Stereoscopic Digital Particle Image Velocimetry and Digital Particle Image Thermometry analysis of the mixed convection phenomena in the system similar to the Czochralski method of single crystal production

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Abstract. This paper presents the results from experimental analysis of mixed convection phenomena in the model of Czochralski system for single crystal production. During the studies the influence of such parameters as: crucible and crystal rotation rate and temperature difference level have been shown. The Stereoscopic Digital Particle Image Velocimetry (SDPIV) and Digital Particle Image Thermometry (DPIT) methods have been used to calculate three dimensional velocity vector fields and temperature distribution in the system. Measurements have been made in a transparent model of Czochralski configuration, where all significant parameters have been kept at the proper level according to the theory of similarity. The results below are presented in the form of contour-vector velocity maps, plots of the particular velocity component in some specific regions of the examined geometry and the contour maps of temperature distribution in the crucible.

1. Introduction
Czochralski crystal growth is the leading method for semiconductor (Si, Ge) and oxide crystals (LiNbO₃, Gd₃Ga₅O₁₂, Bi₁₂SiO₂₀) production. The quality and size of the grown crystal depends on many physical effects, where melt flow plays a major role in the process. Generally, the melt flow is caused by a combination of natural convection, forced convection and Marangoni convection. The natural convection is introduced by buoyancy forces, which are produced by thermal boundary conditions. The forced convection is caused by crystal rotation, crucible rotation, crystal pulling and by reduction of the melt height. The mode of forced convection plays a significant role in Czochralski technique. One of the first works, where authors have been trying to visualize the flow structure inside the crucible in a model systems of Czochralski growth have been performed by Carruthers and Nassau [1], Shiroki [2], Jones [3] and Kimura [4]. Significant contribution in the field of flow visualization of Czochralski growth have been also made by Hintz et al. [5-6]. In those papers, the complex analysis of the flow behaviour under various experimental conditions in the transparent glass crucible has been presented. The use of liquid crystals for visualization of the temperature fields in the Czochralski configuration have been proposed in 1991 by Ozoe [7], where the silicon oil and thermochromic liquid crystals (TLC) with temperature sensing range from 24.0 °C to 30.5 °C have been used for...
measurements. Later on, Mukherjee et al. [8] have used liquid crystals to study the influence of crucible and crystal rotation on the temperature distribution in the Czochralski crucible. TLC measurement technique was also used by Banerjee et al. [9] to investigate the impact of temperature difference and crystal rotation rate on the system stability. Aleksic et al. [10] extended those analysis by measuring with DPIT and 2D PIV techniques, the fluid velocity and temperature distribution inside the Czochralski crucible for natural convection and crystal rotation case. Flow visualization, in close to the operating conditions have been made by Kakimoto et al [11-12] and Krzyminsky and Ostrogorsky[13]. Due to the rapid progress of PC computing power, the problem of mixed convection phenomena in a Czochralski crucible was also broadly studied by numerical calculations [14-16].

In the presented paper the results from Stereoscopic Digital Particle Image Velocimetry (SDPIV) and Digital Particle Image Thermometry (DPIT) analysis of mixed convection phenomena in Czochralski configuration are presented. To the authors knowledge there are lack of results from SDPIV and DPIT measurements in the investigated geometry, thus they may present an important contribution in the field of Czochralski growth studies.

2. Experimental setup and procedure
The experimental setup used for SDPIV measurements is presented in figure 1b-c; The test section consists of a glass crucible with a 0.12 m inner diameter, filled with glycerol water solution. The beaker was immersed in the cubic glass aquarium filled with water that was used to keep the isothermal conditions during measurements. Water to the aquarium was supplied from two constant temperature thermal baths. Temperature inside the aquarium was measured by two type K thermocouples. A copper cylinder which represented a model of growing crystal was placed at the top surface of experimental fluid. The third constant temperature thermal bath was used to control the crystal dummy temperature which was measured by type T thermocouple. Both crucible and cooper cylinder were rotated by the 10 W EZI-STEP PlusR stepping motors. Measurements were performed in the room temperatures (between 20 °C – 30 °C).

![Figure 1](image)

**Figure 1.** (a) DPIT system configuration, (b) Experimental setup, (c) SDPIV system configuration.

The SDPIV system consisted of two Lavision (Imager Pro 4M) CCD cameras and the pulsed Nd:YAG laser (figure 1c). The laser was generating narrow light plane with thickness and energy of about 1 – 3 mm and 400 mJ respectively. The CCD cameras were positioned in about 60 degree angle to the laser light plane. For each measure a set of 400 pairs of images were recorded. Time delay between each pair was varied from 15 – 65 ms. The velocity of the fluid was measured based on the displacement of glass sphere particles with diameter of 9 – 13 µm and density 1100 kg/m³. Corresponding DPIT setup is presented in figure 1a. It consisted of one 3 CCD camera positioned
perpendicularly to the white light plane generated by 1 kW projector. Temperature fields were calculated based on the response of the TLC’s mixed with the experimental fluid. The details of DPIT experimental setup, measurement process and temperature field calculation are presented in authors previous work [17].

2.1. Experimental fluid
In the SDPIV and DPIT measurements the 35 % and 30 % glycerol water solution was used respectively. The small differences in the amount of glycerol for both measurements raised from differences between the tracer particles used in both cases. The physical properties of two experimental fluids at temperature of 25 ºC are presented in table 1.

| Quantity                          | Symbol | Unit       | 30% of glycerol | 35% of glycerol |
|-----------------------------------|--------|------------|-----------------|-----------------|
| Volume fraction                   | --     | --         | 0.70/0.30       | 0.65/0.35       |
| Mass fraction                     | --     | --         | 0.65/0.35       | 0.60/0.40       |
| Density                           | $\rho$ | kg/m$^3$   | 1089            | 1084            |
| Dynamic viscosity                 | $\eta$ | Pa·s       | 2.6·10$^{-3}$   | 3.2·10$^{-3}$   |
| Kinematic viscosity              | $\nu$  | m$^2$/s    | 2.36·10$^{-6}$  | 2.96·10$^{-6}$  |
| Specific heat                     | $c_p$  | J/(kg·K)   | 3546.12         | 3455.82         |
| Thermal conductivity             | $k$    | W/(m·K)    | 0.49            | 0.48            |
| Coeff. of thermal expansion      | $\beta$| 1/K        | 3.76·10$^{-4}$  | 3.92·10$^{-4}$  |

Thermal conductivity for selected glycerol water solution was obtained from the Fillipov equation [18]. Values of specific heat were calculated as a weighted mean of water and glycerol specific heats. In particular, water and glycerol thermal conductivities were calculated with the accuracy of 1.7 % and 4.7 % respectively, based on the method presented in [19]. Similarly, the corresponding values of specific heat, with accuracy of 0.59 % (water) and 4.0 % (glycerol) were calculated from equations presented in [20]. Dynamic viscosity of the experimental liquids were estimated according to the Cheng method [21] which have an accuracy of about 5 %. The coefficient of cubical thermal expansion was assumed to be almost equal in the examined range of temperatures (20 ºC – 30ºC) [22]. Its value in the presented work was estimated based on the data available in [22] showing $\beta$ as a function of glycerol concentration in the solution.

2.2. Similarity
To perform the measurements in any kind of setup that represents the model of specific system it is necessary to ensure the similarity in the geometrical construction as well as in the thermodynamic mechanisms that may be observed during measurements. In the Czochralski configuration, the geometric similarity is acquired when the crucible to crystal radius ratio and fluid height to crucible ratio are kept at the same level as in the real device. To achieve the dynamic similarity and similarity in thermal boundary conditions it is necessary to precisely specify the values of non-dimensional numbers such as $Re_c = (\Omega_c R_c) / \nu$, $Re_r = (\Omega_r R_r) / \nu$, $Gr = (g \beta \Delta T R_c^3) / \nu^2$, $Ra_T = (g \beta \Delta T H^2) / (\nu h)$ and $Pr = \nu / h = (c_p \nu) / k$. In the above equations $\Omega_c$, $\Omega_r$ represents the crystal and crucible rotation rate, $R_c$ and $H$ states for the crucible radius and fluid height, $g$ is the gravitational acceleration, $\Delta T$ defines
Table 2. Experimental parameters.

| $Re_c$ | $Re_x$ | $Gr$ | $Ra_T$ | $Pr$ | $Re_c$ | $Re_x$ | $Gr$ | $Ra_T$ | $Pr$ |
|--------|--------|------|--------|------|--------|--------|------|--------|------|
| 276    | 276    | $2.14 \times 10^5$ | $4.93 \times 10^6$ | 23   | 269    | 269    | $2.41 \times 10^5$ | $4.35 \times 10^6$ | 18   |
| 552    | 552    | $3.21 \times 10^5$ | $7.39 \times 10^6$ | --   | 535    | 535    | $3.62 \times 10^5$ | $6.52 \times 10^6$ | --   |
| 829    | 829    | $4.29 \times 10^5$ | $9.86 \times 10^6$ | --   | 808    | 808    | $4.83 \times 10^5$ | $8.69 \times 10^6$ | --   |
| 1105   | 1105   | $5.36 \times 10^5$ | $1.23 \times 10^7$ | --   | 1078   | 1078   | $6.03 \times 10^5$ | $1.09 \times 10^7$ | --   |
| 1382   | 1382   | --    | --     | --   | 1348   | --    | --    | --     | --   |
| --     | 1796   | --    | --     | --   | --     | --     | --    | --     | --   |

the temperature difference between the crucible walls and crystal surface. Finally, $h$ shows the coefficient of thermal diffusivity. Remaining variables are described and presented in table 1. In the presented paper all above quantities were calculated separately for SDPIV and DPIT measurements (table 2). Experimental parameters chosen for analysis shows good agreement with the literature data [5-6], [9], [13].

3. Results and discussion

The analysis of mixed convection phenomena in the investigated geometry were performed for all parameters from table 2. In the presented paper authors decided to focus on crucible and crystal rotation influence on the mixed convection phenomena as it plays an important role during crystal growth process.

3.1. Velocity fields

In case of a crystal rotation, it was noticed that for the low values of Reynolds number ($Re_c = 276$) (figure 2a, figure 3a) the cold jet characteristic for natural convection circulation [5] starts to oscillate around the crucible axis. Moreover, slowly rotating crystal caused some disturbances in the flow structure of the experimental fluid. Those disturbances could be observed in the vertical velocity component variations. In figure 3a, (○), (*) and (⊳) points represents its values at three sections: 10 cm below fluid surface, 10 cm above crucible bottom and in the middle of fluid height accordingly. Comparing those relationships, it was concluded that the rotating crystal influence was mostly observed in region close to the fluid surface. With the successive increase of crystal rotation rate, part of a fluid under the crystal started to rotate in the direction opposite to the natural convection circulation (figure 2b-c). At the same time the rest of a fluid, that was moving in accordance with natural convection flow, was gradually pushed towards the crucible sidewalls. This effect was confirmed by the variation of vertical velocity component presented in figure 3b-c. Here, middle peak represents the warm jet flow and the natural convection flows reveals as the two almost symmetrical minima. Furthermore, for $Re_c = 829$ and $Re_c = 1796$, two counter rotating vortices just under the crystal surface were observed. One more, that caused some asymmetry in the vertical velocity component (figure 3b) appeared in the bottom right side of the warm jet for $Re_c = 829$. This last vortex vanished for larger crystal rotational rates.

Studying the crucible rotation influence it was found that its impact on the fluid flow was much stronger that in case of a crystal rotation. The in plane velocity distribution presented in figure 2d-f shows an increase in cold jet oscillations with the increase in crucible rotation rate. Furthermore, the flow direction was almost identical in all parts of the crucible and reflected the direction of crucible
Figure 2. Influence of the crystal (a-c) and crucible (d-f) rotation rate on the flow structure in a crucible, $Gr=2.14 \times 10^5$; (a) $Re_x = 279$, (b) $Re_x = 829$, (c) $Re_x = 1796$, (d) $Re_c = 279$, (e) $Re_c = 829$, (f) $Re_c = 1382$.

Figure 3. Influence of the crystal (a-c) and crucible (d-f) rotation rate on the vertical velocity component in a crucible, $Gr=2.14 \times 10^5$; (a) $Re_x = 276$, (b) $Re_x = 829$, (c) $Re_x = 1796$, (d) $Re_c = 276$, (e) $Re_c = 829$, (f) $Re_c = 1382$. 
rotation. Raise in the flow disturbances were also reflected by the behaviour of vertical velocity component what is presented in figure 3d-f.

The main advantage of SDPIV technique is the possibility to measure the spatial velocity component simultaneously with the in plane components. The magnitude of the third velocity component both for the crystal and crucible rotation cases, in figure 2 are presented in the form of two colour contour plots. Analysing the contour maps on figure 2a-c one can easily find that the increase in crystal rotation rate gradually enhance its influence on the fluid motion in a crucible.

![Figure 4](image.png)

**Figure 4.** Influence of the crystal (a, b) and crucible (c, d) rotation rate on the spatial velocity component in a crucible, $Gr=2.14\cdot10^5$; (a, c) Section 10 cm below fluid surface, (b, d) Section 10 cm above crucible bottom.

In figure 4a-b the spatial velocity component for all investigated crystal rotation rates are presented. The left one shows the values of spatial velocity component measured 10cm below the fluid surface. Values presented on the right side were measured 10 cm above the crucible bottom. Comparing those plots, it can be found that for both cases the maximum of spatial velocity for low Reynolds number is close to the rotational axis of the crucible and is shifted towards the crystal edge for larger values of Reynolds numbers. It is also interesting that after reaching its maximum value the spatial velocity is decreasing and its drop is more intense in the upper part of the crucible than in its lower region.

Corresponding crucible rotation case is presented in figures 2d-f and 4c-d. It can be found that due to the crucible rotation, the maximum spatial velocity of the fluid was reached in the vicinity of its sidewalls what looks reasonable and agree with the theory of rotational motion. With the increase of the crucible rotation rate the maximum value of spatial velocity component was increasing and also the parts of the fluid closer to the crucible rotation axis started to rotate. What is worth to notice is the fact that, in contrast to crystal rotation, the distribution and values of spatial velocity component for each crucible rotational speed were the same in both investigated sections (figure 4c-d).
3.2. Temperature fields

The temperature distribution in a Czochralski crucible for crystal and crucible rotation case is presented in figure 5. For a crystal rotation it was found that with the intensification of rotational motion the cold jet observed for the natural convection started to vanish due more efficient mixing of a fluid (figure 5a). When the speed of rotation increased, the crystal started to drawn in the warmer fluid from the crucible bottom what resulted in the formation of the warm jet instead of a cold one (figure 5b-c). Larger crystal rotation velocities caused better mixing of the experimental fluid what in case of temperature measurements resulted in the equalization of the temperature field in a large part of a beaker.

![Figure 5](image)

**Figure 5.** Influence of the crystal and crucible rotation rate on the temperature distribution in a Czochralski crucible, $Gr = 2.41 \cdot 10^5$, (a) $Re_c = 269$, (b) $Re_c = 808$, (c) $Re_c = 1078$, (d) $Re_c = 269$, (e) $Re_c = 808$, (f) $Re_c = 1078$.

For a crucible rotation, the oscillations of the cold jet mentioned during the analysis of the velocity distribution were also observed in the analysis of the temperature distribution (figure 5d-f). However those oscillations were visible till the $Re_c = 1078$. In this case due to the efficient mixing of the fluid the temperature in the major part of the crucible dropped significantly and stabilized causing the cold jet to be undistinguishable in the measurement plane (figure 5f). In the temperature maps it was also found that due to the increase in crucible rotation rate the warmer part of the fluid was pushed toward the upper sidewalls part of the crucible. For both crucible and crystal rotation cases with the increase of the rotation rate the temperature distribution in the crucible was more homogenous. Moreover, the temperature in the main part of the crucible dropped to the values of about 28.6 – 28.8 °C. For high investigated Reynolds numbers the medium temperature levels were only observed in the contact area between the cold and warm fluid.
4. Conclusions
In the presented paper the analysis of the influence of crucible and crystal rotation on the velocity and temperature fields distribution in the system similar to the Czochralski crystal growth process was presented. For both cases the experimental results were calculated based on the optical measurement methods (SDPIV and DPIT). The application of SDPIV technique for velocity measurements allows to visualize the full three dimensional velocity distribution in the measurement system. From the above analysis it may be concluded that due to the increase of crystal and crucible rotation rate the cold jet starts to oscillate around the rotational axis of the crucible. Moreover, in case of crystal rotation the fluid below the crystal started to circulate in opposite direction for larger values of crystal Reynolds numbers. During the results analysis, the differences between spatial velocity component for crucible and crystal rotation were found. In the first case the maximum of the spatial velocity component was measured next to the crucible sidewall and was almost linearly decreasing approaching crucible centre. Moreover the experimental fluid was equally accelerated in every part of the crucible by its rotation. By contrast, crystal rotation strongly influenced the fluid motion in the upper part of the crucible. In this case it was also found that the spatial component of the velocity had a maximum value that was first located below the crystal surface and then shifted to its edge. For both crucible and crystal rotation it was found that the temperature distribution become stabilized and equalized in the measurement region.

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6. Literature
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