Griffiths Physics in an Ultracold Bose Gas

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(Dated: June 21, 2018)

Coupled XY model systems consisting of 3D systems with disordered interlayer physics are of significant theoretical interest. We realize a set of coupled quasi-2D pancakes of $^{87}\text{Rb}$ in the presence of disordered inter-layer coupling. This is achieved with our high bandwidth arbitrary optical lattice to obviate restrictions on the dimensionality of disorder with speckle-generated optical fields. We identify multiple phase crossover regions that comport with the existence of a pair of intermediate Griffiths phases between a thermal state and the emergence of bulk 3D superfluidity.

Introduction: The intersection of dimensionality and disorder is a rich area of study in condensed matter physics. The precise control of disorder available with optical potentials enables the realization of well characterized disordered systems with quantum degenerate atomic gases. Optical speckle has been used to generate disorder for one, two, and three dimensional systems\cite{1,2,3}, exhibiting Anderson localization, a disordered BKT transition\cite{4}, mobility edges in three dimensions\cite{5}, and emergence of a Bose glass\cite{6}. Similarly, quasi-disorder provided by incommensurate lattices has been used to realize the Aubry-André model in the presence of interactions\cite{7}, the role of quasi-disorder in adiabaticity\cite{8}, to study many-body localization\cite{9}, and has been contrasted against uncorrelated disorder in transport\cite{10}.

While 2D Bose gases in isolation are well described by Berzinskii-Kosterlitz-Thouless (BKT) physics (approximated by the XY model), recent theoretical work suggests rich behavior when sets of such systems are randomly coupled to one another. Example layered XY model systems, including stacks of 2D superfluids, cuprate superconductors, and planar magnets, are shown through renormalization group techniques\cite{11,12} and Monte Carlo simulations\cite{13} to exhibit Griffiths physics\cite{14,16} when the interlayer couplings or layer thicknesses are subject to uncorrelated disorder. A pair of phases of matter emerge as the temperature is lowered from a nondegenerate state to bulk 3D order (BEC, magnetization, or superconduction). Each intermediate phase is a Griffiths phase, with properties dominated by the most extreme local deviation in the disordered system. The first is an anomalous Griffiths phase, a class of sliding phase that exhibits 2D order (superfluidity, magnetic susceptibility, or superconduction). The second is a semi-ordered Griffiths phase where order appears in the third dimension. (Magnetic Griffiths phases have recently been observed in bulk metal alloy systems\cite{17,18}, but not in anisotropic systems.)

References\cite{11,12} employ the phase stiffness $\rho^x_{\phi_x}$ along a direction $\epsilon_x$ to characterize the Griffiths phases. Phase stiffness is a measure of the energy required to impose a phase difference $\phi_x$ between two ends of a finite system. As a function of system size $L$ and system energy $E$, the phase stiffness is defined as

$$\rho^x_{\phi_x} = \frac{1}{L} \frac{d^2 E(\phi_x)}{d\phi_x^2}$$

While a computationally convenient parameter, it is not easily measurable in experimental atomic systems. Although it is possible to measure the critical velocities of superfluids\cite{19}, these disordered systems are predicted to have very small superfluid fractions and critical velocities over much of the phase diagram. Instead we use an analysis of the fluctuations in large data sets of time-of-flight (TOF) momentum distributions to gain information about the phases as a function of temperature and lattice depth.
We associate the appearance of a Thomas-Fermi-like distribution in the two in-plane dimensions ($p_\parallel$) paired with a thermal distribution in the third ($p_\perp$) with a 2D superfluid transition at $\sim 200$ nK. As we decrease the temperature we observe the emergence of vibrational modes in fluctuation correlations along $p_\perp$[20], which characterize the expanding length-scales of superfluid puddles in the anomalous Griffiths phase. Finally we interpret the suppression of zero momentum atomic density fluctuations at our lowest temperatures with the Bose statistics of macroscopically occupied non-local state, an observation consistent with 3D superfluidity.

**Experimental Approach:** Optical speckle is a well-developed tool in the atomic gas community for the creation of uncorrelated disorder. When generated by passing an optical beam through a phase diffuser and then focused through a lens, the intensity pattern at the focus replicates the statistics of the phase diffuser. This cannot be used to form 1D disorder in a 3D system, as the limits imposed by diffraction of light sets the minimum dimension of optical speckle to two. Instead, we use stroboscopic techniques to realize a set of quasi-2D planes of trapped Bose gas coupled via a random 1D optical lattice potential.

We create degenerate gases of $^{87}$Rb using a hybrid magnetic trap-optical dipole trap[21], which then loads into a dipole trap whose waist is translated 30 cm into a science chamber. The atoms are finally loaded into a dipole trap whose waist is translated 30 cm into the temperature we observe the emergence of vibrational modes in fluctuation correlations along $p_\perp$[20], which corresponds to energies too small to drive excitations.

Our time-averaged Hamiltonian under the local density approximation (LDA) is

$$
\hat{H} = \sum_i \left( \frac{\hat{p}_i^2}{2m} + V_i - \frac{1}{2} \sum_j J_{i,j} \psi_i^\dagger \psi_j \right) + g \psi_i^\dagger \psi_i^\dagger + \frac{m}{2} \hbar^2 \omega_i^2 (x^2 + y^2)
$$

(2)

where $\frac{\hat{p}_i^2}{2m}$ is the in-plane kinetic energy, $V_i$ is the lattice well depth, $J_{i,j}$ is the hopping strength between two lattice sites $i$ and $j$, $g$ is the scaled quasi-2D self-energy term for each pancake, $\omega_i$ is the in-plane trapping frequency, which includes small variations due to the disorder[22].

We prepare gases ranging in temperature from 300 nK down to 25 nK in potentials with depths of 2.8 kHz and 5.6 kHz. The relative phases of our lattices are arbitrarily tunable. We chose two sets of relative phases to create two different optical potentials with similar statistics. We observe no difference in the results between the two potentials, confirming that our atom cloud is large enough (greater than 70 pancakes) to average over the disorder. Each data set contains two different hold times, 200 ms and 4 s, which show no statistical difference, indicating equilibrium.

We levitate our atoms during 48 ms of TOF with the use of a gravity-canceling magnetic coil with a microsecond switching time to resolve the momentum distributions $n(p_\parallel, p_\perp)$. Gross-Pitaevskii simulations show our distributions along the disordered direction, $p_\perp$, are minimally distorted by interactions during the expansion due to the rapid expansion of each atomic plane. We conclude that our TOF distributions faithfully represent the momentum distribution along the disordered direction.

**Analysis:** The trapping energies along the disorder and in-plane differ by over a factor of 85, so we treat their momentum distributions as separable. We focus on the in-plane and longitudinal momentum distributions, $n_{\parallel}(p) = \sum_{p_\perp} n(p_{\parallel}, p_\perp)$ and $n_{\perp}(p) = \sum_{p_{\parallel}} n(p_{\parallel}, p_\perp)$. At high temperatures $n(p_{\parallel})$ is a Gaussian. As the temperature drops a bimodal distribution emerges, a thermal distribution plus a Thomas-Fermi distribution as expected after expansion[23]. At our lowest temperatures the thermal distribution is no longer evident. We apply a bimodal fit to $n(p_{\parallel})$ for every cloud, and take the fraction of atoms in the Thomas-Fermi distribution to be the coherent fraction as shown in Fig. 2(k). Both lattice depths show the emergence of a coherent fraction in $n(p_{\parallel})$ near 200 nK,
which corresponds to finite in-plane phase stiffness. We identify this as the onset of the BKT transition in isolated pancakes. We expect little difference in the BKT transition temperature between the two lattice depths; the band gap and the compression of the planes scale weakly with lattice depth in our relatively shallow lattices.

At lower temperatures coherence between BKT planes emerges in the form of interference effects in $n_\perp(p)$. Identically prepared clouds look radically different from one another in TOF, exhibiting large differences in the phase fluctuation power spectrum. Inspired by Feng, et al.’s success in identifying phases of matter in 1D Bose gases[24], which also exhibit strong phase fluctuations, we apply similar analysis to our data. We normalize our distributions to $\sum_p n_\perp(p) = 1$, sort them into 25 nK bins and calculate the average distributions $\bar{\pi}_\perp(p,T)$. We then calculate the deviation of each cloud from the average for its bin, $\delta n_{\perp,i}(p,T) = n_{\perp,i}(p,T) - \bar{\pi}_\perp(p,T)$, and calculate the two-body fluctuation correlation function $\alpha_i(p_a,p_b,T) = \delta n_{\perp,i}(p_a,T)\delta n_{\perp,i}(p_b,T)$. The averages of those quantities, $\bar{\alpha}(p_a,p_b,T)$ are displayed in Fig. 3

Along each panel’s diagonal, from lower left to upper right, we display the fluctuation power spectrum $\bar{\pi}(p,T)$. From upper left to lower right we display correlations in opposite momenta $\bar{\alpha}(p,-p,T)$. Fig. 3) normalizes the latter by the former, and is a measure of fractional correlations of opposite momenta, $\beta = 2\bar{\alpha}(p,-p,T)/(\bar{\pi}(p,p,T) + \bar{\pi}(p,-p,T))$. Together these show qualitative changes as a function of temperature and lattice depth. In concert with our measurement of in-plane coherent fraction, we identify those differences with a set of phases of matter.

Our atomic gases are finite in size and harmonically trapped in all three dimensions. While disorder plays a strong role in where the BKT transition occurs for each

![FIG. 2. We display $\bar{\pi}(p_a,p_b,T)$ in panels (a)-(i) and $\beta = 2\bar{\pi}(p,-p,T)/(\bar{\pi}(p,p,T) + \bar{\pi}(p,-p,T))$ in panel (j) for our 2.8 kHz lattice. The features at the edge of the panels are artifacts of our normalization scheme. The temperature bins are 25 nK wide, and range monotonically from 225-250 nK in panel (a) to 25-50 nK in panel (i). Note the emergence of distinct momentum peaks in opposite momenta, which do not correspond to the sublattice recoil momenta. The longest length-scale resolved is 10.9 $\mu$m.](image)

![FIG. 3. The fraction of atoms in the Thomas-Fermi distribution in $n(p_b)$ is plotted as a function of temperature. We remove a baseline value of 0.06, a fit artifact, from the coherent fraction. We see threshold behavior in the coherent fraction, establishing $\rho_\parallel > 0$ below 200 nK.](image)
Our harmonic trap favors emergence of order from its center. Consequently there are likely multiple phases of matter in most of our temperature bins. The 2.8 kHz lattice exhibits several competing varieties of correlation in $\vec{\alpha}$ as a function of temperature. Isotropic thermal correlations dominate along the lattice between 250 and 175 nK as seen in Fig. 3(a)-(c), when the coherent fraction in-plane is insufficient to give rise to interference effects. The lack of any structure in the normalized correlations $\beta$ is apparent in Fig. 3(j). The apparent nonzero correlations in the thermal state derive from the finite width of the temperature bins. Colder clouds have smaller rms widths, and vice versa, so fluctuation amplitudes are symmetric about $p_\perp = 0$ and nonzero. The thermal nature of the correlations imply that $\rho^\perp_\perp = 0$. Below 200 nK, we begin to see the growth of a coherent fraction in Fig. 3, implying that $\rho^\parallel_\parallel > 0$. Concurrent $\rho^\perp_\parallel = 0$ and $\rho^\parallel_\perp > 0$ constitute evidence of a sliding phase, a phase exhibiting 2D superfluidity in a 3D bulk, consistent with an anomalous Griffiths phase.

Correlations in our 2.8 kHz lattice begin to exhibit structure below 175 nK as seen in Fig. 2(d). We use a multi-peak Gaussian fit as a generic localized distribution to analyze our $\beta$ distributions. Strong positive correlations in $(p_\perp, -p_\perp)$ emerge around $p_\perp = 0$, and momenta corresponding near wavelengths of 2.4, 3.0, 4.9 and 10.8 $\mu$m. The 4.9 and 10.8 $\mu$m modes blend together into one broad peak at temperatures below 50 nK, as do those at 2.4 and 3.0 $\mu$m, and at our lowest temperatures 3.0 $\mu$m correlations are suppressed. These emergent length scales are distinct from the periods composing our disordered potential (1.21, 1.37, 1.58, 2.02, 2.50, 2.79, and 4.70 $\mu$m). Both the momenta peaks in the correlation spectra and the lattice recoil momenta were measured with the same optical system, thus there is no scale-factor uncertainty between the two. There is nothing special in our potential about the 2.5 $\mu$m lattice, and the correlations on the 4.9 and 10.8 $\mu$m length scales are larger than any imposed by our disorder. We do not see any peaks in $\pi$ corresponding to our shortest 4 lattice.

Fluctuation correlation peaks emerge in completely incoherent lattice systems due to uncorrelated phase noise, as observed in refs. 23 26. Simulations of noninteracting thermal distributions in our disordered optical lattice do show superficially similar correlations at very low temperature, but they exist for each sub-lattice, do so below 50 nK, and emerge first in high momentum states. In addition their exact length scales depend strongly on the choice of relative phase of each sub-lattice. In contrast, our fluctuation correlations emerge at a few momenta, progress from low momentum to high momentum with decreasing temperature, and are evident at higher temperatures, below 175 nK.

The length scales of fluctuations in our interacting system are determined by two factors. The lattice imposes structure on the coherent fraction. The density of coherent puddles are each modulated by the local disorder. There is a second effect that varies with temperature: length scales are imposed by the distances between puddles as well as the puddle sizes. Once every pancake has undergone the BKT transition the second effect vanishes, and we are left with one fully connected puddle.

We attribute this emerging structure to the population of many-body phonon modes as more pancakes undergo the BKT transition, leading to larger superfluid puddles and increasing overall coherent fraction. This is the process described by 11, as the anomalous Griffiths phase proceeds towards the superfluid transition with decreasing temperature. Fluctuation peaks first emerge in Fig. 2(d), but the normalized fluctuation correlations in 2(j) at discrete momenta do not reach their final form until below 100 nK, with a coherent fraction of 30-40%.

We interpret the settled form of $\beta$ below 100 nK as the establishment of complete connectivity. This region suggests the crossover to a Griffiths superfluid phase from an anomalous Griffiths phase, during which the growth of the phase stiffness is governed by the most depleted regions in the 3D superfluid. Unfortunately, we cannot directly measure phase stiffness, and in this region it is expected to be quite small anyway.

The same trend can be seen in the 5.6 kHz lattice data in Fig. 4, but at much lower temperatures, despite the coherent fraction’s concurrent growth with the 2.8 kHz lattice. The disordered lattice in this data set is twice as deep, so we would expect the most depleted pancake’s phase space density, and thus transition temperature, to drop by 1/e, and we see no evidence it ever leaves the sliding phase.

At temperatures below 70 nK, in our 2.8 kHz lattice, we observe a drop in the fluctuation power spectrum $\pi(p, p)$ at $p = 0$, as shown in Fig. 5. Its contrast increases with decreasing temperature. We take this as evidence of the suppression of fluctuations due to Bose statistics of a macroscopically occupied, nonlocal state. This is consistent with the onset of finite phase stiffness along the disorder. We find it difficult to draw a clear line between a superfluid Griffiths phase and a full 3D superfluid. The literature does not discriminate between the two phases using any accessible parameters 11 12, and ref. 13 does not identify a distinction. If there are two such distinct phases, this macroscopically ordered region is likely the latter.

Conclusion: Using our HiBAL we engineered a novel optical field isotropic in 2D and disordered in the third dimension. We explored the phase diagram of a previously unrealized class of disordered system. Using fluctuation correlations in momentum-space we found evidence of a sliding phase, a probable Griffiths superfluid, and a regular 3D superfluid. A detailed connection between momentum-space fluctuation correlation data and previously calculated parameters, such as phase stiffness,
FIG. 4. Normalized fluctuation correlations \( \beta \) from our deeper 5.6 kHz lattice. Thermal correlations persist to 100 nK, below which peaks develop. In contrast, this occurs in the shallower lattice at 175 nK.

would greatly benefit the precise identification of the multiple phases of this disordered system. Our HiBAL is a flexible platform capable of generating arbitrary sets of optical lattices over two spatial octaves with phase, amplitude, and wavevector control at MHz frequencies. It will enable a large set of experiments, from transport measurements in disordered systems and the production of Hamiltonians for the study of Floquet physics, simultaneous with Bragg spectroscopy of all of the above.

We would like to thank Ian Spielman for the extensive discussions during the preparation of this manuscript.

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[1] J. Billy, V. Josse, Z. Zuo, A. Bernard, B. Hambrecht, P. Lugan, D. Clément, L. Sanchez-Palencia, P. Bouyer, and A. Aspect, Nature 453, 891 (2008).

[2] B. Allard, T. Plisson, M. Holzmann, G. Salomon, A. Aspect, P. Bouyer, and T. Bourdel, Phys. Rev. A 85, 033602 (2012).

[3] S. S. Kondov, W. R. McGeehee, J. J. Zirbel, and B. DeMarco, Science 344, 66 (2011).

FIG. 5. Displayed are the square fluctuations \( \overline{\alpha} (p, p, T) \) for a few choices of temperature in our 2.8 kHz lattice. A drop in fluctuations at \( p = 0 \) emerges below 70 (±2.5) nK.

[4] M. Beeler, M. Reed, T. Hong, and S. Rolston, New Journal of Physics 14, 073024 (2012).

[5] G. Semeghini, M. Landini, P. Castilio, S. Roy, G. Spagnolli, A. Trenkwalkder, M. Fattori, M. Inguscio, and G. Modugno, Nat. Phys. 11, 554 (2015).

[6] C. Meldgin, U. Ray, P. Russ, D. Chen, D. M. Ceperley, B. DeMarco, and E. Mueller, Nat. Phys. 12, 646 (2016).

[7] B. Deissler, M. Zaccanti, C. D’Errico, M. Fattori, M. Modugno, G. Modugno, and M. Inguscio, Nat. Phys. 6, 354 (2010).

[8] E. E. Edwards, M. Beeler, T. Hong, and S. L. Rolston, Phys. Rev. Lett. 101, 260402 (2010).

[9] M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, Science 349, 842 (2015).

[10] B. Gadway, D. Pertot, J. Reeves, M. Vogt, and D. Schneble, Phys. Rev. Lett. 107, 145306 (2011).

[11] D. Pekker, G. Refael, and E. Demler, Phys. Rev Lett.
[12] P. Mohan, P. M. Goldbart, R. Narayanan, J. Toner, and T. Vojta, Phys. Rev Lett. 105, 085301 (2010).
[13] N. Laflorencie, EPL 99, 66001 (2012).
[14] T. Vojta, J. Low Temp. Phys. 161, 229 (2010).
[15] M. Randeria, J. P. Sethna, and R. G. Palmer, Phys. Rev. Lett. 54, 1321 (1985).
[16] R. B. Griffiths, Phys. Rev. Lett. 23, 17 (1969).
[17] S. Ubaid-Kassis, T. Vojta, and A. Schroeder, Phys. Rev. Lett. 104, 066402 (2010).
[18] R. Wang, A. Gebretsadik, S. Ubaid-Kassis, A. Schroeder, T. Vojta, P. J. Baker, F. L. Pratt, S. J. Blundell, T. Lancaster, I. Franke, J. S. Moller, and K. Page, Phys. Rev. Lett. 118, 267202 (2017).
[19] R. Desbuquois, L. Chomaz, T. Yefsah, J. Leonard, J. Beugnon, C. Weitenberg, and J. Dalibard, Nat. Phys. 8, 645 (2012).
[20] L. Mathey, A. Vishwanath, and E. Altman, Phys. Rev. A 79, 013609 (2009).
[21] Y.-J. Lin, A. R. Perry, R. L. Compton, I. B. Spielman, and J. V. Porto, Phys. Rev. A 79, 063631 (2009).
[22] The out-of-plane momentum $p_\perp$ is contained in the $J_{i,j}$ and $\tilde{g}_i$ terms of the Hamiltonian.
[23] Y. Castin and R. Dum, Phys. Rev. Lett. 77, 5315 (1996).
[24] B. Fang, A. Johnson, T. Roscilde, and I. Bouchoule, Phys. Rev. Lett. 116, 050402 (2016).
[25] S. Folling, F. Gerbier, A. Widera, O. Mandel, T. Gericke, and I. Bloch, Nature 434, 481 (2005).
[26] I. B. Spielman, W. D. Phillips, and J. V. Porto, Phys. Rev. Lett. 98, 080404 (2007).