Nonclassicality of quantum excitation of classical coherent field in photon loss channel

Shang-Bin Li, Xu-Bo Zou, and Guang-Can Guo

1. Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei 230026, China. and
2. Shanghai research center of Amertron-global, Zhangjiang High-Tech Park, 299 Lane, Bisheng Road, No. 3, Suite 202, Shanghai, 201204, P.R. China

We investigate the nonclassicality of photon-added coherent states in the photon loss channel by exploring the entanglement potential and negative Wigner distribution. The total negative probability defined by the absolute value of the integral of the Wigner function over the negative distribution region reduces with the increase of decay time. The total negative probability and the entanglement potential of pure photon-added coherent states exhibit the similar dependence on the beam intensity. The reduce of the total negative probability is consistent with the behavior of entanglement potential for the dissipative single-photon-added coherent state at short decay times.

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Nonclassical optical fields play a crucial role in understanding fundamentals of quantum physics and have many applications in quantum information processing [1]. Usually, the nonclassicality manifests itself in specific properties of quantum statistics, such as the antibunching [2], sub-Poissonian photon statistics [3], squeezing in one of the quadratures of the field [4], partial negative Wigner distribution [5], etc.

When the nonclassical optical fields propagate in the medium, they inevitably interact with their surrounding environment, which causes the dissipation or dephasing [6]. It is well known that the dissipation or dephasing will deteriorate the degree of nonclassicality of the optical fields. A quantitative measure of non-classicality of quantum fields is necessary for further investigating the dynamical behavior of their non-classicality. Many authors have investigated the relations between non-classicality of optical fields and the entanglement and shown that non-classicality is a necessary condition for generating inseparable state via the beam splitter [7]. Based on them, a measure called the entanglement potential for quantifying the non-classicality of the single-mode optical field has been proposed [8]. The entanglement potential is defined as the entanglement achieved by 50:50 beam splitter characterized by the unitary operation \( U_{BS} = e^{\frac{\pi}{4}(ab^\dagger + ba^\dagger)} \) acting on the target optical mode \( a \) and the vacuum mode \( b \). Throughout this paper, log-negativity is explored as the measure of entanglement potential. The log-negativity of a density matrix \( \rho \) is defined by [9]

\[
N(\rho) = \log_2 \|\rho^\Gamma\|,
\]

where \( \rho^\Gamma \) is the partial transpose of \( \rho \) and \( \|\rho^\Gamma\| \) denotes the trace norm of \( \rho^\Gamma \), which is the sum of the singular values of \( \rho^\Gamma \).

Nevertheless, experimental measurement of the entanglement potential is still a challenge task. How to quantify the variation of the nonclassicality of quantum fields based on the current mature laboratory technique is an interesting topic. The Wigner function is a quasi-probability distribution, which fully describes the state of a quantum system in phase space. The partial negativity of the Wigner function is indeed a good indication of the highly nonclassical character of the state. Reconstructions of the Wigner functions in experiments with quantum tomography [10, 11, 12] have demonstrated appearance of their negative values, which can not be explained in the framework of the probability theory and have not any classical counterparts. Therefore, to seek certain possible monotonic relation between the partial negativity of the Wigner distribution and the entanglement potential may be an available first step for experimentally quantifying the variation of nonclassicality of quantum optical fields in dissipative or dephasing environments.

Here, for clarifying the feasibility of this idea, we investigate the nonclassicality of photon-added coherent states in the photon loss channel by exploring the entanglement potential and negative Wigner distribution. The total negative probability defined by the absolute value of the integral of the Wigner function over the negative distribution region is introduced, and our calculations show it reduces with the increase of decay time. The total negative probability and the entanglement potential of pure photon-added coherent states exhibit the similar dependence on the beam intensity. The reduce of total negative probability is consistent with the behavior of entanglement potential for the dissipative single-photon-added coherent state at short decay times.

The photon-added coherent state was introduced by Agarwal and Tara [13]. The single photon-added coherent state (SPACS) is experimentally prepared by Zavatta et al. and its nonclassical properties are detected by homodyne tomography technology [14]. Such a state...
represents the intermediate non-Gaussian state between quantum Fock state and classical coherent state (with well-defined amplitude and phase) \cite{13}. For the SPACS, a quantum to classical transition has been explicitly demonstrated by ultrafast time-domain quantum homodyne tomography technique. Thus, it is timely to analyze how the photon loss affects the non-classicality of such kind of optical fields.

Let us first briefly recall the definition of the photon-added coherent states (PACSS) \cite{13}. The PACSSs are defined by $|\alpha, m\rangle = L_m(|\alpha\rangle)$, where $|\alpha\rangle$ is the coherent state with the amplitude $\alpha$ and $a^\dagger$ is the creation operator of the optical mode. $N(\alpha, m) = m! L_m(-|\alpha|^2)$, where $L_m(x)$ is the $m$th-order Laguerre polynomial. When the PACS evolves in the photon loss channel, the evolution of the density matrix can be described by \cite{6}

$$\frac{dp}{dt} = \frac{\gamma}{2} (2 m p a^\dagger - a^\dagger a p - p a^\dagger a), \quad (2)$$

where $\gamma$ represents dissipative coefficient. The corresponding non-unitary time evolution density matrix can be obtained as

$$\rho(t) = \frac{1}{m! L_m(-|\alpha|^2)} \sum_{k=0}^{\infty} \frac{(1 - e^{-\gamma t})^k}{k!} \hat{L}(t) a^k a^\dagger a^\dagger a^\dagger \hat{L}(t), \quad (3)$$

where $\hat{L}(t) = e^{-\frac{1}{2} \gamma t a^\dagger a}$. For the dissipative photon-added coherent state in Eq.(3), the total output state passing through a 50/50 beam splitter characterized by the unitary operation $e^{i (a^\dagger b + ab^\dagger)}$ with a vacuum mode b can be obtained or

$$\rho_{tot} = D_a(t) D_b(t) \frac{e^{-m \gamma \tau t} e^{-|\alpha|^2 (e^{-\gamma \tau t} - 1)}}{m! L_m(-|\alpha|^2)} \sum_{k=0}^{\infty} \frac{(e^{\gamma \tau t} - 1)^k}{k!} \hat{E}^k \hat{E}^\dagger_{m}(0) \langle 00 \rangle \hat{E}^\dagger_{k} \hat{D}_a(t) \hat{D}_b(t),\quad (4)$$

where $D_a(t) = e^{\frac{\gamma t}{2} a(t)^2} e^{\frac{\gamma t}{2} a(t)^2 a^\dagger(t)^2}$ and $D_b(t) = e^{-\frac{\gamma t}{2} a(t)^2} e^{\frac{\gamma t}{2} a(t)^2 a^\dagger(t)^2}$. The local unitary operators can not change entanglement, therefore, we only need to consider the entanglement of the mixed state given as follows:

$$\rho_{tot} = \frac{e^{-m \gamma \tau t} e^{-|\alpha|^2 (e^{-\gamma \tau t} - 1)}}{m! L_m(-|\alpha|^2)} \sum_{k=0}^{\infty} \frac{(e^{\gamma \tau t} - 1)^k}{k!} \hat{E}^k \hat{E}^\dagger_{m}(0) \langle 00 \rangle \hat{E}^\dagger_{k} \hat{E}^\dagger_{k}. \quad (5)$$

The log-negativity of the above density matrix can be analytically solved for the case of single photon excitation, i.e. $m = 1$, but its expression is still lengthy. The corresponding entanglement potential of the SPACSs quantified by log-negativity is given in Fig.5(b).

On the other hand, the presence of negativity in the Wigner function of the field is also the indicator of non-classicality. The Wigner function, the Fourier transformation of characteristics function \cite{16} of the state can be derived by \cite{17}

$$W(\beta) = \frac{2}{\pi} \text{Tr}[(\hat{O} - \hat{O}_c) \hat{D}(\beta) \rho \hat{D}^\dagger(\beta)], \quad (6)$$

where $\hat{O}_c = \sum_{n=0}^{\infty} |2n\rangle \langle 2n|$ and $\hat{O}_t = \sum_{n=0}^{\infty} |2n+1\rangle \langle 2n+1|$ are the even and odd parity operators respectively. In the photon loss channel described by the master equation (2), the time evolution Wigner function satisfies the following Fokker-Planck equation \cite{18}

$$\frac{\partial}{\partial t} W(q, p, t) = \frac{\gamma}{2} \left( \frac{\partial^2}{\partial q^2} + \frac{\partial^2}{\partial p^2} \right) W(q, p, t) \quad (7)$$

where $q$ and $p$ represent the real part and imaginary part of $\beta$, respectively. Substituting the initial Wigner function of a SPACS \cite{13}

$$W(q, p, 0) = \frac{-2 L_1(2q + 2ip - \alpha^2)}{\pi L_1(-|\alpha|^2)} e^{-2|q+ip-\alpha|^2} \quad (8)$$

and the initial Wigner function of a two photon-added coherent state (TPACS) \cite{13}

$$W(q, p, 0) = \frac{-2 L_2(2q + 2ip - \alpha^2)}{\pi L_2(-|\alpha|^2)} e^{-2|q+ip-\alpha|^2} \quad (9)$$

into the Eq.(7), we can obtain the time evolution Wigner function. In Fig.1, the Wigner function of the SPACS with $\alpha = 0.5$ at three different values of decay time are plotted. The phase space Wigner distribution of the pure SPACSs with $\alpha = 0.5$ loses its circular symmetry and moves away from the origin because of the appearance of a definite phase. The partial negativity of the Wigner function indicates the nonclassical nature of the single quantum excitation of the classical coherent field. The photon loss causes the gradual disappearance of the partial negativity of the Wigner function. The tilted ring-like wings in the distribution gradually start to disappear and the distribution becomes more and more similar to the Gaussian typical of a classical coherent field. In Fig.2, the phase space Wigner distributions at $p = 0$ of the SPACSs in the photon loss channel are depicted for several different values of $\alpha$ and $\gamma t$, from which the influence of photon loss on the partial negativity of the Wigner function is explicitly shown. For the cases of $\alpha = 0.1, 0.5, 1.0, 1.5$, it is found that those curves $W(q, 0)$ at decay times $\gamma t = 0, 0.2, 0.4, 0.6$ exhibit the partial negativity. The further photon loss will completely destroy the partial negativity. One can also find that, the larger the parameter $|\alpha|$, the more rapidly the Wigner function tends to the Gaussian function. The pure TPACSs exhibit more non-classicality than the pure SPACSs when
the entanglement potential is adopted as the measure of nonclassicality [19]. In Fig. 3, the Wigner functions of TPACSs with the parameter $\alpha = 0.5$ at three different values of decay times are depicted. It is also shown that the photon loss deteriorates its partial negativity. For more explicitly observing the details, we plot $W(q, 0)$ of the TPACSs with different values of $\alpha$ and $\gamma t$ in Fig. 4. Different from the cases of SPACSs, here, $W(q, 0)$ of these pure TPACSs with $|\alpha| \leq 1$ have two explicit negative local minimal values. As $|\alpha|$ increases, the absolute value of the negative local minimum at the left more rapidly decreases than the one at the right. As $\gamma t$ increases, the absolute value of negative minimum of $W(q, 0)$ decreases which implies the decreases of the non-classicality of the states. When $\gamma t$ exceeds a threshold value, the partial negativity of the Wigner distribution can not be explicitly observed from this figure. Similarly, the larger the parameter $|\alpha|$, the more rapidly the Wigner function of the TPACSs in the photon-loss channel tends to the Gaussian function. A natural question arises whether the partial negativity of the Wigner function can be used to quantitatively measure the non-classicality of certain kinds of nonclassical fields. It is obvious that the negative minimum of the Wigner function can not appropriately quantitatively measure the nonclassicality. For example, comparing Fig. 2(a, b) with Fig. 4(a, b) respectively, the negative minimum in the Wigner function of the pure SPACSs is smaller than the one of the pure TPACSs. However, there was already the evidence that the pure TPACSs possesses larger nonclassicality than the pure SPACS with the same $|\alpha|$. Nevertheless, the absolute value $P_{NW}$ of the total negative probability of the Wigner function may be a good choice for quantifying the nonclassicality. $P_{NW}$ is defined by

$$P_{NW} = \left| \int_{\Omega} W(q, p) dq dp \right|, \quad (10)$$

where $\Omega$ is the negative Wigner distribution region, and $|x|$ represents the absolute value of $x$. In Fig. 5(a), we plot $P_{NW}$ of the SPACSs with different values of $\alpha$ as the function of $\gamma t$. $P_{NW}$ decreases with $\gamma t$, and becomes invisible after a threshold value of $\gamma t \approx 0.7$. In Fig. 5(b), the entanglement potential quantified by log-negativity of SPACSs in the photon-loss channel is plotted. The entanglement potential also decreases with $\gamma t$. These calculations partially elucidate the consistent between $P_{NW}$ and the entanglement potential of the dissipative SPACSs at short time. Currently, the experimental quantitative investigation of the nonclassicality of the quantum optical fields is
still an open issue, and the experimental measurement of entanglement potential has still several technical difficulties. So, the measurement of $P_{NW}$ may be adopted as a replaced approach to investigate the influence of photon loss on the nonclassicality of the SPACSs. In Fig.5(c), we also investigate the effect of photon loss on $P_{NW}$ of the TPACSs with various values of $\alpha$. At short decay times $\gamma t \ll 1$, the values of $P_{NW}$ of the TPACSs are larger than those of the SPACSs with the same beam intensity $|\alpha|^2$. However, $P_{NW}$ of the TPACSs is more fragile against photon loss than the one of the SPACS. As an illustration, for $\alpha = 0.1$ and $\gamma t \geq 0.34$, the $P_{NW}$ of TPACSs is smaller than the one of SPACS.

From Fig.5(b), we can also find that the entanglement potentials of SPACSs with different values of $|\alpha| < 1$ more rapidly decrease than the ones of SPACSs with $|\alpha| > 1$ in the photon loss channel. Both the pure SPACSs and pure TPACSs are non-gaussian nonclassical states with partial negativity of the Wigner distributions for any large but finite values of $|\alpha|$. In Fig.5(d), the dependence of $P_{NW}$ of the pure SPACSs and the pure TPACSs on $|\alpha|$ are shown. It is found that both the $P_{NW}$ of the pure SPACSs and the pure TPACSs reduce with the increase of $|\alpha|$, and the TPACS possesses larger...
than the SPACS with the same value of $|\alpha|$. This property is also coincident with the dependence of entanglement potentials of the pure SPACSs and the pure TPACSs on $|\alpha|$.

In summary, we have investigated the non-classicality of photon excitation of classical coherent field in the photon loss channel by exploring both the entanglement potential and the partial negativity of the Wigner function. The total negative probability defined by the absolute value of the integral of the Wigner function over the negative distribution region is introduced. Consistent behaviors of the total negative probability and the entanglement potential of single quantum excitation of classical coherent fields are found for the short time photon loss process. Similar dependence of the total negative probability and the entanglement potential on the beam intensity is revealed for few photon-added coherent states. The partial negativity of the Wigner function can not be observed when the decay time exceeds a threshold value, while the entanglement potential always exists for any large but finite decay time. In the future tasks, it is interesting to investigate the variation of nonclassicality of general nonclassical states in gaussian channel or non-gaussian channel.

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