Spin precession in the Dvali-Gabadadze-Porrati braneworld scenario

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Abstract

In this letter we work out the secular precession of the spin of a gyroscope in geodesic motion around a central mass in the framework of the Dvali-Gabadadze-Porrati multidimensional gravity model. Such an effect, which depends on the mass of the central body and on the orbit radius of the gyroscope, contrary to the precessions of the orbital elements of the orbit of a test body, is far too small to be detected.

1 Introduction

The Dvali-Gabadadze-Porrati (DGP) multidimensional gravity model [1] has recently attracted great interest because it not only allows to explain the observed acceleration of the expansion of our Universe but also predicts some tiny post-Einsteinian effects that are testable at local scales and yield information on the global properties of the Universe as the kind of cosmological expansion currently ongoing [2]. For a comprehensive phenomenological overview of the DGP gravity see [3].

Up to now, secular precessions of the longitude of pericentre $\omega$ [4] and of the mean anomaly $M$ [5] of the orbit of a test particle freely falling around a central body have been worked out. Such effects, which depend on the eccentricity $e$ via second-order terms and are independent of the semimajor axis $a$ of the orbiter, amount to $10^{-4} - 10^{-3}$ arcseconds per century ($''$ cy$^{-1}$). The ideal test-bed for them is represented by the inner planets of the Solar System [6, 7]. Such orbital precessions lie at the edge of the present-day precision of the latest planetary data. The DGP features of motion related to the self-acceleration cosmological phase are compatible to them [8].

In this letter we wish to investigate the impact of DGP gravity on the precession of a spin $S$ orbiting a central body of mass $M$. 

1
2 The spin precession

In the DGP picture our Universe is a (3+1) space-time brane embedded in a larger five-dimensional Minkowskian bulk. Contrary to the other forces constrained to remain on the brane, gravity can fully explore the entire bulk getting strongly modified at scales $r \geq r_c$, where $r_c$ is a free-parameter fixed by the observations of the Supernovae Type IA to $r_c \sim 5 \text{ Gpc}$. At much smaller scales $r \ll r_c$ the usual Newton-Einstein gravity is recovered apart from small corrections. By neglecting the effects of the spatial curvature, the modifications to the Newtonian potential can be accounted for as\[1\]

\[(ds)^2 = \left(1 - \frac{R_g}{2r} \pm \sqrt{\frac{R_g r}{2r_c^2}}\right)^2 (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2, \quad (1)\]

where $R_g = 2GM/c^2$ is the Schwarzschild radius of the central mass and the $\pm$ sign is related to the different cosmological phases: the $+$ sign is for the Friedmann-Lemaître-Robertson-Walker (FLRW) phase while the $-$ sign is for the self-accelerated phase.

Let us consider a gyroscope falling along a geodesic of the space-time metric: thus, its spin vector is carried along by parallel transport according to\[9\]

\[\frac{dS^\mu}{d\tau} = -\Gamma^\mu_{\alpha\beta} S^\alpha dx^\beta, \quad (2)\]

where $\Gamma^\mu_{\alpha\beta}$ are the Christoffel symbols\[3\]

\[\Gamma^\mu_{\alpha\beta} = \frac{g^{\mu\nu}}{2} \left(\frac{\partial g_{\alpha\rho}}{\partial x^\beta} + \frac{\partial g_{\beta\rho}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\rho}\right), \quad (3)\]

and $\tau$ is the proper time along the geodesic of the orbiting gyroscope.

By neglecting all the terms proportional to $R_g^2$, $R_g/r_c^2$, $R_g^{3/2}/r_c$ we can pose

\[g_{00} \simeq 1 - \frac{R_g}{r} \pm \sqrt{\frac{2R_g r}{r_c^2}}, \quad (4)\]

\[g^{00} \simeq 1 + \frac{R_g}{r} \mp \sqrt{\frac{2R_g r}{r_c^2}}, \quad (5)\]

The non-vanishing Christoffel symbols for the metric of eq.\[11\] are

\[\Gamma^0_{0i} = \Gamma^0_{i0} \simeq \frac{R_g x^i}{2r^3} \pm \sqrt{\frac{R_g}{8r_c^2}} \frac{x^i}{r^{3/2}}, \quad i = 1, 2, 3. \quad (6)\]
The equations for the spatial components of the spin thus become
\[
\frac{dS^i}{d\tau} = -\Gamma^i_{\alpha\beta}S^\alpha \frac{dx^\beta}{d\tau} = -\Gamma^i_{00}S^0, \quad i = 1, 2, 3. \tag{7}
\]
The time-like component \(S^0\) of the spin four-vector is, from \(g_{\mu\nu}S^\mu \frac{dx^\nu}{d\tau} = 0\)
\[
S^0 = -\frac{1}{g_{00}} \left( S_x g_{11} \frac{dx}{ds} + S_y g_{22} \frac{dy}{ds} + S_z g_{33} \frac{dz}{ds} \right) \simeq \left( 1 + \frac{R_g}{r} \mp \sqrt{\frac{2R_g r}{r_c^2}} \right) \frac{S \cdot v}{c}. \tag{8}
\]
The spin precession due to the DGP correction to the Newtonian potential is thus
\[
\dot{S}_{\text{DGP}} = \mp \sqrt{\frac{R_g}{8r_c^2a^3}} (S \cdot v)r. \tag{9}
\]
By considering a circular planar orbit of radius \(a\) and period \(P = 2\pi\sqrt{a^3/GM} = 2\pi/n\), so that
\[
\begin{align*}
\mathbf{r} &= a \cos nt \mathbf{i} + a \sin nt \mathbf{j}, \tag{10} \\
\mathbf{v} &= -v \sin nt \mathbf{i} + v \cos nt \mathbf{j}, \tag{11}
\end{align*}
\]
and averaging over one orbital period eq. (9) becomes
\[
\langle \dot{S}_{\text{DGP}} \rangle = \pm \sqrt{\frac{R_g}{32r_c^2a}} v (-S_y i + S_x j) = \pm \sqrt{\frac{R_g}{32r_c^2a^3}} (\mathbf{r} \times \mathbf{v}) \times \mathbf{S} \equiv \Omega_{\text{DGP}} \times \mathbf{S}. \tag{12}
\]
Thus, by assuming \(v = na\), the spin of an orbiting gyroscope moving along a circular orbit precesses at a speed
\[
\Omega_{\text{DGP}} = \frac{GM}{4cr_c a}. \tag{13}
\]
Contrary to the secular precessions of the Keplerian orbital elements of a test particle, which is determined from the geodesic equation of motion, such a spin precession depends on the mass of the central body and on the radius of the gyroscope’s orbit. Unfortunately, the size of such an effect amounts to only \(10^{-11} - 10^{-12} \, \text{cy}^{-1}\) for the orbital angular momenta of the inner planets of the Solar System and to \(10^{-12}\) milliarcseconds per year for the gyroscopes of the GP-B spacecraft. It is so because of the \(c^{-1}\) factor in eq. (13) due to the presence of \(dx^0/d\tau\) only once in eq. (7). Instead, in the geodesic equation of motion \(dx^0/d\tau\) appears twice.
3 Conclusions

In this letter we have worked out the behavior of the spin of a gyroscope in geodesic motion around a central mass in the framework of the Dvali-Gabadadze-Porrati braneworld scenario. It turns out that the spin undergoes a precession which depends both on the mass of the central body and on the radius of the gyroscope orbit, contrary to the secular precessions of the Keplerian orbital elements of a test particle. The magnitude of such an effect is too small to be detected in a foreseeable future.

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