Thermalization at lowest energies?  
A view from a transport model

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Abstract

Using the Isospin Quantum Molecular Dynamics (IQMD) model we analyzed the production of pions and kaons in the energy range of 1-2 AGeV in order to study the question why thermal models could achieve a successful description. For this purpose we study the variation of pion and kaon yields using different elementary cross sections. We show that several ratios appear to be rather robust versus their variations.

1 Introduction

In the last decades heavy ion experiments have studied various combinations of heavy ion collisions. Besides the understanding of dynamical observables the production of particles have been of a special focus. Recent experiments at the SIS accelerator at GSI have us provided with high-quality data of the production of pions [1] and of kaons [2, 3]. The yields of particle ratios have been shown of large interest especially for the application of thermal models [4]. Several ratios of measured yields could be interpreted by the use of law-of-mass relations [5, 6]. That idea was later on supported by a detailed analysis from transport models [7, 8].

We use the Isospin Quantum Molecular Dynamics model (IQMD) [9, 10] in order to study the sensitivity of those ratios on the variation of several cross sections. If the system is near a thermal equilibrium, the influence of the cross sections is negligible. The dominant cross section in the energy range under study is besides the elastic nucleon-nucleon cross section the inelastic reaction of two nucleons forming a Δ and the corresponding inverse reaction \( NN \leftrightarrow N\Delta \). The produced Δ may also decay into a nucleon-pion pair and the produced pion can again be absorbed by a nucleon forming a Δ: \( \Delta \leftrightarrow N\pi \).

Figure 1 presents on the left side the parameterizations of the total pp (dashed) and pn cross sections (dotted line) and their respective inelastic cross sections forming a Δ (pp full line, pn thick dotted line). The differences between

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the total and the inelastic cross section is given by the elastic cross section. It should be noted that the cross sections of the absorption of a $\Delta$ are related to their production cross sections via detailed balance. The right hand side shows the cross sections for the production of a $\Delta$ by absorption of a pion. The inverse reaction, the production of a pion by decay of a $\Delta$ is governed by its decay width $\Gamma_\Delta$ (around 120 MeV in the pole). In the following we use these parameterizations fitted to experimental data of free nucleon-nucleon and pion-nucleon reactions and scale them by a global factor. This has been done either by scaling all cross sections or only by scaling a dedicated cross section. It should be noted that the scaling of the cross section for $NN \rightarrow N\Delta$ automatically scales the inverse reaction with the same factor in order to assure detailed balance.

Figure 1: Energy dependence of different cross sections used in IQMD.

Figure 2: Effect of the scaling of the cross sections on the isotropy ratio of the momentum space distribution in central Au+Au collisions.
2 Isotropy and pion production

The first question is whether a dynamical equilibration in nuclear phase space has been reached or not. One criterion is an isotropy of the momentum distribution in the center of mass. We present in Fig. 2 the ratio of transverse energy divided by twice the longitudinal energy, \( E_t/(2E_l) \), as a function a multiplicative scaling factor to the nucleon cross sections. We have chosen central collisions of a heavy system (Au+Au, \( b=0 \) fm at 1.5 AGeV) which has a high degree of stopping. Smaller systems do reach only lower values of this ratio. With the standard cross section we do not reach unity (green thick line), and a much higher cross section is needed to approach this value. We find a significant influence of the scaling of the inelastic NN cross section (dashed line) which is related to the loss of kinetic energy when producing a \( \Delta \). An important part is due to the elastic cross section, while the pion-induced cross section (dashed line) does not show any sensitivity. No saturation is seen when further increasing the cross sections which would also be a possible signature of an equilibrium. We can thus conclude that a global dynamical equilibrium is not reached.

![Figure 3: Left: The total pion yield in central Au+Au collisions at 1.5 AGeV as a function of the scaling parameter to the cross sections. Right: The total pion yield as function of the system size A in central A+A collisions at 1.5 AGeV. Different scaling parameters to the inelastic cross sections have been employed.](image)

Next we study the influence of the cross sections on the pion yield, depicted on the left side of Fig. 3, and no saturation can be seen: The increase of the inelastic NN cross sections (\( NN \rightarrow N\Delta \) and its inverse reaction \( N\Delta \rightarrow NN \), dashed line) does not enhance the pion yield significantly. Here we seem to reach a balancing of both channels. On the other hand the increase of the \( N\pi \rightarrow \Delta \) cross section (dotted line, its reverse reaction is governed by the \( \Delta \) decay width \( \Gamma_\Delta \)) decreases the pion yield. This reaction reinserts the \( \Delta \) and enhances the probability of \( N\Delta \rightarrow NN \), while the decay \( \Delta \rightarrow N\pi \) diminishes it. As a results, we change the balance point of the \( NN \leftrightarrow N\Delta \) system. When applying the
scaling factor to all cross sections (full line), we obtain a maximum of the pion yield for a scaling factor of 1, i.e. for the present standard parametrization.

Let us now study the influence of this scaling parameter to the system-size dependence as shown on the right side of Fig. 3. The pion yield has been divided by the participant number, which is equivalent to the system size for $b = 0$. Only the standard parametrization (factor 1, full line) assures a scaling of the pion number with the system size as confirmed by experiment (see e.g. [1]), while other scaling factors results in increasing or decreasing curves. This special situation comforts the application of thermal models at these low energies, since the pion yield can now be described as an extensive variable.

Figure 4: Comparison of the excitation function of the pion yield in central Au+Au collisions with FOPI data (top left) and of the $K^+$ (top right) and $K^-$ (bottom) yield in with experimental inclusive data of the KaoS collaboration.
3 Production of $K^+$ and $K^-$

Before continuing we would like to state that IQMD calculations are able to explain the experimental pion and kaon excitation functions within about 10-20% precision, as it is demonstrated in Fig. 4. The experimental data stems from the FOPI Collaboration for the pion [1] and for the $K^+$ and $K^-$ from KaoS Collaboration [2].

The kaon production at this energy range is dominated by two-step processes like $N_1N_2 \rightarrow N\Delta \quad N_3\Delta \rightarrow NYK$ [11, 8]. This chain of reactions is sensitive to the yield of $\Delta$s and thus to $\sigma(NN \leftrightarrow N\Delta)$. Since the production occurs in two steps, the production probability depends also on the density reached in the system. Therefore, the kaon yield is related to the degree of stopping indicated in Fig. 2. Furthermore, the kaon yield depends on the life time of a $\Delta$, i.e. the duration when it is available for a collision.

![Figure 5: Influence of a scaling factor to the $\Delta$-decay width $\Gamma_\Delta$ on the final pion and kaon yield (left) and on the contributions to the kaon yield (right)](image)

The influence of the life time of the $\Delta$ on the kaon yield is demonstrated in Fig. 5 left, applying an multiplicative factor the decay width $\Gamma_\Delta$ which is about 120 MeV at the pole. A factor less than 1 decreases the width and enhances the life time. A larger factor reduces the life time and thus penalizes the reaction $N\Delta \rightarrow NN$. The influence of the life time is partially balanced by reabsorption of the produced pion via the $N\pi \rightarrow \Delta$ reaction as can be seen on the left side of Fig. 5 where we show the final yield of pions (dotted line, divide by a factor 400 to compare with the kaons). Only for drastic changes of the decay width the yield changes significantly. The yield of the kaons (full line) changes quite moderately. The reason can be seen on the right side of Fig. 5. The dominant channel $N_1N_2 \rightarrow N\Delta \quad N_3\Delta \rightarrow NYK$ (dashed line) is sensitive to the $\Delta$ lifetime and drops for larger values of $\Gamma_\Delta$. However, this effect is to a large extend balanced by the appearance of another production chain $N_1N_2 \rightarrow N\Delta, \quad \Delta \rightarrow N\pi, \quad N_3\pi \rightarrow YK$ (dotted line). Only a small sensitivity of the total yield (full line) remains.
Figure 6: Excitation function of the $K/\pi$ ratio in Au+Au collisions for different scaling factors applied to $NN \leftrightarrow N\Delta$ and $\pi N \rightarrow \Delta$ at the same time. The fact that the kaon production depends strongly on a scaling of the cross sections while the pions are only effected moderately explains the strong influence of a scaling to the ratio of $K^+/\pi$ shown in Fig. 6. The rising trend of the curves is similar, only the absolute values vary with the scaling factor. We would like to stress that the cross sections for the dominant chain including a $N\Delta \rightarrow NYK$ are experimentally unknown. Therefore, the absolute yield of the kaons contains a large uncertainty. Therefore, any conclusion from the absolute values of the $K^+/\pi$ ratios in this energy range is quite difficult to establish.

Figure 7: Excitation function of the $K^-/K^+$ ratio in Au+Au collisions for different scaling factors to the cross sections and comparison of the $K^-/K^+$ ratio with the $\pi/A$ ratio.

The production of antikaons is dominated by charge exchange of the hyperons: $\pi Y \leftrightarrow N\bar{K}$. Since the yield of the hyperons is related to the yield of
kaons via associate production $BB \rightarrow NY K$ (which has a much lower threshold than the direct production $BB \rightarrow NNK$), there exist a direct link between the yield of kaons and that of antikaons via the hyperons. Here we find first indications for the validity of a law of mass action, as it has been reported in [5, 6]. This idea was supported by IQMD calculations [7] which showed, that the yield of antikaons is rather insensitive on scaling factors to the cross section of the reaction $\pi Y \leftrightarrow N \bar{K}$ if we apply it to both directions. This observation is interpreted as a dynamical balance between pions and hyperons on one side and nucleons and antikaons on the other side.

A scaling of other cross sections (like $N\Delta$ or $N\pi$) does not effect that link between kaons and antikaons. This is demonstrated on the left side of Fig. 7 where a scaling of the cross sections only causes a small effect on the ratio of $K^-/K^+$. The other parameter governing this ratio is the pion yield, as shown on the right side of Fig. 7: experimental data of the ratio $K^-/K^+$ show the same excitation function as the experimental pion yield, scaled with a factor of the participant size [5].

![Excitation function of the $(K^-/K^+)/(\pi^-/\pi^+)$ double ratio in Au+Au collisions for different scaling factors to the cross sections and in comparison to THERMUS results](image)

The values of this ratio correspond to the values obtained by thermal models like THERMUS [12]. Figure 8 shows the comparison of the ratio $(K^-/K^+)/(\pi^-/\pi^+)$ between THERMUS (triangles, compiled from $K^-/\pi^-$ and $K^+/\pi^+$ taken from [13]) with IQMD calculations (lines). Since the ratio $\pi^-/\pi^+$ is mostly sensitive to the isospin composition of projectile and target, the excitation function of the ratio $(K^-/K^+)/(\pi^-/\pi^+)$ corresponds mainly to the ratio $K^-/K^+$. This ratio turn out to be independent from a scaling of the cross sections and the values are comparable to the values obtained with THERMUS. This observable demonstrates well the validity of the law of mass action. Due to this relation, a description with thermal models appears applicable.
4 Conclusion

We have used the IQMD model to study the influence of a scaling of cross sections on the yield of pions, kaons and antikaons. The pion yield is slightly effected by a scaling of the cross sections and yields a maximum for the standard parametrization obtained from the free experimental cross sections. The scaling of the pion yield with the participant number can only be assured when no scaling of the cross sections is done. The ratio of $K/\pi$ depends strongly on the scaling of the cross sections but its absolute value suffer from the uncertainty in unknown production channels of the kaon. The ratio $K^-/K^+$ reflects nicely the properties of the law of mass action. Consequently, thermal models are applicable for some but not all ratios is this energy range.

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