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Abstract
This article is to review, for the benefit of university teachers, the most important arguments concerning the theory of sailing, especially regarding its high-speed aspect. The matter presented should be appropriate for students with basic knowledge of physics, such as advanced undergraduate or graduate. It is intended, furthermore, to put recent developments in the art of sailing in the proper historic perspective. We first regard the general geometric and dynamic conditions for steady sailing on a given course and then take a closer look at the high-speed case and its counter-intuitive aspects. A short overview is given on how the aero-hydrodynamic lift force arises, disposing of some wrong but entrenched ideas. The multi-faceted, composite nature of the drag force is expounded, with the special case of wave drag as a phenomenon at the boundary between different media. It is discussed how these various factors have to contribute in order to attain maximum speed. Modern solutions to this optimisation problem are considered, as well as their repercussions on the sport of sailing now and in the future.

Keywords: physics of sport, sailing, hydrodynamics

(Some figures may appear in colour only in the online journal)

Introduction

Driving a ship by the force of the wind is an ancient technique harking back to the very beginnings of human civilisation. For many centuries, sailing ships were the prime means of long-distance transportation [1]. Although the perils were many, a considerable percentage of
ships were lost and transport was slow, it was commercially very successful. In some instances, moreover, speed became important, as for perishable goods like tea, or for unlawful purposes like smuggling. Making hulls more streamlined and rigs taller, speed could be pushed from a few knots of the ordinary merchantman to more than ten knots, approaching and surpassing twenty knots in some cases (1 knot = 1 nautical mile per hour = 1.853 km h\(^{-1}\)). In any case, even the fastest sailing ships of the classic times could not travel at speeds comparable to the speed of the wind driving them. This and much more can be achieved today, however, and this article will deal with the pre-requisites for this ultra-efficient mode of sailing.

**Sailing from the point of view of physics**

Two questions loom if you start to think about sailing: 1. How is it possible that a sailing ship moves against the direction of the wind, reaching a destination situated to windward; 2. How is it possible that boat speeds are reached in excess of true wind speed, and that both downwind and upwind?

From a physicist’s point of view a sailing ship is a system made up of two interconnected hydrodynamic foils, interacting with media of different density which meet them with different speeds at different angles. Figure 1, depicting a boat sailing close-hauled on the wind (i.e. at an acute angle to the wind direction), illustrates this fact.
Its velocity is $v_s$ and its true course deviates from its heading by a leeway angle $\beta$. From the perspective of an underwater observer at rest with respect to the hull, water arrives with a velocity $-v_s$ at an angle of attack $\beta$. Any body in a real fluid flow experiences a force component in the direction of the incoming flow, called resistance or drag, and a component normal to it called lift. The effect of the water flow on the keel profile is a drag component $D_H$ and a lift component $L_H$ perpendicular to it. These vector components add up to a resultant total hydrodynamic force $R_H$. A vector addition of the wind velocity $v_T$, ‘true’ wind, as seen by an observer at rest to the water, and an additional component $-v_s$ from the head wind due to the boat velocity, results in an ‘apparent’ wind $v_A$ as perceived in the reference frame of the moving boat. The apparent wind by interaction with sails and rigging generates a drag component $D_A$ and a lift component $L_A$, which add up to a resultant total aerodynamic force $R_A$. Alternatively, we may use for the component splitting the direction of boat movement and its normal, giving an aerodynamic side force $S_A$ and an aerodynamic driving force $F_A$. These latter may be calculated from $D_A$ and $L_A$ by the relations

$$S_A = L_A \cos \gamma + D_A \sin \gamma,$$
$$F_A = L_A \sin \gamma - D_A \cos \gamma. \tag{1}$$

Here $\gamma$ is the angle between course and apparent wind. We note from figure 1 that, due to elementary geometry,

$$\gamma = \varepsilon_A + \varepsilon_H. \tag{2}$$

The angles $\varepsilon_A$ and $\varepsilon_H$ are called aerodynamic and hydrodynamic glide angles, respectively. The tangents of these angles are given by the ratios of the respective drag and lift components. Thus the angle $\gamma$ at which a boat can sail to the apparent wind is determined by the hydrodynamic efficiency of the two foils involved. For the boat to steadily move at constant velocity the total forces $R_A$ and $R_H$ must of course balance out to zero, as is also true for their components $F_A$ and $D_H$ in the course direction, and $S_A$ and $L_H$ normal to the course direction, respectively. As the apparent wind does not meet the boat head-on but at a certain angle $\gamma$, it is possible that an aerodynamically efficient sail generates a forward driving force $F_A$, enabling the boat to beat upwind, i.e. move in a zigzag pattern towards a target in the eye of the wind (see equation (1)).

For a steady motion, not only force equilibrium, but also equilibrium of moments has to be maintained. It is convenient, although arbitrary, to consider the moments about three principal axes. The first axis is perpendicular to the water surface. Here it is to be considered that cambered airfoils can themselves develop a considerable moment. A typical additional moment arises if the aerodynamic force $R_A$ and the hydrodynamic force $R_H$ are not in line (figure 2).

This can be due to faulty design or unsuitable distribution of sails, but can also be a consequence of heel in a gust. The line of action of the sail force $R_A$ is not in the symmetry plane of the boat. When heel is increased it comes to lie even farther outboard. As the resultants of the aerodynamic and hydrodynamic forces then are no more in line, a moment is developed which tries to luff the boat (turn it into the wind), couple $R_H, R_A'$ in figure 2. This has to be met in time by rudder action and easing of the sheets. Otherwise uncontrolled luffing may result with possibly catastrophic consequences, especially when sailing under spinnaker.

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1. We designate vector entities by the term ‘velocity’ and speak of ‘speed’ if absolute values are meant.
2. For simplicity, for the time being we understand all forces to be parallel to the surface of the water.
3. According to a well-known theorem of classical mechanics, any distribution of forces working on a rigid body can be replaced by one couple and a resultant force.
The second axis is running lengthwise in the symmetry plane, and a moment around this axis tries to tilt (heel) the boat sideways (figure 3).

This moment is caused by the aerodynamic side force $S_A^*$ and the hydrodynamic side force which is equal to the hydrodynamic lift $L_H^*$. That the boat does not capsize immediately is due to the action of another couple, i.e. the weight force $F_G$ and the buoyancy force $F_B$. They are in
line in the upright boat. In the heeled boat the centre of buoyancy $B$ moves to leeward from its upright position $B_0$. A very important characteristic of the stability of the boat is the righting arm $x_a$ which follows for a heeling angle $\theta$ by

$$x_a = \frac{G}{M} \sin \theta.$$  

Here $G/M$ is the metacentric height, the distance of the centre of mass $G$ of the boat and the metacentre $M$. The metacentre for sufficiently small heeling angles is approximately determined by the point where the line of force of the buoyancy meets the symmetry plane of the hull. Metacentric height is a characteristic parameter of the transverse stability of a boat. The righting arm multiplied by the weight (= displacement) of the boat gives the righting moment.

The third axis is athwartships (normal to the symmetry plane). Moments about this axis cause pitching motion. For instance, the sail drive force $F_A$ acting several meters above the water plane together with hydrodynamic drag $D_H$ tends to submerge the bow. Whereas a conventional boat usually has enough reserve buoyancy in the bow region, this can become critical in catamarans and is the main reason for capsizing in this type of boats. The theoretical treatment is analogous to the side stability, making use of a lengthwise metacentre with a much greater metacentric height.

**How much speed is possible?**

The wind triangle $v_T$, $v_S$, and $v_A$ can have quite different aspects depending on the course sailed and on the ratio between boat speed and true wind speed (figure 4), the apparent wind speed being generally greater than the true wind speed if the latter comes from a front direction and smaller if it comes from astern.

If the boat speed is very high and surpasses the true wind speed (insert d in the lower part of figure 4), the apparent wind meets the boat from a forward direction even if the true wind comes from abaft. Therefore the boat is sailing ‘close-hauled’ on all points of course. This is a well-known fact in ice-sailing and can be observed clearly in the newest generation of America’s Cup foiling catamarans. The fact that apparent wind speed can be several times the true wind speed is sometimes causing bewilderment as it vaguely reminds of a *perpetuum*
mobile, the boat seemingly generating its own wind. The energy law remains untouched, however. As the boat slips along a comparatively large air mass, it takes an ever so small energy amount of each unit volume and converts it into kinetic energy concentrated in a small object, the moving boat. The same effect is seen in a wind turbine like that in figure 5 which is similar in construction to the toy objects you can often buy at fairs. A comparatively slight breeze puts them into fast rotation. Every single blade of the turbine behaves like a high-speed sailing boat, making use of the wind (true wind) together with a tangential wind felt by it due to the rotation. Because the latter can be larger than the true wind, the blades can be set at fairly small angles to the plane of the wheel.

In an ice yacht, the ‘hydrodynamic drag’, corresponding to the resistance of the sled on the ice, can be brought to virtually zero. The angle $\gamma$ between course and the apparent wind is then determined only by the aerodynamic glide angle $\varepsilon_A$ (figure 6).

What is the highest speed $v_S$ we can sail with a given true wind speed $v_T$? From the wind triangle in figures 1 and 4, we have

\[ v_S = \frac{v_T}{\cos \gamma} \]

**Figure 5.** Wind turbine analogue of high-speed sailing. At the right side a schematic view on a single blade from its tip. Axial wind velocity $v_T$ is enhanced by a component $-v_S$ due to tangential motion to apparent wind $v_A$ as perceived by moving turbine blade. This Singhofen, Windrad image has been obtained by the author(s) from the Wikimedia website where it was made available by Peter Kaminsky under a CC BY-SA 4.0 licence. It is included within this article on that basis. It is attributed to Peter Kaminsky.

**Figure 6.** Ice yacht in the approximation $D_H = 0$. Here boat speed is four times true wind speed. [2] Copyright Wiley-VCH Verlag GmbH & Co. KGaA. Reproduced with permission.
It follows that $\gamma - \gamma_T$ should optimally be $90^\circ$. For very small $\gamma \approx \varepsilon_A$, this means that true wind direction should be perpendicular to the course (in nautical language, this is a beam reach). Then the only way to further enhance top speed is to reduce $\gamma \approx \varepsilon_A$. In a more conventional boat where $\varepsilon_H$ cannot be disregarded, it should in any case be as small as possible. Both requirements call for profiles with high lift to drag ratios. If, however, hydrodynamic drag cannot be reduced enough, then the boat is confined to the low- speed range, $v_{f} < v_T$. In this case, it can make sense to choose a rig that generates maximum total aerodynamic force, even at the price of enhanced drag [3]. In any case, the aerodynamic force has to be reconciled with the stability of the boat as excessive heel causes additional resistance, causing the boat to slow down.

Let us now investigate the nature of hydrodynamic (aerodynamic) forces. We think it appropriate to state at this point that for our present discussion on sailboats where all speeds are small as compared to the speed of sound, air may to a very good approximation be regarded as an incompressible fluid [2], as is generally assumed to hold for liquid water. Both can simply be regarded as ‘fluids’.

Hydrodynamic lift

We have defined lift as a force which acts normal to the flow direction on a body, typically an airfoil. What is the origin of such a force and how can we optimise it? For centuries scientists including the greatest minds of their age such as Isaac Newton, Daniel Bernoulli, Christopher Wren, and others [4] have tried to understand this phenomenon, coming up with various explanations which, although sometimes containing some grain of truth, have serious deficiencies.

The first obvious idea was that the air molecules are deflected by a collision with the sail, transferring part of their momentum and thus generating the aerodynamic force. What was overseen was the fact that the molecules in a fluid exchange forces not only with the sail but also among themselves. Therefore a fluid does not behave like a particle shower, and the forces exerted by it on the sail have to be accounted for along the whole surface, specifically also on the leeward side. The whole problem has to be treated in the framework of fluid dynamics. The momentum transfer idea is not completely wrong, however. Indeed we observe that a sail deflects the air flow somewhat. This direction, and therefore momentum, change of the fluid can be observed up to a rather great distance upstream and downstream of the sail (affecting other sails in the vicinity such as a competitor in a regatta). The effect of the sail can therefore be simulated by a virtual deflecting barrier. Its position is not identical, however, with the actual position of the sail. The sail, or rather the complete rig, rather has to be regarded as a ‘black box’ having the same effect on air flow as the virtual deflecting barrier would have on a stream of separate air particles.

The second popular explanation correctly invokes Bernoulli’s theorem:

$$\frac{\rho v_0^2}{2} + p + \rho g z = \text{const.},$$

(5)

$\rho$ being the density of the fluid, $v_0$ the speed of flow, $p$ the pressure, and $g$ the local gravity acceleration. This is an energy conservation theorem, meaning that kinetic energy, pressure energy and potential energy in the gravitation field sum up to a constant along a streamline

4 Streamlines are the field lines of the velocity field in fluid flow. The tangent vector to a streamline is always parallel to the instantaneous local velocity vector.
now that along a typical asymmetrical airfoil profile fluid particles on the upper, more curved side have to negotiate a longer path towards the tail end of the profile, resulting in higher flow speed and consequently lower pressure. The flow is that there is absolutely no reason for two neighbouring fluid particles which become separated at the nose of the profile to reach the tail of the profile at the same time. On the contrary, a correct calculation [5] of the fluid dynamics shows that on a lift-generating airfoil the particle on the upper side reaches the tail much earlier than its buddy travelling along the underside of the profile.

What is then the true mechanism of hydrodynamic lift? Bernoulli’s theorem tells us that there has to be some additional flow component increasing the airspeed on the upper (leeward) side relative to the lower side (windward side) of the profile. It turns out that in order to create positive lift there has to be a sufficient attack angle $\alpha$ by which we mean the angle between the (undisturbed) air flow and the chord of the profile. This angle depends on the form of the profile. If the profile is cambered, this means that the midline of the profile deviates from the chord line by a curve, then positive lift can be reached even at negative attack angles. Conversely, a completely symmetrical profile like e.g. a keel fin, can create positive lift only at a positive attack angle.

When a profile starts to move relative to the fluid, the picture at first resembles very much what we would get in an ideal (non-viscous) fluid ((a) in figure 7). There is a stagnation point (SP₁ in figure 7) near the nose on the underside of the profile where the flow separates. The flows recombine at an aft stagnation point SP₂ near the tail on the upper side. In a viscous fluid, the innermost layer is attached to the body due to adhesion forces. As the flow negotiates the sharp trailing edge towards the aft stagnation point, due to inertia a vortex is created (similar to a car braking on a gravel road in a curve) which finally detaches and is carried off by the surrounding flow. The law of conservation of angular momentum leads to the establishment of a counter-rotating flow around the profile (bound circulation). The bound circulation smooths out the flow at the tail so that upper-side flow and lower-side flow join smoothly (Kutta’s run-off law). In fact this run-off condition is steadily kept up and fine-tuned by left- and right-rotating vortices which are left in the wake. The bound circulation leads to the lift force via Bernoulli’s theorem according to the law of Kutta–Joukowski [7]:

$$L(y) = -\rho \ v_0 \ \Gamma.$$  \hspace{1cm} (6)

In this formula as in figure 7 the undisturbed flow with magnitude $v_0$ is assumed to be in the $x$ direction. The $z$ coordinate points to the top of the sheet and $L(y)$ is the lift force per unit length (of wing in the $y$ direction) in the positive $z$ direction. $\rho$ is the density of the fluid and $\Gamma$ the circulation which mathematically is defined as a closed path integral of the flow velocity field $\mathbf{v} = \mathbf{v}(x, t)$ around the profile:

$$\Gamma = \oint_C \mathbf{v} \cdot d\mathbf{s}.$$  \hspace{1cm} (6a)

The minus sign in equation (6) is due to orientation conventions as the assumed clockwise bound circulation is formally mathematically negative. Equation (6) deals with an essentially 2D situation. It yields the lift per unit length of a wing with infinite length where the profile does not change along the wing. Real wings, however, have properties which vary along the wingspan. Usually a wing is tapered, so the chord length diminishes as we walk outwards on the wing. The profile may change and the wing may be twisted, circulation $\Gamma(y)$ will depend on the position $y$ along the wing. Finally, the wing ends somewhere. This leads to additional vortices and the so-called induced drag (see below). If we sum up the lift of a 3D wing by integrating equation (6) along the (double) wing with total span $s$:

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5 The following explanation is accepted knowledge and can be found for instance in [3].
and make corrections for effects which are due to the three-dimensional situation, we can finally describe lift in the form of an ‘engineer’s formula’:

$$L = \frac{1}{2} \rho v_0^2 c_L A$$  \hspace{1cm} (8)$$

with wing (or sail) area $A$. The expression $\frac{1}{2} \rho v_0^2$ in front we found already in Bernoulli’s theorem equation (5). It corresponds to the kinetic energy content of a unit volume of the flow and is called dynamic pressure. In the form of equation (8) the fact is reflected that the circulation $\Gamma$ needed to guarantee smooth run-off at the tail is proportional to the flow speed $v_0$. The circulation $\Gamma$ also scales with the chord length of the wing. The dependence on attack angle, the properties of the profile design, and the Reynolds number all are contained in the lift coefficient $c_L$. There is a nearly linear relationship between $c_L$ and the attack angle $\alpha$. 

**Figure 7.** Formation of the bound circulation (bound vortex) around an airfoil: (a) initial flow picture similar to ideal fluid (b) emergence of the starting vortex (c) bound circulation around the wing, the starting vortex is left in the wake. [2] Copyright Wiley-VCH Verlag GmbH & Co. KGaA. Reproduced with permission, after experiments by Prandtl [6].
What form of profile has the desired properties, i.e. as much lift and as little drag as possible? A classical sail made of fabric or synthetic foil can provide a large lift force if the camber (curvature) is pronounced and if we can prevent the flow from detaching in the lee for large attack angles. Especially at higher Reynolds numbers, however, it will be definitely limited as to the ratio lift/drag. One reason is that it is deformed by the force of the wind (which can be partially compensated by introducing stiffening battens into the sail). High-performance rigs call for rigid wing profiles which enable higher $L/D$ ratios. The most sophisticated constructions allow one to adjust both camber and twist when sailing as in the recent America’s cup boats which carry a wing sail divided by a slit, a configuration known to enhance lift ([2], compare landing flaps in commercial aircraft).

**Induced drag $D_i$**

Up to now, we have not yet talked about the consequences of the wing (sail, fin keel, centerboard, rudder) having a three-dimensional structure with varying profile properties along the wingspan. As the chord length of the profile diminishes along the wingspan, the bound circulation $\Gamma$ also diminishes. The Helmholtz vortex laws require that vortex strength must be conserved (mathematically, $\text{div(curl } \mathbf{v}) = 0$) so that a vortex may not arise or die anywhere within the bulk volume of a fluid. This means that the bound circulation strength we lose when progressing along the wing must be left as vortex in the wake, analogous to branching electric currents. In figure 8 this situation is depicted schematically.

Thus the 3D wing leaves a trailing vortex sheet behind. These (infinitesimal) vortices together create a downwash $w$ which at the position of the wing is given for a normalised position $\eta = y/(s/2)$ along the wingspan by

$$w(\eta) = K \, \text{CP} \left[ \int_{-1}^{1} \frac{\Gamma'(\eta')}{\eta - \eta'} d\eta' \right],$$

with some constant $K$. The function CP means Cauchy principal value as the integrand has a point of discontinuity at $\eta = \eta'$.

The downwash has a detrimental influence on lift because it adds a vertical component to the incoming flow so that the effective attack angle diminishes by a certain amount. If the same lift as without the downwash shall be obtained, it is necessary to increase the attack...
angle by the same amount by tilting the profile. Now we have the same total aerodynamic force as before, but tilted by the correction angle. This can be interpreted as the original force plus an additional drag component, the induced drag. Figure 9 illustrates in a 2D cut the effect of downwash and the necessary angular correction.

The downwash has in general different values along the wingspan. It turns out that total induced drag is minimal (for a wing of given area and span) if the downwash is constant along the wingspan [2]. This corresponds to a certain function $\Gamma(\eta)$ with which the bound circulation varies along the wingspan. The function is an ellipse for a double wing or a half-ellipse for one wing. This specific distribution of lift is called ‘elliptic loading’. The coefficient of induced drag is in this case inversely proportional to the length to width ratio (aspect ratio) of the wing (see appendix A8 in [2]). If we however regard the induced drag itself, we get the following formula:

$$D_1 = \frac{1}{\pi} \frac{1}{\frac{1}{2} \rho v^2} \frac{L^2}{s^2}.$$  

(10)

The induced drag is proportional to the square of the lift and indirectly proportional to the wingspan $s$. Therefore, high-performance wings should be long (and consequently for a given area also narrow, compare high-performance sailplanes).

From an energy viewpoint, the vortices shed continuously by the wing are an energy expenditure which is equivalent to the action of a drag force, viz. the induced drag. The vortex sheet due to the mutual interaction of the vortices has a tendency to roll itself up into two large vortices behind the wing. Figure 10 illustrates this phenomenon. It can create problems if a smaller plane meets the vortex trail of a big plane, but also in regattas one should beware of the ‘foul air’ due to vortices behind a competitor.

We have seen that in order to minimise induced drag, we can make the wing as long as possible (in a sailboat we always have transverse stability as a limiting condition, though) and give it a favourable planform which according to classical lifting line theory should be nearly elliptic.

6 Elliptic loading (= elliptic lift distribution) is, however, not identical with an elliptic planform, as we also have to make allowance for twist and changing profile along the wing.
Recently, however, the so-called square-top shape (a tapered form which looks like the very tip of the sail is cut off) has become very popular for racing boats. It has proved to be almost equivalent or even superior to the elliptical shape as regards induced drag. At the same time it shows advantages in varying wind strengths, better tolerating gusts.

Another way to deal with induced drag is to impede or modify the flow around wing tips by mounting winglets there, as can be seen regularly in present-day commercial aircraft or at the tips of wind turbine blades. In sailboats, a mast top winglet is not practical, but winglets on a keel have proved very successful: in 1983, an Australian yacht fitted with keel winglets succeeded in winning the America’s cup for the first time in 132 years from the US team.

Drag

Many influences contribute to the resistance (= drag) of a body in a flow. We already talked about induced drag as a concomitant phenomenon to lift of a 3D wing.

From a physicist’s point of view a body can experience drag (or lift) only by forces exerted by the surrounding fluid and acting either parallel to its surface (shear forces due to viscosity) or normal to its surface (pressure). In ship design it has however proved convenient to distinguish drag components according to their causes so as to be able to optimise a design in a systematic way. Thus the whole drag is usually subdivided as

$$D = D_{\text{V, upright}} + D_{\text{W, no leeway}} + D_{\text{Heel}} + D_{\text{I}} + D_{\text{added}}$$

With a further subdivision

$$D_{\text{V}} = D_{\text{F}} + D_{\text{R}} + D_{\text{VP}}.$$  \hspace{1cm} (11a)

The meaning of the indices is: \text{V} vor ‘viscous’, \text{W} for ‘wave’, \text{I} for ‘induced’, \text{F} for ‘friction’, \text{R} for ‘roughness’, and \text{VP} for ‘viscous pressure’.

In the following paragraphs we will discuss these separate components.
In a hypothetical ideal fluid with zero viscosity no forces at all would be exerted on a body immersed in the flow (d’Alembert’s paradox), which of course is not the case in a real, viscous fluid. The viscous force is transferred between two parallel fluid layers moving at different velocities. It acts in a direction parallel to the layers and is proportional to the velocity gradient normal to the layers and a material constant called dynamic viscosity. The dynamic viscosity for an ambient temperature of 20 °C is in the range of $\mu = 1.8 \times 10^{-5}$ Pa s for air and in the range of $\mu = 10^{-3}$ Pa s for water. An important detail is that kinematic viscosity, defined as viscosity divided by density, is larger by a factor of about 14 for air as compared to water. As far as the character of fluid motion is concerned, air is therefore the more viscous medium than water (see table 1).

Kinematic viscosity $\nu$ enters in a dimensionless number which is a critical parameter pertaining to the general character of flow, i.e. Reynolds’s number,

$$Re = \frac{lu}{\nu}. \quad (12)$$

The other two quantities are a characteristic length $l$ and the speed of flow $u$. According to the situation, the length $l$ can have different meanings. For fluid flow in a tube, the radius of the tube will be the appropriate characteristic length, for an airfoil it could be the chord length of the profile. If we want to discuss how the character of the boundary layer evolves as we pass along the surface of a body, then it will be appropriate to consider the distance from the forward end of the body to the point we inspect. As $Re$ surpasses a certain threshold, which for smooth airfoils is in the region of $Re = 5 \times 10^5$ to $10^6$, the flow character changes from laminar to turbulent. In the first case the streamlines are smooth and well-behaved, whereas in turbulent flow vortices of various sizes arise in an irregular way, showing the typical behaviour of chaotic dynamics. Given the same flow speed, the transition to turbulent flow will therefore occur in air at a point farther down the profile than is the case in water.

7 The reasoning presented here can be found in more detail in textbooks on hydrodynamics, e.g. [7] or [8].

| Fluid       | $\mu$ (Pa s) | $\nu$ ($10^{-6}$ m$^2$ s$^{-1}$) |
|-------------|--------------|----------------------------------|
| Water (fresh) | 0.0018   | 0 °C                             |
|             | 0.00152  | 5 °C                             |
|             | 0.001297 | 10 °C                            |
|             | 0.0010  | 15 °C                            |
|             | 0.001    | 20 °C                            |
|             | 0.00891 | 25 °C                            |
|             | 0.0008  | 30 °C                            |
| Air         | $1.8 \times 10^{-5}$ | 14.6                          |
| Ethanol     | 0.0012   | 1.5                              |
| Glycerine   | 1.52     | 1200                             |
| Mercury     | 0.00156  | 0.12                             |
This kinematic phase transition is due to the fact that the Navier–Stokes equations which govern fluid motion as expressed by the velocity field \( \mathbf{v}(\mathbf{x}, t) \), are nonlinear (in the second term in the following equation (13)):

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{v}.
\] (13)

The friction resistance proper \( D_f \) is caused by viscous forces parallel to the surface elements. There is also another contribution \( D_{vp} \) which arises because due to viscosity the flow in the vicinity of the body is altered so that the pressure components in the flow direction at the body surface do not cancel out to zero (as would the case in an ideal fluid) any more. If we strive to reduce drag, we have to carefully balance friction drag and viscous pressure drag. In a typical flow scenario around a foil flow speed is increased from a stagnation point (speed zero) at the entry towards a maximum and then decreases again until it is slowed down so much that it finally separates from the body, generating a vortex zone after this point which contributes to \( D_{vp} \). Making the body very slim and long will shift the separation point towards the tail, reducing the width of the vortex zone and therefore the pressure drag associated with it. The price one has to pay is, however, a larger surface where the flow adheres to the body and therefore generally more friction drag. Friction drag is not directly proportional to the wetted surface, however, but depends on the shape of the body and its curvature (a very important problem in ship design). What is the best solution depends on the speed of the flow, more exactly on the Reynolds number.

Viscous drag is generated within a boundary layer where the flow speed increases from zero at the body surface (where the fluid is attached by adhesion forces) to the speed of the surrounding flow. If the flow character is turbulent, the boundary layer is thicker, and viscous drag is larger than in laminar flow. However, transverse momentum transfer takes place more efficiently in a turbulent boundary layer, so that flow does not become sluggish so soon, and the detachment point is shifted towards the tail. This in turn reduces the width of the vortex zone and therefore pressure drag. An example of this effect is golf balls where the dimples in the surface intentionally cause turbulence within the boundary layer [9].

Of course, the roughness of the surface plays an eminent role in viscous drag. Here the inner structure of the boundary layer comes into play. There is a laminar lower layer under the turbulent boundary layer. Surface roughness becomes felt as soon as it juts out of this lower layer, setting limits on ‘allowed’ roughness, the latter being larger towards the rear of the boat, and generally smaller as boat speed (∼ speed of flow) increases. This is true for irregularly spaced roughness. A special surface nanostructure behaves differently and may reduce viscous drag significantly. The contribution of surface roughness is usually separated from viscous resistance as \( D_R \), see equation (11a).

We will talk about the contribution \( D_W \) below. \( D_I \) means drag due to vortices shed by the underwater body and especially the keel when the boat makes leeway, analogous to the induced drag of an airfoil. \( D_{Heel} \) means added resistance of various nature as the boat sails heeled: more wave drag, more induced drag, more (or less) viscous drag due to the altered wetted surface. \( D_{added} \) means additional drag by dynamic interaction with waves if the boat is sailing in a seaway. Drag components arise both in water and in air, and act on both the hull and the rig. It is clear that the wave resistance \( D_W \), \( D_{Heel} \) and \( D_{added} \) apply only to the hull.

Total hydrodynamic drag on a macroscopic body with turbulent flow (excluding wave drag and \( D_{added} \)) can again be written in terms of an ‘engineer’s formula’ similar to
equation (8):

\[ D = D_v (D_t + D_{\text{heel}}) = \frac{1}{2} \rho v_0^2 c_D A. \] (14)

Here the dynamic pressure \( \frac{1}{2} \rho v_0^2 \) again appears in front. \( A \) is the area, either as seen head-on by the flow, or, when considering an airfoil, its plan area as seen from above. The drag coefficient \( c_D \) contains dependence on the shape of the moving object, roughness of its surface, and Reynolds number (and angles of leeway and heel, if applicable).

**Wave drag \( D_W \)**

In addition to the hydrodynamic drag of a body completely immersed in a fluid, a boat also experiences forces originating in the phenomena at the boundary of the two media in which it moves. As far as wave drag of a ship is concerned, water waves are driven by gravity. Neglecting the influence of viscosity, deep water waves\(^8\) travel at a phase velocity

\[ v_p = \frac{\omega}{k} = \frac{g}{k} = \sqrt{\frac{g \lambda}{2\pi}}. \] (15)

The group velocity is equal to one half that value. Here \( \omega \) is the angular frequency, \( k = 2\pi/\lambda \) the length of the wave vector, \( \lambda \) the wavelength, and \( g \) the local gravity acceleration. A ship moving along its course generates a disturbance of the water surface similar to what we would get by continuously throwing little stones into it. These disturbances interfere and create caustics, resulting in a characteristic wave pattern derived already in the 19th century by Lord Kelvin (William Thomson) (figure 11)\(^9\).

It has a traverse and a diverging system within a wedge structure opening at an angle corresponding to \( 2 \times \arcsin (1/3) \approx 2 \times 19.28' \). It is quite remarkable that this angle does not depend on ship speed, density of the fluid, or the local gravity. It would be essentially the same if we were sailing on the methane sea on Saturn’s moon Titan, for instance. The general form of the Kelvin wave depends only on the dispersion relation equation (15) and the fact

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\(^8\) Deep water means that water depth markedly exceeds the wavelength.

\(^9\) A derivation can be found in [10].
that the wave system, as seen from the ship, is stationary. The distance of the single waves in the pattern depends on the speed of the ship, however. The patterns of two different-sized ships are strictly geometrically similar if the ships themselves are strictly geometrically similar and the dimensionless Froude numbers

\[ Fr = \frac{v}{\sqrt{gL}} \]  

(16)

are equal, \( l \) meaning the waterline length. A specific Froude number means that the ship-generated waves in the far-field have a certain wavelength as measured by the waterline length of the ship. Near the hull, the exact wave form is determined very much by hull shape and also by viscous interactions. If a ship design is to be investigated by towing a model in a tank the towing speed has to be chosen so that the model and the prospective ship travel at the same Froude numbers, making their wave systems geometrically similar.

Wave drag is not a simple function of ship speed, and cannot be represented by the quadratic law equation (14). The disturbances are generated from every point along the hull, and they all interfere. In sailing yachts, the bow and stern yield the most important contributions. Constructive interference of the bow and stern systems lead to higher waves and increased wave drag. Conversely, if the maxima of the bow system meet the minima of the stern system, lower wave drag results.

Total wave drag can be approximately described by an expression like (\( v \) is the boat speed)

\[ D_w = [A + B_1(v) + B_2(v) + B_3(v) + B_4(v)]v^6 \]  

(17)
i.e. a general dependence on \( v^6 \) where the terms in brackets account for the wave interference phenomena, showing minima and maxima at certain Froude numbers. If two geometrically similar ships travel at the same Froude numbers (their wave systems therefore looking geometrically similar), their wave drag scales like \( L^3 \), i.e. with the volumes of the generated waves.

How to reduce wave drag? First of all, a long, narrow boat with sharp ends and little displacement will generally show less wave drag at a given speed than a short, beamy boat with large displacement. Great length helps because a boat is then travelling at a lower Froude number compared to a short boat at the same absolute speed. At a Froude number of about 0.4 (exactly \( (1/(2\pi))^{1/2} \approx 0.4 \)) the second wave peak of the bow system just coincides with the first peak of the stern system, a situation called hull speed. Measuring the waterline length of the ship in meters, and the ship speed in knots, hull speed, which is the phase speed of a wave with \( \lambda = l \), can be calculated by the rule-of-thumb formula

\[ v_h \approx 2.43 \sqrt{l}. \]  

(18)

Any further increase in speed means that the boat has to climb up the bow wave which entails a substantial increase in drag.\(^{10}\) A conventionally built boat with large displacement cannot easily surpass this resistance barrier. If the boat is light and can muster the necessary driving force and stability, then it may climb the bow wave and enter the planing state which is characterised by dynamic lift so that the boat displaces less water than it weighs. At the same time, the stern wave is left astern, the bow wave is flattened, and the drag coefficient diminishes markedly. A planing state is reached, depending on the characteristics of the hull, at Froude numbers between \( Fr = 0.5 \) and \( Fr = 1 \).

\(^{10}\) Actual hull speed, defined as the speed at which wave drag increases sharply, however depends on the shape of the hull, so that some boats may be driven well beyond the speed given by equation (18).
Figure 12 is a schematic representation of the hydrodynamic situation on the centerline of a planing hull. Reproduced with permission from [11] and McGraw-Hill Global Education Holdings.

Figure 12 is a schematic representation of the hydrodynamic situation on the centerline of a planing hull.

As the liquid flow does not surround the hull, obviously lift can be generated only by a pressure increase at the bottom. Water meeting the hull before the stagnation point has to escape sideways as spindrift. A good planing hull is rather wide and flat, although too large beam will unacceptably increase wave drag at speeds below the planing regime. For course stability and in order to avoid excessive slamming into waves, most planing hulls nevertheless carry a certain amount of deadrise (v-shaped sections near the keel), reducing thereby the projected surface somewhat. It is especially important that planing hulls are flat at the stern, with an edge allowing the flow to detach from the hull. For constant lift, an optimal inclination of the hull to the flow of about $7^\circ$ yields the lowest drag [12]. All modern sailing speed records have been obtained by craft in a planing state (at least during extended legs of their course).

**Transverse stability**

The wind force generated by the sails has an important side component which tends to heel the boat (see figure 2 and the discussion there). Since heel reduces sail driving force by diminishing projected sail area, and increases wave and vortex drag ($D_{\text{heel}}$), it is necessary to counteract it. One possibility is large beam which, however, again goes with increased wave drag. Another is the shifting of living or dead weights to windward, the first being realised by crew hiking to windward, often with the help of trapezes or similar contrivances. The second can be realised by water ballast pumped into tanks on the windward side, or by a canting keel (in the 19th century, sand bags were sometimes shifted). A very efficient method to achieve transverse stability and have at the same time a narrow and sharp hull, is to build a multihull boat, such as a catamaran (two hulls joined by beams) or a trimaran (three hulls).

It is with the help of sandwich build and modern materials like epoxy and carbon fibre that in recent times great advances in light-weight construction have been made, enabling easily planing hulls and pushing top speeds.
Dynamic lift by hydrofoils

The ultimate recipe to reduce wave drag is to lift the hull out of the water altogether by horizontal underwater hydrofoils. It is important here not to confound the function of the classical keel or centerboard which can be seen as a hydrofoil installed in a more or less vertical position, and which is generating a side force to counteract the side force of the wind, with hull-lifting hydrofoils, called just ‘foils’ in modern parlance. What blurs the distinction somewhat, however, is that both functions are combined in modern foiling constructions where the foils are curved and have an oblique position. Although the idea is several decades old and had been proved to work in principle, the breakthrough has come only in our day as light-weight and at the same time strong materials have become available.

Figures 13 and 14 offer glimpses at foiling catamarans representing the edge technology as of 2017, realised for the 2017 America’s Cup competition.

The contact with water is established only via three hydrodynamic foils. Port and in front on the leeward side there is a lift-generating surface, placed where a classical centerboard would be. Its function is to lift the hull out of the water and to counteract the sail side force. The two rudders carry additional horizontal foils at their ends. Their attack angles are set for positive lift on the leeward side and for negative lift on the windward side. The reason is that, no hull being in the water, both the heeling and the forward tilting (pitching) moments of the sail (the centre of aerodynamic force is several meters above the water plane) have to be compensated for. In a classical sailboat, the heeling and pitching moments can be met by the buoyancy of the hull. Here everything has to be done by the forces generated by the hydrofoils.

Figure 14 gives a closer look on the forward foils. On the windward side it is drawn up, and the lift-generating horizontal, slightly oblique part is seen. On the leeward side both the side force generating part and the lift-generating part (of which you can see just the tip), joined by a curved
intermediate piece, are effective in the water. They can be lifted, canted (tilted sideways along a lengthwise axis) and rotated about a vertical axis in order to change the attack angle.

How to move fastest over a regatta course

We can plot aerodynamic lift against drag and thus obtain a polar diagram of total aerodynamic force. A typical example is shown in figure 15. Making use of equations (8) and (14) we can also plot directly the lift coefficient $c_L$ against the drag coefficient $c_D$, which is then called a Lilienthal diagram.

The respective attack angles $\alpha$ are indicated at the little circles along the diagram. Lift increases steeply with attack angle, until after reaching a maximum flow separates behind the nose. At very large attack angles, the aerodynamic profile delivers its force mainly in the form of drag instead of lift. Although this is relevant for conventional yachts running before the wind, high-speed sailing always exploits the regime of attack angles left of the maximum. The aerodynamic glide angle $\varepsilon_A$ appears in the diagram as the angle between the $y$ axis and a straight line from the origin to a given point on the curve. The smallest possible glide angle with a given profile is obtained by drawing a tangent to the curve from the origin. As there is always some drag, the polar diagram does not contain the origin. The horizontal offset is increased further by additional resistance by mast, shrouds, spreaders, and the hull (‘parasitic’ resistance). With respect to a sailing boat, the $x$-axis (drag) of this diagram has to be oriented parallel to the apparent wind direction. We can graphically obtain the largest possible driving

Figure 14. U.S. competitor at the 2017 America’s Cup, view from aft. The main lift-generating hydrofoil is working on the port side while it is in a retracted position on the starboard side. The lift-generating foils at the end of the rudders incidentally serve also as winglets for the rudders themselves, diminishing their induced drag. Reproduced with permission from Rick Tomlinson.
force component $F_A$ by laying a tangent to the polar curve normal to the direction of course. This chooses a particular angle of attack $\alpha$. Due to the relation $\gamma = \alpha + \delta + \beta$ we thereby also establish the most favourable sail angle $\delta$ subtended by the chord of the sail and the symmetry plane of the boat. This is, in fact, what the sailor actually chooses by easing or hardening the sheet. Assuming this to be correctly done and all other rigging parameters to be optimally tuned, we can draw a polar curve for boat velocity which shows a characteristic heart shape (figure 16).

In this diagram, true wind comes from above (negative $y$ direction). In all sectors where the polar diagram is convex, the best course direction is that indicated by the diagram. In all sectors where the diagram is concave, the fastest courses are determined by common tangents. Most notably this is the case for beating to windward. The best tacks for upwind sailing are given by joining the origin to the osculation points of the tangent common to the symmetric starboard and port parts of the curve. Thus we obtain the largest velocity component in the windward direction (the $y$ component of the velocity vector), usually called velocity made good (VMG). Mind that this does generally not correspond to the smallest possible angle a boat can sail to the true wind. Especially with high-speed yachts, a comparatively larger angle to the true wind direction is often favoured. This is very pronounced after a tacking maneuver when it is imperative to gain speed quickly in order to reach sufficient side force. Not only upwind, but also downwind high-speed boats show a concave velocity diagram. Here the same principle applies: the best course directions are obtained by laying common tangents. Therefore these boats gybe downwind in successive laps.

\[ \gamma = \alpha + \delta + \beta \]
Figure 17 shows speed polar diagrams for fast boats, an 18 ft skiff and a Tornado catamaran. Grey areas show the concave regions for the 18 ft skiff. We already talked about beating upwind and gybing downwind (sectors I and III). The polar diagram of the 18 footer has a pronounced ‘nose’ for broad reaches as a very high speed can be reached on this point of sailing with a gennaker sail. In sector II this means that e.g. a racing mark is reached fastest by sailing one lap with gennaker and the other lap without gennaker. Note that the downwind effective velocity VMG is greater than the true wind speed.

Conclusions and outlook

At present we experience a sailing speed explosion on water11. The 500 m speed record of 2012 reads 65.45 knots $\approx 121.2 \text{ km h}^{-1}$ established by a sled-like asymmetric planing construction, the Vestas sailrocket. Although this achievement required a very special setting (the runs were made in a narrow canal), speeds in excess of 40 knots have been reached at the recent America’s Cup regatta with foiling catamarans.

True wind speed, in contrast to a long-held popular belief, is not to a limiting barrier to boat speed, which is borne out by present-day sailing performance: you can sail upwind with an effective VMG several times the true wind speed, and you can out-sail downwind a balloon drifting with the wind.

11 To keep abreast of new developments we recommend to follow the website https://sailspeedrecords.com/.
Although the basic physics of high-speed sailing has been known for at least a century, this evolution could only take place as extremely light-weight and strong materials have become available, mostly compounds of epoxy resin and carbon or aramid fibres. At the same time computational fluid dynamics [13] and wind tunnel tests have greatly refined hydro-aerodynamic know-how. It is an ironic twist of history that nowadays when sailing ships as freight transporters are a thing of the past, sailing technique in the realm of sports has been improved to an unprecedented degree.

If we want to sail at high speed, we must reduce aerodynamic and hydrodynamic glide angles, which is equivalent to achieving high lift to drag ratios. Provided one can build efficient wing sails and underwater profiles, the main problem remaining is wave drag. A classical sailing yacht displaces as much water as it weighs and builds up considerable wave mountains, reaching a barrier to further acceleration at hull speed. In a first step, this barrier was overcome when light-weight boats were constructed that were able to plane easily. Thus they could enter a state where part of their weight was carried by hydrodynamic lift and the wave drag was considerably reduced. The second step is foiling: the whole weight of the boat is carried by hydrodynamic lift. It goes without saying that a boat sailing on two to three ‘stilts’ is difficult to stabilise and control. Even in this respect, great progress is being made, and more user-friendly foiling boats are beginning to enter the market [14].

The 2013 edition of the America’s Cup, the oldest and most renowned sailing competition, brought with it a paradigm change in speed and media coverage. For the first time foiling catamarans were employed, and the unfolding drama of the race was brought to the TV spectator in hitherto unseen detail, showing current speeds, expected positions at crossings, VMG, wind shifts, positions with respect to the mark etc. This was partly done also in

Figure 17. Speed polar diagrams for an 18 ft skiff and a Tornado catamaran. Grey: Concave sectors (details: see text). [2] Copyright Wiley-VCH Verlag GmbH & Co. KGaA. Reproduced with permission.
earlier editions of the America’s cup. Thus sailing became a spectator sport. What is more, the improvements in speed, strength to weight ratio, and sophisticated aerodynamics yield many spin-offs for the common yachtsman, and we can confidently expect that many of the features which almost seem out of science fiction today will become commonplace.

Further reading on sailing physics: apart from the author’s book [2], which is available in German only, the works by Larsson, Eliasson and Orych [11] Anderson [15] (rather concise), Garrett [16], Kimball [17] and Fossati [18] are recommended.

### Appendix. A glossary of some nautical terms

| Term | Definition |
|------|------------|
| Apparent wind | Wind as perceived in the reference frame of the moving boat. |
| Beam reach | Point of sailing with the true wind at right angle to the course. |
| Beat (to beat) | A boat is beating if it moves in a zigzag fashion towards a target which cannot be reached directly or which can be reached faster this way. Also: one of the zigzag laps. |
| Bow | The forward end of a boat. |
| Centerboard | Like the keel, a vertical plate in the centerline of the boat put into the water in order to generate hydrodynamic side force and thus prevent excessive leeway. |
| Close-hauled | A boat is close-hauled if it sails with a small angle to the wind so that the largest velocity component to windward can be obtained. |
| Ease (to ease) | To ease a sheet means to release part of it in the direction of the pull, so that the attack angle of the sail will be reduced. |
| Gennaker | Large downwind sail (hybrid of a spinnaker and a genoa) mostly set on a bowsprit (pole extending from the bow). |
| Gybe (or jibe) | Maneuver to change the side from which the boat receives the wind by turning the stern through the wind. |
| Harden (to harden) | To harden a sheet means to pull it tighter so that the attack angle of the sail will be increased. |
| Heel | Sideways tilt of a boat. Also: to heel (over): to tilt sideways. |
| Keel | Besides being a construction element in a boat’s hull also a vertical extension of the hull, mostly in the form of a hydrofoil (fin keel), with ballast attached to its lower end. |
| Leeway | A sideways drifting motion of a boat due to the aerodynamic side force. |
| Pitch (to pitch) | Movement of a boat about a transverse axis, for instance when dipping the bow. |
| Plane (to plane) | A boat is planing if it moves at a speed greater than hull speed, receiving hydrodynamic lift so that it displaces less water than it weighs. |
| Reach | Any course which is not close-hauled or running, especially one where true wind is at a 90° or greater to the course. |
| Rig | Sails and the complete construction supporting them like mast, boom, standing rigging. |
| Rudder | Immersed blade by which a boat is steered. |
| Running | Sailing with the wind from abaft: before the wind. |
| Sheet | A rope serving to control sail as to its position relative to the wind direction. (In the terminus ‘vortex sheet’ the word ‘sheet’ is however used in its commonly known meaning, i.e. a surface-like structure.) |
| Spinnaker | A balloon-like sail set on reaches and running. |
| Stern | The rear part of a boat. |
| Tack (to tack) | Maneuver to change the side from which the boat receives the wind by turning the bow through the wind. Also: the laps of course between tacking maneuvers. |
| True wind | Wind as perceived in a reference frame where the water is at rest. |
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