Thermodynamics of interacting tachyonic scalar field

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Abstract
In this paper we discuss the laws of thermodynamics for interacting tachyonic scalar field. The components of the tachyonic scalar field in the universe are taken to exist in the state of non-equilibrium initially, but due to interaction they undergo a transition towards the equilibrium state. We show that the zeroth law of thermodynamics demands interaction among the components of cosmic field. The second law of thermodynamics is governing dynamics in transfer of energy among the three components of the proposed field with local violation of conservation of energy for individual components.

1 Introduction
The accelerated expansion of the universe reveal by a large number of cosmological observations [1] and can be understood by introduction of repulsive gravity, although the other alternatives [2] may also explain the accelerated expansion. The repulsive gravity demands some different types of entities which have negative pressure. Considering its repulsive character such type of entity is called dark energy.

A class of scalar fields is one of the promising candidates of dark energy. Among itself tachyonic scalar field appearing in the context of string theory [3] is logically more appealing than its counterpart quintessence due to its relativistic Lagrangian as analogue of particles. Cosmological relevance of this field has been studied by several authors during last few decades [4]. Action and Lagrangian of the cosmological tachyonic scalar field is given by

\begin{equation}
A = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} - V(\phi) \sqrt{1 - \partial^i \phi \partial_i \phi} \right)
\end{equation}

with Lagrangian \( L = -V(\phi) \sqrt{1 - \partial_i \phi \partial^i \phi} \) and the equation of motion of the field is found by varying action as

\begin{equation}
\ddot{\phi} + \frac{1 - \dot{\phi}^2}{\dot{\phi}} V'(\phi) + 3H(1 - \dot{\phi}^2) = 0
\end{equation}

The stress energy tensor for this Lagrangian

\begin{equation}
T^{ik} = \frac{\partial L}{\partial (\partial_i \phi)} \partial^k \phi - g^{ik} L
\end{equation}

This gives energy density and pressure for spatially homogeneous field as

\begin{equation}
\rho = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad P = -V(\phi) \sqrt{1 - \dot{\phi}^2}
\end{equation}

respectively.

2 Components of Field
We assume that radiation with equation of state \( w_r = 1/3 \) also exists as one inherent component of same tachyonic scalar field. Due to some physical mechanism not known in detail at present to us, the cosmic

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tachyonic scalar field may be decomposed into several components with the assumption that the field is spatially homogeneous. We can write the expressions for energy density and pressure as

\[ P = -\frac{V(\phi)}{\sqrt{1 - \phi^2}} + \frac{\dot{\phi}^2 V(\phi)}{\sqrt{1 - \phi^2}} + 0 \quad (5) \]

\[ \rho = \frac{V(\phi)}{\sqrt{1 - \phi^2}} + \frac{3\dot{\phi}^2 V(\phi)}{\sqrt{1 - \phi^2}} - \frac{3\phi^2 V(\phi)}{\sqrt{1 - \phi^2}}. \quad (6) \]

From (5) and (6) it is seen that when we include radiation in tachyonic scalar field then one new exotic component also appears (say, exotic matter since its energy density is negative) with zero pressure. Thus, the tachyonic scalar field resolves into three components say, \(a\), \(b\) and \(c\). The pressure and energy density of \(a\) is given as

\[ P_a = -\frac{V(\phi)}{\sqrt{1 - \phi^2}}, \quad \rho_a = \frac{V(\phi)}{\sqrt{1 - \phi^2}} \Rightarrow \bar{w}_a = -1 = w_\lambda. \]

This is nothing but the ‘true’ cosmological constant because of its equation of state being \(w_\lambda = -1\).

The second component with

\[ P_b = \frac{\dot{\phi}^2 V(\phi)}{\sqrt{1 - \phi^2}}, \quad \rho_b = \frac{3\dot{\phi}^2 V(\phi)}{\sqrt{1 - \phi^2}} \Rightarrow \bar{w}_b = 1/3 \]

can be identified as radiation with \(w_r = 1/3\). The last component is characterised by

\[ P_c = 0, \quad \rho_c = \frac{3\phi^2 V(\phi)}{\sqrt{1 - \phi^2}} \Rightarrow \bar{w}_c = 0. \]

This component mimics dust matter but has negative energy density. The exotic matter may include the Dirac fermions as well as the Majorana fermions whence the negative energy states turn into the positive energy states\[5, 6\]. In our earlier work \[7\] we allowed a small time dependent perturbation in the equation of state (EoS) of the cosmological constant with \(w_\lambda = -1 + \varepsilon(t)\). Thus, with the perturbed EoS, the true cosmological constant becomes a shifted cosmological parameter. This has a bearing upon the EoS of radiation and exotic matter, both. Therefore, these two entities turn into shifted radiation and shifted exotic matter respectively. With fixed energy density of field components, the expressions for the energy density and pressure of each component are given as below. For the shifted cosmological constant one has

\[ \bar{\rho}_\lambda = \frac{V(\phi)}{\sqrt{1 - \phi^2}}, \quad \bar{\rho}_\lambda = -\frac{V(\phi)}{\sqrt{1 - \phi^2}} + \frac{\varepsilon V(\phi)}{\sqrt{1 - \phi^2}} \quad (7) \]

and \(\bar{w}_\lambda = -1 + \varepsilon(t)\). For shifted radiation, we have

\[ \bar{\rho}_r = \frac{3\dot{\phi}^2 V(\phi)}{\sqrt{1 - \phi^2}}, \quad \bar{\rho}_r = \frac{(1 + 3\varepsilon)\dot{\phi}^2 V(\phi)}{\sqrt{1 - \phi^2}} \quad (8) \]

In presence of perturbation the zero pressure of exotic matter turns into negative non-zero pressure due to shifted exotic matter which would also accelerate the universe like dark energy. Thus, the energy density and pressure for shifted exotic matter are now, respectively, given as

\[ \bar{\rho}_m = \frac{-3\phi^2 V(\phi)}{\sqrt{1 - \phi^2}}, \quad \bar{\rho}_m = \rho_\phi - \bar{\rho}_\lambda - \bar{\rho}_r = \frac{-\varepsilon(1 + 3\phi^2) V(\phi)}{\sqrt{1 - \phi^2}} \quad (9) \]

with \(\bar{w}_m = \frac{\varepsilon(1 + 3\phi^2)}{3\phi^2}\).

### 3 Thermodynamics Laws for interacting components

Why must the components of cosmic field interact? This is one of the most interesting question about interaction. Interaction might be justify by thermodynamics \[8\]. As shown by obvious observations the cosmic field must include at least three components representing matter, dark energy and radiation (many other components are also possible) and behave as an ensemble of three interacting thermodynamic systems. We apply the
Zeroth law of the thermodynamics to these three systems called as shifted cosmological parameter (SCP), shifted radiation (SR) and shifted exotic matter (SEM) each. If SCP is in equilibrium with SR and SR is in equilibrium with SEM then SEM should be also in equilibrium with SCP. This zeroth law demands interaction among cosmic field components whenever equilibrium gets perturbed for any reasons. If equilibrium is disturbed and the components are in non-equilibrium (thermal, mechanical or else), then to re-attain the equilibrium the components must interact mutually. If the components are in equilibrium then due to interaction perturbation in equilibrium state reacts trying to restore its state or achieve a new one (Le Châtelier-Braun principle) [9, 10]. If all components of tachyonic scalar field are in non-equilibrium, then, to achieve an equilibrium state they must fall into mutual interaction. This motivation provides one justification to study the interaction of these components. Here, we assuming that even though the total energy of the perturbed field (spatially homogeneous) is kept conserved (First law of thermodynamics) yet during interaction it can get reasonably violated for individual components. We consider the three components of cosmic tachyonic scalar field the components are shifted cosmological parameter (SCP), shifted radiation (SR) and shifted exotic matter (SEM). The individual equations of energy conservation for SCP, SR and SEM are respectively given as,

\[ \dot{\rho}_\lambda + 3H(1 + \bar{w}_\lambda)\rho_\lambda = -Q_1 \] (10)

\[ \dot{\rho}_r + 3H(1 + \bar{w}_r)\rho_r = Q_2 \] (11)

\[ \dot{\rho}_m + 3H(1 + \bar{w}_m)\rho_m = Q_1 - Q_2 \] (12)

where \( Q_1 \) and \( Q_2 \) are the interaction strengths and \( H \) is Hubble parameter. Second Law of Thermodynamics is the governing dynamics of interaction and sign of \( Q_1 \) and \( Q_2 \) shows the direction of flow of energy density during interaction among components. The positivity of the quantity \( Q_1 - Q_2 \) implies that \( Q_1 \) should be large and positive. For if \( Q_1 \) had been large and negative, then the second law of thermodynamics would have been violated and the SCP (as the dark energy candidate) would have dominated much earlier withholding the structure formation against the present observations. Also, \( Q_2 \) should be positive and small since if it is negative and large then conservation of energy of tachyonic field is violated.

4 Interaction among the components

The interacting dark energy models have been recently proposed by several authors [11]. We study the interaction of these three components assuming that even though the total energy of the perturbed field (spatially homogeneous) is kept conserved, yet during interaction it can get reasonably violated for individual components.

In the above expressions (10), (11) and (12) the following broad conditions must govern the dynamics.

**Condition(I) \( |Q_1| > |Q_2| \).** This corresponds to the following cases,

(i) If \( Q_1 > 0, Q_2 > 0 \) then the right hand side of (10) is negative while (11) and (12) are positive, respectively. This means that there is energy transfer from shifted cosmological parameter to shifted radiation and shifted exotic matter, respectively. Thermodynamics allows this kind of transfer of energy.

(ii) \( Q_1 < 0, Q_2 < 0 \) implies that there is an energy transfer to shifted cosmological parameter from shifted radiation and shifted exotic matter.

**Condition(II) \( |Q_2| > |Q_1| \).**

(i) \( Q_2 > 0, Q_1 > 0 \) would make the right hand side of (11) as positive and (10) and (12) as negative. This shows that there is an energy transfer to shifted radiation from shifted cosmological parameter and shifted exotic matter. Thermodynamics again does not allow this kind interaction.

(ii) \( Q_1 < 0, Q_2 < 0 \) makes way for the energy transfer from shifted radiation to shifted exotic matter and shifted cosmological parameter.

**Condition(III) If \( Q_2 = Q_1 = Q \) then we have the following possibility**

(i) \( Q > 0 \) leads to an energy transfer to shifted radiation from shifted cosmological parameter, while the shifted exotic matter remains free from interaction with its energy density held conserved. This type of interaction holds compatibility with the laws of thermodynamics.

(ii) If \( Q < 0 \), energy would flow from shifted radiation to shifted cosmological parameter, whereas the shifted exotic matter does not get involved in interaction mechanism. Thus, the conservation of energy for shifted exotic matter holds good.

3
(iii) As an alternative, $Q = 0$ would pull the components of tachyonic scalar field out of mutual interaction like the standard ΛCDM model.

The second case of condition (I) and condition (II) violate the laws of thermodynamics, therefore, we are not interested in these types of interactions. The interaction of type condition (iii) has been discussed for two components of tachyonic scalar field in our earlier work [12]. Due to the lack of information regarding the exact nature of dark matter and dark energy (as the cosmological constant or else) we present the form of interaction term heuristically as function of time rate of change in energy densities as

$$Q_1 = \alpha \dot{\bar{\rho}}_\lambda; \quad Q_2 = \beta \dot{\bar{\rho}}_r$$

(13)

where $\alpha, \beta$ are proportionality constant. While several authors have proposed different forms of $Q$ [11].

From (10), (11) and (12), for the specific dynamical form of interaction strengths (13) one can found the functional form of energy density with redshift $z$ as,

$$\bar{\rho}_\lambda = \bar{\rho}_0 \lambda^{3\varepsilon/(1+\alpha)}$$

(14)

where $\frac{4\varepsilon}{\alpha} = 1 + z = x$

$$\bar{\rho}_r = \bar{\rho}_0 r^{4+3\varepsilon/(1-\beta)}$$

(15)

and

$$\bar{\rho}_m = \bar{\rho}_m^{0}\frac{3\varepsilon\rho^3}{3\varepsilon - \eta - \eta\alpha} [x^{3\varepsilon/(1+\alpha)} - x^\eta] - \frac{\beta \bar{\rho}_r^2 (4 + 3\varepsilon)}{4 + 3\varepsilon - \eta + \eta\beta} [x^{4+3\varepsilon/(1-\beta)} - x^\eta]$$

(16)

where $\eta$ assuming constant (with $\dot{\phi}^2 \approx \text{constant}$) is defined as

$$\eta = \frac{3\dot{\phi}^2 (1 + \varepsilon) + \varepsilon}{\dot{\phi}^2}.$$  

(17)

Thus the cosmic expansion history of the universe is given by Hubble parameter with interaction as,

$$H^2 = \frac{\kappa^2}{3} [\bar{\rho}_\lambda + \bar{\rho}_r + \bar{\rho}_m]$$

(18)

where $\kappa^2 = 8\pi G$.

5 Conclusion

Having the motivation for the relativistic (tachyonic) scalar field, in contrast to quintessence, the single tachyonic scalar field which splits due to some unknown mechanism into three components (cosmological constant, radiation and dust matter). Due to consideration of radiation in this field the dust matter appears with negative energy. A small perturbation allowed in EoS of cosmological constant changes its status from a true cosmological constant to a shifted cosmological parameter (SCP). Similarly, status of radiation and dust matter changes to shifted radiation (SR) and shifted exotic matter (SEM). Thermodynamics laws (Zeroth, First and Second) might be responsible for interaction among components of single cosmic field. We consider the components of the field as thermodynamic systems and they interact to achieve a thermodynamical equilibrium. Particularly Zeroth Law invite interaction among components to maintain thermodynamical equilibrium or get new one, First Law demands the total energy of field stays conserved but the field components mutually interact with interaction strength parameter $Q$ resulting in local violation of energy conservation and Second Law decide the direction of flow of energy during interaction. The entire evolution of the universe arises from this process of interaction.

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