Accuracy of the short fibers reinforced composite material plasticity models

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Abstract. Composite materials reinforced with short high-strength fibers are used in the aerospace industry, due to their high specific stiffness and strength. The anisotropic nonlinear nature of a short fiber reinforced composites requires the usage of anisotropic plasticity model of the material representative volume mechanical characteristics to predict the stress-strain state of the molded products. In this work, was made a comparison of three models of binder hardening – power, exponential and linear-exponential laws. The models were built in the Digimat MF system. The models describe tension of ISO 527-2 samples of 50% short glass-fiber reinforced polyamide PA6, cut out from plate at angles of 0°, 45° and 90° to the molding direction. The models parameters were identified by reverse engineering in the Digimat MX. The paper gives estimates of the accuracy of approximating the plasticity curve in each tension direction by the investigated models. The influence of the Mori Tanaka and the double inclusion homogenization schemes is investigated. It is shown that the exponential hardening law makes the best tension curve approximation with mean relative error less than 3.8% and maximum relative error less than 9.7%.

1. Introduction

Considering the high specific strength and high specific stiffness short fibers reinforced composite materials was used the vehicle body design [1] for the fabrication of numerous parts used in the automotive and aerospace industries [2]. Such short carbon fiber reinforced composites as based on poly-ether-ether-ketone have not only high specific mechanical characteristic but also have excellent thermal properties therefore attract great attention in automobile and aerospace industries [3]. In the aerospace industry, short-reinforced composites can be used to make fittings – products that cannot be made from laminated composites due to their complex shape and delamination problems. To improve the accuracy of manufactured products from short-reinforced composites, it is necessary to use mathematical modeling methods [4,5]. Due to the selection of the composition and structure of the constituent components, it becomes possible to create an optimal product [6]. On the other hand, this possibility is also the main disadvantage, since the properties of the final product can vary in a wide range, because these materials have an anisotropy, which is determined by the technological parameters of production by injection molding [7]. At the moment, there is no single concept and methodology for calculating the strength of products from such materials [8-10]. As a matter of fact, strength analysis involves the use of a certain averaged isotropic curve [11,12], which leads to serious errors at the design stage. Such an error can significantly affect the strength analysis and the choice of stress and strain limits. This approach to design involves the introduction of overestimated safety factors, which also
requires a significant increase in experimental testing to confirm the operational performance of the product, which leads to a sharp decrease in design efficiency. Because of this, it is necessary to build a universal mechanical-strength model of the material that can take into account both the spread of the parameters of the material microstructure and the fundamental decisions on the manufacturing technology of a specific product that is made from this material [13]. The anisotropic nature of the material used assumes the use of a multilevel approach to the process of its modeling.

Modeling a composite reinforced by short fibers requires the use of the approach, in which the material is considered homogeneous and its properties are determined experimentally. In case of composites with a predetermined fiber orientation (laminated composites, continuous fiber reinforced composites), it is necessary to carry out tests to obtain the elastic and strength properties of the material in the main directions [14]. However, in the case of short-reinforced composites, where the mechanical characteristics depend on “locally changing orientation” [10,15], it is necessary to use modeling methods, which are combined with the use of experimental methods.

The work considers the modeling of a composite material, taking into account the multilevel approach. The construction of a material model is investigated using three different models of binder plasticity: power-law, exponential and linear-exponential hardening law. The aim of the work is to check the accuracy of the models by comparing the created material mathematical models with experimental data. This approach will allow choosing a model that can most accurately describe the mechanical characteristics of a short-reinforced composite material.

2. Methods and materials

The multilevel approach implies calculation of the stress-strain state of products based on nonlinear model of mechanical characteristics of a representative volume of anisotropic composite material [16]. The model of the short-reinforced composite material mechanical characteristics is based on the approximation of the experimental dependences of the material samples tension at different angles to the molding direction. Tension test of Polylastic Moscow Armamid PA SV 50-1E polyamide-6 reinforced by 50% of glass fibers ISO 527-2 [17] samples cut out from 4 mm thickness plate at angles of 0°, 45° and 90° to the molding direction is shown at figure 1. The tests are carried out on an MTS 322 universal servo-hydraulic testing machine.

![Figure 1](image)

**Figure 1.** ISO 527 spacemen tension test: (a) specimen geometry [17], (b) testing process, (c) tension of samples cut out from plate at angles of 0°, 45° and 90° to the molding direction.
For material models creation the Digimat MF module is used. Digimat MF implements Mean-field homogenization theory [18] based on Eshelby approach [19], Mori-Tanaka [20,21] and Double inclusion [22] homogenization schemes. Fiber orientation tensor corresponds to the tensor of fiber orientation for flat plate molding case. Calculation of short fibers reinforced flat plate molding a of polyamide held in Moldex3d [16]. As an orientation tensor values (figure 2) are chosen averaged thickness values from the regular part – in its center, in place of samples cut.

![Figure 2. Fiber orientation tensor.](image)

The elasto-plastic binder behavior based on the von Mises equivalent stress formulation:$$\sigma_{eq} = \sigma_Y + R(\varepsilon_p),$$where $\sigma_Y$ – is initial yield stress, $R(\varepsilon_p)$ – hardening stress function from $\varepsilon_p$ – accumulated plastic strain.

In the paper three hardening stress laws was compared:

- power law

$$R(\varepsilon_p) = k(\varepsilon_p)^m,$$

- exponential law

$$R(\varepsilon_p) = R_\infty[1 - e^{-m\varepsilon_p}],$$

- exponential and linear law

$$R(\varepsilon_p) = k\varepsilon_p + R_\infty[1 - e^{-m\varepsilon_p}].$$

For the identification of models parameters Digimat MX reverse engineering module was used (figure 3).

![Figure 3. Identification of model parameters using Digimat MX reverse engineering.](image)
3. Results and discussion

In all cases, the composite material model was build using the binder density 1400 kg·m⁻³ and the following fiber parameter values: mass fraction 0.5, density 2600 kg·m⁻³, Young modulus 74 GPa, Poisson 0.22. The parameters of the binder and fiber aspect ratio (table 1) was identified by Digimat MX reverse engineering using Mori-Tanaka [20,21] and Double inclusion [22] homogenization schemes to ensure the best approximation of the composite tension curves. Binder plasticity curves and short fiber reinforced tension model using different binder hardening laws shown at figures 4-6. Mean and maximum absolute and relative errors of curve fitting in comparison with experimental data are presented in tables 2-5. To exclude the influence of a small base on the value of relative errors, relative errors in tables 4, 5 were calculated at deformations of more than 0.25%.

Table 1. Parameters of the Polyplastic Moscow Armamid PA SV 50-1E material models.

| Parameter                       | Power law | Exponential law | Exponential and linear law |
|---------------------------------|-----------|-----------------|---------------------------|
|                                 | Mori-Tanaka | Double inclusion | Mori-Tanaka | Double inclusion | Mori-Tanaka | Double inclusion |
| Binder                          |            |                 |              |                 |              |                 |
| Young modulus, MPa              | 3842       | 3550            | 3460         | 3453            | 4198         | 3834            |
| Poisson                         | 0.391      | 0.297           | 0.159        | 0.130           | 0.157        | 0.148           |
| Yield stress, MPa               | 7.2        | 7.7             | 24           | 19.5            | 13.2         | 13.1            |
| Hardening modulus Rₐ₀, MPa      | 34.556     | 32.494          | 43.022       | 36.874          |
| Hardening exponent m            | 0.306      | 0.281           | 130.6        | 146.2           | 149.1        |
| Hardening modulus k, MPa        | 134.9      | 109.0           | 31.6         | 31.0            |
| Fiber                           |            |                 |              |                 |              |                 |
| Aspect ratio                    | 8.922      | 7.684           | 8.672        | 7.875           | 8.262        | 7.567           |

Strength limit

|                      | Power law | Exponential law | Exponential and linear law |
|----------------------|-----------|-----------------|---------------------------|
| Axial tensile strength, MPa | 146.22    | 151.00          | 178.88                    | 182.69          | 185.36        | 183.82          |
| Inplane tensile strength, MPa | 97.27     | 94.58           | 81.89                     | 81.15           | 81.00         | 81.10           |
| Transverse shear strength, MPa | 48.46     | 50.21           | 50.65                     | 50.49           | 47.88        | 48.86           |

Figure 4. Power law of binder hardening: (a) binder plasticity model, (b) composite plasticity model.
Figure 5. Exponential law of binder hardening: (a) binder plasticity model, (b) composite plasticity model.

Figure 6. Exponential and linear law of binder hardening: (a) binder plasticity model, (b) composite plasticity model.

Table 2. Mean absolute error, MPa.

| Load direction | Power law | Exponent law | Exponent and linear law |
|----------------|-----------|--------------|-------------------------|
|                | Morii-Tanaka | Double inclusion | Morii-Tanaka | Double inclusion | Morii-Tanaka | Double inclusion |
| 0°             | 0.80       | 0.97         | 2.64        | 2.27         | 4.06       | 3.21           |
| 45°            | 6.61       | 5.51         | 2.20        | 2.49         | 3.30       | 3.09           |
| 90°            | 6.25       | 4.76         | 2.89        | 2.70         | 3.10       | 3.12           |
| Mean by direction | 4.55     | 3.75         | 2.58        | 2.48         | 3.49       | 3.14           |
| Maximum by direction | 6.61     | 5.51         | 2.89        | 2.70         | 4.06       | 3.21           |
The models does not exceed 1.1%. The Exponential hardening law with Double inclusion problem and did not bring an increase in the approximation accuracy. Moreover, the differences between composite tension models did not exceed 3.2% for Power law model case and did not exceed 1.1% for Exponential and linear law model cases.

For the best approximation of the composite tension curves, the Mori Tanaka homogenization scheme assumes the stiffness of the binder is 11.5% higher than the Double inclusion scheme (figures 4-6). On the other hand, if the stiffness of the binder is not experimentally known and the goal is only to best match the stiffness of the composite, the difference between Mori-Tanaka and Double inclusion composite tension models did not exceed 3.2% for Power law model case and did not exceed 1.1% for Exponential and Exponential and linear model cases.

Power law of binder hardening gives better results for composite description at 0° direction (figure 4). Exponential and Exponential and linear laws of binder hardening allows us to most accurately describe loading at 45° and 90° directions and can be recognized as the best choices among the studied models (figures 5, 6). In the present study, it turned out that the complexity of the Exponential and linear law model was exceeding - the addition of a linear term complicated the parameter identification problem and did not bring an increase in the approximation accuracy. Moreover, the difference between the models does not exceed 1.1%. The Exponential hardening law with Double inclusion

### Table 3. Maximum absolute error, MPa.

| Load direction | Power law | Exponent law | Exponent and linear law |
|----------------|-----------|--------------|-------------------------|
|                | Mori-Tanaka | Double inclusion | Mori-Tanaka | Double inclusion | Mori-Tanaka | Double inclusion |
| 0°             | 2.71       | 1.92         | 10.91       | 9.33          | 15.24       | 10.83          |
| 45°            | 12.73      | 10.90        | 6.46        | 6.47          | 6.99        | 6.12           |
| 90°            | 15.23      | 11.97        | 6.55        | 5.86          | 7.59        | 7.36           |
| Mean by direction | 10.22     | 8.26         | 7.97        | 7.22          | 9.94        | 8.10           |
| Maximum by direction | 15.23    | 11.97        | 10.91       | 9.33          | 15.24       | 10.83          |

### Table 4. Mean relative error, %.

| Load direction | Power law | Exponent law | Exponent and linear law |
|----------------|-----------|--------------|-------------------------|
|                | Mori-Tanaka | Double inclusion | Mori-Tanaka | Double inclusion | Mori-Tanaka | Double inclusion |
| 0°             | 0.81       | 1.01         | 2.38        | 1.97          | 3.38        | 2.94           |
| 45°            | 8.23       | 6.65         | 3.12        | 3.32          | 4.35        | 4.00           |
| 90°            | 8.25       | 6.36         | 3.99        | 3.78          | 4.26        | 4.34           |
| Mean by direction | 5.76      | 4.67         | 3.16        | 3.02          | 4.00        | 3.76           |
| Maximum by direction | 8.25     | 6.65         | 3.99        | 3.78          | 4.35        | 4.34           |

### Table 5. Maximum relative error, %.

| Load direction | Power law | Exponent law | Exponent and linear law |
|----------------|-----------|--------------|-------------------------|
|                | Mori-Tanaka | Double inclusion | Mori-Tanaka | Double inclusion | Mori-Tanaka | Double inclusion |
| 0°             | 1.98       | 2.15         | 7.48        | 6.40          | 10.46       | 7.43           |
| 45°            | 15.99      | 13.43        | 9.70        | 9.63          | 11.51       | 9.84           |
| 90°            | 16.62      | 13.06        | 8.82        | 9.27          | 10.64       | 10.42          |
| Mean by direction | 11.53     | 9.55         | 8.67        | 8.43          | 10.87       | 9.23           |
| Maximum by direction | 16.62    | 13.43        | 9.70        | 9.63          | 11.51       | 10.42          |
homogenization scheme turned out to be the best choice of current research and gives maximum relative error less than 9.7% for all loading angle directions.

4. Conclusion
Models of mechanical characteristics of short-reinforced composite materials are used in the design of molded structures, including aerospace applications. Estimation of the approximation error of the mechanical characteristics of the material affects the assignment of strength and stiffness reserves, which is especially important in the design of ultralight structures. Power, Exponential and Exponential and linear laws of binder hardening used to approximate tension curves of 50% short glass fiber reinforced polyamide-6 were shown an error not exceeding 13.43% which allows their use in the design of products. The best was the exponential model using Double inclusion homogenization scheme with mean relative error less than 3.8% and maximum relative error less than 9.7% for all loading angle directions.

The homogenization scheme influences the determination of the characteristics of the composite with a given binder plasticity curve, but its influence can be largely compensated by the selection of parameters at the curve-fitting stage. Taking into account the distribution of the orientation tensor over the thickness of the composite samples and customization of homogenization models can bring the values of the approximated parameters closer to the experimental ones and are the subject of further research.

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