NUCLEON ELECTRIC POLARIZABILITY IN SOLITON MODELS AND THE ROLE OF THE SEAGULL TERMS

Norberto N. SCOCCOLA\textsuperscript{a,b} \footnote{Fellow of the CONICET, Buenos Aires, Argentina.} and Thomas D. COHEN\textsuperscript{c} \footnote{On leave from Department of Physics, University of Maryland, College Park, MD 20742.}

\textsuperscript{a} INFN, Sez. Milano, Via Celoria 16, 20133 Milano, Italy.
\textsuperscript{b} Departamento de Física, Comisión Nacional de Energía Atómica, Av. Libertador 8250, 1429 Buenos Aires, Argentina.
\textsuperscript{c} Department of Physics and Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA.

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ABSTRACT

The full Hamiltonian of the soliton models contains no electric seagull terms. Here it is shown that if one restricts the fields to the collective subspace then electric seagull terms are induced in the effective Hamiltonian. These effective seagull contributions are consistent with gauge invariance. They also reproduce the leading nonanalytic behavior of a large $N_c$ chiral perturbation theory calculation of the electric polarizability.
1 Introduction

The electric and magnetic polarizabilities of the nucleon are important low energy observables. During the past several years there has been significant improvements in the experimental measurement of these quantities\cite{1}. The ability to explain the polarizability is a significant test of any proposed model of the nucleon. Of course, in order to use the experimental data to discriminate between the various models on the market, one needs to be able to calculate the polarizabilities within each of the models. This is not necessarily trivial either technically or conceptually. This paper concerns a central conceptual issue in calculations of the electric polarizability in the topological soliton models \cite{2, 3} and in other hedgehog soliton or bag models based on large $N_c$ QCD such as the chiral-quark meson soliton model \cite{4}, the chiral or hybrid bag model \cite{5} or the soliton approach to the Nambu–Jona-Lasinio (NJL) model \cite{6}. As all these models behave identically with regard to the main issues in this article, for simplicity we will refer to all of them as soliton models.

There have been a number of calculations of the nucleon electric polarizability in soliton models \cite{7}–\cite{12}. In all of these calculations the dominant contribution to the polarizability has been a seagull contribution—\textit{i.e.}, a term in the effective Hamiltonian proportional to the square of the external electric field. In fact, the derivation of this seagull term in all of the calculations to date has been rather naive. The method used was to couple the Lagrangian density for the model to an external electromagnetic potential in the usual way. An external electric field in the $\hat{z}$ direction is introduced by choosing $A_0 = -E \hat{z}$. A collective Lagrangian is obtained by inserting a rotating hedgehog solution into the Lagrangian density and integrating over the spatial coordinates; the collective variables being the angles specifying the rotation and their associated angular velocities. Finally, the collective Hamiltonian is obtained from the collective Lagrangian. The seagull term in the collective Hamiltonian is then simply identified as minus the seagull contribution to the collective Lagrangian since this term contains no time derivatives.
This procedure is essentially the one used to obtain a collective Hamiltonian in the absence of external fields. However, in the present context such a procedure seems quite cavalier in view of the fact that when the Lagrangian contains derivative couplings the identification of the part of the Lagrangian lacking time derivatives with minus the corresponding terms in the Hamiltonian is, in general, not correct. For example, in scalar electrodynamics there is an electric seagull term in the Lagrangian given by \( \mathcal{L} = \phi^* \phi A_0^2 \), while in \( \mathcal{H} \) there are no terms proportional to \( A_0^2 \) \[13\]. Indeed, it is obvious that the existence of such a term in the Hamiltonian would violate gauge invariance.

It has recently been argued that the procedure used in obtaining the electric seagull terms in the collective Hamiltonian of the Skyrme model and other soliton models is more than cavalier; namely, that it is wrong. A general form of this argument is due to L’vov \[14\] who argues that a general theorem of Brown \[15\] shows that existence of electric seagulls violates local gauge invariance. A particularly transparent version of this argument is that if the procedure used to obtain electric seagulls in the Skyrme model were used with a constant \( A_0 \) then one would obtain an energy of the soliton directly proportional to \( A_0^2 \). This would be manifestly absurd since the energy of a charged state in an external constant potential has an electric energy shift directly proportional to \( A_0 \) with no higher powers \[16\]. Moreover, Saito and Uehara \[17\] explicitly demonstrated that the full Hamiltonian of the Skyrme model coupled to an external electric potential has no seagulls.

The arguments in Refs. \[14, 17\] appear to be compelling. Therefore it would seem that the electric seagulls should be absent and the calculations in Refs. \[7–12\] are simply incorrect. On the other hand, there is quite strong evidence that the electric seagull terms as calculated in these works are correct. This evidence comes from the behavior of the electric polarizability \( \alpha \) as one approaches the large \( N_c \) and chiral limits (with the \( N_c \rightarrow \infty \) limit taken first). The polarizability in this limit is

\[ \text{2} \]
completely determined via large $N_c$ chiral perturbation and is given by\textsuperscript{3}

$$
\lim_{m_\pi \to 0} \lim_{N_c \to \infty} \alpha = \frac{5 e^2 g_A^2}{128 \pi^2 f_\pi^2 m_\pi}
$$

Now consider the electric seagull contributions to $\alpha$ using the methods of Refs. [7]–[12]. In general, the result is given by an integral which depends on the details of the model. However, as $m_\pi$ goes to zero the integral is increasingly dominated by the long range tail. The overall strength of the contribution is proportional to the square of the amplitude of the pion field which in the standard treatment of soliton models is directly proportional to $g_A^2 / f_\pi^2$ (where $g_A$ is the appropriate value of $g_A$ for the model in question). Evaluating the integral for $\alpha$ in this limit precisely reproduces Eq. (1). Moreover, apart from the seagull, no contribution to $\alpha$ in the calculations based on soliton models diverges as $m_\pi \to 0$. Indeed it is precisely for this reason that seagulls dominate the result when $m_\pi$ is finite but small as it is in the physical world. Thus, the electric seagull terms appear to be required to get the correct behavior in the large $N_c$ and chiral limits.

Thus there is a paradox: On the one hand, there is both a general argument based on gauge invariance and an explicit calculation showing the absence of seagulls in the Hamiltonian; while on the other hand, these seagulls are apparently necessary to get the correct result in the large $N_c$ and chiral limits. The purpose of the present paper is to resolve this paradox.

The resolution is, in fact, quite simple. The full Hamiltonian for the soliton models does not contain any electric seagulls as is argued in Refs. [14, 17]. However, the model is studied in the large $N_c$ limit in which there is a collective manifold of configurations which dominate the low energy behavior. For the original Skyrme

\textsuperscript{1}This result is precisely three times the naive chiral perturbation result of Ref. [18]. This factor of three comes from the contribution of $\Delta + \pi$ intermediate states which contribute in the large $N_c$ limit since the $\Delta$ becomes degenerate with the nucleon and gives rise to new infrared behavior. In this limit the inclusion of the $\Delta$ contributions always yields three times the naive result of chiral perturbation theory [10, 19].
model, this manifold is simply the space of rotating hedgehogs [20]. For models including explicit fermion or vector meson degrees of freedom the collective manifold includes “cranking” effects as discussed in Ref. [21]. The central point in resolving the paradox is that while the full Hamiltonian has no electric seagulls, the effective Hamiltonian in terms of the collective variables can, and in general will, have such terms. This will be shown explicitly below.

Moreover, it is clear why the effective theory can have electric seagull terms without violating Brown’s theorem on electric seagulls. That theorem depends explicitly on the fact that the field theory must be local. However, the constraint to some collective manifold will depend on spatial integrals and the effective Hamiltonian which emerges does not correspond to a local field theory.

However, there is still a potential problem: namely, even if electric seagulls do exist, the identification of the electric seagull term in $H^{\text{coll}}$ as minus the seagull in $L^{\text{coll}}$ is problematic. In general, this identification is clearly incorrect; the case of a constant $A_0$ provides an explicit example where this procedure fails. We will show, however, that in certain cases the identification is in fact valid. In particular, it is valid when the term in the collective Hamiltonian (or Lagrangian) linear in the external field vanishes for all values of the collective variables. One can only expect this to happen due to symmetry. In fact, this will occur if all the configurations in the collective manifold have the same parity and the external $A_0$ is of odd parity; which is precisely the case of the soliton model collective manifold of states in a constant electric field.

This paper is organized as follows: in Sec. 2 we discuss in the simple model of a charged scalar field the effect of introducing a constraint in the allowed field configuration. We show that this automatically leads to the presence of a seagull term in the effective Hamiltonian. In Sec. 3 we study in all detail a soliton model in an external $A_0$ field, namely we discuss how the electric seagulls do contribute to the nucleon electric polarizability in the Skyrme model. In Sec. 4 we give our conclusions.
2 Electric seagulls for a constrained system of charged scalars

Before discussing the specifics of the soliton models constrained to a collective manifold, it is useful to study a simpler system which raises the same issues. We will consider a system of charged scalars interacting with an external electromagnetic field; the fields are subject to a constraint that they must be in some collective manifold. Here we will assume that the external magnetic field is zero and we will work in a gauge in which $A_i = 0$. We will denote the charge of the particle by $Q$.

The Lagrangian density for the system in the absence of any constraint is

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - iQ A_0 (\phi^* \dot{\phi} - \phi \dot{\phi}^*) + Q^2 A_0^2 \phi^* \phi$$  \hspace{1cm} (2)$$

which leads to the following Hamiltonian density

$$\mathcal{H} = \pi^* \pi + \vec{\nabla} \phi^* \cdot \vec{\nabla} \phi + m^2 \phi^* \phi + iQA_0 (\pi^* \phi - \pi \phi) .$$  \hspace{1cm} (3)$$

As expected, for the full theory there is a seagull in the Lagrangian but not the Hamiltonian.

Suppose we now expand the fields in terms of some (for now arbitrary) orthonormal and complete set of functions, $f_j(x)$, satisfying

$$\int dV f_j^*(x) f_j(x) = \delta_{i,j} , \hspace{1cm} (4)$$

$$\sum_i f_i^*(x) f_i(y) = \delta^3(x - y) . \hspace{1cm} (5)$$

The Hamiltonian becomes

$$H = \sum_{i,j} \left[ \pi_i^* \delta_{i,j} \pi_j + \phi_i^* B_{i,j} \phi_j + iQ (\pi_i^* A_{i,j} \phi_j - \pi_i A_{i,j} \phi_j) \right]$$  \hspace{1cm} (6)$$

where $\phi_i = \int dV f_j^*(x) \phi(x)$ and $\pi_i = \int dV f_j(x) \pi(x)$ are the dynamical variables and their conjugate momenta, respectively. The matrices $A$ and $B$ are defined by the following:

$$A_{i,j} \equiv \int dV A_0(x) f_i^*(x) f_j(x) ,$$  \hspace{1cm} (7)$$

$$B_{i,j} \equiv m^2 \delta_{i,j} + \int dV \vec{\nabla} f_i^*(x) \cdot \vec{\nabla} f_j(x) .$$  \hspace{1cm} (8)$$
At this stage the Hamiltonian in Eq. (6) is completely equivalent to the original Hamiltonian and, of course, there are no electric seagulls. Now, suppose that the dynamics were constrained so that fields had to be in a collective space of dimension \( N \). A simple way to do this is to choose a particular basis set \( f_j(x) \) in which to expand and then to impose the following condition on the field variables:

\[
\dot{\phi}_k = 0 \quad \text{for} \quad k > N .
\]

(9)

Given this constraint the dynamics has \( N \) true degrees of freedom. The equation of motion for \( \dot{\phi}_k \) with \( k > N \) becomes the following equation of constraint for the conjugate momenta:

\[
\pi_k^* = iQ \sum_{j \leq N} A_{k,j} \phi_j .
\]

(10)

Thus, even though \( \phi_j \) is zero for \( j > N \), \( \pi_j \) is in general non-zero; these non-zero conjugate momenta lead directly to electric seagulls in the effective \( N \)-dimensional Hamiltonian. This is straightforward to see. The simplest way is to insert the constraint Eq. (10) into the Hamiltonian. The resulting collective Hamiltonian, \( H^{\text{coll}} \), reads

\[
H^{\text{coll}} = \sum_{i,j \leq N} \left[ \pi_i^* \delta_{i,j} \pi_j + \phi_i^* B_{i,j} \phi_j + iQ(\pi_i^* A_{i,j}^* \phi_j - \pi_i A_{i,j} \phi_i) - \sum_{k>N} Q^2 \phi_i^* A_{i,k} A_{k,j}^* \phi_j \right] .
\]

(11)

The last term is the electric seagull. Of course, it is generally not permissible to insert a solution of the equation of motion back into the Hamiltonian since a variation of the substituted Hamiltonian will not necessarily give the same equation of motion as the original. However it is trivial to show in this case that the equations of motion for the collective Hamiltonian exactly reproduce the equations of motion for the full Hamiltonian for the \( N \) collective degrees of freedom provided the noncollective degrees of freedom satisfy the constraint Eqs. (9) and (10).

Thus we have shown explicitly how constraining the theory leads to an effective electric seagull term. Moreover, it is clear that the seagull term is physically sensible.
in that it corresponds to an induced electric charge density in the presence of a background field. The charge density can be written as \( \rho(x) = -iQ(\pi^*\phi^* - \pi\phi) \). This can be re-expressed as

\[
\rho(x) = \rho_1(x) + \rho_2(x) \quad (12)
\]

\[
\rho_1(x) = -iQ \sum_{i\leq N, j \leq N} f_i(x)f_j^*(x)\pi_i^*\phi_j^* + \text{h.c.} \quad (13)
\]

\[
\rho_2(x) = -iQ \sum_{j \leq N, k > N} f_k(x)f_j^*(x)\pi_k^*\phi_j^* + \text{h.c.} = iQ^2 \sum_{i,j \leq N, k > N} f_k(x)f_j^*(x)A_{k,i}^*\phi_i\phi_j^* \quad (14)
\]

where the last equality follows from the equation of constraint for \( \pi_k \). Clearly \( \rho_2(x) \) is the induced charge due to the electric seagull. Indeed the seagull term can be expressed as

\[
H_{\text{seagull}} = \frac{1}{2} \int dV A(x)\rho_2(x) \quad (15)
\]

Although the preceding argument explicitly demonstrates that electric seagulls can arise from a constraint onto a collective subspace, it should be noted that the procedure used here is not the one used in the soliton model calculations. It is useful to show that, when applied to the present case, the methods used there yield the same electric seagulls in the Hamiltonian as the construction above. As already mentioned, in soliton models the constraint on the fields is imposed at the level of the original Lagrangian thereby obtaining a collective Lagrangian. The collective Hamiltonian is then derived from such a collective Lagrangian by means of Legendre transformation acting on the collective variables. It is not surprising that this gives identical results to the collective Hamiltonian above. Expressed in terms of our functions \( f_i \) the collective Lagrangian reads

\[
L_{\text{coll}} = L_1 + L_2 \quad , \quad (16)
\]

\[
L_1 = \sum_{i,j \leq N} \left[ \dot{\phi}_i^*\dot{\delta}_{i,j}\dot{\phi}_j - \phi_i^*B_{i,j}\phi_j - iQ(\phi_j^*A_{j,i}\dot{\phi}_i + \dot{\phi}_j^*A_{j,i}\phi_i) \right] \quad , \quad (17)
\]
\[ L_2 = Q^2 \left[ \sum_{i,j,l \leq N} A_{i,l} A_{l,j} \phi_i^* \phi_j + \sum_{i,j \leq N, k > N} A_{i,k} A_{k,j} \phi_i^* \phi_j \right] \] (18)

It is a simple exercise to show that the Legendre transform of \( L^{\text{coll}} \) yields \( H^{\text{coll}} \). It is worth observing that the seagull term in \( H^{\text{coll}} \) is only (minus) the second term in \( L_2 \). Thus, in general the procedure used in the soliton model calculations of Refs. [7]–[12]—of identifying the seagull in the collective Hamiltonian with minus the seagull in the collective Lagrangian—is not correct. It is only valid if the first term in \( L_2 \) vanishes, which generally does not happen. However, it is also worth noting that symmetry can make such terms vanish for certain classes of external electrostatic potentials. For example, if the \( A_0 \) distribution is odd under parity and all of the states in the collective subspace have the same parity (either even or odd) then \( A_{i,j} = 0 \) for \( i, j \) members of the collective subspace and the first term in \( L_2 \) vanishes. This is significant since this is precisely the circumstance of the soliton models in a constant external electric field. The electrostatic potential is just \(-E_0 z\), so \( A_0 \) is clearly odd; while all of the configurations in the collective subspace in the Skyrmion, hedgehogs, and rotated hedgehogs are odd under \( x \rightarrow -x \) (making them even under parity since under parity \( x \rightarrow -x \) and \( \pi \rightarrow -\pi \)).

Thus, although the procedure usually used in soliton models of identifying the seagulls in the collective Hamiltonian with the negative of the seagulls in the collective Lagrangian is not generally valid, it is valid in the context in which it is actually used. At this stage, it is worth noting that the objection of L’vov to the procedure used in Refs. [7]–[12] based on the case of a constant \( A_0 \) does not apply to the case of constant electric field. The procedure of Refs. [7]–[12] should not be used for a constant \( A_0 \) since symmetry does not cause the second term of \( L_2 \) to vanish. In fact, it is rather easy to see that for constant \( A_0 \) it is the second term in \( L_2 \) that vanishes. This, of course, leads to a vanishing seagull term in \( H^{\text{coll}} \) as is expected from gauge invariance.

Having shown how seagull terms can be induced by constraining the allowed field configurations in the case of the simple system of charge scalars we will study now how this mechanism works for the actual case of soliton models.
3 Electric seagulls in the Skyrme model

As discussed in the Introduction from the point of view of the electric seagulls the various versions of soliton model behave in a very similar way. In this section we will discuss in detail the particular case of the original Skyrme model \[2\] with quartic term stabilization. However, all our conclusions can be immediately extended to other models based on hedgehog solitons.

We start by introducing the canonical pion field $\Phi_a$ in terms of which the $SU(2)$ chiral field $U(x)$ reads

$$U = \frac{1}{f_\pi} (\Phi_0 + i\tau_a \Phi_a) \quad (19)$$

where

$$\Phi_0^2 = f_\pi^2 - \sum_{a=1}^{3} \Phi_a^2. \quad (20)$$

In terms of these fields the Hamiltonian of the Skyrme model in the presence of an external e.m. field can be expressed as \[17\]

$$\tilde{H}_{Sky} = H_{Sky} + H_1 + H_2, \quad (21)$$

where

$$H_{Sky} = \int dV \left\{ \frac{1}{2} \Pi_a K_{ab}^{-1} \Pi_b + \nu \right\}, \quad (22)$$

$$H_1 = \int dV \ eA_0 \left\{ -\rho - J_a^0 K_{ab}^{-1} \Pi_b \right\}, \quad (23)$$

$$H_2 = \frac{1}{2} \int dV \ e^2 A_0^2 \left\{ -\Gamma^{00} + J_a^0 K_{ab}^{-1} J_b^0 \right\}. \quad (24)$$

Here, $\rho$ is the electric charge density and $\nu = \nu(\Phi_a, \nabla \Phi_a)$ are terms containing no time derivative. The explicit form of these quantities can be found e.g., in Ref. \[22\]. $\Gamma^{00}$ is the seagull term in the Lagrangian as defined in Ref. \[17\], and

$$\Pi_a = K_{ab} \dot{\Phi}_b + eA_0 J_a^0, \quad (25)$$

$$J_a^0 = \epsilon_{3ca} \Phi_c K_{da}, \quad (26)$$

9
\[ K_{ab} = X_{ab} - \frac{1}{e^2 f^4} [X_{ab} X_{cd} - X_{ac} X_{bd}] \partial_i \Phi_c \partial^i \Phi_d , \] (27)

\[ X_{ab} = \delta_{ab} + \frac{\Phi_a \Phi_b}{\Phi_0^2} . \] (28)

In writing Eq. (27) we have considered, as in the previous section, the particular case in which the external magnetic field is zero and have chosen a gauge in which \( A_i = 0 \). Moreover, we have dropped several terms which are not relevant for our arguments\(^2\).

As discussed in Ref. [17], explicit calculation shows that the two terms in \( H_2 \) cancel each other. This is in complete analogy to what we have already seen in the previous section. At the level of the full Hamiltonian there are no electric seagull terms. However, within the Skyrme model, baryons are described in the large \( N_c \) limit as slowly rotating hedgehog configurations, namely, by field configurations given by \(^2\)

\[ \Phi_a = f_\pi \sin F(r) R_{ai} \hat{r}_i , \] (29)

\[ \Phi_0 = f_\pi \cos F(r) . \] (30)

\( F(r) \) is the so-called chiral angle and \( R_{ab} \) is the time-dependent rotation matrix. The introduction of this ansatz implies a strong restriction in the allowed configurations in the \( \Phi \)-field Hilbert space. As in the case of a charged scalar field this will lead to the appearance of an electric seagull term in the restricted (collective) Hamiltonian.

Replacing the Skyrme ansatz in \( \tilde{H}_{Sky} \), Eq. (21), we get

\[ \tilde{H}_{Sky}^{\text{coll}} = M_{sol} + \frac{1}{2} \Theta \Omega^2 - \int dV e A_0 \rho(r) - \frac{1}{16 \pi} \int dV e^2 A_0^2 \Theta(r) \left[ 1 - R_{3n} R_{3p} \hat{r}_n \hat{r}_p \right] . \] (31)

\( M_{sol} \) is the soliton mass (see, e.g., Ref. [21] for its explicit expression), \( \tilde{\Omega} \) is the angular velocity defined by \( R_{an} \dot{R}_{ai} = \epsilon_{nai} \Omega_l \), the charge density \( \rho(r) \) is given by

\[ \rho(r) = - \frac{1}{4 \pi^2} \frac{\sin^2 F}{r^2} F^\prime , \] (32)

\(^2\)They lead to higher order corrections in \( 1/N_c \) once the rotating Skyrme ansatz, Eq. (30), is introduced [17].
and \( \Theta \) is the soliton moment of inertia

\[
\Theta = \int dr \ r^2 \ \Theta(r) \quad (33)
\]

where

\[
\Theta(r) = \frac{8\pi f_\pi^2}{3} \sin^2 F \left[ 1 + \frac{1}{e^2 f_\pi^2} \left( \frac{F''^2}{r^2} + \frac{\sin^2 F}{r^2} \right) \right]. \quad (34)
\]

As we see, although \( H_2 \) vanished, we do have terms proportional to \( e^2 \)—i.e., seagull-like type—in \( \tilde{H}_{Sky}^{\text{coll}} \). They have their origin in \( H_{Sky} \) and \( H_1 \). In order to find the canonical expression of the collective Hamiltonian we have still to eliminate the angular velocity \( \Omega_i \) in terms of the angular momentum operator \( J_i \). The general form of this operator can be obtained from the full Skyrme model Lagrangian in the presence of an external \( A_0 \) field by using

\[
J_i = \int dV \ \epsilon_{ijk} \ r_j \ T_{0k}, \quad (35)
\]

where \( T_{0k} = \Pi_a \partial_k \Phi_a \) are the corresponding components of the energy–momentum tensor. We get

\[
J_i = \int dV \ \epsilon_{ijk} \ r_j \ K_{ab} \ \hat{\Phi}_b \partial_k \Phi_a + \int dV \ eA_0 \ \epsilon_{ijk} \ k_{3cd} \ r_j \ K_{ab} \ \hat{\Phi}_c \partial_k \Phi_a. \quad (36)
\]

The collective form of this operator is then obtained by replacing the Skyrme ansatz, Eq. (30). We obtain

\[
J_i = \Theta \Omega_i + \frac{3}{8\pi} \int dV \ eA_0 \Theta(r) \left( \delta_{in} - \hat{r}_i \hat{r}_n \right) R_{3n}. \quad (37)
\]

Using this equation, we get the final expression of the Hamiltonian in the restricted space of the rotating soliton

\[
\tilde{H}_{Sky}^{\text{coll}} = H_0 + H_L + H_Q, \quad (38)
\]

where

\[
H_0 = M_{sol} + \frac{J^2}{2\Omega}, \quad (39)
\]

\[
H_L = -\int dV eA_0 \left[ \rho(r) + \frac{3}{8\pi\Theta} \Theta(r) \left( \delta_{in} - \hat{r}_n \hat{r}_i \right) R_{3n} J_i \right], \quad (40)
\]
\[ H_Q = \frac{\Theta}{2} \left(\frac{3}{8\pi\Theta}\right)^2 \left[ \int dV eA_0 \Theta(r) (\delta_{in} - \hat{r}_i \hat{r}_n) \right] \left[ \int dV eA_0 \Theta(r) (\delta_{im} - \hat{r}_i \hat{r}_m) \right] R_{3n} R_{3m} \]

\[ -\frac{3}{16\pi} \int dV e^2 A_0^2 \Theta(r) \left[ 1 - R_{3n} R_{3m} \hat{r}_n \hat{r}_m \right] . \] (41)

We can now discuss some particular cases. First, we will consider the situation proposed by L’vov in which \( A_0 \) is a constant field. In this case it is easy to show that

\[ \frac{3}{8\pi\Theta} \int dV eA_0 \Theta(r) (\delta_{in} - \hat{r}_i \hat{r}_n) = eA_0 \delta_{ni} , \] (42)

\[ \frac{3}{16\pi} \int dV e^2 A_0^2 \Theta(r) \left[ 1 - R_{3n} R_{3m} \hat{r}_n \hat{r}_m \right] = \frac{\Theta}{2} e^2 A_0^2 . \] (43)

from where we get

\[ H_L(A_0 = cte) = -eA_0 \left( \frac{1}{2} + I_3 \right) , \] (44)

\[ H_Q(A_0 = cte) = 0 . \] (45)

Here we have used the well-known relation \( I_3 = R_{3i} J_i \) and \( \int dV \rho(r) = 1/2 \). As expected in the case of a constant \( A_0 \) field the quadratic term in \( A_0 \) vanishes and the resulting Hamiltonian corresponds to the interaction of the constant \( A_0 \) field with the baryon electric charge.

Now we turn to the more interesting case of a constant electric field in the \( z \)-direction, namely \( A_0 = -zE \). This is the field configuration that has been used in Refs. [7]–[12] to determine the electric polarizabilities. It is easy to show that in this case

\[ \int dV eA_0 \rho(r) = 0 , \] (46)

\[ \frac{3}{8\pi\Theta} \int dV eA_0 \Theta(r) (\delta_{in} - \hat{r}_i \hat{r}_n) = 0 , \] (47)

\[ \frac{3}{16\pi} \int dV e^2 A_0^2 \Theta(r) \left[ 1 - R_{3n} R_{3m} \hat{r}_n \hat{r}_m \right] = \frac{e^2 E^2}{6} \int drr^4 \Theta(r) \left[ 1 - \frac{2}{5} D_{00}^{(2)} \right] , \] (48)

where we have used \( (R_{33})^2 = \frac{1}{3} + \frac{2}{3} D_{00}^{(2)} \) with \( D_{00}^{(2)} \) being the corresponding \( D \)-matrix. Using these relations we get

\[ H_L(E = cte) = 0 , \] (49)

\[ H_Q(E = cte) = -\frac{1}{2} \gamma_e E^2 \left[ 1 - \frac{2}{5} D_{00}^{(2)} \right] , \] (50)
where
\[
\gamma_e = \frac{e^2}{3} \int dr \, r^4 \Theta(r) .
\] (51)

These expressions coincide with the ones usually used to calculate the electric polarizabilities in the Skyrme model (see e.g. Ref. [9]). In fact, for the case of the nucleon the matrix element of \(D_{00}^{(2)}\) vanishes and we simply get
\[
\alpha_N = \gamma_e .
\] (52)

As already found in the case of the charged scalars, although it is not in general correct to take the interaction terms in the Hamiltonian as minus those in the Lagrangian, this relation does hold for the case of a constant electric field.

4 Conclusions

In conclusion we have shown that although a fundamental bosonic theory has no electric seagulls at the Hamiltonian level the act of constraining the dynamics to a collective subspace induces electric seagulls in the collective Hamiltonian. We have explicitly shown how this mechanism works in a simple model of a charged scalar and in the more relevant case of the Skyrme model. We have argued that this can be immediately generalized to other versions of soliton models. Moreover, although we have only discussed how these seagull terms contribute to the nucleon electric polarizabilities, it is clear that our conclusions can be extended to other baryons as the \(\Delta\) and strange hyperons. We have demonstrated that the correct collective Hamiltonian can be obtained either by imposing the constraint on the Lagrangian to obtain a collective Lagrangian and then taking a Legendre transform, or by first obtaining the full Hamiltonian and then applying the field constraint; both methods leading to the same result. We have also discussed that the Legendre transform for the seagull term in the Lagrangian is simply given by the negative of the term provided certain symmetry conditions are met. In particular, for the case of a constant electric field such conditions are fulfilled. Thus, we conclude that the procedures used in
Refs. [7]–[12] to determine the seagull contributions to the electric polarizabilities are completely valid. On the other hand, for constant $A_0$ this relation does not hold. Explicit calculation shows, as expected, that electric seagulls in $H_{coll}$ vanish in that case.

We should also mention the interesting work of Nikolov, Broniowski and Goeke [23]. This work is based on a Nambu–Jona-Lasinio type model. As the model has no fundamental pion, it manifestly has no fundamental $\pi\gamma\gamma$ seagull terms. Therefore, their calculation is done in terms of the fundamental quark degrees of freedom which, of course, give only dispersive contributions. However, they also show that if one wishes to obtain a Skyrme-type model with pions from the NJL model via a gradient expansion, the leading term in the expansion for the electric polarizability is precisely the seagull term of the $\sigma$ model approaches. Again we see the general point: an effective model written in terms of meson fields restricted to a collective manifold does contain electric seagulls.

Finally, we want to mention that in general the seagull terms considered here are not the only contributions to the electric polarizabilities. In fact, when pion fluctuations around the soliton are taken into account the linear terms $H_L$ can contribute in second order perturbation theory. For the case of the nucleon the corresponding intermediate states are dipole-excited, negative-parity, nucleon resonances. A rough estimation [7] shows that these contributions are small. Similar result is found for the case of the $\Lambda$ [24]. Thus, the total electric polarizability is dominated by the seagull term—as expected by naive $N_c$ counting—and the arguments based in chiral perturbation theory given in the Introduction. Another point to be mentioned is that the techniques discussed in this article can also be applied for the case of a constant magnetic field, i.e., in the determination of the diamagnetic contributions to the magnetic polarizabilities. In that case, as already found in Ref. [8], the seagull terms in the Lagrangian differ from those in the Hamiltonian. Numerically, however, this difference is negligible.
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