Motion induced second order temperature and $y$-type anisotropies after the subtraction of linear dipole in the CMB maps

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Abstract. $y$-type spectral distortions of the cosmic microwave background allow us to detect clusters and groups of galaxies, filaments of hot gas and the non-uniformities in the warm hot intergalactic medium. Several CMB experiments (on small areas of sky) and theoretical groups (for full sky) have recently published $y$-type distortion maps. We propose to search for two artificial hot spots in such $y$-type maps resulting from the incomplete subtraction of the effect of the motion induced dipole on the cosmic microwave background sky. This dipole introduces, at second order, additional temperature and $y$-distortion anisotropy on the sky of amplitude few $\mu$K which could potentially be measured by Planck HFI and Pixie experiments and can be used as a source of cross channel calibration by CMB experiments. This $y$-type distortion is present in every pixel and is not the result of averaging the whole sky. This distortion, calculated exactly from the known linear dipole, can be subtracted from the final $y$-type maps, if desired.

Keywords: cosmic background radiation, cosmology:theory, Sunyaev-Zeldovich effect, CMBR experiments
1 Introduction

Recently South Pole Telescope (SPT) [1] and Atacama Cosmology Telescope (ACT) [2] teams published their power spectra for the thermal $y$-type distortions [3] for high $\ell$ multipoles. There are also many theoretical computations of $y$-type distortion maps going down to $y \sim 10^{-7} - 10^{-6}$ [4–7], where $y$ is the amplitude of the $y$-type distortion, with the expectation that Planck Surveyor’s High Frequency Instrument (Planck-HFI) [8] will be able to create maps of $y$-type distortion for the full sky. Future proposed experiments Cosmic Origins Explorer (CoRE) [9], Lite satellite for the studies of B-mode polarization and Inflation from cosmic background Radiation Detection (LiteBIRD) [10] and Primordial Inflation Explorer (Pixie) [11] will be able to improve the sensitivity by more than an order of magnitude over Planck-HFI and several orders of magnitude compared to Cosmic Background Explorer’s Far Infrared Absolute Spectrophotometer (COBE-FIRAS) [12]. Pixie will have an angular resolution of approximately a degree. The purpose of this short note is to remind the observers about an additional artificial component having a $y$-type spectrum which arises in the usual procedure of subtracting the dipole component from cosmic microwave background (CMB) maps.

It is well known that sum of blackbody spectra is not a blackbody [13] and that the averaging of dipole component of CMB over the whole sky should lead to $y$-type distortion with amplitude $y \approx 2.5 \times 10^{-7}$ [14, 15]. In this note we give the exact formulae for the angular dependence and amplitude of the $y$-type distortions in CMB sky maps after the usual subtraction of the linear dipole component described in the Planck pre-launch and early results papers [16, 17]. It is important that the experimentalists will take special search of two maxima on the $y$-type distortion maps peaked, one towards and the other exactly opposite, in the direction of our motion with respect to CMB with the characteristic dimensions of 38 deg$^2$ for $y > 80\%$ of maximum and 105 deg$^2$ for $y > 50\%$ of maximum. If this feature is discovered in $y$ maps, it would provide a source of calibration for different channels of instruments such as Planck-HFI, in addition to the solar dipole and the orbital dipole of the spacecraft. Relative small angular dimensions of the hot spots open a possibility of their detection by experiments like SPT and ACT or even experiments with much poorer angular resolution. Planck-HFI, CoRE, LiteBIRD and Pixie are unique because they are able to produce full sky $y$-type distortion maps.

In addition to the space missions Relikt [18], COBE [19] and Wilkinson Microwave Anisotropy Probe (WMAP) [20], there have been numerous ground based and airborne measurements of the CMB dipole, see [21] for a complete list. Since we know the dipole amplitude ($3.355 \pm 0.008$ mK) and direction (galactic longitude $263.99 \pm 0.14$ deg, latitude $48.26 \pm 0.03$ deg) at high precision from COBE Differential Microwave Radiometers (DMR) and (WMAP) measurements [19, 20], these artificial second order temperature and $y$-type quadrupoles can be easily subtracted from the CMB maps.

It is important to mention that the evaluation of the terms of second order in $\beta = v/c$, where $v$ is our motion w.r.t. CMB and $c$ is the speed of light, gives us an additional term with blackbody spectrum but with characteristic quadrupolar angular dependence over the sky in the large scale CMB
maps. This term also has two positive maxima coinciding with the axes of dipole and the $y$-type distortion is comparable in magnitude to this well known second order quadrupole effect. The blackbody part of the motion induced quadrupole will add to the primary CMB quadrupole, but both these components would be absent in the $y$-type distortion maps.

This frequency dependence of the second order terms in the Taylor series expansion of motion induced anisotropies in the CMB was studied by several authors [14, 22–26]. However, the connection with the $y$-type distortion, in particular the possibility of the residual $y$-type anisotropy due to our motion in the full sky $y$-type maps produced by highly sensitive experiments like Planck-HFI, was not made in the previous studies. See also [26] on additional aspects of the motion induced quadrupole not discussed here.

2 $y$-type distortion and temperature anisotropy in CMB maps from motion induced dipole

In each direction $\hat{n}$ in the sky, CMB consists of a blackbody with temperature $T_0 + T_1(\hat{n})$, where $T_0 \approx 2.725$ K is the average temperature and $T_0 + T_1(\hat{n}) \equiv T_0 + T_1(\theta) = T_0(1 + \beta \cos(\theta))^{-1}(1 - \beta^2)^{1/2}$ [27, 28] is the velocity induced anisotropy, where $\beta = 1.23 \times 10^{-3} \pm 0.2\%$ is the velocity of sun w.r.t. CMB in units of speed of light in the direction $\hat{n}_1 = (\ell = 263.99, b = 48.26)$ in galactic coordinates [19, 20] and $\cos(\theta) = \hat{n}_1 \cdot \hat{n}$. Taylor expanding to second order in $\beta$, we get for the velocity induced monopole, dipole and quadrupole, $T_1(\theta) \approx T_0 \left[ \beta \cos(\theta) + \beta^2 \left( \cos^2(\theta) - \frac{1}{2} \right) \right]$. Therefore the intensity or equivalently the occupation number ($I_v = 2h\nu^3/c^2n(\nu)$) in each direction in the sky is given by, following [22],

\[
\begin{align*}
n_{\nu}(T_0 + T_1(\hat{n})) & = \frac{1}{\rho_\nu(T_0 + T_1(\hat{n}))} - 1 \\
\approx & n_{\nu}(T_0) + \ln \left[ 1 + \frac{T_1(\hat{n})}{T_0} \right] \frac{\partial n_{\nu}(T_0)}{\partial \ln[T_0]} + \frac{1}{2} \left( \ln \left[ 1 + \frac{T_1(\hat{n})}{T_0} \right] \right)^2 \frac{\partial^2 n_{\nu}(T_0)}{\partial (\ln[T_0])^2} \\
= & n_{\nu}(T_0) + \left( \frac{T_1(\hat{n})}{T_0} \right)^2 T_0 \frac{\partial n_{\nu}(T_0)}{\partial T_0} + \frac{1}{2} \left( \frac{T_1(\hat{n})}{T_0} \right)^2 T_0^4 \frac{\partial^2 n_{\nu}(T_0)}{\partial T_0^2} + \frac{1}{2} n_{\nu}(T_0) \frac{\partial n_{\nu}(T_0)}{\partial T_0} \\
= & n_{\nu}(T_0) + \left( \frac{T_1(\hat{n})}{T_0} \right)^2 T_0 \frac{\partial n_{\nu}(T_0)}{\partial T_0} + \frac{1}{2} n_{\nu}(T_0) \frac{\partial n_{\nu}(T_0)}{\partial T_0} + \frac{1}{2} Y(x) \left( \frac{T_1(\hat{n})}{T_0} \right)^2,
\end{align*}
\]

where $h$ is the Planck’s constant and $k_B$ is the Boltzmann’s constant. Planck-HFI is insensitive to the average intensity and only measures the fluctuating part, $n_{\nu}(T_0 + T_1(\hat{n}))-n_{\nu}(T_0)$. We are therefore subtracting, and mixing, two blackbodies of different temperature which should give us a $y$-type distortion + change in the temperature [13–15].

\[
\frac{c^2}{2h\nu^3} \Delta I_v = \left( \frac{T_1(\hat{n})}{T_0} + \left( \frac{T_1(\hat{n})}{T_0} \right)^2 \right) G(x) + \frac{1}{2} Y(x) \left( \frac{T_1(\hat{n})}{T_0} \right)^2 \\
= \left[ \beta \cos(\theta) + \beta^2 \left( 2 \cos^2(\theta) - \frac{1}{2} \right) \right] G(x) + \frac{1}{2} Y(x) (\beta \cos(\theta))^2,
\]

\[2.2\]
where $x = h\nu/k_B T_0$ is the dimensionless frequency,

$$G(x) = \frac{xe^x}{(e^x-1)^2}$$

and

$$Y(x) = \frac{xe^x}{(e^x-1)^2} \left( \frac{e^x+1}{e^x-1} - 4 \right)$$

is the $y$-type distortion.

We can also subtract, and absorb in $T_0$, an average term, $(\beta^2/6)G(x)$ to make the $\beta^2G(x)$ term proportional to the Legendre polynomial $P_2(\cos(\theta))$. Similarly, we should also subtract average $y$-type distortion. Subtracting the linear dipole thus leaves the following residual in the map,

$$\frac{c^2}{2h\nu^3} T_0 \text{residual} = \beta^2 \left( 2\cos^2(\theta) \frac{2}{3} - \frac{2}{3} \right) G(x) + \frac{1}{2} Y(x) \beta^2 \left( \cos^2(\theta) - \frac{1}{3} \right)$$

The first term is just change in the blackbody temperature, the second term is the $y$-distortion amplitude, and might become visible in the future $y$-distortion maps of Pixie [11] and CoRE [9], and possibly in the Planck-HFI [8] maps. Pixie will also be sensitive to the monopole, $y = \beta^2/6 = 2.5 \times 10^{-7}$. This average distortion is larger than what is expected from reionization [11, 14]. Note that after subtracting the monopole, the quadrupolar $y$-type distortion parameter is negative in part of the sky with minimum around $\theta = \pi/2, 3\pi/2$.

The maximum residual occurs at $\theta = 0, \pi$ and is given by, in CMB temperature units,

$$\frac{c^2}{2h\nu^3} T_0 \text{residual} \left|_{\theta=0,\pi} \right. = T_0 \left[ \frac{4}{3} \beta^2 + \frac{1}{3} \beta^2 Y(x)/G(x) \right] .$$

The maximum change in the blackbody temperature is $T_0 t_{\text{max}} = T_0 \frac{4}{3} \beta^2 = 5.5 \mu\text{K}$. The maximum $y$-distortion amplitude is $y_{\text{max}} = \frac{1}{2} \beta^2 = 5 \times 10^{-7} \pm 0.5\%$. The uncertainties in these values depend only on our knowledge of the dipole which is known with an accuracy of $8 \mu\text{K}$ [20]. Pixie will have absolute sensitivity of few nK [11] and the $y$-distortion quadrupole, also known with a similar precision, might be an important external source of calibration for it. Table 1 gives the maximum signal for different Planck-HFI and Low Frequency Instrument (LFI) channels. Figure 1 shows the linear dipole, additional blackbody temperature anisotropy $\tau = \beta^2 \left( 2 \cos^2(\theta) - \frac{2}{3} \right)$ and $y$-type anisotropy $y = \frac{1}{2} \beta^2 \left( \cos^2(\theta) - \frac{1}{3} \right)$ and their relative amplitude with respect their maxima as a function of angle $\theta$ of the line of sight direction $\hat{n}$ with the dipole $\hat{n}_1$. Figure 2 shows the 3d visualization of the dipole, and second order temperature and $y$-distortion (353 GHz) anisotropies with Mollweide equal area projection on the x-y plane and amplitude in color scale as well as the z axes. The temperature quadrupole ($\beta^2G(x)$ term) in particular has a different orientation than the primary CMB quadrupole [29] but much smaller amplitude, $Q'_{\text{rms}} = (5/(4\pi)C_{l=2})^{1/2} = 4\beta^2/(3 \sqrt{5})T_0 = 2.5 \mu\text{K}$. The area on the sky covered by a spot with $y > 0.8y_{\text{max}}$ is 38 deg$^2$ and the region with $y > 0.5y_{\text{max}}$ covers 105 deg$^2$. Thus even though this signal may not be detectable in individual pixels, the peaks, with more than a 1000 pixels, might be detected. This $y$-type distortion is artificial, resulting from incomplete subtraction of the effect of our motion from the CMB maps, but it still opens a way to calibrate CMB experiments, as was already noted by Refs. [14, 26] but Ref. [26] did not separate out $y$-type part.

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\[ \Delta T = T_0 + \beta^2 (\mu K) \]

| Channel | \( \Delta T \) | y-distortion (\( \mu K \)) | total (\( \mu K \)) |
|---------|--------------|----------------|----------------|
| 30      | 5.5          | -2.7           | 2.8            |
| 44      |              | -2.6           | 2.9            |
| 70      |              | -2.4           | 3.1            |
| 100     |              | -2.1           | 3.4            |
| 143     |              | -1.4           | 4.1            |
| 217     |              | 0.0            | 5.5            |
| 353     |              | 3.1            | 8.6            |

Table 1. Maximum residuals in CMB temperature units in Planck-HFI and LFI channels after removing the linear dipole. The blackbody temperature part is absent from the \( y \)-type distortion maps produced by SPT, ACT, Planck-HFI.

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Figure 1. Temperature and $y$-distortion anisotropy as a function of angle from the CMB dipole axes, $\theta \equiv \hat{n} \cdot \hat{n}_1$. Upper panel shows the angular distribution of dipole, temperature and $y$-type anisotropies relative to their respective maxima while the bottom panel shows their actual values.

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Figure 2. Full sky maps in equal-area Mollweide projection on the x-y plane for the dipole, second order motion induced blackbody temperature quadrupolar anisotropy and motion induced y-type quadrupolar anisotropy is shown. Only the y-type quadrupolar component would be present in the y-type distortion maps produced by CMB experiments such as Planck-HFI.