The Unevolved Main Sequence of Nearby Field Stars and the Open Cluster Distance Scale

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ABSTRACT. The slope and zero point of the unevolved main sequence as a function of metallicity are investigated using a homogeneous catalog of nearby field stars with absolute magnitudes defined with revised Hipparcos parallaxes, Tycho-2 photometry, and precise metallicities from high-dispersion spectroscopy. \((B - V)\)–temperature relations are derived from 1746 stars between \([\text{Fe/H}] = -0.5\) and +0.6 and 372 stars within 0.05 dex of solar abundance; for \(T_{\text{eff}} = 5770\) K, the solar color is \(B - V = 0.652 \pm 0.002\) (SEM). From over 500 cool dwarfs between \([\text{Fe/H}] = -0.5\) and +0.5, \(\Delta(B - V)/\Delta[\text{Fe/H}]\) at fixed \(M_V = 0.213 \pm 0.005\), with a very weak dependence upon the adopted main-sequence slope with \(B - V\) at a given \([\text{Fe/H}]\). At Hyades metallicity, this translates into \(\Delta M_V/\Delta[\text{Fe/H}]\) at fixed \(B - V = 0.98 \pm 0.02\), midway between the range of values empirically derived from smaller and/or less homogenous samples and model isochrones. From field stars of similar metallicity, the Hyades \(([\text{Fe/H}] = +0.13)\) with no reddening has \((m - M)_0 = 3.33 \pm 0.02\) and M67, with \(E(B - V) = 0.041, A_V = 3.1E(B - V)\), and \([\text{Fe/H}] = 0.00\), has \((m - M)_0 = 9.71 \pm 0.02\) (SEM), where the errors quoted refer to internal errors alone. At the extreme end of the age and metallicity scale, with \(E(B - V) = 0.125 \pm 0.025\) and \([\text{Fe/H}] = +0.39 \pm 0.06\), comparison of the fiducial relation for NGC 6791 to 19 field stars with \(B - V\) above 0.90 and \([\text{Fe/H}] = +0.25\) or higher, adjusted to the metallicity of NGC 6791, leads to \((m - M)_0 = 13.07 \pm 0.09\), internal and systematic errors included.

1. INTRODUCTION

Distance determination to open and globular clusters is key to placing them in the proper Galactic evolutionary context and an indispensable component in evaluating stellar evolution as a function of mass, chemical composition, and age. For nearby clusters like the Hyades, Praesepe, the Pleiades, and Coma, Hipparcos parallaxes (ESA 1997), coupled with proper-motion and radial-velocity memberships, have generated precise distances, independent of the cluster’s composition and reddening, though these results have not been without controversy (Pinsonneault et al. 1998; van Leeuwen 1999; Stello & Nissen 2001; Makarov 2002; Soderblom et al. 2005; van Leeuwen 2009). Any remote cluster beyond the reach of parallax with a known chemical makeup comparable to a nearby cluster can be compared differentially using stars on the unevolved main sequence to obtain a reliable distance (see, e.g., Pinsonneault et al. 2004; An et al. 2007). While unevolved main sequences of nearby clusters are ideal reference points due to the uniformity of age and composition among the cluster stars, the range in chemical composition sampled by the nearby clusters is approximately \([\text{Fe/H}] = -0.2\) to +0.2. For open clusters outside this range and for all globular clusters, a traditional fallback procedure is to use cooler field dwarfs with parallaxes and well-defined abundances either to isolate a stellar sample of similar abundance or to interpolate among stars that bracket the metallicity of interest (Twarog et al. 1999; Gratton et al. 2003). Given the broad Gaussian distribution in \([\text{Fe/H}]\) for field stars, this approach becomes more of a challenge for clusters whose metallicity places them in the extended, low-metallicity tail of the field star sample.

A routine alternative to field star comparison has been the construction of theoretical isochrones which can be tuned to any combination of composition and age, though ultimately these models must be linked to the empirical data of the field stars by matching the theoretical model combinations of mass, luminosity, composition, and temperature to the observed values of mass, absolute magnitude, abundance, and color at a given age. For stars of approximately solar mass and higher, there is reasonable agreement among the various isochrone compilations currently available in the literature and online, with most discrepancies among the models understandable in terms of differences in the adopted parameterizations of the internal physics, the model atmospheres, and the relations used to transfer from the theoretical to the observational plane (Girardi et al. 2002; Pietrinferni et al. 2004; Demarque et al. 2004; Vandenberg et al. 2006; Marigo et al. 2008; Dotter et al. 2008). As one might expect, the discrepancies within the observational color-magnitude diagram (CMD) plane grow larger as one moves to stars of lower mass and/or more extreme compositions, reflecting the same paucity of empirical constraints found when
attempting a direct match of distant clusters to nearby field stars with parallaxes.

Issues of reddening and metallicity determination aside, the more contentious discussions of cluster distances generally have focused on metal-poor systems of the thick disk and halo, while the metal-rich end of the distribution has been moderately immune due to the rich sample of nearby stars of solar abundance, the proximity of the Hyades, and the rarity of clusters with compositions significantly higher than the Hyades. For almost 50 yr, the one potential exception has been the old open cluster, NGC 6791, though the extent of its anomaly has been hidden beneath disagreements over its reddening, composition, distance, and age. In recent years, significant progress has been made in constraining the first two parameters, both of which are critical to defining the third using main-sequence fitting, while the fourth has no bearing on the distance if the comparison is made to stars of sufficiently low mass. In every instance where revised reddening and/or metallicity estimates have been obtained, an improved distance modulus has been derived through comparison with theoretical isochrones (Bedin et al. 2008; Carraro et al. 2006; Carney et al. 2005; King et al. 2005; Stetson et al. 2003; Chaboyer et al. 1999; Kaluzny & Rucinski 1995; Tripicco et al. 1995; Garnavich et al. 1994; Montgomery et al. 1994; Demarque et al. 1992). For the apparent distance modulus, the current range from main-sequence fitting extends from a low of 13.1 (Stetson et al. 2003) to a high of 13.6 (Sandage et al. 2003; Anthony-Twarog et al. 2007). While the scatter is partly due to the adoption of different values for [Fe/H] and $E(B - V)$, a large portion is tied to disagreement over the location of the unevolved main sequence among the isochrones at high metallicity and the range of the CMD used to define an adequate fit to the models; in some instances, the unevolved main sequence and giant branch features are used simultaneously to optimize the fit.

To circumvent the issues presented by the discrepancies among metal-rich isochrones, it was decided that the distance to NGC 6791 could best be obtained by an empirical fit to unevolved field stars of similar composition. The feasibility of this option has been enhanced by the more restricted range among recent determinations of the cluster parameters, the availability of revised parallaxes and broadband photometry from Hipparcos/Tycho-2 (ESA 1997; van Leeuwen 2007), and the compilation of a catalog of precise spectroscopic abundances, temperatures, and surface gravities on a common scale for almost 2100 nearby field stars. While the initial motivation for this study was the distance to NGC 6791, the need to define the precise location of the unevolved main sequence at high metallicity using the very restricted field star sample at comparable [Fe/H] generated a more comprehensive investigation of the metallicity dependence of the unevolved main sequence for stars of typical disk metallicity ([Fe/H] = −0.5 to +0.5). This paper builds upon the approach laid out in Percival et al. (2003), using a sample expanded by an order of magnitude to derive the change in $M_V$ for cool dwarf stars at a given $B - V$ as [Fe/H] is varied from −0.5 to +0.5. While it is usually assumed that the ratio, $\Delta M_V / \Delta [\text{Fe/H}]$, is constant with metallicity for unevolved disk dwarfs, derived values from observation and theoretical models vary by more than a factor of 2 (Kotoneva et al. 2002; Percival et al. 2003; Pinsonneault et al. 2004; Karataş & Schuster 2006). The goal of this investigation is to explore the assumption of a constant ratio, derive it, and test the consistency of the field star main sequence when applied to well-studied open clusters. Unless noted otherwise, errors quoted are standard deviations.

The layout of the paper is as follows. In § 2 we will describe the database of astrometry, photometry, and spectroscopy for the nearby stars that is used in § 3 to define the characteristics of the unevolved main sequence as a function of color and metallicity. In § 4, the derived CMD relation is used to estimate distances to the well-studied open clusters, M67 and the Hyades, as well as the extremely metal-rich cluster, NGC 6791. Section 5 contains a summary of our conclusions.

2. THE FIELD STAR SAMPLE

2.1. Fundamental Properties—The Metallicity Scale

Delineation of the unevolved main sequence as a function of metallicity and color requires a significant sample of stars with reliable abundance estimates, homogeneous and precise photometry that can be compared to the cluster data, and accurate absolute magnitudes. The sample of choice to fulfill the first condition is the spectroscopic catalog of abundances compiled by Twarog et al. (2007), expanded by the inclusion of three published spectroscopic surveys that have appeared since 2007. The core data set that defines the metallicity, temperature, and surface gravity scale of the entire catalog is that of Valenti & Fischer (2005), which supplies half of the stars in the final sample. The addition of three new sources of spectroscopic data (Fuhrmann 2008; Sousa et al. 2008; Mishenina et al. 2008) raises the total source number to 29 and the number of stars in the current catalog to 2085. While half of the stars come from the catalog of Valenti & Fischer (2005), that database has been doubled without any significant reduction in the quality of the composite [Fe/H] measures. For details on the selection of the literature sources and the merger process, the reader is referred to Twarog et al. (2007). For the three recent additions, the numbers of stars used in the transformations and the adopted errors for a single abundance estimate are 228 and 0.025 for Sousa et al. (2008), 67 and 0.031 for Fuhrmann (2008), and 87 and 0.060 for Mishenina et al. (2008).

While a concerted effort has been made to ensure the homogeneity of the abundances obtained by merging data from multiple sources, for the purpose of probing potential differences with other derivations of the position of the unevolved CMD with [Fe/H], it is useful to explore how the metallicity scale of Valenti & Fischer (2005) compares to that of other large
abundance surveys. The two primary comparisons of interest are to the photometrically-based abundance estimates of Nordström et al. (2004), updated in Holmberg et al. (2007), and the comprehensive spectroscopic catalog compiled by Taylor (2005). For both sources, our spectroscopic data were cross-matched with the online catalogs; stars were retained if the published \( T_{\text{eff}} \) was between 4500 K and 6500 K, with absolute differences in [Fe/H] below 0.45 dex. For Holmberg et al. (2007) this produced 1519 stars in common while the comparison data set from Taylor (2005) was 898 stars. In each case, residuals (in the sense of the catalogs—this work) were calculated and polynomials fit to the data as a function of the catalog [Fe/H] using a linear–least-squares routine. Due to the small number of metal-poor stars in the current catalog, the metallicity range was limited to stars with [Fe/H] above −1.0. Linear and quadratic fits were made to the entire sample and to the data sorted by \( T_{\text{eff}} \) into four bins 500 K wide between 4500 K and 6500 K. In each case, initial fits were made and, after eliminating points more than three standard deviations away from the mean, repeated. The results were extremely consistent for both catalogs; inclusion of quadratic terms failed to have a statistically significant impact upon the quality of any fit and were eliminated. A similar conclusion was confirmed regarding the removal of potentially deviant points, a byproduct of the large number of stars populating every temperature bin except the coolest, where the numbers ranged from 34 to 43. For the coolest bin, no stars were ever rejected as being too deviant.

As an initial comparison, the average residuals and their standard deviations are \(-0.044 \pm 0.061\) from 871 stars (Taylor 2005) and \(-0.072 \pm 0.084\) from 1487 stars (Holmberg et al. 2007), implying that the Valenti & Fischer (2005) system is slightly more metal-rich than the others. The differences among the catalogs become somewhat more distinct if one adds a linear term to the comparison. For Taylor (2005), the residuals for 871 stars follow the relation

\[
\Delta [\text{Fe/H}] = -0.036(\pm 0.003) + 0.044(\pm 0.008)[\text{Fe/H}]_{\text{TAY}}
\]

with a dispersion of 0.060 dex. For Holmberg et al. (2007), the comparable relationship for 1482 stars is

\[
\Delta [\text{Fe/H}] = -0.084(\pm 0.002) - 0.121(\pm 0.007)[\text{Fe/H}]_{\text{HOL}}
\]

with a dispersion of 0.078 dex. The good agreement with the data of Taylor (2005) is encouraging but not surprising, since the majority of the spectroscopic surveys included in our compilation are included in the earlier comprehensive analysis. However, the catalog of Taylor (2005) does not include the data of Valenti & Fischer (2005), the defining catalog of our metallicity scale, so the agreement does imply that the two metallicity scales are very similar and transformations from one sample to the other are straightforward. The relation also demonstrates that the metallicity range among the disk stars in the catalog of Taylor (2005) is slightly larger than we would claim, but the two systems are in closer agreement at the metal-rich end of the scale. By contrast, the abundance catalog of Holmberg et al. (2007) exhibits a disk metallicity range reduced by one-sixth compared to that of Taylor (2005), with the absolute discrepancy growing larger at higher metallicity. Thus, a star with \([\text{Fe/H}] = +0.5\) in our catalog would have \([\text{Fe/H}] = +0.36\) on the system of Holmberg et al. (2007).

Finally, if one looks at the trends with temperature, there are only minor differences in the linear relations for the data of Taylor (2005); the dispersion in the residuals and the number of stars as one goes from the 6500 K bin to the 5000 K bin are (0.045, 270), (0.060, 416), (0.070, 141), and (0.068, 43), respectively. By contrast, the comparable data for Holmberg et al. (2007) exhibit increasing scatter among the cooler bins, (0.068, 462), (0.070, 756), (0.104, 234), and (0.127, 34), respectively, as well as a significant increase in the slope of the linear relation for the coolest stars. Photometric abundances for stars with \( T_{\text{eff}} \) below 5000 K on the Holmberg et al. (2007) scale are questionable.

2.2. Fundamental Properties—Parallax and Color

Of the 2085 stars in the primary catalog, 2043 have parallaxes available in the revised catalog of *Hipparcos* (van Leeuwen 2007); the metallicity distribution of this sample is shown in Figure 1. While 86 stars are more metal rich than \([\text{Fe/H}] = +0.30\), most are only slightly so; half of the stars fall between \([\text{Fe/H}] = +0.31\) and \(+0.35\). Moreover, preliminary analysis demonstrated that the majority of the stars fall in the vertical turnoff region for stars of the old disk, making them inappropriate for comparison with stars on the unevolved main sequence. For the purpose of deriving the distance to an exceptionally metal-rich cluster, the small sample could be expanded to include stars with \([\text{Fe/H}]\) as low as \(+0.25\), if reliable metallicity-dependent shifts in the CMD could be generated for unevolved main-sequence stars.

![Fig. 1.—Metallicity distribution for stars in the spectroscopic abundance catalog.](image-url)
To derive metallicity-dependent CMD shifts, the field stars must be on a common photometric system that is readily transformable to a significant cluster sample reaching the unevolved main sequence; for now, this restricts the analysis to broadband $BV$ data. Fortunately, fewer than 1.5% of the stars with spectroscopic abundances and *Hipparcos* parallaxes lack reliable $BV$ photometry on the *Tycho-2* system (Hog et al. 2000); the exceptions are dominated by stars that are too bright to have been observed photometrically or too faint to have reliable photometry. The *Tycho-2* $BV$ photometry (Hog et al. 2000) has been converted to the Cousins-Johnson system using the transformation relations of Mamajek et al. (2002), derived by applying polynomial fits to a table of mean points compiled by Bessell (2000). The table of Bessell (2000) was constructed by a direct comparison of the original *Tycho* photometry with $E$-region standards composed primarily of $B-G$ dwarfs and $K-M$ giants. The revised relations exhibit significant structure as a function of $B-V$ in comparison with the original linear relations provided by the *Hipparcos* catalog. The newer *Tycho-2* photometry follows the same relations defined by Mamajek et al. (2002), but with higher internal precision.

A systematic error in the zero-point of the photometry and/or the slope of the unevolved main sequence could cause systematic shifts in the distance scale. Two basic tests of the reliability of the transformed *Tycho-2* data are available. First, Percival et al. (2003) defined the approach of interest in this study to constrain the distance scale for nearby open clusters studied by *Hipparcos*. To ensure a reliable photometric comparison between the clusters and field stars, Percival et al. (2003) obtained $BVRI$ photometry of 54 cool field dwarfs over a range in [Fe/H] from $-0.56$ to $+0.44$, as defined by intermediate-band photometry recalibrated to avoid the issues raised by Twarog et al. (2002). The broadband photometry is on the Cousins $E$-region system; the $V$ magnitudes and $B-V$ indices have mean standard deviations of $\pm 0.007$ mag and $\pm 0.004$ mag, respectively. We have taken the *Tycho-2* $BV$ photometry of these 54 stars and processed it through the same transformation relations as our catalog stars. For 54 stars, the mean residuals in $B-V$ and $V$ (in the sense of the converted *Tycho-2* data—Percival et al. 2003), are $+0.0007 \pm 0.0023$ and $+0.0029 \pm 0.0030$. In $B-V$, if the 2 stars with absolute residuals above $0.04$ mag are excluded, the mean for the 52 stars becomes $+0.0003 \pm 0.0166$. In $V$, if the two stars with residuals in $V$ greater than $0.05$ mag are removed, the mean for the 52 remaining stars becomes $+0.0012 \pm 0.0215$. Note that in all cases the errors are dominated by the uncertainty in the *Tycho-2* photometry.

Second, for reasons that will become apparent in §4, we will compare the $BV$ photometry for stars in the Hyades with the transformed *Tycho-2* data for the same stars. Johnson & Knuckles (1955) was selected as the primary source of broadband cluster data. After eliminating composite systems and variable stars, 100 stars with $B_T-V_T$ redder than 0.4 remained. For these 100 stars, the mean residuals in $B-V$ and $V$ are $-0.011 \pm 0.028$ and $+0.017 \pm 0.028$, respectively. The offset in color, implying that the Johnson & Knuckles (1955) photometry is too blue, is consistent within the errors with the derived offsets of $-0.009 \pm 0.001$ mag (SEM) and $-0.006 \pm 0.004$ (SEM) in $B-V$ found by An et al. (2007) in comparisons with the photometry of Joner et al. (2006) and *Tycho-2* data, respectively. The respective comparisons in $V$ indicate that the Johnson & Knuckles (1955) data are too faint by $+0.026 \pm 0.002$ (SEM) and $+0.018 \pm 0.003$ (SEM), again in good agreement with the results derived here. The $B-V$ color offset between the Hyades and the *Tycho-2* system has also been confirmed by the analysis of Joner et al. (2008).

To test for a color dependence among the residuals, we have applied a zero-point offset of $+0.011$ to the residuals for the Hyades and merged the two samples. Figure 2 shows the trend with color for 154 stars; filled squares are the data of Percival et al. (2003) while circles are the Hyades observations. The error bars for each point are based upon the combined errors in both measures, but again are dominated by the errors in the *Tycho-2* data. The apparent lack of a slope is confirmed by attempts to fit a line through the data; the resulting slope remains decidedly zero whether the data are included with or without weighting by the inverse square of the uncertainties.

The final element in the field star database is the absolute magnitude. Of the 2043 stars with parallax, 61 have parallax errors larger than 12.5% of the parallax; these have been excluded from the sample. For the remaining 1982 stars, distances and $M_V$ have been calculated from the parallax with the inclusion of Lutz-Kelker corrections (Lutz & Kelker 1973). Because of the small uncertainties among the parallaxes, the average Lutz-Kelker shift is below 0.03 mag. Of the parallax stars, 17 are in composite systems or variable, while 25 do not have broadband colors from *Tycho-2*, leaving a final sample of 1940 stars with complete data.

![Fig. 2.—Residuals in $B-V$ (in the sense of the literature—the converted *Tycho-2* data), for Percival et al. (2003) (filled squares) and the Hyades photometry of Johnson & Knuckles (1955) (circles), adjusted by $+0.011$.](image-url)
3. THE $M_V - (B-V) - \text{[Fe/H]}$ RELATION

3.1. The $(B-V)$, $T_{\text{eff}}$ Relations

While our primary interest is the CMD location of unevolved stars as a function of color and metallicity, it is useful to explore first the relationship between effective temperature and color. The temperature scale of the stars is tied to that of Valenti & Fischer (2005), derived from a comparison of high-dispersion spectra to a grid of synthetic spectra constructed using the models of Kurucz (1992) and interpolated to optimize the match between observation and theory. By defining the relation between temperature and color, we can use the spectroscopic temperatures to generate $B-V$ for stars with no $B-V$ photometry or for stars with excessively large photometric errors, a not uncommon issue for the coolest stars in the sample. Additionally, the color-temperature relationship will allow a direct comparison between the current system and that adopted by others using different spectra, analyses, and photometry.

As a simple preliminary test, we have isolated the 375 stars in the final sample with [Fe/H] between $-0.05$ and $+0.05$ and $T_{\text{eff}}$ between 4250 K and 7150 K. Excluding three stars with residuals between the calculated and observed $B-V$ greater than 0.10 mag, a fit between $T_n$, defined as $(T_{\text{eff}} - 5770)/5770$, and $B-V$ weighted by the inverse square of the uncertainty in $B-V$ for the remaining 372 stars produces

$$B-V = 0.651(\pm 0.001) - 1.966(\pm 0.016)T_n + 2.079(\pm 0.140)T_n^2.$$  

Inclusion of a cubic term does not lead to a statistically significant improvement in the fit. The uncertainty estimates for $B-V$ included the photometric uncertainty and the dispersion in $T_{\text{eff}}$, translated to an error in $B-V$ and combined in quadrature with the photometric error. The standard deviation among the residuals in $B-V$ is 0.021 mag. Since the solar temperature of the models used to derive the spectroscopic abundances is set at 5770 K (Valenti & Fischer 2005), the implied $B-V$ color of the Sun for this combination of stellar parameters and photometry is $B-V = 0.651$.

Next, the stellar sample was sorted in bins 0.1 dex wide ranging from [Fe/H] = $+0.55$ to $-0.95$ and a quadratic polynomial fit was made for each bin using the same temperature parameterization as described. For these fits, stars were included if they fell in the temperature range from $T_{\text{eff}} = 4000$ K to 7500 K; points were weighted in the same manner as described. Figure 3 illustrates the morphology of the mean relations plotted differentially relative to the color relation for solar metallicity, in the sense of $(B-V)_{\text{Fe/H}} - (B-V)_{\text{Fe/H}=0}$. For clarity, mean relations are plotted for every other metallicity bin starting with [Fe/H] = $+0.40$ at the top. The curves are plotted only above the $T_{\text{eff}}$ range covered by the sample used to define the curve. The relations exhibit reasonable morphological consistency between [Fe/H] = $+0.5$ and $-0.5$, with the predominant change being a regular shift in the zero point of the curves as the metallicity declines. However, the differential relations for stars with [Fe/H] below $-0.5$ exhibited significant deviations from linearity, as illustrated by the bottom curve in Figure 3. It should be emphasized that this is not a product of small number statistics; while the color range is declining among the lower metallicity stars, the bins centered on [Fe/H] = $-0.5$, $-0.6$, and $-0.7$ contained 59, 54, and 34 stars, respectively. Whether this is an effect tied to real changes in the color-temperature relations at lower [Fe/H] or an artifact of the $T_{\text{eff}}$ merger process for stars of low metallicity remains unknown and, for our purposes, irrelevant. To define our final color-$T_n$ relation, we have restricted the metallicity range to [Fe/H] = $+0.6$ to $-0.5$, including only stars with $T_{\text{eff}}$ between 4200 K and 7000 K. Using $T_n$ as described in this section, polynomial fits to $B-V$ were made to 1759 points using multiple combinations of $T_n$ and [Fe/H]. As expected from the earlier discussion, no terms above a quadratic in $T_n$ were found to be statistically significant. For [Fe/H], only the linear term and no cross terms involving [Fe/H] and $T_n$ were retained. After removing 13 stars with final residuals in $B-V$ greater than 0.10 mag, the remaining 1746 stars produce the following color-$T_n$ relation:

$$B-V = 0.653(\pm 0.001) - 1.995(\pm 0.008)T_n + 1.933(\pm 0.076)T_n^2 + 0.148(\pm 0.003)[\text{Fe/H}].$$

The solar color is found to be $B-V = 0.653$, slightly larger than from the solar metallicity sample alone, but the same within the errors; the standard deviation among the residuals in $B-V$ is again 0.021 mag.
3.2. The $M_V, (B-V) - [\text{Fe/H}]$ Relations

Unlike a cluster CMD, the field star sample at a given metallicity is composed of stars with a wide range in age and therefore an increasing spread in $M_V$ at higher temperatures. To minimize the impact of age, the obvious solution is to select stars of low enough mass and temperature to ensure that evolution has a negligible impact. For stars with metallicity as high as NGC 6791, the predicted turnoff color for an adopted reddening of $E(B-V) = 0.15$ is near $(B-V)_0 = 0.75$; a reddening at 0.10 implies $(B-V)_0$ near 0.80. Therefore, to define the unevolved main sequence at high [Fe/H], we need to restrict our sample to stars redder than $B-V = 0.80$. As the metallicity declines, this boundary will shift to bluer colors. The down side of this restriction is the increasing fraction of stars with larger errors in observed $B-V$ among the cooler dwarfs. To illustrate the effect, we plot in Figure 4 the CMD for all stars between [Fe/H] $= +0.05$ and $-0.05$, including error bars in $M_V$ from parallax measurements and in $B-V$ from the published photometric uncertainties. While the mean relation with increasing $B-V$ is well defined, the fraction of stars with photometric scatter above 0.05 mag also increases.

By contrast, we duplicate the same data in Figure 5 using $B-V$ based upon the transformation between the spectroscopically determined $T_{eff}$ and $B-V$ as derived in the previous section. Note that this more general transformation includes a small adjustment for the individual values of [Fe/H]. The error bars in $B-V$ are now based upon the calculated uncertainties in $T_{eff}$. For the CMD in the range of interest, $B-V$ redder than 0.85, the scatter in color is reduced with no apparent change in the mean relation among the stars along the lower main sequence. Excluding stars with color residuals above 0.10 mag, for 144 stars redder than $B-V = 0.72$, the weighted average of the color residuals, in the sense of 

\[ (B-V)_{\text{OBS}} - (B-V)_{\text{PRE}} \text{ is } +0.006 \pm 0.009. \]

The cut at $B-V = 0.90$ reduces the sample to 62 stars with an average residual of $-0.002 \pm 0.021$. Because the scatter in $B-V$ based upon the converted $T_{eff}$ is inherently smaller than the scatter among the observed colors and because it allows us to include in the CMD analysis stars that do not have reliable, if any, photometric measures on the Tycho-2 BV system, we will use the transformed $T_{eff}$ estimates as our $B-V$ source for the CMD.

As a first probe of the unevolved main-sequence relation, we have attempted a polynomial fit using only the solar metallicity data of Figure 5. The initial color range was restricted to $B-V = 0.72$ to 1.30 and stars with excessive deviations from the mean trend were eliminated. The preliminary cut adopted was 0.50 mag, but this was reduced to 0.35 mag after an initial iteration of the fit. Of 132 stars that initially met the color criterion, 4 were eliminated from the final fit. Tests of the polynomial fit showed that the terms beyond the linear were statistically irrelevant. The final main-sequence relation for the solar metallicity stars is

\[ M_V = 1.75(\pm 0.04) + 4.77(\pm 0.05)(B-V). \]

The dispersion in the residuals about the mean relation is $\pm 0.13$ mag. If we extrapolate the relation to an assumed solar color of $B-V = 0.652$, $M_V$ becomes 4.86, only slightly fainter than our adopted value of $M_{V,0} = 4.84$. Given the age of the Sun, one might expect a larger differential compared to the unevolved main sequence. We will return to this issue in § 4. The linear fit is shown as a solid line in Figure 5.

To derive the unevolved main-sequence relation for a range of [Fe/H], our initial sample of 1982 stars with reliable abundances and parallax measures was restricted to single, non-variable stars with [Fe/H] between $-0.50$ and $+0.50$, $M_V$
fainter than +4.5, $B - V$ bluer than 1.30, and $B - V$ redder than $B - V = 0.2[\text{Fe/H}] + 0.72$, cutting the data set to 533 stars. Binaries were tagged from the high-dispersion spectra obtained to derive the abundances in each study included in the composite catalog, while significant variability was checked through the Tycho catalog. After a preliminary polynomial fit to the data, including tests of up to cubic terms in $B - V$, quadratic terms in [Fe/H], and multiple cross-term combinations, only the linear terms in $B - V$ and [Fe/H] survived, the former result being consistent with what was found for the solar sample. All stars with residuals in $M_V$ greater than 0.35 mag were excluded and the final calibration repeated. The final sample consisted of 501 stars with a standard deviation among the residuals in $M_V$ of 0.135 mag. The derived polynomial function is

$$M_V = 1.55(\pm 0.05) + 5.00(\pm 0.03)(B - V) - 1.07(\pm 0.02)[\text{Fe/H}].$$

For solar metallicity stars, the slope of the unevolved main sequence based upon the derived relation is somewhat steeper than found previously, leading to stars that are 0.03 mag brighter at $B - V = 0.75$, but 0.09 mag fainter at $B - V = 1.25$. If we demand that the main-sequence relation have the same slope at solar metallicity as derived from the solar sample, the relation becomes

$$M_V = 1.75(\pm 0.01) + 4.77(B - V) - 1.04(\pm 0.02)[\text{Fe/H}].$$

The standard deviation among the 502 residuals increases slightly to 0.137. For reasons that will be discussed in §4, it is probable that even the slope of 4.77 is too steep for the true, unevolved main sequence.

### 3.3. Comparison to Previous Determinations

As stated in §1, a primary goal of this investigation is to test the sensitivity of the absolute magnitude of unevolved cooler main-sequence stars as $B - V$ and [Fe/H] are varied. Qualitatively, our data relations confirm that, within the current uncertainties, there is no statistically significant evidence for a variation in the ratio, $\Delta M_V/\Delta [\text{Fe/H}]$, with $B - V$ or [Fe/H] between $+0.5$ and $-0.5$ for truly unevolved stars. The constant ratio is consistent with the work of Percival et al. (2003), using intermediate-band photometric abundances for an order-of-magnitude smaller sample, and the prediction of stellar isochrones used to define the cluster distance scale as detailed in Pinsonneault et al. (2004) and An et al. (2007), but disagrees weakly with the analysis of a comparable dataset of hotter stars using abundances derived from $UBV$ indices (Karataş & Schuster 2006). However, the scatter in the results becomes apparent when the specific values of the slope are compared.

Because the technique adopted is an expanded version of the technique laid out by Percival et al. (2003), we will translate our change in $M_V$ with [Fe/H] at a given $B - V$ to a change in $B - V$ with [Fe/H] at a given $M_V$ to allow a direct comparison with their result. Since the main-sequence slope at a given [Fe/H] is assumed to be linear, these comparisons are equivalent. From 54 stars, Percival et al. (2003) find $\Delta (B - V)/\Delta [\text{Fe/H}] = 0.154$, with a very weak sensitivity of the final value to the adopted main-sequence slope. With a main-sequence slope of 4.77, this translates into $\Delta M_V/\Delta [\text{Fe/H}] = 0.73$. For our two relations above, $\Delta (B - V)/\Delta [\text{Fe/H}] = 0.214$ and 0.218, respectively. Changing the fixed main-sequence slope to 4.5 and 5.5 produces $\Delta (B - V)/\Delta [\text{Fe/H}] = 0.217$ and 0.208, respectively, confirming the insensitivity of the color gradient to the adopted main-sequence slope but clearly indicating that the value of Percival et al. (2003) underestimates the metallicity effect by 29%. Note that because $\Delta (B - V)/\Delta [\text{Fe/H}]$ is constant, $\Delta M_V/\Delta [\text{Fe/H}]$ will vary directly with the adopted slope for the main sequence.

At the other end of the scale, Pinsonneault et al. (2003, 2004) have constructed empirically-adjusted isochrones to use in defining cluster distances through main-sequence fitting. The isochrones are built upon the models of Sills et al. (2000) and transformed to the observational plane using a variety of $T_{eff}$-color relations that are empirically adjusted to ensure an ideal match to the Hyades. Pinsonneault et al. (2004) include a comparison of the impact of the various $T_{eff}$-color relations on the absolute magnitude of a star at a given color as [Fe/H] is varied between $-0.3$ and $+0.2$ over the color range $B - V = 0.40$ to 1.0. For their models, the zero points of the absolute magnitude-[Fe/H] relations vary with the $T_{eff}$-color relation adopted, but the slopes are extremely consistent: $\Delta M_V/\Delta [\text{Fe/H}] = 1.4$, 35% larger than derived in this investigation and almost double that found by Percival et al. (2003). The equivalent $\Delta (B - V)/\Delta [\text{Fe/H}] = 0.29$. It should be emphasized that these numbers are tied to a specific set of models. A check of the many available online sources for theoretical isochrones shows that among the cooler dwarfs, there is significant variation in $\Delta (B - V)/\Delta [\text{Fe/H}]$ from one set of isochrones to another and, among some sets, the slope can be found to vary with both $B - V$ and [Fe/H].

The final comparison is with the recent work of Karataş & Schuster (2006). This study derives a new calibration of metallicity as a function of the color excess, $b(U - B)_{0.6}$, and, using stars with Hipparcos parallaxes on the original system, defines an $M_V$, $(B - V)$, $b(U - B)_{0.6}$ relation. The range in $B - V$ for the sample is bluer than ours, covering approximately $B - V = 0.4$ to 0.9, with evolved stars eliminated using a magnitude cut roughly parallel to the unevolved main sequence in the CMD. The absolute magnitude calibration is constructed and tested with a number of sample variations; relations are derived for $M_V$ as a function of $B - V$ and $b(U - B)_{0.6}$ using all stars with parallax and for $\Delta M_V$ as a function of $B - V$ and $b(U - B)_{0.6}$ using only Hyades dwarfs and field halo dwarfs. The uniform pattern among the various relations is that the slope,
ΔM\textsubscript{V}/Δ[Fe/H], varies with both B − V and [Fe/H]. Using the final differential calibration applied at B − V = 0.70 between [Fe/H] = +0.39 and −0.50, the average ΔM\textsubscript{V}/Δ[Fe/H] = 0.63, smaller than the value found by Percival et al. (2003). The result is even more extreme if the change is translated to a B − V ratio because the average main sequence slope defined by the main-sequence data of Karataş & Schuster (2006), weighted toward the bluer end of the main sequence, is 5.5, leading to Δ(B − V)/Δ[Fe/H] = 0.115.

4. THE OPEN CLUSTER DISTANCE SCALE

4.1. The Hyades and M67

Independent of the sensitivity of M\textsubscript{V} with [Fe/H] at a given B − V, does the absolute scale of the parallax sample generate plausible distance moduli for well-studied clusters? The obvious initial test case is the Hyades. Because of the respectable number of stars with a metallicity approaching that of the Hyades, we can follow an approach similar to that for the solar metallicity sample in the previous section. All stars with [Fe/H] within ±0.05 dex of the Hyades metallicity have been identified, adjusted in M\textsubscript{V} to the metallicity of the Hyades, and then compared to the Hyades main-sequence relation.

What is the Hyades metallicity? We have identified Hyades stars within our catalog through a comparison with two sources of Hyades members. From 18 Hyades stars that overlap with the spectroscopic catalog of Taylor (2005), [Fe/H] = 0.129 ± 0.029 on our system. The average offset of −0.031 dex is essentially identical to the predicted value of −0.032 dex from the relation for the entire catalog as derived in § 2. From 10 stars that overlap with the catalog utilized by Pinsonneault et al. (2004), [Fe/H] = 0.137 ± 0.040. We will adopt [Fe/H] = +0.13 for the Hyades as defined by our spectroscopic scale.

For the Hyades main sequence we have adopted the mean relation derived by Pinsonneault et al. (2004), equivalent to a parallax-defined true modulus of \((m − M)\text{H} = 3.33\), with two modifications. As detailed in § 2, the Pinsonneault et al. (2004) photometric scale exhibits small offsets relative to the converted Tycho-2 BV system that defines our sample. We have adjusted the Pinsonneault et al. (2004) Hyades mean relation to our scale by adding 0.009 mag in B − V and decreasing V by 0.020 mag. This is equivalent to shifting the original relation in M\textsubscript{V} at a given color by −0.06 mag.

Figure 6 shows a plot of the residuals, in the sense of \((M\textsubscript{V}\text{Hyades} − M\textsubscript{V}\text{Field})\), as a function of B − V for parallax stars with [Fe/H] between +0.08 and +0.18, adjusted in M\textsubscript{V} to [Fe/H] = +0.13 using ΔM\textsubscript{V}/Δ[Fe/H] = 1.04. A color-dependent asymmetry among the residuals is obvious. On the negative side of the distribution, there is an extremely sharp edge to the residuals near −0.15 mag which should be indicative of the scatter in the absolute magnitudes due to the combination of parallax errors, photometric errors, and temperature errors translated into errors in B − V. On the positive side of the residuals, there is a clear increase in the range of the scatter as one moves from the red to the blue side of the plot. The asymmetry is real and has two primary sources. First, stars that are increasingly older than the Hyades will be increasingly brighter than the Hyades main sequence at a given color, with the degree of the offset decreasing for redder stars. Second, the color distribution of the parallax sample is more heavily weighted toward bluer stars. While we can set a boundary to the sample at the blue end to exclude stars that are more likely to show the effects of evolution, random errors in the temperature estimates, translated into random errors in B − V, will preferentially shift more of the blue stars toward the red than vice versa. The impact of this asymmetry will be to steepen the derived main-sequence slope obtained by fitting a straight line to field star parallax data, as done in § 3, unless the blue color limit for the sample is set well redward of the expected evolutionary cutoff. This effect, in part, provides a plausible explanation for why the main-sequence relation for solar metallicity stars, as derived in § 3, passes so close to the position of the evolved Sun at solar B − V. For the adjusted Hyades mean relation, a linear fit to the data between B − V = 0.73 and 1.30 produces

\[M\textsubscript{V} = 1.87(\pm0.005) + 4.53(\pm0.025)(B − V).\]

As expected from the trend in Figure 6, the slope is shallower than found for the sample as a whole or for the solar sample. If we adopt the slope of the Hyades relation as the correct match to the unevolved main sequence, the revised relation for field stars as a function of metallicity becomes

\[M\textsubscript{V} = 2.00(\pm0.01) + 4.53(B − V) − 0.98(\pm0.02)[Fe/H].\]

It should be emphasized that while the inclusion of stars that are too blue can have a significant impact on the derived slope of the main-sequence relation, the mean residuals in ΔM\textsubscript{V} are only mildly affected. Using a cutoff of B − V = 0.70, bluer than the value of B − V = 0.75 adopted for stars of Hyades metallicity in defining the main-sequence relation in § 3, and excluding stars with absolute residuals greater than 0.5 mag, the remaining 105 stars generate a weighted average ΔM\textsubscript{V} of +0.017 ± 0.005 (SEM). If we draw the color cut for the sample at B − V = 0.90, the comparable offset is −0.006 ± 0.010 (SEM) from 51 stars, confirming that the revised Hyades relation is an excellent match to the field stars of comparable metallicity. Note that the consistency between the M\textsubscript{V} scales from the Hyades analysis by Pinsonneault et al. (2004) and the current field star sample is equivalent to stating that had we matched the unevolved Hyades apparent main sequence to our field stars of comparable metallicity, the derived modulus would be 3.33, independent of the previous determination.

Before moving on to M67, the trend seen in Figure 6 may supply an explanation for the significantly smaller ratio of ΔM\textsubscript{V} with [Fe/H] found by Karataş & Schuster (2006), 0.63 as opposed to 0.98. The turnover dwarf sample is identified
by a star’s position in the CMD relative to a line that runs roughly parallel to the main sequence between $B - V = 0.3$ to 0.65. For redder stars, the sample limit is defined by a 12 Gyr isochrone with $[\text{Fe/H}] = -0.3$. For the bluer stars, a fixed boundary in $M_V$ at a given color will allow a larger range of evolved turnoff stars to be included in the sample as $[\text{Fe/H}]$ decreases. Thus, the shift in the average $M_V$ of the sample above the unevolved main sequence at a given color will increase as $[\text{Fe/H}]$ decreases. Qualitatively, this should steepen the slope of the mean main sequence with decreasing $[\text{Fe/H}]$ while compressing the range in $M_V$ at a given color for a given range in $[\text{Fe/H}]$, leading to a smaller derived ratio of $\Delta M_V$ with $[\text{Fe/H}]$.

As a second test of the revised main-sequence relation, we have adjusted all parallax stars between $[\text{Fe/H}] = -0.05$ and 0.05 to an assumed $[\text{Fe/H}] = 0.0$ and calculated residuals relative to the adjusted Hyades relation shifted to $[\text{Fe/H}] = 0.0$. Excluding all stars with absolute residuals greater than 0.5 mag, for $B - V$ greater than 0.72, 130 stars produce $\Delta M_V = +0.038 \pm 0.005$ (SEM). Placing the color cut at $B - V = 0.90$ reduces the offset to $+0.033 \pm 0.008$ (SEM) for 57 stars. The conclusion is that application of the mean relation at solar metallicity to a cluster with the same properties as the nearby solar field stars will lead to an apparent modulus that potentially is too small by 0.03 mag. With this caveat in mind, we can now derive the apparent modulus for the well-studied open cluster, M67.

Unlike the field and Hyades stars included in the current study, M67 is reddened and not tied directly to the spectroscopic scale or the Tycho-2 $BV$ system. Within the extensive literature dealing with the fundamental properties of M67, the two most relevant recent studies are An et al. (2007) and Pasquini et al. (2008). The former builds upon the approach of Pinsonneault et al. (2003, 2004) to construct semiempirical isochrones in multiple colors to simultaneously define the reddening, metallicity, and distance for a number of open clusters, including M67. The final values derived for M67 are $E(B - V) = 0.042$, $[\text{Fe/H}] = -0.02$, and $(m - M) = 9.74$. The latter article uses high-resolution spectra and line-depth ratios, $H_\alpha$ lines, and Li measures to identify stars within M67 that resemble solar analogs. With a derived solar color of $B - V = 0.649$, $E(B - V) = 0.041$ and $[\text{Fe/H}] = +0.01$, Pasquini et al. (2008) find $(m - M) = 9.78$. If we fix $E(B - V)$ at 0.041 and set $[\text{Fe/H}] = 0.00$, the two investigations imply $(m - M) = 9.76$ and 9.77, respectively.

The last piece of the puzzle requires a link between the adopted $BV$ photometry for M67 (Sandquist 2004) and that of the transformed Tycho-2 colors. The only direct comparison is that of Joner et al. (2008) where the Hyades, M67, and transformed Tycho-2 data are coupled through SAAO observations, all ultimately tied to SAAO E-region standards. Within the uncertainties, the multiple comparisons imply that the color offsets required to transform the Johnson & Knuckles (1955) Hyades
Data to the *Tycho-2*/SAAO system are similar to those applicable to the Sandquist (2004) *BV* data of M67 tied to the Johnson system through the standards of Stetson (2000) and Landolt (1992). We therefore have added 0.009 mag to the *B − V* indices of M67. For *V*, a direct link between the M67 photometry of Sandquist (2004) and the *Tycho-2*/SAAO system is unavailable. Standards for the M67 CCD calibration are from the systems of Stetson (2000) and Landolt (1992); comparisons with other M67 sources indicate that the Sandquist (2004) *V* magnitudes are a match to this standard system at the ±0.004 mag level. By default, no adjustment is made to the M67 *V* photometry.

From the compilation of Sandquist (2004), we have selected all single-star probable members of M67 with adjusted and reddening-corrected (*B − V*)0 above 0.75. From 38 stars, giving all individual moduli equal weight, the average apparent modulus for M67 is (*m − M*) = 9.807 ± 0.014 (s.e.m). With weighting based upon the inverse square of the uncertainty in *V*, including the effects of the color errors, the apparent modulus rises slightly to 9.820 with the uncertainty cut in half. If we were to fit the M67 main sequence directly to field stars of comparable metallicity, i.e., stars between [Fe/H] = −0.05 and 0.05 adjusted in *M*V to an adopted value of [Fe/H] = 0.00, the moduli would be increased by 0.033 mag, or (*m − M*) = 9.84 for the unweighted average. With *AV* = 3.1 *E*(B − V), this becomes (*m − M*)0 = 9.71 ± 0.02 (SEM), taking the uncertainty in the reddening into account.

Is the offset between the current value for the modulus and those of An et al. (2007) or Pasquini et al. (2008) significant? The errors quoted in the previous paragraph are the internal errors defined by the scatter within the photometry. Because the comparisons are made under the same assumptions for the reddening and metallicity, at a systematic level, the dominant source of uncertainty remains the size and applicability of the color adjustment to transfer the *B − V* system of Sandquist (2004) to the *Tycho-2* system that defines the field stars and the Hyades. If we adopt ±0.009 as a plausible estimate for the uncertainty in the differential color correction, this alone leads to an uncertainty in (*m − M*) of ±0.040.

There is one potential source of a systematic offset between the distance scale of An et al. (2007) and ours. The main sequences used to fit the cluster sequences are tied to theoretical isochrones empirically modified to match the Hyades at an adopted [Fe/H] = +0.13 and (*m − M*)0 = 3.33; the Hyades mean relation transferred to the *Tycho-2* system is an excellent match to field stars of comparable metallicity in our database. When shifted in *M*V at a given *B − V* using Δ*M*V/Δ[Fe/H] = 0.98, the Hyades relation overshoots the field star solar sample by a modest amount, i.e., it is too faint compared to the field stars, implying that our slope over this short distance is too large. However, as discussed previously, the isochrones employed by An et al. (2007) predict Δ*M*V/Δ[Fe/H] = 1.4, though the trends with metallicity for individual clusters in

An et al. (2007) indicate values between 1.2 and 1.4. If the isochrone sequences defining the main-sequence relations predict too great a change in *M*V/ with [Fe/H] relative to the Hyades, for a shift in [Fe/H] of −0.13 dex and Δ*M*V/Δ[Fe/H] = 1.25, the main sequence matched to M67 will be systematically too faint by 0.035 mag relative to our system and 0.065 mag too faint compared to the field stars.

4.2. The Distance to NGC 6791

While the old open cluster, NGC 6791, is not the primary focus of this investigation, it does have a valuable role to play as a test of the more extreme limits of the field star main-sequence calibration. M67 and the Hyades are both well-placed among the richly populated metallicity distribution and differentially separated by only a modest shift in [Fe/H]. By contrast, current metallicity estimates from photometric and spectroscopic techniques (Origlia et al. 2006; Carraro et al. 2006; Gratton et al. 2006; Carretta et al. 2007; Anthony-Twarog et al. 2007; Boesgaard et al. 2009) generate a weighted average of [Fe/H] = +0.39 ± 0.06. Moreover, while the exact age of the cluster remains uncertain due to questions regarding the reddening and metallicity, the reddened color of the stars within the vertical turnoff will lie between (*B − V*)0 = 0.74 and 0.84 for *E*(B − V) = 0.15 to 0.10. Minimally, one should choose only stars with (*B − V*)0 redder than 0.90 to derive the distance to NGC 6791. For [Fe/H] ≥ 0.30, only 10 stars in our sample meet the criterion. Therefore, any attempt to test the cluster distance scale at the metal-rich end will require a shift of field stars from lower [Fe/H] to generate a statistically significant sample.

Figure 7 shows the residuals in a comparison between the field star data, adjusted to [Fe/H] = 0.39, and the shifted Hyades relation using Δ*M*V/Δ[Fe/H] = 0.98. Stars with absolute residuals above 0.50 mag have been excluded. Filled circles are stars with [Fe/H] above 0.29, while open circles are stars between [Fe/H] = 0.25 and 0.29. While the scatter is larger, the pattern that emerges is similar to that of Figure 6. The bluer (*B − V* below 0.90) stars are systematically brighter than predicted from the adjusted Hyades relation, as expected if these stars are exhibiting the effects of evolution away from the main sequence. By contrast, the redder stars approximately scatter around mean relation, though there may be weak evidence for a slope among the residuals, implying that the Hyades relation is too shallow compared to the very metal-rich field stars. We will return to this point in § 5.

As an initial check, the 10 stars between [Fe/H] = 0.30 and 0.47 with (*B − V*)0 above 0.9 have been compared to the adjusted Hyades relation and individual residuals calculated for each field star relative to the Hyades using weighting by the inverse errors in *M*V, including the propagated errors from the uncertainty in the temperatures used to define the colors. For 10 stars, the average residual, in the sense of (*M*V/Hyades − *M*V/Field), is 0.002 ± 0.026 (SEM). If we lower the color limit to *B − V* = 0.85, the sample expands to 16 stars, but the mean
residual increases significantly to $0.091 \pm 0.018$ (SEM), a confirmation of the asymmetry in the distribution with decreasing color. If we keep the color limit at $B - V = 0.90$, but lower the field star limit to $[\text{Fe/H}] = 0.25$, the mean residual is $0.004 \pm 0.019$ (SEM) from 19 stars, indicating that the field stars over the color range of $(B - V)_0 = 0.9$ to 1.1 shifted to the metallicity of NGC 6791 should predict the same distance modulus at the $\pm 0.02$ mag level as a direct comparison of the cluster to the adjusted Hyades relation over the same color range.

With the metallicity set, two key issues remain in establishing the comparison. Estimates of the cluster reddening have ranged from $E(B - V) = 0.08$ to 0.26, but recent work based upon a variety of techniques has limited the plausible range to $E(B - V) = 0.10$ to 0.15 (Anthony-Twarog et al. 2007). As a compromise, we will adopt $E(B - V) = 0.125 \pm 0.025$.

The second issue is the perennial question of the photometric system of the cluster $BV$ data. The most comprehensive and internally reliable photometric sample available to date is that of Stetson et al. (2003) which is tied to the same standard star network adopted by Sandquist (2004) in the photometric calibration of M67. For internal consistency, we will apply the same color offset of $+0.009$ mag to the NGC 6791 photometry as we did for M67 and the Hyades. No adjustment will be made to $V$.

For the cluster CMD relation, we will use the fiducial relation compiled by Sandage et al. (2003) limited to an adjusted $B - V$ below 1.25 since our reddest field star is at $(B - V)_0 = 1.09$ and the reddening is $E(B - V) = 0.125$.

The results are illustrated in Figure 8, which has the same symbol definitions as Figure 7, using the fiducial relation of NGC 6791 with $(m - M) = 13.45$. For the same 19 stars redder than $B - V = 0.90$, the average residual in $M_V$ is 0.005. What is different compared to the Hyades comparison is the removal of the apparent asymmetry in residuals for bluer stars and the weak evidence for a slope at redder colors. This change indicates that the fiducial relation for NGC 6791 is a better match to the CMD position of the average field star of high metallicity than the Hyades, as one would expect if the typical star in the sample is older than the Hyades cluster but younger than NGC 6791. The possibility that the field stars may be younger than NGC 6791 comes from the sharp turn toward negative residuals for $B - V$ bluer than 0.85. With the adopted cluster reddening of $E(B - V) = 0.125$, this color marks the start of the rapid vertical rise in the cluster turnoff, thereby leading to anomalously bright absolute magnitudes relative to less evolved stars.

For a fixed value of the reddening and metallicity, taking into account the potential uncertainty in the color offset applied to the fiducial relation for NGC 6791, the apparent modulus is $(m - M) = 13.46 \pm 0.049$. Sandage et al. (2003) supply no estimate of the uncertainty in their fiducial relations. We have done an independent check on their relation by rederiving

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**Fig. 7**—Residuals between the field star data, adjusted to $[\text{Fe/H}] = 0.39$, and the shifted Hyades relation using $\Delta M_V/\Delta[\text{Fe/H}] = 0.98$. Filled circles are stars with $[\text{Fe/H}]$ above 0.29, while open circles are stars between $[\text{Fe/H}] = 0.25$ and 0.29.
the fiducial curve over the color range of interest using only stars in the core of NGC 6791. Stars between $B - V = 0.96$ and 1.26 were identified and retained if they fell within $\pm 0.30$ mag of the Sandage et al. (2003) relation. Stars between $B - V = 0.96$ and 1.02 were adjusted in $V$ to the central color of the bin using the slope of the main-sequence relation as defined by Sandage et al. (2003) and counted into bins 0.06 mag wide in $V$. The resulting histogram was fit with a Gaussian profile to define the peak of the distribution. The bin was then shifted redward in $B - V$ by 0.04 mag and the process repeated. The fiducial points were compared to those of Sandage et al. (2003); the average difference for the 7 points (in the sense of Sandage et al. 2003—this work), is $0.009 \pm 0.020$, indicating that the Sandage et al. (2003) fiducial relation is not a significant source of uncertainty. If we allow for $\pm 0.025$ uncertainty in $E(B - V)$ and $\pm 0.06$ uncertainty in [Fe/H], the combined internal and external errors imply $(m - M) = 13.46 \pm 0.15$, with the dominant source of error being the reddening uncertainty. If the true modulus is calculated, the error bars are reduced because the reddening effect is partially compensated by the correlated change in $A_V$; the true modulus is $(m - M)_0 = 13.07 \pm 0.09$, internal and external errors included.

How does this result compare with current estimates? The definitive value for NGC 6791 at present is that of Grundahl et al. (2008) based upon analysis of the cluster eclipsing binary, V20. The beauty of the technique is that the masses and radii can be determined independent of the reddening and metallicity and age estimation can be carried out through comparison to isochrones in the mass-radius plane without requiring a transformation of the theoretical parameters to the observational plane. Conversion of the stellar parameters to luminosities and distances still requires an assumed reddening and metallicity so that the observed stellar colors can be translated into effective temperatures and luminosities converted to $M_V$. Adopting $E(B - V) = 0.15$ and [Fe/H] = +0.40, Grundahl et al. (2008) derive $(m - M)_0 = 13.00 \pm 0.10$; if we had adopted the same reddening and metallicity, our result would have been $(m - M)_0 = 13.13 \pm 0.09$.

5. SUMMARY AND CONCLUSIONS

Distance determination for field stars and clusters remains a primary observational objective for those interested in understanding stellar and Galactic evolution. Ideally, parallaxes for a large sample of nearby stars with well-defined abundances and temperatures/colors would allow one to map the impact on the absolute magnitude of varying the abundance of a star of a given temperature/color. With this information in hand, determining the distance to any cluster with reliable abundance and reddening information becomes a straightforward task. Unfortunately, the
reality is somewhat different. While reliable parallaxes are available for a large sample of field stars and most have colors on the Tycho-2 system, the critical component missing from the picture has been a comparable catalog of precise metallicity estimates. The observational approaches to defining the ratio of $\Delta M_V$ with [Fe/H] (Kotoneva et al. 2002; Percival et al. 2003; Karataş & Schuster 2006) have relied upon photometric abundances coupled to a spectroscopic subset of the sample. The slope of the relation is either assumed to be constant with [Fe/H] and color and/or tested through the use of theoretical isochrones. For Kotoneva et al. (2002) and Karataş & Schuster (2006), the baseline used to define the slope is extended to [Fe/H] below $-1$ through the inclusion of halo dwarfs and/or globular clusters. The data set of Karataş & Schuster (2006) extends to stars where evolution off the main sequence is significant and may produce a selection bias in the mean absolute magnitude with [Fe/H].

Starting with a data set of almost 2000 stars with reliable parallaxes, spectroscopic abundances, and homogeneous colors, we have reduced the sample to approximately 500 stars between $[\text{Fe}/\text{H}] = -0.5$ and $+0.5$ with colors red enough that evolution off the main sequence should be negligible. However, as evidenced by the old, metal-rich turnoff stars in NGC 6791, even our metallicity-dependent cutoff allows some significantly evolved stars into the mix. Over the primary color range of interest, $B - V = 0.75$ to $1.15$, the ratio with metallicity, $\Delta M_V / \Delta [\text{Fe}/\text{H}] = 0.98 \pm 0.02$ with no evidence for a color or $[\text{Fe}/\text{H}]$ dependence. Because this value assumes a universal slope of 4.53 for the main sequence with $B - V$, it is probably better to define the relation in terms of $\Delta (B - V) / \Delta [\text{Fe}/\text{H}] = 0.213 \pm 0.005$, which is only weakly dependent upon the adopted slope of the main sequence. The concern that the slope of the main sequence with $B - V$ varies with [Fe/H] is real and not dependent upon the multiple sets of theoretical isochrones which are inconsistent on this point. From the observed cluster sequences analyzed in this investigation, the Hyades has a main-sequence slope of 4.5; over the same color range, the fiducial relation for NGC 6791 has a slope of 5.4.

With the field star relations in hand, reliable estimation of cluster distances should follow. The challenge in this phase is ensuring that the clusters are on the same photometric color and spectroscopic abundance system as the field stars which, for more distant objects, also includes accurate reddening estimation. Because our catalog includes Hyades stars and the Hyades is assumed to be reddening-free, the only potential source of controversy is the photometric scale. Our analysis confirms what has been found by others (Joner et al. 2006; An et al. 2007; Joner et al. 2008). The Johnson & Knuckles (1955) photometric system in the Hyades is offset from the SAAO/E-region standards that define the transformed Tycho-2 system by approximately $-0.009$ mag and $0.02$ mag in $B - V$ and $V$, respectively. When these corrections are applied to the fiducial relations for the Hyades (Pinsonneault et al. 2004), the cluster produces an excellent match to the field stars at the appropriate [Fe/H] with the cluster modulus set at $(m - M)_0 = 3.33$.

The second test of the system is a match to M67. Because this cluster and NGC 6791 are two steps further removed from the field star sample in that there are no cluster stars with spectroscopic abundances in our catalog and the photometric data are not directly comparable to the Tycho-2 system, we have applied the same color offset found for the Hyades to the cluster data and made differential comparisons based upon a commonly adopted reddening and metallicity estimate. If the M67 data of Sandquist (2004) are matched to the Hyades fiducial relation shifted to $[\text{Fe}/\text{H}] = 0.00$, the apparent modulus of the cluster becomes $9.81 \pm 0.02$; if a match is made directly to the field stars of identical [Fe/H], the modulus increases by $0.03$. The comparable value from An et al. (2007) and Pasquini et al. (2008) is $9.765$. Note that part of the offset relative to An et al. (2007) may be a product of the elegant but hybrid approach that defines the main-sequence relations using a mix of theoretical isochrones with color corrections defined by cluster data. The isochrones used by An et al. (2007) define a ratio, $\Delta M_V / \Delta [\text{Fe}/\text{H}]$, over the color range of interest that is typically 1.4, significantly larger than our derived value near 1. With the Hyades CMD position fixed via parallax, differential shifts with metallicity then define the predicted position of clusters like M67. Even for a change in [Fe/H] of 0.13, an overestimate of 25% in the slope would lead to an underestimate of the distance by 0.03 mag.

Finally, at the extreme end of the age and metallicity scales among open clusters, the distance to NGC 6791 is derived using field stars of comparable metallicity. Initial comparisons using the adjusted Hyades relation exhibited evidence for significant evolutionary effects off the main sequence for the bluer stars in our sample, implying that the average star in the comparison is older than the Hyades. This trend virtually disappears when the data are compared to the fiducial relation for the cluster. In fact, the bluest stars in the field are systematically fainter than the stars in the cluster at the same color, indicating that they are, as expected, younger than NGC 6791. Adopting $[\text{Fe}/\text{H}] = 0.40$ and $E(B - V) = 0.15$, we find $(m - M)_0 = 13.13 \pm 0.09$. By comparison, the definitive value tied to analysis of the eclipsing binary, V20, using the same parameters is $(m - M)_0 = 13.00 \pm 0.10$. The significance of the difference is marginal, especially given the underlying question of the photometric zero points in $V$ and $B - V$.

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