Evolution of dynamics of omni-channel pre-sale and spot competition based on consumer heterogeneity

Peiqin Li
East China University of Political Science and Law, Shanghai, China

Corresponding author and e-mail: Peiqin Li, lipeiqin@ecupl.edu.cn

Abstract. The paper constructs the pre-sale competition model, the pre-sale and spot hybrid competition model, and the pure spot competition model under the influence of the pre-sale promotion efforts. In the analysis of complete pre-sale competition model, only when the homogeneity of consumers is high, retailers will launch full pre-sale competition. If the pre-sale competition is to be launched, the greater the difference in the response of consumers’ demand to the pre-sale promotion, the higher the requirement for the homogeneity of consumers is. In the hybrid competition model of presale and spot, it is found that the optimal price equilibrium solution to full pre-sale competition is always less than the optimal price equilibrium solution of both pre-sale market and spot market. In the pure spot market competition model, the conditions are given for breaking the hybrid competition equilibrium between pre-sale and spot market, so as to realize the pure spot market competition. Finally, the paper finds the consumer heterogeneity interval optimized by the above three models through numerical analysis. And when heterogeneity of consumers in a medium range, enterprises 1 need more significant effort level to achieve the established earnings rise, on the other hand, when a consumer heterogeneity, within the range of low or high level interval, enterprise 1 only takes significant promotional effort in general level, the rise of yield can be achieved.

1. Introduction
Under the surging tide of mobile communication technology in recent years, numerous literatures have carried out a series of researches on online and offline channel conflicts and competitive optimization [1-3]. Presently a variety of rich apps, WeChat mini-programs, convenient and unimpeded information flow and payment method of capital flow, etc., highlight the importance of mobile channels in competitive optimization of supply chain [4]. The internal loss caused by multi-channel competition has gradually been fully recognized by some merchants, and the omni-channel sales model is becoming one of the normal conditions in the retail market [5-6].The integration of online and offline price information and promotion information has a positive impact on the rights and interests of the overall retailers [7].

At the same time, under the background of today's highly developed Internet, transaction costs are greatly reduced, and the role of pre-sale is gradually diversified. Aviation, hotel and other industries are characterized by certain supply capacity but uncertain demand, and they first adopted the pre-sale model [8].The seasonal cycle of perishable products that cannot be stored or consumed for a long time is obvious. Pre-sale can help merchants obtain consumers' demand information in advance and avoid...
the problem of information asymmetry [9-11]. Pre-sale demand information has important guiding significance for enterprises' production capacity and inventory [12]. Moreover, the pre-sale period of consumer groups have different preferences. In the hybrid competition mode between pre-sale and spot, the heterogeneity of consumers has great influence on the acceptance degree of pre-sale mode. The pre-sale model is generally applicable to markets with obvious consumer heterogeneity and uncertain demand [13]. The reason why the pre-sale competition model is adopted is more for the need of competition than the pure profit maximization [14]. The pre-sale model is conducive to the realization of optimal discount terms [15], the joint decision of pre-sale and repurchase helps dealers to obtain more revenue, and the extent to which pre-sale can increase revenue [16]. In addition, in the pre-sale stage, such factors as information disclosure [17], consumers' optimal purchase time [18-20], consumers' time preference, and consumers' regret [21], etc. have also been deeply discussed in many literatures.

In particular, the COVID-19 outbreak at the end of 2019 has severely impacted the existing ecology of many industries in China. Retail online consumption differences are further weaken, logistics become the "link of people with the outside world". During the outbreak, originally common mode of presale online gradually becomes a normal consumption trend.

In short, how will the dynamics of retailers' pre-sale and spot competition evolve? The function connotation of pre-sale will be richer, pre-sale's low-price promotion function is still in, but pre-sale should not only be a kind of promotion means, pre-sale itself will also superimpose a variety of promotional means to promote, pre-sale will play a more prominent role in the market competition. Based on the new characteristics of COVID-19 epidemic affecting offline market demand blowout, this paper focuses on the general evolution and trend of "pre-sale and spot mixed competition" under the influence of pre-sale promotion efforts and other factors

2. Model assumptions and notations

The two oligopolistic firms \( f_1 \) and \( f_2 \) produce the same kind of alternative products, which have same pricing strategies for online and offline commodities. Without loss of generality, assume that the total consumer market is 1. It assumes that promotion is mainly carried out in the pre-sale stage, ignoring the promotion cost in the spot market. The basic notations of this paper are shown in Table 1, including \( i = 1, 2 \).

| Symbol | Meaning |
|--------|---------|
| \( \alpha \) | Number of consumers purchasing from \( f_1 \) in pre-sale period |
| \( (1 - \alpha) \) | Number of consumers purchasing from \( f_2 \) in pre-sale period |
| \( v_j \) | The sale price in spot market \( (j = H, M, L, V, \geq 0) \) |
| \( \theta_j \) | The probability of sale price in spot market |
| \( \bar{v}(\cdot) \) | The average expected value of the pre-sale market |
| \( \delta \) | Degree of heterogeneity in consumer preferences |
| \( 1 - F(\cdot) \) | The PDF of \( f_1 \) |
| \( f(\cdot) \) | The PDF of \( f_2 \) |
| \( p_i \) | The pre-sale price of \( f_i \) |
| \( \sigma_m \) | The total revenue of \( f_i \) in Model \( m_i \), \( (i = I, II, III, \sigma_m > 0) \) |
| \( \gamma \) | The degree of response to their own pre-sale promotion effort |
| \( \lambda \) | The degree of response to competitor’s pre-sale promotion effort, \( (\gamma \geq \lambda) \) |
$\phi_i$ Pre-sale promotion effort level of $f_i$.

$\rho(\phi_i)$ Pre-sale promotion effort cost function of $f_i$.

$\varphi_i$ The coefficient of pre-sale promotion effort cost of $f_i$.

3. Model and analysis

3.1. Participation constraints and incentive constraints

The participation constraints of $f_i$ is $aE[p] - \rho_i \geq 0$ and the incentive constraints of $f_i$ is $aE[p] - \rho_i \geq (1-a)e[p] - \rho_i$ , it concludes that $\alpha \geq \max \left\{ \frac{\rho_i}{E[p]^2} \cdot \frac{1}{2} \left[ 1 + \frac{\rho_i}{E[p]} \right] \right\}$ ; Similarly it finds that when $\alpha \leq \min \left\{ \frac{\rho_i}{E[p]^2} \cdot \frac{1}{2} \left[ 1 + \frac{\rho_i}{E[p]} \right] \right\}$, $f_i$ will join in the presale competition.

We define $\tilde{\alpha} = \frac{1}{2} \left( 1 + \frac{\rho_i}{E[p]} \right)$, $\alpha = 1 - \frac{\rho_i}{E[p]}$ and get that, all consumers with $\alpha > \tilde{\alpha}$ purchase from $f_i$, and all consumers with $\alpha < \tilde{\alpha}$ purchase from $f_i$. Consumers with $\alpha \leq \alpha \leq \tilde{\alpha}$ with purchase from neither $f_i$ nor $f_i$, however, they will wait for the spot.

3.2. Model I: Pure presale competition model

In model 1 all consumers purchase from presale market of $f_i$ and $f_i$. Both of their profit functions are as follows:

$$\pi_{11} = \left\{ \frac{1}{\delta} \times \left( 1 + \frac{\delta}{2} - \hat{\alpha} \right) + \gamma e_1 - \lambda e_2 \right\} p_1 - \frac{\varphi_1}{2} e_1^2$$

$$\pi_{12} = \left\{ \frac{1}{\delta} \times \left( \hat{\alpha} - \frac{1}{2} \right) + \gamma e_2 - \lambda e_1 \right\} p_2 - \frac{\varphi_2}{2} e_2^2$$

And two firms charge

$$p_{11}^* = \delta E[p] \left\{ \frac{1}{3} \left( \gamma + \lambda \right) e_1 - e_2 \right\} + \left[ 1 + \left( \gamma - \lambda \right) e_1 + e_2 \right]$$

$$p_{12}^* = \delta E[p] \left\{ \left[ 1 + \left( \gamma - \lambda \right) e_1 + e_2 \right] - \frac{1}{3} \left( \gamma + \lambda \right) e_1 - e_2 \right\}$$

Two conditions that $p_{11}^* + p_{12}^* \leq E[p]$ & $p_{12}^* > 0$ need to be satisfied, and theorem 1 describes the equilibrium presale price and the corresponding consumer behavior, corollary 1 comes out sequentially.

**Theorem 1** Under the condition $e_1 \geq e_2$, $\gamma > \lambda$, if

$0 < \delta \leq \frac{1}{3} \left( 1 - \lambda \right)$ and $\frac{1}{3} \left( \gamma - \lambda \right) e_1 + e_2 > 0$, both $f_i$ and $f_i$ will compete in presale market totally, all consumers won’t purchase in spot market.

**Corollary 1** When consumer homogeneity is high, both $f_i$ and $f_i$ will launch full pre-sale competition. The greater the difference between consumer demand and pre-sale promotion, that is, $\gamma - \lambda > 0$, the higher the demand for consumer homogeneity will be. If there is no difference in the response of consumer demand to pre-sale promotion, that is, $\gamma - \lambda = 0$, the heterogeneous range of consumers in which $f_i$ and $f_i$ launch complete pre-sale competition is the largest, which implies $0 < \delta \leq \frac{1}{2}$.
3.3. Model II: Presale and spot hybrid competition model

In model II the condition \( p_1 + p_2 > E[V_Y] \) is satisfied, consumers may choose \( f_1 \) or \( f_2 \) to buy in the pre-sale stage, or wait to buy in the spot market.

\[
\pi_{\Pi_1} = p_1 \times \left[ \frac{1}{\delta} \times \left( \frac{1}{2} + \frac{\delta}{2} - \alpha \right) + \gamma e_1 - \lambda e_2 \right] + \left[ \theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l} \right] \int_{-a}^{x} \alpha dF(\alpha) - \frac{\theta_1}{2} e_1^2
\]

\[
\pi_{\Pi_2} = p_2 \times \left[ \frac{1}{\delta} \times \left( \alpha - \frac{1}{2} - \frac{\delta}{2} \right) + \gamma e_2 - \lambda e_1 \right] + \left[ \theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l} \right] \int_{-a}^{x} (1 - \alpha) dF(\alpha) - \frac{\theta_2}{2} e_2^2
\]

And two firms charge

\[
P_{\Pi_1} = \frac{1}{2} \left[ \frac{1 + (\gamma - \lambda)(e_1 + e_2)}{v_{y} + v_{l}} + \frac{\theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l}}{v_{y} + v_{l}} \right]
\]

\[
P_{\Pi_2} = \frac{1}{2} \left[ \frac{1 + (\gamma - \lambda)(e_1 + e_2)}{v_{y} + v_{l}} + \frac{\theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l}}{v_{y} + v_{l}} \right]
\]

Two conditions that \( p_{\Pi_1}^* + p_{\Pi_2}^* > E[V_Y] \) and \( p_{\Pi_1}^* + p_{\Pi_2}^* > 0 \) need to be satisfied, and theorem 2 describes the equilibrium price, corollary 2 comes out sequentially.

**Theorem 2** If \( \frac{1}{2} \left[ \frac{1 + (\gamma - \lambda)(e_1 + e_2)}{v_{y} + v_{l}} + \frac{\theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l}}{v_{y} + v_{l}} \right] \geq \frac{1}{2} \) and \( \frac{1}{2} \left[ \frac{1 + (\gamma - \lambda)(e_1 + e_2)}{v_{y} + v_{l}} + \frac{\theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l}}{v_{y} + v_{l}} \right] > 0 \), both \( f_1 \) and \( f_2 \) can launch the mixed competition between pre-sale and spot. Consumers either buy in the pre-sale stage or wait to buy in the spot market.

**Corollary 2** The conditions \( p_{\Pi_1}^* - p_{\Pi_1}^* > 0 \) and \( p_{\Pi_2}^* - p_{\Pi_2}^* > 0 \) are always established. In other words, the optimal price equilibrium of complete pre-sale competition between \( f_1 \) and \( f_2 \) is always smaller than that of the optimal price equilibrium of both pre-sale market and spot market, and the mixed competition between pre-sale and spot market is a better competitive state.

When new online spending habits gradually formed, even after the outbreak of the era of market consumption status back to normal, many have emergency transition or the cable on the enterprise business, will face more competition in the pre-sale and spot mixed state, pre-sale promotion race for online consumers will come to a new stage.

3.4. Model III: Pure spot market competition model

If both \( f_1 \) and \( f_2 \) give up the pre-sale competition mode and adopt full spot market sales, the profit expectations of \( f_1 \) and \( f_2 \) are (ignoring the spot market promotion efforts) as follows. Theorem 3 and corollary 3 comes out sequentially.

\[
\pi_{III_1} = \left[ \theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l} \right] \int_{-a}^{x} \alpha dF(\alpha) = \frac{1}{2} \left[ \theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l} \right]
\]

\[
\pi_{III_2} = \left[ \theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l} \right] \int_{-a}^{x} (1 - \alpha) dF(\alpha) = \frac{1}{2} \left[ \theta_1 v_{y} + \theta_2 v_{y} + \left( 1 - \theta_1 - \theta_2 \right) v_{l} \right]
\]
Theorem 3 When \( f_1^* \) and \( f_2^* \) compete completely in the spot market, the expected profit of both firms is 
\[
\left[ \left( \sigma_x \rho + \rho_y (1 - \sigma_x \rho) \right) \right].
\]

Corollary 3 When \( \pi_{x1}^* - \pi_{x2}^* \geq 0 \) or \( \pi_{mx1}^* - \pi_{mx2}^* \geq 0 \) is established, \( f_1^* \) and \( f_2^* \) will break the mixed competitive equilibrium of pre-sale and spot market, and turn to the spot market competition, and presale competition as auxiliary means.

4. Numerical analysis
Suppose the parameters are assigned as follows:
\[
\gamma = 0.4, \delta = 0.5, \nu_x = 0.7, \nu_y = \beta, \theta_1 = 0.3, \theta_2 = 1 \times 10^{-5}, \nu = 0.5 \times 10^{-5}; \varphi_1 = 0.002, \varphi_2 = 0.004, \nu = 200.\] This parameter is suitable for the daily consumer goods industry with strong heterogeneity of consumers and low cost of enterprise promotion efforts. The profit function unit of the enterprise is 100 million yuan, and the corresponding market demand is 100 million pieces.

4.1. Comparative analysis between model ‘s and model III’s expected revenue
In order to compare pre-sale competition with pure spot competition, the expected revenue of pure spot competition is \( \pi_{mx1}^* - \pi_{mx2}^* = 51 + 40\beta \). Take \( \pi_1^* \) as an example, the result is shown in Figure 1, in which the spatial curve \( O(0,0,0) \) corresponds to the equation \( \pi_{mx1}^* - \pi_{mx2}^* = 0 \). And corollary 4 comes out sequentially.

![Figure 1](image)

**Figure 1.** The optimal revenue’s difference changes with \( \delta \) and \( \beta \) between Model I and Model III.

Corollary 4 Comparing model I with model III, when consumer heterogeneity is very high \((0, 91 < \delta \leq 1)\), the lower the spot market expected sale price is, the more suitable for pure presale competition will be in model I; On the contrary, taking pure spot market competition model will be better. Further, when the heterogeneity of consumers is within the range \( 0 < \delta \leq 0.91 \), the pure spot market competition model is always better than the pure pre-sale market competition model.

Corollary 4 means that when the heterogeneity of consumers is very high, the difference of consumer preferences is very large, and the market competition is already very low. At this time, if the lowest price in the spot market is expected to be high, it is better to adopt the pure pre-sale model. For some segmented industries that require a high degree of customer customization and have a wide range of consumer audiences, such as auto parts customization, pre-sale mode is the best to prevent the loss of income caused by clearance of excessive inventory, which has been widely recognized in management practice.

4.2. Comparative analysis between model II’s and model III’s expected revenue
It still takes \( \pi_1^* \) as an example, comparing pre-sale and spot hybrid competition with pure spot competition the result is shown in Figure 2, in which the spatial curve \( E(1,0,0) \) corresponds to the equation \( \pi_{mx1}^* - \pi_{mx2}^* = 0 \). And corollary 5 comes out sequentially.
Figure 2. The optimal revenue’s difference changes with $\delta$ and $\beta$ between Model II and Model III.

**Corollary 5** Comparing model II with model III, when consumer heterogeneity is high ($0.823 < \delta \leq 1$), the presale and spot hybrid competition model is always better than pure spot competition model; On the contrary, when consumer heterogeneity is low ($0 < \delta \leq 0.669$), the pure spot competition model is always better than presale and spot hybrid competition model; when consumer heterogeneity is moderate ($0.699 < \delta \leq 0.823$), the lower the minimum expected sale price in the spot market, the more suitable it is to adopt the hybrid model of pre-sale and spot, otherwise, the pure spot market competition model should be adopted.

4.3. **Comparative analysis between model II’s and model I’s expected revenue**

Let’s continue to compare pre-sale and spot hybrid competition with pure presale competition, taking $f_i$ as an example, and the result is shown in Figure 3, in which the spatial curve $E_{3}E_{3}$ corresponds to the equation $\pi_{ii} - \pi_{ii}^* = 0$. And corollary 6, corollary 7& corollary 8 comes out sequentially.

![The difference between model II and model III’s optimal revenue](image)

Figure 3. The optimal revenue’s difference changes with $\delta$ and $\beta$ between Model II and Model I.

**Table 2.** The numerical results of optimal variables under parameter assignment.

| $\theta$ | $\sigma_1$ | $\sigma_2$ | $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | $\pi_5$ | $\pi_6$ |
|---------|------------|------------|---------|---------|---------|---------|---------|---------|
| 0.30    | 1.06       | 61.26      | 122.73  | 385.23  | 262.50  | 1.03    | 3495.03 | 1.39    |
| 0.30    | 1.05       | 36.47      | 73.15   | 335.65  | 262.50  | 25.82   | 2968.72 | 238.94  |
| 0.30    | 1.03       | 15.82      | 31.84   | 294.34  | 262.50  | 46.48   | 2546.26 | 420.88  |
| 0.30    | 1.033      | 0.60       | 0.21    | 262.71  | 262.50  | 62.29   | 2232.69 | 550.35  |
| 0.30    | 1.03       | -4.8       | -9.47   | 253.03  | 262.50  | 67.13   | 2138.48 | 588.26  |
| 0.30    | 1.025      | -15.2      | -30.13  | 232.38  | 262.50  | 77.46   | 1940.09 | 666.49  |
| 0.30    | 1.019      | -27.6      | -54.91  | 207.59  | 262.50  | 89.85   | 1706.86 | 755.56  |
| 0.35    | 1.062      | 96.37      | 139.0   | 420.25  | 281.25  | 1.13    | 3866.86 | -11.46  |
| 0.35    | 1.05       | 69.33      | 84.92   | 366.17  | 281.25  | 28.17   | 3284.09 | 257.80  |
Corollary 6 Comparing model II with model I, when consumer heterogeneity is high (0.781 < δ ≤ 1), the presale and spot hybrid competition model is always better than pure presale competition model; On the contrary, when consumer heterogeneity is low (0 < δ ≤ 0.583), the pure presale competition model is always better than presale and spot hybrid competition model; when consumer heterogeneity is moderate (0.583 < δ ≤ 0.781), the lower the minimum expected sale price in the spot market, the more suitable it is to adopt the hybrid model of pre-sale and spot, otherwise, the pure presale competition model should be adopted.

On the basis of corollary 4 to corollary 6, and Fig. 1 to Fig. 3, corollary 7 and corollary 8 can be showed as follows:

Corollary 7 When consumer heterogeneity is low (0 < δ ≤ 0.669), the optimal expected revenue of pure spot competition market model is the highest; When the heterogeneity of consumers is high (0.823 < δ ≤ 1), the hybrid competition model of pre-sale and spot has the highest optimal expected revenue. When the heterogeneity of consumers is within the range (0.669 < δ ≤ 0.823), if the minimum expected sale price of spot market is higher, the optimal expected profit of pure spot market competition model is the largest; otherwise, the optimal expected profit of the hybrid pre-sale and spot competition model is the largest.

Corollary 8 The pure pre-sale competition model may be better than the pure spot market competition model only when the heterogeneity of consumers is very high (0.91 < δ ≤ 1), but it is still lower than the hybrid competition model of pre-sale and spot. Only when the heterogeneity of consumers is low (0 < δ ≤ 0.583), the pure pre-sale competition model may be better than the hybrid pre-sale and spot market competition model, but it is still lower than the pure spot market competition model.

4.4. The influence of presale promotion efforts

Suppose the parameters are assigned as follows: \( \gamma = 16, \lambda = 8; \) 
\( r_x = 0.7r_y, r_i = 0.3r_y, \sigma_i = 1, \beta_1 = 0.2, \beta_2 = 0.4, v_x = 500, \delta = 0.55, \theta_1 = 0.56 \), all of these parameters are suitable for market segments with remarkably heterogeneity of consumers, high cost of enterprise promotion efforts, and relatively high unit price of products. Kinds of promotion activities are rich and diverse, and the cost of promotion is relatively high. The unit of profit function is ten thousand yuan, and the corresponding market demand is ten thousand pieces. This segment mainly discusses the relationship between the behavior of price reduction promotion and the cost of promotion efforts, and the table 2, corollary 9 & corollary 10 are drawn sequentially.

Corollary 9 Under the constraint of \( p_i + \rho_i \geq \frac{r_2 + r_1}{2} \), only when \( \theta_i \in (1.019,1.062) \), \( f_1 \) and \( f_2 \) can be motivated to carry out the hybrid competition between pre-sale and spot. In other words, the increase of promotion efforts of \( f_2 \) will lead to a significant decline in profits of \( f_2 \), until \( f_2 \) exits from hybrid competition and only participates in the spot market. If \( f_2 \) wants to survive in the hybrid competition of pre-sale and spot, it should either increase promotion efforts or survive at a lower pre-sale price.

Corollary 10 With the increase of promotion efforts, the pre-sale price and total profit of \( f_1 \) gradually increase, and its pre-sale price is gradually higher than the average price of spot market. The pre-sale price and total profit of \( f_2 \) gradually decrease, and the motivation of \( f_2 \) to participate in pre-sale competition gradually disappears. However, at the same level of promotion efforts, as the sales
probability of the medium price of the product increases, that is, the pre-sale price and total profit of \( f_i \) and \( f_j \) both rise, the average price of the spot market also rises.

4.5. The influence of consumer heterogeneity

Based on the assumption of segment D’s numerical value, by observing Fig. 4 & Fig. 5, the corollary 11 and corollary 12 can be drawn as follows:

![Figure 4](image_url)

**Figure 4.** Online promotional efforts change when \( f_i \)'s revenue is certain.

![Figure 5](image_url)

**Figure 5.** Online promotional efforts change when \( f_i \)'s revenue is certain.

**Corollary 11** The higher the heterogeneity of consumers, the lower the level of promotion efforts of \( f_i \) can achieve the established revenue; The lower the heterogeneity of consumers, the more promotion efforts \( f_i \) needs to make in order to achieve the established revenue. Moreover, when the consumer heterogeneity is in the medium range of \( \delta \in (0.3, 0.6) \), \( f_i \) needs to pay a relatively significant level of effort to achieve the increase of the established revenue, such as from \( \pi_{12} = 1000 \) to \( \pi_{13} = 2000 \). On the contrary, when the heterogeneity of consumers is at a lower level \( \delta \in (0, 0.3) \) or a higher level \( \delta \in (0.6, 1) \), \( f_i \) only needs to pay a general significant level of promotion efforts to achieve an increase in the established revenue, such as from \( \pi_{12} = 1000 \) to \( \pi_{13} = 2000 \).

**Corollary 12** When consumer heterogeneity is constant, the higher the level of pre-sale promotion efforts of \( f_i \), the lower the expected revenue of \( f_i \). The higher the consumer heterogeneity, the smaller the influence of \( f_i \)'s pre-sale promotion efforts on \( f_i \)'s revenues. Moreover, when consumer heterogeneity is in the medium range \( \delta \in (0.3, 0.6) \), \( f_i \) needs to pay more efforts to make \( f_i \)'s established revenue significantly lower, such as from \( \pi_{22} = 1000 \) to \( \pi_{23} = 500 \). On the contrary, when consumer heterogeneity is at a lower level \( \delta \in (0, 0.3) \) or a higher level \( \delta \in (0.6, 1) \), \( f_i \) only needs a general level of effort to achieve a decline in \( f_i \)'s established revenue, such as from \( \pi_{24} = 2500 \) to \( \pi_{25} = 2000 \).

5. Conclusions

Recently, Chinese governments have stepped up the pace of investment in new infrastructure and other fields. 5G, industrial Internet and other fields will develop faster in the future. The online market
has a bright prospect and the omni-channel wave is unstoppable. The pre-sale competition model, which has been popularized in various industries, is no longer limited to the initial functions such as simple demand forecasting and price reduction promotion, and the promotion efforts of pre-sale itself are also increasingly valued.

For monopolistic enterprises, when the heterogeneity of consumers is low, pure pre-sale competition can achieve a low-price equilibrium state of the game, and pre-sale becomes an important means for them to gain more market share. With less pre-sale promotion cost, they can gain obviously competitive advantage. For competitive enterprises, pure spot market sales mode could gain the most revenue.

With the gradual increase of consumer heterogeneity, the optimal equilibrium state among monopoly enterprises keeps changing, from the existence of double mixed competitive equilibrium to the short-term non-existence of equilibrium state, then to the constant existence of pre-sale and spot hybrid competitive equilibrium state. However, for competitive enterprises without monopoly power, when the heterogeneity of consumers is very high, the market competition is very small, the pre-sale model, or the hybrid competition model of pre-sale and spot can be adopted to obtain the best profits. When the consumer heterogeneity is in the medium level range, the hybrid competition among monopolistic enterprises is the most intense, requiring more cost of pre-sale promotion efforts to obtain an obvious competitive advantage.

Further research will be directed in the future including the deep integration of omni-channel, the new changes of consumer characteristics, and the influence of the richer pre-sale function on the competitive equilibrium, etc.

Acknowledgment
Supported by Youth Program of East China University of Political Science and Law (PN: 401-B-6001-16-0022409-20); Scientific Research Program of East China University of Political Science and Law (PN: 20HZK009)

References
[1] Wei—yu Kevin Chiang D C, James D Hess. Direct Marketing, Indirect Profits: A Strategic Analysis of Dual—channel Supply Chain Design[J]. Management Science, 2003: 1—20.
[2] Liz Giblert S M, Lai G. Supplier encroachment under asymmetric information[J]. Management Science, 2014, 60(2): 449—462
[3] Wang W, Li G, Cheng T C E. Channel selection in a supply chain with a multi-channel retailer: The role of channel operating costs[J]. International Journal of Productin Economics, 2016, 173(1): 54—65
[4] Verhoef P C, Kannan P K, Inman J J. From Multi-Channel Retailing to Omni-Channel Retailing[J]. Journal of Retailing, 2015, 91(2)
[5] Bell D R, Gallino S, Moreno A. Offline showrooms in omni-channel retail: Demand and operational benefits Management Science[J]. 2017, 3: 1-23
[6] Gao Fei, Su Xuanming. Omnichannel retail operation with buy online and pickup in store[J]. Management Science, 2017a, 638: 2478-2492
[7] Gallino S, Moreno A. Integration of online and offline channels in retail: The impact of sharing reliable inventory availability information [J]. Management Science, 2014, 60(6): 1434-1451
[8] Gale I L, Holmes T J. Advance-Purchase Discounts and Monopoly Allocation of Capacity[J]. American Economic Review, 1993, 83(01):135-146
[9] Moe W., W., S. Fader. Using Advance Purchase Orders to Forecast New Product Sales[J]. Marketing Science, 2002, 21(3): 347-364.
[10] Fay S, Xie J. The Economics of Buyer Uncertainty: Advance Selling vs. Probabilistic Selling[J]. Marketing Science, 2011, 29(06):1040-1057
[11] Boyaci T, zer. Information acquisition for capacity planning via pricing and advance selling: When to stop and act?[J]. Operations Research, 2010, 58(5) : 1328 — 1349.
[12] Özalp zer, Wei W. Invertory Control with Limited Capacity and Advance Demand Information[J]. Operations Research, 2004, 52(06) : 988 — 1000.
[13] Gale I L, Holmes T J. The Efficiency of Advance purchase Discounts in the Presence of Aggregate Demand Uncertainty[J]. International Journal of Industrial Oragnization, 1992,10(03):413-437
[14] Cachon G P, Feldman P. Is advance selling desirable with competition[J]. Marketing Science, 2017, 36(2):214-231
[15] Mccardle K, Rajaram K, Tang C S. Advance Booking Discount Programs Under Retail Competition[J]. Management Science, 2004, 50(05): 701-708.
[16] Xie J, Shugan S M. Electronic Tickets, Smart Cards, and Online Prepayments: When and How to Advance Sell[J]. Marketing Science, 2001, 20(03):219-243
[17] Chu L Y, Zhang H. Optimal Preorder Strategy with Endogenous Information Control[J]. Management Science, 2011, 57(06):1055-1077.
[18] Su X, Zhang F. Strategic Customer Behavior, Commitment, and Supply Chain Performance[J]. Management Science, 2008, 54(10):1759-1773.
[19] Liu, Q., G. van Ryzin. 2008. Strategic capacity rationing to induce early purchases. [J]. Management Science, 2008, 54(06):1115-1131.
[20] Swinney R. Selling to Strategic Consumers When Product Value is Uncertain: The Value of Matching Supply and Demand[J]. Management Science, 2011, 57(10):1737-1751.
[21] Nasiry, J., I. Popescu. Advance selling when consumers regret[J]. Management Science, 2012, 58(6):1160-1177.