Interferometric determination of thermal expansion coefficient of piston/cylinder unit – preliminary investigation

L Grgec Bermanec¹ and M Katic²

¹ Croatian Metrology Institute/Faculty of Mechanical Engineering and Naval Architecture, Laboratory for Process Measurement, 10000 Zagreb, Croatia
² Croatian Metrology Institute/Faculty of Mechanical Engineering and Naval Architecture, Laboratory for Length Measurement, 10000 Zagreb, Croatia

E-mail: lovorka.grgec@fsb.hr

Abstract. This paper describes laboratory method for experimental determination of the pressure balance piston and cylinder coefficients of thermal expansion (CTE) using interferometric technique with internally developed software. Measurements were carried out on one gas standard piston/cylinder units in the Croatian National Length (HMI/FSB-L PMD) and Pressure Laboratory (HMI/FSB-LPM). Measurement method, equipment, procedure and CTE results are given, as well as an evaluation of the measurement uncertainty. Results were compared with other relevant data from literature.

1. Introduction

Coefficients of thermal expansion (CTE) are defined as the fractional increase in length per unit rise in temperature and are material properties of the piston and cylinder used in pressure measurements. The effective area of a piston-cylinder changes with temperature depending on the thermal expansivity of the piston-cylinder materials. The magnitude of this change with temperature, thermal expansion coefficient $\alpha + \beta$ is usually given in the manufacturer technical documentation or calibration certificate and it is not measured. Standard piston–cylinder units used in Croatian National Pressure Laboratory – HMI/FSB-LPM are made from tungsten carbide and stainless steel. The materials of the units and the CTE values for piston and cylinder ($\alpha + \beta$) are given in Table 1. The reference temperature for pressure measurements at LPM is 20°C.

| Piston-cylinder unit | Piston material | Cylinder material | $\alpha + \beta$ in 10⁻⁶ °C⁻¹ |
|----------------------|-----------------|-------------------|-----------------------------|
| TLVAG-09             | Tungsten carbide| Tungsten carbide  | 9.0 ± 1.0                   |
| TLVAG-08             | Steel           | Tungsten carbide  | 15.5 ± 1.5                 |

The uncertainties of $\alpha + \beta$ coefficients from Table 1 affects the uncertainty in the calculation of the cross sectional area when there are deviations from the reference temperature. For tungsten carbide pistons and cylinders whose expansion coefficients are 4.5 -10⁻⁶ per °C change in effective area per °C is 9 ppm, therefore , the effect of an uncertainty in CTE of 10% is an uncertainty of ≈ 0.9 ppm in pressure [1], [2].

Effective area ($A_{tp}$) at the piston-cylinder temperature $t$ and the measured pressure $p$ is expressed by the following equation [3]:

$$A(t; p) = A_0(20; 0)[1 + (\alpha + \beta)(t - 20 °C)] \cdot (1 + \lambda \cdot p) \quad (1)$$
Where \((\alpha + \beta)\) is the superficial thermal expansion coefficient of the piston-cylinder unit and \(\lambda\) is the distortion coefficient.

The temperature dependence of the effective area \(A_e(t)\) of the pressure balance is described by the equation:

\[
A_e(t) = A_0(tr)[1 + (\alpha + \beta)(t - tr)]
\]

(2)

Where \(A_0(t_r)\) is effective area of assembly measured at reference temperature \(t_r\), \(\alpha_k\) and \(\alpha_c\) are CTEs of piston and cylinder, and \(t\) is actual temperature of piston-cylinder assembly.

The following equation results from application of theory of linear temperature expansion derived using theory of zero-order pressure scales:

\[
A_e(t) = \pi \cdot \left[ r(t) + R(t) \right]^2
\]

(3)

Coefficients \(\alpha\) and \(\beta\) are defined by the following expressions:

\[
r(t) = r(t_r)[1 + \alpha(t - 20^\circ C)] \quad \text{and} \quad R(t) = R(t_r)[1 + \beta(t - 20^\circ C)]
\]

(4)

After substitution into eq. 3 and by using eq. 2, and developing resulting relation into a series, the following expression is obtained:

\[
A_e(t) \approx A_0(t_r) \left[ 1 + (t - t_r) \left( \frac{2\alpha r(t_r) + 2\beta R(t_r)}{r(t_r) + R(t_r)} \right) \right]
\]

(5)

2. CTE measurement setup

Interferometric measurement of CTE was applied in this work, where CTE is calculated from changes in length (measured using an interferometer) which result from measured changes in temperature. This is similar in principle to measurement of gauge block length (Figure 2), where a gauge block is placed on an optical flat and phase difference between reflections from optical flat and top of gauge block is measured. Typically, gauge block CTE is known in advance and is used to correct the length of gauge block to 20 °C. In this case, the length of piston is assumed to be known, and changes in its length at different temperatures are used to measure its CTE.
Analysis of interferogram data for gauge block measurement is well known (Figure 2), and can be directly applied to the setup proposed in this research. Since only differences in piston length (at different temperatures) need to be known, only fringe fractions for each temperature were measured from interferograms. Integer number of half-wavelengths was calculated from initial estimate of piston length, and increased accordingly for each temperature level.

![Figure 2. Left: Integer (N) and fractional number (F) of half-wavelengths than span the measurement object. Right: simulated interferogram showing the method of measurement of fractional part, where F = a / b.](image)

However, because the top surface of piston is not polished, specular reflection occurs and standard gauge block interferometry was found to be inapplicable. In order to provide adequate reflection from the top surface, a broadband dielectric mirror was placed on top of piston, as can be seen from Figure 3. The piston, with mirror on top, was then placed on a high precision optical flat and aligned until sufficient parallelism between mirror and optical flat was achieved. This procedure resulted with adequate visibility of interference fringes both from the optical flat and the mirror placed on the piston, with residual parallelism deviation visible from interferograms (Figure 3, right image). The CTE of one piston was measured in this work (TLVAG-09 assembly), which is made of tungsten carbide with nominal CTE of 4,5 /10⁻⁶ K⁻¹ and nominal length of 70 mm. Measurements were made with HMI/FSB-LPMD gauge block interferometer, a modified Kosters interferometer design with several laser sources and digital interferogram acquisition and analysis.

![Figure 3. Left: Piston in gauge block interferometer, with mirror on top, placed on a precision optical flat. Right: Interferogram acquired at one measurement step.](image)

### 3. Results

Several stable temperatures were established in interferometer chamber, ranging from 17.71 °C to 23.50 °C, and change in piston length (together with mirror thickness of 3,400 mm) was measured at each temperature as shown in Table 2. Average CTE was calculated from these results according to the following equation [4]:

\[
\text{Average CTE} = \frac{\sum_{i=1}^{n} \Delta L_{i}}{\sum_{i=1}^{n} \Delta T_{i}}
\]
where 
\[ \bar{\ell}_i = \frac{1}{2} (\ell_i + \ell_{i+1}) \]
\[ \bar{L}_i = \frac{1}{2} (L_i + L_{i+1}) \]
and \[ \Delta L_i = (L_{i+1} - L_i) \] is the length difference which arises due to temperature difference \[ \Delta t_i = (t_{i+1} - t_i) \].
Nominal length of piston-mirror assembly was taken to be exactly 73.4 mm (measured with a micrometre caliper), from which an integer value of half-wavelength was calculated at initial temperature level. Increase of integer half-wavelength values was then calculated for each temperature level, and measured fractional parts of half-wavelengths were added to determine the change of length for each measurement step.

Table 2. Measurement results.

| Piston temperature, °C | \( \dot{\ell}_i \), mm | \( \bar{L}_i \), mm | \( \Delta L_i \), mm | \( \Delta t_i \), °C | \( \bar{\ell}_i \), °C | \( \alpha(\dot{\ell}_i) \times 10^{-6} K^{-1} \) |
|------------------------|----------------|----------------|----------------|----------------|----------------|-----------------------------|
| 17.71                  | 73.3994        |                |               |                |                |                             |
| 19.68                  | 73.4001        | 73.3997        | 0.0007        | 1.97           | 18.70          | 4.79                        |
| 19.87                  | 73.4001        | 73.4000        | 0.0001        | 0.19           | 19.78          | 4.71                        |
| 20.27                  | 73.4003        | 73.4002        | 0.0001        | 0.40           | 20.07          | 4.48                        |
| 20.76                  | 73.4004        | 73.4003        | 0.0002        | 0.49           | 20.52          | 4.17                        |
| 23.50                  | 73.4013        | 73.4008        | 0.0008        | 2.74           | 22.13          | 4.19                        |

Figure 4. Graph of CTE variation over temperature range.

4. Conclusions
Preliminary results presented in this paper indicate that well-known methods from dimensional metrology can be used for determination of CTE values of highest accuracy piston-cylinder assemblies. However, because pistons and cylinders cannot be directly used to reflect light in a gauge block interferometer, additional optics must be used; in our case, a mirror placed on top of measured object. That leads to additional questions about the measurement uncertainty budget of the method presented here- it is not advisable to simply apply existing measurement uncertainty used for gauge block measurement. Further research will be focused on determination of an uncertainty budget for the proposed method, and will include a larger range of temperatures.

References
[1] M Bair and P Delajoud, Uncertainty analysis for pressure defined by a PG7601, PG7102, PG7202 or PG7302 Piston Gauge, Technical Note
[2] Deadweight Pressure Gauges, Recommended Intrinsic Standards Practice (RISP-4), National Conference of Standards Laboratories, 1998.
[3] Dadson R.S.,Lewis S.L.,Peggs G.N., The pressure balance: Theory and Practice, Ed.1, HMSO, London, 1982.
[4] R Schödel, Ultra-high accuracy thermal expansion measurements with PTB’s precision interferometer, 2008 Meas. Sci. Technol. 19 08400