Sagnac interferometer-enhanced particle tracking in optical tweezers

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Abstract
A set-up is proposed to enhance tracking of very small particles, by using optical tweezers embedded within a Sagnac interferometer. The achievable signal-to-noise ratio is shown to be enhanced over that for standard back-focal-plane interferometry. The enhancement factor increases asymptotically as the interferometer visibility approaches 100%, but is capped at a maximum given by four times the ratio of the trapping field intensity to the detector saturation threshold. For an achievable visibility of 95%, the signal-to-noise ratio can be enhanced by a factor of up to 158 and the minimum trackable particle size is 2.3 times smaller than without the interferometer. This technique is particularly useful for optical tweezers which require counter-propagating trap beams.

Keywords: Sagnac interferometer, optical tweezers, particle tracking, shot noise limited sensing

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Optical tweezers are devices which trap and detect small particles in a tightly focused laser beam [1]. Radiation pressure draws particles towards higher light intensities and traps them in the focus of the beam. The effect of a particle on the trapping beam profile can be analyzed to extract information about the position of, and force exerted on, the trapped particle with subnanometer and subpiconewton detection sensitivity [2, 3]. This has become an important technique in a range of applications, particularly high-precision manipulation of biological samples. Optical tweezers have been used to manipulate viruses and bacteria [4], unfold single RNA molecules [5], study the motion of the biological motor protein kinesin [6] and muscle myosin molecules [7], and sequence DNA [8].

Particles trapped in optical tweezers are tracked via the interference between the trapping beam and the light which scatters off the particle [9, 10]. A quadrant photodiode can be used to infer both the position of the particle and the force exerted on it [11, 12]. Detecting the particle becomes much more difficult as it becomes smaller, because small particles can scatter very little light, with the amplitude of Rayleigh scattering scaling as the particle radius to the power of six [13, 14]. Particles which have been successfully trapped and detected in optical tweezers include 26 nm dielectric particles [15] and 9.5 nm gold nanoparticles [16]. The sensitivity of such measurements is limited by low scattered light levels.

In almost all cases, and especially when trapping small particles, the trapping field intensity used in optical tweezers is much brighter than the saturation threshold of the detector used to track particle position. Two typical methods are used to avoid saturation. A second beam, which is much less bright, may be added to the optical tweezers set-up, and this beam can be used to detect the particle position. This second beam may be orthogonally polarized to the trapping beam [17], or it may be at a different wavelength [12]. Alternatively an attenuator is placed in the beam between the optical tweezers and the detector. This attenuates the trapping beam, but it also

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attenuates the light scattered from the particle, degrading our ability to detect the particle. The signal-to-noise ratios (SNRs) achieved with these two approaches are identical if we assume optical linearity throughout the system.

Several methods to improve the sensitivity of optical tweezers exist, using, for example, two orthogonally polarized beams [6] or spatial homodyne detection [18, 19]. Sophisticated techniques have also been proposed to surpass the shot noise detection limit using quantum states of light [22–24]. Both spatial homodyne detection and the use of quantum states of light could be integrated into the technique proposed in this paper to further enhance the particle tracking capability.

A particularly common method to improve the sensitivity of optical tweezers is to track the particle by detecting the back-scattered rather than forward-scattered light. This can substantially reduce the detected trapping field intensity, reducing the shot noise without degrading the detected signal [20, 21]. However, such techniques are incompatible with optical tweezers with counter-propagating trapping fields, which are necessary in some instances. One situation is when trapping high refractive index particles [25, 26], which are strong scatterers. The use of counter-propagating traps cancels the radiation pressure in the direction of propagation, enabling axial trapping. Another situation where this trap type is necessary is when imaging the sample from the side [27]. In this case, lower numerical aperture (NA) lenses are required to increase the sample–objective separation. This reduces the trapping force, particularly in the axial direction. In addition to the radiation pressure force canceling, axial trapping can be strengthened by establishing a standing wave in the optical trap, greatly increasing the field gradient in the axial direction.

We propose the combined use of optical tweezers and Sagnac interferometry to enhance the detection of particles in traps with counter-propagating trap beams, extending a recent demonstration of Sagnac interferometer-based phase plate characterization [28]. With optical tweezers embedded in a Sagnac interferometer, selective interference attenuates the trapping field and hence reduces the detection shot noise, substantially improving the detection SNR when compared to using a standard attenuator. The particle tracking SNR is enhanced by a factor which increases as the interferometer visibility approaches 100%, up to a maximum enhancement defined by four times the ratio of the trapping field power to the detector saturation threshold. If, for example, the Sagnac interferometer had a visibility of 95%, the signal-to-noise ratio could be enhanced by a factor of up to 158, which would consequently enable tracking of 2.3 times smaller particles than the equivalent standard optical tweezers scheme.

2. Theory

A schematic of the Sagnac interferometer-embedded optical tweezers proposed here is shown in figure 1. The input optical field \( E_0 \) is split by a beamsplitter, resulting in two optical fields propagating through the Sagnac interferometer, \( E_{cw} \) traveling counterclockwise and \( E_{ccw} \) traveling clockwise. These fields form an optical trap at the focus of the objective lenses. When a particle is trapped, it will scatter light from both fields, modifying their spatial profiles. The fields then recombine at the beamsplitter, with the trapping field \( E_{ccw} \) traveling counterclockwise around the Sagnac interferometer, and the reflected field \( E_{cw} \) traveling clockwise. Once the fields reach the beamsplitter again they recombine and interfere. The quadrant photodiode detects the light, producing sum and difference photocurrents \( i_s \) and \( i_d \).

![Figure 1. Layout of the optical tweezers detection scheme. The trapping field is split at the beamsplitter, with the transmitted field \( E_{cw} \) traveling counterclockwise around the Sagnac interferometer, and the reflected field \( E_{ccw} \) traveling clockwise. Once the fields reach the beamsplitter again they recombine and interfere. The quadrant photodiode detects the light, producing sum and difference photocurrents \( i_s \) and \( i_d \).](image)

where \( n \) is an arbitrary subscript and \( u_n \) is normalized such that

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |u_n(x, y)|^2 \, dx \, dy = 1.
\]
The transmittance and reflectance of the beamsplitter are given by \( T \) and \( R \), respectively, so that the transmitted and reflected fields, \( E_{ccw} \) and \( E_{cw} \), are given by

\[
E_{ccw}(x, y) = \sqrt{T}E_0(x, y)
\]

and

\[
E_{cw}(x, y) = -\sqrt{R}E_0(x, y),
\]

where the negative sign is due to a hard boundary reflection at the beamsplitter. The counterclockwise traveling field propagates a distance of \( L_1 \) to the optical tweezers and picks up a phase shift of \( e^{i\kappa L_1} \), where \( k = \frac{2\pi}{\lambda} \) is the wavenumber and \( \lambda \) is the optical wavelength. Likewise the clockwise propagating field picks up a phase shift of \( e^{i\kappa L_2} \) as it travels a distance of \( L_2 \) to the optical trap.

Since the clockwise and counterclockwise fields travel the same optical path in opposite directions through the set-up, they will form a standing wave. At the trapping position the effect of this will be to greatly increase the light intensity gradient in the axial direction. Standing waves have previously been used in optical tweezers to improve the axial trapping of sub-wavelength particles [29]. Although not strictly necessary, for simplicity we assume that this is achieved, such that the particle is trapped in the \( \hat{z} \) direction at an antinode of the standing wave. This constraint on the axial position of the particle will ensure that the trapping fields from both directions are in phase with each other. This means that the phase terms due to beam propagation cancels out because both beams are equal, e\( ^{i\kappa L_1} \) = \( -e^{i\kappa L_2} \), so that the electric field at the trapping point is

\[
E_{OT}(x, y) = e^{i\kappa L_1}(\sqrt{T} + \sqrt{R})E_0(x, y).
\]

To first order, sufficiently small trapped particles leave the trapping field unchanged except for the introduction of a component \( E_p \) which is scattered from the particle [9]. This component will be equal in the two directions of propagation, as forward- and back-scattering are equal for Rayleigh scattering. After interacting with the particle the counterclockwise and clockwise propagating fields are

\[
E_{ccw}\text{OT}(x, y) = \sqrt{T}E_0(x, y)e^{i\kappa L_1} + E_p(x, y),
\]

\[
E_{cw}\text{OT}(x, y) = -\sqrt{R}E_0(x, y)e^{i\kappa L_2} + E_p(x, y).
\]

The scattered field \( E_p \) can be separated without loss of generality into symmetric and antisymmetric parts \( E_s \) and \( E_a \), so that

\[
E_p(x, y) = E_s(x, y) + E_a(x, y),
\]

which, due to their symmetry, have the properties

\[
E_s(x, y) = E_s(-x, y)
\]

\[
E_a(x, y) = -E_a(-x, y).
\]

This is useful because for small particle displacements all of the particle position information is found in the antisymmetric part of the scattered field [19]. The amplitude of the scattered field is proportional to the trapping field amplitude, such that \( A_s = (\sqrt{T} + \sqrt{R})A_0e^{i\kappa L_1} \) and \( A_a = (\sqrt{T} + \sqrt{R})A_0e^{i\kappa L_2} \), where \( \xi \) and \( \xi_a \), respectively, denote the proportion of the trapping field which scatters from the particle into symmetric \( u_s(x, y) \) and antisymmetric \( u_a(x, y) \) modes. Here we work in the experimentally relevant limit that the proportion of the trapping field which is scattered is very small or, equivalently, \( |\xi|, |\xi_a| \ll 1 \). Substituting these expressions into equations (6) and (7), we find

\[
E_{ccw}\text{OT}(x, y) = A_0e^{i\kappa L_1}[\sqrt{T}u_0(x, y) + (\sqrt{T} + \sqrt{R})(\xi u_s(x, y) + \xi_a u_a(x, y))],
\]

\[
E_{cw}\text{OT}(x, y) = -A_0e^{i\kappa L_2}[\sqrt{R}u_0(x, y) + (\sqrt{T} + \sqrt{R})(\xi u_s(x, y) + \xi_a u_a(x, y))].
\]

Each reflection off a mirror causes a reflection of the beam profile in the \( x \) direction. This is shown graphically in figure 2. As is seen in equation (10), this results in a change in the sign of the antisymmetric scattered field \( E_a \), but does not affect either the trapping field \( E_0 \), which we assume to be symmetric as is typical in optical tweezers, or the symmetric scattered field \( E_s \). As a result the antisymmetric scattered field, which contains the particle position information, picks up an additional phase shift on each reflection compared to the trapping field and symmetric scattered field.

For the sake of brevity the explicit spatial dependence is now dropped, with \( u_0(x, y) \) written as \( u_0 \) throughout. After interaction with the particle, both counterclockwise and clockwise fields propagate back to the beamsplitter. The phase shift due to beam propagation cancels out because both beams travel the same total distance. The counterclockwise and clockwise fields experience \( g \) and \( f \) reflections, respectively, before reaching the beamsplitter, with each reflection inducing a \( \pi \) phase shift on their antisymmetric components. The fields at the beamsplitter are then

\[
E'_{ccw} = A_0[\sqrt{T}u_0 + (\sqrt{T} + \sqrt{R})\xi u_s
\]

\[\hspace{1cm} + (-1)^g(\sqrt{T} + \sqrt{R})\xi_a u_a],\]

\[
E'_{cw} = -A_0[\sqrt{R}u_0 + (\sqrt{T} + \sqrt{R})\xi u_s
\]

\[\hspace{1cm} + (-1)^f(\sqrt{T} + \sqrt{R})\xi_a u_a].\]

The field leaving the light port is given by

\[
E_L = -\sqrt{R}E'_{ccw} + \sqrt{T}E'_{cw},
\]

where the negative sign in the first expression is due to the reflection of the counterclockwise field from a hard
boundary at the beamsplitter. Similarly, the field leaving the dark port is given by
\[ E_D = A_0(T - R)(u_0 + \xi u_s) + A_0(-1)^f T - R)(\sqrt{T} + \sqrt{R})\xi u_a. \] (15)

Notice that the components of both the trapping and symmetric scattered fields which exit through the dark port suffer destructive interference due to the prefactor \((T - R)\), and cancel exactly when \(T = R\), which corresponds to perfect interferometer visibility. In contrast, constructive interference can be achieved for the antisymmetric scattered field through an appropriate choice of \(g\) and \(f\). The term in equation (15) relating to the antisymmetric scattered field can be simplified by defining the difference in the number of reflections experienced by the clockwise and counterclockwise fields after interaction with the particle, \(m = f - g\), so that
\[ E_D = A_0(T - R)(u_0 + \xi u_s) + (-1)^f A_0 \times (\sqrt{T} - (-1)^m \sqrt{R})(\sqrt{T} + \sqrt{R})\xi u_a. \] (16)

It is apparent that the sign of the antisymmetric coefficient will depend on \(g\). The only effect this has is to alter the sign of the detected photocurrent \(i_x\), with the sensitivity of the measurement left unchanged. Hence, without loss of generality we set \(g = 2\) as in figure 1. We can then find
\[ E_{D,m\text{ odd}} = A_0(T - R)(u_0 + \xi u_s) + A_0(\sqrt{T} + \sqrt{R})^2\xi u_a, \] (17)
\[ E_{D,m\text{ even}} = A_0(T - R)(u_0 + \xi u_s) + A_0(T - R)\xi u_a. \] (18)

In the case that \(m\) is odd, the antisymmetric part of the scattered field constructively interferes at the dark port, as shown by the presence of a \((\sqrt{T} + \sqrt{R})^2\) prefactor on \(u_a\). In contrast, if \(m\) is even the antisymmetric part of the scattered field destructively interferes, as shown by the \((T - R)\) prefactor. SNR enhancement requires constructive interference of the antisymmetric terms. If the total number of mirrors in the interferometer is odd, \(m\) is odd and this condition is met. Henceforth we only consider this case, with \(A_0\) set to be real without loss of generality. The mean photon number flux reaching each position in the detector is given by \(\langle n_D(x, y) \rangle = \frac{\omega_0^2}{2h} E_0^2 E_D\), where \(h\) is Planck's constant and \(\omega_0\) is the vacuum permittivity. Substituting equation (17) into this expression we find
\[ \langle n_D \rangle = \frac{\omega_0^2}{2h} A_0^2 u_0(T - R)[(T - R)(u_0 + (\xi + \xi^*)u_4) + (\sqrt{T} + \sqrt{R})^2(\xi + \xi^*)u_4], \] (19)
where terms of \(O(2)\) in \(\xi_a\) and \(\xi_s\) have been neglected since \(|\xi_a, \xi_s| \ll 1\). This is detected on a quadrant detector, and subtraction of the resulting photocurrents is performed in the standard manner to infer the position. We assume that the detector size is large compared to the beam size. The sum and difference photocurrents are then given by
\[ \langle i_N \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle n_D(x, y) \rangle \, dx \, dy \] (20)
and
\[ \langle i_x \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle n_D(x, y) \rangle \, dx \, dy - \int_{-\infty}^{\infty} \int_{-\infty}^{0} \langle n_D(x, y) \rangle \, dx \, dy, \] (21)
where the photocurrents \(\langle i_N \rangle\) and \(\langle i_x \rangle\) are in units of electrons per second. \(\langle i_N \rangle\) is the mean total photocurrent generated by the light hitting the detector. This also gives the shot noise variance \(\Delta^2 i_N\), which for shot-noise-limited detection is the noise in the position measurement. Using the normalization property of mode functions given in equation (2), and now neglecting terms of \(O(1)\) in \(\xi_a\) and \(\xi_s\), we find the mean total photocurrent:
\[ \Delta^2 i_N = \langle i_N \rangle = 2 \omega_0^2 (T - R)A_0^2. \] (22)

The mean photocurrent difference can be found in a similar manner. Since it is obtained by subtracting the flux on one half of the detector from that on the other, the intrinsically symmetric terms \(u_0\) and \(u_0 u_4\) in equation (19) can be ignored. The result is that
\[ \langle i_x \rangle = \frac{\omega_0^2}{2h} \frac{(T - R)(\sqrt{T} + \sqrt{R})^2(\xi + \xi^*)A_0^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_d u_0 \, dx \, dy - \int_{-\infty}^{0} \int_{-\infty}^{0} u_d u_0 \, dx \, dy}. \] (23)
Due to the symmetry of \(u_0\) and \(u_a\), this simplifies to
\[ \langle i_x \rangle = \frac{\omega_0^2}{2h} \frac{(T - R)(\sqrt{T} + \sqrt{R})^2(\xi + \xi^*)A_0^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_d u_0 \, dx \, dy}. \] (24)
We can define an overlap integral \(\eta_{x0}\) as
\[ \eta_{x0} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_d u_0 \, dx \, dy, \] (25)
which quantifies the similarity between the mode containing particle position information \(u_d\) and the detected mode, given by \(\vec{\mathbf{u}}(x) \times u_0(x, y)\) [19]. Substituting this into equation (24) we find
\[ \langle i_x \rangle = \frac{\omega_0^2}{2h} \frac{(T - R)(\sqrt{T} + \sqrt{R})^2(\xi + \xi^*)A_0^2}{\eta_{x0}}. \] (26)
Using this expression and equation (22) for the shot noise variance, we find the shot-noise-limited SNR for particle tracking in the \(x\) direction to be
\[ \text{SNR}_x = \frac{(\langle i_N \rangle)^2}{\Delta^2 i_N} = \frac{\omega_0^2}{2h} \frac{(T - R)(\sqrt{T} + \sqrt{R})^2(\xi + \xi^*)^2A_0^2 \eta_{x0}}{2}. \] (27)
case of $R_{\text{dir}} = 1$, $T_{\text{dir}} = 0$, because then there is only one trapping beam. The total detected photocurrent would then be

$$
\langle i_N \rangle = \frac{\epsilon_0 \lambda}{2 h} A_0^2.
$$

(28)

This is a factor of $(T - R)^2$ brighter than for the Sagnac interferometer case, where $R$ and $T$ remain the reflection and transmission parameters describing the beamsplitter in the Sagnac interferometry case. Since typical trapping powers are of the order of 1 mW (for example, see [4]), and typical photodiodes used for detection have saturation thresholds below 10 mW,\(^5\) the optical field is attenuated prior to detection. To enable a fair comparison, we attenuate the light by a factor of $(T - R)^2$, so that it is the same brightness as in the interferometer case. We find that

$$
\Delta^2 i_N = \frac{\epsilon_0 \lambda}{2 h} (T - R)^2 A_0^2 = \Delta^2 i_N
$$

(29)

and

$$
\langle i_s \rangle = \frac{\epsilon_0 \lambda}{2 h} (T - R)^2 (\xi_s + \xi_s^*)^2 A_0^2 \eta_{a0},
$$

(30)

where we have again set $A_0$ to be real without loss of generality. This results in an SNR of

$$
\text{SNR}_{\text{dir}} = \frac{\epsilon_0 \lambda}{2 h} (T - R)^2 (\xi_s + \xi_s^*)^2 A_0^2 \eta_{a0}^2
$$

(31)

which is identical in form to the Sagnac SNR except that the $(\sqrt{T} + \sqrt{R})^2$ term in equation (27) becomes $(T - R)^2$ here, meaning the Sagnac interferometry substantially enhances the SNR when $T \approx R$. Explicitly, the SNR enhancement factor $\mathcal{E}$ for the Sagnac over direct detection is

$$
\mathcal{E} = \frac{\text{SNR}}{\text{SNR}_{\text{dir}}} = \frac{(\sqrt{T} + \sqrt{R})^2}{(T - R)^2}
$$

(32)

which is shown as a function of $R$ in figure 3(a), assuming a lossless beamsplitter such that $T = 1 - R$. Note that $\mathcal{E}$ tends to infinity as $(T - R)$ goes to zero. This is clearly unphysical since it corresponds to perfect interference on the Sagnac beamsplitter, which requires perfect polarization and spatial overlap as well as $R = T$. A physically useful parameter which includes all non-ideal effects is the interferometer visibility $\text{VIS}$, given by

$$
\text{VIS} = \frac{\langle i_a \rangle - \langle i_{aD} \rangle}{\langle i_a \rangle + \langle i_{aD} \rangle} = 1 - 2 \frac{(T - R)^2}{(T + R)^2}.
$$

(33)

The visibility quantifies the mode overlap between the two beams in the interferometer, with a visibility of 1 indicating perfect mode matching. Using this and equation (32), we can express the enhancement factor in terms of the visibility as

$$
\mathcal{E} = \frac{2}{(1 - \text{VIS})^2} \left(1 + \sqrt{1 + \text{VIS}} \frac{2}{2}\right)^2.
$$

(34)

where we have assumed a lossless beamsplitter, such that $T + R = 1$. The enhancement $\mathcal{E}$ as a function of VIS is shown in figure 3(b). We see that, as the mode overlap goes to unity, the enhancement again approaches infinity.

In reality the enhancement is limited by the ratio of the trapping field intensity to the detector saturation threshold. In order to compare Sagnac interferometer detection to standard detection, the optical intensity in standard detection is attenuated by a factor of $(T - R)^2$. However, once the optical power is below the saturation threshold of the detector, it is no longer sensible to apply more attenuation. Once this limit has been reached, no further advantage can be had from improving the visibility of the Sagnac, since there is no requirement to further reduce the optical power reaching the detector.

In practice, the maximum enhancement conferred by the Sagnac interferometer is achieved when it is used to reduce the detected light intensity to the point that the total photocurrent defined in equation (22) is just within the saturation threshold $\langle i_{\text{sat}} \rangle$:

$$
\langle i_N \rangle = \frac{\epsilon_0 \lambda}{2 h} (T - R)^2 A_0^2 = \langle i_{\text{sat}} \rangle.
$$

(35)

Rearranging this, we find

$$
(T - R)^2 = \frac{2 h \langle i_{\text{sat}} \rangle}{\epsilon_0 \lambda A_0^2}.
$$

(36)

If we substitute this into equation (32), and model a lossless ideal beamsplitter with $T = R = \frac{1}{2}$, we find that the maximum enhancement is

$$
\mathcal{E}_{\text{max}} = \frac{4 \langle i_N \rangle}{2 h \langle i_{\text{sat}} \rangle} = \frac{4 \langle i_{N_{\text{sat}}} \rangle}{\langle i_{\text{sat}} \rangle},
$$

(37)

where $\langle i_{N_{\text{sat}}} \rangle$ is the mean photocurrent that would result from the trapping field if there was no attenuation. The condition that $T = R = \frac{1}{2}$ is not critical for this calculation, as this limit will be approximately accurate for any splitting ratio which is close to 50/50. The value of this limit will depend on the trapping field intensity and the specific detector used. Supposing a Thorlabs PDQ30C quadrant detector was

\(^5\) For example, the commonly used Thorlabs PDQ30C quadrant detector has a 1 mW saturation threshold.
used with a trapping intensity of 1 W as in [4], the absolute maximum enhancement possible would be approximately 4000.

To assess the usefulness of this technique we consider a specific example. If optical tweezers are set up in a Sagnac interferometer with a visibility of 95%, equation (34) indicates that the SNR would be enhanced by a factor of approximately 158. As shown in equation (27), the SNR is proportional to the real part of the antisymmetric scattered field intensity, given by \((\zeta_a + \zeta_s)^2 A_0^2\), and is therefore proportional to \(r^6\), where \(r\) is the particle radius [13, 14]. A 158 times increase in SNR would therefore allow a reduction in the minimum detectable particle size when compared to standard detection of 158\(\ast\), or approximately 2.3 times. As will be discussed further in section 3, due to the high numerical aperture of optical tweezers objectives it may be challenging to achieve high interferometer visibility. If a lower visibility of 80% is achieved, a similar calculation yields a 1.8 times reduction in minimum detectable particle size.

In the particular implementation of [30], for example, where 100 nm particles were tracked with standard back-focal-plane detection, inserting the optical tweezers into such an interferometer would allow tracking of 55 nm particles with the same SNR as the original measurements. The sensitivity could be further improved by using spatial homodyne detection instead of the quadrant detector, as quadrant detection has been shown to perform sub-optimally [18, 19].

Finally, we note that the described configuration of the interferometer will only enhance the \(x\) position detection because the interfered beams are only flipped in the \(x\) direction on reflection against interferometer mirrors. Reflections of both the \(x\) and \(y\) directions are required in order to extend this technique to enhance the \(x-y\) position detection. This can be achieved with a three-dimensional layout of mirrors.

3. Experimental feasibility

The theory presented in this paper shows that antisymmetric fields generated within a Sagnac interferometer are selectively directed to the dark port of the interferometer. By implementing optical tweezers within a Sagnac interferometer, this property can be used to enhance optical-tweezers-based particle tracking. To achieve significant enhancement, it is critical to obtain high interferometer visibility. Since optical tweezers require the trap beam to be strongly focused in high NA objectives, this is a significant technical challenge.

A primary challenge is to ensure that the phase fronts of the clockwise and counterclockwise fields remain matched after transmission through both objectives. The mismatch can be minimized by placing the optical trap in the center of the Sagnac so each beam propagates an equal distance from the trap to the beamsplitter, and by accurately ensuring the separation of the objectives is very close to twice their working distance. An additional concern is clipping within the objectives, which has the effect of altering the intensity distributions of the two counter-propagating fields. The resulting degradation of interferometer visibility can be minimized by ensuring that both fields are centered on the optical axis of the objectives and therefore experience the same clipping. The clipping will then reshape the mode without affecting the visibility. The loss of intensity due to clipping affects the SNR in Sagnac-based detection and back-focal-plane detection in the same manner and therefore has no effect on the level of enhancement.

Back-reflections from the sample cover slip can present an additional complication to experiments. Here we show that this effect is, for realistic levels of back-reflection, fully accounted for as a degradation of the visibility in equation (34) and a consequential degradation in the SNR enhancement. For experimentally realistic levels of back-reflection only modest degradation occurs.

Let us define \(r\) as the proportion of each trapping field’s intensity back-reflected from the cover slips. Since much of the back-reflected light will not re-enter the trapping beam mode but rather diverge in space and not be detected, we also define \(C\) as the proportion of back-reflected light collected at the detector. Typically both \(r\) and \(C\) will be less than 10%, so we can treat the reflected light as a perturbation on the total light intensity. In general, the spatial modes of the back-reflected fields will overlap with those of the optical tweezers trapping and scattered fields. The interference caused by this overlap substantially increases the complexity of the results from our model, but even in the worst-case scenario, only has a modest effect on the achievable SNR. Furthermore, given the high numerical aperture of optical tweezers objectives, one would expect the spatial overlap to be negligible. Hence, for the sake of simplicity, we here treat the back-reflected fields as being spatially orthogonal to both the trapping and the scattered fields.

Within these constraints the back-reflected fields can be treated as a separate additional Michelson interferometer which introduces some light to the dark port. Each trapping field has back-reflections with an amplitude of \(\sqrt{\frac{rC}{2}}A_0\). The additional light intensity detected is then \(\phi rCA_0^2(1 + \eta_\phi \cos \phi)\), where \(\phi\) is the phase between the two back-reflected fields and \(\eta_\phi\) is the spatial overlap of the two back-reflected fields. In the most destructive scenario, the overlap is perfect (\(\eta_\phi = 1\)) and the back-reflected fields constructively interfere (\(\cos \phi = 1\)). To provide a worst-case scenario we take this condition, causing an interferometer visibility \(\text{VIS}_r\) reduction to

\[
\text{VIS}_r = \text{VIS} - 2rC. \quad (38)
\]

A separate calculation of the SNR, including the increase in shot noise due to the back-reflected intensity, gives

\[
\text{SNR}_r = \frac{\text{SNR}}{1 + rC(1 - T + R)^2}, \quad (39)
\]

which for the experimentally realistic case of \(rC \ll 1\), gives the same change in SNR enhancement as does substituting the modified visibility in equation (38) into (34). This means that the expression for SNR enhancement as a function of interferometer visibility in equation (34) already accounts for back-reflections. Let us now consider some examples to illustrate the magnitude of degradation due to back-reflections.
In a set-up with a visibility of $\text{VIS} = 80\%$, adding back-reflections with $r = 10\%$ and $C = 10\%$ reduces the SNR enhancement by $9\%$ from 38.0 to 34.5. However, starting from a higher visibility of $\text{VIS} = 95\%$, the same level of back-reflection reduces the SNR enhancement by $29\%$ from 158.0 to 112.8. The back-reflected light in general will have a much larger amplitude than the scattered light from the particle, but is an essentially stationary background to the signal. As the visibility of the interferometer increases, the detected trapping field reduces, which increases the relative contribution of the back-reflected light to the shot noise. This is the cause of the increased importance of back-reflections at higher visibilities. However, as seen in the examples above, large enhancement is possible even in the presence of significant back-reflections. To further improve the enhancement, it is possible to reduce back-reflections via antireflection coatings on sample slides, or through the use of index matching fluid between the objectives and the sample.

The theory presented here assumes that the axial position of the trapped particle is fixed at the peak of the standing wave in the optical tweezers. In practice, the particle will deviate from that position, depending on the stiffness of the trap. As a result, the standing wave amplitude will reduce, which correspondingly reduces the scattered light. More critically, the antisymmetric mode will no longer have completely constructive interference at the detector, because the path lengths to the beamsplitter will no longer be ideal. It is therefore apparent that the particle must be trapped close to the antinode to achieve SNR enhancement. This can be ensured via strong axial trapping using bright trapping fields, or with active feedback control of the beam path within the Sagnac to keep the antinode following the particle.

For simplicity, this paper uses a scalar theorem for optical trapping. This scalar paraxial model was derived for standard back-focal-plane interferometry [9], and has been shown to be very precise for back-focal-plane interferometry [3, 16, 32], including with counter-propagating traps [26]. A more accurate description could be found by deriving the detected signal using three-dimensional vector diffraction theory [31]. However, the principle behind this paper is that antisymmetric scattered modes will constructively interfere at the interferometer dark port, which also holds for vector fields. No assumptions were made about mode shapes, except that the trapping beam profile is symmetric. The technique should therefore remain valid even in regimes where the high NA objectives make three-dimensional models necessary.

A particularly relevant application area of the technique presented here is optical tweezers incorporating side tracking of the $z$ position of the particle [27]. Here low NA objectives are required, which will allow for good interferometer visibility and ensure that the scalar diffraction theory is sufficient; while the low NA enforces the use of two counter-propagating trap fields to achieve sufficient axial trapping, precluding the possibility of particle tracking via detection of back-scattered light. Towards this goal, we have begun experimental implementation of the system described here. The laser source is a 700 mW Innolight Prometheus laser. This laser produces the two counter-propagating 250 mW 1064 nm trapping fields for a vertically oriented optical tweezers trap within a Sagnac interferometer. The trap itself is formed by a pair of long working distance, $\text{NA} = 0.4$ objectives (OFR LMH-20X-YAG). The long working distance, and hence low NA, of the objectives allows side imaging of the trapped sample. To date, we have measured an interferometer visibility of 80% without optimizing the system. This should enable an SNR enhancement of approximately 38, according to equation (34).

4. Conclusion

By using a Sagnac interferometric detection scheme, the signal-to-noise ratio for particle tracking in optical tweezers with counter-propagating trap beams is enhanced by a factor which increases as the interferometer visibility approaches 100%, up to a maximum enhancement defined by four times the ratio of the trapping field intensity to the detector saturation threshold. This improvement comes about because the interferometer causes destructive interference of the trapping field at the dark port without affecting the information carrying part of the scattered field. If optical tweezers were set up in a Sagnac interferometer with a visibility of 95%, the signal-to-noise ratio could be enhanced by a factor of up to 158, which would consequently enable tracking of 2.3 times smaller particles than the equivalent standard optical tweezers scheme.

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References

[1] Afzal R S and Treacy E B 1991 Optical tweezers using a diode laser Rev. Sci. Instrum. 63 2157
[2] Lang M J and Block S M 2003 Resource letter: Lbot-1: laser-based optical tweezers Am. J. Phys. 71 201–15
[3] Neuman K C and Nagy A 2008 Single-molecule force spectroscopy: optical tweezers, magnetic tweezers and atomic force microscopy Nat. Methods 5 491–505
[4] Ashkin A and Dziedzic J M 1987 Optical trapping and manipulation of viruses and bacteria Science 235 1517–20
[5] Bustamante C 2005 Unfolding single RNA molecules: bridging the gap between equilibrium and non-equilibrium statistical thermodynamics Q. Rev. Biophys. 38 291–301
[6] Svoboda K, Schmidt C F, Schnapp B J and Block S M 1993 Direct observation of kinesin stepping by optical trapping interferometry Nature 365 721
[7] Finer J T, Simmons R M and Spudich J A 1994 Single myosin molecule mechanics: piconewton forces and nanometre steps Nature 368 113–9
[8] Greenleaf W J and Block S M 2006 Single-molecule, motion-based DNA sequencing using RNA polymerase Science 313 801
[9] Gittes F and Schmidt C F 1998 Interference model for back-focal-plane displacement detection in optical tweezers Opt. Lett. 23 7–9
[10] Harada Y and Asakura T 1996 Radiation forces on a dielectric sphere in the rayleigh scattering regime Opt. Commun. 124 529–41
[11] Pralle A, Prummer M, Florin E-L, Stelzer E H K and Hörber J K 1999 Three-dimensional high-resolution particle tracking for optical tweezers by forward scattered light Microsc. Res. Tech. 44 378–86
[12] Simmons R M, Finer J T, Chu S and Spudich J A 1996 Quantitative measurements of force and displacement using an optical trap Biophys. J. 70 1813–22
[13] Berne B J and Pecora R 2003 Dynamic Light Scattering (New York: Dover)
[14] Van De Hulst H C 1982 Light Scattering by Small Particles (New York: Dover)
[15] Ashkin A, Dziedzic J M, Bjorkholm J E and Chu S 1986 Observation of a single-beam gradient force optical trap for dielectric particles Opt. Lett. 11 288–90
[16] Hajizadeh F and Reihani S N S 2010 Optimized optical trapping of gold nanoparticles Opt. Express 18 551–9
[17] Sehgal H, Aggarwal T and Salapaka M 2007 Characterization of dual beam optical tweezers system using a novel detection approach Proc. ACC 4234
[18] Hsu M T L, Delaubert V, Lam P K and Bowen W P 2004 Optimal optical measurement of small displacements J. Opt. B: Quantum Semiclass. Opt. 6 495
[19] Tay J W, Hsu M T L and Bowen W P 2009 Quantum limited particle sensing in optical tweezers Phys. Rev. A 80 063806
[20] Lindfors K, Kalkbrenner T, Stoller P and Sandoghdar V 2004 Detection and spectroscopy of gold nanoparticles using supercontinuum white light confocal microscopy Phys. Rev. Lett. 93 037401
[21] Zukurov Evers H, Müller C, Renn A, Helenius A and Sandoghdar V 2009 High-speed nanoscopic tracking of the position and orientation of a single virus Nat. Methods 6 923
[22] Treps N, Andersen U, Buchler B, Lam P K, Maître A, Bachor H-A and Fabre C 2002 Surpassing the standard quantum limit for optical imaging using nonclassical multimode light Phys. Rev. Lett. 88 203601
[23] Treps N, Grosse N, Bowen W P, Fabre C, Bachor H-A and Lam P K 2003 A quantum laser pointer Science 301 940
[24] Treps N, Grosse N, Bowen W P, Hsu M T L, Maître A, Fabre C, Bachor H-A and Lam P K 2004 Nano-displacement measurements using spatially multimode squeezed light J. Opt. B: Quantum Semiclass. Opt. 6 S664
[25] Vossen D L J, van der Horst A, Dogterom M and van Blaaderen A 2004 Optical tweezers and confocal microscopy for simultaneous three-dimensional manipulation and imaging in concentrated colloidal dispersions Rev. Sci. Instrum. 75 2960
[26] van der Horst A, van Oostrom P D J, Moroz A, van Blaaderen A and Dogterom M 2008 High trapping forces for high-refractive index particles trapped in dynamic arrays of counterpropagating optical tweezers Appl. Opt. 47 3196
[27] Perch-Nielsen I R, Rodrigo P J and Glückstad J 2005 Real-time interactive 3D manipulation of particles viewed in two orthogonal observation planes Opt. Express 13 2852
[28] Tay J W, Taylor M A and Bowen W P 2009 Sagnac-interferometer-based characterization of spatial light modulators Appl. Opt. 48 2236
[29] Zemánek P, Jonáš A, Šrámek L and Liška M 1998 Optical trapping of Rayleigh particles using a gaussian standing wave Opt. Commun. 151 273–85
[30] Reihani S N S and Oddershede L B 2007 Optimizing immersion media refractive index improves optical trapping by compensating spherical aberrations Opt. Lett. 32 1998–2000
[31] Rohrbach A and Stelzer E H K 2002 Three-dimensional position detection of optically trapped dielectric particles J. Appl. Phys. 91 5474–88
[32] Sanii B and Ashby P D 2010 High sensitivity deflection detection of nanowires Phys. Rev. Lett. 104 147203