Vacuum Cerenkov Radiation in Lorentz-Violating Theories Without CPT Violation

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Abstract

In theories with broken Lorentz symmetry, Cerenkov radiation may be possible even in vacuum. We analyze the Cerenkov emissions that are associated with the least constrained Lorentz-violating modifications of the photon sector, calculating the threshold energy, the frequency spectrum, and the shape of the Mach cone. In order to obtain sensible results for the total power emitted, we must make use of information contained within the theory which indicates at what scale new physics must enter.
In recent years, there has been a great deal of interest in the possibility of Lorentz violation, since string theory and many other candidate theories of quantum gravity may predict deviations from Lorentz invariance in certain regimes. If Lorentz violation were to be observed experimentally, it would be a discovery of momentous importance and a profound clue regarding the structure of the universe at the most fundamental level. Experimental searches for Lorentz violation—which have thus far not yielded any compelling positive results—have included studies of matter-antimatter asymmetries for trapped charged particles [1, 2, 3, 4] and bound state systems [5, 6], determinations of muon properties [7, 8], analyses of the behavior of spin-polarized matter [9, 10], frequency standard comparisons [11, 12, 13, 14], Michelson-Morley experiments with cryogenic resonators [15, 16, 17], Doppler effect measurements [18, 19], measurements of neutral meson oscillations [20, 21, 22, 23, 24, 25], polarization measurements on the light from distant galaxies [26, 27, 28, 29], analyses of the radiation emitted by energetic astrophysical sources [30, 31], and others.

Significant work has also been done on the theoretical side. A Lorentz- and CPT-violating effective field theory, the standard model extension (SME), has been developed in detail [32, 33, 34]. The theory’s stability, causality [35], and one-loop renormalizability [36], have all been examined. Recent work has also probed the question of how generic Lorentz violation is within quantum field theory [37]. The SME contains coefficients parameterizing all possible observer-independent Lorentz violations. The results of experimental Lorentz tests can be used to place bounds on the coefficients of the minimal SME, which contains only gauge invariant and renormalizable parameters. Some of the minimal SME coefficients are extremely tightly bounded, but the bounds on many other coefficients are weak or even nonexistent.

Scattering and decay processes may be affected in unexpected ways by Lorentz violation. One especially interesting process is vacuum Cerenkov radiation, $e^- \rightarrow e^- \gamma$, which is forbidden in Lorentz-symmetric theories, since the speeds of charged particles are always less than the speed of light propagation. Since vacuum Cerenkov radiation could be an extremely important energy loss process for the highest energy particles [38, 39, 40], a better understanding of this process is needed. We shall look at this kind of radiation in detail, in the presence of one particular type of photon-sector Lorentz violation—the type for which the experimental bounds are the weakest. For definiteness, we shall consider a matter sector with a single fermion field, so that the Lagrange density, omitting the Lorentz violation, is

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} \gamma^\mu (i \partial_\mu - e A_\mu) - m \bar{\psi}. \quad (1)$$

However, the detailed structure of the matter sector is actually unimportant; our calculations would still be valid if the charged particles were bosons.

Lorentz-violating modifications of the electromagnetic sector fall into two categories—those which generate photon birefringence and those which do not. A CPT-odd Chern-Simons term $\mathcal{L}_{AF} = \frac{1}{2} k^\mu_A \epsilon_{\mu\nu\rho\sigma} F^{\nu\rho} A^\sigma$ will always generate birefringence, as will ten of the
nineteen independent coefficients in the CPT-even \( \mathcal{L}_F = -\frac{1}{4} k_{F}^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \). Birefringence has been ruled out very strongly by polarization measurements made on photons that have traversed cosmological distances \([26, 27, 28, 29]\). The remaining nine coefficients, which are contained in the symmetric, traceless \( k_{F\alpha}^{\mu\alpha\nu} \), are much less strongly constrained, at the \( 10^{-16} \) level or worse (compared to \( 10^{-32} \) or better for the birefringent terms). Therefore, if we are interested in looking for potential signatures of actual Lorentz violation in the photon sector, it is most natural to look for effects associated with these particular coefficients.

Ordinary Cerenkov radiation occurs when a charged particle moving in a medium exceeds the speed of light in that medium. Something similar can occur in the vacuum if there is Lorentz violation. When a charged particle is moving faster than the photon signal speed in a given direction, we expect the charge to radiate. This radiation field has been studied using both microscopic \([41]\) and macroscopic \([42, 43]\) electrodynamics, for the situation in which the source of the Lorentz violation is a \( k_{AF} \). However, Cerenkov radiation in the presence of \( k_{F} \) has not been studied in the same detail.

If the ten components of \( k_{F} \) which generate birefringence are set to zero, then \( k_{F} \) takes the form

\[
\tilde{k}_{\mu\nu} = \frac{1}{2} \left( g^{\mu\rho} k_{F \alpha}^{\nu\alpha\rho} - g^{\mu\sigma} k_{F \alpha}^{\nu\alpha\sigma} - g^{\nu\rho} k_{F \alpha}^{\mu\alpha\rho} + g^{\nu\sigma} k_{F \alpha}^{\mu\alpha\sigma} \right).
\] (2)

\( \tilde{k}_{\mu\nu} \equiv k_{F\alpha}^{\mu\alpha\nu} \) is symmetric and traceless in \( (\mu, \nu) \). It is invariant under both C and PT. Since \( \tilde{k} \) should be small, we shall evaluate expressions only to leading order in this parameter.

We shall exploit a duality between the theory with a \( k_{F} \) as in (2) and a different Lorentz-violating theory. The original theory has Lorentz violation in the electromagnetic sector only; the fermion sector is conventional. A coordinate transformation \( x^\mu \rightarrow x'^\mu = x^\mu - \frac{1}{2} \tilde{k}^{\mu} x^\nu x^\nu \) moves all the Lorentz violation into the matter sector \([44]\). To leading order, the transformation of the Lagrange density is

\[
\mathcal{L}_0 + \mathcal{L}_F \rightarrow \mathcal{L}_0 - \frac{1}{2} \tilde{k}^{\mu\nu} \bar{\psi} \gamma_\nu (i \partial_\mu - e A_\mu) \psi.
\] (3)

The \( \tilde{k} \) becomes a \( c \) term in the fermion sector. If the initial theory contained Lorentz violations in both sectors, the effective \( c \) would simply be the sum of the initial fermionic \( c \) and the induced \( c \) coming from the gauge sector. However, we shall neglect this possibility for simplicity. In what we shall call the “original” coordinates, all the Lorentz violation is in the photon sector; and we shall refer to the coordinates in which the Lorentz violation has been moved entirely into the matter sector as the “primed” coordinates, although we shall not write the primes explicitly. Most of the tightest bounds on the combined electron \( c \) and photon \( \tilde{k} \) coefficients come from observations of astrophysical synchrotron and inverse Compton radiation, combined with the lack of observed vacuum Cerenkov radiation \([31]\).
Figure 1: Geometry of Cerenkov shocks in a theory with a direction-dependent speed of light. The charge $e$ is moving with speed $v$, and the the ellipses represent the signal fronts. $c$ denotes the signal speed in the indicated direction.

If the Lorentz violation is all in the photon sector, the physical picture is as follows. The speed of electromagnetic wave propagation is generally direction dependent, and it may be smaller than one. Then a charge moving with a velocity very close to one may be moving faster than the physical speed of light in the same direction, and Cerenkov radiation results. In the dual theory, in the primed coordinates, the fermions’ maximum speeds may be greater than one in certain directions. It is easier to consider this version, with the charges moving superluminally, because when the electromagnetic sector is conventional, standard results for Cerenkov emission may be applied directly.

However, ordinary Cerenkov radiation in matter is emitted only up to some cutoff frequency. Above that frequency, the dielectric constant becomes close enough to one that the signal speed exceeds the charge’s speed. This effect ensures that the total power radiated by the charge is finite. Something similar occurs when the Cerenkov radiation induced by a $k_{AF}$ Chern-Simons term is considered; yet with a $k_F$ that is independent of frequency, there is no such cutoff, and the radiated power would appear to diverge. This is a significant but not insurmountable complication. As we shall see, the theory with $k_F$ quite naturally contains some information about the scale at which new physics must come into play, and this will largely allow us to resolve the issue.

The basic situation—in the original coordinates—is shown in Figure 1. The picture is similar to the conventional one describing Cerenkov radiation, except that the speed of light is not isotropic. The envelope of the ellipses forms the Mach cone—the wave front of the emitted radiation. In general, the cone is neither right angled nor circular; the opening
angles on either side of the charge’s trajectory depend on the Lorentz violation. However, the figure is exaggerated, in that we expect the deviation of the light speed from one in any given direction to be a very small correction, of $O(k)$; and the smallness of the $k$ will simplify the situation significantly. In the primed coordinates, where the electromagnetic sector is conventional, the figure would be different; the signal fronts become circles, and the shocks arise because the charge’s speed is greater than one.

Since $k$ is small, any particle emitting Cerenkov radiation must have a velocity $\tilde{v}$ with magnitude very close to one. (In what follows, $\tilde{v}$ and $v$ will always denote the velocity and speed in the original coordinates where the Lorentz violation is purely electromagnetic.) When the Lorentz violation is moved entirely into the fermion sector (so that we are using primed coordinates), the maximum particle speed in a direction $\hat{e}$ is

$$v + \frac{1}{2} \left[ \tilde{k}_{jk} \hat{e}_j \hat{e}_k + \tilde{k}_{(0j)} \hat{e}_j + \tilde{k}_{00} \right],$$

where $\tilde{k}_{(0j)} = \tilde{k}_{0j} + \tilde{k}_{j0}$. The speed $v$ becomes

$$v + \frac{1}{2} \left[ \tilde{k}_{jk} \hat{v}_j \hat{v}_k + \tilde{k}_{(0j)} \hat{v}_j + \tilde{k}_{00} \right],$$

where $\hat{v}$ is a unit vector in the direction of $\tilde{v}$, and we have neglected the deviation of $v$ from one in the explicitly $\tilde{k}$-dependent terms. The condition for Cerenkov emission is therefore that $1 - v < \frac{1}{2} \left[ \tilde{k}_{jk} \hat{v}_j \hat{v}_k + \tilde{k}_{(0j)} \hat{v}_j + \tilde{k}_{00} \right]$. At high energies, the Lorentz factor is

$$\gamma \approx \frac{m}{\sqrt{\tilde{k}_{jk} \hat{v}_j \hat{v}_k + \tilde{k}_{(0j)} \hat{v}_j + \tilde{k}_{00}}}.$$  \hspace{1cm} (4)

If the square root in (4) is imaginary, the speed of light in the direction $\hat{v}$ is greater than one, and charges moving in that direction will never emit Cerenkov radiation.

In the primed coordinates, the Cerenkov angle $\theta_C$ (the angle between $\hat{v}$ and the emitted radiation) is given by the standard formula $\cos \theta_C = \left\{ v + \frac{1}{2} \left[ \tilde{k}_{jk} \hat{v}_j \hat{v}_k + \tilde{k}_{(0j)} \hat{v}_j + \tilde{k}_{00} \right] \right\}^{-1}$, or

$$\theta_C^2 = \tilde{k}_{jk} \hat{v}_j \hat{v}_k + \tilde{k}_{(0j)} \hat{v}_j + \tilde{k}_{00} - 2(1 - v).$$  \hspace{1cm} (5)

Since $\theta_C^2$ is already $O(k) - (1 - v)$ being at most $O(k)$ when there is Cerenkov emission—the Cerenkov angle is effectively the same in either set of coordinates. Thus, when higher order corrections are neglected, the Cerenkov emission still occurs along a right circular cone; the Mach cone is symmetric about the direction $\hat{v}$, although the width of the cone does depend on which direction the charge is moving. Because the speed of the moving charge can be only very slightly greater than the signal speed, the cone is very broad. The situation is therefore simpler than is depicted in the exaggerated Figure 11, and the shape of the Mach cone is almost completely determined by the local radii of curvature of the signal fronts where they intersect the charge’s straight line path.

All the radiation is emitted at the Cerenkov angle $\theta_C$. The energy radiated per unit frequency per unit time is

$$P(\omega) = \frac{e^2}{4\pi} \theta_C^2 \omega = \frac{e^2}{4\pi} \left[ \tilde{k}_{jk} \hat{v}_j \hat{v}_k + \tilde{k}_{(0j)} \hat{v}_j + \tilde{k}_{00} - 2(1 - v) \right] \omega.$$  \hspace{1cm} (6)
This frequency spectrum is unambiguous, at least at lower frequencies. However, at higher frequencies, this result is somewhat problematic. Since there is a shock front with zero thickness, the electromagnetic field contains Fourier components at arbitrarily short wavelengths, and the total rate of energy emission appears to diverge. Some sort of cutoff is required if we are to obtain a sensible result.

Fortunately, the theory itself contains a natural indication of the correct cutoff scale. The electromagnetic sector alone does not specify any energy scale; however, the matter sector contains the mass $m$. The theory in the primed coordinates runs into causality problems at a scale $O \left( m \tilde{k}^{-1/2} \right)$ [36]. Since electromagnetic and fermionic Lorentz violations mix under renormalization, the Lorentz-violating electromagnetic theory coupled to fermions will fail at roughly the same scale. Some new physics must emerge at this scale, and one of the roles they must play will be to cut off the power spectrum $P(\omega)$ at a scale $\Lambda_{\tilde{k}} \sim m \tilde{k}^{-1/2}$. With multiple species, the relevant mass $m$ is that of the lightest charged particles (i.e., electrons).

Then the total power emitted becomes

$$P = \frac{e^2 m^2}{8\pi} \left( \frac{\theta_C^2 \Lambda_{\tilde{k}}^2}{m^2} \right),$$

most of the emission coming around $\omega \sim \Lambda_{\tilde{k}}$. The expression in parentheses in (7) is dimensionless and $O(1)$. The new physics at the cutoff scale may be Lorentz violating, so $\Lambda_{\tilde{k}}$ could depend on $\hat{v}$. However, if Lorentz violation is small at low energy scales, it is likely to be small at high scales also [15], so the dominant contribution to $\Lambda_{\tilde{k}}$ may be direction independent. Not surprisingly, the threshold energy $E_T$ is at the same scale as $\Lambda_{\tilde{k}}$. Since the new physics only appear at energies above $\Lambda_{\tilde{k}}$, any particle emitting vacuum Cerenkov radiation must be at least that energetic, so that the new physics which are necessary to make the total emission finite can come into play.

Back reaction on the charge is relatively simple. The charge loses energy at the rate $P$, and because $\theta_C$ is small, all the Cerenkov photons are beamed into a narrow pencil of angles around $\hat{v}$. The particle therefore loses energy and momentum at essentially the same rate, in accordance with its ultrarelativistic dispersion relation. As the energy falls close to $E_T$, the emission rate slackens, and the particle slows to the terminal speed $v = 1 - \frac{1}{2} \left[ \tilde{k}_{jk} \hat{v}_j \hat{v}_k + \tilde{k}_{(0j)} \hat{v}_j + \tilde{k}_{00} \right]$.

At energies far above the threshold for emission, we may neglect $(1 - v)$, and the Cerenkov angle approaches $\theta_C^2 = \tilde{k}_{jk} \hat{v}_j \hat{v}_k + \tilde{k}_{(0j)} \hat{v}_j + \tilde{k}_{00}$. According to (7), the rate of energy loss becomes independent of energy, and the lifetime of a particle with energy $E \gg E_T$ is $\tau \sim \frac{8eE}{c\theta_Cb}$. For an electron with an energy of $10^8$ GeV, we find $\tau \sim 10^{-6}$ s. So particles with energies above $E_T$ should lose their excess energies quite quickly.

It may seem paradoxical that the total power emitted is not suppressed by any power of $\tilde{k}$. However, the power per unit frequency is so suppressed, and the overall Cerenkov emission effect is strongly suppressed by the smallness of the Lorentz violation, because as $\tilde{k}$ decreases in size, the threshold $E_T$ for there to be any emission at all is pushed to
higher and higher energies. So low-energy physics will be negligibly affected if $\tilde{k}$ is small enough.

The experimental bounds on the Lorentz-violating but CPT-even $\tilde{k}$ coefficients are by far the weakest of any in the electromagnetic sector. This makes these coefficients potentially among the most interesting in the SME. Analyses of vacuum Cerenkov radiation in the presence of $\tilde{k}$ have previously been stymied by the observation that the total emitted power would probably diverge. However, since the theory automatically contains information about the scale at which new physics must enter, we have been able to circumvent that difficulty, at least enough to obtain order of magnitude estimates; and with these results, it is now possible in principle to search quantitatively for evidence of vacuum Cerenkov radiation in the emissions from the highest-energy particles.

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