Two SVDs produce more focal deep learning representations

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Abstract

A key characteristic of work on deep learning and neural networks in general is that it relies on representations of the input that support generalization, robust inference, domain adaptation and other desirable functionalities. Much recent progress in the field has focused on efficient and effective methods for computing representations. In this paper, we propose an alternative method that is more efficient than prior work and produces representations that have a property we call focality – a property we hypothesize to be important for neural network representations. The method consists of a simple application of two consecutive SVDs and is inspired by (Anandkumar et al., 2012).

In this paper, we propose to generate representations for deep learning by two consecutive applications of singular value decomposition (SVD). In a setup inspired by (Anandkumar et al., 2012), the first SVD is intended for denoising. The second SVD rotates the representation to increase what we call focality – a property we hypothesize to be important for neural network representations. The method consists of a simple application of two consecutive SVDs and is inspired by (Anandkumar et al., 2012).

In Section 1, we introduce the two methods SVD1 and SVD2 and show that SVD2 generates better (in a sense to be defined below) representations than SVD1. In Section 2, we compare 1LAYER and 2LAYER SVD2 representations and show that 2LAYER representations are better. Section 3 discusses the results. We present our conclusions in Section 4.

1 SVD1 vs. SVD2

Given a base representation of n objects in \( \mathbb{R}^d \), we first compute the first k dimensions of an SVD on the corresponding \( n \times d \) matrix \( C \). \( C_k = U S V^T \) (where \( C_k \) is the rank-k approximation of \( C \)). We then use \( US \) to represent each object as a k-dimensional vector. Each vector is normalized to unit length because our representations are count vectors where the absolute magnitude of a count contains little useful information – what is important is the relative differences between the counts of different dimensions. This is the representation SVD1. It is motivated by standard arguments for representations produced by dimensionality reduction: compactness and noise reduction. Denoising is also the motivation for the first SVD in the method proposed by (Anandkumar et al., 2012).

We then perform a second SVD on the resulting matrix \( C' \) of dimensionality \( n \times k \), \( C' = U' S' V'^T \) (full-rank, no dimensionality reduction). We again use \( U' S' \) to represent each object as a k-dimensional vector. Each vector is normalized to unit length. This is the representation SVD2.
Figure 1: 1LAYER architecture. Distributional word vectors form the bottom layer. A trigram – represented as a triple of word vectors – is transformed by the first SVD into a 100-dimensional vector (layer “SVD$^1$”). These vectors are then rotated (layer “SVD$^2$”).

Note that the operation we are applying is not equivalent to a single linear operation because of the length normalization that we perform between the two SVDs.

SVD$^2$ is intended to be a rotation of SVD$^1$ that is more “focal” in the following sense. Consider a classification problem $f$ over a $k$-dimensional representation space $R$. Let $M_0(f, R)$ be the size $k'$ of the smallest subset of the dimensions that support an accuracy above a threshold $\theta$ for $f$. Then a representation $R$ is more focal than $R'$ if $M_0(f, R) < M_0(f, R')$. The intuition is that good deep learning representations have semantically interpretable hidden units that contribute input to a decision that is to some extent independent of other hidden units. We want the second SVD to rotate the representation into a “more focal” direction.

The role of the second SVD is somewhat analogous to that of the second SVD in the approach of Anandkumar et al. (2012), where the goal also is to find a representation that reveals the underlying structure of the data set.

The architecture of the 1LAYER setup is depicted in Figure 1.

**Experimental setup.** We use a corpus of movie review sentences (Pang and Lee, 2004). Following Schütze (1995), we first compute a left vector and a right vector for each word. The dimensionality of the vectors is 250. Entry $i$ for the left (right) vector of word $w$ is the number of times that the word with frequency rank $i$ occurred immediately to the left (right) of $w$. Vectors are then tf-idf weighted and length-normalized. We randomly select 100,000 unique trigrams from the corpus, e.g., “tangled feelings of” or “as it pushes”. Each trigram is represented as the concatenation of six vectors, the left and right vectors of the three words. This defines a matrix of dimensionality $n \times d$ ($n = 100000$, $d = 1500$). We then compute SVD$^1$ and SVD$^2$ on this matrix for $k = 100$.

**Analysis of correlation coefficients.** Figure 2 shows histograms of the 10,000 correlation coefficients of SVD$^1$ (left) and SVD$^2$ (right). Each correlation coefficient is the correlation of two columns in the corresponding 100000 $\times$ 100 matrix and is transformed using the function $f(c) = \log_{10} |c|$ to produce a histogram useful for the analysis. The histogram of SVD$^2$ is shifted by about 0.5 to the left. This is a factor of $10^{0.5} \approx 3$. Thus, SVD$^2$ dimensions have correlations that are only a third as large as SVD$^1$ correlations on average.

We take this to indicate that SVD$^2$ representations are more focal than SVD$^1$ representations because the distribution of correlation coefficients would change the way it changes from SVD$^2$ to SVD$^1$ if we took a focal representation (in the most extreme case one where each dimension by itself supported a decision) and rotated it.

**Discrimination task.** We randomly selected 200 words in the frequency range [25, 250] from the corpus; and randomly arranged them into 100 pairs. An example of such a pair is (documentary, special). For each pair, we first retrieved the SVD$^1$ and SVD$^2$ representations of all triples from the set of 100,000 in which one of the two words was the central word. For the example, “typical documentary footage”, “good documentary can”, and “really special walk” are such triples. Then we determined for each dimension $i$ of the 100 dimensions (for both SVD$^1$ and SVD$^2$) the optimal discrimination value $\theta$ by exhaustive search; that is, we determined the threshold $\theta$ for which the accuracy of the classifier $\vec{v}_i > \theta$ (or $\vec{v}_i < \theta$) was greatest – where the discrimination task was to distinguish triples that had one word vs the other as their central word. So for “typical documentary footage” and “good documentary can” the classifier should predict class 1 (“documentary”), for “re-
ally special walk” the classifier should predict class 2 (“special”). Finally, of the 100 discrimination accuracies we chose the largest one for this word pair.

On this discrimination task, SVD$^2$ was better than SVD$^1$ 55 times, the two were equal 15 times and SVD$^2$ was worse 30 times. On average, discrimination accuracy of SVD$^2$ was 0.7% better than that of SVD$^1$. This is evidence that SVD$^2$ is better for this discrimination task than SVD$^1$.

This indicates again that SVD$^2$ representations are more focal than SVD$^1$ representations: each dimension is more likely to provide crucial information by itself as opposed to only being useful in conjunction with other dimensions.

To illustrate in more detail why this discrimination task is related to focality, assume that for a particular 100-dimensional representation $r$ of trigrams $t$, the decision rule “if $r(t)_{27} > 0.2$ then ‘documentary’ else ‘special’” (i.e., if the value of dimension 27 is greater than 0.2, then the trigram center is predicted to be “documentary”, else “special”) has an accuracy of 0.99; and that the decision rule “if $r(t)_{27} > 0.2$ and $r(t)_{91} < -0.1$ then ‘documentary’ else ‘special’” has an accuracy of 1.0. Then $M_{0.99}(f, 	ext{documentary-vs-special}) = 1$, $M_{1.00}(f, 	ext{documentary-vs-special}) = 2$ and we can view the representation $r$ as highly focal since a single dimension suffices for high accuracy and two dimensions achieve perfect classification results.

2 1LAYER vs. 2LAYER

We compare two representations of a word trigram: (i) the 1LAYER representation from Section 1 and (ii) a 2LAYER representation that goes through two rounds of autoencoding, which is a deep learning representation in the sense that layer 2 represents more general and higher-level properties of the input than layer 1.

The architecture of the 2LAYER is depicted in Figure 3.

To create 2LAYER representations, we first create a vector for each of the 20701 word types occurring in the corpus. This vector is the concatenation of its left vector and its right vector. The resulting $20701 \times 500$ matrix is the input representation to SVD$^1$. We again set $k = 100$. A trigram

Figure 2: Histograms of $\log_{10} |c|$ of the 10,000 correlation coefficients of SVD$^1$ (above) and SVD$^2$ (below).
Figure 3: 2LAYER architecture. Distributional word vectors form the bottom layer. Each word vector is transformed by SVD into a 100-dimensional vector (first layer “SVD\(^1\)”). This layer constitutes the 1LAYER part of this architecture. A triple of vectors of three consecutive words is transformed by SVD into a 100-dimensional vector (second layer “SVD\(^2\”)}. These vectors are then rotated (layer “SVD\(^3\)”). The last two layers constitute the 2LAYER part of this architecture.

is then represented as the concatenation of three of these 100-dimensional vectors. We apply the SVD\(^2\) construction algorithm to the resulting \(100000 \times 300\) matrix and truncate to \(k = 100\).

We now have – for each trigram – two SVD\(^2\) representations, the 1LAYER representation from Section 1 and the 2LAYER representation we just described. We compare these two trigram representations, again using the task from Section 1: discrimination of the 100 pairs of words.

2LAYER is better than 1LAYER 64 times on this task, the same in 18 cases and worse in 18 cases. This is statistically significant (\(p < .01\), binomial test) evidence that 2LAYER SVD\(^2\) representations are more focal than 1LAYER SVD\(^2\) representations.

3 Discussion

3.1 Focality

One advantage of focal representations is that many classifiers cannot handle conjunctions of several features unless they are explicitly defined as separate features. Compare two representations \(\vec{x}\) and \(\vec{x}'\) where \(\vec{x}'\) is a rotation of \(\vec{x}\) (as it might be obtained by an SVD). Since one vector is a rotation of the other, they contain exactly the same information. However, if (i) an individual “hidden unit” of the rotated vector \(\vec{x}'\) can directly be interpreted as “is verb” (or a similar property like “is adjective” or “takes NP argument”) and (ii) the same feature requires a conjunction of several hidden units for \(\vec{x}\), then the rotated representation is superior for many upstream statistical classifiers.

Focal representations can be argued to be closer to biological reality than broadly distributed representations (Thorpe, 2010); and they have the nice property that they become categorical in the limit. Thus, they include categorical representations as a special case.

A final advantage of focal representations is that in some convolutional architectures the input to the top-layer statistical classifier consists of maxima over HU (hidden unit) activations. E.g., one way to classify a sentence as having positive/negative sentiment is to slide a neural network whose input is a window of \(k\) words (e.g., \(k = 4\)) over it and to represent each window of \(k\) words as a vector of HU activations produced by the network. In a focal representation, the hidden units are more likely to have clear semantics like “the window contains a positive sentiment word”. In this type of scenario, taking the maximum of activations over the \(n - k + 1\) sliding windows of a sentence of length \(n\) results in hidden units with interpretable semantics like “the activation of the positive-sentiment HU of the window with the highest activation for this HU”. These maximum values are then a good basis for sentiment classification of the sentence as a whole.

The notion of focality is similar to disentanglement (Glorot et al., 2011) – in fact, the two notions may be identical. However, Glorot et al. (2011) introduce disentanglement in the context of domain adaptation, focusing on the idea that “disentangled” hidden units capture general cross-domain properties and for that reason are a good basis for domain adaptation. The contributions of this paper are: proposing a way of measuring “entanglement” (i.e., measuring it as correlation), defining focality
in terms of classification accuracy (a definition that covers single hidden units as well as groups of hidden units) and discussing the relationship to convolution and biological systems.

It is important to point out that we have not addressed how focality would be computed efficiently in a particular context. In theory, we could use brute force methods, but these would be exponential in the number of dimensions (systematic search over all subsets of dimensions). However, certain interesting questions about focality can be answered efficiently; e.g., if we have $M(f, R) = 1$ for one representation and $M(f, R') > 1$ for another, then this can be shown efficiently and in this case we have established that $R$ is more focal than $R'$.

3.2 mSVD method

In this section, we will use the abbreviation $mSVD$ to refer to a stacked applications of our method with an arbitrary number of layers even though we only experiment with $m = 2$ in this paper (2LAYER, 2-layer-stacking).

SVD and other least squares methods are probably the most widely used dimensionality reduction techniques for the type of matrices in natural language processing that we work with in this paper (cf. (Turney and Pantel, 2010)). Stacking a second least squares method on top of the first has not been considered widely because these types of representations are usually used directly in vector classifiers such as Rocchio and SVM (however, see the discussion of (Chen et al., 2012) below). For this type of classifier, performing a rotation has no effect on classification performance. In contrast, our interest is to use SVD\(^2\) representations as part of a multilevel deep learning architecture where the hidden unit representations of any given layer are not simply interpreted as a vector, but decisions of higher layers can be based on individual dimensions.

The potential drawback of SVD and other least squares dimensionality reductions is that they are linear: reduced dimensions are linear combinations of original dimensions. To overcome this limitation many nonlinear methods have been introduced: probabilistic latent semantic indexing (Hofmann, 1999), kernel principal component analysis (Schölkopf et al., 1998), matrix factorization techniques that obey additional constraints – such as non-negativity in the case of non-negative matrix factorization (Lee and Seung, 1999) – , latent dirichlet allocation (Blei et al., 2003) and different forms of autoencoding (Bengio, 2009; Chen et al., 2012). All of these can be viewed as dimension reduction techniques that do not make the simplistic assumptions of SVD and should therefore be able to produce better representation if these simplistic assumptions are not appropriate for the domain in question.

However, this argument does not apply to the mSVD method we propose in this paper since it is also nonlinear. What should be investigated in the future is to what extent the type of nonlinearity implemented by mSVD offers advantages over other forms of nonlinear dimensionality reduction; e.g., if the quality of the final representations is comparable, then mSVD would have the advantage of being more efficient.

Finally, there is one big difference between mSVD and deep learning representations such as those proposed by Hinton et al. (2006), Collobert and Weston (2008) and Socher et al. (2012). Most deep learning representations are induced in a setting that also includes elements of supervised learning as is the case in contrastive divergence or when labeled data are available for adjusting initial representations produced by a process like autoencoding or dimensionality reduction.

This is the most important open question related to the research presented here: how can one modify hidden layer representations initialized by multiple SVDs in a meaningful way?

The work most closely related to what we are proposing is probably mDA (Chen et al., 2012) – an approach we only became aware of after the initial publication of this paper. There are a number of differences between mDA and mSVD. Non-linearity in mDA is achieved by classical deep learning encoding functions like tanh() whereas we renormalize vectors and then rotate them. Second, we do not add noise to the input vectors – mSVD is more efficient for this reasons, but it remains to be seen if it can achieve the same level of performance as mDA. Third, the mSVD architecture proposed here, which changes the objects to be represented from small frequent units to larger less frequent units when going one layer up, can be seen as an alternative (though less general since it’s customized for natural language) way of extending to very high dimensions.
4 Conclusion

As a next step a direct comparison should be performed of SVD$^2$ with traditional deep learning (Hinton et al., 2006). As we have argued, SVD$^2$ would be an interesting alternative to deep learning initialization methods currently used since SVD is efficient and a simple and well understood formalism. But this argument is only valid if the resulting representations are of comparable quality. Datasets and tasks for this comparative evaluation could e.g. be those of Turian et al. (2010), Maas et al. (2011), and Socher et al. (2011).

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