Pion Suppression in Nuclear Collisions

Marek Gaździek
Institut für Kernphysik, Universität Frankfurt
August–Euler–Strasse 6, D - 60486 Frankfurt, Germany

Mark I. Gorenstein
Bogolyubov Institute for Theoretical Physics
UK - 252143 Kiev, Ukraine

Stanisław Mrówczyński
Soltan Institute for Nuclear Studies,
ul. Hoża 69, PL - 00-681 Warsaw, Poland
and Institute of Physics, Pedagogical University,
ul. Leśna 16, PL - 25-509 Kielce, Poland

The pion multiplicity per participating nucleon in central nucleus–nucleus collisions at the energies 2–15 A·GeV is significantly smaller than in nucleon–nucleon interactions at the same collision energy. This effect of pion suppression is argued to appear due to the evolution of the system produced at the early stage of heavy–ion collisions towards a local thermodynamic equilibrium and further isentropic expansion.

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Pions, which are copiously produced in high energy interactions, play a key role in the collision dynamics. However, in spite of hard efforts, the pion production in nucleus–nucleus interactions is far from being completely understood (see, e.g., [1]). Thus, it is of particular interest to compare the properly normalized data on pion multiplicity from nucleus–nucleus and nucleon–nucleon collisions.

One would expect that inelastic secondary interactions produce additional pions and therefore their number per participating nucleon should be larger in nucleus–nucleus than in nucleon–nucleon collisions at the same initial energy per nucleon. The experimental data on pion multiplicities amazingly contradicts this intuitive expectation. Indeed, it has been recently found [2, 3] for BNL AGS energies and below that the number of produced pions per participating nucleon, \( \langle \pi \rangle / \langle N_P \rangle \), in central collisions of identical nuclei \((A+A)\) is lower (pion suppression) than in inelastic nucleon–nucleon \((N+N)\) interactions.

In Fig. 1 we show the ratio \( \langle \pi \rangle / \langle N_P \rangle \) as a function of \( \langle N_P \rangle \) at three initial momenta 2.1, 4.5, and 15 A·GeV/c. The data is taken from the compilation [2], where the pion multiplicities from various experiments are recalculated to obtain the total multiplicities independent of the rapidity and/or transverse momentum cuts. In all three cases the relative pion production decreases when going from \(N+N\) interactions \((\langle N_P \rangle = 2)\) to central \(A+A\) collisions. At 2.1 A·GeV/c the pion yield per nucleon is smaller by a factor of about 3. At all energies the pion suppression is approximately independent of the size of (sufficiently large) colliding nuclei (see Fig. 1 and the review [2]). Further, the pion suppression factor defined as

\[
\Delta \frac{\langle \pi \rangle}{\langle N_P \rangle} = \frac{\langle \pi \rangle_{AA}}{\langle N_P \rangle_{AA}} - \frac{\langle \pi \rangle_{NN}}{\langle N_P \rangle_{NN}},
\]

appears to be approximately independent of the collision energy (up to BNL AGS energies) [2, 3]. As seen in Fig. 2, it equals about \(-0.35\).

The aim of this paper is to discuss the mechanism leading to pion suppression. We try to connect the scaling properties of the suppression factor (1) – its approximate independence of the size of colliding nuclei and the initial energy – with the hypothesis supported by the existing experimental data that the system created in nucleus–nucleus collisions approaches the local thermodynamical equilibrium [4, 5, 6, 7].

We assume that the system produced at the early stage of \(A+A\) collision is formed due to the superposition of \(N+N\) interactions. At this stage the chemical composition of hadronic matter is expected to be the same as in the nucleon–nucleon collisions. The system however evolves towards thermodynamic equilibrium and we assume that local equilibrium is reached before the system disintegrates into the final state free hadrons. Then, the difference between the properly normalized pion multiplicities in \(A+A\) and \(N+N\) collisions appears as a result of

- the chemical equilibration of the initially nonequilibrium hadronic matter,
- the hydrodynamic expansion preceding the system freeze–out.

We analyse the two mechanisms of pion suppression separately. Let us start with the equilibration one.
A large fraction (50–100 \%) of baryons emitted in inelastic $N + N$ interactions is known to be in the state of nucleon isobars (deltas and heavier baryonic resonances), which successively decay into pions and nucleons [8, 9, 10, 11, 12, 13]. On the other hand, it is also known, see e.g. [14], that in the equilibrated system at temperatures smaller than 150 MeV, which are characteristic for the energy domain of interest, the fraction of baryon number carried by deltas does not exceed 50 \%. Therefore, when the system created in $A + A$ collisions evolves towards equilibrium, the initial surplus of deltas has to be reduced causing the suppression of the final state pions. The microscopic process responsible for the $\Delta$ absorption is $\Delta + N \rightarrow N + N$ [15]. Multi–nucleon reactions are also discussed in this context, see e.g. [16]. So far the qualitative argument and now we move to a model formulation.

An equilibrium state of the system of nucleons, deltas and pions (heavier mesons and baryons are neglected in our considerations) is controlled by two parameters: baryon density ($\rho_B$) or baryon chemical potential ($\mu_B$) and temperature ($T$). The system, which is formed at the early stage of central $A + A$ collision, is assumed to be close to thermal equilibrium. This assumption can be justified by short (relatively to the evolution time) thermal equilibration time \[17, 18\]. The fraction of baryon charge carried by deltas, however, exceeds its chemical equilibrium value. Thus, we describe such a system in terms of thermodynamics but an additional parameter, which measures the delta surplus ($DS$), is introduced. Specifically, we define

$$\lambda_\Delta \equiv \frac{\bar{\rho}_\Delta - \rho_\Delta}{\rho_B},$$

where $\bar{\rho}_\Delta$ and $\rho_\Delta$ are the initial and equilibrium (corresponding to the initial temperature) densities of deltas.

Keeping in mind that in the final state there are direct pions and those originating from the delta decays, the pion multiplicity per participating nucleon is

$$\frac{\langle \pi \rangle}{\langle N_P \rangle} = \frac{\rho_\Delta + \rho_\pi}{\rho_B},$$  \hspace{1cm} (2)

with $\rho_\pi$ being the pion density. The suppression factor (1) due to the chemical equilibration of initial DS matter produced in $A + A$ collisions by a superposition of $N + N$ interactions then reads

$$\frac{\Delta \langle \pi \rangle}{\langle N_P \rangle}_{DS} = \frac{\rho_\Delta (\mu_B, T) + \rho_\pi (T)}{\rho_B} - \frac{\rho_\Delta (\mu^i_\Delta, T^i) + \rho_\pi (T^i)}{\rho^i_B},$$  \hspace{1cm} (3)

where $\mu_B$ and $T$ describe the equilibrium state while $\mu^i_\Delta, T^i$ and $\rho^i_B$ the initial nonequilibrium one formed in the early stage of heavy–ion collision. The particle and energy densities used later are given by the well known formulas:

$$\rho_j (\mu_j, T) = \int \frac{d^3p}{(2\pi)^3} e^{\beta (\sqrt{p^2 + m^2_j} - \mu_j)} \pm 1,$$

$$\varepsilon_j (\mu_j, T) = \int \frac{d^3p}{(2\pi)^3} e^{\beta (\sqrt{p^2 + m^2_j} - \mu_j)} \pm 1,$$

where $m_j$ and $\mu_j$ are particle masses and chemical potentials with $j = \pi, N, \Delta; \beta \equiv 1/T$. The numbers of internal degrees of freedom are: $g_\pi = 3, g_N = 4$, and $g_\Delta = 16$. The
chemical potential of pions $\mu_\pi$ is zero. Due to the strong interaction the above ideal
gas formulae are probably not very realistic at $T$ larger than, say, 100 MeV and $\rho_B$
significantly exceeding normal nuclear density. The most important effect i.e. the short
range repulsion in the hadron system can be taken into account via the van der Waals
correction. Then, the particle number ratios, which are of our particular interest, could
be not far away from their ideal gas values (see e.g. Ref. [7]).

The pion density depends solely on the temperature while the delta density is a func-
tion of the temperature and delta chemical potential. The values of chemical potentials
in Eq. (3) are chosen in such a way that

$$\rho_B = \rho_\Delta(\mu_B, T) + \rho_N(\mu_B, T),$$
$$\rho_B' = \rho_\Delta(\mu_B', T') + \rho_N(\mu_B', T'),$$

where $\rho_N$ is the nucleon density. Since we use the parameter $\lambda_\Delta$ to control the delta
surplus, we require that

$$\frac{\rho_\Delta(\mu_B', T') - \rho_\Delta(\mu_B', T)}{\rho_B'} = \lambda_\Delta,$$  \hfill (4)

where $\rho_\Delta(\mu_B', T')$ is the equilibrium value of the delta density at the temperature $T'$
and baryon density $\rho_B'$. This equilibrium density is found from the equation

$$\rho_B' = \rho_\Delta(\mu_B', T') + \rho_N(\mu_B', T').$$  \hfill (5)

A complete treatment of the pion suppression requires a simultaneous study of col-
lective expansion and chemical equilibration processes. To estimate the role of the two
phenomena we however discuss them separately. Therefore, we assume that the hydro-
dynamic expansion does not develop essentially at the time of chemical equilibration and
the latter process is studied at the constant volume. Therefore, the baryon and energy
densities are the same for initial and equilibrium phases:

$$\rho_B' = \rho_B,$$

$$\varepsilon_\Delta(\mu_B', T') + \varepsilon_N(\mu_B', T') + \varepsilon_\pi(T') = \varepsilon_\Delta(\mu_B, T) + \varepsilon_N(\mu_B, T) + \varepsilon_\pi(T).$$

After these two equations are solved simultaneously with the additional conditions (4)
and (5), the suppression factor (3) is a function of three free parameters choosen to be $\rho_B$, $T$ and $\lambda_\Delta$. We have calculated numerically the pion suppression for the temperatures
and baryon densities which cover the values characteristic for the hadronic matter formed
in $A + A$ collisions. The temperature $T$ then varies from 50 to 150 MeV while the baryon
density $\rho_B$ from $2\rho_0$ to $5\rho_0$ with $\rho_0 = 0.16 \text{ fm}^{-3}$ being the normal nuclear density.

The pion suppression (3) disappears when the parameter $\lambda_\Delta$ goes to zero. To esti-
mate the maximal value of $\lambda_\Delta$ we note that at high energies of colliding nuclei, where the
temperature approaches 150 MeV, the equilibrium value of $\rho_\Delta/\rho_B$ is about 0.5, and conse-
quently one has $\lambda_\Delta < 0.5$. At the lowest energies of interest, where $\langle \pi \rangle_{NN}/\langle N_P \rangle_{NN} \approx 0.5$, the majority of pions in $N + N$ collisions comes from the delta decays while the equi-
librium delta density in $A + A$ is close to zero. Therefore, we again have $\lambda_\Delta < 0.5$ and
consider $\lambda_\Delta = 0.5$ as the largest value.
In Fig. 3 we show the pion suppression as a function of the equilibrium temperature $T$ at $\rho_B = 2\rho_0$ and $5\rho_0$ for the extreme $\lambda_\Delta$ value. We also present the results for $\lambda_\Delta = 0.3$. Fig. 3 covers the whole physically reasonable domain of $T$, $\rho_B$ as well as $\lambda_\Delta$. The suppression factor is seen to range from $-0.2$ to $-0.4$. Keeping in mind how strongly the pion multiplicity varies with $T$, $\rho_B$ and $\lambda_\Delta$, one finds very striking a very weak dependence of the pion suppression on the mentioned parameters. This in turn agrees with an approximate independence of the suppression factor of the participant number and collision energy (cf. Figs. 1, 2). More than that, the numerical values of the pion suppression due to the equilibration process are close to the experimentally measured mean suppression equal of $-0.35$.

The chemical equilibration leads to an increase of the system temperature ($T > T^i$), and therefore the number of direct pions increases as well. The initial nonequilibrium number of deltas however strongly decreases. In Fig. 4 we show $\rho_\pi/\rho_B$ and $\rho_\Delta/\rho_B$ ratios before (dotted lines) and after (solid lines) the chemical equilibration. The sum of these ratios defines the total pion multiplicity per participating nucleon ($\frac{s}{\rho_B}$).

The suppression remains almost unchanged when direct pions are removed from our calculations by setting $g_\pi = 0$. Therefore, the pion suppression occurs in our model due to the equilibration of the baryon subsystem. Consequently, the assumption that the direct pions are in equilibrium is not very important and can be relaxed. We return to this point at the end of our paper.

Note also that the entropy per baryon, $\langle S \rangle / \langle N_P \rangle \equiv s/\rho_B$, increases due to the chemical equilibration. This is shown in Fig. 5. The entropy density $s$ is calculated from ideal gas formulae for chemical nonequilibrium initial state with parameters $T^i, \mu^i_N, \mu^i_\Delta$ (dotted line) and for chemical equilibrium with parameters $T, \mu_B$ (solid line).

Let us now estimate the effect of the second mechanism of the pion suppression i.e. the absorption of pions due to the system hydrodynamic expansion. We call it a delayed freeze–out ($DF$) effect in $A+A$ collisions: the hadronic system produced in these collisions is larger than that from $N+N$ interactions and therefore the freeze–out density is expected to be smaller. We consider an isentropic evolution of the locally equilibrated hadron matter. It was observed a long time ago [19, 20] that number of pions indeed decreases in the course of such an expansion of the pion–baryon gas.

The locally equilibrium system starts with $\rho_B, T$ and then expands until the freeze–out values $\rho_B^f, T^f$ are reached. The pion suppression then reads

$$
\left( \frac{\Delta \langle \pi \rangle}{\langle N_P \rangle} \right)_{DF} = \frac{\rho_\Delta (\mu_B^f, T^f) + \rho_\pi (T^f)}{\rho_B^f} - \frac{\rho_\Delta (\mu_B, T) + \rho_\pi (T)}{\rho_B}.
$$

Since the entropy is assumed to be conserved during the expansion, the ratio of the entropy density to the baryon density is constant. The freeze–out temperature $T^f$ can be thus expressed as a function of $\rho_B, T$ and $\rho_B^f$. Consequently, the pion suppression (6) is controlled by the same three parameters.

We have found a remarkable ‘scaling’ property of the pion suppression due to the isentropic expansion which, as far as we know, has not been noticed before. At fixed $T$ the suppression (6) becomes a function of the ratio $\rho_B^f/\rho_B$ only. If the thermal pion contribution to the system entropy is neglected, the temperature $T^f$, which is a solution of
an equation of the isentropic expansion, depends at fixed initial temperature $T$ on the ratio $ho_f^f / \rho_B$ only. The delta part of the pion suppression (3), $\rho_D(\mu_B, T^f) / \rho_B - \rho_D(\mu_B, T) / \rho_B$, manifests then the same scaling property as $T^f$. The thermal pion break the exact scaling, but still the scaling for $T^f$ holds with a high accuracy. The thermal pion part of the suppression (6) $(\rho_\pi(T^f) / \rho_B - \rho_\pi(T) / \rho_B)$ does not scale, but the effect is numerically small. Therefore, the corrections to the ‘scaling law’ for the total pion suppression factor (3) are very minor (less than a few percent) as long as the initial baryon density is sufficiently large, say $\rho_B > 0.5 \rho_0$, and the initial temperature is not too big, $T < 150$ MeV. In Fig. 6 we show $\rho_\pi / \rho_B$ and $\rho_\Delta / \rho_B$ ratios for $T = 150$ MeV and $\rho_B = 2 \rho_0$. In physical terms, the scaling tells us that the pion suppression due to the isentropic expansion mainly results from the delta absorption.

Our numerical calculations of the $DF$ pion suppression (3) are shown as a function of $\rho_f^f / \rho_B$ in Fig. 7. One sees that the pion suppression due to the expansion is very small for the initial temperature $T = 50$ MeV and reaches a value of about $-0.4$ at $T = 150$ MeV and $\rho_f^f / \rho_B = 0.1$. Thus, it is expected to increase with growing collision energy (large $T$) and the size of the colliding nuclei (small $\rho_f^f / \rho_B$ due to the delayed freeze–out).

The final pion suppression in $A + A$ collisions combines the effects caused by the equilibration and expansion and can be calculated as a simple sum of the factors (3) and (6). As follows from the results presented in Figs. 3 and 7, the sum varies between $-0.2$ and $-0.7$ in the whole physically acceptable domain of the hadronic matter parameters. As mentioned above, we expect the pion absorption to increase with growing size of the colliding nuclei and collision energy. This is indeed consistent with data: the independence of the pion suppression of the size of colliding nuclei breaks down for nuclei as heavy as gold. Then, the pion suppression (3) equals about $-0.6$ at 11.6 A·GeV/c [2, 3]. The same trend is found in the recent GSI SIS results at lower energies [4].

The suppression caused by $DS$ and $DF$ mechanisms was calculated under assumption that direct pion component is in equilibrium i.e. its chemical equilibration time is much smaller than the system evolution time. However, as pointed above, the suppression remains unaffected when direct pion component is removed from the calculations. This implies that our results are valid also for the case when chemical equilibration time of direct pions is much larger than the evolution time i.e. the number of direct pions is effectively frozen.

We summarize our considerations as follows. The suppression of the pion production per participating nucleon, which is observed in central $A + A$ collisions at the energies of BNL AGS and below, has been discussed within a thermodynamical approach. An approximate independence of the suppression factor (3) on the collision energy and the participant number (see Figs. 1 and 2) as well as its numerical value agree with a scenario of the heavy–ion collision which distinguishes the following three stages:

1. The initial preequilibrium stage when the nonequilibrium hadronic system is formed by a superposition of $N + N$ interactions.

2. The equilibration stage when the number of deltas decreases to the equilibrium value leading to the reduction of the total number of pions (see Fig. 3).
3. The expansion stage of locally equilibrated hot hadronic matter which causes an additional pion suppression (see Fig. 7).

For the initial energies of BNL AGS and below this scenario is checked to be qualitatively correct after adding more mesonic and baryonic resonances and going beyond the ideal gas approximation applied here. It would be also interesting to check our picture of the pion suppression against microscopic transport calculations.

As a final remark we should stress that at the CERN SPS energies (160–200 A·GeV) one observes a pion *enhancement* instead of the suppression when going from \( N + N \) to \( A + A \) collisions \[3\]. This qualitatively different behaviour can not be understood within the model presented here. A novel feature of A+A collisions at SPS energy is a role of meson resonances which becomes much more important than that at AGS energy and below: a number of mesons at the freeze–out in A+A collisions at SPS is several times larger than number of baryons. The chemical equilibration and hydrodynamical expansion in such a system may lead to a change of the suppression pattern observed at low energies and deserve a special study. It is also possible that the explanation of the pion *enhancement* effect requires the introduction of new mechanisms. A formation of the Quark–Gluon Plasma at CERN SPS energies has been considered as an obvious candidate \[3\], however other mechanisms are also discussed \[22\].

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Figure 1: The pion multiplicity per participant nucleon $\langle \pi \rangle / \langle N_P \rangle$ as a function of the participant number for nucleon–nucleon interactions (square) and central collisions of identical nuclei (circles) at 2.1, 4.5, and 15 A·GeV/c. The lines are shown to guide the eye.
Figure 2: The experimentally measured suppression factor $\Delta(\langle \pi \rangle / \langle N_p \rangle)$ as function of the collision energy which is expressed through the Fermi variable defined as $F \equiv (\sqrt{s_{NN}} - 2m_N)^{3/4} / s_{NN}^{1/8}$. The dashed line shows the mean value equal $-0.35$. 
Figure 3: The suppression factor (3) as a function of the temperature $T$. The extreme cases of $\rho_B = 5 \rho_0$ and $\rho_B = 2 \rho_0$ are shown by, respectively, the dotted and solid lines. The upper pair of the dotted and solid lines corresponds to $\lambda_\Delta = 0.3$ while the lower one to $\lambda_\Delta = 0.5$. 
Figure 4: $\rho_\pi/\rho_B$ and $\rho_\Delta/\rho_B$ ratios in the initial state (dashed lines) and in the chemical equilibrium state (solid lines). The equilibrium baryonic density is chosen as $2\rho_0$ and $\lambda_\Delta = 0.5$. 
Figure 5: The entropy per participating nucleon in the initial state (dotted lines) and in the chemical equilibrium state (solid lines). The equilibrium baryonic density is chosen as $2\rho_0$ and $\lambda_\Delta = 0.5$. 
Figure 6: $\rho_{\pi}/\rho_B^f$ (solid line) and $\rho_{\Delta}/\rho_B^f$ (dashed line) ratios in an isentropic expansion as a function of $\rho_B^f/\rho_B$. The parameters are $T = 150$ MeV and $\rho_B = 2\rho_0$. 
Figure 7: The suppression factor \( \Delta(\langle\pi\rangle/\langle N_p\rangle)_{df} \) as a function of \( \rho_f^B/\rho_B \). The dotted, dotted–dashed and solid line corresponds to \( T \) equal 50, 100 and 150 MeV, respectively.