Theoretical study of short pile effect in tunnel excavation

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Abstract. The Misaki Sato Go ideal elastoplastic model is adopted and the two stage analysis theory is used to study the effect of tunnel excavation on short pile effect in this paper. In the first stage, the free field vertical displacement of the soil at the corresponding pile location is obtained by using empirical formula. In the second stage, the displacement is applied to the corresponding pile location. The equilibrium condition of micro physical differential equation settlement of piles. Then through logical deduction and the boundary condition expressions of the settlement calculation, obtain the pile side friction resistance and axial force of the week. Finally, an engineering example is used to analyze the influence of the change of main parameters on their effects.

1. Introduction
At present, the research on the effect of pile foundation in underground engineering has been carried out. The conventional methods include model test method, theoretical analysis method and numerical simulation method. Loganathan and so on \cite{1} based on the indoor physical model test of centrifugal field, studied the influence of the excavation of the cohesive soil foundation tunnel on the pile foundation. Effect of pile group effect of horse Shaokun \cite{2} with centrifugal model test study of the Guangxi tunnel is located in different positions, and through the method of drainage control stratum loss to constrain ground settlement. MROUEH \cite{3} uses three-dimensional numerical simulation to study the effect of tunnel excavation on pile group effect\cite{4-6}. The aforementioned research results are concentrated in the long pile effect, but when the pile tunnel distance is constant, the short pile is almost entirely located in the tunnel crack surface, the negative friction affected by the excavation, this paper mainly from the theoretical study of effects of tunnel excavation on short pile effect, and the sensitivity analysis of parameters, according to the the corresponding protective measures quantified results, put forward the theory guidance for similar engineering.

2. Establish the basic differential equations and solve

2.1 Differential equation of settlement
The basic differential equation of pile foundation settlement induced by tunnel excavation is introduced: (1) The pile is assumed to be an elastic foundation beam; (2) there is no slip between pile and soil, and the interaction is simulated with a continuously distributed spring.
The differential equation of settlement of pile foundation is obtained by the balance condition of infinitesimal force:

\[ \frac{d u}{dz^2} (x, z) = \frac{u_p}{E_p A_p} \tau(x, z) \]  \hspace{1cm} (1)

When there is no relative slip between pile and soil, Randolph and Worth treat the soil around the pile as a concentric cylinder, and give the relation between the soil deformation at the site of Z and the soil friction at the R of the pile shaft:

\[ u(z, r) = \begin{cases} 0 & r > r_m \\ r \cdot r_0 \left[ \ln(r_m) - \ln(r_0) \right] / G_s & r_0 \leq r \leq r_m \\ \end{cases} \]  \hspace{1cm} (2)

\[ r_m = 2.5 L (1 - \mu) \]  \hspace{1cm} (3)

\[ \tau(x, z) \] Misaki Sato Go’s ideal elastoplastic model is adopted:

\[ \tau(x, z) = k \left[ s(x, z) - w(x, z) \right] \]  \hspace{1cm} (4)

\( k \) is the coefficient of proportionality: cohesive soil: 4–6, mm, sand soil: 6–10, mm.

Simultaneous expressions (2) and (4) obtained:

\[ \frac{d^2 w(x, z)}{dz^2} - \lambda^2 w(x, z) = -\lambda^2 \frac{d^2 s(x, z)}{dz^2} \]  \hspace{1cm} (5)

\( \tau(x, z) \) pile side friction; \( u_p, L, A_p, r_0, E_p \): pile perimeter, pile length, cross-sectional area, pile diameter, elastic modulus of pile; \( w(x, z) \) displacement; and, the uniform pile side soil vertical spring stiffness, pile body of the soil maximum radius, shear modulus; \( k, r_0, G_s \): The free soil vertical displacement field, by means of empirical method.

\[ s(x, z) \bigg|_{z=0} = 0.313 \exp \left[ -x^2 / 2i(0)^2 \right] / i(0) \]  \hspace{1cm} (6)

\( i(0) \):Width of ground settlement trough; \( V \):Formation loss rate; \( D \):Tunnel diameter.

For the soil below the ground surface, the width coefficient of the trough curve will decrease with its depth.

\[ i(z) = i(0) \left( 1 - 0.3 z/z_0 \right)^n \]  \hspace{1cm} (7)

\( n \) is related to tunnel radius and soil condition, \( n = 0.35 \text{~} 0.85 \).

### 2.2 Solving differential equations

Can get the equation by means of variation of constants (8) solution:

\[ w(z) = c_1 e^{\xi z} + c_2 e^{-\xi z} + w(z) = c_3 e^{\xi z} + c_4 e^{-\xi z} + \frac{e^{\xi z}}{2} \int s(z) dz e^{-\xi z} + \frac{e^{-\xi z}}{2} \int s(z) dz e^{\xi z} \]  \hspace{1cm} (9)
\( w(z) \) is a special solution, where \( G_1 \) and \( C_2 \) are determined by boundary conditions:

\[
-E_p A_p \frac{d w(0)}{d(0)} = N(0) \quad -E_p A_p \frac{d w(L)}{d(L)} = N(L) = w(L) + \epsilon_0 \cdot G \cdot (1 - \mu)
\]

Side friction of pile:

\[
\tau(z) = \frac{E_p A_p}{A_p} \frac{d w^2(z)}{d z^2} \tag{10}
\]

Additional axial force:

\[
N(z) = -E_p A_p \frac{d w(z)}{d z} \tag{11}
\]

3. Example analysis

Take tunnel diameter 6m, tunnel buried depth 25m, formation loss ratio 1%; pile: C35, diameter 0.8m; 
\( N(0) = 100kN \); soil \( \theta = 22^\circ \), \( m = 0.45 \), \( n = 0.5 \), study the influence of tunnel excavation on short pile effect at \( x = 9.0m \) location.

Fig.2 soil displacement and pile foundation effect diagram

Known by the figure 2: when the pile distance tunnel center horizontal distance 9m, for short pile, pile sinking pile formation with almost synchronous, the shorter the settlement is larger, with the increase of the length of pile settlement decreases, and the pile-soil relative displacement increases, pile friction increases, the pile axial force increases. And the relative displacement between pile and soil is 0 to pile side friction resistance is 0, the maximum axial force of pile pile in tunnel; most rupture surface, which mainly bear negative friction of pile foundation.

To avoid further details, the following study investigates the effect of tunnel parameters on the results.

From Figure 3 that the depth and the width of the groove on the settlement of the tunnel, the greater depth, width of the settlement trough; the distance closer to the tunnel axis, the displacement of soil
with depth decreases, and with the increase of the distance, the displacement of soil with depth increases with the increase of the burial depth of pile foundation displacement the increase of negative skin friction of pile with the depth increases, friction resistance is 0 at the maximum axial force of the pile; depth of tunnel increases, pile-soil displacement is more and more close, the pile-soil relative displacement is smaller, the frictional resistance is close to 0, the axial force along the pile is almost the same, that tunnel the greater the depth of influence of pile foundation is small, so pay attention to the impact on the surrounding environment of the shallow buried tunnel in construction.

Fig. 3 Effect of tunnel depth on results
Fig. 4 Effect of tunnel diameter on results

We can see from Figure 4, the settlement of soil increases with the diameter of the tunnel was proportional to the increasing of ratio, the larger diameter, the ground loss is bigger, the pile settlement increases, pile side friction is bigger, the pile in the tunnel is broken more on the surface, the axial force of pile is bigger. If the tunnel diameter is 7m, the pile body is full of negative frictional resistance.

Fig. 5 Effect of formation loss rate on results

We can see from Figure 5: soil settlement and pile settlement increases with soil loss rate was proportional to the increasing of ratio, the loss rate is greater, the greater the negative frictional resistance of the pile, the pile in the tunnel is broken more on the surface of the pile, the additional axial force is bigger, and the negative friction increases with soil the depth first increases and then decreases with the depth increasing positive friction increases gradually; such as stratum loss rate is 1.5%, the pile for negative friction, so the stratum loss rate of soil must be reinforced.
4. Conclusion
In this paper, theoretical analysis is used to study the effect of tunnel excavation on short pile effect. The conclusions are as follows:

(1) based on the Kerr foundation model and the elastic foundation beam theory, the differential equation of the settlement of a single pile under tunneling is established.

(2) according to the relative displacement of pile and soil, the influence of tunnel excavation on side friction and axial force of short pile is studied:

(3) according to the parameters analysis of tunnel excavation effects on short pile effect change: the shorter the pile, the settlement is large; with the increase of the length of the pile, the pile-soil relative displacement increases, the negative friction of the pile increases; when the level of the properties of soil and pile tunnel distance is constant, the diameter of the tunnel, tunnel depth and stratum the loss rate of influence on its effect.

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