Study on the coupled relationship between AE accumulative ring-down count and damage constitutive model in soil unconfined compression test

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Abstract: Indoor acoustic emission tests can be used to obtain acoustic emission characteristics and damage evolution rules under uniaxial compression conditions during the damaging process of soil body. Soil damage is characterized by ringing acoustic emission count by analyzing the variation relation between AE parameters and stress. The results show that the acoustic emission distribution can be classified into 3 different stages. Starting from the time domain, the coupling relation between the cumulative acoustic emission ring-down count and strain is derived, which is based on the time parameter as an intermediate variable. Further, the coupling relation between the cumulative acoustic emission ring-down count and stress and damage is deduced, which is based on distribution of the Weibull damage constitutive model. Results indicated that theory curves and test dates matched well. Proved by the test data, the precision of constitutive model is higher, which can provide accurately the basis for soil damage assessment.

1. Introduction

Acoustic emission refers to the phenomenon that occurs when material or structure undergoes deformation or suffer damage. This phenomenon is mainly presented as the object interior rapidly produces the energy after receiving the external action, and then the energy released in the form of elastic wave [1]. Acoustic emission represents micro-damage of the material and reflects the microscopic rupture of the material. Hence, the parameters (acoustic emission ring count, energy, etc.) may be used to characterize the level of damage of the material, and should also fall into this category just like the stress, strain constitutive parameters of constitutive equation. As a tool that reflects the internal dynamic changes of the material, the strength of the characteristic parameters of acoustic emission indicates the degree of soil damage, and has a direct relationship with evolution of the internal soil damage. Therefore, there is an inevitable inner link between the acoustic emission characteristics of the soil and damage variable of the soil. In the 1880s to 1990s, the Japanese scholar Ohtsu [2] and American scholars Dai [3] are proposed empirical and theoretical formula of acoustic emissions and stress, meanwhile, they have verified the suitability of the formula. Since then, many scholars put forward a lot of constitutive damage models of acoustic emission under different parameters and conditions [4-6].

Discreteness of soil particles or soil clumps and non-uniformity of soil adhesion reflect the essence of soil structure, and strength of soil non-uniformity determines that attenuation law of soil strength must be considered in order to further the research of soil damage. Based on this, macro-analysis method
of soil is used to assume soil structure of discrete structure as continuous medium model, the space in the model is completely filled by the particle. The physical quantities of the particle such as deformation and damage are continuous functions in space, and comply with the Newton's mechanics laws and thermodynamics laws. But in practical research of soil unit, the real qualitative behavior of the material of discrete form are ignored and will not be considered, only take the average effect into consideration to reflect the structure damage. Then a model that may reflect derived development of the soil structure damage based on the continuous medium mechanics can be established. Many scholars [7-9] discussed the soil damage model based on this. In addition, as theories of acoustic emission technology fall behind its practice, the usage in the soil structure is facing the problem of how to establish a substantive link between acoustic emission characteristic parameters and soil mechanical parameters to provide evidence for instability failure of soil engineering. In this paper, the uniaxial unconfined compression test of the soil containing the different moisture content is carried out to obtain the emission characteristics of the parameters and mechanical parameters of the soil under uniaxial unconfined compression. And then the damage model of acoustic emission parameters and mechanical parameters for the soil are derived, and provide the theoretical support for the research of the soil damage using acoustic emission technology in the engineering.

2. Indoor uniaxial compressive acoustic emission test

2.1 Soil sample information

The soil samples used in the test are silty clay which collected in Harbin, China, and the properties of the soil can be found in Table 1. According the "standard of geotechnical test method" (GB-T50123-1999) for the preparation of soil samples to taking the soil placed in a well-ventilated place to dry, and then screening the soil by 2mm sieve after grinding it. The soil below the 2mm is retained, and the soil samples is divided into 3 groups according to the optimum water content of 15.38% measured by test: group A as the moisture content is 13% of the specimen; group B as the moisture content is 15% of the specimen; the group C as the moisture content is 17% of the sample, each set of specimens of five, A total of 15 specimens. The sample compacted by Multifunctional light electric compaction instrument is made into a cylinder with a radius of 5 cm and a height of 12.7 cm.

| Moisture content | Wet density(g/cm3) | Dry density(g/cm3) | Void ratio | Liquid limit | Plastic limit | Plasticity index |
|------------------|--------------------|--------------------|------------|--------------|---------------|-----------------|
| 25.17            | 1.89               | 1.54               | 0.72       | 40.89        | 25.30         | 15.59           |

2.2 Test method

The test uses the Sensor High-way II all-weather structural health monitoring (SHM) system produced by PAC company of America and AEwin software to acquire and analyze the signals of acoustic emission. Unconfined compressive strength test was carried out by using the model of electro hydraulic servo testing machine of TYJ-500KN. In this experiment, the signal is collected by setting 4 channels, the loading rate is controlled at 0.042mm/s, the sampling rate is 20MHz, the gain of the preamplifier is set to 40dB and the threshold value is set to 45dB. Uniaxial compression and acoustic emission signals were collected simultaneously.

2.3 Characteristics analysis of acoustic emission signal in uniaxial compression test

Ring count the most common evaluation technique of acoustic emission. When an event impacts a sensor, it allows the sensor to produce rings. Each of the formed oscillation waves of electric signal exceeding the threshold is recorded as a ring count. The total number of the rings is a cumulative value of ring count of acoustic emission in a process of acoustic emission. It is a general reflect the accumulated condition of internal damage of the material, and is the external manifestation of change accumulation of the internal state. In this paper, acoustic emission test of the uniaxial unconfined
compression is respectively carried out for the three groups of test specimens, the count-time curve and stress-time curve of ring count of acoustic emission for each group are as shown in Figures 1, 2, 3 and 4. The following can be found through graphical expression comparison of acoustic emission test under the different moisture contents:

(1) It can be seen from the count-time curve and stress-time curve of ring count of acoustic emission for test of three groups, the stress variation and failure process of the soil is basically consist with the energy parameters of acoustic emission, which is to say that the number of times of ring count of acoustic emission continuously increases with the increase of the stress, and it gradually decreases after the peak stress. Parameter variation of acoustic emission ring is similar to the failure process of the soil, it can be used to reflect the evolution process of soil damage.

(2) Acoustic emission characteristics for three groups of the test specimens are substantially the same which can be roughly divided into three stages: ①Compaction stage. At this stage, micro cracks of the soil are constantly compressed and reduced, the acoustic emission ring count uniformly appears throughout the specimen. There are slightly more ring events on both ends of the curve than any other places, showing that the end effect exists at the compaction stage. Stress curve growth rate is slow; ② Elastic-plastic stage. At this stage, voids in the soil are substantially compacted, the specimen begins to produce microscopic rupture damage, acoustic emission ring events become less and without a traceable pattern. With further compression, macroscopic cracks appear in the soil specimen both internally and externally. The quantity of acoustic emission ring events increases and are rapidly distributed throughout the specimen before gradually concentrated to the rupture surface. The soil stress increased sharply at this stage, and reached a peak stress, the curve shows inflection point and with slow growth rate.③ Failure stage. The cracks of the soil specimen are further increased, interconnected, and begin to form rupture surface, the rupture damage finally occurs. When the load reaches a peak, the number of the acoustic emission ring events increased dramatically and concentrated on rupture surface. The soil stress reduced after the specimen is damaged, and the acoustic emission ring events decrease and eventually disappear.

(3) It can be found through comparison of the specimen curves of three groups that the acoustic emission ring count and stress of group B are higher than groups A and C, indicating that the closer to the optimum moisture content is, the stronger the acoustic emission activity is. This is because that when the optimum moisture content is reached, the compaction degree of the soil is greater, the occlusion between the soil particles is better, so that the inner friction angle between the soil particles increases. Meanwhile, the tightly-compacted moisture content of the soil is suitable to allow the gap between the particles in the soil smaller and the contact between the soil particles increasingly closer, and the cohesion of the soil also increased to enhance the soil strength, so more energies are required to be consumed and released in the process of the crack damage.
3. Ring cumulative number of acoustic emission and model building of mechanical parameters

3.1 Coupling relationship between ring cumulative number of acoustic emission and strain

It can be seen from acoustic emission test of different moisture contents of the soil that internal voids and structure of the soil have a large impact on its mechanical properties. The test analysis results show that the ring cumulative number of acoustic emission of the soil is closely related to the evolution of the soil structure damage, so the ring accumulation of acoustic emission is intrinsically associated with the mechanical properties of the soil. By fitting the test data of the relationship between ring cumulative number \( N \) of acoustic emission of the soil and time \( t \), it can be found that the function relationship between the ring cumulative number \( N \) of acoustic emission and time \( t \) is as follows according to the fitting diagram shown in Figure 5:

\[
N = N_0 + A \exp(ct) \tag{1}
\]

Where: \( N_0 \), \( A \), and \( c \) are constants, and shall be determined by test data fitting.

The instantaneous deformation of the specimen under load is not considered, and the linear relationship between strain and time is known by fitting the strain and time from figure 6.

\[
\varepsilon = at \tag{2}
\]

Where: \( \varepsilon \) is the strain, \( a \) is the rate of strain.
Simultaneous equation (1) and (2),
\[ N = N_0 + A \exp\left(\frac{c \varepsilon}{a}\right) \] (3)

Equation (3) is the coupled-relationship between ring cumulative number of acoustic emission and strain under unconfined compression.

3.2 Coupling relationship between ring cumulative number of acoustic emission, strain and damage variable

According to the strain equivalence hypothesis of Lemaitre, the damage constitutive relation of material under uniaxial compression can be known,
\[ \varepsilon = \frac{\sigma}{E} = \frac{\sigma}{E (1 - D)} \] (4)
\[ \text{or} \quad \sigma = E (1 - D) \varepsilon \] (5)

Where: \( \varepsilon \) is the strain, \( \sigma \) is the effective stress, \( \sigma \) is the nominal stress, \( E \) is the elastic modulus of soil, and \( D \) is the damage variable.

Assuming the micro unitary strength of the clay meet Weibull distribution, so it can be considered that damage variable of the soil meet Weibull distribution, too. The strength probability density of soil micro element in its distribution law is the following equation,
\[ f(\varepsilon) = \frac{m}{\alpha} \left(\frac{\varepsilon}{\alpha}\right)^{m-1} \exp\left[-\left(\frac{\varepsilon}{\alpha}\right)^m\right] \] (6)

Where: \( m \) is the shape parameter of Weibull distribution, and \( \alpha \) is the scale parameter of Weibull distribution.

Taking into account the random destruction of micro-unit, therefore the damage variable as the probability of failure of the micro unit, it can be expressed as equation (7),
\[ D = \int_0^\varepsilon f(\varepsilon)dx = 1 - \exp\left[-\left(\frac{\varepsilon}{\alpha}\right)^m\right] \] (7)

Simultaneous equations (5) and formula (7) for the constitutive equation of the stress and strain of soils under unconfined compression,
\[ \sigma = E \varepsilon \exp\left[-\left(\frac{\varepsilon}{\alpha}\right)^m\right] \] (8)

Xie Xing[11], Yang Minghui[12] obtained the expressions of the model parameters of the rock and soil material damage model by assuming the geometric boundary conditions,
\[ \alpha = \left( \frac{\epsilon_p}{m} \right)^{1/m}, \quad m = \frac{1}{\ln\left(\frac{E \epsilon_p}{\sigma_p}\right)} \tag{9} \]

Where: \( \sigma_p \) is the peak stress, and \( \epsilon_p \) is the peak strain.

By formula (3),

\[ \epsilon = \frac{a}{c} \ln \frac{N - N_0}{A} \tag{10} \]

Simultaneous equation (8) and (10),

\[ \sigma = a \frac{\ln\left(\frac{N - N_0}{A}\right)}{A} \exp\left[\frac{\ln\left(\frac{N - N_0}{A}\right)}{c\alpha}\right] \tag{11} \]

Equation (11) is the coupled-relationship between ring cumulative number of acoustic emission and stress under unconfined compression.

Simultaneous equation (7) and (10),

\[ D = 1 - \exp\left[\frac{\ln\left(\frac{N - N_0}{A}\right)}{c\alpha}\right] \tag{12} \]

Equation (12) is the coupled-relationship between ring cumulative number of acoustic emission and damage variable under unconfined compression.

### 3.3 Model validation

Combine the data obtained from the test, and respectively make the appropriate function fitting on strain-time relationship and cumulative number of acoustic emission ring-time relationship to verify the validity and rationality of the ring cumulative number of the above established soil with the strain model, stress model and damage variable model, and the model parameters table obtained through a large quantity of test data analysis as it is shown in Table 2. The fitting parameters from test data are brought into formulas (3), (11) and (12) to determine the relationship between the ring cumulative number \( N \) of the soil under unconfined uniaxial compression and the stress \( \sigma \), strain \( \epsilon \), damage variable \( D \), the relationship is shown in Figures 7-9.

With the comparative validation of test data and fitting model shows and in accordance with Figure 7, the ring cumulative number of acoustic emission-curve of strain model results share better consistency with the test data in the whole occurring process of the strain. Figure 8 shows that the ring cumulative number of acoustic emission-stress curve is highly fit in respect of model results and test data before the stress peak and during the peak, and the model trend after the peak is consistent with test data, but the coincidence degree is not very high. Figure 9 shows that the model curve of ring cumulative number of acoustic emission-damage variable curve is consistent with the trend of test data curve, the whole data fits well. Overall, the correlation and coincidence degree of the fitted curve of the parameters damage model of the acoustic emission for the soil established in this test under the unconfined axial compression is better related to the test data, thus further demonstrate the effective rationality of the model.

| Parameter | Fitting parameters | The parameters of Weibull distribution |
|-----------|--------------------|---------------------------------------|
| a         | N0                 | A                                     |
| Fitting result | 0.0331           | -1689.4271   | 2397.8759   | 0.00745 | 1.0156 | 10.827 |
4. Conclusions

By studying the characteristics of acoustic emission parameters of soil under uniaxial compression, and analyzing the relationship between acoustic emission parameters, mechanical parameters of the soil and damage variables, this paper can draw the following conclusions:

(1) Test of acoustic emission of the soil under unconfined compressive condition in the process of the soil damage shows that the characteristic curve of the acoustic emission of the soil containing the different moisture contents can be divided into three stages. These stages are corresponding to the process of the soil damage and fracture. When the moisture content of the soil is closer to the optimum moisture content, the acoustic emission activity is stronger, and the ringing count is higher.

(2) From the perspective of time, and use it as a "bridge" connecting the ring-cumulative number of acoustic emission and damage variable, establish coupling model of the ring cumulative number of the soil under unconfined uniaxial compression with the strain according to the fitting relationship between the ring cumulative number \( N \) of acoustic emission of the soil and time \( t \), strain \( \varepsilon \), time \( t \); further introduce the damage model of the ring cumulative number of acoustic emission with the stress and
damage variable based on Weibull distribution and constitutive equation of continuous damage mechanics.

(3) The form of damage model of the derived acoustic emission is simple and several related parameters are easier to obtain. By comparison, analysis and verification, the ring cumulative number of acoustic emission is more orderly in terms of the test data obtained from the stress, strain, and relatively high in terms of the model fitting accuracy; but when the cumulative number of acoustic emission is made and damage variable is fitted, the fitting accuracy is not very high probably due to the mechanical properties of the soil affected by many factors. Overall, the established damage model of acoustic emission is more reasonable and effective, and can provide a new way of thinking and theoretical basis for further research of the soil.

(4) In this paper, the test is carried out under unconfined axial compression and at different moisture contents of the soil. Many factors have a great influence on acoustic emission characteristics of the soil, including the different types of soil and whether there is lateral confinement for loading method. It is recommended that the test with other factors can be carried out in the future to verify and improve the model in this paper.

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