Research Article

A Nonlinear Strength Criterion of Frozen Saline Sandy Soil at High Pressures

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The strength of frozen saline soil is influenced by various factors, such as confining pressure, salt content, and temperature. To investigate the influences of these factors on strength parameters, a series of triaxial tests were performed for frozen saline samples. According to the test results, as the mean principal stress increases, the failure strength increases firstly and then reaches a peak value, followed by a decreasing tendency. The pressure corresponding to ice melting is higher for frozen samples at a lower temperature. A strength function, which consists of cohesion and friction components, is developed in $p-q$ plane. Then, the relationships between temperature and strength parameters are given by fitting the test data. The envelope between those for the SMP and von Mises criteria is employed to describe the strength properties in $\pi$ plane. By comparing the predicted data with test results, the developed strength criterion can well reproduce the strength variation of frozen sandy samples considering the effects of confining pressure, salt content, and temperature.

1. Introduction

Frozen soil, accounting for about 50% of continent area in the world, is a special kind of geomaterial which contains soil particles, unfrozen water, gaseous inclusions, and ice crystals. In China, frozen soil is mostly spread across high mountains in the northwest, Qinghai-Tibet Plateau, northern part of the northeast, Songnen Plain, and north of the Qinling Mountain-Huaihe River line [1]. Besides the general compositions of frozen soil, frozen saline soil contains salt solution and salt crystals. As more and more engineering projects are constructed in cold areas, study on mechanical properties of frozen soils is important. The concentration of salt solution can have an impact on freezing point, which will influence the mechanical response of saline soil. Besides, salt crystals fill in the pores between soil particles, causing the variation of soil structures. In addition to salt concentration, some other factors, including stress state and temperature, will also affect the mechanical characteristics of frozen saline soil. Seasonal variation of temperature and variation induced by global warming may have considerable effect upon the behavior of frozen soil. The high stress concentration conditions are widely encountered in some typical engineering activities (e.g., mining project, tunnels, and underground power stations), and they also exert a considerable impact on mechanical features of frozen geomaterials [2]. Thus, further study on the strength properties of frozen saline soil at various complicated conditions is needed.

Given common geomaterials, many studies have been performed to investigate their strength properties [3–9] and some strength criteria are proposed [10, 11]. Mohr-Coulomb criterion, which can be described by a linear function, is widely employed to describe the shear strength characteristics of soils. The strength of a geomaterial depends on the stress state, and it is also affected by the cementation, reinforcement, and soil structure. In general, the strength behavior may be nonlinear when the pressure is over a large range. Various nonlinear strength criteria have been proposed for different kinds of solid materials [12–18]. To illustrate the strength properties of concrete, Du et al. proposed a
nonlinear unified criterion [19]. A unified formula was employed to describe the nonlinear strength properties in meridian and deviatoric planes. The failure curve in deviatoric plane considered the effect of intermediate principal stress while that in meridian plane considered the influences of cohesion and mean principal stress. Xiao et al. developed a new strength criterion of frictional geomaterials and applied it to the modified Cam-Clay model [20]. The nonlinear and cross-anisotropic properties of strength for different types of soils can be captured by the developed strength criterion.

As for frozen soils, the strength characteristics can be rather complicated since the pore ice crystals can be crushed and melt at relatively high stresses. Many investigations were performed to research the strength of frozen geomaterials [21–28]. Compared with general geomaterials, the influence factor of frozen soil strength is more complicated, such as pressure, dry density, and temperature. Typically, the strength increases firstly and decreases afterwards as confining pressure increases [29]. When the confining pressure is relatively high, pressure melting of ice may occur in frozen soils, which will result in a decrease of strength [30–32]. As the temperature reduces, the strength shows an increasing tendency and various relationships between strength and negative temperature have been proposed [1]. The salt type and salt content in soil samples affect the freezing point, unfrozen water content, and cohesion. Similar to unfrozen fine-grained soils, the strength of frozen geomaterials is also comprised of cohesion and friction components [33].

Numerous scholars studied the strength properties of frozen geomaterials based on experimental data rather than theoretical analysis [34–37]. Alkire and Andersland pointed out that the cohesive component of strength of sand-ice materials is affected by ice crystals while the frictional component exhibits similar features as unfrozen sand at high confining pressures [38]. Considering that the confining pressure affects both cohesion and friction strength of frozen soils, Qi and Ma developed a new strength criterion at different temperatures [39]. Compared with the experimental data of frozen sand, the compressive strength of frozen sand can be well captured by the proposed criterion. However, the proposed criterion cannot reflect the tensile strength properties. Based on the generalized nonlinear strength theory, Liao et al. established a nonlinear strength criterion for frozen saline geomaterials [40]. The effects of salt content and pressure melting are well reproduced. However, the effect of negative temperature is not considered, and the physical meaning of pressure melting parameters is not clear.

To investigate the effect of temperature on strength of frozen saline soils, conventional triaxial tests at various confining pressures were firstly performed in this study. A strength function, composed of cohesion and friction components, was then developed in $p$-$q$ plane. According to SMP and Mises criteria, the shape curves in $\pi$ plane were given. The proposed strength criterion was verified by comparing the predicted results with test data.

2. Test Results

The triaxial test was performed for frozen sandy soils with the salt contents of 0%, 0.5%, 1.5%, 2.5%, and 3.5% by weight at different temperatures of -4°C, -6°C, -8°C, and -10°C. A coarse sand was employed, and the salt adopted was anhydrous Na$_2$SO$_4$ fine particles. The specific gravity of the tested coarse sand is 2.63, and the grain size composition is shown in Table 1. The freezing temperature test was performed for saline sandy soil. The freezing temperatures of samples with 0%, 0.5%, 1.5%, 2.5%, and 3.5% salt contents are -0.13°C, -0.96°C, -1.35°C, -1.40°C, and -1.60°C, respectively. The confining pressure, ranging between 0 MPa and 16 MPa, was used during the consolidation stage in the test. During the shearing process, the axial strain was chosen as 1.67 × 10$^{-4}$ s$^{-1}$. The main testing procedures of sample preparation and triaxial test can be referred to Chang et al. [41].

Some of the triaxial test results are shown in Figure 1. Confining pressure affects the stress-strain relation considerably. For samples under confining pressure lower than 6 MPa at various temperatures, an obvious strain softening phenomenon is observed. The pattern of stress-strain curve becomes strain hardening gradually with increasing confining pressure.

For frozen sandy samples exhibiting strain softening phenomenon, the strength is equal to the maximum value of deviatoric stress while for those showing strain hardening phenomenon, it is equal to the deviatoric stress when the axial strain is 15%. Figure 2 shows the relationship between failure strength and mean principal stress of samples with different salt contents at various temperatures. With the increase of mean principal stress, the failure strength increases firstly and then reaches a peak value, followed by a decreasing tendency afterwards. The pore ice in frozen samples melts at relatively high pressure, leading to the degradation of strength. The failure strength is also affected by temperature significantly. With the decrease of temperature, the strength of pore ice increases and the ice content in frozen samples also exhibits an increasing tendency, resulting in the increase of failure strength, as shown in Figure 2. On the other hand, the pressure corresponding to ice melting is higher for samples with a lower temperature, which is consistent with the results from [24]. The salt content also affects the failure strength of frozen sandy soil. As the salt content increases, the salt crystallization content increases, which results in the reduction of pore ice content. Thus, the failure strength decreases slightly with increasing salt content.

3. Strength Criterion for Frozen Saline Sandy Soil

In principal stress space, the shape of strength surface of general unfrozen geomaterials is a linear cone, passing
through the origin of stress space. However, when the stress is relatively high, the strength may be overestimated with the linear criterion and the materials exhibit nonlinear strength properties. The strength envelope for frozen saline soils traverses the isotropic tension point other than the origin point due to the cementation effects of salt and ice crystals. Besides, pressure melting of ice in frozen soils occurs at high pressure, resulting in the reduction of strength. Thus, the strength envelope in $p$-$q$ plane should not be a straight line.

In this part, the strength envelopes of frozen saline soils will be discussed.

3.1. Strength Formula in $p$-$q$ Plane. To establish the current strength criterion, the following stress invariants are employed:

$$p = \frac{\sigma_{ii}}{3} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3},$$

$$q = \sqrt{\frac{3}{2} s_{ij} s_{ij} = \sqrt{\frac{1}{2} \left[ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 \right],}$$

(1)

where $\sigma_{ii}$ is principal stress tensor ($i = 1, 2, 3$); $s_{ij}$ is deviatoric stress tensor and can be written as follows:

$$s_{ij} = \sigma_{ij} - \frac{\sigma_{kk} \delta_{ij}}{3},$$

(2)
Figure 2: Failure strength results for frozen saline sandy samples at different temperatures.

Figure 3: Fitting results of the proposed strength function in $p$-$q$ plane at different temperatures.
where $\delta_{ij}$ is the Kronecker delta, $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$.

Given conventional triaxial condition, the mean principal stress and generalized shear stress are written as follows:

$$ p = \frac{\sigma_1 + 2\sigma_3}{3}, $$

$$ q = \sigma_1 - \sigma_3. $$

As confining pressure affects both cohesion and friction strength of frozen soils, Qi and Ma established a nonlinear strength criterion based on Drucker-Prager and Mohr-Coulomb criteria [39]. This strength criterion can reproduce the strength variation under a wide range of confining pressures. The proposed nonlinear criterion in $p$-$q$ plane is expressed as follows:

$$ q = c_0 + \Delta c \log \left( \frac{p}{p_{cr}} \right) + \rho \tan \left( \varphi_0 + \Delta \varphi \log \left( \frac{p}{p_{cr}} \right) \right), $$

where $c_0$, $\Delta c$, $\rho$, and $\varphi_0$ are the cohesion of frozen soils, cohesion increment, confining pressure increment, and angle of friction, respectively. $p_{cr}$ and $\varphi_{cr}$ are the critical confining pressure and critical friction angle for frozen soils, respectively.

### Table 2: Fitting parameters obtained from the test results.

| Salt content | $c_0$ | $\Delta c$ | $\rho$ | $\varphi_0$ | $\varphi_{cr}$ |
|--------------|-------|------------|--------|-------------|----------------|
| 0%           | 4.823 | -0.075     | 8.932  | -0.100      | 0.003          |
| 1.5%         | 5.355 | -0.054     | 10.953 | -0.058      | -0.050         | 2.265 | 0.124 |
where \( c_0 \) and \( \varphi_0 \) are two parameters related to the cohesion and friction properties, respectively, at the maximum value of failure strength; \( \Delta c \) and \( \Delta \varphi \) are the increments when confining pressure is applied by a number of intervals, respectively; \( p_{cr} \) is the critical mean principal stress.

For frozen soils, both cohesion and friction components change with the confining pressure due to the variations of ice crystals and unfrozen water contents during loading process. Compared with the experimental results, the strength criterion denoted by Equation (5) can well characterize the strength variations of samples at different confining pressures. However, as ice crystals have a certain degree of cementation effect, frozen soil has tensile strength. The function expressed by Equation (5) cannot reflect the tensile strength properties of frozen saline soil. Besides, the generalized shear stress \( q \) becomes infinite when the mean principal stress \( p \) approaches zero, which is unreasonable for numerical calculation. Therefore, we develop a modified strength function in \( p-q \) plane after [39], which is given as follows:

\[
q = c_0 \exp \left( \frac{p}{p_{cr}} \right) + p \tan \left( \varphi_1 + \varphi_2 \exp \left( \frac{p}{p_{cr}} \right) \right),
\]

where \( c_0 \) represents the failure strength when \( p \) is equal to zero; \( \varphi_1 \) and \( \varphi_2 \) are the coefficients governing the variation law of friction angle with the mean principal stress \( p \). If \( c_0 = 0 \) and \( \varphi_2 = 0 \), the envelope becomes a straight line over the origin.

Figure 3 shows the fitting curves of the strength envelope predicted by Equation (6) for frozen saline samples at different temperatures. To achieve a reasonable fitting curve, some parameters are set to be in a certain fixed range. Otherwise, the fitting would be poor or even aborted. The least coefficients of determination \( (R^2) \) in Figure 3 are all greater than 0.90, which indicates that Equation (6) can well
reproduce the failure strength evolution for saline samples at different temperatures.

Figure 4 presents the results of model parameters at salt contents of 0% and 1.5%. $T_0$ represents the temperature of 1°C. As the temperature decreases, the cohesion parameter $c_0$ increases, which means the bonding effect between crystals and soil particles is gradually enhanced. Meanwhile, the pressure corresponding to ice melting shows an increasing tendency with the decrease of temperature [24]. The parameter $p_{cr}$ refers to the critical mean principal stress and can reflect the pressure melting characteristics. As temperature decreases, the parameter $p_{cr}$ exhibits an increasing tendency as shown in Figure 4(b). As for friction parameters, both $\phi_1$ and $\phi_2$ demonstrate an increasing trend with the decreasing temperature. All the four model parameters $c_0$, $p_{cr}$, $\phi_1$, and $\phi_2$ vary obviously with the temperature as shown in Figure 4. The relationships between model parameters and temperature are expressed as follows:

$$c_0 = c_1 \exp \left( c_2 \frac{T}{T_0} \right),$$  \hspace{1cm} (7a)

$$p_{cr} = p_{c1} \exp \left( p_{c2} \frac{T}{T_0} \right),$$  \hspace{1cm} (7b)

$$\phi_1 = s_1 + s_2 \frac{T}{T_0} + s_3 \left( \frac{T}{T_0} \right)^2,$$  \hspace{1cm} (7c)

$$\phi_2 = \ln \left( t_1 + t_2 \frac{T}{T_0} \right),$$  \hspace{1cm} (7d)

where $c_1$, $c_2$, $p_{c1}$, $p_{c2}$, $s_1$, $s_2$, $s_3$, $t_1$, and $t_2$ are fitting parameters based on test data. Table 2 shows the fitting parameters at 0% and 1.5% salt contents.

To reveal the influences of different parameters on failure strength, a parametric study is conducted herein. The effects of cohesion parameters $c_0$, friction parameters $\phi_1$ and $\phi_2$, and critical mean principal stress $p_{cr}$ are examined. The strength envelopes are plotted in Figure 5 by considering various values of these parameters. From Figure 5(a), we can see that the slope of strength envelope becomes large when $c_0$ increases. $c_0$ can be regarded as the initial cohesive force when $p$ is equal to zero. The cementation effect between soil particles and ice/salt crystals is enhanced when $c_0$ increases. Thus, the peak strength value exhibits an increasing tendency. The parameters $\phi_1$ and $\phi_2$ mainly affect the internal friction strength. The slope of envelope curve will be greater when the value of $\phi_1$ is larger. As $\phi_1$ increases, the isotropic tensile strength becomes smaller while the peak value of failure strength is getting larger. However, the influence of $\phi_2$ is not obvious when $p$ is relatively low, i.e., $p < 10$ MPa. With the increase of absolute value of $\phi_2$, the peak value exhibits a decreasing tendency and the curvature of the envelope becomes greater. The parameter $p_{cr}$ is the critical mean principal stress and reflects the pressure melting characteristics. The mean principal stress corresponding to pressure melting of ice tends to be greater at a larger $p_{cr}$. Besides, when $p_{cr}$ approaches infinity, the pressure melting phenomenon will hardly happen.

3.2 Internal Friction Angle in $\sigma$-$\tau$ Stress Plane at Different Temperatures. The strength criterion of geomaterials is described by generalized shear stress and mean principal stress as shown in Equation (6). On the other hand, the normal stress and shear stress can also be adopted to express the yield criterion. The strength criterion is determined by the following equation:

$$f(\sigma_1, \sigma_3) = \left( \sigma - \frac{\sigma_1 + \sigma_3}{2} \right)^2 + \tau^2 - \left( \frac{\sigma_1 - \sigma_3}{2} \right)^2 = 0,$$  \hspace{1cm} (8)

where $\sigma$ is the normal stress and $\tau$ represents the shear stress.

\[\text{Figure 6: Relationship between average mean principal stress } \sigma \text{ and internal friction angle } \phi \text{ for frozen saline soil at different temperatures.}\]
Under the conventional triaxial condition, \( p \) and \( q \) are functions of principal stresses \( \sigma_1 \) and \( \sigma_3 \). Combination of Equations (3), (4), and (8) gives the following:

\[
f(p, q) = \left( \sigma - \frac{q}{6} \right)^2 + \tau^2 - \frac{q^2}{4} = 0. \tag{9}
\]

Equation (6) can be rewritten as follows:

\[
g(p, q) = c_0 \exp \left( \frac{p}{P_{cr}} \right) + p \tan \left[ \varphi_1 + \varphi_2 \exp \left( \frac{p}{P_{cr}} \right) \right] - q = 0. \tag{10}
\]

According to the envelope theory, the following relationship is obtained:

\[
\frac{\partial f}{\partial p} \frac{\partial q}{\partial q} - \frac{\partial f}{\partial q} \frac{\partial q}{\partial p} = 0. \tag{11}
\]

Substituting Equations (9) and (10) into Equation (11) yields the following:

\[
\sigma = p + \frac{3 - 4\chi}{18 + 3\chi} q, \tag{12}
\]

where \( \chi \) presents a dimensionless parameter and can be written as follows:

\[
\chi = \frac{c_0 + \varphi_1 p}{P_{cr}} \exp \left( \frac{p}{P_{cr}} \right) + \tan \left[ \varphi_1 + \varphi_2 \exp \left( \frac{p}{P_{cr}} \right) \right] + \frac{\varphi_2 p}{P_{cr}} \tan^2 \left[ \varphi_1 + \varphi_2 \exp \left( \frac{p}{P_{cr}} \right) \right] \exp \left( \frac{p}{P_{cr}} \right). \tag{13}
\]

Substitution of Equation (12) into Equation (9) yields the following:

\[
\tau = \sqrt{\frac{(3 + 2\chi)(3 - \chi)}{6 + \chi}} q. \tag{14}
\]

Based on Equations (12) and (14), \( \sigma \) and \( \tau \) are obtained.
at a given stress state. The friction angle in $\sigma$-$\tau$ plane satisfies the following:

$$\tan \phi = -\frac{\partial f / \partial \sigma}{\partial f / \partial \tau} = -\frac{\sigma - p - 1/6q}{\tau}. \quad (15)$$

By combining Equations (12), (14), and (15), we obtain the following:

$$\phi = \arctan \left[ \frac{3\chi}{2 \sqrt{(3 + 2\chi)(3-\chi)}} \right]. \quad (16)$$

During the loading process in triaxial test, the evolution of friction angle with the mean principal stress $p$ is obtained by referring to Equations (13) and (16). The relationships between $\phi$ and $p$ for frozen saline sandy samples at different temperatures are shown in Figure 6. The friction angle decreases gradually as $p$ increases, which is ascribed to that the interlocking effect between crystals and soil particles are weakened. On the other hand, the unfrozen water content increases as $p$ increases, leading to a reduction in friction strength. The initial friction angle, which equals to $\phi$ when $p = 0$, is little affected by temperature. However, with the increase of $p$, the influence of temperature becomes more and more obvious. The general rule is that the friction angle $\phi$ shows an increasing phenomenon as the temperature varies from -4°C to -8°C. However, with further decrease in temperature, the friction angle of frozen samples at -10°C exhibits a deceasing tendency when $p$ is relatively low, and an increasing tendency when $p$ is relatively high, i.e., $p > 16.4$ MPa at the salt content of 1.5%. The brittleness of ice is enhanced for samples at the temperature of -10°C, resulting in the decrease of friction angle. According to the equation of Clausius-Clapeyron, the ice temperature increases by 1°C as mean principal stress rises by 13.5 MPa [24]. Therefore, the brittleness of ice is weakened when $p$ increases, which leads to the increase of friction angle afterwards. The evolution laws at various salt contents are similar as illustrated in Figure 6.

3.3. Shape Function in $\pi$ Plane. Various criteria have been developed to describe the strength characteristics of geomaterials, such as Lade-Duncan criterion, Drucker-Prager criterion, Mises criterion, and spatially mobilized plane strength criterion. According to a series of directional shear tests, Chen et al. [28] found that generalized nonlinear strength theory (GNST) developed by Yao et al. [42] can well present the strength evolution of frozen samples. The simulation data of GNST was in good agreement with test results in $\pi$ plane. Thus, the GNST shape function is employed in $\pi$ plane for frozen saline sandy soil in our study. In $\pi$ plane, the profile of GNST is a smooth curve between those of SMP and Mises criteria. For isotropic materials, the strength criterion can be written as follows:

$$F = F_1(I_1) + F_2(J_2, \theta_\sigma) = 0, \quad (17)$$

where $I_1$ presents the first stress invariant, $J_2$ presents the second invariants of deviatoric stress tensor, $F_1(I_1)$ and $F_2(J_2, \theta_\sigma)$ are the strength functions in hydrostatic plane and deviatoric plane, respectively; $\theta_\sigma$ is the Lode angle. $F_2(J_2,$
The yield function \(F_2(J_2, \theta_\sigma) = \sqrt{J_2} / g(\theta_\sigma), \) can be expressed as follows:

\[
F_2(J_2, \theta_\sigma) = \frac{\sqrt{J_2}}{g(\theta_\sigma)},
\]

where \(g(\theta_\sigma)\) presents the shape function in deviatoric plane.

As for the shape function of GNST, \(g(\theta_\sigma)\) is written as follows:

\[
g(\theta_\sigma) = \lambda g_1(\theta_\sigma) + (1 - \lambda) g_2(\theta_\sigma),
\]

where \(\lambda\) is a material parameter ranging from 0 to 1 and \(g_1(\theta_\sigma)\) and \(g_2(\theta_\sigma)\) are the shape functions of SMP criterion and Mises criterion, respectively.

The SMP strength criterion in terms of stress invariants and Lode angle is as follows:

\[
f_1(I_1, I_2, I_3) = I_1 I_2 - k I_3 = 0,
\]

where \(I_2\) is second stress invariant, \(I_3\) presents third stress invariant, and \(k\) is related to internal friction angle \(\phi\), which is written as follows:

\[
k = \frac{9 - \sin^2 \phi}{1 - \sin^2 \phi}.
\]

The stress invariants \(I_1\) and \(I_3\) are the functions of \(I_1, I_2,\) and Lode angle \(\theta_\sigma\), which are expressed as follows:

\[
I_2 = \frac{1}{3} I_1^2 - I_2,
\]

\[
I_3 = -\frac{2}{3 \sqrt{3}} I_2^{2/3} \sin 3\theta_\sigma - \frac{1}{3} I_1 I_2 + \frac{1}{27} I_1^3.
\]

Substitution of Equations (22) and (23) into Equation (20) gives the following:

\[
f_1(I_1, I_2, \theta_\sigma) = \frac{9 - k}{27} I_1^2 - \frac{3 - k}{3} I_1 I_2 + \frac{2k}{3 \sqrt{3}} I_2^{3/2} \sin 3\theta_\sigma.
\]

Considering \(I_1 = 3p\) and \(I_2 = q^2/3\), Equation (24) is rewritten as follows:

\[
f_1(p, q, \theta_\sigma) = (9 - k)p^3 - \frac{3 - k}{3} pq^2 + \frac{2k}{27} q^3 \sin 3\theta_\sigma = 0.
\]

By solving Equation (25), we obtain the following:

\[
\frac{p}{q} = \frac{2}{3} \sqrt{\frac{3 - k}{9 - k}} \sin \left\{ \frac{\pi}{3} + \frac{1}{3} \arcsin \left[ k \sqrt{9 - k/(3-k)^3} \sin 3\theta_\sigma \right] \right\}.
\]

Thus, the shape function of SMP in \(\pi\) plane can be given as follows:

\[
g_1(\theta_\sigma) = \frac{\sin \left\{ \frac{\pi}{3} - 1/3 \arcsin \left[ k \sqrt{9 - k/(3-k)^3} \right] \right\}}{\sin \left\{ \frac{\pi}{3} + 1/3 \arcsin \left[ k \sqrt{9 - k/(3-k)^3} \sin 3\theta_\sigma \right] \right\}}.
\]

The Mises strength criterion has a simple form and can be written as follows:

\[
f_2 = \sqrt{J_2} - \chi = 0,
\]

where \(\chi\) is a material parameter.

From Equation (28), we can see the shape of Mises criterion in \(\pi\) plane is a circle, and the shape function is written as follows:

\[
g_2(\theta_\sigma) = 1.
\]

Substituting Equations (27) and (29) into Equation (19), we obtain the following:

From Equation (30), it is concluded that both parameters \(\lambda\) and \(k\) affect the shape function. Figures 7(a) and 7(b) show the shape function curves in deviatoric plane at different values of \(\lambda\) and \(k\), respectively. The shapes of GNST in deviatoric plane are convex curves. When the value of \(\lambda\) varies from 0 to 1, the failure strength criterion changes from the SMP to Mises criteria. The shape function can reflect the tensile strength and compressive strength at
various Lode angles. The parameter $k$ exerts less significant influence. With the decrease of $k$, the shape curve tends to be a circle gradually.

During the loading process in triaxial test, the friction angle varies with $p$ as shown in Figure 6. The shape function curves in $\pi$ plane at -4°C and -10°C are shown in Figure 8. As $p$ increases, the size of failure envelope in $\pi$ plane enlarges firstly and then decreases afterwards, which characterizes pressure melting phenomenon of ice. Besides, the pressure corresponding to ice melting increases with the decreasing temperature.

The failure surfaces in principal stress space for frozen soil samples at various temperatures are presented in Figure 10. The shape and size of failure surface are considerably affected by the temperature. With the decrease of temperature, the failure curve tends to expand outwards gradually.

4. Conclusions

In this study, a series of conventional triaxial tests were conducted for frozen saline sandy soil at -4°C, -6°C, -8°C, and -10°C. The stress-strain relationships and strength properties were analyzed. Some conclusions are obtained as follows:

(1) With the increase of mean principal stress, the failure strength increases firstly and then reaches a peak value, followed by a decreasing tendency. The lower the temperature is, the larger the failures strength will be

(2) A strength function, which incorporates the cohesion and friction components, is proposed in $p-q$ plane. The proposed function can well describe the evolution of failure strength at various temperatures. The relationships between strength parameters and temperature are revealed

(3) In $\pi$ plane, the strength envelope varying between those of SMP and von Mises criteria is employed to illustrate strength characteristics of frozen saline sandy samples. The proposed strength criterion can consider the influence of the temperature and pressure melting of ice

Data Availability

The data used to support the findings of this study are included within the article, and further data or information required is available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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