From Spectral Relaxation to Quantified Decoherence

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Quantum information processing (QIP) requires thorough assessment of decoherence. Atoms or ions prepared for QIP often become addressed by radiation within schemes of alternating microwave-optical double resonance. A well-defined amount of decoherence may be applied to the system when spurious resonance light is admitted simultaneously with the driving radiation. This decoherence is quantified in terms of longitudinal and transversal relaxation. It may serve for calibrating observed decoherence as well as for testing error-correcting quantum codes.

The spectroscopic determination of atomic relaxation is among the earliest applications of laser spectroscopy [1]. Over the decades that have passed since then, the corresponding techniques have been dramatically refined, and the demand for reliable data has expanded. An important aspect is the availability of respective data derived from observations on individual trapped ions [2].

Laser-cooled ions prepared in an electrodynamic or electromagnetic trap and individually addressed are convenient building blocks for the storage and processing of quantum information [3]. In fact, they represent a scalable system, in contrast with certain competitive approaches, as, e.g., spin resonance spectroscopy [4]. The more important is the realization and full understanding of the radiative interaction of such a system: The implementation of the coherent dynamics of an individual atomic particle being radiatively driven is prerequisite for the demonstration of any quantum-logical gate, in particular of the basic Hadamard transformation [5]. The demonstration of the coherently controlled dynamics of trapped ions has been reported with the vibrational sidebands (or "one-phonon lines"), and carrier line ("zero-phonon") of a laser-driven Raman transition between ground-state hyperfine levels [6], with a microwave-driven hyperfine carrier line [7], with the vibrational sidebands of a dipole-forbidden optical line [8], and with a corresponding optical carrier line [9].

Some of these schemes are troubled by imperfectly identified kinds of decoherence that would thwart the performance of a substantial number of successive steps of coherent interaction. This decoherence may range from the residual decay of the involved metastable states to spurious light scattering, and to parasitic phase fluctuations of the applied radiation. For instance, the decoherence on the vibrational dynamics of ions localized in a Paul trap [5] has been suggested recently to result from parasitic spontaneous scattering of the two off-resonant light fields that serve as the pump and Stokes waves for Raman excitation of the hyperfine resonance [10]. Also, various types of reservoirs have been implemented that make the specific motional states decay in characteristic ways [11].

In a microwave-optical double-resonance experiment, we have found decoherence to appear during the coherent evolution of an individual trapped and cooled $^{171}$Yb$^+$ ion that was microwave-driven on the ground-state hyperfine transition. The evolution was monitored via the corresponding spin nutation. For this purpose, the ion was alternately driven and probed by highly monochromatic microwave radiation, and by resonant scattering of laser light, respectively. The observed decoherence was brought about by a minute amount of residual laser light that impinged upon the ion while the microwave drive was applied. Blocking out this spurious light eliminated the decoherence.

![Diagram](image-url)

FIG. 1. An individual measurement includes a pulse of resonant light that prepares the ion in the $F = 0$ state of the ground-state hyperfine doublet, a microwave driving pulse that is $N$ times a unit length $\delta t$, and a probe light pulse. The scattered light is detected by a photon counting (PC) system (top). With each subsequent measurement, $N$ is incremented by a unit, up to $N_{\text{max}}$, and yields a sequence of $N_{\text{max}}$ data, "on" or "off" (center). Accumulation of 50 sequences reveals rf nutation on the ground-state hyperfine transition (bottom).
We shall show that well-defined quantities of longitudinal and/or transversal relaxation may be admitted to the ion by controlling the strength and polarization of the light and the level and direction of the ambient magnetic field. Such a "designed" decoherence seems useful for quantitative comparison with observed decoherence and as a testing ground for the sensibility of quantum algorithms to the action of decoherence, as well as for the function of error-correcting codes.

The double-resonance experiment was performed on a single $^{171}$Yb$^+$ ion in a 2 mm-sized electrodynamic trap. The $F = 0 \rightarrow 1$ hyperfine resonance of the ion’s $S_{1/2}$ ground state was driven, for $\delta t = 100$ or $400\mu s$, by 12.6 GHz microwave radiation. Probing the $F = 1$ state immediately followed, when the ion was illuminated by a 5 ms pulse of 369 nm laser light. The excitation and observation of resonance scattering on the $S_{1/2}(F = 1) \rightarrow P_{1/2}(F = 0)$ line proves the preceding attempt of microwave excitation to have succeeded; the absence of scattering proves this attempt to have failed. A frequency-doubled Ti:sapphire laser generated 369 nm light of about 100 kHz bandwidth. This light was scattered on the ion’s $S_{1/2} - P_{1/2}$ resonance line, such that the ion was cooled deep into the Lamb-Dicke regime. Occasional optical pumping of the ion into its metastable $D_{3/2}$ level was counteracted, by using 935 nm light from a diode laser to repump the ion into the ground state.

Trajectories of measurements have been recorded, each of which includes an initial preparatory light pulse that pumps the ion into state $F = 0$, a microwave driving pulse of length $\tau = N\delta t$, a probe-light pulse, and simultaneous recording of the presumptive fluorescence scattered off the ion (Fig. 1). Within a trajectory, $N$ varied from 1, indicating the initial measurement, through 300, the final one. The recorded signals form quasi-random sequences of "on" and "off" results; however, the superposition of many accumulated trajectories yields the probability $P_1(\theta)$ for occupation of the $F = 1$ ground level by the ion, as a function of the driving time $\tau$. The data of 50 trajectories have been superimposed, and modulation of $P_1$ emerges that is ascribed to Rabi nutation of the ion driven by the microwave pulses of area $\theta = \Omega\tau$, whose duration $\tau$ was stepwise extended (Fig. 1, bottom).

![FIG. 2. Scattered-light response vs. length of microwave driving pulse. (a) Observed, with level of light applied to the ion during the drive: no light (top), 2nW (center), 20nW (bottom). (b) Simulated: Rabi frequency $\Omega = 0$ (top), 50 kHz (center), 500 kHz (bottom).](image)
The nutational oscillation shows high contrast. However, when a spurious level of probe light is admitted to the ion simultaneously with the driving pulse, the evolution of the probability of light scattering is dramatically modified (Fig. 2a): (i) the Rabi nutation becomes damped with its time constant being reduced upon increased intensity of the background light, and (ii) at long times $\tau$, the probability $P_3(\theta)$ saturates to a level somewhere between 1/2 and unity, and not necessarily to level 1/2.

In order to account for these observations, the ion, the light fields, and their interaction have been modelled by four-level Bloch equations (Fig. 2b). The corresponding level scheme is shown in Fig. 3. The Zeeman sublevels $m_F = +1$ and $-1$ have been combined and make up for the effective level 2 not affected by the driving microwave radiation on the hyperfine transition 0-1, with $\Delta m_F = 0$.

**FIG. 3.** Simplified level scheme of the $^{171}\text{Yb}^+$ ion used in the microwave-optical double resonance experiment.

### Diagram

- $P_{1/2}, F=0 \rightarrow |3\rangle$
- $S_{1/2}, F=1 \rightarrow |1\rangle$
- $\Omega_F$
- $\Gamma_3$
- $m_F = \pm 1$
- $S_{1/2}, F=0 \rightarrow |0\rangle$

**The minute light intensity applied to the ion during the driving intervals is far below saturation of the resonance line, $I = \Omega_F^2/\gamma_3 \ll 1$, where $\Omega_F$ is the Rabi frequency of the ion generated by the laser light, $\Gamma_3$ and $\gamma_1 = \Gamma_3/2 + \gamma_{ph}$ are the constants of energy relaxation of the resonance level ($P_{1/2}$), and of the phase relaxation of the laser-excited (electric) dipole, respectively. In the present experiment, the extra rate is small, $\gamma_{ph} \ll \Gamma_3$, and it is neglected. In the above limit it is appropriate, for the interpretation of the ionic evolution, to restrict the modelling of the optical part of the dynamics — , i.e. the part related to the three levels 1, 2, 3 — to optical pumping and decay, in terms of rate equations. This rate evolution is coupled to the microwave-driven coherent evolution, on the two-level system 0-1, via level 1 that represents state $S_{1/2}(F = 1, m_F = 0)$. Now, the latter coherent dynamics is affected by light-generated decoherence of two kinds: (i) a net loss of population, from the driven two-level system 0-1, by optically pumping the ion into state 2, i.e. the $F = 1$ Zeeman sublevels $m_F = \pm 1$, with subsequent repumping, and (ii) additional loss of phase coherence of the driven spin dynamics by Rayleigh scattering into the eigenstate 1, i.e. $F = 1, m_F = 0$.

The loss of some population from state 1 to state 2 makes the probability $P_3$ of finding the ion in the entire probed hyperfine state $F = 1$ (states 1 and 2) saturate above 1/2, since the Zeeman sublevels $\pm 1$ retain part of the population from the coherent transfer — on the condition $\Delta m_F = 0$ — into state $F = 0$. This fractional population may become re-excited, however, by a component of the linearly polarized laser light and taken back to state 1 ($F = 1, m_F = 0$) during the interval of probing, since the width of the resonance line far exceeds the small Zeeman splitting of the $S_{1/2}(F = 1)$ state. Complete pumping to level 2 ($m_F = \pm 1$) makes $P_3$ saturate at unity. The scattering rates are $\beta_i n_i r_i$, where $n_i$ is the population in the initial state, $\beta_f$ is the branching ratio of the decay of state 3 into the final state ($\beta_1 = \frac{1}{3}$, $\beta_2 = \frac{2}{3}$), and the scattering rate per atom is the average population $\langle P_3 \rangle$ of state 3, times the decay rate $\Gamma_3$, such that

$$r_1 = \langle P_3(0) \rangle \Gamma_3$$

$$r_2 = (\langle P_3(+1) \rangle + \langle P_3(-1) \rangle) \Gamma_3$$

Here ($m \equiv m_F$),

$$\langle P_3(m) \rangle = \frac{1}{2} \frac{I(m)L(B,m)}{1 + I(m)L(B,m)}$$

where $I(\pm 1) = I_0 \sin^2 \alpha$, $I(0) = I_0 \cos^2 \alpha$. $I_0$ is the density of light flux at the ion’s location, $\alpha$ is the angle subtended by the direction of the light polarization and magnetic field $B$. $L(B,m)$ is defined by

$$L(B,m) = \frac{(\Gamma_3/2)^2}{(\Gamma_3/2)^2 + (\omega_0 - \omega + m\delta)^2}$$

$\omega$ and $\omega_0$ are the light and resonance frequencies, respectively, $\delta = g_F \mu_B B/\hbar$, $g_F \approx 1$ is the Landé factor of state 2, and $\mu_B$ is the Bohr magneton [12]. Now, let us further restrict the model to the two-level system made up by states 0 and 1, and attribute to it conventional rates per atom of phase and energy relaxation, $\gamma$ and $\Gamma$, respectively. These rates will become identified with quantities taken from the above three-level model: the spin mutation gives rise to oscillation of the population difference in the two-level system that is supposed to damp out exponentially with an effective constant $\gamma$ of phase relaxation [12], such that

$$\gamma = \frac{\Gamma}{2} + \gamma_{ph} \approx r_1$$

where $\gamma_{ph}$ is the contribution of extra phase perturbation not related to intrinsic relaxation. Moreover, the flow equilibrium established by the scattering as well as the quasi-steady state on the microwave-driven line after dephasing require

$$n_0 = n_1 = \frac{1 - n_2}{2}$$
and

\[ \frac{1}{3} n_2 r_2 = \frac{2}{3} n_1 r_1 . \] (6)

Thus, \( n_2 = \frac{r_1}{n_1 + r_2} \), and the normalized probe signal, i.e., the probability of finding the system in the upper state 1 of a relaxing two-level system is

\[ P_1^{(3)} = n_1 + n_2 = \frac{1 + n_2}{2} \] (7)

\[ = 1 - \frac{1}{2} \frac{r_2/r_1}{1 + r_2/r_1} \]

\[ = 1 - P_0^{(3)} . \]

One may identify \( P_0^{(3)} \) with an effective two-level excitation probability \( P_0^{(2)} = \frac{1}{2} \frac{1}{1 + \Gamma} \), where \( \Gamma = \frac{1}{2} \frac{I}{\Delta \gamma \tau} \) and \( \Delta \) is the Rabi frequency of the microwave. This interpretation (i) yields the effective rate constant of energy relaxation,

\[ \Gamma = \frac{\Omega}{r_2} , \] (8)

and (ii) shows that this effective relaxation makes the ion decay into the excited state 1. Note that the two effective rate constants \( \gamma \) and \( \Gamma \) of eqs. 4 and 8 may be set separately by suitable selection of the light polarization and/or of the ambient magnetic field. Fig. 4 shows trajectories calculated with selected values of \( T_0 = \Gamma^{-1} \) and \( T_2 = \gamma^{-1} \). They demonstrate various degrees of damping as well as different levels of saturation of \( P_1(\tau) \).

With a single atom, competing spontaneous decay into non-degenerate levels may preserve coherence in the atom [13]. Therefore, complete modeling of the rate of optical pumping represented by eq. 2 would require one to include an interference term of the transition amplitudes of the back-and-forth optical pumping, via the states \( F = 1, m_F = +1 \) and \(-1 \), that give rise to indiscernible pathways. This term displays a resonance in zero magnetic field and represents what is called "zero-field level crossing" [14], a ground-state Hanle effect. The width of the crossing resonance is determined by the lifetime of the intermediate interfering states. The \( F = 1 \) levels show an effective lifetime on the order of millisecond, or longer. Thus, the interference term in rate \( r_2 \) would make this rate vary across a spectral tuning range of the laser less than 1 kHz wide. This spectral feature is not resolved by the emission bandwidth of the laser.

Historically it is interesting, that the Hanle effect when observed on Ne atoms irradiated by the light field of a HeNe laser was one of the earliest laser-spectroscopic topics; it was dealt with in the diploma thesis of Theo Hänsch [15,16].

The availability of easily quantifiable longitudinal and transversal relaxation that is light-induced upon individual atomic systems displays important advantages when it comes to the application of such a system to manipulations in QIP impaired by loss of coherence. In particular, codes of information processing may be tested for their applicability under this challenging but commonplace condition. On the other hand, codes for error correction may be made to demonstrate their capacity upon increasing levels of decoherence fed into the system. The light-induced decoherence, as demonstrated in this report, is readily applicable to individually addressed quantum systems, it may be switched on and off immediately, and it is reproducible.

In summary, a simple method has been outlined that adds a predetermined degree of decoherence on the coherent radiative interaction of an individual atom or ion that is to be used for information processing. Although, in the present experiment, the coherent drive was mi-
crowave radiation resonant with a ground-state hyperfine transition, the same principle seems to apply to a system where a dipole-forbidden optical transition is driven by laser light.

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