Fractional quantum Hall junctions and two-channel Kondo models

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A mapping between fractional quantum Hall (FQH) junctions and the two-channel Kondo model is presented. We discuss in detail this relation for the particular case of a junction of a FQH state at $\nu = 1/3$ and a normal metal. We show that in the strong coupling regime this junction has a non-Fermi liquid fixed point. At this fixed point the electron Green’s function has a branch cut. Here we also find that at this fixed point there is a non-zero value of the entropy equal to $S = \frac{1}{2} \ln 2$. We construct the space of perturbations at the strong coupling fixed point and find that the dimension of the tunneling operator is 1/2. These behaviors are strongly reminiscent of the non-Fermi liquid fixed points of a number of quantum impurity models, particularly the two-channel Kondo model. However we have found that, in spite of these similarities, the Hilbert spaces of these two systems are quite different. In particular, although in a special limit the Hamiltonians of both systems are the same, their Hilbert spaces are not since they are determined by physically distinct boundary conditions. As a consequence the spectrum of operators in both problems is different.

PACS: 73.40.Hm, 71.10.Pm, 73.40.Gk, 73.23.-b

I. INTRODUCTION

In recent years, advances in nanofabrication techniques have made possible to study in a controlled manner the physics of the two channel Kondo model (2CK) in real low-dimensional materials. Systems like quantum point contact tunnel junctions, quantum dots connected to leads, and quantum Hall tunnel junctions; have been proposed as good candidates to exhibit the full range of 2CK physics by an appropriate variation of the experimental parameters.

All of these experiments can be described in terms of the conceptual setting of the Kondo model, i.e. considering the effects of a local interaction of fermions with a quantum impurity. While in the one-channel Kondo model one deals with a set of effectively one-dimensional (radial) fermions -in practice a Fermi liquid- coupled to a point-like quantum mechanical magnetic impurity, in the multichannel model the situation is complicated either by a band degeneracy or by an orbital impurity. The theoretical description of the systems mentioned above is rather natural in terms of a 2CK model, as it can be seen, for instance, in the case of quantum dots: at resonance the dot contains two degenerate levels that can be associated with the impurity spin states (up and down), while the two leads can be represented by the channels. When the leads are modelled by Fermi liquids, this identification is extremely useful in the analysis of experimental results for the various observed conductance regimes.

Another set of important and exciting new experiments have been carried out in similar settings with systems that are best characterized as Luttinger liquids. Perhaps the cleanest, from the point of view of the physics, are the tunneling experiments in which electrons from an external three-dimensional reservoir are injected into the edges of a fractional quantum Hall state. Just as important are recent noise experiments which, having made possible the study of the properties of quasiparticles in FQH states, provided further support to the Luttinger liquid picture for edge states. Also, Luttinger physics seems to be the appropriate description for transport experiments in carbon nanotubes and nanotube junctions.

Hence, it is tempting to model these systems also with the two-channel Kondo model picture, i.e. two channels locally interacting with a quantum impurity. For example, in the case of FQH junctions, the tunneling experiments involve transfer of electrons from a Fermi liquid external lead to an edge state at a weak tunneling center, or tunneling of quasiparticles between two edges in a Hall bar experiment at a constriction. In the case of nanotubes there are junctions at the endpoints of the tubes or internal junctions due to kinks in the tube itself. Thus, tunneling centers, constrictions and kinks play the role of the quantum impurity, while the channels are not Fermi but Luttinger liquids.

The existence of a connection between quantum impurity problems, (such as Kondo models), and FQH junctions has been known for some time particularly since the work of Kane and Fisher. As it turns out, the universal properties of all these models can be understood in terms of a generic quantum impurity problem. Furthermore, all quantum impurity problems -at least their universal behavior- can be classified in terms of an appropriate set of boundary conditions for a suitable 1+1-dimensional boundary conformal field theory.

Our recent work on quantum Hall tunnel junctions has yielded evidence for a relation between the properties of the junctions in the strong coupling regime and some effective 2CK model. The purpose of this paper is to investigate these analogies further. A crucial problem discussed below is the relation between the description...
of the isolated FQH edges and their Hilbert spaces as given by the chiral Luttinger liquid model proposed by Wen \[3\], and their 2CK model counterparts. Although chiral Luttinger liquids are examples of boundary conformal field theories, the relation with the 2CK model at this level is still far from obvious since it is not clear if the Hilbert spaces can be mapped into each other or not. Thus, in order to make possible such a mapping it is necessary to carry out a detailed comparison between the respective Hilbert spaces. The junction model that looks like the more suitable candidate to compare with the 2CK model is the one between a \( \nu = 1/3 \) FQH state and a normal metal. In fact, analysis of different theoretical models \[1,2\], for this particular junction seem to indicate that, in the strong coupling regime, it possesses several properties that appear in the 2CK physics like:

1. Both systems have Fermi liquid and non-Fermi liquid fixed points.

2. At their respective non-Fermi liquid fixed points they both have a vanishing one-body S-matrix (albeit of physically different operators)

3. They both have a non-integer value for the boundary entropy at their respective non-Fermi liquid fixed points equal to \( S_{2CK} = \frac{1}{2} \ln 2 \). For a general FQH junction of a Laughlin state the entropy is given by \( S_{FQH/NM} = \frac{1}{2} \ln(k+1) \), where \( 2k+1 = \nu^{-1} \). Thus, for \( k = 1 \) corresponding to \( \nu = 1/3 \), the entropy has the 2CK value \[2,3\].

4. In both cases the renormalization group (RG) flow from the non-Fermi liquid to the Fermi liquid fixed point is induced by a relevant operator of scaling dimension 1/2. While in the 2CK, this perturbation corresponds to a potential breaking the channel symmetry \[1,3\]; in the FQH/NM junction it corresponds to the tunneling term at the strong coupling fixed point.

Considering all these similarities, it is tempting to conclude that FQH junctions are in fact equivalent to an effective 2CK system. This is most remarkable given that the 2CK system at the Tolouse point flows to the non-trivial fixed point of the isotropic system and thus the apparent problem with the symmetry is avoided.

The paper is organized as follows. After briefly reviewing the model proposed for a FQH/NM junction (Sec. II-A), we discuss the \( \nu = 1/3 \) junction and discuss the analogies with the two-channel Kondo problem. In Sec. II-B we show that the perturbation produced by a tunneling operator (in the dual picture) has dimension 1/2 as in the 2CK case. In Sec. II-C we show that the value of the entropy at the non-Fermi liquid fixed point coincides with the 2CK boundary entropy. In Sec. II-D we show that at the non-Fermi liquid fixed point the Green’s function for electrons on the normal metal side of the junction has a branch cut singularity. Thus, at this fixed point, the one-body scattering matrix vanishes as it does in the 2CK system. We sketch the steps in the calculation for the junction in equilibrium, \( \nu = 1/3 \) at \( V = 0 \) (zero voltage), and in non-equilibrium (\( V \neq 0 \)). Next, by mapping the electron operator present on the normal metal side of the junction, we discuss in Sec. III the mapping of the FQH/NM junction to the Toulouse limit of the 2CK problem. Here we show that, in spite of the close analogies, both models have different operator content, \( \nu = 1/3 \), their Hilbert spaces are different. The discussion of these results are summarized in Sec. IV.

II. MODEL OF A FQH/NM JUNCTION

A. Model for a FQH junction

We start this section with a review of the model for a FQH/NM junction used in Ref. \[2\]. The Lagrangian for the FQH/NM junction that describes the dynamics on the edge of a FQH liquid, the electron gas reservoirs, and the tunneling between them at a single point-contact is:

\[ \mathcal{L} = \mathcal{L}_{\text{edge}} + \mathcal{L}_{\text{res}} + \mathcal{L}_{\text{tun}}. \]  

The dynamics of the edge of the FQH liquid with a Laughlin filling factor \( \nu = \frac{1}{2k+1} \) is described by a free chiral boson field \( \phi_1 \) with the Lagrangian \[3\]

\[ \mathcal{L}_{\text{edge}} = \frac{1}{4\pi} \phi_1 (\partial_t - \partial_x) \phi_1. \]  

where the units have been chosen so that the velocity of both bosons is \( v = 1 \) (this is consistent for a coupling at a single point in space).
The operators that create electrons and quasiparticles at the edge of a Laughlin state with filling factor $\nu$ are given by

$$
\psi_e \propto \eta_{\text{edge}} : e^{-i \frac{\pi}{2} Q_{\text{res}}} \phi_1(x,t) : ,
$$
$$
\psi_{qp} \propto \eta_{\text{edge}} : e^{-i \sqrt{\nu} \phi_1(x,t)} : ,
$$

(3)

where we introduced the Klein factor $\eta_{\text{edge}}$.

In Eq. (1), $L_{\text{res}}$ describes the dynamics of the electron gas reservoir. As shown in Ref. [19], a 2D or 3D electron gas can be mapped to a 1D chiral Fermi liquid (FL) ($\nu = 1$) when the tunneling is through a single point-contact. This 1D chiral Fermi liquid is represented by a free chiral boson field $\phi_2$. $L_{\text{res}}$ is given by

$$
L_{\text{res}} = \frac{1}{4\pi} \partial_x \phi_2 (\partial_t - \partial_x) \phi_2 .
$$

(4)

In this case, the electron operator is given by

$$
\psi_{\text{res}} \propto \eta_{\text{res}} : e^{-i \phi_2(x,t)} : ,
$$

(5)

where $\eta_{\text{res}}$ is the corresponding Klein factor for the electrons on the metal side of the junction.

The tunneling Lagrangian between the FQH system and the reservoir is

$$
L_{\text{tun}} = \Gamma \delta(x) \eta_{\text{edge}} \eta_{\text{res}} : e^{i[\frac{\pi}{2} Q_{\text{res}}(x,t) - \phi_2(x,t)]} : + \text{h.c.} ,
$$

(6)

where $\Gamma$ represents the strength of the electron tunneling amplitude which takes place at a single point in space $x = 0$, the point contact. In Eqs. (3)-(6) the Klein factors insure that an electron on the FQH edge anticommutes with an electron on the reservoir. The simplest choice [20] is to define them as

$$
\eta_{\text{edge}} = : e^{i \frac{\pi}{2} Q_{\text{edge}}} : ,
$$
$$
\eta_{\text{res}} = : e^{-i \frac{\pi}{2} Q_{\text{res}}} : ,
$$

(7)

which satisfy the correct mutual statistics. Here $Q_{\text{edge}}$ and $Q_{\text{res}}$ are the total charge of the FQH edge and the Fermi liquid reservoir respectively. Since the total charge $Q_{\text{edge}} + Q_{\text{res}}$ is conserved by the tunneling process, the factor $\eta_{\text{edge}} \eta_{\text{res}}$ in Eq. (6) is a constant of motion and as such it can be absorbed in the coupling constant. In Sec. 4 we will find an analogous set of operators in the context of the two-channel Kondo problem.

In what follows, by analogy with quantum impurity problems, we will refer to the point contact as the impurity. Notice however that, in contrast with the impurity as in the 2CK model, the point contact does not have internal degrees of freedom. In spite of this difference we still will find a close analogy between the two problems.

An external voltage difference between the two sides of the junction can be introduced in the model by letting $\Gamma \rightarrow \Gamma e^{-i \omega_0 t}$, where $\omega_0 = eV/\hbar$. This external voltage $V$ can be interpreted as the difference between the chemical potentials of the two systems: $V = \mu_1 - \mu_2$. In what follows the tunneling amplitude will be considered to be complex, in order to deal with the more general non-equilibrium situation. For the equilibrium calculation we will simply put $V = 0$, thus making the coupling constant $\Gamma$ real.

By a suitable rotation the original Lagrangian $\mathcal{L}$ can be mapped into a new one [12,13].

$$
\mathcal{L} = \mathcal{L}_0 + \Gamma \delta(x) e^{-i \omega_0 t} e^{i \sqrt{\nu} \phi_1(x,t) - \phi_2(x,t)} + \text{h.c.}
$$

(8)

where the new fields $\phi_1'$ and $\phi_2'$ have a free dynamics governed by $\mathcal{L}_0$ and $g'$ is an effective Luttinger parameter (which can also be regarded as an “effective filling fraction”) given by:

$$
g' = \frac{(1 + \nu^{-1})}{2} .
$$

(9)

The rotation matrix is given by:

$$
\begin{pmatrix}
\phi_1' \\
\phi_2'
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix} ,
$$

(10)

where the angle $\theta$ is determined by

$$
\cos 2\theta = \frac{2\sqrt{\nu}}{1 + \nu} , \quad \sin 2\theta = \frac{1 - \nu}{1 + \nu} .
$$

(11)

Next, we introduce the fields $\phi_-$ and $\phi_+$ that separate $\mathcal{L}$ into two decoupled Lagrangians $\mathcal{L}_+$ and $\mathcal{L}_-$,

$$
\phi_+ = \frac{\phi_1' + \phi_2'}{\sqrt{2}} , \quad \phi_- = \frac{\phi_1' - \phi_2'}{\sqrt{2}} .
$$

(12)

In terms of the $\phi_\pm$ fields the total Lagrangian reads:

$$
\mathcal{L} = \mathcal{L}_0(\phi_+; \phi_-) + \Gamma \delta(x) e^{-i \omega_0 t} e^{i \sqrt{2g'} \phi_-(x,t)} + \text{h.c.}
$$

(13)

where $\mathcal{L}_0(\phi_+; \phi_-)$ describes the free dynamics. The strong coupling limit of this system is best described in the dual picture with the dual fields $\phi_\pm$. In terms of these dual fields, the effective Lagrangian has a new Luttinger parameter $g' = \frac{1}{g'}$ and an effective tunneling amplitude $\Gamma \sim \frac{\Gamma}{g'^2}$.

The (dual) Lagrangian is

$$
\tilde{\mathcal{L}} = \tilde{\mathcal{L}}_0(\phi_+; \phi_-) + \tilde{\Gamma} \delta(x) e^{-i \omega_0 t} e^{i \sqrt{2g'} \phi_-(x,t)} + \text{h.c.}
$$

(14)

Working exactly at the strong coupling fixed point, the duality transformation can also be written as follows:

$$
\begin{align*}
\phi_1' &= \Theta(-x) + \phi_2' \Theta(x) \\
\phi_2' &= \Theta(x) + \phi_1' \Theta(-x)
\end{align*}
$$

(15)

where $\Theta(x)$ is the step function. The expressions for the dual fields in terms of the original fields correspond to an effective change in the basis used to describe the model. This change of basis can be carried out only at the strong coupling fixed point i.e. $\Gamma = 0$. 

3
III. FIXED POINTS OF THE FQH/NM JUNCTION

In this section we analyze the fixed points of the model for a FQH/NM junction described in the previous section. We begin by calculating the scaling dimension of the tunneling operator in each fixed point, i.e., \( \Gamma = 0 \) and \( \Gamma \rightarrow \infty \). We follow closely the procedure developed by Cardy [21] to obtain the scaling dimension for a boundary operator. At the end of this section we calculate the boundary entropy at the strong coupling fixed point.

A. \( \Gamma = 0 \) Fixed Point

As stated in Sec. [1], the Lagrangian describing the junction is:

\[
\mathcal{L} = \frac{1}{4\pi} \partial_x \phi_+ (\partial_t - \partial_x) \phi_+ + \frac{1}{4\pi} \partial_x \phi_- (\partial_t - \partial_x) \phi_- + 2 \Gamma \delta(x) \cos \left( \sqrt{\frac{2}{g'}} \phi_-(x,t) \right) \tag{16}
\]

where the field \( \phi_-(x,t) = \phi_-(t + ix) \) extends from \( x \rightarrow -\infty \) to \( x \rightarrow \infty \) (see Fig. 1).

![FIG. 1. Chiral boson field \( \phi_-(x,t) \) with a quantum point contact modeled as an impurity at the origin.](image)

We now proceed to transform the chiral theory into a non-chiral one on the semi-line by the transformation:

\[
\phi_L(x,t) = \phi_-(x,t), \quad \phi_R(x,t) = \phi_-(-x,t) \tag{17}
\]

where the fields \( \phi_{R,L}(x,t) \) are defined on the semi-line \( x \geq 0 \) as shown in Fig. 2.

![FIG. 2. Non-chiral theory in terms of the chiral fields \( \phi_{R,L} \) with an impurity in the origin.](image)

It is straightforward to show that the sum of the free chiral Lagrangians in Eq. (16) corresponds to a free theory for the non-chiral field \( \phi \) defined as

\[
\phi(x,t) = \phi_R(x,t) + \phi_L(x,t)
\]

\[
\mathcal{L}_0 = \frac{1}{8\pi} \int_0^L dx \left[ (\partial_t \phi)^2 - (\partial_x \phi)^2 \right] \tag{18}
\]

The tunneling term is transformed in the following way

\[
\mathcal{L}_{tun} = 2 \int_{-\infty}^\infty dx \Gamma \delta(x) \cos \left( \sqrt{\frac{2}{g'}} \phi_-(t + ix) \right)
\]

\[
= 2 \int_0^\infty dx \Gamma \delta(x) \left[ \cos \left( \sqrt{\frac{2}{g'}} \phi_L(t + ix) \right) \right.
\]

\[
\left. + \cos \left( \sqrt{\frac{2}{g'}} \phi_R(t - ix) \right) \right] \tag{19}
\]

By using the fact that \( \phi_R(x = 0,t) = \phi_L(x = 0,t) \), the expression for \( \mathcal{L}_{tun} \) finally reads

\[
\mathcal{L}_{tun} = 2^2 \Gamma \cos \left( \frac{1}{\sqrt{2g'}} \phi(x = 0,t) \right) \tag{20}
\]

Once the chiral theory has been transformed into a non-chiral one, we follow the procedure developed by Cardy to calculate the scaling dimension. By transforming the two-point function of the non-chiral field in a plane with boundaries, into a four-point function of the chiral field in the entire plane, we obtain:

\[
\Delta_{FQH/NM}(\Gamma = 0) = \frac{1}{g'} \tag{21}
\]

for the 1/3 FQH junction, \( g' = 1/2 \) and

\[
\Delta_{FQH/NM}(\Gamma = 0) = 2 \tag{22}
\]

Because the dimension of the boundary space is equal to 1, the value of \( \Delta_{FQH/NM}(\Gamma = 0) \) implies that \( \Gamma = 0 \) is an irrelevant perturbation as expected.

B. \( \Gamma \rightarrow \infty \) Fixed Point

To calculate the dimension of the tunneling term as appearing in the dual picture of the junction, we proceed along the same lines described above. The dual tunneling Lagrangian now looks like

\[
\mathcal{L}_{tun} = 2^2 \Gamma \cos \left( \frac{\sqrt{g'}}{2} \phi(x = 0,t) \right) \tag{23}
\]

In this case we obtain the scaling dimension:

\[
\Delta_{FQH/NM}(\Gamma \rightarrow \infty) = g' \tag{24}
\]

and for the 1/3 FQH junction:

\[
\Delta_{FQH/NM}(\Gamma \rightarrow \infty) = \frac{1}{2} \tag{25}
\]

Since \( \Delta_{FQH/NM} < 1 \), the tunneling operator is a relevant perturbation at the strong coupling fixed point.
Also, it was shown in Ref. [12] that, although exactly at the strong coupling fixed point many scattering processes are forbidden, the main effect of the tunneling operator is to increase the allowed scattering processes that otherwise are suppressed. Hence, in this way the presence of the tunneling operator drives the system from the non-Fermi liquid to the Fermi-liquid fixed point. In analogy with quantum impurity problems, it is useful to introduce an energy scale \( T_k \), associated with the impurity. At frequencies high compared with the crossover scale \( T_k \), the amplitude for these additional scattering channels is exceedingly small and, in this regime, non-Fermi liquid physics should be observable. However, this analysis assumes that the only relevant operator is the tunneling operator and all other operators, such as multiparticle tunneling processes, at the strong coupling fixed point are irrelevant. While at the weak coupling fixed point it is trivial to show that these higher order processes are indeed strongly suppressed, it is not obvious that this should be the case at the strong coupling fixed point. Nevertheless it is reasonable to expect that there should exist a regime in which these higher order processes can be effectively ignored. In this case the model with just one relevant perturbation makes physical sense.

Below we will show by comparison with the 2CK model, that this tunneling operator has the same scaling dimension as the channel symmetry breaking operator in the 2CK model. Interestingly, it is known that the channel symmetry breaking perturbation drives the 2CK from the non-Fermi liquid to a Fermi liquid fixed point. Thus, it is reasonable to conjecture that the tunneling operator in the FQH junction corresponds to the channel symmetry breaking operator in the 2CK model. We will show below that, to an extent, this is true.

### C. Entropy at the non-Fermi liquid fixed point

The dual description describes the physics at strong coupling, i.e., the non-Fermi liquid fixed point. Thus, to obtain the entropy at this fixed point it is necessary to work with the Lagrangian describing the dynamics of the field \( \phi_- \) (Eq. (14)) which corresponds to a boundary sine-Gordon model. In the boundary sine-Gordon model, the interaction term gives a boundary contribution to the energy (the boundary itself can hold energy). As a consequence, energy conservation at the boundary implies a dynamical boundary condition, e.g., the boundary condition can flow from Neumann (\( \partial \phi(0) = 0 \)) to Dirichlet (\( \phi(0) = 0 \)).

In [24], Fendley, Saleur and Warner (FSW) calculated this boundary contribution to the free energy. They showed that in the IR (low energy) limit, this contribution vanishes but it remains finite in the UV (high energy) limit. From the free energy, they obtained the boundary contribution to the entropy which, in the UV limit, is given by \( S = \frac{1}{2} \ln(\lambda + 1) \) where \( \lambda \) is the compactification radius of the bosonic field: \( R = \sqrt{\frac{\lambda}{\pi}} \).

By comparison with these results, we can obtain the value for the entropy in the FQH/NM junction. In this case, the IR limit corresponds to the \( \Gamma = 0 \) fixed point and the UV limit to the \( \Gamma \to \infty \) fixed point. Thus, we obtain a vanishing entropy at \( \Gamma = 0 \) and a finite entropy at \( \Gamma \to \infty \). Since the compactification radius is \( \sqrt{2g} = \sqrt{\frac{k + 1}{2}} \), the value of the entropy is given by \( S = \frac{1}{2} \ln(k + 1) \). Furthermore, for the special case of \( k = 1 \), the value of the entropy is \( S = \frac{1}{2} \ln(2) \). In other words, a FQH/NM junction with an effective \( g' = 1/2 \) i.e., a \( \nu = 1/3 \) junction, has a boundary contribution to the entropy of \( S = \frac{1}{2} \ln(2) \) at the \( \Gamma \to \infty (\Gamma = 0) \). This is precisely the same value found for the two-channel, spin-\( \frac{1}{2} \) Kondo model at the non-trivial fixed point. Thus, we have proved that the \( \nu = 1/3 \) FQH/NM junction and the 2CK model have the same value for the entropy at the non-Fermi liquid fixed point.

### IV. GREEN’S FUNCTION FOR ELECTRONS ON THE NORMAL SIDE OF A FQH/NM JUNCTION

As shown in Ref. [12], the scattering processes allowed in a junction between a normal metal and a FQH liquid are different at the two different fixed points \( \Gamma = 0 \) and \( \Gamma \to \infty \). While for \( \Gamma = 0 \) the scattering matrix corresponds to a Fermi liquid fixed point, for \( \Gamma \to \infty \) the scattering matrix is non-unitary. This non-unitarity is reflected by the vanishing of the electron-electron scattering matrix element. Usually, the manifestation of Fermi or non-Fermi liquid behavior appears in the electron Green’s function. In this section, we present a standard perturbation theory calculation of the Green’s function for electrons on the normal metal side of the junction. In what follows instead of working with the tunneling amplitude \( \Gamma \), it is more convenient to work with the energy scale \( T_k \) introduced earlier. This energy scale is related to the tunneling amplitude by \( T_k = 4\pi |\Gamma|^2 \). Because \( \Gamma = 0 \) is an unstable fixed point and the perturbation is a relevant operator (as we show in Sec. [11]), it is important to remark that the expression obtained for the Green’s function is valid in the regime of high energies or large momenta compared with \( T_k \), i.e., when \( T_k \) is the smallest energy scale. The electron’s propagator in real space-time is given by:

\[
G_e(x, x', t, t') = -i[T: e^{-i\phi_2(x, t)} : e^{i\phi_2(x', t')} :] \tag{26}
\]

i.e. an electron is created at \((x', t')\) and destroyed, past the point contact, at \((x, t)\). Here \( T \) represents the time-ordered product and \( \cdots : \) : represents the normal ordered operator. Notice that this definition presupposes that the electron propagator depends on \( x, x' \) and not necessarily on the difference \( x - x' \), i.e., translation invariance is broken due to the presence of the point contact. The Fourier transform of this quantity is given by:

\[
\]
This expression can be interpreted as the propagator for an electron created in the state \((k_i, \omega_i)\) and destroyed in the state \((k_o, \omega_o)\). Because the presence of the impurity breaks translation invariance, momentum is not a good quantum number for this problem. Nevertheless far away from the impurity site, the normal metal is a Fermi liquid with translation invariance. If we consider the scattering of electrons that are created and destroyed asymptotically far away from the impurity, then we can use the operator \(e^{i\phi_2(x,t)}\) to represent electron states (or use momentum eigenvalues to label electron states).

We can calculate the expression for \(G_e(\omega, k_i; \omega_i, k_i)\) at both fixed points. In particular, at the \(\Gamma = 0\) fixed point, it is straightforward to show that the electron’s Green’s function presents the characteristic pole structure that is the signature of Fermi liquid behavior. To see this, notice that the mean value in this case, is taken with respect to an action \(S = S_1 + S_2\), where \(S_{1,2}\) are the actions of free chiral fermion fields. Thus, we obtain:

\[
G_e(\omega, k_i) \propto \frac{1}{\omega + k_i - i\epsilon}
\]  

\(i.e\) the Green’s function has the pole structure corresponding to a Fermi liquid as expected. The details of the calculation are given in Appendix A.

The tunneling term, represents a perturbation by an irrelevant operator (see Sec. II), and as such, it is not expected to introduce appreciable changes in the low-energy physics of the system. In particular, this pole structure is effectively the leading term in the expression for the Green’s function, when the tunneling perturbation is included.

To calculate \(G_e(\omega_o, k_o; \omega_i, k_i)\) at the strong coupling fixed point, we use Eq. (13). Notice that in this case, the perturbative expansion is around the non-Fermi liquid fixed point (at \(\bar{\Gamma} = 0\)), hence the mean value is taken with respect to the action containing the dual Lagrangian given in Eq. (14). As stated above, this perturbative calculation is legitimate only at high enough energies or temperatures; it can be done in equilibrium conditions, where there is no external voltage applied to the junction, and in non-equilibrium conditions. In what follows we will describe first the equilibrium situation and later we will show that non-equilibrium corresponds only to a shift in frequencies for the incoming electron. We will restrict to the case where \(k_o = k_i = k\), i.e., the incoming and outgoing electrons are in the same momentum state and by convenience we define \(\omega = \omega_i\). By standard perturbative techniques we calculate \(G_e(\omega, k)\) as follows

\[
G_e(\omega, k) = \int_{-L}^{L} dx dx' \int_{-\infty}^{\infty} dt dt' e^{-i(k_o x - \omega t)} G_e(x, x', t, t')
\]

where

\[
S_I = \int_{-L}^{L} dx \int_{-\infty}^{\infty} dt \delta(x) \cos(\sqrt{2g^2}\phi_\pm)
\]

Because the interacting action is given in terms of the dual fields, it is necessary to write the field for the electron operator \(\phi_2(x,t)\) in terms of its duals. Thus, we work with the inverse of the dual transformation:

\[
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix} = \begin{pmatrix}
\Theta(x) & \frac{\cos \theta - \sin \theta}{\sqrt{2}} & \frac{\cos \theta + \sin \theta}{\sqrt{2}} \\
\frac{\cos \theta + \sin \theta}{\sqrt{2}} & \cos \theta + \sin \theta & \cos \theta - \sin \theta
\end{pmatrix} \begin{pmatrix}
\tilde{\phi}_+ \\
\tilde{\phi}_-
\end{pmatrix}
\]

\(\phi_2(x,t)\) in terms of its duals, the mean value factorizes into a product of two terms:

\[
\langle : e^{-i\sqrt{2}g\tilde{\phi}_+(x,t)} :: e^{i\sqrt{2}g\tilde{\phi}_+(x',t')} :\rangle S_0(\tilde{\phi}_+) \\
\times \langle : e^{-i\sqrt{2}g\tilde{\phi}_-(x,t)} :: e^{i\sqrt{2}g\tilde{\phi}_-(x',t')} :\rangle S_0(\tilde{\phi}_-)
\]

\(= 0\)

In terms of a Coulomb gas interpretation, this is nothing else that the consequence of the neutrality condition of the Coulomb gas.

\textbf{B. Order \(n = 0\)}

By using the expansion in Eq. (29) we find that, at first order in \(\bar{\Gamma}\), the expression for \(G_e(x, x', t, t')\) is also given by a product of two factors:

\[
G_e^{(1)} = \bar{\Gamma} \int_{-\infty}^{\infty} dt_1 \langle : e^{-i\sqrt{2}g\tilde{\phi}_+(x,t)} :: e^{i\sqrt{2}g\tilde{\phi}_+(x',t')} :\rangle S_0(\tilde{\phi}_+) \\
\times \langle : e^{-i\sqrt{2}g\tilde{\phi}_-(x,t)} :: e^{i\sqrt{2}g\tilde{\phi}_-(x',t')} :\rangle S_0(\tilde{\phi}_-)
\]

Using the fact that

\[
\langle T : e^{i\alpha_1\phi(x_1,t_1)} : \cdots : e^{i\alpha_n\phi(x_n,t_n)} :\rangle S_0(\phi) = e^{-\frac{1}{2} \sum_{i,j} \alpha_i \alpha_j \langle [\phi(x_i,t_i),\phi(x_j,t_j)] \rangle}
\]

where

\[
\langle [\phi(x_i,t_i),\phi(x_j,t_j)] \rangle = 2\pi \delta(x_i - x_j) \delta(t_i - t_j)
\]
\[ \langle T \{ \phi(x_i, t_i) \phi(x_j, t_j) \} \rangle = -\ln[\delta + \text{sgn}(t_i - t_j)(t_i - t_j - x_i + x_j)] \] (36)

we obtain the following expression for \( G_e(k, \omega) \)

\[ G_e = \tilde{\Gamma} \int_{-\infty}^{\infty} d\omega d^k R_\alpha(\omega, k_1) R_\alpha(\omega_1, -k_1) \times R_{1-\alpha}(\omega - \omega_1, k - k_1) + \text{h.c.} \] (37)

\[ R_\alpha(\omega, k) = \int_{-\infty}^{\infty} dx \, dt \, e^{i(\omega t - kx)} \left[ \frac{\delta + \text{sgn}(t)(t - x)}{1 + \nu} \right] \] (38)

\[ R_{1-\alpha}(\omega, k) = \int_{-\infty}^{\infty} dx \, dt \, e^{i(\omega t - kx)} \left[ \frac{\delta + \text{sgn}(t)(t - x)}{1 + \nu} \right] \] (39)

(\text{where } \alpha = 2\nu/(1 + \nu)). The explicit expression for \( R_\alpha(\omega, k) \) and \( R_{1-\alpha}(\omega, k) \) are given in Appendix \[\text{B}.\] Finally, the expression for the Green’s function depends on the value of \( \nu \) and it is given by:

- For \( \nu > 1/3 \)
  \[ G_e^{(1)}(\omega, k) = \tilde{\Gamma} C(\nu) \frac{1}{(\omega - k + i\epsilon)^2} k^{-\frac{1}{1 + \nu}} \] (40)

where

\[ C(\nu) = 2^\nu \left( \frac{\nu}{1 + \nu} \right)^4 \frac{\Gamma \left[ \frac{1 + \nu}{1 + \nu^2} \right]}{\Gamma^2 \left[ \frac{2\nu}{1 + \nu^2} \right]} \left( \frac{1 + \nu}{3\nu - 1} \right) \] (41)

Here \( \Gamma(x) \) is the Gamma function and \( \epsilon \) is an infrared (IR) cutoff used in the calculation of the chiral fermion propagators \( R_\alpha \) and \( R_{1-\alpha} \).

- For \( \nu = 1/3 \)
  \[ G_e^{(1)}(\omega, k) = \frac{(4\pi)^2}{(\omega - k + i\epsilon)^2} \left( \frac{k}{\pi T_k} \right)^{-\frac{1}{2}} \ln \left[ \frac{k}{T_k} \right] \] (42)

Here we have used the fact that \( T_k \propto \tilde{\Gamma}^2 \). These results hold only at large energies (and momenta) and thus \( T_k \) plays the role of an IR cutoff.

- For \( 0 < \nu < 1/3 \)
  \[ G_e^{(1)}(\omega, k) = \frac{C''(\nu)}{T_k^2} \frac{k^{-\frac{1}{1 + \nu}}}{(\omega - k + i\epsilon)^2} \Phi \left( \frac{3\nu - 1}{1 + \nu}; \frac{4\nu}{1 + \nu}; \frac{T_k}{k} \right) \] (43)

where

\[ C''(\nu) = \frac{(2\pi)^4}{\Gamma^2} \left( \frac{2\nu}{1 + \nu^2} \right)^{\frac{1 - \nu}{1 + \nu}} \left( \frac{1 + \nu}{3\nu - 1} \right)^2 \] (44)

where \( \Phi(x, y, z) \) is the confluent hypergeometric function.

Our results for the Green function at the strong coupling fixed point show that it has a branch cut singularity, in clear contrast with the pole structure that appears at weak coupling. The case \( \nu = 1/3 \) is special in that it has a logarithmic branch cut. Furthermore, the structure of the expressions for the Fermion propagator has the form:

\[ G_e^{(1)}(\omega, k) = G_e^{\text{out, in}} C \] (45)

where \( G_e^{\text{out, in}} \) is the Green’s function for a chiral Fermi liquid. By inspection we see that \( C \) plays the role of a vertex function, and as such it yields the leading order behavior for the one-body S-matrix. Notice that exactly at the strong coupling fixed point the one-body S-matrix is zero. The results of Eqs. (40), (42) and (43) represent the leading non-vanishing behavior close to the strong coupling fixed point. These results apply at frequencies high compared with the crossover scale \( T_k \). (Notice that at this level the frequency dependence enters only through \( G_e^{\text{out, in}} \).

The branch cut singularity of \( \Sigma \) and the vanishing of the one-body S-matrix show that at the strong coupling fixed point scattering is incoherent (i.e., there is a broad continuum of multiparticle scattering processes) which signals the breakdown of the simple particle picture of the decoupled Fermi liquid. This result is superficially reminiscent of scattering processes off a black hole, with the broad continuum seemingly playing the role of Hawking radiation.

An interesting point in this calculation is the dependence of \( G_e \) on the filling fraction \( \nu \). This dependence can be understood in terms of a screening process by using the Coulomb gas interpretation for the expectation values. In terms of the field \( \phi_2 \) we see that the dimensions of the operator representing the insertion of a charge due to the impurity \( e^{-ig\phi_2(x, t)} \) and the operator representing the asymptotically outgoing electron \( e^{-ig\phi_2(x, t)} \) depend on \( \nu \). In particular, for \( \nu = 1/3 \) these dimensions have the same value of 1/2. In terms of a Coulomb gas interpretation the unit charge created at \( (x, t) \) is partially or almost totally screened by the inserted charge representing the effects of the impurity, while the unscreened part of it is the charge seen at \( (x, t) \). For \( \nu > 1/3 \) the inserted charge is very effective in screening the charge created at \( (x', t') \) and thus, the charge seen at \( (x, t) \) is small. As \( \nu \) decreases, the screening diminishes and, in particular at \( \nu = 1/3 \) the inserted charge screens it only by half. In other words, for \( \nu = 1/3 \) these dimensions have the same value of 1/2, as it was observed previously in Ref. [6]. Finally, for \( \nu < 1/3 \) the screening by the inserted charge is almost completely ineffective.
C. Non-equilibrium case

Finally it is interesting to consider a non-equilibrium situation. For this purpose we include a voltage difference between the reservoir and the FQH edge by replacing the original coupling constant $\Gamma$ by $\Gamma e^{-i\omega t}$. Repeating the same procedure outlined above, we find that Eq. (37) is modified as follows:

$$G_e(k, \omega) = \tilde{\Gamma} \int_{-\infty}^{\infty} d\omega_1 \, dk_1 \, R_\alpha(\omega_1, k_1) R_\alpha(-\omega_1 + \omega_0, -k_1) \times R_{1-\alpha}(\omega - \omega_1 + \omega_0, k - k_1) + h.c. \quad (46)$$

Carrying out the rest of the algebra we find that the effect of the voltage is to change the Green’s function of the outgoing electron ($G_{\text{out}}$) by

$$\frac{1}{(\omega - k + ie)^2} \rightarrow \frac{1}{(\omega - k + ie) (\omega + \omega_0 - k + i\epsilon)} \quad (47)$$

Hence, to leading order, the external voltage does not affect the branch cut singularity of the Green’s function.

V. MAPPING OF THE FQH/NM JUNCTION TO THE 2CK PROBLEM: DOES IT WORK?

Given all the similarities between the $\nu = 1/3$ FQH junction and the 2CK model, it is natural to look for an explicit mapping that can relate not just the formal Hamiltonians, but also the operator content of both models. In particular, since in the FQH/NM junction the strong coupling properties can be connected to the weak coupling ones through a duality transformation, it is possible to look for a mapping between the electron operator as defined in the FQH/NM junction at couplings $\Gamma = 0$ and $\Gamma \rightarrow \infty$. It is also natural to ask whether the fermions of the reservoir side of the junction are related in any way to the fermions of the 2CK model. Since the model for the FQH/NM junction is written in the fermionic reservoir, it is useful to work with the 2CK model also in the bosonized version.

We begin our mapping by showing that the (bosonized) Hamiltonian for the FQH/NM junction (in its dual form) can be mapped onto the (bosonized) Hamiltonian of the 2CK model, in the Toulouse limit. Following Ref. [24], the Hamiltonian for the 2CK model after performing the Emery-Kivelson transformation [24] reads:

$$H'_0 = H_0$$

$$H'_\perp = \frac{J_p^{(1)}}{4\pi\alpha} (-1)^{N_{c+} + N_{c-}} [S^+ e^{-i\phi_{s-}(0)} - h.c.]$$

$$+ \frac{J_p^{(2)}}{4\pi\alpha} (-1)^{N_{c-} - N_{c+}} [S^+ e^{i\phi_{s-}(0)} - h.c.]$$

$$H_{\parallel} = (J_z - 2\pi v_F) S_z \partial_x \phi_{s+}(0) + \delta J_z S_x \partial_t \phi_{s+}(0) \quad (48)$$

where $H_0$ is the spin-1/2 free fermion Hamiltonian, written in terms of two chiral boson one for charge and one for spin for each channel denoted by ±. Each chiral boson has compactification radius (Luttinger parameter) equal to one. In Eq. (48), $J_p^{(i)}$ is the component of the coupling constant $J$ on the plane for channel $i$, $J_p^{(1)} + J_p^{(2)}$ and $\delta J_z = (J_p^{(1)} - J_p^{(2)})/2$. Also, since the charge modes decouple, their Hamiltonian has been absorbed in $H_0$.

In order to compare the Lagrangian for the 2CK model, with the Lagrangian for the FQH/NM junction at $\nu = 1/3$, we will rescale time and space so that the constants $\alpha$ and $v_F$ take the values $\alpha = \frac{1}{4\pi}$ and $v_F = 1$. For simplicity we also take $J_p^{(2)} = 0$, i.e. we represent the anisotropy in the channels by taking $J_p^{(1)} \neq 0$. Finally, following the approach of Emery and Kivelson [24], we recognize that both Lagrangians have similar structures at the Toulouse point, where $J_z = 2\pi v_F$ and $\delta J_z = 0$. It is well known that in the 2CK model the Tolouse point flows under the RG to the isotropic 2CK model.

Within these conventions, the Lagrangian for the isotropic 2-channel Kondo model reads:

$$\mathcal{L}_{2CK} = \frac{1}{4\pi} \int_{-\infty}^{\infty} dx \, \partial_x \phi_{s-} (\partial_t - \partial_x) \phi_{s+} - \frac{J_p^{(1)}}{4\pi\alpha} (-1)^{N_{c+}} [S^+ e^{-i\phi_{s-}(x,t)} - h.c. \quad (49)$$

Here we have set $N_{c+} = 0$ because the field $\phi_{c-}$ does not have dynamics and $N_{c-}$ represents the total charge of the field $\phi_{s-}$. 

![FIG. 3. Qualitative phase diagram of the two-channel Kondo model.](image-url)
The Hamiltonian for the FQH/NM junction is:

\[ \mathcal{L}_{FQH} = \frac{1}{4\pi} \int_{-\infty}^{\infty} dx \partial_x V \delta(x) \phi - (\partial_t - \partial_x) \phi - + 2 \Gamma \delta(x) \eta_{\text{edge}}^{\text{res} \uparrow} \left[ e^{-i\phi(x,t)} + e^{i\phi(x,t)} \right] \]  

(50)

In these expressions we have explicitly left out the Lagrangian corresponding to the free field \( \phi_+ \) that decouples from the impurity, after making the appropriate transformations.

Comparing Eqs. (19) and (51), we make the identifications:

\[ \begin{aligned}
J_p^{(1)} &\to 2 \Gamma \\
(-1)^{N_{\phi-}} &\to \eta_{\text{edge}}^{\text{res} \uparrow}
\end{aligned} \]  

(51)

Clearly, the following identification between operators can be also made:

\[ \begin{aligned}
e^{-i\phi(x,t)} &\equiv S^+ e^{-i\phi_+(x,t)} \\
e^{i\phi(x,t)} &\equiv S^- e^{i\phi_+(x,t) + \pi}
\end{aligned} \]  

(52)

From these expressions we see that the dual field \( \tilde{\phi}_- \) is given by a composite operator involving spin and fermion operators of the 2CK. Thus, although the Hamiltonian can be mapped into each other, the degrees of freedom are glued in a particular way. In other words, what in one system has a simple expression it becomes a more complicated object in the other.

These expressions show that, at the level of their Hamiltonians the 2CK model and the FQH/NM junction at \( \nu = 1/3 \) apparently do map into each other. Notice that in reality we have not mapped the junction into the full 2CK model but only to the Tolouse limit of this model. However, since the full 2CK model flows (under RG) to the Tolouse limit (under the RG), then at this level the mapping is established. In particular, the tunneling operator of the junction maps onto the channel anisotropy term in the 2CK. Hence the RG flow of the FQH junction, which is an integrable system \([22]\), maps onto the RG trajectory of the 2CK model from the non-trivial 2CK fixed point to the stable fixed point of the 1CK model. This trajectory has been studied in detail by Andrei and Jerez \([13]\).

However, although these arguments are strongly suggestive, they are not by any means sufficient to establish a mapping. In addition to a mapping between the Hamiltonians, it is necessary to specify the Hilbert spaces on which these Hamiltonians act and how these spaces may (or may not) be related. In other terms, it is necessary to specify what operators are physically allowed in each problem and how they may possibly be related to each other. We will see, however, that although the same bosonized Hamiltonian does describe both systems, the way the physical operators are put together is quite different and that operators that are physical in one system are not physical in the other. Furthermore, even though the Hamiltonians are the same, we will also find that the fields obey different boundary conditions.

Let us begin by seeking an expression for the electron operator on the reservoir (the Fermi liquid side of the junction), as defined at the \( \Gamma = 0 \) fixed point, in terms of the field \( \phi_{\bar{\nu}} \). We can do that by using the transformations between the fields \( \phi_2 \) and \( \phi_{\bar{\nu}} \) (Eq. (51)) and we find,

\[ \begin{aligned}
&\begin{array}{l}
e^{i\phi_2(x<0)} = e^{i\frac{2\pi}{3} \phi_+(x<0)} e^{-i\frac{2\pi}{3} \phi_-(x<0)} \\
e^{i\phi_2(x>0)} = e^{i\frac{2\pi}{3} \phi_+(x>0)} e^{i\frac{2\pi}{3} \phi_-(x>0)}
\end{array}
\]  

(53)

By comparison with Eq. (52), Eq. (53) shows that the electron operator for the reservoir, \( e^{i\phi_2} \), cannot be put in terms of the spin and fermion operators that appear in the two channel Kondo model. Notice that in the mapping of Eq. (53) the field \( \phi_+ \) enters. Furthermore, the operators that appear on the r.h.s. of Eq. (53) are not allowed in the Hilbert space of the 2CK model since they violate the required periodicity conditions. Thus, the operator \( e^{i\phi_2} \) is not contained in the spectrum of the 2CK model. This is most remarkable since each system contains an electron operator whose Green’s function becomes non-trivial at the respective non-trivial fixed point. Nevertheless that by itself does not imply a relation and indeed the mapping between the electron operators does not exist.

Another way to understand the difference between the Hilbert spaces of the two models, is to look at what boundary conditions do their fields satisfy. At the \( \Gamma = 0 \) fixed point the field \( \phi_- \) satisfies Neumann boundary conditions, i.e. \( \partial \phi = 0 \), both at the point contact and at infinity. This condition follows from current conservation: no current flows in or out of the system. When the tunneling perturbation is applied, \( \Gamma \) is not contained in the spectrum of the 2CK model. This is most remarkable since each system contains an electron operator whose Green’s function becomes non-trivial at the respective non-trivial fixed point.
we have also shown that in spite of this closely related behaviors, the two problems are not actually equivalent to each other. We found instead that both models do not share the same operator spectrum. In order to understand the difference we analyze the boundary conditions at the strong coupling fixed point. We showed that while the FQH junction satisfies Neuman at the point contact and Dirichlet at infinity, the boundary conditions for the 2CK model are the opposite. Thus, we conclude that although the two models share many common physical properties, their Hilbert spaces cannot be mapped into each other.

VI. CONCLUSIONS

In this paper we have analyzed in detail the nature of the relation between FQH/NM junctions and the 2CK model. We have found an explicit mapping between their respective Hamiltonians and shown how to construct the operators of interest. We have also shown that both models possess the same value for the residual entropy in the strong coupling regime by using exact results derived for the boundary sine-Gordon model. Furthermore, by performing a perturbative calculation at the strong coupling fixed point, we have found a branch cut singularity in the Green’s function for electrons on the normal side of the junction, making manifest the non-Fermi liquid nature of this fixed point. We also showed that the scaling dimensions and the RG flows followed by the tunneling perturbation in the junction suggest that this tunneling operator could be mapped into the channel symmetry breaking perturbation in the 2CK model. In both cases the dimension is 1/2 and the flow goes from the non-Fermi liquid to the Fermi liquid fixed point. Nevertheless,

VII. ACKNOWLEDGEMENTS

NS acknowledges Hubert Saleur for helpful discussions. We are grateful to Natan Andrei, Claudio Chamon and Andreas Ludwig for many discussions and suggestions. During the completion of this work EF was a participant of the Program on High Tc Superconductivity at the Institute for Theoretical Physics, UCSB. This work is supported in part by NSF grants DMR-99-84471 at Brandeis University (NS), DMR-98-17941 at UIUC (EF) and PHY94-07194 at ITP-UCSB (EF).

APPENDIX A: ELECTRON PROPAGATOR AT THE \( \Gamma = 0 \) FIXED POINT.

In this appendix we give the details of the calculation for the electron Green’s function at the \( \Gamma = 0 \) fixed point. The definition of the electron creation operator is given by

\[
\psi^\dagger(t, \sigma) = \eta : e^{-\frac{i}{\hbar} \phi_L(t, \sigma)} : \quad (A1)
\]

where the definition of the normal ordered exponential is given by \[\text{(2)}\]

\[
: e^{-\frac{i}{\hbar} \sum_n \phi_L(t, \sigma) + \sum_n (t+\sigma)} : = e^{-\frac{i}{\hbar} \sum_n \phi_L(t, \sigma) + \sum_n (t+\sigma)} \times e^{\frac{i}{\hbar} \sum_n \phi_L(t, \sigma) + \sum_n (t+\sigma)} \tag{A2}
\]

Notice that for the electron on the normal metal side of the junction \( \nu = 1 \). To calculate the electron’s propagator we need to evaluate the expression

\[
G_e(t, \sigma) = -i \left< 0 \left| T \left[ : \psi(t, \sigma) \psi^\dagger(0, 0) : \right] 0 \right> \right> \tag{A3}
\]

that is, we need to calculate products of the type

\[
: \psi(t, \sigma) \psi^\dagger(0, 0) : \quad ; : \psi^\dagger(0, 0) \psi(t, \sigma) : \tag{A4}
\]

To normal order the products involving exponentials we make use of the identity

\[
e^A e^B = e^{[A, B]} e^B e^A \tag{A5}
\]
A typical commutator to evaluate looks like
\[ [A, B] = \left( -\frac{1}{\nu} \right)^n \sum_{n=1}^{\infty} \frac{-i}{n} e^{-i\nu(t+\sigma)} \sum_{m=1}^{\infty} \frac{i}{m} [\alpha_n, \alpha_m^\dagger] \tag{A6} \]

After all the steps to normal order these products are carried out, we have to calculate expressions like the following one:
\[ \sum_{m=1}^{\infty} \frac{1}{m} e^{-im(t+\sigma)} \tag{A7} \]

that we evaluate using an analytic continuation on time \( t \rightarrow t(1-i\epsilon) \) with \( \epsilon > 0 \). Thus, we find that
\[ \sum_{m=1}^{\infty} \frac{1}{m} e^{-im(t+\sigma)} = \ln \left( \frac{1}{1-e^{-i(t+\sigma)}} \right) \tag{A8} \]

Finally, we get the expressions:
\[ \langle \psi(t, \sigma) \psi^\dagger(0, 0) \rangle = \eta^2 e^{-i\nu \Theta(t+\sigma)} \left[ 1 - e^{-i(t+\sigma)} \right]^{-\frac{1}{\nu}} \]
for \( t > 0 \)
\[ \langle \psi^\dagger(0, 0) \psi(t, \sigma) \rangle = \eta^2 e^{i\nu \Theta(t+\sigma)} \left[ 1 - e^{i(t+\sigma)} \right]^{-\frac{1}{\nu}} \]
for \( t < 0 \)
\[ \tag{A9} \]
Replacing these expressions in Eq. (A3), we obtain
\[ G_{\nu}(t, \sigma) = -\frac{i}{\nu} \left( \frac{-i}{t+\sigma} \right)^{\frac{1}{\nu}} \Theta(t) - \frac{i}{\nu} \left( \frac{i}{t+\sigma} \right)^{\frac{1}{\nu}} \Theta(-t) \tag{A10} \]

In the thermodynamic limit \( (t, \sigma) \ll 1 \) (in units of \( L = 2\pi \)), and the expression for the electron Green’s function can be approximated by
\[ G_{\nu}(t, \sigma) = -\eta^2 \left( \frac{-i}{t+\sigma} \right)^{\frac{1}{\nu}} \Theta(t) - \eta^2 \left( \frac{i}{t+\sigma} \right)^{\frac{1}{\nu}} \Theta(-t) \tag{A11} \]

Using the fact that on the normal metal side of the junction \( \nu = 1 \), and by Fourier transforming the expression given in Eq. (A11) we finally obtain:
\[ G_{\nu}(\omega, k) = 2\pi \eta^2 \lim_{\delta \rightarrow 0} \frac{\Theta(k)}{\omega + k + i\delta} + \frac{\Theta(-k)}{\omega + k - i\delta} \tag{A12} \]
as corresponds to a chiral Fermi liquid.

**APPENDIX B: CALCULATION OF THE FUNCTIONS \( R_\alpha(\omega, K) \) AND \( R_{1-\alpha}(\omega, K) \)**

In this appendix we give the details of the calculation for the expressions of the functions \( R_\alpha(\omega, K) \) and \( R_{1-\alpha}(\omega, K) \). We represent both functions by a generic function \( Q_\alpha(\omega, k) \) where
\[ \alpha = \frac{2\nu}{1+\nu}; \quad 1-\alpha = \frac{1-\nu}{1+\nu} \tag{B1} \]

Thus,
\[ Q_\alpha(\omega, k) = \int_{-\infty}^{\infty} dx dt \frac{e^{i\omega t}}{[\delta + i\text{sgn}(t)(t-x)]^{\alpha}} \tag{B2} \]

Because of the function \( \text{sgn}(t) \) in the denominator, we need to evaluate integrals like
\[ \int_{-\infty}^{\infty} dx \frac{e^{-ixk}}{(\beta - ix)^{\alpha}} = \frac{2\pi k^{\alpha-1}}{\Gamma(\alpha)} e^{-\beta k} \Theta(k) \tag{B3} \]
and
\[ \int_{-\infty}^{\infty} dx \frac{e^{-ixk}}{(\beta + ix)^{\alpha}} = \frac{2\pi (-k)^{\alpha-1}}{\Gamma(\alpha)} e^{\beta k} \Theta(-k) \tag{B4} \]
where \( \beta = (\delta + it) \).

By integrating these expressions with respect to the variable \( t \) using an IR cutoff \( \mu \), we obtain
\[ Q\alpha = \frac{2\pi k^{\alpha-1}}{\Gamma(\alpha)} \left[ \frac{\Theta(k) e^{-\delta k}}{(\omega - k + i\mu)} + \frac{\Theta(-k) e^{\delta k}}{(\omega - k - i\mu)} \right] \tag{B5} \]

By replacing the values of \( \alpha \) by their corresponding functions of \( \nu \) we obtain the expressions mentioned in Sec. 3.

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