Reconstruction of complex cracks by exterior measurements

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Abstract. In this paper, we deal with the acoustic inverse scattering problem for reconstructing complex cracks from the far field map, which models the diffraction of waves by thin two-sided cylindrical screens. A complex crack is characterized by its shape, the type of boundary data and the boundary coefficients (surface impedance). We give explicit formulas which can be used to reconstruct the shape of the crack, distinguish its type of boundary conditions and reconstruct the possible material coefficients on it by using the far-field map. Some numerical examples are also presented. Similar results could be given using near field measurements.

1. Statement of the problem
To describe the diffraction of acoustic waves by thin two-side cylindrical screens, the scattering problems are governed by the Helmholtz equation by a crack $\Gamma$ in $R^2$. Let $\Gamma$ be a two dimensional open curve of class $C^3$ with a parameterization representation $\Gamma = \{ x := x(s), s \in [a, b] \}$ where $x : [a, b] \rightarrow R^2$ is locally of class $C^3$. We set $P = x(a)$ and $Q = x(b)$ to be the two tips of $\Gamma$, and fix the orientation of $\Gamma$ as follows. Travelling on $\Gamma$ from $P$ to $Q$, we associate to the right side the sign $+$, i.e. $\Gamma^+$, and to the left side the sign $-$, i.e. $\Gamma^-$ and we set $\nu$ to be the unit normal on $\Gamma$ oriented towards $\Gamma^+$. The different boundary conditions specified on $\Gamma$ represent the acoustic properties of the crack. For given incident waves $u^i(x) = e^{i\omega x}$, we consider the following scattering problems for total waves $u(x) = u^t(x) + u^s(x)$:
continuous functions of order $\beta$ with different acoustic properties. Using the asymptotic behavior of the fundamental solution, direct fields, we cite [7] and [8]. These problems describe the scattering problems of the cracks lower bounds. For the well posedness of direct problems (1)-(2)-(3) and the computation of the wave $u$ the direction of incidence of plane wave $u$ where the function $\Phi(\gamma) := \sqrt{\mu} \Phi$ conditions (on both the two faces) and the complex valued surface impedance distributed on it. In this paper, we shall be concerned by reconstructing complex cracks by giving [2], the linear sampling method by Colton and Kirsch has been used to detect the shape of the cracks. In particular in [5] and [6], the probe method by Ikehata has been used where the task is to justify the blowup of the indicator functions when the point source parameter approaches the crack to justify the reconstruction of the shape. Also, in [2], the linear sampling method by Colton and Kirsch has been used to detect the shape of the crack. In this paper, we shall be concerned by reconstructing complex cracks by giving

\begin{align}
(\text{Dirichlet}) \quad \begin{cases}
(\Delta + \kappa^2)u = 0, & \text{in } R^2 \setminus \Gamma, \\
u = 0 & \text{on } \Gamma^\pm, \\
u = u^s + e^{i\kappa x} & \text{in } R^2 \setminus \Gamma, \\
\lim_{r \to \infty} \sqrt{r} (\frac{\partial u^s}{\partial r} - i\kappa u^s) = 0, 
\end{cases} \\
(\text{Mixed}) \quad \begin{cases}
(\Delta + \kappa^2)u = 0, & \text{in } R^2 \setminus \Gamma, \\
u = 0 & \text{on } \Gamma^+, \\
\frac{\partial u^s}{\partial r} - i\kappa \sigma^\pm u^s = 0, & \text{on } \Gamma^-, \\
u = u^s + e^{i\kappa x} & \text{in } R^2 \setminus \Gamma, \\
\lim_{r \to \infty} \sqrt{r} (\frac{\partial u^s}{\partial r} - i\kappa u^s) = 0, 
\end{cases} \\
(\text{Robin}) \quad \begin{cases}
(\Delta + \kappa^2)u = 0, & \text{in } R^2 \setminus \Gamma, \\
\frac{\partial u}{\partial r} \pm i\kappa \sigma^\pm u = 0, & \text{on } \Gamma^+, \\
u = u^s + e^{i\kappa x} & \text{in } R^2 \setminus \Gamma, \\
\lim_{r \to \infty} \sqrt{r} (\frac{\partial u^s}{\partial r} - i\kappa u^s) = 0, 
\end{cases}
\end{align}

where $u^s(x)$ is the scattered wave outside of $\Gamma$, while $\kappa > 0$ is the wave number and $d$ is the direction of incidence of plane wave $u^s(x)$. We assume that $\sigma^\pm$ are complex valued Holder continuous functions of order $\beta \in (0, 1], \sigma^\pm = \alpha^\pm + i\beta^\pm$, and their real parts have positive uniform lower bounds. For the well posedness of direct problems (1)-(2)-(3) and the computation of the direct fields, we cite [7] and [8]. These problems describe the scattering problems of the cracks with different acoustic properties. Using the asymptotic behavior of the fundamental solution, as in [3], we can show that the scattered wave has the asymptotic behavior:

$$u^s(x, d) = \frac{e^{ikr}}{\sqrt{r}} u^\infty(\hat{x}, d) + O(r^{-3/2}), \quad r := |x| \to \infty,$$

where the function $u^\infty(\cdot, d)$ defined on the unit circle $S^1$ is called the far-field of the scattered wave $u^s$ corresponding to incident direction $d$. We introduce a constant $\gamma_2 := \frac{\kappa \pi}{\sqrt{\kappa}}$ and $\Phi(x, y) := i \frac{1}{4} H^{(1)}_0(\kappa|x-y|), x \neq y, x, y \in R^2$, the fundamental solution to the Helmholtz equation in $R^2$, where $H^{(1)}_0$ is the Hankel function of the first kind of order zero.

We call a complex crack, a crack which is characterized by its shape, its type of boundary conditions (on both the two faces) and the complex valued surface impedance distributed on it.

In this paper, we will consider the following:

**Complex crack reconstruction problem.** Given $u^\infty(\cdot, \cdot)$ on $S^1 \times S^1$ for any one of the scattering problems (1) or (2) or (3), reconstruct the shape of the crack $\Gamma$, distinguish its two faces and reconstruct the eventual surface impedances $\sigma^\pm(x)$.

The inverse problems for cracks detection have been studied by many authors. We refer to [1] for some results concerning, in particular, detection of piecewise linear cracks from one or few exterior measurements. We are interested by detection of cracks of general shapes but using meany measurements. Precisely, we use the farfield map and our aim is to reconstruct the whole crack. There were several works devoted to the detection of cracks from many measurements. Among others, we shall cite [1], [2], [5] and [6] where the authors gave reconstruction methods to detect the shape of the cracks. In particular in [5] and [6], the probe method by Ikehata has been used where the task is to justify the blowup of the indicator functions when the point source parameter approaches the crack to justify the reconstruction of the shape. Also, in [2], the linear sampling method by Colton and Kirsch has been used to detect the shape of the crack. In this paper, we shall be concerned by reconstructing complex cracks by giving
the shapes, the type of boundary conditions and the pointwise values of the complex value surface impedances distributed along them. Precisely, we provide direct formulas which link the farfield map to the unknowns. We use also the probe method, or equally the singular sources method by Potthast [11] (see [4] and [10] for the justification why these two methods are the same and for more relations between them and other reconstruction methods). In contrast to the works [5], [6] and [2], we give exact asymptotic expansion of the indicator crack function. This exact asymptotic expansion contains the whole information on the complex crack. In addition to solving mathematically the inverse problem, these formulas can give an insight towards the extend to what these methods can be used for numerical purposes. In this proceedings paper, we announce the theoretical results and show some numerical tests. Their full justifications and more extensive numerical tests are given in [9].

2. Presentation of the results

It is well known, see [3], that the scattered field associated with the Herglotz incident field \( v^i_g \) defined by \( v^i_g(x) := \int_{S^1} e^{i\kappa x \cdot d} g(d) \, ds(d), \ x \in \mathbb{R}^2 \) with \( g \in L^2(S^1) \) is given by \( v^i_g(x) := \int_{S^1} u^s(x, d) g(d) \, ds(d), \ x \in \mathbb{R}^2 \setminus \Gamma, \) and its far field is \( v^\infty_g(\hat{x}) := \int_{S^1} u^\infty(\hat{x}, d) g(d) \, ds(d), \ \hat{x} \in S^1. \)

We will need the following identity, see [3],

\[
\begin{align*}
  u^\infty(\hat{x}, d) &= -\gamma_2 \int_{\partial D} \left\{ \frac{\partial u^s(y, d)}{\partial \nu} e^{-i\kappa \hat{x} \cdot y} - \frac{\partial e^{-i\kappa \hat{x} \cdot y}}{\partial \nu} u^s(y, d) \right\} \, ds(y) \quad (1)
\end{align*}
\]

where \( \partial D \) is a closed curve containing a part of \( \Gamma \) and avoiding the tips (\( P, Q \)). In addition, we assume that the bounded domain surrounded by \( \partial D \), i.e. \( D \), is such that \( \Gamma \subset \overline{D} \). In particular \( D \) contains the tips (\( P, Q \)).

Assume that \( \Gamma \subset \subset \Omega \) for some known \( \Omega \) with smooth boundary. For \( a \in \Omega \setminus \Gamma \), denote by \( \{ z_p \} \subset \Omega \setminus \overline{D} \) a sequence tending to \( a \). For any \( z_p \), set \( D^p_a \) to be a \( C^2 \)-regular domain such that \( \Gamma \subset D^p_a \) with \( z_q \in \overline{\Omega \setminus \overline{D^p_a}} \) for every \( q = 1, 2, \ldots, p \) and that the Dirichlet interior problem on \( D^p_a \) for the Helmholtz equation is uniquely solvable. In this case, the Herglotz wave operator \( H \) defined from \( L^2(S^1) \) to \( L^2(\partial D^p_a) \) by

\[
H[g](x) := v^i_g(x) = \int_{S^1} e^{i\kappa x \cdot d} g(d) \, ds(d) \quad (2)
\]

is injective, compact with dense range, see [3]. Now we consider the sequence of point sources \( \Phi(\cdot, z_p) \). For every \( p \) fixed, we construct two density sequences \( \{ g^p_n \} \) and \( \{ f^p_m \} \) in \( L^2(S^1) \) by the Tikhonov regularization such that

\[
||v^i_{g^p_n} - \Phi(\cdot, z_p)||_{L^2(\partial D^p_a)} \to 0, \ n \to \infty \quad (3)
\]

\[
||v^i_{f^p_m} - \frac{\partial}{\partial x_j} \Phi(\cdot, z_p)||_{L^2(\partial D^p_a)} \to 0, \ m \to \infty. \quad (4)
\]

We choose \( \partial D \) to contain a part of \( \Gamma \) surrounding the fixed point \( a \), such that \( \{ z_p \} \subset \Omega \setminus \overline{D} \) (for \( p \) large enough) and \( \overline{D} \subset D^p_a \). Since both \( v^i_{g^p_n} \) and \( \Phi(\cdot, z_p) \) satisfy the same Helmholtz equation in \( D^p_a \), (3) implies that

\[
||v^i_{g^p_n} - \Phi(\cdot, z_p)||_{H^{1/2}(\partial D)} \to 0, \ n \to \infty \quad (5)
\]

and

\[
||\frac{\partial}{\partial \nu} v^i_{g^p_n} - \frac{\partial}{\partial \nu} \Phi(\cdot, z_p)||_{H^{-1/2}(\partial D)} \to 0, \ n \to \infty \quad (6)
\]
Similarly, it follows from (4) that

\[ \| v_{f_m} - \frac{\partial}{\partial x_j} \Phi(\cdot, z_p) \|_{H^\frac{1}{2}(\partial D)} \to 0, \ m \to \infty \]  \hspace{1cm} (7)

and

\[ \| \frac{\partial}{\partial \nu} v_{f_m} - \frac{\partial}{\partial \nu} (\frac{\partial}{\partial x_j} \Phi(\cdot, z_p)) \|_{H^\frac{1}{2}(\partial D)} \to 0, \ m \to \infty \]  \hspace{1cm} (8)

Multiplying (1) by \( f_{m}^{j,p}(d)g_{n}(\hat{x}) \) and integrating over \( S^1 \times S^1 \), we have

\[ - \int_{S^1} \int_{S^1} u^\infty(-\hat{x},d)f_{m}^{j,p}(d)g_{n}^p(\hat{x}) \ ds(d) = \gamma_2 \int_{\partial D} \left\{ \int_{S^1} \frac{\partial v^s(y, d)}{\partial \nu} f_{m}^{j,p}(d) \ ds(d) \cdot \int_{S^1} e^{i\kappa \hat{x} \cdot y} g_{n}^p(\hat{x}) \ ds(\hat{x}) - \right. \]

\[ \left. \int_{S^1} \frac{\partial e^{i\kappa \hat{x} \cdot y}}{\partial \nu} g_{n}^p(y) \ ds(\hat{x}) \cdot \int_{S^1} u^s(y, d)f_{m}^{j,p}(d) \ ds(d) \right\} \ ds(y) = \gamma_2 \int_{\partial D} \left\{ \frac{\partial v_{f_m}^s(y)}{\partial \nu} f_{m}^{j,p}(y) - \frac{\partial v_{f_m}^s(y)}{\partial \nu} (\frac{\partial}{\partial x_j} \Phi(\cdot, z_p)) \right\} \ ds(y). \]  \hspace{1cm} (9)

From (7), (8) and (9), we have

\[ \lim_{n \to \infty} \int_{S^1} \int_{S^1} u^\infty(-\hat{x},d)f_{m}^{j,p}(d)g_{n}^p(\hat{x}) \ ds(d)ds(d) = \gamma_2 \int_{\partial D} \left\{ \frac{\partial v_{f_m}^s(y)}{\partial \nu} f_{m}^{j,p}(y) - \frac{\partial v_{f_m}^s(y)}{\partial \nu} \Phi(\cdot, z_p) \right\} \ ds(y) = \gamma_2 v_{f_m}^s(z_p) \]  \hspace{1cm} (10)

from the Green formula, where \( v_{f_m}^s(\cdot) \) is the scattered wave corresponding to incident wave \( v_{f_m}^{j,p}(x) = H[f_{m}^{j,p}](x) \).

Denote by \( E_j^s(x, z_p) \) the scattered wave corresponding to the incident wave \( \frac{\partial \Phi(x, z_p)}{\partial x_j} \), which is well defined for every \( x \in R^2 \setminus \bar{D} \). Then it follows from (6), (7), the well posedness of the direct scattering problem and the use of interior estimate that

\[ E_j^s(x, z_p) = \lim_{m \to \infty} v_{f_m}^s(\cdot), \ x \in R^2 \setminus \bar{D}. \]  \hspace{1cm} (11)

Finally, it follows from (10) that

\[ \lim_{m \to \infty} \lim_{n \to \infty} \int_{S^1} \int_{S^1} u^\infty(-\hat{x},d)f_{m}^{j,p}(d)g_{n}^p(\hat{x}) \ ds(d)ds(d) = \gamma_2 E_j^s(z_p, z_p). \]  \hspace{1cm} (12)

We set

\[ I_j(z_p) := \frac{1}{\gamma_2} \lim_{m \to \infty} \lim_{n \to \infty} \int_{S^1} \int_{S^1} u^\infty(-\hat{x},d)f_{m}^{j,p}(d)g_{n}^p(\hat{x}) \ ds(\hat{x})ds(d). \]  \hspace{1cm} (13)

Let us mention that the construction of \( f_{m}^{j,p} \) and \( g_{n}^p \) is independent on the unknown crack. Hence \( I_j(z_p) \) is computable from our data only.
The formulas (16) and (17) can be used to provide the following information on the crack:

2.1. How to use these formulas?

The notation \( a \) is the transformation of coordinates under which the image of \( a \) is \( 0 \) and a function \( f \in C^{3}(-r,r) \) such that

\[
f(0) = \frac{df}{dx}(0) = 0, \quad D \cap B(0,r) = \{(x,y) \in B(0,r); y > f(x)\}
\]

in terms of the new coordinates where \( B(0,r) \) is the 2-dimensional ball of center \( 0 \) with radius \( r \). The answer to the inverse problem is based on the following theorem.

**Theorem 2.1** Assume that \( \Gamma \) is of class \( C^{3} \) and \( \sigma_{\pm} := \sigma_{\pm}^{r} + i\sigma_{\pm}^{i} \) are complex valued Holder continuous functions with positive lower bounds for their real parts \( \sigma_{\pm}^{r} \).

I. Let \( a \in \Omega \setminus \overline{\Gamma} \). Then for all the three types of problems (1)-(2)-(3), we have:

\[
|I_{1}(z_{p})| + |I_{2}(z_{p})| = O(1) \text{ for } z_{p} \text{ near } a. \tag{15}
\]

II. For the points \( a \in \Gamma \), let the sequence \( \{z_{p}\}_{p \in N} \) be included in \( C_{a,\theta} \), where \( C_{a,\theta} \) is a cone with center \( a \), angle \( \theta \in [0, \frac{\pi}{2}) \) and axis \( \nu(a) \). Then we have the following formulas:

II. 1. \( \Re(I_{j}(z_{p})) = \)

\[
\begin{cases}
\frac{\mp \nu_{j}(a)}{4\pi |(z_{p} - a) \cdot \nu(a)|} \pm \nu_{j}(a) \kappa \sigma_{\pm}^{r} (a) \frac{1}{\pi} \ln(|(z_{p} - a) \cdot \nu(a)|) + O(1), & a \in \Gamma^{\pm} \setminus \{P, Q\} \\
(\text{for the impedance boundary conditions}), \\
\frac{\mp \nu_{j}(a)}{2\pi |(z_{p} - a) \cdot \nu(a)|} + O(1), & a \in \Gamma^{\pm} \setminus \{P, Q\}
\end{cases}
\]

II. 2. \( \Im(I_{j}(z_{p})) = \)

\[
\begin{cases}
\frac{\mp \nu_{j}(a)}{\pi} \kappa \sigma_{\pm}^{r} \ln(|(z_{p} - a) \cdot \nu(a)|) + O(1), & a \in \Gamma^{\pm} \setminus \{P, Q\} \\
(\text{for the impedance boundary conditions}), \\
O(1), & a \in \Gamma^{\pm} \setminus \{P, Q\}
\end{cases}
\]

The notation \( a \in \Gamma^{\pm} \) means that the sequence \( \{z_{p}\} \) tends to \( a \) from the right \((+\)) (or, the left \((-\)) side of \( \Gamma \).

2.1. How to use these formulas?

The formulas (16) and (17) can be used to provide the following information on the crack:

- A sample of points on the curve and the normals on these points. The points can be given by numerically solving \( |\Re I_{j}(z)| = C \) for constants \( C \) large. The normals are obtained as follows

\[
\nu(a) = \pm (t \sqrt{\frac{1}{1 + t^{2}}} \sqrt{\frac{1}{1 + t^{2}}}) \text{ where } t := \lim_{z_{p} \to a} \frac{\Re I_{1}(z_{p})}{\Re I_{2}(z_{p})}.
\]
• Distinguish the parts where we have Dirichlet or Impedance type of boundary conditions. This is a consequence of the following identities for \( a \in \Gamma^\pm \setminus \{P, Q\} \) and any given \( s \in (0, 1) \):

\[
\lim_{z_p \to a} \frac{|\Im I_j(z_p)|}{\kappa \ln(|(z_p - a) \cdot \nu(a)|)^s} = \begin{cases} 
\infty, \text{ Impedance boundary,} \\
0, \text{ Dirichlet boundary.} 
\end{cases}
\tag{18}
\]

• In addition, in case of impedance type boundary conditions, we can reconstruct the real and the imaginary parts of the surface impedance \( \sigma_\pm \):

\[
\sigma_\pm^r(a) = \lim_{z_p \to a} \frac{\pi \sum_{j=1}^2 \pm \nu_j(a) \Im I_j(z_p)}{\kappa \ln(|(z_p - a) \cdot \nu(a)|)}
\tag{19}
\]

and

\[
\sigma_\pm^i(a) = -\lim_{z_p \to a} \frac{\pi \sum_{j=1}^2 \pm \nu_j(a) \Re I_j(z_p) + \frac{1}{4(|z_p - a) \cdot \nu(a)|}}{\kappa \ln(|(z_p - a) \cdot \nu(a)|)}.
\tag{20}
\]

• The formulas (19), rewritten as

\[
\pm \sigma_\pm^r(a) = \lim_{z_p \to a} \frac{\pi \sum_{j=1}^2 \pm \nu_j(a) \Im I_j(z_p)}{\kappa \ln(|(z_p - a) \cdot \nu(a)|)}
\tag{21}
\]

enables us to know if \( a \in \Gamma^+ \) or \( a \in \Gamma^- \), i.e to distinguish between the two faces of the crack. Indeed, since \( \sigma^r(a) > 0 \) then if the right hand side of (21) is positive then \( a \in \Gamma^+ \) and if it is negative then \( a \in \Gamma^- \).

We would like to mention that the formulas given in the parts (II.1) and (II.2) of the theorem are valid if \( z_p \) is not tangential to \( \Gamma \). However, these possibilities are very rare in numerical implementations.

3. Numerical tests

![Figure 1](image.png)

**Figure 1.** Construction of two \( D_a^p \)'s. \( z_p \) can approach \( \Gamma \) from its convex side using \( D_a^p \) in the left-hand side, while \( D_a^p \) in the right-hand side is used for \( z_p \) approaching \( \Gamma \) from its concave side. When \( \delta_1 + l \times \delta_0 \to 0 \) with \( l \) the approaching step at each direction, \( z_p \to a \in \Gamma^\pm \) from both sides of \( \Gamma \) along radius direction, respectively.

In our model problem we take the crack as a half semi-circle with the representation

\[
\Gamma = \{ x : x = (x_1(s), x_2(s)) = 1.2 \times (\cos s, \sin s), s \in [0, \pi]\}. \tag{1}
\]
We test our inversion method by showing the reconstruction effect for all unknown ingredients in the model: the crack shape, crack type and surface impedance $\sigma_{\pm}$ in two sides of the crack.

Firstly, we use the blowing-up property of the indicator
\[
\text{Loc}(z_p) := |\Re(I_1(z_p))| + |\Re(I_2(z_p))| \text{ as } z_p \to \Gamma
\]
(2)
to detect the location of crack due to (16). That is, when $\text{Loc}(z_p)$ is large enough, we consider $z_p$ to be almost on $\Gamma$.

Secondly, we use the following equivalent form of the reconstruction formulas (19) and (20)
\[
\lim_{z_p \to a} \frac{\pi \sum_{j=1}^{2} \nu_j(a) \Im(I_j(z_p))}{\kappa \ln(|(z_p - a) \cdot \nu(a)|)} = \begin{cases} 
-\sigma^+_\nu & \text{if } z_p \to a \text{ from } \Gamma^+ \\
-\sigma^-_\nu & \text{if } z_p \to a \text{ from } \Gamma^- 
\end{cases}
\]
for the surface impedance reconstruction.

Finally, the crack type is shown by considering the blowing-up property of the function
\[
\text{Type}(z_p) := \frac{|\Im(I_1(z_p))| + |\Im(I_2(z_p))|}{\ln(|(z_\nu - a) \cdot \nu(a)|)^{1/2}} \text{ as } z_p \to \Gamma
\]
(5)
using the formula (18). That is, $\text{Type}(z_p)$ should increase up to some value (theoretically $\infty$) for the impedance crack.

**Example** In our model problems we take the wave number $\kappa = 1.2$. We take the surface impedance as the complex functions of the forms
\[
\kappa \sigma_- (x) \equiv 1 + 1.5i, \quad \kappa \sigma_+ (x) \equiv 2 + 1.2i
\]
(6)
and use incident plan waves along 64 directions distributed uniformly in $[0, 2\pi]$.

The reconstruction results for crack location are given in the left-hand side of Figure 2. The only a-priori information about the crack is that we know that $z_p$ is in the upper-side of the crack.

The crack type detection are checked using (5) with the numerical performance given in the right-hand side of Figure 2. The blowing-up property are shown obviously, except at the tips of the crack.

Now let us recover the boundary impedances $\sigma_{\pm}$. The reconstruction results are shown in the left hand side of Figure 3. Although there are some oscillations for real part of $\sigma_+$, the reconstruction is satisfactory.

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Figure 2. Construction of $\Gamma$ from the convex side of $\Gamma$ (left). It can be seen that the tips are not easy to identify with satisfactory accuracy. The crack type detection is shown in the right-hand side. The blowing-up property for the boundary type is obvious, except at the tips.

Figure 3. Construction $\sigma_+$ from the convex side of $\Gamma$: Imaginary part $\kappa\sigma_+^i$ (left) and real part $-\kappa\sigma_+^r$ (right).

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