Modeling Cassegrain Antenna Ka Band

S Rizal1*, Z Rainaldy Ardhana2, N S Y Hasanah3
1 Computer Science, Facility and Program Department, PUSTEKSAT LAPAN, Jalan Cagak Satelit, Km. 04 Ranca Bungur, Bogor, 16310, Indonesia
2,3 Analyse Utilization of Technology, Facility and Program Department, PUSTEKSAT LAPAN, Jalan Cagak Satelit, Km. 04 Ranca Bungur, Bogor, 16310, Indonesia

*Corresponding author: sofian.rizal@lapan.go.id

Abstract. As frequency increases wavelength decreases. Hence it becomes possible to construct antennas that are of moderate physical size. This allows the possibility of constructing antennas with a large aperture, high gain and narrow beamwidth. This type of radiation behaviour allows the antenna to be deployed in point to point microwave links. A very convenient way of achieving this behaviour at microwave frequencies is to use shaped metal reflectors. This method use Cassegrain design for subreflector configuration where the subreflector is a section of hyperboloid within the focus of the paraboloidal main reflector. The subreflector has two focal points. One of which is made to coincide with that of the main reflector and the other with the phase center of the feed horn. To facilitate maintenance it is possible to install it at ground level in a building under the antenna by using a system of microwave mirrors to guide the radio waves from the primary feed at ground level to the focus of the reflector. This arrangement permits the high power and low noise amplifiers to be installed at the ground surface. A rotary joint is not required because the beams are guided in free space rather than via waveguide. The perturbation caused by the auxiliary mirror leads to a slight reduction of gain.

1. Introduction

Antenna Gain to noise temperature (G/T) is a figure of merit with regards to the characterization of an antenna’s performance. Where \(G\) is the antenna gain in decibels at the receive frequency and \(T\) is the equivalent noise temperature of the receiving system in Kelvin. Cassegrain Antennas are used at earth stations that require a high G/T ratio and are capable of carrying large numbers of data channels simultaneously [8][9][12]. The formula for calculating G/T as equation (1).

\[
\frac{G}{T} = 10 \log \left( \frac{8 \pi k (y-1)}{S_0 \lambda^2 C_a} \right)^{1/2}
\] (1)

Where

- \(\frac{G}{T}\) = antenna system dB/K
- \(k\) = Boltzmann Constant = 1.38 x 10^{-23} W/K/Hz
- \(y\) = source noise power density to cold sky power density, linear no units
- \(S_0\) = Solar Flux Density, expressed in Solar Flux Units (SFU) 10^{22} W/m²/Hz
- \(\lambda\) = Wavelength in meters
C = Beam Correction Factor, linear no units
α = Atmospheric Attenuation at elevation angle, linear no units

Cassegrain antennas are required to supply gains in excess of 25 dB. This requires the use of an aperture antennas are feed horns and reflector antennas. An aperture antenna achieves gain and a narrow beam by creating an electromagnetic field over the aperture that has uniform phase. It is necessary to control the amplitude distribution of the aperture field to maximize gain and to minimize losses. A Cassegrain antenna is a dual reflector antenna which consist of paraboloid main reflector and hyperboloid subreflector. Smooth-wall horn present problems that can be eliminated by corrugating the walls. The corrugated wall presents the same boundary conditions to the electric and magnetic field when it is capacitive slots \( \lambda/4 \) to \( \lambda/2 \) deep.

2. Methodology
The methodology like in figure 1.

![Cassegrain Antenna Schematic](image)

**Figure 1.** Cassegrain antenna schematic.

3. Results and discussion
A Cassegrain antenna with a beam waveguide feed system assembly to minimize the losses in the transmission lines connecting the high power amplifier and the low noise amplifier to the feed consist of four mirrors supported by a shroud and precisely located to the subreflector, the elevation axis, the azimuth axis and the feed. The shroud assembly acts as a shield against ground noise and provides a rigid structure which maintains the mounting integrity of the mirrors when the antenna is subjected to wind, thermal or other external loading conditions [2-7][11]. Four reflectors in a Cassegrain antenna can be made up of linear parallel reflector which will reflect one polarization and transmit the orthogonal polarization. When they are aligned with the \( F_1 \) dan \( F_2 \) reflector element at 45° to the incoming polarization in figure 1 then half of the energy is passed and half is reflected. The \( P_1 \) and \( P_2 \) reflector will redirect the transmitted energy back through the linear elements with a phase which...
depends on the spacing between \( F_1 \) and \( P_1 \) and \( F_2 \) with \( P_2 \). A spacing of one until eight wavelength create a 90° phase shift and circular polarization. A spacing one until quarter wavelength creates a half wave phase shift and a reflected polarization at 90° from the incident polarization.

The situation \( F_1 \) and \( F_2 \) is illustrated in figure 2 [13]. Which the incident wave direction and position dependent phase are characterized by wavevector \( k'_1 \). The incidence angle is shown as \( \theta_1 \). The reflected wave characterized by wavevector \( k_1 \) will propagate away from the interface at angle \( \theta'_1 \). The transmitted wave \( k_2 \) will propagate into the second region \( \theta_2 \). One would suspect that the incident and reflected angle are equal \( \theta_1=\theta'_1 \). The medium 1 and medium 2 are losless dielectrics characterized by intrinsic impedances \( \eta_1 \) and \( \eta_2 \) and refractive index \( n_1=\sqrt{\varepsilon_{R1}} \) and \( n_2=\sqrt{\varepsilon_{R2}} \). E field is polarized in the plane of the page with \( H \) perpendicular to the page and pointing outward. With \( E \) lying in the plane of incidence the wave is said to have parallel polarization or is \( p \)-polarized. \( H \) is perpendicular to the incidence plane parallel to the interface or transverse to the direction normal to the interface. Another name for this type of polarization is transverse magnetic or TM polarization.

![Figure 2. Incident wave p-polarization (TM).](image)

The situation in which the field directions have been rotated by 90° in figure 3. \( H \) lies in the plane of incidence whereas \( E \) is perpendicular to the plane. Since \( E \) is used to define polarization the configuration is called perpendicular polarization or is \( s \)-polarized. \( E \) is parallel to the interface and \( E \) is also called transverse electric or TE polarization. The reflection and transmission coefficients will differ for the two polarization types but the reflection and transmission angles will not depend on polarization.

![Figure 3. Incident wave s-polarization (TE).](image)
For \( p \)-polarization to begin the incident, reflected, and transmitted fields in phasor form using notation

\[ e^{j(\omega t + \theta)} = \cos(\theta + \omega t), \quad \omega = \frac{2\pi}{\tau}, \quad k = \frac{2\pi}{\lambda} \]

\[ E_{s1} = E_0 e^{j(k^+ x - \omega t)} \]

\[ E_{s1} = E_0 e^{j(k^- x - \omega t)} \]

\[ E_2 = E_0 e^{j(k^2 x - \omega t)} \]

where

\[ k^+ = k_1 (\cos \theta_1 a_x + \sin \theta_1 a_z) \]

\[ k^- = k_1 (-\cos \theta_1' a_x + \sin \theta_1' a_z) \]

\[ k^2 = k_2 (\cos \theta_2 a_x + \sin \theta_2 a_z) \]

and where

\[ r = x a_x + z a_z \]

The boundary condition for continuous tangential electric field.

\[ E_{s1}^+ + E_{s1}^- = E_{s2} \quad (x=0) \]

At the boundary the field amplitudes are related.

\[ H_i + H_r = H_f \]

The wavevector magnitudes are \( k_1 = n_1 \omega / c \) and \( k_2 = n_2 \omega / c \). By operating a conical horn in what is termed a hybrid mode. Which is a non linear combination of transverse electric (TE) and transverse magnetic (TM) modes [10]. For metallic mirror main reflector, \( P_1 \) and \( P_2 \) in figure 1 for detailed in figure 4.

**Figure 4.** Parabolic antenna and its projected effective aperture.
The H plane and E plane gains in figure 4 are

$$g_E(\psi) = \left| \frac{(1+\cos \psi)^2 \sin \left( \frac{\pi b \sin \psi}{\lambda} \right)}{\frac{\pi b \sin \psi}{\lambda}} \right|^2$$

$$g_H(\psi) = \left| \frac{(1+\cos \psi)^2 \cos \left( \frac{\pi a \sin \psi}{\lambda} \right)}{1 - 4 \left( \frac{\pi a \sin \psi}{\lambda} \right)^2} \right|^2$$

where $a$ and $b$ are the dimensions of the waveguide ($\lambda$).

Achieving a hybrid mode is to corrugate the inside wall of the horn is shown in figure 5. The pattern symmetry is improved and the cross polarization is reduced. A more efficient main beam is produced with low sidelobes. It is important to reduced the cross polarization where frequency reuse is employed [1].

**Figure 5.** Corrugated horn (Courtesy of Bruce Elbert).

Various type of horn antenna can be classified as shown in table 1.

**Table 1.** Various type of horn.

| Horn type | Class spater horn | Rectangular spater horn |
|-----------|------------------|------------------------|
| shape     |                  |                        |
| aperture shape |                  |                        |
| frequency characteristics |                  |                        |
| beam symmetry |                  |                        |
| side lobe |                  |                        |
| cross-polarization level |                  |                        |
| power in beam |                  |                        |
| remarks |                  |                        |

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| remarks |                  |                        |
In table 1 that figure 5 not different for multi flare horn modification in figure 6. For distance x along the A side of the horn and distance y along the B side the deviations will be:

\[
\Delta_a(x) = \frac{x^2}{2Ra} \\
\Delta_b(y) = \frac{y^2}{2Rb}
\]

And impedance

\[
\eta = \sqrt{\frac{\mu}{\varepsilon}}
\]

Where

\[
\mu = 8.854 \times 10^{-12} \text{ farad/m} \\
\mu = \mu_0(1+\chi_m) \\
\varepsilon = 4\pi \times 10^{-7} \text{ henry/m} \\
\varepsilon = \varepsilon_0(1+\chi)
\]

The permeability \( \mu \) and the permittivity \( \varepsilon \) are related to the magnetic and the electric susceptibilities of the material. The susceptibilities \( \chi_m \) and \( \chi \) are measures of the magnetic and the electric polarization properties of the material.

For the radiation patterns of the multi flare horn modification define the related quantities

\[
\sigma^2_x = \frac{A^2}{2\pi Ra} \\
\sigma^2_y = \frac{B^2}{2\pi Rb}
\]

The function can be expressed [14]

\[
F_0(v, \sigma) = \frac{ex_0 j \left( \frac{v^2}{\sigma} \right)}{\sigma} \left[ F_0 \left( \frac{v}{\sigma} + \sigma \right) - F_0 \left( \frac{v}{\sigma} - \sigma \right) \right]
\]

Where \( F(x) = C(x) - jS(x) \) is the standar Fresnel integral and

\[
F_1(v, \sigma) = \frac{1}{2} [F_0(v+0.5, \sigma) + F_0(v-0.5, \sigma)]
\]

The normalized wavenumbers have

\[
v_x = \frac{A}{\lambda} \sin \theta \cos \phi
\]
\[ v_y = \frac{B}{\lambda} \sin \theta \sin \phi \]  

(21)

for \( \theta \) and \( \phi \) in equation (20) and (21) like in figure 7.

**Figure 6.** Multi-flare horn modification.

**Figure 7.** Radiation fields from an aperture.
The radiation intensity is
\[ U(\theta, \phi) = \frac{1}{32\pi\lambda^2} \left| E_0 \right|^2 (AB)^2 c^2(\theta) F_1(v_x, \sigma_\alpha) F_0(v_y, \sigma_\beta) \left| F(\theta) \right|^2 \]  
\[ (20) \]

Where
\[ c(\theta) = c_\phi(\theta) = \frac{(1+\cos \theta)}{2} \]  
\[ (21) \]

\[ E_0 = A e^{-jkr} \]  
\[ (22) \]

If \( v_x = v_y = 0 \) have
\[ U_{\text{max}} = \frac{1}{32\pi\lambda^2} \left| E_0 \right|^2 (AB)^2 \left| F_1(0, \sigma_\alpha) F_0(0, \sigma_\beta) \right|^2 \]  
\[ (23) \]

The normalized gain
\[ g(\theta, \phi) = \frac{U(\theta, \phi)}{U_{\text{max}}} \]  
\[ (24) \]

The H plane and E plane gains corresponding to \( \phi = 0^\circ \) and \( \phi = 90^\circ \) are
\[ G_{H}(\theta) = g(\theta, 0^\circ) \]  
\[ (25) \]

\[ G_{E}(\theta) = g(\theta, 90^\circ) \]  
\[ (26) \]

For subreflector in detailed like in figure 8.

Figure 8. Subreflector.

In figure 8 the diffracted field \( H^d \) can be expressed as [15]
\[ H^d(P_2) = \frac{1}{2\sqrt{2\pi}d(4)} e^{-j(d(4) + \frac{\pi}{2})} \frac{1}{\sqrt{1 + \left( \frac{d(4)}{R(1)} \right)^2 \sin \beta}} \left[ \beta D^h H^i(\beta) + \alpha D^s + H^i(\alpha) \right] \]  
\[ (27) \]

Where \( R_1 \) or \( R(1) \) = the radius of curvature of the diffracted wavefront passing through \( O^d \)
\[ \sin \beta = \frac{d_4}{d(4)} \] = the angle between the tangent to the edge and \( d_4 \) or \( d(4) \)
\[ \alpha,\beta \] = the unit vectors of the diffracted ray coordinates
\(D^s, D^d\) = the soft and hard diffraction coefficients
\(H_i^a\) or \(H_i(\alpha)\), \(H_i^\beta\) or \(H_i(\beta)\)= the projections of the \(H^i\) incident field on the ray coordinates at \(O^d\)

In figure 8 the \(E\) can be expressed as
\[
E = ZH \times \hat{r} \tag{28}
\]

Where \(Z\) is the intrinsic impedance of the medium (\(Z=120\pi\) ohms for the free space) and \(\hat{r}\) is the unit vector in the direction of each ray.

4. Conclusion
The Cassegrain antenna offers some advantages like noise temperature, pointing accuracy and flexibility in feed design. A disadvantage of Cassegrain antenna is the masking effect of the auxiliary reflector.

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