Luttinger liquids with curvature: Density correlations and Coulomb drag effect

D. N. Aristov

Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany and
Center for Functional Nanostructures, Universität Karlsruhe, 76128 Karlsruhe, Germany
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We consider the effect of the curvature in fermionic dispersion on the observable properties of Luttinger liquid (LL). We use the bosonization technique where the curvature is irrelevant perturbation, describing the decay of LL bosons (plasmon modes). When possible, we establish the correspondence between the bosonization and the fermionic approach. We analyze modifications in density correlation functions due to curvature at finite temperatures, $T$. The most important application of our approach is the analysis of the Coulomb drag by small momentum transfer between two LL, which is only possible due to curvature. Analyzing the a.c. transconductivity in the one-dimensional drag setup, we confirm the results by Pustilnik et al. for $T$-dependence of drag resistivity, $R_{12} \sim T^2$ at high and $R_{12} \sim T^5$ at low temperatures. The bosonization allows for treating both intra- and inter-wire electron-electron interactions in all orders, and we calculate exact prefactors in low-$T$ drag regime. The crossover temperature between the two regimes is $T_1 \sim E_F \Delta$, with $\Delta$ relative difference in plasmon velocities. We show that $\Delta \neq 0$ even for identical wires, due to lifting of degeneracy by interwire interaction, $U_{12}$, leading to crossover from $R_{12} \sim U_{12}^2 T^2$ to $R_{12} \sim T^5 U_{12}^2$ at $T \sim U_{12}$.

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I. INTRODUCTION

The effects of the curvature of fermionic dispersion for the observables in strongly-correlated one-dimensional (1D) fermionic systems were discussed by several groups recently. One physical effect, which probes both the curvature and interactions is the Coulomb drag effect. In a typical experimental setup, this effect is observed as non-zero drag resistivity, $R_{12} = V_2/I_1$, with a d.c. current $I_1$ flowing in the "active" wire and the voltage bias $V_2$ per unit length applied to the second "passive" wire in order to assure $I_2 = 0$, see review articles Ref. [8].

It is generally agreed that the principal source of the drag effect is the particle-hole asymmetry of the electronic system. This was established in various works considering electrons in higher spatial dimensions, $D > 1$, both in the absence of magnetic field and in the presence of it (see Ref. [10] and references therein). It was shown that the leading contribution to the drag effect is obtained in the second order of the interwire interaction $U_{12}$ and is schematically depicted by the Feynman graph in Fig. [2]. This contribution corresponds to the virtual processes away from the Fermi surface, and is ultimately determined by the curvature of the electronic dispersion.

One naturally would expect that the curvature should be also responsible for the drag effect in one spatial dimension (1D). However, the first study of the drag effect in 1D was devoted to a different mechanism, specific for 1D and requiring some additional conditions, we return to it below. The study of the drag effect in 1D due to curvature of electronic dispersion was initiated only recently, by Pustilnik et al. The reason is that the drag effect depends both on the curvature and the fermionic interaction, whereas it is known that the interaction is very important in 1D and ultimately leads to the notion of Luttinger liquid. The possibility to find a complete solution, i.e. Luttinger liquid, for the interacting system, lies in the crucial simplification of the theory - the linearization of the fermionic dispersion around the Fermi energy. In most cases it is an innocent procedure, and does not influence the final result. The Coulomb drag effect is one example, where this usual theoretical trick with linearization leads to the immediate disappearance of the observable quantity in question. In their work Pustilnik et al. (see also Ref. [12]) hence used the fermionic formalism, and found the drag effect as the function of temperature, for different regimes. Restricting themselves to the lowest order of interwire, as well as intrawire interactions, they showed that $R_{12} \propto T^2$ for equal wires and $R_{12} \propto T^5$ for non-equal wires. In this work we check their results by another method and extend their treatment beyond the lowest order of perturbation theory (PT) in fermionic interactions.

To this end, we use the traditional tool for studying the effects of interaction in 1D, known as bosonization technique, which is devised for effective resummation of the appearing series in PT. Employing the bosonization we take into account the inter- and intrawire interaction in all orders, whereas the curvature of fermionic dispersion is treated as perturbation. We show that it suffices to consider fourth order in curvature when calculating low-temperature drag effect. At higher temperatures we perform the resummation of most singular contributions in PT in curvature, incorporating free-fermion results into bosonization language. In this sense, our method is complementary to the analysis in Ref. [13] where the curvature was treated exactly and the interactions — in the lowest necessary order.

The issue of the curvature of fermionic dispersion in bosonization technique was rarely discussed previously. Therefore, although we are ultimately interested in the
Our Hamiltonian is given by four terms:
\[ \mathcal{H} = \int dx \left( H_1 + H_2 + H_{12} + \mathcal{H}_{\text{cur}} \right), \]
with the linearized interacting fermions in individual wires
\[ H_{j=1,2} = -i v F_j \psi_R^\dagger \partial_x \psi_R + \frac{i v F_j}{2} \partial_x \psi_L^\dagger \partial_x \psi_L + U_j \rho_j^2(x), \]
forward-scattering interaction between wires
\[ H_{12} = \sum_{j=1,2} \frac{1}{2 \hbar m_j} \left( \psi_{Rj}^\dagger(i \partial_x)^2 \psi_{Rj} + \psi_{Lj}^\dagger(i \partial_x)^2 \psi_{Lj} \right), \]
and the terms describing the curvature of fermionic dispersion
\[ \mathcal{H}_{\text{cur}} = \sum_{j=1,2} \frac{1}{2 \hbar m_j} \left( \psi_{Rj}^\dagger(i \partial_x)^2 \psi_{Rj} + \psi_{Lj}^\dagger(i \partial_x)^2 \psi_{Lj} \right). \]

In this bosonization notation, the Hamiltonian of the system is rewritten as follows:
\[ \mathcal{H} = \int dx \left( H_1 + H_2 + H_{12} + \mathcal{H}_{\text{cur}} \right), \]
\[ H_{i=1,2} = \frac{1}{2 \pi} \left( v_{iJ} \left( \partial_x \phi_i \right)^2 + v_{iN} \left( \partial_x \phi_i \right)^2 \right), \]
\[ H_{12} = \frac{U_{12}}{\pi^2} \partial_x \phi_1 \partial_x \phi_2, \]
\[ \mathcal{H}_{\text{cur}} = \frac{1}{6 \pi m_3} \partial_x \phi_1 \left( 3 \left( \partial_x \theta_1 \right)^2 + \left( \partial_x \phi_1 \right)^2 \right) + (1 \leftrightarrow 2). \]

Here and below I use the shorthand notation \( \partial_x \phi_1 = \partial_x \phi_1(x)/\partial x \), etc. The electronic density operator in (7) is given by density correlation functions is discussed in Sec. III, and the drag coefficient is obtained for relatively simple case of non-equal wires. The analysis of nearly-equal wires is performed separately, in most technically involved Sec. VI, the final results for \( R_{12} \) in low temperature regime are discussed here. We present our summary and conclusions in Sec. VII.

II. SETTING UP THE PROBLEM

A. Hamiltonian

We consider spinless (spin-polarized) electrons in two wires, with the forward-scattering short-range interaction both inside and between the wires. The electrons in \( i \)th wire (\( i = 1,2 \)) are conventionally subdivided into right- and left-going species, centered around corresponding Fermi momenta, \( k_{Fi} = \pi n_i \), with the fermionic density \( n_i \). We go beyond the the linearized spectrum approximation, keeping quadratic terms in dispersion.

As usual, we decompose fermionic operator into "slowly oscillating" chiral components
\[ \psi(x) = e^{i k F x} \psi_R(x) + e^{-i k F x} \psi_L(x), \]
so that the smooth part of the fluctuating fermionic density is
\[ \rho(x) = \psi_R^\dagger(x) \psi_R(x) + \psi_L^\dagger(x) \psi_L(x). \]

Our Hamiltonian is given by four terms:
B. Curvature as interaction

In bosonization, the normally ordered parts of chiral fermionic densities become

\[ R_j \equiv \psi_R^\dagger \psi_{Rj} , \quad L_j \equiv \psi_{Lj}^\dagger \psi_{Lj} , \]

with \( \rho_j = R_j + L_j = \pi^{-1} \partial_x \phi_j \). The main statement of (Abelian) bosonization, dating back to Tomonaga,\(^{15}\) is that the free chiral fermions with linear dispersion are equivalent to quadratic form in chiral densities:

\[ -i\psi_R^\dagger \partial_x \psi_R = \pi R^2 . \]

Tomonaga established this relation, studying the equation of motion for the chiral density \( R \) with the fermionic Hamiltonian with linear dispersion. This observation was later corroborated by Schick,\(^{16}\) who found that similar consideration with the quadratic fermionic dispersion yields another important relation:

\[ \psi_R^\dagger \left( i \partial_x \right) \left( i \partial_x \right)^2 \psi_R = \frac{2\pi^2}{3m} R^3 . \]

Another derivation of (11) was shown in the paper by Haldane,\(^{17}\) who started with the fermion representation. Note that using Eq. (6), one arrives also to the term in the Hamiltonian, which constitutes a full derivative,\(^{18}\) Note that using Eq. (6), one arrives also to the term in the Hamiltonian, which constitutes a full derivative,\(^{18}\) is known as the higher-loop cancellation theorem after exactly zero for the linear dispersion law. This statement is that the free chiral fermions with linear dispersion are equivalent to quadratic form in chiral densities:

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This statement was later corroborated by Schick,\(^{16}\) who found that similar consideration with the quadratic fermionic dispersion yields another important relation:

Schematically, it is shown as Feynman diagrams in Fig. 1, where fermion Green’s functions are shown by lines with arrows, and double lines represent bosonic Green’s functions.\(^{21}\) Note that the leading-order drag diagram is given by the "two triangles" fermionic diagrams in Fig. 2 with wavy lines standing for \( U_{12} \). Comparing it to Fig. 1, we see that the drag coefficient should correspond to "two stars" bosonic diagram in the r.h.s. of Fig. 2. Further observing that interwire interaction, \( U_{12} \), is exactly included into the Luttinger liquid formalism, Eq. (7), we eventually arrive to the diagrams depicted in Fig. 4 below. Note that passing from Fig. 2 to Fig. 4 corresponds to the screening of interwire interaction, see e.g. Kamenev and Oreg.\(^{9}\) The RPA resummation in case of LL gives an exact result; however, the simplest bosonic diagram in Fig. 2 corresponds to the optical transconductivity, rather than to d.c. drag effect, as will be shown below.

Before considering a general case of two different wires, let us discuss the modification of density correlations in one wire with the curvature term.

![Fig. 1: Feynman diagrams representing non-linear response of fermionic system. Two three-tail fermion loops correspond to a boson decay process, see the text for discussion.](image1)

![Fig. 2: First Coulomb drag diagrams in the fermionic formalism and their bosonization counterpart, see the text for discussion.](image2)

III. DENSITY CORRELATIONS

Let us first discuss the density correlation function for one wire, with and without the Luttinger-liquid type interaction. Such analysis was previously done in\(^{13}\), we extend it for the case \( T \neq 0 \). The bosonized Hamiltonian

\[ \frac{1}{2\pi m v_F q_1 - \omega_1} \frac{q_2}{v_F q_2 - \omega_2} \frac{q_3}{v_F q_3 - \omega_3} , \]

which corresponds to three density correlators for linear spectrum, \((2\pi)^{-1} q/(v_F \omega)\), attached to above vertex, \(2\pi^2/(3m)\); factor \(3! = 6\) comes from symmetrization.

\[ \Lambda(\omega, q) = \frac{1}{2\pi} \ln \frac{\omega - v_F q + q^2/2m}{\omega - v_F q - q^2/2m} . \]

\[ T = \frac{q_1 q_2 q_3 \left( \sum_{i=1}^{3} \Lambda(\omega_i, q_i) \right)}{(\omega_1 q_2 - \omega_2 q_1)^2 - (q_1 q_2 q_3/2m)^2} . \]

\[ = \frac{q_1}{2\pi m v_F q_1 - \omega_1} \frac{q_2}{v_F q_2 - \omega_2} \frac{q_3}{v_F q_3 - \omega_3} . \]
reads as
\[ \mathcal{H} = \int dx (H_0 + H_{\text{cur}}), \]
\[ H_0 = \frac{1}{2\pi} (v_F (\partial_x \theta)^2 + v_N (\partial_x \phi)^2), \]
\[ = \pi v_F (R^2 + L^2) + \frac{U}{2} (R + L)^2, \]
\[ H_{\text{cur}} = \frac{1}{6\pi m} \partial_x \phi \left( 3(\partial_x \theta)^2 + (\partial_x \phi)^2 \right), \]
\[ = \frac{2\pi^2}{3m} (R^3 + L^3). \]

The commutation rules for the chiral densities are

\[ [R(x), R(y)] = -[L(x), L(y)] = \frac{i}{2\pi} \partial_x \delta(x-y). \]

Our choice of the interaction in the form \( U \) corresponds to the case \( g_4 = g_2 \) in more general form of the Hamiltonian \( g_4 (R^2 + L^2) + 2g_2 RL \).

### A. Free fermions

In the absence of the mixing interaction term \( U \) in \( \mathcal{H} \), each chiral component is considered separately. In the absence of the curvature, \( 1/m = 0 \), the bare boson retarded right (left) Green’s functions is independent of \( T \).

The self-energy part of the form \( \Sigma = \frac{q}{12m^2} q^2 + \frac{(2\pi T/v_F)^2}{q v_F - \omega - i0} \).

Note, that in a quite unusual way\(^{17}\), the spectral weight for the function analytic in upper semiplane of \( z \). In the limit of zero temperature we obtain

\[ \chi = \frac{m}{2q} \Theta(1 - |\Omega|) \]

\[ \chi = \frac{q}{4\pi w_q} \ln \frac{\omega - q v_F - w_q + i0}{\omega - q v_F + w_q + i0} \]

with the step-function \( \Theta(x) = 1 \) at \( x > 0 \).

Let us now discuss how these exact expressions relate to the bosonization theory. Formally, the curvature term \( H_{\text{cur}} \) has a scaling dimension 3, which is strongly irrelevant. The common wisdom then would prescribe to treat it in the lowest order of perturbation theory (PT) in the infrared limit, \( q, \omega \to 0 \). Comparing \( \Sigma \) and the bosonization expression \( \Sigma_{\text{Bos}} \), one can see that the \( \delta \)-function is a good approximation for the spectral weight in this limit. However, the lowest PT contribution, with two bosons in intermediate state, leads to the self-energy part of the form

\[ \Sigma = \frac{2\pi}{12m^2} q^2 + \frac{(2\pi T/v_F)^2}{q v_F - \omega - i0} \]
In this case right and left chiral densities mix, but the quadratic Hamiltonian which is the Luttinger model is still solvable. The corresponding \( u - v \) Bogoliubov transformation diagonalizes the Hamiltonian in terms of new chiral densities, \( \tilde{R}, \tilde{L} \), which obey the same commutation relations. Performing this transformation, one obtains

\[
H_0 = \pi v \left( \tilde{R}^2 + \tilde{L}^2 \right),
\]

\[
H_{\text{car}} = \frac{\alpha_1}{3} (\tilde{R}^3 + \tilde{L}^3) + \frac{\alpha_2}{2} (\tilde{R} \tilde{L} + \tilde{R} \tilde{L}^2),
\]

\[
\alpha_1 = \frac{\pi^2}{m} \frac{3 + K^2}{2\sqrt{K}}, \quad \alpha_2 = \frac{\pi^2}{m} \frac{K^2 - 1}{\sqrt{K}},
\]

with \( v^2 = v_F (v_F + U/\pi) \), \( K^2 = (1 + U/\pi v_F)^{-1} \). One can see from that apart from some rescaling of the old mass term, \( \tilde{R}^3 + \tilde{L}^3 \), the effect of interaction gives rise to a new type of the curvature vertex, which mixes new right and left densities.

The density correlation function \( \langle \rho \rho \rangle_{\omega q} \) is determined through the new variables \( \tilde{R}, \tilde{L} \) as

\[
\langle \rho \rho \rangle_{\omega q} = K \langle (\tilde{R} \tilde{L}) \rangle_{\omega q}.
\]

Note that \( RL \) mixing, which was removed at the level of quadratic action, reappears due to the curvature. The mixed terms \( \langle \tilde{R} \tilde{L} \rangle \) are non zero in case of both \( U \neq 0 \) and \( m^{-1} \neq 0 \), and it necessitates the consideration of the (retarded) matrix Green function

\[
\tilde{G}(\omega, q) = \begin{pmatrix}
\langle \tilde{R} \tilde{R} \rangle_{\omega q} & \langle \tilde{R} \tilde{L} \rangle_{\omega q} \\
\langle \tilde{L} \tilde{R} \rangle_{\omega q} & \langle \tilde{L} \tilde{L} \rangle_{\omega q}
\end{pmatrix}
\]

It is clear that the PT for bosons in the case \( K \neq 1 \) contains processes absent in the free fermion case \( K = 1 \).

Let us calculate the boson self-energy in the second order of PT. Denoting three types of boson loops as \( A_{RR}, A_{RL}, A_{LL} \) (Fig.), one obtains

\[
\Sigma_{RR} = 2 \alpha_1^2 A_{RR} + \frac{1}{2} \alpha_2^2 A_{RL} + \alpha_2^2 A_{RL},
\]

\[
\Sigma_{LL} = 2 \alpha_2^2 A_{LL} + \frac{1}{2} \alpha_2^2 A_{RR} + \alpha_2^2 A_{RL},
\]

\[
\Sigma_{RL} = \Sigma_{LR} = \alpha_1 \alpha_2 (A_{RR} + A_{LL}) + \alpha_2^2 A_{RL}.
\]

Off the light cones, \( \omega \neq \pm vq \), one has \( \text{Im} \ A_{RR} = \text{Im} \ A_{LL} = 0 \) and all components of the imaginary part of the self energy are given by the quantity

\[
\Sigma_{o f f} \equiv \alpha_2^2 A_{RL}.
\]

We list here the form of the appearing (retarded) expressions:

\[
A_{RR} = \frac{1}{48\pi^3} \frac{q^2 + (2\pi T/v^2)^2}{(qv - \omega - i0)}
\]

\[
A_{LL} = \frac{1}{48\pi^3} \frac{q^2 + (2\pi T/v^2)^2}{(qv + \omega + i0)}
\]

\[
\text{Im} \ A_{RL} = \frac{\pi}{2} \frac{\omega^2 - v^2 q^2}{(4\pi v^3)} \left[ \frac{\coth(\omega/vq)}{4T} \right].
\]

Note that the anomalous (appearing for \( K \neq 1 \) only) contribution \( A_{RL} \) is ultra-violet divergent, which will be reflected in the appearance of UV cutoff \( \Lambda \sim E_F \) in real part of \( A_{RL} \). Seeking a function \( A_{RL} \), analytical in the upper semiplane of \( \omega \), one can restore \( \text{Re} A_{RL} \) using the rule

\[
i \coth \beta \omega \rightarrow \frac{2}{\pi} v \left( -i \frac{\beta \omega}{\pi} \right) + \frac{1}{i \beta \omega}.
\]

Such a recipe of analytical continuation delivers \( \text{Re} A_{RL} \) up to a constant \( \sim \ln(T/\Lambda) \) multiplied by \( (\omega^2 - v^2 q^2) \), as will be seen shortly.

In the limit of \( T = 0 \), the expression for \( A_{RL} \) is simplified

\[
\text{Im} A_{RL} = \frac{\pi}{(4\pi v^3)} (\omega^2 - v^2 q^2) \Theta(\omega^2 - v^2 q^2) \text{sign} \omega,
\]

\[
\text{Re} A_{RL} = -\frac{1}{(4\pi v^3)} (\omega^2 - v^2 q^2) \ln \left| \frac{\omega^2 - v^2}{\Lambda^2} \right|.
\]

Importantly, this part of \( \Sigma \) vanishes at the light cone for \( T = 0 \) and remains finite (and purely imaginary) for \( T \neq 0 \), in contrast to the singular contributions \( A_{RR}, A_{LL} \). Notice the appearance of UV cutoff \( \Lambda \) in Eq. (41), which replaces temperature in this limit.

Neglecting for a moment the singular (purely real) parts \( A_{RR}, A_{LL} \), we write the Dyson equation in the form

\[
\tilde{G}^{-1} = \left( \tilde{g}^{-1} - \Sigma_{o f f}, -\Sigma_{o f f} \right),
\]

with \( \tilde{g}_{RL}^{-1} \) given by with the change \( v_F \rightarrow v \). Some calculation shows then that the Green’s function defined for the initial fields \( \phi, \theta \) is given by

\[
\left( \langle \partial \phi, \partial \phi \rangle, \langle \partial \theta, \partial \theta \rangle \right) = \frac{\pi^2}{4} \frac{\langle K v, \omega/q, (v - \Sigma_{o f f}/\pi) \rangle}{\text{Det}(\omega/q, K^{-1}(v - \Sigma_{o f f}/\pi))},
\]

with

\[
\text{Det} = q^2 v^2 - \omega^2 - \frac{q^2 v^2}{\pi} \Sigma_{o f f},
\]

and \( \partial \) standing for spatial derivative in (43).
As a result, the density correlations are given by

\[ \langle \rho \rho \rangle_{\omega q} = \frac{1}{\pi^2} \frac{(\partial \phi \partial \phi)_{\omega q}}{\omega^2 - q^2 v^2 - q^2 v \Sigma_{off}/\pi} = v_F \frac{q^2 Z}{\pi q^2 v^2 - \omega^2}, \]

which is parametrically larger than \[18\] at \[|\omega - vq| \ll \omega \sim T\], and is non-zero also at \[|\omega| < |qv|\]. It should be stressed again, that Eqs. \[18, 52\] are applicable only for \[|\omega - vq| \gtrsim k_F^{-1}|q| \max(T, qv)\].

\[ Z^{-1} = 1 - \frac{q^2 \nu \Sigma_{off}}{\pi(q^2 v^2 - \omega^2)}. \]

IV. A.C. CONDUCTIVITY OF SINGLE WIRE

Let us discuss the coupling to electromagnetic field in bosonization and its modification due to curvature. We start from general expression for the kinetic term in the Hamiltonian,

\[ H_{kin} = \frac{1}{2m} \psi^\dagger (-i \nabla - eA)^2 \psi - \mu \psi^\dagger \psi, \]

with \( h = c = 1 \) and \( \mu \) chemical potential. Using the above representation \( \psi(x) \sim e^{ikFx + \phi_R} + e^{-ikFx - i\phi_L} \), and denoting

\[ \Phi_R = kFx + \phi_R, \quad \Phi_L = kFx + \phi_L \]

one can represent \( H_{kin} \) in the presence of \( eA \) as

\[ H_{kin} = \frac{1}{2\pi} \left( \frac{1}{6m} \left( \partial_x \Phi_R - eA \right)^2 - \mu \partial_x \Phi_R \right) + \frac{1}{2\pi} \left( \frac{1}{6m} \left( \partial_x \Phi_L + eA \right)^2 - \mu \partial_x \Phi_L \right). \]

Slightly digressing here, we note that in this notation \( kFx \) is clearly a zero (classical) mode of the bosonic fields \( \Phi_{R,L} \) which is obtained from \[54\] by variation over these fields. Note that the zero mode \( kFx \) corresponds to a local minimum of the action corresponding to \[54\], while generally speaking the action is unstable with respect to choice \( \partial_x \Phi_{R,L} \rightarrow -\infty \). The latter unphysical choice of "classical" vacuum can be ruled out by imposing the hard-core condition, \( \partial_x \Phi_{R,L} > 0 \) for all \( x \). It means that the local fluctuations of the density should not lead to negative values of total electronic density, \( \partial_x \phi_R > -k_F \) etc.

Note, that we can obtain \[54\] also by including vector potential in the \( U(1) \) phase according to \( \psi(x) \sim e^{ikFx + eA} + e^{-ikFx - eA} \). The cubic terms in \( eA \) generated by two terms in \[54\] cancel each other. In more general case of non-parabolic band dispersion, \( \varepsilon(k) \), the Peierls substitution, \( k \rightarrow k - eA \), leads to higher powers of \( eA \) in the gradient expansion of \( \varepsilon(k - eA) \). The recipe for tackling this case was outlined long ago by Luttinger and Kohm\[2\] and amounts to retaining only \( eA \) and \( (eA)^2 \) terms in such expansion, while discarding higher terms as unphysical.

Upon thermodynamic averaging of \( H_{kin} \), the diamagnetic term, \( \sim (eA)^2 \), gives the usual value, \( k_F/(\pi m) = K_0/\pi \). This is the zero-mode contribution, whereas terms linear in \( \partial_x \phi \) yield zero. The term linear in vector potential has a structure

\[ \frac{eA}{2m} \left( (k_F + \partial_x \phi_L)^2 - (k_F + \partial_x \phi_R)^2 \right) = \pi^{-1} eA(v_F \partial_x \theta + m^{-1} \partial_x \theta \partial_x \phi) \]
The last expression means that the electric current is given by the expression
\[ j = -e\pi^{-1}(v_F \partial_x \theta + m^{-1}\partial_x \theta \partial_x \phi). \tag{55} \]
This expression is confirmed by the immediate application of the continuity equation, \( \partial_x j = -\partial_t \rho = ie\pi^{-1}[\partial_x \phi, H] \), and using the bosonized Hamiltonian. The first term in (55) is usual for LL, the second term appears due to fluctuation of the Fermi velocity when changing the density.

The Kubo formula expresses the conductivity via the retarded Green’s function of the uniform current,
\[ \sigma(\omega) = \frac{\langle jj \rangle_{q=0,\omega=0} - \langle jj \rangle_{q=0,\omega=0}}{i\omega}. \tag{56} \]
The subtraction of the diamagnetic term \( \langle jj \rangle_{q=0,\omega=0} \) should be performed when using direct representation (55). If one calculates first the density correlator and uses the continuity equation afterwards,
\[ \langle jj \rangle_{q=0,\omega} = \lim_{q \rightarrow 0} \omega^2 q^{-2} \langle \rho \rho \rangle_{q,\omega}, \]
then the diamagnetic contribution is subtracted automatically.

Using the above result, Eq. (44), we have
\[ \sigma(\omega) = \frac{v_F}{-i\pi \omega}, \quad \text{Re } \sigma(\omega) = v_F \delta(\omega), \tag{57} \]
i.e. the conductivity of one wire is not affected by the temperature, interaction and curvature, in accordance with Giamarchi and Millis. It should be stressed that this conclusion assumes our above choice \( g_2 = g_4 = U \), i.e. physical charge-charge interaction; we do not consider models with \( g_2 \neq g_4 \) which would correspond to current-current interaction and to modifications of (57).

We also comment here on the equivalence of two ways of calculation of \( \sigma(\omega) \), via Eq. (55) and via continuity equation. Using (55) for \( \langle jj \rangle \) correlation function, one obtains a bosonic propagator \( \langle \partial_x \phi, \partial_x \phi \rangle \), and bosonic loops of the form \( m^{-2} \langle \partial_x \theta \partial_x \phi, \partial_x \theta \partial_x \phi \rangle \). From Eq. (53) the average \( \langle \partial_x \theta, \partial_x \theta \rangle \) is vanishing in the limit \( q \rightarrow 0 \), and subtracting the diamagnetic contribution \( v_F / \pi \), one arrives at (57). The bosonic loop terms give zero, as will be seen shortly.

The mixed terms in \( \langle jj \rangle \) correlator, \( m^{-1} \langle \partial_x \theta, \partial_x \theta \partial_x \phi \rangle \) are zero in all orders of perturbation theory. It is trivial to see in lowest order of \( m^{-1} \), because the mixed average contains three bosons. In the next order, one has a diagram with a bosonic loop and the Green’s function attached to it. This Green’s function bears the zero external momentum and finite \( \omega \) and hence vanishes. The same argument about zero contribution to conductivity can be applied to any diagram containing external lines carrying zero momentum, particularly in the two-wire situation. Hence, apart from the \( \delta \)-function contribution produced by conventional bosonization, we have to discuss only loop diagrams in Fig. 3 and their dressing by the interaction, \( m^{-1} \).

V. A.C. CONDUCTIVITY, TWO WIRES

For the system described by the Hamiltonian (7), we wish to compute the transresistivity, i.e. the voltage drop in one wire, caused by the current in another wire. Technically, we use Kubo formula and calculate the conductivity, not resistivity. We consider the drag effect which is caused purely by electronic interaction, and in the absence of disorder in wires. In this situation, one expects a \( \delta \)-function contribution to the conductivity, which should be separated from the proper drag contribution, seen in the limit \( \omega \rightarrow 0 \). Let us first discuss this small-\( \omega \) limit.

A. d.c. limit

A general form of transconductivity in the limit \( \omega \rightarrow 0 \) can be found, when starting with the transresistivity matrix \( \hat{R}(\omega) = [\hat{\sigma}(\omega)]^{-1} \). Exactly at \( \omega = 0 \) the voltage drop in both wires should be absent when the drift charge velocities, \( \pi j_1 / k_{F1} \), in both wires coincide. In order to see it suffices to make a Galilean transformation to the coordinate frame moving with the drift velocity\( \mathbf{v} \). We combine this observation with the third Newton law, equalling the forces acting on both wires, i.e. \( E_1 k_{F1} = -E_2 k_{F2} \), and represent the complex-valued impedance as a series in \( \omega \):

\[ \hat{R}(\omega) = \pi r_0 \left( \frac{k_{F1}^{-2}}{k_{F1}^{-2} k_{F2}^{-2}}, -rac{k_{F1}^{-1} k_{F2}^{-1}}{k_{F1}^{-1} k_{F2}^{-1}} \right), \tag{58} \]

\[ -i\omega \mathbf{A} + \omega^2 \mathbf{A}_2 + \ldots, \]

with the transresistivity given by \( R_{12} = \pi r_0 / (k_{F1} k_{F2}) \).

In the absence of interwire interaction \( U_{12} \) in (7), Eq. (57) shows that \( r_0 = 0 \), \( \mathbf{A}_2 = 0 \) and \( \mathbf{A} = \)
\[ \pi \text{diag}[v_{F1}^{-2}, v_{F2}^{-2}] \]. We make a natural conjecture, that \( r_0, A_1, A_2 \) allow Taylor expansions in \( u \). It follows then, that for \( \omega \lesssim r_0 v_F k_F^{-2} \) the first two terms in (58) are of the order of \( r_0 k_F^{-2} \), and the third term is \( \sim (r_0 k_F)^{-2} O(u) \), i.e. contains an additional small factor \( O(u r_0) \). Therefore one can invert \( \hat{R}(\omega) \) at small \( \omega \), retaining only first two terms in (58) and taking the above non-perturbed value of \( A \).

The resulting complex-valued conductivity is obtained as

\[
\sigma(\omega) = \sigma_1(\omega) + \sigma_2(\omega),
\]

\[
\sigma_1(\omega) = \frac{\pi^{-1} d^{-1}}{+0i\omega} \left( v_{F1} v_{F2}^{-2} m_1^{-2} m_2^{-1}, \ m_1^{-1} m_2^{-1}, \ v_{F1} v_{F2} m_2^{-1} \right),
\]

\[
\sigma_2(\omega) = \frac{\pi^{-1} d^{-1}}{r_0 d - i\omega} \left( -m_1^{-2}, \ -m_1^{-1} m_2^{-1}, \ m_2^{-2} \right),
\]

\[ d = m_1^{-2} v_{F1} + m_2^{-2} v_{F2}. \]

Interestingly, (59) provides the universal, i.e., independent of details of fermionic interaction, relation between the drag resistivity and the drag conductivity in small-\( \omega \) limit,

\[ R_{12} \text{ Re } [\sigma_{12}](\omega) = (c + c^{-1})^{-2}, \]

with \( c^2 = m_1 k_F/(m_2 k_F) \). The r.h.s. of (61) is always less than \( 1/4 \), which value is achieved for equal wires.

In the continuum media, the integral of \( \sigma(\omega) \) over all frequencies should not depend on the interactions in the system, which statement is known as the optical sum rule (see, e.g., Ref. 21 and references therein). From (59) we restore this statement:

\[ \text{Re } \int_{-O(r_0d)}^{O(r_0d)} \ d\omega \sigma(\omega) \simeq \pi \begin{pmatrix} v_{F1} & 0 \\ 0 & v_{F2} \end{pmatrix}, \]

which means that the spectral weight of the part \( \sigma_2(\omega) \) is large \( \sim v_F \) and is concentrated at the lowest energies. Regrettably, this is the worst accessible region for bosonization analysis, as will be seen shortly.

We can cast (61) in the form

\[
\sigma(\omega) = \begin{pmatrix} v_{F1} & 0 \\ 0 & v_{F2} \end{pmatrix} \left( \frac{\pi^{-1} d^{-1}}{-i\omega} \right) + \frac{\pi^{-1} r_0}{\omega(\omega + ir_0 d)} \left( -m_1^{-2}, \ -m_1^{-1} m_2^{-1} \right),
\]

and identify the first term here with the bare bosons contribution (57), then the second term is entirely the effect of the curvature. At larger \( \omega \), we have the expansion

\[ \text{Re } \sigma \sim \frac{r_0 \omega m_2^{-2}}{\omega^2 + r_0 d^2} \propto \frac{r_0 d^3}{\omega^2} + \frac{r_0 d^3}{\omega^2} + \ldots \]

The drag effect, \( r_0 d \neq 0 \), is absent without curvature, and bosonization treats the curvature as perturbation. It means, particularly, that calculating the d.c. drag resistivity starting from Kubo formula, (50), one should seek a lowest order contribution to \( \text{Re } \sigma(\omega) \) behaves as \( \omega^{-2} \). Such contribution should arise in the higher-than-second order of \( m^{-1} \), and the next important term, \( \omega^{-4} \), is found in even higher order of PT. Our approach in determination of \( r_0 \) as the coefficient before the \( \omega^{-2} \) term in conductivity is thus similar to memory function formalism. 21

Below we prove that apart from the singular \( \omega = 0 \) contribution, the matrix \( \sigma(\omega) \) is given by

\[
\sigma(\omega) = \sigma_d(\omega) \begin{pmatrix} m_1^{-2} - m_1^{-1} m_2^{-1} \\ -m_1^{-1} m_2^{-1} - m_2^{-2} \end{pmatrix}. \]

This degenerate matrix cannot be inverted and one needs the complete representation (59) in order to obtain \( R_{12} \). From now on we discuss the real part of the drag conductivity \( \sigma_d(\omega) \).

### B. Optical Transconductivity

For the Hamiltonian (7), the bare Green’s function defined for the vector \( \Phi = (\partial_x \varphi_1, \partial_x \varphi_2, \partial_x \varphi_1, \partial_x \varphi_2) \) is found as

\[ G = \pi \begin{pmatrix} v_{11} & \omega/q, & 0, & 0 \\ \omega/q & v_{11} & 0, & U_{12}/\pi \\ 0, & 0 & v_{22} \end{pmatrix}. \]

It shows that the excitation spectrum has two branches \( \varepsilon_{+q} = v_+ |q| \) and \( \varepsilon_{-q} = v_- |q| \) with the sound (plasmon) velocities

\[ v_{\pm}^2 = \frac{1}{2} (v_1^2 + v_2^2) \pm \sqrt{\frac{1}{4} (v_1^2 - v_2^2)^2 + u^2 v_1^2 v_2^2}, \]

we assume \( v_+ > v_- > 0 \) below. We also focus on the case of almost identical wires, \( v_1 \simeq v_2 = v \) and small \( u \), when \( v_+ \simeq v_- \simeq v \) and

\[ \Delta \equiv (v_+ - v_-)/v_+ \simeq \sqrt{(v_1 - v_2)^2/v_2^2 + u^2} \ll 1. \]

Let us discuss the first diagrams appearing in bosonization analysis of transconductivities. As shown above, the linear-in-bosons components of the currents (55), \( \partial \theta \), produce the \( \delta \)-function contribution to the conductivity. This is the first term in Eq. (63).

The proper transconductivity part, \( \sigma_d(\omega) \) in (61) is generated by the second term of the current operator in (55). Schematically, it is given by two diagrams, depicted in Fig. 3. These bosonization diagrams correspond to the usual fermionic ones, Fig. 2 after we identify Fig. 2 with Fig. 4 in the approach using the continuity relation, and notice that the external tail Green’s function become constant in the limit \( q \to 0, \omega \neq 0 \).

After a lengthy calculation, sketched in Appendix, we
obtain
\[ \sigma_d(\omega) = u^2\omega \frac{\sinh(\omega/2T) v_+^2 v_-^2 v_+^{-1} v_-^{-1} (v_+ + v_-)^{-2}}{8 \sinh(\frac{\omega}{2T(v_+ + v_-)} \sinh(\frac{\omega}{2T(v_+ + v_-)}))} \]
\[ + (v_+ \rightarrow -v_-), \quad (68) \]

\[ \sigma_d(0) = T \frac{(v_1 v_2)}{(v_1 + v_2)}^2 \frac{2 v_+^2 + 3 v_+ v_-^2}{(2v_+^2 - v_-^2)^2}. \quad (69) \]

First observation about the obtained optical conductivity is that \( \sigma_d(\omega) \) non-zero solely due to the interwire interaction \( u \). Secondly, at \( \omega \gtrsim T \) the second term in \( (68) \) is exponentially small and the first one produces linear-in-\( \omega \) optical conductivity,
\[ \sigma_d(\omega) \sim u^2 v_F^2 \omega / E_F^2, \quad \omega \gtrsim T. \quad (70) \]

Unusually, there is no upper boundary for this regime in the theory, and \( \text{Re} \sigma_d(\omega) \propto |\omega| \) until \( |\omega| \lesssim E_F \). At the same time, the integral contribution of higher frequencies into the optical sum is small, \( \int d\omega \sigma(\omega) \sim u^2 v_F \), i.e. only a fraction of the above value \( (62) \).

In the case of identical wires, \( v_1 = v_2 \equiv v \), coupled by a weak interaction, one has \( u \ll 1 \) and the new velocities \( v_{\pm} = v \sqrt{1 \pm u} \). The optical conductivity takes then the form
\[ \sigma_d(\omega) \simeq \frac{u^2 \omega \coth(\omega/4T)}{128 v^3} + \omega \frac{\sinh(\omega/2T)}{8u^3 \sinh^2(\omega/2uT)}. \quad (71) \]

One sees the appearance of a smaller energy scale \( uT \ll T \) in this case and parametric enhancement \( \sigma_d(0) \sim T/u \), we return to this point below.

Overall, the function \( (68) \) is a smooth function of \( \omega \), not exhibiting the internal crossover at \( \omega \sim r_0 d \sim m^{-4} \) as suggested by \( (69) \). The region of applicability of the expression \( (68) \) can be estimated by using the "broadened" propagators \( (24) \) instead of \( \delta \)-function-like \( (21) \), during calculation of \( \sigma_d(\omega) \). This estimate shows that the finite bosons' linewidth intervenes the formula \( (68) \) at \( \omega \lesssim \omega_0 = T^2/m v^2 \), in accordance with \( (24) \). Using this estimate as a crossover energy from \( (68) \) to the Lorentzian form \( (69) \), we can write \( \sigma_d(\omega) \simeq r_0/\omega_0 \) and thus find \( r_0 \) up to a numerical coefficient. Remarkably, this simple method gives a correct value of \( r_0 \) at lowest \( T \), but it wrongly estimates a crossover to high-temperature regime, discussed below. We see that the estimate of \( r_0 \) requires going beyond the lowest order of curvature and we suggest a way to calculate it in the next subsection.

C. Alternative representation of \( \sigma_d(\omega) \)

In this subsection we derive another representation for \( \sigma_d(\omega) \), which explicitly recovers the matrix structure \( (64) \) and shows the overall prefactor \( u^2 \) in \( \sigma_d(\omega) \). For infinitesimal \( u \) this representation yields the well-known formula for the drag coefficient.

We rewrite the Kubo formula as
\[ \sigma(\omega) = \frac{\langle \partial_j \partial_j \rangle_{q=0} \omega}{i \omega^3}, \quad (72) \]
and use the equation of motion \( \partial_t j = i[H,j] \) and Eq. \( (55) \). Some lengthy, but straightforward calculation verifies that the commutation with the terms \( H_1, H_2, H_{\text{cur}} \) produces full derivatives with respect to \( x \), which vanish for the uniform currents. The only surviving terms come from \( H_{12} \) in \( (7) \) and are
\[ \partial_t j_1(x) = U_{12} m_1^{-1} \rho_1 \partial_x \rho_2, \quad (73) \]
\[ \partial_t j_2(x) = U_{12} m_2^{-1} \rho_2 \partial_x \rho_1 - U_{12} m_2^{-1} \rho_1 \partial_x \rho_2. \quad (74) \]

modulo full derivatives. It is immediately seen that the structure \( (64) \) is reproduced, and the scalar drag conductivity is given by
\[ \sigma_d(\omega) = \frac{U_{12}^2}{i \omega^3} \langle \rho_1 \partial_x \rho_2, \rho_1 \partial_x \rho_2 \rangle_{q=0,\omega}. \quad (75) \]
Comparing to \( (63) \), we obtain at \( \omega > r_0 d \)
\[ r_0 = \frac{U_{12}^2}{\omega} \text{Im} \langle \rho_1 \partial_x \rho_2, \rho_1 \partial_x \rho_2 \rangle_{q=0,\omega}. \quad (76) \]

For the infinitesimal \( U_{12} \) the cross-terms, \( \sim \langle \rho_1, \partial_x \rho_2 \rangle \), in the average \( (75) \) can be neglected, and a comparison with Eq. \( (63) \) at \( \omega \to 0 \) gives
\[ r_0 = \pi U_{12}^4 \int \frac{q^2 dq \, d\omega \, \text{Im} \chi_1(q,\omega) \text{Im} \chi_2(q,\omega)}{2 \pi (2\pi)^2 \sinh^2(\omega/2T)}, \quad (77) \]
which corresponds to previous findings \( (62) \). It is also worth noting that our Eq. \( (70) \) for transresistivity exactly corresponds to Eq. \( (18) \) in \( (22) \). We should stress, however, that the exact formula for the case of clean Luttinger liquids is given by Eq. \( (63) \), and \( (77) \) should be regarded as the high-energy asymptote of the expression \( (63) \).

D. Non-equal wires

We consider now the case of two wires, which are coupled by infinitesimal \( U_{12} \) and are characterized by non-equal plasmon velocities. In this case we can use Eq. \( (77) \) and the results of the Section \( III \B \) In the absence of curvature and intrawire interaction, the formula \( (44) \) with \( Z = 1 \) gives \( r_0 = 0 \) in \( (77) \). The first non-vanishing contribution is given by Eq. \( (52) \), and we obtain
\[ r_0 = \frac{U_{12}^2 v_{F1} v_{F2}}{16\pi T (v_{F1}^2 - v_{F2}^2)} \int \frac{q^5 dq}{\sinh(v_1 q/2T) \sinh(v_2 q/2T)} \times \left[ \frac{\gamma_2}{v_1 k_{F2} \sinh(v_1 q/2T)} + (1 \leftrightarrow 2) \right]. \quad (78) \]

In the important case of almost identical wires, when \( \Delta = (v_1 - v_2)/v \ll 1 \), we can omit the wire index in the
prefactors, expand \( \sinh((v_1 - v_2)q/4T) \approx \Delta v q/4T \) and get

\[
    r_0 \approx \frac{8\pi^2 U_{12}^2 \gamma T^5}{m^2 v^4 \Delta^2 15v^5},
    \tag{79}
\]

with \( \gamma \) given by \( \langle 1 \rangle \) and we used \( \int_{-\infty}^{\infty} dx x^4 / \sin^2 x = \pi^4 / 15 \). Comparing \( \langle 79 \rangle \) to the result by Pustilnik et al., we see that we obtained the same \( T^5 \) dependence but parametrically larger prefactor, \((v_1 - v_2)^{-2}\), rather than \((v_1 - v_2)^{-1}\).

Note that one would reproduce the prefactor obtained in \( \sim \), if one neglected the \( T \)-dependence of density propagator and substituted in \( \langle 77 \rangle \) the expression \( \langle 48 \rangle \) for \( \Im \chi_1(q, \omega) \). In this case a naive substitution \( \Delta \to \langle 80 \rangle \) for \( \Im \chi_1(q, \omega) \) at \( T = 0 \) instead of \( \langle 72 \rangle \). We see that finite \( T \) parametrically enhances the “tail” of density correlator, and it has consequences for the drag effect. The importance of calculation of intermediate polarization operators \( \Im \chi_1(q, \omega) \) in \( \langle 77 \rangle \) at finite \( T \) was also emphasized in \( \langle 10 \rangle \).

Checking the applicability of \( \langle 52 \rangle \), one can conclude that the derivation of \( \langle 80 \rangle \) assumed a condition \( v_1 - v_2 \gtrsim T/k_F \). In other words, the expression \( \langle 79 \rangle \) should be regarded as low-temperature regime of drag resistivity, taking place at

\[
    T \lesssim T_1 \equiv k_F (v_1 - v_2) = E_F \Delta.
\]

Above this temperature scale, the “core” spectral weights of \( \chi^{1/2}_0(q\omega) \) and \( \chi^{3/2}_0(q\omega) \) overlap and one can estimate the drag coefficient, by taking the expression for the “core” of the density propagator for free fermions, Eq. \( \langle 24 \rangle \), to obtain

\[
    r_0 \sim \frac{U_{12}^2 m}{v^3} T^2 F \left( \frac{T_1}{2T} \right),
    \tag{80}
\]

with

\[
    F(z) = \int_0^\infty 2x \sinh(x z) \sinh(x (z - 1)) dx / \sinh(z) = 1, \quad z \to 0,
    \approx 4z^2 e^{-2z}/3, \quad z \gg 1.
\]

The estimate \( \langle 80 \rangle \) shows an activational behavior of the drag for free fermions, similarly to Ref. \( \langle 1 \rangle \).

The obtained estimates for the drag effect at lower and higher temperatures, however, raise a few questions. First, the Eqs. \( \langle 80 \rangle \) and \( \langle 79 \rangle \) apparently describe different contributions to the drag, they do not match at the suggested crossover temperature \( T \sim T_1 \), and Eq. \( \langle 79 \rangle \) is explicitly non-zero only due to intrawire interactions and this feature is absent in \( \langle 80 \rangle \).

Second, in case of initially equal wires, the plasmon velocities are split due to interwire interaction, Eq. \( \langle 77 \rangle \). It means that \( U_{12} \) cannot be regarded as infinitesimal in \( \langle 76 \rangle \) and hence Eq. \( \langle 78 \rangle \) is, generally speaking, invalid. In this case a naive substitution \( \Delta \sim U_{12}/v \) from \( \langle 81 \rangle \) results in \( r_0 \), Eq. \( \langle 79 \rangle \) independent of interwire coupling.

To clarify these issues, we separately consider the case of nearly equal wires below.

VI. DRAG FOR NEARLY EQUAL WIRES IN BOSONIZATION

A. Identical wires, Hamiltonian

When the wires are identical, it is convenient to introduce the symmetrized combinations

\[
    \sqrt{2} \phi_\pm = \phi_1 \pm \phi_2, \quad \sqrt{2} \theta_\pm = \theta_1 \pm \theta_2.
    \tag{81}
\]

A technical convenience of \( \sqrt{2} \) prefactor stems from the same commutation relations for the symmetrized operators, however the zero modes are different, \( \partial_x \phi_+^{(0)} = \sqrt{2} k_F, \partial_x \phi_-^{(0)} = 0 \). We will call \( \phi_+ \) and \( \phi_- \) pseudocharge and pseudospin fields, by obvious analogy with bosonization for one wire in the spinful case.

In terms of densities and momentum operators,

\[
    \rho_\pm = \pi^{-1} \partial_x \phi_\pm, \quad \Pi_\pm = \pi^{-1} \partial_x \theta_\pm,
    \tag{82}
\]

the Hamiltonian \( \langle 4 \rangle \) is rewritten as follows

\[
    \mathcal{H} = \int dx (H_+ + H_+ + H_{\text{cur}}),
    \tag{83}
\]

\[
    H_\pm = \frac{\pi}{2} (v_+ \Pi_\pm^2 + v_N (1 \pm u) \rho_\pm^2),
    \tag{84}
\]

\[
    H_{\text{cur}} = \frac{\pi^2}{6m\sqrt{2}} (\rho_+^2 + 3\Pi_+^2 + 3\Pi_-^2 + 3\rho_-^2) + 6\Pi_+ \rho_- \Pi_-.
    \tag{85}
\]

It is seen that the change in the zero mode of \( \phi_+ \) is compensated by the enhancement of its “mass”. The plasmon velocities and Luttinger parameters are given by

\[
    v_\pm^2 = (1 \pm u) v_N v_J, \quad K_-^2 = (1 \pm u) v_N / v_J.
    \tag{86}
\]

According to our definitions

\[
    K_-^{-2} = 1 + (\pi v_F)^{-1} (U_1 \pm U_{12}),
\]

so that \( K_- = 1 \) for equal intrawire and interwire interactions.

We diagonalize the Hamiltonian \( \langle 83 \rangle \) by transformation

\[
    \rho_\pm = K_\pm^{-1/2} \rho_\pm, \quad \Pi_\pm = K_\pm^{-1/2} \Pi_\pm.
    \tag{87}
\]

In terms of new left and right movers, \( 2 \hat{R}_\pm = \rho_\pm + \Pi_\pm, 2 \hat{L}_\pm = \rho_\pm - \Pi_\pm \), there are no mixed terms \( \hat{R}_\pm \hat{L}_\pm \) in \( \mathcal{H} \), and propagators \( \langle \hat{R}_\pm \hat{L}_\pm \rangle \) vanish.

The curvature term becomes

\[
    H_{\text{cur}} = \frac{\alpha_1}{3} \left( \hat{R}_+^3 + \hat{L}_+^3 + \alpha_2 \hat{R}_+^2 \hat{L}_+ + \hat{R}_+ \hat{L}_+^2 \right) + \frac{\alpha_3}{2} \left( \hat{R}_+ \hat{R}_-^2 + \hat{L}_+ \hat{L}_-^2 \right) + \frac{\alpha_4}{2} \left( \hat{R}_- \hat{R}_-^2 + \hat{L}_- \hat{L}_-^2 \right) + \alpha_5 (\hat{R}_+ + \hat{L}_+) \hat{R}_- \hat{L}_-.
    \tag{88}
\]
modulo full derivatives. We seek the contribution to $r$ in the limit of vanishing interwire interaction we have to real frequencies $\omega$ any external loops, carries the zero external momentum and is zero at case the intermediate propagator, connecting two boson diagrams of the RPA type, Fig. 6, vanish, because in our considering two skeleton graphs, depicted in Fig. 5. The calculation at finite $T$ is further complicated by the necessity to perform the calculation at finite $T$, which allows the essential simplification of this task. Fortunately, there is a method for $T > 0$ which can be obtained by contour deformation in the complex plane. However, this case is largely unknown to the wider audience, we sketch the details of calculation may skip to Eq. (98) below.

With

$$\alpha_1 = \frac{\pi^2}{m} \frac{3 + K^2_+}{2\sqrt{2K_+}}, \quad \alpha_2 = \frac{\pi^2}{m} \frac{(K^2_+ - 1)}{\sqrt{K_+}},$$

$$\alpha_3(4) = \frac{\pi^2}{m} \frac{\sqrt{K_+}}{\sqrt{2K_+}} \left( \frac{1}{K_-} + K_- \pm \frac{2}{K_+} \right),$$

$$\alpha_5 = \frac{\pi^2}{m} \frac{\sqrt{K_+}}{\sqrt{2K_+}} \left( K_- - \frac{1}{K_-} \right),$$

(88)

in the limit of vanishing interwire interaction we have $K_+ = K_-$ and $2\alpha_1 = \alpha_3$ and $\alpha_2 = \alpha_4 = \alpha_5$; this case returns us to Eq. (31). In the absence of interaction, $K_+ = 1$ and $\alpha_2 = \alpha_4 = \alpha_5 = 0$. Note that the vertex $\alpha_1$, describing the decay within the same mode, is always non-zero.

The drag current operator in (75) is

$$\rho_1 \partial_x \rho_2 = \rho - \partial_x \rho^+ = (K_+ K_-)^{1/2} \rho - \partial_x \rho^+,$$

(89)

modulo full derivatives. We seek the contribution to $r_0$ in the next order in $m^{-1}$ which can be obtained by considering two skeleton graphs, depicted in Fig. 6. The diagrams of the RPA type, Fig. 6, vanish, because in our case the intermediate propagator, connecting two boson loops, carries the zero external momentum and is zero at any external $\omega \neq 0$, see (21).

Given ten possible decay processes (87), one has to analyze $\sim 10^2$ diagrams, corresponding to Fig. 6. It is further complicated by the necessity to perform the calculation at finite $T$ and make an analytical continuation to real frequencies $\omega \to 0$. Fortunately, there is a method which allows the essential simplification of this task.

B. Calculation by "Unitarity condition" method

At finite $T$, the analytical continuation of the diagrams and the essential reduction of their number under consideration are simultaneously achieved in the "Unitarity condition" method (22). Since this method for $T > 0$ is largely unknown to the wider audience, we sketch the main points of it below. Readers not interested in the details of calculation may skip to Eq. (98) below.

We define a cross section of the diagram as a line, bisecting this diagrams in two parts, each with the external vertex $\rho - \partial_x \rho^+$ carrying the external frequency $\omega$ (see Fig. 7). We should take the product of spectral weights of the Green’s functions $\text{Im} g(x_i, q_i)$ at the cross section, to divide each individual spectral weight by $2 \sinh(x_i/2T)$ (in our case of bosons), and to multiply the whole product by $2 \sinh(\omega/2T)$. The sum over all possible cross sections, multiplied by the square of the generalized vertex part, gives a value of the imaginary part of the diagram. See Figures 5 and 7 schematically showing this method.

Using this method, all contributions can be classified by the number and types of Green’s functions appearing in the cross section. Formally, we have the contributions with two propagators in the cross section (2-CS) and three propagators in the cross section (3-CS). However the cross sections with two propagators are essentially reduced to the simple loop diagram, with a static vertex correction (except for diagrams with $\alpha_1$ vertex, see below). Given the structure of the external vertex, $\rho - \partial_x \rho^+$, and kinematic properties for linear spectrum, one can verify that the contribution of 2-CS to $\rho$ does not contain $\omega^{-2}$. We may hence exclude this type of cross-section from our analysis.

Each partial 3-CS contribution to the drag resistivity can be represented as (21), (cf. Eq. (77) ) :

$$\delta r_0 = \pi U_{ij}^2 K_+ K_- \int \ldots \ldots [\text{Eq. (77)}]$$

$$\delta r_0 = \frac{\pi^2}{m} \delta \left( \sum \ldots \right) \left( \omega - \sum x_i \right) \prod \frac{|d q_i d x_i|}{(2\pi)^2},$$

(91)

$$S_{jkl} = \frac{g_{ij}''(x_1, q_1) g_{kl}''(x_2, q_2) g_{lj}''(x_3, q_3)}{T \sinh \frac{x_1}{2T} \sinh \frac{x_2}{2T} \sinh \frac{x_3}{2T}},$$

and the spectral weight $g''_{ij}(x, q) = \mp (q/2)\delta(qv_j \pm \omega)$ is imaginary part of the retarded Green’s function for chiral density $j = R_{\pm}, L_{\pm}$, Eq. (21). A symmetrization prefactor $1/2$ should be added to $S_{jkl}$ when values of two indices are coinciding. The total drag resistivity is given by summation over all possible $j, k, l = R_{\pm}, L_{\pm}$ so that

$$r_0 = \pi U_{ij}^2 K_+ K_- \int \ldots \ldots [\text{Eq. (77)}]$$

$$\times \frac{g_{ij}''(x_1, q_1) g_{kl}''(x_2, q_2) g_{lj}''(\omega - x_1 - x_2 - q_1 - q_2)}{2T \sinh \frac{x_1}{2T} \sinh \frac{x_2}{2T} \sinh \frac{x_1 + x_2 - \omega}{2T}},$$

(92)
and we obtain that the corresponding static vertex is
sider only the above three CSs and multiply the result
lead to the absence of such contribution in (92) at
\[ \omega \neq 0. \]
ous symmetry, the latter change together with
\[
\frac{g}{2 \pi \alpha} R^\prime - \frac{g}{2 \pi \alpha} R^+ \quad (95)
\]
where the kinematic restrictions were used again.

Finally, we come to the most problematic term of the cross sections \( g''_R, g''_R, g''_L, g''_L \) and \( g''_R, g''_R, g''_L, g''_L \). The corresponding vertices \( \Gamma \)'s in (90), Fig. 9 explicitly read as
\[
\Gamma_R-R-L^- = \frac{q_1}{2 \pi} \frac{2 \alpha_5 v_+}{v_+^2 - v_-^2},
\]
where \( \alpha_5 \) is the lowest order con-
tribution to the non-vanishing vertex
\[
\Gamma_R-R+R^- = \alpha_1 q_R g_R^+(x_1 + x_2, q_1 + q_2) + \alpha_2 q_R g_R^+(x_1 + x_2, q_1 + q_2) - \alpha_3 q_R g_R^-(x_2 + x_3, q_2 + q_3) - \alpha_5 q_R g_L^-(x_2 + x_3, q_2 + q_3),
\]
\[
\Gamma_R-R+L^- = \alpha_1 q_R g_R^+(x_1 + x_2, q_1 + q_2) + \alpha_2 q_R g_R^+(x_1 + x_2, q_1 + q_2) - \alpha_3 q_R g_R^-(x_2 + x_3, q_2 + q_3) - \alpha_4 q_R g_L^-(x_2 + x_3, q_2 + q_3).
\]

It is seen here, that the term \( \sim \alpha_1 \) is divergent on the assumption of ideal bosons, which happens because of simultaneous conservation of energy and momentum for the linearized spectrum. This divergence in \( g_R^+(x_1 + x_2, q_1 + q_2) \) would not happen if we would assume a damping of boson excitations; moreover, the kinematic restrictions in the cross section (91) lead to prefactor \( q_3 = 0 \) at \( \omega = 0 \), so that the contribution from \( \alpha_1 \) term is zero for any model damping of bosons. In our problem however, this "damping" can only be obtained as the result of resummation of higher order corrections in \( m^{-1} \). The divergent diagrams are depicted in Fig. 10.

The first diagram here, \( \sim \alpha_1^2 \), is the lowest order contribution to the self-energy part, Eq. (28). Evidently it corresponds to the triple pole \( (x - v_+ q)^{-3} \) which resulted in the above divergence. At the same time, as discussed above, this is the first singular diagram in a series, which should ultimately produce the broadened propagator, Eq. (24). Importantly, this type of singular diagrams appears without the properly fermionic interaction, as seen in (88). Therefore, in the application of the formula (92), one should omit the most singular contribution, \( \sim \alpha_1^2 \), in \( \Gamma_R-R^- \) and use the the broadened propagator, Eq. (24) instead.

The subleading singular term, \( \sim \alpha_1 \), is depicted by the right diagram in Fig. 10. We argue here that this diagram should vanish in our consideration, and provide the following heuristic argument. The linear

FIG. 8: Leading contribution to low-T drag coefficient is given by diagrams with three propagators in a cross section.

\[
\begin{align*}
\text{FIG. 9: First-order in curvature contribution to the vertex part in the above diagram.}
\end{align*}
\]

We can distinguish between the cross sections with i) three different chiral densities, ii) two different chiral densities and iii) one sort of chiral density.

I. One sort of chiral density in the cross section can appear only on 3-CS of the type \( g''_R, g''_R, g''_L \) and \( g''_L, g''_L, g''_L \). The kinematic restrictions (91) obviously lead to the absence of such contribution in (92) at \( \omega \neq 0 \).

Having this contribution excluded, we observe that the \( \omega^{-2} \) contribution to \( \sigma \) is delivered by setting \( \omega = 0 \) in the integrand of (92); upon doing this the calculation is simplified.

II. It can be shown that 3-CS with three different propagators may contain only two combinations
\[
\Gamma_R+L+L^- = \frac{q}{2 \pi} \left( 4 \alpha_2 v_+^2 + \alpha_3 (v_+ - v_-)^2 - \alpha_4 (v_+ + v_-)^2 \right),
\]
here \( q \) refers to \( L^- \) boson ; one has \( \Gamma_R+L+R^- = -\Gamma_R+L+L^- \). The contribution to the drag coefficient is proportional to
\[
\int \frac{dq q^3 |\Gamma_R+L+L^-|^2 (v_+^2 - v_-^2) \nu_+^{-3}}{T \sinh \left( \frac{v_+ - v_-}{4T} \right) \sinh \left( \frac{v_+ + v_-}{4T} \right) \sinh \frac{v_+ g}{2T}},
\]
this expression corresponds to (78). However, substituting (88) into (93), one obtains \( \Gamma_R+L+L^- = 0 \) and the absence of this partial contribution to \( \sigma_0 \) (cf. next subsection).

III. The remaining combinations, i.e. 3-CS with two different propagators, are : \( g''_R, g''_R, g''_L \), \( g''_R, g''_R, g''_R \), \( g''_R, g''_R, g''_L \) and the combinations which are obtained from the above ones by the change \( R \leftrightarrow L \). Due to obvious symmetry, the latter change together with \( q_i \rightarrow -q_i \) leaves the expressions for \( \delta \sigma_0 \) intact, so it suffices to consider only the above three CSs and multiply the result by two.
dispersion law for bosons led to the singularity in $\alpha_1$ term \[ \text{(65)}, \] but this singularity is removed on the assumption of finite width of the bosonic pole. In this case the Green’s function $g_{R^+}(x_1 + x_2, q_1 + q_2)$ in \[ \text{(66)}, \] (77) becomes purely imaginary quantity. Considering the purely real $|\Gamma_{R^+R^+R^-}| = |\Gamma_{R^-R^-R^+}|\Gamma_{R^+R^-+R^-}$ one sees that the linear-in-$\alpha_1$ terms disappear. This argument is most obvious for the simple Lorentzian form of $g_{R^+}(x, q)$ and should be generalized for more complicated cases, particularly, for $g_{R^+}(x, q)$ given by \[ \text{(24)}. \] The analytical structure of more complicated diagrams at finite T becomes rather involved, see Ref. \[ \text{(24)}, \] and a general proof of the cancellation of these vertex corrections should be considered elsewhere. Here we make a conjecture that the singular terms represented by the second diagram in Fig. \[ \text{(10)}. \] vanish exactly for the "dressed" propagators, given by \[ \text{(24)}. \]

This conjecture basically amounts to the following convention:

i) one should discard the curvature term $\alpha_1$ from the Hamiltonian,

ii) one should use the dressed propagators \[ \text{(24)}, \] for the modes $R^+, L^+$, with the "effective mass" $1/m^* = \alpha_1/(2\pi^2)$, and

iii) the other curvature terms $\alpha_2, \ldots, \alpha_5$, not leading to singularities, can be treated in the regular PT.

Proceeding this way, we conclude that at higher temperatures, $T \gg T_1$, we can use Eq. \[ \text{(78)}, \] with the change $\Im \chi_1 \Im \chi_2 \to \Im \chi_1 \Im \chi_2$ and consider the "tail" of one propagator in \[ \text{(78)}, \] and the core of another one, approximated by $\delta$-function. At higher temperatures, $T \gg T_1$, propagators in \[ \text{(78)} \] significantly overlap; this is non-perturbative regime for bosonization, which cannot provide a prefactor in the estimate $r_0 \sim T^2$. However, one may argue that all interaction-induced details\[ \text{2.2}\] in the shape of the core of propagators are smeared by higher temperatures and one can safely use the free-fermion estimate \[ \text{(24)} \] here.

At lower temperatures, $T \lesssim T_1$, we obtain the finite contribution $r_0 \sim T^5$, due to vertices $\alpha_2, \ldots, \alpha_5$, absent in the non-interacting situation. A rather long and straightforward calculation gives a prefactor in this $T^5$ temperature dependence:

\begin{equation}
\begin{align*}
r_0 &= U_{12}^2 \gamma T^5, \\
\gamma &= \gamma_1 + \gamma_2 + \gamma_3, \\
\gamma_1 &= \frac{1}{4\pi^2} \frac{\alpha_2 (v_+ + v_-) - \alpha_3 (v_+ - v_-)^2}{30v_+ v_- (v_+ - v_-)^2 v_0^2}, \\
\gamma_2 &= \frac{1}{4\pi^2} \frac{\alpha_3 (v_+ + v_-) - \alpha_4 (v_+ - v_-)^2}{30v_+ v_- (v_+ + v_-)^2 v_0^2}, \\
\gamma_3 &= \frac{1}{4\pi^2} \frac{\gamma_T^2 v_+^2}{15v_+^2 (v_+^2 - v_-^2)^2},
\end{align*}
\end{equation}

with $\alpha_i$ given by \[ \text{(83)}. \] It is seen that at $v_+ - v_- \rightarrow 0$ the leading term in $\gamma$ is given by $\alpha_3^2$ term and is $\propto (v_+ - v_-)^{-3}$, which is different from the above regime $\sim (v_+ - v_-)^{-2}$ non-equal wires, Eq. \[ \text{(79)}. \] To resolve the seeming discrepancy, we perform the analysis of the nearly-equal wires below.

### C. Nearly-Identical wires

We consider now the case when the plasmon velocities $v_{1,2}$ in individual wires are nearly coinciding and their difference is of order of $U_{12}$. The main idea of this subsection is that the coefficients in the canonical transformation diagonalizing the quadratic Luttinger Hamiltonian are sensitive to the ratio $U_{12}/(v_1 - v_2)$. These coefficients, in turn, define the values $\alpha_i$ for the vertices \[ \text{(83)}. \] We can calculate these values and use the previous expressions \[ \text{(14)}, \text{ (58)} \] to obtain the drag in this more general case.

Using dimensionless difference of plasmon velocities \[ \text{(67)}, \] it is convenient to introduce the parameter $\lambda$ as follows:

\begin{equation}
\begin{align*}
v &= \Delta \sin \lambda, \\
v_1 - v_2 &= v \Delta \cos \lambda,
\end{align*}
\end{equation}

where the first equality is the properly definition of $\lambda$.

The general Bogoliubov transformation, diagonalizing the quadratic part of the Hamiltonian \[ \text{(7)}, \] explicitly reads as

\begin{equation}
\begin{pmatrix}
\theta_1 \\
\theta_2 \\
\phi_1 \\
\phi_2
\end{pmatrix}
= S_1 S_2 S_3 
\begin{pmatrix}
\theta_+ \\
\theta_- \\
\phi_+ \\
\phi_-
\end{pmatrix},
\end{equation}

with

\begin{align*}
S_1 &= \text{diag} \begin{pmatrix} v_{12}^{1/2}, v_{12}^{1/2}, v_{12}^{1/2}, v_{12}^{1/2} \end{pmatrix}, \\
S_2 &= \begin{pmatrix}
\cos(\lambda/2), & -\sin(\lambda/2), & 0, & 0 \\
\sin(\lambda/2), & \cos(\lambda/2), & 0, & 0 \\
0, & 0, & \cos(\lambda/2), & -\sin(\lambda/2) \\
0, & 0, & \sin(\lambda/2), & \cos(\lambda/2)
\end{pmatrix},
\end{align*}

\begin{align*}
S_3 &= \text{diag} \begin{pmatrix} v_+^{1/2}, v_-^{1/2}, v_+^{-1/2}, v_-^{-1/2} \end{pmatrix}, \\
S_4 &= \begin{pmatrix}
\cos(\lambda/2), & -\sin(\lambda/2), & 0, & 0 \\
\sin(\lambda/2), & \cos(\lambda/2), & 0, & 0 \\
0, & 0, & \cos(\lambda/2), & -\sin(\lambda/2) \\
0, & 0, & \sin(\lambda/2), & \cos(\lambda/2)
\end{pmatrix},
\end{align*}

where $v_+$ and $v_-$ are given by \[ \text{(67)} \] and correspond to the new modes $\phi_+$ and $\phi_-$, respectively. Using this transformation, we can determine the values of the coefficients $\alpha_i$ in \[ \text{(83)}. \]
Note that the set of the vertices (87) was obtained for the case \( \lambda = \pi/2 \). For other values of \( \lambda \), we have also another set of the vertices which is obtained from (87) by a change \( R_+ \leftrightarrow R_- \) and \( L_+ \leftrightarrow L_- \), explicitly

\[
H'_{\text{cur}} = \frac{\alpha_1'}{3} \left( \tilde{R}_+^2 + \tilde{L}_+^2 \right) + \frac{\alpha_2'}{2} \left( \tilde{R}_+ \tilde{L}_- + \tilde{R}_- \tilde{L}_+ \right)
+ \frac{\alpha_3'}{4} (\tilde{R}_+^2 + \tilde{L}_+^2) + \frac{\alpha_4'}{2} (\tilde{R}_-^2 + \tilde{L}_-^2)
+ \alpha_5' (\tilde{R}_- \tilde{L}_+ + \tilde{L}_- \tilde{R}_+).
\] (101)

For our purposes here, it is sufficient to let \( v_{f1} = v_{f2} \) and \( s_1 \approx s_2 \) in the calculation of \( \alpha_j, \alpha_j' \). Thus we neglect the difference in plasmon velocities in (100). At the same time, we allow for the intrawire interaction, \( K = v_{f1}/s_1 \neq 1 \).

We obtain the expressions for vertices in (87) as

\[
\alpha_1 \approx \frac{\pi^2}{4} + \frac{K^2}{2\sqrt{K}} \left( \cos \frac{3\lambda}{2} + \sin \frac{3\lambda}{2} \right)
\alpha_2 \approx \frac{\pi^2}{4} \frac{K^2 - 1}{\sqrt{K}} \left( \cos \frac{3\lambda}{2} + \sin \frac{3\lambda}{2} \right)
\alpha_3 \approx \frac{\pi^2}{4} \frac{K^2 + 3}{2\sqrt{K}} \cos \frac{\lambda}{2} + \sin \frac{\lambda}{2} \sin \lambda
\alpha_4 \approx \frac{\pi^2}{4} \frac{K^2 - 1}{2\sqrt{K}} \cos \frac{\lambda}{2} + \sin \frac{\lambda}{2} \sin \lambda
\alpha_5 \approx \alpha_4'.
\] (102)

and the set \( \alpha_j' \) is obtained from (102) by changing \( \alpha_j \rightarrow \alpha_j' \) and \( \lambda \rightarrow -\lambda \).

One can verify that despite more complicated structure (100), the drag current vertex is still given by the last expression in (87). Further, the existence of the second set of vertices, \( \alpha_j' \), leads to the same set of diagrams as before, with appropriate change \( R_+ \leftrightarrow R_- \) etc. Importantly, there are no diagrams of the type Fig. 5 containing both \( \alpha_j' \) and \( \alpha_j \). It means that one can use the previous expressions (87), corroborated by the previously vanishing contribution (94) and to add the contribution from the \( \alpha_j' \) set, obtained by \( \lambda \rightarrow -\lambda \).

Combining all terms, we find

\[
r_0 \approx \frac{\pi^3 K U_{12}^2 T^5}{120 m^2 \nu^3}
\left[ \frac{(1 - K^2)^2 \cos^2 \lambda}{\Delta^2} + \frac{(3 + K^2)^2 \sin^2 \lambda}{4 \Delta^3} \right]
\approx \frac{U_{12}^2 T^5}{E_F^3 \nu_F^2}
\left[ \cos^2 \frac{\lambda}{2} O(1) + \frac{\sin^2 \lambda}{\Delta^3} O(1) \right].
\] (103)

This is the central result of this section. Let us discuss it in more detail.

First of all, we see that the term \( \sim \Delta^{-2} \) in (103) is proportional to intrawire interaction strength, \( (1 - K)^2 \), whereas the term \( \sim \Delta^{-3} \) is present even for initially non-interacting wires. This is not surprising, because in the latter case the factor \( \sin^2 \lambda \sim U_{12}^2/(v_1 - v_2)^2 \) means that \( U_{12} \) serves as the Luttinger-type interaction \( (K - 1) \), this term should be obtained for non-equal wires in the order \( U_{12}^3 \) of conventional fermionic formalism.

For identical wires, we have an interesting crossover from \( U_{12}^2 T^2 \) regime at high temperatures to \( T^5/U_{12} \) behavior at lower temperatures. Recalling our definitions of the drag resistivity \( R_{12} = \pi v_{f0}/(k_F^2 k_F) \) and of the small scale \( T_1 = E_F \Delta \sim U_{12} k_F \), we obtain

\[
R_{12} \sim \frac{T^5}{v_F E_F^2 T_1}, \quad T < T_1 \sim U_{12} k_F, \quad (104)
\sim \frac{T_1^2 T^2}{v_F E_F^2}, \quad T > T_1,
\]

which shows a smooth crossover at \( T \approx T_1 \). The parametrically large prefactor, \( 1/U_{12} \), is similar to above Eq. (71). This prefactor i) shows non-perturbative character of the obtained result and ii) does not mean actual enhancement of the drag effect, due to its overall small amplitude \( \sim T^5 \).

For the wires with different plasmon velocities we have \( T_1 \approx (v_1 - v_2) k_F \), and the above Eq. (105) shows the following behavior

\[
R_{12} \sim \frac{T^5}{v_F E_F^2 T_1}, \quad T < T_1, \quad (105)
\sim \frac{U_{12}^2 T^2}{v_F E_F^2}, \quad T > T_1,
\]

and two regimes do not match at \( T = T_1 \). As we saw above, \( T^2 \) regime changes to exponential decrease at \( T < T_1 \), which means that there is a logarithmically narrow crossover region of temperatures, where \( R_{12} \) interpolates between \( T^5 \) and \( T^2 \) in (105).

VII. SUMMARY AND CONCLUSIONS

In this paper we discuss the effects of curvature for the Luttinger liquid. The curvature is irrelevant perturbation, in the sense that it does not change the low-energy of the system and usually can be simply discarded. Bosonization technique regards the curvature as interaction and, at the first glance, a regular perturbation theory is possible. However, a certain care must be exercised with such PT, in spite of evident correspondence between fermionic and bosonization results. We show, that the PT in curvature in bosonization produces both singular and regular contributions to different quantities. The singular contributions arise already for the free fermions and correspond to the non-Lorentzian character of fermionic density propagator. We show that these singular contributions stem from one vertex of boson interaction, and we propose to exclude this vertex from PT analysis, while simultaneously using the free-fermion expression as a "dressed" form of bosonic propagators. For the Coulomb drag problem, the dressed form of propagators becomes important at high temperatures.

All other bosonic vertices appear due to fermionic interaction and yield regular contributions in bosonization
They are responsible for the low-$T$ regime in the Coulomb drag problem.

Comparing bosonization to fermionic approach at the example of Coulomb drag problem, we observe a complementary character of these methods. High-temperature $T^2$ regime is obtained most easily with fermions, but this is essentially non-perturbative regime for bosonization. Conversely, the low-temperature $T^2$ regime for different wires is obtained relatively easily with bosons, whereas the calculation by fermionic approach is cumbersome. We demonstrated that the bosonization is particularly useful in case of Coulomb drag between nearly-equal wires, when it simultaneously resums several RPA channels for fermionic diagrams.

We did not include spin into our consideration. It also requires a separate analysis, because the cubic terms in the fermionic densities violate the spin-charge separation realized at the level of quadratic action. It is evident that spinful electrons in one wire are equivalent to spinless fermions in two wires. In our formalism, the Hamiltonian is given by (83) with identification $\phi_+$ ($\phi_-$) for charge (spin) density. The above case of nearly identical wires corresponds to Zeeman-splitting of the Fermi velocities in the magnetic field by value $\sim B/k_F$.

Finally, we note that in one spatial dimension, the alternative source for the "ideal" drag was proposed by Nazarov and Averin. They considered the "backscattering" between the wires, i.e. $2k_F$ Fourier component of the interwire interaction $U_{12}(2k_F) = U_{bs}$, not considered here. The effect of renormalization leads to the increase $U_{bs} = U_{bs}(E_F/T)^{2-2K}$ with lowering $T$, and the drag resistivity $R_{12} \sim U_{bs}^2$ diverges in the limit $T \to 0$. However, as was noted in [12], for large interwire distances $D$, the backscattering amplitude is small $U_{bs}^{1/2} \sim e^{-2k_F D}$ and the effect of renormalization can compensate this smallness only at exponentially small temperatures. The backscattering drag mechanism is also sensitive to the difference of fermionic densities in two wires. Particularly, a straightforward calculation by Fuchs et al. showed the exponential suppression of $R_{12}$ due to backscattering at $T < v_F(k_{F1} - k_{F2}) = T_1^r$. In view of an obvious similarity between $T_1^r$ and our $T_1$, it is tempting to regard the regime $R_{12} \sim T^5$ as the only one, surviving at low temperatures. The inclusion of the backscattering drag mechanism into our discussion is clearly beyond the scope of this study.

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APPENDIX: OPTICAL CONDUCTIVITY

We calculate optical trans-conductivity according to Kubo formula, \( \text{(80)} \) and have $\text{Re } \overline{\sigma}_{jl} = \omega^{-1}\text{Im } Q_{jl} (\omega)$ where \( Q_{jl} (\omega) \) is a retarded current response function. Combining different contributions to $\overline{Q} (\omega)$, depicted in Fig. 3 and making analytical continuation to real frequencies at finite $T$, we obtain

$$ Q_{jl} = \int dq dx_1 dx_2 F_{jl} \frac{N(x_1) - N(x_2)}{\omega - x_1 + x_2 + i0} W(x_1) W(x_2), $$

$$ F_{12} = F_{21} = \frac{1}{2} e^2 u^2 v_1^2 v_2^2 (x_1 + x_2)^2, \quad (A.1) $$

$$ F_{11} = \frac{1}{2} e^2 u^2 v_1^2 (x_1 x_2 + v_1^2 q^2) + \frac{1}{2} e^2 u^2 v_1^2 v_2^2 (x_1^2 + x_2^2 - 2v_1^2 q^2), \quad (A.2) $$

$$ W(x) = \frac{1}{2x} \left( \frac{e^2}{\epsilon_1} - i\epsilon_1 \right) \left( \delta(x + \epsilon_1) + \delta(x - \epsilon_1) \right), \quad (A.3) $$

and $F_{22}$ is obtained from $F_{11}$ by interchanging $v_1$ and $v_2$; here $N(x)$ is Planck function.

We calculate $\text{Im } Q_{12}$ at $\omega \neq 0$. For non-zero frequencies the coinciding arguments $x_1 = x_2 = \pm \epsilon_{1,2}$ do not contribute and the rest of the integration is simple. After collecting all the terms coming from $A_3$, we obtain the real part of the conductivity $\text{Re } \overline{\sigma} (\omega)$ in the form $[61]$, with $\sigma_d$ given by $[68].$

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* On leave from Petersburg Nuclear Physics Institute, Gatchina 188300, Russia.
1. M. Pustilnik, E.G. Mishchenko, L.I. Glazman, and A.V. Andreev, Phys. Rev. Lett. 91, 126805 (2003).
2. A. G. Abanov and P. B. Wiegmann, Phys. Rev. Lett. 95, 076402 (2005).
3. R. G. Pereira, J. Sirker, J.-S. Caux, R. Hagemans, J. M. Maillet, S. R. White, and I. Affleck, Phys. Rev. Lett. 96, 257202 (2006).
4. S. Teber, Eur. Phys. J. B 52, 233 (2006); cond-mat/0609754
5. M. Pustilnik, M. Khodas, A. Kamenev, and L. I. Glazman, cond-mat/0512494
6. P. Pirozzinia, P. Kopietz, cond-mat/0608448; cond-mat/0612234
7. D.B. Gutman, cond-mat/0608448; cond-mat/0612234
8. A.G. Rojo, J. Phys.: Condens. Matter 11 R31 (1999); P. Debray, V. N. Zverev, V.L. Gurevich, R. Klesse and R. S. Newrock, Semicond. Sci. Technol. 17, R21 (2002).
9. A.-P. Jauho and H. Smith, Phys. Rev. B 47, 4420 (1993); L. Zheng and A.H. MacDonald, Phys. Rev. B 48, 8203 (1993); K. Flensberg, B. Yu-Kuang Hu, A.-P. Jauho and J.M. Kinaret, Phys. Rev. B 52, 14761 (1995); A. Kamenev and Y. Oreg, Phys. Rev. B 52, 7516 (1995).
10. I. V Gornyi, A. D. Mirlin, and P. von Oppen, Phys. Rev.
B 70, 245302 (2004).
11 Yu.V. Nazarov and D.V. Averin, Phys.Rev.Lett. 81, 653 (1998).
12 A.O. Gogolin, A.A. Nersesyan, and A.M. Tsvelik, Bosonization and Strongly Correlated Systems (Cambridge University Press, 1998).
13 F.D.M. Haldane, J.Phys. C, 14, 2585 (1981)
14 M. Schick, Phys. Rev. 166, 404 (1968).
15 S. Tomonaga, Prog. Theor. Phys., 5, 544 (1950).
16 I.E. Dzyaloshinskii and A.I. Larkin, JETP, 38, 202 (1974).
17 K.V. Samokhin, J. Phys. Condens. Matter 10, L533 (1998).
18 J. M. Luttinger and W. Kohn, Phys. Rev. 97, 869 (1955)
19 T. Giamarchi and A. J. Millis, Phys. Rev. B 46, 9325 (1992)
20 D.N. Aristov and R. Zeyher, Phys. Rev. B 72, 115118 (2005).
21 W. Götze and P. Wolffe Phys. Rev. B 6, 1226 (1972)
22 I. D’Amico and G. Vignale, Phys. Rev. B 62, 4853 (2000)
23 S.V. Maleyev, Theor. Math. Phys. 4, 694 (1970)
24 R. Klesse and A. Stern, Phys.Rev. B 62, 16 912 (2000); T. Fuchs, R. Klesse, and A. Stern, ibid. 71, 045321 (2005).