Long-Time Evolution of Gas-Free Disc Galaxies in Binary Systems

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Received ; Accepted

ABSTRACT

We present the results of several detailed numerical N-body simulations of the dynamical interactions of two equal mass disc galaxies. Both galaxies are embedded in spherical halos of dark matter and contain central bulges. Our analysis of the dynamical evolution of the binary system focuses on the morphological evolution of the stellar distribution of the discs. The satellite galaxy has coplanar or polar disc orientation in relation to the disc of the primary galaxy and their initial orbits are prograde eccentric \(e = 0.1, 0.4\), or \(0.7\). Both galaxies have mass and size comparable to the Milky Way. We show that the merger of the two disc galaxies, depending on the relative orientation of the discs, can yield either a disc or lenticular remnant, instead of an elliptical one. These are the first simulations in the literature to show the formation of S0-like galaxies from protracted binary galaxy interactions. Additionally, we demonstrate that the time to merger increases linearly with the initial apocentric distance between the galaxies, and decreases with the initial orbital eccentricity. We also show that the tidal forces of the discs excite transient \(m = 1\) and \(m = 2\) wave modes, i.e., lopsidedness, spiral arms, and bars. However, after the merging of the discs, such instabilities fade completely, and the remnant is thicker and bigger than the original discs. The maximum relative amplitude of these waves is at most about 15 times greater compared to the control case. The \(m = 2\) wave mode is generated mainly by tidal interaction in the outer region of the discs. The \(m = 1\) wave mode depends mostly of an interaction of the inner part of the discs, producing an off-centering effect of the wave mode center relative to the center of mass of the disc. These characteristics produce a time lag among the maximum formation of these two wave modes. Finally, the disc settles down quickly, after the merger, in less than one outer disc rotation period.

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1. Introduction

Seventy percent of galaxies in the nearby universe are characterized by a disc with prominent spiral arms, but our understanding of the origin of these patterns is incomplete, even after decades of theoretical study ([Sellwood 2011]). Several ideas have been proposed to explain the formation of spiral arms.

The latest simulations show that gravitational instabilities in the stars lead to flocculent and multi-armed spirals which persist for many Gyrs ([Oh et al. 2008]). However, the mechanism which produces and maintains two-armed grand design galaxies is still ambiguous.

Grand design galaxies, which exhibit symmetric two-armed spiral structures, represent a significant fraction of spiral galaxies. The production of such a spiral galaxy faces two major obstacles: first, inducing the $m = 2$ spiral structure, and secondly maintaining it.

It is known that the spiral arms of disc galaxies can be excited by tidal interactions with nearby companion galaxies ([Oh et al. 2008]).

Oh and collaborators ([Oh et al. 2008]) have investigated the physical properties of tidal structures in a disc galaxy created by gravitational interactions with a companion using numerical N-body simulations. They have considered a galaxy model consisting of a rigid halo/bulge and an infinitesimally thin stellar disc with Toomre parameter $Q \approx 2$. The perturbing companion was treated as a pointmass moving on a prograde parabolic orbit, with varying mass and pericenter distance. They have shown that tidal interactions produce well-defined spiral arms and extended tidal features, such as bridge and tail, which are all transients.

Dobbs’s et al. (2010) modeled the disc galaxy M51 and its interaction with a companion point-mass NGC 5195, focusing primarily on the dynamics of the gas, and secondly the stellar disc. The halo was represented by a rigid potential. The tidal interaction has produced spiral arms in the stars and in the gas. The resulting spiral structure has shown excellent agreement with that of M51.

The work of Lotz et al. (2010) analyzed the effect of a gas fraction on the morphologies of a series of simulated disc galaxy mergers. Each galaxy was initially modeled as a disc of stars and gas, a stellar bulge and a dark matter halo, with different number of particles and masses for each component. All the simulated mergers had the same orbital parameters. Each pair of galaxies has started on a sub-parabolic orbit with eccentricity 0.95 and an initial pericentric radius of 13.6 kpc. The galaxies have had a roughly prograde-prograde orientation relative to the orbital plane, with the primary galaxy tilted 30° from a pure prograde orientation. Their simulations have predicted that galaxy mergers would exhibit high asymmetries for longer periods of time if they have had high gas fractions.

Struck and collaborators ([Struck et al. 2011]) have discovered long-lived waves in numerical simulations of fast (marginally bound or unbound) flyby galaxy collisions. The main galaxy has had a rigid halo potential, gas and the companion was modeled as a point mass. They have found that none of the simulations has resulted in bar formation. They have also shown that while these
waves propagate through the disc, they are maintained by the coherent oscillations initiated by the
impulsive disturbance.

Snaith et al. (2012) have studied the properties and evolution of a simulated polar disc galaxy.
This galaxy was composed of two orthogonal discs, one of which contains old stars (old stellar
disc) and the other both younger stars and cold gas (polar disc). They have confirmed that the polar
disc galaxy is the result of the last major merger, where the angular moment of the interaction is
orthogonal to the angle of the infalling gas.

In one of our previous works in kinematic and morphology of spiral galaxies we have shown
a deep interaction between the dynamical and morphological properties of this kind of galaxy
(Chan & Junqueira 2003). With continual satellite forcing, the final state was in the form of a
slowly evolving wave pattern, as shown by the existence of pattern speeds for stable $m = 1$ and
$m = 2$ wave modes. The pattern speeds obtained from the density and the three positive velocity
component distributions are the same. This was also true for the negative velocity components.

Kinematic studies of spiral galaxies have revealed a remarkable variety of interesting behav-
ior: some galaxies have large scale asymmetries in their rotation curves as signature of kine-
matic lopsidedness (Junqueira & Combes 1996), while in others the inner regions counter-rotate
with the respect to the rest of the galaxy (Garcia-Burillo et al. 2000). Most of the spiral galax-
ies have asymmetric HI profiles and asymmetric rotation curves (Haynes et al. 2000). Such in-
triguing kinematics could plausibly result if these galaxies are the end-products of minor mergers
(Haynes et al. 2000). Minor mergers and weak tidal interactions between galaxies occur with much
higher frequency than major ones. By weak interactions between galaxies we mean those that do
not destroy the disc of the primary galaxy. However, weak interactions may cause disc heating and
satellite remnants may build up the stellar halo.

Galaxies’ interactions are likely to play a key role in determining the morphology and the dy-
namical properties of disc galaxies. Careful examination shows that most disc galaxies are not truly
symmetric but exhibit a variety of morphological peculiarities of which spiral arms and bars are
the most pronounced. Disc galaxies currently show significant spiral-generating tidal perturbations
by one or more small-mass companions, and nearly all have had tidal interactions at sometime in
the past.

After decades of efforts, we now know that these features may be driven by environmental
disturbances acting directly on the disc, in addition to self-excitation of a local disturbance (e.g.,
by swing amplification) (Toomre 1981). However, the disc is embedded within a halo and, there-
fore, the luminous disc is not dynamically independent (Combes 2008). The dark matter halo is
disturbed by dwarf companions, neighboring galaxies in groups and clusters and the tidal force
from the overall cluster. If the halo can respond globally to such disturbances, it can affect the disc
structure. Thus, because most spirals have dwarf companions, interactions with these companions
are present and the inward propagation of external perturbations by the halo could be a dominant
source of disc structure for all galaxies (Vesperini & Weinberg 2000).
In order to study the dynamical evolution of two disc galaxies and their morphological evolution, this paper explores the picture as follows. First, we assume a disc galaxy with the characteristics of the Milk Way (disc, bulge and halo). Second, we let a secondary galaxy orbit on prograde coplanar or polar disc (orientation in relation to the primary disc galaxy). Although the gas is important in modeling a realistic disc galaxy, in this work we focus our attention only to the morphological stellar properties.

As an example of work in evolution of stellar disc galaxies without gas, we can mention the recent published work by Baba, Saitoh & Wada (Baba et al. 2013). Using N-body simulations, they analyzed the physical mechanisms of non steady stellar spiral arms in disk galaxies, i.e., without the gaseous component. They have studied the growing and damping phases of the spiral arms and they have confirmed that the spiral arms are formed due to a swing amplification mechanism that reinforces density enhancement. The main motivation was that all the previous time-dependent simulation works have not been able to prove the existence of stationary density waves in a disk galaxy without external perturbations and a bar structure.

Thus, the main goal of the present work is, utilizing detailed numerical N-body simulations, to study the dynamical interactions of the two discs of the galaxies. In particular, we have investigated whether interactions can induce a persistent and stable $m = 1$ or $m = 2$ patterns in disc galaxies.

The paper is organized as follows: in Section 2 we describe the numerical method used in the simulations. In Section 3 we present the initial conditions. In Section 4 we show power spectra of the instabilities. Finally, in Section 5 we discuss and summarize the results.

2. The Numerical Method

The full N-body code utilized in the simulations was GADGET (Springel et al. 2001). The code was parallelized and the communication is done by means of the Message Passing Interface (MPI). GADGET evolves self-gravitating collisionless fluids with the traditional N-body approach, and a collisional gas by smoothed particle hydrodynamics. But in our case we use only the particle integration, which uses a tree algorithm to compute gravitational forces. The parallel version has been designed to run on massively parallel supercomputers with distributed memory.

Fortin, Athanassoula & Lambert (2011) published a comparison with different codes for galactic N-body simulations, in particular, the GADGET code and the Dehnen’s algorithm. They have shown that the serial implementation of the Dehnen’s algorithm is slower, in terms of execution time, in comparison with the parallelized implementation of the GADGET code, with 8 to 128 processors.

The GADGET code does not exactly conserve energy or momentum, but the energy is conserved to better than 0.3% over an entire evolution, and the center of the mass moves a distance of at most $82\epsilon$ (the softening parameter) from the initial system center mass, with a time step size $\Delta t = 1.000 \times 10^{-3}$ and $\epsilon = 8.000 \times 10^{-4}$. Too large $\epsilon$ reduces the noise but it increases the error in the calculation of the force due to the failure to resolve the interactions of particles with...
scale lengths less than it. On the other hand, too small value for $\epsilon$ produces a noisy estimation of the force due to the finite number of particles. The value of the optimal softening depends both on the mass distribution and on the number of particles used to represent it. For a higher number of particles the optimal softening is smaller. More concentrated mass distributions necessitate smaller softening. Several works have already analyzed this problem carefully (Merrit 1996) (Athanassoula et al. 2000) (Rodionov & Sotnikova 2005). For example, Athanassoula et al. (2000) have shown that, for a Dehnen sphere, $\epsilon = 4.000 \times 10^{-3}$. Thus, the softening scale is motivated by a tradeoff between accuracy and computational speed. We have run several simulations of the isolated disc galaxy in order to find the optimal $\epsilon$ and maximum integration time step. The criteria were to find the maximum softening parameter and integration time step such that it could minimize the heating of the disk, without changing too much the physical disc parameters, mainly $Z_d$. When the $\epsilon$ was too big then the disc heated up, increasing the width of it. Thus, the chosen softening parameter is smaller than the disc scale height $Z_d$, in order to the disk of the galaxies being followed accurately.

For non zero tolerance parameter $\theta$, the treecode CPU time scales as $O(N \log N)$, but in the limit $\theta \rightarrow 0$ this method scales as $O(N^2)$, approaching to the direct code. At the very beginning of this work we have used the GADGET-1 (Springel et al. 2001). Thus, we have assumed for the tolerance parameter $\theta$ the value 0.577, in order to avoid pathological situations when using the treecode as described by Salmon & Warren (Salmon & Warren 1994). However, this value of $\theta$ is CPU time-consuming but the error in the force calculation is minimized, although many authors studying disc galaxies have used $\theta > 0.7$ (Oh et al. 2008) (Minchev et al. 2012).

3. The Initial Conditions of the Simulations

We have used in the simulations the nearly self-consistent disc-bulge-halo galaxy model proposed by Kuijken & Dubinski (Kuijken & Dubinski 1995). We assumed a model that has mass distributions and rotation curves closely resembling of the Milk Way, i.e., the model MW-A of the Kuijken & Dubinski’s work (Kuijken & Dubinski 1995). This galaxy disc model has a disc-bulge mass ratio of 2:1 and halo-disc mass ratio of 5:1 (see Table 1). The disc is warm with a Toomre parameter $Q = 1.7$ at the disc half-mass radius.

Our simulations are based on a small number of particles in comparison with several works in disc galaxies. However, since none of the works in the literature has tackled before this kind of problem proposed here, we have decided to run modest simulations using the computer clusters that were available, in order to have, at least, an initial dynamical study of these kinds of binary galaxies.

The disc follows approximately an exponential-sech law described by

$$\rho_d(R, Z) = \rho_o \exp \left( -\frac{R}{R_d} \right) \text{sech}^2 \left( \frac{Z}{Z_d} \right),$$

(1)
Table 1. Disc Galaxy Model Properties

| Galaxy | $M_d$ | $N_d$ | $R_d$ | $Z_d$ | $R_t$ | $M_b$ | $N_b$ | $M_h$ | $N_h$ | $m$ | $\epsilon$ |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|---------|
| $G_1$  | 0.871 | 40,000| 1.000 | 0.100 | 5.000 | 0.425 | 19,538| 4.916 | 225,880| 2.176 $\times$ 10$^{-5}$ | 8.000 $\times$ 10$^{-4}$ |

$M_d$ is the disc mass in units of mass, $N_d$ the number of particles of the disc, $R_d$ the disc scale radius in units of length, $Z_d$ the disc scale height in units of length, $R_t$ the disc truncation radius in units of length, $M_b$ the bulge mass in units of mass, $N_b$ the number of particles in the bulge, $M_h$ the halo mass in units of mass, $N_h$ the number of halo particles, $m$ the mass of each particle in units of mass and $\epsilon$ the softening of each particle in units of length.

Fig. 1. The contour plot of the primary galaxy at the beginning of the simulation ($t = 0$) and at the Hubble time of the simulation ($t = t_H$). The smoothing was done, averaging the 25 first and second neighbors of each pixel. Hereinafter, the density levels in the planes XY and XZ at $t = 0$ will be used in all the contour plots, in the planes XY and XZ, respectively.

where $\rho_o$ is the central density that is related to the total mass of the disc. This approximation has been used because the full potential equation obtained by Kuijken & Dubinski (Kuijken & Dubinski 1995) is analytically more complicated.

4. The Results of the Simulations

In the Figure 1 we show the contour plot of the primary galaxy at the beginning of the simulation ($t = 0$) and at the Hubble time of the simulation ($t = t_H$). We note that the central density in the plane XY has increased slightly after one Hubble time of simulation, since the contour levels are the same for the two instants of time. In the XZ plane the scale height apparently has increased due to the 2-body relaxation heating, however we can see that this quantity has changed very little (see Figure 5).

Comparing the Figures 2 and 3 we note from the quantity $< V_z^2 >^{1/2}$ that the self-heating of the initial disc and the particle halo add another significant source of heating in the disc. The gravitational softening can also cause the disc to puff up, this is the reason we have chosen a such small softening parameter, 125 times smaller than the scalar disc height. We can also observe that the total rotation curves $V_c$ and the angular momentum in the Z direction have not changed, after one Hubble time of simulation.
Fig. 2. The rotation curve at the time $t = 0$ of the disc $V_c$, the main component of the angular momentum per unit of mass $J_z$ and the velocity dispersion in the $Z$ direction $<V^2_z>^{1/2}$. The coordinate $R$ is the radius in cylindrical coordinates. The dotted line denotes the disc, the long-dashed line denotes the bulge, the short-dashed line denotes the halo and the solid line denotes the total rotation curve.

Fig. 3. The rotation curve at the time $t = t_H$ of the disc $V_c$, the main component of the angular momentum per unit of mass $J_z$ and the velocity dispersion in the $Z$ direction $<V^2_z>^{1/2}$. The coordinate $R$ is the radius in cylindrical coordinates. The dotted line denotes the disc, the long-dashed line denotes the bulge, the short-dashed line denotes the halo and the solid line denotes the total rotation curve.

In the Figures 4 and 5 we present the time evolution of the scale radius ($R_d$) and the scale height ($Z_d$). We notice that, as expected, due to the heating of the disc the first quantity diminishes with the time while the second increases with the time. The linear fitting parameters of these two quantities are presented in the captions of these figures. Since the scale height has increased less than 0.2%, we have assumed, hereinafter, that this scale has not changed when we analyzed the data of the simulations.
Fig. 4. The time evolution of the scale radius \( R_d \). The projected particle number density on the XY plane was fitted using the approximation given by the Equation (1). The coordinate \( R \) is the radius in cylindrical coordinates. The fitting parameters are: \( R_d = (-0.7042 \times 10^{-1} \pm 0.2840 \times 10^{-3}) (t/t_H) + (0.8819 \pm 0.1620 \times 10^{-1}) \).

Fig. 5. The time evolution of the scale height \( Z_d \). The projected particle number density on the XZ plane was fitted using the approximation given by the Equation (1). The fitting parameters are: \( Z_d = (0.1791 \times 10^{-2} \pm 0.6320 \times 10^{-5}) (t/t_H) + (0.9006 \times 10^{-1} \pm 0.3563 \times 10^{-3}) \).

The units used in the simulations are: \( G = 1 \), [length] = 4.500 kpc, [mass] = 5.100 \( \times 10^{10} \) \( M_\odot \), [time] = 1.993 \( \times 10^{7} \) years \( (H_0 = 100 \text{ km/s/Mpc}) \) and [velocity] = 220.730 km/s. Hereinafter, all the physical quantities will be referred in these units. The particle softening radius was assumed to be 0.0008 or 1/125 of the disc scale height. The critical opening angle was set to \( \theta = 0.577 \) and the forces between the cells and particles used the quadrupole correction. The maximum integration step time was assumed to be 0.001 in units of simulation time. The Hubble time \( t_H \) corresponds to 490 time units.
We have run several simulations, with no satellite in order to check the initial instabilities of the galaxy model. The initial galaxy simulations were run in a SUN FIRE 6800 cluster, with 16 CPU processors. Each simulation has taken about 50 days of CPU time. For the simulations with the primary and satellite galaxies we have used several clusters with a variety of CPU processors: SUN BLADE X6250, SUN FIRE X2200, SGI ALTIX ICE 8200, SGI ALTIX 450/1350, SGI ALTIX-XE 340, IBM P750, INTEL PENTIUM QUAD CORE and INTEL PENTIUM DUAL CORE. The number of CPU processors varied from the minimum of 8 to the maximum of 128. Each simulation has taken 90 days of CPU time in average.

A great number of the works in the literature (Oh et al. 2008, D, L, S, B) simulating interaction between two disc galaxies have used initial conditions as parabolic or hyperbolic orbits. However, none has studied the bounded eccentric orbits in an interval of a Hubble time because is very expensive in terms of CPU time machine. This cost is mainly due to the fact that in this kind of simulation we must integrate adequately the internal dynamic of the disc galaxy during a huge cosmological interval of time. We have decided not to use cosmologically consistent initial conditions from publicly available simulations as done, e.g., by Ruszkowski & Springel (Ruszkowski & Springel 2009) for two reasons. First, we would like to know what happened with the binary galaxies in bound orbits and, secondly, it would cost much more (in number of experiments and in CPU time) if we have begun our simulations from cosmological unbound galaxies to arrive to eccentric bound galaxies.

All the initial conditions of the numerical experiments are presented in Table 2. The orbits of the initial galaxies are eccentric (\( e = 0.1, 0.4 \) or 0.7) and the orientations of the discs are coplanar (\( \Theta = 0 \)) or polar (\( \Theta = 90 \)) to each other. The simulations begin with the two galaxies at the apocentric positions.

In Figure 6 we show the dependence of the time of merging (\( T_M \)) (see Table 3) with the eccentricity (\( e \)) and the initial apocentric distance (\( R_a \)). The time of merging is defined as the time when the centers of mass of the two discs (primary and secondary galaxies) overlap each other (Chan & Junqueira 2001). We notice that the merging time increases linearly with the initial apocentric distance. The \( T_M \) for different eccentricities is obtained extrapolating the linear approximation for each eccentricity and determined \( T_M \) for a fixed value of \( R_a \). We have obtained that the time of merging decreases with eccentricity.

There are two different kinds of merged remnants in our simulations. One of them (coplanar discs) is a disc galaxy with scale radius and height very similar to the initial disc galaxy (see Table 3), but with a tidal radius that is at least five times greater than of the initial galaxy (see Figure 7). The other one (polar discs) resembles a lenticular galaxy, but again with a tidal radius that is greater than the initial galaxy radius (see Figure 7).

In the Figures 7 and 8 we present the contour snapshots of the result of the merger of the primary and secondary galaxies together in the planes XY and XZ, at the Hubble time of the simulation (\( t = t_H \)). We show the simulations: EXP13, 14, 16, 17, 19, 20, 22, 23, 25, 27, 28, 30, 31, 33, 34 and 36. We notice that in the simulations with coplanar disc orbits (EXP13, 14, 16, 17, 25,
Table 2. Primary and Secondary Galaxy Initial Conditions

| EXP | Θ  | Rₚ | e   | Rₐ | Vₐ  |
|-----|----|----|-----|----|-----|
| 00  | 0  | 30 | 0.1 | 36.67 | 0.5521 |
| 01  | 0  | 30 | 0.4 | 70.00 | 0.3263 |
| 02  | 0  | 30 | 0.7 | 170.00 | 0.1480 |
| 03  | 0  | 40 | 0.1 | 48.89 | 0.4782 |
| 04  | 0  | 40 | 0.4 | 93.33 | 0.2826 |
| 05  | 0  | 40 | 0.7 | 226.67 | 0.1282 |
| 06  | 90 | 30 | 0.1 | 36.67 | 0.5521 |
| 07  | 90 | 30 | 0.4 | 70.00 | 0.3263 |
| 08  | 90 | 30 | 0.7 | 170.00 | 0.1480 |
| 09  | 90 | 40 | 0.1 | 48.89 | 0.4782 |
| 10  | 90 | 40 | 0.4 | 93.33 | 0.2826 |
| 11  | 90 | 40 | 0.7 | 226.67 | 0.1282 |
| 12  | 0  | 15 | 0.1 | 18.33 | 0.7808 |
| 13  | 0  | 15 | 0.4 | 35.00 | 0.4614 |
| 14  | 0  | 15 | 0.7 | 85.00 | 0.2094 |
| 15  | 0  | 20 | 0.1 | 24.44 | 0.6762 |
| 16  | 0  | 20 | 0.4 | 46.67 | 0.3996 |
| 17  | 0  | 20 | 0.7 | 113.33 | 0.1813 |
| 18  | 90 | 15 | 0.1 | 18.33 | 0.7808 |
| 19  | 90 | 15 | 0.4 | 35.00 | 0.4614 |
| 20  | 90 | 15 | 0.7 | 85.00 | 0.2094 |
| 21  | 90 | 20 | 0.1 | 24.44 | 0.6762 |
| 22  | 90 | 20 | 0.4 | 46.67 | 0.3996 |
| 23  | 90 | 20 | 0.7 | 113.33 | 0.1813 |
| 24  | 90 | 23 | 0.1 | 28.11 | 0.6306 |
| 25  | 90 | 23 | 0.4 | 53.67 | 0.3726 |
| 26  | 90 | 23 | 0.7 | 107.14 | 0.2564 |
| 27  | 90 | 10 | 0.1 | 28.11 | 0.6306 |
| 28  | 90 | 10 | 0.4 | 53.67 | 0.3726 |
| 29  | 90 | 10 | 0.7 | 107.14 | 0.2564 |

$G_1$ is the primary galaxy, $G_2 = G_1$ the secondary galaxy, $\Theta$ the angle between the two planes of the discs in units of degree, $R_p$ the pericentric distance in units of length, $M_1$ the primary galaxy mass in units of mass, $e$ the eccentricity, $R_a$ the apocentric distance in units of length, $V_a$ the velocity at the apocentric distance in units of velocity, $M_1$ the primary galaxy mass and $M_2 = M_1 = 0.621$ the secondary mass galaxy in units of mass.

27, 28 and 30) the resulting fused galaxies are still disc galaxies. Their fitted scale radii ($R_d(12)$) and heights ($Z_d(12)$) using the Equation (1) are presented in Table 3. We can see that the merged disc galaxies are thicker and bigger than the initial ones.

However, for the simulations with polar disc orbits (EXP19, 20, 22, 23, 31, 33, 34 and 36) the resulting fused galaxies are not disc galaxies anymore. In both planes, XY and XZ, the galaxies resemble to lenticular galaxies. The outer contour level of the merged galaxy in EXP23 is clearly deformed, differently of others merged polar discs, maybe because of the number of orbits (see Table 3). This is the unique simulation among all our experiments with the maximum number
Table 3. Characteristics of the Final Stage of the Orbits and Merged Discs

| EXP | Disc Interaction | Number of Orbits | $T_M$ | $R_{d(12)}$ | $Z_{d(12)}$ | $R_f$ |
|-----|------------------|------------------|------|-------------|-------------|------|
| 01  | Open             | 1.5              |      |             |             |      |
| 02  | Open             | 1.0              |      |             |             |      |
| 03  | Open             |                  |      |             |             |      |
| 04  | Open             | 1.0              |      |             |             |      |
| 05  | Open             | 0.5              |      |             |             |      |
| 06  | Open             |                  |      |             |             |      |
| 07  | Open             | 1.5              |      |             |             |      |
| 08  | Open             | 1.0              |      |             |             |      |
| 09  | Open             |                  |      |             |             |      |
| 10  | Open             | 1.0              |      |             |             |      |
| 11  | Open             | 0.5              |      |             |             |      |
| 12  | Open             |                  |      |             |             |      |
| 13  | Merge            | 1.0              | 0.25 | 0.867 ± 0.041 | 0.116 ± 0.006 | 10   |
| 14  | Merge            | 1.5              | 0.42 | 0.946 ± 0.051 | 0.142 ± 0.010 | 10   |
| 15  | Graze            | 1.0              | 1.60*|             |             |      |
| 16  | Merge            | 1.5              | 0.42 | 0.771 ± 0.042 | 0.128 ± 0.009 | 10   |
| 17  | Merge            | 2.5              | 1.00 | 0.682 ± 0.025 | 0.199 ± 0.018 | 10   |
| 18  | Open             | 0.5              |      |             |             |      |
| 19  | Merge            | 1.0              | 0.25 |             |             |      |
| 20  | Merge            | 1.5              | 0.43 |             |             |      |
| 21  | Graze            | 1.0              | 1.61*|             |             |      |
| 22  | Merge            | 1.5              | 0.42 |             |             |      |
| 23  | Merge            | 2.5              | 1.00 |             |             |      |
| 24  | Open             | 0.5              |      |             |             |      |
| 25  | Merge            | 1.5              | 0.63 | 0.791 ± 0.038 | 0.116 ± 0.006 | 10   |
| 26  | Open             | 1.5              |      |             |             |      |
| 27  | Merge            | 1.5              | 0.50 | 0.871 ± 0.044 | 0.465 ± 0.079 | 10   |
| 28  | Merge            | 1.5              | 0.60 | 0.791 ± 0.038 | 0.116 ± 0.006 | 10   |
| 29  | Open             | 1.5              |      |             |             |      |
| 30  | Merge            | 1.5              | 0.50 | 0.809 ± 0.041 | 0.175 ± 0.007 | 10   |
| 31  | Merge            | 1.5              | 0.60 |             |             |      |
| 32  | Open             | 1.5              |      |             |             |      |
| 33  | Merge            | 1.5              | 0.54 |             |             |      |
| 34  | Merge            | 1.5              | 0.60 |             |             |      |
| 35  | Open             | 1.5              |      |             |             |      |
| 36  | Merge            | 1.5              | 0.54 |             |             |      |

Open means that the two discs do not touch each other during the time of the experiment ($t_H$). Graze means that the two discs touch each other for a while and after they separate. Merge means that the two discs fuse to each other. Number of orbits is the total angular excursion of the companion up to the merger, relative to its starting point. $T_M$ is the time of merging in units of $t_H$ when the two discs fuse to each other. Number of orbits is the total angular excursion of the companion up to the merger, relative to its starting point. $T_M$ is the time of merging in units of $t_H$ when the two discs fuse to each other (the symbols * in the times of merging of the simulations EXP15 and EXP21 denote that these times are estimations, using EXP17). $R_{d(12)}$, $Z_{d(12)}$, and $R_f$ are the fitted scale radius, height and cutoff fitting radius of the unique merged coplanar disc in units of length, respectively, using Equation (1).

In Figures 9 and 10 we can show what happened to these galaxies using the simulation EXP31 at $t = 0.5t_H$ and at $t = t_H$. These figures show the discs of the primary galaxy $G_1$ and secondary $G_2$. We can see that the polar characteristic of the $G_2$ is still there at $t = 0.5t_H$ but this is lost at $t = t_H$. At this time the polar disc is completely disrupted and its debris form a stellar halo. Overlapping the contours of $G_1$ and $G_2$, we get Figure 8 for the simulation EXP31.
Fig. 6. The time of merging with the fitted straight lines, for each eccentricity, where $R_a$ is the apocentric distance. The open circles represent the simulations with $e = 0.1$. The open triangles represent the simulations with $e = 0.4$. The open squares denote the experiments with $e = 0.7$. The best fit parameters are: $\frac{t_M}{t_H} = (0.039 \pm 0.002)R_a + (-0.508 \pm 0.056)$ (for $e = 0.1$), $\frac{t_M}{t_H} = (0.049 \pm 0.005)R_a + (-1.285 \pm 0.020)$ (for $e = 0.4$) and $\frac{t_M}{t_H} = (0.038 \pm 0.006)R_a + (-1.635 \pm 0.038)$ (for $e = 0.7$) (the far two points were obtained extrapolating the time evolution of the distance between the two discs, using the simulation EXP17).

Fig. 7. The contour snapshot of the merger of the primary and secondary galaxies (flat disk merged remnants) together in the planes XY and XZ, at the Hubble time of the simulation ($t = t_H$). Simulations EXP13, 14, 16, 17, 25, 27, 28 and 30.

5. Power Spectrum Analysis

The power spectrum analysis provides a useful and objective tool for the study of the induced waves. This analysis uses the amplitude and the phase of the Fourier components of the surface density of the stars, allow us to evidence the presence of the spiral modes in our simulations. This analysis give us the pattern speed of all present spiral perturbations and their relative positions.
Fig. 8. The contour snapshot of the merger of the primary and secondary galaxies (oblate disk merged remnants) together in the planes XY and XZ, at the Hubble time of the simulation ($t = t_H$). Simulations EXP19, 20, 22, 23, 31, 33, 34 and 36.

Fig. 9. The contour of the snapshot of the merger of the primary and secondary galaxies plotted separately in the plane XY and XZ, at 50% of the Hubble time ($t = 0.5t_H$). Simulation EXP31.

The method of power spectrum, known historically as periodogram, is used to search for periodicities in sparse, noisy unevenly spaced data (Junqueira & Combes 1996).

If we take a N-point sample of the function $c(t)$ at equal intervals of time $t$ and compute its discrete Fourier transform (Press at al. 1992) we get the power spectrum $P(\Omega)$ of $c(t)$.

We have used the grid expansion method in order to analyze the density distribution ($128 \times 128 \times 128$ pixels), for obtaining the power spectrum.

Firstly, using a radial binning of 0.0125, we obtain the amplitude and the phase of the Fourier components. Secondly, using the snapshots of the slab of the disc density in a interval of time of
Fig. 10. The contour of the snapshot of the merger of the primary and secondary galaxies plotted separately in the plane XY and XZ, at 60% of the Hubble time ($t = 0.6 t_H$). Overlapping the contours of $G_1$ and $G_2$, at $t = t_H$, we get the contours of Figure 8 for the simulation EXP31.

0.01$t_H$, we calculate the superposition of all the Fourier amplitude and phase for each radial bin and for each interval of time. Finally, we obtain the power spectrum as a plot of number density for each radial bin. The power spectrum analysis was done by studying the primary galaxy just before the merging time. The orientation of the disc was not followed dynamically because it has not deviated from the initial orientation angle, as we can see in Figure 10.

Since we have a 3D particle disc, we limited the number of the particles within the planes $Z = -Z_{\text{max}}$ and $Z = Z_{\text{max}}$ in order to simplify the application of the grid expansion method (Chan & Junqueira 2003). We have considered this thin slab between these two planes as the plane $Z = 0$ for the grid expansion. Henceforth, this thin slab will be denoted as $Z = 0$ in the equations. The chosen quantity $Z_{\text{max}} = 0.1$ is the value of the scale height of the disc ($Z_d$). There are approximately 40% of the total disc particles ($N_d$) within these two planes. In all the analysis hereinafter it is assumed a maximum radius of 5 length units since we have 95% of the mass of the disc within this radius.

The basic assumption of the density wave theory is that spiral arms are not always composed of the same stars but instead they are the manifestation of the excess matter associated with the crest of a rotating wave pattern. Two further assumptions were introduced from the onset, the linearity and quasi-stationarity of the wave. These assumptions allow us to write any perturbation of the axisymmetric background as superposition waves given by

$$\rho_d(R, \phi, Z = 0, t) = \sum_m \rho_m(R)e^{i(\Omega(m)t - m\phi)}$$

(2)
where \( \rho_d \) is the density. The summation index indicates the symmetry of the component: \( m = 0 \) corresponds to the axisymmetric background; \( m = 1 \) corresponds to the lopsided perturbation and \( m = 2 \) corresponds to the symmetric two arms perturbation (spiral, bar). \( \Omega(m) \) is the pattern speed of the component \( m \).

We can rewrite Equation (2) in the usual wave notation

\[
\rho_d(R, \phi, Z = 0, t) = \sum_m p_m(R) e^{i[\Psi_m(R) + \Omega(m)t - m\phi]},
\]

where \( p_m(R) \) is the amplitude of the wave and \( \Psi_m(R) \) is the phase angle of the mode \( m \).

Now the density mode \( m \) can be obtained from

\[
\rho_m(R, \phi, Z = 0, t) = p_m(R) \cos[\Psi_m(R) + \Omega(m)t - m\phi].
\]

Let us interpret the \( m = 1 \) and \( m = 2 \) plots in Figures 11 and 12. They do not look like a clear one-armed spirals, or a clear asymmetry in the two-arms, like in the Figure 13 of the work of Junqueira & Combes (Junqueira & Combes 1996), because in this present work they represent the Fourier analysis of a transient wave. Junqueira & Combes analyzed only the gaseous disc, but the analysis is similar to a stellar disc.

In Figure 11 we show the transient wave modes \( m = 1 \) and \( m = 2 \) for the simulation EXP15, at two different instants of time \( 0.65t_H \) and \( t_H \). We notice that transient \( m = 1 \) wave modes are mostly present in outer part of the discs. We note also that transient spiral arms (\( m = 2 \)) are formed in the outer regions of the discs, and bars are present in the inner regions. Since they are transient \( m = 2 \) wave modes the power spectrum for EXP15 (see Figure 14) shows an undefined angular velocity for this mode.

In Figure 12 we show the transient wave modes \( m = 1 \) and \( m = 2 \) for the simulation EXP31, at two different instants of time \( 0.5t_H \) and \( t_H \). We notice that the transient \( m = 1 \) wave mode at \( t = 0.5t_H \) is mostly present in outer part of the discs, except at \( t = t_H \). There is a big transient spiral arm (\( m = 2 \)) at \( t = 0.5t_H \) in the outer region of the disc and a prominent bar in the inner region of the disc at \( t = 0.6t_H \). As in the EXP15, here we have transient \( m = 2 \) wave modes the power spectrum for EXP31 (see Figure 14). If we compare the Figures 12 and 13 we can see the Fourier decompositions are very similar with the snapshots of the discs.

In Figures 14 we can see the power spectra for the \( m = 2 \) wave mode for the simulations EXP00, 13, 14, 15, 16, 17, 19, 20, 21, 25, 28 and 31. We have shown only the \( m = 2 \) wave mode because we have not detected any \( m = 1 \) wave mode in any simulation. Others experiments that presented \( m = 2 \) wave mode and that are not shown in this figure are: EXP22, 23, 27 and 33. These simulations had similar behaviors as shown in Figure 14. Others simulations have not shown any sign of \( m = 2 \) wave mode mostly because the primary and secondary halo did not touch each other during their evolution time (open disc interaction: see Table 3).
Fig. 11. The wave modes $m = 1$ and $m = 2$ for the simulation EXP15, at two different instants of time $0.65H$ and $t_H$. The density levels for these plots are the same used in $m = 1$ ($t = 0.65t_H$).

In Figure 14 the first plot shows the power spectrum for the mode $m = 2$, for the simulation EXP00, without the secondary galaxy. This was done in order to analyze the existence of self-excited gravitational instabilities $m = 2$ wave mode in the disc. As we can see, there are not any wave modes.

Note that in Figure 14 the fuzzy small perturbations in the outer radii of the discs (note also that the density levels are three times greater than that used in the experiment EXP00). Most of the experiments in this figure have shown merged discs (see Table 3), except the simulations EXP15 and 21 (grazing discs). There we can also see partial $m = 2$ wave modes in the outer radii of the disc with high clumpy density regions that do not stretch to the inner part of the discs. Thus, we cannot classify them as being $m = 2$ stable wave modes because these characteristics of these power spectra.

There are not discernable and significant substructures in Figure 14. If we had a real stable wave we should have a plot like the Figure 16 of the paper of Junqueira & Combes, a straight line parallel to the radial axis giving the pattern speed of the mode. Instead, since we have a transient wave, we get a fuzzy plot, more concentrated in the outer radial regions.

We have integrated the wave amplitudes radially to get the global Fourier amplitudes $|p_m|$ (Harsono et al. 2011), where $m$ is the Fourier mode:

$$|p_m(t)| = \frac{1}{N_{disc}} \sum_{R=5}^{R=5} |p_m(R)e^{i[\Psi_m(R)+\Omega_m(t)-m\phi]}|,$$

(5)

where $p_m(R)$ is giving by equation 3 and $N_{disc}$ is total number of particles in the disc within $R = 5$. The global amplitude analysis was done in two distinct ways: a) studying the primary galaxy until the merging time, b) analyzing the compound galaxies after the merge, when they occur. The orientation of the disc was not followed dynamically because of the deformation of the disc in the

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Fig. 12. The wave modes $m = 1$ and $m = 2$ for the simulation EXP31, at two different instants of time $0.5t_H$ and $0.6t_H$. The density levels for these plots are the same used in EXP15 ($m = 1, t = 0.65t_H$).

Fig. 13. The snapshots of the slabs $Z = 0$ for the simulation EXP15, at two different instants of time $0.65t_H$ and $t_H$. Also, the snapshots of the slabs $Z = 0$ for the simulation EXP31, at two different instants of time $0.5t_H$ and $0.6t_H$.

some simulations. In Figure 15, we show the temporal evolution of the relative global integrated amplitudes for the simulations EXP13, 14, 15, 16, 17, 19, 20, 21, 25, 28 and 31.

As we can see in Figure 15, there is not a possible kick due to non-cosmological initial conditions. The reason is simply because all the simulations have begun at the apocentric distance, where the tidal interaction between the two binary galaxies is weaker.

Furthermore, all the waves are mostly driven shortly before merger, because the separations have gotten small, even in the cases with no merger.
Fig. 14. The power spectrum for the mode $m = 2$ for the primary galaxy ($G_1$) and for the simulations EXP00, 13, 14, 15, 16, 17, 19, 20, 21, 25, 28 and 31 until the time of merging. The density levels are three times greater than that of the experiment EXP00.

Let us analyze the special case EXP17 in Figure [15]. The simulation EXP17 presents three maxima for $m = 2$ wave mode. The first maximum is due to the formation of a bar, when the two discs are still far apart. The second and third ones are due to the formation of a two-arms spiral, when the discs are already touching and deforming each other.

Lopsided features are preferentially observed in the distribution of gas in late-type spiral galaxies. In several cases these features can be identified as one-armed spirals ($m = 1$ mode). More frequently, nuclei of galaxies are observed displaced with respect to the gravity center, as in M33 and M101. The nucleus of M31 reveals such an off-centering which has been interpreted in terms of an $m = 1$ perturbation. Miller and Smith [Miller & Smith 1992] have studied through N-body simulations of disk galaxies, a peculiar oscillatory motion of the nucleus with respect to the rest of the axisymmetric galaxy. They interpret the phenomenon as a $m = 1$ instability, a density wave in orbital motion around the center of mass of the galaxy. Moreover, Junqueira and Combes [Junqueira & Combes 1996] have shown that stars and gas are off-centered with respect to the center of mass of the system. In Figure [15] we can note that the $m = 1$ and $m = 2$ modes are excited at different times. This is due to the fact that the mode excitation comes from the outer region to the inner region of the disc. Thus, there is a time lag in order to this excitation reaches the inner disc region. Since the $m = 1$ mode needs an off-centered disc mode with respect to the center of mass and since the $m = 2$ mode is mostly excited from the outer disc region, this produces a time delay among the maxima of these two modes.
Fig. 15. The global integrated ratio amplitudes of the modes $m = 1$ (solid lines) and $m = 2$ (dotted lines) for the primary galaxy ($G_1$) for the simulations 15 and 17 during a Hubble time. The $|p_{00}|$ denotes the global amplitude of the control simulation EXP00. For the simulations EXP13, 16, 25 and 28 we show the global ratio amplitudes for $G_1$ until the merger time, denoted by the dashed lines. After the merge we show the global ratio amplitude of the compound galaxy ($G_1 + G_2$). For the experiments EXP19, 20 and 31 (polar disc), we plot the evolution of $G_1$ until the merger of $G_1$ and $G_2$, represented again by the dashed lines.

The Figures 16 and 17 show the evolution of some halo contours containing, for example, about 40% and 90% the halo mass (EXP15). The early merger paper of Barnes and Hernquist (Barnes & Hernquist 1996), which first discussed about the halo mergers, has given attention only to what happened with the remnant halos at the end of the simulation. As we can see in the present Figures, the maximum halo contour deformation (90%) coincides with the maximum disc deformation. After the passage of the secondary galaxy through the primary at the Hubble time, the halos settles down and their contours resemble with the initial ones.

6. Discussion

We have evolved dynamically, using N-body simulations, two disc galaxies with halo and bulge. The initial disc model is stable against any self-excited $m = 1$ or $m = 2$ wave modes. The satellite galaxy has coplanar or polar disc orientation in relation to the disc of the primary galaxy and their initial orbits are prograde eccentric ($e = 0.1$, $e = 0.4$ or $e = 0.7$). Both galaxies have similar mass and size of the Milk Way.
Fig. 16. The $G_1$ halo contours of the EXP15 in the plane XY at three different times: $t = 0$, $t = 0.68 t_H$ at the time of the maximum amplitude of the disc $m = 2$ component (see Figure 15), and $t = t_H$. The $G_1$ disc contour of the EXP15 in the plane XY at the time $t = 0.68 t_H$. The two outer halo contour density levels correspond approximately to 40% of the total halo mass at the radius $R \approx 4$ and 90% of the total halo mass at the radius $R \approx 11$. The halo and the disc of the secondary galaxy $G_2$ can be obtained just reflecting the respective contour image relative to the Y axis at $X = 0$, since the two galaxies have the same mass distribution.

Most of the recent papers that studied the tidal interaction between two galaxies have used a fixed potential for the halo\cite{Oh et al. 2008,D,S}. This condition can mislead the results because the live halos are very important to transmit angular momentum to the disc of the primary galaxy. The halo of the primary galaxy can respond globally to disturbance of the halo of the secondary galaxy, thus it can affect the disc structure in an inward effect. These effects can be clearly seen in the analysis of the power spectra (see Figure 14).

We note that this is the first published work, as far as we know, that has studied the secular evolution of bound disc binary galaxies. Nevertheless, we will only compare our results with the global results of similar papers, since the numerical methods, initial conditions, time of integration, etc., are different from ours.

We have shown that the merger of two coplanar ($\Theta = 0$) pure stellar disc galaxies can result in a disc galaxy, instead of an elliptical one, as it is shown in other papers\cite{Bournaud et al. 2005,B}. If we have the merger of two polar ($\Theta = 90$) disc galaxies we can also have formation of lenticular-like galaxies. These results are new in the literature, as far as we have knowledge.

In fact, none of our simulations resulted in elliptical galaxies. In a recent work Bois et al. (2011) has studied the formation of early-type galaxies through mergers with a sample of high-resolution numerical simulations of binary mergers of disc galaxies. The initial galaxy model had alive halo, bulge, disc and gas. The orbits used in the merge simulations were all parabolic or hyperbolic, corresponding to initially unbound galaxy pairs, differently of our simulations where the galaxy pairs were, from the very beginning, bound in eccentric orbits.
Fig. 17. The $G_1$ halo contours of the EXP15 in the plane XZ at three different times: $t = 0$, $t = 0.68t_H$ at the time of the maximum amplitude of the disc $m = 2$ component (see Figure 15), and $t = t_H$. The $G_1$ disc contour of the EXP15 in the plane XZ at the time $t = 0.68t_H$. The two outer halo contour density levels correspond approximately to 40\% of the total halo mass at the radius $R \approx 4$ and 90\% of the total halo mass at the radius $R \approx 11$. The halo and the disc of the secondary galaxy $G_2$ can be obtained just reflecting the respective contour image relative to the $Y$ axis at $X = 0$, since the two galaxies have the same mass distribution.

Furthermore, we have demonstrated that the time of merging increases linearly with the initial apocentric distance of the galaxies and decreases with the eccentricity (see Figure 6). In their paper Boylan-Kolchin & Quataert (2008) have studied the merging time of extended dark matter haloes using N-body simulations. Each of their simulations consists of a host halo and a satellite halo; the ratio of the satellite to the host mass, varied from 0.025 to 0.3 and initial circularity of the satellite varied from 0.33 to 1, i.e., the initial eccentricity varied from 0 to 0.67. They have found that the merging time decreases exponentially with the eccentricity. This result is in partial agreement with our findings since the $T_M$ decreases with the eccentricity. However, we do not have enough simulations with different eccentricities to confirm the exponential behavior.

We also have shown that the tidal forces of the discs can excite the wave mode $m = 1$ and the wave mode $m = 2$, but they are not stable, i.e., they are transient wave modes (see Figure 14). In a previous work (Chan & Junqueira 2003) we have shown that tidal interaction of a secondary point-mass galaxy could excite stable $m = 1$ and $m = 2$ wave modes in the density distribution as well as in the velocity distribution. In contrast to our previous paper, here we begin the simulations with an apocentric distance where the halos do not touch each other. However, after the merging of the discs, when it has happened, such instabilities have faded away completely and the fused disc has become thicker and bigger.

Many authors (Oh et al. 2008, L.D.S.S) have shown that the tidal interaction can trigger gravitational instabilities, such as spiral arms or lopsidedness. Our results have confirmed the results of
these papers, that it was possible to create spiral arms, bars or lopsidedness through the tidal force, but they were transient phenomena.

The simulations with merger remnants, the waves abruptly disappear after the merger is completed (in less than one outer disc rotation period). This point, illustrated in Figure 15, shows that it is almost the opposite result of Struck et al. (Struck et al. 2011), who have found that weak flybys induce waves that take a long time to nonlinearly break. The maximum relative amplitude of these waves is at most about 15 times greater compared to the control case. The \( m = 2 \) wave mode is generated mainly by tidal interaction in the outer region of the discs. The \( m = 1 \) wave mode depends mostly of an interaction of the inner part of the discs, producing an off-centering effect of the wave mode center relative to the center of mass of the disc. These characteristics produce a time lag among the maximum formation of these two wave modes. The disc settles down quickly after the merger, in less than one outer disc rotation period. Furthermore, though the two discs may spend a long time in orbit, waves are only induced in the short time they are close together. The stellar discs can survive gentle merging, even with a massive companion and the waves abruptly disappear after the merger is completed.

Finally, galaxy discs are born gas-rich, and the key to S0 formation is how to get there from such progenitors. It is theoretically interesting that some form of disc can be preserved through some types of major merger. Practically, however, it is not likely that too many S0s are made as a result of S0+S0 or early Sa+Sa mergers. A related, and more important point is that if stellar discs can survive some gas-free, major mergers, then they are also likely to survive multiple, minor mergers, which may play a more important role in finishing the formation of S0s. The idea that minor mergers play such a role in ellipticals is very well known nowadays, making it for S0s is much more enlightening.

**ACKNOWLEDGMENTS**

One of the authors (RC) acknowledges the financial support from FAPERJ (no. E-26/171.754/2000, E-26/171.533/2002 and E-26/170.951/2006 for construction of a cluster of 16 INTEL PENTIUM DUAL CORE PCs) and the other author (SJ) also acknowledges the financial support from FAPERJ (no. E-26/170.176/2003). The author (RC) also acknowledges the financial support from Conselho Nacional de Desenvolvimento Científico e Tecnológico - Brazil.

We also would like to thank the generous amount of CPU time given by LNCC (Laboratório Nacional de Computação Científica), CESUP/UFRGS (Centro Nacional de Supercomputação da UFRGS), CENAPAD/UNICAMP (Centro Nacional de Processamento de Alto Desempenho da UNICAMP), NACAD/COPPE-UFRJ (Núcleo de Atendimento de Computação de Alto Desempenho da COPPE/UFRJ) in Brazil. Besides, this research has been supported by SINAPAD/Brazil.

The authors would like to thank Dr. Vladimir Garrido Ortega for the useful discussions at the very beginning of this work.

We acknowledge Dr. Curt Struck for the careful reading of the manuscript and giving many suggestions that improved this work.
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List of Objects