Time Response of Arc Driven by Alternating Magnetic Field

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Abstract. The expanded arc has been developed by imposing an alternating magnetic field to the arc. The time response of the arc motion is investigated in the present work. The alternating magnetic field in the form of \( B = B_0 \cos(\omega t + \phi) \) is assumed to be imposed perpendicularly to the arc. Physical/mathematical models on the oscillatory motion of the plasma gas are described. Solving a set of three 3rd order linear ordinary differential equations, the trajectories of the plasma gas from the torch to the anode are obtained for various frequencies of the alternating magnetic field. Numerical analyses reveal that the amplitude of the oscillatory arc motion remains constant for small \( \omega \) but it reduces with the increase of the frequency for large \( \omega \). Theoretical predictions are compared with experimental results. It is confirmed that their agreements are good.

1. Introduction
Transferred arcs have been widely used in various industrial fields such as welding and cutting metals, because of the intense energy concentrated into a small arc-root. However, conventional arcs remain inconvenient for heating or melting over a wide area. Several attempts were made to expand arc profiles using an external magnetic field [1]. Takeda et al developed a magnetically driven arc by imposing an alternating magnetic field perpendicular to a transferred arc [2]. The high speed oscillating arc can be regarded as a broad heat source with the width of the oscillation amplitude as shown in figure 1.

![Figure 1. Typical arc profiles in various magnetic fields; (a) no magnetic field, (b) steady magnetic field and (c) alternating magnetic field.](image_url)

A magnetically driven arc presents the following advantages: the oscillation amplitude can be varied easily by adjusting the magnetic flux density and the space distribution of the heat flux can be controlled by varying the wave form of the alternating field. For example, if the waveform is rectangular, then the heat flows are localized at both ends of the oscillation. Arc profiles and heat flux distributions have been investigated under the alternating magnetic field imposed perpendicularly to
the arc [3-6]. Recently arc motions not only in the perpendicular magnetic field but also in the oblique field have been investigated [7]. However in these analyses, the magnetic field is restricted to a low frequency so that the plasma gas may travel in a quasi-static magnetic flux density during its flight from an arc torch to an anode. In the present work, we focus our attention to the time response of the arc motion. The variation of the arc profiles with the field frequency is investigated.

2. Theoretical consideration of the arc driven by alternating magnetic field

2.1. Description of the model

The schematic illustration of a magnetically driven arc is shown in figure 2. A transferred arc is generated between an arc torch and an anode plate. The origin of the coordinate system (x, y, z) used in the theoretical model is located at the torch exit. Plasma forming gas is introduced to the torch at a certain flow rate. And it is ionized in the torch. An alternating magnetic field is imposed parallel to x-axis. It is uniform in space but its flux density and direction vary with time at a certain frequency. The ionized gas (or partially ionized plasma) is ejected from the orifice of the torch nozzle and travels toward the anode.

The fundamental concept of the present modeling is based on the idea that the streamline of the plasma gas flow after the torch represents the arc profile. This is because electric current cannot exist in a space without plasma gas. Therefore the pass of the plasma gas gives the pass of arc current. The interaction between the arc current and the imposed magnetic field produces the electromagneto force, which drives the plasma in the y-direction. The authors consider a small element of the plasma ejected from the torch is considered for calculations; it is called “fluid parcel”. The fluid parcel travels from the torch to the anode under the influence of the electromagnetic force. In order to understand the trajectory of the fluid parcel, the momentum equation governing the parcel motion is analyzed by a Lagrangian method. Physical parameters and their nomenclature are listed in table 1.

The momentum equation for the fluid parcel is generally expressed as

\[ \rho \frac{d\vec{v}}{dt} = \vec{F} - \nabla p + \mu \Delta \vec{v} + \frac{\mu}{3} \nabla (\nabla \cdot \vec{v}) , \quad (1) \]

where \( \vec{F} \) represents the external force. To simplify the consideration, the following assumptions are used.

1. Pressure is constant, as the plasma moves in an open atmosphere.
2. Compressible and viscous effects are negligible.
3. External force consists of the electromagnetic force (\( \vec{F} = \vec{j} \times \vec{B} \)) only.
4. The arc current is anti-parallel to the motion of the plasma gas. The relation of
\[ j = -\alpha \dot{v} \] is satisfied, where \( \alpha \) is a positive proportional constant. This is because the arc current is along the plasma gas flow as mentioned above. However, it should be noticed that the direction of the arc current is opposite to that of plasma gas. The arc current runs from the anode to the torch, while the plasma gas moves from the torch to the anode.

The validity of these assumptions was confirmed experimentally in the previous work [5].

Table 1. List of main parameters used in the present model.

| symbol | nomenclature |
|--------|--------------|
| \( \tilde{B} \) | magnetic flux density, [T], \( \tilde{B} = (B_x, B_y, B_z) = (B_z, 0, 0) \) |
| \( B_x \) | x-component of \( \tilde{B} \), [T], \( B_x = B_x \cos(\omega t + \phi) \) |
| \( B_0 \) | amplitude of \( B_x \), [T] |
| \( I_a \) | arc current, [A] |
| \( Q_0 \) | mass flow rate of plasma forming gas, [kg/s] |
| \( T_{\text{field}} \) | characteristic changing time of the magnetic field, [s] |
| \( T_{\text{travel}} \) | characteristic traveling time of the parcel, [s] |
| \((X_a, Y_a, Z_a)\) | position of the arc root on the anode, [m] |
| \( f \) | frequency of the alternating magnetic field, [Hz] |
| \( f^* \) | critical frequency |
| \( j \) | arc current density, [A/m²], \( |j| = j \) |
| \( \dot{v} \) | velocity of the plasma gas, [m/s], \( \dot{v} = (v_x, v_y, v_z) = (0, v_y, v_z) \) |
| \( v_0 \) | initial velocity of the plasma gas at \( t=0 \), [m/s] |
| \( \alpha \) | proportional constant, [A s/m], \( \alpha = j/|\dot{v}| \) |
| \( \phi \) | phase shift, [rad] |
| \( \lambda \) | \( \lambda = I_a B_0 / Q_0 \), [A T s/kg] |
| \( \rho \) | density of the plasma gas, [kg/m³] |
| \( \omega \) | angular velocity of the alternating magnetic field, [rad/s], \( \omega = 2\pi / f \) |

Taking these assumptions into account, equation (1) is simplified as

\[ \rho \frac{d\dot{v}}{dt} = -\alpha (\dot{v} \times \tilde{B}). \] (2)

Considering that the imposed magnetic field has no y-component or z-component as \( \tilde{B} = (B_x, 0, 0) \), equation (1) expressed in vector form is decomposed into the following three equations in scalar form.

\[ \rho \frac{dv_x}{dt} = 0 \] (3)
\[ \rho \frac{dv_y}{dt} = -\alpha (v_y B_x) \] (4)
\[ \rho \frac{dv_z}{dt} = \alpha (v_z B_x) \] (5)

Differentiating equation (5) with respect to \( t \) and then substituting equation (4), the following equation is obtained.

\[ \frac{d^2 v_y}{dt^2} - \frac{1}{B_x} \frac{dB_x}{dt} \frac{dv_y}{dt} + \left( \frac{\alpha}{\rho} \right)^2 B_x^2 v_z = 0 \] (6)

In a similar manner, equation (7) is also obtained.

\[ \frac{d^2 v_z}{dt^2} - \frac{1}{B_x} \frac{dB_x}{dt} \frac{dv_z}{dt} + \left( \frac{\alpha}{\rho} \right)^2 B_x^2 v_y = 0 \] (7)

Then, substituting the following relations of equation (8) into equations (3), (6) and (7),
the governing equations for the movement of the fluid parcel are obtained;
\[
\frac{d^2 x}{dt^2} = 0,
\]
(9)
\[
\frac{d^3 y}{dt^3} - \frac{1}{B_z} \frac{dB_y}{dt} \frac{dy}{dt}^2 + \left( \frac{\alpha B_y}{\rho} \right) \frac{dy}{dt} = 0,
\]
(10)
and
\[
\frac{d^3 z}{dt^3} - \frac{1}{B_z} \frac{dB_z}{dt} \frac{dz}{dt}^2 + \left( \frac{\alpha B_z}{\rho} \right) \frac{dz}{dt} = 0.
\]
(11)
The imposed magnetic field is assumed as
\[B_z = B_0 \cos(\omega t + \phi) .\]
(12)
Then, equations (11) and (12) are written respectively as
\[
\frac{d^3 y}{dt^3} - \omega \tan(\omega t + \phi) \frac{dy}{dt}^2 + \lambda^2 \cos(\omega t + \phi) \frac{dy}{dt} = 0 ,
\]
(13)
and
\[
\frac{d^3 z}{dt^3} - \omega \tan(\omega t + \phi) \frac{dz}{dt}^2 + \lambda^2 \cos(\omega t + \phi) \frac{dz}{dt} = 0 .
\]
(14)
Where, \( \lambda \) is defined as
\[
\lambda = \frac{\alpha B_0}{\rho}.
\]
(15)
If the cross sectional area of the plasma flow tube (or the arc current) is represented as \( S \), the following relations are obtained; \( j S = I_a \) and \( \rho \frac{|v|}{S} = Q_0 . \) Substituting these relations and \( \alpha = j / |v| \) into equation (15), the new parameter \( \lambda \) can be expressed by familiar experimental parameters such as the arc current \( (I_a) \), the flow rate of plasma forming gas \( (Q_0) \) and the imposed magnetic flux density \( (B_0) \).
\[
\lambda = \frac{I_a B_0}{Q_0}.
\]
(16)

2.2. Initial conditions
In order to solve the differential equations represented by equations (9), (12) and (13), it is necessary to give the initial conditions for each equation. As the fluid parcel starts from the exit of the arc torch at \( t=0 \), the initial condition for the position is;
\[
(x, y, z) = (0,0,0) \text{ at } t = 0.
\]
(17)
And as the plasma gas is ejected vertically from the torch exit, the initial condition for the velocity is;
\[
\left( \frac{dx}{dt} \frac{dy}{dt} \frac{dz}{dt} \right) = (v_x, v_y, v_z) = (0,0,0) \text{ at } t = 0.
\]
(18)
Substituting the above conditions to equations (4) and (5), the third initial condition is obtained.
\[
\left( \frac{d^2 x}{dt^2} \frac{d^2 y}{dt^2} \frac{d^2 z}{dt^2} \right) = \left( \frac{dv_x}{dt} \frac{dv_y}{dt} \frac{dv_z}{dt} \right) = (0,-\lambda v_y,0), \text{ at } t = 0.
\]
(19)

2.3. Determination of the initial velocity
The plasma forming gas goes into the arc torch at the flow rate of \( Q_0 \) and is ionized there. After drastic expansion of its volume, it is ejected from the torch. Experimentally it is not easy to measure the gas velocity. Theoretically, it is also difficult to predict its value, since the velocity depends on the torch configuration, the magnitude of the arc current and many other experimental parameters.
Apart from the magnetically driven arc, the movement of electrons under a stationary and uniform magnetic field is considered. The momentum equation for the electron is expressed in a similar form to equation (2). Suppose that the magnetic flux density is presented as \( \vec{B} = (B_z, B_y, B_x) = (B_0,0,0) \) and the initial velocity of the electron at \( (x,y,z) = (0,0,0) \) is denoted as \( \vec{v} = (v_x,v_y,v_z) = (0,0,v_0) \). It is known that
the trajectory of the electron is a circle with a Larmor radius described as \( R_L = \frac{mv_0}{qB_0} \), where \( m \) and \( q \) represent mass of electron and its charge respectively.

It is apparent that the fluid parcel in a steady and uniform magnetic field also travels along a circle with a radius \( R \) and its position \((X, Y, Z)\) is expressed in the similar manner to the electron as;

\[
Y = -R + \left( R^2 - Z^2 \right)^{\frac{1}{2}}, \quad (20)
\]

where \( R \) is defined as

\[
R = \frac{v_0}{\lambda} = \frac{Q_0v_0}{I_aB_0}. \quad (21)
\]

If an arc is generated under a stationary magnetic field in the arrangement shown in Figure 2, the position on the anode, \((X_a, Y_a, Z_a)\) satisfies the following relations;

\[
X_a = 0, \quad (22)
\]

and

\[
Y_a = -\frac{Q_0v_0}{I_aB_0} + \left\{ \frac{Q_0v_0}{I_aB_0} - Z_a \right\}^{\frac{1}{2}}. \quad (23)
\]

Rearranging equation (23), the initial velocity is represented as

\[
v_0 = \frac{I_aB_0}{Q_0} \frac{Y_a^2 + Z_a^2}{2Y_a}. \quad (24)
\]

Therefore if the position of the arc root on the anode can be measured by experiment in the conditions of \( I_a, Q_0 \) and \( B_0 \), the unknown value \( v_0 \) can be evaluated by these experimental parameters.

2.4. Critical frequency

Then, we consider how much change of the magnetic flux density the fluid parcel experiences during the travel from the torch to the anode. As the alternating magnetic field changes with time in the form of \( \cos(\omega t) \), the characteristic time for the magnetic field change \( (T_{field}) \) can be estimated as

\[
T_{field} \approx \frac{1}{\omega} = \frac{1}{2\pi f}. \quad (25)
\]

Where, \( f \) represents the frequency of the alternating field. On the other hand, the characteristic time for the fluid parcel to travel from the torch to the anode \( (T_{travel}) \) is approximated as

\[
T_{travel} \approx \left| \frac{Z_a}{v_0} \right|. \quad (26)
\]

Here, we define the critical frequency \( f^* \) as

\[
f^* = \frac{1}{2\pi} \left| \frac{V_0}{Z_a} \right|. \quad (27)
\]

If the frequency of the alternating magnetic field is less than the critical frequency \( (f < f^*) \), then the characteristic time for the magnetic field change is larger than that for the fluid parcel to travel \( (T_{field} > T_{travel}) \). In this case, the fluid parcel travels during its flight under almost constant magnetic field. The frequency change of the magnetic field has small effect on the trajectory of the parcel. The amplitude of the oscillatory arc motion is maintained constant in the frequency range from 0 to \( f^* \). On the contrary, if the frequency of the magnetic field is larger than \( f^* \), the parcel experiences different electromagnetic forces from place to place. In such conditions, the trajectory of the parcel becomes rather complicated.

3. Numerical results

Numerical calculations were conducted. Commercial software (Mathematica) [8] was used to solve the set of differential equations presented by equations (9), (13) and (14) under the initial conditions of equations (17), (18) and (19). The fluid parcel starts from the torch exit at a time \( t=0 \) and travels toward the anode located at \( z=Z_a \). It reaches there at a time \( t=T_{travel} \). At \( t=0 \), the magnetic flux density is \( B=B_0\cos\phi \). The location of the parcel at time \( t \) is given as the set of \( x(t), y(t) \) and \( z(t) \). The trajectory of the parcel is described by the variation of \((x, y, z)\) with time \((0 \leq t \leq T_{travel}) \). The initial velocity of the
fluid parcel is assumed $v_0 = -60 \text{ [m/s]}$. The anode plate locates at $Z_a = 6.0 \times 10^{-2} \text{ [m]}$. The magnitude of $\lambda$ is assumed 800 $\text{[s^{-1}]}$. Under these conditions, the critical frequency, $f^*$ is estimated to be 160 $\text{[Hz]}$.

The trajectories for various frequencies are shown in figure 3, where $\phi$ is fixed at zero. As mentioned in 2.4, for $f < f^*$ the profile of the trajectory is almost the same as that calculated for $f=0$, and for $f > f^*$ its profile varies with the change of the frequency.

![Figure 3](image1.png)

**Figure 3.** Variation of the trajectories with the frequency under the condition of $\phi=0$. The abscissa and the ordinate in each figure represent respectively y-position and z-position in $\text{[m]}$.

The trajectories with various $\phi$ at $f=10\text{Hz}$ are illustrated in figures 4 (a)-(k). Whole profile of the oscillatory motion is depicted in figure 4-(l). As the frequency is much smaller than $f^*$, the fluid parcel travels from the torch exit to the anode in the quasi-stationary magnetic field. The magnitude of the stationary field strength is approximated as $B_0 \cos \phi$. Uniform and steady electro-magnetic force acts on the fluid parcel during the flight and therefore the curvature of the trajectory does not change. Because of the periodic property of the alternating field, the profiles of the trajectory are repeated at the interval of $\phi = 2\pi$.

![Figure 4](image2.png)

**Figure 4.** Variations of the trajectory with $\phi$ (shown in (a) – (k)) and the whole oscillating movement (shown in (l)); the frequency of the imposed magnetic field is at $f=10\text{Hz}$.
And as shown in figure 5, similar calculations are carried out for \( f = 500 \text{Hz} \). In this case, the change of the magnetic field is so fast that the direction of the electro-magnetic force may vary not only in its magnitude but also its direction during the flight of the parcel. The curvature of the trajectory changes from place to place.

![Figure 5](image_url)  
**Figure 5.** Variations of the trajectory with \( \phi \) (shown in (a) – (k)) and the whole oscillating movement (shown in (l)); the frequency of the imposed magnetic field is \( f = 500 \text{Hz} \).

The variation of the amplitude in the oscillatory motion with frequency is depicted in figure 6. The solid line represents the numerical result and the critical frequency, \( f^* \) is also shown in this figure.

![Figure 6](image_url)  
**Figure 6.** Variation of the amplitude in the oscillatory motion with frequency. Critical frequency is represented by \( f^* \).

4. Experiment

4.1. Experimental procedure
Experiments were carried out to assess the theoretical results. Experimental arrangement is illustrated in figure 7. A tungsten cathode in the arc torch and an anode copper tube were connected to a dc power
supply which was operated in a constant current mode. An arc current was fixed at $I_a = 100$ [A]. Argon gas was fed into the arc torch at the flow rate of $Q_o = 1.8 \times 10^{-4}$ [kg/s]. Standoff distance between the torch exit and the anode was adjusted to be $Z_a = 6 \times 10^{-2}$ [m]. A magnetic field was generated by a water cooled rectangular coil through which electric currents were supplied with various frequencies ($f = 0 - 1000$ [Hz]). All the parameters used in the experiment were adjusted to be same as those in the numerical analyses.

4.2. Determination of the initial velocity of the plasma gas
In order to determine the initial velocity of the plasma gas ejected from the torch, the arc was produced under the stationary magnetic field at $B_o = 1.8 \times 10^{-3}$ [T]. Other operating conditions were same as those described in the subsection 4.1. The arc profile obtained under these conditions is shown in figure 8. From the measurement of the anode root position, $Y_o = -4.4 \times 10^{-2}$ [m] was obtained. Substituting $Y_o$ and other known experimental parameters into equation (24), the initial velocity of the fluid parcel (or plasma gas) was evaluated as $v_o = -60$ [m].

4.3. Experimental results
Then the arc was driven by alternating magnetic fields with various frequencies. The magnitude of the magnetic flux density was adjusted at $B_o = 1.4 \times 10^{-3}$ [T]. The variations of the arc profiles with the frequency are shown in figure 9. As was predicted in the theoretical consideration, the arc oscillated in

![Figure 7. Schematic illustration of the experimental arrangement.](image1)
![Figure 8. Deformation of arc by dc magnetic field. Experimental conditions; $B_o=1.8x10^{-3}$ [T], $Z_a=6x10^{-2}$[m], $Q_o=1.8x10^{-4}$[kg/s] and $I_a=100$[A].](image2)
the Y-Z plane two dimensionally. The reduction of the oscillation amplitude was observed with the increase of the frequency.

The amplitudes of the oscillatory motion were measured for various frequencies of the magnetic field. In figure 10, the experimental results on the variation of the amplitude were compared with theoretical ones. Closed circles represent the experimental results and a solid curve depicts the theoretical variation. In spite of various assumptions in the modeling, experimental results are in fairly good agreement with numerical predictions. The critical frequency observed experimentally is in reasonable agreement with that obtained theoretically.

![Figure 9. Variation of the arc profiles with the frequency of the alternating magnetic field.](image)

![Figure 10. Comparison of the amplitude variation with frequency between theoretical results and experimental measurements.](image)

5. Conclusion
The time response of the arc motion driven by alternating magnetic field was investigated. A theoretical model was constructed on the idea that the trajectory of the plasma gas from the arc torch to the anode represented the arc profile. Numerical analyses revealed that the amplitude of the oscillatory arc motion decreased with the increase of the alternating frequency. The critical frequency \( f^* \) was defined to characterize the amplitude reduction. Theoretical consideration predicted that the variation of the amplitude was small if \( f < f^* \) and then it became large if \( f > f^* \). Experiment was conducted to assess the theoretical results. Good agreement was obtained between theoretical predictions and experimental observations.
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