Development of a Size Degradation Model of Coke Particles at the Drum Test and inside the Blast Furnace

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Experimental as well as theoretical studies were made on the basis of the breakage contact model to elucidate the breakage mechanism of cokes in the drum tests and the following equation to express the size distribution of cokes after breakage was obtained.

\[ R(D_p, t) = (1 - \left( \frac{D_p}{D_{p,s}} \right)^{\frac{E}{M}} \left( \frac{D_{p_{max}}}{D_{p_{max}}} \right)^{g} \) \exp \left[ -\left( \frac{E}{M} \right) \left( \frac{D_p}{D_{p_{max}}} \right)^{g} \right] \]

Here in the equation, \( R(D_p, t) \) is the size distribution function at time \( t \) in the cumulative oversize of \( D_p \). The key \( w \) means the specific work rate cokes suffer from the breakage system. \( D_{p,s} \) is the size of sample and \( D_{p_{max}} \) is the maximum size to breakage fragments. The key \( \frac{E}{M} \) means the specific energy loaded per unit mass of breakage fragment and is named apparent fines generation energy. The key \( g \) is the size distribution index to characterize the size distribution profile of the breakage fragments. These two parameters dominate the breakage phenomena and their values are given as functions of the mechanical properties of sample coke and the mechanical load conditions of the concerned breakage system.

The equation was confirmed useful to predict the breakage phenomena throughout the experiments with using three kinds of drum machine.

KEY WORDS: blast furnace; breakage contact; cokes; coke size degradation; coke size distribution; coke fines generation; drum test.; DEM.

1. Introduction

Coke is an indispensable staff in blast furnace and the quality of coke affect strongly on the operational performance of the blast furnace. Therefore, severe checks are being made on the mechanical quality of cokes before and after the gasification reaction. Such indices as drum index DI15 and coke strength after reaction index CSR are commonly used as the mechanical strength index of cokes. However, these indices are nothing but the test results under restricted conditions and don't stand for the basic property to dominate the breakage phenomenon of cokes. There was no measure to connect these indices with the actual coke size degradation or coke fines generation in the blast furnace. This is due to the lack of the basic theory to elucidate the mechanism of the breakage of coke particles in spite that several kinds of study had been performed.1–5

Standing on this viewpoint, a study was made and a theoretical model to quantify the breakage contact phenomena of coke particles was developed as presented in the previous report.6 According to the model, the mode of the breakage of coke particles is surface compressive breakage and the basic property to dominate the coke breakage phenomena is the compressive strength. The mechanical load condition also affects on the breakage phenomena. Therefore, if mechanical load conditions both at the coke strength test systems and inside the blast furnace are given in addition to the compressive strength of cokes, it will become possible to connect coke strength indices such as DI and CSR with the actual coke size degradation and coke fines generation inside blast furnace.

Here in this study, drum tests were performed using three kinds of drum machines and three kinds of coke samples in order to clear the breakage phenomena at drum tests and to check the usefulness of the theoretical model. And based on the experimental results, a theoretical model to predict the breakage of cokes at arbitral breakage system was developed.

2. Experimental Study

The breakage phenomenon is quantified in general as the change in the size distribution of particle assembly before and after the breakage. The study put focus on the determination of the formula of the size distribution of coke particles after drum tests and on the clarification of the influences of drum conditions and coke properties on the size distribution of coke particles after the drum tests.

2.1. Experimental Method

2.1.1. Experimental Apparatus

Three kinds of drum machines were used as listed in Table 1. These drum machines are usually used for the
evaluations of the index RDI for sinters, indices CSR and DI for cokes, respectively. In the experiments, rotating speeds were set at usually adjusted values for respective drum machines. Numbers of rotating times were changed in order to change the total energy loaded on coke samples. Mass of samples was set at constant values of 100 g, 5 kg and 100 g for I-drum, DI-drum and RDI-drum, respectively.

As regards the practical values of the effective dropping height \( h \) and the numbers of drops per a rotation \( t \) were evaluated by three-dimensional DEM calculation for each drum test. In the DEM calculations, samples were assumed with the shape of sphere and with the same diameter of 20 mm. The formulas used to express the normal and tangential force were those derived in the previous study.6) The calculated values are listed on the table. With using obtained effective dropping height \( h \), specific contact energy per a rotation \( e \) and total specific energy loaded on particles \( e_t \) are given as follows:

\[
e = gh = (1/2)V^2 \quad \text{.........(1)}
\]

\[
e_t = en \tau \quad \text{.........(2)}
\]

Here, \( V \) is the collision velocity, \( g \) is the gravity coefficient and \( n \) is the number of rotation times.

2.1.2. Coke Samples

Coke samples used at the experiments are listed in Table 2. Three kinds of cokes with different compressive strength were used after sieved into several size ranges. In this study, the particle diameter means the sieve diameter. The compressive strength this study means is a kind of Brinel hardness. Values for each coke sample were measured using the spherical aluminum oxide particle with the diameter of 15 mm. At the measurement, aluminum oxide particle was compressed on the flat plane of coke sample with the force of 30 kg-f and the diameter of the circular trace marked on the particle was measured to evaluate the compressive strength. Values obtained through this procedure were widely fluctuated. Therefore, measurements were made ten times for each coke sample and the average of five values excluded abnormal ones were selected.

### Table 1. Drum conditions at the experiments.

| Drum     | Diameter | Lifters | Rotation rate | Drops per rotation | Sample Mass | Effective High* | Rotation times  |
|----------|----------|---------|---------------|-------------------|-------------|-----------------|----------------|
| I-Drum   | 0.7m     | Tube    | 20rpm         | 2.0               | 100gr       | 0.400           | 300, 600, 900, 1200 |
| DI-Drum  | 1.5m     | 6 p     | 15rpm         | 1.5               | 5kg         | 1.275           | 30, 90, 150     |
| RDI-Drum | 0.14m    | 2 p     | 30rpm         | 2.0               | 100gr       | 0.074           | 600, 1800      |

*Dem

### Table 2. Coke particles used at the experiments.

| Compressive Strength | Apparent Density | Particle Diameter |
|----------------------|------------------|-------------------|
| A-coke               | 50MPa            | 8-10, 10-12, 15-20, 25-30, 30-35mm |
| B-coke               | 40MPa            | 8-10, 10-12, 15-20mm |
| C-coke               | 20MPa            | 8-10, 10-12mm |

*Brinel Index

2.2. Experimental Results

2.2.1. Size Distribution Profile of Breakage Fragments

All the cokes after drum tests were sieved and their size distributions were examined. Figure 1 shows examples of the obtained size distributions. These patterns are similar to each other and indicate the cokes after drum tests were composed of the breakage fragments and the residues of parent cokes. This means that the mode of breakage at drum tests is the surface compressive breakage as is predicted by the breakage contact model.6)

The size distributions were patterned in the same formula as follows:

\[
D(Dp) = \phi(Dp/Dp_{max})^\alpha + (1 - \phi(Dp/Dp_{max}))^\beta, \quad Dp > Dp_{min}
\]

\[
= \phi(Dp_{min}/Dp_{max})^\gamma(Dp/Dp_{max})^\delta, \quad Dp < Dp_{min} \quad \text{.........(3)}
\]

Here, \( D(Dp) \) is the size distribution function expressed by the cumulative undersize of \( Dp \) and \( \Phi \) is the breakage fragments ratio defined as the ratio of the mass of generated fragments \( M \) to the mass of sample cokes \( M_0 \). The powers \( \alpha, \beta \) and \( \gamma \) represent the size distribution profiles of fine fragments, coarser fragments and residual parent cokes, respectively. \( Dp_{max} \) is the maximum size of the breakage fragments and is thought to be in proportion to the size of coke sample \( Dp_c \). The size of breakage fragments must be less than half of the sample size. Therefore, the value of \( Dp_{max} \) was set at half as much as the coke sample size. \( Dp_{min} \) is the size to discriminate finer fragments from coarser part.

As regards the influences of drum conditions and coke properties on these parameters, \( \alpha \) and \( Dp_{min} \) were confirmed almost constant with the value of about 1.0 and 150 \( \mu \)m, respectively. On the other hand, the parameter \( \beta \)
was more than 5 for all cases and parameters $\Phi$ and $\gamma$ changed with the change of drum conditions and coke properties. The dependences of the parameter $\Phi$ and $\gamma$ on the drum conditions and coke properties are analyzed in the following sections.

2.2.2. Size Distribution Parameter $\gamma$

The influences of the drum conditions and coke properties on $\gamma$ were analyzed as shown in Fig. 2 and the following equation was obtained.

$$
\gamma = 3400e^{-0.3(D_p/10^{-3})^{0.75}}t^{10}(S/10^{6})^{-1} \ldots \ldots (4)
$$

Figure 3 shows the accuracy of the obtained regression equation. The figure confirms the strong correlation between regressed $\gamma$ and actual $\gamma$, which indicates that the parameter $\gamma$ may have strong relations with such factors as collision velocity, particle size and compressive strength.

On the other hand, it was difficult to clarify the dependence of $b$ on drum test conditions or coke properties. But if the mass balance between fragments and parents before and after breakage is considered, the power $b$ should be expressed as follows:

$$
b = \frac{d(M_0/\Phi)}{dn} = E/(E/M) \ldots \ldots (5)
$$

2.2.3. Fines Generation Energy

According to the breakage contact model (see Appendix), fines ratio $\Phi$ is related with energy consumption per a rotation as follows:

$$
d(M_0/\Phi)/dn = E_s/(E_s/M) \ldots \ldots (6)
$$

Here, $M_0$ is the mass of sample used for a test. However, the value of energy consumption can’t be directly measured in case of drum test. On the other hand, contact energy can be evaluated with the help of Eq. (1). Additionally, the following relationship forms between energy consumption and contact energy.

$$
E = E_s/(1 - \eta^2) \ldots \ldots (7)
$$

Therefore, the new parameter defined by the next equation is introduced to take a part of fines generation energy $(E/M)$ in this study.

$$
(E/M) = (E_s/(1 - \eta^2)) \ldots \ldots (8)
$$

With using the apparent fines generation energy, Eq. (6) is rewritten as follows:

$$
d(M_0/\Phi)/dn = E_s/(E_s/M) \ldots \ldots (9)
$$

$$
E = M_d(1 - \Phi)e = M_d(1 - \Phi)\tau V^2/2 \ldots \ldots (10)
$$

This equation has the next analytical solution.

$$
\Phi = 1 - \exp\{-E/(n\tau V^2/2)\} \ldots \ldots (11)
$$

Therefore, the following equation is derived for the evaluation of the apparent fines generation energy from measured fines ratio.

$$
(E/M) = -(\tau V^2/2)n/\ln(1 - \Phi) \ldots \ldots (12)
$$

With the help of Eq. (12), the values of apparent fines generation energy for each test case were evaluated and the influences of the drum test conditions on the apparent fines energy was analyzed. Figure 4 shows the results. The following regression equation was obtained.
The apparent fines generation energy is expressed by the Eq. (8). According to the breakage contact model (see Appendix), the fines generation energy \((E/M)\) is expressed by Eq. (A-8) and restitution coefficient is given by Eq. (A-14).

\[
(E/M) = (S/k \rho) \left\{ (X-1)^2 + 2(X-1/3)k/k \right\} \] ..........................(14)

\[
\eta = (32/15)X^{3/5}/(16/15)(X^2+X^{1.5}) + 2(X-1/3)/k \] ..........................(15)

On the other hand, next relationship forms between the collision velocity \(V\) and the contact energy \(E\).

\[
(1/2)(\pi Dp^3/6\rho)/V^2 = E \] ..........................(16)

Additionally, the contact energy is given by the Eq. (A-6) as follows:

\[
E = \pi S\delta Y \left\{ 16/15 + (X-1)(X+1/3) \right\} \] ..........................(17)

The value of \(E\) when \(X\) equals to 1 corresponds to the critical contact energy against the surface compressive breakage and therefore, the critical velocity \(V_0\) and the normalized contact displacement \(X\) are written as follows:

\[
X = (16/15)^{0.5}(V/V_0) \] ..........................(18)

\[
V_0 = (16/5)^{0.5}(\pi/2)^{5/2}(r/Dp)^{3/5}(S/Y)^3(S/\rho)^{0.5} \] ..........................(19)

In the Eq. (19), the values of \((S/\rho)\) and \((S/Y)\) are around 50 000 and 0.02, respectively. The value of \((r/Dp)\) is 0.25 supposing the shape of cokes sphere. According to these values, the order of \(V_0\) becomes around 0.05 m/s. On the other hand, collision velocities were estimated 5.0 m/s, 2.8 m/s and 1.2 m/s for DI, I-type and RDI drum, respectively. Therefore, the value of \(X\) is far larger than 1 and consequently, the approximate formulas for fines generation energy and restitution coefficient are written as follows:

\[
(E/M) = (S/k \rho) \] ..........................(14’)

\[
\eta = 2(1+X^{0.5}) \] ..........................(15’)

On the basis of the discussions above made, the apparent fines generation energy is expressed as a function of coke properties and drum conditions, as follows:

\[
(E/M) = (S/k \rho) \left\{ 1 + 2X^{-0.5} \right\} \] ..........................(20)

The first brace of the equation shows the fines generation energy and the second brace shows the effect of restitution. The compressive strength to Young’s modulus ratio \((S/Y)\)
is regarded constant because these two values are usually in proportion to each other. The effective curvature radius of contact point $r$ is proportional to the size of sample in case of sphere particle. However, in case of the particle with angular shape, $r$ is supposed rather constant. Then, by setting $(S_c/Y)$ at 0.02 and $r$ at 5 mm, values of apparent fines generation energy was calculated for all the test cases.

Figure 6 compares the calculated $(E/M)$ with those obtained by experiments. The accuracy of the calculation is lower than that of the regression equation. Especially, the influence of the total specific energy $e_t$ isn’t taken into account in the Eq. (20). However, the dependences on the collision velocity, sample size and compressive strength are well elucidated. Therefore, it can be said the model could evaluate the value of apparent fines generation energy.

3.1.2. Size Distribution Index $\gamma$

It is supposed that the energy consumed at the breakage contact equals to the energy supplied to generate cracks near the contact surface and in the case of brittle materials like cokes, most of generated cracks change to the surfaces of fragments. Then, if the generated cracks turn to fragments surfaces in a constant ratio, the energy consumption will be in proportion to the surface energy of fragments and therefore, next equation is formed.

$$(E/M) = (\Gamma/\rho)k\alpha_1, \ldots, \ldots, \ldots, \ldots (21)$$

Here, $\Gamma$ is the specific surface energy of coke matrix and $\kappa$ is the constant to express the ratio of generated cracks to turn into the surfaces of fragments.

The energy consumption per unit mass of fragments is
given by Eq. (14). On the other hand, fragments surface area $a_f$ can be evaluated with the help of fragments size distribution function given by Eq. (3).

$$a_f = \int_{D_{lo}}^{D_{max}} \left( \frac{6}{\pi} \frac{Dp}{Dp} \right) (dN/dDp) dDp = \left( \frac{6}{\pi} \frac{Dp}{Dp} \right) \ln \left( \frac{Dp_{max}}{Dp_{min}} \right)$$

Here, $\psi$ is the shape factor of fragments, $D_{lo}$ is the minimum size of fines. By combining Eqs. (23) and (24), next relationship is formed.

$$\gamma = \ln \left( \frac{6}{\pi} \frac{Dp_{max}}{Dp_{min}} \right) \ln \left( \frac{Dp_{min}}{Dp_{max}} \right)$$

According to the experimental results, $Dp_{min}$ was confirmed almost constant. If $D_{lo}$ is also assumed constant, Eq. (23) expresses the relationship between $\gamma$ and sample properties and gives the following formula to evaluate the value of $\gamma$.

$$\gamma = \ln \left( \frac{6}{\pi} \frac{Dp_{max}}{Dp_{min}} \right) \ln \left( \frac{Dp_{max}}{Dp_{min}} \right)$$

Here, the surface energy of graphite is reported to be about 6 J/m². As for the value of $D_{lo}$, the minimum possible size must be around several times of graphite lattice size of 1.42 Å and therefore, it is assumed around 10 Å. As regards the shape factor of fragments, the values is assumed about 0.3. However, the value of $\kappa$ is unknown. Then, by supposing $\psi$ at 0.3 and $D_{lo}$ at 10 Å, practical values of $\gamma$ were calculated for every cases and the value of $\kappa$ was fitted for the order of calculated $\gamma$ to agree with experiments.

**Figure 7** shows the correlation between $\gamma$ by calculation and by experiments. The value of $\kappa$ fitted was 2.5, which means 40% of the total cracks forms the fragments surface. The figure shows there are big deviations. Especially, the dependence of $\gamma$ on the collision velocity isn't elucidated. However, the dependence of $\gamma$ on the compressive strength is well explained. Here, the key $k$ is the parameter to express the ratio of the breakage fragment volume to the swept-off volume of the breakage contact plane and was regarded constant with the value of 2 in the breakage contact model. However, this value should change in actual affected not only by the coke strength but also by the contact conditions, which is pointed out as the reason why Eq. (24) could not elucidate the effect of collision velocity.

### 3.1.3. Apparent Fine Generation Energy for Arbitrary Size

According to Eq. (8), the apparent fines generation energy for arbitrary size $(E/M)_{Dp}$ is written as follows.

$$(E/M)_{Dp} = S_c(k_{Dp}/(1 - \eta^2))$$

Here, the key $k$ is replaced by $k_{Dp}$. The key $k_{Dp}$ shows the volume ratio of fragments with the size under $Dp$ to the total breakage fragments and $\{S_c(k_{Dp}/\rho)\}$ is the fines generation energy itself for the fragments with the size of under $Dp$.

As for the fines generation energy for $Dp_{min}$, next equation is derived from Eq. (23) by replacing $Dp_{max}$ by $Dp_{min}$.

$$S_c(k_{Dp}/\rho) = \kappa \left( \frac{6}{\pi} \frac{Dp_{min}}{Dp_{max}} \right) \ln \left( \frac{Dp_{min}}{Dp_{max}} \right)$$

In this equation, $\Gamma$ is thought constant. The value of $Dp_{min}$ were confirmed almost constant. Therefore, if the values $\kappa$, $\psi$ and $D_{lo}$ are also supposed constant, the right side of the Eq. (26) becomes constant. This indicates the fine generation energy for $Dp_{min}$ is constant and the value $k_{Dp_{min}}$ depends only on the compressive strength to density ratio of coke. Substituting those parameters as $\kappa$, $\psi$, $Dp_{min}$, $D_{lo}$, $\Gamma$ and $\rho$ by their own values as was mentioned before, the practical value of the fines generation energy for $Dp_{min}$ is calculated as 24 kJ/kg. On the other hand, it is possible to evaluate the values of apparent fines generation energies for $Dp_{min}$ from experimental results. Accordingly, the regression analysis was made on the dependence of apparent fines generation energy for $Dp_{min}$ on coke properties and drum conditions using evaluated values. **Figure 8** summarizes the result of regression and the practical formula obtained was as follows:

$$(E/M)_{Dp_{min}} = 22000(S_c/10^3)^{0.45}(Dp_{min}/10^{-3})^{-0.32}y^{-0.25}$$

The apparent fines generation energy depends on the restitution as shown in Eq. (25) and its influence is written in the form of the second brace in Eq. (20). Here, the influences of $S_c$, $Dp_{min}$ and $V$ on $(E/M)_{Dp_{min}}$, given by the regression Eq. (27) is similar to those predicted by the theoretical Eq. (20). This coincidence demonstrates the supposition above mentioned. Namely, the dependence of $(E/M)_{Dp_{min}}$ on coke properties and collision velocity is due to the effect of restitution and the fines generation energy for $Dp_{min}$ itself is almost constant.

On the other hand, as the $k$ in Eq. (23) equals to $k_{Dp_{min}}$, next equation forms from Eqs. (23) and (26).

$$(E/M)_{Dp_{min}} = k_{Dp_{min}}(Dp_{min}/Dp_{max})$$

The regression Eq. (4) showed $\gamma$ depends both on the coke properties and drum conditions. Therefore, according to the Eq. (28), $k_{Dp_{min}}$ is said to depend both on the coke properties and drum conditions.

Based on the discussion above made, the apparent fines generation energy for arbitrary size $Dp$ within the range between $Dp_{min}$ and $Dp_{max}$ is written by two kinds of formula as follows.

$$(E/M)_{Dp} = (E/M)_{Dp_{min}}(Dp_{min}/Dp)^{\gamma}$$
These two formulas give the same value when coke properties and drum conditions are set within the range as experienced at the experiments.

### 3.2. Development of Coke Size Degradation Model

The parameters necessary to predict the breakage phenomena of coals at drum tests are the apparent fines generation energy and the size distribution index. The dependences of these parameters on drum conditions and coke properties were evaluated through the experiments as written in Eqs. (4) and (13). Additionally, the breakage contact model has elucidated their theoretical meanings. Therefore, it becomes possible to predict the coke size distribution at an arbitrary breakage system. Here in this clause, the coke size distribution at arbitrary breakage system is developed with a degradation model to predict the change in the coke size distribution at drum tests.

According to the population balance theory, the cumulative undersize $D_p$ at an arbitrary time $t$ is usually given as follows:

\[
dD(Dp,t)/dt = \int_{D_{p0}}^{D_p} S(Dp')B(Dp',Dp) \times (dD(Dp',t)/dDp') dDp'
\]

Here, $D(Dp,t)$ is the size distribution function at the time $t$ expressed by the cumulative undersize $D_{p0}$. $S(Dp')$ is the selection function to express the contact frequency and $B(Dp',Dp)$ is the breakage function (or distribution function) to express the size distribution of fragments to be generated from a piece of particle with the size of $Dp'$ after a contact. However, it is also possible to regard the product of selection function and breakage function as a modified breakage function $B'(Dp',Dp)$ to express the size distribution of fragments from a piece of particle with the size of $Dp'$ after an unit interval of time.

\[
B'(Dp',Dp) = S(Dp')B(Dp',Dp)
\]

Based on the experimental result, the modified breakage function is expressed as follows:

\[
B'(Dp',Dp) = \phi(Dp')[(Dp/Dp')^\theta + (1-\phi(Dp'))(Dp/Dp')^\beta]
\]

Here, $(E/M)_{dy}$ is the apparent fines generation energy for the particle size $Dp'$ and given by Eq. (29) or (30). The key $w$ is the specific work rate loading on coke particle and is given for collision contact and for friction contact as follows:

\[
w = \chi e^{\text{collision system}} \quad (35)
\]

\[
w = P_e \{ \mu_s (dL/dt) + \mu_f (dL/dt) \} \quad \text{friction system} \quad (36)
\]

Here, $\chi$ is the collision frequency, $P_e$ is the normal contact force. $\mu_s$ and $\mu_f$ are friction coefficients for slipping and rotating and they are given by Eqs. (A-15) and (A-16).

\[
\mu_s = \frac{2/3\pi(2\delta/r)^{0.5}}{3X^{-0.5}\phi(X-1)^{-1.5}-1} \quad (37)
\]

\[
\mu_f = \frac{2/3\pi(2\delta/r)^{0.5}}{1.5X^{-0.5}(d/\delta y)/k + (X-1)^{1.5}(X-1/3)} \quad (38)
\]

On the other hand, $(dL/dt)$ and $(dL/dt)$ are the increment rate of slipping and rotating distances.

Substituting Eqs. (33) and (34) into Eq. (32), the concrete formula of Eq. (32) becomes as follows:

\[
\phi(Dp') = \frac{w(E/M)_{dy}}{w} \quad (34)
\]
The first term of the right side of this equation corresponds to the size distribution of fragments and the second term corresponds to that of parent cokes. Here, in the case when the second term of the right side is neglected, the analytical solution is given as follows:

\[ R(D_p, t) = \exp\left[ -\left(\frac{w}{(E/M)_{D_p}}\right)\left(\frac{D_p}{D_{p,\text{max}}}\right)^\gamma \right] \] ........(40)

At the integration, the practical formula for \( (E/M)_{D_p} \) given by Eq. (30) is used. \( R(D_p, t) \) is the size distribution function expressed in the cumulative oversize and equals to "1 - \( D(D_p, t) \)". On the other hand, the effect of the second term is limited near the size of initial parent cokes and therefore, following formula is obtained as an approximate solution of Eq. (39).

\[ R(D_p, t) = \left\{1 - \left(\frac{D_p}{D_{p,\text{max}}}\right)^{\gamma(\frac{E}{M}/w)} \right\} \times \exp\left[ -\left(\frac{w}{(E/M)_{D_p}}\right)\left(\frac{D_p}{D_{p,\text{max}}}\right)^\gamma \right] \] ........(41)

Equation (41) is the formula to express the size distribution at arbitrary time in arbitrary drum machine. The size distributions for all the experimental cases were calculated using Eq. (41) and results are compared with measured ones. Figure 9 shows the results obtained with using Eqs. (20) and (24) for \( (E/M) \) and \( \gamma \), respectively. Therefore, this result stands on the perfect theory. Figure 10 shows the results obtained using regression Eqs. (4) and (14) for \( (E/M) \) and \( \gamma \). When these two results are compared, it is obvious that the result by regression equations is better than that of theoretical equations. Figure 11 shows the correlations between the cumulated under 1 mm calculated and measured. Higher accuracy was achieved in the case when regression equations were used. However, it can be said that the theoretical result also elucidates the measurements on the whole.

By the way, Rosin-Rammler’s formula is well known as the expression appropriate for the expression of cokes and coals size distribution functions.

\[ R(D_p, t) = \exp\left[ -(\frac{D_p}{D_{p,\text{max}}})^\gamma \right] \] ........(42)

If the Eq. (42) is compared with the Eq. (40), it is obvious that these two formulas are the same and the model gives the characteristic size \( D_{p,\text{e}} \) in Rosin–Rammler’s formula as follows:

\[ D_{p,\text{e}} = \left(\frac{(E/M)/(w)}{\gamma}\right)^{1/\gamma} D_{p,\text{max}} \] ........(43)

The model shows that the apparent fines generation energy and size distribution index dominate the characteristics of the Rosin–Rammler’s size distribution function.

3.3. Relation between Drum Index and Fines Generation inside Blast Furnace

According to the model above developed, the breakage of cokes is dominated by the compressive strength of cokes and the mechanical load conditions of the breakage system. Therefore, it becomes possible to estimate the breakage of cokes inside blast furnace if the compressive strength of
cokes and mechanical load conditions inside blast furnace are given. As for the mechanical load conditions, many kinds of model have been developed for the elucidation of granular assembly dynamics such as discrete element model and plasticity model. With the help of these models, the mechanical loading conditions inside actual blast furnace can be evaluated. On the other hand, as regards the compressive strength of cokes, it is not easy to measure its actual value directly because measured values will show serious fluctuation due to the heterogeneity of coke matrix. On the contrary, enormous numbers of collision occur during the drum test and the mechanical load conditions at drum tests are comparatively clear. Additionally, the model here developed could evaluate the compressive strength from drum test data. Therefore, it is possible to evaluate the value of compressive strength of cokes by drum test. However, the gasification reaction occurs inside blast furnace that deteriorate the strength of cokes. Therefore, the effect of the gasification reaction must be considered, too.

This model can calculate the relationships between coke compressive strength and breakage fragments generation inside blast furnace. At the calculation, Eq. (30) was selected for the expression of apparent fines generation energy. The particle size of cokes was set at 50 mm. In case of the deadman surface, the slipping distance was set at \( \pi D_p \). The compression force was set at 1 kN according to the analyses on the stress field inside blast furnace made by Katayama. In case of the raceway, the collision velocity and the collision frequency were set at 10 m/s and 10 times, respectively according to the raceway model by the authors.

Figure 12 shows the result of the calculation. As shown in the figure, DI-drum is the hardest loading system. However, cokes are to deteriorate their strength through the gasification reaction. If such case is imaged as the compres-
sive strength drops to the half as much as the original value after gasification reaction, the breakage in raceway becomes significant. Additionally, the small difference in the compressive strength before reaction induce a big difference in fines generation both inside the raceway and near the deadman surface. According to these results, it is inevitable to quantify the mechanism of the strength deterioration of cokes after gasification reaction. This will be the next target of this study.

4. Conclusions

The coke breakage phenomenon at drum tests were investigated based on the breakage contact model and the coke breakage model to predict the size distribution of cokes both in drum tests and inside blast furnace was developed. According to the model, the size degradation of coke is dominated not only by the compressive strength of coke but also by the mechanical loading conditions. Therefore, if the mechanical loading conditions inside blast furnace are evaluated, the actual fines generation from cokes inside blast furnace can be predicted.

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Appendix. Outline of the Breakage Contact Model

1) Basic Viewpoint of the Model

Coke is a particle with small convexes on its surface and when coke contact to the other object, all the convexes within apparent contact plane is broken. Additionally, surface compressive breakage of particle itself occurs in the case when contact force exceeds the compressive strength of coke particle.

2) Breakage Contact Conditions

Breakage contact conditions are expressed with using critical contact force \( P_c \), critical displacement \( \delta_c \) and normalized contact displacement \( X = \delta_c / \delta_c \).

Critical contact force; \( P_c = (4 \pi^3 / 3 \rho \delta_c S_c) \) ..........................(A-1)
Critical displacement; \( \delta_c = (r/2) (\pi^2 / 2)^2 (S_c / \rho)^2 \) ..........................(A-2)
Contact force; \( P = P_c X^{1.5} \quad X < 1 \)

\[ \text{Contact plane radius; } R_p = (2r \delta_c X^{0.5} \quad X < 1 \] ........................(A-3)

\[ \text{Broken plane radius; } R_b = (2r \delta_c (X-1)^{0.5} \quad X < 1 \] ........................(A-4)

\[ \text{Contact energy; } E = \pi \rho S_c \delta_c^2 [16/15 + (X-1)(X+1/3) \] 
\[ + 2X^{-0.5}(X^2-4X^{0.5}/3) + 1/3] \] ........................(A-5)

Here, \( r \) is the equivalent curvature radius of contact points \( (=r, r)/ (r + r_2) \), \( Y \) is the equivalent Young’s modulus \( (=Y_1, Y_2)/(Y_1 + Y_2) \) and \( S_c \) is the compressive strength of cokes and in case of \( X > 1 \), surface compressive breakage occurs.

3) Energy Consumption and Fines Generation

Relationship between energy consumption \( E_i \) and mass of fines generation \( M \) is expressed as follows:

\[ E_i = (E/M)M \] ..........................(A-7)

Constant \( E/M \) is named “fines generation energy” and is principally given as the compressive strength to density ratio \( (S_c / \rho) \) including the constant \( k \) expressing the ratio of breakage volume to swept off volume. However, in case of coke, both convex and particle itself are broken. Therefore, practical value depends on the contact mode. Practical values for normal contact, rotating contact and slipping contact are given by following equations, respectively.

\[ (E/M)_n = (S_c / \rho) [(X-1)^2 + 2(X-1/3) k] / \{ (X-1)^2 + 2(X-1/3) \} \] ..........................(A-8)

\[ (E/M)_o = (S_c / \rho) [3(X-1/3)X^{-0.5}k + (X-1)^{1.5} \] 
\[ / \{ 3(X-1/3)X^{-0.5}k + (X-1)^{1.5} \} \] ..........................(A-9)

\[ (E/M)_s = (S_c / \rho) [1.5(d \delta_c)(X-1/3)X^{-0.5} \zeta k + (X-1)^{1.5} \] 
\[ / \{ 3(d \delta_c)(X-1/3)X^{-0.5} \zeta k + (X-1)^{1.5} \} \] ..........................(A-10)

On the other hand, energy consumptions for each contact mode are written by following equations.

\[ E_n = E \] ..........................(A-11)

\[ E_o = P \mu \] ..........................(A-12)

\[ E_s = P \mu \] ..........................(A-13)

\( L \) is the slipping or rotating distance. Restitution coefficient \( \eta \) and friction coefficients \( \mu \) for slipping and rotating contacts are given as follows:

\[ \eta^2 = (32/15)(X^{1.5}/((16/15)(X^2 + X^{1.5}) + 2(X-1/3)k) \] ..........................(A-14)

\[ \mu_r = (2 \pi)(2 \delta c/r) [3X^{-0.5}k + (X-1)^{1.5} \] 
\[ / (X-1)^{1.5} \} ] ..........................(A-15)

\[ \mu_s = (2 \pi)(2 \delta c/r) [1.5X^{-0.5}(d \delta c) \] 
\[ / k + (X-1)^{1.5} \} ] ..........................(A-16)

Here, \( \zeta = 1 + r / r_2 \), \( \zeta \) is the ratio of fines generation energy of particle itself and convex, \( (E/M)_n / (E/M)_o \) is the height of convex. Suffixes \( n \) and \( s \) represent normal, rotating and slipping contacts, respectively.