Vehicle-bridge Interaction Analysis Based on the ANCF Quasi-conforming Plate Technique

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Abstract. A new plate element is developed for analysis of plate structures in vehicle-bridge interaction analysis based on the combining of absolute nodal coordinate formulation (ANCF) and quasi-conforming technique (QCT). In order to simulate complex contact and large deformation during vehicle-bridge interaction (VBI) for the slender bridge, new curvature strains and explicit formulation of internal forces are developed for the shell elements of the bridge deck. The developed QCT_ANCF shell element is compared with the original ANCF element to verify its locking remedies. Compared with the original model, the new QCT_ANCF element shows better convergence and curvature continuity and is more accurate under the same number of elements. Numerical cases are analyzed using the QCT_ANCF element in comparison to analytical solutions and the original ANCF shell element. Meanwhile, there is less high frequency vibration in the velocity and acceleration curve by comparing with the original model. Furthermore, the vehicle-bridge interaction is parametrically analyzed using the new QCT_ANCF element under series of road roughness index and vehicle speeds. The impact factors based on the displacements and strains over the transverses of the bridge are investigated.

1. Introduction

Bridge-vehicle interaction (VBI) and road surface roughness are very essential to check the true capacity of existing bridges to heavier traffic and for proper design of new bridges. Plate is the most suitable element for these slender bridges among finite element method, which could not only simulate the 3D deformation contrast to the beam element but also save the computation contrast to the solid element. González et al [1] studied the structural deformation based on the Kirchhoff plate element. Kwasniewski et al [2] took 612000 solid elements on LS-DYNA to simulate the bridge on VBI analysis, and the computation had to conduct on a cloud platform. Nassif & Liu [3] simulated the bridge and vehicle interaction by beam element. During VBI analysis, the vehicle structural system and the bridge structural system deform separately, and the contact boundaries between the two systems are changing with time. Therefore, the VBI is a nonlinear procedure which could be taken as a multibody problem. So, it is a natural choice to take absolute nodal coordinate formulation (ANCF) to conduct the VBI analysis. The generalized coordinates of ANCF are absolute displacements and slopes and make no small deformation assumptions [4-6]. The mass matrix of the plate element keeps constant, which will reduce the degree of non-linearity of the dynamic equations [7]. Its discontinuities in bending strain would bring unrealistic strain energy to the element, resulting in an increased stiffness, element distortion, and locking [8, 9]. Many studies were carried out to take care of the locking, such as redefined polynomial expansion [10], selective integration procedure [11-13],
modifying the constitutive relations [14], and using higher order geometry elements. Nowadays, selective integration is the primary method to solve the locking problems. However, it is constrained in boundary conditions. Tang et al. [15] presented the quasi-conforming technique. Instead of displacement fields traditionally, the strain fields are described by the polynomials [16]. As reported, the QCT elements are free from shear and membrane locking phenomena, and spurious kinematic modes.

This study, applying the quasi-conforming techniques, presents an ANCF element, namely QCT_ANCF element. Compared with the original ANCF element, this element matrix could be explicitly integrated or numerically applied by modifying the original code minimally. The element exhibits its computational efficiency and is robust to distortion.

2. ANCF Thin Plate Element
The coordinates at the node of the plate element are defined as \( r_p \):
\[
\mathbf{r}_p = S(x_p, y_p, z_p)\mathbf{e}(t_p)
\]  
(1)

A lower order ANCF plate element (Figure 1) developed by Gerstmayr and Shabana is introduced in this section. For the freedom of each node, there are only three position vector and two gradient vector obtained by differentiating with respect to coordinates in element plane. The displacements are interpolated in x and y by polynomials with three orders.

![Figure 1. Kinematics of the 36 d.o.f. element by Dufva & Shabana](image)

The axial curved strain in axis of element is as follows [17]:
\[
\kappa = \left| \begin{array}{c} \mathbf{r}_x \\ \mathbf{r}_x \end{array} \right| \cdot \left| \begin{array}{c} \mathbf{r}_x \\ \mathbf{r}_x \end{array} \right| \]  
(2)

3. Formulation of absolute nodal coordinate cable/beam element based on the quasi-conforming techniques
In Eq. (2), the curvatures \( r_{ax} \) are discontinuous, which could be the sources of lockings. The Quasi-Conforming technique is taken as the tool to take care of the problems. Described the strain fields by the polynomials, the formulation of Quasi-Conforming cable/beam element, named as QCT_ANCF, is briefly presented.

In Eq. (4) and Eq. (5), the strain resultant is as follows:
\[
\mathbf{r}_{ax} = \frac{\partial^2 \mathbf{r}}{\partial x^2} = \mathbf{a}^T \mathbf{P}_x, \quad \mathbf{P}_x = [2 \quad 6x]^T, \quad \mathbf{a} = \begin{bmatrix} a_x \\ a_y \end{bmatrix}, \quad \mathbf{a} = \begin{bmatrix} a_x \\ a_y \end{bmatrix}
\]  
(3)

Integral domain can be written as \([x_1, x_2]\). Using weighted integration with respect to x by weight \( P_x \), the curvature strain can be computed as:
\[
\int_{S_1} \frac{\partial^2 r}{\partial x^2} P_x dx = a \int_{S_1} P_x P_x^T dx, \quad \int_{S_1} P_x P_x^T dx = A
\]  \hspace{1cm} (4)

To evaluate the right hand side integral of Eq. (4) evaluated, the Green’s theorem is used. As composed by polynomials on the left side of (4), it may be integrated easily.

\[
\int_{S_1} \frac{\partial^2 r}{\partial x^2} P_x dx = P_x \frac{\partial r}{\partial x} \bigg|_{S_1} - \int_{S_1} \frac{\partial r}{\partial x} \frac{\partial P_x}{\partial x} dx = Ce
\]  \hspace{1cm} (5)

The parameters for the assumed strain as in Eq. (3) could be expressed as:

\[
a = A^{-1} Ce
\]  \hspace{1cm} (6)

In order to obtain the strain discretization matrix, B, equations (6) to (3) is substituted into curvature strain term. The element strains could be expressed as:

\[
r_{ee} = P^T a = P^T A^{-1} Ce
\]  \hspace{1cm} (7)

4. Engineering application

In the research, the bridge deck is simulated by QCT_ANCF plate element and the vehicle model is composed by mass-spring-damper. The plate is simple supported. The parameters are shown as Table 1 and Table 2.

| Table 1. Parameters of the bridge deck |
|---------------------------------------|
| element type | QCT_ANCF element type | ANCF mass-spring-damper |
| Dimensions (m) | 24m*12m*0.8m | Load type | track |
| elastic modulus | 3.0E9 | Geometric params | axil distance in longitude 5.5m; axil distance in transverse 2m |
| passion ratio | 0.15 | tire params | elastic modulus 1E6; damper 3E3 |
| density (m) | 1500kg/m³ | weight | vehicle 165kN; front gear 30kN; back gear 60kN |
| dimension | 1.2*1.0 | moment | Ix: 4.5E5, Iy: 4.5E5 |

Time-displacement curve at the midpoint of the span under the speed of 60Km/h is shown in Figure 2. As can be seen, with the large of the road rough index, the displacement of the middle became larger.
5. Conclusion
1) This QCT_ANCF element does not depend on the higher order or assuming strain, and suffers less from locking and enjoys better convergence characteristics in simulating the flexible beams, etc. Therefore, the quasi-conforming technique could be a quick, easy, and cheap way to improve the computing performance for absolute nodal coordinate method.
2) With the element, the analysis suitable for the VBI are conduct under the united platform, and the convergent and accurate are promoted obviously.

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7. References
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