Abstract—A simultaneously transmitting and reflecting reconfigurable intelligent surfaces (STAR-RISs) enhanced uplink non-orthogonal multiple access (NOMA) communication system is proposed. A total power consumption minimization problem is formulated by jointly optimizing the transmit-power of users, receive-beamforming vectors at the base station (BS), STAR-beamforming vectors at the STAR-RIS and time slots. Here, the STAR-beamforming introduced by STAR-RIS consists of transmission- and reflection-beamforming. To solve the formulated non-convex problem, an efficient penalty-based alternating optimization (P-AltOp) algorithm is proposed. Simulation results validate the effectiveness of the proposed scheme and reveal the effect of various system parameters on the total power consumption.

Index Terms—beamforming, non-orthogonal multiple access, reconfigurable intelligent surfaces, simultaneous transmission and reflection.

I. INTRODUCTION

Recently, reconfigurable intelligent surfaces (RISs) have been proposed as a cost-effective technology to enhance the communication signal coverage [1]. By smartly adjusting the amplitude and phase response of the reflecting elements, RIS can effectively reconfigure the wireless propagation environment. Nevertheless, in existing research, conventional RISs can only reflect incident signals within limited angular range. To overcome this limitation, a novel RIS, namely, simultaneously transmitting and reflecting RISs (STAR-RISs) [2] has been proposed. Distinctively different from traditional reflection-only RISs, STAR-RISs can simultaneously enable transmission and reflection of the incident signals at different sides of RIS, which leads to enhanced full-space coverage. The incident signals can be divided into two parts by a STAR-RIS. One part is reflected to the reflection space and the other part is transmitted into transmission space. In addition, STAR-RIS can independently control the angles of transmitted and reflected signals via its transmission- and reflection-beamforming (referred to as STAR-beamforming) [2]. In a word, STAR-RISs can bring a 360° communication coverage into reality.

Although there are many advantages of STAR-RIS, the research on STAR-RIS assisted wireless communication is still in its infancy. In the initial work [2], three practical operating protocols for STAR-RISs, namely, energy splitting (ES), mode switching (MS), and time switching (TS), were proposed, and their representative benefits and drawbacks were analyzed. The sum coverage range maximization problems for a STAR-RIS aided downlink non-orthogonal multiple access (NOMA) and orthogonal multiple access (OMA) systems were studied in [3]. In [4], a STAR-RIS assisted secrecy multiple input single-output (MISO) network was exploited and the weighted sum secrecy rate maximization problem was formulated by jointly designing the beamforming and the transmitting and reflecting coefficients.

Different from the above-mentioned works [2–4], which mainly focus on the downlink communication systems, we consider STAR-RIS empowered uplink communication systems here. To the best of our knowledge, this is the first work to consider STAR-RIS empowered uplink multi-antenna communication systems. This letter advocates a unified framework to solve the joint optimization problem over receive-beamforming and STAR-beamforming for the considered system.

Notations: diag(x) denotes a diagonal matrix whose diagonal elements are the corresponding elements in vector x. \( x_m \) is the \( m \)-th element of vector x. \( \| x \|_2 \) is the \( \ell_2 \)-norm of factor x. The \((m, n)\)-th element of matrix X is denoted as \( X_{m,n} \). \( x^H \) denotes the conjugate transpose of vector x. The notations \( \text{Tr}(X) \) and \( \text{rank}(X) \) denote the trace and rank of matrix X, respectively. \( \| X \|_\ast \) and \( \| X \|_2 \) denote the nuclear norm and spectral norm of matrix X, respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

As shown in Fig 1, a STAR-RIS is deployed in a typical uplink NOMA wireless network, where \( K \) single-antenna users are communicating with a \( N_T \)-antenna BS. Suppose that the STAR-RIS has \( M \) elements. Since the STAR-RIS divides the full space of signal propagation into two parts, namely, reflection space and transmission space. We refer to the users that are located in the reflection space and transmission space as R users and T users, respectively. We further assume that the direct communication links between the R/T users and the BS are blocked by obstacles. The number of R users and T
users are \( K_r \) and \( K_t \), respectively. Furthermore, the sets of R users and T users are denoted by \( K_r = \{1, \cdots, K_r\} \) and \( K_t = \{1, \cdots, K_t\} \), respectively.

**B. STAR-RIS Employing Time Switching Protocol**

By employing time switching protocol [2], STAR-RIS periodically switches all elements between the reflection mode and transmission mode (referred to as R mode and T mode) in two orthogonal time slots, namely R period and T period. Let \( 0 < \lambda_r \leq 1 \) denote the percentage of time slot allocated to the R period. Thus, the time slot allocated to the T period is \( \lambda_t = 1 - \lambda_r \). The corresponding reflection- and transmission-beamforming vectors are given by
\[
\begin{align*}
\Theta_r &= \begin{bmatrix} e^{j\theta_1} & e^{j\theta_2} & \cdots & e^{j\theta_m} \end{bmatrix}^T, \\
v_r &= \begin{bmatrix} e^{j\theta_1} & e^{j\theta_2} & \cdots & e^{j\theta_m} \end{bmatrix}^T,
\end{align*}
\]
where \( \theta_m, \theta_m \in [0, 2\pi) \), \( m \in \{1, 2, \cdots, M\} \).

**C. Signal Model and Problem Formulation**

We denote the channel link from the \( k \)-th T/R user to the STAR-RIS as \( g_{q,k} \), the channel link from the STAR-RIS to the BS as \( F_q \), where \( k \in K_q, q \in \{t, r\} \). During the T/R period, the received signal at the BS for T/R user \( k \) is
\[
y_{q,k} = w_{q,k}^H \left( \sum_{j=1}^{K_q} F_{q,j} g_{q,j} \sqrt{p_{q,j}} x_{q,j} + z_q \right),
\]
where \( x_{q,j} \) is the transmitted signal, \( w_{q,k} \) is the receive-beamforming vector, \( p_{q,j} \) is the transmit power, \( \Theta_q = \text{diag} \{v_q\} \), \( E\{|x_{q,j}|^2\} = 1 \) and \( z_q \in \mathcal{CN}(0, \sigma_0^2) \) is the additive white Gaussian noise with zero mean and a covariance matrix of \( \sigma_0^2 \).

To facilitate NOMA transmission, the \( K_q \) T/R users are ordered as [5]
\[
|w_{q,k}^H F_{q} g_{q,k} |^2 \geq \cdots \geq |w_{q,k}^H F_{q} g_{q,K_q} |^2.
\]

After successfully canceling the signals intended for users \( \{1, 2, \cdots, k-1\} \), the signal-to-interference-plus-noise ratio (SINR) of user \( k \) is given by
\[
\text{SINR}_{q,k} = \frac{p_{q,k} |w_{q,k}^H h_{q,k}|^2}{\sum_{j=k+1}^{K_q} p_{q,j} |w_{q,k}^H h_{q,j}|^2 + \sigma_0^2}, \quad k \in K_q.
\]

where \( h_{q,j} = \Theta_q g_{q,j} \) is the equivalent combined channel. The corresponding achievable data rate is given by
\[
R_{q,k} = \lambda_q \log (1 + \text{SINR}_{q,k}).
\]

Then, the considered total power consumption minimization problem is formulated as
\[
\min_{p_{q,k} \geq 0, w_{q,k}, v_{q,k}, \lambda_q} \sum_{k=1}^{K_q} p_{r,k} + \sum_{k=1}^{K_t} p_{t,k},
\]
s.t. \( \lambda_q \log (1 + \text{SINR}_{q,k}) \geq R_{q,k}^\text{min}, \quad k \in K_q, q \in \{t, r\}, \quad m \in \{1, 2, \cdots, M\} \).

### III. PROPOSED SOLUTION

#### A. Equivalent Transformation of Problem (5)

To facilitate the design, we first define \( V_q \triangleq v_q v_q^H \geq 0 \) and \( W_{q,k} \triangleq w_{q,k} w_{q,k}^H \geq 0 \), where \( \text{rank}(V_q) = 1 \) and \( \text{rank}(W_{q,k}) = 1 \). Let us proceed to define the variable \( A_{q,k} \triangleq F \text{diag}(g_{q,k}) \). Using the new variables as defined above, the term \( |w_{q,k}^H F_{q} g_{q,j}|^2 \) in problem (5) can be written as
\[
|w_{q,k}^H F_{q} g_{q,j}|^2 = \text{Tr} \left( A_{q,j} V_{q} A_{q,j}^H W_{q,k} \right) = H_{j,k}^q.
\]

Plugging (6) into the original problem (5), we have:
\[
\min_{p_{q,k} \geq 0, w_{q,k}, v_{q,k}, \lambda_q} \sum_{k=1}^{K_q} p_{r,k} + \sum_{k=1}^{K_t} p_{t,k},
\]
s.t. \( \lambda_q \log (1 + \text{SINR}_{q,k}) \geq R_{q,k}^\text{min}, \quad k \in K_q, q \in \{t, r\}, \quad m \in \{1, 2, \cdots, M\} \).

#### B. STAR-Beamforming Optimization

In this subsection, we focus on the joint optimization problem over transmission- and reflection-beamforming vectors with given \( \{P_{q,k}\}, \{W_{q,k}\} \) and \( \{\lambda_q\}, \) which is formulated as:
\[
\min_{P_{q,k} \geq 0, V_{q} \geq 0} \sum_{q,k} P_{q,k}^r + P_{q,k}^t \sum_{k=1}^{K_q} P_{q,k}^r \sum_{j=k+1}^{K_q} P_{q,j}^t H_{j,k}^q + \sigma_0^2
\]
s.t. (7c), (7f), (7h).

**Theorem 1.** By defining sum transmit-power \( P_{q,k}^r \triangleq \sum_{k=1}^{K_q} P_{q,k} \) and normalized transmit-power \( \overline{P}_{q,k} \triangleq \frac{P_{q,k}^r}{P_{q,k}^r} \) with constraint \( \sum_{k=1}^{K_q} \overline{P}_{q,k} = 1 \), solving problem (7) is equivalently to solving problem (8).

**Proof:** see Appendix A.

However, problem (8) is still non-convex. In the following, we propose a penalty-based alternating optimization (P-AltOp) algorithm to solve problem (8) by exploiting penalty-based semidefinite programming (P-SDP), successive convex approximation (SCA) and alternating optimization.
For clarity of problem formulation, rearranging the QoS constraint (8b) leads to
\[ P_{q,k} H_{q,k}^T H_{q,k} \geq r_{q,k}^\min \left( \sum_{j=k+1}^{K} P_j H_{j,k}^T + \frac{\sigma^2}{P_{\text{sum}}} \right) \] (10)
where \( r_{q,k}^\min = 2^{-q} - 1, k \in K_q, q \in \{r,t\}.  

Now, the remaining non-convexity in problem (9) lies in the rank-one constraint (7h). According to [6], rank \( V_q \) = 1 can be transformed equivalently as: \( \|V_q\|_s - \|V_q\|_2 = 0 \). Furthermore, according to the SCA method, by utilizing first-order Taylor approximation to \( \|V_q\|_s \), we have:
\[ \|V_q\|_2 \geq \|V_q^{(\tau_1)}\|_2 + \text{Tr} \left( V_q^{(\tau_1)} \left( V_q - V_q^{(\tau_1)} \right) \right) \]
(11)
where \( V_q^{(\tau_1)} \) is the solution obtained in the \( \tau_1 \)-th iteration and \( e^{(\tau_1)} \) denotes the eigenvector corresponding to the maximum eigenvalue of matrix \( V_q^{(\tau_1)} \).

By adding \( \|V_q\|_s - \|V_q^{(\tau_1)}\|_2 \) as a penalized function to the objective function of problem (9), we obtain the following optimization problem:
\[
\begin{align*}
\min_{P_{\text{sum}} \geq 0, V_q \geq 0} & \quad P_{\text{sum}} + \frac{1}{\mu_1} \left( \|V_q\|_s - \|V_q^{(\tau_1)}\|_2 \right), \\
\text{s.t.} & \quad (7c), (7f), (10),
\end{align*}
\] (12a)
where \( \mu_1 \) is a penalty factor.

Problem (12) is an SDP problem and can be solved by the CVX tool [7]. The details of the proposed P-SDP algorithm to solve problem (9) is presented in Algorithm 1, which comprises two loops. The outer-layer iteration is used to update the penalty factor and the inner-layer iteration is used to iteratively solve problem (12). The constraint violation is defined as \( \mathcal{E}_{\text{erro}} = \|V_q\|_s - \|V_q\|_2 \). The proposed P-SDP algorithm is guaranteed to converge to a stationary point of the original problem [6].

**Algorithm 1 Proposed P-SDP Algorithm to Solve Problem (9)**

1. Initialize \( V_q^{(0)} \) and \( \mu_1 \). Set \( 0 < \Delta_1 < 1 \) and convergence tolerance \( 0 < \mathcal{E}_1 \leq 1 \).
2. repeat
3. Set inner-iteration index \( \tau_1 = 0 \);
4. repeat
5. Update \( V_q^{(\tau_1 + 1)} \) by solving problem (12);
6. \( \tau_1 \leftarrow \tau_1 + 1 \);
7. until the objective value of problem (12) converges;
8. Update \( V_q^{(0)} = V_q^{(\tau_1)} \);
9. Update \( \mu_1 \leftarrow \mu_1 \Delta_1 \);
10. until constraint violation \( \mathcal{E}_{\text{erro}} \leq \mathcal{E}_1 \).

**C. Receive-Beamforming Optimization**
The receive-beamforming optimization problem can be formulated as
\[
\begin{align*}
\min_{P_{\text{sum}} \geq 0, V_q \geq 0} & \quad P^q_{\text{sum}} P^q_q, \\
s.t. & \quad (7b), (7e), (7g), (8c)
\end{align*}
\] (13a)
Problem (13) is non-convex and has rank-one constraints, which can be solved similarly to problem (9) and thus omitted for simplicity.

**D. Transmit-Power Optimization**
For given \( \{W_{q,k}\}, \{V_q\} \), and \( \{\lambda_q\} \), combining with the definition \( P_{q,k} \triangleq \frac{P_{q,k}}{P_{\text{sum}}} \), the transmit-power optimization problem in problem (8) can be expressed as
\[
\begin{align*}
\min_{P_{q,k} \geq 0} & \quad \sum_{k=1}^{K} P_{q,k}, \\
s.t. & \quad (7b)
\end{align*}
\] (14a)

**Theorem 2.** The optimal transmit-power solution of problem (14) is given by
\[
\begin{align*}
\min_{P_{q,k}} \frac{r_{q,k} - \sum_{j=K_q-1}^{K_q-2} R_{q,j} H_{q,j}}{H_{q,k-2}^2},
\end{align*}
\] (15)

Proof: See Appendix B.

**E. Time Slot Optimization**
The time slot optimization problem in (8) with fixed \( \{\overline{P}_{q,k}\}, \{W_{q,k}\}, \{V_q\} \) is reduced to
\[
\begin{align*}
\min_{\lambda_q, P_{\text{sum}} \geq 0} & \quad (P_{\text{sum}}^t + P_{\text{sum}}^s), \\
s.t. & \quad \lambda_q = 1 - \lambda_r, \lambda_r \in [0,1],
\end{align*}
\] (16a)
(8b).

To obtain the optimal solution of problem (16), we have the following Theorem.

**Theorem 3.** Problem (16) can be equivalently transformed to the following optimization problem
\[
\begin{align*}
\min & \quad \{P_{\text{sum}} \min_{\lambda_q} (\lambda_q) + P_{\text{sum}} \min_{\lambda_r} (\lambda_r)\}, \\
s.t. & \quad \lambda_{r, \min} \leq \lambda_r \leq \lambda_{r, \max},
\end{align*}
\] (17a)
(17b)
(17c)

with
\[
\begin{align*}
\begin{bmatrix}
\sigma^2 \left( \frac{P_{q,k} H_{q,k}^T}{P_{q,k}^{R_{q,k}}} - 1 \right) - \sum_{j=k+1}^{K_q} H_{q,j}^T & -1 \\
\frac{R_{q,k} H_{q,k}^T}{2 P_{q,k}} & -1
\end{bmatrix}^{-1}
\end{align*}
\] (18)
where \( R_{q,k} = \log_2 \left( 1 + \frac{P_{q,k} H_{q,k}^T}{\sum_{j=k+1}^{K_q} P_{q,j} H_{q,j}^T} \right) \).

Proof: See Appendix C.

It is easy to observe that problem (17) is a non-convex optimization problem with one dimensional variable \( \lambda_r \). The
optimal $\lambda_{opt}$ can be obtained by invoking the exhaust search method.

F. Overall Algorithm, Convergence and Complexity Analysis

Based on the above analysis, the proposed P-AltOp algorithm is summarized in Algorithm 2. In each iteration, the computational complexity for solving the SDP problem (12) and (13) are $O(8M^6 + 2K^2M^2)$ and $O(K^3N_T^2 + K^2N_T^2)$, respectively. Note that the convergence is guaranteed since the total power consumption decreases at each iteration and the total power consumption clearly has an lower bound.

Algorithm 2 Proposed P-AltOp Algorithm to Solve Problem (5)

1: Initialize a decoding order, $V_{q}^{(0)}$, $W_{q,k}^{(0)}$ and $T_{q,k}^{(0)}$, $k \in K_u$, $q \in \{r,t\}$. Set the iteration index $\tau_2 = 1$.
2: repeat
3: Update $\lambda_{q}^{(\tau_2)}$ by using one-dimension search method;
4: Update $V_{q}^{(\tau_2)}$ via Algorithm 1;
5: Update $W_{q,k}^{(\tau_2)}$ by using the proposed P-SDP algorithm;
6: Update $p_{q,k}^{(\tau_2)}$ according to (15) and calculate $T_{q,k}^{(\tau_2)} = \frac{\sum_{k=1}^{K_u} p_{q,k}^{(\tau_2)}}{\lambda_{q}^{(\tau_2)}}$;
7: $\tau_2 \leftarrow \tau_2 + 1$;
8: until the objective value of problem (8) converge.

IV. NUMERICAL RESULTS

In this section, the simulation and the performance results are evaluated to the performance of the STAR-RIS-NOMA empowered uplink NOMA system with the proposed algorithm. The BS and the STAR-RIS are located at (0 m, 0 m, 10 m) and (0 m, 50 m, 10 m), respectively. We set $K_r = K_t = 2$ and assume that the T/R users are randomly and uniformly distributed in a half-circle centered at the STAR-RIS with a radius of 10 m. The distance-dependent channel path loss is modeled as $P(d) = \varepsilon (d/d_0)^{-\varphi}$, where $\varepsilon$ is the path loss at the reference distance $d_0 = 1$ m, $d$ denotes the link distance and $\varphi$ denotes the path loss exponent. We adopt Rician fading to model small-scale fading for all channels involved. Specifically, we set $\varepsilon = -30$ dB, the path loss exponents and Rician factors for the channels are set to 2.2 and 1, respectively. The noise power is $-90$ dBm.

In order to validate the effectiveness of our proposed algorithm, three benchmark schemes are considered, namely, Eq-PAltOp, Fixed-PAltOp and RIS-OMA. In the Eq-PAltOp algorithm, we set $\lambda_r = \lambda_t = 0.5$. For Fixed-PAltOp algorithm, the elements of the STAR-beamforming vectors are set to one. For RIS-OMA algorithm, the BS serves all the users through time division multiple access with the aid of one traditional reflection-only RIS and one traditional transmission-only RIS, the receive-beamforming vectors are obtained via the maximum-ratio transmission (MRT) beamformer and the STAR-beamforming vectors are solved by the successive refinement algorithm [8].

Fig. 2 depicts the convergence of the proposed P-AltOp algorithm. It is observed that the convergence of our proposed algorithm is confirmed in multiple cases, i.e., $M = 10, 15, 20$. In Fig. 3, the total power consumption versus the number of STAR-RIS elements $M$ under different number of BS antennas $N_T$ is plotted. We observe that the total power consumption decreases as $M$ and $N_T$ increases. This is expected since larger $M$ enables higher reflection- and transmission-beamforming gains, which in turn reduces the total power consumption. Furthermore, larger $N_T$ can also achieve a higher receive-beamforming gain. Fig. 4 compares the proposed P-AltOp algorithm with benchmark schemes. It is observed that the proposed algorithm always outperforms the benchmark schemes. In addition, the proposed STAR-RIS enhanced NOMA system can achieve a lower total power consumption than the traditional RIS-OMA system. This is because, compared with RIS-OMA system, the users can be served simultaneously through the NOMA protocol in our proposed system.

![Fig. 2: Total power consumption versus the number of iterations, $N_T = 4$ and $R_{q,k}^{min} = 0.2$ bit/s/Hz](image1)

![Fig. 3: Total power consumption versus the number of STAR-RIS elements $M$ under different number of BS antennas $N_T$, $R_{q,k}^{min} = 0.2$ bit/s/Hz](image2)

![Fig. 4: Total power consumption achieved by various schemes under different QoS requirements, $N_T = 4$](image3)

V. CONCLUSIONS

In this paper, we investigated a STAR-RIS empowered uplink NOMA communication system to achieve full-space coverage. We formulated an optimization problem to minimize the total power consumption and proposed an efficient algorithm to jointly optimize the transmit-power of the users, receive-beamforming vectors at the BS, STAR-beamforming vectors at the STAR-RIS and time slots. Numerical results have shown that the proposed algorithm can achieve better performance than benchmark schemes and the proposed system outperforms traditional RIS-OMA system.
APPENDIX A: PROOF OF THEOREM 1

Since \( \sum_{k=1}^{K} p_{q,k} = P^q_{\text{sum}} \), by substituting this equation into the objective function of problem (7), then problem (7) and problem (8) have the same objective function. Furthermore, according to the definitions of \( \overline{p}_{q,k} \), we have \( p_{q,k} = P^q_{\text{sum}} \overline{p}_{q,k} \) and \( \sum_{k=1}^{K_q} \overline{p}_{q,k} = 1 \). As a result, the QoS constraint (7b) can be equivalently rewritten as (8b). Therefore, problem (7) is equivalent to problem (8). Assume that \( \{ p_{q,k}^{\text{opt}}, q_{k}^{\text{opt}} \}_{k=1}^{K_q} \) is the optimal solution of problem (8), then with \( p_{q,k}^{\text{opt}} = P^q_{\text{sum}} \overline{p}_{q,k}^{\text{opt}} \), \( \{ q_{k}^{\text{opt}}, \overline{p}_{q,k}^{\text{opt}} \}_{k=1}^{K_q} \) is the optimal solution of problem (7).

APPENDIX B: PROOF OF THEOREM 2

Let \( p_{q,k}^{\min} \) denote the minimum transmit-power for user \( k \). If the users transmit their signal to the BS with \( \{ p_{q,k}^{\min} \}_{k=1}^{K_q} \), then all the users will achieve their minimum QoS requirements \( \{ R_{q,k}^{\min} \}_{k=1}^{K_q} \) [9]. Thus, we have the following equations:

\[
\begin{aligned}
\log_2 \left( 1 + \frac{p_{q,k}^{\min} H_{k,k}^q \lambda q}{\sigma^2} \right) &= R_{q,k}^{\min}, \\
\log_2 \left( 1 + \frac{p_{q,k}^{\min} H_{k,k}^q \lambda q}{\sum_{j=k+1}^{K_q} p_{q,j}^{\min} H_{j,k}^q + \sigma^2} \right) &= R_{k}^{\min}, k \in K_q / K_q.
\end{aligned}
\]

(B.1)

By solving the above equations, we can obtain the minimum transmit-power which is expressed as (15).

Note that the SINR of user \( k \) in (3) can be rewritten as

\[
\text{SINR}_{q,k} = \frac{p_{q,k} H_{k,k}^q}{\sum_{j=k+1}^{K_q} p_{q,j} H_{j,k}^q + \sigma^2}.
\]

It is easy to observe that the SINR function \( \text{SINR}_{q,k} \) is strictly monotonically increasing in \( p_{q,k} \), and monotonically decreasing in \( p_{q,j} \) with \( j > k \). In addition, the user \( k \) sees interference only from users with index \( j > k \).

Denote by \( \{ p_{q,k}^{\text{opt}} \}_{k=1}^{K_q} \) the optimal solution of problem (14). Since user \( K_q \) is decoded last and sees no interference after decoding. Therefore, the minimum transmit-power \( p_{q,k}^{\min} \) is required to achieve the target \( R_{q,k}^{\min} \). Then, the optimal transmit-power is \( P_{q,k}^{\text{opt}} = p_{q,k}^{\min} \). If this is not true, there are two cases, namely, \( P_{q,k}^{\text{opt}} < p_{q,k}^{\min} \) and \( P_{q,k}^{\text{opt}} > p_{q,k}^{\min} \). The first case can be easily ruled out because it violates the minimum QoS requirement constraint in (7b). For the second case, since user \( K_q - 1 \) sees interference only from user \( K_q \), the interference of user \( K_q - 1 \) introduced by user \( K_q \) becomes larger with \( P_{q,k}^{\text{opt}} \). This contradicts with our assumption. The contradiction \( p_{q,k}^{\min} < p_{q,k}^{\text{opt}} \) can also be shown.

In the same way, the optimality can be proved for all the other users. As a result, the optimal solution of problem (14) is unique and is given by \( p_{q,k}^{\text{opt}} = p_{q,k}^{\min} \) \( k \in K_q \).

APPENDIX C: PROOF OF THEOREM 3

To facilitate the analysis, we first rewrite the minimum QoS requirement in constraint (8b) as follows

\[
\frac{p_{q,k} H_{k,k}^q}{p_{q,k}^{\min}} \geq \sum_{j=k+1}^{K_q} p_{q,j} H_{j,k}^q \geq \frac{\sigma^2}{P_{\text{sum}}} - 1 \quad (C.1)
\]

We can easily deduce from (C.1) that the following inequality holds

\[
\frac{p_{q,k} H_{k,k}^q}{P_{\text{sum}}} \geq \sum_{j=k+1}^{K_q} p_{q,j} H_{j,k}^q > 0 \quad (C.2)
\]

By keeping the condition (C.2) satisfied, (C.1) can be rewritten as

\[
\frac{p_{q,k} H_{k,k}^q}{p_{q,k}^{\min}} \geq \sum_{j=k+1}^{K_q} p_{q,j} H_{j,k}^q \geq \frac{\sigma^2}{P_{\text{sum}}}. \quad (C.3)
\]

Then, (C.3) can be further reduced as

\[
\frac{p_{q,k} H_{k,k}^q}{p_{q,k}^{\min}} \geq \left\{ \frac{\sigma^2}{P_{\text{sum}}} \right\} - 1 \quad \text{(C.4)}
\]

where \( P_{\text{sum}} \) is the minimum value achieved by \( P_{\text{sum}} \) and is a function of the time slot \( \lambda_q \).

In addition, inequality (C.2) can be equivalently transformed to \( \lambda_q > \frac{p_{q,k} H_{k,k}^q}{P_{\text{sum}}} \), \( k \in K_q \), which can be further reexpressed as

\[
\lambda_q > \max_{k \in K_q} \left\{ \frac{p_{q,k} H_{k,k}^q}{P_{\text{sum}}} \right\} \equiv \lambda_q^{\min} \quad (C.5)
\]

Since \( \lambda_r = 1 - \lambda_q \), we have:

\[
\lambda_r \leq 1 - \lambda_q^{\min} \leq \lambda_r^{\max} \quad (C.6)
\]

As a result, combing with (C.4), (C.5), (C.6) and \( \lambda_r = 1 - \lambda_q \), problem (16) can be equivalently reformulated as problem (17), which completes the proof.

REFERENCES

[1] M. Di Renzo, A. Zappone, M. Debbah, M.-S. Alouini, C. Yuen, J. de Rosny, and S. Tretyakov, “Smart radio environments empowered by reconfigurable intelligent surfaces: how it works, state of research, and the road ahead,” IEEE J. Sel. Areas Commun., vol. 38, no. 11, pp. 2450–2525, July 2020.

[2] Y. Liu, X. Mu, J. Xu, R. Schober, Y. Hao, H. V. Poor, and L. Hanzo, “STAR: Simultaneous transmission and reflection for 3D0-v coverage by intelligent surfaces,” 2021, accept to appear. Available: https://arxiv.org/abs/2103.09104.

[3] C. Wu, Y. Liu, X. Mu, X. Gu, and O. A. Dobre, “Coverage characterization of STAR-RIS networks: NOMA and OMA,” IEEE Commun. Lett., vol. 25, no. 9, pp. 3036–3040, Sept. 2021.

[4] H. Niu, Z. Chu, F. Zhou, and Z. Zhu, “Simultaneous transmission and reflection reconfigurable intelligent surface assisted MISO networks,” IEEE Commun. Lett., 2021 (Early Access).

[5] Y. Liu, Z. Qin, M. Elkashlan, Z. Ding, A. Nallanathan, and L. Hanzo, “Non-orthogonal multiple access for 5G and beyond,” Proceedings of the IEEE, vol. 105, no. 12, pp. 2347–2381, Dec. 2017.

[6] X. Yu, D. Xu, D. W. K. Ng, and R. Schober, “IRS-assisted green communication systems: provable convergence and robust optimization,” IEEE Trans. Commun., 2021 (Early Access).

[7] M. Grant and S. Boyd, “CVX: Matlab software for disciplined convex programming, version 2.1,” http://cvxr.com/cvx, Mar. 2014.

[8] Q. Wu and R. Zhang, “Beamforming optimization for wireless network aided by intelligent reflecting surface with discrete phase shifts,” IEEE Trans. Commun., vol. 68, no. 3, pp. 1838–1851, March 2020.

[9] M. Schubert and H. Boche, “Iterative multiuser uplink and downlink beamforming under SINR constraints,” IEEE Trans. Signal Process., vol. 53, no. 7, pp. 2324–2334, July 2005.