Brane Fluctuation and New Counting Rules for Kaluza-Klein Towers

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Abstract

In models with extra dimensions, branes have been usually treated as solid bodies though they are prohibited by the relativity. In the previous letter, we proposed a method of taking account of brane fluctuation by introducing Nambu-Goldstone bosons, and prove that when a tension of the brane is small, the interaction between boundary fields and Kaluza-Klein modes is suppressed exponentially. In this letter, we further investigate this suppression in more generic configuration, and obtain three counting rules, “AND”, “OR” and “STACK” rules, depending on the softness of branes and the character of fields on the branes four dimensions. The choice determines the number of Kaluza-Klein towers contributing to renormalization group equations, leading to a remarkable change in the running behavior of coupling constants.

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1 Introduction

Over the past years models with extra dimensions have been intensively studied from various viewpoints [1]. Within the framework of these models, our spacetime is thought to be a four dimensional object located in higher dimensional space, and observable quarks and leptons are regarded as either fermion fields defined on the brane or Kaluza-Klein zero modes of bulk fermion fields. Gauge and gravitational fields are usually assumed to propagate in the bulk space. Therefore, the interaction between boundary and bulk fields plays an important rôle when it comes to discussing phenomenology. When constructing Lagrangians with boundary-bulk interactions, we have usually treated the boundary as a membranous solid body. However, the fact that the relativity prohibits the existence of solid bodies suggest that we should take account of the effect of brane fluctuation.

In the previous letter, we propose a method of incorporating brane fluctuation by introducing Nambu-Goldstone bosons originated from the spontaneous breaking of the translational symmetry along extra spatial directions [2]. Consequently, we show the exponential suppression of the interaction between four dimensional electron fields and Kaluza-Klein modes of five dimensional photon fields when the brane tension is sufficiently small. The analysis can be immediately extended to the interaction involving generic boundary-boundary-bulk vertices. If we adopt this prescription, we do not have to introduce an ultraviolet cutoff required to regulate the divergence from an infinite summation of Kaluza-Klein modes because interactions with higher Kaluza-Klein modes are properly suppressed. While the smallness of the interaction unfortunately lessens the possibility of observing extra dimensions, on the other hand it provides an interesting phenomenon of observing the scalar fields describing brane fluctuation [4].

In this letter, we will further investigate the effect of brane fluctuation in more generic situations where fields involving boundary-bulk interactions extend in different dimensional spaces, and according to the softness of the branes we obtain three counting rules for Kaluza-Klein towers, which are provisionally called as “AND”, “OR” and “STACK” rules.

More interestingly, the alteration of counting rules drastically changes a running behavior of coupling constants because it is the number of Kaluza-Klein towers propagating in a relevant loop that determines an exponent of the power-law [5][6][7].

*Similar suppression factor was obtained in a different context in Ref. [3].
2 Solid brane system

In this section, we briefly review usual treatment for boundary-bulk interaction under the solid body approximation, and show the resultant power-law behavior of beta functions \[5\].

Within models with extra dimensions, matter fields such as quarks and leptons are regarded as (i) fermion fields defined only on a brane, or (ii) Kaluza-Klein zero modes of bulk fermion fields. For simplicity, we assume the number of the extra spatial dimension to be one, the generalization being straightforwardly to the case with more extra spatial dimensions. In order for our discussion to be strict, we consider only scalar fields as boundary and bulk fields throughout this article. We expect that one may perform the same analysis with fermion fields and generalize our setup to supersymmetric models.

Let us investigate a concrete interaction involving with a scalar field \(\phi_1(x)\) satisfying the case (i) and \(\phi_2(x, y)\) and \(\phi_3(x, y)\) propagating in five dimensional spacetime. In this model, an interaction term can be described as

\[
S_{\text{int}} = \int d^4x \int_0^{2\pi R} dy \, Y \phi_1(x) \delta(y) \phi_2(x, y) \phi_3(x, y). \tag{2.1}
\]

where \(Y\) is a coupling constant and \(\delta(y)\) is inserted in order to take the boundary values of the bulk fields. Decomposing \(\phi_2\) and \(\phi_3\) into Kaluza-Klein modes \(\phi_2 = \sum_n \phi_2^{(n)}(x)e^{iny/R}\) and \(\phi_3 = \sum_m \phi_3^{(m)}(x)e^{imy/R}\), the equation (2.1) is rewritten in terms of four dimensional field theory as

\[
S_{\text{int}} = \int d^4x \, Y \sum_{n,m=-\infty}^{\infty} \phi_1(x) \phi_2^{(n)}(x) \phi_3^{(m)}(x). \tag{2.2}
\]

Since all Kaluza-Klein modes interact with \(\phi_1(x)\) with an equal coupling constant, the Kaluza-Klein modes of \(\phi_2(x, y)\) and \(\phi_3(x, y)\) contribute equally to renormalization group equations. We should note that this result comes from the \(\delta(y)\) function in Eq. (2.1). In addition, owing to the localization of \(\phi_1(x)\) in the higher dimensional space, we cannot impose a momentum conservation rule along extra spatial directions, thus there appears no restriction about the Kaluza-Klein excitation numbers \(n\) and \(m\). Therefore, two Kaluza-Klein towers parameterized by integers \(n\) and \(m\) contribute independently to the anomalous dimension of \(\phi_1(x)\), and finally we obtain the power-law dependence with the exponent 2.

On the other hand, if the boundary field \(\phi_1(x)\) is assumed to be a Kaluza-Klein zero mode, the action can be written as

\[
S_{\text{int}} = \int d^4x \int_0^{2\pi R} dy \, Y \phi_1^{(0)}(x) \phi_2(x, y) \phi_3(x, y). \tag{2.3}
\]
In terms of four dimensions, we obtain
\[ S_{\text{int}} = \int d^4x \, Y \sum_{n,m=-\infty}^{\infty} \phi_1^{(0)}(x) \phi_2^{(n)}(x) \phi_3^{(m)}(x) e^{i(n+m)\eta}. \] (2.4)

The assumption that \( \phi_1(x) \) is a Kaluza-Klein zero mode requires a momentum conservation rule along the extra spatial direction, thus only the Kaluza-Klein modes satisfying \( n + m = 0 \) contribute to renormalization group equations. Consequently, we obtain the power-law behavior with the exponent 1. It should be noted that the case (ii) implies a momentum conservation rule along extra spatial directions while the case (i) does not because their extra dimensional momenta are indefinite.

Similarly, it is found that when both \( \phi_1 \) and \( \phi_3 \) are boundary fields, the resultant exponent is always one whether they belong to the case (i) or (ii).

Here we recapitulate these results:

- in the case with \( \phi_1 \) belonging to the type (i) fields, the exponent turns out to be the number of fields propagating to the extra direction in a relevant loop;
- in the case with \( \phi_1 \) belonging to the type (ii) fields, the exponent turns out to be one if at least one of the fields \( \phi_2 \) and \( \phi_3 \) expands in the extra dimension; otherwise zero.

We can put these rules in the following way. As for the former, the total number of Kaluza-Klein towers propagating in a relevant loop contributes to the exponent. On the other hand, as for the latter, only the extra dimension where \( \phi_2(x) \) or \( \phi_3(x) \) extends contributes to the total exponent of the power-law. Therefore, we call the former “STACK” rule and the latter “OR” rule. In Ref. [7], we showed that the OR rule prefers democratic Yukawa matrices. On the basis of STACK rule, the exponent of beta functions of Yukawa couplings differs generically depending on their generation, thus it seems viable to realize fermion mass hierarchies [8]. Further investigation on this point has been in progress [8].

In the case with more extra dimensions, the total exponent would be given by the sum of the contribution from each extra direction.

## 3 Soft brane system

Let us follow the prescription proposed in Ref. [2] and introduce a Nambu-Goldstone boson \( \phi(x) \) stemming from the spontaneous breaking of extra dimensional translation symmetry.

\(^1\)In the paper, we did not call this rule as “OR” rule explicitly.
The Lagrangian of this system remains the same as the preceding one (2.1), but there appears a kinetic term of $\phi(x)$ from a determinant of the induced metric, i.e. the Nambu-Goto action $\int d^4x (-\tau_4) \sqrt{-g}$,

$$\int d^4x \left\{-\tau_4 + \frac{\tau_4}{2} \partial_\mu \phi(x) \partial^\mu \phi(x) + \cdots \right\},$$

(3.1)

where $\tau_4$ is a four dimensional tension and the ellipsis stands for negligible higher derivative terms.

By introducing this scalar fields $\phi(x)$ we here take account of brane fluctuation; we substitute $y$ in Eq. (2.3) with $\phi(x)$ and obtain the following boundary-bulk interaction [2],

$$S_{\text{int}} = \int d^4x Y \sum_{n,m} \phi_1(x) \phi_2^{(n)}(x) \phi_3^{(m)}(x) e^{\frac{i(n+m)\phi(x)}{R}}$$

(3.2)

Since our analysis are based on a perturbation theory, it seems appropriate to rewrite the exponential factor $\exp(i \frac{n}{R} \phi(x))$ into the normal ordered form referring to the free kinetic term of $\phi$ in Eq. (3.1),

$$S_{\text{int}} \sim \int d^4x Y \sum_{n,m} \phi_1(x) \phi_2^{(n)}(x) \phi_3^{(m)}(x) e^{-\frac{1}{2} \left( \frac{n+m}{R} \right)^2 \Delta(l_s) \cdot e^{\frac{i(n+m)\phi(x)}{R}}} :$$

(3.3)

where $\Delta$ is the free propagator of $\phi$ defined as

$$\Delta(x-y) \equiv \langle \phi(x) \phi(y) \rangle = \frac{1}{f^4} \cdot \frac{1}{-(x-y)^2},$$

(3.4)

Note that $f^{-1}$ is a characteristic length of brane fluctuation along the extra spatial direction and related to the brane tension through $\tau_4 = f^4 l_s^4$. The limit $f \to \infty$ corresponds to the solid brane approximation. Since we have treated this system as an effective theory valid below the cutoff energy scale $M_s = l_s^{-1}$, the propagator $\Delta(x)$ with $|x| \leq l_s$ should be regarded as $\Delta(l_s) = 1/(f^4 l_s^2)$, and thus we have replaced the infinite $\Delta(0)$ by the value $\Delta(l_s)$ in Eq. (3.3). We can read the effective Yukawa coupling $Y_{n,m}$ from Eq. (3.3),

$$Y_{n,m} = e^{-\frac{1}{2} \left( \frac{n+m}{R} \right)^2 \frac{M_s^2}{f^4} Y.}$$

(3.5)

In the case with a small tension, this effective coupling signifies apparently that only the diagrams satisfying the relation $n + m = 0$ contribute to the anomalous dimension. This restriction on the combination of momenta can be effectively interpreted as an extra dimensional momentum conservation rule. If we take the limit of $f \to \infty$, the suppression
factor of the effective Yukawa coupling disappears and we are led to the same coupling shown in the solid body system.

This effective momentum conservation rule can be interpreted intuitively as follows. In the case with a small brane tension, when a boundary field decays into a boundary field and only one Kaluza-Klein mode of bulk fields, the brane gets deformed by the back-reaction. Accordingly, the overlapping between the wave function of the initial state and that of the final state (i.e. before and after the emission) gets suppressed exponentially. On the other hand, when a boundary field decays into two Kaluza-Klein modes carrying equal momenta along the extra spatial directions with an opposite sign, the brane remains unchanged. As a result, the coupling does not suffer the exponential suppression. This is the reason why only one Kaluza-Klein tower contributes to the wave functional renormalization of $\phi_1(x)$.

In order to complete the classification of our counting rules let us finally investigate the case that the scalar field $\phi_3$ does not extend in the extra spatial direction, i.e. $\phi_3$ is defined only on the four dimensional brane (case (i)) or regarded as a Kaluza-Klein zero mode (case (ii)). In this configuration, whether $\phi_3$ belongs to the case (i) or (ii), the action is written as

$$S_{\text{int}} = \int d^4x \sum_{n,m} \phi_1(x) \phi_2^{(n)}(x) \phi_3(x) e^{-\left(\frac{n}{2}\right)^2 M_s^4 \left(3.6\right)}$$

and it indicates that the couplings between Kaluza-Klein modes of $\phi_3$ and $\phi_1$ is suppressed for the case with a small brane tension.

Taking account of the results obtained in the case with a soft brane, we find that it is only an extra spatial dimension where both $\phi_2$ and $\phi_3$ extend that contributes to an anomalous dimension of $\phi_1$, thus let us call this counting rule as “AND” rule. In table 1, we summarize the counting rules for various cases.

4 Conclusion

We have investigated the effect of brane fluctuation on boundary-bulk interactions on the basis of the method proposed in the previous paper, and obtained three counting rules for Kaluza-Klein towers contributing to renormalization group equations. We have shown that one should adopt an appropriate counting rule judging from a brane tension and configuration

\[\text{OR} \text{ and } \text{AND} \] in the name of the counting rules are analogous to those in the logic circuit and the boolean algebra. We would observe the analogy explicitly in the table after regarding “bulk” and “boundary” as “1” and “0”, respectively.
Table 1: The number of Kaluza-Klein towers contributing to the wave function renormalization of \( \phi_1 \) and the configuration of relevant fields. The number is determined by the property of the fields \( \phi_1 \), an extra spatial configuration of \( \phi_2 \) and \( \phi_3 \) and a brane tension. Note that the index \( \varnothing \) in the first line of the table means that the fields \( \phi_3 \) does not extend in the extra spatial dimension.

| \( \phi_1 \)          | \( \phi_2^{(m)} \) | \( \phi_3^{(n)} \) or \( \varnothing \) | # of KK towers |
|-----------------------|---------------------|--------------------------------------|----------------|
| solid brane           | bulk                | bulk                                 | 2              |
| solid brane           | bulk                | boundary                             | 1              |
| KK zero mode          | bulk                | bulk                                 | 1              |
| KK zero mode          | bulk                | boundary                             | 1              |
| fluc. brane           | bulk                | bulk                                 | 1              |
| fluc. brane           | bulk                | boundary                             | 0              |

Table 2: The relation between the property of \( \phi_1 \) and the resultant counting rule.

| \( \phi_1 \)          | counting rules  |
|-----------------------|-----------------|
| solid brane           | STACK           |
| KK zero mode          | OR              |
| fluc. brane           | AND             |

of scalar fields when it comes to evaluating Feynman diagrams. It should be noted that the alteration of counting rules changes the exponents of renormalization group equations, and thus occurs a drastic change in the running behavior of coupling constants, which plays a significant rôle in discussing phenomenology. Further investigation on this point will be shown in our future paper \([8]\).

Finally, we here make a brief comment on realization of chiral matter fields. As is well known, the standard model fermions such as quarks and leptons have been correctly treated as chiral multiplets. Within the framework of field theories with extra dimensions it is known that such a simple compactification as on a torus unfortunately cannot produce chiral fermions, and that one must take more complicated procedure such as orbifold compactifications with an appropriate projection, where desirable chiral fermions appear on the
fixed brane in the whole spacetime. For example, in the effective description of heterotic M-theory they appear on the fixed brane of the $S^1/Z_2 \times M^4$ manifold after $Z_2$ projection. In general, such an orbifold projection with respect to extra dimensions suggests that the fixed brane should be treated as a rigid object, and the incorporation of the brane fluctuation seems naively incompatible with a chiral projection.

However, since it is clear that no rigid brane exists in the relativistic theory, we expect that even the orbifold brane should fluctuate so far as the tension is finite, where chiral fermions can be realized on the fluctuating brane after an appropriate projection. Thus we expect that the similar counting rules can be applied also to the system with chiral fermions. Further investigations should be performed in order to confirm the validity, which is our next task to be accomplished.

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