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Development of a Neural Network-Based Mathematical Operation Protocol for Embedded Hexadecimal Digits Using Neural Architecture Search (NAS)

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Abstract
It is beneficial to develop an efficient machine-learning based method for addition using embedded hexadecimal digits. Through a comparison between human-developed machine learning model and models sampled through Neural Architecture Search (NAS) we determine an efficient approach to solve this problem with a final testing loss of 0.2937 for a human-developed model.

Keywords: datasets, neural networks, machine learning, hexadecimal

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1 Introduction
Mathematical operations, mainly addition, subtraction, division, and multiplication remain at the very foundation of all mathematical concepts [26]. Neural Networks have also been used extensively in mathematical applications, with most approaches focusing on problems from competitions like AMC 10, AMC 12, and AIME or university-level math problems [25][11][14].

Our approach. We develop a machine learning based method to predict the results of these four operations when given PyTorch embeddings of hexadecimal digits [2, 16]. A variety of model types are tested and ranked based on a loss metric. Three different humans developed PyTorch neural networks are first described, trained, tested, and evaluated. These human-developed networks all have different architectures: one uses a Fully Connected neural network, one uses an LSTM layer, and the final uses self-attention. Following the creation of these networks, Neural Architecture Search (NAS) has been performed on the base linear layer neural network using Microsoft’s Neural Network Intelligence Package [3, 12]. The human-developed neural networks and searched networks were then compared to each other.

For this project three different simulated datasets were used in order to simultaneously observe if the number of hex digits that are represented had an effect on the model’s final loss value. This test was inspired by Nogueira et al.’s findings which indicated that the way a number is represented can affect accuracy in mathematical operations [20].

These experiments were done as part of a larger project that focused on machine learning-based interpretations of human code and is a way to improve efficiency in that program by acting as a subroutine that removes the need for the larger model to recognize and compute mathematical operations.

An outline of this write-up is as follows: Section 2. describes related work in the field of machine learning addition problems, Section 3 describes our methods, Section 4. describes results, and Section 5. communicates our conclusions and future work. Section 3. includes information on all models created as part of this project. These models can be used when normal numerical operations are unavailable, such as when using embeddings that lose information about the original numbers.

2 Related Work
Neural Networks in mathematics Neural networks have been previously used to solve mathematical problems from competitions such as those provided using the MATH dataset proposed by Hendrycks et al. (2021) [14]. With the MATH dataset, problems are taken from competitions such as AMC...
Transformers in Mathematics It has been shown that Transformers can perform difficult calculations similarly to most calculators or computer systems. Amini et al. (2019), and Ling et al. (2017) use plug and chug mathematics problems that are multiple choice to observe sequence-to-program generation [7][18]. Saxton et al. (2019) also developed the DeepMind Mathematics dataset that includes some problems (mainly addition) that are similar to those solved in this project [21]. Henighan et al. (2020) showed that most problems in the DeepMind Mathematics dataset were solvable with large transformers [15]. Lample and Charton also used Transformers to solve symbolic integration problems and reached more than 95% accuracy [17].

Number representations The representation of numbers has also been found to affect results when entered into a neural network. Nogueira et al. (2021) found that introducing position tokens (e.g., “2 10e1 2”) can lead to improved performance [20].

Neural Architecture Search and Hyperparameter Optimization Neural architecture search has been a growing interest in recent times [23][27]. Hyperparameter optimization is especially important for cases where the actual network architecture in terms of layers is relatively optimized and is increasingly being used [22][9][8]. Network search protocols have also been used to create new models [13][23][27]. There have been several different neural architecture search algorithms created for PyTorch-based neural networks such as Microsoft’s Neural Network Intelligence, Auto-PyTorch, and Efficient Neural Architecture Search (ENAS) [3][1][10].

3 Methods

3.1 Overview of Software Components

The software components used in the creation of the machine learning models are Python, PyTorch, torchviz and Neural Network Intelligence.

Python is an object-oriented high-level programming language and is one of the most popular programming languages. Many machine learning libraries are based in Python due to its ease of use, and libraries. For this project we used Python version 3.8.10 [4]. PyTorch is an open-source machine learning framework that has many applications in computer vision and natural language processing. We chose to use PyTorch due to its simplicity and since the larger machine learning project that this subroutine is part of uses PyTorch. Pytorch was used to program all neural networks in this experiment. We used PyTorch version 1.10.2 [5].

Neural Network Intelligence (NNI) is a toolkit that is used to run automated machine learning (AutoML) experiments created by Microsoft. Neural Network Intelligence was chosen for its Neural Architecture Search capability. We used NNI version 2.7 [3]. Torchviz is a package used to “visualize PyTorch execution graphs.” Torchviz version x.xx was used to develop visualizations for our machine learning models [6].

The Visual Studio Code Integrated Development Environment (IDE) was used to write and run all the code described in this publication [19].

3.2 Data Format and Generation

Our primary goal was to predict the result of four arithmetic operations: addition, subtraction, multiplication, and division. All numerical data were originally in the form of 4-digit hexadecimal integers. An example of a conversion table for hexadecimal and decimal for numbers up to 256 can be found in Figure 1.

We created one dataset for each operation that resulted in a total of 4 datasets. Each dataset had 500,000 pairs and 500,000 results that depended on which operation each dataset was based on. In training and testing, a selection of these 500,000 values was taken and inputted into the model in the form of n-grams. All of these values were embedded using PyTorch embeddings of dimension 8 with data values. The final data values submitted to all the models have 16 integer values that make up the embedded 4-digit hexadecimal digits and the real sum in base 10. Figure 2. displays an example of two of these n-grams.

As discussed earlier, a paper found that the way numbers are represented influences accuracy [20]. Therefore, as a separate experiment we decided to change the number of hex digits represented to see if this had any effect on the loss. We did experiments with the full data, 3 hex digits, and 2 hex digits.

3.3 Human-Developed Machine Learning Models

We developed 12 network architectures for use with our embedded hex mathematical operation problems. We had 4 versions (one for each operation) of three different architectures. Each network was trained on each of the three data representations and the evaluation metric was the loss produced by the network after the final epoch training.

3.3.1 Human Generated Model Definitions. Our first machine learning model was a model with 3 PyTorch linear layers and served as the base model for our Neural Network
Intelligence ValueChoice model. Figure 2 shows the architecture of this first model. Our next model used a Long Short-Term Memory layer and our final model used a self-attention layer. Figure 3 shows the architecture for the self-attention neural network and Figure 4 displays the network architecture for the LSTM. All three models were trained three times using different kinds of data encoding for each trial as described in section 3.2.

### 3.4 Neural Architecture Search (NAS) Machine Learning Models

#### 3.4.1 ValueChoice Model

We used Neural Network Intelligence’s ValueChoice feature using our fully connected neural network in order to see if it would be possible to improve the performance of the model by changing the number of connections between neurons. We gave Neural Network Intelligence 4 possibilities for the number of connections: 16, 32, 64, and 128. Our original fully connected neural network used 64 for this parameter. The final goal of this experiment was to produce one network that was the best out of the parameters provided to Neural Network Intelligence. We used “Random” for our search strategy and allowed Neural Network Intelligence to go up to 5 epochs.

#### 3.4.2 LayerChoice Model

We also used Neural Network Intelligence’s LayerChoice feature to see the effects of using Neural Architecture Search to change the network architecture. Similar to our ValueChoice model we used our original fully connected neural network and allowed Neural Network Intelligence to develop networks through that. We gave Neural Network Intelligence the choice between an identity layer and a linear layer and allowed it to find the best one. We used “Random” for our search strategy and allowed Neural Network Intelligence to go up to 5 epochs. Since we wanted to see how well the models extrapolated on larger data, we trained them on 2 digit hex numbers and tested them on 4 digit hex numbers.

### 4 Results

#### 4.1 Comparison between human-generated machine learning models

The models for every operation (addition, subtraction, multiplication, and division) were based on addition (multiplication and division were done using logs), and thus this
Table 1. Final loss after 200 epochs and on testing data for all human-generated networks

| Model                     | Loss after 200 Epochs  | Loss for Testing Data  |
|---------------------------|------------------------|------------------------|
| Fully Connected, 4 Digits | 0.000005605330506      | **0.000005576385969**  |
| Self-Attention, 4 Digits  | 0.0001048754811        | 0.00011305436          |
| LSTM, 4 Digits            | 0.000005764168952      | 0.000005698243726      |
| Fully Connected, 3 Digits | 0.0000351484126        | 0.2713421421           |
| Self-Attention, 3 Digits  | 0.00007848687369       | **0.2675585822**       |
| LSTM, 3 Digits            | 0.00001873845527       | 0.276513088            |
| Fully Connected, 2 Digits | 0.0000005991071621     | 0.2936991373           |
| Self-Attention, 2 Digits  | 0.0000007127712775     | 0.2945040227           |
| LSTM, 2 Digits            | 0.000006743473424      | **0.2730684676**       |

Figure 7. Comparison between human-generated machine learning models for 3-digit hexadecimal values for 200 epochs

Figure 8. Comparison between human-generated machine learning models for 3-digit hexadecimal values for 25 epochs

Figure 9. Comparison between human-generated machine learning models for 2-digit hexadecimal values for 200 epochs

Figure 10. Comparison between human-generated machine learning models for 2-digit hexadecimal values for 25 epochs

Each neural network seemed to have benefits in different situations and the comparisons can be seen in Figures 1–5 and Table 1. Figures 5, 7, and 9 show the loss as the models were training for 200 epochs and figures 6, 7, and 8 show the loss as the models train on the same data for 25 epochs. Table 1 shows the final losses for the networks on the training
Table 2. Loss on testing data after 200 epochs for Neural Architecture Search sourced neural network and human-generated fully connected neural network

|             | NAS   | Human |
|-------------|-------|-------|
| Loss        | 0.2940571394 | 0.2936991373 |

dataset and on testing data. As seen from Figures 5 and 6, and Table 1, the Fully Connected neural network fits the fastest to the training data and has the lowest loss on both the testing and training data after 200 epochs. When the data are reduced to only include embedded versions of 3 hexadecimal digits in Figures 7, and 8, the Self-Attention neural network fits fastest at first but is quickly surpassed by the other two networks. The LSTM network comes out to be the best on the training data around the 15th epoch. However, when the loss on the testing data is viewed, the self-attention network has the lowest loss. It is unclear why this occurs and requires further testing. All the networks were similar on the 2-digit hexadecimal values in Figures 9 and 10 but the Fully Connected network seems to be the slowest to fit to the training data. After running the models on the testing data, the LSTM network had the lowest loss.

4.2 Comparison of Human-Generated Networks and NAS-Searched Networks

After running Neural Network Intelligence using Value-Choice, we found that the best network in the number of epochs we gave it used 32 connections. Through looking at the raw data, we were in fact able to determine that it did have the lowest loss within the epoch range but noticed that it was increasing after each epoch and that 16 and 128 seemed to be converging better.

Using LayerChoice, we found that the architecture with only two linear layers and one identity layer was the best. Figures 11 and 12 show the loss for the Neural Architecture Searched model and the original human-generated model and from the graphs it is obvious that the NAS network converges a lot faster and fits to the data well. However, when testing on the testing data after training for 200 epochs we generated the results in Table 2. The human-generated loss is slightly less but this may be a result of over-fitting in the NAS network.

5 Conclusions and Future Work

All networks in the project worked and generated results that could be useful given a need for embedded hexadecimal digits. The NAS-sourced models were comparable to the human-generated models and did converge faster in the LayerChoice examples. There were some unexpected results in that the human-generated network was better than the NAS sourced network in the testing stage of the LayerChoice experiment. There are several directions for future research in this area, including searching a wider model space in the NAS section of our research and possibly developing a larger dataset if given more processing power.

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