Varying Alpha Monopoles

J. Menezes*, P. P. Avelino†, and C. Santos‡
Centro de Física do Porto e Departamento de Física da Faculdade de Ciências da Universidade do Porto,
Rua do Campo Alegre 687, 4169-007, Porto, Portugal

We study static magnetic monopoles in the context of varying-α theories and show that there is a group of models for which the 't Hooft-Polyakov solution is still valid. Nevertheless, in general static magnetic monopole solutions in varying-α theories depart from the classical 't Hooft-Polyakov solution with the electromagnetic energy concentrated inside the core seeding spatial variations of the fine structure constant. We show that Equivalence Principle constraints impose tight limits on the allowed variations of α induced by magnetic monopoles which confirms the difficulty to generate significant large-scale spatial variation of the fine structure constant found in previous works. This is true even in the most favorable case where magnetic monopoles are the source for these variations.

I. INTRODUCTION

The possibility that at least some of the fundamental "constants" of Nature might be dynamical appears naturally in the context of models with extra-spatial dimensions. The interest in this type of models has recently been increased with results coming from both quasar absorption systems [1, 2] (see however [3, 4, 5]) and the Oklo natural nuclear reactor [6] suggesting a cosmological variation of the fine-structure constant, α = e²/(4 π ħ c) at low red-shifts. Other constraints at low redshift include atomic clocks [7] and meteorites [8] while at high redshifts there are also upper limits to the allowed variations of α coming from either the Cosmic Microwave Background or Big Bang Nucleosynthesis [9, 10, 11, 12, 13, 14, 15].

On the theoretical side some effort has been made on the construction of self-consistent phenomenological models for space-time α variability most of them based on the original Bekenstein model [16]. In some of these models [17, 18, 19, 20, 21, 22, 23, 24, 25] the possibility that the variation of fine structure constant might be coupled to the dark energy equation of state responsible for the recent acceleration of the Universe [25, 26] has also been explored.

Most of these studies have focused mainly on the variation of α with time. The reason for this is that it is very difficult to generate significant large-scale spatial variations of the fine structure constant. This has been shown in [27] (see also [28]) in a model where the spatial variation of α were induced by cosmic strings cosmic and in [29] in a more general context. In the cosmic string case, the electromagnetic energy concentrated along the core of the local string is a source of these spatial variations, which are roughly proportional to the gravitational potential induced by the strings. They are constrained by Equivalence Principle to be small and overall limits for the allowed spatial variations have been calculated.

In this article, we consider the case of spatial variations of α seeded by a static non-Abelian local monopole. We show that the electromagnetic energy in its core can produce spatial variations of the fine-structure constant in the vicinity of the monopole in the context of Bekenstein-type models. We start by solving the standard problem with no α variation and recover the classical solution presented by ‘t Hooft and Polyakov [30, 31] (see for instance Refs. [32, 33, 34, 35]) and then proceed to investigate other solutions with variable α.

The article is organized as follows. In Sec. II we introduce Bekenstein-type models in Yang-Mills theories and obtain the equations that describe a static magnetic monopole. We write the energy density of the monopole and show that the standard electromagnetism is recovered outside the core if α is a constant. In Sec. III we present the numerical technique applied to solve the equations of motion and give results for several choices of the gauge kinetic function. In Sec. IV we study the limits imposed by the Equivalence Principle on the allowed variations of α in the vicinity of the monopole, as a function of the symmetry breaking scale. Finally, in Sec. IV, we summarize and discuss our results. Throughout this paper we shall use units in which ħ = e = 1 and the metric signature +−−−.

II. BEKENSTEIN-TYPE MODELS IN NON-ABELIAN FIELD THEORIES

We first introduce Bekenstein-type models in a non-Abelian Yang-Mills theory. We take the electric charge to be a function of the space-time coordinates, e = e₀(ϕ) in which e is a real scalar field and e₀ is an arbitrary constant charge. Let the Higgs field Φ² be an isovector, where a = 1, 2, 3 are internal indices associated to isospace with SU(2) symmetry.

The Lagrangian density is given by

\[
\mathcal{L} = \frac{1}{2} (D_{\mu} \Phi^a) (D^{\mu} \Phi^a) - V(\Phi^a) - \frac{B_F(\varphi)}{4} f_{\mu \nu} f^{a \mu \nu} + \frac{1}{2} \partial_{\mu} \varphi \partial^{\mu} \varphi, \tag{1}
\]
where

\[ B_F(\varphi) = \epsilon(\varphi)^{-2} \]  

is the gauge kinetic function of a massless scalar field \( \varphi \). This function acts as the effective dielectric permittivity and can phenomenologically be taken as an arbitrary function of \( \varphi \).

Defining an auxiliary gauge field \( a^a_\mu = \epsilon A^a_\mu \) and a new non-Abelian gauge field strength by

\[ f^a_{\mu\nu} = \epsilon F^a_{\mu\nu} = \partial_\mu a^a_\nu - \partial_\nu a^a_\mu + \epsilon_{abc} a^b_\mu a^c_\nu, \]  

the covariant derivatives are written in the usual form

\[ D_\mu \Phi^a = \partial_\mu \Phi^a + \epsilon_{abc} a^b_\mu \Phi^c, \]  

where \( \epsilon^{abc} \) is the Levi-Civita tensor.

We consider that the potential \( V(\Phi^a) \) is given by

\[ V(\Phi^a) = \frac{\gamma}{8} (\Phi^a \Phi^a - v^2)^2 \]  

where \( \gamma > 0 \) is the coupling of the scalar self-interaction, and \( v \) is the vacuum expectation value of the Higgs field. The Lagrangian density in eqn. (1) is then invariant under SU(2) gauge transformations of the form

\[ \delta \Phi^a = \epsilon^{abc} \Phi^b \Lambda^c, \]  

\[ \delta a^a_\mu = \epsilon^{abc} a^b_\mu \Lambda^c + \epsilon_0 \partial_\mu \Lambda^a, \]

where \( \Lambda^a \) is a generic isovector. This symmetry is broken down to \( U(1) \) because there is a non vanishing expectation value of the Higgs field. Thus the two components of the vector field develop a mass \( M_W = v e_0 \), while the mass of the Higgs field \( M_H = v \sqrt{2} \).

It is convenient to define the dimensionless ratio

\[ \zeta = \frac{M_H}{M_W} = \frac{\sqrt{2}}{e_0}, \]  

and to rescale the radial coordinate by \( M_W \), so that distance is expressed in units of \( M_W^{-1} \).

Varying the action with respect to the adjoint Higgs field \( \Phi^a \) one gets

\[ (D_\mu D^\mu \Phi^a) = -\gamma \Phi^a \left( \Phi^b \Phi^b - v^2 \right). \]  

Variation with respect to \( a^a_\mu \) leads to

\[ D_\nu \left[ B_F(\varphi) f^a_{\mu\nu} \right] = j^{a\mu} \]  

with the current \( j^{a\mu} \) defined as

\[ j^{a\mu} = \epsilon_0 \epsilon^{abc} \Phi^b (D_\mu \Phi^c). \]  

Finally, variation with respect to \( \varphi \) gives

\[ \partial_\mu \partial^\mu \varphi = -\frac{1}{4} \frac{dB_F(\varphi)}{\partial r^2}, \]  

in which \( f^2 = f^a_{\mu\nu} f^{a\mu\nu} \).

We are now interested in static, spherically symmetric, magnetic monopole solutions. Therefore we make the “hedhehog” ansatz

\[ \Phi^a(\varphi) = H(\varphi) \frac{x^a}{r}, \]  

\[ a^0_\mu(\varphi) = 0, \]  

\[ a^r_\varphi(\varphi) = \epsilon_{ask} x_k \left[ W(\varphi) - 1 \right], \]  

where \( x^a \) are the Cartesian coordinates and \( r^2 = x_k x_k \). \( H(\varphi) \) and \( W(\varphi) \) are dimensionless radial functions which minimize the self-energy, i.e., the mass of the monopole

\[ E = \frac{4\pi v}{e_0} \int_0^\infty dr \left\{ \frac{r^2}{2} \left( \frac{dH}{dr} \right)^2 + H^2 W^2 + \frac{\zeta^2 r^2}{8} (1 - H^2)^2 \right\} + B_F(\varphi) \left[ \frac{dW}{dr} \right]^2 + \frac{(1 - W^2)^2}{2 r^2} + \frac{r^2}{2} \left( \frac{d\varphi}{dr} \right)^2 \right\}, \]  

where the coordinate \( r \) and the functions \( H \) and \( \varphi \) have been rescaled as

\[ r \rightarrow \frac{r}{e_0 v}, \quad H \rightarrow v H, \quad \varphi \rightarrow v \varphi. \]  

It will prove useful to compute the energy density which is given by

\[ \rho = v e_0 B_F(\varphi) \left[ \left( \frac{W'}{r} \right)^2 + \frac{1}{2} \left( 1 - W^2 \right)^2 \right] \]  

\[ + \frac{v}{e_0} \left\{ \frac{H^2 r^2}{2} + \left( \frac{W H}{r} \right)^2 + \frac{\zeta^2}{8} (1 - H^2)^2 \right\} \]  

with a prime meaning a derivative with respect to the dimensionless coordinate \( r \). Substituting the ansatz given in (13-15) into equations (8-12) one gets

\[ \frac{1}{r^2} \left( r^2 H' \right)' - \frac{\zeta^2}{2} (1 - H^2) + \frac{2 W^2}{r^2} = 0 \]  

\[ [B_F(\varphi) W']' - \frac{B_F(\varphi)}{r^2} (1 - W^2) + H^2 = 0 \]  

\[ \frac{1}{r^2} \left( r^2 \varphi' \right)' - \frac{d B_F(\varphi)}{d \varphi} \left[ W'^2 + \frac{1}{2} \left( 1 - W^2 \right)^2 \right] = 0. \]

Defining

\[ f_{\mu\nu} = \frac{\Phi^a}{|\Phi^a|} f^a_{\mu\nu} + \frac{1}{e_0 |\Phi^a|^3} \epsilon^{abc} \Phi^a (D_\mu \Phi^b) (D_\nu \Phi^c), \]  

and choosing the gauge where \( \Phi^a = 8a^3 |\Phi| \), i.e., with \( \Phi \) pointing in the same direction everywhere, one gets

\[ f_{\mu\nu} = \partial_\mu a^3_\nu - \partial_\nu a^3_\mu. \]  

By writing \( a^3_\mu = a_\mu \), one identifies (28) with the usual electromagnetic tensor which for constant \( \varphi \) satisfies the ordinary Maxwell equations everywhere except in the region where \( H \sim 0 \).
III. NUMERICAL IMPLEMENTATION OF EQUATIONS OF MOTION

In order to solve numerically the equations of motion, we have to reduce them to a set of first order equations. For that purpose we define the variables

\[ V = H', \quad U = W', \quad \eta = \varphi'. \]  \hspace{1cm} (24)

Equations (19-21) can then be written as

\[ V' = - \frac{2V}{r} + \frac{\zeta^2}{2} H(H^2 - 1) + \frac{2W^2 H}{r^2} \]  \hspace{1cm} (25)

\[ U' = \frac{1}{B_F} \left[ \frac{dB_F}{d\varphi} \eta U + \frac{B_F}{r^2} (W^2 - 1)W + H^2 W \right] \]  \hspace{1cm} (26)

\[ \eta' = - \frac{2\eta}{r} + \frac{dB_F}{d\varphi} \left[ U^2 + \left( \frac{1 - W^2}{r^2} \right)^2 \right] \]  \hspace{1cm} (27)

which require at least six boundary conditions.

At far distances from the core \((r \to \infty)\), the Higgs field \(H(r)\) falls off to its vacuum value, \(H = 1\). Using this boundary condition in eqn. (19), one gets that \(W(r)\) must vanish far from the core. On the other hand, since the symmetry at the core is not broken, then the Higgs field vanishes and from regularity of the energy-momentum tensor, one can choose a gauge for which \(W = 1\). The other two boundary conditions come from the normalization of the electric charge at the origin. At the core \(e = e_0\), which means that the gauge kinetic function is equal to one. Finally, by using the Gauss law to solve the eqn. (21) without sources of \(\alpha\) variation other than the monopole, one gets that \(\eta\) must vanish at \(r = 0\).

Therefore, we have at all six boundary conditions, four of them at origin and the other two far away from the core:

\[ \lim_{r \to 0} H(r) = 0 \quad \lim_{r \to \infty} H(r) = 1, \]  \hspace{1cm} (28)

\[ \lim_{r \to 0} W(r) = 1 \quad \lim_{r \to \infty} W(r) = 0, \]  \hspace{1cm} (29)

\[ \lim_{r \to 0} B_F(r) = 1 \quad \lim_{r \to 0} \eta(r) = 0. \]  \hspace{1cm} (30)

As the boundary conditions are at different points of the domain of the functions to be found, we use the relaxation numerical method replacing the set of differential equations by finite-difference equations on a grid of points that covers the whole range of the integration.

A. t’Hooft-Polyakov Standard Solution

In this section we consider as a first example the special case of \(B_F = 1\). This recovers the standard t’Hooft-Polyakov monopole problem \[30\] in which there is no variation of the fine-structure constant with \(\varphi(r) = \text{constant}\).

Let us first consider \(\zeta = 0\), i.e., the case of a massless Higgs field. The energy of the monopole can be written as

\[ E = \frac{4\pi}{e_0} \int_0^\infty \frac{dr}{r} \left[ \frac{1}{2} r H' - \frac{1 - W^2}{r^2} \right]^2 + \frac{1}{r^2} \left[ W' + WH \right]^2 \]  \hspace{1cm} (31)

where we have introduced a new scalar field \(P(r)\) as

\[ P(r) = H (1 - W^2). \]  \hspace{1cm} (32)

Note that for \(H(r)\) and \(W(r)\) that solve the first-order differential equations

\[ H' = \frac{1 - W^2}{r^2} \]  \hspace{1cm} (33)

\[ W' = - \frac{WH}{r^2} \]  \hspace{1cm} (34)

the energy is minimized to

\[ E = 4 \pi \left[ P(r \to \infty) - P(r = 0) \right]. \]  \hspace{1cm} (35)

This procedure was first presented by Bogomol’nyi \[37\] and generalized in \[38, 40, 41\]. Taking into account the boundary conditions for \(H(r)\) and \(W(r)\), one gets that the minimum of energy is equal to \(4 \pi / e_0\). The solutions of equations \[38, 39\] are

\[ H(r) = \frac{r}{\sinh (r)} \]  \hspace{1cm} (36)

\[ W(r) = \frac{1}{\tanh (r)} - \frac{1}{r} \]  \hspace{1cm} (37)

which is the well known BPS (Bogomol’nyi, Prasad, Sommerfield) solution \[37, 38\]. In Figs. 1 and 2 we plot \(H(r)\) and \(W(r)\) given by eqns. (36) and (37).

Let us now consider the opposite limit, \(\zeta \to \infty\). Fixing \(e_0\) one gets in this limit that \(\gamma \to \infty\), i.e., the Higgs
potential is much larger than the kinetic term forcing the Higgs field to be frozen at its vacuum value everywhere except at the origin. Then the only equation of motion is

$$W'' = W - \frac{W(1 - W^2)}{r^2}. \quad (38)$$

In Figs. 1 and 2 we plot $H(r)$ and $W(r)$ for $\zeta \to \infty$.

Of course, the former values of $\zeta$ simplify very much the equations of motion. However, we have found numerically the set of solutions for a generic non vanishing finite $\zeta$. In fact, we achieved very good precision for $\zeta$ ranging the interval $10^{-4} < \zeta < 10^3$.

We also compute the mass of the monopole and verified that it increases monotonically with increasing $\zeta$, remaining finite when $\zeta \to \infty$ (see Fig. 3).

In Figs. 1 and 2, we plot the numerical solution of the scalar field $\psi \equiv \ln \epsilon$ as a function of distance, $r$, to the core of the monopole, for several values of $\omega$. Note that the energy remains finite when $\xi < 0$, $\epsilon \to \infty$ when $r \to \infty$. On the other hand, if $\omega > 0$ then $\epsilon \to 0$ when $r \to \infty$. The dashed line represents the constant-$\alpha$ theory, which corresponds to the limit $\omega \to \infty$.

### B. Bekenstein Model

Consider a gauge kinetic function, $B_F(\phi)$, given by:

$$B_F(\phi) = e^{-\frac{2\sigma}{\omega}} \epsilon, \quad (39)$$

where $\omega$ is a positive coupling constant. This recovers the original Bekenstein model [10].

Defining $\psi = \phi/\sqrt{\omega}$ and using eqn. (39), the Lagrangean density in (11) can be written as

$$\mathcal{L} = \frac{1}{2} (D_\mu \Phi^a)^2 (D^\mu \Phi^a) - V(\Phi^a) - e^{-2\psi} \frac{F_{\mu \nu} f^{a \mu \nu}}{4} + \omega \partial_a \psi \partial^a \psi. \quad (40)$$

Note that despite the fact that the gauge kinetic function in eqn. (39) is only well defined for $\omega > 0$, the model described by the Lagrangean density in eqn. (40) allows for both positive and negative values of $\omega$.

In this model equations (26) and (27) are given by

$$U' = 2 \sigma U + \frac{W(W^2 - 1)}{r^2} + e^2 \psi H^2 W \quad (41)$$

$$\sigma' = -2 \sigma r - 2 \omega \frac{e^{-2\psi}}{r} \left[ \left( \frac{1 - W^2}{r^2} \right)^2 + U^2 \right] \quad (42)$$

where we took $\sigma = \psi'$. First we note that in the limit $\omega \to \infty$ one recovers the t’Hooft-Polyakov classical solution described in the previous subsection. In Fig. 4 we plot the numerical solution of the scalar field $\psi(r) = \ln \epsilon$ for several values of $\omega$.

From Fig. 4 one concludes that if $\omega < 0$ then $\epsilon$ diverges asymptotically away from the core of the monopole, and the energy density

$$\rho = e^{-2\psi} \left[ \left( \frac{W'}{r} \right)^2 + \frac{1}{2} \left( \frac{1 - W^2}{r^2} \right)^2 \right]$$

FIG. 2: The numerical solution of the field $W(r)$ as a function of distance, $r$, to the core of the t’Hooft-Polyakov monopole for three values of $\zeta$.

FIG. 3: The numerical solution of the energy of the t’Hooft-Polyakov $E(\zeta)$ as a function of the ratio of the Higgs to vector field mass, $\zeta$. Note that the energy remains finite when $\zeta \to \infty$.

FIG. 4: The numerical solution of the scalar field $\psi \equiv \ln \epsilon$ as a function of distance, $r$, to the core of the monopole, in the Bekenstein model. Note that if $\omega < 0$, $\epsilon \to \infty$ when $r \to \infty$. On the other hand, if $\omega > 0$ then $\epsilon \to 0$ when $r \to \infty$. The dashed line represents the constant-$\alpha$ theory, which corresponds to the limit $\omega \to \infty$. 

\[\text{FIG. 2: The numerical solution of the field } W(r) \text{ as a function of distance, } r, \text{ to the core of the t’Hooft-Polyakov monopole for three values of } \zeta.\]

\[\text{FIG. 3: The numerical solution of the energy of the t’Hooft-Polyakov } E(\zeta) \text{ as a function of the ratio of the Higgs to vector field mass, } \zeta. \text{ Note that the energy remains finite when } \zeta \to \infty.\]

\[\text{FIG. 4: The numerical solution of the scalar field } \psi \equiv \ln \epsilon \text{ as a function of distance, } r, \text{ to the core of the monopole, in the Bekenstein model. Note that if } \omega < 0, \epsilon \to \infty \text{ when } r \to \infty. \text{ On the other hand, if } \omega > 0 \text{ then } \epsilon \to 0 \text{ when } r \to \infty. \text{ The dashed line represents the constant-} \alpha \text{ theory, which corresponds to the limit } \omega \to \infty.\]
is no longer positive definite. However if $\omega > 0$ then $\epsilon$ vanishes asymptotically when $r \to \infty$, and the energy density is in this case positive definite (See Fig. 5). We also note by observing Fig. 4 that in the large $\omega$ limit the curves for positive and negative $\omega$ are approximately symmetric approaching the dashed line which represents the constant-$\alpha$ model when $\omega \to \infty$.

Finally, in Fig. 6 we plot the numerical solution of the scalar field $H(r)$ and the gauge field $W(r)$ as a function of distance $r$, to the core of the monopole. The dashed-line represents the constant-$\alpha$ solution and the solid line represents the Bekenstein one for $\omega = 2.0$. We verified that even in the $\omega \to 0$ limit, the change in $H(r)$ with respect to t’Hooft-Polyakov solution is still negligible.

### C. Polynomial Gauge Kinetic Function

We now consider another class of gauge kinetic functions given by

$$ B_F(\varphi) = 1.0 + \sum_{i=1}^{N} \beta_i \varphi^i, \quad (44) $$

where $\beta_i$ are dimensionless coupling constants and $N$ is an integer.

Note that by considering

$$ \beta_i = \frac{(-2)^i}{\omega^{i/2}i!} \quad (45) $$

one recovers the Bekenstein coupling given in (39). This relationship between the coupling constants $\beta_i$ and $\omega$ has interesting consequences for the model given by eqn. 44.

First we verified that the behaviour of $\psi$ both for $\beta_1 > 0$ and $\beta_1 < 0$ is similar to that of the Bekenstein model with $\omega > 0$ which is recovered in the limit of small $|\beta_1|$/large $\omega$.

Another feature that we verified is that if one takes $\beta_1 = 0$ one gets the t’Hooft-Polyakov limit, for any of $\beta_i$ for $i > 1$. This means that there is a class of gauge kinetic functions for which the classical static solution is maintained despite the modifications to the model.

Another property can be noted when one substitutes the gauge kinetic function in equation of motion for $\varphi$ (41). One gets

$$ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d \varphi}{dr} \right) = C^2 \left( \sum_{k=1}^{N} (2k-1)\beta_{2k-1} \varphi^{2k-2} \right) + C^2 \left( \sum_{k=1}^{N} (2k)\beta_{2k} \varphi^{2k-1} \right) \quad (46) $$

with

$$ C = \left[ (W')^2 + \frac{1}{2} \left( \frac{1-H^2}{r^2} \right)^2 \right]^\frac{1}{2}. \quad (47) $$

Since that $C^2 > 0$, when $\beta_i \to -\beta_i$ one sees by eqn. 46 that $\varphi(r) \to -\varphi(r)$. However, as $B_F$ in eqn. 44 is kept invariant, $H(r)$ or $W(r)$ do not vary.

Although we have found the set of solutions for several values of $N$, for simplicity we consider that $N = 2$ in eqn. 46, that is

$$ B_F = 1.0 + \beta_1 \varphi + \beta_2 \varphi^2, \quad (48) $$

with two free parameters. We define the models 1 and 2 as $\beta_1 = -3$, $\beta_2 = 0$ (linear coupling) and $\beta_1 = -2$, $\beta_2 = 5$ (quadratic coupling) respectively.

In Fig. 7 we plot the numerical solution of the scalar field $\psi(r)$ as a function of the radial coordinate. As we have shown earlier the model 0 represents any model with
FIG. 7: The numerical solution of the scalar field \( \psi \equiv \ln \epsilon \) as a function of distance, \( r \), to the core of monopole, for a polynomial gauge kinetic function. Models 0, 1 and 2 are defined by \( \beta_1 = 0 \) (\( \beta_2 \) arbitrary), \( \beta_1 = -3, \beta_2 = 0 \) (linear coupling) and \( \beta_1 = -2, \beta_2 = 5 \) respectively.

FIG. 8: The numerical solution of the fields \( H(r) \) and \( W(r) \) as a function of distance, \( r \), to the core of monopole, for models 0, 1 and 2. Note that the change in \( W(r) \) with respect to the standard constant-\( \alpha \) result is much more dramatic than the change in \( H(r) \).

FIG. 9: Plot of \( L_i(r) = \log \left( \frac{H_i}{H_0} \right) \) for the models 0, 1 and 2. One clearly sees that even a small value of \( \beta_1 \) leads to a different vortex solution from the standard t’Hooft-Polyakov one.

One clearly sees in Fig. 8 that the change in \( W(r) \) with respect to the standard constant-\( \alpha \) result is much more dramatic than the change in \( H(r) \), we define the function

\[
L_i(r) = \log \left( \frac{H_i}{H_0} \right),
\]

and plot in Fig. 9 the results for the different models.

Note that even a small value of \( \beta_1 \) will lead to a modification of the magnetic monopole solution with respect to the standard t’Hooft-Polyakov solution.

We have also studied the behaviour of the energy density in this model. In Fig. 10 one sees by comparing with the dashed line, which represents the constant-\( \alpha \) model, that since the fine structure constant varies there is a new contribution due to the field \( \varphi \), to the total energy of the topological defect. In fact the energy density of the monopole can be divided into two components: one that is localized inside the core of the monopole and other related to the contribution of the kinetic term associated with the spatial variations of the fine structure constant.

IV. CONSTRAINTS ON VARIATIONS OF \( \alpha \)

After having discussed in the previous section the numerical solutions for the varying-\( \alpha \) monopole, our next investigating point is to find an overall limit of the spatial variations of the fine-structure constant on monopole networks.

For simplicity we assume

\[
B_P(\varphi) = 1 + \beta \varphi,
\]

i.e., the gauge kinetic function is a linear function in \( \varphi \) that satisfies the spherically symmetric Poisson equation
given by
\[ \nabla^2 \varphi = \frac{1}{4} \beta f^2. \]

(51)

Note that \( \beta \) is constrained by Equivalence Principle tests to be such that \( |\beta| < 10^{-3} G^{1/2} \) (see [42]).

Integrating eqn. (50) from the core up to \( r_{\text{max}} \), which represents a cosmological cut-off scale, one gets
\[ 4\pi r^2 \frac{d\varphi}{dr} = \beta I(r) M(r_{\text{max}}). \]

(52)

In eqn. (52) we used the mass of the monopole which is given by
\[ M(r) = 4\pi \int_0^r \rho(r') r'^2 dr', \]

(53)

and
\[ I(r) = \frac{\pi}{M(r_{\text{max}})} \int_0^r f^2(r') r'^2 dr', \]

(54)

which is a slowly varying function of \( r \) outside the core always smaller than unity. Thus we can take \( I(r) \sim \text{const} \) and integrate eqn. (52) to get
\[ \varphi \sim \frac{\beta I M(r_{\text{max}})}{4\pi} \left( \frac{1}{r} - \frac{1}{r_0} \right), \]

(55)

where \( r_0 \) is a integration constant which could be identified as the core radius. Using \( B_F(\varphi) = \epsilon^{-2} \) one gets
\[ \epsilon \sim 1 - \left( \frac{\beta I M(r_{\text{max}})}{8\pi} \right) \left( \frac{1}{r} - \frac{1}{r_0} \right), \]

(56)

which means that the variation of the fine structure constant away from the monopole core is proportional to the gravitational potential induced by the monopoles.

A. GUT Monopoles

Let us estimate an overall limit for the spatial variation of \( \alpha \) outside the core of GUT monopoles. In this context the mass of the monopole is of order of
\[ M(r_{\text{max}}) \sim 10^{16} \text{ GeV}. \]

(57)

The variation of \( \alpha \) from the core up to infinity is
\[ \frac{\Delta \alpha}{\alpha} = \frac{\alpha(r \to \infty) - \alpha(r_0)}{\alpha(r_0)} = \epsilon^2 - 1 \lesssim 10^{-13}, \]

(58)

where we have used \( \alpha = \alpha_0/B_F(\varphi) \), with \( \alpha = \alpha(r \to \infty) \) and \( \alpha_0 = \alpha(r_0) \).

B. Planck Monopoles

Proceeding as above for the Planck scale symmetry with
\[ M(r_{\text{max}}) \sim 10^{19} \text{ GeV} \]

(59)

one obtains an overall limit for the spatial variation of the fine-structure seeded by magnetic monopoles
\[ \frac{\Delta \alpha}{\alpha} \lesssim 10^{-7}, \]

(60)

which is still very small even for Planck scale monopoles.

V. CONCLUSION

In this paper we investigated static monopole solutions in the context of varying-\( \alpha \) theories based on Bekenstein-type models. First we studied various models with constant \( \alpha \) and reviewed the standard static t’Hooft-Polyakov magnetic monopole solution. Then we considered models with varying-\( \alpha \) and showed that despite of the existence of a class of models for which the t’Hooft-Polyakov standard solution is still valid, in general, our solutions depart from the former one. We showed that Equivalence Principle constraints impose tight limits on the variations of \( \alpha \) induced by magnetic monopoles. This confirms the difficulty to generate significant large-scale spatial variation of the fine structure constant found in previous works, even in the most favorable case where these variations are seeded by magnetic monopoles.

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