Evaluation of the Modernization of Hydraulic Projects Management Compact-Center-Point Triangular Whitenization Weight Function

Li Lijie¹, Wang Xiao², Zhang Lina³
Business School, Hohai University, No. 8 Focheng West Road, Jiangning District, Nanjing, Jiangsu Province, China, Ph./Fax. 15850654585¹, 15151973522², 15996282279³
*Corresponding author, e-mail: lilijie81@126.com¹, wx101@hotmail.com², linazhangv@163.com³

Abstract
In the actual analysis of grey clustering evaluation, it has been found that the length of the grey clustering interval is partially larger, which is determined by the method of grey clustering evaluation based on the center-point triangular whitenization weight function. In order to solve this problem, a new grey evaluation method based on reformative triangular whitenization weight function is researched. The existing end-point triangular whitenization weight function and center-point triangular whitenization weight function are revised, and a new compact-center-point triangular whitenization weight function is constructed. Then the rules for grey category interval of the three triangular whitenization weight functions are compared, and an example about the evaluation of the modernization of hydraulic projects management is proposed for analyzing the three methods to further verify that the improved grey clustering evaluation method based on the compact-center-point triangular whitenization weight function is feasible and effective. The results show that the compact-center-point triangular whitenization weight function is superior to both the end-point triangular whitenization weight function and the center-point triangular whitenization weight function.

Keywords: grey clustering evaluation, triangular whitenization weight function, hydraulic projects, modernization

1. Introduction
This paper is based on the grey system theory which was first put forward by J.L. Deng in 1982 [1]. This theory has been applied in grey clustering evaluation analysis to solve such uncertain problems with poor information and small sample [2]. As one of the most concerned models in recent years, the grey clustering evaluation model based on triangular whitenization weight function has been widely used in such fields as economics, management and engineering technology. And it has attracted many scholars to do related researches, but the studies are still limited. The existing researches mainly emphasize more on application than on theory. At present, the construction method of triangular whitenization weight function is still in a development stage.

Based on the existing studies, there are still some deficiencies of the end-point triangular whitenization weight function (ETWF hereafter) and the center-point triangular whitenization weight function (CTWF hereafter) in practical application, such as complexity of computing and identifying the endpoints of grey clustering intervals. In order to solve these problems and make a contribution to this area, this paper proposes an improved construction model of triangular whitenization weight function, namely the compact-center-point triangular whitenization weight function (CCTWF hereafter). Then taking the evaluation index system of a reservoir in Hubei province as a case study, this new model is further proved to be feasible.

This paper includes five parts. The first part is the introduction of the whole paper. The second part expounds the construction of CCTWF. In the third part, this paper makes a comparison among the three kinds of triangular whitenization weight functions in practical use. In section four, a case study is conducted to further verify the feasibility and validity of the grey clustering evaluation model based on CCTWF. In the last part, this paper summarizes its major findings and conclusions.

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2. Construction of CCTWF

S. F. Liu et al. have constructed the grey clustering evaluation method based on ETWF and that based on CTWF [3,4]. In practical use, the division of grey clustering intervals in these existing triangular whitenization weight functions is short of scientifcality. In response to this shortage, on the basis of analyzing the overlapping properties, the clustering coefficients, the division of grey clustering intervals and the selection of endpoints in these functions, this paper improves the existing functions and constructs CCTWF. The calculation procedure of the grey clustering evaluation model based on CCTWF is as follows:

Assuming there is an object set \( O = \{ O_i | i = 1, 2, \ldots, n \} \), which is clustered into different grey clusters of \( s \), and \( s \in \{1,2,3,4\} \). Then \( g = \{ g_j | j = 1, 2, \ldots, m \} \) is the evaluation index set of an object \( O_i \). \( x_{ij}, i = 1,2, \ldots, n; j =1,2, \ldots, m \) are the observation values of an object \( O_i \) for clustering index \( g_j \). The corresponding object \( O_i \) can be evaluated according to the observation value \( x_{ij} \). In order to describe it correctly, any object \( O_i \in O \) is taken as an example. The following is the procedure:

Step 1: Determine the \( \bar{\lambda}_{ij}^1, \bar{\lambda}_{ij}^2, \ldots, \bar{\lambda}_{ij}^s \) be the grey center points of the clustering index \( g_j \) of the object \( O_i \), and the value range allowed for \( x_{ij} \) is \( [a_{ij}^1, a_{ij}^{s+1}] \), thus we can get the center points \( \bar{\lambda}_{ij}^0, \bar{\lambda}_{ij}^{s+1} \) by extending grey clusters toward different directions.

Step 2: Let \( \Delta_k = \max \{ \lambda_{ij}^k - b_{ij}^k, b_{ij}^{k+1} - \lambda_{ij}^k \} \), \( \beta_k = \min \{ \Delta_k, \lambda_{ij}^k - \lambda_{ij}^{k-1} \} \), then we identify the grey interval of the cluster \( k \) is \( \left( (c_{ij}^k, (c_{ij}^{k+1}) \right] = \left[ \lambda_{ij}^k - \beta_k, \lambda_{ij}^k + \alpha_s \right] \). Special note: if \( \lambda_{ij}^k = \frac{b_{ij}^k + b_{ij}^{k+1}}{2} \), then \( (c_{ij}^k) = b_{ij}^k \), and \( (c_{ij}^{k+1}) = b_{ij}^{k+1} \).

Let the grey interval of the cluster \( k \) be \( \left( (c_{ij}^k, (c_{ij}^{k+1}) \right] \), connect the points \( (c_{ij}^k), (c_{ij}^{k+1}), (c_{ij}^1), 0 \), then we can get the triangular whitenization weight function of the index \( j \) on the grey cluster \( k \) is \( \bigg. f_{ij}^k(x_j), i=1,2,\ldots,n, j=1,2,\ldots,m, k=1,2,\ldots,s \).

For an observation value \( x_{ij} \) of the index \( j \), we can prove that its degree of membership to the grey cluster \( k \) \( (k=1,2,\ldots,s) \) is \( f_{ij}^k(x_j) \) by the following formulas:

\[
f_{ij}^k(x_j) = \begin{cases} 
0, & x_{ij} \not\in \left[ (c_{ij}^k, (c_{ij}^{k+1}) \right] \\
\frac{x_{ij} - (c_{ij}^k)}{\bar{\lambda}_{ij}^k - (c_{ij}^k)}, & x_{ij} \in \left[ (c_{ij}^k), (c_{ij}^k) \right] \\
\frac{(c_{ij}^{k+1}) - x_{ij}}{\lambda_{ij}^{k+1} - (c_{ij}^{k+1})}, & x_{ij} \in \left[ \lambda_{ij}^k, (c_{ij}^{k+1}) \right] 
\end{cases}
\] (1)
Step 3: The integrated clustering coefficients of the object $i(i=1, 2, \ldots, n)$ belonging to the grey cluster $k$ can be calculated by:

$$\sigma_i^k = \sum_{j=1}^{m} f_{ij}^k(x_{ij}) \eta_{ij}$$

(2)

Where $f_{ij}^k(x_{ij})$ is the triangular whitenization weight function of the index $j$ belonging to the grey cluster $k$, and $\eta_{ij}$ is the weight of the object $i$ belonging to the index $j$ in comprehensive clustering.

Step 4: Because of $\max_{1 \leq k \leq s} \sigma_i^k = \sigma_i^{k^*}$, we can say that the object $i$ belongs to the grey cluster $k^*$. It means that when more than one objects belong to the grey cluster $k^*$, we can sort these objects according to the size of the integrated clustering coefficients, and then determine the precedence or quality of each object which belongs to the grey cluster $k^*$.

3. Division of Grey Clustering Intervals

There is no pragmatic way of selecting ETWF’s end points $a_0, a_1, a_2, \ldots, a_s, a_{s+1}, a_{s+2}$, and the division of grey clusters is lack of scientific evidence.

Also, CTWF lets $\lambda_k^*$, which is most likely to belong to the grey cluster $k$, be the end point of that grey cluster, so it’s more apt to get each grey cluster’s triangular whitenization weight functions based on $\lambda_1^*, \lambda_2^*, \lambda_3^*, \ldots, \lambda_{s+1}^*$. [5, 6] In fact, in accordance with their thinking habits, people have more accurate understanding and judgment of grey clustering end points than those of grey clustering intervals, and CTWF is superior to ETWF on endpoints selection. But the division of grey cluster CTWF lacks scientificity.

Let $\lambda_1^*, \lambda_2^*, \ldots, \lambda_s^*$ be the grey center points of the clustering index $g_j$ of the object $O_i$, and the range of value allowed for $x_{ij}$ is $[a_i^j, a_i^{j+1}]$. According to the construction methods of CTWF and CCTWF, the grey intervals of the grey cluster $k$ are $[\lambda_y^{k-i}, \lambda_y^{k+i}]$ and $[c_{ij}^{k-i}, c_{ij}^{k+i}]$, respectively, and it is clear that $[c_{ij}^{k-i}, c_{ij}^{k+i}] = [\lambda_y^{k-i}, \lambda_y^{k+i}]$. Thus for the same grey interval, the grey interval length divided by CCTWF is smaller, the crossing area of grey haze set is diminished, the calculation efficiency is increased, meanwhile it further differentiates index observation value’s membership grade for each grey cluster, and thus ensures the conclusion’s reliability.

4. Case Study

Based on the connotation, features and function of the evaluation of the modernization of hydraulic projects management, and on account of analyzing water conservancy construction’s macroscopic background and understanding the evaluation of the modernization of hydraulic projects management, this paper, using the methods like literature reading, frequency analysis, attribute reduction and reference method, confirms the index system of the evaluation of the modernization of hydraulic projects management. The example of a reservoir in Hubei is proposed, and each evaluation index’s characteristic values and weights are as shown in Table 1 [7].
### Table 1. The Evaluation Index System of the Modernization of Hydraulic Projects Management

| Subgoal Layer                              | Index Level                                                                 | Weight | Evaluation Value % |
|--------------------------------------------|-------------------------------------------------------------------------------|--------|--------------------|
| The modernization level of organizational management | the improvement rate of management system and operational mechanism      | 0.2    | 90                 |
|                                            | the improvement rate of management regulations                              | 0.2    | 97                 |
|                                            | the adaptive rate of personnel number on guard and post setting requirements | 0.15   | 90                 |
|                                            | the target rate of talent proportion                                        | 0.10   | 75                 |
|                                            | the target rate of staff annual training proportion                         | 0.15   | 80                 |
|                                            | the target rate of archives management system                               | 0.2    | 95                 |
|                                            | the target rate of water administration management                           | 0.40   | 93                 |
|                                            | the target rate of flood-prevention ability                                 | 0.30   | 96                 |
|                                            | the target rate of accident anticipated plan formulation and implementation | 0.30   | 98                 |
|                                            | the intact rate of projects facilities                                      | 0.15   | 75                 |
|                                            | the intact rate of observation facilities                                   | 0.10   | 90                 |
|                                            | the target rate of projects inspection standardization                      | 0.10   | 93                 |
|                                            | the target rate of projects maintenance implementation                      | 0.10   | 80                 |
|                                            | the target rate of projects automatic control degree                        | 0.15   | 81                 |
|                                            | the target rate of office automation degree                                 | 0.15   | 86                 |
|                                            | the target rate of waters functional areas' water quality                   | 0.05   | 100                |
|                                            | the control rate of water and soil loss                                     | 0.05   | 97                 |
|                                            | the profit and loss rate of water management units                          | 0.25   | 100                |
|                                            | the implement rate of repair and maintenance funds                          | 0.25   | 100                |
|                                            | the tax rate of reasonable water charges and other fees                     | 0.25   | 89                 |
|                                            | the utilization ratio of developable water and soil resources               | 0.25   | 85                 |
| The modernization level of safety management |                                                                            |        |                    |
| The modernization level of project management |                                                                            |        |                    |
| The modernization level of economic management |                                                                            |        |                    |

1) Confirm the evaluation of grey clusters. According to the evaluation requirements, we divide the evaluation of the modernization of hydraulic projects management into four grey clusters: “poor type”, “general type”, “good type” and “excellent type”.

2) Combining the proposals offered by the experts of hydraulic projects management institutions, we can confirm each grey cluster’s end points, and then calculate its grey clustering intervals according to the calculation formulas of CTWF and CCTWF, which are depicted in Table 2.

### Table 2. The Division of Grey Clustering Intervals Based on CTWF and CCTWF

| Whitenization Weight Function | Poor | General | Good | Excellent |
|------------------------------|------|---------|------|-----------|
| CTWF                         | 55 ≤ x₁₁ ≤ 85 | 70 ≤ x₁₁ < 93 | 85 ≤ x₁₁ < 98 | 93 ≤ x₁₁ < 103 |
| CCTWF                        | 65 ≤ x₁₂ ≤ 75.5 | 75.5 ≤ x₁₂ < 92.5 | 89 ≤ x₁₂ < 97 | 95.5 ≤ x₁₂ < 100.5 |

1) For the grey clustering model based on CTWF, the procedure of the evaluation of the modernization of hydraulic projects management is as follows:

Step 1: According to the construction method of CTWF, we can calculate the whitenization weight function clustering coefficients of each index, and it means we can calculate the membership grade $f_j^k(x_i)$ of each index which belongs to grey cluster $k(k = 1, 2, 3, 4)$.

If $x_{11} = 90$, then $(f_1^1(x_{11}), f_1^2(x_{11}), f_1^3(x_{11}), f_1^4(x_{11})) = (0, 0.375, 0.625, 0)$. Similarly, we can get the whitenization weight function clustering coefficients of the evaluation of the...
modernization of hydraulic projects management, which are depicted in Table 3.

Table 3. The Triangular Whitenization Weight Function of the Evaluation Indexes based on CTWF

| code | \( f_j^1(x_j) \) | \( f_j^2(x_j) \) | \( f_j^3(x_j) \) | \( f_j^4(x_j) \) | \( f_j^5(x_j) \) | \( f_j^6(x_j) \) | \( f_j^7(x_j) \) | \( f_j^8(x_j) \) |
|------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 11   | 0               | 0.375           | 0.625           | 0               | 0               | 1               | 0               | 0               |
| 12   | 0               | 0               | 0.2             | 0.8             | 0               | 0.333           | 0.667           | 0               |
| 13   | 0               | 0.375           | 0.625           | 0               | 0.133           | 0.867           | 0               | 0               |
| 14   | 0.667           | 0.333           | 0               | 0               | 0.267           | 0.733           | 0               | 0               |
| 15   | 0.333           | 0.667           | 0               | 0               | 0               | 0.875           | 0.125           | 0               |
| 16   | 0               | 0               | 0.6             | 0.4             | 0               | 0               | 0               | 0.6             |
| 21   | 0               | 0               | 1               | 0               | 0               | 0               | 0.2             | 0.8             |
| 22   | 0               | 0               | 0.4             | 0.6             | 0               | 0               | 0               | 0.6             |
| 23   | 0               | 0               | 0               | 1               | 0               | 0               | 0               | 0.6             |
| 31   | 0.667           | 0.333           | 0               | 0               | 0.5             | 0.5             | 0               | 0               |
| 32   | 0               | 0.375           | 0.625           | 0               | 1               | 0               | 0               | 0               |

Step 2: According to the integrated clustering coefficient formulas in the methods of CTWF clustering evaluation, we can calculate each criterion layer’s index as well as the integrated clustering coefficients of the evaluation of the modernization of hydraulic projects management, which are depicted in Table 4.

Table 4. The Integrated Whitenization Weight Function of the Evaluation Indexes based on CTWF

| grey cluster | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( x_4 \) | \( x \) |
|--------------|----------|----------|----------|----------|-------|
| poor         | 0.117    | 0        | 0.193    | 0        | 0.12  |
| general      | 0.265    | 0        | 0.525    | 0.375    | 0.372 |
| good         | 0.379    | 0.52     | 0.191    | 0.125    | 0.268 |
| excellent    | 0.24     | 0.48     | 0.07     | 0.3      | 0.2   |

Step 3: By analyzing the clustering results in Table 4, and by \( \max(\sigma^k) = \sigma^2 = 0.372 \), we know that the result of the evaluation of the modernization of hydraulic projects management belongs to “general type”.

2) For the grey clustering model based on CCTWF, the procedure of the evaluation of the modernization of hydraulic projects management is as follows:

Step 1: According to the construction method of CCTWF, we can calculate the whitenization weight function clustering coefficients of each index, and it means we can calculate the membership grade \( f_j^k(x_j^k) \) of each index which belongs to grey cluster \( k(k = 1, 2, 3, 4) \).

If \( x_{11} = 90 \), then \( \left( f_j^1(x_{11}), f_j^2(x_{11}), f_j^3(x_{11}), f_j^4(x_{11}) \right) = (0, 0.333, 0.25, 0) \). Similarly, we can get the whitenization weight function clustering coefficients of the evaluation of the modernization of hydraulic projects management, which are depicted in Table 5.
Table 5. The Triangular Whitenization Weight Function of the Evaluation Indexes based on CCTWF

| code | $f_1^j(x_j)$ | $f_2^j(x_j)$ | $f_3^j(x_j)$ | $f_4^j(x_j)$ | code | $f_1^j(x_j)$ | $f_2^j(x_j)$ | $f_3^j(x_j)$ | $f_4^j(x_j)$ |
|------|--------------|--------------|--------------|--------------|------|--------------|--------------|--------------|--------------|
| $x_{11}$ | 0.333 | 0.25 | 0 | $x_{31}$ | 0 | 0 | 1 | 0 |
| $x_{12}$ | 0 | 0 | 0 | 0.714 | $x_{34}$ | 0 | 0.474 | 0 | 0 |
| $x_{13}$ | 0 | 0.333 | 0.25 | 0 | $x_{35}$ | 0 | 0.789 | 0 | 0 |
| $x_{14}$ | 0.091 | 0 | 0 | 0 | $x_{36}$ | 0 | 0.579 | 0 | 0 |
| $x_{15}$ | 0 | 0.474 | 0 | 0 | $x_{37}$ | 0 | 0.867 | 0 | 0 |
| $x_{16}$ | 0 | 0 | 0.5 | 0 | $x_{38}$ | 0 | 0 | 0 | 0.2 |
| $x_{17}$ | 0 | 0 | 0 | 1 | $x_{39}$ | 0 | 0 | 0 | 0.714 |
| $x_{18}$ | 0 | 0 | 0.25 | 0.2 | $x_{41}$ | 0 | 0 | 0 | 0.2 |
| $x_{19}$ | 0 | 0 | 0 | 0 | $x_{42}$ | 0 | 0 | 0 | 0.2 |
| $x_{21}$ | 0.091 | 0 | 0 | 0 | $x_{43}$ | 0 | 0.467 | 0 | 0 |
| $x_{22}$ | 0 | 0 | 0 | 0 | $x_{44}$ | 0 | 0.333 | 0.25 | 0 |
| $x_{23}$ | 0 | 0 | 0 | 1 | $x_{45}$ | 0 | 0 | 0 | 0.714 |
| $x_{24}$ | 0 | 0 | 0 | 0 | $x_{46}$ | 0 | 0 | 0 | 0.714 |
| $x_{25}$ | 0 | 0 | 0 | 0 | $x_{47}$ | 0 | 0 | 0 | 0.2 |

Step 2: Based on the integrated clustering coefficient formulas in the methods of CCTWF clustering evaluation, we can calculate each criterion layer’s index as well as the integrated clustering coefficient of the evaluation of the modernization of hydraulic projects management, which is depicted in Table 6.

Table 6. The Integrated Whitenization Weight Function of the Evaluation Indexes based on CCTWF

| grey cluster | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x$ |
|--------------|------|------|------|------|----|
| poor | 0.009 | 0 | 0.014 | 0 | 0.009 |
| general | 0.188 | 0 | 0.416 | 0.367 | 0.301 |
| good | 0.188 | 0.475 | 0.125 | 0 | 0.171 |
| excellent | 0.143 | 0.36 | 0.046 | 0.1 | 0.121 |

Step 3: By analyzing the clustering results in Table 6, and by $\max_{i \leq j \leq 4} \sigma_i = \sigma^2 = 0.301$, we know that the result of the evaluation of the modernization of hydraulic projects management belongs to “general type”.

(3) Analysis of the evaluation results

From Table 3, we can see that there is overlap between the adjacent grey clustering intervals for CTWF, while there is no overlap between $f_1^j(x_j)$ and $f_2^j(x_j)$ for CCTWF.

Comparing Table 4 and Table 6, we can see that the clustering coefficient based on CCTWF simplifies crossing function calculation and thus makes it easy and convenient to operate.

According to $\max_{i \leq j \leq 4} \sigma_i = \sigma^2 = 0.301$, we can see from Table 5 and Table 7 that the result of the evaluation of the modernization of hydraulic projects management belongs to “good type” in the modernization level of safety management, “general type” in project management and “general type” in economic management. However, there exist different evaluation results in the modernization level of organizational management, among which the result of the grey clustering evaluation based on CTWF belongs to “good type”, but the result of the grey clustering evaluation based on CCTWF shows that the secondary index of this project has a similar membership grade between “good type” and “general type”. Observing the comprehensive evaluation results, we can see that the distinction between $\sigma^2$ and other clustering coefficients is much clearer when the grey evaluation method is based on CCTWF rather than CTWF. The hydraulic project belongs to “general type” in the comprehensive
evaluation results, has good technical indexes, attaches importance to environment protection and social influence, and it works well in general.

5. Conclusion

On the basis of researching the problems like grey clustering overlap, clustering indexes and grey clustering intervals confirmation in ETWF and CTWF, we construct CCTWF and the conclusions are as follows:

1. From the comparative research of grey clustering overlapping features, we know that CCTWF is superior to ETWF in the overlapping features of grey clusters.
2. From the comparative research of clustering coefficients, we get that ETWF fails to meet the normalization, CTWF meets the weak normalization, and CCTWF, which simplifies crossing function calculation and makes it easy and convenient to operate, meets normality.
3. From the comparative research of the division of grey cluster, we can see that the confirmation of grey clustering intervals based on CCTWF fits in with end points' connotation to a higher degree, diminishes the crossing areas of grey haze set, and further differentiates index observation value’s membership grade for each grey cluster.
4. The evaluation of the modernization of hydraulic projects management further verifies the three above methods’ validity as well as the feasibility and effectiveness of the grey clustering evaluation method. In conclusion, CCTWF is superior to ETWF and CTWF.

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