Performance Evaluation of SC-FDMA Systems Using Wireless Images

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ABSTRACT

Recently, single carrier frequency division multiple access (SC-FDMA) has drawn great attention in wireless communications due to its low peak-to-average power ratio and low sensitivity to carrier frequency offsets. The aim of this paper is to investigate and test the wireless image transmission over SC-FDMA with consideration of different basis functions, different modulation schemes and different subcarriers mapping schemes. Several experiments are carried out and the obtained results show that the wireless images transmission over SC-FDMA systems is possible efficiently. Simulation results also show that the interleaved systems significantly improve the clarity of the received image and their performances are better than that of the localized systems irrespective of the wireless channel or the modulation scheme used. Moreover, the results show that the discrete cosine transform (DCT)-based SC-FDMA (DCT-SC-FDMA) and discrete sine transform (DST)-based SC-FDMA (DST-SC-FDMA) system achieve better performance than the conventional discrete Fourier transform (DFT)-based SC-FDMA (DFT-SC-FDMA) system in terms of Mean Square Error (MSE) and Peak Signal-to-Noise Ratio (PSNR) values.

Keywords: Image transmission, DFT-SC-FDMA, DCT-SC-FDMA, DST-SC-FDMA, QPSK and 16QAM.
Introduction
The rapidly increasing requirements for multimedia wireless communications are growing today and at the future steadily. Many of current wireless standards adopt a multicarrier air interface based on orthogonal frequency division multiple access (OFDMA) and SC-FDMA for high-rate multimedia transmission. Both techniques are modified form of the orthogonal frequency division multiplexing (OFDM) and single carrier frequency division equalization techniques respectively [1, 2].

However, OFDMA has some inherent disadvantages such as the high Peak-to-average power ratio (PAPR) and the sensitivity to carrier frequency offsets [3, 4]. To solve these problems, much attention has been directed recently to SC-FDMA [4, 5]. SC-FDMA inherently provides similar advantages of OFDMA such as throughput performance and robust resistance to multipath. In addition, it has lower PAPR than OFDMA. Because of these advantages, SC-FDMA recently has been recommended to use in the uplink multiple access scheme in the long term evolution (LTE) of cellular systems under consideration by the third generation partnership project (3GPP) [3-5]. The transmitters in an SC-FDMA system use different orthogonal frequencies (subcarriers) to transmit information symbols as in OFDMA. However, all the subcarriers are transmitted in series in contrast to the parallel transmission of subcarriers in OFDMA as shown in Fig. (1). Thus only one subcarrier is present at any particular time and hence it gives lower PAPR characteristics.

There are two different schemes of subcarriers mapping modes in SC-FDMA systems [5-7]. The first mode is the localized SC-FDMA (LFDMA) in which the scheduler assigns consecutive subcarriers to convey information from a particular user. The second mode is interleaved SC-FDMA (IFDMA) in which the users are assigned distributed subcarriers over the entire frequency band.

Figure 1: OFDMA and SC-FDMA systems.

Due to remarkable internet growth, image transmission has emerged as one of the high growth applications of wireless communication technology. Performance comparison of image transmission over MC-CDMA system with two interleaving techniques; Helical interleaving and chaotic interleaving is presented in [8]. In [9], the wireless image transmission over OFDMA
and SC-FDMA systems is investigated over a frequency selective channel for different modulation and subcarriers mapping schemes. The performance of DCT-SC-FDMA system using continuous phase modulation was examined with image transmission in [10]. Images transmission over OFDMA systems has been investigated and evaluated for different wireless channel in [11]. An efficient wireless transmission scheme based on the DST-based multicarrier code division multiple access system has been studied in [12]. Image transmission over a space-time coded OFDM system was suggested and discussed in [13].

In this paper, the issue of image transmission over DFT-SC-FDMA, DCT-SC-FDMA and DST-SC-FDMA systems is investigated and evaluated with consideration of different modulation schemes and different subcarriers mapping schemes. The peak-signal-to-noise ratio (PSNR) and the mean square error (MSE) performances of the received image over the mentioned systems are studied, compared and investigated for different modulation and subcarriers mapping schemes.

The remaining part of the paper is organized as follows: Section II presents the basis functions. Section III describes the SC-FDMA systems models. Section IV discusses simulations results. Section V concludes the paper.

Basis Functions
The transform operation is mathematical operation that is applied to a given signal to convert it from one domain into another domain and vice versa. It is desired to convert the input signal from discrete-time data representation into a discrete-frequency representation and vice and versa. Thus once the signal is converted into frequency domain, it will have various components that can be used to remove specific unwanted frequency components. The most popular transform systems related to signal processing are DFT, DST, and DST.

A. The Discrete Fourier Transform.

The DFT is one of the most important tools that is widely used in digital signal processing and related fields. The forward Fourier transform is defined as the integral:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad (1)$$

The DFT takes $N$ samples in the time-domain and transforms them into $N$ values $X(k)$ in the frequency-domain. It means that DFT operates at a finite number of discrete data points. Thus eq. (1) becomes:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{j\frac{2\pi kn}{N}}, \ k = 0, 1, ..., N - 1 \quad (2)$$

By using Euler's formula than eq. (2) can be rewritten as:

$$X(k) = \sum_{n=0}^{N-1} x(n) \left(\cos\frac{2\pi nk}{N} - j \sin\frac{2\pi nk}{N}\right) \quad (3)$$

It is clear that DFT takes two parts, real part and imaginary part.

B. The Discrete Cosine Transform

DCT is similar to DFT except in the fact that it exploits only the real part of DFT. It expresses the samples of the input signal as the sum of cosinusoidal functions oscillating at different frequencies. There are four types of DCT. However, the type-II DCT is the most commonly used and often called simply the DCT. DCT is given by [14]:

$$X(k) = \frac{2}{\sqrt{N}} \beta(k) \sum_{n=0}^{N-1} x(k) \left(\cos\frac{\pi k(2n+1)}{2N}\right), \ k = 0, 1, ..., N - 1 \quad (4)$$

Where $X(k)$ is the $n^{th}$ sample of the input signal. $\beta(k)$ can be given as what follows [14]:

$$\beta(k) = \begin{cases} \frac{1}{\sqrt{2}}, & k = 0 \\ 1, & k = 0, 1, ..., N - 1 \end{cases} \quad (5)$$

C. The Discrete Sine Transform.
Like any Fourier-related transform, DST expresses the input signal in terms of a sum of sinusoids with different frequencies and amplitudes. However, it uses only real functions instead of the complex functions used in the conventional DFT. There are many types of DST and DST-I type is considered in this paper. DST can be expressed by [15]:

$$y(k) = \sum_{n=1}^{N} X(n) \sin \left( \frac{\pi kn}{N+1} \right), k = 1, 2, ..., N$$

(6)

**SC-FDMA SYSTEMS MODELS**

**A. DFT-SC-FDMA System**

The block diagram of the DFT-SC-FDMA system is shown in Fig. 2. One base station and U uplink users are assumed. At the transmitter side, the encoded data is transformed into a multilevel sequence of complex numbers in one of several possible modulation formats. The resulting modulated symbols are grouped into blocks, each containing N symbols and the DFT is performed. Then, the data symbols are mapped, using the interleaved or the localized subcarriers mapping technique. After that, an M-point inverse discrete Fourier transform (IDFT) is performed and a cyclic prefix (CP) is added to the resulting signal. The transmitted signal from the uth user can be formulated as follows:

$$\hat{x}'' = P_{ad} F_M^{-1} M_T^u F_N d''$$

(7)

where $d''$ is an $N \times 1$ vector containing the modulated symbols of the $u$th ($u=1, 2, ..., U$) user. $F_N$ is the $N \times N$ DFT matrix. $M_T^u$ is an $M \times N$ ($M = Q \times N$) subcarriers mapping matrix of the $u$th user. $Q$ is the bandwidth expansion factor of the symbol sequence. For perfect time and frequency synchronization, if all terminals transmit $N$ symbols per block, the system can handle $Q$ simultaneous transmissions without interference. $F_M^{-1}$ is the $M \times M$ IDFT matrix. $P_{ad}$ is an $(M+Nc) \times M$ matrix, which adds the CP.

![Figure 2: Structure of the uplink DFT-SC-FDMA system.](image-url)
At the receiver side, assuming perfect time and frequency synchronization, the received signal at the base station can be written as follows:

$$\mathbf{r} = \sum_{u=1}^{U} \mathbf{H}_u^* \tilde{x}_u^* + \mathbf{n}$$  \hspace{1cm} (8)

where $\mathbf{H}_u^*$ is an $(M+N) \times (M+N)$ matrix describing the channel of the $u$th user. $\mathbf{n}$ is an $(M+N) \times 1$ vector containing the noise. After the removal of the CP, the received signal becomes:

$$\mathbf{r} = \sum_{u=1}^{U} \mathbf{H}_u^* \tilde{x}_u + \mathbf{n}$$  \hspace{1cm} (9)

where $\mathbf{n}$ is the noise after the CP removal. $\tilde{x}_u^* = \mathbf{F}_M \tilde{x}_u^*$ is an $M \times 1$ vector representing the transmitted samples from the $u$th user after the mapping process. $\mathbf{N}$ is the DFT of $\mathbf{n}$. Finally, the demapping, the equalization, the IDFT, the demodulation and the decoding processes take place.

B. DCT-SC-FDMA System

The structure of the DCT-SC-FDMA system is shown in Fig. 3. At the transmitter part, which is the mobile unit, the structure of the DCT-SC-FDMA is similar to that of the DFT-SC-FDMA except the IDFT and the DFT blocks are replaced by the IDCT and the DCT blocks, respectively. At the receiver part, which is the base station, the structure of the DCT-SC-FDMA still uses the DFT and the IDFT for the one-tap frequency domain equalizer in addition to the DCT and IDCT blocks. On the other hand, the receiver complexity of the DCT-SC-FDMA system is slightly higher than that of the DFT-SC-FDMA system. However, the increase in the receiver complexity in the uplink is tolerable considering the advantages of the DCT-SC-FDMA system.

C. DST-SC-FDMA System

The structure of the DST-SC-FDMA system is similar to that of the DCT-SC-FDMA system in previous subsection. The difference is that the DCT and the IDCT blocks at the transmitter and receiver are replaced by the DST and the IDST blocks, respectively.

In this paper, the interleaved DFT-SC-FDMA is denoted by DFT-IFDMA, the localized DFT-SC-FDMA is denoted by DFT-LFDMA, the interleaved DCT-SC-FDMA is denoted by DCT-IFDMA, the localized DCT-SC-FDMA is denoted by DCT-LFDMA, the interleaved DST-SC-FDMA is denoted by DST-IFDMA, and the localized DST-SC-FDMA is denoted by DST-LFDMA.

Simulation Results

In this section, PSNR and MSE metrics have been used to measure the quality of the transmitted Cameraman image over SC-FDMA system with different basis functions, different subcarriers mapping schemes and different modulation schemes by using MATLAB simulator. PSNR is defined as the ratio between the maximum possible power of a signal and the power of the corrupting noise that affects the fidelity of this signal. It can be given by the following formula

$$PSNR = 10 \log \left( \frac{f_{max}^2}{MSE^2} \right)$$  \hspace{1cm} (11)

Where $f_{max}$ is the maximum pixel value in the image. The $MSE$ is defined as follows:

$$MSE = \frac{\sum_{i=1}^{M} \sum_{j=1}^{M} (I_o(i,j) - I_r(i,j))^2}{M^2}$$  \hspace{1cm} (12)

where $M$ is number of pixels and $I_o$ and $I_r$ are the transmitted and the received images, respectively.
The extensive simulation parameters are shown in Table 1. A convolutional code with memory length seven and octal generator polynomials (133,171) is chosen as the channel coding. The transmitted Cameraman image for all users of size 256 × 256 is shown in Fig. 4.

**Table 1: Simulation Parameters.**

| Parameter                        | Specification                   |
|----------------------------------|---------------------------------|
| Simulation method                | Monte Carlo                     |
| Total number of subcarriers      | \( M = 256 \)                   |
| Number of users                  | \( U = 4 \)                     |
| Modulation type                  | QPSK and 16QAM                  |
| Channel model                    | vehicular A channel             |
| Channel coding                   | Convolutional code with rate \( \frac{1}{2} \) |
| Subcarriers mapping mode         | Interleaved and Localized       |
| Cyclic prefix length             | 20 samples                      |
| Channel estimation               | Perfect                         |
| System bandwidth                 | 5MHz                            |
| Equalization                     | MMSE                            |
A. PSNR Performance:

As mentioned, the first parameter that is used for performance evaluation is PSNR. For this purpose, Cameraman image has been transmitted over the coded SC-FDMA systems. DFT, DCT and DST are used with different subcarriers mapping schemes and different modulation schemes. PSNR values of the received image are calculated for different SNR values from 0 through 35 dB with 5 dB steps. The obtained values of PSNR are tabulated in Table 2 and 3 for QPSK and 16QAM modulation schemes and plotted in Figs. 5 and 6, respectively.

Table 2: PSNR values of the received Cameraman image over the DFT-SC-FDMA, the DCT-SC-FDMA and the DST-SC-FDMA systems when QPSK is used

| SNR(dB) | DFT-SC-FDMA | DCT-SC-FDMA | DST-SC-FDMA |
|---------|-------------|-------------|-------------|
|         | DFT-LFDM    | DFT-IFDMA   | DCT-LFDM    | DCT-IFDMA   | DST-LFDM    | DST-IFDMA   |
| 0       | 10.2080     | 10.2771     | 10.2898     | 10.4563     | 10.2656     | 10.3738     |
| 5       | 12.9874     | 13.3443     | 13.2590     | 13.6281     | 13.2362     | 13.6772     |
| 10      | 17.7587     | 19.2205     | 18.3531     | 20.1733     | 18.3874     | 20.3777     |
| 15      | 25.1867     | 28.7409     | 26.4664     | 31.5331     | 27.3673     | 31.3177     |
| 20      | 31.9197     | 35.7429     | 34.4100     | 36.1237     | 34.4429     | 36.0053     |
| 25      | 38.5854     | 39.5274     | 37.7646     | 38.9335     | 38.1576     | 39.7188     |
| 30      | Inf         | Inf         | 49.3802     | Inf         | 49.3802     | Inf         |
Table 3: PSNR values of the received Cameraman image over the DFT-SC-FDMA, the DCT-SC-FDMA and the DST-SC-FDMA systems when 16QAM is used.

| SNR(dB) | DFT-SC-FDMA | DCT-SC-FDMA | DST-SC-FDMA |
|---------|-------------|-------------|-------------|
|         | DFT-LFDMA   | DFT-IFDMA   | DCT-LFDMA   | DCT-IFDMA   | DST-LFDMA   | DST-IFDMA   |
| 0       | 8.7691      | 8.7008      | 8.7532      | 8.6578      | 8.7435      | 8.6850      |
| 5       | 10.2697     | 9.3899      | 10.0285     | 9.3900      | 10.0044     | 9.3696      |
| 10      | 13.0524     | 11.7599     | 12.8112     | 11.8620     | 12.8034     | 11.8552     |
| 15      | 16.4961     | 15.9245     | 16.1343     | 16.2217     | 16.0954     | 16.0546     |
| 20      | 21.0151     | 22.7711     | 21.2917     | 23.5329     | 21.1310     | 23.4453     |
| 25      | 26.9265     | 30.8800     | 28.6601     | 32.1253     | 28.3922     | 31.8325     |
| 30      | 33.2733     | 35.2852     | 35.2801     | 37.7103     | 35.4358     | 38.9662     |
| 35      | 44.4245     | 56.9953     | 44.3399     | 57.7223     | 47.9389     | 60.7519     |

Fig. 5 and 6 show the relationship between PSNR and SNR when Cameraman image is transmitted through the DFT-SC-FDMA, DST-SC-FDMA and DCT-SC-FDMA systems for different subcarriers mapping schemes when QPSK and 16QAM modulation schemes are used, respectively. As illustrated in both figures, it is observed that PSNR is increased as SNR is increased. Clearly, it is observed that the interleaved systems give better PSNR performance than the localized systems. Moreover, it is also noted that DST-IFDMA and DCT-IFDMA systems provide better PSNR performance than DFT-IFDMA system. Furthermore, both DST-IFDMA and DCT-IFDMA systems provide the same PSNR performance as shown in Fig.5 but PSNR performance of DST-IFDMA is better than DCT-IFDMA beyond 20 dB as shown in Fig. 6.

Figure 5: PSNR against SNR of the Lena image transmission over the DFT-OFDMA, the DCT-OFDMA and the DST-OFDMA systems when the QPSK is used.
A. MSE Performance

In this subsection, the overall performance of MSE is tested and investigated. For that purpose, Cameraman image is transmitted over DFT-SC-FDMA, DST-SC-FDMA and DCT-SC-FDMA for different subcarriers mapping schemes when the QPSK and the 16QAM modulation techniques are used. The obtained values of MSE are presented in Tables 4 and 5 and plotted in Figs. 7 and 8.

Figures 7 and 8 illustrate the relationship between MSE and SNR when Cameraman image is transmitted through the DFT-SC-FDMA, DST-SC-FDMA and DCT-SC-FDMA systems for different subcarriers mapping schemes when QPSK and 16QAM modulation schemes are used. It is observed that the MSE decreases when SNR increases. However, we observed that the interleaved systems give lower values of MSE than the localized systems.

Table 4: MSE values of the received Cameraman image over the DFT-SC-FDMA, DCT-SC-FDMA and the DST-SC-FDMA systems when QPSK is used.

| SNR(dB) | DFT-SC-FDMA | DCT-SC-FDMA | DST-SC-FDMA |
|---------|-------------|-------------|-------------|
|         | DFT-LFDMA   | DFT-IFDMA   | DCT-LFDMA   | DCT-IFDMA   | DST-LFDMA | DST-IFDMA |
| 0       | 8.7691      | 8.7008      | 8.7532      | 8.6578      | 8.7435    | 8.6850    |
| 5       | 10.2697     | 9.3899      | 10.0285     | 9.3900      | 10.0044   | 9.3696    |
| 10      | 13.0524     | 11.7599     | 12.8112     | 11.8620     | 12.8034   | 11.8552   |
| 15      | 16.4961     | 15.9245     | 16.1343     | 16.2217     | 16.0954   | 16.0546   |
| 20      | 21.0151     | 22.7711     | 21.2917     | 23.5329     | 21.1310   | 23.4453   |
| 25      | 26.9265     | 30.8800     | 28.6601     | 32.1253     | 28.3922   | 31.8325   |
| 30      | 33.2733     | 35.2852     | 35.2801     | 37.7103     | 35.4358   | 38.9662   |
| 35      | 44.4245     | 56.9953     | 44.3399     | 57.7223     | 47.9389   | 60.7519   |
As illustrated in both figures, it is also noted that the DST-IFDMA and DCT-IFDMA systems give slightly lower MSE values than DFT-IFDMA system. Additionally, both DST-IFDMA and DCT-IFDMA systems provide the same MSE values as shown in Fig.7 but MSE values of DST-IFDMA are little bit lower than DCT-IFDMA beyond 27 dB as shown in Fig. 8
A. Clarity Investigation

As mentioned earlier, two metrics are used to measure and evaluate the performance of SC-FDMA for different basis functions, different modulation schemes and different subcarriers mapping schemes. To clarify the quality of the received image over the system, the reconstructed image at an SNR= 15 and QPSK dB are selected and shown in Fig. 9. By comparing the received images with the original images shown in Fig. 4, it is concluded that the quality of the received image by using DST-SC-FDMA and DCT-SC-FDMA systems are better than those images that are received by using the conventional DFT-SC-FDMA system for different subcarriers mapping schemes.

CONCLUSION

In this paper, the performance of SC-FDMA for different basis functions, different modulation schemes and different subcarriers mapping schemes has been investigated and tested. For that purpose, wireless transmission of gray-scale images over SC-FDMA system has been studied. It is concluded that transmits wireless images over SC-FDMA systems is possible for different basis functions. In addition, all of the simulation result achieved over DST-SC-FDMA and DCT-SC-FDMA systems show noticeable improvement in term of PSNR and MSE metrics. Moreover, the interleaved scheme provides better MSE and PSNR performances than the localized schemes for all systems.
Figure 9: Image for different SC-FDMA Systems at SNR = 20 dB and QPSK.

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