On the uncertainty factor in approximation of the heat transfer coefficient in the problem of modelling the interaction of meteor body with the atmosphere

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Abstract. The problem of modelling the interaction of meteor body with the Earth’s atmosphere is considered. The motion, ablation, and energy deposition of the meteoroid and its fragments moving as a single body are modelled by numerically solving the meteor physics equations. For the heat transfer coefficient, which is a governing parameter in these equations, the proposed approximate expression is used, depending on the meteoroid velocity and size and the atmospheric density, as well as a constant value accepted in the literature. An uncertainty parameter is introduced into the formula for the heat transfer coefficient to make allowance for the existing uncertainty in the radiative heat flux computations in the meteoric range of flow parameters. The influence of uncertainty of the heat transfer coefficient on the meteoroid mass loss, velocity, trajectory, and energy deposition is evaluated.

1. Introduction
The high velocities of cosmic bodies entering the Earth’s atmosphere, significantly exceeding the spacecraft velocities, determine the specific nature of their interaction with it. One of the main processes affecting the interaction of cosmic objects with the atmosphere is their ablation. Due to strong heating, mostly radiative for large bodies, the meteoroid and its fragments (in the case of destruction) lose most of their mass. Most studies on radiation gas dynamics consider the heating of reentry spacecraft and probes in the range of flow parameters limited by the values of velocities and sizes characteristic of this problem. This is one of the difficulties when modelling the ablation of cosmic bodies. The other one is following. To solve the meteor physics equations, which determine the meteoroid deceleration due to aerodynamic drag, and the mass loss due to intense heating, it is necessary not only to know the heat flux, but also to represent it in the form of analytical dependence on the body velocity and size, and the atmospheric density. Overview of correlation formulae for the radiative heat flux in the range of parameters typical for spacecraft, at velocities less than 12 km/s, is given in [1]. In a wider range of flow parameters, approximation formulae for the radiative heat flux at the stagnation point on a non-destructive surface are given in [2].

In recent years, in connection with the actualization of the problem of the asteroid hazard after the Chelyabinsk event, numerical approaches have been developed to calculate the radiative heat flux for the flow parameters characteristic of the atmospheric entry of large meteoroids, taking into account the effect of ablation on the flow field in the shock layer [3–5]. The numerical model [5] was applied to model the radiation flux reaching the ground in the case of the Tunguska event [6]. Note that the
results of calculating the radiative heat flux by different authors in the case of high velocities and sizes and low altitudes are very different from each other [5]. There is considerable uncertainty in the results of radiative heating simulations because of many factors are not taken into account in computations or are not known. For example, estimates of the study [7] showed that the uncertainty in calculations of the radiative heat flux at the stagnation point for a spacecraft entering the Earth’s atmosphere at a velocity of 15 km/s is approximately of +81 and −52%.

Due to insufficient data on the heat flux in the meteoric range of parameters and its uncertainty, in almost all studies that solve the meteor physics equations, the heat transfer coefficient is assumed to be constant (or a step function). Sometimes it is adjusted so that the calculation results are consistent with observations of the flight and luminosity of cosmic bodies, and sometimes it is set arbitrary, most often equal to 0.1.

Here we use for the heat transfer coefficient both a constant value and an approximate expression depending on the meteoroid velocity and size, and atmospheric density, and introduce in this expression the uncertainty factor. To model the interaction of a meteoroid and its fragments (after destruction) with the atmosphere the meteor physics equations are numerically solved. The effect of the heat transfer coefficient uncertainty on the results of modelling the meteoroid mass loss, velocity, trajectory and energy deposition is studied for various values of the initial meteoroid mass.

2. Equations and heat transfer coefficient approximation

The interaction of a meteor body with the atmosphere is modeled in the framework of the generalized meteor physics equations [8]. The equations of motion (deceleration), ablation (mass loss) and trajectory of the meteoroid moving as a single body, and the relation for the isothermal atmosphere have the form

\[
\frac{dV}{dt} = -\frac{\pi}{2} R_0^2 C_D \rho V^2 + M g \sin \theta, \quad \frac{dM}{dt} = -\frac{\pi}{2} R_0^2 C_H \rho V^3,
\]

\[
\frac{dh}{dt} = -V \sin \theta, \quad \frac{d\theta}{dt} = \frac{g \cos \theta}{V} \cdot \frac{V \cos \theta}{R_0 + h}, \quad \rho = \rho_0 \exp \left( -\frac{h}{h'} \right).
\]

Here \( t \) is the time, \( V, M \) and \( R_0 \) are the meteoroid velocity, mass and midsection radius, \( \theta \) is the angle of inclination of the trajectory relative to the horizon (to the tangent to the Earth’s surface), \( C_D \) is the drag coefficient, \( C_H \) is the heat transfer coefficient per unit midsection area, \( Q \) is the effective heat of mass loss, \( g \) is the gravitational acceleration, \( R_0 \) is radius of the Earth, \( \rho \) is the atmospheric density, \( \rho_0 = 1.29 \text{ kg/m}^3, h' = 7 \text{ km} \). Equations (1) differ from the equations of the simple physical meteor theory [9], in that they take into account the gravity force and the curvilinearity of the trajectory, i.e., change of the angle \( \theta \).

The meteor body kinetic energy \( E \) deposited along its trajectory per unit height is determined by the equation

\[
\frac{dE}{dh} = -\frac{1}{V \sin \theta} \frac{d}{dt} \left( \frac{MV^2}{2} \right) = -\frac{1}{V \sin \theta} \left( \frac{V^2}{2} \frac{dM}{dt} + MV \frac{dV}{dt} \right).
\]

In this paper, it is assumed that the meteor body has a spherical shape, the drag coefficient \( C_D \) for the sphere is set equal to 1. The governing parameter in equations (1) is the heat transfer coefficient \( C_{Ht} \). At high velocities typical of meteor bodies, the radiative heat flux significantly exceeds the convective flux over most of the trajectory in the continuum flow regime, where ablation of large bodies entering the atmosphere occurs. However, we include the convective heat flux, because it plays the key role for small fragments in the last part of their trajectory, when the body is significantly decelerated.

For the radiative heat transfer coefficient \( C_{Hr} \) we use the approximate formula

\[
C_{Hr} = \varphi C_{Hr0}(V, R, \rho).
\]
Here \( C_{h0} \) is the radiative heat transfer coefficient at the stagnation point of a spherical body with an indestructible density \( \rho \) and is a function of the velocity \( V \), the nose radius \( R \), and the atmospheric density \( \rho \). Parameter \( \phi \) characterizes change of the radiative heat flux along the surface and is set equal to 0.7. For \( C_{h0} \), we use the following formula, which is some modification of the correlation obtained in [2]

\[
C_{h0}(V, R, \rho) = \frac{2 \times 10^{-3} q}{\rho V^3}, \quad q = 0.5(q_1(V, R, \rho) + q_2(V, R, \rho)) K_1(V) K_2(R) K_3(\rho),
\]

\[
K_1(V) = \begin{cases} 
1, & V \geq 14 \\
\left(\frac{14}{V}\right)^2, & V < 14
\end{cases}, \quad
K_2(R) = \begin{cases} 
1 - 0.22 \frac{2^{1/3}}{1.5}, & R \geq 8.94 \\
1 - 0.17 \left| \frac{\rho^*}{\rho} \right|^{1/3}, & R < 8.94
\end{cases}, \quad
K_3(\rho) = \begin{cases} 
1 - 0.22 \log \rho^* - 0.7, & \rho^* > 0.2 \\
1 - 0.17 \log \rho^* - 0.5, & \rho^* \leq 0.2
\end{cases}.
\]

Here \([\rho] = \text{kg/m}^3, [R] = \text{m}, [V] = \text{km/s}, [q] = \text{Wt/cm}^2\). The functions \( q_1 \) and \( q_2 \) are approximations of numerical calculations of the radiative heat flux of [10] \( (q_1) \) and of [11] \( (q_2) \) and are given in [2]; \( \rho^* \) is the atmospheric density at an altitude of \( 50 + 15(V - 12)/V \) km. In the formula (4), in comparison with [2], improving corrections are made for \( V \) and \( \rho \) to extend the range of its applicability to lower velocities and lower altitudes.

For the convective heat transfer coefficient per unit midsection area \( C_{hc} \), we use the formula [12] obtained taking into account results of the study [13]

\[
C_{hc} = 3.6 \times 10^{-4} (R \rho)^{-1/2} K, \quad K = \begin{cases} 
\frac{1 - \sigma^*}{1 + 0.3(V^2 - 8.94^2)/8} + \sigma^*, & V \geq 8.94 \\
1, & V < 8.94
\end{cases}, \quad \sigma^* = 0.2.
\]

For the total heat transfer coefficient \( C_h \) we use the following formula

\[
C_h = \eta(C_{h0} + C_{hc}) \quad \text{(6)}.
\]

The uncertainty parameter \( \eta \) is introduced into the expression (6) for \( C_h \) to estimate the effect of the heat transfer coefficient uncertainty on the results of modelling the meteoroid mass loss, velocity, trajectory, and energy deposition. The \( C_h \) uncertainty is due to unaccounted influence on the heat flux of the precursor absorption, turbulence, absorption by a meteoroid vapour layer, uncertainty in the optical properties of hot air and vapors, inaccuracy of the models of radiation transport and flow field, and other factors.

3. Results and discussion

Equations (1), (2) were solved by the Runge-Kutta method using formulas (3) – (6) for the heat transfer coefficient \( C_h \) to simulate the meteoroid mass loss, velocity, energy deposition, trajectory angle relative to the horizon and location of falling non-evaporated mass (meteorite) on the ground. Calculations were carried out under different initial conditions, the values of the uncertainty parameter \( \eta \) varied from 1.2 to 0 (no ablation). The calculation results at the values of the initial meteoroid mass \( M_e = 1.2 \times 10^4 \text{ kg}, 1.2 \times 10^5 \text{ kg}, 1.2 \times 10^6 \text{ kg} \) (the initial radius \( R_e = 0.954 \text{ m}, 0.443 \text{ m}, 0.206 \text{ m} \)) are shown in Figures 1 and 2. Other parameters corresponding the presented results are: the initial meteoroid velocity \( V_e = 19 \text{ km/s} \), its bulk density \( \delta = 3.3 \times 10^3 \text{ kg/m}^3 \), the angle of entry into the atmosphere \( \theta_e = 18^\circ \), the effective heat of mass loss \( Q \) is set equal to 6 km/s². The conditions for \( \theta_e, \delta \) and \( V_e \) correspond to the conditions for the entry of the Chelyabinsk asteroid into the atmosphere [14], and bodies of different initial masses can be considered as hypothetical fragments after its destruction at an altitude of about 50 km.

The heat transfer coefficient \( C_{hc} \), the meteoroid normalized mass \( M/M_e \), the normalized velocity \( V/V_e \), the energy deposition \( dE/dh \) and the trajectory angle \( \theta \) calculated at different values of the
parameters $M_e$ and $\psi$, are shown in Figure 1. Calculation results obtained at constant $C_H$ values are also given for comparison in the case of $M_e = 1.2 \times 10^3$ kg.

Figure 1. Heat transfer coefficient $C_H$, meteoroid trajectory angle relative to the horizon $\theta$, normalized mass $M/M_e$, normalized velocity $V/V_e$, energy deposition $dE/dh$ at various values of the uncertainty parameter $\psi$. $M_e = 1.2 \times 10^2$ kg (left column), $1.2 \times 10^3$ kg (middle), $1.2 \times 10^4$ kg (right).
The top graphs in Figure 1 describe a behavior of the heat transfer coefficient $C_H$ along the meteoroid trajectory. With increase in body size, the radiative part of the coefficient $C_H$, which dominates on the first part of the meteoroid trajectory, also increases. However, on the last part of the trajectory, when the meteoroid is significantly decelerated, the convective part of $C_H$ dominates, which decreases with increasing size. On the first part of the trajectory, when the heat flux is mainly radiative, $C_H$ changes slightly. But at some altitude, which depends on the body size, because small fragments are decelerated faster, the radiative part of $C_H$ begins to decrease rapidly due to essential decrease of the velocity, and the convective part becomes the main one.

The next two series of graphs in Figure 1 show the effect of the heat transfer coefficient on the trajectory angle relative to the horizon $\theta$ and on the meteoroid velocity. The trajectory angle $\theta$ first decreases very slowly and slightly with decreasing the meteoroid velocity until it reaches 7.9 km/s. With a further decrease of the velocity, the angle $\theta$ begins to rapidly increase, and the trajectory is noticeably curved at low altitudes. The smaller fragment mass, the earlier it is decelerated, and the greater the altitude, where a significant increase of the angle $\theta$ begins. The altitude where the angle $\theta$ starts to grow depends also on the value of the uncertainty factor $\psi$. The higher the heat transfer coefficient, the earlier the trajectory angle begins to grow. The smallest, but noticeable effect of a change in the heat transfer coefficient has on the deceleration process, on the meteoroid velocity decrease.

The uncertainty of the heat transfer coefficient has the greatest influence on the meteoroid mass loss, which is natural, because $C_H$ is directly included in the ablation equation. The difference between the mass loss calculations at different values of the factor $\psi$ is slightly larger for larger bodies.

The lower graphs in Figure 1 demonstrate the effect of the heat transfer coefficient on the meteoroid energy deposition along the trajectory. Energy deposition or a part of it, observed as luminosity, is the main information about a cosmic body entering the atmosphere, received by the ground-based and satellite observation systems. The magnitude of energy deposition directly depends on the body mass, which is natural. With an increase of initial meteoroid mass, the maximum of energy deposition (peak of the bolide brightness) is attained at the lower altitude. Increase of the heat transfer coefficient gives a greater altitude of the energy deposition maximum, i.e., an earlier peak brightness, and a greater magnitude of this peak. The distance between the predicted peak brightness altitudes at the considered values of $M_e$ is up to 8–10 km in the range of parameter $\psi$ from 1.2 to 0 and up to 4 km in the most reasonable range of $\psi$ from 1.2 to 0.4. The peak magnitude changes up to 10% in this range of $\psi$.

![Figure 2. Meteoroid trajectory: correlation between the flight altitude $h$ and the distance along the horizontal Earth's surface $l$ ($l=0$ at $h=50$ km) at various values of the parameter $\psi$.](image)

Calculations were also carried out at constant $C_H$ values. We chose $C_H = 0.1$ as the most commonly used in the literature and $C_H = 0.054$, corresponding to the value of the heat transfer coefficient at $\psi = 0.8$ and $M_e = 1.2 \times 10^3$ kg on the first part of the trajectory, where it changes slightly. Figure 1 shows
that calculation results obtained with a well-chosen constant \( C_H = 0.054 \) are in satisfactory agreement with results obtained with using formulas (3) – (6) at altitudes, corresponding to the first part of the trajectory, until \( C_H \) begins to rapidly decrease. At lower altitudes, at the last part of trajectory, there is a significant difference in the results of calculating the meteorite mass loss, the trajectory angle, the energy deposition and the location of meteorite fall obtained using constant \( C_H \) value and formulas (3) – (6).

The influence of the heat transfer coefficient (uncertainty parameter \( \psi \)) on the calculated trajectories of meteor bodies with initial masses of 1200 and 120 kg is shown in Figure 2. The distance along the horizontal Earth's surface \( l \) determines the location of falling non-evaporated mass (meteorite) on the ground, initial value of \( l \) is set equal to 0 at \( h = 50 \) km. Fragments with lower mass are decelerated faster and fall to the ground before fragments with higher mass; fragments with small masses approach the earth's surface at an angle close to 90\(^\circ\). Figure 2 shows that the calculated location of the meteoroid fragment falling on the ground depends on the setting of the heat transfer coefficient. The distance between the predicted locations of meteorite fall at the considered values of \( M_f \), can be up to 5 km and up to 8–9 km when using values of parameter \( \psi \) from 1.2 to 0.4 and from 1.2 to 0 or more when using \( C_H = 0.1 \). This should be taken into account when predicting approximate locations of meteorites fall.

4. Conclusions
The effect of the uncertainty of the heat transfer coefficient on various characteristics of the interaction of a meteoroid or its fragments moving as a single body with the atmosphere is estimated. It is shown that the setting of the heat transfer coefficient has a significant influence on the calculation of such practically important characteristics as the energy deposition of the meteoroid along the trajectory, its terminal mass and the locations of meteorites fall.

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