Influence of position and parameters of inhomogeneities on vortex structure in long Josephson junctions

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Abstract. Numerical experiment results on long Josephson junction with one and two rectangular inhomogeneities in the barrier layer are presented. We demonstrate the effect of the shifting of the inhomogeneity and the value of the Josephson current on the vortex structure. The disappearance of mixed fluxon-antifluxon states is shown when the position of inhomogeneity shifted to the end of the junction. A change of the amplitude of Josephson current at the end makes a strong effect on the stability of the fluxon states and smoothes the maximums of the dependence “critical current-magnetic field”.

1. Motivation and model
Variety of the applications of the long Josephson junctions (JJ) are related to its vortex structure. But the inhomogeneity can make a strong effect on the vortex structure and change it essentially. So, the investigation of the effect of the inhomogeneity and its parameters is very interesting and actual problem today.

Single inhomogeneity in the form of the narrow rectangle in the barrier layer of a Josephson junction with length $L$ is characterized by its width $\Delta < L$, localization of its center $\zeta \in [L - \Delta/2, L + \Delta/2]$ and portion of Josephson current $\kappa$ through it [1, 2]. The existence of the inhomogeneity leads to local change of the amplitude $j_C(x)$ of the Josephson current, which can be modelled by $j_C(x) = 1 + \kappa$ inside of the inhomogeneity and $j_C(x) = 1$ outside of it. When $\kappa > 0$ the inhomogeneity is of shunt type (SJJ), at $\kappa \in [-1, 0)$ it is a microresistor type (RJJ).

In order to investigate the possible fluxon states and their stability in inhomogeneous overlap JJ, we solve the following non-linear eigenvalue problem [3, 4]

\begin{align}
-\varphi_{xx} + j_C(x) \sin \varphi - \gamma &= 0, \quad (1a) \\
\varphi_x(0) - h_e &= 0, \quad \varphi_x(L) - h_e &= 0, \quad (1b) \\
-\psi_{xx} + [\lambda - j_C(x) \cos \varphi] \psi &= 0, \quad (1c) \\
\psi_x(0) &= 0, \quad \psi_x(L) &= 0, \quad (1d) \\
\int_0^L \psi^2(x) \, dx - 1 &= 0, \quad (1e)
\end{align}
with respect to the triplet \( \{ \varphi(x), \psi(x), p \} \) for given \( \lambda \). Here \( \varphi(x) \) represents the static magnetic distribution in JJ, \( h_e \) is the external magnetic field, \( \gamma \) — external current, \( p \) — one of the parameter \( h_e \) and \( \gamma \). The curve “critical current — external magnetic field” of JJ as whole consists of the pieces of bifurcation curves

\[
\lambda_{\text{min}}(h_e, \gamma) = 0,
\]

(2)
corresponding to different distributions \( \varphi(x) \) with critical current \( \gamma \) being maximal for given field \( h_e \) (see details in [3, 4]).

2. Shift of the inhomogeneity

Results of numerical solution of non-linear eigenvalue problem (1) are presented in Fig. 1 and Fig. 2, where we show the bifurcation curves (2) for different magnetic flux distributions \( \varphi(x) \) present in long \( (L = 12) \) homogeneous JJ. Here \( \Phi^n, n = \pm 1, \pm 2, \ldots \) denotes “pure” \( n \)–fluxon (antifluxon) state in the junction [4].

Figure 1. Bifurcation curves for long \( (L = 12) \) homogeneous \( (\Delta = 0) \) JJ

The existence of the attractive resistive inhomogeneity \( (\Delta = 0.7) \) situated in the junction center \( (\zeta = 3.5) \) leads (see Fig. 2 — left) to stabilization of mixed states like \( \Phi^n\Phi^m (n, m = \pm 1, \pm 2, \ldots, n \neq m \) and \( nm < 0 \) [4]. Note that in case of homogeneous JJ the mixed states are unstable [4]. The shift of the inhomogeneity from the center (Fig. 2 — right) demonstrates

Figure 2. Bifurcation curves for long \( (L = 12) \) inhomogeneous \( (\Delta = 0.7) \) JJ: left — with centered inhomogeneity \( (\zeta = 6) \), right — with shifted inhomogeneity \( (\zeta = 7) \).
a tendency to the disappearance of mixed states and to the monotonic decrease of maxima of bifurcation curves for pure fluxon states when the external magnetic field $h_e$ increases.

At sufficient length $L$ in contact except of main stable fluxon, pinned at the inhomogeneity, there exist right $\Phi_{r}^{\pm 1}$ and left $\Phi_{l}^{\pm 1}$ fluxons too. Some distributions of magnetic field $\varphi_x(x)$ along the RJJ for $\Phi_{l}^1$ at different position of inhomogeneity are shown in Fig. 3 (left). The shift of the inhomogeneity leads to the disappearance the bifurcation curve for displaced fluxon states, which is demonstrated in Fig. 3 (right). This effect follows from the decreasing of $[\zeta + \Delta/2, L]$-interval when $\zeta$ increase. For example, at $\zeta = 8$ the region of stability by change of $h_e$ is essentially smaller in compare with the case $\zeta = 6$.

![Figure 3](image1.png)

**Figure 3.** The distributions (left) and bifurcation curves (right) for $\Phi_{l}^1$ state at different position of inhomogeneity

Different position of the inhomogeneity makes effect on the bifurcation curves and distributions of magnetic field along the junction for states $\Phi_{l}^n$, $n > 1$, too. But as we can see in Fig. 4, in this case the influence of the shifting on fluxon state $\Phi_{l}^2$ at small $\zeta$ is not so crucial as for $\Phi_{l}^1$ — the “domain of existence” $[0, \zeta - \Delta/2]$ for $\Phi_{l}^2$ grows when $\zeta \to L$.

![Figure 4](image2.png)

**Figure 4.** Bifurcation curves (left) and distributions of magnetic field along the junction for $\Phi_{l}^2$ at different position of inhomogeneity

3. **Change of the Josephson current value**

The variation of the parameter $k$, which characterizes the amplitude of Josephson current through the inhomogeneity has a strong influence on the bifurcation curves of the fluxon states. For example, Fig. 5 (left) shows the bifurcation curves for $\Phi^1$ in SJJ at different values of this
Figure 5. Bifurcation curves for $\Phi^1$ in shunted JJ at different values of parameter $k$.

parameter. The curve 1 correspond to homogeneous JJ, 2 — to shunt type overlap JJ. It can be seen that the change of $k$ leads to the change of stability region of the $\Phi^1$ and the appearance of the so called created by current states (CbC states) [5].

4. Two inhomogeneities

The bifurcation curves for long enough JJ ($L = 12$) with single resistive inhomogeneity at the end of the junction are shown in Fig. 6 (left). The value of the Josephson current amplitude is $j_{C1} = 0.3$. It shows that even in the case when the inhomogeneity is in the end of JJ, the variation of the $j_c$ through it leads to the appearance of the CbC pure fluxon states. Nevertheless the maxima and minima of bifurcation curves corresponding to different states decrease monotonically as in case of homogeneous JJ (see Fig. 1) and all of possible mixed states are unstable.

Figure 6. Critical curves for the junction $L = 12$ with inhomogeneity at the end and $j_{C1} = 0.3$ (left) and for JJ with $L = 12$ with two inhomogeneities: one at the center with $j_{C1} = 0$ and another one at the end of junction with $j_{C2} = 0.3$ (right).

Fig. 6 (right) presents the critical curves for JJ with $L = 12$ with the strong inhomogeneity ($j_{C1} = 0$) at the center and the small one ($j_{C2} = 0.3$) at the end of the junction. It can be seen that the existence of small boundary inhomogeneity reduces the domains of stability for mixed states and thus smooths the critical curve of JJ out [2, 5].
We thank Prof. I.V. Puzynin (JINR, Dubna, Russia) and Prof. A. Irie (Utsunomiya University, Japan) for helpful discussions.

This research was partially supported by Russian Foundation for Basic Research, Grant 08-02-00520-a and by Sofia University Scientific foundation under Grant No 135/2008.

5. References
[1] E.G. Semerdjieva et al., Coordinate transformation in long Josephson junctions: geometrically equivalent junctions, *Physics of low temperatures* (Russian), vol. 31 (10), 2005.
[2] T.L. Boyadjiev et al., Equivalent Josephson junctions, *Technical Physics*, 2008, Vol. 53, No. 1, pp. 712.
[3] T.L. Boyadjiev et al., Newtonian algorithm for calculation of critical curves in one-dimensional inhomogeneous Josephson junction, Comm. JINR P11-88-409, Dubna, 1988.
[4] I.V. Puzynin et al., Methods of computational physics for investigation of models of complex physical systems, In *Physics of Particles and Nuclei*, 2007, Vol. 38, No. 1, pp. 7016.
[5] T.L. Boyadjiev, O.Yu. Andreeva, E.G. Semerdjieva and Yu.M. Shukrinov, Created-by-current states in long Josephson junctions, 2008 *EPL*, 83 47008.