Direct Numerical Simulation of Low and Unitary Prandtl Number Fluids in Reactor Downcomer Geometry

Cheng-Kai Tai,a* Tri Nguyen,b Arsen S. Iskhakov,a Elia Merzari,b Nam T. Dinh,a and Igor A. Bolotnov,a

aNorth Carolina State University, Department of Nuclear Engineering, Campus Box 7909, Raleigh, North Carolina 27695-7909
bThe Pennsylvania State University, Department of Nuclear Engineering, 206 Hallowell Building, University Park, Pennsylvania 16802-4400

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Abstract — Mixed convection of low and unitary Prandtl fluids in a vertical passage is fundamental to passive heat removal in liquid metal and gas-cooled advanced reactor designs. Capturing the influence of buoyancy in flow and heat transfer in engineering analysis is hence a cornerstone to the safety of the next-generation reactor. However, accurate prediction of the mixed convection phenomenon has eluded current turbulence and heat transfer modeling approaches, yet further development and validation of modeling methods is limited by a scarcity of high-fidelity data pertaining to reactor heat transfer. In this work, a series of direct numerical simulations was conducted to investigate the influence of buoyancy on descending flow of liquid sodium, lead, and unitary Prandtl fluid in a differentially heated channel that represents the reactor downcomer region. From time-averaged statistics, flow-opposing/aiding buoyant plumes near the heated/cooled wall distort the mean velocity distribution, which gives rise to promotion/suppression of turbulence intensity and modification of turbulent shear stress and heat flux distribution. Frequency analysis of time series also suggests the existence of large-scale convective and thermal structures rising from the heated wall. As a general trend, fluids of lower Prandtl number were found to be more susceptible to the buoyancy effect due to stronger differential buoyancy across the channel. On the other hand, the effectiveness of convective heat transfer of the three studied fluids showed a distinct trend against the influence of buoyancy. Physical reasoning on observation of the Nusselt number trend is also discussed.

Keywords — Direct numerical simulation, vertical mixed convection, low-Prandtl-number fluids, downcomer.

Note — Some figures may be in color only in the electronic version.

I. INTRODUCTION

Mixed convection is the flow regime where the buoyancy effect prevails in the flow, yet it is not merely linear superposition of forced convection and the buoyancy effect. Depending on the system thermal boundary conditions and orientation of the flow to gravity, the buoyancy effect could give rise to a great variety of sophistication in the flow. In the context of thermal-hydraulic advanced nuclear reactors, transition from a forced to a mixed convection regime would appear during, for instance, low-flow transients or accidental scenarios. Accurately capturing mixed convection behavior is therefore crucial to the safety of advanced reactor designs.

Computational fluid dynamics (CFD) simulation based on Reynolds-averaged Navier Stokes (RANS) formulation remains the workhorse for flow and heat transfer analysis in engineering systems owing to relatively less intense effort than with high-fidelity simulations,
such as direct numerical simulation (DNS) or large-eddy simulation (LES). With that being said, the credibility of RANS results largely hinges on the performance of the closures for Reynolds shear stress (RS) and turbulent heat flux (THF). Such practice is currently challenged by the incorporation of nonunitary Prandtl fluids, namely, liquid metals (low Prandtl) and molten salts (high Prandtl), in advanced reactors due to their utterly distinct heat transfer characteristics and mixed convection behavior from those of water.

One of the broadly applied measures is the simple gradient diffusion hypothesis (SGDH) coupled with Reynolds analogy and the unitary turbulent Prandtl number, \( Pr_t = \frac{\nu_t}{\alpha_t} \), where \( \nu_t \) and \( \alpha_t \) = turbulent eddy diffusivity and turbulent thermal diffusivity, despite its robustness in the cases of unitary Prandtl fluids, the breakdown of which is observed for both the forced and the mixed convection regimes of the nonunitary Prandtl fluids due to the underlying assumptions on the similarity between turbulent momentum and heat transfer.\(^{3,4}\) Underperformance of the current turbulence and heat transfer models hence spurred the exploration of higher-order modeling techniques, namely, algebraic heat flux models\(^5\) and the four-equation model,\(^6\) to accurately capture mixed convection flow and strong anisotropic THF. On the other hand, resorting to more sophisticated modeling approaches also requires an abundant supply of data on high-order flow behaviors.

Fundamental understanding of mixed convection has been previously studied in experiments of heated air\(^7\) and liquid metals (sodium-potassium,\(^8\) sodium)\(^9,10\) in a heated pipe. It was found that buoyancy would distort the mean velocity profile as well as modify the near-wall shear stress distribution. The resulting alternation in velocity fluctuation impaired/enhanced convective heat transfer in the buoyancy-aligned/opposed setup for airflow.

Recent advances in high-performance computing has made DNS an affordable option for investigating complicated mixed convection phenomena. DNS allows direct access to detailed flow behavior, such as distribution of higher-order statistics, that facilitates the development of modeling approaches. Existing DNS efforts have largely been devoted to the prototypical Poiseuille-Rayleigh-Bénard (PRB) flow (horizontal turbulent channel with unsteady temperature stratification created by isothermal wall of different temperature). For the widely studied PRB flow of air, the influence of buoyancy facilitates the eruption of plumes from the heated wall. Consequently, these plumes facilitate the emergence of a large-scale roller in the velocity field and thermal structures.\(^11,12\) The buoyancy effect also significantly promotes turbulence intensity on the wall-normal direction, which reportedly has an influence on the Nusselt number. With decreasing Prandtl number, the buoyancy promotion of the wall-normal turbulence fluctuation intensity and heat flux becomes more evident due to the strong thermal diffusivity of liquid metal. Also, large spatial scale eruption of hot plumes that reaches the opposite end can be clearly identified, signifying a stronger buoyancy effect.\(^13,14\)

However, high-fidelity data for mixed convection in a reactor heat transfer context remains scarce. The limited availability of data has hampered the development and validation of turbulence models. Despite more DNS research effort being dedicated to reactor-relevant mixed convection setups, such as mixed convection of lead-bismuth eutectic (LBE) over a vertical backward-facing step\(^14\) and vertical mixed convection of LBE over heated bare bundles,\(^15\) there remains a need for broader understanding of fundamental mixed convection behavior.

In this paper, we present DNS of low-flow mixed convection of liquid metals and unitary Prandtl fluid in a vertical turbulent channel. The studied problem is fundamental to passive heat removal in the reactor downcomer and is crucial for advanced reactor resilience to operational transients or accidents. This work is a part of the research campaign in the integrated research project (IRP) of the Nuclear Energy Advanced Modeling and Simulation (NEAMS) program.\(^16\) The rest of the paper is organized as follows. Section II outlines the problem specification and the numerical methods, and Sec. III discusses the results of the simulation, including analysis of flow regime transition as well as fully developed mixed convection. The work is concluded in Sec. IV.

## II. METHODOLOGY

### II.A. Problem Specification

We consider mixed convection of low and unitary Prandtl fluids in a long vertical turbulent channel, as shown in Fig. 1a. The dimension of the channel is \( (L_x, L_y, L_z) = (2\pi\delta, 2\delta, 240\delta) \), where \( \delta \) = half of the wall spacing, and \( x, y, \) and \( z \) = spanwise, wall-normal, and streamwise directions, respectively. The specified geometry is deemed an approximation to a sector of the annular downcomer region, whose radius is large in general. On the streamwise direction, the channel is divided into three subsections, namely, the upper and the lower adiabatic and the region with heat flux applied to the wall. The two adiabatic subsections \( (L_{z1}, L_{z2}) \) are used for numerical treatments and flow establishments, and the main research interest lies in the middle subsection \( (L_{z2}) \).
The velocity and temperature boundary conditions are summarized in Table I. The flow is driven by the isothermal velocity inlet at the top boundary of the channel, whose velocity profile is recycled from the cross section \( L_{z0} = 3D_h < L_{z1} \) downstream from the inlet boundary and normalized by its magnitude. This is to create sustainable inlet turbulence while conserving the Reynolds number. The stabilized outlet by Dong et al.\cite{17} is adopted to prevent breakdown of the simulation because of backflow. In between the two adiabatic sub-sections, the walls are applied with heat flux of the same magnitude but different signs, i.e., differential heating. This setup aims to mimic the situation when heat is removed from the core to the outside of the reactor pressure vessel, such as postshutdown decay heat removal. Details of the problem formulation will be further discussed in Sec. II.B.

### II.B. Governing Equations and Numerical Solver

The dimensionless incompressible Navier-Stokes (INS) equations and energy equations are solved in this study, as shown in Eqs. (1), (2), and (3):

**TABLE I**

Summary of the Velocity and Temperature Boundary Conditions

| Boundary         | Field Variables          | Velocity                      | Temperature                      |
|------------------|--------------------------|-------------------------------|----------------------------------|
| ±x               | Periodic                 | Periodic constant heat flux, \( \hat{q}_{wall}^* = \pm \frac{2}{Pe} \)  |
| ±y, heated section | No-slip wall             | Insulated, \( \frac{\partial T}{\partial z} = 0.0 \)                   |
| ±y, adiabatic section | No-slip wall             | Dirichlet, \( T^* = 0.0 \)               |
| z                | Recycling inlet, \( \tilde{u} = \frac{\tilde{u}(L_{z0})}{\tilde{u}(z=L_{z0})} \) | Stabilized outlet\cite{17}       | Insulated, \( \frac{\partial T}{\partial z} = 0.0 \) |
| +z               |                          |                                |                                  |
\[ \nabla \cdot \mathbf{u}^* = 0 \]  
\[ \frac{\partial u^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla u^* = -\nabla p^* + \nabla \cdot \frac{1}{Re} \nabla u^* + Ri_q T^* \]  
\[ \frac{\partial T^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla T^* = \frac{1}{Pe} \nabla \cdot \nabla T^* \]  
\[ T^* = \frac{T - T_{ref}}{\Delta T_{ref}} \],

where \( \mathbf{u} = (u, v, w) \) = velocity vector, \( t = \) time, \( p = \) pressure, 
\( Re = \frac{|u| D_h}{v} \) = Reynolds number, \( D_h = 2L_v = 4\delta \) = hydraulic diameter, \( v = \) kinematic viscosity, \( Ri_q = \frac{G_m}{Re} \) = modified Richardson number, \( Gr_q = \frac{g \delta \Delta T_{ref} D_h^3}{v^2} \) = modified Grashof number, \( g = \) gravitational acceleration, \( \beta = \) coefficient of thermal expansion, \( \Delta T_{ref} = \) reference temperature difference between the heat sink and the heat source, \( T = \) temperature, \( Pe = RePr = \frac{\text{Péclet}}{\alpha} \) = Prandtl number, and \( \alpha = \) thermal diffusivity.

The adoption of the modified definition is in response to the fact that the temperature gradient is fixed on the walls but the temperature itself. In other words, wall temperature is part of the solution, and hence, the conventional definition of the Gr is not applicable here. The temperature difference between the heat source and the sink is hence expressed as a function of the heat flux:

\[ Gr_q = \frac{g \delta \Delta T_{ref} D_h^3}{v^2} = \frac{g \beta \phi \Delta T_{ref} D_h^3}{2 \lambda_{ref} v^2} \]

\[ q_{wall}^* = \frac{\Delta T_{ref}}{\Delta T_{ref}} \phi \]

where \( \lambda = \rho c_p \alpha = \) thermal conductivity of the fluid, \( \rho = \) fluid density, and \( c_p = \) isobaric specific heat. Equation (6) shows that the reference temperature difference \( \Delta T_{ref} \) is essentially the conduction limit, i.e., the temperature difference when convective heat transfer is excluded. The reference scales (with subscript \( \phi_{ref} \)) for the problem nondimensionalization are summarized in Table II. Note that for an arbitrary quantity \( \phi \), its nondimensional form (with asterisk) is related to the dimensional one by \( \phi^* = \phi \phi_{ref} \).

Combining Eq. (6) and \( q_{wall}^* \), the dimensionless wall heat flux can be expressed as

\[ q_{wall}^* = \frac{q_{wall}}{(\rho c_p)_{ref} u_{ref} \Delta T_{ref}} = \frac{2 \lambda_{ref}}{(\rho c_p)_{ref} u_{ref} L_{ref}} \]

\[ = \frac{2}{RePr} = \frac{2}{Pe} \].

The governing equations are solved using the open-source Navier-Stokes solver NekRS maintained by Argonne National Laboratory,[18] NekRS, as well as its predecessor Nek5000,[19] is based on the spectral element method,[20] featuring high-order accuracy and low numerical diffusion. NekRS is the GPU-oriented version of Nek5000 that can leverage the accelerators in modern high-performance computing facilities and achieve considerable speedup compared to CPU-only runs. Both NekRS and Nek5000 have been broadly applied and validated in CFD simulations for thermal-hydraulic analysis of advanced reactors, such as LES of a bare and wire-wrapped bundle,\[21\] modeling bypass flow between reflectors of molten salt pebble bed reactors,[22] and full SMR core RANS simulation with mixing vein-representing momentum source.[23]

In NekRS, the computational domain is discretized into second-order hexahedral spectral element \( N_e \), within which the solution is represented by the expansion of the Lagrange polynomial of order \( P_N \) based on the \((P_N + 1) \)

| TABLE II | Reference Scales for Problem Nondimensionalization |
|---|---|
| Reference velocity, \( u_{ref} \) (m/s) | Bulk velocity, \( u_{bulk} \) |
| Reference length scale, \( L_{ref} \) (m) | Hydraulic diameter, \( D_h \) |
| Reference timescale, \( t_{ref} \) (s) | \( L_{ref} / u_{ref} \) |
| Reference temperature, \( T_{ref} \) (K) | Inlet temperature, \( T_{in} \) |
| Reference density, \( \rho_{ref} \) (kg/m³) | \( \rho(T_{in}, 1\text{atm}) \) |
| Reference pressure, \( p_{ref} \) (Pa) | \( \rho_{ref} u_{ref} \) |
| Reference temperature difference, \( \Delta T_{ref} \) (K) | Conduction temperature difference limit, \( \frac{q_{ref} u_{ref}}{2 \lambda_{ref}} \) |
| Reference heat capacity, \( (\rho c_p)_{ref} \) (J/K) | \( \rho_{ref} c_p \) |
| Reference scale of diffusivity, \( v_{ref} \) (m²/s) | \( v_{ref} \) |
| Reference wall heat flux, \( q_{wall,ref} \) | \( (\rho c_p)_{ref} u_{ref} \Delta T_{ref} \) |
Gauss-Lobatto-Legendre quadrature points in each direction. Hence, the total number of degrees of freedom per field variable is \( N_c(P_N + 1)^3 \). Meshing details are given in Sec. II.C.

II.C. Simulation Matrix and Mesh Design

Table III shows the simulation matrix of this work and the corresponding mesh configuration. The specifications of the global simulation parameters, i.e., Re, Gr, and Pr, are all based on the conditions relevant to the practice of the respective reactor types. From the specified simulation matrix, we aim to acquire the knowledge base of the influence of buoyancy on the flow as well as the role of each of the input parameters on the flow.

The DNS of mixed convection must resolve the smallest spatiotemporal scale of the turbulent eddies for both temperature and velocity fields. For velocity and temperature fields, the smallest length scales are characterized by the Kolmogorov scale and Batchelor scale, respectively:

\[
\eta = \left( \frac{\nu^3}{\epsilon} \right)^{0.25},
\]

\[
\eta_T = \left( \frac{\alpha^2 \nu}{\epsilon} \right)^{0.25},
\]

where \( \eta = \) Kolmogorov length scale, \( \epsilon = \) dissipation rate of the turbulent kinetic energy (TKE), and \( \eta_T = \) Batchelor scale. For the low and the unitary Prandtl number fluids, the limiting length scale is the Kolmogorov scale since \( \alpha \geq \nu \) in general.

In this work, the spatial resolution requirement is specified as \( (\Delta x^+, \Delta y_{wall}^+, \Delta y_{e}^+, \Delta z^+) = (9.0, 0.5, 6.0, 12.0) \). Since the influence of buoyancy on turbulence is not exactly known beforehand, the element layout and the polynomial order of each case is determined a posteriori. A precursor run is conducted for each case to obtain an estimate of the wall shear stresses as well as the dissipation rate distribution. Then, different element layouts combined with different polynomial orders are adopted to capture the turbulence under different simulation conditions. The resulting mesh configuration is summarized in Table III. The \((N_{ex}, N_{ey}, N_{ez})\) denotes the number of spectral elements along the x-, y-, and z-directions, respectively.

The method proposed by Gray and Giorgini\cite{24} is employed to further examine the error introduced by Boussinesq approximation corresponding to the cases shown in Table III. Upon the Navier-Stokes equation with the linear variability of the fluid properties to temperature and pressure retained, the INS with Boussinesq approximation can be obtained if the following criteria are satisfied: (1) relative property change to the reference state is negligible, (2) pressure work and viscous dissipation are negligible, or

\[
\beta_q \Delta T_{ref} \leq \epsilon
\]

\[
\frac{g \beta L_{ref} \Pr}{c_p} \leq \epsilon
\]

**TABLE III**
Element Layout and Polynomial Orders of the Presented Cases*

| Re    | Gr<sub>e</sub> | 0       | 5 × 10<sup>6</sup> | 10<sup>7</sup> |
|-------|---------------|---------|------------------|----------------|
| 5000  |               | □, ●, ▼ | □, ●, ▼         | □, ●, ▼        |
|       |               | Layout 1 | Layout 2        | Layout 2       |
|       |               | P<sub>N</sub> = 6 | P<sub>N</sub> = 8 | P<sub>N</sub> = 8 |
|       |               | R<sub>i</sub> = 0.0 | R<sub>i</sub> = 0.2 | R<sub>i</sub> = 0.4 |
| 7500  |               | □, ●, ▼ | □, ●, ▼         | □, ●, ▼        |
|       |               | Layout 1 | Layout 3        | Layout 3       |
|       |               | P<sub>N</sub> = 8 | P<sub>N</sub> = 8 | P<sub>N</sub> = 8 |
|       |               | R<sub>i</sub> = 0.0 | R<sub>i</sub> = 0.089 | R<sub>i</sub> = 0.178 |
| 10 000 |               | □, ▼   | □, ▼            | □, ▼           |
|       |               | Layout 1 | Layout 3        | Layout 3       |
|       |               | P<sub>N</sub> = 8 | P<sub>N</sub> = 8 | P<sub>N</sub> = 8 |
|       |               | R<sub>i</sub> = 0.0 | R<sub>i</sub> = 0.05 | R<sub>i</sub> = 0.1 |

*□: unitary Prandtl (Pr = 1.0), ●: lead (Pr = 0.0169), and ▼: sodium (Pr = 0.0048). Element layout \((n_{ex}, n_{ey}, n_{ez})\): Mesh 1: (24, 26, 360), Mesh 2: (24, 30, 360), and Mesh 3: (24, 30, 480).
\[ \frac{g^\beta L_{ref} T_{ref}}{c_p \Delta T_{ref}} \leq \epsilon, \]

(12)

where \( \beta = \) linear rate of change of fluid property \( \phi = (\rho, \mu, \lambda, c_p) \) with respect to temperature and \( \epsilon_{lim} = \) tolerance limit. Here, the properties, temperature correlation, and associated uncertainties of the lead and sodium properties are from Sobolev,\(^{25}\) and \( T_{ref} = 723 \, \text{K} \) and \( p_{ref} = 10^5 \, \text{Pa} \). The results of the error bound map for the lead and sodium cases are shown in Fig. 2. The cases in the simulation matrix are well within the error bounds (gray area) with tolerance of 5%. Hence, the error introduced by the approximation is regarded as insignificant for the presented cases.

### III. RESULTS AND DISCUSSION

In this section, we present selected results of the mixed convection of low to unitary Prandtl fluids in a vertical channel, including the flow regime transition behavior, effect of the Prandtl number on time-averaged statistics at the fully developed position, and discussion of the effect of buoyancy on convective heat transfer.

#### III.A. Flow Visualization and Frequency Analysis

Figure 3 presents a snapshot of the temperature and streamwise velocity of the \( \text{Ri}_q = 0.2 \) and 0.4 cases of the three fluids for qualitative characterization of the roles of \( \text{Pr} \) and \( \text{Ri}_q \) in vertical mixed convection. Sharp contrast of the thermal boundary layer behavior of the three fluids is observed due to the order of the magnitude difference in thermal diffusivity. For the low \( \text{Pr} \) extreme of sodium, a thick thermal boundary layer is rapidly established, and a temperature stratified temperature field is seen owing to conduction-dominated heat transfer within. In contrast, the temperature distribution of the unitary Prandtl fluid...
is largely driven by turbulent heat transfer, and the buildup of the thin thermal boundary layer spanned across a roughly half-length of the heated section. It is also notable that the temperature distribution of lead showed transitional behavior between the conduction-dominated and the convection-dominated heat transfer regime.

Based on distinctions in the thermal field characteristics, buoyancy rendered different extents of alternation to the velocity field. The extensive thermal boundary layers of the liquid metal cases allowed a stronger buoyancy effect to be established than that in the unitary Prandtl fluid. Near the heated side, the flow was retarded by the buoyant plume, and broad low-velocity streaks appeared. Near the cooled wall, the flow-aligning buoyant plumes accelerated the flow. The flow distortion was enhanced with increasing $Ri_y$.

Power density spectra of the streamwise velocity time signal were captured near both walls at transition zones ($z = 10$) and fully developed zones ($z = 50$), as shown in Fig. 4. Compared to the power spectra between the transition and the fully developed region in the liquid metal cases, the energy content in the low- to medium-frequency range increased considerably in the downstream. This implies the enhancement of a large timescale turbulence structure owing to the influence of buoyancy. For the unitary Prandtl fluid, however, the influence of buoyancy reflects on the gaining discrepancies between the heated and the cooled wall spectra. In the fully developed region, the power density near the cooled wall is lower than that of the heated wall in the frequency range shown. The observed gap can be attributed to the acceleration of the flow that laminarizes the flow adjacent to the cooled wall and the enhanced shear production of turbulence near the heated wall.

Figure 5 shows the power spectral density (PSD) of temperature signals captured in the corresponding location. Across the Prandtl number range covered in this work, the liquid metal temperature fluctuation is dominated by large timescale fluctuations whereas the unitary Prandtl fluid shows more energy content in the intermediate-frequency range. This could be credited to the fact that high thermal diffusivity in the liquid metals smears out high-frequency thermal eddies. In the liquid lead and sodium profiles, considerable increase of low-frequency power density is identified compared to the two axial positions, implying an enhanced large temporal scale thermal structure due to the effect of buoyancy. In contrast to the liquid metals, the promotion of power density by buoyancy in the unitary Prandtl case also covers the frequency range of $[1, 40]$Hz, implying potentially enhanced turbulent transport of heat by the buoyant plume. Another notable feature common to all temperature PSDs of the three fluids is the peaking of energy content around 35Hz. It is currently deduced to be rooted from the burst of the thermal plumes near the wall, yet further investigation is needed to reach that conclusion.

Wavelet transform is further performed on the temperature time series at several positions to gain insight on the spatiotemporal evolution of the power spectrum. Details of the continuous wavelet transformation are given in the Appendix. Figures 6, 7, and 8 show the temperature signals and corresponding wavelet spectrograms near the heated wall ($y = -0.15$), channel centerline $y = 0.0$, and near the cooled wall ($y = 0.15$) at $z = 50$. With decreasing Prandtl numbers, the temperature signal near the cooled wall and the channel centerline showed higher correlation with the heated wall signal, and their corresponding spectrograms exhibited coherent variation with time. This observation suggests the existence of large spatial scale thermal structures in the channel due to perturbation of hot buoyant plumes emerging from the heated wall. For the unitary Prandtl fluid, the strength of the fluid ejection is weaker, and hence, only lower coherence is seen from the channel centerline.

### III.B. Time-Averaged Flow Statistics

In this section, mainly the time-averaged statistics of the $Ri_y = 0.2$ and 0.4 cases of the three fluids are selected for discussion of mixed convection behaviors in different conditions. Note that the time-averaged statistics are collected on the whole channel basis over one-flow-through time (60 convective units, $0.6R_e$). Since the studied flow is homogeneous across the spanwise direction, the collected statistics are further averaged across the $x$-direction to reduce the uncertainty in the data.

The capability of collecting time-averaged flow statistics in NekRS is first benchmarked against the forced convection DNS data set by Kasagi and Iida and Kasagi et al., a Poiseuille flow with $Re = 150$, and isothermal walls. Figure 9 summarizes the comparison of the mean and root-mean-squared (RMS) velocity, RS, streamwise THF, budgets of the RMS streamwise velocity (main component of TKE), and wall-normal THF. The budgets for Reynolds stresses can be expressed as

$$\frac{\partial}{\partial t} \langle u_i u_j \rangle + \frac{\partial}{\partial x_k} T_{ij} = P_{ij} + R_{ij} - \epsilon_{ij}$$ (13)
Fig. 3. Snapshots of (a) temperature and (b) streamwise velocity of the selected mixed convection cases. Note that the channel length (z-direction) is scaled by a factor of 0.5 and different temperature scales are used for unitary Prandtl fluid and liquid metals for better visibility.

\[
T_y = T_y^t + T_y^v + T_y^p = \left< u'_i u'_j u'_k \right> - \nu \frac{\partial \left< u'_j \right>}{\partial x_k} + \frac{1}{\rho} \left( \left< u'_j p' \right> \delta_{ik} + \left< u'_i p' \right> \delta_{jk} \right)
\]

\[
\mathcal{P}_y = -\left< u'_i u'_k \right> \frac{\partial \left< u'_j \right>}{\partial x_k} - \left< u'_j u'_k \right> \frac{\partial \left< u'_i \right>}{\partial x_k}
\]
\[
\mathcal{R}_{ji} = -\frac{1}{\rho} \left< u_i \frac{\partial p'}{\partial x_j} + u_j \frac{\partial p'}{\partial x_i} \right>
\]

\[
\Pi_{ij} = \mathcal{R}_{ji} - \frac{\partial T_{kj}}{\partial x_j}
\]

\[
\epsilon_{ij} = 2\nu \left( \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \right)
\]

where \( \frac{\partial}{\partial t} \) = material derivative with advection by mean velocity, \( \left< X \right> \) = time-average operator, \( T_{kj} \) = the turbulent transport tensor, \( P_{ij} \) = production tensor, \( \mathcal{R}_{ij} \) = pressure-rate-of-strain tensor, \( \Pi_{ij} \) = velocity-pressure-gradient tensor, and \( \epsilon_{ij} \) = dissipation tensor. Similarly, for the transport equation for THF,

Fig. 4. Power spectral density of streamwise velocity signals collected near heated wall \((y = -0.15)\) and cooled wall \((y = 0.15)\) at transition region \((z = 10)\) and fully developed region \((z = 50)\) of the channel. (a) Na, (b) Pb, and (c) unitary Prandtl.
Fig. 5. Power spectral density of temperature signals collected near heated wall ($y = 0.15$) and cooled wall ($y = 0.15$) at transition region ($z = 10$) and fully developed region ($z = 50$) of the channel. (a) Na, (b) Pb, and (c) unitary Prandtl.

\[
\frac{\partial\langle u'_iT' \rangle}{\partial t} + \frac{\partial}{\partial x_j} T_{iT} = P_{iT} + \Phi_{iT} - \epsilon_{iT} \tag{19}
\]

\[
T_{iT} = T_{iT}^{'} + T_{iT}^{''} = \frac{\partial}{\partial x_j} \langle u'_iT'_u' \rangle + \frac{\partial}{\partial x_j} \left( \alpha \langle u'_T \frac{\partial T}{\partial x_j} \rangle + \nu \langle T' \frac{\partial u'_i}{\partial x_j} \rangle \right) \tag{20}
\]

\[
P_{iT} = -\langle u'_iT' \rangle \frac{\partial\langle u_i \rangle}{\partial x_j} - \langle u'_i \rangle \frac{\partial\langle T \rangle}{\partial x_j} \tag{21}
\]
\[ \phi_{IT} = \frac{1}{\rho} \left\langle \frac{T'}{c_{\xi_j}} \right\rangle \]

\[ \epsilon_{IT} = (\alpha + v) \left\langle \frac{\partial T'}{\partial x_j} \frac{\partial u_j'}{\partial x_j} \right\rangle, \]

where \( T_{IT}, P_{IT}, \phi_{IT}, \) and \( \epsilon_{IT} \) = turbulent transport tensor, production tensor, temperature-pressure gradient correlation, and dissipation tensor, respectively. Good agreement can be found between the NekRS and the reference data set.

**III.B.1. Statistics in the Fully Developed Region**

Figure 10 shows the comparison of the law-of-the-wall profiles at the fully developed region \((z = 50)\) of the \( \text{Re}_{\text{L}} = 0.2 \) and 0.4 cases along with the forced convection profile of the same \( \text{Re} \). In this scenario, the velocity profile is distorted by the buoyancy with direction varying across the walls. Near the cooled wall, the flow-aligning buoyancy accelerates the flow within the thickness of the thermal boundary layer and results in an increase in wall shear and shrinkage of the viscous boundary layer thickness. The intensive flow acceleration
in the liquid metal cases can also be observed from the inclusion of the velocity peak position (where zero viscous shear stress is) in the cooled wall profile. For the heated wall, the flow-opposing buoyancy decelerated the mean flow in the vicinity and expanded the viscous boundary layer region, resulting in a shift of the maximum velocity position toward the cooled side. Across the Ri$_q$ and Pr range shown, the distortion of the flow exacerbates with increasing Ri$_q$ as well as decreasing Pr owing to a larger Ri$_q$ΔT product locally in the thermal boundary layer range.

Figure 11 shows the RMS streamwise velocity and its production and dissipation rates. In the differential heating scenario, the turbulence intensity is significantly enhanced near the heated wall and channel centerline and is slightly suppressed near the cooled side. The increased shear (velocity gradient along the y-direction) contributes to the enhancement of the velocity fluctuation as a consequence of expansion of the heated wall viscous boundary layer. On the other hand, local acceleration near the cooled wall caused laminarization of the turbulent flow. This can be identified from the reduced RMS velocity in the unitary and Ri$_q$ = 0.2 of the liquid metal cases. In the Ri$_q$ = 0.4 case for both liquid metals, the turbulence intensity near the cooled wall is potentially restored due to further expansion of the increased shear production zone emerged from the heated wall.

Time-averaged and RMS temperature profiles of the Ri$_q$ = 0.2 and 0.4 cases are shown in Fig. 12. In general,
Pr has a dominating effect on the distribution of temperature. The mean temperature distribution in the liquid metal cases is nearly linear across the walls. This corresponds to the range of the thermal boundary layer, where heat is mainly transferred by heat conduction (molecular diffusion). In the lead $\text{Ri}_q = 0.4$ case, a slight departure from the linear distribution appeared beyond $y^+ = 10$, which is believed to be caused by the buoyancy-promoted turbulent mixing near the channel centerline.

With the lower thermal diffusivity in the unitary Prandtl fluid, the thermal boundary layer is considerably thinner than that of the liquid metals. With the stronger buoyancy (increasing $\text{Ri}_q$), the temperature distribution from both walls branched beyond $y^+ = 10$. The increasing temperature gradient can be explained by the locally altered turbulence conditions previously discussed. The laminarization of the flow locally suppressed the turbulent heat transfer and rendered the temperature profile to retreat toward the linear distribution. Near the heated wall, the enhanced turbulent fluctuation promoted turbulent mixing, and hence, a flatter profile is observed with higher $\text{Ri}_q$.

Figure 12b shows the corresponding distribution of temperature fluctuation. For the liquid metal cases, the magnitude of the temperature fluctuation is promoted with increasing buoyancy despite their rather insignificant magnitude. The unitary Prandtl mixed convection case, on the other hand, showed increased/reduced...

Fig. 8. Wavelet spectrogram of temperature at transition and fully developed region of the unitary Prandtl $\text{Ri}_q = 0.4$ case.
temperature fluctuation near the cooled/heated wall, which is possibly caused by the emerging wall-normal temperature gradient around the channel centerline (Fig. 12a) that promoted the production of temperature fluctuation.

Figure 13 shows the distribution of the RS and its production and dissipation rates. Briefly, distribution of RS in the mixed convection cases highlights significant growth of the shear stress near the heated wall and shift of the zero-shear stress position (where shear is zero).
Fig. 10. Profiles of time-averaged streamwise velocity at fully developed region of Ri₉ = 0.2 and 0.4 cases. Superscript + stands for normalization by the forced convection friction velocity $w_τ = \sqrt{\frac{\tau_{vw}}{\rho}}$ of the corresponding Re, where $\tau_{vw}$ is the wall shear stress.

Fig. 11. Profiles of fully developed (a) variance of streamwise velocity, (b) production, and (c) dissipation rate of $w^{+\text{rms}}$ of $\text{Ri}_9 = 0.2, 0.4$ cases. Superscript + stands for normalization by the forced convection friction velocity $w_τ$ and $\nu^+ / τ$ (for the budget terms) of the corresponding Re. See Fig. 10 for legend.
overall enhancement of turbulent fluctuation that perturbed the temperature field and promoted turbulent mixing. The wall-normal THF also increases seemingly monotonically with ascending Ri_q. The change of THF in unitary Prandtl fluid agrees with the alternation of turbulence intensity discussed in Fig. 11. At Ri_q = 0.2, slight enhancement/impairment of THF is observed due to buoyancy promotion/suppression of velocity fluctuation. At Ri_q = 0.4, however, overall enhancement of THF is seen. This could be explained by stronger temperature distortion that creates higher temperature fluctuation near the cooled wall, which also promoted THF.

### III.C. Turbulent Prandtl Number

In this section, we examine the SGDH and Reynolds analogy based on the obtained DNS data. As shown previously, DNS provides access to closure terms for the RANS framework, namely, RS and THF, and hence can be used to inspect the validity of those assumptions in the studied vertical mixed convection cases.

The definition of the turbulent Prandtl number Pr_t is indicated in Eqs. (24), (25), and (26):

\[
Pr_t = \frac{\nu_t}{\alpha_t} \quad (24)
\]

\[
\nu_t = \frac{-\langle v'w' \rangle}{\left( \frac{\partial \omega}{\partial y} + \frac{\partial \omega}{\partial z} \right)} \quad (25)
\]

\[
\alpha_t = \frac{\langle v' T' \rangle}{\frac{\partial (\rho T)}{\partial y}} \quad . \quad (26)
\]

Figure 15 shows the wall profiles of Pr_t, \nu_t, and \alpha_t near the heated and cooled sides of the channel for the Re = 5000 mixed convection cases, and Fig. 16 shows the Pr_t profile of the Re = 7500 cases. As a general trend, the Pr_t increases as the molecular Prandtl number decreases while Re and Gr_q are fixed and decrease with increasing Re while Pr and Gr_q are fixed. For the unitary Prandtl fluid, the widely adopted assumption of Pr_t~1.0 does not deviate from the DNS results by much. For the liquid metals, Pr_t departs from the unitary value beyond the viscous sublayer, and the maximum value falls on the order of O(10^0) or O(10^1) depending on the fluid and Ri_q.
Fig. 13. Profiles of (a) RS and its (b) production and (c) dissipation at the fully developed region of $\text{Ri}_q = 0.2, 0.4$ cases. Superscript $^+$ stands for normalization by the forced convection $w_\tau^2$ and $\nu/\tau$ (for the budget terms) of the corresponding $\text{Re}$. See Fig. 10 for legend.

Fig. 14. Profiles of (a) wall-normal THF and its (b) production and (c) dissipation at the fully developed region of $\text{Ri}_q = 0.2, 0.4$ cases. Superscript $^+$ stands for normalization by the forced convection $w_\tau^2$ and $\nu/\tau$ (for the budget terms) of the corresponding $\text{Re}$. See Fig. 10 for legend.
In all liquid metal cases listed, Pr$_t$ profiles are considerably lower in the mixed convection cases near both walls. Near the heated wall, the enhanced RS and THF and slightly decreased velocity and temperature gradient lead to increased turbulent diffusivities. Near the cooled side, reduction of $\nu_t$ corresponds to higher wall shear due to acceleration/laminarization and slightly increased $\alpha_t$ owing to enhanced THF and a more gentle temperature gradient.

III.D. Nusselt Number in the Fully Developed Region

The impact of buoyancy on the local effectiveness of convective heat transfer is examined in this section. The local Nusselt number is calculated on the cross-sectional slice basis:

$$\text{Nu}(z) = \frac{h(z)D_h}{\lambda},$$

(27)
where $h(z)$ is the local convective heat transfer coefficient and can be calculated by

$$h(z) = \frac{q_{wall}^*}{< T >_{wall} - < T >_{bulk}},$$

(28)

where $< T >_{wall}$ = mean temperature of heated or cooled wall and $< T >_{bulk}$ = slice bulk fluid temperature, which is averaged by velocity weighting.

Figure 17 shows the trend of the normalized Nusselt number against the buoyancy number $Bo$, which is defined as in Eq. (29)$^{29}$:

$$Bo = \frac{8 \times 10^4 Gr_\mu / Re^{1.425 Pr^{0.8}}}{Pr_0}.$$
The raw Nu of the mixed convection cases is normalized by the forced convection value of the identical Re (Nu0) so as to highlight the influence of buoyancy. Also note that the Nu/Nu0 of existing works on air mixed convection is included for comparison with the unitary Prandtl fluid.130–33

In the parameter range covered in this work, convective heat transfer in the unitary Prandtl fluid is monotonically enhanced/impaired on the buoyancy-opposed/aided side. The observed alternation on convective heat transfer is deeply connected to the local turbulence condition since turbulent transport of heat is a major component of heat flux as opposed to conduction flux. As reported in Figs. 11 and 14, the enhanced shear production of velocity fluctuation and laminarization near the heated and cooled walls hence led to the observed trend in Nu. Compared to the existing mixed convection data for air (Pr~0.71), the obtained data roughly follow the trend in that of the existing work in Bo = [0, 0.2]. Critical impairment of Nu (vertical decline and spread of Nu at around Bo~0.3) cannot be concluded from the presented data set.

For the sodium data set, reversed branching behavior compared to the unitary Prandtl cases is identified; i.e., convective heat transfer is enhanced/impaired for the buoyancy-opposed/aided side. Also note that the deviation of Nu/Nu0 is relatively minor despite a significant buoyancy effect. This can be explained from the fact that turbulent heat transfer in sodium plays only a minor role due to dominating conduction. Convective heat transfer depends on mean velocity rather than turbulent transport. Hence, enhancement/suppression of convective heat transfer is seen as due to the acceleration/deceleration near the cooled/heated wall.

The presented lead data points form a unique trend compared to the other two fluids. In the lead cases, only enhancement of convective heat transfer is seen, and the extent of Nu increase is higher for the cooled side. It can also be noted that Nu for lead has seemingly higher dependence on Re as data from Re = 5000 and 7500 formed their separate branch. It is currently believed that convective heat transfer in lead is the result of complicated interaction between the base forced convection condition and the buoyancy promoted THF as well as the acceleration near the cooled wall due to its intermediate Pr and should be further investigated.

IV. CONCLUSIONS

In this work, series of DNS have been conducted to study the low-to-unitary Prandtl mixed convection phenomenon in a differentially heated vertical channel, which is highly relevant to low-flow heat transfer in the downcomer of advanced reactor designs. The datasets obtained hence serve as an important basis for the next-generation reactor and advanced modeling development.

In the intermediate Ri, studied in this work, differential heating rendered the buoyancy varying across the channel and distorted the flow, and fluids with a lower Prandtl number were more susceptible to this effect. From the spectral analysis of the velocity and temperature time series, we highlight the existence of large time and potentially spatial scale convective and thermal structures that emerged from the heated wall. From time-averaged statistics, the flow-opposing plumes near the heated wall decelerated the flow and promoted shear production of velocity fluctuation. In contrast, flow-aligned buoyancy near the cooled wall accelerated and laminarized the flow. The respective effect of buoyancy also reflects on the trend of Nusselt number against strength of buoyancy.

Direct access of RS and THF allows us to check the validity of the Reynolds analogy. It was found that the assumption of the unitary turbulent Prandtl number is not a faithful representation beyond the viscous sublayer for the liquid metal cases, yet it remains acceptable for the unitary Prandtl fluid.

APPENDIX

WAVELET TRANSFORM

Wavelet transform allows further insight into the temporal evolution of the time series spectrum.34,35 In the continuous wavelet transform, the time signals are convoluted with the parent wavelet of scale a and shift b, \( \psi(\frac{t-b}{a}) \):

\[
\phi(f, t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi(\frac{t-b}{a}) \phi(t')dt'.
\]

Features of different characteristic timescales can then be extracted based on the response of the original time signal to the parent function of different scales. Here, the Mexican hat wavelet with zero shift \( b = 0 \) is adopted:

\[
\psi(t, a) = \frac{2}{3^{0.5} a^{0.25}} e^{-\frac{\rho^2}{2}} (1-t^2).
\]

The scale of wavelets adopted in this work is uniformly distributed in the log space \( a = [10^{0.2}, 10^{3.2}] \), which is determined by trial and error. The distribution of the wavelet equivalent frequency is shown in Fig. A.1.
Fig. A.1. Distribution of the equivalent frequency of wavelets used in the continuous wavelet transform in this work.

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ORCID

Cheng-Kai Tai http://orcid.org/0000-0001-5949-7646
Tri Nguyen http://orcid.org/0009-0001-2470-1456
Arsen S. Iskakov http://orcid.org/0000-0003-4064-9894
Igor A. Bolotnov http://orcid.org/0000-0003-1382-5442

References

1. G. GRÖTZBACH, “Challenges in Low-Prandtl Number Heat Transfer Simulation and Modelling,” Nucl. Eng. Des., 264, 41 (2013); https://doi.org/10.1016/j.nucengdes.2012.09.039.

2. F. ROELOFS et al., “Status and Perspective of Turbulence Heat Transfer Modelling for the Industrial Application of Liquid Metal Flows,” Nucl. Eng. Des., 290, 99 (2015); https://doi.org/10.1016/j.nucengdes.2014.11.006.

3. H. KAWAMURA et al., “DNS of Turbulent Heat Transfer in Channel Flow with Low to Medium-High Prandtl Number Fluid,” Int. J. Heat Fluid Flow, 19, 482 (1998); https://doi.org/10.1016/S0142-727X(98)10026-7.

4. D. D. SANTIS et al., “The Influence of Low Prandtl Numbers on the Turbulent Mixed Convection in an Horizontal Channel Flow: DNS and Assessment of RANS Turbulence Models,” Int. J. Heat Mass Transfer, 127, 345 (2018); https://doi.org/10.1016/j.ijheatmasstransfer.2018.07.150.

5. A. SHAMS et al., “Assessment and Calibration of an Algebraic Turbulent Heat Flux Model for Low-Prandtl Fluids,” Int. J. Heat Mass Transfer, 79, 589 (2014); https://doi.org/10.1016/j.ijheatmasstransfer.2014.08.018.

6. S. MANSERVISI and F. MENGHINI, “A CFD Four Parameter Heat Transfer Turbulence Model for Engineering Applications in Heavy Liquid Metals,” Int. J. Heat Mass Transfer, 69, 312 (2014); https://doi.org/10.1016/j.ijheatmasstransfer.2013.10.017.

7. J. JACKSON and J. LI, “Influences of Buoyancy and Thermal Boundary Conditions on Heat Transfer with Naturally-Induced Flow,” University of Manchester.

8. H. O. BUHR, E. A. HORSTEN, and A. D. CARR, “The Distortion of Turbulent Velocity and Temperature Profiles on Heating, for Mercury in a Vertical Pipe,” J. Heat Transfer, 96, 2, 152 (1974); https://doi.org/10.1115/1.3450156.

9. J. D. JACKSON, B. P. AXCELL, and A. WALTON, “Mixed-Convection Heat Transfer to Sodium in a Vertical Pipe,” Exp. Heat Transfer, 7, 71 (1994); https://doi.org/10.1080/08916159408946473.

10. J. JACKSON, “Turbulent Mixed Convection Heat Transfer to Liquid Sodium,” Int. J. Heat Fluid Flow, 2, 107 (1983); https://doi.org/10.1016/0142-727X(83)90011-5.

11. S. PIROZZOLI et al., “Mixed Convection in Turbulent Channels with Unstable Stratification,” J. Fluid Mech., 821, 482 (2017); https://doi.org/10.1017/jfm.2017.216.

12. O. IIDA and N. KASAGI, “Direct Numerical Simulation of Unstably Stratified Turbulent Channel Flow,” J. Heat Transfer, 119, 1, 53 (1997); https://doi.org/10.1115/1.2824100.

13. W. GUO et al., “Influence of Buoyancy in a Mixed Convection Liquid Metal Flow for a Horizontal Channel Configuration,” Int. J. Heat Fluid Flow, 85, 108630 (2020); https://doi.org/10.1016/j.ijheatfluidflow.2020.108630.

14. P. ZHAO et al., “DNS of Turbulent Mixed Convection over a Vertical Backward-Facing Step for Lead-Bismuth Eutectic,” Int. J. Heat Mass Transfer, 127, 1215 (2018); https://doi.org/10.1016/j.ijheatmasstransfer.2018.08.116.
15. D. ANGELIA, A. FREGNI, and E. STALIOB, “Direct Numerical Simulation of Turbulent Forced and Mixed Convection of LBE in a Bundle of Heated Rods with P/D = 1.4,” Nucl. Eng. Des., 355, 110320 (2019); https://doi.org/10.1016/j.nucengdes.2019.110320.

16. I. BOLOTNOV et al., “Challenge Problem 1: Benchmark Specifications for the Direct Numerical Simulation of Canonical Flows,” U.S. Department of Energy, Office of Scientific and Technical Information (2021).

17. S. DONG, G. E. KARNIADAKIS, and C. CHRYSSOSTOMIDIS, “A Robust and Accurate Outflow Boundary Condition for Incompressible Flow Simulations on Severely-Truncated Unbounded Domains,” J. Comput. Phys., 261, 83 (2014); https://doi.org/10.1016/j.jcp.2013.12.042.

18. P. FISCHER et al., “NekRS, A GPU-Accelerated Spectral Element Navier-Stokes Solver,” Arxiv Preprint (2021); https://arxiv.org/abs/2104.05829.

19. P. F. FISCHER, J. W. LOTTES, and S. G. KERKEMEIER, “Nek5000 Version 19.0 Release Date,” Argonne National Laboratory (Dec. 28, 2019); https://nekt5000.mcs.anl.gov.

20. A. PATERA, “A Spectral Element Method for Fluid Dynamics: Laminar Flow in a Channel Expansion,” J. Comput. Phys., 54, 468 (1984); https://doi.org/10.1016/0021-9991(84)90128-1.

21. E. MERZARI, A. OBABK, and P. FISCHER, “Spectral Element Methods for Liquid Metal Reactors Applications,” Argonne National Laboratory (2017).

22. B. FENG et al., “Development of Advanced Reactor Models for Virtual Test Bed Using NEAMS Tools,” Argonne National Laboratory (2021).

23. J. FANG et al., “Feasibility of Full-Core Pin Resolved CFD Simulations of Small Modular Reactor with Momentum Sources,” Nucl. Eng. Des., 378, 111143 (2021); https://doi.org/10.1016/j.nucengdes.2021.111143.

24. D. GRAY and A. GIORGINI, “The Validity of the Boussinesq Approximation for Liquid and Gases,” Int. J. Heat and Mass Transfer, 19, 545 (1976); https://doi.org/10.1016/0017-9310(76)90168-X.

25. V. SOBOLEV, “Database of Thermalphysical Properties of Liquid Metal Coolants for GEN-IV (BLG–1069),” Belgian Nuclear Research Center SCK-CEN Belgium (2010).

26. N. KASAGI and O. IIDA, “Progress in Direct Numerical Simulation of Turbulent Heat Transfer,” Proc. 5th ASME/JSME Joint Thermal Engineering Conf., San Diego, California, 1999.

27. N. KASAGI et al., “DNS Database of Turbulence and Heat Transfer”; https://htltab.jp/.

28. S. B. POPE, Turbulent Flows, Cambridge University Press, New York (2000).

29. W. B. HALL and J. D. JACKSON, “Laminarization of a Turbulent Pipe Flow by Buoyancy Forces,” ASME Paper 69-HT-55, ASME (1969).

30. J. BYRNE and E. EJOOGU, “Combined Free and Forced Convection Heat Transfer in a Vertical Pipe,” Proc. Symp. Heat and Mass Transfer by Combined Forced and Natural Convection, Manchester, United Kingdom, 1971.

31. A. STEINER, “On the Reverse Transition of a Turbulent Flow Under the Action of Buoyancy Forces,” J. Fluid Mech., 47, 503 (1971); https://doi.org/10.1017/S0022112071001198.

32. J. P. EASBY, “The Effect of Buoyancy on Flow and Heat Transfer for Gas Passing Down a Vertical Pipe at Low Turbulent Reynolds Number,” Int. J. Heat Mass Transfer, 21, 6, 791 (1978); https://doi.org/10.1016/0017-9310(78)90041-8.

33. M. A. COTTON and J. D. JACKSON, “Vertical Tube Air Flows in the Turbulent Mixed Convection Regime Calculated Using a Low-Reynolds-Number k-epsilon Model,” Int. J. Heat Mass Transfer, 275 (1989).

34. C. MENEVEAU, “Analysis of Turbulence in Orthonormal Wavelet Representation,” J. Fluid Mech., 232, 469 (1991); https://doi.org/10.1017/S0022112091003786.

35. G. R. LEE et al., “PyWavelets: A Python Package for Wavelet Analysis,” J. Open Source Softw., 4, 36, 1237 (2019); https://doi.org/10.21105/joss.01237.