Breakdown of Energy Equipartition in a 2D Binary Vibrated Granular Gas

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We report experiments on the equipartition of kinetic energy between grains made of two different materials in a mixture of grains vibrated in 2 dimensions. In general, the two types of grains do not attain the same granular temperature, \( T_g = 1/2m < v^2 > \). However, the ratio of the two temperatures is constant in the bulk of the system and independent of the vibration velocity. The ratio depends strongly on the ratio of mass densities of the grains, but is not sensitive to the inelasticity of grains. Also, this ratio is insensitive to compositional variables of the mixture such as the number fraction of each component and the total number density. We conclude that a single granular temperature, as traditionally defined, does not characterize a multi-component mixture.

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When two different gases are put in thermal contact with each other they eventually reach thermal equilibrium, with equipartition of kinetic energy between the gas molecules and a single temperature for the system as mandated by the zeroth law of thermodynamics. In this article we present an experiment designed to test the validity of energy equipartition in a mixture of two kinds of macroscopic grains driven into a non-equilibrium steady state by external vibration. Kinetic theories for granular systems have long used the idea of a granular temperature defined as the average kinetic energy of the particles, \( T_g = 1/2m < v^2 > \); the value of \( T_g \) in a given system is determined by a balance between the power input by external driving sources and the energy lost in interparticle collisions. The results of such calculations of transport in single-component systems have been validated in both simulations and experiments. It is natural to ask whether the variable \( T_g \) obeys a zeroth law, and in particular, whether a mixture of grains is characterized by a single, shared, granular temperature.

The earliest theoretical developments of kinetic theories for binary mixtures of spheres or of discs, start with a clearly identified assumption that a single temperature variable, \( T_g \), characterizes the entire mixture. Most subsequent work preserves this assumption. An important exception is the work of Garzo and Dufty who study theoretically the “cooling” of a binary granular mixture from an initial distribution of particle velocities. They conclude that the cooling rate of each species in the mixture is the same, although their temperatures are different throughout the cooling process. Experiments and simulations on binary mixtures of granular materials have primarily focused on segregation and mixing of different types of grains, and for the most part, have not considered the issue of equipartition. However, an experiment on a vibrated monolayer of grains by Losert et al. found that grains of two different sizes and masses do not attain the same value of \( T_g \). In this article, we show that such differences persist in the bulk of a 2D system and that in general, the two components of a mixture arrive at a ratio of granular temperatures, \( \gamma \neq 1 \). We explore the dependence of the temperature ratio, \( \gamma \), on particle properties as well as on compositional parameters of the mixture. Our data argue for a redefinition of the granular temperature in order to accommodate extension to a multi-component system.

We make binary mixtures of spherical glass balls with aluminum, steel or brass balls. The two relevant material properties being varied are mass and inelasticity (see Table I), and the two compositional parameters being varied are the number fraction of each component, \( x \), and the number density, specified in terms of the average occupied area fraction, \( \rho_{avg} \), of the particles in the cell.

To determine the importance of particle material properties we hold average number density and number fraction fixed and compare mixtures of steel (heavy-elastic) and glass (light-elastic), brass (heavy-inelastic) and glass (light-elastic), and aluminum (light-inelastic) and glass (light-elastic) in terms of the \( T_g \) of each component and the temperature ratio \( \gamma \). Next, to determine whether failure of equipartition is a bulk effect, we hold the number fraction fixed for a steel-glass mixture and measure the

TABLE I: Some material properties of the spheres used in the experiment.

| Particle | Mass [mg] | Effective inelasticity a | Mass ratio w/ glass |
|----------|-----------|--------------------------|---------------------|
| Glass    | 5.24      | 0.17                     | -                   |
| Alum     | 5.80      | 0.31                     | 0.92                |
| Steel    | 15.80     | 0.21                     | 0.33                |
| Brass    | 18.00     | 0.39                     | 0.28                |

aThe effective inelasticity quoted is the fraction of energy lost in a collision in single component system, averaged over all collisions. Since relative velocity of the spheres, the impact parameter, and rotational motion all affect the inelasticity of a given collision, the number quoted above is the mean of a wide distribution.
temperature ratio as the number density of balls is varied. Finally, to test the dependence of the temperature ratio on the relative fraction concentration, we hold the number density fixed for a brass-glass pair, and measured the temperature ratio of the components as the number fraction of brass is varied.

The balls (diameter: \( d = 1.600 \pm 0.002 \, \text{mm} \)) are confined to a vertical, rectangular, Delrin cage (32 \( d \) high x 48 \( d \) wide x 1.1 \( d \) thick) sandwiched between two parallel plates of anti-static coated Plexiglas. An electromechanical shaker (LDS 500L) vibrates the cage vertically at a frequency of 60 Hz and amplitudes up to 2.4 \( d \), producing maximum accelerations, \( \Gamma \), and velocities, \( v_0 \), of 56 \( g \) and 1.45 \( m/s \) respectively. The motions of the balls are recorded with a monochrome high-speed camera (Kodak Motioncorder) at a rate of 2000 frames/s. Each ball is located with a precision of 0.03 \( d \) (\( \approx 40 \, \mu m \)). Glass balls are unambiguously distinguished from their metallic partners by using a combination of reflected and transmitted light, and by thresholding appropriately. Most of the results we discuss here are taken in a rectangular (10 \( d \) x 20 \( d \)) window centered in the geometrical center of the cell.

The data presented in this article are all taken at relatively large values of excitation, corresponding to \( \Gamma > 32 \, g \) and \( v_0 > 0.86 \, m/s \). Beyond this scale of excitation, the vertical density profile does not vary much and tends to an asymptotic profile that is symmetric about the center of the cell, as shown in Fig. 1. The density profile is identical for two different species when their masses are matched, even if they have different inelasticity, as in the case of aluminum-glass mixture shown in Fig. 1(a). In contrast, the profiles of the other two mixtures (steel-glass and brass-glass) in Fig. 1(b), show a slightly higher concentration of heavy particles in the center of the cell and a more uniform distribution of light particles throughout the cell. However, in all three mixtures, while there are differences in the overall density distribution, the particles in the mixture are locally well-mixed, with no obvious tendency for balls of a particular type to cluster when vigorously excited as in this case. Finally, we have not observed any gradients of velocity or density in the horizontal direction.

The principal observation of our experiment is that the two components of a binary mixture do not always equilibrate to the same granular temperature. Figure 2(a) shows \( T_g \) of three pairs of materials as a function of the squared vibration velocity of the cell, \( v_0^2 \), with the number density being held fixed at \( \rho_{avg} = 0.096 \) and \( \rho_{avg} = 0.096 \) and \( x = 1/2 \) for all mixtures. (b) Ratio between glass temperature and partner temperature, \( \gamma = T_{glass}/T_{partner} \), versus the squared vibration velocity, \( v_0^2 \).
0.29, $m_{\text{glass}}/m_{\text{brass}} = 0.33$). Even when the temperatures of the two components do not equilibrate, the ratio of the temperatures do not vary as a function of $v_0^2$ within the error bars, as shown in Fig. 3(b). Furthermore, the ratio of temperatures are very close to one another $(\gamma_{g-s} = T_{\text{glass}}/T_{\text{steel}} = 0.66 \pm 0.06$ and $\gamma_{g-b} = T_{\text{glass}}/T_{\text{brass}} = 0.69 \pm 0.09$), even though the inelasticity of the metal partner in each case is quite different (Table 1). We conclude that the temperature ratio differs from one when the mass ratio departs significantly from one, but does not depend sensitively on the relative value of the inelasticity of the two components.

The failure of equipartition is not merely a boundary effect, nor is the value of $\gamma$ that we report in Fig. 3(b), an inhomogeneous average over different regions of the cell. This is shown in Fig. 3 where the temperature profiles of steel and glass are plotted as a function of the height of the cell, $z [d]$, for $v_0 = 1.45 \text{ m/s}$. In a collision with a moving wall which is effectively infinitely massive, both species pick up the same velocity, which implies that the heavier species (steel) picks up greater kinetic energy. The temperature ratio $\gamma$ is thus different from one at the walls; this is analogous to the previous observations of Losert et al [9]. Remarkably, however, the temperature ratio quickly reaches a constant value $(\gamma = 0.69)$ in the interior of the cell, even as the $T_g$ of each component continues to decrease toward the middle of the cell as a result of losses from inelastic collisions. Therefore the effect of inelastic collisions is to drive the value of $\gamma$ in the interior of the cell toward a value different from one, which as we mentioned above, depends on the mass ratio of components, but is insensitive to their inelasticity, and to the value of $v_0$.

We pursue further the possibility that the value of $\gamma$ obtained is due to boundary effects by increasing the number density, $\rho_{\text{avg}}$, of a steel-glass mixture holding the number fraction fixed at $x = 1/2$. While this is not exactly equivalent to increasing the system size, it does tune the ratio of system size to mean free path, so that we vary the number of collisions experienced by balls in transferring energy from the boundary to the interior. The temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{steel}}$, is plotted against of the square vibration velocity, $v_0^2$, in Fig. 3 for mixtures whose number density, $\rho_{\text{avg}}$, varies by a factor of 4. Once again, the ratio of the two temperatures are essentially independent of $v_0^2$. Importantly, the temperature ratio maintains an average value of $\gamma = 0.60 \pm 0.06$, with no obvious systematic trend as $\rho_{\text{avg}}$ changes. This result further confirms that the failure of equipartition is a bulk effect, and also suggests that the value of $\gamma$ is insensitive to the number density of the mixture.

Finally, we study the effect on equipartition of varying number fraction in a mixture at fixed density. Figure 3 shows the temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{brass}}$, of a brass-glass mixture as a function of $v_0^2$, for three different number fractions of brass, $x$: 1/2, 1/4, and 1/8. The temperature ratios for the different relative concentrations are all the same within experimental errors. This demonstrates that the components do not reach thermal equilibrium, even though the relative number of collisions with glass balls increase for a given brass ball, as the number fraction of brass is decreased. It has been predicted [10] that equipartition does not generally occur even in the tracer limit of a single foreign particle in a sea of other particles. This is consistent with our observation that the temperature ratio $\gamma$ does not depend on number fraction.

As mentioned earlier, Garzo and Dufty [6] study a freely-cooling binary system, and find a temperature ratio dependence on material parameters as well as on compositional parameters. Since there are no comparable predictions for a driven system, we have compared our data with these predictions and find quite good agreement with our results as a function of material parameters. However, they predict a strong dependence on number fraction of the components, where none is observed. The comparison between theory and experiment encour-

![FIG. 3: Vertical temperature profiles for a steel-glass mixture. Dotted and dot-dash lines represent $T_{\text{steel}}$ and $T_{\text{glass}}$ and the full line represents the ratio between the glass and steel temperatures. Dashed vertical lines represent the limits of the vibrating walls. For this system $\rho_{\text{avg}} = 0.096$, $x = 1/2$ and $\Gamma = 56 \text{ g}$.](image1)

![FIG. 4: Temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{steel}}$, in a steel-glass mixture with number fraction $x = 1/2$, plotted against squared vibration velocity, $v_0^2$. Different markers represent different number densities of the mixture.](image2)
FIG. 5: Ratio between glass temperature and brass temperature, \( \gamma = \frac{T_{\text{glass}}}{T_{\text{brass}}} \), versus squared vibration velocity of the cell, \( v_0^2 \). Different markers represent different number fractions of brass for the same total number of particles \( (\rho_{\text{avg}} = 0.049) \).

ages further theoretical development in driven systems.

In conclusion, we have observed that energy equipartition does not generally hold for a binary vibrated granular system. However, the ratio between the steady state values of the granular temperatures of the two components does robustly satisfy some general trends. The temperature ratio is constant in the interior of a system, and is insensitive to vibration velocity and inelasticity, while depending strongly on mass density. Furthermore, the ratio is insensitive to compositional parameters such as number fraction of each component and average number density. This opens the possibility that just as in the work of Garzo and Dufty \[7\] for a cooling system, there is a well-defined and calculable ratio of temperatures achieved in a steady state system. Thus a modified definition of granular temperature with a prefactor depending chiefly on material properties, may adequately describe a multi-component system. Numerical simulations could play a useful role, in that material properties may be continuously tuned, as opposed to experiments in which only a limited pool of materials is readily available.

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