Energy spectrum of simply constant chromoelectric flux tubes

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Abstract

In this article we obtain the energy spectrum of colored, spinor particles in chromoelectric flux tubes. The chromoelectric field of the flux tubes considered here comes from simply constant gauge potentials rather than from covariantly constant gauge potential, as is usually the case. The energy spectrum of the simply constant flux tubes is different than that of the covariantly constant flux tubes. The spectrum is discrete due to the walls of the tube and with a plus/minus constant shift depending on the magnitude of the constant chromoelectric background. This goes against the classical intuition where one would expect a charged particle in a uniform “electric” field to accelerate with ever increasing velocity/energy i.e. there would be no constant energy eigenvalue.

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I. INTRODUCTION

Flux tubes – both “electric” and “magnetic” – play or are thought to play a role in many physical systems: (i) In type II superconductors Abrikosov magnetic flux tubes are an experimentally observed phenomenon [1]; (ii) in Yang-Mills-Higgs theory Nielsen-Olesen flux tubes are interesting theoretical solutions to the classical field equations [2]; (iii) in the dual superconducting model of QCD, chromoelectric flux tubes are thought to lead to quark confinement [3]; (iv) in the glasma state which is thought to occur in the early stages of high energy collisions of ultra heavy ions [4].

In this article we find the energy spectrum of color charged, spinor particles inside a flux tube with a constant chromoelectric field. The constant chromoelectric field we consider comes from a constant gauge potential rather than usual case of a covariantly constant gauge potential [5]. There have been a few other works which have studied constant color electric or color magnetic fields coming from simply constant vector potentials [6] [7]. The new features of the present work are: (i) we consider tubes of such constant color fields, instead of having the constant color fields permeate all space; (ii) we study the energy spectrum of spinor particles inside these tubes. The energy spectrum is not the same as that produced by a covariantly constant gauge potential, as is to be expected since the covariantly constant vector potential is gauge inequivalent to the simply constant vector potential. The energy spectrum of the simply constant potential has the unusual feature that the energy levels are those of a particle inside a cylindrical tube but shifted up or down by some constant value which depends on the strength of the chromoelectric field. Since the energy spectrum is different for the two types of potentials which yield constant chromoelectric fields there might be some situations where this could yields experimentally distinguishable results. One such possible case might be the glasma state of matter which occurs in high energy heavy ion collision such as at RHIC or ALICE. One could ask whether the flux tubes of the glasma state come from covariantly constant potentials or simply constant potentials. In the conclusion we will discuss the possible relevance of our results to the glasma state.

Previous work studying the instability of such simply constant field configurations can be found in [8–10]. Related studies of the motion of color charged particles in chromomagnetic and chromoelectric fields produced by simply constant gauge potentials are given in the references [9, 11–16]. In the present work we focus on the system of a spinor, colored particle
in a classical, chromoelectric flux tube produced by a simply constant gauge potential. This problem is solved within the framework of ordinary quantum mechanics i.e. we solve the energy eigenvalue problem of the Dirac equation for a colored spinor in the classical background of a simply constant vector potential. The confinement property of QCD is taken into account by taking the wave function of the spinor particle to vanish at the cylindrical walls of the flux tube.

II. QUARKS IN THE CHROMOELECTRIC FLUX TUBE

Quarks belong to the fundamental representation of the color group $SU_c(3)$. The Dirac equation for spinor quarks in an external color field is obtained from the free Dirac equation by minimal coupling – one shifts the momentum as $P_\mu = p_\mu + gA_\mu + gA_\mu^a \lambda^a / 2$. This gives the Dirac equation for a quark with mass $M$:

$$(\gamma^\mu P_\mu - M) \Psi = 0,$$  \hspace{1cm} (1)

where $\gamma^\mu$ are Dirac matrices, $A_\mu^a$ is the external gauge field, $\lambda^a$ are the Gell-Mann matrices, $g$ is the color interaction coupling constant, and the color index $a$ runs $a = 1, \ldots, 8$. The spinor $\Psi$ has two Majorana components $\phi$ and $\chi$,

$$\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}.$$  \hspace{1cm} (2)

Each of these Majorana components are split into two spin components

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix},$$  \hspace{1cm} (3)

where $\psi$ stands for both spinors $\phi$ and $\chi$. The components $\psi_\pm$ transform under the fundamental representation of the color group $SU_c(3)$ and have three color spin components corresponding to the eigenvalues of color spin operator $T^3 = \lambda^3 / 2$:

$$\psi_\pm = \begin{pmatrix} \psi_\pm (\lambda^3 = 1) \\ \psi_\pm (\lambda^3 = -1) \\ \psi_\pm (\lambda^3 = 0) \end{pmatrix} = \begin{pmatrix} \psi_\pm^{(1)} \\ \psi_\pm^{(2)} \\ \psi_\pm^{(3)} \end{pmatrix}.$$  \hspace{1cm} (4)

Now we study of motion of quarks in a constant color electric field coming from a simply, constant non-Abelian vector potential as introduced in [6]. (Note in an Abelian theory it
is not possible to have constant electric or magnetic fields coming from a constant vector potential). The field strength tensor for a color electric/magnetic field coming from constant gauge potentials, $A_\mu^a$ is given by $F^c_{\mu\nu} = g f^{abc} A_\mu^a A_\nu^b$. The $f^{abc}$ are the structure constants of $SU_c(3)$. The $i^{th}$ component of the color electric field corresponds to one of the indices $\mu, \nu = 0$ while the other index $\nu, \mu = i$. Without loss of generality we will take the chromoelectric field to point in the $i = 1$ or $x$-direction.

A constant chromoelectric field can be given by the following choice of constant components of a non-Abelian vector potential $A_\mu^a$:

$$A_\mu^1 = (\sqrt{\tau}, 0, 0, 0), \quad A_\mu^2 = (0, \sqrt{\tau}, 0, 0), \quad \text{all others } A_\mu^a = 0,$$

where $\tau$, $\tau_1$ are constants. The field strength tensor $F^a_{\mu\nu}$ has one non-zero component:

$$F^3_{01} = g \sqrt{\tau_1 \tau} = E^3_x. \quad (6)$$

As mentioned above the chromoelectric field is along the $x$-axis in ordinary space. Also without loss of generality we have taken the chromoelectric field to point along the 3-axis in color space. We note again that the constant color fields produced by covariantly constant as studied in [5] are gauge inequivalent to the simply constant potentials studied in (5). Thus one should be able to experimentally distinguish between these two options for obtaining a constant color field flux tube.

In this background, chromoelectric field the squared Dirac equation has the form:

$$\left[ P^\mu P_\mu + \frac{i}{4} g \sigma^{\mu\nu} F^c_{\mu\nu} \chi^c - M^2 \right] \Psi = 0, \quad (7)$$

where $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$. The corresponding equations for the $\phi$ and $\chi$ spinors are:

$$\left[ p^2 - M^2 + \frac{1}{4} (G^2 - G_1^2) I_2 + \mathcal{G} P_1 \chi^2 - \mathcal{G}_1 E \chi^1 + \frac{i}{2} \sigma^x \mathcal{E}^3 \chi^3 \right] \phi, \chi = 0. \quad (8)$$

where here $\mathcal{G} = g \tau^{1/2}$, $\mathcal{G}_1 = g \tau_1^{1/2}$ and

$$I_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (9)$$
is a color matrix. From (8), only the $\psi^{(1,2)}_\pm$ states interact via the last three terms of the color interaction i.e.

$$G p_1 \lambda^2 - G_1 E \lambda^1 \mp \frac{i}{2} g \sigma^x \mathcal{E}^3 \lambda^3 \equiv \lambda^a I_a^\pm,$$  \hspace{1cm} (10)

since $(\lambda^a)_3 = 0$ for $\psi^{(3)}_\pm$ when $a = 1, 2, 3$. Here we have written the interaction term using the color vector $I_a^\pm$, which has components $(-G_1 E, \frac{1}{2} g \sigma^x \mathcal{E}_x^3, 0, 0, 0, 0, 0, 0)$. The interaction term is a conserved operator, since it commutes with equation (8) for the $\phi, \chi$ spinors. Squaring this operator $(\lambda^a I_a^\pm)^2$ gives the following color diagonal form:

$$(\lambda^a I_a^\pm)^2 = \left[ G_1^2 E^2 + G^2 p_1^2 - \frac{1}{4} (\mathcal{E}_x^3)^2 \right] I_2. \hspace{1cm} (11)$$

By separating the coupled equations in (8) one finds that $\psi^{(1,2)}_\pm$ obey the same fourth order equation:

$$\left[ \left( E^2 - p^2 - M^2 - \frac{1}{4} (G^2 - G_1^2) \right)^2 - \left( G_1^2 E^2 + G^2 p_1^2 - \frac{1}{4} (\mathcal{E}_x^3)^2 \right)^2 \right] \psi^{(1,2)}_\pm = 0. \hspace{1cm} (12)$$

Solving this last equation leads to two relations between the energy/momentum of the spinor particle and the parameters of the chromoelectric field:

$$E^2 - p^2 - M^2 - \frac{1}{4} (G^2 - G_1^2) = \pm \sqrt{G_1^2 E^2 + G^2 p_1^2 - \frac{1}{4} (\mathcal{E}_x^3)^2}. \hspace{1cm} (13)$$

These relations are defined by the two eigenvalues of the $\lambda^a I_a^\pm$ operator (i.e. $\pm \sqrt{G_1^2 E^2 + G^2 p_1^2 - \frac{1}{4} (\mathcal{E}_x^3)^2}$). These two relations lead to the two branches of the continuous spectrum [9, 11]:

$$E_{1,2}^2 = p^2 + M^2 + \frac{1}{4} (G^2 + G_1^2) \pm \sqrt{G_1^2 (p^2 + M^2) + G^2 p_1^2}^{1/2}. \hspace{1cm} (14)$$

The long range confinement property of QCD is dealt with as in [12] by taking the flux tubes to have a finite radius. This is done by placing a cylindrical, hard wall at some radius, $r_0$. In principle we should also place two hard walls at some positions along the $x$-axis. Doing this does not really change the overall analysis so we not carry this out. Note the operator $p^2$ commutes with equation (12). Thus we can solve the eigenfunction/eigenvalue problem for $p^2$ and then combine these results with (12). In cylindrical coordinates with the axis of the cylinder along the $x$-axis the eigenvalue equation for $p^2$ is

$$p^2 \psi^{(i)}_\pm (\mathbf{r}) = -\nabla^2 \psi^{(i)}_\pm (\mathbf{r}) = p^2 \psi^{(i)}_\pm (\mathbf{r}) \hspace{1cm} (15)$$
For (15) the separation ansatz is \( \psi_\pm^i (r) = R(r) \cdot u(\varphi) \cdot \psi(x) \xi_\pm^i \) where \( \xi_\pm^i \) contains the spin and color spin parts of wave function and can be chosen as the eigenvectors of the corresponding \( \sigma^3 \) and \( \lambda^3 \) operators:

\[
\xi_\pm^i = \xi_\pm \xi^i ,
\]

where

\[
\xi_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \xi_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} ,
\]

and

\[
\xi^1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \xi^2 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \xi^3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} .
\]

The separation ansatz divides (15) into three independent equations:

\[
\left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \left( p^2 - p_1^2 - m^2 \right) \right] R(r) = 0 \tag{16}
\]

\[
- \frac{\partial^2}{\partial \varphi^2} u(\varphi) = m^2 u(\varphi) \tag{17}
\]

\[
\left[ \frac{\partial^2}{\partial x^2} + p_1^2 \right] \psi(x) = 0 \tag{18}
\]

where \( r = \sqrt{y^2 + z^2} \). The solution of (17) is \( u(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi} \) \( (m = 0, \pm 1, \pm 2,...) \). Here \( m \) is the chromomagnetic quantum number and is defined as the projection of the chromomagnetic moment onto the field direction i.e. the axis of the cylinder or the \( x \)-axis. The chromomagnetic moment is connected with the orbital motion of colored particles [12, 14].

Next, the eigenvalue equation (18) has the general solution:

\[
\psi(x) = B e^{ip_1 x} \tag{19}
\]

where \( p_1 \) is the momentum of the particle along the \( x \)-axis. Finally the solutions to (16), under the requirement that the wave function be finite at the origin, are Bessel functions of the first kind

\[
R(r) = C J_m(p_\perp r) \tag{20}
\]

where \( p_\perp = \sqrt{p^2 - p_1^2} \) is the transverse momentum. The interaction of the chromomagnetic moment of the quarks with the background field is characterized by the index \( m \). However,
the energy spectrum (14) does not depend on this quantum number and thus, up to this point, we have a degeneracy over the quantum number $m$.

For the radial direction we still need to take into account the boundary condition that the wave function of the quark should vanish as the cylinder wall i.e.

$$R (r = r_0) = C J_m (p_\perp r_0) = 0.$$  \hfill (21)

This yields the condition $p_\perp r_0 = \alpha_m^{(N)}$ where $\alpha_m^{(N)}$ is the $N^{th}$ zero of the $m^{th}$ Bessel function of the first kind. This gives the following quantization condition on $p_\perp$:

$$(p_\perp)_m^{(N)} = \frac{\alpha_m^{(N)}}{r_0}.$$  \hfill (22)

Here $N$ is the radial quantum number. Inserting the quantized value transverse momenta (22) and the free momentum $p_1$ along the $x$-axis into the spectrum (14) we obtain the energy spectrum for the quark:

$$\left( E_m^{(N)} \right)_{1,2}^2 = p_1^2 + \left( \frac{\alpha_m^{(N)}}{r_0} \right)^2 + M^2 + \frac{1}{4} \left( G^2 + G_1^2 \right) \pm \sqrt{G_1^2 \left( p_1^2 + \left( \frac{\alpha_m^{(N)}}{r_0} \right)^2 + M^2 \right) + G^2 p_1^2}.$$  \hfill (23)

The energy spectrum (23) now depends on $m$ through $\alpha_m^{(N)}$ and thus the interaction with the boundary eliminates the degeneracy of the energy spectrum over the $m$ quantum number. As noted earlier the states $\psi_\pm^{(3)}$ do not interact with the color background thus the spectrum for these states is simply [12]:

$$\left( E_m^{(N)} \right)_3^2 = p_\perp^2 + p_1^2 + M^2.$$  \hfill (24)

However, these states do interact with the boundaries thus the solutions for the $\psi_\pm^{(3)}$ states are like those for the $\psi_\pm^{(1,2)}$ states but without the color background term which shifts the energy states up/down. Imposing the flux tube boundary conditions for the $\psi_\pm^{(3)}$ states we get the quantization condition (22). Inserting these into (24) gives the spectrum for the $\psi_\pm^{(3)}$ states

$$\left( E_m^{(N)} \right)_3^2 = \left( \frac{\alpha_m^{(N)}}{r_0} \right)^2 + p_1^2 + M^2.$$  \hfill (25)

This spectrum for the $\psi_\pm^{(3)}$ states is completely independent of the color fields/color interaction terms.
One can simplify the energy spectra in (23) with the following two assumptions: (i) We take \( \tau = \tau_1 \) and thus \( G = G_1 = g\sqrt{\tau} \). (ii) For light quarks \((u,d,s)\) the mass, \(M\) can be neglected with respect to the QCD field strength/energy scale set by \( g\sqrt{\tau} \). With these assumptions (23) becomes

\[
\left( E^{(N)}_{m} \right)_{1,2}^2 = p_1^2 + \left( \frac{\alpha_m}{r_0} \right)^2 + \frac{1}{2}g^2\tau \pm \sqrt{2g^2\tau p_1^2 + \left( \frac{\alpha_m}{r_0} \right)^2}.
\]

(26)

The energy spectrum is split into three distinct levels: (i) An upper level, \( (E^{(N)}_{m})_1 \); (ii) a middle level \( (E^{(N)}_{m})_3 \); (iii) a lower level \( (E^{(N)}_{m})_2 \). The splitting into the three levels given by (26) and (25) is due to the chromoelectric field. This has similarities to the splitting of the \( n = 2 \) level of hydrogen by an ordinary electric field in the Stark effect – of the four \( n = 2 \), electron energy levels one is shifted up, one is shifted down and two are un-shifted. The middle level, \( (E^{(N)}_{m})_3 \), comes simply from the confining boundary conditions, while the upper and lower levels, \( (E^{(N)}_{m})_{1,2} \), are split due to the constant chromoelectric field. Again this type of behavior is not what one would expect for a constant “electric” field which should accelerate the “charged” particle continuously and have now energy eigenvalue.

III. SPECTRUM OF EMISSION IN THE CHROMOELECTRIC FLUX TUBES

The transition between the different energy levels given by (25) and (26) can occur either via gluons or photons since the quarks carry both color and electric charge. However since the characteristic time scale for the electromagnetic interaction is many orders of magnitude longer (\( \approx 10^{-16} \) seconds) as compared to the strong interaction (\( \approx 10^{-24} \) seconds) the transitions will occur essentially only via gluons which will manifest themselves as jets.

The explicit expressions for the energy spectrum of quarks in a chromoelectric flux tube given in (25) and (26) may be studied by measuring the spectrum of gluons (jets) emitted by this tube. The energies of the emitted gluons are given by

\[
\omega = E^{(N')}_{m',n'} - E^{(N)}_{m,n} ,
\]

(27)

where \( N', m', n' \) are the quantum numbers of the initial state and \( N, m, n \) are the quantum numbers of the final state. There are many different permutations of initial and final quantum numbers, but there are some restrictions coming from conservation of certain operators.
First, the color vector $\lambda^a I_\pm$ of (10) is a conserved quantity as discussed in section II. The states corresponding to the three different energy branches, $E_{1,2,3}$, have different values of $\lambda^a I_\pm$ and so transitions between these three branches are forbidden by conservation of this operator. Second there is the restriction that $\Delta m = m' - m = 0, \pm 1$ for transitions between the levels within each of the three branches, $E_1, E_2, E_3$. This is the usual selection rule associated with the operator $L_z$ which is the conserved component of angular momentum along the special axis of the problem. In this case the special direction is called $x$ and so the operator is actually $L_x$.

By studying the spectrum of emitted gluons (which will manifest as jets), (27), and taking into account the selection rules associated with the operators $\lambda^a I_\pm$ and $L_x$, one should be able to observe the three energy branches $E_1, E_2$ and $E_3$. In order for the emitted gluons (jets) to be observable they must have energies in the $10 \times \text{GeV}$ range so that one is in the pertubative regime of QCD. The energy scale between adjacent levels is set by the radius of the flux tube, $r_0$. Taking $r_0$ to be of a size typical of strongly interacting systems namely $1 \text{GeV}^{-1}$ then implies that the energy between adjacent levels is of order $1 \text{GeV}$. In order to have the emitted gluons have energies of $10 \times \text{GeV}$ or greater means we are looking for transitions where the final $N$ and/or $n$ are of the order 1 while the initial $N'$ and/or $n'$ should be of order 10 or larger. Since the quantum number $m$ is restricted by $\Delta m = m' - m = 0, \pm 1$ it can not give the needed splitting between final and initial energy levels. Having $N' - N \geq 10$ would give a difference between the energy levels of $10 \times \text{GeV}$ or larger and the emitted gluon would appear as a jet. As mentioned in section II there is a gauge inequivalent way of obtaining constant color fields via covariantly constant non-Abelian vector potentials [5]. The energy spectrum of such constant color fields coming from covariantly constant non-Abelian vector potentials would be different from the energy spectrum obtained here and one could experimentally distinguish between these two types of chromoelectric flux tubes.

IV. DISCUSSION AND CONCLUSION

The central result of this paper is the explicit energy spectrum given in (23) for a spinor, color charged particle (i.e. a quark) inside a chromoelectric flux tube of radius $r_0$, and its application to obtaining the spectrum of emitted gluons (jets) in (26) (25) (27). This result may have applications to the glasma state [4] which is thought to occur during the initial
stages of heavy ion collisions such as those studied at RHIC or to be studied at ALICE. As the heavy ions collide and pass through one another, chromoelectric and chromomagnetic flux tubes are thought to form. The fields of these color flux tubes of the glasma state are assumed to behave like classical, color field flux tubes. Thus the results presented here might be used to probe for the presence of these color field flux tubes by looking for gluons/jets which exhibit the discrete spectrum of \( E_1, E_2, E_3 \) (23). For a flux tube with a non-confining boundary the continuous spectrum (14) will be useful. In this case for comparison with experiments the energy of the moving quark jet should be measured directly.

The spectrum given by (23) depends on several quantum number, \( n, m, N \), and as such yields a complex spectrum for emitted gluons/jets (27). The spectrum is restricted by two selections rules: (i) No transitions occur between the three different energy branches, \( E_1, E_2, E_3 \) due to the conservation of the color projection operator \( \lambda^a T_a^i \). (ii) Due to conservation of the angular momentum projection operator, \( L_z \), one has the standard selection rule \( \Delta m = 0, \pm 1 \) between the energy levels of each branch, \( E_1, E_2, E_3 \). There is also a practical requirement that the observable transitions are those which occur between widely spaced levels. For example, the initial \( N' \) and/or \( n' \) should be of order 10 or greater and the final \( N \) and/or \( n \) should be of order 1. This requirement comes about since for the gluons to appear as jets they should be in an energy range where QCD is perturbative i.e. they should have energies of the order or greater than \( 10 \times \text{GeV} \) rather than \( 1 \text{ GeV} \). This spectrum for a quark in the chromoelectric flux tube is similar to molecular spectrum with several quantum numbers (electronic, rotational, and vibrational quantum numbers) rather than the simple hydrogen atom spectrum with its single, radial quantum number. At present our results already allow us to say that the spectrum of gluons from the glasma state might be complex and the energy of the emitted gluons would be in the \( 10 \times \text{GeV} \) range or larger. By experimentally measuring the gluon/jet spectrum and thus the energy spectrum one could determine physical characteristics of the glasma such as the chromoelectric field strength \( E_3 = g \sqrt{\tau T_1} \).

In this article we have only investigated chormoelectric flux tubes. In practice the chromomagnetic flux tubes arise at the same time with chromoelectric ones [4]. One can calculate the energy spectrum of the chromomagnetic flux tubes in a manner exactly similar to the procedure used in this article for chromoelectric flux tubes and one will arrive at a spectrum similar to (23). However in the case of a color electric charged quark placed in a color
magnetic flux tube one finds that this system develops a field angular momentum, $L_{QCD}$ coming from the color electric and color magnetic fields. This field angular momentum is given by

$$L_{QCD} = \frac{1}{4\pi} \int \mathbf{r} \times \left( \mathbf{E}^a \times \mathbf{B}^a \right) d^3 x ,$$

(28)

where $\mathbf{E}^a$ and $\mathbf{B}^a$ are the color electric field of the quark and color magnetic field of the flux tube. This field angular momentum coming from a color electric charge in combination with a color magnetic dipole was conjectured to play a role in explaining the “missing” spin of the proton. The relevant color fields in regard to calculating the QCD field angular momentum of (28) are a longitudinal cylindrical chromomagnetic field and Coulomb chromoelectric field. The field angular momentum of the electromagnetic version of such a system (longitudinal magnetic field and Coulomb electric field) was calculated in where it was found that $L_{EM}$ was proportional to the radius of the flux tube, the strength of the magnetic flux and the strength of the electric charge. In the approximation we have used in this paper, where the fields are treated classically, the QCD field angular momentum will also have the same kind of dependence (i.e. $L_{QCD} \propto r_0, g$). This field angular momentum will shift the total angular momentum of the system of chromomagnetic flux tube plus quark. Naively the angular momentum of this system should just come from the quark, but taking (28) into account means the system will have a total angular momentum of $L_{total} = L_{quark} + L_{QCD}$ where $L_{quark}$ is the spin plus orbital angular momentum of the quark. Additionally, the field angular momentum of (28) can have an indirect influence on the energy spectrum of the chromomagnetic flux tube plus quark through the introduction of an additional restriction on the parameters that determine the energy spectrum. In this work the color fields have been treated as classical fields and therefore the field angular momentum as well is classical. However, it is known that quantum mechanically angular momenta must be quantized in integer steps of $\hbar/2$, thus we should have $L_{QCD} = \hbar n'/2$ where $n'$ is an integer. This condition gives a further quantization of the parameters, $r_0, g$, which are used in calculating $L_{QCD}$ (this is the same procedure used in deriving the Dirac quantization condition for monopoles). Thus in addition to the shift of the angular momentum of the chromomagnetic flux tube plus quark system due to (28), the energy spectrum will have an additional restriction due to the quantization condition $L_{QCD} = \hbar n'/2$. The investigation of the details of the chromomagnetic flux tube plus quark will be the topic of a future paper.
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