Bounding the Number of Light Neutrinos Species in a Left-Right Symmetric Model

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(Dated: March 26, 2022)

Abstract

Using the experimental values for the rates $R^{\text{LEP}}_{\text{exp}} = \Gamma_{\text{inv}}/\Gamma_{\text{l}} = 5.942 \pm 0.016$, $R^{\text{Giga-Z}_1} = \Gamma_{\text{inv}}/\Gamma_{\text{l}} = 5.942 \pm 0.012$ (most conservative) and $R^{\text{Giga-Z}_1} = \Gamma_{\text{inv}}/\Gamma_{\text{l}} = 5.942 \pm 0.006$ (most optimistic) we derive constraints on the number of neutrino light species $(N_\nu)_{\text{LRSM}}$ with the invisible width method in the framework of a left-right symmetric model (LRSM) as a function of the LR mixing angle $\phi$. Using the LEP result for $N_\nu$ we may place a bound on this angle, $-1.6 \times 10^{-3} \leq \phi \leq 1.1 \times 10^{-3}$, which is stronger than those obtained in previous studies of the LRSM.

PACS numbers: 14.60.Lm,12.15.Mm, 12.60.-i

Keywords: Ordinary neutrinos, neutral currents, models beyond the standard model.

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I. INTRODUCTION

The number of fermion generations, which is associated to the number of light neutrinos, is one of the most important predictions of the Standard Model of the electroweak interactions (SM) \[1\]. In the SM the decay width of the $Z_1$ boson into each neutrino family is calculated to be $\Gamma_{\nu\bar{\nu}} = 166.3 \pm 1.5 \, MeV$ \[2\]. Additional generations, or other new weakly interacting particles with masses below $M_{Z_1}/2$, would lead to a decay width of the $Z_1$ into invisible channels larger than the SM prediction for three families while a smaller value could be produced, for example, by the presence of one or more right-handed neutrinos mixed with the left-handed ones \[3\]. Thus the number of light neutrino generations $N_\nu$, defined as the ratio between the measured invisible decay width of the $Z_1$, $\Gamma_{inv}$, and the SM expectation $\Gamma_{\nu\bar{\nu}}$ for each neutrino family, need not be an integer number and has to be measured with the highest possible accuracy.

The most precise measurement of the number of light ($m_\nu < 45 \, GeV$) active neutrino types, and therefore the number of associated fermion families, comes from the invisible $Z_1$ width $\Gamma_{inv}$, obtained by subtracting the observed width into quarks and charged leptons from the total width obtained from the lineshape. The number of effective neutrinos $N_\nu$ is given by \[4\]

$$N_\nu = \frac{\Gamma_{inv}}{\Gamma_{l}} \left(\frac{\Gamma_{l}}{\Gamma_{\nu}}\right)_{SM},$$

where $\left(\frac{\Gamma_{l}}{\Gamma_{\nu}}\right)_{SM}$, the SM expression for the ratio of widths into a charged lepton and a single active neutrino, is introduced to reduce the model dependence. The experimental value from the four LEP experiments is $N_\nu= 2.9841 \pm 0.0083$ \[2, 5\], excluding the possibility of a fourth family unless the neutrino is very heavy or sterile. $N_\nu$ is the effective number of light neutrino generations deduced from the $Z_1$ invisible width based on the expected partial width for one light neutrino generation ($N_\nu = \Gamma_{inv}/\Gamma_{\nu}^{SM}$). This result is in agreement with cosmological constraints on the number of relativistic species around the time of Big Bang nucleosynthesis, which seems to indicate the existence of three very light neutrino species \[6\]. On the other hand, the LEP result measures precisely the slight deviations of $N_\nu$ from three. In particular, the most precise LEP numbers can be translated into $N_\nu = 2.9841 \pm 0.0083$ \[2\], about two sigma away from the SM expectation, $N_\nu= 3$. While not statistically significant, this result suggests that the $Z\nu\bar{\nu}$-couplings might be suppressed with respect to the SM
value $4, 7$.

Using the experimental value for \( R_{\text{exp}}^{\text{LEP}} = \frac{\Gamma_{\text{inv}}^{\text{LRSM}}}{\Gamma_{\text{ll}}^{\text{LRSM}}} = 5.942 \pm 0.016 \) [5], we will determine the allowed region for \((N_\nu)_{\text{LRSM}}\) as a function of the mixing angle \(\phi\) and estimate bounds for the number of light neutrinos species in the framework of a left-right symmetric model (LRSM) [8, 9]. We will also use the LEP results to get a constraint on the LR mixing angle \(\phi\). On the other hand, if one assumes that the results for \(\Gamma_{\text{inv}}\) and \(\Gamma_{\text{ll}}\) for a future TESLA-like Giga-Z\(_1\) experiment agree with the central values obtained at LEP, one would measure \((\frac{\Gamma_{\text{inv}}}{\Gamma_{\text{ll}}})_{\text{Giga-Z}_1} = 5.942 \pm 0.012\) (most conservative) or \((\frac{\Gamma_{\text{inv}}}{\Gamma_{\text{ll}}})_{\text{Giga-Z}_1} = 5.942 \pm 0.006\) (most optimistic) [4], in this case we estimate also a limit for the number of light neutrinos species.

This paper is organized as follows: In Sec. II we present the expressions for the decay widths of \(Z_1 \rightarrow l\bar{l}\) and \(Z_1 \rightarrow \nu\bar{\nu}\) in the LRSM. In Sec. III we present the numerical computation and, finally, we summarize our results in Sec. IV.

II. WIDTHS OF \(Z_1 \rightarrow l\bar{l}\) AND \(Z_1 \rightarrow \nu\bar{\nu}\)

In this section we calculate the partial widths for \(Z_1 \rightarrow l\bar{l}\) and \(Z_1 \rightarrow \nu\bar{\nu}\) using the transition amplitude given in Ref. [8] in the context of the LRSM. The expression for the transition amplitude for the channel \(Z_1 \rightarrow l\bar{l}\) is given by

\[
M(Z_1 \rightarrow l\bar{l}) = \frac{g}{\cos\theta_W} \frac{1}{2} (a g'_{\nu} - b g'_{A} \gamma_5) [\bar{u}(l) \gamma^\mu v(\bar{l})] \varepsilon_\mu^\lambda(Z_1),
\]

where \(u(\nu)\) is the lepton (antilepton) spinor and \(\varepsilon_\mu^\lambda\) is the \(Z_1\) boson polarization vector and the expressions for the couplings \(a\) and \(b\) in the LRSM are:

\[
a = \cos\phi - \frac{\sin\phi}{\sqrt{\cos 2\theta_W}} \quad \text{and} \quad b = \cos\phi + \sqrt{\cos 2\theta_W} \sin\phi,
\]

where \(\phi\) is the mixing angle of the LRSM [7, 11].

After applying some of the trace theorems of the Dirac matrices and of sum and average over the initial and final spin the square of the matrix elements becomes

\[
\Sigma_s |M|^2 = \frac{g^2 M_{Z_1}^2}{3 \cos^2 \theta_W} \left[a^2 (g'_{\nu})^2 (1 + \frac{2 m_l^2}{M_{Z_1}^2}) + b^2 (g'_{A})^2 (1 - \frac{4 m_l^2}{M_{Z_1}^2})\right].
\]
Our next step, now that we know the square of the Eq. (3) transition amplitude, is to calculate the partial width of the reaction $Z_1 \rightarrow l\bar{l}$:

$$\Gamma_{ll} = \frac{G_F M_{Z_1}^2}{6\pi\sqrt{2}} \sqrt{1 - 4\eta_l[a^2(g_L)^2 + b^2(g_A)^2 + 2\eta_l(a^2(g_L)^2 - 2b^2(g_A)^2)],}$$

(4)

where $\eta_l = \frac{m_l^2}{M_{Z_1}^2}$.

For the $Z_1$-decay width into $\nu\bar{\nu}$ we obtain

$$\Gamma_{\nu\bar{\nu}} = \frac{G_F M_{Z_1}^3}{12\pi\sqrt{2}} \sqrt{1 - 4\eta_\nu\left[\frac{1}{2}(a^2 + b^2) + \eta_\nu(a^2 - 2b^2)\right]},$$

(5)

where $\eta_\nu = \frac{m_\nu^2}{M_{Z_1}^2}$.

The partial widths Eqs. (4) and (5) are applicable to all charged leptons and all neutrinos respectively.

III. RESULTS

In order to compare the respective expressions Eqs. (4) and (5) with the experimental result for the number of light neutrinos species $N_\nu$, we will use the definition for $N_\nu$ in a SM analysis [10],

$$N_\nu = R_{exp} \left( \frac{\Gamma_{ll}}{\Gamma_{\nu\bar{\nu}}} \right)_{SM},$$

(6)

where the quantity in parenthesis is the standard model prediction and the $R_{exp}$ factor is the experimental value of the ratio between the widths $\Gamma_{inv}$ and $\Gamma_{ll}$ [2, 5],

$$R_{exp}^{LEP} = \left( \frac{\Gamma_{inv}}{\Gamma_{ll}} \right) = 5.942 \pm 0.016.$$  

(7)

This definition replaces the expression $N_\nu = \frac{\Gamma_{inv}}{\Gamma_{\nu\bar{\nu}}}$ since (7) reduces the influence of the top quarks mass. To get information about what is the meaning of $N_\nu$ in the LRSM we should define the corresponding expression [7],

$$(N_\nu)_{LRSM} = R_{exp} \left( \frac{\Gamma_{ll}}{\Gamma_{\nu\bar{\nu}}} \right)_{LRSM}.$$  

(8)
This new expression is a function of the mixing angle $\phi$, so in this case the quantity defined as the number of light neutrinos species is not a constant and not necessarily an integer. Also, $(N_{\nu})_{LRSM}$ in formula (8) is independent from the $Z_2$ mass and therefore depends only of the mixing angle $\phi$ of the LRSM. Experimental values for $\Gamma_{inv}$ and for $\Gamma_{\bar{u}l}$ are reported in literature which, in our case, can give a bound for the angle $\phi$. However, we can look to those experimental numbers in another way. The partial widths $\Gamma_{inv} = 499.0 \pm 1.5$ MeV and $\Gamma_{\bar{u}l} = 83.984 \pm 0.086$ MeV were reported recently [2], but we use the value given by (8) for the $R_{exp}$ rate of Ref. [5]. All these measurements are independent of any model and can be fitted with the LRSM parameter $(N_{\nu})_{LRSM}$ in terms of $\phi$.

In order to estimate a limit for the number of light neutrinos species $(N_{\nu})_{LRSM}$ in the framework of a left-right symmetric model, we plot the expression (8) to see the general behavior of the $(N_{\nu})_{LRSM}$ function, Fig. 1. For the mixing angle $\phi$ between $Z_1$ and $Z_2$, we use the reported data of Maya et al. [7]:

$$- 9 \times 10^{-3} \leq \phi \leq 4 \times 10^{-3},$$

with a 90% C.L. Other limits on the mixing angle $\phi$ reported in the literature are given in Refs. [11, 12]. In this figure we observed that for the mixing angle $\phi$, around 0.65 rad, $(N_{\nu})_{LRSM}$ can be as high as 5.9, and for values of $\phi$ around -0.95 rad, $(N_{\nu})_{LRSM}$ is as low as 0. This shows a strong dependence in $\phi$ for leptonic decays of the $Z_1$ boson. Therefore, according to the above discussion, if we consider $(N_{\nu})_{LRSM}$ as the number of neutrinos, the restriction on the number of species can be “softened” if we consider a LRSM. In Fig. 2, we show the allowed region for $(N_{\nu})_{LRSM}$ as a function of $\phi$ with 90% C.L. The allowed region is the inclined band that is a result of both factors in Eq. (8). In this figure $(\frac{\Gamma_{\bar{u}l}}{\Gamma_{\bar{\nu}\nu}})_{LRSM}$ gives the inclination while $R_{exp}$ gives the broading. This analysis was done using the experimental value given in Eq. (7) for $R_{exp}$ reported by [5] with a 90% C.L. In the same figure we show the SM ($\phi = 0$) result at 90% C.L. with the dashed horizontal lines. The allowed region in the LRSM (dotted line) for $(N_{\nu})_{LRSM}$ is wider that the one for the SM, and is given by:

$$2.925 \leq (N_{\nu})_{LRSM} \leq 3.02 \quad \text{or} \quad (N_{\nu})_{LRSM} = 2.987^{+0.033}_{-0.062}, \quad 90\% \ C.L.,$$

whose center value is quite close to the standard model of three active neutrino species.

In the case of a future TESLA-like Giga-$Z_1$ experiment we obtain the limits
2.926 \leq (N_{\nu})_{LRSM} \leq 3.019 \text{ or } (N_{\nu})_{LRSM} = 2.987^{+0.032}_{-0.061}, \text{ 90\% C.L. (most conservative)} \quad (11)

2.929 \leq (N_{\nu})_{LRSM} \leq 3.016 \text{ or } (N_{\nu})_{LRSM} = 2.987^{+0.029}_{-0.058}, \text{ 90\% C.L. (most optimistic)}, \quad (12)

which are consistent with those reported in the literature \text{[4]}. 

Finally, and just for completeness, we reverse the arguments that is, we fix the number of neutrinos in the LRSM to be three then the theoretical expression for $R$ will be given by

$$R_{LRSM} = \frac{3\Gamma_{\nu\nu}}{\Gamma_{ll}}.$$ \quad (13)

The plot of this quantity as function of the mixing angle $\phi$ is shown in Fig. 3. The horizontal lines give the experimental region at 90\% C.L. From the figure we observed that the constraint for the $\phi$ angle is:

$$-1.6 \times 10^{-3} \leq \phi \leq 1.1 \times 10^{-3},$$ \quad (14)

which is about one order of magnitude stronger than the one obtained in previous studies of the LRSM \text{[7, 11, 12]}. 

In the case of a future TESLA-like Giga-Z_{1} experiment we would obtain the following bounds for the mixing angle $\phi$:

$$-1.1 \times 10^{-3} \leq \phi \leq 0.9 \times 10^{-3}, \text{ (most conservative)},$$ \quad (15)

$$-0.8 \times 10^{-3} \leq \phi \leq 0.33 \times 10^{-3}, \text{ (most optimistic)}. \quad (16)$$

IV. CONCLUSIONS

We have determined a bound on the number of light neutrinos species in the framework of a left-right symmetric model as a function of the mixing angle $\phi$, as shown in Eq. (10) and Fig. 2. Using this result and the LEP values obtained for $N_{\nu}$, we were able to put a limit on the LR mixing angle $\phi$ which is better than the one obtained in previous studies of these models.

In summary, we conclude that in the LRSM it is possible to obtain from the experimental results a value for $N_{\nu}$ different from 3 (not necessarily an integer number). In particular for
the left-right symmetric model with Dirac neutrinos, \((N_\nu)_{LRSM}\) is in the neighborhood of three. However, if new precision experiments find small deviations from three, this model may explain very well these deviations with a small value of \(\phi\). We have shown that new data of \(R_{\text{exp}} = \frac{\Gamma_{\text{inv}}}{\Gamma_{\text{l}}}\) can considerably shrink the allowed region of \((N_\nu)_{LRSM}\). In the limit of \(\phi = 0\), our bounds takes the value previously reported in the literature [2, 4, 5].

Acknowledgments

We would like to thank O. G. Miranda for useful discussions. This work was supported by CONACyT and SNI (México).

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FIG. 1: \((N_\nu)_{LRSM}\) as a function of the mixing angle \(\phi\).

FIG. 2: Allowed region for \((N_\nu)_{LRSM}\) as a function of the mixing angle \(\phi\) with the experimental value \(R_{\text{exp}}^{\text{LEP}}\). The dashed line shows the SM allowed region for \(N_\nu\) at 90% C.L., while the dotted line shows the same result for the LRSM.
FIG. 3: The curve shows the shape for $R_{LRSM}$ as a function of the mixing angle $\phi$. The dashed line shows the experimental region for $R_{\text{exp}}^{\text{LEP}}$ at 90% C.L.