Oscillations of a flexible beam in cavity flow with tonal sound

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Abstract. To establish the methodologies for utilization of oscillation energy in a cavity flow, the self-sustained oscillations in a flow over a cavity with and without a flexible beam are investigated, where stainless steel and aluminium beams are utilized. The coupled simulations for aeroacoustic phenomena and bending vibration of the beam are conducted along with wind tunnel experiments. The measurement of sound pressure level with changing the freestream Mach number shows that intense oscillations occur for the second acoustic mode for both the cavities with a stainless and an aluminium beam along the upstream edge of the cavity while the oscillations with the first mode occur for the cavity without a beam. The computational results show that the beam is vibrated at the natural frequency of each beam, where this frequency can be different from the fundamental frequency of the radiating sound. Also, it is shown that the vibrations become more intense for the cavity flow with the beam along the downstream edge, where the acoustic radiation also becomes more intense.

1. Introduction

It is well known that the acoustic radiation occurs from the self-sustained oscillations in the cavity flow as shown in figure 1. The cavity flow exists in various industrial products such as the car gap of a high-speed train, sunroof of an automobile, and pipe arrangement in a plant. Under the current circumstances, the oscillation energy in these cavity flows is wasted without utilization. The final objective of this research is to establish the methodologies of utilization of the oscillation energy in the cavity flow.

Many researchers over the past 50 years have investigated the mechanism of self-sustained oscillations with acoustic radiation (cavity tone) in cavity flows. Rossiter [1] described an oscillation mechanism similar to that presented for edge tones by Powell [2]. In this mechanism, the interactions of vortices with the downstream edge of the cavity radiate acoustic waves, which cause the formation of new vortices at the upstream edge.

Rockwell et al. classified the oscillations into three categories: fluid-dynamic oscillations, fluid-resonance oscillations and fluid-elastic oscillations [3]. The fluid-dynamic oscillations occur mainly due to the unsteadiness of the shear layer of the cavity flow with acoustic feedback [4], the fluid-resonance oscillations occur due to the coupling of this unsteadiness and acoustic resonance [5], and the fluid-elastic oscillations occur with vibrating components. The oscillations due to the coupling of
all these oscillations (fluid-resonance-elastic oscillations) can occur intensely in a deep cavity with a vibrating beam as shown in figure 1. By installing a piezoelectric material as the vibrating beam, the oscillation energy can be converted to electric energy. To establish the methodology for the utilization of the oscillation energy, the flow field, acoustic radiation, beam vibrations in the fluid-resonance-elastic oscillations are necessary to be clarified.

The objective of this paper is to clarify the conditions of intense self-sustained oscillations in the flow over a cavity with a flexible beam. Particularly, the effects of the material of the beam and installation position on the oscillations are focused on. The direct aeroacoustic simulations coupled with the computation of the beam vibrations are performed along with wind tunnel experiments.

2. Methodologies

2.1. Flow configurations

The experimental setup is shown in figure 2. The coordinate origin is set at the spanwise center of the upstream edge of the cavity as shown in the figure. The cavity length is \( L = 20 \text{ mm} \), and the depth-to-length ratio is \( D/L = 2.5 \).

For the above-mentioned cavity geometry, the acoustic resonance for the first mode with one-quarter wavelength mode and the second mode with three-quarter wavelength mode were estimated to be 1300 Hz and 3900 Hz, respectively, with reference to the literature [5]. A stainless steel beam with the thickness of \( t = 0.2 \text{ mm} \) and an aluminium beam of \( t = 0.5 \text{ mm} \) were selected as a flexible beam to fit the natural frequency of the plate estimated by equation (1) with the frequencies of first and second acoustic modes, respectively.

\[
f_c = \frac{\lambda^2}{2\pi L_y^2} \left( \frac{EI}{\rho_y A} \right)^{\frac{1}{2}},
\]

where \( \lambda = 1.875 \) is a constant for the first vibration mode, \( \rho_y \) is the density of the material, \( E \) is the modulus of elasticity, and \( I \) is the second moment of area. The beam length was set to be \( L_y/L = 0.5 \).

The beam was attached along the upstream edge of the cavity in the experiment. To clarify the effects of the installation position of the beam, the computation was also performed for the installation of the beam along the downstream edge.

The freestream Mach number based on the freestream velocity, \( M \), was changed from 0.03 to 0.14 in the experiment, while the computations were performed at \( M = 0.13 \). The incoming boundary layer is laminar, and the boundary layer thickness was \( \delta/L = 0.053 \) at \( M = 0.13 \) as shown in figure 3.
The distance of the upstream edge of the cavity from the imaginary origin of the boundary layer was estimated to be 4.1$L$.

2.2. Experimental methodologies
The experiments were carried out using a suction-type, low-noise wind tunnel with a rectangular test section having a cross-section with dimensions of 150 mm × 75 mm. The intensity of freestream turbulence was less than 0.5%, and the background-noise level was 55.2 dB (A) at $M = 0.087$. The test section of the cavity was terminated in the spanwise direction by end walls composed of porous plates, which suppressed acoustic resonance in the spanwise direction. The width of the test section was $W/L = 7.5$. The beam and test section in the upstream of the cavity were carved out from one block so that the beam can be vibrated as a cantilever.

The magnitude of the acoustic pressure in the far field ($x/L = 6.75$, $y/L = 21.5$) was measured with a non-directional 1/2-inch microphone (UC-53A, RION, Tokyo, Japan) and a precision sound-level meter (NL-52, RION, Tokyo, Japan). The measurement time required for data acquisition was 30 s, and the sampling frequency was 80 kHz.

2.3. Numerical simulations
In order to simulate the interactions between flow and acoustic fields, the three-dimensional compressible Navier-Stokes equations with mass and energy conservation laws were directly solved using a finite difference scheme. The spatial derivatives were evaluated by the six-order-accurate compact finite difference scheme (the forth-order-accurate at the boundary) [6]. The time integration was performed using the third-order-accurate Runge–Kutta method. The detailed numerical methodologies such as computational domain and boundary conditions for the simulations of the cavity flow are described in the previous paper [7].

To reproduce the vibrating beam on rectangular grids, the volume penalization (VP) method [8] was utilized. The VP methods utilized in this paper are the same as those described in the previous papers [9–11]. Also, while the beam vibrations are given as a sinusoidal wave in the previous paper, the computation of the beam vibration was also coupled with the direct aeroacoustic simulations in the present paper. The following governing equations of the beam vibration was calculated at each time step of the aeroacoustic simulation.


\[ \frac{\partial^2}{\partial x_p^2} \left[ EI(x_p) \left( 1 + \eta \frac{\partial}{\partial t} \right) \frac{\partial^2 y_p}{\partial x_p^2} (x_p, t) \right] + \rho_p S(x_p) \left( \frac{\partial^2 y_p}{\partial t^2} (x_p, t) + \gamma_p \frac{\partial y_p}{\partial t} (x_p, t) \right) = F(x_p, t), \quad (2) \]

where \( t \) is time and \( x_p \) is the horizontal position along the beam length, \( y_p(x_p, t) \) is the vertical position of the top-surface of the beam, and \( S(x_p) \) denotes the cross-sectional area. The coefficient \( \eta \) represents the magnitude of the internal viscoelastic losses, and \( \gamma_B \) accounts for damping of the surrounding fluid. In this paper, the values were set to be \( \eta = 6 \times 10^{-7} \) s and \( \gamma_B = 100 \) s\(^{-1} \) referring to the literature [12]. \( F(x_p, t) \) is a driving force per unit length, which was calculated from the fluid pressure around the beam at each time step. Equation (2) was solved by using the implicit-\( \theta \)-scheme [12], which is a specific class of implicit schemes known as the “variable-weighted implicit approximation method” [13]. Finally, clamped-free boundary conditions lead to the constraints.

\[ y_p(0, t) = \frac{\partial y_p}{\partial x_p}(0, t) = 0, \quad (3) \]

\[ \frac{\partial^2 y_p}{\partial x_p^2}(L_p, t) = \frac{\partial^3 y_p}{\partial x_p^3}(L_p, t) = 0 \quad (4) \]

3. Numerical validation

Figure 4 shows the predicted and measured sound pressure spectra for the cavity flow with the aluminium beam along the upstream edge of the cavity at \( M = 0.13 \). The frequency resolution is 150 Hz, and the spectra are averaged 15 times and 9400 times in computation and experiment, respectively. As shown in the figure, the intense tonal noise occurs at the fundamental frequency around

![Figure 4](image)

**Figure 4.** Predicted and measured sound pressure spectra for the cavity with the aluminium beam along the upstream edge of the cavity at \( M = 0.13 \) (\( x/L = 6.75, y/L = 21.5 \)).

4. Results and discussions

4.1. Effects of installation of beam on acoustic radiation

Figure 5 shows the measured sound pressure spectra for the cavity flows with and without a beam at \( M = 0.13 \), where the frequency resolution is 5 Hz. The tonal sound at the fundamental frequency around
4000 Hz becomes more intense by attaching the beam. While the tonal sound at a lower frequency around 2000 Hz becomes weaker, the tones at higher harmonic frequencies become more intense.

Figure 6 shows the variation of the sound pressure level at the fundamental frequency with the freestream Mach number. While a peak appears at $M = 0.10$ in the case without a beam, two peaks appear around $M = 0.06–0.08$ and $M = 0.12–0.14$ in the case with a beam. The Mach number for the maximum peak is $M = 0.134$ and 0.128 for the stainless steel and aluminium beams, respectively.

Figure 5. Measured sound pressure spectra with and without a beam at $M = 0.134$.

Figure 6. Variation of sound pressure level at the fundamental frequency with freestream Mach number.

Figure 7 shows that variation of the non-dimensionalized fundamental frequency (Strouhal number) with the Mach number, where the Strouhal number is based on the freestream velocity and the opening length of the cavity, $L–L_p$ and $L$, for the cavity flow with and without a beam. Also, the curves of the frequencies for the first and second acoustic modes predicted by the semi-empirical equation in the literature [5] are shown. Figure 7(a) shows that the measured fundamental frequency for $M = 0.05–0.12$ is close to that for the first acoustic mode. This indicates that the tonal sound occurs due to the acoustic resonance of the first mode in the cavity at the above-mentioned peak at $M = 0.10$. Also, figure 7(b) shows that the measured fundamental frequency at $M = 0.11–0.14$ is close to that for the second mode in the cavity flow with a beam. This means that the above-mentioned peak around $M = 0.13$ occurs due to the acoustic resonance of the second acoustic mode. These results show that the installation of the beam can vary the dominant acoustic mode in the cavity.
4.2. Predicted flow and acoustic fields

Figure 8 shows predicted vortices and acoustic fields by the iso-surfaces of second invariant of velocity gradient tensor and contours of fluctuation pressure for the cavity flow with the aluminium beam at $M = 0.13$. The time of $t = 0$ corresponds to the time when the vortex is collided with the downstream edge of the cavity. As shown in the figure, the apparent two vortices are convected in the free shear layer, which indicates the occurrence of the second Rossiter mode [1]. In the cavity, the pressure fluctuations near the opening of the cavity and those near the bottom show anti-phase behaviour. This supports the above-mentioned occurrence of the acoustic resonance with the second (three-quarter wavelength) mode.

Figure 8(a) shows that the expansion wave is radiated into the cavity when the vortex is collided with the downstream edge. After the half fundamental period, the compression wave is radiated into the cavity as shown in figure 8(b). The acoustic radiation sustains the acoustic resonance in the cavity.
4.3. Vibrations of beam
Figure 9(a) shows the time variation of the beam displacement at the tip \((M = 0.13)\). The peak-to-peak amplitude is 13 \(\mu m\) and 11 \(\mu m\) for the stainless and aluminium beams, respectively. The power spectra of the displacement fluctuations are shown in figure 9(b). The stainless and aluminium beams were found to be vibrated at the 1400 and 4100 Hz, which approximately agree with the natural frequencies of these beams. As shown in figure 5, for both the cases, the acoustic radiation occurs around 4000 Hz, which is corresponding to the frequency of the second acoustic mode. The result for the cavity flow with the stainless steel beam present that the beam can be vibrated at the natural frequency even if the frequency is different from the fundamental frequency. This is possibly because the pressure fluctuations occur not only at the fundamental frequency but in the broadband frequency range, which causes the source for the beam vibrations at each natural frequency.

4.4. Effects of installation position
The effects of the installation position of the beam on the oscillations are discussed. Figure 10 shows the predicted vortices and acoustic fields for the cavity flow with the aluminium beam along the downstream edge of the cavity. As shown in the figure, the acoustic radiation into the outward of the cavity becomes more intense compared with the case with the beam along the upstream edge of the cavity (figure 8). Also, the vortices are found to be deformed by the collision with the tip of the beam.
and convected along the beam. The deformation of the vortices can lead to the intensification of the acoustic radiation [12].

Figure 11 shows the time variation of the beam displacement at the tip. The peak-to-peak amplitude for the downstream installation is 32 μm, which is threefold larger compared with that for the upstream installation. This is because the deformed vortices and intense acoustic resonance cause the pressure fluctuations around the beam.

Figure 10. Predicted iso-surfaces of the second invariant and contours of the fluctuation pressure for the cavity flow with the aluminium beam along the downstream edge of the cavity (t = 0).

Figure 11. Predicted displacement of the beam tip for the cavity flow with the aluminium beam along the downstream edge of the cavity compared with those along the upstream edge.

5. Conclusions

In order to clarify the conditions of intense fluid-resonance-elastic oscillations in the flow over a cavity with a flexible metal beam, wind tunnel experiments and the direct aeroacoustic simulations coupled with the computations of beam vibrations were conducted.

The measured variation of sound pressure level of cavity tone with the freestream Mach number shows that intense oscillations occur with the second acoustic mode for the cavity flows with a beam along the upstream edge of the cavity while the oscillations occur with the first acoustic mode for the cavity without a beam. The predicted acoustic field confirms the occurrence of the second acoustic mode in the cavity with the beam.
The predicted power spectra of the beam displacement show that the beam can be vibrated at the natural frequency of each beam independently of the fundamental frequency of the radiating sound. Also, the amplitude of the beam displacement become threefold larger in the cavity flow with the beam along the downstream edge compared with that along the upstream edge.

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