Spin Half-Adder

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A new proposal is given to design a spin half-adder in a nanojunction. It is well known that at finite voltage a net circulating current (known as “circular current”) appears within a mesoscopic ring under the asymmetric ring-to-electrode interface configuration. This circular current induces a finite magnetic field at the center of the ring. Herein, this phenomenon is utilized to construct a spin half-adder. The circular current–induced magnetic field is used to regulate the alignments of local free spins; by their orientations the output states of the “sum” and “carry” are specified. All the outputs are spin based; therefore, the results get atomically stored in the system. The experimental possibilities of the proposed model are also illustrated.

1. Introduction

The ultimate goal of modern technology is to make atomic-scale devices. The continuous shrinking in the size of the channel length of a transistor has driven the industry from the first four-function calculators to the modern laptops.[4] The functionality of these atomic-scale devices is based on the quantum nature of electrons. But the movement of charge within an information-processing device always associates dissipation, which makes the device energy inefficient.[5] Replacement of “electrons” by “spin” has been found to be the most suitable way to resolve these problems. In 1990,[6] Datta and Das came out with a proposal of a spin-field-effect transistor (SFET), where they used the spin degree of freedom of channel electrons instead of the charge. Starting from the idea of the SFET, till now various proposals have been reported using spin degrees of freedom, such as spin injection into semiconductors from ferromagnetic metals[7,8] and the development of diluted ferromagnetic semiconductors.[9,10] These devices have several advantages, such as low power consumption and speedy processing, compared to the commonly used semiconducting devices.[11–13] Apart from these advantages, the most important factor in the context of computation is that these devices are nonvolatile in nature. Therefore, unlike a charge-based microprocessor, spintronic devices can store the output itself and we do not need any extra memory device. For example, using magnetoresistive elements, AND, OR, NAND, and NOR gates have been constructed with nonvolatile output.[14] Dery et al. have designed a logic gate consisting of a semiconductor structure with multiple magnetic contacts.[15] In a recent work, Datta and co-workers[16] have proposed all-spin logic devices along with a storage mechanism. Spin–orbit interaction has been used to perform a universal logic operation utilizing minimum possible devices.[17] In another work Khajetoorains et al.[18] have combined bottom-up atomic fabrication with spin-resolved scanning tunneling microscopy to construct and read out atomic-scale model systems performing logic operations.

It is always important to design a spin-based combinational digital circuit at atomic scale. In this article, we propose a spin half-adder using a circular current–induced magnetic field in a conformational interface. A usual half-adder consists of AND and XOR gates that are independently composed of various transistors, resistors, capacitors, etc, whereas our model has a strikingly simple design consisting of a couple of loops, where the orientations of spins denote the low and high states of the inputs and outputs.

Under finite bias condition, a net current[20–24] appears within the ring, along with the transport current (or drain current). This current is known as “circular current,” I. The circular current is analogous to the persistent current in an Aharonov–Bom ring, where the driving force is a magnetic field. The circular current produces a net local magnetic field, B at the ring center. In some cases the magnitude of B reaches ~millitesla (mT), and even up to the order of ~tesla (T). Such a high local magnetic field can be used to manipulate the alignment of a local spin embedded at the center of the ring or at any point on the axis (e.g., the Z-axis) passing through the center of the ring.[21,23,25] Using bias-induced circular current, here we design a half-adder where all the inputs and outputs are spin based. The system is composed of a nanochannel sandwiched between electrodes, namely, a source and a drain (as shown in Figure 1). The electrodes are semi-infinite and nonmagnetic. The channel consists of multiple loops. The spin orientations (viz. up and down) of sites 4 and 6 are taken as inputs for the entire operation. The outputs of the half-adder, i.e., “sum” and “carry,” are specified by the spin orientations of the two free spins S and C, respectively, which are attached at the center of the top loop containing atomic sites 3–4–5–6 and at the center of the whole system, respectively. The sum and carry are not orientated in the Z-direction and make an angle θC (e.g., θC = 30°). This can be done by the application of a constant external magnetic field. We use the circular current–induced magnetic field to tune these free spins. Under finite bias condition, the circular current and the magnetic field vanish when the loops are symmetric, and they

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reappear for asymmetric loop geometry. This is the key idea behind the logical operations. When a large magnetic field is produced in the loop, the corresponding free spin changes its orientation toward the $Z$-direction, whereas when the circular current (and the associated magnetic field) vanishes, the free spin again moves back by an angle $\theta_C$ due to the applied external constant magnetic field. In the system, the asymmetry is introduced by the different spin orientations of sites 4 and 6.

As the Boolean operation is based on the manipulation of local spin by means of circular current, this device is expected to have negligible power consumption and delay time, and very high endurance ($>10^{15}$ cycle) which exceeds the requirements of various memory-use cases, including high-performance applications such as CPU level-2 and level-3 caches, as we discuss in Section 4 where we use a bias-induced magnetic field to regulate the input states. 7) The results are valid at nonzero temperature, which is very crucial for practical applications.

We have arranged the study in the following manner. The theoretical method is discussed in Section 2. In Section 3 we present all the results. In Section 4, various other logical operations are demonstrated in the same setup. An experimental proposal is demonstrated in Section 5 and finally, we give an overview in Section 6.

2. Theoretical Prescription

To calculate the circular current in a nanojunction, we use the wave-guide theory. We start by writing the tight binding Hamiltonian of the model as shown in Figure 1. The system consists of a quantum channel connected to the electrodes. Therefore, the Hamiltonian for the entire system becomes

$$H = H_C + H_S + H_D + H_T$$

(1)

Here, $H_C$, $H_S$, and $H_D$ are the Hamiltonians for the channel (C), source (S), and drain (D), and they read as

$$H_{\alpha} = \sum \epsilon_{\alpha,\sigma} c_{\alpha,\sigma}^\dagger c_{\alpha,\sigma} + \sum (\epsilon_{\alpha+1,\sigma} t_{\alpha,\sigma} c_{\alpha,\sigma}^\dagger h.c.)$$

(2)

where $\alpha = C, S, D$. For the electrodes, the on-site energy and atom-to-atom coupling become $\epsilon_{\alpha,\sigma} = \epsilon_0$ and $t_{\alpha,\sigma} = t_0$, respectively. However, for the channel they are $\epsilon$ and $t$, respectively. For example, spin readout of nitrogen sites are taken as inputs, and the sum and carry are specified by the alignment of the free spin, namely, S and C, respectively.

Figure 1. Model of the half-adder where a quantum channel containing multiple loops is connected to the electrodes. The spin orientations of sites 4 and 6 are taken as inputs, and the sum and carry are specified by the alignment of the free spin, namely, S and C, respectively.
In the channel, the atomic sites 4, 6, 11, and 13 are magnetic. We need to calculate all spin-dependent components of the circular current $I_C$. In one of our recent works,\textsuperscript{[24]} we have put forward the methodology to calculate the spin components of circular current. Here we follow the same prescription. The detailed calculation of the bond current density $I_{i,i+1}$ between sites $i$ and $i+1$ is described in Appendix A. The current at bias voltage $V$ can be written as\textsuperscript{[36,37]}

$$I_{i,i+1}(V) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_S(E) - f_D(E) |E| dE$$

(3)

where $f_S(E) = \left[ 1 + e^{|E|/k_B T} \right]^{-1}$ is the Fermi function ($k_B$ is the Boltzmann constant and $T$ is the temperature) corresponding to the source and drain and $\mu_S$ is the corresponding chemical potential.

In the present system, we need to calculate the circular current for two loops: One is for the top loop containing atomic sites 3, 4, 5, and 6, and the other is for the center loop containing atomic sites 1–2–3–6–5–7–8–9–10–13–12–14. After calculating the corresponding bond currents, we calculate the net circular current of these two loops as

$$I_1 = \frac{1}{4} (I_{1,4} + I_{4,5} + I_{5,6} + I_{6,1})$$

(4)

and

$$I_2 = \frac{1}{12} (I_{1,2} + I_{2,3} + I_{3,6} + I_{6,5} + I_{5,7} + I_{7,8} + I_{8,9} + I_{9,10} + I_{10,13} + I_{13,12} + I_{12,14} + I_{14,1})$$

(5)

respectively. If the current goes in anticlockwise direction, then we consider it to be positive, and vice versa.

Net local magnetic fields are established as the circular currents flow within the rings. Using the Biot–Savart law, we can calculate the magnetic fields as

$$\vec{B}_{n}(\vec{r}) = \sum_{i,j} \left( \frac{\mu_0}{4\pi} \right) \int I_{ij} \frac{\vec{r} \times (\vec{r} - \vec{r}_i)}{|(\vec{r} - \vec{r}_i)|^3}$$

(6)

where $n = 1, 2$ and $\mu_0$ is the magnetic constant.

Now we consider a free spin is embedded at the ring center, as shown in Figure 1. The spin is initially misaligned with the $Z$ direction. With the appearance of $I_C$ and associated $B$, the spin tries to align itself along the $Z$ direction. We calculate the spin angle of rotation $\theta_C$\textsuperscript{[23,35,38]} by the magnetic field $B$ for a time $\tau$ as

$$\theta_i = g\mu_B Br/2h$$

(7)

where $i = 1, 2$, $g(\approx 1)$ is the Landé $g$-factor, and $\mu_B$ is the Bohr magneton.

### 3. Numerical Results and Discussion

As the functionality of the half-adder depends on the appearance of a circular induced magnetic field in the asymmetric situation, we want to examine the dependence of the induced magnetic field on the system asymmetry. In Figure 2, we plot the magnetic field produced at the center of the device with the angle of rotation of the inputs A and B. We consider spin 11 and 13 to be down. All the $h_i$ are 0.25 eV. Here we find that a large magnetic field is produced when the system has the most asymmetry and it smoothly varies toward zero as the orientations of A and B become similar to spins 11 and 13, and this is the key factor of our proposal. For the execution of logic operation, the appearance of zero circular current is a prime requirement. This demands an ideal symmetric condition, which seems to be unrealistic in a real situation. But we can see in Figure 2 that the produced magnetic field is insufficient to rotate the spin along $Z$ for a considerable range around the symmetry point ($\theta_A = \theta_B = 180^\circ$). Therefore, we can argue that even if there is a little asymmetry due to manufacturing imperfection and other factors, this proposal will still be equally valid.

Now we explain the half-adder operation.

In Figure 3 we show the circuit diagram and the truth table for the spin half-adder. The input states are specified by the spin orientations of spins A and B as $|0\rangle \rightarrow 0$ and $|1\rangle \rightarrow 1$, whereas the output conditions are specified by the spin alignment of free spin $S$ (representing sum) and $C$ (representing carry). The mechanisms of sum and carry are as follows (see Sections 3.1 and 3.2).

### 3.1. Sum

We assume that the free spins initially are not aligned along the $Z$-direction. The output for the sum is defined as follows: If the free spin $S$ is in its initial position, then the output is 0, and if it is aligned along $Z$, it represents 1. The alignment of the individual free spin depends on the appearance of the current-induced magnetic field in each loop. Therefore, there is no magnetic field when $A$ and $B$ are parallel, i.e., when both are either up or down (see Figure 3c,d). In these cases, $S$ remains in its initial position such that the logical output becomes 0. When they are antiparallel (as shown in Figure 3e,f), the corresponding loop becomes asymmetric and a net magnetic field is produced along $Z$ direction and the free spin $S$ follows the field. These situations imply 1. So, this part of the circuit behaves like an XOR gate, which is the sum of the half-adder.
3.2. Carry

To generate carry, we consider the full system and place a free spin C at the center of the whole system. The output for the carry is assumed to be 0 when C is aligned along $Z$; otherwise it is 1. As the lower loop contains two up spins, the whole system becomes symmetric only when both the inputs (i.e., A and B in upper loop) are up. Therefore, no magnetic field is developed at the center of the circuit and C remains in its initial position (Figure 3f). This situation implies 1. For all three input conditions, the system is asymmetric and a net magnetic field is produced at the center, which turns the free spin C along the $Z$ direction (as shown in Figure 3c–e); therefore, the output becomes 0 in these cases. So AND behavior is accomplished at C and hence the construction of the half-adder is accomplished.

In Figure 4 we plot the produced magnetic fields $B_1$ and $B_2$ associated with the operations sum and carry, respectively, as a function of voltage for four different input conditions (i.e., when inputs A and B are ($\uparrow$, $\uparrow$), ($\downarrow$, $\uparrow$), ($\uparrow$, $\downarrow$), and ($\downarrow$, $\downarrow$); shown in Figure 4a–d, respectively). Here we set all site energies to zero, the hopping integral in contacting leads at 1 $\text{eV}$, and all the $t_i$ in the ring at 0.5 $\text{eV}$ and the ring-to-lead couplings at 0.5 $\text{eV}$. The magnitude of the net magnetization at sites 4, 6, 11, and 13 is 0.5 $\text{eV}$. The calculations are done considering 250 $\text{K}$ temperature. The average atomic distance $a$ is considered to be 1 Å. The red curve represents the magnetic field corresponding to sum (i.e., $B_1$) and the black one represents $B_2$, which is the magnetic field associated with carry. The free spins corresponding to S and C are initially set at 30°. When the loops become asymmetric, circular currents and therefore net magnetic fields will be produced in each loop, which will turn the free spins S and C toward the $Z$ direction. Considering the desired operation time as $\tau = 5$ ns, we can calculate the desired magnetic field to align the spin along $Z$ as $\approx 2.4$ mT (as follows from Equation (7)). So we need at least $\approx 2.4$ mT to execute all the logic operations. For all four cases of Figure 4 we find that large enough magnetic fields are produced which are more than sufficient to turn S and C in appropriate cases. On the other hand, for the proper cases the magnetic fields are exactly zero, which will leave the S and C in their initial positions. As the results remain valid for large ranges of voltage and temperature, we can expect that the proposed model might be implemented in the laboratory.

The quantitative representation of the half-adder is shown in Table 1. Here all the parameters are chosen to be the same as Figure 4 and magnetic fields are evaluated at bias voltage 0.5 $\text{V}$.

4. Reprogrammable Spin Logic Gate

In this study, our main motivation is to construct a spin half-adder, though other logical operations can be accomplished by reprogramming the same system. For example, in Figure 5 we have shown sketches for NAND and NOT gates. The spins shown in green represent the inputs and O represents the output. As the logical operations follow the symmetry conditions of the ring, we accordingly set the spin orientations of the other sites in the system. For the NAND gate (Figure 5a) spins 11 and 13 are set to be down. Therefore, only for the ($\uparrow$, $\uparrow$) input condition no circular current appears at the center of the system and the output becomes zero. But for the other three cases the outputs are 1, which is a NAND gate response. For the NOT gate (Figure 5b), there is one input, i.e., A, and other spins are considered to be up. So if A is down, the output becomes 1 and vice versa.

In a similar fashion, we can reprogram this model to have other logical operations also, which definitely implies the versatility of our proposal.
5. Experimental Setup of Half-Adder

To have the input conditions 0 and 1, we need to align the spins in the magnetic ring selectively. Several prescriptions are available to control single electron spin. For instance, using radiofrequency pulses these spins can be manipulated,\(^{[39-41]}\) though in this case a relatively larger time scale is required to operate the spins. On the other hand, the manipulations can be made much faster, such as in the picosecond or femtosecond time scale, with the help of optical pulses.\(^{[42-44]}\) In another pioneering work, Press et al. have shown that the selective tuning of electron spins is possible within the spins’ coherence times by means of ultrafast laser pulses.\(^{[45]}\) With the availability of these various sophisticated prescriptions, we strongly believe that the alignment of selective spins (i.e., 4 and 6) can be properly adjusted.

Apart from the aforementioned proposals, here we present another suitable method for the proper regulation of the spin-specifying input conditions. We make use of the circular current–induced magnetic field to regulate the inputs and for this tuning of the external magnetic field is required. We take

![Figure 4](image1.png)

**Figure 4.** a–d) Produced magnetic fields \(B_1\) (red curve) and \(B_2\) (black curve) associated with the sum and carry, respectively, for the four input conditions. Here, we set \(T = 250\,\text{K}\).

### Table 1. Truth tables for different parallel logical operations.

| Input | Sum \(|B_1|\) [mT] | Carry \(|B_2|\) [mT] |
|-------|-----------------|-----------------|
| A  B  | \(S\)            | \(C\)            |
| 1  1  | 0               | 62.7            |
| 1  0  | 101             | 30.3            |
| 0  1  | 101             | 41.5            |
| 0  0  | 0               | 0               |

![Figure 5](image2.png)

**Figure 5.** Layouts of different logic gates.
a nanoring with two side-attached metallic electrodes (S and D) connected to the two adjacent sites of the ring as shown in Figure 6 (left). As the electrodes are connected at the two neighboring sites of the ring, with a finite probability, the electrons can directly hop from S to D. Let \( t_C \) be the related hopping integral between the electrodes. By changing the relative positions of the electrodes, we can tune \( t_C \), and tuning \( t_C \) we can regulate the circular current–induced magnetic field \( B \) for a large scale. This proposal has already been discussed in one of our previous works.\(^{[49]}\) For example, taking a 20-site ring we show the variation of the magnetic field with source-to-drain coupling \( t_C \) in Figure 6 (right), where we follow the same theoretical prescription for the calculation of the current and magnetic field as described in Section 2. From the result, we can conclude that by regulating the tunneling between the side-attached electrodes the local magnetic field \( B \) can be tuned at a large scale, which is required for flipping of spin states (i.e., up or down). Such nanojunctions needed to be put at sites 4 and 6 (as shown in Figure 6, right, the distance of the ring center to sites 4 and 6 will be 20 Å) and in every case changing the shunting paths between the source and drain we could specify the required input conditions.

### Appendix A. Circular Current Density

The wave guide formalism\(^{[24,46–49]}\) involves a set of linear coupled equations that are obtained from the Schrödinger equation \( H|\psi\rangle = E|\psi\rangle \) with \( |\psi\rangle = [\sum A_{n,\sigma}a_{n,\sigma} + \sum B_{n,\sigma}b_{n,\sigma} + \sum C_{n,\sigma}c_{n,\sigma}^{\dagger}]|0\rangle \). The coefficients \( A_{n,\sigma}, B_{n,\sigma}, \) and \( C_{n,\sigma} \) are the wave amplitudes corresponding to the \( n \)th site of the electrodes (namely, the source and drain), and \( i \) represents the site index of the ring. Let an up spin be injected from the source to the channel as a plane wave with unit amplitude. For our setup as shown in Figure 1, we have the following equations:

\[ I(t) = \frac{2e}{h} \int \sum_{\sigma} A_{i,\sigma}^* \epsilon_{\sigma} \rho_{\sigma} \, dt \]

\[ \rho_{\sigma} = \frac{\partial^2}{\partial \epsilon_{\sigma}^2} \delta(E - \epsilon_{\sigma}) \]

\[ \delta(E - \epsilon_{\sigma}) = \begin{cases} 1 & \text{if } E = \epsilon_{\sigma} \\ 0 & \text{otherwise} \end{cases} \]

\[ \sum_{\sigma} A_{i,\sigma}^* \epsilon_{\sigma} \rho_{\sigma} \]

\[ I(t) = \frac{2e}{h} \int \sum_{\sigma} A_{i,\sigma}^* \epsilon_{\sigma} \rho_{\sigma} \, dt \]

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\[ \delta(E - \epsilon_{\sigma}) = \begin{cases} 1 & \text{if } E = \epsilon_{\sigma} \\ 0 & \text{otherwise} \end{cases} \]

\[ \sum_{\sigma} A_{i,\sigma}^* \epsilon_{\sigma} \rho_{\sigma} \]
The parameters $t_0$ and $t_D$ represent the coupling between the source and channel and the channel and drain, respectively. $\rho$ and $\tau$ are the reflection and transmission probabilities, respectively. $\theta_1$ is the polar angle and $\varphi$ is the azimuthal angle. $k$ is the wave vector and $a$ is the atomic length. By solving Equation (8), we get bond current densities for sites $i$ and $i+1$ of the channel as

$$J_{i,i+1} = \frac{(2e/\hbar)Im[C_{11,11}t_C^{11,11}]}{(2e/\hbar)(1/2)t_0 \sin(ka)}$$

(A2)

In the aforementioned expressions, $\sigma$ is used for the incident spin, and $\sigma'$ represents the transmitting spin.

Similarly, for the down-spin incidence, we get a set of equations like Equation (8) and we evaluate $J_{i,i+1,11}$ and $J_{i,i+1,11}$. With all these components of circular bond current densities, now we define the net bond current density $J_{i,i+1}$ as

$$J_{i,i+1} = J_{i,i+1,11} + J_{i,i+1,11} + J_{i,i+1,11} + J_{i,i+1,11}.$$  

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Conflict of Interest

The author declares no conflict of interest.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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