Abstract

We present a model for dilepton production of proton-proton collisions using a realist $T$-matrix that by incorporating $\Delta$ degrees of freedom fits the $NN$ scattering data up to 2 GeV. The results we find differ in details from earlier work that use less sophisticated interactions but the overall agreement with these calculations is good.
I. INTRODUCTION

Recently various scenarios related to in-medium modifications of the properties of the \( \rho \)-meson have been proposed \cite{1,2} to explain the apparent increase in the production of dilepton pairs with an invariant mass around the \( \rho \) meson in heavy ion collisions \cite{3}. To really be able to extract these signals, possibly due to a decrease in the \( \rho \) mass as a result of chiral symmetry restoration, the description of the fundamental processes has to be on a firm ground and its inherent approximations have to be well understood. In this paper we will investigate the elementary process of dilepton pair production off nucleon-nucleon collisions. Two ingredients are of importance here. First there is the question of the nucleon-nucleon form factor. Vector Meson Dominance suggests a form which naturally gives a significant increase in the cross-section around the \( \rho \) mass. However, while the concept is well established for the pion formfactor, for the nucleon formfactor its validity is much less clear \cite{4}. In this paper we will focus on a second point, the off-shell character of the \( NN \) interaction. From real photon bremsstrahlung one knows that already at pion threshold off-shell effects are crucial to describe the data. For virtual photon bremsstrahlung one might expect the effects to be even larger since the typical energies involved are much higher than for real photon bremsstrahlung and one thus can reach kinematics which are further off shell. In this paper we will present a calculation based on a \( T \) matrix which includes \( \Delta \) degrees of freedom and fits the \( NN \) data up to 2 GeV. We will compare our results with a previous calculation that uses a One Boson Exchange fit to the \( NN \) on-shell data to find its off-shell behaviour \cite{5}.

II. THEORETICAL FRAMEWORK

The quantity of interest is the differential cross-section for production of lepton pairs with an invariant mass \( M \). It is most easily calculated in the c.m. frame of the incoming nucleon pair.
\[
\frac{d\sigma}{dM} = \frac{1}{p_{cm}\sqrt{s}} \frac{e^4 m_N^4 m_e^2}{8(2\pi)^7 M^4} \int_0^{P_{\gamma}^{\text{max}}} dP_{\gamma} \frac{M P_{\gamma}^2}{P_{0\gamma}^2} d\Omega_{\gamma} \\
\times \int d\Omega_{pN} \frac{2P_N^2}{-P_{\gamma} \cos(\theta_{pN}) + 2p_N(E^N_+ + E^N_-)} M_{\mu} M_{\nu} \int d\Omega_{pe} \frac{2P_e^2}{P_{\gamma} \cos(\theta_{pe}) + 2p_e(E^e_+ + E^e_-)} L^\mu\nu
\]

with \( P_{\gamma}^{\text{max}} = \left[ \frac{1}{16s} \left( 2s - 8m^2 - M^2 \right) - M^2 \right]^{\frac{1}{2}} \).

In this expression \( e \) is the elementary charge, \( m_N, m_e \) the nucleon and electron masses respectively, \( s \) the total incoming invariant mass and \( p_{cm} \) the momentum of the incoming nuclei. Also, \( p_N \) is the momentum of the outgoing nucleons relative to each other, its value is determined by energy conservation. The final nucleon pair thus has momenta \( p^+_N = -\frac{P_{\gamma}}{2} + p_N, \quad p^-_N = -\frac{P_{\gamma}}{2} - p_N \), the energies of the final nucleon pair are denoted by \( E^+_N, E^-_N \). In the integration over the solid angle of \( p_N \), \( \theta_{pN} \) is taken relative to the photon momentum, \( \vec{P}_\gamma \). A similar notation is used for the dilepton pair, their momenta are \( p^+_e = \frac{P_{\gamma}}{2} + p_e, p^-_e = -\frac{P_{\gamma}}{2} - p_e \), with corresponding energies \( E^+_e, E^-_e \). Furthermore, \( P^0_\gamma \) is the zero-th component of the photon momentum, \( P^0_\gamma = \sqrt{P_{\gamma}^2 + M^2} \). \( L^\mu\nu \) is the leptonic tensor describing the decay of the virtual photon into the dilepton pair,

\[
L^{\mu\nu} = \frac{1}{m_e^2} [p^\mu_e p'^\nu_{e,-} + p^\mu_{e,-} p'^\nu_{e,+} - g^{\mu\nu}(p_+ \cdot p_- + m_e^2)].
\]

\( M^\mu \) is the matrix element for producing a virtual photon with rest mass \( M \) off a nucleon-nucleon collision. The diagrams we include in the calculation of this amplitude are shown in Fig. 1. We have the nucleon single-scattering (1a), rescattering (1b) and \( \Delta \) single-scattering (1c) and rescatter diagrams (1d). We include the same diagrams as in a previous study of real-photon bremsstrahlung [6], the expressions given there are readily generalized to the case of virtual photon production.

For the \( N N \gamma \) vertex we again use the standard form without formfactors

\[
\Gamma^{NN\gamma}_\mu = -ie\gamma_\mu - eK_\mu k^\nu \frac{\sigma_{\mu\nu} k^\nu}{2m_N},
\]

where we defined the photon momentum \( k = p_i - p_f \) to be outgoing. The form of the \( N \Delta \gamma \) vertices is less well established. Following Jones and Scadron [7] we have the general form
\[ \Gamma_{\mu\nu}^{N\Delta\gamma} = K_{\mu\nu}^1 + K_{\mu\nu}^2 + K_{\mu\nu}^3 \]
\[ \Gamma_{\mu\nu}^{\Delta N\gamma} = -K_{\mu\nu}^1 + K_{\mu\nu}^2 + K_{\mu\nu}^3 \]
with
\[ K_{\mu\nu}^1 = ieG_1 (g_{\mu\nu} k - k_{\mu}\gamma_{\nu})\gamma^5 T_z \]
\[ K_{\mu\nu}^2 = ieG_2 (g_{\mu\nu} P \cdot k - k_{\mu}P_{\nu})\gamma^5 T_z \]
\[ K_{\mu\nu}^3 = ieG_3 (g_{\mu\nu} k^2 - k_{\mu}k_{\nu})\gamma^5 T_z. \] (2.4)

In these expressions the index \( \mu \) is to be contracted with an index of the \( \Delta \) propagator, \( P = p_\Delta^\mu + p_N^\mu \) is the total momentum and \( T_z \) is third component of the isospin transition matrix for coupling an isospin 3/2 to an isospin 1/2 particle. The \( K^3 \) component does not contribute in the case of a real photon \( (k^2 = 0) \) and thus experimental input on this quantity is very limited. One attempt is the calculation of Nozawa and Lee [8] of electroproduction of pions off the nucleon. However, using the values Nozawa and Lee employ we found that results were insensitive to the contribution from this vertex part and we can safely ignore this contribution.

The most elaborate determinations of the \( G_1 \) and \( G_2 \) coupling constants come from fitting the \( M1^+ \) and \( E1^+ \) multipole data on the photoproduction of pions off nucleons [7,9–13]. In judging these numbers we have to realize that in our calculation we treat the nucleon background differently. It is therefore the most realistic to take values from calculations where the nucleon background contribution in the \( P_{33} \) channel is implicitly included in the coupling constants. In such an approach Jones and Scadron find \( G_1 = 2.68 \text{ (GeV}^{-1}) \) and \( G_2 = -1.84 \text{ (GeV}^{-2}) \), which we will use in our calculation. This is very close to the values found in a recent analysis of Lee [14]: \( G_1 = 2.89 \text{ (GeV}^{-1}) \) and \( G_2 = -2.18 \text{ (GeV}^{-2}) \). With these numbers we have to bear in mind that the dominant contribution of the vertex comes from the \( K_1 \) part and we can order the sets of values according to the value of \( G_1 \), the accompanying value of \( G_2 \) has not much influence on the results. An independent method to find the values is assuming vector-dominance. Then the values are determined by the ratio \( g_{NN\rho}/g_{N\Delta\rho} \). Using the values of ter Haar [15] we find \( G_1 = 2.0 \text{ (GeV}^{-1}) \) and \( G_2 = 0.0 \text{ (GeV}^{-2}) \). This value is on the low end of the range of values found in pion-photoproduction.
In Ref. [5] a value taken from the decay width of the \( \Delta \) into a photon and nucleon is used: 
\[ G_1 = 2.3 \text{ (GeV}^{-1}) \] \text{ and } \[ G_2 = 0.0 \text{ (GeV}^{-2}) \] .

In calculating the diagrams we use a \( T \)-matrix that is based on the Paris potential and additionally contain \( \Delta \)-degrees of freedom. Including \( \Delta \) degrees of freedom is essential for describing the inelastic channels and resonant structures in the phase-shifts. The resulting \( T \)-matrix describes the nucleon-nucleon data well up to 2 GeV [16].

Other calculations of dilepton production use a less sophisticated description of the \( NN \) interaction. One approach is to develop a Soft Photon Approximation for virtual photon bremsstrahlung as is done in Ref. [17,18]. Another possibility is to parametrize the on-shell nucleon scattering data by means of an One Boson Exchange interaction, and use this parametrization to extrapolate to the off-shell matrix elements which enter in the calculation of the matrix element \( M^\mu \) [5,19,20]. This method has certain limitations, first of all, the \( NN \)-scattering amplitude is not unitary due to its OBE form. Also, in the fit to the \( NN \) data in the approach of Ref. [5] the on-shell equivalence of the pseudo-scalar and pseudo-vector coupling makes the parameters independent of whether one chooses a pseudo-scalar or pseudo-vector pion-nucleon-nucleon vertex. However, the results for dilepton production do depend on this choice. Although the contribution from the electric part of the vertex remains the same [21], the contribution from the magnetic part of the vertex is affected by this choice. Using a \( T \)-matrix one has a more realistic description of the off-shell character of the \( NN \) interaction, since the off-shell amplitudes enter in the \( T \)-matrix equation and these are thus implicitly constrained by the fit to the \( NN \) data. For example, the degeneracy of the pseudo-scalar/pseudo-vector choice for the pion coupling is resolved to a large extent: one has to use different parameters to obtain the same fit.

Using a sophisticated description of the \( NN \) interaction has ramifications on other parts of the model. The current arising from the nucleonic diagrams (Figs. [1],b) is not gauge-invariant since we do not include negative energy states in the intermediate propagators of the \( T \)-matrix. Note that the \( \Delta \)-decay diagrams are gauge-invariant due to the gauge invariant \( NV\Delta\gamma \) vertex. For real photon bremsstrahlung this is not too big a problem. In
that case the current is gauge invariant up to order \( k \) in the photon momentum. Moreover, an actual calculation of the negative energy state contributions shows them to be small \[22\].

Due to the fact that the matrix element for virtual photon bremsstrahlung depends on two independent quantities (the mass of the virtual photon and its momentum) the situation for virtual photon-bremsstrahlung is less favourable. Only in the c.m. frame of the virtual photon one can show that the matrix element is model independent only up to order \( \frac{1}{M_\gamma} \). One also finds rather large differences between the various low energy theorems \[17\]. This all implies that the issue of gauge invariance is more important for dilepton pair production than it is for real photon bremsstrahlung.

The problem of a non-gauge invariant current is also encountered in models of \((e, e'p)\) reactions. Various schemes have been devised to render a calculated current gauge invariant \[23–26\]. Of course neither scheme is unique and will lead to different results which can be related to the choice of a particular gauge \[27\]. Having noticed that our \( M_0 \) is particularly large as compared to the spatial components we decided to use the method of Refs. \[24,25\] and express \( M_0 \) in terms of \( M_i \) to obtain a conserved current:

\[
M_0 = \frac{P_i M_i}{P_0 \gamma}.
\] (2.5)

The resulting amplitude then trivially fulfills the gauge-invariance condition. This choice has the advantage that in this scheme the results for real-photon bremsstrahlung are unaffected since this process is independent of \( M_0 \).

### III. RESULTS

With the model described before we calculated the differential cross-section for production of dilepton pairs with invariant mass \( M \) for proton-proton collisions at 1 GeV laboratory energy. We restrict ourselves to proton-proton collisions since in neutron-proton collisions meson-exchange-currents usually dominate, but are very elaborate to calculate in a \( T \)-matrix approach. We think that studying the pp reaction provides a clearer picture of the additional
dynamics introduced by the use of a $T$-matrix. We choose a laboratory energy of 1 GeV to still be in the range where the $T$-matrix provides a detailed description of the phase-shifts. At higher energies the total and inelastic cross-sections are still very well reproduced, but the reproduction of more sensitive observables like the polarization cross-sections is less satisfactory [16]. Since the latter is also true for other models that fit the $NN$ data to high energies using a $T$-matrix formalism [28], one might argue that the description of $NN$ scattering in terms of nucleon and meson degrees of freedom in terms of a $T$-matrix starts to break down and one has to include additional physics, like explicit quark degrees of freedom.

The results are displayed in Fig. 2. Comparing our nucleonic contribution (dash-dot-dot) with the one of Schäfer et al. (dash-dot) we see a rather large difference. At low $M$ our cross-section is around a factor 2 larger than the one of Schäfer. At higher $M$ the situation is reversed. There the cross-section of Schäfer is larger. This can be largely attributed to the choice of a pseudo-scalar pion vertex. Taking a pseudo-vector pion coupling the result of Schäfer is reduced by a factor 10 and is again smaller than our result. The Soft-Photon result of Ref. [18] (short-dashed line) is even lower than the Schäfer result, and we can conclude that in our calculation the additional off-shell information we include leads to an enhancement of the nucleonic part of the cross-section. At the energies under consideration the total cross-section is dominated by the $\Delta$ contributions, even more than we found for the real-photon bremsstrahlung case [1]. To show the dependence of the total results on the strength of the $N\Delta\gamma$ vertex we performed calculations with a strong coupling $G_1 = 2.68$ (GeV$^{-1}$) and $G_2 = -1.84$ (GeV$^{-2}$, dotted line) and with the relatively weak vector dominance value $G_1 = 2.0$ (GeV$^{-1}$) and $G_2 = 0.0$ (GeV$^{-2}$, dashed line). The result of Schäfer $G_1 = 2.3$ (GeV$^{-1}$) and $G_2 = 0.0$ (GeV$^{-2}$) is represented by the solid line. The result with the weak coupling is about 25% smaller than the one with the strong coupling. As stated before, these values are on the high and low end of the spectrum found in various pion-photoproduction models and thus represent a natural measure of the theoretical uncertainty. The results of Schäfer et al. nicely fall in between and the overall agreement between both models is reassuring. It remains to be seen whether the similarities hold when looking at
more refined observables, like e.g. anisotropies of the dilepton pairs \cite{19}. Another point of additional investigation is the influence of form-factors. A straightforward application of vector dominance will give us simply an upward shift in the cross-section, increasing with increasing invariant mass $M$. However, a more detailed calculation of the form-factors, as performed in Ref. \cite{4} shows that the vector dominance assumption is too general for the nucleonic form-factor.

In conclusion, we presented a calculation of dilepton production at 1 GeV laboratory energy using a $T$ matrix that includes $\Delta$ degrees of freedom and provides an excellent fit to the $NN$ scattering data. We argued that the use of a $T$ matrix gives a more reliable description of the off-shell behaviour of the effective $NN$ interaction, the price to be paid however is the non-gauge invariance of the calculated current. We showed that after repairing gauge invariance in one of the well-known schemes, we find results that are similar to the ones obtained in models that use less sophisticated $NN$ interactions. In particular, we verified that the dominant contribution to virtual bremsstrahlung in this energy range comes from the $\Delta$-resonance decay. In detail we found significant differences: the nucleonic contribution is larger than found in models which only include on-shell $NN$ scattering information.

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FIGURES

FIG. 1. The various diagrams included in the current.

FIG. 2. Differential cross-section for dilepton pair production off pp-collisions at 1 GeV laboratory energy as a function of the invariant mass $M$. The solid line and dash-dot line are the full and nucleonic result of Schäfer et al. The dotted line represents our full result calculated with the strong $N\Delta \gamma \gamma$ coupling constant, the long-dashed line the one with the weak coupling constant. The dash-dot-dot line stands for our nucleonic result. The short-dashed line is the Soft-Photon result of Eq. 8 of [18].
The graph shows the differential cross section $d\sigma/dM$ in units of $\mu b/GeV$ as a function of $M$. The axes are labeled as $d\sigma/dM (\mu b/GeV)$ and $M$. The graph includes multiple curves, each representing different values or conditions.