THE EFFICIENCY ANALYSIS OF THE AUTOMATED PLANTS

I. Kovalev, P. Zelenkov, S. Ognerubov, K. Bahmareva, E. Denisova
Siberian State Aerospace University named after Academician M.F.Reshetnev
31 “KrasnoyarskiyRabochiy” prospect, Krasnoyarsk, 660014, Russia.
E-mail: zelenkov@sibsau.ru

Abstract. The approach to analyze the efficiency of the technical systems by DEA method is considered. DEA method and its modification are described. The modification allows analyzing the plants by bad outputs. The recycling factories are researched. Finding results are analyzed.

Introduction

In recent years an estimation problem of efficient operation of business arises in spheres of product manufacture and production distribution. Frequently appears the problem of comparison and ranking organization departments and firms or organizations on the whole by some characteristics that can’t be directly measured [1]. And a general idea of demonstration the degree of analyzing latent quality is formed as a result of particular measured characteristics this quality depends on. Beyond all question efficiency is the main concept. “Efficiency is a most general property of any purposeful activity that from the cognitive point of view uncovers itself by way of the target category and is objectively expressed by the degree of a goal achievement taking into account time and resources expenses.” That’s why the efficiency estimation of business and organizations operation is very important for making right management decisions.

The efficiency of the garbage recycling companies is analyzed in this paper. Three companies are considered: "Novosibirskiymusoropererabativaushiyzavod"(Novosibirsk city), "Kazanskiyekozologicheskiy complex" (Kazan city), "Stroy plus" (Krasnoyarsk city).

A Swedish equipment line "Presona" (some assembly lines (conveyors), two presses and a plastic bottles recycling device) for garbage sorting and pressing is installed at the plant in Novosibirsk. A sorting technology allows to significantly reduce the amount of solid waste. For example, the plant produces 2 cubic meters of waste after processing of 10 cubic meters of garbage. The plant in Krasnoyarsk can sort up to 600 cubic meters of garbage a day. 240 cubic meters of waste remain after the sorting and the separation of useful materials. This waste usually has a bio-decaying nature and much safer than unprocessed garbage.

The plant in Kazan has a garbage sorting installment which was built using Imabelberica company's technologies. The sorting process reduces the volume of garbage by 50 percent. The pressing process adds an additional 25-30% reducing.

DEA-method and its modification and realization for an analysis of the technological and organizing aspects of presented companies are considered in this paper.

The feature of the realization of DEA-method is that the estimation of the functioning efficiency of companies task is divided by three subtasks. The first subtask is a problem of the
company's efficiency evaluating, which provides the maximum output of useful products and materials. The second subtask is a problem of the company's efficiency evaluating, using the sum of the amounts of useful materials after the recycling process. The third subtask is a problem of the company's efficiency evaluating, which provides the minimum output of waste materials after the recycling process. The DEA method modification is used to solve the latter problem.

The efficiency criterion is producing more materials using less input resources. But according to saving environment paradigm, output waste such as air pollution or hazardous waste are being recognized as more and more dangerous and therefore unwanted. This unwanted output should be considered while company's efficiency estimating. Thus companies with a better good (desirable) output and a lesser bad (unwanted) output should be recognized as effective.

**DEA-method usage**

The features of garbage recycling companies can be considered and estimated using DEA methodology. These features are:
- The input parameters - raw materials;
- The output parameters - recycled useful product, waste, ecological influence;
- The algorithms for the technological processes can be created using different approximations such as a system which interacts with an environment and factors and many more;
- The multicomponent mass is usually used as a raw material;
- As a result of the automated sorting the components, which cannot be used, remain;
- The efficiency of plant is evaluated using some criterions; therefore it fits DEA method and its lag's bound modification.

**DEA-method**

The given method estimates the production function that in reality is unknown. DEA-method is based on constructing the efficiency frontier that is an analogy of the production function in the case when the output is not a scalar but a vector, i.e. when there exist some kinds of output. This frontier has a form of a convex hall or a convex con in the space of input and output variables describing each DMU in the system. As follows from method’s name, the efficiency frontier envelopes or covers over a scatter of points in the multidimensional space. The efficiency frontier is used as a standard (or a peer) to obtain numerical value of efficiency measure of each object in the researching set. The efficiency degree for the i-th DMU is determined with degree of proximity to the efficiency frontier in the multidimensional space of inputs and outputs. The way of constructing an efficiency frontier is to solve the linear programming problem \(N\) times.

Let’s describe the main idea of the DEA-method on the example. A firm uses two inputs \((x_1 \text{ and } x_2)\) to produce a single output \((y)\).

If we assume constant returns to scale, we can represent the technology by a unit production possibility curve on a two-dimensional plot. The axes reflect costs for a unit, i.e. the volume of resources \(x_1\) and \(x_2\) per a unit of the output. Consequently, we have a unit isoquantum.

Let the given firm uses quantities of inputs, corresponding to the point \(P\), to produce a unit of the output. Then the technical inefficiency of that firm could be represented by the segment \(QP\). A point \(Q\) is a point \(P\) projection on the efficiency frontier. At the same time the projection is made towards the coordinate origin. The segment \(QP\) is a number by which all
inputs \( (x_1 \text{ and } x_2) \) could be proportionally reduced without a reduction in the output \((y)\). This approach of the efficiency measurement is called input-oriented.

![Figure 1. Production technology with two inputs and a single output.](image)

The technical efficiency (TE) of the firm is measured as follows:

\[
T E_P = \frac{0Q}{0P}
\]

In Figure 1 A, B, C and D are efficiency points. They form the efficiency frontier. A point P is not technically efficient because it doesn’t lie on the efficient isoquantum.

If an object A can produce the definite volume of output from the definite volume of resources, an object B can also produce the same volume of output from the same volume of resources. That’s why it is right to project the points corresponding to inefficient objects on the efficiency frontier.

Figure 1 shows that the value of technical efficiency can’t exceed 1. Projecting the inefficient object on the efficiency frontier, a hypothetic target object is formed. It is to be efficient. In mathematical sense this hypothetic target object corresponds to a linear combination of the real efficient objects (in this case a real object is a point in multidimensional space). The number of objects included by this combination depends on a number of factors like the input and output variables describing the objects, and values of these variables. The input and output variables of the target object are targets for the inefficient objects.

Let’s describe an idea of DEA-method using an example. Assume there exist data for \( K \) inputs and \( M \) outputs for each of \( N \) objects or DMU’s (they could be firms, banks, universities). For the \( i \)-th DMU they are represented by vectors \( x_i \) and \( y_i \), respectively. The model is formulated as are line a programming problem:

\[
\begin{align*}
\min_{\theta, \lambda} & \quad (\theta), \\
-y_i + Y\lambda & \geq 0, \\
\theta x_i - X\lambda & \geq 0, \\
\lambda & \geq 0,
\end{align*}
\]
The same linear programming problem must be solved N times, once for each DMU in the sample.

The presented model (1) is constructed under assumption of constant returns to scale and after its N times solving the efficiency frontier is formed as a convex cone.

**DEA method modification**

Consider the DEA method's model, which is built assuming constant returns to scale. The efficiency bound passes through points which correspond to efficient objects. Let's build the bound but on the contrary. For that, the points which produce the minimum output using the maximum inputs should be found. The new bound will pass through the points which correspond to unprofitable (inefficient) objects. Therefore those objects should be considered as inefficient.

The DEA model for the inefficient bound, which is built assuming constant returns to scale:

\[
\begin{align*}
\min & \quad \theta y_i - Y \lambda \\
\text{s.t.} & \quad \theta y_i - Y \lambda \geq 0 \\
& \quad -x_i - X \lambda \geq 0 \\
& \quad \lambda \geq 0
\end{align*}
\]

where \(\theta\) is the lag measurement for an object. \(\theta\) equal 1 means that object is a part of the bound and therefore is inefficient. \(\theta\) varies from 0 to 1.

Using this bound we can evaluate inefficiency degree for all considered objects and find the most inefficient of them.

Figure 2 a) shows the efficiency bound and figure 2 b) shows the inefficiency bound.

The modification of DEA-method can be used to estimate the amount of waste for the recycling plants. It is important to find plants which produce the minimum amount of waste using the maximum amount of input (unsorted garbage).

Here are the criterions which can be obtained using DEA method and the modification to considered sample.

Figure 3 shows schematics with inputs and outputs which are used for DEA based analysis. The inputs and outputs are marked as:

- \(x_1\) – the amount(volume) of unsorted raw materials(garbage);
- \(y_1, \ldots, y_7\) – the amount(volume) of the sorted, useful materials (paper, carton, glass, plastic bottles, polyethylene, aluminum cans, tin cans);
- \(y_8\) – the sum of all outputs;
- \(y_9\) – the amount(volume) of waste which cannot be used using given technology and installments.
According to given structure of the subtasks, here are their definitions.

Subtask 1. The efficiency of a plant evaluation for every output (glass, paper, carton and so on). It is shown on figure 3a.

Subtask 2. The efficiency of a plant evaluation for the sum of all outputs. It is shown on figure 3b.

Subtask 3. The minimum unwanted waste output evaluation. It is shown on figure 3c.

DEA method and the modification are used for schematics 3a and 3b. According to the methodology, the data sample of several objects (in our case, the recycling plants) is used and then the sets of their inputs \( x_1 \) and outputs \( y_1 \ldots y_9 \) are analyzed. After that, the efficient objects are defined and then the efficiency frontier is built. Next, this frontier can be used as a standard for other objects. Using this standard, it is possible to provide the inefficient objects with the parameter correction recommendations.

Using model (2) output \( y_9 \) we can obtain the inefficiency bound.
Calculation results

Subtask 1.

Table 1. Input, output and efficiency values for subtask 1

| Id of a plant | Vm³, cubic meters per year | paper | carton | glass | Plastic bottles | polyethylene | Tin cans | Aluminium cans | result, θ | ω |
|---------------|----------------------------|-------|--------|-------|-----------------|--------------|---------|---------------|-----------|---|
| 1             | 149400                     | 9960  | 12450  | 12450 | 7470            | 2490         | 2490    | 249           | 0.44      |   |
| 2             | 124500                     | 15017 | 29238  | 12816 | 19861           | 6724         | 8722    | 840           | 1         | 1  |
| 3             | 60310                      | 9660  | 19920  | 16814 | 6105            | 2044         | 3499    | 651           | 1         | 1  |

Table 1 shows the data for three plants. They are the plants in Krasnoyarsk, Novosibirsk and Kazan and they are marked as 1, 2 and 3 accordingly.

Plants 2 and 3 are efficient according to the method (θ = 1). Plant 1 is inefficient (θ< 1). It is impossible to show a seven parameter function as a two dimension graph. So let's show ratio of input to each output separately.

Since plants 2 and 3 are efficient, they will be a standard for plant 1. Figure 4 shows that plant 1 does not have any minimum value, only maximum values. It is possible to use the efficiency frontier to get recommendations how to increase efficiency.
**Figure 4.** The ratio of input to output: a) input/paper, b) input/carton, c) input/glass, d) input/plastic bottles, e) input/polyethylene f) input/tin cans, g) input/aluminum cans
Subtask 2.

**Table 2.** Input, output and efficiency values for subtask 2.

| Id of a plant | input  | output | result |
|---------------|--------|--------|--------|
|               | $V m^3$, cubic meters per year | Summary output in tons per year | $\theta_{seff}$ |
| 1             | 149400 | 76613  | 0.499  |
| 2             | 124500 | 111718 | 0.785  |
| 3             | 60310  | 68953  | 1      |

According to table 2, the efficiency bound can be built using plant 3 because its $\theta = 1$. It has the best ratio of the summary useful output to the summary input. Table 2 shows that plant 2 has a worse result than plant 3. It should be noticed that plant 1 is more competitive using the summary output than any single output.

Subtask 3.

**Table 3.** Input, output and ecological efficiency for subtask 3.

| Id for a plant | input  | output | result |
|---------------|--------|--------|--------|
|               | $V m^3$, cubic meters per year | Waste in tons per year | $\theta_{waste}$ |
| 1             | 149400 | 35019  | 1      |
| 2             | 124500 | 30000  | 0.973  |
| 3             | 60310  | 18000  | 0.785  |

Table 3 shows that plant 1 is the most environment friendly. Plants 2 and 3 produce more waste after the recycling process and shows less ecological efficiency.