Abstract—In robot swarms operating under highly restrictive sensing and communication constraints, individuals may need to use direct physical proximity to facilitate information exchange. However, in certain task-related scenarios, this requirement might conflict with the need for robots to spread out in the environment, e.g., for distributed sensing or surveillance applications. This paper demonstrates how a swarm of minimally-equipped robots can form high-density robot aggregates which coexist with lower robot densities in the domain. We envision a scenario where a swarm of vibration-driven robots—which sit atop bristles and achieve directed motion by vibrating them—move somewhat randomly in an environment while colliding with each other. Theoretical techniques from the study of far-from-equilibrium collective and statistical mechanics clarify the mechanisms underlying the formation of these high and low density regions. Specifically, we capitalize on a transformation that connects the collective properties of a system of self-propelled particles with that of a well-studied molecular fluid system, thereby inheriting the rich theory of equilibrium thermodynamics. This connection is a formal one and is a relatively recent result in studies of motility induced phase separation; it is previously unexplored in the context of robotics. Real robot experiments as well as simulations illustrate how inter-robot collisions may precipitate the formation of non-uniform robot densities in a closed and bounded region.

I. INTRODUCTION

Swarm robotic systems are comprised of robots that, though individually simple, aim to produce useful group-level behaviors via local interactions (see surveys [1], [2] and references within). For such swarm systems, a significant research focus has been on achieving emergent and self-organized collective behaviors via strictly local rules, e.g. [3].

Designing collective behaviors becomes especially challenging when considering scenarios where robots with limited sensing and communication capabilities perform tasks which require them to spread across an environment, e.g., for distributed sensing or environmental surveillance applications [4], [5]. Under these circumstances, the robots might require spatial proximity to facilitate information exchange among themselves, while simultaneously performing task related actions [6], [7].

In this paper, we highlight a mechanism to achieve co-existing regions of low and high robot density in swarms with severe constraints on sensing and communication. We demonstrate these behaviors on a team of vibration-driven robots, called brushbots [8], which achieve directed locomotion by vibrating bundles of flexible bristles. In particular, these robots do not possess sensors to detect other robots and simply traverse the environment while colliding with other robots. We illustrate that the mechanisms underlying the obtained density distributions can be explained using results from statistical mechanics, which investigates how macroscopic phenomena observed in physical systems can be related to the microscopic behaviors of constituent particles [9].

While equilibrium statistical mechanics provides an extensive vocabulary to describe macroscopic behaviors, much of the classical theory deals with idealized interactions among particles, limiting its applicability in swarm robotic systems [10]. However, the study of physical systems that are far from equilibrium has generated a great deal of recent excitement [11], given its ability to systematically analyze complex collectives in nature [12]. In these active matter systems, an interplay between self-propulsion, inter-particle effects, and environmental forces leads to a wide-variety of emergent behaviors [13], [14]. This paper takes advantage of a lesser-known formal connection between certain types of active matter systems and equilibrium thermodynamics to develop a microscopic description for a team of self-propelled brushbots, while retaining the extensive benefits of the classical thermodynamic theory.

We envision a team of brushbots moving somewhat randomly in a closed domain, while colliding with each other. The simultaneous formation of regions with lower and higher robot density is intuitively supported by two observations. Firstly, a given robot’s speed decreases with increasing robot density around it—a direct consequence of the inter-robot collisions experienced by the robot. Secondly, a system of particles—which are embodied by robots in this context—tend to accumulate in regions where they move more slowly [15]. Such a phenomenon is known as motility induced phase separation (MIPS) in the physics literature [16] and allows the formation of single or multiple high-density robot clusters which contain only a subset of the total robots in the domain. Recently established connections between such observations in active particle systems and equilibrium thermodynamics [16] allow us to impose design constraints on the motion characteristics of the robots displaying such behaviors.

A team of brushbots provides an ideal platform for achiev-
ing such variable density behaviors, since their minimalist construction makes them robust to the force experienced during inter-robot collisions even at relatively high speeds [8]. Additionally, the inherently noisy dynamics of the brushbots ensures that—similar to active matter systems—high-density robot clusters do not last infinitely long.

The outline of the paper is as follows. The next section makes more detailed connections to relevant literature. Section III introduces the stochastic differential equation describing the dynamics of each brushbot, and uses existing results on inter-robot collisions to derive a density dependent speed profile for each robot. In Section IV this model is leveraged to discuss the conditions under which motility induced phase separation can occur by drawing connections with an equivalent system of particles at thermal equilibrium. Simulations confirm the formation of robot aggregations for varying parameter ranges. In Section V the mechanism is deployed on a team of real differential-drive–like brushbots to illustrate the formation of high and low robot densities. Section VI concludes the paper.

II. RELATED WORK

A. Aggregation in Swarm Robotics

Within the robotics literature, several threads of work have examined how to get robots, under a variety of circumstances, to aggregate in certain places or at particular densities [17]–[20]. The problem of aggregation becomes especially relevant when the robots have basic sensing capabilities and limited computational resources, since physical proximity is then essential to enable more sophisticated swarm behaviors, e.g., [21]. In [22], the authors investigate the ability of robots with access to minimal but unrestricted range information to aggregate. [23] investigates how robots can achieve different packing arrangements based on the concept of different radii of interaction.

In contrast to the above methods which cluster robots into high-density groups, this paper investigates the maintenance of two different densities which must coexist over time. Thus, our approach emphasizes the notion of different phases, with distinct densities, for which intermediate densities are dynamically unstable and hence vanish. We demonstrate that the physical effects of inter-robot collisions cause the robots to slow down, precipitating the formation of such regions.

B. Active Matter Systems

Active matter physics deals with the study of self-driven particles which convert stored or ambient energy into organized movement, and can be used to describe systems as diverse as fish schools, colloidal-suspensions, and bacterial colonies (see surveys [12], [14] for a discussion on collective phenomena in active matter systems).

In particular, the study of motility induced phase separation focuses on how active matter systems consisting of self-propelled particles under purely repulsive interactions can spontaneously phase separate. In particular, it has been shown that, under suitable modeling assumptions, a direct connection can be made between such a system and a passive simple fluid at equilibrium with attractive interactions among the molecules. Equilibrium systems like these are well understood [24], thus allowing a large body of analysis to be transferred for the study of active self-propelled systems, as discussed in [16], [25], [26]. In this paper, we leverage this connection to demonstrate how a swarm of brushbots [8] with severely constrained sensing capabilities can phase separate into regions of high and low robot density.

III. MOTION AND COLLISION MODEL

A. Motion Model

As introduced in Section I the brushbot [8] does not possess sensors to detect other robots and is robust to collisions, making it an ideal platform on which to study the formation of collision-induced variable density aggregates. In this section, we first briefly motivate the dynamical model for this robot operating in an environment without the presence of other robots. Following this, we analyze the effects of interactions between multiple robots and characterize the velocity of the robots as a function of swarm density.

The differential-drive–like construction of the brushbots allows the motion of the robot to be expressed using unicycle dynamics with linear and angular velocity as the inputs. For a team of N robots, let \( z_i = (x_i, y_i) \) and \( \theta_i \) denote the position and orientation of robot \( i \in \{1, \ldots, N\} \), respectively. Each robot has a circular footprint of radius \( r \) and operates in a closed and bounded domain \( V \subset \mathbb{R}^2 \).

As discussed in [8], the presence of manufacturing differences as well as the nature of bristle-based movement implies that the motion of the brushbots is somewhat stochastic. This can be modeled in the form of additive noise terms affecting the translational and rotational motion of the robots, with diffusive coefficients \( D_t \) and \( D_r \), respectively. Under this model, the state \((z_i, \theta_i)\) evolves according to standard Langevin dynamics, which is a stochastic differential equation, given componentwise as

\[
\begin{align*}
    dx_i &= v_0 \cos \theta_i + \sqrt{2D_t} \Delta W_x, \\
    dy_i &= v_0 \sin \theta_i + \sqrt{2D_t} \Delta W_y, \\
    d\theta_i &= \sqrt{2D_r} \Delta W_\theta,
\end{align*}
\]

where \( v_0 \) is the constant self-propelled speed of the robot and \( \Delta W_x, \Delta W_y, \Delta W_\theta \) denote Wiener process increments representing white Gaussian noise. The total distance traveled by the robot will depend on the diffusion parameters \((D_t, D_r)\) and the speed \( v_0 \). The effective diffusion coefficient of the robot [12], measured on time scales longer than the time required for the orientation of the robot to uncorrelate (defined as \( \tau_r = D_r^{-1} \)), can be quantified as

\[
D = \frac{v_0^2}{2D_r} + D_t.
\]

Figure I shows the trajectories of a robot for different values of the diffusivity parameter, \( D \), illustrating its impact on the displacement of the robot over time.
An important predictor of phase separation behavior in interacting systems is the activity parameter, e.g., [25], [26], which we define here as

$$A = \frac{v_0}{2rD_r}.$$  \hspace{1cm} (3)$$

The activity parameter is proportional to the distance traveled by a robot before its direction uncorrelates completely [25], and will be used to characterize the proportion of high and low robot density regions in Section IV. In Section IV we use robotic experiments to empirically determine the diffusion coefficients $D_l$ and $D_r$ of the brushbot, but for now, we simply assume that these parameters are available to us and are constant throughout the swarm.

B. Inter-Robot Interaction Models

In the previous section, we described the dynamics of a single robot moving with a self-propelled speed $v_0$ through the domain. Given a team of $N$ brushbots moving randomly according to these dynamics in the domain $\mathcal{V}$, we now develop a model to describe the effects of inter-robot collisions on the average speed of the robots.

For a team of colliding robots, average robot speeds reduce with increasing density in the region around the robot—a direct consequence of the density-dependent collision rates [21]. The distribution of robots over the domain can be described in terms of the "coarse-grained" density $\lambda : \mathcal{V} \to \mathbb{R}_+$, obtained by overlaying a suitable smoothing function, e.g. [25], over the microscopic density operator,

$$\sum_{k=1}^{N} \delta(z - z_k),$$ \hspace{1cm} (4)$$

defined at each point $z \in \mathcal{V}$. Here $\delta$ denotes the Dirac delta function. We defer the actual computational details of the coarse-grained density to Section III-C which discusses the simulation results.

A given robot $i$ will experience varying collision rates depending on the density of robots surrounding it. The developed model is agnostic to which robot we pick since the following mean-field analysis applies to any robot in the domain [27]. Then, the speed of a given robot $i$ is a function of its location as well as the robot density over the domain $\lambda$. This dependence is formalized as $v(g(\lambda, z_i)) \in \mathbb{R}_+$, where $g : \mathcal{F} \times \mathcal{V} \to \mathbb{R}$, and $\mathcal{F}$ denotes the space of functions to which $\lambda$ belongs. The following assumption simplifies the dependence of the robot speed on the robot density.

**Assumption 1.** Assume that the coarse-grained density of robots $\lambda$ varies slowly over the domain,

$$\left| \frac{\partial \lambda}{\partial z} \right| \ll 1, \forall z \in \mathcal{V}.$$  \hspace{1cm} (1)$$

Furthermore, let the speed of robot $i$ depend only on the densities in some neighborhood of the robot location $z_i$. Then, the speed of the robot can be assumed to depend only on the density at the robot’s location, i.e., the function $g$ is simplified as

$$g(\lambda, z_i) \approx \lambda(z_i).$$

Consequently, we denote the speed of robot $i$ as simply $v(\lambda(z_i))$. This assumption allows us to develop an analytical expression for the speed of a robot at a given location, as a function of the robot density at that location. Using the inter-robot collision model developed in our previous work [21], the expected time between collisions experienced by robot $i$ at its current location $z_i$ is given as,

$$\tau_c(\lambda(z_i))^{-1} = 4r\frac{4}{\pi}v_0\lambda(z_i),$$ \hspace{1cm} (5)$$

where $r$ denotes the radius of each robot. The modified speed term $(4/\pi)v_0$ is equal to the mean relative speed between all the robots [21].

Furthermore, let $\tau_m$ denote the expected time spent by a robot in a collision with another robot before re-attaining its self-propelled speed $v_0$. This process occurs purely via forces acting on the robots as well as the rotational diffusion of each robot (as dictated by the rotational diffusion coefficient $D_r$ in (1)). For instance, the rotational diffusion might cause the robots to eventually move in different directions effectively resolving the collision. In Section III-C for a choice of diffusion parameters, we use a team of simulated brushbots to empirically compute the average speed and estimate $\tau_m$ as the parameter value which best fits the speed data.

The robot is expected to travel at speed $v_0$ between collisions, and effectively remain stationary during the collision. Consequently, the average speed can be expressed as

$$v(\lambda(z_i)) = v_0 \left( 1 - \frac{\tau_m}{\tau_c(\lambda(z_i)) + \tau_m} \right),$$  \hspace{1cm} (6)$$

Under the simplifying assumption that the time to resolve collisions is smaller than inter-collision time intervals, i.e., $\tau_m \ll \tau_c$, (which is valid at intermediate densities), and substituting from (5) the expression for $\tau_c$, (6), can be cast into the general form

$$v(\lambda(z_i)) = v_0 \left( 1 - \frac{\lambda(z_i)}{\lambda^*} \right),$$  \hspace{1cm} (7)$$
where
\[ \lambda^* = \left(4r^4 \frac{4}{\pi} \tau_m \right)^{-1} \] (8)
represents the extrapolated density at which the speed becomes zero, and can be interpreted as the packing density of robots in the domain [16]. Next, we introduce a simulation setup which verifies the linear dependency of velocity on local robot density (as predicted by (5), (7) and (8)), and gives us numerical estimates for the collision resolution time \( \tau_m \).

C. Simulation Setup

We validate the density-dependent speed model using a simulated team of brushbots operating in a closed and bounded rectangular domain \( \mathcal{V} \). The simulation consists of \( N \) disks of a given radius \( r \) moving according to the dynamics described by (1). During collisions, excluded volume constraints—a consequence of the fact that robots are not inter-penetrable—are imposed via a force acting on the robots, as is common in the physics literature, e.g., [26]. In order to avoid the influence of boundary effects, periodic boundary conditions are applied, i.e., when robots exit from one side of the domain, they reappear on the other side. In Section V when performing experiments on the actual brushbots, we allow the robots to turn around when they reach the boundaries of the domain.

For varying robot densities, Fig. 2 plots the speed of the robots averaged over all the robots. This simulation is performed for a uniform distribution of robots over the domain to prevent the averages from being affected by variable high and low density regions. A uniform distribution was ensured by selecting the activity parameter \( \mathcal{A} \), defined in (3), well below the values which would lead to phase separation (see supplementary material for [25]). The average speed of the swarm at each point in time was computed by projecting the instantaneous velocity of each robot along its current orientation vector \( \Theta_i = [\cos \theta_i, \sin \theta_i]^T \),
\[ \hat{v}(t) = \frac{1}{N} \sum_{i=1}^{N} z_i(t)^T \Theta_i(t). \]

As seen in Fig. 2, the average robot speed decreases linearly with density, as predicted by (5) and (7). Deviations at high densities can be justified by the violation of the assumption that \( \tau_m \ll \tau_c \). In the next section, we illustrate how the slowdown of robots due to collisions can actually lead to phase separated regions of high and low robot densities, as predicted by equilibrium thermodynamics.

IV. MOTILITY-INDUCED PHASE SEPARATION

In the previous section, we analyzed the velocity profile of robots interacting purely via inter-robot collisions. In this section, we provide a brief summary of important results in the active matter literature which establish an equivalence between such a swarm of robots and a passive Brownian molecular system at equilibrium (see [16] for further details). This equivalence allows us to specify the system parameters under which the swarm can be expected to phase separate and form regions of unequal robot density.

A. Theoretical Analysis

We begin the analysis by studying the probability distribution of a single particle whose speed, denoted as \( v(\lambda(z)) \), varies spatially over the domain according to the robot density. For a single robot obeying the Langevin dynamics in (1), the probability density \( \phi \) describing the location of a robot over the domain evolves according to the following equation, as given in [28],
\[ \phi(z) = \nabla \cdot J, \]
\[ J = -D(\lambda(z)) \nabla \phi(z) + V(\lambda(z)) \phi(z), \forall z \in \mathcal{V}, \]
where \( \nabla \cdot \) denotes the divergence operator, \( \nabla \) denotes the gradient, and \( D \) is now the spatially varying diffusion coefficient of the robot, modified from (2) as
\[ D(\lambda(z)) = \frac{v^2(\lambda(z))}{2D_{t}} + D_t. \]

The drift-velocity \( V \) satisfies the relation
\[ \frac{V(\lambda(z))}{D(\lambda(z))} = -\left(1 + \frac{2D_tD_t}{v^2(\lambda(z))}\right)^{-1} \nabla \ln v(\lambda(z)). \] (9)

From these equations, the coarse-grain density of robots interacting with each other has been shown [16] to evolve according to the following stochastic differential equation
\[ \dot{\lambda}(z) = -\nabla \cdot \left( -D\nabla \lambda(z) + V \lambda(z) + \sqrt{2D\lambda(z)} \Lambda \right). \] (10)
where \( \Lambda \) represents white Gaussian noise interpreted in the Itô sense [16]. We suppress the explicit dependence of \( D \) and \( V \) on \( \lambda(z) \) for readability.

The steady state probability distribution of the density, \( P_{\text{eq}}(\lambda) \), can be obtained by solving for the equilibrium solution of the corresponding Fokker-Planck equation,
\[ \left[ V \lambda - D \nabla \lambda - D \lambda \left( \nabla \frac{\partial}{\partial \lambda} \right) \right] P_{\text{eq}}(\lambda) = 0. \] (11)
The following theorem, initially presented in [29] and summarized in [16], illustrates how a swarm of robots interacting via collision interactions can be mapped to a system of passive Brownian particles at equilibrium.

**Theorem 1.** ([16]) A team of robots operating in a domain \( \mathcal{V} \), satisfying the dynamics in (11) and interacting via purely inter-robot collisions, can be expressed as a system of passive Brownian particles with an attractive potential if the following conditions are satisfied:

1. Assumption 1 is valid.
2. The coarse-graining of the microscopic density operator given by (4) is valid and satisfies the dynamics given by (10).

Under these conditions, the free energy density of the system can be expressed as,

\[
 f(\lambda(z)) = \lambda(z)(\ln(\lambda(z)) - 1) + \int_0^{\lambda(z)} \frac{1}{2} \ln \left( \frac{v^2(s)}{D_r} + 2D_t \right) ds, \quad \lambda(z) \leq \lambda^*.
\]  

(12)

where \( \lambda^* \) is given by (3).

**Proof Sketch.** An equivalence between the self-propelled robot swarm and a passive Brownian particle system at equilibrium is made by expressing the steady-state solution of (11) as obeying the equilibrium Boltzmann distribution [24] over the domain, given as

\[
 \mathcal{P}_{eq} \propto \exp(-\mathcal{F}),
\]

(13)

where \( \mathcal{F} = \mathcal{F}_{ent} + \mathcal{F}_{vel} \) is the free energy of the system, expressed in terms of the free energy density functional \( f \),

\[
 \mathcal{F} = \int_{\mathcal{S}} \mathcal{F}(\lambda(z)) \, dz.
\]

The free energy density, given by,

\[
 f(\lambda(z)) = \lambda(z)(\ln \lambda(z) - 1) + f_{vel}(\lambda(z)),
\]

comprises of two parts: an ideal entropy contribution (contributing to \( \mathcal{F}_{ent} \)) and an excess energy density \( f_{vel} \) (contributing to \( \mathcal{F}_{vel} \)). The latter, which would stem from the attractive potential in a passive Brownian system, in fact stems from the density dependent velocity profiles of the robots. To satisfy (13), the following integrability condition results (discussed in [29]),

\[
 V(\lambda(z)) = \frac{\partial \mathcal{F}_{vel}}{\partial \lambda}.
\]

Substituting from (9), and observing that \( \nabla \ln v = v^{-1} \nabla v \), this condition can be re-written as,

\[
 \frac{v \nabla v \tau_r}{v^2 \tau_r + 2D_t} = \frac{\partial \mathcal{F}_{vel}}{\partial \lambda},
\]

(14)

where \( \tau_r = D_r^{-1} \). Given Assumption 1 which implies that the speed of each robot only depends on the local density, (14) is satisfied by the following speed-derived free energy density

\[
 f_{vel}(\lambda) = \int_0^{\lambda} \frac{1}{2} \ln \left( \frac{v^2(s)}{D_r} + 2D_t \right) ds,
\]

under the constraint that the density \( \lambda \) can never exceed the close packed value given by (8). This leads to the desired result.

Given the equivalence established by Theorem 1 we can analyze the phase separation properties of the robot swarm by analyzing the free-energy density functional in (12). The following observations summarize some theoretical characterizations of the conditions under which such a system can be expected to phase separate.

**Observation 1.** In classical thermodynamics [24], spontaneous separation of a particle system into regions of high and low density occurs when concavities exist in the free energy density \( f \) defined in (12). More specifically, when the local density \( \lambda(z) \) at a point \( z \in \mathcal{V} \) is such that \( f''(\lambda(z)) < 0 \) where \( '' \) denotes a derivative with respect to the density \( \lambda \), small fluctuations cause the system to phase separate into regions with densities which result in a reduction in free energy density. This process is called spinodal decomposition.

Given (12), it is a simple exercise to verify the conditions on the density-dependent velocity of the robots which favor occurrence of spinodal decomposition in the system [16],

\[
 f''(\lambda(z)) < 0 \iff \frac{v(\lambda(z))^2}{D_r} \left( 1 + \lambda(z) \frac{v'(\lambda(z))}{v(\lambda(z))} \right) < -2D_t.
\]

Substituting the expression of density dependent speed from (7), we can furthermore make the following observation regarding the conditions on robot density and parameters under which the swarm can be expected to achieve non-uniform density distributions.

**Observation 2.** For a team of brushbots with velocity dependent speed given in (7), the spinodal densities (at which \( f''(\lambda) = 0 \), represented as \( \lambda_s^\pm \), are given as

\[
 \lambda_s^\pm = \frac{\lambda^*}{4} \left( 3 \pm \sqrt{1 - 16D_tD_r/v_0^2} \right),
\]

where \( \lambda^* \) is specified in (8). Consequently, these spinodal points only exist when the following condition is satisfied by the robot parameters:

\[
 v_0 > \sqrt{16D_tD_r},
\]

(15)

Since robot densities cannot exceed the packing density \( \lambda^* \), the following ranges of densities,

\[
 \lambda_s^- \leq \lambda(z) \leq \min(\lambda_s^+, \lambda^*), \quad z \in \mathcal{V}
\]

(16)

favor spontaneous phase separation. That is, when the local density in a region of the domain lies between these values, the swarm can be expected to spontaneously phase separate into regions with low and high robot densities.

**B. Simulations**

We observe the formation of dynamic robot clusters in a rectangular domain with periodic boundary conditions using the simulation setup described in Section III-C. Fig. 3 illustrates the intermediate-time snapshots of the phase separating
swarm robotic system. The robots are placed according to a uniform distribution in the domain at time $t = 0$ (see Fig. 3a) and simply move according to the Langevin dynamics described in Section III-A while experiencing collisions among each other. Figure 3b depicts how a uniformly distributed swarm of robots can form regions of high and low robot density, primarily caused by the density-dependent slow down of robots which precipitates the formation of dynamic high-density robot clusters.

We characterize the simultaneous existence of regions with higher and lower robot densities by computing the coarse-grained density $\lambda$ over the domain,

$$\lambda(z) = \frac{1}{N} \sum_{i=1}^{N} w(||z - z_i||), \forall z \in V,$$

where the weighting function $w$ is given as,

$$w(d) = \exp(-d^2/(d_c^2 - d^2)).$$

Here, $d_c$ is the cutoff distance at which $w(r) \rightarrow 0$. In simulation, the coarse-grained density was evaluated on a grid of $l^2$ lattice points, with cutoff distance $d_c = 0.8l$. For a grid size $l = 10$, Fig. 4 plots the empirically obtained distribution of the coarse-grained densities evaluated at the lattice points. These values were obtained by applying a Gaussian kernel-smoothing on the histogram of coarse-grained densities over the grid. For each set of activity parameters, data points were collected from multiple simulations of the robot swarm, to average out effects due to initial conditions. As seen, the distribution of robot densities is unimodal at low activity levels and becomes distinctly bimodal at higher activities, indicating the formation of low and high robot densities in the domain.

To quantify the formation of clusters in the swarm, we measure the time-averaged fraction of robots that belong to high-density aggregations in the swarm. An aggregation is defined as robots in physical contact with each other, whose total size exceeds a minimum cut-off value $N_c$. Figure 5 illustrates the impact of the activity parameter $A$ of the robots, by plotting the average aggregation fraction for increasing activity values and for different mean robot densities in the domain (denoted as $\lambda = N/|V|$ where $|V|$ denotes the area of domain V). As seen, no significant robot aggregation occurs below a certain activity threshold of the robots, regardless of the density. This observation is in agreement with the lower activity thresholds for observed phase separation in major studies, e.g., [16]. Figure 6 illustrates the need for a sufficient robot density to see phase separation behaviors, as predicted by (16).

V. DEPLOYMENT AND EXPERIMENTS

In this section, we first identify the noise parameters of a single brushbot and discuss the implications of these
parameters on the phase separation properties of the swarm. Following this, we illustrate the emergence of dynamic regions of low and high robot density using a team of 26 brushbots operating in a confined rectangular environment.

In order to identify the translational and rotational diffusion coefficient of the brushbots, we excited one brushbot with a constant self-propelled linear speed and zero angular velocity in a confined rectangular space. Position and orientation data was collected using an overhead tracking system. When a robot encountered the walls of the domain, the data collection was terminated until the robot turned around to traverse the environment again. Data was collected for a total time of 300s at a self-propelled speed \( v_0 = 6 \text{ cm/sec} \).

The rotational diffusion coefficient can be computed by measuring the mean-square variation in the orientation of a robot [30],

\[
D_r = \frac{1}{2} \lim_{t \to \infty} \frac{d}{dt} \langle (\theta(t) - \theta(0))^2 \rangle,
\]

where \( \langle (\theta(t) - \theta(0))^2 \rangle \) represents the mean-square orientation change in a time duration \( t \), averaged over multiple experimental runs. We numerically estimate \( D_r \) by performing a linear regression on the mean-square variations in the orientation of the robot for varying time-intervals (estimated value \( D_r = 0.0041 \)). Noticeably, the value for the translational diffusion coefficient \( D_t \) as computed from these experiments was negligible within numerical precision (computed using the Green-Kubo method [30]).

These estimates of the noise characteristics of a brushbot, allows us to make the following observation regarding the ability of a swarm of brushbots to spontaneously phase separate

**Observation 3.** Based on the robot noise parameters identified \( (D_r = 0.014, D_t = 0) \), the swarm of brushbots can be expected to spontaneously form regions of low and high robot density as long as the number of robots are high enough to achieve the local densities described in [16].

It should be noted that the identification of diffusion parameters for the brushbots performed above is not meant to serve as a quantitative analysis of the noise properties of the brushbots, but to understand the qualitative effects of these values on the formation of lower and higher density regions in the swarm. Indeed, since the effects from translational noise are minimal, the exact values of the noise parameters do not play a role in the prediction of motility-induced phase separation.

Figure 7 illustrates intermediate-time snapshots of 26 brushbots achieving regions of varying robot density in an enclosed square domain. A constant speed and zero angular velocity input was supplied to the robots. Specular reflection boundary conditions were imposed by injecting the robots with an angular velocity when they hit the boundary of the domain, reorienting them towards the interior of the domain.

As expected, collisions caused the robots to slow down, which precipitated the formation of high density regions. Robot clusters were identified based on physical contact among the robots, beyond a minimum threshold size (chosen as 4). We quantitatively analyzed the formation of these clusters by plotting the aggregation fraction in the swarm over time. This represents the fraction of robots which belong to a cluster at any given point in time. As seen in Fig. 8 at least 20% of the robots were clustered in aggregations at any point of time, while the rest of the robots moved freely through the domain.

**VI. Conclusions**

This paper demonstrates a mechanism to achieve regions of non-uniform density on a team of brushbots which possess no sensors to detect other robots but simply traverse the environment while colliding with each other. Analysis of similar self-propelled particle systems in the physics literature provide a theoretical basis for the formation of higher and lower robot density regions. We characterize the average speed of a robot as a function of the swarm density, and illustrate that certain conditions on this speed profile can lead to distinct regions of robot density. This emergent behavior was demonstrated on a team of 26 brushbots operating in a confined space while colliding with each other.

**References**

[1] E. Şahin, “Swarm robotics: From sources of inspiration to domains of application,” in *International workshop on swarm robotics*. Springer, 2004, pp. 10–20.

[2] M. Brambilla, E. Ferrante, M. Birattari, and M. Dorigo, “Swarm robotics: a review from the swarm engineering perspective,” *Swarm Intelligence*, vol. 7, no. 1, pp. 1–41, 2013.

[3] E. Bonabeau, M. Dorigo, and G. Theraulaz, *Swarm Intelligence: From Natural to Artificial Systems*. New York, NY, U.S.A.: Oxford University Press, Inc., 1999.

[4] K. H. Low, J. M. Dolan, and P. Khosla, “Active Markov Information-Theoretic Path Planning for Robotic Environmental Sensing,” in *Proceedings of International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, Taipei, Taiwan, May 2011.

[5] J. Elston, M. Stachura, E. Frew, and U. Herzelfeld, “Toward Model Free Atmospheric Sensing by Aerial Robot Networks in Strong Wind Fields,” in *Proceedings of the International Conference on Robotics and Automation (ICRA09)*, Kobe, Japan, May 2009, pp. 369–374.
N.S.F., "Condensed-matter and materials physics: The science of world randomly in the domain while colliding with each other. The reduction in beyond a cut-off size in physical contact with each other. The robots travel a team of real brushbots. An aggregation is defined as a collection of robots travel a team of 26 brushbots propelled at a constant speed in a domain. Reflective boundary conditions were applied by injecting an different regions of the domain.

Fig. 7. Snapshots for a team of 26 brushbots propelled at a constant speed in a domain. Reflective boundary conditions were applied by injecting an artificial life effect.

Fig. 8. Fraction of robots belonging to high density robot aggregations for a team of real brushbots. An aggregation is defined as a collection of robots beyond a cut-off size in physical contact with each other. The robots travel randomly in the domain while colliding with each other. The reduction in speed caused due to collisions precipitates the formation of simultaneous regions of low and high robot density as predicted by the phase separation theory in Section IV. As seen, the fraction of robots in aggregations remains fairly significant throughout the experiments.

[6] B. Tovar, L. Freda, and S. M. LaValle, “Using a robot to learn geometric information from permutations of landmarks,” Contemporary Mathematics, vol. 438, pp. 33–46, 2007.

[7] Y. Diaz-Mercado and M. Egerstedt, “Multi-robot mixing using braids,” in 52nd IEEE Conference on Decision and Control, IEEE, 2013, pp. 2001–2005.

[8] G. Notomista, S. Mayya, A. Mazumdar, S. Hutchinson, and M. Egerstedt, “A Study of a Class of Vibration-Driven Robots: Modeling, Analysis, Control and Design of the Brushbot,” arXiv e-prints, p. arXiv:1902.0380, Feb 2019.

[9] D. Chandler, “Introduction to modern statistical mechanics,” Introduction to Modern Statistical Mechanics, 1987.

[10] W. M. Spears, D. F. Spears, R. Heil, W. Kerr, and S. Hettiarachchi, “An Overview of Physicomimetics,” in Swarm Robotics. Springer Berlin Heidelberg, 2005, pp. 84–97.

[11] N.S.F. “Condensed-matter and materials physics: The science of world around us,” 2010.

[12] C. Bechinger, R. Di Leonardo, H. Löwen, C. Reichhardt, G. Volpe, and G. Volpe, “Active particles in complex and crowded environments,” Reviews of Modern Physics, vol. 88, no. 4, p. 045006, 2016.

[13] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, “Novel type of phase transition in a system of self-driven particles,” Physical review letters, vol. 75, no. 6, p. 1226, 1995.

[14] M. C. Marchetti, J.-F. Joanny, S. Ramaswamy, T. B. Liverpool, J. Prost, M. Rao, and R. A. Simha, “Hydrodynamics of soft active matter,” Reviews of Modern Physics, vol. 85, no. 3, p. 1143, 2013.

[15] M. J. Schnitzer, “Theory of continuum random walks and application to chemotaxis,” Physical Review E, vol. 48, no. 4, p. 2553, 1993.

[16] M. E. Cates and J. Tailleur, “Motility-induced phase separation,” Annu. Rev. Condens. Matter Phys., vol. 6, no. 1, pp. 219–244, 2015.

[17] M. Cates, R. Gross, T. H. Labelle, E. Sahin, and M. Dorigo, “Evolving aggregation behaviors in a swarm of robots,” in European Conference on Artificial Life. Springer, 2003, pp. 865–874.

[18] O. Soysal and E. Sahin, “Probabilistic aggregation strategies in swarm robotic systems,” in Proceedings IEEE Swarm Intelligence Symposium (SIS’05), 2005, pp. 325–332.

[19] W. Ren, R. W. Beard, and E. M. Atkin, “A survey of consensus problems in multi-agent coordination,” in Proceedings of the American Control Conference (ACC’05). IEEE, 2005, pp. 1859–1864.

[20] S. Garnier, C. Jost, R. Jeanson, J. Gautrais, M. Asadpour, G. Caprari, and G. Theraulaz, “Aggregation behaviour as a source of collective decision in a group of cockroach-like-robots,” in Advances in artificial life. Springer, 2005, pp. 169–178.

[21] S. Mayya, P. Pierpaoli, G. Nair, and M. Egerstedt, “Localization in densely packed swarms using interrobot collisions as a sensing modality,” IEEE Transactions on Robotics, vol. 35, no. 1, pp. 21–34, Feb 2019.

[22] M. Gauci, J. Chen, W. Li, T. J. Dodd, and R. Groß, “Self-organized aggregation without computation,” International Journal of Robotics Research, vol. 33, no. 8, pp. 1145–1161, Jul. 2014.

[23] J. Chen, M. Gauci, M. J. Price, and R. Gross, “Segregation in swarms of e-puck robots based on the Brazil nut effect,” in International Conference on Autonomous Agents and Multiagent Systems (AAMAS’12), Jun. 2012, pp. 163–170.

[24] P. Atkins and J. De Paula, Physical chemistry for the life sciences. Oxford University Press, USA, 2011.

[25] J. Stenhammar, A. Tiribocchi, R. J. Allen, D. Marenduzzo, and M. E. Cates, “Continuum theory of phase separation kinetics for active brownian particles,” Physical review letters, vol. 111, no. 14, p. 145702, 2013.

[26] Y. Fily, S. Henkes, and M. C. Marchetti, “Freezing and phase separation of self-propelled disks,” Soft matter, vol. 10, no. 13, pp. 2132–2140, 2014.

[27] S. Mayya, P. Pierpaoli, G. Nair, and M. Egerstedt, “Collisions as information sources in densely packed multi-robot systems under mean-field approximations,” in Proceedings of Robotics: Science and Systems, Cambridge, Massachusetts, July 2017.

[28] M. Cates and J. Tailleur, “When are active brownian particles and run-and-tumble particles equivalent? consequences for motility-induced phase separation,” EPL (Europhysics Letters), vol. 101, no. 2, p. 20010, 2013.

[29] J. Tailleur and M. Cates, “Statistical mechanics of interacting run-and-tumble bacteria,” Physical review letters, vol. 100, no. 21, p. 218103, 2008.

[30] R. Marino, “Dynamics and thermodynamics of translational and rotational diffusion processes driven out of equilibrium,” Ph.D. dissertation, KTH Royal Institute of Technology, 2016.