Solving transportation problem using modified ASM method

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Abstract. Transportation problems are related to activities aimed at minimizing the cost of distributing goods from a source to a destination. One of the methods used to solve transportation problems is the ASM Method as a method capable of producing optimal direct solutions without having to determine the initial basic feasible solution first. Determination of the allocation of goods in the ASM Method uses a reduced cost of 0 by calculating the maximum amount in the allocation of goods. Then the ASM method is modified so that the iteration used is simpler in obtaining the optimal direct solution without calculating the maximum number of row and column elements. The method is called Modified ASM Method. This method also provides more optimal results than the ASM method. This research aimed to solve transportation problems using the Modified ASM method to produce optimal solutions directly. This research shows that the Modified ASM method successfully solves the problem of balanced and unbalanced transportation by producing optimal solutions in a simpler way than the ASM method.

1. Introduction
Every company must strive for minimum costs in the transportation process, so a strategy in solving problems was needed to minimize the costs of the transportation process. The transportation problem is a part of linear programming where the goal is to minimize the cost of distribution from a source to a destination [1]. Basically, the procedure for solving transportation problems consists of formulating a mathematical model of transportation problems, finding an initial basic feasible solution that can be done using several methods, such as the North West Corner (NWC) method, Allocation Table Method (ATM), Least Cost (LC), then optimizing the initial basic feasible solution obtained by using the Modified Distribution (MODI) or Stepping Stone methods [1,2].

In addition to going through these stages, several studies have introduced methods that can immediately produce an optimal solution so that the steps to find an initial basic feasible solution are not needed. Examples are the ASM Method, Mean Arithmetic. One of them is the research of [1] which introduced the Modified ASM method, which can also produce optimal direct solutions. The Modified ASM method is a modification of the ASM method [1]. The Modified ASM method focuses on the elements resulting from the reduction of rows and columns. The Modified ASM method has shorter iterations than the ASM method because it does not need to find the maximum number of reduced costs of 0 [1] in the element of the i-row and the jth column so that this method will be allocated as many goods at the result of the reduced cost of 0. However, if the settlement has not met the fruit of the base
variable, then the allocation of goods is done due to the smallest cost of a subsequent reduction in rows and columns that have not been met both supply and demand. While in the ASM Method of goods, as much as will only be allocated to the result of reduced costs of 0 only.

2. Method
This research will be conducted through the following stages.

a) We are identifying and forming the transportation model regarding decision variables, objective functions, and constraint functions.

b) We are identifying whether the transportation problem is balanced or unbalanced.

c) We are solving the transportation problem with the steps of the Modified ASM method.

d) We are giving a numerical illustration to solve transportation problems using the Modified ASM method.

3. Results And Discussion
3.1 Forming the Transportation Problem Model
The transportation model is a method used to regulate the allocation of goods from sources that provide products to destinations [3,4]. The purpose of this model is to determine the amount that must be sent from each source to each destination to obtain the minimum total transportation cost [3]. The cost of transportation of goods from \(i\) sources to \(j\) destinations is expressed in Equation 3.1, which can be written as follows.

\[
Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}
\]  

(1)

With the constraint functions as follows.

Constraint function of supplies:

\[
X_{11} + X_{12} + \cdots + X_{ij} = S_i
\]  

(2)

\[
X_{21} + X_{22} + \cdots + X_{ij} = S_j
\]  

(3)

\[
X_{1i} + X_{2i} + \cdots + X_{ij} = S_i
\]  

(4)

Constraint function of demands:

\[
X_{11} + X_{21} + \cdots + X_{ij} = D_i
\]  

(5)

\[
X_{12} + X_{22} + \cdots + X_{ij} = D_j
\]  

(6)

\[
X_{1j} + X_{2j} + \cdots + X_{ij} = D_j
\]  

(7)

Where

\(Z\) = the total cost of allocating goods to be minimized

\(X_{ij}\) = the capacity for allocating goods from the \(i\)th source to the \(j\)th destination, 2\(^{nd}\) source to the 2\(^{nd}\) destination, and so on until the \(n\)th source to the \(n\)th destination

\(C_{ij}\) = the cost for allocating goods from the \(i\)th source to the \(j\)th destination, 2\(^{nd}\) source to the 2\(^{nd}\) destination, and so on until the \(n\)th source to the \(n\)th destination

\(S_1, S_2, ..., S_i\) = the 1\(^{st}\) supply, the 2\(^{nd}\) supply, and so on until the \(i\)th supply, where \(i = 1, 2, ..., m\).

\(D_1, D_2, ..., D_j\) = the 1\(^{st}\) demand, the 2\(^{nd}\) demand, and so on until the \(j\)th demand, where \(j = 1, 2, ..., n\).

After forming the transportation problem model, the basic assumptions in the transportation problem model for this research was made, namely that the amount of transportation costs for allocating goods on the route is proportional to the number of goods distributed, the goods to be distributed are similar goods, and the amount of demand and supply is known. Next was solving transportation problems by obtaining an optimal direct solution using the Modified ASM method to calculate the cost of distributing goods.
3.2 The Steps of Solving Balanced and Unbalanced Transportation Problems Using the Modified ASM Method

The following are steps to solve transportation problems with a flowchart:

![Flowchart](image)

**Figure 1.** flowchart: steps to solve transportation problems using the modified ASM

The steps to solve transportation problems are both balanced and unbalanced in detail are as follows:

1) Form a preliminary table of transportation problems. If the amount of supply is equal to the amount of demand, proceed to the next step. However, if the amount of supply is not equal to the amount of demand, it needs to be balanced first by adding a dummy variable.

2) Swap even rows with even rows and odd rows with odd rows. This step is carried out until row \(m\), if row \(m\) is an even row then row \(m\) will be swapped with other even rows, namely rows \(2, 4, ..., m - 2\) and odd rows will be swapped for other odd rows, namely rows \(1, 3, ..., m - 1\), so that the order of the row swap in the table is \(m - 1, m, 1, 2, ..., m - 2\). However, if row \(m\) is an odd row, then row \(m\) will be swapped with another odd row, namely rows \(1, 3, ..., m - 2, m\), and even rows will be swapped with other even rows, namely rows \(2, 4, ..., m - 1\), so that the order of the row swap in the table is \(m, m - 1, 1, 2, ..., m - 2\).

3) Swap even columns with even columns and odd columns with odd columns. This step is carried out until column \(n\), if column \(n\) is an even column then column \(n\) will be swapped with another even column, namely column \(2, 4, ..., n - 2\), and odd column will be swapped for another odd column, namely column \(1, 3, ..., n - 1\) so that the order of the results of swapping columns in the table is \(n - 1, n, 1, 2, ..., n - 2\). However, suppose column \(n\) is an odd column. In that case, \(n\) then column \(n\) will be exchanged with another odd column, namely columns \(1, 3, ..., n - 2, n\) and...
even columns will be exchanged with other even columns, namely columns 2, 4, ..., \( n - 1 \), so that the order of the results of swapping columns in the table is \( n, n-1, 2, ..., n-2 \).

4) Identify the least cost per row and subtract each row element from the transportation problem table using the least cost per row. The transportation table will not change when a dummy variable is added to the column because the smallest cost is 0.

5) Identify the least cost per column and subtract each column element from the transportation table with the least cost in each column. The transportation table will not change when a dummy variable is added to the row because the smallest cost is 0.

6) Selects the elements in the rows of the transport table that have a reduced cost of 0 and allocates goods of \( X_{ij} \) with \( \min (S_i, D_j) \) on those elements.

7) Delete rows or columns for which supply or demand has been met. If the result of reduced cost 0 has been filled in completely, then the remaining supply or demand is allocated to the element that has the next smallest reduction cost value in the rows and columns that have not been fulfilled for both supply and demand until \( \sum S_i = \sum D_j \) is fulfilled.

8) The filling table of transportation problems produced the optimal solution for the minimum total cost.

3.3 Numerical Illustration of Transportation Problems Using the Modified ASM Method

The following is a numerical illustration of [1] for assumptions so that it is easy to explain more clearly about the solution of transportation problems using the Modified ASM Method as follows:

It is known from the transportation problem that there are three sources \( (i) \) and four destinations \( (j) \), where these sources and destinations are interrelated and have transportation costs \( (C_{ij}) \) and the number of goods distributed \( (X_{ij}) \). In this case, the cost from each source to the destination, the amount of supply from the source, and the amount of demand from the destination is known, illustrated in Table 1 as follows.

| Source \((i)\) | Destination \((j)\) | Supply \((S_i)\) |
|---------------|-------------------|----------------|
| \( S_1 \)  | \( D_1 \) | 1 | 30 |
| \( S_1 \)  | \( D_2 \) | 2 |
| \( S_1 \)  | \( D_3 \) | 1 |
| \( S_1 \)  | \( D_4 \) | 4 |
| \( S_2 \)  | \( D_1 \) | 3 |
| \( S_2 \)  | \( D_2 \) | 3 |
| \( S_2 \)  | \( D_3 \) | 2 |
| \( S_2 \)  | \( D_4 \) | 1 |
| \( S_3 \)  | \( D_1 \) | 4 |
| \( S_3 \)  | \( D_2 \) | 2 |
| \( S_3 \)  | \( D_3 \) | 5 |
| \( S_3 \)  | \( D_4 \) | 9 |
| Demand \((D_j)\) | 20 | 40 | 30 | 10 | 100 |

The Steps of Solving Transportation Problems Using the Modified ASM Method

**Step 1**

Form the initial table of transportation problems as shown in Table 2 with three sources, namely \( S_1, S_2, \) and \( S_3 \), and four destinations, namely \( D_1, D_2, D_3, \) and \( D_4 \). The transportation problem above the amount of supply is equal to the number of demand, so proceed to the next step.

**Step 2**

Swap even rows with even rows and odd rows with odd rows. In Table 2, even rows will not be exchanged because there is only one even row, namely row \( S_2 \). And for odd rows will be exchanged, namely rows \( S_1 \) and \( S_3 \).

**Step 3**

Swap even columns with even columns and odd columns with odd columns. Even columns will be swapped, namely columns \( D_2 \) and \( D_4 \). And odd columns will be swapped, namely columns \( D_1 \) and \( D_3 \). Thus, Table 3 was formed as follows.
Table 2. Results of Swapping Columns

| Source (i) | Destination (j) | Supply (S_i) |
|------------|----------------|--------------|
|            | D_3 | D_4 | D_1 | D_2 |       |
| S_3        | 5   | 9   | 4   | 2   | 20    |
| S_2        | 2   | 1   | 3   | 3   | 50    |
| S_1        | 1   | 4   | 1   | 2   | 30    |
| Demand (D_j)| 30  | 10  | 20  | 40  | 100   |

Step 4
Identify the least cost per row and reduce each row element of the transportation table to the least cost in each row. Suppose the smallest costs are 2 in a row S_3, 1 in row S_2. And 1 in row S_3. Thus, Table 4 was formed as follows.

Table 3. Results of Reducing Row Cost

| Source (i) | Destination (j) | Supply (S_i) |
|------------|----------------|--------------|
|            | D_3 | D_4 | D_1 | D_2 |       |
| S_3        | 3   | 7   | 2   | 0   | 20    |
| S_2        | 1   | 0   | 2   | 2   | 50    |
| S_1        | 0   | 3   | 0   | 1   | 30    |
| Demand (D_j)| 30  | 10  | 20  | 40  | 100   |

Step 5
Identify the least cost per column and reduce each column element of the transportation table with the least cost in each column. The smallest cost in all columns in the above table equals 0, so there will be no change in the cost reduction for each column.

Step 6
Selects the elements in the rows of the transport table that have a reduced cost of 0 and allocates goods of X_{ij} with min(S_i, D_j) on those elements. Because the smallest cost of the reduction in each row is 0, namely row S_3 in element S_3D_2, row S_2 in element S_2D_4, and row S_1 in elements S_1D_3 and S_1D_1. The allocation starts from the element S_3D_2 which has a reduced cost of 0 with an allocation of 20 goods. Thus, Table 5 was formed as follows.

Table 4. Allocating to Element S_3D_2 on Cost Reduction

| Source (i) | Destination (j) | Supply (S_i) |
|------------|----------------|--------------|
|            | D_3 | D_4 | D_1 | D_2 |       |
| S_3        | 3   | 7   | 2   | 0   | 20    |
| S_2        | 1   | 0   | 2   | 2   | 50    |
| S_1        | 0   | 3   | 0   | 1   | 30    |
| Demand (D_j)| 30  | 10  | 20  | 40  | 100   |
Step 7
Delete the row or column where supply from the source or demand from the destination was met. The allocation in step 6, which is 20 elements in $S_3D_2$, thus the supply from the source is fulfilled, where $X_{S_3D_2} = S_3$ so that in the next step, in row $S_3$ there will be no allocation of goods.

Step 8
Repeat steps 6 and 7, which is allocating goods of $X_{ij}$ at the costs reduced by 0 until the amount of supply and demand is met. The smallest cost resulting from a reduction of 0 will be allocated, but supply and demand have not been met, so the allocation of goods will be made to the cost of the next smallest reduction, which is in elements $S_2D_3$ with a reduced cost of 1 and $S_2D_2$ with a reduced cost of 2, thus the goods will be allocated on the element $S_2D_3$ as many as 20. Thus, Table 6 was formed as follows.

| Sumber ($S_i$) | Tujuan ($D_j$) | Supply ($S_i$) |
|---------------|---------------|---------------|
| $S_3$         | $D_2$         | 20            |
| $S_2$         | $D_3$         | 20            |
| $S_1$         | $D_1$         | 30            |
| Demand ($D_j$)| 30            | 10            |

Step 9
The filling table of transportation problem above produces the optimal solution for the minimum total cost. Hence, the total cost is

$$Z = \sum_{i=1}^{3} \sum_{j=1}^{4} C_{ij}X_{ij}$$

$$= 2(20) + 1(20) + 1(10) + 2(20) + 3(20) + 1(10)$$

$$= 180$$

The above transport problems have filled 20 items in $D_2S_3$ cells, as many as 20 in $D_3S_2$ cells, as many as 10 in $D_4S_2$ cells, as many as 20 in $D_3S_2$ cells, as many as 10 in $D_3S_1$ cells and as many as 20 in $D_4S_4$ cells. Thus, the value of the optimal solution for solving the transportation problem is 180.

4. Conclusions
This research concludes that the Modified ASM Method can solve the problem of balanced and unbalanced transportation. Based on research, this method produces an optimal solution and has been tested for optimality using the UV method. The results obtained are either equal to the ASM method or even less than that. An initial basic feasible solution is calculated before finding the optimal solution. Still, in this Modified ASM method, as proved in this research, the problem can be solved directly, and the optimal solution is obtained.
References

[1] Sirisha J and Viola A 2018 *Int. J. Pure Appl. Math.* **118**
[2] Maharana M 2017 *J. Res.* **3**
[3] Affandi P, Fadhilah I, Iftitah N and Mardiah H 2020 *Kapita Selekta Seri 1 Metode Transportasi* (Banjarbaru: Yayasan Cipta Cerdas)
[4] Taha H A 2007 *Operation Research An Introduction Edisi ke-8* (New Jersey: Upper Saddle River)