Event-by-event study of DCC-like fluctuation in ultra-relativistic nuclear collisions

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Abstract

A method based on sliding window scheme is developed to search for patches in the pseudorapidity-azimuth plane, on an event-by-event basis, having unusual fluctuation in the neutral pion fraction which may arise due to the formation of Disoriented Chiral Condensates (DCC) in high energy nuclear collisions. The efficiency of the method to extract the patches and the purity of the extracted sample are studied for possible experimental situations.

Key words: Disoriented Chiral Condensates, non-statistical fluctuation, sliding window method, ultra-relativistic nuclear collisions

Disoriented Chiral Condensates (DCC) have been predicted to be formed in high energy hadronic and nuclear collisions when the chiral symmetry is temporarily restored at high temperatures. As the matter cools and expands, the vacuum may relax into a state that has an orientation different from the normal vacuum. This may lead to the formation of localized domains of DCC, emitting low momentum pions in a single direction in isospin space \cite{1}. This would lead to event-by-event fluctuation in the number of charged particles and photons in a given phase space\textsuperscript{1}, since majority of photons originate from $\pi^0$ decay and charged particles are mostly charged pions. It has been estimated that the neutral pion fraction($f$), corresponding to the DCC domain, follows the probability distribution $P(f) = \frac{1}{2\sqrt{f}}$, where $f = \frac{N_{\pi^0}}{N_{\pi^0} + N_{\pi\pm}}$. This distribution is markedly different from the standard binomial distribution (peaking at $f = \frac{1}{3}$) for generic pion production in nuclear collisions. The observed Centauro($N_{ch} > N_\gamma$) and Anti-Centauro($N_{ch} < N_\gamma$) events in cosmic ray collisions \cite{2} may be due to the formation of large domains of DCC.

\textsuperscript{1} A section of the pseudorapidity($\eta$)-azimuth($\phi$) phase space where DCC pions may be localized will be called a patch.
There is very little theoretical guidance about the nature and probability of DCC formation in nucleon-nucleon and nucleus-nucleus collisions. Crude theoretical estimates suggest that in central lead-lead collisions at the CERN SPS, DCC formation probability may be \( \lesssim 10^{-3} \) [3]. The results of DCC search in cosmic ray [4] and \( \bar{p}p \) experiments [5] have been largely inconclusive. In nucleus-nucleus collisions, upper limits at 90% confidence level have been set in the analysis of data from the WA98 and NA49 experiments at the SPS [6–9] within the context of a simple DCC model and under various assumptions. For domains localized in \((\eta, \phi)\) the most strict limit set so far, for the top 5% central Pb-Pb collision events in the WA98 experiment [8], are \( \sim 10^{-2} \) and \( \sim 3 \times 10^{-3} \) for azimuthal domain sizes 45°-90° and 90°-135° respectively.

Several methods have been proposed to detect the DCC formation in high energy hadron and heavy ion collisions and also used in the analysis of experimental data. For details see the recent review by Mohanty and Serreau [1]. The sensitivity of the methods has been studied using a simple DCC model and generating simulated data containing a mixture of generic and DCC type events. The Discrete Wavelet Transform method [10] is found to be sensitive if the fraction of DCC events is at least \( 2 \times 10^{-3} \) for domain size in the range of 40°-90° in a scenario where all the pions within the domain are of DCC origin. Considering only pions with \( p_T < 300 \) MeV/c being of DCC origin, the sensitivity is found to be at \( 10^{-2} \) level. The fluctuation measure [11] has similar sensitivity. \( \Phi \)-measure [12] is useful if each event contains 25% pions from DCC source and the DCC event fraction in the sample is \( 10^{-2} \). It is thus found that the present methods, including those used in the analysis of data from the WA98 experiment, have sensitivity in the same range as the possible DCC formation probability. In this letter we describe a method which allows direct observation of DCC-like patches by examining the event structure and its sensitivity can be better than \( 10^{-4} \), being limited only by the available statistics.

We use a simple DCC model where charged neutral fluctuation is introduced in generic events produced by the VENUS event generator [13], keeping initially the identity of \( \pi^0 \) intact. It is assumed that the DCC pions survive till freezeout, and that they also remain localized in a \((\eta, \phi)\) patch. To introduce DCC-like fluctuation the identity of charged and neutral pions is interchanged pairwise \( (\pi^+ \pi^- \leftrightarrow \pi^0 \pi^0) \) according to the \( \frac{1}{2 \sqrt{f}} \) probability distribution within a randomly selected \((\eta, \phi)\) domain. Two scenarios are considered for DCC-like fluctuation in the selected domain:

- **DCC-I**: all the pions are of DCC origin;
- **DCC-II**: only the pions having \( p_T \leq 250 \) MeV/c are of DCC origin.

For the present study the DCC domain size is kept as one unit in pseudo-rapidity and 60° in azimuth. \( \pi^0 \)'s are allowed to decay after introducing the
DCC-like fluctuation. Considering experimental situations where one measures the multiplicities of charged particles \( N_{ch} \) and photons \( N_{\gamma} \) in an event, the neutral pion fraction may be approximately written as \( f = \frac{N_{\gamma}}{N_{\gamma}/2 + N_{ch}} \). We consider typical nuclear systems and collision centralities such that after \( \pi^0 \)-decay, the mean multiplicities of photons and charged particles are \( \sim 400 \) within one unit of pseudorapidity and with full azimuthal coverage in the generic event. Such multiplicities are typical for detector systems available in SPS and RHIC experiments.

Neutral pion fraction \( f \) distribution for DCC-I patches before \( \pi^0 \)-decay is shown in Fig. 1. The \( f \)-distribution after \( \pi^0 \)-decay, is also shown in Fig. 1 for various cases. In general the \( f \)-distributions shrink inward for both scenarios of DCC formation after \( \pi^0 \) decay [14], the effect being more pronounced for DCC-II. It is clear that in the central part of \( f \)-distribution the study of DCC-like fluctuation is difficult due to overwhelming background from the binomial distribution of generic particle production mechanism. We therefore concentrate on the low-\( f \) and high-\( f \) regions, which will be referred to as charge-excess and photon-excess type fluctuations respectively.

Different ensembles of simulated DCC-like events are generated by adding events having DCC-like fluctuation and generic events in different proportions,
reflecting the DCC formation probability. Such ensembles of events will be referred to as 'simulated data'. The fraction of DCC-like events ($\alpha$) in various ensembles of simulated data varies from $2 \times 10^{-4}$ to 1. The total number of events in each ensemble is $10^5$.

The search for any non-statistical fluctuation requires a reference for comparison which can describe statistical fluctuation in a model-independent manner and is free from inherent correlations. For the analysis of experimental data the technique of 'mixed events' provides such a reference set where correlations among particles are completely destroyed. The mixed events are generated using the standard procedure of taking large number of events (at least equal to the multiplicity of particles) and constructing new events having particles picked up randomly, one particle from one event. These events are created with the same multiplicity distribution as the 'simulated data', similar statistics and have similar charge-neutral correlation on a global scale.

For the study of charged neutral fluctuation, we assume that there are detectors measuring the photon and charged particle multiplicities in overlapping part of $(\eta, \phi)$ phase space, with or without the $p_T$-information of particles. The common coverage of the two detectors is assumed to be one unit in $\eta$ and $2\pi$ in $\phi$, the introduced DCC domain being fully contained within this region. We develop a simple but powerful 'sliding window' method (SWM) in which a window in azimuthal plane, say of size $\Delta \phi$, is chosen in the common coverage of the two detectors in which the neutral pion fraction $f$ is calculated. The entire azimuthal range of common coverage is scanned by continuously sliding the window, shifting each time by a small amount, say $\delta \phi$, to search for a patch having a neutral pion fraction several standard deviations away from the mean value. This method utilizes the full advantage of azimuthal resolution of the detectors and allows direct observation of patches having large (or small) $f$-values. The value of $\delta \phi$ depends on the azimuthal resolution of the two detectors. For the present work, we have chosen $\delta \phi = 2^\circ$. A preliminary version of the SWM is described in [15].

The SWM provides a set of $f$-values in each event. We focus on the maximum and minimum $f$-values ($f_{\max}$ and $f_{\min}$) to search for photon-excess and charge-excess type fluctuations. The $f_{\max}$ and $f_{\min}$ distribution of simulated data for $\alpha=0.01$ and 0.5 are shown in Fig. 2 for a typical window size $\Delta \phi=60^\circ$ along with those for mixed events generated from those samples. The $f_{\max}$ and $f_{\min}$ distributions for mixed events are almost Gaussian in shape for lower value of $\alpha$ but become skewed for large values of $\alpha$.

It is observed that the signals of non-statistical fluctuation can be distinguished only in the outer regions. We estimate the mean ($\mu$) and r.m.s. deviation ($\sigma$) of the mixed event distributions. A cut on the $f_{\max}$ and $f_{\min}$ distribution of simulated data is applied by $\mu \pm n\sigma$, where $n$ ranges from 3 to 5,
Fig. 2. $f_{\text{max}}$(left) and $f_{\text{min}}$(right) distributions for a $\Delta \phi = 60^\circ$ window size for two values of $\alpha$ in DCC-I scenario as indicated. Continuous lines : simulated events, dashed lines : mixed events generated from corresponding simulated events.

positive sign being applied for $f_{\text{max}}$ and negative sign for $f_{\text{min}}$ distributions. The events having patches where the $f_{\text{max}}$ or $f_{\text{min}}$ values are beyond the cut are labeled as 'extracted'.

The sensitivity of the SWM to extract DCC-like fluctuation is decided by the limit of statistical background that the mixed event sample gives for any given extraction procedure. The patches in mixed events having very large (or very small) $f$-values, arising due to statistical fluctuation, constitute the background to the study of DCC-like fluctuation. The statistical fluctuation depends on the multiplicity of particles in the patches and hence also on the window size in the SWM.

For mixed events produced from generic events ($\alpha=0$) Fig. 3(a) displays the fraction of events, which constitutes the background, versus window size for $\mu \pm 4\sigma$ cut on mixed event $f_{\text{max}}$ and $f_{\text{min}}$ distributions. The background is almost independent of window size for $\Delta \phi \geq 60^\circ$ and is found to be $4\times10^{-4}$ and $1\times10^{-4}$ respectively for photon-excess and charge-excess cases. For subsequent investigations we have used $\Delta \phi=60^\circ$.

The background due to statistical fluctuation has also been investigated using the mixed events generated from different ensembles of events (different $\alpha$
Fig. 3. Fraction of events versus window size for $\alpha=0$ (top) and versus $\alpha$ for $\Delta \phi=60^\circ$ (bottom) for $f_{\text{max}}$ distribution with $\mu+4\sigma$ cut (filled circles) and for $f_{\text{min}}$ distribution with $\mu-4\sigma$ cut (filled squares) for mixed events generated from different ensembles of events. Filled triangles in the bottom panel represent fraction of events for $f_{\text{max}}$ distribution with $\mu+5\sigma$ cut.

Fig. 3(b) shows the fraction of events as a function of $\alpha$ for $\mu \pm 4\sigma$ cuts on mixed event $f_{\text{max}}$ and $f_{\text{min}}$ distributions. The background is of similar magnitude as that in Fig. 3(a) for lower values of $\alpha$. With increasing $\alpha$, the background increases and then decreases for very high values. This apparent decrease at very high $\alpha$ is due to the distorted shape of the corresponding mixed event distributions and the resulting high values of $\sigma$. Fig. 3(b) also displays the background for photon-excess cases with $\mu + 5\sigma$ cuts. The background values are now an order of magnitude smaller, being as low as $5 \times 10^{-5}$ for smaller values of $\alpha$.

In the present case of simulation study, where the character of each event and patch is known, we can estimate the 'background' (B) which arises from the following sources: (a) the extracted patch is from a generic event and (b) the extracted patch, in the DCC-like event, is not from the azimuthal region where DCC was actually introduced. Extracted patches which belong to the DCC-like events and are found around the same azimuthal location are labeled as 'extracted signal' (S). In order to characterize the SWM we estimate the efficiency of extraction S/N, where N is the number of DCC-like events present in the ensemble depending on $\alpha$ value, and the 'signal purity
The effect of window size on the results of SWM analysis has been investigated further for the case of simulated events with $\alpha=0.01$. The efficiency of extraction and the purity of the extracted patches are plotted for the case of both charge-excess and photon-excess fluctuations as a function of window size in Fig. 4, using $\mu \pm 4\sigma$ cut as the extraction criteria. It is found that the efficiency and purity are not much affected by varying the window size over a wide range around the azimuthal domain where fluctuation was originally introduced. The results are affected only at very small window sizes, which, as discussed earlier, get dominated by statistical fluctuation due to small number of particles.

Fig. 5 shows the efficiency and purity as a function of $\alpha$ for the entire data set analyzed in the present work for both charge-excess and photon-excess type fluctuations in DCC-I and DCC-II scenarios. The extraction criteria applied is a cut of $\mu \pm n\sigma$ on the $f_{\text{max}}$ and $f_{\text{min}}$ distributions obtained from the corresponding mixed events for each $\alpha$. Results for DCC-I scenario are presented for $n=3,4,5$ for photon-excess and for $n=3,4$ for charge-excess cases.

As a function of increasing $n$, the signal falls very slowly whereas the back
Fig. 5. Efficiency and purity versus $\alpha$. The left panels are for photon-excess and the right panels for charge-excess fluctuations. Filled circles: $3\sigma$ cut for DCC-I, filled square: $4\sigma$ cut for DCC-I, filled triangles: $5\sigma$ cut for DCC-I, open stars: $4\sigma$ cut for DCC-II with multiplicity measurement only, filled stars: $4\sigma$ cut for DCC-II with the addition of charged particle $p_T$ measurement.

The background falls sharply. The efficiency is almost constant for lower values of $\alpha$. The decrease in efficiency for larger values of $\alpha$ is due to distortions in the mixed event $f_{\text{min}}$ and $f_{\text{max}}$ distributions and the apparently large $\sigma$-values. The efficiency for extraction of patches for both the charge-excess and photon-excess cases in DCC-I scenario is found to be in the range of 10-20%, lower values of efficiency corresponds to higher purity. Even for $\alpha$ as low as $3\times10^{-4}$, the purity is more than 50%, indicating that the background is less than the signal. It is clear that if the statistics is high, one can make even tighter cuts ($n > 5$) to further improve the purity with only marginal loss in efficiency.

For the analysis of data having fluctuation in DCC-II scenario, two approaches are used. First we assume that only multiplicity information is available. The analysis proceeds in a manner identical to what was done for DCC-I case. The efficiency of extracting DCC-II type patches and the purity of such extracted patches are shown in Fig. 5 using $\mu \pm 4\sigma$ cut as the extraction criteria. The efficiency is now quite low for both charge-excess and photon-excess type patches. This is understandable as the fraction of DCC pions in the patches is now reduced to less than half.
In the second approach we assume that the $p_T$ information of charged particles is available. Such a possibility exists in the STAR experiment at RHIC [16]. The neutral pion fraction $f$ is now calculated by taking only the charged particles having $p_T < 250$ MeV/c. Using the new $f$-values both $f_{\text{min}}$ and $f_{\text{max}}$ distributions are generated. The efficiency of extraction and the purity of extracted samples, shown in Fig. 5 using the $\mu \pm 4\sigma$ cut as the extraction criteria, are found to increase by a factor of about 5-10 over the case when only multiplicity information was used.

In summary the SWM provides a simple and elegant method for the study of charge-neutral fluctuation in ultra-relativistic nuclear collisions. The sensitivity of the method, decided by the background due to statistical fluctuation, can be improved to a great extent by making suitable extraction criteria so that the background is minimized. In the present analysis, with just $10^5$ events, it has been possible to reduce the background to $5 \times 10^{-5}$. This is almost two orders of magnitude better than the previous methods. At higher multiplicity, as is expected at the LHC experiments, background due to statistical fluctuation may be further reduced.

The efficiency of extracting the DCC-like patches varies from a few percent to about 40% (combining the photon-excess and charge-excess cases). This depends on the extraction criteria, DCC formation probability and the $p_T$ distribution of DCC pions. The purity of the extracted patches also depends on those parameters and can reach 60-80% even for low values of $\alpha$. For high values of $\alpha$ this reaches close to 100%. For a given $\alpha$ and extraction criteria the purity is higher for photon excess cases than for charge-excess cases as observed earlier [14]. In DCC-II scenario the efficiency is much lower for charge-excess cases than for photon-excess cases. Considering limitations due to detector effects [14] it is advantageous to look for photon-excess type charge-neutral fluctuation.

The SWM is a general method and can be utilized not only in azimuthal space but also in pseudorapidity space or even in any combined phase space with multi-dimensional windows and using any suitable physical observable which can be computed over the window. The only important point is to slide the window over the acceptable phase space region to compute a number of values of the observable under investigation and then study their distributions. This allows for direct observation of unusual structures in the event and draw conclusions which are model-independent.

The method developed here can be directly applied to the analysis of experimental data using mixed event as the reference set without recourse to any DCC model. With improved statistics, which is now-a-days available in collider experiments, it should be possible to reduce the background to any desired level and improve the sensitivity of the SWM. This should also allow one to
employ tight cuts which further improves the signal purity to the estimated level to confirm or deny the occurrence of exotic phenomena.

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