Recent developments in chiral and spin polarization effects in heavy-ion collisions

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Abstract We give a brief overview of recent theoretical and experimental results on the chiral magnetic effect and spin polarization effect in heavy-ion collisions. We present updated experimental results for the chiral magnetic effect and related phenomena. The time evolution of the magnetic fields in different models is discussed. The newly developed quantum kinetic theory for massive fermions is reviewed. We present theoretical and experimental results for the polarization of $\Lambda$ hyperons and the $\rho_{00}$ value of vector mesons.

Keywords Relativistic heavy-ion collisions · Chiral magnetic effect · Chiral kinetic theory · Spin polarization

1 Introduction

In relativistic heavy-ion collisions, two charged nuclei collide to produce a hot, dense state of matter known as a quark–gluon plasma (QGP). Very high magnetic fields and orbital angular momenta (OAMs) are generated in these collisions. The magnetic fields are on the order of $10^{17}$–$10^{18}$ Gs [1–4] and are the strongest magnetic fields observed in nature. The QGP is also found to be the most vortical fluid [5], where a huge OAM is transferred to the fluid in the form of vorticity fields. These novel phenomena open a new window for the study of the QGP in heavy-ion collisions. These novel phenomena are quantum in nature and are usually negligible in classical fluids.

The chiral magnetic effect (CME) and chiral separation effect (CSE) [6–8] are two quantum effects on the magnetic field in a chiral fermion system. In the CME, a charge current is induced along the magnetic field:

$$j = \frac{e^2}{2\pi^2} \mu_5 B,$$

where $\mu_5$ is the chiral chemical potential. The magnetic field can also generate a chiral current

$$j_5 = \frac{e^2}{2\pi^2} \mu B,$$

where $\mu$ is the charge chemical potential. These two quantum effects are related to the chiral anomaly, which is absent from classical theories. The chiral chemical potential represents a chirality imbalance. The nonzero $\mu_5$ arises from the

$\mu_5$ field.
from topological fluctuations in quantum chromodynamics (QCD), which are related to the local violation of parity and charge parity. Therefore, the observation of the CME in heavy-ion collisions implies local parity and charge parity violation. The collective modes associated with the CME and CSE are called the chiral magnetic wave (CMW). In addition, the CME has been observed in condensed matter [9] and can be applied in quantum computing [10]. These phenomena are discussed in more detail in recent reviews [11–20] and references therein.

In non-central heavy-ion collisions, part of the very large OAM of colliding nuclei is converted to a vorticity field in the fluid. The vorticity can be regarded as the local OAM of the fluid and can polarize particles with spins through spin–orbit coupling. The global spin polarization of particles is along the direction of the global OAM of colliding nuclei (or the direction perpendicular to the reaction plane). The global polarization of Λ and ¯Λ hyperons has been measured in the STAR experiment, and the average angular velocity or vorticity was estimated as $\omega \sim 10^{22} s^{-1}$ [5]. The data on the global polarization of Λ and ¯Λ hyperons can be well described by various phenomenological models.

The STAR Collaboration has also measured the local spin polarization effect in Au+Au collisions at 200 GeV [21]. The data show a decreasing trend in the global spin polarization from the in-plane to the out-of-plane direction. In addition, the STAR Collaboration measured the azimuthal angle dependence of the spin polarization along the beam direction. These experimental results are still difficult to understand in terms of phenomenological models.

This paper is structured as follows. In Sect. 2.1, we review recent progress in theoretical studies of the CME and related effects in heavy-ion collisions. In Sect. 2.2, we present experimental results on the CME and CMW. In Sect. 3, we give a brief overview of recent developments in the quantum kinetic theory for massive fermions. In Sect. 4.1, we discuss the decomposition of the OAM and spin from the total angular momentum as well as the spin hydrodynamics. In Sect. 4.2, we discuss experimental results on the global and local polarization effects and spin alignments of vector mesons. We summarize the paper in Sect. 5.

2 Chiral magnetic and related effects in heavy-ion collisions

2.1 Theoretical progress

To study the magnetic-field-related effects in heavy-ion collisions, we need to know the magnetic field as a function of time. The electromagnetic field can be estimated using the Lienard–Wiechert potential [1–4]. Although the peak value of the magnetic field can be as large as a few times $m_e^2$, where $m_e$ is the mass of an electron, it decays very rapidly in vacuum [7]. Such a magnetic field in vacuum could not provide sufficient time to generate the CME and other chiral transport effects; thus, intermediate effects must be present [4, 22–24].

Magnetohydrodynamics (MHD) is widely used in astrophysics to investigate the coupling between a charged medium and a magnetic field. Ordinary MHD consists of the hydrodynamic conservation equations coupled with Maxwell’s equations. Very recently, several MHD studies [25–29] showed that the magnetic fields decay as $1/\tau$ in the infinite electrical conductivity limit, where $\tau$ is the proper time. The analytic solution for the anomalous MHD with the CME and chiral anomaly in a Bjorken flow has been derived [30, 31]. The magnetic fields decay approximately as $\sim 1/\tau$ or $\sim e^{-\sigma \tau}/\tau$, where $\sigma$ is the electrical conductivity. For numerical simulations of the ideal MHD, see Ref. [32, 33]. Although the electrical conductivity of the QGP is not infinite, it is important to study the coupling between the electromagnetic field and QGP. The ratio of the magnetic density to the initial fluid density can reportedly be greater than 1 in event-by-event simulations of $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions [3], i.e., the magnetic field may also affect other physical quantities [25, 29, 34]. Further study of the relativistic MHD with a temperature-dependent electrical conductivity is needed.

The ideal MHD framework has been extended to the second order using the self-consistent Grad moment expansion [35, 36]. In this framework, the electromagnetic fields are coupled with normal dissipative terms, such as the shear viscous tensor and bulk viscous pressure. One can also use a similar approach based on the kinetic theory of massless fermions [37].

By comparing phenomenological studies and experimental data, one can constrain the lifetime of the magnetic field, $t_B \simeq 0.5 - 1$ fm/c [38] or $t_B = A/\sqrt{s_{NN}}$, where $A = 115 \pm 16$ GeV fm/c [39].

It has been proposed that the magnetic field can be produced by a charged rotating fluid [39]. By using Maxwell’s equations in a charged fluid, $\tilde{\epsilon}_\mu F^{\mu\nu} = j^\nu$, and assuming the particle number density $n$ is homogeneous or changes very slowly, the relationship between the vorticity $\omega$ and the magnetic field $B$ is $\omega = e B / m_e c$, where $e$ is the electric charge and $c$ is the speed of light.
and the magnetic field in the local rest frame, \( \omega = (\nabla \times B) / en \), can be obtained. By introducing the average vorticity \( \bar{\omega} \), the average magnetic field per transverse area can be expressed as \( \bar{B} \approx \frac{e}{4\pi} An \bar{\omega} \), where \( A \) is the transverse area of the vortex. In heavy-ion collisions, the evolution of both the vorticity and the charge density at freeze-out can be extracted from AMPT simulations. (The AMPT model is a very good tool for simulating the CME [40–44]; see Sect. 2.2.) For a transverse area of the vorticity of \( 16\pi \text{fm}^2 \), and assuming a centrality of 20–50% for Au+Au collisions at 10–200 GeV at the RHIC, the magnetic field and its evolution can be estimated. Then, the splitting in the polarization of \( \Lambda \) and \( \bar{\Lambda} \) hyperons can be estimated from the averaged magnetic field, \( P_{\Lambda} - P_{\bar{\Lambda}} \approx 2 |\mu_\Lambda| [\bar{B} / T_0] \), where \( |\mu_\Lambda| = 0.613e / (2M_N) \), \( M_N = 938 \) MeV, and \( T_0 = 155 \) MeV. The numerical results are close to the STAR data [5].

Another theoretical model is the anomalous-viscous fluid dynamics (AVFD) [45–47], which is the hydrodynamical realization of the CME in relativistic heavy-ion collisions. The latest event-by-event version of the AVFD includes the local charge conservation (LCC) effect [48] and the introduction of partilization, which may be the best way to quantify the LCC [49]. It also gives predictions for isobaric collisions [50]. For the event plane, AVFD predicts \( \phi_{\text{isobar}} \equiv \frac{\partial \text{OS-SS}}{\partial \text{Ru-Zr}} \parallel \text{EP} = \frac{\partial \text{OS-SS}}{\partial \text{Ru-Zr}} \parallel \text{EP} \sim -0.41 \pm 0.27 \), which yields \( \langle \cos(2\Psi_B - 2\Psi_{\text{EP}}) \rangle \sim 0.46 \); for the reaction plane, the ratio is \( \phi_{\text{RP}} \equiv \frac{\partial \text{OS-SS}}{\partial \text{Ru-Zr}} \parallel \text{RP} \sim -0.90 \pm 0.45 \), which yields \( \langle \cos(2\Psi_B - 2\Psi_{\text{EP}}) \rangle \sim -0.95 \). According to the calculations, these ratios are independent of the initial axial charge.

As a natural extension to the CME, it is interesting to consider how large the mass correction is. This question can be addressed using a perturbation method in quantum kinetic theory, as described in Sect. 3. However, as noted in Ref. [51], the mass correction to the CME is related to another well-known phenomenon, Schwinger pair production (which is determined by the impact parameter and beam axis), and \( V_n \) and \( \alpha_n \) are the coefficients of the \( P \)-odd and \( P \)-odd Fourier terms, respectively. An azimuthal three-particle correlator [61], \( \gamma_{112} \), was proposed to explore the first coefficient, \( a_1 \), of the \( P \)-odd Fourier terms characterizing the charge separation due to the CME; it is given by

\[
\gamma_{112} \equiv \langle \cos(\phi_2 + \phi_B - 2\Psi_{\text{RP}}) \rangle \sim \langle \cos(\phi_2 + \phi_B - 2\Psi_2) \rangle, \tag{7}
\]

where \( \alpha \) and \( \beta \) denote particles with the same or opposite electric charge, respectively; angle brackets denote the averages over particles and events, and \( \Psi_{\text{RP}} \) can be approximated by the azimuthal angle of the second-order event plane, \( \Psi_2 \). Similarly, \( \gamma_{123} \), a charge-dependent
correlator with respect to the azimuthal angle of the third-order event plane, $\Psi_3$, is defined as

$$\gamma_{123} \equiv \langle \cos(\phi_x + 2\phi_y - 3\Psi_3) \rangle$$

(8)

and can reflect the charge-dependent background effects unrelated to the CME, because $\Psi_3$ is not correlated with the magnetic field direction. Other types of azimuthal correlators $\gamma_{ijk}$ for charged particles can also be defined similarly for different purposes. Except in ambiguous cases, we use the shorthand notation $\gamma$ to represent $\gamma_{112}$ hereafter.

From the CME expectation, the difference between the opposite-sign and same-sign correlation for charged particles $\Delta\gamma = \gamma_{\text{opp}} - \gamma_{\text{same}}$ is expected to be proportional to $B^2$ and $\cos[2(\Psi_3 - \Psi_2)]$ [1, 42, 64]:

$$\Delta\gamma \propto \langle B^2 \cos[2(\Psi_3 - \Psi_2)] \rangle.$$  

(9)

Because the magnitude of the magnetic field is proportional to the collision energy [2, 65], the CME should produce large differences between the $\gamma$ correlators at very different energies, such as the RHIC and LHC energies. However, Fig. 1 shows a very weak energy dependence of the $\gamma$ correlator in a wide energy range from the RHIC energy to the LHC energy.

As Eq. (9) shows, the $\Delta\gamma$ resulting from the CME is proportional to both the squared magnetic field and the correlation in the magnetic field direction with respect to the event plane. The left panel of Fig. 2 shows the simulation results of $\Delta\gamma$ due to the CME as a function of the centrality for Pb+Pb collisions at 5.02 TeV and Xe+Xe collisions at 5.44 TeV. They are very different except in the most central collisions. However, the right panel of Fig. 2 shows the preliminary ALICE data for Xe+Xe collisions, which show a centrality dependence that is very similar to that for Pb+Pb collisions in Fig. 1. The observed weak independence of $\Delta\gamma$ from the collision energy indicates that the dominant component of $\Delta\gamma$ is most likely due to the background.

One may argue, however, that the CME signal also depends on the lifetime of the magnetic field. Because of the shorter magnetic field lifetime at higher collision energies, the signal is very weak at very high energies. Fortunately, experimentalists are searching for a possible signal by comparing Au+Au collisions and U+U collisions at the RHIC. The top-left panel of Fig. 3 shows the expected CME contribution to $\Delta\gamma$ as a function of $N_{\text{part}}$ for Au+Au collisions at 200 GeV and U+U collisions at 193 GeV [66], which are multiplied by a factor $N_{\text{part}}/c_\text{B}$ to scale the effect from the elliptic flow and transverse momentum conservation. The CME contributions for Au+Au and U+U collisions are different except in very peripheral collisions. The difference is sizable for $N_{\text{part}}$ larger than 150. In addition, the background contribution to $\Delta\gamma$ is simulated using a hydrodynamics model with and without maximal LCC, as shown in the bottom-left panel of Fig. 3. For each case, the expected contribution is almost the same for Au+Au collisions at 200 GeV and U+U collisions at 193 GeV, although local charge conservation can significantly increase the magnitude of $\Delta\gamma$. To check these results, the STAR Collaboration has measured three correlators, $\Delta\gamma_{112}$, $\Delta\gamma_{123}$, and $\Delta\gamma_{132}$, in Au+Au collisions at 200 GeV and U+U collisions at 193 GeV [66]. Their $N_{\text{part}}$ dependence is presented in the right panel of Fig. 3; they are normalized by a factor of $N_{\text{part}}/c_\text{B}$ for the above reason. It is observed that the mixed harmonic correlations do not follow the background-only expectations. Differences between these correlators in Au+Au collisions and U+U collisions appear only for very central collisions with $N_{\text{part}}$ larger than 300. The illustrations on the far right side of Fig. 3 show that $\Delta\gamma_{132}$ is not correlated with the magnetic field direction, in contrast to $\Delta\gamma_{112}$ and $\Delta\gamma_{132}$; it is hard to see whether this difference is due to the signal or the

![Fig. 1](Color online) Measured centrality dependence of the $\gamma_{112}$ correlator for Au+Au collisions at 200 GeV and Pb+Pb collisions at 2.76 TeV [62] (left panel) and for Pb+Pb collisions at 2.76 TeV and 5.02 TeV (right panel) [63]
A detailed study of these correlators is needed to clarify their nature.

To extract the contribution of the CME to the observable $D_{c}$, the $D_{c}$ measured in the STAR experiment was decomposed into the $v_{2}$ background and the CME signal:

$$D_{c}(w_{TPC}) = D_{c}^{CME}(w_{TPC}) + D_{c}^{Bkg}(w_{TPC});$$
$$D_{c}(w_{ZDC}) = D_{c}^{CME}(w_{ZDC}) + D_{c}^{Bkg}(w_{ZDC});$$

where $w_{TPC}$ and $w_{ZDC}$ are the event planes measured for mid-rapidity particles in the time projection chamber (TPC) and for spectator neutrons in the zero degree calorimeter (ZDC), respectively. Assuming that the CME is proportional to the magnetic field squared and the background is proportional to $v_{2}$ [67], one obtains

$$\Delta \gamma^{CME}(\psi_{TPC}) = a \Delta \gamma^{CME}(\psi_{ZDC}),$$
$$\Delta \gamma^{Bkg}(\psi_{ZDC}) = a \Delta \gamma^{Bkg}(\psi_{TPC}),$$

where $a = \langle \cos(2(\psi_{ZDC} - \psi_{TPC})) \rangle$ and can be obtained from the $v_{2}$ measurement:

$$a = \frac{v_{2}(\psi_{ZDC})}{v_{2}(\psi_{TPC})}.$$

The CME signal relative to the inclusive $\Delta \gamma(\psi_{TPC})$ can be determined as

Fig. 2 (Color online) Results of simulation of $\Delta \gamma$ as a function of centrality for the CME in Pb+Pb collisions at 5.02 TeV and Xe+Xe collisions at 5.44 TeV (left panel); ALICE measurements of the $\gamma_{112}$ correlator for Xe+Xe collisions at 5.44 TeV (right panel) [63]

Fig. 3 (Color online) $N_{\text{part}}$ dependence of the CME contribution (top left) and flow background (bottom left) projected onto the correlator $\gamma_{112}$ for Au+Au collisions at 200 GeV and U+U collisions at 193 GeV. STAR measurements of $N_{\text{part}}$ dependence of the correlators $\Delta \gamma_{112}, \Delta \gamma_{123},$ and $\Delta \gamma_{132}$ multiplied by a factor of $N_{\text{part}}/v_{2}$ for Au+Au collisions at 200 GeV and U+U collisions at 193 GeV (right panel) [66]
\[ f_{\text{CME}} = \frac{\Delta \gamma_{\text{CME}}(\psi_{\text{TPC}})}{\Delta \gamma(\psi_{\text{TPC}})} = \frac{Aa - a^2}{1 - a^2}, \]

where \( A \) is defined as

\[ A = \frac{\Delta \gamma(\psi_{\text{ZDC}})}{\Delta \gamma(\psi_{\text{TPC}})}. \]

It can be rewritten as

\[ A = f_{\text{CME}} \frac{(1/\alpha) \Delta \gamma_{\text{CME}}(\psi_{\text{TPC}}) + a \Delta \gamma_{\text{BG}}(\psi_{\text{TPC}})}{\Delta \gamma_{\text{CME}}(\psi_{\text{TPC}})} = a + \left( \frac{1}{\alpha} - a \right) f_{\text{CME}}, \]

which one can solve for \( f_{\text{CME}} \) to obtain the last equality of (13). Note that \( A \) can also be measured experimentally. The left panel of Fig. 4 shows the STAR preliminary data on the centrality dependence of \( \Delta \gamma \) with respect to the ZDC and TPC event planes in \( \text{Au+Au} \) collisions at 200 GeV and \( \text{U+U} \) collisions at 193 GeV. We see that \( \Delta \gamma(\psi_{\text{ZDC}}) \) is consistently lower than \( \Delta \gamma(\psi_{\text{TPC}}) \), which indicates that \( A \) is less than unity. This result indicates that the flow background contributes less to \( \Delta \gamma(\psi_{\text{ZDC}}) \) than to \( \Delta \gamma(\psi_{\text{TPC}}) \). Applying the above method, the STAR Collaboration extracted the CME fraction \( f_{\text{CME}} \) from different datasets for \( \text{Au+Au} \) collisions at 200 GeV and \( \text{U+U} \) collisions at 193 GeV; their results are summarized in the right panel of Fig. 4. The combined result for the CME fraction for \( \text{Au+Au} \) at 200 GeV and \( \text{U+U} \) at 193 GeV is \( f_{\text{CME}} = 8 \pm 4 \pm 8\% \) [66]. Note that the CMS Collaboration has systematically measured different types of correlators \( \gamma_{ij} \) in \( \text{p+Pb} \) collisions at 5.02 and 8.16 TeV and \( \text{Pb+Pb} \) collisions at 5.02 TeV, which provide constraints on the upper limit of the CME fraction, i.e., 13% for \( \text{p+Pb} \) and 7% for \( \text{Pb+Pb} \) collisions at the 95% confidence level [68, 69].

Many other methods of searching for the CME have been proposed, such as event shape engineering [69], the use of the H factor [71], and the invariant mass method [72]. See [11, 73] for recent reviews. A new observable, the signed balance function, was proposed recently; it is defined as [74]

\[ B_{P_{\gamma}}(S_{\gamma}) = \frac{N_{++}(S_{\gamma}) - N_{+-}(S_{\gamma})}{N_{+}} \]

and

\[ B_{N_{\gamma}}(S_{\gamma}) = \frac{N_{-+}(S_{\gamma}) - N_{--}(S_{\gamma})}{N_{-}}. \]

Note that \( S_{\gamma} = +1 \) if particle \( \alpha \) is leading particle \( \beta \) (i.e., \( p_{\gamma}^x > p_{\gamma}^y \)), and \( S_{\gamma} = -1 \) otherwise. \( N_{++}(S_{\gamma}) \) denotes the number of positive–negative charge pairs with sign \( S_{\gamma} \) in an event. \( N_{++}(S_{\gamma}) \), \( N_{--}(S_{\gamma}) \), and \( N_{+-}(S_{\gamma}) \) have similar definitions. \( N_{-+}(S_{\gamma}) \) is the number of positive (negative) charge pairs in an event. Here, the x-axis is along the reaction plane, the z-axis is along the beam direction, and the y-axis is perpendicular to both the x- and z-axes. One can calculate an event using the event difference between \( B_{P_{\gamma}} \) and \( B_{N_{\gamma}} \):

\[ \delta B_{\gamma}(\pm 1) = B_{P_{\gamma}}(\pm 1) - B_{N_{\gamma}}(\pm 1) \]

and

\[ \Delta B_{\gamma} = \delta B_{\gamma}(+1) - \delta B_{\gamma}(-1). \]

Note that \( \Delta B_{\gamma} \) can be defined similarly. When the CME is absent, for a positive–negative charge pair, the probability of the positive particle leading the negative one is equal to the probability of the opposite case. Thus, \( B_{P_{\gamma}(x)} \) and \( B_{N_{\gamma}(x)} \) measure the same quantity, in principle, and the distribution of \( \Delta B_{\gamma}(x) \) is subject only to statistical fluctuation. When the CME is present, the two probabilities become unbalanced within an event; thus, more pairs of particles of one charge type lead the other type. Therefore, for each event, \( B_{P_{\gamma}} \) and \( B_{N_{\gamma}} \) tend to differ; consequently, \( \Delta B_{\gamma} \) has a wider

![Fig. 4](color online) STAR measurements of the centrality dependence of \( \Delta \gamma_{112} \) with respect to the ZDC and TPC event planes (left panel) and extracted CME fractions \( f_{\text{CME}} \) (right panel) from \( \text{Au+Au} \) collisions at 200 GeV and \( \text{U+U} \) collisions at 193 GeV [66]
distribution. By contrast, the distribution of $\Delta B_x$ is not broadened, as there is no charge separation in the $x$ direction. To cancel out the statistical fluctuation, one can define the ratio of the width of $\Delta B_x$ to that of $\Delta B_y$ as

$$r = \frac{\sigma_{\Delta B_y}}{\sigma_{\Delta B_x}}$$

(20)

to characterize the magnitude of the CME, because $r$ will be greater than unity or unity with or without the CME, respectively. That is, the strength of the CME will be positively correlated with the deviation of $r$ from unity. The ratio $r$ can be calculated either in the laboratory frame ($r_{\text{lab}}$) or in the rest frame ($r_{\text{rest}}$) of the pair. One can take the ratio of these two cases:

$$R_B \equiv \frac{r_{\text{rest}}}{r_{\text{lab}}}$$

(21)

where the subscript $B$ indicates a balance function. It has been found that $r_{\text{lab}}$, $r_{\text{rest}}$, and $R_B$ are sensitive not only to the strength of the CME, but also to the elliptic flow of primordial pions and $\rho$ resonances, and even to the global spin alignment of the resonances [74]. Figure 5 presents the recent STAR measurement of the centrality dependence of $r_{\text{lab}}$, $r_{\text{rest}}$, and $R_B$ from the signed balance functions in Au+Au collisions at 200 GeV [70]. In the 30–40% centrality bin, $r_{\text{rest}}$, $r_{\text{lab}}$, and $R_B$ are all larger than the AVFD result without the CME and larger than unity, which supports the presence of the CME. However, none of the AVFD results for CMEs of different strengths can describe all three of these observables simultaneously. Therefore, more experimental and theoretical studies are needed.

Another new correlator, $R_{\Psi_2}(\Delta S) = C_{\Psi_2}(\Delta S)/C_{\Psi_2}^+(\Delta S)$, has been designed to measure the magnitude of the CME-induced charge separation parallel to the $B$ field [76, 77], where $C_{\Psi_2}(\Delta S)$ and $C_{\Psi_2}^+(\Delta S)$ are used to quantify the charge separation $\Delta S$, i.e., the difference between the averaged $\langle \sin(\Delta \phi_2) \rangle$ values of negatively and positively charged particles emitted near the estimated $\Psi_2$ plane, $\Delta \phi_2 = \phi - \Psi_2$. Because the CME can result in charge separation along the $B$ field, one expects a concave $R_{\Psi_2}(\Delta S)$ distribution having a width that reflects the magnitude $a_1$ of the CME-induced charge separation. Because the reaction plane angle $\Psi_2$ is very weakly correlated with the $B$ field in small collision systems [78], the corresponding measurement in small systems can provide a baseline for the background contributions. The STAR Collaboration recently released the results for the $R$ correlator [75]. Figure 6 compares the $R_{\Psi_2}(\Delta S)$ correlators at $\langle N_{ch} \rangle \sim 20$ for p+Au, d+Au, and Au+Au collisions at 200 GeV. The convex or flat distributions shown for p(d)+Au collisions are consistent with the approximately random $B$ field relative to $\Psi_2$ expected in small collisions. By contrast, the Au+Au measurement relative to $\Psi_2$ shows a concave distribution, which is consistent with the CME-driven charge separation according to the AVFD [76] and AMPT models [43, 44].

In addition to the CME, a gapless CMW could be formed by the interplay between the CME and CSE. The CMW results in an electric quadrupole moment in the initial coordinate space of the QGP [82, 83]. It can ultimately be transformed into a charge-asymmetry-dependent elliptic flow of pions by collective expansion [84]. Therefore, the elliptic flow of positively and negatively charged pions is given by

$$v_2(\pi^\pm) = v_2^{\text{base}} + \frac{r(\pi^\pm)}{2} A_{\text{ch}},$$

(22)

where

$$A_{\text{ch}} = \frac{N^+ - N^-}{N^+ + N^-}$$

(23)

is the charge asymmetry. The elliptic flow difference between positive and negative pions, $[\Delta v_2 = v_2(\pi^-) - v_2(\pi^+)]$, can then be fitted using the relation $\Delta v_2 = rA_{\text{ch}} + \Delta v_2(0)$. The left panel of Fig. 7 presents the results for Au+Au collisions at 200 GeV (30–40%) measured in the STAR experiment [79], where the slope parameter $r$ is expected to reflect the strength of the CMW. In addition, because the third order of the event plane is not correlated with the magnetic field direction, measuring the slope parameter $r$ from the triangular flow $v_3$ can provide a reference from the background in comparison with that from elliptic flow $v_2$. However, care should be taken, because flow measurements by the Q-cumulant method using all the charged particles as a reference can introduce a trivial linear term to $\Delta v_2(A_{\text{ch}})$ owing to non-flow correlations. When all the charged hadrons are used as reference particles, as is typically done in data analysis, the two-particle cumulant can be rewritten as [85]

$$d_n\{2; \pi^\pm h\} = \frac{d_n\{2; \pi^+ h^+\} + d_n\{2; \pi^\pm h^-\}}{2} + \frac{d_n\{2; \pi^\pm h^+\} - d_n\{2; \pi^\pm h^-\}}{2} A_{\text{ch}}.$$

(24)

The second term on the right-hand side of (24) is proportional to $A_{\text{ch}}$ and is opposite in sign for $\pi^+$ and $\pi^-$. These characteristics result directly in a trivial contribution to the CMW-sensitive slope parameter. Because non-flow correlations are always present in experimental data, the flow coefficients $d_n\{2; \text{pairs}\}$ from like-sign pairs and opposite-sign pairs differ. Therefore, the second term on the right-hand side of (24) is always finite as a trivial term, and it should be removed to detect the possible CMW signal. To eliminate the trivial linear $A_{\text{ch}}$ term in practice, one can use
hadrons of a single charge sign instead of all the charged hadrons as reference particles. One may use positive and negative particles separately as reference particles to obtain

\[
v_p n f_2^2; h^+ g^+ \text{ and } v_p n f_2^2; h^+ / C_{0g}^2\text{ and then take an average:}
\]

\[
v_p n / C_{17}^2
\]

Previous STAR results showed significant negative slopes for \( v_3 \) [86], which were thought to support the presence of the CMW. By applying the new method of removing the trivial term, the slope \( r \) can be corrected to some extent. As shown in the right-hand plot in Fig. 7, the normalized \( v_3 \) slopes are consistent with positive values (1.5\( \sigma \) above zero for 20–60% centrality), which are similar to the normalized \( v_2 \) slopes in terms of the relative magnitudes. In addition, it has been found that the non-flow correlations give rise to additional background in the slope of \( \Delta v_n(A_{ch}) \) owing to competition among different pion sources and the large multiplicity dilution of \( \pi^+ (\pi^-) \) at positive (negative) \( A_{ch} \) [80, 81]. Therefore, more detailed studies are needed to search for the CMW in the future.

### 3 Quantum kinetic theory

In recent years, chiral kinetic theory (CKT) has been developed significantly [87–102] to describe various chiral effects in heavy-ion collisions. Numerical simulations based on the CKT have been developed [103–110]. However, with the discovery of global polarization at relatively low energies [5, 111], it is necessary to go beyond the chiral limit and develop a more general and practical quantum kinetic theory to describe spin effects for massive fermions. In this brief review, we consider only some of the works that were reported at the 2019 Quark Matter conference.

Most of these works are based on the Wigner functions but use slightly different realizations. The methods used in Refs. [112–114] are based on early works on the covariant Wigner functions [115–117]. The covariant Wigner function \( W(x, p) \) for the Dirac fermion is defined as a two-point function:
The kinetic equations for \( W_s(p, x) \) between \( \pi^- \) and \( \pi^- \) as a function of charge asymmetry \( A_{ch} \) for Au+Au collisions at 200 GeV (30–40%) (left plot) [79]; centrality dependence of the slope parameter \( r \) of normalized \( \Delta v_n(A_{ch}) \) with and without trivial term correction in Au+Au collisions at 200 GeV [80, 81] together with the subsidiary condition \( p \cdot \mathbf{A} \delta(p^2 - m^2) = 0 \), where \( \nabla\mu = \partial\mu^\nu - F^{\mu\nu} \partial\nu \), and \( \mathbf{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} / 2 \). The integrated kinetic equation in three-vector form can be obtained by integrating the zero component of the momentum:

\[
(\nabla_x + v \cdot \nabla) \mathbf{A} = \mathbf{B} \times \mathbf{A} - \mathbf{E}(v \cdot \mathbf{A})
\]

(32)

\[
(\nabla_x + v \cdot \nabla) \mathbf{A} = \mathbf{B} \times \mathbf{A} - \mathbf{E}(v \cdot \mathbf{A})
\]

(33)

where \( \mathbf{v} = p / E_p \), \( E_p = \sqrt{p^2 + m^2} \), \( \nabla_t = \partial_t + \mathbf{E} \cdot \nabla_p \), \( \nabla_x = \nabla_x + \mathbf{B} \times \nabla_p \), and the left arrow over \( \nabla_x \) denotes that it acts only on the electromagnetic fields on its left. When this method is used, various spin effects such as the chiral anomaly, CSE, quantum magnetization effect, and global polarization effect emerge naturally. It can be shown that the Dirac sea or vacuum contribution originating from the anti-commutation relations between antiparticle field operators in the Wigner function without a normal-order operator plays a crucial role in generating the chiral anomaly in quantum kinetic theory for both massive and massless fermions [118]. The coefficient of the chiral anomaly derived in this way is universal and is independent of the phase space distribution function at zero momentum.

Weickgenannt et al. [113, 114] also derived the kinetic theory for massive spin-1/2 particles in the covariant Wigner function formalism. They chose the scalar and tensor components, \( \mathcal{F} \) and \( \mathcal{F}_{\mu\nu} \), as the basis from which the other components can be derived; these components are
related to the distribution function $V$ and tensor distribution or dipole moment $\Sigma_{\mu\nu}$ as independent variables of $O(\hbar)$:

$$\mathcal{F} = \delta(p^2 - m^2)mV - \frac{\hbar}{2}mF_{\mu\nu}\Sigma^\nu_\mu \delta'(p^2 - m^2),$$  \hspace{1cm} (34)

$$\mathcal{G}_{\mu\nu} = \delta(p^2 - m^2)m\Sigma_{\mu\nu} - hF_{\mu\nu}V\delta'(p^2 - m^2),$$  \hspace{1cm} (35)

where $\delta'(x) \equiv d\delta(x)/dx$, $V = V_0 + hV_1$, and $\Sigma_{\mu\nu} = \Sigma_{\mu\nu}(0) + h\Sigma_{\mu\nu}(1)$. In addition, $V_0$ and $\Sigma_{\mu\nu}(0)$ are scalar functions of the phase space and can be expressed as particle number distributions [113, 114]. The kinetic equations for these functions are given by

$$0 = \delta(p^2 - m^2) \left[ p \cdot \nabla V + \frac{\hbar}{4}(\bar{\alpha}_\mu F^\mu_\nu)\bar{\delta}_\Sigma^{\nu_\mu} \right]$$

$$- \frac{\hbar}{2} \delta'(p^2 - m^2)F^\mu_\nu p \cdot \nabla \bar{\Sigma}^{\nu_\mu},$$

$$0 = \delta(p^2 - m^2) \left[ p \cdot \nabla \Sigma_{\mu\nu} - F^\nu_\mu \bar{\Sigma}^{\nu_\mu}_\rho + \frac{\hbar}{2}(\bar{\alpha}_\mu F^\mu_\nu)\bar{\delta}_p V \right]$$

$$- \hbar \delta'(p^2 - m^2)F^\mu_\nu p \cdot \nabla \Sigma_{\mu\nu},$$  \hspace{1cm} (36)

where the superscripts $+$ and $-$ correspond to the massless case. The kinetic equations yield a special single-particle distribution in global equilibrium with rigid rotation. When this distribution is used, analogs of various spin effects appear, such as the CME and CSE, for massive fermions. A smooth connection between the kinetic theory of massive spin-1/2 particles and the CKT was found recently [119] by introducing a frame-dependent decomposition of the dipole-moment tensor $\Sigma_{\mu\nu}$.

All the components of the covariant Wigner function can be derived from $\gamma^\mu$ and $\mathcal{G}^\mu_\nu$ for massive fermions in the Schwinger–Keldysh formalism [120]. The CKT for the massless case can be recovered from the formula for

massive fermions. The kinetic equation has been generalized to include collision terms in the Schwinger–Keldysh formalism [121].

A covariant kinetic equation for massive fermions in curved space-time and an external electromagnetic field can also be derived. Liu et al. [122] use a general covariant Wigner function formalism not only under the $U(1)$ gauge and local Lorentz transformation but also under diffeomorphism, which is compatible with general relativity. The spin polarization in the presence of Riemann curvature and an electromagnetic field in both local equilibrium and global equilibrium has also been studied.

Integration of the covariant Wigner function over $p_0$ yields the equal-time Wigner function [123–125]. Compared to the covariant form, the equal-time form loses obvious Lorentz covariance, but it is more convenient for time evolution problems such as pair production in a strong electric field [126, 127]. The kinetic equation can also be derived from the equal-time Wigner functions. The equal-time Wigner function $W(x, p)$ can be obtained from the covariant one $W(x, p)$:

$$W(x, p) = \int dp_0 W(x, p)p^0.$$  \hspace{1cm} (39)

It can be decomposed into the following components:

$$W = \frac{1}{4}[f_0 + \gamma^0 f_1 - i\gamma^0 \gamma^2 f_2 + \gamma_0 f_3 + \gamma_0 \gamma^0 \gamma^2 \cdot g_0]$$

$$+ \gamma_0 \gamma^2 \cdot g_1 - i\gamma^0 \gamma^2 \cdot g_2 - \gamma^0 \gamma^2 \cdot g_3.$$  \hspace{1cm} (40)

The fermion number density $f_0$ and spin current $g_0$ are chosen to be independent components; their kinetic equations are derived as [128]

$$\left( \nabla \mp \frac{p}{E_p} \cdot \nabla \right) f_0^\pm = \frac{\hbar E}{2E_p^3} \cdot \nabla$$

$$\times g_0^\pm + \frac{\hbar}{2E_p^2} B \cdot (p \times g_0^\pm)$$

$$+ \frac{\hbar B \times p}{E_p^2} \cdot E \times g_0^\pm,$$

$$\left( \nabla \mp \frac{p}{E_p} \cdot \nabla \right) g_0^\pm = \frac{1}{E_p^2} \left[ p \times (E \times g_0^\pm) \mp E_p B \times g_0^\pm \right]$$

$$\mp \hbar \left( \frac{B}{2E_p^2} \pm \frac{E \times p}{2E_p^2} \right) \cdot \nabla f_0^\pm$$

$$\mp \hbar \left( \frac{p \cdot E}{E_p^2} \pm \frac{E \times p}{2E_p^2} \right) \cdot g_0^\pm.$$  \hspace{1cm} (41)

where the superscripts $+$ and $-$ indicate the particle and antiparticle, respectively. The small mass expansion can be performed. The mass correction changes only the structure of the chiral kinetic equation in terms of the effective
collision terms, which is only a first-order quantum correction to the CME.

All the works mentioned above are valid only up to the first order of $\hbar$ and without collision terms for particle scattering. In addition to these works based on Wigner functions, Li and Yee [129] derived the kinetic equations for the spin-density matrix in the Schwinger–Keldysh formalism. Specifically, they formulated collision terms for fermions from their interactions with the QGP medium. However, they consider only the spatial homogeneity limit; thus, the collisions are local and do not involve the vorticity. The time evolution equations are derived for the particle number distribution $f(p,t)$ and the spin polarization density $S(p,t)$ in the leading log order of $g^4 \log(1/g)$:

$$\frac{\partial f(p,t)}{\partial t} = C_2(F) \frac{m_p^2 g^2 \log(1/g)}{(4\pi)^{3/2}} \frac{1}{2pE_p} \Gamma_f,$$

$$\frac{\partial S(p,t)}{\partial t} = C_2(F) \frac{m_p^2 g^2 \log(1/g)}{(4\pi)^{3/2}} \frac{1}{2pE_p} \Gamma_S,$$

(43)

where $C_2(F) = (N^2_c - 1)/2N_c$, and $\Gamma_f$ and $\Gamma_S$ are diffusion-like differential operators in momentum space that contain up to the second-order derivatives of the momentum. The explicit forms of $\Gamma_f$ and $\Gamma_S$ are given in Eq. (4.70) of Ref. [129].

Note that the quantum kinetic equation for massive fermions is very different from the chiral kinetic equation for massless fermions. The kinetic equation for massive fermions includes four coupled independent functions; one of them gives the particle distribution function, and the other three give the spin polarization vector. By contrast, the kinetic equation for massless fermions contains only one distribution function. The reason is that the spin direction coincides with the momentum of the fermion and is not a dynamical quantity at the chiral limit; however, for the massive case, the spin does not coincide with the momentum and becomes dynamical. It was thought that the chiral kinetic equation could be obtained from the quantum kinetic equation for massive fermions by naively taking the massless limit; however, this limit was found to be more important than it appears. The most recent works on this problem can be found in Refs. [119, 130].

4 Spin polarization effects

4.1 Theoretical progress

In early works [131, 132], Liang and Wang proposed that particles can be polarized as a result of the global OAM in non-central heavy-ion collisions. The formation of the vorticity in heavy-ion collisions was subsequently studied [133]. Becattini and his collaborators [134–136] performed a systematic study of statistical models of relativistic spinning particles. In addition, recent review articles are cited in Refs. [137–139].

The initial global OAM is estimated to be as large as $J_0 \sim 10^5 h$ in 200 GeV Au+Au collisions with the impact parameter $b = 10$ fm. Consequently, the QGP is found to be the most vortical fluid ever observed in nature. The vorticity properties can be studied using the AMPT and HIJING [140–142], UrQMD [143], and hydrodynamic models [144–149]. The vorticity is found to be more strongly suppressed at higher collision energies. One can find further discussion in Refs. [150–152] and reference therein.

The spin polarization per particle for spin-1/2 fermions with momentum $p$ at freeze-out can be derived using the statistical model for relativistic spinning particles [136] and the Wigner functions [153]:

$$S^\mu(p) = - \frac{1}{8m} \frac{e^{\mu\nu\rho\sigma} u_{\nu} f_{\mu\rho} \int d\Sigma(p) (u \cdot p)^{-1} f_{FD}(1 - f_{FD}) \theta_{\mu\rho\sigma}(x)}{\int d\Sigma(p)f_{FD}}$$

(44)

where $\Sigma$ denotes the freeze-out hypersurface, $S^\mu$ is the Pauli–Lubanski pseudo-vector, $m$ is the fermion mass, $f_{FD} \equiv f_{FD}(x,p)$ is the Fermi–Dirac distribution. In addition,

$$\theta_{\mu\rho\sigma} = \frac{1}{2} (\partial_{\mu} \beta_{\rho} - \partial_{\rho} \beta_{\mu})$$

(45)

is the thermal vorticity, where $u^\mu$ is the fluid velocity, and $\beta_{\mu} = u_{\mu}/T$ is the temperature four-vector. Equation (44) has been widely used in hydrodynamics models and transport models to calculate the spin polarization of hyperons.

In quantum field theory, the decomposition of the total angular momentum into the orbital and spin components for massive particles with spin is not unique. Different decompositions are related by a pseudo-gauge transformation. Although the energy–momentum tensor $T^{\mu\nu}$ and spin tensor $S^{\mu\nu}$ depend on the decomposition, the total energy–momentum and total angular momentum are invariant under the pseudo-gauge transformation. A special pseudo-gauge transformation is the transformation of the canonical definitions of $T^{\mu\nu}_C$ and $S^{\mu\nu}_C$ into the Belinfante $T^{\mu\nu}_B$, which absorbs $S^{\mu\nu}_C$, i.e., $S^{\mu\nu}_B = 0$. Thus, the total angular momentum consists only of the orbital component. The Belinfante energy–momentum tensor is symmetric with respect to its two indices: $T^{\mu\nu}_B = T^{\nu\mu}_B$.

The thermal properties of a quantum field system can be described by the density operator $\hat{\rho}$. The density operator in local equilibrium can be determined from the maximal entropy principle [154–156] under given densities of conserved currents on the space-like hypersurface $\Sigma$. This is
done by maximizing \( S = -\text{Tr}(\hat{\rho} \log \hat{\rho}) \) under the conditions

\[
\begin{align*}
\mu \text{Tr} \left( \hat{\rho} T^{\mu \nu} \right) &= n_{\mu} T^{\mu \nu}, \\
\mu \text{Tr} \left( \hat{\rho} f^{\mu} \right) &= n_{\mu} f^{\mu},
\end{align*}
\]

where \( n_{\mu} \) is the normal vector of \( \Sigma \). The constraint on the total angular momentum density can generally be included as well:

\[
n_{\mu} \text{Tr} \left( \hat{\rho} \mathbf{J}^{\mu_{\lambda_{\nu}}} \right) = n_{\mu} \mathbf{J}^{\mu_{\lambda_{\nu}}}. 
\]

For the Belinfante energy–momentum tensor, however, the above condition is automatically satisfied by using (46). In this case, the density operator at local equilibrium has the form

\[
\hat{\rho}_{LE} = \frac{1}{Z} \exp \left[ -\int \Sigma(\tau) \left( \hat{T}^{\mu \nu} \beta_{\nu} - \zeta^{\mu} \right) \right],
\]

where \( \beta^{\mu} \) and \( \zeta \) are the Lagrange multiplier functions of space-time; the former is the temperature four-vector, and the latter is the ratio of the chemical potential to the temperature [156].

In general, \( \hat{\rho}_{LE} \) in (48) depends on the time \( \tau \) as \( \Sigma_{\mu}(\tau) \). One can rewrite the exponent of \( \hat{\rho}_{LE} \) as

\[
\begin{align*}
\int \Sigma(\tau) \left( \hat{T}^{\mu \nu} \beta_{\nu} - \zeta^{\mu} \right) \\
= \int \Sigma(\tau_{0}) \left( \hat{T}^{\mu \nu} \beta_{\nu} - \zeta^{\mu} \right) \\
- \int_{\theta} d\theta \left( \hat{T}^{\mu \nu} \partial_{\mu} \beta_{\nu} - \right) \hat{q} \partial_{\mu} \zeta,
\end{align*}
\]

where \( \Theta \) is the space-time volume bounded by \( \Sigma(\tau) \), \( \Sigma(\tau_{0}) \), and the time-like boundary connecting \( \Sigma(\tau) \) and \( \Sigma(\tau_{0}) \). Here, we used

\[
\partial_{\mu} \left( \hat{T}^{\mu \nu} \beta_{\nu} - \zeta^{\mu} \right) = \hat{T}^{\mu \nu} \partial_{\mu} \beta_{\nu} - \hat{q} \partial_{\mu} \zeta,
\]

where we used the conservation of energy–momentum and current. To make \( \hat{\rho}_{LE} \) independent of the choice of \( \Sigma_{\mu}(\tau) \), i.e., to obtain the global equilibrium density matrix \( \hat{\rho}_{GE} \), the following conditions must be met:

\[
\partial_{\mu} \beta_{\nu} + \partial_{\nu} \beta_{\mu} = 0, \\
\partial_{\mu} \zeta = 0,
\]

according to which \( \zeta \) is constant and \( \beta^{\mu} \) is a Killing vector:

\[
\beta^{\mu} = \omega_{\mu}^{\alpha \beta} x_{\alpha} + b^{\mu},
\]

where \( \omega_{\mu}^{\alpha \beta} \) is a constant antisymmetric tensor given by (45), and \( b^{\mu} \) is a constant vector. The local equilibrium density operator in (48) can be expressed in terms of the canonical energy–momentum and spin tensors:

\[
\hat{\rho}_{LE} = \frac{1}{Z} \exp \left[ -\int \Sigma(\tau) \left( \hat{T}^{\mu \nu} \beta_{\nu} - \zeta^{\mu} - \frac{1}{2} \Omega_{\mu}^{\alpha \beta} q_{\alpha \beta} + \frac{1}{2} \omega_{\mu}^{\alpha \beta} q_{\alpha \beta} \right) \right].
\]

If we substitute (51) and (52) in the above expression for \( \hat{\rho}_{LE} \), we obtain \( \hat{\rho}_{GE} \):

\[
\hat{\rho}_{GE} = \frac{1}{Z} \exp \left[ -b^{\mu} P_{\mu}^{\nu} + \frac{1}{2} \omega_{\mu}^{\alpha \beta} J^{\mu \nu} + \zeta Q \right],
\]

where \( P^{\mu} \), \( J^{\mu \nu} \), and \( Q \) are given by

\[
\begin{align*}
P^{\mu} &= \int \Sigma(\tau) \hat{T}^{\mu \nu}, \\
J^{\mu \nu} &= \int \Sigma(\tau) \mathbf{J}^{\mu \nu}, \\
Q &= \int \Sigma(\tau) \hat{q}.
\end{align*}
\]

If we work with the canonical energy–momentum and spin tensors, we have to impose the constraint in (47), which also constrains the spin tensor:

\[
n_{\mu} \text{Tr} \left( \hat{\rho} \mathbf{S}^{\mu \lambda_{\nu}} \right) = n_{\mu} \mathbf{S}^{\mu \lambda_{\nu}}.
\]

This introduces a spin chemical potential, the antisymmetric tensor \( \Omega_{\lambda \nu} \), as a Lagrange multiplier into the equation for \( \hat{\rho}_{LE} \):

\[
\hat{\rho}_{LE} = \frac{1}{Z} \exp \left[ -\int \Sigma(\tau) \left( \hat{T}^{\mu \nu} \beta_{\nu} - \frac{1}{2} \Omega_{\mu}^{\alpha \beta} q_{\alpha \beta} - \zeta^{\mu} \right) \right].
\]

Comparing (57) with (53), we see that \( \hat{\rho}_{LE} \) is the same for the Belinfante and canonical tensors under the following conditions: (a) the field \( \beta^{\mu} \) is the same in both cases; (b) \( \Omega_{\lambda \nu} = \omega_{\lambda \nu}^{bh} \); and (c) \( \partial_{\lambda} \beta_{\mu} + \partial_{\nu} \beta_{\mu} = 0 \). We can see that if \( \Omega_{\lambda \nu} = \omega_{\lambda \nu}^{bh} \) and \( \partial_{\lambda} \beta_{\mu} = \partial_{\nu} \beta_{\mu} = 0 \), then \( \hat{\rho}_{LE} \) for the canonical tensors becomes the same as \( \hat{\rho}_{GE} \) for the Belinfante tensor in (54). Therefore, one can conclude that the density operator in global equilibrium is pseudo-gauge-invariant, but in local equilibrium, it depends on the pseudo-gauge [157].

If we choose the Belinfante tensors or \( \hat{\rho}_{LE} \) in (48) or (53), the spin relaxation time is microscopically small, and
the value of the spin potential agrees with the thermal vorticity almost immediately. If we choose the canonical tensors or $\hat{\rho}_{\text{LE}}$ in (57) with the spin chemical potential, the spin density slowly reaches global equilibrium, just as a conserved charge density or energy density does, and finally, the spin chemical potential should converge to the thermal vorticity [157].

Similar problems involving the decomposition of the total angular momentum of quarks and gluons in the proton spin (see Ref. [158] for a review) have been widely debated in the QCD community for years. Two well-known decomposition methods are that of Jaffe and Manohar and that of Ji, which are close to the canonical and Belinfante tensor forms, respectively. A review article [159] discusses that of Ji, which are close to the canonical and Belinfante decomposition methods are that of Jaffe and Manohar and in the QCD community for years. Two well-known equations including the spin degrees of freedom. There have been a few attempts in spin hydrodynamics [37, 157, 161–165]. Another approach using Lagrangian techniques can also provide some information about the local equilibrium [166–169].

Spin degrees of freedom can be introduced into the phase space distribution for spin-1/2 fermions by generalizing the scalar function to a $2 \times 2$ Hermitian matrix [136]:

$$f_{rs}^{\pm}(x, p) = \frac{1}{2m} \epsilon_{rs}^\alpha(p) X^\alpha u_s(p),$$

$$f_{rs}^{-}(x, p) = -\frac{1}{2m} \epsilon_{sr}^\alpha(p) X^\alpha v_r(p),$$

where $r, s = \pm 1$; $u_r$ and $v_r$ are Dirac spinors normalized by $\bar{u}_r(p) u_r(p) = 2m \delta_{rs}$ and $\bar{v}_r(p) v_r(p) = -2m \delta_{rs}$, respectively; and $X^\pm$ are $4 \times 4$ matrices defined as

$$X^\pm = \exp\left(-\beta \mu p^\mu \pm \zeta \pm \frac{1}{4} \Omega_{\mu \nu} \sigma^{\mu \nu}\right).$$

The Wigner function in equilibrium for fermions and anti-fermions are given by

$$W_{\pm}(x, k) = \sum_{r,s} \int d^4p \theta(p_0) \delta(p^2 - m^2) \times \delta^{(4)}(k - p) u_r(p) \bar{u}_s(p) f_{rs}^{\pm}(x, p)$$

$$= \frac{1}{2m} \int d^4p \theta(p_0) \delta(p^2 - m^2) \times \delta^{(4)}(k - p)(p_{\mu} \gamma^\mu + m) X^\pm(p_{\mu} \gamma^\mu + m),$$

$$W_{=}(x, k) = -\sum_{r,s} \int d^4p \theta(-p_0) \delta(p^2 - m^2) \times \delta^{(4)}(k + p) v_r(p) \bar{v}_s(p) f_{rs}^{-}(x, p)$$

$$= \frac{1}{2m} \int d^4p \theta(-p_0) \delta(p^2 - m^2) \times \delta^{(4)}(k - p)(p_{\mu} \gamma^\mu - m) X^- (p_{\mu} \gamma^\mu - m).$$

The total Wigner function is the sum of them:

$$W_{\text{eq}}(x, k) = W^{+}_{\text{eq}}(x, k) + W^{-}_{\text{eq}}(x, k).$$

The components of $W_{\text{eq}}(x, k)$ are given in (27) and can be extracted by taking the traces of $\Gamma_i W_{\text{eq}}(x, k)$ with $\Gamma_i = 1, \gamma^5, \gamma^\mu, \gamma^5 \gamma^\mu, \sigma^{\mu \nu}.$

The energy–momentum and spin tensors proposed by de Groot, van Leeuwen, and van Weert (GLW) [170] can be extracted by taking the traces of $\Gamma_i W_{\text{eq}}(x, k)$ with $\Gamma_i = 1, \gamma^5, \gamma^\mu, \gamma^5 \gamma^\mu, \sigma^{\mu \nu}.$

In equilibrium, we use $W = W_{\text{eq}}$ in (63) and obtain $T^{\mu \nu}_{\text{GLW}} = T^{\mu \nu}_{\text{eq}}$ and $S^{\mu \nu}_{\text{GLW}} = S^{\mu \nu}_{\text{eq}}$. Because the GLW energy–momentum tensor is symmetric, the GLW spin tensor should be conserved separately. Thus, the conservation laws read

$$\partial_\mu T^{\mu \nu}_{\text{GLW}} = 0, \quad \partial_\mu S^{\mu \nu}_{\text{GLW}} = 0, \quad \partial_\mu X^{\mu \nu}_{\text{GLW}} = 0.$$ (64)

The canonical forms of the energy–momentum and spin tensors can be obtained from the Dirac Lagrangian [113, 117]:

$$\partial_\mu T^{\mu \nu} = 0, \quad \partial_\mu S^{\mu \nu} = 0, \quad \partial_\mu X^{\mu \nu} = 0.$$
\[ T^\text{inv}_C = \int d^4 k k^\nu \gamma^\nu \mu, \]
\[ S^\text{inv}_C = \frac{1}{4} \int d^4 k \text{Tr} \left\{ \{ \sigma^{\mu \nu}, \gamma^\lambda \} W \right\} = -\frac{1}{2} \epsilon^{\mu \nu \rho \sigma} \int d^4 k \epsilon_{\mu \nu \rho \sigma}. \]

(65)

One can verify that the canonical tensor forms are related to the GLW forms by the pseudo-gauge transformation \[ \text{(163)}. \] The explicit relation between the two tensor forms can be written as
\[ T^\text{inv}_C = T^\text{GLW}_{C} - \frac{\hbar}{2m} \int d^4 k k^\nu \partial_{\nu} S^\text{GLW}_C = T^\text{inv}_C - \partial_{\nu} S^\text{GLW}_C. \]
\[ S^\text{inv}_C = S^\text{GLW}_C + S^\text{inv}_C + S^\text{GLW}_C. \]

(66)

One can verify that the conservation laws for \( T^\text{inv}_C \) and \( S^\text{inv}_C \) follow those for \( T^\text{GLW}_C \) and \( S^\text{GLW}_C \) in (64).

However, it is still an open question whether the QGP reaches a local equilibrium of the spin degrees of freedom. A microscopic model of the spin polarization generated by spin–orbit coupling in particle collisions has been proposed \[ \text{(171)}. \] Without assuming a local equilibrium of spins, it uses an effective method of wave packets to handle particle scattering for specified impact parameters. The spin-vorticity coupling naturally emerges from the spin–orbit coupling encoded in the polarized scattering amplitudes of collisional integrals when a local equilibrium of the momentum is assumed. First, the collision rate is calculated:
\[ R_{AB-12} = n_A n_B |v_A - v_B| \sigma \]
\[ = \frac{d^3 p_A d^3 p_B}{(2\pi)^3} f_A(x_A, p_A) f_B(x_B, p_B) \int |v_A - v_B| \Delta \sigma, \]

(67)

where \( v_A = |p_A|/E_A \) and \( v_B = -|p_B|/E_B \) are the longitudinal velocities, with \( p_A = -p_B \) in the center-of-mass frame of colliding particles; \( f_A \) and \( f_B \) are the phase-space distributions of the incident particles \( A \) and \( B \), respectively; and \( \Delta \sigma \) denotes the infinitesimal element of the cross section. After incorporating the components of the wave packets, the matrix elements of 2-to-2 scattering, the spin projection, and the proper Lorentz transformations, one obtains the polarization production rate per unit volume:
\[ \frac{d^3 P_{AB-12}(X)}{dX^4} = \frac{1}{(2\pi)^4} \frac{\partial \langle \psi | \rho \rangle}{\partial X^4} \sum_{A \neq B} \int d^3 p_A d^3 p_B \]
\[ \times |v_{c, A} - v_{c, B}| \left\{ \Lambda^{-1} \right\}_{c} e_{c} d_{A B} P_{AB}^k h_{A} \]
\[ \times f_A(X, p_A) f_B(X, p_B) \left( \langle p_A | - p_B \rangle \right) \Theta_{c} (p_c A), \]

(69)

which is proportional to the thermal vorticity. Here, the tensor \( \mathbf{W}^\text{pp} \) contains 64 components, each of which involves a 16-dimensional integration. The numerical
calculation of $W^\omega$ is challenging owing to the very large number of scattering amplitudes and high-dimensional collision integrals. To tackle this problem, a new Monte Carlo integration algorithm, ZMCIntegral, which can handle 16-dimensional integration, has been developed for use on multiple GPUs [172, 173]. The most recent application of this algorithm is the solution of the Boltzmann equations of a partonic system [174]. The numerical result shows that $W^\omega$ has an antisymmetric form:

$$W^\omega = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & We_z & -We_x \\ 0 & -We_x & 0 & We_z \\ 0 & We_z & -We_x & 0 \end{pmatrix},$$

(70)

where $W$ is approximately constant. Note that $W$ depends on the cutoff of the impact parameter, as shown in Fig. 8. Finally, the polarization rate per unit volume for one quark flavor can be expressed compactly:

$$\frac{dN}{d^2\theta^\prime} = \frac{1}{2}(1 + z_H p_H \cos \theta^\prime),$$

(72)

where $z_H$ is the decay constant of the hyperon, $p_H$ is the polarization of the hyperon (a fraction), and $\theta^\prime$ is the angle between the momentum of the daughter baryon and the polarization direction in the hyperon rest frame. The experimental results for the global polarization of $\Lambda$ and $\bar{\Lambda}$ hyperons measured by the STAR Collaboration [5, 111] are shown in the left panel of Fig. 9. $P_H(\Lambda)$ and $P_H(\bar{\Lambda})$ decrease with the collision energy. From the data, the fluid vorticity can be estimated as $\omega \simeq k_B T (p_\Lambda + p_A) / \hbar$ [176]. It also appears that $P_H(\bar{\Lambda}) > P_H(\Lambda)$, although the difference is not significant; this result may suggest a possible contribution from the magnetic field or other effects [177, 178]. The data can be described by various phenomenological models using (44) [105, 142, 143, 179–181].

The right panel of Fig. 9 shows the recent STAR Collaboration measurement [21] of the longitudinal polarization as a function of the azimuthal angle $\phi$ relative to the second-order event plane $\Psi_2$. The longitudinal spin polarization data show positive $\sin(2\phi - 2\Psi_2)$ behavior, whereas the theoretical results of the relativistic hydrodynamics model [182] and transport models [143, 180, 183] show the opposite sign along the beam line direction. A simulation using CKT [110] and the results obtained using a simple phenomenological model [184] give the correct sign as the data. The azimuthal angle dependence of the spin polarization in the direction of the global OAM has been measured by the STAR Collaboration. Some phenomenological models do not well describe the STAR data for the azimuthal angle dependence of the global polarization.

Note that the result of Ref. [171] or Eq. (69) does not include the back reaction to contain the growing polarization with increasing vorticity in the absence of a cutoff. A systematic derivation of the spin polarization from the vorticity through non-local collisions with back reactions has been performed [175] by expanding the collision terms in the Planck constant for massive fermions [121].

4.2 Experimental results

The global polarization of $\Lambda$ hyperons can be measured through their weak decays. The angular distribution of the daughter baryons is [5, 111]

![Fig. 8](Color online) Numerical results for $W$ as a function of $b_0$, the cutoff for $|b_0|$
temperature vorticity, and thermal vorticity. They calculated the local polarization of hyperons corresponding to each type of vorticity. By using the (3 + 1)-dimensional hydrodynamic model with the AMPT initial conditions encoding the global OAM, they found a few remarkable differences between the results for different vorticities. First, although all four types of vorticity give the correct sign and magnitude of the polarization along the global OAM direction, only the temperature vorticity agrees with the STAR preliminary result in showing a decreasing trend of the azimuthal angle dependence. Second, only the temperature vorticity can simultaneously provide the correct sign and magnitude of the longitudinal polarization. This result suggests the possibility of spin–temperature vorticity coupling, analogous to magnetic moment–magnetic field coupling. It is also possible that the agreement may be a coincidental result of the main assumption that the spin vector depends on the temperature vorticity in the same way as it does on the thermal vorticity. Further investigation is needed to clarify why the results for the temperature vorticity are the most satisfactory.

The vector meson spin alignment is another recent research topic. The spin alignment of a vector meson is described by the 3\times 3 Hermitian spin-density matrix \( \rho_{00} \). The 00-component of the spin-density matrix enters the angular distribution of its decay daughter as

\[
\frac{dN}{d\cos \theta'} \propto \left[ (1 - \rho_{00}) + (3\rho_{00} - 1) \cos^2 \theta' \right],
\]

where \( \theta' \) is the angle between the decay daughter and the spin quantization direction in the vector meson’s rest frame. Thus, the \( \rho_{00} \) value of the vector meson can be measured using the angular distribution of its decay daughter. The vector meson (\( K^{*0} \) and \( \phi \)) spin alignments have been measured by the ALICE Collaboration [189]. The \( \rho_{00} \) value is consistent with 1/3 for both \( K^{*0} \) and \( \phi \) mesons in p+p collisions. In Pb+Pb collisions, the \( \rho_{00} \) value of \( K^{*0} \) is approximately 1/3 at high \( p_T \) and less than 1/3 at low \( p_T \).

Theoretical calculations using the statistical-hydro model [176] and quark coalescence model [190] give

\[
\rho_{00}^\phi \approx \frac{1}{3} - \frac{4}{9} P_\Lambda P_\Lambda \approx \frac{1}{3}.
\]

Using the STAR data, \( P_\Lambda \approx (1.08 \pm 0.15 \pm 0.11)\% \) and \( P_\Lambda \approx (1.38 \pm 0.30 \pm 0.13)\% \) [5, 111], one can estimate \( (4/9) P_\Lambda P_\Lambda \approx 6.6 \times 10^{-5} \), which may suggest that \( \rho_{00}^\phi \) cannot be significantly larger than 1/3 and thus contradict the STAR preliminary data for \( \rho_{00}^\phi \).

To solve this puzzle, it is proposed that a strangeness current can exist in heavy-ion collisions and give rise to a nonvanishing mean \( \phi \) field [191]. Like the magnetic field, the magnetic part of the \( \phi \) field can also polarize \( s \) and \( \bar{s} \) through their magnetic moments; this polarization contributes to the polarization of \( \Lambda \) and \( \bar{\Lambda} \), whereas the contribution from the electric part of the \( \phi \) field is absent and therefore is not constrained by the polarization of \( \Lambda \) and \( \bar{\Lambda} \). However, the electric part of the \( \phi \) field contributes significantly to \( \rho_{00}^\phi \), which is positive definite [191]:

\[
\rho_{00}^\phi \approx \frac{1}{3} + \frac{8}{9} \frac{\langle s \bar{s} \rangle^2}{m_s^2 T_{\text{ch}}^2 \langle \rho^2 \rangle_{\phi} \langle E_{\phi, z}^2 + E_{\phi, x}^2 \rangle}.
\]
where \( E_\phi \) is the electric part of the \( \phi \) field, \( m_S \) is the strange quark mass, \( \langle p^2 \rangle_\phi \) is the mean value of \( p^2 \) for \( s \) or \( \bar{s} \) in the \( \phi \) meson wave function, \( g_\phi \) is the coupling constant of the \( s \) quark to the \( \phi \) meson in the quark-meson model, and \( T_{\text{eff}} \) is the average temperature of the fireball. The contribution originates from the spin–orbit coupling term that polarizes \( s \) and \( \bar{s} \). Thus, one can see that \( \rho_{00} \) for the \( \phi \) meson is a good tool for analyzing the \( \phi \) mean field even if it may strongly fluctuate in space-time [191]. The theoretical prediction of \( \rho_{00}^\phi \) as a function of the collision energy is shown in Fig. 10.

However, this theory does not work for another vector meson, \( K^{*0} \), for several reasons. First, because of the unequal masses of \( \bar{s} \) and \( d \), one cannot derive the same formula as that for \( \phi \) mesons, in which the contributions from the vorticity and those from the electric and magnetic fields are decoupled. Second, the interaction of \( K^{*0} \) with the surrounding matter is much stronger than that of the \( \phi \) meson. The above points are supported by preliminary data from ALICE [189].

### 5 Summary

We gave a brief overview of recent theoretical developments and experimental results on the effects of chirality and vorticity on heavy-ion collisions. We focus on works reported at the Quark Matter conference of 2019.

We discussed the time evolution of magnetic fields in various models of the QGP, for example, the Lienard-Wiechert potential and MHD models. Macroscopic models such as the second-order dissipative MHD and AVFD models are applied in phenomenological studies.

It is still experimentally challenging to obtain a clear CME signal against the dominant backgrounds. The non-flow correlations for the CMW give rise to additional backgrounds in the slope of \( \Delta v_2(A_h) \). More detailed studies of the CME and CMW are needed.

The kinetic theory of massive fermions has been developed for the covariant and equal-time Wigner functions and the Schwinger–Keldysh formalism. The collision terms have been studied at the leading and next-to-leading order in an expansion of the Planck constant.

The decomposition of the total angular momentum into the OAM and spin has been debated for quite a long time. Another important issue that remains to be resolved is the relationship between the theoretical spin and the results of experimental measurements.

For the global polarization of \( \Lambda \) and \( \bar{\Lambda} \) hyperons, various phenomenological models give consistent descriptions of the experimental data. For the local polarization, the nature of the spin sign problem is still unclear. It has been proposed that a significant positive deviation of 1/3 for \( \rho_{00}^\phi \) may indicate the existence of a mean \( \phi \) field.

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