Vacuum stress around a topological defect
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We show that a dispiration (a disclination plus a screw dislocation) polarizes the vacuum of a scalar field giving rise to an energy momentum tensor which, as seen from a local inertial frame, presents non-vanishing off-diagonal components. The results may have applications in cosmology (chiral cosmic strings) and condensed matter physics (materials with linear defects).

It is fairly well known that a needle solenoid carrying a magnetic flux makes virtual charged particles to run around the solenoid inducing a non-vanishing current density (see e.g. Ref. [1]). We wish to consider what seems to be a gravitational (geometric) analogue of this Aharonov-Bohm effect, by computing the vacuum expectation value of the energy momentum tensor of a massless and neutral scalar field far away from a dispiration.

Let us begin by presenting the geometry of the background (units are such that $c = \hbar = 1$),

$$ds^2 = dt^2 - dr^2 - \alpha^2 r^2 d\theta^2 - (dz + \kappa d\theta)^2,$$  \hspace{1cm} (1)

where the points labeled by $(t, r, \theta, z)$ and $(t, r, \theta + 2\pi, z)$ are identified [2,3]. When $\alpha = 1$ and $\kappa = 0$ Eq (1) becomes the line element of the flat space-time written in cylindrical coordinates. Borrowing terminologies in condensed matter physics, the parameters $\alpha$ and $\kappa$ correspond to a disclination and a screw dislocation, respectively. We should remark that Eq (1) may be associated with the gravitational background of certain chiral cosmic strings [4] (as has been suggested in Ref. [2]), as well as can describe (in the continuum limit) the effective geometry around a dispiration in an elastic solid (see Ref. [5] and references therein).

The definitions $\varphi := \alpha \theta$ and $Z := z + \kappa \theta$ lead to

$$ds^2 = dt^2 - dr^2 - r^2 d\varphi^2 - dZ^2,$$  \hspace{1cm} (2)

which should be considered together with the peculiar identification

$$(t, r, \varphi, Z) \sim (t, r, \varphi + 2\pi \alpha, Z + 2\pi \kappa).$$  \hspace{1cm} (3)

Although Eq. (2) expresses the fact that the background is locally flat, due to Eq. (3) we cannot use Eq. (2) (which is a local statement) to infer that the global symmetries of the background are the same as those of the Minkowski space-time (in this sense Eq. (2) is singular). In fact, Eq. (2) disguises a curvature singularity on the symmetry axis [2] (when $\kappa \neq 0$, in the context of the Einstein-Cartan theory, there is also a torsion singularity at $r = 0$ [3,4]).

The vacuum expectation value of the energy momentum tensor is obtained by applying a differential operator to the renormalized scalar propagator around a dispiration (see e.g. Ref. [7]),

$$\langle T_{\mu\nu} \rangle = i \lim_{x' \rightarrow x} D_{\mu,\nu}(x, x') \ D^{(\alpha,\kappa)}(x, x').$$  \hspace{1cm} (4)

We have recently obtained $D^{(\alpha,\kappa)}(x, x')$ (classical propagators have been considered in Ref [8]) by using the Schwinger proper time prescription combined with the completeness relation of the eigenfunctions of the d’Alembertian operator [9]. Such eigenfunctions have the form $R(r) \chi(\varphi) \exp\{i(\nu Z - \omega t)\}$ which, by observing Eq. (2), leads to

$$\chi(\varphi + 2\pi \alpha) = e^{-i2\pi \nu \kappa} \chi(\varphi).$$  \hspace{1cm} (5)

This boundary condition is typical of the Aharonov-Bohm set up where $\nu \kappa$ is identified.
with the flux parameter $e\Phi/2\pi$. If we carry over to the four-dimensional context lessons from gravity in three dimensions \[10,11\], it follows that the charge $e$ and the magnetic flux $\Phi$ should be identified with the longitudinal linear momentum $\nu$ and $2\pi\kappa$, respectively \[2\].

When $\kappa/r \to 0$, Eq. (4) yields for the diagonal components the expressions of the vacuum fluctuations around an ordinary cosmic string ($\kappa = 0$) \[12\]. Regarding the other components, the prescription in Eq. (4) kills off the dominant contribution in the renormalized propagator \[9\], resulting that the subleading contribution yields two non vanishing off-diagonal components,

$$
\langle T^e Z \rangle = \frac{i}{r^4} \lim_{x' \to x} \partial_{x'Z} D^{(\alpha \kappa)}(x,x') = \frac{\kappa}{r^6} B(\alpha), \tag{6}
$$

and

$$
\langle T^Z \varphi \rangle = \frac{\kappa}{r^4} B(\alpha), \tag{7}
$$

where $B(\alpha)$ depends on the disclination parameter only \[9\]. Unlike the diagonal components, $\langle T^e Z \rangle$ and $\langle T^Z \varphi \rangle$ do not depend on the coupling parameter $\xi$.

When $\alpha = 1$, $B = 1/60\pi^2$ which corresponds approximately to the value of $\alpha$ in the physics of formation of ordinary cosmic strings \[13\].

It is instructive to display both disclination and screw dislocation effects in a same array. When $\xi = 1/6$ (conformal coupling), for example, $\langle T^\mu \nu \rangle$ with respect to the local inertial frame [cf. Eq. \[2\]] can be cast into the form

$$
\langle T^\mu \nu \rangle = \frac{1}{r^4} \begin{pmatrix}
-A & 0 & 0 & 0 \\
0 & -A & 0 & 0 \\
0 & 3A & \kappa B/r^2 & 0 \\
0 & \kappa B & -A & 0
\end{pmatrix}, \tag{8}
$$

where $A(\alpha) := (\alpha^{-4} - 1)/1440\pi^2$, and which holds far away from the defect (and for $\alpha \neq 1$, when $\kappa \neq 0$). [When $\kappa \neq 0$, by setting $\alpha = 1$ in Eq. \[8\], $A$ vanishes and subleading contributions depending on $\kappa$ take over.]

Before closing we should report a discrepancy in the literature. Recently, works \[14,15\] have appeared stating that, for $\alpha = 1$, the subleading contribution in $\langle T^0 0 \rangle$ [cf. Eq. \[8\]] is $\lambda \kappa^2/r^6$, where $\lambda$ is a certain constant. We have performed a rather detailed study showing that such a subleading contribution is $f(\kappa/r) \kappa^2/r^6$ instead, where $f(\kappa/r)$ diverges as $\kappa \to 0$ [although $\lim_{\kappa \to 0} \kappa^2 f(\kappa/r) = 0$].

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