IMPROVED CRYSTAL METHOD FOR PHOTON BEAM LINEAR
POLARIZATION MEASUREMENT AT HIGH ENERGIES

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Abstract

A method for photon linear polarization determination based on the measurement of the asymmetry of pairs produced by polarized photons in single crystals within the optimal intervals of pair particles energies is proposed. In difference to the well known methods the asymmetry in this case is essentially larger. The optimal orientation of crystal is found which provides the maximal values for analyzing power and figure of merit as well as minimal measurement time.

1. Introduction

At present there is a demand for production of linearly and still more of circularly polarized photons with energies $\omega \geq 20$ GeV because after the beautiful series of the SLAC [1] and few others photoproduction experiments carried out with linearly and circularly polarized photon beams at $\omega \leq 10$ GeV no published experimental data exist besides those obtained at CERN SPS on $\phi$- photoproduction with the help of linearly polarized photons with $\omega = 20$-75 GeV [2]. The SLAC beam was produced by Compton scattering of polarized laser photons on SLAC, 20 GeV electrons, and there was no necessity to measure its high polarization $P \approx 90\%$, while the polarization $P \approx 30\%$ of the CERN beam produced by coherent bremsstrahlung on Si single crystal has been measured [3] with the help of the asymmetry of the decay $\rho^0 \rightarrow \pi^+ + \pi^-$ because some experimental factors could influence and change the expected value of $P$. Now the operation of these as well as of another polarized photon beam used for few QED experiments only [4] is stopped.

On the other hand there are many proposed experiments (see [5]) devoted to QCD and nucleon form factors problems which can be realized with the help of polarized photon

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beams with $\omega \geq 20$ GeV produced at SLAC and Fermilab as it is described in [6,7] and [8], respectively. In this connection the measurement of the photon beam polarization becomes important. Unfortunately, the methods of polarization measurements at $\omega \leq 10$ GeV (see [9]) are not applicable at higher energies and low intensities. The $\rho$-decay asymmetry method used in [3] is connected with difficulties, due to the relatively low yield of $\rho$ mesons (about $10^{-5}$ per photon, see [5]) and the necessary $\pi/e$ angular discrimination to eliminate the high intensity $e^+ e^-$-background.

Taking into account the above said and the perspectives of the realization of the polarization conversion with linear and circular polarization measurements by the $\rho$-decay method according to the proposal [5] it is necessary to revise the possibilities of the methods using single crystal. Let us note that as the preliminary measurements [10] have shown one of the recently proposed method [11] measuring the asymmetry of total pair yield with $y = \varepsilon_+/\omega = 0 -1$ ($\varepsilon_\pm$ is the energy of produced pair particles) without magnetic spectrometer has not provided practical results at CERN SPS because of the presence of low energy photons accompanying high energy photons with multiplicity 1.5-3 in the beam [12].

In this work to escape the difficulties connected with the photon beam multiplicity it is suggested to use magnetic spectrometer, and since the method is for relatively low intensity beams it is proposed to use asymmetric pairs of the coherent pair production in a much wider y-interval in contrast to the well known method [13] when one uses only symmetric pairs with $y \approx 0.5$ within a narrow y-interval with $\Delta y / y \approx 1 - 2\%$. The method can be tested and used in the experiment [5] devoted to the conversion of the linear polarization into circular in a way described in [5,14]. Though diamond crystals provide better results all the calculations are carried out for silicon single crystals because large diamond crystals covering the electron beam cross section are not available and are expensive.

2. The Physics and Formulae of the Method

To illustrate the physics of the proposed method Fig.1 shows the y-dependence of the differential absorption coefficients of the coherent pair production in Si single crystal by 100 GeV photons in case of ”point effect” orientation when polar entrance angle with
respect to the axis [110] is $\theta = 40$ mrad and the azimuthal angle between the plane (110) and the plane containing the photon momentum $\vec{k}$ and axis [110] is $\alpha = 21.8$ mrad. As it follows from Fig.1 the differential analyzing power

$$
\rho = \frac{dw_{\parallel}/dy - dw_{\perp}/dy}{dw_{\parallel}/dy + dw_{\perp}/dy},
$$

(1)

where $dw_{\parallel,\perp}/dy$ are the differential pair absorption coefficients with photon polarization vector $\vec{e}$ parallel and perpendicular to the plane $(\vec{k}, [110])$, is sufficiently high and approximately constant within a wide $y$-interval

$$
y_1 \leq y \leq y_2.
$$

(2)

Here $y_{1,2}$ are the values of $y$ when the differential absorption coefficients $dw_{\parallel,\perp}/dy$ have a jump. The calculations show that the value of the asymmetry for chosen pairs within the interval (2) appears to be significantly larger than the asymmetry for pairs within the interval $y=0-1$. Therefore in this work we propose to use the experimental asymmetry

$$
a = \frac{n_{\parallel} - n_{\perp}}{n_{\parallel} + n_{\perp}},
$$

(3)

with the numbers of pairs $n_{\parallel,\perp}$ produced in the interval (2). The degree of linear polarization is determined by the formula $P = a/r$ where

$$
r = \frac{w_{\parallel} - w_{\perp}}{w_{\parallel} + w_{\perp}},
$$

(4)

and $w_{\parallel,\perp}$ are the theoretical integral absorption coefficients, i.e. the integrals of the $dw_{\parallel,\perp}/dy$ over the interval (2). In difference from the notations of [11] W, R and F calculated for the full interval $(0,1)$ of $y$ in this paper the pair production characteristics integrated over the interval (2) are denoted by small letters $w$, $r$ and $f$ for absorption coefficients, asymmetry and figure of merit, respectively. The values of $y_{1,2}$ for given reciprocal lattice vector $\vec{g}$ are determined from the condition $g_{\parallel} = \delta$

$$
y_{1,2}(g) = \frac{1}{2}(1 \pm \sqrt{1 - \tau(g)}),
$$

(5)

where $g_{\parallel}$ is the component of $\vec{g}$ parallel to the photon momentum $\vec{k}$, $\delta = m^2/2\omega y(1 - y)$, $\delta_m = 2m^2/\omega$ is the minimal momentum transfer and $\tau(g) = \delta_m/g_{\parallel}$, ($\hbar = c = 1$). In the case of “point effect” when the reciprocal lattice vector (220) gives the main contribution
we have $\tau(g) = bm/4\sqrt{2}\pi\lambda_c\omega\theta\sin\alpha$ (b is the lattice constant, $\lambda_c$ is the electron Compton wavelength). If the momentum acceptance ($y_{\text{min}}$, $y_{\text{max}}$) of the pair spectrometer is not sufficiently large only pair components with $y_{\text{min}} \geq y_1$, and/or $y_{\text{max}} \leq y_2$ can be detected, hence the limits of integrations of the differential absorption coefficients $y_1$ and/or $y_2$ should be replaced by $y_{\text{min}}$ and/or $y_{\text{max}}$. In the following calculations we assume that the pair spectrometer acceptance is large and all pairs within the interval (2) can be accepted.

Following [11] we present the polarization ($\vec{e}$) dependent integral absorption coefficients $w$ in the form

$$w(\omega, \vec{e}) = w_{\text{am}} + w_{\text{coh}}^1(\omega) + w_{\text{coh}}^2(\omega, \vec{e}).$$

(6)

The amorphous part is independent of $\vec{e}$ and is equal to

$$w_{\text{am}} = w_0[B_1(y_2) - B_1(y_1)]\psi_1 - [B_2(y_2) - B_2(y_1)],$$

(7)

where in the exponential and Moliere screening approximation

$$\psi_{\text{exp}}^1 = 4[0.5 + \ln(m/\beta) - B(s)],$$

$$\psi_{\text{Mol}}^1 = 4[0.5 + \ln(111Z^{-1/3}) - \sum_{i=1}^3 a_i^2B(s_i) \sum_{i \neq j} a_i a_j b_i^2 - b_j^2 \exp(s_i) E_i(s_i)].$$

(8)

(9)

Here $\beta^{-1} = R$ is screening radius, $u$ is mean square thermal oscillation amplitude, $a_i$ and $b_i$ are parameters of the Moliere potential, $B(x)$ is a well known function [15] of $s = \beta^2 u^2$ and $s_i = \beta_i u_i^2$, $\beta_i = mb_i Z^{1/3} / 121$ and $E_i(x)$ is the exponential integral. The functions $B_{1,2}$ are given by the expressions

$$B_1(x) = x - 2x^2/3 + 4/3x^3/9, \quad B_2(x) = 2(x^2 - 2x^3/3)/9.$$  

(10)

The first of the coherent parts depends on $\omega$ and is equal to

$$w_{\text{coh}}^1(\omega) = w_0 \delta_m \sum_{\vec{g}} D(\vec{g}) \frac{g^2}{g^2} \{[A_1(y_2) - A_1(y_1)]$$

$$+ \tau(g) [A_2(y_2) - A_2(y_1)] - \tau^2(g) [A_3(y_2) - A_3(y_1)]/4$$

(11)

and the second coherent part which depends on $\omega$ and $\vec{e}$ is equal to

$$w_{\text{coh}}^2(\omega, \vec{e}) = w_0 \delta_m \sum_{\vec{g}} D(\vec{g}) \frac{g^2}{g^2} \frac{\tau^2(g)}{8} [1 - 2|\hat{g} \cdot \vec{e}|^2] [A_3(y_2) - A_3(y_1)].$$

(12)
where

\[ D(\vec{g}) = \frac{2\pi^2 S(\vec{g})}{V_0} n_0 \exp(-g^2 u_1^2) \left[ 1 - \frac{1 - F(\vec{g})}{g^2} \right]^2, \]  

(13)

\[ A_1(x) = \ln \frac{x}{1 - x} - 2x, \quad A_2(x) = \ln \frac{x}{1 - x}, \quad A_3(x) = \frac{1}{1 - x} - \frac{1}{x} + \ln \frac{x}{1 - x}. \]  

(14)

In the above expressions \( w_0 = n\sigma_0, \sigma_0 = Z^2 r_0^2 / 137 \), \( n \) and \( Z \) are the atomic density and nucleus charge of the crystal, \( r_0 \) is classical electron radius, \( m \) is electron mass, \( \vec{g}_\perp \) is the component of \( \vec{g} \) perpendicular to \( \vec{k} \), \( \hat{\vec{g}}_\perp = \vec{g}_\perp / g_\perp \). In the formulae for \( w_{\text{coh}}^{1,2} \) the summations are carried out for all vectors \( \vec{g} \) with \( g_\parallel \geq \delta_m \) and the limits of integration \( y_1,2 \) must be replaced by (5) if \( y_1,2(g) \) are less than \( y_1,2 \). In the case of integration over full interval (0,1) substituting \( y_1 = 0 \) and \( y_2 = 1 \) in \( w_{\text{am}} \) and replacing \( y_1 \) and \( y_2 \) by (5) we obtain the total absorption coefficients \( W_{\text{am}} \) and \( W_{\text{coh}} \) and corresponding \( R = (W_\parallel - W_\perp) / (W_\parallel + W_\perp) \).

In conclusion of this section let us write the expression for the number of primary photons \( n_\gamma \) necessary to measure the beam linear polarization \( P \) with an accuracy \( \Delta P \) [11].

\[ n_\gamma = 1 / (tf(\Delta P)^2) \]  

(15)

where \( t \) is the crystal thickness in cm and \( f = r^2(W_\parallel + W_\perp) \) is the figure of merit.

3. Numerical Results and the Measurement of the Linear Polarization

To obtain numerical results with the help of the formulae (6-15) first we calculate the dependence of \( w_{\parallel,\perp} \) upon \( \theta_y = \theta \sin \alpha \) (see Fig.2 for a [110] Si crystal for \( \omega = 100 \text{ GeV} \)). The variation of the angle \( \theta_x = \theta \cos \alpha \) in a wide interval does not change the results. Then we calculate the figure of merit \( f \), entering in (15), depended on the angle \( \theta_y \) (see Fig. 3 for \( \omega = 100 \text{ GeV} \)). The optimal orientation angle \( \theta_{yo} \) corresponding to the maximal value of the figure of merit \( f_{\text{max}} \) gives the orientation for which minimal measurement time is required for the given accuracy \( \Delta P / P \) of the polarization measurement.

Fig.4 shows the energy dependencies of the optimal \( \theta_{yo} \) and \( r_{yo} \) corresponding to the maximal figure of merit \( f_{\text{max}} \). For comparison in Fig.4 is shown also optimal \( R_{yo} \) calculated by the use \( W_{\parallel,\perp}^{\text{opt}} \) [11]. In this case the asymmetry is low \( R_{yo} \approx 20\% \) in comparison to the \( r_{yo} \approx 40\% \) for photon energy \( \omega = 100 \text{ GeV} \) and at the same crystal orientation (\( \theta_x = 40\text{mrad}, \theta_y = 0.873\text{mrad} \)). As the calculations show the optimal angles \( \theta_{yo} \) corresponding to the \( f_{\text{max}} \) and the \( F_{\text{max}} \) coincide. The \( \omega \)-dependencies of \( f_{\text{max}} \) and
corresponding \( w_{\|,\perp}^{\text{opt}} \) as well as \( w_{\text{am}} \) are shown in Fig.5. For comparison in Fig.5 is also shown \( y \) dependence of \( F_{\text{max}} \). Having the results of the Figs.4 and 5 one can choose the optimal parameters of the crystal polarimeter. Indeed, coming out from the photon energy and the energy interval in the spectral distribution one firstly chooses the crystal angle \( \theta_{y}^{\text{opt}} \) using the corresponding curve of Fig.4. Then using the value \( r^{\text{opt}} \) from Fig.4 and the values of \( w_{\|,\perp}^{\text{opt}} \) from Fig.5 and giving the desired measurement accuracy \( \Delta P/P \) one finds the number of necessary photons for the given thickness of crystal with the help of formula (15). The choice of \( t \) is conditioned by the compromise between available thickness, fast gathering of necessary statistics and allowed radiation energy losses of the produced pair particles in the crystal polarimeter. Determining the polarization with the help of the formula \( P = a/r \) one must use the asymmetry measured only for pairs with \( y \) values in the interval (2) and (5) for the given photon energy and orientation.

4. Summary

The obtained formulae (6)-(15) and numerical results allow to use the described method for the measurement of polarization when the photon beam intensity is low as in the case of experiment [5]. The large value of the figure of merit and the reduced measurement time is the advantage of the proposed method.

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Figure Captions

Fig.1 The differential absorption coefficients $dw_{∥}/dy$ (solid line) and $dw_{⊥}/dy$ (dashed line) as a function of $y$ for [110] Si at $ω = 100$ GeV. The incidence angles are $θ_{x}$= 40 mrad and $θ_{y}$=0.873 mrad ($y_{1,2} = 0.37, 0.63$).

Fig.2 The absorption coefficients $w_{∥}$ and $w_{⊥}$ and $w = (w_{∥} + w_{⊥})/2$ as a function of $θ_{y}$ for [110] Si at $ω = 100$ GeV.

Fig.3 The figure of merit $f = r^{2}(w_{∥} + w_{⊥})$ as a function of $θ_{y}$ for [110] Si at $ω = 100$ GeV ($θ_{x} = 40$ mrad).

Fig.4 The energy dependencies of $θ_{y}^{opt}$, $r^{opt}$ and $R^{opt}$ calculated with the use of absorption coefficients $w_{∥,⊥}^{opt}$ and $W_{∥,⊥}^{opt}$, respectively.

Fig.5 The energy dependencies of $f^{max}$, $F^{max}$, $w_{∥,⊥}^{opt}$ and $w^{am}$. 
Fig. 3

\[ f \text{ (cm}^{-1}\text{)} \]

\[ \theta_y \text{ (m rad)} \]

Fig. 4

\[ \theta_y^\text{opt} \]

\[ \omega \text{ (GeV)} \]
Fig. 5