LARGE-SCALE CURVATURE PERTURBATIONS
WITH SPATIAL AND TIME VARIATIONS
OF THE INFLATON DECAY RATE

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Abstract

We present a gauge-invariant formalism to study the evolution of the curvature and entropy perturbations in the case in which spatial and time variations of the inflaton decay rate into ordinary matter are present. During the reheating stage after inflation curvature perturbations can vary with time on super-horizon scales sourced by a a gauge-invariant inflaton decay rate perturbation. We show that the latter is a function not only of the spatial variations of the decay rate generated during inflation, as envisaged in a recently proposed scenario, but also of the time variation of the inflaton decay rate during reheating. If only the second source is present, the final curvature perturbation at the end of the reheating stage is proportional to the curvature perturbation at the beginning of reheating with a coefficient of proportionality which can be either smaller or larger than unity depending upon the underlying physics governing the time variation of the inflaton decay rate. As a consequence, we show that the standard consistency relation between the amplitude of curvature perturbations, the amplitude of tensor perturbations and the tensor spectral index of one-single field models of inflation is violated and there is the possibility that the tensor-to-curvature amplitude ratio is larger than in the standard case.

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1 Introduction

One of the basic ideas of modern cosmology is that there was an epoch early in the history of the universe when potential, or vacuum, energy associated to a scalar field, the inflaton $\phi$, dominated other forms of energy density such as matter or radiation. During such a vacuum-dominated era the scale factor grew exponentially (or nearly exponentially) in time. During this phase, dubbed inflation [1, 2], a small, smooth spatial region of size less than the Hubble radius could grow so large as to easily encompass the comoving volume of the entire presently observable universe. If the universe underwent such a period of rapid expansion, one can understand why the observed universe is so homogeneous and isotropic to high accuracy.

Inflation has also become the dominant paradigm for understanding the initial conditions for structure formation and for Cosmic Microwave Background (CMB) anisotropy. In the inflationary picture, primordial density and gravity-wave fluctuations are created from quantum fluctuations “redshifted” out of the horizon during an early period of superluminal expansion of the universe, where they are “frozen” [3, 4, 5, 6, 7]. Perturbations at the surface of last scattering are observable as temperature anisotropy in the CMB which was first detected by the Cosmic Background Explorer (COBE) satellite [8, 9, 10]. The last and most impressive confirmation of the inflationary paradigm has been recently provided by the data of the Wilkinson Microwave Anistropy Probe (WMAP) mission which has marked the beginning of the precision era of the CMB measurements in space [11]. The WMAP collaboration has produced a full-sky map of the angular variations of the CMB, with unprecedented accuracy. WMAP data confirm the inflationary mechanism as responsible for the generation of curvature (adiabatic) super-horizon fluctuations.

Despite the simplicity of the inflationary paradigm, the mechanism by which cosmological adiabatic perturbations are generated is not yet fully established. In the standard picture, the observed density perturbations are due to fluctuations of the inflaton field itself. When inflation ends, the inflaton oscillates about the minimum of its potential and decays, thereby reheating the universe. As a result of the fluctuations each region of the universe goes through the same history but at slightly different times. The final temperature anisotropies are caused by the fact that inflation lasts different amounts of time in different regions of the universe leading to adiabatic perturbations. Under this hypothesis, the WMAP dataset already allows to extract the parameters relevant for distinguishing among single-field inflation models [12].
An alternative to the standard scenario is represented by the curvaton mechanism [13,14,15] where the final curvature perturbations are produced from an initial isocurvature perturbation associated to the quantum fluctuations of a light scalar field (other than the inflaton), the curvaton, whose energy density is negligible during inflation. The curvaton isocurvature perturbations are transformed into adiabatic ones when the curvaton decays into radiation much after the end of inflation.

Recently, another mechanism for the generation of cosmological perturbations has been proposed [16,17,18]. It acts during the reheating stage after inflation and it was dubbed the “inhomogeneous reheating” mechanism in Ref. [18]. The coupling of the inflaton to normal matter may be determined by the vacuum expectation value of fields $\chi$’s of the underlying theory. If those fields are light during inflation, fluctuations $\delta \chi \sim H^2 \frac{\pi}{2}$, where $H$ is the Hubble rate during inflation, are left imprinted on super-horizon scales. These perturbations lead to spatial fluctuations in the decay rate $\Gamma$ of the inflaton field to ordinary matter, $\frac{\delta \Gamma}{\Gamma} \sim \frac{\delta \chi}{\chi}$, causing adiabatic perturbations in the final reheating temperature in different regions of the universe.

Interestingly, these different scenarios have different observational predictions. The curvaton scenario and the one based on variation of the decay rate allows to generate the observed level of density perturbations with a much lower scale of inflation and thus generically predicts a smaller level of gravitational waves. Furthermore, because the field responsible for the fluctuations is not the inflaton, it can have significantly larger self couplings and thus density perturbations could be non-Gaussian. The non-Gaussianity can be large enough to be detectable by CMB and Large Scale Structure observations contrary to what predicted in the traditional one-single field model of inflation, where the level of non-Gaussianity is very small [19].

Contrary to the standard picture, both the curvaton and the inhomogeneous reheating mechanism exploit the fact that the total curvature perturbation (on uniform density hypersurfaces) $\zeta$ can change on arbitrarily large scales due to a non-adiabatic pressure perturbation which may be present in a multi-fluid system [20,21,22,23]. While the entropy perturbations evolve independently of the curvature perturbation on large scales, the evolution of the large-scale curvature is sourced by entropy perturbations.

In this paper, we present a gauge-invariant formalism to study the evolution of the gauge-invariant curvature and entropy perturbations in the inhomogeneous reheating mechanism. To do so, we extend the gauge-invariant formalism first introduced in Ref. [24], which is
appropriate to describe the evolution of curvature and entropy perturbations in multi-fluid cosmologies when energy transfer between fluids is included. We show that the curvature perturbation during the reheating stage can vary with time on super-horizon scales sourced by a gauge-invariant inflaton decay rate perturbation

$$\delta \Gamma^{\text{GI}} = \delta \Gamma - \frac{\dot{\delta \rho}_\phi}{\rho_\phi}$$

where $\rho_\phi$ is the inflaton energy density and $\delta \rho_\phi$ its perturbation. The gauge-invariant inflaton decay rate perturbation is a function not only of the spatial variation of the decay rate $\delta \Gamma$ generated during inflation, but also of the time variation of the inflaton decay rate $\dot{\Gamma}$. To our knowledge, this new source proportional to $\dot{\Gamma}$ has never been discussed in the literature.

As an application, we study the evolution of the curvature perturbation in the inhomogeneous reheating scenario where fluctuations of the inflaton decay rate $\delta \Gamma$ are induced by the fluctuations of some light scalar field during inflation and confirm the results of Refs. [16, 18]. We extend their findings showing that variation of the total curvature perturbation $\zeta$ on super-horizon scales take place even when the inflaton decay rate changes with time during reheating. This new effect is proportional to the total curvature perturbation generated during the inflationary period $\zeta_{\text{in}}$ and causes either an increase or a depletion of $\zeta_{\text{in}}$ depending upon the underlying model responsible for the time variation of $\Gamma$. Furthermore, we show that the standard consistency relation between the amplitude of curvature perturbations, the amplitude of tensor perturbations and the tensor spectral index is violated. Finally, for completeness, we extend our analysis to the case in which the classical inflaton field $\phi$ does not release its energy perturbatively, but very rapidly (explosively) decays into either its own quanta or into other bosons due to broad parametric resonance, the so-called preheating stage.

2 The perturbative inflaton decay

Our starting point are the equations governing the evolution of cosmological perturbations during the reheating stage. We follow the gauge-invariant approach developed in Ref. [24] for the general case of an arbitrary number of interacting fluids in general relativity.

At the end of inflation, once the Hubble rate drops below the mass of the inflaton field $\phi$, the inflaton starts oscillating around the minimum of its potential. Averaged over several
oscillations, the effective equation of state is \( \langle P_\phi / \rho_\phi \rangle = 0 \), where \( P_\phi \) and \( \rho_\phi \) are the inflaton pressure and energy density, respectively. The coherent oscillations of the inflaton field are equivalent to a fluid of non-relativistic particles \([25]\). The vacuum energy during inflation is transformed into the energy density of the coherent inflaton oscillations. Assuming the inflaton is unstable and decays into light particles ("radiation") with a decay rate \( \Gamma \), this represents an energy transfer from the pressureless inflaton fluid to the radiation fluid with energy density \( \rho_\gamma \).

The evolution of the background FRW universe during the reheating stage is governed by the Friedmann constraint

\[
H^2 = \frac{8\pi G}{3} \rho, \quad (2.1)
\]

\[
\dot{H} = -4\pi G (\rho + P), \quad (2.2)
\]

and the continuity equation

\[
\dot{\rho} = -3H (\rho + P), \quad (2.3)
\]

where the dot denotes differentiation with respect to the coordinate time \( t \), \( H \equiv \dot{a}/a \) is the Hubble parameter, and \( \rho \) and \( P \) are the total energy density and the total pressure of the system. The total energy density and the total pressure are related to the energy density and pressure of the inflaton field and radiation by

\[
\rho = \rho_\phi + \rho_\gamma, \\
P = P_\phi + P_\gamma, \quad (2.4)
\]

where \( P_\gamma \) is the radiation pressure. The inflaton field \( \phi \) and the radiation component have energy-momentum tensor \( T^{\mu\nu}_{\phi} \) and \( T^{\mu\nu}_{\gamma} \), respectively. The total energy momentum tensor \( T^{\mu\nu} = T^{\mu\nu}_{\phi} + T^{\mu\nu}_{\gamma} \) is covariantly conserved, but we allow for energy transfer between the fluids,

\[
\nabla_\mu T^{\mu\nu}_{\phi} = Q^{\nu}_{\phi}, \\
\nabla_\mu T^{\mu\nu}_{\gamma} = Q^{\nu}_{\gamma}, \quad (2.5)
\]

where \( Q^{\nu}_{\phi} \) and \( Q^{\nu}_{\gamma} \) are the generic energy-momentum transfer to the inflaton and radiation sector respectively and are subject to the constraint

\[
Q^{\nu}_{\phi} + Q^{\nu}_{\gamma} = 0. \quad (2.6)
\]
The continuity equations for the energy density of the inflaton field $\rho_\phi$ and radiation $\rho_\gamma$ in the background is thus \cite{26} ($Q_\phi = Q^{0}_\phi$, $Q_\gamma = Q^{0}_\gamma$)

\begin{align}
\dot{\rho}_\phi & = -3H (\rho_\phi + P_\phi) + Q_\phi, \\
\dot{\rho}_\gamma & = -3H (\rho_\gamma + P_\gamma) + Q_\gamma.
\end{align}

(2.7)

From now on, we parametrize, the energy transfer from the inflaton to radiation by \cite{24}

\begin{align}
Q_\phi & = -\Gamma \rho_\phi, \\
Q_\gamma & = \Gamma \rho_\phi,
\end{align}

(2.8)

where $\Gamma$ is the decay rate of the inflaton into radiation. The positions (2.8) are valid in the case in which the inflaton decays into light states through a perturbative process. However, if the classical inflaton field $\phi$ very rapidly (explosively) decays into either its own quanta or into other bosons due to broad parametric resonance, the so-called preheating stage \cite{28}, Eqs. (2.8) should be modified. The corresponding equations will be discussed in Section 3.

The energy conservation equations are therefore

\begin{align}
\dot{\rho}_\phi & = -\rho_\phi (3H + \Gamma), \\
\dot{\rho}_\gamma & = -4H \rho_\gamma + \Gamma \rho_\phi, \\
\dot{\rho} & = -H (3\rho_\phi + 4\rho_\gamma).
\end{align}

(2.9-2.11)

It is convenient to work in terms of the dimensionless density parameters \cite{24}

\begin{align}
\Omega_\phi & \equiv \frac{\rho_\phi}{\rho}, \\
\Omega_\gamma & \equiv \frac{\rho_\gamma}{\rho},
\end{align}

(2.12)

and the dimensionless “reduced” decay rate \cite{24}

\begin{align}
g & \equiv \frac{\Gamma}{\Gamma + H},
\end{align}

(2.13)

which varies monotonically from 0 to 1 in an expanding universe.

The background equations (2.9-2.11) can then be written as an autonomous system

\begin{align}
\Omega'_\phi & = \Omega_\phi \left( \Omega_\gamma - \frac{g}{1 - g} \right), \\
\Omega'_\gamma & = \Omega_\phi \left( \frac{g}{1 - g} - \Omega_\gamma \right), \\
g' & = \frac{1}{2} (4 - \Omega_\phi) (1 - g) g + \frac{\Gamma'}{\Gamma} g (1 - g),
\end{align}

(2.14-2.16)
where the prime denotes differentiation with respect to the number of e-foldings \( N \equiv \ln a \), and we have allowed for a time variation of the inflaton decay rate. The density parameters are subject to the constraint

\[
\Omega_\phi + \Omega_\gamma = 1.
\]

(2.17)

2.1 Gauge-invariant perturbations

Linear scalar perturbations about a spatially-flat FRW background model are defined by the line element

\[
ds^2 = -(1 + 2\varphi)dt^2 + 2aB_i dt dx^i + a^2 [(1 - 2\psi)\delta_{ij} + 2E_{ij}] dx^i dx^j,
\]

(2.18)

where we have used the notation of Ref. [21] for the gauge-dependent curvature perturbation, \( \psi \), the lapse function, \( \varphi \), and scalar shear, \( \chi \equiv a^2 \dot{E} - aB \).

The zero-th component of the perturbed energy transfer vectors, Eq. (2.5), including terms up to first order, are written as [26]

\[
Q_{(\phi)0} = -Q_\phi (1 + \varphi) - \delta Q_\phi, \\
Q_{(\gamma)0} = -Q_\gamma (1 + \varphi) - \delta Q_\gamma,
\]

(2.19)

where the gravitational redshift (time-dilation) factor \( 1 + \varphi \) has been made manifest.

Both the density perturbations, \( \delta \rho_\phi \) and \( \delta \rho_\gamma \), and the curvature perturbation, \( \psi \), are in general gauge-dependent. Specifically they depend upon the chosen time-slicing in an inhomogeneous universe. The curvature perturbation on fixed time hypersurfaces is a gauge-dependent quantity: after an arbitrary linear coordinate transformation, \( t \to t + \delta t \), it transforms as \( \psi \to \psi + H\delta t \). For a scalar quantity, such as the energy density, the corresponding transformation is \( \delta \rho \to \delta \rho - \dot{\rho} \delta t \). However a gauge-invariant combination can be constructed which describes the density perturbation on uniform curvature slices or, equivalently the curvature of uniform density slices.

The curvature perturbation on uniform total density hypersurfaces, \( \zeta \), is given by [27]

\[
\zeta = -\psi - H \frac{\delta \rho}{\dot{\rho}},
\]

(2.20)

while the curvature perturbation on uniform inflaton energy density and radiation energy
density hypersurfaces are respectively defined as

\[ \zeta_\phi = -\psi - H \frac{\delta \rho_\phi}{\bar{\rho}_\phi}, \]
\[ \zeta_\gamma = -\psi - H \frac{\delta \rho_\gamma}{\bar{\rho}_\gamma}. \]  \hspace{1cm} (2.21)

The total curvature perturbation (2.20) is thus a weighted sum of the individual perturbations

\[ \zeta = \frac{\dot{\rho}_\phi}{\bar{\rho}} \zeta_\rho + \frac{\dot{\rho}_\gamma}{\bar{\rho}} \zeta_\gamma, \]  \hspace{1cm} (2.22)

while the difference between the two curvature perturbations describes a relative gauge-invariant entropy (or isocurvature) perturbation

\[ S_{\phi\gamma} = 3(\zeta_\phi - \zeta_\gamma) = -3H \left( \frac{\delta \rho_\phi}{\bar{\rho}_\phi} - \frac{\delta \rho_\gamma}{\bar{\rho}_\gamma} \right). \]  \hspace{1cm} (2.23)

From the definitions of the total curvature perturbation (2.22) and the entropy perturbation (2.23), we get for instance that

\[ \zeta_\phi = \zeta + \frac{1}{3} \frac{\dot{\rho}_\gamma}{\bar{\rho}} S_{\phi\gamma}. \]  \hspace{1cm} (2.24)

On wavelengths larger than the horizon scale, the perturbed energy conservation equations for the inflaton energy density and the radiation energy density can be written, including energy transfer, as

\[ \dot{\delta \rho}_\phi + 3H(\delta \rho_\phi + \delta P_\phi) - (\rho_\phi + P_\psi) 3\dot{\psi} = Q_\phi \phi + \delta Q_\phi, \]
\[ \dot{\delta \rho}_\gamma + 3H(\delta \rho_\gamma + \delta P_\gamma) - (\rho_\gamma + P_\psi) 3\dot{\psi} = Q_\gamma \phi + \delta Q_\gamma. \]  \hspace{1cm} (2.25)

The inflaton field and radiation have fixed equations of state (\(\delta P_\phi = 0\) and \(\delta P_\gamma = \delta \rho_\gamma/3\)) and hence there cannot be intrinsic non-adiabatic pressure perturbations. Using the perturbed (0 – i)-component of Einstein’s equations for super-horizon wavelengths \(\dot{\psi} + H \varphi = -\frac{H}{2} \frac{\delta \rho}{\bar{\rho}},\) we can re-write Eq. (2.25) in terms of the gauge-invariant curvature perturbations \(\zeta_\phi\) and \(\zeta_\gamma\) [21]

\[ \dot{\zeta}_\phi = -\frac{H}{\rho_\phi} (\delta Q_{\text{intr.,}\phi} + \delta Q_{\text{rel.,}\phi}), \]
\[ \dot{\zeta}_\gamma = -\frac{H}{\bar{\rho}_\gamma} (\delta Q_{\text{intr.,}\gamma} + \delta Q_{\text{rel.,}\gamma}). \]  \hspace{1cm} (2.26)
where

\[
\delta Q_{\text{intr},\phi} \equiv \delta Q_{\phi} - \frac{\dot{Q}_\phi}{\rho_\phi} \delta \rho_\phi,
\]

\[
\delta Q_{\text{intr},\gamma} \equiv \delta Q_{\gamma} - \frac{\dot{Q}_\gamma}{\rho_\gamma} \delta \rho_\gamma,
\]

(2.27)

are the gauge-invariant intrinsic non-adiabatic energy transfer perturbations, which are automatically vanishing if the local energy transfer functions \(Q_i (i = \phi, \gamma)\) are functions of the local energy density \(\rho_i\), and

\[
\delta Q_{\text{rel},\phi} = \frac{Q_\phi \dot{\rho}}{2\rho} \left( \frac{\delta \rho_\phi}{\rho_\phi} - \frac{\delta \rho}{\rho} \right) = -\frac{Q_\phi}{6H \rho} \dot{\rho}_\gamma S_{\phi\gamma},
\]

\[
\delta Q_{\text{rel},\gamma} = \frac{Q_\gamma \dot{\rho}}{2\rho} \left( \frac{\delta \rho_\gamma}{\rho_\gamma} - \frac{\delta \rho}{\rho} \right) = -\frac{Q_\gamma}{6H \rho} \dot{\rho}_\phi S_{\gamma\phi}.
\]

(2.28)

are the gauge-invariant relative non-adiabatic energy transfer due to the presence of relative entropy perturbations [24].

**2.2 Perturbing the inflaton decay rate**

Allowing for a perturbed decay rate \((\delta \Gamma \neq 0)\) and for a possible time variation of \(\Gamma (\dot{\Gamma} \neq 0)\), the perturbed energy transfer is simply given by

\[
\delta Q_{\phi} = -\Gamma \delta \rho_\phi - \delta \Gamma \rho_\phi,
\]

\[
\delta Q_{\gamma} = \Gamma \delta \rho_\phi + \delta \Gamma \rho_\phi.
\]

(2.29)

(2.30)

The corresponding intrinsic non-adiabatic energy transfer terms from the inflaton are given by

\[
\delta Q_{\text{intr},\phi} = -\delta \Gamma \rho_\phi + \Gamma \frac{\dot{\rho}_\phi}{\rho_\phi} \delta \rho_\phi,
\]

\[
\delta Q_{\text{intr},\gamma} = \delta \Gamma \rho_\phi + \Gamma \delta \rho_\phi - \frac{\dot{\rho}_\phi}{\rho_\gamma} \delta \rho_\phi - \frac{\dot{\rho}_\phi}{\rho_\gamma} \delta \rho_\gamma.
\]

(2.31)
The relative non-adiabatic energy transfer terms are given by

$$
\delta Q_{\text{rel},\phi} = -\frac{\Gamma_{\phi}}{2\rho} \left( \frac{\delta \rho_{\phi}}{\hat{\rho}_{\phi}} - \frac{\delta \rho}{\hat{\rho}} \right),
$$

$$
\delta Q_{\text{rel},\gamma} = \frac{\Gamma_{\phi}}{2\rho} \left( \frac{\delta \rho_{\gamma}}{\hat{\rho}_{\gamma}} - \frac{\delta \rho}{\hat{\rho}} \right).
$$

(2.32)  

(2.33)

Thus the evolution equations (2.26) for the curvature perturbation on uniform inflaton density hypersurfaces, $\zeta_{\phi}$, and uniform radiation density hypersurfaces, $\zeta_{\gamma}$, are given by

$$
\dot{\zeta}_{\phi} = -\frac{\Gamma_{\phi}}{6} \frac{\hat{\rho}_{\gamma}}{\rho_{\phi}} S_{\phi\gamma} + H \frac{\rho_{\phi}}{\rho_{\phi}} \delta \Gamma_{G\phi}^{G\phi},
$$

$$
\dot{\zeta}_{\gamma} = \frac{\Gamma_{\phi}}{3} \frac{\hat{\rho}_{\phi}}{\rho_{\gamma}} \left( 1 - \frac{\rho_{\phi}}{2\rho} \right) S_{\phi\gamma} - H \frac{\rho_{\phi}}{\rho_{\gamma}} \delta \Gamma_{G\phi}^{G\phi},
$$

where

$$
\delta \Gamma_{G\phi}^{G\phi} = \delta \Gamma - \frac{\hat{\Gamma}}{\hat{\rho}_{\phi}} \delta \rho_{\phi},
$$

$$
\delta \Gamma_{G\gamma}^{G\gamma} = \delta \Gamma - \frac{\hat{\Gamma}}{\hat{\rho}_{\gamma}} \delta \rho_{\gamma} = \delta \Gamma_{G\phi}^{G\phi} + \frac{\hat{\Gamma}}{H \hat{\rho}_{\gamma}} \left( \zeta - \zeta_{\phi} \right)
$$

(2.34)  

(2.35)

are the gauge-invariant perturbations of the inflaton decay rate. As anticipated in the Introduction, the gauge-invariant decay rate perturbation receives contribution from two sources: the fluctuation of the decay rate $\delta \Gamma$ (generated during the inflationary stage) and the time variation of the decay rate $\dot{\Gamma}$ during reheating.

Writing the total curvature perturbation on uniform total density hypersurfaces (2.22) as

$$
\zeta = f \zeta_{\phi} + (1 - f) \zeta_{\gamma},
$$

(2.37)

where

$$
f = \frac{3\Omega_{\phi} + 4\Omega_{\gamma}}{4\Omega_{\gamma} + 3\Omega_{\phi}},
$$

(2.38)

we can obtain the following system governing the evolution of the total curvature perturbation on uniform total density hypersurfaces and the curvature perturbation on uniform inflaton density hypersurfaces.
\[
\frac{\Omega}{(1-g)}(2-3)(1-g)(1-\Omega g) + \frac{(1-g)\Omega}{4(1-g)(1-\Omega g)-g\Omega}\right](\zeta-\zeta) \nonumber \]
\[
\zeta' = \frac{\Omega}{(1-g)}(2-3)(1-g)(1-\Omega g) + \frac{(1-g)\Omega}{4(1-g)(1-\Omega g)-g\Omega}\right](\zeta-\zeta) \nonumber \]
\[
\zeta' = \frac{g(4-\Omega)}{2(3-2g)}(\zeta-\zeta) - \frac{1-g}{3+g} \frac{\delta \Gamma}{H} . \tag{2.39} \]

In the standard inflationary scenario, \( \rho_\gamma \) is supposed to dominate the initial energy density after reheating and is assumed to be unperturbed at the end of inflation (beginning of the reheating stage) \( \zeta_{\gamma,\text{in}} = 0 \). Thus the curvature perturbation is initially (right after the end of inflation) an adiabatic density perturbation in the inflaton field. At this stage some comments are in order.

- From the set of equations (2.39), it is clear that during reheating, if \( \delta \Gamma_{\phi} = 0 \), the solution \( \zeta = \zeta_{\phi} \) is a fixed point attractor. Therefore, if at the end of inflation the total curvature perturbation \( \zeta \) is entirely provided by the curvature perturbation on uniform inflaton density hypersurfaces, \( \zeta_{\text{in}} = \zeta_{\phi,\text{in}} \), the total curvature perturbation \( \zeta \) remains constant on super-horizon scales. At the end of the reheating stage and beginning of the radiation phase, \( \zeta = \zeta_{\gamma} = \zeta_{\phi,\text{in}} \).

- If the inflaton decay rate \( \Gamma \) depends only upon the inflaton field, \( \Gamma = \Gamma(\phi) \), then the gauge-invariant perturbation of the decay rate reads

\[
\delta \Gamma_{\phi} = \frac{\partial \Gamma}{\partial \phi} \left( \frac{\delta \phi}{\dot{\phi}} - \frac{\delta \rho_\phi}{\dot{\rho}_\phi} \right) \tag{2.40} \]

Since the long-wavelength solutions for the vacuum fluctuations in the inflaton field obey the adiabatic condition \( \delta \phi/\dot{\phi} = \delta \phi/\ddot{\phi} \), the gauge-invariant perturbation of the decay rate vanishes identically. This implies that the source term proportional to \( \delta \Gamma_{\phi} \) for \( \zeta' \) in Eqs. (2.39) vanishes identically and \( \zeta = \zeta_{\phi} \) is a fixed point attractor during the reheating stage. The total curvature perturbation \( \zeta \) remains constant on super-horizon scales during reheating.

The same conclusion can be drawn if the decay rate is a function of the temperature associated to radiation. Suppose that the inflaton field is coupled to some fermion \( \psi \) (radiation) through the Yukawa coupling \( L_Y = h \bar{\psi} \psi / \phi \). This coupling allows the
inflaton field to decay into fermions with a decay rate \( \Gamma = \frac{h^2}{8\pi} M_\phi \sqrt{1 - 4M_\psi^2/M_\phi^2} \), where \( M_\phi \) and \( M_\psi \) are the inflaton mass during the coherent oscillations and the fermion mass, respectively. During the reheating stage, at very early times, \( t \ll \Gamma^{-1} \), the energy density of the universe is dominated by the scalar field \( \phi \) and the radiation density is negligible. As the scalar field decays into fermions, the decay products rapidly thermalize forming a plasma with temperature \( T \). The latter grows until it reaches a maximum value \( T_{\text{MAX}} \) and then decreases as \( T \propto a^{-3/8} \) up to the temperature \( T_{\text{RH}} \) at the time \( t \simeq \Gamma^{-1} \) which determines the end of reheating [29, 30].

The thermalized fermions, produced during the first stages of reheating, acquire a plasma mass of the order of \( gT \), where \( g \) is the typical (gauge) coupling governing the fermion interactions [31]. This happens because forward scatterings of fermions do not change the distribution functions of particles, but modify their free dispersion relations, producing a plasma mass. The decay rate is therefore a function not only of the inflaton field \( \phi \), but also of the temperature \( T \sim \rho_\gamma^{1/4} \). The gauge-invariant decay rate can be written as

\[
\delta \Gamma_{\phi}^{\text{GI}} = \delta \Gamma - \frac{\delta \rho_\phi}{\rho_\phi} \delta \rho_\gamma \left( \frac{\dot{\rho}}{\rho_\gamma} - \frac{\delta \rho_\phi}{\rho_\phi} \right) = \frac{\partial \Gamma}{\partial \rho_\gamma} \frac{\dot{\rho}}{H} (\zeta - \zeta_\phi) ,
\]

which shows that the source term coming from \( \delta \Gamma_{\phi}^{\text{GI}} \) for \( \zeta_\phi' \) in Eqs. (2.39) is proportional to \( (\zeta - \zeta_\phi) \) and therefore \( \zeta = \zeta_\phi \) remains a fixed point attractor during the reheating stage. The total curvature perturbation remains constant on super-horizon scales.

- If the inflaton decay rate depends upon the vacuum expectation value of another field \( \chi \) whose quantum fluctuations are excited during inflation and whose time variation during reheating is negligible, \( \dot{\chi} \simeq 0, \) one recovers the inhomogeneous reheating scenario [16, 18] with \( \delta \Gamma_{\phi}^{\text{GI}} = \delta \Gamma = (\partial \Gamma/\partial \chi) \delta \chi \) and \( \dot{\Gamma} = 0 \). During the reheating stage, when the inflaton field is still oscillating around the minimum of its potential and \( t \ll \Gamma^{-1} \), we can set \( \Omega_\phi \simeq 1, g \simeq 0 \). Eqs. (2.39) reduce to

\[
\zeta' \simeq - (\zeta - \zeta_\phi) ,
\]

\[
\zeta_\phi' \simeq - \frac{1}{3} \frac{\delta \Gamma}{H} ,
\]

(2.42)
which is easily solved to give (going back to cosmic time)

\[ \zeta \simeq \zeta_{\text{in}} - \frac{1}{3} \int_{t_{\text{in}}}^{t} dt' \frac{H(t')}{a(t')} \int_{t_{\text{in}}}^{t'} dt'' \frac{H(t'') a(t'')}{H(t'')} \frac{\delta \Gamma}{(t'')} , \quad (2.43) \]

where \( t_{\text{in}} \) denotes the initial time of the reheating stage. Since on super-horizon scales we may consider \( \delta \Gamma \) as a constant and \( a \propto t^{2/3} \) during the inflaton coherent oscillation phase, we obtain at the end of the reheating phase (which we set to be at \( t = \Gamma^{-1} \))

\[ \zeta \simeq \zeta_{\text{in}} - \frac{2}{15} \frac{\delta \Gamma}{\Gamma} . \quad (2.44) \]

Since deep in the radiation phase the gravitational potential is \( \psi_{\gamma} = -\frac{2}{3} \zeta \), we obtain

\[ \psi_{\gamma} = \frac{4}{45} \frac{\delta \Gamma}{\Gamma} \simeq \frac{1}{9} \frac{\delta \Gamma}{\Gamma} , \quad (2.45) \]

which confirms the findings of Refs. [16, 18], in the case where the initial curvature perturbation \( \zeta_{\text{in}} \) is tiny.

### 2.3 Effects of a time variation of the inflaton decay rate

Let us now analyze the new source of the variation of the total curvature perturbation on large scales proportional to \( \dot{\Gamma} \). We suppose that the decay rate is a function of time during the reheating stage, \( \dot{\Gamma} \neq 0 \), and that its fluctuations during inflation are vanishing, \( \delta \Gamma = 0 \). Under these circumstances, the gauge-invariant decay rate perturbation is given by

\[ \delta \Gamma_{\phi}^{\text{GI}} = -\dot{\Gamma} \frac{\delta \rho_{\phi}}{\rho_{\phi}} . \quad (2.46) \]

Solving Eqs. (2.39) (going back to cosmic time), we find the total curvature perturbation at the end of the reheating stage is given by

\[ \zeta_f = \zeta_{\text{in}} - \int_{t_{\text{in}}}^{t} dt' f(t') e^{\int_{t_{\text{in}}}^{t'} (f-g) dt'} \int_{t_{\text{in}}}^{t'} dt'' S(t'') e^{-\int_{t_{\text{in}}}^{t'} (f-g) dt''} , \quad (2.47) \]

where
\[ f = H \left[ \frac{\Omega_\phi (2g - 3)}{(1 - g) (4 - \Omega_\phi)} + \frac{\dot{\Gamma}}{H^2 4(1 - g)(1 - \Omega_\phi) - g \Omega_\phi} \right], \]
\[ g = \frac{g (4 - \Omega_\phi)}{2 (3 - 2g)} H, \]
\[ S = \frac{\dot{\Gamma} (1 - g)}{3 + g} \frac{\delta \rho_\phi}{\dot{\rho}_\phi}. \]  

Let us give an example. Suppose again that the inflaton field decays into some fermion \( \psi \) (radiation) through the Yukawa coupling \( \mathcal{L}_Y = \hbar \bar{\psi} \gamma^\mu \psi \phi \). The fermion mass \( M_\psi \) is a function of a scalar field \( S \),

\[ M_\psi = \lambda S. \]  

The potential of the scalar field \( S \) reads \( V(S) = \frac{1}{2} M_S^2 S^2 + C^2 \frac{H^2}{2} (S - S_0)^2 \), with \( C \gg 1 \). If during inflation \( H \gg M_S \), the field \( S \) is sitting at the position \( S = S_0 \) and its quantum fluctuations are not excited since its effective mass \( \sim CH \) is much larger than the Hubble rate. At the end of inflation, the inflaton starts oscillating with mass \( M_\phi \). If \( \lambda S > M_\phi / 2 \), the inflaton cannot decay since the mass of the fermion \( M_\psi \) is larger than \( M_\phi / 2 \) and the decay channel into fermions is kinematically forbidden. However, during the coherent oscillation phase, the Hubble parameter drops down as \( a^{-3} \). If the Hubble rate becomes smaller than \( C^{-1} M_S \), the scalar field \( S \) may start rolling down towards the minimum at \( S = 0 \) and settle there, thus decreasing the fermion mass. As soon as \( M_\psi = \lambda S \) becomes smaller than \( M_\phi / 2 \), say at some time \( t = t_* \), the inflaton decay channel into fermions becomes accessible and the decay rate rises from zero to a nonvanishing value \( \Gamma_0 = \frac{k^2}{8 \pi} M_\phi \). We can safely approximate the decay rate as \( \dot{\Gamma} = \Gamma_0 \theta (t - t_*) \) corresponding to \n
\[ \dot{\Gamma} = \Gamma_0 \delta (t - t_*) . \]  

From Eqs. (2.39), we expect that the total curvature perturbation suffers a jump at \( t = t_* \). Inserting this expression in Eq. (2.37) and working in the limit \( \Gamma_0 / H_* \lesssim 1 \), we find that the total curvature perturbation jumps from the initial value \( \zeta_{in} \) to the final value

\[ \zeta_f \simeq \left( 1 + \frac{4}{45} \frac{\Gamma_0}{H_*} \right) \zeta_{in}. \]  

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In the opposite limit $\Gamma_0/H_s \gtrsim 1$, we find

\[ \zeta_t \simeq \left( 1 + \frac{4}{5} \frac{H_s}{\Gamma_0} \right) \zeta_{\text{in}}. \tag{2.52} \]

One can envisage the alternative possibility that during the reheating stage the Hubble rate remains always larger than $C^{-1}M_S$. Under these circumstances, the field $S$ does not roll towards $S = 0$, but remains stuck to the minimum of its potential at $S = \frac{C H^2}{C^2 H^2 + M^2 S^2} S_0$. The location of the minimum, however, changes adiabatically with time, $\dot{S}/S \sim -6(M^2 S^2/C^2 H^2)$. Correspondingly, the decay rate $\Gamma = \Gamma_0 \sqrt{1 - 4 M^2 \psi M^2 \phi}$ changes with time as $\dot{\Gamma}/\Gamma_0 \sim 48 \lambda^2 S^2 M^2 \phi^2 C^2 M^2 S H$. Solving Eq. (2.47) gives

\[ \zeta_t \simeq \left[ 1 - \frac{2}{15} \left( \frac{\dot{\Gamma}}{\Gamma_0} \right) \right] \zeta_{\text{in}} \simeq \left[ 1 - \frac{96}{15} \frac{\lambda^2 S^2 M^2 \phi^2}{C^2 M^2 \phi^2 \Gamma_0} \right] \zeta_{\text{in}}. \tag{2.53} \]

We conclude that during the reheating stage the total curvature perturbation may be altered on super-horizon scales if the decay rate of the inflaton changes with time. Even though the amount of change is model dependent, the shift in the total curvature perturbation is always proportional to the value of the total curvature perturbation at the end of inflation and the beginning of the reheating stage $\zeta_{\text{in}}$.

### 2.3.1 Violation of the consistency relation

During inflation both scalar and tensor perturbations are generated. As we have seen in the previous section, the total curvature perturbation can change on super-horizon scale during reheating if the inflaton field decay rate is a function of time. The difference between the values of the total curvature perturbation at the end of reheating and at the beginning of inflation can be parametrized by $\Delta \equiv (\zeta_t - \zeta_{\text{in}})/\zeta_{\text{in}}$. The power spectrum of scalar curvature perturbations at the end of the reheating stage and the beginning of the radiation phase is therefore given by

\[ P_\zeta(k) = \frac{k^3}{2\pi^2} |\zeta_t|^2 = \frac{k^3}{2\pi^2} (1 + \Delta)^2 |\zeta_{\text{in}}|^2 = \frac{1}{8\pi^2} \frac{(1 + \Delta)^2}{\epsilon} \left( \frac{H(k)}{M_P} \right)^2, \tag{2.54} \]
where $M_P$ is the Planck mass, $H(k)$ indicates the value of the Hubble parameter when a
given wavelength $\lambda = 2\pi/k$ crosses the horizon, i.e., when $k = aH$, and $\epsilon = -\dot{H}/H^2$ is a
slow-roll parameter accounting for the time variation of the Hubble rate during inflation.

The primordial spectrum of gravitational waves is given by

$$P_T(k) = \frac{2}{\pi^2} \left( \frac{H(k)}{M_P} \right)^2.$$  
(2.55)

From expressions (2.54) and (2.55), we can predict a consistency relation between the
amplitude of the scalar perturbations, $P_\zeta(k)$, the amplitude of the tensor perturbations,
$P_T(k)$, and the tensor spectral index, $n_T \equiv d\ln P_T(k)/d\ln k$. Indeed, since $P_T(k) \propto H^2(k)$,
$n_T$ is given by $n_T = d\ln H^2(k)/d\ln k = -2\epsilon$ and the consistency relation, for $|\Delta| \ll 1$,
reads

$$\frac{P_T(k)}{P_\zeta(k)} = 16 (1 - 2\Delta) \epsilon = -8 (1 - 2\Delta) n_T.$$  
(2.56)

This consistency relation differs from the traditional one, $P_T/P_\zeta = -8n_T$, obtained for
one-single field models of inflation [32] when the inflaton decay rate during reheating does
not change with time. Notice that, if $\Delta < 0$, the tensor-to-scalar amplitude ratio can
be larger than for one-single field models of inflation. Departures from the traditional
consistency relation can be caused by other reasons: higher-order terms in the expansion in
slow-roll parameters [32], quantum loop corrections [33] or the presence of multiple fields
during inflation [34]. Given CMB B-mode polarization measurements, departures from the
traditional consistency relation can be detected if the tensor-to-scalar amplitude ratio is
larger than about $10^{-3}$ [35] and would rule out the simplest case of one-single field models
of inflation with no variation of the decay rate.

3 The nonperturbative inflaton decay

As we already mentioned, if the classical inflaton field $\phi$ very rapidly (explosively) decays
into either its own quanta or into other bosons due to broad parametric resonance, the
so-called preheating stage [28], Eqs. (2.8) should be modified.

Suppose the inflaton is coupled to a light scalar $\chi$ with coupling $\mathcal{L} = \frac{1}{2} g^2 \phi^2 \chi^2$. During
the coherent oscillations of the inflaton field, the $\chi$-quanta satisfy the so-called Mathieu
equation which leads to the existence of exponential instabilities $\chi_k \propto e^{\mu^{(n)}_k M_\phi t}$ within the set of resonance bands of frequencies $\Delta k^{(n)}$ labelled by an integer $n$ and (for the first few bands)

$$\mu^{(n)} = \frac{M_\phi}{2n} \frac{q^n}{(2^{n-1}(n-1)!)^2},$$

$$q = \frac{g^2 \phi^2}{M_\phi^2}.$$ (3.1)

For $q \lesssim 1$, preheating occurs mainly in the first resonance band with $\mu^{(1)} = q M_\phi / 2$ and a width $\Delta k^{(n)} \sim \mu^{(1)}$. For $q \gg 1$, however, many resonance bands are excited and preheating occurs in the broad resonant regime, with $\mu^{(n)}_k \sim 0.17$, independent of $n$. At the beginning of the preheating stage, the so-called linear stage, the $\chi$-quanta grow exponentially in time till back-reaction effects set in. They are originated by the fact that the inflaton effective mass squared $M_{\text{eff}}^2 = M_\phi^2 + g^2 \langle \chi^2 \rangle$ becomes dominated by the second term, thus decreasing the parameter $q$, and by the scattering of the produced particles off the zero mode \[36, 37, 38\].

During the linear stage, it is a good approximation to write the continuity equations (2.7) as

$$\dot{\rho}_\phi = -3H (\rho_\phi + P_\phi) - \mu \rho_\gamma,$$

$$\dot{\rho}_\gamma = -3H (\rho_\gamma + P_\gamma) + \mu \rho_\gamma,$$ (3.2)

where we have indicated by $\rho_\gamma$ the energy density of light degrees of freedom, the $\chi$-particles generated during the first stage of preheating and by $\mu$ the rate of production. Notice that $\mu$ is a function of time as clear from Eqs. (3.1).

The background equations (2.7) can be re-written as

$$\Omega_\phi' = \Omega_\gamma (\Omega_\phi - r),$$ (3.3)

$$\Omega_\gamma' = r \Omega_\gamma \Omega_\phi,$$ (3.4)

$$r' = \frac{r \mu'}{\mu} + \frac{r}{2} (4 - \Omega_\phi),$$ (3.5)
where we have set

\[ r = \frac{\mu}{H}. \quad (3.6) \]

The local energy transfer functions are now given by \( Q_\phi = -\mu \rho_\gamma \) and \( Q_\gamma = \mu \rho_\gamma \). Proceeding as in subsection 2.1, we can write the equations for the change of the curvature perturbation on uniform inflaton energy density and radiation energy density on super-horizon scales

\[
\dot{\zeta}_\phi = \frac{\mu}{3} \frac{\dot{\rho}_\gamma}{\rho_\phi} \left( 1 - \frac{1}{2} \frac{\rho_\gamma}{\rho} \right) S_{\rho\gamma} + H \frac{\rho_\gamma}{\rho_\phi} \delta \mu_{\phi}^{\text{GI}}, \quad (3.7)
\]

\[
\dot{\zeta}_\gamma = -\frac{\mu}{6} \frac{\rho_\gamma}{\rho} \frac{\dot{\rho}_\phi}{\rho_\gamma} S_{\rho\gamma} - H \frac{\rho_\gamma}{\rho_\phi} \delta \mu_{\gamma}^{\text{GI}}, \quad (3.8)
\]

where

\[
\delta \mu_{\phi}^{\text{GI}} = \delta \mu - \frac{\dot{\mu}}{\dot{\rho}_\phi},
\]

\[
\delta \mu_{\gamma}^{\text{GI}} = \delta \mu - \frac{\dot{\mu}}{\dot{\rho}_\gamma} \delta \mu_{\phi}^{\text{GI}} + \frac{\dot{\mu}}{H} \frac{\dot{\rho}_\phi}{\rho_\phi} (\zeta - \zeta_\phi) \quad (3.9)
\]

are the gauge-invariant perturbations of the inflaton decay rate during the linear stage of preheating.

The equations for the total curvature perturbation and the curvature perturbation on uniform inflaton energy density become

\[
\zeta' = \left\{ \Omega_\phi \left( 2r - \frac{3}{2} \right) - \frac{3(\Omega_\phi - r)(r - 4) + \frac{1}{2} r (4 - \Omega_\phi) - \frac{\rho_\phi}{\rho} [\Omega_\gamma (\Omega_\phi - 4) - 1]}{(r - 4)(4 - \Omega_\phi)} \right\} (\zeta - \zeta_\phi),
\]

\[
\zeta'_\phi = -\frac{r (4 - \Omega_\phi)}{(3 - r) \Omega_\phi + r} \left( 1 - \frac{1}{2} \frac{\Omega_\gamma}{\Omega_\phi} \right) (\zeta - \zeta_\phi) - \frac{\Omega_\gamma}{(3 - r) \Omega_\phi + r} \frac{\delta \mu_{\phi}^{\text{GI}}}{H}. \quad (3.10)
\]

From Eqs. (3.10), we conclude that if the resonance parameter \( \mu \) has either spatial or time variations, then the total curvature perturbation can be modified on large-scales. As a matter of fact, the parameter \( \mu \) is far from being constant during preheating. This
fact was not appreciated in previous studies of the effects of preheating on super-horizon perturbations [39]. If the nonperturbative inflaton decay rate $\mu$ depends only upon the inflaton field, $\mu = \mu(\phi)$, then the gauge-invariant perturbation of the decay rate $\delta \mu_{\phi}^{GI}$ vanishes identically and $\zeta = \zeta_{\phi}$ is a fixed point attractor during the preheating stage. The total curvature perturbation $\zeta$ remains constant on super-horizon scales during preheating. This happens, for instance, if Eq. (3.1) holds and $\mu \sim q^n \sim \phi^{2n}$. However, one can envisage other possibilities. For instance, as in the inhomogeneous reheating scenario, the coupling constant $g^2$ between the inflaton field $\phi$ and the light bosons $\chi$ can be a function of some other field which remains light during inflation. This leads to spatial fluctuations of the $q$-parameter. Tiny variations of $q$ lead to large variations of the efficiency in extracting energy out of the inflaton zero mode [37] and may have a dramatic effect on the final temperature fluctuations after the system thermalizes. This analysis will be presented in a separate publication [40].

4 Conclusions

We have studied the evolution of large-scale curvature perturbations during the reheating stage after inflation in the case in which spatial and temporal variations of the inflaton decay rate are present. We have shown that the total curvature perturbation $\zeta$ can change on large scales due to either spatial variations of the decay rate originated during inflation – the so-called inhomogeneous reheating scenario [16, 18] – or to a time variation of the decay rate during reheating. If only the latter source is present, the final curvature perturbation at the beginning of the radiation phase can be either smaller or larger than the curvature perturbation at the beginning of the reheating stage. This result leads to a violation of the consistency relation for one-single field models of inflation and to the observationally promising possibility that the tensor-to-scalar amplitude ratio is larger than in the standard scenario. Finally, we have presented the equations for the evolution of the total curvature perturbation and the curvature perturbation on uniform inflaton energy density hypersurfaces in the case in which the vacuum energy driving inflation is released into radiation through a (linear) stage of preheating.

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