Finite element analysis of Bi-stable plates for piezoelectric energy harvesting

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Abstract. In order to increase the output voltage of piezoelectric energy harvesting, this paper mainly discusses bi-stable plates for piezoelectric energy harvesting, and its property is analyzed with finite element analysis. The device changes kinetic energy into electrical one, whose core part comprises a substrate, a piezoelectric layer, mass block and the electrode. The research focuses on firstly examining the effect of the area of piezoelectric layer, the temperature and the mass block on the natural frequency of the system, and then analyzing the effect of laying type and area of piezoelectric layer on the output voltage of the system. Eventually a new structure of bi-stable piezoelectric energy harvesting is put forward to improve its harvesting efficiency. This conclusion can be used to provide a theoretical basis for application of piezoelectric technology in engineering.

1. Introduction
Piezoelectric materials have the ability to transform mechanical vibration into electrical energy and vice versa. At present, researchers have researched the application of piezoelectric technology in various fields, and designed various structures of piezoelectric energy harvesting. The bi-stable plates is widely used in engineering because of its structure with good nonlinear characteristics, the very first application on the wings of unmanned aerial vehicle(UAV) warplane and micro air vehicle. In 2008, Cezar.[1] adopted dynamics method to study bi-stable composite laminates, and then especially analyze its bi-stable characteristics under the influence of geometric nonlinearity. Later researches on bi-stable plates in the application of piezoelectric energy harvesting have been widely concerned that bi-stable plate in the non-resonant frequency situation can produce large-scale vibration with its snap-through characteristics. In 2010, Arrieta et al. [9] made non-linear experiments on bi-stable plates of piezoelectric. Experiments showed that the bi-stable plates for piezoelectric energy harvesting can realize the response in a wide range of frequency and achieve high-efficiency power generation. In 2014, Weaver et al. [10] performed a nonlinear analysis of the bi-stable plates with finite element analysis, and the calculated results are very close to the experimental results. The conclusion is drawn that the laying type and the material property of the plates have a great influence on the bi-stable characteristics. In 2014, MA Yepeng et al. [2] analyzed the factors affecting the output voltage of the bi-stable plates for piezoelectric energy harvesting, as well as the relationship between the output voltage and the nonlinear dynamic behavior of piezoelectric energy harvesting, which provides a theoretical basis for the application of bi-stable plates for piezoelectric energy harvesting.

Bi-stable plates for piezoelectric energy harvesting not only increase the bandwidth of the harvesting, but also save the trouble of many additional structures, making it easier to manufacture and more widely used in engineering. Based on the bi-stable plate model, this paper focuses on analyzing...
the effect of the laying type and the area of piezoelectric layer on the output voltage of the piezoelectric energy harvesting with finite element analysis. In addition, the effect of the temperature and the mass block on the natural frequency of the system also is analyzed to find ways to improve the efficiency of the piezoelectric energy harvesting.

2. The System Model of Bi-Stable Plates for Piezoelectric Energy Harvesting
The piezoelectric energy harvesting developed currently generally has the problem of a single operating frequency. Therefore there is not a high efficiency of collecting the vibration energy. Designing a wide-frequency piezoelectric energy harvesting is an effective solution to that problem, in order to collect more energy from vibrating situation and make the operating frequency of the piezoelectric energy harvesting more close to the vibration frequency in the situation.

So far there have been many studies on piezoelectric energy harvesting based on bi-stable plates to increase the frequency bandwidth of the piezoelectric energy harvesting. The transition between the two steady-states can be easily achieved because of its steady state characteristics, without requirements of additional devices compared to other bi-stable structures. In addition, it is also possible to adjust the frequency of the Bi-stable plates for piezoelectric energy harvesting appropriately by adding a mass block to the plates, thus solving some problems of narrow frequency bandwidth and low efficiency of the piezoelectric energy harvesting. Improving the collection efficiency of piezoelectric energy harvesting constantly is the new direction to piezoelectric energy harvesting technology, namely that it is necessary to improve the output voltage of the system by designing a high-efficient piezoelectric energy harvesting.

The piezoelectric energy harvesting is based on a bi-stable plate that was two-layered square plates fixed in the critical midpoint of the plates and free at four edges. The core part of the device comprises a substrate, a piezoelectric layer mass block and the electrode. A schematic diagram of the device was presented in Figure 1.

The principle of this energy harvesting comes from the piezoelectric effect of the piezoelectric layer attached to the substrate, when the system is stimulated by external excitation. The piezoelectric layer deforms under the excitation and the polarization direction of the piezoelectric layer produces a voltage difference, meanwhile the mechanical energy is changed into electrical energy.

3. The Natural Frequency of the Structure
This section mainly studies the influence of temperature and mass blocks on the natural frequency of piezoelectric energy harvesting. ABAQUS was used to analyze the structure of the piezoelectric energy harvesting, since ABAQUS is a finite element analysis tool whose non-linearity is very powerful. The substrate is composed of a square steel plate with a side length of 100 mm and a thickness of 0.5 mm. The piezoelectric layer is also made of a square plate with a thickness of 0.2 mm. And it is amade of PZT-4 and its side length of 10mm~100mm. The material parameters of the substrate and the piezoelectric layer are shown in Table 1, and the piezoelectric properties of the piezoelectric layer are shown in Table 2.
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Table 1. The material parameters

| Parameters       | Piezoelectric | Substrate |
|------------------|---------------|-----------|
| $\rho$(kg/m$^3$) | 7500          | 7800      |
| E(GPa)           | 76            | 200       |
| Poisson ratio    | 0.32          | 0.3       |
| $a$(m/°C)        | $1.0 \times 10^{-5}$ | $1.2 \times 10^{-5}$ |

Table 2. The piezoelectric constant and the dielectric constant

| Piezoelectric constant(C/m$^2$) | Dielectric constant (*$10^{-9}$F/m) |
|----------------------------------|-------------------------------------|
| $e_{31}$                         | $\varepsilon_{11}$ 6.5             |
| $e_{32}$                         | $\varepsilon_{22}$ 6.5             |
| $e_{33}$                         | $\varepsilon_{33}$ 5.6             |

4.1 Effect of Temperature on Natural Frequency

Temperature will change in the process of the laminated plate. Correspondingly thermal stress is generated in the structure with the change of external temperature due to expansion caused by heat and contraction caused by cold. In particular, the laminate leads to large deflection under the influence of temperature due to the two different thermal expansion coefficients. Therefore, it is necessary to consider the effect of temperature on deflection and natural frequency.

Firstly, we study the impact of temperature on the natural frequency. Structural modal analysis with thermal stress using ABAQUS, changing temperature of the structure is used to simulate the process of environmental temperature changes. The negative sign in the table is used to indicate a decrease in
temperature. To make it clearer, a parameter is introduced and defined as the ratio of the length of the piezoelectric layer to the length of the substrate ($\eta = \frac{L_p}{L_s}$). On condition that substrate length is constant, the larger the $\eta$ is, the longer the piezoelectric layer is and the larger the area of the piezoelectric layer is.

**Table 3. Deformation of structures at different temperature (mm)**

| $\eta$ | -200°C | -100°C | 0    | 100°C | 200°C |
|--------|--------|--------|------|-------|-------|
| 0.1    | -0.179 | -0.068 | 0    | 0.0422| 0.0697|
| 0.2    | -0.538 | -0.254 | 0    | 0.166 | 0.314 |
| 0.3    | -0.834 | -0.460 | 0    | 0.315 | 0.48  |
| 0.4    | -1.144 | -0.698 | 0    | 0.506 | 0.736 |
| 0.5    | -1.46  | -0.898 | 0    | 0.685 | 1.024 |
| 0.6    | -1.767 | -1.110 | 0    | 0.882 | 1.300 |
| 0.8    | -2.170 | -1.422 | 0    | 1.211 | 1.763 |
| 1      | -2.727 | -1.889 | 0    | 1.893 | 2.740 |

**Table 4. Natural frequencies of structures at different temperature (Hz)**

| $\eta$ | -200°C | -100°C | 0    | 100°C | 200°C |
|--------|--------|--------|------|-------|-------|
| 0.1    | 126.48 | 135.19 | 146.35| 157.34| 167.48|
| 0.2    | 172.25 | 153.07 | 155.76| 171.25| 187.45|
| 0.3    | 221.7  | 181.26 | 166.05| 190.85| 216.99|
| 0.4    | 264.76 | 211.83 | 170.35| 210.17| 245.63|
| 0.5    | 282.93 | 231.20 | 177.98| 225.92| 262.29|
| 0.6    | 303.62 | 248.05 | 182.46| 239.46| 280.95|
| 0.8    | 282.93 | 231.20 | 177.98| 225.92| 262.29|
| 1      | 346.99 | 281.19 | 169.67| 281.27| 347.20|

The structures of different $\eta$ values are considered respectively, meanwhile the deflection of each structure at different temperatures is obtained (Take the deflection of the node with the most deformation in all nodes as the deflection of the structure) as shown in Table 3. In addition, the first natural frequency of each structure at different temperatures is calculated as shown in Table 4.

In order to make the data more intuitive, the curve graph is drawn as shown in Figure 2 and Figure 3. Figure 2 is the curve graph of the deflection at different temperatures and Figure 3 depicts the natural frequency of the structure.

![Figure 2. Deformation of structures at different temperature](image)
Figure 3. Natural frequencies of structures at different temperature

According to Figure 2: the deflection of the structure increases with temperature and area of the piezoelectric layer increment. The influence of temperature on the structure is more obvious with the increasing area of the piezoelectric layer.

According to Figure 3: when the piezoelectric layer area is relatively small (such as $\eta = 0.1$), the main types of deformation caused by temperature changes are tensile and compression in the plane. The plate is stretched when the temperature rises and the plate is compressed when the temperature decreases. At this point the natural frequency curve with the temperatures is close to a straight line. The main deformation type is bending with the increment of the area of the piezoelectric layer. The deflection of the structure increases with the rising temperature changes. Then the natural frequency curve is close to a parabola.

4.2 The Impact of Mass Blocks on Natural Frequency
The section investigates the effect of the additional mass blocks on the natural frequency of the system. There are four identical masses mounted on the substrate, with their density $8.0 \times 10^{-3}$ g/mm$^3$. The bottom of mass blocks is a square with a side length of 25 mm and its thickness is 10 mm, 20 mm, 30 mm respectively. The location of the mass block is shown in Figure 4.

Figure 4. The location of mass blocks
The system natural frequencies are calculated as shown in Table 5 for both models of $\eta_1 = 0.2$ and $\eta_2 = 0.5$. It can be seen that mass blocks decrease the natural frequency of the system, and the natural frequency of the system decreases with the weight increment of the mass. However stress concentration occurs in the fixed area of the substrate if the mass is overweight. And consequentially the system will be damaged because of its instability. Therefore, the system should be checked for strength and rigidity before its usage, and its maximum stress should be less than the permissible stress of the substrate material ($\sigma_{\text{max}} < [\sigma]$).

5. Output Voltage Research

5.1 The Effect of Laying Type on the Output Voltage

Currently, surround-laying types of piezoelectric layer are often used in many studies, as shown in Structure II in Figure 6. This structure is relatively common due to its ease of manufacture and installation; however, the output voltage and the energy efficiency are not necessarily the highest.

This paper attempts to design a new intermediate-laying type of the piezoelectric layer, and calculates its output voltage compared with the traditional laying type, as shown in Structure I in Figure 5. Structure I and Structure II are exactly same except for the laying type of the piezoelectric layer.

The same concentration force $F$ is loaded on the four corner points of the substrate, and these two different structures will deform under the influence of the concentration force $F$. After calculation, the output voltage of Structure I and Structure II is shown in Figure 7.

Comparing the output voltage of two different structures, the result is as follows: structure I has higher output voltage than Structure II when they are under the same force and the total area of the piezoelectric layer.

In consideration of efficiency of collecting energy, the laying type of structure I is adopted by this device because of its higher output voltage.

| thickness (mm) | $\eta_1 = 0.2$ | $\eta_2 = 0.5$ |
|----------------|---------------|---------------|
| 0              | 114.35        | 146.88        |
| 10             | 57.35         | 71.756        |
| 20             | 43.143        | 53.309        |
| 30             | 35.557        | 43.685        |

Table 5. Effect of mass block on the natural frequency

![Figure 5. Structure I](image1)

![Figure 6. Structure II](image2)
5.2 The Influence of the Area of Piezoelectric Layer on the Output Voltage

The output voltage of the energy harvesting increases with the increase of the piezoelectric layer area. However, the stiffness and the natural frequency of the bi-stable plate will increase due to the increase of the piezoelectric layer area while the deflection of energy harvesting will reduce thanks to the increased stiffness of the bi-stable plate in the same vibrational environment. The output voltage of the energy harvesting system does not increase linearly if the area of the piezoelectric layer increases only. Therefore, both the effect of piezoelectric layer area and the effect of the deflection on the output voltage should be considered when designing a piezoelectric energy harvesting.

This section focuses on the effect of the piezoelectric layer laying area on the output voltage, based on the laying type of Structure I, the maximum output voltage of the piezoelectric layer calculated under the same external force as shown in Figure 8.

As shown in Figure 8, the voltage reaches its maximum when \( \eta = 0.48 \), and the length of the piezoelectric layer is 48 mm.

Figure 7. The output voltage of two laying types

Figure 8. The influence of the area of piezoelectric layer on the output voltage
6. Conclusion
In this paper, the impact of laying type and area of piezoelectric layer on the output voltage of bi-stable plates piezoelectric energy harvesting was analyzed, and the influence of external temperature and additional mass block on its natural frequency is investigated. In contrast to some bi-stable plates for piezoelectric energy harvesting based on surround-laying without mass block in the reference, results are as follows:
1) The intermediate-laying type is better than the surround-laying type of piezoelectric layer in consideration of the output voltage.
2) The device must avoid excessive temperature differentials when it is manufactured because its natural frequency is susceptible to the temperature difference.
3) The natural frequency of the energy harvesting system will reduce if the mass block is installed
4) When the ratio of the length of the piezoelectric layer to the length of the substrate $\eta = 0.48$ and the intermediate-laying type is adopted, the output voltage of the energy harvesting system is the highest.

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