Abstract

We initiate the analysis of the Kaluza–Klein mass spectrum of massive IIA supergravity on the warped AdS$_6 \times w S^4$ background, by deriving the linearised equations of motion of bosonic and fermionic fluctuations, and determining the mass spectrum of those of spin-2. The spin-2 modes are given in terms of hypergeometric functions and a careful analysis of their boundary conditions uncovers the existence of two branches of mass spectra, bounded from below. The modes that saturate the bounds belong to short multiplets which we identify in the representation theory of the $f(4)$ symmetry superalgebra of the AdS$_6 \times w S^4$ solution.




## Contents

1 Introduction                           .............................................................. 1

2 Massive IIA supergravity and its AdS\(_6\) solution                        3
   2.1 Equations of motion ............................................................... 3
   2.2 The AdS\(_6\) \(\times w\) S\(_4\) solution ...................................... 4

3 Linearised equations of motion      ......................................................... 5
   3.1 Bosonic sector ....................................................................... 6
   3.2 Fermionic sector ................................................................. 9

4 Spin-2 mass spectrum                ............................................................ 9

5 Conclusions                         ................................................................. 14

A Identities                           ............................................................... 15
   A.1 Metric perturbations ............................................................. 15
   A.2 Conformal transformations ...................................................... 15

B The operators \(L^{(k)}\)............... 17

1 Introduction

Gauge field theories in five dimensions are non-renormalizable, however string theory predicts the existence of strongly coupled superconformal field theories for certain gauge groups and matter content \([1, 2]\). Such an example appears in type I\(’\) string theory from a system of D4-branes probing an O8-plane with a stack of D8-branes on top of it. This system admits a supergravity description in massive IIA supergravity \([3, 4]\), and in the near-horizon limit the geometry becomes a warped product of six-dimensional anti-de Sitter spacetime AdS\(_6\), and a four-sphere S\(_4\). The warp factor is singular at the equator of S\(_4\), due the presence of the O8-plane, and the internal space is therefore actually a hemisphere. The near-horizon background has an exceptional \(f(4)\) symmetry superalgebra, which is the unique superconformal algebra in five dimensions. The bosonic subalgebra of \(f(4)\) is \(so(2, 5) \oplus su(2)\), with \(so(2, 5)\) realised as the isometry algebra of AdS\(_6\) and the (R-symmetry algebra) \(su(2)\) as an isometry of S\(_4\). The dual superconformal field theory arises as the UV fixed point of \(\mathcal{N} = 1\) supersymmetric USp(2\(N\))
Yang–Mills theory coupled to $N_f < 8$ hypermultiplets in the fundamental representation, and one hypermultiplet in the antisymmetric representation.\(^1\)

In view of the AdS/CFT correspondence, there is the motivation to study this supergravity background in order to learn about the dual field theory. Examples include the calculation of the holographic entanglement entropy \([6]\), and the action of probe branes \([7, 8]\). Another such study is that of the Kaluza–Klein mass spectrum, which corresponds to the spectrum of the dual field theory operators. Complete Kaluza–Klein mass spectra have been obtained for anti-de Sitter compactifications whose geometry is a direct product, and the internal space a coset space, typically a sphere; for example \([9, 10, 11, 12]\). In this note we progress towards obtaining the Kaluza–Klein mass spectrum of a warped compactification, by completing the task of obtaining the linearised equations of motion for small fluctuations around the background. Furthermore, we analyse the spectrum of massive spin-2 particles or gravitons in AdS\(_6\), uncovering some interesting features\(^2\).

The analysis of the spectrum is complicated by the presence of the warp factor as it modifies the differential operators which determine it. These are differential operators on the internal manifold, in the present case a four-sphere, and turn out to be warped versions of the Laplace operator on $S^4$. Hence the standard spherical harmonic analysis is not readily available, a fact that is also due to the presence of the singularity at the equator. In the case of the spin-2 modes we reduce the problem to solving a hypergeometric ordinary differential equation with the mass spectrum determined by imposing appropriate boundary conditions. Although a rather modest task compared to the analysis of the full spectrum, it already reveals some interesting features: we find two branches of spin-2 mass spectra one of which is rather exceptional in that a certain derivative of the modes is singular. Both branches are bounded from below, and the bound is saturated by modes belonging to short multiplets which we have identified in the work of \([18, 19]\) on representations of the $\mathfrak{f}(4)$ superconformal algebra\(^3\).

The remainder of this note is as follows. In section 2 we briefly review massive IIA supergravity and its AdS\(_6\) $\times_w S^4$ solution. In section 3 we present the linearised equations of motion for fluctuations around this solution. In section 4 we determine the mass spectrum of fluctuations of spin-2. We end in section 5 with a discussion and

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\(^1\)Orbifold generalizations of this system were introduced and studied in \([5]\).

\(^2\)Spin-2 excitations of flux compactifications with anti-de Sitter, Poincaré or de Sitter invariance obey the massless scalar wave equation in ten dimensions \([13]\). For other anti-de Sitter backgrounds of massive IIA supergravity, this fact has been exploited in order to study their mass spectrum without deriving the full set of linearised equations of motion \([14, 15, 16, 17]\).

\(^3\)See also \([20]\).
comments on future work. Certain technical details are included in appendices.

2 Massive IIA supergravity and its AdS$_6$ solution

In this section we collect the equations of motion of massive IIA supergravity and review the AdS$_6 \times S^4$ solution [3].

2.1 Equations of motion

Massive IIA supergravity$^4$ in ten dimensions consists of the following bosonic fields: the metric $g$, the dilaton $\Phi$, the field strengths $H_3$, $F_2$, $F_4$, with the subscript denoting their form rank, and the constant “Romans mass” $F_0$. The field strengths satisfy the Bianchi identities

$$dH_3 = 0, \quad dF_2 = F_0 H_3, \quad dF_4 = H_3 \wedge F_2,$$

and are given in terms of the potentials $B_2$, $A_1$ and $A_3$ by

$$H_3 = dB_2,$$
$$F_2 = dA_1 + F_0 B_2,$$
$$F_4 = dA_3 + A_1 \wedge H_3 + \frac{1}{2} F_0 B_2 \wedge B_2.$$

The equations of motion of the bosonic fields are:$^5$

$$0 = R_{MN} - \frac{1}{2} \partial_M \Phi \partial_N \Phi - \frac{1}{16} F_0^2 e^{5\Phi/2} g_{MN} - \frac{1}{2} e^{3\Phi/2} (F_{MP} F_N^P - \frac{1}{16} g_{MN} (F_2)^2)$$
$$- \frac{1}{12} e^{\Phi/2} (F_{MPQR} F_N^{PQR} - \frac{3}{32} g_{MN} (F_4)^2) - \frac{1}{4} e^{-\Phi} (H_{MPQ} H_N^{PQ} - \frac{1}{17} g_{MN} (H_3)^2),$$

$$0 = \nabla^M \nabla_M \Phi - \frac{5}{4} F_0^2 e^{5\Phi/2} - \frac{3}{8} e^{3\Phi/2} (F_2)^2 - \frac{1}{96} e^{\Phi/2} (F_4)^2 + \frac{1}{12} e^{-\Phi} (H_3)^2,$$

$$0 = \nabla^M (e^{-\Phi} H_{MNP}) - F_0 e^{3\Phi/2} F_{NP} - \frac{1}{2} e^{\Phi/2} F_{NPQR} F^Q R + \frac{1}{2 \cdot 4 \cdot 6} \epsilon_{M_1 \ldots M_8 N P} F^{M_1 \ldots M_4} F^{M_5 \ldots M_8},$$

$$0 = \nabla^M (e^{3\Phi/2} F_{MN}) + \frac{1}{6} e^{\Phi/2} F_{PQR} H^{PQR},$$

$^4$We use the conventions of [21] for the formulation of the theory, with some changes in notation.

$^5$In what follows we will suppress the subscript denoting the rank of a form field whenever its indices appear.
\[ 0 = \nabla^M (e^{\Phi/2} F_{MNPQ}) - \frac{1}{144} \epsilon_{M_1 \ldots M_7 NPQ} F^{M_1 \ldots M_4} H^{M_5 \ldots M_7}. \] (2.7)

In the above \( R_{MN} \) is the Ricci tensor and \( \epsilon_{M_1 \ldots M_7} \) the totally antisymmetric tensor. For a \( p \)-form \( \alpha_p \) we have used \( (\alpha_p)^2 \) to denote the contraction \( \alpha_{M_1 \ldots M_p} \alpha^{M_1 \ldots M_p} \). By taking the trace of the Einstein equation (2.3) and substituting into (2.4) we find an alternate equation for the dilaton:

\[ 0 = \nabla^M \nabla_M \Phi - 2 R + g^{MN} \partial_M \Phi \partial_N \Phi + \frac{1}{6} e^{-\Phi} (H_3)^2, \] (2.8)

where \( R \) is the Ricci scalar.

In addition to the bosonic fields, massive IIA supergravity contains the gravitino \( \Psi_M \), and the dilatino \( \Lambda \) which are both 32-component Majorana spinors. Their equations of motion are respectively

\[ 0 = \Gamma^{MNP} D_N \Psi_P - \frac{1}{4} d\Phi \cdot \Gamma^M \Lambda + \frac{1}{4} F_0 e^{5\Phi/4} \Gamma^{MN} \Psi_N + \frac{5}{16} F_0 e^{5\Phi/4} \Gamma^M \Lambda \\
- \frac{1}{8} e^{3\Phi/4} (2 \Gamma^{[M} F_2 \cdot \Gamma^{N]} \Gamma_{11} \Psi_N - \frac{3}{2} F_2 \cdot \Gamma^M \Gamma_{11} \Lambda) \\
- \frac{1}{8} e^{-\Phi/2} (2 \Gamma^{[M} H_3 \cdot \Gamma^{N]} \Gamma_{11} \Psi_N - H_3 \cdot \Gamma^M \Gamma_{11} \Lambda) \\
+ \frac{1}{8} e^{\Phi/4} (2 \Gamma^{[M} F_4 \cdot \Gamma^{N]} \Psi_N + \frac{1}{2} F_4 \cdot \Gamma^M \Lambda), \] (2.9)

\[ 0 = \Gamma^M \nabla_M \Lambda - \frac{5}{16} e^{3\Phi/4} F_2 \cdot \Gamma_{11} \Lambda + \frac{3}{8} e^{3\Phi/4} \Gamma^M F_2 \cdot \Gamma_{11} \Psi_M \\
+ \frac{1}{4} e^{-\Phi/2} \Gamma^M H_3 \cdot \Gamma_{11} \Psi_M + \frac{3}{16} e^{\Phi/4} F_4 \cdot \Lambda - \frac{1}{8} e^{\Phi/4} \Gamma^M F_4 \cdot \Psi_M \\
- \frac{1}{2} \Gamma^M d\Phi \cdot \Psi_M - \frac{21}{16} F_0 e^{5\Phi/4} \Lambda - \frac{5}{8} F_0 e^{5\Phi/4} \Gamma^M \Psi_M, \] (2.10)

where \( \nabla_M \) is the usual spin-covariant derivative acting on (vector)-spinors, and \( \cdot \) denotes the Clifford product: \( \alpha_p \cdot \Lambda := \frac{1}{p!} \epsilon_{\alpha_{M_1 \ldots M_p}} \Gamma^{M_1 \ldots M_p} \Lambda \). The matrices \( \Gamma_M \) generate the Clifford algebra \( C\ell(1,9) \) and satisfy \( \{ \Gamma_M, \Gamma_N \} = 2 g_{MN} \). The constant chirality operator is defined as \( \Gamma_{11} = \Gamma_0 \ldots \Gamma_9 \).

2.2 The AdS\(_6 \times S^4\) solution

The AdS\(_6 \times S^4\) solution of massive IIA supergravity was found [3] by considering the near-horizon limit of a system of D4–D8-branes in the presence of an O8 orientifold plane. In this background (we use \( \ast \) above a field to denote its background value) the
metric is a warped product of \( \text{AdS}_6 \) and \( S^4 \) given by\(^6\)

\[
d s_{10}^2 = e^{2A(y)} \left[ \frac{9}{4} d s_{\text{AdS}_6}^2(x) + d s_{S^4}^2(y) \right]. \tag{2.11}
\]

Here \( x, y \) denote the external and internal coordinates respectively, and the line elements on \( \text{AdS}_6 \) and \( S^4 \) are of unit radius. In what follows we will use the external metric \( g_{\mu\nu} \), and internal metric \( g_{mn} \) defined by

\[
\frac{9}{4} d s_{\text{AdS}_6}^2 = g_{\mu\nu}(x) d x^\mu d x^\nu, \quad d s_{S^4}^2 = g_{mn}(y) d y^m d y^n = d \theta^2 + \sin^2 \theta d s_{S^3}^2. \tag{2.12}
\]

The warp factor is

\[
e^{2A} = \left( \frac{3}{2} F_0 \cos \theta \right)^{1/12}. \tag{2.13}
\]

The remaining non-zero fields of the solution are the dilaton and the 4-form field strength given by:

\[
e^\Phi = \left( \frac{3}{2} F_0 \cos \theta \right)^{-5/6}, \quad \hat{F}_4 = -\frac{10}{3} \left( \frac{3}{2} F_0 \cos \theta \right)^{1/3} \text{vol}_{S^4}. \tag{2.14}
\]

The coordinate \( \theta \) lies in the interval \([0, \pi/2]\), and at \( \theta = \pi/2 \), where the warp factor diverges, the geometry has a boundary corresponding to the location of the \( O_8 \)-plane. The internal space is therefore more accurately a hemisphere \( HS^4 \) with an \( S^3 \) boundary at \( \theta = \pi/2 \).

### 3 Linearised equations of motion

In this section we consider small fluctuations around the \( \text{AdS}_6 \times_{w} S^4 \) solution outlined in the previous section, determine the equations of motion linearised in fluctuations, and reorganise them as field equations for massive free fields propagating in \( \text{AdS}_6 \).

We perturb the bosonic fields around their background values as:

\[
g_{MN} = \hat{g}_{MN}(x, y) + e^{2A} h_{MN}(x, y), \quad \Phi = \hat{\Phi}(y) + \phi(x, y),
\]

\[
H_3 = \hat{H}_3(x, y) + \delta H_3(x, y), \quad F_2 = \hat{F}_2(x, y) + \delta F_2(x, y), \quad F_4 = \hat{F}_4(y) + \delta F_4(x, y), \tag{3.1}
\]

\(^6\)We work in the Einstein frame. An overall scale related to the “trombone symmetry” of the equations of motion has been set to one. It can be reinstated by \( d \hat{s}_{10}^2 \to L^2 d s_{10}^2, \hat{F}_4 \to L^3 \hat{F}_4, F_0 \to L^{-1} F_0 \).
and similarly for the fermionic fields:

\[ \Psi_M = \hat{0} + e^A \psi_M(x,y), \quad \Lambda = \hat{0} + \lambda(x,y). \quad (3.2) \]

The Bianchi identities (2.1) allow us to introduce potentials \( b_2, a_1 \) and \( a_3 \) such that

\[ \delta H_3 = db_2, \quad \delta F_2 = da_1 + F_0 b_2, \quad \delta F_4 = da_3, \quad (3.3) \]

and it is in terms of these potentials that we will write the equations of motion.

In what follows all geometric quantities, in particular covariant derivatives, and contractions are with respect to the \( g_{\mu\nu} \) and \( g_{mn} \) metrics defined by (2.12). In order to keep the equations covariant we will not substitute for the value of the function \( A(\theta) \), given by (2.13). Also, where it occurs, we will replace the background dilaton by its equivalent value \( \hat{\Phi} = -20A \). The Laplace–de Rham operators acting on 0-, 1-, 2- and 3-forms on \( \text{AdS}_6 \) are defined as

\[ \Delta_0 \alpha := \nabla^\mu \nabla_\mu \alpha \quad (3.4a) \]
\[ \Delta_1 \alpha_\nu := \nabla^\mu \nabla_\mu \alpha_\nu - R_\nu^\mu \alpha_\mu, \quad (3.4b) \]
\[ \Delta_2 \alpha_{\nu\rho} := \nabla^\mu \nabla_\mu \alpha_{\nu\rho} - 2R_\rho^\mu \nu^\mu_\alpha_{\mu\rho} + 2R_\rho^\mu \alpha_{\rho|\mu} + 2R_{[\nu[\mu}\alpha_{\rho]]\mu} - 3R_{[\nu[\rho}\alpha_{\sigma]})_{\mu]}^{\alpha_\mu} \quad (3.4c) \]
\[ \Delta_3 \alpha_{\nu\rho\sigma} := \nabla^\mu \nabla_\mu \alpha_{\nu\rho\sigma} - 6R_{\nu[\rho^\mu_\alpha_{\mu\sigma]}_{\mu1\mu2} - 3R_{[\nu[\rho^\mu_{\alpha\rho\sigma]}_{\mu]}_{\rho}}, \quad (3.4d) \]

where \( R_{\mu\nu} = -\frac{20}{9}g_{\mu\nu} \) and \( R_{\mu\nu\lambda\rho} = \frac{4}{9}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) \) are the Ricci and Riemann tensor of the \( \text{AdS}_6 \) metric \( g_{\mu\nu} \). Finally, we will introduce the following notation for the warped Laplace operators on \( S^4 \) which appear in the linearised equations:

\[ \mathcal{L}^{(k)} := e^{-8A}\nabla^\nu(e^{8A}\nabla_\nu) = \partial^2_\theta + \left(3 \cot \theta - \frac{k}{3} \tan \theta \right) \partial_\theta + \frac{1}{\sin^2 \theta} \Delta_{S^3}, \quad (3.5) \]

where \( \Delta_{S^3} \) is the \( S^3 \) Laplace–Beltrami operator.

### 3.1 Bosonic sector

**Einstein equation**

We start with the Einstein equation (2.3) which splits into three subequations:

\[ 0 = \nabla^\lambda \nabla_\lambda h_{\mu\nu} - 2\nabla_{(\mu} \nabla^\lambda h_{\nu)\lambda} + \nabla_\mu \nabla_\nu h^\lambda_\lambda + \frac{8}{5}h_{\mu\nu} + \mathcal{L}^{(1)} h_{\mu\nu} \]
\[ - 2\nabla_{(\mu}[e^{-8A}\nabla^\nu(e^{8A}h_{\nu)\nu})] + \nabla_\mu \nabla_\nu h^\rho_\rho + (t^1 + t^2)g_{\mu\nu}, \quad (3.6a) \]
Hence, we can recast the linearised dilaton equation in a simpler form:

\[ 0 = \nabla^\lambda \nabla \lambda h_{\mu \nu} - 2 \nabla (\mu \nabla \lambda h_{\nu}) + \nabla \mu \nabla \nabla \lambda h_{\mu} - \frac{50}{9} h_{\mu \nu} + \mathcal{L}^{(1)} h_{\mu \nu} \]

\[ - 2 \nabla (\mu [e^{-8A} \nabla ^p (e^{8A} h_{\nu})]) + \nabla \mu \nabla h_{\nu}^p - 3 \cdot 2^6 A^p A_{\lambda} h_{\mu \nu} \]

\[ - 20 A_{\mu} \phi_{,\nu} - \frac{5}{9} e^{-8A} \epsilon_{\nu \rho \sigma} (\nabla \rho a_{\mu \sigma} - 3 \nabla \rho a_{\mu \lambda}) \]  

(3.6b)

where

\[ t^1 := A_{(m} [-2 \nabla \lambda h_{m \lambda} - 2 e^{-8A} \nabla ^n (e^{8A} h_{mn}) + \nabla \lambda h_{m} + \nabla m h_{n}] ; \]

\[ t^2 := \frac{5}{7} \left[ \frac{1}{2^4} e^{-8A} \epsilon_{mn} a_{pq} \nabla m a_{pq} - \frac{16}{45} h_{m} + \frac{17}{10} h_{m} + \frac{96}{5} A^{m} A^{n} h_{mn} + (32 A^{m} A_{m} - \frac{7}{9}) \phi \right] , \]

\[ t^3 := - \frac{25}{6} \left[ \frac{1}{2^4} e^{-8A} \epsilon_{mn} a_{pq} \nabla m a_{pq} + \frac{7}{6} h_{m} - \frac{288}{25} A^{m} A^{n} h_{mn} - (\frac{96}{5} A^{m} A_{m} - \frac{4}{5}) \phi \right] . \]  

(3.7)

A comma denotes partial differentiation e.g. \( A_{m} := \partial_{m} A \) and \( \epsilon_{mpq} \) is the Levi–Civita tensor.

**Dilaton equation**

Next we linearise the alternate form of the dilaton equation (2.8):

\[ 0 = \Delta_0 (\phi + 2 h_{\nu} + 2 h^n) + \mathcal{L}^{(0)} (\phi + 2 h_{\nu} + 2 h^n) - 16 (\nabla^m \nabla^n A + 24 A^{m} A^{n}) h_{mn} \]

\[ + 2 h_{\mu \nu} R_{\mu \nu} + 2 h_{mn} R_{mn} - 2 \nabla^\mu \nabla^\mu h_{\mu \nu} - 4 \nabla^\mu \nabla^n h_{\mu \nu} - 2 \nabla^m \nabla^n h_{mn} \]

\[ - 16 A^{m} (2 \phi_{,m} + \nabla^\nu h_{\mu \nu} + \nabla^n h_{mn} - \frac{1}{2} \nabla m h_{\mu \nu} - \frac{1}{2} \nabla m h_{\mu \nu} ) . \]  

(3.8)

The Ricci tensors of \( g_{\mu \nu} \) and \( g_{mn} \) are \( R_{\mu \nu} = - \frac{20}{3} g_{\mu \nu} \) and \( R_{mn} = 3 g_{mn} \). Furthermore, the expression \( \nabla_{m} \nabla_{n} A + 24 A_{m} A_{n} \) evaluates to

\[ \nabla_{m} \nabla_{n} A + 24 A_{m} A_{n} = - \frac{1}{24} g_{mn} . \]  

(3.9)

Hence, we can recast the linearised dilaton equation in a simpler form:

\[ 0 = \Delta_0 (\phi + 2 h_{\nu} + 2 h^n) + \mathcal{L}^{(0)} (\phi + 2 h_{\nu} + 2 h^n) \]

\[ - \frac{40}{9} h_{\nu} + \frac{20}{3} h^n - 2 \nabla^\mu \nabla^\nu h_{\mu \nu} - 4 \nabla^\mu \nabla^n h_{\mu} - 2 \nabla^m \nabla^n h_{mn} \]

\[ - 16 A^{m} (2 \phi_{,m} + \nabla^\nu h_{\mu \nu} + \nabla^n h_{mn} - \frac{1}{2} \nabla m h_{\mu \nu} - \frac{1}{2} \nabla m h_{\mu \nu} ) . \]  

(3.10)
2-form field strength equation

Equation (2.6) for $F_2$ splits into two subequations:

$$0 = \Delta_1 a_\nu - \nabla_\nu \nabla^\mu a_\mu + \mathcal{L}^{(-3)} a_\nu - \nabla_\nu [e^{24A} \nabla^m (e^{-24A} a_m)]$$
$$+ F_0 \nabla^\mu b_{\mu
u} + F_0 e^{24A} \nabla^m (e^{-24A} b_{mn}) , \quad (3.11a)$$

$$0 = \Delta_0 a_n - \nabla_n \nabla^\mu a_\mu + (\mathcal{L}^{(-3)} - 3) a_n - 24(\nabla_n \nabla^m A)a_m - \nabla_n [e^{24A} \nabla^m (e^{-24A} a_m)]$$
$$+ F_0 \nabla^\mu b_{\mu n} + F_0 e^{24A} \nabla^m (e^{-24A} b_{mn}) + \frac{5}{3} e^{24A} \epsilon_{nm1m2m3} \nabla^{m1} b^{m2m3} . \quad (3.11b)$$

3-form field strength equation

Equation (2.5) for $H_3$ splits into three subequations:

$$0 = \Delta_2 b_{\nu\rho} + 2 \nabla_\nu \nabla^\mu b_{\rho\mu} + (\mathcal{L}^{(3)} - F_0^2 e^{-48A}) b_{\nu\rho} + 2 \nabla_\nu [e^{-24A} \nabla^m (e^{24A} b_{\rho m})]$$
$$- 2 F_0 e^{-48A} \nabla_\nu [a_{\rho}] - \frac{10}{3} e^{-16A} \epsilon_{\nu\rho\mu1...\mu4} \nabla^\mu_1 a_{\mu2\mu3\mu4} , \quad (3.12a)$$

$$0 = \Delta_1 b_{\nu\rho} + 2 \nabla_\nu \nabla^\mu b_{\rho\mu} + (\mathcal{L}^{(3)} - 3 - F_0^2 e^{-48A}) b_{\nu\rho} - 24(\nabla_n \nabla^m A)b_{\rho m}$$
$$+ 2 \nabla_\nu [e^{-24A} \nabla^m (e^{24A} b_{\rho m})] - 2 F_0 e^{-48A} \nabla_\nu [a_{\rho}] ; \quad (3.12b)$$

$$0 = \Delta_0 b_{\nu\rho} + 2 \nabla_\nu \nabla^\mu b_{\rho\mu} + (\mathcal{L}^{(3)} - 4 - F_0^2 e^{-48A}) b_{\nu\rho} - 2 \cdot 24(\nabla_n \nabla^m A)b_{\rho m}$$
$$+ 2 \nabla_\nu [e^{-24A} \nabla^m (e^{24A} b_{\rho m})] - 2 F_0 e^{-48A} \nabla_\nu [a_{\rho}] + \frac{5}{3} e^{-24A} \epsilon_{nm1m2} (2 \nabla^1 a^{m2} + F_0 b^{m1m2}) . \quad (3.12c)$$

4-form field strength equation

Equation (2.7) for $F_4$ splits into four subequations:

$$0 = \Delta_3 a_{\nu\rho\sigma} - 3 \nabla_\nu \nabla^\mu a_{\rho\sigma\mu} + \mathcal{L}^{(-1)} a_{\nu\rho\sigma} - 3 \nabla_\nu [e^{8A} \nabla^m (e^{-8A} a_{\rho\sigma m})]$$
$$- \frac{10}{3} e^{16A} \epsilon_{\nu\rho\sigma\mu1\mu2\mu3} \nabla^{\mu1} b^{\mu2\mu3} , \quad (3.13a)$$

$$0 = \Delta_2 a_{\nu\rho\sigma} - 3 \nabla_\nu \nabla^\mu a_{\rho\sigma\mu} + (\mathcal{L}^{(-1)} - 3) a_{\nu\rho\sigma} - 8(\nabla_n \nabla^m A)a_{\rho\sigma m}$$
$$- 3 \nabla_\nu [e^{8A} \nabla^m (e^{-8A} a_{\rho\sigma m})] , \quad (3.13b)$$

$$0 = \Delta_1 a_{\nu\rho\sigma} - 3 \nabla_\nu \nabla^\mu a_{\rho\sigma\mu} + (\mathcal{L}^{(-1)} - 4) a_{\nu\rho\sigma} - 2 \cdot 8(\nabla_n \nabla^m A)a_{\rho\sigma m}$$
$$- 3 \nabla_\nu [e^{8A} \nabla^m (e^{-8A} a_{\rho\sigma m})] + \frac{10}{3} e^{8A} \epsilon_{\nu\rho\sigma\mu} e^{-A} \nabla^m (e^{-A} h^\rho\sigma) , \quad (3.13c)$$

$$0 = \Delta_0 a_{\nu\rho\sigma} - 3 \nabla_\nu \nabla^\mu a_{\rho\sigma\mu} + (\mathcal{L}^{(-1)} - 3) a_{\nu\rho\sigma} - 3 \cdot 8(\nabla_n \nabla^m A)a_{\rho\sigma m}$$
$$- 3 \nabla_\nu [e^{8A} \nabla^m (e^{-8A} a_{\rho\sigma m})] - \frac{5}{3} e^{8A} \phi_m e_{nm1m2} .$$
\[ + \frac{10}{3} e^{8A} (\nabla^{\lambda} h_{\lambda}^m + \nabla^{p} h_{p}^m - \frac{1}{2} \nabla^m (h_{\lambda}^\lambda + h_{p}^p)) \epsilon_{mnra} + 10 e^{8A} \nabla^m h_{[n}^p \epsilon_{|m|p|r|a]} . \] (3.13d)

### 3.2 Fermionic sector

The equation of the gravitino (2.9) yields two subequations:

\[
0 = \Gamma_{\mu\nu\rho} \nabla_{\mu} \psi_{\nu} + \Gamma_{\mu} \Gamma_{m} (\nabla_{m} \psi_{n} + \frac{9}{2} A_{m} \psi_{n}) \\
+ \Gamma_{\mu} \Gamma_{n} (\nabla_{n} \psi_{\mu} - \nabla_{\mu} \psi_{n} - \frac{9}{2} A_{n} \psi_{\mu}) - 4 A_{n} \Gamma_{\mu} \psi_{n} \\
+ 5 A_{n} \Gamma_{m} \Gamma_{\mu} \lambda + \frac{1}{4} F_0 e^{-24A} \Gamma_{\mu\nu} \psi_{\nu} + \frac{1}{4} F_0 e^{-24A} \Gamma_{\mu\nu} (\frac{5}{4} \lambda + \Gamma_{n} \psi_{n}) \\
- \frac{5}{288} \epsilon_{n_{1}n_{2}n_{3}n_{4}} \Gamma_{n_{1}n_{2}n_{3}n_{4}} (2 \Gamma_{\mu\nu} \psi_{\nu} + \frac{1}{2} \Gamma_{\mu} \lambda) ,
\] (3.14a)

\[
0 = \Gamma_{m} \Gamma_{\mu\nu\rho} \nabla_{\mu} \psi_{\nu} + \Gamma_{mn} \Gamma_{\mu} \psi_{n} + \Gamma_{m} \Gamma_{mn} \psi_{n} \\
+ \Gamma_{mn} \Gamma_{\mu} (\nabla_{\mu} \psi_{n} + \nabla_{n} \psi_{\mu} + \frac{9}{2} A_{n} \psi_{\mu}) + 4 A_{n} \Gamma_{mn} \psi_{n} \\
+ 5 A_{n} \Gamma_{m} \Gamma_{mn} \lambda + \frac{1}{4} F_0 e^{-24A} \Gamma_{mn} \psi_{\nu} + \frac{1}{4} F_0 e^{-24A} \Gamma_{mn} (\frac{5}{4} \lambda + \Gamma_{\nu} \psi_{\nu}) \\
- \frac{5}{288} \epsilon_{n_{1}n_{2}n_{3}n_{4}} \Gamma_{n_{1}n_{2}n_{3}n_{4}} (- 2 \Gamma_{mn} \psi_{n} + \frac{1}{2} \Gamma_{m} \lambda) ,
\] (3.14b)

and that of the dilatino (2.10) the following:

\[
0 = \Gamma_{m} \nabla_{\mu} \lambda + \Gamma_{mn} \nabla_{\mu} \lambda - \frac{5}{288} \epsilon_{m_{1}m_{2}m_{3}m_{4}} \Gamma_{m_{1}m_{2}m_{3}m_{4}} (\frac{3}{2} \lambda - \Gamma_{\mu} \psi_{\mu} + \Gamma_{n} \psi_{n}) \\
+ 10 A_{m} \Gamma_{m} (\frac{21}{16} \lambda - \Gamma_{\mu} \psi_{\mu} - \Gamma_{n} \psi_{n}) + 20 A_{m} \psi_{m} - \frac{5}{8} F_0 e^{-24A} (\frac{21}{10} \lambda + \Gamma_{\mu} \psi_{\mu} + \Gamma_{m} \psi_{m}) .
\] (3.15)

In these linearised equations the gamma matrices satisfy \{\Gamma_{\mu}, \Gamma_{\nu}\} = 2 g_{\mu\nu} and \{\Gamma_{m}, \Gamma_{n}\} = 2 g_{mn}.

### 4 Spin-2 mass spectrum

In this section we look at the the spectrum of massive gravitons or spin-2 particles propagating in AdS\(_6\). These are the transverse and traceless parts of the metric fluctuation \(h_{\mu\nu}\), which we will denote by \(h_{\mu\nu}^{tt}\):

\[
\nabla^{\mu} h_{\mu\nu}^{tt} = 0 , \quad g^{\mu\nu} h_{\mu\nu}^{tt} = 0 . \quad (4.1)
\]

From the linearised Einstein equation we see that it satisfies

\[
\frac{9}{4} \nabla^{\lambda} \nabla_{\lambda} h_{\mu\nu}^{tt} + 2 h_{\mu\nu}^{tt} + \frac{9}{4} \mathcal{L}^{(1)} h_{\mu\nu}^{tt} = 0 ,
\] (4.2)
where recall $\mathcal{L}^{(1)} h_{\mu\nu}^{tt} := e^{-8A} \nabla^m \left(e^{8A} \nabla_m h_{\mu\nu}^{tt}\right)$. Taking into account the fact that the anti-de Sitter metric has radius $\frac{3}{2}$, we recognize the above equation as the equation of motion of a massive graviton of mass squared $M^2$, given by the eigenvalues of $\mathcal{L}^{(1)}$:

$$\mathcal{L}^{(1)} h_{\mu\nu}^{tt} = -\frac{4}{9} M^2 h_{\mu\nu}^{tt}.$$  

We proceed to solve the eigenvalue problem by factorizing $h_{\mu\nu}^{tt}$ as

$$h_{\mu\nu}^{tt}(x, y) = h_{\mu\nu}^{tt}(x) \Upsilon(y)$$  

and further expanding $\Upsilon$ in terms of $S^3$ scalar spherical harmonics:

$$\Upsilon = \sum_{\ell=0}^{\infty} (\sin \theta)^\ell f_\ell(\theta) Y_\ell,$$

where $Y_\ell$ are $S^3$ spherical harmonics of eigenvalue $-\ell(\ell+2)$. The differential operator $\mathcal{L}^{(1)}$ takes the form

$$\mathcal{L}^{(1)} = \partial_\theta^2 + \left(3 \cot \theta - \frac{1}{3} \tan \theta \right) \partial_\theta + \frac{1}{\sin^2 \theta} \Delta_{S^3},$$

where $\Delta_{S^3}$ is the $S^3$ Laplace–Beltrami operator. The equation $\mathcal{L}^{(1)} h_{\mu\nu}^{tt} = -\frac{4}{9} M^2 h_{\mu\nu}^{tt}$ thus reduces to an ordinary differential equation (ODE) for $f_\ell$:

$$9 \sin(2\theta) f''_\ell + 6[9 + 6\ell - (10 + 6\ell) \sin^2 \theta] f'_\ell + [4M^2 - 3\ell(3\ell + 10)] \sin(2\theta) f_\ell = 0,$$

where a prime denotes differentiation with respect to $\theta$. We now make a change of variables to

$$z = \sin^2 \theta, \quad z \in [0, 1]$$

and the ODE becomes the hypergeometric differential equation (henceforth dropping the $\ell$ subscript):

$$z(1-z) \frac{d^2 f}{dz^2} + (a - (a+b+1)z) \frac{df}{dz} - abf = 0,$$

with

$$a = \frac{5}{6} + \frac{\ell}{2} - \frac{1}{3} \sqrt{M^2 + \left(\frac{5}{2}\right)^2}, \quad b = \frac{5}{6} + \frac{\ell}{2} + \frac{1}{3} \sqrt{M^2 + \left(\frac{5}{2}\right)^2}, \quad c = 2 + \ell.$$

In order to define a space of admissible solutions, we will recast the hypergeometric
equation in a Sturm–Liouville form:

\[ Sf = -\lambda w(z)f, \]  

(4.10)

where

\[ S := \frac{d}{dz} \left( p(z) \frac{d}{dz} \right), \quad p(z) := z^{\ell+2}(1-z)^{2/3}, \quad w(z) := z^{\ell+1}(1-z)^{-1/3}, \]  

(4.11)

and

\[ \lambda := \frac{1}{9} \left[ M^2 - \frac{3}{2} \ell \left( \frac{3}{2} \ell + 5 \right) \right]. \]  

(4.12)

We introduce the weighted inner product

\[ (f_1, f_2)_w := \int_0^1 f_1(z)f_2(z)w(z)dz \]  

(4.13)

and impose boundary conditions such that two eigenfunctions \( f_1, f_2 \) of distinct eigenvalues \( \lambda_1, \lambda_2 \) are orthogonal. We compute

\[ (\lambda_2 - \lambda_1)(f_1, f_2)_w = \int_0^1 (Sf_1)f_2 - f_1(Sf_2) = [p(f'_1f_2 - f_1f'_2)]_0^1, \]  

(4.14)

and hence we will impose

\[ [pf'f]_0^1 = 0. \]  

(4.15)

Given the above condition we can derive a bound on the mass spectrum, as follows:

\[ \lambda(f, f)_w = \int_0^1 (-Sf)f dz = \int_0^1 pf'^2 dz \geq 0, \]  

(4.16)

and hence \( \lambda \geq 0 \) or

\[ M^2 \geq \frac{3}{2} \ell \left( \frac{3}{2} \ell + 5 \right). \]  

(4.17)

Returning to the hypergeometric equation, near \( z = 0 \), the solution is a linear combination of \( _2F_1(a, b; c; z) \) and \( (1 - z)^{1-c}_2F_1(a-c+1, b-c+1; 2-c; z) \), but since \( 2-c = -\ell \in \mathbb{Z}_{\leq 0} \), the latter needs to be replaced by a more complicated expression\(^7\) which is however singular at \( z = 0 \) and thus we discard it. We conclude:

\[ f = C_2F_1(a, b; c; z). \]  

(4.18)

\(^7\)See for example [22].
In order to check regularity near \( z = 1 \) we employ the identity
\[
2F_1(a, b; c; z) = \frac{\Gamma(c)\Gamma(c - a - b)}{\Gamma(c - a)\Gamma(c - b)} 2F_1(a, b; 1 + a + b - c; 1 - z) \\
+ (1 - z)^{c - a - b}\frac{\Gamma(c)\Gamma(a + b - c)}{\Gamma(a)\Gamma(b)} 2F_1(c - a, c - b; 1 - a - b + c; 1 - z).
\]

(4.19)

Since \( c - a - b = 1/3 \), we conclude that \( f \) is regular at \( z = 1 \), with
\[
\lim_{z \to 1} f = C \frac{\Gamma(2 + \ell)\Gamma(1/3)}{\Gamma(c - a)\Gamma(c - b)}.
\]

(4.20)

A similar check for \( f' = C \frac{ab}{c} 2F_1(a + 1, b + 1; c + 1; z) \) shows that it is singular at \( z = 1 \):
\[
\lim_{z \to 1} f' = C \frac{ab}{c} \frac{\Gamma(3 + \ell)\Gamma(2/3)}{\Gamma(a + 1)\Gamma(b + 1)} (1 - z)^{-2/3}.
\]

(4.21)

Given that \( f \) and \( f' \) are constant at \( z = 0 \), we have \( p f' f \big|_0 = 0 \). On the other hand
\[
\lim_{z \to 1} p f' f = C^2 \frac{ab}{c} \frac{\Gamma(3 + \ell)\Gamma(2/3)}{\Gamma(a + 1)\Gamma(b + 1)} \frac{\Gamma(2 + \ell)\Gamma(1/3)}{\Gamma(c - a)\Gamma(c - b)}.
\]

(4.22)

Notice that the singularity of \( f' \) at \( z = 1 \) is of the same order as the zero of \( p \), so they cancel. We would like to make \( \lim_{z \to 1} p f' f \) vanish. There are two ways to do so:

A. \( a = -j \in \mathbb{Z}_{\le 0} \) in which case \( f \) is a polynomial and \( f' \) is regular at \( z = 1 \). Imposing so, we derive the mass spectrum:
\[
M^2 = \left( \frac{3}{2}\ell + 3j \right) \left( \frac{3}{2}\ell + 3j + 5 \right).
\]

(4.23)

\( f \) becomes proportional to the Jacobi polynomial \( P_j^{(\ell+1,-1/3)}(1 - 2z) \).\(^8\)

B. \( c - b = -j \in \mathbb{Z}_{\le 0} \) in which case \( \lim_{z \to 1} f = 0 \) and \( f' \) is singular at \( z = 1 \). Imposing so, we derive the mass spectrum:
\[
M^2 = \left( \frac{3}{2}\ell + 3j + 1 \right) \left( \frac{3}{2}\ell + 3j + 6 \right).
\]

(4.24)

\(^8\)The Jacobi polynomials are defined in terms of the hypergeometric function as \( P_n^{(\ell_1, \ell_2)}(x) = \binom{\ell_1 + \ell_2}{n} 2F_1(-n, \ell_1 + \ell_2 + n + 1; \ell_1 + 1; \frac{1}{2}(1 - x)) \).
$f$ becomes proportional to $(1 - z)^{1/3} P_j^{(1/3, \ell + 1)}(2z - 1)$.

We will refer to the above two branches of the mass spectrum as branch A and branch B. One might be sceptical about branch B, since in that case $f'$ is singular, but as we will see the representation theory of the f(4) superconformal algebra supports its existence.

In particular, the Kaluza–Klein excitations should organize in multiplets of f(4), the symmetry of the AdS$_6 \times$ $S^4$ solution, that include states of highest spin 2. The states lying at the bottom of the spectrum, at $j = 0$, are expected to belong to short multiplets with masses determined by their $\text{su}(2)_R$ R-symmetry charge (spin), which for states corresponding to $S^3$ harmonics $Y_\ell$ is $\ell/2^9$.

According to the AdS/CFT dictionary, the scaling dimension $\Delta$ of the operator dual to a bulk graviton excitation is given by the relation

$$M^2 = \Delta(\Delta - 5).$$

(4.25)

First note that via this relation the bound (4.17) for the mass spectrum maps to a unitarity bound for the dimension of the dual field theory operators: $\Delta \geq \frac{3}{2}\ell + 5$. Furthermore, (4.25) gives the following dimensions for the two branches:

branch A: $\Delta = \frac{3}{2}\ell + 3j + 5$, \hspace{1cm} branch B: $\Delta = \frac{3}{2}\ell + 3j + 6$.

(4.26)

We thus expect short multiplets of f(4) with a state of (highest) spin 2 and dimension $\Delta = \frac{3}{2}\ell + 5$ for branch A and dimension $\Delta = \frac{3}{2}\ell + 6$ for branch B. This is indeed the case as shown in the work of [18, 19]: in [18] the corresponding multiplets are $B[0, 0; k]$ and $A[0, 0; k]$ in Table 1 respectively, and in [19] they are $B_2$ and $A_4$ in Table 22, given explicitly in section 4.7 there.

The dual five-dimensional superconformal field theory has no Lagrangian description, but arises as the UV fixed point of a Yang–Mills theory coupled to hypermultiplets. In particular the gauge group is USp(2N), and the matter content comprises $N_f < 8$ hypermultiplets in the fundamental representation and one hypermultiplet in the antisymmetric representation. We can employ the fields of this theory for a schematic description of the operators dual to the graviton modes. The operator dual to the massless graviton is of course the stress-energy tensor $T_{\mu\nu}$ with dimension $\Delta = 5$. For branch A, we can construct operators $O_{\mu\nu}^A$ by multiplying $T_{\mu\nu}$ with the scalars in the hypermultiplets which transform as a doublet under SU(2)$_R$ and have dimension $\Delta = \frac{3}{2}$.

---

$^9$The R-symmetry resides in the $\mathfrak{so}(4)$ isometry of $S^3$. 

13
In particular we have:

\[ O^A_{\mu\nu} := \text{Tr} \left( (\epsilon^{IJ} A_I^J A_{(I_1 A_{I_2} \ldots A_{I_\ell})}) T_{\mu\nu} \right), \tag{4.27} \]

where \( A_{ab}^I \) is the hypermultiplet in the antisymmetric representation of \( \text{USp}(2N) \), with \( I \) an \( \text{SU}(2)_R \) index and \( a, b \) \( \text{USp}(2N) \) gauge indices. The trace \( \text{Tr} \) refers to the contraction of the gauge indices of \( A \) which are contracted with \( J_{ab} \), the \( \text{USp}(2N) \) invariant antisymmetric tensor, while \( \epsilon_{IJ} \) is the \( \text{SU}(2)_R \) one. The description of the operators dual to the graviton modes of branch B, in terms of the fields of the IR theory is less clear, if possible. We expect these operators to have the form \( O^B_{\mu\nu} = O^A_{\mu\nu} O \), where \( O \) is a scalar operator of dimension one and R-charge zero. However, \( O \) doesn’t admit a straightforward representation by the IR fields. A potential candidate for \( O \) would be the scalar in the vector multiplet, however the latter is not a representation of the superconformal algebra \( \mathfrak{f}(4) \).

5 Conclusions

In this note we have taken a first step towards obtaining the Kaluza–Klein mass spectrum of massive IIA supergravity on warped \( \text{AdS}_6 \times S^4 \). In particular, we have derived the linearised equations of motion for fluctuations (bosonic and fermionic) around the background and determined the mass spectrum of the spin-2 ones. By a careful analysis of the boundary conditions of the latter at the singularity of the background solution, we have uncovered the existence of two branches of mass spectra. These are bounded from below and the excitations that saturate the bound belong to short supermultiplets, which we have identified from the representation theory of the symmetry algebra of the solution. For one of the two branches we have provided an effective description of the dual field theory operators, in terms of the fields of the theory which at the UV gives rise to the strongly coupled supersymmetric fixed point. For the second branch we lack such a description, and it would be interesting to investigate more the nature of these spin-2 operators.

The next step in this endeavour is to determine the mass spectrum for the rest of the fluctuations. This is a challenging task as the warped nature of the background complicates the equations of motion, and the harmonic expansion of the modes on the internal manifold. A convenient gauge for the modes has to be chosen, in which the equations of motion simplify, and the form of the latter suggest a warped generalization of the transverse gauge that is usually used for Kaluza–Klein theories on spheres. Ulti-
mately, we expect that the mass spectrum will be determined by the eigenvalue problem of the warped Laplace operators $L^{(k)}$, defined in (3.5). As was the case for the spin-2 modes, for which $k = 1$, this eigenvalue problem can be mapped to a hypergeometric differential equation; see appendix B.

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**A Identities**

**A.1 Metric perturbations**

Under a small perturbation of the metric $g_{MN} \to \hat{g}_{MN} = \tilde{g}_{MN} + \delta g_{MN}$, the inverse metric transforms as $g^{MN} = \tilde{g}^{MN} - \delta g^{MN}$. The Christoffel symbols $\Gamma^P_{MN}$, Laplace operator $\nabla^2 := \nabla^M \nabla_M$, and Ricci tensor $R_{MN}$ transform as:

\[
\Gamma^P_{MN} = \hat{\Gamma}^P_{MN} + \frac{1}{2} \left( \tilde{\nabla}_M \delta g_N^P + \tilde{\nabla}_N \delta g_M^P - \tilde{\nabla}^P \delta g_{MN} \right),
\]

\[
\nabla^2 = \tilde{\nabla}^2 - \delta g^{MN} \tilde{\nabla}_M \partial_N - \left( \tilde{\nabla}^M \delta g_N^P - \frac{1}{2} \tilde{\nabla}^N \delta g_M^P \right) \partial_N,
\]

\[
R_{MN} = \tilde{R}_{MN} + \frac{1}{2} \hat{\Delta} L \delta g_{MN} + \tilde{\nabla}_N \left( \tilde{\nabla}^P \delta g_M^P \right) - \frac{1}{2} \tilde{\nabla}_M \tilde{\nabla}_N \delta g^P_P,
\]

where the Lichnerowicz operator is defined as

\[
\hat{\Delta} L \delta g_{MN} := -\tilde{\nabla}^2 \delta g_{MN} - 2\tilde{R}_{MNPQ} \delta g^{PQ} + 2\tilde{R}_{(M} P \delta g_{N)P},
\]

and all indices are raised and lowered using the metric $\tilde{g}$.

**A.2 Conformal transformations**

The Christoffel symbols $\hat{\Gamma}^P_{MN}$ associated with the warped background metric $\tilde{g}_{MN}$ can be expressed in terms of the Christoffel symbols $\Gamma^P_{MN}$ associated with the unwarped
metric $g_{MN} = e^{-2A} g_{MN}$ and derivatives of the warp factor $A$ as follows:\footnote{A comma denotes a partial derivative: $A_{,M} := \partial_M A$.}

$$\hat{\Gamma}_{MN}^P = \Gamma_{MN}^P + \delta_{M}^P A_{,N} + \delta_{N}^P A_{,M} - g_{MN} A_{,P}.$$ (A.5)

Indices on the right-hand side are raised using $g_{MN}$. For the Riemann and Ricci tensors the analogous expressions are:

$$\hat{R}^{M}_{NPQ} = R^{M}_{NPQ} + 2\delta^{M}_{[P} A_{,Q]} A_{,N} + 2 g_{N[P} \nabla_{Q]} A^{M} + 2 \delta^{M}_{[P} g_{Q]N} A_{,R} A^{R},$$ (A.6)

$$\hat{R}_{MN} = R_{MN} + 8 (A_{,M} A_{,N} - \nabla_{M} \nabla_{N} A) - (8 A_{,P} A_{,P} + \nabla^{2} A) g_{MN}.$$ (A.7)

The relations between warped and unwarped Laplace operators and covariant spinor derivatives are:

$$\hat{\nabla}^{2} = e^{-2A} \left( \nabla^{2} + 8 A^{M} \partial_{M} \right),$$ (A.8)

$$\hat{\nabla}_{M} = \nabla_{M} + \frac{1}{2} A_{,N} \Gamma_{M}^{N},$$ (A.9)

where the gamma matrices $\Gamma_{M}$ satisfy \{ $\Gamma_{M}, \Gamma_{N}$ \} = $2 g_{MN}$.

A warped covariant derivative of the metric perturbation $\delta g_{MN} = e^{2A} h_{MN}$ decomposes as

$$e^{-2A} \hat{\nabla}_{Q} \delta g_{MN} := T_{QMN} = \nabla_{Q} h_{MN} - 2 A_{,(M} h_{N)Q} + 2 A_{,R} g_{Q(M} h_{N)R}.$$ (A.10)

Using the above tensor $T$ we find:

$$e^{-2A} \hat{\nabla}_{P} \hat{\nabla}_{Q} \delta g_{MN} = \nabla_{P} T_{QMN} - 2 A_{,(P} T_{Q)MN} - A_{,M} T_{QPN} - A_{,N} T_{QPM} + g^{RS} A_{,S} (g_{PQ} T_{RMN} + g_{PM} T_{QRN} + g_{PN} T_{QRM}).$$ (A.11)

Employing these results we can derive expressions for all warped quantities in (A.2), (A.3) and (A.4) in terms of unwarped ones.

Finally, we record the “unwarping” of the following terms that appear in the equations of motion of the fermions:

$$\hat{\Gamma}^{M}_{M} \nabla_{M} \Lambda = e^{-A} \left( \Gamma^{M}_{M} \nabla_{M} \Lambda + \frac{9}{2} A_{,M} \Gamma^{M}_{M} \Lambda \right),$$ (A.12)

$$\hat{\Gamma}^{MNP}_{N} \nabla_{P} \Psi_{P} = e^{-3A} \left( \Gamma^{MNP}_{N} \nabla_{P} \Psi_{P} + \frac{7}{2} A_{,N} \Gamma^{MNP}_{N} \Psi_{P} + 8 A_{,[M} \Gamma^{N]} \Psi_{N} \right).$$ (A.13)
where on the left-hand side the gamma matrices $\Gamma^M$ satisfy $\{\Gamma^M, \Gamma^N\} = 2\delta^M_N$.

**B  The operators $L^{(k)}$**

The eigenvalue equation

$$L^{(k)} \varphi = -\frac{4}{9} M^2 \varphi, \quad (B.1)$$

for the operators $L^{(k)}$ defined in (3.5), upon introducing $f(\theta)$ such that $\varphi = (\sin \theta)^\ell f$ and switching variables to

$$z = \sin^2 \theta, \quad z \in [0, 1], \quad (B.2)$$

becomes the hypergeometric differential equation

$$z(1-z) \frac{d^2 f}{dz^2} + [c - (a + b + 1)z] \frac{df}{dz} - abf = 0, \quad (B.3)$$

where $c = 2 + \ell$ and

$$a = \frac{9 + k}{12} + \frac{\ell + 1}{3} \sqrt{M^2 + \left(\frac{9 + k}{8}\right)^2}, \quad b = \frac{9 + k}{12} + \frac{\ell - 1}{3} \sqrt{M^2 + \left(\frac{9 + k}{8}\right)^2}. \quad (B.4)$$
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