Gas permeability measurement in porous graphite under steady-state flow

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Keywords: permeability, graphites, computational fluid dynamics (CFD), compressible-darcy-forchheimer

Abstract

Porous graphite is employed for the development of air bearings used in the ultraprecision machine tools. The static performance of this new bearing type depends on the permeability and inertia coefficient of the porous graphite inserted in it. In this study, an experimental fitting method was used in conjunction with computational fluid dynamics (CFD) modeling to determine the permeability and inertia coefficient of three types of porous graphite with porosity of 16%, 13%, and 8% respectively. The experimental results show that the Compressible-Darcy-Forchheimer equation can fit the experimental mass flow rate and pressure drop well. The average permeability of SG-3, SG-5 and SG-8 porous graphite are $5.74 \times 10^{-14}$ m$^2$, $3.65 \times 10^{-15}$ m$^2$ and $1.85 \times 10^{-15}$ m$^2$ respectively. SG-5 and SG-8 porous graphite have good permeability consistency and can be used to make porous media air bearing. The PPSM (number of pores per square millimeter) of SG-3 and SG-8 are similar, but the permeabilities are very different. For low permeability porous graphite, samples with different sizes from the same material should be tested and the averaged inertia coefficient can be used. The accuracy of the pore-scale FVM (finite volume method) is highly dependent on the quality of the pore microstructure image.

1. Introduction

Porous media widely exist in engineering materials, organisms, and formation media [1, 2]. Porous graphite is a typical porous media with the features of lubricity, superior machining performance. Porous graphite is often used in engineering applications, such as air bearing [3–5]. Permeability is one of the most important properties which represents the ability to transmit fluid. The static performance of porous air bearing is highly dependent on the permeability and inertia coefficient of the porous graphite. Therefore, it is critical to accurately determine the permeability and inertia coefficient of airflow through porous graphite. The two most widespread methods are pore-scale Navier–Stokes’s equations (NSEs) and Lattice Boltzmann Method (LBM) [6–20]. However, the accuracy of these two pore-scale methods is related to the quality of CT images [21]. It is difficult to obtain high-quality CT images of tight porous graphite due to the pore is usually on the micro-scale. Pore network modeling (PNM) is an alternative method characterized by many benefits [22–27]. However, this method greatly simplifies the real pore microstructure of porous media. For low porosity porous media, the performance of PNM is unstable. The fractal model and Kozeny-Carmen equation [28–36] also can be used to determine the permeability of porous media. However, these two methods can’t estimate the inertia coefficient.

The experiment fitting method has always been consistent. The traditional Darcy-Forchheimer equation has been developed as the most notable curve fitting method [37–45] that can simultaneously determine permeability and inertia coefficient. However, the traditional Darcy-Forchheimer equation does not consider the compressibility of air. Another problem is that Darcy-Forchheimer is a curve fitting method based on experimental data. The fitting has great uncertainty, and the permeability and inertia coefficient need to be verified by another method.
In this work, a Compressible-Darcy-Forchheimer equation based on the mass flow rate that accounts for the compressibility of air was presented. All physical quantities in the Compressible-Darcy-Forchheimer equation have clear and consistent definitions. The computational fluid dynamics (CFD) was performed at the macroscale to obtain the theoretical mass flow rate based on the experimental permeability and inertia coefficient. The results show that the Compressible-Darcy-Forchheimer equation can fit the experimental results well and CFD is a powerful verification tool.

2. Theory

2.1. Darcy-Forchheimer equation

The plate experiment is often used to measure the pressure drop and mass flow rate through porous media. The principle is shown in Figure 1.

The pressure drop across the porous zone can be given by a polynomial of velocity:

$$\Delta p = C_1 U + C_2 \frac{1}{2} \rho U^2$$

(1)

where $U$ is the superficial velocity.

The Darcy-Forchheimer equation can be used to describe the correlation between the steady-state unidirectional pressure drop and flow velocity of isotropic porous media that is fully saturated with Newtonian incompressible fluid.

$$\frac{-d\rho}{dx} = \frac{\mu}{K} U + C_2 \frac{1}{2} \rho U^2$$

(2)

where $\rho$ is the pressure, $\rho$ is the air density, $C$ is the inertia coefficient, $\mu$ is the air dynamic viscosity, $U$ is the superficial velocity, and $K$ is the permeability. The right terms of equation (2) represent viscous drag and inertial drag, applied by the solid wall to the flowing fluid, respectively. These two effects are always present in the flow. The quadratic term is very small when the fluid is characterized by the creeping flow. Thus, the viscous effect will dominate. Equation (2) can also be regarded as a macroscopic momentum equation which is added to the NSEs as a source term. Along the length direction, flow velocity varied with air density and is expressed as follows:

$$U = \frac{G}{\rho A}$$

(3)

where $G$ is the mass flow rate and $A$ is the cross-sectional area of porous media. Assuming that air is an ideal gas, the ideal gas state equation is introduced. Thus, parameter $\rho$ is expressed as:

$$\rho = \frac{p}{R_{\text{specific}} T}$$

(4)

$$R_{\text{specific}} = \frac{R}{M}$$

(5)

where $R$ is the universal gas constant equal to 8.3145 J/mol k and $M$ is the molar mass of air equal 0.0289 kg/mol. 

**Figure 1.** Schematic representation of plate experiment.
By substituting equations (3) – (5) into equation (2), the following equation is obtained:

\[
\frac{-dp}{dx} = \frac{\mu RT}{KPMA} G + \frac{CRT}{2PMA^2} G^2
\]

(6)

By integrating the pressure along the length, the following equation is obtained:

\[
-\frac{p^2}{2} = \left( \frac{\mu RT}{KMA} G + \frac{CRT}{2MA^2} G^2 \right) x + d
\]

(7)

Considering the inlet boundary condition, \( x = 0, \ p = p_1 \), the constant \( d \) is obtained:

\[
d = \frac{p_1^2}{2}
\]

(8)

Considering the outlet boundary condition, \( x = L, \ p = p_2 \), a Compressible-Darcy-Forchheimer equation can be written as:

\[
\frac{p_1^2 - p_2^2}{2L} = \frac{\mu RT}{KMA} G + \frac{CRT}{2MA^2} G^2
\]

(9)

2.2. Computational fluid dynamics (CFD) theory

The physical model of air flow through porous media at the macro-scale is schematically illustrated in figure 2. CFD can be used to determine the theoretical mass flow rate with the permeability and inertia coefficient obtained by the experiment. The fluid through porous media is governed by the mass, momentum, and energy conservation equation [46].

2.2.1. Mathematical description of physical phenomena

The conservation equation for a general scalar variable \( \phi \) can be expressed as:

\[
\frac{\partial (\rho \phi)}{\partial t} + \nabla \cdot (\rho \mathbf{v}\phi) = \nabla \cdot (\Gamma \phi \nabla \phi) + Q^\phi
\]

(10)

The steady-state form of the above equation is obtained by dropping the transient term. Therefore, the equation for conservation of mass, or continuity equation, can be written as follow:

\[
\nabla \cdot (\rho \mathbf{U}) = 0
\]

(11)

where \( \rho \) is the gas density, \( \mathbf{U} \) is the velocity vector.

The porous media model adds a momentum sink in the momentum equation.

\[
\nabla \cdot (\rho \mathbf{U}\mathbf{U}) = \nabla \cdot \{ \mu \mathbf{U}\nabla \mathbf{U} \} - \nabla p + \nabla \cdot \{ \mu (\nabla \mathbf{U})^2 \} - \frac{2}{3} \nabla (\mu \nabla \cdot \mathbf{U}) + \mathbf{S}
\]

(12)

where \( \mathbf{S} \) is the source term representing the effect of porous media.

Considering the x component of equation (1), the source term has unit of force/volume.

\[
\Delta p_x = C_1 U_x + C_2 \frac{1}{2} \rho |\mathbf{U}| U_x
\]

(13)

\[
F_x = - \Delta p_x A = - \left( C_1 U_x + C_2 \frac{1}{2} \rho |\mathbf{U}| U_x \right) A
\]

(14)
where \( L \) is the thickness of the porous media in the flow direction. The thickness of the media is incorporated into the coefficients \( C_1 \) and \( C_2 \):

\[
S_x = \left( \frac{C_1}{L} U_x + \frac{C_2}{L} \rho |U| U_x \right) \frac{1}{L}
\]

By comparing equation (16) with equation (2), the following expressions can be written:

\[
\frac{C_1}{L} = \mu K
\]

\[
\frac{C_2}{L} = C
\]

The equation for conservation of energy can be written as:

\[
\nabla \cdot [\rho c_p \nabla T] = \nabla \cdot [k \nabla T] + \rho T \frac{D \phi}{D t} + \frac{D \phi}{D t} - \frac{2}{3} \mu \Psi + \mu \Phi + \Phi\
\]

The above set of equations should be appended by an equation of state relating density to pressure and temperature, which for an ideal gas is given by:

\[
\rho = \frac{p}{RT}
\]

2.2.2. The finite volume mesh
A 3D model of the porous graphite was created as shown in figure 3(a). The computational domain was spatially discretized using hexahedron grids. The cell-centroid scheme is adopted and all variable values are stored in the center of the element. To ensure valid results, a mesh sensitivity study was performed, resulting in an optimal volume mesh containing 40000 hexahedron elements as shown in figure 3(b).

2.2.3. Discretization of the diffusion term
The general steady-state diffusion term is given by:

\[
- \nabla \cdot (\Gamma \nabla \phi) = Q^\phi
\]

Where \( \phi \) denotes a scalar variable (e.g., temperature, velocity, pressure), \( Q^\phi \) denotes source term. \( \Gamma \phi \) denotes the diffusion coefficient. To demonstrate the diffusion term discretization process, taking the two-dimensional Cartesian mesh as an example as shown in figure 4. Following the volume integral and divergence theorem, equation (21) can be written as:

\[
(- \Gamma^\phi \nabla \phi)_x \cdot S_x + (- \Gamma^\phi \nabla \phi)_w \cdot S_w + (- \Gamma^\phi \nabla \phi)_n \cdot S_n + (- \Gamma^\phi \nabla \phi)_s \cdot S_s = Q^\phi_{\text{LC}}
\]
To obtain the linear iterative equation, the surface gradient is replaced by cell center value interpolation. The above equation can finally be transformed into the following form:

\[
\begin{align*}
    a_C \phi_C + \sum_{F \sim \text{nb}(C)} a_F \phi_F &= b_C \\
    a_F &= \text{Flux}_F \\
    a_C &= \sum_{f \sim \text{nb}(C)} \text{Flux}_f \\
    b_C &= Q_C \phi_C - \sum_{f \sim \text{nb}(C)} \text{Flux}_f
\end{align*}
\]

(23)

where \( \phi_C \) denotes the variable stored in the element C. \( \phi_F \) denotes the variables stored in the neighboring element (i.e., E, N, W, S).

2.2.4. Discretization of the convection Term

One-dimensional convection–diffusion equation is used to illustrate the discrete process of convective term.

\[
\frac{d(\rho u \phi)}{dx} = \frac{d}{dx} \left( \Gamma \frac{d\phi}{dx} \right) = 0
\]

(24)

The discretization of equation (24) also starts by integrating the conservation equation over one dimensional mesh shown in figure 5 to yield:
\[ \int_{V_c} [\nabla \cdot (\rho u \phi) - \nabla \cdot (\Gamma \phi \nabla \phi)] \, dV = 0 \]  \hspace{1cm} (25)

Then, using the divergence theorem, the volume integral is transformed into a surface integral giving:
\[ \int_{\partial V_c} [\rho \phi \mathbf{i} - \Gamma \phi \frac{d\phi}{dx} \mathbf{i}] \cdot dS = 0 \]  \hspace{1cm} (26)

Replacing the surface integral by a summation of fluxes over the element faces, equation (26) becomes:
\[ \sum_{f \sim nb(C)} \left( \rho \phi \mathbf{i} - \Gamma \phi \frac{d\phi}{dx} \mathbf{i} \right) \cdot S_f = 0 \]  \hspace{1cm} (27)

The upwind scheme is compatible with the advection process, which is schematically displayed in Figure 5. The upwind scheme mimics the basic physics of advection in that the cell face value is dependent on the upwind nodal value.

\[ \phi_E = \begin{cases} 
\phi_C & \text{if } \dot{m}_e > 0 \\
\phi_E & \text{if } \dot{m}_e < 0
\end{cases} \quad \text{and} \quad \phi_W = \begin{cases} 
\phi_C & \text{if } \dot{m}_w > 0 \\
\phi_W & \text{if } \dot{m}_w < 0
\end{cases} \]  \hspace{1cm} (28)

where \( \dot{m}_e \) and \( \dot{m}_w \) are the mass flow rates at faces \( e \) and \( w \) given by:

\[ \dot{m}_e = (\rho \mathbf{v} \cdot \mathbf{S})_e = (\rho u S)_e = (\rho u \Delta y)_e \]
\[ \dot{m}_w = (\rho \mathbf{v} \cdot \mathbf{S})_w = -(\rho u S)_w = -(\rho u \Delta y)_w \]  \hspace{1cm} (29)

The advection flux at face \( e \) can be written as:

\[ \dot{m}_e \phi_e = |\dot{m}_e| \phi_C - |\dot{m}_w| \phi_E = \text{Flux}_{C_{\text{Conv}}} \phi_C + \text{Flux}_{E_{\text{Conv}}} \phi_E + \text{Flux}_{V_{\text{Conv}}} \]  \hspace{1cm} (30)

where \( \text{Flux}_{C_{\text{Conv}}} = |\dot{m}_e| \phi_C, \text{Flux}_{E_{\text{Conv}}} = -|\dot{m}_w| \phi_E, \text{Flux}_{V_{\text{Conv}}} = 0. \)

In equation (30), the term \( |a,b| \) represents the maximum of \( a \) and \( b \). Moreover, a similar relationship can be derived for the advection flux at face \( w \) and is given by:

\[ \dot{m}_w \phi_w = |\dot{m}_w| \phi_C - |\dot{m}_w| \phi_W = \text{Flux}_{C_{\text{Conv}}} \phi_C + \text{Flux}_{W_{\text{Conv}}} \phi_W + \text{Flux}_{V_{\text{Conv}}} \]  \hspace{1cm} (31)

where \( \text{Flux}_{C_{\text{Conv}}} = |\dot{m}_w| \phi_C, \text{Flux}_{W_{\text{Conv}}} = -|\dot{m}_w| \phi_W, \text{Flux}_{V_{\text{Conv}}} = 0. \)

Finally, equation (24) can be expressed as:

\[ a_C \phi_C + a_E \phi_E + a_W \phi_W = 0 \]  \hspace{1cm} (32)

with

\[ a_E = \text{Flux}_{E_{\text{Conv}}} + \text{Flux}_{D_{\text{Diff}}} \]
\[ = -|\dot{m}_e| - \Gamma \phi \frac{S}{\delta x} \]
In the above step, the governing equations, are transformed into a set of algebraic equations, one for each element in the computational domain. Finally, these algebraic equations are assembled into a global matrix as shown in figure 6.

\[ \mathbf{a} \phi = \mathbf{b} \quad (33) \]

### 2.2.5. Initial and boundary conditions

The boundary conditions used in this paper are shown in the following table 1:

| Boundary name | Physical quantity | Boundary type |
|---------------|-------------------|---------------|
| Inlet         | Pressure          | Dirichlet     |
| Outlet        | Pressure          | Dirichlet     |
| Wall          | Velocity temperature | Dirichlet     |
| Symmetry      | flux              | Neumann       |

\[ a_w = \text{Flux}^\text{Conv}_w + \text{Flux}^\text{Diff}_w = -\|\mathbf{m}_w, 0\| - \Gamma_w \frac{S_w}{\delta x_w} \]

\[ a_C = \sum_f (\text{Flux}^\text{Conv}_f + \text{Flux}^\text{Diff}_f) \]

\[ b_C = -\sum_f (\text{Flux}^\text{Conv}_f + \text{Flux}^\text{Diff}_f) = 0 \]

In the above step, the governing equations, are transformed into a set of algebraic equations, one for each element in the computational domain. Finally, these algebraic equations are assembled into a global matrix as shown in figure 6.

\[ \mathbf{a} \phi = \mathbf{b} \quad (33) \]
Figure 8. A flow chart of the SIMPLE algorithm for compressible fluid flow.

Figure 9. (a) Experimental setup, (b) test section.
Take the diffusion term as an example,

\[ \text{Flux}_b = -\Gamma_b^\phi (\nabla \phi)_b \cdot S_b \]

\[ = -\Gamma_b^\phi \left\| S_b \right\| \left( \phi_b - \phi_C \right) \]

\[ = \text{Flux}_C \phi_C + \text{Flux}_V b \]

Yielding

\[ \text{Flux}_C b = a_b \]

\[ \text{Flux}_V b = -a_b \phi_b \]

\[ \text{Flux}_V b \text{ is transferred to } b \text{ matrix.} \]

If the flux (or normal gradient to the face) of \( \phi \) is specified at the boundary, then the boundary condition is denoted by a Neumann boundary condition as shown in figure 7(b). Take the diffusion term as an example;

\[ - (\Gamma^\phi \nabla \phi)_b \cdot i = q_b \]

\[ - (\Gamma^\phi \nabla \phi)_b \cdot \left\| S_b \right\| i = q_b \left\| S_b \right\| = \text{Flux}_C b \phi_C + \text{Flux}_V b \]

Where \( \text{Flux}_C b = 0, \text{Flux}_V b = q_b S_b \). \( \text{Flux}_V b \) is transferred to \( b \) matrix.

A symmetry boundary condition is equivalent to a Neumann boundary condition with the value of flux set to zero (i.e., \( \text{Flux}_C b = \text{Flux}_V b = 0 \)).

The above linear algebraic equations can be solved by SIMPLE (Semi-Implicit Method for Pressure Linked Equations) algorithm. The solution process of SIMPLE algorithm is shown in figure 8:

### 3. Experimental setup

The experiment is designed to determine mass flow rate, pressure drop, and temperature across the porous graphite schematically shown in figure 9(a). The air dryer is used to remove the moisture in the compressed air. The air source triple piece is used to further filter impurities in the air and control the air mass flow rate in the test section. The gas tank is used to stabilize the air pressure at the front of the test section.

The test section is an aluminum block shown in figure 9(b). To prevent air leakage, porous graphite was bonded with a thin coating of T1401 adhesive to the internal surface of the test channel. A small step was

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**Table 2. Instruments used in the experiment.**

| Instruments          | Model    | Accuracy  |
|----------------------|----------|-----------|
| Mass flow meter      | M-250SLPM| ±0.6%     |
| Thermocouples        | PT100A   | ±0.2°C    |
| Pressure Differential| 3051DP4  | ±0.075%   |
machined to avoid the porous graphite falling in the condition of high pressure. The adhesive bonding is very quick, which greatly reduces the possibility of adhesive intrusion through the graphite pore. Two thermocouples allow the temperature to be measured before and after the porous graphite. A pressure differential transmitter was employed to measure the steady-state unidirectional pressure drop. For each pressure drop, the mass flow rate was measured using a laminar differential flow meter. The accuracy of the main instruments used in the experiment is shown in table 2.

The materials used to determine the permeability and inertia coefficient are shown in figure 10(a). There are three types of porous graphite labeled by SG-3, SG-5, and SG-8. SG-5 and SG-8 are called isostatic graphite. Compared with other manufacturing techniques, isostatic graphite has good isotropy. SG-3 was manufactured by pressure molding. To facilitate the experiment, the original porous graphite was machined to the diameter of 40, 45, and 50 mm with a tolerance of ±0.1 mm. The thickness of all samples was machined to 5 mm. The test samples are shown in figure 10(b). The remaining porous graphite parameters are provided in table 3.

Table 3. Porous graphite parameters.

| Sample No. | Porosity (%) | Volume density (g/cm³) | Elastic modulus (Gpa) |
|------------|--------------|------------------------|----------------------|
| SG-3       | 16           | 1.75                   | 9                    |
| SG-5       | 13           | 1.85                   | 11.8                 |
| SG-8       | 8            | 1.91                   | 12                   |

Figure 11. Experimental system of Micro-CT.

Figure 12. Schematic representation of PPSM calculation, (a) grayscale image slices of porous graphite, (b) binary image slices, white regions denote pores, (c) isolated pore of slices labeled by a yellow rectangle box.
4. Results and discussion

PPSM (number of pores per square millimeter) is used to characterize the pore microstructure. A pore is defined as an isolated pore region on the two-dimensional slice of x-ray image. The x-ray scanning experiment is shown in figure 11. The samples were scanned with an x-ray set to 60kV voltage and a current of 50 μA at the target. During a rotation of 360° of sample stage, 1440 projections were obtained, corresponding to a 0.25° step-size. The scanning resolution is 1 μm.

The calculation process of PPSM is shown in figure 12. Firstly, CT images are denoised and cut into 6003 sizes, as shown in figure 12(a). Next, threshold segmentation is performed on the images to obtain the pore structure, as shown in figure 12(b). Finally, the isolated recognition is carried out on the slices, and the number of isolated regions on each slice is counted. Each isolated region is regarded as a pore.

According to table 4, SG-3 and SG-8 have similar PPSM values.

Table 4. Permeability and inertia coefficient of the porous graphite.

| Sample No. | Average PPSM (1/mm²) | Porosity (%) | Diameter (mm) | Equivalent diameter (mm) | Permeability (m²) | Inertia coefficient (1/m⁴) |
|------------|-----------------------|--------------|---------------|--------------------------|------------------|--------------------------|
| SG-3       | 680                   | 16           | 40            | 34.9                    | 6.50 x 10⁻¹⁴     | 1.02 x 10⁶              |
|            |                       |              | 45            | 39.94                   | 4.62 x 10⁻¹⁴     | 1.99 x 10⁶              |
|            |                       |              | 50            | 45.47                   | 6.10 x 10⁻¹⁴     | 1.07 x 10⁶              |
| SG-5       | 1187                  | 13           | 40            | 36.15                   | 3.65 x 10⁻¹⁵     | 2.3 x 10¹⁰             |
|            |                       |              | 45            | 41.66                   | 3.67 x 10⁻¹⁵     | 1.22 x 10¹⁰             |
|            |                       |              | 50            | 46                      | 3.65 x 10⁻¹⁵     | 1.25 x 10¹⁰             |
| SG-8       | 718                   | 8            | 40            | 36.12                   | 1.79 x 10⁻¹⁵     | 2.1 x 10¹⁰             |
|            |                       |              | 45            | 40.72                   | 1.91 x 10⁻¹⁵     | 3.13 x 10¹⁰             |
|            |                       |              | 50            | 45.78                   | 1.86 x 10⁻¹⁵     | 4.89 x 10¹⁰             |

4. Results and discussion

Figure 13 show the mass flow rate versus pressure drop for the three types of porous graphite. It can be observed that the relationship between the mass flow rate and the pressure drop is nonlinear. With the increase of the pressure drop, the experiment mass flow rate increases non-linearly. Then, the experimental data was
rearranged according to the format of equation (9) and the coefficients of first and second-order terms are obtained from the fitting equation. The coefficients then were used to simultaneously calculate the permeability and inertia coefficient. The curve fitting results are shown in figure 14. The data of SG-3 can be fitted by a quadratic equation. The data of SG-5 and SG-8 seem almost linear. But the quadratic equation also can be used to fit the data with higher $R^2$ compared with linear equation fitting. However, this will cause large fluctuations in the inertia coefficient. The permeability and inertia coefficient determined by the compressible Darcy-Forchheimer equation for each sample are shown in table 4. The three sizes of the test samples cut from SG-5 and SG-8 material have almost the same permeability respectively. The inertia coefficient shows significant variations. The difference in the inertia coefficient seems to occur due to the curve fitting uncertain. The permeability of SG-3 varies greatly, indicating that the uniformity of SG-3 is poor. Furthermore, we found that SG-3 was characterized by aerodynamic noise when the pressure drop is large. The difference in permeability of the same material is caused by poor material consistency.

In the following section, CFD simulation at the macro scale was performed to verify the permeability and inertia coefficient obtained by the experiment fitting method. The simulation mesh is shown in figure 2(b). The mesh is structured with good quality, which can ensure the convergence of numerical simulation. A comparison of mass flow rate between the experiment and CFD simulation can be seen in figure 15. The errors between CFD and experimental mass flow rate are within 10%. The mass flow rate of the CFD simulation is always larger than that of the experiment. Figure 13(a) shows that the SG-3 cure of mass flow rate versus pressure drop is close to a straight line. The SG-5 and SG-8 cures are highly nonlinear. This is mainly due to the small inertia coefficient of SG-3 and large inertia coefficient of SG-5, SG-8. The influence of inertia coefficient on mass flow rate is second order, which will lead to the bending of the curve. Viscous drag and inertial drag always exist inside porous media. With the increase of pressure drop, the influence of the inertial drag increases gradually, resulting in greater curve bending.

When the permeability of porous graphite is very low (such as SG-5 and SG-8), the inertia coefficient is difficult to accurately determine. Therefore, it is necessary to discuss the influence of the inertia coefficient on the mass flow rate. SG-8 is taken as an example. The permeability is set to $1.86 \times 10^{-15}$ m² and the inertial
The inertia coefficient is set to be \(4.89 \times 10^{10}\), \(3.13 \times 10^{10}\), and \(2.1 \times 10^{10}\) m\(^{-1}\) respectively. The SG-8 porous graphite used in CFD simulation has a diameter of 50 mm and a thickness of 5 mm. Then the changes in the mass flow rate can be observed in Figure 16. It can be observed that the mass flow rate change is only 4% while the inertia coefficient changes by more than 100%. Since the velocity of the flow through low permeability porous graphite is very small, the influence of the inertia term is small as well. In the practical application, the inertia coefficient

Figure 15. Comparison of mass flow rate between experiment and CFD simulation, (a) SG-3, (b) SG-5, (c) SG-8.

Figure 16. The influence of inertia coefficient on the mass flow rate.
can be measured multiple times with different sizes of samples from the same material. The average value can be taken to represent the porous graphite.

Some literature \cite{30,47} suggest that the permeability within the limits of \(3.13 \times 10^{-15} \text{ m}^2\) to \(8.44 \times 10^{-14} \text{ m}^2\) is appropriate for porous media air bearing (PMAB). On the other hand, some of the existing investigations \cite{48,49} suggest that the permeability of porous graphite in the order of \(10^{-15} \text{ m}^2\) is usually adopted. Through experimental investigation conducted in this paper, it is suggested that the permeability of porous graphite for the PMAB should be less than \(10^{-14} \text{ m}^2\).

The pore-scale finite volume method (FVM) also was used to calculate the permeability. The pore microstructure images were obtained by the Micro-CT as shown in figure 17. The porosity comparison between CT image and experiment is shown in table 5. We can see that the porosity obtained by CT provides close approximations to the experiment. Gas is considered incompressible when the pressure drop is small. Therefore, the equations used for numerical simulation are conservation of mass and momentum as shown in equations \cite{39}. To reduce the computational complexity, the momentum equation is simplified. Finally, Darcy’s law was used to calculate the permeability based on the flow rate obtained by the FVM simulation. The permeability comparison between FVM and the method proposed in this paper is shown in figure 18. As can be

\begin{table}[h]
\centering
\begin{tabular}{c|c|c}
\hline
Sample & CT & Experiment \\
\hline
SG-3 & 16.7\% & 16\% \\
SG-5 & 12.8\% & 13\% \\
SG-8 & 7\% & 8\% \\
\hline
\end{tabular}
\caption{Porosity comparison between CT and experiment.}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure17.png}
\caption{3D pore microstructure of the porous graphite. The colorful region is pore and the other region is solid matrix, (a) SG-3, (b) SG-5, (c) SG-8.}
\end{figure}
seen from figure 16, the permeability estimated by the pore-scale FVM and Compressible-Darcy-Forchheimer has a big difference. This is mainly because the scanning resolution is large than the pore size, which makes CT unable to capture sub-micro pores and also makes image segmentation difficult.

\[ \nabla \cdot U = 0 \]
\[ \mu \nabla^2 U - \nabla P = 0 \]
\[ \frac{Q}{S} = \frac{K \Delta P}{\mu L} \]

5. Conclusions

Porous graphite holds considerable promise for the development of ultraprecision air bearing. The results of the current study contribute the determination of permeability and inertia coefficient of porous graphite. We find that the compressible Darcy-Forchheimer equation can fit the mass flow rate and pressure drop very well. CFD is a useful tool that can be used to verify the permeability and inertia coefficient obtained by the fitting method. The average permeability of SG-3, SG-5 and SG-8 porous graphite are \(5.74 \times 10^{-14} \text{ m}^2\), \(3.65 \times 10^{-15} \text{ m}^2\) and \(1.85 \times 10^{-15} \text{ m}^2\) respectively. SG-5 and SG-8 porous graphite have good permeability consistency and can be used to make porous media air bearings. The PPSM of SG-3 and SG-8 are similar, but the permeabilities are very different. For low permeability porous graphite, samples with different sizes from the same material should be tested and the averaged inertia coefficient can be used. The accuracy of the pore scale FVM is highly dependent on the quality of the pore microstructure image. Further work will include the use of SG-5 and SG-8 to make porous graphite air bearings, and the capacity performance study.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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