On Brane Stabilization and the Cosmological Constant

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Abstract

We address the problem of the cosmological constant within the Randall-Sundrum scenario with a brane stabilization mechanism. We consider brane tensions of general form. We examine the conditions under which a small change of the positive tension of the first brane can be absorbed in a small modification of the two-brane configuration, instead of manifesting itself as a cosmological constant. We demonstrate that it is possible to have a cosmological constant in the range predicted by recent observational data, if there is an ultraviolet cutoff of order 10 TeV in the contributions to the brane tension from vacuum fluctuations.
**Introduction:** The smallness of the cosmological constant has defied explanation despite a long history of attempts [1]. The problem is further complicated by the recent observational evidence that the present cosmological constant may be of the order of the critical density of the Universe [2]. An interesting proposal for the resolution of the problem was made in ref. [3]: If our four-dimensional world is embedded in a higher-dimensional space-time, the effect of non-zero vacuum energy may affect only the curvature in the extra dimensions, allowing for a flat four-dimensional metric.

The possible existence of large extra dimensions [4, 5, 6] provides a setup in which to realize the above idea. The Standard Model fields are assumed to be localized on a four-dimensional surface, the “brane”, and only gravitons can propagate in the “bulk” of the extra dimensions [6]. For a class of geometries characterized as “warped” the low energy gravitons are also localized on the brane [7]. This framework provides a new opportunity to confront the cosmological constant problem and several attempts have been made in this direction [8]-[10]. We are interested in the possibility that changes of the vacuum energy of the brane, the brane tension, can be absorbed in modifications of the bulk geometry so that the effective four-dimensional constant remains zero or almost zero. Existing scenarios have undesirable features, such as the presence of naked singularities in the metric, or very specific assumptions about the form of the couplings in the effective action of the theory [8].

Our setup is that of ref. [7], as generalized in refs. [11, 12]: We consider a five-dimensional system with two four-dimensional branes, of positive and negative tension respectively. These are located at the boundaries of a compact fifth dimension with anti-deSitter bulk metric. The geometry is warped, in a way that the low-energy gravitons are localized near the positive-tension brane. The distance between the two branes is not arbitrary as in ref. [7]. The presence of a bulk field with a non-trivial potential permits only static configurations with specific values for the size of the fifth dimension [11]. The backreaction of the field on the metric is taken into account along the lines of ref. [12].

We consider brane tensions of general form, which also depend on the value of the bulk field at the location of the brane. Our starting point is a configuration with zero four-dimensional cosmological constant. We then discuss under what conditions small modifications of the positive tension of the first brane, which we identify with our low-energy world, can be absorbed into small displacements of the branes, instead of manifesting themselves as a cosmological constant.

**The fine-tuning:** We consider a system of two branes in the background of a bulk scalar field $\phi$. The action is given by

$$S = \int d^4x \, dy \sqrt{|\det g_{\mu \nu}|} \left[ -2M^3 R + \frac{1}{2} (\partial \phi)^2 - V(\phi) \right] - \sum_{\alpha = 1, 2} \int d^4x \sqrt{|\det g_{ij}|} \lambda_\alpha(\phi). \quad (1)$$

By rescaling all dimensionful quantities by $2M$ (which is of the order of the fundamental Planck’s constant) we obtain

$$S = \int d^4x \, dy \sqrt{|\det g_{\mu \nu}|} \left[ -\frac{1}{4} R + \frac{1}{2} (\partial \phi)^2 - V(\phi) \right] - \sum_{\alpha = 1, 2} \int d^4x \sqrt{|\det g_{ij}|} \lambda_\alpha(\phi), \quad (2)$$

consistently with the notation of ref. [12]. We emphasize that all quantities in eq. (2) are dimensionless, even though they are denoted by the same symbols as in eq. (1).
For the metric we assume the ansatz
\[ ds^2 = e^{2A(y)} \eta_{ij} \, dx^i \, dx^j - dy^2 \] (3)
with space-time topology \( \mathbb{R}^{3,1} \times S^1 / \mathbb{Z}_2 \). The two branes are located at the boundaries of the fifth dimension. Einstein’s equations and the equation of motion of the field are [12]

\[ \phi'' + 4A' \phi' = \frac{\partial V(\phi)}{\partial \phi} + \sum_{\alpha=1,2} \frac{\partial \lambda_\alpha(\phi)}{\partial \phi} \delta(y-y_\alpha) \] (4)

\[ A'' = -\frac{2}{3} \phi'^2 - \frac{2}{3} \sum_{\alpha=1,2} \lambda_\alpha(\phi) \delta(y-y_\alpha) \] (5)

\[ A'^2 = -\frac{1}{3} V(\phi) + \frac{1}{6} \phi'^2. \] (6)

Primes denote derivatives with respect to \( y \). We set \( y_1 = 0 \) and \( y_2 = R \).

The solutions of the above equations for general potentials \( V(\phi) \) predict a fixed distance \( R \) between the two branes. They generalize the stabilization mechanism of ref. [11] by taking into account the backreaction of the scalar field on the gravitational background. The functions \( \lambda_\alpha(\phi) \) are characterized as the brane tensions. Their form is determined by the vacuum energy of the fields that live on the brane. We have also allowed for an interaction of these fields with the bulk field, so that the tensions depend on \( \phi \). The presence of the branes imposes boundary conditions for \( A'(y) \) and \( \phi(y) \) at \( y = 0, R \). The integration of eq. (4), (5) around the \( \delta \)-functions and use of the \( \mathbb{Z}_2 \) symmetry leads to

\[ y = 0 \quad \phi' = \frac{1}{2} \frac{\partial \lambda_1(\phi)}{\partial \phi} \quad A' = -\frac{1}{3} \lambda_1(\phi) \] (7)

\[ y = R \quad \phi' = -\frac{1}{2} \frac{\partial \lambda_2(\phi)}{\partial \phi} \quad A' = \frac{1}{3} \lambda_2(\phi). \] (8)

By imposing these conditions, we have only to solve eqs. (4)-(6), neglecting the \( \delta \)-function contributions.

The three equations (4)-(6) are not independent, as they are related through the Bianchi identities. We look for a solution of eqs. (4), (6), which automatically satisfies eq. (3). Our ansatz for the metric, eq. (3), indicates that we should expect a fine-tuning for the existence of a static solution [12]. The reason is that our choice of four-dimensional Minkowski metric \( \eta_{ij} \) requires the vanishing of the effective cosmological constant on the branes.

A simple way to understand the fine-tuning is the following: We can substitute \( A' \) as given by eq. (3) into eq. (4). Without loss of generality we choose the negative root for \( A' \) and \( A(y = 0) = 0 \). We assume that our low-energy Universe corresponds to the positive-tension brane located at \( y = 0 \) (with \( \lambda_1(\phi) > 0 \)). This means that \( A < 0 \) in the bulk. Eq. (4) now becomes a non-linear second-order differential equation for \( \phi(y) \), whose solution requires two boundary conditions. These are obtained by substituting eqs. (7) into eq. (4). The resulting algebraic equation in general has a discrete number of solutions that give the allowed values of \( \phi \) at the location of the first brane. For each of them the corresponding value of \( \phi' \) is given by the first of eqs. (7). Let us denote generically these solutions by \( (\phi_1, \phi'_1) \). Now we can integrate eq. (4), with \( A' \) expressed in terms of eq. (3) and the initial conditions \( \phi(0) = \phi_1, \phi'(0) = \phi'_1 \). The resulting trajectory \( (\phi(y), \phi'(y)) \) determines the form of the field and the metric in the bulk.
In analogy with above, the substitution of the conditions $[8]$ into eq. $[9]$ leads to a discrete number of possible values of $\phi$ and $\phi'$ at the location of the second brane. Let us denote them generically by $(\phi_2, \phi'_2)$. The fine-tuning is now apparent: The trajectory $(\phi(y), \phi'(y))$ must pass through $(\phi_2, \phi'_2)$. This can be achieved only through a careful choice of $\lambda_2(\phi)$ (assuming that $\lambda_1(\phi)$, $V(\phi)$ are chosen arbitrarily). However, a possible change of $\lambda_1(\phi)$, through a phase transition on the first brane for example, destabilizes the solution. The trajectory corresponding to the new initial condition $(\tilde{\phi}_1, \tilde{\phi}'_1)$ does not pass through $(\phi_2, \phi'_2)$. Some unknown mechanism must modify the tension $\lambda_2(\phi)$ of the second brane for a new static solution to exist.

However, there is another possibility: A new static configuration can still exist without modification of $\lambda_2(\phi)$ if the new boundary conditions $(\tilde{\phi}_1, \tilde{\phi}'_1)$ lie on the initial trajectory $(\phi(y), \phi'(y))$. Then the new solution of eqs. $[4]$, $[5]$ is the part of the original one between $(\tilde{\phi}_1, \tilde{\phi}'_1)$ and $(\phi_2, \phi'_2)$. Physically it corresponds to a small displacement of the positive-tension brane in a way that the second brane remains unaffected. Since the values of $(\tilde{\phi}_1, \tilde{\phi}'_1)$ are determined by the initial function $\lambda_1(\phi)$ and its change through a phase transition, it seems that this scenario is not possible in general. In the following we discuss how it may work.

**A first attempt:** We start by assuming an initial configuration with a metric given by eq. $[3]$. As we explained above this requires an initial fine-tuning of the brane tensions $\lambda_1(\phi)$, $\lambda_2(\phi)$. We do not attempt to address this issue in this work, even though we comment on its possible resolution later on. We are concerned with the requirement of a new fine-tuning every time $\lambda_1(\phi)$ changes. We consider a small change $\lambda_1(\phi) \rightarrow \lambda_1(\phi) + c(\phi)$, with $|c(\phi)| \ll 1$. There are no constraints on the form of the function $c(\phi)$, apart from the assumption that it is small. If we restore the dimensions of the brane tension (see eq. $[3]$), our assumption is that $|c(\phi)| \ll (2M)^4$ for the relevant values of $\phi$.

For the new brane tension $\lambda_1(\phi) + c(\phi)$, the boundary conditions $[3]$ when substituted into eq. $[3]$ lead to a new algebraic equation for $\phi$. We denote the solution of this equation by $\tilde{\phi}_1 = \phi_1 + \delta \phi_1$, where $\phi_1$ is the original solution when the tension is given by $\lambda_1(\phi)$. Assuming $\delta \phi_1 = \mathcal{O}(c(\phi_1))$ and keeping terms up to order $c(\phi_1)$, we find

$$
\frac{\partial}{\partial \phi} \left[ V - \frac{1}{8} \left( \frac{\partial \lambda_1}{\partial \phi} \right)^2 + \frac{1}{3} \lambda_1^2 \right] (\phi_1) \delta \phi_1 = -\frac{2}{3} \lambda_1(\phi_1) c(\phi_1) + \frac{1}{4} \frac{\partial \lambda_1}{\partial \phi} (\phi_1) \frac{\partial c}{\partial \phi} (\phi_1) + \mathcal{O}(c^2(\phi_1)).
$$

(9)

For the field derivative we find from the first of eqs. $[7]$

$$
\delta \phi'_1 = \frac{1}{2} \frac{\partial^2 \lambda_1}{\partial \phi^2} (\phi_1) \delta \phi_1 + \frac{1}{2} \frac{\partial c}{\partial \phi} (\phi_1) + \mathcal{O}(c^2(\phi_1)).
$$

(10)

The second of the boundary conditions $[3]$ provides the new value of $A'$, which we do not need explicitly$^3$.

As we discussed above, we are looking for modifications of the brane tension that lead to a new solution lying on the initial trajectory from $(\phi_1, \phi'_1)$ to $(\phi_2, \phi'_2)$. We consider this trajectory at a small distance $\delta y = \mathcal{O}(c(\phi_1))$ from the initial location of the positive-tension brane. By integrating the differential equation $[3]$ up to order $\delta y$, we find

$$
\delta \phi'(y) = \phi''(0) \delta y + \mathcal{O}(y^2) = \left[ -4A'(\phi_1) \phi'_1 + \frac{\partial V}{\partial \phi}(\phi_1) \right] \delta y + \mathcal{O}(c^2(\phi_1))
$$

$^3$For this section $A$ may be eliminated as an independent variable. $A'$ can be seen as a function of $\phi$ and $\phi'$ defined in eq. $[3]$.  

3
\[ \delta \phi(y) = \phi'(0) \delta y + \mathcal{O}(y^2) = \frac{1}{2} \frac{\partial \lambda_1}{\partial \phi}(\phi_1) \delta y + \mathcal{O}(c^2(\phi_1)). \]  

(12)

We would like to identify \( \delta \phi(y) \) and \( \delta \phi'(y) \) with \( \delta \phi_1 \) and \( \delta \phi_1' \) respectively. The parameter \( \delta y \) can be adjusted so as to achieve part of our purpose. By solving eq. (12) for \( \delta y \) and substituting into eq. (11), we can express \( \delta \phi'(y) \) in terms of \( \delta \phi(y) \). If we require that the resulting expression be consistent with eq. (10) (after the identification of \( \delta \phi(y), \delta \phi'(y) \) with \( \delta \phi_1, \delta \phi_1' \)) we find

\[ \frac{\partial}{\partial \phi} \left[ V - \frac{1}{8} \left( \frac{\partial \lambda_1}{\partial \phi} \right)^2 + \frac{1}{3} \lambda_1^2 \right](\phi_1)^{\delta \phi_1} = \frac{1}{4} \frac{\partial \lambda_1}{\partial \phi}(\phi_1) \frac{\partial c}{\partial \phi}(\phi_1) + \mathcal{O}(c^2(\phi_1)). \]  

(13)

For the last expression to be combatible with (11) we expect a constraint on \( \lambda_1(\phi_1), c(\phi_1) \). This turns out to be simply \( \lambda_1(\phi_1) c(\phi_1) = 0 \) up to order \( c(\phi) \). As we would like to keep the form of \( c(\phi) \) arbitrary, we are led to the condition \( \lambda_1(\phi_1) = 0 \). The interpretation is the following: If the brane is initially located at a position where the value of the bulk field is such that the brane tension vanishes, subsequent small modifications of the brane tension can be absorbed in small displacements of the brane with the metric retaining its four-dimensional Minkowski form. We emphasize that it is not necessary for \( \lambda_1(\phi) \) to vanish for any \( \phi \). On the contrary, it may be of order 1 in general. Only the presence of a zero is necessary. This is possible because the constraint we derived is independent of the derivatives of \( \lambda_1(\phi) \), contrary to the naive expectation.

There are two unsatisfactory elements in our solution: Firstly, it seems inconsistent to introduce the brane tension as a \( \delta \)-function source in Einstein’s equations and then require it to...
Positive (negative) values of $\Lambda$ correspond to deSitter (anti-deSitter) four-dimensional metrics. We remove this ambiguity by setting a non-zero effective cosmological constant $\Lambda$, which leads to the replacement of eq. (6) by (12). Notice, however, that the derivatives of $\lambda$ specifically, we consider the possibility that the four-dimensional tensions and the potential of the bulk field that plays the role of the bulk cosmological constant.

The limit $\Lambda \rightarrow 0$ reproduces the Minkowski metric we considered earlier.

Within this framework, no fine-tuning is required. General choices of $\lambda_1(\phi)$, $\lambda_2(\phi)$ are expected to lead to a solution with some value of $\Lambda$ (12). There is a graphic way to see this: Substitution of eqs. (12) into eq. (17) with $\Lambda = \Lambda_1$ and $A_1 = 0$ gives the value $\phi_1$ at the location of the first brane between the brane tension and the potential of the bulk field that plays the role of the bulk cosmological constant. The limit $\Lambda \rightarrow 0$ is expected to lead to a solution with some value of $\Lambda$ (12).

Positive (negative) values of $\Lambda$ are determined by the mis-match at the location of the first brane between the brane tension and the potential of the bulk field that plays the role of the bulk cosmological constant.

The boundary conditions (7), (8) remain unaffected. Eqs. (7), (17) then demonstrate that the set of initial conditions forms a curve $C_1$ on the $A = 0$ plane, parametrized by $A_1$. We assume that $A_1$ grows in the direction of the arrow in fig. 1. For given $A_1$, the initial conditions $(\phi_1, \phi_1', 0)$ result in a unique solution for eqs. (4), (17). This corresponds to a trajectory in $(\phi, \phi', A)$ space. Varying $A_1$ results in a surface formed by the various trajectories.

At the location of the second brane, eq. (17) must be satisfied after substitution of eqs. (8). For a given $\Lambda = \Lambda_2$, the possible values $(\phi_2, \phi_2', A_2)$ form a curve in $(\phi, \phi', A)$ space. In general, this curve meets the surface of trajectories at some point $P$. The trajectory going through $P$ corresponds to a value $A_1$ that is not necessarily equal to $A_2$. However, identification of $A_1$ and $A_2$ is expected to be possible in general through variation of $A_2$. The point $P$ moves with changing $A_2$, in a way that it creates a curve $C_2$ on the trajectory surface. On $C_2$ trajectories characterized by $A_1$ meet boundary conditions characterized by $A_2$. For large families of functions $\lambda_2(\phi)$, the identification of $A_1$ and $A_2$ should be possible at some point on $C_2$ without the necessity of

\[ A'^2 - \Lambda e^{-2A} = \frac{1}{3}V(\phi) + \frac{1}{6}\phi'^2. \]
fine-tuning. For example, one may consider some function \( \lambda_2(\phi) \) for which \( \Lambda_2 \) increases on \( C_2 \) in the direction indicated by the arrow in fig. 1, opposite to the direction of increase of \( \Lambda_1 \). The location of \( P \) with \( \Lambda_1 = \Lambda_2 = \Lambda \) determines the trajectory that corresponds to a static solution of eqs. (14, 17) under the boundary conditions (7), (8).

The presence of the term proportional to \( \Lambda \) in eq. (17) modifies the solutions relative to the case with four-dimensional Minkowski metric. The nature of the change can be seen by solving eq. (17) for a flat potential to the case with four-dimensional Minkowski metric. The nature of the change can be seen by solving eq. (17) for a flat potential \( V(\phi) = -\Lambda \text{=const.} \) and \( \phi = \text{const.} \). For \( 0 < |\Lambda| \ll 1 \) one finds that \( \Lambda' \) remains constant (as in the case with \( \Lambda = 0 \)), but then quickly diverges or goes to zero for \( \Lambda > 0 \) or \( \Lambda < 0 \) respectively, at a distance

\[
R_1 \approx -\frac{\sqrt{3}}{2\sqrt{\Lambda}} \ln |\Lambda| = \mathcal{O} (|\ln |\Lambda||) \tag{18}
\]

from the positive-tension brane (12). The negative-tension brane must exist at \( y = R < R_1 \) in both cases. For a \( y \)-dependent \( \phi \), one expects that the solutions with \( \Lambda = 0 \) will not be modified significantly if \( |\Lambda| \) is sufficiently small for the second term to be negligible relative to the first one in the lhs of eq. (17). This implies that the second brane must be located at a distance \( y = R < R_1 \approx \mathcal{O} (|\ln |\Lambda||) \). This is confirmed by numerical studies. If the negative-tension brane in the solution with \( \Lambda = 0 \) was located at \( R > R_1 \), the solution for \( \Lambda \neq 0 \) must change drastically in order to accommodate the second brane much closer to the first one than before (6). This observation may provide a link between the cosmological constant and the brane location. Configurations with branes far apart seem to be possible only if the effective cosmological constant is exponentially small.

**Small cosmological constant:** We now return to the problem of finding a static configuration after the tension of the first brane has changed from \( \lambda_1(\phi) \) to \( \lambda_1(\phi) + c(\phi) \). With the new brane tension the boundary conditions at \( y = 0 \) are given by a new curve \( \tilde{C}_1 \) on the \( A = 0 \) plane. Let us consider some point \((\tilde{\phi}_1(\tilde{\Lambda}_1), \tilde{\phi}'_1(\tilde{\Lambda}_1))\) on \( \tilde{C}_1 \) (denoted by \( \tilde{O}' \)) close to the point \((\phi_1(\Lambda_1 = 0), \phi'_1(\Lambda_1 = 0))\) on \( C_1 \) (denoted by \( O \)). Writing \( \tilde{\phi}_1 = \phi_1 + \delta \phi_1 \) and repeating the calculation that led to eqs. (9), (10) we find

\[
\frac{\partial}{\partial \phi} \left[ V - \frac{1}{8} \left( \frac{\partial \lambda_1}{\partial \phi} \right)^2 + \frac{1}{3} \lambda_1^2 \right] (\phi_1) \delta \phi_1 - 3\tilde{\Lambda}_1 = -\frac{2}{3} \lambda_1(\phi_1)c(\phi_1) + \frac{1}{4} \partial \lambda_1 \frac{\partial c}{\partial \phi}(\phi_1) + \mathcal{O}(c^2(\phi_1)) \tag{19}
\]

and

\[
\delta \phi'_1 = \frac{1}{2} \frac{\partial^2 \lambda_1}{\partial \phi^2}(\phi_1) \delta \phi_1 + \frac{1}{2} \frac{\partial c}{\partial \phi}(\phi_1) + \mathcal{O}(c^2(\phi_1)). \tag{20}
\]

Let us consider the variations of \((\phi, \phi')\) on the trajectory that starts at the point \( \Lambda_1 = 0 \) on \( C_1 \). They are given by eqs. (11), (12). It is possible now to identify them with \((\delta \phi_1, \delta \phi'_1)\) of eqs. (19), (20) if

\[
\tilde{\Lambda}_1 = \frac{2}{9} \lambda_1(\phi_1)c(\phi_1) + \mathcal{O}(c^2(\phi_1)) \tag{21}
\]

For \( \lambda_1(\phi_1) = \mathcal{O}(c(\phi_1)) \), we obtain \( \tilde{\Lambda}_1 = \mathcal{O}(c^2(\phi_1)) \). This means that the trajectory that starts at the point \( \Lambda_1 = 0 \) on \( C_1 \) passes through the curve \( \tilde{C}_1 \) at some point with \( \tilde{\Lambda}_1 = \mathcal{O}(c^2(\phi_1)) \). We

\footnote{More precisely, the trajectory does not go exactly through \( \tilde{C}_1 \). As \( A'(y = 0) = -\lambda_1(\phi_1)/3 \) and \( \delta y = \mathcal{O}(c(\phi_1)) \), we have \( A(y = \delta y) = \mathcal{O}(c^2(\phi_1)) \). However, this small deviation of \( A \) from 0 can be neglected in our considerations. This is obvious from eq. (13), if we assume \( \Lambda = \mathcal{O}(c^2(\phi_1)) \). A shift of \( A \) at some point by \( \mathcal{O}(c^2(\phi_1)) \) gives an effect \( \mathcal{O}(c^4(\phi_1)) \).}


also know where this trajectory ends: on the curve $C_2$ at $\Lambda_2 = 0$. (This is the initial fine-tuning that we assumed to have been achieved.)

If we concentrate only on the part of the trajectory from $\tilde{C}_1$ to $C_2$, we have the situation we analyzed earlier. A slight mis-match between $\Lambda_1$ and $\Lambda_2$. But, as we argued before, there should be a nearby trajectory for which $\Lambda_1$ and $\Lambda_2$ can be matched, especially if $\Lambda_1$ and $\Lambda_2$ grow in opposite directions on $C_2$. For the case of interest $\Lambda_1 = O(c^2(\phi_1))$ and $\Lambda_2 = 0$, this trajectory is expected to have $\Lambda_f = O(c^2(\phi_1))$.

The upshot of this complicated reasoning is that we identified a static solution of Einstein’s equations with tensions $\lambda_1(\phi) + c(\phi)$ for the first brane and $\lambda_2(\phi)$ for the second. It is a solution with an effective four-dimensional constant $\Lambda$. Contrary to expectations, $\Lambda$ is not of order $c(\phi_1)$, but of order $c^2(\phi_1)$. The reason is that the location of the first brane has been shifted by an amount $\delta y = O(c(\phi_1))$, so as to absorb the leading effect from the change of the brane tension. The necessary condition is $\lambda_1(\phi_1) = O(c(\phi_1))$ or smaller. In order not to change the positivity of the tension for any sign of $c(\phi_1)$, it is preferable to take $\lambda_1(\phi_1)$ somewhat larger than $|c(\phi_1)|$.

Discussion: The basic objective of our approach was to compensate a possible change in a brane tension with a slight modification of the two-brane configuration in a way that keeps the effective cosmological constant $\Lambda$ small. We considered variations of the positive tension $\lambda_1(\phi)$ of the first brane. The reason is that we identify our Universe with the positive-tension brane, while we view the negative-tension one as a regulator that cuts off possible singularities in the solutions of Einstein’s equations. We allowed small arbitrary changes $c(\phi)$ of $\lambda_1(\phi)$, while we kept fixed the form of the negative tension $\lambda_2(\phi)$ and the potential $V(\phi)$ of the bulk field. We view $\lambda_1(\phi)$, $\lambda_2(\phi)$ and $V(\phi)$ as effective low-energy quantities. In particular, $\lambda_1(\phi)$ represents the vacuum energy on the first brane, induced by the fields localized on it (such as the Standard Model fields) and their interactions with the bulk field. The changes $c(\phi)$ originate in variations of the characteristic energy scale on the first brane, or possible phase transitions on the brane. We assumed that, apart from isolated points where it may approach zero, $\lambda_1(\phi)$ is of order 1 in units of the fundamental Planck’s constant, while $|c(\phi)| \ll 1$.

We started from an initial configuration with zero effective cosmological constant. This requires an initial fine-tuning of $\lambda_1(\phi)$ and $\lambda_2(\phi)$ for which we do not have a convincing explanation. We can only speculate that, since the two branes can exist far apart only if $\Lambda$ is exponentially small, the fine-tuning may be a consequence of the initial location of the branes. Another possibility is that the initial configuration has exact supersymmetry that forces the effective cosmological constant to be zero. This situation should correspond to a large energy scale on the first brane. When the energy scale is lowered, supersymmetry gets broken and an effective cosmological constant may appear. Our concern was the destabilization of the initial configuration every time $\lambda_1(\phi)$ changes. We looked for possible new static configurations, similar to the initial one. We did not address the question of the evolution of the system from one configuration to the other. This requires a time-dependent solution of Einstein’s equations, with ansätze for the metric much more general than the ones we employed. The technical difficulties involved in obtaining such solutions are a significant obstacle in this direction.

We demonstrated in a graphic way the known fact that, for general $\lambda_1(\phi)$, $\lambda_2(\phi)$, a static configuration exists with some value of the cosmological constant. Consequently, every change $c(\phi)$ of $\lambda_1(\phi)$ from its initial fine-tuned form results in a new static configuration with non-zero $\Lambda$. In general, one expects $\Lambda = O(c(\phi_1))$, where $\phi_1$ is the value of the bulk field at the initial location of the first brane. We showed that this is not always the case. Our main result is that
one can have $\Lambda = \mathcal{O}(c^2(\phi_1))$. The only requirement is that $\phi_1$ is near a zero of $\lambda_1(\phi)$, so that $\lambda_1(\phi_1) = \mathcal{O}(c(\phi_1))$. We point out that the derivatives of $\lambda_1(\phi)$ at $\phi_1$ are in general of order 1. If $\lambda_1(\phi)$ can become negative for a certain range of $\phi$ we assume that $\lambda_1(\phi_1)$ is somewhat larger than $|c(\phi_1)|$, so that the positivity of the tension is maintained for any sign of $c(\phi_1)$. Another possibility is that $\lambda_1(\phi)$ has minimum near $\phi_1$, where $\lambda_1(\phi)$ and $c(\phi)$ are comparable.

The implications of our result for a possible realistic scenario are interesting. In order to be more specific, we return to dimensionful quantities. We take the fundamental constant $M$ defined in the beginning to be $M = \mathcal{O}(10^{19} \text{ GeV})$. For the four-dimensional Planck’s constant we expect

$$\frac{M_4^2}{M^2} \sim \int_0^R dr \, e^{2A(r)} = \mathcal{O}(1). \tag{22}$$

Recent observations are consistent with a cosmological constant of the order of the critical density of the Universe $\Lambda$

$$\frac{\Lambda}{M^4} = \mathcal{O}\left(10^{-120}\right). \tag{23}$$

If we assume an initial two-brane configuration with zero effective cosmological constant, subsequent changes in the tension of the first brane are consistent with the above constraints if

$$c(\phi_1) = \mathcal{O}\left(10^{-60} M^4\right) = \mathcal{O}\left(10 \text{ TeV}^4\right). \tag{24}$$

Let us consider the possibility that for a certain value $\phi_1$ of the bulk field there is an ultraviolet cutoff of order 10 TeV for the vacuum energy associated with the fields of the first brane. If at the initial location of the brane $\phi \simeq \phi_1$, then $\lambda(\phi_1)$ is of order $(10 \text{ TeV})^4$. Our results imply that if the effective cosmological constant was zero at the beginning, any subsequent modification $c(\phi)$ of the brane tension may result in a cosmological constant consistent with the observational data as long as $c(\phi) \sim \mathcal{O}\left(10 \text{ TeV}^4\right)$. In this scenario, smaller modifications of the tension, such as those caused by cosmological phase transitions at scales below 10 TeV, do not modify the cosmological constant substantially. All phase transitions predicted by known physics (such as the electroweak or the QCD phase transitions) fall in this category. The nature of the cutoff cannot be specified by our considerations. It is possible that supersymmetry provides the necessary mechanism for the cancellation of quantum contributions to the energy density at energy scales above 10 TeV.

Before concluding, we would like to emphasize that the mechanism we presented should be viewed only as a simple example of how a non-trivial topology can ameliorate the cosmological constant problem. We find that its most important merit relative to alternative proposals within the same framework is the absence of singularities and strong assumptions about the form of the interactions of the brane fields with the bulk field. Our initial ansatz for the effective action of the system is general and our only assumption about the changes of the brane tension is that they are small in units of Planck’s constant. Moreover, our scenario allows for a non-zero cosmological constant of the right order of magnitude.

As a final remark, we point out that in eqs. (9), (13), (19) we assumed that the bulk potential cannot be given identically by

$$V(\phi) = \frac{1}{8} \left( \frac{\partial \lambda_1(\phi)}{\partial \phi} \right)^2 - \frac{1}{3} [\lambda_1(\phi)]^2. \tag{25}$$

This interesting expression could be favoured by gauged supergravity. (The example of ref. [13], based on the construction of ref. [14], fulfills this relation.) If eq. (25) holds, any value of $\phi$ can
be taken as the value of the bulk field at the location of the first brane. As a result, a continuous range of possible values for the tension of the first brane could lead to solutions with the same tension for the negative brane. This can be viewed as an improvement with respect to the case of arbitrary $V(\phi), \lambda_1(\phi)$, for which only a discrete number of $\phi$ values is possible. However, our approach is much more general. We allow for arbitrary changes of the form of $\lambda_1(\phi)$, not just its value. Such changes would invalidate the relation (25).

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References

[1] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[2] N. Bahcall, J.P. Ostriker, S. Perlmutter and P.J. Steinhardt, Science 284, 1481 (1999).
[3] V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. B 125, 136 (1983); ibid. 125, 139 (1983).
[4] I. Antoniadis, C. Bachas, D. Lewellen and T. Tomaras, Phys. Lett. B 207, 441 (1988); I. Antoniadis, Phys. Lett. B 246, 377 (1990).
[5] K. Akama, “Pregeometry” in Lect. Notes Phys. 176, 267 (1982), hep-th/0001113.
[6] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429, 263 (1998); Phys. Rev. D 59, 086004 (1999).
[7] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); Phys. Rev. Lett. 83, 4690 (1999).
[8] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, Phys. Lett. B 480, 193 (2000); S. Kachru, M. Schulz and E. Silverstein, Phys. Rev. D 62, 045021 (2000); C. Csaki, J. Erlich, C. Grojean and T. Hollowood, Nucl. Phys. B 584, 359 (2000); J. Chen, M.A. Luty and E. Ponton, JHEP 0009, 012 (2000); P. Brax and A.C. Davis, Phys. Lett. B 497, 289 (2001); A. Kehagias and K. Tamvakis, hep-th/0011006; J.E. Kim, B. Kyae and H.M. Lee, hep-th/0011118.
[9] S.H. Tye and I. Wasserman, hep-th/0006068; A. Krause, hep-th/0006226.
[10] C. Burges, R. Myers and F. Quevedo, hep-th/9911164; C. Schmidhuber, Nucl. Phys. B 580, 140 (2000); Z. Kakushadze, Phys. Lett. B 489, 207 (2000).
[11] W.D. Goldberger and M.B. Wise, Phys. Rev. Lett. 83, 4922 (1999).
[12] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, Phys. Rev. D 62, 046008 (2000).
[13] U. Ellwanger, hep-th/0001126.
[14] A. Lukas, B.A. Ovrut, K. Stelle and D. Waldram, Phys. Rev. D 59, 086001 (1999), hep-th/9803233.