Bearing Life Prediction Based on SPSS and Grey Prediction Model

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Abstract. Many sophisticated weapons and equipment have bearings, which can play a vital role of reducing friction to improve the efficiency of equipment. The performance of CV can quantify the processing of the bearing, it will make the staff have a reference standard to evaluate the working condition of the bearing. Primarily, according to the known data, this paper calculated the real CV value with the maximum likelihood estimation method. Both SPSS curve estimation and the method of cumulative grey prediction model were used to train the part of the real data. Then, the failure moment of bearings failure moment can be simulated. The final step was to calculate the relative error of predicted CV value. Compared with the predicted results of the original grey prediction model (GM (1,1)), the three relative errors were 6.9502 percent, 4.6433 percent and 10.5938 percent. Three methods above all have certain practical value in engineering. However, the effect of cumulative grey prediction is better obviously.

1. Introduction
Most of the weapons and equipment have bearings [1], such as the ear shaft of the launching arm on the missile launch vehicle’s rotates with bearings, and the medium and small size electrical motors adopt bearings. However, the bearings will be damaged and finally lose their function after the failures occurring [2]. In article 3, it takes rolling bearings as an example; it mentions that the defects on the inner and outer ring will cause the metal of contacting layer surface falling off flakily, which will form the bearing surface pits [3]. A variety of contaminants such as viscose and particulate matter caused by the strike of the rolling body on one or both sides of the roller will enter the rolling track between the rolling body and the roller, which will cause wear or bruising, etc. In this paper, it takes maximum likelihood method to have the bearing’s value of CV to quantify the working status of the bearing; it has used three kinds of method to predict the failure time of the bearing and it has made a detailed comparative analysis.

2. Calculation method of bearing’s real confidential value
CV value can be seen as a dichotomous probability [4], so it can take CV (confidential value) as a measurement of whether the bearing is in failure, and CV value is adopted as the indices of the performance of the bearing. CV value can quantify the failure of the rolling bearing, and the value range is 0 to 1. It is noted as 0 when the state of the bearing is complete in failure, and it is noted as 1 when the bearing in in perfect working performance. If the CV value is equal or less than 0.05, it will
note that the bearing has no longer working capacity, the CV value is expressed as the following formula [5]:

\[
CV = P(cv = 1 | x_i) = \frac{1}{1 + e^{-\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k}}
\]  

(1)

In which, \( x \) is a vector of k-dimension, \( \beta_0 \) is the regression intercept, \( \beta_i \) is the regression coefficient. In this paper, it tests the accuracy of the performance of three different methods of prediction with the bearing data from the University of Cincinnati in United States [6]. Firstly, it needs to calculate the CV value throughout the bearing working life cycle, the data of this type of bearing consists of testing data of 984 hours, and the interval of sampling is 10 minutes. Then the CV value is calculated with the maximum likelihood method.

\[
\begin{pmatrix}
X_1 & X_2 & \ldots & X_{984} \\
x_{11} & x_{12} & \ldots & x_{1,984} \\
x_{21} & x_{22} & \ldots & x_{2,984} \\
\vdots & \vdots & \ddots & \vdots \\
x_{20480,1} & x_{20480,2} & \ldots & x_{20480,984} \\
\downarrow & \downarrow & \downarrow & \downarrow \\
cv_1 & cv_2 & \ldots & cv_{984}
\end{pmatrix}
\]  

(2)

\[
\ln L = \sum_{i=1}^{984} \ln \left[ \frac{e^{\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_k x_{ki}}}{1 + e^{\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_k x_{ki}}} \right] + (1 - cv_i) \ln \left[ \frac{1}{e^{\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_k x_{ki}}} \right] 
\]

(3)

If \( \ln L = 0 \), we can get:

\[
\begin{align*}
\frac{\partial \ln L}{\partial \beta_0} &= 0 \\
\vdots \\
\frac{\partial \ln L}{\partial \beta_{984}} &= 0
\end{align*}
\]

(4)

It can solve out \( \beta_0 \) to \( \beta_{984} \) with formula (4), it means that it can get all the 984 CV values of the whole bearing working cycle, as Fig.1.

Figure 1 The real CV value of entire life cycle

Figure 2 SPSS curve estimate forecast results

3. Prediction method of bearing’s real confidential value

3.1 Prediction of remaining working life by SPSS

The software, SPSS, integrates numerous mathematical models to do data analysis, it can use to solve big data problems [7-8]. When doing the prediction of the bearing life that is the prediction of CV
value, it needs to choose the appropriate mathematical model according to the CV value just found for this type of bearing. Because the real CV value of bearing may have non-positive value, that it will have a decreasing trend, and the scatter chart will be a mess, so it is better to take the curve estimation function in regression analysis. After having compared the mathematical models of linear, quadratic curve fitting and cubic curve fitting, it shows that $R(0.781)$ curve fitting is the best method to fit the data, as shown in Table 1, so the cubic curve fitting is selected for prediction.

Table 1. Linear model, quadratic model, cubic curve model parameters

| Equation   | $R$  | Constant | $b_1$  | $b_2$  | $b_3$  |
|------------|------|----------|--------|--------|--------|
| Linear     | 0.524| 0.871    | 0.000  |        |        |
| Quadratic  | 0.748| 0.699    | 0.001  | -1.357E-6|
| Cubic      | 0.781| 0.731    | 0.001  | 1.446E-6|-1.145E-10|

The estimation parameters of the model are $0.731, 0.001, 1.446 \times 10^{-6}, -1.145 \times 10^{-10}$, the fitted curve as Fig.2(up).

With using the above-mentioned model to predict the CV values after time 910, it finds when the CV value decreases to 0.05, the corresponding x-axis value is 979, which means that from the time of 979, the bearing is in failure, and the remaining life $S$ (min) is:

$$s = 10 \times (979 - n_0)$$

In which, $n_0$ is the known detection time.

3.2 Prediction of remaining working life by grey model

With observing Fig.1, it finds that the CV values change trend is not stable, especially from time 700 to time 900, it exists a mutation phenomenon. In purpose to verify the remaining working life of the bearing, it performs the prediction respectively with dividing the working life into three period, and at the last period (901-984) it gets the bearing failure moment.

3.2.1 The principle of GM (1,1) prediction

To predict CV value with adopting GM (1,1) grey prediction model [9], it needs to add the original data $x^0(t)$ to $x^0(n)$, and build a model for $Y^0(n)$:

$$\frac{dx^0(n)}{dn} + ax^0(n) = u \tag{6}$$

Then to solve the grey parameter $\hat{a}$ with least square method:

$$\hat{a} = (B^T B)^{-1} B^T x_s \tag{7}$$

In which, the grey parameter $\hat{a}$ is the matrix of the ratio of $a$ to $u$, the expression of $B$ and $x_s$ are as following:

$$B = \begin{bmatrix} 
\frac{x^0(1) + x^0(2)}{2} & 1 \\
\frac{x^0(2) + x^0(3)}{2} & 1 \\
\vdots & \vdots \\
\frac{x^0(n-1) + x^0(n)}{2} & 1 \\
\end{bmatrix} \tag{8}$$

$$x_s = \begin{bmatrix} 
x^0(2) \\
x^0(3) \\
\vdots \\
x^0(n) \\
\end{bmatrix} \tag{9}$$
After getting $\hat{a}$, then substitute it in the formula, it can get the cumulative result of the $n+1$ items:

$$\hat{x}^{(0)}(n + 1) = |x^{(0)}(1) - \hat{a}f + \hat{a}$$

(10)

Then get the prediction value of $\hat{x}^{(0)}(n + 1)$ with calculating the difference between $\hat{x}^{(0)}(n + 1)$ and $\hat{x}^{(0)}(n)$.

3.2.2 Principle of improved GM (1,1) prediction model with cumulative method

When predicting CV values, it adopts the model of GM (1,1) based on cumulative method, the cumulative method is a method of fitting, it combines with the original GM (1,1) model can improve the performance of the prediction model [10].

Calculating the background values, and $x^{(0)}(i) = \sum_{j=1}^{i} x^{(0)}(j), (i = 2, 3, \ldots, n)$:

$$z^{(0)}(i) = \frac{x^{(0)}(i - 1) + x^{(0)}(i)}{2}$$

(11)

The equation set consists of first-order operator and second-order operator can solve the prediction parameters $a$ and $u$:

$$\sum_{i=2}^{n} x^{(0)}(i) + a \sum_{i=2}^{n} z^{(0)}(i) = u(n - 1)$$

$$\sum_{i=2}^{n}(n - i + 1)x^{(0)}(i) + a \sum_{i=2}^{n}(n - i + 1)z^{(0)}(i) = u \frac{n(n - 1)}{2}$$

(12)

We can get:

$$a = \frac{n(n - 1)}{2} \sum_{i=2}^{n} x^{(0)}(i) - (n - 1) \sum_{i=2}^{n} z^{(0)}(i)$$

$$u = \frac{\sum_{i=2}^{n}(n - i + 1)x^{(0)}(i) + \sum_{i=2}^{n}(n - i + 1)z^{(0)}(i)}{2}$$

(13)

Substitute $a$ and $u$ in grey equation of GM (1,1) model, it can get the CV value prediction formula:

$$\hat{x}^{(0)}(i) = \frac{2(2 - a)^{1/2}(u - ax^{(0)}(i))}{(a + 2)^{1/2}}$$

(14)

4. Conclusion and analysis

4.1. Conclusion of simulation

With observing the bearing CV value characteristic of the whole working life, it takes moment 1 to moment 30 as the training data of first period, the moment 701 to moment 710 as the training data of second period and moment 901 to moment 910 as the training data of third period. It can get the values of $a$ and $u$ in the process of the two prediction methods after section calculating respectively, as in Table.2(a) or 2(b)

| Table 2(a) $A$ value of the two models |
|----------------------------------------|
| $a$ | 1-700 moments | 701-900 moments | 901-984 moments |
| Cumulative | 0.0016 | 0.0027 | 0.0387 |
| Original | 0.0001 | 0.0015 | 0.0213 |

| Table 2(b) $U$ value of the two models |
|----------------------------------------|
| $u$ | 1-700 | 701-900 | 901-984 |

moments & moments & moments \\
\hline
Cumulative & 0.5901 & 0.4852 & 0.2317 \\
Original & 0.7946 & 0.6564 & 0.2508 \\
\hline

Then, it can write two kinds of prediction equation and make a graphic according to the values of \(a\) and \(u\). The matlab simulation comparison figure is as Fig.3.

When \(CV = 0.05\), the moment predicted with cumulative GM (1,1) model corresponds to 960, the one predicted with original GM (1,1) model corresponds to 949, so the remaining working life of the bearing \(S\) (min) is:

\[ s = 10 \times (x_{CV=0.05} - n_0) \tag{15} \]

In which, \(x_{CV=0.05}\) is the x-axis coordinate when \(CV\) is 0.05 (it means the moment of failure).

\[ q^n(i) = \frac{x^n(i) - \hat{x}^n(i)}{x^n(i)} \tag{16} \]

As in Fig.4, the mean value of the relative error is 6.9502\% predicted by SPSS, 10.5938\% with GM (1,1) prediction model, 4.6433\% with cumulative method improved GM (1,1) prediction model.
From the simulation results, it shows that the mean value of relative error of cumulative method improved GM (1,1) prediction model is the smallest, less than 5 percent, even the largest mean value of relative error obtained by original GM (1,1) model is only 10 percent. Therefore, these three methods will have a good practice in the maintenance and troubleshooting of weapons and equipment.

5. Conclusion
In this paper, it takes the bearing fault data of University of Cincinnati to do SPSS curve prediction and improved prediction model GM (1,1) based on cumulative method, and it compares the results with the results obtained with original GM (1,1) prediction model in article 8, it calculates the relative error with the three methods. Finally, it shows that SPSS prediction and cumulative grey prediction are better organized. The two methods (SPSS and cumulative grey prediction) are more practical in the maintenance of weapons and rotary machine, it will facilitate the actual operation of staff.

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