Analytical model of ear dynamics and conditions for efficient grain extraction

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Abstract. The paper is devoted to studying the motion dynamics of stem and ear, as well as particular wheat grains. The purpose of this consideration is to establish parameters of the effect on the plant as a whole, and in particular on the ear, for the efficient extraction of grains from the ear. For this, a mathematical model of the interaction between the elements of the stem-ear-grain system and the conditions for the separation of grain from the ear is built. In a first approximation, a simple stem model consisting of absolutely rigid bodies interconnected by elastic bonds is taken as such a model. Stems with a given moment of inertia act as such bodies, and spiral springs act as elastic bonds.

1. Introduction

Agriculture is the most important sector of the national economy of our country. Modern agriculture is unthinkable without the cultivation and production of crops. Among the crops, the central place belongs to wheat and barley. They occupy more than half of the sown area. For the full harvesting of the grown grain crop, it is necessary to harvest it before it is shed and forcefully separate the grain from the ear [1, 2].

For this, grain harvesters, which are the main machine for harvesting grain crops, are now widely used. In their designs, a method of threshing and primary separation of the harvested heap is implemented. This method, invented more than a century ago, assumes that the whole grown mass is cut off and fed to a threshing device, where it is subjected to the impact of the whips of the drum with simultaneous dragging along a rigid lattice. In this case, up to 80% of the power is spent on threshing and grinding of straw and about 7% of the energy is spent on separating grain from the receptacle.

This mode of influence on the processed mass in combines, necessary for the separation of grain and its separation through the deck from the ears of plants, has serious drawbacks. Firstly, it is energy consuming. Secondly, it leads to an overall high level of grain injury. Thirdly, due to the insufficient connection of grain with an ear during untimely harvesting, a process of self-shedding can occur.

In the paper [3], the distribution of dry matter was optimized to increase the spike growth, the amount of grain, and the spring wheat crop index. In [4], an analysis was made of grain losses during harvesting based on logistic regression. The feasibility of measuring moisture on a combine for controlling harvesting and post-harvesting operations and planning capacity when harvesting grain was determined in [5]. A model for predicting grain losses after harvesting was studied in [6]. The paper [7] examined the
quality of wheat grains harvested with different moisture contents and stored in a sealed and traditional system.

Thus, the urgent task of today is the task of providing low-energy and low-traumatic forced extraction of grain from the ear.

For this, it is necessary not only to further improve the existing technology and methods of harvesting grain [8-12], but also to develop new highly effective physical and mechanical methods of harvesting it and to design the appropriate equipment.

The system under consideration (Figure 1) has \(2 + N\) degrees of freedom, where \(N\) is the number of grains (the \(k\)-th grain is shown in the figure).

Figure 1. Stem model.

Choose as generalized coordinates the angles of deviation of the stem, ear and each of the grains from the vertical:

\[
q_1 = \psi_1, \\
q_2 = \psi_2, \\
q_{k+2} = \phi_k, \text{ where } k = 1, \ldots, N.
\]

Then the system of Lagrange equations of the second kind in the generalized coordinates (1) will take the following form (\(k = 1, \ldots, N\)):

\[
\frac{d}{dt} \frac{\partial T}{\partial \psi_1} - \frac{\partial T}{\partial \psi_1} = \bar{Q}_1 \\
\frac{d}{dt} \frac{\partial T}{\partial \psi_2} - \frac{\partial T}{\partial \psi_2} = \bar{Q}_2
\]
\[ \frac{d}{dt} \frac{\partial T}{\partial \dot{\varphi}_k} - \frac{\partial T}{\partial \varphi_k} = Q_k \]

The kinetic energy of the system is:

\[ T = T_1 + T_2 + \sum_{k=1}^{N} T_k \] (3)

For the stem, performing a rotational motion, the kinetic energy has the form:

\[ T_1 = \frac{I_{01} \dot{\psi}_1^2}{2} \] (4)

where the moment of inertia relative to point 0 is \( J_{01} = \frac{M_1 L_1^2}{3} \).

The ear performs a plane-parallel motion. By setting the motion of its center of mass in a coordinate way, using Koenig's theorem we find the kinetic energy of the ear:

\[ T_2 = \frac{m_k V_k^2}{2} = \frac{m_k}{2} \left[ L_1 \dot{\psi}_1^2 + b_k \dot{\psi}_2^2 + l_k \dot{\varphi}_k^2 + 2 L_1 b_k \dot{\psi}_1 \dot{\psi}_2 \cos(\psi_2 - \psi_1) \right] + \frac{1}{2} \frac{M_2 L_2^2}{12} \dot{\psi}_2^2 \] (5)

and the kinetic energy of the k-th grain:

\[ T_k = \frac{m_k V_k^2}{2} = \frac{m_k}{2} \left[ L_1 \dot{\psi}_1^2 + a_k^2 \dot{\psi}_2^2 + l_k \dot{\varphi}_k^2 + 2 L_1 a_k \dot{\psi}_1 \dot{\psi}_2 \cos(\psi_2 - \psi_1) \right] + \frac{1}{2} \frac{M_2 L_2^2}{12} \dot{\psi}_2^2 \] (6)

Summing up (4)-(6), we find the kinetic energy of the system:

\[ T = \frac{1}{2} \frac{M_1 L_1^2}{3} \dot{\psi}_1^2 + \frac{M_2}{2} \left[ L_1 \dot{\psi}_1^2 + a_k^2 \dot{\psi}_2^2 + L_1 a_k \dot{\psi}_1 \dot{\psi}_2 \cos(\psi_2 - \psi_1) \right] + \frac{1}{2} \frac{M_2 L_2^2}{12} \dot{\psi}_2^2 + \sum_{k=1}^{N} \frac{m_k}{2} \left[ L_1 \dot{\psi}_1^2 + b_k \dot{\psi}_2^2 + l_k \dot{\varphi}_k^2 + 2 L_1 b_k \dot{\psi}_1 \dot{\psi}_2 \cos(\psi_2 - \psi_1) \right] + \frac{1}{2} \frac{M_2 L_2^2}{12} \dot{\psi}_2^2 \] (7)

Choose the vertical position of the stem and ear (zero equipotential surface) for the zero position and find the potential energy of the system for potential forces (gravity and elasticity):

\[ P = -M_1 g a_1 (1 - \cos \psi_1) - M_2 g [L_1 (1 - \cos \psi_1) + a_2 (1 - \cos \psi_2)] \]

\[ - \sum_{k=1}^{N} m_k g [L_1 (1 - \cos \psi_1) + b_k (1 - \cos \psi_2) + l_k (1 - \cos \varphi_k)] + \frac{k_1}{2} \dot{\psi}_1^2 \]

\[ + \frac{k_2}{2} (\psi_2 - \psi_1)^2 + \sum_{k=1}^{N} \frac{c_k (\varphi_k - \psi_1 - \psi_2)^2}{2} \] (8)

For the non-potential forces (dissipative forces) find the Rayleigh function \( \beta_1, \beta_1, \alpha_k \) are the corresponding damping coefficients:

\[ R = \frac{\beta_1 \dot{\psi}_1^2}{2} + \frac{\beta_2 (\psi_2 - \psi_1)^2}{2} + \sum_{k=1}^{N} \frac{\alpha_k (\varphi_k - \psi_1 - \psi_2)^2}{2} \] (9)

By virtue of (8) - (9), write out the generalized forces:
\[ \dot{Q}_1 = -\frac{\partial P}{\partial \psi_1} - \frac{\partial R}{\partial \psi_1} - F d \psi_1 \]
\[ = M_1 g a_1 \sin \psi_1 + M_2 g L_1 \sin \psi_1 + \sum_{k=1}^{N} m_k g L_1 \sin \psi_1 - k_1 \psi_1 + k_2 (\psi_2 - \psi_1) + \sum_{k=1}^{N} c_k (\varphi_k - \psi_1 - \psi_2) - \beta_1 \psi_1 \]
\[ + \beta_2 (\psi_2 - \psi_1) + \sum_{k=1}^{N} \alpha_k (\varphi_k - \psi_1 - \psi_2) - F d \psi_1 \]

\[ \dot{Q}_2 = -\frac{\partial P}{\partial \psi_2} - \frac{\partial R}{\partial \psi_2} \]
\[ = M_2 g a_2 \sin \psi_2 + \sum_{k=1}^{N} m_k g b_k \sin \psi_2 - k_2 (\psi_2 - \psi_1) + \sum_{k=1}^{N} c_k (\varphi_k - \psi_1 - \psi_2) \]
\[ - \beta_2 (\psi_2 - \psi_1) + \sum_{k=1}^{N} \alpha_k (\varphi_k - \psi_1 - \psi_2) \quad (10) \]

\[ Q_k = -\frac{\partial P}{\partial \psi_k} - \frac{\partial R}{\partial \psi_k} = m_k g l_k \sin \varphi_k - c_k (\varphi_k - \psi_1 - \psi_2) - \alpha_k (\varphi_k - \psi_1 - \psi_2) \]

Substituting the found values (7), (10) into equations (2), we obtain a nonlinear system of 2 + N Lagrange differential equations of the second kind describing the motion of the stem and spike with grains:

\[ \frac{M_1 L_1^2}{3} \ddot{\psi}_1 + M_2 L_1^2 \ddot{\psi}_1 + M_2 L_1 a_2 \ddot{\psi}_2 \cos(\psi_2 - \psi_1) - M_2 L_1 a_2 \ddot{\psi}_2 \sin(\psi_2 - \psi_1)(\dot{\psi}_2 - \dot{\psi}_1) \]
\[ + \sum_{k=1}^{N} m_k [L_1^2 \ddot{\psi}_1 \psi_1 + L_1 b_k \psi_2 \cos(\psi_2 - \psi_1) - L_1 b_k \psi_2 \sin(\psi_2 - \psi_1)(\dot{\psi}_2 - \dot{\psi}_1)] \]
\[ + L_1 b_k \ddot{\psi}_2 \cos(\varphi_k - \psi_1) - L_1 l_k \ddot{\psi}_2 \sin(\varphi_k - \psi_1) (\ddot{\varphi}_k - \ddot{\psi}_1) \]
\[ - M_2 L_1 a_2 \ddot{\psi}_1 \dot{\psi}_2 \sin(\varphi_k - \psi_1) \]
\[ - \sum_{k=1}^{N} m_k [L_1 b_k \ddot{\psi}_1 \dot{\psi}_2 \sin(\psi_2 - \psi_1) + L_1 l_k \ddot{\psi}_1 \dot{\psi}_2 \sin(\varphi_k - \psi_1)] \]
\[ = M_1 g a_1 \sin \psi_1 + M_2 g L_1 \sin \psi_1 \]
\[ + \sum_{k=1}^{N} m_k g L_1 \sin \psi_1 - k_1 \psi_1 + k_2 (\psi_2 - \psi_1) + \sum_{k=1}^{N} c_k (\varphi_k - \psi_1 - \psi_2) - \beta_1 \psi_1 \]
\[ + \beta_2 (\psi_2 - \psi_1) + \sum_{k=1}^{N} \alpha_k (\varphi_k - \psi_1 - \psi_2) - F d \psi_1 \]
\[
\sum_{k=1}^{N} m_k \left[ L^2 \ddot{\psi}_k \right] + L_1 b_k \dot{\psi}_2 \cos(\psi_2 - \psi_1) - L_1 b_k \dot{\psi}_2 \sin(\psi_2 - \psi_1) \left( \dot{\psi}_2 - \dot{\psi}_1 \right) \\
+ L_1 l_k \dot{\psi}_2 \cos(\varphi_k - \psi_1) - L_1 l_k \dot{\psi}_2 \sin(\varphi_k - \psi_1) \left( \dot{\varphi}_k - \dot{\psi}_1 \right) \\
+ M_2 L_1^2 \ddot{\psi}_1 + M_2 L_2^2 \ddot{\psi}_1
\]

\[
\sum_{k=1}^{N} m_k \left[ L^2 \ddot{\psi}_k \right] + L_1 b_k \dot{\psi}_2 \sin(\psi_2 - \psi_1) + b_k l_k \dot{\psi}_2 \dot{\varphi}_k \sin(\varphi_k - \psi_2) \\
= M_2 g a_2 \sin(\psi_2) + \sum_{k=1}^{N} m_k g b_k \sin(\psi_2) - k_2 (\psi_2 - \psi_1) + \sum_{k=1}^{N} c_k (\varphi_k - \psi_1) - \beta_2 (\psi_2 - \psi_1) \\
+ \sum_{k=1}^{N} a_k (\dot{\varphi}_k - \dot{\psi}_1 - \dot{\psi}_2)
\]

(k=1,...,N)

Since the analytical solution of this system (11) is fundamentally impossible, and the numerical solution is very difficult, we proceed to linearize the resulting system.

To do this, we first calculate the generalized inertia coefficients:

\[
a_{ij} = \frac{\partial^2 T}{\partial \dot{q}_i \partial \dot{q}_j} \bigg|_{q_1=q_2=\ldots=q_{2+N}=0} (i, j = 1, \ldots, 2 + N)
\]

and generalized stiffness factors:

\[
c_{ij} = \frac{\partial^2 P}{\partial \dot{q}_i \partial \dot{q}_j} \bigg|_{q_1=q_2=\ldots=q_{2+N}=0} (i, j = 1, \ldots, 2 + N)
\]

The expressions for the kinetic and potential energies will take the form:

\[
T = \frac{1}{2} \sum_{i=1}^{2+N} \sum_{j=1}^{2+N} a_{ij} \dot{q}_i \dot{q}_j \\
P = \frac{1}{2} \sum_{i=1}^{2+N} \sum_{j=1}^{2+N} c_{ij} \dot{q}_i \dot{q}_j
\]

(12)

In a first approximation, the influence of dissipative forces will be neglected further. Substituting (12) into (2), we obtain a linearized system of Lagrange equations of the second kind of size 2 + N by 2 + N:

\[
a_{11} \ddot{q}_1 + a_{12} \ddot{q}_2 + \ldots + a_{1,2+N} \ddot{q}_{2+N} = c_{11} q_1 + c_{12} q_2 + \ldots + c_{1,2+N} q_{2+N} = 0 \\
a_{21} \ddot{q}_1 + a_{22} \ddot{q}_2 + \ldots + a_{2,2+N} \ddot{q}_{2+N} = c_{21} q_1 + c_{22} q_2 + \ldots + c_{2,2+N} q_{2+N} = 0 \\
\ldots \\
a_{2+N,1} \ddot{q}_1 + a_{2+N,2} \ddot{q}_2 + \ldots + a_{2+N,2+N} \ddot{q}_{2+N} = c_{2+N,1} q_1 + c_{2+N,2} q_2 + \ldots + c_{2+N,2+N} q_{2+N} = 0
\]

(13)
Since system (13) is a system of ordinary linear differential equations with constant coefficients, we will seek its solution in the form:

\[ h_f = v_f \frac{\partial w}{\partial x} + y_0, \]

where \( p = 1, \ldots, 2 + r \) (14)

Substituting (14) into (13), we obtain an algebraic system with respect to \( v_f \):

\[
\begin{align*}
0 &= \begin{vmatrix} c_{11} - \omega^2 a_{11} & c_{12} - \omega^2 a_{12} & \cdots & c_{1,2+N} - \omega^2 a_{1,2+N} \\
0 &= \begin{vmatrix} c_{21} - \omega^2 a_{21} & c_{22} - \omega^2 a_{22} & \cdots & c_{2,2+N} - \omega^2 a_{2,2+N} \\
0 &= \begin{vmatrix} c_{2+N,1} - \omega^2 a_{2+N,1} & c_{2+N,2} - \omega^2 a_{2+N,2} & \cdots & c_{2+N,2+N} - \omega^2 a_{2+N,2+N} \\
\end{vmatrix}
\end{align*}
\]

Equation (16) is solved numerically and allows finding all eigenfrequencies \( \omega_k (k = 1, \ldots, 2 + N) \).

Since the system of equations (15) is homogeneous and its determinant is zero, this system of equations is linearly dependent. Therefore, its coefficients can be expressed, for example, through \( v_f = v_f + \frac{\partial w}{\partial x} + y_0 \).

Here the index (k) shows which eigenvalue \( \omega_k \) this coefficient corresponds to.

We introduce into consideration the values:

\[
E_f = -E_f - \omega \frac{\partial w}{\partial x} + \omega \frac{\partial w}{\partial x} + y_0
\]

Then the final solution to be written in the form:

\[ q_i = \sum_{k=1}^{2+N} q_i^{(k)}, \quad \text{where} \quad q_i^{(k)} = \mu_i^{(k)} A_i^{(k)} \sin(\omega_k t + \varepsilon_k) \]

here \( q_i^{(k)} \) is the eigenmodes corresponding to the k-th frequency.

Numerical calculations were carried out for winter wheat varieties "Dmitry" [1].

The ratio (8) includes the stiffness of coil springs \( c_k \), modeling the elastic properties of the elements of the ear and stem. The first step of numerical analysis is the identification of these parameters, which is carried out on the basis of experimental data. Thus, table 1 presents data on the dependence of bending forces of grain with scales from the angle obtained in ten tests.
Table 1. Bending grain scale.

| S/# | Angle, ° | 5  | 10  | 15  | 20  | 25  | 30  | 35  | 40  | 45  | 50  |
|-----|----------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   |          | 0.12 | 0.22 | 0.31 | 0.41 | 0.53 | 0.68 | 0.74 |     |     |     |
| 2   |          | 0.09 | 0.20 | 0.34 | 0.45 | 0.6  | 0.70 | 0.81 | 0.90 | 1.06 |     |
| 3   |          | 0.10 | 0.21 | 0.34 | 0.47 | 0.62 | 0.78 | 0.85 | 0.96 |     |     |
| 4   |          | 0.12 | 0.17 | 0.27 | 0.37 | 0.50 | 0.60 | 0.74 | 0.82 |     |     |
| 5   |          | 0.12 | 0.17 | 0.26 | 0.38 | 0.53 | 0.70 | 0.50 | 0.63 | 1.18 | 1.34 |
| 6   |          | 0.10 | 0.25 | 0.32 | 0.54 | 0.70 | 0.82 | 0.92 | 1.02 | 0.86 | 0.93 |
| 7   |          | 0.02 | 0.06 | 0.14 | 0.22 | 0.34 | 0.42 | 0.56 | 0.69 |     |     |
| 8   |          | 0.14 | 0.23 | 0.32 | 0.44 | 0.57 | 0.63 | 0.73 | 0.84 |     |     |
| 9   |          | 0.04 | 0.16 | 0.27 | 0.31 | 0.52 | 0.65 | 0.45 | 0.39 |     |     |
| 10  |          | 0.08 | 0.17 | 0.25 | 0.35 | 0.43 | 0.50 | 0.31 | 0.24 |     |     |

The data in table 1 was processed using the least squares method. As a result, a linear dependence is obtained, shown by a straight line in Figure 2. In this case, the stiffness coefficient of the spiral spring modeling the bending stiffness of the flakes is \( c = 0.00962 \text{ Nm} \).

Figure 2. Dependence of the force on angle (bending flakes).

To build continuum models, identification of the elastic properties of stem elements with an ear is required. The elastic modulus of the materials of stem \( E_s \) and spike \( E_c \) were found from the solution of the problem of bending a cantilever beam of length \( L \), with an inertia moment \( J \) under the action of \( P \) at its end from the differential equation:

\[
E_s J \left( \frac{d^4}{dx^4} y(x) \right) = 0
\]  

(17)

A solution that under these conditions of fastening and loading has the form

\[
y_p(x) = -\frac{P x^2}{6 E_s J} + \frac{L P x^2}{2 E_s J}
\]  

(18)

Whence the equation for the deflection at the end of the \( Y \_L \) beam of circular cross section with a diameter \( d \) and wall thickness \( h \) is:
The elastic modulus is found by the formula:

$$E_s = \frac{64PL^3}{3\pi(4d^3-6d^2h+4dh^2-h^3)}$$

(19)

And the elastic modulus is found by the formula:

$$E_s = \frac{64PL^3}{3Y_L(h(4d^3-6d^2h+4dh^2-h^3))}$$

(20)

For the spike, a beam with a rectangular section with sides $a$, $b$ was used, and the formula for determining Young's modulus has the form:

$$E_s = \frac{4PL^3}{Y_Lab^3}$$

(21)

As a result of calculations, the average values of the elastic modulus of stem and spike were taken

$$E_s = 1.0 \times 10^{10} \frac{H}{m^2}, E_c = 0.2 \times 10^{10} \frac{H}{m^2}$$

(22)

Table 2 presents the geometric characteristics of wheat. On a spikelet rod, one grain with scales on a leg is geometrically modeled.

Table 2. Geometric characteristics of wheat

| Characteristic                                | Meaning |
|----------------------------------------------|---------|
| Plant height, m                              | 0.89    |
| Stem length, m                               | 0.80    |
| Spike length, m                              | 0.09    |
| Stem diameter in the middle part, mm          | 3.65    |
| Wall thickness in the middle part of the stem, mm | 0.20 |
| Grain length mm                              | 8       |
| Grain width mm                               | 3       |
| Length of grain leg in the middle line, mm   | 1.3     |
| Angle of inclination of grain to the ear, degree | 35    |

The next step of the study is modal analysis. Knowing the natural resonant frequencies and modes of oscillations, allows you to choose the mode of mechanical action on the spike for the effective extraction of grain.

Table 3 shows the obtained resonant oscillation frequencies of the stem with an ear.

Table 3. Resonant frequencies of stem with ear.

| Natural frequency number | Resonant frequency value, Hz |
|--------------------------|----------------------------|
| 1                        | 1.57                       |
| 2                        | 9.34                       |
| 3                        | 18.12                      |
| 4                        | 31.68                      |
| 5                        | 59.03                      |
| 6                        | 93.79                      |

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