An Experimental Research on Design and Development Diversified Controllers for Tri-copter Stability Comparison

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Abstract. The drones have become the focus of researchers’ attention because they enter into many details of life. The Tri-copter was chosen because it combines the advantages of the quadcopter in stability and manoeuvrability quickly. In this paper, the nonlinear Tri-copter model is entirely derived and applied three controllers; Proportional-Integral-Derivative (PID), Fractional Order PID (FOPID), and Nonlinear PID (NLPID). The tuning process for the controllers’ parameters had been tuned by using the Grey Wolf Optimization (GWO) algorithm. Then the results obtained had been compared. Where the improvement rate for the Tri-copter model of the nonlinear controller (NLPID) if compared with the linear controller is (81.69%). Also, compared the paper results with the previous work results, where the greatest improvement in the stability of the Tri-copter model was (94.2 %) in the nonlinear controller and for traditional PID controller was (79.22).

1. Introduction
Presently, researches in the field of engineering of control concentrate on the drone, like helicopters, hexacopter, quadcopter, and Tri-copter, because they have many applications, mostly in the field of defence. Also, cover surveillance, a monitor to the environmental, media coverage, photography, producing movies, border patrols, and agriculture [1]. Generally, the first quadcopter introduced during World War I, but the first introduced of Tri-copter during 1948. In 2002, BRAUN MODELLTECHNIK Inc. had made and traded a small Tri-copter called Tribelle [2]. The design of a small drone with three rotors appears the right solution because it joins the advantages of helicopter and quadcopter drones in this configuration. Nevertheless, there is a problem in a yawing moment because the number of rotors is odd for the Tri-copter [3], to solve this problem, one of the two designs should be taken. The first plan was called (single Tri-copter). In this design, the tail rotor was tilt by an exact angle to control the yawing moment. The tail angle changed and controlled by a servo motor [4], as shown in Figure 1, and the other design called (coaxial Tri-copter) or Dragan-flyer X6. It has six motors. Two motors that had inverse rotating put on the spindle to void the reaction torque naturally. Also, stability improvement, as given in Figure 2 [5]. The Tri-copter drone is a nonlinear system, the static body that can be known by body structure reference and global (earth) reference and this drone type has three rotors, each motor is put on each spindle. Each spindle has a similar length to the middle point [6]. The servo motor is putting on the tail axis to redress the generated reaction torque by tilt the tail motor. This model made the Tri-copter movement superb, especially in back turns. Tri-copter has an extensive studies area as it has significant variations to other multi-rotor in model and control due to its singular yaw movement trouble rise by the unpaired motor in the tail [3]. In this paper, the single Tri-copter will be modelled and test the stability with three different controllers (PID, FOPID and NLPID). The controllers’ parameters will tune by GWO algorithm.
2. Modelling and control literature survey

In this section, the researches dealing with the Tri-copter model have been listed as:

- Ryś et al. (2014) [7] focused on hardware of Tri-copter, the mechanical design, control system architecture, and the practical part of the control. Various PID controllers were applied and used algorithms with a modified loop of systems. Also, they proposed a control system to give more stabilization and an implant microcontroller-based navigation.

- Z. Song et al. (2016) [8] resolve three types of rotor arranging for the Tri-copter vehicle, and he chose the top performance design of them. Then applied a PID linear controller to control attitude. However, the outcomes were not good with oscillation and high overshoot. So, a nonlinear backstepping mode control was used to win a stable manoeuvring flight.

- Z. Ali et al. (2017) [9] used Model Reference Adaptive Control (MRAC) for the path tracking of a Tri-copter model. The mathematical model of the drone was used Newton–Euler formula and the parameters of the rising control subsystem were tuned by applied Fuzzy Proportional Derivatives (F-PD) control. In contrast, the (F-PID) control was applied to tuning the position control subsystem. The model was applied in trajectory tracking, and the results were satisfactory with errors in tracking the square path.

- M. Mehndiratta and E. Kayacan (2018) [10] applied the learning-based cascade Nonlinear Model Predictive Control (NMPC) algorithm for the square path tracking of a Tri-copter drone. Where the drag-moment and thrust coefficients, are evaluated online incorporating nonlinear moving horizon estimation method.

- N. M. A. Ahmed and M. H. Saleh (2019) [11] used the mathematical model derived by Newton-Euler equations. The author using the Proportional-Integral and Derivative (PID) controller, and Integral Backstepping Sliding Mode controller (IBSM) to control the model of Tri-copter. The parameters of PID are tuned by particle swarm optimization (PSO) method to minimize the error of position and angles. Still, the linearization and approximations in the mathematical model are used.

- A. Houari and et Al. (2020) [12] used a PID controller and Linear Quadratic Regulator (LQR) controller to control a nonlinear system for Tri-rotor UAV. They applied several tests such as vertical take-off and landing. After they compared the simulation results for both controllers, the results had appeared the performance of LQR controller was better than the PID controller in their model.

In this paper a derivation of the nonlinear Tri-copter model and applying three different controllers, as well as the model, will be simulated without approximation or linearization. Also, in this paper, we will compare the results with previous works.
3. The Tri-copter mechanism

Tri-copter drone mechanical body includes a frame, three motors arms, a radio transmitter, and three cells of a battery. The tail rotor is tilted by a fit angle using a servo motor to remove the reaction torque problem. Tri-copter has a useful feature of owning a quick movement created by tilt its tail motor; also, it is a difficulty in this system because it needs an exact angle of inclining the tail for the stability of the drone [8]. The Tri-copter as any drone has six Degrees of Freedom (DOF), and the control can by four control actions (Roll, Pitch, Yaw, and Altitude). The Altitude can be controlled by speed up or slow the angular velocities for three Brushless Direct Current Motor (BLDC motor), as shown in Figure 3. The Roll moment can be controlled by making a different angular velocity between the front rotors (M2 and M3). Also, tail motor (M1) work to fix the moment of the drone, as shown in Figure 4. The Pitch controlled by change the angular speed of M1 and setting the angular velocities of front motors, as shown in Figure 5. The yaw moment is created from the sum of motors moment (M1, M2, and M3) and the horizontal compound of the tail motor. Yaw angle can be controlled by changing the tail angle, in which a servo motor is controlled on it, as shown in Figure 6.

![Figure 3. The altitude control](image1)

![Figure 4. The Roll control](image2)

![Figure 5. The pitch control](image3)

![Figure 6. The yaw control](image4)

4. The mathematical model for Tri-copter

The Tri-copter has six Degrees of Freedom (6DOF), like any flying copter. To transform the body coordinate to earth coordinate need to the rotation matrix to Earth-frame, the geometric rotation matrix, according to Figure 7 as shown.

![Figure 7. Tri-copter Schematic](image5)
Where \( c \) is \( \cos \), \( s \) is \( \sin \), \((\phi)\) is the rotational angle about the x-axis, \((\theta)\) is the rotational angle about the y-axis, \((\psi)\) is the rotational angle about the z-axis, the \((x, y, z)\) are coordinate. And \( Q_{\phi \theta \psi}^{xyz} \) the translation transformation matrix between body frame and earth frame. And in Figure 7. \( T \) is torque, \( \omega \) is angler velocity, \( l \) is length, \( X_{lc}, Z_{lc}, Z_{lc} \) are coordinate of body.

The mathematical model of Tri-copter is concluded by using Newton-Euler formulation. Furthermore, the force equations of the Tri-copter drone are finding, as shown expressed in Eq. (2) With observance to Figure 7, and moment equations of the Tri-copter as shown expressed in Eq. (4).

\[
\overline{F}_B = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} 0 \\ F_1. \sin \mu \\ F_2 + F_3 + F_1. \cos \mu \end{bmatrix}
\]  

Where \( \mu \) is the tail angle and \( \overline{F}_B \) is a force of the body.

The force \((f_y)\) is a small value it is founded to remove the moment reaction of the tail rotor, and the result of the force is zero.

\[
\overline{F}_B = \begin{bmatrix} 0 \\ 0 \\ F_2 + F_3 + F_1. \cos \mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ U_1 \end{bmatrix}
\]  

\[
\overline{T} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} l_2 (F_2 - F_3) \\ T_1. \sin \mu + l_1. F_1. \cos \mu - l_3 (F_2 + F_3) \\ -l_1. F_1. \sin \mu + T_1. \cos \mu + T_2 - T_3 \end{bmatrix} = \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix}
\]  

\( U_1 \) is Altitude, \( U_2 \) is Roll, \( U_3 \) is Pitch and \( U_4 \) is Yaw.

Where \( F = K_f. \omega^2 \) and \( T = K_m. \omega^2 \). And from newton’s equation [11]:

\[
m. \ddot{v} = f_e + f_g
\]  

Where \( \ddot{v} = [\ddot{x} \ \ddot{y} \ \ddot{z}]^T \), \( f_e = Q_{\phi \theta \psi}^{xyz} \overline{F}_B \), \( f_g = [0 \ 0 \ g]^T \)

, \( m \) is mass, \( f_e \) is the force of drone with respect to earth and \( f_g \) is the earth force.

\( \ddot{v} \) is acceleration, and \( g \) is ground acceleration. After derivations of the equations above the expressions became as:

\[
\ddot{x} = \frac{\sin \theta}{m} U_1
\]  

\[
\ddot{y} = \frac{\sin \phi \cos \theta}{m} U_1
\]  

\[
\ddot{z} = \frac{\cos \phi \cos \theta}{m} U_1 - g
\]  

And from [13] \( T = I. \dot{\Omega} + \Omega \times I. \Omega \)

Where \((I)\) is a matrix Moment of inertia for the drone and \( \Omega \) is a matrix of angle velocity

\[
I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad , \quad \dot{\Omega} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}
\]
After derivation of the equations above and add the effect of motors torque the expressions became as following: [13]

\[
\ddot{\phi} = \frac{I_{y}-I_{z}}{I_{x}} \dot{\psi} \dot{\phi} + \frac{U_{2}}{I_{x}} \dot{\phi} + \frac{J_{rot}}{I_{x}} \dot{\theta} \tag{10}
\]

\[
\ddot{\theta} = \frac{I_{z}-I_{x}}{I_{y}} \dot{\psi} \dot{\theta} + \frac{U_{3}}{I_{y}} \dot{\psi} + \frac{J_{rot}}{I_{y}} \dot{\phi} \tag{11}
\]

\[
\ddot{\psi} = \frac{I_{x}-I_{y}}{I_{z}} \dot{\theta} \dot{\psi} + \frac{U_{4}}{I_{z}} \dot{\theta} + \frac{J_{rot}}{I_{z}} \dot{\phi} \tag{12}
\]

Where \( J_{r} \) is the Moment of inertia for the BLDC motors and \( \omega r \) sum of angler velocity of BLDC motors.

5. Controllers Design

The design of the three controllers of the Tri-copter model is similar in case of the inner loop and the outer loop, where the Tri-copter model contains six controllers that control six degrees of freedom (positions and angles) as shown in Figure 8.

The equations that represent the controls are:

5.1. Traditional PID Controller Design

The general equation for the PID controller is [11]:

\[
u(t) = K_{p} e(t) + K_{i} \int_{0}^{t} e(\tau) d\tau + K_{d} \frac{de(t)}{dt} \tag{13}\]

Where \( u(t) \) is a control input, \( e(t) \) is the error between desired input and output, the proportional parameter \( K_{p} \), the integral parameter \( K_{i} \), and the derivative parameter \( K_{d} \). The equations for the inner loop are:
• Roll angle control equation $U_\phi$

$$U_\phi = K_{P\phi}e_\phi + K_{I\phi} \int_0^t e_\phi \, dt + K_{D\phi} \frac{d}{dt} e_\phi$$  \hspace{1cm} (14)$$

Where $K_{P\phi}, K_{I\phi}$ and $K_{D\phi}$ are parameters of Roll’s PID controller. $e_\phi$ is the error in $\phi$ angle, it is the difference between the desired $\phi$ angle ($\phi_d$) and the actual ($\phi$) that mean ($e_\phi = \phi_d - \phi$).

• Pitch angle control equation $U_\theta$

$$U_\theta = K_{P\theta}e_\theta + K_{I\theta} \int_0^t e_\theta \, dt + K_{D\theta} \frac{d}{dt} e_\theta$$  \hspace{1cm} (15)$$

Where $K_{P\theta}, K_{I\theta}$ and $K_{D\theta}$ are parameters of Pitch’s PID controller. $e_\theta$ is the error in $\theta$ angle, it is the difference between the desired $\theta$ angle ($\theta_d$) and the actual ($\theta$) that mean ($e_\theta = \theta_d - \theta$).

• Yaw angle control equation $U_\Psi$

$$U_\Psi = K_{P\Psi}e_\Psi + K_{I\Psi} \int_0^t e_\Psi \, dt + K_{D\Psi} \frac{d}{dt} e_\Psi$$  \hspace{1cm} (16)$$

Where $K_{P\Psi}, K_{I\Psi}$ and $K_{D\Psi}$ are parameters of Yaw’s PID controller. $e_\Psi$ is the error in $\Psi$ angle, it is the difference between the desired $\Psi$ angle ($\Psi_d$) and the actual ($\Psi$) that mean ($e_\Psi = \Psi_d - \Psi$).

• Equation of Altitude control signal ($U_z$)

$$U_z = K_{Pz}e_z + K_{Iz} \int_0^t e_z \, dt + K_{Dz} \frac{d}{dt} e_z$$  \hspace{1cm} (17)$$

Where $K_{Pz}, K_{Iz}$ and $K_{Dz}$ are parameters of the Altitude PID controller. $e_z$ is the error in Altitude, it is the difference between the desired Altitude ($z_d$) and the actual Altitude ($z_e$) that mean ($e_z = z_d - z_e$).

Where the ($U_z$) represent to altitude signal in Global coordinate ($Z_Gc$); therefore, the Trust signal ($U_\zeta$) concerning Local coordinate of the Tri-copter will be: from Eq 8.

$$m.\ddot{z} = U_\zeta.\cos\phi\cos\theta - g.m = U_z$$  \hspace{1cm} (18)$$

$$U_\zeta = \frac{U_z + g.m}{\cos\phi\cos\theta}$$  \hspace{1cm} (19)$$

Moreover, the outer loop equations are:

• Equation of $x$-position control signal ($U_x$)

$$U_x = K_{Px}e_x + K_{Ix} \int_0^t e_x \, dt + K_{Dx} \frac{d}{dt} e_x$$  \hspace{1cm} (20)$$

Where $K_{Px}, K_{Ix}$ and $K_{Dx}$ are parameters of the $x$-position PID controller. $e_x$ is the error in $x$-position, it is the difference between the desired value of $x$ ($x_d$) and the actual $x$-position ($x$) that mean ($e_x = x_d - x$). With referring to Eq 6:

$$\dot{x}.m = U_\zeta.\sin\theta_d = U_x$$  \hspace{1cm} (21)$$

$$\theta_d = \sin^{-1}\frac{U_x}{U_\zeta}$$  \hspace{1cm} (22)$$

• Equation of $y$-position control signal ($U_y$)

$$U_y = K_{Py}e_y + K_{Iy} \int_0^t e_y \, dt + K_{Dy} \frac{d}{dt} e_y$$  \hspace{1cm} (23)$$

Where $K_{Py}, K_{Iy}$ and $K_{Dy}$ are parameters of the $y$-position PID controller. $e_y$ is the error in $y$-position, it is the difference between the desired value of $y$ ($y_d$) and the actual $y$-position ($y$) that mean ($e_y = y_d - y$), and with referring to Eq 7:
\[
\ddot{y}.m = -U_{\zeta} \cdot \sin \phi \cdot \cos \theta = U_y
\]
\[
\phi_d = \sin^{-1}\left(\frac{-U_y}{U_{\zeta} \cdot \cos \theta}\right)
\]

5.2. FOPID Controller Design

The general equation for the FOPID controller is:

\[
U(t) = K_p e(t) + K_i D^{-\lambda} e(t) + K_d D^\alpha e(t)
\]

where

\[
D^\alpha = \begin{cases} 
\frac{d^\alpha}{dt^\alpha} & \text{if } \alpha > 0 \\
1 & \text{if } \alpha = 0 \\
\int (dt)^{-\alpha} & \text{if } \alpha < 0 
\end{cases}
\]

The inner and outer loop equations are similar to equations for the PID controller with a difference in the order of derivatives and integrations.

5.3. NL PID Controller Design

The design of the Tri-copter model using the NL PID controller also comes in two-part: the inner loop and the outer loop. The design of the NL PID controller has been used a (tanh) function to get compatible with the nonlinear Tri-copter model, the general equation is:

\[
U(t) = K_p e(t) + \int_0^t e(\tau) d\tau + K_d e(t) \frac{de(t)}{dt}
\]

\[
\begin{align*}
K_p(e) &= k_{p1} + k_{p2} \tanh(k_{p3} \cdot |e(t)|) \\
K_i(e) &= k_{i1} + k_{i2} \tanh(k_{i3} \cdot |e(t)|) \\
K_d(e) &= k_{d1} + k_{d2} \tanh(k_{d3} \cdot |e(t)|)
\end{align*}
\]

Where \(k_{p1,2,3}, k_{i1,2,3}\) and \(k_{d1,2,3}\) are positive constants, and the inner and outer loop equations are similar to equations for the PID controller with a difference in the order of derivatives and integrations.

All parameters for PIDs, FOPIDs and NL PIDs have been tuned by using the GWO algorithm, and the algorithm equations had been illustrated in [14].

6. Simulation Results

The simulation has been done in MATLAB 2020a program. The first step is tuning the controllers’ parameters by using GWO, and values for the parameters used in Tri-copter model has been listed in Table 1. The best results for the parameters for the controllers that achieve the best stability are listed in Table 2 for PID, Table 3 for FOPID, and Table 4 for NL PID.

| Symbol | Value | Description |
|--------|-------|-------------|
| m      | 0.85 Kg | Mass of Tri-copter |
| L      | 0.180 m | Arm length |
| 1_{xx} | 0.0430 Kg. m^2 | Moment of Inertia for Roll |
| 1_{yy} | 0.0480 Kg. m^2 | Moment of Inertia for Pitch |
| 1_{zz} | 0.0770 Kg. m^2 | Moment of Inertia for Yaw |
| 1_{M}  | 1.97 x 10^{-6} Kg. m^2 | Rotational Moment of Inertia for motor |
| g      | 9.806 m/sec^2 | Ground acceleration |
| K_f    | 1.970 x 10^{-6} N. s^2 | Aerodynamic force constant |
| K_t    | 2.88 x 10^{-7} N.m. s^2 | Aerodynamic moment constant |
| tout   | 20 s   | Time of simulation |
Moreover, stability results for the Tri-copter model responses have been illustrated in Figures (8 – 13).

**Table 2.** List of PIDs’ Parameters that tuning by GWO

| parameters | X   | Y   | Z   | \( \phi \) | \( \theta \) | \( \psi \) |
|------------|-----|-----|-----|------------|------------|----------|
| K_p        | 19.920792 | 47.913898 | 43.51231 | 30.2599    | 16.59948   | 12.36146 |
| K_i        | 0.0113547  | 0.0052894  | 2.170821 | 1.439645   | 0          | 15.39798 |
| K_d        | 6.4772787  | 10.365471  | 28.4701  | 10.10089   | 4.695909   | 3.505297 |

**Table 3.** List of FOPIDs’ Parameters that tuning by GWO

| parameters | X   | Y   | Z   | \( \phi \) | \( \theta \) | \( \psi \) |
|------------|-----|-----|-----|------------|------------|----------|
| K_p        | 29.960957  | 24.226776  | 21.217157 | 1.0180293  | 44.397167  | 10.381401 |
| K_i        | 5.0308938   | 37.447647   | 112.91746 | 23.414280   | 2.3072553   | 2.3882184 |
| K_d        | 9.3964539   | 12.301300   | 116.24931 | 8.3654346   | 9.7158289   | 60.156002 |
| \( \lambda \) | 0.0868086   | 0.0032461   | 0.3798824 | 0.0456872   | 0.3443585   | 0.6737137 |
| \( \alpha \) | 0.9461064   | 0.9704718   | 0.9899971 | 1.0763948   | 1.0457079   | 0.7790650 |

**Table 4.** List of NLPIDs Parameters that Tuning by GWO

| parameters | PID controller for: |
|------------|---------------------|
| K_p\((e)\) | X   | Y   | Z   | \( \phi \) | \( \theta \) | \( \psi \) |
| k_p1       | 128.8932  | 129.6503 | 100.800 | 59.3471   | 126.1876   | 104.088   |
| k_p2       | 4.84305   | 128.7254 | 7.19421 | 1.118218  | 48.63723   | 67.6775   |
| k_p3       | 62.50555  | 44.8141  | 99.4813 | 8.035404  | 126.2086   | 1.35251   |
| k_i1       | 0.033057  | 0  | 5.02599 | 0.237625  | 90.15234   | 0.45397   |
| K_D\((e)\) | X   | Y   | Z   | \( \phi \) | \( \theta \) | \( \psi \) |
| k_d1       | 14.7312   | 23.83013 | 130  | 51.60179  | 80.45607   | 69.84799  |
| k_d2       | 3.783679  | 0.802225 | 1.607524 | 0.236955  | 0.163798   | 12.32227  |
| k_d3       | 124.2145  | 60.42259 | 22.61133 | 0.023668  | 1.359732   | 93.30147  |

Moreover, stability results for the Tri-copter model responses have been illustrated in Figures (8 – 13).
Figure 12. Pitch ($\theta$) response for unit step

Figure 13. Yaw ($\psi$) response for unit step

The results in figures above show that the nonlinear model of Tri-copter shows a good response in all controllers, but the superiority is clear for the nonlinear controller. At the same time, the time-domain measurements (Rising time $t_r$, Settling time $t_s$ and Overshoot $M_p$%) has been listed in Table 5 that shows the comparison of the results in time-domain, in addition to comparing them with previous work.

Table 5. The Stability Comparison Results with Previous Works [11]

| Axes | Criterion | Previous Works [11] | This paper |
|------|-----------|---------------------|------------|
|      |           | PID     | IBSM     | PID    | FOPID  | NLPID |
| x    | $M_p$%    | 2.4154 | 1.1753 | 0.7    | 0.5    | 0.3   |
|      | $t_s$     | 1.0016 | 2.9374 | 0.4971 | 0.3224 | 0.235 |
|      | $t_r$     | 0.73   | 3.35   | 0.4798 | 0.3328 | 0.24  |
| y    | $M_p$%    | 7.6143 | 1.0192 | 0.2    | 0      | 0.8   |
|      | $t_s$     | 1.2823 | 2.6728 | 0.3561 | 0.2717 | 0.1534|
|      | $t_r$     | 0.61   | 3      | 0.335  | 0.2795 | 0.191 |
| z    | $M_p$%    | 0.1414 | 0      | 4.737  | 0.6    | 0.5   |
|      | $t_s$     | 1.1541 | 1.1001 | 0.5092 | 0.1329 | 0.1825|
|      | $t_r$     | 1.3    | 1.3    | 0.041  | 0.011  | 0.013 |
| $\phi$ | $M_p$%   | 0.0032 | 0      | 36.3   | 3      | 0.1   |
|      | $t_s$     | 1.1838 | 1.8942 | 0.6539 | 0.1517 | 0.002 |
|      | $t_r$     | 1.404  | 2.2    | 0.008  | 0.004  | 0.0014|
| $\theta$ | $M_p$%  | 0.0172 | 0      | 21.34  | 3.3    | 0.2   |
|      | $t_s$     | 2.444  | 2.8407 | 0.172  | 0.0095 | 0.0012|
|      | $t_r$     | 3.1    | 3.3    | 0.0155 | 0.005  | 0.0009|
| $\psi$ | $M_p$%   | 0.0776 | 0.0032 | 10.6   | 23.5   | 0.3   |
|      | $t_s$     | 1.1141 | 1.1009 | 0.4717 | 0.019  | 0.0024|
|      | $t_r$     | 1.4    | 1.3    | 0.0295 | 0.0049 | 0.0016|
| $\Sigma$ ITAE |       | 0.679  | 2.311  | 0.1411 | 0.0927 | 0.0394|

Enhancement percentage with respect to this paper: negative - 34.3 % 72.08 %
Enhancement percentage with respect to previous works: negative - 79.22 % 86.35 % 94.2 %

Where $\Sigma$ ITAE is the summation of Integration Time Absolute Error, and it is the cost function for measurement the stability in this model. The enhancement percentage can calculate with referring to the PID controller in the previous works and apply the following equation.

\[
\text{Enhancement percentage} = 1 - \frac{\text{controller's cost function}}{\text{PID's cost function}} \times 100\% \quad (24)
\]
7. Conclusions
The Tri-copter model is a complex nonlinear model with six degrees of freedom that represents to x, y, z axes and angle of each axis. So, it required mathematical derivation and nonlinear simulation, where three different controllers were applied. The NLPID controller surpass all controllers and got great results, with the improvement rate reaching if compared with the linear controller is (81.69%). Controllers' parameters were tuned by using GWO algorithm and achieved excellent results compared with results in previous works that tuned by PSO, as the improvement ratio for traditional PID controller (79.22 %). The percentage of improvement for the model in case of NLPID controller is (94.2 %) compared to previous works and FOPID (86.35 %) compared with traditional PID controller in previous work.

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