ABSTRACT

Combining heavy quark effective theory and the chiral Lagrangian approach we investigate D meson radiative decays. First, we reanalyse $D^* \to D\gamma$ decays using heavy quark spin symmetry, chiral symmetry Lagrangian, but including also the light vector mesons. Then, we calculate branching ratios $D \to V\gamma$. We make some comments on the Cabbibo suppressed decays $D \to \rho/\omega\gamma$ and the branching ratios of $D \to PP\gamma$ decays.
Experimentally radiative decays of $D$ mesons have not been measured, while the known branching ratios of $D^*$ radiative decays [1, 2] can be described using the combination of heavy quark effective theory and chiral Lagrangians [3]-[15].

The strong interaction meson Lagrangian for the light pseudoscalar octet and heavy pseudoscalar and vector triplets in the chiral and heavy quark limits was first written down by Wise [16] (see also [4]). The electromagnetic interactions between these mesons was described in [1, 2, 3]. The octet of light vector mesons was included in the Wise Lagrangian [16] later by Casalbuoni, et al. [12] as the gauge particles associated with the hidden symmetry group SU(3)$_H$ [17]. The light pseudoscalar mesons are described by the $3 \times 3$ unitary matrix $u = \exp(i \Pi/f)$, ($f \simeq 132$ MeV) with $\Pi$ the usual pseudoscalar matrix as in ref. [14], while the octet of light vector mesons is described by the $3 \times 3$ matrix $\hat{\rho}{}_{\mu} = i \frac{g_V}{\sqrt{2}} \rho{}_{\mu}$ where $g_V$ ($\simeq 5.8 \sqrt{2/a}$ with $a = 2$ in the case of exact vector dominance) is the coupling constant of the vector meson self–interaction [17] and $\rho{}_{\mu}$ the vector meson matrix. The heavy mesons are $Q q^a$ ground states, where $Q$ is a $c$ quark and $q^1 = u$, $q^2 = d$ and $q^3 = s$. In the heavy quark limit they are described by $4 \times 4$ matrix $H_a$ ($a = 1, 2, 3$) [16]

$$H_a = \frac{1}{2} (1 + \slashed{\gamma})(P_{a\mu}^* \gamma^\mu - P_a \gamma_5)$$

where $P_{a\mu}^*$ and $P_a$ annihilate, respectively, a spin-one and spin-zero meson $Q q^a$ of velocity $v_\mu$. Following the procedure of [14] we write down the even parity strong and electromagnetic Lagrangian for heavy and light pseudoscalar and vector mesons:
\[
\mathcal{L}_{\text{even}} = \mathcal{L}_{\text{light}} - \frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + i Tr(H_\alpha v_\mu D^\mu \bar{H}_a) \\
+ \ ig Tr[H_b \gamma_\mu \gamma_5 (A^\mu)_{ba} \bar{H}_a] + i \beta Tr[H_b v_\mu (\gamma_\mu - \hat{\rho}_\mu)_{ba} \bar{H}_a] \\
+ \frac{\beta^2}{2 f^2 a} Tr(\bar{H}_b H_a \bar{H}_a H_b)
\]

with

\[
\mathcal{L}_{\text{light}} = -\frac{f^2}{2} \{ tr(A_\mu A^\mu) + a \ tr[(V_\mu - \hat{\rho}_\mu)^2] \} \\
+ \frac{1}{2g^2} tr[F_{\mu\nu}(\hat{\rho}) F^{\mu\nu}(\hat{\rho})]
\]

where \( B_\mu, A_\mu, V_\mu, D_\mu \) are defined in ref. [14]. In equation (2) \( g \) and \( \beta \) are constants which should be determined from experimental data [1, 2, 11, 12, 13]. The constant \( a \) in (2)-(3) is in principle a free parameter, but we shall fix it by assuming exact vector dominance [17], for which \( a = 2 \). The electromagnetic field can couple to the mesons also through the anomalous interaction; i.e., through the odd parity Lagrangian. We write down the two contributions of the odd parity Lagrangian which are significant for our calculation [18, 19]

\[
\mathcal{L}_{\text{odd}}^{(1)} = -4 C_{VV\Pi} \frac{f}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} Tr(\partial_\mu \rho_\nu \partial_\alpha \rho_\beta \Pi)
\]

\[
\mathcal{L}_{\text{odd}}^{(2)} = -4 e \sqrt{2} C_{V\pi\gamma} \frac{f}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} Tr(\{ \partial_\mu \rho_\nu, \Pi \} Q \partial_\alpha B_\beta)
\]

Equation (3) and equation (4), together with vector dominance couplings
\[ \mathcal{L}_{V-\gamma} = -m_V^2 \frac{e}{g_V} B_\mu (\rho_\mu^0 + \frac{1}{3} \omega_\mu - \frac{\sqrt{2}}{3} \Phi_\mu) \] (6)

which come from the second term in (3), describe the anomalous type electromagnetic interactions in the light sector. The interactions of light vector mesons (and of photons via vector-dominance (3)), heavy pseudoscalars or heavy vector $D$ mesons is introduced through the higher dimensional invariant operator

\[ \mathcal{L}_1 = i \lambda T [H_a \sigma_{\mu\nu} F^{\mu\nu}(\hat{\rho})_{ab} \bar{H}_b] \] (7)

The heavy quark-photon interaction is generated by the term

\[ \mathcal{L}_2 = -\lambda' e T [H_a \sigma_{\mu\nu} F^{\mu\nu}(B) \bar{H}_a] \] (8)

According to quark models the parameter $|\lambda'|$ can be approximately related to the charm quark magnetic moment via $1/(6 m_c)$ [1-4]. The experimentally measured [20, 21] branching fractions are $R_0^{\gamma} = \Gamma(D^{*0} \to D^0 \gamma)/\Gamma(D^{*0} \to D^0 \pi^0) = 0.572 \pm 0.057 \pm 0.081$. and $R_0^{+} = \Gamma(D^{*+} \to D^+ \gamma)/\Gamma(D^{*+} \to D^+ \pi^0) = 0.035 \pm 0.047 \pm 0.052$. With our Lagrangians these branching ratios are calculated to be

\[ R_0^{\gamma} = 64 \pi f^2 \alpha_{EM} \left( \frac{\lambda'}{g} + \frac{2}{3} \frac{\lambda}{g} \right)^2 \left( \frac{p^0_\gamma}{p^0_\pi} \right)^3 \] (9)

\[ R_0^{+} = 64 \pi f^2 \alpha_{EM} \left( \frac{\lambda'}{g} - \frac{1}{3} \frac{\lambda}{g} \right)^2 \left( \frac{p^+_\gamma}{p^+_\pi} \right)^3 \] (10)
From the experimental data, we have found\[14\]

\[
\left| \frac{\lambda'}{g} + \frac{2}{3} \frac{\lambda}{g} \right| = (0.863 \pm 0.075) \text{GeV}^{-1}
\]

and

\[
\left| \frac{\lambda'}{g} - \frac{1}{3} \frac{\lambda}{g} \right| = (0.089 \pm 0.178) \text{GeV}^{-1}
\]

In our approach both $\lambda$ and $\lambda'$ are considered as purely phenomenological.

The part of the weak Lagrangian for the pseudoscalar and vector, light and heavy mesons, which we will use, can be written as \[22, 23\]

\[
L_{W}^{eff}(\Delta c = \Delta s = 1) = -\frac{G}{\sqrt{2}} V_{ud} V_{cs}^* \left[ a_1(\bar{u}d)^{\mu}_{V-A}(\bar{s}c)_{V-A,\mu} + a_2(\bar{s}d)^{\mu}_{V-A}(\bar{u}c)_{V-A,\mu} \right] \tag{13}
\]

where $V_{ud}$, etc. are the relevant CKM mixing parameters, while $a_1$ and $a_2$ are the QCD Wilson coefficients, which depend on a scale $\mu$. One expects the scale to be the heavy quark mass and we take $\mu \simeq 1.5 \text{ GeV}$ which gives $a_1 = 1.2$ and $a_2 = -0.5$, with an approximate 20% error. Many heavy meson weak nonleptonic amplitudes \[22, 23, 24\] have been calculated using the factorization approximation. The quark currents are approximated by the corresponding meson currents \[25\] defined later in eqs. (18) and (17):

\[
(\bar{q}_a Q)_V^{\mu} \equiv \bar{q}_a \gamma^{\mu} (1 - \gamma^5) Q \simeq J_{q_a}^{\mu} \tag{14}
\]

\[
(\bar{q}_a q_a)_V^{\mu} \equiv \bar{q}_a \gamma^{\mu} (1 - \gamma^5) q_a \simeq J_{q_{ab}}^{\mu} \tag{15}
\]
In order to describe weak currents we use the "bosonised" versions like [14, 12, 16]. For the quark current with one heavy quark we will use

\[ J_{Q_i}^\mu = \frac{1}{2} i f_H \sqrt{m_H} Tr[\gamma^\mu (1 - \gamma_5) H_i u_{b_i}^\dagger] \]  \hspace{1cm} (16)

The light meson part of the weak current is

\[ J_{q_i}^\mu = i f^2 u [A^\mu + a(V^\mu - \bar{\rho}^\mu)] u^\dagger \]  \hspace{1cm} (17)

The simplest radiative decays of D mesons are into a light meson and a photon. Since the process \( D \rightarrow P \gamma \) (\( P \) is a light pseudoscalar) is forbidden due to the requirement of gauge invariance and chiral symmetry [26], as well as angular momentum conservation, we will concentrate on the \( D \rightarrow V \gamma \) (\( V \) is a light vector meson) decays. We consider the only two processes which are possible at tree level and are not Cabibbo supressed, namely \( D^0 \rightarrow \bar{K}^{*0} \gamma \) and \( D^{s+} \rightarrow \rho^+ \gamma \). Both processes have contributions from the odd-parity interaction Lagrangian. The second one has, in addition, a direct emission term, due to the charged initial and final mesons.

In our numerical calculations we used the following numerical values \( C_{VVV} = 0.423, C_{VII} = -3.26 \times 10^{-2}, g_V = 5.8 \) [12], \( f \approx f_\pi = 132 \text{ MeV} \), and the other decay constants \( f_{P,V} \) were taken from [24]. It is straightforward to calculate the decay widths. The result, of course, depends on which numerical value we take for \( (\lambda' + 2\lambda/3) \) and \( (\lambda' - \lambda/3) \).

Computing these decay widths using \( \lambda \) and \( \lambda' \) we have to point out that the \( \frac{1}{m_c} \) corrections, coming from light-quark current, effectively included into the \( \lambda' \) parameter, are not necessarily the same as in the case \( D^* \rightarrow D \gamma \)
decay. Of course, this uncertainty unfortunately increases the theoretical and experimental uncertainties already present in the calculation of the $D \to V\gamma$.

On figs. 1 and 2 the decay widths for $D^0 \to K^{*0}\gamma$ and $D^{*+} \to \rho^+\gamma$ are shown as functions of the combinations $\lambda' + 2\lambda/3$ and $\lambda' - \lambda/3$ respectively. The full (dashed) lines denotes the values for these combinations, which are allowed (forbidden) by experimental constraints (11) – (12) together with $|g| = 0.57 \pm 0.13$.  

An interesting feature can be seen from fig. 1: a not very precise measurement of the $D^0 \to K^{*0}\gamma$ decay width is sufficient to differentiate between positive or negative solutions for $\lambda' + 2\lambda/3$, which are predicted to be of one order of magnitude different. Given the total $D^0$ decay width $\Gamma_{TOT}(D^0) \approx 0.0016$ eV [20] the branching ratio is constrained by $2 \times 10^{-4} < B < 4 \times 10^{-4}$ for positive and $B < 0.4 \times 10^{-4}$ for negative values of $\lambda' + 2\lambda/3$. Our prediction is consistent with the nonrelativistic quark model [29] and with the order of magnitude estimate of ref. [30], but not with the analysis of Burdman et al. [31], who get a branching ratio somewhere between our allowed regions.

A less dramatic situation is obtained for the decay $D^{*+} \to \rho^+\gamma$ in fig. 2, where a not precise determination of the partial width is not enough to further constrain the combination $\lambda' - \lambda/3$. From $\Gamma_{TOT}(D^{*+}) \approx 0.0014$ eV [20] we see that the branching ratio for this decay is in the range $2 \times 10^{-4} < B < 7 \times 10^{-4}$. The result is similar to ref. [31], but it is one order of magnitude larger than the one in [29].

In ref. [32], [33] it was noticed that a nice bonus can be obtained in measuring the charm meson decay $D^0 \to \rho^0(\omega)\gamma$, which can get contributions from New Physics by $c \to u\gamma$ transitions, while the same contributions are absent
in the $D^0 \to K^{*0}\gamma$ decay. The authors of [33] claim that a discrepancy between the experimental measured ratio of the two decays and the theoretical expectation, i.e. the violation of

$$R_{\rho(\omega)} = \frac{B(D \to \rho(\omega)\gamma)}{B(D \to K^{*}\gamma)} = \frac{\tan \theta_c^2}{2}$$

would be a clear sign of non-Standard Model physics. Equation (18), in which we have included a factor of $1/2$ that was overlooked in refs. [32], [33], can be derived from eq. (13) in the factorization approximation.

In our approach, which we believe to describe effectively the low-energy physics of the Standard Model, relation (18) is correct in two cases:

1) in the U(3) limit, where the masses of the light pseudoscalar nonet are equal, as well as the masses and the decay constants of the light vector nonet;
2) in the case that the diagram with the photon emission by the heavy meson dominates, i.e. for positive values of the experimentally allowed value for the combination $(\lambda' + 2\lambda/3)$ (see eq. (11)), providing that the vector decay constants are approximately equal.

In terms of quark diagrams, there are also penguin contributions, for example, to $D \to \rho/\omega\gamma$, but not to $D \to K^{*}\gamma$. In the following we will neglect them, since they were found to be rather small [31].

Our conclusion is, that the eq. (18) may not be satisfied even in the Standard Model, as is shown on fig. 3: if the combination $(\lambda' + 2\lambda/3)$ turns out to be negative, the ratio (18) can be anything between 0 and $\infty$. As shown in the case $D \to K^{*}\gamma$, the negative values of $(\lambda' + 2\lambda/3)$ can cause a destructive interference between the photon emission by the heavy and the light meson.

A similar effect is possible also in the decay $D \to \rho(\omega)\gamma$, only that the
zero can be achieved at a different value of $(\lambda' + 2\lambda/3)$, because of the U(3) breaking. It is clear that such a situation does not allow us to conclude anything about some New Physics. Actually the situation would be in this case even worse, because negative values of $(\lambda' + 2\lambda/3)$ would mean a very small branching ratio (see fig. 1) and so an extremely difficult measurement. If, on the other side, $(\lambda' + 2\lambda/3)$ turns out to be positive, the decays are much easier to detect experimentally, and also the theoretical situation is clearer, since the curve is approaching the ideal theoretical value 1 (fig. 3). A large disagreement with the theoretical prediction (18) would give in this case some sign of New Physics. But even here one should take care, since the amplitudes are approximately proportional to the decay constants of the final vector meson. This can be seen, if we calculate the decay $D^0 \rightarrow \omega \gamma$ with the values of the light vector decay constants taken from [24]: $f_{K^*} = f_{\rho} = 221$ MeV and $f_{\omega} = 156$ MeV. In this case we get a similar curve as in fig. 3 for $D^0 \rightarrow \rho^0 \gamma$, but for large positive values of $(\lambda' + 2\lambda/3)$ the ratio is approaching a value of approximately 0.5 instead of 1. The fact can be explained by the difference in the decay constants, i.e. $(f_{\omega}/f_{K^*})^2 \approx 0.5$.

The radiative decays $D \rightarrow PP\gamma$ have not been measured yet. In ref. [32] it was pointed out that branching ratios of D mesons are determined with 5 – 10% accuracy. We analyse the branching ratios of $D \rightarrow PP\gamma$ decays in order to see if they are measurable. As an example we investigate the decay $D \rightarrow K^-\pi^+\gamma$. In this decay there are charged particles in the final state and therefore we expect that the dominant contribution will come out from the "bremsstrahlung". We simply take the amplitude $|A(D \rightarrow K^-\pi^+)|$ from the corresponding decay and write
\[
A(D(P) \to K^-(k)\pi^+(p)\gamma(q)) = -eA(D \to K^-\pi^+)(\frac{\epsilon \cdot k}{k \cdot q} - \frac{\epsilon \cdot p}{p \cdot q}) \tag{19}
\]

The integration over the phase space is done making the cut in the photon energy \(\omega_{\text{min}}\). The result is presented on fig. 4. For the photon energy cut \(\omega_{\text{min}} = 10\) MeV we find \(B(D \to K^-\pi^+\gamma) = 1.6 \times 10^{-3}\), higher than the calculated \(B(D^0 \to \bar{K}^{*0}\gamma)\). Contrary to the bremsstrahlung in \(D^0 \to K^-\pi^+\gamma\), there are only neutral pseudoscalar mesons in the final state of the decay \(D \to \bar{K}^0\pi^0\gamma\), and therefore the decay amplitude is the so-called direct emission \([34]\). Usually the branching ratio is much smaller in decays with direct emission than in decays where the bremsstrahlung dominates \([34]\).

We can conclude that our framework - the combination of heavy quark symmetry and chiral symmetry - builds the effective strong, weak and electromagnetic Lagrangian. Within this scheme the calculated \(D \to V\gamma\) decay widths provide some guidance. One should be careful in experimental searches, since the bremsstrahlung for \(D^0 \to K^-\pi^+\gamma\) leads to higher branching ratio than for \(D^0 \to \bar{K}^{*0}\gamma\). The hope for new physics in \(D \to \rho/\omega\gamma\) coming from \(c \to u\gamma\) transitions is lost since the result obtained using our approach might screen possible signals coming from nonminimal SUSY.

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Fig. 1: The decay width for $D^0 \to \bar{K}^{*0}\gamma$ as function of the combination $\lambda' + 2\lambda/3$. The full (dashed) lines denote the experimentally allowed (forbidden) values for this combination.

Fig. 2: The decay width for $D^* \to \rho\gamma$ as function of the combination $\lambda' - \lambda/3$. The full (dashed) line denotes the experimentally allowed (forbidden) values for this combination.

Fig. 3: The ratio of the decay rates for $D \to \rho\gamma$ and $D \to K^{*}\gamma$ times a constant factor $2/\tan\theta_c^2$ as function of the combination $(\lambda' + 2\lambda/3)$. The full (dashed) line denotes the experimentally allowed (forbidden) values for this combination. In the $U(3)$ limit of the Standard Model such a ratio should be equal to 1.

Fig. 4: The branching ratio of $D^0 \to \pi^+K^-\gamma$ as function of the photon energy cut.
The figure shows the decay width $\Gamma(D^0 \rightarrow K^0 \gamma)$ as a function of $(\lambda' + 2\lambda/3)$ in GeV$^{-1}$. The graph has a solid line and a dashed line, representing different theoretical predictions or experimental data sets. The y-axis is labeled \( \Gamma(D^0 \rightarrow K^0 \gamma) \times 10^{-7} \text{eV} \), and the x-axis is labeled $(\lambda' + 2\lambda/3)$ in GeV$^{-1}$. The figure is titled "Fig. 1."
Fig. 2

$\Gamma(D_s^+ \rightarrow \rho^0) \left[10^{-7} \text{ eV}\right]$ vs $\lambda'/\lambda^3 \left[\text{GeV}^{-1}\right]$
Fig. 3
Fig. 4

The figure shows the branching fraction $B[D^0 \to K^+\pi^+]$ as a function of $\omega_{\text{MIN}}$ in MeV. The branching fraction decreases monotonically as $\omega_{\text{MIN}}$ increases.