Reliability Studies on FRP Pultruded Profiles

S. Souryalachumy
M.E. Structural Engineering
Department of Civil & Structural Engineering
Annamalai University
Chidambaram, India

Dr. R. Sivagamasundari
Assistant Professor
Department of Civil & Structural Engineering
Annamalai University
Chidambaram, India

Dr. G. Kumaran
Professor
Department of Civil & Structural Engineering
Annamalai University
Chidambaram, India

Abstract --- FRP sections such as rods, flats, angles, channels, beams, H-Sections, I-sections, W-sections, square and rectangular hollow tube sections, round tubes, studs and nuts are used in making non-corrosive industrial structures such as walkway platforms, staircase, crossover bridge, support etc. A range of FRP sections are available in Polyester and Vinyl ester resins for various structural applications. These FRP composite shapes have many advantages over current structural materials including high strength-to-weight ratios, fire retardant, UV resistant, low weight, corrosion resistance, good fatigue properties, freeze-thaw resistance, low maintenance, long life, and low life-cycle cost. Among these sections, FRP Pultruded profiles can be shaped for custom applications, which is not always possible with other structural materials. The objective of the study is to calibrate the behaviour of FRP-Pultruded profiles using a reliability-based technique that accounts for the randomness in important design variables. Monte-Carlo simulation is performed on each of the desired profile and random set of values is obtained. Finally, the reliability index and probability of failure of the chosen sections are determined.

Keywords - FRP, Pultruded Profiles, Monte Carlo Simulation, Reliability Analysis

1. INTRODUCTION

FRP pultruded sections are strong, lightweight, corrosion resistant and non-conductive. It allows for better safety measures and more flexible design capabilities than steel, aluminum and structural timber. The standard shapes of structural pultrusions are readily available in 70 different shapes. They include: Pultruded Fiberglass Rod, Pultruded Fiberglass Channel, Pultruded Fiberglass Flat Sheet, Pultruded Fiberglass Angle, Pultruded Fiberglass Tube/Tubing, Pultruded Fiberglass Beam, Pultruded Fiberglass Plate and Pultruded Fiberglass Bar. Fig.1 and Fig.2 shows the different shapes of FRP-pultruded sections.

They are typically produced though the pultrusion process, which is a continuous process for the manufacture of products having a constant cross section, such as rod stock, structural shapes, beams, channels, pipe, tubing, fishing rods, and golf club shafts. Pultrusion produces profiles with extremely high fiber loading, thus pultruded products have high structural strength. The manufacturing of pultruded profiles is the most efficient method of forming composite materials with consistent high quality and low scrap. Availability of recognized code for FRP composite section and design are very rare .The design needs more number of researches for the formation of a recognized code. Load and Resistance Factor Design (LRFD) was developed to estimate the loads applied to the structure as well as the Factor Design capacity of the structure. LRFD defines the limit state as the point where the structure no longer performs adequately under the design requirements. In LRFD design the probability of exceeding the limit state is equal to or less than a certain predetermined value.

Reliability-based techniques are used to account for the randomness in design variables affecting the strength of FRP-Pultruded profiles. The development of structural reliability methods during the last four decades has provided a more logical basis for developing structural design codes.

Fig.1: W-section and I-section
The overall aim of structural reliability analysis is to quantify the reliability of structures under consideration of the uncertainties associated with the load and resistance. The present work focuses on the reliability analysis of FRP-Pultruded profiles. Thus safety is usually expressed in terms of reliability index, \( \beta \), obtained from reliability analysis based on the theory of probability.

2. LITERATURE REVIEW

Many researches have been carried out in the area of reliability analysis of FRP sections. Vanevenhoven (2010) [8], developed a standardization of production and design of pultruded profiles to enable mainstream use of these profiles in structural engineering practice and demonstrated that a unified design equation for pultruded columns can be developed for LRFD with reliability indices that are similar to those used for conventional materials. Ellingwood (2003) [5], has shown that it is feasible to develop practical probability-based limit states design criteria for engineered FRP composite pultruded structures in an LRFD format. A load and resistance factor design standard for composites would facilitate their use in civil infrastructure, creating a market for new FRP building materials by providing a basis for structural design that is comparable with existing LRFD standards for other common construction materials. Peter Bjørager (1990) [10], outlined and evaluated the state of the art of reliability computation methods for structural engineering based on the classification of the mathematical reliability models.

From a vast literature survey, the following objectives have been derived.

1. Calibrating the behaviour of FRP- Pultruded profiles using a reliability-based technique that accounts for the randomness in important design variables.
2. Based on the randomly generated set of values the reliability index for FRP-pultruded profiles is calculated.
3. This research focuses exclusively on FRP-Pultruded Wide flanged (W) profiles under concentric axial loading.

3. RELIABILITY ANALYSIS

Reliability analysis is defined as the probability of success. Reliability analysis is used to analyze the performance of any system. The analysis gives information about their quality, resistance and probability of failure. The reliability engineering emphasizes a system or component to function under stated conditions for a specified period of time. The reliability index \( \beta \) is a measure of the structural reliability and safety of a system or component. The reliability index value depends on load factor, resistance factors, material properties, and member dimensions. Any change in the parameters will correspond to change of reliability index value. The probability of failure of a system can be determined using reliability index and resistance factor. It is to be noted that the reliability index for a 6x6x1/4 inch column will not be the same as the reliability index for a 4x4x1/4 inch column, even though the samples have same material properties.

To determine the reliability index for a whole structure or a group of components such as columns, beams etc. for every component of the structure the reliability index is calculated separately. From the calculated values the least value is taken as the reliability index for the structure or the components. Hence a large sample size with varying parameters is required for a profile. The reliability index can be used to represent the probability of failure of a profile. The value of \( \beta \) is related to probability of failure with a resistance factor. If the value of the reliability index is high, the value for probability of failure obtained is low, which indicates a highly reliable or safe profile. If the reliability index value is less, the probability of failure is high, indicating a less reliable or unsafe profile. The relationship between the probability of failure and \( \beta \) can be seen in Table 1 [4]. As shown from Table 1, a small change in the reliability index corresponds to a large change in the probability of failure. For example, a difference in the reliability index from 1.28 to 2.33 indicates a difference in the probability of failure from 10\(^{-4}\) (one failure out of 10 samples) to 10\(^{-2}\) (one failure out of 100 samples).

| Probability of Failure (\(P_f\)) | Reliability Index (\(\beta\)) |
|----------------------------------|-------------------------------|
| 10\(^{-1}\)                      | 1.28                          |
| 10\(^{-2}\)                      | 2.33                          |
| 10\(^{-3}\)                      | 3.09                          |
| 10\(^{-4}\)                      | 3.71                          |
| 10\(^{-5}\)                      | 4.26                          |
| 10\(^{-6}\)                      | 4.75                          |
| 10\(^{-7}\)                      | 5.19                          |
| 10\(^{-8}\)                      | 5.62                          |
| 10\(^{-9}\)                      | 5.99                          |
A) Definition of $\beta$:

The reliability index, $\beta$ is defined as the shortest distance from the origin of reduced variables to the line $g(Z_R, Z_Q) = 0$. Here, $Z_R$ and $Z_Q$ represents the reduced variables of the random variables resistance($R$) and load demand ($Q$). It is graphically represented in Fig.3 [4]. In probability based design, the structural performance or the structural reliability is defined by a limit state function, which can be generally expressed as,

$$G(X) = 0$$ (1)

Where $X$ is the vector of resistance or load random variables. The safety of a structural component depends on its resistance ($R$) and load demand ($S$). It is expressed as the limit state function as the difference between the resistance of the profile and the load demand acting on the profile.

$$G = R - S$$ (2)

If $G > 0$ the structure or the profile is said to be safe or reliable and for lesser values it is said to be unsafe or the probability of failure is more.

The probability of failure, $P_f$ is equal to:

$$P_f = \text{Prob} (R - S < 0)$$ (3)

It is very difficult to determine the probability of failure directly by laboratory tests or by other methods. Hence the probability of failure or the structural safety is determined in terms of the reliability index ($\beta$) along with resistance factor ($\Phi$).

$$P_f = \Phi (-\beta)$$ (4)

![Fig.3: Reliability index](image)

4. MONTE CARLO SIMULATION

Monte Carlo Simulation is an efficient and useful method which saves more time and money by avoiding the extra laboratory works to be carried out to determine the strength parameters. This simulation is carried out by randomly selecting results from tests that have been executed before. The parameters included in selection of random variables are material properties, section dimensions, and load magnitudes with well-defined statistical distributions. In the present work the Monte Carlo Simulation method is used as an approach to determine the structural reliability of FRP Pultruded profiles. It is necessary that the sample space should be optimum level because the accurate result of probability of failure is obtained only by increasing the sample space.

A significant feature of Monte Carlo method is that infinite number of simulations can be performed for a single profile. There is no need of extra work to be performed to increase the number of simulations. In many cases the samples which cannot be tested in laboratory are predicted or assumed and checked for reliability by the simulation process. The parameters taken as input data are tabulated in Table 2.

| Parameter | Symbol |
|-----------|--------|
| Profile Number | $\#$ |
| Dead Load | $D$ |
| Live Load | $L$ |
| Longitudinal Compressive Modulus, Flange | $E_{Lc}$ |
| Longitudinal Compressive Modulus, Web | $E_{Lw}$ |
| Transverse Compressive Modulus, Flange | $E_{Tc}$ |
| Transverse Compressive Modulus, Web | $E_{Tw}$ |
| Poisson’s Ratio, Flange | $\nu_{L}$ |
| Poisson’s Ratio, Web | $\nu_{W}$ |
| Shear Modulus | $G_b$ |
| Flange Thickness | $t_f$ |
| Web Thickness | $t_w$ |
| Section Depth | $D$ |
| Section Breadth | $B$ |
| End Restraint Coefficient | $K$ |
| Specimen Length | $L$ |

The input values for the material properties of the FRP- pultruded profiles are taken from the Cooling technology Institute (CTI) code book [6]. The values of the material properties chosen are listed in Table 3. The intensity of dead load is taken to be 40psf (per square feet) by referring to ASCE 7-05 code book [1]. The intensity of live load is taken in the form of ratios corresponding to dead load. The ratios for dead and live loads are taken to be 1:1, 1:2, 1:3 such as the dead load remains constant as 40psf and the live load varies in the range 40 psf, 80psf and 120psf. A set of profiles selected with varying cross sectional dimensions and lengths are listed in Table 4.
Table 3: Input Values for Material Properties

| Material Property                  | Values          |
|-----------------------------------|-----------------|
| Longitudinal Compressive Modulus, Flange $E_{LCf}$ | $2.6 \times 10^6$ psi |
| Longitudinal Compressive Modulus, Web $E_{Lcw}$      | $2.6 \times 10^6$ psi |
| Transverse Compressive Modulus, Flange $E_{TCf}$      | $1 \times 10^6$ psi |
| Transverse Compressive Modulus, Web $E_{TCw}$        | $1 \times 10^6$ psi |
| Poisson’s Ratio, Flange $\nu_{LF}$                   | 0.33            |
| Poisson’s Ratio, Web $\nu_{LW}$                      | 0.33            |
| Shear Modulus $G_{b}$                      | $0.425 \times 10^6$ psi |
| End Restraint Coefficient $k$                | 1               |

Table 4: Dimensions of Profiles chosen

| S.No | Size in inches | Length 1 in Inches | Length 2 in Inches |
|------|----------------|--------------------|--------------------|
| 1.   | W 6x6x1/4      | 35                 | 25                 |
| 2.   | W 6x4x1/4      | 35                 | 25                 |
| 3.   | W 6x3x1/4      | 35                 | 25                 |
| 4.   | W 4x4x1/4      | 35                 | 25                 |
| 5.   | W 4x3x1/4      | 35                 | 25                 |
| 6.   | W 3x3x1/4      | 35                 | 25                 |

The output of the Monte Carlo Simulation consists of the nominal value, bias, mean, COV and standard deviation (STD). The results are tabulated in Table 5.

The nominal values is the design values obtained from the simulation of material properties. The bias is the ratio between the nominal and mean values. From the above tabulation it is clear that a change in depth of the profile leads to change of statistical values.

Table 5: Output from the Simulation Process

| Profile     | Nominal Value x $10^6$ psi | Bias | Mean | COV % | STD % |
|-------------|----------------------------|------|------|-------|-------|
| W 6x6x1/4   | 5.12                       | 7.68 | 8.32 | 9.6   | 10.24 |
| W 6x4x1/4   | 3.41                       | 5.12 | 5.55 | 6.4   | 6.8   |
| W 6x3x1/4   | 2.6                        | 3.84 | 4.16 | 4.8   | 5.12  |
| W 4x4x1/4   | 3.41                       | 5.12 | 5.55 | 6.4   | 6.8   |
| W 4x3x1/4   | 2.6                        | 3.84 | 4.16 | 4.8   | 5.12  |
| W 3x3x1/4   | 2.6                        | 3.84 | 4.16 | 4.8   | 5.12  |

5. RELIABILITY INDEX CALCULATION

The Nominal value, Bias, Mean, COV and STD determined for the profiles by Monte Carlo Simulation method corresponds to the resistance demand ($M_R$) of those profiles. Hence for each load considered a load demand should be developed for the profiles. The Nominal value, Bias, Mean, COV and STD should also be calculated for the load demand of each profile. For dead load it is referred to as $M_D$ and for live load it is referred to as $M_L$.

For the value of dead load and live load taken, the nominal value of the load demands are same. According to ACI 440.1R-05 code book [2], the nominal value for load demands can be calculated using the following formula.

$$M_D = M_L = \frac{\Phi M_R}{\gamma_D + \gamma_L}$$

Where $\Phi$ = resistance factor for FRP-Pultruded profile

$M_R$ = resistance demand of each profile

$\gamma_D$ = load factor for dead load

$\gamma_L$ = load factor for live load

According to ACI 318 [3], the load factors for dead and live load are takes as 1.2 and 1.6. The nominal value and mean value varies for each profile according to their cross section. From the nominal value and mean value of the loads the bias, COV and STD are calculated for each load and each profile.

A) Steps for Calculating Reliability Index:

1) Calculate the Nominal values, Bias, Mean, COV and STD for each profile and for each load.
2) Tabulate those specified values for each profile separately.
3) Determine the reduced variable for the resistance demand and load demands for each profile using the formula,

$$Z_{R_\sigma} = \frac{X_R - M_R}{\sigma_R}$$  \hspace{1cm} (6)

Where $Z_{R_\sigma}$ is set of values of reduced variable including $Z_{MBR}$, $Z_{MID}$ and $Z_{ML}$.

$X_R$ = nominal value of each parameter for each profile

$M_R$ = mean value of each parameter for each profile

$\sigma_R$ = standard deviation of each parameter for each profile

4) Determine the column vector $G$ which is the negative value of the standard deviation of the resistance demand and load demand.

$$G = -\{\sigma\}$$  \hspace{1cm} (7)

5) Calculate the value of $\{G\}^T$ which is the transpose of the matrix $G$.

6) Finally the reliability index $\beta$ can be found using the formula,

$$\beta = \frac{\{G\}^T\{Z\}}{\sqrt{\{G\}^T\{G\}}}$$  \hspace{1cm} (8)

Hence the value of the reliability index calculated for each profile by the above procedure is tabulated in Table 6.

**Table 6: Reliability Index Value for Each Profile**

| Profile     | Reliability Index ($\beta$) |
|-------------|----------------------------|
| W 6x6x1/4   | 1.79                       |
| W 6x4x1/4   | 1.77                       |
| W 6x3x1/4   | 1.76                       |
| W 4x4x1/4   | 1.52                       |
| W 4x3x1/4   | 1.50                       |
| W 3x3x1/4   | 1.27                       |

6. CONCLUSIONS & RECOMMENDATIONS

In the present work carried out the reliability index value of the FRP-Pultruded profiles are found. Among the profiles chosen, the reliability index value ranges between 1.28 to 2.33 according to Table 1. This explains that there is an expected probability of failure of one sample among hundreds of samples. It is to be noted that the reliability index value of each sample differs slightly. It clearly specifies that the profiles with decreasing depth has lesser reliability index value. Hence the depth of the section should be more than the optimum depth level to obtain higher reliability index value and less probability of failure.

It can be concluded that among the six samples the most reliable sample is W6X6X1/4 and the least reliable sample is W3X3X1/4. The influence of randomness in cross sectional dimensions of the profiles and the load intensities has lesser impact with the reliability index. This is due to the minimum number of samples chosen due to availability conditions. If large number of samples are chosen when the availability is more and also by predicting random variables for more samples their impact will be shown clearly with high reliability index value.

Based on the present work, it is recommended that for a FRP-Pultruded profile the reliability index value is found to be acceptable. Hence if those pultruded profiles are used in a building as a whole structure like built up section their reliability index will be more than the value obtained for a profile. It may lead to more reliable structures and reduces failure. The reliability analysis can also be performed for steel sections, FRP, GFRP profiles and also for RCC components or a RC building itself. It is very useful in predicting the performance of a component or structure before being practiced at site.

7. REFERENCES

[1] ASCE 7-05 (2003): Minimum Design Loads for Buildings and Other Structures.

[2] ACI 440.1R-05 (2006): Guide for the Design and Construction of Structural Concrete Reinforced with Fiber-Reinforced Polymer (FRP) Bars.

[3] ACI 318-11 (2011): Building Code Requirements for Structural Concrete and Commentary.

[4] A.S. Nowak and K.R. Collins, “Reliability of Structures”, McGraw-Hill, 2000.

[5] Bruce R. Ellingwood, “Toward Load and Resistance Factor Design for Fiber-Reinforced Polymer Composite Structures”, Journal of Structural Engineering, Vol.129, pp. 449-458, 2003.

[6] Cooling technology institute (CTI) code Tower (2009) Standard specifications: Fiberglass Pultruded Structural Products for use in Cooling Towers.

[7] Jeremy McNutt, “Reliability Analysis of FRP Composite Columns”, University of Tennessee Academic Press, Knoxville, 1998.

[8] Linda M. Vanevenhoven, Carol K. Shield and Lawrence C. Bank, “LRFD Factors for Pultruded Wide-Flange Columns”, Journal of Structural Engineering, Vol. 136, pp. 554-564, 2010.

[9] M. Di Sciuva and D. Lomario, “A comparison between Monte Carlo and FORMs in calculating the reliability of a composite structure”, Journal of Composite Structures, Vol. 59, pp. 155–162, 2003.

[10] Peter bjerajer, “On computation methods for structural reliability analysis”, Journal of Structural Safety, Vol. 9, pp. 79-96, 1990.

[11] P.D. Gosling, Faimun, O. Polit, “A high-fidelity first-order reliability analysis for shear deformable laminated composite plates”, Journal of Composite Structures, Vol. 115, pp. 12–28, 2014.

[12] Richard E. Chambers, “ASCE Design Standard for Pultruded Fiber-Reinforced-Plastic (FRP) Structure”, Journal of Composite Construction, Vol. 1, pp. 26-38, 1997.

[13] Sujata Kulkarni, “Calibration of Flexural Design of Concrete Members Reinforced With FRP Bars”, Academic Press, University of Pune, India, 2006.