Multipolar Conversion Induced Subwavelength High-Q Kerker Supermodes with Unidirectional Radiations

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The two-mode coupling model, with energy splitting and formation of supermodes with different life times, has been pervasive in almost every discipline of physics. This fundamental model is revisited from a different perspective of multipolar expansions, and a hidden dimension of it is revealed, by establishing a subtle connection between the two seemingly unrelated properties of Q-factors and far-field angular radiation patterns. It is discovered that in both regimes of negative and positive couplings, significant Q-factor enhancement can be attributed to dramatic redistribution of radiation that originates from multipolar conversions from lower to higher orders. Relying on this connection and generalized Kerker effects of interferences among different multipoles, the two outstanding features of high-Q factor and unidirectional radiation are synchronized into one subwavelength supermode. The implications of this study are not confined to optics and photonics, and can potentially shed new light on coupling between resonances of mechanical, phononic, electronic, or other hybrid natures.

1. Introduction

The elegant model of two-mode coupling has been serving as one of the most fundamental frameworks for different branches of physics and many other interdisciplinary fields. This model is generically related to lots of exotic phenomena including Fano resonances, electromagnetically induced transparencies, bound states in the continuum, scarred states in open systems, non-Hermitian and topological effects, and so on. For the simplest case of two coupled resonances supported by open and passive resonators, generally two supermodes with different life times would emerge. The longer-lived higher-Q supermode that originates from out-of-phase superposition of the two original states can locate on the lower- or higher-energy branch, depending on the sign (positive or negative, respectively) of the coupling strength. On one hand, the formation of the higher-Q supermode can be intuitively attributed to the radiation loss suppression as a result of far-field destructive interference between the two original states. On the other hand, however, the description “destructive interference” is itself, to some extent still hazy, but rather we do not know exactly what really happens in the far field that has induced the Q-factor enhancement.

To more accurately grasp the far-field properties of the higher-Q supermode and to provide new insights, we revisit this fundamental two-mode coupling model in photonic resonators and manage to reveal a hidden dimension of it. Our study here is conducted from a different perspective of multipolar expansions, which serve as a fundamental tool for far-field scattering or radiation analysis. In sharp contrast to previous studies that mostly treat Q-factors and angular radiation distributions as unrelated properties, here we try to establish a connection between them. It is discovered that in the anti-crossing regions of strong mode coupling with both negative and positive coupling strengths, the Q-factor enhancement is intrinsically connected with multipolar conversions from lower to higher orders. Based on this subtle connection and generalized Kerker effects of interferences among multipoles of different natures (electric or magnetic) and orders to produce directional radiations, we manage to synchronize three features of subwavelength mode volume, high-Q factor, and unidirectional radiation pattern into one supermode. We show that such subwavelength Kerker supermode can be realized with simple dielectric subwavelength resonators with broken symmetries, which is vitally important for nanoscale lasers, sensors, and single-photon sources.

2. Two-Mode Coupling and Supermode Energy Splitting within a Dielectric Rod

As is shown schematically in Figure 1a, we start with this widely employed configuration (refer to Section V of the Supporting Information for the coupling type classification and extra evidence that such a connection is also manifest in other scenarios, including both strong and weak coupling regimes). The resonator is a nonmagnetic dielectric rod of height $h$, radius...
Figure 1. a) Schematic of a dielectric rod resonator of height $h$, radius $R$, and relative permittivity $\varepsilon$. The exciting normally incident plane wave propagating along $z$ can be $p$- or $s$-polarized. The scattering cross section spectra for the rod are shown in (b) and (c), for $p$ and $s$ polarizations respectively. Four strong mode coupling induced anti-crossing regions characterized by different azimuthal quantum numbers are marked, and within each region two supermode branches are pinpointed by circles. For $p$ polarization, the supermodes locate on the right branches experience significant Q-factor enhancement; while for $s$ polarization, in contrast, the supermode locate on the left branch experiences significant Q-factor enhancement. This indicates negative (positive) coupling strength for $p$ and $s$ polarizations, respectively.

Figure 2. a–c) Evolution of $Q$ factors, and d–f) the radiated power from all multipoles that are not negligible, for the supermodes located on the right branches in the three anti-crossing regions marked in Figure 1b, with $v_p = 0, 1, 2$ respectively. For each case in (a–c), three supermodes are indicated by points $M_0, 1, 2, H_0, 1, 2$, and $F_0, 1, 2$, and the corresponding far-field radiation patterns of those supermodes are shown respectively in (g–i). Specific parameters for the indicated points are: $\beta_M = 0.65, 0.4, 0.5, \beta_F = 0.703, 0.541, 0.641$, and $\beta_F = 0.75, 0.7, 0.8$, for $v_p = 0, 1, 2$ respectively.

$R$, and relative permittivity $\varepsilon = 40$. Certain eigenmodes of interest supported by the resonator can be excited with a plane wave with wave-vector $k$ along $z$ and perpendicular to the rod axis $y$. The incident wave can be either $p$-($E||x$) or $s$-polarized ($E||y$). The plane wave scattering cross section spectra for this resonator with respect to normalized radius $\alpha = kR$ and aspect ratio $\beta = R/h$ are shown in Figure 1b,c, for $p$ and $s$ polarizations, respectively (refer to Section I of the Supporting Information for the detailed methods). Altogether four representative anti-crossing regions of strong two-mode coupling are marked: within each region the two branches of supermodes, which are induced by the coupling between Mie-type and Fabry–Perot-type modes that share the same azimuthal quantum number $\nu$, are pinpointed by circles. The supermodes are quasi-normal modes, which can be characterized by complex eigenfrequencies $\tilde{\omega} = \tilde{\omega}_1 + i\tilde{\omega}_2$; the circles are centered at $\tilde{\omega}_1 / (\tilde{\omega}_2 R / c, \beta)$, where $c$ is the speed of light and the Q factor can be obtained through $Q = \tilde{\omega}_1 / (2\tilde{\omega}_2)$. As will be further verified in Figures 2 and 3, for $p$ polarization,
tured, which correspond respectively to two sets of spherical harmonics with expansion coefficients of \( a_{\nu m} \) and \( b_{\nu m} \) (refer to Section II of the Supporting Information for more information). To be more specific, the radiated power of each electric multipole and magnetic multipole of order \( n \) is proportional respectively to \( \sum_{\nu=-n}^{n} |a_{\nu m}|^2 \) and \( \sum_{\nu=-n}^{n} |b_{\nu m}|^2 \), with \( n = 1, 2, 3 \) corresponding to dipole, quadrupole, octupole, and so on and so forth. The expansion coefficients \( a_{\nu m} \) and \( b_{\nu m} \) can be calculated through either current integrations or direct expansions of the radiated fields.\(^{[23–25]}\) It is worth mentioning that modes and multipoles are two different concepts, though there are direct correspondences between them in highly symmetric structures such as spheres.\(^{[23–26]}\) For most open photonic systems without symmetry, there is no such direct mapping between them and instead each eigenmode can be expanded into a series of multipoles and not every multipole has an eigenmode correspondence.\(^{[23–26]}\) For investigations related to far-field radiation properties, the language of multipoles is superior, as the eigenmode itself is not directly linked to angular radiation pattern.\(^{[27]}\)

4. Connections between Multipolar Conversions and Q-Factor Enhancement

The results obtained for the supermodes that can be excited with the \( p \)-polarized plane wave are summarized in Figure 2, with all the three right branches marked in Figure 1b investigated. We emphasize here that the eigenmodes of a resonator are determined solely by the consisting materials and geometric configuration, while they have nothing to do with the exciting source, be it plane wave, dipole emitter, or source of any other forms. The plane wave scattering spectra shown in Figure 1 provide a direct guide as to where the Q-factor enhanced supermodes locate, whereas we have to keep in mind that such a guide is incomplete, as no information of many other modes that cannot be excited by plane waves can be extracted from such spectra. For all three branches investigated, the Q factors of the supermodes are significantly enhanced in anti-crossing regions and for each case there is an optimum point (indicated by H point) where the Q factor reaches the maximum (see Figure 2a–c). The corresponding results (radiated power from each contributing multipole with the total radiated power normalized, as is also the case in Figures 3 and 4) obtained through multipolar expansions of the supermodes are shown in Figure 2d–f, where the radiated power of the multipoles that are not shown is negligible. It is clear that for all cases of different \( \nu_p \), the Q-factor enhancement is intrinsically accompanied by the multipolar conversions from lower to higher orders: mainly MD to MO (magnetic octupole) for \( \nu_p = 0 \) (Figure 2d), ED to MQ for \( \nu_p = 1 \) (Figure 2e), and EQ to MO for \( \nu_p = 2 \) (Figure 2f). Moreover, the positions of maximum Q factors always coincide with the peaks of the dominating higher-order multipoles, and away from those optimum points, multipoles are reversely converted from higher to lower orders with decreasing Q factors. Moreover, at the optimum points, the higher orders the dominating multipole are of, the higher Q factors can be achieved.

To further verify the results from multipolar expansions, along each supermode branch we have selected three supermodes indicated by \( M_{0,1,2} \), \( H_{0,1,2} \), and \( F_{0,1,2} \) (the subscript indicates the

3. Multipolar Expansion for Each Supermode

The radiated fields of each eigenmode can be fully expanded into two categories of multipoles of electric and magnetic

![Figure 3](image)

\( \nu_p = 0, 1, 2... \) corresponds to the magnetic dipole (MD), electric dipole (ED), electric quadrupole (EQ).... \( \nu_s = 0, 1, 2... \) corresponds to the electric dipole (ED), magnetic dipole (MD), magnetic quadrupole (MQ)....\(^{[42,43]}\)

Next we examine the higher Q-factor supermodes, which locate on the higher-energy right (negative coupling) and lower energy left (positive coupling) branches for \( p \) and \( s \) polarizations respectively, with mode cross coupling strengths of opposite signs.\(^{[14,22]}\) We investigate in detail the process of Q-factor enhancement for those modes, while noting that the supermodes on the other branches experience significant Q-factor suppression (refer to Section IV of the Supporting Information for more details). In the following, we categorize the high-Q supermodes into two groups of \( p \) and \( s \) polarizations. It basically means that only those supermodes can be excited by the normally incident plane waves of corresponding polarizations respectively. In conventional studies it is taken for granted that the Q-factor of a resonance is inextricably linked to the total radiation loss, while its relevance to radiation distributions along different directions is basically disregarded (refer to Section V of the Supporting Information for more general discussions based on the two mode coupling model, clarifying why the connection we reveal is generally hidden). In contrast, here through the approach of multipolar expansions,\(^{[23–27]}\) we manage to reveal that the Q factor and radiation pattern are not segregated properties of resonances but rather they are subtly connected with each other.

![Figure 3](image)
respectively). The corresponding radiation patterns at those points are shown in (d).

The results of Q-factor evolutions, multipolar radiation spectra, and the corresponding far-field radiation patterns of those supermodes are shown correspondingly in Figure 2a–c. This includes not only highest Q-factor modes \( H_{0,1,2} \), but also lower Q-factor modes \( M_{0,1,2} \) and \( F_{0,1,2} \) that bear more resemblances respectively to Mie-type and Fabry–Perot-type modes, due to weak coupling between them at regions relatively far from the optimum points.\(^{33,34}\) The corresponding near-field distributions of those supermodes, which agree well with what is shown in Figure 2d–f. At \( H_{0,1,2} \) points there are more radiation lobes (the lobe number is proportional to the multipole order) than at the other two points, confirming the conversions between multipoles of different orders.

We proceed to discuss the supermodes that can be excited with the \( s \)-polarized plane wave, as is marked in Figure 1c with \( \nu_s = 1 \). The results of Q-factor evolutions, multipolar radiation spectra, and the corresponding far-field radiation patterns of the selected supermodes are shown in Figure 3a–c, respectively. The same conclusion with respect to the connection between Q-factor enhancement and multipolar conversions can be drawn. We note here that: on one hand, the multipolar conversion mechanism can be widely employed to enhance Q factors, not only for individual particles as we show here, but also for more sophisticated photonic crystal cavities\(^{44,45}\) and probably many other sorts of resonators\(^{46,47}\) on the other hand however, we have to keep in mind that not all sorts of Q-factor enhancements can be attributed to such a mechanism, such as those obtained relying on resonators consisting of materials of unusual effective parameters where there is no effective mode coupling effect\(^{48-50}\) (refer also to Section IV of the Supporting Information, where it is shown that the left-branch supermodes marked in Figure 1b and the right-branch supermodes marked in Figure 1c experience significant Q-factor suppressions without multipolar conversions).

5. Kerker Supermodes with Unidirectional Radiations

Here we have gained a deeper insight into the fundamental two-mode coupling model, by successfully bridging the two features of Q-factor enhancement and multipolar conversions from lower to higher orders. Now the question is: what can we do with it? Up to now, we have already obtained supermodes of simultaneous high-Q factors and subwavelength mode volumes with symmetric dielectric rods. Besides high-Q factors and subwavelength mode volumes, for many applications relying on subwavelength elements such as low-threshold lasers, antennas, sensors, and single-photon sources,\(^{7,8}\) modes with asymmetrically directional radiation patterns are even more desired. This leads to a fundamental but also technologically interesting question: is it feasible to synchronize all three desirable features of subwavelength mode volume, high-Q factor and unidirectional far-field radiation within one subwavelength resonator?

Our next goal is to add the third desired feature to the supermodes achieved to make their radiations unidirectional. We term such supermodes as “Kerker supermodes”, as according to the generalized Kerker theory the unidirectional radiations should consist of a series of multipoles of different natures and/or different orders.\(^{27,31,32}\) Though it is shown in lots of previous studies that many symmetric structures may exhibit highly directional scattering (radiation) patterns, it should be noted that such directionality always comes from the co-excitations and interferences of several eigenmodes, and thus it is generically dependent on external sources.\(^{27,32}\) To obtain an eigenmode with intrinsic directional radiation that is independent on the exciting source, the basic requirement is to break the rotational symmetry of the resonator.\(^{51-56}\) Our further explorations for Kerker supermodes are still based on the dielectric rod shown in Figure 1a, the symmetry of which is broken by introducing an extra eccentric cylindrical air hole of radius \( r = 0.25\text{R} \) and center-to-center
displacement $d = 0.675R$ along the $x$ direction, as is shown in the inset of Figure 4a. Following our previous approach, we also show in Figure 4a the scattering spectra with a p-polarized incident plane wave ($k|x$ and $E|x$), from which the supermodes with enhanced Q factors can be identified. We have further pinpointed by circles one supermode branch in the scattering spectra, for which the dependence of Q factors and multipolar radiation spectra on aspect ratios are shown respectively in Figure 4b,c.

For this asymmetric resonator however, there is no well-defined azimuthal quantum number to characterize such supermode branch, nor is it easy to identify which sets of modes couple and hybridize with one another to induce it. Nevertheless, almost identical to the results shown in Figures 2 and 3, for the asymmetric case the Q-factor enhancement is also accompanied by multipolar conversions (see Figure 4b,c). Along the supermode branch we have pinpointed three points [the corresponding far-field radiation patterns are shown in Figure 4d]. The highest Q factor is obtained at point $H$ and the radiation power from ED and MD equal to each other at points $K_1$ and $K_2$. At point $H$, the coexistence of ED, MQ, MO, and interference among them can only render the radiation slightly asymmetric along the $x$ direction (see Figure 4d), since here the contribution from ED is still dominant over other multipoles. At points $K_1$ and $K_2$, the interference among the multipoles excited (mainly ED and MD) renders the radiation highly asymmetric (see Figure 4d) (refer to Section III of the Supporting Information for the corresponding near-field distributions of those supermodes, which are also asymmetric due to geometric symmetry breaking). These Kerker supermodes share the same mechanism as that of the Kerker scattering of the first and second kinds, with suppressed forward or backward scatterings that originate from destructive interferences between EDs and MDs of close magnitudes. $^{[27,28,31,32]}$

6. Conclusions and Discussions

In conclusion, we have discovered a hidden dimension of the fundamental two-mode coupling model, revealing the subtle connection between Q-factor enhancement and redistributions of far-field angular radiations that originate from multipolar conversions from lower to higher orders. Based on this discovery and the generalized Kerker effect that can result in radiation directionality, we have achieved with asymmetric subwavelength dielectric resonators the Kerker supermodes with simultaneously high-Q factors, subwavelength made volumes, and unidirectional radiation patterns. Here we have confined our discussions of this fundamental model within passive and nonmagnetic photonic systems, and it is expected that a natural extension to more sophisticated configurations (periodic, quasi-periodic, or random that involves many-mode coupling) with active materials (with gain, chirality or nonlinearity$^{[19,57,58]}$) would render much broader freedom for manipulations of strong light–matter interactions at the subwavelength regime. We believe that the neglected new dimension we have revealed here can spur many optical resonator related fundamental explorations of cavity electrodynamics and incubate a wide range of applications related to nanoscale lasers, sensors, single-photon sources and many other optical elements. Our discovery can also shed new light on lots of other interdisciplinary fields (such as optomechanics, exciton-polaritons, phonon-polaritons and so on) that involve interactions and hybridizations among modes of mechanical, phononic, electronic or other hybrid natures.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

Kerker supermodes, multipolar conversions, subwavelength high-Q resonances, two-mode coupling

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