Quantifying invariant features of within-group inequality in consumption across groups

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We study unit-level expenditure on consumption across multiple countries and multiple years, in order to extract invariant features of consumption distribution. We show that the bulk of it is lognormally distributed, followed by a power law tail at the limit. The distributions coincide with each other under normalization by mean expenditure and log scaling even though the data is sampled across multiple dimension including, e.g., time, social structure and locations. This phenomenon indicates that the dispersions in consumption expenditure across various social and economic groups are significantly similar subject to suitable scaling and normalization. Further, the results provide a measurement of the core distributional features. Other descriptive factors including those of sociological, demographic and political nature, add further layers of variation on the this core distribution. We present a stochastic multiplicative model to quantitatively characterize the invariance and the distributional features.

Keywords: Inequality, invariance, consumption distribution, power law, lognormal distribution.

Any city, however small, is in fact divided into two, one the city of the poor, the other of the rich . . . - Plato

I. INTRODUCTION

Plato’s remark as stated at the beginning refers to the intuitive notion that even though the reason for inequality could be vastly different, but the dispersion in affluence is always present. Seminal work by [30] shows that the right tail of the wealth distribution has a power law tail. Further explorations have shown that this feature is invariant across countries although the origin of the power law is not settled yet, either theoretically and empirically. The existence of a fat tail constitutes one such invariance even though the exponent and the share accruing to the top income classes are seen to be fluctuating substantially across countries and time ([4], see also [9]). The bulk of the distributions are also described well by lognormal or gamma distributions ([9]), which again is susceptible to substantial variation in parameters even though the functional form remains the same. Thus there is hardly any precise and specific quantitative feature which is common across samples.

Quantifying inequality has been one of the most important factors in devising economic policies targeted towards mitigating the same. Theoretical tools developed for that purpose are equipped to find out the level of inequality based on a vector of income or wealth from a sample of units. Depending on the case, the unit could be an individual or a household (or something else). Keeping track of the level of inequality for the same sample over time or the same across different samples collected at the same point in time, allows us to make comparative judgments about the dynamics of inequality. In this paper, we ask the question: is there any fundamental feature of inequality that is invariant across samples (both across time and geographic boundary)?

We argue that in case of consumptions, the mean of the distribution is an important scaling factor. Once the distributions are normalized by their respective mean values, the inequality of the normalized sample show reasonable agreement in terms of numerical values. We study data with large sample size across three countries (India, Brazil and Italy) and a number of years (distinct waves when the data were collected). Each country is a fiscal and monetary union of smaller states and/or other units e.g. religious or ethnic groups. The financial markets are also more integrated within each country than across countries. Both of these imply that the consumption decisions faced by households within a country are made in an environment much similar than households across countries. We show that within each country, the consumption distribution across different economic or social identities (states or religions or...
locations) show almost identical features once normalized by the respective mean values. The choice of set of countries (India, Brazil and Italy) under study stems from data availability.

To account for the distributional features, we provide a small-scale heterogeneous households model to quantify the dispersion and the existence of both lognormal bulk and power law tail. Previous models had either focused on the bulk which is lognormally distributed e.g. the literature that builds upon the approach proposed by Kalecki or the tail, which is power law distributed (see [9]). In the present paper, we propose a mixture model that is able to generate both simultaneously. In the model, we assume that households’ consumption decisions are affected by habit. They interact through a capital market and receive idiosyncratic shocks in labor income that they cannot smooth out. Such incompleteness in the market along with heterogeneity in habits across households, generate a distribution of consumption. Using tools from distributional analysis, we show that the distribution has a dominant power law component in the limit and a lognormal bulk.

[10] was an initial attempt to study if there is any invariance in consumption. However, the scope of that study was very limited due to data availability (India 66th round, year 2009-10). In the present context, we have analyzed a much bigger data set from multiple countries spanning over multiple years. This paper is related to two strands of literature. One, we invoke the idea of invariance in distributions of economic quantities like income or wealth ([34]). We differentiate our work from [27] who stressed the evolution of inequality across time due to evolution of market institutions. In a similar vain, [11] proposed a theory of historical evolution of inequality as a reflection of the evolution of political institutional features. A different version was proposed by [10] which emphasized development of institutions, specially educational sector being an important factor. Our approach is complementary in that we propose that there always exists a substantial level of inequality conditional on the state of the economy which is captured by the average affluence. On the technical side, there are multiple attempts to model the power law structure which is the most commonly known invariant feature of income/wealth distributions. [9] contains a number of models in that direction. [6] showed that it is possible to generate a power law in the tail by using an overlapping generation framework with incomplete markets. As we have discussed in the modeling section in details, we use the specifications in [15] and [25] for analytical purpose.

In the next section, we describe the data and summary statistics of consumption distribution. In the following section, we present the key results regarding variation in consumption across countries and time. Finally, to account for the robust pattern we see in data, we present a simple stochastic model of consumption distribution.

II. DATA DESCRIPTION

We use the data for Household Consumer Expenditure 68th Round (2011-2012) from the National Sample Survey Office [29] of India. It contains information about expenditure incurred by households on consumption goods and services during the reference period. These sample surveys are conducted using households as unit of the economy. This ignores heterogeneity in household size but the data contains information about monthly per capita consumer expenditure (MPCE) in Indian Rupee (INR). Data is available for all sampled households in the different states and Union territories (UT), across several parameters like castes, religions and rural-urban divide. [10] studied Household Consumer Expenditure 66th Round (2009-2010) collected by the National Sample Survey Office (NSSO) which collected data for multiple definitions of expenditure and used multiple definitions of inequality. The important conclusions we draw from that study is that the results are robust to such changes in definitions. So we focus on very specific and standard definitions only, in the present work. There are 101717 households in this data set (see tables IV and V). To study the inequality structure, we use two kinds of data which provides two perspectives.

Data from Brazil is procured from IBGE- Instituto Brasileiro de Geografia e Estatística. The Consumer Expenditure Survey data from two rounds ([22] with 48470 households and [23] with 55970 households) are used here (table I). The data contains information about household size, geographical location (state) and consumer expenditure in Brazilian Real (BRL) for different expenditure sectors, among other things.

For Italy, we use microdata provided by [5] and has information about household consumer expenditure in Euro (EUR). We analyze 10 years of data (1980-1984, 1986, 1987,1989, 1991, 1993, 1995, 1998, 2000, 2002, 2004, 2006, 2008, 2010, 2012). See App. VI A for description and summary statistics.

III. RESULTS

In this section, we discuss the main results of data analysis. First feature is that the normalized data collapses on one single distribution. The second feature is that the distribution is lognormal followed by power law tail.
A. Invariance

The available consumption data shows a scaling property across time and countries. Consider a variable $x$ which denotes consumption expenditure. Suppose it has a distribution $p_{it}(x)$ in cross section, in the $i-$th country, $t-$th period. We show that by choosing a suitable scaling parameter, the data collapses into one single aggregate distribution across different countries and years upon taking log transformation. That is, the scaled variable

$$X_{it} = \log \left( \frac{x_{it}}{\tau_{it}^{1/\kappa}} \right)$$

has a distribution

$$p_{it}(X) = p(X)$$

for all $i$ and $t$. The parameters we chose, are mean consumption expenditure ($\tau_{it} = E(x_{it})$ where $E(.)$ denotes expectation operator) and $\kappa = 1$. Collapse of data onto a single distribution indicates that there is a core inequality process which is generated and described by mechanisms similar across geographic boundary and time.

B. Normalized distributions

Consider a variable $x$ following a lognormal distribution,

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp \left( -\frac{(\log x - \mu_x)^2}{2\sigma_x^2} \right).$$

The mean of this distribution is

$$E(x) = e^{\mu_x + \sigma_x^2/2}$$

and the variance is given by

$$V(x) = (e^{\sigma_x^2} - 1)e^{2\mu_x + \sigma_x^2}.$$  

Thus upon normalization by the mean, the new distribution has a mean of

$$E^{norm}(x) = e^0 = 1$$

and variance,

$$V^{norm}(x) = (e^{\sigma_x^2} - 1).$$

By linearizing the exponential term in the variance we get

$$V(x) = \sigma_x^2 + 2\mu_x^2 + \sigma_x^4$$

whereas for the normalized variable we have

$$V^{norm}(x) = \sigma_x^2.$$  

The tail of the distribution is found to be power law or a Pareto distribution which can be represented as

$$p(x) \sim x^{-(1+\gamma)}.$$  

By the nature of the distribution, power law is scale-free i.e. normalization of data which is power law distributed, leaves the distributional features unchanged.

The scaled distribution in all cases show a lognormal bulk and a power law asymptotically. This form has also been argued to be extracted from income and wealth data ([9]). The difference is that here, the exponent of the power law tail is much higher than that of income or wealth distributions indicating much faster rate of decay and lower inequality. The finding that consumption is less dispersed than income is consistent with the available evidence ([11]). Finally, we note that Eqn. 1 is a linear transformation of the original expenditure variable and hence the
distributional features remain intact. Only the moments change due to this transformation. See Sec. III B for a short description of the parametric features of the normalized distribution.

We present the distribution of the scaled expenditure variable in Fig. 1 in for Indian states and also for other dimensions including caste (panel b), religion (panel c) and urbanity (panel d). This figure with superimposition of the cross-sectional data shows that the distributions coincide under normalization which is consistent with the preliminary findings made by [10] for different wave of data collection. The bulk of the data fits with lognormal

FIG. 1: Collapse of consumption expenditure data for Indian states. Data collected for round 68 (2011-2012). Panel (a): Normalized data for all states. Fitted with a lognormal distribution and a power law at the right tail. Panel (b): Normalized data across caste categorization. Panel (c): Normalized data across religious categorization. Panel (d): Normalized data across urban and rural population.

FIG. 2: Consumption expenditure data across Brazilian states normalized with respect to the respective mean expenditure across states; Panel (a): year 2002-03, Panel (b): year 2008-09. See table I for details.
distribution and the tail fits with a power law. Fig. 2 shows a similar data collapse in case of Brazil across all states in two given years and Fig. 3 shows for multiple years across all states. The data is fitted with a lognormal distribution. In Fig. 4 we present normalized Italian data across years. Similar to the Indian data set, the bulk fits with lognormal distribution and the tail is fitted with a power law.

We do formal tests of how close the distributions are across states or across years, after proper normalization. The results have been shown in Fig. 5. It should be noted that the variable under consideration is \( \log(x/E(x)) \) (Eqn. 1). We show that for a substantial number of cases, the normalized and log-transformed variables across states within a country (India and Brazil) or across time for the same country (Italy) are actually distributionally identical. Thus not only the broad algebraic forms of the distributions (lognormal bulk and power law tail) coincide, but also the parameters describing the distributions are very similar. In particular the Brazilian data establishes our claim. One interesting point is that such tests could be sensitive to existence of fat tails. Given that both India and Italy show prominent fat tails, it is not surprising that there are many cases where the distributions are identical according to the test. Essentially this could be attributed to the existence of outliers (data on extreme right) whereas the bulk of the data do fall on a single distribution.
IV. AN HETEROGENEOUS AGENT MODEL

Here we propose a brief model of evolution of the consumption distribution. The basic goal would be to account for two observations, viz., the bulk of the distribution is seen to be following a lognormal distribution and there is a Pareto tail. Several assumptions are necessary to simplify the exposition. Time is discrete and goes till infinity i.e. \( T = 1, 2, \ldots \). There are \( N \) dynasties who are producing and consuming. With a little abuse of notation, we will also use the same \( N \) to denote the set of agents as well where no confusions arise. Each dynasty can be thought of as an unit of observation in the present context. We do not attempt to provide any microfoundation of their consumption decision and construct our model based on the approach recently introduced in [15] and [25]. They consider firm growth process resulting from interconnection among a large number of firms. The dynamical properties are developed from the proposed set of interconnections. In this case, we follow a similar route and assume that the growth rate of consumption expenditure at the unit level (the unit could be individual or family or household depending on the case) admits interconnections between agents who differ in their attitude towards consumption. At the same time, we keep our model general enough to incorporate aggregate effects like long-term growth which can potentially affect inequality (positively or negatively). Hence, the growth rate of consumption expenditure is assumed to be a function of the level of present expenditure, household specific factors and the state of the macroeconomy.

In particular, we propose the following behavioral form of growth rate of expenditure of the \( i \)-th unit at any generic time-point \( t \)

\[
\hat{x}_i(t) = \frac{\Delta x_i(t)}{x_i(t-1)} = \lambda_i(t)(x_i(t-1))^{\alpha_i(t)} + \frac{\eta_i(t)m_i(t) + r(t)b_i(t)(\sum_{j=1}^{N} w_j(t)) + \chi_i(t)}{x_i(t-1)} - 1 \tag{11}
\]

where \( \alpha_i \) is agent specific shock, \( \eta_i(t), m_i(t) \) is the total contribution of all unit-specific behavioral factors (e.g., religion, sex, geographic location etc.) that can potentially affect the level of consumption expenditure, \( \chi_i(t) \) is a noise term with mean \( \mu \) and variance \( \sigma^2 \). The term in the middle requires elaboration. We assume that consumption expenditure is affected at the business cycle frequency. We are agnostic about the preferences of agents who participate in the same process and introduce a parameter \( b_i(t) \) that captures the effects on the \( i \)-th dynasty. The rate of return is given by \( r(t) \) and the aggregate wealth (capital) is given by \( \int w_j(t) dj \).

There are some classic studies on the growth rates of consumption. In particular, [19] provided a framework to study the growth rates of consumption expenditure in the following form,

\[
\hat{x}_i(t) = \frac{\chi_i(t)}{x_i(t-1)}. \tag{12}
\]

In App. [VIIB] we provide the basic framework that gives rise to such a growth rate. However, empirical studies have rejected such models. [24] presents evidence that the theory is rejected when tested with U.S. data. [20] shows that
the discrepancy might come from time aggregation bias (see also [28]). In a separate field of study, the firm growth rates had been described by similar functional forms. In particular, [15] starts describing a granular economy where each firm has a growth rate of

\[ \hat{x}_i(t) = \chi_i(t) \]  

(13)

which is known to generate a lognormal distribution (see below). Such specifications are known as Gibrat’s law ([17]). However, empirical estimations show that such a growth equation is incorrect ([18], [12], [7]). [25] expands this simple framework to incorporate relationships between growth rates as follows,

\[ \hat{x}_i(t) = f(\{\hat{x}_j(t)\}_j, W_{i,j \in N}(t), \chi_i(t)) \]  

(14)

where \( f(.) \) captures a linear evolution of growth rates across size and \( W \) captures the interaction matrix. We combine the above mechanisms to propose Eqn. [11]

Imagine that the production function is given by the following simple equation

\[ Y(t) = s(t)K(t) \]  

(15)

which says output is a linear function of capital \( (K) \) and a productivity shock \( s \). One can incorporate labor. But for simplicity of exposition, we ignore it and assume that households (or individuals) supply labor inelastically. The rental payment in the competitive market exhausts the total output. Noting that wealth acts as capital, we see that the income is given by \( Y(t) = r(t) \int w_j(t)dx \) in absence of wage income. Note that this also introduces a coupling among the agents through the capital market. This is the only source of direct interaction among agents. Effectively such a factor induces an aggregate shock (rate or return \( r \) is a function of the aggregate shock \( s \)) to the dynamic process. This type of approach to link the dynamic processes of multiple agents can also be seen in [32] although the links in the generalized Lotka-Volterra type models were considered for scaling purposes (and not for representing any aggregate shock).

We can decompose the income as the sum of a trend component \( (Y^{T}(t)) \) and a transitory component \( (Y^{C}(t)) \):

\[ Y(t) = Y^{T}(t) + Y^{C}(t). \]  

(16)

The transitory component captures the purely fluctuating part. Note that by construction

\[ E(Y^{C}(t)) = 0. \]  

(17)

Since we have shown that there is an invariance in the distribution of expenditure once the data is normalized with respect to the household specific microeconomic factors, we ignore their contribution for modeling the core inequality process. Also, the trend part can be ignored as it contributes to very long-term inequality. By taking all of the above into consideration, we arrive at the following equation,

\[ \hat{x}_i(t) = \lambda_i(t) \left(x_i(t-1))^{\alpha_i(t)} + \frac{b_i(t)(Y^{C}(t) + \chi_i(t))}{x_i(t-1)} \right) - 1 \]  

(18)

Therefore, we can use the definition of the growth rate and rewrite the above equation as

\[ x_i(t) = \lambda_i(t) (x_i(t-1))^{1+\alpha_i(t)} + b_i(t)Y^{C}(t) + \chi_i(t). \]  

(19)

The business cycle component induces a distortion on the mechanism. Note that it is a common factor to all agents. Thus if for a sustained period (a few quarters), the economy is either hit by very high or very low shocks expenditure growth rate is affected exacerbating inequality. On the other hand when the economy returns to the baseline (zero shock), the growth rates are diminished reducing inequality. Business cycle can be taken to be an exogenous factor as usually it affects inequality and the converse is unlikely. Thus due to the business cycle inequality might wax and wane ([24]).

Eqn. [19] forms the basis of the subsequent analysis. Below we show known solutions of the above dynamic equation in three limits.

**Case I.** Let the transitory component and the agents’ idiosyncratic shocks to expenditure be identically equal to zero i.e. \( Y^{C}(t) = 0, \alpha_i(t) = 0 \) and \( \lambda_i(t) = 1 \) for all \( t \) in Eqn. [19]. Then the equation boils down to the following form

\[ \log(x_i(t)) = \log(x_i(t-1)) + \chi_i(t). \]  

(20)
This random walk in logarithm is known to generate a lognormal distribution,
\[
f(x,t) = \frac{1}{x(t)\sqrt{2\pi\sigma^2(t+1)}} \exp \left( -\frac{(\log x(t) - (t + 1)\mu)^2}{2\sigma^2(t+1)} \right).
\]
(21)
Due to the explicit time dependence, there does not exist a steady state distribution for this process. Standard deviation increases without bound over time at the rate \(\sqrt{t+1}\) (see for example [9]).

**Case II.** Suppose the transitory component and the noise term are zero and the idiosyncratic term has a distribution over \([\alpha_{\text{min}}, \alpha_{\text{max}}]\) where \(0 < 1 + \alpha < 1\) for all \(\alpha \in [\alpha_{\text{min}}, \alpha_{\text{max}}]\) and all moments exist. Then we have
\[
\log(x_i(t)) = (1 + \alpha_i(t)) \log(x_i(t-1)) + \log \lambda_i(t).
\]
(22)
By solving it recursively and using the lag operator \(L\), we can rewrite it as
\[
x_i(t) = \exp([1 + (1 + \alpha_i)L + (1 + \alpha_i)^2L^2 + \ldots].\lambda_i(t)).
\]
(23)
For simplicity of exposition we assumed \(\alpha_i(t) = \alpha\) which can be relaxed without changing the basic result. Thus the bracketed term becomes the sum of an infinite series of noise terms with standard deviation going to zero in limit. Hence, this process reaches a steady state described by a lognormal distribution. This process was formulated and proposed as a model of income evolution by Kalecki in 1945 (see [9] for a detailed exposition).

**Case III.** Consider Eqn. 19 with the idiosyncratic term distributed over \([\alpha_{\text{min}}, \alpha_{\text{max}}]\) where \(E(\alpha) = 0\) and \(\sigma_\alpha \to 0\). We make two additional assumptions, (a) \(E(\log \lambda_i(t)) < 0\) and (b) \(\eta_i = b_i(t)Y^C(t) + \chi_i(t)\), is distributed over \(\mathbb{R}_+\). [34] shows that under such assumptions, the steady state distribution is power law. Such a dynamics is called Kesten process
\[
x(t) = \lambda(t)x(t-1) + \xi(t),
\]
(24)
which is known to generate power laws in the limit (26). [13] uses such a mechanism to generate a power law in the city size distribution. See also [33] for a textbook treatment.

### A. Heterogeneity of agents

We introduce heterogeneity among the agents along one dimension viz., the upper range of the multiplicative factor \((\lambda_{\text{max}})\). Let us assume without loss of generalization that \(0 < \lambda_{i,\text{max}} \leq \lambda_{j,\text{max}}\) for all \(i \leq j\) and there exists some agent \(1 < k < N\) for whom \(\lambda_{k,\text{max}} = 0\). For all agents, \(-1 < \alpha_{\text{min}} \leq \alpha_{\text{max}} < 0\). Thus effectively there are two types of agents. Fraction \(f\) of total number of agents have \(0 < \lambda_{i,\text{max}} < 1\) for all \(i \in N_f\) where \(N_f\) is the set of all such agents. The evolution of the expenditure is given by Eqn. 19 which we can rewrite as
\[
\log(x_i(t)) = (1 + \alpha_i(t)) \log(x_i(t-1)) + \hat{\lambda}_i(t)
\]
(25)
ignoring the noise factor and the business cycle variation. Thus we are back to Case II above.

The second type is described by \(\lambda_{1,\text{max}} > 1\) with the condition that \(E(\log \lambda_i) < 0\) as mentioned in Case III above. To gain intuition about why this process converges to a power law, assume that \(E(\alpha) \to 0\) and \(\sigma_\alpha \to 0\). We assume that \(b_i(t)\) is highly procyclical i.e. \(E(b_iY^C) > 0\). The first assumption allows us to maintain the exact parametric requirement of the Kesten process. Strong procyclicality of consumption share effectively induces a lower bar on the expenditure even when the variable receives consecutive bad shocks through \(\alpha\). Thus this becomes a reflective barrier and we can apply the methodology devised in the literature to find out the steady state distribution.

[14] provides a very simple proof that the mechanism generates a power law. Assuming the existence of the lower (reflective) boundary through the business cycle effects, we know that the variable can never be less than that. Hence, we consider the other extreme and study the right tail when the variable is far from the boundary making the additive terms relatively unimportant. Let us assume that the multiplicative factor \(\lambda_i\) is distributed according to \(f(\lambda)\). Then we can write the evolution equation of the expenditure variable \(X\) as
\[
\text{Prob.}(X_i(t) < x) = \text{Prob.} \left( X_i(t) < \frac{x}{\lambda_i(t)} \right)
\]
(26)
Letting the left hand side be denoted as $M_t(e)$, we have a recursive equation

$$M_{t+1}(x) = \int_{R^+} M_t\left(\frac{x}{\lambda}\right) f(\lambda) d\lambda.$$  \hspace{1cm} (27)

The trick is to apply the criteria that when the system converges, the above equation would be time independent and one can guess and verify the functional forms. In particular, [14] shows that $M_\infty(X) \propto 1/X^\gamma$ solves the equation and the condition reduces to

$$E(\lambda^\gamma) = 1.$$  \hspace{1cm} (28)

The same can also be shown using techniques developed by [34]. Eqn. 28 describes the relationship between the distribution of the multiplicative factor and the exponent of the distribution.

In Fig. 6, we present numerical simulations results for the evolution of the Pareto exponent in the more general context (Eqn. 19). Eqn. 28 gives the solution in only one limit ($\alpha \to 0$). The left panel shows the determination of the exponent following Eqn. 28 in the limit. We use Monte Carlo simulation to find the Pareto exponent $\gamma$ in the general case. The right panel shows the estimated exponents for $(\alpha, E(\lambda))$ pairs on the parameter plane. For the purpose of simulation, $\alpha$ is taken to be a constant. For each combination of $\alpha$ and $E(\lambda)$, we simulate Eqn. 19 and estimate the exponent. The estimated exponents are averaged over $O(10)$ realizations in order to arrive at stable values. The surface indicates the exponents for different pairs of $\alpha$ and $E(\lambda)$.

**B. Shape of the ensemble distribution**

As we have described in the model above, there are essentially two types of agents. The first type generates a lognormal bulk whereas the second type generates a power law distribution for the tail. Here we want to show that for the aggregate distribution the tail is indeed described by a power law.

For this purpose, we use a result (see [14] for a review) on the sum of two variables both distributed according power laws with potentially different exponents. Let the variables be $v_1$ and $v_2$. We assume that $x_i \sim C_i x_i^{-(1+\gamma_i)}$ with $\gamma_1 \neq \gamma_2$. Then the sum these two variables ($x = x_1 + x_2$) will be distributed as $x \sim \tilde{C} x^{-(1+\min(\gamma_1, \gamma_2))}$.  \hspace{1cm} (30)
The intuition is simple: the fatter tail dominates the distribution. Note that the tail of a lognormal distribution can be approximated well by a power law with high exponent. Thus the tail of the aggregate distribution can be modeled as the sum of two power laws with different exponents. Since the exponent of the distribution that approximates a lognormal distribution is typically quite large, the other distribution dominates (following Eqn. 30).

V. DISCUSSION AND SUMMARY

In this paper, we describe two robust features of consumption inequality across time and countries. One, if consumption data is normalized with a proper scaling factor, all data collapses on one single aggregate distribution. Two, the distribution has a lognormal bulk and a power law at the limit with high exponent (compared to income and wealth). Finally we provide a stochastic model to account for the basic distributional features.

In the present work, we differentiate between long run versus short run inequality. We focus exclusively on the latter in order to study cross sectional properties of inequality.

Throughout our analysis, we have considered nominal data. There are two reasons for it. One, the available data is in nominal terms. Two, in our cross-sectional analysis, we normalize the data first with respect to a scaling factor. As long as within region dispersion in price-levels are not significantly high, such a normalization takes care of the pricing factors. All the subsequent analysis including cross-time and cross-country comparisons are based on normalization with respect to respective scaling factor. Hence, such comparisons are free of biases due to between-region or between-time periods variations in general price levels.

The way we have described the consumption growth process, a number of additional implications can be presented. First, the power law arises due to the effects of business cycle implying that inequality in cross-section can be affected at business cycle frequencies. documents that in U.S. mean wealth is negatively correlated with macroeconomic volatility. One can argue that the changes in mean wealth is also accompanied by redistribution of purchasing power affecting inequality, thus corroborating the prior implication. makes the point that there is a relationship between fluctuations of macroeconomic fundamentals and inequality. Secondly, in terms of the generative mechanism, our approach has a parallel with the method used by which also generates a power law in income in an overlapping generations framework. However, they provided a microfounded framework for consumption-savings decision even though the essential mechanism is similar. Earlier empirical works show that volatility of the business cycle is negatively related to the total income of the country. In terms of the model presented above, such a linkage would contribute to lower consumption volatility. In a fully specified utility-maximization framework this would imply higher welfare. Finally, all other non-economic factors are seen to affect the mean of the expenditure distribution. This has a corollary that the spread of the core inequality process is independent of the social, political and geographic factors. and also made a similar observation by considering U.S. data for specific social groups and conditioned on specific (e.g. racial or educational) factors. This is complementary to our approach where we focus exclusively on the idea of core features of the distribution.

VI. APPENDIX

In this section, we present the additional figures and tables. A simple derivation of random walk model in consumption is also presented.

A. Data summary

In Fig. we present the available cross-sectional data for India for all states for three waves of data collection (2004-05, 2009-10 and 2011-12). In the main text, we have analyzed data for 2011-12. presents some complementary results on the data set from 2009-10. Fig. shows the general shift of the consumption density indicating both inflation and rising consumption power. Fig. shows the available Brazilian data for all states for two waves of data collection (2002-03 and 2008-09).

Details of the data and summary statistics have been tabulated in table (across states) and (urban-rural). Table contains summary statistics for the Italian data. Finally, table (across states) and (social and other dimensions) contains summary statistics for the Indian data.
B. Random walk in consumption

In a standard utility-maximizing framework with representative agent, the Euler equation would be

\[ u'(x_t) = \beta R(t) E_t(u'(x_{t+1})) \]

(31)

where \( u(.) \) is the utility function defined over consumption good \( x \) at time \( t \). The discount factor is denoted by \( \beta \) and the rate of return by \( R \). \( E_t(.) \) denotes expectation with the information set \( s \). The simplest framework to derive random walk (19) is to assume

\[ u(x) = -\frac{(\bar{x} - x)^2}{2} \]

(32)

where \( \bar{x} \) is the bliss point. Also assume \( R\beta = 1 \) to solve the above equation to get

\[ E_t(x_{t+1}) = x_t. \]

(33)

Thus the consumption growth equation is

\[ x_{t+1} = x_t + \chi_{t+1}, \]

(34)
where $\chi_{t+1}$ is the innovation term. Thus the growth rate is

$$
\ddot{x}(t) = \frac{\chi(t)}{x(t-1)}
$$

(35)

[1] D. Acemoglu and J. Robinson. *Why Nations Fail: The Origins of Power, Prosperity and Poverty*. Crown Business, 2013.
TABLE I: Number of households, average per capita consumption expenditure $E(x)$ and Gini indices for Brazil, for 2 rounds 2002-2003 and 2008-2009. Tabulated across states.

| ID | State          | 2002-2003 |           | 2008-2009 |           |
|----|----------------|-----------|-----------|-----------|-----------|
|    |                | #Households | $E(x)$   | Gini      | #Households | $E(x)$   | Gini      |
| 1  | Rondônia       | 1112      | 10870.00  | 0.535     | 907        | 15090.8  | 0.498     |
| 2  | Acre           | 960       | 8361.90   | 0.570     | 863        | 12854.8  | 0.484     |
| 3  | Amazonas       | 1075      | 7388.29   | 0.549     | 1344       | 11011.6  | 0.504     |
| 4  | Roraima        | 554       | 9221.05   | 0.529     | 644        | 12965.1  | 0.558     |
| 5  | Pará           | 1666      | 6645.31   | 0.509     | 1894       | 12063.7  | 0.538     |
| 6  | Amapá          | 568       | 7163.82   | 0.510     | 689        | 14428.5  | 0.537     |
| 7  | Tocantins      | 933       | 8027.05   | 0.569     | 1270       | 12306.8  | 0.498     |
| 8  | Maranhão       | 2231      | 4749.73   | 0.502     | 2562       | 8725.61  | 0.524     |
| 9  | Piauí          | 2222      | 6263.12   | 0.557     | 2056       | 9844.81  | 0.498     |
| 10 | Ceará          | 2017      | 6351.92   | 0.571     | 1861       | 8553.52  | 0.514     |
| 11 | Rio Grande do Norte | 1548 | 6819.38   | 0.558     | 1342       | 11016.7  | 0.501     |
| 12 | Paraíba        | 2367      | 5614.04   | 0.538     | 1628       | 10990.6  | 0.543     |
| 13 | Pernambuco     | 1674      | 7126.29   | 0.558     | 2367       | 11360.2  | 0.532     |
| 14 | Alagoas        | 2965      | 6601.53   | 0.583     | 2712       | 9138.34  | 0.541     |
| 15 | Sergipe        | 1143      | 6705.08   | 0.518     | 1654       | 12289.4  | 0.512     |
| 16 | Bahia          | 2457      | 7678.18   | 0.584     | 3050       | 11997    | 0.539     |
| 17 | Mímaros Gerais | 3094      | 11283.4   | 0.528     | 5028       | 17065.5  | 0.508     |
| 18 | Espírito Santo | 2337      | 12148.5   | 0.535     | 3489       | 16538.2  | 0.511     |
| 19 | Rio de Janeiro | 1285      | 17931.3   | 0.591     | 1938       | 22063.4  | 0.531     |
| 20 | São Paulo      | 2617      | 16074.3   | 0.516     | 3023       | 22959.2  | 0.486     |
| 21 | Paraná         | 2265      | 12733.5   | 0.519     | 2477       | 17829.8  | 0.470     |
| 22 | Santa Catarina | 1989      | 12169.9   | 0.646     | 2029       | 23417.8  | 0.498     |
| 23 | Rio Grande do Sul | 1850  | 14370.6   | 0.534     | 2210       | 20394.7  | 0.482     |
| 24 | Mato Grosso do Sul | 2541 | 9788.09   | 0.505     | 2247       | 17071.5  | 0.498     |
| 25 | Mato Grosso    | 2355      | 9214.06   | 0.513     | 2423       | 14363.5  | 0.485     |
| 26 | Golas          | 2356      | 9160.95   | 0.505     | 2686       | 16997.5  | 0.523     |
| 27 | Distrito Federal | 981  | 26497.2   | 0.590     | 977        | 26081.3  | 0.564     |
|    | all Brazil     | 48470     | 9626.94   | 0.568     | 55070      | 12777    | 0.533     |

TABLE II: Number of households, average per capita consumption expenditure $E(x)$ and Gini indices for Brazil, for 2 rounds 2002-2003 and 2008-2009. Tabulated according to location.

| Location                | 2002-2003 |           | 2008-2009 |           |
|-------------------------|-----------|-----------|-----------|-----------|
|                         | #Households | $E(x)$   | Gini      | #Households | $E(x)$   | Gini      |
| rural                   | 48357      | 18352.6   | 0.514     | 43193       | 16658.7  | 0.528     |
| urban                   | 114        | 20352.3   | 0.478     | 12777       | 9847.44  | 0.507     |

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[4] A. B. Atkinson and T. Piketty. *Top income: a global Perspective*. Oxford University Press, 2010.

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[13] J. Angle. The inequality process and the distribution of income to blacks and whites. *The Journal of Mathematical Sociology*, 17(1):77–98, 1992.
| ID | year | #Households | $E(x)$ | Gini |
|----|------|-------------|--------|------|
| 1  | 1980 | 2980        | 8529.86 | 0.307 |
| 2  | 1981 | 4091        | 10373.1 | 0.298 |
| 3  | 1982 | 3967        | 12304.9 | 0.296 |
| 4  | 1983 | 4107        | 15474.4 | 0.302 |
| 5  | 1984 | 8022        | 17315.6 | 0.297 |
| 6  | 1986 | 8024        | 23652.2 | 0.333 |
| 7  | 1987 | 8024        | 23652.2 | 0.333 |
| 8  | 1989 | 8274        | 24392.9 | 0.289 |
| 9  | 1991 | 8186        | 26334.6 | 0.285 |
| 10 | 1993 | 8088        | 29087.4 | 0.297 |
| 11 | 1995 | 8135        | 33631.8 | 0.305 |
| 12 | 1998 | 7146        | 36157.1 | 0.316 |
| 13 | 2000 | 8001        | 38089.7 | 0.308 |
| 14 | 2002 | 8010        | 20466.6 | 0.317 |
| 15 | 2004 | 8011        | 22419.9 | 0.305 |
| 16 | 2006 | 7768        | 23674.8 | 0.290 |
| 17 | 2008 | 7976        | 23817.4 | 0.279 |
| 18 | 2010 | 7950        | 25261.1 | 0.294 |
| 19 | 2012 | 8149        | 25408.3 | 0.291 |

TABLE III: Number of households, average per capita consumption expenditure $E(x)$ and Gini indices for Italy, for several years.
| ID | Geographic location       | #Households | $E(x)$ | Gini |
|----|--------------------------|-------------|--------|------|
| 1  | Jammu & Kashmir          | 3382        | 1846.749 | 0.310 |
| 2  | Himachal Pradesh          | 2040        | 2105.473 | 0.336 |
| 3  | Punjab                   | 3118        | 2571.475 | 0.334 |
| 4  | Chandigarh               | 312         | 3577.070 | 0.378 |
| 5  | Uttarakhand              | 1784        | 2073.443 | 0.350 |
| 6  | Haryana                  | 2589        | 2575.453 | 0.356 |
| 7  | Delhi                    | 999         | 3653.659 | 0.382 |
| 8  | Rajasthan                | 4127        | 1824.600 | 0.332 |
| 9  | Uttar Pradesh             | 9018        | 1414.226 | 0.357 |
| 10 | Bihar                    | 4581        | 1243.082 | 0.286 |
| 11 | Sikkim                   | 768         | 1850.008 | 0.243 |
| 12 | Arunachal Pradesh        | 1674        | 1863.248 | 0.371 |
| 13 | Nagaland                 | 1024        | 2185.466 | 0.241 |
| 14 | Manipur                  | 2560        | 1388.841 | 0.290 |
| 15 | Mizoram                  | 1536        | 2129.177 | 0.259 |
| 16 | Tripura                  | 1856        | 1609.395 | 0.290 |
| 17 | Meghalaya                | 1260        | 1759.212 | 0.263 |
| 18 | Assam                    | 3440        | 1417.833 | 0.309 |
| 19 | West Bengal              | 6317        | 1886.182 | 0.387 |
| 20 | Jharkhand                | 2737        | 1349.307 | 0.341 |
| 21 | Orissa                   | 4029        | 1246.751 | 0.347 |
| 22 | Chattisgarh              | 2173        | 1464.659 | 0.367 |
| 23 | Madhya Pradesh           | 4718        | 1449.213 | 0.366 |
| 24 | Gujarat                  | 3430        | 2143.533 | 0.345 |
| 25 | Daman & Diu              | 128         | 2196.510 | 0.273 |
| 26 | Dadra & Nagar Haveli     | 192         | 1901.413 | 0.335 |
| 27 | Maharaashtra             | 8041        | 2323.568 | 0.391 |
| 28 | Andhra Pradesh           | 6898        | 2094.464 | 0.345 |
| 29 | Karnataka                | 4096        | 2117.983 | 0.399 |
| 30 | Goa                      | 448         | 2700.791 | 0.306 |
| 31 | Lakshadweep              | 192         | 3094.633 | 0.396 |
| 32 | Kerala                   | 4460        | 3014.732 | 0.341 |
| 33 | Tamil Nadu               | 6647        | 2122.480 | 0.357 |
| 34 | Pondicherry              | 576         | 3086.998 | 0.339 |
| 35 | Andaman & Nicobar Is.    | 567         | 3937.967 | 0.347 |
|    | all India                | 101717      | 1939.779 | 0.378 |

**TABLE IV**: Number of households, average per capita consumption expenditure $E(x)$, Gini index for India. Data available for different states for 68st round (2011-2012).

| Filter | #Households | $E(x)$ | Gini |
|--------|-------------|--------|------|
| ST     | 13403       | 1601.763 | 0.338 |
| SC     | 15652       | 1507.782 | 0.335 |
| OBC    | 39721       | 1800.488 | 0.360 |
| Other castes | 32938 | 2150.539 | 0.301 |
| Hinduism | 77036 | 1935.365 | 0.384 |
| Islam  | 13274       | 1698.742 | 0.347 |
| Christianity | 6950 | 2223.477 | 0.357 |
| other religions | 4477 | 2291.247 | 0.359 |
| Rural  | 59693       | 1525.498 | 0.322 |
| Urban  | 42024       | 2528.214 | 0.386 |

**TABLE V**: Number of households, average per capita consumption expenditure $E(x)$, Gini index for India. Data available religions, caste as well as urban-rural divide for 68th round (2011-2012).