RELATIVISTIC ELECTROMAGNETIC MASS MODELS: CHARGED DUST DISTRIBUTION IN HIGHER DIMENSIONS

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Abstract.

Electromagnetic mass models are proved to exist in higher dimensional theory of general relativity corresponding to charged dust distribution. Along with the general proof a specific example is also sited as a supporting candidate.

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1. Introduction

In many theories higher dimensions play an important role, specially in super string theory (Schwarz, 1985; Weinberg, 1986) which demands more than usual four dimensional space-time. This is also true in studying the models regarding unification of gravitational force with other fundamental forces in nature. In the case of a simple solution to the vacuum field equations of general relativity in 4 + 1 space-time dimensions Chodos and Detweiler (1980) have shown that it leads to a cosmology which at the present epoch has 3 + 1 observable dimensions in which the Einstein-Maxwell equations are obeyed. Lorenz-Petzold (1984) has studied a class of higher dimensional Bianchi-Kantowski-Sachs space-times of the Kaluza-Klein type whereas Ibanez and Veraguer (1986) have examined radiative isotropic cosmologies with extra dimensions related to FRW models.

In this connection it is interesting to note that electromagnetic mass models where all the physical parameters, including the gravitational mass, are arising from the electromagnetic field alone have been extensively studied (Tiwari et al., 1984; Gautreau, 1985; Grøn, 1986; Ponce de Leon, 1987; Tiwari and Ray, 1991; Ray and Bhadra, 2004) in the space-time of four dimensional general relativity. Thus it is believed that study of electromagnetic mass models in higher dimensional theory will be physically more interesting.

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Under this motivation we have considered here a static spherically symmetric charged dust distribution corresponding to higher dimensional theory of general relativity. It is proved, as a particular case, from the coupled Einstein-Maxwell field equations that a bounded and regular interior static spherically symmetric charged dust solution, if exists, can only be of purely electromagnetic origin. An example, which is already available, is examined in this context and is shown that the solution set satisfies the condition of being electromagnetic origin.

2. The Einstein-Maxwell Field Equations

The Einstein-Maxwell field equations for the case of charged dust distribution are given by

$$G^{ij} = R^{ij} - \frac{1}{2} g^{ij} R = -8\pi [T^{ij}_m + T^{ij}_e],$$  \hspace{1cm} (1)

$$[(-g)^{1/2} F^{ij}]_j = 4\pi J^i (-g)^{1/2},$$  \hspace{1cm} (2)

$$F_{[ij,k]} = 0$$  \hspace{1cm} (3)

where $F^{ij}$ is the electromagnetic field tensor and $J^i$ the current four vector which is equivalent to

$$J^i = \sigma u^i$$  \hspace{1cm} (4)

$\sigma$ being the charge density and $u^i$ is the four velocity of the matter satisfying the relation

$$u_i u^i = 1.$$  \hspace{1cm} (5)

The matter and electromagnetic energy momentum tensors are, respectively, given by

$$T^{(m)}_{ij} = \rho u^i u_j,$$ \hspace{1cm} (6)

$$T^{(em)}_{ij} = \frac{1}{4\pi} [-F_{jk} F^{ik} + \frac{1}{4} g_{ij} F^{kl} F^{kl}],$$ \hspace{1cm} (7)

where $\rho$ is the proper energy density.

Now we consider the $(n + 2)$ dimensional spherically symmetric metric

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 X_n^2$$  \hspace{1cm} (8)
where
\[ X_n^2 = d\theta_1^2 + \sin^2\theta_1d\theta_2^2 + \sin^2\theta_1\sin^2\theta_2d\theta_3^2 + \ldots + \prod_{i=1}^{n-1} \sin^2\theta_i \] d\theta_n^2.

The convention adopted here for coordinates are
\[ x^1 = r, \quad x^2 = \theta_1, \quad x^3 = \theta_2, \ldots x^{n+1} = \theta_n, \quad x^{n+2} = t \] (9)
and also
\[ g_{11} = -e^\lambda, \quad g_{22} = -r^2, \quad g_{33} = -r^2\sin^2\theta_1, \]
\[ g_{44} = -r^2\sin^2\theta_1\sin^2\theta_2, \ldots g_{(n+1)(n+1)} = -r^2 \prod_{i=1}^{n-1} \sin^2\theta_i, \]
\[ g_{(n+2)(n+2)} = e^{\nu/2}, \] (10)
As we have considered here a static fluid, so
\[ u^i = [0, 0, 0, \ldots (n + 1)\text{times}, \quad e^{-\nu/2}], \] (11)
\[ J^1 = J^2 = J^3 = \ldots J^{n+1} = 0, \quad J^{n+2} \neq 0 \] (12)
so that the only non-vanishing components of \( F_{ij} \) of equations 2 and 3 are \( F_{1(n+2)} \) and \( F_{(n+2)1} \).

In view of the above, the Einstein-Maxwell field equations for static spherically symmetric charged dust corresponding to the metric (8) are
\[ e^{-\lambda}[n\nu'/2r + n(n - 1)/2r^2] - n(n - 1)/2r^2 = -E^2, \] (13)
\[ e^{-\lambda}[\nu'' + \nu^2/4 - \nu'\lambda'/4 + (n - 1)(\nu' - \lambda')/2r + (n - 1)(n - 2)/2r^2] - (n - 1)(n - 2)/2r^2 = E^2, \] (14)
\[ e^{-\lambda}[n\lambda'/2r - n(n - 1)/2r^2] + n(n - 1)/2r^2 = 8\pi \rho + E^2, \] (15)
\[ [r^nE]' = 4\pi r^n\sigma e^{\lambda/2} \] (16)
where \( E \), the electric field strength, is defined as \( E = -e^{-(\nu + \lambda)/2}\phi' \), the electrostatic potential \( \phi \) being related to the electromagnetic field tensor as \( F_{(n+2)1} = -F_{1(n+2)} = \phi' \).
3. Higher Dimensional Electromagnetic Mass Models

From the field equations (13) and (15), we have

\[ e^{-\lambda}(\nu' + \lambda') = 16\pi r\rho/n. \]  

(17)

Now, we make use of the conservation equations \( T_i^j; i = 0 \) which yield

\[ \rho\nu' = \left[ q^2 \right]'/4\pi r^4 + (n - 2)E^2/2\pi r \]  

(18)

where the charge, \( q \), is related with the electric field strength, \( E \), through the integral form of the Maxwell’s equation (16), which can be written as

\[ q = Er^n = 4\pi \int_0^r \sigma e^{\lambda/2} r^n dr. \]  

(19)

Again, equation (15) can be expressed in the following form as

\[ e^{-\lambda} = 1 - 4M/4\pi r^{n-1} \]  

(20)

where the active gravitational mass, \( M \), is given by

\[ M = 4\pi \int_0^r \left[ \rho + E^2/8\pi \right] r^n dr. \]  

(21)

Hence, following the technique of Tiwari and Ray (1991) we see that for vanishing charge density, \( \sigma \), via equation (19) one gets from equation (18) the unique relation

\[ \rho\nu' = 0. \]  

(22)

Thus, we have the following two cases:

**Case I:** \( \rho \neq 0, \quad \nu' = 0 \)

For this case, from equations (13) and (14), we have \( \lambda \) to be a constant. This in turn makes \( \rho \) equal to zero and hence by virtue of equation (22) space-time becomes flat.

**Case II:** \( \rho = 0, \quad \nu' \neq 0 \)

In this case, from equations (20) and (21), one can see that \( \lambda \) becomes zero. Then from equation (17), we have the metric potential as a constant and again the space-time becoming flat.

We are not considering here the third case, viz. \( \rho = \nu' = 0 \), which is quite a trivial one. However, from the above cases (i) and (ii) it is evident that, at least at a particular case, all the charged dust models are of electromagnetic origin, viz., all the physical parameters originating
purely from electromagnetic field. This type of models are known as electromagnetic mass models in the literature (Lorentz, 1904; Feynman, 1964).

**An example:**

The solution set obtained by Khadekar et al. (2001) for the static spherically symmetric charged dust is as follows:

\[ e^\nu = Ar^{2N}, \quad (23) \]
\[ e^{-\lambda} = \left[ \frac{(n-1)}{N + (n-1)} \right]^2, \quad (24) \]
\[ \rho = \frac{Nn(n-1)^2}{8\pi r^2[N + (n-1)]^2}, \quad (25) \]
\[ \sigma = \frac{N(n-1)^2[n(n-1)]^{1/2}}{4\pi^{1/2}r^2[N + (n-1)]^2}, \quad (26) \]

where \( A \) and \( N \) both are constants with the restriction that \( N \geq 0 \).

The total charge and mass of the sphere in terms of its radius, \( a \), are respectively given by

\[ q = \frac{N[n(n-1)]^{1/2}a^{n-1}}{2^{1/2}[N + (n-1)]}, \quad (27) \]
\[ m = \frac{Na^{n-1}}{N + (n-1)}. \quad (28) \]

The charge and mass densities in the present case take the relationship

\[ \sigma = [2(1 - 1/n)]^{1/2} \rho. \quad (29) \]

Therefore, the charge and mass densities are proportional to each other with the constant of proportionality \([2(1 - 1/n)]^{1/2}\) which takes the value unity for \( n = 2 \) i.e. in the four dimensional case and the relation (29) reduces to the usual form

\[ \sigma = \pm \rho \quad (30) \]

which is known as the De-Raychaudhuri (1968) condition for equilibrium of a charged fluid.

Thus, from equations (23)-(29), it is evident that all the physical quantities including the effective gravitational mass vanish and also the spherically symmetric space-time becomes flat when the charge density vanishes implying \( N = 0 \). The solution here, therefore, satisfies the criteria of being of purely electromagnetic origin.
4. Discussions

The present paper is, in general, higher dimensional analogue of the work of Tiwari and Ray (1991) whereas the example given here (Khadekar et al., 2001) is the higher dimensional analogue of the paper of Pant and Sah (1979). Thus, we have presented here a model which corresponds to spherically symmetric gravitational sources of purely electromagnetic origin in the space-time of higher dimensional theory of general relativity. It is already proved in the four dimensional presentation of the present paper (Tiwari and Ray, 1991) that a bounded continuous static spherically symmetric charged dust solution, if exists, can only be of electromagnetic origin. Hence, this is also true in the higher dimension of general theory of relativity.

In this regard we would like to discuss briefly the role of higher dimensions in different context. It have been shown by Ibanez and Veraguer (1986) that for the open models related to FRW cosmologies the extra dimensions contract as a result of cosmological evolution whereas for flat and closed models they contract only when there is one extra dimension. Fukui (1987) recovers Chodos-Detweiler (1980) type solutions, as mentioned in the introduction, where the Universe expands as $t^{1/2}$ by the percolation of radiation into 4D space-time from the fifth dimension, mass, although the 5D space-time-mass Universe itself is in vacuum as a whole. It is interesting to note that considering mass as fifth dimension a lot of other works also have been done by several researchers (Wesson 1983; Banerjee, Bhui and Chatterjee 1990; Chatterjee and Bhui 1990) which contain Einstein’s theory embedded within it. In one of such investigations it is argued that a huge amount of entropy can be produced following shrinkage of extra-dimension which may account for the very large value of entropy per baryon observed in 4D world (Chatterjee and Bhui 1990). Kaluza-Klein type higher dimensional inflationary scenario have been discussed by Ishihara (1984) and Gegenberg and Das (1985) where it is shown that the contraction of the internal space causes the inflation of the usual space.

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