A method to reanalyze Dark Matter experimental results in different theoretical scenarios

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Abstract. There are a number of papers that calculate how the limits or positive results of current experiments would be if some specific twist is applied to the standard interpretation framework (e.g., SI interactions with $f_p \neq f_n$). These works are usually not performed by members of the experiments, and therefore make very simple assumptions on experimental details like efficiencies. Nevertheless, it is possible to retain this type of information without actually knowing it, by starting from the final exclusion plots and working backwards. This possibility is discussed and exemplified.

1. Introduction

Direct Dark Matter (DM) search results are usually reported within a standard interpretation framework, that converts the measured rates or rate limits to values or limits on the interaction cross sections of the nucleons with the Dark Matter Particle (DMP). This conversion necessarily involves a model for the incident DMP current, hence a dark halo model, and a DMP-matter interaction model. Quantitatively, the differential rate $dR/dE$ per unit target mass is modeled as:

$$\frac{dR}{dE} = \epsilon(E) \sum_A \frac{f_A}{A} \int \frac{d\sigma_A}{dE} F F_A(E) \rho_{DM} v f(\vec{v}) d\vec{v} = \epsilon(E) \frac{\rho_{DM}}{M} \sum_A \frac{\sigma_A}{2\mu_A^2} F F_A(E) f_A \int \frac{f(v)}{\vec{v}} d\vec{v}$$

where $\epsilon(E)$ is the detection efficiency of the recoiling nuclei as a function of recoil energy $E$, $f_A$ the abundance of the isotope with mass number $A$ in the target, $\rho_{DM}$ the local DM mass density from halo modeling, $M$ the DMP mass, $f(\vec{v})$ the halo velocity distribution. The zero momentum transfer differential cross section for elastic scattering off the isotope $A$ is $d\sigma_A/dE = \sigma_A A/(2\mu_A^2 v^2)$, with $\sigma_A$ the integral cross section from 0 to the maximum recoil momentum $2\mu_A v$. $FF_A(E)$ denotes the form factor of nucleus $A$.

In the standard framework, the dark halo model is a non-rotating spherical isothermal halo reproducing the $1/r$ gravity needed for the galactic disk rotation velocity to be constant. This implies that $f(\vec{v})$ is, relative to the galactic center, a Maxwellian. There are, of course, studies on the impact of different (more realistic or more exotic) halo models (e.g., [1, 2, 3, 4, 5]) on the direct search results, but the experiments usually employ $f(\vec{v}) \propto exp[(\vec{v} - \vec{v}_E)^2/v_E^2]$, where $\vec{v}_E$, the detector’s velocity relative to the galactic center, has an effectively constant component (the Sun’s velocity), a component rotating with a 1 year period (the Earth’s velocity relative to...
the Sun), and a component rotating with a 1 day period (the Earth’s rotation velocity around its axis). While directional detectors hope to detect a diurnal modulation in their rates, to date there is only one claim of a clear (9.3σ C.L. [6], increasing over the years) annual modulation, which no other experiment could, so far, reproduce with a C.L. sufficient for a discovery claim (the annual modulation observed by CoGeNT hasn’t, so far, reached a high C.L. [7]).

In addition, there are also studies on the effect of changing the interaction of the DMP with matter. Although the DM could be a mixture of multiple DMPs, but the standard interpretation framework assumes just one, and it assumes it to have GeV to TeV scale mass, and weak interactions. These characteristics define the well known Weakly Interacting Massive Particles (WIMPs), that, if thermally generated in the early Universe, would naturally have a relic density of the observed order of magnitude.

Another assumption is that the main channel of interaction is the elastic scattering off nuclei, which is key in the data analysis, since every event identified as not a nuclear recoil is rejected, greatly reducing the background. Nevertheless, occasionally analyses are performed of the low dE/dx signals to explore possibilities like an Axion-like particle signal or that the observed annual modulation is due to leptophilic DM producing primarily $e^-$ recoils. Some techniques, like SDDs and bubble chambers [10, 11, 12, 9], allow to even set the detector insensitive to low dE/dx events, so they just don’t see such events as $\beta$s, $\gamma$s, etc. Once the events selected for analysis are recoiling nuclei, the leading order interactions are the Spin-Independent (SI) and Spin-Dependent (SD) scattering, and experiments typically declare themselves primarily sensitive to one of these channels. For SD interactions, the nuclear spins tend to pair such that an even number of protons (neutrons) has a very small total spin relative to an odd group. On the other hand, odd-odd nuclei are very rare, and usually none of the isotopes employed by a Direct DM search is of this type. So, SD results are traditionally reported ignoring the contributions from the even group of nucleons of each sensitive isotope. For SI results, it is generally assumed that the DMP coupling with the nucleon does not depend on its isospin,

1 This may have significant implications for accelerator searches. It is possible, e.g., that the first DMP candidate produced by an accelerator is a tiny fraction of the DM composition. In this scenario, it would be hard for direct searches to confirm it as a component of the Milky Way’s DM halo.
i.e., the coupling strengths $f_{p,n}$ of the DMP with the proton and the neutron are equal\textsuperscript{2}. This provides a coherent enhancement of the sensitivity $\propto A^2$ relative to SD results, and allows to formulate an SI result simply as a nucleon cross section $\sigma_N$ (here $N$ means nucleon) and a DMP mass. Of course, the $\propto A^2$ scaling of the sensitivity favors the choice of heavy target nuclei like Xe \cite{14, 15, 16} and W \cite{17}. In the common case of limits, this becomes a $\sigma_N$ vs $M_{DMP}$ plot, as exemplified in Fig. [1]. The most straightforward generalization of this interaction model is to allow for $f_p \neq f_n$, which makes the limits 3D plots, and the positive results elliptical coronas. The latter circumstance highlights the necessity of multiple positive results to pinpoint the nature of “the” DMP, once it is found. If there are multiple types of DMP, different experiments may be primarily sensitive to different types, further calling for multiple detections, in order to investigate the DM. For the SD sector, when single experiments do not neglect the even group of nucleons, the results are also 3D limits or elliptical crown \cite{18, 19} positive claims.

Other two popular generalizations of the standard interaction model of the DM with baryons are the inelastic scattering \cite{20, 21, 22} and the inclusion of velocity-dependent operators, which are normally sub-leading because the typical velocities of halo particles are of order $10^{-3}$ of the speed of light, but may become dominant in case of an unexpected suppression of the leading SI and SD operators. As $\vec{v} = \frac{\vec{P}}{M} = \frac{\vec{P}}{\vec{r}E}$, these sub-leading interactions also depend on the nucleons’ orbital angular momenta.

Now, when people other than the experiments themselves want to constrain a non-standard interaction, they have the problem they do not have an accurate knowledge of the experimental details which are generally summarized in parameters like detection efficiencies, quenching factors and effective exposures. The same difficulties are faced by an experiment trying to show that its performance relative to others is better in a non-standard scenario. The common practice is to assume these parameters to be constants, though they are known to depend on recoil energy and possibly other factors (e.g., if the detector slightly degrades but not so much to compel to discard the run altogether, a time dependence is possible), or to adopt some simple approximation \cite{23, 24}. But it is possible to retain the original information, by starting from the published cross sections rather than the raw rates. To illustrate this point, I shall consider the case of SI limits given by the experiment for $f_p = f_n$, and convert these to the general case $f_p \neq f_n$ without losing the experimental details included in the original analysis. This type of approach has been used also for the generalization of SD results, and can, in principle be used for other departures from the standard interpretation, as long as it is possible to “go backwards” without being stopped by a non-invertible step of the original analysis.

\textsuperscript{2} This is certainly verified when the leading interaction channel is the higgs exchange, as is the case for the neutralino, since the lower bounds on the $s$-quark masses are far above the Higgs mass, and the strange quark component has the same weight in both nucleons\cite{13}.
2. Example: SI reanalysis of PICO-2L and CDMSlite run 2 with \( f_p \neq f_n \)

The SI limits published by CDMSlite [8] and PICO-2L [9], as available from the repository hosted by Brown University [25] are shown in Fig. 1. The selected colors are the same used in the original papers for the same datasets (in both cases the second run of the detector).

In the standard model for SI interaction with \( f_p = f_n \), the nucleus cross section relates to the nucleon’s as \( \sigma_A = \sigma_N A^2(\mu_A/\mu_p)^2 \), which substituted in Eq (1) yields:

\[
\frac{dR}{dE} = \frac{\sigma_N}{2\mu_p^2} \sum_A A^2 f_A FF_A(E) \frac{\rho_{DM}}{M} \int \frac{f(v)}{v} dv = \frac{\sigma_N}{2\mu_p^2} \sum_A A^2 f_A FF_A(E) J_{DMP}
\]

where \( J_{DMP} \) denotes, with abuse of language, the factors calculated by the experiment that we don’t need to alter. Although related, \( J_{DMP} \) is not the incident DM current. Thanks to the low recoil energies of both results considered in this example, \( FF_A \approx 1 \), so there is no need to take into account the change in form factor relative to the standard framework [26]. If we drop the \( f_p = f_n \) assumption, and retain in \( \sqrt{\sigma_{p,n}} \) the sign of \( f_{p,n} \), Eq (2) becomes:

\[
\frac{dR}{dE} = \sum_A \frac{[\sqrt{\sigma_p Z} + \sqrt{\sigma_n (A - Z)}]^2}{2\mu_p^2} f_A FF_A(E) J_{DMP}
\]

so, equating the right hand sides of Eq.s (2) and (3), we get:

\[
\sum_A [\sqrt{\sigma_p Z} + \sqrt{\sigma_n (A - Z)}]^2 f_A FF_A(E) = \sigma_N \sum_A A^2 f_A FF_A(E) \leq \sigma_N \sum_A A^2 f_A FF_A(E)
\]

where \( \sigma_N^+ \) is the upper limit on \( \sigma_N \) published by the experiment, and the weak dependence of \( J_{DMP} \) on \( A \) is neglected. Then, defining the auxiliary cross section limits

\[
\left\{
\begin{array}{l}
\sigma_p^A = \sigma_N \sum_A A^2 f_A FF_A(E) \\
\sigma_n^A = \sigma_N \sum_A (A-Z)^2 f_A FF_A(E)
\end{array}
\right.
\]

we get the \( f_p \neq f_n \) allowed region:

\[
\sum_A (\sqrt{\sigma_p^A} + \sqrt{\sigma_n^A})^2 \leq 1
\]

Note that if \( \sigma_p = 0 \) (\( \sigma_n = 0 \)) the limit on \( 1/\sigma_n \) (\( 1/\sigma_p \)) becomes \( \sum_A 1/\sigma_n^A (\sum_A 1/\sigma_p^A) \). These traditional-looking exclusion plots are shown in Fig. 2 and are less stringent than the standard \( f_p = f_n \) limits because the DMP is assumed to interact still coherently, but with only approximately half of the nucleons. The general case could be plotted in 3D, but unfortunately the elliptical cross section of these plots is very eccentric, making the 3D rendering quite poor. The alternative is to select some interesting mass values, and plot the (signed) \( \sqrt{\sigma_p} \) vs \( \sqrt{\sigma_n} \), as in Fig. 3.

In the given example, the mass of 10 GeV was arbitrarily chosen, for being in the approximate middle of the range published by the two experiments. The interior of the ellipses is, of course, allowed, while the exterior is excluded. In the case of a positive signal, which excludes the origin \( (\sqrt{\sigma_p} = \sqrt{\sigma_n} = 0) \), the allowed region is an elliptical crown. The combined limits or positive results are, instead, the intersection of the ellipses (elliptical crowns) allowed by each experiments. In the chosen example, the largest allowed cross sections are for \( f_p/f_n \approx -1.3 \), with \( \sigma_p \lesssim 3.3 \text{ fb} \) and \( \sigma_n \lesssim 1.9 \text{ fb} \).
Figure 3. Region allowed by both experiments for a DMP mass of about 10 GeV/c². The dash-dotted line shows the $f_n = -f_p$ locus. The extreme $f_n = -f_p$ points of PICO-2L are excluded by CDMSlite, while the corresponding points of CDMSlite are allowed by PICO-2L.

3. Conclusions

As shown in the example for the case of a generalized SI interaction, it is generally possible to calculate the constraints of an experiment on a non-standard scenario by starting from the results published by the experiment within the standard framework. All that is required is to invert the late steps of the calculation of the original limits back to the point where the new scenario departs from the standard one, and then work forward in the new model. The advantage is to retain the detailed experimental information on the detector performance and whatever may have happened during the runs, as they were included by the experiment itself, without a need to know them.

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