Simulation of flux penetration into square superconducting networks

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Abstract. Flux penetration into a superconducting film with holes is simulated by considering nonlinear current-voltage dependence for flux creep. The film has a pattern of a square superconducting network (SCN) which has square lattice of square holes. The simulated flux penetration into SCN with the ratio of \( w/a = 0.5 \) shows a pattern extending along the parallel and diagonal direction where \( a \) is lattice constant and \( w \) is line width. The simulated pattern is similar to some of the experimentally observed the flux penetration patterns.

1. Introduction

The motion of vortices in superconductor is governed by the geometry of the sample as well as temperature, magnetic field, and current. The recent rapid development of nano-fabrication technique has encouraged many researchers to study ways to control vortices by artificial potential, such as rectified vortex motion [1]. In superconducting anti-dot array, vortices easily penetrate along a direction between nearest anti-dots, so called vortex channeling model [2]. However, our group has reported an anomalous flux penetration patterns which extend along the diagonal direction of the lattice in square superconducting networks (SCNs) [3, 4, 5]. We have concluded that the most plausible origin of this phenomenon is the concentration of shielding current at the intersections of the SCNs. In addition to the experimental results, a time-dependent Ginzburg-Landau (TDGL) simulation has reproduced the phenomenon [6]. However, in the TDGL simulation, the vortex pinning and demagnetizing effect caused by finite film thickness are completely neglected. In this paper, we simulate the flux penetration into SCNs by taking into account the effect of pinning and the thin film approximation.

2. Model

The simulation is mainly based on the model for multiply-connected superconductors as described in Refs. [7, 8]. Suppose a superconducting thin film with a thickness \( d \) placed in a transverse magnetic field. Shielding current flows especially near sample edges. The vortices starts to penetrate into the film when Lorentz force on a vortex due to the current exceeds the pinning force. In the Bean model, the vortices move only when current density is higher than the critical current density, \( j_c \). In the power-law model, flux creep even with a current density
The Biot-Savart law can be formulated as
\[ \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}_0 \times \mathbf{J}(\mathbf{r}) \]
where \( \mathbf{E} \) is electric field, \( U_0 \) is activation energy, and \( \mathbf{J} \) is sheet current density which is calculated as 
\[ \mathbf{J}(\mathbf{r}) = \int_{-d/2}^{d/2} dz j_z(\mathbf{r}, z) \cong d j_z(\mathbf{r}, 0) \]
with thin film approximation. Since current is conserved, the sheet current density can be described as
\[ \mathbf{J} = \nabla \times \mathbf{g}(\mathbf{r}) \]
where \( \mathbf{g}(\mathbf{r}) \) is the local magnetization [10]. The Biot-Savart law can be formulated as
\[ \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}_0 \times \mathbf{J}(\mathbf{r}) \]
where \( H_0 \) is the applied magnetic field and \( S \) indicates the sample area. The integral kernel \( Q(\mathbf{r}, \mathbf{r}') \) means the magnetic field at \( \mathbf{r} \) made by magnetic dipole at \( \mathbf{r}' \). Thus, \( Q \) in the plane \( z = 0 \) is obtained as
\[ Q(\mathbf{r}, \mathbf{r}') = -1/4\pi |\mathbf{r} - \mathbf{r}'|^3 \]
Note that integral of \( Q(\mathbf{r}, \mathbf{r}') \) over the plane \( z = 0 \) should become zero because the total magnetic flux of a dipole is zero in any plane. When \( Q(\mathbf{r}, \mathbf{r}') \) is discretized on an equidistant grid with grid points \( \mathbf{r} \), and weight \( w \),
\[ Q(\mathbf{r}_i, \mathbf{r}_j) = Q_{ij} = (\delta_{ij} - 1) q_{ij} \left( \sum_{l \neq i} q_{il} w_i + C_i \right) / w_j \]
where \( q_{ij} = 1/4\pi |\mathbf{r}_i - \mathbf{r}_j|^3 \) for \( i \neq j \) and \( q_{ii} = 0 \). \( C \) is an integral of \( -Q \) over outside of the film \( S \) described as
\[ C(\mathbf{r}) = \int_{\bar{S}} 1/4\pi |\mathbf{r} - \mathbf{r}'|^3 d^2 \mathbf{r}' \]
For an infinite strip film filling \( |x| \leq l \), \( C \) can be solved as
\[ C(x) = l/\pi (l^2 - x^2) \]
The time evolution of \( \mathbf{g} \) can be calculated from Eq. (2), hence we get,
\[ \dot{\mathbf{g}}(\mathbf{r}_i) = \sum_j K_{ij} \left[ \frac{\dot{B}(\mathbf{r}_j)}{\mu_0} - \dot{H}_a \right] \]
where \( K_{ij} = (Q_{ij} w_j)^{-1} \) and \( \dot{B}(\mathbf{r}) \) is given by Maxwell equation,
\[ \dot{B}(\mathbf{r}) = - (\nabla \times \mathbf{E}(\mathbf{r}))_z = \nabla \cdot \left( \frac{\rho(\mathbf{r})}{\mu_0} \nabla g(\mathbf{r}) \right) \]
For multiply connected superconductors, Eq. (4) can also be applied. However, \( \mathbf{J}(\mathbf{r}) \) is required to be \( 0 \) in the holes, thus \( g(\mathbf{r}) \) in the holes should become equivalent. There are some approaches to satisfy this requirement: (1) \( g(\mathbf{r}) \) is set to the lowest value of \( g(\mathbf{r}) \) around the holes [11, 12], (2) the area of holes has a large Ohmic resistance and small \( J_c \) [13], and (3) an iteration scheme to adjust flux density in the holes \( B_h \) by Faraday’s law [8] as,
\[ B_h^{(n+1)}(\mathbf{r}) = B_h^{(n)}(\mathbf{r}) - \mu_0 \int_{\text{hole}} d^2 \mathbf{r}' Q(\mathbf{r}, \mathbf{r}') \dot{g}_h^{(n)}(\mathbf{r}') + K^{(n)} \]
where \( K^{(n)} \) is a constant chosen with the condition that \( \dot{B}_h(\mathbf{r}) \) satisfies Faraday’s law,
\[ \int_{\text{hole}} d^2 \mathbf{r} B(\mathbf{r}) = - \oint_{\text{hole edge}} d\mathbf{l} \cdot \mathbf{E} \]
We have used the approach (3) to calculate the time development of \( g(\mathbf{r}) \).

3. Experimental
Nb square SCNs are fabricated as described in Ref. [3]. We use magneto-optical (MO) imaging technique in which Faraday effect in the in-plane magnetic garnet film is used to detect the distribution of flux density. All MO images are differential images which are subtracted by a zero field background image.
Figure 1. A schematic drawing of the sample geometry. The whole sample shape is an infinite strip film with a width $2l$. The simulated area is a part of the strip highlighted with darker color. A slit is placed on a edge. Square holes on a square lattice are distributed with a lattice constant $a$ and line width $w$. 

Figure 2. Simulated flux density map of flux penetration into square SCN from the end of slit with $l = 60 \, \mu m$, $a = 11 \, \mu m$, $w = 6 \, \mu m$, $d = 50 \, nm$, at $H_a = (a) \, 8.7 \times 10^{-6} \, (b) \, 1.1 \times 10^{-5} \, (c) \, 2.1 \times 10^{-5} \, (d) \, 7.6 \times 10^{-8} \, (e) \, 1.2 \times 10^{-7} \, (f) \, 2.0 \times 10^{-7} \, T$ after ZFC, with (a-c) $n = 3$ and $J_c = 5 \times 10^{-2} \, A/m$ and (d-f) $n = 10$ and $J_c = 1.5 \, A/m$. The red line shows flux front.

4. Results and Discussions

The sample geometry is shown in Fig. 1. The whole shape is an infinite strip film with a width of $2l$. Slits are placed on both edges to concentrate magnetic field in it and promote flux penetration from the end of the slit. Square holes on a square lattice are distributed around the slit with a lattice constant $a$ and a line width $w$. In each hole, $g_h(r)$ is flattened by the iteration scheme with Eq. (6) on every step of the time evolution.

Figure 2 shows a field development of simulated flux density map with increasing field after zero-field-cooling (ZFC). The first and second rows have different values of $n$ and $J_c$. The red line shows the flux front which is determined by a boundary between cells in which the total flux is finite and nearly zero. At the initial stage of vortex penetration as in Fig. 2(a) and (d), clear anisotropy is not observed. As the field is increased, vortices penetrate along parallel directions as shown in Figs. 2(b) and (e). Further increase in field results in the flux front extending along both parallel and diagonal directions as shown in Figs. 2(c) and (f). The evolution of the flux front does not depend on $n$ or $J_c$ though the flux penetration pattern changes at different temperatures in the earlier experiment [3]. Besides, the pattern extending along both parallel and diagonal directions are not produced with $w/a = 0.5$ in the TDGL simulation [6]. The patterns may be due to large shielding currents at intersections as shown in Fig. 3.

Figures 3(a) and (b) show the calculated magnitude of the current density $|J|$ and contour plots of $g$ in the same conditions as in Figs. 2 (c) and (f), respectively. The current density at the intersection is enhanced along the diagonal direction between holes in Fig. 3(b) than in Fig. 3(a). The contour plots of $g$, which correspond to current-flow lines, makes steeper turns when $n$ is large as shown in 3(b). With $n = 10$, the current turns steeply enough to consider it as the Bean model ($n = \infty$). Therefore, the shape of flux front does not change at larger $n$.

Figures 4(a) and (b)-(d) show an optical micrograph of the Nb SCN and MO images of flux penetration patterns under adequate magnetic field at 4.2 K, 6 K and 7.5 K, respectively. These MO images have the same length scale and the red rectangles corresponds to the slit.
5. Summary

Flux penetration into a SCN with square lattice of square holes is simulated by considering nonlinear current-voltage dependence for flux creep. The simulated flux penetrations into SCN with the ratio of $w/a = 0.5$ extend both along parallel and diagonal directions. The simulated shape of the flux front does not change much with $n$ although the flux penetration pattern changes at different temperature in experiment. Compared with the experimental results, the simulated pattern is similar to the experimental results at 6 K in the sense that the both diagonal and parallel penetrations are observed. Further simulations of flux penetration at different temperatures, with different $n$, $w/a$ and $J_c$ are required to understand systematically the variation of flux penetration patterns at different temperatures.

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