Fitting BOOMERANG and MAXIMA-1 data with a
Einstein-Yang-Mills Cosmological Model

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Abstract

We analyse the implications of recent Cosmic Microwave Background (CMB) data for a specific cosmological model, based on the higher-dimensional Einstein-Yang-Mills system compactified on a $R \times S^3 \times S^d$ topology and conclude that the model parameters are tightly constrained by CMB spectra. Moreover, the model predicts a relationship between the deceleration parameter at present, $q_0$, and some characteristic features of CMB spectra, namely the height of the first peak and the the location of the second peak, that is consistent with the observations and which can be further tested by future CMB and other experiments.

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Recent BOOMERANG data suggest that, within the framework of inflation-motivated adiabatic Cold Dark Matter (CDM) models, the spatial curvature is close to flat and the primordial fluctuation spectrum is nearly scale invariant [1]. Moreover, the data prefer a baryon density $\Omega_b h^2$ somewhat higher than previous studies indicated, though consistent with estimates from Big Bang nucleosynthesis. When combined with large scale structure observations, this provides evidence for both dark matter and dark energy contributions to the total energy density $\Omega_{\text{tot}}$, in agreement with Supernova observations [2]. On the other hand, MAXIMA-1, another balloon-borne experiment, finds new constraints on a seven-dimensional space of cosmological parameters within the class of inflationary adiabatic models [3]: $\Omega_{\text{tot}} = 0.90 \pm 0.15$, $\Omega_b h^2 = 0.025 \pm 0.010$, $\Omega_c h^2 = 0.13 \pm 0.10$ ($\Omega_c \equiv \Omega_{\text{cdm}}$), $n_s = 0.99 \pm 0.09$, all at the 95% confidence level. By combining MAXIMA-1 results with high-redshift supernovae measurements, further constraints are obtained on the value of the cosmological constant and the fractional amount of matter in the Universe: $0.4 < \Omega_\Lambda < 0.76$ and $0.25 < \Omega_m < 0.50$.

In view of the increasingly tight constraints on cosmological parameters, fitting CMB data with particle physics-motived cosmological models has become a major goal for theoretical cosmology. Such models necessarily have to account for the negative pressure dark energy component of the energy density of the Universe, as revealed by data. Although the simplest and most obvious candidate for the missing energy is a vacuum density that contributes a fraction $\Omega_\Lambda \simeq 0.7$ of closure density, there are alternative, possibly theoretically advantageous candidate theories. Quintessence, as a substitute for the cosmological constant, is a slowly varying component with a negative equation of state. An example of quintessence is the energy associated
with a scalar field slowly evolving down its potential \[4\] or a scalar field coupled non-minimally with gravity \[5\]. Another possibility for the nature of dark energy is the vacuum energy density of scalar and internal-space gauge fields arising from the process of dimensional reduction of higher dimensional gravity theories \[6\].

The purpose of this article is to fit CMB spectra, notably data recently obtained by the BOOMERANG and MAXIMA-1 experiments, within the framework of a cosmological model based on the multidimensional Einstein-Yang-Mills system, compactified on a \( R \times S^3 \times S^d \) topology \[6\]. In particular, we analyse constraints coming from the measurement of the height and positions of the first and second peaks in the CMB spectrum.

The model is derived from the multidimensional Einstein-Yang-Mills-Inflaton system

\[
S = \frac{1}{16\pi k} \int_{M^D} d^Dx \sqrt{-g} \left[ R - 2\Lambda + \frac{1}{8e^2} \Tr F_{\mu\nu}F^{\mu\nu} - \frac{1}{2} (\partial_\mu \chi)^2 - U(\chi) \right] 
\]

(1)

where \( g \) is \( \det (g_{\mu\nu}) \), \( g_{\mu\nu} \) is the \( D \)-dimensional metric, \( R \), \( F_{\mu\nu} \equiv F^a_{\mu\nu} \tau_a \), \( e \), \( k \) and \( \Lambda \) are, respectively, the scalar curvature, gauge field strength (\( \tau^a \) being the generators of the gauge group that we assume to be \( \text{SO}(N) \), \( N \geq d + 3 \)), gauge coupling, gravitational and cosmological constants in \( D \) dimensions.

We have included a scalar field, the inflaton \( \chi \), with a potential \( U(\chi) \), responsible for the inflationary expansion of the external space and generation of the primordial energy density fluctuations.

After compactification on a \( R \times S^3 \times S^d \) topology and setting the relevant fields to their vacuum configurations, the equations relevant for the resulting cosmological model are the following \[6\]:

3
\[
\left( \frac{\dot{a}}{a} \right)^2 = -\frac{1}{4a^2} + \frac{8\pi k}{3} \left[ \frac{\psi^2}{2} + W(a, \psi) + \rho \right],
\]
with
\[
\ddot{\psi} + 3 \left( \frac{\dot{a}}{a} \right) \dot{\psi} + \frac{\partial W}{\partial \psi} = 0.
\]

Different values for the cosmological constant \( \Lambda \) correspond to different compactification scenarios (see Ref. [7] and, for a quantum mechanical analysis, also Ref. [8]). If \( \Lambda > c_2/16\pi k \), where \( c_2 = \left(\frac{(d+2)^2(d-1)/(d+4)}{64\pi k}\right)\frac{e^2}{16v_2} \), there are no compactifying solutions and for \( \frac{c_1}{16\pi k} < \Lambda < \frac{c_2}{16\pi k} \) with \( c_1 = d(d-1)e^2/16v_2 \), a compactifying solution exists which is classically stable but semiclassically unstable. Alternatively, if \( \Lambda < c_1/16\pi k \), the effective 4-dimensional cosmological constant, \( \Lambda^{(4)} = 8\pi kW(a \to \infty, \psi) \), is negative. Since \( \Lambda^{(4)} \) is required to satisfy the order of magnitude observational bound \( \Lambda^{(4)} \approx 10^{-120}/16\pi k \), we fine-tune the multidimensional cosmological constant as
\[
\Lambda = \frac{c_1(1 + \delta)}{16\pi k},
\]
so that \( \delta \) is clearly proportional to \( \Lambda^{(4)} \) and \( c_1 \) is determined by choosing \( \Lambda \) such that \( \psi = 0 \) corresponds to the absolute minimum of the potential in...
Eq. (4), where \( \langle b \rangle^2 = 16\pi kv^2/e^2 \). Hence

\[
\Lambda = \frac{d(d-1)}{16\langle b \rangle^2} (1 + \delta) .
\]

(6)

Substituting Eq. (6) into Eq. (4), yields, in the large \( a \) limit (implying that the radiation term can be neglected)

\[
W = \frac{d(d-1)}{128\pi k \langle b \rangle^2} \delta .
\]

(7)

Notice that, although a non-vanishing \( \delta \) induces a semiclassical instability in the compactification solution, the decompactification time exceeds the age of the Universe by many orders of magnitude.

The deceleration parameter at present can be computed differentiating Eq. (2) and substituting the resulting term in \( \ddot{\psi} \) by Eq. (3) \[6\]

\[
q_0 = -\delta_1 + \frac{\epsilon}{\frac{1}{4} + \delta_1 + \epsilon} ,
\]

(8)

where \( \delta_1 \equiv d(d-1)\alpha_0\delta_0/48 \), \( \alpha_0 \) being an order one constant defined by

\[
\left( \frac{\alpha}{\langle b \rangle} \right)^2 = \alpha_0 10^{120}, \quad \delta = \delta_0 10^{-120}
\]

and

\[
\epsilon \equiv \frac{8\pi k}{3}\rho_0 a_0^2 = \frac{3.2\pi}{3}\alpha_0 \Omega_m h^2 ,
\]

(9)

where \( 0.4 \lesssim h \lesssim 0.7 \) parametrizes the observational uncertainty in the Hubble constant, \( H_0 = 100 \, h \, km \, s^{-1} \, Mpc^{-1} \).

A bound on \( \delta_0 \) can be obtained from \( a_q \equiv a(t_q) = \alpha a_0 \), where \( t_q \) is the time when the vacuum contribution started dominating the dynamics of the Universe and \( \alpha \equiv \frac{a_q}{a_0} \) is a constant, equating the contributions of \( W \) and \( \rho(a_q) \) and using the observational bound \( \Omega_m \lesssim 0.3 \) \[9\]. Thus, we get:

\[
\alpha^3 \delta_0 \lesssim \frac{15.36 \pi}{d(d-1)} h^2 ,
\]

(10)
Since the red-shift of the supernovae data indicating the accelerated expansion of the Universe is \( z \geq 0.35 \), then \( \alpha \leq 0.74 \) and, for \( d = 7 \) and \( h = 0.5 \), we obtain

\[
\delta_0 \approx 0.71 ,
\]

which implies, for e.g. \( \delta_0 = 0.7, \alpha_0 = 5 \), that

\[
q_0 = -0.59 ,
\]

Note that we have corrected numerical values in Eqs. (9) and (10) of Ref. [6], but the main result, Eq. (12), remains within the most likely region of values for \( q_0 \), as revealed by observational data [2].

We show, in Figure 1, contours of \( q_0 \) in the two-dimensional parameter space \((\Omega_\Lambda, H_0)\), \( \Omega_\Lambda \) being the vacuum energy density, in which it varies most strongly. Indeed, we have checked that variation with \( \Omega_m \) and \( \alpha_0 \) is modest.

A distinct feature of the model is that, in spite of having a closed topology, a phase of accelerated expansion can take place [6]. We would like to stress that (slightly) closed models are actually favoured by recent BOOMERANG data. Very closed models work by increasing \( n_s \), the scalar spectral index, and \( r \), the gravity waves contribution [10].

In Figure 2, we show the CMB power spectra that correspond to the four sample CDM models that fit B98+COBE data. These are best-fit theoretical models defined by the value of six parameters using successively more restrictive “prior probabilities” on the parameters [1]. The parameters are: \((\Omega_{\text{tot}}, \omega_b, \omega_c, \Omega_\Lambda, n_s, \tau_C)\), where \( \omega_{b,c} = \Omega_{b,c} h^2 \) are the cosmological baryon and CDM densities, \( \tau_C \) is the optical depth to Thompson scattering from the epoch at which the Universe reionized to the present. Hence, \( \Omega_{\text{tot}} \equiv \Omega_b + \Omega_c + \Omega_\Lambda = 1 - \Omega_k \) where \( \Omega_k \) is the curvature density. Model 1 fixes these parameters.
at (1.3, 0.10, 0.80, 0.6, 0.80, 0.025), Model 2 at (1.15, 0.03, 0.17, 0.44, 0.925, 0), Model 3 at (1.05, 0.02, 0.06, 0.90, 0.825, 0) and Model 4 at (1.0, 0.03, 0.27, 0.60, 0.975, 0). For \( d = 7 \), these models fix parameter \( \delta_0 \) in our model as: 1.88, 0.26, 1.17 and 1.096, respectively. On the other hand, our EYM-motivated scenario predicts the corresponding \( q_0 \) values to be, respectively, \(-0.03, 0.13, -0.71, -0.23\), for \( \alpha_0 = 5 \); hence, model 2 would be excluded in this scenario, for reasonable values of \( \alpha_0 \).

Next, we study the dependence of \( q_0 \) on certain characteristic features of CMB spectra which are being increasingly constrained by CMB experiments, mainly the height of the first acoustic peak, \( A_1 \), its position, \( l_1 \) and the location of the second peak, \( l_2 \).

The height of the primary peak is controlled mainly by the baryon-to-photon ratio, varying as \( \Omega_b h^2 \), the dark matter-to-photon ratio, varying as \( \Omega_c h^2 \) and the cosmological constant \([11]\). In fact, \( \Omega_b h^2 \) and \( \Omega_c h^2 \) produce competing effects: when \( \Omega_b h^2 \) increases \( A_1 \) increases and when \( \Omega_c h^2 \) increases, \( A_1 \) decreases. The effect of varying \( \Lambda \), holding \( \Omega_b h^2 \) and \( h \) fixed, is that the largest values of \( \Lambda \) correspond to the largest Doppler peaks. On the other hand, as \( h \) increases, \( A_1 \) decreases and \( l_1 \) shifts to larger scales. Observations indicate that \( l_1 \simeq 200 \). The spectral index \( n_s \) also changes the amplitude of the peaks such that, as \( n_s \) increases, \( A_1 \) also increases. The low second peak found by BOOMERANG and MAXIMA-1 can be fit by either decreasing the tilt \( n_s \) or by increasing the baryon density compared to the usually assumed values \( n_s \approx 1, \omega_b \approx 0.02 \), although both of these solutions have problems of their own \([10]\).

The location of the second peak in the CMB power spectrum depends mainly, if the geometry is fixed, on the expansion rate of the Universe at the epoch of recombination \([12]\), and this depends on the nonrelativistic matter
density and the Hubble constant. Of course, the precise location of the second peak can change upon variations in several other parameters; however, it changes very little as each of these parameters is allowed to vary within its acceptable range \[13\]. We consider $\Omega_b h^2$ in the range advocated in Ref. \[14\], from measurements of the deuterium abundance, $\Omega_b h^2 = 0.019 \pm 0.001$. On the other hand, allowable variations in $n_s$ ($n_s = 0.99 \pm 0.010$ \[3\]) lead to even smaller uncertainties in the second-peak location than those from uncertainty in the baryon density.

Figure 3 shows contours of $l_2$, the multipole moment at which the second peak in the CMB power spectrum occurs, in the two-dimensional parameter space $(\Omega_m, H_0)$, in which it varies most strongly, for $\Omega_b h^2 = 0.019$, in two slightly closed models $\Omega_{\text{tot}} = 1.01, 1.05$ (B98 data with medium priors suggests $0.88 < \Omega_{\text{tot}} < 1.12$ at 95\% confidence level).

In Figure 4, we show contourplots of $q_0 = -0.25, -0.5, -0.75$ in the $(A_1, l_2)$ parameter space, for $\Omega_{\text{tot}} = 1.01, 1.05$ and $\Omega_b h^2 = 0.019$. Once these latter parameters are fixed, we see that our Einstein-Yang-Mills model can be greatly constrained by CMB data as knowledge of the values $(A_1, l_2)$ determines $q_0$. We find that, for $l_2 \approx 500$, our results do not vary significantly for different $\Omega_b h^2$ values. In turn, once the value of $q_0$ is known, the relevant parameters of our model, e.g. $\alpha_0$ and $\delta_0$, become fixed (see Figure 1). Furthermore, as expected, only for flat or slightly closed models the observational constraints on $A_1$ ($4500 \mu K^2 \lesssim A_1 \lesssim 5500 \mu K^2$) and $l_2$ ($450 \lesssim l_2 \lesssim 600$) can be satisfied; indeed, already for $\Omega_{\text{tot}} = 1.05$ the corresponding curves are broken, meaning that observational constraints cannot be met for the full range of $l_2, A_1$ values considered, and the situation worsens as $\Omega_{\text{tot}}$ increases.

We conclude that recent CMB data tightly constrain our Einstein-Yang-Mills-inspired cosmological model. The strongest prediction of the model is
the connection that can be inferred between the acceleration parameter and the height of the first peak together with the location of the second peak in the CMB spectrum. This connection can be further tested in the near future, as upcoming CMB data put tighter constraints on $A_1$, $l_2$ and $q_0$ becomes also more constrained via supernovae observations and other experiments (see [9] and references therein). In particular, we find that our model is consistent with available CMB data, favouring flat or very slightly closed models and $q_0 \simeq -0.5$.

We have used CMBFAST [15] to calculate the CMB power spectra.
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Fig. 1: Contours of $q_0$, from $-0.3$ (right) to $-0.7$ (left), at 0.1 intervals, in the two-dimensional parameter space $\Omega_\Lambda, H_0$ (in units km/sec/Mpc), for $d = 7, \Omega_m = 0.3$ and $\alpha_0 = 5$, in our compactified Einstein-Yang-Mills system model.
Fig. 2: CMB angular power spectra, $C_l = l(l+1)\langle |T_{lm}|^2 \rangle /(2\pi)$, where the $T_{lm}$ are the multipole moments of the CMB spectra. The three curves correspond, respectively, to Models 1, 2 and 3, defined by specific values of $\Omega_{tot}, \omega_b, \omega_c, \Omega_\Lambda, n_s, \tau_C$ (see text), considered to be good fits to B98+COBE data.
Fig. 3: Contours of $l_2$, the multipole moment at which the second peak in the CMB power spectrum occurs, in the two-dimensional parameter space $\Omega_m, H_0$ (in units km/sec/Mpc), for $\Omega_b h^2 = 0.019$, in two slightly closed models $\Omega_{tot} = 1.01$ (full curve), 1.05 (dashed).
Fig. 4: Contourplots of $q_0 = -0.25$ (bottom), $-0.5$ and $-0.75$ (top) in the $(l_2, A_1)$ parameter space, for $\Omega_{tot} = 1.01$ (full curve), 1.05 (dashed) and $\Omega_b h^2 = 0.019$ ($\alpha_0 = 5$).