Perturbative QCD correlations in multi-parton collisions

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Abstract We examine the role played in double-parton interactions (DPI) by the parton–parton correlations originating from perturbative QCD parton splittings. Also presented are the results of the numerical analysis of the integrated DPI cross sections at Tevatron and LHC energies. To obtain the numerical results the knowledge of the single-parton GPDs gained by the HERA experiments was used to construct the non-perturbative input for generalized double-parton distributions. The perturbative two-parton correlations induced by three-parton interactions contribute significantly to a resolution of the longstanding puzzle of an excess of multi-jet production events in the back-to-back kinematics observed at the Tevatron.

1 Introduction

The field of multiple hard parton interactions (MPI) is an important element of the picture of strong interactions at high energies. The issue attracts a lot of attention. A series of theoretical studies were carried out in the last decade [1–18]. Attempts have been made to incorporate multi-parton collisions into event generators [19–21].

Multi-parton interactions can serve as a probe for non-perturbative correlations between partons in the nucleon wave function and are crucial for determining the structure of the underlying event at LHC energies. They constitute an important background for new physics searches at the LHC. A number of experimental studies were performed at the Tevatron [22–24]. New measurements are under way at the LHC [25,26].

Double hard parton scattering in hadron–hadron collisions (DPI) can contribute to production of four hadron jets with large transverse momenta $p_\perp \gg \Lambda_{\text{QCD}}$, of two electroweak bosons, or “mixed” ensembles comprising three jets and $\gamma$, two jets and $W$, etc. In this paper we present the numerical results for a variety of final states. However, for the sake of definiteness, in the following we refer to production of four final state jets in the collision of partons 1 and 2 from one incident hadron with partons 3 and 4 from the second hadron: $1 + 3 \to J_1 + J_3$, $2 + 4 \to J_2 + J_4$.

The double hard interaction is a process difficult to approach theoretically. Formally speaking, it calls for analysis of four-parton operators, which emerge in the squared matrix element describing a two-parton state in a hadron. Relevant objects—quasi-parton operators—were introduced and classified, and evolution of their matrix elements studied by Bukhvostov et al. in the 1980s in a series of papers [27–29].

An approach to MPI based on the operator product expansion and on the notion of transverse momentum dependent parton distributions (TMD) is being developed in [7–9].

MPI is a multi-scale problem, not only because the separate parton–parton interactions may differ in hardness. More importantly, each single hard interaction possesses two very different hardness scales. The distinctive feature of DPI is that it produces two pairs of nearly back-to-back jets, so that

$$\delta_{13}^2 \equiv (\vec{J}_{1\perp} + \vec{J}_{3\perp})^2 \ll Q_1^2 = J_{1\perp}^2 \simeq J_{3\perp}^2, \quad (1a)$$

$$\delta_{24}^2 \equiv (\vec{J}_{2\perp} + \vec{J}_{4\perp})^2 \ll Q_2^2 = J_{2\perp}^2 \simeq J_{4\perp}^2. \quad (1b)$$

Hence, in the collision of partons 1 and 3 the first (larger) scale is given by the invariant mass of the jet pair, $Q_1^2 = 4J_{1\perp}^2 \simeq 4J_{3\perp}^2$, while the second scale is the magnitude of the total transverse momentum of the pair: $\delta^2 = \delta_{13}^2$.

It is important to stress that in the MPI physics there is no factorization in the usual sense of the word. The cross sections do not factorize into the product of the hard parton interaction cross sections and the multi-parton distributions depending on momentum fractions $x_i$ and the hard scale(s).
In [17] we have introduced the necessary theoretical tools for approaching the problem by introducing the general- ized double parton distributions (2GPDs) in the momentum space. The double-parton GPD depends on one extra variable as compared to the double-parton distribution—the transverse momentum mismatch \( \Delta \) between the partons in the wave function and the wave function conjugated. In the mixed space of longitudinal momenta and transverse coordinates, an object equivalent to 2GPD has been introduced by Treleani and Paver in the early-1980s [30] and has long since been present in the literature; see, in particular, [7,31,32].

In order to construct a viable model for the two-parton distribution stemming from the non-perturbative parton wave function of the proton and, in particular, for the distribution stemming from the non-perturbative parton integrated cross section momentum imbalances, give rise to the expression for the differential distribution, being integrated over transverse momentum imbalances, 2 processes. Here we reexamine the role of the so-called “short split” term in 1 \( \otimes \) 2 subprocesses. We show here that the “short split” contribution to the integrated cross section is actually contained in \( \sigma \) that one obtains integrating the simple DDT-like formula derived in the complementary kinematical region \( \delta_{13}^2 \ll \delta_{24}^2 (\delta_{13}^2 \gg \delta_{24}^2) \). Thus we correct (and simplify) the expression for the integrated DPI cross section (the original Eq. (28) of [18] was plagued by double counting).

In Sect. 6 we discuss the independent parton model for the non-perturbative part of two-parton correlations. The numerical results for Tevatron and LHC energies are presented in Sect. 7. We conclude and discuss the results and perspectives of DPI studies in Sect. 8.

2 Hidden reefs of DPI physics

An important question is whether MPI admits an intuitive probabilistic parton interpretation as do the classical single hard interaction processes. When one considers inclusive one-parton distribution in a hadron (pdf), the quantum state of the parton in the light-cone hadron wave function (w.f.) coincides with that in the wave function conjugated (w.f.c.). This lays down the foundation for the probabilistic QCD improved parton picture.
In the case of the two-parton correlation the situation is different. Here only the overall quantum characteristics of the parton pair as a whole (its total energy-momentum, its spin and color states) should be identical in the w.f. and w.f.c. As a result, the momentum, spin and color state of a single parton in the pair do not necessarily match in the wave function and in the wave function conjugated, thus endangering the very probabilistic interpretation of the process under consideration.

A general approach to double hard interactions has been developed in [17]. It turned out that the transverse momentum \( \mathbf{p} \) of the parton in the w.f. and that of its counterpart in the w.f.c. are indeed necessarily different, with their difference \( \mathbf{\Delta} \) being conjugate to the relative transverse distance between the two partons in the hadron. This has led to introduction of the notion of the generalized double-parton distribution, \( 2\text{GPD} \), which depends on a new momentum parameter \( \mathbf{\Delta} \). It is important to stress that the cross section of the double hard process does not factorize into the product of the hard cross section and parton distributions. Instead, it contains a convolution of the product of two \( 2\text{GPDs} \) over \( d^2 \mathbf{\Delta} \).

Non-diagonality in the longitudinal momentum fractions also occurs. Representing the incident partons as plain waves with definite momenta, one ignores the fact that the two partons originate from one and the same finite size hadron. Therefore, when one picks two partons with momenta \( x_1, x_2 \) from the hadron wave function, one has to integrate over \( x_1 - x_2 \) at the level of the amplitude, which integration ensures that the longitudinal separation between the partons does not exceed the size of the parent hadron. When one considers independent hard interactions of two pairs of partons, \((x_1, x_3)\) and \((x_2, x_4)\), taken from colliding hadrons, the integrals over \( r = x_1 - x_2 \) and \( r' = x_3 - x_4 \) do not manifest themselves, since all four longitudinal momentum fractions \( x_i \) are uniquely determined by the kinematics of the two produced hard systems.

This is not in order for \( 1 \otimes 2 \) processes. In this case a flow of large momenta between the two hard vertices is possible, and the integration over \( r' \) is instrumental in getting rid of the fake singularity of the scattering matrix element (a simple explanation of the origin of this unphysical singularity can be found in [37]; see also [18]).

As a result, a small offset between the parton longitudinal momenta in the w.f. and w.f.c. emerges, \( |x_3 - x'_3| \propto \delta^2/Q^2 \). However, this mismatch turns out to be negligible in the dominant kinematical region where the squared total transverse momentum \( \delta^2 \) of the jet pair is much smaller than the overall hardness \( Q^2 \) of the process: \( \delta^2_{13} \equiv (\mathbf{p}_{\perp 1} + \mathbf{p}_{\perp 3})^2 \ll Q_1^2 \simeq 4p_{\perp 1}^2 \).

As noticed by Gaunt [38], the non-diagonality in the longitudinal momentum space is likely to get induced in \( 1 \otimes 2 \) processes by higher-order QCD effects. This happens when incoming partons, in the w.f. and w.f.c., exchange (real or virtual) gluons with transverse momenta in the interval \( \Lambda_{\text{QCD}}^2 \ll k_2^2 \ll \delta^2 \). In [38, 39] arguments were raised in favor of smallness of the crosstalk effects in the DPI cross section. Otherwise, this would have led to an unwelcome complication of the problem: one would be forced to deal with an unknown function of four longitudinal momentum fractions (three independent variables) in place of the two of \( 2\text{GPD}(x_1, x_2) \). Hence for the time being we disregard this complication.

We do not dwell on the potential non-diagonality in color and in spin variables either. One may argue that such non-diagonal configurations are likely to be suppressed, as they can be related with form factors for proton transition between two states with different quantum numbers of the proton constituents. For example, consider swapping inside the proton the colors of two partons that sit at distances of the order of the nucleon radius. This corresponds to an excitation that can be visualized as adding an extra piece of color string whose energy, \( \mathcal{O} \) (1 GeV), would excite and destroy the proton.

### 3 Generalized two-parton distribution

In [17,18] we have developed a formalism to address the problem of multi-parton interactions. The QFT description of double hard parton collisions calls for introduction of a new object—the generalized two-parton distribution, \( 2\text{GPD} \).

Defined in the momentum space, it characterizes two-parton correlations inside a hadron [17]:

\[
D_h(x_1, x_2, Q_1^2, Q_2^2; \mathbf{\Delta}).
\]

Here the index \( h \) refers to the hadron, \( x_1 \) and \( x_2 \) are the light-cone fractions of the parton momenta, and \( Q_1^2, Q_2^2 \) are the corresponding hard scales. As has been mentioned above, the two-dimensional vector \( \mathbf{\Delta} \) is Fourier conjugate to the relative distance between the partons 1 and 2 in the impact parameter plane. The distribution obviously depends on the parton species; we suppress the corresponding indices for brevity.

The double hard interaction cross section (and, in particular, that of production of two dijets) can be expressed through the generalized two-parton distributions \( 2\text{GPD} \).

\( 2\text{GPDs} \) enter the expressions for the differential distributions in the jet transverse momentum imbalances \( \delta_{ik} \) in the kinematical region (1), as well as for the “total” DPI cross section (integrated over \( \delta_{ik} \)). In the latter case the hardness parameters of the \( 2\text{GPDs} \) are given by the jet transverse momenta \( Q_1^2 \), while in the differential distributions—by the imbalances \( \delta_{ik}^2 \) themselves. The corresponding formulas derived in the leading collinear approximation of pQCD can be found in Ref. [18].
It is important to bear in mind that the DPI cross section does not factorize into the product of the hard parton interaction cross sections and the two two-parton distributions depending on momentum fractions \( x_i \) and the hard scales, \( Q_1^2, Q_2^2 \). Instead,

\[
\frac{d\sigma_{\text{DPI}}}{d\Delta d_{T24}} = \frac{d\sigma}{d\Delta d_{T24}} \times \frac{1}{\sigma_{\text{eff}}},
\]

(2a)

\[
\frac{1}{\sigma_{\text{eff}}} = \int \frac{d^2\Delta}{(2\pi)^2} \frac{D_{h_1}(x_1, Q_1^2; \Delta)D_{h_2}(x_2, Q_2^2; \Delta)}{D_{h_1}(x_1, Q_1^2)D_{h_2}(x_2, Q_2^2)D_{h_1}(x_3, Q_1^2)D_{h_2}(x_4, Q_2^2)}.
\]

(2b)

The effective interaction area \( \sigma_{\text{eff}} \) is given by the convolution of the 2GPDs of incident hadrons over the transverse momentum parameter \( \Delta \) normalized by the product of single-parton inclusive pdfs.

For the expression (2) for the DPI cross section to make sense, the integral over \( \Delta \) has to be convergent. This is the case when the two partons are taken from the non-perturbative (NP) proton wave function. Indeed, a typical inter-parton distance in the proton is large, of the order of the hadron size \( R \). Accordingly, one expects the corresponding correlator in the momentum space to be concentrated at a finite NP scale \( \Delta^2 \sim R^{-2} \) and to fall fast at large \( \Delta^2 \) (exponentially or as a sufficiently high power of \( \Delta \)).

However, there is another source of two-parton correlations. This is the purely perturbative (PT) mechanism when the two partons emerge from perturbative splitting of one parton taken from the hadron wave function. In this scenario the production of the parton pair is concentrated at much smaller distances. As a result, the corresponding contribution to 2GPD turns out to be practically independent on \( \Delta^2 \) in a broad range, up to the hard scale(s) characterizing the hard process under consideration (\( \Delta^2 \) only affects the lower limit of transverse momentum integrals in the parton cascades, causing but a mild logarithmic dependence).

Given the essentially different dependence on \( \Delta \), one has to treat the two contributions separately by casting the 2GPD as a sum of two terms:

\[
D_h(x_1, x_2, Q_1^2, Q_2^2; \Delta) = [2]D_h(x_1, x_2, Q_1^2, Q_2^2; \Delta) + [1]D_h(x_1, x_2, Q_1^2, Q_2^2; \Delta).
\]

(3)

Here subscripts \([2]\) \( D \) and \([1]\) \( D \) denote the first and the second mechanisms, respectively: two partons from the wave function versus one parton that perturbatively splits into two.

### 4 Perturbative two-parton correlations

In this section we discuss the role of the PT parton correlations and show that, given a sufficiently large scale of hard interactions, they turn out to be as important as NP ones.

#### 4.1 Estimate of the PT correlation

Let us choose a scale \( Q_{\text{sep}}^2 \) that separates NP and PT physics to be sufficiently low, so that parton cascades due to evolution between \( Q_{\text{sep}}^2 \) and \( Q_1^2 \) are well developed. To get a feeling of relative importance of the PT correlation, as well as to understand its dependence on \( x \) and the ratio of scales, \( Q_2^2 \) vs. \( Q_{\text{sep}}^2 \), the following lowest-order PT estimate can be used.

Imagine that at the scale \( Q_{\text{sep}}^2 \) the nucleon consisted of \( n_q \) quarks and \( n_g \) gluons (“valence partons”) with relatively large longitudinal momenta, so that triggered partons with \( x_1, x_2 \ll 1 \) resulted necessarily from the PT evolution. In the first logarithmic order, \( \alpha_s \log(Q_2^2/Q_{\text{sep}}^2) \equiv \xi \), the inclusive spectrum can be represented as

\[
D \propto (n_q C_F + n_g N_c) \xi,
\]

where we suppressed the \( x \)-dependence as irrelevant. If both gluons originate from the same “valence” parton, then

\[
[1]D \propto \frac{1}{2} N_c \xi \cdot D + (n_q C_F^2 + n_g N_c^2) \xi^2,
\]

(4a)

while independent sources give \([2]D\):

\[
(n_q(n_q-1)C_F^2 + 2n_qn_gC_FN_c + n_g(n_g-1)N_c^2)\xi^2 = D^2 - (n_q C_F^2 + n_g N_c^2) \xi^2.
\]

(4b)

Recall that the \( \Delta \)-dependence is different in (4a) and (4b). However, at \( \Delta = 0 \) the second terms cancel in the sum and we get for the correlator

\[
\frac{D(x_1, x_2; 0)}{D(x_1)D(x_2)} - 1 \simeq \frac{N_c}{2(n_q C_F + n_g N_c)}.
\]

(5)

The correlation is driven by the gluon cascade—the first term in (4a)—and is not small (being of the order of unity). It gets diluted when the number of independent “valence sources” at the scale \( Q_{\text{sep}}^2 \) increases. This happens, obviously, when \( x_i \) are taken smaller. On the other hand, for large \( x_i \sim 0.1 \) and increasing, the effective number of more energetic partons in the nucleon is about 2 and decreasing, so that the relative importance of the \( 1 \otimes 2 \) processes grows.

We conclude that the relative size of PT correlations is of order one, provided \( \xi = \mathcal{O}(1) \).

Moreover, the PT parton correlations cannot be disregarded without running a risk of violating general principles. This can be illustrated by looking at the momentum sum rule for double-parton distributions.

#### 4.2 Momentum sum rule

An obvious momentum sum rule should be satisfied; namely, that the integral over \( dx_2 \) with the weight \( x_2 \) (summing over all parton species) should in the end produce \( (1 - x_1) \) times the inclusive one-parton distribution \( D(x_1) \)—that is, the total

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longitudinal momentum carried by all the partons but the
triggered one:

\[
\sum_{i_1} \int dx_2 x_2 D_{h_2}^{(1,1)}(x_1, x_2, Q_1^2, Q_2^2; \Delta = 0)
\]

\[
= (1 - x_1) \cdot D_{h_1}^{i_1}(x_1, Q_1^2)
\]

(here we have explicitly restored the parton species indices
\(i_1, i_2\)). This sum rule, together with other ones concerning
the valence quantum numbers, has been discussed by Gaunt
and Stirling in \[14\] as a means for restricting the form of
double-parton distributions. Setting the \(\Delta_1\) and Stirling in \[14\] as
a means for restricting the form of the valence quantum
numbers, has been discussed by Gaunt \[14\].

\section{5 1 \(\otimes\) 2 DPI process}

Actually, the NP and PT contributions do not enter the
physical DPI cross section in arithmetic sum (3), driving one
even farther from the familiar factorization picture based
on universal (process independent) parton distributions. As
explained in \[18\], the double hard interaction of two pairs
of partons that both originate from PT splitting of a single
parton from each of the colliding hadrons does not produce
back-to-back dijets. In fact, such an eventuality corresponds
to a one-loop correction to the usual 2 \(\rightarrow\) 4 jet production
process and should not be looked upon as a multi-parton inter-
action. The term \[1|D\}_{h_1} \times [1|D\]_{h_2} has to be excluded from
the product \(D_{h_1} \times D_{h_2}\), a conclusion we share with Gaunt
and Stirling \[33\].

So, we are left with two sources of genuine two-parton
interactions: four-parton collisions described by the product
of (PT-evolved) 2GPDs of NP origin (2 \(\otimes\) 2),

\[
[2]D_{h_1}(x_1, x_2, Q_1^2, Q_2^2; \Delta) [2]D_{h_2}(x_3, x_4, Q_1^2, Q_2^2; -\Delta),
\]

(7a)

and three-parton collisions due to an interplay between the
NP two-parton correlation in one hadron and the two partons
emerging from a PT parton splitting in another hadron (1 \(\otimes\) 2),
described by the combination

\[
[2]D_{h_1}(x_1, x_2, Q_1^2, Q_2^2; \Delta) [1]D_{h_2}(x_3, x_4, Q_1^2, Q_2^2; -\Delta)
\]

\(+ [1]D_{h_1}(x_1, x_2, Q_1^2, Q_2^2; \Delta) [2]D_{h_2}(x_3, x_4, Q_1^2, Q_2^2; -\Delta).
\]

(7b)

Given that \([2]D\) falls fast at large \(\Delta\), the mild logarithmic \(\Delta\)-
dependence of \([1]D\) can be neglected in the product in (7b).

5.1 On separation of NP and PT parts of 2GPD:

parameter \(Q_0^2\)

Separation of PT and NP contributions is a delicate issue.

By definition of the perturbative correlation function, \([1]D\)
vanishes when \(Q_1^2, Q_2^2\) are taken equal to the separation scale
\(Q_{\text{sep}}^2\) that one chooses to set the lower limit for applicabil-
ity of the pQCD calculations. Strictly speaking, \(Q_{\text{sep}}^2\) can
be chosen arbitrarily: both the NP input function \([2]D\) and
the PT-calculable correlation \([1]D\) contain \(Q_{\text{sep}}^2\)-dependence,
but their sum does not depend on this formal parameter. At
the same time, the character of the \(\Delta\)-dependence of \([2]D\)
depends, obviously, on the choice of the \(Q_{\text{sep}}^2\) scale. Indeed,
by increasing the value of \(Q_{\text{sep}}^2\) one will shuffle a part of the
perturbative splitting contribution from \([1]D\) into \([2]D\). As a
result, the “NP correlator” \([2]D\), contaminated by a short-
range PT correlation, would acquire a “tail” at large \(\Delta^2\),
which would spoil convergence of the \(\Delta\) integration in (2).

Thus, in order to preserve the logic of the NP–PT sep-
eration, one is led to introduce a specific resolution scale,
\(Q_{\text{sep}}^2 = Q_0^2\), at which scale the NP correlation \([2]D\) falls fast
with increase of \(\Delta^2\). So defined, \(Q_0^2\) is no longer an arbi-
trary “factorization scale” but a phenomenological param-
eter whose value (which one expects to be of order of 1GeV)
should be established from the data.

5.2 Composition of the 1 \(\otimes\) 2 DPI cross section

In order to derive the DPI cross section, one has to start with
examination of the double differential transverse momentum
distribution and then integrate it over jet imbalances \(\delta_{ik}\). Why
is this step necessary?

The parton distribution \(D(x, Q^2)\)—the core object of the
QCD-modified parton model—arises upon logarithmic inte-
gration over the transverse momentum up to the hard
scale, \(k_1^2 < Q^2\). Analogously, the double-parton distribu-
tion \(D(x_1, x_2, Q_1^2, Q_2^2; \Delta)\) embeds independent integrations
over parton transverse momenta \(k_{1\perp}^2, k_{2\perp}^2\) up to \(Q_1^2\) and \(Q_2^2\),
respectively. However, the 1 \(\otimes\) 2 DPI cross section contains
a specific contribution (“short split”; see below) in which the
transverse momenta of the partons 1 and 2 are strongly cor-
related (nearly opposite). This pattern does not fit into the
structure of the pQCD evolution equation for 2GPD where
\(k_{1\perp}\) and \(k_{2\perp}\) change independently. Given this subtlety,
the legitimate question arises whether the expression for the inte-
grated 1 \(\otimes\) 2 cross section (7b) based on the notion of the
two-parton distribution \([1]D\) takes the short split into account.
Below we demonstrate that in fact it does.

The differential distribution over jet imbalances was
derived in \[18\] in the leading collinear approximation of
pQCD. It resembles the “DDT formula” for the Drell–Yan
spectrum \[49\] and contains two derivatives of the product of
GPDs (7) that depend on the corresponding \( \delta_{ik} \) as hardness scales, and the proper Sudakov form factors depending on (the ratio of) the \( Q_i^2 \) and \( \delta_{ik}^2 \).

In particular, in the region of **strongly ordered** imbalances,

\[
\frac{\pi^2 d\sigma^{\text{DPI}}}{d^2\delta_{13} d^2\delta_{24}} \propto \frac{\alpha_s^2}{\delta_{13}^2 \delta_{24}^2}; \quad \delta_{13}^2 \gg \delta_{24}^2, \quad \delta_{13}^2 \ll \delta_{24}^2, \quad (8)
\]

the differential \( 1 \otimes 2 \) cross section reads

\[
\frac{\pi^2 d\sigma_{1 \otimes 2}}{d^2\delta_{13} d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{d^2\delta_{13} d^2\delta_{24}} \int \frac{d^2 \Delta}{(2\pi)^2} \{ \alpha_s (\delta^2) \}
\]

\[
\times S_1 \left( Q_1^2, \delta_{13}^2 \right) S_2 \left( Q_2^2, \delta_{24}^2 \right) \{ h_1 \leftrightarrow h_2 \}.
\]

The differential distribution for the \( 2 \otimes 2 \) DPI mechanism has a similar structure; see Eq. (25) of [18].

In addition to (9), there is another source of double collinear enhancement in the differential \( 1 \otimes 2 \) cross section. It is due to the kinematical region where the two imbalances nearly compensate one another,

\[
\delta^2 = (\delta_{13} + \delta_{24})^2 \ll \delta^2 = \delta_{13}^2 \mp \delta_{24}^2, \quad (10)
\]

and the dominant integration region is complementary to that of (8):

\[
\frac{\pi^2 d\sigma_{\text{short}}^{\text{DPI}}}{d^2\delta_{13} d^2\delta_{24}} \propto \frac{\alpha_s^2}{\delta^2 \delta'; \delta^2 \ll \delta'^2}. \quad (11)
\]

This enhancement characterizes the set of \( 1 \otimes 2 \) graphs in which there is no accompanying radiation with transverse momenta exceeding \( [\delta^2 \delta'] \).

In this situation, the parton that compensates the overall imbalance, \( \vec{k}_1 \rightarrow -\vec{k}_2 \) is radiated off the incoming, quasi-real, parton legs as shown in Fig. 1. At the same time, the virtual partons after the core splitting “0” → “1” + “2” enter their respective hard collisions without radiating any off-spring along the way.

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leading collinear approximation is given by Eq. (18) of [18]:

\[ P_h^{(a,b)}(x_1, x_2; q_1^2, q_2^2; \Delta) = \sum_{a',b',c} \frac{d^2 \alpha_s(k^2)}{k^2} \int \frac{dy}{k^2} D_h^a(x, k^2) \times \int \frac{dz}{z(1-z)} \rho^{a',b'}(z) D_{a'}^b \left( \frac{x_1}{z}, q_1^2, q_2^2 \right). \]

Here \( a, b \) denote the registered partons, and \( a', b', c \) are the indices of the partons involved in the splitting \( c \to a'(z) + b'(1-z) \) with the DGLAP probability \( P_c^{a',b'}(z) \). The distribution \( D_h^a(y, k^2) \) describes the standard probability of finding a parton \( c \) inside the incident hadron \( h \) at the transverse momentum scale \( k^2 \), and the functions \( D_{a'}^b(x, q_1^2; k^2) \) in the second line stand for the distribution of parton \( i \), probed at scale \( q_2^2 \), in the initial parton \( i' \) at lower virtuality scale \( k^2 \).

Let us examine the structure of the terms one gets applying two derivatives to (13) substituted into (9):

\[ \frac{d}{dq_1^2} \frac{d}{dq_2^2} \left[ \alpha_s \alpha_s \frac{d^2 \alpha_s(k^2) \mathcal{F}'}{k^2} \left( q_1^2, q_2^2 \right) \right]. \]

Taking the derivative over \( q_2^2 \) of the product \( D(x, q_1^2) \times S(Q_2^2, q_2^2) \) corresponds to picking up from the parton chain an accompanying parton \( \ell \) (with the largest transverse momentum) which compensates the imbalance, \( \ell = -q_1 \).

Apart from logarithmic dependences of \( D \) and \( S \), the derivative in (9) may act upon the upper limit of the \( k^2 \)-integration in (13). We have

\[ \frac{d}{dq_1^2} \frac{d}{dq_2^2} \left[ \alpha_s \alpha_s \frac{d^2 \alpha_s(k^2) \mathcal{F}'}{k^2} \left( q_1^2, q_2^2 \right) \right]. \]

Differentiating the upper limit of the \( k^2 \)-integral describes another legitimate situation, i.e. when one of the partons “1” and “2” does not radiate before entering the hard interaction. In this case the smaller of the jet imbalances is determined by the splitting momentum \( k \):

\[ \frac{d}{dq_1^2} \frac{d}{dq_2^2} \left[ \alpha_s \alpha_s \frac{d^2 \alpha_s(k^2) \mathcal{F}'}{k^2} \left|_{k^2=q_1} \right. + \frac{d}{dq_1^2} \frac{d}{dq_2^2} \left|_{k^2=q_2} \right. \right]. \]

Finally, applying both derivatives to the integration limit (13) gives rise to

\[ \frac{d}{dq_1^2} \frac{d}{dq_2^2} \left[ \alpha_s \alpha_s \frac{d^2 \alpha_s(k^2) \mathcal{F}'}{k^2} \left|_{k^2=q_1} \right. + \frac{d}{dq_1^2} \frac{d}{dq_2^2} \left|_{k^2=q_2} \right. \right]. \]

Contrary to the first two contributions (15a) and (15b), the last term (15c) obviously violates the condition of applicability (8) of the DDT formula. Instead, in the region \( \delta_{13}^2 \sim \delta_{24}^2 \) it is the short split that contributes in the leading collinear approximation, so that (12) has to be used to describe the differential spectrum in place of (15c).

However, as far as the total cross section is concerned, the integrals over imbalances of the short split and of the fake singular term (15c) turn out, as if by miracle, to be the same. Indeed, integrating the short split (12) over \( \delta_{13}^2 \) up to \( \delta_{24}^2 \), we get

\[ \sum_c^{(1,2)} \frac{P_c^{(1,2)}(x_1)}{x_1+x_2} \int \frac{d^2 \alpha_s(\delta^2) \mathcal{F}_h^c(x_1+x_2, \delta^2)}{\delta^2} \times \prod_{i=1}^{4} S_i \int \frac{d^2 \Delta}{(2\pi)^2} [2] D_{h_2}(x_3, x_4, \delta^2, \delta^2; \Delta), \]

where \( \prod_{i=1}^{4} S_i = S_1(Q_2^2, \delta^2)S_2(Q_2^2, \delta^2)S_3(Q_2^2, \delta^2)S_4(Q_2^2, \delta^2) \).

On the other side, taking the integrand of (11) \( D \) (13) in the point \( q_1^2 = q_2^2 = k^2 \), and using

\[ D_{a'}^b \left( \frac{x_1}{z}, q_1^2, k^2 \right) \left|_{q_1^2=k^2} = \delta_{a'b} \delta \left( 1 - \frac{x_1}{z} \right), \right. \]

we evaluate the momentum integrals,

\[ \int \frac{dz}{z(1-z)} \int \frac{d^2 \alpha_s(k^2) \mathcal{F}'}{k^2} \left|_{k^2=q_1^2} \frac{d^2 \mathcal{A}}{(2\pi)^2} \left[ 2 \right] \right| D_{h_2}(x_3, x_4, \delta^2, \delta^2; \Delta), \]

we arrive at the very same expression (16).

It is worth noticing that this correspondence does not depend on the precise form of the upper integration limit in (13). The result does not change, within the leading logarithmic accuracy, if one replaces a sharp \( \delta \)-function cut by a smooth damping factor that cuts the logarithmic \( k^2 \) integration at \( k^2 \sim \min(q_1^2, q_2^2) \).

Thus, for the integrated DPI cross section we obtain two contributions to the effective interaction area:

\[ \prod_{i=1}^{4} D(x_i) \prod_{i=1}^{4} D(x_i) = \int \frac{d^2 \mathcal{A}}{(2\pi)^2} \left[ 2 \right] \prod_{i=1}^{4} D_{h_1}(x_1, x_2, Q_1^2, Q_2^2; \Delta) \left[ 2 \right] \prod_{i=1}^{4} D_{h_2}(x_3, x_4, Q_1^2, Q_2^2; \Delta), \]

Let us stress in conclusion that a compact and intuitively clear expression containing the product of the zGPDs \( \prod \) and \( [1] \) in (17b) applies only to the integrated \( 1 \otimes 2 \) cross section.
When addressing the differential distributions, one has to employ the “DDT-like formula” (9) in the region of strongly ordered transverse momenta (8), and the quite different expression (12) in the kinematical region of nearly opposite jet pair imbalances (11).

6 Modeling [2]D

To proceed with the quantitative estimates, one needs a model for the non-perturbative two-parton distributions in a proton.

A priori, we know next to nothing about them. The first natural step to take is an approximation of independent partons. It allows one to relate 2GPD with known objects, namely [17]

\[ [2]D(x_1, x_2, Q_1^2, Q_2^2; \Delta) \simeq G(x_1, Q_1^2; \Delta^2)G(x_2, Q_2^2; \Delta^2). \]  

(18a)

Here \( G \) is the non-forward parton correlator (known as the generalized parton distribution, GPD), which determines, e.g., hard vector meson production at HERA and which enters in our case in the diagonal kinematics in \( x_1 = x_1' \), see Fig. 2.

The modeling by (18a) is not perfect. First of all, it does not respect the obvious restriction \( D(x_1 + x_2 > 1) = 0 \). So, \( x_i \) have to be taken not too large (say, \( x_i < 0.5 \)). Actually, \( x_i \) must be taken even smaller. The GPD in Fig. 2 is an elastic amplitude, while the corresponding block in the DPI represents the inclusive cross section (the cut-through amplitude). For the analogy to hold, the interaction amplitude has to be close to imaginary. This condition calls for \( x_i < 0.1 \).

On the other hand, \( x_i \) should not be too small to stay away from the region of the Regge–Gribov phenomena where there are serious reasons for parton correlations to be present at the non-perturbative level (see the discussion in [40]).

Thus, we fix the domain of applicability of the model (18a) for 2GPD as \( 10^{-1} > x_i > 10^{-3} \).

The GPD, in turn, can be modeled as

\[ G(x_1, Q_1^2; \Delta^2) \simeq D(x_1, Q_1^2) \times F_{2g}(\Delta^2), \]  

(18b)

with \( D \) the usual one-parton distribution determining DIS structure functions and \( F \) the so-called two-gluon form factor of the hadron. The latter is a non-perturbative object; it falls fast with the “momentum transfer” \( \Delta^2 \). This form factor can be parametrized differently. For example, by a dipole formula

\[ F_{2g}(\Delta^2) = \left( 1 + \frac{\Delta^2}{m_g^2} \right)^{-2}, \]  

(18c)

where the effective parameter \( m_g^2 \) extracted from the FNAL and HERA \( J/\psi \) exclusive photoproduction data lies in the ballpark of \( m_g^2(x \sim 0.03, Q^2 \sim 3 \text{ GeV}^2) \approx 1.1 \text{ GeV}^2 \) and decreases with further decrease of \( x \) [41].

Substituting (18) into (2) gives

\[ \sigma_{\text{eff}}^{-1} = \int \frac{d^2\Delta}{(2\pi)^2} F_{2g}(\Delta^2) = \frac{m_g^2}{28\pi}, \quad \sigma_{\text{eff}} \approx 32 \text{ mb}. \]  

(19)

It is about a factor of 2 larger than the value measured by the Tevatron experiments [22,23].

pQCD induced parton correlations (1 \( \otimes \) 2 DPI processes) are capable of explaining this discrepancy.

Turning to the 1 \( \otimes \) 2 term, we neglect a mild logarithmic \( \Delta \)-dependence of [11]D in (17b) and use the model (18) for [2]D to obtain

\[ \sigma_{3}^{-1} \approx \frac{7}{3} \left[ [1][1]D(x_1, x_2) \right] + \frac{[1][1]D(x_3, x_4) D(x_1) D(x_2)}{\sigma_{28}^{-1}}, \]  

(20)

where we substituted the value of the integral (cf. (19))

\[ \int \frac{d^2\Delta}{(2\pi)^2} F_{2g}(\Delta^2) = \frac{m_g^2}{12\pi}. \]

We will parametrize the result in terms of the ratio

\[ R \equiv \frac{\sigma_{1 \otimes 2}}{\sigma_{2 \otimes 2}} = \frac{\sigma_{4}}{\sigma_{3}}. \]  

(21)

For the effective interaction area,

\[ \sigma_{\text{eff}}^{-1} = \sigma_{3}^{-1} + \sigma_{3}^{-1}, \]  

(22)

we then have

\[ \sigma_{\text{eff}} = \frac{28\pi}{m_g^2} \left( \frac{1}{1 + R} \right) = \frac{35 \text{ mb}}{[\text{GeV}]^2}, \quad \frac{1}{1 + R} \approx \frac{32 \text{ mb}}{1 + R} \]  

(23)

(the phenomenological value \( m_g^2 = 1.1 \text{ GeV}^2 \) was used).

Within the framework of the NP two-parton correlations model (18a), there is but one free parameter \( Q_0^2 \). The DPI theory applies to various processes and holds in a range of energies and different kinematical regions. Therefore, having fixed the \( Q_0^2 \) value, say, from the Tevatron data, one can consider all other applications (in particular, to LHC processes) as parameter-free theoretical predictions.

![Fig. 2 GPD in the vector meson electroproduction amplitude](Image)
7 Numerical results

7.1 Calculation framework

In numerical calculations we used the GRV92 parametrization of gluon and quark parton distributions in the proton [42]. We have checked that using more advanced GRV98 and CTEQ6L parametrizations does not change the numerical results. The explicit GRV92 parametrization is speed efficient and allows one to start the PT evolution from rather small virtuality scales. The combination of gluon and quark parton distributions in the proton is very similar to that of the CDF experiment shown in Figs. 4, 5 the enhancement factor $\sigma_{\text{eff}}$ was used in dependence on top of the relevant power of the two-gluon form factor $F_{2g}(\Delta^2)$.

To quantify the role of the $1 \otimes 2$ DPI subprocesses, we calculated the ratio $R$ (21) in the kinematical region $10^{-3} < x_1 < 10^{-1}$ for Tevatron ($\sqrt{s} = 1.8 \pm 1.96 \text{ TeV}$) and LHC energies ($\sqrt{s} = 7 \text{ TeV}$). We chose to consider three types of ensembles of colliding partons:

1. $u(\bar{u})$ quark and three gluons, relevant for “photon plus three-jets” CDF and D0 experiments,
2. four gluons (two pairs of hadron jets),
3. $ud$ plus two gluons, illustrating $W^+ jj$ production.
4. $ud$ plus $d\bar{u}$, corresponding to the $W^+ W^-$ channel.

7.2 Perturbative $1 \otimes 2$ correlation at the Tevatron

7.2.1 CDF experiment

In Fig. 3 we show the profile of the $1 \otimes 2$ to $2 \otimes 2$ ratio $R$ for the $\gamma$+three-jets process in the kinematical domain of the CDF experiment [22]. The calculation was performed for the dominant “Compton scattering” channel of the photon production: $g(x_2) + u(\bar{u})(x_4) \rightarrow \gamma + u(\bar{u})$. The longitudinal momentum fractions of two gluons producing second pair of jets are $x_1$ and $x_3$. The typical transverse momenta were taken to be $p_{1,1,3} \simeq 5 \text{ GeV}$ for the jet pair, and $p_{1,2,4} \simeq 20 \text{ GeV}$ for the photon–jet system. In Fig. 3 $R$ is displayed as a function of rapidities of the photon–jet, $\eta_2 = \frac{1}{2} \ln(x_2/x_4)$, and the two-jet system, $\eta_1 = \frac{1}{2} \ln(x_1/x_3)$.

We observe that the enhancement factor lies in the ballpark of $1 + R \simeq 1.5 \pm 1.8$. Using (23), it translates into $\sigma_{\text{eff}} \simeq 18 \div 21 \text{ mb}$. This expectation has to be compared with the CDF finding $\sigma_{\text{eff}} = 14.5 \pm 1.7 \pm 1.7 \text{ mb}$. A recent reanalysis of the CDF data points at an even small value: $\sigma_{\text{eff}} = 12.0 \pm 1.4 \pm 1.3 \pm 1.5 \text{ mb}$, [43].

In our previous report [40] we have included a plot of the $x$-dependence of $R$ for central production at Tevatron and LHC energies. That plot turned out to be confusing: a rather sharp $x$-dependence it has demonstrated seemed to contradict the CDF findings of approximate constancy of $\sigma_{\text{eff}}$. In fact, this variance with $x$ was in the major part resulting from the kinematical link between $x$ and $Q^2$ for a given collision energy ($Q^2 = x^2 s/4$).

The results of a numerical calculation for the fixed hardness $Q^2$ shown in Fig. 3 for the CDF kinematics exhibit a very mild $x$-dependence of the $R$ factor and thus of the $\sigma_{\text{eff}}$.

7.2.2 D0 experiment

The ratio $R$ is practically constant in the kinematical domain of the D0 experiment on photon+three-jets production [23, 24] and is very similar to that of the CDF experiment shown above in Fig. 3. So, for the D0 kinematics we instead display in Figs. 4, 5 the enhancement factor $1 + R$ in dependence on $p_\perp$ of the secondary jet pair for photon transverse momenta 10, 20, 30, 50, 70, and 90 GeV (from bottom to top).

The corresponding prediction for $\sigma_{\text{eff}}$ is shown in Fig. 6 in comparison with the D0 findings.

Both the absolute value and (a hint at) the $p_\perp$-trend look satisfactory.
Fig. 4 Central rapidity photon+three-jets production in $u(\bar{u})$–gluon collisions in the D0 kinematics

Fig. 5 Same as Fig. 4 for $Q_0^2 = 1\text{GeV}^2$

7.3 LHC energies

In Fig. 7 we show the $1 \otimes 2$ to $2 \otimes 2$ ratio for production of two pairs of back-to-back jets with transverse momenta $50\text{GeV}$ produced in collision of gluons at the LHC energy $\sqrt{s} = 7\text{TeV}$.

The dependence on the hardness parameters of the DPI process of double gluon–gluon collisions is illustrated in Fig. 8. For the sake of illustration, we have chosen the value of the PT cutoff parameter $Q_0^2 = 0.5\text{GeV}^2$, and we have calculated the enhancement factor $1 + R$ for five values of the transverse momenta of the jets in one pair, $p_{\perp 1} = 20, 40, 60, 80, 100\text{GeV}$.

Figure 8 demonstrates its dependence on the transverse momenta of jets in the second pair, $p_{\perp 2} \equiv p_{\perp 2}$.

We observe that within the chosen range, $R$ increases by about 15–25 % with increase of the hardness of one of the jet pairs. This corresponds to approximately 10 % drop in $\sigma_{\text{eff}}$.

Finally, in Fig. 9 we show the rapidity profile of the $R$ ratio for the process of production of the vector boson, $u\bar{d} \to W^+$, accompanied by an additional pair of (nearly back-to-back) jets with transverse momenta $p_{\perp} = 30\text{GeV}$ produces in a gluon–gluon collision.

It is interesting to notice that the effect of perturbatively induced parton–parton correlations is maximal for equal rapidities of the $W$ and the jet pair, and it diminishes when they separate. This feature is more pronounced when the cutoff parameter $Q_0^2$ is taken larger, so that the PT correlation becomes smaller and, at the same time, exhibits a stronger rapidity dependence.

The recent ATLAS study [44] reported for this process the value $\sigma_{\text{eff}} = 15 \pm 3^{+5}_{-3}\text{mb}$, which is consistent with the expected enhancement due to contribution of the $1 \otimes 2$ DPI channel; see (21).

Figure 10 shows a slight variation of $\sigma_{\text{eff}}$ with the jet scale in $Wjj$ production.

7.4 $Q_0$-dependence

The dependence of the enhancement factor $1 + R$ on the $Q_0$ parameter is shown for the typical kinematics of the CDF photon+three-jets experiment in Fig. 11, and for central production of two pairs of $p_{\perp} \simeq 50\text{GeV}$ jets at the LHC ($\eta_1 = \eta_2$) in Fig. 12.

8 Conclusions and discussion

In the previous paper [18] we have analyzed the perturbative correlation that arises due to $1 \otimes 2$ splitting of the parton in one of the colliding hadrons and derived the corresponding expressions in the leading collinear approximation of the pQCD. Here we presented the results of the numerical evaluation of this contribution to the DPI cross section measured at the Tevatron and found the theoretical results to be con-
Fig. 7 Rapidity dependence of the $R$ factor for two pairs of $p_\perp = 50$ GeV jets produced in gluon–gluon collisions.

Fig. 8 $1 + R$ for two dijets at LHC: $p_\perp = 20, 40, 60, 80, 100$ GeV (from bottom to top).

Fig. 9 Ratio $R$ for production of $W$ plus a pair of $p_\perp \simeq 30$ GeV gluon jets.

Fig. 10 $\sigma_{\text{eff}}$ for the $Wjj$ process at LHC energy as a function of jet transverse momentum.

Consistent with the data for the value of the model parameter $Q_0^2 \simeq 0.5$ GeV$^2$. With $Q_0^2$ fixed, theoretical expectations for certain exemplary DPI processes at LHC energies become parameter-free predictions.

The theoretical derivation of the effective interaction area $\sigma_{\text{eff}}$ ("effective cross section") relied on certain assumptions and approximations. Our approach to perturbative QCD effects in DPI developed in [18] was essentially probabilistic. In particular, we did not discuss the issue of a possible interference between $1 \otimes 2$ and $2 \otimes 2$ two-parton amplitudes. One can argue that such an eventuality should be strongly suppressed. Indeed, the spatial properties of the accompany-
DPI cross section within the LLA accuracy, one integrates the differential distribution over jet imbalances, \( \delta_{ik} \ll Q_i^2 \), up to the scale given by the transverse momenta of the jets, \( Q_i^2 \). In reality, due to experimental cuts that are imposed in order to extract jets in the back-to-back kinematics the true hard scale of the DPI cross section is lower. Being formally a subleading \( O(\alpha_s) \) correction, it will affect both the 1 \( \otimes \) 2 and the 2 \( \otimes \) 2 cross sections. In which way the subleading pQCD effects will change the ratio is so far unknown. To establish the true hard scales of the parton distributions entering the DPI cross section formula, one has to carry out the NLLA analysis which would include taking into consideration concrete details of the jet finding algorithms employed in the experimental setup.

Finally, our prediction for the DPI cross sections was based on a model assumption of the absence of NP two-parton correlations in the proton. This assumption is arbitrary. One routinely makes it for lack of any first-hand information as regards such correlations. In [40] we have pointed out a source of genuine non-perturbative two-parton correlations that should come onto the stage for very small \( x \) values, \( x \ll 10^{-3} \), and we estimated its magnitude via inelastic diffraction in the framework of the Regge–Gribov picture of high energy hadron interactions. Also it was argued in [45] that strong quark–antiquark correlations may arise from dynamical chiral symmetry breaking.

In order to be able to reliably extract the DPI physics, one has to learn how to theoretically predict 1 \( \otimes \) 1 parton collision processes with production of two hard systems (four jets in particular). This is the dominant channel, and it is only in the back-to-back kinematics that the 2 \( \otimes \) 2 and 1 \( \otimes \) 2 DPI processes become competitive with it. Among the first subleading pQCD corrections to the 1 \( \otimes \) 1 amplitude, there is a loop graph that looks like two-by-two parton collision. This resemblance is deceptive, though. Unlike the 2 \( \otimes \) 2 and 1 \( \otimes \) 2 contributions, this specific correction does not depend on the spatial distribution of partons in the proton (information encoded in \( \sigma_{\text{eff}} \)), is not power enhanced in the region of small transverse momenta of hard systems, and therefore does not belong to the DPI mechanism [18,33]. Treating the 1 \( \otimes \) 1 amplitude at the one-loop level corresponds to the two-loop accuracy for the cross section. Until this accuracy is achieved, the values of \( \sigma_{\text{eff}} \) extracted by experiments should be considered as tentative.

Our first conclusion is that in the kinematical region explored by the Tevatron experiments, the \( x \)-dependence of \( \sigma_{\text{eff}} \) turns out to be rather mild. This by no means implies, however, that \( \sigma_{\text{eff}} \) can be looked upon as any sort of universal number. On the contrary, we see that the presence of the perturbative correlation due to the 1 \( \otimes \) 2 DPI mechanism results in the dependence of \( \sigma_{\text{eff}} \) not only on the parton momentum fractions \( x_i \) and on the hardness parameters, but also on the type of DPI process.
For example, in the case of the golden DPI channel of production of two same-sign $W$ bosons [46] the discussed mechanism leads to the expectation of significantly larger $\sigma_{\text{eff}}$ than for, say, $W$ plus two-jets process. Indeed, the comparison of the values of $R$ for central production of two-gluon jet pairs, $Wjj$ and $W^+W^+$ (with jet transverse momenta $p_\perp \sim M_W/2$), gives ($\sqrt{s} = 7 \text{TeV}, \eta_1 = \eta_2 = 0$)

$$R(jj + jj) = 1.18 \ (0.81)$$
$$R(W + jj) = 0.75 \ (0.45)$$
$$R(W^+W^+) = 0.49 \ (0.26)$$

(24)

As a result of the varying magnitude of the perturbative correlation, the effective interaction areas $\sigma_{\text{eff}}$ turn out to be significantly different for the three processes:

$jj + jj: \quad \sigma_{\text{eff}} = 14.6 \div 17.6 \text{mb},$

$W + jj: \quad \sigma_{\text{eff}} = 18.3 \div 22.0 \text{mb},$

$W^+W^+: \quad \sigma_{\text{eff}} = 21.5 \div 25.4 \text{mb},$

The smaller value for each effective interaction area corresponds to more developed perturbative parton cascades than the larger one ($Q_0^2 = 0.5 \text{GeV}^2$ versus $Q_0^2 = 1.0 \text{GeV}^2$).

Contrary to the $W^+W^+$ channel, the double Drell–Yan process favors the $1 \otimes 2$ mechanism, $g \to u\bar{u}$. As a result, the effective interaction area here turns out to be even smaller.

For central production of two $Z$ bosons at $\sqrt{s} = 7 \text{TeV}$ we get

$$R(ZZ) = 1.03 \ (0.73),$$

$$ZZ: \quad \sigma_{\text{eff}} = 15.9 \div 18.5 \text{mb}.$$

(26)

An important feature of the $1 \otimes 2$ mechanism is its dependence on the hardness of the process. With increase of $Q_i^2$, the $1 \otimes 2$ to $2 \otimes 2$ ratio $R$ should increase rather fast thus pushing $\sigma_{\text{eff}}$ to smaller values. At the same time, with decrease of the $p_\perp$ of the jets this contribution decreases. As we have seen above, such a trend is consistent with the D0 data for $x \sim 10^{-2}$.

By pushing the hardness scales down to $p_\perp \sim 3 \div 4 \text{GeV}$, one enters the domain of the physics of minijets. Here one should have $\sigma_{\text{eff}} \sim 25 \text{mb}$ for $x_i \sim 10^{-2}$, a much larger value than the $Q$-independent $\sigma_{\text{eff}}$ which had been assumed in Monte Carlo models like PYTHIA for a long time.

It would be interesting to implement in the MC models a more realistic account of MPI in which $\sigma_{\text{eff}}$ would decrease with increase of $p_\perp$.

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Appendix A: Momentum sum rule

The proof of the sum rule (6) involves two types of graphs; see Fig. 13.

A.1 Two partons from the wave function

Consider first two partons 1 and 2 whose parents are taken from the hadron wave function at some scale $Q_{\text{sep}}^2$ that separates NP and PT stages. Distribution of each parton then independently evolves up to the hard scale $Q^2$ according to the standard pQCD rules.

The single-parton distribution is presented in the form of a convolution:

$$D_i^f(x, Q^2) = \sum_B \int \frac{dy}{y} D_f^i(\frac{x}{y}, Q^2; Q_{\text{sep}}^2) w_B^i(y; Q_{\text{sep}}^2)$$

of the NP input function $w$ with the parton evolution function $D_f^i(x, Q^2; Q_{\text{sep}}^2)$ obeying the initial condition

$$D_f^i(x, Q_{\text{sep}}^2; Q_{\text{sep}}^2) = \delta_i^f \cdot \delta(1 - x).$$

Applying the momentum integral (6) to the single parton distribution $D_i^f(x_2, Q^2)$ gives

$$\sum_{i_2} \int d x_2 x_2 D_{i_2}^i(x_2) = \sum_B \bar{y}_B,$$

with $\bar{y}_B$ the average energy fraction of the proton carried by the initial parton $B$ (parent of $i_2$) at the scale $Q_{\text{sep}}^2$.

$$\bar{y}_B = \int dy_B y_B w_B^i(y_B; Q_{\text{sep}}^2).$$

Due to the momentum sum rule for the parton wave function of the proton, summing over parton species $B$ produces the total energy carried by all initial partons but the parent of the second registered parton $i_1$:

![Fig. 13 Independent and correlation diagrams](image-url)
\[
\sum_B \hat{y}_B = 1 - y_1.
\]

The parent parton energy \( y_1 \) is not observable. What is fixed by the measurement is \( x_1 \), while \( y_1 \) is being integrated over:

\[
\sum_{i z} \int dx_2 x_2 D_{h}^{i z}(x_1, x_2)
= \sum_A \int_1^1 \frac{dy_1}{y_1} \left[ 1 - y_1 \right] w^A(y_1) D^A_i \left( \frac{x_1}{y_1}, Q^2; Q_0^2 \right)
= \left[ 1 - \langle y_1 \rangle \right] \cdot D^i_h \left( x_1, Q^2 \right),
\]

where we have introduced an average conditional parent parton energy \( \langle y_1 \rangle \) which depends on \( x_1 \) and \( Q^2 \). It is important to notice that this quantity depends also on the unphysical scale \( Q^{2 \text{sep}} \) which separates the domains of the PT and NP descriptions (unlike the physical inclusive parton distribution in the proton \( D \) on the r.h.s. of (27)).

A.2 One parton splitting into two

For the perturbative correlation we have

\[
\sum_{A, B} \int \frac{dy}{y^2} \int \frac{dz}{z(1-z)} P_A^B(z) D^A_h(y) D^i_B \left( \frac{x_1}{y z} \right) \times (1-z)^2 y^2
= \sum_{A, B} \int dy \int \frac{dz}{z} P_A^B(z) D^A_h(y) D^i_B \left( \frac{x_1}{y z} \right) \times (1-z).
\]

We split the factor \((1 - z)\) into two pieces, \((1) + (-z)\). The first one gives

\[
\frac{\alpha_s}{2\pi} \sum_B \int dy \int \frac{dz}{z} P_A^B(z) D^A_h(y) D^i_B \left( \frac{x_1}{y z} \right)
= - \int dy S_A(k^2) D^A_h(y, k^2)
- \frac{\partial}{\partial \ln k^2} \left[ S_A^{-1}(k^2) D^A_i \left( \frac{x_1}{y}, Q^2; k^2 \right) \right].
\]

Here we have used the evolution equation for the second D function differentiated over the smaller scale \( k^2 \):

\[
\frac{\alpha_s(k^2)}{2\pi} \sum_B \int dz \frac{P_A^B(z)}{z} D^B_h \left( \frac{x}{z}, Q^2; k^2 \right)
= - S_A(k^2) \frac{\partial}{\partial \ln k^2} \left[ S_A^{-1}(k^2) D^A_h \left( x, Q^2; k^2 \right) \right],
\]

where \( S_A \) is the Sudakov form factor of the parton \( A \) depending on the two scales, the overall \( Q^2 \) and the floating splitting scale \( k^2 \) [47–49].

An alternative evolution equation where the derivative is applied to the upper scale of the parton distribution in the proton reads

\[
\frac{\alpha_s(k^2)}{2\pi} \sum_B \int dz \frac{P_A^B(z)}{z} D^B_h \left( \frac{x}{z}, Q^2; k^2 \right)
= S_B^{-1}(k^2) \frac{\partial}{\partial \ln k^2} \left[ S_B(k^2) D^B_h \left( x, k^2 \right) \right].
\]

This equation allows us to analogously represent the second piece \((-z)\) as the derivative of the first D-function over the upper scale:

\[
\frac{\alpha_s}{2\pi} \sum_A \int dy (-z) \int \frac{dz}{z} P_A^B(z) D^A_h(y) D^i_B \left( \frac{x_1}{y z} \right)
= - \int dy \int dz \frac{d}{dz} \left[ S_B^{-1}(k^2) \right] \frac{\partial}{\partial \ln k^2} \left[ S_B(k^2) D^B_h(y', k^2) \right],
\]

with \( y' \equiv z y \). Combining (28) and (31), we get a full logarithmic scale derivative of the product of the D-functions:

\[
- \int dy \int \frac{dz k^2}{k^2} \frac{d}{dz} \ln k^2 \left[ D^A_h(y, k^2) D^i_B \left( \frac{x_1}{y}, Q^2; k^2 \right) \right].
\]

Now we integrate over the intermediate virtuality and make use of the boundary conditions,

\[
D^A_h \left( \frac{x}{y}, Q^2; Q^2 \right) = \delta^A \delta \left( 1 - \frac{x}{y} \right), \quad D^A_h(y, Q^{2 \text{sep}}) = w^A_h(y),
\]

with \( w^A_h \) as the NP input parton distribution. We obtain

\[
\sum_A \int \frac{dy_1}{y_1} w^A(y_1) D^A_i \left( \frac{x_1}{y_1}, Q^2; Q^{2 \text{sep}} \right)
- x_1 D^i_h \left( x_1, Q^2 \right) \equiv \left[ \langle y_1 \rangle - x_1 \right] \cdot D^i_h \left( x_1, Q^2 \right).
\]

Once the two- and one-parton contributions (27) and (32) are taken together, the unphysical quantity \( \langle y_1 \rangle \) cancels out, and we arrive at the desired sum rule (6).

We conclude that for consistency of the DPI picture, the perturbative parton correlation (and thus the \( \otimes \otimes \) subprocesses) should be taken into full consideration at that very moment when one allows for distributions of partons picked from the hadron wave function to evolve with the hard scale(s).

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