Indonesian Earth’s Lithospheric Magnetic Field modelling using Spherical Cap Harmonic Analysis Method

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Abstract. The earth’s lithospheric magnetic field is part of the main earth’s magnetic field. The lithospheric field has a very small value compared to the Earth’s main magnetic field, approximately less than 1%, and this field is generated at the earth’s crust and upper mantle. Modelling of lithospheric field is useful mainly for predicting the distribution of the value of lithospheric fields and to determine the magnetic anomaly. In this research, modelling the Earth’s lithospheric magnetic field uses Spherical Cap Harmonic Analysis (SCHA) method and this method can do modelling using regional magnetic data. The data used for the modelling are magnetic repeat station data in Indonesia region (BMKG’s Epoch) and SWARM satellite data. The results of the modelling using integrated SWARM satellite and repeat station data produce RMSE values of 64.0834 nT and the expansion of index K is 70. In addition, the results of the modelling resolution is 1.5°. The value’s range of modelling’s result are \(-987.192 \text{–} 998.239 \text{ nT}\) for X component, \(-968.189 \text{–} 949.438 \text{ nT}\) for Y component, \(-981.266 \text{–} 608.676 \text{ nT}\) for Z component, and \(-904.151 \text{–} 997.389 \text{ nT}\) for total intensity are.

Keywords: Lithosphere, SCHA, Modelling

1. Introduction
The lithospheric field is a part of the geomagnetic field that has small value comparing to the main field, about 1% of the main field [1]. This field are generated at the earth’s crust and the upper mantle. In order to know the value of the lithospheric field over a region, some modelling method are used. One of the modelling method used for modelling lithospheric field in a region is Spherical Cap Harmonic Analysis (SCHA) [2]. There are two types of data that used as input for this model. The first one is repeat station data measured by BMKG (Meteorological Climatological and Geophysics Agency of Indonesia) and this type of data is used before for mapping the main field [3]. The second one is SWARM satellite data [4]. In order to knowing the local anomaly magnetic field cover some regions, the lithospheric field can be modelled. The local anomaly is caused by the rock and mineral that available there. Furthermore, the result of modelling can interpret the geological feature in the subject region [4]. By knowing the local anomaly in the subject place, the result can be used for some purposes (e.g. prospecting, determination of the kiblah direction).
2. Data and methods

This study uses data from the magnetic repeat station measurement which has been used for mapping the Indonesian main magnetic field for the year of 2015 [3], and SWARM satellite data in the same year. Based on [3], repeat stations are measured at the place that relatively free from geomagnetic disturbance. There are 68 stations all over Indonesian region that can be seen in Figure 1.

The repeat station’s data are relatively free from magnetic field disturbance, while the satellite data are not. The satellite data are easily affected by the external source of geomagnetic field such as solar wind [5]. In order to minimize the effect of external disturbance, the data are selected based on Dst index and Kp index [4]. Based on [4] and [6], data selected with respect to \( K_p \leq 2 \), \( |D_{st}| \leq 50 \) nT, and \( |dD_{st}/dt| = 3 \) nT.

The lithospheric field of each repeat stations must be known by reducing the measurement result by the main field. In this study, BMKG’s magnetic calculator data [3] are used as the main field. This process mathematically described by

\[
\partial B = B_m - B_c
\]  

Where \( \partial B \) is the residual value or lithospheric field for each stations, \( B_m \) is the measurement result, and \( B_c \) is the main field at the same position and time as the measurement point.

Spherical Cap Harmonic Analysis (SCHA) is method for modelling earth’s magnetic field in regional scale. This method introduced by Haines [2] by solving Laplace equation in spherical coordinate. The Equations stated in [1], described by

\[
\Delta X = \sum_{k=1}^{K_{max}} \sum_{m=0}^{k} \left( \frac{a}{r} \right)^{n_k+2} (g_{k}^m \cos m\Lambda + h_{k}^m \sin m\Lambda) \frac{dP_{n_k}^m(\cos\Theta)}{d\Theta}
\]  

\[
\Delta Y = \sum_{k=1}^{K_{max}} \sum_{m=0}^{k} \left( \frac{m}{\sin\Theta} \right)^{n_k+2} (g_{k}^m \cos m\Lambda - h_{k}^m \sin m\Lambda) P_{n_k}^m(\cos\Theta)
\]  

\[
\Delta Z = \sum_{k=0}^{K_{max}} \sum_{m=0}^{k} (n_k + 1) \left( \frac{a}{r} \right)^{n_k+2} (g_{k}^m \cos m\Lambda + h_{k}^m \sin m\Lambda) P_{n_k}^m(\cos\Theta)
\]
\[ n_k = \left(2k + \frac{1}{2}\right) \frac{\pi}{2\theta_0} - \frac{1}{2} \]  

(5)

Where \( a \) is the earth mean radius (6371.2 Km), \( \Theta \) is colatitude, \( \Lambda \) is longitude, \( \theta_0 \) is the regional radius, \( r \) is the radius of station to the earth’s centre, \( g_k^m \) and \( h_k^m \) are model’s parameter determined through inversion. Then, the resolution of model’s result is determined by

\[ l = \frac{360^\circ}{\left(\frac{K_{\text{max}} + 0.5}{\theta_0}\right) - 0.5} \]  

(6)

After the model’s parameter and the coordinate based on resolution are calculated, Equation (2), (3), and (4) are applied again with the model’s parameter and the new coordinate as input in order to know the model’s result of lithospheric field.

3. Results and discussion

In this study, we do the inversion process two times with the expansion of SCHA’s order \((K_{\text{max}})\) reach 30. Firstly, the inversion process use repeat station data only. Secondly, the inversion process is applied by integrated repeat station and satellite data. The results are shown in Table 1. In the first process, the RMS value is still high (285.10367 nT). However, in the second process, the RMS value is 64.03337 nT and this value is reasonable enough according having similarity with the research were done by [1] and [7]. Both processes show huge difference of RMS value caused by amount of data being used. We repeat the second inversion process with expanding \(K_{\text{max}}\) to 70. The results show that RMS values decrease significantly until \(K_{\text{max}} = 30\) and they are mostly stable beyond those point. However, if \(K_{\text{max}} = 30\) is being used, the resolution of the map becomes wider, as can see in Table 2. Consequently, \(K_{\text{max}} = 70\) is elected for better resolution.

Table 1. The difference of RMS value based of amount data used

| Data                  | Number of data | RMSE (nT)    |
|-----------------------|----------------|--------------|
| Repeat station        | 68             | 285.10367    |
| with satellite data   | 1806           | 64.03337     |

Table 2. Resolution and RMS of \(K_{\text{max}} = 30\) and \(K_{\text{max}} = 70\)

| \(K_{\text{max}}\) | RMSE (nT) | Resolution |
|--------------------|-----------|------------|
| 30                 | 64.03337  | 3.66°      |
| 70                 | 64.0834   | 1.58°      |

Figure 2. The X component of lithospheric magnetic field
Figure 3. The Y component of lithospheric magnetic field

Figure 4. The Z component of lithospheric magnetic field

Figure 5. Total intensity of lithospheric magnetic field
Those pictures above are the map result of the lithospheric modelling in this study. Figure 2 shows the map of X component of the lithospheric field which have range -987.192 to 998.239 nT. Figure 3 shows the map of Y component of the lithospheric field which have range 968.189 to 949.438 nT. Figure 4 shows the map of Z component of the lithospheric field which have range -981.266 to 608.676 nT. Lastly, Figure 5 shows the map of total intensity of the lithospheric field which have range -904.151 to 997.389 nT.

The lithospheric field contour in oceanic region has shorter wavelength, higher amplitude, and higher frequency than in continent region [8]. The lithospheric contour in the west of Sumatera and the south of Java which have the oceanic contour characteristic is depicted in Figure 5. In the continent region has contour pattern of continental crust. Based on the result this study has similar finding who stated by [8].

Figure 6 shows the total intensity map of the lithospheric field in Indonesia region from EMAG2 model [9]. EMAG2 model is the global modelling of lithospheric field. This model do not use data from repeat station only, but they use both the shipborne data and aeromagnetic data. They have more complex data and smaller total intensity value’s range than this study. EMAG2 model has resolution about 2 minutes degree, this caused by the expand of Legendre equation’s order (K-index in regional modelling) reach 150, meanwhile this study only has resolution about 1.5 degree and the expand of K-index just reach 70. However, both EMAG2 and this study’s result agree that Indonesia region mostly has negative total intensity of lithospheric field.

![Figure 6. EMAG2 model (global modelling of lithospheric field)](image)

4. Concluding remark
This study shows the regional modelling result of lithospheric field as we can see in Figure 2 to Figure 5. The results of the modeling resolution is 1.5°. The range of lithospheric magnetic field values of component X, Y, Z, and the total magnetic field intensity are -987.192 – 998.239 nT, -968.189 – 949.438 nT, -981.266 – 608.676 nT, and -904.151 – 997.389 nT respectively. These results are considered reasonable by comparing them with the previous result from [1], [7], and the statement of [8]. These results still need improvement and it would be possible by including shipborne and aeromagnetic data, and expanding the number of K-index to achieve better resolution.
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