Method of determination of parameters of scattering proton-nucleus potential from the experimental bremsstrahlung data

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New approach for determination of radius parameters of proton-nucleus potential from analysis of photon bremsstrahlung accompanying scattering of protons off nuclei is presented. On the example of reaction $p + ^{208}\text{Pb}$ at proton incident energies of 140 and 145 MeV sensitivity of the spectra in dependence of the studied parameters is shown, that allows to extract information about the parameters of such a potential.

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Methodology for determining the parameters of the interacting potential between proton and nucleus in scattering of these objects is based on successful agreement between experimental and calculated cross sections. However, theoretical estimations are depended on the chosen model of the proton-nucleus scattering. In particular, description of the tunneling, imposition of boundary conditions, inclusion of more accurate calculating techniques, taking non-local quantum properties (which may not be so small, see [1]) and other effects into account are different in different models. Some variations of all these parameters can lead to the coincident calculated cross-sections, that causes uncertainties in the resulting parameters of the proton-nucleus potential.

In such regards, alternative methods for extraction of new information about the parameters of proton-nucleus potential cause increased interest. This is especially promising in cases when they involve own independent experimental data. It turns out that bremsstrahlung emission accompanying the scattering of proton off nucleus is one such a way. It includes both experimental cross-sections of the emitted photons, and own models.

In spite of naturalness of such an approach, it is difficult to realize (that was noted by some authors, see [2]). This is explained by difficulty in achievement of converging calculation of the bremsstrahlung spectra. Absence of any information in literature about determination of the potential by such a way is some indication also (while history of the bremsstrahlung topic is extremely long). However, the bremsstrahlung emission involves new useful parameters (influencing on the spectra), that indicates on perspective. In this paper we show realization of such a way.

We shall start from generalization of the non-stationary Pauli equation on the system composed from $A + 1$ nucleons, describing scattering of proton off nucleus, where Hamiltonian can be constructed as

\begin{equation}
H = \sum_{i=1}^{A+1} \left\{ \frac{1}{2m_i} \left( \mathbf{p}_i - \frac{z_i e}{c} \mathbf{A}_i \right)^2 - \frac{z_i e \hbar}{2m_i c} \mathbf{\sigma} \cdot \mathbf{rot} \mathbf{A}_i + z_i e \mathbf{A}_i,0 \right\} + V(\mathbf{r}_1 \ldots \mathbf{r}_{A+1}). \tag{1}
\end{equation}

Let us transform to the center-of-masses frame. Introducing coordinate of centers of masses for the nucleus $\mathbf{R}_A = \sum_{j=1}^{A} m_j \mathbf{r}_{A_j}/M$, coordinate of centers of masses of the complete system $\mathbf{R} = (\mathbf{M} \mathbf{R}_A + m_p \mathbf{r}_p)/(M + m_p)$, relative coordinates $\mathbf{\rho}_{A_j} = \mathbf{R}_A - \mathbf{r}_j$ and $\mathbf{r} = \mathbf{r}_p - \mathbf{R}_A$, we obtain new independent variables $\mathbf{R}$, $\mathbf{r}$ and $\mathbf{\rho}_{A_j}$ ($j = 1 \ldots A - 1$)

\begin{equation}
\mathbf{R} = \frac{1}{M + m_p} \left\{ \sum_{j=1}^{A} m_{A_j} \mathbf{r}_{A_j} + m_p \mathbf{r}_p \right\}, \quad \mathbf{r} = \mathbf{r}_p - \frac{1}{M} \sum_{j=1}^{A} m_{A_j} \mathbf{r}_{A_j}, \quad \mathbf{\rho}_{A_j} = \mathbf{r}_{A_j} - \frac{1}{M} \sum_{k=1}^{A} m_{A_k} \mathbf{r}_{A_k}, \tag{2}
\end{equation}

and calculate operators of corresponding momenta

\begin{equation}
\mathbf{p}_p = \frac{m_p}{M + m_p} \mathbf{P} + \mathbf{p}, \quad \mathbf{p}_{A_j} = \frac{m_{A_j}}{M + m_p} \mathbf{P} - \frac{m_{A_j}}{M} \mathbf{p} + \frac{M - m_{A_j}}{M} \mathbf{\tilde{P}}_{A_j} - \frac{m_{A_j}}{M} \sum_{k=1, k \neq j}^{A-1} \mathbf{\tilde{P}}_{A_k}. \tag{3}
\end{equation}

Let us study leading emission operator of the system composed from proton and nucleus in the laboratory frame:

\begin{equation}
\hat{H}_\gamma = -\frac{z_p e}{m_p c} \mathbf{A}(\mathbf{r}_p, t) \hat{\mathbf{p}}_p + \sum_{j=1}^{A} \frac{z_j e}{m_j c} \mathbf{A}(\mathbf{r}_j, t) \hat{\mathbf{p}}_j. \tag{4}
\end{equation}
Here, $A(r_s, t)$ describes emission of photon caused by nucleon with number $s$ ($s = p$ for proton, $s = j$ for nucleons of nucleus). Using its presentation in form (5) of [3], for the emission operator in the center-of-mass frame we obtain:

$$
\hat{H}_\gamma = - e \frac{2\pi \hbar}{w_{ph}} \sum_{\alpha=1,2} \left( - \frac{m_p}{M + m_p} \frac{e^{(\alpha)\ast}}{r} \right) \left( \frac{1}{M + m_p} \sum_{j=1}^{A} e^{-i k \mathbf{r} \mathbf{r}_{z} + \sum_{j=1}^{A} z_{A_j} e^{-i k \mathbf{r} \mathbf{r}_{A_j}} \mathbf{p} + \sum_{j=1}^{A-1} z_{A_j} e^{-i k \mathbf{r} \mathbf{r}_{A_j}} \mathbf{p}}{M + m_p} \right) \left( e^{-i k \mathbf{r} \mathbf{r}_{z} + \sum_{j=1}^{A} z_{A_j} e^{-i k \mathbf{r} \mathbf{r}_{A_j}} \mathbf{p} + \sum_{j=1}^{A-1} z_{A_j} e^{-i k \mathbf{r} \mathbf{r}_{A_j}} \mathbf{p}} \right) \mathbf{P},
$$

(5)

where star denotes complex conjugation. Let us represent the wave function in the form:

$$
\Psi_s(R, r, \rho_{A_1} \cdots \rho_{A_{A-1}}) = e^{i K_s \cdot R} \psi_{nucl,s}(r) \psi_{nucl,s}(\rho_{A_1} \cdots \rho_{A_{A-1}}),
$$

(6)

where $s = i$ or $f$ (indexes $i$ and $f$ denote the initial state (i.e. the state before emission of photon) and the final state (i.e. the state after emission of photon)), $K_s$ is full momentum of the total system, $e^{i K_s \cdot R}$ is wave function describing motion of the full system in laboratory frame, $\psi_{nucl,s}(r)$ is wave function describing relative motion (with tunneling) of proton concerning nucleus, $\psi_{nucl,s}(\rho_{A_1} \cdots \rho_{A_{A-1}})$ is wave function describing internal states of nucleus connecting with relative motion of its nucleons.

Now we shall calculate the matrix element on the basis of leading operator of emission [6]. We study scattering in the center-of-mass frame, neglect by possible response in result of emission and neglect by possible photon emission in result of deformation of nucleus in reaction. In result, we obtain:

$$
\langle f | \hat{H}_\gamma | i \rangle = - \frac{e^2}{\mu} \frac{Z_{eff}}{2 \pi \hbar} \sum_{\alpha=1,2} \left( - \frac{m_p}{M + m_p} \frac{e^{(\alpha)\ast}}{r} \right) \left( \sum_{j=1}^{A} z_{A_j} e^{-i k \mathbf{r} \mathbf{r}_{A_j}} \mathbf{p} + \sum_{j=1}^{A-1} z_{A_j} e^{-i k \mathbf{r} \mathbf{r}_{A_j}} \mathbf{p} \right) \left( e^{-i k \mathbf{r} \mathbf{r}_{z} + \sum_{j=1}^{A} z_{A_j} e^{-i k \mathbf{r} \mathbf{r}_{A_j}} \mathbf{p} + \sum_{j=1}^{A-1} z_{A_j} e^{-i k \mathbf{r} \mathbf{r}_{A_j}} \mathbf{p}} \right) \mathbf{P},
$$

(7)

where we introduce the effective charge of the system composed from proton and nucleus

$$
Z_{eff}(r) = e^{i k \mathbf{r} \mathbf{r}_{z}} \frac{m_p}{M + m_p} \left( \frac{M Z_p(r) - e^{i k \mathbf{r} \mathbf{r}_{A_j}} Z_A(r)}{M + m_p} \right),
$$

(8)

and the charged form-factor of the nucleus as

$$
Z_A(r) = \left( \psi_{nucl,l}(\rho_{A_1} \cdots \rho_{A_{A}}) \sum_{j=1}^{A} z_{A_j} e^{-i k \mathbf{r} \mathbf{r}_{A_j}} \right) \psi_{nucl,l}(\rho_{A_1} \cdots \rho_{A_{A}}).
$$

(9)

Here, $\mu = m_p M/(m_p + M)$ is reduced mass and $Z_{eff}^{(dip)}$ is the dipole effective charge obtained in the first approximation of $\exp(ik \mathbf{r}) \rightarrow 1$ (i.e. at $\mathbf{k} \mathbf{r} \rightarrow 0$).

We shall define wave function $\psi_{nucl,s}(r)$ in form of bilinear combination of eigenfunctions of orbital and spinor subsystems as eq. (12) in Ref. [3] which is characterized by quantum numbers $l$ and $j$ of the orbital and total momenta. In such a case, formalism in Ref. [3] can be applied for determination of the leading matrix element [7] and we do not repeat it in this paper. In result, we obtain the bremsstrahlung probability as

$$
\frac{d^2 \sigma(\varphi_f, \theta_f)}{d \varphi_f d \cos \theta_f} = \frac{Z_{eff}^2 e^2}{2 \pi \hbar} \frac{w \mu E_i}{\mu^2 k_i} \left\{ p(k_i, k_f) \frac{d p}{d \cos \theta_f} \right\} (\sin \theta d \theta f) + c.c.,
$$

(10)

where c. c. is complex conjugation, $p(k_i, k_f)$ is defined as electrical component $p_{el}$ in eqs. (36) and $d p/d \cos \theta_f$ is defined as summation of differential electrical components $dp_{el}^\mu/\sin \theta d \theta_f$ over $\mu$ in eq. (43) in [3].

We calculate these wave functions numerically concerning to the chosen potential of interaction between the proton and the spherically symmetric core. For description of proton-nucleus interaction we use the single-particle potential as $V(r) = v_{c}(r) + v_{N}(r) + v_{so}(r) + v_{l}(r)$, where $v_{c}(r), v_{N}(r), v_{so}(r)$ and $v_{l}(r)$ are Coulomb, nuclear, spin-orbital and centrifugal components having the form (see eqs. (5) in [4]):

$$
v_{N}(r) = - \frac{V_R}{1 + \exp \left( \frac{r - R_R}{a_R} \right)},
$$

$$
v_{c}(r) = \left\{ \begin{array}{ll}
\frac{Z e^2}{2 R_c}, & \text{at } r \geq R_c, \\
\frac{Z e^2}{R_c} \left( 3 - \frac{r^2}{R_c^2} \right), & \text{at } r < R_c.
\end{array} \right.
$$

(11)
We choose the parametrization proposed by Becchetti and Greenlees in [4]:

\[ V_R = 54.0 - 0.32 E + 0.4 Z/A^{1/3} + 24.0 I, \quad V_{so} = 6.2, \]
\[ R_R = r_R A^{1/3}, \quad R_c = r_c A^{1/3}, \quad R_{so} = r_{so} A^{1/3}, \]
\[ r_{so} = 1.01 \text{ fm}, \quad a_R = 0.75 \text{ fm}, \quad a_{so} = 0.75 \text{ fm}. \]  

(12)

Here, \( I = (N - Z)/A \), \( A \) and \( Z \) are mass and proton numbers of the daughter nucleus, \( E \) is incident lab energy, \( V_R \) and \( V_{so} \) are strength of nuclear and spin-orbital components defined in MeV, \( R_c \) and \( R_R \) are Coulomb and nuclear radiiuses of nucleus, \( a_R \) and \( a_{so} \) are diffusion parameters. The criterion function of the theoretical fit is taken to be

\[ \varepsilon = \frac{1}{n_{\text{max}}} \sum_{n=1}^{n_{\text{max}}} \left| \sigma^{(\text{theor})}(E_n) - \sigma^{(\text{exp})}(E_n) \right|, \]

(13)

where \( \sigma^{(\text{theor})}(E_n) \) and \( \sigma^{(\text{exp})}(E_n) \) — theoretical and experimental values of the bremsstrahlung cross-sections for the chosen nucleus at energy \( E_n \), and summation is performed over all values of experimental data. We shall look for the value for \( r_R \) (in the first calculations we shall restrict ourselves by approximation \( r_c = r_R \)) when this error \[ (13) \] is the minimal (for simplicity, we shall call such approach as method of minimization) [8].

\[ p + ^{208}\text{Pb}\]

\[ E_p = 140 \text{ MeV} \]

\[ \theta = 90^{\circ} \]

For analysis we shall choose reaction \( p + ^{208}\text{Pb} \), which was subject of intensive experimental study and discussions [5–7]. In particular, authors of [5, 6] found experimental difference from typical exponential shape of the bremsstrahlung spectrum previously measured by Edington and Rose in [5]. But, we shall put emphasis of development of our tools which should be working and applicable for analysis (i.e. this approach should be able to extract information about radius parameter \( r_R \) of proton-nucleus potential) of even conflicting experimental data. So, we shall choose two experimental data sets [5] and [4, 7] at the corresponding incident proton energies of 140 MeV and 145 MeV. We shall clarify if the calculated spectrum is changed in dependence on variation of parameter \( r_R \) (for simplicity, we shall restrict ourselves by fixing \( r_c = r_R \)). Results of such calculations at 140 MeV are presented in Fig. II (a). From here one can see that the spectra are slowly decreased with decreasing of this parameter. In order to estimate more accurately, which value of this parameter will be the most proper, we shall compare the calculated spectra with experimental data [5] and calculate error by formula \[ (13) \]. Here, we normalize each calculated curve on the same experimental point (for simplicity, we chose experimental data of 643 nb/(sr · MeV) at energy 45 MeV and angle of 90° taken from table 8 in [5], see p. 544). In Next Fig. II (b) one can see presence of a visible minimum in tendency of such errors. This clearly indicates on existence of some stable value for parameter \( r_R \), for which agreement between theory and experiment should be the most appropriate. Results for analysis of the bremsstrahlung in \( p + ^{208}\text{Pb} \) at the incident proton energy 145 MeV are presented in next Fig. II. We obtain sensitivity of the calculated spectra on the parameter \( r_R \) (see figure (a)), and clear minimum in dependence of function \( \varepsilon \) given by \[ (13) \] on \( r_R \) (see figure (b)).
So, we see that our method is working in analysis of different experimental data \[ and \[. in each case it allows to obtain optimal value for \( r_R \), and these two data are enough close to one to other. We find \( r_R = 0.95 \text{ fm} \) in analysis of \[ at \( E_p = 140 \text{ MeV} \) and \( r = 1.17 \text{ fm} \) in analysis of \[ at \( E_p = 145 \text{ MeV} \) (last result is more logical and with better agreement with obtained in \[ from in fitting procedure in scattering, which is \( r_R = 1.17 \text{ fm} \) and \( r_c = 1.22 \text{ fm} \). This confirms ability of idea and approach described above for determination of the parameters of the proton-nucleus potential from analysis of the experimental bremsstrahlung spectra. Results presented above answer on question in \[ on whether there is a sense to put forces and develop potential models (taking into account many nucleons, collective effects) for description of bremsstrahlung in proton-nucleus scattering and nucleus-nucleus collisions: such a way with improvement of accuracy and stability of calculations shall give proper tools for obtaining new detailed information about proton-nucleus interactions and mechanisms of photon emission.

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\[ \text{FIG. 2: (Color online) (a) The calculated bremsstrahlung cross-sections for } p + ^{208}\text{Pb} \text{ at } 145 \text{ MeV at different values of the parameter } r_R \text{ in comparison with experimental data \[ at } \theta = 90^\circ. \text{ Once again we obtain slow sensitivity of the spectra on the parameter } r_R. \text{ One can see that there is visible minimum which indicates on presence of optimal values for } r_R, \text{ at which agreement between theory and experimental data are the highest.} \]

\[ \text{FIG. 2: (b) Estimated errors obtained by method of minimization in dependence on values of the parameter } r_R. \text{ One can see that there is visible minimum which indicates on presence of optimal values for } r_R, \text{ at which agreement between theory and experimental data are the highest.} \]

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