Two Novel Characterizations of the DE Flip Flop
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Abstract. Modern digital circuits, especially those based on large-scale integration devices employ DE flip flops, which are an extension of the D type with the capacity to store an input value only upon request or enabling. The DE flip flop could possibly be described algebraically by its characteristic equation or tabularly by its next-state table (used for analysis purposes) and its excitation table (used for synthesis purposes). This paper explores two novel characterizations of DE flip flops. First, equational and implicational descriptions are presented, and the Modern Syllogistic Method is utilized to produce complete statements of all propositions that are true for a general DE flip flop. Next, methods of Boolean-equation solving are employed to find all possible ways to express the excitations in terms of the present state and next state. The concept of Boolean quotient plays a crucial role in exposing the pertinent concepts and implementing the various desired derivations. This paper is expected to be of an immediate pedagogical benefit, and to facilitate the analysis and synthesis of contemporary sequential digital circuits.

Keywords: Flip flop, Characterization, Modern Syllogistic Method, Boolean-equation solving, Boolean quotient, Excitation.

1. Introduction

An essential building block of a sequential digital circuit is an elementary memory cell called a flip flop, a one-bit register, a bistable or a bistable multivibrator[1]. A flip-flop is a circuit that has two stable states and can be used to store a single bit (binary digit) of data; with one of the two states of the flip flop representing a "one" and the other representing a "zero". A flip flop is usually named in terms of its inputs or excitations. In the following, we use the symbols $y_i$ and $Y_i$, to denote the present and next states of flip flop number $i$ in a given circuit. We also use upper-case subscripted letters to designate the excitations of a flip flop.

There are many well-known types of flip flops, including those with a single input (such as the D flip flop and the T flip flop), those with two inputs (such as the SR flip flop, the JK flip flop, and the DE flip flop), and those with three or more inputs. Out of these, we single out the JK flip-flop as a very versatile device that was commonly used to construct circuits with discrete components. The JK flip flop is discussed in many publications on digital design (See, e.g., [1-8]) and its various characterizations are detailed in [1]. However, it has been mostly replaced by a variation of the D flip flop, which is conveniently called the DE flip flop (or occasionally, the E-type flip flop)[9-12]. This variation is simply a D flip flop that is equipped with an enable input $E$. 


This paper is a tutorial exposition about flip flops in general, and about $DE$ flip flops, in particular. After surveying conventional characterizations of $DE$ flip flops (in comparison with those of $JK$ flip flops), we employ novel mathematical methods of logic deduction and Boolean-equation solving for further characterization of the $DE$ flip flop.

The organization of the remainder of this paper is as follows. Section 2 offers an introductory characterization of the $DE$ flip flop in comparison with the $JK$ flip flop. Section 3 uses the Modern Syllogistic Method (MSM) of logic deduction to ferret out all that can be said about the $DE$ flip flop in the most compact form. Section 4 applies methods of 'big' Boolean-equation solving to find all possible solutions of the excitations of the $DE$ flip flop in terms of its present and next states. Section 5 concludes the paper. To make the paper self-contained, it is supplemented with an appendix on the “Boolean Quotient”, a crucial concept for some of the current derivations.

2. The DE Flip Flop Versus the JK Flip Flop

We recall that the $DE$ flip flop is a variation of the $D$ flip flop, which is simply a $D$ flip flop that is equipped with an enable input $E$ \cite{1,9-12}. Despite the contemporary widespread use of the $DE$ flip flop, it is definitely less familiar (to most readers) than the $JK$ flip flop. We devote this section to a brief introduction of the $DE$ flip flop, in comparison to the $JK$ flip flop. Figure 1 presents the characteristic maps of both types of flip flops, in both conventional-map form and variable-entered form. Figure 2 displays the transition maps for both types of flip flops. The transition variable $\delta y_i$ is a four-valued variable defined to be $\delta y_i = 0$ when $y_i = Y_i = 0$, $\delta y_i = 1$ when $y_i = Y_i = 1$, $\delta y_i = \Delta$ when $y_i = 0$ and $Y_i = 1$, and finally $\delta y_i = \nabla$ when $y_i = 1$ and $Y_i = 0$ \cite{9}. Figure 3 presents the excitation maps for both the $JK$ and the $DE$ flip flops. Of course, the excitation map of the $DE$ flip flop (as shown in Fig. 3(b)) is virtually unknown in the literature, but it will be verified and formally derived in the forthcoming sections.

We now digress a little bit to explain why the $JK$ flip flop is superior from the theoretical point of view, and why it was dominantly used in the era of discrete components. A quick glimpse at Fig. 1-3 demonstrates clearly that the $JK$ flip flop has a more balanced behavior and a more versatile capability than the $DE$ flip flop. In fact, the $JK$ flip flop enjoys the following distinctive advantages:

- By contrast to the $SR$ flip-flop, which has an indeterminate outcome when $S_i = R_i = 1$, (or must, otherwise, be operated under the constraint $S_i R_i = 0$), the $JK$ flip-flop has the merit of guaranteed determined outcome under unconstrained operation.

- The $JK$ flip-flop is intimately related to other famous flip flops. It is exactly an $SR$ flip flop with an additional external feedback provided via the relations $S_i = f_i \bar{y}_i$ and $R_i = K_i y_i$. These relations ensure the satisfaction of the aforementioned constraint ($S_i R_i = 0$) and allow the designer, if he/she wishes, to dispense with external feedback from the outputs of a particular flip flop to its own inputs. The $JK$ flip-flop also combines the delay function of a $D$ flip-flop and the toggle function of a $T$ flip-flop. The $JK$ flip-flop reduces to a $D$ flip-flop by making its excitations complementary ($f_i = D_i$, $K_i = \bar{D}_i$), and reduces to a $T$ flip-flop by making its excitations equal ($f_i = K_i = T_i$).

- The $JK$ flip-flop possesses the minimum number of excitations that can control its next-state variable to assume any of the four possible values that might be taken by
a Boolean function of a single variable representing the present-state value, namely: the present state itself, its complement, logic 0, and logic 1. In fact the algebraic characteristic equation (next-state equation) of the \(JK\) flip-flop

\[
Y_i = J_i \bar{y_i} \lor \bar{K}_i y_i,
\]

might be rewritten in the minterm-expansion form

\[
Y_i = (y_i) J_i \lor \bar{K}_i y_i \lor (0) J_i \lor K_i \lor (\bar{y}_i) J_i \lor K_i \lor (1) J_i \lor \bar{K}_i.
\]

Equations (1) and (2) can be readily deduced from Fig. 1. Note that \(Y_i\) for a \(DE\) flip flop can assume any of the three values 0, 1, and \(y_i\), while for a \(JK\) flip flop, it can assume any of the four values 0, 1, \(y_i\), and \(\bar{y}_i\). Though the \(DE\) flip flop seems, theoretically, less versatile than the \(JK\) flip flop, it is by no means inferior to it within the modern circuits in which it is used such as an FPGA, in which a look-up table (LUT) is used rather than conventional logic gates [9-12].

3. Equational and Implicational Descriptions

In this section, we use the Modern Syllolgistic Method (MSM) [13-23] to ferret out from the characteristic Equation (4) (viewed as a premise) all that can be concluded from it, with the resulting conclusions cast in the simplest or most compact form. A similar study was conducted in [1] for the \(JK\) flip flop.

First, we reformulate (4) as an equation whose R.H.S. is 0, i.e.,

\[
f = f(Y_i, y_i; D_i, E_i) = 0,
\]

where

\[
f = Y_i \oplus (y_i E_i \lor D_i E_i)
\]

\[
= Y_i(y_i E_i \lor D_i E_i) \lor \bar{Y}_i(y_i E_i \lor D_i E_i)
\]

\[
= Y_i(\bar{y}_i E_i \lor D_i E_i) \lor \bar{Y}_i(y_i E_i \lor D_i E_i).
\]

Then we replace \(f\) in (7) by its complete-sum form using the Blake-Tison Method [13-27]. We note that (7) involves four variables, which are all biform. There is no consensus \(w.r.t.\) any of the three variables \(Y_i, y_i,\) and \(D_i\). However, there are two consensuses \(Y_i \bar{y}_i D_i\) and \(\bar{Y}_i y_i D_i\), which are obtained \(w.r.t.\) \(E_i\). The resulting formula is absorptive (i.e., it has no term that can absorb another) [13], and hence it represents the

“Positive-Edge-Triggered”. Equation (4) might be rewritten as a minterm expansion in the form

\[
Y_i = (y_i) \bar{D}_i E_i \lor (0) \bar{D}_i E_i \lor (1) D_i E_i \lor (y_i) D_i E_i
\]

Again, Equations (4) and (5) can be deduced from Fig. 1. Note that \(Y_i\) for a \(DE\) flip flop can assume any of the three values 0, 1, and \(y_i\), while for a \(JK\) flip flop, it is by no means inferior to it within the modern circuits in which it is used such as an FPGA, in which a look-up table (LUT) is used rather than conventional logic gates [9-12].
complete sum $CS(f)$ of $f$, and by virtue of (6), we obtain

$$CS(f) = Y_i(\bar{y}_i \bar{E}_i \lor \bar{D}_i \bar{E}_i \lor \bar{y}_i \bar{D}_i) \lor \bar{y}_i (y_i \bar{E}_i \lor D_i E_i \lor y_i D_i) = 0.$$  \hspace{1cm} (8)

The expression in (8) is a disjunction of six terms equated to zero. This is exactly equivalent to each of the terms in (8) being individually equated to zero. These six equations (See Table 1) constitute all the propositions that can be stated about the DE flip flop. However, we might use the equivalence \cite{13}

$$\{A \rightarrow B\} \equiv \{AB = 0\}, \hspace{1cm} (9)$$

to convert each of the six equational statements in Table 1 to any of eight equivalent implicational forms, as shown in Table 1.

Complete information about the DE flip flop is possible by using one of the nine equivalent statements given in each of the six main (major or double) rows in Table 1. Though Table 1 provides a wealth of facts about the DE flip flops, many of these facts are redundant as they are deducible from other facts. In fact, only four independent statements suffice (albeit with some inconvenience occasionally) to characterize the DE flip flop. By the "independence" requirement we rule out the following cases:

a) Selection of the first three successive equations $Y_i \bar{y}_i \bar{E}_i = 0$, $Y_i \bar{D}_i E_i = 0$, and $Y_i \bar{y}_i \bar{D}_i = 0$ or the next three consecutive equations $\bar{y}_i y_i \bar{E}_i = 0$, $\bar{y}_i D_i E_i = 0$, and $\bar{y}_i y_i D_i = 0$. In each case, the third equation is simply the consensus of the former two, and is deducible from them.

b) Selection of two statements that belong to the same major double row, since the nine statements in the same major double row are simply equivalent.

c) Selection of three statements that belong to the first three major rows or to the last three major rows.

The simplest characterization is naturally the characterization \textit{via} the equational forms in major rows 1, 2, 4, and 5, (highlighted in blue in Table 1). These equations are deducible from the original characteristic function (7) equated to zero, and they are neutral about utility to analysis or synthesis. By contrast, there are six redundant analysis-oriented implicational characterizations (highlighted in yellow), in which the antecedents depend on the excitations and the consequents depend on the present and next states. There are also six redundant synthesis-oriented implicational characterizations (highlighted in green), in which the antecedents depend on the present and next states and the consequents depend on the excitations. Let us consider the four analysis-oriented implicational statements selected from major rows 1, 2, 4, and 5, namely

$$\bar{E}_i \rightarrow \bar{Y}_i \lor y_i, \hspace{1cm} (10a)$$

$$\bar{D}_i E_i \rightarrow \bar{Y}_i, \hspace{1cm} (10b)$$

$$E_i \rightarrow Y_i \lor \bar{y}_i, \hspace{1cm} (10c)$$

$$D_i E_i \rightarrow Y_i, \hspace{1cm} (10d)$$

The implicational statements (10) have the excitation $E_i$ and (possibly) the excitation $D_i$ in the antecedents of the implications, and have the next state $Y_i$ and (possibly) the present state $y_i$ in the consequents. Using techniques of Boolean reasoning, we can view Equations (10) as a precise translation of the characteristic map of the DE flip flop (Figs. 1(b) and 1(d)). The implication $\bar{D}_i E_i \rightarrow \bar{Y}_i$ in (10b) is equivalent to the second column in Fig. 1(b) or Fig. 1(d), which can be read as

$$\{D_i = 0, E_i = 1\} \rightarrow \{Y_i = 0\}. \hspace{1cm} (11)$$
Note that this implication keeps silent about \( y_i \), which is its way of saying that \( y_i \) is a don't care and could be either 0 or 1, when \( D_i = 0 \) and \( E_i = 1 \). Likewise, the implication \( D_i E_i \rightarrow Y_i \) in (10d) is equivalent to the third column in Fig. 1(b) or Fig. 1(d), which can be read as

\[
\{D_i = 1, E_i = 1\} \rightarrow \{Y_i = 1\}. \tag{12}
\]

Finally, the two implications in (10a) and (10d) can be combined as

\[
\{E_i = 0\} \rightarrow \{(\overline{Y_i} \lor y_i)(Y_i \lor \overline{Y_i}) = 1\} = \{(Y_i \equiv Y_i) = 1\}, \tag{13}
\]

which is equivalent to the first and fourth columns combined in Fig. 1(b) or Fig. 1(d).

We now consider the four synthesis-oriented implicational statements selected from major rows 1, 2, 4, and 5, namely

\[
Y_i \overline{Y_i} \rightarrow E_i, \tag{14a}
\]
\[
Y_i \rightarrow D_i \lor E_i, \tag{14b}
\]
\[
\overline{Y_i} Y_i \rightarrow E_i, \tag{14c}
\]
\[
\overline{Y_i} \rightarrow D_i \lor E_i. \tag{14d}
\]

The implications in (14) are the converses of those in (10) and hence they have antecedents involving the next state \( \overline{Y_i} \) and (possibly) the present state \( y_i \) and consequents involving the excitation \( E_i \) and (possibly) the excitation \( D_i \). The implications in (14) are precisely equivalent to the excitation map of the \( DE \) flip flop (Fig. 3(b)). The conditions (14b) and (14d) might be expanded as

\[
Y_i \rightarrow \overline{D_i} \overline{E_i} \lor D_i \overline{E_i} \lor D_i E_i, \tag{14b1}
\]
\[
\overline{Y_i} \rightarrow \overline{D_i} \overline{E_i} \lor D_i \overline{E_i} \lor D_i E_i. \tag{14d1}
\]

These mean that if \( Y_i = 1 \) then \((D_i, E_i)\) belongs to \( S_1 = \{(0,0), (1,0), (1,1)\} \). The condition (14a) means that if further to \( Y_i = 1 \), we have \( y_i = 0 \) then \( E_i = 1 \), and hence \( S_1 \) reduces to \( \{(1,1)\} \), i.e., \( D_i = 1 \). If \( Y_i = 0 \) then \((D_i, E_i)\) belongs to \( S_2 = \{(0,0), (0,1), (1,0)\} = \{(0,d), (d,0)\} \).

The condition (14c) means that if further to \( Y_i = 0 \), we have \( y_i = 1 \) then \( E_i = 1 \), and hence \( S_2 \) reduces to \( \{(0,1), (1,0)\} \). These arguments suffice (albeit with difficulty) to verify all entries in the two excitation maps of Fig. 3(b). It would have been more convenient if we complete the picture by augmenting Equations (14) by the two synthesis-oriented implicational statements in major rows 3 and 6, namely

\[
Y_i \overline{Y_i} \rightarrow D_i, \tag{14e}
\]
\[
\overline{Y_i} Y_i \rightarrow \overline{D_i}. \tag{14f}
\]

4. Boolean-Equation Solving for Excitations

In this section, we employ methods of Boolean-equation solving \(^{[27-35]}\) to obtain all possible solutions for the excitations of a \( DE \) flip flop in terms of its present state and next state. We consider a single flip flop \( i \) and seek solutions of its excitations \( D_i \) and \( E_i \) in terms of its present state \( y_i \) and next state \( Y_i \). The characteristic equation of this flip flop (4) is our starting point. To solve for \( D_i \) and \( E_i \) in terms of \( Y_i \) and \( y_i \), we first convert (4) to the form of a single equation of a function equated to 1, i.e., to the complement of (6), namely

\[
g(Y_i, y_i; D_i, E_i) = 1, \tag{15}
\]

where \( g \) is the complement of \( f \) in (7), and is given by

\[
g = Y_i \circ (y_i E_i \lor D_i E_i) = Y_i (y_i \overline{E_i} \lor D_i E_i) \lor \overline{Y_i} (y_i \overline{E_i} \lor \overline{D_i} E_i) = (Y_i y_i \lor \overline{Y_i} \overline{y_i}) D_i E_i \lor (Y_i y_i \lor \overline{Y_i} \overline{y_i}) \overline{D_i} E_i \lor (Y_i \overline{D_i} E_i). \tag{16}
\]

The function \( g \) in (16) can be viewed as \( g(D_i, E_i) \) where \( g : B^2 \rightarrow B \), and \( B = B_{16} = FB(Y_i, y_i) \) is the free Boolean algebra with 2 generators \( Y_i \) and \( y_i \), \( 2^2 = 4 \) atoms given by \( Y_i \overline{y_i}, \overline{Y_i} y_i, \overline{Y_i} \overline{y_i} \), and \( Y_i y_i \), and \( 2^4 = 16 \) elements. These elements can be identified as all the switching functions of the two variables \( Y_i \) and \( y_i \). Figure 4(a) is the natural (also
called variable-entered) map for \(g(D_l, E_i)\). Figure 4(b) is a replica of Fig. 4(a) with its map entries being expanded as disjunctions of minterms of \(Y_l\) and \(y_i\) (i.e., as atoms of \(FB(Y_l, y_i)\)). The four atoms \(\bar{Y}_l\bar{y}_i, \bar{Y}_lY_i, Y_l\bar{y}_i,\) and \(Y_lY_i\) make their appearances in the cells of the map of Fig. 4(b) 3, 1, 1, and 3 times, respectively. Since none of the four atoms is absent in Fig. 4(b), the consistency condition for Equation (15) is satisfied trivially as an identity \(\{0 = 0\}\). This means that Equation (15) is unconditionally consistent. It has a number of particular solutions equal to \(3 \times 1 \times 1 \times 3 = 9\) \([11, 30, 32, 33]\).

Figure 4(b) can be used to construct the auxiliary function \(G(D_l, E_i, p)\) in Fig. 5. Each of the appearances of the four atoms \(\bar{Y}_l\bar{y}_i, \bar{Y}_lY_i, Y_l\bar{y}_i,\) and \(Y_lY_i\) in Fig. 4(b), is appended in Fig. 5 by a distinguishing binary tag selected from the orthonormal sets \(\{\bar{p}_1p_2, \bar{p}_1p_3, \bar{p}_3p_4, p_3, p_4\}\) respectively \([32, 33]\). For example, Fig. 5 shows the atom \(\bar{Y}_l\bar{y}_i\) appended (ANDed with) \(\bar{p}_1p_2\) in the cell \(\bar{D}_l\bar{E}_i\), appended with \(\bar{p}_1p_2\) in the cell \(\bar{D}_l\bar{E}_i\), and appended with \(p_1\) in the cell \(\bar{D}_l\bar{E}_i\).

Finally the solution for \(D_l\) and \(E_i\) is written as \([32, 33]\)

\[
D_l = \text{Disjunction of entries of the domain } D_l \text{ in Fig. 5} = \bar{Y}_l\bar{y}_i\bar{p}_1p_2 \lor Y_l\bar{y}_i p_3p_4 \lor Y_l\bar{y}_i(1) \\
\lor Y_l\bar{y}_i p_3 = \bar{y}_l\bar{p}_1p_2 \lor Y_l\bar{y}_1 \lor Y_l(1), (17a)
\]

\[
E_i = \text{Disjunction of entries of the domain } E_i \text{ in Fig. 5} = \bar{y}_l\bar{p}_1p_2 \lor \bar{y}_l(1) \lor Y_l p_3 \lor y_l(1), (17b)
\]

Equations (17) are a parametric solution for \(D_l\) and \(E_i\), where each of the four independent parameters \(p_1, p_2, p_3\) and \(p_4\) belongs to \(B_2 = \{0, 1\}\), and hence (17) can be used to deduce the nine particular solutions of (15). These nine particular solutions for \(D_l\) and \(E_i\) in terms of \(Y_l\) and \(y_i\) are shown in algebraic and map forms in Figs. 6 and 7, respectively.

We might elect to replace the two parameters \(p_3\) and \(p_4\) by the two parameters \(p_1\) and \(p_2\) (provided we let these parameters \(p_1\) and \(p_2\) belong to the underlying Boolean algebra \((B_{16} = FB(Y_l, y_i))\)). In this case, the solution becomes

\[
D_l = \bar{y}_l\bar{p}_1p_2 \lor Y_l\bar{y}_1 \lor Y_l(1), (18a) \\
E_i = p_1 \lor Y_i p_2, \lor Y_l\bar{y}_i, (18b)
\]

Equations (18) are another parametric solution for (15) that uses a minimum number of parameters (two) belonging to \(B_{16} = FB(Y_l, y_i)\). The solutions (17) and (18) are equivalent, since they produce the same set of nine particular solutions shown in Figs. 7 and 8.

An alternative way to solve for \(D_l\) and \(E_i\) is to use the concept of atomic decomposition \([35]\). Figures 8(a) and 8(b) present the atomic decompositions of the variables \(D_l\) and \(E_i\), namely

\[
D_l = (D_{l0})\bar{Y}_l\bar{y}_1 \lor (D_{l1})\bar{Y}_l y_1 \lor (D_{l2}) Y_l\bar{y}_1 \lor (D_{l3}) Y_l y_1, (19a) \\
E_i = (E_{i0})\bar{y}_l y_1 \lor (E_{i1})\bar{y}_l y_1 \lor (E_{i2}) y_l \lor (E_{i3}) y_l, (19b)
\]

where the four atomic components of \(D(D_{l0}, D_{l1}, D_{l2}, D_{l3})\) and those of \(E(E_{i0}, E_{i1}, E_{i2}, E_{i3})\) are arbitrary binary values. Substituting these decompositions into (16) we obtain the atomic decomposition of \(g(D_l, E_i)\) as

\[
g(D_l, E_i) = (E_i \lor \bar{D}_l E_i) \bar{Y}_l\bar{y}_1 \lor \bar{(D_{l0})} \bar{Y}_l\bar{y}_1 \lor \bar{(D_{l1})} \bar{Y}_l y_1 \lor \bar{(D_{l2})} Y_l\bar{y}_1 \lor \bar{(D_{l3})} Y_l y_1 \\
\lor (E_{i0}) \lor \bar{(D_{l0})} \bar{Y}_l\bar{y}_1 \lor (E_{i1}) \bar{Y}_l y_1 \lor (E_{i2}) \bar{y}_l \lor (E_{i3}) \bar{y}_l \\
\lor ((E_{i0}) \lor \bar{(D_{l0})}) \bar{Y}_l\bar{y}_1 \lor ((E_{i1}) \lor \bar{(D_{l1})}) \bar{Y}_l y_1 \lor ((D_{l2}) \lor E_{i2}) \bar{y}_l \lor ((D_{l3}) \lor E_{i3}) \bar{y}_l \\
\lor ((E_{i0}) \lor \bar{(D_{l0})}) \bar{Y}_l\bar{y}_1 \lor ((E_{i1}) \lor \bar{(D_{l1})}) \bar{Y}_l y_1 \lor ((E_{i2}) \lor \bar{(D_{l2})}) \bar{y}_l \lor ((E_{i3}) \lor \bar{(D_{l3})}) \bar{y}_l \\
\lor (E_{i0}) \lor \bar{(D_{l0})} \bar{Y}_l\bar{y}_1 \lor (E_{i1}) \bar{Y}_l y_1 \lor (E_{i2}) \bar{y}_l \lor (E_{i3}) \bar{y}_l \\
\lor ((E_{i0}) \lor \bar{(D_{l0})}) \bar{Y}_l\bar{y}_1 \lor ((E_{i1}) \lor \bar{(D_{l1})}) \bar{Y}_l y_1 \lor ((D_{l2}) \lor E_{i2}) \bar{y}_l \lor ((D_{l3}) \lor E_{i3}) \bar{y}_l \\
\lor (E_{i0}) \lor \bar{(D_{l0})} \bar{Y}_l\bar{y}_1 \lor (E_{i1}) \bar{Y}_l y_1 \lor (E_{i2}) \bar{y}_l \lor (E_{i3}) \bar{y}_l \\
\lor ((E_{i0}) \lor \bar{(D_{l0})}) \bar{Y}_l\bar{y}_1 \lor ((E_{i1}) \lor \bar{(D_{l1})}) \bar{Y}_l y_1 \lor ((D_{l2}) \lor E_{i2}) \bar{y}_l \lor ((D_{l3}) \lor E_{i3}) \bar{y}_l \\
\lor (E_{i0}) \lor \bar{(D_{l0})} \bar{Y}_l\bar{y}_1 \lor (E_{i1}) \bar{Y}_l y_1 \lor (E_{i2}) \bar{y}_l \lor (E_{i3}) \bar{y}_l \\
\lor ((E_{i0}) \lor \bar{(D_{l0})}) \bar{Y}_l\bar{y}_1 \lor ((E_{i1}) \lor \bar{(D_{l1})}) \bar{Y}_l y_1 \lor ((D_{l2}) \lor E_{i2}) \bar{y}_l \lor ((D_{l3}) \lor E_{i3}) \bar{y}_l.
\]

\[(20)\]
Figure 8(c) illustrates the atomic decomposition of \( g(D_i, E_i) \) as given by (20). The equation to be solved forces each individual entry in the map of Fig. 8(c) to be 1. Hence, we obtain

\[
\begin{align*}
\bar{E}_{10} \lor \bar{D}_{10}E_{10} &= 1, \\
\bar{D}_{11}E_{11} &= 1, \\
D_{12}E_{12} &= 1, \\
\bar{E}_{13} \lor D_{13}E_{13} &= 1.
\end{align*}
\]

The solutions of Equations (21) are precisely those reported in Fig. 7. For convenience, we follow Brown \cite{36} in expressing the parametric solutions (18) for \( D_i \) and \( E_i \) in terms of Boolean quotients (See Appendix A), namely

\[
\begin{align*}
D_i &= (\bar{D}_i/\bar{y}_i)\bar{y}_i \lor (D_i/y_i)y_i, \\
E_i &= (E_i/\bar{y}_i)\bar{y}_i \lor (E_i/y_i)y_i.
\end{align*}
\]

Here, the Boolean quotients are

\[
\begin{align*}
D_i/\bar{y}_i &= \bar{p}_1p_2 \lor Y_i, \\
D_i/y_i &= Y_i(p_1 \lor p_2), \\
E_i/\bar{y}_i &= p_1 \lor Y_i, \\
E_i/y_i &= p_1 \lor \bar{y}_i.
\end{align*}
\]

5. Conclusions

This paper is a tutorial exposition about two widely used two-input types of flip flops, namely, \( DE \) and \( JK \) flip flops, with a stress on the \( DE \) type. The paper contributes two novel characterizations of \( DE \) flip flops, using the methods of logic deduction and Boolean-equation solving. The immediate benefit to be gained from this paper is that it might help facilitate the analysis and synthesis of sequential digital circuits. Future work in support and continuation of the present analysis might include the provision of some simulation and implementation results.

| Table 1. All possible statements that can be made about \( DE \) flip flops (arranged in six major double rows). Four irredundant equational characterizations are highlighted in blue. Six redundant analysis-oriented implicational characterizations are highlighted in yellow. Six redundant synthesis-oriented implicational characterizations are highlighted in green. |
|---|---|---|---|---|---|
| Equational form | Implicational form |
| \( Y_i\bar{y}_iE_i = 0 \) | \( Y_i\bar{y}_iE_i \lor 0 \) | \( Y_i\bar{y}_iE_i \lor E_i \) | \( Y_i\bar{y}_iE_i \lor 0 \) | \( Y_i\bar{y}_iE_i \lor 0 \) |
| \( Y_i\bar{D}_iE_i = 0 \) | \( Y_i\bar{D}_iE_i \lor 0 \) | \( Y_i\bar{D}_iE_i \lor D_i \) | \( Y_i\bar{D}_iE_i \lor 0 \) | \( Y_i\bar{D}_iE_i \lor 0 \) |
| \( Y_i\bar{y}_iD_i = 0 \) | \( Y_i\bar{y}_iD_i \lor 0 \) | \( Y_i\bar{y}_iD_i \lor D_i \) | \( Y_i\bar{y}_iD_i \lor 0 \) | \( Y_i\bar{y}_iD_i \lor 0 \) |
| \( Y_i\bar{y}_iE_i = 0 \) | \( Y_i\bar{y}_iE_i \lor 0 \) | \( Y_i\bar{y}_iE_i \lor E_i \) | \( Y_i\bar{y}_iE_i \lor 0 \) | \( Y_i\bar{y}_iE_i \lor 0 \) |
| \( Y_i\bar{D}_iE_i = 0 \) | \( Y_i\bar{D}_iE_i \lor 0 \) | \( Y_i\bar{D}_iE_i \lor D_i \) | \( Y_i\bar{D}_iE_i \lor 0 \) | \( Y_i\bar{D}_iE_i \lor 0 \) |
| \( Y_i\bar{y}_iD_i = 0 \) | \( Y_i\bar{y}_iD_i \lor 0 \) | \( Y_i\bar{y}_iD_i \lor D_i \) | \( Y_i\bar{y}_iD_i \lor 0 \) | \( Y_i\bar{y}_iD_i \lor 0 \) |
Fig. 1. Characteristic maps for the JK and DE flip flops.

(a) The JK flip flop (conventional map)  
(b) The DE flip flop (conventional map)

(c) The JK flip flop (with present state as entered variable)  
(d) The DE flip flop (with present state as entered variable)

Fig. 2. Transition maps for the JK and DE flip flops.

(a) The JK flip flop  
(b) The DE flip flop
Two Novel Characterizations of the DE Flip Flop

Fig. 3. Excitation maps for the JK and DE flip flops.

(a) The JK flip flop

(b) The DE flip flop

Fig. 3. Excitation maps for the JK and DE flip flops.
Fig. 4. Natural map for the function \( g(D_i, E_i) \) as obtained from (16) and with minterm-expanded entries.

Fig. 5. Natural map for the auxiliary function \( G(D_i, E_i, p) \).
Fig. 6. The nine particular solutions for $E_i$ and $D_i$ expressed algebraically in terms of $Y_i$ and $y_i$. One of these solutions ($E_i = 1$, $D_i = Y_i$) asserts that upon enabling, the DE flip flop behaves as a standard D flip flop.

Fig. 7. The nine particular solutions for $D_i$ and $E_i$ expressed in terms of maps of map variables $Y_i$ and $y_i$. This figure is an expanded version of the excitation map of the DE flip flop in Fig. 3(b).
Fig. 8. Atomic decompositions of the variables $D_i$ and $E_i$ as well as of the function $g(D_i, E_i)$ equated to 1.

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Appendix A

Boolean Quotients and the Boole-Shannon Expansion

The concept of a Boolean quotient is a switching-algebraic concept that can be conveniently used to facilitate Boolean manipulations. Given a two-valued Boolean function \( f \) and a term \( t \), the Boolean quotient of \( f \) with respect to \( t \), denoted by \( (f/t) \) or \( (f|t) \), is defined to be the function formed from \( f \) by explicitly imposing the condition \( \{t = 1\} \) (See Brown \(^{[13]}\), Rushdi and Rushdi \(^{[37]}\), or Rushdi \(^{[38]}\), i.e.,

\[
f/t = [f]_{t=1}, \tag{A.1}
\]

The Boolean quotient is also known as a ratio \(^{[3]}\), a subfunction \(^{[39, 40]}\) or a restriction \(^{[41]}\). An important feature of Boolean quotients is that the conjunction of a term with a function is equal to the conjunction of the term with the Boolean quotient of the function with respect to the term, \( \text{v.i.,} \)

\[
t \land f = t \land (f/t). \tag{A.2}
\]

If the term \( t \) is implied by the function \( f \) (i.e., \( f \leq t, f \rightarrow t, f = t \land f \)), then (A.2) reduces to

\[
f = t \land (f/t). \tag{A.3}
\]

The concept of the Boolean quotient has a striking similarity to that of conditional probability \(^{[37, 42, 43]}\), but perhaps the most important utilization of the Boolean quotient is its use in the Boole-Shannon Expansion, which constitutes the most fundamental theorem of Boolean algebra (See Brown \(^{[13]}\), Rushdi and Rushdi \(^{[37]}\), or Rushdi and Ghaleb \(^{[44]}\))

\[
f(X) = (X \land (f(X)/\neg X)) \lor (X \land (f(X)/X)), \tag{A.4}
\]

For example, the next state \( Y_i \) of flip flop number \( i \) can be expressed in terms of the present state \( y_i \) of the same flip flop as

\[
Y_i = (Y_i/\neg y_i) \neg y_i \lor (Y_i/y_i) y_i. \tag{A.5}
\]

The two Boolean quotients \((Y_i/\neg y_i)\) and \((Y_i/y_i)\) in (A.5) are independent of the state of flip flop \( i \). They are functions of other variables of the circuit, including inputs to the excitation logic and (possibly) the present states of other flip flops. For the DE flip flop, they are given by \( D_iE_i \) and \( E_i \lor D_i \), respectively.
توصيفان مبتكران لبدال تمكين التأخير (م خ)

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المستخلص:
تستعمل الدوائر الرقمية الحديثة، وخاصة تلك المبنية على البانرات ذات القدم، الدوائر الكهربائية البديلة التي يُعرف باسم بدالات تمكين التأخير (م خ) تُمثل امتداداً لبدالات التأخير، يتم التمكن فيها لتخزين قيمة مدخلة بحيث يكون هذا التخزين متقراً أو التمكين. يتم وصف هذا البديل جبرياً بواسطة معادلاته المميزة، أو جدولياً بواسطة جدول الحالات التالية (المستعمل لأغراض التحليل) أو جدول الاستئناف (المستعمل لأغراض الترتيب والاصطناع). تستكشف ورقة البحث هذه توصيفين مبتكرين لبدال تمكين التأخير (م خ)، بداية، يتم وصف البدال بتوصيفات ضامنية أو معادلية، وتستعمل الطريقة الاستدلالية الحديثة لإنتاج تعليقات كاملاً عن كل الأخبار التي تصح بالنسبة لبدال تمكين التأخير. يلي ذلك توزيع طرق حل المعادلات البولانية لإيجاد جميع الأساليب الممكنة للتعبير عن مدخل الاستئناف (التغذية) بدالة الحالات الراهنة والحالة التالية. وهنا يلعب مفهوم "خارج القسمة البولانية" دوراً هاماً في توضيح المفاهيم المعنية وفي إنجاز الإشكالات المطلوبة المختلفة. ويُرجى لهذه الورقة أن تكون ذات نفع تعليمي مباشر وأن تيسر تحليل وتركيب الدوائر الرقمية التالية.

كلمات مفتاحية: البدال، التوصيف، الطريقة الاستدلالية الحديثة، حل المعادلات البولانية، خارج القسمة البولانية، الاستئناف (التغذية).