Finiteness of the universe and computation beyond Turing computability

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We clarify the confusion, misunderstanding and misconception that the physical finiteness of the universe, if the universe is indeed finite, would rule out all hypercomputation, the kind of computation that exceeds the Turing computability, while maintaining and defending the validity of Turing computation and the Church-Turing thesis.

Through private communication with some individuals, we have encountered some confusion, misunderstanding and misconception that the physical finiteness of the universe, if the universe is indeed finite, would rule out all hypercomputation, the kind of computation that exceeds the Turing computability. And now this misleading thinking has somehow made its way to formal presentation in [1]. We would like to take this as an opportunity to publicly present our arguments, for the record, against such misconception. For that purpose, we pose below three questions and then give our answer to each one.

Is the universe finite?

We do not know for sure, even though it would not surprise us if the universe is finite. This is an important physics question and will surely be investigated and debated thoroughly in the years to come. However, in encountering this finiteness presumption or any (yet to be confirmed) model of quantum measurement which implies such finiteness in a discussion of hypercomputation, one should keep it in mind that this is only an assumption or an unconfirmed model, and not a fact.

Let us recall that we have explicitly assumed that the universe is infinite in discussing our quantum algorithm for Hilbert’s tenth problem [2]. This assumption is only for the convenience in presenting our algorithm, so that we could avoid the need of introducing unnecessary distractions. However, we have also stated elsewhere in the same paper that it is sufficient to have the dimensions of the underlying Hilbert space finite but unbounded.

Would such finiteness maintain the status quo of the Church-Turing thesis?

No. This is clearly seen by taking the arguments of [1] which lead to the result that Chaitin’s Ω [3] (see also [4]) is not computable because a physically finite universe would allow us to physically compute (by some unspecified means) only a finite number of binary digits of the number (once a programming language has been specified). Such arguments are of course correct but, unfortunately, are also applicable to more ‘normal’ and ‘ordinary’ numbers such as π or e: with finite physical resources, any Turing machine can physically compute only some finite number of binary digits of any real number! In this way, we would have to conclude that π, for example, is noncomputable too! Also, ‘most’ rational numbers would have been classified noncomputable! Clearly, this is too restrictive and not very useful a discussion of computable numbers. In fact, with such restriction, one would not need the concept of effective computation, of recursive functions in general. And neither one would need the thesis of Church Turing at all–let alone hoping that the physical finiteness of the universe would support the thesis itself as wishfully presented in [1]. After all, with finite physical resources one can physically represent, in binary form say, only some large but finite number/integer, whether it is in Turing computation or hypercomputation. Full stop. For any number larger than this physical limit, only abstract mathematical representations can exist.

The point we want to draw attention to here is that such use of physical finiteness of the universe is not in the spirit of even mathematical Turing computation–let alone hypercomputation–and not at all fruitful in the context of mathematical computability.

This leads us to a more useful and relevant question next.

Would such finiteness render all hypercomputation ineffective?

No, in as much as physical finiteness would not render Turing computation ineffective.
Recall that Turing machines are abstract constructs in which finite but unbounded tapes are required for the operation. The tapes can be lengthened as much as necessary during the computation. Parallely similar to the lengths of these tapes in Turing machines are the dimensions of the underlying Hilbert spaces in our quantum adiabatic algorithm for Hilbert’s tenth problem [5]. Given any Diophantine equation, the algorithm looks for the global minimum of the square of the Diophantine polynomial (since knowing this minimum, we can then decide if the equation has a non-negative integer solution—i.e. when and only when this global minimum is zero). It is easily seen that the global minimum for the square of any given Diophantine polynomial has to take place at some \textit{finite} values for the polynomial variables. This fact is also reflected in the \textit{finite} energy of the ground state to be obtained in our quantum adiabatic algorithm. As a result, the dimension of the underlying Hilbert space need be only finite (but sufficiently large). We demonstrate in [6] how to find such sufficiently large dimensions.

The physical finiteness of the universe would of course impose some upper limit on the number of dimensions one can physically realise. But as we know when in a Turing computation the end of a Turing tape has been reached and cannot be lengthened further due to lack of resources, we would also know when the upper dimensions of the computation Hilbert space have been physically arrived at. At that point, the computation has to be abandoned before we can obtain the final result. At no time, however, the physical finiteness of the universe should lead us to the wrong computation result; it simply would not allow us to complete the computation for some group of Diophantine equations.

In summary, the physical finiteness of the universe should not impose any limitations on hypercomputation more than those which it would already impose on Turing computation since, in the end, \textit{all} computation is physical. Because of this indiscrimination, it is logically inconsistent and wrong to use the finiteness arguments to rule out hypercomputation while still maintaining and defending the validity of Turing computation and the Church-Turing thesis. On the other hand, the probable physical finiteness should not and cannot stop us from investigating hyper-computation as it has not deterred us from studying Turing computation (or mathematics in general).

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[5] T.D. Kieu, Quantum adiabatic algorithm for Hilbert’s tenth problem: I. The algorithm, arXiv:quant-ph/0310052 (2003).
[6] T.D. Kieu, Numerical simulations of a quantum algorithm for Hilbert’s tenth problem, in Proceedings of SPIE Vol. 5105 \textit{Quantum Information and Computation}, eds. Eric Donkor, Andrew R. Pirich and Howard E. Brandt, pp. 89-95 (SPIE, Bellingham, WA, 2003).

Notes added on 16 April 2004:
After my posting of this paper, Prof Srikanth has sent me a revised version of \textit{quant-ph/0402128} and pointed out to me that his work concerned with a different kind of finiteness which limits the number of terms allowed in a quantum coherent linear superposition. This finiteness of quantum parallelism, however, is non-standard and constitutes another postulate different from the usual von Neumann postulate of measurement in quantum mechanics.