HYDROGEN BURNING ON MAGNETAR SURFACES

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ABSTRACT

We compute the rate of diffusive nuclear burning for hydrogen on the surface of a “magnetar” (Soft Gamma-Ray Repeater or Anomalous X-Ray Pulsar). We find that hydrogen at the photosphere will be burned on an extremely rapid timescale of hours to years, depending on composition of the underlying material. Improving on our previous studies, we explore the effect of a maximally thick “inert” helium layer, previously thought to slow down the burning rate. Since hydrogen diffuses faster in helium than through heavier elements, we find this helium buffer actually increases the burning rate for magnetars. We compute simple analytic scalings of the burning rate with temperature and magnetic field for a range of core temperature. We conclude that magnetar photospheres are very unlikely to contain hydrogen. This motivates theoretical work on heavy element atmospheres that are needed to measure effective temperature from the observed thermal emission and constrains models of AXPs that rely on magnetar cooling through thick light element envelopes.

Subject headings: diffusion — stars: abundances, interiors, magnetic fields — stars: neutron

1. INTRODUCTION

The surface composition of young, isolated neutron stars is not well constrained by theory. Since the spectrum is formed in the outermost ~ 10^{-17} M_☉, it is difficult to predict the surface composition from supernova simulations, especially in light of uncertainties in the amount of fallback, nuclear burning in the nascent star, and subsequent accretion and spallation reactions. On the observational side, the absence of well-understood atomic spectral lines, and the extreme physical conditions at the photosphere impede progress using spectral modeling, though some inferences have been made (Pavlov, Sauwal & Zavlin 2002).

Diffusive nuclear burning (DNB) is a mechanism by which the composition of young neutron star (NS) surface layers can evolve with time (Chang & Bildsten 2003; 2004, and Chang, Arras & Bildsten 2004, hereafter paper I, II and III respectively). The simple picture of DNB (Chin & Salpeter 1964, Rosen 1968) can be understood as follows (see Fig. 1 of paper I). Rapid sedimentation (~1 sec) allows H to settle above heavier elements. While nuclear burning timescales for H at the surface are far too long to be of interest, a diffusive tail readily extends to depths at which proton captures occur, increasing the burning rate by many orders of magnitude.

Anomalous X-Ray Pulsars (AXP’s) and Soft Gamma-Ray Repeaters (see e.g. Woods & Thompson 2004 for a review), are a class of strongly magnetized (5 × 10^{14}–15)G), hot (T_{bh} ∼ (5 - 7) × 10^{6}K) 3 young (P/2P ~ 10^{3-4}yr), and slowly rotating (P ~ 5 – 10 sec) neutron stars. We will refer to them as “magnetars”, following Duncan and Thompson (1992). In this Letter, we show that the high temperatures and substantial alteration of the equation of state (EOS) by super-strong magnetic fields combine to increase the rate of DNB by many orders of magnitude as compared to other observed neutron stars. This conclusion is robust for a variety of heavy proton capturing elements, even when a He layer exists between H and the proton-capturing element. Consequently, magnetar photospheres are most likely composed of heavy elements, possibly helium.

2. REVIEW OF DNB

The rate at which a column y_H = \int \rho_H dz of H is burned by the proton-capturing substrate is (Papers I and II)

\[ \dot{y}_H = \frac{y_H}{\tau_{col}} = \int dz \frac{n_H m_p}{\tau_H n_H n_{pc} T}, \]

where \( z \) is the depth, \( n_H \) and \( n_{pc} \) are the number density of H and proton-capturing substrate, respectively, \( T \) is the temperature, and \( \tau_H = (\bar{\sigma} n_H n_{pc})^{-1} \) is the local lifetime of a proton. The two key ingredients in this formula are the temperature profile and concentration, \( f_H = n_H/n_{pc} \), of H in the diffusive tail. The temperature profile for strongly magnetized NS envelopes is reviewed in Ventura & Potekhin (2002).

The concentration of a trace particle with charge \( Z_t \) and mass number \( A_t \) in a background \( Z_b \) and \( A_b \) depends on the thermal \( (k_B T) \), Fermi \( (E_F) \), and Coulomb interaction \( (E_C) \) energies. In the non-degenerate limit, \( k_B T \gg E_F \), the energy gained by separating the two species across a pressure scale height is \( \sim k_B T \). The resulting concentration profile \( f = n_t/n_b \) is a power-law with column (paper I) \( f \propto y^\delta \) with exponent \( \delta = A_t(Z_b + 1)/A_b - Z_t - 1 \). In the degenerate limit, if \( Z_t/A_t \neq Z_b/A_b \), the separation energy is \( \sim E_F \) over a scale height, leading to an exponential concentration profile (paper I) \( f \propto \exp[-(Z_t - Z_b A_t/A_b)E_F/k_B T] \). However, if \( k_B T \ll E_F \) and \( Z_t/A_t = Z_b/A_b \), the dominant separation energy is provided by Coulomb interactions \( E_C \sim Z^2/ae \) where the mean electron spacing is \( a_e = (4\pi n_e/3)^{-1/3} \). The resulting concentration profile
is (paper III) \( f \propto \exp \left[ -0.9Z(Z^{2/3} - Z'^{2/3})e^2/\alpha_{e}kB T \right] \). The composition profile drops off much more rapidly in the degenerate limit (exponential) as compared to the nondegenerate limit (power-law).

For hot NS’s \( (T_e \gtrsim 10^{6.2} \text{ K}) \) such as magnetars, DNB is “diffusion limited” (see paper II), meaning the burning occurs in a thin layer at which the nuclear burning time becomes comparable to the downward diffusion time \( \tau_B = \tau_{\text{diff}} \) breaking the assumption of diffusive equilibrium (paper I, II). The hydrogen concentration profile is rapidly cut off in this region. Cool NS \( (T_e \lesssim 10^{6.2} \text{ K}) \) have “nuclear limited” DNB with \( \tau_B \lesssim \tau_{\text{diff}} \), even in the burning layer. In this case, burning occurs in a degenerate layer where the temperature profile becomes isothermal and hydrogen fraction is exponentially dropping (paper I).

Paper II studied the effect of a “standard” pulsar strength \( (B = 10^{12} \text{ G}) \) magnetic field on the compositional structure. The effect of electron quantization into Landau levels was taken into account in the opacity and EOS. For \( B = 10^{12} \text{ G} \), it was found that the burning rate increased by more than an order of magnitude. Most of this decrease is due to the smaller temperature at fixed column for a given core temperature \( T_c \). In addition, \( \tau_B \) in the burning layer decreased compared to the \( B = 0 \) case. The reason, clearly seen in Fig. 4 of Paper II, is that the electric field above the burning layer to be weaker for standard radio pulsar fields.

3. MAGNETIC FIELD DEPENDENCE OF DNB

Magnetic fields significantly modify the opacity and EOS when electrons occupy only the ground Landau level, i.e. \( \hbar \omega_{e,c} \gtrsim \text{max}(k_B T, E_F) \) where \( \hbar \omega_{e,c} = 11.57 B_{12} \text{ keV} \) is the cyclotron energy. There are two main effects. First, electron scattering and free-free R"osseland opacities are reduced by a factor \( \sim (k_B T/\hbar \omega_{e,c})^2 \). Hence, for fixed \( T_c \), larger \( B \) implies lower temperature at a fixed column (paper II, Potekhin et al. 2003), and a smaller burning rate. The photosphere also moves to higher density, and the \( \tau_c(T_c) \) relation becomes highly dependent on \( B \) and \( \theta \), the angle between the field and normal. Second, the gas becomes more nondegenerate. The ratio of the \( B = 0 \) Fermi energy \( E_{F,AD} \) to the ground Landau level Fermi energy \( E_{F,1D} \) is \( E_{F,1D}/E_{F,AD} \sim (E_{F,1D}/\hbar \omega_{e,c})^{2/3} \ll 1 \), pushing the point \( k_B T = E_F \) deeper into the envelope. This dramatically alters the composition profile, allowing the slower power-law decrease all the way to the burning layer, increasing the burning rate. In effect, the parameter space for “diffusion-limited” DNB is made larger.

The opacity change decreases burning while the degeneracy change increases burning. Which one wins? The answer is that in the high temperature limit, the burning rate decreases with \( B \), and in the low temperature limit it increases with \( B \) (in the large \( B \) limit), as we show in this section.

We extend the methodology and results of paper II to higher magnetic fields and core temperatures. We calculate DNB timescales assuming a vertical magnetic field, \(^{12}\text{C}\) as the proton-capturing element, a hydrogen column of \( y_H = 10^4 \text{ g cm}^{-2} \), and a fixed surface gravity \( g = 2.43 \times 10^{14} \text{ cm s}^{-2} \). Varying the magnetic field direction was discussed in Paper II. The ions are assumed to be an unmagnetized ideal gas. We use the results of Chabrier & Potekhin (1998, 2000) for the electron equation of state and Potekhin et al. (1999) for the electron conductivity. We ignore Coulomb interactions in this section for simplicity, but include it in the next (see Paper III for details). Radiative opacities are from Potekhin & Yakovlev (2001). The composition profile is computed including diffusion and nuclear burning (see paper II). The ion-ion diffusion coefficient in the nondegenerate regime is from Alcock & Illarionov (1980) and in the liquid regime from Brown, Bildsten & Chang (2002). We refer the interested reader to paper I and II for additional details.

Fig. 1 shows the DNB timescale, \( \tau_c = y_H/\dot{y}_H \) as a function of \( T_c \) for a range of \( B \) at fixed \( y_H \). The low field \( (B = 10^9 \text{ G}) \) case (solid line) is included as a baseline. In the low \( T_c \) limit, \( \tau_c \) initially increases with \( B \) until \( B \sim 10^{12-13} \text{ G} \), after which it decreases with \( B \). In the high \( T_c \) limit, \( \tau_c \) increases with \( B \). The dependence on core temperature is power law for high \( T_c \) and exponential for low \( T_c \). The rate at which hydrogen is depleted from the photosphere, \( \dot{y}_{ph} \approx 500 \text{ g cm}^{-2} \) for \( B_{15} \geq 10^5 \), can be found using the scaling relation \( \tau_c \propto y_H^{-5/12} \). The column lifetime at the photosphere is far shorter than the magnetar age \( (\sim 10^3-10^4 \text{ yr}) \).

We estimate the burning rate in the high temperature limit...
limit following paper II. Using free-free opacity with electrons in the ground Landau level and ideal gas pressure from electrons and ions gives the temperature profile (Ventura & Potekhin 2001) \( T_7 = 1.1(T_{\mathrm{e}}^{2/3} y_{\mathrm{H}}/B_{15})^{4/13} \). The diffusion coefficient for nondegenerate ions gives a diffusion time \( \tau_{\text{diff}} \simeq 2.5 \times 10^5 \text{ sec}(B_{15}/T_{\mathrm{e}}^{2/3})^{9/4} \). The lifetime of a proton to nonresonant capture on C is \( \tau_{\text{H}} \simeq 1.4 \times 10^{13} \text{ sec}(T_{\mathrm{e},6}^2/B_{15})^{1/3} \), expanded around \( T_7 = 1 \). Setting \( \tau_{\text{diff}} = \tau_{\text{H}} \) gives the temperature, concentration, column, and capture rate in the burning layer. Placing these results into eq. (1) gives the rough analytic formula

\[
\tau_{\text{col}} \simeq 0.1 \text{ yr} \left( \frac{10^4 \text{ g cm}^{-2}}{y_{\text{H}}} \right)^{5/12} \left( \frac{B}{10^{15} \text{ G}} \right)^{1.2} \left( \frac{10^6 \text{ K}}{T_{\text{e}}} \right)^{2.4}
\]

in the high \( T_{\text{e}} \) limit.

In the low temperature nuclear-limited regime, we estimate the burning rate following the analytic calculation in Paper I. In this case, the burning occurs in a degenerate and nearly isothermal region, so the hydrogen fraction is \( f_\text{H} \simeq (y_{\text{H}}/y_{\text{deg}})^{17/12} \exp[-1/E_F/2k_B T_{\text{e}}] = (y_{\text{H}}/y_{\text{deg}})^{17/12} \exp[-(y/y_{\text{deg}})^2/3], \) where the column at which electrons in the ground Landau level become degenerate is \( y_{\text{deg}} \simeq 3.9 \times 10^6 \text{ g cm}^{-2} B_{15} T_{\text{e}}^{3/2}. \) The first factor in \( f_\text{H} \) accounts for the decrease in the outer nondegenerate envelope, while the second factor gives the exponential decrease in the degenerate regime. The temperature is nearly \( T_{\text{e}} \), with a small correction scaling as \( y^{-1/3} \). Inserting this into the p-12C rate gives \( \exp[-137/T_{\text{e}}^{1/3}] \simeq \exp[-137/T_{\text{e}}^{1/3}] \exp[-(y_{\text{deg}}/y)^{2/3}], \) where the burning rate changes over a column \( y_{\text{deg}} \simeq 7.2 \times 10^6 \text{ g cm}^{-2} B_{15} T_{\text{e},6}^{13/2}. \) Using the method of steepest descents (Paper I) for the integral in eq.(4) the dominant scalings for the column lifetime in the low \( T_{\text{e}} \) limit are

\[
\tau_{\text{col}} \propto B_{15}^{-7/12} y_{\text{H,4}}^{-5/12} \exp(29.5/T_{\text{e}}^{1/3}).
\]

The scaling of \( \tau_{\text{col}} \) with \( B \) can be explained as two factors of \( B \) from the width of the burning layer and the density in the burning layer, and the decrease in composition in the nondegenerate envelope \( y_{\text{deg}}^{17/12} \propto B_{15}^{-17/12}. \)

4. HEAVIER P-CAPTURING ELEMENTS AND A HELIUM BUFFER

In section K we found that the (diffusion-limited) burning rate for \( p-^{12}\text{C} \) is fast enough to deplete H from the photosphere of a magnetars in a few hours. We now repeat the calculation for heavier elements Mg, Si and Fe. The Mg and Si rates are from NACRE (http://pntpm.uml.ac.be/Nacre/nacre.htm) and Fe from Schatz (2004, private communication). In spite of the large Coulomb barrier for these elements, we find that H is easily consumed. For Mg, Si and Fe, the column lifetimes for \( y_{\text{H}} = 10^4 \text{ g cm}^{-2} \) at \( B = 10^{14} \text{ G} \) are, respectively, \( \tau_{\text{col}} \simeq 0.005, 0.05 \) and 2 years for \( T_{\text{e}} = 7 \times 10^6 \text{ K}, 0.01, 0.1 \) and 5 years for \( 5 \times 10^6 \text{ K} \), and 0.03, 0.7, and 100 years for \( 3 \times 10^6 \text{ K} \). These timescales are all much shorter than magnetar spin down ages \( \sim 10^4 \text{ yrs}. \)

For sufficiently large \( Z \), the reaction rate is so small that DNB is nuclear-limited instead of diffusion-limited. Column lifetimes then increase exponentially with temperature, as opposed to the slower power-law increase in the diffusion limited regime. The transition between nuclear and diffusion limited burning is given by the condition \( \tau_{\text{diff}} \sim \tau_{\text{H}} \) evaluated at the burning layer. Since the nuclear reaction rate is \( \propto \exp(E_0/k_B T_{\text{e}}) \) where \( E_0 \propto Z^{2/3} T_{\text{e}}^{2/3} \) is the Gamow energy (Clayton 1983), and \( T_{\text{e}} \propto T_{\text{e}}^{1/2} \) (Ventura & Potekhin 2002), the transition temperature \( T_{\text{e, tr}}, \propto T_{\text{e, tr}}^{1/2} \propto Z. \) For \( T_{\text{e, tr}} > 5 \times 10^6 \text{ K}, \) DNB occurs only in the diffusion limited, and hence rapid, regime for elements up to and including Fe.

We now consider the effect of a He buffer between H and the underlying p-capturing elements. The H concentration decreases into the He layer as \( f_\text{H} \propto 2(1/5)^{1/3} \), and the C concentration as \( f_\text{C} \simeq (y_{\text{H}})^{2/3} \) in a nondegenerate layer and exponentially in a degenerate layer, due to Coulomb interactions. Hence an arbitrarily thick He buffer would strongly suppress DNB. However, thermal stability for the 3α reaction sets a maximum He column \( y_{\text{He}} \lesssim 10^8 \text{ g cm}^{-2} \) for \( T \gtrsim 2 \times 10^8 \text{ K}. \) In Fig. 4 we plot the H/He/C profile for a magnetar \( (B = 10^{14} \text{ G}) \) with an effective temperature of \( T_{\text{e}} = 4 \times 10^6 \text{ K}. \) Because of the high temperatures, even a maximally thick He buffer makes little difference in the column lifetime, which is set by the diffusion time down to the burning layer. A pure H/C envelope at this \( T_{\text{e}} \) has \( \tau_{\text{col}} \simeq 30 \text{ hrs}. \) Including the He buffer actually reduces the lifetime to \( \tau_{\text{col}} \simeq 5 \text{ hrs}. \) The higher burning rate is the result of faster diffusion of H through He rather than C in the diffusion limited regime. To get the rough factor, consider the diffusion time \( \tau_{\text{diff}} \propto Z_\nu^2 A_\nu^{-1/2}, \) where the scaling is found from the diffusion coefficient in the nondegenerate limit. We find \( \tau_{\text{diff}} \) is shorter by a factor of 5 for He compared to C. Hence for the high \( T_{\text{e}} \) in magnetars, our estimates for the DNB burning rate are not slowed by the presence of a He buffer and in the case of diffusion-limited DNB may be enhanced.
5. SUMMARY AND CONCLUSIONS

We have shown that diffusive nuclear burning removes hydrogen from the photospheres of magnetars on a timescale much shorter than the age. This conclusion holds for a range of proton capturing elements up to Fe. Our new understanding of stratification in Coulomb liquids allows us to include a maximally thick He buffer, which increases the burning rate in the diffusion limited regime.

We conclude that magnetar photospheres will not contain hydrogen unless it can be supplied at a rate

$$M \gtrsim 2 \times 10^{-16} M_{\odot} \text{yr}^{-1} \left( \frac{y_{\text{H}}}{10^2 \text{g cm}^{-2}} \right) \left( \frac{\tau_{\text{col}}}{1 \text{day}} \right)^{-1}. \quad (4)$$

The burning time is so short that the supply of hydrogen must be continuous over the timescale of the observations. The relativistic outflow expected from magnetars likely precludes accretion of hydrogen. However, spallation reactions due to magnetospheric currents may generate fresh hydrogen.

The absence of hydrogen will modify our understanding of both the thermal evolution and the thermal spectrum of magnetars. Atomic physics (e.g. Lai 2001) and spectral modelling of heavy element atmospheres in $B \sim 10^{14} \text{G}$ fields is relatively unexplored. The light atmosphere models of Heyl & Hernquist (1997), Ozel (2001) and Ho & Lai (2001) assume fully ionized H and may be significantly different if the photospheric material is He or heavy elements. Even after $T_e$ has been determined, the question remains as to whether the data require an additional heat source (e.g. field decay; Heyl & Kulkarni 1998) or whether the apparent “hotness” is due to a thick layer of low opacity material (i.e. helium; Heyl & Hernquist 1997). Models of AXP’s that rely on magnetar cooling with a H envelope thicker than even a photosphere are difficult to realize given the rapid timescales we have found for H consumption. Thermal stability restricts hot ($T > 2 \times 10^8 \text{K}$) He envelopes to a maximum column of $y_{\text{He}} \lesssim 10^8 \text{g cm}^{-2}$. Since $T_e \approx 2 - 3 \times 10^8 \text{K}$ (Yakovlev, private communication) for a superfluid NS between the ages of $10^3 - 10^4 \text{yr}$, the effective temperature of such a maximal envelope ($y_{\text{He}} = 10^8 \text{g cm}^{-2}$ with C below) is $T_e \approx 3 \times 10^6 \text{K}$ at $B = 10^{15} \text{G}$. In comparison a thermally unstable pure H or pure He envelope would have $T_e \approx 5 \times 10^6 \text{K}$ and $T_e \approx 4 \times 10^6 \text{K}$, respectively. Therefore, passively cooling neutron stars without internal heat sources (such as magnetic field decay; Heyl & Kulkarni 1998 and references therein; also see Arras, Cumming & Thompson 2004) fall short of the observed luminosity of AXPs by a factor of 3 to 30 (Perna et al. 2001).

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