Optimization technique for allocation scheme and low head hydroelectric power plant parameters

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Abstract The article deals with the formalized description of low head hydroelectric power plants and their cascades with the choice of technical and geometrical parameters’ optimization, optimality criteria and constraints that are imposed at optimization parameters. The goal of the work is to develop a method for optimizing the parameters of low head hydroelectric power plants, including determination of hydroelectric stations’ location in the cascade, and to create a computer model to solve the problem. Methods of mathematical modeling and methods of multicriteria optimization have been used. Formalized description of the problem of optimal placement of hydroelectric power plants on the river network is given. The following criteria are considered as optimality criteria: economic criterion (net present value, construction and operation costs), environmental criterion and energy criterion. The main parameters of low head hydroelectric power plants have been identified; the interrelations of these parameters with optimization criteria have been established. The method has been developed for optimizing the parameters of a hydroelectric power plant at their determined location on a watercourse. The following results have been obtained. 1) The problem of optimization of hydroelectric stations on a watercourse, which takes into account the energy, economic, social, environmental, water-energy and resource features of the region, is formulated and presented in the form of mathematical model. 2) Using the graph network and the corresponding vector values of the function, Pareto and minimax solutions of this problem have been found. 3) With the determined location of hydroelectric power station on a watercourse, method and algorithm have been developed for optimizing technical and geometrical parameters. Technical and economic models of the energy process at the hydroelectric power plant, as well as technical and economic models of the main and auxiliary equipment and facilities (dams, hydroelectric power station buildings, etc.) have been developed.

1. Introduction

Hydropower engineering development including small HPP has a high social and economic importance for Russia. According to the government energy strategy it is necessary to keep the level of non-fuel energy sector in the structure of the generating capacities up to 2035. For further development of this energy sector it is necessary to take into account reasonably additional effects resulting from construction of water power facilities. This influences also on their parameters.

HPP is characterized by a complex of the characteristics that determine efficiency of its functioning in various aspects. In common case these characteristics reflect economic efficiency, quality and reliability of electric power production, environmental impact, economic and social consequences [1, 3, 4, 5, 8, 9, 13, 18].

Validation of the HPP parameters is a complex problem. Natural, social, ecological, economic, technical and other factors make influence on this problem. Different types of the resources (limited
ones) generating different flows of products are involved in the process of creation and operation of hydro power plant. Their estimation (if achieving the main goal – power supply) determines justification of their parameters choice.

It is necessary to take into account many factors (economic, resource, energy, social, environmental, etc.) when choosing the most advantageous locations for HPP construction and evaluating their social and economic efficiency. The key criteria include the following: economic, resource, social and ecological significance of the object [1, 3].

Based on these factors integral importance of HPP construction is estimated. The preferable objects for financing and their efficiency are determined by means of comparison of HPP integral importance with alternative power supply technologies.

Problems of development of small hydropower in Russia are described in [17]. Some issues of optimizing the parameters of hydroelectric power plants are presented in papers [1, 4, 6, 10, 18].

2. Statement of problem (model)
Let’s consider an open water passage or its part with known cadastre of water power resources. Hydrological information related to characteristic sites or the complete water course for several years is known. Morphometric characteristics for characteristic sites of the water course are specified (capacity and areas curves depending on water surface elevation in the site and level-flow dependencies curves for various seasons). The requirements of social–environmental nature and requirements of water consumers and water users are specified in the form of limitations (kind of inequality) related to HPP operation modes [1, 4, 5].

For the considered water course the graphs of water consumption by different parties of hydrologic system of the given region and return water flow rate are specified. It is assumed that a part of social and environmental requirements are taken into consideration in the form of the so called “red line”. Herewith upper and bottom marks of “the red line” are determined by the equations:

\[ Z^H (L) = \min (Z_1^{\text{max}} (L), Z_2^{\text{max}} (L), ... Z_l^{\text{max}} (L)), \]

\[ Z^T (L) = \max (Z_1^{\text{min}} (L), Z_2^{\text{min}} (L), ... Z_l^{\text{min}} (L)), \]

where \( L \) is a longitudinal coordinate counted along the river bed.

HPP together with its reservoirs can be situated only inside the admissible area limited by the mentioned lines. Mathematically the problem is formulated by the following way. The water power cadastre is given (Figure 1). We shall find: the vectors \( L \) and \( H \) that marking the points of HPP cascade site location and water heads in these points. We shall optimize: \( f (L, H) \) provided the following limitations:

\[ Z^T (L) \leq Z(L) \leq Z^H (L). \]

The objective vector-function being maximized consists of three components. Each function of the objective is determined for the cascade variant \( j \) by the following formula:

\[ f_i^j = \sum_{k=1}^{m_i^j} f_{ik}^j, \quad i = 1, 2, 3, \]

where \( m_i^j \) is a number of HPPs in the cascade in the variant \( j \). The components of \( f_{ik}^j \) are determined by the following way.
Figure 1. The results of water-energy cadastre researches of the watercourse.

Criterion of environmental acceptability:

\[ f_{1k}^j = \frac{C_{1k}^j}{C_{MAX} - C_{MIN}}, \]

where \( C_{1k}^j \) is ecology capacity of the HPP in the variant \( j \), \( C_{MAX}, C_{MIN} \) - maximal and minimal HPP ecology capacity. Ecology capacity means ecological costs related to the usage of the environment while making a product unit [3]. Ecology capacity value allows estimating of the relative “hazard” of the production, power in particular. Common formula of ecology capacity can be presented as follows:

\[ C_{1k}^j = (k_y \cdot C_{1k}^{D,j} + k_{II} \cdot C_{1k}^{II,j}) \cdot \mathcal{C}_{1k}^j, \]

where \( \mathcal{C}_{1k}^j \) is electric power generation by \( k \)th HPP in the cascade in the variant \( j \), \( C_{1k}^{D,j} \) - damage capacity, \( C_{1k}^{II,j} \) - nature capacity of the variant, \( k_y, k_{II} \) - constant coefficients.

Damage capacity is a value of the economic damage per product unit. Nature capacity is a cost estimate of the natural resources per product unit. Here we should mention that damage capacity value determines expenses related to environmental protection measures, and nature capacity value determines compensation expenditures and residual damages. HPP economic efficiency - criterion \( f_{2k}^j \)

\[ f_{2k}^j = \frac{E_{2k}^j}{E_{MAX} - E_{MIN}}, \]

where \( E_{2k}^j \) - HPP expenses in the variant \( j \), \( E_{MAX}, E_{MIN} \) - maximal and minimal HPP expenses.

Criterion of HPP energy efficiency \( f_{3k}^j \) is calculated by the formula 8.

\[ f_{3k}^j = \frac{\mathcal{C}_{3k}^j}{\mathcal{C}_{MAX}}, \]
where $E_{kj}$ is electric power generation on the $k$\textsuperscript{th} HPP in the variant $j$, $E_{MAX}$ - maximal electric power generation.

We shall find such variant of the HPP that maximizes objective functions. Let’s assume that we have generated the variants of the HPP cascades taking into account limitations. These variants of partition form a certain network (Figure 2). Graph correspondent to the cascade of HPPs variants given in [1].

Figure 2. Cascade of HPPs variants.

Let us formulate the problem as a multicriterion optimization problem on the network [1]. Let $S = \langle N, R, \vec{f} \rangle$ denote a network whose graph $G = \langle N, R \rangle$ belongs to the class of directed and acyclic graphs. Here $N$ is a set of nodes, $R$ a two argument relation defined on the set $N$, and $\vec{f}$ a vector function defined on the arcs of the graph $G$. The two argument relation $R$ defines which nodes are connected by arcs and in which direction. The vector function $\vec{f}$ is defined as follows. A vector

$$\vec{f}(n_r, n_s) = [f_1(n_r, n_s), ..., f_k(n_r, n_s)]^T$$

is associated with any pair of nodes $n_r, n_s \in N$ for which $n_r R n_s$. The components $f_i$ ($i \in I$) of the vector $\vec{f}(n_r, n_s)$ which are given as the 'weights' assigned to the arcs connecting the pairs of nodes $n_r, n_s$.

We denote $p_j = [n_1, ..., n_j, ..., n_e]$ the $j$ path in the graph joining the initial node $n_1$ to the terminal node $n_e$, $P = \{p_j\}$ a set of all $p_j$ paths in the graph, where $j = 1, 2, ..., J$. The problem is formulated as follows. Find a path $a^*_j = [n_1^*, ..., n_j^*, ..., n_e^*]^T$ in the network $S = \langle N, R, \vec{f} \rangle$ which optimizes a vector function $\vec{f}(p_j) = [f_1(p_j), ..., f_k(p_j)]^T$ defined in $k$ -dimensional space $E^k$. The $i$ component of the vector is determined from

$$f_i(p_j) = \pm \sum_{n_r, n_s \in p_j} f_i(n_r, n_s),$$

where sign '+' refers to the functions which are to be minimized, and sign '-' to the functions which are to be maximized.
The ideal vector \( f^0 = [f^0_1, f^0_2, \ldots, f^0_k] \) can be determined after finding the extreme paths for each criterion problem separately. Let \( p_{j}^{0(i)} = [n^0_{r_1}, \ldots, n^0_{r_s}, n^0_{c}] \) be the path which minimizes or maximizes the \( i \)-th objective function \( f_i(p_j) \). In other words, the path \( p_j^{0(i)} \in P \) is such that, for the objective functions which are to be minimized we have

\[
\sum_{j \in s} f_i(n_{r}, n_{c}) = f_i(p_j) = \min_{p_j \in P, n_{r}, n_{c} \in p_j} \sum_{j \in s} f_i(n_{r}, n_{c}),
\]

(11)

and for the objective functions which are to be maximized we have

\[
f_i(p_j) = \max_{p_j \in P, n_{r}, n_{c} \in p_j} \sum_{j \in s} f_i(n_{r}, n_{c}).
\]

(12)

In network terminology the path which satisfies (13) is called the shortest path and at which satisfies (14) the longest path.

3. Algorithm of optimization

3.1. Pareto optimal paths

A path \( p^* \in P \) is Pareto optimal if for every \( p_j \in P \) either

\[
\bigcap_{i \in I} f_i(p_j) = f_i(p^*)
\]

(13)

or, there is at least one \( i \in I \) such that

\[
f_i(p_j) \neq f_i(p^*).
\]

(14)

Obviously, we have a variety of Pareto optimal paths. We use \( P^{OP} \) to denote this set and \( \tilde{f}^{OP} \) to denote the map \( P^{OP} \) in the space of objectives \( E^k \).

3.2. Min-max optimal path

Let \( z(p_j) = [z_1(p_j), \ldots, z_k(p_j)] \) be a vector of relative increments of the objective functions for the path \( p_j \). The components of the vector \( z(p_j) \) are defined as follows:

\[
\bigcap_{i \in I} z_i(p_j) = \max \{ z_i^+(p_j), z_i^-(p_j) \}
\]

(15)

where

\[
z_i^+(p_j) = \frac{|f_i(p_j) - f_i^0|}{|f_i^0|}, \quad z_i^-(p_j) = \frac{|f_i(p_j) - f_i^0|}{|f_i(p_j)|}.
\]

(16)

A path \( p^* \in P \) is optimal in the min-max sense if for every \( p_j \in P \) the recurrence formulas is satisfied.

3.3. Parameters to optimize (technical and geometric)

These parameters include (Figure 3): reservoir live storage \( V_E \), number of hydraulic units \( m \), turbine type \( R_H \), turbine runner diameter \( D_1 \), small HPP power house type \( R_H \), etc. \([1, 4, 11, 12, 14-16, 18-20]\). Vector of these parameters is defined as \( X \).
Figure 3. Hydro power plant cross section with optimized parameters indicated.

For the purpose of HPP parameters optimization the target function is described as follows:

\[ E_{2k}^j(X) = CRF \cdot K_k^j(X) + U_k^j(X), \]

where \( X \) is vector of optimized HPP parameters; \( E_{2k}^j(X) \) is total cost of HPP with \( X \) parameters; \( K_k^j(X) \) is HPP construction capital cost; \( CRF \) is a conversion factor to recalculate non-recurrent capital cost into equivalent annual expenses during estimated service life \( T_{SL} \) with real interest rate \( r \); and \( U_k^j(X) \) is HPP running cost. For determination of the expenses the following model is used [2].

\[ K_k^j = \alpha \cdot \theta \cdot \left( \frac{N_{Y_k}^j}{(H_k^j)^\gamma} \right)^\beta, \]

\[ U_k^j = \varepsilon \cdot K_k^j, \]

where \( N_{Y_k}^j \) \( k \)th HPP installed capacity in the variant \( j \), \( H_k^j \) - water head on the \( k \)th HPP in the variant \( j \), \( \alpha, \theta, \gamma, \beta, \varepsilon \) - some constants.

Let’s assume that we have generated HPP variant with \( H_k, Q_k \) parameters. Water-power calculations are made for these parameters and installed water power is determined - \( N_{Y_k}^j \).

3.4. Limitations

\[ N_{HPP}(i) = g \cdot Q_T(i) \cdot H_T(i) \cdot \eta_{HU}(i) \cdot m \leq N_{Y_k}^j, \]

\[ K(X) \leq K_{MAX}, \]
where \( g \) is gravity acceleration; \( N_{\text{HPP}} \) is HPP capacity in \( i \) mode; \( Q_i \) hydraulic turbine flow in \( i \) mode; \( H_i \) is hydraulic turbine head in \( i \) mode; \( \eta_{\text{HU}} \) is hydraulic unit efficiency; \( K \) is HPP construction capital cost; \( K_{\text{MAX}} \) is maximal HPP construction capital cost.

4. Computation scheme
When HPP dam site location is defined, variants of structures at HPP site are considered. For run-of-river or reservoir HPP, the power site includes a spillway structure and a fixed dam.

The spillway structure may use the HPP power house (combined with surface or bottom spillways), spillway dike, shore surface or bottom spillway. The HPP power house dimensions are calculated in accordance with parameters of equipment to be installed. The length of the HPP power house determines what part of waterfront might not be used by spillway structures.

The spillway structures are calculated in consideration of necessary discharge flow. The fixed dam length should provide cross section closure. In course of design works there shall be determined parameters of all structures and volumes of concreting and earthworks. Stability of structures is checked, and capital cost is calculated.

Layout of structures at the HPP site depends on its parameters \( X \). Let’s check a typical situation where waterfront is created by HPP power house, spillway dike and fixed dam. The design process requires choosing location of these principal structures, and calculating their main dimensions.

Layout process is iterative. First, a designer arrange structures at topographic map and geological section (without any dimensional calculations yet). That provides information for following calculations, including level marking, geological features, etc.

After that, HPP power house view plan is prepared and waterfront length is checked. Then it is checked if the HPP power house and spillway dike fit the dimensions. The following design process shall be used for different structures at the site: next, fixed weir shall be designed, and construction capital cost for the chosen site shall be calculated; next, spillway dike is designed. To confirm the crest width, it is necessary to consider its decreasing by higher unit discharge and longer fixed dam.

5. Formal description
Let’s describe the task formally. HPP constructions include: a fixed weir, spillway dike, HPP power house itself, headrace and/or spillway channel or upstream pipeline. Let’s name parameter vectors of these structures as \( n \), \((X)_S \), \((X)_H \), \((X)_C \), \((X)_P \) correspondingly. Let’s call control (optimized) parameters of the structures as vectors \((Y)_D \), \((Y)_S \), \((Y)_H \), \((Y)_C \), \((Y)_P \). The following additional assumptions have also been taken: 1) The HPP is equipped with similar hydraulic machines with the same characteristics; 2) Number of power house blocks is equal to number of power units. Vector elements \((Y)_i \) are limited by some factors described in [11, 12, 14-16, 18-23].

In accordance with basic HPP structures, basic parameter vectors are defined. Basic parameters are calculated using mathematic models of HPP structures. The mathematic model of HPP structures includes a combination of mathematic relations that establish links between basic properties and controlling variables.

Hydraulic power calculations take a major role in HPP parameters argumentation. For fixed HPP parameters, hydraulic power calculations are carried out, hydro capability chart is drawn up, and guaranteed HPP capacity is calculated. The HPP equipment and structure parameters \((R_H, m, Q_{\text{IM}}, R_{\text{HU}}, D_1, n)\) affect HPP basic properties [18].

With hydraulic power calculations, the following optimization task is solved. Let’s take \( t_0 \) and \( t_k \) for initial and final time points. For this time period \( T = [t_0; t_k] \) divided into \( k \) non-overlapping intervals \( \Delta t_i = [t_{i-1} ; t_i], \ i \in (1, k), \) and planned dependence \( a(V_i) \) and \( b(Q_i) \) it is necessary to find maximum of functional [5, 18]:

\[\text{maximize} \quad f(V_i, Q_i) \quad \text{subject to} \quad a(V_i) = b(Q_i) \]
\[ W = \sum_{i=1}^{N} N_i \Delta t_i \rightarrow \max, \quad N_i \leq N = \text{const} \quad \Delta t_i = t_i - t_{i-1}, \quad (22) \]

For
\[ N_i = 9.81 Q_i H_i \eta_i, \quad H_i = \frac{H_{i-1} + H_i}{2}, \quad H_i = a(V_i) - b(Q_i), \quad (23) \]
\[ V_i = V_{i-1} + (S_i - Q_i - P_i) \Delta t_i, \quad (24) \]
with the following limitations
\[ Q_i \leq Q_{iHM}, \quad V_{\text{min}} \leq V_i < V_{\text{max}} (V_{NW} V_{\text{min}}), \quad V_E = V_{\text{max}} (V_{NW} V_{\text{min}}) \quad (25) \]

Formulas (22) - (25) for \( i \) mode use the following symbols: \( N_i \) is HPP capacity, \( Q_i \) is HPP flow, \( H_i \) is average head, \( H_i \) is HPP net head, \( V_i \) reservoir volume, \( P_i \) water losses from reservoir (idle discharge, evaporation losses, filtration, water consumption from reservoir by third parties), \( S_i \) water intake to reservoir, \( Q_{iHM} \) HPP maximum throughput, \( V_{\text{min}} \), \( V_{\text{max}} \) minimum and maximum water volume in reservoir.

Process in HPP water-conveyance system determines directly HPP capacity in different modes. Mathemetic model of process in HPP water-conveyance system is described by the following equations [18].

Annual power production by HPP is:
\[ W_T(j) = \sum_{i=i_{ij}}^{i_{nj}} N_H(i) \cdot \Delta t_i, \quad (26) \]
where \( i_{ij} \) and \( i_{nj} \) are numbers of initial and final operating modes of HPP during hydro-economic year \( j \). Each mode \( i \) is active for a certain period and the sum of these periods equals a year.

HPP capacity in \( i \) mode is:
\[ N_H(i) = g \cdot Q_T(i) \cdot H_T(i) \cdot \eta_{\text{Hu}}(i) \cdot m \leq N, \quad (27) \]
where
\[ \eta_{\text{Hu}}(i) = \frac{H(i) - \nabla H(i)}{H(i)} \cdot \eta_T(i) \cdot \eta_G(i), \quad (28) \]
where \( g \) is gravity acceleration; \( \eta_T(i) \) is hydraulic turbine efficiency in \( i \) mode, \( \eta_G(i) \) is generator efficiency in \( i \) mode.

Turbine discharge \( Q_T(i) \) is calculated by formula:
\[ Q_T(i) = \frac{Q_H(i)}{m}. \quad (29) \]

Head losses in HPP water-conveyance system in \( i \) mode [5, 11, 12, 18]:
\[ \nabla H(i) = \sum_k \Delta h_{mk}(i) + \nabla h_T(i) + \sum_k \Delta h_{tk}(i) + \Delta h_{n}(i), \quad (30) \]
where \( \nabla h_T(i) \) represents losses in hydraulic turbine flow section:
\[ \nabla h_T(i) = (1 - \eta_T(i)) \cdot H_T(i). \quad (31) \]
Head at the turbine in \( i \) mode:

\[
H_T(i) = H(i) - \sum_k \Delta h_{mk}(i) - \sum_k \Delta h_{rk}(i) - \Delta h_w(i),
\]

where \( H(i) = \nabla Z_{iHw}(i) - \nabla Z_{iTw}(i) \) is static head in \( i \) mode; \( \nabla Z_{iHw}(i), \nabla Z_{iTw}(i) \) water levels on headrace and tail-bay side accordingly.

Headrace water levels at given time moments are determined by schedule of reservoir water levels; tail-bay levels depend on water discharge to tail-bay (regardless of the fact that this water is discharged through HPP turbines, spillway duct or other structures) in accordance with curve \( Q = f(z) \).

Head losses \( \nabla h_w(i) \) in intake channel with a grate are calculated by the formula \([15, 20]\):

\[
\nabla h_w(i) = \xi_w \cdot \frac{\vartheta_{wi}^2}{2 \cdot g},
\]

where \( \vartheta_{wi} \) is average flow rate in the channel in \( i \) mode, \( \xi_w \) is factor of losses at the grate. Head losses at the channel length \( \nabla h_{lk}(i) \) are calculated by Weisbach – Darsi formula (for diversion HPP pipeline) \([15, 20]\):

\[
\nabla h_{lk}(i) = \lambda \cdot \frac{L_k}{D_p} \cdot \frac{\vartheta_{rk}^2}{2 \cdot g},
\]

where \( \vartheta_{rk} \) is average flow rate in \( k \) section of the pipeline in \( i \) mode, \( \lambda \) is a factor of losses by length, \( L_k \) is length of \( k \) section of the pipeline. Local losses \( \nabla h_{mk}(i) \) depend on type and configuration of local restrictions.

Average net head is taken for HPP design head:

\[
H_{Wv} = \frac{\sum H_T(i) \cdot N_H(i) \cdot t_i}{\sum N_H(i) \cdot t_i},
\]

where \( H_T(i) \) head at hydraulic turbine in \( i \) mode, \( t_i \) operating time in \( i \) mode.

Generator rotor rotation speed is taken the same as turbine rotation speed. If a step-up gear is used, the step-up factor should be used. To determine hydraulic generator main parameters the following initial data is used: hydraulic turbine nominal capacity \( N_T \), hydraulic unit type \( HU \), runner diameter \( D_1 \), and nominal rotation speed \( n \).

HPP hydraulic units should be used, where practical, along with serial induction generators or motors for generators, in some cases step-up gear should be used. In special cases, serial synchronous generators should be used.

The HPP building mathematic model allows calculating roughly HPP power house dimensions and volume of concreting depending on SHPP parameters. Usability of different HPP power house types is also indicated.

For cavitation-free operation level of hydraulic turbine runner \( \nabla PK_j \) in \( i \) mode should satisfy the next condition \([11, 12, 20]\):

\[
\nabla R_j = \nabla Z_{iTw} + H_{Si},
\]

where \( H_{Si} \) is permissible draught head in \( i \) mode that is mostly limited by cavity factor. Taking into account dependence \( \nabla ZH(Q) \) and formula for calculation of permissible draught head as well as
hydraulic turbine universal characteristic, dependence $\nabla PK = f(Q)$ is calculated for various heads and number of running units. Calculated minimum level is taken for operating level:

$$VR = \min(\nabla R_i).$$ (37)

Weight of serial hydraulic turbines is indicated by manufacturers.

Specific volume of concreting is calculated by the next formula [18]:

$$w_0 = \varphi(\bar{n}_S, D_1, H).$$ (38)

where $\varphi$ - polynomial function; $\bar{n}_S$ is factor of hydraulic turbine specific speed (r.p.m.) that is calculated by formula [11, 20]:

$$\bar{n}_S = D_1 \cdot N_T^{0.5} \cdot H^{-1.25}.$$ (39)

Once specific volume of concreting is calculated, it is possible to calculate HPP construction capital cost and then calculate running expenditures and working cost of energy production at the HPP during service life. These evaluations are done for each variant of parameters $X$; from many variants, by using criterion (19), optimal variant of the HPP structures and equipment is taken.

6. Conclusion

In this article we formulated and presented in the form of mathematical model the problem of cascade HPP sites' location optimization with regard for energetic, economical, social, ecological, water-energetic and resource peculiarities of the region.

The optimization algorithm for solving the problem was worked out. Using networks graph and corresponding vector-valued function, Pareto and min-max solution of the problem was found.

Basing on suggested models of energy generating process, equipment, and structures HPP parameters optimization algorithms were developed which take into account some assumptions. The developed algorithm is used for optimized location of HPP on site.

References

[1] Sidorenko G I, Alimirzoev A S 2015 Multicriterion optimization of power sites location on the watercourse Applied Mechanics and Materials 725 293-8.
[2] Gordon J L 1983 Hydropower cost estimates Water Power and Dam Construction 30-7
[3] Khrisanov N I, Arefiev N V 1992 Ekologicheskoe obosnovanie gidrotekhnicheskogo stroitelstva. St. Petersburg: Polytechnic university publishing house 167.
[4] Sidorenko G I, Alimirzoev A S 2014 Optimization Technique for Allocation Sheme and Hydro Power Plant Parameters with Respect Regional Peculiarities Applied Mechanics and Materials 672-674 472-6.
[5] Arbuzov J D, Bezrukikh P P 2002 Resursy i effektivnost ispolzovaniya vozobnovlyaemykh istochnikov energii v Rossii, Executive editor Bezrukikh, P.P. St. Petersburg.: Nauka, 314
[6] Heinsson E B, Eliasson J 2002 Optimal Design and Cost/Benefit Analysis of Hydroelectric Power Systems by Genetic Algorithms, VIII Symposium of Specialists in Electrical Operations and Expansion Planning, May, 19-23, 2002. Brasilia, Brasil, pp.1-7.
[7] Haizhang Zhang, Jun Zhang 2013 Vector-valued reproducing kernel Banach spaces with applications to multi-task learning. Journal of Complexity 29195-215.
[8] Harb G, Schneider J, Zenz G, Bäumel E, Lesky U, Rechberger A 2013 EU-Projekt SEE HydroPower, Wasserland Steiermark, pp. 34 – 38
[9] Taele B M, Mokhutsoane L, Hapazari I 2012 An overview of small hydropower development in Lesotho: Challenges and prospects Renewable Energy 44 448-52
[10] Heinsson E B 1990 Optimal Sizing of Projects in a Hydro-based System. IEEE Transactions on Energy Conversion 5(1) 32-8.
[11] Shchavelev D S, Vasilev Yu S, Pretro G A 1981 Gigroenergeticheskie ustanovki 2nd edition, revised and enlarged, Leningrad: Energoizdat 520.
[12] Yu.S.Vasilev Ispolzovanie vodnoy energii 1995 M.: Energoatomizdat, 608.
[13] European Small Hydropower Association: Energy Recovery in existing infrastructures with small hydropower plants, Sixth Framawork Programme, Publication by Mhylab, Switzerland, June, 2010.
[14] Voznesenskiy A 1967 Energeticheskie resursy USSR. Gidroenergetika. The USSR Academy of Sciences. Nauka 600.
[15] Malinin N K 1985 Teoreticheskie osnovy gidroenergetiki, M.: Energoatomizdat.
[16] Mihailov L P, Feldman B N, Markanova T K et al 1989 Malaya gidroenergetika, Moscow: Energoatomizdat 184.
[17] Bliashko I I 2011 Problemy maloy gidroenergetiki v Rossii Malaya energetika 3-4 21-5
[18] Vasilev Y S, Sidorenko G I, Frolov V V 2012 K metodike obosnovaniya parametrov malykh GES Nauchno-tekhnicheskie vedomosti Saint Petersburg State Polytechnical University 4(70), 12-22.
[19] Karelin V Ya, Volshanik V V 1986 Sooruzheniya i oborudovaniya malykh gidroelektrostantsiy, M.: Energoatomizdat, 200.
[20] Giesecke J, Mosonyi E 1997 Wasserkraftanlagen. Plunning, Bau und Betrieb, Berlin Heidelberg, 623.
[21] Davydov R, Antonov V, Molodtsov D, Trebukhin A 2018 Mathematical Simulation of Flood Management by Hydro Systems with Temporarily Flooded Reservoirs Advances in Intelligent Systems and Computing 692 915-920 DOI: 10.1007/978-3-319-70987-1_99
[22] Nikonova O, Skvortsova O, Ivanov T, Terleev V, Nikonorov A, Togo I, Volkova Y, Pavlov S 2017 Assessment of the Investment Appeal of Hydropower Construction Based on the Analytic Hierarchy Process MATEC Web of Conferences 106 08049 DOI: 10.1016/j.energy.2016.06.102
[23] Badenko V, Badenko N, Nikonorov A, Molodtsov D, Terleev V, Lednova J, Maslikov V 2016 Ecological Aspect of Dam Design for Flood Regulation and Sustainable Urban Development MATEC Web of Conferences 73 03003 DOI: 10.1051/matecconf/20167303003