On the emergence of duration from quantum observation and some consequences

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Abstract
We show that by the act of quantum measurement on a system there emerges a notion of duration and a corresponding time flow which turns out to be the thermal flow connected to the system. We show that, under some quasi-classical assumptions on the observer, this flow shows relevant properties of empirical time and some interesting consequences for Special Relativity are drawn.

Keywords: Special Relativity; Quantum Information; Quantum Measurement

1 Introduction
In both theories, Special Relativity (SR) and Quantum Mechanics (QM), the observer plays an important role. In SR this has, amongst others, the known implication that the question ”which clock goes faster?” is not well posed, if one considers the clocks of two observers in relative uniform motion. In SR time and space merge to a continuum whose isometry group (Lorentz transformations) mixes space and time coordinates. SR finds it as difficult as Newtonian physics does, to explain some empirical features of time like, for example, its direction and duration. It can be shown in classical physics, including General Relativity, that coordinate time can be made disappear altogether as a fundamental quantity and what is left is change of physical quantities in relation to other physical quantities [1,8].Traditional QM is even more radical in establishing a dependency on the observer. An observer must interact with a system to form correlations which then determine probabilities for different outcomes of a specific observable. This view is systematically developed in [2]. In the corresponding relational interpretation of QM correlations between systems and observers through interaction are considered a complete description of the world. In relation to time, however, QM makes in a way a step back and uses the absolute time concept of Newtonian physics.

In the present paper we want to make use of the relational approach to QM and develop a notion of time, and in particular duration, which emerges from
(quantum) observation. The corresponding time flow will be shown to have some properties of empirical time and to allow interesting connections to the notion of time in SR.

2 Duration

QM is about establishing probabilities for measurement-outcomes of observable physical properties. It is an empirical fact that repeated measurements result in a chain of single valued outcomes, i.e., manifestations, of these properties. Time is not part of the quantum mechanical framework but enters the theory as an external parameter. It is known not to be an observable represented by an operator under realistic physical assumptions, i.e., a bound on energy from below. To include time, we take the route to ask the question, how long exactly does it take to complete a measurement. An answer to the question about the duration of a measurement has been given in the spirit of QM in [3]. This will be our starting point.

For the sake of simplicity, we assume in the sequel to be in a two-dimensional space-time with one real “time” coordinate \( t \in \mathbb{R} \) and one real space coordinate \( x \in \mathbb{R} \). We also make the assumption of a (global) temperature \( T \).

Let a quantum observer \( O \) have knowledge, through prior measurement, of the initial state \( \psi_0 \) of a system \( S \) and of an initial state \( A_0 \) of an apparatus \( A \). The observer \( O \) must interact with \( A \) to read off the information which the apparatus has about \( S \). By doing that, \( O \) makes a measurement and will observe only one branch of the evolution of the \( AS \)-system to get a definite result. Without interaction, \( O \) will describe a unitary evolution \( \Xi \) of correlations between \( S \) and apparatus \( A \), \( \Xi := \psi_0 \rangle \otimes A_0 \rangle \rightarrow \psi_I \rangle \otimes A_I \rangle, 0 \rightarrow \Xi \), where \( A_I \rangle \) and \( \psi_I \rangle \) are a respective orthogonal basis of apparatus and system, relating to some fixed observable, and \( I \) denotes the set of indices over the spectrum. By reduction over \( A \), \( O \) assigns to \( S \) at \( \Xi \) the density matrix \( \Psi \)

\[
S = -k \cdot \text{tr}(\Psi \log \Psi).
\]

The question "within what time has the measurement happened?" can be answered in an operative fashion [3]. Define the operator

\[
M := \sum_I (\psi_I \rangle \otimes A_I \rangle) (\psi_I \rangle \otimes A_I \rangle).
\]

There holds \( M (\psi_J \rangle \otimes A_J \rangle) = \delta_{IJ} \cdot \text{Id} \). \( M \) is a self-adjoint operator on the Hilbert space of the coupled system \( AS \) and therefore represents an observable. \( M = 1 \) does imply that the pointer indicates the correct value of \( S \), where \( M = 0 \) means it does not. We now say that, if the pointer correctly indicates the value, the measurement has happened, else it has not. In particular, if we follow the Schrödinger evolution \( \Xi \) of the coupled system from 0 to \( \Xi \), we can define the probability \( P \) that the measurement has happened by \( P(t) = \langle \Xi(t) | M | \Xi(t) \rangle \). Correspondingly, there is entropy \( S = -k \cdot \text{tr}(\Psi \log \Psi) \) with probability \( P(t) \). The question "within what time has the measurement happened?" is therefore not well posed in the classical sense but finds an answer in the probabilistic sense of QM. We realize that this "duration" is a relational
quantity between two different states of the system-apparatus complex $AS$. It is invariant under unitary transformations of $AS$ and is hence a full observable, namely measurable and predictable.

The stream of measurement outcomes is realized in the spectrum of the observable and a one-dimensional parameter, called (coordinate) time, which just represents the basic (empirical) fact that there is single valued change of spectral values which can be labelled by this parameter. The main challenge of the above answer to the "within what time"-question is, that duration is not well defined. For an observer $O$ a "second" resulting from one measurement needs not be a "second" resulting from the next one. Another challenge is that the time interval not only depends upon the system $S$ but also on the apparatus $A$ and the interaction between the two. To overcome these difficulties we have to define a specific observer.

### 3 The quasi-classical observer

**Definition 1** Let for some measurement evolution $\Theta(t)$, $P(t) = (\Theta(t)|M|\Theta(t))$. With respect to the measurement of a specific observable on a quantum system $S$ with evolution $\Theta(t)$, observer $O$ is quasi-classical, if i) $P \in \{0, 1\}$, ii) $OS$ is a separable state.

We will now construct a specific quasi-classical observer. Let us look at the entropy balance sheet of the total system $AOS$. As mentioned above, the correlation between $A$ and $S$ leads, at $t$, with certainty to entropy of the amount $S = -k \cdot \text{tr}(\Psi \log \Psi)$, where $k$ is the Boltzmann factor and $\Psi$ the mixed state which $O$ assigns to $S$ by reduction over $A$. For any intermediate $0 \leq t \leq T$ it exists with a certain probability $P(t)$. This entropy must also be there from the perspective of the inner observer $A$ (apparatus), after having registered a definite result, in order to preserve the second law of thermodynamics [4]. For this reason it was possible after all to discover the second law in the 19th century, because it relates to the "stream of consciousness" of an inner observer who experiences the world being in definite states. From the perspective of $O$, there is entropy created because of the correlations between $A$ and $S$. From the perspective of $A$ there is entropy creation because the former state of the register has been erased (forgotten) by acquiring a new definite result. Given the environment $\Omega$ at temperature $T$, this erasure induces an average dissipation of energy $\overline{E} = -kT \text{tr}(\Psi \log \Psi)$ into the environment $\Omega$. By construction, $O$ does not know the full dynamics $H$ related to the dissipation. What $O$ does know, however, it the corresponding average energy $\overline{E}$ which is sufficient for our purposes as we will see below. The Hamiltonian $H$ acts on $\Omega \otimes AS$ leading to the sequence

$$\omega_0 \otimes AS_0 \rightarrow \omega_I \otimes AS_I, \quad \langle \omega_0, \omega_I \rangle = 0.$$  

We now define $O \subset \Omega$ to be quasi-classical with respect to the dissipation-induced evolution (1). By Def 1. i), it follows that $O$ has perceived the mea-
surement, exactly if the state $\omega_I \rangle$ is reached. The time $\Delta t$ it takes to do this we chose to be the minimal coordinate-time interval $\Delta t_{\text{min}}$ it takes $\Omega$ to evolve from its initial state $\omega_0 \rangle$ indicating "nothing has happened" into an orthogonal state $\omega_I \rangle$ indicating "measurement completed with certainty". So we actually look at the (minimal) time that elapses until $O$ has perceived that there is a definite correlation between apparatus and system. The necessary interaction between $AS$ and the environment is triggered by the dissipation of entropy/energy once $A$ has erased a former state $A_0$.

Is there a way to have an indication how long this takes? The answer is "yes" because of a well-established fact by Margolus and Levitin,\[6\], that a general quantum system with average energy $E$ (assuming a ground state $E_0 = 0$) takes at least $\Delta t_{\text{min}} = \frac{\hbar}{4E}$ to evolve into an orthogonal state ($\hbar$ is Planck’s constant). Therefore we derive

$$\Delta t_{\text{min}} = \frac{\hbar}{4E} = \frac{\hbar}{4kTS}. \tag{2}$$

The minimum is achievable if $2E$ is an element of the energy spectrum of $H$ and will nearly be attained in macroscopic systems, where we would expect spectral values to come close to $2E$. Because $E = -kT tr (\Psi \log \Psi)$ we realize, that the time interval which observer $O$ experiences in connection with the measurement on $S$ can be derived from the Schrödinger flow generated by the Hamiltonian $H = -kT \log \Psi$ induced by the system. This flow and its time quantum $\Delta t_{\text{min}}$ only depend on the system $S$. The flow of time is hence a series of these time quanta. There is no time elapsing, if there is nothing "happening" and hence no further dissipation to process states.

Since $O$ is defined to be quasi-classical, the $OS$ system is separable. Assume it has the general form of a classical-quantum state

$$\Psi_{OS} = \sum_{i \in I} p^O (i) i \rangle \langle i \otimes \Psi^S_i, \tag{3}$$

where $p^O (i)$ are the classical probabilities of some distinguishable states $i$ of $O$. This is a plausible assumption, especially in the light of SR. We may now look at the time flow as seen by a further observer $P$ who knows $S$ and $O$. In order to determine the time flow from this perspective, we use the formula of conditional entropy for a classical-quantum state \[7\]

$$S (S \mid O) := S (S, O) - S (O) = \sum_{i \in I} p^O (i) S (\Psi^S_i), \tag{4}$$

where $S (S, O)$ denotes the joint entropy. There always holds $S (S \mid O) \leq S (S)$.

If the inequality is strict, then there is a sort of time dilation between the time flows $\Delta t^{S \mid O}_{\text{min}} > \Delta t^S_{\text{min}}$. As soon as the notion of separability is abandoned, there are challenges to be expected. If the correlation between $O$ and $S$ were purely

\[1\]We could not simply have set $\Delta t_{\text{min}} = t$ and focused only on the $AS$ interaction, since this would have implied a dependence on the apparatus.
quantum, then the relative entropy might become negative \( S(S \mid O) < 0 \). In this case the expected energy of dissipation is negative \( \mathcal{E} < 0 \). We will dedicate the last paragraph 5 to the case of these anti-qubits.

To summarize, we have constructed a quasi-classical observer who (passively) notices what is happening to separated physical systems by registering definite outcomes of experiments. This observer experiences a time flow moving in discrete steps, defined by the system \( S \) with correspondig density matrix \( \Psi \). This flow turns out to be the thermal time flow for \( \Psi \), with “quanta” \( \Delta t_{\min}^S \), which get smaller, if the average energy per bit of information \( kT = \frac{\partial E}{\partial S} \) gets bigger. Since the dissipation of information into the environment, which triggers the flow, is irreversible, the time flow has a direction. The time quanta \( \Delta t_{\min}^S \) are time steps which remain the same over repeated measurements and therefore allow constant ratios

\[
\frac{\Delta t_{\min}^{S_1}}{\Delta t_{\min}^{S_2}}
\]

between thermal flows with respect to two systems \( S_1, S_2 \). The marching in step of “natural clocks” in that sense is thus possible. We will now choose a quasi-classical position observer, construct the corresponding thermal-flow and discuss some consequences.

4 Consequences for space-time

4.1 Bound on velocity

For a non-interacting observer with (coordinate) time parameter \( t \), the system \( S \), with initial state \( \psi_0 \), undergoes a unitary evolution \( \psi(t), t \geq 0 \). Entropy is an invariant quantity under the time evolution and there holds \( S_0 := S(\psi_0) = S(\psi(t)) \). It is now possible to introduce the velocity of the process of repeated measurement over a period \([0, t]\). If we denote by \( \theta = \frac{4kT S_0}{\hbar} \cdot t \) the number of orthogonal states which the system passes over an interval of (coordinate) time \([0, t]\), we can define the “velocity” by

\[
v = \frac{\theta}{t} = \frac{4kT}{\hbar} \cdot S_0. \tag{5}
\]

This corresponds to how fast the observer is registering consecutive measurement outcomes of the system with initial field \( \psi_0 \). Please note that for the time parameter in (5) we may use any (coordinate) time parameter \( s \).

We now want to apply (5) to the special situation where position is measured. Remember, we assumed to be in a two dimensional space-time with one real “time” coordinate \( t \in \mathbb{R} \) and one real space coordinate \( r \in \mathbb{R} \). We choose \( \psi_0 \) to be a free particle, represented by a Gaussian wave package, with mean wave number \( \langle k \rangle = k_0 \) and particle source \( \langle r \rangle \) at the origin.
\[
\psi_0 = \sqrt{\frac{2\sigma_0}{\sqrt{\pi}}} \exp\left(-\frac{\sigma_{k_0}^2 r^2}{2}\right) \exp(ik_0 r).
\]

A quasi-classical observer, asking whether the particle is inside or outside an interval \([0, R]\), decomposes \(\psi_0\) into two orthogonal fields

\[
\psi_0 = \psi_0 \cdot \text{Id}_{[0,R]} + \psi_0 \cdot \text{Id}_{[R,\infty)} = \psi_R + \psi^\perp_R.
\]

The two fields are eigenvectors to the corresponding projectors \(\text{Id}_R\) and \((1-\text{Id}_R)\). The corresponding density matrix is

\[
\Psi_R = \begin{pmatrix}
|\psi_R|^2 & 0 \\
0 & |\psi^\perp_R|^2
\end{pmatrix}.
\]

For entropy we get

\[
S(\Psi_R) = - \left[ \left( \int_0^R |\psi_R|^2 \, dr \right) \log \left( \int_0^R |\psi_R|^2 \, dr \right) + \right]

+ \left( \int_R^\infty |\psi_R|^2 \, dr \right) \log \left( \int_R^\infty |\psi_R|^2 \, dr \right) \right].
\]

Note that the probability to find a particle within \([0, R]\) is increasing with \(R\), since \(\int_R^\infty |\psi_R|^2 \, dr \to 0, R \to \infty\).

We introduce the error function \(\text{erf}(r) := \frac{2}{\sqrt{\pi}} \int_0^r e^{-t^2} \, dt\) to get

\[
S(\Psi_R) = - \left[ \text{erf}\left( \frac{R}{\sigma_{k_0}} \right) \log \left( \text{erf}\left( \frac{R}{\sigma_{k_0}} \right) \right) + \right]

+ \left( 1 - \text{erf}\left( \frac{R}{\sigma_{k_0}} \right) \right) \log \left( 1 - \text{erf}\left( \frac{R}{\sigma_{k_0}} \right) \right) \right]

\]

\[
: = -G\left( \frac{R}{\sigma_{k_0}} \right).
\]

In the function \(G\) the parameter \(\sigma_{k_0}\) only appears in the argument and the maximum \(C\) over \([0, R]\) is therefore independent of \(\sigma_{k_0}\). It can be computed and turns out to be \(C = \log 2\), over \([0, \infty)\). Therefore a repeated measurement of the particle within any interval \([0, R]\) cannot go faster in the sense of (5) than

\[
v_{\text{max}} \leq 4 \log 2 \cdot \frac{kT}{h}.\tag{6}
\]

\(^2\)The particle could lie in \([R, \infty)\), albeit with a small probability for large \(R\).
Estimate (6) is independent of the line element $dr$. A quasi-classical observer will thus “see” the particle moving with at most a process velocity of $v_{\text{max}}$.

If we chose the measure to express the number of orthogonal states, i.e. positions in an interval, to be the distance $\Delta x$, we can go a step further and consider not only process velocity but classical (average) velocity as defined by $v := \frac{\Delta x}{\Delta t} \leq \frac{R}{\Delta t_{\text{min}}} = \frac{4kT}{h} S(\Psi_R) \cdot R$, if the particle materializes within $[0, R]$. We again make use of the error function to get

$$S(\Psi_R) \cdot R = - \left[ \text{erf} \left( \frac{R}{\sigma_{k_0}} \right) \log \left( \text{erf} \left( \frac{R}{\sigma_{k_0}} \right) \right) + \left( 1 - \text{erf} \left( \frac{R}{\sigma_{k_0}} \right) \right) \log \left( 1 - \text{erf} \left( \frac{R}{\sigma_{k_0}} \right) \right) \right] \cdot R$$

$$v_{\text{max}} \leq 1.832 \cdot \frac{kT}{h} \cdot \sigma_{k_0}.$$  \hspace{1cm} (7)

With the uncertainty relation $\sigma_{x_0} \cdot \sigma_{k_0} = 1$, we have

$$v_{\text{max}} \leq \frac{kT}{h \sigma_{x_0}}.$$  \hspace{1cm} (8)

For free (Gaussian) particles there is therefore a bound on classical velocity independent of the initial average momentum $k_0$. In fact we see that estimate (8) implies that there is a universal bound on velocity if and only if there is a lower bound on the resolution of space $\sigma_{x_0} \geq \Lambda_0$ and that

$$v_{\text{max}} \leq \frac{kT}{h \Lambda_0}.$$  \hspace{1cm} (9)

4.2 Consistency

Classically, space-time coordinates serve the dual purpose to label individual objects and to represent physical properties of these. A change in observer equals a change in gauge (co-ordinates). The experimentally observed invariance of a specific speed must then be reflected by invariance under the transformation group of the co-ordinate system with regard to moving frames. The corresponding transformation group is the Lorentz group $\Lambda$.\footnote{The existence of a frame-independent velocity $v_{\text{max}}$ is only one postulate of SR, the other one being the general covariance of physical laws with respect to change of Lorentz frame.} Consistency would leave us to expect that $v_{\text{max}}$ (7), which we have logically derived, not empirically found and which is a composite quantity, is invariant under Lorentz transformations.
Remember the set up: a quasi-classical observer $O$ measures position within an interval $[0, R]$ of a system $S$ with field $\psi_0$ in form of a Gaussian wave packet. The corresponding density matrix is $\Psi_R$. Assume there is a frame with another quasi-classical observer $O'$ that moves with velocity $v$ in the negative $x$-direction. From the perspective of $O'$ the $OS$ system moves in the positive $x$-direction and is, for any $R$, a classical-quantum system of the form $\Phi_{OS} = v \langle v \otimes \Psi_R$. By (4) we derive that

$$S (S | O) = S (\overline{\Psi}_R).$$

(10)

A field function $\overline{\psi} = \overline{\psi (x)}$ must reflect that motion and the $O'$ co-ordinates, $\overline{x}$, are correlated with the moving ones, $x$, by Lorentz-transformations $x = \gamma \overline{x} + a$, where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and $a \in \mathbb{R}$. The length of any object, $r$, is contracted and there holds the transformation $\overline{r} = \gamma^{-\frac{1}{2}} r$. We want to fall back to the original way of looking at relativistic thermodynamics in [9], where temperature transforms according to

$$T = \gamma^{-\frac{1}{2}} T.$$

(11)

For entropy $S$ we calculate the transformation law by plugging the transformations into the definition of $S$ to get

$$S := S (r) = \overline{S} (\gamma r) := \overline{S}.$$  

(12)

With (4) there holds

$$\Delta t_{\text{min}} = \frac{\hbar}{4 kTS} = \frac{\gamma h}{4 kTS} = \gamma \Delta t_{\text{min}}.$$

(13)

Note that we assume that the fundamental constants of nature $\hbar, k$ are invariant. By the length-contraction $\overline{r} = \gamma^{-1} r$, observer $O'$ thus derives

$$\frac{\overline{r}}{\Delta t_{\text{min}}} = \frac{\gamma^{-1} \overline{r}}{\gamma^{-1} \Delta t_{\text{min}}} \leq 1.832 \frac{kT}{h} \cdot \sigma_{ko}.$$  

(14)

The bound is indeed invariant.

### 4.3 Simultaneity

So far we addressed the question of duration which turned out to be a full observable. We found that there is indeed an element of minimal duration or time quantum related to a thermal time flow in case an observer is quasi-classical. Another question is the one of simultaneity of two events. If a quasi-classical observer $O$ measures two systems and two different observables it is very difficult to give an operative definition of simultaneity, since we compare pairs with apples. If the measurements relate to one single observable and there is maximal process velocity $v_{max}$, then there is the possibility of the following operational definition. An event corresponds to the passing of one state into a
different, orthogonal one. Given two time intervals $[0, t_1], [0, t_2]$, the number of orthogonal states which are processed by repeated events within the intervals are $\theta_1 = \frac{4kTS_1}{h} \cdot t_1$ and $\theta_2 = \frac{4kTS_2}{h} \cdot t_2$, respectively. Assuming that the process velocity was maximal, we can define the two starting points of the two processes to be simultaneous, if the time difference $\Delta t$ of their measured end by $O$ is

$$\Delta t = \frac{\theta_2 - \theta_1}{v_{\text{max}}}.$$  \hspace{1cm} (15)

In case of position measurement and classical velocity, (15) just says that two distant events are simultaneous with respect to $O$, if the difference of arrival times of two light beams sent at the moment of the events amount to the difference of their respective distance from $O$ divided by the speed of light.

## 5 Anti-qubits

In paragraph 2 we shortly mentioned the case of anti-qubits which we will now follow up in some detail. In this paragraph we denote the density matrix $\Psi$ of a state by $\rho_{\Psi}$. In the case of a single state $\psi$ the density matrix $\rho_{\psi} \geq 0$, and entropy is well defined. In case of a composite system consisting of two states $\Psi$ and $\Phi$ and with corresponding joint density matrix $\rho_{\Psi\Phi}$ we can define the generalized conditional entropy

$$S(\Psi | \Phi) := -k \cdot tr \left( \rho_{\Psi\Phi} \log \rho_{\Psi | \Phi} \right),$$

where

$$\rho_{\Psi | \Phi} = \lim_{n \to \infty} \left[ \rho_{\Psi\Phi} (id_{\Psi} \otimes \rho_{\Phi})^{-\frac{1}{n}} \right]^n.$$  \hspace{1cm} (16)

It turns out,\cite{[5]}, that conditional entropy is well defined for any composite system and that $S(\Psi | \Phi) = S(\Psi, \Phi) - S(\Phi)$. If the two states are separable,

$$\rho_{\Psi\Phi} = \sum_{i \in I} w_i \rho_i \otimes \rho_i, 0 \leq w_i \leq 1, \sum_{i \in I} w_i = 1,$$

then there holds that the conditional entropy is positive $S(\Psi | \Phi) \geq 0$. This ensures that the concept of “thermal” time works for a quasi-classical observer. In case of entangled systems, however, there might hold $S(\Psi | \Phi) < 0$. The average energy dissipated would then be negative and, in analogy to the situation with anti-particles, the question arises whether such anti-qubits $\rho_{\Psi | \Phi}$ move backwards in the thermal-time defined by the Hamiltonian $H = -kT \log \rho_{\Psi | \Phi}$.

It turns out that the Margolus-Levitin theorem is “robust” with respect to density matrices with spectrum bounded from below. Assume that the eigenvalues $E_i, i \in I$ of $H$ are bounded from below, i.e. $E_i \geq E_0$, which is the only physically realistic situation. There holds

$$\Delta t_{\text{min}}^\rho \geq \frac{h}{4(E - E_0)} \geq 0.$$  \hspace{1cm} (16)
Inequality (16) results directly from the proof of the Margolus-Levitin theorem [3]. Assume that 
\[ \rho_t = \sum_n c_n e^{-i E_n t} \ket{E_n}. \]

There holds
\[ \langle \rho_{\Psi|\Phi} | \rho_t \rangle = \sum_n |c_n|^2 e^{-i E_n t} = e^{i E_0 t} \sum_n |c_n|^2 e^{-(E_n - E_0) t}. \]

To find the smallest value of \( t \) such that \( \langle \rho_{\Psi|\Phi} | \rho_t \rangle = 0 \), we write

\[ \text{Re} \left( \langle \rho_{\Psi|\Phi} | \rho_t \rangle \right) = \sum_n |c_n|^2 \cos \left( \frac{(E_n - E_0)}{h} \right) \geq 1 - \frac{2}{\pi} (E - E_0) t + \frac{2}{\pi} \text{Im} \left( \langle \rho_{\Psi|\Phi} | \rho_t \rangle \right). \]

Inequality (17) follows because of \( \cos (x) \geq 1 - \frac{2}{\pi} (x + \sin x) \), \( x \geq 0 \).

If \( \langle \rho_{\Psi|\Phi} | \rho_t \rangle = 0 \), then both \( \text{Re} = 0 \) and \( \text{Im} = 0 \), and there follows (16).

Therefore, anti-qubits also move forwards in thermal-time and for the process velocity (5) there holds

\[ \theta t = \frac{4kT}{\hbar} \left( S (\Psi | \Phi) - \left( \log \rho_{\Psi|\Phi} \right)_0 \right). \]

Here \( \left( \log \rho_{\Psi|\Phi} \right)_0 \) is the first (lowest) eigenvalue of \( H = - \log \rho_{\Psi|\Phi} \).

6 Conclusion

In the thermal time flow which we derived by defining a quasi-classical observer, there exists a quantum of duration which gives the “now” an extension. The "quasi-classicality" consists of the idea that the end-observer is only able to perceive an indirect confirmation that the measurement has happened in form of distinguishable states of the environment which he is part of. The transition between these orthogonal states happens by a Schroedinger-evolution governed by entropy dissipation which is triggered by the measurement. The resulting time flow only depends on the original system which is an important feature of the classical description.

What are the prerequisites to derive the (thermal) flow? The flow results from the perception of change of some physical property of a system. If no (external) observer can be defined, like in the case where the system is the
whole universe, there can be no single time flow in the above sense to describe
the change of the whole system. The question whether this is a limiting condition
is another one, since it is known that the state of a system is uniquely defined
by the correlations of its subsystems [10]. The flow further depends upon the
information content (entropy) of the system. Its ability to potentially take on
different values is key. If nothing changes, there is no time. We note that for
one observer there might potentially be more than one time-flow because there
is more than one system being measured. This might lead to the perception
that, even if nothing changes with respect to a specific system, time is flowing
nevertheless because the feeling relates to another flow. Likewise, if there is no
energy available (in form of temperature in our model) to process information
there is no thermal time either.

Finally, imagine a universe with a very high temperature and without any
interactions yet. The beginning of (thermal) time would then coincide with the
first measurement induced erasure of information and would be accompanied by
a dissipation of very big amounts of energy. There should also be a measurable
background radiation caused by all the continuing measurements in the uni-
verse. Note that if the observer is not assumed to be quasi-classical, then a very
strange world emerges, where change is registered in random time intervals and
potentially instantaneous action is possible, and where clocks could not march
in step, not even with themselves.

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