The CMB bispectrum from bouncing cosmologies

Paola C. M. Delgado, a Ruth Durrer b and Nelson Pinto-Neto c

a Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, ul. prof. Stanisława Łojasiewicza 11, 30-348 Krakow, Poland
b Département de Physique Théorique and CAP, Université de Genève, 24 quai Ernest-Ansermet, CH-1211 Genève, Switzerland
c Department of Cosmology, Astrophysics and Fundamental Interactions, Centro Brasileiro de Pesquisas Físicas, rua Dr. Xavier Sigaud 150, 22290-180, Rio de Janeiro, Brazil
E-mail: paola.moreira.delgado@doctoral.uj.edu.pl, ruth.durrer@unige.ch, nelsonpn@cbpf.br

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Abstract. In this paper we compute the CMB bispectrum for bouncing models motivated by Loop Quantum Cosmology. Despite the fact that the primordial bispectrum of these models is decaying exponentially above a large pivot scale, we find that the cumulative signal-to-noise ratio of the bispectrum induced in the CMB from scales $\ell < 30$ is larger than 10 in all cases of interest and therefore can, in principle, be detected in the Planck data.

Keywords: non-gaussianity, CMBR theory, Inflation and CMBR theory, quantum cosmology

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1 Introduction

The standard cosmological model, solidly grounded in General Relativity theory (GR) and a variety of cosmological observations (the cosmic microwave background radiation and its anisotropies [1], the abundance of light elements [2], the features of the distribution of large scale structures and cosmological red-shifts [3, 4], among others), asserts that the Universe is expanding from a very hot era dominated by radiation, when the geometry of space was highly homogeneous and isotropic, with very small deviations from this special, symmetric state. However, extrapolating the standard cosmological model back to the past using GR, one necessarily encounters a singularity, where physical quantities diverge. Hence, the model is incomplete: GR is pointing us to its own limits, requiring new physics to understand these extreme situations, which is still under debate.

Assuming that the Universe had a beginning immediately followed by a hot expanding phase implies some important new puzzles, related to initial conditions. The size of regions in causal contact in the Universe is given by the Hubble radius, \( R_H \equiv |1/H| \equiv |a/\dot{a}| \), where \( a \) is the scale factor, and the overdot represents a derivative with respect to cosmic time. The Hubble radius evolves with respect to the scale factor as

\[
\frac{d \ln(R_H)}{d \ln(a)} = 1 - \frac{\ddot{a} a}{\dot{a}^2}.
\]  

(1.1)

If the cosmic fluid has non-negative pressure in the hot era, the Friedmann equations imply \( \ddot{a} < 0 \), so that \( d \ln(R_H)/d \ln(a) > 1 \). Hence, in the past of an expanding universe the size of cosmological scales we are able to see today, which evolve with the scale factor, were much larger than the Hubble radius. This implies that the basic properties of the cosmic microwave background radiation (CMB), its temperature isotropy and its tiny anisotropies, cannot be explained by causal physics, as the scales presenting these observed properties contained hundreds of causally disconnected regions on the last scattering surface. This is called the ‘horizon problem’. Furthermore, the observed matter and dark energy density of
the Universe today, $\rho_0$, is very close to the total energy density of a Universe with flat spatial sections, $\rho_c$. Using again the Friedmann equations, the ratio $\Omega(t) \equiv \rho(t)/\rho_c(t)$ evolves as

$$\frac{d}{dt} \left| \frac{\Omega(t) - 1}{\Omega(t)} \right| = -\frac{\ddot{a}}{a^3}. \quad (1.2)$$

As $\dot{a} > 0$ and $\ddot{a} < 0$, if $\Omega(t)$ is close to unity today, it must have been much closer to unity at earlier times, a spectacular fine tuning of initial conditions. This is the so called ‘flatness problem’.

A simple solution to these puzzles is to evoke that at some early stage a new field dominates the Universe evolution, for which $p/\rho < -1/3$ which implies $\ddot{a} > 0$, such that $d \ln(R_H)/d \ln(a) < 1$ and $d |\Omega(t) - 1|/dt < 0$. This primordial phase is called inflation [5–8], usually driven by a simple scalar field, which can be investigated in the framework of standard quantum field theory in curved spacetime. It not only solves the above puzzles, but it also predicts the observed almost scale invariant marginally red-tilted spectrum of primordial cosmological scalar perturbations [9]. Inflation became an essential part of any cosmological model in which the Universe has a beginning.

However, there is an alternative simple solution to the above puzzles if one assumes that the Universe had a very long contracting decelerating phase before the present expanding era. In this case, eq. (1.1) implies that the Hubble radius was much bigger than any cosmological scale of physical interest in the far past of the contracting phase, because a contracting universe running backwards in time implies larger scale factors. Also, as $\dot{a} < 0$, $\ddot{a} < 0$, and $\ddot{a}/\dot{a}^3 > 0$, eq. (1.2) implies that flatness becomes an attractor in the contracting phase, rather than a repeller. Hence, the Universe looks spatially flat to us now because it has not expanded long enough in comparison with the very long contracting phase it experienced in the past.

In realistic models with a contracting phase, there must be a bounce connecting it to the present expanding phase. In GR contraction generally leads to the time reversal of the Big Bang singularity, the Big Crunch. To avoid this, bouncing models must necessarily involve new physics. In other words, bouncing models must face the cosmological singularity problem, which is not addressed by inflation. A realistic bouncing model then not only solves the above puzzles related to initial conditions, but it is also complete, i.e., free of singularities.

The new physics required for bouncing models can be classical extensions of GR [10–17], or quantum gravity effects [18–29]. Different models have been investigated in the last decades, and it has been shown that many of them [30–34] satisfy the constraints imposed by cosmological observations on the properties of primordial cosmological perturbations, and other cosmological features, without the need of an inflationary phase. Nevertheless, bouncing models are not incompatible with inflation. Indeed, in some scenarios, bouncing models lead to the appropriate initial conditions for inflation [35].

The next step in this investigation is to find observable ‘fingerprints’, which indicate the existence of a previous contracting phase and a bounce. In ref. [36], in the context of bouncing models with inflation, it was shown that non-Gaussianities originating from the contracting era before inflation [37] can substantially alleviate the large scale anomalies detected in the CMB [38]. The model contains a canonical scalar field with potential $V(\varphi)$, and the scale factor around the bounce is generically parametrized as

$$a(t) = a_b(1 + bt^2)^n, \quad (1.3)$$

where $t$ is cosmic time, the bounce happens at $t = 0$, $a_b$ is the scale factor at the bounce, and $b$ is a constant parametrizing the Ricci scalar at the bounce, $R_b = 12nb$. The parameter
controls the way the model enters and leaves the bouncing phase and starts classical expansion. For \( n \approx 1/6 \), the scalar field energy density just after the bounce is concentrated in the kinetic term (this is the case of Loop Quantum Cosmology (LQC) models [28]), and inflation starts later, while for larger values of \( n \) the scalar field potential is already relevant at the bounce, and inflation starts earlier. Hence, the features of this class of bouncing models are controlled by the bounce Ricci scalar \( R_b \), and by \( n \).

The initial quantum state for the perturbations is chosen to be the adiabatic (Minkowski) vacuum in the far past of the contracting phase. Therefore the quantum state of cosmological perturbations at the onset of inflation deviates from the Bunch-Davies vacuum. In terms of the modes one can write

\[
v_k(\eta) = \alpha_k v_k^{BD}(\eta) + \beta_k v_k^{*BD}(\eta),
\]

where \( \eta \) is conformal time, implying that the ratio between the primordial dimensionless power spectrum \( P_R(k) \) and the pure Bunch-Davies primordial dimensionless power spectrum \( P_R^{BD}(k) \) reads

\[
\frac{P_R(k)}{P_R^{BD}(k)} = |\alpha_k + \beta_k|^2.
\]

This class of models has two fundamental scales: the bounce scale given by the comoving wave number \( k_b = a_b \sqrt{R_b/6} \), and the (comoving) inflation scale \( k_i = 2\pi a_i \sqrt{R_i/6} \), where \( a_i \) and \( R_i \) are the scale factor and Ricci scalar at the beginning of inflation, respectively. As the energy scale of the bounce is larger than the energy scale of inflation, \( k_i < k_b \). One has three different regimes for the power spectrum: \( k > k_b \), \( k_i < k < k_b \), and \( k < k_i \) (corresponding to length scales smaller than the bounce length scale, bigger than the bounce length scale but smaller than the inflation length scale, and bigger than the inflation length scale, respectively). Scales smaller than the bounce scale do not feel the bounce, hence they will not deviate from inflationary (Bunch-Davies) results. However, the two other scales are affected by the bounce, leading to different physical effects. Indeed, it is shown in ref. [36] that non-Gaussianities arise, correlating super-horizon modes with infrared scales, enhancing the probability of the appearance of CMB anomalies at large scales. If the duration of inflation is very long, such effects are suppressed, yielding the constraint \( n < 1/4 \) for these effects to be significant. The scales which contribute most to the non-Gaussianity are in the range \( k_i < k < k_b \), which is larger for a bounce closer to the Plank scale. Hence, models with bounce phases occurring at length scales larger than the Planck length need a larger \( f_{nl} \) parameter to yield the desired effects, some of them require \( f_{nl} \) of order \( 10^4 \). As the non-Gaussian correlations obtained in ref. [36] are restricted mostly to super-horizon modes, the authors suggest that these large values of \( f_{nl} \) should not be directly observed. However, the CMB bispectrum of these models is not calculated, and it is not clear under which conditions the model satisfies the Planck constraints on it [39].

The aim of this short paper is to fill this gap. We calculate the bispectrum for the two representative models mostly studied in ref. [36]: the Loop Quantum Cosmology case \( n = 1/6 \) [28], and the \( n = 0.21 \) case, which seems to best mitigate the CMB anomalies according to ref. [36]. For these two cases, we consider the minimum and maximum values of \( f_{nl} \) allowed (3326 and 8518 for \( n = 1/6 \), and 959 and 4372 for \( n = 0.21 \)). We test their viability against Planck measurements, and we calculate the signal-to-noise ratio (SNR) of the bispectrum in order to decide whether it can be measured in a CMB experiment which is cosmic variance limited at low multipoles, \( \ell < 30 \), like the Planck experiment [40].
The numerical calculations are presented in section 2. We find that the bispectrum can indeed be quite large at large scales. However, for all values (ℓ₁, ℓ₂, ℓ₃) with ℓᵢ ≥ 4, i = 1, 2, 3, it remains smaller than cosmic variance. Nevertheless, we find that the cumulative SNR for a cosmic variance limited CMB experiment with 70% sky coverage becomes larger than 10 for all models considered here. In section 3 we end with a discussion and conclusions.

2 The bispectrum in a bouncing model

2.1 The theoretical expressions

The bouncing model discussed in [36] has the following dimensionless power spectrum, \( \mathcal{P}_R(k) \), and bispectrum, \( B(k_1, k_2, k_3) \), of the curvature fluctuations in Fourier space.

\[
\mathcal{P}_R(k) = A_s \left\{ \begin{array}{ll}
(k/k_i)^2 (k_i/k_b)^q & \text{if } k \leq k_i \\
(k/k_b)^q & \text{if } k_i < k \leq k_b \\
(k/k_b)^{n_s-1} & \text{if } k > k_b .
\end{array} \right. \tag{2.1}
\]

\[
B(k_1, k_2, k_3) = \frac{3}{5} (2\pi)^2 f_{nl} \left[ \frac{\mathcal{P}_R(k_1) \mathcal{P}_R(k_2)}{k_1^3 k_2^3} + \frac{\mathcal{P}_R(k_1) \mathcal{P}_R(k_3)}{k_1^3 k_3^3} + \frac{\mathcal{P}_R(k_2) \mathcal{P}_R(k_3)}{k_2^3 k_3^3} \right] \times \exp \left( -\frac{\gamma}{k_b} k_1 + k_2 + k_3 \right). \tag{2.2}
\]

Here \( n_s = 0.9659 \) and \( A_s = 2.3424 \times 10^{-9} \), corresponding to the Planck values [41]. The inflation and bounce (pivot) scales are, respectively, \( k_i = 10^{-6}\text{Mpc}^{-1} \) and \( k_b = 0.002\text{Mpc}^{-1} \). The parameters \( q, f_{nl} \) and \( \gamma = \sqrt{n_\pi/2T[1-n]} / \Gamma[3/2-n] \) depend on the details of the bounce, being related to the parameter \( n \) in eq. (1.3). See [36] for details. The values of \( q \) and \( f_{nl} \) used in this work are shown in table 1. The power spectrum for \( n = 1/6 \) is shown in figure 1.

The bispectrum (2.2) is decaying exponentially for \( k > k_b \). Since \( k_b \) is close to the horizon scale, the authors of ref. [36] argue that the model is not excluded by observations, even for quite large values of \( f_{nl} \). Apart from this exponential decay, which is of course crucial to render a bispectrum with such a large value of \( f_{nl} \) viable, the bispectrum (2.2) is actually just the local bispectrum. Due to the strong exponential decay, however, its overlap with the local bispectrum is small. In appendix B we determine numerically the overlap of the LQC bispectrum with the local, equilateral and orthogonal shapes. We find that the overlap with all of them is very small even though the first is somewhat larger than the latter two. The values requested for \( f_{nl} \) given in table 1 are much larger than the Planck limit for the local shape which is \( f_{nl} \lesssim 10 \), see [42]. This is possible since most of the Planck constraint comes from smaller scales, where the LQC bispectrum is exponentially suppressed.

In this work we check quantitatively whether the LQC bispectrum with the parameters given in table 1 is compatible with observations. Clearly, this non-Gaussianity is best
Figure 1. The power spectrum $P_R(k)$ versus $k$ for $n = 1/6$. Note the three different regimes separated by the inflationary and the bounce scales.

constrained on very large scales corresponding to $k \lesssim k_b$. This motivates us to compute the CMB bispectrum induced by it, concentrating on the largest angular scales. Expanding the CMB temperature fluctuations in spherical harmonics,

$$\Delta T \big( n \big) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(n),$$

the bispectrum is defined by

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle = G_{\ell_1 \ell_2 \ell_3}^{\ell_1 \ell_2 \ell_3} b_{\ell_1 \ell_2 \ell_3}.$$  

Here $G_{\ell_1 \ell_2 \ell_3}^{\ell_1 \ell_2 \ell_3}$ is the so called ‘Gaunt factor’ which can be expressed in terms of the Wigner $3j$-symbols as

$$G_{\ell_1 \ell_2 \ell_3}^{\ell_1 \ell_2 \ell_3} = \sqrt{\prod_{j=1}^{3} (2\ell_j + 1) \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right)} b_{\ell_1 \ell_2 \ell_3}.$$  

The $m_i$-dependent prefactor is a consequence of statistical isotropy, see [43] for details. The model dependent quantity $b_{\ell_1 \ell_2 \ell_3}$ is called reduced bispectrum. It depends only on the values $\ell_1, \ell_2, \ell_3$ and vanishes if these do not satisfy the triangle inequality, $|\ell_1 - \ell_2| \leq \ell_3 \leq \ell_1 + \ell_2$, or if the sum $\ell_1 + \ell_2 + \ell_3$ is odd.

Within linear perturbation theory, the reduced CMB bispectrum is entirely determined by the primordial bispectrum of the curvature fluctuations in Fourier space. More precisely,

$$b_{\ell_1 \ell_2 \ell_3} = \left( \frac{2}{\pi} \right)^3 \int_0^\infty dx x^2 \int_0^\infty dk_1 \int_0^\infty dk_2 \int_0^\infty dk_3 \times \prod_{j=1}^{3} T(k_j, \ell_j) j_{\ell_j}(k_j x) (k_1 k_2 k_3)^2 B(k_1, k_2, k_3),$$

where $T(k, \ell)$ is the CMB transfer function and $j_\ell$ is the spherical Bessel function of index $\ell$, see [43] for more details. On large scales, considering only the Sachs Wolfe term, we can approximate the transfer function by

$$T(k, \ell) \simeq \frac{1}{5} j_\ell(k(t_0 - t_{\text{dec}})).$$
Our bispectrum in $k$-space is separable, i.e., it can be written as a sum of products of functions of $k_i$,

$$(k_1k_2k_3)^2 B(k_1,k_2,k_3) = B_0 \left[ f(k_1)f(k_2)g(k_3) + f(k_1)f(k_3)g(k_2) + f(k_2)f(k_3)g(k_1) \right]$$

where

$$B_0 = \frac{3}{5}(2\pi^2)^2 f_{nl}$$

$$f(k) = \frac{\gamma_2}{k} \exp(-\gamma k/k_b)$$

and

$$g(k) = k^2 \exp(-\gamma k/k_b)$$

Setting

$$X_\ell(x,k) = T(k,\ell) j_\ell(kx)f(k)$$

and

$$Z_\ell(x,k) = T(k,\ell) j_\ell(kx)g(k)$$

with eq. (2.6) we obtain

$$b_{\ell_1\ell_2\ell_3} = \left( \frac{2}{\pi} \right)^3 B_0 \int dx x^2 \int_0^\infty dk_1 \int_0^\infty dk_2 \int_0^\infty dk_3 \left[ X_{\ell_1}(x,k_1)X_{\ell_2}(x,k_2)Z_{\ell_3}(x,k_3) + X_{\ell_1}(x,k_1)X_{\ell_3}(x,k_3)Z_{\ell_2}(x,k_2) + X_{\ell_2}(x,k_2)Z_{\ell_1}(x,k_1) \right]$$

This is the sum of three separable $k$-integrals. We introduce

$$X_\ell(x) = \int_0^\infty dk X_\ell(x,k)$$

and

$$Z_\ell(x) = \int_0^\infty dk Z_\ell(x,k)$$

such that the bispectrum becomes the following integral:

$$b_{\ell_1\ell_2\ell_3} = \left( \frac{2}{\pi} \right)^3 B_0 \int dx x^2 \left[ X_{\ell_1}(x)X_{\ell_2}(x)Z_{\ell_3}(x) + X_{\ell_1}(x)X_{\ell_3}(x)Z_{\ell_2}(x) + X_{\ell_2}(x)Z_{\ell_1}(x) \right]$$

2.2 Numerical calculations

The functions $X_\ell(x,k)$ and $Z_\ell(x,k)$ are heavily oscillating as functions of $k$ (containing products of two Bessel functions of different arguments) and difficult to integrate. However when $x = t_0 - t_{\text{dec}}$ the product $j_\ell(k(t_0 - t_{\text{dec}}))j_\ell(kx)$ becomes a square and both $X_\ell(x,k)$ and $Z_\ell(x,k)$ are positive definite functions of $k$ for this value of $x$. We therefore expect that the integrals over $k$, $X_\ell(x)$ and $Z_\ell(x)$ peak at $x = t_0 - t_{\text{dec}}$. As an example, see the functions $X_\ell(x)$ and $Z_\ell(x)$ in figure 2 for $n = 1/6$ and in figure 3 for $n = 0.21$. We see that both functions decay rapidly for growing $|x - (t_0 - t_{\text{dec}})|$ for the two values of $n$ considered. One might hope that due to this feature, which is the basic idea behind the Limber approximation,
the Limber approximation might be relatively good. However, as we show in appendix A this is not the case. The Limber approximation actually overestimates the signal by more than one order of magnitude at low \( \ell \).

The bispectrum (2.18) can be computed numerically if one takes into account the peaked behavior described above, restricting the integration range for the integral over \( x \) to an interval around \( x = t_0 - t_{\text{dec}} \). As we can see in figures 2 and 3, the width of the central peak is larger for low values of the multipole \( \ell \). For this reason we choose an \( x \)-range around \( t_0 - t_{\text{dec}} \) of \( 10^3 \) Mpc for \( \ell_1 + \ell_2 + \ell_3 \leq 90 \) and \( 2000 \) Mpc for \( \ell_1 + \ell_2 + \ell_3 > 90 \), encompassing a large percentage of the total contribution. For the latter cases, the difference with respect to

Figure 2. The functions \( X_\ell(x) \) and \( Z_\ell(x) \) for \( n = 1/6 \) with the integration over \( k \) performed up to \( k_{\text{max}} = 10^{-2} \) Mpc\(^{-1} \). Note the sharp peaks at \( x = t_0 - t_{\text{dec}} \) and the oscillatory behavior that is especially visible for higher multipoles. Left: \( \ell = 10 \) and \( \ell = 15 \); right: \( \ell = 95 \) and \( \ell = 100 \).

Figure 3. The functions \( X_\ell(x) \) and \( Z_\ell(x) \) for \( n = 0.21 \) with the integration over \( k \) performed up to \( k_{\text{max}} = 10^{-2} \) Mpc\(^{-1} \). Again we see the sharp peaks at \( x = t_0 - t_{\text{dec}} \) and the oscillatory behavior for higher multipoles. Left: \( \ell = 10 \) and \( \ell = 15 \); right: \( \ell = 95 \) and \( \ell = 100 \).
The bispectrum $b_{\ell_1 \ell_2 \ell_3}$ as a function of $\ell_1$ and $\ell = \ell_2 = \ell_3$ for $n = 1/6$ (left) and $n = 0.21$ (right). The dots correspond to numerical results for $f_{\text{nl}} = 8518$ in the case $n = 1/6$ and $f_{\text{nl}} = 4372$ in the case $n = 0.21$. For the other values of $f_{\text{nl}}$ in table 1, the plots are re-scaled by the ratios of the $f_{\text{nl}}$’s, the factors of $0.390$ and $0.219$ for $n = 1/6$ and $n = 0.21$, respectively. The gray planes correspond to the product fits obtained in section 2.4, eqs. (2.26) and (2.28).

We want to compare the present model and the bispectrum for the local shape. To do so, we first fix $\ell_1 = 4$, require $\ell_2 = \ell_3 = \ell$ and perform the numerical computations for the bispectrum of the current work. Then, recalling that the reduced bispectrum of the local shape is given by [43]

$$b_{\ell_1 \ell_2 \ell_3}^{(\text{local})} = \frac{3f_{\text{nl}}(2\pi^2 A_s)^2}{4 \times 5^4} \left( \frac{1}{\ell_1(\ell_1 + 1)\ell_2(\ell_2 + 1)} + \frac{1}{\ell_1(\ell_1 + 1)\ell_3(\ell_3 + 1)} + \frac{1}{\ell_2(\ell_2 + 1)\ell_3(\ell_3 + 1)} \right),$$

we substitute $\ell_1 = 4$, $\ell_2 = \ell_3 = \ell$ and $f_{\text{nl}} = 5.0$, this value of $f_{\text{nl}}$ is chosen based on the Planck constraint on local non-Gaussianity [39]. In figure 5, we plot the bispectra versus $\ell$.

The bispectrum of the bounce followed by an inflationary phase is larger than the local bispectrum for all the low multipoles considered here. However, this does not mean that it is ruled out by the Planck observations, as most of the observational power from Planck limiting the bispectrum comes from higher values of $\ell$ which are not present in this plot and for which $b^{(\text{local})}$ is much larger than the bispectrum from our bouncing models.
In order to decide whether the bispectrum of the bouncing models discussed here can be ruled out, we consider its signal-to-noise by adding cosmic variance as the dominant noise source on large scales. This is the minimal error on $b_{\ell_1 \ell_2 \ell_3}$. Here we follow [44]. Let us introduce the random variable

$$\hat{B}_{\ell_1 \ell_2 \ell_3} = \sum_{m_1 m_2 m_3} \binom{\ell_1 \ell_2 \ell_3}{m_1 m_2 m_3} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}. \quad (2.20)$$

This is an estimator of $B_{\ell_1 \ell_2 \ell_3}$ defined in (2.4), i.e., $\langle \hat{B}_{\ell_1 \ell_2 \ell_3} \rangle = B_{\ell_1 \ell_2 \ell_3}$. In the same way as

$$\hat{C}_\ell = (2\ell + 1)^{-1} \sum_m |a_{\ell m}|^2 \quad (2.21)$$

is an estimator of the power spectrum $C_\ell = \langle |a_{\ell m}|^2 \rangle$. Using identities of the Wigner 3j symbols and neglecting terms involving the bispectrum, which are much smaller than the terms from the power spectrum to the third power, one finds for the variance of our estimator $\hat{B}_{\ell_1 \ell_2 \ell_3}$

$$\text{var}(B_{\ell_1 \ell_2 \ell_3}) \simeq \langle \hat{B}_{\ell_1 \ell_2 \ell_3}^2 \rangle \simeq C_{\ell_1} C_{\ell_2} C_{\ell_3} \left(1 + \delta_{\ell_1 \ell_2} + \delta_{\ell_1 \ell_3} + \delta_{\ell_2 \ell_3} + 2\delta_{\ell_1 \ell_2} \delta_{\ell_2 \ell_3}\right). \quad (2.22)$$

For the reduced bispectrum this yields

$$\text{var}(b_{\ell_1 \ell_2 \ell_3}) \simeq \langle \hat{g}_{\ell_1 \ell_2 \ell_3}^2 \rangle \simeq C_{\ell_1} C_{\ell_2} C_{\ell_3} \left(1 + \delta_{\ell_1 \ell_2} + \delta_{\ell_1 \ell_3} + \delta_{\ell_2 \ell_3} + 2\delta_{\ell_1 \ell_2} \delta_{\ell_2 \ell_3}\right). \quad (2.23)$$

Of course this equality is only valid when $g_{\ell_1 \ell_2 \ell_3} \neq 0$, i.e., for values of $\ell_1, \ell_2, \ell_3$ which satisfy the triangle inequality and are such that $\ell_1 + \ell_2 + \ell_3$ is even, since otherwise $b_{\ell_1 \ell_2 \ell_3} = 0$ with vanishing variance.

The minimal error on $b_{\ell_1 \ell_2 \ell_3}$ an experiment measuring all $a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$ with negligible instrumental noise is (the square root of) the cosmic variance. The latter is computed using the values of $C_\ell$ obtained with the Cosmic Linear Anisotropy Solving System (CLASS) [45, 46] and compared to the amplitude of the bispectrum of the bouncing model in figure 5. Clearly, the amplitude of the square root of the cosmic variance is larger than the bispectrum for all values of $\ell$ considered, i.e. $\ell_i \geq 4$, $i = 1, 2, 3$. This precludes a measurement of the individual $b_{\ell_1 \ell_2 \ell_3}$’s, but in order to investigate whether the model can be ruled out due to its non-Gaussianity, we have to go on and compute the total signal-to-noise ratio.

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure5_left}
\includegraphics[width=0.45\textwidth]{figure5_right}
\caption{The bispectrum of the current work with $n = 1/6$ (left) and $n = 0.21$ (right) and the local bispectrum with $f_{\text{nl}} = 5.0$, considering multipoles such that $\ell_1 = 4$ and $\ell_2 = \ell_3 = \ell$. The dots correspond to $f_{\text{nl}} = 8518$ for $n = 1/6$ and to $f_{\text{nl}} = 4372$ for $n = 0.21$. For the other values of $f_{\text{nl}}$ in table 1, the bispectrum is re-scaled by factors of 0.390 for $n = 1/6$ and 0.219 for $n = 0.21$. The local bispectrum and the square root of cosmic variance are depicted as the black and gray lines, respectively.}
\end{figure}
2.4 Signal-to-noise ratio

For each individual triple \((\ell_1, \ell_2, \ell_3)\) with \(\ell_i \geq 4, \ i = 1, 2, 3\), the square root of cosmic variance is larger than the value of the bispectrum. However, this does not mean that such a bispectrum is not detectable. To decide this, we estimate the cumulative signal-to-noise ratio (SNR) of the entire bispectrum for \(\ell_i \leq \ell_{\text{max}}\). We choose \(\ell_{\text{max}} = 80\) to make sure that the Sachs-Wolfe term calculated here really is the dominant contribution. However, as we shall see, the SNR saturates already at \(\ell_{\text{max}} \sim 30\).

\[
\left( \frac{S}{N} \right)^2 (\ell_{\text{max}}) = \sum_{\ell_1\ell_2\ell_3=2}^{\ell_{\text{max}}} \frac{b_{\ell_1\ell_2\ell_3}^2}{\text{var}(b_{\ell_1\ell_2\ell_3})}. \tag{2.24}
\]

In order to perform this computation, we fit the numerical results of the bispectrum obtained for different sets of multipoles by a product ansatz, including the ones where \(\ell_1 \neq \ell_2 \neq \ell_3\). The fits of our product approximation read

\[
\ln(b_{\ell_1\ell_2\ell_3}) = -3.727 \times 10^{-6}(\ell_1\ell_2\ell_3) - 2.225 \ln(\ell_1\ell_2\ell_3) - 25.607 \tag{2.25}
\]

for \(n = 1/6\) and \(f_{nl} = 3326\),

\[
\ln(b_{\ell_1\ell_2\ell_3}) = -3.727 \times 10^{-6}(\ell_1\ell_2\ell_3) - 2.225 \ln(\ell_1\ell_2\ell_3) - 24.667 \tag{2.26}
\]

for \(n = 1/6\) and \(f_{nl} = 8518\),

\[
\ln(b_{\ell_1\ell_2\ell_3}) = -3.204 \times 10^{-6}(\ell_1\ell_2\ell_3) - 2.661 \ln(\ell_1\ell_2\ell_3) - 22.491 \tag{2.27}
\]

for \(n = 0.21\) and \(f_{nl} = 959\) and

\[
\ln(b_{\ell_1\ell_2\ell_3}) = -3.204 \times 10^{-6}(\ell_1\ell_2\ell_3) - 2.661 \ln(\ell_1\ell_2\ell_3) - 20.974 \tag{2.28}
\]

for \(n = 0.21\) and \(f_{nl} = 4372\). Two of them are shown in figure 6, while the other two are simply re-scaled by the ratios of the \(f_{nl}\) values.

The variance is obtained from eq. (2.23), as before. Summing all the terms corresponding to allowed sets of multipoles, i.e. with \(\ell_1 + \ell_2 + \ell_3\) even and \(|\ell_1 - \ell_2| \leq \ell_3 \leq \ell_1 + \ell_2\), we obtain the cumulative SNR as a function of the maximum multipoles \(\ell_{\text{max}}\).

Our purpose it to investigate whether we achieve a value of order \(\mathcal{O}(10)\) within the low \(\ell\) regime, i.e. \(\ell \ll 200\), which corresponds to the validity of the transfer function given in eq. (2.7), and which is also the regime in which the non-Gaussianity of these models is larger. The results are shown in figure 7. In order to consider a sky coverage of 70\%, we multiply the cosmic variance by 1/0.70. Clearly, the SNR saturates very fast, namely roughly at \(\ell_{\text{max}} = 30\), but it achieves a value larger than 25, both for \(n = 1/6\) and \(n = 0.21\) for the larger value of \(f_{nl}\). For the smaller values of \(f_{nl}\) the cumulative SNR for \(n = 1/6\) is 10.3 while for \(n = 0.21\) it is 10.6. In all cases these bispectra should be detectable in the Planck data.

3 Conclusions

In this paper we have investigated the CMB bispectrum induced by bouncing models which are motivated by Loop Quantum Cosmology. These models have been proposed in order to address the large scale anomalies of the CMB data. The hope was, that due to the exponential decay of the bispectrum on subhorizon scales, this non-Gaussianity is not substantial. In this
work we show, however, that in all cases with sufficient non-Gaussianity to mitigate the large scale anomalies of CMB data, the bispectrum should be detectable in the Planck data. Even though the individual $b_{\ell_1 \ell_2 \ell_3}$ are below cosmic variance if $\ell_i \geq 4$, $i = 1, 2, 3$, the cumulative SNR of the bispectrum with $\ell_{\text{max}} = 30$ is larger than 10 for all the models proposed, when assuming a sky coverage of 70% and considering only temperature data. Note that the largest contributions to the SNR come from triples $(\ell_1, \ell_2, \ell_3)$ where at least one multipole is smaller than 4, for which the signal is larger than or comparable to the square root of the variance. For the higher values of $f_{\text{nl}}$ the cumulative SNR is about 26.5 ($n = 1/6$) and 48.2 ($n = 0.21$) respectively. To get an impression of the amplitude of the SNR of these models, one may want to compare it to the one of CMB lensing, which is about 40 in the Planck 2015 data, see [47]. Note that already at $\ell_{\text{max}} = 5$ the SNR is larger than 15 ($n = 1/6$) and 35 ($n = 0.21$). But also for the two models with the lower value of $f_{\text{nl}}$, the cumulative SNR is actually just slightly above 10, so that the bispectrum can in principle be detected. In order to obtain an undetectable bispectrum one would have to reduce the $f_{\text{nl}}$ by about a factor of 10, so that the cumulative SNR would become of order unity. However, when reducing $f_{\text{nl}}$ to these values, the CMB large scale anomalies can no longer be resolved efficiently by these
Figure 8. The Limber approximation (dashed) is compared with our product approximation (solid) for $\ell_2 = \ell_3 = 30$ fixed, as a function of $\ell_1$. Even though the Limber approximation is probably better for $\ell > 50$, it is much worse than our excellent product approximation in the relevant regime, $\ell_1 < 30$.

models and they lose one of their main attractive features. Furthermore, adding polarisation data may well enhance the SNR by about a factor of two.

These findings motivate us to perform a search for this bispectrum in the actual Planck data.

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A Limber approximation

We also considered to solve the integrals (2.16) and (2.17) with $T(k, \ell) = j_\ell(k(t_0 - t_{\text{dec}}))/5$ using the Limber approximation \[48\]. This yields

$$X_\ell(x) = \int dk T(k, \ell) j_\ell(kx) f(k) \approx \frac{1}{5} \int dk j_\ell(k(t_0 - t_{\text{dec}})) j_\ell(kx) f(k)$$

$$\simeq \frac{\pi}{10} \delta(t_0 - t_{\text{dec}} - x) \frac{1}{(\ell + 1/2)^2} f \left( \frac{\ell + 1/2}{t_0 - t_{\text{dec}}} \right) e^{-\gamma_b \frac{\ell + 1/2}{b}} \tag{A.1}$$

$$Z_\ell(x) = \int dk T(k, \ell) j_\ell(k) g(k) \approx \frac{1}{5} \int dk j_\ell(k(t_0 - t_{\text{dec}})) j_\ell(kx) g(k)$$

$$\simeq \frac{\pi}{10} \delta(t_0 - t_{\text{dec}} - x) \frac{1}{(\ell + 1/2)^2} g \left( \frac{\ell + 1/2}{t_0 - t_{\text{dec}}} \right) e^{-\gamma_b \frac{\ell + 1/2}{b}} \tag{A.2}$$
Here we used the functions $f$ and $g$ defined in eqs. (2.11) and (2.12). Of course, the product of three delta functions cannot be integrated, but replacing them by narrow Gaussians we obtain up to an unknown constant $A$, related to the width of the Gaussian,

\[ b_{\ell_1,\ell_2,\ell_3} = \left(\frac{2}{\pi}\right)^3 B_0 \int dxx^2 X_{\ell_1}(x)X_{\ell_2}(x)Z_{\ell_3}(x) \]

\[ \sim \frac{AB_0}{125} \exp\left(-\frac{\ell_1 + \ell_2 + \ell_3}{k_b(t_0 - t_{dec})}\right) \left[ \mathcal{P}_R \left(\frac{\ell_1 + 1/2}{t_0 - t_{dec}}\right) \mathcal{P}_R \left(\frac{\ell_2 + 1/2}{t_0 - t_{dec}}\right) + \mathcal{O}\right]. \]  

(A.3)

Here $\mathcal{O}$ indicates that the two permutations $(\ell_1, \ell_2) \rightarrow (\ell_1, \ell_3)$ and $(\ell_1, \ell_2) \rightarrow (\ell_3, \ell_2)$ have to be added. We have tested this approximation with our numerical computations and found that for the relevant values of $\ell$, $\ell < 30$, which contribute most to the SNR, it is much less accurate than the product approximation which we use in the main text, see figure 8.

B Overlap with standard bispectrum shapes

In this appendix we determine the overlap of the LQC bispectrum with other standard shapes as they are studied, e.g., in the Planck analysis [42]. For this purpose we define the dimensionless shape function

\[ S(k_1, k_2, k_3) = \alpha(k_1, k_2, k_3)^2 B(k_1, k_2, k_3), \]  

(B.1)

where $\alpha$ is a normalization factor such that $S(k_c, k_c, k_c) = 1$ for a characteristic scale $k_c$. For the model analyzed in this work, we choose $k_c = k_b$ so that

\[ \alpha = \frac{5}{9} \frac{e^{\gamma}}{f_{nl}(2\pi)^2 P_R^2(k_b)}. \]  

(B.2)

For the standard cosmological scenario, one usually defines the local, equilateral and orthogonal shapes, which are given respectively by

\[ S^{(\text{local})}(k_1, k_2, k_3) = \frac{1}{3}\left(\frac{k_1^2}{k_2k_3} + \frac{k_2^2}{k_1k_3} + \frac{k_3^2}{k_1k_2}\right), \]  

(B.3)

\[ S^{(\text{equi})}(k_1, k_2, k_3) = \frac{(k_1 + k_2 - k_3)(k_2 + k_3 - k_1)(k_3 + k_1 - k_2)}{k_1k_2k_3}, \]  

(B.4)

\[ S^{(\text{ortho})}(k_1, k_2, k_3) = \frac{-3k_1^2 + 3[k_1^2 - (k_2 - k_3)^2](k_3 + k_2) + 3k_2^2 + k_2(3k_2 - 8k_3)}{k_1k_2k_3}. \]  

(B.5)

Since these shapes are not orthogonal to each other in general, one may construct a scalar product

\[ \langle S_1, S_2 \rangle = \int_V S_1(k_1, k_2, k_3)S_2(k_1, k_2, k_3)w(k_1, k_2, k_3)dk_1dk_2dk_3 \]  

(B.6)

within a region $V$ with $|k_1 - k_2| \leq k_3 \leq k_1 + k_2$, where the shape functions $S_1$ and $S_2$ do not diverge and the weight function $w(k_1, k_2, k_3)$ is an arbitrary non-negative function. One can then define an “angle” $\theta$ between the shapes $S_1$ and $S_2$ by

\[ \cos \theta_{12} = \frac{\langle S_1, S_2 \rangle}{\sqrt{\langle S_1, S_1 \rangle \langle S_2, S_2 \rangle}}. \]  

(B.7)

\footnotetext[1]{The orthogonal shape is orthogonal to the equilateral one for a weight function $(k_1k_2k_3)^{-1}$.}
If this angle is large, \( \cos \theta_{12} \ll 1 \), there is a small overlap. We choose a weight function given by

\[
W(k_1, k_2, k_3) = \begin{cases} 
(k_1 k_2 k_3)^4 & \text{if } 10^{-4} \leq k_i \leq 0.04 \\
0 & \text{else}
\end{cases},
\]

where \( i = 1, 2, 3 \). We first perform the integration over \( k_3 \) with lower and upper limits given respectively by \( |k_1 - k_2| \) and \( k_1 + k_2 \). The step function is chosen in order to avoid \( k_i = 0 \), since the shape functions diverge for \( k_i \to 0 \), and to consider the interesting range for the bispectrum under investigation, i.e. low values of \( k \). The power law was chosen so that the expression to be integrated is regular for all values of \( k_i \) considered. Performing the integrations

\[
\langle S^{(\text{bounce})}, S^{(j)} \rangle = \int_{10^{-4}}^{4 \times 10^{-2}} dk_1 \int_{10^{-4}}^{4 \times 10^{-2}} dk_2 \int_{|k_1 - k_2|}^{k_1 + k_2} dk_3 S^{(\text{bounce})} S^{(j)} w, \quad (B.9)
\]

where \((j) = (\text{local}), (\text{equi}), (\text{ortho})\), and normalizing the results, we obtain for \( n = 1/6 \)

\[
\begin{align*}
\cos \theta^{(\text{bounce, local})} &= 2.369 \times 10^{-4}, \\
\cos \theta^{(\text{bounce, equi})} &= 2.364 \times 10^{-4}, \\
\cos \theta^{(\text{bounce, ortho})} &= -3.985 \times 10^{-5},
\end{align*}
\]

and for \( n = 0.21 \)

\[
\begin{align*}
\cos \theta^{(\text{bounce, local})} &= 7.117 \times 10^{-5}, \\
\cos \theta^{(\text{bounce, equi})} &= 7.071 \times 10^{-5}, \\
\cos \theta^{(\text{bounce, ortho})} &= -1.206 \times 10^{-5}.
\end{align*}
\]

The results do not depend on \( f_{nl} \), as it cancels in the expression of the shape function. The projection is only slightly larger for the local than for the equilateral shape, but it is very small for all three standard shapes. This is a consequence of the rapid exponential decay of the bouncing bispectrum.

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\(^2\) The integration limits for \( dk_1 \) and \( dk_2 \) are chosen to encompass the non-negligible region after integration over \( k_3 \).
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