WCNNM Algorithm for Wideband DOA Estimation

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Abstract. This paper presents a novel direction-of-arrival (DOA) estimation method for multiple wideband (WB) signals impinging on a nested array, named as WCNNM algorithm. Making use of the proposed method, the DOA estimation problem can be effectively transformed into a convex nuclear norm minimization problem, by solving it to surrogate to recover the signal component from the observed matrix so that the number of incoming signals and signal subspace can be obtained. By the signal subspace, we can obtain the DOA estimation. Compared with the related works, the presented method can not only improve the DOA estimation performances, such as higher resolution, smaller root-mean-square-error (RMSE), but also increase the maximal number of detectable signals. Simulation results are presented to verify the efficiency of the pro-posed method.

1. Introduction
The direction-of-arrival (DOA) estimation problem has been intensively studied in the past few decades. It has been widely applied in radar, sonar and wireless communication, as it is a crucial parameter for sorting and recognition of sources, directing jamming and passive location, etc. Under the key assumption that the number of active sources is less than the number of sensors, several high-resolution algorithms, such as multiple signal classification (MUSIC) [1] and estimation of signal parameter via rotation invariance techniques (ESPRIT) [2] are proposed to resolve the uncorrelated signals. However, when the number of sources is larger than the number of sensors, often referred to as the underdetermined case, the above high-resolution algorithms will fail in accurately estimating the DOAs. Several methods are presented to resolve the above difficulty. For examples, the DOA estimation algorithms are based on higher-order cumulants [3-5]. A Khatri-Rao (KR) subspace based algorithm is proposed for nonstationary sources in underdetermined case [6].

Recently, many sparse signal recovery (SSR) based DOA estimation methods are proposed [7-9]. For examples, a covariance sparsity-aware DOA estimation method is presented for the nonuniform noise [7]. The presented approach can handle more sources and obtain better performance. A co-prime array is use to estimate DOA [8]. To overcome the grid mismatch problem, a new method is proposed for the co-prime array [9]. Thus, it is of considerable interest to determine the DOAs for the WB signals [10-11]. An effective DOA estimation method is proposed for multiple WB signals [10]. This approach exploits the interpolation technique to form a set of virtual arrays. A subspace-based method is presented [11]. It can achieve higher-resolution by formulating a combined signal subspace. The KR-DOA method [6] is extended to the case of WB non-stationary signals by using the incoherent signal-subspace (ISS) processing. However, its estimation effectiveness is degraded for low signal-to-noise ratio (SNR) scenarios [11].
In this paper, an efficient DOA estimation approach is developed for the WB signals impinging on a nested array. The WB DOA estimation problem is transformed into a convex nuclear norm minimization problem. And then, the DOAs are estimated via a one-dimensional search. The rest of the paper is organized as follows. The data model is described in Section 2. Section 3 introduces the proposed algorithm. Section 4 shows some simulation. Finally, the conclusion is given in Section 5.

2. Data Model

Consider a two-level linear nested array with $M$ sensors, including two concatenated uniform linear arrays (ULAs)-inner and outer, where the inner ULA has $M_i$ sensors with spacing $d_i$ and the outer ULA has $M_o$ sensors with spacing $\frac{d_i}{2}$, $M = M_i + M_o$. More precisely, it is a linear array with sensors locations given by the union of the sets $S_{inner} = \{ m d_i, m = 1, 2, \ldots, M_i \}$ and $S_{outer} = \{ n (M_i + 1), n = 1, 2, \ldots, M_o \}$, where the first sensor is set to be the reference, as shown in figure 1.

![Figure 1. Nested array configuration](image)

Assume that the $Q$ WB signals have a common bandwidth $B$ (Hz) with center frequency $f_0$ (Hz). The observed $q$ th bandpass signal $x_q(t)$ at a reference point can be given by:

$$x_q(t) = s_q(t)e^{j2\pi f_0 t}$$

where $s_q(t)$ denotes the $q$ th baseband signal. If the signal is observed over a duration $[t_a, t_b + T_a]$, the baseband signal can be written as follows

$$s_q(t) = \sum_{i=1}^{I} S_q(f_i)e^{j2\pi f_i t}, \quad t_0 \leq t \leq t_0 + T_a$$

where the Fourier coefficients $S_q(f_i) = \frac{1}{T_a} \int_{t_a}^{t_b + T_a} s_q(t)e^{-j2\pi f_i t} \, dt$ with $f_i = (i - (I + 1)/2)/T_a$, $i = 1, \ldots, I$. $I$ stands for the number of frequency components. Consider the propagation delay $\tau_{q,m}$ of the $q$ th signal at the $m$ th sensor, the modulated bandpass signal at the reference point can be expressed as

$$x_q(t + \tau_{q,m}) = \sum_{i=1}^{I} S_q(f_i)e^{j2\pi f_i (t + \tau_{q,m})}$$

where $\tau_{q,m} = m d_i \sin(\theta_q)/c$, $c$ denotes the propagation speed. The demodulated signal can be expressed as

$$x_q(t, \tau_{q,m}) = x_q(t + \tau_{q,m})e^{-j2\pi f_0 t}$$

Let $X_q(t) = [x_q(t, \tau_{q,1}), x_q(t, \tau_{q,2}), \ldots, x_q(t, \tau_{q,M})]^T$. Then, we have the following representation

$$X_q(t) = \sum_{i=1}^{I} a_q(f_i + f_0)S_q(f_i)e^{j2\pi f_i t}.$$
where \( a(\theta_q, f_0 + f_i) \) denotes the \( M \times 1 \) steering vector of the \( q \)th signal, it has the following form

\[
a(\theta_q, f_0 + f_i) = \left[ e^{j2\pi(f_0 + f_i)\tau_{1,\theta}}, \ldots, e^{j2\pi(f_0 + f_i)\tau_{M,\theta}} \right]^T
\]

(6)

The received data vector has the form

\[
X(i) = \sum_{i=1}^{I} A(\theta, f_i) S(f_i) + N(f_i)e^{j2\pi f_i}
\]

(7)

where \( A(\theta, f_i) = [a(\theta_1, f_0 + f_i), \ldots, a(\theta_q, f_0 + f_i)] \) stands for \( M \times Q \) array steering matrix, \( S(f_i) = [s_i(f_0 + f_1), \ldots, s_i(f_0 + f_Q)]^T \) denotes the \( Q \times 1 \) signal vector, \( N(f_i) = [N_1(f_0 + f_1), \ldots, N_Q(f_0 + f_i)]^T \) represents the \( M \times 1 \) noise Fourier coefficient vector, and define \( y(i) \) as the \( M \times 1 \) Fourier coefficient vectors of \( X(i) \):

\[
y(i) = A(\theta, f_i) S(f_i) + N(f_i), \quad i = 1, \ldots, I
\]

(8)

The common assumptions are listed as follows:

A1) The source DOAs \( \theta_q \) \( (q = 1, \ldots, Q) \), are distinct from one another.

A2) The source signals \( S(t)(t = 1, \ldots, Q) \), are statistically independent. This guarantees that the covariance matrix of the source signals is diagonal.

A3) The noise is spatially and temporally white, and uncorrelated with the source signals.

Based on the above assumptions, the covariance matrix can be represented as

\[
R_y = E(y(i)y^H(i)) = A_i A^H_i + \sigma^2_n I
\]

(9)

where \( E(\cdot) \) denotes the expectation operator, \( R_y = \text{diag}(\rho_{11}, \rho_{22}, \cdots, \rho_{ii}) \), \( \rho_{ii} \) and \( \sigma^2_n \) represent the signal power and the noise power, respectively. \( I \) is an \( M \times M \) identity matrix, for brevity, we substitute \( A_i \) for \( A(\theta, f_i) \).

\[
z = \text{vec}(R_y) = (A_i \odot A_i) p_i + \sigma^2_n I
\]

(10)

Let where \( \text{vec}(\cdot) \) denotes the vectorization operation. We can see that \( z \) in (10) behaves like the received signal at an array whose manifold is given by \( A_i \odot A_i \), \( p_i = [p_{i1}, p_{i2}, \cdots, p_{ii}]^T \) denotes the equivalent source signal vector, it consists of the powers \( p_{ij} \), of the actual sources, and these powers behave like fully coherent sources, \( \rho_{ij} = [e_1^T, e_2^T, \cdots, e_Q^T] \) with \( e_i \) being a column vector of all zeros except a 1 at the \( i \)th position. In addition, \( (\cdot)^* \) stands for complex conjugation without transposition and symbol \( \odot \) denotes the KR product.

3. Algorithm Formulation

3.1 Spatial Smoothing

Since the difference co-array of a two level nested array has sensors located from \((-M^2/4 - M/2 + 1)d_i \) to \((M^2/4 + M/2 - 1)d_i \). We firstly construct a new matrix \( \Lambda_i \) of size \((M^2/2 + M/2 + 1) \times Q \) from \( A_i \odot A_i \), where we have removed the repeated rows and sort them so that the \( j \)th row corresponds to the sensor location \((-M^2/4 - M/2 + j)d_i \) in the difference co-array of the two level nested array. This is
equivalent to removing the corresponding rows from the observation vector \( z \) and sorting them to get a new vector \( \tilde{z}_i \) given by
\[
\tilde{z}_i = \bar{A}_i p_i + \sigma^2 \epsilon_i
\]
(11)
where \( \bar{A}_i \in \mathbb{C}^{(M^2/2 + M/2 \times 1)} \) is a vector of all zeros except a 1 at the \((M^2/2 + M/2)th\) position. We now divide these \((M^2/2 + M/2 - 1)\) sensors into \((M^2/2 + M/2)n\) overlapping subarrays, in which case the \( j \)th subarray has sensors located at \((-j + 1 + m)_{d_i} \), \( m = 0, \ldots, M^2/2 + M/2 - 1 \). The \( j \)th subarray corresponds to the \((M^2/2 + M/2 - j + 1)th\) rows of \( z \), which we denote as
\[
\tilde{z}_j = \bar{A}_j p_j + \sigma^2 \epsilon_j
\]
(12)
We can check that \( \tilde{z}_j = \bar{A}_j \Phi_j p_j + \sigma^2 \epsilon_j \), where \( \Phi_j = \text{diag}(\epsilon^{-j(2\pi/\lambda)d_1}, \ldots, \epsilon^{-j(2\pi/\lambda)d_{4 \times 2 \times 4} \times 4}) \). Viewing \( \tilde{z}_j \) as a newly received vector, the corresponding covariance matrix \( \overline{R}_j = \tilde{z}_j \tilde{z}_j^H \) can be obtained. Taking the average of \( \overline{R}_j \) over all \( j \), we get
\[
\overline{R}_{\text{ave}} = \frac{1}{(M^2/2 + M/2)} \sum_{j=1}^{M^2/2 + M/2} \overline{R}_j
\]
(13)
According to [12], \( \overline{R}_{\text{ave}} \) can also be expressed as \( \overline{R}_{\text{ave}} = \overline{R}_j^2 \), where \( \overline{R}_j \) has the form
\[
\overline{R}_j = \frac{1}{\sqrt{M^2/2 + M/2}} \overline{A}_{j, \overline{A}_j, \overline{A}_j}^H + \sigma^2 I
\]
(14)
The matrix \( \overline{R}_j \) in (14) has the same form as the conventional covariance matrix used in subspace based DOA estimation techniques when applied on a ULA with \( M^2/2 + M/2 \) sensors whose array manifold is represented by \( \bar{A}_j \), \( \bar{A}_j \in \mathbb{C}^{(M^2/2 + M/2 \times 1)} \). As long as \( Q \leq M^2/2 + M/2 \), \( \overline{R}_j \) has a low rank component \( \overline{A}_{j, \overline{A}_j, \overline{A}_j}^H \). In practice, since the covariance matrix \( \overline{R}_j \) of the received signal is usually unavailable, we will use the sample-average estimated array auto-covariance matrix given by
\[
\overline{R}_y = \frac{1}{L} \sum_{i=1}^{L} y(i)y^H(i)
\]
(15)
In this case, the smoothed matrix has the form
\[
\overline{R} = \frac{1}{\sqrt{M^2/2 + M/2}} \overline{A}_{y} \overline{R}_y \overline{A}_{y}^H + E
\]
(16)
where \( \overline{R}_y \) is a diagonal matrix containing the diagonal of \( \frac{1}{L} \sum_{i=1}^{L} S(f_i)S^H(f_i) \) and \( E \) represents the noise, due to finite samples effect, signal and noise cross correlation terms are not 0. Here the matrix \( \overline{A}_{y} \overline{R}_y \overline{A}_{y}^H \) is still low rank, there is no easy way to find its rank.

3.2 Low Rank Matrix Recovery and DOA Estimation
In order to address the above problem, low rank matrix recovery is performed by nuclear norm minimization, moreover, it can also avoid spurious peaks for DOA estimation. Assume that the matrix \( B = A + C \) in which \( A \) is low rank and \( C \) has bounded entries, for some sufficiently small \( \varepsilon \), the
following in equation $\|C\|_F \leq \varepsilon$ is satisfied, where $\|\cdot\|_F$ denotes Frobenius norm. For reasonably large samples, the noise $\varepsilon$ is considered to have a small Frobenius norm since the cross correlation terms become smaller with the number of samples increasing.

Due to the matrix $\widetilde{A}_i R_s \widetilde{A}_i^u$ is inherently low rank, thus it can be estimated by the following convex nuclear norm minimization given by

$$
\min_{R} \|R\|_n \\
\text{s.t.} \|R - R^*\|_F \leq \varepsilon
$$

(17)

where $\|\cdot\|_n$ denotes the nuclear norm (sum of singular values). The optimal solution $R^*$ is an estimation for the matrix $\widetilde{A}_i R_s \widetilde{A}_i^u$, and it is a positive semidefinite matrix. Then we can perform the singular value decomposition (SVD) of $R^*$, namely, $\text{SVD}(R^*) = U \Lambda V^\dagger$. If the rank $\text{rank}(R^*) = r$, we can order the singular values such that the last $M^2/4 + M/2 - r$ singular values are equal to 0. Denote the corresponding columns of $V$ as $V_r = [v_r, \ldots, v_{M^2/4 + M/2 - r}]$, and $\tilde{a}_i$ as the steering vector of the co-array corresponding to the direction $\theta_q$. Combining the resulting measurements for all the different frequencies, we construct the new combined DOA estimation spectrum:

$$
M(\theta) = \frac{1}{I} \frac{1}{\sum_{i=1}^r a_i V_i V_i^\dagger a_i^\dagger}
$$

(18)

the estimated DOAs are corresponding to the largest values of the spectrum $M(\theta)$.

It is clear that the low rank recovery stage succeeds in recovering $\widetilde{A}_i R_s \widetilde{A}_i^u$, true DOAs will be recovered. When we choose $\varepsilon = \sqrt{M^2/4 + M/2 - 2\kappa}$, the estimated covariance matrix $\hat{R}$ can be replaced by the exact covariance matrix $R^*$. Suppose that total observation time is $T$ and divide it into $L$ segments, there are $I$ samples within each segment. Therefore, we have $I \times L$ samples $x = x_{1, \ldots, I}$, for each segment $i$, we employ DFT to get the $M \times I$ corresponding frequency coefficient matrix:

$$
y = [y_i(1), \ldots, y_i(I)], \ i = 1, \ldots, L
$$

(19)

Consider all the segments, we can get the $M \times L$ coefficient matrix for each frequency index $i$:

$$
Y^i = [y_i(1), \ldots, y_L(i)], \ i = 1, \ldots, L
$$

(20)

The resulting sample covariance matrix for frequency index $i$ can be written as $R^i = \frac{1}{I} \sum_{i=1}^r Y^i V_i V_i^\dagger$, following the spatial smoothing technique and low rank matrix recovery in the previous process, the correspondingly optimal solution $R^*$ can be obtained, we can conduct DOA estimation based on the optimal solution.

4. Identifiability

To obtain better DOA estimation and derive the maximal number of identifiability signals, the identifiability of the proposed method is presented below. Since the rank of the matrix $(\widetilde{A}_i \odot \widetilde{A}_i)_{\text{smooth}}$ affects the identifiability, it is necessary to further investigate the rank of the matrix $(\widetilde{A}_i \odot \widetilde{A}_i)_{\text{smooth}}$. We use the following lemma (see property 2 in [13]).
Lemma 1. For two matrices $A \in \mathbb{C}^{m \times k}$ and $B \in \mathbb{C}^{n \times l}$, with \( \text{rank}(A) \geq 1 \) and \( \text{rank}(B) \geq 1 \), it holds true that

\[
\text{rank}(A \odot B) \geq \text{rank}(A \odot B) \geq \min\{\text{rank}(A) + \text{rank}(B) - 1\}.
\]

The rank properties of KR product have a strong connection to a concept called Kruskal rank, or k-rank for short. The k-rank of a matrix \( A \), denoted by \( \text{krank}(A) \), is supposed to be equal to \( r \) when every collection of \( r \) columns of \( A \) is linearly independent, but there exists a collection of \( r + 1 \) linearly dependent columns. However, the rank of \( A \) is the maximal number of linearly independent columns, the relationship between them is

\[
\text{rank}(A) \geq \text{krank}(A).
\]

Theorem 1. Let \( A \) be a van der monde matrix as follows:

\[
A = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
1 & a_1 & \cdots & a_q \\
\end{bmatrix} \in \mathbb{C}^{m \times q}
\]

where \( a_q = e^{\frac{j2\pi \text{rank}(A)}{m+q-1}} \), all \( a_q \neq a_q \) for all \( q \neq k \).

The KR product \( A^{*} \odot A \) via \( m^2 / 4 + m / 2 \) factors spatial smoothing in a two level nested array with \( m \) sensors, \( (A^{*} \odot A)_{\text{smoo}} \) is of full column rank if \( q \leq m^2 / 4 + m / 2 \).

Proof: for a van der monde matrix \( A \), it is easy to know that \( \text{krank}(A^*) = \text{rank}(A) = \min\{m, q\} \). And for distinct \( a_1, \ldots, a_q \), it must hold true that \( \text{rank}(A) = \min\{m, q\} \). Moreover, \( \text{krank}(A^*) = \text{rank}(A) = \min\{m, q\} \). Since the dimension of the matrix \( A^{*} \odot A \) is \( m^2 \times q \), but it has precisely as many distinct rows as the number of degree of freedoms (DOFs) of the difference co-array which is \( (m^2 - 2) / 2 + m \) for two level nested array. After \( m^2 / 4 + m / 2 \) factors spatial smoothing, each subarray has \( m^2 / 4 + m / 2 \) distinct rows. As a consequence, we have \( \text{rank}((A^{*} \odot A)_{\text{smoo}}) = \text{rank}((A^{*} \odot A)_{\text{smoo}}) = \min\{q, m^2 / 4 + m / 2\} \), where the last inequality is due to Lemma 1 and spatial smoothing. For \( 1 \leq q \leq m^2 / 4 + m / 2 \), \( \text{rank}((A^{*} \odot A)_{\text{smoo}}) = q \), \( (A^{*} \odot A)_{\text{smoo}} \) has full column rank.

This concludes the proof.

Consider \( \tilde{K}^{n} = \hat{K}^n \), where \( \hat{K} \) has the form \( \hat{K} = \frac{1}{\sqrt{M^2 / 4 + M / 2}}(\tilde{\mathcal{K}}_\theta, \tilde{\mathcal{K}}_\theta^* + \sigma_\theta^2 I) \). Moreover, \( \tilde{\mathcal{K}}_{\theta} = (A^{*} \odot A)_{\text{smoo}} \),

\[
\tilde{\mathcal{K}}_{\theta} = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
1 & a_1 & \cdots & a_q \\
\end{bmatrix}
\]

where \( a_q = e^{\frac{j2\pi \text{rank}(A)}{m+q-1}} \). A direct consequence of theorem 1 is that the virtual array response matrix \( \tilde{\mathcal{K}}_{\theta} = (A^{*} \odot A)_{\text{smoo}} \) has full column rank if \( q \leq m^2 / 4 + m / 2 \).

Remark:

1) To correctly estimate the DOAs of the WB signals, \( q \leq m^2 / 4 + m / 2 \) must be satisfied, \( \tilde{\mathcal{K}}_{\theta}, \tilde{\mathcal{K}}_{\theta}^* \) is inherently low rank matrix with rank \( q \) and we can correctly use the convex programming method to estimate it. Thus the proposed method can identify the maximal number of the WB signals is \( m^2 / 4 + m / 2 - 1 \). However, the augmented matrix approach cannot identify more than
sources, because it uses only one half of co-array [14], and the conventional subspace-based methods can only handle \( M^{-1} \) sources.

2) From difference co-array perspective of the nested array, it is a filled ULA with \( 2M - (M_i + 1) - 1 \) elements whose positions are given by the set \( \{ x_n = -N, \ldots, N, N + M_j(M_i + 1) - 1 \} \). In this paper, the spatial smoothing technique only uses \( M^{-1}/4 + M / 2(M_i + M_j) \) DOFs offered by the difference co-array.

5. Simulation Results
In this section, we construct several simulations to evaluate the proposed method for WB sources DOA estimation in underdetermined case by using a linear nested array. In the examples, we consider a nested array with \( N = 6 \) sensors and \( Q = 8 \) WB sources impinging on it from different directions \([-40, -30, -20, -10, 5, 20, 30, 60] \). Suppose the WB sources have the same center frequency \( f_0 = 100 \text{Hz} \) and the same bandwidth \( B = 40 \text{Hz} \). By using the singular value thresholding approach [14] to solve (17) and recover the low rank matrix, we use the routine singular-value-rpca [15] under the svt package available online. The routine uses a parameter \( \lambda \), which is related to the parameter \( \varepsilon \) in (17), in the simulation experiments, we set \( \lambda = 5 \). The demodulated data is sampled at a frequency of 300Hz.

The array output is divided into \( I = 33 \) narrowband components via DFT and we choose the segment number to be \( L = 100 \). Therefore we use a total of \( I \times L = 3300 \) samples. In the following simulations, \( K \) Monte Carlo runs are performed to calculate the average result for each experiment and the input signal-to-noise ratio (SNR) is defined as \( 10 \log_{10} \left( \frac{\rho_{s,i}^2}{\sigma_m^2} \right) \), where \( \rho_{s,i}^2 \) denotes the power of signal source and \( \sigma_m^2 \) stands for the noise power.

Figure 2 shows the representative DOA estimation spectrum obtained using the proposed method, with respect to various angles at a SNR of 0dB. We can see that the proposed method can use 6 sensors to resolve the 8 sources well.

We consider the performance of our proposed method by studying the root mean square error (RMSE) of the angle estimates as a function of the SNR and the snapshot number. By choosing the reasonable value for which the RMSE is minimized, the RMSE is defined as follows [16]

\[
\text{RMSE} = \sqrt{\frac{1}{K \times Q} \sum_{k=1}^{K} \sum_{q=1}^{Q} \left( \theta_{q,k} - \hat{\theta}_{q,k} \right)^2}
\]

where \( \theta_{q,k} \) and \( \hat{\theta}_{q,k} \) denote the true and estimation DOAs, respectively.

Figure 2. DOA estimation spectrum.

Figure 3. RMSE curves versus the SNR, using nested array, traditional ULA with 6 sensors and 15 sensors.
In the first simulation, since the nested array has 6 sensors and 11 DOFs, we also consider the corresponding RMSE for conventional MUSIC applied to 6-sensor and 15-sensor ULAs. We plot the RMSE for the source at $30^\circ$, the performance is similar for the other sources, and we fix the number of snapshots is 256. Figure 3 shows the RMSE of the three methods as a function of SNR, we can see that the performance of all three methods improve with the SNR increasing. In addition, the proposed method performs reasonably better than the corresponding ULA with same number of sensors, and performs close to the much longer ULA with 15 sensors. For the purpose of comparison, we also plot the Cramer-Rao Bound (CRB) for the WB signals, which can be applied to the underdetermined case. The RootCRB (N) represents the square root of CRB bound with N sensors, we compare the proposed method with the RootCRB (6) and RootCRB (15). Figure 4 gives RMSE curves with SNR from -10dB to 10dB, we fix the number of snapshots is 256. The proposed method shows a more satisfactory performance and the RMSE is reasonably close to RootCRB (15) at the same SNR. When SNR is set to 10dB and the snapshot number varies from 50 to 500 with a step of 50, the RMSE curves of the proposed method, RootCRB (6) and RootCRB (15) are shown in Figure 5, respectively. The Figure 5 shows that the performance of the proposed method is better than the ULA with 6 sensors, but still not good as the ULA with 15 sensors at the same snapshot number.

6. Conclusions
In this paper, a WCNNM algorithm is proposed for multiple WB signals impinging on a nested array. The proposed method can improve the DOA estimation performances, as well as increase the maximal number of detectable signals. Simulation results validate the effectiveness of the presented approach and it still has better performance even when the total number of incident sources exceeds that of array elements.

7. Acknowledgment
This work was supported by the Program for New Century Excellent Talents in University (NCET-13-0105), and by the Support Program for Hundreds of Outstanding Innovative Talents in Higher Education Institutions of Hebei Province, under Grant No.BR2-259, and by Natural Science Foundation of Hebei Province (No. F2016501139), and by the Specialized Research Fund for the Doctoral Program of Higher Education of China (No.20130042110003), and by the Fundamental Research Funds for the Central Universities under Grant No. N142302001, and by the Fundamental Research Funds for the Central Universities under Grant No.N162304002.

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