Undrained Shear Strength Anisotropy of Cohesive Soils Caused by the Principal Stress Rotation

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Abstract. The construction of geotechnical structures, e.g. diaphragm walls, tunnels, pad foundations or embankments, on cohesive soils, requires assessing the bearing capacity in undrained conditions. For this purpose, it is necessary to determine the value of undrained shear strength. Undrained shear strength is not the same in the subsoil, because it depends on many factors. One of these factors is principal stress rotation caused by changing the stress state due to loading or unloading the subsoil. During determination of undrained shear strength in laboratory tests the influence of principal stress rotation on undrained shear strength is often overlooked because of difficulty of reflecting this phenomenon in laboratory research. A device that allows to determine the anisotropy of undrained shear strength in soils due to the principal stress rotation is a Torsional Shear Hollow Cylinder Apparatus (TSHCA). The paper presents test results performed in a TSHCA on undisturbed sandy silty clays (sasiCl) characterized by different index properties. The main objective of the research was determining the undrained shear strength at wide range of angle of the principal stress rotation. The results of laboratory tests allow assessing the influence of the principal stress rotation on the value of undrained shear strength that should be used to determine the bearing capacity of the subsoil.

1. Introduction

Undrained shear strength is one of the basic parameter that determine mechanical properties of the cohesive soils. The value of this parameter depends on many factors, e.g. soil type, consistency, stress history, stress state [1-3]. Loading or unloading the subsoil changes the stress state and thereby causes the principal stress rotation compared to the initial state. These rotations arise when principal stress increment directions do not coincide the current principal stress directions, as illustrated in figure 1. One of the ways to determine principal stress rotation due to changes in the load of subsoil is numerical analysis [4,5]. Studies carried out so far show a significant impact of the principal stress rotation on undrained shear strength [6-13]. Taking the value of the undrained shear strength determined at one angle of the principal stress rotation for the entire subsoil most often leads to the underestimation or overestimation of the actual value of the undrained shear strength of the subsoil, and thus its bearing capacity [14].

During determination of undrained shear strength of cohesive soils in laboratory tests the conditions prevailing in the field should be as accurate as possible. However, the influence of principal stress rotation on undrained shear strength in laboratory tests is often overlooked because of the
difficulty to achieve this phenomenon in laboratory research. Available laboratory devices most often determine the undrained shear strength at one value of angle of the principal stress rotation, usually at \( \alpha = 0°, 45° \) or \( 90° \). The change in the undrained shear strength due to the principal stress rotation can also be determined by using equations with empirical coefficients depending on the soil properties [15,16]. However, the use of these equations requires first conducting a series of soil strength tests characterized by similar index properties in order to determine empirical coefficients.

Figure 1. Principal stress rotation [17]

One of the most advanced laboratory device for the determination of the undrained shear strength is the Torsional Shear Hollow Cylinder Apparatus (TSHCA) [17]. TSHCA allows to determine undrained shear strength at any angle of the principal stress rotation. In this device, hollow cylindrical samples are tested. Hollow cylindrical samples are subjected to axial load \( P \), torque \( M_T \), inner and outer pressures, \( p_i \) and \( p_o \). The torque \( M_T \) develops shear stresses \( \tau_{xy} \) and \( \tau_{yz} \) in vertical and horizontal planes, the axial load \( P \) contributes to a vertical stress \( \sigma_z \), differences between \( p_o \) and \( p_i \) establish a gradient of radial stress \( \sigma_r \) across the cylindrical wall. Then, the circumferential stress \( \sigma_\theta \) depends on radial stress \( \sigma_r \) and sample radius \( r \) and is determined according to the equation:
\[ \sigma_\theta = \sigma_r + r \frac{d\sigma_r}{dr} \]  

The hollow cylinder specimen with particular stresses is presented in figure 2.

**Figure 2.** Stresses acting on a hollow cylinder specimen [17]

The major principal stress \( \sigma_1 \), intermediate principal stress \( \sigma_2 \) and minor principal stress \( \sigma_3 \) are defined as follows:

\[
\sigma_1 = \frac{\sigma_z + \sigma_\theta}{2} + \sqrt{\left(\frac{\sigma_z - \sigma_\theta}{2}\right)^2 + \tau_{\theta z}^2}
\]

\[ \sigma_2 = \sigma_r \]  

\[
\sigma_3 = \frac{\sigma_z + \sigma_\theta}{2} - \sqrt{\left(\frac{\sigma_z - \sigma_\theta}{2}\right)^2 + \tau_{\theta z}^2}
\]

Values of angle of the principal stress rotation \( \alpha \) and parameter of intermediate principal stress directions \( b \) are determined from the following equations:

\[
\alpha = \frac{1}{2} \tan^{-1} \left( \frac{2\tau_{\theta z}}{\sigma_z - \sigma_\theta} \right)
\]

\[ b = \frac{\sigma_2 - \sigma_3}{\sigma_2 - \sigma_3} \]  

Testing the soil sample in the shape of a hollow cylinder causes the stress non-uniformity in the sample. To reduce impact of stress non-uniformity on test results the length of soil sample \( l \) and the ratio of the inner radius \( r_i \) to the outer radius \( r_o \) should be as follows [18]:

\[
l \geq 5.44 \sqrt{r_o^2 - r_i^2}
\]

\[
\frac{r_i}{r_o} \geq 0.65
\]
The analyses showed that in the case of a ratio of the inner radius $r_i$ to the outer radius $r_o$ equal to 0.65, the maximum level of stress non-uniformity occurring in the sample is 16% which is an acceptable value. Research shows that, the higher value of this ratio, the more stress non-uniformity occurs at a lower level [18].

The paper presents test results performed in a TSHCA on undisturbed sandy silty clays ($sasiCl$) characterized by different index properties. The main objective of the tests was the determination of the undrained shear strength at wide range of angles of the principal stress rotation. The results of laboratory tests allow assessing the influence of the principal stress rotation on the value of undrained shear strength that should be used to determine the bearing capacity of the subsoil.

2. Laboratory tests
The tests to determine the undrained shear strength were carried out in the Water Centre Laboratory of Warsaw University of Life Sciences – SGGW using the Torsional Shear Hollow Cylinder Apparatus (Figure 3). The research was performed on undisturbed sandy silty clays ($sasiCl$) characterized by different index properties. The index properties of the tested soils are presented in table 1.

![Figure 3. Torsional Shear Hollow Cylinder Apparatus (TSHCA) in the Water Centre Laboratory of Warsaw University of Life Sciences - SGGW; 1 – TSHC apparatus cell, 2 – pressure and volume controllers, 3 – electronic measuring device, 4 – computer.](image)

| Soil No. | OCR (-) | $w_o$ (%) | $w_L$ (%) | $w_P$ (%) | $I_p$ (%) | $I_L$ (%) | $I_C$ (%) | Fraction [17,18] (%) |
|---------|---------|-----------|-----------|-----------|-----------|-----------|-----------|---------------------|
| 1       | 2.4     | 31.9      | 52.3      | 21.8      | 30.5      | 0.33      | 0.67      | 0 25 47 28          |
| 2       | 2.7     | 28.8      | 59.0      | 24.3      | 34.7      | 0.13      | 0.87      | 0 21 50 29          |
| 3       | 3.2     | 31.1      | 49.5      | 23.9      | 25.6      | 0.28      | 0.72      | 0 28 48 24          |
| 4       | 3.5     | 28.5      | 42.6      | 22.5      | 20.1      | 0.30      | 0.70      | 0 34 46 20          |
| 5       | 4.0     | 27.1      | 43.9      | 17.3      | 26.6      | 0.37      | 0.63      | 0 23 51 26          |
| 6       | 4.4     | 25.3      | 50.1      | 22.9      | 27.2      | 0.09      | 0.91      | 0 30 43 27          |

Notes: OCR – overconsolidation ratio, $w_o$ – water content, $w_L$ – liquid limit, $w_P$ – plastic limit, $I_p$ – plasticity index, $I_L$ – liquidity index, $I_C$ – consistency index, Gr – gravel, Sa – sand, Si – silt, Cl – clay.
The undrained shear strength for particular soil type were determined at following angles of the principal stress rotation $\alpha$: 0°, 15°, 30°, 45°, 60°, 75°, 90°. Soil sample in the shape of hollow cylinder prepared for testing in a Torsional Shear Hollow Cylinder Apparatus is presented in figure 4.

Figure 4. Soil sample in the shape of hollow cylinder prepared for testing in a Torsional Shear Hollow Cylinder Apparatus

The TSHCA tests were performed in the following stages: flushing, saturation, anisotropic consolidation, change of parameter of intermediate principal stress directions $b$, change of angle of the principal stress rotation $\alpha$, shearing in undrained conditions. Flushing was carried out to remove air and gases having the largest dimensions from the samples and tubes. Saturation of soil samples was performed using the back pressure method. This stage lasted until the value of the Skempton’s parameter $B$ exceeded 0.95. Anisotropic consolidation was performed to dissipate excess of pore water pressure.

After dissipation of excess pore water pressure, parameter $b$ started to change to a value of 0.5. In the next step, the value of angle $\alpha$ changed to the determined value in a particular test. Finally, the process of sample shearing was carried out in the stress path involving the increase in the deviator stress $q$ and constant value of the total mean stress $p$. During the entire shearing process of the soil samples, constant values of parameter $b$ and angle $\alpha$ were kept.

### 3. Results and discussions

Performed laboratory tests allowed to obtain the values of the undrained shear strength at a selected angle of the principal stress rotation for particular sandy silty clays (Tables 2 and 3, Figure 5 and 6). To determine these parameters, the maximum deviator stress was used as the failure criterion.

| Soil No. | Angle of the principal stress rotation $\alpha$ (°) | Undrained shear strength $\tau_{fu}$ (kPa) |
|----------|-----------------------------------------------|--------------------------------------|
| 1        | 0 | 15 | 30 | 45 | 60 | 75 | 90 |
| 1        | 102.4 | 100.1 | 95.4 | 94.8 | 92.1 | 91.4 | 90.3 |
| 2        | 129.3 | 125.8 | 117.7 | 106.6 | 101.4 | 99.7 | 98.4 |
| 3        | 141.5 | 133.4 | 131.2 | 120.5 | 118.9 | 116.3 | 116.0 |
| 4        | 168.9 | 164.6 | 160.2 | 150.8 | 144.8 | 141.7 | 135.6 |
| 5        | 111.3 | 110.9 | 106.4 | 101.3 | 99.7 | 96.5 | 96.1 |
| 6        | 172.5 | 170.4 | 162.3 | 150.4 | 144.5 | 142.8 | 138.7 |
Table 3. Normalized undrained shear strength $\tau_{fu}/\sigma'_{vo}$ for tested soils

| Soil No. | Angle of the principal stress rotation $\alpha$ (°) | Normalized undrained shear strength $\tau_{fu}/\sigma'_{vo}$ (kPa) |
|----------|---------------------------------|--------------------------------------------------|
| 1        | 0  | 0.640 | 0.626 | 0.596 | 0.593 | 0.576 | 0.571 | 0.564 |
| 2        | 15 | 0.588 | 0.572 | 0.535 | 0.485 | 0.461 | 0.454 | 0.447 |
| 3        | 30 | 0.615 | 0.580 | 0.570 | 0.524 | 0.517 | 0.506 | 0.504 |
| 4        | 45 | 0.734 | 0.716 | 0.697 | 0.656 | 0.630 | 0.616 | 0.590 |
| 5        | 60 | 0.530 | 0.528 | 0.507 | 0.482 | 0.475 | 0.460 | 0.458 |
| 6        | 75 | 0.750 | 0.741 | 0.706 | 0.654 | 0.628 | 0.621 | 0.603 |

Each value of the undrained shear strength was obtained at an adequate value of axial strain. All the obtained values of the undrained shear strength were normalized based on the in situ vertical effective stress component $\sigma'_{vo}$ to obtain comparable values of the normalized undrained shear strength independent on the value of the in situ effective stress (Table 3, Figure 6).
Axial strains corresponding to the obtained values of the undrained shear strength are in the range of 7.1–16.5%. Based on the test results it can be stated that the value of undrained shear strength for particular sandy silty clays decreases with the increasing angle of the principal stress rotation. However, the decrease trend is different in particular soils. Analysis of the test results shows that in case of sandy silty clay a higher decrease in the undrained shear strength occurs for angles $\alpha$ between 0º and 45º, whereas the decrease in the undrained shear strength is much smaller for angles above 45º. The undrained shear strength at $\alpha = 45^\circ$ compared to $\alpha = 0^\circ$ is lower in the range of 7 – 15% while the undrained shear strength at $\alpha = 90^\circ$ compared to $\alpha = 0^\circ$ is lower in the range of 12 – 24% for tested soils. The biggest difference in undrained shear strength is in case of soil No. 2. The undrained shear strength at $\alpha = 90^\circ$ is about 24% lower than at $\alpha = 0^\circ$. In case of soil No. 2 the difference in undrained shear strength is the smallest and is equal to 12%.

4. Conclusions

The anisotropy of undrained shear strength caused by the principal stress rotation due to difficulty to achieve in laboratory tests is often overlooked during determination the bearing capacity of the subsoil. Taking the value of the undrained shear strength determined at one angle of the principal stress rotation for the entire subsoil most often leads to the underestimation or overestimation of the actual value of the undrained shear strength of the subsoil.

Tests performed in the Torsional Shear Hollow Cylinder Apparatus showed that principal stress rotation causes the anisotropy of undrained shear strength in the subsoil. The value of the undrained shear strength $\tau_{fu}$ in sandy silty clays ($sasiCl$) decreases with increase of angle of the principal stress rotation $\alpha$. In case of tested sandy silty clays the decrease in the undrained shear strength is higher for the angle $\alpha$ between 0º and 45º in comparison to values for the angle $\alpha$ between 45º and 90º. The undrained shear strength at $\alpha = 90^\circ$ compared to $\alpha = 0^\circ$ is lower in the range of 12 – 24% for tested soils.

In order to determine the influence of the principal stress rotation on undrained shear strength further research on soils characterized by different index properties should be carried out.

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