Study of Extra Space Dimensions in Vector Boson Pair Production at LEP

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Abstract
Recent theoretical scenarios propose that quantum gravity effects may manifest at LEP energies by means of gravitons that couple to Standard Model particles and propagate into extra space dimensions. These predictions are checked against the most recent experimental results on photon, W and Z pair production. No deviations from the Standard Model expectations are found and limits of the order of 1 TeV on the scale of these models are set.

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1 Introduction

One of the great unsolved questions of contemporary physics is the wide difference between the scales of two fundamental interactions of nature, the gravitational and the electroweak. Denoting with $G_N$ the gravitational constant it indeed follows that the Planck ($M_{Pl} \sim G_N^{-1/2} \sim 10^{19} \text{GeV}$) and the electroweak ($M_{ew} \sim 10^2 \text{GeV}$) scales differ by seventeen orders of magnitude.

The Standard Model [1] (SM) successfully describes the electroweak interactions but leaves this difference unexplained. While the SM is tested by the present colliders at distances comparable to $M_{ew}^{-1}$, the experimental knowledge of the gravitational force reach only distances around a centimetre [2], thirty three orders of magnitude above its characteristic distance $M_{Pl}^{-1}$.

A recent theoretical scenario [3], proposes a modification of the present description of the gravitational force in this large unexplored domain. A scale $M_S$ of the order of $M_{ew}$ is postulated for quantum gravity, then referred to as Low Scale Gravity (LSG). The known behaviour of the gravitational force is recovered by the existence of $n$ new space dimensions of size $R$ such that:

$$M_{Pl}^2 \sim R^n M_S^{n+2}. \quad (1)$$

A single extra dimension with $M_S \sim M_{ew}$ is ruled out as it implies values of $R$ comparable to the dimensions of the solar system. Two or more extra dimensions correspond to $R < 0.1 - 1 \text{mm}$, in the unexplored regime of gravity. Severe limits are derived for $n = 2$ from SN1987A [4].

Spin two gravitons are predicted to propagate in $4 + n$ dimensions and interact with SM particles with a sizeable strength. The effects of graviton exchange diagrams in vector boson pair production are predicted to be experimentally accessible [5, 6] at the CERN $e^+e^-$ collider LEP. Data collected by the four LEP experiments up to July 1999 on photon, W and Z pair production are investigated to search for these effects.

2 $\gamma\gamma$ production

The four LEP experiments have studied [7, 8] the differential distribution of $e^+e^- \rightarrow \gamma\gamma$ events collected above the Z pole to extract limits on the QED cut–off parameters $\Lambda_+$ and $\Lambda_-$. They are defined by an additional term in the $e^+e^- \rightarrow \gamma\gamma$ cross section [9]:

$$\frac{d\sigma(e^+e^- \rightarrow \gamma\gamma)}{d \cos \theta} = \frac{2\alpha^2 \pi}{s} \left(1 + \cos^2 \theta\right) \left(\frac{1}{1 - \cos^2 \theta}\right) \pm \frac{\alpha^2 \pi s}{\Lambda_4^4}(1 + \cos^2 \theta), \quad (2)$$

$\theta$ is the polar photon production angle, $s$ the square of the centre–of–mass energy and $\alpha$ the electromagnetic coupling.

No signals of deviation from QED are observed by any of the experiments that quote the 95% confidence level (CL) limits presented in Table 1. These limits are obtained with a maximum likelihood fit to the distribution of $\cos \theta$ in data with Equation (2) with $1/\Lambda_4$ as a free parameter. The limits $\Lambda_{95}^+$ and $\Lambda_{95}^-$ follow from the integration of the likelihood functions $\mathcal{L}$ over the physical region $1/\Lambda_4 > 0$:

$$\int_0^{\Lambda_{95}^+} \mathcal{L}(x)dx = 0.95 \int_0^{+\infty} \mathcal{L}(x)dx \quad \text{and} \quad \int_{-\Lambda_{95}^-}^0 \mathcal{L}(x)dx = 0.95 \int_{-\infty}^0 \mathcal{L}(x)dx. \quad (3)$$

The investigated data samples are large enough to expect a normal distribution of the likelihood functions. Hence Equations (3) can be solved numerically for each of the pair of
Table 1: Reported limits on the QED cut-off parameters $\Lambda_+$ and $\Lambda_-$ at 95% CL

| Experiment | $\sqrt{s}$ (GeV) | $\Lambda_+$ (GeV) | $\Lambda_-$ (GeV) |
|------------|-----------------|------------------|------------------|
| ALEPH      | 189             | 269              | 308              |
| ALEPH      | 161–183         | 270              | 230              |
| DELPHI     | 130–189         | 284              | 278              |
| L3         | 130–196         | 323              | 294              |
| OPAL       | 183–196         | 271              | 331              |
| OPAL       | 130–172         | 195              | 210              |

entries of Table 1 inferring the original likelihood functions. As a cross check, limits on $\Lambda_+$ and $\Lambda_-$ for each of the data samples are derived from the inferred likelihood functions and found to be in agreement with those in Table 1 within all the quoted digits.

A combined likelihood function is built from the sum of the individual inferred ones. It shows no deviations from the QED expectations and yields the following limits at 95% CL:

$$\Lambda_+ > 343 \, \text{GeV}, \quad \Lambda_- > 367 \, \text{GeV},$$

these limits improve all those reported by the single collaborations.

The differential cross section for photon pair production in $e^+e^-$ collisions is modified by $s-$channel graviton exchange \cite{5,6}. From the formula in Reference \cite{6} it follows:

$$\frac{d\sigma(e^+e^- \to \gamma\gamma)}{d\cos\theta} = \frac{2\alpha^2\pi}{s} \left(1 + \cos^2\theta\right) - \frac{\alpha\lambda s}{M_S^4} \frac{\lambda^2 s^3}{8\pi M_S^8} \left(1 + \cos^2\theta\right)(1 - \cos^2\theta). \quad (4)$$

The LSG contributions are weighted by a factor $\lambda$ \cite{10} that include the dependence on the full theory. In the following $\lambda = \pm 1$ is chosen to allow for the different signs of the interference. The pure gravitational part in the third term never exceeds 1% of the second term, the interference one, and can be neglected. From a comparison of Equations (4) and (2) it then follows:

$$\frac{\lambda}{M_S^4} = \pm \frac{\pi\alpha}{\Lambda_+^4}. \quad (5)$$

The combined likelihood function described above can therefore be translated in terms of $\lambda/M_S^4$. Figure 1 displays this likelihood function that agrees with the SM expectations. Limits on the scale $M_S$ of LSG are then extracted as listed in Table 2.

A similar analysis based on a reduced data sample is described in Reference \cite{11}.

| Process          | $\lambda = -1$  | $\lambda = +1$  |
|------------------|-----------------|-----------------|
| $e^+e^- \to \gamma\gamma$ | 0.88            | 0.94            |
| $e^+e^- \to W^+W^-$  | 0.85            | 0.68            |
| $e^+e^- \to ZZ$      | 0.62            | 0.63            |
| Combined           | 0.96            | 0.93            |

Table 2: Lower limits on $M_S$ at 95% C.L.
Since 1996 LEP is running above the W pair production threshold. The combined [13] results of the four experiments for the $e^+e^- \rightarrow W^+W^-$ cross section at different $\sqrt{s}$ are reported in Table 3. SM prediction obtained with the KORALW [12] Monte Carlo program are also listed.

The LSG contributions to W pair is described at Born level in Reference [6]. To take into account higher order corrections the following procedure is applied. First the SM $e^+e^- \rightarrow W^+W^-$ cross section is calculated from the Born level amplitudes [14]. Then a correction factor $A$ is calculated as the ratio of the KORALW cross section to this Born level calculation at different energies. The values of $A$ are reported in Table 3. This correction factor is assumed to hold for LSG diagrams as well.

| $\sqrt{s}$ (GeV) | $\sigma(e^+e^- \rightarrow W^+W^-)$ (pb) | $\sigma^{SM}(e^+e^- \rightarrow W^+W^-)$ (pb) | $A$ |
|-----------------|------------------------------------|-----------------------------------|-----|
| 172             | 12.0 ± 0.7                          | 12.40                             | 0.82|
| 183             | 15.83 ± 0.36                        | 15.70                             | 0.89|
| 189             | 16.05 ± 0.22                        | 16.65                             | 0.92|
| 192             | 16.5 ± 0.5                          | 16.97                             | 0.93|
| 196             | 17.2 ± 0.5                          | 17.28                             | 0.95|

Table 3: Measured and expected W pair production cross sections at the different LEP energies. Correction factors to Born level calculations are also given.

The $e^+e^- \rightarrow W^+W^-$ cross section in presence of LSG is calculated from the matrix elements [6,14] and multiplied by $A$. From the comparison of this value with data at the different energies a likelihood function is built. It takes into account the experimental uncertainties quoted in Table 3 and a 2% uncertainty on the theoretical treatment of the initial state radiation. This likelihood function is displayed in Figure 1 in terms of $\lambda/M_S^4$ and shows no significant deviations from the SM. Lower limits on $M_S$ are derived by integrating the likelihood over the physical region and are summarised in Table 2.

The sensitivity of the WW channel to LSG effects is limited by the initial state radiation error. In the hypothesis of its reduction to 0.5% [15] the tightest of these limits would improve to 0.99 TeV.

### 4 ZZ production

In 1997 the pair production of Z bosons became accessible at LEP. The four collaborations reported a combined measured cross section of $0.17\pm0.09$ pb at $\sqrt{s} = 183$ GeV and $0.70\pm0.08$ pb at $\sqrt{s} = 189$ GeV [16]. The SM predictions are 0.26 pb and 0.65 pb respectively, as calculated with YFSZZZ [17].

The LSG matrix element for Z pair production is similar to the W pair one [6]. The same procedure described above is used to extract limits on $M_S$. The values of the correction factor $A$ to the Born level predictions for the $e^+e^- \rightarrow ZZ$ cross sections with respect to the YFSZZZ ones are 1.12 and 0.80 at 183 GeV and 189 GeV, respectively.

Figure 1 presents the likelihood function used to determine the limits in Table 2. The impact of the theory uncertainty in this channel is negligible when compared to the experimental one.
5 Combined results

Assuming that no higher order operators contribute to Equation (1) and to the LSG ZZ and WW matrix elements, and the meaning of $M_S$ is the same for all the investigated channels, the three likelihood functions described above can be added. The combined likelihood function is found to be in agreement with the SM predictions as shown in Figure 1. Lower limits on $M_S$ at 95% CL can be derived as 0.92 TeV and 0.96 TeV for $\lambda = +1$ and $\lambda = -1$ respectively. The second of this limits does not improve the $e^+e^- \to \gamma\gamma$ one as the effect of the $e^+e^- \to W^+W^-$ likelihood is to shift the combined maximum toward higher values without a major narrowing of its width.

The LEP data on Bhabha scattering were also recently analysed in terms of possible LSG contributions [18]. The original likelihood is inferred from the quoted limits with the same procedure used for the $e^+e^- \to \gamma\gamma$ process and is added to the combined likelihood described above. The $\lambda = -1$ 95% CL lower limit on $M_S$ reads then 1.01 TeV, improving the limits of both the boson and Bhabha analyses. The $\lambda = +1$ limit derived from the Bhabha scattering dominates this combination that does not improve it.

In conclusion the first limits on LSG from combined LEP data for all vector boson pair production processes are derived, improving those reported by the single collaborations [8, 19]. A combined analysis of boson and fermion pairs improves the sensitivity to LSG effects.

References

[1] S. L. Glashow, Nucl. Phys. 22 579 (1961); A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8) edited by N. Svartholm, (Almqvist and Wiksell, Stockholm, 1968), p. 367; S. Weinberg, Phys. Rev. Lett. 19 1264 (1967).

[2] J. C. Long et al., Nucl. Phys. B 539 23 (1999).

[3] N. Arkani–Hamed et al., Phys. Lett. B 429 263 (1999).

[4] N. Arkani–Hamed et al., Phys. Rev. D 59 086004 (1999).

[5] G. F. Giudice et al., Nucl. Phys. B 544 3 (1999).

[6] K. Agashe and N. G. Deshpande, Phys. Lett. B 456 60 (1999).

[7] ALEPH Collab., R. Barate et al., Contributed paper #6,429 to the EPS-HEP99, Tampere, Finland, 1999, R. Barate et al., Phys. Lett. B 429 201 (1998); DELPHI Collab., P. Abreu et al., Contributed paper #6,364 to the EPS-HEP99, Tampere, Finland, 1999; L3 Collab., M. Acciarri et al., Contributed paper #7,233 to the EPS-HEP99, Tampere, Finland, 1999.

[8] OPAL Collab., K. Ackerstaff et al., Contributed paper #1,880 to the EPS-HEP99, Tampere, Finland, 1999, Eur. Phys. J. C 21 (1998).

[9] F. E. Low, Phys. Rev. Lett. 14 238 (1965); R. P. Feynman, Phys. Rev. 74 939 (1948); F. M. Renard, Phys. Lett. B 116 264 (1982); S. Drell, Ann. Phys. 4 75 (1958).

[10] J. Hewett, Phys. Rev. Lett. 82 4765 (1999).
[11] K. Cheung, Preprint hep-ph/9904266.

[12] M. Skrzypek et al., Comp. Phys. Comm. 94 216 (1996); M. Skrzypek et al., Phys. Lett. B 372 289 (1996).

[13] The LEP Electroweak Working Group, D. Abbaneo et al., Preprint CERN-EP/99-15; F. Cavallari, XXXIVth Rencontres de Moriond, Electroweak Interactions and Unified Theories, 1999, to appear in the Proceedings; A. Barczyk, EPS-HEP99, Tampere, Finland, 1999, to appear in the Proceedings.

[14] W. Beenaker and A. Denner, Int. J. Mod. Phys. A9 4837 (1994).

[15] A. Ballestrero, EPS-HEP99, Tampere, Finland, 1999, to appear in the Proceedings.

[16] E. Sanchez, EPS-HEP99, Tampere, Finland, 1999, to appear in the Proceedings.

[17] S. Jadach et al., Phys. Rev. D56 6939 (1997).

[18] D. Bourilkov, JHEP 08 006 (1999).

[19] ALEPH Collab., R. Barate et al., Contributed paper #7.252 to the EPS-HEP99, Tampere, Finland, 1999; L3 Collab., M. Acciarri et al., Contributed paper #7.233 to the EPS-HEP99, Tampere, Finland, 1999; OPAL Collab., G. Abbiendi et al., Preprint CERN-EP/99-088, Preprint CERN-EP/99-097.
Figure 1: Difference with respect to the minimum of the negative logarithm of the individual and combined likelihood functions in terms of $\lambda/M_s^4$. 