Plasmonic amplification and suppression in nanowaveguide coupled to gain-assisted high-quality plasmon resonances

Song-Jin Im and Gum-Song Ho

Department of Physics, Kim Il Sung University, Daesong District, Pyongyang, Democratic People’s Republic of Korea

E-mail: ryongnam7@yahoo.com

Received 28 February 2014, revised 9 February 2015
Accepted for publication 9 February 2015
Published 20 March 2015

Abstract
We study transmission in a nano waveguide coupled to high-quality plasmon resonances for which the metal loss is overcompensated by gain. The on-resonance transmission can vary widely from lower than −20 dB to higher than 20 dB for a range of gain coefficient. A reversible transition between the high-quality amplification and the suppression can be induced by a quite small change of gain coefficient for a moderately increased distance between the waveguide and the resonator. It is expected that in practice a small change of gain coefficient can be made by flexibly controlling pumping rate or utilizing nonlinear gain. Additionally, based on the frequency-dependant model for gain-transition susceptibility, it is shown that the wide variation of the on-resonance transmission can also be observed for different detuning of the gain-transition line-center. Such a widely controllable on-resonance transmission is promising for applications such as well-controlled lumped amplification of surface plasmon-polariton as well as plasmonic switching.

Keywords: surface plasmons, surface plasmon-polariton, nanostructures, optical amplifiers

(Some figures may appear in colour only in the online journal)

1. Introduction
Plasmonic devices provide a means of guiding and manipulating light at deep subwavelength scales [1–3] and have found applications such as spectroscopy [4], imaging [5], biosensing [6, 7], circuitry [8], cancer treatment [9, 10] and solar energy conversion [11]. Surface plasmons are fundamentally lossy due to metal losses such as the ohmic losses and interband transitions in the metals, limiting their practical application. The use of gain media where the population inversion is created optically or electrically has been suggested for compensating the metal loss, surface plasmon amplification or spasing [12, 13] and experimentally demonstrated [14–18]. Stimulated emission of surface plasmon polaritons has been theoretically [19] and experimentally [20, 21] studied.

In [22], they proposed gain-assisted plasmonic switch consisting of a gold-air-gold plasmonic waveguide coupled to a resonator filled with gain medium for gain coefficient exactly compensating for the metal loss.

In this letter we study transmission in such a similar structure for gain coefficient larger than the exactly metal-loss-compensating one. The transmission is widely changeable from smaller than a hundredth to larger than a hundred for a range of gain coefficient. A reversible transition between the high-quality amplification and the suppression can be induced by a quite small change of gain coefficient. Such a transition can also be induced by changing a detuning of the gain-transition frequency. For calculations we introduce a model for frequency-dependant gain coefficient.

2. Gain-assisted high-quality plasmon resonance
Quality factor of plasmon resonance can be expressed by [12]

\[ Q = \frac{\omega}{\Delta\nu} \frac{\text{Re}(\omega)}{2 \text{Im}(\omega)}. \]  \hspace{1cm} (1)
Equation (6) is essentially $g_{\text{gain}} = \frac{\varepsilon_b}{\varepsilon_b - \varepsilon_m(\omega)}$, where $s(\omega)$ is Bergman’s spectral parameter [23], $\varepsilon_m(\omega)$ is the permittivity of embedded metal and $\varepsilon_b(\omega)$ is the permittivity of embedding dielectric host medium. If we introduce a gain material in the host medium, 

$$\varepsilon_b = \varepsilon_a + \chi_g$$

where $\varepsilon_a$ is permittivity of embedding dielectric excluding the gain transition contribution and $\chi_g$ is the gain-transition susceptibility.

If we reasonably assume

$$|\varepsilon_a| >> |\varepsilon_b|, |\text{Re}\varepsilon_m(\omega)| >> |\text{Im}\varepsilon_m(\omega)|,$$  

the imaginary part of Bergman’s parameter is approximately expressed as

$$\text{Im} s \approx \frac{\varepsilon_a |\text{Re}\varepsilon_m(\omega)|}{[\varepsilon_a + |\text{Re}\varepsilon_m(\omega)|]^2} = \frac{|\text{Im}\varepsilon_m(\omega)|}{|\text{Re}\varepsilon_m(\omega)|} = \frac{|\text{Im}\chi_g|}{\varepsilon_a}.$$  

From equations (1) and (5), we can obtain the gain coefficients for high-quality plasmon resonances

$$g_0(\omega) = \frac{\sqrt{\varepsilon_a} \omega |\text{Im}\varepsilon_m(\omega)|}{c |\text{Re}\varepsilon_m(\omega)|}.$$  

In the derivation of equation (6), we considered that the gain coefficient is given in general by $g = 2\text{Im} \sqrt{\varepsilon a} \omega / \text{Re} \approx \omega |\text{Im}\varepsilon_m(\omega)| / \text{Re} \approx \omega \text{Im} \chi_g / \sqrt{\varepsilon_a}$. Equation (6) is essentially same as the spasing criterion $g_0$ in [24] and the condition for a large quality factor in [25] under the assumption (4). Figure 1(a) shows $g_0$ according to wavelength for silver and gold assuming the dielectric permittivity $\varepsilon_a = 12.25$. The experimental data for the permittivity spectra of silver and gold [26] are directly used. As shown in figure 1(a), silver is better than gold for high-quality plasmon resonance. Moreover, resonance wavelengths near 1.1 $\mu$m for silver are promising if we consider that a gain coefficient greater than 3000 cm$^{-1}$ is technically difficult to realize [24].

We take account of the frequency dependency of the gain-transition susceptibility $\chi_g$. The frequency-dependant gain-transition susceptibility $\chi_g(\omega)$ can be expressed in the two-level approximation [27] by

$$\chi_g(\omega) = g_{\text{gain}} = \frac{\varepsilon_b - \varepsilon_m(\omega)}{\varepsilon_b - \varepsilon_m(\omega) - i(\omega - \omega_0)^2/\Gamma^2},$$  

where $\omega_0$ is the gain-transition line-center, $\Gamma$ is the line-center gain coefficient and $\Gamma$ is the gain-transition line width. For a sufficiently strong electric field [27],

$$g_{\text{gain}} = \frac{\varepsilon_b}{1 + |\text{Re}\varepsilon_m(\omega)|^2} = \frac{\Gamma}{1 + \text{Re}\varepsilon_m(\omega)^2/\Gamma^2} = \frac{\Gamma}{1 + |\text{Im}\varepsilon_m(\omega)|^2/\Gamma^2},$$  

where $E_0$ is the line-center saturation field strength and $g_{\text{gain}} = g_{\text{gain}}^0 N (\phi_0 - \rho_{\text{gain}}) = g_{\text{gain}}^0 \mu_2 |\text{Re}\varepsilon_m(\omega)|^2 (\text{Im}\varepsilon_m(\omega))$ [27] is the line-center gain coefficient for a weak electric field compared to $E_0$.

Figure 1(b) shows gain-assisted improvement of plasmon resonance quality. The quality factor is calculated by equation (1). The embedded metal is silver. For the gain medium, $\varepsilon_a$ is the same as in figure 1(a), the gain-transition line-center in terms of the wavelength $\lambda_{\text{gain}} = 1.1$ $\mu$m, the dipole dephasing rate $\Gamma = 5 \times 10^{13}$ s$^{-1}$ [24] and the line-center gain coefficient $g_{\text{gain}}^0 = 2000$ cm$^{-1}$ corresponding to $g_0(\lambda_{\text{gain}})$. It is assumed that the electric field is sufficiently weak and $\Gamma = \Gamma_{\text{gain}}$. As
expected, in the narrow spectral range around 1.1 \( \mu \)m a peak of quality factor appears.

We choose a simple two-dimensional rectangular nanostructure with dimensions of 70 nm and 80 nm filled with dielectric with the permittivity \( \varepsilon_d = 12.25 \) embedded in silver (shown in the insert of figure 2) as a plasmonic resonator with resonance wavelength \( \lambda_{\text{res}} = 1.1 \) \( \mu \)m. When solving directly the Maxwell equation in the frequency domain as an eigenfrequency problem taking account of retardation effects, the eigenfrequency is \( \omega_{\text{res}} - i\gamma \), where \( \omega_{\text{res}} \) and \( \gamma \) means a resonance frequency and a relaxation rate due to the loss in metal, respectively. For the above nanostructure, the numerical calculation shows \( \omega_{\text{res}} = 1.73 \times 10^{15} \) s\(^{-1} \) (\( \lambda_{\text{res}} = 1.09 \mu \)m), \( \gamma = 9.8 \times 10^{12} \) s\(^{-1} \) and \( Q = \omega_{\text{res}}/2|\gamma| = 88 \). The quality factor \( Q \) of the plasmon resonance can be increased by introducing a gain material to the dielectric rectangle as shown in figure 2. For the gain material, \( \lambda_{21} = \lambda_{\text{res}} \) and \( \Gamma_{12} \) is the same as in figure 1(b). The line-center gain coefficient \( g_{21} = 1080 \) cm\(^{-1} \) for the maximum \( Q \) and the zero \( \gamma \), where the metal loss is exactly compensated, is significantly smaller than the \( g_0(\lambda_{\text{res}}) = 2080 \) cm\(^{-1} \) expected in figure 1(a). This discrepancy is attributed to retardation effects which are ignored in the quasistatic approximation in the derivation of equation (1). Near the maximum \( Q \), a sign of relaxation rate \( \gamma \) is changed. For a positive relaxation rate \( \gamma \) the metal loss relaxes the local field to a stationary state, while the negative sign mathematically means that the local field is exponentially enhanced for \( t \to +\infty \) by a gain overcompensating the metal loss, which is physically impossible. In fact, equation (8) shows a sufficiently enhanced field decreases the gain coefficient so that the local field goes to a stationary state with a great quality factor \( Q \). Thus, for a line-center gain coefficient \( g_{21} \) greater than the threshold value of of 1080 cm\(^{-1} \), the rectangular nanoresonator can act as a two-dimensional spaser.

3. Amplification and suppression in nanowaveguide coupled to gain-assisted high-quality plasmon resonances

We calculate transmission \( T \) and reflection \( R \) coefficients in a metal-insulator-metal (MIM) nanowaveguide side coupled to the above gain-assisted plasmonic resonator (shown in figure 3(a)) by numerically solving the Maxwell equation in a frequency domain. Our calculations agree well with the coupled-mode theory [22, 28].

In [22] a quite similar structure was studied mainly focusing on the range between the high-loss state without pumping and the exact metal-loss-compensation state with pumping. Figures 3(b)–(d) shows essentially the same facts as in [22] that the metal-loss-free state (shown in figure 3(b)) neglecting the imaginary part of the permittivity of silver can be reproduced for a certain gain coefficient (shown in figure 3(d)), while figure 3(c) shows the full-loss state without gain. The \( g_{21} = 1100 \) cm\(^{-1} \) (shown in figure 3(d)) almost coincides with

![Figure 3](https://example.com/image.png)
the $g_{21} = 1080$ cm$^{-1}$ for the exact metal-loss-compensation (shown in figure 2). The difference between these two gain coefficients is attributed to a metal loss in the MIM waveguide. Whereas the rectangular plasmonic resonator with a $g_{21}$ larger than the exactly metal-loss-compensating one has no physically stable state as shown in figure 2, the resonator with such larger $g_{21}$ coupled to the waveguide has stable states as shown in figure 4, where an excess energy produced by overwhelming gain can leak from the resonator through the waveguide. For $g_{21} = 1190$ cm$^{-1}$, broadband full transmission $T = 1$ and on-resonance amplified reflection $R(\lambda_{\text{res}}) = 4$ can be seen in figure 4(a). A further larger gain coefficient can give rise to a high-quality amplification of both transmission and reflection as shown in figure 4(b).

Transition between the suppression (figure 3(d)) with the zero of on-resonance $T$ and the high-quality amplification (figure 4(b)) with a large on-resonance $T$ can be induced by a change of gain coefficient as shown in figure 5(a). The interval between the gain coefficients for the high-quality amplification and the suppression rapidly decreases with the increase of the distance $d$ between the resonator and the waveguide and can be quite small for a moderately increased $d$. The longer distance $d$ results in a weaker coupling between the waveguide and the resonator and thus the less leakage from the resonator which can be compensated by the smaller increment of gain coefficient. Here, it is noted that the position of gain coefficient for the suppression is independent on the distance $d$. The transition can be also induced by changing the normalized detuning of the gain-transition line-center $\delta = (\omega_{21} - \omega_{\text{res}})/\Gamma_{12}$ as can be seen in figure 5(b).

4. Conclusions

The transmission in the nano waveguide coupled to high-quality plasmon resonances for which the metal loss is overcompensated by gain can vary in a wide range from lower than $-20$ dB to higher than 20 dB for a range of gain coefficients, the width of which can be quite small for a moderately increased distance between the waveguide and the resonator. We expect that in practice a small change of gain coefficient can be made by flexibly controlling pumping rate or utilizing nonlinear gain. The widely variable on-resonance transmission is promising for applications such as plasmonic switching and lumped amplification of surface plasmon-polariton.
References

[1] Maier S A 2007 Plasmonics: Fundamentals and Applications (New York: Springer)
[2] Barnes W L, Dereux A and Ebbesen T W 2003 Surface plasmon subwavelength optics Nature 424 824–30
[3] Gramotnev D K and Bozhevolnyi S I 2010 Plasmonics beyond the diffraction limit Nat. Photon. 4 83–91
[4] Stiles P L, Dieringer D J, Shah N C and Van Duyne R P 2008 Surface-enhanced Raman spectroscopy Ann. Rev. Anal. Chem. 1 601–26
[5] Kawata S, Inouye Y and Verma P 2009 Plasmonics for near-field nano-imaging and superlensing Nat. Photon. 3 388–94
[6] Anker J N 2008 Biosensing with plasmonic nanosensors Nat. Mater. 7 442–53
[7] Homola J 2008 Surface plasmon resonance sensors for detection of chemical and biological species Chem. Rev. 108 462–93
[8] Ebbesen T W, Genet C and Bozhevolnyi S I 2008 Surface plasmon circuitry Phys. Today 61 44–50
[9] Lal S, Clare S E and Halas N J 2008 Nanoshell-enabled photothermal cancer therapy: impending clinical impact Acc. Chem. Res. 41 1842–51
[10] Huang X H, Neretina S and El-Sayed M A 2009 Gold nanorods: from synthesis and properties to biological and biomedical applications Adv. Mater. 21 4880–910
[11] Atwater H A and Polman A 2010 Plasmonics for improved photovoltaic devices Nat. Mater. 9 205–13
[12] Bergman D J and Stockman M I 2003 Surface plasmon amplification by stimulated emission of radiation: quantum generation of coherent surface plasmons in nanosystems Phys. Rev. Lett. 90 027402
[13] Stockman M I 2011 Spaser action, loss compensation and stability in plasmonic systems with gain Phys. Rev. Lett. 106 156802
[14] Oulton R F 2009 Plasmon lasers at deep subwavelength scale Nature 461 629–32
[15] Hill M T 2007 Lasing in metallic-coated nanocavities Nat. Photon. 1 589–94
[16] Noginov M A 2009 Demonstration of a spaser-based nanolaser Nature 460 1110–3
[17] Seidel J, Grafstrom S and Eng L 2005 Stimulated emission of surface plasmons at the interface between a silver film and an optically pumped dye solution Phys. Rev. Lett. 94 177401
[18] van Exter M P, Tenner V T, van Beijnum F, de Dood M J A, van Veldhoven P J, Geluk E J and ’t Hooft G W 2013 Surface plasmon dispersion in metal hole array lasers Opt. Express 21 27422–37
[19] Noginov M A 2008 Stimulated emission of surface plasmon polaritons Phys. Rev. Lett. 101 226806
[20] De Leon I and Berini P 2010 Amplification of long-range surface plasmons by a dipolar gain medium Nat. Photon. 4 382–7
[21] De Leon I and Berini P 2011 Measuring gain and noise in active long-range surface plasmon-polariton waveguides Rev. Sci. Instrum. 82 033107
[22] Yu Z, Veronis G, Fan S and Brongersma M L 2008 Gain-induced switching in metal-dielectric-metal plasmonic waveguides Appl. Phys. Lett. 92 041117
[23] Bergman D J and Stroud D 1992 Properties of Macroscopically Inhomogeneous Media Solid State Physics vol 46 ed H Ehrenreich and D Turnbull (San Diego: Academic) pp 148–270
[24] Stockman M I 2011 Nanoplasmonics: past, present and glimpse into future Opt. Express 19 22029–106
[25] Wang F and Shen Y R 2006 General properties of local plasmons in metal nanostructures Phys. Rev. Lett. 97 206806
[26] Johnson P B and Christy R W 1972 Optical constants of noble metals Phys. Rev. B 6 4370–9
[27] Boyd R W 2008 Nonlinear Optics 3rd edn (San Diego: Pergamon)
[28] Haus H A and Lai Y 1992 Theory of cascaded quarter wave shifted distributed feedback resonators IEEE J. Quantum. Electron. 28 205