$Z'Z', K^+K^-$ and $K^0\bar{K}^0$ boson production with definite helicity amplitudes through $e^+e^-$ collisions in the 3-3-1 models without exotic electric charges

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Abstract

A detailed analysis of $Z'Z', K^+K^-$ and $K^0\bar{K}^0$ pair production in $e^+e^-$ collisions is presented by using helicity amplitudes. The trilinear bosons couplings in the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ models without exotic electric charges are also calculated. We carry out the mentioned analysis for two models, one of them is a one family model which is an $E_6$ subgroup [1] and the other one is a three family model with right handed neutrinos[2, 3]. These models do not contain exotic electric charges. For them, we give explicit formulae and the corresponding numerical estimates of the cross-sections and angular distributions occurred in the processes $e^+e^- \rightarrow Z'Z'$, $e^+e^- \rightarrow K^+K^-$ and $e^+e^- \rightarrow K^0\bar{K}^0$ present in our models. We suppose these processes are invariant under $C$, $P$ and $T$ transformation.
1 Introduction

As a consequence of the existence of neutral currents any successful model requires that $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge group be embedded in a larger gauge group. At the present time an interesting class of models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (hereafter called 3–3–1 models) gauge group has been proposed [1, 2, 3]. There are many versions 3–3–1 models, however all them have seventeen gauge bosons, twelve of them are the usual eight gluons, $W^\pm$, $Z$ and the photon. The remaining five bosons contain a neutral $Z'$ boson and four bosons: charged, double charged, neutral, depending of the 3–3–1 model chosen. To get results consistent with experiments is assumed that the last five bosons are heavy. Theoric results also emphasized this result [1].

The masses obtained there, for these exotic bosons, are at the order of $V$, a vacuum expectation value with $V \gg v \sim 250$ GeV, the electroweak breaking scale. The article [2], for example, provides us a lower limit $V \geq 1.5$ TeV.

We require decisive and clear test about these extensions. The existence and properties of new gauge bosons, Higgs bosons, exotics fermions, possible signatures of $C$, $P$ and $T$ violation and so forth, play a central role in these models. $e^-e^+$ collisions appears as one of the best tools for studying the production of each of these particles and its properties. The physical interest of this collisions is because the neutral character of the initial state allows in each case a complete transformation of the energy into new particles, which is no possible in processes like $e^-e^-$ or $e^+e^+$ collisions.

The main purpose of this work is to study $Z^0$, $K^\pm$ and $K^0$ boson production processes through $e^-e^+$ collisions associated to two special $SU(3)_L \otimes U(1)_X$ models without exotic electric charges [2, 3]. Specifically, we refer models $A$ and $D$ as they are called in paper [3]. There, model $A$ is a one family model which has been partially analyzed in Ref.[1] and the three family model $D$ has been analyzed in the literature in Ref.[3]. We add to these studies, our original additional phenomenology analysis such as the trilinear gauge boson couplings, the charged and neutral currents, the helicity amplitudes as well the angular dependence of the density matrix and the cross sections for the processes $e^-e^+ \rightarrow Z^0Z^0$, $e^-e^+ \rightarrow K^-K^+$ and $e^-e^+ \rightarrow K^0\bar{K}^0$ present in our models. We believe that these amplitudes will be helpful in extracting the physical consequences from the measurements of the processes.

For similar works to this you can see, for example, paper [3]. But, in this article, the authors used models with exotic electric charges, they do not consider polarized collision beams for the initial and final particles, which is important to check the validity of the special constraints on the $\gamma KK$, $Z'KK$ and $Z'\bar{K}K$ vertices imposed by gauge theories, and also do not calculate the total cross section as function of energy collision.

This paper is organized as follows. In the section 2 we make a remembrance of the main mathematical topics previously studied in the literature for $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ models without exotic electric charges, which allows us to find the trilinear gauge boson couplings, the neutral and charged currents for the models $A$ and $D$ previously defined. Section 3 will be dedicated to present details of our models, specially their leptonic content, currents and trilinear gauge boson couplings. In the section 4 the helicity amplitudes in general form are found. In the section 5 the helicity amplitudes for the processes $e^-e^+ \rightarrow Z^0Z^0$, $e^-e^+ \rightarrow K^-K^+$, $e^-e^+ \rightarrow K^0\bar{K}^0$ for the mentioned models are given, we also check the unitarity for these amplitudes. In the last section we present our conclusions.

2 Main topics for 3-3-1 models

2.1 The gauge boson sector

As we do not consider exotic electric charges in our models to study, the parameter $b = 1/2$ [2], and thus the covariant derivate takes the form: Rep 3

$$D_\mu = I_{3\times3} \partial_\mu - i \frac{1}{2} \begin{pmatrix} D^1_\mu + 2g XB_\mu & g\sqrt{2}W^+_\mu & g\sqrt{2}K^{+}_\mu \\ g\sqrt{2}W^-_\mu & D^2_\mu + 2g XB_\mu & g\sqrt{2}K^{0}_\mu \\ g\sqrt{2}K^-_\mu & g\sqrt{2}K^{0}_\mu & D^3_\mu + 2g XB_\mu \end{pmatrix}. \tag{1}$$
Inverting expressions (4) and (5), to express the original no physical gauge fields in terms of the final physical
φ in general, it corresponds to a small quantity, omitted in all almost cases. So is a good approximation to put

\[
\phi = 0 \quad \text{(no mixing)} \quad \text{[7]},
\]

and then

\[
A_\mu^3 = g(A_\mu^3 + A_\mu^8), \quad D_\mu^2 = g(-A_\mu^3 + A_\mu^8) \quad \text{and} \quad D_\mu^3 = g(-2A_\mu^8).
\]

After breaking the gauge symmetry through Higgs mechanism we find the following fields:

- The physical charged gauge bosons given by:

\[
W^{\pm}_\mu = \frac{A^{1}_\mu \mp iA^{2}_\mu}{\sqrt{2}}, \quad K^{\pm}_\mu = \frac{A^{4}_\mu \mp iA^{5}_\mu}{\sqrt{2}}, \quad K^0_\mu (K^0_\mu) = \frac{A^{6}_\mu \mp iA^{7}_\mu}{\sqrt{2}}. \quad \text{(3)}
\]

- The massless photon field

\[
A_\mu = S_w A^{3}_\mu + C_w \left[ \frac{T_w}{\sqrt{3}} A^{8}_\mu + \left( 1 - \frac{T_w^2}{3} \right) B_\mu \right]. \quad \text{(4)}
\]

- And two neutral gauge bosons \(Z^0\) and \(Z'^0\) given by:

\[
Z_\mu = C_w A^{3}_\mu - S_w \left[ \frac{T_w}{\sqrt{3}} A^{8}_\mu + \left( 1 - \frac{T_w^2}{3} \right) B_\mu \right], \quad \text{(5)}
\]

\[
Z'_\mu = T_w B_\mu - \left( 1 - \frac{T_w^2}{3} \right) \frac{1}{2} A^{8}_\mu.
\]

where \(S_W\) and \(C_W\) are the sine and cosine of the electroweak mixing angle respectively (similar way for \(T_W = S_W/C_W\) defined by \(S_W = \sqrt{3g'/\sqrt{3g^2 + 4g'^2}}\), and \(g,g'\) are the structure constants for the symmetry groups \(SU(3)_L\) and \(U(1)_X\) respectively.

In reality the bosons defined in \([3]\) are not eigenstates of the mass matrix mixing the no-physical neutral
gauge bosons after symmetry breaking is applied. The true neutral vector bosons are \(Z^1_\mu\) and \(Z^2_\mu\) which are related with \(Z\) and \(Z'\) through a rotation model dependent angle \(\phi\) as follows:

\[
Z^1_\mu = Z^\mu \cos \phi + Z'^\mu \sin \phi,
\]

\[
Z^2_\mu = -Z^\mu \sin \phi + Z'^\mu \cos \phi. \quad \text{(6)}
\]

Although the mixing angle \(\phi\) depends of the scalar fields chosen and the way as the symmetry breaking occur,
in general, it corresponds to a small quantity, omitted in all almost cases. So is a good approximation to put
\(\phi = 0\) (no mixing) \([7]\), and then

\[
Z^1_\mu = Z^\mu,
\]

and

\[
Z^2_\mu = Z'^\mu.
\]

Inverting expressions \([4]\) and \([5]\), to express the original no physical gauge fields in terms of the final physical
gauge fields useful for computations below, we have

\[
A_\mu^3 = S_w A_\mu + C_w Z_\mu,
\]

\[
A_\mu^8 = \frac{1}{\sqrt{3}}(S_w A_\mu - \frac{S_w^2}{C_w} Z_\mu - \frac{a}{C_w} Z'_\mu), \quad \text{(7)}
\]

\[
B_\mu = \frac{1}{\sqrt{3}}(\frac{1}{C_w} Z'_\mu + a A_\mu^8).
\]
Also, it is convenient to write $Z'$ in terms of the new constants to give:

$$Z'_\mu = \frac{1}{\sqrt{3}C_w}(-a A^\mu + S_w B^\mu),$$

with $a = \sqrt{3 - 4S^2_w}$. Also, in the next section we need to use the diagonal components of matrices $[1]$ and $[2]$, for that the following identities:

Rep 3*

\begin{align*}
gD_{1\mu} - 2g' X B^\mu &= g\{2S_w A_\mu(2/3 - X) + \frac{Z^\mu}{C_w}(C^2_w - S^2_w[1/3 - 2X]) - \frac{Z'_\mu}{aC_w}(6XC^2_w + a^2[1/3 - 2X])\}, \\
gD_{2\mu} - 2g' X B^\mu &= -g\{S_w A_\mu(1/3 + X) - \frac{Z^\mu}{C_w}(C^2_w + S^2_w[1/3 - 2X]) - \frac{Z'_\mu}{aC_w}(6XC^2_w - a^2[1/3 - 2X])\}, \\
gD_{3\mu} - 2g' X B^\mu &= -2g\{S_w A_\mu(1/3 + X) - \frac{S^2_w}{C_w}Z_\mu(1/3 + X) + \frac{Z'_\mu}{aC_w}(3XC^2_w - a^2[1/3 + X])\}.
\end{align*}

(8)

Rep 3

\begin{align*}
gD_{1\mu} + 2g' X B^\mu &= g\{2S_w A_\mu(2/3 + X) + \frac{Z^\mu}{C_w}(C^2_w - S^2_w[1/3 + 2X]) + \frac{Z'_\mu}{aC_w}(6XC^2_w - a^2[1/3 + 2X])\}, \\
gD_{2\mu} + 2g' X B^\mu &= g\{2S_w A_\mu(-1/3 + X) - \frac{Z^\mu}{C_w}(C^2_w + S^2_w[1/3 + 2X]) + \frac{Z'_\mu}{aC_w}(6XC^2_w - a^2[1/3 + 2X])\}, \\
D_{3\mu} + 2g' X B^\mu &= 2g\{S_w A_\mu(-1/3 + X) - \frac{S^2_w}{C_w}Z_\mu(-\frac{1}{3} + X) + \frac{Z'_\mu}{aC_w}(3XC^2_w - a^2[-1/3 + X])\}.
\end{align*}

(9)

U(1)$_X$ abelian group,

$g' X B^\mu = X\{S_w A_\mu - \frac{S^2_w}{C_w}Z_\mu + \frac{S^2_w}{aC_w}Z'_\mu\},$

the hypercharge $X$ will take a value according to the model to study.

2.2 Trilinear gauge bosons couplings

The 3-3-1 gauge theory makes especially predictions for the gauge boson interactions, the reason is that in any gauge theory the three gauge boson-interactions should obey C, P and T invariance (to lowest order), and have definite values for the magnetic dipole and electric quadrupole moments. We can find these couplings by using self-interactions kinetic Lagrangian for gauge fields:

$$\mathcal{L} = -gf_{abc}\partial_\mu A^a_\mu A^{b\mu} A^{c\nu} \quad a, b, c = 1, 2..., 8.$$
Expressing each $A^a$ in terms of the physical fields, see (7), we find:

$$
\mathcal{L} = ie\{[A^\mu (W^\mu - W^\mu) + A^\mu W^\mu W^\mu] +
+ T_w^{-1}[Z^\mu (W^\mu W^\mu) - W^\mu W^\mu] + Z^\mu W^\mu W^\mu] +
+ 1[A^\mu (K^\mu - K^\mu - K^\mu K^\mu) + K^\mu K^\mu K^\mu] +
+ T_w^{-1}[Z^\mu (K^\mu - K^\mu - K^\mu K^\mu) + Z^\mu K^\mu K^\mu] +
- \frac{a}{S_{2w}}[Z^\mu (K^\mu - K^\mu - K^\mu K^\mu) + Z^\mu K^\mu K^\mu] +
- S_{2w}^{-1}[Z^\mu (K^\mu - K^\mu - K^\mu K^\mu) + Z^\mu K^\mu K^\mu] +
+ \frac{1}{\sqrt{2}S_{2w}}[K^\mu (K^\mu W^\mu - W^\mu K^\mu) + K^\mu W^\mu K^\mu] +
+ \frac{1}{\sqrt{2}S_{2w}}[K^\mu (K^\mu W^\mu - W^\mu K^\mu) + K^\mu W^\mu K^\mu]
$$

where any $X_{\mu\nu} = \partial_\mu X_{\nu} - \partial_\nu X_{\mu}$. Until now, taking account that the mixing angle $\phi$ be negligible (6), the results obtained are independent of the model without exotic electric charges chosen. Next, we treat two models.

3 Models

3.1 Model A (only leptons) as an $E_6$ subgroup

This is a one family model where the anomaly cancellations occur independently in each family particle content, widely studied in the literature [1]. The multiplet structure for this model is given by:

$$
\psi_L = \begin{pmatrix} e^- \\ \nu_e \\ N_1^L \end{pmatrix}_L \quad \psi_{1L} = \begin{pmatrix} E^- \\ N_2^L \end{pmatrix}_L \quad \psi_{2L} = \begin{pmatrix} N_3^L \\ e^+ \end{pmatrix}_L
$$

(1, 3, -1/3) \quad (1, 3, -1/3) \quad (1, 3, 2/3)

where the numbers inside the parenthesis refers to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ quantum numbers. For our analysis below, the exotic particles $E^\pm, N^\pm_1, (i = 1, 2, 3, 4)$, will be heavy particles with masses approximate of $\approx 1$ TeV. It because its exoticity and value orders handled actually in experiments. By using the former identities (3) and (9) we find the following currents (in $e$ units):

Charged currents:

$$
\mathcal{L}^{CC} = -\frac{1}{\sqrt{2}S_w}(W^\mu (\bar{\nu}_L \gamma^\mu e_L^- + \bar{N}_{2L}^0 \gamma^\mu E_L^- + E_L^+ \gamma^\mu N_{2L}^0) +
+ K^+(\bar{N}_{1L}^0 \gamma^\mu e_L^- + \bar{e}_L^+ \gamma^\mu N_{4L}^0 + \bar{N}_{3L}^0 \gamma^\mu E_L^-) +
+ K^+(\bar{e}_L^+ \gamma^\mu E_L^+ + \bar{N}_{1L}^0 \gamma^\mu \nu e_L + \bar{N}_{2L}^0 \gamma^\mu N_{2L}^0 + H.c)).
$$

Neutral currents:

$$
\mathcal{L}^{NC} = A_\mu(-\bar{e}^- \gamma^\mu e^- - \bar{E}^- \gamma^\mu E^-) +
+ Z_\mu(T_w^{-1}(-\bar{e}_L^- \gamma^\mu e_L^- - \bar{E}_L^- \gamma^\mu E_L^- - \bar{E}_R^- \gamma^\mu E_R^-) +
+ S_{2w}^{-1}(2S_{2w}^{-1} \bar{e}_L^- \gamma^\mu e_L^- + \bar{e}_L^+ \gamma^\mu \nu e_L + \bar{N}_{2L}^0 \gamma^\mu N_{2L}^0 - \bar{N}_{4L}^0 \gamma^\mu N_{4L}^0)) +
+ \frac{Z'}{\alpha}(T_w^{-1}(\bar{e}_L^- \gamma^\mu e_L^- + \bar{E}_L^- \gamma^\mu E_L^- + \bar{E}_R^- \gamma^\mu E_R^- + \bar{e}_L^+ \gamma^\mu \nu e_L + 2\bar{e}_L^- \gamma^\mu e_L^-) +
+ S_{2w}^{-1}(\bar{N}_{1L}^0 \gamma^\mu N_{1L}^0 + \bar{N}_{3L}^0 \gamma^\mu N_{3L}^0) + S_{2w}^{-1}(\bar{N}_{4L}^0 \gamma^\mu N_{3L}^0 - \bar{E}_R^- \gamma^\mu E_R^-)).
$$
3.2 Model D (only leptons) with right handed neutrinos

Also, this model has had a special attention in the literature \[5\]. But here the anomaly cancellations occur among families. The multiplet structure for this model is:

\[
\psi_\alpha_L = \begin{pmatrix}
\alpha^-

\mu_\alpha

N^0_\alpha
\end{pmatrix}_L
\]

\((1,3^*, -1/3) \ (1, 1, 1)\)

for \(\alpha = e, \mu, \tau\). The exotic particles \(N_\alpha\), considered to be the right handed neutrinos, we assume they have masses of order of 1 TeV. The currents are:

\[
\mathcal{L}^{CC} = \frac{1}{\sqrt{2}S_w} \{ W^+_\mu \bar{\nu}_\alpha L \gamma^\mu \alpha_L + K^+_\mu \bar{N}^0_\alpha L \gamma^\mu \alpha_L + K^0_\mu \bar{N}^0_\alpha L \gamma^\mu \nu_{\alpha L} + H.c \},
\]

\[
\mathcal{L}^{NC} = A_\mu \{ -\bar{\alpha}^- \gamma^\mu \alpha^- \} +
+ Z_\mu \{ S^{-1}_{2w} \bar{\nu}_\alpha L \gamma^\mu \nu_{\alpha L} - T^{-1}_{2w} \bar{\alpha}^- L \gamma^\mu \alpha^- L + T_w \bar{\alpha}_R \gamma^\mu \alpha_R \} +
+ \frac{Z'_\mu}{a} ( T^{-1}_{2w} (\bar{\nu}_\alpha L \gamma^\mu \nu_{\alpha L} + \bar{\alpha}_L^- \gamma^\mu \alpha_L^-) + \frac{T^{-1}_w}{2} \bar{N}^0_\alpha L \gamma^\mu \lambda_\alpha L )) \}.
\]

Finally, with the help of results above, we make a summary of the couplings constants:
Table 1

| Trilinear gauge bosons couplings vertex | coupling/e |
|----------------------------------------|------------|
| $\gamma W^+ W^-$                         | $1$        |
| $ZW^+ W^-$                              | $t_w^{-1}$ |
| $\gamma K^+ K^+$                        | $1$        |
| $ZK^+ K^-$                              | $t_w^{-1}$ |
| $Z' K^+ K^-$                            | $-\sqrt{3} - 4 s_w^2 / s_w$ |
| $ZK^0 K''$                              | $-1 / s_w$ |
| $Z' K^0 K''$                            | $-\sqrt{3} - 4 s_w^2 / s_w$ |
| $K^- \nu K^0 W^+$                       | $1 / \sqrt{2} s_w$ |
| $K^+ \nu K^0 W^-$                       | $1 / \sqrt{2} s_w$ |

Table 2

| Couplings $Z(\bar{Z}')-\epsilon_{L(R)}$ vertex | Model A | Model C | Model D |
|-----------------------------------------------|---------|---------|---------|
| $Ze_L^-$                                      | $-T_w^{-1}$ | $-T_w^{-1}$ | $-T_w^{-1}$ |
| $Ze_R^-$                                      | $T_w$ | $T_w$ | $T_w$ |
| $Z' e_L^-$                                    | $T_w^{-1}$ | $-S_w^{-1}$ | $T_w^{-1}$ |
| $Z' e_R^-$                                    | $-T_w^{-1}$ | $T_w$ | $-T_w^{-1}$ |

4 Helicity Amplitudes

The reactions

$$e^+(p') e^- (p) \rightarrow V(k_+, \lambda_1) \bar{V}(k_-, \lambda_2)$$

serve as "theoretical laboratories" for to study these models. For the following processes we have used the next conventions: The momenta of the incoming $e^-$ and $e^+$ with helicities $L$ and $R$ are denoted by $p$ and $p'$; The outgoing $V$ bosons with helicities $\epsilon_+$ and $\epsilon_-$ are associated with momenta $k_+$ and $k_-$. As the threshold of these processes are sufficiently high, we neglect the electron mass in comparation with the beam energy. In this limit, the momenta in c.m. (center of mass) system become:

$$p^\mu = (E, 0, 0, E) \quad k_-^\mu = (E, |k| \sin \theta, 0, |k| \cos \theta)$$

$$p'^\mu = (E, 0, 0, -E) \quad k_+^\mu = (E, -|k| \sin \theta, 0, -|k| \cos \theta)$$

where $E$ is the beam energy, $|k_+| = |k_-| \equiv |k|$ and $\theta$ is the angle between $p$ and $k_-$. The helicity wave functions are given by:

$$\epsilon_1^\mu = (0, \cos \theta, 0, -\sin \theta) \quad \epsilon_1'^\mu = (0, -\cos \theta, 0, \sin \theta)$$

$$\epsilon_2^\mu = (0, 0, 1, 0) \quad \epsilon_2'^\mu = (0, 0, 1, 0)$$

$$\epsilon_3^\mu = \frac{1}{M_V} (|k|, E \sin \theta, 0, E \cos \theta) \quad \epsilon_3'^\mu = \frac{1}{M_V} (|k|, -E \sin \theta, 0, -E \cos \theta)$$

For the calculation of helicity amplitudes we have used the technique described in Ref. [5]. The explicit forms for these amplitudes are:
Helicity amplitudes $A_{\pm \pm}^{L(R)t}$ for the $t$ channel

$$
A_{11}^{L(t)} = -2s \sin \theta (2 \cos \theta - \beta) \\
A_{12}^{L(t)} = -2is \sin \theta \\
A_{21}^{L(t)} = 2is \sin \theta \\
A_{22}^{L(t)} = -2s \beta \sin \theta \\
A_{13}^{L(t)} = \frac{2}{M} s^2 \left[ (\beta - \cos \theta) \cos \theta + \frac{2M^2}{s} \right] \\
A_{31}^{L(t)} = -\frac{2}{M} s^2 \left[ (\beta - \cos \theta) \cos \theta + \frac{2M^2}{s} \right] \\
A_{23}^{L(t)} = \frac{2i}{M} s^2 \left[ (\beta - \cos \theta) + \frac{2M^2}{s} \cos \theta \right] \\
A_{32}^{L(t)} = -\frac{2i}{M} s^2 \left[ (\beta - \cos \theta) + \frac{2M^2}{s} \cos \theta \right] \\
A_{33}^{L(t)} = -2s \sin \theta \left[ \frac{s}{2M^2} (\beta - \cos \theta) + \beta \right]
$$

where $M$ and $\beta = \frac{|\vec{v}_{\perp}|}{E} = \frac{|\vec{v}_{\perp}^*|}{E} = \sqrt{1 - 4M^2/s}$ are the mass and the velocity of boson respectively, and $s = (p + p')^2 = (k_+ + k_+)^2$. The other Mandelstam variables are:

$$
t = M^2 - \frac{s}{2} (1 - \beta \cos \theta) \\
u = M^2 - \frac{s}{2} (1 + \beta \cos \theta)
$$

Helicity Amplitudes $A_{\pm \pm}^{L(R)u}$ for the $u$ channel

$$
A_{11}^{L(u)} = -2s \sin \theta (2 \cos \theta + \beta) \\
A_{12}^{L(u)} = -2is \sin \theta \\
A_{21}^{L(u)} = 2is \sin \theta \\
A_{22}^{L(u)} = 2s \beta \sin \theta \\
A_{13}^{L(u)} = \frac{2}{M} s^2 \left[ (\beta + \cos \theta) \cos \theta - \frac{2M^2}{s} \right] \\
A_{31}^{L(u)} = \frac{2}{M} s^2 \left[ (\beta + \cos \theta) \cos \theta - \frac{2M^2}{s} \right] \\
A_{23}^{L(u)} = \frac{2i}{M} s^2 \left[ (\beta + \cos \theta) + \frac{2M^2}{s} \cos \theta \right] \\
A_{32}^{L(u)} = \frac{2i}{M} s^2 \left[ (\beta + \cos \theta) + \frac{2M^2}{s} \cos \theta \right] \\
A_{33}^{L(u)} = 2s \sin \theta \left[ \frac{s}{2M^2} (\beta + \cos \theta) + \beta \right]
$$

$$
A_{11}^{R(u)} = -A_{11}^{L(u)} \\
A_{12}^{R(u)} = A_{12}^{L(u)} \\
A_{21}^{R(u)} = A_{21}^{L(u)} \\
A_{22}^{R(u)} = -A_{22}^{L(u)} \\
A_{13}^{R(u)} = -A_{13}^{L(u)} \\
A_{31}^{R(u)} = -A_{31}^{L(u)} \\
A_{23}^{R(u)} = A_{23}^{L(u)} \\
A_{32}^{R(u)} = A_{32}^{L(u)} \\
A_{33}^{R(u)} = -A_{33}^{L(u)}
$$
Helicity Amplitudes $C_{\epsilon - \epsilon'}^L(R)$ for the $s$ channel

\[
\begin{align*}
C_{11}^L &= -2s\beta \sin \theta d_2 \\
C_{12}^L &= 2s \sin \theta d_5 \\
C_{13}^L &= \frac{s^{3/2}}{M} [id_4 + \beta \cos \theta d_3] \\
C_{21}^L &= 2s \sin \theta d_5 \\
C_{22}^L &= 2s\beta \sin \theta d_2 \\
C_{23}^L &= \frac{s^{3/2}}{M} [i\beta d_3 - \cos \theta d_4] \\
C_{31}^L &= C_{13}^L \\
C_{32}^L &= C_{23}^L \\
C_{33}^L &= 2s\beta \sin \theta d_1
\end{align*}
\]

where:

\[
\begin{align*}
d_1 &= 1 + \frac{s}{2M^2}\kappa \\
d_2 &= 1 + \frac{s}{2M^2}\lambda \\
d_3 &= 1 + \kappa + \lambda + if_4 \\
d_5 &= 4\beta^2 s f_7 + f_6
\end{align*}
\]

Independently of the gauge group considered, they demand a separate conservation of $C$, $P$ and $T$ as well as specific values for the magnetic dipole and electric quadrupole parameters. Explicitly, gauge theories demand:

\[
\begin{align*}
f_i &= 0 \quad (i = 4, ..., 9) \\
\kappa_{\gamma} &= \kappa_{Z} = 1 \\
\lambda_{\gamma} &= \lambda_{Z} = 0
\end{align*}
\]

irrespective of the gauge group considered.

## 5 Processes for boson productions at high energy colliders

The production of bosons in the 3 - 3 - 1 model without exotic electric charges is particularly relevant for colliders in the TeV range since the models predict new gauge bosons at the same scale.

For unpolarized $e^+e^-$ beam and averaging over the initial beam we have the differential cross sections in terms of amplitude polarizations:

\[
\left. \frac{d\sigma}{d\cos \theta} \right|_{\epsilon - \epsilon'} = C \frac{\beta}{s} |\mathcal{M}_{\epsilon - \epsilon'}|^2. \tag{10}
\]

Summing all polarizations of the final state particles, the total cross section is:

\[
\left. \frac{\partial\sigma}{\partial\cos \theta} \right|_T = \sum_{\epsilon - \epsilon'} \left. \frac{\partial\sigma}{\partial\cos \theta} \right|_{\epsilon - \epsilon'}, \tag{11}
\]

where $C$ is a process dependent constant.

We assume for all cases that the weak-mixing angle takes a value $\sin^2 W = 0.25$. For the boson productions $Z^0$, $K^\pm$ and $K^0(K^0)$ and when its masses be considered fixed in the processes their values are of order of 1 TeV.

The methodology that we carry out, in this section, for study the processes will be the following:
• First we give the model,
• then we study each processes, specifically the productions $Z^0, K^\pm$ and \( K'^0(\bar{K}'^0) \) in $e^+e^-$ collisions,
• we calculate the amplitude dispersion for each processes, and verify its unitarity,
• we plot cross section for each process from several points of view:

- fixing energy collision and taking each polarization of the initial and final particles to see the differential cross section at different angles: for that we use the differential cross section \( (10) \) and the amplitude formula for each processes giving ahead. We calculate the different differential cross sections with definite polarization of the initial and final states particles as a function of the c.m scattering angle $\theta$ at a specific energy collision.

There are two state polarizations of the initial particles: $e^- e^+$ and $e^- e^+$. While the final boson particles possessing nine different polarizations, from which, due the symmetry, only six polarizations 11,12,13,21,22,23 and 33 are sufficient for our study.

- or fixing energy, but now the mass is the variable, to plot the total cross section: we take two scale energy collision $\sqrt{s} = 3$ TeV and $\sqrt{s} = 5$ TeV. Thus, the mass boson $Z^0$ will be spanned between 0.5 TeV and 1.5 TeV or 0.5 TeV and 2.5 TeV respectively.

- or finally, plotting the total cross section at different energy collision, maintaining fixes the mass: in the graphs the collision energy goes to a maximum value of 5 TeV.

5.1 Model A as an $E_6$ subgroup
For this model the fine-structure constant has a value $\alpha = 1/124.6$. Now we study each processes.

5.1.1 $e^+e^- \rightarrow Z^0Z^0$
Using Feynman diagrams for this process we compute the amplitude as function of polarization at initial and final particles. The helicities are given in section (4), its masses are of that particle produced in the collision. In this case the mass of neutral gauge boson $Z^0$. The helicity amplitudes for this process are:

$$|\mathcal{M}_{\epsilon e^+}|^2 = \frac{1}{t} \left( A^{L(t)}_{\epsilon e^+} + 4 A^{R(t)}_{\epsilon e^+} \right) + \frac{1}{u} \left( A^{L(u)}_{\epsilon e^+} + 4 A^{R(u)}_{\epsilon e^+} \right)^2. \quad (12)$$

With

$$C = \frac{\alpha}{2} \left( \frac{T_{2w}^2}{4 (3 - 45 s^2_w)} \right)^2$$

Unitarity
The unitarity guarantee us that the total cross section converge to a constant finite value at high energy collision. By using the identity:

$$\sum_{\epsilon e^+} (A^{L(t)}_{\epsilon e^+} + A^{R(t)}_{\epsilon e^+}) = \sum_{\epsilon e^+} (A^{L(u)}_{\epsilon e^+} + A^{R(u)}_{\epsilon e^+}) = 0$$

we get:

$$|\mathcal{M}_{\epsilon e^+}|^2 = 9 \left| \frac{1}{t} A^{R(t)}_{\epsilon e^+} + \frac{1}{u} A^{R(u)}_{\epsilon e^+} \right|^2$$

When $s \rightarrow \infty$ we have $\beta \rightarrow 1, t \rightarrow -s/2(1-\cos \theta), u \rightarrow -s/2(1+\cos \theta)$. The structure of the helicity amplitudes $A^{R(t)}_{33}$ and $A^{R(u)}_{33}$ could have trouble because they are of the form:

$$A^{R(t)}_{33} = s^2 \sin \theta \frac{\theta}{M^2} (1 - \cos \theta) + 2s \sin \theta$$

$$A^{R(u)}_{33} = -s^2 \sin \theta \frac{\theta}{M^2} (1 + \cos \theta) - 2s \sin \theta$$
however the term:

$$\frac{1}{t} A_{33}^{R(t)} + \frac{1}{u} A_{33}^{R(u)}$$

cancels exactly the contribution of the terms containing $s^2$.

### 5.1.2 Differential cross sections for $e^+ e^- \rightarrow Z^0 \bar{Z}^0$ polarized collisions

With the help of (12) we be able to draw figure (1) which shows a plot for each helicity:

![Differential cross section graph](image)

Figure 1: Differential cross section for $e^+ e^- \rightarrow Z^0 \bar{Z}^0$ in the 3-3-1 model $A$ with definite polarizations of the initial and final particles, as a function of the c.m. scattering angle $\theta$. Solid (broken) lines correspond to $e_L^- e_R^+ (e_R^- e_L^+)$ initial states. The polarization of the final gauge bosons is indicated on the subfigures. $e^+ e^-$ beam energy $E = \sqrt{s} = 2.5 \times 10^3$ GeV.

### 5.1.3 Total cross section for the process $e^+ e^- \rightarrow Z^0 \bar{Z}^0$ as function of mass of boson $Z^0$

In the following we calculate the total cross section for the process $e^+ e^- \rightarrow Z^0 \bar{Z}^0$ varing the mass boson $Z^0$. For that we summing over all the polarization initial and final particles states given in the formula (10).

**Energy collision $\sqrt{s} = 3$ TeV**

Figure (2) shows the total cross section as function of mass of boson $Z^0$ at energy collision $\sqrt{s} = 3$ TeV.

**Energy collision $\sqrt{s} = 5$ TeV**

Figure (3) shows the total cross section as function of mass of boson $Z^0$ at energy collision $\sqrt{s} = 5$ TeV.

### 5.1.4 Total cross section for the process $e^+ e^- \rightarrow Z^0 \bar{Z}^0$ as function of c.m energy

In this section we calculate the total cross section for the process $e^+ e^- \rightarrow Z^0 \bar{Z}^0$ as function of c.m energy $\sqrt{s}$ maintaining fixed masses of boson gauge at expecting sensate values. Its corresponding graph is given in figure (4).
Figure 2: $\sigma(e^+e^- \rightarrow Z^0 Z^0)$ in the 331 model A without exotic electric charges as a function of mass of neutral $Z^0$ boson gauge from 0.5 TeV to 1.5 TeV at a C.M energy $\sqrt{s} = 3$ TeV.

Figure 3: $\sigma(e^+e^- \rightarrow Z^0 Z^0)$ in the 331 A model without exotic electric charges as a function of mass of neutral $Z^0$ boson gauge from 0.5 TeV to 2.5 TeV at a C.M energy $\sqrt{s} = 5$ TeV.

Figure 4: Total cross section $\sigma(e^+e^- \rightarrow Z^0 Z^0)$ in the 3-3-1 model A as function of $\sqrt{s}$. 
5.1.5 \( e^+e^- \rightarrow K^+K^- \)

The helicity amplitudes for this process are:

\[
|\hat{M}_{\epsilon^-\epsilon^+}|^2 = \left| \frac{1}{4xD_{N_{14}^0}}A_{\epsilon^-\epsilon^+}^{L(t)} + \frac{1}{4xD_{N_{14}^0}}A_{\epsilon^-\epsilon^+}^{R(t)} - \frac{1}{s} \left( C_{\epsilon^-\epsilon^+}^L + C_{\epsilon^-\epsilon^+}^R \right) + \right.
\]
\[
+ \left. \frac{1}{s-M_Z^2} \left( -T_{2w} C_{\epsilon^-\epsilon^+}^L + \frac{T_w}{T_{2w}} C_{\epsilon^-\epsilon^+}^R \right) + \right.
\]
\[
- \left. \frac{1}{s-M_Z^2} (T_{2w} S_{2w})^{-1} \left( C_{\epsilon^-\epsilon^+}^L + 2C_{\epsilon^-\epsilon^+}^R \right) \right|^2 ,
\]

(13)

with \( C = \frac{s}{16} \alpha^2 \) and \( D_{N_{14}^0} = t - M_{Z_{14}^0}^2 \).

Unitarity

When \( s \rightarrow \infty \) \( D_{N_{14}^0} \rightarrow t, s - M_Z^2 \rightarrow s \). By using the identities:

\[
\sum_{\epsilon^-\epsilon^+} \left( A_{\epsilon^-\epsilon^+}^{L(t)} + A_{\epsilon^-\epsilon^+}^{R(t)} \right) = 0
\]

and

\[
1 + T_{2w}^2 + \frac{T_{2w}^{-1}}{S_{2w}} = 1 - T_{2w}^{-1} + 2 \frac{T_{2w}^{-1}}{S_{2w}} = \frac{1}{S_{2w}^2}
\]

we find:

\[
\sum_{\epsilon^-\epsilon^+} \hat{M}_{\epsilon^-\epsilon^+} = \frac{1}{2} \frac{1}{s} \frac{S_{2w}^2}{S_{2w}^2} \left[ C_{\epsilon^-\epsilon^+}^L + C_{\epsilon^-\epsilon^+}^R \right]
\]

\[
= -\frac{4i}{S_{2w}^2} \frac{s^{1/2}}{M_K}.
\]

We have no problem with unitarity. This process is different to standard model process since in the former process each channel cancel independently.

5.1.6 Differential cross sections for \( e^+e^- \rightarrow K^+ K^- \) polarized collisions

Using the amplitude formula (13) and the differential cross section (10), we calculate the different differential cross sections with defined polarization of the initial and final states particles as a function of the c.m scattering angle \( \theta \) at a specific energy collision. Figure (5) shows the corresponding diagram for each helicity:

5.1.7 Total cross section for the process \( e^+e^- \rightarrow K^+ K^- \) as function of mass of charged boson \( K^+(K^-) \)

In the following we calculate the total cross section for the process \( e^+e^- \rightarrow K^+ K^- \) varying the mass boson \( K^+(K^-) \). For that we adding over all the polarization initial and final particles states given in the formula (10). We maintain fixed the mass of boson \( Z^0 \) at a constant value of 1 TeV.

Energy collision \( \sqrt{s} = 3 \text{ TeV} \)

Figure (6) shows the total cross section as function of mass of boson \( K^\pm \).

Energy collision \( \sqrt{s} = 5 \text{ TeV} \)

Figure (7) shows the total cross section as function of mass of boson \( K^\pm \).
Figure 5: Differential cross section for $e^+ e^- \rightarrow K^+ K^-$ in the 3-3-1 model $A$ with definite polarizations of the initial and final particles, as a function of the c.m. scattering angle $\theta$. Solid (broken) lines correspond to $e^- e^+$ ($e^+ e^-$) initial states. The polarization of the final gauge bosons is indicated on the subfigures. $e^+ e^-$ beam energy $E = \sqrt{s} = 2.1 \times 10^3$ GeV.

Figure 6: $\sigma(e^+ e^- \rightarrow K^+ K^-)$ in the 331 model $A$ without exotic electric charges as a function of mass of charged $K^+$ ($K^-$) boson gauge from 0.5 TeV to 1.5 TeV at a C.M energy $\sqrt{s} = 3$ TeV.
Figure 7: $\sigma(e^+ e^- \to K^+ K^-)$ in the 331 model A without exotic electric charges as a function of mass of charged $K^+$ ($K^-$) boson gauge from 0.5 TeV to 2.5 TeV at a C.M energy $\sqrt{s} = 5$ TeV.

5.1.8 Total cross section for the process $e^+e^- \to K^+ K^-$ as function of c.m energy

In this section we calculate the total cross section for the process $e^+e^- \to K^+ K^-$ as function of c.m energy $\sqrt{s}$ maintaining fixed the masses of boson gauge at expecting sensate values. We maintain fixed the mass of boson $Z^{0'}$ at a constant value of 1 TeV. Its corresponding graph is given in figure 8.

Figure 8: Total cross section $\sigma(e^+ e^- \to K^+ K^-)$ in the 3-3-1 model A as function of $\sqrt{s}$. Observe as $s \to \infty$, $\sigma \to C/(8\pi \cdot 125^2 \cdot \sin^4 W) = 1.261 \cdot 10^{-36}$ cm$^2$, a constant value.

5.1.9 $e^+e^- \to K^0\bar{K}^0$

The helicity amplitudes for this process are:

$$|\mathcal{M}_{\epsilon^-\epsilon^+}|^2 = \left| \frac{1}{4xD_{E^-}} A_{\epsilon^-\epsilon^+}^{L(u)} + \frac{1}{s - M_{E^-}^2} \left( (T_{2w}s_{2w})^{-1} C^{L}_{\epsilon^-\epsilon^+} \frac{T_{w}}{s_{2w}} - \frac{T_{w}}{s_{2w}} C^{R}_{\epsilon^-\epsilon^+} \right) \right|^2 +$$

$$- \left( \frac{1}{s - M_{E^-}^2} (T_{2w}s_{2w})^{-1} \left( C^{L}_{\epsilon^-\epsilon^+} + 2C^{R}_{\epsilon^-\epsilon^+} \right) \right)^2,$$

with $C = \frac{3}{32} \alpha^2$ and $D_{E^-} = u - M_{E^-}^2$.

Unitarity

When $s \to \infty$ $D_{E^-} \to u$, $s - M_{E^-}^2 \to s$ we have:

$$\sum_{\epsilon^-\epsilon^+} |\mathcal{M}_{\epsilon^-\epsilon^+}|^2 \propto \left[ \frac{1}{2u} A_{\epsilon^-\epsilon^+}^{L(u)} - \frac{1}{s} C_{\epsilon^-\epsilon^+}^{R} \right]^2$$
and again the terms $A_{33}^{L(u)}$ and $C_{33}^{R} = -2s \sin \theta (1 + \frac{s}{2M_{K^0}^2})$ may have trouble, however the terms that contain $s^2$ cancel exactly.

**5.1.10 Differential cross sections for $e^+ e^- \rightarrow K^0 \bar{K}^0$ polarized collisions**

Using the amplitude formula (14) and the differential cross section (10), we calculate the different differential cross sections with defined polarization of the initial and final states particles as a function of the c.m. scattering angle $\theta$ at a specific energy collision. A diagram for each helicity is given in figure (9).

![Differential cross section](image)

Figure 9: Differential cross section for $e^+ e^- \rightarrow K^0 \bar{K}^0$ in the 3-3-1 model A with definite polarizations of the initial and final particles, as a function of the c.m. scattering angle $\theta$. Solid (broken) lines correspond to $e_L^- e_R^+ (e_R^- e_L^+)$ initial states. The polarization of the final gauge bosons is indicated on the subfigures. $e^+ e^-$ beam energy $E = \sqrt{s} = 2.1 \times 10^3$ GeV.

**5.1.11 Total cross section for the process $e^+ e^- \rightarrow K^0 \bar{K}^0$ as function of mass of neutral gauge boson $K^0(\bar{K}^0)$**

In the following we calculate the total cross section for the process $e^+ e^- \rightarrow K^0 \bar{K}^0$ varying the mass of neutral gauge boson $K^0(\bar{K}^0)$. For that we summing over all the polarization initial and final particles states given in the formula (10). We maintain fixed the mass of boson $Z^0$ at a constant value of 1 TeV.

Energy collision $\sqrt{s} = 3$ TeV

Figure (10) shows its diagram for this process as a function of mass boson $K^0$.

Energy collision $\sqrt{s} = 5$ TeV

Figure (11) shows its diagram for this process as a function of mass boson $K^0$.
Figure 10: $\sigma(e^+ e^- \rightarrow K^0 \bar{K}^0)$ in the 331 model without exotic electric charges as a function of mass of neutral $K^0 (\bar{K}^0)$ boson gauge from 0.5 TeV to 1.5 TeV at a C.M energy $\sqrt{s} = 3$ TeV.

Figure 11: $\sigma(e^+ e^- \rightarrow K^0 \bar{K}^0)$ in the 331 model without exotic electric charges as a function of mass of neutral $K^0 (\bar{K}^0)$ boson gauge from 0.5 TeV to 2.5 TeV at a C.M energy $\sqrt{s} = 5$ TeV.
5.1.12 Total cross section for the process $e^+ e^- \rightarrow K^0 \bar{K}^0$ as function of c.m energy

In this section we calculate the total cross section for the process $e^+ e^- \rightarrow K^0 \bar{K}^0$ as function of c.m energy $\sqrt{s}$ maintaining fixed the masses of boson gauge at expecting sensate values. We maintain fixed the mass of boson $Z^{0}$ at a constant value of 1 TeV. The diagram for this process is given in figure (12).

![Figure 12: Total cross section $\sigma(e^+ e^- \rightarrow K^0 \bar{K}^0)$ in the 3-3-1 model A as function of $\sqrt{s}$.](image)

5.2 Model D with right handed neutrinos

The treatment is similar as was done with model A, the differences are present in their fermionic content. For this model the fine-structure constant has a value $\alpha = 1/125.9$. Now we study each processes.

5.2.1 $e^+ e^- \rightarrow Z^0 Z^0$

Using Feynman diagrams for this process we compute the amplitude as function of polarization at initial and final particles. The helicities are given in section (4), its masses are of that particle produced in the collision. The helicity amplitudes for this process are:

$$|\mathcal{M}_{\epsilon^- \epsilon_+}|^2 = \left| a'_L \left( \frac{1}{t} A^{L(t)}_{\epsilon^- \epsilon_+} + \frac{1}{u} A^{L(u)}_{\epsilon^- \epsilon_+} \right) + a'_R \left( \frac{1}{t} A^{R(t)}_{\epsilon^- \epsilon_+} + \frac{1}{u} A^{R(u)}_{\epsilon^- \epsilon_+} \right) \right|^2,$$

where $C = \frac{\pi}{8} \alpha^2$, $a'_L = \frac{T^2}{4 \alpha^2}$ and $a'_R = \frac{T^2}{4 \alpha^2}$.

5.2.2 Differential cross sections for $e^+ e^- \rightarrow Z^0 Z^0$ polarized collisions

Using the amplitude formula (15) and the differential cross section (10), we calculate the different differential cross sections with defined polarization of the initial and final states particles as a function of the c.m scattering angle $\theta$ at a specific energy collision.

5.2.3 Total cross section for the process $e^+ e^- \rightarrow Z^0 Z^0$ as function of mass of boson $Z^{0}$

In the following we calculate the total cross section for the process $e^+ e^- \rightarrow Z^0 Z^0$ varying the mass boson $Z^{0}$. For that we summing over all the polarization initial and final particles states given in the formula (10).

Energy collision $\sqrt{s} = 3$ TeV

Figure (14) shows the total cross section as function of mass of boson $Z^{0}$ at energy collision $\sqrt{s} = 3$ TeV.

Energy collision $\sqrt{s} = 5$ TeV

Figure (15) shows the total cross section as function of mass of boson $Z^{0}$ at energy collision $\sqrt{s} = 5$ TeV.
Figure 13: Differential cross section for $e^+ e^- \rightarrow Z'^0 Z'^0$ in the 3-3-1 model $D$ with definite polarizations of the initial and final particles, as a function of the c.m. scattering angle $\theta$. Solid (broken) lines correspond to $e_L^+ e_R^- (e_R^+ e_L^-)$ initial states. The polarization of the final gauge bosons is indicated on the subfigures. $e^+ e^-$ beam energy $E = \sqrt{s} = 2.5 \times 10^3$ GeV. Note that both polarization incident particles $e^\pm$ have the same differential cross section for all helicities of the final particles.

Figure 14: $\sigma(e^+ e^- \rightarrow Z'^0 Z'^0)$ in the 331 model $D$ without exotic electric charges as a function of mass of neutral $Z'^0$ boson gauge from 0.5 TeV to 1.5 TeV at a C.M energy $\sqrt{s} = 3$ TeV.
Figure 15: $\sigma(e^+ e^- \rightarrow Z'^0 Z'^0)$ in the 331 model without exotic electric charges as a function of mass of neutral $Z'^0$ boson gauge from 0.5 TeV to 2.5 TeV at a C.M energy $\sqrt{s} = 5$ TeV.

5.2.4 Total cross section for the process $e^+ e^- \rightarrow Z'^0 Z'^0$ as function of c.m energy

In this section we calculate the total cross section for the process $e^+ e^- \rightarrow Z'^0 Z'^0$ as function of c.m energy $\sqrt{s}$ maintaining fixed the mass of boson gauge at expecting sensate values. Its corresponding graph is given in figure (16).

![Figure 16: Total cross section $\sigma(e^+ e^- \rightarrow Z'^0 Z'^0)$ in the 3-3-1 model as function of $\sqrt{s}$.
](image)

5.2.5 $e^+ e^- \rightarrow K^+ K^-$

The helicity amplitudes for this process are:

\[
|M_{\epsilon_-\epsilon_+}|^2 = \left| \frac{1}{4xD_N^0} A^{L(t)}_{\epsilon_-\epsilon_+} - \frac{1}{s} \left( C^L_{\epsilon_-\epsilon_+} + C^R_{\epsilon_-\epsilon_+} \right) + \right.
\]

\[
\left. + \frac{1}{s - M_2^T} \left[ -T_2^{-2} C^{L}_{\epsilon_-\epsilon_+} + \frac{T_w}{T^2_2} C^{R}_{\epsilon_-\epsilon_+} \right] + \right. \left. \frac{1}{s - M_2^T} \left[ -(S_2wT_2)^{-1} C^{L}_{\epsilon_-\epsilon_+} + \frac{T_w}{S^2_2} C^{R}_{\epsilon_-\epsilon_+} \right]^2 \right| ^2,
\]

and $C = \frac{\pi}{32} \alpha^2$.

5.2.6 Differential cross sections for $e^+ e^- \rightarrow K^+ K^-$ polarized collisions

Using the amplitude formula (16) and the differential cross section (10), we calculate the different differential cross sections with defined polarization of the initial and final states particles as a function of the c.m scattering angle $\theta$ at a specific energy collision. Figure (17) shows the corresponding diagram for each helicity.
Figure 17: Differential cross section for $e^+e^- \rightarrow K^+ K^-$ in the 3-3-1 model $D$ with defined polarizations of the initial and final particles, as a function of the c.m. scattering angle $\theta$. Solid (broken) lines correspond to $e^-_L e^+_R (e^-_R e^+_L)$ initial states. The polarization of the final gauge bosons is indicated on the subfigures. $e^+e^-$ beam energy $E = \sqrt{s} = 2.1 \times 10^3$ GeV.

5.2.7 Total cross section for the process $e^+e^- \rightarrow K^+ K^-$ as function of mass of charged boson $K^+ (K^-)$

In the following we calculate the total cross section for the process $e^+e^- \rightarrow K^+ K^-$ varying the mass boson $K^+ (K^-)$. For that we summing over all the polarization initial and final particles states given in the formula (10).

Energy collision $\sqrt{s} = 3$ TeV

Figure 18 shows the total cross section as function of mass of boson $K^\pm$.

Figure 18: $\sigma(e^+e^- \rightarrow K^+ K^-)$ in the 331 model without exotic electric charges as a function of mass of charged $K^+ (K^-)$ boson gauge from 0.5 TeV to 1.5 TeV at a C.M energy $\sqrt{s} = 3$ TeV.
Energy collision $\sqrt{s} = 5$ TeV

Figure (19) shows the total cross section as function of mass of boson $K^\pm$.

Figure 19: $\sigma(e^+ e^- \to K^+ K^-)$ in the 331 model without exotic electric charges as a function of mass of charged $K^+(K^-)$ boson gauge from 0.5 TeV to 2.5 TeV at a C.M energy $\sqrt{s} = 5$ TeV.

5.2.8 Total cross section for the process $e^+ e^- \to K^+ K^-$ as function of c.m energy

In this section we calculate the total cross section for the process $e^+ e^- \to K^+ K^-$ as function of c.m energy $\sqrt{s}$ maintaining fixed the mass of boson gauge at expecting sensate values. We maintain fixed the mass of boson $Z^0$ at a constant value of 1 TeV. Its corresponding graph is given in figure (20).

Figure 20: Total cross section $\sigma(e^+ e^- \to K^+ K^-)$ in the 3-3-1 model $D$ as function of $\sqrt{s}$.

5.2.9 $e^+ e^- \to K^0 \bar{K}^0$

The helicity amplitudes for this process are:

$$|M_{\epsilon^- \epsilon^+}|^2 = \left| \left( T_{2w}^{-1} C_{\epsilon^- \epsilon^+} - T_w C_{\epsilon^- \epsilon^+}^R \right) \left( \frac{1}{s - M_2^2} - \frac{1}{s - M_2^2'} \right) \right|^2, \quad (17)$$

with $C = \frac{\pi}{32} \left( \frac{\alpha}{8\pi^2} \right)^2$.

5.2.10 Differential cross sections for $e^+ e^- \to K^0 \bar{K}^0$ polarized collisions

Using the amplitude formula (17) and the differential cross section (10), we calculate the different differential cross sections with defined polarization of the initial and final states particles as a function of the c.m scattering angle $\theta$ at a specific energy collision. A diagram for each helicity is given in figure (21).
5.2.11 Total cross section for the process $e^+ e^- \rightarrow K^0 \bar{K}^0$ as function of mass of neutral gauge boson $K^0(\bar{K}^0)$

In the following we calculate the total cross section for the process $e^+ e^- \rightarrow K^0 \bar{K}^0$ varying the mass of neutral gauge boson $K^0(\bar{K}^0)$. For that we summing over all the polarization initial and final particles states given in the formula (10). We maintain fixed the mass of boson $Z^0$ at a constant value of 1 TeV.

Energy collision $\sqrt{s} = 3$ TeV

Figure (22) shows its diagram for this process as a function of mass boson $K^0$.

Energy collision $\sqrt{s} = 5$ TeV

Figure (23) shows its diagram for this process as a function of mass boson $K^0$.

5.2.12 Total cross section for the process $e^+ e^- \rightarrow K^0 \bar{K}^0$ as function of c.m energy

In this section we calculate the total cross section for the process $e^+ e^- \rightarrow K^0 \bar{K}^0$ as function of c.m energy $\sqrt{s}$ maintaining fixed the masses of boson gauge at expecting sensate values. We maintain fixed the mass of boson $Z^0$ at a constant value of 1 TeV. The diagram for this process is given in figure (24).

In the model $D$ the unitarity is proved similarly as was done in the model $A$.

6 Conclusions

The $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ models are among the various gauge theory models that have been proposed to unify weak and electromagnetic interactions. These models makes predictions of the existence of new particles,
Figure 22: $\sigma(e^+ e^- \rightarrow K^0 \bar{K}^0)$ in the 331 model without exotic electric charges as a function of mass of neutral $K^0 (\bar{K}^0)$ boson gauge from 0.5 TeV to 1.5 TeV at a C.M energy $\sqrt{s} = 3$ TeV.

Figure 23: $\sigma(e^+ e^- \rightarrow K^0 \bar{K}^0)$ in the 331 model without exotic electric charges as a function of mass of neutral $K^0 (\bar{K}^0)$ boson gauge from 0.5 TeV to 2.5 TeV at a C.M energy $\sqrt{s} = 5$ TeV.

Figure 24: Total cross section $\sigma(e^+ e^- \rightarrow K^0 \bar{K}^0)$ in the 3-3-1 model $D$ as function of $\sqrt{s}$. 
such as five new vector bosons. We need to study experimentally many aspects of these models. In this paper we get information about the trilinear couplings, neutral and charged currents and its respectively couplings for the models $A$ and $D$ described above. With these information we find explicit formulae for the processes $e^+e^- \rightarrow Z'Z'$, $e^+e^- \rightarrow K^+K^-$ and $e^+e^- \rightarrow K^0\bar{K}^0$. It is a simple task to compute the cross section for production of any pair of vector bosons for a given helicity state.

However we have shortcomings with the unknown quantities like masses of $Z'$, $K^+$, $\bar{K}^0$ and masses for exotic leptons. In consequence is no possible to get values for $\alpha(M_{V}^2)$ and $S_w \equiv \sin(\theta_w)$ exactly. By using different values for $M_{Z'}$, $M_{K^+}$, $M_{K^0}$ and $\alpha(M_{V}^2)$ and $S_w \equiv \sin(\theta_w)$ at TeV scales according to models proposed [6, 7, 9] we hope that the formulae presented here be helpful for new experiments.

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