Foot terrain impact modeling and motion performance research of hexapod robot based on passive elastic leg

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Abstract. In order to improve the performance of the multi-legged robot, the impact of foot-terrain is an important target to measure the performance of the robot. In order to ensure the compliance contact of foot-terrain for the legged robot, it is essential to add passive spring between the calf and foot. The mechanical structure makes the leg of hexapod robot retractable passive elastic structure. For multi-legged robots with passive elastic legs, the foot-terrain impact model is presented to measure the dynamic performance and provide theoretical support for the low impact force motion planning. The traditional planning method will cause additional impact for the switching case between the swing and stance phase. This paper presents a motion planning method that considers the deformable leg structure and impact performance. The experiments have been carried out, and the results have demonstrated the effectiveness of the foot terrain impact model and motion planning method in comparison with a conventional method.

1. Introduction

Legged robots have potential for maneuverability over uneven terrains. The robots are mainly classified into two groups: hydraulically driven and electrically driven robots [1]. Adaptive Suspension Vehicle [2] is an excellent hydraulically driven hexapod robot in discrete terrains. Irawan [3, 4] adopts an impedance control method to traverse uneven and sloped terrains, which was applied in COMET-IV. The BigDog is developed by Boston Dynamics to transport military supplies over complex terrains [5]. Kalakrishnan [6] presents architecture for controlling quadruped robot over challenging terrains. The electrically driven robot ATHLETE has been equipped with motion optimization algorithms to ensure walking with maximum stability margin [7, 8]. Electrically driven robot Rhex achieves fast locomotion to traverse different terrains with six actuators and it can move on complex terrains [9].

Legged robots have broad application prospects in the fields of rescue and disaster relief, however, the robots are currently not practically applicable. Legged robots have been greatly restricted by rugged terrain, impact and energy consumption. It is important to reduce the foot-ground impact force through the planning of the foot trajectory. The kinematics and motion planning methods of legged
robots are the prerequisites for achieving control, and the motion performance by optimizing the foot trajectory. According to the joint control input obtained from the foot end trajectory planning, the desired trajectory of the robot body is obtained, thereby realizing the autonomous movement of the robot.

Chettibi [10] proposes an optimal method to develop the motion trajectory for robotic systems; this method is effective in solving the optimal free motion planning problem (OFMPP) for robotic ankles, and the algorithms are applicable for different legged robots. Sakakibara [11] proposes a compound cycloid foot trajectory planning method, which solved the motion planning problem of a quadruped robot walking on uneven terrain. The study also discusses the issue of reducing the impact force during initial contact, and the trajectory is applied to the dynamic walking of a quadruped robot. Ruina [12] analyzes the impact force of the leg due to the collision during terrain contact. The energy loss generated by the collision is assumed to be proportional to the joint force impact.

Impact force is identified as a cause of energy loss in locomotion of both robots and animals [13]; it also has great effect on the stability locomotion of robots. Foot-terrain impact will cause impact changes for foot velocity and energy consumption increasing, but there are few studies about foot trajectory planning methods that consider foot-terrain mechanics [14]. The impact of passive elastic leg on the motion planning of the foot tip has been rarely studied, and there are no reports considering impact force modeling, in the locomotion of heavy-duty legged robots.

The aim of the study is to propose foot terrain impact model and develop a low impact force method for the hexapod robot with a passive compliant ankle. The main contributions of the work are as follows: 1) a foot terrain impact model is developed with consideration of the length of passive elastic leg; 2) a new algorithm for reducing impact force has been developed using a polynomial equation subject to velocity and acceleration constraints; and 3) extensive experimental studies of the proposed method have been carried out, and it has been compared with a conventional method in terms of various terrains.

This paper is organized as follows: Section 2 introduces the hexapod prototype, including the kinematics and foot terrain impact model. The foot terrain interaction mechanics and process analysis are formulated in Section 3. Finally, comprehensive experiments are presented in Section 4. The conclusions are given in Section 5.

2. Robot Kinematics and Foot Terrain Impact Model

2.1. Kinematics model

The kinematics model of the hexapod robot is based on the characteristics of legged insect animals, and is designed to achieve ability of omni-directional movement. The kinematics equations of three links including the coxa, thigh and shank are formulated based on the velocity and acceleration of the joints. As shown in figure 1, the robot is about 1.9m long, 2.1m wide, and 0.5m height, weighting approximately 330kg. The robot parameters are: radius of the trunk body(0.4m), coxa link(0.18 m), thigh link(0.5m), shank link(0.5m).

![Figure 1. The hexapod robot platform.](image)
The kinematics model of the six-legged robot is shown in figure 2 and figure 3. $\Sigma G$, $\Sigma B$ and $\Sigma L$ are the global, body, and leg coordinates of the hexapod robot. $\mathbf{X}B$ and $\mathbf{Y}B$ are the forward and lateral directions. As shown in figure 3, the parameters of joints are defined as angle of coxa link ($\alpha$), angle of thigh link ($\beta$), and angle of shank link ($\gamma$). The height between the coxa link and terrain is $H$; the lengths of the coxa, thigh, and shank are $l_c$, $l_t$, and $l_s$, respectively; and the length of the compliant ankle is $l_b$.

Frame of reference leg coordinates $\Sigma L$ and global robot coordinates $\Sigma B$ are obtained using the transformation matrix $BTL$ presented below:

$$
B^B_{TL} = \begin{pmatrix}
B^B_R & B^B_P \\
O & 1
\end{pmatrix} \in \mathbb{R}^{4\times4}
$$

(1)

The position of the foot tip $^L_{B}P_L$ can be determined by forward kinematics with respect to the values of joint angles as follows:

$$
^B_{PL} = ^B_{TL} \begin{pmatrix}
((l_s + l_b)\cos(\gamma - \beta) + l_t\cos\beta + l_c)\sin\alpha \\
((l_s + l_b)\cos(\gamma - \beta) + l_t\cos\beta + l_c)\cos\alpha \\
H + l_s\sin\beta - (l_s + l_b)\sin(\gamma - \beta) \\
1
\end{pmatrix}
$$

(2)
Depending on the position of foot tip, the joint velocity $\dot{\theta}$ and acceleration $\ddot{\theta}$ equations for the locomotion can be expressed as follows:

$$
\begin{align*}
\begin{bmatrix} \dot{\theta} \end{bmatrix} &= \left[J\right]^{-1} \begin{bmatrix} \dot{\theta}_p \end{bmatrix}, \\
\begin{bmatrix} \ddot{\theta} \end{bmatrix} &= \left[J\right]^{-1} \begin{bmatrix} \ddot{\theta}_p \end{bmatrix} + \left[J\right]^{-1} \begin{bmatrix} \dddot{\theta}_p \end{bmatrix}
\end{align*}
$$

where the position velocity vector is $\begin{bmatrix} \dot{\theta}_p \end{bmatrix} = \begin{bmatrix} -v_x & -v_y & 0 \end{bmatrix}^T$, and the joint velocity vector is $\begin{bmatrix} \dot{\theta} \end{bmatrix} = \begin{bmatrix} \alpha & \beta & \gamma \end{bmatrix}^T$.

2.2. Impact model

Foot terrain interaction is mainly divided into normal and tangential directions, but the previous researches mainly focused on static foot terrain loading experiments. The contact model is analyzed for the foot terrain dynamic contact, and the impact model of the foot terrain mechanics is obtained. When the leg contacts the ground with velocity, impact will cause due to the velocity of the landing stance phase. The process of foot terrain contact impact is shown in figure 4.

![Figure 4. Foot terrain interaction model.](image)

When the foot contacts with terrain, the fixed ground gives the leg a restraining force to prevent the foot from sinking into the ground. Normal contact force $F_n$ along the vertical contact direction, a scalar $\lambda$ is added in the normal direction, which is defined as the unknown ground contact force level.

$$
F_n = n\lambda
$$

The reaction force is used to maintain foot contact, and it needs to meet the constraint: $\lambda \geq 0$. Define a variable $\zeta$ that represents the velocity of contact separation in the normal direction, which can be expressed as:

$$
\zeta = n \cdot v
$$

Since hexapod robot with a passive elastic leg needs continuously lift and land during walking, the foot terrain contact is not always stable during the contact process. First, the foot with a passive spring falls from the air to the ground and collides and contacts for stance phase. Second, when the mass center of robot is in locomotion, the foot is subjected to different forces at different load positions. It will lead to uneven distribution of foot force or cause slippage between the foot and the ground; finally, the foot lifts from the ground back to the air for swing phase, and completes the entire process of the swing phase and support phase of the robot locomotion. The acceleration of foot terrain contact separation can be computed as:
\[ \dot{\zeta} = \frac{d}{dt}(n \cdot v) = n \cdot a + \dot{n} \cdot v \]  

(7)

The velocity of foot terrain separation is determined by the velocity of the trunk body and the different shapes of the contact surface. Then the simplified separation acceleration can be obtained as:

\[ \ddot{\zeta} = n \cdot \Gamma^{-1}(F_c + f - v \times I v) + \dot{n} \cdot v \]  

(8)

If the terrain is hard material, a large impact force will be generated in a short time \( \delta t \to 0 \), and the impact force \( F_c / \delta t \) will become large in a short time:

\[ I = \lim_{\delta t \to 0} \left[ t \right] t_{t + \delta t}^{t + \delta t} \frac{F_c(t)}{\delta t} dt \]  

(9)

The locomotion velocity of the foot changes step by step, which results in foot contact impact. The next step is to derive the locomotion and impact equation of the robot's single leg. The foot terrain impact force is generated by the inertia \( I \) and velocity \( v \) of the leg movement. \( \Delta v \) is velocity change for the impact force \( t \):

\[ \Delta v = \lim_{\delta t \to 0} v(t + \delta t) - v(t) \]

\[ = \lim_{\delta t \to 0} \int_{t}^{t+\delta t} a(t) dt \]

\[ = \lim_{\delta t \to 0} \int_{t}^{t+\delta t} \Gamma^{-1}(F_c(t) - v \times I v) dt \]

\[ = \lim_{\delta t \to 0} \Gamma^{-1}(I \int_{t}^{t+\delta t} F_c(t) dt - \lim_{\delta t \to 0} \int_{t}^{t+\delta t} (I^{-1}v \times I v) dt) \]

\[ = I^{-1} - \lim_{\delta t \to 0} \int_{t}^{t+\delta t} (I^{-1}v \times I v) dt \]  

(10)

The relation between foot terrain impact force and foot velocity is obtained as:

\[ t = I \Delta v + \lim_{\delta t \to 0} \int_{t}^{t+\delta t} (v \times I v) dt \]  

(11)

Finally, foot terrain impact model based on foot velocity is obtained, and the relationship between the impact force and the foot terrain velocity is established.

3. Foot Terrain Interaction Process Analysis

Foot-terrain interaction model is for analyzing the relationship between the terrain property parameters and the feet parameters for the normal force and tangential force. Ding et al. [14] proposed a three-dimensional foot-terrain interaction mechanics model, which was used for the normal and tangential forces between the foot and terrain. The soft ground is subsided and deformed under foot terrain locomotion. In order to represent the force-sinkage relationship on the soft terrain, the Bekker pressure model is used to meet the pressure characteristics of the soft terrain:

\[ \sigma = (k_c / b + k_p) \cdot z^n \]  

(12)

where \( \sigma \) is normal force, \( k_c \) and \( k_p \) are equivalent stiffness coefficients of the foot and terrain. \( b \) is radius of disc-shaped flat feet, \( n \) is the exponent of the damping terms, \( z \) is the summation deformation of foot and terrain in the normal direction.

The traction generated by legged robot when walking on a soft ground can be affected by the shear strength of terrain. Therefore, the shear characteristics of terrain are important factors that affect the
robot's walking and foot slippage. The relationship between shear force and shear displacement can be expressed by the Janosi terrain shear model. According to the model, the relationship between the tangential force and shear displacement can be obtained as:

$$\tau = (c + \sigma \tan \phi) \cdot (1 - e^{-j/K})$$  \hspace{1cm} (13)

where $\tau$ is tangential force, $j$ is shearing trajectory order, $z$ is shape of foot velocity constant at $z = 0$, $\sigma$ is the modulus of shearing displacement.

The foot shape of the hexapod robot used in this paper is a disc-shaped flat foot. The normal and tangential forces can be obtained according to the bearing and shearing force:

$$\begin{cases}
F_N(t) = \sigma(t)S = (k_e \pi b + \pi b^2) \cdot (z(t) - z(0))^a \\
F_T(t) = \tau(t)S = (\pi b^2 c + F_N(t) \tan \phi) \cdot (1 - e^{-j(t-t(0))/K})
\end{cases}$$  \hspace{1cm} (14)

The purpose of introducing the interaction model of tangential direction is to illustrate that the shearing displacement of the foot on the terrain will bring impact in the tangential plane. As the deformable foot slips on the terrain, the tangential force increases with the shearing displacement of foot and its velocity, and changes to a stable value when the velocity decreases to constant value. The method considers the slipping of the deformable foot to avoid impact force in the normal and tangential direction. And the process of interaction mechanics with elastic leg is formulated based on the foot-terrain interaction model. The process of foot-terrain interaction with elastic leg is shown in figure 5.

![Figure 5. Process of foot–terrain interaction with elastic leg.](image)

As figure 5 showed, the velocity of the foot in normal and tangential direction are $V_N$ and $V_T$. When the deformable foot contacts the terrain, there is still velocity in the tangential direction and makes the slipping of the foot. The lifting of the foot experiences the slipped owing to the elastic leg, since the foot will have moved forward before the deformation of the foot decreases to zero, and it will experience the impact force. Therefore, it is essential to consider the deformation of the foot for dealing with the impact force.

4. **Foot Motion Planning Considering Impact Characteristics**

Motion planning in the normal direction obtains the required trajectory of lifted and contacted of the legs. In order to reduce the impact force at the time of switching phases and control the velocity and acceleration of the foot tip. $Z_{L_i}$ is the trajectory of the foot tip in the normal direction. In order to avoid the slipping impact in the normal and tangential directions of the foot, length of the passive spring should be considered in the normal foot trajectory planning.

The position constraints of the foot trajectory in the normal direction are defined as:

$$\begin{cases}
Z_{L_i}(0) = H_0 \\
Z_{L_i}(T_i) = h \\
Z_{L_i}(T / 2) = H_S
\end{cases}$$  \hspace{1cm} (15)
where $H_s$ is the starting position for swing phase; $h$ is the length of passive spring at instant $T_1$, and $H_s$ is the swing height. The velocity and acceleration constraints of the normal trajectory are given by:

$$
\begin{align*}
\dot{Z}_{Li}(0) &= \dot{Z}_{Li}(T/2) = \dot{Z}_{Li}(T) = 0 \\
\dot{Z}_{Li}(T_1) &= v_p \\
\dot{Z}_{Li}(T_2) &= -v_p
\end{align*}
$$

where $v_p$ is the velocity when the foot reaches the passive spring; $T_1$ and $T_2$ are the instants when the foot reaches the position of elastic leg.

The normal trajectory with the polynomial coefficients for swing phase $(0, T/2)$ is expressed as:

$$
Z_{Li}(t) = a_0^i + a_1^i t + a_2^i t^2 + a_3^i t^3 + a_4^i t^4 + a_5^i t^5 + a_6^i t^6 \quad t \in (0, T/2)
$$

The coefficients of the polynomial in normal direction are calculated as:

$$
\begin{bmatrix}
a_0^i \\
a_1^i \\
a_2^i \\
a_3^i \\
a_4^i \\
a_5^i \\
a_6^i \\
\end{bmatrix} = 
\begin{bmatrix}
0 \\
0 \\
0 \\
\frac{hT^6 - 15H_iT^2T_1^4 + 24HT_iT_1^5 - 10HT_i^6}{T^3T_1(T-T_1)^3} \\
\frac{3(-hT^6 + 5HT_1^2T_1^3 - 9HT_iT_1^5 + 5HT_i^6)}{T^4T_1(T-T_1)^3} \\
\frac{-3(-hT^6 + 8HT_1^2T_1^3 - 9HT_iT_1^4 + 2HT_i^6)}{T^5T_1(T-T_1)^3} \\
\frac{-hT_1^3T_1(T-T_1)^3}{T^5T_1(T-T_1)^3} \\
\frac{hT^3 + 15HT_1^2T_1^3 - 15HT_iT_1^4 + 6HT_i^5}{T^4T_1(T-T_1)^3}
\end{bmatrix}
$$

$T$ is the gait cycle for the swing and stance phase. Combining the above equations, the curve of the foot position for swing and stance phase in the normal direction is as shown in figure 6. The velocity and acceleration equations for swing phase of the foot tip in normal direction can be calculated.

The authors compares the conventional and proposed method, and the foot acceleration plots are plotted as shown in figure 7.

![Figure 6. Foot trajectory in normal direction.](image-url)
5. Experimental Validation

5.1. Experimental Setup
The hexapod prototype walking on blanket terrain and passive elastic leg are shown in figure 8. A spring mechanism is mounted to make the leg elastic.

![Hexapod robot walking on blanket terrain and spring mechanism.](image)

Figure 8. Hexapod robot walking on blanket terrain and spring mechanism.

The robot is equipped with manipulation and locomotion computers. A foot is mounted on an F/T sensor with a tangential full-scale of 660 N and normal full-scale of 1980 N. This sensor is used to measure the normal and tangential forces.

5.2. Comparison of interaction forces between with and without elastic leg
The tripod gait is the fast gait for the hexapod robot, and the tripod gait has adopted for the comparison of forces on tile terrain. The swing phase is classified into two groups as \{leg1, leg3, leg5\} and \{leg2, leg4 leg6\}. A comparison of the normal force with and without spring mechanism is shown in figure 9.

![Comparison of normal forces between with and without elastic mechanism.](image)

Figure 9. Comparison of normal forces between with and without elastic mechanism.
It can be seen that the normal force without elastic leg at switching case is 1401 N. However, the normal force with elastic leg at switching case is 1200 N. The decreased rates of normal force is 14%. Thus the elastic leg is essential for the legged robot to reduce additional impact force.

5.3. Comparison of tangential forces
The impact forces in the tangential direction are compared between the two methods on the hard and deformable terrains. The proposed method considers the elastic leg into the algorithm.

The comparison of tangential forces for hard terrain are as shown in figure 10. Figure shows that the tangential forces in the switching phase with the two methods are 74 N and 50 N, respectively.

![Figure 10. Comparison of tangential forces on hard terrain.](image1)

The comparison of tangential forces for deformable terrain are as shown in figure 11. The tangential forces in the switching phase with the two methods are 260 N and 120 N.

![Figure 11. Comparison of tangential forces on deformable terrain.](image2)

5.4. Comparison of normal forces
Experiments have been carried out on various terrains. Tripod gait is adopted for comparing the normal forces between two methods on the tile, rubber and soft blanket terrains.

![Figure 12. Comparison of normal forces on tile terrain.](image3)
Figure 12 shows the normal forces in the switching phase on tile terrain between the two methods are 858 N and 740 N, respectively.

![Figure 13](image_url)

**Figure 13.** Comparison of normal forces on tile terrain.

Figure 13 shows the normal forces in the switching phase on rubber terrain between the two methods are 690 N and 620 N, respectively.

![Figure 14](image_url)

**Figure 14.** Comparison of normal forces on soft blanket terrain.

Figure 14 shows the normal forces in the switching phase on soft blanket terrain between the two methods are 564 N and 545 N, respectively. Since the stiffness coefficients of the tile, rubber and soft blanket terrains are different; the decreased rates of the normal forces on hard terrain is reduced greater than the deformable terrains.

6. Conclusion
In this study, we have investigated foot terrain impact model and motion planning considered passive elastic leg aimed at reducing the impact force of a hexapod robot on different terrains. Foot terrain interaction process has been analyzed. The validity of the proposed method is evaluated by using a hexapod prototype. Several conclusions are drawn as follows: (1) Foot terrain impact model is proposed with consideration of the interaction process for passive elastic leg; (2) Motion planning has been developed for hexapod robot with elastic leg to avoid normal and tangential impact forces; (3) Comprehensive experiments for impact modeling and motion planning are carried out, and the results validate the theoretical analysis. The high tracking performance for the contact interaction process will be considered in the future.

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