Warped QCD without the Strong CP Problem

Akito Fukunaga and Izawa K.-I.

Department of Physics, University of Tokyo,
Tokyo 113-0033, Japan

Abstract

QCD in a five-dimensional sliced AdS bulk with chiral extra-quarks on the boundaries is generically free from the strong CP problem. Accidental axial symmetry is naturally present except for suppressed breaking interactions, which plays a role of the Peccei-Quinn symmetry to make the strong CP phase sufficiently small. Breaking suppression and enhancement due to AdS warping are considered in addition to naive boundary separation effects.
1 Introduction

The standard model of elementary particles as an effective theory including gravity has two apparent fine-tuning problems which are hard to be undertaken directly by additional (gauge) symmetries: the cosmological constant and the strong CP problems. The presence of extra dimensions might serve as an alternative to symmetry which naturally affects such fine-tuned parameters.

In this paper, following a previous one, we proceed to consider QCD in a five-dimensional sliced AdS bulk with chiral extra-quarks on the boundaries and confirm that it is generically free from the strong CP problem. We have adopted the AdS bulk as a natural curved spacetime background without the restriction to bulk flatness.

For definiteness, let us suppose that there is a pair of extra-quarks in addition to the standard-model quarks: a left-handed colored fermion $\psi_L$ and a right-handed one $\psi_R$. We assume an extra-dimensional space which separates them from each other along the extra dimension. If the distance between them is sufficiently large, the theory possesses an axial $U(1)_A$ symmetry

$$\psi_L \rightarrow e^{i\alpha} \psi_L, \quad \psi_R \rightarrow e^{-i\alpha} \psi_R$$

approximately, whose breaking is suppressed at a fundamental scale. This accidental global symmetry, which is actually broken by a QCD anomaly, naturally plays a role of the Peccei-Quinn symmetry, making the effective strong CP phase to be sufficiently small.

The point is that the presence of such an approximate symmetry is not an artificial requirement, but a natural result stemming from the higher-dimensional geometry, which might well be stable even against possible quantum gravitational corrections.

2 Bulk color gauge theory

Let us consider the four-dimensional Minkowski spacetime $M_4$ along with one-dimensional extra-space $S^1$, whose coordinate $y$ extends from $-l$ to $l$ (that is, two points at $y = l$ and $y = -l$).
\( y = -l \) are identified. The SU(3)_C gauge field is assumed to propagate on the whole spacetime \( M_4 \times S^1 \) equipped with the AdS-slice metric

\[
ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2; \quad \sigma = k|y|,
\]

where \( \mu, \nu = 0, \ldots, 3 \) and \( k \) denotes a positive or negative constant which determines the AdS curvature.

The action of the five-dimensional gauge field is given by

\[
S_A = \int d^4x \int_{-l}^{l} dy e^{-4\sigma} \frac{M_*}{4g^2(y)} \text{tr}(F_{MN}F^{MN}) + \int h(y) \text{tr}(AF^2 - \frac{1}{2} A^3 F + \frac{1}{10} A^5) \tag{3}
\]

where \( F_{MN} = \partial_M A_N - \partial_N A_M + [A_M, A_N] \) \((M, N = 0, \ldots, 4; x^4 = y)\), \( A = A_M dx^M \), and \( F = dA + A^2 = (1/2) F_{MN} dx^M dx^N \). Here, \( M_* \) is supposed to be a cutoff scale in the higher-dimensional theory and \( g(y) \) and \( h(y) \) are gauge and Chern-Simons coupling functions, respectively.

Kaluza-Klein reduction to the four-dimensional spacetime, however, yields a massless color-octet scalar which is undesirable in the low-energy spectrum. Hence we consider an \( S^1/Z_2 \) orbifold instead of the \( S^1 \). The five-dimensional gauge field \( A_M(x, y) \) is now under a constraint

\[
A_\mu(x, y) = A_\mu(x, -y), \quad A_4(x, y) = -A_4(x, -y), \tag{4}
\]

which eliminates the scalar zero mode and only yields a vector field at low energies.

In order to define the theory on the orbifold consistently, the action Eq.(3) on the \( S^1 \) should be invariant under the \( Z_2 \) transformation. This invariance is achieved as long as \( g(y) = g(-y) \) and \( h(y) = -h(-y) \). Note that the background metric Eq.(2) itself has been chosen to be invariant under the \( Z_2 \) transformation. We take the \( g(y) \) to be \( y \)-independent and the \( h(y) \) as

\[
h(y) = c \frac{y}{|y|}, \tag{5}
\]

where \( c \) is a constant to be determined in the next section.\(^3\) The gauge symmetries are

\(^3\)From a five-dimensional perspective, eigenstates corresponding to massive Kaluza-Klein modes have energies larger than \( l^{-1} \), which are to be integrated out in the reduction process. Note that their four-dimensional apparent masses depend on the location of each mode in the extra dimension and could appear as a modified CFT on a boundary \([6]\). We assume that the effects of the modified CFT to the four-dimensional QCD be phenomenologically viable, though they are not fully understood due to their nonperturbative nature (see also the localization arguments cited in the Discussion).
left unbroken in the bulk: the infinitesimal $\text{SU}(3)_C$ gauge transformation parameter is restricted to satisfy $\varepsilon(x, y) = \varepsilon(x, -y)$.

3 Boundary extra-quarks

There are two fixed points in the $S^1/Z_2$ orbifold: $y = 0$ and $y = l$. Let us put chiral extra-quarks on the fixed-point boundaries: a left-handed extra-quark $\psi_L$ at $y = 0$ and a right-handed one $\psi_R$ at $y = l$. The action of the split extra-quarks contains

$$S_\psi = \int_{y=0} d^4x \bar{\psi}_L i\not{D} \psi_L + \int_{y=l} d^4x e^{-3\sigma} \bar{\psi}_R i\not{D} \psi_R$$

$$= \int_{y=0} d^4x \bar{\psi}_L i\not{D} \psi_L + \int_{y=l} d^4x \bar{\tilde{\psi}}_R i\not{D} \tilde{\psi}_R,$$

where we have defined the canonically normalized field

$$\tilde{\psi}_R = e^{-\frac{3}{2}k} \psi_R.$$ (7)

Under an infinitesimal gauge transformation, this fermionic sector provides a gauge anomaly

$$\delta S_{\text{eff}} = \frac{i}{24\pi^2} \int_{y=0} \text{tr} \left( \varepsilon d(AdA + \frac{1}{2} A^3) \right) - \frac{i}{24\pi^2} \int_{y=l} \text{tr} \left( \varepsilon d(AdA + \frac{1}{2} A^3) \right)$$

due to its chirality, though the fermion content is vector-like from a four-dimensional perspective.

On the other hand, the bulk action yields

$$\delta S_A = \int h(y) \text{tr} \left( (d\varepsilon) d(AdA + \frac{1}{2} A^3) \right)$$

$$= - \int (dh(y)) \text{tr} \left( \varepsilon d(AdA + \frac{1}{2} A^3) \right)$$

under the gauge transformation. Gauge anomaly cancellation with the fermionic sector implies

$$c = \frac{i}{48\pi^2}.$$ (10)

4 The standard-model quarks (and leptons) are assumed to be on the fixed-point boundary at $y = 0$. We also include QCD $\theta$ and (possibly dominant) Yang-Mills terms implicitly on the boundary 3-brane.

5 In fact, the extra-quarks may be directly connected through superheavy modes (besides gauge interactions) with masses of order $m (\sim M_\ast)$ in the higher dimensions. This effect is exponentially suppressed when $ml$ is large enough, which we estimate in the Appendix.
4 Anomalous quasi-symmetry

The gauge-invariant theory is given by the total action $S = S_A + S_\psi$. Then, there is an approximate axial $U(1)_A$ symmetry given by Eq.(1).

4.1 Spontaneous breaking

The extra-quarks should be decoupled from the low-energy spectrum to escape from detection. Thus, we introduce hypercolor gauge interactions in the bulk to confine the extra-quarks at high energies. Such new gauge interactions would simultaneously induce a chiral condensate $\langle \psi_L \bar{\psi}_R \rangle$ to break down the axial $U(1)_A$ symmetry and provide a corresponding Nambu-Goldstone (NG) boson called an axion [7]. Nonvanishing anomaly $U(1)_A[SU(3)_C]^2$ induces a potential of the axion field.

We adopt $SU(3)_H$ as the hypercolor gauge group and assume that the chiral fermions on each boundary transform as $\psi_L(3, 3^*)$ and $\psi_R(3, 3^*)$ under the $SU(3)_C \times SU(3)_H$ gauge group. The $SU(3)_H$ interaction is supposed to be confining at an intermediate scale $F_a (< M_* )$ and develop a chiral condensate $\langle \psi_L \bar{\psi}_R \rangle \simeq F_a^3$. Note that the gauge anomalies due to the fermionic sector can be canceled by bulk Chern-Simons terms in a similar way as in the previous section.

$SU(3)_H$-charged particles are confined and only massless NG bosons are left at low energies. If one switches off the $SU(3)_C$ gauge interaction, there is the $U(3)_L \times U(3)_R$ flavor symmetry that acts on $\psi_L$ and $\psi_R^\dagger$. The flavor symmetry $U(3)_L \times U(3)_R$ is spontaneously broken down to a diagonal $U(3)$ symmetry. However, there is not the $U(3)_L \times U(3)_R$ symmetry actually, since a diagonal SU(3) is gauged as the $SU(3)_C$ gauge group. Thus, the NG bosons due to such chiral symmetry breaking transform as $3 \times 3^* = \text{adj.} + 1$ under the $SU(3)_C$. Moreover, the adjoint-part of the NG bosons acquire masses due to the $SU(3)_C$ radiative corrections. What remains massless is only the color-singlet NG boson, which corresponds to the axial $U(1)_A$ symmetry in Eq.(1).

The axial symmetry discussed above, however, also has $U(1)_A[SU(3)_H]^2$ anomaly. Therefore, the color-singlet NG boson obtains a large mass and it cannot play a role.

---

6Extensions to larger gauge groups and fermion representations are straightforward, which are touched upon at the end of this section.
of the axion for the color SU(3)$_C$. Hence we further introduce an additional pair of chiral fermions on each boundary: $\chi_L(1, 3^*)$ at $y = 0$ and $\chi_R(1, 3^*)$ at $y = l$. The global symmetry is now U(4)$_L \times$ U(4)$_R$ if the SU(3)$_C$ gauge interaction is neglected. The strong dynamics of the SU(3)$_H$ gauge group lead to chiral symmetry breakdown

$$\langle \psi_L \bar{\psi}^R \rangle \simeq F_3^3$$

and

$$\langle \chi_L \bar{\chi}^R \rangle \simeq F_3^3.$$ Two color-singlets would remain massless if it were not for anomalies. In this case, one of them does play a role of the axion that makes the effective strong CP phase to be sufficiently small.

### 4.2 Explicit breaking

The accidental chiral symmetry discussed above is broken by effective operators of the chiral fermions. When the condensation

$$\langle \psi_L \psi^R \rangle \simeq e^{2kl} F_3^3$$

is less than $M_3^3$, dominant breaking is expected to come from the lowest dimension operators, which we concentrate on in this subsection.

The operators involving both $\psi_L$ or $\chi_L$ and $\psi_R$ or $\chi_R$ may be highly suppressed. In view of an example of the mediator interactions investigated in the Appendix, the breaking term is estimated as

$$\frac{e^{-2kl}}{e^{ml} - e^{-ml}} M_s (\psi_L \bar{\psi}^R + \psi_R \bar{\psi}^L),$$

where $m$ denotes the mediator mass. This results in an axion potential term

$$V_{\text{bulk}}(a) \simeq \frac{e^{-\frac{1}{2}kl} M_s F_3^3}{e^{ml} - e^{-ml}} f_{\text{bulk}}(a/F_a),$$

where $f_{\text{bulk}}(a/F_a)$ is a function of order unity, whose minimum is generically different from that of the potential induced exclusively by the QCD effects. This finally yields an effective QCD $\theta$ parameter of order

$$\theta_{\text{bulk}} \simeq \frac{e^{-\frac{1}{2}kl} M_s F_3^3}{(e^{ml} - e^{-ml}) \Lambda_{QCD}^4},$$

in the case with sufficient suppression.

On the other hand, the operators involving either $\psi_L$ and $\chi_L$ or $\psi_R$ and $\chi_R$ are expected to be suppressed only by powers of $1/M_s$. Such operators also induce an additional potential of the axion, though this correction does not necessarily spoil the Peccei-Quinn
mechanism: Axial symmetry breaking operators on each boundary may have coupling coefficients of order one. Such operators with the lowest mass dimension are given by
\[
\int_{y=0} d^4x \frac{1}{M_*^5}(\psi_L)^3(\psi_L)^3 + \int_{y=l} d^4x e^{-4\phi} \frac{1}{M_*^5}(\psi_R^\dagger)^3(\psi_R^\dagger)^3 + \text{h.c.} \quad (14)
\]
Integration of heavy particles with masses of order \( F_a \) due to the SU(3)\(_H\) interaction induces an additional potential of the axion field \( a \) as
\[
V_\partial(a) \simeq \frac{F_a^{14}}{(e^{-\frac{1}{2}kl}M_*)10^4} f_\partial(a/F_a). \quad (15)
\]
The resulting shift in the QCD \( \theta \) parameter is expected to be of order
\[
\theta_\partial \simeq \frac{F_a^{14}}{(e^{-\frac{1}{2}kl}M_*)10^4 \Lambda_{\text{QCD}}^4} \quad (16)
\]
again in the case with sufficient suppression. We note that the axial symmetry breaking operators on each boundary can be made to have higher mass dimensions if we adopt a larger hypercolor gauge group instead of the SU(3)\(_H\).

Combining with the expression for the gravitational scale \( M_G \simeq 10^{18}\text{GeV} \) in four dimensions
\[
M_G^2 = \frac{M^3}{k} (1 - e^{-2kl}) \quad (17)
\]
given by the one \( M \ (\sim M_*) \) in five dimensions \( \text{[4]} \), the above results restrict possible values of the parameters to circumvent the strong CP problem. For example, \( \theta_\partial < 10^{-9} \) is realized for \( |kl| \lesssim 15 \) when \( F_a \simeq 10^{10}\text{GeV} \) and \( M_*^2 \simeq 2lM^3 \).

5 Discussion

When \( F_a \) is larger than \( e^{-\frac{1}{2}kl}M_* \), the analysis based on an operator power expansion (as in the previous section) does not seem reliable. However, even in such a case, the framework of effective theory implies that the potential energy coupled to the original five-dimensional metric (based on the proper time) be less than the cutoff scale. Hence, instead of Eq.(13), we obtain
\[
\theta_{\text{bulk}} \simeq \frac{(e^{-\frac{1}{2}kl}M_*)^4}{\Lambda_{\text{QCD}}^4} \quad (18)
\]
as a conservative estimate. We also obtain
\[
\theta_\partial \simeq \left( e^{-\frac{1}{2} kl M_*} \right)^8 \simeq \left( e^{-\frac{1}{2} kl M_*} \right)^4 \frac{\theta_{\text{bulk}}}{F_a^4 \Lambda^4_{\text{QCD}}} \theta_{\text{bulk}} < \theta_{\text{bulk}}
\] (19)

instead of Eq.(16).

For example, \( \theta_{\text{bulk}} < 10^{-9} \) with \( M_* \simeq 10^{18} \text{GeV} \) is realized for \( kl \gtrsim 100 \). This result might suggest a possible dual role played by a common bulk with spacetime inflationary background which simultaneously achieves a tiny cosmological constant in four dimensions for \( kl \gtrsim 140 \) [8]. Then, the quantum dynamics of gluons should be localized (due to the Yang-Mills term [9]) at the \( y = 0 \) boundary, so that they would be only partly affected by the background curvature.

**Acknowledgments**

We would like to thank K. Fujikawa, J. Hisano, T. Watari, and T. Yanagida for valuable comments and discussions.

**A Appendix**

This appendix deals with five-dimensional bulk mediator fermions and their solvable mixing with boundary fermions.

The kinetic term for a bulk fermion \( \Psi \) on the AdS-slice orbifold is given by [12]
\[
S_\Psi = \int d^4x \int_{-l}^l dy \ e^{-4x} \bar{\Psi} D\Psi; \quad D = ie^\sigma \partial - 2 \gamma_5 \sigma' + \gamma_5 \partial_y
\] (20)

with the restriction either \( \Psi(x,-y) = +\gamma_5 \Psi(x,y) \) or \( \Psi(x,-y) = -\gamma_5 \Psi(x,y) \). Here, the spin connection including \( \sigma' \) has been taken into account and the prime denotes differentiation with respect to \( y \).

Let us introduce two fermions with opposite chiralities
\[
\Psi_1(x,-y) = \gamma_5 \Psi_1(x,y),
\]
\[
\Psi_2(x,-y) = -\gamma_5 \Psi_2(x,y)
\] (21)

\[\text{7Alternatively, yet higher-dimensional}\] [10] analogues or an \( y \)-dependent [11] gauge coupling \( g(y) \) may be considered.
along with a mass term:

\[ S_\Psi = \int d^4x \int_{-l}^l dy \ e^{-4\sigma} \bar{\Psi} \left( \begin{array}{c} D \\
\frac{m}{D} 
\end{array} \right) \Psi; \quad \Psi = \left( \begin{array}{c} \Psi_1 \\
\Psi_2 \end{array} \right). \quad (22) \]

Natural mixing with the boundary fermions \( \psi_L \) and \( \psi_R \) is expected through such terms of order one coefficients as

\[ \int_{y=0} d^4x M^\frac{1}{2} (\bar{\Psi}_2 \psi_L + \bar{\psi}_L \Psi_2) + \int_{y=l} d^4x e^{-4\sigma} M^\frac{1}{2} (\bar{\Psi}_1 \psi_R + \bar{\psi}_R \Psi_1), \quad (23) \]

when \( \Psi \) is properly charged. Integrating out the bulk fermions \( \Psi \), we obtain, among others, U(1)\( _A \)-breaking nonderivative terms of the form

\[ \int d^4x \frac{1}{2} \left( \frac{e^{(2k+m)|y|}}{e^{2ml} - 1} - \frac{e^{(2k-m)|y|}}{e^{-2ml} - 1} \right) y \ e^{-4kl} M_s (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R) \]

\[ = \int d^4x \frac{e^{-2kl}}{e^{ml} - e^{-ml}} M_s (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R) = \int d^4x \frac{e^{-\frac{1}{2}kl}}{e^{ml} - e^{-ml}} (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R), \quad (24) \]

which are exponentially suppressed for large values of \( ml \).

References

[1] J.E. Kim, Phys. Rep. 150 (1987) 1; H.-Y. Cheng, Phys. Rep. 158 (1988) 1.

[2] H.-C. Cheng and D.E. Kaplan, [arXiv:hep-ph/0103346]; M. Chaichian and A.B. Kobakhidze, [arXiv:hep-ph/0104158]; G. Hiller and M. Schmaltz, [arXiv:hep-ph/0105254, arXiv:hep-ph/0201251]; K.S. Babu, B. Dutta, and R.N. Mohapatra, [arXiv:hep-ph/0107100]; S.L. Glashow, [arXiv:hep-ph/0110178]; C.T. Hill and A.K. Leibovich, [arXiv:hep-ph/0205237]; G. Aldazabal, L.E. Ibáñez, and A.M. Uranga, [arXiv:hep-ph/0205250]; H. Collins and R. Holman, [arXiv:hep-ph/0210110]; A.G. Dias, V. Pleitez, and M.D. Tonasse, [arXiv:hep-ph/0210172, arXiv:hep-ph/0211107]; K.S. Babu, I. Gogoladze, and K. Wang, [arXiv:hep-ph/0212339].
[3] Izawa K.-I., T. Watari, and T. Yanagida, arXiv:hep-ph/0202171.

[4] L. Randall and R. Sundrum, arXiv:hep-ph/9905221.

[5] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16 (1977) 1791.

[6] Z. Chacko and E. Pontón, arXiv:hep-ph/0301171 and references therein.

[7] S. Weinberg, Phys. Rev. Lett. 40 (1978) 223; F. Wilczek, Phys. Rev. Lett. 40 (1978) 279.

[8] Izawa K.-I., arXiv:hep-ph/0007079; T. Shiromizu and D. Ida, arXiv:hep-th/0102035.

[9] G. Dvali, G. Gabadadze, and M. Shifman, arXiv:hep-th/0010071; E.Kh. Akhmedov, arXiv:hep-th/0107223.

[10] I. Oda, arXiv:hep-th/0006203; S.L. Dubovsky, V.A. Rubakov, and P.G. Tinyakov, arXiv:hep-ph/0007179.

[11] A. Kehagias and K. Tamvakis, arXiv:hep-th/0010112.

[12] Y. Grossman and M. Neubert, arXiv:hep-ph/9912408; S. Chang, J. Hisano, H. Nakano, N. Okada, and M. Yamaguchi, arXiv:hep-ph/9912498; B. Bajc and G. Gabadadze, arXiv:hep-th/9912232.