Constraints on the density distribution of type Ia supernovae ejecta inferred from late-time light-curve flattening

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Abstract

The finite time, $\tau_{\text{dep}}$, over which positrons from $\beta^+$ decays of $^{56}\text{Co}$ deposit energy in type Ia supernovae ejecta lead, in case the positrons are trapped, to a slower decay of the bolometric luminosity compared to an exponential decline. Significant light-curve flattening is obtained when the ejecta density drops below the value for which $\tau_{\text{dep}}$ equals the $^{56}\text{Co}$ lifetime. We provide a simple method to accurately describe this "delayed deposition" effect, which is straightforward to use for analysis of observed light curves. We find that the ejecta heating is dominated by delayed deposition typically from 600 to 1200 day, and only later by longer lived isotopes $^{57}\text{Co}$ and $^{55}\text{Fe}$ decay (assuming solar abundance). For the relatively narrow $^{56}\text{Ni}$ velocity distributions of commonly studied explosion models, the modification of the light curve depends mainly on the $^{56}\text{Ni}$ mass-weighted average density, $\langle \rho \rangle t_2$. Accurate late-time bolometric light curves, which may be obtained with JWST far-infrared (far-IR) measurements, will thus enable to discriminate between explosion models by determining $\langle \rho \rangle t_2$ and the $^{57}\text{Co}$ and $^{55}\text{Fe}$ abundances. The flattening of light curves inferred from recent observations, which is uncertain due to the lack of far-IR data, is readily explained by delayed deposition in models with $\langle \rho \rangle t_2 \approx 0.2 M_{\odot} (10^4 \text{ km s}^{-1})^{-3}$, and does not imply supersolar $^{57}\text{Co}$ and $^{55}\text{Fe}$ abundances.

Key words: plasmas – radiation mechanisms: general – supernovae: general

1 INTRODUCTION

It is widely accepted that the light curves of both Type Ia supernovae (Ia SNe) and, at least at late times, core collapse supernovae (CCSNe, including type Ib/c), are powered by the decay of radionuclides synthesized in the explosion. The most important power source is the decay-chain (Pankey 1962; Colgate & McKee 1969)

$$^{56}\text{Ni} \quad t_{1/2}=6.07 \text{d} \quad \rightarrow \quad ^{56}\text{Co} \quad t_{1/2}=77.2 \text{d} \quad \rightarrow \quad ^{56}\text{Fe}. \quad (1)$$

Part of the decay energy of this chain is released in the form of $\sim 1 \text{ MeV } \gamma$-rays, while the second stage of the chain includes also a $\beta^-$ decay channel in which $\sim 1 \text{ MeV}$ positrons are released. These positrons lose their kinetic energy and heat the plasma mainly by ionization, and then release their rest-mass energy by annihilation. The contribution of this chain to the energy deposition rate is given at late times by (Swartz, Sutherland & Harkness 1995; Jeffery 1999)

$$Q_{56} \approx \frac{M_{\text{Co56}}}{M_{\odot}} \times t^{-1/1.14} \times \left[ 138 \left( \frac{t_0}{t} \right)^2 + 4.64 \right] \times 10^{41} \text{ erg s}^{-1}, \quad (2)$$

where $M_{\text{Co56}}$ is the mass of $^{56}\text{Co}$ and all its radioactive parents at the time of explosion, $t$ is the time since explosion and $t_0$ is the time at which the column density of the ejecta is comparable to the mean free path of $\gamma$-rays produced in the decay ($t_0 \approx 35 \text{ day for Ia SNe}$, Wygoda, Elbaz & Katz 2019). The first term in the square brackets of Equation (2) represents the $\gamma$-ray energy deposition, valid at late times, $t > t_0$, when the probability of a $\gamma$-ray to experience a single (multiple) Compton collision before escaping is small (negligible). The second term represents the positron energy deposition, under the approximation that it is instantaneous. For times $t \gg t_0$, the main heating source of the ejecta is the kinetic energy loss of the positrons (Arnett 1979; Axelrod 1980).

Additional heating is provided by the decay-chains (Seitenzahl, Taubenberger & SIM 2009)

$$^{57}\text{Ni} \quad t_{1/2}=35.6 \text{h} \quad \rightarrow \quad ^{57}\text{Co} \quad t_{1/2}=271.2 \text{d} \quad \rightarrow \quad ^{57}\text{Fe},$$

$$^{55}\text{Co} \quad t_{1/2}=17.53 \text{h} \quad \rightarrow \quad ^{55}\text{Fe} \quad t_{1/2}=999.7 \text{d} \quad \rightarrow \quad ^{55}\text{Mn}. \quad (3)$$

While the abundance of these isotopes is low, their contribution may be significant at times longer than the $^{56}\text{Co}$ lifetime, due to their longer life times. Although positrons are not emitted in the second stages of these chains, low-energy internal conversion (IC) and Auger electrons are emitted, along with their (less energetic) associated X-ray cascades. The contribution of these chains to the bolometric luminosity, assuming instantaneous energy deposition

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by the emitted particles, is
\[ Q_{57} \approx \frac{M_{57\text{Co}}}{M_\odot} e^{-t/391.3} \times 2.22 \times 10^{40} \text{ erg s}^{-1}, \]
\[ Q_{55} \approx \frac{M_{55\text{Fe}}}{M_\odot} e^{-t/1442.2} \times 1.52 \times 10^{39} \text{ erg s}^{-1}, \]
where \( M_{57\text{Co}}(M_{55\text{Fe}}) \) is the mass of \( ^{57}\text{Co}(^{55}\text{Fe}) \) and all its radioactive parents at the time of explosion.

At the late times of interest here, the radioactive heating of the ejecta is dominated by the energy deposition of electrons and positrons, and the bolometric luminosity is well approximated by the energy deposition rate (see discussion at § 2.4). As a result, the observed light curve depends on the electron/positron transport, which in turn depends on the magnetic field within the expanding ejecta. In the presence of a sufficiently strong tangled magnetic field, the electrons/positrons are "fully trapped," that is, confined to the fluid element in which they were released. In the absence of such a field, electrons/positrons may escape the ejecta.

The dependence of the light curves on the transport of electrons/positrons was studied in the context of Ia SNe using Monte-Carlo simulations for some specific ejecta and magnetic field configurations (Colgate, Petschek & Kriese 1980; Milne, The & Leising 1999, 2001). A few approximate calculation methods were suggested as well (Chan & Lingenfelter 1993; Cappellaro, et al. 1997; Ruiz-Lapuente & Spruit 1998). Milne, The & Leising (1999) found that for fully trapped positrons the finite positron energy deposition time, \( \tau_{\text{dep}} \), combined with the exponential decline in the number of newly created positrons, leads to a flattening of the light curve, as the positrons’ kinetic energy is stored and contributes to ejecta heating at later times (the delayed deposition effect, the effect is also briefly discussed by Axelrod 1980). On the other hand, if positrons escape the ejecta, then the kinetic energy loss leads to a steepening of the light curve. The results of Monte-Carlo simulations were compared to optical light-curve observations in a few cases (Colgate, Petschek & Kriese 1980; Milne, The & Leising 1999, 2001; Milne & Wells 2003), with the conclusion that there is evidence for a significant escape of positrons. However, with the additional measurements of redder bands (Lair, et al. 2006a,b) and especially infrared (IR) (Sollerman, et al. 2004; Spyromilio, et al. 2004), it was discovered that the bolometric light curves do not show a steepening as would be expected in the case of positron escape, but rather an increase with time of the fraction of light emitted in the redder bands (see also Bryngelson 2012; Graur, et al. 2020). With a lack of evidence for positron escape, we consider in what follows only the fully trapped positrons scenario.

In the last few years, a few Ia SNe were observed to late times with a more complete spectral coverage, and were found to exhibit a late-time flattening of the light curve (Graur, et al. 2016; Dimitriadis, et al. 2017; Kerzendorf, et al. 2017; Shappee, et al. 2017; Yang, et al. 2018; Graur, et al. 2018a,b; Graur 2019; Li, et al. 2019). Assuming that the observed flattening is due to heating by \( ^{57}\text{Co} \) and \( ^{55}\text{Fe} \) decays, and ignoring delayed deposition, abundances of \( ^{57}\text{Co} \) and \( ^{55}\text{Fe} \) were derived, typically assuming fully trapped positrons. As we show here, delayed deposition by trapped positrons competes with, and may dominate, plasma heating by \( ^{57}\text{Co} \) and \( ^{55}\text{Fe} \) decays. Thus, ignoring the contribution of the delayed deposition effect to the flattening of the light curve would lead to a systematic overestimate of the abundances.

In this paper we provide a simple and accurate method to calculate the delayed deposition effect, bypassing the use of Monte-Carlo simulations that makes the scan of a large model parameter space difficult. The method was developed in the context of "kilonovae", i.e. for calculating light curves produced by the ejecta of neutron-star mergers (Waxman, et al. 2018; Waxman, Ofek & Kushnir 2019). Given the large uncertainties regarding the structure and composition of neutron-star mergers’ ejecta, results were derived for general simple distributions of the electrons/positrons’ energy and of the ejecta velocity. Here we apply this method for the study of positrons from \( \beta^+ \) decays of \( ^{56}\text{Co} \), that are fully trapped in an iron Ia SN ejecta. This enables us to derive exact results for ejecta velocity distributions of commonly studied explosion models.

Two comments are in place here regarding approximations adopted in our analysis. First, we calculate the energy loss of the positrons/electrons assuming that they are embedded in a plasma dominated by Iron group elements (for which the energy loss is well approximated by that of an Iron plasma). This is a valid approximation since most of the \( ^{56}\text{Ni} \) mass produced in common Ia SN models is contained in a plasma dominated by Iron group elements, and since the energy loss depends only weakly on the plasma composition (see Section 2.2). Expanding our analysis method to include energy loss in a plasma of different composition is straight forward. Secondly, we calculate energy losses assuming a time-independent low ionization, with a number of free electrons per atom \( \chi_e \sim 1 \). This is a valid approximation since, as we show in Section 2, the results are not sensitive to the exact value of \( \chi_e \), and since at the relevant times the plasma ionization cannot be significantly larger. Here too, expanding our analysis method to include a more accurate description of \( \chi_e \) is straight forward.

The structure of the paper is as follows. In Section 2, we describe our calculation method, derive the late-time bolometric light curves and discuss the effect of delayed deposition. We consider radioactive heating by the decay of \( ^{56}\text{Co} \), \( ^{57}\text{Co} \), and \( ^{55}\text{Fe} \), and assume that the bolometric luminosity equals the heating rate, as appropriate at the late times of interest. We focus on ejecta velocity distributions of commonly studied Ia SNe explosion models ("Chandra", "sub-Chandra" and direct-collision models, see Maoz, Mannucci & Nelemans 2014, for a review), but address also the modifications to the light curves that will arise in the presence of wider velocity distributions. In Section 3, we demonstrate how ejecta properties may be inferred from observations by analyzing the late-time light curve of SN 2011fe using our method. Our conclusions are summarized and discussed in Section 4, where we also comment on the differences in the role played by delayed deposition in the different types of ejecta produced in Ia SNe, CCSNe, and kilonovae.

2 A SIMPLE AND ACCURATE METHOD TO CALCULATE THE DELAYED DEPOSITION EFFECT

We consider the bolometric luminosity produced by a radioactively heated expanding ejecta. We are interested in the evolution at late times, at which the contribution of \( \gamma \)-rays to the energy deposition is small and given by the first term in the square brackets of Equation (2). The equations describing the evolution of the kinetic energy \( E \) of electrons/positrons released in radioactive decays are given in Section 2.1. The equations describing the heating of the ejecta by confined electrons/positrons are given in Section 2.2. In Section 2.3, we discuss the effects of delayed deposition on the late-time light curves. In our analysis, we approximate the bolometric luminosity as given by the energy deposition rate, \( L = Q \). We show
in Section 2.4 that this is a valid approximation for the time range in which we are interested.

### 2.1 The evolution of electron/positron energy

We assume that electrons/positrons produced in radioactive decays are "fully trapped", that is, confined to the fluid elements in which they were released. The trapped particles lose energy by ionization, free electron scattering, and bremsstrahlung, that contribute to plasma heating, and by adiabatic losses due to plasma expansion, that do not lead to heating but rather to the acceleration of the ejecta. The kinetic energy $E$ of an electron/positron evolves according to

$$\frac{dE}{dt} = -\nu_e \left( \frac{\rho_e}{\gamma} + \rho \frac{dE}{dX} \right).$$

Here, $\nu_e$ and $\rho_e$ are the electron/positron’s velocity and momentum, $\rho$ is the plasma density, and $dX = \rho dN$ is the column density traversed by the electron/positron in the time interval $dt$. The second term describes the energy losses due to ionization, scattering, and bremsstrahlung, while the first term describes adiabatic losses assuming that (in the absence of other losses) adiabatic expansion leads to $\rho_e \propto t^{-1/2}$. The velocity, $v$, of each fluid element in the ejecta is constant, and its density decreases as $\rho \propto t^{-3}$. For spherical symmetry, the density is related to the mass–velocity distribution of the ejecta by

$$4\pi^2 \rho(v, t) = v^{-2} \frac{dm}{dv},$$

where $m(v)$ is the mass of the ejecta with velocity $v$.

The ionization, scattering and, bremsstrahlung loss processes are briefly described below.

(i) **Ionization** – The energy loss per unit column density (grammage) traversed by the electron/positron, $dX = \rho dx$, is given by (e.g., Longair 1992)

$$\frac{dE}{dx}_{\text{ion}} = -\frac{4\pi^4 Z}{m_e m_p v_e^2} \frac{A}{\bar{A}} \left[ \ln \left( \frac{\gamma_e^2 m_e v_e^2}{\gamma^2} \right) - \frac{1}{2} \ln(1 + \gamma_e) \right]$$

$$- \left( 2\gamma_e + \gamma_e^2 - \frac{1}{2\gamma_e} \right) \ln 2 + \frac{1}{2\gamma_e} + \frac{1}{16} \left( 1 - \frac{1}{\gamma_e^2} \right)^2$$

where $\gamma_e$ is the Lorentz factor of the electron/positron, $Z$ and $A$ are the atomic and mass numbers of the plasma nuclei, and $\bar{A}$ is the effective average ionization energy of the atoms, which is empirically determined and approximately given by $\bar{A} = 10 Z$ eV for heavy nuclei (e.g., Mukherji 1975). In general, for ionized plasma one should use the number of bound electrons and the effective $\bar{A}$ for the ionized state. However, since we consider only low-ionization plasma, and since the dependence on $\bar{A}$ is logarithmic, these corrections are small.

(ii) **Plasma losses** – Electrons/positrons also lose energy by scattering free plasma electrons, with a rate given by (Solodov & Betti 2008)

$$\frac{dE}{dx}_{\text{plasma}} = -\frac{4\pi^4}{m_e m_p v_e^2} \chi_e \frac{\bar{A}}{A} \left[ \ln \left( \frac{1}{\omega_p} \right) ^2 \frac{\gamma_e + 1}{2\gamma_e} + 1 \right]$$

$$+ \frac{1}{8} \left( \frac{\gamma_e - 1}{\gamma_e} \right) - \frac{1}{2\gamma_e^2} \ln 2,$$

where $\chi_e$ is the number of free electrons per atom, $\omega_p = (4\pi e^2 n_e / m_p)^{1/2}$ is the plasma frequency, and $n_e = \chi_e e / (\bar{A} m_p)$ is the number density of free electrons.

![Figure 1. The energy loss rate of electrons/positrons propagating in a singly ionized, $\chi_e = 1$, iron plasma ($Z = 26$, $A = 56$), as a function of their kinetic energy. Blue: ionization losses, Equation (7); red: plasma losses; Equation (8); green: bremsstrahlung losses, Equation (9); black: the total loss rate. Also shown is the energy spectrum of positrons released in $\beta^+$ decay of $^{56}$Co, Equation (10). The energy loss rate for a typical $\beta^+$ positron is dominated by ionization losses with some tens of percent contribution from plasma losses, leading to a total energy loss rate of $(\nu_e/c)dE/dX = 2\text{MeV}/(\text{g cm}^{-2})$.](image)

(iii) **Bremsstrahlung** – At highly relativistic energies, bremsstrahlung losses dominate, with a rate given by (e.g., Longair 1992)

$$\frac{dE}{dx}_{\text{Brem}} = \frac{4\pi^4}{m_e m_p c^3 \gamma_e} \frac{Z^2 e^2}{\bar{A} \bar{A} c^2} \left[ \ln \left( \frac{183}{Z^2} \right) + \frac{1}{8} \right].$$

The energy-dependent energy loss rate of electrons/positrons propagating in a singly ionized, $\chi_e = 1$, iron plasma ($Z = 26$, $A = 56$) is shown in Figure 1. Also shown is the energy spectrum of positrons released in $\beta^+$ decay of $^{56}$Co, given by (e.g., Nadyozhin 1994)

$$\frac{dN}{dE} \propto \left( E + m_e c^2 \right)^2 (1.4589 \text{MeV} - E^2) G(E),$$

where the function $G(E)$ is tabulated in Rose et al. (1955). As can be seen, the energy loss rate for a typical $\beta^+$ positron is dominated by ionization losses with some tens of percent contribution from plasma losses, leading to a total energy loss rate of $(\nu_e/c)dE/dX = 2\text{MeV}/(\text{g cm}^{-2})$. For lower energy (Auger and IC) electrons, the energy loss rate is higher.

We next define an effective electron/positron ‘opacity’

$$\kappa_e = [(dE/dx)/E]_{E=1\text{MeV}} = 2 \text{cm}^2 \text{g}^{-1}.$$
Let us consider first electrons/positrons produced with a single energy $E_i$, and denote by $E(t_i, t)$ the time-dependent energy of a positron produced at time $t_i$. The heating rate of the plasma at time $t$ is then given by

$$Q_d(t, E_i) = \rho(t) \int dE \frac{dn(E, t)}{dE} \nu_e \left( \frac{dE}{dx}_{\text{ion}} + \left( \frac{dE}{dx} \right)_{\text{plasma}} \right),$$

where $dn/dE$ is the number of electrons/positrons per unit energy. We neglect the bremsstrahlung contribution, which is negligible for the relevant electron/positron energies (see Figure 1).

The differential electron/positron number is given by

$$\frac{dn(E, t)}{dE} = \frac{1}{\tau_i} \left[ \frac{\partial n}{\partial E} \right]_{E_i} \left( t_i, t \right),$$

where $\tau_i$ is the life time of the positron.

The energy distribution of the positrons produced in the $^{56}$Co decay, the positron energy distribution is given by

$$i_{E} \propto \text{dotacc}$$

The delayed deposition of positron energy at $t_\tau$ leads to a flattening of the light curve, that is, to an enhancement of the emission above an exponential decline following the decline of the $^{56}$Co decay rate, that would be obtained for instantaneous energy deposition, $Q_{\text{inst}}(t)$. To see this, note that when $t > t_\tau$ we have $\tau_{\text{dep}}(t) > t/\tau$, and positrons produced at time $t$ deposit their energy over a time longer than the exponential decay time of the $^{56}$Co decay rate. Thus, at time $t + \tau_{\text{dep}}(t)$ the population of positrons that heat the plasma is dominated by positrons produced at time $t$, the number of which is larger than that of positrons produced at $t + \tau_{\text{dep}}(t)$ by a factor $\exp[\tau_{\text{dep}}(t)/t_{1/2}]$. While the kinetic energy $E$ of the positrons produced earlier is lower, they dominate the heating since their number is larger and the ionization/plasma loss rate is nearly independent of $E$ (see Figure 4). The enhanced late-time deposition $t_{\text{dep}}$ using Equation (12)

$$\tau_{\text{dep}} = E_{\text{MeV}}/(\rho c k_c g(E_i)),$$

where $E_i = 1 E_{\text{MeV}}$ MeV is the average initial positron energy, we define $t_\tau$ as the time at which $\tau_{\text{dep}} = t_{1/2}$.

$$t_\tau = \left( \frac{E_{\text{MeV}}}{c k_c g(E_i)} \right)^{1/3}$$

Here, $\langle \rho \rangle$ is the $^{56}$Ni (and its daughter nuclei) mass-averaged density, and we measure $\langle \rho \rangle^{1/3}$ in units characteristic of the density of Ia SN ejecta

$$\langle \rho^{1/3} \rangle_{\text{SNNe}} = \frac{\langle \rho \rangle^{1/3}}{1 M_{\odot}/(10^{4} \text{ km s}^{-1})^3} = \frac{\langle \rho \rangle^{1/3}}{4 \pi M_{\odot}/(10^{4} \text{ km s}^{-1})^3}.$$
deposition and luminosity, compared to that produced by instantaneous deposition, is produced by the energy that is "stored" in the positrons that did not lose all their energy instantaneously at earlier time. Due to the fast exponential decrease in the decay rate, the small fractional reduction of the deposition (and luminosity) at early time, obtained for a finite deposition time compared to instantaneous deposition, translates to a large fractional contribution to the deposition and luminosity at late time. The small fractional deficit at earlier times will be typically difficult to detect (see below).

In Figure 3 we show the time $\Delta(t_{1/2})$ over which a positron of initial energy $E_i$ produced at time $t$ loses all its energy, obtained by numerically solving Equation (5) for a few initial energies and two values of the plasma density. Solving this equation, the positron reaches zero energy at a finite time. In reality, its energy loss is suppressed when it reaches the thermal plasma energy. Since the thermal energy is much smaller than $E_i$, the implied corrections to the deposited energy and to the deposition rate at late time are small. For the average energy of $^{56}$Co $\beta^+$ decay positrons, 0.63 MeV, we find that the time at which the energy loss time equals the $^{56}$Co half life time is $t_r \approx 700(1000)$ day for $\rho^3 = 0.2(1.1)M_\odot/(10^4$ km s$^{-1})^3$, in agreement with the estimate of Equation (17). The modification of the light curve is significant at $t > t_r$. For $t_r$ smaller than $t_{\Delta}$, the time at which the instantaneous energy deposition rate by $^{57}$Co and $^{56}$Co are equal, delayed deposition dominates the contribution to the plasma heating by $^{57}$Co decay over a significant time.

![Figure 3](image-url)
thermally high velocity, with correspondingly low values of $\rho t^3$, may have a large effect on the delayed deposition rate and bolometric luminosity, since the time at which delayed deposition would become important for such fast components would be significantly earlier than $t_\gamma$ corresponding to the average density. For example, consider the double-detonation model of Polin, Nugent & Kasen (2019) for a 1.4 $M_\odot$ WD with a helium shell of 0.08 $M_\odot$ (presented in their figure 1). In this model, a few percent of the total $^{56}$Ni is synthesized in the helium shell, with a high, $\approx 20 \times 10^3$ km s$^{-1}$, velocity, corresponding to $\rho t^3 \approx 10^{-3} M_\odot/(10^4$ km s$^{-1}$)$^3$. To demonstrate the effect of such a fast component, we added to our 1 $M_\odot$ model a high velocity component of $^{56}$Co, around $\rho t^3 = 10^{-3} M_\odot/(10^4$ km s$^{-1}$)$^3$, with 10% of the mass of the original model. The $\gamma$-rays from the fast component escape at early times ($t_q \approx 13$ day), so that the $^{56}$Ni mass deduced from the light curve would not include its contribution. At early times, there is a suppression of a few percent of the positron heating rate due to the finite time it takes positrons to deposit their energy. These positrons times, there is a suppression of a few percent of the positron heating rate due to the finite time it takes positrons to deposit their energy. These positrons at the time $t = t_\gamma$, where $t_\gamma$ is the light crossing time of the ejecta. T

\[ t_\gamma = (v/c) \tau_c \approx 20 d < t_1/2 \text{ (using Eq. (17)) and } v/c = 1/30, \]

\[ \text{at later time } \tau_\text{dep} \propto \rho^{-1} \propto r^3. \]

\[ \text{Let us consider next the former condition. As long as } t_\text{cool} \ll \text{max}[t_1/2, \tau_\text{dep}], \text{ the cooling rate equals the heating rate and is approximately given by } t_\text{cool} \approx 1 \text{eV} / q, \text{ where } q \text{ is the energy deposition rate per atom (Axelrod 1980). Using the instantaneous deposition value, i.e. neglecting the delayed deposition effect, of } q \text{ for } ^{56}\text{Co } \beta^+ \text{ positrons, given by Equation (2), we have } t_\text{cool} \approx 73 \text{exp} (t/111.4 d) s \text{ and } t_\text{cool} < t_1/2 \text{ at } t \lesssim 1300 \text{ day. This implies that in fact the cooling time is much smaller than max}[t_1/2, \tau_\text{dep}] \text{ at } t = 1300 \text{ day, since at this time } t_1/2 \ll t_\text{dep} \text{ and } q \text{ is larger than the instantaneous value given by Equation (2) due to the delayed deposition. The approximation of } L=Q \text{ holds therefore well beyond 1300 day, typically to } t \gtrsim 2000 \text{ day.}

A comment is in place here also regarding the recombination energy. Since the plasma cannot be assumed to be in an ionization equilibrium at $t \gtrsim 600$ day (Axelrod 1980), the recombination energy fraction of the deposited energy may be stored in the plasma without being radiated away (reducing $L$ below $Q$). However, the recombination energy is typically only a few percent of the effective average ionization energy, $I$ (see Equation (7) and the text below). Most of the deposited energy is therefore transferred to the plasma by the kinetic energy loss of the secondary electrons, and the correction to our $L=Q$ approximation is small.

\section{The Case of SN 2011fe}

The best Type Ia supernova for late-time studies is SN 2011fe (see discussion in Shappee, et al. 2017), which was observed over a wide range of wavelengths at late time. However, even in this case, a significant fraction of the flux is predicted to be emitted outside of the observed wavelength range (Fransson & Jerkstrand 2015), complicating the comparison to models. For this supernova, the bolometric correction is more uncertain at $\sim 600 – 800$ days (Dimitriadis, et al. 2017), where NIR data are not available. We therefore use only the $t \gtrsim 900$ day data (black and blue circles in Figure 5 showing the quasi-bolometric, 4000 – 17000 Å, light curve, Kerzendorf, et al. 2017; Shappee, et al. 2017, respectively). At these times, the $\gamma$-ray contribution to the light curve may be neglected (see Figure 4).

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The best-fitting solution, which fits the data well, is shown in Figure 5. We obtain \( \langle \rho t \rangle_{\text{SND}} \approx 0.14 \), consistent with the sub-Chandra and direct-collision models, and an observed flux fraction of \( f \approx 0.37 \). The delayed deposition (red dashed line) is significantly larger than the instantaneous deposition (black dashed line) and is the dominant contribution to the light curve for \( t \lesssim 1200 \) day. At later times the heating and the bolometric light curve are dominated by \(^{57}\text{Co}\) (dotted black line) and by \(^{55}\text{Fe}\) (dotted red line) decays. The effect of the delayed deposition is to decrease the inferred amount of \(^{57}\text{Co}\) and \(^{55}\text{Fe}\). However, a reliable determination of the parameters would require a more complete measurement of the bolometric light curve.

4 SUMMARY AND DISCUSSION

We have calculated the effect of the finite time, \( t_{\text{dep}} \), given by Equation (16), over which positrons from \( \beta^+ \) decays of \(^{56}\text{Co}\) deposit their energy in the ejecta of type Ia supernovae on the late-time light curves of these explosions, assuming that the positrons are trapped within the ejecta. The equations describing the deposition of energy are given in Sections 2.1 and 2.2. A Matlab code that solves these equations, and is straightforward to use for the analysis of observed light curves, is provided (see footnote 1).

We have shown that a significant light-curve flattening is obtained due to delayed deposition at \( t \gtrsim t_{\tau} \), where \( t_{\tau} \) is given by Equation (17), is the time at which the ejecta density drops below the value for which \( t_{\text{dep}} \) equals the \(^{56}\text{Co}\) life time, \( t_{1/2} \). We find that the ejecta heating is dominated by delayed deposition typically from 600 to 1200 day, and only later by the decay of the longer lived isotopes \(^{57}\text{Co}\) and \(^{55}\text{Fe}\) (assuming solar abundance), see Figures 4 and 5.

The modification of the light curve is determined by the density distribution of the ejecta. We have shown (see Figure 4) that for the relatively narrow velocity distributions of commonly studied explosion models, including Chandra, sub-Chandra, and direct-collision models, the effect depends mainly on the \(^{56}\text{Ni}\) mass-weighted average density \( \langle \rho t \rangle^3 \). Nevertheless, the strong dependence of \( t_{\tau} \) on the velocity implies that the presence of a relatively small amount of \(^{56}\text{Co}\) within fluid elements with relatively high velocity, with correspondingly low values of \( \langle \rho t \rangle^3 \), may have a large effect on the delayed deposition rate and bolometric luminosity, see Figure 4. This is due to the fact that the time at which delayed deposition would become important for such fast components would be significantly earlier than \( t_{\tau} \) corresponding to the average density.

Accurate late-time bolometric light curves, which may be obtained with far-IR measurements as would be provided by JWST, will thus enable one to discriminate between explosion models by determining \( \langle \rho t \rangle^3 \) and the \(^{57}\text{Co}\) and \(^{55}\text{Fe}\) abundances, and by constraining the width of the velocity distribution of the synthesized \(^{56}\text{Ni}\).

In Section 3, we have analysed the late-time light-curve observations of SN 2011fe, for which the most complete late-time wavelength coverage is available. We have shown that the observed flattening is consistent with \( \langle \rho t \rangle^3 \approx 0.15 \ M_\odot \ (10^4 \text{ km s}^{-1})^3 \) and solar abundance \(^{57}\text{Co}\) and \(^{55}\text{Fe}\) (see Figure 5). In general, the contribution of delayed deposition to the heating of the ejecta at \( t > 600 \) day implies that the recent observations suggesting late-time light-curve flattening are readily accounted for by delayed deposition, and do not imply supersolar \(^{57}\text{Co}\) and \(^{55}\text{Fe}\) abundances.

While the delayed deposition effect is important for Ia SNe, it is unlikely to be observed in CCSNe, due to their larger values of \( \langle \rho t \rangle^3 \), as shown in Figure 2. The larger densities imply larger values of \( t_{\tau} \), so that by the time delayed deposition of \(^{56}\text{Co}\) positrons...
becomes important, the energy deposition in the ejecta is already dominated by the decay of the longer lived isotopes $^{57}$Co and $^{55}$Fe, see Figure 3 (for a solar metallicity mass ratio, $^{57}$Fe/$^{56}$Fe = 0.023, Lodders 2019, the instantaneous energy deposition rates of $^{56}$Co and $^{57}$Co are equal at $t_{\gamma} \approx 1050$ day). We note that the energy deposition by $^{57}$Co and $^{55}$Fe may be considered as instantaneous, since the energy of the electrons produced is lower, with energy loss rate significantly higher, compared to that of $^{56}$Co positrons, and since the life times of $^{57}$Co and $^{55}$Fe are significantly longer than that of $^{56}$Co. The combination of the shorter loss time and longer decay time implies that, at the relevant times, the energy deposition time is small compared to the $^{57}$Co and $^{55}$Fe life times, and the effect of delayed energy deposition is small.

We note in this context that the escape of positrons, in case they are not fully trapped, may affect the light curves of CCSNe only at very late time. In the extreme case of the absence of magnetic fields, the ejecta crossing time of the escaping positrons is $t_{\text{cross}} \sim vt/v_{e}$, where $v \approx 10^{4}$ km s$^{-1}$ is the typical velocity of supernovae ejecta. The suppression of energy deposition due to escape is significant only if positions escape before losing their energy to heating, i.e. only if $t_{\text{cross}} < t_{\text{dep}}$. Using Equation (19), we find that this condition holds at $t > t_{\text{esc}} \approx (v/v_{e})^{1/2}t_{e}$, where $t_{\text{esc}}$ is the time at which $t_{\text{dep}} = t_{\text{cross}}$. For the densities relevant for Type Ib/c SNe, $t_{\text{esc}} \approx 1$ to 10 (see Figure 2), $t_{\text{esc}} \gtrsim 700$ day, so that considering positron escape at much earlier times for these SNe (e.g. Wheeler, Johnson & Clocchiatti 2015) is unrealistic. For SNe of type II the densities are expected to be higher, and $t_{\text{esc}}$ is expected to be still larger.

Several comments are in place here regarding the differences between the IA SN case discussed in this paper and the kilonova case discussed by Waxman, et al. (2018); Waxman, Ofek & Kushnir (2019). First, the characteristic density of the ejecta is some four orders of magnitude lower in the kilonova case, due to the smaller ejecta mass and its higher velocity. Second, in the kilonova case the radioactive energy release is dominated at early times $t$ by the decay of isotopes with $t_{1/2} \sim t$ so that $t_{e}$ is similar to the time $t_{e}$, given by Equation (19), at which adiabatic losses of the electrons/positrons become significant (for IA SNe, the two times are distinct, with $t_{e} < t_{e}$). As a result, the characteristic times-scales in the kilonova case are $\sim$ 10 day, instead of $\sim$ 1000 day in the IA SN case. Finally, the velocity distribution of radioactive material is wide in the kilonova case, and the light-curve modifications depend not only on the average density but also on the shape of the density distribution. In particular, due to the strong dependence of $t_{e}$ on $v_{e}$, the presence of a component with small mass at high velocity may strongly affect the light curve at times earlier than those estimated using $(\rho) t^{3}$. The behavior of the kilonova light curves at $t > t_{e}$ was analysed in detail by Waxman, Ofek & Kushnir (2019). For the IA SN discussed here, where $t_{e} < t_{e}$, we were interested mainly in the regime $t_{e} < t_{e}$ since at $t > t_{e}$ the contribution to the heating of the plasma is dominated by the decays of $^{57}$Co and $^{55}$Fe.

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