Reconstructing the dark energy potential

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Abstract. Dark energy equation of state can be effectively described by that of a barotropic fluid. The barotropic fluid model describes the background evolution and the functional form of the equation of state parameter is well constrained by the observations. Equally viable explanations of dark energy are via scalar field models, both canonical and non-canonical; these scalar field models being low energy descriptions of an underlying high energy theory. In this paper, we attempt to reconcile the two approaches to dark energy by way of reconstructing the evolution of the scalar field potential. For this analysis, we consider canonical quintessence scalar field and the phantom field for this reconstruction. We attempt to understand the analytical or semi-analytical forms of scalar field potentials corresponding to typical well behaved parameterisations of dark energy using the constraints from recent observations.

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In the nineties, observations of Supernovae of type Ia proved that the present day expansion of the universe is accelerating [1, 2]. These observations presented convincing evidence of the presence of an unknown component, namely the dark energy. This component, with negative pressure, dominates the energy budget of the universe at present. This has lead to proposals of a large number of theoretical models, formulated to explain the observed accelerated expansion. The acceleration is explained by the presence of a either a cosmological constant or by an alternative models of dark energy.

The cosmological constant model is consistent with observations and is also a preferred theoretical description of dark energy by way of its simplicity. However, this model has the fine tuning problem, as the value of cosmological constant, required by observations, is smaller by a factor of $10^{-121}$ than that one computed as the vacuum energy density in quantum field theory (A detailed discussion on cosmological constant can be found in [3, 4]). The theoretical issues related to the cosmological constant lead to formulation of various dark energy models, based mainly on isotropic fluids or on scalar fields. Though the observations are pointing more and more towards a cosmological constant model being a good description of dark energy, the allowed range of dark energy equation of state parameter allows significant deviations from a cosmological constant.

In general, the equation of state parameter can be different from that of a cosmological constant and can also be a function of time. The background evolution with a varying dark energy parameters is described equally well by scalar fields and by fluid models. This is not the case when the perturbations in the energy density are considered and it has been shown that including dark energy perturbations affects how structures formed in the universe [5–12]. Since the distance measurements depend on the background evolution, it is safe to assume that both fluid and homogeneous scalar field models are viable descriptions for an accelerated expansion as far as cosmological parameter determination is concerned. In a detailed review [16], Bamba et al. have studied varied classes of scalar field and fluid dark energy models and also studied modified theories of gravity and point out the equivalence of different dark energy models, including construction of scalar field models corre- sponding to cosmological constant and other cosmological scenarios. More reviews where different aspects of dark energy have been exhaustively discussed are [13–15, 17].

In the lack of a theoretical explanation of models involving the form of a scalar field potential, an attempt can be made to ‘reverse engineer’ a scalar field potential. This type of ‘reverse engineering’ is termed the reconstruction of the scalar field potential. For a given expansion history, one can reconstruct a potential which will reproduce the evolution. Pioneering work in this direction was by Ellis and Madsen [17] and by Starobinsky [18]. Reconstruction of dark energy equation of state from
cosmological distance measurement has been discussed by Huterer and Turner [19] and by Saini et al. [20].

The present work is an attempt to reconstruct the scalar field potential from different parameterisations of dark energy equation of state parameter. Rubano and Barrow [21] have found an exact form of the scalar field for a two fluid model. Recently Nojiri et al. [22] have discussed about singular cosmological evolution using canonical and phantom scalar fields. For the present study, we consider a constant equation of state parameter and also consider the case where the equation of state parameter is expanded in Taylor series in terms of the scale factor and the case in which the equation of state parameter is a logarithmic function of the redshift. The series expansion of the dark energy equation of state parameter up to the first order has been proposed by Chevallier, Polarski and Linder [23, 24] which is the well studied CPL parameterisation. Scherrer [25] mapped the CPL parameterisation onto physical dark energy, namely the quintessence and barotropic models. Some earlier work on the reconstruction of scalar field potential is reported in [18–20, 26–38].

The paper is organized as follows. In section 2, we discuss the background cosmological equations and also the scalar field description of dark energy which are required to understand the concept of reconstruction of scalar field potential. In section 3, the reconstruction of the scalar field potential for three different parameterisations of dark energy equation of state parameter have been discussed. The results obtained from the observational constraints on the model parameters are presented in section 4. Finally, in section 5, we present the conclusion and summary of the work.

2 Dark energy cosmology

For a spatially flat, homogeneous and isotropic universe, the cosmological evolution is described by the Friedmann equations given by

\[
\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho, \tag{2.1}
\]

\[
2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p, \tag{2.2}
\]

where \(a\) is the scale factor, \(\rho\) is the total energy density and \(p\) is the pressure. The total energy density \(\rho\) at a given epoch is

\[
\rho = \rho_R(a) + \rho_m(a) + \rho_{DE}(a), \tag{2.3}
\]

where the subscripts \(m\), \(R\) and \(DE\) correspond to the non-relativistic, the relativistic and the dark energy components respectively. The radiation energy density falls rapidly with the expansion of the universe as \(\rho_r \sim \frac{1}{a^4}\), therefore, the contribution of the relativistic particles can be neglected at late times. Observations suggest that the energy of the present universe is dominated by dark energy, where less than one-third contribution is due to the energy density from non-relativistic matter.

Equation (2.1) can, therefore, be written in terms of density parameter as,

\[
\frac{\dot{a}^2}{a^2} = H^2 = H_0^2 \left[ \Omega_m + \Omega_{DE}(a) \right], \tag{2.4}
\]

where \(H_0\) is the present day value of the Hubble parameter \(H\) and \(\Omega_{m0}\) is the present day matter density parameter, given by \(\Omega_m = \rho_m/\rho_c\), with the critical energy density of the universe given by \(\rho_c = 3H_0^2/8\pi G\). The present value of the scale factor has been scaled to be unity. The dark energy density parameter \(\Omega_{DE}(a)\) is in general a function of the scale factor. In case of the cosmological constant model of dark energy, \(\rho_{DE}\) remains a constant.

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The equation of state for a barotropic fluid is given by \( p = w \rho \), where \( w \) is the equation of state parameter. For dark energy, with a constant \( w \), the dark energy density evolves as a function of scale factor as \( a^{-3(1+w)} \). In case of the cosmological constant, the equation of state parameter is given by \( w = -1 \) and any deviation of \( w \) from \(-1\) imposes time evolution of dark energy. This is the \( w \)CDM (constant \( w \) with cold dark matter) dark energy model. In general, \( w \) can be a function of time and its behaviour can be approximated by way of assuming a functional form for its evolution. A simple parameterisation is an expansion of the energy equation state in a Taylor series suggested by [23]

\[
w(a) = w_0 + w'(1-a).
\] (2.5)

In this parameterisation, namely the CPL parameterisation, \( w_0 \) is the present value of equation of state parameter and \( w' \) is its first derivative. This functional form is used in most studies of varying dark energy models. This parameterisation allows a slow variation of the dark energy density at late times. The asymptotic or early time value of the dark energy equation of state is \( w_0 + w' \) and the present day value (i.e. at \( a = 1 \)) is \( w_0 \). In this case, the variation of the dark energy density as a function of scale factor is given by

\[
\frac{\rho_{DE}}{\rho_{DE_0}} = a^{-3(1+w_0+w')} \exp \left[-3w'(1-a)\right].
\] (2.6)

Another description of a varying equation of state parameter, is the logarithmic parameterisation given as,

\[
w(a) = w_0 - w'\log(a).
\] (2.7)

In this case, the equation of state increases monotonically [39] and the variation of energy density with scale factor is given by

\[
\frac{\rho_{DE}}{\rho_{DE_0}} = a^{-3(1+w_0-\frac{w'}{2}\log(a))}.
\] (2.8)

All the parameterisation discussed above are appropriate to fit thawing models of dark energy as shown in [26]. We attempt for find explicit form of the scalar field potential which has the same background evolution as described by these parameterisations. Since all scalar field models of dark energy are largely phenomenological, it is reasonable to fit functional forms of scalar fields with the fluid parameterisations.

A detailed analysis cosmological parameters allowed by Supernovae Type Ia (SNIa) data [1, 2, 40–46], Baryon Acoustic Oscillation (BAO) data [47–53] and direct measurements of Hubble parameter (H(z) data) [54–56] and a combination of these data sets for different models is given in [57]. The allowed ranges of cosmological parameters, at the 3\(\sigma\) confidence level for \( w \)CDM model, CPL and the logarithmic parameterisation from [57] are listed in table 1. In the present work, the nature and evolution of the scalar field dark energy potentials are reconstructed for the evolution history allowed by these three parameterisations.

### 3 Reconstruction of scalar field potential

Dark energy is equivalently described by scalar fields, both canonical and non canonical. In this paper, we consider the canonical, quintessence field and the phantom field. For models which are of ‘quintessence’ type scalar fields [15, 58–69], \( w > -1 \) and on the other hand, \( w < -1 \) for ‘phantom’ like models [15, 70–78]. The phantom scalar fields have a negative kinetic energy and are the same as the c-fields proposed by Hoyle and Narlikar [79]. These c-fields are massless scalar fields and generate negative gravitational field because of negative energy density.
|               | SNIa | BAO | H(z) | SNIa+BAO+H(z) |
|---------------|------|-----|------|---------------|
| wCDM model    |      |     |      |               |
| -1.57 \leq w \leq -0.66 | -1.13 \leq w \leq -0.95 |
| 0.05 \leq \Omega_m \leq 0.43 | 0.25 \leq \Omega_m \leq 0.31 |
| -2.19 \leq w \leq -0.42 | -1.78 \leq w \leq -0.72 |
| 0.19 \leq \Omega_m \leq 0.36 | 0.2 \leq \Omega_m \leq 0.35 |
| -1.57 \leq w \leq -0.66 | -1.13 \leq w \leq -0.95 |
| 0.05 \leq \Omega_m \leq 0.43 | 0.25 \leq \Omega_m \leq 0.31 |

\begin{align*}
\text{Table 1.} & \text{ This table shows the 3} \sigma \text{ confidence limit for various data sets for the } wC\text{DM model, CPL parameterisation and logarithmic parameterisation. These constraints on parameters are as in [57].} \\
\text{The pressure and energy density for quintessence and phantom scalar field are given by} \\
p &= \pm \frac{\dot{\phi}^2}{2} - V(\phi) \\
\rho_{DE} &= \pm \frac{\dot{\phi}^2}{2} + V(\phi), \\
\text{where } \phi \text{ denotes the scalar field and } V(\phi) \text{ is the scalar field potential. In the above expressions, the} \\
\text{plus sign corresponds to a quintessence field and the negative sign corresponds to a phantom field dark energy i.e., for a negative kinetic energy term. Therefore, the scalar field potential which is} \\
\text{emulated by the parameterisation given in equation 2.5 can be reconstructed as} \\
V(a) &= \frac{1}{2}(1-w)\rho_{DE}(a). \\
\text{for the scalar field. Here, } w \text{ can be a constant or a function of the scale factor. The variation of the} \\
\text{scalar field with time for a quintessence field is given as} \\
\left[ \frac{d\phi}{dt} \right]^2 &= (1+w)\rho_{DE} \\
\text{which, in turn, can be written as} \\
\left[ \frac{d\phi}{da} \right]^2 &= \frac{\sqrt{(1+w)\rho_{DE}}}{a H(a)}. \\
\text{We mainly consider the positive sign in the above expression for our discussion. For completeness, we} \\
\text{discuss the results for the negative sign branch within the quintessence scenario for the case of a} \\
\text{constant equation of state parameter. The effective dynamics are the same for both the negative and} \\
\text{positive branch as the energy density depends on } \dot{\phi}^2. \\
\text{For a phantom like scalar field, since the kinetic energy is negative, the variation in the field } \phi \\
\text{as a function of time is given as} \\
\left[ \frac{d\phi}{dt} \right]^2 &= -(1+w)\rho_{DE}, \\
\end{align*}
which, in terms of the scale factor, is given by,

$$\frac{d\phi}{da} = \sqrt{-(1+w)\rho_{DE}} \frac{1}{aH(a)}. \quad (3.3)$$

For a universe with dark energy as its sole constituent, the scalar field potential for a constant dark energy equation of state is given by (see also [80])

$$V(\phi) = \frac{1}{2}(1-w)\rho_{DE} \exp \left[ -\sqrt{24\pi G(1+w)}(\phi - \phi_0) \right],$$

which can be rewritten as

$$\tilde{V}(\tilde{\phi}) = \frac{1}{2}(1-w)\rho_{DE} \exp \left[ -\sqrt{3(1+w)}(\tilde{\phi} - \tilde{\phi}_0) \right] \quad (3.4)$$

where $\tilde{V} = V/\rho_{DE}$, $\tilde{\phi} = \sqrt{8\pi G}\phi$ and $\tilde{\phi}_0$ is the value of field at $a = 1$. And for a phantom dark energy, the potential is of the form

$$V(\phi) = \frac{1}{2}(1-w)\rho_{DE} \exp \left[ \sqrt{24\pi G(1+w)}(\phi - \phi_0) \right].$$

We scale the potential with the present day value of dark energy density and $\phi$ by $\sqrt{8\pi G}$ and then the above equation takes the form

$$\tilde{V}(\tilde{\phi}) = \frac{1}{2}(1-w)\rho_{DE} \exp \left[ \sqrt{-3(1+w)}(\tilde{\phi} - \tilde{\phi}_0) \right]. \quad (3.5)$$

The slope of the potential and its amplitude are determined by the equation of state parameter of dark energy. The exponential potential belongs to the ‘thawing’ class of scalar fields, where the early times scalar field equation of state is like that of a cosmological constant with $w = -1$ and at late times begins to deviate from this value. This potential has been employed extensively for dark energy studies and as an inflaton potential [81].

If the contribution of matter density is significant, the solutions for the quintessence scalar field for $w = constant$ are given by

$$\tilde{\phi} - \tilde{\phi}_0 = \frac{\sqrt{3(1+w)}}{3w} \ln \left( \frac{\sqrt{\sqrt{1+r_0 a^{3w}} - 1}}{\sqrt{\sqrt{1+r_0 a^{3w}} + 1}} \right), \quad (3.6)$$

where $r_0 = \rho_{m_0}/\rho_{DE_0}$. If $d\phi/da$ is negative, the expression for field is same as this with an overall negative sign and the expression of the quintessence scalar field potential can be written as,

$$\tilde{V}(\tilde{\phi}) = \frac{(1-w)}{2} \left( \frac{r_0 \sinh^2 \left( \frac{3w\tilde{\phi}}{2\sqrt{1+w}} \right)}{\sqrt{1+w}} \right)^{\frac{1}{1+w}}. \quad (3.7)$$

Similarly, we obtain an expression for phantom scalar field, which is given by

$$\tilde{\phi} - \tilde{\phi}_0 = \frac{\sqrt{-3(1+w)}}{3w} \ln \left( \frac{\sqrt{\sqrt{1+r_0 a^{3w}} - 1}}{\sqrt{\sqrt{1+r_0 a^{3w}} + 1}} \right), \quad (3.8)$$
Figure 1. The figure represents 3σ allowed regions for the reconstructed potential, scaled by the present day dark energy density \( \rho_{DE}(a = 1) \) as a function of scale factor for the \( w_{\text{CDM}} \) model. From left, the plots in the rows are the results obtained from the analysis of SNIa, BAO, H(z) and combined datasets respectively. The plots in first row are for a quintessence potential and the second row represents plots for a phantom potential.

Figure 2. The plots show 3σ allowed regions for field \( \phi \) as a function of scale factor reconstructed from the \( w_{\text{CDM}} \) model. The order in which the plots are presented is the same as in 1.

The functional form of the potential for the phantom field is same as that for a quintessence potential except for a negative sign in the argument \( \sqrt{-\left(1 + w\right)} \), and is given as

\[
\tilde{V}(\tilde{\phi}) = \left(1 - w\right) \frac{r_0}{2} \sinh^2 \left( \frac{\sqrt{3}w\tilde{\phi}}{\sqrt{-\left(1 + w\right)}} \right) \frac{1}{1 + w}.
\]

Here, the scalar field \( \phi \) is scaled by \( \sqrt{8\pi G} = M_{\text{pl}}^{-1} \). For a large value of the scalar field \( \phi \), this potential takes the exponential form. The functional form of this potential (equation 3.9), take the the
Figure 3. The plots show allowed regions at the 3σ level for the scalar field potential $V(\phi)$ as a function of the field $\phi$ reconstructed from the wCDM model. We have taken the envelope of the family of curves corresponding to different values of the equation of state parameter.

Figure 4. The plots show 3σ allowed regions for field potential $V(\phi)$ versus field $\phi$ reconstructed from wCDM model for the branch where $d\phi/da$ is negative. The sequence is same as in figure 3.

same form as in a purely dark energy universe. Therefore, safe to assume that the potential can be reconstructed in a dark energy only universe.

We now consider the models where the equation of state parameter is a function of time. We first consider the CPL parameterisation (2.5) which is the parameterisation employed in most dark energy studies. It has been pointed out that barotropic fluids are not consistent with a freezing type behaviour [25, 26] in general and in particular for the CPL parameterisation which is the scenario we will discuss next.

The variation of the scalar field ($\phi$) as a function of the scale factor $a$ for the CPL parameterisa-
Figure 5. The plots in the rows represent 3σ allowed regions for potential reconstructed from $w(a) = w_0 + w'(1-a)$ parameterisation as a function of scale factor. As before, the potential is scaled by present value of the dark energy density $[\rho_{DE}(a = 1)]$. From the left, the plots in both the rows are from the analysis of SNIa, BAO and H(z) data sets respectively. The first and second row correspond to quintessence and phantom field respectively.

Figure 6. The figure represents 3σ allowed regions for field $\phi$ versus scale factor reconstructed from $w(a) = w_0 + w'(1-a)$ parameterisation.

d\phi / da = \pm \frac{[1 + w_0 + w'(1-a)]\rho_{DE}}{a^2 H^2}.

(3.10)

Here again, the plus sign is for a quintessence field and the negative sign is for a phantom field. For further discussion we have considered $d\phi / da$ to be positive. The conditions for the CPL parameterisation to emulate quintessence like behaviour are $w_0 + w' \geq -1$ and $w_0 > -1$. These conditions ensure that the equation of state parameter, $w(a)$ is always greater than $-1$ at all times. On the other hand, the
Figure 7. The plots in the rows show 3σ allowed regions for field potential $V(\phi)$, scaled by the present dark energy density $[\rho_{DE}(a = 1)]$ versus field $\phi$ reconstructed from $w(a) = w_0 + w'(1 - a)$ parameterisation. The plots in both the rows represent the results obtained from SNIa, BAO, H(z) and combined data sets respectively.

Condition $w_0 + w' \leq -1$, along with $w_0 < -1$, ensures that the equation of state parameter, $w(a)$, is less than $-1$, for all values of $a$ and hence the equation of state parameter is phantom like at all times.

In the low redshift regime, when the dark energy density is the dominant factor in the total energy of the universe, the scalar field potential can be expressed as,

$$V(a) = \frac{1}{2} [1 - w_0 - w'(1 - a)] a^{-3(1+w_0+w')} e^{-3w'(1-a)}$$

and scalar field is given as

$$\tilde{\phi} - \tilde{\phi}_0 = 2\sqrt{3} \left[ \sqrt{\pm (1 + w_0 + w'(1 - a))} - \sqrt{\pm (1 + w_0)} \right]$$

$$+ \frac{\sqrt{\pm (1 + w_0 + w')}}{2} \ln \left\{ \frac{\sqrt{\pm (1 + w_0 + w'(1 - a))} - \sqrt{\pm (1 + w_0 + w')}}{\sqrt{\pm (1 + w_0 + w'(1 - a))} + \sqrt{\pm (1 + w_0 + w')}} \right\}$$

$$- \frac{\sqrt{\pm (1 + w_0 + w')}}{2} \ln \left\{ \frac{\sqrt{\pm (1 + w_0)} - \sqrt{\pm (1 + w_0 + w')}}{\sqrt{\pm (1 + w_0)} + \sqrt{\pm (1 + w_0 + w')}} \right\}.$$ 

Since the equation of state parameter is an expansion about its present day value, it is expected that the reconstructed potential is close to that of the case with a constant $w$ with a slight increase in the allowed range of parameters. We explore this aspect in the next section.

We now consider the scenario where the equation of state parameter is a function of the logarithm of redshift or the scale factor. The variation of the scalar field $\phi$ with the scale factor in this case is expressed as,

$$\left[\frac{d\phi}{da}\right]^2 = \pm \frac{[1 + w_0 - w' \log(a)] \rho_{DE}}{a^2 H^2}$$

Since dark is dominant in the low redshift regime, we have neglected the contribution of matter, and for the scalar field potential can then be expressed as

$$V(a) = \frac{1}{2} [1 - w_0 + w' \log(a)] a^{-3(1+w_0-w' \log(a)/2)}$$
Figure 8. The plots in the rows represent 3σ allowed regions for potential, scaled by the present dark energy density $\rho_{DE}(a = 1)$, reconstructed from $w(a) = w_0 - w' \log(a)$ parameterisation versus scale factor. The sequence of plots is the same as before.

and quintessence scalar field is given as

$$\tilde{\phi} - \tilde{\phi}_0 = - \frac{2}{\sqrt{3}} \left[ \frac{(1 + w_0 - w' \log(a))^{3/2}}{w'} - \frac{(1 + w_0)^{3/2}}{w'} \right], \quad (3.15)$$

with the corresponding expression for a phantom scalar field given by

$$\tilde{\phi} - \tilde{\phi}_0 = \frac{2}{\sqrt{3}} \left[ \frac{(w' \log(a) - w_0 - 1)^{3/2}}{w'} - \frac{(-w_0 - 1)^{3/2}}{w'} \right]. \quad (3.16)$$

In this case, we can obtain a closed form for the scalar field potential, and the expression for quintessence scalar field potential is given by

$$\tilde{V}(\tilde{\phi}) = \frac{1}{2} \left[ 2 - \left( (1 + w_0)^{3/2} - \frac{3w'}{2\sqrt{3}} (\tilde{\phi} - \tilde{\phi}_0) \right)^{2/3} \right] \exp \left[ -\frac{3}{2w'} \left( (1 + w_0)^2 - \left( (1 + w_0)^{3/2} - \frac{3w'}{2\sqrt{3}} (\tilde{\phi} - \tilde{\phi}_0) \right)^{4/3} \right) \right] \quad (3.17)$$

and phantom scalar field potential in terms of $\tilde{\phi}$ is given by

$$\tilde{V}(\tilde{\phi}) = \frac{1}{2} \left[ 2 + \left\{ \frac{3w'}{2\sqrt{3}} (\tilde{\phi} - \tilde{\phi}_0) + (-w_0 - 1)^{3/2} \right\}^{2/3} \right] \exp \left[ -\frac{1}{2w'} \left( (1 + w_0)^2 - \left[ \frac{3w'}{2\sqrt{3}} (\tilde{\phi} - \tilde{\phi}_0) + (-w_0 - 1)^{3/2} \right]^{4/3} \right) \right]. \quad (3.18)$$
In this section, we discuss the observational constraints on the variation of the reconstructed scalar field potential from different data sets and on the reconstructed scalar field potentials as a function of the field corresponding to a fluid dark energy equation of state. The individual data sets allow a higher range of variation and when combined, the resulting allowed range is significantly narrower as a result of tighter constraints on parameters.

In Figure 1, we have plotted the $3\sigma$ allowed regions for the reconstructed potential as a function of the scale factor $a$, for the constant equation of state parameter ($w_{\text{CDM}}$) model. We have plotted $V(a)/\rho_{DE_0}$ vs $a$ for quintessence (the first row) and phantom (the second row). For quintessence, the allowed range lies below unity when $a > 0.8$ and for $a < 0.8$, this range lies above $V(a)/\rho_{DE_0} = 1$.
line. As the value of \( a \) decreases from one, the allowed range begins to get narrower till it reaches \( a \sim 0.8 \), where it is narrowest. With further decrease in \( a \), the allowed range starts increasing again. The phantom potential shows a behaviour opposite to that of quintessence field but with a similar switch in allowed range at \( a \sim 0.8 \).

In figure 3, we show the 3\( \sigma \) allowed regions for reconstructed potentials as a function of scalar field, \( \phi \), for the \( w \)CDM model. We have plotted \( V(\phi)/\rho_{DE0} \) vs \( \phi \), where \( \phi \) is in units of \( \sqrt{8\pi G} \). In Figure 2, we show the 3\( \sigma \) allowed regions for scalar field \( \phi \) as a function of scale factor, \( a \), for \( w \)CDM model. The value of the scalar field is in units of \( M_{pl}^{-1} \). The plots in the two rows of figure 2 shows the results obtained from the analysis of SNIa, BAO and H(z) data sets respectively. As before, the plots in first row are for a quintessence field and plots in the second row shows allowed range for a phantom field and the results from the combined analysis. In the case of a varying equation of state parameter, the allowed range of the dark energy density variation increases as compared to the \( w \)CDM model. The family of curves representing the scalar field potential as a function of the field have a fairly restricted range of variation for the quintessence models. For phantom like models, the allowed range increases as compared to the \( w \)CDM case. Here we have plotted the envelope of family of curves corresponding to the allowed range in \( w \), the curve is defined by the function \( V(\phi) \) and the constants.

In figure 7 we show the variation of scalar field potential \( V(\phi) \) as a function of the field \( \phi \) for the CPL scenario. These solution are valid only under the assumption that the scale factor is very close to its present day value, i.e., valid at late times. In figure 5, 3\( \sigma \) allowed region is shown for potentials reconstructed from CPL model. The phantom potential shows a similar behaviour as in the \( w \)CDM case. The figure 8 shows the allowed range of potential for the logarithmic parameterisation. In this case also, the narrowest range is obtained from BAO data sets which when combined with other data sets restricts the range further. The corresponding field versus scale factor plots for scalar field, derived from CPL model, are shown in figure 6.

Figure 7 represents the 3\( \sigma \) allowed regions for the reconstructed potential as a function of scalar field, \( \phi \), for the CPL model. Figure 9 shows the results obtained for logarithmic parameterisation. The allowed ranges obtained for the logarithmic parameterisation from the individual datasets and combined analysis are shown in figure 10. The plots show that the profiles of uncertainty associated with the best fit curves of the scalar field for these two models are different. The dark energy potential shows similar behaviour for these models.

In all the models considered above, the results in general are similar to each other. The most stringent constraints on the variation of the scalar field as a function of scale factor are due to the BAO data and as a result the combination of different datasets allows for a limited range too. More data at different redshifts will further limit this range.

5 Summary and Conclusion

In the present work, we attempt to connect two alternate explanations of dark energy, namely barotropic fluid models and scalar field models by way of reconstructing scalar field potentials which emulate the barotropic equation of state. We assume a constant dark energy equation of state parameter, a slowly varying function of redshift and a logarithmic growth with respect to the the redshift for this reconstruction. The assumptions are reasonable as a combination of low redshift observations, restrict the allowed range of the evolution of dark energy density. Since it is straightforward to parameterise the dark energy equation of state and constrain its parameters, therefore, we constrain the cosmological parameters and using parameters allowed by individual and combined datasets, we obtain a range in variation of the scalar field potential. We study quintessence and phantom nature of dark energy
and reconstruct the respective potentials for these models and obtain semi-analytical forms for the scalar field potentials. In this context, it is worth mentioning that for fluid models, a transition from quintessence to phantom like behaviour is straightforward as both the behaviours are described by the same equation of state. This is not the case for scalar field models as the equations describing the dynamics are fundamentally different from each other. Because the dynamics of the two scalar fields are different, we use different priors for quintessence and phantom field, namely we assume the parameter sets such that the evolution of the equation of state parameter does not cross over the \( w = -1 \) (the phantom) divide. The energy density for quintessence scalar field decays with the scale factor, and in the case of phantom field the behaviour is opposite to that of a quintessence field.

The evolution of the scalar field has very similar behaviour for both, quintessence and phantom. The uncertainty in the reconstructed potential is much higher when the analysis is carried out with individual data sets and the evolution of the potential is well constrained in the combined analysis with the data sets, namely SNIa, BAO and \( \text{H}(z) \) data. The allowed range is obtained to be minimum at \( 0.8 < a < 0.9 \), and it slightly increases at \( a \sim 1 \). This profile of uncertainty of the reconstructed potential is very similar for all the three models considered in this paper.

The potential for the \( w \text{CDM} \) model (scaled by its present day value) remains close to the value of unity, which is the boundary between the quintessence and phantom class of dark energy. The constant equation of state parameter model accommodates an exponential potential, belonging to thawing class of models. The slope of the potential and its amplitude depends on the equation of state of the dark energy fluid. If the matter contribution is also taken into account, the potential also accommodates a slow-rolling nature. For both the scenarios, namely the quintessence and phantom models of dark energy, the evolution of the potential tends to converge to a narrow range. For scenarios with varying dark energy parameterisations, the observations restrict the variation significantly. To study dark energy perturbations, the sound speed is considered as a parameter in fluid models of dark energy. Since the pressure gradients are more easily computed in scalar field models, the reconstructed potentials are hence of help in studying perturbations in these scenarios. The large scale structure data would further rule out models using data in addition to distance measurements. Fluid models are effectively used as a representation for dark energy, and analytical connection between common parameterisations and scalar field models is therefore of significance for further studies.

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