Design of open rectangular and trapezoidal channels

C P González¹, P E Vera¹, G Carrillo¹ and S Garcia¹
¹ Universidad Pontificia Bolivariana, Bucaramanga, Colombia

E-mail: claudia.gonzalez@upb.edu.co

Abstract. In this work, the results of designing open channels in rectangular and trapezoidal form are presented. For the development of the same important aspects were taken as determination of flows by means of formula of the rational method, area of the surface for its implementation, optimal form of the flow to meet the needs of that environment. In the design the parameter of the hydraulic radius expressed in terms of the hydraulic area and wet perimeter was determined, considering that the surface on which the fluid flows is the product of the perimeter of the section and the length of the channel and where shear is generated by the condition of no slippage.

1. Introduction

The use of surface drainage systems is of great importance within the scope of civil engineering because it provides stabilization solutions on areas affected by the adverse effects of water flow, which are captured, transported and unloaded at specific points as sources. natural under optimum hydraulic conditions [1-2].

That is why in hydraulic there are devices such as landfills. The design and construction of these is of great importance for the experimental and practical teaching of the engineering student since it brings him closer to a better understanding of its operation for which he needs to determine: flow meters, as well as the limits of application depending on the ground conditions or place of execution.

In this paper, the results of analyzing thin-walled landfills in rectangular and trapezoidal section design, determination of flow rates using hydraulic formulas are reported.

2. Mathematical development

2.1. Calculation of the flow

The flow calculation was determined by means of the formula of the rational method.

\[ Q = C \times I \times A \]  

Where, \( Q \) = Flow (L/s), \( C \) = Runoff coefficient, is based on the soil and a value of 0.3 corresponding to slopes with vegetation according to the table D.4.5 RAS 2000 [3-4]. The \( I \) = Intensity (L/(s×ha)), is established by curves IDF (Intensity, Duration, and Frequency) [5] for a given return period, assuming a value of 361.11(L/(s × ha)). \( A \) = Area (ha), which corresponds to the surface exposed to rainfall and where surface runoff will occur. Expressing the equation (1) depending on the drainage area:

\[ Q = \frac{dv}{dt} = C \times I \times dA \]
Where, $A = x \times y$. Starting from a change ratio of the surface area of a rectangle with the conditions: $x = 4000m, y = 6000m$, possible area where the canal will be built.

$$\frac{dx}{dt} = -\frac{1cm}{s}, \quad \frac{dy}{dt} = \frac{10cm}{s}, \quad \frac{dA}{dt} = y \frac{dx}{dt} + x \frac{dy}{dt} = 340m^2.$$  \hspace{1cm} (3)

This differential equation area (3) is integrated for an interval equal to $t_2=1s \ y \ t_1=0s$.

$$\int dA = \int^1_0 340 \frac{m^2}{s} dt = 340m^2$$  \hspace{1cm} (4)

Thus, the formula of the flow is:

$$Q = 3.68 \times 10^{-3} \text{ m}^3/s$$  \hspace{1cm} (5)

2.2. Calculation of the cross-section of the channel with rectangular geometry

The rectangular landfill is one of the simplest to build and for this reason is one of the most used. It is a landfill with a section of flow in the form of rectangle with thin walls, metal, wood or some resistant polymer, with a beveled or cut crest in slope, in order to obtain a thin edge. The accuracy of the reading it offers is determined by its error level ranging between 3 and 5% [6-7]. To determine the dimensions of the section of the channel, we use the continuity formula:

$$Q = A \times V$$  \hspace{1cm} (6)

The Manning equation:

$$V = \frac{1m^{(i/3)}}{n} \times (R^{(2/3)}) \times (S^{(1/2)})$$  \hspace{1cm} (7)

Where, $R=Hydraulic \ radio \ (Hydraulic \ area/Wet \ perimeter)$, $S=Channel \ slope$, $n=Coefficient \ of \ roughness$. Assuming the following variables as initial conditions: $b = 15cm, n = 0.009, S = 0.07\%$. We have:

$$A = b \times yn,$$  \hspace{1cm} (8)

The objective equation is established.

$$x^{(5/2)} - 0.01x - 7.62 \times 10^{-4} = 0$$  \hspace{1cm} (9)

To estimate $X$, the Newton Rapshon iterations method is applied, using the following equation:

$$X_{n+1} = X_n - \frac{f(x)}{f'(x)}$$  \hspace{1cm} (10)

Table 1 reports the interactions found by the Newton Rapshon iterations Method. It was evidenced that the variable $X$ of all the evaluated ones closest to generating the equality is presented in the point is $X = 0.0743$.
Table 1 Interaction found by the Newton Rapshon.

| No | X   | \( f(x) \) \( X^{5/2} \) | \( g(x) \) \( 0.01X+7.62*10^{-4} \) | \( h(x) = f(x) - g(x) \) |
|----|-----|----------------|----------------|----------------|
| 1  | 0   | 0              | 0              | -0.000762      |
| 2  | 0.01| 0.000001       | 0.000862       | -0.000852      |
| 3  | 0.02| 5.6569E-05     | 0.000962       | -0.000905      |
| 4  | 0.03| 0.00016        | 0.001062       | -0.000906      |
| 5  | 0.04| 0.00032        | 0.001162       | -0.000842      |
| 6  | 0.05| 0.00056        | 0.001262       | -0.000703      |
| 7  | 0.06| 0.00088        | 0.001362       | -0.000480      |
| 8  | 0.07| 0.00130        | 0.001462       | -0.000166      |
| 9  | 0.071| 0.00134    | 0.001472       | -0.000129      |
| 10 | 0.072| 0.00139   | 0.001482       | -9.09868E-05   |
| 11 | 0.073| 0.00144   | 0.001492       | -5.21835E-05   |
| 12 | 0.074| 0.00149   | 0.001502       | -1.23669E-05   |
| 13 | 0.0741| 0.00149 | 0.001503       | -8.32930E-06   |
| 14 | 0.0742| 0.00150  | 0.001504       | -4.28145E-06   |
| 15 | 0.0743| 0.00150  | 0.001505       | -2.23382E-07   |
| 16 | 0.076| 0.00159    | 0.001522       | 7.03333E-05    |
| 17 | 0.077| 0.00165    | 0.001532       | 0.000113       |
| 18 | 0.078| 0.00170    | 0.001542       | 0.000157       |
| 19 | 0.079| 0.00175    | 0.001552       | 0.000202       |
| 20 | 0.080| 0.00181    | 0.001562       | 0.000248       |

Demonstrated that the value of the fifth iteration satisfies the equality it is determined that the height of the sheet of water \( (y_n) \) of the channel for the proposed conditions is 7.40cm and it assumes a free edge of 2.60cm. In the Figure 1, a scheme with the dimensions of the rectangular design is presented.

Figure 1. Rectangular design of the channel.

2.2.1. Type of flow. Fluid flows are classified by two types of subcritical and supercritical regimes, for the flow under study was determined under the conditions [8-9]:

\[
\text{Si } y_c > y_n \text{ supercritical flow, Si } y_c < y_n \text{ subcritical flow} \tag{12}
\]
y.

corresponds to the critical height that will be calculated by the first derivative of the specific energy
of the flow as a function of the normal height.

\[ E_s = Z + Y n + \frac{V^2}{2g} = 0.0755m \]  (13)

Where, \( Z \)=Load by position, \( Y \)=Pressure loading, \( V^2/2g \): Load by speed: Hydrostatic pressure,
\( g \)=Gravitational acceleration.

\[ V = \frac{Q}{A} \quad A = b \times y \quad \text{for rectangular section} \quad V = 0.33m/s \]  (14)

Critical height

\[ y_c = \sqrt{\frac{(3.68 \times 10^{-3})^2}{(0.15)^2 + (9.81)}} = 0.039441m \]  (15)

With this critical height value \( (y_c) \) the condition is met \( S_1 \geq y_c < y_n \) subcritical flow
0.039441m < 0.074m. Therefore the flow presents a subcritical regime.

2.3. Calculation of the channel with trapezoidal section

A weir with a trapezoidal section compensates for the decrease in flow due to lateral contractions
through its triangular parts; this offers the advantage of avoiding correction in the calculations. For the
calculation of the trapezoidal section, the dimensions and optimum angle in the elevation of its slopes
were determined. In the calculation, the hydraulic area and the constant angle are maintained [10-11].

\[ P = b + \frac{2y}{s e n\theta}, \quad A = \left( b + \frac{y}{t a n\theta}\right)y \]  (16)

Calculation of the elevation angle of the channel slopes.

\[ \theta = \cos^{-1}\left(\frac{1}{2}\right) \quad \theta = 60^\circ \]  (16)

Calculation of the trapezoidal section dimensions for the maximum efficiency section with the
optimum angle.

\[ R = \frac{A}{P} = \frac{y}{2^n}, \quad b = \frac{2y(1-\cos60^\circ)}{s e n60^\circ} = 0.0911m \]  (17)

In the Figure 2 shows the dimensions for the section of maximum trapezoidal efficiency.

2.4. Comparison of the maximum rectangular and trapezoidal efficiency section

This parallel is made according to the length of the perimeter of the channel for each of the two sections
analyzed, taking into account that an equal channel length is assimilated for the two alternatives [12-
13]. Rectangular section perimeter.

\[ 2(y + c) + b \rightarrow 2(0.074m + 0.026m) + 0.15m = 0.35m \]  (18)

Perimeter trapezoidal section.

\[ b + \frac{2(y+c)}{s e n\theta} \rightarrow 0.09m + \frac{2(0.08m+0.03m)}{s e n60^\circ} = 0.34m \]  (19)
According to this, the implementation of the trapezoidal section is better since it is equivalent to 97.14% of the perimeter of the total rectangular section.

![Figure 2 Maximum trapezoidal efficiency section.](image)

**3. Conclusions**

The method of iterations as a numerical solution is a tool of great importance to find the height of the water sheet of a rectangular section channel for certain operating conditions.

For the cross-section it was evidenced that the variable $X$ of all the evaluated ones closest to generating the equality is presented in the point, verified in the fifth iteration, determining that the height of the water sheet ($y_n$) of the channel for the raised conditions is $7.40\text{cm}$ with a free edge of $2.60\text{cm}$.

It is determined that the critical height of the rectangular section is $0.039441\text{m}$, with which it is obtained that the flow presents a subcritical regime.

According to the calculations made, it was found that the trapezoidal section is the most effective in terms of design and implementation since it is equivalent to 97.14% of the perimeter of the total rectangular section.

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