Two-photon transitions of charmonia on the light front

Meijian Li
University of Santiago de Compostela, Spain
In collaborations with: Yang Li, and James P. Vary

DIS2022: XXIX International Workshop on Deep-Inelastic Scattering and Related Subjects
May 2-6, 2022, Santiago de Compostela
Outline

- **Introduction**: two-photon transition of charmonium
- **Method**: light-front Hamiltonian formalism
- **Results**: transition width and transition form factors
- **Summary**

*Based on: Y. Li, M. Li and J.P. Vary, Phys. Rev. D 105 (2022) 7, L071901; arXiv:2111.14178 [hep-ph]*
Introduction: two-photon transition

- Charmonium provides an ideal testing ground for various investigations to understand QCD
  - Challenging: relativistic, non-perturbative effects
- The two-photon transition, $H_{c\bar{c}} \rightarrow \gamma^* + \gamma$, provides a clean and important probe to hadron states
- Experimental measurements
  - Diphoton width $\Gamma_{H \rightarrow \gamma\gamma}$: extensive measurements for $\eta_c, \eta'_c, \chi_{c0}, \chi_{c2}$
  - Transition form factors $F_{H\gamma}(Q^2)$: $F_{\eta_c\gamma}(Q^2)$ by BABAR 2010; $F_{\chi_{cj}\gamma}(Q^2)$ by Belle 2017

![Graph showing diphoton width and transition form factors over time]
Method: light-front Hamiltonian formalism

Light-front Hamiltonian formalism is a natural framework for addressing relativistic bound-state and scattering problems in QCD

- Light-front quantization
  - The quantum field is quantized on the equal light-front time $x^+ = 0$

\[
\begin{align*}
\text{instant form} & \quad \text{front form} & \quad \text{point form} \\
\text{time variable} & \quad t = x^0 & \quad x^+ & \equiv x^0 + x^3 \\
\text{quantization surface} & \quad t=0 & \quad x^+ = 0 \\
\text{Hamiltonian} & \quad H = P^0 & \quad P^- & \equiv P^0 - P^3 \\
\text{kinematical} & \quad \vec{P}, \vec{J} & \quad \vec{P} \perp, P^+, \vec{E} \perp, E^+, J^- \\
\text{dynamical} & \quad \vec{K}, P^0 & \quad \vec{F} \perp, P^- \\
\text{dispersion relation} & \quad p^0 = \sqrt{\vec{p}^2 + m^2} & \quad p^- = (\vec{p} \perp + m^2) / p^+ \\
\end{align*}
\]

\[
\begin{align*}
\tau & \equiv \sqrt{t^2 - \vec{x}^2 - a^2} \\
\end{align*}
\]
Method: light-front Hamiltonian formalism

- Hamiltonian formalism
  - the invariant masses and the boost invariant wavefunctions can be obtained directly by solving the eigenvalue equation
    \[
    \left(P^+\hat{P}^- - \hat{P}_\perp^2\right)|\psi_h(P,j,m_j)\rangle = M_h^2|\psi_h(P,j,m_j)\rangle
    \]
  - the light-front wavefunction encodes the information of the system, and provides direct access to observables

- Basis representation
  - basis can encode an analytical approximation to the solution
  - optimal basis is the key to numerical efficiency

→ Basis Light-Front Quantization (BLFQ)
The charmonium light-front wavefunction by BLFQ

- The charmonium light-front wavefunction is solved using the BLFQ approach in the $|q\bar{q}\rangle$ sector\(^1\),

\[
H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m^2_q}{x} + \frac{\vec{k}_\perp^2 + m^2_{\bar{q}}}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \frac{\partial}{\partial x} \left( x(1-x) \frac{\partial}{\partial x} \right) + V_g
\]

\[\text{LF kinetic energy} \quad \text{confinement} \quad \text{one-gluon exchange}\]

\[
\begin{align*}
x &= p^+_q / P^+ \\
\vec{k}_\perp &= \vec{p}_q - x \vec{P}_\perp
\end{align*}
\]

- Confinement
  - Transverse (QCD holography)\(^2\)
  - Longitudinal (completes the transverse confinement, and produces desirable distribution amplitudes)
- One-gluon exchange

\[
V_g = -\frac{4}{3} \frac{4\pi\alpha_s(Q^2)}{Q^2} \bar{u}_\sigma \gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}
\]

- Basis representation: basis functions are eigenfunctions of $H_0$

\(^1\) Y. Li, P. Maris, and J. P. Vary, Phys. Rev. D96, 016022 (2017).
\(^2\) S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and J. Erlich, Phys. Rept. 584, 1 (2015)
The charmonium light-front wavefunction by BLFQ

- The charmonium light-front wavefunction is solved using the BLFQ approach in the \(|q\bar{q}\rangle\) sector\(^1\),
  - Light-front wavefunctions:

    e.g., \(\eta_c(1S)\)

- Access to a variety of observables:
  - Form factors [Li, PRD '18; Mondal, PRD '20],
  - PDFs/GPDs [Lan, PRL '19, PRD '20; Adhikari, PRC '18, '21],
  - Radiative transitions [M. Li, PRD '18 & '19],
  - Diffractive production [Chen, PLB '17 & PRC '18]

\(^1\) Y. Li, P. Maris, and J. P. Vary. Phys. Rev. D96, 016022 (2017).
The two-photon transition

- The amplitude of the hadron-to-two-photon transition, $H_{cc}(j, p, \lambda) \rightarrow \gamma^*(q_1, \lambda_1) + \gamma(q_2, \lambda_2)$, is related to the hadron matrix element,

$$
\epsilon^*_\mu(q_1, \lambda_1)\epsilon^*_\nu(q_2, \lambda_2)e_\alpha(p, \lambda)\mathcal{M}^{\mu\nu\alpha} = \mathcal{H}_{\lambda_1, \lambda_2; \lambda}(q_1, q_2) = \epsilon^*_\nu(q_2, \lambda_2)\langle \gamma^*(q_1, \lambda_1)|J^\gamma(0)|H(p, \lambda)\rangle
$$

The EM current

\[\text{The photon light-front wavefunction (by LO perturbation)}\]

\[\text{The hadron light-front wavefunction (by BLFQ)}\]

- The decay width, can be measured from experiments

$$
\Gamma_{H\rightarrow\gamma\gamma} = \frac{1}{2} \frac{1}{16\pi} \frac{1}{m_H} \frac{1}{2j + 1} \sum_{\lambda=-j}^{j} \sum_{\lambda_1, \lambda_2 = \pm 1} |H_{\lambda_1, \lambda_2; \lambda}|^2
$$

- Light-cone dominance is manifest in the frame $q_2^+ = q_1^- = 0$
Results: two-photon decay widths

- Without any parameter tuning, our results have reasonable agreement with experimental data.¹

¹ Y. Li, M. Li and J.P. Vary, Phys. Rev. D 105 (2022) 7, L071901; arXiv:2111.14178 [hep-ph]
Results: two-photon decay widths

Without any parameter tuning, our results have reasonable agreement with experimental data\(^1\)

\[^1\] Y. Li, M. Li and J.P. Vary, Phys. Rev. D 105 (2022) 7, L071901; arXiv:2111.14178 [hep-ph]
Results: the transition form factor (1) pseudoscalar $0^{+-}$

- The pseudoscalar transition amplitude is parameterized as
  \[
  \mathcal{M}^{\mu\nu} = 4\pi\alpha_{em}\epsilon^{\mu\nu\rho\sigma}q_1^\rho q_2^\sigma F_{\gamma\gamma}(q_1^2, q_2^2)
  \]
  
- Single-tagged transition form factor: $F_{\gamma\gamma}(Q^2, 0) = F_{\gamma\gamma}(0, Q^2)$

- The transition form factor in the light-front wavefunction representation reads
  \[
  F_{\gamma\gamma}(Q^2) = 2Q^2_f \sqrt{2N_c} \int \frac{d^2k_\perp}{(2\pi)^3} \int_0^1 dx \frac{\psi_{\uparrow\downarrow-\uparrow/p}(x, \vec{k}_\perp)}{k_\perp^2 + m_f^2 + x(1-x)Q^2}
  \]
  
- At large $Q$, it reduces to the partonic interpretation in terms of light-cone distribution amplitude (LCDA)
  \[
  F_{\gamma\gamma}(Q^2) = \frac{e_f^2 f_p}{Q^2} \int_0^1 dx \frac{\phi_p(x, Q)}{x(1-x)}
  \]
Results: the transition form factor
(1) pseudoscalar $0^{+-}$

- $\eta_c$

- $\eta_c'$

- The calculated transition form factors are in a reasonable agreement with experimental data
  - BLFQ (this work), uncertainty is calculated from basis sensitivity
  - BLFQ/DA, using the LCDA obtained from the BLFQ wavefunction
  - Monopole fit, vector meson dominance model
Results: the transition form factor
(2) scalar $0^{++}$

- The transition amplitude is parameterized as

$$\mathcal{M}^{\mu\nu}(q_1, q_2) = 4\pi\alpha_{em}\left\{[(q_1 \cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu]F_{S\gamma\gamma,1}(q_1^2, q_2^2) + \frac{1}{m_s^2}[q_1^2 q_2^2 g^{\mu\nu} + (q_1 \cdot q_2)q_1^\mu q_2^\nu - q_2^\mu q_1^\nu - q_2^\mu q_1^\nu]F_{S\gamma\gamma,2}(q_1^2, q_2^2)\right\}$$

- Single-tagged transition form factor: $F_{S\gamma}(Q^2 = -q^2) = F_{S\gamma\gamma,1}(q^2, 0) = F_{S\gamma\gamma,1}(0, q^2)$

- The transition form factor in the light-front wavefunction representation reads

$$F_{S\gamma}(Q^2) = e_f^2 2\sqrt{2N_c} \int_0^1 \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2k_\perp}{(2\pi)^3} \left\{\psi_{\uparrow\downarrow+\downarrow/\uparrow/\downarrow/s}(x, \vec{k}_\perp) \frac{(1-2x)[x(1-x)Q^2 + m_f^2]}{[k_\perp^2 + x(1-x)Q^2 + m_f^2]^2} + \psi_{\uparrow\uparrow/\downarrow/s}(x, \vec{k}_\perp) \frac{\sqrt{2m_f(k_x + ik_y)}}{[k_\perp^2 + x(1-x)Q^2 + m_f^2]^2}\right\}$$

- At large $Q$, in terms of distribution amplitude (LCDA)

$$F_{S\gamma}(Q^2) = e_f^2 f_S \int_0^1 dx \frac{(1-2x)\phi_S(x, \mu)}{x(1-x)Q^2 + m_f^2}$$
Results: the transition form factor (2) scalar $0^{++}$ and tensor $2^{++}$

- $\chi_{c0} (0^{++})$

- $\chi_{c2} (2^{++})$

- The calculated transition form factors are in a reasonable agreement with experimental data
  - BLFQ (this work), uncertainty is calculated from basis sensitivity
  - BLFQ/DA, using the LCDA obtained from the BLFQ wavefunction
Summary and outlook

- We investigated the **two-photon transitions** of charmonia, $\eta_c, \eta_c', \chi_{c0},$ and $\chi_{c2},$ in the light-front Hamiltonian approach
  - We derived the **formulas** of transition form factors in the light-front wavefunction representation
    - Universal for other hadron light-front wavefunctions
  - We computed the decay widths, and the transition form factors, **both in good agreements** with experimental measurements
    - Reveal relativistic nature of charmonia

*Based on: Y. Li, M. Li and J.P. Vary, Phys. Rev. D 105 (2022) 7, L071901; arXiv:2111.14178 [hep-ph]. LFWFs available on Mendeley Data*

- Ongoing and future works on radiative transitions
  - A comprehensive study on different leptonic and radiative transitions
  - Extension to bottomonia, heavy-light mesons, and light mesons

Thank you!