Pseudoclassical mechanics and hidden symmetries of 3d particle models

Khazret S. Nirov

Physikalisches Institut, Universität Bonn, Nussallee 12, D-53115 Bonn, Germany

Abstract

We discuss hidden symmetries of three-dimensional field configurations revealed at the one-particle level by the use of pseudoclassical particle models. We argue that at the quantum field theory level, these can be naturally explained in terms of manifest symmetries of the reduced phase space Hamiltonian of the corresponding field systems.

Talk presented at the Meeting on
Quantum Aspects of Gauge Theories, Supersymmetry and Unification
Neuchâtel, Switzerland, 18–23 September 1997

1E–mail: nirov@dirac.physik.uni-bonn.de (Alexander von Humboldt fellow)
2On leave from the Institute for Nuclear Research of the Russian Academy of Sciences
1 Introduction

Pseudoclassical mechanics has been basically understood in terms of the principles formulated by J.L. Martin (1959) [1], F.A. Berezin and M.S. Marinov (1975-77) [2], and R. Casalbuoni (1976) [3]. The underlying idea of refs. [1, 2, 3] was to consider elements of a Grassmann algebra as classical dynamical variables (functions on the phase space), so generalizing the concept of classical mechanics to a purely algebraic construction of the ring and the graded Lie algebra, in order to describe spin dynamics. Such a formalism was implemented also by A. Barducci, R. Casalbuoni and L. Lusanna, and L. Brink, S. Deser, B. Zumino, P. Di Vecchia and P. Howe [4], when constructing various models of supersymmetric spinning particles.

There, as the starting point, the well-known covariant supersymmetric differential 1-form was reconsidered. It was observed that this object can be realized in different ways. Indeed, instead of considering the superspace parameterized by the usual space-time coordinates and pairs of Weyl spinors, one can introduce real anticommuting variables, Grassmannian odd (pseudo)vectors and (pseudo)scalars, together with the even classical coordinates forming a set of configuration space variables. Upon quantization, these new variables turn to be the generators of real Clifford algebras with corresponding spin indices, but not the Fermi variables as it was formerly, so that no contradiction with the spin-statistics relation emerges.

Having all this at hand, one can construct various superparticle (pseudoclassical) Lagrangians, thus explicitly describing spin degrees of freedom of the corresponding quantum-mechanical and field systems. Further, if the latter possess some dynamical (hidden) symmetries, for which the spin dynamics is responsible, then the formalism of pseudoclassical mechanics certainly provides an appropriate basis for revealing such properties and understanding their nature.

Exactly this task was in the program of refs. [5, 6], where hidden symmetries of three-dimensional field configurations were revealed by means of the corresponding pseudoclassical models. So, the spin dynamics of the \( P, T \)-invariant systems of planar massive fermions [3] and topologically massive U(1) gauge fields [4] have been described by the Lagrangians \( L_F \) and \( L_T \), respectively with

\[
L_F = \frac{1}{2e} \left( \dot{x}_\mu - i\nu \varepsilon_{\mu\nu\lambda} \xi^\nu \xi^\lambda \right)^2 - \frac{1}{2} \epsilon m^2 + 2iqm\nu\theta_1 \theta_2 - \frac{i}{2} \xi^\mu \dot{\xi}_\mu + \frac{i}{2} \theta_a \dot{\theta}_a, \tag{1.1}
\]

\[
L_T = \frac{1}{2e} \left( \dot{x}_\mu - \frac{i}{2} \nu \varepsilon_{\mu\nu\lambda} \xi^\nu \xi^\lambda \right)^2 - \frac{1}{2} \epsilon m^2 - iqmv \xi^\mu \xi_2 \mu + \frac{i}{2} \xi^\mu \xi_2 \mu. \tag{1.2}
\]

The configuration spaces of the systems were thus represented by the sets of even, \( x_\mu, e, v \), and odd, \( \xi^\mu, \theta_a, \xi_2 \), variables, \( \mu = 0, 1, 2, a = 1, 2 \). In this, \( x_\mu \) are the space-time coordinates of the particles, \( e \) and \( v \) make sense as Lagrange multipliers, with \( e \) representing the worldline metrics, whereas the odd variables \( \xi^\mu, \theta_a \) and \( \xi_2 \) are incorporated for the spin degrees of freedom of the three-dimensional field theories expected as quantum counterparts of the pseudoclassical models; \( q \) denotes a real \( c \)-number parameter, hereafter called the model parameter; the metric and the totally antisymmetric tensors of the 3d Minkowski space-time are fixed by the conditions \( \eta_{\mu\nu} = \text{diag}(-1, +1, +1) \) and \( \varepsilon^{012} = 1 \).
The models (1.1) and (1.2) have rich sets of symmetries – discrete, gauge and continuous global. It was observed [5, 6] that $L_F$ and $L_T$ hold parity and time-reversal invariance (discrete symmetries) for any value of the model parameter. However, the cases of $q = -1, 0, +1$ in (1.1) and $q = -2, 0, +2$ in (1.2) turned out to be special both at the classical and the quantum levels of the theories. The case of $q = 0$ is dynamically degenerated, with some of the spin variables being trivial integrals of motion, and this value of the model parameter is excluded at the quantum level. As well, the values $q = -1, +1$ and $q = -2, +2$ are separated from the point of view of dynamics of (1.1) and (1.2), respectively. However, in the latter cases, the classical systems get maximal global symmetries, with sets of generators including nontrivial integrals of motion which exist only at these special values of the model parameter. Exactly the same values reveal themselves at the quantum level too; but quantum mechanically they are singled out not only by the requirement for the continuous global symmetries to be maximal. Indeed, only for these special values of the model parameter the discrete symmetries of the initial classical systems are conserved upon quantization. We see that if a system has a quantized parameter, then its special discrete values may in principle reveal themselves just at the classical level, from the point of view of dynamics. This interesting phenomenon, called in ref. [8] the classical quantization, implies some hidden relationships between continuous and discrete symmetries. Note that (1.1) and (1.2) belong to the class of gauge systems having quadratic in Grassmann variables nilpotent constraints of the form [8]

$$\left(A_{ij}\delta^{ab} + qB_{ij}e^{ab}\right)\zeta_i^a\zeta_j^b \approx 0,$$

where $A_{ij}$ and $B_{ij}$ being functions of even variables fulfil the relations $A_{ij} = -A_{ji}, B_{ij} = B_{ji}$, $\delta^{ab}$ is the Kronecker symbol, $a, b = 1, 2$, $e^{ab} = -e^{ba}$. It can be shown that in this general case, the special values of $q$ are singled out by the following quantization condition:

$$\{q = q_k \mid \det \parallel 2ik A_{ij} + q_k (kB_{ij} - \frac{1}{2}B\eta_{ij}) \parallel = 0\}, \quad k = 1, \ldots, K,$$

$B = \eta^{ij}B_{ij}$ and $\eta_{ij}$ is regarded as the metric tensor of the $K$-dimensional linear space spanned by the indices $i, j = 1, 2, \ldots, K$. The quantization condition for the model parameter means actually consistency of classical and quantum dynamics.

Quantization of the systems with the Lagrangians $L_F$ and $L_T$ has led to three-dimensional parity and time-reversal conserving systems of massive fermions and Chern-Simons U(1) gauge fields, respectively [4, 5]. When analyzing algebras of the integrals of motion of the pseudoclassical models, hidden U(1,1) symmetry and S(2,1) supersymmetry of the corresponding field configurations were elucidated [4, 6]. It has also been shown that these field systems realize irreducible representations of a non-standard super-extension of the (2+1)-dimensional Poincaré group, ISO(2,1|2,1), labelled by the zero eigenvalue of the corresponding superspin operators.

In this talk, we shall discuss dynamical U(1,1) symmetry of the $P, T$-invariant system of topologically massive gauge fields, related to (1.2), and show that this hidden symmetry can be naturally understood in terms of manifest symmetries of the corresponding reduced phase space Hamiltonian. Similar analysis for the double fermion system can analogously be provided. With this investigation, we start solving the problem of generalization of the hidden symmetries, revealed at the one-particle level [4, 6], onto the level of quantum field theory.
In the section 2, we recall the structure of the field system in question and stress on similarity to the string field theory construction; then, in the section 3, we carry out the Hamiltonian description of the system. The paper is concluded by discussion of some related problems to be further investigated.

2 Constructing the $P,T$-invariant field system

In order to find the field system corresponding to the pseudoclassical model (1.2), its general quantum state $\Psi(x)$ was realized over the vacuum as an expansion into the complete set of eigenvectors of the fermion number operator $[6]$. The coefficients of this expansion, initially supposed to be square-integrable functions of the space-time coordinates, turned in fact to be belonging to the Schwartz space, which is a rigged Hilbert space $[9]$. The latter is due to the mass-shell, $\phi \approx 0$, and quadratic in spin variables nilpotent, $\chi \approx 0$, constraints following from the Lagrangian $L_T$. The quantum counterparts of these first class constraints were used to single out the physical subspace of the theory, $\hat{\phi}\Psi = \hat{\chi}\Psi = 0$. As the consistent solution to the quantum constraints, the doublet of vector fields $F_\mu^+, F_\mu^-$ satisfying the linear differential equations

$$L_{\mu\nu}^+ F_{\nu}^+ = 0, \quad L_{\mu\nu}^- F_{\nu}^- = 0,$$

(2.1)

was obtained. This is the subspace of the total state space connected with the special values $|q| = 2$ of the model parameter. It is easy to see that, due to the basic equations (2.1), the fields $F_\mu^\pm$ obey also the Klein-Gordon equation ($-\partial^2 + m^2)F_\mu^\pm = 0$ and the transversality condition $\partial_\mu F_\mu^\pm = 0$. Therefore, the physical subspace of the theory is described by the vector fields $F_\mu^\pm$, respectively carrying massive irreducible representation of the spin $s = \mp 1$ of the 3d Poincaré group.

To construct the action functional, that would reproduce the field equations (2.1) by the variational principle, the average value of the constraint operator $\hat{\chi}$ over the general state was considered, $\langle \hat{\chi} \rangle = \Psi^\dagger(x)\hat{\chi}\Psi(x)$. It was however observed that the metric on the state space was indefinite in the doublet $\Phi = (F_+^+, -F_-^-)$. Actually, restricting the scalar product $\langle \cdot, \cdot \rangle$ onto the physical subspace, one gets $\langle \Psi, \Psi \rangle = \Phi\Phi$, $\Phi = (F_+^+, -F_-^-)$. And so, to have the norm of the state vectors defined from a positive-definite scalar product and give the physical states with spins +1 and −1 equal treatment, the scalar product was modified to $\langle\langle \cdot, \cdot \rangle\rangle = \bar{\Phi}\sigma_3\Phi$. For the constraint operator $\hat{\chi}$ this gave the following average value restricted onto the physical subspace:

$$\langle\langle \hat{\chi} \rangle\rangle = \varepsilon^{a}_{\mu\beta} (F_{+\alpha}^a \partial^\mu F_{+\gamma}^\beta + F_{-\alpha}^a \partial^\mu F_{-\gamma}^\beta) + m (F_{+\gamma}^\gamma F_{+\gamma}^\gamma - F_{-\gamma}^\gamma F_{-\gamma}^\gamma).$$

(2.2)

The space-time integral of this quantity has finally resulted in the desirable action functional

$$A = \int d^3 x \langle\langle \hat{\chi} \rangle\rangle = \int d^3 x \left( L_{\mu\nu}^+ F_{\nu}^+ + L_{\mu\nu}^- F_{\nu}^- \right)$$

(2.3)

of the parity and time-reversal conserving system of Chern-Simons U(1) gauge fields $[10]$, given in terms of a self-dual free massive field theory $[11]$. Dynamical (super)symmetries of the field theory (2.3) are generated by the average values of the quantum counterpart of the integrals of motion of the system (1.2) with $|q| = 2$. In this, the corresponding quantum mechanical nilpotent operators $[6]$ realize mutual transformation.
of the physical states of spins +1 and −1. It is crucial that these physical operators turned out to be Hermitian exactly with respect to the scalar product \( \langle \langle \cdot, \cdot \rangle \rangle \).

The procedure we have implemented is actually reminiscent of that suggested by Siegel \[12\] for constructing a string field theory and subsequently developed by Witten \[13\]. There, an object of the form \( A = \int d\mu \langle \Psi | \Omega | \Psi \rangle \), with a BRST operator \( \Omega \) singling out physical states and \( d\mu \) being an integration measure, was treated as a string field theory action. Also, a scalar product \( \langle || \rangle \) was proposed to ensure hermiticity for the BRST operator. The underlying idea was taken from the observation that the functional \( A \) is extremal on the physical subspace: the variational principle applied to the “action” \( A \) reproduces “quantum equations of motion” encoded in the BRST condition \( \Omega | \Psi \rangle = 0 \), and besides, it keeps symmetries of the initial first-quantized theory. Obviously, here we dealt with an analogous construction, while having the constraint operator \( \hat{\chi} \) instead of the BRST-charge \( \Omega \) and the finite-mode decomposition of the general state vector \( \Psi(x) \) where the expansion coefficients were, in particular, topologically massive U(1) gauge fields. As well as in the string field theory \[12, 13\], the action \( A \) detains all symmetries of its first-quantized counterpart \[6\].

3 Hamiltonian on the reduced phase space

To construct the Hamiltonian description of the field theory (2.3), let us introduce, as usual, the phase space with the generalized momenta \( P^\mu_\epsilon(x, t) \) canonically conjugate to the generalized coordinates \( F^\mu_\epsilon(x, t) \), \( \epsilon = +, - \),

\[
\{ F^\mu_\epsilon(x, t), P^\nu_\epsilon'(y, t) \} = \delta_{\epsilon\epsilon'} \eta^{\mu\nu} \delta(x - y). \tag{3.1}
\]

There are primary constraints in the system,

\[
P^0_\epsilon \approx 0, \quad P^i_\epsilon + \epsilon^{0ij} F^j_\epsilon \approx 0, \tag{3.2}
\]

and hence, the total Hamiltonian \( H_T(t) = \int d^2x h_T(x, t) \) contains Lagrange multipliers \( \lambda^0_\epsilon \) and \( \lambda^i_\epsilon \); we have

\[
h_T = h + \sum_{\epsilon = +, -} \lambda^0_\epsilon P^0_\epsilon + \lambda^i_\epsilon \left( P^i_\epsilon + \epsilon^{0ij} F^j_\epsilon \right),
\]

where the density of the canonical Hamiltonian \( h(x, t) \) is given by the expression

\[
h = \sum_{\epsilon = +, -} \epsilon^{0ij} \left( F^0_\epsilon \partial_i F^0_\epsilon - F^0_\epsilon \partial_i F^0_\epsilon \right) + \epsilon m \left( F^0_\epsilon F^0_\epsilon - F^i_\epsilon F^i_\epsilon \right).
\]

Under subsequent application of the Dirac-Bergmann formalism \[14\], secondary constraints arise in the system,

\[
\epsilon^{0ij} \partial_i F^j_\epsilon - \epsilon m F^0_\epsilon \approx 0, \tag{3.3}
\]

and the Lagrange multipliers get fixed, \( \lambda^0_\epsilon = -\partial_i F^i_\epsilon, \lambda^i_\epsilon = -\partial_i F^0_\epsilon + \epsilon m \epsilon^{0ij} F^j_\epsilon \). The complete set of the constraints (3.2), (3.3) is of the second class. We see that the initial phase space can be reduced to the corresponding constraint surface. Restricting the Poisson brackets (3.1) onto this reduced phase space, one obtains the related Dirac brackets. The physical
degrees of freedom, i.e. the points of the reduced phase space, may now be described by the set of the field variables $\mathcal{F}_\epsilon^i(\vec{x}, t)$, $i = 1, 2$, $\epsilon = +, -$ , having the Dirac brackets

$$\{ \mathcal{F}_\epsilon^i(\vec{x}, t), \mathcal{F}_{\epsilon'}^j(\vec{y}, t) \}^* = \frac{1}{2} \delta_{\epsilon\epsilon'} \delta^i_j \delta(\vec{x} - \vec{y})$$

and satisfying the equations of motion

$$\dot{\mathcal{F}}_\epsilon^i = -\epsilon \epsilon' \delta_{\epsilon\epsilon'} \Delta_{jk} \mathcal{F}_\epsilon^k,$$

$$\Delta_{ij} = m \delta_{ij} - \frac{1}{m} \epsilon_{ijk} \partial^k \epsilon_{0jl} \partial^l.$$  

It is worthwhile noting that one has also the equations $\dot{\mathcal{F}}_0^i = -\partial_i \mathcal{F}_0$ confirming the above mentioned transversality condition. In terms of these variables, converted into the representation

$$\mathcal{F}_\epsilon^i(\vec{x}, t) = \frac{1}{2\pi} \int d^2\vec{p} \ e^{i\vec{p}\vec{x}} \tilde{\mathcal{F}}_\epsilon^i(\vec{p}, t), \quad \tilde{\mathcal{F}}_\epsilon^i(\vec{p}, t) = \mathcal{F}_\epsilon^i(-\vec{p}, t),$$

the Hamiltonian of the system on the reduced phase space takes the form

$$H^* = -\int d^2\vec{p} \ \sum_{\epsilon = +, -} \epsilon \tilde{\mathcal{F}}_\epsilon^i \tilde{\Delta}_{ij} \tilde{\mathcal{F}}_{\epsilon'}^j, \quad \tilde{\Delta}_{ij} \equiv m \delta_{ij} + \frac{1}{m} \epsilon_{0ik} p^k \epsilon_{0jl} p^l.$$  

Seeing that the operator $\tilde{\Delta}_{ij}$ allows for the representation

$$\tilde{\Delta}_{ij} = \omega_{ik} \omega_{kj}, \quad \omega_{ij} \equiv \sqrt{m} \delta_{ij} + \frac{1}{\sqrt{m}} \frac{\epsilon_{0ik} p^k \epsilon_{0jl} p^l}{\sqrt{m^2 + \vec{p}^2 + m}},$$

so that the new fields $u_\epsilon^i = \omega_{ij} \tilde{\mathcal{F}}_{\epsilon'}^j$ are well-defined, the Hamiltonian $H^*$ can be given another convenient form

$$H^* = \int d^2\vec{p} \left( u_\epsilon^- u_\epsilon^- - u_\epsilon^+ u_\epsilon^+ \right).$$

Finally, passing on to the complex variables $z_\epsilon = u_1^\epsilon + i u_2^\epsilon$ with the canonical brackets

$$\{ z_\epsilon(p), \bar{z}_{\epsilon'}(q) \}^* = -i |p^0|^2 \delta_{\epsilon\epsilon'} \delta(\vec{p} + \vec{q}), \quad |p^0| \equiv \sqrt{m^2 + \vec{p}^2},$$

we get for the reduced phase space Hamiltonian the expression

$$H^* = \int d^2\vec{p} \left( |z_-|^2 - |z_+|^2 \right).$$

The latter is exactly the well-known [15] defining quadratic form for the symmetry group U(1,1) with the non-compact group manifold $\mathcal{M}(U(1,1)) \cong S^1 \times \mathbb{R}^2$. Note that if one rescales the complex variables $z_\pm$ as $|p^0|^{1/2} z_\epsilon \rightarrow z_\epsilon$, they become the classical counterparts of the oscillator annihilation-creation operators obeying the standard commutation relations

$$[\tilde{z}_\epsilon(p), \tilde{z}_{\epsilon'}(q)] = \delta_{\epsilon\epsilon'} \delta(\vec{p} + \vec{q}).$$

Let us introduce a positive-definite scalar product $(,)$ on the corresponding state space and denote the respective Hermitian conjugation by the dagger $\dagger$. Then the operators $\tilde{z}_\epsilon$ and $\tilde{\bar{z}}_\epsilon$ are mutually conjugate with respect to the scalar product $(,)$. We have

$$(\tilde{z}_\epsilon)\dagger = \tilde{\bar{z}}_\epsilon.$$
The explicit forms of the dynamical symmetry group generators can easily be found (see e.g. ref. [15]). They are simply the conserved charges of the field theory with the Hamiltonian being the quantum counterpart of $H^\ast$. Their densities are given by the expressions

$$ u = \frac{1}{2} (\bar{z}_- z_- - \bar{z}_+ z_+), \quad q_0 = \frac{1}{2} (\bar{z}_- z_- + \bar{z}_+ z_+ + 1), \quad q_+ = \bar{z}_+ z_-, \quad q_- = \bar{z}_- \bar{z}_+. \quad (3.10) $$

In this, $u$ is the density of the U(1) generator, while the rest fulfil the usual $su(1, 1)$ algebra

$$ [q_+, q_-] = 2q_0, \quad [q_\pm, q_0] = \pm q_\pm. $$

The operators $u$ and $q_0$ are Hermitian, and $q_+$ and $q_-$ are mutually conjugate with respect to the scalar product $(,)$, $(u)^\dagger = u$, $(q_0)^\dagger = q_0$, $(q_\pm)^\dagger = q_\mp$. We have thus reproduced the dynamical $U(1, 1) = U(1) \times SU(1,1)$ symmetry of the $P,T$-invariant system of topologically massive vector fields at the quantum field theory level. Certainly, one could gain the same ends while using not the auxiliary representation (3.5), but the ordinary decomposition of the fields $\mathcal{F}^i$ over annihilation and creation operators.

Instead of dealing with the action $\mathcal{A}$, one might consider another action functional

$$ \mathcal{A}' = \int d^3x \left( -\mathcal{F}^i_+ \mathcal{L}^+_{\mu\nu} \mathcal{F}^i_\nu + \mathcal{F}^i_- \mathcal{L}^-_{\mu\nu} \mathcal{F}^i_\nu \right) \quad (3.11) $$

leading to the same equations (2.1) for the Chern-Simons vector fields. Constructing the reduced phase space Hamiltonian reads:

$$ H^\ast = \int d^2\bar{p} \sum_{\epsilon = +,-} \bar{\mathcal{F}}^i_\epsilon \tilde{\Lambda}_{ij} \mathcal{F}^j_\epsilon = \int d^2\bar{p} \left( |z_-|^2 + |z_+|^2 \right). \quad (3.12) $$

In contrast with the preceding analysis, one should thus find the dynamical symmetry group of the field theory with $\mathcal{A}'$ to be U(2) having the compact group manifold $\mathcal{M}'(U(2)) \cong S^3$, as it is obvious from the defining quadratic form (3.12). However, the dynamical symmetry group is again U(1,1). This result on continuous global symmetries hidden in (2.3) and (3.11) looks somewhat enigmatic, if one takes into account only the equations of motion and ignores the corresponding discrete symmetries. Indeed, the action $\mathcal{A}$ is odd under the parity and time-reversal transformations, $P,T : \mathcal{A} \to -\mathcal{A}$, although it describes physical states with the spins $-1$ and $+1$, being thus respective to a $P,T$-invariant system. The price for such a disorder is that the reduced phase space Hamiltonian $H^\ast$ is not positive-definite. On the other hand, $\mathcal{A}'$ is parity and time-reversal invariant, and its reduced phase space Hamiltonian $H'^\ast$ is positive-definite, but now the oscillator-like operators have the following commutation relations:

$$ [\hat{z}_\epsilon(p), \hat{z}_{\epsilon'}(q)] = -\epsilon \delta_{\epsilon \epsilon'} \delta(p + q). \quad (3.13) $$

As a consequence, we get

$$ (\hat{z}_\epsilon)^\dagger = -\epsilon \hat{z}_\epsilon \quad (3.14) $$

for the Hermitian conjugation with respect to our positive-definite scalar product $(,)$.

The last two relations are essentially different from the corresponding eqs. (3.3) and (3.9). Taking into account these properties, we obtain the densities of the dynamical symmetry group $U(1,1)$ generators of the system (3.11), (3.12) to be of the form

$$ u' = \frac{1}{2} (\bar{z}_- z_- + \bar{z}_+ z_+), \quad q'_0 = \frac{1}{2} (\bar{z}_- z_- - \bar{z}_+ z_+ + 1), \quad q'_+ = i \bar{z}_+ z_-, \quad q'_- = i \bar{z}_- \bar{z}_+. \quad (3.15) $$
They satisfy the same relations as do the above described (non-primed) U(1,1) generators.

Note that, analogous to (2.2) and (2.3), the action $A'$ is related to the constraint operator $\hat{\chi}$ of the pseudoclassical model (1.2), only that averaged over the state space with the metric $\langle , \rangle$. Going back to the one-particle level of (3.11) and seeking for the new U(1,1) dynamical symmetry group generators to be Hermitian with respect to the scalar product $\langle , \rangle$, one gets the desirable set of the physical operators by appropriate redefinitions of the U(1,1) quantum-mechanical generators $Q_\alpha, \alpha = 0, 1, 2, \text{ and } U$ from ref. [6]. The new operators $Q'_\alpha$ related to the system (3.11) form $\text{su}(1,1)$ symmetry and $s(2,1)$ supersymmetry [16] algebras, as it was the case of ref. [6].

We have observed that the hidden symmetries of the planar free field systems revealed when explicitly modelling the underlying spin dynamics at the one-particle level, appeared to be the manifest (dynamical) symmetries of the corresponding field theories’ reduced phase space Hamiltonian. And again, as well as for the dynamical picture of the pseudoclassical gauge systems [1, 8], the discrete symmetries turned out to be of crucial importance for the continuous global symmetries of these field theories. However, we must mention that it is rather difficult, if only feasible, to explain the hidden supersymmetries of the systems under consideration working on exclusively the field theory level, say, through some quirky fermionization. It seems very likely that in order to see the dynamical symmetries to be accompanied by the supersymmetries leading to non-standard super-extensions of the Poincaré group [3, 6], it is necessary to investigate the corresponding pseudoclassical models.

4 Concluding remarks

We have analyzed hidden (dynamical) symmetries revealed in various field configurations by means of the corresponding pseudoclassical particle models. The motivation for the research into these field systems is usually based on the claim about their relevance to critical phenomena in planar physics. Therefore, the main problem to be further investigated in view of the present analysis is naturally to find any possible development of the discussed dynamical (super)symmetries in application to real physical processes. For this purpose, it worths to consider models with matter couplings to the free systems investigated here.

Acknowledgement

This work has been supported by the Alexander von Humboldt Fellowship and the European Commission TMR Programme ERBFMRX–CT96–0045 and ERBFMRX–CT96–0090.

References

[1] J. L. Martin, Generalized classical mechanics, and the “classical analogue” of a Fermi oscillator, Proc. Roy. Soc. A251 (1959) 536, The Feynman principle for a Fermi system, Proc. Roy. Soc. A251 (1959) 543.

[2] F. A. Berezin and M. S. Marinov, Classical spin and Grassmann algebra, JETP Lett. 21 (1975) 320, Particle spin dynamics as the Grassmann variant of classical mechanics, Ann. Phys. (NY) 104 (1977) 336.
[3] R. Casalbuoni, On the quantization of systems with anticommuting variables, Nuovo Cim. 33A (1976) 115, The classical mechanics for Bose–Fermi systems, Nuovo. Cim. 33A (1976) 389, Relativity and supersymmetries, Phys. Lett. 62B (1976) 49.

[4] A. Barducci, R. Casalbuoni and L. Lusanna, Supersymmetries and the pseudoclassical relativistic electron, Nuovo Cim. 35A (1976) 377; L. Brink, S. Deser, B. Zumino, P. DiVecchia and P. Howe, Local supersymmetry for spinning particles, Phys. Lett. 64B (1976) 435; L. Brink, P. DiVecchia and P. Howe, A Lagrangian formulation of the classical and quantum dynamics of spinning particles, Nucl. Phys. B118 (1977) 76.

[5] G. Grignani, M. Plyushchay and P. Sodano, A pseudoclassical model for P,T-invariant planar fermions, Nucl. Phys. B464 (1996) 189 hep-th/9511072.

[6] Kh. S. Nirov and M. S. Plyushchay, Pseudoclassical model for topologically massive gauge fields, JHEP 02 (1998) 015 hep-th/9712097; P, T-invariant system of Chern–Simons fields: Pseudoclassical model and hidden symmetries, Nucl. Phys. B512 (1998) 295 hep-th/9803221.

[7] J. L. Cortés, M. S. Plyushchay and L. Velázquez, A pseudoclassical model for the massive Dirac particle in d dimensions, Phys. Lett. B306 (1993) 34.

[8] Kh. S. Nirov and M. S. Plyushchay, Symmetries and classical quantization, Phys. Lett. B405 (1997) 114 hep-th/9707070.

[9] N. N. Bogolubov, A. A. Logunov, A. I. Oksak and I. T. Todorov, General Principles of Quantum Field Theory (Kluwer, 1990).

[10] W. Siegel, Unextended superfields in extended supersymmetry, Nucl. Phys. B156 (1979) 135; R. Jackiw and S. Templeton, How superrenormalizable interactions cure their infrared divergences, Phys. Rev. D23 (1981) 2291; J. Schonfeld, A mass term for three-dimensional gauge theories, Nucl. Phys. B185 (1981) 157; S. Deser, R. Jackiw and S. Templeton, Three-dimensional massive gauge theories, Phys. Rev. Lett. 48 (1982) 975, Topologically massive gauge theories, Ann. Phys. (NY) 140 (1982) 372, Erratum: ibid. 185 (1988) 406.

[11] P. K. Townsend, K. Pilch and P. van Nieuwenhuizen, Selfduality in odd dimensions, Phys. Lett. 136B (1984) 38, Addendum: ibid. 137B (1984) 443; S. Deser and R. Jackiw, ‘Selfduality’ of topologically massive gauge theories, Phys. Lett. 139B (1984) 371.

[12] W. Siegel, Covariantly second quantized string, 3. Phys. Lett. 149B (1984) 157 (corrected in Phys. Lett. 151B (1985) 391), Covariantly second quantized string, 3. Phys. Lett. B149B (1985) 162 (corrected in Phys. Lett. 151B (1985) 396).

[13] E. Witten, Noncommutative geometry and string field theory, Nucl. Phys. B268 (1986) 253, Interacting field theory of open superstrings, Nucl. Phys. B276 (1986) 291.

[14] P. A. M. Dirac, Lectures on Quantum Mechanics (Yeshiva University, New York, 1964).

[15] A. O. Barut and R. Raczka, Theory of Group Representations and Applications (World Scientific, 1986).

[16] M. de Crombrugghe and V. Rittenberg, Supersymmetric quantum mechanics, Ann. Phys. (NY) 151 (1983) 99.