A natural cure for causality violations in Newton–Schrödinger equation

Filippo Maimone¹, Giovanni Scelza¹ and Adele Naddeo²,³

¹Associazione Culturale ‘Velia Polis’, via Capo di Mezzo, I-84078 Vallo della Lucania, Italy
²INFN, Sezione di Napoli, C. U. Monte S. Angelo, Via Cinthia, I-80126 Napoli, Italy
³E-mail: filippo.maimone@gmail.com, lucasce73@gmail.com and anaddeo@na.infn.it

Received 2 November 2018, revised 15 January 2019
Accepted for publication 30 January 2019
Published 15 April 2019

Abstract
It is explicitly shown that a one-family parameter model reproducing the nonlinear Newton–Schrödinger equation as the parameter goes to infinity is free from any causality violation problem for any finite value of it. This circumstance arises from the intrinsic mechanism of spontaneous state reduction of the model, absent in the Newton–Schrödinger limit. A specific ideal EPR experiment involving a superposition of two distinct CM’s position states of a massive lump is analyzed, showing recovered compatibility with quantum mechanics. Besides, the new framework suggests a soft version of the many worlds interpretation, in which the typical indiscriminate proliferation of Everett branches, together with the bizarre inter-branches communications made possible by nonlinearity, are strongly suppressed in the macroscopic world by the same mechanism of state reduction.

Keywords: Newton–Schrödinger equation, causality violation, nonunitary Newtonian gravity

(Some figures may appear in colour only in the online journal)

1. Introduction

Causality principle lies the foundations of all the science [1]. That a cause precedes its effect is strictly linked, in special relativity, to the impossibility of superluminal communications (no-signaling condition). In quantum mechanics (QM) this principle is ensured by the kinematic structure of the theory and by the linearity of the dynamics, a circumstance known as ‘peaceful coexistence between QM and special relativity’. As a consequence, entanglement between two space-like separated regions cannot be exploited for superluminal communication. This consideration led some authors to include the no-signaling condition in the basic set of axioms of QM [2]. Recently a further generalization has been introduced as well, known as the principle of information causality: it states that there cannot be more information available than was transmitted [3].

Notwithstanding, the issue of a possible modification of non-relativistic QM in a nonlinear sense has a long-standing history, starting from Wigner’s suggestion [4]. The motivation for this search is twofold. On one side, there was a need to understand whether linearity is a foundational aspect of QM or should be considered just an approximation, though a very good one. On the other side, the measurement problem, and the seemingly subjective border separating quantum and classical domain, led people to explore nonlinear modifications of the Schrödinger equation. This subject, however, should not be confused with the issue of the nonlinear versions of the Schrödinger equation considered on completely different grounds. In fact there exists a vast literature on what is called ‘nonlinear Schrödinger equation’, as the famous Gross–Pitaevskii equation describing Bose–Einstein condensates, which has to be considered as a mean field limit in the framework of the standard theory and, at the best, only a test-case in nonlinear dynamics.

Despite some serious efforts to introduce nonlinear corrections [5] assuming that the observables include all the usual linear Hermitian operators, stringent limitations on the allowed nonlinearities became soon evident on the basis of the no-signaling condition [6, 7]. Nowadays this latter is widely recognized to be a very powerful condition. In fact, for example, it can be used alone to find the maximum fidelity in the copy of quantum states [8].

Interestingly, as Polchinski showed [7], while one can construct a theory free from causality problems by means of the stringent requirement that the Hamiltonian of each subsystem
depends (in the formalism of [5]) only on its density matrix, unusual (Everett) communications take place among the different branches of the wave function. That is, the state of a system at any given instant of time and in a specific world (or mind!) depends also on what happened or is happening in some other worlds (minds). Incidentally, this circumstance could be used, in principle, to devise an experiment to test the existence of parallel worlds, independently of a detailed knowledge of human brain perception’s physiology (required, for example, in the conceptual experiment discussed by Deutsch [9]).

We note that these foundational problems of QM are not isolated, but appear in close connection with the still elusive theory of quantum gravity. In particular, the way in which gravitational fields are produced by quantum matter is still controversial. Indeed it is not even clear if gravity has to be quantized at all, in which case the existence of a quantum superposition of space-times is implied (see [10] for some related conceptual problems); or gravity is intrinsically classical, and should be treated consequently. A natural candidate for this latter hypothesis is the semi-classical gravity, which (at least in its simplest form) prescribes to take as the source of the field (appearing on the right hand side of the Einstein field equation) the expectation of the quantum energy-momentum tensor. Its Newtonian limit, the famous Newton–Schrödinger equation, turns out to be a nonlinear quantum mechanical equation [11, 12]. While showing some interesting features, such as the existence of self-localized stationary solutions [13], this equation is plagued by the causality problems mentioned above [11].

To give a further chance to semiclassical gravity, various attempts have been made, inspired and connected with the phenomenological collapse models, to add ad hoc stochastic terms to the Newton–Schrödinger term in such a way that non-classical (delocalized) states become unstable, rapidly collapsing to well-localized states [11, 14, 15].

In the present work we take a somehow different viewpoint, by looking at the Newton–Schrödinger equation as a mean field approximation of more fundamental quantum mechanical equations, technically in a quite similar way in which Gross–Pitaevski nonlinear equation is derived from standard QM. In particular, a single-particle $N$–$S$ equation is regarded as the mean field approximation of an equation of $N$ identical copies of the particle, gravitationally interacting among them, as $N$ tends to infinity. Indeed, it was shown that the limit of $N \to \infty$ gives back the Newton–Schrödinger equation [16]. More specifically, within this model, physical observations are referred to only one of these particles, while the remaining ones are considered to belong to an hidden system. The $N$ particles interact uniquely via gravitational interaction, while the global state of the system is constrained to be symmetric with respect to particles state permutations. (Incidentally, it is interesting in this respect the observation by Adler pointing to an interpretational problem with particles self-interaction within the Hartee approximation of the Newton–Schrödinger equation [17].) The evolved physical state is obtained by tracing out the unobservable degrees of freedom of the $N - 1$ particles (see appendix B for a self-contained second-quantization general formulation of the model).

The model, known as nonunitary Newtonian gravity (NNG, from now on), has been studied in some detail, in particular the limit $N = 2$, showing the interesting property of (entropic) dynamical self-localization for masses above the (sharp) threshold of $10^{11}$ proton masses ($m_p$ from now on), with precise signatures susceptible to future experimental tests [18–20].

Since generally pure states evolve into mixed states even for isolated systems [18, 21], the fundamental description of physical reality cannot be associated to the wave function. Instead, density matrix has to be considered as a fundamental description of the nature (see, concerning this last point, the latest approach of Weinberg to the foundations of QM [22]).

While it can be inferred directly by the well-posedness of the model that it is free from causality problems, in the following we will show this fact explicitly, unfolding its physical basis. In particular we will elucidate how the basic mechanism operates and guarantees the no-signaling condition. It turns out that this mechanism is strictly related to the dynamical mechanism of state reduction naturally embedded within the model and completely suppressed in the limit $N \to \infty$ (see appendix B.1 for an explicit demonstration).

This is the same effect operating in the stochastic versions of Newton–Schrödinger, but here it emerges naturally and is not an ad hoc prescription, while state reduction appears to be a necessary built-in consequence of the no-signaling condition.

The plan of the present work is as follows. In section 2, we present in the simplest possible setting the Newton–Schrödinger limit, helping to elucidate its physical rationale. Then, in section 3 we analyze a specific EPR situation, showing how in the simplest $N = 2$ case the oddities of causality violations are effectively cured. In section 4, we show how a communication among Everett worlds is instead possible, though strongly limited by the model dynamics itself. Some concluding remarks, in particular on a variant of the Everett many worlds interpretation suggested by our results, end the paper.

2. Newton–Schrödinger limit

Let us begin by seeing how a superposition of states looks like within the model. Consider in the ordinary QM setting a superposition of two states of a body corresponding to two different locations of its CM. Its representation in the theory goes as follows. Given $p, q > 0$, with $p + q = 1$, the CM state of the system and of its hidden counterparts, i.e. the metastate, is

$$
|\Psi^{(N)}\rangle = \otimes_{k=0}^{N} (\sqrt{p}|x^{k}\rangle + \sqrt{q}|y^{k}\rangle)
$$

$$
= \sum_{k=0}^{\infty} \sqrt{\binom{N}{k}} p^k q^{N-k} |x^{k}\rangle |y^{N-k}\rangle.
$$

(1)
where

\[ |x\rangle^{\otimes k} |y\rangle^{\otimes (N-k)} \equiv \frac{1}{\sqrt{N^p}} \sum_{p} P[x^1 \ldots x^k y^{k+1} \ldots y^N]. \]

(A proof of equation (1) can be found in appendix A.)

Now, starting from the normalization

\[ \langle \Psi^{(N)} | \Psi^{(N)} \rangle = \sum_{k=0}^{N} \binom{N}{k} p^k q^{N-k}, \]

for \( N \) sufficiently large the binomial term can be approximated by a gaussian

\[ B_p(k, N) \sim N(Np, Np(1-p)), \]

and, putting \( \alpha = \frac{k}{N} \), we get

\[ \langle \Psi^{(N)} | \Psi^{(N)} \rangle \rightarrow \int_{0}^{1} \frac{1}{\sqrt{2\pi p(1-p)}} e^{-\frac{(\alpha - p)^2}{2p(1-p)}} d\alpha \]

\[ \rightarrow \lim_{N \to \infty} \int_{0}^{1} \delta(\alpha - p) d\alpha. \] (2)

Then, in the limit \( N \to \infty \) the only surviving contribution of the original sum is that with \( k = Np \), i.e. the huge superposition of states reduces to the single (central) term:

\[ \Psi^{(N)} \rightarrow \Psi^{\infty} \propto \lim_{N \to \infty} |x\rangle^{\otimes N} |y\rangle^{\otimes (N-1-p)}. \]

In other words, the superposition reduces to the state in which a fraction \( p \) of meta-matter is displaced to \( x \) position and the remaining part to position \( y \).

3. Analysis of an ERP situation

Following the simple argument given in [14], let us consider a sphere of matter of ordinary density of radius \( R \), and let the state \( |Z\rangle \) denote the sphere with center on the \( z \)-axis at \( z = Z \), and suppose the state of the sphere is, within the ordinary QM setting, \((|+Z\rangle + |-Z\rangle)/\sqrt{2}\). A probe mass moving along the \( x \)-axis will, according to the non-relativistic quantum theory of gravity, become entangled with the state of the sphere, resulting in the state vector \((|+Z\rangle |up\rangle + |-Z\rangle |down\rangle)/\sqrt{2} \), where \(|up\rangle \) (|down\rangle) means that the probe is deflected in the positive (negative) \( z \)-direction. According to semi-classical gravity, the probe mass should be undeflected. This was experimentally tested under the hypothesis that QM continues to hold in the macroscopic domain [23], with the (not unexpected) result that the mass is deflected. Indeed, semi-classical gravity implies precisely a modification of QM in this domain, so the hypothesis is self-contradictory.

Instead, a serious theoretical objection to semi-classical gravity is, as said above, that it allows superluminal communication and then causality violation. To see this, consider the entangled state \((|+Z\rangle |0\rangle + |-Z\rangle |1\rangle)/\sqrt{2} \), where the state \(|0\rangle \) and \(|1\rangle \) denote orthogonal states of a two state system (qbit) which is at a large distance from the sphere, but close to a `sender’. A probe mass is then used as before.

If the sender chooses not to measure the system, the ‘receiver’, who is close to the sphere and uses the probe mass as described above, finds it undeflected. If, on the other hand, the sender chooses to measure the system, thereby finding it to be in the state \(|0\rangle \) or \(|1\rangle \), the sphere will immediately be in the state \(|+Z\rangle \) or \(|-Z\rangle \) respectively. Then the receiver will be able to see this because the probe mass will now be deflected up or down.

Let us translate now this conceptual experiment into our model of replicas.

The entangled state of the sphere+q-bit is analogous to equation (1)

\[ \otimes_{m=1}^{N} \left( |+Z\rangle |0\rangle + |-Z\rangle |1\rangle \right)/\sqrt{2} \]

\[ = \sum_{k=0}^{N} \sqrt{\binom{N}{k}} \left( \frac{1}{2} \right)^{N/2} |+Z; 0\rangle^{\otimes k} - |Z; 1\rangle^{\otimes (N-k)}, \]

(3)

where the tensor product terms are defined in the same way as in equation (1).

Consider now the probe particle (supposed to have a mass much more smaller than the lump) shut just over the superposition. Including the probe in the system’s description, we have that an initially unentangled (global) state of the probe and of the composite sphere+q-bit system evolves towards an entangled one

\[ \otimes_{m=1}^{N} |+Z; 0\rangle^{\otimes k} - |Z; 1\rangle^{\otimes (N-k)}, \]

\[ \times \otimes_{m=1}^{N} |+Z; 0\rangle^{\otimes k} - |Z; 1\rangle^{\otimes (N-k)}, \]

(4)

where \(|\phi_{N}^{(N)}\rangle = \otimes_{m=1}^{N} |\phi_{N}^{m}\rangle \) is the product of identical copies of the probe’s state (it is assumed that probe’s mass is so small that gravity-induced internal entanglement among copies is irrelevant; see the example of the particle in Earth’s gravitational field in [18]). The meaning of the index \( k \) is clarified in figure 1.

For simplicity let us consider the simplest case with \( N = 2 \), for which we have only three branches in the superposition (downward, central and upward trajectories). Generalization to a generic \( N \) is straightforward.

\[ \text{Figure 1. Seen from above, deflection branches of the probe particle passing over the massive sphere in superposition of equation (4).} \]
First of all, observe that in order to detect a whatever deflection in the probe particle’s trajectory, the size of the wave packets describing the particle’s states in the superposition should be smaller than the deflection itself. Moreover, their spreading along the path have to be taken into account before the position measurement of the particle’s position along the z-axis. Denoting with \( v_z \) the velocity of the particle, \( m \) its mass, and \( M \) the mass of the sphere, the time during which the sphere’s gravitational attraction is effective is of the order of \( T \sim R/v_z \). Then it should be

\[
\frac{GMT}{R^2} \gtrsim \frac{h}{mZ}.
\]

On the other hand, a peculiar dynamical signature of the models (studied in [18]; see also the calculation below in the limiting case \( N = 2 \)) is that, for a lump of mass \( M \) above a threshold of \( 10^3 m_p \), a superposition of the CM wave packets separated by a distance much smaller than the body’s size undergoes a rapid state reduction after a characteristic time

\[
\tau_r \sim \frac{\hbar G^{-1} M^{-5/3}}{\rho^{-1/3}} \quad \text{(see appendix B.1 for a self-contained derivation),}
\]

leading from the initial superposition to an ensemble of localized states (see footnote 3 on page 7). The additional condition for the detection of a significant deflection of the probe is that the measurement time should be smaller than this reduction time, which together with the constraint above gives the set of conditions that must be satisfied simultaneously:

\[
\begin{align*}
T &\gtrsim \left( \frac{\hbar}{m} \right)^{1/3} \left[ \frac{4}{3} \pi \rho \right]^{1/3} G M T^{1/3} \quad (5) \\
T &\ll \tau_r = \frac{\hbar G^{-1} M^{-5/3}}{\rho^{-1/3}}.
\end{align*}
\]

As said previously, the generalization to a generic \( N \) is straightforward. In other words, our conclusion can be stated by saying that gravity-induced state reduction is so rapid to forbid a sufficiently long measurement, which would otherwise permit a deflection discrimination with respect to the spreading of the wave packet. These conditions have been depicted in figure 2, from which it is clear that they cannot be satisfied together.

In other words the ‘peaceful coexistence’ between (deterministic) QM and special relativity indeed imply linearity (see for example [2]), unless a certain amount of predictability loss is present.

4. Everett phone

As mentioned above, one of the drawbacks of the introduction of nonlinearities in a theory with a properly restricted observational algebra is the appearance of bizarre communications among Everett branches of the wave function. This circumstance appears also within the nonlinear model described here. At variance with the other nonlinear modifications of the theory, in this case predictability loss strongly limits such communication possibilities as far as macroscopic (mass) cat states are involved, as we are going to show. Let us start by looking at the interaction among branches; then we turn to the possibility to use these interactions to construct, in principle, an Everett phone.

The existence of effective interactions among Everett branches can be immediately inferred by considering the following physical situation. Assume an initial superposition of localized states of an isolated lump of matter (whose mass we will assume just above the localization mass threshold). Branches or worlds independence means that, following the evolution of each of the states independently from each other and, then, forming the final superposition of the evolved states at some later time, is equivalent to considering the time evolution of the global state from the beginning. This is clearly not true within the present model, since the state reduction dynamics rely precisely on the existence of all the other states in the superposition, while the exact reduction time depends also on the spatial distribution of the localized states. (For example, a long cigar-shaped matter distribution would be reduced more slowly in comparison with a ball-like distribution occupying the same volume.)

Said in other words, the evolution of a superposition of localized states does not coincide with the superposition of the evolved localized states. This is a statement of the existence of interactions among the branches. Incidentally, we stress that Everett branches should not be confused with the copy/copies of the physical system, which represent just a useful and easy way to formulate the model.

Let us now illustrate, following Polchinski’s proof of principle, how an Everett phone could be constructed on the basis of the theory. Consider a two-level system \( S \) initially in the state \( |\Phi_0\rangle \) and a system \( M \), initially in the state \( |M_0\rangle \), that we can imagine composed in general of environment + recording device + (eventually) an observer’s mind.

Measuring the \( q \)-bit state in the \(|\{1\}, |0\}\rangle \) basis, we get

\[
|\Phi_0\rangle \otimes |M_0\rangle \longrightarrow \frac{1}{\sqrt{2}} (|1\rangle |M_1\rangle + |0\rangle |M_2\rangle).
\]

Calling \( \sigma_3 \) the third Pauli matrix, if we obtain 1, it is \( \rho' = \frac{1 + \sigma_3}{4} \); otherwise \( \rho' = \frac{1 - \sigma_3}{4} \). In this second case, the observer (or the recording automatic system) can follow one
of two actions: (a) nothing; (b) rotates the $q$-bit state into the $\hat{i}$ direction. We analyze these cases in the NNG representation, starting from the second case.

• **Case (b).**

We write our (meta-)state $|\psi(t)\rangle$ as

$$
\frac{1}{2} (|1\rangle M_1 + |i\rangle M_2) \otimes (|1\rangle \tilde{M}_1 + |i\rangle \tilde{M}_2)
$$

with

$$
\frac{1}{2} (|1\rangle |\hat{i}\rangle \langle \hat{i}| M_1 \tilde{M}_1 + |1\rangle |i\rangle \langle i| M_1 \tilde{M}_2
$$

and

$$
\frac{1}{2} (|i\rangle |\hat{i}\rangle \langle \hat{i}| M_2 \tilde{M}_1 + |i\rangle |i\rangle \langle i| M_2 \tilde{M}_2)
$$

$$
\longrightarrow \frac{1}{2} (|1\rangle |\hat{i}\rangle \langle \hat{i}| M_1 \tilde{M}_1 + |1\rangle |i\rangle \langle i| M_1 \tilde{M}_2)
$$

(Remember that $|i\rangle = \frac{1}{\sqrt{2}} (|1\rangle + |0\rangle)$.) Suppose that, at the beginning, state $|1\rangle$ was measured; in (7), part of the wave function not living in the branch of $M_1$ has to be disregarded. Writing the most general time-evolved relevant states in the form

$$
|1\rangle M_1; \tilde{M}_1 \rangle = \sum_{(\mu,\nu,\rho)} c_{(\mu,\nu,\rho)} (t) |\mu; \tilde{\mu}\rangle
$$

and

$$
|1\rangle M_1; \tilde{M}_2 \rangle = \sum_{(\mu,\nu,\rho)} c_{(\mu,\nu,\rho)} (t) |\mu; \tilde{\mu}\rangle
$$

the general expression for the evolved $q$-bit state is ($\tilde{S}$ is the hidden $q$-bit)

$$
\rho(t) = \text{Tr}_{M_1} \left[ \langle \psi(t) \rangle \langle \psi(t) \rangle \right] = \sum_{\alpha,\beta,\gamma,\delta} \langle \alpha, \beta, \gamma | \psi(t) \rangle \langle \psi(t) | \alpha, \beta, \gamma \rangle
$$

$$
= \frac{1}{4} \sum_{(\alpha,\beta,\gamma,\delta)} \left( c_{(\alpha,\beta,\gamma,\delta)}^{(1)} c_{(\alpha,\beta,\gamma,\delta)}^{(1)*} + c_{(\alpha,\beta,\gamma,\delta)}^{(2)} c_{(\alpha,\beta,\gamma,\delta)}^{(2)*} + c_{(\alpha,\beta,\gamma,\delta)}^{(1)*} c_{(\alpha,\beta,\gamma,\delta)}^{(2)} + c_{(\alpha,\beta,\gamma,\delta)}^{(2)*} c_{(\alpha,\beta,\gamma,\delta)}^{(1)} \right) |\tilde{\alpha}\rangle \langle \tilde{\alpha}|
$$

$$
= \frac{1}{4} \sum_{(\alpha,\beta,\gamma,\delta)} C_{\alpha,\beta,\gamma,\delta} |\tilde{\alpha}\rangle \langle \tilde{\alpha}|
$$

from which

$$
\langle \sigma_3 \rangle = \text{Tr} [\sigma_3 \rho(t)] = C_{1,1} - C_{2,2}
$$

• **Case (a).**

In this case our meta-state is

$$
\frac{1}{2} (|1\rangle M_1 + |0\rangle M_2) \otimes (|\hat{i}\rangle \tilde{M}_1 + |\hat{i}\rangle \tilde{M}_2).
$$

With a bit of algebra we can verify that the above expression remains formally unchanged, but with values of $c_{(\alpha,\beta,\gamma,\delta)}^{(2)}$ generally changed.

To summarize the above scheme, the observer measuring 0 at the beginning is able to send a bit of information (say, ‘a’ if he chooses action (a), ‘b’ if he chooses action (b)) to the observer who originally measured 1. In this way a procedure to communicate between two Everett branches is set up.

In order to ensure the consistency of our argument, we can verify that in the absence of gravitational interactions, cases (a) and (b) coincide. For this purpose it is convenient to use ‘mixed basis states’ (in which one factor of the product tensor is taken at the initial time as before, while the other is considered at time $t$) to represent the time evolved states, i.e.

$$
|1\rangle M_1; \tilde{M}_1 \rangle = \sum_{(\mu,\nu,\rho)} c_{(\mu,\nu,\rho)} (t) |\mu; \tilde{\mu}\rangle
$$

and

$$
|1\rangle M_1; \tilde{M}_2 \rangle = \sum_{(\mu,\nu,\rho)} c_{(\mu,\nu,\rho)} (t) |\mu; \tilde{\mu}\rangle
$$

Remembering that $0\langle \tilde{\mu} | M_1 \rangle = \langle \tilde{\mu} | M_2 \rangle = \hat{\delta}_{\mu,\nu} \delta_{\tilde{\mu},\tilde{\mu}}$, it is immediate to prove the identity of $\langle \sigma_3 \rangle$ for the two cases, given by

$$
\langle \sigma_3 \rangle = 2 \sum |\alpha| \langle \alpha | 1, \alpha, M_1 | \alpha \rangle^2 - |\alpha| \langle \alpha | 1, \alpha, M_1 | \alpha \rangle^2.
$$

It is important to note that, apart from formal consistency, the above result show that an Everett phone cannot work for truly microscopic systems.

As the last point concerning branch communications, it is easy to see that if the superposition $M_1 + M_2$ involves sufficiently spatially-separated massive systems, then the expectation values $\langle \sigma_3 \rangle$ for cases (a) and (b) become identical in a very short time, meaning that after that time Everett Universes’ communication possibilities are strongly suppressed.

Suppose, in fact, that $M_1$ and $M_2$ are approximate CM’s position eigenstates with a mass above threshold $m > 10^{11} m_p$ at a distance $\sim (m_p/m)^{1/2} cm$ from each other. As before, the dynamics in this situation lead to a rapid reduction of the state within the characteristic time $\tau_g$. For the sake of simplicity, let us suppose that the spin dynamics is slower than $\tau_g$; then, with respect to the case (b), the state $\frac{1}{\sqrt{2}} (|1\rangle M_1 + |i\rangle M_2)$ evolves in a time $t \gtrsim \tau_g$ to the almost diagonal state

$$
\rho(t) \approx \frac{1}{2} (|1\rangle M_1 + |i\rangle M_2) \langle 1 | M_1 | i \rangle \langle i | M_2 | 1 \rangle,
$$

from which, after tracing out system $M$, we see that there is a given probability to obtain 1 associated to $M_1$, and the complementary probability to obtain $\hat{i}$ associated to (mind, recording device, etc) $M_2$ without any ‘transfer’ from branch of $M_2$ to branch $M_1$ after a time $t \gtrsim \tau_g$.

In conclusion, to set-up an efficient Everett communication, we need macroscopic mass superpositions (because

---

4 By ‘superposition of two approximate position eigenstates’ we have meant, throughout the text, a superposition of two contiguous clusters of localized states; as a matter of fact, a necessary condition for a complete state reduction within the characteristic time $\tau_g$ is that the superposition is composed by a large number of localized states, of width $\sim (m_p/m)^{1/2} cm$ (see [19] for an explicit numerical simulation of this case). Otherwise, a superposition of two really separated localized states would lead to a rapid oscillation of coherences in the basis of positions. Indeed, for all practical purposes, for sufficiently massive bodies, nonunitary gravity acts to reduce the overall quantum state at the very beginning of the (unitary) process of superposition formation.
gravity is responsible for nonlinearity, the basic ingredient for Everett communication!). At the same time, the macroscopic nature of the required quantum superposition implies its rapid decay, putting a severe trade-off between efficiency and practical feasibility of the Everett phone.

5. Conclusions

We have explicitly shown how the well-known causality problems of the Newton–Schrödinger equation can be cured in a natural and satisfactory way at the price of introducing a certain amount of predictability loss into the theory. The character of this extra-level of indeterminism is such that it is completely irrelevant for microscopic systems, while prompting large entropy production when macroscopic bodies are involved. Being the ensuing theory, equivalent to NNG, a fully consistent QM model, we have shown in an explicit analysis of an ensemble of localized states through gravitational self-interaction. This has been accomplished by analyzing an (ideal) EPR-type experiment involving the superposition of two distinct CM position states of a massive body. It has been shown that this circumstance arises from the intrinsic mechanism of spontaneous state reduction of the model, which is completely suppressed in the Newton–Schrödinger limit. As a matter of fact, the present analysis can be considered as an independent argument for the introduction of NNG.

As to the experimental consequences of our calculations, one could not design an experimental setting to detect the average gravitational field of a whatever quantum superposition of localized states, even if semiclassical gravity is the correct description for all practical purposes. Turning it the other way around, one could not use semiclassical gravity to design an apparatus to send faster-than-light signals.

Besides, since within NNG density matrix plays a fundamental role, amounting to the most complete characterization of a physical system’s state, the Everett many world interpretation appears to be the most natural conceptual framework of that theory.

As in other approaches in which nonlinearity was introduced at a fundamental level in QM, the possibility of constructing an Everett phone connecting different branches of the wave function emerges, though the mechanism it is based on appears to be strongly inhibited. In fact the theory gets rid of the huge number of branches continuously forming by turning each time the macroscopic states superposition into ensembles of localized states through gravitational self-interaction.

The severe restriction to branching proliferation would be responsible for keeping the process confined within the microscopic-to-mesoscopic realm. To further clarify this point, let us consider the famous Schrödinger’s cat thought experiment in whatever of its countless versions. The main point concerning this thought experiment is that a microscopic system’s dynamics (described by the Schrödinger equation) is amplified up to the level of a macroscopic object (like a gun or a bottle of poison), and from it to a poor cat, turning the world split into two macroscopically well distinct branches: one in which the cat stands up happy and alive, and one in which the same cat is lying stretched out on the floor. Decoherence theory adds to this picture the participation of the surrounding environment in this branching, but the principle remains exactly the same. What happens when non-unitary gravity is acting? That as soon as the gun’s trigger (or the poison vapor) begins to form superpositions of mesoscopically different mass displacement states, the overall (cat+apparatus+mesoscopic system+surrounding environment) pure state is rapidly converted into an ensemble of classical-like states, meaning that each state has its own probability of occurrence. It is important to stress that this would not correspond to the subjective coarse graining, usually given by tracing out the environmental degrees of freedom. It would, instead, be associated with a fundamental coarse graining, the adjective ‘fundamental’ meaning that one cannot resort even in principle to a more complete description, regardless of how precise and technologically sophisticated his measuring apparatuses would be.

Notwithstanding, of course in the microscopic realm the branching of the wave function continues undisturbed, while some remote echoes of other parallel worlds could eventually take a role in the biological processes involved in the brain’s functioning.

As a final remark of moral order, a soft version of the Everett formulation like the one just described, if found to conform to physical reality, would presumably sound less horrible, and even psychologically/morally more acceptable than the quite disturbing notion of an infinite number of replicas of ourselves doing who knows what, who knows where.

Note added

During the completion of this paper we became aware of a work by S De Filippo, which also points out the ability of NNG model in avoiding superluminal communications [24].

Appendix A. Proof of equation (1)

Let us proceed by induction. For \( N = 1 \), equation (1) is trivially verified. Assuming that it is verified for \( N \), it is sufficient to prove that it is verified for \( N + 1 \), i.e. we have to prove the equivalence of

\[
\sum_{k=0}^{N} \binom{N}{k} p^k q^{N-k} |x^k\rangle \langle y|^{\otimes (N-k)}
\times \left( \sqrt{p} |x^{N+1}\rangle + \sqrt{q} |y^{N+1}\rangle \right)
\]

(A.1)
we introduce a color quantum number such a way that each couple of creation-annihilation operators, i.e. one or molecules, according to the energy scale. For notational simplicity, \( \psi \) denote the whole set of creation-annihilation operators, i.e. one couple per particle species and spin component. This Hamiltonian includes the usual electromagnetic interaction between particles of one and the same color, the resulting state space does not contain states that can distinguish between operators of different color. The only way to accommodate a faithful representation of the physical algebra within the metastate space is to restrict the algebra to that of \( \psi \) operators. Note that the resulting constrained theory is, by construction, a fully consistent QM theory.

**B.1. State reduction**

The evolution operator in the interaction representation mapping an initial physical state \( \rho(0) = \langle \Phi(0) | \Phi(0) \rangle \) into the evolved physical state \( \rho(t) \) can be written, according to [16, 24], as

\[
H_G = \sum_{\alpha=1}^{N} H[\psi^\alpha, \psi] - \frac{G}{N - 1} \sum_{j<k} m_j m_k \times \int \frac{d\alpha}{N} \int dy \psi_{\alpha,i}^* (x) \psi_{\alpha,i} (x) \psi_{\beta,j}^* (y) \psi_{\beta,j} (y), \quad \text{[B.1]}
\]

where here and henceforth Greek indices denote color indices, \( \psi_{\alpha,i,\gamma} \equiv (\psi_{\alpha,1,\gamma}, \psi_{\alpha,2,\gamma}, \ldots, \psi_{\alpha,N,\gamma}) \) and \( m_i \) denotes the mass of the \( i \)th particle species, while \( G \) is the gravitational constant. While the \( \psi_{\alpha,i} \) operators obey the same statistics as the original operators \( \psi \), we take advantage of the arbitrariness pertaining to distinct operators and, for simplicity, we chose them commuting with one another: \( \alpha = \beta \Rightarrow [\psi_{\alpha,i}, \psi_{\alpha,i}^*] = [\psi_{\beta,i}, \psi_{\beta,i}^*] = 0 \). The metaphysical state space \( S \) is identified with the subspace of \( \otimes_i F_\alpha \) including the metastate obtained from the vacuum \( \langle 0 \rangle = \otimes \alpha |0\rangle \) applying operators built in terms of the product \( \prod_{\alpha=1}^N \psi_{\alpha,i}^* (x_\alpha) \) and symmetrical with respect to arbitrary permutations of the color indices, which, as a consequence, for each particle species, have the same number of metaphysical states of each color. This is a consistent definition since the time evolution generated by the overall Hamiltonian is a group of (unitary) endomorphism of \( S \). If we prepare a pure \( n \)-particle state, represented in the original setting, excluding gravitational interactions, by

\[
|g\rangle \equiv \int d^nx \, g(x_1, x_2, \ldots, x_n) \psi_{1,1}^* (x_1) \psi_{1,2}^* (x_2) \ldots \psi_{1,n}^* (x_n) |0\rangle ,
\]

its representative in \( S \) is given by the metastate

\[
|g^{\otimes N}\rangle \equiv \prod_{\alpha=1}^N \left[ \int d^nx \, g(x_1, x_2, \ldots, x_n) \psi_{1,\alpha}^* (x_1) \psi_{1,\alpha}^* (x_2) \ldots \psi_{1,\alpha}^* (x_n) |0\rangle \right].
\]

As for the physical algebra, it is identified with the operator algebra of say the \( \alpha = 1 \) metaworld. In view of this, expectation values can be evaluated by previously tracing out the unobservable operators, namely with \( \alpha > 1 \), and then taking the average of an operator belonging to the physical algebra. It should be made clear that we are not prescribing an ad hoc restriction of the observable algebra. Once the constraint restricting \( \otimes_i F_\alpha \) to \( S \) is taken into account, in order to get an effective gravitational interaction between particles of one and the same color, the resulting state space does not contain states that can distinguish between operators of different color. The only way to accommodate a faithful representation of the physical algebra within the metastate space is to restrict the algebra to that of \( \psi \) operators.
\[ M(t) = \int [D[\varphi]] \prod_{\alpha} D[\dot{\varphi}_\alpha] D[\varphi'] \prod_{\alpha} D[\dot{\varphi}'_\alpha] \exp \left[ \frac{ic^2}{2\hbar} \int dt \, dx \, \varphi \nabla^2 \varphi \right. \]
\[ \left. - \sum_{\alpha} \dot{\varphi}_\alpha \nabla^2 \varphi - \varphi^2 \varphi' + \sum_{\alpha} \varphi'_\alpha \nabla^2 \varphi'_\alpha \right] \times N \bigotimes_{\alpha=2} \delta(\varphi(0)) \bigg] \bigg[ T^{-1} \exp \left[ \frac{2mc}{\hbar} \sqrt{\frac{\pi G}{N-1}} \sum_{\alpha=2}^N \int dt \, dx \, \varphi' \right. \right. \]
\[ \left. \times \frac{1}{2h} \int dt \, dx \, \varphi' \right] \left. + \varphi(x, t) \right| v(x, t) v(x, t) \left| \varphi_1(x, t) \right) \bigg] \]
\[ + \varphi'_1(x, t) \left| v(x, t) v(x, t) \right| \left| \varphi'_1(x, t) \right) \bigg] \]

(B.2)

Let us consider localized states \(|\zeta_j\rangle\), which are approximate eigenstates of the density operator \(\psi_1^\dagger(x, t) \psi_1(x, t) |\zeta_j]\) ≈ \(n(x - z) |\zeta_j]\) that are quasi-stationary, apart from a slow spreading proportional to \(N\) (associated to the center of mass \(N m\) spreading). Taking as initial state a superposition of a large number \(N'\) of localized states,

\[ |\Phi(0)\rangle = \frac{1}{\sqrt{N'}} \sum_{j=1}^{N'} |\zeta_j]\rangle, \]

where \(N'\) is the number of states in superposition.

We want to evaluate explicitly the matrix element

\[ \langle \zeta_1 | M(t) | \zeta_1 \rangle \]
\[ = \int [D[\varphi]] \prod_{\alpha} D[\dot{\varphi}_\alpha] D[\varphi'] \prod_{\alpha} D[\dot{\varphi}'_\alpha] \exp \left[ \frac{ic^2}{2\hbar} \int dt \, dx \, \varphi \nabla^2 \varphi \right. \]
\[ \left. - \sum_{\alpha} \dot{\varphi}_\alpha \nabla^2 \varphi - \varphi^2 \varphi' + \sum_{\alpha} \varphi'_\alpha \nabla^2 \varphi'_\alpha \right] \times N \bigotimes_{\alpha=2} \delta(\varphi(0)) \bigg] \bigg[ T^{-1} \exp \left[ \frac{2mc}{\hbar} \sqrt{\frac{\pi G}{N-1}} \sum_{\alpha=2}^N \int dt \, dx \, \varphi' \right. \right. \]
\[ \left. \times \frac{1}{2h} \int dt \, dx \, \varphi' \right] \left. + \varphi(x, t) \right| v(x, t) v(x, t) \left| \varphi_1(x, t) \right) \bigg] \]
\[ + \varphi'_1(x, t) \left| v(x, t) v(x, t) \right| \left| \varphi'_1(x, t) \right) \bigg] \]

(B.3)

\[ \left. \times T^{-1} \exp \left[ \frac{2mc}{\hbar} \sqrt{\frac{\pi G}{N-1}} \sum_{\alpha=2}^N \int dt \, dx \, \varphi' \right. \right. \]
\[ \times \frac{1}{2h} \int dt \, dx \, \varphi' \right] \left. + \varphi(x, t) \right| v(x, t) v(x, t) \left| \varphi_1(x, t) \right) \bigg] \]
\[ + \varphi'_1(x, t) \left| v(x, t) v(x, t) \right| \left| \varphi'_1(x, t) \right) \bigg] \]

Now, using the property of the multistep distributions for Multi(\(n, \mathbf{p}\)) ≈ \(\mathcal{N}(n \mathbf{p}, n \Sigma)\), where \(\mathcal{N}\) is the normal multivariate distribution and \(\Sigma = P - \mathbf{pp}^T\), with \(P\) being the
diagonal matrix whose diagonal is formed by the probability vector \( \mathbf{p} \), we can rewrite (B.3) as

\[
\frac{1}{N^N} \sum_{k_1, k_2, \ldots, k_N} \left( N - 1 \right)^{\frac{1}{N}} \left[ e^{\frac{A_{k_1}}{N^N}} \right] \left[ e^{\frac{A_{k_2}}{N^N}} \right] \cdots \left[ e^{\frac{A_{k_N}}{N^N}} \right],
\]

which, introducing the rescaled variables \( \chi_i \equiv \frac{k_i}{N-1} \) and passing from sum to integral, transforms into

\[
\frac{e^{iA_{\chi}}}{N} \int_{\chi_i > 0, \chi_i = 1, \ldots, \chi_N = 1} d\chi_1 \cdots d\chi_N \prod_{i=1}^{N-1} \left( \chi_i = \frac{1}{N} \right) \times 
\]

\[
\lim_{N \to \infty} \int_{\chi_i > 0, \chi_i = 1, \ldots, \chi_N = 1} d\chi_1 \cdots d\chi_N \delta\left( \chi_1 - \frac{1}{N} \right) \times 
\]

\[
\ldots \times \delta\left( \chi_{N-1} - \frac{1}{N} \right) \exp\left( i(A_{\chi} - A_{\chi})\chi_1 \right).
\]

Then

\[
\left| \langle \chi_{1}|M(\tau)|\chi_{2} \rangle \right| \to \frac{1}{N}, \quad \text{consistent with} \quad N \to \infty.
\]

while, in the same limit, the spreading time tends to infinity, from which we conclude that the mechanism of random phase cancelation, leading for finite \( N \) to a rapid decoherence of the superposition of localized states on one side, and the spreading of the wave function on the other side, are completely suppressed in the limit \( N \to \infty \).

It is worth noting that, for the simplest case of \( N = 2 \) treated in the main text, we get easily the expression for the characteristic state reduction time. Starting from the intermediate expression B.3 specialized for \( N = 2 \)

\[
\frac{1}{N^2} \sum_{j=1}^{N} \exp\left[ \frac{im^2G}{2\hbar} \right] \times 
\]

\[
\int dx \frac{[n(x - z_j)n(y - z_k) - n(x - z_j)n(y - z_k)]}{[x - y]},
\]

we see that phase cancelation occurs when exponentials reach values of order 1, which happens in a characteristic time \( \tau_g = \frac{IG^2M^{1/2}}{\rho^{1/3}} \). [18]. For example, for a very fine grain of sand, of mass \( 10^{-3} \) g., we have \( \tau_g \sim 10^{-10} \) s.

**ORCID iDs**

Adele Naddeo https://orcid.org/0000-0002-0723-9343

**References**

[1] Pearl J 2009 Causality: Models, Reasoning and Inference (Cambridge: Cambridge University Press)
[2] Simon C, Bužek V and Gisin N 2001 Phys. Rev. Lett. 87 170405
[3] Pawłowski M, Paterek T, Kaszkiel D, Scarani V, Winter A and Żukowski M 2009 Nature 461 1101
[4] Wigner E P 1961 Remarks on the mind-body questions The Scientist Speculates ed I J Good (London: Heineman) pp 284–302
[5] Weinberg S 1989 Ann. Phys., NY 194 336
[6] Gisin N 1989 Helv. Phys. Acta 62 363
[7] Polchinski J 1991 Phys. Rev. Lett. 66 397
[8] Bužek V and Hillery M 1996 Phys. Rev. A 54 1844
[9] Deutsch D 1986 Quantum Concepts in Space and Time ed R Penrose and C J Isham (New York: Oxford University Press) pp 215–25
[10] Penrose R 1996 Gen. Relativ. Gravit. 28 581
[11] Bahrami M, Großardt A, Donadi S and Bassi A 2014 New J. Phys. 16 115007
[12] Giuliani D and Großardt A 2012 Class. Quantum Grav. 29 215010
[13] Moroz I M, Penrose R and Tod K P 1998 Class. Quantum Grav. 15 2733
[14] Pearle P and Squires E 1994 Phys. Rev. Lett. 73 1
[15] Nimmrichter S and Hornberger K 2015 Phys. Rev. D 91 024016
[16] De Filippo S 2001 The Schrödinger–Newton model as \( N \to \infty \) limit of a N color model arXiv:gr-qc/0106057
[17] De Filippo S and Maimone F 2006 Quantum Gravity Research Trends ed A Reimer (New York: NOVA Science Publishers) pp 191–221
[18] Adler S L 2007 J. Phys. A: Math. Theor. 40 4
[19] De Filippo S and Maimone F 2002 Phys. Rev. D 66 044018
[20] De Filippo S, Maimone F and Robustelli A L 2003 Physica A 330 459
[21] Maimone F, Szelc G, Naddeo A and Pelino V 2011 Phys. Rev. A 83 062124
[22] Szelc G, Maimone F and Naddeo A 2018 J. Phys. Commun. 2 015014
[23] Weinberg S 2014 Phys. Rev. A 90 042102
[24] Page D N and Geiller C D 1981 Phys. Rev. Lett. 47 979
[25] De Filippo S 2018 Nonunitary Newtonian Gravity Makes Semiclassical Gravity Viable for All Practical Purposes arXiv:1807.10469