Negative radiation pressure on an ensemble of atoms in free space

Navdeep Arya,1 Navketan Batra,2 Kinjalk Lochan,1 and Sandeep K. Goyal1,2

1Department of Physical Sciences, Indian Institute of Science Education & Research (IISER) Mohali, Sector 81 SAS Nagar, Manauli PO 140306 Punjab India.
2Department of Physics, Brown University, Providence, RI 02912, USA

Light is known to exert a pushing force through the radiation pressure on any surface it is incident upon, via the transfer of momentum from the light to the surface. For an atom, the interaction with light can lead to both absorption as well as emission of photons, leading to repulsive and attractive forces, respectively. For classical light, these two processes occur at the same rates. Therefore, a thermal ensemble of atoms at a finite temperature always experiences a net pushing force. In this paper, we show that when treated quantum mechanically the pulsed electromagnetic field interacting with the thermal ensemble of atoms leads to unequal transition rates, again resulting in a non-zero net force. However, the signature and the magnitude of the force depends upon the intensity of the light, the number of atoms, and the initial temperature of the ensemble. Thus, even at finite temperature, controlling the parameters of the electromagnetic pulse and the number of particles in the ensemble, the net force can be changed from repulsive to attractive, generating negative radiation pressure in the process. Quite counterintuitively, this negative radiation pressure arising out of pure quantum character of light gets stronger for higher temperatures.

In this paper, we present a novel attractive optical force arising on a thermal ensemble of atoms when we shine light on it. This force arises due to accounting for the quantum properties of the electromagnetic field as well as due to the conjunction between the spontaneous and the stimulated emission processes leading to unequal absorption and emission rates. This process makes the total rate of emission larger than the sum of the spontaneous and the stimulated emission rates. The additional contribution to the emission rate originates from first adding the transition amplitudes due to spontaneous and stimulated emissions and then calculating the transition probability [23]. The net optical force is proportional to the intensity of light, the number of atoms and the initial temperature of the ensemble. Consequently, the ensemble can be made to accelerate towards the light source, as opposed to moving away from it, as is the case with the standard radiation pressure.

Interestingly, the presence of this new force is tied to the symmetry of the emission profile of the atom interacting with the electromagnetic field in a given quantum state. We illustrate that certain classes of quantum states, e.g., optical coherent state and Fock states, of electromagnetic field result in non-symmetric emission profile, thus leading to the non-zero net force. On the other hand, states leading to symmetric emission, such as thermal radiations, do not cause any net acceleration. Therefore, in vacuum state of the electromagnetic field, there is no velocity change of the atom. Thus, the emission profile provides a straightforward way of identifying quantum states of the field leading to zero and non-zero accelerations both [12, 24].

Let us consider a two-level atom with \( \{|g\rangle, |e\rangle\} \) representing its ground and excited states, and the free Hamiltonian \( H_0 = \hbar \omega_0 |e\rangle \langle e| \). The atom is also interacting with electromagnetic field, with the interaction Hamiltonian \( H_I \) given by \( H_I(\tau) = d^\mu E_\mu = -\mathbf{D}(\tau) \cdot \mathbf{E}(\tau) \) in the rest frame of the atom identified by its proper time \( \tau \).

\[ \text{skgoyal@iisermohali.ac.in} \]
Here

\[ E(\tau) = \sqrt{\hbar} \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda=1}^{2} \epsilon_{k\lambda} \hat{e}^{-i\omega_k \tau} - H.c., \]

(1)

and \( D(\tau) = d_{\sigma} e^{-i\omega_\sigma \tau} + d^*_{\sigma} e^{+i\omega_\sigma \tau} \), are the electric field and electric dipole moment operators, respectively, \( d = \langle g| D(\tau = 0)| e \rangle \) is the transition dipole moment which we assume to be non-zero only along the \( z \)-axis, and \( a_{k\lambda} \) annihilates (creates) an excitation in the mode \( \langle k, \lambda_k \rangle \), with polarisation denoted by \( \lambda_k \), with \( \sigma_+ = \sigma_1^\dagger = |e\rangle \langle g| \) being the step up operator for the atom.

If the atom is prepared initially in the excited state \( |e\rangle \), and the field is in a state \( |\Psi\rangle \), then the probability of atomic transition over a long time from excited state \( |e\rangle \) to ground state \( |g\rangle \), to the leading order in the interaction Hamiltonian, is given by

\[ P(\omega_0) = \frac{1}{\hbar^2} \sum_{\text{out}} \int_{-\infty}^{\infty} \ d\tau e^{i\omega_0 \tau} \ |\langle e, \Psi| d \cdot E(\tau) | g, \text{out} \rangle|^2, \]

(2)

where the sum is over all the possible final states \( \{ |\text{out}\rangle \} \) of the field. The most significant contribution in the set of final states \( \{ |\text{out}\rangle \} \) of the field comes from single photon added state \( |\Psi_{q\lambda_n}\rangle = a_{q\lambda_n}^\dagger |\Psi\rangle / \sqrt{\langle a_{q\lambda_n} a_{q\lambda_n}^\dagger \rangle \Psi} \). Therefore, we will restrict our discussions to these states only, where \( P(\omega_0) \) will mark the transition probability due to such single-photon emission events. The probability density of emission of an additional photon in mode \( (q, \lambda_q) \) can be written as

\[ P(\Psi_{q\lambda_n}, \omega_0) = \frac{1}{\hbar \epsilon_0} \int_{-\infty}^{\infty} d\tau e^{i\omega_0 \tau} \int \frac{d^3k}{2ck(2\pi)^3} \sum_{\lambda_k} d \cdot \epsilon_{k\lambda} \langle \Psi \rangle \left(a_{k\lambda} e^{-ikc\tau} - a_{k\lambda}^\dagger e^{ikc\tau}\right) \frac{a_{q\lambda_n}}{\sqrt{\langle a_{q\lambda_n} a_{q\lambda_n}^\dagger \rangle \Psi}}^2. \]

(3)

From (3) we can obtain the decay-profile, i.e., \( dP(\omega_0)/d\Omega \) of the atom, which is given by

\[ P(\theta, \phi, \omega_0) = \int dq \ q^2 \sum_{\lambda_q} \bar{P}(q, \lambda_q, \omega_0). \]

(4)

The emission of a photon with wave-vector \( q \) imparts a recoil of momentum \( -\hbar q \) to the atom. Therefore, the expectation value of net momentum change as a cumulative result of such atomic transitions can be written as

\[ \Delta p = -\hbar \int d^3q \ q \bar{P}(q, \lambda_q, \omega_0), \]

(5)

which in turn gives rise to the force expressed by the four-vector

\[ K^\mu = \frac{dp^\mu}{d\tau} = \left( \frac{1}{c} \frac{dp}{d\tau} \cdot \frac{d\tau}{c} \right). \]

(6)

This is the force experienced by the atom originating from its interaction with the quantum electromagnetic field. The force four-vector in the lab-frame can be obtained from Lorentz transformation \( \Lambda_{\mu}^\nu \) on \( K^\mu \), i.e., \( K_{\mu}^ab = \Lambda_{\mu}^\nu K^\nu \). Note that the force we have calculated is due to the recoil caused by the stimulated emission from the atom. A force due to absorption, having similar kind of expression, can be obtained for the case when the atom is in the ground state (see Appendix). The net force on the atom over an extended period of time will be the integration of these forces due to the changing state of the atom. To get more insight in to the nature of this force, we calculate the force for some specific states of the field.

\[ \text{Force from } N \text{-photon state of the field: We first consider the electromagnetic field to be in } N \text{-photon wavepacket propagating in the } x \text{-direction, which is expressed as} \]

\[ |\psi_N\rangle = \left( \frac{A^\dagger_{s}}{\sqrt{N!}} \right) |0\rangle, \]

(7)

where \( A^\dagger_{s} = \int dx \mathcal{F}(s) a_{s \lambda_0}^\dagger \) creates a single photon wavepacket (a pulse), and \( \lambda_0 = 1, 2 \) denotes the two orthogonal polarizations. The distribution \( \mathcal{F}(s) \) is chosen as an appropriately normalized Gaussian [26],

\[ \mathcal{F}(s) = \left( \frac{1}{2\pi \sigma^2} \right)^{3/4} \exp \left( -\frac{1}{4\sigma^2} (s - \tilde{k}_0)^2 \right), \]

(8)

where \( \sigma \) is the bandwidth of the wavepacket and \( \tilde{k}_0 = \frac{\omega_0}{c} \) is the center of the Gaussian wavepacket.

For the resonant case, i.e., \( k_0 = \omega_0/c \) and in the limit when \( \omega_0 \gg \sigma, \) with \( \sigma \neq 0, \) the expression for the net momentum change takes the form (see Appendix)

\[ \Delta p = -\frac{2 |d|^2}{\sqrt{2\pi \epsilon_0 c}} \left( \frac{\omega_0}{c} \right)^2 N(N + 2)\sigma^2 \hat{x}. \]

(9)

Since \( c \sigma \) is the spectral width of the incoming photon, the average time \( \Delta t \) for which the atom and the photon interact is proportional to \( (c\sigma)^{-1} \). Therefore, the approximate expression for the spatial force reads [27]

\[ F \equiv \frac{\Delta p}{\Delta \tau} \Delta p \sigma \hat{x} \sim -\frac{2 |d|^2}{\sqrt{2\pi \epsilon_0 c}} \left( \frac{\omega_0}{c} \right)^2 N(N + 2)\sigma^2 \hat{x}. \]

(10)
Similarly, the average temporal component of the force can be obtained as
\[ F^0 = \frac{1}{c} \Delta E \sim -\sigma \hbar \omega_0. \quad (11) \]

From this analysis, we find that the \( N \)-photon state \( |\psi_N\rangle \) results in a non-zero force, on an excited atom, in the comoving frame. The three-acceleration of the atom, in the lab frame, can be calculated to be (see Appendix)
\[ \mathbf{a} = -\frac{1}{\gamma m_0} \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0} \left( \frac{\omega_0}{c} \right)^2 N(N+2)\sigma^2 \hat{x}. \quad (12) \]

A similar calculation yields the force on the atom in the ground state due to absorption as (see Appendix)
\[ \mathbf{F} = \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0} N \left( \frac{\omega_0}{c} \right)^2 \sigma^2 \hat{x}. \quad (13) \]

This force is due to absorption of a photon by the atom resulting in exciting the atom in the process. From Eqs. (10) and (13) we can see that the two forces are not equal. The atom in the excited state experiences a force towards the source of light whereas the atom in the ground state experiences a weaker force away from the source of light.

From Eq. (10), we can see that the force due to the presence of a \( N \)-photon state is in the direction opposite to the direction of the photon wavepacket. Therefore, this force is not due to the momentum transfer of the incident photons to the atom, but due to the stimulated emission, which makes it different from the usual radiation pressure. Furthermore, there are two competing processes of momentum change which are at work, one is due to the transition from the excited state to the ground state while the other is the transition from the ground state to the excited state. Therefore, if we have an ensemble of atoms, we can write an effective rate of change of population of the excited state \( n_e \) and the ground state \( n_g \) as \( \dot{n}_e = -\Gamma_{e \rightarrow g} n_e + \Gamma_{g \rightarrow e} n_g \) where \( \Gamma_{i \rightarrow j} \) is the rate of transition from state \( |i\rangle \) to \( |j\rangle \).

If the system attains an equilibrium configuration, then the number of upward transitions will be equal to the number of downward transitions. Since, the forces in these two transitions are not exactly the same, there will be a net force in the newly acquired equilibrium which can formally be written as
\[ \mathbf{F}_{net} = n_e \mathbf{F} + n_g \tilde{\mathbf{F}} \quad (14) \]
\[ = \Xi \left( 1 - 3 \frac{n_e}{n} \right) N - \frac{n_e}{n} N^2 \hat{x}, \quad (15) \]
where \( \Xi = n \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0} \left( \frac{\omega_0}{c} \right)^2 \sigma^2 \). Here we can see that the net force is proportional to \( \sigma^2 \) and there are two competing terms, one proportional to \( N \) and other proportional to \( N^2 \). The net force has an additional dependency on the fraction \( \chi \equiv \frac{n_e}{n} \). The force changes its sign from + to − when \( \chi \) becomes greater than (3 + \( N \))\(^{-1} \). For example, in an initial thermal distribution, the temperature \( T \) and \( N \) decide the fraction \( \chi \) in the new equilibrium and is given by (see Appendix)
\[ \chi = \frac{\Gamma_0 N(\omega_0) + \Gamma(-\omega_0)}{\Gamma_0 \coth \left( \frac{\hbar \omega_0}{2k_B T} \right) + \tilde{\Gamma}(\omega_0) + \Gamma(-\omega_0)}, \quad (16) \]
where \( N(\omega_0) = 1/(e^{\beta \hbar \omega_0} - 1) \) is the Planck distribution at the transition frequency \( \omega_0 \),
\[ \Gamma(\omega_0) = \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0} (N^2 + 2N) \frac{\omega_0 \sigma^2}{c} + \Gamma_0 \equiv \tilde{\Gamma}(\omega_0) + \Gamma_0, \quad (17) \]
\[ \Gamma(-\omega_0) = \frac{2|d|^2 N \omega_0 \sigma^2}{\sqrt{2\pi} \epsilon_0 c}, \quad (18) \]
and \( \beta = k_B T \).

From the discussion above, we note that there are two interesting control parameters; (i) number of photons \( N \) and (ii) the initial temperature of the ensemble. Here, it is important that we remain in a parameter regime in which first-order perturbative calculations remain trustworthy. This means that \( N \) cannot be made arbitrarily large, for example, for \( \sigma = 10^{-4} m^{-1} \), \( N \) should be much less than \( 10^9 \). At a given temperature \( T \), controlling \( N \) can change the force from repulsive to attractive. More interestingly, for a fixed \( N \), increasing the temperature can also turn the force attractive. This may appear surprising as quantum effects typically get weaker at high temperatures, however, at large temperature \( T \) the \( n_e \) increases which coupled with large transition rate leads to enhancement of the attractive force. This is a novel feature which can readily be tested in an experimental setting and may have interesting experimental implications. In Fig. 1, below, we plot the atomic acceleration as a function of \( N \) for different values of \( T \).

FIG. 1. Acceleration as a function of number of photons (N) for different values of temperature (T).

**Force in the vacuum state of the bath.** A direct consequence of this result is that, for \( N = 0 \), i.e., electromagnetic field is in the vacuum state, the spatial components of the force on the excited as well as on the ground state atom will be zero, and only the time-like component of the four-force \( dE/d\tau = \Gamma_0 \hbar \omega_0 / c \) will be non-zero. Here
The thermal state of the electromagnetic field gives rise to zero acceleration (see Appendix). The four-force in the lab frame (after Lorentz transformation) reads,

$$K'_{\mu} = \Lambda_{\mu}^\nu K^\nu = \left(-\frac{\Gamma_0\hbar\omega_0}{c}, -\frac{\Gamma_0\hbar\omega_0}{c^2}v, 0, 0\right),$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$. An interesting observation from this above expression is that in the lab frame $\frac{dp}{dt} = -(\Gamma_0\hbar\omega_0/c^2)v$ is non-zero and the *Newtonian* force is proportional to the velocity of the atom. In other words, a lab observer will see a decaying atom experiencing a “friction-like” force [12]. The question now arises is whether this force as seen from the lab frame has any effect on the motion of the atom. Writing $p = \gamma m_0 v$ we can see easily that the acceleration $a \equiv d\vec{v}/dt$, as seen from the lab frame

$$a = \frac{1}{\gamma m_0}\frac{dp}{dt} - \frac{v}{\gamma m_0 c^2}\frac{dE}{dt},$$

turns out to be zero. Therefore, the friction-like force acting on the decaying atom in the lab frame does not cause any acceleration. Similar calculation performed for the thermal state of the electromagnetic field gives rise to zero acceleration (see Appendix).

One can see from Eq. [5] that the symmetric nature of the emission profile, i.e., $P(q) = P(-q)$ results in $\Delta p = 0$. In states where this symmetry is broken, e.g., $N$-photon states we have considered, there is a net momentum transfer to the atom. In Fig. 2 we show the schematic decay-profile of an excited atom stimulated by a single-photon wavepacket (see Appendix). The asymmetry of the decay-profile along $\hat{x}$ is a consequence of the asymmetry of the incident photon-wavepacket profile [Eq. (8)]. Optical coherent states constitute another class of such interesting states (see Appendix). Therefore, lasers become viable candidates to study these forces on an ensemble of atoms in laboratory settings.

In this paper, we have studied the effects of electromagnetic field on the motion of a two-level atom. If the atom initially is in the excited state, the electromagnetic field exerts a force towards the source of light. This attractive force is fundamentally different from the standard radiation pressure. On the other hand, if the atom is prepared in the ground state then both the direction and the magnitude of the force change. We treat light as quantum field and as a result the absorption and stimulated decay rates are unequal. Therefore, in an ensemble of atoms initially in a thermal equilibrium, there will be a net force which will be determined by the temperature and the field intensity Fig. 1. For a given intensity, the nature of the force can be turned from repulsive to attractive by controlling the temperature. Similarly, at a fixed temperature the intensity of light as well as the number of atoms in the ensemble determine the nature of the force. These effects can be studied in the standard laboratory settings. One of the interesting application of this force could be to trap and/or isolate atoms with certain velocity in an atomic ensemble. This provides an additional control for optical tweezers and laser cooling techniques.

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**Appendix A: Force on an atom initially in the excited state interacting with $N$-photon wavepacket**

If initial and final states of the atom-field composite system are $|e, \Psi\rangle$ and $|g, \text{out}\rangle$, respectively. The transition probability from $|e, \Psi\rangle$ to $|g, \text{out}\rangle$ is given by the Born’s rule:

$$P(\omega_0) = \sum_{\text{out}} |\langle g, \text{out}| U |e, \Psi\rangle|^2$$

(A1)
where $U$ is the unitary time-evolution operator for the composite system. Using $U$ to first order and using $\langle g | D(\tau) | e \rangle = d e^{-i\omega_0 \tau}$, we can write

$$P(\omega_0) = \frac{|d|^2}{\hbar^2} \sum_{\text{out}} \left| \int_{-\infty}^{\infty} d\tau e^{i\omega_0 \tau} \langle \Psi | E_z(\tau) | \text{out} \rangle \right|^2,$$  \hspace{1cm} (A2)

where we have assumed that $d = |d| \hat{z}$. We are interested in knowing the decay-probability of the atom to a field mode $(q, \lambda_q)$ via single-photon decay processes, therefore, we write $|\text{out}\rangle = B a_{q\lambda_q}^\dagger |\Psi\rangle$, where $B$ is the normalization of $|\text{out}\rangle$. For a real $F(q)$, the normalization of $|\text{out}\rangle$ demands that

$$B = \frac{1}{\left[ N |F(q)|^2 \delta_{\lambda_0 \lambda_q} + \delta(0) \delta_{\lambda_0 \lambda_q} \right]^{1/2}} = \frac{V^{-1/2}}{|n |F(q)|^2 + 1}^{1/2},$$  \hspace{1cm} (A3)

where $n \equiv N/V$, and $V$ is the volume of the box that we imagine in order to box-normalize $|\text{out}\rangle$. In case of continuous field theory without boundaries, $V \rightarrow \infty$. The correlation function for the N-photon wavepacket is $\langle N | a_{q\lambda_q} a_{k\lambda}^\dagger |N\rangle = N F^*(k) F(q) \delta_{\lambda_0 \lambda_q} + \delta(k - q) \delta_{\lambda_0 \lambda_q}$. Equation (A2) leads to

$$P(q, \omega_0) = \sum_{\lambda_q=1}^2 \frac{4\pi^2 |d|^2}{\hbar c} |B|^2 \left[ -\frac{\pi^2}{2} \frac{N F^*(q) \delta_{\lambda_0 \lambda_q}}{\sqrt{2c(2\pi)^3}} \left( \frac{\omega_0}{c} \right)^{5/2} e^{-\frac{1}{4\sigma^2} \left( \frac{q^2}{c^2} + k_0^2 \right)} \left[ I_0^2 \left( \frac{\omega_0 k_0}{4c\sigma^2} \right) + I_1^2 \left( \frac{\omega_0 k_0}{4c\sigma^2} \right) \right] + \frac{qc}{\sqrt{2c(2\pi)^3}} \delta(\omega_0 - qc) \epsilon^z(q, \lambda) \right]_0^{\infty} \delta(\omega_0 - qc) \epsilon^z(q, \lambda).$$  \hspace{1cm} (A4)

The total momentum transferred to the atom, by the field is given by $\Delta p = -\hbar \int dq \ q P(q, \omega_0)$. Noting that whenever $\sigma \neq 0$, we can take the normalization (A3) of $|\text{out}\rangle$ to be 1 because, as mentioned above, in continuous field theory without boundary, $V \rightarrow \infty$. Then, for $\sigma \neq 0$, using (A4) we get

$$\Delta p = -\frac{|d|^2 \pi^2}{8\sqrt{2\pi} c \sigma} \left( \frac{\omega_0}{c} \right)^5 e^{-\frac{1}{2\sigma^2} \left( \frac{q^2}{c^2} + k_0^2 \right)} \left[ I_0^2 \left( \frac{\omega_0 k_0}{4c\sigma^2} \right) + I_1^2 \left( \frac{\omega_0 k_0}{4c\sigma^2} \right) \right] + 8N I_1 \left( \frac{\omega_0 k_0}{4c\sigma^2} \right) \left[ \frac{\omega_0}{2c\sigma^2} I_0 \left( \frac{\omega_0 k_0}{4c\sigma^2} \right) - \frac{1}{k_0} I_1 \left( \frac{\omega_0 k_0}{4c\sigma^2} \right) \right] \delta(\omega_0 - qc)e^z(q, \lambda).$$  \hspace{1cm} (A5)

We note that the net momentum transfer is in the negative $\hat{x}$-direction which is a consequence of the asymmetry in the atom’s decay profile which in turn results from the asymmetry in stimulating photon-wavepacket’s profile as given in (8). Using the large $x$ limit [28], $I_1^2 \left( \frac{\omega_0}{c} \right) \rightarrow \frac{1}{(2\pi)^{1/2} \sigma} \frac{\pi}{\omega_0 k_0} \sigma$, of the modified Bessel function we get an expression for net momentum transferred valid for $\frac{\omega_0 k_0}{2c\sigma^2} \gg 1$:

$$\Delta p = -\frac{|d|^2 \pi^2}{8\sqrt{2\pi} c \sigma} \left( \frac{\omega_0}{c} \right)^5 \frac{8c \sigma^2}{\pi^2 \omega_0 k_0} \exp \left( -\frac{1}{2\sigma^2} \left( \omega_0/c - k_0 \right)^2 \right) \frac{2N}{\omega_0} \left[ \frac{k_0}{\omega_0} N + 2 \right] \delta(\omega_0 - qc).$$  \hspace{1cm} (A6)

From (A6) we see that the maximum momentum transfer occurs for $k_0 = \omega_0/c$ and is given by,

$$\Delta p = -\frac{2|d|^2}{\sqrt{2\pi} c \sigma} \left( \frac{\omega_0}{c} \right)^2 N(N + 2) \delta(\omega_0 - qc).$$  \hspace{1cm} (A7)

The net momentum transferred to the atom when the field is in the optical coherent state $|\alpha\rangle$ is obtained by replacing $N$ by $|\alpha|^2$ in (A7). A similar calculation can be performed for the case when the atom is initially in the ground state. In the limit $\sigma \neq 0$, $\frac{\omega_0 k_0}{2c\sigma^2} \gg 1$ the maximum net momentum transferred to the atom is obtained to be

$$\Delta g = \frac{2|d|^2}{\sqrt{2\pi} c \sigma} \left( \frac{\omega_0}{c} \right)^2 N \sigma \delta(\omega_0 - qc).$$  \hspace{1cm} (A8)

The results for the optical coherent state

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} (A_0^\dagger)^n |0\rangle, \quad \alpha \in \mathbb{C}$$  \hspace{1cm} (A9)

can be obtained by replacing $N$ in above expressions by $|\alpha|^2$. 


1. Decay-profile of the excited atom

Equation (A1) can be rewritten as

\[
P(q, \omega_0) = \frac{|d|^2}{4\pi \hbar c} \left[ N^2 F^2(q) h^2(\omega_0, k_0, \sigma) + q \bar{\delta}(q - \omega_0/c) \sin^2 \theta + 2N h(\omega_0, k_0, \sigma) F(q) q^{1/2} \delta(q - \omega_0/c) \right], \tag{A10}
\]

where for brevity we have defined

\[
h(\omega_0, k_0, \sigma) = \pi^2 \left( \frac{1}{2\pi \sigma^2} \right)^{3/4} \left( \frac{\omega_0}{c} \right)^{5/2} e^{-\frac{1}{4\pi^2} \left( \frac{k_0^2}{\sigma^2} + \frac{k_0^2}{\sigma^2} \right)} \left[ I_0^2 \left( \frac{\omega_0 k_0}{4\sigma^2} \right) + I_1^2 \left( \frac{\omega_0 k_0}{4\sigma^2} \right) \right]. \tag{A11}
\]

From (A10) we obtain the decay-probability per unit solid-angle \(d\Omega_q\), around \(q\), to be

\[
P(\theta, \phi, \omega_0) = \frac{|d|^2}{4\pi \hbar c} \int dq \int d\Omega_q \left[ N^2 q F^2(q) h^2(\omega_0, k_0, \sigma) + q \bar{\delta}(q - \omega_0/c) \sin^2 \theta + 2N h(\omega_0, k_0, \sigma) F(q) q^{1/2} \delta(q - \omega_0/c) \right].
\]

Evaluating the integrals we get the decay-profile of the excited atom to be

\[
P(\theta, \phi, \omega_0) = \frac{|d|^2}{4\pi \hbar c} \left[ N^2 e^{-\frac{k_0^2}{2\sigma^2}} \left( \frac{N^2}{2\pi} e^{-\frac{1}{2\sigma^2} \left( \frac{k_0^2}{\sigma^2} + \frac{k_0^2}{\sigma^2} \right)} \right) \left[ \frac{\bar{\delta}(0) \sin^2 \theta + 2N h(\omega_0, k_0, \sigma) F(\omega_0, k_0, \sigma) \left( \frac{\omega_0}{c}, \theta, \phi \right) \left( \frac{\omega_0}{c} \right)^{5/2}}{\omega_0^2} \right] \right].
\]

Figure (A1) shows the logarithm of the decay-profile of the atom for stimulation by a single-photon wavepacket given by (7) with \(\sigma = 1 \text{m}^{-1}\).

2. Estimation of the Decay-rates in the continuous-mode limit

Using the probability-density given in (A10) we can estimate the stimulated decay-rate. First we calculate the total decay probability using

\[
\bar{P}(\omega_0) = \int d^3q \sum_{\lambda_q} \bar{P}(q, \lambda_q, \omega_0) = \int d^3q \bar{P}(q, \omega_0).
\]

Recall that apart from the single-mode limit, the normalization of \(|\text{out}\rangle\) can be approximated to be 1.

Evaluating the integrals we arrive at,

\[
\bar{P}(\omega_0) = \frac{|d|^2}{4\hbar c} \left( \frac{\omega_0}{c} \right)^{5/2} \left( N^2 + 2N \right) \left( \frac{1}{2\pi \sigma^2} \right)^{3/2} \left( \frac{N^2}{2\pi} e^{-\frac{1}{2\sigma^2} \left( \frac{k_0^2}{\sigma^2} + \frac{k_0^2}{\sigma^2} \right)} \right) \left[ I_0^2 \left( \frac{\omega_0 k_0}{4\sigma^2} \right) + I_1^2 \left( \frac{\omega_0 k_0}{4\sigma^2} \right) \right] + \frac{|d|^2}{3\pi \hbar c} \int_{-\infty}^{\infty} dt.
\]

Though the \(\tau\) integration formally runs from \(-\infty\) to \(\infty\), in the limit \(\omega_0/\sigma \gg 1\) the integration limits can safely be assumed to be \(\pm (\sigma \tau)^{-1}\) without making much of an error, as the stimulated decay dominantly takes place over a time scale \(\Delta \tau = (\sigma \tau)^{-1}\), i.e., the temporal-width of the incident photon-wavepacket. Therefore, the decay rate can be approximated as

\[
\Gamma(\omega_0) = \frac{\pi^2 |d|^2}{8\sqrt{2} \pi \hbar c} \left( \frac{\omega_0}{c} \right)^{5/2} \left( \frac{N^2 + 2N}{\sigma^2} \right) e^{-\frac{1}{2\sigma^2} \left( \frac{k_0^2}{\sigma^2} + \frac{k_0^2}{\sigma^2} \right)} \left[ I_0^2 \left( \frac{\omega_0 k_0}{4\sigma^2} \right) + I_1^2 \left( \frac{\omega_0 k_0}{4\sigma^2} \right) \right] + \frac{|d|^2}{3\pi \hbar c^2}.
\]
FIG. 3. Logarithm of the decay-profile, i.e., $\ln P(\theta, \phi, \omega_0)$, of an excited atom due to stimulation by a single-photon wavepacket with $\sigma = 1 \text{m}^{-1}$.

Expression in (A16) can be further simplified to

$$\Gamma(\omega_0) = \sqrt{\frac{2}{\pi \hbar \epsilon_0 c}} \left| d \right|^2 \left( N^2 + 2N \omega_0 \sigma^2 + \frac{\left| d \right|^2 \omega_0^3}{3 \pi \hbar \epsilon_0 c^3} \right),$$

(A17)

using the asymptotic forms of the modified Bessel functions and setting $\tilde{k}_0 = \omega_0 / c$ to get the maximum decay rate. Note that we are not requiring $\sigma \to 0$. A similar calculation for the case when the atom is initially in the ground state leads to the excitation-rate given by

$$\Gamma(-\omega_0) = \sqrt{\frac{2}{\pi \hbar \epsilon_0 c}} N \omega_0 \sigma^2.$$  

(A18)

Appendix B: Vacuum as the bath

For $|\Psi\rangle = |0\rangle$, we obtain

$$\bar{P}(\omega_0, q, \lambda_q) = \frac{|d|^2}{8 \pi^2 \hbar \epsilon_0} \int_{-\infty}^{\infty} d\tau_+ q \delta \left( q - \frac{\omega_0}{c} \right) (e^{z(q, \lambda_q)})^2,$$

(B1)

where we have assumed that $d = |d| \hat{z}$ and have effected the following coordinate transformation $\tau_+ \equiv (\tau_1 + \tau_2) / 2, \tau_- \equiv \tau_1 - \tau_2$. The decay-probability into a solid angle $d\Omega$ around $q$ is obtained to be

$$\bar{P}(\omega_0, \theta, \phi) = d\Omega_q \frac{|d|^2 \omega_0^3}{8 \pi^2 \hbar \epsilon_0 c^3} \int_{-\infty}^{\infty} d\tau_+ \left( 1 - \cos^2 \theta \right).$$

(B2)
This immediately gives the decay rate per unit solid angle,

\[ \Gamma_{q}(\theta, \phi, \omega_0) = \frac{|d|^2 \omega_0^3}{8\pi^2 \hbar \epsilon_0 c^3} (1 - \cos^2 \theta) \tag{B3} \]

and the total decay-rate,

\[ \Gamma = \int d\Omega \Gamma_{q}(\theta, \phi, \omega_0) = \frac{|d|^2 \omega_0^3}{3\pi \hbar \epsilon_0 c^3} \equiv \Gamma_0. \tag{B4} \]

Using (B3, B4) we can calculate the spacelike components of the tensorial force in atom’s rest frame

\[ K_{\mu} = \frac{dP_{\mu}}{d\tau} = \left( \frac{1}{c} \frac{dE}{d\tau}, \frac{dp}{d\tau} \right) \tag{B5} \]

where \( p \) is the three-momentum of the atom in atom’s rest frame. Uniform angular decay rate implies that \( \frac{dp}{d\tau} = 0 \).

Therefore,

\[ K_{\mu} = \left( \frac{1}{c} \frac{dE}{d\tau}, 0, 0, 0 \right) \equiv \left( -\frac{\Gamma_0 \hbar \omega_0}{c}, 0, 0, 0 \right). \tag{B6} \]

Lorentz-transforming \( K_{\mu} \) to the lab-frame (w.r.t. which the atom is moving with \( v = v \hat{x} \)) we get

\[ K_{\mu}^{\text{lab}} = \Lambda_{\mu}^{\nu} K_{\nu} = \left( -\gamma \frac{\Gamma_0 \hbar \omega_0}{c}, -\gamma \frac{\Gamma_0 \hbar \omega_0}{c^2}, v, 0, 0 \right). \tag{B7} \]

As the velocity of the atom does not change, using (B7) and \( p(t = 0) = \gamma m_0 v_0 \), we can write

\[ \frac{dp}{dt} = -\frac{\Gamma_0 \hbar \omega_0}{c^2} \frac{p_0}{\gamma m_0}. \tag{B8} \]

**Appendix C: Decay of an excited atom to the thermal field state**

The density matrix for the multimode thermal state of the electromagnetic field is given by [29],

\[ \rho_B = \prod_{k \lambda} (1 - e^{-\beta \hbar \omega_k}) e^{\beta \hbar \omega_k a_k^\dagger a_k}. \tag{C1} \]

This can be rewritten as

\[ \rho_B = \sum_{\eta} P_\eta |\eta\rangle \langle \eta| \tag{C2} \]

where,

\[ P_\eta = \prod_{k \lambda} (1 - e^{-\beta \hbar \omega_k}) e^{-\beta \hbar \omega_k n_{k \lambda} k}. \tag{C3} \]

Here we have defined \( \eta \equiv \{ n_{k \lambda} \} \) as \( n \)-photon state for all the field modes combined. Using the relation

\[ \langle \eta | a_{k \lambda} a_{q \lambda}^\dagger | \eta \rangle = (n_{q \lambda} + 1) \delta (k - q) \delta_{\lambda \lambda}, \tag{C4} \]

we can calculate the probability \( \bar{P}(q, \omega_0) \) as

\[ \bar{P}(q, \omega_0) = \frac{|d|^2}{16\pi^3 \hbar^2} \frac{q c \delta^2 (\omega_0 - qc) \sin^2 \theta}{\sum_\eta P_\eta (n_{q \lambda} + 1)^2}. \tag{C5} \]

We note that like the case for vacuum state, the decay-probability to a given mode \( (q, \lambda) \) is symmetric about the \( z \)-axis and that \( P(q, \omega_0) = P(-q, \omega_0) \). Which means that the atom will not experience any space-like force. Notably, both the vacuum and the thermal states have inherent isotropy in them which can be witnessed in terms of the field correlators in these states.
Appendix D: Acceleration of a collection of atoms as a function of temperature in the limit $\frac{\omega_{\text{trans}}}{2\pi c T} \gg 1, \sigma \neq 0$

Consider a collection of two-level atoms, with a total number of $n$ atoms in it, interacting simultaneously with a thermal bath and with either N-photon wavepacket or a coherent state electromagnetic field. The atoms are assumed to be not interacting with each other. The rate equation for the population of the excited atomic level can be written as:

$$\dot{n}_e(\beta) = -\Gamma_{e\rightarrow g} n_e + \Gamma_{g\rightarrow e} n_g$$

(D1)

In steady state $\dot{n}_e = 0$ and we get,

$$n_e(\beta) = \frac{\Gamma_0 N(\omega_0) + \Gamma(-\omega_0)}{\Gamma_0 \coth\left(\frac{\beta \omega_0}{2}\right) + \tilde{\Gamma}(\omega_0) + \Gamma(-\omega_0)} n_s$$

(D2)

where $N(\omega_0) = (e^{\beta \hbar \omega_0} - 1)^{-1}$ is the Planck distribution at the transition frequency,

$$\Gamma(\omega_0) = \frac{2|d|^2}{\sqrt{2\pi} \hbar \epsilon_0} (N^2 + 2N) \frac{\omega_0 \sigma^2}{c}$$

(D3)

and

$$\Gamma(-\omega_0) = \frac{2|d|^2 N \omega_0 \sigma^2}{\sqrt{2\pi} \hbar \epsilon_0}$$

(D4)

When $\frac{\omega_{\text{trans}}}{2\pi c T} \gg 1$, the average force on an atom due to a $|e\rangle \rightarrow |g\rangle$ transition is

$$\Delta F = -\frac{2|d|^2}{\sqrt{2\pi} \epsilon_0} \left(\frac{\omega_0 \sigma^2}{c}\right)^2 N(N + 2) \hat{x},$$

(D5)

and due to a $|g\rangle \rightarrow |e\rangle$ transition is

$$\Delta G = \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0} \left(\frac{\omega_0 \sigma^2}{c}\right)^2 N \hat{x}.$$  

(D6)

The net force on the atomic ensemble can be written as,

$$F_{\text{net}} = n_e(\beta) \Delta F + n_g(\beta) \Delta G$$

$$= -\hat{x} n_e \left[ \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0} \left(\frac{\omega_0}{c}\right)^2 N(N + 2) \sigma^2 + \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0} \left(\frac{\omega_0}{c}\right)^2 N \sigma^2 \right] \left(\frac{\omega_0}{c}\right)^2 + \hat{x} n \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0} \left(\frac{\omega_0}{c}\right)^2 N \sigma^2$$

(D7)

$$\Rightarrow \mathbf{a} = \frac{F_{\text{net}}}{m_a n} = \hat{x} \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0 m_a} \left(\frac{\omega_0}{c}\right)^2 \left[1 - 3 \frac{n_e(\beta)}{n} \right] N \sigma^2 - \hat{x} \frac{2|d|^2}{\sqrt{2\pi} \epsilon_0 m_a} \left(\frac{\omega_0}{c}\right)^2 \frac{n_e(\beta)}{n} N \sigma^2$$

(D8)

[1] A. Ashkin, Physical Review Letters 24, 156 (1970).
[2] A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, and S. Chu, Optics Letters 11, 288 (1986).
[3] V. G. Veselago, Soviet Physics Uspekhi 10, 509 (1968).
[4] V. G. Shvedov, A. V. Rodé, Y. V. Izdebskaya, A. S. Desyatnikov, W. Krolikowski, and Y. S. Kivshar, Physical Review Letters 105, 118103 (2010).
[5] S. Sukhov and A. Dogariu, Optics Letters 35, 3847 (2010).
[6] A. Mizrahi and Y. Fainman, Optics Letters 35, 3405 (2010).
[7] J. Chen, J. Ng, Z. Lin, and C. Chan, Nature Photonics 5, 531 (2011).
[8] H. B. Casimir and D. Polder, Physical Review 73, 360 (1948).
[9] W. G. Unruh, Physical Review D 14, 870 (1976).
[10] S. W. Hawking, Nature 248, 30 (1974).
[11] K. Lochan, H. Ulbricht, A. Vinante, and S. K. Goyal, Physical Review Letters 125, 241301 (2020).
Alternatively, the force can also be calculated by obtaining transition rate from Eq. 3 and getting the momentum change per unit time.