Hyperon Radiative Decays in the $1/N_c$ Expansion

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Abstract

Using a recent calculation of transition magnetic moments in the $1/N_c$ expansion and a calculation showing the suppression of $E2/M1$ by powers of $N_c$, we compute the widths for the radiative decays $\Sigma^* \to \Sigma \gamma$, $\Sigma^* \to \Lambda \gamma$, and $\Xi^* \to \Xi \gamma$.

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In this Report we combine the results of two calculations using the 1/N_c expansion of QCD to predict radiative widths of strange spin-3/2 baryon resonances. The first, by the current authors [1], predicts all unmeasured diagonal and transition magnetic moments to relative accuracy ε^2/N_c^2, where ε is the dimensionless scale of SU(3) flavor breaking (shown in Ref. [1] to be at most ≃ 1/3). The second, by Jenkins, Ji, and Manohar [2], shows that M (spin-3/2) → m (spin-1/2) γ decays have an E2/M1 amplitude ratio of O(1/N_c^2). Only Δ → Nγ is considered in Ref. [2], but the same conclusion holds for almost the full SU(3) baryon multiplets, with the exceptions noted below. This conclusion follows because ΔJ=2 quadrupole operators require one more quark interaction, and hence an additional factor of 1/N_c, than ΔJ=1 magnetic moment operators [10].

The full radiative width is given by

\[ \Gamma(M \rightarrow m \gamma) = \frac{k_\gamma^2}{2M} \frac{m}{\pi} \left[ |M1|^2 + 3|E2|^2 \right], \]

(1)

where

\[ M1 = \frac{e}{2m} k_\gamma^{1/2} \mu_{Mm} \]

(2)

defines the transition moment \( \mu_{Mm} \), and the photon momentum is

\[ k_\gamma = \frac{M^2 - m^2}{2M}. \]

(3)

From Eq. (1), one sees that completely neglecting the E2 amplitude introduces only a relative O(1/N_c^4) correction to the width for O(N_c^1) transition magnetic moments. Since this is comparable to the effect of the calculated uncertainty of the transition moments in \( |M1|^2 \), we include it in the stated uncertainties; however, the relative E2 contribution to the width (3/N_c^4 ≃ 4%) is somewhat special in that it is positive definite. We accommodate this effect by shifting the predicted width values upward by this amount as well as adding in quadrature an additional uncertainty of this magnitude: In other words, the central value chosen for the E2 contribution is given by its natural size according to the 1/N_c expansion, and the uncertainty allows it to range between zero and twice this size.

An exception to this N_c scaling occurs for Σ^*-→Σ^-γ or Ξ^*-→Ξ^-γ, which vanish in the limit of SU(3) symmetry for N_c→3. Since the U-spin subgroup of SU(3) does not change electric charge, the photon is a U-spin scalar. On the other hand, for N_c = 3, Σ^*- and Ξ^*- belong to a U = 3/2 quartet, while Σ^- and Ξ^- form a U = 1/2 doublet, which forbids the transition in the SU(3) limit. Although this multiplet structure is somewhat modified
for $N_c > 3$, the final step of a calculation in the $1/N_c$ expansion sets $N_c = 3$. Therefore, the smallness of the corresponding transition moments, due to $U$-spin conservation, is still respected by the $1/N_c$ expansion: All SU(3)-conserving contributions to these processes are proportional to $(N_c - 3)$. In fact, the leading operators for both the transition magnetic and electric quadrupole moments in these cases have $O(\varepsilon N_c^0)$ matrix elements \[^{11}\] meaning that the only nontrivial scaling of $E2/M1$ comes from kinematic factors of the photon momentum \[^{2}\] $k_{\gamma}^{3/2}/k_{\gamma}^{1/2} = O(1/N_c)$, so that we estimate a relative $3/N_c^2 \simeq 33\%$ uncertainty from neglecting the $E2$ width contribution to $\Sigma^+ \rightarrow \Sigma^- \gamma$ or $\Xi^+ \rightarrow \Xi^- \gamma$.

Note that we only estimate the quadrupole moments according to their magnitude and $1/N_c$ scaling; it would also be possible to carry out a more detailed analysis using, for example, the measured $E2, \Delta^+ \rightarrow p \gamma$ transition and the results of a $1/N_c$ analysis among the quadrupole moments \[^{3}\] to determine the others. However, since the $E2$ contributions are seen to be smaller than $M1$ in all cases—often much smaller—this refinement is not essential for our purposes.

Combining Eqs. (1), (2), and (3), and using the predictions in Table XI of Ref. \[^{1}\] gives the results in Table I. While a number of calculations to predict these widths have been carried out in the past (using such varied methods as quark models \[^{4}\], the MIT bag model \[^{5}\], Skyrme models \[^{6}\], algebraic models \[^{7}\], heavy baryon chiral perturbation theory \[^{8}\], and lattice QCD \[^{9}\]), we point out that this calculation is not only model independent, but depends upon experimental input only from the 11 measured magnetic moments \[^{1}\], and upon expansions in $1/N_c = 1/3$ and $\varepsilon \lesssim 1/3$.

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\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Transition & Width (keV) & Reference \\
\hline
$\Sigma^+ \rightarrow \Sigma^+ \gamma$ & $118 \pm 10$ & \[^{1}\] \\
$\Sigma^0 \rightarrow \Lambda \gamma$ & $298 \pm 25$ & \[^{1}\] \\
$\Sigma^+ \rightarrow \Sigma^0 \gamma$ & $24.9 \pm 4.1$ & \[^{1}\] \\
$\Sigma^- \rightarrow \Sigma^- \gamma$ & $0.58 \pm 0.70$ & \[^{1}\] \\
$\Xi^+ \rightarrow \Xi^0 \gamma$ & $135 \pm 12$ & \[^{1}\] \\
$\Xi^- \rightarrow \Xi^- \gamma$ & $0.68 \pm 0.82$ & \[^{1}\] \\
\hline
\end{tabular}
\caption{Predicted values of the radiative widths (in keV).}
\end{table}
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[10] It should be noted that this suppression can in principle be wiped out if the additional quark contribution adds coherently for all $N_c$ quarks in the baryon. For the quadrupole case, the only operator for which this holds is $G^{iQ}G^j_3/N_c$, where $G$ is a combined spin-flavor operator (Eq. (2.1) of Ref. [1]), $i$ and $j$ are spin indices, $Q$ is the flavor index corresponding to coupling to a photon according to the quark charges $(q_u, q_d, q_s)$, and the flavor 3 index indicates a contribution along the $\Delta I = 1, \Delta I^3 = 0$ direction. In fact, such an operator appears with the usual $O(10^{-3})$ coefficient characteristic of isospin breaking, and therefore is negligible here.

[11] The leading magnetic moment operators are $\varepsilon q_s J^3_s$ and $\varepsilon Q J^3_s/N_c$, where $q_s = -\frac{1}{3}$ and $J_s$ is the angular momentum operator acting only on the strange quarks. The leading quadrupole operator in this notation is $\varepsilon \{G^{iQ}, J^j_s\}/N_c$. 

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