Matrix Orientifolding and Models with Four or Eight Supercharges

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Abstract

The conditions under which matrix orientifolding and supersymmetry transformations commute are known to be stringent. Here we present the cases possessing four or eight supercharges upon $\mathbb{Z}_3$ orbifolding followed by matrix orientifolding. These cases descend from the matrix models with eight plus eight supercharges. There are fifty in total, which we enumerate.

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I. Introduction

Continuing attention has been paid to matrix models which are proposed to enable non-perturbative studies of strings beyond their perturbative and semiclassical regimes. The objects playing a central role are, of course, discretized string coordinates represented by matrices taking values of appropriate Lie algebras. Their diagonal entries represent spacetime points, while off-diagonal ones mediate interactions between blocks which may be identified as D-objects. A few ideas on the formation of our spacetime such as the one via branched polymers and the one via generalized monopoles have appeared and approximation schemes have been devised. (See for more references.)

Not only the string coordinates but also algebraic operations in the first quantized string theory have natural matrix counterparts even when the size of the matrices is kept finite. In particular, the matrix counterpart of twist operation or orientifolding is easily obtained as any Lie algebra valued matrix splits into a direct sum of the adjoint representation and the antisymmetric representation of \( USp(2k) \) or \( SO(2k) \) Lie algebra. Selecting one of these two representations for each of the original matrix coordinates is referred to as matrix orientifolding in this paper.

Realizing the twist operation of matrices this way has turned out to put stringent conditions on the number of supercharges: the supersymmetry transformations in the Wess-Zumino gauge are non-linear and requiring that they commute with the projectors materializing matrix orientifolding yields nontrivial algebraic conditions. In the case of 8 + 8 supercharges, these conditions are successful in selecting the two known cases which corresponds to the \( USp \) matrix model relevant to type I superstrings and the matrix model of heterotic M theory. In the light of assessing these algebraic conditions further and of hoping to find principal matrix configurations leading to \( \mathcal{N} = 1 \) vacua in four dimensions, it is interesting to find out how many cases of matrix orientifolding one can construct which possess fewer supercharges. To put this question more concrete, consider the matrix analog of \( C^3 / \mathbb{Z}_3 \) and subsequently operate matrix orientifolding. In this paper, we focus upon the problem of enumerating all possible such cases with supersymmetries, namely, the ones obtained by \( \mathbb{Z}_3 \) orbifolding followed by matrix orientifolding.

In the next section, we recall the two cases of matrix orientifolding with 8 + 8 supercharges. After introducing \( \mathbb{Z}_3 \) orbifolding acting upon three complex matrix coordinates and its prototypical example in section III, we carry out the matrix orientifolding of this example in section IV. We show that there are two consistent possibilities with respect to supersymmetries and that there are in total five cases: the one possesses 4 + 0 supersymmetries while the remaining four possess 2 + 2 supersymmetries. The problem to enumerate
all cases obtained upon an arbitrary $\mathbb{Z}_3$ orbifolding and subsequently matrix orientifolding while keeping some supersymmetries intact is addressed in section V. $\mathbb{Z}_3$ orbifolding leaves either $4+4$ supersymmetries or $8+8$ supersymmetries intact. We show that, to each of the four possibilities belonging to the former, there is one case of consistent matrix orientifolding with $4+0$ supersymmetries and four with $2+2$ supersymmetries. As for each of the six possibilities belonging to the latter ($8+8$ supersymmetries), we show that there is also one with $8+0$ supersymmetries and four with $4+4$ supersymmetries. The total number of such cases is fifty. This number is considered to be small in the light of an innumerable number of perturbative superstring vacua.

II. Matrix Orientifolding with 8+8 Supercharges

The action of the IIB matrix model is

$$ S = -\frac{1}{g^2} \text{Tr} \left( \frac{1}{4} [A_N, A_M] [A^N, A^M] + \frac{1}{2} \bar{\psi} \Gamma^N [A_N, \psi] \right). $$ (2.1)

Here $\psi$ is a ten-dimensional Majorana-Weyl spinor, and $A_I$ and $\psi$ are $N \times N$ Hermitian matrices. The action has dynamical supersymmetry

$$ \delta^{(1)} \psi = \frac{i}{2} [A_N, A_M] \Gamma^{NM} \epsilon, $$ (2.2)
$$ \delta^{(1)} A_N = i \bar{\epsilon} \Gamma^N \psi, $$ (2.3)

and kinematical supersymmetry

$$ \delta^{(2)} \psi = \xi, $$ (2.4)
$$ \delta^{(2)} A_N = 0. $$ (2.5)

As is mentioned in the introduction, any $U(2k)$ Lie algebra valued matrix splits into a direct sum of the two matrices which are respectively the adjoint representation and the antisymmetric representation of $USp(2k)$ Lie algebra and this is schematically drawn as

$$ U(2k) \text{ adjoint} \xrightarrow{\varphi_+} \text{USp adjoint} \xleftarrow{\varphi_-} \text{USp antisymmetric} $$

adj $X : X^t F + FX = 0$ (2.6)

asy $Y : Y^t F - FY = 0.$ (2.7)
Here $F$ is the matrix counterpart of the twist operation

$$F = \begin{pmatrix} 0 & I_k \\ -I_k & 0 \end{pmatrix},$$  

(2.8)

and $\hat{\rho}_\mp$ are the projectors

$$\hat{\rho}_\pm \cdot = \frac{1}{2}(\cdot \mp F^{-1} \cdot^t F).$$  

(2.9)

Let

$$v_M \equiv \delta_M^{(N)} \hat{\rho}_b^{-} A_N,$$

$$\Psi_A \equiv \delta_A^{(B)} \hat{\rho}_f^{-} \psi_B,$$  

(2.10)

where $\hat{\rho}_b^{(N)}$ and $\hat{\rho}_f^{(B)}$ are either $\hat{\rho}_-$ or $\hat{\rho}_+$ for each $N$ and for each $B$ respectively. More explicitly

$$\hat{\rho}_b^{(M)} \equiv \Theta(M \in \mathcal{M}_-) \hat{\rho}_- + \Theta(M \in \mathcal{M}_+) \hat{\rho}_+,$$

$$\hat{\rho}_f^{(A)} \equiv \Theta(A \in \mathcal{A}_-) \hat{\rho}_- + \Theta(M \in \mathcal{A}_+) \hat{\rho}_+,$$  

(2.11)

where

$$\mathcal{M}_- \cup \mathcal{M}_+ = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, \quad \mathcal{M}_- \cap \mathcal{M}_+ = \emptyset,$$  

(2.12)

$$\mathcal{A}_- \cup \mathcal{A}_+ = \{1, 2, 5, 6, 9, 10, 13, 14, 19, 20, 23, 24, 27, 28, 31, 32\}, \quad \mathcal{A}_- \cap \mathcal{A}_+ = \emptyset.$$  

(2.13)

By construction, each component of $v_M$ and that of $\Psi_A$ belong either to the adjoint or to the antisymmetric representation of USp(2k). We impose eq.\,(2.10) on $A_N$ and $\psi_B$. The condition $[\hat{\rho}_b^{(N)}, \delta_1] A = 0$ gives

$$\sum_A (\epsilon \Gamma_M)_A (\hat{\rho}_f^{(A)} - \hat{\rho}_b^{(M)}) \psi_A = 0$$  

(2.14)

with $M$ not summed, while the condition $[\hat{\rho}_f^{(A)}, \delta_1] \psi|_{v_M \rightarrow \hat{\rho}_b v_M} = 0$ gives

$$(1 - \hat{\rho}_f^{(A)}) [\hat{\rho}_b^{(M)} A_M, \hat{\rho}_b^{(N)} A_N] (\Gamma^{MN} \epsilon)_A = 0.$$  

(2.15)

The condition $[\hat{\rho}_b^{(N)}, \delta_1] A = 0$ does not give us anything new while $[\hat{\rho}_f^{(A)}, \delta_2] \psi = 0$ gives

$$\xi_A \mathbf{1} = \xi_A \hat{\rho}_f^{(A)} \mathbf{1}.$$  

(2.16)

Eq.\,(2.14) gives

$$(\epsilon \Gamma_{M_+})_{A_+} = (\epsilon \Gamma_{M_-})_{A_-} = 0,$$  

(2.17)
while eq. (2.15) gives
\[ (\Gamma^{M-N^+} \epsilon)_{A_-} = 0, \quad (\Gamma^{M-N} \epsilon) = (\Gamma^{M+N^+} \epsilon)_{A_+} = 0, \] (2.18)
and eq. (2.16) gives
\[ \xi_{A_-} = 0. \] (2.19)

Let
\[ \epsilon = (\epsilon_0, 0, \epsilon_1, 0, 0, 0, 0, 0, \epsilon_0, 0, \epsilon_1, 0, 0, 0)^t. \] (2.20)

The strategy to find solutions to eq. (2.17), (2.18) under eq. (2.14), (2.15), namely, that of finding two pairs of nonintersecting sets \( \mathcal{M}_- \) and \( \mathcal{M}_+ \) and \( \mathcal{A}_- \) and \( \mathcal{A}_+ \) are fully described in [3] and we will not repeat it here. The solution is
\[ \mathcal{M}_- = \{0, 1, 2, 3, 4, 7\}, \quad \mathcal{M}_+ = \{5, 6, 8, 9\}, \] (2.21)
\[ \mathcal{A}_- = \{1, 2, 5, 6, 19, 20, 23, 24\}, \quad \mathcal{A}_+ = \{9, 10, 13, 14, 27, 28, 31, 32\}, \] (2.22)
and this leads to the one of the two known cases of possessing \( 8+8 \) supercharges. The corresponding projectors are
\[ \hat{\rho}_{b+} = \text{diag}(\hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_-, \hat{\rho}_+), \]
\[ \hat{\rho}_{f+} = \hat{\rho}_- I_4 \otimes \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix} + \hat{\rho}_+ I_4 \otimes \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}. \] (2.23)

The other solution with \( 8+8 \) supercharges is
\[ \mathcal{M}_- = \{4, 7\}, \quad \mathcal{M}_+ = \{0, 1, 2, 3, 5, 6, 8, 9\}, \] (2.24)
\[ \mathcal{A}_- = \{1, 2, 5, 6, 27, 28, 31, 32\}, \quad \mathcal{A}_+ = \{9, 10, 13, 14, 19, 20, 23, 24\}. \] (2.25)

The corresponding projectors are
\[ \hat{\rho}_{b+} = \text{diag}(\hat{\rho}_+, \hat{\rho}_+, \hat{\rho}_+, \hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_-, \hat{\rho}_+, \hat{\rho}_+), \]
\[ \hat{\rho}_{f+} = \hat{\rho}_- I_4 \otimes \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix} + \hat{\rho}_+ I_4 \otimes \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}. \] (2.26)

*For further developments of the USp matrix model, see [13, 14]. The complete construction of this matrix model includes the \( n_f = 16 \) sectors belonging to the (anti-)fundamental representation. The use of USp Lie algebra is required by the SO(2n_f) Chan-Paton factor realized by open loop variables. [13, 14]*
III. $\mathbb{Z}_3$ Orbifolding

We now describe $\mathbb{Z}_3$ orbifolding of the IIB matrix model. Let $A_N = (A_\mu (\mu = 0, \ldots, 3), B_1 = A_4 + i A_5, B_2 = A_6 + i A_7, B_3 = A_8 + i A_9)$. The complex coordinates $B_i$ are postulated to transform under $\mathbb{Z}_3$ as

$$B_i \rightarrow \omega^{a_i} B_i,$$  \quad (3.1)

where $a_i$ are integers and $\omega$ is a cubic root of unity. We introduce the 'tHooft matrices

$$U = \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix}, \quad V = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$  \quad (3.2)

which satisfy $UV = \omega VU$. The $\mathbb{Z}_3$ transformation is given by $M \rightarrow UMU^\dagger$. The $\mathbb{Z}_3$ invariant bosonic matrices thus satisfy the conditions;

$$A_\mu = U A_\mu U^\dagger, \quad B_i = \omega^{a_i} U B_i U^\dagger.$$  \quad (3.3)

In order to find the conditions for the fermionic matrices, let us note that ten dimensional chirality operator $\Gamma^{10}$ can be thought of as the product of the lower dimensional chirality operators $\Gamma^{10} = (i \Gamma^0 \cdots \Gamma^3) \cdot (i \Gamma^4 \Gamma^5) \cdot (i \Gamma^6 \Gamma^7) \cdot (i \Gamma^8 \Gamma^9)$ and that $\psi$ is expanded by a set of eigenfunctions $\psi_i \sim \psi_3$, $\psi_0^c \sim \psi_3^c$

$$\psi = \sum_{i=0}^3 (\psi_i + (\psi_i)^c).$$

The eigenfunctions satisfy the conditions:

$$\psi_i = \omega^{b_i} U \psi_i U^\dagger,$$  \quad (3.4)

where $b_i$ are given by the table

| $\Gamma^{10}$ | $i \Gamma^{0123}$ | $i \Gamma^{45}$ | $i \Gamma^{67}$ | $i \Gamma^{89}$ | $b_i$ |
|---------------|-----------------|----------------|----------------|----------------|-------|
| +             | +               | +             | +             | +             | $\psi_0$ | $- (a_1 + a_2 + a_3)/2$ |
| +             | +               | -             | -             | -             | $\psi_1$ | $- (a_1 - a_2 - a_3)/2$ |
| +             | -               | +             | -             | -             | $\psi_2$ | $- (-a_1 + a_2 - a_3)/2$ |
| +             | -               | -             | -             | +             | $\psi_3$ | $- (-a_1 - a_2 + a_3)/2$ |
| -             | -               | -             | -             | -             | $(\psi_0)^c$ | $- (-a_1 - a_2 - a_3)/2$ |
| -             | -               | +             | +             | -             | $(\psi_1)^c$ | $- (-a_1 + a_2 + a_3)/2$ |
| -             | +               | -             | -             | +             | $(\psi_2)^c$ | $- (a_1 - a_2 + a_3)/2$ |
| -             | +               | +             | -             | -             | $(\psi_3)^c$ | $- (a_1 + a_2 - a_3)/2$ |
The bosonic part of the action is

\[ S_b = -\frac{1}{4g^2} \text{Tr} \left( [A_\mu, A_\nu]^2 + 2 \sum_{i=1}^3 [A_\mu, B_i][A^\mu, B_i^\dagger] + \frac{1}{2} \sum_{i,j=1}^3 \left( [B_i, B_j^\dagger][B_i^\dagger, B_j] + [B_i, B_j][B_i^\dagger, B_j^\dagger] \right) \right), \]

and the fermionic part is

\[ S_f = -\frac{1}{2g^2} \text{Tr} \left( \sum_{i=0}^3 \bar{\psi}_i \Gamma^\mu [A_\mu, \psi_i] + 2 \sum_{i=1}^3 (\bar{\psi}_i) \Gamma^{(i)} [B_i^\dagger, \psi_0] + \sum_{i,j,k=1}^3 |\epsilon_{ijk}| (\bar{\psi}_i) \Gamma^{(j)} [B_j, \psi_k] + h.c. \right), \]

where \( \Gamma^{(1)} = \frac{1}{2}(\Gamma^4 - i\Gamma^5) \), \( \Gamma^{(1)} = \frac{1}{2}(\Gamma^4 + i\Gamma^5) \) and so on.

A prototypical example is

\[ a_i = 2 \quad \text{for } i = 1, 2, 3, \]

and

\[ b_0 = 0 \quad \text{and } b_i = -2 = 1 \mod 3 \quad \text{for } i = 1, 2, 3. \]

Using the ’tHooft matrices, we can represent \( \mathbb{Z}_3 \) invariant matrices \( A_\mu, B_i, \psi_0, \) and \( \psi_i \) as

\[ A_\mu = \sum_{a=0}^2 A_\mu^a \otimes U^a, \quad B_i = \sum_{a=0}^2 B_i^a \otimes (U^a V), \quad \psi_0 = \sum_{a=0}^2 \psi_0^a \otimes U^a, \quad \psi_i = \sum_{a=0}^2 \psi_i^a \otimes (U^a V^{-1}). \]

The dynamical supersymmetry is

\[ \delta^{(1)} \psi_0 = \frac{i}{2} \left( [A_\mu, A_\nu] \Gamma^{\mu\nu} \epsilon_0 + [B_i, B_i^\dagger] \epsilon_0 \right), \]

\[ \delta^{(1)} \psi_i = \frac{i}{2} \left( \epsilon_{ijk} [B_j, B_k] \Gamma^{(j)} \Gamma^{(k)} \epsilon_0 + 2 [A_\mu, B_i^\dagger] \Gamma^{(i)} \bar{\epsilon}_0 \right), \]

\[ \delta^{(1)} A_\mu = i \bar{\epsilon}_0 \Gamma^\mu \psi_0 + i \bar{\epsilon}_0 \Gamma^\mu \psi_i, \]

\[ \delta^{(1)} B_i = 2i \bar{\epsilon}_0 \Gamma^{(i)} \psi_i, \]

while the kinematical supersymmetry is

\[ \delta^{(2)} \psi_0 = \xi_0, \]

and zero otherwise. This is a model with 4+4 supercharges.

**IV. Matrix Orientifolding with Four Supercharges**

Having the discussion of the preceding sections in mind, we turn to constructing cases with four supercharges upon matrix orientifolding, which descends from the case leading to
the USp matrix model. In this section, we restrict our attention to the prototypical example of $Z_3$ orbifolding discussed in section [III]. From the condition $[\hat{\rho}_{f\mp}, \delta^{(1)}] \psi_0 = 0$, we obtain

$$
(\Gamma^{\mu-\nu} \epsilon_0)_{A_+} = (\Gamma^{\mu+\nu} \epsilon_0)_{A_+} = 0 = (\epsilon_0)_{A_+}, (\Gamma^{\mu-\nu} \epsilon_0)_{A_-} = 0.
$$

(4.1)

Similar equation holds for $\epsilon_0^\dagger$. The condition $[\hat{\rho}_{b\mp}, \delta^{(1)}] A = 0$ leads to

$$
(\epsilon_0 \Gamma^{\mu-})_{A_+} = 0 = (\epsilon_0^\dagger \Gamma^{\mu-})_{A_+}, (\epsilon_0 \Gamma^{\mu+})_{A_-} = 0 = (\epsilon_0^\dagger \Gamma^{\mu+})_{A_-}.
$$

(4.2)

Similarly $[\hat{\rho}_{b\mp}, \delta^{(1)}] B = 0$, $[\hat{\rho}_{f\mp}, \delta^{(1)}] \psi_0 = 0$ and $[\hat{\rho}_{f\mp}, \delta^{(2)}] \psi_0 = 0$ respectively yield

$$
(\bar{\epsilon}_0 \bar{\Gamma}^{(i-)}A_+ = 0 = (\bar{\epsilon}_0 \bar{\Gamma}^{(i-)}A_-,
$$

\[\begin{cases} (\Gamma^{(j-)} \Gamma^{(k-)} \epsilon_0)_{A_-} = (\Gamma^{\mu+} \bar{\Gamma}^{(i-)} \epsilon_0)_{A_-} = 0, \\ (\Gamma^{(j-)} \Gamma^{(k+)} \epsilon_0)_{A_+} = (\Gamma^{(j+)} \Gamma^{(k+)} \epsilon_0)_{A_+} = (\Gamma^{\mu-} \bar{\Gamma}^{(i-)} \epsilon_0)_{A_+} = 0, \end{cases}\]

(4.3)

and $$(\xi_0)_{A_-} = 0.$$

(4.4)

(4.5)

Eqs. (4.1)-(4.5) define a set of conditions satisfied by the anticommuting parameters $\epsilon_0, \xi_0$.

Let us find solutions to these equations. The spinor $\epsilon_0$ is $\psi_0$-type and must be of the form

$$
\epsilon_0 = (a, 0, a, 0, ia, 0, -a, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t,
$$

(4.6)

where $a = (\alpha, \beta)^t$. Similarly

$$
\epsilon_0^\dagger = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ib, 0, ib, 0, b, 0, -ib)^t,
$$

(4.7)

where $b = (\gamma, \delta)^t$. The spinor and vector indices are grouped into nonintersecting sets $A_+, A_-, M_+, M_-, I_+ \text{ and } I_-$ such that

$$
A = A_+ \cup A_- = \{1, 2, 5, 6, 9, 10, 13, 14, 19, 20, 23, 24, 27, 28, 31, 32\}
$$

$$
M = M_+ \cup M_- = \{0, 1, 2, 3\}, \quad I = I_+ \cup I_- = \{1, 2, 3\}.
$$

Let us first classify the possibilities by the division of $M$ into $M_+$ and $M_-$. This is done by using eq. (4.2) and by following the procedure given in [4]. It turns out that there are three distinct possibilities for the division:

poss. 1. $(\alpha \neq \beta, \alpha, \beta \neq 0; \gamma \neq \delta, \gamma, \delta \neq 0); \{0, 1, 2, 3\}, \emptyset$,

poss. 2. $(\alpha = \pm \beta \neq 0; \gamma = \pm \delta \neq 0); \{0, 1\}, \{2, 3\},$

poss. 3. $(\alpha \neq 0, \beta = 0 \text{ or } \alpha = 0, \beta \neq 0; \gamma \neq 0, \delta = 0 \text{ or } \gamma = 0, \delta \neq 0); \{0, 3\}, \{1, 2\},$
Let us see each possibility more closely.

- **poss. 1:** From eq. (4.1), we see
  \[ \mathcal{A}_- = \mathcal{A}, \quad \mathcal{A}_+ = \emptyset, \]  
  \[ (4.8) \]
  while \((\bar{\epsilon}_0 \Gamma^\mu)_{\mathcal{A}_-} = 0\) in eq. (4.2) implies
  \[ \mathcal{M}_- = \mathcal{M}, \quad \mathcal{M}_+ = \emptyset. \]  
  \[ (4.9) \]
  From eq. (4.3), we conclude
  \[ \mathcal{I}_- = \mathcal{I}, \quad \mathcal{I}_+ = \emptyset. \]  
  \[ (4.10) \]
  Finally eq. (4.5) tells us that the kinematical supersymmetry is broken completely:
  \[ \xi_0 = 0. \]  
  \[ (4.11) \]
  This case has \(4 + 0\) supersymmetries.

- **poss. 2:** Following the same procedure as that of poss. 1, we conclude that this possibility does not lead to a consistent solution.

- **poss. 3:** This possibility leads to four different solutions.
  i) Choosing \(a = (\alpha, 0)^t, b = (\gamma, 0)^t\), from eq. (4.1), we conclude
  \[ \mathcal{A}_- = \{1, 5, 9, 13, 19, 23, 27, 31\}, \quad \mathcal{A}_+ = \{2, 6, 10, 14, 20, 24, 28, 32\}, \]  
  \[ (4.12) \]
  while from eq. (4.2) and from eq. (4.3), we conclude respectively
  \[ \mathcal{M}_- = \{0, 3\}, \quad \mathcal{M}_+ = \{1, 2\}, \]  
  \[ (4.13) \]
  and \(\mathcal{I}_- = \{1, 2, 3\}, \quad \mathcal{I}_+ = \emptyset. \)  
  \[ (4.14) \]
  Finally eq. (4.5) is solved by
  \[ \xi_0 = (c, 0, c, 0, ic, 0, -c, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \]  
  \[ \xi_0^c = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \]  
  where \(c = (0, \beta)^t, d = (0, \delta)^t.\)
  ii) Choosing \(a = (0, \alpha)^t, b = (0, \gamma)^t\), from eq. (4.1), we conclude
  \[ \mathcal{A}_- = \{2, 6, 10, 14, 20, 24, 28, 32\}, \quad \mathcal{A}_+ = \{1, 5, 9, 13, 19, 23, 27, 31\}, \]  
  \[ (4.15) \]
while from eq.(4.2) and from eq.(4.3), we respectively conclude
\[ M_- = \{0, 3\}, \quad M_+ = \{1, 2\}, \quad (4.16) \]
and \[ I_- = \{1, 2, 3\}, \quad I_+ = \emptyset. \quad (4.17) \]

Finally eq.(4.5) is solved by choosing \( c = (\beta, 0)^t, d = (\delta, 0)^t \).

iii) Choosing \( a = (\alpha, 0)^t, b = (0, \gamma)^t \), from eq.(4.1), we conclude
\[ A_- = \{1, 5, 9, 13, 20, 24, 28, 32\}, \quad A_+ = \{2, 6, 10, 14, 19, 23, 27, 31\}, \quad (4.18) \]
while from eq.(4.2) and from eq.(4.3), we respectively conclude
\[ M_- = \{0, 3\}, \quad M_+ = \{1, 2\}, \quad (4.19) \]
and \[ I_- = \emptyset, \quad I_+ = \{1, 2, 3\}. \quad (4.20) \]

Finally eq.(4.5) is solved by choosing \( c = (0, \beta)^t, d = (\delta, 0)^t \).

iv) Choosing \( a = (0, \alpha)^t, b = (\gamma, 0)^t \), from eq.(4.1), we conclude
\[ A_- = \{2, 6, 10, 14, 19, 23, 27, 31\}, \quad A_+ = \{1, 5, 9, 13, 20, 24, 28, 32\}, \quad (4.21) \]
while from eq.(4.2) and from eq.(4.3), we respectively conclude
\[ M_- = \{0, 3\}, \quad M_+ = \{1, 2\}, \quad (4.22) \]
and \[ I_- = \emptyset, \quad I_+ = \{1, 2, 3\}. \quad (4.23) \]

Finally eq.(4.5) is solved by choosing \( c = (\beta, 0)^t, d = (0, \delta)^t \).

These four cases have \( 2 + 2 \) supersymmetries.

V. Enumerating the Cases with Four or Eight Supercharges

Let us generalize the results obtained in the last section. We would first need to rewrite supersymmetry transformations in the new variables \( A_{\mu}, B_i, \psi_0 \) and \( \psi_i \), but we will not spell out its explicit form here. As we have seen in the last section, the condition \([\rho^{(\mu)}_{\nu_+}, \delta^{(1)}]A_{\mu} = 0\) yields

\[
(\bar{\epsilon}_0 \Gamma_{\mu}^-)_{A_+} = (\bar{\epsilon}_0 \Gamma_{\mu}^-)_{A_+} = (\bar{\epsilon}_0 \Gamma_{\mu}^-)_{A_+} = (\bar{\epsilon}_0 \Gamma_{\mu}^-)_{A_+} = 0, \\
(\bar{\epsilon}_0 \Gamma_{\mu}^-)_{A_-} = (\bar{\epsilon}_1 \Gamma_{\mu}^+)_{A_-} = (\bar{\epsilon}_0 \Gamma_{\mu}^+)_{A_-} = (\bar{\epsilon}_1 \Gamma_{\mu}^+)_{A_-} = 0, \\
\quad (5.1)\]

These four cases have \( 2 + 2 \) supersymmetries.
and the condition \([\hat{\rho}_D^{(i)}, \delta^{(1)}]B_i = 0\) leads to
\[
(\varepsilon_0 \bar{\Gamma}^{(i_+)} )_{A_+} = (\varepsilon_{i_+} \bar{\Gamma}^{(i_+)} )_{A_+} = (\varepsilon_j \bar{\Gamma}^{(i_+)} )_{A_+} = 0, \\
(\varepsilon_0 \bar{\Gamma}^{(i_-)} )_{A_-} = (\varepsilon_{i_-} \bar{\Gamma}^{(i_-)} )_{A_-} = (\varepsilon_j \bar{\Gamma}^{(i_-)} )_{A_-} = 0,
\] (5.2)

where \(j \neq i\). Here the repeated indices are not to be summed over unless stated explicitly. Similarly, from \([\hat{\rho}_D^{(i)}, \delta^{(1)}]B_j = 0\) we obtain
\[
(\varepsilon_0 \Gamma^{(i_-)} )_{A_+} = (\varepsilon_{i_-} \Gamma^{(i_-)} )_{A_+} = (\varepsilon_j \Gamma^{(i_-)} )_{A_+} = 0, \\
(\varepsilon_0 \Gamma^{(i_+)} )_{A_-} = (\varepsilon_{i_+} \Gamma^{(i_+)} )_{A_-} = (\varepsilon_j \Gamma^{(i_+)} )_{A_-} = 0.
\] (5.3)

The condition \([\hat{\rho}_f^{(0)}(A), \delta^{(1)}] (\psi_0)_A = 0\) yields
\[
(\Gamma^{\mu_+ \nu_+} \varepsilon_0 )_{A_-} = 0, \\
(\Gamma^{\mu_- \nu_-} \varepsilon_0 )_{A_+} = (\Gamma^{\mu_+ \nu_+} \varepsilon_0 )_{A_+} = 0,
\] (5.4)

\[
|\varepsilon_{i_- j+ k}| (\bar{\Gamma}^{(i_-)} \bar{\Gamma}^{(j_+)} \varepsilon_k )_{A_-} = 0, \\
|\varepsilon_{i_- j- k}| (\bar{\Gamma}^{(i_-)} \bar{\Gamma}^{(j_-)} \varepsilon_k )_{A_+} = |\varepsilon_{i_+ j+ k}| (\bar{\Gamma}^{(i_+)} \bar{\Gamma}^{(j_+)} \varepsilon_k )_{A_+} = 0,
\] (5.5)

\[
(\Gamma^{\mu_-} \bar{\Gamma}^{(i_+)} \varepsilon_k )_{A_-} = (\Gamma^{\mu_+} \bar{\Gamma}^{(i_-)} \varepsilon_k )_{A_-} = 0, \\
(\Gamma^{\mu_-} \bar{\Gamma}^{(i_-)} \varepsilon_k )_{A_+} = (\Gamma^{\mu_+} \bar{\Gamma}^{(i_+)} \varepsilon_k )_{A_+} = 0,
\] (5.6)

\[
(\varepsilon_0 )_{A_-} = 0,
\] (5.7)

while \([\hat{\rho}_f^{(0)}(A), \delta^{(1)}] (\psi_0^c )_A = 0\)
\[
(\Gamma^{\mu_- \nu_+} \varepsilon_0^c )_{A_-} = 0, \\
(\Gamma^{\mu_- \nu_-} \varepsilon_0^c )_{A_+} = (\Gamma^{\mu_+ \nu_+} \varepsilon_0^c )_{A_+} = 0,
\] (5.8)

\[
|\varepsilon_{i_- j+ k}| (\bar{\Gamma}^{(i_-)} \bar{\Gamma}^{(j_+)} \varepsilon_k^c )_{A_-} = 0, \\
|\varepsilon_{i_- j- k}| (\bar{\Gamma}^{(i_-)} \bar{\Gamma}^{(j_-)} \varepsilon_k^c )_{A_+} = |\varepsilon_{i_+ j+ k}| (\bar{\Gamma}^{(i_+)} \bar{\Gamma}^{(j_+)} \varepsilon_k^c )_{A_+} = 0,
\] (5.9)

\[
(\Gamma^{\mu_-} \bar{\Gamma}^{(i_+)} \varepsilon_k^c )_{A_-} = (\Gamma^{\mu_+} \bar{\Gamma}^{(i_-)} \varepsilon_k^c )_{A_-} = 0, \\
(\Gamma^{\mu_-} \bar{\Gamma}^{(i_-)} \varepsilon_k^c )_{A_+} = (\Gamma^{\mu_+} \bar{\Gamma}^{(i_+)} \varepsilon_k^c )_{A_+} = 0,
\] (5.10)

\[
(\varepsilon_0^c )_{A_-} = 0.
\] (5.11)
The condition \([\tilde{\rho}^{(ii)(a)}_{f_{\mp}}, \delta^{(1)}](\psi_i)\_A = 0\) leads to
\[
(\Gamma^{\mu-\nu_+} \epsilon_i)_{A_+} = 0,
(\Gamma^{\mu-\nu_-} \epsilon_i)_{A_-} = (\Gamma^{\mu+\nu_+} \epsilon_i)_{A_+} = 0,
(\Gamma^{\mu-\nu_-} \epsilon_i)_{A_+} = (\Gamma^{\mu+\nu_+} \epsilon_i)_{A_-} = 0,
\]
(5.12)
\[
|\epsilon_{ij_+}k| (\Gamma^{\mu-} \Gamma^{(j_+)} \epsilon_k)_{A_-} = |\epsilon_{ij_-}k| (\Gamma^{\mu+} \Gamma^{(j_-)} \epsilon_k)_{A_-} = 0,
|\epsilon_{ij_-}k| (\Gamma^{\mu-} \Gamma^{(j_-)} \epsilon_k)_{A_+} = |\epsilon_{ij_+}k| (\Gamma^{\mu+} \Gamma^{(j_+)} \epsilon_k)_{A_+} = 0,
|\epsilon_{ij_+}k| (\Gamma^{\mu-} \Gamma^{(k_-)} \epsilon_0)_{A_-} = 0,
|\epsilon_{ij_-}k| (\Gamma^{\mu+} \Gamma^{(k_+)} \epsilon_0)_{A_+} = 0,
(5.13)
\]
\[
|\epsilon_{ij_+}k| (\Gamma^{(j_+)} \Gamma^{(k_+)} \epsilon_0)_{A_+} = |\epsilon_{ij_-}k| (\Gamma^{(j_-)} \Gamma^{(k_-)} \epsilon_0)_{A_+} = 0,
|\epsilon_{ij_-}k| (\Gamma^{(j_-)} \Gamma^{(k_-)} \epsilon_0)_{A_-} = |\epsilon_{ij_+}k| (\Gamma^{(j_+)} \Gamma^{(k_+)} \epsilon_0)_{A_-} = 0,
|\epsilon_{i-j_+}k| (\Gamma^{(i_-)} \Gamma^{(j_+)} \epsilon_{i_+})_{A_-} = |\epsilon_{i-j_-}k| (\Gamma^{(i_+)} \Gamma^{(j_-)} \epsilon_{i_+})_{A_-} = 0,
(5.14)
|\epsilon_{i-j_+}k| (\Gamma^{(i_-)} \Gamma^{(j_+)} \epsilon_{i_+})_{A_+} = |\epsilon_{i-j_-}k| (\Gamma^{(i+)} \Gamma^{(j_-)} \epsilon_{i_+})_{A_+} = 0,
|\epsilon_{i-j_-}k| (\Gamma^{(i_-)} \Gamma^{(j_-)} \epsilon_{i_-})_{A_+} = |\epsilon_{i-j_+}k| (\Gamma^{(i+)} \Gamma^{(j_+)} \epsilon_{i_-})_{A_+} = 0,
(5.15)
\]
\[
(\epsilon_i)_{A_+} = 0.
(5.17)
\]
The condition \([\tilde{\rho}^{(iii)(a)}_{f_{\mp}}, \delta^{(1)}](\psi_i^c)\_A = 0\) leads to
\[
(\Gamma^{\mu-\nu_+} \epsilon_i^c)_{A_-} = 0,
(\Gamma^{\mu-\nu_-} \epsilon_i^c)_{A_+} = (\Gamma^{\mu+\nu_+} \epsilon_i^c)_{A_+} = 0,
(\Gamma^{\mu-\nu_-} \epsilon_i^c)_{A_+} = (\Gamma^{\mu+\nu_+} \epsilon_i^c)_{A_-} = 0,
(5.18)
\]
\[
|\epsilon_{ij_+}k| (\Gamma^{\mu-} \Gamma^{(j_+)} \epsilon_k)_{A_-} = |\epsilon_{ij_-}k| (\Gamma^{\mu+} \Gamma^{(j_-)} \epsilon_k)_{A_-} = 0,
|\epsilon_{ij_-}k| (\Gamma^{\mu-} \Gamma^{(j_-)} \epsilon_k)_{A_+} = |\epsilon_{ij_+}k| (\Gamma^{\mu+} \Gamma^{(j_+)} \epsilon_k)_{A_+} = 0,
|\epsilon_{ij_+}k| (\Gamma^{\mu-} \Gamma^{(k_-)} \epsilon_0)_{A_-} = 0,
|\epsilon_{ij_-}k| (\Gamma^{\mu+} \Gamma^{(k_+)} \epsilon_0)_{A_+} = 0,
(5.19)
\]
\[
|\epsilon_{ij_+}k| (\Gamma^{(j_+)} \Gamma^{(k_+)} \epsilon_0)_{A_+} = |\epsilon_{ij_-}k| (\Gamma^{(j_-)} \Gamma^{(k_-)} \epsilon_0)_{A_+} = 0,
|\epsilon_{ij_-}k| (\Gamma^{(j_-)} \Gamma^{(k_-)} \epsilon_0)_{A_-} = |\epsilon_{ij_+}k| (\Gamma^{(j_+)} \Gamma^{(k_+)} \epsilon_0)_{A_-} = 0,
(5.20)
\]
\[
(\Gamma^{\mu-} \Gamma^{(i_+)} \epsilon_0)_{A_-} = (\Gamma^{\mu+} \Gamma^{(i_-)} \epsilon_0)_{A_-} = 0,
(\Gamma^{\mu-} \Gamma^{(i_-)} \epsilon_0)_{A_+} = (\Gamma^{\mu+} \Gamma^{(i_+)} \epsilon_0)_{A_+} = 0,
(5.21)
\]
\begin{align*}
&|\varepsilon_{i-j-k}|(\Gamma^{(i-})\Gamma^{(j+})\varepsilon_i^{c})_{A-} = |\varepsilon_{i+j-k}|(\Gamma^{(i+})\Gamma^{(j-})\varepsilon_i^{c})_{A-} = 0, \\
&|\varepsilon_{i-j-k}|(\Gamma^{(i-})\Gamma^{(j-})\varepsilon_i^{c})_{A+} = |\varepsilon_{i+j-k}|(\Gamma^{(i+})\Gamma^{(j+})\varepsilon_i^{c})_{A+} = 0, \quad (5.22)
\end{align*}

\begin{align*}
(\varepsilon_i^{c})_{A_+} = 0. \quad (5.23)
\end{align*}

In addition, from $[\hat{\rho}^{(A)}_f, \delta^{(2)}](\psi_0)_A = 0$ and $[\hat{\rho}^{(A)}_f, \delta^{(2)}](\psi_i)_A = 0$, we obtain

\begin{align*}
(\xi_0)_{A_-} = (\xi_i)_{A_-} = 0. \quad (5.24)
\end{align*}

Upon $\mathbb{Z}_3$ orbifolding, the number of surviving supersymmetries is related to the number of $b_i$ such that $b_i = 0$ is satisfied. The cases with $4 + 4$ supercharges have only one such $b_i$, and we obtain the following four possibilities with $4 + 4$ supercharges:

- $b_0 = 0, b_1 = -a_1, b_2 = -a_2, b_3 = -a_3 \quad (a_1 + a_2 + a_3 = 0)$
- $b_0 = -a_1, b_1 = 0, b_2 = a_3, b_3 = a_2 \quad (a_1 - a_2 - a_3 = 0)$
- $b_0 = -a_2, b_1 = a_3, b_2 = 0, b_3 = a_1 \quad (a_1 - a_2 + a_3 = 0)$
- $b_0 = -a_3, b_1 = a_2, b_2 = a_1, b_3 = 0 \quad (a_1 + a_2 - a_3 = 0)$.

The first one is the model which we already treated in the last section.

Similarly we construct the models with $8 + 8$ supercharges, which have two of vanishing $b_i$. There are six possibilities:

- $b_0 = b_1 = 0, a_1 = 0, b_2 = -a_2 = a_3$
- $b_0 = b_2 = 0, a_2 = 0, b_1 = -a_3 = a_1$
- $b_0 = b_3 = 0, a_3 = 0, b_1 = -b_2 = a_2 = -a_1$
- $b_1 = b_2 = 0, a_3 = 0, b_0 = -b_3 = -a_1 = -a_2$
- $b_1 = b_3 = 0, a_2 = 0, b_0 = -b_2 = -a_1 = -a_3$
- $b_2 = b_3 = 0, a_1 = 0, b_0 = -b_1 = -a_2 = -a_3$. 

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We collect these possibilities in the table:

| supersymmetry | \(b_0\) | \(b_1\) | \(b_2\) | \(b_3\) |
|---------------|--------|--------|--------|--------|
| **4 + 4**     | 0      | \(-a_1\) | \(-a_2\) | \(-a_3\) |
|               | \(-a_1\) | 0      | \(a_3\) | \(a_2\) |
|               | \(-a_2\) | \(a_3\) | 0      | \(a_1\) |
|               | \(-a_3\) | \(a_2\) | \(a_1\) | 0      |
| **8 + 8**     | 0      | 0      | \(-a_2\) | \(a_2\) |
|               | 0      | \(-a_1\) | 0      | \(a_1\) |
|               | 0      | \(-a_1\) | \(a_1\) | 0      |
|               | \(-a_1\) | 0      | \(a_1\) | 0      |
|               | \(-a_2\) | \(a_2\) | 0      | 0      |

In each possibility, we need only to keep \(\epsilon_i\) such that \(b_i = 0\) is satisfied. Note that the individual forms of \(\epsilon_i\) are written as

\[
\epsilon_0 = (a, 0, a, 0, 0, -a, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \tag{5.25}
\]
\[
\epsilon_1 = (b, 0, b, 0, -ib, 0, -b, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \tag{5.26}
\]
\[
\epsilon_2 = (c, 0, -c, 0, ic, 0, c, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \tag{5.27}
\]
\[
\epsilon_3 = (d, 0, d, 0, -id, 0, d, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \tag{5.28}
\]
\[
\epsilon_0^c = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \tag{5.29}
\]
\[
\epsilon_1^c = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \tag{5.30}
\]
\[
\epsilon_2^c = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \tag{5.31}
\]
\[
\epsilon_3^c = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t, \tag{5.32}
\]

where \(a, b, c, d, e, f, g, h\) are two component real vectors. Consequently we consider the conditions on these remaining parameters.

It is noted that eqs. (5.26)-(5.28) become proportional to eq. (5.25) once we flip signs in one or two entries. The same is true for eqs. (5.30)-(5.32), which become proportional to eq. (5.29) with one or two sign flips. This means that the calculation in the last section is also applicable to the remaining possibilities. From each of the four possibilities of \(\mathbb{Z}_3\) orbifolding with \(4 + 4\) supercharges, we obtain one case with \(4 + 0\) supercharges and four cases with \(2 + 2\) supercharges upon matrix orientifolding. There are in total twenty such cases.
Likewise, to each of the six possibilities of $\mathbb{Z}_3$ orbifolding with $8 + 8$ supercharges, we first find an appropriate intersection of above eqs. (5.25)–(5.28), (5.29)–(5.32) and impose the conditions of matrix orientifolding. In this way, we are able to exhaust all cases with either $8 + 0$ supersymmetries or $4 + 4$ supersymmetries upon $\mathbb{Z}_3$ orbifolding followed by matrix orientifolding. To each of the six possibilities, there exist one case with $8+0$ supersymmetries and four cases with $4+4$ supersymmetries. There are thirty such cases in total.

We conclude that there are in total fifty cases carrying four or eight supercharges upon $\mathbb{Z}_3$ orbifolding followed by matrix orientifolding.
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