Scaling properties of the cosmic background plasma and radiation

A. Bershadskii

March 20, 2022

ICAR, P.O. Box 31155, Jerusalem 91000, Israel

Abstract

Scaling properties of the cosmic microwave background (CMB) radiation are studied using satellite (COBE-DMR maps), balloon-borne and ground-based (combined QMASK map) data. Quantitative consistency is found between the multiscaling properties of the COBE-DMR and QMASK CMB maps. Surprisingly, it is found that the observed CMB temperature multiscaling resembles quantitatively the multiscaling properties of fluid turbulence, that indicates primordial plasma turbulence as an origin of the CMB temperature space anisotropy.

PACS numbers: 98.70.Vc, 98.80Es, 95.30Qd
1 Introduction

In the last few years the interest in nonlinear (turbulent) processes in primordial plasma has been renewed in relation to the origin of magnetic fields observed in galaxies (see, for instance [1]-[6] and references therein). It has been suggested that primordial magnetic fields might arise during the early cosmic phase transitions. The plasma of the early universe has a high conductivity so that a primordial magnetic field would be imprinted on the co-moving plasma and would dissipate very slowly. Such a field could then contribute to the seed needed to understand the presently observed galactic magnetic fields, which have been measured in both the Milky Way and other spiral galaxies, including their halos. At earliest times, the magnetic fields are generated by particle physics processes, with length scales typical of particle physics. If the inflation hypothesis is correct, long correlation lengths can be expected following the inflation. It is shown in [1] that turbulence with its cascade processes is operative, and hence the scale of magnetic fields is considerably larger than would be the case if turbulence were ignored.

The turbulent nature of the magnetic field may have interesting effects on various phase transitions in the early universe. Also, the inherent shift of energy from small to large scales may be of interest in connection with density fluctuations due to the magnetic energy. In particular, it is shown in [3],[4] that rotational velocity perturbations, induced by a tangled magnetic field can produce significant angular scale anisotropies in cosmic microwave background (CMB) radiation through the Doppler effect. The conclusions are relevant to arcminute scales [3] and to $l > 1000$ [4]. These scales will be available for observation with the new MAP and Planck space missions. On the other hand, the authors of [5] and [6] consider early universe turbulence and its imprint on the cosmic background on larger angular scales via a tensor mode contribution. The question then is: Can one use the modern data on the CMB radiation (see for recent reviews [7],[8]) to find out the fingerprints of the primordial turbulence? The first attempt to find such fingerprints was performed in [9]. However, a CMB map used in Ref. [9] was one of the earliest of the COBE-satellite CMB maps and, therefore, the preliminary investigation of Ref. [9] gave a qualitative picture only. In present paper to obtain quantitative results we will use the improved four-year COBE-satellite CMB maps. Moreover, we have now a new generation of balloon-borne and ground-based CMB data obtained with angular resolution of the order of a degree (the angle resolution for COBE data, in contrast, was $\sim 7^\circ$). We
choose so-called QMASK map representing the data in a form suitable for our purpose. The QMASK map is a combination of QMAP (balloon-borne) and Saskatoon (ground-based) data [10]-[13]. Using the three CMB maps (two COBE-DMR and one QMASK) we study the structure functions of the CMB temperature space increments and moments of CMB dissipation rate for frequency ranges: 31.5, 53, and 90 GHz (COBE-DMR data), 26-46 GHz (QMASK map).

The main results of our investigation are:
1) The CMB temperature structure functions and dissipation rate moments are found exhibiting multiscaling properties.
2) The observed multiscaling resembles quantitatively the multiscaling properties of fluid turbulence, that indicates its primordial turbulence origin.
3) Comparison of the results obtained for the COBE-DMR and for the QMASK shows quantitative consistency of their CMB maps.

2 Structure functions of QMASK map

Let us start from the QMASK CMB map. The QMAP (balloon) and Saskatoon (ground-based) data were used to construct this map [10]-[13]. The observations were made in Ka and Q bands (26-36 GHz and 36-46 GHz respectively) for different polarization channels. QMAP was designed to measure the CMB anisotropy by direct mapping. The Saskatoon data set is different in the sense that it does not contain simple sky temperature measurements. Instead, it contains different linear combinations of sky temperatures with complex set of weighting functions. The QMAP and Saskatoon data were combined to produce a CMB map named QMASK. The map was generated by subdividing the sky into square pixels of side $\Theta \simeq 0.3^\circ$ and consists of 6495 pixels. The data are represented using three coordinates $x, y, z$ of a unit vector $\mathbf{R}$ in the direction of the pixel in the map (in equatorial coordinates). Since some pixels are much noisier than others and there are noise correlations between pixels, Wiener filtered maps are more useful than the raw ones. Wiener filtering suppresses the noisiest modes in a map and shows the signal that is statistically significant.

The main tool for extracting cosmological information from the CMB maps is the angular power spectra. However, such spectra use only a fraction
of the information on hand. Taking advantage of the good angular resolution of the QMASK map, we can study statistical properties of angular increments of CMB temperature for different values of angular separation \[14\]. The increments are defined as

\[ \Delta T_r = (T(R + r) - T(R)) \]  

where \( r \) is dimensionless vector connecting two pixels of the map separated by a distance \( r \), and the structure functions of order \( p \) as \( \langle |\Delta T_r|^p \rangle \) where \( \langle . \rangle \) means a statistical average over the map.

Figure 1 shows, in log-log scales, the dependence of the moments of different orders, \( p = 0.2, 0.5, 0.7, 1, 2, 3 \) of \( \langle |\Delta T_r|^p \rangle \) against \( r \) for the Wiener-filtered QMASK map. The best linear fits are drawn to indicate the scaling

\[ \langle |\Delta T_r|^p \rangle \sim r^{\zeta_p}. \]  

Figure 2 shows the scaling exponents \( \zeta_p \) extracted from the QMASK data shown in Fig. 1 (circles). We also show in this figure (as dashed curve) the scaling exponents \( \zeta_p \) corresponding to velocity increments in fluid turbulence. For this purpose we use the She-Leveque model \[15\], which is in very good agreement with the numerous data for velocity increments obtained in inertial interval of scales for isotropic fluid turbulence:

\[ \zeta_p = p/9 + 2[1 - (2/3)^{p/3}] \]  

The correspondence seen in Figure 2 is rather remarkable (especially for low moments for which the calculations are also the most reliable).

It appears that for the angular interval between 0.9° and 4°—this being the most reliable range for the data \[13\]—the temperature increments in QMASK map, scale essentially as velocity increments in fluid turbulence.

It should be noted that under certain circumstances scaling of the moments (2) for \( p = 2 \) can be related to scaling of corresponding spectrum. One of the necessary conditions for such relation is sufficiently fast decrease of the spectrum with increasing of wave numbers \( k > \eta^{-1} \), where \( \eta \) is the lower edge of the scaling interval of \( r \). For fluid turbulence such fast decay of the spectral asymptotic indeed takes place (\( \eta \) is so-called Kolmogorov or viscous scale there). However, it seems that for the CMB data this is not the case and the strong relation between fluid turbulence and the CMB space fluctuations takes place in a restricted interval of scales only (and does not
determine the spectral asymptotic).

Background signal can be distinguished from the atmospheric and foreground contamination signals by their frequency dependence, their frequency coherence and their spatial power spectra. Employing these methods, it has been shown in a series of papers [10]-[13], [16], [17], [18] that all known atmospheric and foreground sources give a minor contribution to the QMASK map. We are thus led to conclude that the background (primordial plasma) turbulence is the most plausible reason why the CMB map resembles fluid turbulence in its intermittency properties. It should be noted that remarkable correspondence between the scaling laws in the near space plasma and in isotropic fluid turbulence is a well known puzzle (see, for instance, [19], [20] and references therein).

3 Dissipation rate of CMB temperature from the four-year COBE-DMR data maps

The above discovered modulation of the CMB temperature by a fluid turbulent velocity fluctuations can be explored further. Dissipation rate of kinetic energy corresponding to the velocity field $\mathbf{u}$ is [21]:

$$\varepsilon \sim \sum_{ij} \left( \frac{\partial u_i}{\partial x_j} \right)^2$$  \hfill (4)

Local space averaging of the dissipation rate is used to construct a space dissipation measure [21]:

$$\varepsilon_r = \frac{\int_{v_r} \varepsilon dv}{v_r}$$  \hfill (5)

where $v_r$ is a subvolume with space-scale $r$. Scaling law of this measure moments,

$$\langle \varepsilon_r^p \rangle \sim r^{-\mu_p}$$  \hfill (6)

(where $\langle ... \rangle$ means an average) is an important characteristic of the dissipation rate [21]. For isotropic fluid turbulence there exists a simple relation between scaling exponents $\zeta_p$ for the velocity increments (see previous section) and the scaling exponents $\mu_p$ (6) [21]:

$$\mu_p = p - \zeta_{3p}$$  \hfill (7)
This relationship allows to calculate $\mu_p$ using the She-Leveque representation (3).

Let us now calculate the dissipation rate of the CMB temperature fluctuations using the COBE-DMR CMB maps. The COBE satellite was launched on 1989 into 900 km altitude sun-synchronous orbit. The Differential Microwave Radiometers (DMR) operated for four years of the COBE mission and mapped the full sky. The instrument consists of six differential microwave radiometers, two nearly independent channels that operate at each of three frequencies: 31.5, 53 and 90 GHz. Each differential radiometer measures the difference in power received from two directions in the sky separated by $60^0$, using a pair of horn antennas. Each antenna has a $7^0$ beam. In this investigation we will use a four-year DMR data map of the CMB temperature. Most of the Galactic emission has been removed from the map using so-called ”combination technique” in which a linear combination of the DMR maps is used to cancel the Galactic emission [23]. We will also use a DMR four-year sky map for frequency 31.5 GHz only. In the last case we will remove the sky regions with violently large temperature fluctuations to remove roughly the Galactic contamination (see below and Fig. 4). The first detail interpretations of the four-year COBE/DMR data one can find in [24]-[28].

To organize the DMR data in the temperature maps the sky was divided into 6144 equal area pixels. These are formed by constructing a cube with each face divided into $32 \times 32 = 1024$ squares, projected onto a celestial sphere in elliptic coordinates. The projection is adjusted to form equal area pixels having a solid angle of $4\pi/6144$ sr or 6.7 square degrees. Because the $7^0$ beamwidth of the sky horn is greater than separation between pixels ($2.6^0$ average), this binning oversamples the sky.

Let us introduce a dissipation rate for CMB temperature $T$ in following way (cf (4),(5)):

$$\chi_r = \frac{\int_{v_r} (\nabla T)^2 dv}{v_r}$$

(8)

Technically, using the above described pixel map, we will calculate the CMB temperature dissipation rate using summation over pixel sets instead of integration over subvolumes $v_r$. So that in the multiscaling

$$\langle \chi_s^p \rangle \sim s^{-\mu_p}$$

(9)

the metric scale $r$ is replaced by number of the pixels - $s$, characterizing size.
of the summation set (cf (6)). The $\chi_s$ is a surrogate of the real 3D dissipation rate $\chi_r$. It is believed that the surrogates can reproduce quantitative multiscaling properties of the 3D dissipation rates [22].

Figure 3 shows scaling of the CMB temperature dissipation rate moments $\langle \chi_p^s \rangle$ calculated for the DMR maps (Fig 3a: for combination of frequencies 31.5, 53 and 90 GHz; Fig. 3b: for frequency 31.5 GHz). Log-log scales are chosen in the figure for comparison with scaling equation (9). The straight lines (the best fit) are drawn to indicate the scaling.

Figure 4 shows the scaling exponents $\mu_p$ extracted from figure 3a (open circles) and from figure 3b (crosses), the dashed curve in figure 4 corresponds to the intermittency exponents $\mu_p$ calculated using the She-Leveque model [15] for velocity (kinetic energy) fluctuations of fluid turbulence: equations (3) and (7).

Thus, for the satellite data we also observe good agreement with fluid turbulence modulation of the CMB temperature. Applicability of the fluid turbulence relationship (7) to the CMB data can be itself considered as an additional indication of this modulation. Moreover, comparing Figures 2 and 4 (through the relationship (7)) one can conclude that the QMASK data represented in Figure 2 are quantitatively consistent to the COBE-DMR data represented in Figure 4. In this respect it should be noted that the QMASK and the COBE-DMR data were obtained in about the same range of frequencies. Very recent suggestions to detect the primordial turbulence using the relic gravitational waves [6], [29], [30] might give a complementary experimental tool for the present observations in the nearest future.

The author is grateful to the QMAP, Saskatoon, COBE-DMR instruments teams and to the NASA Goddard Space Flight Center for providing the data and support. Discussions on the subject with C.H. Gibson, A. Kosowsky and K.R. Sreenivasan as well as the Referee’s comments and information were useful for this investigation.
References

[1] A. Brandenburg, K. Enqvist and P. Olesen, Phys. Rev. D 54, 1291 (1996).

[2] J.D. Barrow, P.G. Ferreira and J. Silk, Phys. Rev. Lett. 78, 3610 (1997).

[3] K. Subramanian and J.D. Barrow, Phys. Rev. Lett. 81, 3575 (1998).

[4] K. Subramanian and J.D. Barrow, MNRAS, 335, 3 (2002).

[5] R. Durrer, P.G. Ferreira and T. Kahniashvili, Phys. Rev. D, 61, 043001 (2000).

[6] A. Kosowsky, A. Mack, and T. Kahniashvili, Phys. Rev. D, 66, 024030 (2002).

[7] E. Gawiser and J. Silk, Phys. Reports, 333-334, 245 (2000).

[8] W. Hu and S. Dodelson, Ann. Rev. Astr. and Astrophys. in press (2002); astro-ph/0110414

[9] A. Bershadskii, Phys. Rev. D, 58, 127301 (1998).

[10] Y.Z. Xu, M. Tegmark, A. de Oliveira-Costa, M.J. Devlin, T. Herbig, A.D. Miller, C.B. Netterfield and L. Page, Phys. Rev. D 63, 3002 (2001).

[11] M.J. Devlin, A. de Oliveira-Costa, T. Herbig, A.D. Miller, C.B. Netterfield, L. Page and M. Tegmark, ApJ., 509, L69, (1998).

[12] A. de Oliveira-Costa, M.J. Devlin, T. Herbig, A.D. Miller, C.B. Netterfield, L. Page and M. Tegmark, ApJ., 509, L77 (1998).

[13] Y. Xu, M. Tegmark and A. de Oliveira-Costa, Phys. Rev. D (in press), astro-ph/0104419 (2002).

[14] A. Bershadskii and K.R. Sreenivasan, Phys. Lett. A, 299, 149 (2002).

[15] Z.-S. She and E. Leveque, Phys. Rev. Lett., 72, 336 (1994).

[16] M. Tegmark, D.J. Eisenstein, W. Hu and A. de Oliveira-Costa, ApJ. 30, 133 (2000).
[17] A.D. Miller, J. Beach, M.J. Devlin, et al., astro-ph/0108030 (2001).

[18] A. de Oliveira-Costa, M. Tegmark, M.J. Devlin, et al., ApJ. 542, L5 (2000).

[19] M.L. Goldstein, Astrophys. Space Sci. 227, 349 (2001)

[20] J. Cho, A. Lazarian and E.T. Vishniac, ApJ, 564, 291 (2002).

[21] A.C Monin and A.M.Yaglom, Statistical fluid mechanics, Vol. 2 (MIT Press, Cambridge, 1975).

[22] K.R. Sreenivasan and R.A. Antonia, Annu. Rev. Fluid Mech, 29, 435 (1997).

[23] COBE-DMR Four-Year Analysed Science Data Sets, ed. C.L. Bennett, D. Leisawitz, and P.D. Jackson, COBE Ref. Pub. 96-B ( Greenbelt, MD: NASA/GSFC, 1996).

[24] C.L. Bennet et al., ApJ, 464, L1 (1996).

[25] E.L. Wright et al., ApJ, 464, L21 (1996).

[26] A. Kogut et al., ApJ, 464, L29 (1996).

[27] A.J. Banday et al., ApJ, 475, 393 (1997).

[28] D.J. Fixen et al., ApJ, 486, 623 (1997).

[29] A.D. Dolgov, D. Grasso and A. Nicolis, Relic Backgrounds of Gravitational Waves from Cosmic Turbulence, astro-ph/0206461 (2002).

[30] A.D. Dolgov and D. Grasso, Phys. Rev. Lett. 88, 011301 (2002).
Figure Captions

Figure 1. Logarithm of the structure functions $\langle |\Delta T_i|^p \rangle$ against $\log_{10} r$ for the Wiener-filtered QMASK map. The CMB temperature $T$ is measured in $\mu K$. The straight lines (the best fit) are drawn to indicate the scaling (2).

Figure 2. The scaling exponents $\zeta_p$, corresponding to Eq. (2), for the QMASK map (circles). The dashed curve corresponds to the scaling exponents $\zeta_p$ calculated using the She-Leveque model (3) [15] for the turbulent fluid velocity increments.

Figure 3. a) The CMB dissipation rate moments $\langle \chi_s^p \rangle$ against $s$ (the COBE-DMR four-year map for combination of frequencies: 31.5, 53 and 90 GHz). The dissipation rate $\chi_s$ is measured in $10^{-2} \mu K^2$; b) Analogous data, but now for the CMB sky four-year map corresponding to frequency 31.5 GHz.

Figure 4. The scaling exponents $\mu_p$ extracted from figure 3 against $p$. Open circles correspond to the CMB four-year map for combination of frequencies: 31.5, 53 and 90 GHz, and crosses correspond to the CMB four-year sky map for frequency 31.5 GHz. The dashed curve corresponds to the intermittency exponents $\mu_p$ calculated using the She-Leveque model for the kinetic energy dissipation in fluid turbulence (equations (3),(7)) [15].
\[ <|\Delta T_r|^p> \]

QMASK (Wiener-filtered)

- \( p=3 \)
- \( p=2 \)
- \( p=1 \)

\[ <|\Delta T_r|^p> \]

QMASK (Wiener-filtered)

- \( p=0.7 \)
- \( p=0.5 \)
- \( p=0.2 \)

\( r \)
\[ \zeta_p \]

- QMASK (Wiener-filtered)
- velocity increments in fluid turbulence
