An accretion disk and radio spectra of pulsars

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ABSTRACT
On the basis of the unified model of compact radio sources, the dependence of a turnover frequency in the smoothed radio spectrum of a pulsar upon the ratio of the dispersion measure D to the period P of a pulsar is obtained. This relation is produced by the radial density wave in the accretion disk surrounding a pulsar. The unified model of compact radio sources gives also the smoothed spectral indices of radio emission from pulsars as $\alpha = 2$ for the gaseous disk with the temperature profile $T = \text{const}$ and $\alpha = 3$ for the gaseous disk with the pressure profile $P = \text{const}$ ($F_\nu \propto \nu^{-\alpha}$). The transverse density wave in the magnetosphere of a pulsar can be responsible for the polarisation of optical radiation from pulsars.

Key words: accretion, accretion disks - radio continuum: pulsars

1 INTRODUCTION

The mechanism of radio emission from pulsars has remained so far an unsolved problem (Edwards & Stappers 2003; Vivekanand 2001). The difficulties which have been encountered by plasma mechanisms of radio emission from pulsars clearly show that the radiation is produced by a low-energy medium (Gedalin et al. 2002). Such a medium is the gaseous disk surrounding the central star. The accretion disk surrounding a radio pulsar is fed by the fallback material left over after the original supernova explosion (Chakrabarty et al. 2001). Note that ordinary, not recycled pulsars normally are not binaries (Edwards & Stappers 2003). Pulsar wind (Gaensler 2003) may be responsible both for a stationary convection in the accretion disk surrounding a pulsar and for outflows of gas in the gaseous disk.

This approach allows us to consider radio pulsars as representatives of a wide class of compact radio sources which includes also active galactic nuclei and maser complexes (Prigara 2003). The unified model of compact radio sources, as it is shown below, gives such characteristics of the smoothed radio spectra of pulsars as the spectral indices, a short-wavelengths cut-off and turnover frequencies.

In the unified model of compact radio sources, radio emission from pulsars is treated as thermal radiation from an accretion disk amplified by the maser mechanism (Prigara 2003). A maser amplification of thermal radio emission in continuum produces the high brightness temperatures of compact radio sources and a rapid variability of total and polarised flux density, that is characteristic for non-saturated maser sources. In particular, pulsars signals show a variability on every observable timescale up to nanoseconds (Edwards & Stappers 2003; Vivekanand 2001).

2 GASEOUS DISK MODEL

It was shown recently (Prigara 2003) that thermal radio emission has a stimulated character. According to this conception thermal radio emission from non-uniform gas is produced by an ensemble of individual emitters. Each of these emitters is a molecular resonator the size of which has an order of magnitude of mean free path $l$ of photons

$$l = \frac{1}{n\sigma}$$

where $n$ is the number density of particles and $\sigma$ is the absorption cross-section.

The emission of each molecular resonator is coherent, with the wavelength

$$\lambda = l,$$

and thermal radio emission of gaseous layer is incoherent sum of radiation produced by individual emitters.

The condition (2) implies that the radiation with the wavelength $\lambda$ is produced by the gaseous layer with the definite number density of particles $n$.

The condition (2) is consistent with the experimental results by Looney and Brown on the excitation of plasma waves by electron beam (Alexeev 2003; Chen 1984). The wavelength of standing wave with the Langmuir frequency of oscillations depends on the density as predicted by equation (1). The discrete spectrum of oscillations is produced by the non-uniformity of plasma and the readjustment of the wavelength to the length of resonator. From the results...
of experiment by Looney and Brown the absorption cross-section for plasma can be evaluated.

The product of the wavelength by density is weakly increasing with the increase of density. This may imply the weak dependence of the size of elementary resonator in terms of the wavelength upon the density or, equivalently, wavelength.

In the gaseous disk model, describing radio emitting gas nebulae (Prigara 2003), the number density of particles decreases reciprocally with respect to the distance \( r \) from the energy centre

\[ n \propto r^{-1} . \]  

(3)

Together with the condition for emission \( \nu \) the last equation leads to the wavelength dependence of radio source size:

\[ r_\lambda \propto \lambda . \]  

(4)

The relation \( \lambda \) is indeed observed for sufficiently extended radio sources. For example, the size of radio core of galaxy M31 is 3.5 arcmin at the frequency 408 MHz and 1 arcmin at the frequency 1407 MHz (Sharov 1982).

3 DENSITY PROFILE OF COMPACT RADIO SOURCES

In the case of compact radio sources instead of the relationship \( \lambda \propto \lambda^2 \)

\[ r_\lambda \propto \lambda^2 \]  

(5)
is observed (Lo 1982; Lo et al. 1993). This relationship may be explained by the effect of a gravitational field on the motion of gas which changes the equation \( \lambda \) for the equation

\[ n \propto r^{-1/2} . \]  

(6)

The mass conservation in an outflow or inflow of gas gives \( \nu r = \text{const} \), where \( \nu \) is the velocity of flow. In the gravitational field of a central energy source the energy conservation gives

\[ \nu = \left( v_0^2 + c^2 r_s / r \right)^{1/2} \]  

(7)

where \( r_s \) is the Schwarzschild radius. Therefore, at small values of the radius the equation (6) is valid, whereas at the larger radii we obtain the equation \( \lambda \). 

It is well known (Shklovsky 1984) that the delay of radio pulses from pulsars at low frequencies is proportional to \( \lambda^2 \). This fact is a mere consequence of Eq. (5), if we only assume the existence of the radial density wave travelling across the radius with a constant velocity and triggering the pulse radio emission. In this treatment the pulsars also obey the \( \lambda^2 \) dependence of compact source size. Note that the wavelength dependence of a pulse duration is a similar effect.

The spatial distribution of SiO, water, and OH masers (each of which emits in its own wavelength) in the maser complexes also is consistent with the \( \lambda^2 \) dependence of compact source size (Bochkarev 1992; Eisner et al. 2002).

To summarise, extended radio sources are characterised by the relation \( \lambda \), and compact radio sources obey the relation \( \lambda^2 \).

4 RADIO EMISSION FROM THE GASEOUS DISK

The spectral density of flux from an extended radio source is given by the formula

\[ F_\nu = \frac{1}{a^2} \int_0^R B_\nu(T) \times 2\pi r dr \]  

(8)

where \( a \) is a distance from radio source to the detector of radiation, and the function \( B_\nu(T) \) is given by the Rayleigh-Jeans formula

\[ B_\nu = \frac{2kT\nu^2}{c^2} . \]  

(9)

where \( \nu \) is the frequency of radiation, \( k \) is the Boltzmann’s constant, and \( T \) is the temperature.

The extended radio sources may be divided in two classes. Type 1 radio sources are characterised by a stationary convection in the gaseous disk with an approximately uniform distribution of the temperature \( T = \text{const} \) giving the spectrum

\[ F_\nu \propto \text{const} . \]  

(10)

Type 2 radio sources are characterised by outflows of gas with an approximately uniform distribution of gas pressure \( P = nkT = \text{const} \). In this case the equation (9) gives

\[ T \propto r, \]  

(11)

so the radio spectrum, according to the equation (7), has the form

\[ F_\nu \propto \nu^{-1} . \]  

(12)

Both classes include numerous galactic and extragalactic objects. In particular, edge-brightened supernova remnants (Kulkarni & Frail 1993) belong to the type 2 radio sources in accordance with the relation \( \lambda^2 \), whereas centre-brightened supernova remnants belong to the type 1 radio sources.

The relationship between linear size and turnover frequency in type 2 radio sources (gigahertz-peaked spectrum sources and steep-spectrum sources) (Nagar et al. 2002) is a consequence of the wavelength dependence of radio source size. The turnover frequency is determined by the equation \( \nu_T = R \), where \( R \) is the radius of a gaseous disk. The same equation determines a turnover frequency for planetary nebulae (Pottasch 1984; Prigara 2003; Sidniak & Tylenda 2001).

5 THE SPECTRAL INDEX OF RADIO EMISSION

The flux density is determined by Eq. (5). In the case of pulsars the relation \( \lambda^2 \) is valid. Thus, a stationary convection in the accretion disk with the temperature profile \( T = \text{const} \) now produces the spectrum

\[ F_\nu \propto \nu^{-2} . \]  

(13)

An outflow or inflow of gas with the pressure profile \( P = nkT = \text{const} \) gives rise to the temperature profile \( T \propto r^{1/2} \), according to Eq. (6). The flux density in this case is given by the formula
A combination of a stationary convection and outflows or inflows gives the intermediate values of the spectral index \(2 < \alpha < 3\), where \(F_\nu \propto \nu^{-\alpha}\). The spectrum (13) seems to be the case for the Crab pulsar and many other pulsars (Shklovsky 1984). The spectrum (18) has also been detected in many pulsars (Lipunov 1987).

The range of frequencies in which the spectra (13) or (18) are valid is confined by a short-wavelengths cut-off, \(\nu_0\), and a turnover frequency, \(\nu_t\). In the range \(\nu < \nu_t\) the radio spectrum is flat or slightly inverted (Lipunov 1987). This effect is similar to those in active galactic nuclei (gigahertz-peaked spectrum sources and steep-spectrum sources) (Nagar et al. 2002).

6 A TURNOVER FREQUENCY

Consider now in more detail the delay of radio pulses from pulsars at low frequencies discussed in Sec.3. Making use of Eq. (15), the time of the delay of pulses may be written in the form

\[
\tau = r_s/u = (r_0/u) \left(\frac{\lambda^2}{\lambda_0^2}\right) = \mu P \lambda^3 / (2\lambda_0^3),
\]

(15)

where \(u\) is the velocity of the radial density wave in the accretion disk, \(P\) is the period of a pulsar, \(P = 2\pi v r_0\), \(r_0\) is the radius of the neutron star, \(v\) is the velocity of the radial density wave inside the neutron star, and \(\mu = v/u\).

On the other hand, the delay of pulses is normally expressed in terms of the dispersion measure, \(D\), as follows

\[
\tau = e^2 D\lambda^2 / (2\pi mc^2),
\]

(16)

where \(e\) is the charge and \(m\) is the mass of electron, respectively (Bochkarev 1992).

Comparing these relations, we obtain the expression for the frequency \(\nu_0 = c/\lambda_0\) in the form

\[
\nu_0 = \sqrt{\left(e^2 / \pi m c^2\right)} (D/P).
\]

(17)

The frequency \(\nu_0\) may be interpreted as a short-wavelengths cut-off in the smoothed radio spectrum of a pulsar. To evaluate a cut-off frequency, we assume \(\mu = 1\), though this assumption is not likely to be valid in actual cases. Then Eq. (17) gives

\[
\nu_0 = 0.1 \sqrt{D/P},
\]

(18)

where the dispersion measure, \(D\), is in units of \(cm^{-3}pc\), the period of a pulsar, \(P\), is in \(s\), and the turnover frequency, \(\nu_0\), is in \(GHz\).

The Crab pulsar has the dispersion measure of 56.8 \(cm^{-3}pc\) and the period of 33.1 \(ms\) (Lang 1974). The equation (18) gives \(\nu_0 = 4.2 GHz\) and \(\lambda_0 = 7 cm\), that seems to be a good estimation of the short-wavelengths cut-off.

To obtain a turnover frequency, \(\nu_t\), we should replace the radius of the neutron star, \(r_0\), by the radius of the accretion disk, \(R\), in Eq. (15). Then we find

\[
\nu_t = \nu_0 \sqrt{r_0/R}.
\]

(19)

If we assume \(R \approx 10^3 r_0\), then for the Crab pulsar the turnover frequency will be \(\nu_t \approx 140 MHz\), and, respectively, \(\lambda_t \approx 2m\). This result is in good agreement with the observed radio spectrum of the pulsar.

7 CONCLUSIONS

The unified model of compact radio sources, applied to radio pulsars, reproduces the observed spectral indices of radio emission from pulsars. The unified model gives also the dependence of the turnover frequency and short-wavelengths cut-off in smoothed radio spectra of pulsars upon the ratio of the dispersion measure to the period of a pulsar. This relation is produced by the radial density wave in the gaseous accretion disk surrounding a pulsar. Along with the radial density wave in the accretion disk, the transverse density wave in the magnetosphere of a pulsar can exist. The last produces the temporal profile of polarisation in optical.

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