Singlino-driven Electroweak Baryogenesis in the Next-to-MSSM

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ABSTRACT

We explore a new possibility of electroweak baryogenesis in the next-to-minimal supersymmetric standard model. In this model, a strong first-order electroweak phase transition can be achieved due to the additional singlet Higgs field. The new impact of its superpartner (singlino) on the baryon asymmetry is investigated by employing the closed-time-path formalism. We find that the $CP$ violating source term fueled by the singlino could be large enough to generate the observed baryon asymmetry of the Universe without any conflicts with the current constraints from the non-observation of the Thallium, neutron and Mercury electric dipole moments.

KEYWORDS: Baryon asymmetry of the Universe, Supersymmetry, Higgs, $CP$ violation
The origin of the baryon asymmetry of the Universe (BAU) is one of the longstanding problems in particle physics and cosmology. The BAU is observed from the primordial abundances of the light elements (D, $^3$He, $^4$He, and $^7$Li) induced by the big-bang nucleosynthesis (BBN) and the cosmic microwave background (CMB) data. From these data the baryon-to-entropy ratio is found to be \[ Y_B = \frac{n_B}{s} = \begin{cases} (7.2 - 9.2) \times 10^{-11} & \text{BBN} \\ (8.61 - 9.09) \times 10^{-11} & \text{CMB} \end{cases}. \] (1)

The baryon asymmetry of the Universe must arise after the inflation if it happened, and before the BBN era. As pointed out by Sakharov [2] the following three conditions must be satisfied in order to generate the BAU: (i) baryon-number violation, (ii) $C$ and $CP$ violation, and (iii) the departure from thermal equilibrium. The last condition could be evaded if the $CP$ symmetry is violated. Although the standard model (SM) of particle physics can in principle satisfy all the above three conditions, it turns out that the $CP$ violation coming from the Cabbibo-Kobayashi-Maskawa matrix [3] is too feeble to generate sufficient BAU [4]. Furthermore, the electroweak phase transition (EWPT) in the SM is a smooth crossover for a Higgs boson with a mass above the LEP bound ($\sim 115$ GeV) [5], and therefore renders the condition of departure from the thermal equilibrium infeasible. Thus, physics beyond the SM is indispensable to address this issue.

Supersymmetry (SUSY) is an attractive framework for new physics since it can provide elegant explanations for a number of key questions that cannot be accommodated in the SM [6]. Supersymmetry can stabilize the electroweak-symmetry breaking scale by providing a natural cancellation mechanism of the quadratic divergences thus solving the gauge-hierarchy problem, offer a good candidate for the cold dark matter, enable the gauge-coupling unification, break electroweak symmetry dynamically, and generate sufficient BAU. The minimal realization of SUSY, known as the minimal supersymmetric standard model (MSSM), is very attractive because of its simplicity. It is, however, known to suffer from the so-called $\mu$ problem [7] and the little-hierarchy problem as noted more recently. Moreover, the electroweak baryogenesis (EWBG) scenario [8] in the MSSM is highly restricted by current experimental data [9–11]. Especially, a Higgs boson which has escaped the detection at LEP 2 [12] and/or perhaps weighs around 125 GeV as hinted by the recent ATLAS/CMS results [13] has pushed the MSSM to the edge of the allowed parameter space that can be consistent with a first-order EWPT [11]. This tension, however, could be relaxed if the MSSM is extended [14–20].

In this Letter, we consider the next-to-minimal supersymmetric standard model (NMSSM) [21], in which the $\mu$ problem is solved naturally. It has been shown that a strong first-order EWPT can happen in the NMSSM much more easily than in the MSSM [14,15]. In the presence of the additional gauge-singlet field, the NMSSM Higgs potential contains explicit/spontaneous $CP$ violation even at the tree level [22]. Furthermore, this new ad-
ditional source of $CP$ violation may give rise to sufficient BAU even when the MSSM fails.

The NMSSM superpotential is

$$W_{\text{NMSSM}} = \hat{U}_C \hat{h}_u \hat{Q}_H \hat{H}_u + \hat{D}_C \hat{h}_d \hat{Q}_H \hat{H}_d + \lambda S \hat{H}_u \hat{H}_d + \frac{\kappa}{3} S^3,$$

where $\hat{S}$ denotes the singlet Higgs superfield, $\hat{H}_{u,d}$ are the two $SU(2)_L$ doublet Higgs superfields, and $\hat{Q}, \hat{L}$ and $\hat{U}_C, \hat{D}_C, \hat{E}_C$ are the matter doublet and singlet superfields, respectively, related to the up- and down-type quarks and charged leptons. We note that the superpotential respects an extra discrete $Z_3$ symmetry.

The tree-level Higgs potential includes the terms coming from the soft SUSY-breaking terms:

$$V_{\text{soft}} = m_2^2 H_d^\dagger H_d + m_2^2 H_u^\dagger H_u + m_3^2 |S|^2 + \left( \lambda A \lambda S H_u H_d - \frac{1}{3} \kappa A \kappa S^3 + \text{h.c.} \right),$$

in which $\lambda, \kappa, A_\lambda$, and $A_\kappa$ may contain nontrivial $CP$ phases. After the neutral components of the two Higgs doublets and the singlet develop their vacuum-expectation-values (VEVs), one may have three rephasing-invariant combinations of the $CP$-odd phases in the tree-level potential [22]:

$$\phi'_\lambda - \phi'_\kappa; \quad \phi'_\lambda + \phi_{A_\lambda}; \quad \phi'_\lambda + \phi_{A_\kappa},$$

where $\phi'_\lambda \equiv \phi_\lambda + \theta + \varphi$ and $\phi'_\kappa \equiv \phi_\kappa + 3\varphi$ with $\theta$ and $\varphi$ parameterizing the overall $CP$ phases of the doublet $H_u$ and the singlet $S$, respectively. It turns out that the latter two $CP$ phases are fixed up to a two-fold ambiguity by the two $CP$-odd tadpole conditions, while the difference $\phi'_\lambda - \phi'_\kappa$ remains as an independent physical $CP$ phase. This tree-level $CP$ violation is a distinctive feature of the NMSSM compared to the MSSM.

The stringent constraint on the tree-level $CP$ phase $\phi'_\lambda - \phi'_\kappa$ may come from the non-observation of the electric dipole moments (EDMs) for Thallium [23], the neutron [24], and Mercury ($^{199}$Hg) [25, 26]. The detailed study shows that the maximal $CP$ phase $\phi'_\lambda - \phi'_\kappa \sim 90^\circ$ could still be compatible with the current EDM constraints taking account of the uncertainties in the calculations of the Mercury EDM when the sfermions of the first two generations are heavier than about 300 GeV [27].

In this Letter, we address the question whether sufficient BAU can be generated by the $CP$ phase $\phi'_\lambda - \phi'_\kappa \sim \pm 90^\circ$ when the $CP$ phases appearing in all the other soft SUSY-breaking terms are not large enough for the BAU because of the tight EDM constraints with the SUSY particles within the reach of the LHC. Explicitly, we are taking $\sin(\phi'_\lambda + \phi_{A_f}) = \sin(\phi'_\lambda + \phi_i) = 0$ with $\phi_{A_f}$ and $\phi_i$ denoting the $CP$ phases of the soft trilinear parameters $A_f$ and the three gaugino mass parameters $M_{i=1,2,3}$, respectively.
The scenario we are considering has an intermediate value of $\tan \beta$ with small $v_S \sim v(T = 0) \simeq 246 \text{ GeV}$:

$$
\tan \beta = 5, \quad v_S = 200 \text{ GeV},
$$

$$
|\lambda| = 0.81, \quad |\kappa| = 0.08; \quad |A_\lambda| = 575 \text{ GeV}, \quad |A_\kappa| = 110 \text{ GeV};
$$

$$
\phi'_\lambda - \phi'_\kappa = \pm 90^\circ, \quad \text{sign} \left[ \cos(\phi'_\kappa + \phi_A) \right] = \text{sign} \left[ \cos(\phi'_\lambda + \phi_A) \right] = +1.
$$

(5)

The other parameters are chosen as

$$
M_{\tilde{Q}_3} = 1 \text{ TeV}, M_{\tilde{U}_3} = 150 \text{ GeV} - 1 \text{ TeV}, M_{\tilde{D}_3} = 250 \text{ GeV} - 1 \text{ TeV}; \quad |A_t| = |A_b| = 1 \text{ TeV}.
$$

(6)

We find $M_A \simeq 600 \text{ GeV}$ for the parameters chosen and have taken $M_1 = M_2 = -200 \text{ GeV}$ to fix the neutralino and chargino sectors. We find that a first-order phase transition could occur in some regions of the parameter space of this scenario which is needed for the EWBG. Actually, the diverse patterns of the EWPT in the NMSSM have been investigated in Ref. [15]. Among the patterns the so-called type-B transition opens a new possibility for the strong first-order EWPT. In this type of transitions, the lighter stop could be heavier than the top quark, in contrast to the MSSM EWPT. Instead, the singlet Higgs field plays an essential role. In a typical parameter-space point, during the EWPT the VEVs $(v, v_S)$ change from $(0, 600 \text{ GeV})$ to $(208 \text{ GeV}, 249 \text{ GeV})$ at the critical temperature $T_C = 110 \text{ GeV}$. The dramatic change of $v_S$ is the most important feature of the type-B transition. As discussed in Ref. [15], the relatively small $\kappa$ is required to ensure the local minimum in the $v_S$ direction as in the scenario Eq. (5).

For the calculation of the baryon density, the Closed-Time-Path (CTP) formalism is employed [28, 29] and we closely follow Refs. [29, 30]. The baryon density in the broken phase is given by

$$
n_B = \frac{n_F \Gamma_{ws}}{2} A \left[ r_1 + r_2 \frac{v_w^2}{\Gamma_{sq} D} \left( 1 - \frac{D_q}{D} \right) \right] \frac{2Dq}{v_w} [Dv_w + (2D_q - D) \sqrt{v_w^2 + 4RD_q}] + 4RDq,
$$

(7)

where $n_F = 3$ is the number of fermion families, $\Gamma_{ws} = 6 \kappa \alpha_2^5 T \sim 0.5 \times 10^{-5} T$ with $\kappa \simeq 20$ and $\alpha_2 \simeq 1/30$, and $\Gamma_{sq} = 16\kappa' \alpha_s^4 T$ with $\kappa' = \mathcal{O}(1)$. We are taking the bubble wall velocity $v_w = 0.04$. The relaxation term $\mathcal{R} = \Gamma_{ws} \left[ \frac{3}{4} (1 + n_{sq}/6)^{-1} + \frac{3}{2} \right]$ with $n_{sq}$ for the number of light squark flavors. The diffusion constant is given by

$$
D = \frac{(9k_Qk_T + k_Bk_Q + 4k_Tk_B)D_q + k_H(9k_T + 9k_Q + k_B)D_h}{9k_Qk_T + k_Bk_Q + 4k_Tk_B + k_H(9k_T + 9k_Q + k_B)},
$$

(8)

*For the more precise calculation, the profiles of the bubble wall and the profile-dependent masses and widths should be taken into account when solving the quantum transport equations during the EWPT.*
with \( D_q = 6/T \) and \( D_h = 110/T \) \[^{31}\]. The \( k \) factors are given by the sum \( k_{Q,T,B} = k_{4L,t_R,b_R} + k_{4L,t_R,b_R} \) and \( k_H = k_{H_d} + k_{H_u} + k_{H_d} \) where

\[
k_i = g_i \frac{6}{\pi^2} \int_{m_i/T}^{\infty} dx \frac{e^x}{(e^x - 1)^2} x \sqrt{x^2 - m_i^2/T^2}.
\]  

Note \( g_i = 1 \) for a chiral fermion and a complex scalar. For example, \( g_i(t_L) = g_i(t_L) = 3 \) taking account of the 3 colors, \( g_i(H) = 2 \) for a Higgs doublet, and \( g_i(H) = 2 \) for a Dirac Higgsino. We further note that \( k_i = g_i \) and \( k_i = 2g_i \) for a chiral fermion and a complex scalar, respectively, in the zero-mass limit. The coefficients \( r_{1,2} \) are given by

\[
r_1 = \frac{9k_Qk_T - 5k_Qk_B - 8k_Tk_B}{k_H(9k_Q + 9k_T + k_B)}, \quad r_2 = \frac{k_R^2(5k_Q + 4k_T)(k_Q + 2k_T)}{k_H(9k_Q + 9k_T + k_B)^2}.
\]  

Note that, in the limit of very heavy stops and sbottoms, the sfermion contributions to their corresponding \( k \) factors become negligible and one may have \( k_Q = 6 \) and \( k_T = k_B = 3 \), which leads to vanishing \( r_1 \) to which the baryon density \( n_B \) is directly proportional. We also observe that the coefficient \( r_1 \) vanishes when the stops and sbottoms are all degenerate.

In the calculation of the \( k \) factors, we have taken the thermal masses given in Ref. \[^{32}\] with appropriate modifications. Finally, the parameter \( \mathcal{A} \) is given by

\[
\mathcal{A} \simeq k_H L_w \sqrt{\frac{r_T}{D}} \frac{S_{CPV}^{SH_0}}{\sqrt{\Gamma^\tau_M + \Gamma_h}},
\]  

where \( L_w \) denotes the bubble wall width and \( r_T = \bar{\Gamma}/(\Gamma_M^- + \Gamma_h) \). The \( CP \)-conserving particle number changing rate is

\[
\bar{\Gamma} = \frac{(9k_Q + 9k_T + k_B)(\Gamma_M^- + \Gamma_h)}{9k_Qk_T + k_Bk_Q + 4k_Tk_B + k_H(9k_T + 9k_Q + k_B)},
\]  

where

\[
\Gamma_M^- = \frac{6}{T^2} \left( \Gamma_t^- + \Gamma_\tilde{t}^- \right), \quad \Gamma_h = \frac{6}{T^2} \left( \Gamma_H^- \tilde{W}^\pm + \Gamma_{H^0\tilde{W}^0} + \Gamma_{H^0\tilde{B}^0} + \Gamma_{H^0\tilde{S}} \right),
\]  

with \( \Gamma_H^- \tilde{X} = \Gamma_H \tilde{X} - \Gamma_H \tilde{X} \). We refer to Ref. \[^{29}\] for details of the calculations of the rates \( \Gamma_t^- \) and \( \Gamma_\tilde{t}^- \).

Here we present the analytic expression for the singlino-driven \( CP \)-violating source term appearing in the parameter \( \mathcal{A} \) in Eq. \(^{11}\) as follows:

\[
S_{CPV}^{SH_0} = -2|\lambda|^2 |M_S||\mu_{eff}|v^2\beta \sin(\phi_\lambda - \phi_\kappa) \mathcal{T}_{SH_0}^f,
\]  

where \( |\mu_{eff}| = |\lambda|v_S/\sqrt{2} \) and

\[
|M_S(T)| = \left[ \frac{2|\kappa|^2v_S^2 + \frac{|\lambda|^2 + 2|\kappa|^2}{8}T^2}{T^2} \right]^{1/2}
\]  

\(^{11}\)
including the singlino thermal mass term. We assume that there is no spontaneous CP violation or \( \theta = \varphi = 0 \). We note that the source term vanishes when \( \sin(\phi_\lambda - \phi_\kappa) = 0 \) or \( \dot{\beta} = 0 \). If \( \beta \) has a kink-type profile, \( \dot{\beta} \simeq v_w \Delta \beta / L_w \) and \( A \) becomes independent of \( L_w \). In the MSSM, \( \Delta \beta \) was found to be \( O(10^{-2} - 10^{-3}) \) [10, 33]. We are taking \( \Delta \beta = 0.02 \) in our estimation of the source term \( \hat{T} \). The fermionic source function \( I_{ij} \) takes the generic form of

\[
I_{ij} = \frac{1}{4\pi^2} \int_0^\infty dk \frac{k^2}{\omega_i \omega_j} \left[ (1 - 2\text{Re}(n_j)) I(\omega_i, \Gamma_i, \omega_j, \Gamma_j) + (1 - 2\text{Re}(n_i)) I(\omega_j, \Gamma_j, \omega_i, \Gamma_i) \right. \\
\left. - 2(\text{Im}(n_i) + \text{Im}(n_j)) G(\omega_i, \Gamma_i, \omega_j, \Gamma_j) \right],
\]

(16)

where

\[
n_i \equiv \frac{1}{e^{(\omega_i - i\Gamma_i)/T} + 1}
\]

(17)

with \( \omega_i = \sqrt{k^2 + m_i^2} \). We note that the thermal width \( \Gamma_i \) at finite temperature is given by the imaginary part of its self-energy which is nonvanishing independently of whether the particle \( i \) is stable or not [34]. Specifically, for the calculation of the source function, we have taken the thermal widths given in Ref. [35].

The thermal functions \( I \) and \( G \) are defined by

\[
I(a, b, c, d) = \frac{1}{2} \left[ \frac{1}{[(a + c)^2 + (b + d)^2]} \sin \left[ 2 \text{arctan} \left( \frac{a + c}{b + d} \right) \right] \\
+ \frac{1}{2} \left[ \frac{1}{[(a - c)^2 + (b + d)^2]} \sin \left[ 2 \text{arctan} \left( \frac{a - c}{b + d} \right) \right] \right],
\]

(18)

\[
G(a, b, c, d) = -\frac{1}{2} \left[ \frac{1}{[(a + c)^2 + (b + d)^2]} \cos \left[ 2 \text{arctan} \left( \frac{a + c}{b + d} \right) \right] \\
+ \frac{1}{2} \left[ \frac{1}{[(a - c)^2 + (b + d)^2]} \cos \left[ 2 \text{arctan} \left( \frac{a - c}{b + d} \right) \right] \right].
\]

(19)

We note the thermal functions lead to \( I_{ij} \propto \Gamma_i + \Gamma_j \) when \( m_i \sim m_j, T \gg \Gamma_i \).

In Fig. 1 we show our predictions for the singlino-driven \( Y_B / Y_B^{\text{ob}} \) as functions of \( v_S(T_C) \) taking three values of \( M_{\tilde{D}_3} = 250 \text{ GeV}, 400 \text{ GeV}, \) and \( 1 \text{ TeV} \) with \( Y_B^{\text{ob}} \) being the averaged value of the two BAU data in Eq. (1). Here \( v_S(T_C) \) denotes the singlino VEV at \( T_C \) and it can take on any value between \( 250 \text{ GeV} \) and \( 600 \text{ GeV} \) in the type-B EWPT:

\[
(v, v_S) = (0, 600 \text{ GeV}) \rightarrow (208 \text{ GeV}, 249 \text{ GeV})
\]

(20)

\(^1\)Note the source term grows linearly with \( \Delta \beta \) and our choice is optimal. We observe that the variation depending on the choice of \( \Delta \beta \) is to be regarded as the theoretical uncertainty and the precise determination of \( \Delta \beta \) in the NMSSM is beyond the scope of this Letter.

\(^2\)For the thermal width of the singlino, we are taking \( \Gamma_{\tilde{S}} = 0.03T \) considering the large coupling \( |\lambda| = 0.81 \). We find \( Y_B \) is affected by the amount of about \( (25-35)\% \) as \( \Gamma_{\tilde{S}} \) varies between \( 0.003T \) and \( 0.03T \).
Figure 1: The singlino-driven $Y_B$ as functions of the singlino VEV at the phase transition which, in principle, can take any value between 250 GeV and 600 GeV in the type-B EWPT. We are normalizing our predictions to $Y^{ob}_B = 8.8 \times 10^{-11}$ for three values of $M_{\tilde{D}_3} = 250$ GeV, 400 GeV, and 1 TeV. We fix $M_{\tilde{Q}_3} = M_{\tilde{U}_3} = 1$ TeV.

at $T_C = 110$ GeV [15]. We find that the singlino-Higgsino mass difference becomes smaller as $v_S$ decreases, leading to the larger $Y_B$. The more accurate determination of $Y_B$ requires the knowledge of the profiles of the bubble wall and the treatment of the diffusion equation beyond the formalism developed in Refs. [29,30]. We leave the more precise determination of $Y_B$ in the NMSSM framework for future work [36].

From Fig. 1 we can see that $Y_B$ is much suppressed when $M_{\tilde{D}_3} = 1$ TeV since the ratio $r_1$ is almost vanishing when both the stops and sbottoms are heavy and/or degenerate. However, $Y_B$ grows quickly as $M_{\tilde{D}_3}$ decreases. When $M_{\tilde{D}_3} = 400$ GeV, the ratio $Y_B/Y^{ob}_B$ is larger than 1 in the region $v_S(T_C) \lesssim 440$ GeV. Furthermore, in the case with $M_{\tilde{D}_3} = 250$ GeV, sufficient BAU can be generated via the singlino-driven mechanism, irrespective of the nonlinear dynamics during the EWPT. We found the similar behavior by fixing $M_{\tilde{Q}_3}$ and $M_{\tilde{D}_3}$ and varying $M_{\tilde{U}_3}$, as shown in Fig. 2. We observe the same mass $M_{\tilde{U}_3}$ would give a slightly smaller $Y_B$ compared to the same mass of $M_{\tilde{D}_3}$. Summarizing these results, we conclude that a sizable mass splitting either in the stop sector or the sbottom sector is needed for the successful singlino-driven BAU. In passing, we note that the resonance
Figure 2: The same as in Fig. 4 but varying $M_{\tilde{U}_3}$: $M_{\tilde{U}_3} = 150$ GeV, 250 GeV, 400 GeV and 1 TeV. We fix $M_{\tilde{Q}_3} = M_{\tilde{D}_3} = 1$ TeV.

enhancement of the widths $\Gamma_{\tilde{H}^{\pm}\tilde{W}^{\pm}, \tilde{H}^0\tilde{W}^0, \tilde{H}^0\tilde{B}^0}$ induces the dip around $v_S(T_C) = 350$ GeV where $|M_1| = |M_2| = |\mu_{\text{eff}}|$.

Lastly, the EWBG scenario considered here includes the two light Higgs states well below 100 GeV escaping LEP constraints $^{22}$ and the lightest neutralino of about 45 GeV with the singlino fraction of $\sim 40\%$. The light Higgs bosons and the lightest neutralino deserve further studies in connection with the current LHC Higgs searches and the abundance of dark matter in the Universe.

In this Letter, we have examined a new possibility of a singlino-driven mechanism for the BAU in the NMSSM framework. In contrast to the MSSM, explicit and/or spontaneous $CP$ violation can occur in the NMSSM Higgs potential even at the tree level. We emphasize that this new source of $CP$ violation may solely give rise to the sufficient BAU without any conflicts with the current EDM constraints, as long as there is a sizable mass splitting in the stop and/or the sbottom sectors with the lighter stop and/or sbottom weighing below $\sim 500$ GeV $^{30}$. 
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