Implementation of Elliptic Curve Cryptography in Binary Field

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Abstract. Currently, there is a steadily increasing demand of information security, caused by a surge in information flow. There are many ways to create a secure information channel, one of which is to use cryptography. In this paper, we discuss the implementation of elliptic curves over the binary field for cryptography. We use the simplified version of the ECIES (Elliptic Curve Integrated Encryption Scheme). The ECIES encrypts a plaintext by masking the original message using specified points on the curve. The encryption process is done by separating the plaintext into blocks. Each block is then separately encrypted using the encryption scheme.

1. Introduction
Information security has become one of the oft-discussed problem in the wake of the 21st century. Most information channels that we use are insecure: they are vulnerable to leakage. Therefore, it is essential that we look for ways to secure the insecure information channel. Cryptography is one of the many ways to secure the channels.

Neal Koblitz and Victor Miller independently introduced elliptic curve cryptography (ECC) [5]. ECC uses the set of points on an elliptic curve along with an addition rule. The unique mathematical structure of the points with the addition rule enables us to perform encryption and decryption of plaintexts. Another reason that supports the feasibility of ECC is the fact that it uses significantly smaller key sizes than the RSA Cryptosystem. For a comparison between RSA and ECC key sizes, see [6].

In the previous research [3], we presented the implementation of binary field arithmetic operation algorithms. In this paper, we discuss the implementation of elliptic curve cryptography using elliptic curves over binary field. We will address several issues, among them will be case-handling for the point operations, the key generation process, and the encryption.

2. Elliptic Curves and the ECIES
An elliptic curve over $GF(2^n)$ is defined by the simplified Weierstrass equation $y^2 + xy = x^3 + ax^2 + b$, where $a \neq 0$ and $b \neq 0$ [2]. It is possible to define several operations on the points of the elliptic curve, namely point negation, addition, and doubling.

We define the point operations as follows. First, let $E$ be an elliptic curve over $GF(2^n)$ and $P(x, y)$ a point on $E$. The negative of $P$ is defined to be $-P(x, x + y)$. Now let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be distinct points on $E$. The result of adding $P$ and $Q$ is the point $P + Q (x_3, y_3)$ where $x_3 = \lambda^2 + \lambda + x_1 + x_2 + a$, $y_3 = \lambda(x_1 + x_3) + x_3 + y_1$, and $\lambda = (y_2 + y_1)/(x_2 + x_1)$. In
the case that \( P \) and \( Q \) are not distinct, the operation is called a point doubling. The double of point \( P \) is the point \( 2P \) \((x_3, y_3)\) where \( x_3 = \lambda^2 + \lambda + a \), \( y_3 = x_1^3 + \lambda x_3 + x_3 \), and \( \lambda = x_1 + y_1 / x_1 \).

It is also possible to define a scalar multiplication on elliptic curves. Given a positive integer \( n \) the scalar multiple of a point is defined as

\[
nP = P + P + \cdots + P.
\]

The set of all points of an elliptic curve \( E \) over \( GF(2^n) \) forms a commutative group with respect to point addition. Furthermore, according to the Mordell-Weil Theorem, this group is finitely generated \([7]\) \([8]\) . The group structure of the elliptic curves over \( GF(2^n) \) allows its use for cryptography. In this paper, we discuss the implementation of the simplified version of Elliptic Curve Integrated Encryption Scheme (ECIES).

The simplified ECIES is a modification of the ElGamal scheme, which is based on the elliptic curve discrete logarithm problem described as follows. \([9]\)

**Definition 2.1.** Let \( E \) be an elliptic curve over \( GF(2^n) \), \( P \) be a point on \( E \) with order \( n \), and \( Q \in \langle P \rangle \), the finite group generated by \( P \). The discrete logarithm problem is the problem of determining an integer \( m \), \( 0 \leq m \leq n - 1 \) which satisfies \( Q = mP \). The integer \( m \) is then called the discrete logarithm of \( Q \).

Let \( \mathcal{P} \) the set of all possible plaintexts, \( \mathcal{C} \) the set of all possible ciphertexts, and \( \mathcal{K} \) the set of all keys in the encryption scheme. We now define the simplified ECIES as follows. \([9]\)

**Definition 2.2.** Let \( E \) be an elliptic curve over \( GF(2^N) \) such that \( E \) contains a cyclic subgroup \( H = \langle P \rangle \) of order \( n \) in which the elliptic curve discrete logarithm problem is infeasible.

(i) The plaintext space is \( \mathcal{P} = GF(2^N) \) and the ciphertext space is \( \mathcal{C} = E(GF(2^N)) \times GF(2^N) \).

(ii) The key space is \( \mathcal{K} = \{(E, P, Q, m, n) : Q = mP\} \), where \( E, P, Q, \) and \( n \) are public keys and \( m \) a private key.

(iii) For every \( K \in \mathcal{K} \), plaintext \( x \in \mathcal{P} \), and ciphertext \((U, c) \in \mathcal{C} \), and a (secret) random number \( k \in [0, n - 1] \), define the encryption and decryption as follows.

(a) The encryption function is \( e_K(x, k) = (kP, x \cdot x_0) \), where \( kQ = (x_0, y_0) \), and \( x_0 \neq 0 \).

(b) The decryption function is \( d_K(U, c) = c(x_0)^{-1} \), where \( (x_0, y_0) = mU \).

3. Implementation
In this section we will describe several algorithms to perform elliptic curve operations, as well as some modifications made to the simplified ECIES.

**Elliptic Curve Operations**
We first describe the algorithm for point negation. Given a point \( P(x, y) \) on an elliptic curve \( E \), the algorithm simply takes the coordinates of \( P \) as input, then returns \(-P(x, x + y)\) (see Algorithm 1).

**Algorithm 1** Point negation

**Input:** point \( P(x_1, y_1) \).

**Output:** point \(-P\).

1: \( X \leftarrow x_1 \)
2: \( Y \leftarrow x_1 \oplus y_1 \)
3: \textbf{return} \((X, Y)\)
Next, we describe the algorithm for point doubling. We first consider the case when \( P = -P \) and \( P \) is the identity. In this case, the algorithm returns the identity. Otherwise, we compute \( 2P \) using the doubling formula (see Algorithm 2).

**Algorithm 2 Point doubling**

| INPUT: | point \( P(x_1, y_1) \). | OUTPUT: | point \( 2P \). |
|--------|------------------|---------|------------------|
| 1:     | if \( P = -P \) or \( P = O \) then | 4:     | \( \lambda \leftarrow x_1 \oplus y_1 / x_1 \) |
| 2:     | return \( O \) | 5:     | \( X \leftarrow \lambda^2 \oplus \lambda \oplus a \) |
| 3:     | else | 6:     | \( Y \leftarrow x_1^2 \oplus \lambda \cdot X \oplus X \) |
| 7:     | end if | 8:     | \( \text{return } (X, Y) \) |

We now describe the point addition algorithm. We first consider whether \( P = Q \). If this is the case, we utilize the point doubling algorithm. Otherwise, we have several sub-cases to consider. The first sub-case is the case when \( P = -Q \). In this case, return \( O \). The second sub-case is the case when \( P = O \) or \( Q = O \). If \( P = O \), return \( Q \). If \( Q = O \), return \( P \). Should all of these sub-cases fail, we conclude that \( P \) and \( Q \) are distinct and are not the identity. Therefore, we compute \( P + Q \) using the addition formula.

**Algorithm 3 Point addition**

| INPUT: | points \( P(x_1, y_1) \), \( Q(x_2, y_2) \). | OUTPUT: | point \( P + Q \). |
|--------|------------------|---------|------------------|
| 1:     | if \( P \neq Q \) then | 4:     | else |
| 2:     | if \( P = -Q \) then | 5:     | \( \lambda \leftarrow (y_2 + y_1) / (x_2 + x_1) \) |
| 3:     | \( \text{return } O \) | 6:     | end if |
| 4:     | else if \( P = O \) then | 7:     | \( \text{return } Q \) |
| 5:     | \( \text{else if } Q = O \) then | 8:     | \( \text{else} \) |
| 6:     | \( \text{return } P \) | 9:     | \( \text{return } 2P \) |
| 10:    | \( X \leftarrow \lambda^2 \oplus \lambda \oplus x_1 \oplus x_2 \oplus a \) |
| 11:    | \( Y \leftarrow \lambda \cdot (x_1 \oplus X) \oplus X \oplus y_1 \) |
| 12:    | \( \text{return } (X, Y) \) |

Finally, to calculate the scalar multiple of a point, we use the double-and-add method. This method is analogous to the square-and-multiply algorithm for calculating powers. Let \( P \) be a point on an elliptic curve and let \( s \) be a positive integer. First, let \( b_n b_{n-1} \cdots b_0 \) be the binary representation of \( s \). We observe the value of \( b_0 \). If \( b_0 = 1 \), we begin with an initial value of \( P \). Otherwise, we begin with \( O \). Then, for each integer \( i \), where \( 1 \leq i \leq n \), observe \( b_i \). If \( b_i = 1 \), then add \((2^i)P\). For example, suppose we want to compute \( 5P \). The binary representation of \( 5 \) is 101. Therefore, using this method, instead of calculating \( P + P + P + P + P \), we calculate \( 4P + P \). The double-and-add method is more efficient than the naive repeated addition method to compute scalar multiples [9].

**ECIES Parameter Generation**

To begin the encryption and decryption process using the simplified ECIES, we first generate the curve. This is simply done by choosing a random number between 1 and \( 2^N - 1 \), where \( N \) is the degree of the chosen binary field. We then take the binary representation of the random number as the curve parameters \( a \) and \( b \).

Next, we generate the point \( P \). We first randomly choose the \( x \)-coordinate for the point using a similar procedure to the curve parameter generation. Substituting for \( x \) in the curve equation yields an equation in the form \( y^2 + by + d = 0 \). We then determine whether the equation has a solution using Theorem 3.1 [4].
Algorithm 4 Scalar multiple

| Line | Description                                      |
|------|--------------------------------------------------|
| 1    | $A \leftarrow P$                               |
| 2    | $R \leftarrow O$                               |
| 3    | while $n > 0$ do                                |
| 4    | if $n \equiv 1 \mod 2$ then                     |
| 5    | $R \leftarrow R + A$                           |
| 6    | end if                                          |
| 7    | $n \leftarrow n \gg 1$                         |
| 8    | $A \leftarrow 2A$                              |
| 9    | end while                                       |
| 10   | return $R$                                      |

Theorem 3.1. The polynomial $t^2 + t + d \in GF(2^N)[t]$ has a root $u$ in $GF(2^N)$, if and only if $Tr(d) = 0$. $Tr(d)$ is called the trace of an element $d$, which is defined to be

$$Tr(d) = d + d^2 + (d^2)^2 + \cdots + (d^2)^{2^{N-1}}.$$ 

If the equation has a root, we are then able to calculate the roots using Theorem 3.2 [1]. Otherwise, select another $x$-coordinate and repeat the equation checking process.

Theorem 3.2. (Kugurakov) Let $\delta \in GF(2^N)$ and $Tr(\delta) = 0$. Then the equation $y^2 + y = \delta$ has the explicit root $y = 1 + \sum_{j=1}^{m-1} \left( (\delta^2)^j \left( \sum_{k=0}^{j-1} u^{2^k} \right) \right)$ where $u \in GF(2^N)$ and $Tr(u) = 1$. The other root is $y + 1$.

To choose the element $u$ such that $Tr(u) = 1$, we select a random element of $GF(2^N)$, then calculate its trace. This process is repeated until an element with trace 1 is found.

The next step is to calculate the order of $P$. We use Hasse’s theorem [8].

Theorem 3.3. (Hasse) Let $E/F_q$ be an elliptic curve over $GF(q)$. Then,

$$\#E(F_q) = q + 1 - t$$

where $|t| \leq 2\sqrt{q}$.

As a consequence of this theorem, we have $(\sqrt{q} - 1)^2 \leq \#E(F_q) \leq (\sqrt{q} + 1)^2$. Therefore, there exists an integer $M \in [(\sqrt{q} - 1)^2, (\sqrt{q} + 1)^2]$ such that $MP = O$. We can then compute $n$, the order of $P$, as the factor of $M$.

Finally, we can generate the private key $m$ and point $Q = mP$. The private key $m$ is randomly chosen from integers between 1 and $n - 1$, inclusive. Since, by Definition 2.2, the point $P$ generates a cyclic group of order $n$, it suffices to choose $m$ in this manner. After the private key is determined, we compute $Q = mP$ using the double-and-add algorithm.

Encryption and Decryption

Encryption is accomplished by first converting the plaintext into an element, or elements, of $GF(2^N)$. We take the binary representation of the ASCII code of each character in the plaintext. In this paper, it is assumed that every character has an ASCII value of $0 – 127$.

The next step is to group the characters, which are already in their binary representation, into blocks. To do this, first take note that due to the assumption of the ASCII value, the size of the ASCII binary representation is at most 7 bits. We then do a padding process so that every character has a 7-bit binary representation. Once padded, the binary representations are concatenated, then split into blocks where each block has a maximum of $[N/7]$ characters of the original plaintext.

The last step is to encrypt each block of the plaintext using the encryption function defined before.

We provide the proof of correctness for the encryption algorithm (Algorithm 5) as follows.
Algorithm 5 Encryption with the simplified ECIES

INPUT: plaintext \( x \).

OUTPUT: ciphertext \((U(x_1, y_1), y)\).

1: for char \(\in x\) do
2: \(\text{char} \leftarrow \text{BINARYASCII}(\text{char})\)
3: \(\text{char} \leftarrow \text{PADDING(\text{char})}\)
4: \(\text{APPEND(x',char)}\)
5: end for
6: \(\text{blocklength} \leftarrow \lfloor N/7 \rfloor\)
7: \(x' \leftarrow \text{BLOCK}(x', \text{blocklength})\)
8: \(k \leftarrow \text{RANDOM}([1, n-1])\)
9: \(U(x_1, y_1) \leftarrow kP\)
10: \(V(x_2, y_2) \leftarrow kQ\)
11: for char \(\in x'\) do
12: \(\text{cipher} \leftarrow \text{char} \cdot x_2\)
13: \(\text{APPEND}(y, \text{cipher})\)
14: end for
15: return \((U, y)\)

Theorem 3.4. The encryption algorithm with the simplified ECIES is correct.

Proof. We first split the algorithm as described into three blocks: lines 1 – 5, 6 – 10, and 11 – 15. We will establish the correctness of each block.

**Lines 1 – 5.** Let \(x\) be a string and \(x'\) be an array of binary strings of length 7. For the purposes of the algorithm, we allow \(x'\) to be empty. We also note that a string can be viewed as an array of characters. We assert that \(x'\) is a loop invariant. Note that at the beginning of the loop, \(x'\) is an empty array, which we allow. At each iteration, the algorithm will convert the \(n\)th character of \(x\) into its ASCII value in binary form by the \(\text{BINARYASCII}\) function. Its binary form is then padded by the \(\text{PADDING}\) function. Therefore, the character is now a binary string of length 7. This binary string is then appended to \(x'\), which establishes the fact that by the end of the execution of the loop, \(x'\) is an array of binary strings of length 7, verifying our assertion that \(x'\) is indeed a loop invariant. Finally, since we have a 'for' loop, and the length of \(x\) is finite, the loop is guaranteed to terminate. We have established the correctness of this block.

**Lines 6 – 10.** For this block, let \(x'\), the array of binary strings of length 7 and the points \(P\) and \(Q\), the public keys, be our initial assertions. The final assertion will be \(x'\), the array of binary strings of length at most \(N\), and points \(kP\) and \(kQ\). The algorithm on this block first calculates the block length, which is the maximum of characters of the original plaintext that can be concatenated by the \(\text{BLOCK}\) function. Next, the \(\text{BLOCK}\) function manipulates \(x'\) by replacing each of the elements with blocks of concatenated binary representations of at most \(\lfloor N/7 \rfloor\) characters from the original plaintext. Since \(\lfloor N/7 \rfloor \leq N/7\), each block can only have a binary string of length at most \(N\). The algorithm then randomly computes an integer within the \([1, n-1]\) interval. Finally, by the point scalar multiple algorithm, we obtain \(kP\) and \(kQ\). We have completed the proof of correctness for this block.

**Lines 11 – 15.** Let \(x'\) be an array of blocks as previously described, \(U\) be \(kP\), the first part of the ECIES ciphertext, and \(y\) be an array of ciphertexts which make up the second part of the ECIES ciphertext, which we allow to be empty. We first consider the 'for' loop. As with the first block, we assert that \(y\) is a loop invariant. We note the fact that \(y\) is empty at the beginning of the loop. At the execution of the loop, each element of the array is multiplied with \(x_2\), the \(x\)-coordinate of \(kQ\). It is then appended to \(y\). Therefore, at the end of the execution of the loop, \(y\) is indeed an array of ciphertexts. Again, since we have a 'for' loop, the loop is guaranteed to terminate. Finally, the algorithm returns \((U, y)\), which is the complete form of an ECIES ciphertext, establishing the correctness of this block.

Since we have established the correctness of the blocks, we have also established the correctness of the encryption algorithm. □

The decryption process is essentially the reverse of the encryption process. We first decrypt
each block of the ciphertext using the decryption function. Then, we make sure that the length of each block of the decrypted ciphertext is divisible by 7, padding in if necessary. Each block is then split into sub-blocks of length 7. We can then convert each sub-block into the original character of the plaintext.

**Algorithm 6** Decryption with the simplified ECIES

**INPUT:** ciphertext \((U(x_1, y_1), y)\).

**OUTPUT:** plaintext \(x\).

1. \(V(x_2, y_2) \leftarrow mU\)
2. for \(cipher \in y\) do
3. \(char \leftarrow cipher/x_2\)
4. \(char \leftarrow \text{Padding}(char)\)
5. \(\text{Append}(x, char)\)
6. end for
7. \(x \leftarrow \text{Split}(x)\)
8. for \(char \in x\) do
9. \(char \leftarrow \text{DeASCII}(char)\)
10. \(\text{Append}(x', char)\)
11. end for
12. return \(x'\)

We now provide the proof of correctness for the decryption algorithm (Algorithm 6).

**Theorem 3.5.** The decryption algorithm with the simplified ECIES is correct.

**Proof.** We now split the algorithm into two blocks: lines 1 – 6 and 7 – 12.

**Lines 1 – 6.** Let \((U(x_1, y_1))\) be the ECIES ciphertext and \(m\) be the private key. The first line of this block calculates \(V = mU\), which is used to decrypt the ciphertext. Next, we observe the 'for' loop. Let \(x\) be an array of binary representation blocks of the plaintext which length is divisible by 7 and has a maximum length of \(N\). We allow \(x\) to be empty. We assert that \(x\) is a loop invariant. At the beginning of the loop, \(x\) is indeed empty. At each iteration, we divide an element of \(y\) with \(x_2\), the \(x\)-coordinate of \(V\). By Definition 2.2, this decrypts that element to a block of binary representation of the plaintext. The block is then padded so that each block has a length that is divisible by 7 and appended to \(x\). Therefore, at the end of the execution of this loop, we have an array of binary representation blocks of the plaintext which length is divisible by 7 and has a maximum length of \(N\). Again, since we are dealing with a 'for' loop, this loop is guaranteed to terminate, completing this block’s proof of correctness.

**Lines 7 – 12.** The first line of this block splits the array of plaintext blocks, \(x\), into binary strings of length 7. Next, again we face a 'for' loop. Let \(x'\) be the original plaintext. Since \(x'\) is a string, we can treat \(x'\) as an array of characters, which, again, we allow to be empty. We assert that \(x'\) is a loop invariant. On loop entry, \(x'\) is empty. At each iteration, each element of the array is translated back from a binary string ASCII code into the character represented by the ASCII code. This character is appended into \(x'\). Therefore, \(x'\) is an array of characters of the original plaintext, proving that \(x'\) is indeed a loop invariant. Since we are dealing with a 'for' loop, the loop is guaranteed to terminate, and by the termination of the loop, we have recovered all characters of the original plaintext. We have established the correctness of this block.

Since we have established the correctness of each block, we have also established the correctness of the decryption algorithm.

**4. Results**

We implemented the aforementioned algorithms in an encryption-decryption simulation program written in Python. A sample output of the program is given in Figure 1. Basically, the program first randomizes the curve parameters, then picks a point and calculates its order. Afterwards, the user is asked to provide a plaintext. Once the plaintext is set, the program encrypts the plaintext, then displays the resulting ciphertext. Next, the program prompts the user to enter
the ciphertext. Once the ciphertext is set, the program decrypts the ciphertext, then displays the recovered plaintext.

Several encryption and decryption simulations were conducted using the program. During the simulation, we attempt to encrypt the following plaintext:

Lo...
| No. | Curve Parameters | Point chosen | Order | PtGenTime | PtOrdTime | EncTime | DecTime |
|-----|----------------|--------------|-------|-----------|-----------|---------|---------|
| 1   | 11100000100001 | x            | 1048870 | 0.0102    | 22.1138   | 0.3178  | 0.2799  |
| 2   | 11100000100001 | x            | 1048392 | 0.0102    | 17.9539   | 0.3177  | 0.2693  |
| 3   | 11100000100001 | x            | 1049464 | 0.0099    | 17.9539   | 0.3238  | 0.2721  |
| 4   | 11100000100001 | x            | 1050416 | 0.0926    | 38.0370   | 0.2978  | 0.3185  |
| 5   | 11100000100001 | x            | 1047226 | 0.1215    | 9.1655    | 0.3061  | 0.4407  |
| 6   | 11100000100001 | x            | 1048222 | 0.1661    | 28.4010   | 0.3173  | 0.3036  |
| 7   | 11100000100001 | x            | 1048202 | 0.0127    | 30.2198   | 0.2818  | 0.3142  |
| 8   | 11100000100001 | x            | 1047440 | 0.1303    | 15.7834   | 0.3129  | 0.4970  |
| 9   | 11100000100001 | x            | 1049842 | 0.0118    | 33.2842   | 0.2566  | 0.3056  |
| 10  | 11100000100001 | x            | 1047130 | 0.0968    | 7.0309    | 0.2690  | 0.3076  |
| 11  | 11100000100001 | x            | 1050166 | 0.0121    | 37.1911   | 0.2341  | 0.2961  |
| 12  | 11100000100001 | x            | 1049396 | 0.0218    | 29.2950   | 0.2611  | 0.3748  |
| 13  | 11100000100001 | x            | 1046846 | 0.0208    | 3.7891    | 0.2569  | 0.3157  |
| 14  | 11100000100001 | x            | 116202  | 0.1288    | 1.2743    | 0.2677  | 0.2636  |
| 15  | 11100000100001 | x            | 116328  | 0.0309    | 4.6491    | 0.2794  | 0.3814  |
| 16  | 11100000100001 | x            | 349762  | 0.0954    | 29.8363   | 0.2478  | 0.2640  |
| 17  | 11100000100001 | x            | 209482  | 0.1013    | 8.5569    | 0.2518  | 0.2582  |
| 18  | 11100000100001 | x            | 95332   | 0.0238    | 26.3968   | 0.2518  | 0.2466  |
| 19  | 11100000100001 | x            | 13978   | 0.0963    | 19.3744   | 0.2600  | 0.2874  |
| 20  | 11100000100001 | x            | 349548  | 0.1195    | 21.5548   | 0.2499  | 0.2529  |
| 21  | 11100000100001 | x            | 1047328 | 0.0992    | 7.3525    | 0.2393  | 0.2555  |
| 22  | 11100000100001 | x            | 14918   | 0.1355    | 25.2752   | 0.2933  | 0.2811  |
| 23  | 11100000100001 | x            | 1048438 | 0.1017    | 21.1759   | 0.2564  | 0.2529  |
| 24  | 11100000100001 | x            | 209364  | 0.1075    | 2.6863    | 0.2926  | 0.2808  |

Table 1. Experiment results with the encryption-decryption program. The PtGenTime column shows the time required to generate a valid point in the curve. The PtOrdTime column shows the time required to calculate the chosen point’s order. The EncTime and DecTime column shows the time required to process the encryption and decryption, respectively.
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