Research on radio imaging method (RIM) multifrequency information fusion methods

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Abstract. Radio imaging method (RIM) also called cross-hole electromagnetic wave method, is widely used in mineral exploration, engineering exploration and other fields. The RIM system can collect multi-frequency information, but the use of multi-frequency information is still limited to qualitative analysis or only one type of frequency information is used. By summarizing the multi-frequency information utilization methods appearing in a variety of geophysical methods, and the feasibility of RIM multi-frequency utilization is proved through theoretical analysis, three multi-frequency information fusion methods suitable for RIM are proposed, namely multi-frequency inversion strategies, Multi-frequency data fusion and multi-frequency image fusion. An example application analysis shows that the three multi-frequency information fusion methods make full use of the advantages of different frequencies and can improve the imaging result.

1. Introduction
Radio imaging method (RIM) refers to a geophysical method that transmits or receives electromagnetic wave signals in two (or more) boreholes, and uses the electromagnetic wave signals for imaging to obtain the physical properties between boreholes. The electromagnetic wave signal includes information such as amplitude, travel time, and phase. However, domestic radio imaging instruments generally can only measure amplitude information [2]. Therefore, in practice, only amplitude information is used for attenuation tomography. All RIM systems are multi-frequency, but it is unclear how to weight and combine data from different frequencies to form an image reconstruction algorithm [13]. When these frequency data are processed in actual production, they are usually analyzed by their actual field strength maps, and the optimal frequency is artificially selected for inversion. However, this method can only use the data of one frequency, and the data of other frequencies is not used to cause waste. Besides, the method of artificial selection is also very dependent on the experience of the processing staff. Therefore, RIM multi-frequency utilization is an issue that is easily overlooked but urgently needs to be resolved. The use of multi-frequency information has appeared in a variety of geophysical methods, which can be broadly divided into analytical methods and fusion methods. The multi-frequency information analysis method refers to a method of analyzing inversion results of different frequency information and obtaining different geological information therefrom. For example, in ground penetrating radar, Utsi [5] analyzed the data collected by the 400MHz and 4GHz antennas, and comprehensively revealed the characteristics of the underground buildings, showing the advantages of antennas with more than one frequency in the research of historical buildings. In the radio wave perspective, Xiao et al. [10] analyzed different frequency detection results to comprehensively delineate geological anomalies.
The multi-frequency information fusion method refers to merging multi-frequency data or images together through some combination methods or calculation methods to obtain a result that simultaneously reflects the multi-frequency information. For example, in multi-frequency electromagnetic logging, Shen [4] used both low-frequency and high-frequency information, that is, the inversion results using low-frequency data served as the initial model for high-frequency data inversion. In seismic traveltime tomography, Yang et al. [12] further used data of two or more frequencies. In the initial iteration, a lower frequency data was used to accelerate the convergence rate so that it can reflect the general velocity structure as soon as possible. During the iteration, the frequency is gradually increased to improve the inversion accuracy. In ground penetrating radar, Xu et al. [11] integrated the advantages of detection depth and detection accuracy of different frequency signals by fusing multi-frequency data, and improved the radar imaging quality and the accuracy of image interpretation. In the transient electromagnetic method, Xia [9] uses fusion methods such as weighted linear regression, piecewise linear interpolation, and cubic spline interpolation to fuse data of different frequencies and take full advantage of various single-frequency data. The data of different frequencies are fused, and the advantages of various single-frequency data are fully utilized to improve the overall vertical resolution of the detection results, avoiding the phenomenon that the resistivity response is different when multiple single-frequency imaging is interpreted separately. In the drilling sonar, Wang et al. [6] considered the contribution of different frequencies to the actual detection object and synthesized the detection data of multiple frequencies, effectively solving the problem of the opposition between the detection scale and detection accuracy.

Because the analysis method relies on the experience of the interpreter, and when there are many types of frequencies, it is difficult to interpret the inversion results of each frequency, and even the interpretation results may be inconsistent. Therefore, this paper mainly studies fusion method, which is to try to apply fusion method to RIM.

2. Radio imaging method principle

According to the theory of electromagnetic wave radiation [8], assuming that the electromagnetic wave propagates along a straight line, when the transmitting antenna uses a half-wave dipole antenna, the field strength \( E \) received by the receiving antenna in the far field can be expressed as:

\[
E = E_0 e^{-\beta r} f(\theta_1) \sin \theta_2,
\]

(1)

Where \( E_0 \) is the initial field strength (uV); \( \beta \) is the attenuation constant (Np / m); \( f(\theta_1) \) represents the direction factor of the transmitting antenna; \( \theta_1 \) is the angle between the observation ray and the transmitting antenna; \( \theta_2 \) is the angle between the observation ray and the receiving antenna; \( r \) is the distance between transmitting and receiving points (m). In actual production, the unit of field strength is generally expressed in dB, \( E(\text{dB}) = 20 \log(E(\text{uV})) \). For RIM, a half-wave dipole antenna is often used, and its direction factor is \( f(\theta) = \cos(\frac{\pi}{2} \cos(\theta_1)) / \sin(\theta_1) \).

Move (1) and take the logarithm to get:

\[
\beta r = \ln \frac{E_0 f(\theta_1) \sin \theta_2}{E_r}
\]

(2)

Written in discretized form:

\[
\sum \beta_j r_j = d_i
\]

(3)

\[
d_i = \ln \frac{E_0 f(\theta_1) (\sin \theta_2)}{E_{r_i}}
\]

(4)
Where $r_{ij}$ represents the length of ray $i$ in cell $j$; $\beta_j$ represents the attenuation constant in cell $j$; $d_i$ can be calculated from the data of ray $i$. By solving this matrix equation, the attenuation constant distribution of the study area can be obtained. The attenuation constant is also called the absorption coefficient and can be expressed as:

$$\beta = \omega \sqrt{\frac{\mu \sigma}{2 \varepsilon}} (1 + \frac{\sigma}{\omega \varepsilon})^{-1}$$  \hspace{1cm} (5)$$

Where $\omega$ is the circular frequency, $\omega = 2\pi f$; $\sigma$ is the electrical conductivity, $\varepsilon$ is the dielectric constant, $\varepsilon = \varepsilon_0 \varepsilon_r$; and $\mu$ is the magnetic permeability, $\mu = \mu_0 \mu_r$. It can be seen that the attenuation constant is affected by frequency, etc. The change in frequency will cause the attenuation constant to change, so the attenuation constants measured at different frequencies are inconsistent, which is also the basis for multi-frequency data utilization. When the frequency is low, the detection distance is large; when the frequency is high, the detection accuracy is high, and the detection results of different frequencies reflect different information underground. Therefore, the fusion frequency inversion method, the result obtained is a comprehensive reflection of the underground information, which not only maintains the high resolution of the detection but also guarantees the detection depth.

2.1. Multi-frequency utilization method

This paper mainly attempts to apply the multi-frequency information fusion method to RIM. Multi-frequency information fusion methods can be roughly divided into multi-frequency inversion strategies, multi-frequency data fusion, and multi-frequency image fusion. The application methods of these three methods in RIM are explained in detail below.

2.2. Multi-frequency inversion strategy

The multi-frequency inversion strategy is to use multiple frequencies in the inversion process. When only two types of frequency data are used, the low frequency data can be used for inversion first, and the obtained inversion results serve as the initial model of high frequency. When using more than two types of frequency data, lower frequency data can be used at the beginning of the iteration. Speed up the convergence speed to make the general outline as fast as possible, and in subsequent iterations, gradually increase the frequency and improve the accuracy of the inversion.

2.3. Multi-frequency data fusion

The concept of data fusion originated in the 1970s. Data fusion technology originally designed for military applications can also be applied to industry and agriculture, such as resource management, urban planning, climate, crops, and geological analysis [1]. Multi-frequency data fusion is one of the forms of data fusion. Multi-frequency data fusion refers to the method of transferring as much information as possible to the fused data by combining, correlating, and combining different frequency data.

There are many methods for multi-frequency data fusion. In this paper, an easy-to-implement weighted average method is used to process multi-frequency data. The key of the weighted average method is to obtain the weighting coefficient. This paper proposes a method to calculate the weighting coefficient based on the best measurement frequency. The first step is to calculate the optimal measurement frequency, which mainly depends on the electrical properties of the detection area and the detection distance and other parameters. According to the formula (2), we can get:

$$\beta = \frac{1}{r} \int r f(\theta) \sin \theta \frac{E_r}{Er} \hspace{1cm} (6)$$

For the common karst detection area of RIM, the lithology is mainly limestone, and the conductivity is generally $0.001 \sim 0.01 \ S \cdot m$, which belongs to a good conductive formation, so formula (5) can be simplified as
According to formulas (6) and (7), the calculation formula of the optimal frequency can be obtained:

\[
f = \frac{1}{\pi \mu \sigma} \left( \frac{1}{r} \ln \frac{E_r f(\theta)}{E_r \sin \theta} \right)
\]

(8)

The second step is to calculate the weighting coefficient. The calculation method is based on the research results of Wang et al. [6]. Assuming \( n \) frequencies are used, the weighting factor for frequency \( i \) is:

\[
\phi_i = \frac{1}{(n-1)} \frac{|f_i - f| + \cdots + |f_{i+1} - f| + \cdots + |f_n - f|}{|f_i - f| + \cdots + |f_{i+1} - f| + \cdots + |f_n - f|}
\]

(9)

Where \( f_i \) represents frequency \( i \); \( f \) represents the optimal frequency; and \( \phi_i \) is the contribution of frequency \( i \) to the weighted average. The third step is to weight the average of the measurement data of different frequencies according to the weighting coefficient. The fourth step is to use the weighted averaged data for inversion.

### 2.4. Multi-frequency image fusion

Image fusion is the fusion from the perspective of the image, which can be divided into pixel-based fusion, feature-based fusion, and decision-level fusion [3]. Multi-frequency image fusion is the fusion of images using multi-frequency data. Specifically, the inversion image is obtained by inverting data of different frequencies, and then the inversion image is fused to finally obtain an image containing multiple frequency information. The difference between multi-frequency image fusion and multi-frequency data fusion lies in that the multi-frequency image fusion has gone through an inversion step, from the fusion of data points to the fusion of images. This method is closer to the method of artificially analyzing images, but the method is based on computer fusion instead of the experience of interpreters.

The multi-frequency image fusion method used in this paper is based on pixel fusion. The difference from multi-frequency data fusion is that it undergoes an inversion process. Multi-frequency image fusion can also use the weighted average method, which also uses the weighting coefficient calculation method based on the best measurement frequency. The first and second steps are the same as the first two steps in the second section. The third step is to invert different frequencies first. The fourth step is to use the weighted average method for fusion based on the results of different frequency inversions.

### 3. Calculation results and effect analysis

In a detailed survey of a subway in a certain place, in order to investigate the development of karst in the underground space, an electromagnetic wave measurement was carried out between two boreholes with a horizontal distance of 15.66m. Using the fixed transmission mode, the transmitting antenna is located in the elevation range of HS192 boreholes from -16.64m to 15.36m, and the receiving antenna is located in the elevation range of HS190 boreholes from 23.59m to 15.41m. A total of 33 sets of data were collected, and the observation rays basically covered the target area between the holes.

The data of 4MHz and 8MHz are selected as the goals of this study, which represent low frequency and high frequency, respectively. The inversion method uses the ray method, the initial amplitude \( E_0 = -5dB \), and the joint iteration method (SIRT) is used to iterate 20 times. The inversion results are shown in Figure 1 (a) and Figure 1 (b). Two karst anomalies were found in this section, which are located at the left elevation of -12m and the right elevation of -8m. Comparing the inversion results of the two frequencies, it can be seen that at high frequencies, the cave on the right is more clearly portrayed, and the delineated cave range is more accurate, which reflects the high-frequency resolution. However, the left karst cave is relatively insignificant at high frequencies and is easily missed, because the detection distance is small at high frequencies. In addition, some linear false anomalies can be seen in the area 5m above the elevation in Figure 1 (b), which indicates that a single frequency is easily affected by interference factors. Due to the unknown nature of the detection stratum, the above qualitative analysis
is difficult to comprehensively consider the detection results of different frequencies, and it is impossible to accurately determine the location of the cave. Therefore, it is necessary to introduce a multi-frequency fusion method. The following uses the three RIM multi-frequency fusion methods proposed in this paper for calculation.

![Inverted image at 4MHz](image1) ![Inverted image at 8MHz](image2)

**Figure 1.** Inversion of detailed survey data of a subway in a certain place (The red dotted line surrounds the cave area, and the black dotted line surrounds the false anomaly area.)

![Multi-frequency inversion strategy](image3) ![Multi-frequency data fusion](image4) ![Multi-frequency image fusion](image5)

**Figure 2.** Inversion results of multi-frequency information fusion method (The red dotted line surrounds the cave area, and the black dotted line surrounds the false anomaly area.)

### 3.1. Multi-frequency inversion strategy

The multi-frequency inversion strategy adopted in this paper is to first perform inversion using data with a frequency of 4 MHz to obtain the approximate distribution of the attenuation constant, and then use this as the initial model for inversion using a frequency of 8 MHz. It can be seen from Figure 2 (a) that the left cave is more accurately circled than that in Figure 1 (a) and more prominent than that in Figure
1 (b); the right cave is circled more than in Figure 1 (a). Accurately, in addition to suppressing the false anomaly in Figure 1 (b). Therefore, the inversion result under the multi-frequency inversion strategy absorbs the advantages of low and high frequencies, and also suppresses the disadvantages of low and high frequencies. The inversion result is the result of combining the two frequencies.

3.2. Multi-frequency data fusion
For the detection profile in this article, r is 15.66m, E0 is -5dB, and the instrument measurement threshold E is -140dB. Considering the main ray ranges, \( \theta_1 \) and \( \theta_2 \) are taken at 45 degrees, and \( \sigma \) is taken at 0.01, and the optimal measurement frequency \( f \approx 7 \text{MHz} \) can be calculated. For the 4MHz and 8MHz data used, the weighting factors are 1/4 and 3/4, respectively. The weighted averaged data is used for inversion, and the final result is shown in Figure 2 (b).

Since the optimal measurement frequency is closer to 8MHz, the inversion results after multi-frequency data fusion are very similar to the 8MHz inversion results (Figure 1 (b)), but the details have been improved, mainly reflected in the clearer cave on the left. Because the measurement data and frequency are not a simple linear relationship, the weighting coefficient calculation method used in this paper is not optimal, and further research is needed in this regard.

3.3. Multi-frequency image fusion
The weighted average is based on the inversion results of 4MHz and 8MHz data. The weighting coefficient is the same as that in multi-frequency data fusion. The final result is shown in Figure 2 (c). The inversion effect of multi-frequency image fusion is similar to that of multi-frequency data fusion, and it also improves the imaging effect of the left cave. In addition, false anomalies above 5m were suppressed. Multi-frequency data fusion is first fusion and then inversion, while multi-frequency image fusion method is first inversion and then fusion. In principle, the relationship between image results and frequencies is more complicated than the relationship between measured data and frequencies. Simple weighted average methods may not optimally fuse images of multiple frequencies, which needs to be further studied in this regard.

3.4. Conclusions
1. A single frequency is easily affected by interference factors. A fusion method combining multiple frequencies can reduce the influence of interference and improve the accuracy of detection results.
2. The multi-frequency information fusion method fuses information of different frequencies, guarantees effective perspective at low frequencies, and is sensitive and obvious at high frequencies. It makes full use of the advantages of different frequencies to improve the imaging effect to a certain extent. In addition, the phenomenon that the attenuation constant is different during the interpretation of multiple frequencies is avoided.
3. The weighting factor in the weighted average method is the focus. Secondly, we should consider the non-linear relationship between data or image and frequency to design weighting method.

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