Preheating
in Supersymmetric Theories

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Abstract

We examine the particle production via preheating at the end of inflation in supersymmetric theories. The inflaton and matter scalars are now necessarily complex fields, and their relevant interactions are restricted by holomorphy. In general this leads to major changes both in the inflaton dynamics and in the efficiency of the preheating process. In addition, supersymmetric models generically contain multiple isolated vacua, raising the possibility of non-thermal production of dangerous topological defects. Because of these effects, the success of leptogenesis or WIMPZILLA production via preheating depends much more sensitively on the detailed parameters in the inflaton sector than previously thought.
1 Introduction.

The inflationary paradigm [1] has been remarkably successful in explaining the observed large-scale features of the universe. Apart from its prediction of a flat universe with $\Omega_{\text{tot}} = 1$, inflation naturally produces primordial density fluctuations with a Harrison-Zeldovich power spectrum. These fluctuations in turn give rise to the angular pattern of the cosmic microwave background radiation (CMBR) consistent with recent observations [2].

On the theoretical front, one of the most important recent developments in inflationary cosmology was the realization [3, 4, 5] that non-perturbative particle production may play an important role in the inflaton decay process. The process of the inflaton decay through non-perturbative, parametrically enhanced particle production has been called preheating, to distinguish it from the usual reheating scenario where the inflaton decay is treated perturbatively. An interesting feature of the preheating scenario is that the production of particles with masses greater than the inflaton mass is possible. For example, it was claimed that in the simplest chaotic inflation model with the inflaton mass $m \sim 10^{13}$ GeV, the scalars with masses of up to $10^{14} - 10^{15}$ GeV and fermions with masses of up to $10^{17} - 10^{18}$ GeV can be copiously produced. In particular, the non-thermally produced particles could include the baryon number violating gauge and Higgs bosons of grand unified theories (GUTs) or right-handed neutrinos, resurrecting the possibility of GUT-scale baryogenesis or leptogenesis [6, 7]. The non-thermally produced superheavy particles could also play the role of dark matter [8], and their decays could explain the observed super-GZK cosmic ray events [9].

The magnitude of the observed CMBR anisotropies suggests that the energy scale at which inflation takes place is substantially lower than the Planck scale. Any microscopic model of inflation should explain this hierarchy of scales and its radiative stability. From theoretical point of view, the most well-established and appealing way to do this is to assume that the physics of inflation is supersymmetric. This assumption is independently motivated by the need to resolve the usual gauge hierarchy problem of the Standard Model, and by the consistency considerations in string theory. Supersymmetry puts non-trivial constraints on the spectrum of the theory: bosons and fermions necessarily come in pairs, and matching the bosonic and fermionic degrees of freedom is only possible if all the scalar fields are complex. Moreover, the interaction terms possible in a supersymmetric theory are restricted by holomorphy. However, most of the studies of the preheating process to date concentrate on the case of a real scalar inflaton field, do not take into account the holomorphy constraints on its couplings, and consider production of fermions and bosons independently. In this paper, we would like to examine the inflaton dynamics and the process of preheating after the end of inflation, taking into account the constraints of supersymmetry. We will show that many interesting and important new features arise in supersymmetric models, which can substantially modify

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1Heavier scalar particles could be produced via a slightly different mechanism, “instant” preheating [4].
the conventional preheating scenarios.

Let us mention that the parametric resonance for the case of a complex inflaton coupled to a scalar field was considered in Ref. [11]. However, the structure of the scalar potential assumed in that study is quite different from the one studied here, which we believe is generic in supersymmetric theories. Also, we discuss preheating of fermions which was not considered in [11].

The paper is organized as follows. We begin by reviewing the well-known qualitative results for preheating of scalar particles and fermions in the case of a real inflaton (which we refer to as “non-supersymmetric”) in Section 2. We then proceed to discuss the general structure of the inflaton interactions in supersymmetric models, and explain the assumptions of our analysis (see Section 3.) In Sections 4 and 5, we discuss the preheating of scalar and fermion particles, respectively, in the supersymmetric case. We highlight the similarities and differences between this case and the more familiar, non-supersymmetric case. We conclude in Section 6. Certain more technical points of our analysis are relegated to the Appendices.

2 Review of Non-Supersymmetric Results.

Before discussing the supersymmetric case, let us recall the standard theory of particle production via preheating in non-supersymmetric models. To be concrete, we consider the case of chaotic inflation with the inflaton potential \( V(\phi) = \frac{m^2 \phi^2}{2} \), where the inflaton mass \( m = 10^{-6} M_p \approx 10^{13} \text{GeV} \) and the value of the inflaton immediately after the end of slow-roll inflation is \( \phi_0 \approx M_p/3 \). We will review the non-thermal production of scalars and fermions in turn.

2.1 Production of Scalar Particles.

The production of scalar particles during preheating has been studied in detail in [5]. Let us briefly summarize the results. Assume that the inflaton is coupled to a real scalar field \( \chi \) via

\[
\mathcal{L}_s = \frac{1}{2} \left( M_0^2 + g^2 \phi^2 \right) \chi^2 + V(\phi). \tag{1}
\]

In a flat FRW universe with a scale factor \( a(t) \), the equation for the mode of the field \( \chi \) with a comoving momentum \( k \) reads,

\[
\ddot{\chi}_k + 3H \dot{\chi}_k + \omega_k^2 \chi_k = 0, \tag{2}
\]

where

\[
\omega_k^2 = \frac{k^2}{a(t)^2} + M_0^2 + g^2 \phi^2(t) \tag{3}
\]

is the time-dependent effective frequency of the mode and \( H = \dot{a}/a \). It is useful to first consider particle creation in Minkowsky space, with \( a \equiv 1 \) and \( \phi = \phi_0 \cos mt \). The qualitative features of the process depend crucially on the parameter \( q \equiv \frac{g^2 \phi_0^2}{4m^2} \).
Typically \( q \gg 1 \) during the preheating epoch. In this case, the \( \chi \) particles are created via broad parametric resonance. In this regime, the necessary condition for particle production is non-adiabaticity in the change of the effective frequency,

\[
\frac{d\omega}{dt} \gtrsim \omega^2.
\]

This condition can be satisfied if

\[
M_0^2 + k^2 \lesssim \frac{mg\phi_0}{2}.
\]

That is, particles with bare masses of up to

\[
M_{\text{max}} \sim \sqrt{mg\phi_0/2} = m q^{1/4}
\]

are produced. For example, for \( g \sim 1 \), we find \( M_{\text{max}} \sim 1.5 \times 10^{15} \) GeV, more than two orders of magnitude above the inflaton mass. The maximum momentum for a particle of bare mass \( M_0 \) is of order \( k_{\text{max}} \sim \sqrt{mg\phi_0/2 - M_0^2} \). Note that for \( M \) close to \( M_{\text{max}} \), only particles with very low momenta can be produced, so that the production is suppressed; however, once \( M \lesssim M_{\text{max}}/2 \), the typical momenta of the produced particles become of order \( \sqrt{mg\phi_0/2} \), and the number density of the produced particles becomes of the same order as in the case \( M_0 = 0 \).

The condition (4) is satisfied and the \( \chi \) particles are produced only for a short period of time during each oscillation, when the inflaton is close to the minimum of its potential:

\[
|\phi| \lesssim \sqrt{\frac{m\phi_0}{g}} = \frac{\phi_0}{\sqrt{2}} q^{-1/4} \ll \phi_0.
\]

For the rest of the oscillation period, the number of \( \chi \) particles remains constant. Because of this feature, the process of particle creation can be treated analytically [5].

The expansion of the universe brings in two interesting new features. First, the number of particles with a given wavelength may both increase and decrease during a particular passage of the inflaton through its minimum, depending on the phase of \( \chi_k \) at that time. Nevertheless, the number of \( \chi \) particles is growing exponentially with time when averaged over many oscillation periods (this phenomenon has been called “stochastic” resonance [3]) so preheating does occur. Secondly, the amplitude of the inflaton oscillations is damped with time due to Hubble friction, so that effectively the \( q \) parameter decreases with time. Particle production ceases when the condition (5) is no longer satisfied for any \( k \), that is, when \( q = q_{\text{min}} \sim (M_0/m)^4 \). (If \( M_0 \lesssim m \), the preheating stops when \( q \sim 1 \).) As we have mentioned above, for \( q \gtrsim (2M_0/m)^4 = 16q_{\text{min}} \) the rate of particle production is of the same order as in the case \( M_0 = 0 \); for \( q_{\text{min}} \lesssim q \lesssim 16q_{\text{min}} \) there is some production, but the rate is suppressed.

In the above discussion, we have neglected the backreaction of the produced \( \chi \) particles on the inflaton motion. If \( M_0 = 0 \), this approximation breaks down when [3]

\[
q \gtrsim q_2 = \frac{2g\mu M_p}{3m} \ln^{-1} \frac{10^{12} m}{g^5 M_p}.
\]
where $\mu$ is a parameter of order 0.1 which characterizes the efficiency of particle production. For most reasonable parameter values, the backreaction becomes important after $\mathcal{O}(10)$ inflaton oscillations [3]. The same estimate is true for massive particles, provided that their production is unsuppressed for all $q > q_2$. This occurs if $q_2 \gtrsim 16q_{\text{min}}$, or, assuming $g \sim 1$,

$$M_0 < \frac{m}{2} q_2^{1/4} \approx 4m. \quad (9)$$

If, on the other hand, $q_2 < q_{\text{min}}$, the backreaction can always be neglected: this is the case for $M_0 \gtrsim 8m = 8 \times 10^{13}$ GeV. Thus, there is a reasonably wide range of values of $M_0$ (roughly between $10^{14}$ and $10^{15}$ GeV) where some particles are produced, but not enough for the backreaction to become an important issue.

If the condition (9) is satisfied, preheating proceeds in two stages. During the first stage, $q > q_2$ and the backreaction can be neglected. During the second stage, the backreaction has to be included. The “effective” $q$-parameter at this stage is given by $q_{\text{eff}} = \phi_0^2/4 \langle X^2 \rangle$. This parameter is decreasing as more $X$ particles are created: approximate energy conservation implies $q_{\text{eff}} \propto (\langle X^2 \rangle)^{-2}$. Once $q_{\text{eff}} \sim q_{\text{min}}$, particle production stops.

### 2.2 Fermion Production.

Several authors [8, 12, 13] have studied the fermion production during preheating using the Lagrangian

$$\mathcal{L}_f = (M_0 + g\phi) \bar{\psi}\psi. \quad (10)$$

They found that fermions as heavy as $10^{17} - 10^{18}$ GeV could be produced. This is not surprising: with the Lagrangian (10), as long as the inflaton oscillation amplitude $\phi_0$ is greater than $M_0/g$, there is an instant of time during each oscillation when the effective mass of the fermion vanishes, and so the condition (4) is automatically violated at least for very large wavelengths. Thus, the upper bound on $M_0$ is, for $g \sim 1$, of order of the initial value of the inflaton, $0.3M_p$.

### 3 Inflaton Interactions in Supersymmetric Models

Now, let us discuss how the above discussion of preheating generalizes to the supersymmetric case. Interactions of the inflaton with other fields are of two types: those coming from the Kahler potential and the superpotential. While the Kahler potential couplings such as $\int d^4\theta SX^\dagger X/M_p$ can play a role in perturbative reheating, they do not lead to resonant, non-perturbative particle production that we are interested in. Therefore, we will concentrate on superpotential interactions. The simplest supersymmetric generalization of (4) is a model with two gauge-singlet chiral superfields, the “inflaton” $S$ and
the “matter field” $X$, whose interactions are described by the superpotential

$$W(S, X) = \frac{1}{2} mS^2 + \frac{1}{3} \epsilon S^3 + \frac{1}{2} (M + gS)X^2.$$  

(11)

This is the most general renormalizable superpotential for two singlet fields with a discrete symmetry $X \to -X$. Without loss of generality, we will choose the parameters $M$, $m$ and $g$ to be real and positive. The scalar potential is given by

$$V(S, X) = |mS + \epsilon S^2 + \frac{1}{2} gX^2|^2 + |M + gS|^2 |X|^2,$$  

(12)

where $S$ and $X$ are complex scalar fields. Note that this potential generically has four degenerate minima: $\{S = X = 0\}$, $\{S = -m/\epsilon, X = 0\}$ and $\{S = -M/g, X = \pm \sqrt{2(mM/g^2 - \epsilon M^2/g^3)}\}$. For $\epsilon = 0$, the number of minima is reduced to three: $S = X = 0$ and $\{S = -M/g, X = \pm \sqrt{2mM/g}\}$. The presence of multiple, disconnected degenerate vacua, which will have important consequences for the dynamics of preheating, is completely generic in supersymmetric theories with multiple singlets. Indeed, the scalar potential in such theories has a form

$$V(\phi_i) = \sum_{i=1}^{n} \left| \frac{\partial W(\Phi_i)}{\partial \phi_i} \right|^2.$$  

(13)

The vacua are found by solving a system of algebraic equations $\partial_i W(\Phi_i) = 0, i = 1 \ldots n$. For a renormalizable superpotential, each of these equations is at most quadratic. Generically, the system has $2n$ different complex solutions, each of which corresponds to a vacuum with zero energy. One should keep in mind, however, that the degeneracy of these vacua is lifted by supersymmetry breaking unless the theory possesses additional discrete symmetries relating different vacua. (The model defined by (11) happens to have such a symmetry, the $X \to -X$ reflection.)

The expression (12) can be thought of as the leading terms in the Taylor expansion of the full inflaton potential around $S = 0$. This potential is sufficient to describe the (p)reheating phase considered here, and is independent of the details of the slow roll phase of inflation. The only assumptions we make concern the values of the $S$ and $X$ fields at the end of the slow roll phase, which set the initial conditions for the process we are considering. Specifically, we assume that the initial value of the inflaton field $S_0$ is somewhat lower, but not too far from, the Planck scale $M_p$, while the initial value of $X$ is zero. The latter assumption is reasonable since the mass of $X$ is large (Planckian) during inflation. We treat the initial phase of the complex field $S$ as arbitrary, which is

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2 Supersymmetric models of chaotic inflation were constructed in Ref. [14].

3 We expect the qualitative features of our analysis to hold in both chaotic inflation and new inflation models. Parametric resonance in supersymmetric hybrid inflation models was considered in [15]; however, it was subsequently shown that in these models the inflaton typically decays via tachyonic preheating [16], without developing a resonance.
a good approximation as long as the potential in the phase direction is sufficiently flat during slow-roll inflation. We will study the dependence of the results on this phase.

The remaining freedom concerns the choice of parameters in the superpotential (11). Throughout the analysis, we will assume that \( m \ll |S_0| \); for numerical estimates, we will use \( m \sim 10^{13} \text{ GeV} \) and \( |S_0| \sim M_p/3 \) to facilitate the comparison with the non-supersymmetric analyses reviewed in Sec. 2. (These particular choices are motivated by the models of chaotic inflation [1].) In this paper, we will primarily be interested in the production of particles that are too heavy to be produced through thermal reheating or perturbative inflaton decay; therefore, we will assume \( M > m \). The size of parameter \( \epsilon \) in chaotic inflation models is constrained to be at most about \( 10^{-6} \) [17]. Even such small coupling, however, can have profound effects on the inflaton evolution during the (p)reheating epoch. We will therefore consider two qualitatively different, physically interesting cases: \( \epsilon = 0 \) and \( \epsilon \sim 10^{-6} \). In the first case, the complex phase of the inflaton is conserved during the evolution [1] and the motion of the inflaton is identical to the non-supersymmetric case. Still, there are some important differences as far as production of particles (especially fermions) is concerned. In the second case, the inflaton generally acquires angular momentum and moves along an elliptical spiral trajectory in the complex plane. As a result, the preheating process is quite different from the non-supersymmetric case.

4 Scalar Particle Production in Supersymmetric Models.

In this section, we will consider the resonant production of scalar \( X \) particles during the reheating era in the model defined by the superpotential (11).

4.1 \( \epsilon = 0 \).

We start by examining the situation where the coefficient of the \( S^3 \) term in the superpotential (11) vanishes. As long as the backreaction of the \( X \) particles can be neglected, the motion of the inflaton is described by the potential \( V(S_1, S_2) = m^2|S|^2 \). Since at the end of inflation the inflaton is slowly rolling, i.e. \( \dot{S} \approx 0 \), its subsequent evolution is given by

\[
S(t) = S_0 e^{i\theta} \cos mt,
\]

where \( \theta \) is a constant phase, determined by the initial conditions which we will assume to be completely random. In an expanding universe, the amplitude of the oscillations \( S_0 \) becomes time-dependent; assuming \( S_0 \approx M_p/3 \) at \( t = 0 \), after the first few oscillations \( S_0 \approx M_p/3mt \approx M_p/20N \), where \( N \) is the number of oscillations that have

\[4\text{Strictly speaking, this is only true as long as the backreaction of the produced } X \text{ particles can be neglected - see Sec. 3.}\]
be completed. The motion of the inflaton described by (14) is identical to the non-supersymmetric case, see Sec. 2.

The mass terms for the $X$ field can be read from (12):

$$\left( (M + gS_1)^2 + g^2 S_2^2 + gmS_1 \right) X_1^2 + \left( (M + gS_1)^2 + g^2 S_2^2 - gmS_1 \right) X_2^2 + 2mgS_2X_1X_2,$$

where we have decomposed $X = X_1 + iX_2$ and $S = S_1 + iS_2$. Diagonalizing this mass matrix gives

$$\left( (M + gS_1)^2 + g^2 S_2^2 + gm|S| \right) \tilde{X}_1^2 + \left( (M + gS_1)^2 + g^2 S_2^2 - gm|S| \right) \tilde{X}_2^2,$$

where $\tilde{X}_{1,2}$ are the mass eigenstates. (In what follows, we will drop the tilde to avoid cluttering.) The mode equations for the fields $X_{1,2}$ have the same form as (2), with the effective frequencies

$$\omega_k^2 = \frac{k^2}{a(t)^2} + (M + gS_1)^2 + g^2 S_2^2 \pm gm|S|,$$

where the upper (lower) sign applies to the mode $X_1$ ($X_2$). This equation has some important differences from its non-supersymmetric counterpart, Eq. (3). In particular, the right-hand side of (17) is not positive-definite. Since we are principally interested in heavy particle production, let us assume that $m \ll M$. Then, the right-hand side of (17) for the field $X_2$ is always positive, whereas for the field $X_1$ it can become negative for sufficiently small $k$. The region in the $S_1 - S_2$ plane in which this is possible (the “instability region”) is given by

$$(S_1 - S_*)^2 + S_2^2 \leq \frac{mM}{g^2},$$

where $S_* = -M/g$, and terms of order $m/M$ have been dropped. The instability region and a typical inflaton trajectory in the $\epsilon = 0$ case are shown in Fig. 1.

Just like in the non-supersymmetric case, resonant production of $X$ bosons occurs whenever the condition (4) is violated. For the mode $X_2$, this can happen if

$$M^2(\sin^2 \theta + \frac{m}{M} \cos \theta) \lesssim \frac{1}{2} gmS_0.$$

For the mode $X_1$, the analysis depends on the phase $\theta$. If $\theta \gtrsim \sqrt{m/M}$, the condition is similar to (19):\n
$$M^2(\sin^2 \theta - \frac{m}{M} \cos \theta) \lesssim \frac{1}{2} gmS_0.$$
Figure 1: Motion of the inflaton in the $S_1 - S_2$ plane in the case $\epsilon = 0$. The red circle indicates the instability region $\langle |S| \rangle$.

X$_1$ particles will occur. Thus, scalar particles with bare masses as high as $gS_0$ (i.e. up to about $10^{18}$ GeV, assuming $g \sim 1$) can in principle be produced in our model. However, production of particles substantially heavier than the limit indicated by the non-supersymmetric analysis of Sec. 3, Eq. (6), is only possible if $\theta < \sqrt{m/M}$. Since $m \ll M$, the range of $\theta$ for which this is true is quite small; in other words, production of superheavy scalar particles requires fine-tuned initial conditions.

The resonant production of $X$ particles can be described analytically using the technique developed in Ref. [5]. The relevant calculations are presented in Appendix A. For generic initial conditions on the inflaton, the production rates are identical (up to $O(m/M)$ corrections) to those found in the non-supersymmetric case with the potential (1) and $M_0 = M \sin \theta$. For special initial conditions, when the inflaton passes through the instability region, the production of $X_1$ particles is enhanced due to their locally imaginary mass. As long as the inflaton oscillation amplitude is large, $S_0 \gg |S_\star|$, the $X_1$ production rate is close to the rate estimated in [5] for non-supersymmetric particles with zero bare mass (see Eq. (1) with $M_0 = 0$). On the other hand, when $S_0 \sim S_\star$, the $X_1$ production rate can be dramatically enhanced. These results are derived in Appendix A.

To summarize our discussion so far, for generic initial conditions on the inflaton field,

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5Let us comment on the relation of our analysis in the case of “special initial conditions” to the tachyonic preheating scenario considered in [6]. In both cases, the field coupled to the inflaton develops an instability. However, in our case, the instability occurs only for a very short time period during each inflaton oscillation. As a result, the fractional energy loss by the homogeneous component of the inflaton in each passage is small. (This is true as long as $S_0 \gg S_\star$, a condition that is typically satisfied for a large number of inflaton oscillations at the beginning of the reheating era.) The backreaction of the produced $X$ particles does not become important until at least several inflaton oscillations have taken place, and the language of stochastic resonance is applicable.
\( \theta \gg \sqrt{m/M} \), both \( X_1 \) and \( X_2 \) particles are resonantly produced provided that

\[
M \lesssim \frac{1}{|\sin \theta|} \sqrt{\frac{g m S_0}{2}}.
\]  

(21)

This condition is similar to the one obtained in [5] for the non-supersymmetric case, Eq. (6). Indeed, if (6) is satisfied, preheating occurs regardless of the initial phase of the inflaton field. Preheating of particles with higher bare masses is possible, but requires special initial conditions. The absolute upper bound on \( M \) is achieved for the case when the inflaton is real (\( \theta = 0 \)) and is of order \( g S_0 \sim 10^{18} \) GeV for \( g \sim 1 \). Moreover, for the same special initial conditions, the production of \( X_1 \) bosons can be significantly enhanced.

The above discussion neglected the backreaction of the produced \( X \) particles on the motion of the inflaton. Let us now include this effect. First, consider generic initial conditions, \( \theta \gg \sqrt{m/M} \). In this case, the rough estimate of the produced \( X \) number density is the same as in the non-supersymmetric case discussed in Sec. 2, with the replacement \( M_0 \rightarrow M \sin \theta \). Thus, the backreaction can be neglected for \( M \sin \theta \gg 8 \approx 8 \times 10^{13} \) GeV. For smaller values of \( M \sin \theta \), the \( X \) particles are still being produced at the time when their backreaction becomes important. Including it leads to an effective inflaton potential of the form

\[
V(S) = g^2 \langle |X|^2 \rangle \left( (S_1 - S_\ast)^2 + S_2^2 \right) + m^2 |S|^2.
\]  

(22)

The phase of \( S \) is not conserved by the first term in this potential, and will start changing (tending towards zero) when the backreaction becomes important. As a result, \( S_1 \) and \( S_2 \) oscillations will be out of phase. The resonant \( X \) particle production will quickly terminate, since it is possible only when \( S_1 \) and \( S_2 \) are both close to the minimum of the potential. For the modes \( X_{1,2}(k) \) with large occupation numbers, the subsequent evolution can be considered classically. However, it is still very complicated, with all the four coupled fields \( X_{1,2} \) and \( S_{1,2} \) undergoing large oscillations in the potential

\[
V(S, X) \approx g^2 |X|^2 \left( (S_1 - S_\ast)^2 + S_2^2 \right) + \frac{1}{4} g^2 |X|^4,
\]  

(23)

where we have assumed \( g^2 \langle |X|^2 \rangle \gg m^2 \). The oscillations are damped by Hubble friction. When the oscillation amplitudes become sufficiently small, \( g^2 \langle |X|^2 \rangle < m^2 \) or \( \langle (S_1 - S_\ast)^2 \rangle < mM \), the terms in the scalar potential which were neglected in Eq. (23) become important. It is at this stage that the system chooses which of the three vacua will be the final point of the evolution. This is a very important question. If the vacua with non-zero \( X \) and \( S \) are preferred, the phenomenology will crucially depend on the exact nature of the \( X \) field. For example, if the \( X \) field carries \( R \) parity, these vacua are clearly undesirable. Moreover, in our model there are two vacua with non-zero \( X \). The \( X \) fields whose evolution is considered here are position-dependent, since modes in a wide range of wavelengths obtain large occupation numbers during preheating. It
is therefore very likely that different vacua will be chosen in different spatial regions, leading to formation of domain walls. (This non-thermal defect production mechanism is similar to the one considered, for example, in [18].) Because of the $Z_2$ symmetry relating the two vacua, they will remain degenerate even after supersymmetry is broken, and the domain walls of our model are stable. Therefore, the model is only consistent with standard cosmology if the system ends up in the vacuum $S = X = 0$ throughout the space. It would be very interesting to study the evolution in more detail and understand which vacuum is chosen for various values of the parameters and initial conditions. This will likely require a numerical study using the methods developed in [19, 20].

For special initial conditions, $\theta \lesssim \sqrt{m/M}$, the backreaction is expected to be very important. Indeed, the $X_1$ particle production rate in the instability region (18) is at least as high as the rate in the non-supersymmetric, $M_0 = 0$ case considered in [3]. Based on the analysis of [3], we conclude that the backreaction of $X_1$ particles will become important after just a few passages of the inflaton through the instability region. Moreover, in this case the $X_1$ production will continue even after the backreaction becomes important, and only stop when $\langle |X|^2 \rangle \sim \langle (S_1 - S_s)^2 \rangle$. After that, the evolution of the modes $X_1(k)$ with large occupation numbers can be described as classical motion of the fields $X_1$ and $S_1$ in the potential (23), with $X_2 \approx S_2 \approx 0$, damped by Hubble friction. A quick estimate suggests that the evolution is more likely to end in the vacua with non-zero $X$, leading to domain wall production.

Summarizing, we have found that for generic initial conditions, $\theta \gg \sqrt{m/M}$, and sufficiently high masses, $M \sin \theta \gtrsim 8m$, preheating shuts down before the backreaction of the produced particles on the inflaton motion becomes important, and the picture is quite similar to the non-supersymmetric case studied in [3]. On the other hand, for $M \sin \theta \lesssim 8m$ the backreaction becomes an important effect. Once it is taken into account, the evolution of the system is rather more involved than in the non-supersymmetric case, and may lead to unacceptable consequences such as production of domain walls. Clearly, further investigation of this situation is necessary. Furthermore, for special initial conditions, $\theta \lesssim \sqrt{m/M}$, we found that the upper bound on the masses of the $X$ bosons that can be reheated is substantially higher than in the non-supersymmetric case. In this case, backreaction is expected to be important, and it seems likely that domain walls are produced at the final stages of the process. Again, a more detailed numerical study is necessary to check this conclusion.

4.2 $\epsilon \neq 0$.

The cubic term $S^3$ in the superpotential (11) has important consequences for the motion of the inflaton after the end of the slow-roll inflation and, therefore, for preheating. In the presence of this term, the motion of the inflaton is governed (neglecting backreaction)

6In more general models, supersymmetry breaking can lift the vacuum degeneracy, leading to unstable domain walls. In this case, the cosmological bounds could be avoided, but a more careful examination of the domain wall dynamics is necessary.
by the potential

\[ V(S_1, S_2) = m^2(S_1^2 + S_2^2) + 2\epsilon m(S_1^2 + S_2^2)S_1 + \epsilon^2(S_1^2 + S_2^2)^2. \]  \tag{24}

The second term in this potential violates the symmetry \( S \rightarrow e^{i\theta}S \). Due to this term, the motion of the inflaton for generic initial condition is no longer described by \( (14) \), but is much more complicated. We will restrict our analysis to the case when the terms proportional to \( \epsilon \) in \( (24) \) are of the same order as the first term at the end of the slow-roll inflation. (With our standard assumption \( m \sim 10^{13} \text{ GeV} \) this implies \( \epsilon \sim 10^{-6} \), close to its upper bound in chaotic inflation models \([17]\).) The evolution of the inflaton during the reheating era can be broken into two stages. During the first stage, the effect of the cubic and quartic terms in the potential \( (24) \) is significant, and the motion is complicated. The potential \( (24) \) has two vacua: \( S = 0 \) and \( S = -m/\epsilon \). Because of the Hubble friction, the energy of the system is decreasing and it will eventually settle down into one of these vacua. Let us assume that the system ends up in the vacuum with \( S = 0 \). (This is always the case when \( m/\epsilon \gg |S_0| \). For the case considered here, \( m/\epsilon \sim |S_0| \), the choice of vacuum depends on the initial conditions; domains of attraction of the two vacua have more or less the same size.) In this case, the average magnitude of the inflaton field \( |S| \equiv \sqrt{S_1^2 + S_2^2} \) decreases with time. Eventually the quadratic term in \( (24) \) becomes dominant, and the second stage of the inflaton evolution begins. At this stage, the potential can be approximated by setting \( \epsilon = 0 \). However, the problem does not reduce to the case studied in the previous subsection, since the inflaton field generically has non-zero angular momentum. Its motion can be described as independent, out-of-phase oscillations of \( S_1 \) and \( S_2 \) fields:

\[
\begin{align*}
S_1 &= S_{1,0}(t) \sin mt, \\
S_2 &= S_{2,0}(t) \sin(mt + \delta),
\end{align*}
\]  \tag{25}

where \( S_{1,0} \) and \( S_{2,0} \) are slowly decreasing with time as a result of the Hubble expansion. Thus, during the second stage, the inflaton is approaching the minimum of its potential along an elliptical spiral. As we show in Appendix B, with the assumptions made here, the ellipticity of the orbit is generally of order one. A typical trajectory of the inflaton is sketched in Fig. 2.

The resonant production of \( X \) particles is only possible if the non-adiabaticity condition \( (4) \) is satisfied. For low-momentum modes, the frequency can be approximated by

\[ \omega^2 = (M + gS_1)^2 + g^2S_2^2 \mp g\epsilon|S|. \]  \tag{26}

During the first stage of the reheating era, the resonant \( X \) production is very unlikely. Indeed, approximate energy conservation implies that the velocity of the inflaton satisfies \( |\dot{S}| \lesssim m|S_0| \). Since \( m \ll g|S_0| \), the non-adiabaticity condition can be satisfied only if the frequency \( \omega \) is much smaller than its typical value, which is order \( g|S_0| \). If \( M \ll g|S_0| \), this can only occur when both \( S_1 \) and \( S_2 \) are close to zero, \( |S| \lesssim M \); if \( M \sim g|S_0| \), this requires that the inflaton pass through (or close to) the instability region \( (18) \). The area
Figure 2: A typical trajectory of the inflaton in the case $\epsilon \neq 0$. The first part of the trajectory depends on the initial conditions and does not have a universal shape. The second part of the trajectory, corresponding to $\epsilon |S| \ll m$, is an elliptical spiral (unless the phase of $S$ vanishes at the end of inflation.)
Figure 3: Depending on the step of the spiral, the inflaton either passes through the instability region (left panel) or misses it (right panel).

of these “production regions” in the $S_1 - S_2$ plane is quite small compared to the range of the inflaton motion during the first stage. Since the inflaton motion is highly non-periodic, it is very unlikely that it will pass through the “production regions” repeatedly, as required for the buildup of the number density in the resonant production scenario.

Now, let us consider the second stage of the preheating era, when the motion of the inflaton is approximately described by (25). For simplicity, let us set $S_{1,0} = S_{2,0}$ and $\delta = \pi/2$, so that the orbit is a circular spiral. (The physical results of our analysis are insensitive to these assumptions.) In this case, the condition (4) can be studied analytically. The bottomline is that the $X$ particle production only occurs when the inflaton passes through the “production region” in the $S_1 - S_2$ plane. This production region coincides (up to order-one factors) with the instability region defined by Eq. (18). This is not surprising: it is clear that the adiabaticity condition is always violated in the instability region. In the $\epsilon = 0$ case studied in the previous subsection, we found that $X$ particles can be produced even without the inflaton passing through the instability region as long as their mass is sufficiently low, see Eq. (21). This production occurs twice per oscillation period when the inflaton passes the origin, provided that its velocity $|\dot{S}| \sim mS_0$ is greater than (roughly) $M/g$. In the case of spiral motion considered here, however, the inflaton does not get close to the origin until it loses most of its energy. Thus, the only possible production mechanism in this case is by passing through the instability region.

The inflaton will pass through the instability region regardless of the initial conditions only if the step of its spiral trajectory $\delta$ at the time when $|S| \approx S_*$ is much smaller than the extent of the instability region along the $S_1$ axis, $\Delta S \approx \sqrt{mM/g}$. (This is illustrated in Fig. 3.) Estimating $\delta \approx S_*^2/M_p$, we find that this requirement implies an upper bound
on the bare mass of the $X$ particles that can be produced:

$$M \lesssim (g^2 m M_P^2)^{1/3}. \quad (27)$$

This bound is somewhat higher than the corresponding bound in the non-supersymmetric case, Eq. (14), and the bound for generic initial conditions in the case $\epsilon = 0$, Eq. (21). For our standard parameter values, $g \sim 1$ and $m \sim 10^{13}$ GeV, Eq. (27) implies $M \lesssim 10^{17}$ GeV.

Particle production by a spiralling inflaton passing through the instability region is considered quantitatively in Appendix A. The production is extremely efficient: in fact, the backreaction of the produced $X_1$ particles on the inflaton may become important already after two or three production events, terminating the process. This explosive production is analogous to the so-called tachyonic preheating [16], which frequently occurs in hybrid inflation models. The subsequent evolution of the system can be studied classically, but is very complicated due to a large number of fields involved. It seems likely that this evolution will involve domain wall formation.

In summary, we have found that if the cubic term is present in the inflaton superpotential, the dynamics of preheating is qualitatively different from the non-supersymmetric case. If the condition (27) is satisfied, non-perturbative particle production occurs when the inflaton trajectory is spiral, and does not require fine-tuned initial conditions. Production in this case tends to be explosive, with the inflaton losing a large part of its energy already during the first two or three production events. For larger values of $M$, particle production may in principle occur during the initial, irregular phase of the inflaton motion; however, for generic initial conditions, this possibility is highly unlikely.

### 5 Fermion Production in Supersymmetric Models.

Preheating of fermionic particles in supersymmetric models has important differences from the non-supersymmetric case reviewed in Sec. 2.4. The matter chiral superfield $X$ contains two two-component (Weyl) spinors, which we will denote by $\psi_L$ and $\psi_R$. The interactions of these fields follow from the superpotential (11):

$$\mathcal{L}_f = -(M + gS)\bar{\psi}_R \psi_L - (M + gS^*)\psi_L^\dagger \psi_R. \quad (28)$$

Combining the fields $\psi_L$ and $\psi_R$ into a four-component (Dirac) spinor $\Psi = (\psi_L, \psi_R)^T$, Eq. (28) can be rewritten as

$$\mathcal{L}_f = -(M + g S_1) \bar{\Psi} \Psi + ig S_2 \bar{\Psi} \gamma^5 \Psi, \quad (29)$$

where $S = S_1 + i S_2$, $\bar{\Psi} = \Psi^\dagger \gamma^0$, and we have chosen the Weyl basis in which $\gamma^5 = \text{diag}(-1, -1, 1, 1)$. Comparing Eq. (29) with its non-supersymmetric counterpart, Eq. (14), indicates an important difference: the appearance of an additional, pseudoscalar mass term in the supersymmetric case. This term plays a significant role in the analysis of preheating.
The evolution of the fermion wavefunctions is governed by the Dirac equation of the form
\[ i \partial_t \Psi(t) = A(t) \Psi(t) \],
where
\[ A(t) = \begin{pmatrix} \pm k & M + gS(t) \\ M + gS^*(t) & \mp k \end{pmatrix} \]  
(30)
In this equation, \( k \) is the momentum of the mode and we are working in the helicity basis,
\[ i \sigma^i \partial_i \psi = \pm k \psi \]. The frequencies of the fermionic modes are given by the eigenvalues of
the matrix \( A \), which are in general time-dependent:
\[ \omega = \pm \sqrt{k^2 + |M + gS(t)|^2} \]. A necessary condition for the adiabatic evolution of the system is \( \dot{\omega} \ll \omega^2 \); when this
condition is violated, the evolution is non-adiabatic and particle production is expected
to occur. Below, we will use this condition to obtain a qualitative picture of fermion
preheating in supersymmetric models.

5.1 \( \epsilon = 0 \)
As expected in a supersymmetric theory, the analysis of fermionic preheating in this case
is very similar to the scalar preheating analysis of Sec. 4.1. For \( m \ll M \lesssim \sqrt{mgS_0/2} \),
fermions can be produced for any value of the inflaton phase, whereas for \( \sqrt{mgS_0/2} \lesssim M \lesssim gS_0 \), the production occurs if this phase satisfies
\[ |\sin \theta| \lesssim \frac{1}{M} \sqrt{\frac{gmS_0}{2}} \]  
(31)
The production of very heavy fermions (up to \( M \sim gS_0 \)) is possible but only if the
initial phase of the inflaton is fine-tuned to be close to 0. (This fine-tuned case is identical
to the non-supersymmetric model of Sec. 2.2.) For generic initial conditions, the upper
bound on the mass of the fermions that can be effectively preheated is of order \( \sqrt{gmS_0/2} \),
well below the non-supersymmetric estimate.

The condition \( \dot{\omega} \ll \omega^2 \) is necessary for adiabaticity, but it is not sufficient. Particle
production could occur even when \( \dot{\omega} = 0 \) if the eigenvectors of the matrix \( A \) have strong
time-dependence. For example, a rapid change of sign of the effective mass \( M + gS \),
even without a change in its magnitude, would lead to particle production. In the case \( \epsilon = 0 \), however, it is possible to show that the eigenvector rotation is adiabatic whenever
the condition \( \dot{\omega} \ll \omega^2 \) is satisfied. Thus, fermion production does not occur outside of
the parameter regions discussed in the previous paragraph.

It is important to keep in mind that in supersymmetric models, fermion and boson
production cannot be consired separately: the fermion and scalar masses and their cou-
plings to the inflaton are all expressed in terms of just three superpotential parameters,
\( M, m \) and \( g \). As we emphasized in Section 4.1, some regions of this parameter space

\footnote{Naively, the expression (31) seems to imply that infinitely heavy fermions can be produced for \( \theta = 0 \). This is of course incorrect; the bound (31) is not applicable for \( \theta \lesssim \sqrt{m/M} \). The correct bound in that case is \( M \lesssim gS_0 \).}
may not lead to consistent cosmology, for example due to non-thermal production of topological defects. If this is the case, additional restrictions would be imposed on the masses of the fermions that can be preheated. In particular, the production of super-heavy fermions, \( M > \sqrt{g m S_0} \), requires a passage of the inflaton through (or close to) the instability region, which results in enhanced scalar particle production and may be accompanied by domain wall formation.

Even if the fermion is light enough to be preheated, we expect the number of produced particles for generic initial conditions to be smaller than in the non-supersymmetric case. Indeed, the effective frequency of the modes in the supersymmetric case is limited from below by \( M \sin \theta \), leading to a suppression in the efficiency of particle production for non-zero \( \theta \). The magnitude of this suppression could be estimated by a numerical integration of Eq. (30). Such an investigation is outside the scope of this paper.

5.2 \( \epsilon \neq 0 \).

Again, the analysis is completely analogous to the corresponding analysis of the scalar production presented in Section 4.2. There are two possibilities for fermion production. First, it can occur when the inflaton passes close to the origin in the \( S_1 - S_2 \) plane, with a sufficiently large velocity. The second possibility is when the inflaton passes through the “instability region” around the point \( S_1 = -M/g, S_2 = 0 \). (The fermion effective mass vanishes at that point.) To obtain resonantly enhanced production, the inflaton should pass through either one of these special regions repeatedly.

For non-zero \( \epsilon \), the reheating epoch can be subdivided into two stages. During the first stage, the inflaton motion is highly non-periodic and irregular (see Fig. 2.) Therefore, repeated passage through either one of the two relatively small production regions is unlikely. During the second stage, the inflaton approaches its minimum along an elliptically spiral trajectory. Fermion production is possible if this trajectory passes through the instability region, given by (18). As shown in Section 4.2, this puts an upper bound on the bare mass of the particles that can be preheated, \( M < (g^2 m M_p^2)^{1/3} \). However, if this condition is satisfied, the production of scalars in the instability region is explosive, generically leading to undesirable consequences such as domain wall formation. Thus, obtaining a cosmologically consistent scenario of fermionic preheating in a supersymmetric model with \( \epsilon \neq 0 \) is difficult. Given this conclusion, we will not attempt to calculate the number density of fermions produced in this case.

6 Conclusions.

In this paper, we have studied the implications of supersymmetry for the inflaton dynamics in the reheating era and the process of preheating. The emerging picture shows important differences from the previous, non-supersymmetric analyses. This is due to several robust, generic features of supersymmetric theories. First, all the scalars in supersymmetric theories are necessarily complex, meaning that an extra degree of freedom,
the inflaton phase, has to be considered. Second, the couplings between the inflaton and
the “matter” fields (the fields that are being preheated) are constrained by holomor-
phy. Third, supersymmetric models usually possess multiple degenerate vacua, which
can become relevant (and lead to cosmological difficulties) if the density of particles
produced during preheating is large. Finally, the preheating of bosons and fermions in
a supersymmetric model cannot be considered in isolation: both kinds of particles are
necessarily present, and their couplings are related.

Several applications of preheating were suggested in the literature, and have been
argued (using non-supersymmetric models) to be successful. Do these optimistic con-
cclusions survive in supersymmetric theories? Our analysis demonstrates that the answer
to this question is not straightforward. For example, consider a scenario of leptogene-
sis where heavy (10^{14} GeV) right-handed neutrinos are produced via preheating. This
process is described by our toy model, Eq. (11), if we identify \( X \) with the right-handed
neutrino chiral superfield. Generically, the parameter \( \epsilon \) is non-zero, and the analysis of
Sections 4.2 and 5.2 applies. Eq. (27) shows that sufficiently heavy right-handed neu-
trinos can indeed be produced. However, at the same time, their scalar superpartners
are produced explosively (see Section 4.2 and Appendix A.2.) As a result, the system is
likely to end up in the vacua with non-zero right-handed sneutrino vacuum expectation
value, either in some regions of space or everywhere. Both possibilities are phenomeno-
logically disastrous. Therefore, the overall success of the leptogenesis scenario is not
guaranteed, and may involve significantly more parameter fine-tuning and/or additional
model building than previously thought.

Our analysis is only the first step in the quantitative investigation of preheating after
inflation in supersymmetric models. It needs to be extended in several directions. For
example, in many cases, preheating is so efficient that the backreaction of matter should
be taken into account at the end of this process. We have neglected this effect throughout
the analysis, giving only qualitative estimates for the parameter ranges over which it
becomes important. Studying the evolution of the inflaton-matter system after the end
of preheating, including the backreaction, is crucial since it will allow one to answer the
all-important questions about the vacuum selection and domain wall formation. Also,
we have not attempted to describe quantitatively the regions of parameter space that
allow for successful applications of preheating, e.g. to leptogenesis or WIMPZILLA
production. Further work in this direction is necessary.

Acknowledgments

We thank Gary Felder and Patrick Greene for fruitful discussions. The authors are
supported by the Director, Office of Science, Office of High Energy and Nuclear Physics,
of the U. S. Department of Energy under Contract DE-AC03-76SF00098, and by the
National Science Foundation under grant PHY-00-98840.
A Analytic Description of Scalar Preheating.

In this appendix, we will present a semi-analytic treatment of the production of scalar particles $X$ during inflaton oscillations. The qualitative aspects of this phenomenon have been discussed in Section 4. The calculation presented here is very similar to the one performed in [5] for the case of real scalars. However, our results are more general; in particular we will consider the case when the inflaton field passes through the instability region (18), and show that this generally leads to enhanced particle production.

A.1 $\epsilon = 0$.

Let us first concentrate on the case $\epsilon = 0$, discussed in Section 4.1. Consider production of the quanta of the field $X$. Defining $\Theta(t) \equiv a^{-3/2}(t)X_1(t)$, the mode equation reads

$$\ddot{\Theta}_k + \omega^2_k \Theta_k = 0$$

with the effective frequency

$$\omega^2_k = \frac{k^2}{a^2(t)} + g^2(S_1 - S_\ast)^2 + g^2S_2^2 - gm|S| + \Delta,$$

where $S_\ast = -M/g$, and terms of order $m/M$ are neglected. The correction term $\Delta = -\frac{3}{4}(\dot{a}/a)^2 - \frac{3}{2}(\ddot{a}/a)$ is also typically very small and will be neglected throughout the analysis.

The motion of the inflaton field for $\epsilon = 0$ is described by Eq. (14). Particle production occurs only when the non-adiabaticity condition (4) is satisfied. For generic initial conditions, this requires $M \ll S_0$, see Eq. (21), and production occurs twice during each inflaton oscillation cycle, when $S$ is close to the origin. For special initial conditions production occurs when the inflaton passes through the instability region (18). In both cases, each of the production periods is short compared to the inflaton oscillation period. This leads to the following approximation. Let $t_j$ denote the moments of time when the $S$-dependent terms in $\omega_k$ are minimized:

$$\cos mt_j = -\frac{M \cos \theta}{gS_0}.$$  

Particle production occurs at $t_j - \Delta t \lesssim t \lesssim t_j + \Delta t$, where $m\Delta t \ll 1$. Outside of these time intervals, the mode equation (32) can be solved by making the adiabatic approximation. For $t_{j-1} + \Delta t \lesssim t \lesssim t_j - \Delta t$, we write

$$\Theta_k(t) = \frac{\alpha^j_k}{\sqrt{2\omega}} \exp\left(-i \int_0^t \omega dt\right) + \frac{\beta^j_k}{\sqrt{2\omega}} \exp\left(+i \int_0^t \omega dt\right),$$

where the coefficients $\alpha^j_k$ and $\beta^j_k$ are constant. In the next adiabatic time period, $t_j + \Delta t \lesssim t \lesssim t_{j+1} - \Delta t$, the expression for $\Theta_k(t)$ has the form

$$\Theta_k(t) = \frac{\alpha^{j+1}_k}{\sqrt{2\omega}} \exp\left(-i \int_0^t \omega dt\right) + \frac{\beta^{j+1}_k}{\sqrt{2\omega}} \exp\left(+i \int_0^t \omega dt\right).$$
With our normalization, the coefficients $\alpha$ and $\beta$ are just the coefficients of the Bogolyubov transformation of the creation and annihilation operators. The comoving density of $X_1$ particles at time $t$ is given by

$$n_{X_1}(t) = \frac{1}{2\pi^2a^3} \int_0^\infty dk k^2 |\beta_k(t)|^2. \quad (37)$$

When the adiabaticity condition holds, the number density in (37) is constant, since $\beta_k$ does not change. When the condition is violated, the number density can change: in general $\beta_k^{j+1} \neq \beta_k^j$. Thus, to estimate the change in particle density during a single production event, we need to find the Bogolyubov transformation matrix $B$ corresponding to this event:

$$\begin{pmatrix} \alpha_k^{j+1} \\ \beta_k^{j+1} \end{pmatrix} = B^j_k \begin{pmatrix} \alpha_k^j \\ \beta_k^j \end{pmatrix}. \quad (38)$$

To achieve this, we perform a Taylor expansion of the effective frequency (33) for $t \approx t_j$:

$$\omega_k^2 = \frac{k^2}{a^2(t_j)} + \omega_{k,\text{min}}^2 + g^2 |\dot{S}(t_j)|^2(t - t_j)^2, \quad (39)$$

where $\omega_{k,\text{min}}^2 = M^2 \sin^2 \theta - mM \cos \theta$. For generic initial conditions, the production occurs when $S \approx 0$, and therefore $|\dot{S}(t_j)| \approx mS_0$; whereas for special initial conditions, we have $|\dot{S}(t_j)| \approx m\sqrt{S_0^2 - S_*^2}$. Defining $k_* = \sqrt{g|\dot{S}(t_j)|}$, $\tau = k_*(t - t_j)$, $\kappa = k/a(t_j)k_*$ and using (39), the mode equation at $t \approx t_j$ becomes

$$\frac{d^2 \Theta_k}{d\tau^2} + (\lambda_k^j + \tau^2) \Theta_k = 0 \quad (40)$$

where

$$\lambda_k^j = \kappa^2 + \frac{1}{k_*^2} \left( M^2 \sin^2 \theta - mM \cos \theta \right). \quad (41)$$

Note that the parameter $\lambda_k^j$ can be negative for modes with low momenta if the inflaton passes through the instability region (18). The exact analytic solution of (40) is given by parabolic cylinder functions [21]:

$$\Theta_k(\tau) = C_1 D_p((1 + i)\tau) + C_2 D_p(-(1 + i)\tau), \quad (42)$$

where $p = -(1 + i\lambda_k^j)/2$ and $C_{1,2}$ are arbitrary coefficients. The form (42) is valid for both positive and negative $\lambda$. The asymptotic forms of $\Theta_k(\tau)$ for $\tau \to -\infty$ and $\tau \to +\infty$ should match the adiabatic solutions before and after the production event, Eqs. (35) and (36) respectively. Performing this matching allows us to express the four Bogolyubov coefficients, $\alpha_k^{j,j+1}$ and $\beta_k^{j,j+1}$, in terms of only two constants $C_1$ and $C_2$, and therefore to find the transformation matrix $B^j_k$ in (38). The result is

$$B^j_k = \begin{pmatrix} \sqrt{1 + e^{-\pi\lambda_k^j} e^{-i\varphi}} & i e^{-\pi\lambda_k^j/2 + 2\theta_k^j} \\ -i e^{-\pi\lambda_k^j/2 - 2\theta_k^j} & \sqrt{1 + e^{-\pi\lambda_k^j} e^{i\varphi}} \end{pmatrix}, \quad (43)$$
where $\lambda \equiv \lambda_k^j$, $\theta_k^j = \int_0^{t_j} \omega(t)dt$ is the phase accumulated by the moment $t_j$, and the angle $\varphi$ is given by

$$\varphi = \arg \Gamma \left( \frac{1 + i\lambda}{2} \right) + \frac{\lambda}{2} \left( 1 + \log \frac{2}{\lambda} \right).$$  (44)

The results in section VII of the paper [5] correspond to setting $\lambda = \kappa^2$ in these formulas.

Let us define the density of particles with definite momentum $k$, $n_k = |\beta_k(t)|^2$. Using Eqs. (38) and (43) and the normalization condition $|\alpha_k^j|^2 - |\beta_k^j|^2 = 1$, we find the change in this density during the production event at $t_j$:

$$n_k^{j+1} = e^{-\pi\lambda} + (1 + 2e^{-\pi\lambda})n_k^j - 2e^{-\pi\lambda/2} \sqrt{1 + e^{-\pi\lambda}} \sqrt{1 + n_k^j} \sin \vartheta,$$  (45)

where $\vartheta = \varphi + 2\theta_k^j + \arg \beta_k^j - \arg \alpha_k^j$. When the occupation numbers are large, (45) can be approximated by

$$n_k^{j+1} = \exp(2\pi \mu_k^j) n_k^j$$  (46)

where the “growth index” is given by

$$\mu_k^j = \frac{1}{2\pi} \ln \left( 1 + 2e^{-\pi\lambda} - 2e^{-\pi\lambda/2} \sqrt{1 + e^{-\pi\lambda}} \sin \vartheta \right).$$  (47)

This growth index characterizes the “efficiency” of each particle production event. It was argued in [5] that in an expanding universe the phases $\vartheta$ corresponding to different production moments $t_j$ are practically uncorrelated, and therefore it is a reasonable approximation to treat the phase as a random variable. The effective growth index $\mu_k$ can then be obtained by averaging over the phase $\vartheta$, leading to the expression

$$n_k(t) \approx \frac{1}{2} \exp(2m\tilde{\mu}kt).$$  (48)

It was demonstrated in [5] that the results of this approach agree reasonably well with more exact numerical results.

It is clear from (47) that successful particle production requires $\pi\lambda \lesssim 1$. For generic initial conditions on the inflaton, $\theta \gg \sqrt{m/M}$, this condition practically coincides with the condition obtained from the more qualitative analysis of section 4.1. For special initial conditions, $\theta \lesssim \sqrt{m/M}$, the inflaton passes through the instability region and the quantity $\lambda$ becomes negative at sufficiently low momenta. Since the growth index increases with decreasing $\lambda$, this leads to enhanced particle production rate. (In the non-supersymmetric case, the maximal production rate is achieved when the bare mass of the matter scalar is set to zero, corresponding to $\lambda = \kappa^2 \geq 0$. This is the case considered in [5].) For example, if $\sin \theta = 0$, the minimal value of $\lambda$ is about $-mM/k^2 \approx 8$. Note that for some values of $\vartheta$ the growth index could become negative, corresponding to a decrease of the particle number at time $t_j$.  

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−M/\sqrt{g^2 S_0^2 − M^2}$. If $M \ll g S_0$, the enhancement is insignificant. For $g S_0 \sim M$, however, this effect can become very important.

The rates of $X_2$ particle production can be derived in the same way. The only difference is the sign change in the last term of (41). As a result of this change, $\lambda$ is positive-definite in this case. At any rate, as long as $M \ll g S_0$ and $m \ll M$, the produced number densities of $X_1$ and $X_2$ particles are nearly identical.

Finally, let us note that the above analysis could also be performed in the non-supersymmetric model of Section 2; one just has to use the corresponding expression for the effective frequency, Eq. (3), instead of (33). The results are identical to the ones presented here with the replacement $M^2 \sin^2 \theta − mM \cos \theta \rightarrow M_0$. (Of course this replacement does not make sense for the special initial conditions, $\theta \lesssim \sqrt{m/M}$, in the supersymmetric model.) In particular, for generic initial conditions, $\theta \gg \sqrt{m/M}$, the production rates of $X_{1,2}$ bosons are the same as for a particle of the bare mass $M \sin \theta$ in the non-supersymmetric model.

A.2 $\epsilon \neq 0$.

Now, consider scalar preheating in the case $\epsilon \neq 0$. As we discussed in Section 4.2, non-adiabatic particle production can only occur when the spiral inflaton trajectory passes through the instability region (18). Let us consider a circular trajectory satisfying this property: $S_1 = |S_s| \sin mt$, $S_2 = |S_s| \cos mt$. The effective frequency for the field $X_1$, defined in Eqs. (32) and (33), is given by

$$\omega_k^2 = \frac{k^2}{a^2(t)} + 2M^2 (1 − \sin mt) − mM, \quad (49)$$

where we have used $S_s = −M/g$. The inflaton passes through the center of the instability region at times $t_j = m^{-1}(\pi/2 + j\pi)$. Since the instability region is small, each of the particle production periods is short compared to the inflaton oscillation period $m^{-1}$. This allows us to Taylor expand the sine function in (19) for $t \approx t_j$:

$$\omega_k^2 = \frac{k^2}{a^2(t_j)} - mM + M^2 m^2 (t - t_j)^2. \quad (50)$$

Defining $\tau = \sqrt{Mm}(t - t_j)$, the mode equation (32) at $t = t_j$ becomes identical to (10), with

$$\lambda_j^k = -1 + \frac{k^2}{a^2(t_j)mM}. \quad (51)$$

Thus, the analysis of particle production in the $\epsilon \neq 0$ case effectively reduces to the analysis for $\epsilon = 0$ performed in the previous subsection. The change in the occupation numbers $n_k$ during a single production event at time $t_j$ is given by Eq. (13); assuming large $n_k$ we obtain

$$n_k^{j+1} \approx (1 + 2e^{-\pi \lambda/2} \sqrt{1 + e^{-\pi \lambda} \sin \vartheta})n_k^j, \quad (52)$$
where $\vartheta$ is a random phase and $\lambda \equiv \lambda^j_k$. This formula and Eq. (51) make it clear that particle production by an inflaton on a spiral trajectory is extremely efficient: the occupation numbers of states with low momenta typically grow by a factor of $e^{\pi} \approx 25$ in a single production event. A rough estimate of the energy density of the produced $X_1$ particles yields

$$\rho(t) \approx \frac{m^2 M^2}{16 \pi^5 N^2} e^{\pi N},$$

where $N$ is the number of production events before $t$. This estimate suggests that this process is not, in fact, accurately described by the stochastic resonance picture: after just two or three production events, the energy density of the $X$ particles becomes comparable to that of the inflaton, and their backreaction becomes crucially important. Such explosive particle production in the instability region is similar to the tachyonic preheating [16] which occurs in hybrid inflation models.

**B  Ellipticity of the Inflaton Trajectory.**

When $\epsilon = 0$ in the superpotential (11), the complex phase of the inflaton field $S$ is conserved, and its trajectory in the complex $S$ plane is a straight line. When $\epsilon$ is turned on, the phase is no longer conserved, and the trajectory is more complicated. As we have argued in section 4.2, the evolution of the inflaton can be divided into two stages: during the first stage the phase-violating terms are important, while during the second stage they can be neglected. The inflaton trajectory during the second stage is an elliptical spiral; its ellipticity is determined by the amount of angular momentum accumulated in the first stage. In this appendix we would like to argue that even for small violations of phase invariance ($\epsilon \sim 10^{-6}$) the accumulated angular momentum is sufficient to obtain order-one ellipticity.

The inflaton potential has the form

$$V = |mS + \epsilon S^2|^2 = m^2 |S|^2 + \epsilon m |S|^2 (S + S^*) + \epsilon^2 |S|^4.$$  \hspace{1cm} (54)

The Noether current for the phase invariance is

$$n_S = \int d^3x i(S^* \dot{S} - \dot{S}^* S).$$ \hspace{1cm} (55)

Both the first and the third terms in the potential (54) preserve the phase invariance, while the second term explicitly breaks it. In the presence of the expansion of the universe, the Noether current satisfies the equation

$$\dot{n}_S + 3H n_S = i \left( \frac{\partial V}{\partial S} - \frac{\partial V}{\partial S^*} S^* \right) = i \epsilon m |S|^2 (S - S^*).$$ \hspace{1cm} (56)

In the case $\epsilon = 0$, the motion of the inflaton is described by

$$S(t) = \frac{t_0}{t} S_0 \sin mt = \frac{t_0}{t} |S_0| e^{i\theta} \sin mt,$$ \hspace{1cm} (57)
where \( \theta \) is determined by initial conditions. (Here we assumed that the motion of \( S \) has virialized, which is true when the amplitude is less than the reduced Planck scale.)

For sufficiently small \( \epsilon \), the violation of the phase conservation is small, and the inflaton trajectory is well approximated by (57). In this case, (56) can be integrated:

\[
n_S a^3(\infty) = \int_{t_0}^{\infty} dt i \epsilon m |S|^2 (S - S^*) a^3(t) = 2 \epsilon m |S_0|^3 a^3 t_0 \sin \theta \int_{t_0}^{\infty} dt \frac{1}{t} \sin^3 mt, \tag{58}
\]

where \( a \) is the scale factor. The last integral is order unity. (If the integration is taken from \( t_0 = 0 \), it is \( \pi/4 \).)

The quantity determining the ellipticity of the inflaton orbit is

\[
\frac{n_S}{m |S|^2} \approx 2 \epsilon m |S_0|^3 a^3 t_0 \sin \theta \frac{1}{m |S|^2 a^3} = 2 \epsilon |S_0| t_0 \sin \theta. \tag{59}
\]

Because \( t_0 \) is defined to be the moment where \( S \) starts oscillating around the origin following the harmonic oscillator potential, \( t_0 \approx M_s/(m |S_0|) \) \( (M_s = M_p/\sqrt{8 \pi}) \), we find

\[
\frac{n_S}{m |S|^2} \approx 2 \epsilon \frac{M_s}{m} \sin \theta. \tag{60}
\]

The above analysis is valid only if the ellipticity is small. For generic initial conditions on the inflaton and \( m \sim 10^{13} \) GeV, this implies \( \epsilon \ll 10^{-6} \). The analysis breaks down for \( \epsilon \sim 10^{-6} \), indicating that the ellipticity of the orbit in the case studied in Section 4.2 is of order one unless the initial conditions are tuned so that \( \theta \approx 0 \).

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