Optimum design of a small wind turbine blade for maximum power production

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Abstract. The rotor blades of a wind turbine are critical components that consist of aerodynamic shapes called airfoils. The interaction of these airfoils with the wind allows converting the power in the wind to mechanical power. The structure of the rotor blades must be able to extract maximum energy from the wind while being strong enough to stand steady, periodic and randomly changing loads. For maximum power extraction, an optimum design of the rotor blades is necessary. This paper presents a typical design methodology of the rotor blades of a small wind turbine with a power generation of 11 kW (rotor radius of 3.5 m). First, the design parameters were presented. Then, optimum blade geometry (chord length and twist angle distribution) was determined using optimum rotor theory. Finally, the wind turbine blade performance (power coefficient and power production) was assessed using Q-blade software.

1. Introduction

The rotor blades are the most important components of a wind turbine system. The rotor assembly consists of several blades (usually three blades) that are joined to a common hub, a nose cone and fasteners. Generally, the blades are designed to capture maximum energy from wind and convert it into rotating mechanical energy. To increase the reliability of a wind turbine system, two fundamental parameters must be considered: the aerodynamic performance of the rotor and the structural response of the blades under normal/extreme wind conditions. For example, after the overall shape of the blade is aerodynamically optimized, it is common that the blade root must be redesigned to meet structural requirements. Aerodynamically advantageous features such as nonlinear distribution of the chord length and twist angle are often difficult to build. For this reason, some compromises must be considered so that the blade design can be manufactured with minimum investment cost (low cost).

The efficiency of the turbine system is strongly dependent on the aerodynamic performance of the rotor blades. The section view of these components is basically defined by an airfoil shape (one or more airfoils) that is designed to generate high lift forces and low drag forces. In designing a small wind turbine blade for maximum power extraction, several design methodologies have been used in the literature. Dias do Rio Vaz et al. [1] have presented a model to optimize the distribution of the chord and the twist angle of Horizontal Axis Wind Turbine (HAWT) blades, taking into account the
influence of the wake, and using a Rankine vortex. Wang L. et al. [2] have performed a practical methodology for an optimum blade design of Fixed Pitch Fixed Speed (FPFS) small wind turbines.

The optimum blade design is based on the aerodynamic characteristics of the airfoil which are the lift and drag coefficients, and the annual mean wind speed. The design parameters for this blade optimisation include design wind speed, design tip speed ratio, and design attack angle. A detailed review of the current state-of-art for wind turbine blade design has been presented by P.J. Schubel et al. [3]. This review provides a complete description of the wind turbine blade design methodologies of modern HAWTs. M. S. Campobasso et al. [4] have conducted an optimization strategy for the aerodynamic design of horizontal axis wind turbine rotors by including the variability of the annual energy production. In this study, the ultimate objective is to develop a conceptual design of a small wind turbine blade with an electrical generation of 11 kW. This is a direct drive, Fixed Pitch Variable Speed (FPVS) wind turbine for remote communities (to cover the individual electricity needs of a typical isolated household).

2. Rotor design parameters

The chord and the twist angle distributions in each blade section are strongly dependent on both the blade airfoil and the operating design parameters. Some of these key parameters are the design wind speed, the operating range of Reynolds numbers (low Reynolds numbers for small wind turbines), the design angle of attack, and the blade tip speed ratio. Once these parameters have been defined, the optimum rotor theory can be used to determine the optimum blade shape.

2.1. Design wind speed

The design wind speed is defined as the wind speed at which the wind turbine rotor spins with a maximum power coefficient [5]. According to the IEC61400-2 standard, the design wind speed, \( V_{\text{design}} \), should be 1.4 times the Annual Mean Wind Speed, \( V_{\text{AMWS}} \) [6]. In the present study, a wind turbine class III was selected for the wind turbine design. This corresponds to a \( V_{\text{AMWS}} \) of 7.5 m/s. Therefore the design wind speed can be easily calculated using the following equation [7]:

\[
V_{\text{design}} = 1.4 \times V_{\text{AMWS}} = 10.5 \text{ m/s} \tag{1}
\]

Given that the wind turbine is designed to operate at its maximum power coefficient \( C_p \), from cut-in speed to rated wind speed, the design wind speed is selected to be the same as the rated wind speed.

2.2. Design Reynolds number

The Reynolds number is defined as the ratio of the inertial force to the viscous forces of the air. This can be determined using the following equation [8]:

\[
R_e = \frac{U_{\text{rel}} \cdot C}{\mu} \tag{2}
\]

Where \( \mu \) is the kinematic viscosity of the air, \( U_{\text{rel}} \) is the relative wind velocity to the airfoil, and \( C \) is the chord length of the airfoil. By using a chord length of 0.25 m (an average value from the literature) at the middle section of an 11 kW wind turbine blade, the design Reynolds number can be obtained from the relative wind velocity [9]. For sufficiently high tip speed ratios, the relative wind speed can be approximated using the following formula [7]:

\[
U_{\text{rel}} = \lambda_{0.5} \cdot V_{\text{design}} \tag{3}
\]

The tip speed ratio of the middle blade section, \( \lambda_{0.5} \), can be determined using equation (4).

\[
\lambda_{0.5} = \frac{r}{R} \tag{4}
\]
Where \( r \) is the blade radius along the radial direction and \( R \) is the rotor radius which is taken as 3.5 m. As the majority of small wind turbines are usually designed to achieve maximum efficiency at tip speed ratios between 6 to 8 [10], the corresponding Reynolds numbers are shown in Table 1.

| Tip speed ratio | \( C \) (m) | \( U_{rel} \) (m/s) | \( R_e \) |
|-----------------|-------------|------------------|---------|
| 6               | 0.25        | 31.5             | 525 000 |
| 7               | 0.25        | 36.75            | 612 500 |
| 8               | 0.25        | 42               | 700 000 |

### 2.3. Airfoil selection
The airfoil selection is a critical step of the design process of a small wind turbine blade. The aerodynamic characteristics associated with low Reynolds number regime can severely degrade the performance of the blade if the selected airfoil is not suitable for low Reynolds number applications [11]. Most of the unusual aerodynamic characteristics at low Reynolds numbers can be justified by the presence of a laminar separation bubble on the airfoil. Several airfoils have been used in the small wind energy industry. These include the SG604x airfoil family that has been developed specifically for small variable-speed wind turbines sized from 1 to 5 kW [12], the S series airfoils (especially the S823 and the S822) that have been designed for small stall-regulated wind turbines ranging from 2 to 20 kW [13], the SD7062 airfoil which started its life as an aerofoil for model gliders, and the DU series airfoils (Delft University of Technology) which are popular in middle and high Reynolds wind turbine blades [14].

For the selection process of an airfoil, there is no single consistent database or airfoil selection method in the literature. For the present study, the selection process of the airfoil was conducted based on following criteria: the airfoil should be thick enough to provide a good reserve of strength in bending while maintaining good lift and drag characteristics. The DU 93-W-210 airfoil was selected. The DU 93-W-210 airfoil (thickness of 21%) is a thick airfoil providing more options for strengthening the blade structure. It has good lift to drag coefficient, at typical design Reynolds numbers for small wind turbines.

### 2.4. Design tip speed ratio
Blade tip speed ratio is a critical parameter to be considered in a blade design procedure. This parameter is chosen between 6 and 8, for modern wind turbine systems. The optimum value remains uncertain for different airfoil shapes and blade numbers. For an initial selection of the design tip speed ratio, an empirical relation between the power coefficient and the tip speed ratio was used [15]. This gives the maximum power coefficient of a wind turbine rotor with a finite number of blades.

\[
C_{p_{max}} = \frac{16}{27} \lambda \left[ \lambda + \frac{1.32 + \left( \frac{\lambda - 8}{20} \right)^2}{B^2} \right]^{-1} - \frac{0.57 \lambda^2}{C_{d_{max}}} \left( \lambda + \frac{1}{2B} \right)
\]  

(5)

This multivariable relationship can be summarized to a high degree of accuracy of 0.5% for tip speed ratios from 4 to 20, lift to drag ratios (C\(C_d\)) from 25 to infinity, and from one to three blades (parameter B). Based on XFOIL solver, the maximum lift to drag coefficients for the selected DU93-W-210 airfoil is 93.76. This maximum value occurs at an attack angle of 5°. The plot of the power coefficients against different tip speed ratios is given in Figure 1.
High tip speed ratios for small wind turbines presents many drawbacks such as noise generation in the audible and non-audible ranges, excessive rotations of the rotor, and problems of vibration. As the maximum efficiency does not vary sharply between 6 and 10, a value of 6 was selected as the design tip speed ratio in this study.

2.5. Design angle of attack
The design angle of attack is often selected at the critical angle of attack where the lift to drag ratio is maximum. However, the design angle of attack should be selected to be slightly lower than the optimum angle of attack (which gives the maximum lift to drag coefficient). This practice allows the blade to operate at angles of attack, near the optimum value while avoiding the airfoil to come rapidly into the stall regime when the wind speed increases. For the selected DU 93-W-210 airfoil, the plot of the lift and the drag coefficients against different angles of attack is shown in Figure 2.

The maximum lift-to-drag ratio occurs at an angle of attack of 5°. However, the design angle of attack should be selected to be slightly lower than the optimum angle of attack (which gives the maximum lift to drag coefficient). Therefore, the design angle of attack, α_{design}, is chosen to be 4.5°.
3. Rotor blade design

For the aerodynamic design of the turbine blades, the optimum rotor theory (for blade shape design) and the BEM theory (for performance prediction) are widely used [16]. The BEM theory divides the blade into 30 sections from root to the tip and the total power coefficient is calculated by integrating the power coefficients at these sections, as described by J.F. Manwell et al. [17].

\[
C_p = \left(\frac{8}{\lambda}\right) \int_{\lambda_{kn}}^{\lambda} F_t \sin^2 \phi \cdot \cos \phi \cdot (\cos \phi - \lambda_r \sin \phi) \cdot \sin \phi + \lambda_r \cos \phi) \cdot (1 - \left(\frac{C_d}{C_l}\right) \cdot \cot \phi \cdot \lambda_r ^2) \cdot d_x \lambda_r \tag{6}
\]

Where \(F_t\) is the blade tip losses factor. The most straightforward approach to calculate the blade tip losses is the one developed by Prandtl. This tip loss factor is an approved procedure for the correction of profile data to get a better agreement between measured and computed data. Based on Prandtl’s method, the blade tip losses can be modeled using the following equation [18].

\[
F_t = \left(\frac{2}{\pi}\right) \cdot \cos^{-1} \left[ \exp \left( - \left( \frac{B \cdot (1 - \frac{R}{r})}{\frac{R}{r} \sin \phi} \right) \right) \right] \tag{7}
\]

If the main part of the equation (6) is at its maximum, the total power coefficient is maximized.

\[
\text{Max} \left[ F_t \sin^2 \phi \cdot (\cos \phi - \lambda_r \sin \phi) \cdot \sin \phi + \lambda_r \cos \phi) \cdot (1 - \left(\frac{C_d}{C_l}\right) \cdot \cot \phi \cdot \lambda_r ^2) \cdot d_x \lambda_r \right] \tag{8}
\]

Ignoring the drag-to-lift coefficient ratio and setting the partial derivative of the main part to zero, the chord, \(C_i\), and the twist angle, \(\theta_i\), can be obtained by the following equations [19]:

\[
\lambda_{ri} = \lambda_{design} \cdot \left(\frac{r_i}{R}\right) \tag{9}
\]

\[
\phi_i = \left(\frac{2}{3}\right) \cdot \tan^{-1} \left(\frac{1}{\lambda_{ri}}\right) \tag{10}
\]

\[
C_l = \frac{8 \cdot \pi \cdot r_i}{B \cdot C_{design}} (1 - \cos \phi_i) \tag{11}
\]

\[
\theta_i = \phi_i - \alpha_{design} \tag{12}
\]

Where \(i\) indicates the \(i^{th}\) blade section, \(\lambda_{ri}\) is the speed ratio of the \(i^{th}\) blade section, \(r_i\) is the distance from the \(i^{th}\) blade section to the rotor center, \(\phi_i\) is the angle of relative wind at the \(i^{th}\) blade section. \(C_l_{design}\) and \(\alpha_{design}\) are the design lift coefficient and the design angle of attack at the \(i^{th}\) blade section, respectively. Figures 3 and 4 give the chord and twist angle distributions based on equations (9)-(12).
Figure 3. Blade chord length distribution

Figure 4. Blade twist angle distribution

The chord length is larger at the inner sections of the blade (closer to the root) and smaller at the outer sections of the blade. The twists of the sections close to the root are larger than those close to the tip. The chord at the blade root section (0.14 R) is around 0.62 m, and the twist angle varies from 28.43° at the blade root to 1.8° at the blade tip. Figure 5 gives the blade geometry using the optimum rotor theory.

Figure 5. Optimum rotor blade shape
4. Wind turbine rotor performance analysis
The chord and twist distributions obtained from the above equations are just initial design values, and 
an iterative process should be followed normally. The most widely used procedure in this case is 
summarized in the following five steps [17,19]:

1) Estimate initial values of the axial induction factor, $a$, and angular induction factor, $a'$. The first 
   values can be taken from the initial design blade:

   $$\phi_{1,1} = \left(\frac{2}{3}\right)\tan^{-1}\left(\frac{1}{\lambda_{r,1}}\right)$$  \hspace{1cm} (13)

   $$a_{i,1} = \frac{1}{1 + \frac{4 \sin^2 \phi_{1,1}}{\sigma_{i,1} C_{L,design} \cos \phi_{1,1}}}$$  \hspace{1cm} (14)

   $$a'_{i,1} = \frac{1 - 3a_{i,1}}{4a_{i,1} - 1}$$  \hspace{1cm} (15)

   Where $\sigma_{i,1}$ is the local solidity of the $i^{th}$ blade section expressed as:

   $$\sigma_{i,1} = \frac{B_{L,i}}{2\pi r_{i}}$$  \hspace{1cm} (16)

2) Start the iterative process for the $j^{th}$. For the first iteration $j = 1$, the following equations are used:

   $$\tan \phi_{ij} = \frac{U(1 - a_{ij})}{\Omega r_0 (1 + a'_{i,j})} = \frac{1 - a_{ij}}{(1 + a'_{i,j})\lambda_{r,i}}$$  \hspace{1cm} (17)

   $$F_{u,j} = \left(\frac{2}{\pi}\right) \cos^{-1} \left[ \exp \left(-\frac{B}{2} \cdot \left(\frac{1 - R_j}{R_j} \right) \cdot \sin \phi_{ij} \right) \right]$$  \hspace{1cm} (18)

3) Calculate the new angle of attack, $\alpha_{ij}$, and determine the corresponding lift coefficient, $C_{L,ij}$, and 
   the drag coefficient, $C_{D,ij}$ from the polar data of the used airfoil.

   $$\alpha_{ij} = \phi_{ij} - \theta_{ij}$$  \hspace{1cm} (19)

   Where $\theta_{ij}$ is the twist angle of the blade the $i^{th}$ blade section.

4) Calculate the local thrust coefficient by using the following equation:

   $$C_{Th,i,j} = \frac{\sigma_{i,j} (1 - a_{i,1})^2 (C_{L,ij} \cos \phi_{ij} + C_{D,ij} \sin \phi_{ij})}{\sin^2 \phi_{ij}}$$  \hspace{1cm} (20)
5) Update the values of $a$ and $a'$ for the next iteration. If $C_{Th,i} < 0.96$, then:

$$a_{i,j+1} = \frac{1}{1 + \frac{4F_{ui,j} \sin^2 \phi_{i,j}}{\sigma_i C_{Li,j} \cos \phi_{i,j}}}$$  \hspace{1cm} \text{(21)}$$

If $C_{Th,i} > 0.96$, then the axial induction factor is computed using the following equation:

$$a_{i,j+1} = \left( \frac{1}{F_{ui,j}} \right) \left[ 0.143 + \sqrt{0.0203 - 0.6427(0.889 - C_{Th,i})} \right] \hspace{1cm} \text{(22)}$$

$$a'_{i,j+1} = \frac{1}{\frac{4F_{ui,j} \cos \phi_{i,j}}{\sigma_j C_{Li,j}}} - 1 \hspace{1cm} \text{(23)}$$

If the difference between the $j^{\text{th}}$ and $j^{\text{th}}$ iteration is within the acceptable convergence criteria, then the process converges, and the power performance can be computed. The convergence criterion, $\varepsilon$, defines when an iteration has converged. The maximum of the difference of axial and angular induction factor between the last and the current iteration has to be below $\varepsilon$. A typical value that can be used is given by [20]:

$$\max\left( |a_{i,j+1} - a_{i,j}|, |a'_{i,j+1} - a'_{i,j}| \right) < 10^{-5} \hspace{1cm} \text{(24)}$$

In this study, the iterative process in the blade design was conducted using Q-blade software. This design tool is realized being a part of the wind energy group at the Berlin Technical University Department of Experimental Fluid Mechanics. Q-blade is free-of-charge wind turbine calculation software that enables the user to rapidly design the turbine blades by determining the chord and the twist angle distribution. Q-blade software is based on BEM and Double-Multiple Streamtube (DMS) model. These are integrated into an XFOIL graphical user interface. Figures 6 and 7 give the chord length and the twist angle distribution along the blade span using both the optimum rotor theory and Q-blade software.

![Figure 6. Optimized chord length distribution](image-url)
The chord distribution was optimized according to Schmitz method (equation 11). The twist angle distribution was selected to keep the lift to drag ratio at its maximum along the blade span. The evolution of the power coefficient as a function of the blade tip speed ratio is given in figure 8.

The maximum achievable power coefficient is 0.45, which corresponds to a tip speed ratio of 6. The value of $C_p$ was computed for an ideal rotor by taking into account wake rotation, drag effects and tip losses (Prandtl correction factor). Figure 9 shows the power output of the wind turbine rotor using the optimized rotor blades.
From figure 9, three main conclusions can be drawn:
1) The power output from the turbine increases with the wind speed. At a design wind speed of 10.5 m/s, the rotor power generation is 12.3 kW. This high power production can be justified by the fact that the turbine simulation was performed for a rotor blades having a maximum achievable aerodynamic efficiency ($C_p=0.45$).
2) The starting time of the rotor is relatively high (5 m/s). This makes the wind turbine design not suitable for geographical areas where the mean wind speed is very low (the design cut-in wind speed is usually set to 3 m/s). For an optimum design of the rotor blades, starting time should be optimized too.
3) For relatively high wind speeds, the power production increases and exceeds the maximum overloading limit of the generator which is usually set around 120 %. This problem can be solved using an appropriate control strategy of the wind turbine: below the rated wind speed, the control system maximizes the energy production from the rotor, and above the rated wind speed, the control system limits the generated power using both passive stall regulation and a control algorithm (e.g. limitation of the rotational speed of the rotor using a damp load through a PWM modulation).

5. Conclusion
The aim of this paper was to define an optimum blade shape, at a certain design tip speed ratio, for maximum power extraction. The proposed methodology does not take into account other constraints such as the manufacturing considerations, the blade mass and the blade starting performance. The nonlinear distribution of the chord length and the twist angle are often difficult to build. For this reasons, linearization of the blade geometry should be considered so that the blade can be manufactured with minimum cost. The starting capabilities of the blade must also be considered. Improving the starting performance would reduce the amount of time needed to reach the turbine rated power. This would increase the overall energy yield from the turbine. Additionally, the blade mass must be reduced as much as possible while maintaining the required structural requirements. This will significantly reduce the cost of the used blade material. Therefore, determination of an optimum blade shape is a nonlinear exercise that should be solved using multi-objective optimizations such as genetic algorithm.
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