcal{L}_B + \mathcal{L}_{\text{int}} \), with \( \mathcal{L}_F = \bar{\psi} \left( i \gamma_\mu \partial_\mu - M \right) \psi \) corresponding to the fermion sector, \( \mathcal{L}_B \) to the free bosons and \( \mathcal{L}_{\text{int}} \) being any interaction which can written as \( \mathcal{L}_{\text{int}} = \Gamma_\alpha(\hat{\rho}) \bar{\psi} \Theta_\alpha \psi V^\alpha \), the summation running over the bosons, \( \Theta_\alpha \) containing Dirac and/or isospin matrices and \( V^\alpha \) being any boson field (scalar, vector-isovector, vector-isoscalar, etc).

As usual, the field equations of motion follow from the Euler-Lagrange equations. Here some care has to be taken since the coupling operators are now dependent on the baryon fields \( \bar{\psi} \) and \( \psi \) through \( \hat{\rho} \). Hence, if \( \xi \) stands for a generic field, the Euler-Lagrange equations are:

\[
\frac{\partial \mathcal{L}}{\partial \xi} - \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \xi)} \right) + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \hat{\rho})} \frac{\partial \hat{\rho}}{\partial (\partial_\mu \xi)} = 0 .
\]
When the partial derivatives of $\mathcal{L}$ are performed relatively to the fields $\bar{\psi}$ and $\psi$, they yield extra terms due to the functional dependence of the coupling operators in $\hat{\rho}(\bar{\psi}, \psi)$. These new terms are absent in the usual Quantum Hadrod Dynamic (QHD) model \cite{6}. So, the equations of motion for the fields read:

\begin{align*}
\left( \partial_\mu \partial^\mu + m_\phi^2 \right) \phi &= \Gamma_\phi(\hat{\rho})\bar{\psi}\psi, \\
\partial_\mu \omega^{\mu\nu} + m_\omega^2 \omega^{\mu} &= \Gamma_\omega(\hat{\rho})\bar{\psi}\gamma^{\mu} \psi, \\
\partial_\mu \bar{\rho}^{\mu\nu} + m_\rho^2 \bar{\rho}^{\mu} &= \frac{\Gamma_\rho(\hat{\rho})}{2}\bar{\psi}\gamma^{\mu} \psi, \\
\partial_\mu \Gamma^{\mu\nu} &= \frac{e}{2}\bar{\psi} (1 + \tau_3) \gamma^{\mu} \psi,
\end{align*}

\begin{equation}
[\gamma_\mu \left( i \partial^\mu - \Sigma^\mu(\hat{\rho}) \right) - M^*] \psi = 0 ,
\end{equation}

where $M^* = M - \Gamma_\phi(\hat{\rho})\phi$. Notice that in the equation of motion for the baryon field $\psi$ the vector self-energy consists of two terms, $\Sigma_\mu = \Sigma_\mu^{(0)} + \Sigma_\mu^R$, where:

\begin{align*}
\Sigma_\mu^{(0)} &= \Gamma_\omega(\hat{\rho}) \omega_\mu + \frac{\Gamma_\rho(\hat{\rho})}{2} \bar{\rho}_\mu + \frac{e}{2} (1 + \tau_3) A_\mu, \\
\Sigma_\mu^R &= \left( \frac{\partial \Gamma_\omega(\hat{\rho})}{\partial \hat{\rho}} \omega_\nu j_\nu + \frac{1}{2} \frac{\partial \Gamma_\rho(\hat{\rho})}{\partial \hat{\rho}} \bar{\psi}\gamma^{\nu} \bar{\psi} - \frac{\partial \Gamma_\phi(\hat{\rho})}{\partial \hat{\rho}} \bar{\psi}\psi \bar{\psi} \right) u_\mu,
\end{align*}

where $\Sigma_\mu^{(0)}$ is the usual vector self-energy, $\hat{\rho} u_\mu = j_\mu$ with $u^2 = 1$ and, as a result of the derivative of the Lagrangian with respect to $\hat{\rho}$ a new term appears, $\Sigma_\mu^R$, which is called \textit{rearrangement self-energy} and will play an essential rôle in the applications of the theory. This term guarantees the thermodynamical consistency and the energy-momentum conservation, i.e., $\partial_\mu T^{\mu\nu} = 0$ of the density dependent effective models, where the energy-momentum tensor is given by \cite{1}:

\begin{equation}
T^{\mu\nu} = -g^{\mu\nu} \left[ \bar{\psi} \gamma^\lambda \Sigma_\lambda^{R} \psi + \frac{1}{2} (\partial_\lambda \phi \partial^\lambda \phi - m_\phi^2 \phi^2) - \frac{1}{4} \omega_{\lambda\mu}\omega^{\lambda\mu} + \frac{1}{2} m_\omega^2 \omega^{\lambda\mu} \right. \\
\left. - \frac{1}{4} \bar{\rho}_{\lambda\eta} \cdot \bar{\rho}^{\lambda\eta} \right. \\
\left. + \frac{1}{2} m_\rho^2 \bar{\rho}_\lambda \cdot \bar{\rho}^\lambda - \frac{1}{4} F_{\lambda\eta} F^{\lambda\eta} \right] \\
+ \bar{\psi} i \gamma^\mu \partial^\nu \psi + \partial^\mu \phi \partial^\nu \phi + \partial^\mu \omega_\lambda \omega^{\lambda\mu} + \partial^\nu \bar{\rho}_\lambda \cdot \bar{\rho}^\lambda + \partial^\nu A_\lambda F^{\lambda\mu}.
\end{equation}

The Hamiltonian operator follows from the $T_{00} \equiv \mathcal{H}$ component of the energy-momentum tensor in the form:

\begin{align*}
\mathcal{H} &= \int d^3 x \mathcal{T}_{00} \\
&= \int d^3 x \left\{ \bar{\psi} \left\{ -\hat{\alpha} \cdot \left( i \nabla + \Sigma(\hat{\rho}) \right) + \beta (M - \Gamma_\phi(\hat{\rho})\phi) \right\} \psi - \bar{\psi} \gamma^\mu \Sigma_\mu R \psi + \frac{1}{4} F_{\lambda\eta} F^{\lambda\eta} \\
&\quad - \frac{1}{2} \left( \partial_\mu \phi \partial_\mu \phi - m_\phi^2 \phi^2 \right) + \frac{1}{4} \omega_{\lambda\mu}\omega^{\lambda\mu} - \frac{1}{2} m_\omega^2 \omega^{\lambda\mu} + \frac{1}{4} \bar{\rho}_{\lambda\eta} \cdot \bar{\rho}^{\lambda\eta} - \frac{1}{2} m_\rho^2 \bar{\rho}_\lambda \cdot \bar{\rho}^\lambda \right\}.
\end{align*}
Formally, the thermodynamics of the DDHFT follows from the calculation of the grand partition function:

$$Z_G = Tr \exp \left[ -\beta \left( H(\hat{\rho}) - \sum_{i=p,n} \mu_i \hat{B}_i \right) \right] \equiv \exp [-\beta \Omega],$$

(12)

where the trace goes over a complete set of states in the Fock space, $\beta$ is the inverse of the temperature and $\hat{B}_p$ and $\hat{B}_n$ correspond to the proton and neutron number operator respectively, such that the baryon number operator is $\hat{B} = \hat{B}_p + \hat{B}_n$. Although the equations of motion, eqs.(3-7), seem to be similar to the usual QHD ones [6], it should be noticed that the source terms for the meson fields contain in-medium correlations through the density dependence of the vertices and the rearrangement term contributes explicitly for the baryon field equation. Of course the solution of these field equations, as they stand, is a formidable task. For this reason, we are forced to consider approximations in order to obtain a tractable problem. Hence, we consider the DDHFT in the mean field approximation for nuclear matter at finite temperature.

3 DDHFT for nuclear matter in the mean field approximation

We assume in this work, for the description of the nuclear matter, static meson fields, rotational and translational symmetry, charge conservation and no Coulomb field [9]. Therefore, all time and spatial derivatives vanish and only time-like components of the $\phi$, $\omega$ and $\rho$ meson fields remain. In the mean field approximation, the meson fields are replaced by classic condensed fields and the time-like components of the $\phi$, $\omega$ and $\rho$ condensed meson fields are called $\phi_0$, $\omega_0$ and $b_0$ respectively. In this approximation, only $\bar{\psi}$ and $\psi$ are taken to be quantized field operators. The meson fields are obtained by thermal averages of the corresponding sources. The involved approximations are discussed later. In the nuclear rest frame, consistently with the MFA, we have $\hat{\rho} = \bar{\psi}\gamma_0\psi$. Under these approximations the Hamiltonian of eq.(11) reads:

$$H = \int d^3x \mathcal{H}$$

$$= \int d^3x \left\{ \psi^\dagger \left[ -i\vec{\alpha} \cdot \nabla + \beta(M - \Gamma_0(\hat{\rho})) + \Sigma_0^{(0)}(\hat{\rho}) \right] \psi ight.$$  

$$+ \frac{1}{2} m_0^2 \phi_0^2 - \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} m_\rho^2 b_0^2 \} \right.$$  

(13)
where

\[ \Sigma_0^{(0)} = \Gamma_\omega(\hat{\rho})\omega_0 + \frac{\Gamma_\rho(\hat{\rho})}{2}\tau_3 b_0. \]  

(14)

It is important to emphasize that in the MFA the rearrangement term cancels out in the energy density, as a consequence of the cancellation of this term in the exact Hamiltonian of equation (11), since the density dependent interaction terms in the Lagrangian density of equation (1) involves no derivatives with respect to \(\hat{\rho}\).

Now we proceed to calculate the partition function. In order to perform the trace calculation in (12), we follow the same procedure used in the calculation of quantum gases with density dependent interactions [10] and in the molecular field approximation to the Heisenberg Hamiltonian [11]. For the evaluation of the trace in eq.(12) it is necessary that, in a convenient basis of states, the exponential function decomposes itself in a sum of independent terms. As it is seen next, this can be done only when we approximate the Hamiltonian operator in eq.(13) by a one-body operator, obtaining a linear term in the baryon number operator. Analogously to ordinary statistical mechanics problems [10] we take the Hamiltonian expanded around the equilibrium mean density \(\rho_0 = \langle \hat{\rho} \rangle\). We then keep only terms up to the first order in \(\hat{\rho} - \rho_0\). Higher order terms are going to be neglected.

In the DDHFT it is assumed that the functional form for the density dependent coupling operators is sufficiently well behaved as a function of \(\rho\). In eqs. (13-14) we expand the vertices around \(\rho_0\) in the form,

\[ \Gamma_i(\hat{\rho}) = \Gamma_i(\rho_0) + \left. \frac{\partial \Gamma_i}{\partial \hat{\rho}} \right|_{\hat{\rho} = \rho_0} (\hat{\rho} - \rho_0) + \cdots \]  

(15)

and substitute the normal ordered scalar and isovector density by their thermal averages

\[ \bar{\psi}\psi \to \langle \bar{\psi}\psi \rangle = \rho_s , \]

\[ \bar{\psi}\tau_3\gamma_0\psi \to \langle \bar{\psi}\tau_3\gamma_0\psi \rangle = \rho_3 . \]

In this way we obtain, for the Hamiltonian density up to first order in \(\hat{\rho} - \rho_0\): 

\[ \mathcal{H} = \mathcal{H}(\rho_0) + \Sigma_0^R(\rho_0)(\hat{\rho} - \rho_0) + \cdots \]  

(16)

where

\[ \Sigma_0^R(\rho_0) = \left. \frac{\partial \Gamma_\omega(\hat{\rho})}{\partial \hat{\rho}} \right|_{\hat{\rho} = \rho_0} \omega_0\rho_0 + \left. \frac{1}{2} \frac{\partial \Gamma_\rho(\hat{\rho})}{\partial \hat{\rho}} \right|_{\hat{\rho} = \rho_0} b_0\rho_3 - \left. \frac{\partial \Gamma_\phi(\hat{\rho})}{\partial \hat{\rho}} \right|_{\hat{\rho} = \rho_0} \phi_0\rho_s . \]
To calculate the trace in eq. (12), we use the single particle basis associated with the Dirac field equation, eq. (7), calculated at density $\rho_0$, where $\rho_0 = B/V$ is the equilibrium nuclear matter density. This equation can be solved exactly if one seeks stationary solutions of the plane wave form, $\psi = \psi(p) \exp(i\mathbf{p} \cdot \mathbf{x} - i\varepsilon(p)t)$, where $\psi(p)$ is a four component spinor.

Such procedure is well known in the literature and details can be found in references [6, 9].

So, in the MFA, the baryon spinors are eigenvectors of the stationary Dirac equation:

$$(-i\vec{\alpha} \cdot \nabla + \beta(M - \Gamma_0(\rho_0)\phi_0) + \Sigma_0(\rho_0)) \psi = \varepsilon \psi, \quad (17)$$

and the baryon energy is given by:

$$\varepsilon(p) \equiv \varepsilon^{(\pm)} = \Gamma_0 \omega_0 + \frac{\Gamma_0}{2} \tau_3 b_0 + \Sigma_0^R \pm \left(p^2 + M^*\right)^{1/2},$$

where $M^* = M - \Gamma_0(\rho_0)\phi_0$. Going through the usual steps to solve the Dirac equation, one finds that $\varepsilon^{(+)}(\varepsilon^{(-)})$ corresponds to the baryon $u(p)$ (anti-baryon $v(p)$) four component spinor. Proceeding as in the ordinary Walecka model [6], both positive and negative energy states $u$ and $v$ are found and the field operator, in the Schrödinger picture, can be expanded as:

$$\psi = \sum_{p\lambda} \left( u(p, \lambda) \ a_{p\lambda}^\dagger + v(p, \lambda) \ b_{p\lambda}^\dagger \right).$$

In the summation $\lambda$ stands for the spin and isospin projections. The operators $a_{p\lambda}^\dagger$ and $b_{p\lambda}^\dagger$ are interpreted as creation operators for baryons and anti-baryons and satisfy anticommutation relations [6]. Thus, substituting the field operators into eq. (13) and taking into account the expansion given in eq. (16) the normal ordered Hamiltonian takes the form:

$$H = \sum_{p\lambda} \left( p^2 + M^*\right)^{1/2} \left( a_{p\lambda}^\dagger a_{p\lambda} + b_{p\lambda}^\dagger b_{p\lambda} \right) + \left( \Gamma_0 \omega_0 + \Sigma_0^R(\rho_0) \right) \hat{B} + \frac{\Gamma_0}{2} b_0 \hat{B}_3 + V \left( \frac{1}{2} m_0^2 \phi_0^2 - \frac{1}{2} m_0^2 \omega_0^2 - \frac{1}{2} m_0^2 b_0^2 - \Sigma_0^R(\rho_0) \rho_0 \right), \quad (18)$$

where $\hat{B}_3 = \hat{B}_p - \hat{B}_n$, $V$ is the volume of the system and the nucleon number operator is given by

$$\hat{B}_i = \sum_{ps} \left( a_{ps\tau_3(i)}^\dagger a_{ps\tau_3(i)} - b_{ps\tau_3(i)}^\dagger b_{ps\tau_3(i)} \right), \quad i = p, n.$$

Since in the present approximation all operators in the exponential function defining the partition function, eq. (12), are diagonal in the basis of eigenstates of the baryon and
antibaryon number operators, the grand partition function can be exactly calculated. The results are analogous to those for the Walecka model reading

\[
\Omega(\beta, \mu_p, \mu_n; \phi_0, \omega_0, b_0) = -\frac{1}{\beta} \ln Z_G
\]

\[
\Omega = V \left( \frac{1}{2} m^2_\phi \phi_0^2 - \frac{1}{2} m^2_\omega \omega_0^2 - \frac{1}{2} m^2_P b_0^2 - \rho_0 \Sigma_R^0 \right)
\]

\[
- \frac{V}{\beta (2\pi)^3} \sum_{i=p,n} \sum_{\mathbf{p}} \left[ \ln \left( 1 + e^{-\beta (E^*(\mathbf{p}) - \mu_i^*)} \right) \right] + \ln \left( 1 + e^{-\beta (E^*(\mathbf{p}) + \mu_i^*)} \right) ,
\]

(19)

where \( E^*(\mathbf{p}) = (\mathbf{p}^2 + M^*^2)^{1/2} \) and the effective chemical potentials, \( \mu^*_p \) and \( \mu^*_n \), are defined as

\[
\mu^*_p = \mu_p - \Gamma_\omega \omega_0 - \frac{\Gamma_\rho}{2} b_0 - \Sigma_R^0 ,
\]

\[
\mu^*_n = \mu_n - \Gamma_\omega \omega_0 + \frac{\Gamma_\rho}{2} b_0 - \Sigma_R^0 .
\]

(20)

Notice the explicit contribution of the rearrangement term in the above expressions. The pressure is obtained through the relation \( P = -\Omega/V \) and the energy density by:

\[
\mathcal{E} = \frac{E}{V} = \frac{1}{V} \frac{\partial (\beta \Omega)}{\partial \beta} + \mu_p \rho_p + \mu_n \rho_n .
\]

(21)

The finite temperature meson field equations can be determined by assuming that the thermodynamic potential \( \Omega \) be stationary for variations of these fields. This is to be expected for a system in thermodynamical equilibrium. So,

\[
\frac{\partial \Omega}{\partial \xi} = 0, \quad \xi = \omega_0, b_0, \phi_0
\]

yield the coupled equations

\[
m^2_\phi \phi_0 - \Gamma_\phi (\rho_0) \rho_s = 0 ,
\]

(22)

\[
m^2_\omega \omega_0 - \Gamma_\omega (\rho_0) \rho_0 = 0 ,
\]

(23)

\[
m^2_b b_0 - \Gamma_\rho (\rho_0) \rho_3 = 0 ,
\]

(24)

where the thermal scalar and baryonic densities are defined as

\[
\rho_s = \langle \bar{\psi} \psi \rangle = 2 \sum_{i=p,n} \int \frac{d^3 p}{(2\pi)^3} \frac{M^*}{E^*(\mathbf{p})} (f_{i+} + f_{i-}) ,
\]

(25)

\[
\rho_0 = \langle \bar{\psi} \gamma^0 \psi \rangle = \rho_p + \rho_n ,
\]

(26)

\[
\rho_3 = \langle \bar{\psi} \gamma^3 \tau_3 \psi \rangle = \rho_p - \rho_n ,
\]

(27)
with the distribution functions given by
\[ f_{i\pm} = \frac{1}{1 + \exp[(E^*(p) \mp \mu_i^*)/T]} , \quad i = p, n \] (28)
and the proton and neutron densities \( \rho_p \) and \( \rho_n \) reading
\[ \rho_i = 2 \int \frac{d^3p}{(2\pi)^3} (f_{i+} - f_{i-}) , \quad i = p, n. \] (29)

The equations of motion, eqs.(22-24), are completely consistent with the MFA. They follow from eqs.(3-7) when the sources are approximated by their thermal averages as discussed above. For example, for the scalar field source, \( \Gamma_\phi(\hat{\rho}) \bar{\psi} \psi \rightarrow \Gamma_\phi(\rho_0) \rho_S \), etc.

Next, we reobtain, within the Thomas-Fermi approximation, the same expressions just obtained for the pressure, energy density and meson fields, demonstrating the consistency of our approximations.

4 DDHFT in the Thomas-Fermi approximation

We first define the thermodynamical potential, following thenotation in [12], as
\[ \frac{\Omega}{V} = \mathcal{E} - T S - \mu_p \rho_p - \mu_n \rho_n , \] (30)
where \( \mathcal{E}, S \) are the energy and entropy density respectively, \( T \) is the temperature, \( \mu_p, \mu_n \) is the proton (neutron) chemical potential and \( \rho_p, \rho_n \) are respectively the proton and neutron densities as given in eq.(29), calculated in such a way that \( \rho = \rho_p + \rho_n \). Here, the distribution functions \( f_{i+} \) and \( f_{i-} \) for particles and anti-particles have to be derived in order to make the thermodynamic potential stationary for a system in equilibrium. The entropy density \( S \) is obtained from
\[ S = -2 \sum_{i=p,n} \int \frac{d^3p}{(2\pi)^3} \left( f_{i+} \ln \left( \frac{f_{i+}}{1 - f_{i+}} \right) + \ln(1 - f_{i+}) + (f_{i+} \Leftrightarrow f_{i-}) \right) . \] (31)

Next, we obtain the energy density in the TFA, where the conserved energy-momentum tensor, eq.(10), becomes:
\[ T^{\mu\nu} = \bar{\psi} \gamma^\mu \partial^\nu \psi - g^{\mu\nu} \left[ -\frac{1}{2} m^2_\phi \phi^2_0 + \frac{1}{2} m^2_\omega \omega^2_0 + \frac{1}{2} m^2_\rho \rho^2_0 + \bar{\psi} \gamma^0 \Sigma^R \psi \right] . \] (32)

From the energy-momentum tensor one easily obtains the Hamiltonian operator, which is the same as already displayed in equation (13) and from it, the energy density operator,
which reads [6]:

\[
\mathcal{E} = \frac{1}{V} \sum_{p\lambda} \left( p^2 + M^{*2} \right)^{1/2} \left( a_{p\lambda}^+ a_{p\lambda} + b_{p\lambda}^+ b_{p\lambda} \right) + \Gamma_0 \omega_0 \frac{1}{V} \sum_{p\lambda} \left( a_{p\lambda}^+ a_{p\lambda} - b_{p\lambda}^+ b_{p\lambda} \right) \\
+ \frac{\Gamma_0^2}{2} b_0 \frac{1}{V} \sum_{p\tau_3} \tau_3 \left( a_{p\tau_3}^+ a_{p\tau_3} - b_{p\tau_3}^+ b_{p\tau_3} \right) + \left( \frac{1}{2} m_\phi^2 \phi_0^2 - \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} m_\rho^2 b_0^2 \right), \tag{33}
\]

where \( \lambda = \{ s, \tau_3 \} \) are the spin and isospin indexes. In obtaining the above equation we have assumed that the expectation values of the meson fields are constant classical fields. Moreover the Hamiltonian density is simply \( \mathcal{H}(\rho_0) \), contrary to the expression (16) used in the calculation performed in the last section. The energy density in the TFA is obtained through the substitution of the baryon and antibaryon number operators by their thermal averages. So, the energy density can be written in the semi-classical TFA as

\[
\mathcal{E} = 2 \sum_{i=p,n} \int \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + M^{*2}} (f_{i+} + f_{i-}) + \Gamma_0 \omega_0 \rho_0 + \frac{\Gamma_0}{2} b_0 \rho_3 \\
+ \frac{m_\phi^2}{2} \phi_0^2 - \frac{m_\omega^2}{2} \omega_0^2 - \frac{m_\rho^2}{2} b_0^2. \tag{34}
\]

After straightforward substitutions, eq.(30) becomes

\[
\Omega = 2 \sum_{i=p,n} \int \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + M^{*2}} (f_{i+} + f_{i-}) + \Gamma_0 \omega_0 \rho_0 + \frac{\Gamma_0}{2} b_0 \rho_3 \\
+ \frac{m_\phi^2}{2} \phi_0^2 - \frac{m_\omega^2}{2} \omega_0^2 - \frac{m_\rho^2}{2} b_0^2 \\
+ 2T \sum_{i=p,n} \int \frac{d^3p}{(2\pi)^3} \left( f_{i+} \ln \left( \frac{f_{i+}}{1 - f_{i+}} \right) + \ln(1 - f_{i+}) + f_{i-} \ln \left( \frac{f_{i-}}{1 - f_{i-}} \right) + \ln(1 - f_{i-}) \right) \\
- 2 \sum_{i=p,n} \int \frac{d^3p}{(2\pi)^3} \mu_i(f_{i+} - f_{i-}). \tag{35}
\]

For a complete demonstration of the above shown expressions obtained in a Thomas-Fermi approximation for the non-linear Walecka model, please refer to [12]. At this point, eq.(35) is minimized in terms of the distribution functions for fixed meson fields, i.e.,

\[
\frac{\delta \Omega}{\delta f_{i+}} \bigg|_{f_{i-}, f_{j \pm}, \phi_0, \omega_0, b_0} = 0 \quad i \neq j. \tag{36}
\]

For the proton distribution function, the above calculation yields

\[
E^*(p) + \Sigma_0^R - \mu_p + \Gamma_0 \omega_0 + \frac{\Gamma_0}{2} b_0 = -T \ln \left( \frac{f_{p+}}{1 - f_{p+}} \right). \tag{37}
\]

Similar equations, with some sign differences are obtained for the anti-proton, neutron and anti-neutron distribution functions. The effective chemical potentials are the same as
defined in eq. (20) and from the minimization we reobtain for the distribution functions the expression given in eq. (28). In other words, with the distribution functions given in eqs. (26) and (27) substituted into equation (35), equation (19) is exactly reproduced. In the above calculation we have used \( \rho_0 \) and \( \rho_3 \) as defined in eqs. (26) and (27) respectively. Within the Thomas-Fermi approach the pressure becomes

\[
P = \frac{1}{3 \pi^2} \sum_{i=p,n} \int dp \frac{p^4}{\sqrt{p^2 + M^2}} (f_i^+ + f_i^-) - \frac{m^2}{2} \phi_0^2 \left( 1 + 2 \frac{\rho}{\Gamma_\phi} \frac{\partial \Gamma_\phi}{\partial \rho} \right) \\
+ \frac{m^2}{2} \omega_0^2 \left( 1 + 2 \frac{\rho}{\Gamma_\omega} \frac{\partial \Gamma_\omega}{\partial \rho} \right) + \frac{m^2}{2} b_0^2 \left( 1 + 2 \frac{\rho}{\Gamma_\rho} \frac{\partial \Gamma_\rho}{\partial \rho} \right).
\]

(38)

Again we recover our previous expressions for the pressure and the energy density. It is also important to stress that the thermodynamical consistency, discussed in [1], which requires the equality of the pressure calculated from the thermodynamical definition \( \Omega = -PV \), where \( \Omega \) is given in equation (19) or (35) and from the energy-momentum tensor, is also obeyed by the temperature dependent DDHFT. This can be easily verified calculating the pressure from the thermal average of the energy-momentum tensor (32) in the MFA as

\[
P = \frac{1}{3} \sum_{i=1}^{3} \langle T_{ii} \rangle.
\]

(39)

5 Conclusions

In this paper we have incorporated temperature effects in the DDHFT formalism introduced in [2, 4]. Two different approximations were performed, namely, a mean field approximation where the coupling operators are expanded in powers of the density and the Thomas-Fermi approximation. In the first case, the Hamiltonian is expanded in powers of \( \hat{\rho} - \rho_0 \) and the Dirac equation is solved yielding the baryon energies. The grand partition function is calculated and the thermodynamic quantities are derived. In the second case, the thermodynamical potential is the starting point, where the energy density is calculated in the semi-classical TFA. The thermodynamical potential is minimized in terms of the unknown distribution functions, which are then obtained. Thermodynamic consistency is proved.

As a final remark we discuss briefly another possible application of the formalism we have derived above, i.e., the study of effective chiral lagrangians with Brown-Rho (BR) scaling. In 1991 Brown and Rho (BR) [13] proposed an in-medium scaling law for the
masses and coupling constants for effective chiral lagrangians. In ref. [14] the authors proposed an effective Lagrangian whose parameters scale in nuclear medium according to the BR scaling. In its simplest version the Lagrangian in the MFA is [14, 15]

\[ \mathcal{L} = \bar{\psi} \left[ \gamma_\mu (i \partial^\mu - g_\nu^\rho (\rho) \omega^\mu) - M^\ast (\rho) + h \phi \right] \psi \\
+ \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - \frac{m_\phi^2 (\rho)}{4} \phi^2 \right) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 (\rho) \omega_\mu \omega^\mu. \] (40)

where in the notation of ref. [15] \( \psi \) is the nucleon field, \( \omega_\mu \) the isoscalar vector field, \( \phi \) an isoscalar scalar field and the masses with asterisk are BR-scaled as introduced in ref. [13]. The scaling of the vector coupling constant is left arbitrary and \( h \) is taken constant. So, the Lagrangian in (40) is of the form of a Walecka-like Lagrangian and all the finite temperature formalism that we have developed for the DDHFT can be immediately applied to these lagrangians. The thermodynamics of effective lagrangians with BR scaling has been studied in [16] for zero temperature. The study of the validity of the BR scaling hypothesis for the non-zero temperature case is currently under investigation.

Acknowledgments

This work was partially supported by CNPq - Brazil. We would like to thank Dr. Marcelo Henrique Romano Tragtenberg and Dr. Constança Providência for very useful suggestions and productive discussions related with this work.

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