Mixture Modeling of Exponentiated Pareto Distribution in Bayesian Framework With Applications of Wind-Speed and Tensile Strength of Carbon Fiber

AMMARA NAWAZ CHEEMA, MUHAMMAD ASLAM, IBRAHIM M. ALMANJAHIE, AND ISHFAQ AHMAD

1Department of Mathematics, Air University Islamabad, Islamabad 44000, Pakistan
2Department of Mathematics and Statistics, Riphah International University, Islamabad 44000, Pakistan
3Department of Mathematics, College of Science, King Khalid University, Abha 62529, Saudi Arabia
4Statistical Research and Studies Support Unit, King Khalid University, Abha 62529, Saudi Arabia
5Department of Mathematics and Statistics, Faculty of Basic and Applied Sciences, International Islamic University, Islamabad 44000, Pakistan

Corresponding author: Ammara Nawaz Cheema (ammara.au@gmail.com)

This work was supported in part by the Deanship of Scientific Research at King Khalid University, Saudi Arabia, through the Research Groups Program, under Grant RGP-2/67/41.

ABSTRACT Mixture modelling has stunning applications to explain the composite problems in simple way. Bayesian demonstration of 3-Component mixture model of Exponentiated Pareto distribution in right-type-I censoring scheme is presented in this article. The posterior densities of the parameter(s) are attained supposing the non-informative (uniform, Jeffreys) priors. The symmetric and asymmetric Loss Functions (Squared Error, Precautionary, Quadratic and DeGroot Loss Function) are assumed to get the Bayes estimator(s) and posterior risk(s). The presentation of the Bayes estimator(s) over posterior risk(s) in the studied loss functions is examined over simulation practice. Two real-data sets, wind speed and tensile strength of carbon fiber, are also analyzed for mixture to complete the performance of Bayes estimator(s). To enhance the study, the limiting forms are also derived for Bayes estimator(s) and posterior risk(s). The results reveal that for the component parameter(s), the Bayes Estimator(s) have their risks accordingly: DeGroot Loss Function < Precautionary Loss Function < Squared Error Loss Function < Quadratic Loss Function, and whereas for the proportion parameter(s) these are classified as: Squared Error Loss Function < Precautionary Loss Function < DeGroot Loss Function < Quadratic Loss Function. Therefore, in this study, DeGroot Loss Function performs efficient and the most preferable non-informative prior is the Jeffereys prior for estimation of 3-Component mixture of Pareto distribution.

INDEX TERMS Bayes estimators, exponentiated Pareto distribution, limiting expressions, loss functions, non-informative priors, posterior risks, real data, simulation.

I. INTRODUCTION

The Pareto distribution, is the power law probability density and extensively used in geographical, social, actuarial, scientific and many other areas. To model the earthquakes and forest fire areas [1] used Pareto distribution. Reference [2] discussed applications of Pareto distribution to estimate the model for disk drive errors. Pareto distribution extended as beta Pareto studied by [3]. Kuaraawamy Pareto explored by [4], beta exponentiated Pareto introduced by [5], Gamma Pareto presented by [6]. Reference [7] introduced an exponentiated Weibull Pareto distribution and discussed its several characteristics comprising reliability and hazard function. Reference [8], used exponentiated Weibull distribution to model the bathtub-data. Reference [9] studied Weibull Pareto distribution with its applications. Reference [10] suggested new Weibull Pareto distribution. Reference [11] explored exponentiated Pareto distribution. Reference [12] studied the behavior of different methods of parameters estimation of exponentiated Pareto distribution. Reference [13] explored...
mixture of exponentiated Pareto and exponential model under type-II censoring scheme.

Mixture modelling is widely used in many situations, particularly whenever we have more than one sub population. The number of component in a mixture modeling is in line for to heterogonous property of the parental population. It is therefore limited to be finite, while in several cases the elements may be infinite. As compare to simple composition, it delivers more appropriate explanation of many analytical frame works. Mixture modeling has extensive uses in survival analysis. The 3-Component mixture is important than 2-Component mixture in the context to fix and determine the fault of more than 2-Components. Already in literature Bayesian mixture work on many distributions is done [see: 14-16]. Recently, [17] introduced 3-Component mixture model of exponentiated Weibull distribution among Bayesian paradigm. Moreover, [18] discussed 3-Component mixture for Pareto distribution by using right type-I censoring pattern. Reference [19] explored 3-Component scheme under Bayesian approach for exponentiated inverted Weibull distribution. We considered 3-Component mixture modelling of exponentiated Pareto distribution in right type-I censoring procedure using non-informative priors in this study.

Encouraged by the above stated researches of 3-Component mixture, we examine the Bayesian analysis mixture of 3-Component model of EPD in this article. The key attention of this study is to list the competent Bayes Estimators (BEs) of Component and Proportion parameter(s). For the sack, two asymmetric and two symmetric LFs are used with (Non-Informative Priors) NIPs, UP (Uniform Prior) and JP (Jeffreys Prior) to achieve such results. The estimator(s) are explored under the type-I right censoring procedure. The study algorithm is illustrated in the following FIGURE 1.

The remaining study is intended as bellows. The 3-Component mixture model of EPD is designed in the next section, with the suggested BEs of several parameter(s) assuming different LFs among the limiting expressions. Simulation and Real data applications are elaborated in section III. To finish in section IV conclusions are provided.

II. MATERIAL AND METHODS

A. THE 3-COMPONENT MIXTURE OF EPD

For a random variable $X$, the pdf (probability density function) for the shape parameter with the cdf (cumulative distribution function) of EPD can be illustrated as:

$$f(x, \alpha_i) = \frac{\alpha_i x^{\alpha_i - 1}}{(1 + x)^{\alpha_i + 1}}, \text{ where } x \geq 0, \text{ and } \alpha_i > 0, i = 1, \ldots, 3.$$  \hspace{1cm} (1)

$$F(x, \alpha_i) = \left\{1 - (1 + x)^{-\alpha_i}\right\}^{\alpha_i}, \text{ where } x \geq 0, \text{ and } \alpha_i > 0, i = 1, \ldots, 3.$$  \hspace{1cm} (2)

Here for EPD the shape parameter is denoted by $\alpha_i$.

With the $w_1$ and $w_2$ mixing fraction, a defined 3-Component mixture model can be stated as:

$$f(x) = f_1(x)w_1 + f_2(x)w_2 + f_3(x)(1 - w_1 - w_2),$$  \hspace{1cm} where $w_1, w_2 \geq 0, \text{ and } w_1 + w_2 \leq 1$$

$$f(x; \varphi) = w_1 \left(\frac{\alpha_1 x^{\alpha_1 - 1}}{(1 + x)^{\alpha_1 + 1}}\right)^{\alpha_1} + w_2 \left(\frac{\alpha_2 x^{\alpha_2 - 1}}{(1 + x)^{\alpha_2 + 1}}\right)^{\alpha_2} + (1 - w_1 - w_2) \left(\frac{\alpha_3 x^{\alpha_3 - 1}}{(1 + x)^{\alpha_3 + 1}}\right)^{\alpha_3}$$

For mixing proportion parameter(s) and some component points, a 3-Component mixture of the EPD is displayed in the following FIGURE 2.

For 3-Component mixture the cdf is stated as:

$$F(x) = F_1(x)w_1 + F_2(x)w_2 + F_3(x)(1 - w_1 - w_2)$$

$$F(x; \varphi) = w_1 \left\{1 - (1 + x)^{-\alpha_1}\right\}^{\alpha_1} + w_2 \left\{1 - (1 + x)^{-\alpha_2}\right\}^{\alpha_2} + (1 - w_1 - w_2) \left\{1 - (1 + x)^{-\alpha_3}\right\}^{\alpha_3}$$

B. THE POSTERIOR DISTRIBUTION USING THE NON-INFORMATIVE PRIORS

The prior evidence has main part to differentiate the Bayesian and Classical study. The prior is defined as the characterization of the uncertainty about the parameter, existing to the prior knowledge. Prior distribution categorized into two
classes as non-informative and informative. A prior distribution differentiated as NIP, if it is even relative to likelihood function. Whereas, an IP (Informative Prior) is distinct as a P (prior) which has a link concerning the posterior distribution and is not the result by the probability function. In this segment, posterior distributions through the likelihood are estimated under the NIPs.

Assuming that from the 3-Component mixture modelling of EPD n are obsessive in a life testing process with fixed (test termination time) t. Assume that the designated outcome exposed that n components from r failed till t fixed and the n-r which are remaining units are still at in running stage. Here important to note that because of the failures, from r, r1 categorized as related to subpopulation-I, r2 belongs to subpopulation-II and r3 are connected to subpopulation-III. The total uncensored sample elements are categorized as r = r1, r2 and r3. Whereas, the rest of the n-r sample points are consider as censored. Now we have distinct the time-to-failure, of the ith-unit connecting to lth sub-population as xoi, 0 < Xoi ≤ t, where l = 1, ..., 3; and i = 1, ..., r1.

The likelihood of the 3-Component model can be defined as:

\[
L(\Phi | x) \propto (1 - F(t))^{n-r} \left\{ \prod_{i=1}^{r1} w_{f1}(x_{i1}) \right\} \left\{ \prod_{i=1}^{r2} w_{f2}(x_{i2}) \right\} \times \left\{ \prod_{i=1}^{r3} (1 - w_{f1} - w_{f2}) f_3(x_{i3}) \right\}
\]

After solving the above term, we get likelihood of 3-Component mixture of EPD as:

\[
L(\Phi | x) \propto S e^{-\alpha_1 B_{11}} e^{-\alpha_2 B_{21}} e^{-\alpha_3 B_{31}} \left( \prod_{i=1}^{r1} w_{f1}(x_{i1}) \right) \left( \prod_{i=1}^{r2} w_{f2}(x_{i2}) \right) \times \left( \prod_{i=1}^{r3} (1 - w_{f1} - w_{f2}) f_3(x_{i3}) \right)
\]

where

\[
A_{11} = 1 + r_1, \quad A_{31} = 1 + r_3, \quad A_{21} = 1 + r_2,
\]

\[
B_{11} = \sum_{k=1}^{r1} \ln (1 + x_{1k}) - \sum_{k=1}^{r1} \ln x_{1k} - (i-j) \ln \left\{ 1 - (1+t)^{-1} \right\},
\]

\[
B_{21} = \sum_{k=1}^{r2} \ln (1 + x_{2k}) - \sum_{k=1}^{r2} \ln x_{2k} - (i-j) \ln \left\{ 1 - (1+t)^{-1} \right\},
\]

\[
B_{31} = \sum_{k=1}^{r3} \ln (1 + x_{3k}) - \sum_{k=1}^{r3} \ln x_{3k} - \ln \left\{ 1 - (1+t)^{-1} \right\},
\]

\[
A_{01} = r_1 + i - j + 1, \quad B_{01} = r_2 + j - l + 1,
\]

\[
C_{01} = r_3 + l + 1,
\]

\[
w_3 = 1 - w_1 - w_2.
\]

\[
S = \sum_{i=0}^{n-r} \sum_{j=0}^{i} \sum_{k=0}^{j} (-1)^{n-r} \left( \begin{array}{c}
  n-r \\
  i
\end{array} \right) \left( \begin{array}{c}
  i \\
  j
\end{array} \right)
\]

and \(x = (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33})\) for the uncensored values are the failure time and \(\Phi = (\alpha_1, \alpha_2, \alpha_3, w_1, w_2)\).

The widely used NIPS includes UP and JP. Most of the researchers cited that UP is the foremost studied prior, for the assessment, of unknown parameter(s) of concern (see [20]–[22]). The improper UP is considered for the component parameter(s) of EPD as:

\[
\pi_1(\Phi) = 1; \quad \alpha_1, \alpha_2 \text{ and } \alpha_3 > 0; \quad w_1 \text{ and } w_2 \geq 0; \quad w_1 + w_2 \leq 1
\]

So, for the UP the joint posterior density of parameter(s) \(\alpha_1, \alpha_2, \alpha_3, w_1, w_2\) can be written as:

\[
g_1(\Phi | x) = \Omega_1^{-1} S \left( \alpha_{11}^{-1} \alpha_{21}^{-1} \alpha_{31}^{-1} e^{-\alpha_1 B_{11}} e^{-\alpha_2 B_{21}} e^{-\alpha_3 B_{31}} \right) \times w_{A_{01}}^{-1} w_{B_0}^{-1} w_{C_{01}}^{-1}
\]

where,

\[
\Omega_1 = \Gamma (A_{31}) \Gamma (A_{21}) \Gamma (A_{11}) \sum_{i=0}^{n-r} \sum_{j=0}^{i} \sum_{k=0}^{j} (-1)^{n-r} \times \left( \begin{array}{c}
  n-r \\
  i
\end{array} \right) \left( \begin{array}{c}
  i \\
  j
\end{array} \right) \left( \begin{array}{c}
  j \\
  k
\end{array} \right)
\]

and \(B (A_{11}, B_{01}, C_{01})\) is the Beta function and is expended as \(B (B_{01}, C_{01})\). The Jeffreys suggested a rule of thumb to demonstrate NIP for parameter \(\theta\) as: If \(\theta [a, b]\) then \(g (\theta) = constant\) and if \(\theta [0, \infty]\) then \(g (\theta) = \frac{1}{\theta}\), under transformation of the variables (see [23]–[26]). While, for proportion parameter(s) \(w_1\) and \(w_2\) the JP is classified as \(w_1 \sim U (0, 1)\) and \(w_2 \sim U (0, 1)\). The joint prior density of the parameter(s), under the theory of independence of the studied parameter(s) \(\alpha_1, \alpha_2, \alpha_3, w_1\) and \(w_2\) is specified as:

\[
\pi_2(\Phi) \propto \frac{1}{\alpha_1 \alpha_2 \alpha_3}, \quad \alpha_1 \alpha_2 \alpha_3 > 0, \quad w_1, \quad w_2 \geq 0, \quad w_1 + w_2 \leq 1
\]
Then the joint posterior distribution turns out to be:

\[ g_2(\Phi|x) = \Omega_{2}^{-1} \times \alpha_1^{A_{12} - 1} \alpha_2^{A_{22} - 1} \alpha_3^{A_{32} - 1} e^{-\alpha_1 B_{12} - \alpha_2 B_{22} - \alpha_3 B_{32}} \times w_1^{B_{02} - 1} w_2^{B_{02} - 1} w_3^{C_{02} - 1} \]  

(10)

where,

\[ A_{12} = -2 + r_1, \quad A_{22} = -2 + r_2, \quad A_{32} = -2 + r_3, \]

\[ B_{12} = \sum_{k=1}^{r_1} \ln(1 + x_{1k}) - \sum_{k=1}^{r_2} \ln(x_{1k} - (i - j) \ln \{1 - (1 + t)^{-1}\}, \]

\[ B_{22} = \sum_{k=1}^{r_2} \ln(1 + x_{2k}) - \sum_{k=1}^{r_3} \ln(x_{2k} - (i - j) \ln \{1 - (1 + r)^{-1}\}, \]

\[ B_{32} = \sum_{k=1}^{r_3} \ln(1 + x_{3k}) - \sum_{k=1}^{r_1} \ln(x_{3k} - l \ln \{1 - (1 + t)^{-1}\}, \]

\[ A_{02} = r_1 + i + j, \quad B_{02} = r_2 + 1 + j - l, \quad C_{02} = r_3 + 1 + l, \]

\[ w_1 = 1 - w_1 - w_2, \]

\[ \Omega_2 = \Gamma(A_{32}) \Gamma(A_{22}) \Gamma(A_{12}) \sum_{i=0}^{n-r} \sum_{j=0}^{r_2} \sum_{l=0}^{r_3} (-1)^{n-r} \]

\[ \times \binom{n-r}{i} \binom{i}{j} \binom{j}{l} \times B(A_{02}, B_{02}, C_{02}) B_{12}^{A_{12}} B_{22}^{A_{22}} B_{32}^{A_{32}} \]  

(11)

The graphs of marginal posterior distribution of component parameter(s) are depicted in the following FIGURES 3(a-c).

FIGURES 3(a-c), represent graphs of the marginal posterior densities of component parameter(s) assuming the NIP for the 3-component mixture of EPD. From FIGURES 3(a-c) it is observed that, the graphs of marginal posterior densities for the shape parameter of the mixture of EPD for component parameter(s) indicates symmetrical pattern with slight variation for both t. The graph of the first component tends to be more peaked than the second and third component(s).

C. BAYES ESTIMATORS AND POSTERIOR RISKS UNDER LOSS FUNCTIONS

The existent valued function which represents estimator loss among the particular value of parameter is identified as LF (Loss Function). The present unit deliberated, BEs and PRs over four different symmetric and asymmetric LFs, i.e.; Squared Error Loss Function (SELF), Precautionary Loss Function (PLF), Quadratic Loss Function (QLF) and DeGroot Loss Function (DLF). Here, we discuss theses LFs for 3-Component mixture of EPD one by one.

For \( d \), PR \( \rho (d) \) can be written as: \( \rho (d) = E \{ L(\beta, d) \} \) where \( L(\beta, d) = (d - \beta)(d - \beta)^T \) is the SELF.

We get BEs and PRs supposing NIPs for the component as well as for proportion parameter(s) \( \alpha_1, \alpha_2, \alpha_3, w_1 \) and \( w_2 \) under SELF as:

\[ \hat{\alpha}_{1v} = \Omega_{v}^{-1} \Gamma(A_{3v}) \Gamma(A_{1v} + 1) \Gamma(A_{2v}) SB(A_{0v}, B_{0v} + C_{0v}) \]

\[ \times B(B_{0v}, C_{0v}) B_{1v}^{A_{1v} + 1} B_{2v}^{A_{2v}} B_{3v}^{A_{3v}} \]  

(12)

\[ \hat{\alpha}_{2v} = \Omega_{v}^{-1} \Gamma(A_{3v}) \Gamma(A_{1v} + 1) \Gamma(A_{2v} + 1) SB(A_{0v}, B_{0v} + C_{0v}) \]

\[ \times B(B_{0v}, C_{0v}) B_{1v}^{A_{1v} + 1} B_{2v}^{A_{2v} + 1} B_{3v}^{A_{3v}} \]  

(13)

\[ \hat{\alpha}_{3v} = \Omega_{v}^{-1} \Gamma(A_{1v} + 1) \Gamma(A_{2v}) \Gamma(A_{3v} + 1) SB(A_{0v}, B_{0v} + C_{0v}) \]

\[ \times B(B_{0v}, C_{0v}) B_{1v}^{A_{1v} + 1} B_{2v}^{A_{2v}} B_{3v}^{A_{3v} + 1} \]  

(14)

\[ \hat{\alpha}_{1v} = \Omega_{v}^{-1} \Gamma(A_{3v}) \Gamma(A_{1v}) \Gamma(A_{2v}) SB(A_{0v}, B_{0v} + C_{0v}) \]

\[ \times B(B_{0v}, C_{0v}) B_{1v}^{A_{1v} + 1} B_{2v}^{A_{2v}} B_{3v}^{A_{3v}} \]  

(15)

\[ \rho(\hat{\alpha}_{1v}) = \Omega_{v}^{-1} \Gamma(A_{3v}) \Gamma(A_{1v} + 2) \Gamma(A_{2v}) SB_{3v}^{A_{3v}} B_{2v}^{A_{2v}} B_{1v}^{A_{1v} - 2} \]

\[ \times B(A_{0v}, B_{0v} + C_{0v}) B(B_{0v}, C_{0v}) - \hat{\alpha}_{1v}^2 \]  

(16)

\[ \rho(\hat{\alpha}_{2v}) = \Omega_{v}^{-1} \Gamma(A_{3v}) \Gamma(A_{1v} + 2) \Gamma(A_{2v}) SB_{3v}^{A_{3v}} B_{2v}^{A_{2v}} B_{1v}^{A_{1v} - 2} \]

\[ \times B(A_{0v}, B_{0v} + C_{0v}) B(B_{0v}, C_{0v}) - \hat{\alpha}_{2v}^2 \]  

(17)
\[ \rho(\mathbf{v}) = \sum_{i=1}^{v} \Gamma( \mathbf{A}_{3v} ) \Gamma( \mathbf{A}_{2v} ) \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v} - 2 \]

where \( v = 1 \) for the UP and for the JP \( v = 2 \).

QLF is classified as symmetric LF and have applications in least square scheme. It desired additional attention due to its property of variance and symmetric. Where \( L(\beta, d) = \alpha (\beta - d)^2 \) is the QLF. We express the BE and the PR among QLF as:

\[ E(\beta | \beta - 1) = \frac{1}{E(\beta | \beta - 2)} \]

By utilizing this idea under UP and JP the derivation of BEs and PRs is (21)–(30), as shown at the bottom of the page, where \( v = 1 \) for the UP and \( v = 2 \) for the JP.

The next, studied LF is the PLF an asymmetric LF, which first explored by [27]. The simplified form of PLFs is a special case expressed as: \( L(\beta, d) = \frac{\alpha (\beta - d)^2}{d} \). Under PLF the results of BEs and PRs are derived by:

\[ d = \left[ E_{\beta | \beta} (\beta^2) \right]^{0.5} \] and \[ \rho(\mathbf{d}) = 2 \left[ E_{\beta | \beta} (\beta^2) \right]^{0.5} - 2 E_{\beta | \beta} (\beta) \]

The resulting BEs and PRs under the studied Ps and LF are obtained as:

\[ \mathbf{\hat{a}}_{1v} = \left[ \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{1v}} + 2) \Gamma(\mathbf{A_{2v}}) \times \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right]^{0.5} \]

(31)

\[ \mathbf{\hat{a}}_{2v} = \left[ \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{2v}} + 2) \Gamma(\mathbf{A_{1v}}) \times \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right]^{0.5} \]

(32)

\[ \mathbf{\hat{a}}_{3v} = \left[ \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{2v}}) \Gamma(\mathbf{A_{1v}} + 2) \Gamma(\mathbf{A_{3v}}) \times \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right]^{0.5} \]

(33)

\[ \mathbf{\hat{w}}_{1v} = \left[ \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{2v}}) \Gamma(\mathbf{A_{1v}}) \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right]^{0.5} \]

(34)

\[ \mathbf{\hat{w}}_{2v} = \left[ \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{2v}}) \Gamma(\mathbf{A_{1v}}) \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right]^{0.5} \]

(35)

\[ \rho(\mathbf{\hat{a}}_{1v}) = 2 \left[ \mathbf{\hat{a}}_{1v} - \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{1v}} + 1) \Gamma(\mathbf{A_{2v}}) \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right] \]

(36)

\[ \rho(\mathbf{\hat{a}}_{2v}) = 2 \left[ \mathbf{\hat{a}}_{2v} - \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{2v}} + 1) \Gamma(\mathbf{A_{1v}}) \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right] \]

(37)

\[ \rho(\mathbf{\hat{a}}_{3v}) = 2 \left[ \mathbf{\hat{a}}_{3v} - \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{2v}} + 1) \Gamma(\mathbf{A_{3v}} + 2) \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right] \]

(38)

\[ \rho(\mathbf{\hat{w}}_{1v}) = 2 \left[ \mathbf{\hat{w}}_{1v} - \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{1v}}) \Gamma(\mathbf{A_{2v}}) \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right] \]

(39)

\[ \rho(\mathbf{\hat{w}}_{2v}) = 2 \left[ \mathbf{\hat{w}}_{2v} - \Omega_v^{-1} \Gamma(\mathbf{A_{3v}}) \Gamma(\mathbf{A_{2v}}) \Gamma(\mathbf{A_{1v}}) \mathbf{S}_{3v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{1v}^{-1} \mathbf{B}_{2v}^{-1} \mathbf{B}_{3v}^{-1} \times \{ \mathbf{B}(\mathbf{A_{ov}}, \mathbf{B_{ov}} + \mathbf{C_{ov}}) \mathbf{B}(\mathbf{B_{ov}}, \mathbf{C_{ov}}) \} \right] \]

(40)
\[ \rho (\hat{z}_{2v}) = 1 - \frac{\Gamma (A_{1v}) \Gamma (A_{3v}) \Gamma (A_{2v} + 1) SB^{-A_{3v}}_{3v} B^{-A_{1v}}_{2v}}{(B_{1v} B_{2v} + C_{ov}) B (B_{1v}, C_{ov})} \times B^{-A_{1v}}_{2v} B (A_{ov}, B_{ov} + C_{ov}) B (B_{ov}, C_{ov}) \]  

(47)

\[ \rho (\hat{z}_{3v}) = 1 - \frac{\Gamma (A_{2v}) \Gamma (A_{3v}) \Gamma (A_{3v} + 1) SB^{-A_{3v}}_{3v} B^{-A_{3v}}_{2v}}{(B_{1v} B_{2v} + C_{ov}) B (B_{1v}, C_{ov})} \times B^{-A_{3v}}_{2v} B (A_{ov}, B_{ov} + C_{ov}) B (B_{ov}, C_{ov}) \]  

(48)

\[ \rho (\hat{w}_{1v}) = 1 - \frac{\Gamma (A_{1v}) \Gamma (A_{3v}) \Gamma (A_{2v}) SB^{-A_{1v}}_{1v} B^{-A_{2v}}_{2v}}{(B_{2v} B_{2v} + C_{ov}) B (B_{2v}, C_{ov})} \times B^{-A_{1v}}_{2v} B (A_{ov} + 1, B_{ov} + C_{ov}) B (B_{ov}, C_{ov}) \]  

(49)

**D. LIMITING EXPRESSIONS**

In contents of uncensored sampling technique limiting terms have extensive uses. When test-termination-time \( t \rightarrow \infty \), it may be noted that uncensored outcomes \( r \) approaches to \( n \) (sample size) and \( t_{i} \) approaches to \( n_{i} \), where \( l = 1, \ldots, 3 \). The censored elements become to be uncensored and also, the information mentions in the sample have also increased here. In end result, the efficiency of the BEs is also increases rapidly for to the contribution of all the values in sample. Therefore, limiting terms for the NIPs is easily estimated.

The limiting derivations assuming UP and JP for the BEs and PRs of 3-Component exponentiated Pareto distribution are reported in Table 1.
TABLE 1. Limiting expressions.

| Parameter(s) | UP | Bes | JP |
|--------------|----|-----|----|
| \( \hat{\alpha}_1 \) | \( \frac{1+n_1}{\sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij}} \) | \( \frac{n_1}{\sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij}} \) |
| \( \hat{\alpha}_2 \) | \( \frac{1+n_2}{\sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij}} \) | \( \frac{n_2}{\sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij}} \) |
| \( \hat{\alpha}_3 \) | \( \frac{1+n_3}{\sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij}} \) | \( \frac{n_3}{\sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij}} \) |
| \( \hat{\omega}_1 \) | \( \frac{1+n_1}{3+n} \) | \( \frac{1+n_1}{3+n} \) |
| \( \hat{\omega}_2 \) | \( \frac{1+n_2}{3+n} \) | \( \frac{1+n_2}{3+n} \) |

PRs

| Parameter(s) | UP | Bes | JP |
|--------------|----|-----|----|
| \( \hat{\alpha}_1 \) | \( \frac{1+n_1}{\left( \sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij} \right)^2} \) | \( \frac{n_1}{\left( \sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij} \right)^2} \) |
| \( \hat{\alpha}_2 \) | \( \frac{1+n_2}{\left( \sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij} \right)^2} \) | \( \frac{n_2}{\left( \sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij} \right)^2} \) |
| \( \hat{\alpha}_3 \) | \( \frac{1+n_3}{\left( \sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij} \right)^2} \) | \( \frac{n_3}{\left( \sum_{i=1}^{n} \ln (1+x_{ij}) - \sum_{i=1}^{n} \ln x_{ij} \right)^2} \) |
| \( \hat{\omega}_1 \) | \( \frac{\left( n_1 + n_2 + 2 \right) \left( n_1 + 1 \right)}{(n+4)(n+3)^2} \) | \( \frac{\left( n_1 + n_2 + 2 \right) \left( n_1 + 1 \right)}{(n+4)(n+3)^2} \) |
| \( \hat{\omega}_2 \) | \( \frac{\left( n_1 + n_2 + 2 \right) \left( n_1 + 1 \right)}{(n+4)(n+3)^2} \) | \( \frac{\left( n_1 + n_2 + 2 \right) \left( n_1 + 1 \right)}{(n+4)(n+3)^2} \) |

**III. RESULTS**

**A. SIMULATION BASED RESULTS**

Simulation is organized to evaluate the performance of BEs under UP and JP and with some symmetric and asymmetric LFs for the 3-Component mixture of the EPD. The computational code of simulation analysis is mention in Appendix.

- The first step, we need to create few sample sizes \( n = 50, 100, 200, 500 \) supposing some parameter values fixed as \( (\alpha_1, \alpha_2, \alpha_3, w_1, w_2) = (3, 2, 1, 0.5, 0.3), (3, 3, 0.4, 0.4), (1.2, 3, 0.3, 0.5) \).
- Outcomes are averaged with the help of Mathematica software, by replication of the simulation process 1000 intervals.
- Randomly selected sample-sizes \( w_1n, w_2n \) and \( (1 - w_1 - w_2)n \) are belongs to I, II, and the III factor densities, respectively.
- To determine the effect of \( t \) on BEs the right-type-I censoring technique, is used.
- All points are known as censored which were greater than \( t \).
- Two fixed censoring times \( t \) are applied to measure the behaviour of censoring rate upon the estimates.
- In favour of the fixed \( t \), values are as: \( t = 25 \) and \( 30 \).
- The point(s) which are greater than \( t \) are considered as the censored observations.
- Important to note here, that simply failures may be classified as an associate of subpopulation - I, II and III of the 3-Component mixture model of EPD.

In TABLE 2, simulation analysis is carried out for different \( n = 50, 100, 200, 500 \) and \( (\alpha_1, \alpha_2, \alpha_3, w_1, w_2) = (3, 2, 1, 0.5, 0.3) \).

From Table 2, it is clear that BEs assuming all stated LFs and NIPs is greater for the lesser \( n \) over to the greater \( n \) for both \( t \). It is also noted, for both \( t \) the variation of the BEs from supposed components near to zero with the increase in \( n \). Though, the PRs supposing the stated priors and LFs decrease with the raise in \( n \). FIGURE 4(a) explains the performance of BEs and indicates that for both \( t \) BEs performs best. FIGURE 4(b) illustrate the PRs and depicts that DLF have minimum PR and consider best LF among the other LFs for 3-Component mixture of EPD. From FIGURE 4(a-b) it is also clear that DLF have minimum risk for the component parameter(s). In favors of the valuation of component measure of the parameter(s) PRs gives smaller outcomes among the DLF; over to the results revealed under the SELF, QLF and PLF at different \( n \) and \( t \). For proportion parameter(s) estimation, it is described that SELF illustrates minimum PRs among QLF, PLF and at last DLF. Therefore, results direct that JP is the most suitable prior than the UP for present study. In view of simulation results under the mentioned LFs; the DLF is considered best for assessing the component parameter(s) and SELF observed efficient for proportion parameter(s).

![FIGURE 4. BEs and PRs for 3-Component mixture of EWD under NIP.](image-url)
### TABLE 2. Simulated results of 3-Component mixture of EPD.

| T   | N   | LFs | \(\hat{\alpha}_1\) | \(\hat{\alpha}_2\) | \(\hat{\alpha}_3\) | \(\hat{\psi}_1\) | \(\hat{\psi}_2\) |
|-----|-----|-----|----------------------|----------------------|----------------------|----------------------|----------------------|
| 25  | 50  | SELF | 2.142 1.534 1.558 0.478 0.292 | 0.242 0.147 0.187 0.005 0.004 | 2.633 1.344 1.297 0.448 0.286 | 0.043 0.067 0.100 0.024 0.047 | 3.549 1.583 1.654 0.465 0.327 | 0.149 0.094 0.141 0.011 0.013 | 3.505 1.642 1.756 0.478 0.333 | 0.043 0.063 0.091 0.023 0.042 |
| 100 | SELF | 2.596 1.492 1.498 0.471 0.330 | 0.169 0.079 0.089 0.003 0.002 | 3.060 1.391 1.361 0.478 0.292 | 0.012 0.034 0.052 0.011 0.024 | 3.191 1.951 1.559 0.512 0.295 | 0.064 0.052 0.078 0.005 0.007 | 2.811 1.953 1.588 0.467 0.326 | 0.023 0.033 0.047 0.012 0.022 |
| 200 | SELF | 2.619 1.464 1.471 0.474 0.299 | 0.082 0.037 0.046 0.001 0.001 | 2.742 1.413 1.409 0.469 0.306 | 0.011 0.017 0.026 0.006 0.012 | 2.936 1.784 1.455 0.485 0.307 | 0.032 0.025 0.036 0.003 0.004 | 2.742 1.941 1.512 0.489 0.304 | 0.010 0.016 0.023 0.005 0.011 |
| 500 | SELF | 2.767 1.449 1.451 0.484 0.303 | 0.038 0.015 0.017 0.001 0.000 | 2.597 1.572 1.424 0.482 0.305 | 0.004 0.007 0.010 0.002 0.005 | 3.044 1.942 1.461 0.489 0.299 | 0.013 0.010 0.014 0.001 0.002 | 2.950 1.954 1.468 0.486 0.301 | 0.004 0.007 0.010 0.002 0.005 |
| 30  | 50  | SELF | 2.876 1.538 1.553 0.473 0.308 | 0.344 0.147 0.172 0.005 0.004 | 2.433 1.329 1.296 0.478 0.250 | 0.042 0.077 0.100 0.012 0.057 | 2.845 1.762 1.529 0.473 0.326 | 0.125 0.101 0.155 0.011 0.014 | 2.527 1.726 1.360 0.448 0.347 | 0.045 0.059 0.083 0.025 0.039 |
| 100 | SELF | 2.411 1.489 1.517 0.479 0.316 | 0.124 0.071 0.115 0.002 0.002 | 2.786 1.386 1.367 0.467 0.283 | 0.023 0.037 0.050 0.012 0.026 | 3.555 1.715 1.541 0.492 0.299 | 0.072 0.051 0.071 0.005 0.007 | 3.147 1.545 1.592 0.499 0.306 |

### TABLE 2. (Continued.) Simulated results of 3-Component mixture of EPD.

| t   | n   | LFs | PRs | QLF | PLF | DLF | PRs | QLF | PLF | DLF | PRs |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 200 | SELF | 2.762 1.466 1.477 0.489 0.299 | 0.020 0.033 0.047 0.010 0.023 | 2.536 1.415 1.403 0.486 0.291 | 0.080 0.037 0.053 0.001 0.001 | 2.938 1.774 1.497 0.486 0.316 | 0.011 0.018 0.026 0.005 0.013 | 2.816 1.984 1.513 0.484 0.314 | 0.031 0.023 0.038 0.003 0.004 | 0.010 0.016 0.024 0.005 0.011 |
| 500 | SELF | 2.892 1.459 1.455 0.482 0.308 | 0.037 0.014 0.021 0.000 0.000 | 2.601 1.730 1.425 0.486 0.301 | 0.004 0.007 0.010 0.002 0.005 | 3.105 1.755 1.462 0.488 0.302 | 0.013 0.010 0.015 0.001 0.001 | 2.940 1.946 1.469 0.493 0.300 | 0.004 0.007 0.010 0.002 0.004 |

### B. APPLICATION

Mixture model of EPD has wide applications in engineering. Wind power is one of the major sources to get the clean energy bases compare to vestige gasoline. It is in the form of solar vitality, fixated by the uneven boiler of the earth’s outward. Wind speed is the most general and an important factor of
TABLE 2. (Continued.) Simulated results of 3-Component mixture of EPD.

| DLF | BEs | PRs | QLF | BEs | PRs | PLF | BEs | PRs | DLF | BEs | PRs |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 30  | 50  | SELF | BEs | 2.954 | 1.944 | 1.453 | 0.490 | 0.302 |
|     |     | SELF | PRs | 0.004 | 0.007 | 0.010 | 0.002 | 0.005 |
|     |     | SELF | QLF | 2.198 | 1.752 | 1.152 | 0.437 | 0.291 |
|     |     | SELF | PRs | 0.047 | 0.071 | 0.111 | 0.025 | 0.472 |
|     |     | SELF | PLF | 2.831 | 1.862 | 1.511 | 0.495 | 0.300 |
|     |     | SELF | PRs | 0.114 | 0.101 | 0.140 | 0.009 | 0.013 |
|     |     | SELF | DLF | 2.625 | 1.753 | 1.583 | 0.459 | 0.340 |
|     |     | SELF | PRs | 0.045 | 0.062 | 0.090 | 0.023 | 0.039 |
| 100 | SELF| BEs | 3.987 | 1.741 | 1.440 | 0.468 | 0.312 |
|     |     | SELF| PRs | 0.361 | 0.071 | 0.103 | 0.002 | 0.002 |
|     |     | SELF| QLF | 2.685 | 1.683 | 1.297 | 0.479 | 0.285 |
|     |     | SELF| PRs | 0.021 | 0.035 | 0.052 | 0.010 | 0.024 |
|     |     | SELF| PLF | 3.178 | 1.745 | 1.476 | 0.472 | 0.319 |
|     |     | SELF| PRs | 0.069 | 0.047 | 0.071 | 0.005 | 0.007 |
|     |     | SELF| DLF | 2.954 | 1.945 | 1.511 | 0.499 | 0.291 |
|     |     | SELF| PRs | 0.021 | 0.037 | 0.047 | 0.010 | 0.025 |
| 200 | SELF| BEs | 2.844 | 1.934 | 1.439 | 0.494 | 0.298 |
|     |     | SELF| PRs | 0.087 | 0.037 | 0.054 | 0.001 | 0.001 |
|     |     | SELF| QLF | 2.872 | 1.883 | 1.364 | 0.483 | 0.296 |
|     |     | SELF| PRs | 0.011 | 0.018 | 0.027 | 0.005 | 0.012 |
|     |     | SELF| PLF | 2.905 | 1.765 | 1.458 | 0.479 | 0.310 |
|     |     | SELF| PRs | 0.027 | 0.025 | 0.036 | 0.002 | 0.004 |
|     |     | SELF| DLF | 3.220 | 1.997 | 1.447 | 0.486 | 0.309 |
|     |     | SELF| PRs | 0.010 | 0.017 | 0.025 | 0.005 | 0.011 |
| 500 | SELF| BEs | 2.740 | 1.766 | 1.440 | 0.496 | 0.295 |
|     |     | SELF| PRs | 0.032 | 0.015 | 0.012 | 0.000 | 0.000 |
|     |     | SELF| QLF | 2.658 | 1.914 | 1.410 | 0.484 | 0.299 |
|     |     | SELF| PRs | 0.004 | 0.007 | 0.010 | 0.002 | 0.005 |
|     |     | SELF| PLF | 2.730 | 1.840 | 1.447 | 0.497 | 0.302 |
|     |     | SELF| PRs | 0.011 | 0.010 | 0.015 | 0.001 | 0.001 |
|     |     | SELF| DLF | 2.999 | 1.987 | 1.250 | 0.477 | 0.315 |
|     |     | SELF| PRs | 0.004 | 0.006 | 0.010 | 0.002 | 0.004 |

TABLE 3. Real data analysis of 3-Component mixture of EPD.

| NIP | LFs | Wind Speed | \(\hat{\alpha}_1\) | \(\hat{\alpha}_2\) | \(\hat{\alpha}_3\) | \(\hat{\omega}_1\) | \(\hat{\omega}_2\) |
|-----|-----|------------|----------------|----------------|----------------|----------------|----------------|
| UP  | SELF | BE         | 3.151 | 1.476 | 1.493 | 0.500 | 0.276 |
|     | PR   | 0.283 | 0.104 | 0.131 | 0.003 | 0.003 |
| QLF | BE   | 3.887 | 1.655 | 1.317 | 0.487 | 0.256 |
|     | PR   | 0.026 | 0.050 | 0.062 | 0.013 | 0.036 |
| PLF | BE   | 4.219 | 1.952 | 1.537 | 0.503 | 0.282 |
|     | PR   | 0.131 | 0.069 | 0.086 | 0.006 | 0.009 |
| DLF | BE   | 3.286 | 1.876 | 1.582 | 0.506 | 0.285 |
|     | PR   | 0.025 | 0.045 | 0.055 | 0.012 | 0.032 |
| JP  | SELF | BE         | 3.021 | 1.906 | 1.405 | 0.500 | 0.276 |
|     | PR   | 0.669 | 0.099 | 0.124 | 0.003 | 0.002 |
| QLF | BE   | 3.051 | 1.625 | 1.229 | 0.487 | 0.256 |
|     | PR   | 0.027 | 0.052 | 0.066 | 0.013 | 0.036 |
| PLF | BE   | 4.087 | 1.641 | 1.449 | 0.503 | 0.281 |
|     | PR   | 0.131 | 0.069 | 0.086 | 0.006 | 0.009 |
| DLF | BE   | 3.154 | 1.876 | 1.493 | 0.506 | 0.285 |
|     | PR   | 0.025 | 0.047 | 0.059 | 0.012 | 0.032 |

To establish the suggested approach, we have partitioning the given uncensored/complete data observations into the 3-Component portions giving to type-I right censoring structure with \(r = 91\). It is unidentified that, which parameter component(s) fails till a failure happens at \(t = 9\). To design the programme in Mathematica, the total tests are elaborated 100 times.

\[
n = 100, \quad \sum_{k=1}^{r_1} x_{1k} = 6.40, \quad \sum_{k=1}^{r_3} x_{2k} = 8.39, \quad \sum_{k=1}^{r_3} x_{3k} = 6.94, \quad n - r = 9, \quad r_1 = 46, \quad r_3 = 18, \quad r_2 = 27, \quad r = 91.
\]

Almost here 9% censored points are considered. BEs and the PRs supposing the NIPs are described in Table 3.

3-Component portions giving to type-I right censoring structure with \(r = 91\). It is unidentified that, which parameter component(s) fails till a failure happens at \(t = 9\). To design the programme in Mathematica, the total tests are elaborated 100 times.

\[
n = 100, \quad \sum_{k=1}^{r_1} x_{1k} = 6.40, \quad \sum_{k=1}^{r_3} x_{2k} = 8.39, \quad \sum_{k=1}^{r_3} x_{3k} = 6.94, \quad n - r = 9, \quad r_1 = 46, \quad r_3 = 18, \quad r_2 = 27, \quad r = 91.
\]
Another important application of EPD is in the area of tensile strength of carbon fiber. Hence, in this assessment of study we have taken the dataset of 100 sample observations of tensile strength of carbon fiber. Formerly this dataset also studied by [32], which later has been considered by [33]. This dataset is based on the tensile strength of 100 points of carbon fiber and are as:

\[ 3.7, 3.11, 4.42, 3.28, 3.75, 2.96, 3.39, 3.31, 3.15, 2.81, 1.41, 2.76, 3.19, 1.59, 2.17, 3.51, 1.84, 1.61, 1.57, 1.89, 2.74, 3.27, 2.41, 3.09, 2.43, 2.53, 2.81, 3.31, 3.25, 2.77, 2.68, 4.91, 1.57, 2.00, 1.17, 2.17, 0.39, 2.79, 1.08, 2.88, 2.73, 2.87, 3.19, 1.87, 2.95, 2.67, 4.20, 2.85, 2.55, 2.17, 2.97, 3.68, 0.81, 1.22, 5.08, 1.69, 3.68, 4.70, 2.03, 2.82, 2.50, 1.47, 3.22, 3.15, 2.97, 2.93, 3.33, 2.56, 2.59, 1.36, 1.84, 5.56, 1.12, 2.48, 1.25, 2.48, 2.03, 1.61, 2.05, 3.60, 3.11, 1.69, 4.90, 3.39, 3.22, 2.55, 3.56, 2.38, 1.92, 0.98, 1.59, 1.73, 1.71, 1.18, 4.38, 0.85, 1.80, 2.12, 3.65. 

To establish the suggested procedure, we have portioning the given uncensored data observation towards the 3-Component segments as mentioned in the right type-I censoring technique using the rate of failure \( r = 91 \). It is unidentified that, which parameter component fails till a failure occurs at \( t = 9 \). Formulate the programme in Mathematica, the total tests are run 100 times, and results are as:

\[
\begin{align*}
\text{n} &= 100, \quad \sum_{k=1}^{r_3} x_{1k} = 5.98, \\
\sum_{k=1}^{r_3} x_{3k} &= 3.09, \quad \sum_{k=1}^{r_2} x_{2k} = 4.14, \\
\text{n} - r &= 9, \quad r_1 = 46, \quad r_2 = 27, \quad r_3 = 18, \quad r = 91.
\end{align*}
\]

Practically here 9% censored rate are applied. BEs and the PRs supposing the NIPs are described in Table 3.

Neutrosophic statistics is the generalization of classical statistics. The current study can be extended using neutrosophic statistics [34]–[39].

C. COMPARISON

From TABLE 2-3, it is distinguished that simulation analyses are compatible with both (Wind Speed and Carbon Fiber) real data sets. There seem few concessions which are due to the causes of the small data set. In both, simulation as well as real data applications, outputs under the JP are most precise as compare to the UP. Though, DLF as compare to other studied LFs (SELF, QLF, PLF) demonstrates better results for the proportion segment of parameter(s).

IV. CONCLUSION

In this article, the Bayesian construction mixture of the 3-Component model of EPD under right type-I censoring procedure is considered. The comprehensive simulation technique is formulated to evaluate and establish some significant characteristics about the BEs of the 3-Component mixture of EPD supposing the NIPs under the four symmetric and asymmetric LFs. Over-estimation and under-estimation of proportional mixture are inversely connected to the sample size and are directly relative to censoring percentage. A lesser sample size, a huge censoring degree reasons the higher level of over-estimation. But this consequence may be condensed by assuming a large sample size. Posterior densities are estimated and reported that they are in closed forms. The other objective of this article was the choice of suitable LF and P for the conclusion of mixture parameter(s) at various n and t. To conclude the performance, we estimated several posterior quantities, like BEs with their PRs by considering diverse n and t. The limiting formulation for the BEs and PRs are also derived under the studied LFs (SELF, DLF, QLF, PLF) among the NIPs (JP, UP). The well-matched outcomes are noted for simulation as well as for real data observations study, to investigate the presentation of BEs. Figure 4 also demonstrate the performance of BEs and PRs and conclude that BEs and LFs are greater for small sample size for both t as compare to large sample sizes. While, the PRs using the NIPs and LFs are decreased by increasing the sample size for both t. The contact of several n and t are estimated for BEs. The outcomes revealed that, for the component parts the direction of best BEs are as DLF < PLF < SELF < QLF, and on other hand for the proportion segment analysis this classifying is noticed: SELF < PLF < DLF < QLF. Therefore, we conclude that the efficient and most suitable P is JP over the UP due to the minimum risk. DLF perform efficient for 3-Component mixture model of EPD among the PLF, SELF and QLF.

APPENDIX

Computational code

\[
\begin{align*}
\text{n} &= 50; \quad p_1 = 0.50; \quad p_2 = 0.30; \quad t = 25; \quad \text{Alpha 1} = 3; \\
\text{Alpha 2} = 2; \quad \text{Alpha 3} = 1; \quad u_1 = \text{RandomReal}[\text{UniformDistribution}[\{0, 1\}], \text{Round}[n^{p_1}]]; \\
\text{u_2} &= \text{RandomReal}[\text{UniformDistribution}[\{0, 1\}], \text{Round}[n^{p_2}]]; \quad u_3 = \text{RandomReal}[\text{UniformDistribution}[\{0, 1\}], \text{Round}[n^{p_1} - p_2]]; \\
\text{x_1} &= u_1^{1/\text{Alpha 1}}(1-u_1^{1/\text{Alpha 1}}); \quad \text{x_2} = u_2^{1/\text{Alpha 2}}(1-u_2^{1/\text{Alpha 2}}); \quad \text{x_3} = u_3^{1/\text{Alpha 3}}(1-u_3^{1/\text{Alpha 3}}); \\
\text{sx_1} &= \text{Select[x_1, \# <= 0.50 \&]; \quad \text{sx_2} = \text{Select[x_2, \# <= 0.50 \&];} \quad \text{sx_3} = \text{Select[x_3, \# <= 0.50 \&];} \quad \text{r_1} = \text{Length}[\text{sx_1}]; \\
\text{r_2} &= \text{Length}[\text{sx_2}]; \quad \text{r_3} = \text{Length}[\text{sx_3}]; \quad \text{sum_1} = \text{Sum}[\text{Log}[1+(\text{sx_1}[\text{m}])/(\text{sx_1}[\text{m}])]; \quad \text{m, r_1}] ; \quad \text{sum_2} = \text{Sum}[\text{Log}[1+(\text{sx_2}[\text{m}])/(\text{sx_2}[\text{m}])]; \quad \text{m, r_2}] ; \quad \text{sum_3} = \text{Sum}[\text{Log}[1+(\text{sx_3}[\text{m}])/(\text{sx_3}[\text{m}])]; \quad \text{m, r_3}] ; \quad \text{r} = \text{r_1} + \text{r_2} + \text{r_3}; \quad v = -\text{Log}[1-(1 + t)^{-1}];
\end{align*}
\]

REFERENCES

[1] S. M. Burroughs and S. F. Tebbens, “Upper-truncated power law distribution,” Fractals, vol. 9, no. 2, pp. 209–222, 2001.
[2] B. Schroeder, S. Damouras, and P. Gall, “Understanding latent sector errors and how to protect against them.” ACM Trans. Storage, vol. 6, no. 3, pp. 1–23, Sep. 2010.
[3] A. Akinsete, F. Famoye, and C. Lee, “The beta-Pareto distribution,” Statistics, vol. 42, no. 6, pp. 547–563, Dec. 2008.
[4] M. Bourguignon, R. B. Silva, L. M. Zea, and G. M. Cordeiro, “The Kumaraswamy Pareto distribution,” J. Stat. Theory Appl., vol. 12, no. 2, pp. 129–144, 2013.
IBRAHIM M. ALMANJAHIE was born in Saudi Arabia, in 1979. He received the B.Sc. degree in mathematics from King Khalid University, Saudi Arabia, in 2002, and the M.Sc. degree in mathematical and statistical science and the Ph.D. degree in probability and statistical modeling from the University of Western Australia, Australia, in 2008 and 2015, respectively.

He is currently an Associate Professor with the Department of Mathematics, College of Science, King Khalid University. He is also the President of the Saudi Association for Statistical Sciences. His main research interests include modeling and analysis of ion channel data, hidden Markov models, the EM algorithm, finite mixture models, MCMC, computational methods in statistics, time series modeling, applied statistics, sampling, and functional statistics.

ISHFAQ AHMAD was born in Pakistan, in March 1981. He received the B.Sc. degree in statistics, physics, and mathematics from Bahudin Zakaria University, Multan, Pakistan, in 1999, the M.Sc. and M.Phil. degrees in statistics from Quaid-e-Azam University, Islamabad, Pakistan, in 2003 and 2005 respectively, and the Ph.D. degree in probability and mathematical statistics from the Institute of Applied Mathematics, University of Chinese Academy of Science (UCAS), Beijing, China, in 2010.

He is currently working as an Associate Professor of statistics with the Department of Mathematics and Statistics, Faculty of Basic and Applied Sciences, International Islamic University, Islamabad. Before this, he has worked as an Assistant Professor with King Khalid University, Saudi Arabia. He was also invited as a Research Associate with the City University of Hong Kong, in 2009, and the Shanghai University of Finance and Economics. He is also a HEC approved Ph.D. Supervisor since 2010. He has supervised more than 30 M.S. students in statistics and many of M.S. and Ph.D. students are under supervision. His main research interests include extreme value theory, statistical inference, survey sampling, optimization, Bayesian analysis, and functional analysis. He has published more than 47 articles in journals of international repute, such as the International Journal of Climatology, Scientific Reports, and many others.

Dr. Ahmad became a regular member (M) of the International Statistics Institute (ISI), in 2020. He was awarded with the Higher Education Commission (HEC) Scholarship for overseas countries in 2007.