Rare semileptonic $B \rightarrow K(\pi)l_i^- l_j^+$ decay in vector leptoquark model

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Abstract

We investigate the consequence of vector leptoquarks on the rare semileptonic lepton flavour violating decays of $B$ meson which are more promising and effective channels to probe the new physics signal. We constrain the resulting new leptoquark parameter space by using the branching ratios of $B_{s,d} \rightarrow l^+ l^-$, $K_L \rightarrow l^+ l^-$ and $\tau^- \rightarrow l^- \gamma$ processes. We estimate the branching ratios of rare lepton flavour violating $B \rightarrow K(\pi)l_i^- l_j^+$ processes using the constrained leptoquark couplings. We also compute the forward-backward asymmetries and the lepton non-universality parameters of the LFV decays in the vector leptoquark model. Furthermore, we study the effect of vector leptoquark on $(g - 2)_\mu$ anomaly.

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I. INTRODUCTION

The discovery of Higgs boson at LHC completes the standard model (SM) picture of particle interactions, which is quite successful in describing all the observed experimental data so far below the electroweak scale. Still we need physics beyond it in order to solve the hierarchy and flavour problems. In this context the study of rare $B$ decay modes involving the flavour changing neutral current (FCNC) transitions, $b \to s/d$, are more captivating. The FCNC processes are highly suppressed in the SM and occur via one-loop level only. It should be noted that the current measured data by LHCb collaboration on angular observables in rare $B$ decays show significant deviation from the SM predictions. Especially, the discrepancy of $3\sigma$ in the famous $P_5^\prime$ angular observable \cite{1, 2} and the decay rate \cite{3} of rare $B \to K^*\mu^+\mu^-$ processes have become a tension in recent times. In addition the ratio $R_K = \text{Br}(B \to K\mu^+\mu^-)/\text{Br}(B \to Ke^+e^-)$, cancelling the hadronic uncertainties to a very large extent, has also $2.6\sigma$ deviation from the SM prediction \cite{4, 5}, thus indicates the violation of the lepton flavour universality (LFU). The decay rate of $B_s \to \phi\mu^+\mu^-$ process is also low ($3\sigma$ deviation) compared to its SM value \cite{6}.

Within the SM of electroweak interactions, the generation lepton number is exactly conserved, since the neutrinos are deemed as massless particles. Nonetheless, the observation of neutrino oscillation has provided unambiguous evidence for lepton number violation in the neutral sector. The observation of lepton non-universality by the LHCb collaboration generically implies the existence of lepton flavour violating (LFV) decay processes. Since the observed data on lepton non-universality is due to $25\%$ deficit in the muon channel, thus LFV is more for muonic processes than for electronic processes \cite{7}. The branching ratio of $h \to \tau\mu$ LFV decay is found to be $\text{Br}(h \to \tau\mu) = 0.84^{+0.39}_{-0.37}$ by CMS collaboration \cite{8}, which has a $2.6\sigma$ deviation from the SM value, thus boosted the interest of physicists to study more LFV decay processes in charged sector such as $l_i \to l_j\gamma$, $l_i \to l_jl_k\bar{l}_k$, $B_s \to l_i^\pm l_j^\mp$ and $B \to K^{(*)}l_i^\pm l_j^\mp$ etc. Theoretically, the LFV processes are free from the non-perturbative hadronic effects and significantly contribute some additional operators in comparison with the lepton flavour conserving (LFC) processes. In the literature, there are many attempts to analyze the LFV decays in the $B$-sector in terms of various beyond the standard model scenarios \cite{9, 12}. Even though there is no direct experimental measurement on such LFV processes, but there exist upper bounds on some of these decays \cite{13}. The observation of
lepton flavour violating decays in the upcoming and/or future experiments would provide evidence of new physics beyond the SM.

To settle the observed anomalies at LHCb using a specific theoretical framework, we extend the SM by adding a single vector leptoquark (LQ), which is a color triplet boson and arises naturally from the unification of quarks and leptons. LQs carry both baryon and lepton numbers and can be characterised by their fermion no., spin and charge. Since 1980’s LQs had been enthusiastically searched for, yet without any positive results, though LQs could be produced directly at the colliders. The existence of LQ can be found in many new physics (NP) models, such as the grand unified theories [14, 15], Pati-Salam model, quark and lepton composite model [16] and the technicolor model [17]. The lepton and baryon number violating LQs are very heavy to avoid proton decay bounds. Nevertheless, the LQs having the baryon and lepton number conserving couplings do not allow proton decay and could be light enough to be seen in the current experiments. The interaction of LQ with the SM fermions could be due to a scalar LQ doublet with representation \((3, 2, 7/6)\) and \((3, 2, 1/6)\) or a vector LQ triplet \(V^3(3, 3, 2/3)\), singlet \(V^1(3, 1, 2/3)\) or doublet \(V^2(3, 2, 5/6)\) under the SM \(SU(3)_C \times SU(2)_L \times U(1)_Y\) gauge group. In this work, we consider the vector LQ model which can produce both scalar and pseudoscalar operators in addition to the vector currents. We assume that the LQs conserve \(B\) and \(L\) quantum numbers and do not induce proton decay. We investigate the LFV \(B \rightarrow ql^-l_+\) processes in the context of vector LQ model. Even though the LFV processes occur at loop level with the presence of massless neutrinos in one of the loop or proceed via box diagrams, these can occur at tree level in the LQ model and are expected to have significantly large branching ratios. We compute the branching ratios and forward-backward asymmetries in these LFV processes. In addition, we also check the existence of lepton non-universality in the LQ model. The complete LQ phenomenology and the additional new physics contribution to the \(B\)-sector has been investigated in the literature [10, 11, 18–24].

The paper is organized as follows. In section II, we present the effective Hamiltonian describing the \(b \rightarrow ql^-l_+\) transitions, where \(q = s, d\). The angular distribution and the decay parameters of the semileptonic lepton flavour violating decays are described in section III. In section IV, we discuss the new physics contribution due to the exchange of vector LQ and the constraints on LQ couplings from \(B_{s,d} \rightarrow l^+l^-\), \(K_L \rightarrow l^+l^-\) and \(\tau^- \rightarrow l^-\gamma\) processes are computed in section V. The branching ratios, forward-backward asymmetries and the
lepton non-universality of $B \to K(\pi) l_i^- l_j^+$ LFV decays are computed in section VI. Finally in section VII we explain the muon $g-2$ anomaly and the conclusions are summarized in section VIII.

II. EFFECTIVE HAMILTONIAN FOR $b \to q l_i^- l_j^+$ PROCESSES

In this section we discuss the effective Hamiltonian describing the FCNC $b \to q(=d, s) l_i^- l_j^+$ transitions. Here we will focus mainly on the $b \to s l_i^- l_j^+$ Hamiltonian as the $b \to d l_i^- l_j^+$ Hamiltonian can be obtained from it with the obvious replacements. The effective Hamiltonian for the quark-level transition $b \to s l_i^- l_j^+(l = e, \mu, \tau)$ in the SM is mainly given by [25]

$$
\mathcal{H}_{\text{eff}}^{\text{SM}} = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \left[ \sum_{i=1}^6 C_i(\mu) \mathcal{O}_i(\mu) + C_7^{\text{SM}} \frac{e}{16\pi^2} \left[ \bar{s} \gamma_{\mu} (m_s P_L + m_b P_R) b \right] F^{\mu\nu} 
+ C^{\text{SM} \alpha_{em}}_V \frac{G_F}{4\pi} (\bar{s} \gamma_{\mu} P_L b) L^{\mu \nu}_{ij} + C^{\text{SM} \alpha_{em}}_A \frac{G_F}{4\pi} (\bar{s} \gamma_{\mu} P_L b) L^{5}_{ij} \right],
$$

where $L^{\mu}_{ij} = \bar{l}_i \gamma_{\mu} l_j$ and $L^{5}_{ij} = \bar{l}_i \gamma_{5} \gamma_{\mu} l_j$. Here $G_F$ denotes the Fermi constant, $V_{qq'}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $\alpha_{em}$ is the fine structure constant and $P_{L,R} = (1 \mp \gamma_5)/2$ are the chirality projection operators. The operators $\mathcal{O}_i (i = 1, ..., 6)$ correspond to the tree level current-current operators ($O_{1, 2}$), QCD penguin operators ($O_{3-6}$) and $C_i$’s are the Wilson coefficients. For $i = j$, $C_i^{\text{SM} V, A}$ represent the SM Wilson coefficients $C_{7,9,10}$ and for $i \neq j$ they will vanish.

The total effective Hamiltonian for processes involving $b \to s l_i^- l_j^+$ transition, in the presence of new physics operators with all the possible Lorentz structure, can be expressed as

$$
\mathcal{H}_{\text{eff}} (b \to s l_i^- l_j^+) = \mathcal{H}_{\text{eff}}^{\text{SM}} + \mathcal{H}_{\text{eff}}^{\text{VA}} + \mathcal{H}_{\text{eff}}^{\text{NP}} + \mathcal{H}_{\text{eff}}^T,
$$

where $\mathcal{H}_{\text{eff}}^{\text{SM}}$ is the SM effective Hamiltonian as given in Eqn. [1], and the NP contributions are given as

$$
\mathcal{H}_{\text{eff}}^{\text{VA}} = -N_F \left[ C_V (\bar{s} \gamma_{\mu} P_L b) L^{\mu}_{ij} + C_A (\bar{s} \gamma_{\mu} P_L b) L^{5\mu}_{ij} + C'_V (\bar{s} \gamma_{\mu} P_R b) L^{\mu}_{ij} 
+ C'_A (\bar{s} \gamma_{\mu} P_R b) L^{5\mu}_{ij} \right],
$$

$$
\mathcal{H}_{\text{eff}}^{\text{NP}} = -N_F \left[ C_S (\bar{s} P_R b) L_{ij} + C_P (\bar{s} P_R b) L^{5}_{ij} + C'_S (\bar{s} P_L b) L_{ij} + C'_P (\bar{s} P_L b) L^{5}_{ij} \right],
$$

$$
\mathcal{H}_{\text{eff}}^T = -N_F \left[ 2C_T (\bar{s} \gamma_{\mu} b) L^{\mu\nu}_{ij} + i2C_T (\bar{s} \gamma_{\mu} b) L^{5\mu\nu}_{ij} \right],
$$

4
where \( N_F = \frac{G_F g_{\text{em}}}{\sqrt{2} \pi} V_{tb} V_{ts}^* \), \( L_{ij}^5 = \bar{t}_i \gamma_5 l_j \), and \( L_{ij}^{\mu \nu 5} = 2 \bar{t}_i \sigma_{\mu \nu} \gamma_5 l_j \). Here we use \( \sigma^{\mu \nu \gamma_5} = -\frac{i}{2} \epsilon^{\mu \nu \beta \alpha} \sigma_{\alpha \beta} \) to calculate \( L_{ij}^{\mu \nu 5} \). In the above expressions \( C_i^{(t)} \), where \( i = V, A, S, P \), and \( C_T^{(5)} \) are the NP effective couplings which are negligible in the SM and can only be generated using new physics beyond the SM.

### III. THEORETICAL FRAMEWORK FOR \( \bar{B} \to \bar{K}(\pi)l_i l_j \) DECAY PROCESSES

The semileptonic \( \bar{B} \to \bar{K} l_i l_j \) decay involves the quark level \( b \to s l_i^- l_j^+ \) transitions as mediated by the effective Hamiltonian of the form in Eqn. (2). The relevant kinematical variables describing this three-body decay are the invariant mass squared of the lepton pair \( q^2 = (P_B - P_K)^2 \), and the polar angle \( \theta_l \). Here \( P_B \) and \( P_K \) are the four-momenta of the \( B \) meson and \( K \) meson respectively and \( \theta_l \) is the angle between the \( K \) and lepton \( l_i \) in the \( l_i - l_j \) rest frame. The polar angle differential decay distribution in the momentum transfer squared \( q^2 \) for the process \( \bar{B} \to K l_i l_j \) can be written in the form

\[
\frac{d^2 \Gamma}{dq^2 d\cos \theta_l} = \frac{G_F^2 g_{\text{em}}^2 \beta_i j \sqrt{\lambda} |V_{tb} V_{ts}|^2}{2^{12} \pi^5 M_B^2} \sum_{i=1}^{12} I_i (\cos \theta_l),
\]

where \( \beta_i j = \sqrt{\frac{1 - (m_i + m_j)^2}{q^2} \frac{1 - (m_i - m_j)^2}{q^2}} \) and the kinematical factor \( \lambda = M_B^4 + M_K^4 + q^4 - 2 (M_B^2 M_K^2 + M_B^2 q^2 + M_K^2 q^2) \). The twelve angular coefficients \( I_i (\cos \theta_l) \) appearing in the angular distribution depend on the couplings, kinematic variables, form factors and the polar angle \( \theta_l \), which are defined as

\[
I_1 = 2 \left[ \left( 1 - \frac{(m_i - m_j)^2}{q^2} \right) (q^2 - (q^2 - (m_i + m_j)^2) \cos^2 \theta_l) |H_V^0|^2 \right. \\
+ \left. 4k \frac{(m_i^2 - m_j^2)}{\sqrt{q^2}} \text{Re} \left[ H_V^0 H_V^* \right] \cos \theta_l + \frac{(m_i - m_j)^2}{q^2} (q^2 - (m_i + m_j)^2) |H_V^l|^2 \right] ,
\]

\[
I_2 = 2 \left[ \left( 1 - \frac{(m_i + m_j)^2}{q^2} \right) (q^2 - (q^2 - (m_i - m_j)^2) \cos^2 \theta_l) |H_A^0|^2 \right. \\
+ \left. 4k \frac{(m_i^2 - m_j^2)}{\sqrt{q^2}} \text{Re} \left[ H_A^0 H_A^* \right] \cos \theta_l + \frac{(m_i + m_j)^2}{q^2} (q^2 - (m_i - m_j)^2) |H_A^l|^2 \right] ,
\]

\[
I_3 = 2 \left( q^2 - (m_i + m_j)^2 \right) |H_S|^2 ,
\]

\[
I_4 = 2 \left( q^2 - (m_i - m_j)^2 \right) |H_P|^2 ,
\]

\[
I_5 = 8 \left( 1 - \frac{(m_i - m_j)^2}{q^2} \right) \left( (m_i + m_j)^2 + (q^2 - (m_i + m_j)^2) \cos^2 \theta_l \right) |H_T^{0l}|^2 ,
\]

5
\[ I_6 = 32 \left( 1 - \frac{(m_i + m_j)^2}{q^2} \right) \left( (m_i - m_j)^2 + (q^2 - (m_i - m_j)^2) \cos^2 \theta_i \right) |H_{TE}^{0t}|^2, \quad (12) \]
\[ I_7 = 4 \text{Re} \left[ 2k (m_i + m_j) H_V^0 H_V^* \cos \theta_i + \frac{(m_i - m_j)}{\sqrt{q^2}} (q^2 - (m_i + m_j)^2) H_V^t H_V^* \right], \quad (13) \]
\[ I_8 = 4 \text{Re} \left[ 2k (m_i - m_j) H_A^0 H_P^* \cos \theta_i + \frac{(m_i + m_j)}{\sqrt{q^2}} (q^2 - (m_i - m_j)^2) H_A^t H_P^* \right], \quad (14) \]
\[ I_9 = -8 \text{Re} \left[ 2k (m_i - m_j) H_V^t H_{TE}^{0t} \cos \theta_i + \frac{(m_i + m_j)}{\sqrt{q^2}} (q^2 - (m_i - m_j)^2) H_V^0 H_{TE}^{0t} \right], \quad (15) \]
\[ I_{10} = 16 \text{Re} \left[ 2k (m_i + m_j) H_A^t H_{TE}^{0t} \cos \theta_i + \frac{(m_i - m_j)}{\sqrt{q^2}} (q^2 - (m_i + m_j)^2) H_A^0 H_{TE}^{0t} \right], \quad (16) \]
\[ I_{11} = -16k \sqrt{q^2} \text{Re}[H_S H_{TE}^{0t}] \cos \theta_i, \quad (17) \]
\[ I_{12} = 32k \sqrt{q^2} \text{Re}[H_P H_{TE}^{0t}] \cos \theta_i. \quad (18) \]

Here \( k = (\beta_i \sqrt{q^2})/2 \) is the lepton momentum and the expressions for the helicity amplitudes are given as

\[
H_V^0 = \sqrt{\frac{\lambda}{q^2}} \left( (C_V^{SM} + C_V + C_V') f_+(q^2) + 2C_7^{SM} m_b \frac{f_T}{M_B + M_K} \right),
\quad (19)
\]
\[
H_V^t = \frac{M_B^2 - M_K^2}{\sqrt{q^2}} (C_V^{SM} + C_V + C_V') f_0(q^2),
\quad (20)
\]
\[
H_A^0 = \sqrt{\frac{\lambda}{q^2}} \left( (C_A^{SM} + C_A + C_A') f_+(q^2) \right),
\quad (21)
\]
\[
H_A^t = \frac{M_B^2 - M_K^2}{\sqrt{q^2}} (C_A^{SM} + C_A + C_A') f_0(q^2),
\quad (22)
\]
\[
H_S = \frac{M_B^2 - M_K^2}{m_b} (C_S + C_S') f_0(q^2),
\quad (23)
\]
\[
H_P = \frac{M_B^2 - M_K^2}{m_b} (C_P + C_P') f_0(q^2),
\quad (24)
\]
\[
H_{TE}^{0t} = -2C_T \frac{\sqrt{\lambda}}{M_B + M_K} f_T(q^2),
\quad (25)
\]
\[
H_{TE}^{0t} = -2C_{T5} \frac{\sqrt{\lambda}}{M_B + M_K} f_T(q^2). \quad (26)
\]

The above expressions are calculated by using the parametrizations of matrix elements of the various hadronic currents between the initial \( B \) meson and the final \( K \) meson, in terms of the form factors \( f_0, f_+ \) and \( f_T \) as \[ 5 \]

\[
\langle \bar{K} (P_K) | \bar{s} \gamma^\mu b | \bar{B} (P_B) \rangle = f_+ (q^2) (P_B + P_K)^\mu + \left[ f_0 (q^2) - f_+ (q^2) \right] \frac{M_B^2 - M_K^2}{q^2} q^\mu, \quad (27)
\]
\[
\langle \bar{K} (P_K) | \bar{s} \sigma^{\mu \nu} b | \bar{B} (P_B) \rangle = i \frac{f_T (q^2)}{M_B + M_K} \left[ (P_B + P_K)^\mu q^\nu - q^\mu (P_B + P_K)^\nu \right]. \quad (28)
\]
It should be noted that in general the angular coefficients of semileptonic decays take the form

\[ I_i (\cos \theta_i) = a_i + b_i \cos \theta_i + c_i \cos^2 \theta_i. \]  

(29)

The differential decay rate for the decay \( \bar{B} \to \bar{K} l_i l_j \) can be found by integrating over the polar angle in Eqn. (6) to get

\[ \frac{d\Gamma}{dq^2} = \frac{G_F^2 \alpha^2_{em} \beta_{ij} \sqrt{\lambda} |V_{ul} V_{us}^*|^2}{2^{12} \pi^5 M_B^3} \sum_{i=1}^{10} J_i, \]

(30)

where the coefficients \( J_i = \int_{-1}^{1} I_i (\cos \theta_i) d \cos \theta_i \) are given below as

\[ J_1 = 4 \left[ \left( 1 - \frac{(m_i - m_j)^2}{q^2} \right) \frac{1}{3} \left( 2q^2 + (m_i + m_j)^2 \right) |H_V^0|^2 \right. \\
+ \left. \frac{(m_i - m_j)^2}{q^2} \left( q^2 - (m_i + m_j)^2 \right) |H_V^t|^2 \right] , \]

(31)

\[ J_2 = 4 \left[ \left( 1 - \frac{(m_i + m_j)^2}{q^2} \right) \frac{1}{3} \left( 2q^2 + (m_i - m_j)^2 \right) |H_A^0|^2 \right. \\
+ \left. \frac{(m_i + m_j)^2}{q^2} \left( q^2 - (m_i - m_j)^2 \right) |H_A^t|^2 \right] , \]

(32)

\[ J_3 = 4 \left( q^2 - (m_i + m_j)^2 \right) |H_S|^2, \]

(33)

\[ J_4 = 4 \left( q^2 - (m_i - m_j)^2 \right) |H_P|^2, \]

(34)

\[ J_5 = 16 \left( 1 - \frac{(m_i - m_j)^2}{q^2} \right) \frac{1}{3} \left( 2(m_i + m_j)^2 + q^2 \right) |H_T^{0t}|^2, \]

(35)

\[ J_6 = 64 \left( 1 - \frac{(m_i + m_j)^2}{q^2} \right) \frac{1}{3} \left( 2(m_i - m_j)^2 + q^2 \right) |H_{TE}^{0t}|^2, \]

(36)

\[ J_7 = 8 \frac{(m_i - m_j)}{\sqrt{q^2}} \left( q^2 - (m_i + m_j)^2 \right) \text{Re}[H_V^0 H_S^*], \]

(37)

\[ J_8 = 8 \frac{(m_i + m_j)}{\sqrt{q^2}} \left( q^2 - (m_i - m_j)^2 \right) \text{Re}[H_A^0 H_P^*], \]

(38)

\[ J_9 = -16 \frac{(m_i + m_j)}{\sqrt{q^2}} \left( q^2 - (m_i - m_j)^2 \right) \text{Re}[H_V^0 H_T^{0t}], \]

(39)

\[ J_{10} = 32 \frac{(m_i - m_j)}{\sqrt{q^2}} \left( q^2 - (m_i + m_j)^2 \right) \text{Re}[H_A^0 H_{TE}^{0t}], \]

(40)

Here the coefficients \( J_{11} = J_{12} = 0 \). Next we define the forward-backward asymmetry \( A_{FB} \) for the leptons by integrating over \( \cos \theta_i \) in Eqn. (6) as

\[ A_{FB} (q^2) = \left( \int_0^1 d \cos \theta_i \frac{d^2 \Gamma}{dq^2 d \cos \theta_i} - \int_{-1}^0 d \cos \theta_i \frac{d^2 \Gamma}{dq^2 d \cos \theta_i} \right) / \frac{d\Gamma}{dq^2}. \]

(41)
After integration, we obtain

\[ A_{FB}(q^2) = \frac{X}{\sum_{i=1}^{10} j_i}, \]

where the quantity \( X \) is defined as

\[ \begin{align*}
X &= 8k\text{Re}\left[ \frac{(m_i^2 - m_j^2)}{\sqrt{q^2}} \left( H_V^0 H_V^{*} + H_A^0 H_A^{*} \right) + (m_i + m_j) \left( H_V^0 H_S^* + 4H_A^i H_T^{0*} \right) \\
&\quad + (m_i - m_j) \left( H_A^0 H_P^* - 2H_V^i H_T^{0*} \right) - 2\sqrt{q^2} \left( H_S^0 H_T^{0*} - 2H_P H_T^{0*} \right) \right] .
\end{align*} \] (43)

Another interesting observable is the lepton non-universality parameter, which has been recently observed by LHCb in \( B^+ \to K^+ l^+ l^- \) process and has a 2.6\( \sigma \) discrepancy from the SM prediction in the dilepton invariant mass bin \( (1 \leq q^2 \leq 6) \text{ GeV}^2 \). Analogously we would like to see whether it is possible to observe non-universality in the LFV decays. Hence, we define the ratios of branching ratios of various LFV decays as

\[
\begin{align*}
R_{\mu e}^{\tau} &= \frac{\text{Br}(\bar{B} \to \bar{K} \mu^- e^+)}{\text{Br}(\bar{B} \to \bar{K} l^+ l^-)}, \\
R_{\tau e}^{\tau} &= \frac{\text{Br}(\bar{B} \to \bar{K} \tau^- e^+)}{\text{Br}(\bar{B} \to \bar{K} l^+ l^-)}, \\
R_{\tau \mu}^{\tau} &= \frac{\text{Br}(\bar{B} \to \bar{K} \tau^- \mu^+)}{\text{Br}(\bar{B} \to \bar{K} l^+ l^-)}, \\
R_{\mu \mu}^{\tau} &= \frac{\text{Br}(\bar{B} \to \bar{K} \mu^- \mu^-)}{\text{Br}(\bar{B} \to \bar{K} e^+ e^-)}, \\
R_{\tau \tau}^{\tau} &= \frac{\text{Br}(\bar{B} \to \bar{K} \tau^- \tau^-)}{\text{Br}(\bar{B} \to \bar{K} l^+ l^-)},
\end{align*}
\]

where \( l = \mu, e \). Similarly, one can obtain the branching ratios and other physical observables in \( B \to \pi l^- l^+_l \) processes by incorporating the appropriate CKM matrix elements, form factors and the NP effective couplings. Recently LHCb has measured the ratio of branching fractions of \( B^+ \to \pi^+ l^+_l l^-_j \) over \( B^+ \to K^+ l^+_l l^-_j \) processes [26], given as

\[ \frac{\text{Br}(B^+ \to \pi^+ l^+_l l^-_j)}{\text{Br}(B^+ \to K^+ l^+_l l^-_j)} = 0.053 \pm 0.014 \text{(stat)} \pm 0.001 \text{ (syst)}. \] (49)

In the same context, we also define the ratio of branching fractions of \( B^+ \to \pi^+ l^- l^+_j \) and \( B^+ \to K^+ l^- l^+_j \) LFV processes as

\[ R_{\pi l_j}^{l_l} = \frac{\text{Br}(B^+ \to \pi^+ l^- l^+_j)}{\text{Br}(B^+ \to K^+ l^- l^+_j)}. \] (50)
IV. NEW PHYSICS CONTRIBUTIONS DUE TO THE EXCHANGE OF VECTOR LEPTOQUARK

There are 10 different LQ multiplets under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ SM gauge group [22], of these one half have scalar nature and the rest have vectorial nature under the Lorenz transformation. Vector LQs have spin 1 which exist in grand unified theories, $SO(10)$ including Pati-Salam color $SU(4)$ and larger gauge groups. The scalar and vector LQ multiplets are differ by their weak-hypercharge and fermion number. The strongest bounds on the vector LQs can be avoid by demanding chirality and diagonality of the coupling and diquark coupling have to be forbidden to evade proton decay. There are three relevant vector LQ multiplets, $(3, 3, 2/3)$, $(3, 1, 2/3)$ and $(3, 2, 5/6)$ [23], out of which only $(3, 3, 2/3)$ leptoquark conserves both baryon and lepton numbers.

1. $Q = 2/3$ vectors

There are two vector LQ multiplets $V^3(3, 3, 2/3)$ and $V^1(3, 1, 2/3)$ having fermion number zero and electric charge $Q = 2/3$. The interaction Lagrangian of isotriplet state $V^{(3)}$ with the SM fermions is given by [23]

$$\mathcal{L}^{(3)} = g_L \bar{Q} \tau \gamma_{\mu} L + h.c.,$$

which conserves both lepton and baryon number and contributes new Wilson coefficients, $C_{V, A}^{LQ}$ as

$$C_V^{LQ} = -C_A^{LQ} = \frac{\pi}{\sqrt{2} G_F V_{tb} V_{ts}^{*} \alpha_{em}} \frac{(g_L)_{sl} (g_L)_{bl}^{*}}{M_{V^{(3)}}^2}.$$ 

(52)

Here $Q(L)$ is the left handed quark (lepton) doublet, $g_L$ is the LQ coupling having left handed quark current and $\tau$ represents the Pauli matrices.

The Lagrangian for isosinglet state, $V^{(1)}$ is given by

$$\mathcal{L}^{(1)} = (g_L \bar{Q} \gamma_{\mu} L + g_R \bar{d}_R \gamma_{\mu} l_R) V^{(1)}_{\mu} + h.c.,$$

where $d_R$ and $l_R$ are the right handed down quark and lepton singlets respectively and $g_R$ is the LQ coupling with down quarks and right handed leptons. This LQ violates baryon number and has the coupling to both left and right handed fermions i.e. it is a non-chiral
LQ. In addition to $C_{V,A}$ new Wilson coefficients, these non-chiral LQ contributes scalar and pseudoscalar operators given by

$$C_{NP}^V = -C_{NP}^A = \frac{\pi}{\sqrt{2}G_F V_{tb} V_{ts}^* \alpha_{em}} \frac{(g_L)_{sl}(g_L)_{sl}^*}{M^2_V V_{(1)}},$$  \hspace{1cm} (54a)

$$C_{NP}^{\prime V} = C_{NP}^{\prime A} = \frac{\pi}{\sqrt{2}G_F V_{tb} V_{ts}^* \alpha_{em}} \frac{(g_R)_{sl}(g_R)_{bl}^*}{M^2_V V_{(1)}},$$  \hspace{1cm} (54b)

$$-C_{NP}^P = C_{NP}^{\prime P} = \frac{\sqrt{2}\pi}{G_F V_{tb} V_{ts}^* \alpha_{em}} \frac{(g_L)_{sl}(g_L)_{sl}^*}{M^2_V V_{(2)}},$$  \hspace{1cm} (54c)

$$C_{NP}^P = C_{NP}^{\prime P} = \frac{\sqrt{2}\pi}{G_F V_{tb} V_{ts}^* \alpha_{em}} \frac{(g_R)_{sl}(g_R)_{bl}^*}{M^2_V V_{(2)}}.$$  \hspace{1cm} (54d)

2. $Q = 4/3$ vectors

The vector LQ with charge $Q = 4/3$ has one isospin doublet state $V^2(3, 2, 5/6)$, whose coupling with fermion bilinear is given by [23]

$$\mathcal{L}^{(2)} = g_R Q^C \bar{i} \tau_2 V_{\mu}^{(2)} \gamma^\mu l_R + g_L \bar{d}^C_R \gamma^\mu \bar{\nu}_{\mu}^{(2)\dagger} l + h.c.$$  \hspace{1cm} (55)

This LQ also has both left handed and right handed lepton couplings and violates baryon number. Now performing the Fierz transformation, the additional Wilson coefficients contribution to the $b \rightarrow q l^- l^+$ processes as

$$C_{NP}^V = C_{NP}^{\prime V} = \frac{-\pi}{\sqrt{2}G_F V_{tb} V_{ts}^* \alpha_{em}} \frac{(g_R)_{bl}(g_R)_{sl}^*}{M^2_V V_{(2)}},$$  \hspace{1cm} (56a)

$$-C_{NP}^{\prime V} = C_{NP}^V = \frac{\pi}{\sqrt{2}G_F V_{tb} V_{ts}^* \alpha_{em}} \frac{(g_L)_{bl}(g_L)_{sl}^*}{M^2_V V_{(2)}},$$  \hspace{1cm} (56b)

$$C_{NP}^P = C_{NP}^{\prime P} = \frac{\sqrt{2}\pi}{G_F V_{tb} V_{ts}^* \alpha_{em}} \frac{(g_R)_{sl}(g_R)_{bl}^*}{M^2_V V_{(2)}},$$  \hspace{1cm} (56c)

$$-C_{NP}^{\prime P} = C_{NP}^P = \frac{\sqrt{2}\pi}{G_F V_{tb} V_{ts}^* \alpha_{em}} \frac{(g_L)_{sl}(g_L)_{bl}^*}{M^2_V V_{(2)}}.$$  \hspace{1cm} (56d)

V. CONSTRAINT ON THE LEPTOQUARK COUPLINGS

After having an idea about all possible new physics contributions to the SM, we now proceed to constrain the new Wilson coefficients by comparing the theoretical and experimental branching ratios of various rare decay processes.
A. $B_{s,d} \rightarrow l^+l^-$ processes

The rare leptonic $B_{s,d} \rightarrow \mu^+\mu^-$ processes are mediated by the FCNC $b \rightarrow (s,d)$ transitions and in the SM the branching ratios depend only on the Wilson coefficient $C_A$. In addition to $C^{(t)}_{V,A}$ Wilson coefficients, vector LQ also contributes scalar and pseudoscalar ($C^{(t)}_{S,P}$) Wilson coefficients to the SM. However, there is no additional contributions of tensor Wilson coefficients $C_{T,T_5}$ due to the exchange of vector LQ.

The branching ratio of $B_q \rightarrow \mu^+\mu^-$ process in the LQ model is given by [27, 28]

$$\text{Br}(B_q \rightarrow \mu^+\mu^-) = \frac{G_F^2}{16\pi^3} \frac{\tau_B q \alpha_{em}^2 f_B^2 M_{B_q} m_\mu^2 |V_{tb} V_{tq}^*|^2}{2 m_\mu} \frac{1}{1 - \frac{4 m_\mu^2}{M_{B_q}^2}} \left| |P|^2 + |S|^2 \right|.$$  (57)

where

$$P \equiv \frac{C_{SM}^{SM} + C_{LQ}^{LQ} - C^{tLQ}_A}{C_{SM}^{SM}} + \frac{M_{B_q}^2}{2 m_\mu} \left( \frac{m_b}{m_b + m_s} \right) \left( \frac{C_P^{LQ} - C_{SM}^{LQ}}{C_{SM}^{SM}} \right) \equiv |P|e^{i\phi_P},$$

$$S \equiv \frac{1 - \frac{4 m_\mu^2}{M_{B_q}^2} \frac{M_{B_q}^2}{2 m_\mu} \left( \frac{m_b}{m_b + m_s} \right) \left( \frac{C_{SM}^{SLQ} - C_{LQ}^{SLQ}}{C_{SM}^{SM}} \right)}{1 - \frac{4 m_\mu^2}{M_{B_q}^2} \frac{M_{B_q}^2}{2 m_\mu} \left( \frac{m_b}{m_b + m_s} \right) \left( \frac{C_{SM}^{SLQ} - C_{LQ}^{SLQ}}{C_{SM}^{SM}} \right)} \equiv |S|e^{i\phi_S}.\quad (58)$$

Here $C^{(t)LQ}_A$ and $C^{(t)LQ}_{S,P}$ Wilson coefficients are generated due to the vector LQ exchange and are negligible in the SM, which implies $P^{SM} = 1$ and $S^{SM} = 0$. The experimental result is related to the theoretical predictions as [28]

$$\text{Br}^{th}(B_q \rightarrow \mu^+\mu^-) = \frac{1 - y_q^2}{1 + A_{\Delta\Gamma} y_q} \text{Br}^{exp}(B_q \rightarrow \mu^+\mu^-),\quad (59)$$

where $y_q = \tau_B \Delta \Gamma_q / 2$ and the observables $A_{\Delta\Gamma}$ is the mass eigenstate rate asymmetry equals to +1 in the SM. For calculational convenience, we define the parameter $R_q$ as

$$R_q = \frac{\text{Br}^{th}(B_q \rightarrow \mu^+\mu^-)}{\text{Br}^{SM}(B_q \rightarrow \mu^+\mu^-)} = |P|^2 + |S|^2.\quad (60)$$

If we apply chirality on vector LQ, then the $C^{(t)LQ}_{S,P}$ Wilson coefficients will vanish and there will be additional contribution of only $C^{(t)LQ}_{V,A}$ Wilson coefficients to the SM. Hence, the $R_q$ parameter can be given as [11, 18]

$$R_q = \left| 1 + \frac{C_{LQ}^{LQ} - C^{tLQ}_A}{C_{SM}^{SM}} \right|^2 \equiv \left| 1 + r e^{i\phi^{NP}} \right|^2,\quad (61)$$

where the parameters $r$ and $\phi^{NP}$ are related to the new Wilson coefficients as

$$r e^{i\phi^{NP}} = \frac{C_{LQ}^{LQ} - C^{tLQ}_A}{C_{SM}^{SM}}.\quad (62)$$
Now comparing the theoretical \[29\] branching ratios of \(B_q \to \mu^+\mu^-\) processes with the 1\(\sigma\) range of experimental values \[30\], the constraint on \(r\) and \(\phi^{NP}\) is computed for scalar LQ model in our previous work \[11, 18\]. If we assume that both the scalar and vector LQs have same order mass, \(M_{LQ} = 1\) TeV, one can use the same constraint on \(r\) and \(\phi^{NP}\) parameters to study the processes mediated via vector LQ. For \(B_s \to \mu^+\mu^-\) process, the constraints are found to be \[11\]

\[
0 \leq r \leq 0.35 , \quad \text{with} \quad \pi/2 \leq \phi^{NP} \leq 3\pi/2 ,
\]

(63)

and for \(B_d \to \mu^+\mu^-\) process \[11\]

\[
0.5 \leq r \leq 1.3 , \quad \text{for} \quad (0 \leq \phi^{NP} \leq \pi/2) \quad \text{or} \quad (3\pi/2 \leq \phi^{NP} \leq 2\pi) .
\]

(64)

Using Eqns. \[52, 54a, 54b\], this can be translated to obtain the bounds on LQ couplings (for \(M_{LQ} = 1\) TeV) as

\[
0 \leq |(g_L)_{se}(g_L)_{be}^*| \leq 2.3 \times 10^{-3} ,
\]

(65)

\[
0.7 \times 10^{-3} \leq |(g_L)_{de}(g_L)_{be}^*| \leq 1.81 \times 10^{-3} .
\]

(66)

Similarly using the theoretical predictions \[29\] and the experimental upper limits \[31, 32\] on \(B_q \to e^+e^- (\tau^+\tau^-)\) processes, the constraint on the product of scalar LQ couplings are presented in Table I, which are found to be rather loose as the measured branching ratios of \(B_{d,s} \to \tau^+\tau^- (e^+e^+)\) are not very precise.

TABLE I: Constraints on leptoquark couplings obtained from various leptonic \(B_{s,d} \to l^+l^-\) decays.

| Decay Process | Couplings involved | Upper bound of the couplings |
|---------------|--------------------|------------------------------|
| \(B_s \to e^\pm e^\mp\) | \(|(g_L)_{se}(g_L)_{be}^*| \) | \(< 11.8\) |
| \(B_s \to \tau^\pm \tau^\mp\) | \(|(g_L)_{st}(g_L)_{bt}^*| \) | \(< 0.4\) |
| \(B_d \to e^\pm e^\mp\) | \(|(g_L)_{de}(g_L)_{be}^*| \) | \(< 8.0\) |
| \(B_d \to \tau^\pm \tau^\mp\) | \(|(g_L)_{dr}(g_L)_{bt}^*| \) | \(< 0.593\) |

For simplicity we can neglect the NP contributions to the \(C_{V,A}^{LQ}\) Wilson coefficients, as the \(C_{S,P}^{LQ}\) Wilson coefficients are enhanced by the factor \(M_{B_q}^2/m_l\). Now using Eqns. \[75\],

\[\text{...}\]
(54c), (54d) and (60), the $R_q$ parameter becomes

$$R_q = \frac{|C_S^{LQ} - C_S'^{LQ}|^2}{r_q^2} + \left|1 - \frac{|C_S^{LQ} + C_S'^{LQ}|}{r_q}\right|^2$$  \hspace{1cm} (67)

where

$$r_q = \frac{2m_t(m_b + m_q)C_{SM}}{M_{B_q}^2}.$$  \hspace{1cm} (68)

FIG. 1: Constraint on the combination of scalar Wilson coefficient from $B_s \to \mu^+\mu^-$ process. The left panel is for real $C_S^{LQ} \pm C_S'^{LQ}$ Wilson coefficients and right panel is for complex Wilson coefficients.

FIG. 2: Constraint on $C_S^{LQ} \pm C_S'^{LQ}$ Wilson coefficients from $B_d \to \mu^+\mu^-$ process. The left panel is for real Wilson coefficients and right panel is for complex Wilson coefficients.
Now comparing the theoretical and experimental values of $B_q \rightarrow l^+l^-$ decays, we calculate the allowed region of $C_{S}^{LQ} \pm C_{S}^{\prime LQ}$ Wilson coefficients. If the Wilson coefficients are real, Eqn. (67) will be a circle of radius $|r_q| \sqrt{R_{q}^{\text{expt}}}$ with center at $(C_{S}^{LQ} + C_{S}^{\prime LQ}, C_{S}^{LQ} - C_{S}^{\prime LQ}) = (r_q, 0)$. The left panel of Fig. 1 represents the constraint on real $C_{S}^{LQ} \pm C_{S}^{\prime LQ}$ Wilson coefficients from $B_s \rightarrow \mu^+\mu^-$ process and the right panel is for complex Wilson coefficients. Similarly in Fig. 2, we show the constraint on real (left panel) and complex (right panel) Wilson coefficients for $B_d \rightarrow \mu^+\mu^-$ process. The allowed range of real Wilson coefficients from $B_s \rightarrow e^+e^-$ (left panel) and $B_d \rightarrow e^+e^-$ (right panel) processes are shown in Fig. 3. In Fig. 4, we present the constraint obtained from $B_s \rightarrow \tau^+\tau^-$ (left panel) and $B_d \rightarrow \tau^+\tau^-$ (right panel) processes. The allowed region of $C_{S}^{LQ} \pm C_{S}^{\prime LQ}$ real Wilson coefficients obtained from $B_q \rightarrow l^+l^-$ processes are presented in Table II. Now using the constrained Wilson coefficients, one can calculate the bound on the product of various LQ couplings from Eqns. (54c, 54d).

![Image](image_url)

FIG. 3: The allowed region of $C_{S}^{LQ} - C_{S}^{\prime LQ}$ and $C_{S}^{LQ} + C_{S}^{\prime LQ}$ Wilson coefficients from $B_s \rightarrow e^+e^-$ (left panel) and $B_d \rightarrow e^+e^-$ (right panel) processes.

B. $K_L \rightarrow \mu^+\mu^-(e^+e^-)$ process

The constraint on the product of various LQ couplings from the rare leptonic decays of $K$ meson are discussed in this subsection. The rare $K_L \rightarrow \mu^+\mu^-$ decay mode has both the long and short distance contributions and the dominant contribution comes from the long-distance two photon intermediates state $K_L \rightarrow \gamma^*\gamma^* \rightarrow \mu^+\mu^-$. Only the short distance (SD) part can be calculated reliably and the estimated branching ratio of the SD part
FIG. 4: The allowed region of $C_{S}^{LQ} - C_{S}'^{LQ}$ and $C_{S}^{LQ} + C_{S}'^{LQ}$ Wilson coefficients from $B_{s} \to \tau^{+}\tau^{-}$ (left panel) and $B_{d} \to \tau^{+}\tau^{-}$ (right panel) processes.

TABLE II: Constraint on combinations of $C_{S}^{(t)LQ}$ Wilson coefficients from various leptonic $B_{s,d} \to l^{+}l^{-}$ decays.

| Decay Process  | Bound on $C_{S}^{LQ} + C_{S}'^{LQ}$ | Bound on $C_{S}^{LQ} - C_{S}'^{LQ}$ |
|----------------|------------------------------------|-------------------------------------|
| $B_{s} \to \mu^{+}\mu^{-}$ | $0.0 \to 0.32$                     | $0.1 \to 0.18$                     |
| $B_{s} \to e^{+}e^{-}$     | $-1.4 \to 1.4$                    | $-1.4 \to 1.4$                    |
| $B_{s} \to \tau^{+}\tau^{-}$ | $-150 \to 150$                | $-150 \to 150$                |
| $B_{d} \to \mu^{+}\mu^{-}$ | $-0.16 \to 0.44$                | $0.2 \to 0.36$                   |
| $B_{d} \to e^{+}e^{-}$     | $-4 \to 4$                       | $-4 \to 4$                       |
| $B_{d} \to \tau^{+}\tau^{-}$ | $-1000 \to 1000$              | $-1000 \to 1000$              |

is $\text{Br}(K_{L} \to \mu^{+}\mu^{-})|_{\text{SD}} < 2.5 \times 10^{-9}$ [33]. In the SM the effective Hamiltonian for the $K_{L} \to \mu^{+}\mu^{-}$ process is given by [34]

$$H_{\text{eff}} = \frac{G_{F}}{\sqrt{2} \frac{2\pi}{2\sin^{2}\theta_{W}}} \left( \lambda_{c} Y_{NL} + \lambda_{t} Y(x_{t}) \right) \left( \bar{s}\gamma^{\mu}(1 - \gamma_{5})d \right) \left( \bar{\mu}\gamma_{\mu}(1 - \gamma_{5})\mu \right),$$

(69)

$$= \frac{G_{F}}{\sqrt{2} \frac{2\pi}{2\sin^{2}\theta_{W}}} \lambda_{u} C_{K}^{SM} \left( \bar{s}\gamma^{\mu}(1 - \gamma_{5})d \right) \left( \bar{\mu}\gamma_{\mu}(1 - \gamma_{5})\mu \right),$$

(70)

where $\lambda_{i} = V_{id}V_{is}^{*}$, $x_{t} = m_{t}^{2}/M_{W}^{2}$ and $\sin^{2}\theta_{W} = 0.23$ and $C_{K}^{SM}$ is the SM Wilson coefficient given as

$$C_{K}^{SM} = \frac{\lambda_{c} Y_{NL} + \lambda_{t} Y(x_{t})}{\sin^{2}\theta_{W} \lambda_{u}}.$$  

(71)
The functions $Y_{NL}$ and $Y(x_t)$ are the contributions from charm and top quark respectively and the $Y(x_t)$ function in the next-to-leading order (NLO) is \[35\]

$$Y(x_t) = \eta Y \frac{x_t}{8} \left( \frac{4 - x_t}{1 - x_t} + \frac{3x_t}{(1 - x_t)^2} \ln x_t \right). \quad (72)$$

The branching ratio for the SD part of $K_L \rightarrow \mu^+\mu^-$ process in the SM is given by

$$\text{Br}(K_L \rightarrow \mu^+\mu^-)|_{SD} = \tau_K \frac{G_F^2}{2\pi} |\lambda_u|^2 \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} f_K^2 M_K m_\mu^2 |C_{SM}^K|^2. \quad (73)$$

Now including the contribution of $V^{(1)}(3, 1, 2/3)$ leptoquark, the total branching ratio of $K_L \rightarrow \mu^+\mu^-$ process is given by

$$\text{Br}(K_L \rightarrow \mu^+\mu^-) = \frac{G_F^2}{8\pi^3} \tau_K \alpha_{em} f_K^2 M_K m_\mu^2 |\lambda_u|^2 |C_{SM}^K|^2 \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} \times (|P_K|^2 + |S_K|^2), \quad (74)$$

where

$$P_K \equiv \frac{C_{SM}^K + C_{A}^{LQ} - C_{A}^{LQ}}{C_{SM}^K} + \frac{M_K^2}{2m_\mu} \left( \frac{m_s}{m_s + m_d} \right) \left( \frac{C_{LQ}^{LQ} - C_{LQ}^{LQ}}{C_{SM}^K} \right),$$

$$S_K \equiv \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} \frac{M_K^2}{2m_\mu} \left( \frac{m_s}{m_s + m_d} \right) \left( \frac{C_{LQ}^{LQ} - C_{LQ}^{LQ}}{C_{SM}^K} \right). \quad (75)$$

It should be noted that for $K_L \rightarrow \mu^+\mu^-$ decay process, CP violation in $K - \bar{K}$ mixing is irrelevant and $K_L$ can be treated as a pure CP-odd state. Therefore, we have to take into account the contributions of both $K^0$ and $\bar{K}^0$ amplitudes, which can be done by replacing the leptoquark couplings $(g_L)_{d\mu}(g_L)_{s\mu}^* \rightarrow \sqrt{2} \text{Re}[(g_L)_{d\mu}(g_L)_{s\mu}^*]$. Thus, the new $C_i^{LQ}$ coefficients arise due to the exchange of vector leptoquark and are defined as

$$C_A^{LQ} = -\frac{\pi}{G_F \alpha_{em} \lambda_u} \frac{\text{Re}[(g_L)_{d\mu}(g_L)_{s\mu}^*]}{M_V^{(1)}}, \quad (76a)$$

$$C_A'^{LQ} = -\frac{\pi}{G_F \alpha_{em} \lambda_u} \frac{\text{Re}[(g_R)_{d\mu}(g_R)_{s\mu}^*]}{M_V^{(1)}}, \quad (76b)$$

$$C_S^{LQ} = -C_P^{LQ} = \frac{\pi}{2G_F \alpha_{em} \lambda_u} \frac{\text{Re}[(g_L)_{d\mu}(g_L)_{s\mu}^*]}{M_V^{(1)}}, \quad (76c)$$

$$C_P'^{LQ} = C_P'^{LQ} = \frac{\pi}{2G_F \alpha_{em} \lambda_u} \frac{\text{Re}[(g_R)_{d\mu}(g_R)_{s\mu}^*]}{M_V^{(1)}}. \quad (76d)$$

In the presence of $V^{(3)}(3, 3, 2/3)$ leptoquark, the branching is given by

$$\text{Br}(K_L \rightarrow \mu^+\mu^-) = \frac{G_F^2}{8\pi^3} \tau_K \alpha_{em} f_K^2 M_K m_\mu^2 |\lambda_u|^2 \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} \times \frac{C_{SM}^K + C_A^{LQ}}{2}. \quad (77)$$
For muonic decay the experimentally measured branching ratio is \( \text{Br}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9} \) \cite{13} and for \( K_L \to e^+e^- \) process the branching ratio is \( \text{Br}(K_L \to e^+e^-) = 9_{-4}^{+6} \times 10^{-12} \) \cite{13}. If we apply chirality on the leptoquark, then only \( C_A^{(LQ)} \) Wilson coefficients will contribute. Now comparing Eqn. (74) with the experimental branching ratio of \( K_L \to \mu^+\mu^-(e^+e^-) \) processes, the constraint on the leptoquark couplings for \( M_{LQ} = 1 \) TeV are given by

\[
1.3 \times 10^{-3} \leq \text{Re}[(g_L)_{de}(g_L)_{se}^*] \leq 2.35 \times 10^{-3}, \tag{78}
\]

\[
1.4 \times 10^{-4} \leq \text{Re}[(g_L)_{d\mu}(g_L)_{\mu\mu}^*] \leq 1.5 \times 10^{-4}. \tag{79}
\]

Now by neglecting the \( C_A^{LQ} \) coefficients, the constraints on \( (C_S^{LQ} \pm C_S^{\mu LQ}) \) Wilson coefficients from \( K_L \to e^+e^- \) (left panel) and \( K_L \to \mu^+\mu^- \) (right panel) are shown in Fig. 5. From the figure the allowed regions of LQ couplings for \( K_L \to e^+e^- \) process are given by

\[
-2 \times 10^{-4} \leq C_S^{LQ} + C_S^{\mu LQ} \leq 2 \times 10^{-4}, \tag{80}
\]

\[
1.25 \times 10^{-4} \leq C_S^{LQ} - C_S^{\mu LQ} \leq 2 \times 10^{-4}, \tag{81}
\]

and for \( K_L \to \mu^+\mu^- \) process

\[
-6 \times 10^{-3} \leq C_S^{LQ} + C_S^{\mu LQ} \leq 3 \times 10^{-3}, \tag{82}
\]

\[
5 \times 10^{-5} \leq C_S^{LQ} - C_S^{\mu LQ} \leq 5.6 \times 10^{-3}. \tag{83}
\]

![Graph](image)

**FIG. 5:** The allowed region of \( C_S^{LQ} \pm C_S^{\mu LQ} \) Wilson coefficients from \( K_L \to e^+e^- \) (left panel) and \( K_L \to \mu^+\mu^- \) (right panel) processes.
C. $\tau^- \to \mu^- \gamma$ process

In this subsection we compute the constraint on $(3, 1, 2/3)$ vector LQ couplings from the charged lepton flavour violating processes like $\tau^- \to l^- \gamma$, where $l = \mu, e$. These radiative decays provide an important testing ground for many new physics beyond the SM. The similar analysis in the context of scalar LQs can be found in the literature [24]. In the Ref. [36], the authors have given the general loop formulas for the radiative decay modes. The effective Hamiltonian for $\tau^- \to \mu^- \gamma$ process is given by

$$\mathcal{H}_{\text{eff}} = e \left( C_L \bar{\mu} R \sigma^{\mu \nu} F_{\mu \nu} \tau_L + C_R \bar{\mu} L \sigma^{\mu \nu} F_{\mu \nu} \tau_R \right),$$

where $\sigma^{\mu \nu}$ is the photon field strength tensor and the Wilson coefficients $C_{L,R}$ are expressed as

$$C_L = \frac{N_c}{16\pi^2 M^2_{V(1)}} \left( -\frac{1}{3} \left[ (g_L)_{b\tau} (g_L)_{b\mu}^* f_2(x_b) + (g_R)_{b\tau} (g_R)_{b\mu}^* f_1(x_b) 
\right.ight.

$$

\[\left. + (g_L)_{b\tau} (g_R)_{b\mu}^* f_3(x_b) + (g_R)_{b\tau} (g_L)_{b\mu} f_4(x_b) \right]

\[+ 2 \left[ (g_L)_{b\tau} (g_L)_{b\mu}^* \bar{f}_2(x_b) + (g_R)_{b\tau} (g_R)_{b\mu}^* \bar{f}_1(x_b) \right.
\]

\[\left. + (g_L)_{b\tau} (g_R)_{b\mu} f_3(x_b) + (g_R)_{b\tau} (g_L)_{b\mu} f_4(x_b) \right] \right) + \frac{N_c}{16\pi^2 M^2_{V(3)}} (g_L)_{b\tau} (g_L)_{b\mu}^* \left[ -\frac{1}{6} \bar{f}_2^{(3)}(x_b) + \frac{1}{3} \bar{f}_1^{(3)}(x_b) \right],

(85)

$$C_R = \frac{N_c}{16\pi^2 M^2_{V(1)}} \left( -\frac{1}{3} \left[ (g_L)_{b\tau} (g_L)_{b\mu}^* f_1(x_b) + (g_R)_{b\tau} (g_R)_{b\mu} f_2(x_b) \right.ight.

\[\left. + (g_L)_{b\tau} (g_R)_{b\mu}^* f_3(x_b) + (g_R)_{b\tau} (g_L)_{b\mu}^* f_4(x_b) \right]

\[+ 2 \left[ (g_L)_{b\tau} (g_L)_{b\mu}^* \bar{f}_1(x_b) + (g_R)_{b\tau} (g_R)_{b\mu}^* \bar{f}_2(x_b) \right.
\]

\[\left. + (g_L)_{b\tau} (g_R)_{b\mu} \bar{f}_3(x_b) + (g_R)_{b\tau} (g_L)_{b\mu} \bar{f}_4(x_b) \right] \right) + \frac{N_c}{16\pi^2 M^2_{V(3)}} (g_L)_{b\tau} (g_L)_{b\mu} \left[ -\frac{1}{6} \bar{f}_1^{(3)}(x_b) + \frac{1}{3} \bar{f}_2^{(3)}(x_b) \right].

(86)
Here $N_c = 3$ is the color factor, $x_b = m_b^2/M_{LQ}^2$ (where $M_{LQ} = M_{V(1)}$ or $M_{V(3)}$) and the loop functions are given as [36]

\[
\begin{align*}
    f_1(x_b) &= m_\tau \left[ \frac{-5x_b^3 + 9x_b^2 - 30x_b + 8}{12(x_b - 1)^3} + \frac{3x_b^2 \ln x_b}{2(x_b - 1)^4} \right], \\
    f_2(x_b) &= m_\mu \left[ \frac{-5x_b^3 + 9x_b^2 - 30x_b + 8}{12(x_b - 1)^3} + \frac{3x_b^2 \ln x_b}{2(x_b - 1)^4} \right], \\
    f_3(x_b) &= m_b \left[ \frac{x_b^2 + x_b + 4}{2(x_b - 1)^2} - \frac{3x_b \ln x_b}{(x_b - 1)^3} \right], \\
    f_4(x_b) &= -\frac{m_\tau m_\mu m_b}{m_{V(1)}^2} \left[ \frac{-2x_b^2 + 7x_b - 11}{6(x_b - 1)^3} + \frac{\ln x_b}{(x_b - 1)^4} \right], \\
    \bar{f}_1(x_b) &= m_\tau \left[ \frac{-4x_b^3 + 45x_b^2 - 33x_b + 10}{12(x_b - 1)^3} - \frac{3x_b^3 \ln x_b}{2(x_b - 1)^4} \right], \\
    \bar{f}_2(x_b) &= m_\mu \left[ \frac{-4x_b^3 + 45x_b^2 - 33x_b + 10}{12(x_b - 1)^3} - \frac{3x_b^3 \ln x_b}{2(x_b - 1)^4} \right], \\
    \bar{f}_3(x_b) &= m_b \left[ \frac{x_b^2 - 11x_b + 4}{2(x_b - 1)^2} + \frac{3x_b^2 \ln x_b}{(x_b - 1)^3} \right], \\
    \bar{f}_4(x_b) &= \frac{m_\tau m_\mu m_b}{m_{V(1)}^2} \left[ \frac{x_b^2 - 5x_b - 6 - 6x_b(1 + x_b) \ln x_b}{6(x_b - 1)^3} + \frac{x_b^3 \ln x_b}{(x_b - 1)^4} \right].
\end{align*}
\]

The branching ratio of $\tau^- \to \mu^- \gamma$ process is given by

\[
\text{Br}(\tau^- \to \mu^- \gamma) = \frac{\tau_\tau (m_\tau^2 - m_\mu^2)^3}{16\pi m_b^2} \left[ |C_L|^2 + |C_R|^2 \right].
\]

(88)

This expression can be applied to study other LFV radiative decays like, $\tau^- \to e^- \gamma$ and $\mu^- \to e^- \gamma$. The current upper bounds on the branching ratios of $\tau^- \to \mu^- (e^-) \gamma$ is given by [13]

\[
\begin{align*}
    \text{Br}(\tau^- \to \mu^- \gamma) &< 4.4 \times 10^{-8}, \\
    \text{Br}(\tau^- \to e^- \gamma) &< 3.3 \times 10^{-8}.
\end{align*}
\]

(89)

Comparing Eqn. (88) with the current experimental bounds (89), the allowed regions of $(g_{L(R)}(g_{L(R)})_{bl})$ couplings from $\tau^- \to e^- \gamma$ (left panel) and $\tau^- \to \mu^- \gamma$ (right panel) are shown in Fig. 7, and the constraints on the $(g_{L(R)}(g_{R(L)})_{bl})$ leptoquark couplings from $\tau^- \to e^- \gamma$ (left panel) and $\tau^- \to \mu^- \gamma$ (right panel) are presented in Fig. 8. The numerical values of the constraints on leptoquark couplings are given in Table III. These bounds are
rather weak in comparison to $B_{s,d} \rightarrow \mu^+\mu^-$ processes and they also involve the coupling only to $b$ quark in both the LQ coupling-parameters.

![Plot 6](image6.png)

**FIG. 6:** The constraint on $(g_L)_{bl_i}(g_L)_{bl_j}^*$ and $(g_R)_{bl_i}(g_R)_{bl_j}^*$ leptoquark couplings from $\tau^- \rightarrow e^-\gamma$ (left panel) and $\tau^- \rightarrow \mu^-\gamma$ (right panel) processes.

![Plot 7](image7.png)

**FIG. 7:** The constraint on $(g_L)_{bl_i}(g_R)_{bl_j}^*$ and $(g_R)_{bl_i}(g_L)_{bl_j}^*$ leptoquark couplings from $\tau^- \rightarrow e^-\gamma$ (left panel) and $\tau^- \rightarrow \mu^-\gamma$ (right panel) processes.

**VI. NUMERICAL ANALYSIS OF LFV DECAYS**

After having detailed knowledge about the observables and the bound on new Wilson coefficients, we now proceed for numerical analysis of LFV decays in the LQ model. Though LFV decays are extremely rare in the SM due to loop suppression and the presence of tiny neutrino mass in the loop, still they can occur at tree level and are expected to have significantly large branching ratios in the LQ model. There will be no contributions from
TABLE III: Constraints on leptoquark couplings obtained from $\tau^- \to l^- \gamma$ processes.

| Couplings involved | $\tau^- \to e^- \gamma$ process | $\tau^- \to \mu^- \gamma$ process |
|--------------------|---------------------------------|-----------------------------------|
| $(g_L)_{b\tau} (g_L)_{bl}^*$ | $-0.14 \to 0.14$ | $-0.16 \to 0.16$ |
| $(g_R)_{b\tau} (g_R)_{bl}^*$ | $-0.14 \to 0.14$ | $-0.16 \to 0.16$ |
| $(g_L)_{b\tau} (g_R)_{bl}^*$ | $-0.04 \to 0.04$ | $-0.05 \to 0.05$ |
| $(g_R)_{b\tau} (g_L)_{bl}^*$ | $-0.04 \to 0.04$ | $-0.05 \to 0.05$ |

SM Wilson coefficients in the LFV decays of $B$ meson.

FIG. 8: The variation of branching ratios of $B^+ \to K^+ \mu^- e^+$ (top left panel), $B^+ \to K^+ \tau^- e^+$ (top right panel) and $B^+ \to K^+ \tau^- \mu^+$ (bottom panel) processes (in units of GeV$^{-2}$) in the $V^{1,3}$ vector leptoquark model. Here purple bands represent the contribution from $V^1$ leptoquark model and green solid lines are for $V^3$ leptoquark.

In the presence of LQ, the modified helicity amplitudes are given as

$$H_V^{0LQ} = \sqrt{\frac{\lambda}{q^2}} \left( C_V^{LQ} + C_V^{\prime LQ} \right) f_+(q^2),$$  

(90)
\[ H_{V}^{LQ} = \frac{M_{B}^{2} - M_{K}^{2}}{\sqrt{q^{2}}} \left( C_{V}^{LQ} + C_{V}^{dLQ} \right) f_{0}(q^{2}), \]  
\[ H_{A}^{LQ} = \frac{\lambda}{q^{2}} \left( C_{A}^{LQ} + C_{A}^{dLQ} \right) f_{+}(q^{2}), \]  
\[ H_{A}^{LQ} = \frac{M_{B}^{2} - M_{K}^{2}}{\sqrt{q^{2}}} \left( C_{A}^{LQ} + C_{A}^{dLQ} \right) f_{0}(q^{2)}, \]  
\[ H_{S}^{LQ} = \frac{M_{B}^{2} - M_{K}^{2}}{m_{b}} \left( C_{S}^{LQ} + C_{S}^{dLQ} \right) f_{0}(q^{2}), \]  
\[ H_{P}^{LQ} = \frac{M_{B}^{2} - M_{K}^{2}}{m_{b}} \left( C_{P}^{LQ} + C_{P}^{dLQ} \right) f_{0}(q^{2}). \]  

FIG. 9: The variation of forward-backward asymmetries of $B^{+} \to K^{+} \mu^{-} e^{+}$ (top left panel), $B^{+} \to K^{+} \tau^{-} e^{+}$ (top right panel) and $B^{+} \to K^{+} \tau^{-} \mu^{+}$ (bottom panel) processes in the leptoquark model.

For numerical analysis we have taken the particle masses and life times of $B_{q}$ mesons from [13]. The form factors ($f_{0,+}$) for kaon and pion are taken from [37] and [38] respectively. In order to compute the required LQ couplings, we use the values of the couplings extracted from $B_{s,d} \to l^{+}l^{-}$, as given in Table I and II. Although the bounds obtained from $K_{L} \to l^{+}l^{-}$ processes [79] are little stronger than the bounds obtained from $B_{s,d} \to l^{+}l^{-}$,
FIG. 10: The plots for lepton non-universality parameters, $R_{Ke}^{\mu e}$ (top right panel), $R_{Ke}^{\tau e}$ (bottom left panel) and $R_{Ke}^{\tau\mu}$ (bottom right panel) in high $q^2$ region. Here the top left panel shows the non-universality $R_{Ke}^{\mu e}$ in low $q^2 \in [1, 6]$ region.

only the Real part of the couplings can be constrained there. Therefore, in our analysis, we consider the constraints from Table I and II as basis values and assume that the LQ couplings between different generation of quark and lepton follow the simple scaling law, i.e.,

$$(g_{L(R)})_{ij} = (m_i/m_j)^{1/2}(g_{L(R)})_{ii}$$

with $j>i$. This ansatz has taken from the Ref. [39], which can explain the decay width of radiative LFV $\mu \to e\gamma$ decay. Now using the constrained LQ parameter space, we calculate the branching ratios, forward-backward asymmetries and lepton non-universality in $B \to K(\pi)l_i^-l_j^+$ processes. In Fig. 9, we show the variation of branching ratios of $B^+ \to K^+\mu^-\pi^+$ (top left panel), $B^+ \to K^+\mu^-\pi^+$ (top right panel) and $B^+ \to K^+\mu^-\pi^+$ (bottom panel) processes with respect to $q^2$ in both $V^{1,3}$ leptoquark model. Here the purple bands represent the predictions in the $V^1$ vector LQ model and green solid lines are for $V^3$ leptoquark. The predicted branching ratios of $B^+ \to K^+l_i^-l_j^+$ processes in both the LQ model are presented in Table IV. The plot for forward-backward asymmetries of $B^+ \to K^+\mu^-\pi^+$ (top left panel), $B^+ \to K^+\tau^-e^+$ (top right panel) and $B^+ \to K^+\tau^-\mu^+$ (bottom panel) processes in the LQ model are given in Fig. 10. Fig. 11 shows the $q^2$
FIG. 11: The plots for lepton non-universality parameters, $R_{K\mu}^{\mu e}$ (top right panel), $R_{K\mu}^{\tau e}$ (bottom left panel) and $R_{K\mu}^{\tau\mu}$ (bottom right panel) in high $q^2$ region. Here the top left panel shows the non-universality $R_{K\mu}^{\mu e}$ in low $q^2 \in [1, 6]$ region.

FIG. 12: The variation of lepton non-universality parameter $R_{K\mu}^{\mu e}$ for low $q^2 \in [1, 6]$ (left panel) and high $q^2$ (right panel) regimes.

The variation of $R_{K\mu}^{\mu e}$ (top right panel), $R_{K\mu}^{\tau e}$ (bottom left panel) and $R_{K\mu}^{\tau\mu}$ (bottom right panel) parameters in the high $q^2$ region. The variation of $R_{K\mu}^{\mu e}$ (top right panel), $R_{K\mu}^{\tau e}$ (bottom left panel) and $R_{K\mu}^{\tau\mu}$ (bottom right panel) observables are presented in Fig. 12. The variation of $R_{K\mu}^{\mu e}$ and $R_{K\mu}^{\mu e}$ parameters in low $q^2 \in [1, 6]$ region are also shown in the top left panel of
FIG. 13: The $q^2$ variation of branching ratios of $B^+ \rightarrow \pi^+\mu^-e^+$ (top left panel), $B^+ \rightarrow \pi^+\tau^-e^+$ (top right panel) and $B^+ \rightarrow \pi^+\tau^-\mu^+$ (bottom panel) processes in the $V^{1,3}$ vector leptoquark model.

Fig. 11 and Fig. 12 respectively. The integrated values of forward-backward asymmetries and lepton non-universality parameters such as $R_{K_l}^{\mu\ell_i}$, $R_{Ke}^{\mu\ell_i}$, $R_{K\mu}^{\mu\ell_i}$ are given in Table V. In Fig. 13 we show the plot for lepton non-universality parameters $R_{K}^{\mu\mu}$ in low $q^2$ (left panel) and high $q^2$ (right panel) and the predicted values are presented in Table V. Here the solid red lines denote the SM contributions and integrated values in the SM are given by

$$R_{K}^{\mu\mu}|_{q^2 \in [1, 6]} = 1.001, \quad R_{K}^{\mu\mu}|_{q^2 \geq 14.18} = 1.003, \quad R_{Ke}^{\tau\tau} = 1.144, \quad R_{K\mu}^{\tau\tau} = 1.14.$$

(96)

Analogously we show the variation of the branching ratios of LFV $B^+ \rightarrow \pi^+\mu^-e^+$ (top left panel), $B^+ \rightarrow \pi^+\tau^-e^+$ (top right panel) and $B^+ \rightarrow \pi^+\tau^-\mu^+$ (bottom panel) decay processes with respect to $q^2$ in Fig. 14 and the predicted branching ratios are given in Table IV. Fig. 15 shows the variation of forward-backward asymmetries in $B^+ \rightarrow \pi^+\mu^-e^+$ (top left panel), $B^+ \rightarrow \pi^+\tau^-e^+$ (top right panel) and $B^+ \rightarrow \pi^+\tau^-\mu^+$ (bottom panel) processes. The lepton non-universality parameters $R_{\pi\mu}^{\mu\mu}$ (top right panel), $R_{\pi\mu}^{\tau\tau}$ (bottom left panel) and $R_{\pi\mu}^{\tau\tau}$ (bottom right panel) are presented in Fig. 16. Also, we present the behaviour of $R_{\pi\mu}^{\mu\mu}$ parameter (top left panel) in the region $1 \leq q^2 \leq 6$ GeV$^2$. In Table VI, we present the
predicted values of forward-backward asymmetries and lepton non-universality parameters. The non-universality predictions of $B \rightarrow \pi l^+l^-$ processes in the SM are

$$R^\mu_\pi|_{q^2 \in [1,6]} = 1.001, \quad R^\mu_\pi|_{q^2 \geq 14.18} = 1.003, \quad R^{\tau\tau}_\pi = 1.149, \quad R^{\tau\tau}_\pi = 1.146, \quad (97)$$

and the values for the parameter $R^\mu_\pi$ in the LQ model are listed in Table VI.

The predicted values of $R^i_{ij}$ in $V^{1,3}$ LQ model respectively are

$$R^\mu e|_{V^1 LQ} = 0.525 - 53.34, \quad R^\mu e|_{V^3 LQ} = 0.536, \quad (98)$$

$$R^{\tau\mu}|_{V^1 LQ} = 0.536 - 64.1, \quad R^{\tau\mu}|_{V^3 LQ} = 0.578, \quad (99)$$

$$R^{\tau e}|_{V^1 LQ} = 0.443 - 0.56, \quad R^{\tau e}|_{V^3 LQ} = 0.56. \quad (100)$$

| Decay process      | Values in $V^1$ LQ model     | Values in $V^3$ LQ model     | Expt. upper limit [13] |
|--------------------|-------------------------------|-------------------------------|------------------------|
| $B^+ \rightarrow K^+ \mu^−e^+$ | $(0.009 - 6.16) \times 10^{-10}$ | $< 2.98 \times 10^{-10}$ | $< 9.1 \times 10^{-8}$ |
| $B^+ \rightarrow K^+ \tau^−e^+$ | $(0.118 - 1.882) \times 10^{-10}$ | $< 7.12 \times 10^{-11}$ | $< 4.3 \times 10^{-5}$ |
| $B^+ \rightarrow K^+ \tau^−\mu^+$ | $(0.0064 - 5.58) \times 10^{-9}$ | $< 2.52 \times 10^{-9}$ | $< 4.5 \times 10^{-5}$ |
| $B^+ \rightarrow \pi^+ \mu^−e^+$ | $(0.48 - 3.23) \times 10^{-10}$ | $< 1.6 \times 10^{-10}$ | $< 6.4 \times 10^{-3}$ |
| $B^+ \rightarrow \pi^+ \tau^−e^+$ | $5.23 \times 10^{-12} - 1.51 \times 10^{-6}$ | $< 7.55 \times 10^{-7}$ | $< 7.4 \times 10^{-5}$ |
| $B^+ \rightarrow \pi^+ \tau^−\mu^+$ | $(0.41 - 2.99) \times 10^{-9}$ | $< 1.46 \times 10^{-9}$ | $< 6.2 \times 10^{-5}$ |

VII. $(g - 2)_\mu$

The recent experimental measurement [40] of the anomalous magnetic moment of muon, i.e., $(g - 2)_\mu$ has about 3σ discrepancies from the SM prediction and has set off a flurry of excitement amongst theorists. The experimental result of anomalous magnetic moment of muon is given by [41]

$$a^\text{exp}_\mu = 116592080(63) \times 10^{-11}, \quad (101)$$

which when compared to the SM value

$$a^\text{SM}_\mu = 116591785(61) \times 10^{-11}, \quad (102)$$
FIG. 14: The variations of forward-backward asymmetries of $B^+ \to \pi^+ \mu^- e^+$ (left panel), $B^+ \to \pi^+ \tau^- e^+$ (middle panel) and $B^+ \to \pi^+ \tau^- \mu^+$ (right panel) processes in leptoquark model.

has the discrepancy

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (295 \pm 88) \times 10^{-11}. \quad (103)$$

The absolute magnitude of the discrepancy is small and can be accommodate by adding the new physics contributions. The vector LQ contribution to $a_\mu$ is given by

$$\Delta a_\mu = -2 N_c m_\mu \left[ \langle g_L \rangle_{b\mu} \langle g_R \rangle_{b\mu} \left(- \frac{1}{3} (f_3(x_b) + f_4(x_b)) + \frac{2}{3} (\bar{f}_3(x_b) + \bar{f}_4(x_b)) \right) + \left( \langle |g_L \rangle_{b\mu}|^2 + \langle |g_R \rangle_{b\mu}|^2 \right) \left(- \frac{1}{3} f_1(x_b) + \frac{2}{3} \bar{f}_1(x_b) \right) \right]$$

$$-2 N_c m_\mu \left( \langle |g_L \rangle_{b\mu}|^2 + \langle |g_R \rangle_{b\mu}|^2 \right) \left(- \frac{1}{6} f_1(x_b) + \frac{1}{3} \bar{f}_1(x_b) \right), \quad (104)$$

where the loop functions are given in sec. V C. Now using the constrained leptoquark couplings from $\tau^- \to \mu^- \gamma$ process with the scaling law as discussed in section VI, $\Delta a_\mu$ in the leptoquark model is found as

$$2.38 \times 10^{-9} \leq \Delta a_\mu \leq 2.95 \times 10^{-9}, \quad (105)$$

which is within its $1\sigma$ range.
TABLE V: The predicted values of forward backward asymmetries and lepton non-universality parameters in $B^+ \to K^+ l_i^- l_j^+$ process in the $V^{1,3}$ vector leptoquark model. Also the predicted values of $R^{\mu\mu}_K$, $R^{l_i l_j}_K$, $R^{l_i l_j}_{K\mu}$ parameters for muonic (electronic) processes in low $q^2 \in [1,6]$ region.

| Observables | Values in $V^1$ LQ model | Values in $V^3$ LQ model |
|-------------|--------------------------|--------------------------|
| $\langle A^{K\mu\nu}_{FB} \rangle$ | $7.65 \times 10^{-3}$ | $2.82 \times 10^{-3}$ |
| $\langle A^{K\tau\nu}_{FB} \rangle$ | $0.285$ | $0.285$ |
| $\langle A^{K\tau\mu}_{FB} \rangle$ | $(0.039 - 0.105) \times 10^{-3}$ | $4.8 \times 10^{-3}$ |
| $\langle R^{\mu\nu}_{Ke} \rangle_{|q^2\in[1,6]}$ | $(0.0038 - 3.5) \times 10^{-4}$ | $2.03 \times 10^{-4}$ |
| $\langle R^{\mu\nu}_{Ke} \rangle$ | $(0.033 - 3.36) \times 10^{-4}$ | $2.04 \times 10^{-4}$ |
| $\langle R^{\tau\nu}_{Ke} \rangle$ | $0.526 \times 10^{-4} - 2.52$ | $1.63$ |
| $\langle R^{\tau\mu}_{Ke} \rangle$ | $(0.285 - 5.45) \times 10^{-3}$ | $3.04 \times 10^{-3}$ |
| $\langle R^{\mu\nu}_{K\mu} \rangle_{|q^2\in[1,6]}$ | $(0.0039 - 6.1) \times 10^{-4}$ | $3.2 \times 10^{-4}$ |
| $\langle R^{\mu\nu}_{K\mu} \rangle$ | $(0.0454 - 6.44) \times 10^{-4}$ | $3.3 \times 10^{-4}$ |
| $\langle R^{\tau\nu}_{K\mu} \rangle$ | $0.72 \times 10^{-4} - 4.83$ | $2.58$ |
| $\langle R^{\tau\mu}_{K\mu} \rangle$ | $(0.039 - 0.1) \times 10^{-3}$ | $4.8 \times 10^{-3}$ |
| $\langle R^{\mu\nu}_K \rangle_{|q^2\in[1,6]}$ | $0.57 - 0.9688$ | $0.63$ |
| $\langle R^{\mu\nu}_K \rangle$ | $0.521 - 0.73$ | $0.64$ |

VIII. CONCLUSION

In this work, we have studied the rare lepton flavour violating semileptonic $B$ meson decays in the vector leptoquark model. These decays occur at loop level with a tiny neutrino mass in one of the loop, thus extremely rare in the SM. Whereas these processes can occur at tree level in the vector leptoquark model. There are three vector leptoquarks which are relevant to study the processes mediated via $b \to (s,d)$ transitions. Of these we consider $(3,3,2/3)$ and $(3,1,2/3)$ vector leptoquarks in our analysis and constrained the leptoquark couplings from $B_{s,d} \to l^+ l^-$, $K_L \to l^+ l^-$ and $\tau^- \to l^- \gamma$ processes, where $l$ can be any charged lepton. Using such constrained parameters, we estimated the branching ratios and forward-backward asymmetries of $B \to Kl_i^- l_j^+$ and $B \to \pi l_i^- l_j^+$ processes in the vector leptoquark model. We also computed some parameters like $R^{l_i l_j}_{K(e)}, R^{l_i l_j}_{K(\mu)}$ and $R^{l_i l_j}_+ \mu$ (the ratios of vari-
FIG. 15: The \( q^2 \) variations of \( R_{\mu e}^{\pi \mu} \) (top right panel), \( R_{e \tau}^{\pi \mu} \) (bottom left panel) and \( R_{\tau \mu}^{\pi \mu} \) (bottom right panel) parameters in high \( q^2 \) region in the leptoquark model. Here \( R_{\mu e}^{\pi \mu} \) (top left panel) shows the lepton non-universality for low \( q^2 \in [1, 6] \) region.

ous combination of rare decays) in order to inspect the presence of lepton non-universality. We also study the effect of vector leptoquark on the muon \( g - 2 \) anomaly. We found that our predicted values are sizeable and within the reach of currently running/upcoming experimental limits, the observation of which in the LHCb experiment would provide univocal signal of new physics.

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TABLE VI: The predicted values of forward backward asymmetries and lepton non-universality parameters in $B \to \pi l_i^- l_j^+$ process in the $V^{1,3}$ vector leptoquark model. Also the predicted values of $R_{\pi\mu}^{\mu\mu}$, $R_{\pi l}^{l l l}$, $R_{\pi l}^{l l l}$ parameters for muonic (electronic) processes in low $q^2 \in [1,6]$ region.

| Observables | Values in $V^1$ LQ model | Values in $V^3$ LQ model |
|-------------|--------------------------|--------------------------|
| $\langle A_{FB}^{\pi\mu\epsilon} \rangle$ | $(0.432 - 4.42) \times 10^{-3}$ | $2.4 \times 10^{-3}$ |
| $\langle A_{FB}^{\pi\epsilon\epsilon} \rangle$ | 0.2632 | 0.263 |
| $\langle A_{FB}^{\pi\mu\mu} \rangle$ | 0.263 - 0.271 | 0.26 |
| $\langle R_{\pi\mu}^{\mu\mu} \rangle_{|q^2|\in[1,6]}$ | $1.084 \times 10^{-7} - 1.95$ | $1.655 \times 10^{-7}$ |
| $\langle R_{\pi\epsilon}^{\epsilon\epsilon} \rangle_{|q^2|\in[1,6]}$ | $2.48 \times 10^{-10} - 1.8 \times 10^{-3}$ | $2.46 \times 10^{-10}$ |
| $\langle R_{\pi\epsilon}^{\epsilon\epsilon} \rangle$ | $2.5 \times 10^{-10} - 0.5 \times 10^{-4}$ | $2.47 \times 10^{-10}$ |
| $\langle R_{\pi\epsilon}^{\epsilon\epsilon} \rangle$ | $(0.0187 - 1.53) \times 10^{-4}$ | $1.86 \times 10^{-6}$ |
| $\langle R_{\pi\epsilon}^{\epsilon\epsilon} \rangle$ | $3.75 \times 10^{-9} - 7.33 \times 10^{-3}$ | $3.6 \times 10^{-9}$ |

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