Energy Efficiency Optimization for MIMO Broadcasting Channels

Jie Xu, Student Member, IEEE, and Ling Qiu, Member, IEEE

Abstract

Optimizing the energy efficiency (EE) for the MIMO broadcasting channels (BC) is addressed in this paper, taking into account the transmit independent power which is related to the active transmit antenna number. A new optimization framework is proposed, in which transmit covariance optimization under fixed active transmit antenna sets is first performed and active transmit antenna selection (ATAS) is utilized then. To optimize the EE under a fixed transmit antenna set, we propose an energy efficient iterative waterfilling scheme according to the block-coordinate ascent algorithm, through transforming the problem into a concave fractional optimization via uplink-downlink duality. It is proved that the proposed scheme converges to the global optimality. After that, ATAS is employed to determine the active transmit antenna set and to turn off the rest inactive antennas. ATAS can balance the active transmit antenna number related EE gain with higher capacity gain and the EE loss with more transmit independent power wasting. During the ATAS, the optimal exhaust search and norm based successive selection schemes are borrowed. Through simulation results, we discuss the effect of different parameters on the EE of the MIMO BC.

Index Terms

This work is supported by National Basic Research Program of China (973 Program) 2007CB310602.

This work has been submitted to the IEEE for possible publication. Copyright may be transferred without notice, after which this version may no longer be accessible.

The source of this paper is partly presented in IEEE Wireless Communications and Networking Conference (WCNC) 2012

The authors are with the Personal Communication Network & Spread Spectrum Laboratory, Department of Electrical Engineering and Information Science, University of Science and Technology of China Hefei, Anhui, 230027, China (email: suming@mail.ustc.edu.cn, lqiu@ustc.edu.cn).

Corresponding author: Ling Qiu, lqiu@ustc.edu.cn.
Energy efficiency, spectral efficiency, MIMO broadcasting channels, iterative waterfilling, antenna selection.

I. INTRODUCTION

Wireless communication turns to the era of green. This is not only because of the exponential traffic growth with the popularity of the smart phone but also the limited energy source with ever higher prices. Energy efficiency (EE), as a result, becomes one of the major topics in the research of wireless communications [2] and plenty of research projects either government funded or industrial funded start to investigate the energy efficient solutions for the wireless network as well as the sustainable future for the wireless communications. Meanwhile, multiple input multiple output (MIMO), especially downlink multiuser MIMO (also called MIMO broadcasting channels, BC), is becoming the key technology in the next generation cellular networks due to its significant spectral efficiency (SE) performance. Although the SE for the MIMO BC is well known, the EE still remains an open issue. Thus, studying the EE of the MIMO BC is important for the design of the future green wireless networks.

The capacity of the MIMO BC has been widely studied in the literatures, e.g. [3]–[5] and it is well known that dirty paper coding (DPC) can achieve the capacity region of the MIMO BC. Through transforming the capacity of MIMO BC into the dual MIMO multi-access channel (MAC) [4], the capacity can be efficiently optimized through the convex optimization and a well structural solution is called iterative waterfilling [5], [6]. However, the EE of the MIMO BC has been rarely studied. The EE is in general defined as the capacity divided by the power consumption, which denotes the delivered bits per-unit energy measured in bits per-Joule. For the MIMO BC, when the optimization objective transfers from the capacity to the EE, there is a two-fold impact. On the one hand, the optimization varies from a convex optimization problem to a fractional programming problem [7], when the uplink-downlink duality is applied. The previous spectral efficient schemes are not practical any longer. On the other hand, activating all available transmit antennas and radio frequency (RF) chains for transmission is not always energy efficient although spectral efficient, as more transmit antennas correspond to higher transmit-independent power, which induces a tradeoff between the EE gain with higher multiplexing/diversity and the EE loss with more transmit independent power consumption wasting.

The EE of the MIMO BC is studied in this paper, where a practical power model with transmit
independent power \cite{8,9} is taken into account. The transmit independent power contains the
cost of signal processing, circuit power, etc. at the BS, and is highly related to the active
transmit antenna number. A new optimization framework with transmit covariance optimization
and active transmit antenna selection (ATAS) is proposed to maximize the EE of the MIMO BC.
At first, we find that the EE optimization problem under fixed active transmit antenna set is a
concave fractional programming, and propose an efficient energy efficient iterative waterfilling
scheme to obtain the optimal covariances, which is proven to be globally optimal. After that,
exhaust search and norm-based successive selection are borrowed in ATAS to determine the
active transmit antenna set and turn off the inactive ones.

A. Contributions

We observe that the EE is affected by both the transmit covariances (total transmit power) and
the active transmit antenna sets. On one hand, the active transmit antenna sets are related to the
transmit antenna selection diversity and thus affect the capacity. On the other hand, the size of
the active transmit antenna sets is equal to the active transmit antenna number, which is related
to the transmit independent power. Based on this observation, a new optimization paradigm with
transmit covariances optimization and ATAS is proposed.

Under fixed active transmit antenna sets, we derive the optimal energy efficient transmit
covariances at first. Employing the famous uplink-downlink duality, the nonconcave MIMO BC
capacity is transformed into the dual concave MIMO MAC. Correspondingly, the EE becomes a
quasi-concave function. After that, we propose a novel well structured energy efficient iterative
waterfilling scheme based on the block-coordinate ascent algorithm to solve the EE optimization
problem efficiently. During each iteration, the transmit covariance matrices optimization is formu-
lated as a concave fractional program, which is solved through relating it to a parametric concave
program and applying the Karush-Kuhn-Tucker (KKT) optimality conditions. Interestingly, the
solution of each iteration has a feature of waterfilling. We prove the convergence of the proposed
scheme and validate it through simulations.

A novel ATAS procedure is proposed to compromise between active transmit antenna number
related EE gain with increasing multiplexing/diversity and EE loss with increasing transmit
independent power consumption. Exhaust search and norm based successive selection schemes
are borrowed to choose the energy efficient active transmit antenna sets. A unique feature of the
ATAS is that the inactive antennas should be switched off to save power, e.g. employing micro-sleep \[10\] or discontinuous transmission (DTX) \[11\]. Furthermore, the invisible channel state information at transmitter (CSIT) problem during the implementation is discussed. Simulation results give us insights about the effect of different system parameters on the EE.

B. Related Works

There are a lot of literatures discussing the EE of the point to point MIMO channels with transmit covariances optimization without antenna selection \[12\]–\[17\]. The point to point MIMO channels can always be separated into parallel sub-channels through singular value decomposition (SVD) or after detection. In this case, only power allocation across the sub-channels needs to be optimized to maximize the EE \[13\]–\[15\]. As the sub-channels are parallel, the solution is similar with the energy efficient power allocation in OFDM systems \[18\], \[19\]. The optimization for point to point MIMO channels is not applicable for the MIMO BC, as the MIMO BC cannot be simply transformed into parallel sub-channels. \[1\] There are few literatures discussing the EE for the MIMO BC. To the best of the authors’ knowledge, only \[20\] and our previous work \[8\] addressed the EE of the MIMO BC, but they both assumed linear precoding design and equal transmit power allocation for simplification. These assumptions make the both works far away from the optimal solution. To optimize the EE of the MIMO BC, deriving the well structured transmit covariances is a challenge.

Antenna selection is a widely discussed technology in spectral efficient MIMO systems, both in transmitter side and the receiver side, e.g. in \[21\]–\[25\]. However, the spectral efficient transmitter antenna selection \[21\]–\[23\] is always performed to choose the active antennas when the number of radio frequency (RF) chains is larger than the number of the antennas. Meanwhile, the receive antenna selection \[24\], \[25\] is always performed for the linear precoding schemes to approach the asymptotic optimal performance. In our scenario, as DPC is employed, the receive antenna selection is unnecessary. Moreover, as we consider the case when the transmit antenna number is equal to the RF chain, the purpose of the transmit antenna selection is saving power through turning off the inactive RF chains. Although the selection procedure with exhaust search and

\[1\] Although after zero-forcing (ZF) precoding, for example, the MIMO BC can be separated into parallel sub-channels, the ZF scheme is far away from the optimal solution \[3\].

February 17, 2012 DRAFT
Norm based successive scheme is borrowed from the previous literatures, it is in some sense different from the spectral efficient ones. Especially, there exists another challenge of invisible CSIT problem, which is also discussed in this paper.

C. Organization and Notation

The rest of this paper is organized as follows. Section II introduces the system model and the problem formulation. Section III proposes the transmit covariances optimization to maximize the EE under fixed active transmit antenna set. Section IV proposes the energy efficient ATAS and discusses the implementation issue in the realistic systems. The simulation results and discussions are given in Section V. Finally, section VI concludes this paper.

Regarding the notation, bold face letters refer to vectors (lower case) or matrices (upper case). The superscript $H$ and $T$ represent the conjugate transpose and transpose operation, respectively. $\text{Tr}(\cdot)$ denotes the trace of the matrix.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The system consists of a single BS with $M$ antennas and $K$ users each with $N$ antennas, which is shown in Fig. I. We assume that the number of RF chains is equal to the number of antennas. Denote the channel matrix from the BS to all users as $\mathbf{H} \in \mathbb{C}^{NK \times M}$ with $\mathbf{H} = [\mathbf{H}_1^T, \mathbf{H}_2^T, \ldots, \mathbf{H}_K^T]^T$, where $\mathbf{H}_i \in \mathbb{C}^{N \times M}$ is the channel matrix from the BS to the $i$th user. As active antenna number at the BS has significant affect on the EE, selecting the active transmit antennas is important. Consider that the selected transmit antenna set is $\mathcal{T} \subseteq \{1, \ldots, M\}$ with active transmit antenna number $M_a = |\mathcal{T}|$, and denote the channel matrix from the BS’s active transmit antennas to the users as $\mathbf{H}_\mathcal{T} \in \mathbb{C}^{NK \times M_a}$, with $\mathbf{H}_\mathcal{T} = [\mathbf{H}_\mathcal{T,1}^T, \mathbf{H}_\mathcal{T,2}^T, \ldots, \mathbf{H}_\mathcal{T,K}^T]^T$, where $\mathbf{H}_{\mathcal{T},i} \in \mathbb{C}^{N \times M_a}$ is the channel matrix from the BS’s active transmit antennas to the $i$th user.

The downlink channel can be denoted as

$$ y_i = \mathbf{H}_{\mathcal{T},i} \mathbf{x} + \mathbf{n}_i, \ i = 1, \ldots, K, $$

2The results here can be extended to the general case with different antenna number at each user and are also applicable to the multi-cell scenario with BS cooperation. Moreover, if the number of RF chains is smaller than the antennas, our results can be simply extended after some modifications of the ATAS.
where $x \in \mathbb{C}^{M_a \times 1}$ is the transmitted signal on the downlink, $x_i \in \mathbb{C}^{N \times 1}$ is the transmitted signal of user $i$ on the uplink, $n_i \in \mathbb{C}^{N \times 1}$ and $n \in \mathbb{C}^{M_a \times 1}$ are the independent Gaussian noise with each entry $\mathcal{CN}(0, \sigma^2)$. Frequency flat fading channels with bandwidth $W$ is considered and the channel state information (CSI) is assumed to be perfectly known at the transmitter and receivers.

About the power model, as BSs take the main power consumption in the cellular networks, the users’ consumed power is not considered here. The power radiated to the environment for signal transmission is only a portion of its total power consumption [9], so the practical transmit independent power including circuit power, signal processing power, cooling loss etc. at the BS should be taken into account. The transmit independent power for the BS deployed with multiple antennas is mainly related to the active transmit antenna number. Thus, as a good approximation of the practical power model, we consider a general model given as

$$P_{\text{total}} = f(P, M_a),$$

which is mainly related to the transmit power and antenna number. We can assume that $f(P, M_a)$ is monotonously increasing as a function of $P$ and $M_a$, respectively and based on [26], we assume that $f(P, M_a)$ is affine or convex as a function of $P$. Motivated by [8], [9], more specifically, we consider an affine power consumption model, which can be denoted as

$$P_{\text{total}} = \frac{P}{\eta} + M_aP_{\text{dyn}} + P_{\text{sta}},$$

where $\eta$ denotes the power amplifier (PA) efficiency; $M_aP_{\text{dyn}}$ denotes the dynamic power consumption proportional to the number of radio frequency (RF) chains, e.g. circuit power of RF chains which is always proportional to $M_a$; and $P_{\text{sta}}$ accounts for the static power independent of both $M_a$ and $P$ which includes power consumption of the baseband processing, battery unit etc.. $M_aP_{\text{dyn}} + P_{\text{sta}}$ is referred to as the transmit independent power.

Note that although the optimization procedure is performed based on the affine model, the idea can be simply extended to other convex power model case, e.g. considering the non-ideal PA efficiency like [27] or rate dependent $P_{\text{sta}}$ like [28]. Meanwhile, note that we omit the effect of the complexity of the algorithms related signal processing power in the power model, as it is practical that the $P_{\text{sta}}$ and $P_{\text{dyn}}$ would be significantly larger than the signal processing power of the algorithms.
A. Problem Formulation

The EE is defined as the sum capacity divided by the total power consumption. Considering the maximum transmit power constraint at the BS and the minimum sum capacity constraint, we can define the optimization problem as

$$
\max_{\mathcal{T}, P, M_a} \xi = \frac{C_{BC}(\mathbf{H}_{\mathcal{T},1}, \ldots, \mathbf{H}_{\mathcal{T},K}, P)}{\frac{P}{\eta} + M_a P_{\text{dyn}} + P_{\text{sta}}}
$$

$$
\text{s.t.} \quad 0 \leq P \leq P_{\text{max}}
C_{BC}(\mathbf{H}_{\mathcal{T},1}, \ldots, \mathbf{H}_{\mathcal{T},K}, P) \geq C_{\text{min}}.
$$

(4)

In the above problem, the optimization should be performed via optimizing $P, M_a$ and $\mathcal{T}$. Look at the three parameters. $P$ affects the input transmit covariances directly. As $M_a = |\mathcal{T}|$, the chosen of $\mathcal{T}$ would affect not only the channel matrices, but the dynamic power consumption. Therefore, the problem should be solved by optimizing the transmit covariances and selecting the optimal transmit antenna set. Rewrite the optimization problem as

$$
\max_{\mathcal{T}, P} \xi = \frac{C_{BC}(\mathbf{H}_{\mathcal{T},1}, \ldots, \mathbf{H}_{\mathcal{T},K}, P)}{\frac{P}{\eta} + |\mathcal{T}| P_{\text{dyn}} + P_{\text{sta}}}
$$

$$
\text{s.t.} \quad 0 \leq P \leq P_{\text{max}}
C_{BC}(\mathbf{H}_{\mathcal{T},1}, \ldots, \mathbf{H}_{\mathcal{T},K}, P) \geq C_{\text{min}}.
$$

(5)

Interestingly, the structure of the above optimization problem is similar to the capacity maximization of the MIMO BC with linear precoders, e.g. [24], [25], [29], where power allocation with precoding and user/receive antenna selection are performed as two steps to find the optimal solution. Motivated by this idea, to get the optimal solution, the following two-step structure with transmit covariances optimization and the ATAS should be employed. At first, under fixed set $\mathcal{T}$, optimal transmit covariances can be derived. And then we should choose the most energy efficient $\mathcal{T}$.

It is worthwhile to note that the user/receive antenna selection is unnecessary here. As during the optimization of transmit covariances under DPC, the selected users/receive antennas are correspondingly determined. If we consider the linear precoding schemes, e.g. zero-forcing precoding, joint transmit/receive antenna selection and precoding with power allocation should be taken into account, an example can be found in our previous work [30].
B. Achievable Sum Capacity and Energy Efficiency

The sum capacity of the MIMO BC is achieved by DPC, which can be denoted as follows [3] given a total transmit power $P$.

\[
C_{BC}(H_{T,1}, \ldots, H_{T,K}, P) = \max_{\{\Sigma_{T,i}\}_{i=1}^{K}, \Sigma_{T,i} \geq 0, \sum_{i=1}^{K} \text{Tr}(\Sigma_{T,i}) \leq P} W \log \left| \frac{I + \frac{1}{\sigma^2} H_{T,2} (\Sigma_{T,1} + \Sigma_{T,2}) H_{T,2}^H}{I + \frac{1}{\sigma^2} H_{T,2} (\Sigma_{T,2}) H_{T,2}^H} \right| + \cdots + W \log \left| \frac{I + \frac{1}{\sigma^2} H_{T,K} (\Sigma_{T,1} + \cdots + \Sigma_{T,K}) H_{T,K}^H}{I + \frac{1}{\sigma^2} H_{T,K} (\Sigma_{T,K}) H_{T,K}^H} \right|
\]

where the optimization is performed to choose the optimal downlink transmit covariance matrices $\Sigma_{T,i} \in \mathbb{C}^{M \times M}$, $i = 1, \ldots, K$.

Taking (6) into the optimization problem (5). As (6) is nonconcave, even optimizing (5) under fixed $T$ is nontrivial. Fortunately, motivated by [7], [26], [31], we find out that the following property. If the numerator (sum capacity) can be transformed into a convex function, the EE can be formulated as a quasiconcave function, because the denominator (total power consumption) is affine (also holds for the convex case). Based on this observation and the famous uplink-downlink duality, we can transform the EE into a quasiconcave function under fixed $T$.

The dual uplink channel is denoted as

\[
y_{MAC} = \sum_{i=1}^{K} H_{T,i}^H x_i + n.
\]

Applying the uplink-downlink duality [4], the MIMO BC sum capacity (6) is equal to the concave sum capacity of the MIMO MAC with sum transmit power constraint, which can be denoted as

\[
C_{MAC}(H_{T,1}^H, \ldots, H_{T,K}^H, P) = \max_{\{Q_{T,i}\}_{i=1}^{K}, Q_{T,i} \geq 0, \sum_{i=1}^{K} \text{Tr}(Q_{T,i}) \leq P} W \log \left| \frac{I + \frac{1}{\sigma^2} \sum_{i=1}^{K} Q_{T,i} H_{T,i} H_{T,i}^H}{I + \frac{1}{\sigma^2} Q_{T,K} H_{T,K} H_{T,K}^H} \right|
\]

where the uplink transmit covariance matrices $Q_{T,i} \in \mathbb{C}^{N \times N}$, $i = 1, \ldots, K$ need to be optimized.

Thus, the dual MAC optimal EE should be rewritten as

\[
\xi_{MAC}(H_{T,1}, \ldots, H_{T,K}) = \max_{T,P} \frac{C_{MAC}(H_{T,1}^H, \ldots, H_{T,K}^H, P)}{P + |T| P_{dy} + P_{con}},
\]

s.t. $0 \leq P \leq P_{\text{max}}$

\[
C_{MAC}(H_{T,1}, \ldots, H_{T,K}, P) \geq C_{\text{min}}.
\]

According to the duality and the mapping between $\Sigma_{T,i}$ and $Q_{T,i}$ [4], optimal $\Sigma_{T,i}, i = 1, \ldots, K$ and $P$ can be obtained if we can get the optimal $Q_{T,i}, i = 1, \ldots, K$ and $P$ in (9). Furthermore,
\[ \sum_{i=1}^{K} \text{Tr} \left( Q_i \right) = P \] is always required for optimizing (8) [5], (9) can be simplified and rewritten as

\[ \xi_{\text{MAC}} \left( H^T_{T,1}, \ldots, H^T_{T,K} \right) = \max_{T} \max_{\left\{ Q_{T,i} \right\}_{i=1}^{K}, Q_{T,i} \geq 0} \begin{vmatrix} W \log |I + \frac{1}{\sigma^2} \sum_{i=1}^{K} H^H_{T,i} Q_{T,i} H_{T,i} | \end{vmatrix} \]

s.t. \[ 0 \leq \sum_{i=1}^{K} \text{Tr} \left( Q_{T,i} \right) \leq P_{\text{max}} \]

\[ C_{\text{MAC}} \left( H^T_{T,1}, \ldots, H^T_{T,K}, \sum_{i=1}^{K} \text{Tr} \left( Q_{T,i} \right) \right) \geq C_{\text{min}}. \]

Finally, the optimization of (5) is transformed into optimizing (10). If we can obtain the optimal \( Q_{T,i}, i = 1, \ldots, K \) for (10), the optimal \( P \) can be decided correspondingly based on \( \sum_{i=1}^{K} \text{Tr} \left( Q_{T,i} \right) = P \). As (10) is a concave fractional programming problem under fixed \( T \), we will separate the optimization into two steps. First, we can find the optimal transmit covariances under fixed \( T \) and then applying the ATAS to choose the optimal \( T \).

III. EE OPTIMIZATION UNDER FIXED TRANSMIT ANTENNA SET

In this section, we will solve the optimal energy efficient transmit covariances under fixed \( T \).

Let us look at (10) with fixed \( T \) again. Since the numerator is concave and the denominator is affine, (10) is a quasiconcave optimization, which can be solved through the bisection method or interior-point methods [32]. However, the numerical methods would be still too complex when the user number becomes significantly large. Motivated by [5], [6], an energy efficient iterative waterfilling is proposed in the next section to solve it more efficiently. For ease of description, we omit \( T \) in the subscript in this section.

We will first consider the solution of (10) without the minimum capacity and maximum capacity constraints in subsection III-A, and then we will extend it to the general constrained case in subsection III-B.

A. Unconstrained Case

1) Motivation: As the EE is distinct from the capacity, the spectral efficient iterative waterfilling [5] is not applicable for the EE any longer. Nevertheless, we notice that the basic idea of the spectral efficient iterative algorithms are based on the block-coordinate ascent algorithm [33, Sec. 2.7]. That is to say, if we can write the EE as the similar structure with the block-coordinate
ascent algorithm and then prove it satisfies the condition of [33, Sec. 2.7], we can also obtain an iterative solution of the problem (10).

For ease of description, we define the following function \( g(\cdot) \) at first.

\[
g(Q_1, \ldots, Q_K) = \frac{\log |I + \frac{1}{\sigma^2} \sum_{i=1}^K H_i^H Q_i H_i|}{\sum_{i=1}^K \text{Tr} (Q_i) + MP_{\text{dyn}} + P_{\text{sta}}},
\]

(11)

For the block-coordinate ascent algorithm, given the current iterate \( Q^{(k)} = (Q_1^{(k)}, \ldots, Q_K^{(k)}) \), the next iterate \( Q^{(k+1)} = (Q_1^{(k+1)}, \ldots, Q_K^{(k+1)}) \) can be generated as

\[
Q_i^{(k+1)} = \arg \max_{Q_i \geq 0} g(Q_1^{(k+1)}, \ldots, Q_i^{(k+1)}, \ldots, Q_K^{(k+1)}).
\]

(12)

However, to apply the iterative algorithm efficiently, there are conditions need to be satisfied. For one thing, the solution of (12) should be uniquely attained [33, Proposition 2.7.1]. For another, the solution should be simple and easy to employ.

Very fortunately, the two conditions both fulfill and the solution can be obtained following an energy efficient waterfilling feature. We are interested to show it in the next subsection.

2) Energy Efficient Waterfilling: Based on [5], [6], it is fulfilled that

\[
\log |I + \frac{1}{\sigma^2} \sum_{i=1}^K H_i^H Q_i H_i| = \log |I + \frac{1}{\sigma^2} \sum_{j \neq i} H_j^H Q_j H_j + \log |I + \left(\sigma^2 I + \sum_{j \neq i} H_j^H Q_j H_j\right)^{-1/2}|
\]

\[
= \log |Z_i| + \log |I + G_i^H Q_i G_i|,
\]

(13)

where \( Z_i = I + \frac{1}{\sigma^2} \sum_{j \neq i} H_j^H Q_j H_j \) and \( G_i = H_i \left(\sigma^2 I + \sum_{j \neq i} H_j^H Q_j H_j\right)^{-1/2} \). By denoting

\[
a_i = \frac{\sum_{j \neq i} \text{Tr} (Q_j)}{\eta} + MP_{\text{dyn}} + P_{\text{sta}},
\]

\[
b_i = W \log |Z_i|
\]

and substituting (13) into (11) we have that

\[
g(Q_1, \ldots, Q_K) = \frac{b_i + W \log |I + G_i^H Q_i G_i|}{\sum_{i=1}^K \text{Tr} (Q_i) + \eta a_i}
\]

(14)
Therefore, we can redefine the problem (12) by removing the iteration number as to

$$\max_{Q_i:Q_i \geq 0} \ g(Q_i, \ldots, Q_{i-1}, Q_i, Q_{i+1}, \ldots, Q_K) = \frac{b_i + W \log |I + G_i^H Q_i G_i|}{\frac{\trace(Q_i)}{\eta} + a_i}$$  \hspace{2cm} (15)$$

by treating $Q_1, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_K$ as constant. Based on section [III-A1], we need to solve the above problem and prove that the solution is unique.

Since the numerator and denominator in (14) are concave and affine respectively, (15) is a concave fractional program [7]. Define a non-negative parameter $\lambda$, (15) is related to the following convex function separating numerator and denominator with help of $\lambda$.

$$F(Q_i, \lambda) = b_i + W \log |I + G_i^H Q_i G_i| - \lambda \left(\frac{\trace(Q_i)}{\eta} + a_i\right)$$  \hspace{2cm} (16)$$

And then define a convex optimization problem as

$$Y(\lambda) = \max_{Q_i:Q_i \geq 0} F(Q_i, \lambda).$$  \hspace{2cm} (17)$$

We will try to solve (17), and then the solution of (15) can be obtained correspondingly based on the following Theorem.

**Theorem 1:** The optimum feasible transmit covariance matrix $Q_i^*$ achieves the maximum value of (15) if and only if $Y(\lambda^*) = F(Q_i^*, \lambda^*) = \max F(Q_i, \lambda^*|Q_i \geq 0) = 0$.

**Proof:** See Appendix A.

Theorem [1] gives us insights to solve (15). We should optimizing (17) at first under a given $\lambda$ and then solve the equation $Y(\lambda) = 0$ to get the optimal $\lambda$.

To solve (17), we can denote

$$G_i^H G_i = U D_i U^H$$  \hspace{2cm} (18)$$

based on the eigenvalue decomposition at first, where $D_i \in \mathbb{C}^{M \times M}$ is diagonal with nonnegative entries and $U \in \mathbb{C}^{M \times M}$ is unitary. We assume that $D_i$ has $L$ non-zero diagonal entries ($1 \leq L \leq M$), which means $[D_i]_{kk} > 0$ for $k = 1, \ldots, L$ and $[D_i]_{kk} = 0$ for $k = L + 1, \ldots, M$.

And then we have the following equation based on $I + AB = I + BA$ [5]:

$$\log |I + G_i^H Q_i G_i| = \log |I + Q_i G_i^H G_i| = \log |I + Q_i U D_i U^H| = \log |I + U^H Q_i U D_i|$$  \hspace{2cm} (19)$$

Define $S_i = U^H Q_i U$. As $U$ is unitary, we have that $\trace(S_i) = \trace(Q_i)$. Thus, (16) can be rewritten as

$$G(S_i, \lambda) = b_i + W \log |I + S_i D_i| - \lambda \left(\frac{\trace(S_i)}{\eta} + a_i\right)$$  \hspace{2cm} (20)$$
As each $S_i$ corresponds to a $Q_i$ via the invertible mapping $S_i = U^H Q_i U$, solving (17) is equivalent to solving the following convex optimization problem.

$$Y(\lambda) = \max_{S_i: S_i \geq 0} G(S_i, \lambda)$$

(21)

It is proved in the [5, Appendix II] that the optimal $S_i^\ast$ to solve (21) is diagonal with $[S_i^\ast]_{kk} > 0$ for $k = 1, \ldots, L$ and $[S_i^\ast]_{kk} = 0$ for $k = L + 1, \ldots, M$. Thus, $G(S_i, \lambda)$ with diagonal $S_i$ is

$$G(S_i, \lambda) = b_i + W \sum_{k=1}^{L} \log \left( 1 + [S_i]_{kk} [D_i]_{kk} \right) - \lambda \left( \frac{\sum_{k=1}^{L} [S_i]_{kk}}{\eta} + a_i \right).$$

(22)

As (22) is concave in $S_i$, the problem (21) can be solved for a given $\lambda$ by solving the KKT optimality conditions, and the solution can be denoted as

$$[S_i^\ast]^\lambda_{kk} = \left[ \frac{\eta}{\ln(2)\lambda} - \frac{1}{[D_i]_{kk}} \right]^+, k = 1, \ldots, L,$$

(23)

where $[x]^+ = \max(x, 0)$. Then the water level $\lambda^\ast$ can be decided by setting $Y(\lambda^\ast) = 0$ based on Theorem 1 as

$$b_i + \sum_{k=1}^{L} \log \left( 1 + \left[ \frac{\eta}{\ln(2)\lambda^\ast} - \frac{1}{[D_i]_{kk}} \right]^+ \left[ D_i \right]_{kk} \right) - \lambda^\ast \times \left( \frac{\sum_{k=1}^{L} \left[ \frac{\eta}{\ln(2)\lambda^\ast} - \frac{1}{[D_i]_{kk}} \right]^+}{\eta} + a_i \right) = 0.$$  

(24)

Based on (24) and (23), the optimal $S_i^{\ast\lambda^\ast}$ is derived. Based on the mapping between $S_i$ and $Q_i$, finally, the optimal solution of (15) can be derived as

$$Q_i^{\ast} = US_i^{\ast\lambda^\ast} U^H.$$  

(25)

To prove that the solution of (15) is unique, we only need to prove that $\lambda^\ast$ is unique. We give the following Theorem and the proof is given in the Appendix B.

**Theorem 2:** The derived water level $\lambda^\ast$ in (24) is unique and globally optimal for each iteration.

To make the description more clearly, we summarize the energy efficient waterfilling algorithm for optimizing (14) in TABLE I.

3) Iterative Algorithm: Based on the derivation in section III-A2 and the block-coordinate ascent algorithm, the energy efficient iterative waterfilling scheme can be derived as shown in TABLE II, and the proof of converge is given as follows.

**Proof of converge:** Firstly, during each step, the energy efficient waterfilling can achieve an global maximum EE treating the other users’ transmit covariance matrices as constant, the EE is non-decreasing with each step. As the EE is bounded, the EE converges to a limit.
Secondly, according to Theorem 1 and Theorem 2 the derivation of each step is unique. Based on [33, Sec. 2.7], the set of \( Q_1, \ldots, Q_K \) also converge to a limit. \( \square \)

Note that as the proof does not depend on the starting point, we can start the algorithm from any starting values of \( Q_1, \ldots, Q_K \). To show the efficiency of the proposed scheme, we give the simulation results in Fig. 2. The converge behavior of the proposed scheme is shown and it is set that \( d = 1 \text{km}, M = 4, N = 4 \) and \( K = 10 \). We can see that our proposed iterative scheme converges very fast. It can achieve the optimal EE under nearly five iterations.

**B. Constrained EE Optimization**

After deriving the unconstrained optimal transmit covariances, we can extend it to the general constrained case (10) with fixed \( T \). Look at the constrained case, the sum capacity is constrained and the sum transmit power is limited at the BS. We rewrite them as follows.

\[
\sum_{i=1}^{K} \text{Tr} (Q_i) \leq P_{\text{max}} \\
C_{\text{MAC}} \left( H_1, \ldots, H_K, \sum_{i=1}^{K} \text{Tr} (Q_i) \right) \geq C_{\text{min}}.
\]

(26)

Define two related problems. The first one is capacity maximization under power constraint.

\[
\max_{\{Q_i\}_{i=1}^{K}} W \log \left| \mathbf{I} + \frac{1}{\sigma^2} \sum_{i=1}^{K} H_i^H Q_i H_i \right|, 
\]

(27)

And the second one is total transmit power minimization under capacity constraint.

\[
\min_{\{Q_i\}_{i=1}^{K}} \sum_{i=1}^{K} \text{Tr} (Q_i) \\
\text{s.t.} W \log \left| \mathbf{I} + \frac{1}{\sigma^2} \sum_{i=1}^{K} H_i^H Q_i H_i \right| \geq C_{\text{min}}.
\]

(28)

The first problem (27) can be solved by the spectral efficient iterative waterfilling schemes efficiently in [5], while the second power minimizing problem (28) can be solved efficiently based on [34]. We denote the solution as \( \hat{Q}_i \) and \( \bar{Q}_i, i = 1, \ldots, K \), respectively. To help the description, we denote that \( P^* = \sum_{i=1}^{K} \text{Tr} (Q_i^*) \), \( \bar{P} = \sum_{i=1}^{K} \text{Tr} (\bar{Q}_i) \) and \( P_{\text{max}} = \sum_{i=1}^{K} \text{Tr} (\hat{Q}_i) \).

As the numerator of the EE is concave and monotonously increasing and the denominator is affine, the EE is quasiconcave, monotonously increasing as a function of the \( P \) when \( P < P^* \) and monotonously decreasing as a function of the \( P \) when \( P > P^* \). We can get the solution of the constrained problem as follows.
\[
Q_{\text{opt}}^i = \begin{cases} 
Q_i^*, & \text{if } \bar{P} \leq P^* \leq P_{\text{max}} \\
\hat{Q}_i, & \text{if } P^* \geq P_{\text{max}} \geq \bar{P} \\
\bar{Q}_i, & \text{if } P^* \leq \bar{P} \leq P_{\text{max}} \\
\text{infeasible, if } \bar{P} \geq P_{\text{max}} 
\end{cases}, i = 1, \ldots, K. \tag{29}
\]

Note that when the solution is infeasible, we will choose \(Q_{\text{opt}}^i = \hat{Q}_i, i = 1, \ldots, K\) as the optimal solution in the simulation. This choice has practical significance, as when the rate constraint is infeasible, the best choice is to transmit in the available maximum capacity.

IV. ACTIVE TRANSMIT ANTENNA SELECTION

To optimize \(T\), we apply the ATAS procedure in this section to determine it.

From the standpoint of SE, activating all transmit antennas is always optimal, as higher multiplexing and diversity gain can be acquired. However, things change under the energy efficient scenario. As multiple antennas consumes higher dynamic power, there exists a tradeoff between the power consumption loss and the capacity gain. Thus, ATAS is necessary here. The ATAS here is different from the spectral efficient transmit antenna selection, as the conventional transmit antenna selection is always utilized in the scenario when the transmit antennas is more than the RF chain and the purpose is to employ the selection diversity. Although the ATAS can also acquire the selection diversity gain here, its main objective is to compromise the EE gain with higher multiplexing/diversity and the EE loss with more transmit independent power consumption of multiple antennas. After selecting the active transmit antennas in ATAS, the inactive antennas should be turned off through micro-sleep or DTX.

It is intuitive to see that the exhaust search ATAS is the optimal selection scheme. For each possible \(T \subseteq \{1, \ldots, M\}\), the exhaust search ATAS calculates the EE based on section III and then choose the optimal transmit antenna set as follows after comparing the EE.

\[
T_{\text{opt}} = \arg \max_{T \subseteq \{1, \ldots, M\}} \xi(T),
\]

Finally, the antennas in \(T_{\text{opt}}\) activated and the inactive ones are switched off. However, the complexity of the exhaust search ATAS is too high to implement. Thus, developing low complex scheme is important. Let us look at the problem formulation again. As the channel matrices are only related to the capacity, the capacity maximizing antenna selection scheme can be borrowed.
As shown in [21], [23], the capacity is highly dependent of the norm of the channel matrices and selecting antenna based on norm is highly recommended due to its good performance and low complexity. Thus, it is borrowed here, which is shown in Table III. In the algorithm in Table III the EE with selected antenna set under different active transmit antenna set size (the transmit antenna number) is compared. That is because the power is highly related to the active transmit antenna number. For each antenna number, we can choose the channel matrices with largest norm. After that, the EE with different antenna number should be compared and then determine the best antenna number.

A. Implementation Issue in Realistic Scenario

There exists an invisible CSIT problem here. During performing the ATAS, channel matrices of all antennas are needed for calculating the Norm and determine the best active antennas. However, as the inactive BS antennas should be switched off to save energy, the channel estimation for these channels with inactive antennas is impossible. Thus, there might exist time slots in which the channel matrices of the inactive BS antennas are not visible at the BS.

In order to combat this drawback, a possible way is to add one dedicated training period to switch on all the BS antennas to obtain the full CSIT of all antennas to help the antenna selection. In this case, the power consumption of the training period would decrease the EE. Thus, the above definition of EE performs as an upper bound. Nevertheless, there should be other low complex schemes to combat the invisible CSIT problem. For example, the statistical CSIT can be applied for the ATAS. The performance and cost tradeoff of these schemes should be left for the future work.

V. Simulation Results

We evaluate the performance under different scenarios to show the effect of parameters. In the simulation, pathloss and Rayleigh fading are considered. The parameters are set based on [8]. It is set that $W = 5\text{MHz}$, the noise power is $-110\text{dBm}$, $P_{\text{dyn}} = 83\text{W}$, $P_{\text{Sta}} = 45.5\text{W}$, $\eta = 0.38$, $P_{\text{max}} = 46\text{dBm}$ and pathloss is $128.1 + 37.6 \log_{10} d$ with distance $d = 1\text{km}$ ($d$ in kilometers and all users are with the same distance). We use “EE w Exh-AS” to denote the optimal energy efficient transmission with exhaust ATAS, “EE w Norm-AS” to denote the energy efficient transmission with low complexity norm-based successive ATAS, “EE wo AS” to denote the energy efficient
transmission with activating all available BS antennas and ’SE’ to denote the spectral efficient transmission with activating all available BS antennas. Here, schemes with “EE” perform the transmit covariances optimization determined according to section III and schemes with “SE” perform the transmit covariances optimization based on the capacity maximization 5.

The EE versus pathloss is first evaluated in Figs. 3 and 4, where unconstrained case and constrained case with $P_{\text{max}} = 46$ dBm are both considered. We can see that schemes under “EE” with ATAS are better than “EE wo AS”, and the performance gain comes from the ATAS. In Fig. 3, the distance varies from 0.1km to 1km. When the distance is short, e.g. 0.1km, it is shown that the schemes with “EE” are all superior to “SE 46dBm”, whose performance gain comes from the transmit covariance optimization, while the three schemes with maximum power constraint perform the same as the corresponding unconstrained ones, whose reason is that the globally optimal $P^*$ is smaller than 46dBm and thus the constrained and unconstrained scenarios can both achieve the global optimality. When the distance gets larger, the “EE wo AS 46dBm” degenerates to “SE 46dBm”. The reason can be explained as follows. As the distance increases, the global optimal energy efficient sum power increases. For “EE wo AS 46dBm”, the derived $P^* > 46$ dBm, and then 46dBm should be applied. Look at Fig. 4 then. When the distance is larger than 1km, “EE wo AS” is superior to “EE wo AS 46dBm”. This is because when the distance becomes larger, the optimal sum power $P^*$ would be greater than 46dBm, and thus the performance gain comes from the transmit covariance optimization. Look at the case when $d = 5$ km, due to the similar reason above, we have that “EE w Exh-AS” and “EE w Norm-AS” are better than “EE w Exh-AS 46dBm” and “EE w Norm-AS 46dBm”, respectively.

We simulate the effect of capacity constraints in Fig. 5, 6, 7 and the case with $M = 4$, $N = 2$, $K = 2$, $d = 1$ km, $P_{\text{max}} = 46$ dBm is considered. We can see that the “EE w Exh-AS” always achieves the maximum EE, and “EE w Norm-AS” performs very close to “EE w Exh-AS”. Meanwhile, “EE wo AS” has smaller EE than “EE w Norm-AS”, while “SE” has the worst EE. The performance gain between “EE” with ATAS and “EE wo AS” comes from the ATAS gain, as after selective antennas are determined, turning off the inactive antennas can save the dynamic power $M_a P_{\text{dyn}}$ and then improve the EE. Meanwhile, the gain between “EE wo AS” and “SE” shows the efficiency of the energy efficient transmit covariances optimization in section III.

The EE gap between different schemes are becoming smaller as the capacity constraint
increases. When the capacity constraint is larger than 33 bps/Hz, the four schemes perform the same. Look at Fig. 6, the simulated capacity is fixed at 32.2 bps/Hz, and in this case, the maximum power is utilized to transmit. Another observation is that the performances of “EE w Norm-AS” and “EE w Exh-AS” both have the multi-stage feature. This feature can be explained according to Fig. 7. In order to fulfill the increasing capacity constraint, the active antenna number increases, and the active antenna number has a similar multi-stage feature.

Fig. 8 depicts the EE under different BS antenna configuration, where \( N = 2, K = 2, P_{\text{max}} = 46\text{dBm}, C_{\text{min}} = 0, d = 1\text{km} \). “EE w Norm-AS” and “EE w Exh-AS” are both monotonously increasing as a function of the transmit antenna number \( M \) at the BS. The performance gain comes from the transmit antenna selection diversity gain with suitable active transmit antenna number \( M_a \). When the transmit antenna number increases, the probability of choosing channels of active antennas with better channel conditions increases. However, look at “SE” and “EE wo AS”. Their performances degenerate seriously when the antenna number is more than four. The reason can be explained by the multiplexing gain of the DPC. There are \( \min(N \times K, M) \) spatial dimensions for DPC and thus the capacity can scales as only \( N \times K \) for the case with \( M \geq N \times K \). For the “SE” with maximum power transmission, the capacity can increase linear as the antenna number when \( M \leq 4 \), that is why “SE” has the best EE performance when the antenna number is four. Meanwhile, the dynamic power increase linearly with \( M_a \) in the power part, and then the EE loss with increasing dynamic power is significantly larger than the EE gain with capacity increasing when \( M \geq 4 \). Thus, the EE would decrease significantly for “SE” and “EE wo AS” then. Furthermore, from the fact that schemes with “EE” and ATAS are much superior to “EE wo AS”, we can conclude that choosing \( M_a = M \) is not optimal for the limited user number case here.

Although it is demonstrated that more antenna number is benefit for the EE with higher selection diversity under ATAS, it is worthwhile to note that configuring more antenna would cost higher Capital expenditures (CAPEX). In the design of the realistic systems, the tradeoff between the EE gain and the CAPEX loss should be taken into account.

We are interested in discussing the multiuser diversity finally through Fig. 9 which depict the EE under different user number, where \( M = 4, N = 2, P_{\text{max}} = 46\text{dBm}, C_{\text{min}} = 0, d = 1\text{km} \).

\(^3\text{In this case, the capacity constraint is infeasible, and the maximum power is employed.}\)
We can see that “EE w Norm-AS” and “EE w Exh-AS” degenerate into “EE wo AS”, where
the all four transmit antennas should be active. Moreover, about the multiuser diversity, we can
see that from Fig. 9 that a similar $M \log \log(NK)$ scaling law can be acquired. Indeed, in our
another work [36], we analyze the EE scaling law with the help of the Lambert $\omega$ function, and
it is shown that when $M_a P_{\text{dyn}} + P_{\text{sta}} > 0$ in this paper, the multiuser diversity of
$\frac{M \log \log(NK)}{M_a P_{\text{dyn}} + P_{\text{sta}}}$
always holds. We can see that $M_a = M$ is optimal for the large user number case, which is
distinct from the limited user number case.

VI. CONCLUSION

We propose a novel optimization framework with transmit covariance optimization and ATAS
to improve the EE in the MIMO BC. Under fixed active transmit antenna set, we transform the EE
of MIMO BC based on uplink-downlink duality into a concave fractional programming. Based
on this feature, we propose an energy efficient iterative waterfilling scheme to maximize the EE
for the MIMO BC according to the block-coordinate ascent algorithm. We prove the converge
of the proposed scheme and validates it through simulations. After determining the transmit
covariance under fixed antenna set, we discuss the ATAS to further improve the EE, where
exhaust search and Norm-based successive schemes are borrowed to select the active antennas
and switch off the inactive ones. Through simulation results, the effect of system parameters on
the EE is discussed.

APPENDIX A

As in (15) the numerator is concave and differentiable, and the denominator is convex and
differentiable, Theorem 1 can be directly obtained based on [7 Proposition 6].

APPENDIX B

As in (15) the numerator is concave and continuous, and the denominator is convex and
continuous, $F(\lambda)$ is strictly decreasing and continuous based on [7]. Look at (24), we have that
$F(0) = \infty$ and $F(\infty) = -\infty$. Therefore, there exists a unique $\lambda^*$ with $F(\lambda^*) = 0$. Furthermore,
as shown in [7], in a concave fractional program, any local maximum is a global maximum.
Therefore, the derived $\lambda^*$ is global optimal for each iteration.
REFERENCES

[1] J. Xu, L. Qiu, and S. Zhang, “Energy Efficient Iterative Waterfilling for the MIMO Broadcasting Channels,” in IEEE proc. of WCNC 2012, accepted. [Online]. Available: http://home.ustc.edu.cn/~suming

[2] Y. Chen, S. Zhang, S. Xu, and G. Y. Li, “Fundamental tradeoffs on green wireless networks,” IEEE Communications Magazine, vol. 49, no. 6, pp. 30–37, June 2011.

[3] G. Caire and S. S. (Shitz), “On the Achievable Throughput of a Multiantenna Gaussian Broadcast Channel,” IEEE Trans. Inf. Theory, vol. 49, no. 7, pp. 1691–1706, Jul. 2003.

[4] S. Vishwanath, N. Jindal, and A. Goldsmith, “Duality, achievable rates, and sum-rate capacity of MIMO broadcast channels,” IEEE Trans. Inf. Theory, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.

[5] N. Jindal, W. Rhee, S. Vishwanath, S. Jafar, and A. Goldsmith, “Sum power iterative water-filling for multi-antenna Gaussian broadcast channels,” IEEE Trans. Inform. Theory, vol. 51, no. 4, pp. 1570–1580, Apr. 2005.

[6] W. Yu, W. Rhee, S. Boyd, and J. Cioffi, “Iterative water-filling for Gaussian vector multiple-access channels,” IEEE Trans. Inform. Theory, vol. 50, no. 1, pp. 145–152, Jan. 2004.

[7] S. Schaible, “Fractional programming,” Zeitschrift für Operations Research, vol. 27, no. 1, pp. 39–54.

[8] J. Xu, L. Qiu, and C. Yu, “Improving Energy Efficiency Through Multimode Transmission in the Downlink MIMO Systems,” EURASIP J. Wireless Commun. and Net., vol. 2011, no. 1, p. 200, 2011.

[9] O. Arnold, F. Richter, G. Fettweis, and O. Blume, “Power Consumption Modeling of Different Base Station Types in Heterogeneous Cellular Networks,” in Proceedings of the ICT MobileSummit (ICT Summit'10), Florence, Italy.

[10] O. Blume, H. Eckhardt, S. Klein, E. Kuehn, and W. M. Wajda, “Energy savings in mobile networks based on adaptation to traffic statistics,” Bell Labs Technical, vol. 15, no. 2, pp. 77–94, Sep. 2010.

[11] 3rd Generation Partnership Project, “Extended cell DTX for enhanced energy-efficient network operation,” in 3GPP TSG-RAN WG1 #59, R1-094996, Nov. 9-13, 2009.

[12] S. Cui, A. J. Goldsmith, and A. Bahai, “Energy-Efficiency of MIMO and cooperative MIMO Techniques in Sensor Networks,” IEEE J. Select. Areas Commun., vol. 22, no. 6, pp. 1089–1098, Aug. 2004.

[13] Z. Chong and E. Jorswieck, “Energy-efficient Power Control for MIMO Time-varying Channels,” in Proc. of IEEE Online Green Communications Conference (GreenCom), Online, 2011.

[14] R. S. Prabhu and B. Daneshfar, “Energy-Efficient Power Loading for a MIMO-SVD System and Its Performance in Flat Fading,” in IEEE Proc. of GLOBECOM'10, 2010, pp. 1–5.

[15] H. Kim, C.-B. Chae, G. Veciana, and R. Heath, “A Cross-Layer Approach to Energy Efficiency for Adaptive MIMO Systems Exploiting Spare Capacity,” IEEE Trans. Wireless Commun., vol. 8, no. 8, pp. 4264–4275, Aug. 2009.

[16] E. Belmega and S. Lasaulce, “Energy-efficient precoding for multiple-antenna terminals,” IEEE Trans. Signal Process., vol. 59, no. 1, pp. 329–340, Jan. 2011.

[17] F. Héliot, O. Onireti, and M. Imran, “An accurate closed-form approximation of the energy efficiency-spectral efficiency trade-off over the mimo rayleigh fading channel,” in proc. of IEEE ICC 2011 Workshop on Green Communications.

[18] G. Miao, N. Himayat, and G. Y. Li, “Energy-Efficient Link Adaptation in Frequency-Selective Channels,” IEEE Transactions on Communications, vol. 58, no. 2, pp. 545–554, Feb. 2010.

[19] R. S. Prabhu and B. Daneshfar, “An energy-efficient water-filling algorithm for OFDM systems,” in IEEE Proc. of ICC'10, 2010.

[20] Z. Chong and E. Jorswieck, “Energy Efficiency in Random Opportunistic Beamforming,” in Proc. of IEEE 73rd Vehicular Technology Conference, Budapest, Hungary (VTC 2011 Spring), 2011.
[21] S. Sanayei and A. Nostratinia, “Antenna selection in MIMO systems,” *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 68–73, Oct. 2004.

[22] ——, “Capacity maximizing algorithms for joint transmit-receive antenna selection,” in *Proc. 38th Asilomar Conf on Signals, Systems and Computers*, vol. 2, Nov. 2004, pp. 1773–1776.

[23] R. Chen, Z. Shen, J. Andrews, and R. H. Jr., “Efficient transmit antenna selection for multiuser MIMO systems with block diagonalization,” in *Proc. of IEEE GLOBECOM’07*, Nov. 2007, pp. 3499–3503.

[24] Z. Shen, R. Chen, J. Andrews, R. H. Jr., and B. Evans, “Low complexity user selection algorithms for multiuser MIMO systems with block diagonalization,” *IEEE Trans. Signal Processing*, vol. 54, no. 9, pp. 3658–3663, Sep. 2006.

[25] R. Chen, Z. Shen, J. Andrews, and R. H. Jr., “Multimode transmission for multiuser MIMO systems with block diagonalization,” *IEEE Trans. Signal Processing*, vol. 56, no. 7, pp. 3294–3302, Jul. 2008.

[26] C. Isheden, Z. Chong, E. Jorswieck, and G. Fettweis, “Framework for Link-Level Energy Efficiency Optimization with Informed Transmitter,” *IEEE Trans. Wireless Commun.*, submitted. [Online]. Available: http://arxiv.org/abs/1110.1990

[27] A. He, S. Srikanteswara, K. Bae, T. Newman, J. Reed, W. Tranter, M. Sajadieh, and M. Verhelst, “Power consumption minimization for MIMO systems: A cognitive radio approach,” *IEEE J. Select. Areas Commun.*, vol. 29, no. 2, pp. 469–479, Feb. 2011.

[28] C. Isheden and G. Fettweis, “Energy-Efficient Multi-Carrier Link Adaptation with Sum Rate-Dependent Circuit Power,” in *IEEE Proceeding of Globecom 2010*.

[29] T. Yoo and A. Goldsmith, “On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming,” *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 528–541, Mar. 2006.

[30] J. Xu, L. Qiu, and C. Yu, “Link Adaptation and Mode Switching for the Energy Efficient Multiuser MIMO Systems,” *IEICE Trans. Commun.*, submitted. [Online]. Available: http://home.ustc.edu.cn/~suming/

[31] Z. Chong and E. Jorswieck, “Analytical Foundation for Energy Efficiency Optimisation in Cellular Networks with Elastic Traffic,” in *Mobile Lightweight Wireless Systems: 3rd Internationaln ICST Conference (MobiLight 2011)*.

[32] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge University Press, 2004.

[33] D. Bertsekas, *Nonlinear Programming*. Belmont, MA: AthenaScientific, 1999.

[34] C. Fung, W. Yu, and T. Lim, “Multi-antenna downlink precoding with individual rate constraints: power minimization and user ordering,” in *The Ninth International Conference on Communications Systems (ICCS 2004)*, Sep. 2004, pp. 45–49.

[35] N. Jindal and A. Goldsmith, “Dirty-paper coding versus tdma for MIMO broadcast channels,” *IEEE Trans. Inform. Theory*, vol. 51, no. 5, pp. 1783–1794, May 2005.

[36] J. Xu and L. Qiu, “Energy Efficiency Scaling Law for MIMO Broadcasting Channels,” *IEEE Wireless Commun. Letters*, submitted. [Online]. Available: http://home.ustc.edu.cn/~suming/
TABLE I
ENERGY EFFICIENT WATERFILLING ALGORITHM

1) Calculate \( Z_i = I + \frac{1}{\sigma^2} \sum_{j \neq i} H_i^H Q_j H_j \), \( G_i = H_i \left( \sigma^2 I + \sum_{j \neq i} H_i^H Q_j H_j \right)^{-1/2} \), \( a_i = \frac{\sum_{j \neq i} \text{Tr}(Q_i)}{\eta} + M P_{\text{dy}} + P_{\text{sta}}, b_i = W \log |Z_i|; \)
2) Define the related parametric convex program in (16) and (17);
3) Transform the parametric convex program into diagonal forms (20) and (21) by performing eigenvalue decomposition in (18);
4) Solve (21) by solving the Karush-Kuhn-Tucker optimality conditions and obtain the solution \( S_i^{* \lambda} \) in (23);
5) Calculate the energy efficient water level \( \lambda^* \) based on (24) and determine the optimal \( S_i^{* \lambda^*} \);
6) Obtain \( Q_i^* \) based on the mapping (25) finally;

TABLE II
ENERGY EFFICIENT ITERATIVE WATERFILLING SCHEME

**Initialization:** Set \( Q_i = 0, i = 1, \ldots, K, \)

**Repeat:**

For \( i = 1 : K \)

1) Calculate \( Q_i^* \) based on the energy efficient waterfilling algorithm in TABLE II;
2) Refresh \( Q_i \) as \( Q_i^*; \)
End

Until the EE converges.
 TABLE III  
NORM BASED ATAS 

| Algorithm |
|-----------|
| Initialization: Set $\xi_{\text{temp}} = 0$. Sorting the columns of $H$ as $\|H(:, \pi(1))\| \geq \ldots \geq \|H(:, \pi(M))\|$. |
| For: $M_a = 1 : M$ |
| 1) Transmit antenna selection: Choose $T_{M_a} = \{\pi(1), \ldots, \pi(M_a)\}$, and the active channel matrix of $M_a$ selected transmit antennas is denoted as $H_{M_a}$. |
| 2) Compute the EE: Calculate the EE based on section III as $\xi(M_a)$. |
| 3) Compare the EE: If $\xi_{\text{temp}} < \xi(M_a)$, $\xi_{\text{temp}} = \xi(M_a)$, set $T = T_{M_a}$. |
| End For |
| Activating the antennas in $T$ and switching off the inactive ones. |

Fig. 1. System model of the MIMO BC.
Fig. 2. EE converge behavior of the proposed scheme.

Fig. 3. The EE versus distance from 0.1km to 1km, where $M = 4, N = 2, K = 2, P_{\text{max}} = 46\text{dBm}, C_{\text{min}} = 0$. 

$\text{February 17, 2012 DRAFT}$
Fig. 4. The EE versus distance from 1km to 5km, where $M = 4, N = 2, K = 2, P_{\text{max}} = 46\text{dBm}, C_{\text{min}} = 0$.

Fig. 5. The EE under different capacity constraints, where $M = 4, N = 2, K = 2, P_{\text{max}} = 46\text{dBm}$, distance is 1km.
Fig. 6. The corresponding capacity under different capacity constraints, where $M = 4, N = 2, K = 2, P_{\text{max}} = 46\text{dBm}$, distance is 1km.

Fig. 7. Corresponding active transmit antenna number versus capacity constraints.
Fig. 8. The EE under different BS antennas, where $N = 2, K = 2, P_{\text{max}} = 46\text{dBm}, C_{\text{min}} = 0$, distance is $0.5\text{km}$.

Fig. 9. The EE under different user number, where $M = 4, N = 2, P_{\text{max}} = 46\text{dBm}, C_{\text{min}} = 0$. 