Kinematics Modeling and Analysis of Leveling Mechanism of Orchard Work Platform Based on Screw Theory

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Abstract. In view of the previous development of the orchard work platform leveling mechanism, the terrain change was introduced and simplified into a five-degree-of-freedom series linkage mechanism. The kinematics model based on the spin theory was established, and the Jacobian matrix of the workbench was derived. The key factors affecting the movement of the workbench was analysed, the experimental model was built. The results show that the maximum error is only 0.65°, which indicates that the kinematics model is accurate and reliable, and provides a theoretical basis for real-time control of the leveling mechanism.

1. Introduction
The complex working environment of the orchard makes the orchard work platform leveling system essentially a multi-input and multi-output nonlinear uncertain system. It is difficult to establish accurate mathematical models, external random disturbances and dynamic changes in system parameters and mechanism friction. There are uncertainties and nonlinear couplings in the factors, which leads to inaccurate models. In the modeling process, the calculation amount is large and the real-time performance is poor, so that the control effect cannot meet the work requirements, and even affect the leveling performance of the whole system. The kinematics and dynamics model is the basis for achieving high precision and real-time control, and the control problem of uncertain nonlinear systems has always been a difficult problem in the control field. For institutions, kinematics issues include forward kinematics and inverse kinematics [1-4]. Kinematics analysis is the basis of real-time control. Therefore, in order to improve the leveling response and leveling accuracy, it is necessary to perform kinematic modeling and analysis on the orchard work platform leveling mechanism.

The current common method used to describe the motion of the mechanism is the D-H parameter method, which was proposed by Denavit and Hartenberg in 1955. A coordinate system is established at each hinge, and the motion relationship of the adjacent two links passes through a 4×4 homogeneous transformation. The matrix description sequentially transforms the kinematics model of the final establishment mechanism, but the method only describes the motion with four parameters, and the coordinate transformation is complex and the geometric meaning is not obvious. The product of exponential (POE) based on the spin theory only needs to establish the inertial coordinate system and the object coordinate system. The modeling process is simple, with geometric intuitiveness and algebraic abstraction [5-6].
2. Spin theory

Chasles (1830) considers the motion of any rigid body as a combination of the rotation of a straight line and the movement of the line. This motion is called a spiral motion, and its infinitesimal amount is called the motion spin.

\[ \xi = \left[ \begin{array}{c} \dot{\omega} \\ \omega \times v \end{array} \right] \in se(3) = \left\{ \begin{pmatrix} R & t \\ 0 & 0 \end{pmatrix} : R \in SO(3), t \in \mathbb{R}^3 \right\} \] (1)

With \( \dot{\omega} \) is the antisymmetric matrix of the unit vector \( \omega = [\omega_x \ \omega_y \ \omega_z]^T \); \( v \) is the linear velocity, \( v = -\omega \times r \); \( \land \) is the operator that maps the 4\times4 matrix to the 6-dimensional vector; and \( se(3) \) is an algebra of the Lie group; \( R \) is a 3\times3 transferable rotation matrix; \( t \) is a position vector of the object system \( T \) in the inertial system \( S \); \( SO(3) \) is a special orthogonal group, and all 3\times3 orthogonal matrices of the determinant are 1.

The rigid body pose transformation matrix calculated by the Rodriguez formula is:

\[ g = \exp(\xi \theta) = \begin{pmatrix} \exp(\xi \theta) & (I + \exp(\xi \theta))(\omega \times v) + \omega^T v \theta \\ 0 & 1 \end{pmatrix} \] (2)

With \( \exp(\xi \theta) \) is the corresponding matrix index of \( \xi \theta \); \( I \) is a unit matrix of 3\times3.

For the n-degree-of-freedom series mechanism, the spinor exponential product equation of motion\[7\] is:

\[ g_{st}(\theta) = \exp(\xi_1 \theta_1) \exp(\xi_2 \theta_2) \ldots \exp(\xi_n \theta_n) g_{st}(0) \] (3)

With \( g_{st}(0) \) is the initial pose transformation of the coordinate system \( T \) relative to the inertial coordinate system \( S \); \( g_{st}(\theta) \) is the final pose of the object system \( T \) with respect to the inertial frame \( S \); \( \xi_n \) is the rotational momentum of the \( n_{th} \) link; \( \theta_n \) is the \( n_{th} \) link The angle of rotation; \( \exp(\xi_n \theta_n) \) is the corresponding matrix index of \( \xi_n \theta_n \).

Therefore, the rotational motion of the rigid body is expressed in the form of exponential product, which has a simple structure and obvious physical meaning.

3. Kinematics Modeling of Orchard Work Platform Leveling Mechanism

3.1 Orchard work platform overall structure

The overall structure of the orchard work platform is shown in figure 1, including the power unit, the swing mechanism, the lifting mechanism, the leveling mechanism and so on. The lifting cylinder expands and moves the table up or down. The leveling mechanism is mainly controlled by the leveling cylinder, the leveling hydraulic circuit and the leveling control system composition. The parallelogram structure is formed by the beam, the leveling cylinder, the column, and the worktable to keep the table horizontal\[8\].

\[ \] Figure 1 Structure diagram of orchard platform

\[ \] Figure 2 Simplified model of leveling mechanism of orchard platform
3.2 Leveling mechanism simplified model
To describe the movement of the leveling mechanism of the orchard work platform, the following is simplified: (1) The ground is rigid; (2) The components are assumed to be rigid and massless; (3) The hinge clearance and friction are ignored the impact of force; (4) The influence of platform orientation is not considered. The leveling mechanism can be regarded as a five-degree-of-freedom series open-chain linkage mechanism. The simplified structure and the coordinate system are as shown in figure 2, wherein \( T_i \) is an object coordinate system fixed to the centroid of each connecting rod, and the \( q_i \) connecting rod hinge point, \( l_i \) is the length of the link, \( r_i \) is the distance from the hinge point of the link to the centroid of each link. \((i = 1, 2, \ldots 5 \) in above symbols)

3.3 Kinematics model of leveling mechanism based on spin amount theory

![Figure 3](image)

A simplified model of the orchard work platform leveling mechanism is projected on a plane, as shown in figure 3, \( \theta_i \) is the rotation angle of each link. \((i = 1, 2, \ldots 5 \) in above symbols)

It can be seen from figure 3 that when the corners of the respective links are 0, the initial posture of table is:
\[
g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & c \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]
\(c = l_4 + r_5\), \(d = l_1 + l_2 + l_3\)

(4)

The rotation unit vector of each link obtained from figure 3 is:
\[
\omega_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \omega_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \omega_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad \omega_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

(5)

The coordinates of the hinge point on the rotating axis in the coordinate system \( S \) are:
\[
q_1 = 0, \quad q_2 = \begin{bmatrix} 0 \\ l_1 \end{bmatrix}, \quad q_3 = \begin{bmatrix} 0 \\ l_1 + l_2 \end{bmatrix}, \quad q_4 = \begin{bmatrix} 0 \\ l_1 + l_2 + l_3 \end{bmatrix}, q_5 = \begin{bmatrix} 0 \\ l_4 \end{bmatrix}
\]

(6)

According to formula (1), the motion rotation of each link is obtained:
\[
\xi_i = -\frac{\omega_i \times q_i}{\omega_i} \quad (i = 1, 2, 3, 4, 5)
\]

(7)

Substituting equations (6) and (7) into equation (8) respectively, the unit motion of each link is:
\[
\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \xi_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \xi_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \xi_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \xi_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]

(8)

Substituting each motion spin into equation (3) to obtain the kinematics index product form of the orchard work platform leveling mechanism:
\[ g_{st}(\theta) = \exp(\xi_1 \theta_1) \exp(\xi_2 \theta_2) \exp(\xi_3 \theta_3) \exp(\xi_4 \theta_4) \exp(\xi_5 \theta_5) \]

\[ g_{st}(0) = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10) \]

With \( n, o, a \) are the cosine of the object coordinate system to the direction of the inertial coordinate system \( S \), and \( p \) is the position vector of the origin of the end coordinate system in the coordinate system \( S \).

4. Orchard Work Platform Leveling Mechanism Motion Analysis

4.1 Jacobian matrix transformation

From equation (1), the instantaneous spatial velocity of the table in the inertial coordinate system \( S \) is the derivative of the pose with respect to time:

\[ V_{st}^s = g_{st}(\theta) g_{st}^{-1}(\theta) = \sum_{i=1}^{n} \left[ \frac{\partial g_{st}}{\partial \theta_i} \right] g_{st}^{-1}(\theta) \Delta \theta = \sum_{i=1}^{n} \left[ \frac{\partial g_{st}}{\partial \theta_i} \right] g_{st}^{-1}(\theta) \Delta \theta_i = J_{st}^s(\theta) \dot{\theta} \quad (11) \]

Substituting equation (10) into equation (11), according to the accompanying transformation:

\[ A_{st}(\exp(\xi_1 \theta_1) \exp(\xi_2 \theta_2) \ldots \exp(\xi_{n-1} \theta_{n-1}))(\theta) \]

Set \( \xi_i^T = A_{st}(\exp(\xi_1 \theta_1) \exp(\xi_2 \theta_2) \ldots \exp(\xi_{n-1} \theta_{n-1})) \xi_i \), then:

\[ V_{st}^s = J_{st}^s(\theta) \dot{\theta} = [\xi_1^T \xi_2^T \ldots \xi_{n-1}^T][\dot{\theta}_1 \ \dot{\theta}_2 \ \ldots \ \dot{\theta}_n]^T \quad (13) \]

It can be seen from the above that \( V_{st}^s \) represents the space velocity of the table in the inertial coordinate system, \( \dot{\theta} = [\dot{\theta}_1 \ \dot{\theta}_2 \ \ldots \ \dot{\theta}_n]^T \) is the velocity of each member of the leveling mechanism, \( J_{st}^s(\theta) \) is the Jacobian matrix of the space velocity of the table, and \( \xi_i^T \) represents the object coordinate system of the bar which number \( i \) ( \( i = 1, 2, \ldots 5 \) ) from the initial pose through \( \exp(\xi_1 \theta_1) \exp(\xi_2 \theta_2) \ldots \exp(\xi_{n-1} \theta_{n-1}) \) to the current pose.

4.2 Workbench motion analysis

Given a corner \([\theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ \theta_5]^T\) from \([0 \ 0 \ 0 \ \pi/2]^T\) to rotate at a constant speed for 10 s to \([\pi/12 \ \pi/4 \ \pi/18 \ \pi/6 \ \pi/4]^T\), according to equations (10) and (11), the displacement, velocity and acceleration changes in the vertical direction of the center of mass of the table are obtained by Matlab programming, as shown in figure 4, figure 5 and figure 6 is shown.

![Figure 4 Displacement curve of platform with rotation angle of rods](image1.png)  
![Figure 5 Velocity curve of platform with rotation angle of rods](image2.png)
It can be seen from figure 4, figure 5 and figure 6 that the change of the angle of the rod affects the centroid displacement, velocity and acceleration of the table. When $\theta_1$ (the angle of the terrain change), $\theta_2$ (the angle of rotation), $\theta_3$ (the horizontal slope leveling cylinder) are unchanged, $\theta_4$ (beam corner) change increases the maximum displacement of the center of mass of the table, about 1.45m, only 0.05m away from the design requirement (1.5m), the deviation rate is 3.33%, $\theta_5$ (vertical slope leveling cylinder angle) changes to the workbench. The influence of the centroid displacement is also large, and at the same time, the influence of the change on the velocity and acceleration of the center of mass of the table is also large.

5. Conclusion
This paper introduces the motion spin, the rigid body pose transformation and the kinematics model based on the exponential product form in the spin theory, expounds the overall structure and working principle of the orchard work platform, and then simplifies the establishment of the spin coordinate system for the leveling mechanism. The topographical variation is introduced, and the 5-DOF spin-exponential product kinematics model of the leveling mechanism is established. The Jacobian matrix are derived. The displacement and speed of the table caused by the change of the corners of each member are obtained by Matlab. And the acceleration curve, the analysis shows that the beam and the leveling cylinder movement have a great influence on the position of the table.

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