Traceable measurements for beam propagation ratio $M^2$

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Abstract. This paper reports an investigation into measurement techniques and infrastructure suitable for underpinning traceable calibration of devices for beam propagation parameter measurement. The accurate knowledge of the $M^2$ parameter is required for a diverse range of applications including semiconductor manufacture, process control, laser machining, thin-film transistors (TFT), fundamental research into frequency doubling, and the production and characterisation of optical components. Characterisation of the size and position of the beam waist is essential for the correct hazard classification of extended source laser systems using the IEC 60825 series of standards. A system for measuring beam propagation factor measurement has been developed at NPL based on the methodology outlined in the ISO 11146 series of standards. This methodology uses spatial intensity profile measurements of the beam at defined points along the direction of propagation of the beam through a beam waist produced by a convex lens. The beam widths obtained using a CCD and a converging second moment method, are fitted to a hyperbolic propagation envelope and the beam propagation coefficients are obtained from this shape. The traceability of this method has been provided by using graticules calibrated against length standards and carefully measured CCD camera parameters.

1. Introduction
The recent proliferation of laser applications has driven a need to characterise the propagation characteristics of laser beams. These characteristics allow effective prediction of optical system parameters, removing the reliance on “trial and error” to accomplish the requisite performance. A number of commercially available beam propagation measurement systems display results to three significant figures for beam parameters, implying a 1% uncertainty that is not necessarily reflected in practice.

Through consultation with industry, a need for traceability for these devices was established. This need has been met by development of a measurement system which uses the National Physical Laboratory (NPL) primary length standards to provide traceability for beam propagation parameters, and uses the accepted methodology described in the ISO 11146[1][2][3] series of standards.

2. Theory
Figure 1 is a schematic diagram of the measurement method for the determination of the beam propagation ratio and other beam characteristics of lasers. A CCD diode array camera system is placed on a movable carriage in front of the laser source. The relay lens of the camera system allows the CCD to capture a spatial intensity profile of the beam at a particular plane. The beam width is then calculated using a modified second moment technique [1](see Appendix A). In order to allow an
accurate determination of the beam width it is necessary to ensure that vignetting has not occurred. To address this problem a self-converging width measurement technique (CSM)[4] is used to estimate the beam width at each measurement plane and represent the true value to an acceptable level of uncertainty. This measurement is repeated at a number of locations along the test beam axis until enough data points have been obtained to allow the fitting of a hyperbola using a least squares fitting technique. The coefficients of the fitted hyperbola allow the beam propagation parameters of the source to be determined.

![Diagram](https://example.com/diagram.png)

**Figure 1.** Methodology to characterise propagation of laser beams.

If the beam waist is not accessible for direct measurement then a convex lens with minimal aberrations can create an artificial waist. This situation may arise if, for example, the beam waist is formed within the laser or there is insufficient space to perform the required number of measurements either side of the waist. The position and diameter of this artificial waist can then be used, along with the known properties of the transform lens, to calculate the location and size of the original beam waist.

### 3. Beam measurement methods

Measurement of the optical constants of the propagation envelope of a beam [5][6] has been the subject of considerable research over the last 14 years. A consequence of this work is the evolution of ISO standards for the measurement of the diameter and divergence of a beam. ISO 11146:1999. “Test methods for laser beam parameters: beam widths, divergence angle and beam propagation factor” [1]

There are a number of methods available for measurement of the diameter of a beam as well as its far-field divergence. The basic principles for those methods have been established in an ISO standard[1][3]. They are applicable to laser beams with a relatively small beam propagation ratio, $M^2$.

Recent research has demonstrated that adequate steps have to be taken to counter the effects of noise and offset errors when measuring the transverse irradiance distribution of a beam. When these steps are taken, the propagation behaviour of incoherent broadband beams as well as high-quality laser beams can be predicted reproducibly with considerable precision.[3][4][7]

To accurately measure the second moment beam diameter both the number of pixels and the level of digitisation of the signal received on each pixel has to be considered. For beams with a rapidly changing beam diameter the number of bits in the digitisation process becomes more critical. The measurement process establishes a system zero by subtraction of an image produced with the measured beam blocked [3]. Noise on the resultant image acquired by the camera both from electrical and optical sources must be removed by setting a discrimination level, this effectively reduces the
dynamic range of the camera a favouring cameras with an inherently large dynamic range enabled by a larger number of bits in the digitisation electronics.

The methods leading to estimates of the diameter of a beam use a procedure known as the Converging Second Moment (CSM) diameter or width measurement [4]. The schematic of this method is shown in figure 2.

![Diagram of CSM software illustrating the calculation of second moment values.](image)

**Figure 2.** CSM software illustrating the calculation of second moment values.

**4. Description of the system and calibration**

The system layout is deliberately kept as simple as possible to reduce stray light effects and aberrations caused by optical elements. The CCD camera is mounted on an adjustable stage with sufficient degrees of freedom to allow the system to be setup to be collinear with the beam propagation direction. This stage is mounted on a stepper motor driven slide controlled by the measurement system software. An optical encoder is used to allow relative measurement of translation in the beam propagation direction. Before measurements can be made the system is calibrated to NPL length standards. To ensure accuracy in the determination of the beam width the CCD array in combination with the analysis software is calibrated using a photoetched transparent graticule with traceable calibration.

To ensure accurate determination of the position to be measured along the beam waist the stepper motor driven stage fitted with an optical encoder is calibrated against a length measuring interferometer, resulting in a positional accuracy of ±0.01 mm.

![Diagram of M² System layout.](image)

**Figure 4.** M² System layout.
5. Making a measurement

Once the system is aligned and the calibration procedures performed, the following steps are required to predict the relative position of the beam waist with respect to the output aperture of the laser.

a) A combination of ND filters is used to attenuate the beam irradiance so that the full dynamic range of the CCD system is exploited. This is accomplished by locating the position of maximum irradiance in the beam propagation direction (z-direction), and then placing filters in the beam path so that the signal just saturates the CCD pixels. The number of filters used is kept to a minimum by the use of higher neutral densities to replace multiples of lower values. As readings are taken either side of the maximum, the exposure time of the camera can be adjusted to maintain the signal level at the full dynamic range of camera.

b) The image acquisition software is used to capture at least 10 different beam images along the z-direction. Approximately half of the measurements are distributed within one Rayleigh length either side of the beam and approximately half distributed beyond two Rayleigh lengths. Each image has an associated image of background optical noise captured at the same time.

c) The background frame is subtracted from the beam image frame before the digital width analysis process is performed.

d) The corrected image is processed using the convergent second moment (CSM) method to limit the dimensions of the CCD window that is subsequently analysed and hence reduce noise contribution to the second moment evaluation. The CSM values of the beam in the laboratory (CCD array) vertical and horizontal axes are calculated. A cross-moment of the beam distribution in the converged window is used to calculate the azimuth of the principal axes of potentially non-circular distributions. This figure enables the calculation of the dimensions of the beam along its principal axes. The ratio of the principal dimensions (ellipticity), the azimuth angle of the principal axes relative to the laboratory axes and the calibrated linear magnitude of the principal dimensions are recorded. The convergence of the 2nd moment calculations can be seen in figure 3. The program then outputs the final 2nd moment measurements in the X and Y axes.
e) A software routine process is used to discover the best-fit hyperbolic envelope to the propagating beam in each of its principal planes. The coefficients of the hyperbolas are processed to reveal:
   i) locations of the beam waists relative to the vertex of the laser
   ii) transverse dimensions of the waists
   iii) values of the Rayleigh Lengths of the beam along their principal planes
   iv) far-field divergences in those planes

f) If the beam is found to be astigmatic (i.e. the ellipticity of the beam is found to be greater than 1.15 or less than 0.83) and there is a monotonic variation in the azimuth of the principal planes of the propagating beam (twist) then the beam is deemed to suffer from general astigmatism and no further investigation can be justified without a more detailed analysis procedure as outlined in ISO11146-2.

**Figure 3.** An illustration of the CSM process applied to a real spatial intensity profile image.

**Figure 4.** Beam demonstrating general astigmatism.

g) If the beam is identified as stigmatic or simple astigmatic the beam propagation parameters can be calculated.
6. Measurements made using the system

Initial measurements were made using a laser source conditioned to provide a near TEM$_{00}$ Gaussian beam. This source was a flashlamp pumped yttrium lithium fluoride laser (YLF) that had been fibre coupled and then collimated to provide the test beam. The measured $M^2$ ratio using our ISO compliant system was 1.06.

To explore the linearity of the system with respect to higher $M^2$ values a stable laser source within this range was required. The optimal method to achieve this was through the use of a variable $M^2$ source. This was achieved by using a nominally collimated VCSEL (Vertical-Cavity Surface-Emitting Laser) where the control current could be varied. By varying the supply current to the VCSEL it was found that the $M^2$ of the device could be varied. The source was found to have repeatability in power, beam width and hence $M^2$ for a given current. Collimation of the VCSEL output was achieved by the use of a Shack-Hartmann wavefront sensor [8] to allow alignment to be accomplished with minimal aberration. Each source required testing using cylindrical lenses to reject generally astigmatic laser diodes [1].

![Hyperbolic plot giving an $M^2$ of 3.46 achieved using a VCSEL.](image)

**Figure 5.** Hyperbolic plot giving an $M^2$ of 3.46 achieved using a VCSEL.

The VCSEL device was used to test a prototype commercial $M^2$ system against our traceable $M^2$ setup. This has allowed for further improvement of the prototype device by the manufacturer. Sufficient data is required to remove ambiguity in far field divergence and beam waist determination.[9]
Figure 6. Prototype wavefront sensor utilising a multiple distorted diffraction grating [10]

This test meter was compared with the NPL system by utilizing the demonstrated stability of the VCSEL source to allow substitution measurements.

Figure 7. Comparative measurement of test meter vs. NPL ISO 11146 system. [11][12]

7. Conclusion
We have demonstrated a traceable approach to allow calibration of beam propagation ratio measurement devices. The use of a variable M² source has allowed improved linearity calibration of test meters by comparison with the ISO method. This compares to relying on interpolation between discrete M² measurements made with a range of laser sources. The utility of the system has been shown by calibration of prototype commercial devices. The expanded uncertainty achieved at a confidence level of 95% was at the 1% level when M² was near unity and increased monotonically with increasing M². The system could be improved by incorporating the ability to deal with generally
astigmatic sources using Wigner distribution functions [2][3][13]. This would allow the production of a measurement system with an increased range of application.

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9. Appendix A
The reduced second order moments can be determined by a measurement of the energy density distribution over a limited area or window:

\[ \sigma_x^2(z) = \frac{\sum_{y_1}^{y_2} \sum_{x_1}^{x_2} (x - \bar{x})^2 \cdot I(x, y, z)}{\sum_{y_1}^{y_2} \sum_{x_1}^{x_2} I(x, y, z)} \equiv \langle x^2 \rangle \]

\[ \sigma_y^2(z) = \frac{\sum_{y_1}^{y_2} \sum_{x_1}^{x_2} (y - \bar{y})^2 \cdot I(x, y, z)}{\sum_{y_1}^{y_2} \sum_{x_1}^{x_2} I(x, y, z)} \equiv \langle y^2 \rangle \]

where the summations are carried out over a rectangle parallel to the x- and y-axes and:

\[ x_1 = \bar{x} - \frac{3}{2} \frac{d_{\alpha \chi}}{d_{\alpha \chi}} \]
\[ x_2 = \bar{x} + \frac{3}{2} \frac{d_{\alpha \chi}}{d_{\alpha \chi}} \]

and

\[ y_1 = \bar{y} - \frac{3}{2} \frac{d_{\beta \chi}}{d_{\beta \chi}} \]
\[ y_2 = \bar{y} + \frac{3}{2} \frac{d_{\beta \chi}}{d_{\beta \chi}} \]

The concept of second moment measurements is extended to include the “mixed moments” of the spatial and divergence properties of the beam. For example, the spatial mixed moment is:

\[ \sigma_{xy}^2(z) = \frac{\sum_{y_1}^{y_2} \sum_{x_1}^{x_2} (x - \bar{x})(y - \bar{y}) \cdot I(x, y, z)}{\sum_{y_1}^{y_2} \sum_{x_1}^{x_2} I(x, y, z)} \equiv \langle xy \rangle \]

The three spatial moments describe the lateral extent of the power density distribution of the beam in the reference plane. The directions of minimum and maximum extent are called principal axes which are always orthogonal to each other. Any power density distribution is characterized by the extents along its principal axes and the orientation of the principal axes. The beam width along the direction of that principal axis, which is closer to the x-axis of the laboratory system, is given by:

\[ d_{\alpha \chi}(z) = 2\sqrt{2 \left( \left( \langle x^2 \rangle \right)^2 + \left( \langle y^2 \rangle \right)^2 + \left( \langle xy \rangle \right)^2 + 4\left( \langle xy \rangle \right)^2 \right)^{1/2}} \]

and the beam width along the direction of that principal axis, which is closer to the y-axis by:
\[ d_{\varphi}(z) = 2\sqrt{2} \left( \left( x^2 + y^2 \right) \right)^{-\frac{3}{2}} \left[ x^2 - y^2 \right]^{\frac{1}{2}} + 4 \left( xy \right) \]

where \( \gamma = \text{sgn} \left( x^2 - y^2 \right) \)

Finally, the azimuthal angle between the principal axis that is closer to the X-axis and the X-axis is:

\[ \varphi = \frac{1}{2} \arctan \left( \frac{2xy}{x^2 - y^2} \right) \]

10. References

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