String Loop Threshold Corrections for N=1 Generalized Coxeter Orbifolds

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ABSTRACT

We discuss the calculation of threshold corrections to gauge coupling constants for the, only, non-decomposable class of abelian (2,2) symmetric N=1 four dimensional heterotic orbifold models, where the internal twist is realized as a generalized Coxeter automorphism. The latter orbifold was singled out by earlier work as the only N=1 heterotic Z_N orbifold that satisfy the phenonelogical criteria of correct minimal gauge coupling unification and cancellation of target space modular anomalies.
The purpose of this paper is to examine the appearance of one-loop string threshold corrections in the gauge couplings of the four dimensional generalized non-decomposable \( N = 1 \) orbifolds of the heterotic string. In 4D \( N = 1 \) orbifold compactifications the process of integrating out massive string modes, causes the perturbative one-loop threshold corrections\(^1\), to receive non-zero corrections in the form of automorphic functions of the target space modular group. At special points in the moduli space previously massive states become massless and contribute to gauge symmetry enhancement. As a result the appearance of massless states in the running coupling constants appears in the form of a dominant logarithmic term \([1, 2]\).

The moduli dependent threshold corrections of the \( N = 1 \) 4D orbifolds receive non-zero one loop corrections from orbifold sectors for which there is a complex plane of the torus \( T^6 \) left fixed by the orbifold twist \( \Theta \). When the \( T^6 \) can be decomposed into the direct sum \( T^2 \oplus T^4 \), the one-loop moduli dependent threshold corrections (MDGTC) are invariant under the \( SL(2, \mathbb{Z}) \) modular group and are classified as decomposable. Otherwise, when the action of the lattice twist on the \( T_6 \) torus does not decompose into the orthogonal sum \( T_6 = T_2 \oplus T_4 \) with the fixed plane lying on the \( T_2 \) torus, MDGTC are invariant under subgroups of \( SL(2, \mathbb{Z}) \) and the associated orbifolds are called non-decomposable. The \( N = 1 \) perturbative decomposable MDGTC have been calculated, with the use of string amplitudes, in [3]. The one-loop MDGTC integration technique of [3] was extended to non-decomposable orbifolds in [4]. Further calculations of non-decomposable orbifolds involved in the classification list of \( N = 1 \) orbifolds of [5] have been performed in [6].

Here we will perform the calculation of one-loop threshold corrections for the class of \( \mathbb{Z}_8 \) orbifolds, that can be found in the classification list of [5], defined by the Coxeter twist, \( \Theta = \exp[\frac{2\pi i}{8}(1, -3, 2)] \) on the root lattice of \( A_3 \times A_3 \). This orbifold was missing from the list of calculations of MDGTG of non-decomposable orbifolds of [4, 6] and consequently its one-loop moduli dependent gauge coupling threshold corrections were not calculated in [4, 6]. In [7, 8] we found that this orbifold is non-decomposable and it is the only one that possesses this property from the list of generalized Coxeter orbifolds given in [5]. Its twist can be equivalently realized through the generalized Coxeter automorphism \( S_1 S_2 S_3 P_{35} P_{36} P_{45} \) on the root lattice.

Moreover in [8], where a classification list of the non-perturbative gaugino condensation

\(^1\)which receive non-zero moduli dependent corrections from the \( N = 2 \) unrotated sectors.
generated superpotentials and \( \mu \)-terms of all the \( N = 1 \) four dimensional non-decomposable heterotic orbifolds was calculated, its non-perturbative gaugino condensation generated superpotential was given. In this work we will calculate its MDGTC following the technique of \[4\]. Our calculation completes the calculation of the threshold corrections for the classification list of four dimensional Coxeter orbifold compactifications with \( N = 1 \) supersymmetry of \[5\].

The generalized Coxeter automorphism is defined as a product of the Weyl reflections\(^2\) \( S_i \) of the simple roots and the outer\(^3\) automorphisms, the latter represented by the transposition of the roots. An outer automorphism represented by a transposition which exchanges the roots \( i \leftrightarrow j \), is denoted by \( P_{ij} \) and is a symmetry of the Dynkin diagram.

In string theories the one-loop gauge couplings below the string scale evolves according to the RG equation

\[
\frac{1}{g_a^2(p^2)} = \frac{k_a}{g_{M_{\text{string}}}^2} + \frac{b_a}{16\pi^2} \ln \frac{M_{\text{string}}^2}{p^2} + \frac{1}{16\pi^2} \Delta_a, \tag{2}
\]

where \( M_{\text{string}} \) the string scale and \( b_a \) the \( \beta \)-function coefficient of all orbifold sectors. For decomposable orbifolds \[3\] the MDGTC \( \Delta_a \) associated with the gauge couplings \( g_a^{-2} \) corresponding to the gauge group \( G_a \), are determined in terms of the \( N = 2 \) sectors, fixed under both \( (g, h) \) boundary conditions, of the orbifold, namely

\[
\Delta = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \sum_{(g,h)} b_{a}^{(h,g)} Z_{(h,g)}(\tau, \bar{\tau}) - b_{a}^{N=2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2}. \tag{3}
\]

Here, \( b_{a}^{N=2} \) is the \( \beta \)-function coefficient of all the \( N = 2 \) sectors of the orbifold, \( b_{a}^{(h,g)} \), \( Z_{(h,g)} \) the \( \beta \)-function coefficient of the \( N = 2 \) sector untwisted under \( (g, h) \) and its partition function (PF) respectively. The integration is over the fundamental domain \( \mathcal{F} \) of the \( PSL(2, Z) \). For the case of non-decomposable orbifolds the situation is slightly different, namely

\[
\Delta = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \sum_{(g_0,h_0)\in \mathcal{O}} b_{a}^{(h_0,g_0)} Z_{(h_0,g_0)}(\tau, \bar{\tau}) - b_{a}^{N=2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2}. \tag{4}
\]

\(^2\)The Weyl reflection \( S_i \) is defined as a reflection

\[
S_i(x) = x - 2 \frac{x \cdot e_i}{e_i \cdot e_i} e_i,
\]

with respect to the hyperplane perpendicular to the simple root.

\(^3\)an automorphism is called outer if it cannot be generated by a Weyl reflection.
The difference with the decomposable case now is that the sum, in the first integral of (4) is over those $N = 2$ sectors that belong to the $N = 2$ fundamental orbit $O$ and the integration is not over $F$ but over the fundamental domain $\bar{F}$. Because the PF $Z_{(g_0,h_0)}$, for non-decomposable orbifolds, is invariant under subgroups of the modular group i.e $\bar{\Gamma}$, the domain $\bar{F}$ is generated by the action of those modular transformations that generate $\bar{\Gamma}$ from $\Gamma$. In the example that we examine later in this work, $\bar{\Gamma} = \Gamma_0(2)$ and $\bar{F} = \{1,S,ST\}F$. In turn the fundamental orbit $O$ is generated by the action of $\bar{F}$ on the fundamental element of this orbit.

For the orbifold $Z_8$ there are four complex moduli fields. There are three $(1,1)$ moduli due to the three untwisted generations $27$ and one $(2,1)$-modulus due to the one untwisted generation $\bar{27}$. The realization of the point group is generated by

$$Q = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}, \quad (5)$$

If the action of the generator of the point group leaves some complex plane invariant then the corresponding threshold corrections have to depend on the associated moduli of the unrotated complex plane. There are three complex untwisted moduli: three $(1,1)$–moduli and no $(2,1)$-modulus due to the three untwisted $27$ generations and non-existent untwisted $\bar{27}$ generation. The metric $g$ (defined by $g_{ij} = <e_i | e_j>$) has three and the antisymmetric tensor field $B$ an other three real deformations. The equations $gQ = Q^*g$ and $bQ = Q^*b$ determine the background fields in terms of the independent deformation parameters.

Solving the background field equations one obtains for the metric

$$G = \begin{pmatrix}
R^2 & u & v & -u & -2v - R^2 & -u \\
u & R^2 & u & v & -u & -2v - R^2 \\
v & u & R^2 & u & v & -u \\
-u & v & u & R^2 & u & v \\
-2v - R^2 & -u & v & u & R^2 & u \\
-u & -2v - R^2 & -u & v & u & R^2
\end{pmatrix}, \quad (6)$$

\(^4\text{By definition }()^* \text{ mean } ((()^T)^{-1}).\)
with $R, u, v \in \mathbb{R}$ and the antisymmetric tensor field:

$$B = \begin{pmatrix}
0 & x & z & y & 0 & -y \\
-x & 0 & x & z & y & 0 \\
-z & -x & 0 & x & z & y \\
-y & -z & -x & 0 & x & z \\
0 & -y & -z & -x & 0 & x \\
y & 0 & -y & -z & -x & 0 \\
\end{pmatrix},$$

(7)

with $x, y, z \in \mathbb{R}$.

The N=2 orbit is given by these sectors which contain completely unrotated planes, $O = (1, \Theta^4), (\Theta^4, 1), (\Theta^4, \Theta^4)$.

The element $(\Theta^4, 1)$ can be obtained from the fundamental element $(1, \Theta^4)$ by an $S$–transformation on $\tau$ and similarly $(\Theta^4, \Theta^4)$ by an $ST$–transformation. The partition function for the zero mode parts $Z_{\text{torus}}^{(g,h)}$ of the fixed plane takes the following form

$$Z_{\text{torus}}^{(1,\Theta^4)}(\tau, \bar{\tau}, G, B) = \sum_{P \in (\Lambda_N^\perp)^*} q^{{\frac{1}{2}}P_L^2 + \frac{1}{2}P_R^2},$$

$$Z_{\text{torus}}^{(\Theta^4, 1)}(\tau, \bar{\tau}, G, B) = \frac{1}{V_{\Lambda_N^\perp}} \sum_{P \in (\Lambda_N^\perp)^*} q^{{\frac{1}{2}}P_L^2 + \frac{1}{2}P_R^2},$$

$$Z_{\text{torus}}^{(\Theta^4, \Theta^4)}(\tau, \bar{\tau}, G, B) = \frac{1}{V_{\Lambda_N^\perp}} \sum_{P \in (\Lambda_N^\perp)^*} q^{{\frac{1}{2}}P_L^2 + \frac{1}{2}P_R^2} q^{i\pi(P_L^2 - P_R^2)},$$

(8)

where with $\Lambda_N^\perp$ we denote the Narain lattice of $A_3 \times A_3$ which has momentum vectors

$$P_L = \frac{p}{2} + (G - B)w, \quad P_R = \frac{p}{2} - (G + B)w$$

(9)

and $\Lambda_N^\perp$ is that part of the lattice which remains fixed under $Q^4$ and $V_{\Lambda_N^\perp}$ its volume. The lattice in our case is not self dual in contrast with the case of partition functions $Z_{(g,h)}^{\text{torus}}(\tau, \bar{\tau}, g, b)$ of [3]. Stated differently the general result is - for the case of non-decomposable orbifolds - that the modular symmetry group is some subgroup of $\Gamma$ and as a consequence the partition function $\tau_2 Z_{(g,h)}^{\text{torus}}(\tau, \bar{\tau}, g, b)$ is invariant under the same subgroup of $\Gamma$.

The subspace corresponding to the lattice $\Lambda_N^\perp$ can be described by the following winding
and momentum vectors, respectively:

\[
\begin{pmatrix}
  n^1 \\
  n^2 \\
  0 \\
  0 \\
  n^1 \\
  n^2
\end{pmatrix}, \quad n^1, n^2 \in \mathbb{Z}
\quad \text{and} \quad
\begin{pmatrix}
  m_1 \\
  m_2 \\
  -m_1 \\
  -m_2 \\
  m_1 \\
  m_2
\end{pmatrix}, \quad m_1, m_2 \in \mathbb{Z}.
\]

They are determined by the equations \( Q^4 w = w \) and \( Q^* p = p \). The partition function \( \tau_2 Z^{torus}_{(1, \Theta^4)}(\tau, \bar{\tau}, g, b) \) is invariant under the group \( \Gamma_0(2) \), congruence subgroup of \( \Gamma \). Before we discuss the calculation of threshold corrections let us give some details about congruence subgroups. The homogeneous modular group \( \Gamma' \equiv SL(2, \mathbb{Z}) \) is defined as the group of two by two matrices whose entries are all integers and the determinant is one. It is called the "full modular group and we symbolize it by \( \Gamma' \). If the above action is accompanied with the quotient \( \Gamma \equiv PSL(2, \mathbb{Z}) \equiv \Gamma' / \{ \pm 1 \} \) then this is called the 'inhomogeneous modular group' and we symbolize it by \( \Gamma \). The fundamental domain of \( \Gamma \) is defined as the set of points which are related through linear transformations \( \tau \rightarrow \frac{a\tau + b}{c\tau + d} \). If we denote \( \tau = \tau_1 + \tau_2 \) then the fundamental domain of \( \Gamma \) is defined through the relation \( \mathcal{F} = \{ \tau \in \mathbb{C} | \tau_2 > 0, |\tau_1| \leq \frac{1}{2}, |\tau| \geq 1 \} \). One of the congruence subgroup of the modular group \( \Gamma \) is the group \( \Gamma_0(n) \). The group \( \Gamma_0(2) \) can be represented by the following set of matrices acting on \( \tau \) as \( \tau \rightarrow \frac{a\tau + b}{c\tau + d} \):

\[
\Gamma_0(2) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid ad - bc = 1, (c = 0 \mod 2) \right\}
\]

(11)

It is generated by the elements \( T \) and \( ST^2S \) of \( \Gamma \). Its fundamental domain is different from the group \( \Gamma \) and is represented from the coset decomposition \( \tilde{\mathcal{F}} = \{ 1, S, ST \} \mathcal{F} \). In addition the group has cusps at the set of points \( \{ \infty, 0 \} \). Note that the subgroup \( \Gamma^0(2) \) of \( SL(2, \mathbb{Z}) \) is defined as with \( b = 0 \mod 2 \).

The integration of the contribution of the various sectors \( (g, h) \) is over the fundamental domain for the group \( \Gamma_0(2) \) which is a three fold covering of the upper complex plane. By taking into account the values of the momentum and winding vectors in the fixed directions
we get for $Z_{(1,\Theta)}^{\text{torus}}$

$$Z_{(1,\Theta)}^{\text{torus}}(\tau, \bar{\tau}, g, b) = \sum_{(P_L,P_R) \in \Lambda_N^2} q^{\frac{1}{2}P_L^tG^{-1}P_L + \frac{1}{2}P_R^tG^{-1}P_R} \sum_{p,w} e^{2\pi i p^t w - \pi \tau_2 (\frac{1}{4}p^t G^{-1} p - 2p^t G^{-1} B w + 2w^t G w - 2w^t B G^{-1} B w - 2p^t w)}.$$  \hspace{1cm} (12)

Consider now the following parametrization of the torus $T^2$, namely define the $(1,1)$ $T$ modulus and the $(2,1)$ $U$ modulus as:

$$T = T_1 + iT_2 = 2(b + \sqrt{\det g_\perp}),$$
$$U = U_1 + iU_2 = \frac{1}{\sqrt{\det G}} (G_{112} + \sqrt{\det G}),$$  \hspace{1cm} (13)

where $g_\perp$ is uniquely determined by $w^t G w = (n_1^1 n_2^1) G (n_1^1 n_2^1)$ and $b$ the value of the $B_{12}$ element of the two-dimensional matrix $B$ of the antisymmetric field. This way one gets

$$T = 4(x - y) + i 8 v,$$
$$U = i.$$  \hspace{1cm} (14, 15)

Even if we have said that we expect that this $Z_8$ orbifold does not have a $h^{(2,1)}$ U-modulus field, the $T^2$ torus has a $U$-modulus. However its value for the $Z_8$ orbifold is fixed. The partition function $Z_{(1,\Theta)}^{\text{torus}}(\tau, \bar{\tau}, g, b)$ takes now the form

$$Z_{(1,\Theta)}^{\text{torus}}(\tau, \bar{\tau}, T, U) = \sum_{m_1, m_2 \in \mathbb{Z}} \sum_{n_1, n_2 \in \mathbb{Z}} e^{2\pi i (m_1 n_1 + m_2 n_2)^t |T Un^2 + Tn^1 - Um_1 + m_2|^2}.$$  \hspace{1cm} (16)

By Poisson resummation on $m_1$ and $m_2$, using the identity:

$$\sum_{p \in \Lambda} e^{-\pi(p+\delta)^t C (p+\delta) + 2\pi i p^t \phi} = V_A^{-1} \frac{1}{\sqrt{\det C}} \sum_{l \in \Lambda} e^{-\pi(l+\phi)^t C^{-1} (l+\phi) - 2\pi i \delta^t (l+\phi)},$$  \hspace{1cm} (17)

we conclude

$$\tau_2 Z_{(1,\Theta)}^{\text{torus}}(\tau, \bar{\tau}, T, U) = \frac{1}{4} \sum_{\mathcal{M} \in \mathcal{M}} e^{-2\pi i T \det A} T_2 e^{-\pi T_2 (1, U) A (1, U)^t},$$  \hspace{1cm} (18)

where

$$\mathcal{M} = \begin{pmatrix} n_1 & \frac{1}{2} l_1 \\ n_2 & \frac{1}{2} l_2 \end{pmatrix}$$  \hspace{1cm} (19)
and $n_1, n_2, l_1, l_2 \in \mathbb{Z}$.

From (18) one can obtain $\tau Z_{\theta^4,1}^{torus}(\tau, \bar{\tau})$ by an $S$-transformation on $\tau$. After exchanging $n_i$ and $l_i$ and performing again a Poisson resummation on $l_i$ one obtains

$$Z_{\theta^4,1}^{torus}(\tau, \bar{\tau}, T, U) = \frac{1}{4} \sum_{m_1, m_2 \in \mathbb{Z}} e^{2\pi i \tau (m_1 \frac{T^1}{2} + m_2 \frac{T^2}{2})} e^{\frac{\pi i}{2T^2} |T U^1 + T^2 U^2 - U m_1 + m_2|^2}. \quad (20)$$

The factor 4 is identified with the volume of the invariant sublattice in (20). The expression $\tau Z_{\theta^4,1}^{torus}(\tau, \bar{\tau}, T, U)$ is invariant under $\Gamma^0(2)$ acting on $\tau$ and is identical to that for the $(\Theta^4, \Theta^4)$ sector.

Thus the contribution of the two sectors $(\Theta^4, 1)$ and $(\Theta^4, \Theta^4)$ to the coefficient $b^{N=2}_a$ of the $\beta$–function is one fourth of that of the sector $(1, \Theta^4)$, thus

$$b^{N=2}_a = \frac{3}{2} b^{(1, \Theta^4)}_a. \quad (21)$$

The coefficient $b^{N=2}_a$ is the contribution to the $\beta$ functions of the $N = 2$ orbit. Including the moduli dependence of the different sectors, we conclude that the final result for the threshold correction to the inverse gauge coupling reads

$$\Delta_a(T, \bar{T}, U, \bar{U}) = -b^{N=2}_a \ln \left| \frac{8\pi e^{1-\gamma_E}}{3\sqrt{3}} T^2 \eta \left( \frac{T}{2} \right) \right|^4 |U_2\eta((U)|^4. \quad (22)$$

The value of $U_2$ is fixed and equal to one as can be easily seen from eqn.(15). In general for $Z_N$ orbifolds with $N \geq 2$ the value of the $U$ modulus is fixed. The final duality symmetry of (22) is $\Gamma^0(2)$ with the value of $U$ replaced with the constant value 1.

Let us use (4), (22) to deduce some information about the phenomenology of the $Z_8$ orbifold considered in this work. We want to calculate the one-loop corrected string mass unification scale $M_X$, that is when two gauge group coupling constants become equal, i.e. $\frac{1}{k_a g_a} = \frac{1}{k_b g_b}$. We further assume that the gauge group of our theory at the string unification scale if given by $G = \oplus G_i$, where $G_i$ a gauge group factor. Taking into account (22) we get

$$M_X = M_{string}[T^2 \eta \left( \frac{T}{2} \right)^4 |U_2\eta(U)|^4]^{-\frac{b^{N=2}_a - b^{N=2}_b}{2(k_a g_a - k_b g_b)}},$$

$$M_{string} \approx 0.7 g_{string} 10^{18} \text{ GeV}. \quad (23)$$

where $k_i$ the Kac-Moody level associated to the gauge group factor $G_i$. 

We will give now some details about the integration in (4) of the integral that we used so far to derive (22). The integration of eqn. (16) is over a \( \Gamma_0(2) \) subgroup of the modular group \( \Gamma \) since (16) is invariant under a \( \Gamma_0(2) \) transformation \( \tau \to \frac{a \tau + b}{c \tau + d} \) (with \( ad - bc = 1, c = 0 \) mod 2). Under a \( \Gamma_0(2) \) transformation (16) remains invariant if at the same time we redefine our integers \( n_1, n_2, l_1 \) and \( l_2 \) as follows:

\[
\begin{pmatrix}
  n'_1 & n'_2 \\
  l'_1 & l'_2
\end{pmatrix} = \begin{pmatrix}
a & c/2 \\
2b & d
\end{pmatrix} \begin{pmatrix}
n_1 & n_2 \\
l_1 & l_2
\end{pmatrix}
\]

(24)

The integral can be calculated based on the method of decomposition into modular orbits. There are three sets of inequivalent orbits under the \( \Gamma_0(2) \), namely:

a.) The degenerate orbit of zero matrices, where after integration over \( \bar{\mathcal{F}} = \{1, S, ST\} \mathcal{F} \) gives as a total contribution \( I_0 = \pi T_2/4 \).

b.) The orbit of matrices with non-zero determinants. The following representatives give a non-zero contribution \( I_1 \) to the integral:

\[
\begin{pmatrix}
k & j \\
0 & p
\end{pmatrix}, \quad \begin{pmatrix}
0 & -p \\
k & j
\end{pmatrix}, \quad \begin{pmatrix}
0 & -p \\
k & j + p
\end{pmatrix}, \quad 0 \leq j < k, \quad p \neq 0 ,
\]

(25)

where \( I_o + I_1 = -(3/2) \cdot 4 \Re \ln \eta(\frac{T}{2}) \).

c.) The orbits of matrices with zero determinant,

\[
\begin{pmatrix}
0 & 0 \\
j & p
\end{pmatrix}, \quad \begin{pmatrix}
j & p \\
0 & 0
\end{pmatrix}, \quad j, p \in \mathbb{Z}, \quad (j, p) \neq (0, 0) .
\]

(26)

The first matrix in (26) has to be integrated over the half–band \( \{ \tau \in \mathbb{C} \; \tau_2 > 0 , \; |\tau_1| < h \} \) while the second matrix has to be integrated over a half–band with the double width in \( \tau_1 \). The total contribution from the modular orbit \( I_3 \) gives,

\[
I_3 = -4 \Re \ln \eta(U) - \ln (T_2 U_2) + \left( \gamma_E - 1 - \ln \frac{8 \pi}{3 \sqrt{3}} \right)
- \frac{1}{2} \times 4 \Re \ln \eta(U) - \frac{1}{2} \times \ln (T_2 U_2) + \frac{1}{2} \times (\gamma_E - 1 - \ln \frac{8 \pi}{3 \sqrt{3}}).
\]

Putting \( I_o, I_1, I_2 \) together we get (22).

All \( N = 1 \) four dimensional orbifolds have been tested in [3] as to whether they satisfy several phenomenological criteria, involving
a) correct unification of the three gauge coupling constants at a scale $M_X \approx 10^{16}$ Gev, assuming the minimal supersymmetric, Standard Model gauge group $G = SU(3) \times SU(2) \times U(1)$, particle spectrum with a SUSY threshold close to weak scale, in the two cases of i) a single overall modulus in the three complex planes $T = T_1 = T_2 = T_3$ and ii) the anisotropic squeezing case $T_1 >> T_2, T_3$,

b) anomaly cancellation with respect to duality transformations of the moduli in the planes rotated by all the orbifold twists.

The only orbifold from this study that satisfy all the phenomenological criteria, set out by Ibáñez and Lüst, is the $Z_8$ orbifold that we examined in this work.

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