Giant Unification Theory of the Grand Unification and Gravitation Theories

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Abstract

Because the grand unification theory of gauge theories of strong, weak and electromagnetic interactions is based on principal bundle theory, and gravitational theory is based on the tangent vector bundle theory, so people cannot unify these four basic interactions in principal bundle theory. This Letter discovers and gives giant unification theory of the grand unification theory and gravitation theory, i.e., the giant unification theory of strong, weak, electromagnetic and gravitational interactions according to the general fiber bundle theory, symmetry, quantitative causal principle (QCP) and so on. Consequently, the research of this Letter is based on the rigorous scientific bases of mathematics and physics. The Lagrangians of the well-known fundamental physics interactions are unifiedly deduced from QCP and satisfy the gauge invariant principle of general gauge fields interacting with Fermion and/or boson fields. The geometry and physics meanings of gauge invariant property of different physical systems are revealed, and it is discovered that all the Lagrangians of the well-known fundamental physics interactions are composed of the invariant quantities in corresponding spacetime structures. The difficulties that fundamental physics interactions and Noether theorem are not able to be unifiedly given and investigated are overcome, the unified description and origin of the fundamental physics interactions and Noether theorem are shown by QCP, their two-order general Euler-Lagrange Equations and corresponding Noether conservation currents are derived in general curved spacetime. Therefore, using the new unification theory, a lot of research works about different branches of physics etc can be renewedly done and expressed simpler with clear quantitative causal physical meanings.
1 Introduction

Causal principle should be satisfied in expressing physical laws. In quantum field theory, the causal principle is reflected in the results that if the square of the distance of spacetime coordinates of two boson (or fermion) operators is timelike, the canonical commutator (or anticommutator) of the boson (or fermion) field does not vanish, i.e., their measures are coherent, but not coherent for spacelike case [1]. Dispersion relations are deduced by means of the causal principle etc [2]. Ref. [3] studied the relationships of causal principle and symmetry principle.

It is well known that in physics the well-admitted physical law not satisfying causal principle has not been found. In terms of the condition that causal principle is quantitative, i.e., quantitative causal principle (QCP), Ref. [4] gives a unified theory of all differential variational principles, and a unified theory of all integral variational principles are achieved [5]. The investigations about the generalization from classical statistical mechanics to quantum mechanics still satisfy quantitative causal principle [6]. Refs. [4] 5 7 show the quantitative causal principle of gain, loss and transformation of any system, and prove that the equivalent causes must result in the equivalent results, which is just the invariant property of some operations, namely, symmetry properties [8, 9]. In fact, the symmetric properties in physics and mathematics are just the invariant properties under some kinds of operations of systems [9], almost all physics processes, e.g., Refs.[10, 11], satisfy causal relations because causal relations should be satisfied in the universe, the relation between symmetries and QCP is given [4][5], and using QCP, Ref. [8] gives unification theory of different causal algebras and its applications to theoretical physics, in which group theory is included as a special subset.

Utilizing the no-loss-no-gain homeomorphic map transformation satisfying QCP, Ref. [7] overcomes the non-perfect properties of the Volterra process, gains the exact strain tensor formulae in condensed theory, and solves the hard difficulty to give the really serious one-to-one correspondences between the two theories of dislocations in Euclidean space and Weitzenböck Material Manifolds. Ref. [8] gives the unified origin of different variational principle and Noether theorem in terms of QCP. Now it is well known that Noether theorem is deduced from variational principle and group theory [12][13][14], while variational principle and Noether theorem may be derived by QCP [5]. It can be seen that this Letter and Refs. [4] [5] [7] are essential developments of the investigations of Ref. [8] about the relationships of causal principle and symmetry principle. It is well known that physical fundamental interactions and Noether theorem are used to be regarded as independent fundamental laws being not able to be unifiedly proved and studied up to now, but in the following investigations we show that the two laws are just the deductions of QCP and give the unified
description of the two laws.

Many Refs. [15, 16, 17, 18, 19] studied causal field propagation that led to neat results, and these results are interesting and also satisfy QCP.

In fact, superstring theory has gotten many important developments [20, 21, 22], and Ref. [23] very well investigated Quantum modification of general relativity and so on. But it is well known that there are still some key problems in various kinds of the grand or supersymmetric unification theories up to now, e.g., there are too many adjustable free parameters etc in various kinds of the grand or supersymmetric unification theories, which are not natural, even superstring theories and M-theory have themselves unsolved key problems, e.g., we still cannot seriously break the high dimensional theories down to four dimensions and do rigorous phenomenology etc. There are different unification theories, e.g., from up to down (superstring) or from down to up (phenomenological) unification theories, unification theory of this Letter belongs to from down to up unification theory. Therefore, exploring the other possible unification theories is actually needed, which may activate to understand and solve all the problems finally, that is, we may try to solve all the problems from different aspects, which may finally activate development and perfection of the various kinds of the unification theories, e.g., superstring theories, M-theory and so on.

The arrangement of this Letter is: Sect. 2 is a new unification theory of fundamental physics interactions, Sect. 3 is the unified description of fundamental physics interactions and Noether theorem, Sect. 4 is applications of this theory, and Sect. 5 is summary and conclusion.

2 A New Unification Theory of Fundamental Physics Interactions

For the convenience of research, we simply review the proof of QCP. In physics, quantitative action (cause) of some quantities must lead to the equal action (result), i.e., how much lose (cause), how much gain (result), which is just QCP deduced from the no-loss-no-gain principle in the universe [4, 5] and it may be concretely expressed as

$$DS - CS = 0$$ (1)

Eq.(1) means that any quantitative action produced by operator set D acting on S must lead to appearance of set C acting on S so that DS equate CS, where D and C may be different operator sets, the whole process satisfies QCP so that right hand side of Eq.(1) keeps zero, i.e. satisfies the no-loss-no-gain principle in the universe [4]. Thus, Eq.(1) is viewed as a mathematical expression of QCP. Eq.(1) is very useful for the following studies. Eq.(1)’s nonlinear expression refers to Ref. [8].

All physics laws must satisfy causal principle and the causal principle must be quantitative. Furthermore, up to now, all the physics laws violating causal principle have not been found, it can be seen, using the quantitative causal
principle we shall be able to give the giant unification theory. Consequently, the principle is useful and important.

For a 4-dimensional physical general curved spacetime $M^4$ (there naturally may exist the constructions of vector bundle $E(M^4, F, \pi, G)$ or associate vector bundle $E(M^4, F, \pi_v, G, P(M^4, G, \pi))$ of principal bundle $P(M^4, G, \pi)$. Readers, not familiar with manifold and fibre bundle, don’t need to read the Italic type parts in parenthesis, which will not affect their understanding on the Letter, as the same below), taking $S_V$ as a general basic vector field in any open neighborhood $U_V$ in $M^4$, $D$ as differential connection operator, $C$ as connection $\omega_V$ in Eq.(1), it follows that [24, 25]

$$DS_V = \omega_V S_V$$  \hspace{1cm} (2)

Eq.(2)’s physical meaning is that the quantitative physical effect operator $D$’s acting on $S_V$ must equate that the gauge field or connection $\omega_V$ times $S_V$ (in which $S_V$ may be a basic vector of principal bundle whose base manifold is $M^4$ or a basic vector on $M^4$ of vector bundle).

Substituting the transformation $S_V = A_{VU}S_U$ into Eq.(2), it follows that [24]

$$DS_V = dA_{VU}S_U + A_{VU}\omega_US_U = \omega_V S_V$$  \hspace{1cm} (3)

where $DA_{VU} = dA_{VU}$, because $A_{VU}$ is the matrix function transforming $S_U$ into $S_V$ between two open neighborhoods $U_U$ and $U_V$ ($U_U \cap U_V \neq 0$) on $M^4$.

Substituting $A_{VU}^{-1}S_V = S_U$ into Eq.(3), we obtain the relation transforming connection $\omega_U$ into connection $\omega_V$ as follows [24]

$$\omega_V = dA_{VU}A_{VU}^{-1} + A_{VU}\omega_UA_{VU}^{-1}$$  \hspace{1cm} (4)

For gauge field (in principal bundle), it is just gauge transformation between gauge fields $\omega_U$ and $\omega_V$ in two open neighborhoods $U_U$ and $U_V$ ($U_U \cap U_V \neq 0$) in gauge field theory; for connection (in vector bundle), it is the transferring relation between connections $\omega_U$ and $\omega_V$ in two open neighborhoods $U_U$ and $U_V$ ($U_U \cap U_V \neq 0$) in curved spacetime, i.e., Eq.(4) is just the unified transformation expression of gauge field and connection in two open neighborhoods $U_U$ and $U_V$, thus we can generally call the unified transformation as general gauge transformation.

Using Eq.(4), we have $\omega_V A_{VU} - A_{VU}\omega_U = dA_{VU}$, and further acting the differential connection operator $D$ on Eq.(3), we obtain not only the unification expression of curvature tensor 2-form (in tangent vector bundle) and field strength tensor 2-form (in principal bundle) but also the unification expression of general coordinate transformation of curvature tensor 2-form (in tangent vector bundle) and gauge transformation of field strength tensor 2-form (in principal bundle) as follows

$$\Omega_V = A_{VU}\Omega_U A_{VU}^{-1} = d\omega_V - \omega_V \Lambda \omega_V$$  \hspace{1cm} (5)

In fact, Eqs.(4 & 5) show that gravity is also a kind of gauge theory.
Multiplying Eq.(5)'s component quantity expression of the curvature tensor 2-form (in tangent vector bundle)

\[
\frac{1}{2} \Omega^b_{a ij} dx^j \Lambda dx^i = \frac{1}{2} A^t_a \Omega^t_{i j} \varepsilon^{-1} b_{i j} \Lambda dx^j
\]  

(6)

with \( g^{nk} \varepsilon_{b kl m} dx^l \Lambda dx^m \) from right hand side, in which the a, b, c, \( \cdots \), i, j, k, \( \cdots \) and \( a', b', c', \cdots, i', j', k', \cdots \) are in \( U_V \) and \( U_U \) respectively, defining \( \varepsilon_{1234} = (-g)^{\frac{1}{2}} \) \cite{26} & \( x^4 = i c t \) and using \( \varepsilon_{i1234} \varepsilon^{j123j4} = s g n (g) \delta_{i1234} \) \cite{20}, we prove

\[
\Omega^b_{a ij} (-g)^{\frac{1}{2}} dx^l \Lambda dx^i \Lambda dx^4 = \Omega^b_{a ij} (-g)^{\frac{1}{2}} dx^l \Lambda dx^j \Lambda dx^3 \Lambda dx^4
\]  

(7)

Namely, Eq.(7) is invariant or doesn’t depend on coordinates of different \( U_V \) and \( U_U \).

Adding integral sign to Eq.(7) and, as usual, defining \( \Omega^b_{a ij} = g^{bc} R^a_{cab} = R_V, \Omega^a_{b ij} = R_U \) and \( d \tau = dx^l \Lambda dx^j \Lambda dx^3 \Lambda dx^4 \), we obtain the invariant gravitational action \cite{27}

\[
A = \int_{U_U \cap U_V} R_U (-g)^{\frac{1}{2}} d \tau_U = \int_{U_U \cap U_V} R_V (-g)^{\frac{1}{2}} d \tau_V
\]  

(8)

Thus, Eq.(8) keeps invariant in the whole spacetime (manifold \( M^4 = \bigcup U_V \)), because the two open neighborhoods \( U_U \) and \( U_V \) are two arbitrary open overlapped neighborhoods in the global spacetime (manifold in the vector bundle). The above condition (8) is just the condition that Eq.(8) may be taken as invariant gravitational action in the whole spacetime, because the physical consistence demands that Eq.(8) must take the invariant formulation in every open neighborhood of the whole spacetime, which satisfies just the general gauge invariant principle of gravitational gauge fields. Then geometry and physics meanings of gauge invariant property are directly, whole and seriously revealed.

A Noether invariant quantity is obtained from Eq.(5) in terms of 2-form field strength tensor (in principal bundle) as follows \( tr(\Omega_V \Lambda \Omega_V) = tr(\Omega_U \Lambda \Omega_U) \), which satisfy just the general gauge invariant principle \cite{28} of general gauge fields. Then geometry and physics meanings of gauge invariant property of general gauge fields are directly, whole and seriously revealed.

Adding integral sign to \( tr(\Omega_V \Lambda \Omega_V) \) and using \( tr(T^a T^b) = -2 \delta^{ab} \), we have the invariant action with dual gauge field strength tensor in the global curved spacetime as follows \cite{27}

\[
A = \int \frac{1}{2 \pi^2 \kappa} tr(\Omega \Lambda \Omega) = - \int \frac{1}{2 \pi^2 \kappa} \Omega_{aij} \Omega^{aij} (-g)^{\frac{1}{2}} d \tau
\]  

(9)

Thus the Lagrangian density is \( \mathcal{L} = \frac{(-g)^{\frac{1}{2}}}{2 \pi^2 \kappa} \Omega_{aij} \Omega^{aij} \) = \(-\frac{(-g)^{\frac{1}{2}}}{4q^2} \Omega_{aij} \Omega^{aij} \) ( \( \Omega = \frac{1}{2} \varepsilon^{ijkl} \Omega_{ijkl} \) is dual gauge field strength tensor, taking \( \kappa = 2q^2/\pi^2 \) ) with dual gauge field strength tensor existing in the global spacetime \cite{25, 27}. As Eq.(9)’s
natural generalization, for higher dimensions, we may naturally deduce $A = \int \frac{1}{2 \pi \kappa} \text{tr} (\Omega \Lambda \Omega \ldots \Lambda \Omega)$.

In terms of modern differential geometry, we can define the inner product $< \partial_{\mu}, \partial_{\nu}> = \langle e_{\mu}, e_{\nu} > = g_{\mu\nu}$ of natural (or called intrinsical) tangent basic vectors and the inner product $< dx^\mu, dx^\nu > = \langle e^\mu, e^\nu > = g^{\mu\nu}$ of natural (or called intrinsical) cotangent basic vectors, they are consistent, see Appendix A, then we obtain an important invariant $< \omega_{i \mu} dx^\mu, \omega_{j \nu} dx^\nu > = \omega_{i \mu} \omega_{j \nu} g^{\mu\nu} = \omega_{i \mu} \omega_{j \mu}$ of inner product of two 1-forms (e.g., $\omega_V = \omega_V^\mu dx^\mu$), i.e., cotangent vectors, under coordinate transformation. Similar to deducing the important invariant $\omega_{i \mu} \omega_{j \mu}$ of inner product of two 1-forms, see Appendix B we finally achieve the invariant action of general gauge fields in the global curved spacetime as follows

\[
A = \int \frac{1}{2 \pi \kappa} \text{tr} < \Omega, \Omega > (-g)^{\frac{1}{2}} d\tau = \int \frac{1}{4 \pi \kappa} \Omega_{ij}^a \Omega^{aij} (-g)^{\frac{1}{2}} d\tau \quad (10)
\]

Eq.(10) satisfies just the general gauge invariant principle in the global spacetime.

When $S_{Ui} \ (i = 1, 2, 3, \ldots, k; U$ is subscript of the open neighborhood $U_U)$ are $k$ linearly independent basic vectors of the vector space (of vector bundle or associate vector bundle $E(M^4, F, \pi_V, G, P(M^4, G, \pi)))$, or equivalently the orthonormal basic vectors of the Group G’s representation vector space $F$. Therefore, any vector field $S$ may be expressed as

\[
S = S_U \Psi_U \quad (11)
\]

where $\Psi_U$ is complex column matrix function of the corresponding components. It follows from Eq.(2) that

\[
D' S_{Ui} = \omega'_{Uij} S_j = -S_j \omega'_{Uji} \quad (12)
\]

in which we have remarked

\[
D' = \gamma^\mu D_\mu, \quad d' = \gamma^\mu \partial_\mu, \quad \omega'_{Uij} = \gamma^\mu \omega_{U\mu ij} \quad (13)
\]

where $\gamma^\mu$ is $\gamma$ matrix with Lorentz superscript $\mu$ in Eq.(13), and $dx^\mu = a^\mu_{\nu} dx^\nu$ ($\mu, \nu = 0, 1, 2, 3$) have the same transformation law as $\gamma' = a_{\mu}^\nu \gamma^\nu \quad [29], \quad a^\mu_{\nu}$ is Lorentz group matrix element. Because the $i, j, \ldots, k$ belong to the different freedoms' groups, thus, their matrixes corresponding to different groups are commutable. From Eqs.(2), (11) and (12) we have

\[
D' S = \omega'_{U} S_{U} \Psi_U + S_{U} d' \Psi_U = S_{U} (d' \Psi_U - \omega'_{U} \Psi_U) \quad (14)
\]

Using the orthonormal relation (we can choose the orthonormal basic vectors of the vector space)

\[
(S_{Ui}, S_{Uj}) = \delta_{ij} \quad (15)
\]

we obtain
\[ (\mathcal{S}, D'S) = (\nabla_U S_U, S_U(d\Psi_U - \omega_U\Psi_U)) = \nabla_U(d'\Psi_U - \omega'U\Psi_U) \] (16)

Now consider the expression of Eq.(16) in an open neighborhood \( U_W \). It follows from \( \Psi_U = A_{UW}\Psi_W \) that

\[ d'\Psi_U = d'A_{UW}\Psi_W + A_{UW}d'\Psi_W \] (17)

Using Eqs.(4), (16), (17) and \( \Psi_U = A_{UW}\Psi_W \) (\( A_{UW} \) is transformation matrix), we have

\[ \nabla_U(d'\Psi_U - \omega'U\Psi_U) = \nabla_W A_{UW}^+[d'A_{UW}\Psi_W + A_{UW}d'\Psi_W] \]

where we have used \( A_{UW}^+A_{UW} = I \). Namely Eq.(18) is the invariant quantity in the whole spacetime (manifold in the bundle), and the physical consistence demands that Eq.(18) must take the invariant formulation in the every open neighborhood of the whole spacetime, which satisfies just the general gauge invariant principle of the gauge fields interacting with Fermi fields. Then geometry and physics meanings of gauge invariant property of gauge fields interacting with Fermion fields are directly, whole and seriously revealed.

Inserting Eq.(13) into Eq.(18), we obtain the general topological invariant Lagrangians of matter field \( \Psi \) interacting with gauge field \( \omega_{\mu} \), which keeps effective in the whole spacetime as follows

\[ L_{\Psi} = \nabla_{\gamma^\mu}(\partial_{\mu}\Psi - \omega_{\mu}\Psi) \] (19)

Now we consider another invariant.

Similar to the discussion of Eqs.(14 & 18), taking \( D = dx^\mu D_{\mu} \), \( d = dx^\mu\partial_{\mu} \), \( \omega_{Uij} = dx^\mu\omega_{Uij} \), replacing spinor field \( \Psi_U \) with scalar field \( \varphi \) in Eq.(14), it follows that \( DS = S_U(d\Psi_U - \omega_U\Psi_U) \), further taking Hermite conjugation of \( DS \), and making an inner product, relevant to \( (S_{Uj}, S_{Uj}) = \delta_{ij} \) and \( <dx^\mu, dx^\nu> \), of both \( DS \) and its conjugation, we get

\[ <(\overline{DS}, DS) >= <(\overline{\nabla_U(d - \omega_U))S_U, S_U(d\varphi_U - \omega_U\varphi_U))> >= \nabla_U(\partial_{\mu} - \omega_{\mu}U)dx^\mu, dx^\nu(\partial_{\nu}\varphi_U - \omega_{\nu}U\varphi_U) > \] (20)

Analogous to the discussion of Eq.(18), it is easy to prove that Eq.(20) is the invariant quantity existing in the whole spacetime.

In terms of modern differential geometry, we again use the inner product \( <dx^\mu, dx^\nu> =<e^\mu, e^\nu> = g^{\mu\nu} \) of natural cotangent basic vectors, then we achieve an important invariant Lagrangian of scalar fields interacting with gauge fields
\[ \mathcal{L}_\varphi = \langle (DS, DS) \rangle = \overline{\varphi} (\partial_\mu - \omega_\mu) g^{\mu\nu} (\partial_\nu - \omega_\nu) \varphi \]

\[ = \overline{\varphi} (\partial_\mu - \omega_\mu) (\partial_\nu - \omega_\nu) \varphi \]  
(21)

Thus, Eq.(21) satisfies the physical invariant consistent demand in the every open neighborhood of the global spacetime. That is, Eq.(21) satisfies just the general gauge invariant principle of the gauge fields interacting with scalar fields. Then geometry and physics meanings of gauge invariant property of gauge fields interacting with Boson fields are directly, whole and seriously revealed.

On the other hand, due to

\[ (\overline{S}, S) = (\overline{\varphi U} S U, S U \varphi U) = \varphi U \varphi U \]  
(22)

and using \( \varphi U = A_{UW} \varphi W \), it is easy to prove that Eq.(22) is invariant quantity existing in the whole spacetime. Thus we may multiply Eq.(22) with mass parameter \( m^n \) ( \( n = 2 \), taking \( \varphi \) as boson function; \( n = 1 \), taking \( \varphi \) as fermion function ) to construct the mass part of the Lagrangian of a system.

\[ \mathcal{L}_{m\varphi} = m^n \overline{\varphi} \varphi, \]  
(23)

About potential functional of \( (\overline{\varphi U}, \varphi U) \) due to the invariance of \( (\overline{\varphi U}, \varphi U) \) under the general gauge transformation, it must satisfy

\[ V_U (\overline{\varphi U}, \varphi U) = V_W (\overline{\varphi W}, \varphi W) \]  
(24)

which is the invariant condition in the global spacetime, i.e., Eq.(24) satisfies just the general gauge invariant principle of potential energy. For example, Eq.(22)’s arbitrary combinations may satisfy condition (24), i.e., we may generally take Eq.(22) as the variable to construct scalar potential functional. e.g., the potential functional \( V \) of SU(2) complex scalar fields

\[ V (\overline{\varphi \varphi}) = \lambda^2 (\overline{\varphi \varphi} - \mu^2)^2. \]  
(25)

Using Eqs.(8), (10), (19), (21), (23) and (24), we achieve gravitational action, all general actions of matter fields, spinor fields and scalar fields interacting with gauge fields in the curved spacetime as follows

\[ A = \int_{M^4} (\alpha R + \mathcal{L}_m)(-g)^{\frac{1}{2}} d\tau \]  
(26)

\[ A = \int_{M^4} [\alpha R + \overline{\Psi} \gamma^\mu (\partial_\mu \Psi - \omega_\mu \Psi) + m \overline{\Psi} \Psi + U (\overline{\Psi}, \Psi) - \frac{1}{4q^2} F_{\mu\nu} \lambda a F^{\lambda a \mu\nu} ](-g)^{\frac{1}{2}} d\tau \]  
(27)

\[ A = \int_{M^4} [\alpha R + \overline{\varphi} (\partial_\mu - \omega_\mu) (\partial_\nu - \omega_\nu) \varphi + m^2 \overline{\varphi} \varphi + V (\overline{\varphi}, \varphi) - \frac{1}{4q^2} F_{\mu\nu} \lambda a F^{\lambda a \mu\nu} ](-g)^{\frac{1}{2}} d\tau \]  
(28)
where $\alpha$ is a parameter, $\mathcal{L}_m$ is the Lagrangian of general matter fields, $g^{\mu\nu} = g^{\mu\nu}$ and $\gamma^\mu = g^{\mu\nu}\gamma_\nu$. (In the case that the fibre $G$ of principal bundle is a semisimple group of a general form,) the Lagrangian contains $r$ arbitrary constants $q_\lambda$, $\lambda = 1, 2, ..., r$, in which $r$ is the number of invariant simple factors.

The fundamental physics interactions, e.g., strong, weak, electromagnetic and gravitational interactions, can be described by Eqs. (26-27), these actions are unifiedly deduced by QCP, i.e., Eq. (1), thus the new unification theory of fundamental physics interactions is given in terms of modern differential geometry. The more concrete examples are that the known grand unification theories may be SU(5), S(10) or $E_6$ gauge theories, and it is very easy that their relative more fermion mass terms etc can similarly be deduced by using the expression which is relevant to spinor’s general gauge invariant quantities.

3 The Unified Description of Fundamental Physics Interactions and Noether Theorem

In the expression (1) of QCP, which has deduced all the physical fundamental interactions in this Letter, for general field variables $X(x) = \{\Psi(x), \varphi(x), \omega_\mu(x), g_{\mu\nu}(x), \ldots\}$ above, when $S$ is the actions (26-28), $C$ is unit element and $D$ is infinitesimal transformation operator of continuous Lie group [5, 13, 14]

$x^\mu \rightarrow x'^\mu = x^\mu + \varepsilon_\sigma \tau^{\mu\sigma}(x, X, X_\mu)$ (29)

$X^a(x) \rightarrow X'^a(x') = X^a(x) + \varepsilon_\sigma \xi^{a\sigma}(x, X, X_\mu)$ (30)

in which $\varepsilon_\sigma$ ($\sigma = 1, 2, \cdots, m$) are infinitesimal parameters of Lie group $D_m$ in Eq. (1). Thus, we get the unified expression of their variational principles, as follows

$$\Delta A = DA - A = A' - A = 0 \quad (31)$$

Not losing generality, under the transformations of Eqs. (29) and (30), we can take ad-hoc

$L'(x', X'(x'), X'_{\mu}(x'), X'_{\mu\nu}(x')) = L(x', X'(x'), X'_{\mu}(x'), X'_{\mu\nu}(x')) + \partial_\mu \Omega^\mu$ (32)

where $\partial_\mu \Omega^\mu = \varepsilon_\sigma \partial_\mu \Omega^{a\mu}$ and the Ricci Scalar $R$ contains $g_{\alpha\beta\mu\nu}$, and $\Omega$ may be rapidly decreasing to fit the usual physics experiments.

Using the unified expression of the variational principles, we have

$$\Delta A = \int_{M^4} \left[ \left( \frac{\partial L}{\partial X^a} - \partial_\mu \frac{\partial L}{\partial X^a_{\mu}} + \partial_\mu \partial_\nu \frac{\partial L}{\partial X^a_{\mu\nu}} \right) \delta X^a + \partial_\mu \left( L \delta x^\mu + \left( \frac{\partial L}{\partial X^a_{\mu}} - \partial_\nu \frac{\partial L}{\partial X^a_{\mu\nu}} \right) \delta X^a + \frac{\partial L}{\partial X^a_{\mu\nu}} \delta X^{a\nu} + \Omega^{\mu} \right) \right] d^4x$$ (33)
in which \( \delta X^a = \Delta X^a - X^a, \Delta x^\nu \). Using the above discussions, we can obtain their two-order general Euler-Lagrange Equations and corresponding Noether conservation currents as follows

\[
\frac{\partial L}{\partial X^a} - \partial_\mu \left( \frac{\partial L}{\partial X^a_{\mu}} \right) + \partial_\mu \partial_\nu \left( \frac{\partial L}{\partial X^a_{\mu \nu}} \right) = 0 \tag{34}
\]

\[
\partial_\mu J^{\mu \sigma} = 0 \tag{35}
\]

\[
J^{\mu \sigma} = L^{\mu \sigma} + \left( \frac{\partial L}{\partial X^a_{\mu}} - \partial_\nu \left( \frac{\partial L}{\partial X^a_{\mu \nu}} \right) \right) (\xi^{\alpha \sigma} - X^{a, \alpha} \tau^{\alpha \sigma}) + \frac{\partial L}{\partial X^a_{\mu \nu}} \partial_\nu (\xi^{\alpha \sigma} - X^{a, \alpha} \tau^{\alpha \sigma}) + \Omega^{\mu \sigma} \tag{36}
\]

Therefore, Noether theorem of the general physical system is deduced by QCP.

In all, using QCP, i.e., Eq.(1), we derive not only all the well-known fundamental physics Lagrangians and their variational principles and further Noether theorem, but also their two-order general Euler-Lagrange Equations and corresponding Noether conservation currents. Using the above studies, variation laws of the different physical systems are naturally determined. Therefore, it is essential to deduce effectively unified expressions of elementary physical laws by QCP.

4 Applications

We can use the new unification theory to different physical systems. For example:

(1) As Eq.(24)’s natural generalization, when the invariant scalar product \( \overline{\varphi} \varphi \) is extended by the invariant scalar curvature \( R \), we may naturally deduce \( f(R, \overline{\varphi} \varphi) \) gravity, i.e., \( A = \int f(R, \overline{\varphi} \varphi)(-g)^{\frac{1}{2}} d\tau \), because \( (-g)^{\frac{1}{2}} d\tau \) is invariant volume element. Further adding the first term \( \alpha R \) of Eq.(28), we get \( A_1 = \int F(R, \overline{\varphi} \varphi)(-g)^{\frac{1}{2}} d\tau + \alpha R \) \( (-g)^{\frac{1}{2}} d\tau \) [30].

(2) As Eq.(24)’s direct application, when the invariant scalar product \( \overline{\varphi} \varphi \) is taken as the invariant scalar product \( H + H \) of Higgs field \( H \)’s \{5\} multiplets, we may naturally deduce \( V(H) = \frac{\mu_0^2}{2} H^2 + \frac{\lambda}{2}(H + H)^2 \) (\( \mu_0 \) and \( \lambda \) are parameters), which is just the potential of Higgs field \( H \)’s \{5\} multiplets for grand unification theory [31]; for Higgs field \( \phi \)’s \{24\} multiplets that are expressed by a \( 5 \times 5 \) matrix, the matrix satisfies Eq.(5)’s general gauge invariant property, i.e., \( \phi_V = A_{VU} \phi_U \) or \( tr \phi_V = tr \phi_U \) and more generally it follows that \( tr \phi^0_V = tr \phi^0_U \), and as Eq.(24)’s natural generalization, we have \( V(\phi) = -\frac{1}{2} \mu^2 tr \phi^2 + \frac{c}{4} (tr \phi^2)^2 + \frac{d}{2} tr \phi^4 \) (\( \mu \), \( a \) and \( b \) are parameters); for mixed invariants of \( H \)’s \{5\} and \( \phi \)’s \{24\} multiplets, similarly, people can deduce \( V(H, \phi) = c H + H (tr \phi^2) + d H + \phi^2 H \) (\( c \) and \( d \) are parameters), thus we totally deduce \( V(H, \phi) = V(H) + V(\phi) + V(H, \phi) \), which is the total Higgs potential of grand unification theory and the same as that of Ref. [31].
Generalizing $R$ as an invariant functional $f(R)$ and using Eq.(27) deduced by QCP, people can deduce Fermion field Lagrangian interacting with Non-Abelian gauge field and general $f(R)$ gravitational field as follows

$$A = \int_{M^4} [\alpha f(R) + \bar{\Psi}[\gamma^\mu(\partial_\mu \Psi - \omega_\mu \Psi) + m] \Psi + U(\bar{\Psi}, \Psi) - \frac{1}{4q^2_\lambda} F^{\lambda\mu} F^{\lambda\mu}] (-g)^{\frac{1}{2}} d\tau.$$  

(37)

Eq.(37) is just a general generalization of Eq.(27), and further utilizing the deduced Eqs.(34) and (35) by QCP, people can naturally gives the new unification theory of the fundamental Non-Abelian gauge field, Fermion field and general $f(R)$ gravitational field interactions and Noether theorem by using QCP, and can concretely derive their Euler-Lagrange Equations and corresponding Noether conservation currents as done in Sect. 3. Thus the relative books and articles may be renewedly and systematically rewritten, which will help people to understand and express the fundamental Non-Abelian gauge field, Fermion field and general $f(R)$ gravitational field interactions simpler and with clear quantitative causal physical meanings.

(4) Generalizing $R$ as an invariant functional $f(R)$ and using Eq.(28) deduced by QCP, people can deduce Boson field Lagrangian interacting with Non-Abelian gauge field and general $f(R)$ gravitational field as follows

$$A = \int_{M^4} [\alpha f(R) + \bar{\varphi}(\partial^\mu - \omega^\mu)\varphi + m^2 \varphi \varphi + V(\varphi, \varphi) - \frac{1}{4q^2_\lambda} F^{\lambda\mu} F^{\lambda\mu}] (-g)^{\frac{1}{2}} d\tau.$$  

(38)

Eq.(38) is just a general generalization of Eq.(28), and further utilizing the deduced Eqs.(34) and (35) by using QCP, people can naturally gives the new unification theory of the fundamental Non-Abelian gauge field and general $f(R)$ gravitational field interactions and Noether theorem by using QCP, and can concretely derive their Euler-Lagrange Equations and corresponding Noether conservation currents. Thus the relevant books and articles may be renewedly and systematically rewritten, which will help people to understand and express the fundamental Non-Abelian gauge field, Boson field and general $f(R)$ gravitational interactions simpler and with clear quantitative causal physical meanings. When $f(R) = 0$, we naturally give electric and color superconduction theories corresponding Abelian $-\frac{1}{4q^2} F^{\mu\nu} F^{\mu\nu}$ and non-Abelian $-\frac{1}{4q^2_\lambda} F^{\lambda\mu\nu} F^{\lambda\mu\nu}$ in Eq.(38), respectively [31], thus the relevant books and articles may be renewedly and systematically rewritten, which will help people to understand and express the fundamental Non-Abelian gauge field and Boson field interactions simpler and with clear quantitative causal physical meanings.

(5) Using QCP, people can more generally deduce

$$A = \int_{M^4} [\alpha f(R) + \bar{\Psi}[\gamma^\mu(\partial_\mu \Psi - \omega_\mu \Psi) + m] \Psi + \bar{\varphi}(\partial^\mu - \omega^\mu)\varphi + m^2 \varphi \varphi + M(U(\bar{\Psi}, \Psi), V(\varphi, \varphi)) + V_t(H, \phi) - \frac{1}{4q^2_\lambda} F^{\lambda\mu} F^{\lambda\mu}] (-g)^{\frac{1}{2}} d\tau.$$  

(39)

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Eq.(39) is just a general generalization of Eqs.(37) and (38), and further utilizing the deduced Eqs(34) and (35) by QCP, people can naturally gives the new unification theory of the fundamental Non-Abelian gauge field, Fermion field, Boson field, general $f(R)$ gravitational field interactions and Noether theorem by using QCP, and can concretely derive their Euler-Lagrange Equations and corresponding Noether conservation currents. Thus the relevant books and articles may be renewedly and systematically rewritten, which will make people understand and express the fundamental Non-Abelian gauge field, Fermion field, Boson field, and general $f(R)$ gravitational field interactions simpler and with clear quantitative causal physical meanings. Specially, Eq.(39) is just the action of the theory of unifiedly describing all known physical fundamental interactions, further further utilizing the deduced Eqs(34) and (35) by QCP, consequently the new unification theory of the fundamental physics interactions and Noether theorem is given by using QCP for the first time.

Because of length limitation of the Letter, the other Lagrangians in physics may be analogously derived, their detailed applications can be given easily by means of the research of this Letter and which will be written in our following papers.

5 Summary and Conclusion

This Letter naturally gives the new unification theory of both all the fundamental physics interactions and Noether theorem by utilizing QCP, i.e., Eq.(1). The invariant quantities covering all the parts of the Lagrangians of the well-known fundamental physics interactions are deduced by QCP, all these invariant quantities satisfy general gauge invariance of the general gauge fields interacting with Fermion and/or boson fields. These different invariant quantities are constructed (in vector bundle or principal bundle, whose base manifolds are a 4-dimensional physical general curved spacetime manifold $M^4$). Thus, using these invariant quantities, all the Lagrangians of the well-known physical fundamental interactions are given and satisfy the general gauge invariant principle of general gauge fields interacting with Fermion and/or boson fields, and geometry and physics meanings of gauge invariant property of different physical systems are directly, whole and seriously revealed.

Actually it is in terms of QCP that their variational principles and corresponding Noether theorem in field theory are derived. Therefore, the very hard difficulty (that the fundamental physics interactions and variational principle, further Noether theorem, previously regarded as independent fundamental laws, were not able to be unifiedly proved and investigated in the past [32]) is naturally overcome in this Letter. Namely, the unified description and origin of fundamental physics interactions and the Noether theorems is shown by QCP, furthermore, their two-order general Euler-Lagrange equations and corresponding Noether conservation currents are obtained.

In fact, general physics process [4, 5, 8] and different physics processes
all satisfy QCP with no-loss-no-gain character, the above investigations satisfy QCP, and are consistent. Especially, using the theory of this Letter, a lot of research works about different branches of physics etc can be renewedly done and expressed simpler with clear quantitative causal physical meanings, e.g., people can deduce a lot of different parts of different Lagrangians, which will naturally give Lagrangians of different physics systems according to QCP and different symmetries etc, and then using Eqs.(34-36) people can give all corresponding concrete physics laws, i.e., find new Euler-Lagrange equations and conservation laws, and discover new concrete physics laws and so on. Thus the theory of this Letter is useful and will be broad utilized and cited in different branches of physics and so on. Therefore, all articles and books relevant to the fundamental physics interactions and Noether theorem may be supplemented with the new conclusions and this Letter will be cited by using the new unification theory in different physical systems, e.g., condensed physics, atomic physics, molecular physics, quantum optics, nuclear physics, particle physics and so on.

Because the standard model of strong, weak and electromagnetic interactions can be included in a unification interaction model of a big gauge group, thus people get the grand unified theory, and because the gravitational interaction related to the structures of spacetime, and the spacetime is the stage performing the laws of strong, weak and electromagnetic interactions, and gravitational interaction cannot be included into a unification interaction model of a larger gauge group. This is because the grand unification theory of gauge theories of strong, weak and electromagnetic interactions is based on principal bundle theory in differential geometry, and gravitational theory is based on the tangent vector bundle theory in differential geometry, so people cannot unify the these four basic interactions in principal bundle theory in differential geometry theory, so for their unification, people have met great difficulties.

In order to overcome these difficulties, people have to increase the dimensional number of spacetime etc to solve these problems, the most promising success theory is superstring theory, but in return to low four-dimensional case etc, superstring theory encounters the big difficulties, up to now, there is no way that, in the case of four dimensions, strong, weak, electromagnetic and gravitational interactions are unified as giant unification theory.

This Letter, for the first time, discovers and gives the giant unified theory of strong, weak, electromagnetic and gravitational interactions by investigating the general fiber bundle theory (in differential geometry theory) of unified describing both principal bundle theory and tangent bundle theory and by studying important laws in physics, e.g., relations between symmetry and QCP etc. Consequently, the research of this Letter is based on the rigorous scientific bases of mathematics and physics, therefore, people can essentially and really understand the four basic interaction laws of physics as well as the relations between them in terms of the research in this Letter, for their further development builds up the scientific solid foundation.

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Appendix A

For any natural coordinate \( x^\mu \) and local orthogonal coordinate \( x^a \) in any 4-dimensional general spacetime, any tangent vector along any curve line \( t \) can be formally written as \( X = \frac{d}{dt} = \frac{dx^\mu}{dt} \frac{\partial}{\partial x^\mu} = \frac{dx^a}{dt} \frac{\partial}{\partial x^a} \), then the inner product of the tangent vector equals

\[
<X, X> = \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \eta_{\mu\nu} = \frac{dx^a}{dt} \frac{dx^b}{dt} \eta_{ab} \tag{A1}
\]

where we may define the inner product \( <\partial_{\mu}, \partial_{\nu}> = \eta_{\mu\nu} \) (i.e., natural covariant metric) of natural tangent basic vectors and the inner product \( <\partial_a, \partial_b> = \eta_{ab} \), \( \eta_{ab} = -1, \) as \( a = b = 0; \eta_{ab} = 1, \) as \( a = b = 1, 2, 3; \eta_{ab} = 0, \) as \( a \neq b, \) (i.e., orthonormal contravariant metric) of orthonormal tangent basic vectors, so we can deduce the infinitesimal line element square

\[
ds^2 = <X, X> (dt)^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ab} dx^a dx^b \tag{A2}
\]

Similar to \( <\partial_{\mu}, \partial_{\nu}> = g_{\mu\nu} \) and \( <\partial_a, \partial_b> = \eta_{ab}, \) we define the inner product of natural (or called intrinsic) (cotangent basic vectors as \( <dx^\mu, dx^\nu> = <e^\mu, e^\nu> = g^{\mu\nu} \) (i.e., natural contravariant metric) and the inner product of orthonormal cotangent basic vectors as \( <dx_a, dx_b> = <e_a, e_b> = \eta^{ab} \) \((\eta^{ab} = -1, \) as \( a = b = 0; \eta^{ab} = 1, \) as \( a = b = 1, 2, 3; \eta^{ab} = 0, \) as \( a \neq b, \) (i.e., orthonormal contravariant metric),) thus, we prove

\[
g_{\mu\nu}g^{\mu\alpha} = <\frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial x^\nu}> <dx^\mu, dx^\alpha>
\]

\[
= \frac{\partial x^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial x^\alpha} = \frac{dx^\nu}{dx^\alpha} \frac{dx^\alpha}{dx^\mu} <dx^\nu, dx^\mu> = dx^\nu dx^\mu \eta^{\nu\mu} = \delta^\nu_\mu \tag{A3}
\]

Furthermore, using Eq.(A2) we deduce \( ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ab} \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} dx^\mu dx^\nu = \eta_{ab} e^a_\mu e^b_\nu dx^\mu dx^\nu, \) i.e., we prove

\[
g_{\mu\nu} = \eta_{ab} \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} = \eta_{ab} e^a_\mu e^b_\nu \tag{A4}
\]

where \( \frac{\partial x^a}{\partial x^\mu} = e^a_\mu \) is vierbein. On the other hand, we may directly prove Eq.(A4) by using \( g_{\mu\nu} = <\frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial x^\nu}> = \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} = \eta_{ab} e^a_\mu e^b_\nu, \) and we may have \( g^{\mu\nu} = <dx^\mu, dx^\nu> = \frac{dx^a}{dx^\mu} \frac{dx^b}{dx^\nu} = \eta^{ab} e^a_\nu e^b_\mu, \) Thus this method is very convenient.

We still need to prove the consistence of indexes’ raising and lowing by using covariant and contravariant metrics as follows
\[ A^\mu = g^{\mu\nu} A_\nu = < dx^\mu, dx^\nu > A_\nu = \frac{dx^\mu}{dx^c} \frac{dx^\nu}{dx^d} < dx^c, dx^d > A_\nu = e_\mu^\alpha e_\nu^\beta \eta^{\alpha\beta} A_\nu = e_\mu^a A^a \]  

(A5)

Analogously, we may prove \[ A_\mu = g_{\mu\nu} A^\nu = < \frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial x^\nu} > A_\nu = e_\mu^a A^a. \] Thus, these studies are consistent.

Appendix B

Similar to deducing the important invariant \( \omega_\mu \omega^\mu \) of inner product of two 1-forms, therefore, we deduce an important invariant of inner product of two 2-form \( \Omega \) under coordinate transformation, as follows

\[ << \Omega, \Omega >> = \frac{1}{2} \Omega_{\mu\nu} dx^\mu \Lambda dx^\nu, \frac{1}{2} \Omega_{\mu'\nu'} dx^{\mu'} \Lambda dx^{\nu'} >> \]

\[ = \frac{1}{4} \Omega_{\mu\nu} \Omega_{\mu'\nu'} g^{\mu'\nu'} < dx^\mu, dx^{\mu'} > = -\frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu}, \]  

(B1)

substituting Eq.(5) into the deduced inner product expression (B1) and taking trace, then we have

\[ tr << \Omega, \Omega >> = -\frac{1}{4} tr(\Omega_{\mu\nu} \Omega^{\mu\nu}) = tr << \Omega, \Omega >> = -\frac{1}{4} tr(\Omega_{\mu\nu} \Omega^{\mu\nu}), \]  

(B2)

which satisfies gauge invariant principle \[ \text{[28]} \] of general gauge field’s invariant, further using the invariant volume elements \( (-g_U)^{1/2} d\tau_U = (-g_V)^{1/2} d\tau_V \text{[26]} \), we finally achieve the invariant action Eq.(10) of general gauge fields in the global curved spacetime.

References

[1] G. Sterman, An Introduction to Quantum Field Theory, Cambridge University Press, New York 35 (1993).

[2] L. Klein, Dispersion Relations and Abstract Approach to Field Theory, International Science Review Series, Vol. I, Gordon and Breach Publishers Inc., New York 147 (1961).

[3] P. Curie, Journal de physique, (Paris), 3rd series, 3(1894)395.

[4] Yong-Chang (Y. C.) Huang, Mechanics Research Communications, 30 (2003) 567.

[5] Y. C. Huang, X. G. Lee and M. X. Shao, Mod. Phys. Lett., A21(2006) 1107.

[6] Y. C. Huang, F. C. Ma and N. Zhang, Mod. Phys. Lett., B18 (2005) 1367.

[7] Y. C. Huang and B. L. Lin, Phys. Letts. A299 (2002) 644.
[8] Y. C. Huang, C. Huang et al, International Journal of Theoretical Physics, 49 (2010) 2320-2333.

[9] H. Weyl, Symmetry, Princeton University Press, Princeton, (1952).

[10] Y. J. Du, F. Teng, Y. S. Wu, JHEP 09 (2016) 171; X. Luo, Y. S. Wu, Y. Yu, Phys. Rev. D 93, 125005 (2016).

[11] F. M. Chen, Y. S. Wu, Phys.Rev.D82:106012,2010; M. Sato, M. Kohmoto, Y. S. Wu, Phys. Rev. Lett. 97, 010601 (2006).

[12] E. Nöther, Nachr. Akad. Wiss., Gottingen, Math. Phys., Kl., II.235(1918).

[13] D. S. Djukic, Int. J. Non-linear Mech., 8(1993)479.

[14] Z. P. Li, Int. J. Theor. Phys., 26, 853(1987); Z. P. Li, J. H. Jiang, Symmetries in Constrained Canonical Systems, Science Press, New York, 2002.

[15] I. L. Buchbinder, T. V. Snegirev, Yu. M. Zinoviev, Nucl. Phys. B864 (2012) 694.

[16] M. Henneaux, R. Rahman, Phys. Rev. D88 (2013) 064013.

[17] I. L. Buchbinder, P. Dempster, M. Tsulaia, Nucl. Phys. B877 (2013) 260.

[18] C. de Rham, Living Rev. Relativity 17 (2014) 7.

[19] X. O. Camanho, G. L. Gomez, R. Rahman, Phys. Rev. D96 (2017) 084007.

[20] K. Becker, M. Becker, John H. Schwarz, String Theory and M-Theory, Cambridge University Press, 2006.

[21] Gary Shiu, Pablo Soler, and Fang Ye, Phys. Rev. Lett. 110, 241304 (2013); Gary Shiu, Bret Underwood, Kathryn M. Zurek, and Devin G. E. Walker, Phys. Rev. Lett. 100, 031601 (2008).

[22] Mirjam Cvetić, Gary Shiu, and Angel M. Uranga, Phys. Rev. Lett. 87, 201801 (2001); Fernando Marchesano and Gary Shiu, Phys. Rev. D 71, 011701(R) (2005); Y. J. Du, F. Teng, Y. S. Wu, JHEP 11 (2016) 088; F. M. Chen, Y. S. Wu, JHEP 1302: 016, 2013.

[23] Novikov, Evgeny A., MODERN PHYSICS LETTERS A.31 (2016) 1650092; Novikov, Evgeny A., Quantum modification of general relativity, Elect. J. Theoretical Physics, 13 (2016) 79-90.

[24] S. S. Chern, Vector Bundles with a Connection, in Studies in Global Differential Geometry, Mathematical Association of America, 1989.

[25] C. Nash and S. Sen, Topology and Geometry for Physicists, Academic Press, London 200 (1983).
[26] B. Y. Hou etc, Differential Geometry for Physicists, World Scientific Publishing Co Pte Ltd, Singarore (1997).

[27] M. Carmeli, Classical Fields: General Relativity and Gauge Theory, A Wiley-Interscience Publication, John Wiley & Sons, New York 589 (1982).

[28] X. S. Chen et al, Phys. Rev. Lett., 100 (2008) 232002.

[29] D. Lurie, Particles and Fields, John Wiley & Sons Inc, Bristol 19 (1968).

[30] Thomas P. Sotiriou and Valerio Faraoni, REVIEWS OF MODERN PHYSICS, 82, 2010, 451.

[31] Review of Particle Physics, Phys. Rev. D 86 (2012) 010001.

[32] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory (Beijing World Publishing Corp., 2006).

[33] Y. C. Huang and C. X. Yu, Phys. Rev. D 75 (2007)044011; J. C. Hua and Y. C. Huang, Europhysics Letters, 85 (2009) 30007; Y. C. Huang and Q. H. Huo, Physics Letters, B662(2008) 290-296.

[34] Y. C. Huang, L. Liao and X. G. Lee, The European Physical Journal, C60 (2009) 481-487; L. Liao and Y. C. Huang, Phys. Rev. D 75(2007)025025; Y. C. Huang and L. X. Yi, Annals of Physics (New York), 325 (2010) 2140-2152; B. H. Zhou, and Y. C. Huang, Phys. Rev. D 84, 047701 (2011).

[35] M. Wasay Abdul, Y. C. Huang, D. F. Zeng, Quantization and spectrum of RNS supersymmetric open 2-brane, Nuclear Physics, B892 (2015) 353-363.

[36] G. R. Chen, Y. C. Huang, Recovering information of tunneling spectrum from Weakly Isolated Horizon, European Physical Journal, C75 (2015)47.