$H^0 \rightarrow \bar{\ell}_i \ell_j$ through TeV sterile neutrinos

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Abstract

The presence of massive sterile neutrinos $N$ mixed with the active ones induces flavor violating processes in the charged lepton sector at the loop level. In particular, the amplitude of $H^0 \rightarrow \bar{\ell}_i \ell_j$ is expected to be proportional to the product of heavy-light Yukawa couplings $y_i y_j = 2 s_{\nu_i} s_{\nu_j} m_N^2 / v^2$, where $s_{\nu_i,j}$ express the heavy-light neutrino mixings. Here we revisit these Higgs decays in the most generic extension of the neutrino sector, focusing on large values of $y_i$. We show that decoupling effects and a cancellation between the two dominant contributions to these processes makes the amplitude about one hundred times smaller than anticipated. We find that perturbative values of $y_i$ giving an acceptable contribution to the $\rho$ parameter imply $\mathcal{B}(H^0 \rightarrow \bar{\ell}_i \ell_j) < 10^{-8}$ for any lepton flavors, a rate that is not accessible at current colliders.
1 Introduction

The nature of the neutrino masses remains as one of the most intriguing questions in particle physics. Neutrinos are different from the other fermions in that the $SU(2)_L$ singlet required to give them an electroweak (EW) mass is not protected by chirality. The possible mass of this singlet will then define a new scale that, if very large, would explain the tiny value of the neutrino masses ($m_\nu < 1 \text{ eV}$) deduced from flavor oscillations. Indeed, the so called type-I seesaw mechanism provides a minimal and very appealing way to complete the lepton sector of the SM.

There are, however, other non-minimal possibilities that may be considered as well. Notice that gauge singlets, if present, can have any mass. From a phenomenological point of view, the origin of their interactions are arbitrary Yukawa couplings that mix them with the active neutrinos, so they could be very weakly coupled to matter and thus easily avoid all experimental bounds. From a model building point of view, they appear naturally in extensions of the SM with a cutoff much lower than the seesaw scale. This is the case, for example, in little Higgs models [1–3], TeV gravity models [4,5] or composite Higgs models [6], where neutrino masses must be explained relying on physics at or below the TeV scale. In the end, it is the data on neutrino oscillations and charged-lepton flavor physics what decides about the motivation for these sterile neutrinos.

The appearance of non-EW terms in the extended neutrino mass matrix and the different gauge charges of active and sterile neutrinos will imply that the rotation defining the mass eigenstates does not diagonalize, respectively, the Higgs nor the $Z$ coupling to the neutrinos. At the loop level these flavor-changing neutral currents (FCNC), and also the charged currents coupled to the $W$ boson, induce flavor violating processes involving the charged leptons (cLFV) [7–13]. Here we will be interested in these processes. In particular, we will study the cLFV decays $H^0 \rightarrow \bar{\ell}_i \ell_j$ in the presence of the generic heavy sterile neutrinos that appear in the context of low-scale seesaw models. These decay channels are currently searched at the LHC; at 95% C.L., ATLAS [14] and CMS [15,16] find

\begin{align}
B(H^0 \rightarrow \mu e) &< 6.1 \times 10^{-5} \text{ (ATLAS); } 3.5 \times 10^{-4} \text{ (CMS)}, \\
B(H^0 \rightarrow \tau e) &< 2.8 \times 10^{-3} \text{ (ATLAS); } 6.1 \times 10^{-3} \text{ (CMS)}, \\
B(H^0 \rightarrow \tau \mu) &< 4.7 \times 10^{-3} \text{ (ATLAS); } 2.5 \times 10^{-3} \text{ (CMS)},
\end{align}

(1.1)

where $H^0 \rightarrow \ell_i \ell_j$ stands for $H^0 \rightarrow \bar{\ell}_i \ell_j$, $\ell_i \bar{\ell}_j$. Our objective is to establish the maximum rate for these processes that could possibly be caused by the heavy sterile neutrinos.
Previous literature reports approximate results \cite{17,18} or detailed computations \cite{19–22} in the context of inverse seesaw models for neutrino masses. Here we will introduce a minimal set-up \cite{23} that contains just two heavy neutrinos but that is able to capture all the flavor effects relevant in these processes. The simplicity of the parametrization lets us understand the limit with large (top-quark like) Yukawa couplings for the singlets, where one may expect branching ratios near the current bounds. We show that the contribution from such couplings to the $\rho$ parameter may be acceptable (actually, we find remarkable that $\Delta \rho$ from the singlet fermions may have any sign), but that the appearance of a cancellation and of decoupling effects push the decay modes well below these bounds.

2 The set-up

Flavor oscillation experiments are able to access the tiny value of the neutrino masses by combining two very different scales, $L^{-1}E_\nu \approx \Delta m^2_\nu$. In cLFV experiments, however, the lowest available scale is $m_\ell$, so these experiments are not sensitive to $m_\nu$. Any observable effects will then depend on the possibly much larger masses of additional fermion singlets that mix with the active flavors. It turns out that to capture all the cLFV effects in a consistent way it will suffice to consider two massive 2-spinors that may be defining a single Dirac fermion or two Majorana fields of different mass. Although these singlets will not be responsible for the masses of the active neutrinos, the key point is that all the extra ingredients required to complete the neutrino sector will have no effect on cLFV observables.

Let us be more specific (see \cite{23} for details). Consider five self-conjugate spinors $\chi_i = \chi_{Li} + \chi_{Li}^c$ that include the three active neutrinos ($i = 1, 2, 3$) plus two sterile spinors of opposite lepton number ($i = 4, 5$). We will assume that in the basis of the charged-lepton mass eigenstates the only new terms in the Lagrangian are

$$-\mathcal{L} \supset \left( \sum_{i=1}^{3} y_i H L_i P_R \chi_5 + M \bar{\chi}_4 P_R \chi_5 + \frac{1}{2} \mu \bar{\chi}_5 P_R \chi_5 \right) + \text{h.c.} \quad (2.1)$$
Once the Higgs gets a v.e.v. the Majorana mass matrix for the 5 flavors reads

\[
\mathcal{M} = \begin{pmatrix}
0 & 0 & 0 & 0 & m_1 \\
0 & 0 & 0 & 0 & m_2 \\
0 & 0 & 0 & 0 & m_3 \\
m_1 & m_2 & m_3 & M & \mu \\
\end{pmatrix}.
\] (2.2)

Notice that we have ordered the fields according to their lepton number (positive for the first four neutrinos), that the Majorana mass \(\mu\) corresponds to the neutrino with negative lepton number and that \((m_i, M)\) are Dirac masses. The diagonalization of this matrix yields two states \(N_{1,2}\) of mass

\[
m_{N_1} = \frac{1}{2} \left( \sqrt{4 (m_1^2 + m_2^2 + m_3^2 + M^2) + \mu^2} - \mu \right),
\]
\[
m_{N_2} = \frac{1}{2} \left( \sqrt{4 (m_1^2 + m_2^2 + m_3^2 + M^2) + \mu^2} + \mu \right),
\] (2.3)

plus three massless neutrinos \(\nu_i\). It is straightforward to find that these three neutrinos have a component along the (2-dim) sterile flavor space (a heavy-light mixing)

\[
s_{\nu_i} = \frac{m_i}{\sqrt{m_{N_1} m_{N_2}}}.
\] (2.4)

For \(\mu = 0\) the two massive modes will define a Dirac field \((m_{N_1} = m_{N_2})\); in this case, a small entry \(\mu'\) in position \(\mathcal{M}_{44}\) would give a mass \(m_\nu \approx \mu'(m/M)^2\) to one of the standard neutrinos, as proposed in inverse-seesaw models \([24, 25]\). In the opposite limit, if \(M = 0\) and \(\mu \rightarrow 10^{10}\) GeV the configuration describes a type-I seesaw mechanism, with one of the active neutrinos massive, \(m_{N_1} \approx (m_1^2 + m_2^2 + m_3^2)/\mu\), while the second singlet \((\chi_4)\) is massless but decoupled. For \(\mu\) in the TeV range, as long as \(M > 10\sqrt{m_1^2 + m_2^2 + m_3^2}\) (i.e., the mixings are below 0.1) the model may be viable. At any rate, \(\mathcal{M}\) is a rank-2 matrix with three zero mass eigenvalues. As we argued above, the extra spinors and couplings required to generate light neutrino masses will have no effect on cLFV observables. The 5 parameters in \(\mathcal{M}\) are enough to describe all cLFV effects caused by heavy Dirac or Majorana singlets mixed with the three active families.

One should also notice, however, that if \(\mu \neq 0\) the matrix above is not stable under radiative corrections \([26]\): the breaking of lepton number will contribute to all the entries in \(\mathcal{M}\) at the loop level, which would give mass to a linear combination of the three \(\nu_i\). If this breaking is small the mass will be acceptable (i.e., below 1 eV), but if \(\mu\) is large the model will require a fine tuned cancellation of these loop contributions.
summary, the texture that we propose in Eq. (2.2) must be understood as approximate and established at the loop level where we work. Despite the fine tune that this involves, we will consider TeV values of $\mu$ in order to understand the genuine Majorana effects on cLFV observables and on the contribution to the $\rho$ parameter from heavy singlets.

3 Large Yukawa couplings and $\Delta \rho$

The Yukawa couplings $y_i$ in Eq. (2.1) are the origin of any interactions of the heavy singlets, and the rate of $H^0 \rightarrow \bar{\ell}_i \ell_j$ will certainly grow with them. In our model, their relation with the masses and mixings is

$$y_i = \sqrt{2} \frac{m_i}{v} = \sqrt{2} \frac{\sqrt{m_{N_1} m_{N_2}}}{v} s_{\nu_i}.$$  \hspace{1cm} (3.1)

The expression above shows that, for a fixed value of the mixings consistent with current constraints, large singlet masses will probe large values of $y_i$. These couplings, however, break the custodial symmetry of the SM and will contribute to the $\rho$ parameter (or to the Peskin-Takeuchi parameter $T = (\rho - 1)/\alpha$). These oblique corrections can be easily obtained from the contribution of the heavy neutrinos to the gauge boson self-energies at $q^2 = 0$,

$$\Delta \rho = \frac{(\Pi_{WW})_{N_1,2}}{M_W^2} - \frac{(\Pi_{ZZ})_{N_1,2}}{M_Z^2},$$ \hspace{1cm} (3.2)

and they are constrained to be $|\Delta \rho| \lesssim 0.0005$ [27]. At one loop and neglecting charged lepton masses, we find (see the couplings to gauge and Goldstone bosons in Appendix A)

$$\Delta \rho = \frac{g^2}{32\pi^2 M_W^2} \left( \sum_{k=1}^{3} s_{\nu_k}^2 \right)^2 \frac{m_{N_1}^2 m_{N_2}^2}{(m_{N_1} + m_{N_2})^2} \left( 3 - 2 \frac{m_{N_1}^2 + m_{N_2}^2 - m_{N_1} m_{N_2}}{m_{N_2}^2 - m_{N_1}^2} \ln \frac{m_{N_2}}{m_{N_1}} \right).$$ \hspace{1cm} (3.3)

This result presents some interesting features. Let us assume for simplicity mixing with just $\nu_\tau$ and consider first the case with a Dirac singlet field ($\mu = 0$). The contribution is then obtained from Eq. (3.3) by taking the limit $m_{N_1}, m_{N_2} \rightarrow m_N$:

$$\Delta \rho = \frac{g^2}{64\pi^2 M_W^2} s_{\nu_\tau}^4 m_N^2 = \frac{g^2}{64\pi^2 M_W^2} s_{\nu_\tau}^2 \left( y_3 \frac{v}{\sqrt{2}} \right)^2.$$ \hspace{1cm} (3.4)

If we compare this with the correction from the top quark,

$$\Delta \rho_t = 3 \frac{g^2}{64\pi^2 M_W^2} \left( y_t \frac{v}{\sqrt{2}} \right)^2 \approx 0.009,$$ \hspace{1cm} (3.5)
we see an extra suppression by a decoupling factor of \( s_{\nu_r}^2 \). Obviously, if the heavy neutrino were a sequential doublet with a purely EW mass this suppression would be absent; in this case the contribution should be canceled by restoring the custodial symmetry with a very similar Yukawa coupling of the charged lepton in the same doublet. But here, for \( s_{\nu_r} < 0.1 \) and \( y_3 < \sqrt{4\pi} \) we have that \( \Delta \rho < 0.00038 \) is within the experimental bounds.

Another interesting limit goes in the opposite direction: a Majorana mass \( \mu \) much larger than \( M \) and then \( m_{N_2} \gg m_{N_1} \). It is easy to see that if

\[
m_{N_2}^2 > 30 m_{N_1}^2 \quad \text{(or } \mu > 2.1 M) \quad \text{(3.6)}
\]

the second term in Eq. (3.3) dominates and the contribution to \( \Delta \rho \) is negative, something remarkable as multiplets of non-degenerate Dirac fermions always give \( \Delta \rho > 0 \). For \( s_{\nu_r} < 0.1 \) and \( y_3 < \sqrt{4\pi} \) we obtain \(-\Delta \rho < 0.00012\). The correction for a type-I seesaw mechanism \((M = 0, \mu \gg 1 \text{ TeV})\) is just

\[
\Delta \rho \approx -\frac{g^2}{32\pi^2 M_W^2} m_{\nu_r}^2 \left( 2 \ln \frac{\mu}{m_{\nu_r}} - 3 \right),
\]

with \( m_{\nu_r} = y_3^2 v^2 / (2\mu) \). Our results for \( \Delta \rho \) from TeV fermion singlets are consistent with the generic ones in [28].

4 \( H^0 \to \bar{\ell}_i \ell_j \)

The one-loop amplitude for \( H^0 \to \bar{\ell}_i \ell_j \) is mediated in the Feynman-’t Hooft gauge by the diagrams in Fig. 1. One can see that all these diagrams are at least proportional to

\[
y_i y_j y_{\ell_i} = 2\sqrt{2} s_{\nu_r} \frac{m_{N_1} m_{N_2} m_{\ell_i}}{v^3},
\]

where \( \ell_i \) above refers to the heavier final lepton. In addition, diagrams \( W \chi \chi \), \( \chi W G \) and \( W \chi \) are proportional to \( g^2 \), \( \chi W W \) is proportional to \( g^4 \), and \( \chi G G \) to the Higgs quartic coupling \( \lambda \). Of course, each diagram will also depend on the mass and spin of the particles inside the loop, but one may expect that \( G \chi \chi \) and \( G \chi \) dominate with a contribution of order \( \mathcal{M} \approx y_i y_j y_{\ell_i} / (16\pi^2) \).

Using this rough estimate, we can deduce the maximum branching ratio in Higgs decays by comparing with \( \mathcal{B}(H^0 \to \bar{b}b) \simeq 0.6 \). For the decay \( H^0 \to \tau e \) we expect

\[
\mathcal{B}(H^0 \to \tau e) = \mathcal{B}(H^0 \to \bar{b}b) \frac{2 \Gamma(H^0 \to \bar{b}b)}{\Gamma(H^0 \to \tau e)} \approx \mathcal{B}(H^0 \to \bar{b}b) \frac{2}{3} \left( \frac{y_3 y_1 y_\tau}{y_b 16\pi^2} \right)^2.
\]

(4.2)
Taking $y_i < \sqrt{4\pi}$ this gives $\mathcal{B}(H^0 \to \tau e) < 4 \times 10^{-4}$, a value that could be accessible once the LHC reaches its highest luminosity. However, a precise calculation will show that this is not the case.

First of all, although their sum is finite, the diagrams $G\chi\chi$ and $G\chi$ are both divergent. In addition, there is a value of the heavy neutrino mass that exactly cancels the sum of both contributions. For $m_{N_1} = m_{N_2}$ this is

$$\bar{m}_N \approx 0.57 \frac{M_H}{\sqrt{s_{\nu_e}^2 + s_{\nu_\mu}^2 + s_{\nu_\tau}^2}}.$$  \hfill (4.3)$$

Finally, at masses of the heavy neutrinos above $\bar{m}_N$ there are decoupling effects, like the extra factor of $s_{\nu_\tau}^2$ in $\Delta \rho$ found in the previous section.

Let us be more definite. We write the decay amplitude

$$\mathcal{M}(H^0 \to \bar{\tau}e) = \bar{u}(p_2) \frac{f_{\tau e}}{v} [m_\tau P_R + m_e P_L] v(p_1)$$  \hfill (4.4)$$

and will give the results in terms of $m_{N_1}$ and the ratio

$$r \equiv \frac{m_{N_2}^2}{m_{N_1}^2} \geq 1.$$  \hfill (4.5)$$
Figure 2: Contribution to $|f^{\tau e}|$ from the dominant diagrams $G_{\chi\chi} + G_{\chi}$ for maximal mixings and $r = 1, 100$. We have included $|f^{\tau e}|$ from the estimate in Eq. (4.2) for $r = 1$ (gray dots) as well as the contribution from a massive neutrino with an active left-handed component (red dashes).

Constraints from flavor-diagonal processes [29,30] together with

\[ \mathcal{B}(\mu \rightarrow e\gamma) \approx \frac{3\alpha}{8\pi} s_{\nu_e}^2 s_{\nu_{\mu}}^2 s_{\nu_{\tau}}^2 \leq 4.2 \times 10^{-13} \]  

(4.6)

imply [23]

\[ s_{\nu_e}^{\text{max}} = 0.05, \quad s_{\nu_{\mu}}^{\text{max}} = 4.5 \times 10^{-4}, \quad s_{\nu_{\tau}}^{\text{max}} = 0.075. \]  

(4.7)

In Fig. 2 we plot the contribution to $|f^{\tau e}|$ from the $G_{\chi\chi} + G_{\chi}$ for these maximal mixings and $r = 1, 100$. We see that it grows with the heavy-light Yukawas, then there appears the cancellation at $\tilde{m}_N$ discussed above, and finally the amplitude reaches a regime where it grows again with the Yukawas but is suppressed by a (decoupling) factor of $s_{\nu_e}^2 + s_{\nu_{\mu}}^2 + s_{\nu_{\tau}}^2 \approx 0.01$ for maximal mixings. The curves finish at $y_i = \sqrt{4\pi}$, that is $m_{N_1} = 8.2$ (2.6) TeV for $r = 1$ (100). The plot also shows that Majorana effects ($r = 1$ corresponds to a Dirac heavy neutrino) do not change the qualitative behavior of the amplitude and are not able to increase the maximum value of $|f^{\tau e}|$. In the same plot we have included the amplitude for a heavy neutrino in a $SU(2)_L$ doublet: a Dirac field with an active left handed component. Such a neutrino does not decouple for large values of $m_N$, which is purely EW; the plot reveals that in this case $G_{\chi\chi} + G_{\chi}$ follows

\[ \text{This case requires a charged lepton of similar mass to cancel } \Delta R \text{ as well as extra EW fermions to cancel anomalies (e.g., to complete the whole sequential 4th family) that are excluded by the LHC.} \]
Figure 3: Contribution to $|f^{r e}|$ from the different diagrams in Fig. 1 for maximal mixings and $r = 1, 100$. The thick line corresponds to the sum of all the diagrams.

In Fig. 3 we plot the modulus of each contribution and of the sum of all diagrams for $r = 1, 100$. The dominant contribution comes from $\chi GG$ except at very large Yukawa couplings, i.e., maximal mixings and heavy neutrino masses above 2 TeV, when diagrams $G_{\chi \chi} + G_{\chi}$ take the lead despite the decoupling factor, yielding a maximum value that is two orders of magnitude smaller than the naive guess given before. In Appendix B we present expressions for the form factors and give further details of our computation.

5 Summary and discussion

Vectorlike fermions at the TeV scale are a possibility with interesting phenomenological consequences. If they are quarks or charged leptons that mix with the active families, their different EW numbers will induce tree-level FCNCs that are very constrained experimentally. If they are neutrinos, however, collider effects appear at the loop level and the bounds are weaker. Here we have focused on cLFV decays of the Higgs boson. These processes have been studied by several groups, with results that sometimes appear as contradictory. In this work we have proposed a set-up with two sterile fields that
captures all flavor effects and lets us understand the results in a simple way. The model reveals, for example, that in generic low-scale seesaw models Majorana singlets with TeV mass and unsuppressed mixings with the active neutrinos are indeed possible, although they require a fine-tuned cancellation of loop corrections so that the observed neutrinos have sub-eV masses (notice that inverse seesaw models the heavy neutrinos are quasi-Dirac). Or that large values of the heavy-light Yukawa couplings in these models have an impact on $\Delta \rho$ for large enough heavy-light mixings.

Our analysis shows that the Higgs decay modes $H^0 \to \bar{\ell}_i \ell_j$ are not accessible at colliders. The rate of these decays is expected to grow with the Yukawa couplings that mix active and sterile neutrinos, but a cancellation of different contributions and decoupling effects proportional to the sum of squared mixings damp the final result. These two features are clearly shown in Fig. 2. We see that for a fixed mixing and a relatively light neutrino mass the amplitude grows with the Yukawa couplings (which are proportional to the mass) as expected, until the scale in Eq. (4.3) where the dominant amplitude goes to zero and changes sign. At heavier neutrino masses the amplitude grows again with the couplings, however, all but a component of order $(s^2_{\nu_e} + s^2_{\nu_\mu} + s^2_{\nu_\tau})^{1/2}$ is decoupled: the amplitude $\mathcal{M} \approx y_i y_j y_{\ell_i}/(16 \pi^2)$ at low singlet masses becomes of order $(s^2_{\nu_e} + s^2_{\nu_\mu} + s^2_{\nu_\tau}) y_i y_j y_{\ell_i}/(16 \pi^2)$ in this decoupled regime. As a consequence, we find that the largest branching ratio consistent with the maximal mixings summarized in Eq. (4.7) would correspond to the channel $H^0 \to \tau e$ and is

$$B(H^0 \to \tau e) < 1.4 \times 10^{-8}. \quad (5.1)$$

We conclude that the observation of cLFV in Higgs decays at the LHC would involve a different type of new physics.

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10
A Flavor-changing vertices and mixing matrices

The neutrino mass eigenstates come from the interaction eigenstates by the replacement

$$\chi_L \rightarrow \sum_{j=1}^{5} U_{ij}^{\nu} \chi_L,$$  \hspace{1cm} (A.1)

where $U^{\nu}$ is the unitary matrix diagonalizing $M$ (2.2) into real and positive mass eigenvalues. The Lagrangian for charge-current interactions reads

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} W_{\mu}^{-} \sum_{i=1}^{3} \sum_{j=1}^{5} B_{ij} \bar{\ell}_i \gamma^\mu P_L \chi_j + \text{h.c.},$$  \hspace{1cm} (A.2)

where we have used the convention $D_\mu = \partial_\mu - ig\tilde{W}_\mu$ for the covariant derivative and

$$B_{ij} = \sum_{k=1}^{3} \delta_{ik} U_{kj}^{\nu}$$  \hspace{1cm} (A.3)

is a rectangular $3 \times 5$ mixing matrix. In the Feynman-'t Hooft gauge one also needs

$$\mathcal{L}_{G^\pm} = -\frac{g}{\sqrt{2}M_W} G_{\mu} \sum_{i=1}^{5} \sum_{j=1}^{5} B_{ij} \bar{\ell}_i (m_\ell P_L - m_\chi P_R) \chi_j + \text{h.c.},$$  \hspace{1cm} (A.4)

where $G^{\pm}$ is the charged would-be-Goldstone field. The matrix $U^{\nu}$ introduces tree-level flavor-changing interactions with the $Z$ and the Higgs field:

$$\mathcal{L}_Z = \frac{g}{4c_W} Z_\mu \sum_{i,j=1}^{5} \bar{\chi}_i \gamma^\mu (C_{ij} P_L - C_{ij}^* P_R) \chi_j,$$  \hspace{1cm} (A.5)

$$\mathcal{L}_H = -\frac{g}{4M_W} H \sum_{i,j=1}^{5} \bar{\chi}_i [(m_\chi C_{ij} + m_\chi C_{ij}^*) P_L + (m_\chi^* C_{ij}^* + m_\chi C_{ij}) P_R] \chi_j,$$  \hspace{1cm} (A.6)

where

$$C_{ij} = \sum_{k=1}^{3} (U_{ki}^{\nu})^* U_{kj}^{\nu}.$$  \hspace{1cm} (A.7)

A symmetry factor of 2 must be added in the Feynman rule for vertices including two (self-conjugate) Majorana fermions [31, 32]:

$$Z_\mu \rightarrow \frac{g}{2c_W} \gamma^\mu \left( C_{ij} P_L - C_{ij}^* P_R \right),$$  \hspace{1cm} (A.8)
\[ H^0 \longrightarrow \chi_j (\chi_i) \rightarrow \chi_i (\chi_j) \]
\[-i \frac{g}{2M_W} [(m_{\chi_i} C_{ij} + m_{\chi_j} C^*_{ij}) P_L + (m_{\chi_i} C^*_{ij} + m_{\chi_j} C_{ij}) P_R]. \quad \text{(A.9)}\]

One can recover the case of active Dirac neutrinos by replacing \( C_{ij} \rightarrow \delta_{ij}, \ C^*_{ij} \rightarrow 0. \)

The mixing matrix elements involving heavy neutrinos can be expressed in terms of heavy-light mixings and the squared mass ratio \( r = m_{N_2}^2/m_{N_1}^2 \) as

\[
B_{kN_1} = -\frac{ir_1^1}{\sqrt{1 + r_1^1}} s_{\nu_k}, \quad B_{kN_2} = \frac{1}{\sqrt{1 + r_1^1}} s_{\nu_k}, \quad \text{(A.10)}
\]

\[
C_{N_1N_1} = \frac{r_1^1}{1 + r_1^1} \sum_{k=1}^3 s_{\nu_k}^2, \quad C_{N_2N_2} = \frac{1}{1 + r_1^1} \sum_{k=1}^3 s_{\nu_k}^2, \quad C_{N_1N_2} = -C_{N_2N_1} = \frac{ir_1^1}{1 + r_1^1} \sum_{k=1}^3 s_{\nu_k}^2. \quad \text{(A.11)}
\]

### B Form factors

The form factors \( f^{\ell\nu} \) receive contributions from the one-loop diagrams of Fig. 1 in the Feynman-’t Hooft gauge. Neglecting charged lepton masses we find:

\[
f^{\ell\nu}_{WW} = \frac{g^2}{16\pi^2} \sum_{i,j=1}^5 B^*_i B_{e'j} \left\{ C_{ij} \sqrt{x_i x_j} \left[ c_0 + 2 c_1 \right] \right.
\]
\[+ C^*_{ij} \left[ x_j c_0 + (x_i + x_j) c_1 \right] \right\}, \quad \text{(B.1)}
\]

\[
f^{\ell\nu}_{\chi W} = \frac{g^2}{16\pi^2} \sum_{i=1}^5 B^*_i B_{e'1} \left[ -2 \bar{c}_1 \right], \quad \text{(B.2)}
\]

\[
f^{\ell\nu}_{G\chi} = \frac{g^2}{16\pi^2} \sum_{i,j=1}^5 B^*_i B_{e'j} \left\{ C_{ij} \sqrt{x_i x_j} \left[ \frac{1}{4} - 2 c_0 + \frac{1}{2} (x_i + x_j) c_1 + \frac{1}{2} x_Q c_{12} \right] \right.
\]
\[+ C^*_{ij} x_j \left[ \frac{1}{4} - 2 c_0 + x_i c_1 + \frac{1}{2} x_Q c_{12} \right] \right\}, \quad \text{(B.3)}
\]

\[
f^{\ell\nu}_{\chi G} = \frac{g^2}{16\pi^2} \sum_{i=1}^5 B^*_i B_{e'1} \left[ -\frac{1}{2} x_H x_i (\bar{c}_0 + \bar{c}_1) \right], \quad \text{(B.4)}
\]

\[
f^{\ell\nu}_{\chi W} = \frac{g^2}{16\pi^2} \sum_{i=1}^5 B^*_i B_{e'1} \left[ \frac{1}{4} - 2 \bar{c}_0 + \frac{1}{2} x_i (\bar{c}_0 + 2 \bar{c}_1) + \frac{1}{2} x_Q (2 \bar{c}_1 + \bar{c}_{12}) \right], \quad \text{(B.5)}
\]

\[
f^{\ell\nu}_{W\chi} = 0, \quad \text{(B.6)}
\]
where we have introduced the following dimensionless functions in terms of the standard Passarino-Veltman loop functions [33]:

\[
b_0(x_i) \equiv B_0(0; M_W^2, x_i M_W^2, M_W^2), \quad c_{00}(x_i, x_j) \equiv C_{00}(0, Q^2, 0; M_W^2, x_i M_W^2, x_j M_W^2),
\]

\[
c_{(0,1,12)}(x_i, x_j) \equiv M_W^2 C_{(0,1,12)}(0, Q^2, 0; M_W^2, x_i M_W^2, x_j M_W^2), \quad \tau_{00}(x_i) \equiv C_{00}(0, Q^2, 0; x_i M_W^2, M_W^2), \quad \tau_{(0,1,12)}(x_i) \equiv M_W^2 C_{(0,1,12)}(0, Q^2, 0; x_i M_W^2, M_W^2),
\]

with \( x_i \equiv m_{\chi_i}^2/M_W^2, \ x_Q \equiv Q^2/M_W^2, \ x_H \equiv M_H^2/M_W^2 \approx 2.4 \), and \( Q^2 = M_W^2 \) for an on-shell Higgs. We use the conventions of [34]. The functions \( b_0, c_{00} \) and \( \tau_{00} \) are ultraviolet divergent but, thanks to relations between \( B \) and \( C \) matrix elements [23], the divergences in \( f^{\ell\ell}_{G\chi} \) and \( f^{\ell\ell}_{G\chi} \) cancel each other, and \( f^{\ell\ell}_{\chi WG} \) is finite when summing over all neutrino states. The other diagrams are finite.

It turns out convenient to cast the contributions to the form factor (B.1–B.7) into mixing-independent functions \( F, \ G, \ H \):

\[
f^{\ell\ell} = \frac{g^2}{16\pi^2} \sum_{i,j=1}^{5} B_{i1}^* B_{j1} \left[ \delta_{ij} F(x_i) + C_{ij} G(x_i, x_j) + C_{ij}^* H(x_i, x_j) \right],
\]

(B.13)

In this way, the form factor can be expressed in terms of massive neutrinos only [23] as:

\[
f^{\ell\ell} = \frac{g^2}{16\pi^2} \sum_{i,j=1}^{2} B_{iN_i}^* B_{jN_j} \left\{ \delta_{ij} [F(x_{N_i}) - F(0)] + \delta_{ij} [G(x_{N_i}, 0) + G(0, x_{N_j}) - 2G(0, 0)] + \delta_{ij} [H(x_{N_i}, 0) + H(0, x_{N_j}) - 2H(0, 0)] + C_{N_iN_j} [G(x_{N_i}, x_{N_j}) - G(x_{N_i}, 0) - G(0, x_{N_j}) + G(0, 0)] + C_{N_iN_j}^* [H(x_{N_i}, x_{N_j}) - H(x_{N_i}, 0) - H(0, x_{N_j}) + H(0, 0)] \right\},
\]

(B.14)

where, in our particular case,

\[
G(0, 0) = G(0, N_i) = G(0, N_j) = 0,
\]

\[
H(0, 0) = H(0, 0) = 0.
\]

(B.15) (B.16)

Then, substituting (A.10) and (A.11), the form factor has two terms,

\[
f^{\ell\ell} = \frac{g^2}{16\pi^2} s_{\nu\ell} s_{\nu\ell} \left[ f^{(1)} + \sum_{k=1}^{3} s^2_{\nu_k} f^{(2)} \right],
\]

(B.17)
where \( f^{(1)} \) and \( f^{(2)} \) do not depend on mixings. Only diagrams containing the flavor-changing vertex \( H_{\chi_i \chi_j} \) contribute to \( f^{(2)} \), but we treat \( G_{\chi \chi} \) and \( G_{\chi} \) together since they cancel the ultraviolet divergences of one other, present in the part \( f^{(1)} \). In the case of diagrams \( G_{\chi \chi} + G_{\chi} \), the part \( f^{(2)} \), subdominant at low neutrino masses in any case, cancels \( f^{(1)} \) at some point and, for large neutrino masses, becomes the dominant contribution despite the \( s^2_{\nu_\alpha} \) suppression. This is because it keeps growing like \( m_{N_1} m_{N_2} \).

The case of one Dirac singlet (two equal-mass Majorana neutrinos) corresponds to:

\[
\begin{align*}
f^{(1)} &= F(x_N) + G(x_N, x_N) + H(0, x_N) - F(0), \\
f^{(2)} &= H(x_N, x_N) - H(0, x_N).
\end{align*}
\tag{B.18}
\tag{B.19}
\]

The case of one active Dirac neutrino (sequential) can be recovered from:

\[
f_{\ell \ell}^{\text{seq}} = \frac{g^2}{16\pi^2} s_{\nu_\ell} s_{\nu_{\ell'}} \left[ F(x_N) + G(x_N, x_N) - F(0) \right].
\tag{B.20}
\]

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