Circumbinary Accretion from Finite and Infinite Disks

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Abstract

We carry out 2D viscous hydrodynamics simulations of circumbinary disk (CBD) accretion using AREPO. We resolve the accretion flow from a large-scale CBD down to the streamers and disks around individual binary components. Extending our recent studies, we consider circular binaries with various mass ratios (0.1 ≤ qb ≤ 1) and study accretion from “infinite,” steady-supply disks and from finite-sized, viscously spreading tori. For “infinite” disks, a global steady state can be reached, and the accretion variability has a dominant frequency ∼0.2Ωb for qb > 0.5 and Ωb for qb < 0.5 (Ωb is the binary angular frequency). We find that the accretion “eigenvalue” I0—the net angular momentum transfer from the disk to the binary per unit accreted mass—is always positive and falls in the range (0.65–0.85)a2Ωb (with a0 the binary separation), depending weakly on the mass ratio and viscosity. This leads to binary expansion when qb ≥ 0.3. Accretion from a finite torus can be separated into two phases: an initial transient phase, corresponding to the filling of the binary cavity, followed by a viscous pseudostationary phase, during which the torus viscously spreads and accretes onto the binary. In the viscous phase, the net torque on the binary per unit accreted mass is close to I0, the value derived for “infinite” disks. We conclude that binaries of similar mass accreting from CBDs gain angular momentum and expand over long timescales. This result significantly impacts the coalescence of supermassive binary black holes and newly formed binary stars. We offer a word of caution against conclusions drawn from simulations of transient accretion onto empty circumbinary cavities.

Unified Astronomy Thesaurus concepts: Binary stars (154); Black hole physics (159); Supermassive black holes (1663); Orbital evolution (1178); Stellar accretion disks (1579); Star formation (1569); Gravitational wave sources (677)

1. Introduction

Circumbinary disk (CBD) accretion plays an important role in the evolution of many types of binary systems, ranging from young binary stars to massive binary black holes (MBBHs). In these systems, the combined effects of accretion and binary–disk gravitational interaction dictate the long-term evolution of the binary orbit. Numerical simulations are required to understand the accretion process, since the flow is complex and covers a wide range of scales (from the outer CBD, through accretion streams, to circumsingle disks (CSDs) onto individual binary components). Moreover, to determine the secular effect of accretion on the orbital evolution of the binary, long-term simulations are needed in order to average out the rapid flow variability and transient features. For these reasons, numerical simulations of circumbinary accretion are challenging, and only recently have consistent results on the long-term evolution of accreting binaries begun to emerge (see Muñoz et al. 2019; Moody et al. 2019).

The most important byproduct of binary–disk interaction is the change in the binary’s semimajor axis a0. The early theoretical and computational works of Artymowicz & Lubow (1994, 1996) concluded that binaries surrounded by gas disks evolve toward coalescence. These works, however, ignored the effect of accretion, assuming that the cavity carved by the tidal potential was empty enough to partially or totally suppress accretion onto the central objects. Subsequent work expanded upon the original findings of Artymowicz and Lubow, always concluding that the binary migrates inward (e.g., MacFadyen & Milosavljević 2008; Farris et al. 2014).

Cosmological hierarchical structure formation predicts the formation of MBBHs (e.g., Begelman et al. 1980; Volonteri et al. 2003), but observations have not been able to discern whether these binaries merge, or “stall” at some finite separation. The stalling of MBBHs has been dubbed the “final parsec problem” (Milosavljević & Merritt 2003a, 2003b), and occurs when all the dynamical mechanisms that extract angular momentum have been exhausted. One potential solution to this “problem” is the incorporation of dissipative gas dynamical processes, such as the interaction with a circumbinary gas disk (Haiman et al. 2009; Hayasaki 2009; Roedig et al. 2012), giving rise to an increased fraction of gas-assisted MBBH mergers (e.g., Kocsis & Sesana 2011; Kelley et al. 2017a). Such mergers would generate gravitational waves (GWs) in the low-frequency band (Hahnen 1994; Wyithe & Loeb 2003), and thus, understanding the coupling of accreting binaries with surrounding gas disks is essential for making meaningful predictions for GW background and event rates.

Recently, Miranda et al. (2017) carried out 2D viscous hydrodynamical simulations of circumbinary accretion using the PLUTO code (Mignone et al. 2007); through a careful analysis of the angular momentum balance in the CBD (keeping track of the viscous, advective, and gravitational torques), they showed that the central binary gains angular momentum from the gas. Since the simulations of Miranda et al. did not capture the entirety of the gas dynamics inside the
binary cavity (a circular region containing the binary was excised from the computational domain), their results should be considered as tentative.

In Muñoz et al. (2019), the problem was examined using the moving-mesh code AREPO (Springel 2010; Pakmor et al. 2016; Weinberger et al. 2019); the simulations fully captured the flow inside the cavity and the CSDs, resolving the flow down to separations of 0.02a0 from the individual binary components (see also Muñoz & Lai 2016). While the angular momentum transfer rate of CBDs around circular binaries was consistent with Miranda et al. (2017), that of eccentric binaries exhibited significant discrepancies, highlighting the limitations of simulations that exclude the binary cavity. Most importantly, Muñoz et al. (2019) showed that the (time-averaged) angular momentum current through the CBD $\langle J_0 \rangle$ is in agreement with the total torque acting directly on the binary $\langle J_0 \rangle$ (including both gravitational and accretion torques), confirming that their simulations are in (quasi-)steady state and that accreting binaries gain angular momentum from the disk. These results have been subsequently confirmed (and extended to 3D inclined disks) in an independent study by Moody et al. (2019) using ATHENA++ (Stone et al. 2008; White et al. 2016).

Our previous works (Muñoz & Lai 2016; Miranda et al. 2017; Muñoz et al. 2019) focused on equal-mass binaries and accreting from “infinite” disks, where we imposed an outer boundary condition at $R_{\text{out}} \gg a_0$ that supplied gas at a constant rate $M_0$. We have shown that such a disk can reach a quasi-steady state, in which the time-averaged mass and angular momentum transfer rates across the CBD are constant. It is natural to ask what happens to a binary that accretes from a finite disk/torus. In Muñoz et al. (2019), we hypothesized that the secular angular momentum transfer rate $\langle J_0 \rangle$ should still follow the net mass accretion rate $\langle M_0 \rangle$, provided that the latter changes slowly in time.

This work is organized as follows. In Section 2, we present simulations of accretion onto circular binaries of different mass ratios when accreting from “infinite” disks. In Section 3, we present analogous simulations of binaries supplied by finite disks or “tori.” In Section 4, we examine the dependence of these results on the assumed disk viscosity. Finally, in Section 5, we discuss the implications of our work and summarize our key results.

2. Accretion from “Infinite” Disks

The motivation behind simulating “infinite” disks (i.e., those with steady mass supply rate $M_0$ at the outer boundary) is the search for a (quasi-)steady state (Muñoz & Lai 2016; Miranda et al. 2017; Dempsey et al. 2019; Muñoz et al. 2019). The existence of a quasi-steady state allows for the effective erasure of the (arbitrary) initial conditions, providing the means to truly explore the secular behavior of accreting binaries. Once such a steady state is reached, the mass supply rate $M_0$ becomes a free scaling parameter. In practice, evolving a system until a steady state is reached can take several viscous times. Consequently, it is useful to choose an initial condition that resembles a steady-state configuration as closely as possible.

2.1. Setup and Initial Conditions

Our setup for CBDs is similar to Muñoz et al. (2019, Section 2). The binary is of total mass $M_b = M_1 + M_2$ and mass ratio $q_b = M_2/M_1$. We use an $\alpha$ prescription for the kinematic viscosity $\nu$, and locally isothermal equation of state (EOS) such that the disk aspect ratio $A_0 = H/R$ is a constant. The initial disk model is described by the density profile

$$\Sigma(R) = f_{\text{cav}}(R) \Sigma_0 \left( \frac{R}{a_0} \right)^{-1/2} \left[ 1 - 0.7 \left( \frac{a_b}{R} \right) \right],$$

where $f_{\text{cav}}(R)$ is a rapidly rising function in $R$, which mimics a cavity of size $R_{\text{cav}} \sim O(a_0)$. The precise shape of $f_{\text{cav}}(R)$ is unimportant, although it is desirable that $f_{\text{cav}}(R) \rightarrow 1$ when $R \gtrsim 5a_0$, to guarantee that the CBD is indeed in a viscous steady state throughout most of its radial extent. The profile (1) is depicted by the blue curve in Figure 1. The viscous accretion rate associated to this profile (blue dotted line in the figure) satisfies $\dot{M}(R) \approx M_0$ for $R \gtrsim 5a_0$, indicating that the outer regions of the CBD are in an approximate steady state from the beginning.

The binary affects the initial conditions of the CBD via the usual correction to the azimuthal velocity profile:

$$v_r^2(R) = \Omega_b^2 a_0^2 \left( \frac{a_b}{R} \right) \left[ 1 + \frac{3 \Omega_b}{R} \right] - c_s^2(R) \left[ 1 - \frac{R}{\int d\Sigma/dR} \right],$$

where $\Omega_b = (GM_b/a_0^3)^{1/2}$ is the mean motion of the binary and $Q \equiv \frac{1}{2} a_0^2 \Omega_b (1 + q_b)^2$ is its quadrupolar moment. In this work, we focused on circular binaries and explored different values of the mass ratio $q_b$. To limit the scope of this work, we explore mass ratios above $q_b = 0.1$.

For the disk properties, we fix the vertical aspect ratio $h_0 = 0.1$ and choose a fiducial disk viscosity $\alpha = 0.1$ (the dependence of results on disk viscosity is discussed in Section 4). The density scaling $\Sigma_0$ in Equation (1) is determined by such a choice of parameters:

$$\Sigma_0 \equiv \frac{M_0}{3 \pi \alpha h_0^2 \Omega_b a_0^2},$$

where, in internal code units, $a_0 = \Omega_b = M_0 = 1$.

2.1.1. Numerical Methods

As in Muñoz & Lai (2016) and Muñoz et al. (2019), we carry out hydrodynamical simulations using the moving-mesh code AREPO (Springel 2010; Pakmor et al. 2016) in its Navier–Stokes version (Muñoz et al. 2013, 2014). As an initial condition, the resolution elements (mesh-generating points) are placed in a quasi-polar distribution in a nested fashion: from $R = a_0$ to $R = 45a_0$, $N_R = 475$ points are placed logarithmically in radius and $N_\phi = 720$ points along the azimuthal direction; from $R = 45a_0$ to $R = 95a_0$, $N_R = 62$ and $N_\phi = 480$. Consecutive annuli of cells are interleaved, resulting in an approximately centroidal Voronoi mesh (Muñoz et al. 2014). We impose an inflow boundary (constant accretion rate) at $R_{\text{out}} = 95a_0$. In the vicinity of the binary, the resolution is smoothly switched over to a mass-based criterion, with the targeted mass resolution of $m_{\text{target}} = 5.3 \times 10^{-7} \Sigma_0 a_0^2$. Accretion onto the individual binary components is carried out within a sink region of outer edge $r_{\text{acc}} = 0.03a_0$, taken to be equal to the gravitational softening length of each Keplerian potential.
The viscous time at a distance $R$ is

$$t_v = \frac{4 R^2}{9 \nu} = \frac{2^{5/2} P_b \left( \frac{R}{2a_b} \right)^{3/2}}{9 \gamma \sigma_0 c_0^3},$$

where $P_b = 2\pi/\Omega_b$ is the orbital period of the binary. With the fiducial values of $h_0 = \alpha = 0.1$, we find $t_v = 200P_b$ at $R = 2a_b$ (the edge of the cavity), which means that after a few hundred binary periods, the system should begin to approach a steady state. Figure 2 (top panel) shows the gas distribution in the vicinity of a binary with $q_b = 0.5$ at four different times: $t = 10, 100, 200, 750 \times P_b$. At $t = 10P_b$, the cavity is still being filled up by viscous accretion from the CBD, and the CSDs are beginning to form. After $t = 200P_b$, the amount of mass in the CSDs is approximately constant, as is the mean surface density at the cavity edge.

2.2. Quasi-steady Accretion, Variability, and Angular Momentum Transfer

After a few thousand binary orbits, the time-averaged accretion rate $\langle M_0 \rangle$ matches the supply rate $M_0$ (Muñoz & Lai 2016; Miranda et al. 2017; Muñoz et al. 2019). The top panel of Figure 3 shows $M_0$ for a $q_b = 0.4$ binary as a function of time over 500 binary orbits. The bottom panel of Figure 3 shows $\langle M_0 \rangle_{30}$, the total accretion rate after performing a running average, where the subscript denotes the width of the averaging window (30$P_b$). Similarly, $\langle M_1 \rangle_{30}$ and $\langle M_2 \rangle_{30}$ correspond to the running averages for the primary and secondary accretion rates, respectively. Once the high-frequency variability is removed, these time series are constant, demonstrating the steady-state nature of our simulations.

We are interested in the steady-state behavior of the angular momentum transfer rate onto the binary $\dot{J}_b$. This is given by

$$\dot{J}_b = \dot{L}_b + \dot{S}_1 + \dot{S}_2,$$

where $\dot{S}_{1,2}$ is the spin torque onto the primary/secondary and $\dot{L}_b$ is the rate of change of orbital angular momentum. Since $L_b = \mu_b\dot{L}_b$, where the reduced mass $\mu_b = M_1M_2/(M_1 + M_2) = q_b\mu_b/(1 + q_b)^2$ and $\dot{L}_b = a_b^2\Omega_b$ is the specific angular momentum of a circular binary, we compute $\dot{L}_b$ via

$$\dot{L}_b = \frac{1}{1 + q_b^2} \left[ q_b M_b \frac{d\dot{L}_b}{dt}_{\text{ext}} + (M_2 + q_b^2 \dot{M}_1) \dot{M}_1 \right]$$

where the specific torque due to external forces $d\dot{L}_b/dt_{\text{ext}}$ (including both gravity and accretion) is computed directly from simulation output, as are $\dot{M}_1$ and $\dot{M}_2$ (see Muñoz et al. 2019 for more details).

Just like the binary accretion rate $\dot{M}_b$, the transfer rate of angular momentum $\dot{J}_b$ also reaches a stationary behavior without long-term trends. We show the stationary time series in Figure 4, where the normalized accretion rates onto the primary and secondary, $\dot{M}_1/\langle M_b \rangle$ and $\dot{M}_2/\langle M_b \rangle$, and the normalized angular momentum transfer rate $\dot{J}_b/\langle M_b \rangle$ are plotted as functions of time for different values of $q_b$. The time-averaged accretion rates, $\langle M_1 \rangle$ and $\langle M_2 \rangle$, differ from each other, with the lower-mass body receiving more mass (e.g., Bate et al. 2000; Farris et al. 2014).

2.2.1. Short-term Variability

The stationarity of the time series in Figure 4 allows us to carry out spectral analysis of $\dot{M}_b$. Using the Lomb–Scargle periodogram, we compute the power spectral density (PSD, in units of $M_0^2/\Omega_b$) for each of the $M_b$ time series in Figure 4, and normalize them by their peak values. These normalized PSDs are shown in Figure 5. For large mass ratios, the dominant frequency is $\omega_{\text{peak,max}} \approx \frac{1}{5}\Omega_b$ as has been noticed by several previous studies (MacFadyen & Milosavljević 2008; Shi et al. 2012; Farris et al. 2014; Muñoz & Lai 2016; Miranda et al. 2017), although the earlier works prior to 2016 did not reach quasi-steady state in the simulations. The other peaks in the spectral density are attributed to harmonics of $\frac{1}{5}\Omega_b$ and to the binary orbital frequency $\Omega_b$ and its harmonics. For $q_b \lesssim 0.6$ (not shown in the figure), the frequency $\approx \Omega_b$ sits at 20% of the maximum power; for $q_b = 0.5$ it sits at nearly 80% of the maximum power; when $q_b = 0.4$, the maximum power shifts to $\omega_{\text{peak,max}} \approx \Omega_b$ (see Figure 6). This switch in the fundamental frequency from $\frac{1}{5}\Omega_b$ to $\Omega_b$ at $q_b \lesssim 0.4$ is roughly consistent with the spectral analysis of Farris et al. (2014).

2.2.2. Secular Behavior

The accretion “eigenvalue,” defined by

$$l_0 = \frac{\langle J_b \rangle}{\langle M_b \rangle},$$

provides the angular momentum received by the binary per unit of accreted mass. We show the numerical results of $l_0$ for a range of values of $q_b$ in the top panel of Figure 7; the values of $l_0$ lie in the range [0.65, 0.85]$a_b^2\Omega_b$ for all mass ratios explored. As in Muñoz et al. (2019), we find that the time-averaged spin torques $\langle \dot{S}_1 \rangle$ and $\langle \dot{S}_2 \rangle$ are much smaller than $l_0$ (the contribution of $\langle \dot{S}_{1,2} \rangle$ to $\langle J_b \rangle$ is 3%–5% for $r_{\text{acc}} = 0.03a_b$).

For circular binaries, the change in orbital angular momentum can be written as

$$\frac{\dot{L}_b}{L_b} = \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} - \frac{1}{2} \frac{\dot{M}_1}{M_1} + \frac{1}{2} \frac{\dot{M}_2}{M_2} = \frac{1}{2} \frac{\dot{L}_b}{L_b} - \frac{1}{2} \frac{\dot{L}_b}{L_b} + \frac{1}{2} \frac{\dot{L}_b}{L_b} .$$
Since \( \langle \dot{L}_b \rangle \simeq \langle \dot{J}_b \rangle \), we find that the eigenvalue \( l_0 \) and the secular migration rate \( \langle \dot{a}_b \rangle \) are related via

\[
\frac{\langle \dot{a}_b \rangle}{a_b} = 2 \frac{\langle M_b \rangle}{M_b} \left\{ \frac{(1 + q_b)^2}{q_b} \frac{l_0}{a_b^2 \Omega_b} - (1 - \eta) \left( 1 + q_b \right) - \frac{1 + q_b}{q_b} \right\} + \frac{1}{2} \left( \frac{1 + q_b}{q_b} \right) l_0 - l_{0, \text{crit}}
\]

where we have defined the “preferential accretion rate ratio” \( \eta = \langle M_2 \rangle / \langle M_b \rangle \). Note from Equation (9) that \( l_0 \) needs to be greater than a critical value \( l_{0, \text{crit}} \) to result in \( \langle \dot{a}_b \rangle > 0 \). This threshold quantity is also shown in the top panel of Figure 7. We see that \( l_0 \) is significantly above this threshold that leads to binary expansion for \( q_b \gtrsim 0.3 \). When \( q_b \lesssim 0.2 \), the value of \( l_0 \) is very close to \( l_{0, \text{crit}} \), because \( \eta \) is larger. In Figure 7, we also show the values of \( \eta \) (middle panel) as a function of \( q_b \). If \( l_0 \lesssim l_{0, \text{crit}} \), then inward migration is possible, even if \( l_0 \) is positive.

We can measure the migration rate directly from the instantaneous change in the binary’s specific orbital energy

\[
\frac{\dot{a}_b}{a_b} = - \frac{\dot{\Omega}_b}{\Omega_b} + \frac{\dot{M}_b}{M_b}
\]

(10)

The change in orbital energy is \( \dot{\mathcal{E}}_b = - \frac{1}{2} \mathcal{G} M_b / a_b \):
and accretion kicks, which are computed on-the-fly for each time step of the simulation.

The secular migration rate \( \langle \dot{a}_b \rangle / a_b \) (Equation (10)) is shown as a function of \( q_b \) in the bottom panel of Figure 7. For comparison, we also include the migration rate computed from Equation (9). As implied by the corresponding values of \( l_0 \), binaries with \( q_b \gtrsim 0.2 \) exhibit positive migration rates, and those with \( q_b \lesssim 0.2 \) exhibit a sharp drop in the migration rate down to \( \dot{a}_b \approx 0 \). These results extend the findings of Muñoz et al. (2019) to binaries of different mass ratios.

3. Accretion from Finite “Torii”

The simulations presented in Section 2 pertain to “infinite” disks with a constant supply rate \( M_0 \). Realistic systems may not have a constant gas supply, and a true steady state—in a strict sense—is not possible. Nevertheless, if the supply rate changes slowly (e.g., by a CBD that is running out of mass), this quantity may still act as a scaling parameter that is being “adiabatically” dialed down/up. In such case, the amount of angular momentum transferred to the binary would decrease/grow in proportion to the supply rate, but the angular momentum transferred per unit accreted mass—i.e., \( l_0 \)—would remain unchanged. To test whether this is indeed the case, here we consider finite disks/torii, in which the supply rate onto the binary is self-consistently set by the viscous evolution of a finite reservoir of mass.

To model an accretion torus, we modify the initial density profile (Equation (1)) by multiplying by an exponential tapering function, i.e.,

\[
\Sigma(R) \rightarrow \Sigma(R)[1 + \exp(R - R_{\text{disk}})]^{-1}, \tag{11}
\]

where the term in square brackets forces the gas density to drop exponentially away from the central binary. The density profile (11) peaks at \( R \leq R_{\text{disk}} \), and we set \( R_{\text{disk}} = 6a_b \). The red curve in Figure 1 depicts this modified density profile, highlighting the outer region of the disk that will spread due to viscous stresses (e.g., Lynden-Bell & Pringle 1974; Hartmann et al. 1998).

The simulation setup is analogous to the one detailed in Section 2.1.1 except for the outer boundary condition: we choose the computational domain to be a square of side \( 65a_b \) with a background density of \( 10^{-10} \Sigma_0 \) into which the disk is allowed to viscously expand; that is, the outer boundary condition of a viscously expanding disk is trivially captured by the quasi-Lagrangian nature of the AREPO scheme.

3.1. Viscous Evolution of Finite Torii

Due to viscous spreading, the supply rate onto the central binary eventually decreases. We provide an example of such evolution in the lower panels of Figure 2. The initial transient phase (the filling of the cavity, \( t = 10P_b \)) matches that of the “infinite” disk case (shown in the upper panels). The CSDs are progressively filled until they reach a maximum mass (\( t = 100P_b \)). Once the transient phase ends, the circumbinary cavity can only be replenished by a CBD with ever-decreasing mass, resulting in a gas morphology that closely resembles that of the “infinite” disk case (\( t = 200P_b \) and \( t = 750P_b \) but at a
uniformly reduced density scale (compare upper and lower panels). Eventually \( t \sim 1000 P_b \), the circumbinary torus becomes severely depleted, and the accretion rate onto the binary drops to a few per cent of its peak value at earlier times. We show the evolution of an accreting torus in Figure 8. During the transient phase \( t \sim 100 P_b \), the accretion rate rises rapidly as the cavity is filled; later on \( t \sim 1000 P_b \), the accretion rate decreases steadily (top panel). A clearer picture of the long-term trend is provided by the running average \( \bar{\dot{M}} \) (thicker line). At later times, \( \bar{\dot{M}} \) can be compared to the accretion rate onto a central point mass \( \dot{M}_c \) from a viscously evolving disk of initial mass \( \dot{M}_{d,0} \) and initial characteristic radius \( R_{d,0} \) (Hartmann et al. 1998; Andrews et al. 2009),

\[
\dot{M}_c = 3(2 - \gamma) \nu_0 M_{d,0} (R_{d,0})^{-5+2\gamma} \left( 1 + \frac{t}{t_{v,0}} \right)^{-\frac{4-2\gamma}{4-2\gamma}},
\]

where the prescribed viscosity profile is \( \nu = \nu_0 (R/R_{d,0})^\gamma \) and \( t_{v,0} \) is the viscous time at \( R = R_{d,0} \). At later times, and for

\[ \gamma = 1/2, \] we have \( \dot{M}_c \sim t^{-4/3} \), which is in rough agreement with the evolution of \( \langle \dot{M}_{b,30} \rangle \) obtained from infinite disks in a steady state (see Figure 4). As evidenced by the positive values of \( \langle \dot{a}_b \rangle \), binaries with \( q_b \gtrsim 0.3 \) expand while accreting.
we show the accretion rates onto the primary $M_1$ (in blue) and secondary $M_2$ (in red) and their corresponding running averages $\langle M_1 \rangle_{30}$ and $\langle M_2 \rangle_{30}$, respectively (thicker lines). As the net accretion rate $\dot{M}_b$ diminishes, so do $M_1$ and $M_2$, although the dominance of accretion onto the secondary is preserved throughout the duration of the simulation. The bottom panel of Figure 8 depicts the density profile $\Sigma(R)$ of the torus at different times. When $t \sim 500P_b$, the peak surface density has decreased by a factor of $\sim$2.5.

### 3.2. Angular Momentum Transfer

The evolution of finite disks is qualitatively different to that of infinite disks, as evidenced by the decaying accretion rate of Figure 8. Note, however, that while $\dot{M}_1$ and $\dot{M}_2$ also decay in time, they track the evolution $\dot{M}_b$, with the ratio $\dot{M}_1/\dot{M}_2$ remaining roughly constant. Indeed, after normalizing by the running average of $\dot{M}_b$ (top panel in Figure 9), the quantities $\langle \dot{M}_1 \rangle_{30}/\langle \dot{M}_b \rangle_{30}$ and $\langle \dot{M}_2 \rangle_{30}/\langle \dot{M}_b \rangle_{30}$ remain roughly constant for $t \gtrsim 100P_b$. Furthermore, the angular momentum transfer per unit accreted mass $\langle \dot{a}_b \rangle_{30}/\langle \dot{M}_b \rangle_{30}$ (bottom panel in Figure 9) is also constant after the transient phase has ended. Similarly, the running average of the migration rate per unit accreted mass $\langle \dot{a}_b \rangle_{30}/\langle \dot{M}_b \rangle_{30}$ rises from a negative value at the beginning to a nearly constant value.

Remarkably, the values at which $\langle \dot{a}_b \rangle_{30}/\langle \dot{M}_b \rangle_{30}$ and $\langle \dot{a}_b \rangle_{30}/\langle \dot{M}_b \rangle_{30}$ saturate are very close to the nominal values of $l_0$ and $\dot{a}_b/\dot{M}_0$ obtained from infinite disks (for $q_b = 0.5$, $l_0 \approx 0.75a_b^0\Omega_b$, and $\dot{a}_b/\dot{M}_0 \approx 2.4a_b/\dot{M}_0$; see Figure 7). The same behavior is observed for other values of $q_b$ (see Figures 10 and 11). Therefore, binaries accreting from finite mass reservoirs, while being subject to decreasing accretion supplies, still accrete a consistent amount of angular momentum per unit accreted mass.

### 4. The Role of Viscosity

In Section 2 and in our previous works (Muñoz & Lai 2016; Muñoz et al. 2019), we used a high viscosity coefficient...
\( \alpha = 0.1 \) in order to reach a steady state of “infinite” disks for a small number of binary orbits. The results of Section 3 suggest that a true steady state is not required to compute the angular momentum transfer rate per unit of accreted mass, and thus high viscosities may not be necessary. In this section, we provide an initial exploration of accretion at lower viscosities.

4.1. Low-viscosity Circumbinary Tori

We setup a torus in the same way as described in Section 3, imposing a surface density profile as in Equation (11) but rescaling \( \Sigma_0 \) for each value of \( \alpha \) according to Equation (3). We fix \( q_b = 0.5 \) for all simulations and set \( \alpha \) to be 0.01, 0.03, and 0.06. In Figure 12, we compare the scaled surface density field at \( t = 0.5 \nu_{cav} \) for the four different viscosities explored (\( t = 100, 160, 320, \) and \( 1000 \times P_b \) for \( \alpha = 0.1, 0.06, 0.03, \) and 0.01 respectively). In all cases, the outer structure of the CBD is essentially the same, as expected from viscous disk evolution. The binary cavity, on the other hand, appears to depend sensitively on the viscosity coefficient. Low values of \( \alpha \) also lead to larger cavities, as expected from torque balance arguments (Artymowicz & Lubow 1994; Miranda & Lai 2015). In addition, the lower the value of \( \alpha \), the more complex the inflow structure, with multi-arm inflows being evident for \( \alpha = 0.01 \) and \( \alpha = 0.03 \) (see Farris et al. 2014; D’Orazio et al. 2016; Mösta et al. 2019). A multi-ring structure in the CBD becomes increasingly prominent as viscosity is lowered. A similar behavior was noticed by D’Orazio et al. (2013, see their Figure 10) and is attributed to the inability of local viscosity to damp away these disturbances. We have run an additional simulation with twice the number of cells and have obtained the same ringed structure.

The fact that the bulk of the CBD evolves mostly in a viscous fashion implies that, to leading order, the mass transfer onto the binary depends on \( \alpha \) only through a rescaled time coordinate

\[
T = 1 + t/\nu_{cav}
\]

(Lynden-Bell & Pringle 1974; Hartmann et al. 1998). We show the evolution of \( \langle J_b \rangle_{30}/\langle M_b \rangle_{30} \) for \( \alpha = 0.06, \alpha = 0.03, \) and \( \alpha = 0.03 \) in Figure 13, where the time coordinate is shown in conjunction with the dimensionless time \( T \). In these three cases, the evolution of the angular momentum transfer rate per unit mass is nearly homologous to the fiducial case with \( \alpha = 0.1 \): there is a transient phase that lasts until \( T \approx 1.5 \), followed by a stationary phase in which \( \langle J_b \rangle_{30}/\langle M_b \rangle_{30} \) is constant. For \( \alpha = 0.1 \), this constant is only slightly smaller than the reference value of \( l_b = 0.75 \) for an infinite disk (Figure 7), although this asymptotic value is large enough in all cases to guarantee \( d_b > 0 \) (see Equation (9)).

It is interesting to note that \( \langle J_b \rangle_{30}/\langle M_b \rangle_{30} \) is barely changed as \( \alpha \) is reduced. The CBD and CSDs are responsible for negative and positive gravitational torques, respectively (Muñoz et al. 2019), and tidal truncation at low \( \alpha \) suggests larger circumbinary cavities and smaller CSDs (Artymowicz & Lubow 1994; Miranda & Lai 2015). Perhaps surprisingly, these results suggest that, while the specific angular momentum transfer rate may depend on \( \alpha \), this dependence is weak.

5. Summary and Discussion

We have carried out 2D viscous hydrodynamical simulations of CBD accretion using the finite-volume, moving-mesh code AREPO. Focusing on circular binaries, we have considered various binary mass ratios \( (0.1 \leq q_b \leq 1) \), and studied accretion from both “infinite” disks (with steady mass supply at large radii) and finite-sized, viscously spreading disks (tori).
AREPO allows us to follow the mass accretion through a wide radial extent of the CBD, through accretion streams inside the binary cavity, and onto CSDs around the binary components. This paper extends our recent studies of circumbinary accretion (Muñoz & Lai 2016; Muñoz et al. 2019; see also Miranda et al. 2017), which focused on equal-mass (but generally eccentric) binaries accreting from "infinite" disks. Our key findings are:

(i) Binaries accreting from CBDs generally gain angular momentum over long timescales (see Figures 7, 9–11, and 13). The quasi-steady accretion “eigenvalue” \( l_0 \), defined as the angular momentum transfer from the disk onto the binary per unit accreted mass, is robust against the radial extent of the surrounding disk/torus, lying in the range \((0.65–0.85)a_0^2\Omega_0\) for all mass ratios explored in this paper \((q_b = 0.1–1.0)\) and depending weakly on the disk viscosity (for \(\alpha\) between 0.01 and 0.1 and disk aspect ratio \(h_0 = 0.1\)). The corresponding migration rate \(\dot{a}_b\) depends on \(l_0\) and the “preferential accretion rate ratio” \(\eta = \langle M_2 \rangle / \langle M_b \rangle\). For \(q_b > 0.2\), we find that the binary expands in separation \((\dot{a}_b > 0)\). For \(q_b \lesssim 0.2\), \(l_0 \approx l_{0,\text{crit}}\), which leads to a substantially reduced rate of outward migration, and possible inward migration.

(ii) Starting from initial conditions with an empty binary cavity, all simulations exhibit an initial transient phase, after which the inner accretion flow and binary cavity settle into a quasi-steady state. The duration and the inner flow structure of the transient phase are sensitive to the initial conditions and the fluid viscosity. This phase may be accompanied by angular momentum loss from the binary, but it does not represent the long-term behavior of the binary–disk system.

(iii) Even after the flow has reached the global quasi-steady state, the accretion onto the central binary components is highly variable on short timescales. The dominant variability frequency changes from \(0.2\Omega_0\) at \(q_b > 0.5\) to \(\Omega_0\) at \(q_b < 0.5\) (see Figures 6 and 7).

(iv) The low-mass component \(M_2\) of the binary generally accretes more mass (see Figures 7, 9, and 10). In the quasi-steady state, the time-averaged accretion fraction \(\langle M_2 \rangle / \langle M_b \rangle\) increases from 50% at \(q_b = 1\) to 90% at \(q_b = 0.1\).

5.1. Comparison to Previous Works

The findings presented in this paper and in Muñoz et al. (2019) (using AREPO), as well as in Miranda et al. (2017)
Regarding point (ii) above: our simulations of accretion from finite disks (tori) (see Section 3) clearly identified a transient phase where the binary loses angular momentum and undergoes inward migration, followed by a quasi-steady phase with outward migration. This transient phase lasts longer for low-viscosity disks. This explains why some low-viscosity simulations appear to induce binary shrinkage, simply because the CBD has not had time to fill in the cavity (e.g., Ragusa et al. 2016). Regarding (iii): our simulations (as well as those in Muñoz & Lai 2016; Miranda et al. 2017; Moody et al. 2019) considered non-self-gravitating disks and required that the local disk mass (at a few times $a_0$) be much less than the binary mass. There may be situations where this condition is not satisfied. The massive CBDSs with empty initial cavities explored by Cuadra et al. (2009) (in the context of MBBHs following galaxy mergers) could be an example: the binary runs away from the initial configuration before the CBD can fill the cavity (see also Escala et al. 2005; Roedig et al. 2011).

While it is known that 3D effects will alter the excitations of waves in the CBD and CSDs via Lindblad resonances (at least for small $q_b$, e.g., Tanaka et al. 2002; Bate et al. 2003), it is unlikely that the 2D nature of our simulations plays an important role in the sign of $(\delta n_b)$. Moody et al. (2019) have already shown that, for a 3D CBDS coplanar with the binary, the measured value of $l_0$ is consistent with the 2D results of Muñoz et al. (2019); for misaligned CBDSs, their eigenvalue is significantly larger, i.e., outward migration is even faster. We acknowledge, nevertheless, that Moody et al. (2019) focused on the $q_b = 1$ case; thus, the qualitative consistency between 2D and 3D remains untested for $q_b \neq 1$. The EOS assumed could have a moderate effect on our results by modifying the angular momentum currents (Lee 2016; Miranda & Rafikov 2019), although it is unclear whether a more realistic EOS (see, e.g., Kley et al. 2019) would make $l_0$ larger or smaller. Additional physics, such as the launching of a magnetized wind (e.g., Blandford & Payne 1982), could modify the angular momentum transfer rate. While an intriguing possibility, winds also depress the net accretion rate by carrying away mass, and their net effect on the net value of $(\delta n_b) / (M_b)$ is difficult to anticipate. Finally, we note that during the revision stage of this manuscript, a preprint by Duffell et al. (2019) was posted online, carrying out a similar parameter-space exploration as this work and finding qualitatively similar results as ours.

5.2. Implications for Supermassive Binary Black Hole Coalescence

Our finding that binaries gain angular momentum from circumbinary accretion casts doubt on the commonly accepted role of gas disks in the orbital evolution of MBBHs and coalescence (see Dotti et al. 2012; Colpi 2014, for comprehensive reviews on the topic). It also impacts the GW emission from possible mergers of MBBHs in galactic nuclei. Such mergers produce GWs anywhere from the LISA band ($f = \Omega_b / \pi \sim 10^{-4}$ Hz for $M_b \lesssim 10^7 M_\odot$) to the Pulsar Timing Array and Square Kilometre Array bands ($f \sim 10^{-7}$ Hz for $M_b \gtrsim 10^9 M_\odot$). Since binaries such as the ones explored in this work ($q_b \gtrsim 0.4$) dominate the GW background at frequencies $\lesssim 10^{-7}$ Hz (Kelley et al. 2017b), the expected GW background would be significantly altered if such MBBH merging events are suppressed by orbital stalling. It has been argued that losscone filling in the most massive galactic merger remnants is efficient enough to bring MBBHs into the GW regime.
(Khan et al. 2011), or that multiple mergers might assist binary coalescence via triple interactions (e.g., Hoffman & Loeb 2007). A quantitative study of all competing effects acting simultaneously on the MBBHs would be useful.

If MBBHs of comparable mass are stilling or expanding due to CBD torques, we may expect a (as yet undetected) population of binary/dual black holes in galaxies that have undergone major mergers. Possible MBBH candidates include the active galactic nucleus in 0402+379 (Rodriguez et al. 2006) and the quasar OJ 287 (Valtonen et al. 2008). Observational campaigns to search for signatures of binarity (or “duality”) using photometric variabilities (e.g., Graham et al. 2015a, 2015b; Charisi et al. 2016; Liu et al. 2016) may reveal more MBBH candidates.

5.3. Implications for Stellar Binary Formation

In contrast to MBBHs, stellar binaries are likely to have formed within a massive disk, in a process of gravitational fragmentation followed by migration (e.g., Bonnell & Bate 1994; Bate & Bonnell 1997; Kratter et al. 2008). The positive migration rates found for $q_b \gtrsim 0.3$ represent an obstacle to this qualitative picture of binary formation. On the other hand, outward migration might help explain why binaries stop hardening before complete coalescence. The drastic reduction in the magnitude of $\dot{a}_b$ found at $q_b \approx 0.2$ (Figure 7) may offer a solution to this conundrum. A transition in the sign of $\dot{a}_b$ is to be expected at small enough mass ratios, because it is known from linear theory that migration is inward for $q_b < 1$ and there is no secondary accretion (e.g., Ward 1986, 1997).

We underscore that $l_0 > 0$ is not synonymous with outward migration: the fact that $\dot{a}_b > 0$ in Figure 7 results from $l_0 > l_{0,\text{crit}}$ (Equation (9)). Since $l_0$ is roughly constant for $0.1 < q_b < 1$, and $l_{0,\text{crit}}$ increases for smaller $q_b$ (due to increasing $\eta$), inward migration may be possible for $q_b \lesssim 0.2$. An additional transition from $l_0 > 0$ to $l_0 < 0$ may occur at an even smaller mass ratio. Indeed, for companions of planetary mass ($q_b \ll 1$, $\eta \approx 0$), Dempsey et al. (2019) reported negative eigenvalues $l_0 \approx -4 (q_b/\alpha) (h/0.05)^{-5} \dot{a}_b^2$ for massive planets ($2 \times 10^{-3} \lesssim q_b \lesssim 10^{-4}$) embedded in low-mass steady-state disks, which evolve in a “modified” Type-II migration scenario (Dürmann & Kley 2015; although see Duffell et al. 2014).

Alternatively, a plausible solution to the positive torque conundrum is that the binary separation is largely set early on during the Class 0 phase of star formation (Bate et al. 2000). Quasi-spherical accretion from an envelope of low specific angular momentum will naturally allow for large values of $M_b$ while keeping $\dot{J}_b$ low, resulting in $\dot{a}_b \lesssim 0$ (Equation (9)). Our simulations would represent a later, disk-dominated viscously evolving phase. The transition from envelope-dominated accretion to disk-dominated accretion might result in a sign reversal in $\dot{a}_b$ (see Figure 3 in Bate et al. 2000).

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