171 Yb+ System Stability, 5D3/2 Hyperfine State Detection Efficiency and F=2 Lifetime

October 10, 2013

Abstract

A data acquisition system is described that is designed to stabilize cooling and probe rates to maximize detection sensitivity and minimize possible systematic errors due to correlations between drifting experimental conditions and varying drive parameters. Experimental parameters that affect the Yb\textsuperscript{171} 5D3/2 hyperfine state preparation and detection efficiency are characterized and optimized. A set of wait times for optimal sampling of the D3/2 (F = 2) lifetime is chosen and used to measure that lifetime with high statistical sensitivity. A systematic variation in this lifetime seems to be apparent. The source of the variation was not identified, but ion number and cooling rate appear to be ruled out. A net determination is made of $\tau = 61.8 \text{ ms} \pm (0.6)_{\text{stat}} \pm (6.4)_{\text{sys}}$ which is significantly longer than other measurements of the same quantity. An alternate shelving scheme is proposed that would provide S-D state discrimination for Yb even isotopes as well as improved sensitivity for D state hyperfine discrimination in odd isotopes.

1 Introduction

Trapped ions provide an excellent platform for making a variety of precision measurements. In particular, there are renewed prospects for a trapped ion atomic parity violation experiment\textsuperscript{4} using, for example, light shifts of the S1/2 and D3/2 hyperfine states of 171 Yb\textsuperscript{+} generated by driving the S-D transition\textsuperscript{2}. Measuring these shifts with sufficient sensitivity requires efficiently preparing and detecting an ion’s spin state. State detection using shelving in 171 Yb\textsuperscript{+} is typically done using D3/2 hyperfine states and this state’s shorter lifetime and relatively small hyperfine splitting compromise sensitivity. By carefully characterizing pump and probe rates, experimental parameters such as pump and probe times can be chosen to optimize the sensitivity of detecting transitions into or out of particular states to subsequently improve the sensitivity of light shift measurements.

Connecting the measurements from such an experiment to the quantity $Q_W^{(171 \text{ Yb})}$ of interest for evidence of physics beyond the standard model of particle physics will require a number of precise atomic structure parameters including the S-D quadrupole reduced matrix element. This could be determined from a measurement of the D3/2 lifetime and reinforced by theoretical calculations. This lifetime has been measured in 174 Yb\textsuperscript{+} using other methods to be $52.7 \pm 2.4 \text{ ms}$\textsuperscript{3}. This is not yet sufficiently precise and the uncertainty appears to be underestimated. This measurement also only considers collisional quenching for possible systematic errors, and assumes Poisson counting statistics which may give a systematic shift in the fit lifetime. The $S - D$ matrix element has not
yet been the target of recent precision calculation methods and existing calculations of the lifetime vary widely, 41ms[4] and 74ms[5]. A more precise experimental measurement may motivate further theoretical studies.

The importance of the shelved state lifetime to sensitivity motivates considering states other than the $D_{3/2}(F = 2)$ that is commonly used. The very long lived $F_{7/2}$ state would be an excellent alternative and initial work to drive such a shelving transition has been done.

2 Measurement Cycle

Schematically a lifetime measurement of a relatively long-lived excited state in any system is straight-forward: prepare the system in the excited state, wait some period of time, then probe the system to determine if a decay transition occurred. Repeat such a trial as necessary to measure a transition probability, and repeat the entire procedure for a set of different wait times to find the time dependence of the transition probability and determine the lifetime. For trapped ions this probe is done by using shelving to determine if the ion is in some particular state that can be connected to the initial or final state of the decay transition.

2.1 Shelving

A doppler cooled ion scatters photons from the cooling beam at rates of 10’s of MHz, some fraction of which can be detected with a photo-multiplier tube (PMT) providing a cooling signal typically on the order of a few thousand counts per second (kcps), $r_c$. The PMT signal will also generally include a low background rate, $r_b$, giving a net total signal corresponding to $r_T = r_b + r_c$. While being cooled the ion cycles through all the states involved in the cooling process.

Shelving consists of driving the ion to some relatively long-lived state that is not part of this cooling cycle.[6] When the ion is in this shelved state the PMT count rate drops to the background rate, $r_b$, which ideally is easily distinguished from the total count rate. This provides a means of efficiently determining if a single ion is or is not in the shelved state in a time fundamentally limited only by counting statistics. For long-lived states, fairly high cooling rates, and low background rates practically perfect detection can be done very quickly.

For $^{171}$Yb$^+$ the $D_{3/2}(F = 2)$ state that is being studied here can itself be used as the shelved state. For other states or other kinds of measurements, the transition to the shelved state can be done in such a way that only an ion initially in some particular state ends up being shelved, while a different initial state would remain somewhere in the cooling cycle.

To help illustrate the details of using these methods in $^{171}$Yb$^+$, a partial energy level diagram is provided in figure 1. $^{171}$Yb$^+$ is Doppler cooled using the $6S(F = 1) \rightarrow 6P(F = 0)$ transition driven with a 370nm laser. The P state decays primarily back to the ground state and has a lifetime of about 8ns so that this transition when saturated yields a fluorescence rate of about 100MHz. The light collection system detects about $5 \times 10^{-4}$ of these scattered photons giving a detected cooling rate of between 2000 and 10,000 counts per second depending on laser and PMT alignment and the cleanup rates described presently.

The $P$ state can also decay to the $5D$ state. The $D$ state has a relatively long lifetime, $\tau \approx 60$ ms, which would considerably reduce the cooling and detection rate if permitted only to decay naturally, so a 935nm laser is used to excite the ion from the $D$ state to a $3[3/2]_{1/2}$ state from which it decays to the ground state. Cooling can then resume. This 935nm laser is nominally tuned to drive transitions between the $5D(F = 1)$ and $3[3/2]_{1/2}(F = 0)$ hyperfine levels. The branching ratio of
Figure 1: $^{171} \text{Yb}^+$ partial level diagram. The states involved in cooling and pump/probe sequences are in the first two columns. States involved in the proposed alternate shelving scheme are in the third column. Driven transition are shown with solid lines. Transitions are E1 unless marked otherwise.

the P decay to the 6S or 5D state is about 142:1, so the D state must be cleaned out at a rate greater than about 100 MHz/142 $\lesssim$ 1 MHz to avoid significantly reducing the cooling rate.

Similarly, though the 6P($F = 0$) state can not quickly decay to the 6S($F = 0$) hyperfine level because of angular momentum selection rules, the 370nm laser may non-resonantly drive the 6S($F = 1$) to 6P($F = 1$) transition. The 6P($F = 1$) can then decay to the 6S($F = 0$) state also removing it from the cooling cycle. A 7.4GHz electro-optic modulator (EOM) is used on the 370nm laser to provide (second order) side-bands at 12.6+2.2=14.8 GHz to couple the 6S($F = 0$) and 6P($F = 1$) states to drive any ion in the former state back into the cooling cycle. Likewise, the 6P($F = 0$) state will not quickly decay to the 5D($F = 2$) state, but an ion in the 6P($F = 1$) state, through off-resonant excitation from the directly driven 6S($F = 1$)→6P($F = 0$) transition, or directly from the 6S($F = 0$) state via the EOM induced side-bands, will quickly decay to the 5D($F = 2$) state. Again, an EOM is used to generate 2.24 + 0.86 = 3.1 GHz side-bands on the 935nm laser to couple this state to the $^3\!\!^3[3/2]_{1/2}(F = 1)$ state and keep the ion in the cooling cycle.

2.2 Pump and Probe

The 6S($F = 0$) and 5D($F = 2$) hyperfine states are relatively dark when their corresponding EOMs are switched off, making either suitable for use as a shelved state. Consider the 5D($F = 2$) state in particular. With the 935nm laser’s 3.1GHz EOM switched off, this state becomes (nominally) isolated from the cooling cycle. An ion in this state will yield a count rate equal to the background rate rather than the much larger cooling rate, providing a probe of the ion’s state. An ion can be driven into this state in the same way though the rate would be small if relying only on off-resonant couplings, so in practice a 2.2GHz EOM is also employed on the 370nm laser, coupling
the 6S\(F = 1\) state to the 6P\(F = 1\) state from which it can decay quickly to this 5D\(F = 2\) shelved state, providing the pump step.

A sequence of these particular beam combinations then simply allows a measurement of the \(D_{3/2}\) lifetime. The ion is pumped to the 5D\(_{3/2}(F = 2)\) state using the combination of EOM states described, then all the beams are turned off and the ion is left in the dark for some period of time during which it may decay to the 6S state. Probe beams are then applied and the number of counts is recorded.

2.3 Measurement Sequence

Figures 2 and 3 show the basic building blocks of such a sequence. The pump and probe procedures just described are referred to as D2Pump and D2Probe, and all beams off during the wait as Off. Each of these units may be further defined by some parameters, such as the pump, probe and wait times. Where parameters vary, or are otherwise of interest they will be included explicitly so that the full lifetime sequence can be denoted \(D2\) Pump\((t_{pump})/\) Off\((t_{wait})/D2\) Probe\((t_{probe})\). This entire sequence is in turn denoted simply as \(d_0(t_{wait})\).

Note that a particular unit is not necessarily a single set of beam states. D2Pump for example must shut the beams off in a particular way to ensure that the ion is left in a well defined state. Ideally beams are shut off instantaneously and simultaneously, in practice each change in beam state includes delays and transition times that will effectively guarantee that they are not changed simultaneously. If the 370nm laser is shut off before the 935nm laser, the ion may be driven back to the ground state no matter what the steady state pumping probability is. Instead the 370nm beam is explicitly shut off after the 935nm beam in a way that accounts for acousto-optic modulator (AOM) transition times and shutter lags and all other relevant complications, and the resulting pump state is thus well defined and stable. Similar considerations are required for other units and these are reflected in the timing diagrams for each unit but will not be explicitly discussed here.

A number of other sequences of states are also used in actual experiments to provide complementary information or to monitor or stabilize various experimental parameters. For example, SPump is used to drive the ion to some ground state hyperfine level for the block \(d_1(t_{wait}) = SPump/\) Off\((t_{wait})/D2\) Probe to check for possible perturbations that result in 6S \(\rightarrow 5D\) excitations even when all the beams are supposed to be off. And \(c_1 = CoolCount(t_{coolcount})\) is used to stabilize the frequency of the 370nm laser and provide an independent measure of the cooling rate and its fluctuations. Others blocks will be discussed as they are used. The counts resulting from a sequence block are referred to by the name of the block, so that \(d_0\) is the counts resulting from the probe step of a \(d_0(t_{wait})\) block. A complete experiment is a multiplexed sequence of these blocks with parameters and relative frequencies appropriate to maximize the overall sensitivity for the parameter being measured[8] and minimize the possible systematic effects of other varying experimental conditions. In most cases here, a fixed chain of sequences is repeated, such as \(c_1/n_0/n_1/d_0/d_1/c_0\), and the freedom to change the relative frequencies of the blocks is not yet used.

2.4 State Detection and Sensitivity

Most sequences end with a \(D2\) Probe unit which yields a PMT count. Ideally these counts from a single probe could be used to determine with near certainty if a transition was made. After a number of trials the probability, \(s\), that the ion is in the shelved state at the beginning of the probe
Figure 2: $^{171}\text{Yb}^+$ measurement sequence units. These units are combined in blocks, shown in figure 3, that each constitute a single basic measurement. In practice the PMT signal is replaced by a pulse at each edge as a PMT is triggered rather than gated, and the actual leading and trailing edges of the 370nm laser gate are shifted to compensate for the lag of the shutter that actually implements this switch. The on and off delays are measured in a separate procedure.

Figure 3: $^{171}\text{Yb}^+$ measurement sequence blocks formed from units shown in figure 2. Trials are fixed sequences of these blocks. Parameters such as pump/probe/wait times are chosen before each set of trials and a digital waveform is computed which is written to a set of digital outputs timed by a hardware clock to control the various gates.

can be measured, and the lifetime $\tau_D$ determined from

$$s(t_{wait}) = s_{\text{pump}} e^{-t_{wait}/\tau_D}$$

with $s_{\text{pump}}$ the probability for the ion to be in the shelved state after pumping and $t_{wait}$ the wait time between the pump and probe stages. The uncertainty in the measurement of $s$ would be given
by binomial statistics.

For the case of shelving using the $5D_{3/2}(F = 2)$ state this is less straight-forward for reasons that will be considered in detail below. Instead, the average number of photons $n$ collected during the probe as a function of wait time is measured. $n$ can be linearly related to $s$ through two parameters $\bar{n}_b$ and $\bar{n}_c$. With $\bar{n}_T \equiv \bar{n}_b + \bar{n}_c$

$$n = s\bar{n}_b + (1 - s)\bar{n}_T = \bar{n}_T - \bar{n}_c s$$

$\bar{n}_c$ and $\bar{n}_b$ are modified cooling and background count rates and reduce to the corresponding actual count rates $n_c$ and $n_b$ in the the limit of probe times short compared to the probe coherence time. The exact values can be related to various experimental parameters such as these actual cooling and background rates, probe time, and pump, probe and decay rates, but in practice they are more conveniently directly measured experimentally.

Take $\sigma_s$ to be the uncertainty with which $s$ can be determined in a single trial, and $\sigma_{s(N)}^2$ the uncertainty of $s$ determined from $N$ independent trials so that $\sigma_{s(N)}^2 = \sigma_s/\sqrt{N}$. This can be related to $\sigma_n$, the variance of $n$ for a single trial measurement with simple error propagation

$$\sigma_{s(N)}^2 = \sigma_n = \frac{1}{\sqrt{N}} \frac{\sigma_{n_b}}{\bar{n}_c}$$

For a total observation time $T$, and $t_{trial}$ the time needed to do one trials, the total number of trials would be $N = T/t_{trial}$ giving

$$\sigma_{s(N)}^2 = \sqrt{\frac{t_{trial}}{T \bar{n}_c}} \sigma_{n_b}$$

The single trial measurement uncertainty, $\sigma_n$, is given by multi-modal statistics. $\bar{n}_c$ and $\sigma_n$ and $t_{trial}$ all generally depend on the probe time $t_p$ in a non-trivial way but $\sigma_s$ can be approximately minimized by choosing $t_p \approx \tau_{probe}/2$ where $\tau_{probe}$ is a probe coherence time that can be determined experimentally.

For the particular case of a lifetime measurement the precision with which the lifetime $\tau_D$ is determined can be estimated from

$$\sigma_{\tau} \approx \left| \frac{\partial s}{\partial \tau_D} \right|^{-1} \sigma_s = f_{\tau} \tau_D \sigma_s$$

$$f_{\tau} = \frac{1}{s_{pump}} \frac{t_{wait}/\tau_{D}}{\tau_{D}}$$

for a single trial, or with $\sigma_s \to \sigma_{s(N)}^2$ for $N$ trials. The lifetime $\tau_D$ has been included in $\sigma_{\tau}$ explicitly to exhibit its general scale and make $f_{\tau}$ dimensionless. This result is strictly correct only when the only parameter to be determined is $\tau$. An optimal sampling analysis that accounts for the need to also determine the $\bar{n}_i$ can be used to provide an exact result for $f_{\tau}$. For present purposes note only that $\sigma_{\tau}$ is improved as $s_{pump}$ increases so that $s_{pump}$ should be made as large as possible.

### 2.5 System Stability and Calibration

The probe counts are linearly dependent on $s$ so that they can be used to determine the dynamical quantities of interest. These average result of these probe counts will evolve between $\bar{n}_T$ and $\bar{n}_b$. It will be seen in detail that these quantities and others like $s_{pump}$ are determined by a number
of pump and loss rates, all of which depend on the intensity and tuning of the cooling lasers. Variations in these experimental conditions can result in extra contributions to the uncertainties of parameters measured from these quantities which reduces sensitivity or give possible systematic shifts of those measured parameters. As a result these rates should be made as stable as possible, and what can’t be stabilized must be monitored either for use in compensating for the remaining fluctuations and possibly correcting possible resulting systematic errors, or for screening out data taken during excessively fluctuating conditions.

Laser intensity will depend on laser power and losses during delivery, both of which are fairly stable; and positioning of the beam on the ion, which is stable, but not easily controlled. Similarly frequencies of the cooling and re-pump lasers are stabilized but the absolute tuning is not controlled. This results in different, arbitrary but stable, values of \( \bar{n}_c \) and \( \bar{n}_b \) so that probe results for blocks like \( d_0(t_{\text{wait}}) \) or \( n_0(t_{\text{pump}}) \) can not be compared directly between different runs when beam position is adjusted or frequencies are re-tuned and re-locked, but generally once \( \bar{n}_i \) has been determined for each run, comparisons are valid.

For these purposes every measurement sequence includes a number of blocks that provide information that can be used to determine the \( \bar{n}_i \) and assess or improve their stability. The timing sequences of these additional blocks are also shown in figure ???. Among them are \( c_1 \) giving a direct measure of the cooling rate which is some combination of 370nm and 935nm rates; \( c_0 \) giving the background rate; \( n_1 \) which will be seen to directly provide \( \bar{n}_c \); and \( n_0 \) giving an upper bound for \( \bar{n}_b \). Particular measurements may also include still other blocks that provide complementary information about other possible systematic effects, as will be seen in the lifetime measurement. These values are monitored on strip chart plots like the example shown in figure 4a with either a cumulative average, as shown, or a shorter timescale boxcar average as appropriate to facilitate detection of significant variations. The core of the system used to make these measurements is described in [9].

Pump and probe rates are most sensitive to the cooling laser frequencies which are also the most inherently volatile. These are stabilized to the ion itself using \( c_1 \). The 370nm laser frequency is updated after every set of trials to give some fixed average for \( c_1 \) by re-tuning by an amount proportional to the difference between \( c_1 \) and the target value. This can not distinguish between 370nm or 935nm laser frequency fluctuations, so every 100s or so the 935nm laser is tuned independently outside of the measurement cycle by setting the frequency to that which maximizes the cooling rate. This stabilization scheme results in \( c_1 \) distributions that are almost perfectly Poissonian as will be seen when considering the probe count distributions (figure 6a).

This approach yields a very stable overall system that can be used to take data up to 30 hours unattended. The principle limit to this measurement time is currently 370nm laser mode quality which seems to deteriorate with time and temperature and eventually results in insufficient power at the cooling frequency. When this occurs the system is unable to restore \( c_1 \) to its target value by re-tuning the 370nm laser frequency. The data sequence then stops until the laser diode current and grating piezo voltage are manually adjusted to restore the cooling rate.

This scheme still allows for certain collective variations of the related relative rates, such as a 370nm laser power reduction compensated for by re-tuning, that maintains \( c_1 \) but yield changes in \( \bar{n}_i \), so \( n_0 \) and \( n_1 \) are continuously measured. These are not yet actively stabilized, but are monitored for sudden or large variations that merit intervention.

Slow drifts of these values can generate systematic errors in certain measurement. Consider a lifetime or pump/probe rate measurement during which \( \bar{n}_c \) is slowly decreasing due to a cooling laser frequency drift. If measurements for each particular wait or drive time are all done at once,
and in order of increasing time, probes made during the end of the run would show a relative reduction having nothing to do with the dynamics of the process being studied. If this variation is large enough it might be detected by a poor fit to the expected functional form, but the drift might also combine with the dynamics to give the same functional form, or just be too small to identify in this way, and instead lead to a shifted fit parameter. This effect might be mitigated by normalizing the probe results for each time by the \( n_0 \) and \( n_1 \) collected during the same time, but this approach is avoided for precise measurement due to some lingering uncertainties in the detailed understanding of the probe process.

More effective is to vary the way the data is collected. By doing smaller sets of trials for each time, but repeating that sequence many times during the measurement in a generalized AB pattern, systematic shifts due to linear drifts will average out. For more general variations, more sophisticated higher order alternations could be made, e.g. (ABBA)(BAAB), but just as effective and simpler is to pick a parameter value randomly for each trial. In this case the measurement times are completely uncorrelated with any long timescale drifts and all systematic shifts at any order should average out. Measurements presented here are all done in this way. Parameter values are chosen randomly from a corresponding predefined set before each set of trials and a digital waveform that drives all the instruments is generated from these values. Trial times for a complete sequence of blocks are around 50-200ms and \( \sim 10 \) trials are made for each set of parameters so that an actual measurement consists of a long sequence of 1-2s long sets of trials.

There remains at least one more systematic problem that is occasionally encountered. Clouds consisting of large numbers of ions experience more driven motion than small clouds due to the applied radio-frequency trapping field. During long wait, pump or probe times, when the ions are not being actively cooled, they may heat up enough that the cooling count rate is reduced due to increased cloud volume and cooling transition doppler width, and through this the \( \bar{n}_i \) are also reduced or otherwise changed. This gives exactly the kind of spurious \( t_{\text{wait}} \) dependence just described. Figure 4a shows just this sort of effect in a pump time measurement. This can be dealt with in way similar to the case of drifting parameters. The \( n_0(t_{\text{pump}}) \) data can be re-normalized using \( \bar{n}_i(t_{\text{pump}}) \) and the resulting profile fit. Figure 4c shows the resulting \( \tau_{\text{pump}} \) for both cases, illustrating the possible systematic error.

But again, given the unknown, and probably unstable, details of the effects of the increased heating, in preference to this the system is tweaked until this variation is eliminated and the \( n_0(t_{\text{pump}}) \) are fit directly. In this case the number of ions trapped was reduced, the 370nm laser re-tuned to provide a lower \( c_1 \), and the measurement was repeated. Figure 4b shows that the \( t_{\text{pump}} \) dependence of \( n_0 \) has apparently been eliminated and \( c_1 \) and \( n_1 \) in figure 4a are clearly much more stable. The laser re-tuning likely changed pump and probe rates as well, so the resulting \( \tau \) can not be compared directly to the previous case.

### 3 Pump Efficiency

After pumping, the probability that the ion is in the shelved state is determined by the pump time, \( t_{\text{pump}} \) and the pump and loss rates, \( \Gamma_{\text{pump}} \) and \( \Gamma_{\text{loss}} \), into and out of the shelved state. The intermediate states involved are short-lived, and the system can be described with a first order rate equation

\[
 s_{\text{pump}} = s_0 e^{-t_{\text{pump}}/\tau_{\text{pump}}} + s_{\infty} (1 - e^{-t_{\text{pump}}/\tau_{\text{pump}}})
\]
indicating insufficient cooling rate. By recording $c_1(t_{\text{pump}})$ this effect is detected and can be used to re-normalize $n_0$ and possibly correct for this effect. This is not done for precision measurements. Instead adjustments are made until $c_1$ is independent of pump time, as seen in d), the previous data is discarded, and the experiment is restarted.

with $\tau_{\text{pump}} = 1/(\Gamma_{\text{loss}} + \Gamma_{\text{pump}})$ and $s_\infty = \Gamma_{\text{pump}}/(\Gamma_{\text{pump}} + \Gamma_{\text{loss}})$. The quantity $s_0$ is the probability to be in the shelved state when pumping begins. Pumping is usually done immediately after cooling. During cooling, transitions into the $5D_{3/2}(F = 2)$ state are made only through off-resonant couplings, so $s_0$ should be almost zero. In this limit $s_{\text{pump}}$ monotonically increases to $s_\infty$ which approaches 1 for $\Gamma_{\text{pump}} \gg \Gamma_{\text{loss}}$. This limiting value is obtained for $t_{\text{pump}} \gg \tau_{\text{pump}}$ where $t_{\text{pump}}$ is the length of time the pump drive is applied to the ion.

With the 935nm laser’s 3GHz EOM off, the losses from the $5D_{3/2}(F = 2)$ state are due to this state’s own lifetime, $\Gamma_{\text{decay}} = 1/\tau_D \approx 1/50\,\text{ms} = 20\,\text{Hz}$, and off-resonant couplings to the $3/2_{1/2}(F = 1)$, $\Gamma_{\text{off - resonant}}$, giving $\Gamma_{\text{loss}} = \Gamma_{\text{decay}} + \Gamma_{\text{off - resonant}}$. In the example shown below
\(\Gamma_{\text{off-resonant}}^{35} \) is also comparable to \(\Gamma_{\text{decay}}\) resulting in \(\Gamma_{\text{loss}} \approx 50\) Hz.

With normal cooling beams, the pump rate is also due only to off-resonant couplings to the \(6P_{3/2}(F = 1)\) state, \(\Gamma_{\text{off-resonant}}^{370}\). As will be seen below, this rate is about \(25\) Hz. This would yield only a relatively small \(s_{\infty} \approx 25/75 = 1/3\). By using the 370nm laser’s previously described 2GHz EOM, \(\Gamma_{\text{pump}}\) can be increased. This directly drives transitions to the \(6P_{1/2}(F = 1)\) state from which the ion will decay to either of the \(5D_{3/2}\) hyperfine states with similar probabilities. This enhances the pump rate, \(\Gamma_{\text{EOM}}\), by an amount proportional to 2GHz EOM side-band amplitude so that \(\Gamma_{\text{pump}} = \Gamma_{\text{off-resonant}}^{370} + \Gamma_{\text{EOM}}\)

### 3.1 Pump Profile

\(\tau_{\text{pump}}, \ s_{\text{pump}}, \) and their dependence on \(t_{\text{pump}}\) and the EOM power \(P_{\text{EOM}}\) can all be determined with a sequence block like \(n_0\) which consists of a pump followed by the previously described probe sequence. The probe step will yield some average count \(n(t_{\text{pump}}) = \bar{n}_T - s_{\text{pump}}(t_{\text{pump}})\bar{n}_c\) linearly related to \(s_{\text{pump}}\) as described previously. With \(s_{\text{pump}}(t_{\text{pump}})\) as determined previously,

\[
\begin{align*}
 n(t_{\text{pump}}) &= \bar{n}_T - (s_{\infty} - (s_{\infty} - s_0)e^{-t_{\text{pump}}/\tau_{\text{pump}}}t_{\text{pump}})\bar{n}_c \\
 &= (\bar{n}_T - s_{\infty}\bar{n}_c) + (s_{\infty} - s_0)\bar{n}_c e^{-t_{\text{pump}}/\tau_{\text{pump}}}
\end{align*}
\]

This exponential functional form has three actually independent parameters, so a fit of \(n(t_{\text{pump}})\) in this form directly gives \((\bar{n}_T - s_{\infty}\bar{n}_c)\), \((s_{\infty} - s_0)\bar{n}_c\) and \(\tau_{\text{pump}}\), but can not determine the \(\bar{n}_i\) and \(s_i\) independently. As mentioned \(s_0 \approx 0\) so it can be eliminated, allowing \(s_{\infty}\bar{n}_c\) to be determined directly. This can then be used to determine \(\bar{n}_T\) as the same product appears in \((\bar{n}_T - s_{\infty}\bar{n}_c)\).

\(\bar{n}_T\) may also be determined directly from an independent measurement using an \(n_1 = \text{SPump}/\text{D2 Probe}\) block. \(n_1\) is a pump to the ground state followed by a probe. \(\text{SPump}\) should give \(\text{s_{pump}} = 0\) so that a probe gives \(\bar{n}_T\) directly. Ideally a pump step that yields \(s_{\text{pump}} = 1\) could be done so that \(\text{D2 Probe}\) would directly give \(\bar{n}_T - \bar{n}_c \equiv \bar{n}_b\) in the same way, and then \(\bar{n}_c = \bar{n}_T - \bar{n}_b\). Such a procedure was not available for these measurements, though \(c_0\) can provide a lower bound on \(\bar{n}_b\).

For a single profile, still only the combination \(s_{\infty}\bar{n}_c\) is available. The following examples are profiles that will share the same \(\bar{n}_i\) and \(\Gamma_{\text{loss}}\), but have varying \(\Gamma_{\text{pump}}\). Using \(\tau_{\text{pump}}\) and \(s_{\infty}\) as a function of \(\Gamma_{\text{loss}}\) and \(\Gamma_{\text{pump}}\) relates them in a way that if a fit is made to all profiles simultaneously, the \(\bar{n}_i\), \(s_{\infty}\), \(\Gamma_{\text{loss}}\) and the various \(\Gamma_{\text{pump}}\) can be determined independently.

Figure 3 shows \(n_0\) as a function of \(t_{\text{pump}}\) for various 2GHz EOM powers. This data was taken with the same experimental parameters, most importantly the cooling and background rates, except for the pump time and EOM oscillator amplitude. An \(n_0\) block using each of two EOM powers is included in each trial and the pump time was changed from trial to trial to better ensure that possible variations of any other parameters affected all cases in the same way.

The entire sequence used in this measurement was \(c_1/n_0(t_{\text{pump}}, P_{\text{EOM}}^{(1)})/n_0(t_{\text{pump}}, P_{\text{EOM}}^{(2)})/n_1/c_0\), where \(t_{\text{probe}}\) was randomly chosen from the set \{1, 2, 5, 10, 20, 50\}ms before each set of trials, used for 10 trials, and then re-selected for the next set of trials. The time dependence shows the expected exponential behavior with rates and asymptotic values determined by \(\Gamma_{\text{pump}}\) and \(\Gamma_{\text{loss}}\).

With the instrumentation then at hand, only two EOM powers could be used during a particular set of trials constituting a single measurement. So two measurements were made using different pairs of EOM powers, \{-35, -25\} dBm and \{-30, -20\} dBm. The stability and consistency of \(n_1\) and \(c_1\) between these two runs as seen in the strip chart (not shown) is a good justification for considering the cooling and probe rates to be the same between the different sets of EOM power pairs so that
the fits can be made to all profiles simultaneously allowing only $\Gamma_{\text{pump}}$ to change. Thus $\bar{n}_e$, $\bar{n}_b$, $\Gamma_{\text{loss}}$ and $\Gamma_{\text{pump}}(P_{\text{EOM}})$, and the derived $s_\infty$ and $\tau$, can all be determined independently. This example exhibits a range of $\Gamma_{\text{pump}}$ from about $40 - 130$ Hz and $\Gamma_{\text{loss}} \approx 50$ Hz. With $\Gamma_{\text{decay}} \approx 20$ Hz this provides a measure of $\Gamma_{\text{off} - \text{resonant}}^{935} \approx 30$ Hz. Note also that $\bar{n}_T \approx n_1$, and $\bar{n}_b \approx c_0$ as expected.

![Figure 5: Number of probe counts as a function of pump time for various 2GHz EOM powers, and the resulting pump rate as a function of that power. Error bars on the left are determined from the standard deviation of the probe counts and are smaller than the plot symbols used.](image)

### 3.2 Pump Efficiency and $P_{\text{EOM}}$ Dependence

With $\bar{n}_b$ and $\bar{n}_e$ determined the actual $s_{\text{pump}}$ can be estimated. These values are used to set the scale on the right-hand axis of figure 5a and show a maximal $s_\infty$ of 0.72 for the highest pump rate. Here optimizing $s_{\text{pump}}$ is simply a matter of maximizing $\Gamma_{\text{pump}}/\Gamma_{\text{loss}}$ as that both maximizes $s_\infty$ and reduces $\tau_{\text{pump}}$ allowing for a shorter pump time, a shorter trial time and more trials for a given total observation time $T$. This is mostly a matter of increasing the 2GHz EOM as much as is possible and practical. Figure 5d shows $\Gamma_{\text{pump}}$ as a function of $P_{\text{EOM}}$ fitted to a model that assumes that the $\Gamma_{\text{EOM}}$ contribution to $\Gamma_{\text{pump}}$ is proportional to the EOM side-band amplitude $\Gamma_{\text{pump}} = a \sqrt{10^{P_{\text{EOM}}/10}}$ for $P_{\text{EOM}}$ measured in dBm, $\Gamma_{\text{pump}} = \Gamma_{\text{off} - \text{resonant}}^{935} + a 10^{P_{\text{EOM}}/10}$. This provides the $\Gamma_{\text{off} - \text{resonant}}^{935} \approx 25$ Hz referred to previously.

The total $\Gamma_{\text{pump}} \approx 130$ Hz turns out to be about the largest possible in practice. Power in the side-bands comes at the expense of power in the fundamental. When the fundamental power is lowered beyond that which saturates the cooling transition that overall rate drops and that loss soon becomes greater than the increases gained from $\Gamma_{\text{EOM}}$. At $P_{\text{EOM}} = -5$ dBm $s_{\text{pump}}$ is already reduced to 0.5. Subsequent measurements in the $D$ state then use an approximately optimal $P_{\text{EOM}} = -20$ dBm $\rightarrow -15$ dBm. This gives $\tau_{\text{pump}} \lesssim 5$ ms so $t_{\text{pump}} = 15 \rightarrow 20$ ms is used.

### 3.3 $\Gamma_{\text{loss}}$ Dependence

$s_{\text{pump}}^{\infty}$ might be further increased by reducing $\Gamma_{\text{off} - \text{resonant}}^{935}$ and through it $\Gamma_{\text{loss}}$ by reducing the 935nm laser power during the pump. The 935nm laser is switched by using the first deflected beam after passing the beam through an acousto-optic modulator (AOM), so the laser power is essentially controlled by the AOM's RF power input. By applying reduced RF power to the AOM, the resulting switched laser power is reduced. $\Gamma_{\text{loss}}$ is ultimately limited by $\Gamma_{\text{decay}}$, so this could
increase $\Gamma_{\text{pump}}/\Gamma_{\text{loss}}$ by about a factor of two. But since we already have $\Gamma_{\text{pump}}/\Gamma_{\text{loss}} \gtrsim 2.5$ this would only improve $s_\infty^{\text{pump}}$ to about 0.85. It would also reduce the clean up rate which may result in a larger probability to be in the $5D_{3/2}(F = 1)$ level at the end of the probe and negate any gains from increased $\Gamma_{\text{pump}}/\Gamma_{\text{loss}}$. In practice there seems to be no benefit. A sequence $c_1/n_1(P^{(1)}_{935})/n_1(P^{(2)}_{935})/n_0/c_0$ was run with $F_{935}^{(2)} \approx 0.5F_{935}^{(1)}$ with no difference observed between the $n_1$ for both cases. Possibly in this case $\Gamma_{\text{decay}}$ already dominated, or the $5D_{3/2}(F = 1)$ state population became significant.

4 Probe Efficiency and Sensitivity

Off-resonant couplings and the finite lifetime of the $5D_{3/2}$ state also complicate probe sensitivity. A probed ion can be considered to be in either the shelved state, with probability $s$, or somewhere in the cooling cycle with the complementary probability $1 - s$. The probe will yield either $n_b$ or $n_T$ counts, respectively, with $n_i = r_i t_{\text{probe}}$ where $r_T = r_c + r_b$, $r_c$ and $r_b$ are the cooling and background rates as before, and $t_{\text{probe}}$ the probe time. The probe time must be long enough that enough photons are collected that statistics. To the probe time. In this case $t_{\text{probe}}$ can be made arbitrarily long, and $n_i$ can be made a large as necessary to allow the initial state to be determined with almost perfect accuracy.

4.1 Non-ideal Couplings and Finite Probe Time

For the $^{171}$Yb$^+$ $5D_{3/2}(F = 2)$ state, the same $\Gamma_{\text{off-resonant}}^{370}$ contributing to $\Gamma_{\text{pump}}$ will result in the ion being driven to the shelved state after long probe times thereby reducing the total number of probe photons collected for an ion beginning in the $S$ state resulting in $n_T \rightarrow \bar{n}_T < r_T t_{\text{probe}}$. Similarly $\Gamma_{\text{loss}}$ from $\Gamma_{\text{decay}}$ and $\Gamma_{\text{off-resonant}}^{935}$ result in an ion initially in the $D$ state to be driven to the $S$ state resulting in $n_b \rightarrow \bar{n}_b > r_b t_{\text{probe}}$. The probe beams effectively become a set of pump beams so that $s$ is determined by the exponential profile given before as a function of $t_{\text{probe}}$, but in this case with a time scale $\tau_{\text{probe}} = 1/\Gamma_{\text{probe}}$ and $\Gamma_{\text{probe}}$ now given by $\Gamma_{\text{pump}} = \Gamma_{\text{off-resonant}}^{370}$ and $\Gamma_{\text{loss}} = \Gamma_{\text{off-resonant}}^{935} + \Gamma_{\text{decay}}$. For very long probe times, $t_{\text{probe}} \gg \tau_{\text{probe}}$, the ion’s probability to be in the $D$ state is given by the corresponding steady state value $s_\infty^{\text{probe}} = \Gamma_{\text{pump}}/(\Gamma_{\text{pump}} + \Gamma_{\text{loss}})$ yielding a probe count of $n \approx n_T - s_\infty^{\text{probe}} n_c$ independent of the initial state of the ion.

All the information about the initial state of the ion was gathered during the beginning of the probe, during $t < \tau_{\text{probe}}$. Beyond this there will be only a fixed number of extra counts, $\Delta n$, for an ion beginning in the $S$ state compared to beginning in the $D$ state, independent of the probe time. Increasing the probe time increases the counts collected, but not $\Delta n$. For $t > \tau_{\text{probe}}$, as the counts increase so do the fluctuations in that count, $\sigma_n$. At some point $\sigma_n > \Delta n$ and the information about the initial state is effectively lost, or at best not improved while further increasing the probe time only increases the trial time and reduces the number of trials that can be made.

$\tau_{\text{probe}}$ is then effectively a probe coherence time. The actual probe time $t_{\text{probe}}$ must be made less than $\tau_{\text{probe}}$ for the probe to be sensitive to the initial state. This limits the ability to determine the state of an ion using a single trial. For the characteristic rates seen in the previous pumping data $\Gamma_{\text{probe}} \approx 75 \text{ Hz}$ so that $\tau_{\text{probe}} \approx 10 - 15 \text{ ms}$. For cooling rates around 2-4kcps, and background rates 500cps, a probe time of $t_{\text{probe}} = 5 \text{ ms}$ would give $n_c = 15$ and $n_b = 5$. Poisson distributions with
these means overlap considerably and in real experiments fluctuating rates, and especially multiple ions, further obscure the difference.

4.2 Probe Count Distribution

Figure 6 shows typical histograms of \( c_1 \), \( c_0 \), and the \( n_0 \) resulting from a probe for various \( s \) for a single ion. The first two are described very accurately by the expected Poisson distribution, indicating a stable cooling rate. The remaining should follow a bi-modal distribution\(^7\) given by,

\[
P_n(s) = (1 - \bar{s})p_n(n_T) + \bar{s}p_n(n_b)
\]

where

\[
\bar{s} = \gamma(s - s_\infty) + s_\infty
\]

\[
\gamma = \frac{1 - e^{-t_{\text{probe}}/\tau_{\text{probe}}}}{t_{\text{probe}}/\tau_{\text{probe}}}
\]

\( p_n(n_i) \) is a Poisson distribution with mean \( n_i \), and \( s_\infty \) corresponds to that given by the probe beams. Note that for \( t_{\text{probe}}/\tau_{\text{probe}} \rightarrow 0 \), \( \gamma \rightarrow 1 \) and \( \bar{s} \rightarrow s \). This explicitly includes the effects of all the non-ideal couplings that have been discussed. The data used here is from one set that is later used to determine the \( D \) state lifetime so the various \( s \) are generated by a variety of wait times after pumping. \( P_n(\bar{s}) \) is fit to all profiles collectively using a common \( n_c \) and \( n_b \) with a profile dependent \( \bar{s_i} \). \( n_T \) and \( n_b \) should be given by \( c_1 = 7.4 \) and \( c_0 = 1.3 \) respectively. The fits yield \( n_b = 1.4 \) but a slightly lower \( n_T = 6.5 \).

The models reproduce most of the general features of all the distributions including the variances. For a Poisson-distributed random variable the variance is given simply by \( \sigma^2 = n \). For this single ion bi-modal distribution there is an additional contribution\(^7\),

\[
\sigma^2 = n + \bar{s}(1 - \bar{s})n_c^2
\]

Figure 6d shows the measured widths, computed from the standard deviation of the data, as a function of \( \bar{s} \). The variances are clearly larger than for a Poisson distribution. To compare to the bi-modal distribution width, uncertainties in the width are estimated by regarding the standard deviation as any other function and propagating fluctuations in \( n \) through, giving simply \( \sigma_\sigma = \sigma/\sqrt{\text{trials}} \). The predicted error also generally follows the measured standard deviation but misses the precise details.

For \( N_{\text{ions}} > 1 \) it turns out\(^7\) that

\[
\sigma^2 = n + \bar{s}(1 - \bar{s})n_c^2
\]

and the distribution gradually converges to a Poisson. Figure 7 shows a similar series of probe distributions for various \( s \) for cases with \( N_{\text{ions}} = 20 \) - 30 ions with the Poisson distribution that corresponds to their means. The distributions are clearly narrower, though in this case still slightly larger than a Poisson. The prediction for the width is not as accurate as before. This is likely mostly due to an inaccurate estimate of \( n_T \) or \( n_b \) and the resulting \( \bar{s} \), or the number of ions.

The remaining differences between the observations and predicted widths and distributions for this and the \( N = 1 \) differences might be due to probe rate fluctuations, but a fit using a convolution
Figure 6: Histograms of the counts collected for various probes. a) shows the cooling and background rates and the simple Poisson distributions that they are expected to follow with parameters determined directly from the mean of the distributions. b) are distributions from D2Probe with different initial probabilities $s$ to be in the $5D_{3/2} (F = 2)$ state. The solid lines are from the expected bi-modal distributions with parameters determined by fits to the histograms. $c_1$ and $c_0$ are shared between all, and $\bar{s}$ is allowed to vary for each case. c) shows $\bar{s}(s)$ using the $\bar{s}$ determined from the histogram fits and $s$ from a fit to the lifetime profile that this data was intended to measure. d) shows the actual variation of the probe counts for each $\bar{s}$ as determined from the standard deviation of the probe counts along with that expected for both a Poisson distribution, and a bi-modal distribution each with parameters as determined from the data. The uncertainties of the widths, as determined from the data are smaller than the plot points.

of $p_n$ over a uniformly distributed variation of $n_c$ over $\pm 20\%$ does not qualitatively improve the results and that range is already more than can be justified in light of the stability of $c_1$. The discrepancies may also be due to the assumption of simple first order rate equations being invalid due to non-negligible populations in disregarded intermediate states. Further attempts to account for these disparities have not yet been made.

For present purposes note only that the probe count histograms indicate that there is no well-defined distinction between the results of probing a shelved or un-shelved ion, even for a single ion, so the results of a single probe can not be used to determine the initial state. Again, the mean is used instead, 

$$n(t_{\text{probe}}) \equiv \langle n \rangle_{p_n(s)} = (1 - \bar{s})n_T + \bar{s}n_b = n_T - \bar{s}n_c$$
Which can be written \( n(t_{\text{probe}}) = \bar{n}_T - s\bar{n}_c \) as before with

\[
\begin{align*}
\bar{n}_b &= n_b - n_c(1 - s_\infty^{\text{probe}})(1 - \gamma) \\
\bar{n}_c &= n_c\gamma
\end{align*}
\]

Note that \( \gamma = \bar{n}_c/n_c \) is a direct proportional measure of how much of the difference between cooling and background rates is traversed by \( n \) over the range of \( 0 < s < 1 \). This makes \( \gamma \) a measure of probe efficiency. For \( t_{\text{probe}}/\tau_{\text{probe}} \to 0 \) these reduce to, \( \bar{n}_i \to n_i \). For \( t_{\text{probe}} \gg \tau, \bar{n}_c \to 0 \) and both \( \bar{n}_T, \bar{n}_b \to n_b - n_c(1 - s_\infty^{\text{probe}}) \) and the resulting probe count becomes insensitive to the initial state.

\( n(t_{\text{probe}}) \) is a linear function of \( s \) and so can be used to reliably determine at least the dependence of \( s \) on the wait time \( t_{\text{wait}} \) for this measurement, and even \( s \) itself if \( \bar{n}_c \) and \( \bar{n}_b \) can also be determined. For the lifetime data used in the previous histograms \( \bar{n}_T \) can be determined from the decay profile to be \( \bar{n}_T = 6.25 \pm 0.02 \). As usual, \( \bar{n}_b \) can not be determined directly, but it might be estimated by \( \bar{n}_b \approx c_0 + (c_1 - \bar{n}_T) \). The \( s_{\text{start}} \) can then be determined from \( s_{\text{start}} = (\bar{n}_T - n_{\text{start}})/n_c \). The \( s_i \) are then determined from the histogram fits and the resulting \( \bar{s}(s) \) fit to get \( \gamma \) and \( s_\infty^{\text{pump}} \). These can then be used to get a new estimate for \( \bar{n}_b \) and the fit repeated. This procedure quickly converges to \( \bar{n}_b = 2.54, \gamma = 0.60, s_\infty = 0.34 \). The resulting final \( \bar{s}(s) \) and fit are shown in figure 6b. A direct calculation from the \( n_i \) and \( n_s \) give \( \gamma = \bar{n}_c/n_c = 0.73, s_\infty^{\text{pump}} = 0.20 \) which should be, but are not particularly consistent with the values obtained from the fit to \( \bar{s}(s) \), suggesting some further lack of understanding. Either result gives \( \tau_{\text{probe}} \approx 1.2 t_{\text{probe}} \). Here the probe time was 5 ms so that \( \tau_{\text{probe}} \approx 6 \) ms.

### 4.3 Probe Profile and Sensitivity

With the probe distribution and mean understood the \( t_{\text{probe}} \) dependence of the parameters can be investigated. Figure 8 shows the results of \( n_0(t_{\text{probe}}) \) for various initial \( s \) generated by varying the pump times so that \( s = s_{\text{pump}}(t_{\text{pump}}) \). The complete sequence is \( c_1/c_1a/n_1(t_{\text{probe}})/n_0(t_{\text{pump}}, t_{\text{probe}})/c_0 \) where as usual the times are chosen randomly from \( t_{\text{pump}} = \{1, 2, 5, 10\} \) ms and \( t_{\text{probe}} = \{1, 2, 5, 10, 20, 50, 100, 200\} \) ms.
Three fits are shown, the first is an individual fit to the profile for each probe time to n as a function of \(t_{\text{probe}}\) where the \(\bar{n}_i\) are now regarded as functions of \(t_{\text{probe}}\)

\[
n(t_{\text{probe}}) = \bar{n}_T(t_{\text{probe}}) - s\bar{n}_c(t_{\text{probe}})
\]

This is implicitly a function of \(n_1, s_0, s_\infty\) and \(\tau_{\text{probe}}\), but the form again reduces to a three parameter exponential. \(c_1\) and \(c_0\) can be used to determine \(n_T\) and \(n_b\) respectively, and the remaining quantities derived from these and the fit parameters.

These fits are qualitatively good, but still show apparent statistically significant systematic deviations from the expected behavior, especially for short probe times. The resulting fit parameters are fairly consistent giving \(s_\infty \approx 0.9\), \(\bar{n}_b \gtrsim c_0\), and \(\tau_{\text{probe}} = 45 - 60\) ms approximately less then \(\tau_{\text{decay}}\), though not by as much as expected, and with the exception of \(\tau_{10\text{ms}} \approx 200\) ms.

The short time fit can be improved by including one more imperfection. The fit data are the effective rates, the probe counts divided by the probe time, rather than the probe counts directly. This requires that the actual real probe time is known. While the gates that direct the beams states can be timed to better than \(\mu s\), shutter lags and gate delays can result in the actually exposure time during the probe to be different. Suppose that the difference is \(t_0\) independent of \(t_{\text{probe}}\). Then the counts should be normalized by \(t_{\text{probe}} + t_0\) rather than just \(t_0\), so the effective count rate is wrong by a factor \(t_{\text{probe}}/(t_{\text{probe}} + t_0)\). This correction should be included with \(t_{\text{probe}}\) in the exponentials that appear in \(\gamma\) as well, though they will just effectively modify \(\bar{n}_c\) and \(\bar{n}_b\) and not give qualitatively different functional behavior.

Alternately the \(\bar{n}_T\) as measured by \(c_1\) may not be accurate. \(c_1\) is measured during a cooling period when all cooling beams have been on for a while, while the \(n_0\) probe starts with all beams off and then the beam blocks and PMT gates are all switched simultaneously. Again, incorrectly calibrated shutter lags may result in the beams being on shorter, or longer, than intended so that \(c_1\) is not equal to the maximum possible \(n_1\) by a similar factor. In this case \(c_1\) and \(c_0\) should be modified by this factor, but not \(t_{\text{probe}}\). The result is the same functional form, but the resulting \(s_1\) will be slightly different. In this case \(t_0\) turns out to be small enough that these choices are indistinguishable.

With this modification most of the short pump probe time behavior matches precisely with a reasonable \(t_0 \approx 0.1\) ms, though there are still statistically significant variations from the model for longer probe times, and the fit parameters are even more inconsistent. Two of the \(\tau\) yield a much closer to expectations 20 ms, but the two others are unreasonably large. Similarly \(s_\infty\) probe \(\approx 0.9\) except for \(t_{\text{pump}} = 5\) ms where it is greater than 1.

A collective fit to all profiles simultaneously allowing only \(s\) to vary with \(t_{\text{probe}}\), makes matters worse. In this case the fit is poor for almost all probe times, and \(\tau_{\text{probe}}\) is an unreasonably long 80 ms. A pump time analysis could be done to independently determine \(\Gamma_{\text{loss}}\) and \(\Gamma_{\text{pump}}\) and compare the resulting \(s_{\text{pump}}\) to the \(s\) observed from the probe time fits. That can’t be done with this particular data though it was taken with different pump times, because the pump stage uses the 2 GHz EOM and a full power 935 nm laser, while the probe uses the complement, thus providing \(\Gamma_{\text{pump}} = \Gamma_{\text{off - resonant}} + \Gamma_{\text{EOM}}\), and \(\Gamma_{\text{loss}} = \Gamma_{\text{decay}} + \Gamma_{\text{off - resonant}}\) rather than the \(\Gamma_{\text{pump}} = \Gamma_{\text{off - resonant}}\) and \(\Gamma_{\text{loss}} = \Gamma_{\text{decay}} + \Gamma_{\text{off - resonant}}\) required here. Similarly the \(n_0(t_{\text{pump}}, t_{\text{probe}})\) count distributions could be used to determine \(s\) in each case.

Such efforts may highlight an important discrepancy, but the results would not yield a better fit and the model’s functional dependence is clearly wrong. It is not immediately clear how to account for the disparity. Non-negligible intermediate state populations may again be a factor. Such consideration have not yet been pursued as they are becoming too far removed from the
Figure 8: Average probe counts per time as a function of probe time for various initial $s$. The dashed lines are individual fits to each profile assuming $t_0 = 0$, the dotted lines are the same but allowing $t_0 \neq 0$. The solid lines are the best fit to all profile simultaneously, taking $t_0 = 0$, all sharing the same $\tau$ and $s^\infty_{\text{probe}}$. The table shows the fit parameters corresponding to each case.

4.4 Optimal Probe

Though the model doesn’t precisely predict the details, the data is sufficient to confirm an approximate probe coherence time of $\tau_p \approx 10 - 20$ ms. As discussed previously, a probe time much longer than this loses sensitivity because the probe count rate becomes independent of the initial state of the ion. Conversely, a short probe time has poor sensitivity because of low counting statistics. An intermediate time will minimize the uncertainty in determining $s$. The $\sigma_s$ given by the kind of multi-modal count distributions exhibited here turns out to result in an approximately minimal $\sigma_s$ at $t_{\text{probe}} \approx \tau_{\text{probe}}/2$ [7]. With the preceding determination of $\tau_{\text{probe}} \approx 10 - 15$ ms, an optimal probe for this system would have $t_p = 5 - 10$ ms. At this point $\sigma_s$ is given by

$$
\sigma_{Ns} = \sigma_s \sqrt{\frac{t_0}{T}}
$$

$$
\sigma_s \approx 1.27(2a_{\text{max}} + 1)^{1/2}/\sqrt{4N_{\text{ions}}}
$$

$$
a_{\text{max}} = \frac{r_T}{r_c^2 \tau_{\text{probe}}} 4N_{\text{ions}}
$$

For the parameters determined in the previous example $r_T \approx 6.52/\text{ms}$, $r_b \approx 1.43/\text{ms}$, give $a_{\text{max}} \approx 0.05$ corresponding to fairly good counting statistics. For $t_0 \approx 135$ ms this gives $\sqrt{t_0} =
\[ 6 \times 10^{-3} \sqrt{\text{hr}}, \sigma_s = 0.66 \text{ and an overall sensitivity of order } \]
\[ \sigma_{Ns} \sim 4 \times 10^{-3}/\sqrt{T/\text{hr}} \]

about 0.4% \sqrt{\text{hr}}.

### 4.5 935nm Laser Power

\(\tau_{\text{probe}}\) depends on the 935nm laser power through \(\Gamma_{\text{loss}} = \Gamma_{\text{decay}} + \Gamma_{\text{off-resonant}}\). Data presented previously indicated \(\Gamma_{\text{off-resonant}} \sim 20 \to 30 \text{ Hz}\). \(\tau_{\text{probe}}\) could be increased, and \(\sigma_s\) reduced, by reducing \(\Gamma_{\text{off-resonant}}\) with reduced 935nm laser power. This will improve the sensitivity as long as the resonant 935nm transition remains saturated. If 935nm intensity is reduced below saturation then \(r_c\) drops as well, but linearly compared to \(\sqrt{\tau_{\text{probe}}}\) which then increases \(\sigma_s\). When a further decreasing \(\Gamma_{\text{off-resonant}}\) then becomes much less than \(\Gamma_{\text{decay}}\), \(\tau_{\text{probe}}\) doesn’t increase as quickly and \(\sigma_s\) increases even more quickly with lower 935nm laser power. So gains in sensitivity may be realized by reducing 935nm laser power to the point that the resonant transition is just saturated, though that gain may be modest if \(\Gamma_{\text{loss}}\) is already dominated by \(\Gamma_{\text{decay}}\).

As when discussing pump efficiency, the 935nm laser power can be controlled by changing the switch AOM’s RF drive amplitude. Figure 9 shows the results of a sequence that includes the block \(n_0(P_{935}^1)\) and \(n_0(P_{935}^2)\), with \(P_{935}^2 \approx 0.5 P_{935}^1\). The probe using the higher power does indicate a slightly shorter \(\tau_{\text{probe}}\), but also a slight higher \(r_c\) so that in this case there is little difference in sensitivity between the two.

In other cases modest improvements have been seen and so are sometimes used. Generally the reduced power is set by observing the cooling rate and reducing 935nm laser power until this rate just starts to decrease. When the reduced probe rate is used a \(c_{1a}\) block is also included which measures the cooling rate with the reduced 935nm laser power to provide \(n_c\). In all the examples presented here, when this is the case, it is implied that \(c_1\) refers to this attenuated result.

### 4.6 General Considerations

The most important factor affecting probe sensitivity is \(\tau_{\text{probe}}\). Using \(t_{\text{probe}}\) much different than \(\tau_{\text{probe}}/2\) significantly reduces sensitivity. The rates that determine \(\tau_{\text{probe}}\) are very stable during a particular run, but are not easily controlled and require some work to determine. They are always of the same general size, so for new experiments typical values of \(t_{\text{probe}} = 10 - 15 \text{ ms}\) are used and rates are only remeasured to provide better \(t_{\text{probe}}\) if \(\gamma\) becomes less than about 0.3. \(\gamma\) can be determined from \(\bar{n}_c/n_c, \bar{n}_T, n_c,\) and \(n_b\) can all be determined directly from \(n_1, c_1\) and \(c_0\) respectively, but \(\bar{n}_b\) can’t currently be measured directly due to the difficulty in preparing a state with \(s = 1\) using the pumping procedure discussed, but it should be about as close to \(c_0\) as \(\bar{n}_c\) is to \(c_1\), depending on what \(s_{\infty}\) for the probe turns out to be. So knowing \(n_1\) and \(c_1\) provides enough information to estimate \(\gamma\).

There remain some uncertainties about the details of the probe dynamics, in particular the profile of \(n_0\) as a function of \(t_{\text{probe}}\), and its distribution and resulting \(\bar{s}(s)\), each of which fails to precisely match the statistical model, but for a fixed probe time it appears justified to consider the results of a probe to be very accurately linearly related to \(s\).
Figure 9: Probe profile for two different 935nm laser powers, $P_1 \approx 2P_2$.

5 $^{171}$Yb$^+ 5D_{3/2}(F = 2)$ Lifetime

With these details about the pump and probe steps determined and optimized, a lifetime measurement becomes completely straightforward. After the pump step the ion is in the $5D_{3/2}(F = 2)$ state with probability $s_{\text{pump}}$. At time $t_{\text{wait}}$ after the end of the pump step that probability becomes $s_{\text{pump}}e^{-t_{\text{wait}}/\tau_D}$. A probe then yields an average count of

$$n(t_{\text{wait}}) = \bar{n}_T - s_{\text{pump}}\bar{n}_c e^{-t_{\text{wait}}/\tau_D}$$

$$= \bar{n}_T - \bar{n}_c e^{-t_{\text{wait}}/\tau_D}$$

The sequence block $d_0 = D2\text{ Pump / Off}(t_{\text{wait}})/D2\text{ Probe}$ yields this particular $n(t_{\text{wait}})$.

5.1 Sensitivity and Optimal Sampling

Pump and probe parameters can be chosen to minimize $\sigma_s$, but the uncertainty, $\sigma_{\tau}$, in determining $\tau$ from the results of $n(t_{\text{wait}})$, can also be affected by the choice of wait times $t_i$ and the relative frequencies with which they are used. Some sets of wait times can give an $n$ that is more sensitive to $\tau_D$ than others. Since the $\bar{n}_i$ cannot be determined independently, they must be also determined from the data along with the lifetime. There are three effective parameters in $n$ so data from trials with at least three different wait times must be used.

19
The uncertainty in the $\tau_D$ determined in this way can be written in the same general form as used previously:

$$\sigma_\tau = f_\tau \tau_D \sigma_s = f_\tau \tau_D \sigma_s \sqrt{\frac{t_{\text{trial}}}{T}}$$

The trial time $t_{\text{trial}}$ will depend on the wait time used for a particular trial, but will be of order $\tau_D$. Grouping the exact trial time with $f_\tau$ gives

$$\sigma_\tau = \bar{f}_\tau \tau_D \sigma_s \sqrt{T/\text{hour}}$$

The previously determined parameters $\sigma_s \approx 0.66$ and $\tau_D \approx 60 \text{ ms}$ then gives

$$\sigma_\tau = \bar{f}_\tau \cdot 0.16 \text{ ms} \sqrt{T/\text{hour}}$$

A general optimal sampling analysis gives $\bar{f}_\tau$ as a function of a chosen set of wait times $t_i$ and the fraction of the total number of trials that each particular wait time is used, $f_i$. These $t_i$ and $f_i$ can then be chosen to minimize $\bar{f}_\tau$. The optimal results depend implicitly on other details of the measurement sequence through the total trial time. In practice the trial time will be given by the wait time plus some fixed overhead $t_0$ that includes pump and probe times, between trial cooling times, and the other fixed time blocks described previously that are used for calibration and stability, giving $t_{\text{trial}} = t_0 + t_{\text{wait}}$. For these measurements $t_0 \approx 120 \text{ ms} \approx 2\tau_D$.

For the uniformly sampled case $f_i = 1/3$ the optimal result for the parameters of this system turns out to be

$$t_{i_{\text{optimal}}} = \{0.89, 5.16\} \tau_D$$

$$\bar{f}_\tau \lesssim 12.3$$

This can be improved slightly by allowing arbitrary $f_i$ and the best optimal sampling set is

$$t_{i_{\text{optimal}}} = \{0.80, 5.69\} \tau_D$$

$$f = \{0.28, 0.54, 0.18\}$$

$$\bar{f}_\tau \lesssim 11.0$$

where $t = 0.72\tau_D$ is sampled more often than the other cases.

Though only three times are strictly required, such a set would give no extra information that could be used to check for, or correct for systematic errors, such as parasitic couplings to the $5D_{3/2}(F = 2)$ that result in a non-exponential decay profile. With only three sample times such a deviation could never be detected. To check for such things a more than minimal set of wait times is sampled that (almost) covers the overall range of times indicated by the optimal set. In all the data presented this set is $t_i = \{0, 5, 10, 20, 50, 100, 200\}$. In some of the data there are also trials of $d_1$ which is a block like $d_0$ but beginning with an SPump step. As a result the trial time becomes $t_{\text{trial}} = t_0 + 2t_{\text{wait}}$, $\bar{f}_\tau \rightarrow 19.53$ and gives

$$\sigma_\tau = \frac{4.5 \text{ ms}}{\sqrt{T/\text{hour}}}$$
Such a sensitivity is sufficient for this measurement but could be improved. First the $d_1$ block can be eliminated, giving again $t_{\text{trial}} = t_0 + t$, and $f_\tau \rightarrow 17.2$. Similarly a typical measurement also includes the usual $c_1$, $c_{1a}$, $n_0$, $n_1$, $c_0$. This provides useful complementary information for stabilization and systematics, but could be sampled less often, or omitted if the effects they are used for can be eliminated or determined by other means. These extra blocks account for most of the non-$t_{\text{wait}}$ trial time. A $d_1$-block-only sequence would have $t_0 \approx 10\text{ ms} = 0.17\tau_D$ which would further reduce $f_\tau \rightarrow 9.8$, improving sensitivity by about a factor of 2 over the currently used measurement scheme.

A more important factor is $n_c$, which directly affects $\sigma_s$. Increasing $n_c$ by a factor of two decreases $\sigma_s$ by the same factor. The cooling transition is saturated, so $n_c$ is determined by PMT detection efficiency and the number of ions. PMT detection efficiency is given mostly by solid angle and PMT quantum efficiency. Neither is easily improved, but for future measurements it could be worth the effort if a factor of a few could be gained. With the present system the number of trapped ions could be increased, and is for a few cases. But it is not known if multiple ions could affect this $D$ state lifetime, so most data is taken with a single ion.

5.2 Lifetime

Figure 10a shows $d_1(t_{\text{wait}})$ for data already presented in the optimal probe analysis. The $n_i$ and $c_i$ are independent of $t_{\text{wait}}$ indicating a very stable system. The error derived from the standard deviation of the data collected are smaller than the plot points used in the profiles, but apparent in the residuals. The profile fits an exponential decay very precisely. The fit residuals exhibit no systematic variation from the functional form of the fit model. This particular data consists of 46690 trials taken over the course of 2.4 hours. The uncertainty of 4 ms indicates a sensitivity of $5.7\text{ ms}/\sqrt{T/\text{hr}}$, similar to the estimate for the sensitivity given above, indicating that the statistics are generally well understood and that all the possible sources of variation are identified and accounted for.

Figure 10b shows a second example for the case of the longest run achieved, and so having the best statistics. This data consists of 165150 trials taken over the course of 13.1 hours. The uncertainty of 2 ms also gives a similar sensitivity of $6.5\text{ ms}/\sqrt{T/\text{hr}}$, which in this case is slightly larger than the $5.0\text{ ms}/\sqrt{T/\text{hr}}$ that might be expected using the same considerations as before.

Both of these fits individually are very good, but their results are not completely consistent with each other, each lying about $1.1\sigma$ away from their weighted average. This is not an unreasonable disparity, but larger than expected, especially given the otherwise very good quality of the statistics, and more data shows even larger variations. Figure 11a shows the results from all the lifetime data collected for this measurement including 25 separate runs totaling over $10^6$ trials.

These give a weighted mean of $\bar{\tau} = 61.8 \pm 0.6\text{ ms}$. Figure 11b shows the fit $\tau$ uncertainty as a function of the total measurement time for a particular run. The best fit to $\sigma = a/\sqrt{T}$ gives a sensitivity of $4.8\text{ ms}/\sqrt{T/\text{hour}}$ which is in very good agreement with the statistical analysis.
Figure 10: Decay profiles from two separate data sets with fit residuals.

Figure 11: Lifetime and uncertainty for all lifetime data sets with weighted mean, and error as a function of observation time and fit to $\sigma \propto 1/\sqrt{T}$

5.3 Systematics

A number of sets exhibit a significant deviation from this mean. An average of the relative variations from the mean gives

$$\chi^2 = \frac{1}{N} \sum_i \left( \frac{\tau_i - \bar{\tau}}{\sigma_i} \right)^2$$

$$\chi = 2.0$$

Again, not completely unreasonable but large, and some possible systematic errors should be considered.

One possibility might be residual couplings from the cooling lasers while they are nominally off during the wait time that give small extra rates of excitation to or losses from the $5D_{3/2}(F = 2)$ state. These rates have already been considered in the pump and probe analysis and also result in an exactly exponential decay profile with particular rates. Since these rates have been seen to
vary between measurements, they may then give a shift to the lifetime that is arbitrary between measurements but stable during a single measurement. An estimation of their possible size seems to preclude this possibility. The general sizes of these rates have already been determined to be on the order of $20 - 30$ Hz when the beams are on. The 370nm laser is switched by a mechanical shutter that completely blocks the beam when off so no residual rate extending the lifetime should be expected.

To confirm this, some data was taken with a $d_1 = SP_{\text{ump} / \text{Off}(t_{\text{wait}})} / D_{\text{Probe}}$ block. This block begins with the ion in the $S$ state. Leaking 370 nm light would excite the ion to the $D$ state and the $d_1(t_{\text{wait}})$ profile should show a decaying time dependent probe count with the same time constant as the $d_0$ profile. The asymptotic value of this decay gives the ratio of this possible parasitic excitation rate to the total rate. This sort of measurement indicates that this possible spurious excitation rate is statistically consistent with zero and at worst can be no more than a few 0.1% of the decay rate.

The 935nm laser is switched by an AOM that is known to be imperfect but still provides at least 30 dB contrast between states. This would allow for an extra loss rate of $\lesssim 20 - 30$ mHz, shortening the lifetime an undetectable 0.1 ms and not enough to account for the apparent systematic variation.

Pressure effects should similarly be negligible. Collisional quenching has been determined to be on the order of $10^7$ Hz/torr\textsuperscript{3}. These experiments were all done with pressures in the low $10^{-11}$ torr range giving possible reductions of the lifetime on the order of a few mHz relative to the order 1/50ms=20Hz radiative decay lifetime, which would also be undetectable.

Other possibilities include the number of trapped ions and their temperature. The mechanism is not clear, but coulomb interactions between the trapped ions, or details of the micro-motion or secular motion of the ion, or even super-radiance might individually or in combination provide some coupling to the ground state that reduces the lifetime. Figure 12 shows the derived lifetime from each data set as a function of the number of ions in the trap during that run or the cooling rate $c_1$ as a measure of temperature.

Figure 12a shows a possible dependence on the number of ions of $-2.3$ ms/Ion, but it is barely statistically significant, and the variations within results for the same number of ions are just as large as the overall variation. $\chi$ improves insignificantly to 1.9. Figure 12b shows no dependence on cooling rate.

These variations might still be statistical variations and disappear with more data, but it is just as likely that there is a real systematic variation between runs. If there is such a shift it appears to be stable during a single run as sensitivities are completely consistent with statistics, so it would have to be something that changes when a new ion is loaded. Candidates for such an effect beyond
those few already seemingly ruled out are not apparent. The variation remains with a size estimated by

\[ \sigma^2_{\text{sys}} = \frac{\sum_i \left( \bar{\tau}_i - \bar{\tau} \right)^2}{\sum_i \sigma_i^2} \]

and gives

\[ \sigma_{\text{sys}} = 6.4 \text{ ms} \]

\[ \tau = 61.8 \text{ ms} \pm (0.6)_{\text{stat}} \pm (6.4)_{\text{sys}} \]

6 F State Shelving

Detection sensitivity is somewhat limited by the shelved state lifetime which is a modest \( \sim 50 \text{ ms} \), rather shorter than the 100s+ seen in some states in other alkali-like systems. \(^{171}\) Yb\(^+\) has a \( 5F_{7/2} \) state with a lifetime in excess of 6 years (figure 1), which would be an enormous improvement if it could be used as a shelved state.

This \( F \) state has an even lower energy than the \( 5D \) state, but various combinations of angular momentum and parity selection rules, and small energy differences prevent a quick transition to it from either the \( 5D \) state or the \( 6P \) state. It is believed that transitions to it have been observed through collisional couplings to an intermediate state in systems with higher pressures\(^{10}\). In such cases a 638nm laser is used to clean out the \( F \) state through an intermediate \( ^1[5/2]_{5/2} \) state.

Driving shelving transitions to this \( F \) state can be done in a similar way. Direct transitions from the ground state would be very difficult, and slow given they very small coupling indicated by the lifetime. A 410nm laser can drive a transition from the \( 5D \) state to a different intermediate \( ^1[5/2]_{5/2} \) state that decays via an E2 transition quickly and principally to the \( F \) state.

A 410nm diode laser was built for this purpose and when applied to the ion yielded the expected results. While monitoring the cooling signal, the 410nm laser is applied and the cooling signal is observed to disappear almost immediately suggesting a successful transition to the \( F \) state. The long \( F \) state lifetime prevents seeing the radiative decay, but the cooling signal is immediately restored when the 638nm laser is applied which drives the ion back into its cooling cycle.

Since this \( F \) state is very weakly coupled to any part of the cooling cycle, the resulting \( \tau_{\text{probe}} \) should be determined completely by the \( F \) state lifetime, \( \tau_F \). This gives \( \sigma_{\text{max}} \rightarrow 0 \) and \( \sigma_s \) decreases from 0.66 to 0.5its smallest possible value corresponding to binomial statistics, a modest 25% improvement. Such effort for this gain would probably not be justified for a lifetime measurement which is of limited interest. But the improvement would be welcome in a parity non-conservation experiment that is statistics-limited where this would correspond directly to a 25% improvement in precision. It would also allow for a threshold probe that is less sensitive to fluctuating experimental parameters. More importantly \( F \) state shelving would also make it practical to do these kinds of measurements in the isotopes of Yb with zero nuclear spin where the lack of hyperfine structure prevents the use of the measurement scheme presented in this article.

7 Conclusion

Characterization of the probe coherence times, and careful choices for probe times give a probe sensitivity close to the maximal possible for an ideal system even for the fairly poor counting
statistics exhibited here. The specific results shown here for the $D$ state, and the same methods applied to the $S$ state would allow for significantly improved sensitivity for measuring the resonance transition frequencies and lightshifts needed for an Atomic Parity Violation experiment.

For some kinds of measurements, sensitivity to determining a desired quantity is also affected by pump efficiency and the choice of drive parameters. Sensitivity can be maximized by characterizing pump times and by using optimal sampling to determine and optimal set of drive parameters.

Applying these methods to the $D$ state lifetime gives

$$\tau = 61.8 \text{ ms} \pm (0.6)_{\text{stat}} \pm (6.4)_{\text{sys}}$$

significantly longer than previous measurements and with about 15 times better statistical precision. The sensitivity this represents of $4.5 \text{ ms/}\text{hour}$ is also likely very good but this can not be directly compared to other experiments as total observation time is not commonly reported.

This kind of precision would be an excellent target to precise atomic structure calculations, but a larger systematic variation is exhibited that is very likely to have appeared in previous measurements as well. Though the source of this apparent systematic variation remains unidentified, the result for the lifetime is still clearly much longer than previous measurements. As systematic effects are most likely to reduce the lifetime it is possible that the these measurements were shorter due to the same effects and didn’t have sufficient sensitivity to resolve similar systematic variations.

In the case of [3], which finds $\tau = 52.7 \pm 2.4 \text{ ms}$ for $^{174}\text{Yb}$, the estimated systematic error here is larger than the $2.4 \text{ ms}$ quoted uncertainty, but this uncertainty appears to be underestimated. Using data from Figure 4 of [3] a weighted fit gives a similar $\tau = 52.4 \text{ ms}$. An estimate of the variance that neglects the errors of the individual data points gives $\sigma \approx 2 \text{ ms}$, also consistent with the published result, but including those errors in the variance estimate gives $\sigma \approx 9.5 \text{ ms}$. This is well outside the range of the estimated systematic variations, and so this previous measurement may include similar effects without having been able to resolve them. In that case many trials were made using a single wait time and experimental parameters could have changed while switching to different wait times.

If all possible effects are assumed to shorten the lifetime than the best estimate might more correctly taken to be the largest statistically significant value found $\tau > \tau_{\text{max}} = 71.4 \pm 1.8 \text{ ms}$. Which is approaching the largest calculated result of $74 \text{ ms}$[5].

$F$ state shelving would allow for similar state detection lifetime measurement sensitivity in isotopes with nuclear spin zero, and provide a direct comparison to previous results as well as testing for possible isotope dependence.

Acknowledgments. This work was supported by the Laboratory Directed Research and Development program at Los Alamos National Laboratory, operated by Los Alamos National Security, LLC for the NNSA U.S. Department of Energy under contract No. DE-AC52-06NA25396

8 References

1. N. Fortson
   “Possibility of measuring parity non-conservation with a single trapped atomic ion”
   19 April 1993, Physical Review Letters 70(16):2383-2386

2. J. Torgerson, M. Schacht, J. Zhang
“Measurement of Parity Violation with Single Yb+ Ions”
24 July 2010, Variations of Constants and Violations of Symmetries Workshop, Cairns

3. N. Yu and L. Maleki
“Lifetime measurements of the $4f^{14}5d$ metastable states in single ytterbium ions”
12 January 2000, Physical Review A 61(2):022507

4. B.C. Fawcett, M. Wilson
“Computer Oscillator Strengths, Lande g values, and lifetimes in Yb II”
March 1991
Atomic Data and Nuclear Data Tables 47(2):241-317

5. Ch. Gerz, J. Roths, F. Vedel, G. Werth
“Lifetime and collisional depopulation of the metastable 5D 3/2-state of Yb+”
1988
Zeitschrift für Physik D Atoms, Molecules and Clusters 8(3):235-237

6. W Nagourney, J Sandberg, H Dehmelt
“Shelved optical electron amplifier: Observation of quantum jumps”
30 June 1986, Physical Review Letters 56(26):2797-2799.

7. M. Schacht, M. Schauer
“Shelving and Probe Efficiency in Trapped Ion Experiments”

8. M. Schacht,
“Sensitivity and Optimal Sampling in Precision Experiments”

9. M M Schauer, J R Danielson, D Feldbaum, M S Rahaman, L.-B Wang, J Zhang, X Zhao, J R Torgerson
“Isotope-selective trapping of doubly charged Yb ions”
27 December 2010, Physical Review A 82(6):062518

10. M M Schauer, J R Danielson, A.-T Nguyen, L.-B Wang, X Zhao, J R Torgerson
“Collisional population transfer in trapped Yb + ions”
5 June 2009, Physical Review A 79(6):062705