Two-photon interference with thermal light

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The study of entangled states has greatly improved the basic understanding about two-photon interferometry. Two-photon interference is not the interference of two photons but the result of superposition among indistinguishable two-photon amplitudes. The concept of two-photon amplitude, however, has generally been restricted to the case of entangled photons. In this letter we report an experimental study that may extend this concept to the general case of independent photons. The experiment also shows interesting practical applications regarding the possibility of obtaining high resolution interference patterns with thermal sources.

The superposition principle is probably the most mysterious and fascinating concept of the theory of quantum mechanics [1]. In Young’s double-slit experiment, a light quantum has two indistinguishable alternative amplitudes that result in a photo-electron event at space-time point \((r, t)\). The superposition of the two indistinguishable amplitudes produces the interference of the light quantum itself [2]. Quantum theory may never identify through which slit (or both slits) the light quantum passed, however, it accurately predicts the counting rate as a function of the relative delay between the two amplitudes.

The experimental observations of Hanbury-Brown and Twiss introduced the concept of second order coherence [3]. Quantum theory of the second order interferometry describes the physical process of a joint photo-electron event at space-time point \((r_1, t_1)\) and \((r_2, t_2)\), produced by two light quanta with distinguishable and/or indistinguishable amplitudes [4]. Two-photon optics is a complex subject involving optical coherence, photon statistics, and, the nonlocal physics associated with the Einstein-Podolsky-Rosen two-particle system. Besides probing the fundamental issues of quantum theory, the massive study of entangled states, especially the experimental and theoretical research on the entangled two-photon state of Spontaneous Parametric Down Conversion (SPDC) has provided great insights on two-photon interferometry [5]. In particular, we have a better understanding of the troubling statement of Dirac: “Each photon interferes only with itself. Interference between two different photons never occurs.” The question whether two individual photons can or cannot interfere with each other has been answered experimentally based on the study of two-photon interferometry of SPDC: two-photon interference cannot simply be described in terms of interference of two independent photons but must be envisioned as an actual two-photon phenomenon in which the indistinguishable alternatives are two-photon amplitudes contributing to the final joint photo-electron events [6-8]. The concept of two-photon amplitude is somehow troubling, probably because of the nonlocality that it implies, and so, if accepted, it has generally been considered peculiar of entangled photons.

In this letter we wish to report an experimental study that may extend the concept of two-photon interference as the result of superposition of indistinguishable two-photon amplitudes to the general case of two independent photons. The result is intriguing also in a practical sense because \(N\)-photon interferometry with entangled states has been proven to represent a great potential for imaging and metrological applications [9,10]. In particular it has been proposed [11,12] and experimentally shown with two-photon entangled states from SPDC [13] that it is possible to do quantum lithography beyond the classical diffraction limit. In this experiment we simulated the experiment of D’Angelo et al. [13] with a pseudo-thermal source of light [14]. The experiment involves the measurement of second order interference of pseudo-thermal light through a standard Young’s double slit interferometer. In order to provide a clear physical picture of the phenomenon, we will discuss similar and different aspects between the thermal state and the entangled state of SPDC in this regard. It must be noted that a similar source, with a similar setup has been used for different purposes in a historical experiment [15].

The experimental setup is schematically shown in Fig. 1: the pseudo-thermal light source (or entangled two-photon light source of SPDC for comparison) illuminates a double slit of slit-width \(a\) with slit-distance \(d\); the joint photo-detection occurs in the far field plane with two photon counting detectors.

Let’s start by noticing that in both cases of pseudo-thermal light and SPDC radiation there is no observable first order interference effect in this experimental setup. The absence of the first order interference precludes the possibility of the second order interference to be a consequence of interference of the first order or the result of “partial coherence” between the fields at slit A and slit B. In this particular experiment any observable interference is not the result of each photon interfering with itself.

The quantity that governs the probability of joint photodetection and therefore the rate of coincidence counts is the second order Glauber correlation function [16]:

\[
G^{(2)}(t_1, r_1; t_2, r_2) \equiv Tr[\hat{\rho}E^{(-)}_1(t_1, r_1)E^{(-)}_2'(t_2, r_2)E^{(+)}_2(t_2, r_2)E^{(+)'}_1(t_1, r_1)] 
\]

(1)

where \(E^{(\pm)}_{1,2}(r_j, t_j)\), \(j = 1, 2\), are positive-frequency and the negative-frequency components of the field at detectors \(D_1\) and \(D_2\), and \(\hat{\rho}\) represents the density matrix of
the quantum state under consideration. The field operators, in both thermal light and SPDC cases, can be written as the superposition of earlier fields at slit A and B:

\[ E_1^{(+)}(r_1, t_1) = E_A^{(+)}(r_A, t_1 - \frac{r_{A1}}{c}) + E_B^{(+)}(r_B, t_1 - \frac{r_{B1}}{c}) \]  
\[ E_2^{(+)}(r_1, t_1) = E_A^{(+)}(r_A, t_1 - \frac{r_{A2}}{c}) + E_B^{(+)}(r_B, t_1 - \frac{r_{B2}}{c}) \]  

with \( r_{Aj} \) (\( r_{Bj} \)) defining the optical path length from slit \( A \) (\( B \)) to the \( j \)th detector. If \( G^{(2)} \) is different in the case of thermal light and SPDC, the difference must come from the intrinsic property of the light, as expected.

Let’s first briefly review the known physics behind the experiment that uses SPDC as the light source [13]. SPDC is a nonlinear process in which an entangled pair of photons, signal and idler, are simultaneously created. Therefore a joint photodetection is almost always the result of the detection of the signal-idler pair. In ref. [13], the double slit was placed very close to the SPDC crystal so that the source is divided into two regions: upper slit (\( A \)) and lower slit (\( B \)). Due to the entangled nature, a signal-idler pair is generated either from slit \( A \) or slit \( B \), but never from different ones. It is very intuitive, then, to write the two-photon state that would lead to a joint detection measurement in the following way:

\[
|\Psi\rangle \simeq |a\rangle|a\rangle e^{i\phi_A} + b_1^\dagger b_2^\dagger e^{i\phi_B} |0\rangle
\]  

here \( a^\dagger \) and \( b^\dagger \) stand for the photon creation operators at the upper and lower slit respectively, and \( \phi_A \) and \( \phi_B \) are the phases of the two-photon modes in correspondence to the upper slit (\( A \)) and the lower slit (\( B \)). The spatial coherence of the pump beam of the SPDC justifies the assumption: \( \phi_A - \phi_B = \text{constant} \). Under these conditions the expected second order correlation function is calculated as:

\[
G^{(2)} = |e^{ik(r_{A1} + r_{A2})} + e^{ik(r_{B1} + r_{B2})}|^2 
\]

\[
\propto 1 + \cos[k(r_{A1} + r_{A2} - r_{B1} - r_{B2})]
\]

In the far field zone, taking into account the finite size of the slits, the final interference-diffraction pattern is expected as:

\[
G^{(2)} \propto \sin^2\left[\frac{\pi a(x_1 + x_2)}{\lambda z}\right] \cos^2\left[\frac{\pi d(x_1 + x_2)}{\lambda z}\right]
\]

where \( x_1 \) and \( x_2 \) are the horizontal displacement of \( D_1 \) and \( D_2 \), respectively, and \( z \) is the common distance from the detectors to the double slit. Besides opening the road towards quantum lithography, this experiment clearly demonstrated that the second order interference is the result of the superposition between the “upper-upper” (\( A \rightarrow D_1 \) with \( A \rightarrow D_2 \)) and the “lower-lower” (\( B \rightarrow D_1 \) with \( B \rightarrow D_2 \)) two-photon amplitudes.

In the present experiment, we substituted the SPDC light with a pseudo-thermal source [17]. The source is basically composed by a He-Ne laser beam focused on a rotating ground glass diffuser disk: the radiation is randomly scattered in all possible directions. It has been shown theoretically and experimentally [18] that the scattered radiation has the same statistical and optical properties as standard thermal sources. During the experiment, the intensity of the light was operated in a very low counting rate regime to achieve the condition in which only two photons were present in the setup within the joint detection time window. It is reasonable then to restrict our analysis at the level of two photons. Let’s use a simple physical model for the process: it can be shown that one possible basis of the physical state space that describes the two-photon system may be composed by the following three normalized states [19]:

\[
|\alpha\rangle = a_k^\dagger a_k^\dagger |0\rangle
\]
\[
|\beta\rangle = b_k^\dagger b_k^\dagger |0\rangle
\]
\[
|\gamma\rangle = \frac{1}{\sqrt{2}}(a_k^\dagger b_k^\dagger + b_k^\dagger a_k^\dagger) |0\rangle.
\]

Here \( a^\dagger \) and \( b^\dagger \) stand for creation operators of the photons generated at the upper and lower slit respectively; while \( k \) and \( k' \) corresponds to the modes of the radiation leading to detectors \( D_1 \) and \( D_2 \) respectively. Notice that the three basis vectors correspond to the three intuitive alternatives of joint photodetection, i.e. (\( \alpha \)) the two photons both come from the upper slit \( A \); (\( \beta \)) both come from the lower slit \( B \); or (\( \gamma \)) one comes from \( A \) and the other from \( B \). It is important to emphasize that (\( \gamma \)) will lead to the interference feature. We will treat our system as a statistical mixture of the three basis vectors:

\[
\hat{\rho} = |\alpha|^2 |\alpha\rangle \langle \alpha| + |\beta|^2 |\beta\rangle \langle \beta| + |\gamma|^2 |\gamma\rangle \langle \gamma|
\]

where \( |\alpha|^2 \), \( |\beta|^2 \), and \( |\gamma|^2 \), are the probabilities of having the system in one of the basis vectors. In the thermal light case, the three probabilities are equal (1/3). Thus, the second order correlation function can be expressed as follows:

\[
G^{(2)} \propto \langle \alpha| E_1^{(-)} E_2^{(-)} E_1^{(+)} |\alpha\rangle + \langle \beta| E_1^{(-)} E_2^{(-)} E_1^{(+)} |\beta\rangle + \langle \gamma| E_1^{(-)} E_2^{(-)} E_1^{(+)} |\gamma\rangle.
\]

Substituting the field operators of Eq. (2) into Eq. (8) we obtain:

\[
G^{(2)} \propto |e^{ik(r_{A1} + r_{A2})}|^2 + |e^{ik(r_{B1} + r_{B2})}|^2 + \frac{1}{2}|e^{ik(r_{A1} + r_{B2})} + e^{ik(r_{A2} + r_{B1})}|^2.
\]

The superposition of the indistinguishable two-photon amplitudes “upper-lower” (\( A \rightarrow D_1 \) with \( B \rightarrow D_2 \)) and “lower-upper” (\( A \rightarrow D_2 \) with \( B \rightarrow D_1 \)) are responsible for the interference. In the far field zone and considering the finite size of the slits, the interference-diffraction pattern is thus:
Comparing with the SPDC case of (Eq. 5), we obtain a similar interference-diffraction pattern, hinting to the similar two-photon physics behind the two effects.

In the actual experiment we had an attenuated He-Ne laser beam impinging on a double slit, 10cm after the slit we put a converging lens \((f = 25\,mm)\) and placed the rotating ground glass disk at 33.5mm from the lens. Basically we imaged the double slit onto the ground glass in order to produce an effective double slit illumination \((a = 0.043mm\) and \(d = 0.135mm)\) on the disk, that is our source, as indicated in Fig. 1. The radiation scattered by the ground glass was then divided by a beam splitter and sent to two horizontally displaceable fibers, connected to single photon counting modules.

Fig. 2 reports the measured two-photon interference-diffraction pattern. The solid line represents the theoretical fit using Eq. 10. It is interesting to see that, similarly to the SPDC case, the interference-diffraction pattern is twice as narrow as the standard interference-diffraction pattern of He-Ne light (shown in the lower plot of Fig. 2 for comparison) and with interference modulation twice as large as the standard pattern, as if it was produced by a source of light with half the wavelength of the He-Ne laser. The visibility of the pattern, however, is about 28% whereas SPDC allows 100% visibility. Fig. 3 reports the single counts of detector \(D_1\) and \(D_2\) when the detectors are scanned in the horizontal direction. It is apparent that the single counts are flat over the entire analyzed range. This demonstrates the absence of any first order interference phenomena, i.e. no "partial coherence" existing in this experiment. In order to ensure that the source we used was indeed "thermal", before proceeding to the actual measurement, we repeated the historical experiment performed by Arecchi et al [20]. Fig. 4 reports the result of the second order correlation measurements. Basically we plot a histogram of number of coincidence counts versus the time difference of the clicks from the two detectors. This measurement is the principal evidence of thermal light statistics of a pseudothermal source. This result was also used to "calibrate" the coincidence time window. In all the measurements of this experiment, we set a time window of about 600 ns around the peak of Fig. 4 and measured the number of coincidences within that window.

The experimental results therefore confirm the expected similarities and differences between the thermal light and the entangled state regarding to the two-photon interference. As was shown in the theoretical derivation, the analogy between the thermal case and the SPDC case is due to the similar physics: the superposition of the two-photon alternative amplitudes leads to the interference. However there are two main differences: (1) the joint detection counting rate is a function of \(x_1 - x_2\) in the thermal case instead of \(x_1 + x_2\). The reason is that in the SPDC case, the two-photon amplitudes leading to the interference are the "upper-upper" \((A \rightarrow D_1\) with \(A \rightarrow D_2)\) and the "lower-lower" \((B \rightarrow D_1\) with \(B \rightarrow D_2)\) alternatives; in the case of thermal light, instead, the interference is produced by the superposition of the "upper-lower" \((A \rightarrow D_1\) with \(B \rightarrow D_2)\) and the "lower-upper" \((A \rightarrow D_2\) with \(B \rightarrow D_1)\) alternatives; (2) the visibility in the thermal light case is limited to 1/3, in fact in the case analyzed in this experiment the "upper-upper" and "lower-lower" alternatives do not contribute to the interference, but they do contribute to the constant background of the pattern. In the SPDC case, instead, due to entanglement, the only existing two-photon amplitudes are the indistinguishable "upper-upper" and "lower-lower" alternatives.

From a practical point of view it is important to notice that with a pseudo-thermal source we achieved the same doubling in spatial resolution that was obtained with entangled two-photon states. Recently it has been shown that with entangled four-photon states it is possible to achieve resolutions four times better than the standard limit [10]. However notice that since in our experiment the source of light does not involve any nonlinear process (i.e.: the wavelength of the two photons is the same of the pump, while in SPDC the entangled photons have twice the wavelength of the laser pump), the increase in spatial resolution is twice as large as the increase obtained in the analogous (same \(N\), number of entangled photons) case with entangled photons. Moreover notice that in the historical experiment similar to ours [15] the increase in spatial resolution was not obtained, however the similarities between the two experiments and the fact that in [15] the intensity of light was much higher, lead us to think that with this source of light it can be overcome the main limitation presented by entangled photon sources at the moment, i.e.: the low counting rates. The practical applicability of the method shown here could be precluded by the low visibility of the pattern. For certain measurements, however, it may be possible to implement a detection scheme insensitive to the constant background noise that could restore high visibilities.

In conclusion, we have experimentally studied a second order interference phenomenon with thermal light. By comparing this experiment with the analogous one performed with entangled photons, we have justified the physical interpretation of the phenomenon in terms of interference between indistinguishable two-photon amplitudes. Paraphrasing Dirac, we may summarize the physics as follows: although each photon did not interfere with itself in this experiment, the observed interference is the result of each pair of independent photons interfering with itself.

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**FIG. 1.** Sketch of the experimental setup. An attenuated He-Ne laser beam illuminates a double slit, 10cm after the slit there is a converging lens (f = 25mm) and a the rotating ground glass disk is placed at 33.5mm from the lens. Basically the double slit is imaged onto the ground glass producing an effective double slit illumination (a = 0.043mm and d = 0.135mm). The radiation scattered by the ground glass is then divided by a beam splitter and sent to two horizontally displacable fibers, connected to single photon counting modules.

**FIG. 2.** (a) Normalized second order interference diffraction pattern vs position of the detectors. The dots are the experimental data while the solid line is a theoretical fit from Eq. 10. The actual counting rate corresponds to about 1000 coincidence counts per second in the peak. The single counts are about 45000 per second in D1 and 25000 per second in D2.(b) Equivalent first-order interference diffraction pattern.

**FIG. 3.** Single detector counts vs positions of the detector D1 (filled circles) and D2 (hollow circles). The low level counting rate shows that the experiment was performed in the two-photon regime. The flatness of the graphs shows the absence of first order interference.

**FIG. 4.** Histogram of number of joint detection counts vs time difference of the two photo-electron events. The size of each channel is 0.3 ns. The graph is useful to “calibrate” the coincidence time window: in all the measurements of this experiment, the coincidence were counted in a time window of about 600 ns around the peak of the figure while the noise background was verified by shifting the coincidence window of 4000 ns towards the region of the accidental coincidences.

Figure 1. Giuliano Scarcelli, Alejandra Valencia, and Yanhua Shih.
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Figure 4. Giuliano Scarcelli, Alejandra Valencia, and Yanhua Shih.