Extracting amplitudes from photoproduction data

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Abstract. We consider the problems associated with amplitude extraction, from meson photoproduction data, over the first resonance regions. The notion of a complete experiment has motivated the FROST program at Jefferson Lab. Exercises applied to pion photoproduction data illustrate the problems to be confronted in any attempt to extract underlying resonance signals from these data (without introducing a model for the resonant process).

1. Introduction
Experiments at Jefferson Lab, MAMI, and other facilities worldwide are now producing a wealth of meson photoproduction data, utilizing polarized beams and targets. The concept of a complete experiment is often used to motivate the measurement of single- and double-polarization quantities. It has been shown [1] that at least eight independent measurements are required at each energy-angle value to extract the helicity or transversity amplitudes up to an unmeasurable overall phase, \( \phi(E, \theta) \), if a single pseudoscalar meson is produced. If one is interested in extracting resonance signals, these helicity amplitudes must be projected into multipole amplitudes. However, this involves an angular integral which cannot be performed, since \( \phi(E, \theta) \) is an unknown function of \( \theta \). Some further information is required to determine the overall phase.

Another approach [2], focused on the direct extraction of multipoles, was developed and applied to pion photoproduction data in the \( \Delta(1232) \) resonance region. Possible ambiguities were considered in Refs. [3] and [4]. As multipole amplitudes cannot be determined from the data alone, it is worth considering how much theory input is required to produce results that are as unbiased as possible.

In the following, we compare amplitude and multipole analysis in a simple case, explaining how ambiguities are removed. The old analysis of Ref. [2] is compared to recent data, showing some problems. Finally, the program of Ref. [2] is applied to a more modern database. Some potential pitfalls in the fitting procedure are mentioned, and a possible extension beyond the first resonance region is motivated.

2. Multipole vs transversity amplitude analysis
Grushin [2] determined a set of multipole amplitudes for pion photoproduction in the \( \Delta(1232) \) resonance region using only \( \sigma(\theta) \) (unpolarized) and \( P, S, \) and \( T \) [5] (single-polarization) data. This was done without Watson’s theorem, allowing a check against the known \( \pi N \) phase shifts. Grushin assumed that the high partial waves were given by real (electric) Born terms, with only
a few (4) multipoles fitted to the data. Born terms for \( \pi^+ n \) photoproduction determined the overall phase. Isospin symmetry, and a requirement that multipoles such as \( E_{3/2}^{1+} \) and \( M_{3/2}^{1+} \) should have the same phase (though the value of this phase was not required), removed an overall phase for the \( \pi^0 p \) production multipoles, leaving just a single discrete ambiguity.

With these assumptions (the theory input), additional measurements required for a complete experiment (at least 8 in total at each energy and angle) provide no further information, apart from one double-polarization measurement to remove the above mentioned discrete ambiguity. These additional measurements do, however, serve a purpose - they allow a test of the above assumptions. If the obtained amplitudes cannot predict further quantities, either additional waves must be fitted or the Born assumption for high partial waves is inadequate. Here we are, of course, implicitly assuming the data are correct. In figure 1, we show the Grushin multipoles, combined with the MAID (real Born + \( \rho \) exchange) amplitudes for \( \ell \geq 2 \), describing \( \pi^+ n \) data. The agreement is good for both the older data and the more recent \( S \) data set.

We re-fit the \( \pi^+ n \) data at 360 MeV, finding results in qualitative agreement with Grushin [6]. In table 1, results for the \( S \)-wave multipole are compared to Grushin [6], MAID [7], SAID [8], and a Bonn-Gatchina fit [9]. The energy-dependent fits [7-9] do not reproduce \( \text{Im} \ E_{0^+}^{1+} \) very well. The single-energy SAID and MAID fits (not shown) do only marginally better, though the
Table 1. S-wave multipoles for $\pi^+n$ at 360 MeV in units of $10^{-3}/m_\pi$.

|        | Re $E_{0+}^{\pi^+n}$ | Im $E_{0+}^{\pi^+n}$ |
|--------|-----------------------|-----------------------|
| Grushin [6] | 10.89(0.53)           | 2.22(0.44)            |
| Re-fit  | 11.18(0.5)            | 1.39(0.5)             |
| MAID [7]  | 14.47                 | 0.024                 |
| SAID [8]  | 11.7                  | -0.10                 |
| BoGa [9]  | 12.6                  | -0.25                 |

Figure 2. SAID real (solid) and imaginary (short-dash) parts of the $E_{1/2}^{1/2}$ multipole compared to MAID real (long-dash) and imaginary (dot-dash) values. SAID single-energy fits plotted with error bars.

MAID single-energy result has a value for Re $E_{0+}^{\pi^+n}$ closer to the SAID value. The larger MAID value for Re $E_{0+}^{\pi^+n}$ is reflected in figure 2, displaying $E_{1/2}^{1/2}$.

3. Testing the method

In figure 3, a refit to data at 340 MeV is displayed, showing the influence of new $S$ measurements. This suggests a repeat of the Grushin fits using more modern data. However, precise sets of $P$, $S$ and $T$ are not available for both $\pi^+n$ and $\pi^0p$ photoproduction, apart from a few energies, as would be required for an isospin decomposition. A connection to the $E2/M1$ ratio has motivated precise new $S$ measurements around the $\Delta(1232)$, but only a few new $T$ measurements have been made, and the $P$ database is largely the same as was available to Grushin.

As a result, several fits have been made using data generated to scatter around a known solution, such as SAID or MAID. If very precise data are generated (1% errors), it is generally possible to recover the underlying multipoles in a fit. This, however, is not a very stringent test, as the assumption of real high partial waves, given by Born terms, is used in almost every energy-dependent fit. Goodness of fit can also be misleading if only a minimal number of angular values are fitted. For example, we fitted back angle points and were able to predict the more forward cross section values. Taking instead an incorrect Born prediction of the high partial waves, an equally good fit to the back angle points was possible, but forward cross sections were not properly predicted. This demonstrates the need for precise data with a wide angular coverage. In figure 4, we show a test of the 340 MeV fit. A set of Mainz cross sections [10] covering intermediate angles (along with $P$, $S$, and $T$) were fitted. The result was then compared to an older Bonn set [11] covering a wider angular range. Within errors, this validates the fit and tests the given high partial wave component. A more stringent test is the prediction of double polarization quantities not fitted. In figure 5, a set of $G$ (beam-target polarization) data [12] is compared to the fit prediction. The data are well described. As the above method of direct multipole determination bypasses the overall phase problem associated with amplitude reconstruction, it provides a straightforward method to extract multipoles with
Figure 3. Grushin result (modified Born contribution for \( \ell \geq 2 \)) blue curve and refit (red curve) for \( \pi^+n \) data at 340 MeV. Recent MAMI data in red.

Figure 4. Comparison of fitted (red) and predicted (black) cross sections at 340 MeV for \( \pi^+n \).
Figure 5. Prediction of $G$ for $\pi^+n$ at 340 MeV.

minimal theoretical bias, at least for pion photoproduction. As the method in Ref. [2] does not employ Watson’s theorem, an extension to higher energies may be possible.

A fit to $\pi^0p$ photoproduction, and the construction of isospin amplitudes, is considerably more complicated. Multiple solutions are possible and the extracted amplitudes tend to have large errors, as was also found in Ref. [2]. One further serious obstacle is the quality of available $P$ data. For example, the problem associated with $\text{Im} \ E_{0^+}^{\pi^+n}$ in Table 1 can be removed if a slightly worse fit to $P$ data is allowed. Given the scatter and large uncertainties of the $P$ data plotted in Figs. 1 and 3, and the larger uncertainties associated with $\pi^0p$ measurements, more precise sets of this quantity would be very useful.

For eta photoproduction the problem is similar to $\pi^0p$ photoproduction, with both discrete symmetries and an overall unknown phase. Initial SAID fits to this reaction contained an overall unknown phase which masked the resonant contribution. A more recent multi-channel fit [13], based on the Chew-Mandelstam approach, has managed to resolve the low-energy $S$-wave amplitude, which now displays resonant behavior.

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