Numerical Simulation of Solar Magnetic Flux Emergence Using the AMR–CESE–MHD Code

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Abstract

Magnetic flux emergence from the solar interior to the atmosphere is believed to be a key process in the formation of solar active regions and driving solar eruptions. Due to the limited capabilities of observations, the flux emergence process is commonly studied using numerical simulations. In this paper, we develop a numerical model to simulate the emergence of a twisted magnetic flux tube from the convection zone to the corona, using the AMR–CESE–MHD code, which is based on the conservation-element solution-element method, with adaptive mesh refinement. The results of our simulation agree with those of many previous studies with similar initial conditions, but by using different numerical codes. In the early stage, the flux tube rises from the convection zone, being driven by magnetic buoyancy, until it reaches close to the photosphere. The emergence is decelerated there, and with the piling up of the magnetic flux, the magnetic buoyancy instability is triggered, which allows the magnetic field to partially enter into the atmosphere. Meanwhile, two gradually separated polarity concentration zones appear in the photospheric layer, transporting the magnetic field and energy into the atmosphere through their vortical and shearing motions. Correspondingly, the coronal magnetic field is also reshaped into a sigmoid configuration, containing a thin current layer, which resembles the typical pre-eruptive magnetic configuration of an active region. Such a numerical framework of magnetic flux emergence as established will be applied to future investigations of how solar eruptions are initiated in flux emergence active regions.

Unified Astronomy Thesaurus concepts: Solar corona (1483); Solar magnetic fields (1503); Magnetohydrodynamics (1964); Solar active regions (1974); Magnetohydrodynamical simulations (1966)

1. Introduction

Coronal mass ejections (CMEs), flares, and jets are the major forms of solar eruptions, and the physical mechanisms of their triggers and drivers are important research topics in solar physics. Numerous observational studies have reported that these eruptive activities frequently occur in solar active regions, and it is generally believed that the core structure of the pre-eruptive field takes the form of either a twisted flux tube, i.e., a magnetic flux rope (MFR), or a strongly sheared magnetic arcade (Green et al. 2011; Patsourakos et al. 2013). The entire pre-eruption configuration consists of the core field (either an MFR or a sheared arcade) and an envelope field (an overlying field) that confines the core field, while eruptions occur when various instabilities destabilize their force balance (Archontis & Hood 2012).

It is currently accepted that solar active regions are formed by magnetic flux emergence—the process of the magnetic fields that are generated by the solar dynamo entering the solar atmosphere from the depths of the convection zone, which is also considered to be one of the key mechanisms for producing solar eruptive activity (Chen 2011; van Driel-Gesztelyi & Green 2015). Although it has been thought that the emerging magnetic field is sufficient in itself to generate an eruption (Démoïlin et al. 2002; Nindos et al. 2003), in many cases it acts as a trigger for a pre-existing eruptive configuration (Feynman & Martin 1995; Williams et al. 2005). In a stable pre-eruption configuration, the upward magnetic pressure of the internal flux rope is in equilibrium with the downward tension of the envelope field (Archontis & Hood 2012; Leake et al. 2013). When a new flux emerges in the vicinity of the pre-existing eruption configuration, their interaction causes magnetic reconnection, which could reduce the tension of the envelope field and lead to an eruption (Chen & Shibata 2000). There are two possible ways for reconnection to operate in this process, which are tether cutting (Moore & Roumeliotis 1992) and breakout reconnection (Antiochos et al. 1999). In other cases, the pre-eruption configuration is associated with the ideal instability. Continuous flux emergence may push the magnetic configuration higher, and when the envelope field decays too fast with height, the MFR will run into the torus instability and erupt (Kliem & Török 2006). Flux emergence can also increase the degree of twist of the MFR, and when a certain value is exceeded, it triggers kink instability and an eruption (Ulrich 1968; Török et al. 2004).

Without having a direct observational probe of the dynamics of magnetic flux emergence from below the solar surface (i.e., the photosphere), many efforts have been devoted to numerical magnetohydrodynamic (MHD) simulations of flux emergence. As pioneered by the early work of Shibata and colleagues (Shibata et al. 1989), a large number of works involving flux emergence simulation (FES) have been carried out, particularly to mimic simulations of a twisted flux tube emerging into the solar atmosphere (Moreno-Insertis & Emonet 1996; Arber et al. 2001; Magara & Longcope 2001; Archontis et al. 2004; Manchester et al. 2004; Leake & Arber 2006; Murray et al. 2006; Toriumi & Yokoyama 2010; Cheung & Isobe 2014; Syntelis et al. 2017; Toriumi & Wang 2019; Fan 2001, 2021). These simulations have successfully reproduced some of the observed phenomena—such as the vertical motion of the emerging polarities on the photosphere and the sigmoid-shaped coronal MFR—and these comparative results confirm the reliability of MHD simulations.

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To simulate a flux tube emerging from the convection zone into the corona (and to further study how it erupts) it is necessary for a numerical model to incorporate a highly stratified solar plasma, including all the different layers, from below the surface to the photosphere, the chromosphere, the transition region, and the corona, which have physical behaviors that are rather different from one another. Therefore, a major challenge in self-consistent simulations of magnetic flux emergence to eruption is to resolve the multiple spatial and temporal scales into a single model. For example, near the photosphere, the scale heights of gas pressure are only about 100 km, and the gas density varies by more than 8 orders of magnitude within a few megameters, while in the corona the scale height is tens (or even hundreds) of megameters.  

The aim of this paper is to develop a new numerical model of magnetic flux emergence by using our AMR–CESE–MHD code (Jiang et al. 2010); in particular, by utilizing the features of adaptive mesh refinement (AMR; Berger & Colella 1989). The technique of AMR was rapidly developed in computational fluid dynamics and is becoming a standard tool for treating problems with multi-orders of spatial or temporal scales, which fits well for FES. By automatically adapting the computational mesh to the solution of the governing partial differential equations, methods based on AMR can assign more mesh points for regions demanding high resolution (e.g., high-gradient regions) and, at the same time, give fewer mesh points to other less interested regions (low-gradient regions), thereby providing the required spatial resolution, while also minimizing memory requirements and CPU time. Although many classical numerical MHD solvers based on either finite-difference or finite-volume methods have been used in previous FESs—such as the ZEUS-3D code (Stone et al. 2008), the modified Lax–Wendroff method (Magara & Longcope 2001; Toriumi & Yokoyama 2010), and the Lagrangian remap scheme (Lare3d; Arber et al. 2001)—few of these FESs have been implemented with AMR. There are only two simulations that have used AMR (Cheung et al. 2006; Martinez-Sykora et al. 2015), but both of these early simulations only study the evolution of the flux tube below the photosphere, and in Cheung et al. (2006) the simulation is carried out in 2.5D, rather than in 3D. On the other hand, the conservation-element solution-element (CESE) method is distinct from the classical numerical methods of finite-difference or finite-volume schemes, as it has much greater simplicity in terms of mathematics, without a Riemann solver or eigendecomposition, yet it can also achieve higher accuracy at equivalent grid points, which is also desirable for FES. The AMR–CESE–MHD code has achieved many excellent results in other simulations, such as in analyses of the fundamental initiation mechanisms of solar eruptions (Jiang et al. 2021b; Bian et al. 2022), data-driven active region evolutions and eruptions (Jiang et al. 2016, 2021a, 2022), and solar wind modeling (Feng et al. 2012).  

In this paper, we report the first steps in applying the AMR–CESE–MHD code to FES, by simulating the emergence of a twisted flux tube in a simply stratified solar atmosphere, from the convection zone to the corona. In the following, Section 2 describes the details of the model and the numerical methods. In Section 3, we show the process and the key features of the 3D magnetic flux emergence, which are overall consistent with those of previous FESs. In Section 4, we summarize and offer an outlook for future studies based on the new FES model.  

2. Model

2.1. Initial Conditions

The initial settings of our model are similar to those used in typical simulations of the emergence of a twisted flux tube from below the photosphere to the corona, and in particular the parameters are mostly close to the values used in Fan (2009). The simulation volume is a Cartesian box of $-14.4 \text{ Mm} \leq x \leq 14.4 \text{ Mm}$, $-14.4 \text{ Mm} \leq y \leq 14.4 \text{ Mm}$, and $0 \leq z \leq 28.8 \text{ Mm}$, where the $z$-axis is the height, with $z = 0$ denoting the lower boundary, which is a depth of $4.5 \text{ Mm}$ below the photosphere.

The initial conditions consist of a plasma in hydrostatic equilibrium stratified by solar gravity, with a characteristic temperature profile from the top layer of the convection zone to the corona, which is given by a piecewise function of height:

$$T(z) = \begin{cases} T_{\text{ph}} - \frac{\gamma - 1}{\gamma} \frac{g_0}{R} (z - z_{\text{ph}}) & : z \leq z_{\text{ph}} \\ T_{\text{ph}} & : z_{\text{ph}} < z \leq z_{\text{ch}} \\ T_{\text{ch}} \left( \frac{z_{\text{ch}}}{z} \right) & : z_{\text{ch}} < z \leq z_{\text{cor}} \\ T_{\text{cor}} & : z \geq z_{\text{cor}} \end{cases},$$

where the photospheric temperature is $T_{\text{ph}} = 5 \times 10^3 \text{ K}$ and the coronal temperature is $T_{\text{cor}} = 10^6 \text{ K}$. The heights $z_{\text{ph}} = 4.5 \text{ Mm}$, $z_{\text{ch}} = z_{\text{ph}} + 1.5 \text{ Mm}$, and $z_{\text{cor}} = z_{\text{ch}} + 1.5 \text{ Mm}$ are the heights of the photosphere, the chromosphere, and the base of the corona, respectively. As the modeling region is a small range around the solar surface, we assume that the solar gravity is constant, with a value of $g_0 = 274 \text{ m s}^{-2}$ on the solar surface, and that $R = 8.25 \times 10^3$ is the gas constant, such that $p = \rho RT$. By assuming a balance of pressure gradient and gravity,

$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT/g},$$

the pressure is

$$p(z) = \begin{cases} \rho_{\text{ph}} \left[ T(z) \right]^{\gamma/\gamma - 1} & : z \leq z_{\text{ph}} \\ \rho_{\text{ph}} \exp \left[ \frac{g_0(z_{\text{ph}} - z)}{RT_{\text{ph}}} \right] & : z_{\text{ph}} < z \leq z_{\text{ch}} \\ \rho_{\text{ch}} \exp \left[ \frac{z_{\text{cor}} - z}{RT_{\text{cor}} / g_0 \ln(T_{\text{cor}}/T_{\text{ph}})} \right] \left( \frac{T_{\text{ph}}}{T(z)} - \frac{T_{\text{ph}}}{T_{\text{cor}}} \right) & : z_{\text{ch}} < z \leq z_{\text{cor}} \\ \rho_{\text{cor}} \exp \left[ \frac{g_0(z_{\text{cor}} - z)}{RT_{\text{cor}}} \right] & : z \geq z_{\text{cor}} \end{cases}.$$
where

\[
P_{ph} = p_{cor} \exp \left[ -\frac{z_{cor} - z_{ch}}{RT_{ph}/g_0 \ln(T_{cor}/T_{ph})} \left( 1 - \frac{T_{ph}}{T_{cor}} \right) \right] \times \exp \left[ \frac{g_0(z_{ch} - z_{ph})}{RT_{ph}} \right].
\]

We then place a uniformly twisted magnetic flux tube below the photosphere. It is oriented along the \( x \)-direction, with a straight axis located at \( y = 0 \) and \( z = 2.4 \) Mm (i.e., \( 2.1 \) Mm below the photosphere surface). In the local cylindrical coordinate system centered at the tube axis, its magnetic field is given by

\[
B = B_x(r) \hat{x} + B_y(r) \hat{y},
\]

where

\[
B_x(r) = B_0 \exp(-r^2/a^2), \quad B_y(r) = qr B_x(r).
\]

In the above equations, \( \hat{x} \) is the tube axial direction, \( \hat{\theta} \) is the azimuthal direction in the tube cross section, and \( r \) denotes the radial distance to the tube axis. For the specific values of the parameters, we set \( B_0 = 3400 \) G, and the radius \( a = 375 \) km. The twist parameter is set as \( q = -1/a \), which is the threshold value for the kink instability (Linton et al. 1996). The plasma \( \beta \) (defined as the ratio of the plasma pressure to the magnetic pressure) is 8.99 at the axis of the tube.

In Figure 1, we show vertical profiles of the plasma pressure, density, temperature, and the magnetic pressure through the central vertical line \((x, y) = (0, 0)\). Note that all the parameters in the figures of this paper are divided by their values in the coronal base \((z = z_{cor})\).

Since the flux tube is not force-free, to make it in equilibrium, we modify the plasma pressure (without changing the density) in the tube by a difference of

\[
p_1(r) = \frac{B_0^2(r)}{2} \left[ q^2 \left( \frac{a^2}{2} - r^2 \right) - 1 \right],
\]

which ensures that the Lorentz force is balanced by the gas pressure gradient. Then to make the flux tube buoyant, we further add a density change \( \rho_1 \) within the tube:

\[
\rho_1 = -\rho_0(z) \frac{B_0^2(r)}{2p_0(z)} [(1 + \epsilon) \exp(-x^2/\lambda^2) - 1],
\]

where \( \rho_0 \) is the background density, \( \epsilon = 0.2 \), and \( \lambda = 1.2 \) Mm. This makes the middle portion of the flux tube buoyant, since the modified density is lower than the background one (see Figure 2). Note that the buoyancy declines with horizontal distance from \( x = 0 \), following a Gaussian profile with an e-folding length of \( \lambda \), and the two ends of the tube are slightly antibuoyant. This can result in the tube emerging more vertically when it crosses the photosphere.

2.2. MHD Equations

We numerically solve the full set of MHD equations with the above initial conditions. Before describing the model equations in the code, it is necessary to specify the quantities used for nondimensionalization. In contrast to many other papers on FES, here we use typical values in the coronal base \((z = z_{cor})\) for normalization, since our future application of this model will mainly be devoted to investigating the eruptions in the corona that are driven by flux emergence. The specific values for all the variables and parameters are listed in Table 1.

In the rest of the paper, all the variables and quantities are written in nondimensionalized form, if they are not specified. As such, the full set of MHD equations is given as:

\[
\frac{\partial p}{\partial t} = -\nabla \cdot (\rho v),
\]

\[
\rho \frac{Dv}{Dt} = -\nabla p + J \times B + \rho g + \nabla \cdot (\nu p \nabla v) - B \nabla \cdot B,
\]

\[
\frac{\partial B}{\partial t} = \nabla \times (\nu \times B) + \nabla (-\mu \nabla \cdot B) - \nu \nabla \cdot B,
\]

\[
\frac{DT}{Dt} = (1 - \gamma)T \nabla \cdot v - \nu_T (T - T_0),
\]

(9)
where \( J = \nabla \times B \) is the current density and \( \gamma = 5/3 \) is the adiabatic index. Note that we artificially add a source term \( -\nu_T(T - T_0) \) to the equation for temperature, where \( T_0 \) is the temperature at the initial time \( t = 0 \) and \( \nu_T \) is a prescribed coefficient given by

\[
\nu_T = \frac{1}{2} \left[ 1 - \tanh \left( \frac{T - T_{\text{min}}}{T_{\text{min}}} \right) \right],
\]

where \( T_{\text{min}} = T_{\text{th}}/2 = 2.5 \times 10^{-3} \). This source term is a Newton relaxation of the temperature to its initial value by a time of \( 1/\nu_T \), and the specific choice of \( \nu_T \) is aimed at avoiding the overcooling of the plasma during the fast expansion of the flux tube after it passes through the photosphere into the corona. As our code has rather small numerical diffusion, we need some additional kinetic viscosity to dissipate the small-scale disturbances that arise in the simulation. We use a small viscosity coefficient \( \nu = 0.1 \Delta v_{\text{max}} \), which is given according to the local grid size \( \Delta x \) and the largest local wave speed \( v_{\text{max}} \):

\[
v_{\text{max}} = v + \sqrt{c_s^2 + v_A^2},
\]

where \( v \), \( c_s = \sqrt{\gamma p/\rho} \), and \( v_A = B/\sqrt{\rho} \) are the motion speed, the sound speed, and the Alfvén speed, respectively. It corresponds to a grid Reynolds number of

\[
R_g = \frac{\Delta x^2/\nu}{\Delta x v_{\text{max}}} = 10.
\]

In the MHD equation, all the terms associated with \( \nabla \cdot B \) are employed to eliminate the \( \nabla \cdot B \), or the magnetic monopole, which should be exactly zero, but arises due to numerical errors. The diffusion coefficient for the magnetic monopole is given by \( \mu = 0.4(\Delta x)^2/\Delta t \), according to the local grid size and time step. Finally, we note that in the magnetic induction equation, no explicit resistivity is used, but magnetic reconnection is still allowed through numerical diffusion, when a current layer is sufficiently narrow, such that its thickness is close to the grid resolution (see also Jiang et al. 2021b).

### 2.3. Numerical Scheme and Grid Setting

The full set of MHD Equations (9) above is solved by the CESE–MHD code (Jiang et al. 2010). The CESE method treats time as being a dimension, similar to the three dimensions in space, when solving the 3D time-dependent governing equations, by reasonably introducing the conservation element and the solution element in the 4D space of space and time, and by using the conservation law to compute the spacetime flux to obtain the information about the next time. In contrast to many other numerical schemes, the CESE method is simple in its mathematics, since it does not need a Riemann solver or eigendecomposition, yet it can achieve higher accuracy, with an equal number of grid points. More details about the scheme can be found in Feng et al. (2006, 2007) and Jiang et al. (2010, 2012).

The simulation volume is resolved by an AMR grid of cubic cells. The AMR is designed to automatically and dynamically resolve, with the highest resolution, the large-gradient regions in plasma variables that arise during the magnetic flux emergence, as well as the strong–magnetic field regions, which are mainly in the flux tube. Specifically, the whole volume is divided into blocks of different sizes, and each block consists of \( 8^3 \) cells; after each time step of advancing the solution, we check whether the blocks need to be refined or coarsened, using a set of physics-based criteria, defined as

\[
\chi_\rho = \Delta_c \frac{\nabla \rho}{\rho}, \quad \chi_p = \Delta_c \frac{\nabla p}{p},
\]

\[
\chi_B = \Delta_c \frac{\nabla (p_B)}{p_B}, \quad \chi_T = \Delta_c \frac{(B \cdot \nabla) B}{p_B},
\]

where \( p_B = B^2/2 \) is the magnetic pressure and \( \Delta_c \) is the length of the cell. If any of the four quantities in any cell of a block are larger than the threshold for refinement, which is given as 0.25 for both \( \chi_\rho \) and \( \chi_p \) and 0.1 for both \( \chi_B \) and \( \chi_T \), this block will be refined. On the other hand, if all the four quantities in all the cells of a block are smaller than the threshold for coarsening, which is 0.1 for both \( \chi_\rho \) and \( \chi_p \) and 0.04 for both \( \chi_B \) and \( \chi_T \), this block will be coarsened. After the refinement and coarsening, the variables on the new grid will be interpolated from the old grid, then the solution will be advanced in the new time step. Note that the application of the two criteria \( \chi_B \) and \( \chi_T \), which are associated with the magnetic field \( B \) only, is restricted within the strong-field regions satisfying \( p_B/\rho > 1 \times 10^{-3} \) and \( p_B/\rho > 1 \times 10^{-5} \). Figure 3 shows the evolution of the block distribution (which contains \( 8^3 \) cells), with these criteria applied during the simulation. We use three levels of AMR, with a highest resolution of 45 km and a lowest resolution of 180 km. We then employ the PARAMESH software package (MacNeice et al. 2000) to manage the AMR procedure and the paralleling computing.

Since the spatial resolutions and the wave speeds of the blocks within the computational domain vary significantly, the time steps that are computed using a fixed Courant number \( C \sim 1 \), \( \Delta t = C \Delta x/w \), where \( w \) is the maximal wave speed in the block, will also vary significantly. A simple approach is to use a uniform time step for all the blocks, which is defined as \( \Delta t_{\text{g}} = C \Delta x/w_{\text{max}} \), where \( w_{\text{max}} \) is the maximal wave speed over the entire computation domain. However, this will significantly increase the numerical diffusion of the coarser blocks and in the low-wave speed areas, which will be especially evident when contrasting the wave speeds (mainly the sound speed) in the photosphere and in the corona, since the local time step \( \Delta t \) is much larger than the global one \( \Delta t_{\text{g}} \); or, in other words, the local Courant number, defined as \( C_l = w \Delta t_{\text{g}}/\Delta x \), is much smaller than unity. This problem is especially serious for the CESE scheme, which is sensitive to the local Courant number. To overcome this
problem, we use time marching, with block-based variable time steps, in which different time steps are used for different blocks, with the time steps being defined as $\Delta t = C \Delta x / \omega_{\text{max}}$—thus being directly proportional to the resolutions of the blocks. Furthermore, we use the Courant number insensitive approach (Chang 2005), which can substantially reduce the numerical dissipation, in the event that the local Courant number is small.

3. Results

3.1. General Evolution

The whole process of the subsurface twisted magnetic flux tube emerging in the atmosphere is consistent with previous simulations (Archantis et al. 2004; Magara 2004; Manchester et al. 2004; Leake & Arber 2006; Murray et al. 2006; Fan 2001, 2009; Leake et al. 2013; Syntelis et al. 2017). The middle section of the flux tube starts to rise upward from the convection zone, due to the magnetic buoyancy caused by the density deficit, while the two ends of the tube sink slightly, because of the artificial anti-buoyancy. The middle part of the tube continues to rise and expand with height, until its apex touches the surface. The accumulation of the magnetic field under the surface then triggers the magnetic buoyancy instability, allowing part of the flux to enter the photosphere/chromosphere and expand rapidly in the corona.

Figures 4(a)–(l) show three-perspective views of the 3D structure and evolution of the magnetic flux tube during the emergence. The black line in these panels, which represents the axis of the initial flux tube, is obtained by tracing the O-point ($B_0$ minimum) on a vertical cross section of the flux tube at different times. Here, the cross section is selected as being the right $x$ boundary, since the flux tube evolves much more slowly at its two ends, and this part is more regular than the middle part that emerges into the atmosphere. The yellow lines are the field lines through four points that are evenly distributed on this cross section, with a small radial distance of $0.02L_s$ from the O-point. Note that the two ends of the flux tube also expand and evolve (very slightly) during the emergence of the central portion. Therefore, these field lines are not exactly the same sets of field lines in the different panels (or times). Nevertheless, they are a good approximation of the same sets of field lines, and they can reflect the topology and evolution of the magnetic field. The horizontal slice in each panel represents the solar surface, while the color indicates the $z$-component of the magnetic field ($B_z$).

The first column of Figure 4 is a snapshot at $t = 10$, when the middle of the flux tube displays a bulge, in an $\Omega$ shape, while the second column in Figure 4 is a snapshot at $t = 15$, when the front of the $\Omega$-shaped flux has emerged into the atmosphere in a simple arcade configuration, with the central-axis magnetic line (the black field line) being in a weakly forward S shape. As time goes on, the emerging flux rapidly expands into the higher corona, while the magnetic field structure becomes more complex, and eventually more fluxes emerge, forming a strongly reverse S-shaped—i.e., a sigmoid-shaped—magnetic structure.

Figures 4(m)–(p) show the isosurface of the flux tube with magnetic field strength $B = 0.1B_0$. At $t = 10$, the apex of the convex part of the flux tube reaches the height of the surface. Then, at $t = 15$, part of the magnetic field enters the atmosphere in a flattened spherical shape, which indicates that the lateral expansion of the emerging flux is faster than the vertical expansion. With the emergence of the flux, the coronal magnetic field also expands wider and higher, eventually forming a mushroom shape.

3.2. Vortical and Shearing Motion

The gradual separation of the two photospheric magnetic polarities as the flux tube emerges can be observed in the horizontal slices (surfaces) in Figure 4. Figures 5(a)–(c) show the evolution of the tangent velocity in this slice. These snapshots reveal a counterclockwise vortical and shearing motion in each polarity as the flux emerges. It has been suggested that this vortical motion is caused by the difference in the degree of the twist $q$ between the subsurface flux tube and the emerged field (Longcope & Welsch 2000; Fan 2009). The expansion and stretch of the emerged flux in the corona causes its $q$ to decrease rapidly, and the vortical motion of the two polarities transports the twist of the subsurface flux tube into the atmosphere, until the $q$-value equilibrates.

During the evolution of the coronal magnetic field, the combined effect of the vortical motion of the two polarities and the shearing flow distorts the field lines of the emerging flux, turning it from an initial S shape to a reverse S shape. The photospheric shearing flow squeezes the bottom of the coronal magnetic field toward its middle, and it has been suggested that magnetic reconnection occurs directly under the sheared field, to produce a coronal MFR (Fan 2009). Figures 5(d)–(f) show the distribution of the shear angle $\theta$ (indicating the angle between the magnetic field and the $y-z$ plane) of the emerging magnetic field on the central vertical plane ($x = 0$). We find that the distorted magnetic field gradually separates from the magnetic field that remains below the photosphere, eventually forming a coronal magnetic structure with a sigmoid-shaped inner core of the MFR. The newly formed coronal MFR at $t = 26$ is shown in Figures 5(g)–(i) (the blue and red lines).
The shearing motion of the polarities offers an important way for the magnetic energy to enter the atmosphere through the photosphere, along with the direct upward injection of the magnetic field. To quantify the different contributions from these effects, we calculate the total magnetic field energy above the photospheric surface ($z = 0.39$), as well as the Poynting flux through the surface, for the shear term and the vertical injection term (or emergence term), respectively, using the formula as derived in Kusano et al. (2002) and Démoulin & Berger (2003).

As can be seen in Figure 6, the total magnetic energy above the photosphere first increases quite quickly in time, from $t = 10$–$15$, in agreement with the fast increase of the unsigned magnetic flux through the photosphere. After that, the total magnetic energy becomes slower, and it eventually saturates near the end of the simulation (the top panel of Figure 6). The mismatch between the total magnetic energy and the injection magnetic energy means that the contribution from the dissipation and reconnection of magnetic fields is significant in the later phase, accounting for 22.4% of the injection magnetic energy. In the middle panel of Figure 6, the early injection of magnetic energy is mainly contributed by the emergence term, which decays quickly after $t = 13$, however, with the shear term dominating afterward. In the later stage of emergence, i.e., when the unsigned magnetic flux has nearly saturated, the emergence term decreases to a value close to—or even below—zero at the end of the simulation. This suggests that a small submergence of the magnetic energy occurs. The shear term also decays, but with a much slower rate than that of the emergence term. The net contribution of these two terms eventually stabilizes the total magnetic energy. This is consistent with the simulation of Magara & Longcope (2003).

Figure 4. (a)–(l) Three-perspective views of the 3D structure and evolution of the magnetic flux tube during its emergence. (m)–(p) The isosurface of the flux tube with magnetic field strength $B = 0.1B_0$. 

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3.3. Two-step Emergence

Our simulation agrees with many previous simulations, in that the emergence of the flux from the convection zone into the corona undergoes a two-step process, known as a “two-step emergence” mode (Matsumoto et al. 1993; Magara 2001). The first step involves the rise of the flux tube in the convection zone, due to magnetic buoyancy. During this period, the rising speed of the flux tube initially increases, then decelerates, as the flux approaches the surface. The second step involves the evolution of the emerging field into the atmosphere. Toriumi & Yokoyama (2010) have tested the effects of the amount of flux and the initial field strength on the flux emergence in 2D numerical simulations, dividing the results into “two-step emergence,” “direct emergence,” and “failed emergence.”

Direct emergence means that the rise of the flux tube is not reduced before it breaks through the photospheric surface. Failed emergence is when the flux tube eventually fragments in the convection zone and cannot enter into the atmosphere. The work of Murray et al. (2006) shows that the twist degree $q$ is also a factor affecting the emergence of the flux tube, with larger values of $q$ favoring emergence. Toriumi & Yokoyama (2011) point out that the criterion for failure emergence is $q = 0.05$ in 2D simulations.

Figure 7 shows the evolution of the height (top panel) and velocity (bottom panel) of the apex of the flux tube and the two O-points in the convection zone and corona on the central vertical plane ($x = 0$). Here, the apex (the black line) is defined as the highest point where the magnetic field strength $B$ is greater than $0.1B_0$. The evolution of the magnetic flux tube in

Figure 5. (a)–(c) The evolution of the $z$-component of the magnetic field ($B_z$, colors) and the tangent velocities (arrows) on the surface. Panels (d)–(f) show the distribution of the shear angle $\theta$ of the emerging magnetic field on the central vertical plane ($x = 0$). (g)–(i) The yellow and black lines are the same as in Figure 4, while the blue and red lines show the coronal MFR at the position of minimal $\theta$ on the central vertical plane.
its middle section is more complicated than that at its boundary, since multiple positions of very small $B_0$ are generated during emergence, and thus the center of the magnetic flux tube on this plane cannot be defined in the same way as in Figure 4. We consider the location with the largest $B_0$ positions on this cross section to be the O-point of the flux tube in the convection zone, while the highest position with minimal $B_0$ is considered to be the O-point of the coronal MFR, denoted by the Ocon and Ocor points, respectively.

The velocity at the apex of the flux tube (the black line) in the bottom panel of Figure 7 undergoes a process of increase, then decrease, then again increase. The position where the velocity decreases is near the solar surface ($z = 0.39$), thus our simulation falls under “two-step emergence,” as defined in Toriumi & Yokoyama (2010). The difference in position between the red and black lines in the top panel of Figure 7 can also reflect the first slow rise, the flux pileup near the photosphere, and the rapid expansion of the upper part of the flux tube in the corona. Figures 4 (m)–(p) show that the emerging magnetic field exhibits a significant horizontal expansion, which is one of the key features of “two-step emergence.” However, in the top panel of Figure 7, the “pileup” of the apex of the flux tube (the black line) near the surface is not obvious, and we consider that it is due to the relatively large $q$ and $B_0$.

In the first step of the emergence, the buoyancy of the flux tube is suppressed near the photosphere, due to the convective stability of the stratification there, which has a much smaller temperature gradient than that for convection instability (Cheung et al. 2007). Consequently, more and more magnetic fluxes with frozen plasma that rise from below accumulate near the photosphere, eventually resulting in an unstable configuration, in which the heavy plasma (as supported by the magnetic pressure gradient) overlays the lighter flux tube. Such an unstable configuration is called magnetic buoyancy instability (Matsumoto et al. 1993).

Archontis et al. (2004) and Hood et al. (2012) have given the following critical condition for this instability:

$$-H_p \frac{\partial}{\partial z} (\log B) > -\frac{\gamma}{2} \beta \delta + k_f^2 \left(1 + \frac{k_x^2}{k_y^2} \right). \tag{14}$$

where $H_p$, $\gamma$, $\beta$, $\delta$, and $\delta$ denote the local pressure scale height at the photosphere, the height, the magnetic field strength, the ratio of the specific heats, and the ratio of the plasma pressure to the magnetic pressure, respectively. $\delta$ is the superadiabatic index, which is $\sim 0.4$ for a strong stabilization of the atmosphere. $k_x$, $k_y$, and $k_z$ are the three components of the local perturbation wavevector. The left side of the equation describes the variation of the magnetic field strength of the flux tube with height, the first term on the right side indicates the stratification effect of the atmosphere, and the second term indicates the effect of the perturbation. This criterion helps us to determine the time of the appearance of the flux tube on the surface, since the magnetic flux can only emerge across the surface with the criterion being satisfied. We calculate the criterion for the front of the tube at each moment and plot the results in Figure 8. The equation perturbation term is not shown in the figure, since it is a small quantity, which has already been included in the criterion (the red line). We find that Equation (14) is met at $t = 12$, indicating that the buoyancy instability is triggered at around this moment, and indeed the magnetic flux first appears above the photosphere between $t = 11$ and 12. It is worth noting that the actual height of the solar surface is lifted up by the rising flux tube, thus at $t = 11$ the magnetic flux tube exceeds the initial height of the photosphere, but is still suppressed by the stability of the stratification.

3.4. Partial Emergence

Our simulation also agrees with the existing theory that the magnetic flux tube in the convection zone can only partially emerge into the atmosphere, with the field lines behaving on
the central vertical plane ($x = 0$) as described in Leake et al. (2013); i.e., the up-concave part can expand into the corona, while the down-concave part under the original tube axis remains mostly trapped under the surface. To give more details, Figure 9 shows the evolution of the field lines traced at 17 points within $0.04L_s$ of the O-point on the cross section of the right $x$ boundary. These points are the O-point and four uniform points in each direction along the positive and negative directions of $y$ and $z$, respectively, from the O-point. The black line in each panel is obtained by tracing the O-point on the right $x$ boundary. The red lines indicate the field lines in the center part of the tube, while the yellow lines indicate the outer field lines.

Similar to the simulation of Magara (2004), the outer field lines of the emerging flux tube spread out in a wide fan, after breaking through the surface, and some field lines even have a downward trend, such as the yellow field lines at $t = 24$ (Figure 9(c)). The lateral expansion of the inner field lines is restrained by the adjacent twisted field lines, meaning that the inner field lines tend to expand vertically. With time, the internal field of the flux rises higher than the corona, to form an MFR, and remains well connected to the convection zone flux tube.

### 3.5. Current Sheet

Figures 10(a)–(c) show the field lines that are traced at 20 points that are uniformly distributed over the height range $0.3L_s$ to $0.6L_s$ on the central vertical line $(x, y) = (0, 0)$, at $t = 26$. The green lines are the field lines above the black line (the same as in Figure 9), which have a reverse S shape in the corona, with the middle part being concave downward. The red lines indicate the magnetic field between the surface and the black line, and the blue lines are the field lines that do not fully emerge. Archontis et al. (2004) have pointed out that the plasma moves along the magnetic line of motion toward the lower part of the field line, and the heavy plasma that is gathered in the lower part increases the plasma $\beta$, pulls the field lines toward the surface (becoming the structure of the red line in Figure 9), and reduces the magnetic field gradient, which can cause the convective stability of the stratification to increase.

Figure 8. The criterion of magnetic buoyancy instability (the red line) for the front of the tube at each moment. The black line displays the variation of the magnetic field strength of the flux tube with height, while the blue line shows the stratification effect of the atmosphere.

This further restrains the emergence of the flux tube in the middle region between the two polarities, resulting in the subsurface field lines in this region not breaking through the photospheric surface.

Although further emergence of the flux tube is suppressed, the magnetic field can still enter the atmosphere through the motion of the coronal MFR footpoints, which creates the structure of the field lines so that they are like the top two blue lines in Figure 10. The blue and red field lines constitute an X-shaped magnetic field structure in the middle of the two polarity concentration regions, which induces a transverse current sheet. This current sheet comes into contact with the current sheet of the subsurface original magnetic flux tube, to form a ring current sheet (Figure 10(c)), with Figure 10(d) showing the streamline of the electric current (the cyan line) of the ring current sheet. We find that its induced magnetic field is in the same direction as the original flux tube; i.e., it will reduce the tendency of the original magnetic field decay.

Figure 11 shows the evolution of $B_z$ with time at the O-point of the convection zone flux tube, where $B_z$ is hardly decreasing after $t = 20$, which is significantly different from the rapid decrease in the earlier period. This process implies that in the absence of a covered coronal field, the axial direct current is enhanced during flux emergence, with no return current being observed (for more details of the study of current sheets in simulations of the covering field, see Török et al. 2014).

Figure 12 shows the evolution of the current sheet at isosurface $J = 8000$, with the transparent horizontal slice representing the solar surface. We find that in the second step of flux emergence, the evolution of the current sheet is divided into two stages. The first stage is before $t = 20$, when the rapid emergence of partial fluxes causes the subsurface current density to decrease. The second stage is when the current sheet starts to reform, and the subsurface current sheet reforms more rapidly. We believe that the formation of the subsurface current sheet is due to the suppression of the heavy plasma in the middle of the two polarity concentration regions, which causes the convection zone magnetic field to accumulate heavily under the surface. The current sheet above the surface is due to the
X-shaped magnetic field structure in Figure 10. These two current sheets eventually form a cavity configuration.

In addition, the red field lines in Figure 10 are pulled by the shear flow along the polarity reversal line and the heavy plasma, causing the sides of the coronal magnetic field to squeeze toward the middle, forming a vertical current sheet, as shown in Figure 13. In the real case, the resistivity in the corona is extremely low, and it is difficult for reconnection to occur, which leads to a close reverse magnetic field on both sides, forming a thinner and thinner current sheet that accumulates more and more energy. Once reconnection has occurred, a rapid eruption might be produced in the same way as shown in Jiang et al. (2021b): a continuously sheared bipolar arcade can initiate an eruption by tether-cutting reconnection.

4. Summary

In this paper, we have implemented an FES using the AMR–CESE–MHD code, and it has achieved results that are consistent with many previous FESs of similar configuration, but using different numerical codes. The AMR–CESE–MHD method is unique in that its algorithm is much simpler than traditional numerical MHD solvers, but it can achieve higher resolution. Further aided by AMR, it can well handle drastic variations of many orders of magnitudes on both spatial and time scales in the computational domain, including the convection zone and the different layers of the solar atmosphere. The computational cost is moderate, taking around 31 hr on 480 CPUs of 3 GHz.

The simulation follows the whole process of a twisted flux tube rising into the corona, having initially been placed in the convection zone. Driven by magnetic buoyancy, the center part of the tube rises, until it reaches the photospheric layer. At this position, the reduced gradient of the background temperature produces a stratification stabilization effect, which inhibits the further rising of the flux tube, and the magnetic flux starts to pile up near the surface. When the accumulated magnetic field is sufficient to trigger magnetic buoyancy instability, the upper part of the flux tube begins to emerge into the solar atmosphere and expands rapidly. The emerged magnetic field also suppresses the emergence of the following magnetic field, causing only a portion of the original flux tube to emerge.

During the evolution of the emerging magnetic field in the corona, the vortical and shearing motions of the magnetic polarities in the photosphere play an important role in transporting the magnetic energy and nonpotentiality into the atmosphere. To store this energy, the coronal magnetic field has also been reshaped into a sigmoid configuration (containing a weakly twisted rope), from the simple arcade at the early time of the emergence. Due to the strong lateral expansion of the coronal field, the entire 3D profile of the coronal field resembles the shape of a mushroom.

In addition, we also analyze the formation of the current sheet. The shear flow of the photospheric layer squeezes the sides of the coronal magnetic field toward the middle, and the reversed magnetic field (as seen in the central cross section) gets closer and closer, leading to the formation of a vertical current sheet. We also find that below this vertical current sheet, the horizontal current sheet on the surface forms a cavity structure with the current sheet in the convection zone, and the presence of the toroidal current increases the magnetic field in the convection zone, which may lead to the re-emergence of the magnetic field (Syntelis et al. 2017).

The present work has developed a framework for numerical experiments involving magnetic flux emergence and its role in producing solar eruptions, which will be the focus of our future works. For example, with an ultra-high-accuracy MHD simulation, Jiang et al. (2021b) established a fundamental mechanism behind solar eruption initiation: a bipolar field driven by a slow shearing motion in the photosphere can form an internal current sheet in a quasi-static way, which is followed by fast magnetic reconnection (in the current sheet) that triggers and drives the eruption. However, their model domain only includes the corona by assuming the lower layers of the atmosphere below the coronal base (i.e., the photosphere and chromosphere) as a line-tied boundary surface, plus the surface driving velocity is also specified in an ad hoc way. We are inspired by this to perform a higher-resolution FES, to investigate whether the same mechanism could also operate to
produce eruptions during the evolution of the emerging flux in the corona, with the shearing motion in the photosphere being generated in a more self-consistent way. In another study, Bian et al. (2022) have shown that due to the continuous shearing of the same polarity inversion line, the fundamental mechanism can effectively produce homologous CMEs via the recurring formation and disruption of the internal current sheet. Such homologous eruptions will be also investigated with longer-term FESs, to verify whether a second emergence will occur after the first emergence of the same flux tube. And the FESs have other important applications in studying solar eruptions, too—in particular, to explore what the key parameters are that can be used to predict eruptions. One such application has been demonstrated by Pariat et al. (2017), who used the FESs of Leake et al. (2013) to find that the ratio of the magnetic helicity of the current-carrying magnetic field to the total relative helicity can potentially be used to predict eruptions. This merits further studies using our FES model.

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Figure 12. The evolution of the current sheet at isosurface $J = 8000$, with the transparent horizontal slice representing the solar surface.

Figure 13. (a) The distribution of the current sheets on the central vertical plane ($x = 0$). (b) The isosurface of the current sheet at $J = 300$.
