Shape and Rotational Motion Models for Tumbling and Monolithic Asteroid 2012 TC₄:
High Time Resolution Light Curve with the Tomo-e Gozen Camera

Seitaro Urakawa¹, Ryou Ohsawa², Shigeyuki Sako², Shin-ichiro Okumura¹, Yuri Sakurai³, Jun Takahashi⁴, Kazuyoshi Imamura⁵, Hiroyuki Naito⁶, Fumitake Watanabe⁶, Ryoma Nagayoshi⁶, Yasuhiro Murakami⁶, Ryo Okazaki⁶, Tomohiko Sekiguchi⁷, Masateru Ishiguro⁸, Tatsuhiro Michikami⁹, and Makoto Yoshikawa¹⁰

¹ Japan Spaceguard Association, Bisei Spaceguard Center 1716-3 Okura, Bisei, Ibara, Okayama 714-1411, Japan; urakawa@spaceguard.or.jp
² Institute of Astronomy, Graduate School of Science, The University of Tokyo, 2-21-1 Osawa, Mitaka, Tokyo 181-0015, Japan
³ Department of Earth Science, Okayama University, 1-1-1 Kita-ku Tsurumihara, Okayama 700-8530, Japan
⁴ Center for Astronomy, University of Hyogo 407-2 Nishiguchi, Sayo, Hyogo 679-5313, Japan
⁵ Anan Science Center, 8-1 Nagakawa Kamikatsu Minami-Kawabuchi, Anan, Tokushima 779-1243, Japan
⁶ Nayoro Observatory, 157-1 Nishin, Nayoro, Hokkaido 096-0066, Japan
⁷ Asahikawa Campus, Hokkaido University of Education, 9 Hokumon, Asahikawa, Hokkaido 070-8621, Japan
⁸ Department of Physics and Astronomy, Seoul National University, 1 Gwanak-ro, Gwank-gu, Seoul 08826, Republic of Korea
⁹ Faculty of Engineering, Kindai University, Hiroshima Campus, 1 Takaya Umenobe, Higashi-Hiroshima, Hiroshima 739-2116, Japan
¹⁰ Institute of Space and Astronautical Science, Japan Aerospace Exploration Agency, 3-1-1 Yoshinodai, Chuo-ku, Sagamihara, Kanagawa, 252-5210, Japan

Received 2018 November 15; revised 2019 February 7; accepted 2019 February 21; published 2019 March 26

Abstract

We present visible and near-infrared observations of a near-Earth object (NEO), 2012 TC₄. The NEO 2012 TC₄ approached close to Earth at a distance of about 50,000 km in 2017 October. This close approach provided a practical exercise for planetary defense. This apparition was also an appropriate opportunity to investigate 2012 TC₄, which is a monolithic asteroid. We conducted the observation campaign of 2012 TC₄ using six small- and medium-sized telescopes. The multiband photometry analysis showed the taxonomic class of 2012 TC₄ to be an X type. In particular, we successfully obtained the high time resolution light curve of 2012 TC₄ with the Tomo-e Gozen camera, which is the world’s first wide-field CMOS camera, mounted on the 1.05 m Schmidt telescope at Kiso Observatory. The shape and rotational motion models of 2012 TC₄ were derived from the light curve. When 2012 TC₄ was assumed to be a triaxial ellipsoid, the rotational and precession periods were 8.47 ± 0.01 minutes and 12.25 ± 0.01 minutes, respectively, with the long-axis mode. This indicates that 2012 TC₄ is a tumbling and monolithic asteroid. The shape models showed the plausible axial lengths to be 6.2 ± 0.01 m and 8.0 ± 0.01 minutes, respectively, with the long-axis mode. This indicates that 2012 TC₄ is a fragment produced by an impact event. We also estimated the excitation timescale, which implied that the impact event happened within ~3 × 10⁵ yr and 2012 TC₄ has a fresh surface.

Key words: instrumentation: detectors – minor planets, asteroids: individual (2012 TC₄) – techniques: photometric

Supporting material: animation

1. Introduction

“Planetary Defense” or “Spaceguard” refers to a number of efforts against asteroid impact hazard. A spaceguard effort discovers near-Earth objects (NEOs), seeks their trajectory, and judges whether they will collide with Earth. Representative NEO survey projects are Pan-STARRS (the Panoramic Survey Telescope and Rapid Response System; Wainscoat et al. 2014; Chambers & Pan-STARRS Team 2016), Catalina Sky Survey (Larson et al. 1998; Christensen et al. 2014), and NEOWISE (Mainzer et al. 2011a). There are many other ground-based survey projects and future space plans, including ATLAS (Tonry 2011) and NEOCam (Mainzer & NEOCam Science Team 2017). In addition to survey observations, it is also important to reveal the physical properties of each NEO by obtaining information regarding the rotational period, rotational motion, shape, and taxonomic class. In the event of an impact hazard, such information can assist in the development countermeasure, such as a kinetic impactor (Cheng et al. 2018). Moreover, fostering better understanding of NEOs helps to elucidate the planetary formation processes, because NEOs have reflected the history of collision, destruction, and coalescence of small solar system bodies from the planet formation era. NEOs are also practically accessible objects by spacecraft. The physical properties that were estimated by the ground-based observations of asteroid (25143) Itokawa and asteroid (162173) Ryugu became essential information for the Hayabusa and Hayabusa-2 projects (Kaasalainen et al. 2003; Ostro et al. 2005; Kim et al. 2013; Ishiguro et al. 2014; Müller et al. 2017). Furthermore, the technological progress brought on by explorations provides new prospects, such as manned explorations and resource collections of NEOs (Abell et al. 2016). Exploration technology will also return to the spaceguard efforts as an impact avoidance technology. Hayabusa, Hayabusa-2, NEAR, and OSIRIS-Rex are representative NEO spacecraft. The exploration of Itokawa by Hayabusa revealed that Itokawa was covered with numerous boulders and possessed rubble-pile structures due to weak gravity constraints (Fujiiwara et al. 2006). An asteroid that consists of a single boulder is sometimes called a monolithic asteroid. The physical properties of a monolithic asteroid, which could be the smallest unit constituting a rubble-pile asteroid, can provide clues to clarify the formation process of boulders in destructive collisions. An asteroid’s rotational period is an important indicator to distinguish monolithic and rubble-pile asteroids. Although cohesive force might prevent the rotational breakup (Rozitis et al. 2014), most asteroids
rotating shorter than the period of 2.2 hr are considered to be monolithic asteroids because the fast rotation makes it difficult to keep the rubble-pile structure, due to the strong centrifugal force (Pravec & Harris 2000). Almost all monolithic asteroids are NEOs smaller than 200 m in diameter. Such NEOs are confirmed to be monolithic asteroids by measuring their rotational periods, immediately after being discovered by survey observations. Physical properties of monolithic asteroids, such as the taxonomic class and shape, are hardly determined, except for the rotational period and rough diameter. In order to estimate the taxonomic class, spectroscopic observations or multiband photometry is required. However, the small size and faintness of monolithic asteroids make it difficult to conduct spectroscopic observations, which demand sufficient brightness. In the case of the multiband photometry, since the brightness of monolithic asteroids inevitably changes owing to the fast rotation during the switching of the filter, we need to calibrate the brightness change appropriately. The calibration requires obtaining the accurate light-curve data that cover the whole rotational phase of the monolithic asteroids. Otherwise, we need to calibrate the monolithic asteroids with a multiband simultaneous camera without the switching of the filter. Spectroscopic observations and multiband photometry are not carried out immediately after the discovery of NEOs. Furthermore, the estimation of shape requires enough light-curve data that are obtained by observations of the asteroid from various directions. To observationally deduce the taxonomic class and shape of the monolithic asteroids, the closest day of approach of the target asteroid to Earth should be known in advance.

The purpose of our study is to obtain the shape and rotational motion model of an NEO, 2012 TC$_4$, from a high time resolution light curve. Furthermore, we deduce the taxonomic class of 2012 TC$_4$ with the visible and near-infrared color indices by multiband photometry. The NEO 2012 TC$_4$ was discovered by the Pan-STARRS on 2012 October 4 and approached Earth with a distance of $\sim$95,000 km on 2012 October 12. The rotational period and diameter were estimated to be 12.24 ± 0.06 minutes and 7–34 m, respectively (Polishook 2013). Therefore, 2012 TC$_4$ is supposed to be a monolithic asteroid. However, the shape and taxonomic class were not identified. In addition, the light curve of 2012 TC$_4$ was not fully explained by the period of 12.24 minutes. The NEO 2012 TC$_4$ approached Earth again on 2017 October. The closest approach distance was $\sim$50,000 km on 2017 October 12. This apparition was an appropriate observation opportunity to investigate the physical properties of a small monolithic asteroid. In particular, we could use the Tomo-e Gozen camera (Sako et al. 2016, 2018), which was a low-noise, high-quantum-efficiency, and super wide-field CMOS mosaic camera. The quick and contiguous readout capability of the Tomo-e Gozen camera assisted in the observation of 2012 TC$_4$, which is both fast rotating and fast moving. In this paper, we deal with the following. In Section 2, we describe the observations and their data reduction, with particular focus on the Tomo-e Gozen camera. In Section 3, we mention the results of taxonomic class, diameter, shape, and rotational motion. In Section 4, we discuss the impact event that could have happened on the parent object of 2012 TC$_4$ and the excitation and damping timescales. Finally, we summarize the physical properties of 2012 TC$_4$ and mention the significance of elucidating the physical properties of 10 m sized NEOs.

## 2. Observations and Data Reduction

### 2.1. Observations

We conducted the observation campaigns of 2012 TC$_4$ with six small- and medium-sized telescopes from 2017 October 9 to 11. Since 2012 TC$_4$ moved to the day side, we could not observe it on 2017 October 12, the day of the closest approach. The observational circumstances and states of 2012 TC$_4$ are listed in Tables 1 and 2, respectively. The longest observation of this campaign was carried out using the Tomo-e Gozen camera mounted on a 1.05 m f/3.1 Schmidt telescope at Kiso Observatory. The Tomo-e Gozen is an extremely wide field camera equipped with 84 CMOS sensors that consist of four modules with 21 CMOS sensors. The Tomo-e Gozen camera

### Table 1

| Observation Start and End Time$^a$ (JD – 2,458,000) | Exp. Time (s) | Filter | Observatory | Average S/N$^b$ |
|---------------------------------------------------|---------------|--------|-------------|---------------|
| 35.9578326–35.9982278 | 10           | ...    | VISO (1.05 m) | 31.4–31.5  |
| 36.9028528–36.9888035 | 30           | V, R   | Nayoro (0.4 m) | 33.3–34.1  |
| 36.90459–36.91530 | 120          | J, H, K | Nishi-Harima (2.0 m) | 6.77–9.36  |
| 36.9707319–37.1452833 | 60           | g, r, i, z | BSGC (1.0 m) | 38.0–40.7 |
| 37.0073257–37.0808347 | 10/5         | ...    | Kiso (1.05 m) | 4.16–4.54  |
| 37.9282507–37.93755688 | 5            | grism  | Kiso (1.05 m) | 28.17–43.24  |
| 37.9300568–38.0126750 | 2            | ...    | Kiso (1.05 m) | 4.16–4.54  |
| 38.0826590–38.1130069 | 6            | ...    | Anan (1.13 m) | 4.16–4.54  |

### Notes.
$^a$ Center of exposure time. The time is calibrated light-travel time, with the exception of Nishi-Harima.
$^b$ S/N of Nishi-Harima is estimated by an image of seven stacked frames.

### Table 2

| Year mon day (UT) | $\Delta^a$ (au) | $\alpha^b$ (deg) | Sky Motion arcsec minute$^{-1}$ |
|-------------------|-----------------|-----------------|-------------------------------|
| 2017 Oct 9.4578–9.4998 | 0.011–0.010 | 31.4–31.5 | 4.16–4.54 |
| 2017 Oct 10.4029–10.5808 | 0.007–0.0064 | 33.3–34.1 | 6.77–9.36 |
| 2017 Oct 11.4283–11.6130 | 0.0032–0.0025 | 38.0–40.7 | 28.17–43.24 |

### Notes.
$^a$ 2012 TC$_4$ to observer distance.
$^b$ Phase angle (Sun–2012 TC$_4$–observer).
records an approximately 20 deg$^2$ area at a maximum frame rate of 2 Hz (=0.5 s exposure). The field of view (FOV) for one CMOS sensor is 0.24 deg$^2$ with a pixel resolution of 1″2. The Tomo-e Gozen camera was not completed at the time 2012 TC$_4$ approached Earth, but a performance test of the Tomo-e Gozen camera was conducted using a single module with four CMOS sensors. The time control accuracy of the Tomo-e Gozen camera was around 1 s in the performance test. The quick and contiguous readout capability of the Tomo-e Gozen camera is suitable for the observation of a fast-rotating and fast-moving asteroid, such as 2012 TC$_4$. The light curve of 2012 TC$_4$ was obtained with high time resolution during the performance test. The exposure times were 10 s on October 9, 10 and 5 s on October 10, and 2 s on 2017 October 11. To estimate the taxonomic class of 2012 TC$_4$, spectroscopy was also conducted using the grism spectrometer of the Tomo-e Gozen camera with an exposure time of 5 s on 2017 October 11. However, the taxonomic class of 2012 TC$_4$ was not estimated, because it was not possible to carry out the adequate wavelength calibration during the performance test. Despite this, the zeroth-order light curve in the grism spectroscopy was used as the light-curve data. No offsets were obtained, due to the poor weather on 2017 October 10, and 2 s on 2017 October 11. To estimate the magnitude of 2012 TC$_4$; $F_{tc}$, the photometry was conducted using reference stars imaged in the same frame as 2012 TC$_4$:

$$F_{tc}(t) = F_0(t) - F_{tc0}(t),$$

(1)

where $F_{tc}(t)$ is the calibrated light curve of 2012 TC$_4$ under the $ith$ observational condition, namely, each observation, each observation day, and each filter. $F_0(t)$ is the raw magnitude of 2012 TC$_4$; $F_{tc0}(t)$ is the average raw magnitude of the reference stars and represents the change of atmospheric conditions; $t$ is the observational time. There were 20–60 and three reference stars for Kiso Observatory and BSs observations, respectively. One reference star was applied for the observations at Nayoro Observatory and Anan Science Center. Next, offset magnitudes $\Delta F_{fi}$ were calculated to adjust the light curve of each observational condition. Here, we note that the differences of the phase angle and the distances of 2012 TC$_4$ among the observational conditions are not regarded in the relative photometry. The offset magnitudes $\Delta F_{fi}$ were estimated as the difference of the average magnitude $\bar{F}_{fi}$ and the standard average magnitude $\bar{F}_{ksi010}$, obtained on 2017 October 10, at Kiso Observatory:

$$\Delta F_{fi} = \bar{F}_{fi} - \bar{F}_{ksi010},$$

(2)

where the standard average magnitude $\bar{F}_{ksi010}$ was estimated by comparing with the SDSS $g'$ magnitude of reference stars. The standard average magnitude was 17.575 mag. Since the Tomo-e Gozen was not equipped with the SDSS $g'$ filter, the standard average magnitude could be affected by a constant offset due to possible imperfect color correction. The constant offset was, however, estimated to be up to $\sim$0.1 mag, which has little impact on the following discussion. The calibration process above may introduce some systematic error in $\Delta F_{fi}$, since each observation covered a different phase of the light curve and the average brightness should be different in the observational conditions. However, we presume that the systematic errors were sufficiently small, since the observation time of each observational condition was long enough compared with the rotational period as will be described later. We can safely use the average magnitude $\bar{F}_{fi}$ to adjust the light curves obtained in the different observational conditions. Finally, the light curve of 2012 TC$_4$, $F(t)$, could be described as

$$F(t) = F_{ti0}(t) + \Delta F_{fi},$$

(3)

2.3 Data Reduction for Multiband Photometry

The multiband photometry in the visible wavelength region was conducted in BSs. We measured the flux of 17 standard stars from the SDSS Data Release 12 (Alam et al. 2015), whose stars were imaged simultaneously in the same frame as 2012
3. Results

3.1. Light Curve

Assuming a double-peaked light curve, we carried out a periodicity analysis based on the Lomb-Scargle periodogram (Lomb 1976; Scargle 1982). We had a possibility to evaluate the inaccurate rotational period, due to the change in the geometric relationship between Earth, 2012 TC4, and Sun for a few days. To avoid this, only the stable data obtained on 2017 October 10 at Kiso Observatory were used in the periodic analysis. The power spectrum from the periodogram showed a first period of 12.25 ± 0.01 minutes and a second period of 8.47 ± 0.01 minutes (Figure 1). The result was consistent with other observational results (Sonka et al. 2017; Warner 2018; Tan & Gao 2018).11 The appearance of two fast-rotating periods shows that 2012 TC4 is a tumbling and monolithic asteroid. Substituting 12.25 and 8.47 minutes into \( P_1 \) and \( P_2 \), respectively, the period ratio \( P_1 : P_2 \) became approximately 13:9. In order to fit a curve to the light curve of a tumbling asteroid, “Combined Period” \( P_c \) was defined by

\[
P_c = \frac{9P_1 + 13P_2}{2}.
\]

The same surface of 2012 TC4 faces the observer every \( P_c \) = 110.18 minutes. Next, we made the folded light curve with period \( P_c \) for each day. The common specific features appear in the different phases of the folded light curve for each day. The folded light curve covering three observation days was obtained by matching the specific features in the phase (Figure 2). The obtained light curve was fitted to the two-dimensional Fourier series (Pravec et al. 2005):

\[
F^m(t) = C_0 + \sum_{j=1}^{m} \left[ C_{j0} \cos \frac{2\pi j}{P_1} t + S_{j0} \sin \frac{2\pi j}{P_1} t \right] + \sum_{k=1}^{m} \sum_{j=-m}^{m} \left[ C_{jk} \cos \left( \frac{2\pi j}{P_1} + \frac{2\pi k}{P_2} \right) t \right] + S_{jk} \sin \left( \frac{2\pi j}{P_1} + \frac{2\pi k}{P_2} \right) t,
\]

where \( m \) is the order; \( C_0 \) is the mean reduced light flux; \( C_{jk} \) and \( S_{jk} \) are the Fourier coefficients for the linear combination of the two frequency \( P_1^{-1} \) and \( P_2^{-1} \), respectively; and \( t \) is the time. Substituting \( m = 4, P_1 = 12.25 \) minutes, and \( P_2 = 8.47 \) minutes for 2012 TC4, a fitting curve was obtained, as shown by the blue lines in Figure 2. The \( C_0 \) value was 17.578 mag. The brightness was around the same as the standard average magnitude of 17.575 mag described in Section 2.2. This indicates quantitatively that the offset error obtained in Equation (2) is small enough for the purpose of obtaining the fitting curve. The top of Figure 2 shows that the obtained data can cover almost all phases in the light curve. The second top of Figure 2 indicates that the data of multiband photometry on BSGC are distributed evenly to the phase of the light curve. This means that the precise color index can be evaluated by stacking the data for each filter, assuming that 2012 TC4 has a homogeneous surface. The graph legend “Kiso11. Oct/1st” in the bottom of Figure 2 shows the result of zeroth-order photometry by grism spectroscopy. Thus, the photometric accuracy is slightly worse than the result of photometry in the graph legend “Kiso11. Oct/2nd.” A precise and high time resolution light curve was successfully obtained in the graph legend “Kiso11. Oct/2nd” by taking the advantage of a unique feature of the Tomo-e Gozen camera. The high time resolution light curve contributes to the drawing of a precise fitting curve.

3.2. Taxonomic Class and Diameter

The taxonomic class in the visible wavelength region is investigated in terms of a reflectance color gradient and a log reflectance spectrum (Carvano et al. 2010). The reflectance color gradient and log reflectance spectrum are deduced from the color indices of 2012 TC4. Although the adequate wavelength calibration could not be carried out in grism spectroscopy at Kiso Observatory, the spectrum feature did not show the time variation. This indicated that 2012 TC4 had a homogeneous surface and the color indices did not also show the time variation. Assuming the homogeneous surface

11 In addition, see “The 2012 TC4 Observing Campaign,” http://2012tc4.astro.umd.edu/index.shtml.
of 2012 TC\textsubscript{4}, the photometric accuracy could be increased by averaging the flux for each filter. The color indices were \( g' - r' = 0.479 \pm 0.031, \ r' - i' = 0.187 \pm 0.023, \) and \( i' - z' = 0.035 \pm 0.036, \) respectively. The reflectance color is defined as

\[
C_{ij} = -2.5 \log_{10} \frac{R_{ij}}{R_{ij}}.
\]

where \( C_{ij} \) and \( R_{ij} \) are the reflectance color and the reflectance at a given wavelength, \( R_{ij} \) is the reflectance at the reference wavelength, and the subscript \( j \) specifies the wavelength. The wavelengths of \( j = 1, 2, 3, \) and \( 4 \) correspond to the central wavelengths of the SDSS \( g'(0.477 \mu m), \ r'(0.623 \mu m), \ i' (0.763 \mu m), \) and \( z'(0.913 \mu m) \) filters, respectively. When we use the \( g' \) filter as the reference, the reflectance colors of the \( r' \) filter is calculated from the color index as

\[
C_{ij} = (r' - g') - C_{ij}.
\]

where \( C_{ij} \) is the \( r'-g' \) color of the Sun. We adopted the solar colors \( C_{ij} \), \( C_{ij} \), and \( C_{ij} \) (Ivezić et al. 2001). The reflectance colors of the other filters were calculated in the same manner. The reflectance color gradient is defined as

\[
\gamma_{ij} = -0.4 \frac{C_{ij} + 1}{C_{ij}}.
\]

We deduced the reflectance color gradients of \( \gamma_{g} = 0.079 \pm 0.038, \ \gamma_{r} = 0.249 \pm 0.043, \) and \( \gamma_{i} = -0.013 \pm 0.052. \) Figure 3 shows the reflectance color gradients of 2012 TC\textsubscript{4} and asteroids of major taxonomic classes. The rectangles in Figure 3 indicate the range of reflectance color gradients of C-, X-, D-, L-, S-, A-, Q-, O-, and V-type asteroids in the SDSS Moving Object Catalog (SDSS-MOC). The top, middle, and bottom panels correspond to \( \gamma_{g}, \ \gamma_{r}, \) and \( \gamma_{i}, \) respectively. The thick horizontal lines are the average reflectance color gradients of 2012 TC\textsubscript{4}. The reflectance color gradients of 2012 TC\textsubscript{4} are consistent with the range of X-type asteroids. The log reflectance for 2012 TC\textsubscript{4} was normalized at the \( g' \) filter. The normalized log reflectances of the \( r', i', \) and \( z' \) were \( 1.01 \pm 0.01, \ 1.05 \pm 0.01, \) and \( 1.04 \pm 0.02, \) respectively. Figure 4 shows the log reflectance spectra of 2012 TC\textsubscript{4} and the asteroids of the X, S, C, and L types. The log reflectance spectrum of 2012 TC\textsubscript{4} is similar to that of the X type. Both observational results indicate that the taxonomic class of 2012 TC\textsubscript{4} in the visible wavelength region is the X type. Moreover, the color indices in the near-infrared wavelength region were \( J - H = 0.226 \pm 0.041, \) and \( H - K_s = 0.034 \pm 0.045. \) These values were included in the range of C-complex \( J - H = 0.28 \pm 0.08, \ H - K_s = 0.11 \pm 0.08, \) S-complex \( J - H = 0.37 \pm 0.12, \ H - K_s = 0.04 \pm 0.08, \) and X-complex \( J - H = 0.31 \pm 0.12, \ H - K_s = 0.14 \pm 0.07) \) (Popeșcu et al. 2016). Therefore, the taxonomic class of 2012 TC\textsubscript{4} was concluded to be an X type. The color indices are summarized in Table 3.
We estimate the absolute magnitude $H_V$ and effective diameter of 2012 TC$_4$. The average apparent $r'$ magnitude of 2012 TC$_4$ on 2017 October 10, at BSGC, was deduced to be 17.129 ± 0.017 mag. The apparent $V$ magnitude is described in the following form (Fukugita et al. 1996):

$$V = r' - 0.11 + 0.49 \left( \frac{(g' - r') + 0.23}{1.05} \right).$$  \ (9)

Here, for our photometric precision requirements, the difference between the AB magnitude and Vega magnitude in the $V$ band is negligible. The reduced magnitude at the phase angle, $\alpha$, is expressed as

$$H_V = H(\alpha) + 2.5 \log_{10}(R\Delta),$$

where $R$ and $\Delta$ are the heliocentric and geocentric distances in au, respectively. The absolute magnitude is expressed as a so-called $H$-$G$ function (Bowell et al. 1989):

$$H_V = H(\alpha) + 2.5 \log_{10}(1 - G)\Phi(1) + G\Phi(2),$$

where $G$ is the slope parameter dependent on the asteroid’s taxonomy. When we apply $G = 0.20 \pm 0.09$ for $X$ types (Vereš et al. 2015), $H_V$ becomes 28.54 ± 0.03 mag. An effective diameter of asteroids $D$ (in kilometer) is described as

$$D = 1329 \times 10^{-H_V/p_V^{-1/2}},$$ \ (11)

where $p_V$ is the geometric albedo. Assuming an albedo of 0.098 ± 0.081 for the X type (Usui et al. 2013), the effective diameter and range were found to be 8 m and 6 m < $D$ < 20 m, respectively. Since Mainzer et al. (2011b) also showed the albedo of 0.099 ± 0.161 for Tholen X-complex class and the albedo of 0.111 ± 0.143 for Bus-DeMeo X-complex class, the assumption of ~0.1 for the X-type albedo was reasonable. We should note, however, that the X-complex includes the E, M, and P types, whose albedos are 0.454 ± 0.119, 0.169 ± 0.044, and 0.063 ± 0.017, respectively (Usui et al. 2013). The estimated diameter can be affected by the uncertainty of the albedo among the X-complex asteroids.

### Table 3

| Column | Values |
|--------|--------|
| $g - r'$ | 0.479 ± 0.031 |
| $r' - i'$ | 0.187 ± 0.023 |
| $i' - z'$ | 0.035 ± 0.036 |
| $J - H$ | 0.226 ± 0.041 |
| $H - K_s$ | 0.034 ± 0.045 |
3.3. Shape and Rotational Motion

The period analysis revealed that 2012 TC₄ is a tumbling asteroid with a rotational period and precession period. However, period analysis alone cannot conclude whether a given period, P₁ or P₂, corresponds to the rotational period or precession period. Thus, we make the shape and rotational motion models of 2012 TC₄, which is recognized as a force-free asymmetric rigid body, from the dynamic analytical solution. Previous studies have described the equations of motion for a force-free asymmetric rigid body (Samarasinha & A’Hearn 1991; Kaasalainen 2001). The main equations used in this study are detailed in the Appendix. The shape and rotational motion models of 2012 TC₄ were made by substituting the observational result into the equations of this subsection and the Appendix. However, it should be noted that the shape and rotational motion models are representative examples, not unique solutions. Here we define Lₚ (short-axis length), Lₛ (intermediate-axis length), and Lₗ (long-axis length) when 2012 TC₄ is a triaxial ellipsoid body. The axes satisfy the relationship Lₚ < Lₛ < Lₗ. The rotational motions of asteroids are categorized into long-axis modes (LAM) and short-axis modes (SAM). The body of LAM rotates completely around the long axis (ψ in the Appendix) and oscillates around the short axis (φ in the Appendix), as seen by an external observer. On the other hand, the body of SAM oscillates around the long axis (ψ in the Appendix) and rotates fully around the short axis (φ in the Appendix), as seen by an external observer. The shape and rotational motion models of 2012 TC₄ were made for LAM and SAM, respectively.

First, the LAM models were made. The relation between the light-curve amplitude and phase angle is shown as follows:

\[ A(0) = \frac{A(\alpha)}{1 + c\alpha}, \]

where \( A(\alpha) \) is the light-curve amplitude at the phase angle, \( \alpha^\circ \), and \( c \) is the photometric phase slope coefficient. Since the X type of 2012 TC₄ includes the E type, M type, and P type, 0.03 of the M type (Zappalà et al. 1990) was adopted as the \( c \) value. Assuming that the light-scattering cross section of 2012 TC₄ is projected onto the plane of the sky, the light-curve amplitude is described through the lower limit to the true cross section ratio of the body as

\[ A(0) = 2.5 \log_{10}\left(\frac{S_{\text{max}}}{S_{\text{min}}}\right), \]

where \( S_{\text{max}} \) and \( S_{\text{min}} \) are the maximum and minimum light-scattering cross sections, respectively. The maximum amplitude of 2012 TC₄ is 1.43 magnitude and appears at a phase angle of 0.2 in Figure 2 when the phase angle is 39°. Therefore, the relationship \( Lₗ = 2.40Lₘ \) was obtained, assuming \( S_{\text{max}} = \pi LₗLₘ \) and \( S_{\text{min}} = \pi LₗLₘ \). Alternatively, the relationship \( Lₗ = 2.40Lₘ \) could be obtained from \( S_{\text{max}} = \pi LₗLₘ \) and \( S_{\text{min}} = \pi LₗLₘ \), when 2012 TC₄ almost simply rotates around the long axis and an observer sees 2012 TC₄ from the vertical direction for the total rotational angular momentum vector. As described above, period analysis alone is insufficient to determine whether \( P₁ \) or \( P₂ \) corresponds to \( P_φ \) (period of φ) or \( P_ψ \) (period of ψ) in the Appendix. Thus, there were four cases for the LAM models, whose combinations were \( Lₗ = 2.40Lₘ \) or \( Lₗ = 2.40Lₘ \) for \( P_ψ = 12.25 \) minutes and \( P_φ = 8.47 \) minutes or \( P_ψ = 8.47 \) minutes and \( P_φ = 12.25 \) minutes. Substituting the axial ratios and periods into Equation (33), the following limits of \( Lₚ, Lₙ, \) and \( Lₗ \) could be taken for four cases:

1. Case 1: \( Lₗ = 2.40Lₘ, \ P_ψ = 12.25 \) minutes, \( P_φ = 8.47 \) minutes, \( Lₚ \leq 1.88Lₙ \).
2. Case 2: \( Lₗ = 2.40Lₘ, \ P_ψ = 12.25 \) minutes, \( P_φ = 8.47 \) minutes, \( Lₚ \geq 2.97Lₙ \).
3. Case 3: \( Lₗ = 2.40Lₘ, \ P_ψ = 8.47 \) minutes, \( P_φ = 12.25 \) minutes, \( Lₚ \leq 1.39Lₙ \).
4. Case 4: \( Lₗ = 2.40Lₘ, \ P_ψ = 8.47 \) minutes, \( P_φ = 12.25 \) minutes, \( Lₚ \geq 3.69Lₙ \).

The combinations of the average rotational velocities were \( \bar{φ} \sim 42.5 \) minutes⁻¹ and \( \bar{ψ} \sim 29.4 \) minutes⁻¹ or \( \bar{φ} \sim 29.4 \) minutes⁻¹ and \( \bar{ψ} \sim 42.5 \) minutes⁻¹. Moreover, we applied \( Lₕ = 8 \) m as the effective diameter of 2012 TC₄. In addition, the moments of inertia of Equation (26) were given using the total rotational angular momentum \( M \) and total rotational kinetic energy \( E \) as follows:

\[ \frac{M^2}{2E} = \frac{(n_l + l_i)}{n + 1} \text{ or } \frac{(l_i + ml_i)}{m + 1}, \]

where \( n \) is an integer from 1 to 9 and \( m \) is an integer from 2 to 9. The tumbling status was roughly described by changing the integers \( n \) and \( m \). The combination of the \( n \), \( m \), and axial lengths that satisfy the observed velocities \( \bar{φ} \) and \( \bar{ψ} \) was sought. The axial lengths were scanned in steps of 0.1 m for each case. The results were the following combinations:

1. Case 1’: \( Lₚ, \ Lₙ, \ Lₕ = (7.5 \text{ m}, 8.0 \text{ m}, 18.0 \text{ m}), \)
2. Case 2’: \( Lₚ, \ Lₙ, \ Lₕ = (6.2 \text{ m}, 8.0 \text{ m}, 14.9 \text{ m}), \)
3. Case 3’: \( Lₚ, \ Lₙ, \ Lₕ = (3.3 \text{ m}, 8.0 \text{ m}, 14.3 \text{ m}), \)
4. Case 4’: \( Lₚ, \ Lₙ, \ Lₕ = (2.0 \text{ m}, 8.0 \text{ m}, 8.4 \text{ m}), \)

There was solution that satisfied the observed velocities \( \bar{φ} \) and \( \bar{ψ} \) for “Case 2.”

Next, the SAM models were made. The relationship of \( Lₖ = 2.40Lₘ \) was obtained, assuming \( S_{\text{max}} = \pi LₖLₘ \) and \( S_{\text{min}} = \pi LₖLₘ \). Alternatively, the relationship of \( Lₖ = 2.40Lₘ \) could be obtained from \( S_{\text{max}} = \pi LₕLₘ \) and \( S_{\text{min}} = \pi LₕLₘ \), in the case that 2012 TC₄ almost simply rotates around the short axis and an observer sees 2012 TC₄ from the vertical direction for the total rotational angular momentum vector. The combination of the rotational period and precession period is limited to \( P_ψ = 12.25 \) minutes and \( P_φ = 8.47 \) minutes by Equation (42). Substituting the axial ratios and periods into Equation (41), the following limit of \( Lₚ, Lₙ, \) and \( Lₚ \) could be taken for two cases:

1. Case 5: \( Lₕ = 2.40Lₘ, \ P_ψ = 12.25 \) minutes, \( P_φ = 8.47 \) minutes, \( Lₚ \geq 2.97Lₙ \).
2. Case 6: \( Lₕ = 2.40Lₘ, \ P_ψ = 12.25 \) minutes, \( P_φ = 8.47 \) minutes, \( Lₚ \geq 1.81Lₙ \).

The average rotational velocities were \( \bar{φ} \sim 42.5 \) minutes⁻¹ and \( \bar{ψ} \sim 29.4 \) minutes⁻¹. Moreover, we applied \( Lₕ = 8 \) m as the effective diameter of 2012 TC₄. In addition, the moments of
Shade 3D is produced by Shade 3D Co., Ltd. https://shade3d.jp/en/.

The tumbling status was roughly described by changing the axial lengths that satisfy the observed velocities. The tumbling status was roughly described by changing the axial lengths that satisfy the observed velocities $\phi$ and $\psi$ was also sought. The axial lengths were scanned in steps of 0.1 m for each case. The result was the following combination:

1. Case $6^\prime$: $(L_x, L_y, L_z) = (4.2 \text{ m}, 8.0 \text{ m}, 19.2 \text{ m}), M^2/2E = (7L + L)/8$.

There was no solution that satisfied the observed velocities $\phi$ and $\psi$ for “Case 5.”

Finally, the shape and rotational motion models (models 1, 3, 4, and 6) were made for four cases (Cases $1^\prime$, $3^\prime$, $4^\prime$, and $6^\prime$) using a commercial 3DCG (three-dimensional computer graphic) software, Shade 3D, and selecting the plausible models using the artificial light curve produced by the shape and rotational motion models. The rendering models of 2012 TC$_4$ were drawn with the ray-tracing method in the software. The rotational motion was simplified with the fixed rotational velocities of $\phi$ and $\psi$. The rendering models of 2012 TC$_4$ were read out in BMP format in steps of 0.1 minutes. The artificial light curves were produced by the brightness change in the image of each rendering model. The artificial light curves are shown in Figure 5. The artificial light curves do not rigorously reflect the direction of rotational angular momentum or the detailed topography of 2012 TC$_4$. However, the shapes of the light curves help to narrow down the plausible models. When the light curve in Figure 2 is compared to the artificial light curves of Figure 5, it can be seen that the artificial light curves of models 1 and 6 do not match the observed light curve, with respect to the unchanged light-curve amplitude. On the other hand, it can be seen that the artificial light curves of models 3 and 4 created changes in the amplitude, like the observed light curve. Therefore, we concluded that models 3 and 4 were the plausible models of 2012 TC$_4$. For any case of models 3 and 4, the shape of 2012 TC$_4$ is flat and elongated like a pancake. As an example, the shape of model 4 is shown in Figure 6, and the time variation of rotational motion is shown in Figure 7. The rotational motion in model 3 is omitted, since the rotational motion of model 3 is similar to that of model 4. We summarize the shape and rotational motion models of 2012 TC$_4$ in Table 4. The average $\theta$ value is around 29°0 and oscillates within the range of ±0°4 in model 3. The average $\phi$ value is around 29°4 minutes$^{-1}$ and oscillates within the range of ±1°5 minutes$^{-1}$ in model 3. The average $\theta$ value is around 48°5 and oscillates within the range of ±1°5 in model 4. The average $\phi$ value is around 29°4 minutes$^{-1}$ and oscillates within the range of ±2°4 minutes$^{-1}$ in model 4. The $\psi$ is almost constant at around 40°5 minutes$^{-1}$ for both models 3 and 4. The change of $\psi$ is not obvious in the second row of Figure 7.

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Artificial light curves of 2012 TC$_4$ in models 1, 3, 4, and 6.

inertia of Equation (34) were given as follows:

$$M^2 = \frac{(nL + l)}{n + 1} \text{ or } (L + mL)/m + 1.$$  

where $n$ is an integer from 1 to 9 and $m$ is an integer from 2 to 9. The tumbling status was roughly described by changing the integers $n$ and $m$. The combination of the $n$, $m$, and axial lengths that satisfy the observed velocities $\phi$ and $\psi$ was also sought. The axial lengths were scanned in steps of 0.1 m for each case. The result was the following combination:

1. Case $6^\prime$: $(L_x, L_y, L_z) = (4.2 \text{ m}, 8.0 \text{ m}, 19.2 \text{ m}), M^2/2E = (7L + L)/8$.

4. Discussion

We have found that 2012 TC$_4$ is a tumbling, fast-rotating, and monolithic asteroid. Discoveries of fast-rotating asteroids are increasing with asteroid surveys and follow-up observations (Pravec et al. 2000; Hergenrother & Whiteley 2011). The number of fast-rotating asteroids is 84 in the Lightcurve Database (LCDB) with “Quality = 3” (denotes a secure result with no ambiguity and full light-curve coverage)” (Warner et al. 2009). Out of these, the tumbling asteroids are 2000 WL$_{107}$ (Pravec et al. 2005), 2008 TC$_3$ (Betzler et al. 2009; Scheirich et al. 2010), 2004 FH (LCDB), 2013 SU$_{23}$ (Benishek 2014; Warner 2014), 2014 SC$_{324}$ (Warner 2015), 2015 VY$_{105}$
The collisional destruction process is divided into impact cratering fragments and catastrophic disruption. Laboratory impact experiments demonstrated that the impact cratering fragments had an axial ratio of 0.2, and that the catastrophic disruption fragments had an axial ratio of 0.5. Since the impact fragments that are not catastrophic disruption fragments are not captured by the SAM, the transitional timescale of the impact fragments with the LAM is estimated by the excitation and damping timescale of asteroids with the LAM. Nonetheless, the axial ratio of the intermediate axis to the long axis of fast-rotating asteroids (diameter <200 m and rotational period <1 hr) is similar to that of ejecta in laboratory impact experiments and that of boulders on Itokawa and Eros. For example, L₁/L₄, the mean value of axial ratios of boulders larger than 5 m on Itokawa, is 0.61 ± 0.19. Since the light-curve amplitudes of nine tumbling asteroids are larger than 1.0 mag, the shape of tumbling asteroids presumably indicates elongated boulder-like shapes. In particular, the axial ratio L₁/L₄ of 2012 TC₄ is 0.54 in model 3 and 0.56 in model 4, and the axial ratio L₁/L₄ of 2008 TC₃ is 0.54. The NEOs, 2012 TC₄ and 2008 TC₃, will be objects similar to the boulders on Itokawa and Eros. Furthermore, we discuss how the impact event happened to fast-rotating asteroids using the axis ratio, L₄/L₄. The collisional destruction process is divided into impact cratering (low impact energy) and catastrophic disruption (high impact energy). Laboratory impact experiments demonstrated that L₄/L₄ of impact cratering fragments is ~0.2, L₄/L₄ of catastrophic disruption fragments is ~0.5, and L₄/L₄ decreases with decreasing impact energy (Michikami et al. 2016). Numerous impact fragments were generated by the laboratory impact experiments. Despite the catastrophic disruption, a part of the impact fragments will indicate low L₄/L₄. Thus, the collisional destruction process cannot be immediately concluded from the L₄/L₄ of asteroids. Nonetheless, the axial ratio L₄/L₄ of 2012 TC₄ is 0.42 in model 3 and 0.23 in model 4, and the axial ratio L₄/L₄ of 2008 TC₃ is 0.36. The NEO 2012 TC₄ could be generated by catastrophic disruption in model 3 and by impact cratering in model 4. The NEO 2008 TC₃ could have experienced the impact energy between models 3 and 4.

As we discussed above, 2012 TC₄ had possibly experienced an impact event. Here we estimate the excitation and damping timescales of 2012 TC₄. The excitation timescale, especially, helps to deduce the time of the impact event of 2012 TC₄. An impact angle (θ in the Appendix) of asteroids with the LAM increases with dissipating internal energy. Then, the motion of the asteroid transitions to the SAM via an unstable and temporary rotation mode around the intermediate axis. After the transition to the SAM, the excitation angle decreases with time, and the SAM transitions to the pure rotation around the short axis, which is in alignment with the principal axis of the moment of inertia. We call the transition time from the LAM to the SAM the “excitation timescale” and the transition time from the SAM to the pure rotation the “damping timescale.” The excitation and damping timescales (Sharma et al. 2005; Breiter et al. 2012) are expressed as

\[
T_s = D_s(h_1, h_2) \frac{\mu Q}{a^2 \rho \omega_1^2},
\]

where \(D_s(h_1, h_2)\) is a shape parameter, \(\mu\) is the elastic modulus, \(Q\) is the quality factor, \(\rho\) is the density, \(a\) is half of the long-axis length, and \(\omega_1\) is a representative angular velocity around the focusing principal axis. The quantities for the LAM have subscript \(s = 1\), and those for the SAM have subscript \(s = 3\). The shape parameters \(D_s(h_1, h_2)\) for the LAM and SAM are defined as

\[
D_s(h_1, h_2) = \left[ \frac{h_1^2(1 - h_1^2)(1 + h_2^2)^2}{5(1 + h_1^2h_2^2)} \right] \int_{\theta_1}^0 \frac{\rho \sin \theta_1 \cos \theta_1 d\theta_1}{\Psi_1}
\]

\tag{17}

The Astronomical Journal, 157:155 (13pp), 2019 April

Uragawa et al.

Figure 7. Euler angle of $\theta$, $\phi$, and $\psi$ as a function of time for model 4. This figure style is developed by Samarasinha & A’Hearn (1991). The angle $\theta$ is the angle between the long axis and total rotational angular momentum vector $M$. The angles $\phi$ and $\psi$ measure the amount of precession executed by the long axis around $M$ and the amount of rotation around the long axis itself. For model 4, the axial lengths of 3.3 $\times$ 8.0 $\times$ 14.3 m were used with $M^2/2E = (2I_0 + I_1)/3$. The nutation period, $P_n$, is exactly half the rotational period, $P_r$. The variation in the angular velocity, $\psi$, is undetectable in plots of $\psi$ vs. time because the amplitude of variation is negligible. The angle $\phi$ is described based on the constant of $\dot{\phi} \sim 290^\circ$ minutes$^{-1}$.

Table 4

| Column                  | Values       |
|-------------------------|--------------|
| $L_o$                   | 6.2 m (model 3), 3.3 m (model 4) |
| $L_i$                   | 8.0 m (models 3 and 4) |
| $L_d$                   | 14.9 m (model 3), 14.3 m (model 4) |
| $P_\psi$                | 8.47 $\pm$ 0.01 minutes |
| $P_\phi$                | 12.25 $\pm$ 0.01 minutes |
| $\tilde{\psi}$         | 29°0 (model 3), 48°5 (model 4) |
| $\phi$                  | 29°4 minutes$^{-1}$ (models 3 and 4) |
| $\psi$                  | 42°5 minutes$^{-1}$ (models 3 and 4) |

and

$$D_3(h_1, h_2) = \left[ \frac{h_1^2(1 + h_2^2)(1 - h_2^2)}{5(1 + h_1^2 h_2^2)} \right] \times \int_{\theta_0}^{\theta_0'} \sin \theta_3 \cos \theta_3 \Psi_3 d\theta_3,$$  \hspace{1cm} (18)

where $\theta_0$ and $\theta_0'$ are the initial and final maximum wobbling angles, respectively, $h_1 = L_o/L_d$, $h_2 = L_d/L_i$, and $\Psi_3$ and $\Psi_3$ are dimensionless factors of the energy-loss rate (Breiter et al. 2012). According to the manner of Pravec et al. (2014), $\tilde{\omega}_1$ and $\tilde{\omega}_3$ are represented as

$$\tilde{\omega}_1 = \frac{I_o}{I_i} \tilde{\omega}_2 \equiv \frac{1 + h_1^2 h_2^2}{h_1^2(1 + h_2^2)} \tilde{\omega}_2,$$  \hspace{1cm} (19)

and

$$\tilde{\omega}_3 = \frac{I_l}{I_i} \tilde{\omega}_2 \equiv \frac{1 + h_1^2 h_2^2}{1 + h_1^2} \tilde{\omega}_2,$$  \hspace{1cm} (20)

where $I_o$, $I_i$, and $I_l$ are the moments of inertia defined in the Appendix. When we use $\tilde{\omega}_{obs} \equiv 2\pi/P_\phi$ as a proxy for $\tilde{\omega}_2$ and $a \equiv D_m/2h_1$, where $D_m$ is the asteroid mean diameter, the final formulae for the excitation and damping timescales become

$$T_1 = D_1(h_1, h_2) \frac{(h_1^3(1 + h_2^2))^3 h_1^2 \mu Q P_\phi^3}{(1 + h_1^2 h_2^2)^3 2\pi^3 \rho D_m^2}$$  \hspace{1cm} (21)

and

$$T_3 = D_3(h_1, h_2) \frac{(1 + h_1^2 h_2^2)^3 \mu Q P_\phi^3}{(1 + h_1^2 h_2^2)^3 2\pi^3 \rho D_m^2}.$$  \hspace{1cm} (22)

We adopted $D_m = 8$ m, $\mu = 10^3$ Pa, $Q = 100$, and $\rho = 3000$ kg m$^{-3}$ for 2012 TC$_4$. The typical timescale from the impact event to the status of model 3 became $3.1 \times 10^5$ yr when the integration interval of $\theta_1$ was from 0°1 to 29°0 in Equation (17). The status of model 3 transitions to the SAM in the timescale of $2.7 \times 10^5$ yr when the integration interval of $\theta_1$ was from 29°0 to 89°9 in Equation (17). After the transition to the SAM, the damping timescale, $T_3$, became $1.5 \times 10^7$ yr when the integration interval of $\theta_3$ was from 89°9 to 0°1 in Equation (18). In the same way, the typical timescale from the impact event to the status of model 4 became $3.2 \times 10^5$ yr when the integration interval of $\theta_1$ was from 0°1 to 48°5. The status of model 4 transitions to the SAM in the timescale of $1.8 \times 10^5$ yr when the integration interval of $\theta_1$ was from 48°5 to 89°9. The damping timescale, $T_3$, became $3.8 \times 10^5$ yr when the integration interval of $\theta_3$ was from 89°9 to 0°1. On the basis of the excitation and damping timescales, we can make the following scenario of 2012 TC$_4$. Zappalà et al. (2002), Morbidelli & Vokrouhlický (2003), and Granvik et al. (2017) described that impact events and dynamical mechanisms like the Yarkovsky effect continuously supply asteroids to the...
transportation resonances in the asteroid Main Belt. If asteroids once move into the transportation resonances, the orbit dynamically evolves to the NEO region in less than a million years (Morbidelli et al. 2002). After the migration to the NEO region, the dynamical lifetime of a 10 m sized NEO is typically a few million years. In the case of 2012 TC₄, its parent object had experienced an impact event in the asteroid Main Belt within ~3 × 10⁵ yr and the ejected 2012 TC₄ dynamically evolved to the NEO region via the transportation resonances. Even if the derived θ values were underestimated, the ongoing LAM of 2012 TC₄ is evidence that the impact event should have happened less than ~6 × 10⁷ yr ago. The result suggests that 2012 TC₄ should have a fresh surface, since 2012 TC₄ is not exposed to space weathering for more than ~3 × 10⁵ yr. The motion of 2012 TC₄ will transition to the SAM in ~3 × 10⁵ yr and then will reach the dynamical lifetime of the 10 m sized NEOs before the damping timescale of elapses of tens of millions of years.

5. Summary

We investigated the physical properties of 2012 TC₄ by visible and near-infrared photometry. We succeeded in obtaining an unprecedented high time resolution light curve with the Tomo-e Gozen camera. The two fast-rotating periods showed that 2012 TC₄ is a tumbling and monolithic asteroid. The observations demonstrated the Tomo-e Gozen camera to be an extremely suitable instrument to observe fast-rotating and fast-moving asteroids. The multiband photometry indicated the taxonomic class of 2012 TC₄ to be an X type. Assuming the typical albedo of the X-type asteroids, the diameter of 8 m and range of 6–20 m were deduced. Moreover, the shape and rotational motion models of 2012 TC₄ were estimated. The plausible models indicated that 2012 TC₄ has a rotational period of 8.47 minutes and a precession period of 12.25 minutes with the LAM mode. The three axial lengths were 6.2 × 8.0 × 14.9 m or 3.3 × 8.0 × 14.3 m. In any model, the shape of 2012 TC₄ is flattened and elongated like a pancake, which suggests that 2012 TC₄ was produced by a past impact event. We also estimated the excitation and damping timescales. The excitation timescale implies that the impact event happened within ~3 × 10⁵ yr and 2012 TC₄ has a fresh surface that has not been strongly influenced by space weathering.

This study is a detailed observation of small (10 m) NEOs, following the study of 2008 TC₃. Although the impact of a 10 m sized NEO does not cause a catastrophic disaster, the impact happens with a high probability from once a century to once in several decades (Toricarico 2017; Trilling et al. 2017). It will become a crisis close to the Chelyabinsk meteor event (Popova et al. 2013). Furthermore, future space explorations plan to use 10 m sized NEOs as resources. Thus, clarifying the physical properties of 10 m sized NEOs is important for both planetary defense and future space exploration.

This research is supported in part by Japan Society for the Promotion of Science (JSPS) Grants-in-Aid for Scientific Research (KAKENHI) grant Nos. 16K05310, JP18H01261, JP26247074, JP16H02158, JP16H06341, JP2905, 18H04575, JP18H01272, and JP18K13599 and JSPS Program for Advancing Strategic International Networks to Accelerate the Circulation of Talented Researchers grant No. JR2603. This research is also supported in part by the Japan Science and Technology Agency (JST) Precursory Research for Embryonic Science and Technology (PRESTO), the Research Center for the Early Universe (RESCEU), the School of Science, the University of Tokyo, and the Optical and Near-infrared Astronomy Inter-University Cooperation Program.

Appendix

Motion of Force-free Rigid Body

The shape of an asteroid is approximated by a triaxial ellipsoid with the axial lengths \( L_x, L_y, \) and \( L_z \). The tumbling motions are divided into two classes: LAM and SAM. Here the moment of inertia per unit mass for each axis can be described as

\[
I_x = \frac{1}{20} (L_y^2 + L_z^2),
\]

(23)

\[
I_y = \frac{1}{20} (L_z^2 + L_x^2),
\]

(24)

and

\[
I_z = \frac{1}{20} (L_x^2 + L_y^2).
\]

(25)

The motion for LAM can be expressed in terms of the total rotational angular momentum \( \mathbf{M} \) and total rotational energy \( E \) as

\[
I_x \leq \frac{M^2}{2E} < I_z.
\]

(26)

The body approaches pure rotation about the long axis as \( \frac{M^2}{2E} \) approaches \( I_z \). A new independent variable of time \( \tau \) and a constant of the motion \( k^2 (\leq 1) \) are defined by

\[
\tau = t \sqrt{\frac{2E (I_x - I_z) (I_z - \frac{M^2}{2E})}{I_z I_x}}.
\]

(27)

and

\[
k^2 = \frac{(I_x - I_z) \left( \frac{M^2}{2E} - I_z \right)}{(I_z - I_x) \left( I_x - \frac{M^2}{2E} \right)}.
\]

(28)

The motion of a triaxial ellipsoid can be described as the time-series change of Euler angles \( \psi, \phi, \) and \( \theta \). In the case of LAM, \( \psi \) is the rotation about the long axis, \( \phi \) is the precession about the total rotational angular momentum vector, and \( \theta \) is the angle between the long axis and total rotational angular momentum vector \( \mathbf{M} \). \( \psi, \phi, \) and \( \theta \) are described as

\[
\psi = a \tan 2 \left( \frac{I_x}{I_z - I_x} \sin \tau, \frac{I_z}{I_z - I_x} \cos \tau \right),
\]

(29)

\[
\theta = \cos^{-1} \left( \frac{1}{M^2/2E} \left[ \frac{I_x (I_z - \frac{M^2}{2E})}{I_z (I_z - I_x)} \right] \right),
\]

(30)

and

\[
\phi = M \left[ \left( \frac{I_x - I_z}{I_z} \right) + \left( \frac{I_y - I_z}{I_z} \right) \sin^2 \tau \right] \left[ \frac{I_z (I_z - I_x)}{I_x (I_z - I_x) + I_y (I_z - I_y) \sin^2 \tau} \right].
\]

(31)
Here $\text{sn} \tau$, $\text{cn} \tau$ and $\text{dn} \tau$ are Jacobian elliptic functions. In addition, the following relational expressions are established:

$$P_\psi = 4 \sqrt{\frac{I_1I_2}{2E(l_1 - l_2)(l_1 - \frac{M^2}{2E})}} \int_0^{\frac{\pi}{2}} \frac{du}{\sqrt{1 - k^2 \sin^2 u}} \quad (32)$$

and

$$P_\psi \geq \sqrt{\frac{(l_1^2 + l_2^2)(l_1^2 + l_3^2)}{(l_2^2 - l_3^2)(l_1^2 - l_3^2)} - 1}. \quad (33)$$

The integral part of Equation (32) is the complete elliptic integral of the first kind.

On the other hand, the motion for SAM can be expressed as

$$l_i < \frac{M^2}{2E} \leq l_i. \quad (34)$$

The body approaches pure rotation about the short axis as $M^2/2E$ approaches $l_i$. A new independent variable of time $\tau$ and a constant of the motion $k^2 \leq 1$ are defined by

$$\tau = t \sqrt{\frac{2E(l_1 - l_2)(\frac{M^2}{2E} - l_i)}{l_1I_2I_3}} \quad (35)$$

and

$$k^2 = \frac{(l_1 - l_2)(l_1 - \frac{M^2}{2E})}{(l_1 - l_2)(\frac{M^2}{2E} - l_i)}. \quad (36)$$

In the case of SAM, $\phi$ is the rotation about the short axis, $\psi$ is the oscillation about the long axis, and $\theta$ is the angle between the long axis and total rotational angular momentum vector $M$. $\psi$, $\theta$, and $\phi$ are described as

$$\psi = a \tan^{\frac{1}{2}} \left[ \frac{l_1(l_2 - \frac{M^2}{2E})}{l_1 - l_i} \right] \sin \tau, \quad \left[ \frac{l_2(l_1 - \frac{M^2}{2E})}{l_2 - l_i} \right] \cos \tau \quad (37)$$

$$\theta = \cos^{-1} \left[ c_{\frac{1}{n}} \left( \frac{l_1(l_2 - \frac{M^2}{2E})}{l_2 - l_i} \right) \right]. \quad (38)$$

and

$$\dot{\phi} = M \left\{ \frac{\left( \frac{M^2}{2E} - l_i \right)}{l_1(l_2 - \frac{M^2}{2E})} \left( \frac{\left( \frac{M^2}{2E} - l_i \right)}{l_2(l_1 - \frac{M^2}{2E})} \right) - \frac{\sin \tau}{\tau} \right\}. \quad (39)$$

In addition, the following relational expressions are established:

$$P_\phi = 4 \sqrt{\frac{I_1I_2I_3}{2E(l_1 - l_2)(\frac{M^2}{2E} - l_i)}} \int_0^{\frac{\pi}{2}} \frac{du}{\sqrt{1 - k^2 \sin^2 u}}. \quad (40)$$

The integral part of Equation (40) is the complete elliptic integral of the first kind. Moreover, the rotational period $P_\phi$ has the following relationships with the oscillation period $P_\psi$:

$$\frac{P_\psi}{P_\phi} \geq \sqrt{\frac{(l_1^2 + l_2^2)(l_1^2 + l_3^2)}{(l_2^2 - l_3^2)(l_1^2 - l_3^2)} - 1}. \quad (41)$$

and

$$P_\phi \geq \frac{P_\phi}{P_\psi} > 1. \quad (42)$$

ORCID iDs
Seitaro Urakawa © https://orcid.org/0000-0001-7501-8983
Ryou Ohsawa © https://orcid.org/0000-0001-5797-6010
Hiroyuki Naito © https://orcid.org/0000-0001-9067-7653
Masateru Ishiguro © https://orcid.org/0000-0002-7332-2479

References
Abell, P. A., Barbosa, B. W., Chodas, B. W., et al. 2016, in Asteroid IV, ed. P. Michel, F. E. DeMeo, & W. F. Bottke (Tucson, AZ: Univ. Arizona Press), 855
Alam, S., Albareti, F. D., Allende, P., et al. 2015, ApJ, 219, 12
Benishek, V. 2014, MPBu, 41, 257
Betzler, A. S., Novaes, A. B., Beltrame, P., et al. 2009, MPBu, 36, 58
Bowell, E., Hapke, B., Doningue, D., et al. 1989, in Asteroids II, ed. R. P. Binzel et al. (Tucson, AZ: Univ. Arizona Press), 524
Breiter, S., Rożek, A., & Vokrouhlický, D. 2012, MNRAS, 427, 755
Caragounis, A., & Buzzi, L. 2016, MPBu, 43, 115
Carvano, J. M., Hasselman, P. H., Lazzaro, D., & Mothé-Diniz, T. 2010, A&A, 510, 443
Chambers, K. & Pan-STARRS Team 2016, AAS Meeting, 227, 324.07
Cheng, A. F., Rvivkin, A. S., Michel, P., et al. 2018, P&SS, 157, 104
Christensen, E., Lister, T., Larson, S., et al. 2014, in Asteroids, Comets, Meteoroids-Book of Abstracts, ed. K. Muinonen et al. (Helsinki: University of Helsinki), 97
Fujiiwara, A., Kagawachi, Y., Yeomans, D. K., et al. 2006, Sci, 312, 1330
Fukugita, M., Ichikawa, T., Gunn, J. E., et al. 1996, AJ, 111, 1748
Granvik, M., Morbidelli, A., Vokrouhlický, D., et al. 2017, A&A, 598, A52
Hergenrother, C. W., & Whiteley, R. J. 2011, Icar, 214, 194
Ishiguro, M., Kuroda, D., Hasegawa, S., et al. 2014, ApJ, 792, 74
Ivezić, Ž., Tabachnik, S., Rafikov, R., et al. 2001, AJ, 122, 2749
Kaasalainen, M. 2001, A&A, 376, 302
Kaasalainen, M., Kwiatkowski, T., Abe, M., et al. 2003, A&A, 405, L29
Kim, M.-J., Choi, Y.-J., Moon, H.-K., et al. 2013, A&A, 550, 11
Larson, S., Brownlee, J., Hergenrother, C., & Spahr, T. 1998, BAAS, 30, 1037
Lomb, N. R. 1976, Ap&SS, 39, 447
Mainzer, A., Bauer, J., Gray, T., et al. 2011a, ApJ, 731, 53
Mainzer, A., Gray, T., Masiero, J., et al. 2011b, ApJ, 741, 90
Mainzer, A. & NEOCam Science Team 2017, AAS Meeting, 49, 219.01
Michikami, T., Hagermann, A., Kadokawa, T., et al. 2016, Icar, 264, 316
Michikami, T., Nakamura, A. M., & Hirata, N. 2010, Icar, 207, 277
Morbidelli, A., Bottke, W. F., Jr., Froeschlé, Ch., & Michel, P. 2002, in Asteroids III, ed. W. F. Bottke, Jr et al. (Tucson, AZ: Univ. Arizona Press), 409
Morbidelli, A., & Vokrouhlický, D. 2003, Icar, 163, 120
Muller, T. G., Dreuch, J., Ishiguro, M., et al. 2017, A&A, 599, 103
Ostro, S. J., Benner, L. A., Magri, C., et al. 2005, M&P, 40, 1563
Polishook, D. 2013, MPBu, 40, 42
Popescu, M., Licandro, J., Morate, D., et al. 2016, A&A, 591, A15
Popova, O. P., Jenniskens, P., Emel’yanenko, V., et al. 2013, Sci, 342, 1069
Pravec, P., & Harris, A. W. 2000, Icar, 147, 477
Pravec, P., Hergenrother, C., Whiteley, R., et al. 2000, Icar, 147, 277
Pravec, P., Schleicher, P., Dreuch, J., et al. 2014, Icar, 233, 48
Rozitis, B., MacLennan, E., & Emery, J. P. 2014, Natur, 512, 1174
Sako, S., Ohsawa, R., Takahashi, H., et al. 2016, Proc. SPIE, 9908, 99083P
Sako, S., Ohsawa, R., Takahashi, H., et al. 2018, Proc. SPIE, 10702, 107020F
Samarasinha, N. H., & A Hearn, M. F. 1991, Icar, 93, 194
Scargle, D. J. 1982, ApJ, 263, 835
Scheeres, D. J., Marzari, F., & Rossi, A. 2004, Icar, 170, 312
