Planet Formation - Implication of Statistical Properties of Exoplanets

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Abstract. Models of planetary formation are developed based on the observation of our Solar System, star-forming regions and circumstellar disks and on the ever increasing number of exoplanetary systems. The solar nebula theory and the planetesimal hypothesis are discussed. The latter is found to provide a viable theory of the growth of the terrestrial planets, the cores of the giant planets, and the smaller bodies present in the Solar System. The formation of solid bodies of planetary size should be a common event, at least around young stars which do not have binary companions orbiting at planetary distances. Stochastic impacts of large bodies provide sufficient angular momentum to produce the obliquities of the planets. The masses and bulk compositions of the planets can be understood in a gross sense as resulting from planetary growth within a disk whose temperature and surface density decreased with distance from the growing sun.

1. Introduction
The origin of the Solar System is one of the most intriguing problem of science. For more than two centuries, scientific ideas about how the planets came to be were based almost entirely upon theory. There were few constraints on such speculations: astronomers knew that stars rotated and that the Sun’s planet are in almost circular orbits. But beyond these meager knowns, those who speculated on the Solar System’s origin had nowhere to turn except to their own fantasy of how a gigantic amount of cosmic matter might have organized itself into a Sun and planetary system. Astronomers, using the laws of physics as then understood, offered variations on the same answer: the Sun and the planets were born form a rotating disk of cosmic gas and dust. The flattened form of the disk constrained the planets that formed from it to have orbits lying in more or less the same plane, all moving in the same direction the disk had turned. This hypothetical disk, the solar nebula, is where any discussion of the origin of our Solar System must begin.

The idea of the solar nebula was first proposed by Emanuel Swedenborg (1734), and further developed in the course of the 18th century by the Prussian philosopher and physicist Immanuel Kant (1755). Although his treatement of the problem was only qualitative, its precepts were remarkably similar to those considered fundamental today. Kant pictured an early universe evenly filled with thin gas. He thought such a configuration would have been gravitationally unstable, so it must have drawn itself together into many large dense clumps of gas. Kant correctly assumed that these clumps of gas were rotating and also recognized the importance of
this rotation: as they shrank the rotation spun them out into flattened disks. From one of these
disks was our Solar System forged.

The first indirect evidence for disks came from studies of T Tauri stars, which are similar in
mass to our Sun but very young - roughly a million years old. In the 1980s astronomers realized
that about a third of T Tauris stars have “infrared excesses”, that is, the amount of infrared
radiation they emit is too great to be consistent with their output at visible wavelengths. This
can be understood if the stars in question are surrounded by halos of dust kept warm by short-
wave length radiation from the stars; the dust then reradiates the energy it receives at longer
(infrared and radio) wavelengths. However, the strong infrared signatures implied the presence
of enough dust, if distributed evenly in a sphere, to completely block our view of such a star
at visible wavelengths. Only if the dust were arranged in a flattened disk, tilted somewhat to
our line of sight, could we expect to see the star itself.

The observational evidence of disks around young stars that were first directly observed in the
mid 1980s, initially in the form of dusty debris disks such as those seen around Vega (Aumann et
al., 1984) and β Pic (Smith & Terrile, 1984), and later in the form of the gas-rich disks that we
now term “protoplanetary” (Sargent & Beckwith, 1987). Advances in telescope technology in
the intervening two decades has resulted in the detection, both directly and indirectly, of many
more disks around young stars, and these protoplanetary disks, or proplyds, are now understood
to be commonplace. The Hubble Space Telescope could capture such disks at visible wavelengths
in the Orion nebula, an active stellar forming region. The Orion disks are far larger than the
Solar System, and the available material in them are more than enough for planetary systems.
The host stars of these disks are very young, at most a few million years old.

Today we realize that Kant, by and large, got it right, and the widely accepted model,
the Solar Nebula Disk Model, also known as the Planetesimal Hypothesis roots in the same
fundamentals as Kant’s theory. The development of the Solar Nebula Disk Model (hereafter
SNMD) has interwoven a variety of scientific disciplines including astronomy, physics, geology,
and planetary science. Since the dawn of the space age in the 1950s and the discovery of proplyds
and extrasolar planets in the 1990s, the models have been both challenged and refined to account
for new observations.

Until the mid 1990s we had confirmed knowledge of only our planetary system and one around
a pulsar (Wolszczan & Frail 1992). The first published discovery was made by Campbell et al.
(1988), whose radial-velocity observations suggested that a planet orbited the star γ Cep, which
is a sub-giant in 57 year long-period binary system. They remained cautious about claiming a
true planetary detection, and widespread skepticism persisted in the astronomical community for
several years about this and other similar observations. It was mainly because the observations
were at the very limits of instrumental capabilities at the time.

The first unambiguous discovery of an exoplanet around a main sequence star 51 Peg was
announced on October 6, 1995 by Mayor & Queloz (1995). In order to better understand
the physical processes involved in the formation of planetary system it is inevitable to observe
and study a large sample of extrasolar system. By now this is exactly the situation: since
the detection of the planetary companion of 51 Peg the number of exosystems has exceeded
300 providing several constraints and a striking variety of planetary systems for scientist to
formulate a general theory for planet formation. On the other hand theoretical studies of
planetary formation have become another frontline research in the field of astronomy.

1 We note that the search for extrasolar planets since 1995 has evolved into a mature field in astrophysics, and
in 2002 improved techniques allowed to prove the existence of the planet around the star γ Cep (Hatzes et al.
2003).
1.1. Observations of the Solar System

A complete theory of the origin and evolution of our Solar System and the detected exosystems must explain the observed facts. To begin with I list the major ones of the Solar System:

1. **Orbital inclination, eccentricity and direction:** The inclination of most objects with respect to the Sun’s equator is small (coplanar orbits), and they orbit the Sun in the prograde direction (the same sense as the Sun rotates) on nearly circular paths.

2. **Orbital spacing:** The major planets are confined to heliocentric distances of \( \leq 30 \) AU, and the separation between neighboring orbits increases with radial distance. The orbits of the planets do not cross, nor do they approach each other closely. The interplanetary space contains very little debris (aside from the asteroid belt and the Trojans).

3. **Comets (Oort cloud):** Roughly \( 10^{12} \) comets of radius \( R > 1 \) km orbit the Sun at distances beyond \( 10^4 \) AU (Oort 1950); this swarm is isotropically distributed with respect to the Sun, apart from asymmetries produced by the tidal field of the Galaxy (Weissman 1990). There is a second comet reservoir, the Kuiper belt - a flattened disk at \( \geq 40 \) AU (Duncan & Quinn 1993a).

4. **Planetary rotation:** Six of the eight major planets rotate around their axis in the same direction in which they revolve around the Sun, and have obliquities (axial tilts) of \( \leq 30^\circ \). Venus and Uranus rotate in retrograde direction.

5. **Satellite Systems:** Most planets, aside from Mercury and Venus have natural satellites. Most close-in satellites travel on low inclination, low eccentricity orbits, whereas small distant moons tend to have large inclinations and eccentricities.

6. **Planetary masses:** Together the planets account for \( \leq 0.2\% \) of the mass of the Solar System. The most massive planets are Jupiter and Saturn, followed by Neptune and Uranus. The relatively small terrestrial planets orbit closest to the Sun. Between the terrestrial and the giant planets orbit a large number of asteroids. The total mass of the asteroid belt about 1 % that of Mercury.

7. **Angular momentum distribution:** Although the planets represent only a tiny fraction of the total mass, over 98 % of the Solar System’s angular momentum is contained in the orbital motions of the Jovian planets. We note, that in contrast the orbital angular momenta of the satellite systems of the giant planets are far less than the spin angular momenta of the planets themselves.

8. **Bulk composition:** The terrestrial planets are composed primarily of dense, refractory (high condensation temperature) rocky material. The low densities of the largest planets imply \( H \) and \( He \) are the dominant components, yet these planets must still be enriched in high-Z elements (heavier than \( He \)) relative to the Sun or their densities would be even less. Jupiter is \( \approx 90\% \) \( H \) and \( He \) by mass and Saturn is \( \approx 77\% \) (Hubbard & Marley 1989). Orbiting farther from the Sun, Uranus and Neptune have intermediate uncompressed densities (the density a planet would have if its gravity did not compress it), implying compositions dominated by high-Z material, with \( \approx 5 – 20\% \) \( H \) and \( He \) (the \( H \) and \( He \) ratios in these planets are difficult to estimate, as it is not known whether heavier material is primarily very dense rock or lower density melted ”ices” of \( H_2O \), \( NH_3 \) and \( CH_4 \)). The moons of the outer planets consis of rock and ice, in varying proportions. Asteroids are rocky, but their low uncompressed densities, relative to the terrestrial planets (Landgraf 1988), suggest some lighter component, perhaps hydrated silicates. Comet Halley consists of a mixture of ice, rock and organics-rich CHON particles. Aside from extremly volatile elements such as \( H \), \( N \), and the noble gases, Halley’s elemental abundances appear to be nearly solar.

9. **Asteroid belt:** Roughly \( 3 \times 10^{24} \)g of material is spread among countless minor planets orbiting between Mars and Jupiter. The size distribution of asteroids is approximately
a power law with an incremental size index between -2.5 and -3, which has a slope similar to that expected from appropriately adjusted theoretical (Dohnanyi 1969) and experimental (Capaccioni et al. 1986) studies of fragmentation. The eccentricities and inclinations of asteroidal orbits are approximately Rayleigh distributed, with means of 0.14 and 15° respectively. These orbital parameters correspond to random velocities of ≈ 5 km/s, far above the 1.7 km/s escape velocity from 1 Ceres, the largest asteroid.

The SNDM explains several of the above listed observational facts. The nearly circular and coplanar orbits of Solar System planets strongly support a planetary formation in a flattened circumsolar disk. Star formation models suggest that such disks are a neutral byproduct of a gravitationally collapsing and rotating cloud (Terebey et al. 1984). However, many hydrodynamical calculations have attempted to model stellar collapse and its aftermath, but a completely realistic simulation remains an elusive goal. The low angular momentum material of the cloud formed the Sun, while the high angular momentum material ended up in a flattened disk around the Sun. There is no reason to suppose a different composition for the disk and the Sun: the relative abundance in the elements in the nebula is very similar to that of the Sun.

Proplyds contain a mixture of two components: a gas and condensed matter or dust component. Interstellar grains (possibly formed in the nucleosynthesis of former generations of stars) and nebula condensates make up the solid component of the disk. Analysis of interplanetary dust and primitive meteorites suggest grain sizes range from 0.05 – 100µm, i.e. roughly 0.1 micron in average (Kerridge & Anders 1988). For the sake of comparison, I note that, the radii of isolated neutral atoms range between 30 and 300 pm (10⁻¹² m), so average grains are few 1,000 atoms wide.

To give a lower estimation on the mass of proplyds the Minimum Solar Mass Nebula approximation is used: the material in each planet of the Solar System is amended with volatiles to reach the solar composition and the resulted material is then spread out over an annulus centered at the orbit of the planet. The resulting disk would have a mass of 0.01 – 0.02M⊙ and a surface density profile which decreases as ∼ r⁻³/₂, apart from a hole in the region of Mars and the asteroid belt (Weidenschilling 1977a). The bulk of this mass is in the form of gas. Current SNDMs suggest that the mass of the proplyd was several times larger than the Minimum Solar Mass Nebula.

2. Stages of Planet Formation
The process of planetary growth is generally divided for convenience into several distinct stages. The currently accepted sequence of planetary formation is discussed in the following sections.

2.1. Early Stage: From Dust to Planetesimals
In what follows I discuss how dust grains grow from sub-µm sizes to ∼ km-size bodies. The collapse of a fragment of a molecular cloud to Solar System dimension releases a substantial amount of gravitational potential energy. The bulk of this energy is radiated away during the collapse phase, a significant amount is retained as heat. When infall slows and eventually halts, the disk begins to cool.

2.1.1. Condensation of chemical compounds
As the temperature T decreases various chemical compounds condense into microscopic grains (≤ 10⁻⁶ m). In the case of the solar disk, the first condensates are silicates (e.g. olivine, [SiO₄]⁴⁻, pyroxene group [SiₙO₃n]²⁻ etc.). At lower temperature, typically in the outer region of the proplyd, large quantities of water (H₂O), ammonia (NH₃), methane (CH₄) and other ices can condense (for details see Barshay & Lewis, 1976). Preexisting condensates from the interstellar matter may also be present. Growth of particles then proceeds by collision.
2.1.2. Mutual collision of compounds  Observations of dusty disks around T-Tauri stars imply that $\geq 1\%$ of the condensed matter remain in dust phase, which may indicate that collision of particles do not by all means lead to agglomeration of the solid particles. Alternatively it may result from the disruptive collisions between planetesimals, when large amount of dust are ejected.

The mechanical and chemical processes determining dust growth are poorly understood. Experimental data (Forrest & Witten 1979) and numerical models suggest that weakly bounded particles with fractal geometry may be formed. However, the most primitive meteorites, which are thought to have formed in this phase of formation contain chondrules$^2$. The large massratio of chondrules (in some cases $\approx 70\%$) in meteorites implies that a significant amount of the fuzzy solid bodies were rapidly heated and cooled prior to being incorporated into larger bodies. However, the environmental setting, the energy source for the heating, and the precursor material are not known (for details of various models of chondrules formation see Morfill et al. 1993).

The motions of small grains in a proplyd are strongly influenced by the gas. The coupling between the gas and the solids with diameter $\leq 1$ cm is well described by Epstein’s drag law, whereas larger bodies are subject to Stokes drag (Adachi et al. 1976).

The vertical component of the star’s gravity causes dust to fall toward the midplane of the proplyd (this process is also called sedimentation). In gas with density $\rho_g$ and sound speed $c$ a grain with radius $R$ and density $\rho$ at a distance $z$ from the midplane has a settling velocity $v_z$ of

$$\frac{dv_z}{dt} = -\frac{\rho_g c}{R\rho} v_z - \Omega^2 z,$$

where $\Omega$ is the orbital frequency. The equilibrium settling rate is given by:

$$v_z = -\frac{\Omega^2 z}{\rho_g c}.$$

Grains’ settling velocity is linearly proportional to their radius.

At 1 AU from the Sun, the temperature of the proplyd was $\sim 500 - 800$ K and the gas density $\rho_g \sim 10^{-9}$ g/cm$^3$. Eq. (2) yields $e$-folding sedimentation times for $1 \mu$m dust grains of $\sim 10^6$ years under these conditions, and several $e$-foldings are required to produce a thin layer in which the dust density and the gas density is comparable. Since dust grains have different radius they approaching the midplane with different velocities: they collide and agglomerate which further increases their settling velocity. This mechanism shortens the sedimentation time by $\approx 2$ orders of magnitude. Recent models suggest that the majority of solid material was able to agglomerate into macroscopic size, i.e. $\geq 0.1$ mm bodies, and most of these bodies were located to a relatively thin subdisk about the midplane of the proplyd in which the density of solid component was comparable to, or exceeded that of the gas (Weidenschilling & Cuzzi 1993).

The gaseous component of the proplyd is partially supported against the stellar gravity by a pressure gradient in the radial direction, so gas rotates around the star slower than the local Keplerian velocity. The “effective gravity” felt by the gas is

$$g_{\text{eff}} = \frac{k^2 M_*}{r^2} - \left( \frac{1}{\rho_g} \right) \frac{dP}{dr},$$

where $M_*$ is the mass of the star. For estimated protoplanetary disk parameters, the gas rotates roughly 0.5% slower than the local Keplerian speed (Adachi et al. 1976).

Larger particles ($\geq 10$ cm) moving at nearly Keplerian speed thus encounter a headwind, which removes part of their angular momentum and causes them to spiral towards the star.

$^2$ Chondrules are formed by a rapid heating (within minutes or less) to temperatures between 1500°C and 1900°C and subsequent melting. This is followed by a cooling within one to several hours.
Inward drift is greatest for mid-sized particles, which have large surface area to mass ratios and orbit with approximately Keplerian velocities. This inward migration effect is hardly relevant for (i) small particles (≤ 1 mm), which are so strongly coupled to the gas that the headwind they experience is very weak and for (ii) large bodies (≥ 1 km) whose angular momentum is much larger than that of the gas they collide during one revolution. Peak rates for inward drift occur for particles that collide with roughly their own mass of gas in one orbital period. These bodies have a radius of few meters and drift in the terrestrial region of the solar nebula at a rate of up to ∼ 10^6 km/yr, i.e. a body with mass in the range between 10^2 g and 10^8 g has a decay time at 1 AU as short as 10^2 or 10^3 years! Thus, the material that survies to form the planets must completely the transition from cm size to km size rather quickly.

Several hypotheses have been proposed to overcome this difficulty. A popular set of mechanisms for the formation of planetesimals are rooted in the so-called Goldreich-Ward instability (Goldreich & Ward, 1973). In this scenario the vertical settling of small dust grains steadily increases the dust-to-gas ratio at the midplane, until eventually the thin dust sub-disk becomes gravitationally unstable and fragments. We can estimate the conditions at which this occurs by a simple application of the Toomre (1964) criterion, setting

\[ Q = \frac{\sigma \Omega}{\pi G \Sigma_{\text{dust}}} = 1, \]  

(4)

where \( \sigma \) is the velocity dispersion in the dust layer and \( \Sigma_{\text{dust}} \) is the surface density of the dust layer. If we assume that \( \Sigma_{\text{dust}} = \Sigma_{\text{gas}}/100 \approx 10 \text{ g cm}^{-2} \) and consider a 1 M\( \odot \) star, we see that a velocity dispersion of \( \sigma \leq 10 \text{ cm s}^{-1} \) is required in order for gravitational instability to fragment the dust layer. Given that the gas sound speed is typically 1 km s\(^{-1} \), we see that the dust sub-layer must be very thin in order to become unstable. However, this suggestion is attractive because it allows for the formation of planetesimals directly from small (\( R \leq 1 \text{ mm} \)) particles, and thus bypasses the size ranges what are most susceptible to radial drift. If the disk is turbulent, gravitational instabilities are suppressed because the dusty layer remains too thick.

Unfortunately it can be easily shown that, in its simplest form, the Goldreich-Ward mechanism does not result in planetesimal formation. We have seen above that the instability requires that the dust layer be approximately 10^4 times thinner than the gas disk, and consequently at the disk midplane the local dust density dramatically exceeds the local gas density (by a factor ≈ 100). In this layer we therefore expect the solid material to dominate the dynamics, so gas pressure forces are negligible and the dust and gas both orbit at the Kepler speed. Above this layer, however, the gas is significantly sub-Keplerian due to gas pressure, and there is therefore a large velocity shear in the vertical direction. This shear is Kelvin-Helmholz unstable, and the resulting turbulence prevents \( \sigma \) from becoming small enough for instability to set in (Cuzzi et al., 1993).

This problem is the “crux” of the SNDM: How to form planetesimals remains the biggest challenge for modern research in planet formation, and many of the details of this problem still elude us. More recently, several authors have proposed different mechanisms (e.g. aggregation of dust can occur within turbulent eddies) that may provide solutions to the meter-size problem.

Once this meter-size barrier is passed, continued growth via binary accretion leads to the formation the so called planetesimals. In what follows it is supposed that at the end of the early stage a large number of planetesimals orbit the Sun. The settling time of the disk dictates the formation timescale for planetesimals. Most models predict a few 10^4 years.

2.2. Middle Stage: From Planetesimals to Protoplanets

The star’s gravity is the dominant force upon planetesimals. The largest perturbation to the orbit of a planetesimal is the gravitational interaction with other solid bodies. The most important nongravitational forces experienced by a planetesimal is the mutual inelastic collision (kinetic
energy is not conserved) and gas drag. Gravitational torques may lead to significant orbital evolution of bodies larger than a typical planetesimal.

2.2.1. Velocity distribution The distribution of planetesimal velocities play a crucial rôle in planetary growth. The velocity distribution is modified by gravitational interactions and collisions, which convert the ordered motion with Keplerian circular velocity, \( v_K = k(M_*)^{1/2} \) to random motion. Due to gas drag larger than \( v_K \) velocities are decreased, while slower are increased to Keplerian velocity, i.e. gas drag damps the eccentricities and inclinations. In the framework of the kinetic theory of gases the simplest analytic approach for calculating the evolution of planetesimal velocities uses a particle-in-a-box approximation, in which the evolution of the mean square planetesimal velocities is calculated. Since in this phase the number of planetesimals which are participating in planet formation process is \( \geq 10^9 \), only a statistical approach provides a viable method for treating the planetesimal growth.

In the particle-in-a-box approximation the distribution of orbital eccentricities \( e \) and inclinations \( i \) are described by Rayleigh distribution:

\[
f(e, i) = \frac{4 \sigma}{m \langle e^2 \rangle \langle i^2 \rangle} \exp \left( - \frac{e^2}{\langle e^2 \rangle} - \frac{i^2}{\langle i^2 \rangle} \right),
\]

where \( m \) is the mass of the individual planetesimals and \( \sigma \) is the surface mass density of planetesimals with a specific semimajor axis, \( \langle e^2 \rangle \) and \( \langle i^2 \rangle \) are the mean square eccentricity and inclination. Direct \( N \)-body calculations by Ida & Makino (1992a) have verified the form of Eq. (5). The other angular orbital elements have a uniform distribution within their range.

The distribution of random velocities can be derived from Eq. (5) using the epicycle approximation, that is small eccentricities and inclinations are assumed. The result, which is locally (at a specific heliocentric distance \( r \)) equivalent to Eq. (5) is a triaxial Gaussian distribution in cylindrical coordinates:

\[
f_0(z, v) = \frac{\Omega \sigma}{2\pi^2 c_r^2 c_z^2 m} \exp \left[ - \frac{v_r^2}{2c_r^2} + \frac{v_z^2}{2c_z^2} + \frac{\Omega^2 z^2}{2c_z^2} \right],
\]

where

\[
2c_r^2 = \langle e^2 \rangle v_K^2, \quad 2c_z^2 = \langle i^2 \rangle v_K^2,
\]

\( v_\theta \) is the azimuthal component of the velocity relative to the local circular Keplerian velocity, and \( \Omega = v_K/r \) is the orbital frequency.

The \( f_0(z, v) \) distribution is modified by pairwise collisions, gravitational scattering and gas drag. Collisions dissipate some or all of the relative kinetic energy of the colliding bodies while gravitational scattering conserves it but changes the direction of the velocity vectors. Elastic collisions are generally modeled with a Boltzmann collision operator. To account for (i) inelastic collisions and (ii) the gravitational enhancement of collision cross-section the Boltzmann collision operator has been modified; for a detailed discussion see Ida & Nakazawa (1990) and Greenzweig & Lissauer (1990, 1992). The evolution of the distribution function \( f_0(z, v) \) can be described by the collisional Boltzmann equation,

\[
\frac{\partial f_0}{\partial t} + v \cdot \nabla_r f_0 - \left[ \frac{k^2 M_0}{r^3} + (v + v_K) \cdot \nabla v_K \right] \cdot \nabla_v f_0 = \frac{df_0}{dt}_{\text{coll}},
\]

where the velocity is splitted into a Keplerian and a random component. The Boltzmann equation is typically solved by assuming a form for \( f_0 \); in the present case it is given by 6.

Gravitational scattering between planetesimals can be modeled with a Fokker-Planck operator similar to that used in stellar dynamics. Both scattering and collision can superimpose random
motion on the initially ordered differential rotation of the planetesimals, i.e. these processes perturb the Keplerian shear. Inelastic collisions convert some of the energy of the random motion into heat. In addition to generating random motions, gravitational scatterings also tend to equipartition kinetic energy of random motion among planetesimals of different masses (Stewart & Kaula 1980). This process, which is often referred to as dynamical friction, can produce a velocity distribution in which the small particles have a much greater velocity dispersion than do the large ones (Ida & Makino 1992b).

Gas drag damps the orbital eccentricities and inclinations, especially those of smaller bodies. Interested readers are advised to consult Stewart & Wetherill (1988) for the mathematical formulas used to study the evolution of planetesimal velocities.

2.2.2. Runaway growth While gas is still present in the disk eccentricities and inclinations of planetesimals are damped due to aerodynamic gas drag. This counteracts the net effect of collision and scattering and as a result velocities remain low. The simplest accretion picture was first quantified by Safronov (1969). Consider a sphere of radius $R$ moving with velocity $v$ through a uniform medium of density $\rho$. Then the accretion rate is

$$\frac{dm}{dt} = \pi R^2 \rho v.$$  

This rate is the geometrical accretion limit, and $m \sim m^{2/3}$. However, the gravity of the body plays an important role by gravitationally enhancing the cross-section of the body. This effect was first recognized by Safronov: bodies that are larger than the typical size can accelerate their growth rate due to gravitational focusing, i.e. gravitationally enhancing the cross-section of the body (Safronov 1969):

$$\frac{dM}{dt} = \pi R^2 \left[ 1 + \left( \frac{v_{\text{esc}}}{v_{\text{rand}}} \right)^2 \right],$$  

where $v_{\text{esc}} = k \sqrt{2M/R}$ is the escape velocity from the body’s surface, and $v_{\text{rand}}$ represents the velocity dispersion of planetesimals. The second term in the parentheses represent the gravitational enhancement of the accretion cross-section. The quantity $2\theta = v_{\text{esc}}/v_{\text{rand}}$ is known as the Safronov number, and $1 + 2\theta$ is referred to as the gravitational enhancement factor.

While random velocities are small, gravitational focusing can increase the growth rates of bodies by a factor of hundreds, such that $\dot{m} \sim m^{4/3}$, leading to a phase of runaway growth (Ida & Makino, 1992a). The term runaway growth or runaway accretion refers to an evolutionary path where the largest planetesimal in the local region grows much more rapidly than the remainder of the population and therefore becomes detached from the continuous size distribution. These bodies are the progenitors of the protoplanets. The length of this phase depends on the timescale for random velocities of planetesimals to reach the escape speed of the larger bodies. For smaller ($\lesssim 100$ m-sized) planetesimals, gas drag is more efficient such that runaway growth can sustain for a longer time and planetary bodies may be larger.

The runaway growth phenomenon has been seen in numerical simulations (cf. Richardson et al. 2000, Kokubo & Ida 2000). Their results show that the most massive bodies have significantly smaller eccentricities than the remainder of the population, presumably they have been damped by dynamical friction. With runaway accretion the timescale for growth reduce to $\leq 10^5$ years. The simulations indicate that the typical separation between protoplanets formed in this phase is $\approx 4 - 10$ mutual Hill-radii, which is defined as

$$R_{H,m} = \frac{1}{2} \left( a_1 + a_2 \right) \left( \frac{m_1 + m_2}{3M_*} \right)^{1/3},$$  

where $a_1, a_2$, and $M_*$ are the semi-major axis and mass of the central body, respectively.
where $a_1$ and $m_1$ denote the semimajor axis and mass of body 1, etc. (Kokubo & Ida 1996).

The mass of the protoplanets depends on the assumption of planetesimal sizes. In a swarm of $m = 10^{23} - 10^{24}$ g (radius 200 - 400 km) planetesimals, Ida & Makino (1993) calculated $\sim 10^{-3} - 10^{-2} M_\oplus$, where the mass of the Earth, $M_\oplus \approx 6 \times 10^{27}$ g. Considering a perhaps more realistic population of planetesimals with mass $\approx 10^{19}$ g, for the protoplanets $\sim 10^{-5} - 10^{-6} M_\oplus$ are obtained. Thus, although runaway accretion is much more rapid than other modes of accretion (e.g. geometric or oligarchic), it ceases long before protoplanets approaching an Earth mass can form.

2.2.3. Oligarchic growth  As it was discussed above the initial growth mode in a disk of accreting planetesimals is runaway growth, where the mass doubling time for the largest bodies is the shortest. Runaway growth allows relatively short formation times and it is followed by a more lengthy phase of oligarchic growth mode when the largest bodies are still orders of magnitude below an Earth mass. The timescale of oligarchic growth dominates over that of runaway growth.

When these runaway bodies, or protoplanets, become sufficiently massive, it is their gravitational scattering, which is often called viscous stirring which dominates the random velocity evolution of the background planetesimals, rather than the interactions among the planetesimals. Since the accretion cross-section of a protoplanet is smaller among planetesimals with higher random velocities, protoplanet growth now switches to a slower, self-limiting mode, in which the mass ratio of any two protoplanets at adjacent locations in the disk approaches unity over time. Ida & Makino (1993) investigated this transition analytically and through N-body simulations, and Kokubo & Ida (1998, 2000, 2002) studied the subsequent accretion mode, giving it the name oligarchic growth. In this phase just a few dozens of larger bodies, the “oligarchs” dominate the dynamics of the system.

The oligarchic growth phase begins when the dynamical friction of the planetesimals is insufficient to keep the eccentricities and inclinations of the protoplanets very low. In such a configuration protoplanets are not sufficiently isolated from one another to be dynamically stable for a long period of time. Mutual gravitational scattering eventually pump up the relative velocities of the protoplanets which decreases the collisional cross-sections and hence increases the accumulation time. During the oligarchic growth the protoplanets clear their feeding zone by either agglomerating the remaining planetesimals or ejecting them out of the zone.

The final mass of the protoplanets can be approximated by the mass available within the feeding zone. For the case of a protoplanet on a circular orbit the standard theory of the restricted three body problem places an upper bound on the initial semimajor axis separation $b_H$, that may lead to collision. It is convenient to introduce Hill-scaled units for planetesimal with mass $m$, eccentricity $e$, inclination $i$, separation in semimajor axis $a$ from the protoplanet:

$$e_H = \frac{ea}{h}, \quad i_H = \frac{ia}{h}, \quad b_H = \frac{\Delta a}{h},$$

where $h = (m/3M_*)^{1/3}$ is the Hill sphere of the protoplanet and $M_*$ is the mass of the star in solar units. Neglecting gas drag and interactions with other planetesimals, a planetesimal whose orbital elements satisfy the inequality:

$$\frac{3}{4}b_H^2 - e_H^2 - i_H^2 \geq 9$$

cannot enter the protoplanet’s Hill sphere. For $e_H = 0$ and $i_H = 0$ this inequality gives $b_H \geq 2\sqrt{3} \approx 3.46$. Planetesimals on circular orbit with zero inclination and initial separation $b_H$ less than $2\sqrt{3}$ will suffer a close encounter and eventually merge with the protoplanet. Thus the accretion zone, often referred to as the feeding zone embedded in a disk of low random velocity extends over the region

$$b_H \leq B,$$
where $B$ depends on the magnitude of other perturbations on the planetesimals, and typically is $\sim 3.5 - 4$ in a quiescent disk.

The mass of a protoplanet, assuming that it revolves at astrocentric distance $r$ and has accreted all the solid material within an annulus of width $2\Delta r$ is

$$m = \int_{r-\Delta r}^{r+\Delta r} 2\pi r' \sigma(r') dr' \approx 4\pi r \Delta r \sigma(r).$$

(15)

Setting $\Delta r = Bh = Br(m/3M_\ast)^{1/3}$, one obtains from Eq. (15) the isolation mass to which a protoplanet during the oligarchic growth phase may grow:

$$m = \frac{(4\pi Br^2 \sigma)^{3/2}}{(3M_\ast)^{1/2}} = 2.1 \times 10^{-3} \left(\frac{Br^2 \sigma}{2\sqrt{3}}\right) \left(\frac{M_\odot}{M_\ast}\right) M_\oplus$$

(16)

For example, assuming $B = 2\sqrt{3}$, a minimum mass solar nebula with $\sigma = 10$ g cm$^{-2}$ at 1 AU implies protoplanet isolation at 0.066 $M_\oplus$; whereas $\sigma = 3$ g cm$^{-2}$ at 5 AU implies protoplanet isolation at 1.36 $M_\oplus$.

Therefore at the end of this phase only such protoplanets are formed, whose masses are a few perecent of the Earth mass in the terrestrial region, and roughly an Earth mass at Jupiter. One possibility to get larger mass is to start with more than the minimum mass solar nebula. In this case the excess material must be cleared away. This can easily happen in the vicinity of Jupiter as that planet is large enough to eject bodies, but in the terrestrial region, where material resides deep in the potential well of the Sun, it is not evident how the extra material would be lost.

Two plausible mechanisms may contribute to continue the growth of the protoplanets once the isolation mass has been reached:

1. The diffusion of more material into the feeding zone from neighboring regions via scattering between planetesimals, perturbations by close protoplanets and gas drag.
2. Radial motion of a protoplanet may bring it into zones not depleted of planetesimals or enable a low velocity collision and subsequent merge with a neighboring protoplanet.

It is important to realize that terrestrial protoplanets are the same objects as giant planet cores, assuming giant planets to form via the bottom-up core-accretion scenario.

2.3. Late Stage: From Protoplanets to Terrestrial Planets

At the end of the middle stage we have a regular distribution in $a$ of the protoplanets with roughly constant mass. There is also a swarm of smaller planetesimals, which may or may not be relevant. At the beginning of the late stage a.k.a giant impact phase, most of the mass is contained in protoplanets, so dynamical friction is no longer playing an important rôle in their orbital evolution. Even if protoplanets form in circular orbits, mutual gravitational perturbations among several bodies can induce eccentricities of $\sim 0.01$, which is sufficient to enable their orbits to cross so the bodies can suffer close encounters. These are the initial conditions for the last stage of planet formation.

In this stage there are to few bodies to use statistical methods, therefore to track the evolution of the system one must use direct $N$-body integration methods. Once the protoplanets have perturbed one another into crossing orbits, their subsequent orbital evolution is determined by close encounters and collisions. Almost all simulations performed to date assume that the collisions are inelastic and all the material in the coliding bodies end up in the resulting body of the collision. However, giant collisions between protoplanets may not always be accretionary: high-speed or off center collisions can actually erode the body.
For dynamical environments typical of late-stage accretion models Agnor & Asphaug (2004) estimated that more than half of all collisions between same-sized planetary embryos do not result in accumulation into a larger body. While their initial results are limited to a single mass ratio, they suggest that nonaccretionary collisions are typical during the end of terrestrial planet formation. The collisional and dynamical accretion of planets are coupled. For example, the reduced accretion efficiency appears to lengthen the timescale of planet formation by a factor of 2 or more, relative to perfect mergers. The production of significant erosional debris, however, might alter the dynamical environment in ways that remain largely unexplored - for instance, damping the system to lower relative velocities.

Benz et al. (1988) simulated giant collision between a proto-Mercury and a planet one-sixth its size. According to the results it would lead to the loss of most of the silicate mantle of Mercury and thereby account for its anomalously high density. Other simulations in the field lend support to the giant impact hypothesis for the origin of Earth’s Moon; during the final stage of accumulation, an Earth-size planet is typically happen to collide with several objects as large as the Moon and often as massive as Mars. The obliquities of the rotation axes of the planets also provide evidence of the giant impacts during this epoch.

This stage of formation was pioneered by Chambers & Wetherill (1998), who simulated the terrestrial-planet formation using \( N \)-body integrations, in three dimensions, of disks of up to 56 initially isolated, nearly coplanar planetary embryos, plus Jupiter and Saturn. Gravitational perturbations between embryos until their orbits become crossing, allowing collisions to occur. Further interactions produce large-amplitude oscillations in \( e \) and the \( i \) and prevent objects from becoming re-isolated during the simulations. The largest objects tend to maintain smaller \( e \) and \( i \) than low-mass bodies, suggesting some equipartition of random orbital energy, but accretion proceeds by orderly growth. The simulations typically produce two large planets interior to 2 AU, whose time-averaged \( e \) and \( i \) are significantly larger than Earth and Venus. The accretion rate falls off rapidly with heliocentric distance, and embryos in the “Mars zone” (1.2 \( \leq \) \( a \leq \) 2 AU) are usually scattered inward and accreted by “Earth” or “Venus,” or scattered outward and removed by resonances, before they can accrete one another. The asteroid belt (\( a \leq 2 \) AU) is efficiently cleared as objects scatter one another into resonances, where they are lost via encounters with Jupiter or collisions with the Sun, leaving, at most, one surviving object. Accretional evolution is complete after \( 3 \times 10^8 \) years in all simulations that include Jupiter and Saturn. The number and spacing of the final planets, in the simulations, is determined by the embryos’ eccentricities, and the amplitude of secular oscillations in \( e \), prior to the last few collision events.

A large number of simulations were conducted in recent years exploring a wide range of initial conditions. In general the end result of all these simulations is the formation a few (2 - 5) terrestrial planets on a timescale of about \( 10^8 \) years. An important feature is that planetesimal orbits execute a random walk in semimajor axis due to successive gravitational encounters. The resulting widespread mixing of material throughout the terrestrial planet region greatly diminishes any chemical gradients that may have existed in the early stages of planetesimal formation.

The above described planet formation mechanism is unlikely to be as purely sequential as layed out. Grain growth and even the accretion of large planetesimals may well begin during the epoch when a proplyd is still accreting and redistributing material. Given that the theories of each of these epochs are still rather primitive, a sequential study of each stage is probably adequate. However, to the extent that planetary growth depends on, e.g. the initial size distribution of planetesimals, one must recognize that various processes currently being treated as separate events occur simultaneously and may effect one another.
3. Considerations For SNDM From Statistical Properties Of Exoplanets

During the past decade, our understanding of planetary formation had to integrate several new peculiar characteristics. With the number of detected planets regularly increasing, one must continuously reexamine the statistical properties of the derived orbital elements and stellar-host characteristics, and search for constraints for the different planet formation and evolution scenarios. Some confidence to observed trends is brought by the regularly increasing number of candidate detections. The most stunning feature of the sample is the variety of orbital characteristics. This variety challenges the conventional views of planetary formation. The goal now is to interpret the observed orbital distributions in terms of constraints for the planet formation models.

Many studies have discussed from an observational point of view the statistical properties of the extrasolar planets, analyzing various distributions and correlations in order to address the following and several other issues, as reviewed by Udry & Santos (2007).

3.1. Occurrence Rate of Giant Planets

The most direct statistical property of a planet-search program is the fraction of detected planets among the surveyed stars. In the CORALIE planet search sample (1650 FGK stars; Udry et al. 2000), 0.8% of stars have giant planets ($m_2 \sin i \geq 0.2M_{\text{Jup}}$) with separations less than 0.1 AU (hot Jupiters), and $63/1120 = 5.6\%$ of stars have giant planets at separations out to 4 AU. For planets more massive than 0.5 $M_{\text{Jup}}$, in the Lick+Keck+AAT sample (1330 FGKM stars; Marcy et al. 2005) it was found that 1.2% of the stars host hot Jupiters and 6.6% of stars have planets within 5 AU. Within error bars, these two large samples are in good agreement. The planet occurrence frequency is better determined as a function of planet mass and orbital period using Monte Carlo simulations to estimate detection incompleteness. This has been done only in a few cases. For the ELODIE program (Perrier et al. 2003), it is estimated for planets more massive than 0.5 $M_{\text{Jup}}$ a corrected fraction of 0.7 $\pm$ 0.5% for hot Jupiters with $P \leq 5$ days, and of 7.3 $\pm$ 1.5% for planets with periods smaller than 3900 days ($\sim$ 4.8 AU).

3.2. Distribution of planetary masses

Companion minimum mass and separation from the star are the two properties of the system directly obtained from radial-velocity measurements. The known planetary mass distribution, an important constraint for formation models, is discussed in this section separately for giant gaseous planets and for the emerging population of small-mass solid planets.

3.2.1. Giant gaseous planets

It was already clear after the detection of a handful of extrasolar planets that these objects could not be considered the low-mass tail of stellar companions in binary systems (with low $m_2 \sin i$ because of nearly face-on orbital inclinations). The strong bimodal aspect of the secondary-mass distribution to solartype primaries has generally been considered the most obvious evidence of different formation mechanisms for stellar binaries and planetary systems (e.g., Udry et al. 2002). No real clear limit exists as the distribution goes continuously into the brown-dwarf regime but with a decreasing number of candidates. The interval between the two populations (planets and stellar binaries), the brown-dwarf desert, corresponding to masses between $\sim 15M_{\text{Jup}}$ and $\sim 60M_{\text{Jup}}$, contains only a few objects, at least for orbital periods shorter than a decade. There is, however, a probable overlap of the planet and binary distributions in the 10 - 20 $M_{\text{Jup}}$ domain. At this point, it is not easy to differentiate low-mass brown dwarfs from massive planets just from their $m_2 \sin i$ measurements without additional information on the formation and evolution of these systems. Even Hipparcos astrometry has been shown not to reach a sufficient precision to unambiguously probe planetary companions.
Toward the low-mass planets, a clear rise of the distribution is observed. Marcy et al. (2005) proposed a power-law-type function, \( dN/dM \propto M^{-1.05} \), to describe the result from their FGKM sample. This fit is not affected by the unknown \( \sin i \) distribution, which simply scales in the vertical direction. Below the mass of Saturn, the distribution is strongly affected by the detection bias inherent to radial-velocity measurements. We are nevertheless observing the onset of a new population of very light planets.

3.2.2. A new population of solid planets

The low-mass edge of the planet mass distribution is poorly defined because of observational incompleteness. However, although the lowest-mass planets are difficult to detect, in the past years about 40 planets with masses in the Uranus-Neptune range \( \leq 20 M_E \) have been detected. Because of their small masses and locations in the system, close to their parent stars, these light planets may well be composed mainly of a large rocky/icy core (e.g., Brunini & Cionco 2005, Alibert et al. 2006). It is possible that they either lost most of their gaseous atmosphere or simply formed without accumulating a substantial one.

The discovery of very low-mass planets so close to the detection threshold of radial-velocity surveys, and over a short period of time, suggests that this kind of object may be rather common. Moreover, at larger separations, the microlensing technique is finding similar mass objects (the lightest with a minimum mass of \( \sim 2.0 M_E \), Mayor et al. 2009), indicating that smaller-mass planets can be found over a large range of separations. This is in complete agreement with the latest Monte Carlo simulations of accretion-based planet formation models predicting large numbers of solid planets. In this line of thought, we already can note that the detected Neptune-mass planets build a distribution of their own with a gap starting to appear between them and the more common Jupiter-mass planets. Although this result is still strongly observationally biased, it will be interesting to see if the properties of the two populations show further differences, especially when the available statistics increase.

3.3. Orbital Period Distribution of Exoplanets

The distribution of periods of giant exoplanets is characterized by two main features: a peak around 3 days and an increasing distribution with period. The hot Jupiters were completely unexpected before the first exoplanet discoveries. The SNDM (e.g., Mizuno 1980, Pollack et al. 1996) suggests that giant planets form first from ice grains beyond the snowline. Once the solid core is massive enough gas could rapidly accrete (Safronov 1969) over the lifetime of the protoplanetary disk (\( \sim 10^7 \) year, e.g., Haisch, Lada & Lada 2001). During this process, they are also supposed to undergo a migration episode moving from their birth place closer to the central star (see, e.g., Lin, Bodenheimer & Richardson 1996; Papaloizou & Terquem 2006), where they have to stop before falling onto the star. Several stopping mechanisms have been proposed, invoking, e.g., a magnetospheric central cavity of the accretion disk, tidal interactions with the host stars, Roche-lobe overflow by the young inflated giant planet, or photoevaporation. The question is, however, still debated. Alternatives invoke in situ formation, possibly triggered through disk instabilities (Boss 1997). Even in such cases, subsequent disk-planet interactions leading to migration is expected to take place as soon as the planet has formed. The observed pile up of planets with periods around 3 days is believed to be the result of migration and final stopping mechanism (see, e.g., Udry, Mayor & Santos 2003, and references therein, for a more detailed discussion). The exact distance range related to the stopping mechanism is not well defined. A small tail of the distribution points toward the small separations. In particular, three of the transiting planets in the OGLE survey have periods shorter than 2 days (very hot Jupiters). Such short periods, although easy to detect, are not found in the radial-velocity surveys, suggesting that those objects are about 10 times less numerous than hot Jupiters.

Another interesting feature of the period distribution is the rise of the number of planets with increasing distance from the parent star, up to a separation corresponding to the duration...
limit of most of the older surveys (4 - 5 AU). The position of the maximum of the distribution is unknown. However, a conservative flat extrapolation of the present distribution out to larger separations would approximately double the occurrence rate of planets (Marcy et al. 2005). This extrapolation hints that a large population of yet undetected Jupiter-mass planets may exist between 3 - 20 AU. This is of prime importance for the direct-imaging projects under development on large telescopes such as SPHERE, the VLT Planet Finder, or the Gemini Planet Imager, and space-based imaging missions such as NASA’s James Webb Space Telescope (JWST) and Terrestrial Planet Finder, or ESA’s Darwin.

3.3.1. Period distribution of Neptune-mass planets Although the number of known Neptune-mass planets is small, it is interesting to see how their orbital parameters compare with properties of giant extrasolar planets. Because of the tiny radial-velocity amplitude they induce on the primary stars, limiting detections to short orbital periods, a meaningful comparison can only be done for giant planets with periods smaller than \( \sim 20 \) days. In the period distribution of Neptune-mass planets no pile up can be observed, it is rather flat up to 30 days. We even have a candidate with a six-month period. This hints at another difference in the properties of Neptune- and Jupiter-mass planets at short periods.

3.4. The Period-Mass Diagram
The orbital period distribution has highlighted the importance of migration processes to explain the observed configuration of planetary systems. When coupling period and mass, further striking features appear in the distribution: the first noticeable characteristic of the distribution is the lack of massive planets on short-period orbits (Pätzold & Rauer 2002, Udry et al. 2002). Ignoring the multiple-star systems, one completely misses candidates with masses larger than \( \sim 2M_{\text{Jup}} \) and periods smaller than \( \sim 100 \) days. This is not an observational bias as these candidates are the easiest ones to detect. Migration scenarios may naturally result in a paucity of close-in massive planets. Type II migration, when a planet is massive enough \(( \geq 0.5 - 1.0 M_{\text{Jup}} )\) to clear a gap in the disk, has been shown to be less effective for massive planets (Nelson et al. 2000); i.e., massive planets are stranded at wider separations than low-mass planets. Moreover, when a migrating planet reaches small separations from the star, some process related to planet-star interactions could promote mass transfer from the planet to the star, decreasing the mass of the migrating planet, or cause massive planets to fall into the central star (Pätzold & Rauer 2002).

Another interesting feature of the distribution is the rise in the maximum planet mass with planet-star separation (Udry, Mayor & Santos 2003). Massive planets are easily detected at small separations, yet they preferentially reside in more distant orbits. This can be understood in the context of the migration scenario as well. More massive planets are expected to form further out in the protoplanetary disk, where raw materials for accretion are available along a longer orbital path, thus providing a larger feeding zone. Then, migration may be more difficult to initiate as a larger portion of the disk has to be disturbed to overcome the inertia of the planet.

To explain the observed correlation, it has also been suggested that multiplanet chaotic interactions preferentially move low-mass (low-inertia) planets either inward or outward in the system, whereas massive (high-inertia) planets are harder to dislodge from their formation site (Weidenschilling & Marzari 1996, Marzari & Weidenschilling 2002). It is however still an open question whether the correct frequency of hot Jupiters can be reproduced this way.

Simulations of migrating planets in viscous disks are consistent with the observation of a decrease in the efficiency of migration with increasing planet mass (Trilling et al. 1998; Nelson et al. 2000). Therefore, it seems reasonable to expect that a large number of massive planets
may reside on longperiod orbits, some still undetected because of the time duration of the present surveys.

4. Conclusions

The planetesimal hypothesis provides a viable theory of the growth of the terrestrial planets, the cores of the giant planets, and the smaller bodies in the Solar System. The formation of giant planets, which contains significant amounts of H\textsubscript{2} and He, requires a rapid growth of planetary cores so that gravitational trapping of gas can occur prior to the dispersal of the gas from the proplyd.

The summarized theory of planet formation provides excellent or acceptable explanations of the causes of several of the observed Solar System properties listed in the Introduction, but less complete or satisfactory for several others. Many issues remain to be solved: the details of planetesimal formation are still poorly understand, although recent results suggest that several process including turbulence and migration of meter-sized bodies acting simultaneously might be the solution. The details and consequences of giant impacts are not well known, in terms of the fate of collisional debris and compositional changes induced by the impacts. Moreover, a number of the effects of the external parameters could be much more decisive in the formation process than we think it now.

An important challenge in the field of observation is that of directly imaging a planet orbiting a solar-type star. The development of a new generation of adaptive-optics systems, promises a great improvement in this field. All these elements will permit us to better understand the mechanisms leading to the formation of planetary systems like our own, and will thus represent an important step toward the search for life in the Universe. Once earth-like planets orbiting in the habitable zone are known, the search for life in these systems will undoubtedly follow. In the very near future, humanity has to prepare itself to find out that the whole Universe may be teeming with life.

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