On the physical significance of the Effective Independence method for sensor placement

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Abstract: Optimally deploy sparse sensors for better damage identification and structural health monitoring is always a challenging task. The Effective Independence (EI) is one of the most influential sensor placement methods and to be discussed in the paper. Specifically, the effect of the different weighting coefficients on the maximization of the Fisher information matrix (FIM) and the physical significance of the re-orthogonalization of modal shapes through QR decomposition in the EI method are addressed. By analyzing the widely used EI method, we found that the absolute identification space put forward along with the EI method is preferable to ensuring the maximization of the FIM, instead of the original EI coefficient which was post-multiplied by a weighting matrix. That is, deleting the row with the minimum EI coefficient can’t achieve the objective of maximizing the trace of FIM as initially conceived. Furthermore, we observed that in the computation of EI method, the sum of each retained row in the absolute identification space is a constant in each iteration. This potential property can be revealed distinctively by the product of target mode and its transpose, and its form is similar to an alternative formula of the EI method through orthogonal-triangular (QR) decomposition previously proposed by the authors. With it, the physical significance of re-orthogonalization of modal shapes through QR decomposition in the computation of EI method can be obviously manifested from a new perspective. Finally, two simple examples are provided to demonstrate the above two observations.

Key Word: Effective Independence method; Fisher information matrix; sensor placement method; structural health monitoring; QR decomposition
1. Introduction

Sensor placement as an essential issue in dynamic testing of large structures has been widely concerned by many fields in the last decade, with the increasing number of instrumented large structures for health monitoring[1]. Due to the reasons of economy and structural accessibility limitations, the sensors installed in structures are sparse and far less than the actual degrees of freedom in these structures. Furthermore, the identification, feature extraction, and even the health state estimation of a structure are mainly based on the data that obtained from these deployed sensors[2]. Consequently, the optimization of sensor placement scheme is an crucial task for the objective of better structural identification with deploying limited number of sensors.

In the past, a vast amount of literatures on this issue has been produced. And various of methods have been proposed, such as, the modal kinetic energy method selects those sensors with high amplitude of responses[3], the Guyan reduction method which deploys the sensors on master degrees of freedom of the reduced matrix[4], damage sensitivity based methods[5], neural networks and genetic algorithms[6], observability based methods[7], etc. And this paper is based on an influence and commonly used method i.e. the effective independence (EI) method, which is an iterative method proposed by D.C.Kammer in 1991. This method selects candidate sensor positions with the aim that optimizing the linearly independence of the mode shapes and making the mode shapes contain the most information of the target mode[8,9]. To achieve this objective, Kammer maximized the trace[8,10] of the Fisher information matrix (FIM) as the criterion. Although the EI method has been investigated by many scholars and developed very mature, some inner problems are confused yet, for instance, the physical significant of the re-orthogonalization of modal shapes through orthogonal-triangular (QR) decomposition[11] previously proposed by the authors.

In this paper, the EI method is analyzed from the viewpoint of theory. It is found that the traditional EI coefficients which is post-multiolied by a weighting matrix can’t achieve the objective of maximizing the trace of FIM, as a substitution the absolute identification space performs better. Meanwhile, it arrives at the physical significant of the re-orthogonalization of modal shapes through QR decomposition by analyzing the effect of different weighting coefficients on the maximization of the FIM and the property of the absolute identification space in detail. Finally, Two simple examples are employed to demonstrate the above analysis.

2. Theory

In this section, the traditional EI method and the potential re-orthogonalization of target modes in the computation of EI method are presented initially. Subsequently, the effect of different weighting coefficients on the maximization of the FIM is discussed and the physical significance of the re-orthogonalization of target modes can be obviously manifested from a new perspective through the discussion.

2.1. Effective independence method

As mentioned previously in the introduction, the EI method is one of the most influential sensor placement algorithms, Kammer simplified and extended the method proposed by Udwadia[12] which is based on sensitivity analysis from the viewpoint of estimation theory, and thus put forward the EI
method. The theoretical analysis of the EI method starts from an efficient unbiased estimator of the modal coordinate \( q \), the covariance matrix of the estimate error can be formulated as follow:

\[
E[(q - \hat{q})(q - \hat{q})^T] = \left[ \frac{\partial \Psi \hat{q}}{\partial q} \right] \left[ \frac{\partial \Psi \hat{q}}{\partial q} \right]^T \tag{1}
\]

in which \( E \) denotes the expected value, \( \hat{q} \) represents an efficient unbiased estimator of the vector \( q \), \( y \) is a measurement column vector indicating which positions of the structure are measured, and the \( \Psi_{\epsilon}^2 \) is the variance of the stationary Gaussian measurement white noise \( \epsilon \).

In this formulation, because of the assumption that the measurement noise is uncorrelated and possesses identical statistical properties of each sensor, which means the variance of stationary Gaussian measurement is a constant. Therefore, the equation (1) can be simplified as follow:

\[
E[(q - \hat{q})(q - \hat{q})^T] = \left[ \frac{1}{\Psi_{\epsilon}^2} \Phi^T \Phi \right]^{-1} = A^{-1} \tag{2}
\]

where \( A \) is the FIM with the definition of the product of the mode shape matrix and its transpose:

\[
A = \Phi^T \Phi .
\]

Obviously, maximizing the FIM will result in the minimum of the covariance matrix of the estimate error and achieves the best state estimate of \( q \). Therefore, the trace of the FIM is provided as original criterion to maximize the FIM. And the analysis begins by solving the eigenvalue equation:

\[
[\Phi^T \Phi - \lambda I]^T \Psi = 0 \tag{3}
\]

where \( \Psi \) denotes the orthogonal eigenvectors, \( \lambda \) are the corresponding eigenvalues. The eigenvalues of the FIM can be expressed distinctly by forming the absolute identification space which will be called the eigenvalue contribution matrix in this paper. In this matrix each row contains the square of the product of the mode shape matrix \( \Phi \) and the eigenvector matrix \( \Psi \).

\[
G = [\Phi \Psi] \otimes [\Phi \Psi] \tag{4}
\]

in which \( G \) denotes the eigenvalue contribution matrix, i.e. the absolute identification space, \( \otimes \) represents a term-by-term matrix multiplication. Postmultiplying \( G \) by the inverse of the matrix of eigenvalues \( \lambda \), the EI coefficients of the candidate sensors are computed as follow:

\[
E_D = [\Phi \Psi] \otimes [\Phi \Psi] \lambda^{-1} \cdot 1 \tag{5}
\]

in which, \( E_D \) is the EI index.

The traditional EI method computational process is deleting the row with the smallest EI coefficients at each iteration, then repeating the same process until the remained number of the sensors equals to the preconceived value. And those remained sensor locations are regarded as the measurement ones. The details of the EI method is presented in reference [8].

2.2. The re-orthogonalization of target mode

According to the formula of the EI method, target modes \( \Phi \) and its reduced form are employed apparently in the iterative computation of the EI coefficients. Nevertheless, a potential re-orthogonalization of the target modes exists in each iteration of the computation, which essentially changes the directions of the target mode vectors. For a better understanding of the re-orthogonalization
of target modes, an alternative method is provided[13], in which the EI index are computed as the diagonal of the projection matrix.

\[ E_o = \text{diag}\{\Phi^T \Phi\} \]  \hspace{1cm} (6)

where the matrix \( \Phi^T \Phi \) represents the projection matrix of vector space spanned by target modes.

For the objective to reveal the connection between the modal kinetic energy method and the EI method, the authors Li, etc[11,14] disposed the target mode matrix \( \Phi \) through the shiny orthogonal-triangular(QR) decomposition.

\[ \Phi = QR \]  \hspace{1cm} (7)

Then the equation (6) can be written as

\[ E_o = \text{diag}\{QR (R^T Q^T QR)^{-1} R^T Q^T\} = \text{diag}\{QQ\} \]  \hspace{1cm} (8)

in which \( Q \) denotes an \( n \times m \) orthonormal matrix with the same dimensions as \( \Phi \), and \( R \) is an \( m \times m \) upper triangular matrix.

After the first iteration, a new reduced system taken shape. The reduced target mode matrix \( \Phi_1 \) should be re-orthonormalized through the QR decomposition as the original system, and the EI index is computed as follow:

\[ E_{o1} = \text{diag}\{\Phi_1^T (\Phi_1^T \Phi_1)^{-1} \Phi_1^T\} = \text{diag}\{QQ_1\} \]  \hspace{1cm} (9)

Obviously, in the first iteration the target mode matrix is disposed by QR decomposition, then in the next iteration an re-orthogonalization through QR decomposition is applied to the reduced target mode matrix \( \Phi_i \) which is not strictly orthogonal anymore after the certain row of the \( \Phi \) is deleted in the proceeding iteration[15]. The QR decomposition of the reduced matrix \( \Phi_i \) can be regarded as a procedure to re-extract an orthogonal subspace from the matrix \( \Phi_i \). Comparing equation (8) to equation (9), the potential re-orthogonalization of the target modes is manifested distinctly.

2.3. The physical significance of the re-orthogonalization of target modes

As mentioned in the first section of the theory, the aim of the EI method is to select the candidate sensor locations which optimize the linear independence of mode shapes. From the sensitivity analysis of the modal coordinate with estimation theory and equation (2), the objective of optimizing the linear independence of mode shapes can be simplified as the maximization of the FIM. And the trace of the FIM was applied by Kammer as the suitable norm of the FIM to be maximized. For instance, the trace of the FIM can be formulated as[16]:

\[ \text{tr}(FIM) = \lambda_1 + \lambda_2 + \cdots + \lambda_n \]  \hspace{1cm} (10)

where \( \lambda_i \) is the eigenvalue of the FIM.

Therefore, the analysis begins by obtaining the eigenvalues through equation (3). As the eigenvalues of the FIM are positive, and the eigenvectors \( \Psi \) are orthonormal. Then according to the concept of similarity in matrix analysis[16], the FIM is similar to the diagonal matrix of eigenvalues as follow:

\[ \Psi^T (\Phi^T \Phi) \Psi = \lambda \]  \hspace{1cm} (11)

equation (11) can also be written as:

\[ (\Phi \Psi)^T (\Phi \Psi) = \lambda \quad \text{or} \quad \lambda = \text{diag}(\Phi \Psi)^T (\Phi \Psi) \]  \hspace{1cm} (12)
According to equation (12), the elements of the diagonal matrix of eigenvalues are exactly the corresponding norms of the row vectors from the matrix \( \Phi \Psi' \). For distinguishing the contribution of each row of the target modes to the total eigenvalues of the FIM, Kammer made a transformation of equation (12) and arrived at the formula as equation (4), where the matrix \( G \) will be called the eigenvalue contribution matrix here. Equation (4) will be rewritten below with its detailed form:

\[
G = [\Phi \Psi] \otimes [\Phi \Psi]' = \begin{bmatrix}
g_{11} & g_{12} & \cdots & g_{1n} 
g_{21} & g_{22} & \cdots & g_{2n} 
\vdots & \vdots & \ddots & \vdots 
g_{n1} & g_{n2} & \cdots & g_{nn}
\end{bmatrix}
\]  

(13)

where \( g_{ij} \) is the \( i \)th row element of the \( j \)th column of the eigenvalue contribution matrix \( G \). And the sum of each column denotes the corresponding eigenvalue of the FIM, meanwhile each row of the matrix \( G \) sums to the contribution of the corresponding sensor to the total eigenvalues of the FIM.

\[
\lambda_i = g_{nn} + g_{n2} + \cdots + g_{nw} \\
\lambda_{\text{sensor}} = g_{nn} + g_{n2} + \cdots + g_{nw}
\]  

(14a)

\[
\lambda_{\text{sensor}}' = g_{ni} + g_{n2} + \cdots + g_{nw}
\]  

(14b)

where \( \lambda_i \) denotes the \( i \)th eigenvalue, \( \lambda_{\text{sensor}} \) is the contribution of the \( i \)th sensor. And as presented by the original EI method, the eigenvalue contribution matrix \( G \) should be postmultiplied by the inverse of the diagonal matrix of eigenvalues \( \lambda \). The formula will be rewritten with its detailed form:

\[
F_E = [\Phi \Psi] \otimes [\Phi \Psi]'^{-1} = \begin{bmatrix}
\lambda_1^{-1}g_{11} & \lambda_2^{-1}g_{12} & \cdots & \lambda_n^{-1}g_{1n} 
\lambda_1^{-1}g_{21} & \lambda_2^{-1}g_{22} & \cdots & \lambda_n^{-1}g_{2n} 
\vdots & \vdots & \ddots & \vdots 
\lambda_1^{-1}g_{n1} & \lambda_2^{-1}g_{n2} & \cdots & \lambda_n^{-1}g_{nn}
\end{bmatrix}
\]

(15)

From the above formula, the \( F_E \) matrix is equivalent to multiply each column of \( G \) with different weighting coefficients. That is, postmultipling the inverse of the matrix \( \lambda \) can be regarded as weighting matrix \( G \) by a matrix with different coefficients. The \( F_E \) matrix can be analyzed in the same way, the sum of each column of the \( F_E \) matrix represents the fraction rank of the FIM, and each row sums to the contribution of the corresponding sensor position to the rank of the FIM.

\[
e_i = \lambda_i^{-1}(g_{nn} + g_{n2} + \cdots + g_{nw}) \\
e_{\text{sensor}} = \lambda_1^{-1}g_{ni} + \lambda_2^{-1}g_{n2i} + \cdots + \lambda_n^{-1}g_{nwi}
\]  

(16a)

(16b)

For achieving the aim that maximizing the FIM, the row of the matrix \( F_E \) with the minimum summation is removed as originally proposed. However it is found that retaining the rows of the matrix \( F_E \) with large summation in the iterative calculation can’t reflect the eigenvalues well, meanwhile can’t guarantee the maximization of the FIM either. And the matrix \( G \) reflects the eigenvalues more directly in comparison with the matrix \( F_E \) by analyzing equation (14b) and equation (16b). Element of the matrix \( F_E \) represents the proportion of \( i \)th sensor position to the \( j \)th eigenvalue, while the element of the matrix \( G \), i.e. \( g_{ij} \), is the contribution of the \( i \)th sensor position to the \( j \)th eigenvalue. For a more profound understanding, a special situation is discussed which the FIM is a 2×2 matrix with two eigenvalues.

\[
G = \begin{bmatrix} g_{11} & g_{12} 
g_{21} & g_{22} \end{bmatrix} \quad \text{and} \quad G*1 = \begin{bmatrix} g_{11} + g_{12} 
g_{21} + g_{22} \end{bmatrix}
\]  

(17a)

\[
F_E = \begin{bmatrix} \lambda_1^{-1}g_{11} & \lambda_2^{-1}g_{12} 
\lambda_1^{-1}g_{21} & \lambda_2^{-1}g_{22} \end{bmatrix} \quad \text{and} \quad F_E*1 = \begin{bmatrix} \lambda_1^{-1}g_{11} + \lambda_2^{-1}g_{12} 
\lambda_1^{-1}g_{21} + \lambda_2^{-1}g_{22} \end{bmatrix}
\]  

(17b)
where \( 1 \) denotes a \( 2 \times 1 \) column vector with all elements of 1. From above equations, it is assumed that \((g_{11} + g_{12}) > (g_{21} + g_{22})\) in the matrix \( G \) which means the contribution to the total eigenvalues of the 1\(^{st}\) sensor is larger than that of the 2\(^{nd}\) sensor. However, if the proportion of the 1\(^{st}\) sensor to the eigenvalue \( \lambda_1 \) and \( \lambda_2 \) is smaller than that of the 2\(^{nd}\) sensor, i.e. \((\lambda_1^3 g_{11} + \lambda_2^3 g_{12}) < (\lambda_1^3 g_{21} + \lambda_2^3 g_{22})\), the above two criteria arrive different conclusion about maximizing the trace norm of the FIM. According to the matrix \( F_1 \) \* the 2\(^{nd}\) sensor position is remained, oppositely, the matrix \( G \) \*1 remains the 1\(^{st}\) sensor position. And obviously the matrix \( G \) \*1 reflects the trace of the FIM better.

With the analyzing above, another interesting phenomenon is that each element of the matrix \( G \) \*1 is a constant in each iteration. When recalculating the new eigenvalue contribution matrix \( G \) \*1 after the preceding iteration, the only different between \( G \) \*1 and \( G \) \*1 is the removed element which is corresponding to the sensor position selected by using the EI coefficients. For the objective to explain this property of the matrix \( G \), an analysis is made as follow. Because of the matrix \( G \) is related to the eigenvalues of the FIM, and is obtained by equation (13). A transformation of equation (13) is provided as:

\[
G \*1 = diag(\Phi \Psi) \quad (18)
\]

the eigenvectors \( \Psi \) of the FIM have the property that \( \Psi^T \Psi = I \), therefore,

\[
G \*1 = diag(\Psi^T \Psi) \quad (19a)
\]

\[
G \*1 = diag(\Phi \Phi^T) \quad (19b)
\]

where \( \phi_i \) denotes the \( i \)th row of the target mode matrix \( \Phi \). From the equation above, it is obviously that the element of the matrix \( G \) \*1 equals to the row norm of the target mode matrix \( \phi \). Therefore, the elements is constant.

Distinctly, the form of equation (19a) is similar to the equation (9). And the two formulas are compared as follow:

\[
E_0 = F_1 \*1 = diag(\Phi \Psi) \quad (20a)
\]

\[
G \*1 = diag(\Phi \Phi^T) \quad (20b)
\]

The physical significance of the re-orthogonalization of the target mode in each iteration can be regarded as weighing the target mode matrix with the inverse of the eigenvalue matrix \( \lambda \) of the FIM. The effect of different weighting coefficients on the maximization of the FIM and the physical significance of the re-orthogonalization of target modes are demonstrated in the following examples.

3. Two Simple Numerical Examples

A 4-degrees of freedom beam model with concentrated quality is used primarily to illustrate the effect of different weighting coefficients on the maximization of the FIM and the physical significance of the re-orthogonalization of target modes as deduced above. The beam is a simplified model with 4 elements as shown in Figure 1. For the purpose of simplifying the computation, the parameters are provided in a simple form. The total length of the beam is 4 and each element is quartered. The mass of the 4 elements are, \( m_1=1.5 \), \( m_2=1.5 \), \( m_3=1.25 \), \( m_4=0.5 \). The other parameters of the beam are provided as, the cross section is 1, the density of the beam is 1,and the \( E=1 \), \( l=1 \).
The first two orders of the mode shapes are used as target modes to participate in calculation. In the first iteration, computing the FIM and its eigenvector matrix $\Phi$ and the eigenvalue matrix $\lambda$ according to the equation (3). Then the matrix $G^* I$ and the matrix $F_E^* I$, i.e. the matrix of EI index, can be obtained from equation (13) and equation (15) as follow:

$$G^* I = \begin{bmatrix} 0.3739 \\ 0.3243 \\ 0.4494 \\ 0.7818 \end{bmatrix} \quad \text{removed} \quad \text{and} \quad E_o = F_E^* I = \begin{bmatrix} 0.5255 \\ 0.4785 \\ 0.3741 \\ 0.6220 \end{bmatrix}$$

$$\text{trace}_{G^* I} > \text{trace}_{F_E^* I}$$

Obviously, the row with the minimum element of the above two matrix are different. If the 2nd row of the target mode is deleted in this iteration according to the matrix $G^* I$, the trace of the FIM in the next iteration is 1.6051. However, if the 3rd row of the target mode is removed according to the matrix $F_E^* I$, the trace in the next iteration is 1.4800. That is, $\text{trace}_{G^* I} > \text{trace}_{F_E^* I}$ after the first iterative computation.

And as discussed above in equation (17), it is found that the value of the 2nd row surpasses that of the 3rd row by postmultiplying the inverse of the matrix of eigenvalues $\lambda$. Analyzing the 2nd and the 3rd rows of the target modes in details as follow:

$$G^* I = [\Phi \Phi^T]_{\text{re-orth}} * I = \begin{bmatrix} 0.3176 & 0.0067 \\ 0.0242 & 0.4252 \end{bmatrix} * I = \begin{bmatrix} 0.3243 \\ 0.4494 \end{bmatrix}$$

$$F_E^* I = G^* \lambda^* I * I = \begin{bmatrix} \lambda_2^* & \lambda_2^* \times 0.0067 \\ \lambda_2^* & 0.4252 \end{bmatrix} * I = \begin{bmatrix} 0.4785 \\ 0.3741 \end{bmatrix}$$

It demonstrates the deduction of the equation (17) that the large proportion of the 2nd row of the eigenvalues can’t guarantee its contribution to the trace of the FIM is large as well. Therefore, the matrix $G^* I$ is preferable to maximizing the trace of the FIM as well as the FIM itself, instead of the matrix $F_E^* I$ proposed as original.

To illustrate the physical significance of the re-orthogonalization of target modes, the matrix $G^* I$ is analyzed in each iteration. The initial matrix $G^* I$ is provided as:

$$G^* I = \begin{bmatrix} 0.3739 \\ 0.3243 \\ 0.4494 \\ 0.7818 \end{bmatrix}$$

Then in the next iteration with the assumption that one row of the target modes is removed, the new reduced matrix $G^* I$ is computed as follow:
Obviously in the next iteration, comparing to the reduced matrix $G^*1$, the removed element of the initial matrix $G^*1$ is the row corresponding to the one deleted from the target modes in the preceding iteration. And the other rows of the matrix $G^*1$ are constant. In the follow iterations the property of the matrix $G^*1$ is stayed as well. Furthermore, the value of the removed row of the matrix $G^*1$ equals to the reduction of the trace of the FIM. Therefore, the physical significance of the re-orthogonalization of the target modes deduced by equation (20) is demonstrated.

To demonstrate the deduction is generalized, another beam with distributed mass is provided, as shown in the Figure 2. The beam with the length 4 is divided into 4 equal elements. And the simplified parameters are provided as well, whose the cross section is 1, $E=1$ and $I=1$. The density of the 2nd element is 2 while the others are 1.

![Figure 2. Beam model with distributed mass](image)

Table 2. Beam model with distributed mass

The first 4 orders of the mode shapes are remained as the target modes. According to equation (13) and equation (15) the corresponding matrix $G^*1$ and $F_{E^*1}$ can be obtained and the values of each row of the two matrices are listed below in Table 1.

|       | $G^*1$ | $F_{E^*1}$ |
|-------|--------|------------|
| 1st row | 0.6585 | 0.5932     |
| 2nd row | 1.1617 | 0.3663     |
| 3rd row | 0.7624 | 0.4140     |
| 4th row | 1.7599 | 0.5244     |
| 5th row | 1.1239 | 0.4236     |
| 6th row | 1.4843 | 0.4056     |
| 7th row | 3.7408 | 0.5573     |
| 8th row | 10.4685| 0.7157     |
From the Table 1 above, it is obviously that the row with the minimum value of the two matrices are different. Furthermore, the sequence of the row according to the matrix $F_{E^*1}$, i.e. the EI index, is greatly different from that according to the matrix $G_{E^*1}$. Therefore, the removed rows of the target modes in the following iterations according to the two matrices are different either. And the trace of the corresponding FIM in each iteration based on different criterion which defined by the two matrices are shown in Figure 3, as follow,

![Figure 3. The trace of FIM in each iteration according to different criterion](image)

It is observed that the trace of the FIM according to the criterion defined by the matrix $G_{E^*1}$ are larger. Therefore, the deduction is proved that the matrix $G_{E^*1}$ is preferable to maximizing the trace of the FIM as well as the FIM itself, other than purposed originally.

Then the matrices $G_{E^*1}$ in each iteration corresponding to the reducing sequences of the target mode are provided as follow in Table 2. The specific row of the target mode is removed in each iterative computation.

After the 1st iteration, the 1st row of the matrix $G_{E^*1}$ is removed because of the target modes are reduced with the 1st row is removed, and the other rows are remained as initial. So are the following iterations. Therefore the conclusion deduced through equation (20) that the physical significance of the re-orthogonalization of the target mode can be regarded as weighing the target mode matrix with the inverse of the eigenvalue matrix $\lambda$ of the FIM is demonstrated once again.
Table 2. The matrix $G^*1$ in each iteration.

|     | $G^*1$ | $G^*1$ | $G^*1$ | $G^*1$ |
|-----|--------|--------|--------|--------|
| 0.6585 | 1.1617 | 1.1617 | 1.1617 | 1.1617 |
| 0.7624 | 0.7624 |        |        |        |
| 1.7599 | 1.7599 | 1.7599 | 1.7599 |        |
| 1.1239 | 1.1239 | 1.1239 |        |        |
| 1.4843 | 1.4843 | 1.4843 |        |        |
| 3.7408 | 3.7408 | 3.7408 |        |        |
| 10.4685 | 10.4685 | 10.4685 |        |        |

4. Conclusions

Based on the influential sensor placement method, i.e. EI method, the effect of different weighting coefficients on the maximization of the FIM is discussed. It is found that the sum of each row of eigenvalue contribution matrix $G$, i.e. the matrix $G^*1$, is preferable to maximizing the trace of the FIM as well as the FIM itself instead of the EI index purposed originally. Furthermore, the physical significance of the re-orthogonalization of target mode in the EI method is derived by analyzing the eigenvalue contribution matrix $G$ in details. The procedure of re-orthogonalizing the target modes in each iteration can be regarded as weighing the target mode matrix with the inverse of the eigenvalue matrix $\lambda$ of the FIM. And the two simple examples are provided eventually, which demonstrating the theory numerically. Furthermore, an alternative form to compute the matrix $G^*1$ is shown in equation (19a).

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