Satellite cascade attitude control via fuzzy PD controller with active force control under momentum dumping

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Abstract. In this paper, fuzzy proportional-derivative (PD) controller with active force control (AFC) scheme is studied and employed in the satellite attitude control system equipped with reaction wheels. The momentum dumping is enabled via proportional integral (PI) controller as the system is impractical without momentum dumping control. The attitude controllers are developed together with their governing equations and evaluated through numerical treatment with respect to a reference satellite mission. From the results, it is evident that the three axis attitudes accuracies can be improved up to ±0.001 degree through the fuzzy PD controller with AFC scheme for the attitude control. In addition, the three-axis wheel angular momentums are well maintained during the attitude control tasks.

1. Introduction
Small satellites are widely used for various space missions today. Missions like weather forecasting, remote sensing and data or image collections normally require high mission capabilities. Due to that reason, the accuracy and reliability given by the attitude control of satellite system is very important [1]. Satellites equipped with reaction wheels in pyramid configuration and with a wheel momentum dumping system are studied in this paper. As the behavior of the satellite is inherently nonlinear and uncertain, nonlinear controller like fuzzy controller is recommendable for the attitude control. For flexible satellite, adaptive fuzzy sliding mode controls were explained by Liu et al. [2]. An adaptive fuzzy system combined with $H_2/H_{\infty}$ were discussed in detail by Cheng et al. [3]. Fuzzy PID-controllers were investigated in [4]. The comparison between type 1 and type 2 fuzzy logic controller can be seen in [5] and fuzzy on-off controller algorithm based on Takagi-Sugeno was introduced in [6].

Besides fuzzy, AFC control scheme has been widely used in many applications; robotic controls [7], vehicle suspension system [8], vibration controls [9] and space system [10]. It has been demonstrated to be better compared to conventional methods. All those studies have proven that the dynamic system remains stable and robust in the presence of known and unknown disturbance or uncertainties. Thus, it seems that AFC and fuzzy are suitable for a dynamic system such as satellites that are exposed to the harmful space environment. This paper is organized as follows: the satellite modelling of three axis satellite attitude controls are described in Section 2 while satellite attitude controllers are presented in Section 3. Simulation and results are shown and described in Section 4. Finally, conclusions are given in Section 5.
2. Satellite modelling

2.1. Attitude dynamics and kinematics

The dynamics of a satellite as actuated by reaction wheels can be written as follows [11]:

\[
\dot{\omega} = I^{-1} (T_d + T_m - T_w - \omega \times (I \omega + h_w))
\]  

(1)

where \(\dot{\omega}\) is angular velocity of satellite, \(I\) is moments of inertia of the satellite’s body, \(T_w\) is RWs torques, \(T_m\) is magnetic control torques, \(h_w\) is the RWs angular momentum and \(T_d\) is total external disturbance torques.

The attitude parameterization via quaternion is employed in this work. The advantage of using quaternion is that it does not require computationally intensive trigonometric function evaluations when derived from the rotation matrix, as compared to Euler Angle kinematic differential equations. Moreover, unlike Euler Angle, quaternion does not suffer from singularity problem. The kinematics of the satellite is developed by integrating the its angular velocity and the equation is as follows [11].

\[
\dot{q} = \frac{1}{2} \Omega (\omega_{LVLH/B}) q
\]

(2)

where, \(\Omega\) is the skew symmetric matrix defined as:

\[
\Omega = \begin{bmatrix}
0 & \omega_z & -\omega_y & \omega_x \\
-\omega_z & 0 & \omega_x & -\omega_y \\
\omega_y & -\omega_x & 0 & \omega_z \\
-\omega_x & \omega_y & -\omega_z & 0
\end{bmatrix}
\]

(3)

and \(q\) is the attitude quaternion that represents the attitude of the satellite relative to the local-vertical-local-horizontal (LVLH) coordinate system that is defined as:

\[
q = [q_4, q_2, q_3, q_1]^T
\]

(4)

where \(q_1, q_2, q_3\) are imaginary parts and \(q_4\) is the scalar part. The quaternion can be converted into Euler angles and the conversion of quaternion vector into the Euler angles in the sequences of roll (\(\phi\)) - pitch (\(\theta\)) - yaw (\(\psi\)) are as follows:

\[
\begin{bmatrix}
\phi \\
\theta \\
\psi
\end{bmatrix} = \begin{bmatrix}
\arctan(\frac{2(q_1q_3 + q_2q_4)}{q_1^2 + q_3^2 - q_2^2 - q_4^2}) \\
\arcsin(-2(q_1q_3 - q_2q_4)) \\
\arctan(\frac{2(q_1q_2 + q_3q_4)}{q_4^2 - q_3^2 - q_2^2 + q_1^2})
\end{bmatrix}
\]

(5)

2.2. Reaction wheels configuration

Four reaction wheels configuration is often employed to ensure that the satellite can still be controlled by the remaining wheels in case of one or two wheels are malfunctioning. The pyramid configuration is adopted in this research as shown in Figure 1 [12]. In pyramid configurations, the rotational axes of the four wheels have an inclination to X-Y plane by an angle \(\beta\) and each wheel also can induce torques and momentum in the Z-direction.
From the figure, the relation between each reaction wheel torque \( T = [T_1, T_2, T_3, T_4] \) and the torques developed along the three body axes of the satellite \( T = [T_x, T_y, T_z] \) can be detailed as:

\[
\begin{align*}
T_x &= T_1 \cos \beta - T_3 \cos \beta \\
T_y &= T_2 \cos \beta - T_4 \cos \beta \\
T_z &= T_1 \sin \beta + T_2 \sin \beta + T_3 \sin \beta + T_4 \sin \beta
\end{align*}
\] (6)

Thus the wheel distribution matrix for pyramid configuration is:

\[
A_w = \begin{bmatrix}
\cos \beta & 0 & -\cos \beta & 0 \\
0 & \cos \beta & 0 & -\cos \beta \\
\sin \beta & \sin \beta & \sin \beta & \sin \beta
\end{bmatrix}
\] (7)

As the pyramid configuration is rotated by an angle \( \alpha \) around z axis, the wheel distribution matrix as related to the applied wheel torque can be written as:

\[
\begin{bmatrix}
T_{wx} \\
T_{wy} \\
T_{wz}
\end{bmatrix} = \begin{bmatrix}
\cos \beta \sin \alpha & -\cos \beta \sin \alpha & -\cos \beta \sin \alpha & \cos \beta \sin \alpha \\
-\cos \beta \cos \alpha & \cos \beta \cos \alpha & \cos \beta \cos \alpha & \cos \beta \cos \alpha \\
\sin \beta & \sin \beta & \sin \beta & \sin \beta
\end{bmatrix} \begin{bmatrix}
T_{c1} \\
T_{c2} \\
T_{c3} \\
T_{c4}
\end{bmatrix}
\] (8)

3. Satellite attitude controller

In space, the satellite will undergo many situations like detumbling phase, attitude acquisition and 3-axis stabilization state. For 3-axis satellite attitude control using reaction wheels and under momentum dumping mode, AFC is designed to act as disturbance estimator and reject the external disturbance torques while fuzzy PD controller is designed to act as attitude stabilizing controller in this work.

3.1. Active force control

The AFC algorithm is generally simple in structure, fast in computation and robust against unknown disturbances. The implementation of AFC is easy because the AFC components can be cascaded to the baseline controller [14]. Moreover, computational burden of the control system is alleviated in AFC control scheme, owing to the direct force measurements from the sensors that are readily installed in real-time [9]. The AFC idea used in satellite attitude control application relies on the measurement of angular acceleration, which can be translated to the estimated torques via crude approximation of the satellite inertia [10]. The estimated torques are then fed back to the controller to make the actual contact torques equal to the desired control torques, given by the Attitude Control and Determination.
System (ACDS) specification. In space applications, constant external disturbances will cause the accumulation of momentum in the reaction wheel. If the momentum of the wheel is not actively controlled, the momentum will increase infinitely, thus saturating the wheel and hindering its ability to deliver the necessary control torque. This excess momentum must be dumped for the wheel to operate effectively. The block diagram for AFC with momentum dumping control loop is shown in Figure 2. The Fuzzy PD controller is the stabilizing controller.

![Momentum Dumping Control Loop](image)

**Figure 2.** Block diagram of fuzzy PD controller-AFC with momentum dumping control loop

From the diagram, the estimated disturbance torque $T_d'$ is given by:

$$T_d' = T_\Sigma' + T_w - T_m$$

(1)

where the total torques $T_\Sigma'$ acting on the satellite can be estimated using the formula:

$$T_\Sigma' = l' \alpha$$

(2)

and the estimated satellite inertia property, $l'$ can be computed from law of conservation of angular momentum. Because the satellite inertia property is a slow-varying parameter, the ideal inertia model can be used, $l' = l$ for simulation purpose [10]. The angular acceleration, $\alpha$ is assumed to be measured using the angular accelerometer.

In the AFC scheme, the control law $T_c$ is given by:

$$T_c = T_f + T_d'$$

(3)

where $T_f$ is desired control torque from the Fuzzy PD controller. Assuming perfect actuator dynamics, the actual torque delivered by the reaction wheel, $T_w = T_c$ and the disturbance torque, $T_d$ is eliminated through feed-forward compensation:

$$T_\Sigma = T_m - T_w + T_d$$

$$T_\Sigma = T_m - (T_f + T_d') + T_d$$

(4)

$$T_\Sigma = T_m - T_f$$

The magnetic torque, $T_m$ is necessary to remove the excess momentum accumulated in the reaction wheel.
3.2. Fuzzy PD controller
In designing the Fuzzy PD controller, basic control scheme of PD control has to be constructed first.

3.2.1. PD controller. For the 3-axis satellite attitude control, a common quaternion feedback control scheme based on PD control is employed. The control law thus can be represented as follows [11]:

\[ T_{PD} = 2K_p q_e q_4 + K_d \omega \]

where the error quaternion, \( q_e \), is the quaternion difference between the reference quaternion, \( q_r \), and the current quaternion, \( q_c \). \( T_{PD} \) is a \([3 \times 1]\) vector because \( q_e = (q_1, q_2, q_3) \) is a vector and \( q_4 \) is a scalar. The control gains \( K_p \) and \( K_d \) are matrices defined by:

\[ K_p = \omega_r^2 I \]
\[ K_d = 2 \xi \omega_r I \]

These control gains are the functions of dynamic characteristics, i.e., the natural frequency \( \omega_r \) and the damping ratio \( \xi \).

3.2.2. Fuzzy controller. Numbers of research have shown the successfulness of fuzzy applications on satellite attitude control. The four main components of fuzzy are fuzzification, rule base, rule inference and defuzzification. The fuzzification involves converting the input and output data into suitable linguistic values, which may be viewed as labels of fuzzy sets. In this work, the linear Sugeno-type Fuzzy Inference System (FIS) [16] is employed to design the fuzzy controller. The inputs to the fuzzy controller are the error, \( E(s) \) and its derivative change in error, \( CE(s) \). The linear approximation of control torque by fuzzy controller is given as follow:

\[ T_f(s) = [GE * E(s) + GCE * CE(s)] * GU \]
\[ = GE * GU \left[ E(s) + \frac{GCE}{GE} * CE(s) \right] \]

where the error, \( E(s) \) is computed by comparing the desired attitude and actual attitude, whereas the change of error, \( CE(s) \) is generated by the derivation of the error.

By assuming the error is bounded by ±100% of the step input, so \( GE = 100 \). As \( GE \) is known and by comparing Equation 16 with PD characteristic equations, the gain of \( GU \) can be determined by:

\[ GU = \frac{K_p}{GE} \]
\[ GCE = \frac{K_d}{GU} \]

In addition, as in Figure 2, the output variable of FLC is the control torque, \( T_f \) applied to the actuator. Thus the minimum fuzzy decision rules with two inputs consist of the four rules:

- IF \( e(t) \) is Negative (Neg) and \( \dot{e}(t) \) is Negative (Neg), THEN \( u \) is Negative Big.
- IF \( e(t) \) is Negative (Neg) and \( \dot{e}(t) \) is Positive (Pos), THEN \( u \) is Zero.
- IF \( e(t) \) is Positive (Pos) and \( \dot{e}(t) \) is Negative (Neg), THEN \( u \) is Zero.
- IF \( e(t) \) is Positive (Pos) and \( \dot{e}(t) \) is Positive (Pos), THEN \( u \) is Positive Big.

Figure 3 shows the Sugeno-type fuzzy inference method that has been implemented in this work.

The next step after verifying the linear Fuzzy PD controller is making FIS to achieve nonlinear control surface and fine-tune it. For non-linearization, fuzzy sets with Gaussian membership functions
(MFs) are used and it has been chosen because of their curves have the advantage of being smooth and nonzero at all points. The MFS are evenly distributed so that the tuning process of the controller can be easily done. Figure 4 shows the nonlinear control surface. The higher gain is obtained near the center of the error and change of error plane than the linear surface has. This higher gain helps quickly reduce the error when it is small.

3.3. Momentum dumping control
The purpose of momentum dumping is to reject the excess angular momentum accumulated by the reaction wheels. This can be done by exerting secondary external control torques generated by the magnetic torquers. Basically, the magnetic control torques, \( T_m \), can be generated by multiplying the magnetic dipole moments, \( m \), with Earth’s magnetic field strength in body coordinate systems as indicated by Equation 19.

\[
\begin{bmatrix}
T_{mx} \\
T_{my} \\
T_{mx}
\end{bmatrix} =
\begin{bmatrix}
m_y B_x - m_z B_y \\
m_z B_x - m_y B_z \\
m_x B_y - m_z B_x
\end{bmatrix}
\]

(19)

By adopting the Proportional-Integral (PI) controller, the magnetic dipole moment, \( m \) values can be calculated as:

\[
m = -\frac{K_p + K_i}{B^2} (B_{LVLH} \times \Delta h)
\]

(20)

where \( \Delta h \) is the excess wheel angular momentum to be removed. The proportional and integral gains can be obtained through \( K_p = 2\xi \omega_n \) and \( K_i = \omega_n^2 \), respectively. On the other hand, \( B_{LVLH} \) represents the Earth’s magnetic field strength in LVLH coordinate systems. For high inclination orbit, the magnetic moment \( m \) then can be re-written as:

\[
\begin{bmatrix}
m_x \\
m_y \\
m_z
\end{bmatrix} =
\begin{bmatrix}
\frac{\sin \eta (k, \Delta h_{xy})}{-B_s} \\
\frac{\sin \eta (k, \Delta h_{xz})}{B_z} - \frac{\cos \eta (k, \Delta h_{xz})}{B_z} \\
\frac{\cos \eta (k, \Delta h_{xy})}{B_y}
\end{bmatrix}
\]

(21)

where \( \eta \) is the instantaneous satellite angle (\( \eta = \omega_f \)) and note that \( B \) is the magnitude of the vector \( B_{LVLH} \), whereby \( B = \sqrt{B_x^2 + B_y^2 + B_z^2} \).
4. Simulation and results

To have a basis of comparison, the simulation was first run for satellite attitude control system with PD controller only, second for Fuzzy PD controller and then for Fuzzy PD controller with AFC scheme. The simulations have been performed using Matlab®-Simulink™ software in order to evaluate the satellite attitude performances for the three simulations and momentum dumping control is also enabled for the all simulations. The mission parameters of satellite actuated by four pyramidical reaction wheels with the wheel momentum dumping actuated by magnetic torquers are listed in Table 1 and Table 2.

Table 1. Mission parameter

| Inclination, $i$ | Altitude, $h$ | Orbital period, $t$ | External disturbance torques |
|-----------------|--------------|---------------------|-----------------------------|
| 83 [deg]        | 470 [km]     | 22560 [s] (4 orbit) | $T_{dx} = 8 \times 10^{-5}\sin(\omega_0 \cdot t)$ [Nm], $T_{dy} = 8 \times 10^{-6} + 8 \times 10^{-5}\sin(\omega_0 \cdot t)$[Nm] +$5 \times 10^{-5}\cos(\omega_0 \cdot t)$ [Nm] $T_{dz} = 8 \times 10^{-6} + 5 \times 10^{-5}\cos(\omega_0 \cdot t)$ [Nm] |

Table 2. Satellite and gain parameters

| Parameters | Values |
|------------|--------|
| Moments of inertia matrix [kgm$^2$] | $I_{xx} = 4.2, I_{yy} = 4.4, I_{zz} = 4.2$ |
| Wheels configuration matrix | $\beta = 45^\circ, \alpha = 35.64^\circ$ |
| Limit of the reaction wheels torques [Nm] | $T_c = (0.1 \ 0.1 \ 0.1)^T$ |
| Gain parameters of PD | $K_p = 0.458, K_p = 0.573, K_p = 0.332$ |
| $K_p = [Nm/rad]$ | $K_{dx} = 2.775, K_{dy} = 3.213, K_{dz} = 2.363$ |
| Gain parameters of fuzzy | $GU = 0.00458, GCE = 6048035$ |
| Saturation limit of magnetic torquers [Am$^2$] | $m = \pm 1$ |
| Gain parameters PI for wheel momentum dumping | $K_p = 0.00016, K_p = 0.006, K_p = 0.004$ |
| $K_p [rad/s]$ | $K_{ix} = 6.4 \times 10^{-9}, K_{iy} = 9 \times 10^{-8}, K_{iz} = 4 \times 10^{-8}$ |

The results of the simulations of the attitude control with PD, Fuzzy PD Controller and Fuzzy PD Controller with AFC are as depicted in Figure 5, Figure 6, Figure 7 and Figure 8. Comparing Figure 5 and Figure 6, the performance of Fuzzy PD controller is better than the PD controller. As can be seen in these figures, ±0.015º have been achieved by PD controller while ±0.005º have been achieved by attitude control system using Fuzzy PD Controller. Nevertheless, it can be seen in Figure 7 that the attitude accuracies for roll-pitch-yaw axes have been improved by implementing Fuzzy PD controller with AFC scheme, where ±0.001º attitude accuracies can be achieved. Figure 8 shows the graph of the wheel angular momentum. Over the longer times, the result of simulation shows that the wheel angular momentum was maintained within the acceptable limits during the attitude control operation.
5. Conclusion
The satellite attitude control with reaction wheels via Fuzzy PD controller with AFC is described and tested. The results show that satellite attitude control incorporating the momentum dumping mode with Fuzzy PD controller with AFC demonstrated better attitude performances (±0.001°) in controlling all three satellite axes compared to the satellite attitude control with PD controller or Fuzzy PD controller only. For all options, the three-axis wheel angular momentums are well maintained during the attitude control tasks.

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