Big Bang Nucleosynthesis
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A review of Big Bang Nucleosynthesis (BBN) is presented. Observations of deuterium and helium-4 are discussed. Some BBN restrictions on non-standard physics, especially on neutrino properties and time-variation of fundamental constants are given.

1. INTRODUCTION

Big Bang Nucleosynthesis (BBN) is known to be one of three solid pillars on which the Standard Cosmological Model (SCM) stands. The other two include General Relativity (GR) and Cosmic Microwave Background Radiation (CMBR). An agreement of BBN calculations of light element abundances with observations presents the strongest proof in favor of the statement that 12-14 billion years ago the universe was indeed hot with the temperatures in MeV range and that the entropy per baryon is huge, about \(10^9\).

According to the theory, light elements \(^2\)H, \(^3\)He, \(^4\)He, and \(^7\)Li have been created in the early universe during first few hundred seconds of her existence. The abundances of these elements span 9 orders in magnitude and are in excellent, good, or reasonable agreement with the observational data, depending upon the moment when the comparison of theory with the data is taken and upon the personal point of view of a researcher.

The theory of BBN is robust, well defined, and quite precise. The largest uncertainty is introduced by the values of the cross-sections of nuclear reactions. Theoretical accuracy is better than 0.1% for \(^4\)He, better than 10% for \(^2\)H and is about 20-30% for \(^7\)Li. In all the cases theoretical uncertainty is much smaller than observational precision. Observations of light elements encounter two serious problems: systematic errors and evolutionary effects. We will discuss them below.

2. PHYSICS OF BBN

Physical processes which were essential for cosmological creation of light elements took place at the temperatures, \(T\), in the range of a few MeV down to 60-70 keV. The corresponding time, \(t\), interval was from a few×0.1 sec up to \(10^3\) sec. At this time the universe was dominated by relativistic matter and the rate of cosmological cooling was determined by the expression:

\[
\left( \frac{t}{\text{sec}} \right) \left( \frac{T}{\text{MeV}} \right)^2 = 0.74 \left( \frac{10.75}{g_*} \right)^2 \tag{1}
\]

where \(g_*\) is the effective number of particle species in the cosmic plasma. In the standard model the factor \(g_*\) includes contributions from photons equal 2, from \(e^\pm\)-pairs equal 7/2, and the contribution from three neutrino flavors equal 3 · 7/4. Any additional, non-standard form of energy is parametrized in terms of effective number of neutrino species:

\[
g_* = 10.75 + \frac{7}{4} (N_e - 3) \tag{2}
\]

Since the cosmological cooling rate depends upon \(g_*\) it is clear that BBN is sensitive to any form of matter/energy present in the primeval plasma.
in the temperature interval ($\sim\text{MeV}$) - 0.06 keV. This effect was noticed in refs. [3], detailed calculations of the effect were pioneered in papers [3].

The first step in creation of light elements is “preparation” of neutrons. Their number density is determined by the reactions:

\begin{align*}
n + e^+ &\leftrightarrow p + \bar{\nu}_e, \\
n + \nu_e &\leftrightarrow p + e^-
\end{align*}

Since the reaction rate is proportional to $T^5$ and the expansion rate is $H \sim T^2$ thermal equilibrium is maintained at high temperatures and $n/p$-ratio follows the equilibrium curve:

\[ n/p = \exp\left(-\frac{\Delta m + \mu_e}{T}\right) \]

where $\Delta m = 1.293\ \text{MeV}$ is the neutron-proton mass difference and $\mu_e$ is the chemical potential of electronic neutrinos. In the standard model the latter is assumed to be vanishingly small.

At $T = 0.6 - 0.7\ \text{MeV}$ expansion became faster than reactions [3] and the $n/p$-ratio would tend to a constant, if not the neutron decay. Because of the decay the ratio slowly decreases as $\exp(-t/\tau_n)$ with $\tau_n = 889\ \text{sec}$. This behavior lasts approximately till $T_{NS} = 65\ \text{keV}$ when light element formation abruptly begins and all free neutrons quickly disappear from the plasma forming deuterium, helium, and lithium. The value of $T_{NS}$ is determined by the binding energies of light nuclei, in particular deuterium, and by the baryon-to-photon ratio, $\eta = n_B/n_\gamma$.

Practically all neutrons that remained in the plasma to the moment when the temperature dropped down to $T_{NS}$ ended their lives in $^4\text{He}$ because the latter has the largest binding energy.

The mass fraction of the primordial $^4\text{He}$ should be about 25\%. A little deuterium and $^3\text{He}$ survived because they were not able to find a nucleon, $N = p, n$, for further transformation into $^4\text{He}$ through the chain of reactions:

\begin{align*}
N + 2H &\rightarrow (^3\text{H}, \ 3\text{He}) \\
(^3\text{H}, \ 3\text{He}) + N &\rightarrow ^4\text{He}
\end{align*}

As a result the abundance of primordial deuterium would quickly decrease with rising baryon number density or, better to say, parameter $\eta$, and its output with respect to hydrogen would be (a few)$\times 10^{-5}$ by number. A strong sensitivity of deuterium abundance to the baryon-to-photon ratio $\eta$ makes this element a very convenient tool to measure the baryonic charge of the universe. That’s why primordial deuterium is called “baryometer”, the term suggested by David Schramm.

An absence of a (quasi)stable nuclei with $A = 5$ inhibits production of $^7\text{Li}$, because the latter should be produced by fusion of two lighter nuclei and not in a sequence of reactions of free protons or neutrons with nuclei. Correspondingly the fraction of $^7\text{Li}$ is very small, (a few)$\times 10^{-10}$.

The abundances of light elements as functions of the baryon number density, $\eta_{10} = 10^{10}n_B/n_\gamma$, are presented in fig. 1 taken from the review [3].

As we have seen, the abundances of light elements essentially depend upon the following parameters:

1. Baryon-to-photon ratio, $\eta_{10} = 10^{10}n_B/n_\gamma$.

Only two years ago this parameter was determined from BBN itself but now there is an independent way to measure it through the spectrum of angular fluctuations of CMBR. According to the BOOMERANG and DASI measurements:

\[ \Omega_Bh^2 = \begin{cases} 
0.021 \pm 0.004 & [6], \\
0.022^{+0.004}_{-0.003} & [8], 
\end{cases} \]

where $\Omega_B = \rho_B/\rho_{\text{crit}}$ is the fraction of baryon mass density in terms of cosmological critical energy density and $h = H/100\text{km/sec/Mpc}$ is the dimensionless Hubble parameter. $\Omega_Bh^2 = 0.022$ corresponds to $\eta_{10} = 5.7$. A higher value is given by MAXIMA-I measurements [8]. $\Omega_Bh^2 = 0.0325 \pm 0.007$ which disagrees with the other groups by two standard deviations.

2. Rate of the reactions [3]. It is expressed through neutron life-time which is pretty well known now, $\tau_n = 886.7 \pm 1.9$ [4].

3. Total cosmological energy density during BBN. This quantity is usually parametrized as the number of additional neutrino families, $\Delta N = N_\nu - 3$. This is a precise
parametrization if the additional energy is in the form of relativistic particles with the equation of state $p = \rho/3$. However it is not so for matter with a different equation of state, e.g. nonrelativistic matter or vacuum-like energy. In this case the variation of different abundances would be different from that induced by extra neutrinos.

4. Neutrino degeneracy. It is assumed usually that the charge asymmetry of different neutrino flavors is vanishingly small. Thus their chemical potentials are zero or negligible. Strictly speaking their values are unknown and the best way to determined them or obtain an upper bound on their magnitude is BBN. Chemical potentials of $\nu_{\mu}$ and $\nu_{\tau}$ can be described by $\Delta N$, since their role is only to increase the energy density of relativistic matter at BBN. Element abundances are much more sensitive to chemical potential of $\nu_\tau$ because the latter not only changes the canonical energy density but also directly shifts the $n/p$-ratio (4).

3. OBSERVATIONAL DATA

3.1. Observations of Deuterium

The present day abundance of deuterium can be noticeably different from the primordial value because deuterium can be burnt in stars producing $^3$He. Exact evolutionary effects are uncertain. One can rigorously say only that the observed today deuterium presents a lower bound to the primordial one. For more detail see e.g. review (5). Fortunately in recent years observation of deuterium in large Z low metallicity ($<1/50$ of the solar) clouds of cold neutral gas (HI) became possible (10–14). Such clouds are presumably not contaminated by stellar processes and the observed deuterium may be the primordial one; see however ref. (15) where this statement is questioned.

Since the deuterium isotope shift corresponds to velocity of only (-82) km/sec the clouds with a simple velocity structure are necessary for reliable data. In addition to the uncertainty induced by the unknown velocity structure, ionization corrections and a possible “interloper” further increase the systematic errors. A discussion of these effects and a list of references can be found e.g. in (16,17).

Because of these problems, there are only a few measurements of deuterium at high red-shifts available now. They are summarized in Table 1.

The entries in Table 1 are situated in order of increasing the $(D/H)$-values. Entries in columns 3 and 5 correspond to the same system but the result was changed to a larger value after its reanalysis in ref. (21) where a complex velocity structure suggested by S II and $H_2$ absorption lines was taken into account.

The data seem to be grouped around three cen-
Table 1
Abundance of deuterium in high red-shift Lyman alpha absorption systems

| \(z\) | \(10^5(D/H)\) | \(\pm\) |
|---|---|---|
| 3.514 | 1.65 ± 0.35 | 2.24 ± 0.67 |
| 2.0762 | 2.54 ± 0.23 | 3.2 ± 0.4 |
| 3.025 | 3.25 ± 0.3 | 4.0 ± 0.65 |
| 2.536 | 20 ± 5 |
| 2.021 | 3.0572 | 0.701 |
| 2.504 | 2.4 ± 1.0 |
| 2.023 | 6.9 ± 1.5 |
| 2.049 | 6.7 ± 2.3 |
| 2.029 | 6.5 ± 1.5 |
| 2.036 | 6.2 ± 2.1 |
| 2.021 | 6.0 ± 1.8 |

The majority of the data are accumulated around \(10^5(D/H) = 3\). There are two measurements indicating low deuterium about 1.5 and one showing a huge value, 20 ± 5. Taken by the face value, there is an indication that there are systems with high, normal, and low deuterium abundances. If this is true, it would be a confirmation of the model of ref. 25 where spatial variation of primordial abundances are predicted. According to this work, two thirds of the sky should have normal abundance of deuterium, 1/6 should have twice smaller ratio of \(D/H\), and another 1/6 should have about 5 times larger than normal abundance of deuterium. On the other hand, as is argued in ref. 26, the line observed in the \(z = 0.701\) absorption system 24 which indicates a high deuterium abundance most probably is not deuterium. This statement is based on a new measurement of the velocity dispersion of the neutral hydrogen.

3.2. Observations of helium-4

Helium-4 is very strongly binded nuclei so it cannot be destroyed by stellar processes but only produced in stars together with other heavier elements, “metals”, e.g. oxygen (O) and nitrogen (N) - all elements heavier than \(^4\)He are called “metals” in astronomy. Thus the observed mass fraction of \(^4\)He, should be larger than the primordial one, \(Y_p\). \(^4\)He is observed in hot ionized gas regions (H II) in emission optical recombination lines. In contrast to deuterium \(^4\)He has been observed only at relatively small distances, maximum, \(z = 0.045\) 27. This regions are contaminated by stellar processes and to infer primordial abundance from the data one should extrapolate to zero metallicity, i.e. to zero abundances of O or N. A detailed discussion of these issues can be found e.g. in the book 28. The extrapolation to zero metallicity is realized by the linear function with the slope \(\kappa = dY/dZ\) where \(Y\) is the observed mass fraction of \(^4\)He and \(Z\) is that of metals. Analysis by different groups give significantly different values \(\kappa = 6.7 ± 2.3\) 29, 6.9 ± 1.5 30, and \(\kappa = 2.4 ± 1.0\) 31. Though \(\kappa\) is not accurately determined (or it may have different values for different systems), the systems where \(^4\)He is studied have a low metallicity and the net effect on primordial abundance of \(^4\)He is not very strong.

The values of primordial mass fraction of \(^4\)He, \(Y_p\), measured by several different groups are presented in table 2. The discrepancy between the results can be possibly prescribed to different treatment of correction factors: 1) ionization correction factor (icf) which determines how much neutral hydrogen (invisible) is in the object under scrutiny, 2) temperature correction factor (tcf) which describes non-uniform temperature distribution, and 3) density of electrons. A recent analysis of the correction has been performed in ref. 24, where it was shown in particular that combined icf and tcf would diminish the result by 0.002-0.004. It is worth noticing in this connection that the reanalysis of the data of ref. 31 in the paper 33 where icf has been taken into account, has led to a noticeably smaller result (see table 2).

Even the largest mass fraction of \(^4\)He presented in table 2 corresponds to a rather low baryon number density, \(\eta_{10} ≈ 4.2\) which is noticeably smaller than the value determined from CMBR 3. Possibly it means that systematic uncertainties and correction factors are underestimated and more work is necessary to obtain more precise results. This problem is discussed in the recent paper 42. It seems that at the present time the accuracy in determination of primordial mass fraction of \(^4\)He is not sufficiently good.
Table 2
Primingal mass fraction of \(^4\)He according to the results of different groups.

| Ref. | \(Y_p\)       |
|------|---------------|
| 23   | 0.228 ± 0.005 |
| 30   | 0.234 ± 0.002 |
| 32   | 0.2348 ± 0.0025 |
| 31   | 0.2443 ± 0.0015 |
| 33   | 0.238 ± 0.003 |

4. BIG BANG NUCLEOSYNTHESIS AND NON-STANDARD PHYSICS

Many possible modifications of the standard cosmological scenario and/or Minimal Standard Model (MSM) in elementary particle physics can be strongly constrained or even excluded by BBN. As we have already mentioned, BBN permits to restrict the number of neutrino families. The most recent analysis \[30\], based on \(Y_p = 0.238 ± 0.002 ± 0.005\), results in the following 95% CL upper limits:

\[
\Delta N < \begin{cases} 
0.6 & \text{for } \eta_{10} = 5.8 , \\
0.9 & \text{for } \eta_{10} = 2.4.
\end{cases}
\]

\[ (7) \]

though in an earlier paper \[16\] a much more restrictive bound, \(\Delta N < 0.2\), was advocated. At the present stage is safe to assume that \(\Delta N < 1\). Hopefully in the near future one will be able to derive a stronger limit.

Other additional parameters that can influence light element abundances are chemical potentials of different neutrino species, \(\mu_a\), where \(a = e, \mu, \tau\). A possible role of neutrino degeneracy in big bang nucleosynthesis was noted already in the pioneering work by Wagoner, Fowler, and Hoyle \[37\] and after that it was analyzed in a number of papers. A combined analysis of the effect of simultaneous variation of all three chemical potentials on BBN was recently performed in the papers \[38,39\]. Since the effect of positive \(\mu_e\), that diminishes \(n/p\)-ratio, can be compensated by a non-zero \(\mu_\mu\) or \(\mu_\tau\), the limits obtained without any extra information are rather loose, assuming that the mentioned above conspiracy between \(\mu_e\) and \(\mu_\mu, \tau\) exists. However, additional data on the angular spectrum of CMBR permits to obtain \[40\] stronger bounds even if the conspiracy is allowed:

\[-0.01 < \xi_{\nu_e} < 0.2, \quad |\xi_{\nu_e, \nu_e}| < 2.6 \]

\[ (8) \]

These results are derived under assumptions that primordial fraction of deuterium is \(D/H = (3.0 ± 0.4) \cdot 10^{-5}\). Here \(\xi_a = \mu_a/T\) are dimensionless chemical potentials.

These bounds are valid in absence of neutrino oscillations. On the other hand, solar and atmospheric neutrino anomalies present a strong evidence in favor of mixing between \(\nu_e\), \(\nu_\mu\), and \(\nu_\tau\) (see e.g. talks by Y. Totsuka and A. McDonald at this Conference). Oscillations of these neutrinos do not conserve individual leptonic charges and a primordial lepton asymmetries in electronic, muonic, and tauonic charges would be redistributed by the oscillations. In particular, a large \(\mu_\mu\) or \(\mu_\tau\) would create a noticeable \(\mu_e\) of the same sign. An analysis of active-active neutrino oscillations have not yet been accurately performed. Preliminary results \[41\] are \(|\xi_a| < 0.1\) for any flavor \(a = e, \mu, \tau\). Let us note that in the case of vanishing chemical potentials oscillations between active neutrinos would not have a noticeable impact on BBN, because the neutrinos would practically remain in thermal equilibrium independently of oscillations and no deviation from the standard scenario would emerge. There could be a small effect due to spectral distortion of neutrinos by late \(e^+e^-\)-annihilation \[12\]. Oscillations could change the distorted spectrum and might be in principle observable in BBN.

It is still not excluded that sterile companions of active neutrinos exist. Moreover, recent measurements \[13\] seem to confirm creation of \(\bar{\nu}_e\) in \(\bar{\nu}_\mu\) beam. If this is the case, an interpretation of all neutrino data in terms of oscillations demands an existence of one (or several) sterile neutrino(s), \(\nu_s\), mixed with active ones.

In contrast to equilibrium active-active case, mixing of active and sterile neutrinos should have a noticeable impact on BBN. There are three possible effects due to oscillations: 1) production of new relativistic states, \(\nu_s\), and an increase of \(N_e\); 2) distortion of neutrino spectrum, especially important is \(\nu_e\) spectrum; 3) resonance generation of
a large lepton asymmetry from a very small initial leptonic or baryonic one (only the pioneering and most recent papers are quoted; references to many other ones can be found therein).

The effect of creation of additional relativistic species by oscillations is easily estimated and from the condition that BBN allows $\Delta N$ extra neutrino families one can obtain the bound on vacuum mixing angle and the mass difference:

\[
(\delta m^2 / \text{eV}^2) \sin^4 2\theta < \begin{cases} 3.16 \times 10^{-5} (\Delta N)^2 \\ 1.74 \times 10^{-5} (\Delta N)^2 \end{cases}
\]

for $\nu_e \nu_s$ and $\nu_{\mu, \tau} \nu_s$ mixings respectively. This bounds are meaningful only if $\Delta N < 1$.

Discussion of the effects of neutrino spectrum distortion on BBN can be found in ref. [1]. The impact of asymmetry generation on BBN is discussed in ref. [2] (second paper) and in refs. [3]. Resonance case is quite complicated and the effect may have either sign. No simple bounds can be presented here.

BBN permits put stringent bounds on possible time-variation of fundamental constants. If we assume e.g. that the fine structure constant $\alpha$ is different at BBN from its present-day value, $\alpha = 1/137$, we should expect the neutron-proton mass difference should also change with time. The $(n-p)$-mass difference is given by $\delta m \equiv m_n - m_p = m_d - m_u + \alpha m_{em}$, where $m_d$ and $m_u$ are the masses of $u$ and $d$ quarks and the last term describes electromagnetic loop contribution into $\delta m$. As we have seen above the $n/p$-ratio is equal to $n/p = \exp(-\delta m / T_f)$, where $T_f$ is the freezing temperature of reactions (3). This temperature is determined by the competition between the Hubble expansion and weak interaction rates. The latter is proportional to the magnitude of the electroweak coupling constant, which is essentially $\alpha$. Hence $T_f \sim \alpha^{-2/3}$. Demanding that successful results of BBN are not destroyed one can obtain that $(\Delta \alpha / \alpha)_{BBN} < (a few) \times 10^{-2}$.

5. Conclusion

We see that gross features of BBN well agree with observations but the latter are not yet sufficiently accurate to make it really a precise science. Moreover there is a trend to discrepancy between the observations of deuterium which indicate a higher value of $\eta_{10}$ than the observations of $^4\text{He}$. Hopefully it will be clear in a few coming years if this is a real problem or an artifact of systematic and evolutionary effects. We have not discussed above primordial $^7\text{Li}$ because the accuracy of its measurements are rather low now but potentially this element could be very important for verification of the standard model. A recent discussion of $^7\text{Li}$ can be found in ref. [4]. Still even with the existing level of accuracy BBN permit to put powerful constraints on deviations from the standard model. The number of extra neutrino species allowed by the contemporary observations is about unity and with this bound very little can said about mixing parameters between active and sterile neutrinos. However, if $\Delta N$ could be reduced, say, to 0.3 the limit would be meaningful. The restriction on the time $\alpha$ is quite strong and may exclude some of the models predicting such variation.

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