Search for the $D^*\bar{D}^*$ molecular state $Z_c(4000)$ in the reaction of $B^- \to J/\psi \rho K^-$

Yang Zhang,^1 En Wang,^1,* De-Min Li,^1 and Yu-Xiao Li^1

^1School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China

Based on the prediction of a $D^*\bar{D}^*$ molecular state $Z_c(4000)$ with isospin $I = 1$ in the coupled channels unitary chiral approach, and that the existence of the $Z_c(4000)$ is crucial to understand the near threshold enhancement in the $e^+e^- \to (D^*\bar{D}^*)^{\pm}\pi^\mp$ process reported by the BESIII Collaboration, we suggest to search for this state in the reaction of $B^- \to J/\psi \rho K^-$. By taking into account the final state interactions of $J/\psi \rho$ and $D^{(*)}\rho^0$, and the contribution of the $K_1(1270)$ resonance, we find that the $J/\psi \rho$ mass distribution shows a peak around 4000 MeV, which could be associated to the $D^*\bar{D}^*$ molecular state $Z_c(4000)$.

PACS numbers:

I. INTRODUCTION

In the last decades, a lot of charmonium-like states, named as $X$, $Y$, $Z$ states, were discovered experimentally, and can not naturally fit in the expected spectrum of ordinary $c\bar{c}$ meson states [1–3]. Although there are several interpretations for their underlying structures, such as tetraquarks, molecular states, kinematic effects, and so on, it is still difficult to exactly distinguish the exotic states from the conventional $c\bar{c}$ states. One way to identify a resonance as an exotic charmonium state is the observation of its decay mode of a charmonium state plus a light meson with nonzero isospin [4].

The first charged charmonium-like state, $Z_c(4430)$ was reported in the $\pi^-\psi(2S)$ mass distribution of the $B \to K\pi^-\psi(2S)$ by the Belle Collaboration [5, 6], and was confirmed by the LHCb Collaboration seven years later [7]. Since the $Z_c(3900)$ was observed in the $\pi^-J/\psi$ invariant mass distribution of the $e^+e^- \to \pi^+\pi^- J/\psi$ by the BESIII and Belle Collaborations [8, 9], several $Z_c$ states were reported in different processes [1]. The BESIII Collaboration has studied the process of $e^+e^- \to (D^*\bar{D}^*)^{\pm}\pi^\mp$, and observed an enhancement structure near the $D^*\bar{D}^*$ threshold, which was interpreted as the $Z_c(4025)$ with $M = 4026.3 \pm 2.6 \pm 3.7$ MeV and $\Gamma = 24.8 \pm 5.6 \pm 7.7$ MeV [10], and there are many theoretical explanations for its nature, such as a tetraquark state [11–14], or a molecular state [15–20].

It should be pointed out that the interpretation of peaks around threshold is always problematic, and most often an enhancement close to the threshold is an indication of the bound state or resonance below threshold [16].

The interaction of $D^*\bar{D}^*$ and $J/\psi \rho$ with isospin $I = 1$ has been investigated within the local hidden gauge approach in Refs. [16, 21], and a state $Z_c(4000)$ around 4000 MeV with $J = 2$ and $I = 1$ is found. In Ref. [22], it is shown that the state predicted in Ref. [16] is compatible with the reanalysis of the BESIII data for the $e^+e^- \to (D^*\bar{D}^*)^{\pm}\pi^\mp$ reaction [10]. Thus, searching for the predicted state $Z_c(4000)$ in other processes is crucial to understand the enhancement structure near the $D^*\bar{D}^*$ threshold in the $e^+e^- \to (D^*\bar{D}^*)^{\pm}\pi^\mp$. Taking into account that the $Z_c(4000)$, as the $D^*\bar{D}^*$ bound state with $I = 1$, couples to the $J/\psi \rho$ channels [16], and that the large $B$ samples of the LHCb and Belle are the good platform for observing the $X Y Z$ states [3], we suggest to search for the $Z_c(4000)$ in the reaction of the $B^- \to J/\psi \rho K^-$. It should be pointed out that the Belle Collaboration has reported the observation of the exclusive decay process $B \to J/\psi K_1(1270)$, and measured the branching fraction of $Br[B^- \to J/\psi K^+(1270)] = (1.80 \pm 0.34 \pm 0.39) \times 10^{-3}$ [23]. Since the dominant decay channel of the $K_1(1270)$ is $\rho K$ [24], it implies that the reaction of $B^- \to J/\psi \rho K^-$ is accessible experimentally.

This paper is organized as follows. In Sec. II, we will present the mechanism of the reaction $B^- \to J/\psi \rho K^-$, the results and discussions are shown in Sec. III. Finally, the summary is given in Sec. IV.

II. FORMALISM

Analogy to the Refs. [25, 26], the mechanism of the reaction $B^- \to J/\psi \rho K^-$ at the quark level can be depicted in Fig. 1. The $b$ quark first weakly decays into a $c$ quark and a $W^-$ boson, and then the $W^-$ boson couples to a $\bar{c}$ quark and a $s$ quark. Fig. 1(a) shows the internal emission, where the $c$ and $\bar{c}$ go into $J/\psi$, and the $su$ component is hadronized with $\bar{u}u$ pair, created from the vacuum with the quantum numbers of vacuum, to $\rho K^-$. Since the state $Z_c(4000)$ couples strongly to the $D^*\bar{D}^*$, the $D^*\bar{D}^*$ system can be produced primarily, following by the transition to the final state $J/\psi \rho$. Figure 1(b) shows the internal emission mechanism of the reaction $B^- \to D^*\bar{D}^* K^-$, where the $c$ and $\bar{c}$ hadronize with the $\bar{q}q$ pair, created from the vacuum, to the final state $D^*\bar{D}^*$. Because the isospin of the created $\bar{q}q$ is 0, which leads to the isospin $I = 0$ for the $D^*\bar{D}^*$ system, we can not produce the molecular state with $I = 1$, and thus the diagram of Fig. 1(b) has no contribution to the reaction of $B^- \to J/\psi \rho K^-$. In addition, we also have the mechanism of external emission as shown in Fig. 1(c), which is

*Electronic address: wangen@zzu.edu.cn
color-favored with respect to the internal emission. Here the $s\bar{c}$ component from the $W^-$ decay, together with the $\bar{u}u$, is hadronized to produce the $D^{*0}K^-$, and the remaining $c\bar{u}$ leads to the $D^{*0}$.

| FIG. 1: The microscopic quark level production of the $B^-$ decay. (a) The internal emission of the $B^- \rightarrow J/\psi s\bar{u}$ decay and hadronization of the $s\bar{u}$ through $\bar{u}u$ with vacuum quantum numbers. (b) The internal emission of the $B^- \rightarrow K^- c\bar{c}$ decay and hadronization of the $c\bar{c}$ through $\bar{q}q$ with vacuum quantum numbers. (c) The external emission of the $B^- \rightarrow D^{*0}\bar{c}s$ decay and hadronization of the $\bar{c}s$ through $\bar{q}q$ with vacuum quantum numbers. |

Then the tree level diagrams of the $B^- \rightarrow J/\psi \rho K^-$ reaction, and the final state interactions of $J/\psi \rho$ and $D^{*0}\bar{D}^*$, are shown in Figs. 2(a) and (b), respectively. The tree level amplitude for the $B^- \rightarrow J/\psi \rho K^-$ decay in $S$-wave can be expressed as,

$$M^{(a)} = A \times \epsilon_{J/\psi} \cdot \epsilon_{\rho},$$

where the $\epsilon_{J/\psi}$ and $\epsilon_{\rho}$ are the polarization vectors for the $J/\psi$ and $\rho$, respectively, and $A$ stands for the normalization factor of the vertex $B^- \rightarrow J/\psi \rho K^-$. Note that we work on the rest frame of the resonance produced, where the momenta of the $J/\psi$ and $\rho$ are small with respect to their masses, thus we neglect the $c^0$ component. This is actually very accurate for these momenta as can be seen in Appendix A of Ref. [27]. For the final state interactions of the $J/\psi \rho$ and $D^{*0}\bar{D}^*$ final state interaction as shown in Fig. 2(b), we need the $K^-$ in $D$-wave to match the angular momentum of the $B^-$, the amplitude is given by [25, 26],

$$M^{(b)} = B \left( G_{J/\psi \rho} \epsilon_{J/\psi} \cdot \epsilon_{J/\psi} + 3C \frac{1}{\sqrt{2}} G_{D^{*0}\bar{D}^*} t^{D^{*0}\bar{D}^*,J/\psi\rho} \right) \times \left( \epsilon_{J/\psi} \cdot \vec{k} \epsilon_{\rho} \cdot \vec{k} - \frac{1}{3} |\vec{k}|^2 \epsilon_{J/\psi} \cdot \epsilon_{\rho} \right),$$

where $\vec{k}$ is the momentum of the $K^-$ in the $J/\psi \rho$ rest frame. The factor $1/\sqrt{2}$ is the Clebsch-Gordan coefficient for the $D^{*0}\bar{D}^*$ system with isospin $I = 1$. In order to explicitly consider the factor 3 relative to the enhancement of the external emission mechanism of Fig. 1(c), we write $3C$ for the weight of the mechanism relative to $D^{*0}\bar{D}^*$ primary production. We will vary the value of $C$ around unity, but we can anticipate that it hardly changes the shape of the distribution obtained.

The $G_{J/\psi \rho}$ and $G_{D^{*0}\bar{D}^*}$ are the loop functions, and we use the dimensional regularization as,

$$G_i = \frac{1}{10\pi^2} \left\{ \alpha_i + \ln \frac{m_i^2}{m_2^2} + \frac{m_3^2 - m_s^2 + \epsilon}{2s} \ln \frac{m_4^2}{m_1^2} \right\}$$

where the subtraction constants $\alpha_1 = -2.3$ and $\alpha_2 = -2.6$ ($i = 1, 2$ corresponding to the channels of $D^{*0}\bar{D}^*$,

FIG. 2: The mechanisms for the $B^- \rightarrow J/\psi \rho K^-$ reaction. (a) the tree diagram, (b) the $J/\psi \rho$ final state interaction, (c) the term of the intermediate $K_1(1270)$. 

and \( J/\psi p \), \( \mu = 1000 \text{ MeV} \), same as Ref. [16]. \( p \) is the three-momentum of the mesons \( D^* \) or \( J/\psi \) in the rest frame of \( D^* D^* \) or \( J/\psi p \), respectively,

\[
p = \frac{\sqrt{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)}}{2\sqrt{s}},
\]

with \( m_{1,2} \) being the masses of the mesons in the \( i \)th channel.

The transition amplitudes of \( t_{J/\psi p, J/\psi p} \) and \( I_{D^* D^* p, J/\psi p}^f = 1 \) are taken by solving the Bethe-Salpeter equation, as shown in Eq. (8) of Ref. [16].

![Figure 3: The Dalitz plot of the \( B^- \rightarrow J/\psi p K^- \) reaction. The bands colored by blue and red correspond to the energy regions \( (M - \Gamma/2, M + \Gamma/2) \) of the \( Z_c(4000) \) and \( K_1(1270) \) resonances, respectively. Their masses and widths are taken from the PDG [24].](image)

Indeed the \( K^- p \) can undergo the final state interaction. Ref. [23] has observed the \( B \rightarrow J/\psi K_1(1270) \) with \( Br[B^+ \rightarrow J/\psi K_1^*(1270)] = (1.80 \pm 0.34 \pm 0.39) \times 10^{-3} \), and no evidence of the \( B \rightarrow J/\psi K_1(1400) \) is seen. Since the dominant decay channel of the \( K_1(1270) \) is \( \rho K \) (\( Br[K_1(1270) \rightarrow \rho K] = (42 \pm 6) \% \) [24]), we expect the resonance \( K_1(1270) \) will play an important role in the \( \rho K^- \) invariant mass distribution, as shown in Fig. 2(c). Although some theoretical studies show that the \( K_1(1270) \) has a pole structure [28, 29], the contribution of the \( K_1(1270) \) will not affect the peak structure of the \( Z_c(4000) \) in the \( J/\psi p \) invariant mass distribution, according to the Dalitz diagram of the \( B^- \rightarrow J/\psi p K^- \) of Fig. 3. For simplicity, we will include the amplitude for the \( K_1(1270) \) contribution with a Breit-Wigner form,

\[
\mathcal{M}^{(c)} = \frac{A' \times M_{K_1(1270)}^2 \times \epsilon_{J/\psi} \cdot \epsilon_{\rho}}{M_{inv}(K\rho) - M_{K_1(1270)}^2 + iM_{K_1(1270)} \Gamma_{K_1(1270)}},
\]

with \( M_{K_1(1270)} = 1272 \text{ MeV} \), and \( \Gamma_{K_1(1270)} = 90 \text{ MeV} \) [24]. In addition, in order to make the \( A, B \) and \( A' \) with the same dimension, we have to include a factor \( |k_{ave}|^2 \), with \( |k_{ave}| = 1000 \text{ MeV} \) in Eqs. (1) and (5). Now, we can write the full amplitude for the \( B^- \rightarrow J/\psi p K^- \) reaction,

\[
\mathcal{M} = \mathcal{M}^{(a)} + \mathcal{M}^{(b)} + \mathcal{M}^{(c)} = A|k_{ave}|^2 \times \epsilon_{J/\psi} \cdot \epsilon_{\rho} \times \left[ 1 + \frac{\beta M_{K_1(1270)}^2}{M_{inv}(K\rho) - M_{K_1(1270)}^2 + iM_{K_1(1270)} \Gamma_{K_1(1270)}} \right] \frac{3C}{\sqrt{2}} G_{D^* D^*} \frac{I_{D^* D^* p, J/\psi p}^f = 1}{I_{D^* D^* p, J/\psi p}^f = 1} \times \left( \epsilon_{J/\psi} \cdot \vec{k} \epsilon_{\rho} \cdot \vec{k} - \frac{1}{3} |\vec{k}|^2 \epsilon_{J/\psi} \cdot \epsilon_{\rho} \right)
\]

\[
+ B \left( \epsilon_{J/\psi} \cdot \vec{k} \epsilon_{\rho} \cdot \vec{k} - \frac{1}{3} |\vec{k}|^2 \epsilon_{J/\psi} \cdot \epsilon_{\rho} \right) \times (\bar{t}^{(a)} + t^{(c)})
\]
from the $K_1(1270)$ resonance, and the unity value of $C = 1$. Although we don’t know the exact value of the $B/A$, we expect the $B$ has the similar strength with $A$, since the primary production weight of the $J/\psi\rho$ is same as that of the tree diagram of Fig. 2(b), the $K_1(1270)$ resonance (Fig. 2(c)), and the tree diagram (Fig. 2(a)), respectively. The ‘Total’ curve shows the results of the full model.

Up to the arbitrary normalization $A$, we calculate the $J/\psi\rho$ and $\rho K^-$ mass distributions with $B/A = 1$, as shown in Figs. 5 and 6, respectively. For the $J/\psi\rho$ mass distribution, one can see a significant peak structure around 4000 MeV, which is associated to the $D^*\bar{D}^*$ molecular state $Z_c(4000)$. The contributions from the tree diagram of Fig. 2(a) and the resonance $K_1(1270)$ have little effect on the peak position. For the $\rho K^-$ mass distribution of Fig. 6, it shows a narrow peak close to the $\rho K^-$ threshold, which corresponds to the $K_1(1270)$ resonance, and our results are consistent with the $K\rho$ mass distribution of the $B \to J/\psi K\rho$ reported by the Belle Collaboration [23].

Next, we will show the $J/\psi\rho$ mass distribution by varying the values of the three parameters. In Fig. 7, we present the $J/\psi\rho$ mass distribution with the $\beta = 0.3, 0.5, 0.8$, the weight of the contribution from the $K_1(1270)$ resonance. From Fig. 7, we can conclude that contribution from the $K_1(1270)$ resonance does not modify the peak position of the $Z_c(4000)$ resonance markedly, and the peak structure is still clear even with a very large contribution from the $K_1(1270)$ resonance, which is because that the narrow peak structure of the $K_1(1270)$
almost does not contribute to the $J/\psi\rho$ mass distribution of the 3900 $\sim$ 4100 MeV region, as shown in Fig. 3.

The $J/\psi\rho$ mass distributions with the different values $B/A = 0.5, 1.0, 1.5$ are shown in Fig. 8, while the background contributions of the Figs. 2(a) and (c) become larger, the peak structure of the $Z_c(4000)$ will be weaker.

Finally, we show the $J/\psi\rho$ mass distribution with the $C = 0.8, 1.0, 1.2$ in Fig. 9. One can see that it hardly changes the shape of the $J/\psi\rho$ mass distribution for the different values of $C$, the same as the discussion above.

IV. SUMMARY

In this work, we have studied the reaction of $B^- \rightarrow J/\psi\rho K^-$, by considering the contribution from the $K_1(1270)$ resonance, and the $D^{*0}\bar{D}^{0}$ and $J/\psi\rho$ final state interactions within the coupled channels unitary chiral approach, which dynamically generate the $D^* \bar{D}^*$ molecular state $Z_c(4000)$ with isospin $I = 1$. The existence of the $D^* \bar{D}^*$ molecular state $Z_c(4000)$ is crucial to understand the enhancement structure near the $D^* \bar{D}^*$ in the $e^+e^- \rightarrow (D^* \bar{D}^*)^{\pm}\pi^\mp$ reaction reported by the BESIII Collaboration.

Our results show that the $J/\psi\rho$ mass distribution has a peak structure around 4000 MeV, which can be associated to the $D^* \bar{D}^*$ molecular state $Z_c(4000)$. We have also varied the values of the parameters of our model, and the signal of the $Z_c(4000)$ is still clear. On the other hand, one can find that a narrow peak structure close to the $\rho K^-$ threshold in the $\rho K^-$ mass distribution, which corresponding to the $K_1(1270)$ resonance, and the contribution from the $K_1(1270)$ does not affect the peak position of the $Z_c(4000)$.

In summary, the reaction of the $B^- \rightarrow J/\psi\rho K^-$ is accessible experimentally, and we suggest our experimental colleagues to search for the $Z_c(4000)$ state in the reaction of $B^- \rightarrow J/\psi\rho K^-$. 

Acknowledgements

This work is partly supported by the National Natural Science Foundation of China under Grant Nos. 11505158, 11605158, the Key Research Projects of Henan Higher Education Institutions (No. 20A140027), and the Academic Improvement Project of Zhengzhou University.
[8] M. Ablikim et al. [BESIII Collaboration], Observation of a Charged Charmoniumlike Structure in $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ at $\sqrt{s} = 4.26$ GeV, Phys. Rev. Lett. 110, 252001 (2013).

[9] Z. Q. Liu et al. [Belle Collaboration], Study of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ and Observation of a Charged Charmoniumlike State at Belle, Phys. Rev. Lett. 110, 252002 (2013) Erratum: [Phys. Rev. Lett. 111, 019901 (2013)].

[10] M. Ablikim et al. [BESIII Collaboration], Observation of a charged charmoniumlike structure in $e^+e^- \rightarrow (D^*\bar{D}^*)^{\pm}\pi^\mp$ at $\sqrt{s} = 4.26$ GeV, Phys. Rev. Lett. 112, no. 13, 132001 (2014).

[11] Z. G. Wang, Analysis of the Zc(4020), Zc(4025), Y(4360) and Y(4660) as vector tetraquark states with QCD sum rules, Eur. Phys. J. C 74, no. 5, 2874 (2014).

[12] Z. G. Wang, Reanalysis of the Zc(4020), Zc(4025), Z(4050) and Z(4250) as tetraquark states with QCD sum rules, Commun. Theor. Phys. 63, no. 4, 466 (2015).

[13] C. F. Qiao and L. Tang, Interpretation of Zc(4025) as the hidden charm tetraquark states via QCD Sum Rules, Eur. Phys. J. C 74, 2810 (2014).

[14] C. Deng, J. Ping and F. Wang, Interpreting Zc(3900) and Zc(4025)/Zc(4020) as charged tetraquark states, Phys. Rev. D 90, 054009 (2014).

[15] Z. G. Wang, Reanalysis of the Y(3940), Y(4140), Zc(4020), Zc(4025) and Zc(4160) as molecular states with QCD sum rules, Eur. Phys. J. C 74, no. 7, 2963 (2014).

[16] F. Aceti, M. Bayar, J. M. Dias and E. Oset, Prediction of a Zc(4000) $D^*\bar{D}^*$ state and relationship to the claimed Zc(4025), Eur. Phys. J. A 50, 103 (2014).

[17] J. He, X. Liu, Z. F. Sun and S. L. Zhu, Zc(4025) as the hadronic molecule with hidden charm, Eur. Phys. J. C 73, no. 11, 2635 (2013).

[18] W. Chen, T. G. Steele, M. L. Du and S. L. Zhu, $D^*\bar{D}^*$ molecule interpretation of Zc(4025), Eur. Phys. J. C 74, no. 2, 2773 (2014).

[19] C. Y. Cui, Y. L. Liu and M. Q. Huang, Could Zc(4025) be a $J^{PC} = 1^{++}D^*\bar{D}^*$ molecular state?, Eur. Phys. J. C 73, no. 12, 2661 (2013).

[20] K. P. Khemchandani, A. Martinez Torres, M. Nielsen and F. S. Navarra, Relating $D^*\bar{D}^*$ currents with $J^{PC} = 0^+, 1^+$ and $2^+$ to Zc states, Phys. Rev. D 89, no. 1, 014029 (2014).

[21] R. Molina and E. Oset, The Y(3940), Z(3930) and the X(4160) as dynamically generated resonances from the vector-vector interaction, Phys. Rev. D 80, 114013 (2009).

[22] A. Martinez Torres, K. P. Khemchandani, F. S. Navarra, M. Nielsen and E. Oset, Reanalysis of the $e^+e^- \rightarrow (D^*\bar{D}^*)^{\pm}\pi^\mp$ reaction and the claim for the Zc(4025) resonance, Phys. Rev. D 89, no. 1, 014025 (2014).

[23] K. Abe et al. [Belle Collaboration], Observation of $B \rightarrow J/\psi K_1(1270)$, Phys. Rev. Lett. 87, 161601 (2001).

[24] M. Tanabashi et al. [Particle Data Group], Review of Particle Physics, Phys. Rev. D 98, no. 3, 030001 (2018).

[25] L. R. Dai, G. Y. Wang, X. Chen, E. Wang, E. Oset and D. M. Li, The $B^+ \rightarrow J/\psi\phi K^+$ reaction and $D^*\bar{D}^*$ molecular states, Eur. Phys. J. A 55, no. 3, 36 (2019).

[26] E. Wang, J. J. Xie, L. S. Geng and E. Oset, Analysis of the $B^+ \rightarrow J/\psi K_1(1270)$ data at low $J/\psi\phi$ invariant masses and the X(4140) and X(4160) resonances, Phys. Rev. D 97, no. 1, 014017 (2018).

[27] S. Sakai, E. Oset and A. Ramos, Triangle singularities in $B^\pm \rightarrow K^\pm\pi^- D^\mp_{s1}$ and $B^\pm \rightarrow K^\pm\pi^- D^{*\mp}_{s1}$, Eur. Phys. J. A 54, no. 1, 10 (2018).

[28] L. S. Geng, E. Oset, L. Roca and J. A. Oller, Clues for the existence of two K(1270) resonances, Phys. Rev. D 75, 014017 (2007).

[29] G. Y. Wang, L. Roca and E. Oset, Discerning the two K(1270) poles in $D^0 \rightarrow \pi^+VP$ decay, Phys. Rev. D 100, no. 7, 074018 (2019).