Research Article
The Impact of Random Noise on the Dynamics of COVID-19 Epidemic Model

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At the end of 2019, the world knew the propagation of a new pandemic named COVID-19. This disease harmed the exercises of humankind and changed our way of life. For modeling and studying infectious illness transmission, mathematical models are helpful tools. Thus, in this paper, taking into account the effect of the intensity of the noises, we define a threshold value $\Pi_s$ of the model, which determines the extinction and persistence of the COVID-19 pandemic. We give numerical simulations to illustrate the analytical results.

1. Introduction

The world knew the born of a new pandemic named COVID-19 at the end of 2019 in Wuhan, China. It causes by a novel coronavirus family known as SARS-CoV-2. The coronavirus was spread and developed fastly according to the climate of each country. For every period, another strain was appeared, such as Delta and Omicron. This pandemic caused a great degas economics and a high number of deaths of persons of different ages.

On 31st December, 2019, the first case infected by a novel coronavirus (2019-nCoV) had been discovered. Then, in the first two months of 2020, COVID-19 was widely distributed throughout the world. On 30 May, 2020, the WHO declared officially the coronavirus as a pandemic. This disease has rapidly spread around the world and caused a large number of deaths worldwide. According to World Health Organization (WHO) report, the number of deaths and confirmed cases caused by coronavirus, respectively, was 5 224 519 and 262 866 050 from January 13, 2020, to November 29, 2021, worldwide. The new coronavirus is a viral illness that spreads by direct contact with an infected individual or indirectly by connection with a virus-infected environment. The degree of hardness induced by the coronavirus varies depending on immunological strength. Chills, weakness, stiffness, and a loss of smell or taste are common symptoms of this infection. The seriousness of this pandemic is revealed in its ability to fluctuate with rage. A new strain appears after every time. Recently, the World Health Organization (WHO) stated on November 26, 2021 that a new variant of COVID-19 has been discovered in South Africa named Omicron.

The COVID-19 pandemic has caused negative impacts in different domains such as economic, agriculture, and education. In particular, in the education domain, universities and schools were closed in April 2020. Additionally, it is also affected 1 542 412 000 students, according to IAU survey data. The assessments and teaching of students were transferred to the online mode that involves the probability of error in the evaluation of students, in addition to the lack
of means in subdeveloped countries. Therefore, all these factors will cause long-term consequences in levels of study and graduates. In addition to the education field, the COVID-19 pandemic also was damaged the economic area. The sanitary protocol imposed by countries worldwide led to the closure of production centers and some firms. Also, the airline firms have suffered significant losses of halted flight operations and the closure of borders by counties. The global markets had registered a drop in oil prices. Various economic activities stopped during this period. As well, the global gross domestic product growth was reduced in 2020. The agricultural area also did not escape from the impacts of this pandemic. This field has impacted in terms of the demand and the supply of food products. Globally, there was a decrease in food demand due to a fall in purchasing power and limitation on movement.

On March 2, 2020, Morocco discovered the first case infected by the coronavirus in Casablanca city. It is a Moroccan expatriate who came from an endemic country (Italy) to Morocco for visiting her family. After that, the number of infected cases has increased rapidly, which prompted Morocco to decide on the closure of schools, universities, sports halls, restaurants, etc. Precisely, a total confinement has been posed by the Moroccan government on April, 16, 2020. According to World Health Organization (WHO) report, Morocco registered 950088 as confirmed cases of coronavirus disease and 14779 of deaths From January, 3, 2020 to December 2, 2021.

SARS-CoV-2 mainly assaults the respiratory system as it is caused by a virus [1]. As mentioned above, the most common symptoms caused by COVID-19 are dry cough, tiredness, fatigue, running nose, shortness of breath, and fever [2]. This infection can be proliferated basically from individual-to-individual across beads via coughing, sniffling, or talking. Vulnerable people will too be kindled by way of touching contaminated surfaces. A few sufferers may have nasal congestion, aches and pains, a sore throat, and diarrhea. The signs and the symptoms are normally slight, however, can regularly worse [3]. Hand washing, covering of the nose or mouth whilst sneezing or coughing, fending off touching the nostril, mouth or eyes, and social distancing are encouraged preventive measures to keep away from contamination.

A part from almost all nations have implemented strict conditions to curtail the diseases in terms of avoiding public gatherings, closure of institutes, public locations and borders, banning of domestic and international journeys, putting lockdown, and so forth. Lockdown is at the main edge of those limitations. More than half of the world’s populace will be exposed to a lockdown with severe regulation measures by spring 2020. In history, for the first time, such apportions are implemented in a massive scale (Organisation for Economic Co-operation and Development (OECD)), [4]. Besides, following health advices: wearing masks, social distances, awareness programmers, and have benefited maximum that carried out in advance than others. Though those techniques have been beneficial, they caused socioeconomic damages. In reality, during the lockdown, social disconnectedness and fear of an unsure future have fueled intellectual health issues, and domestic violence has multiplied. Many workers have lost their respective jobs. Business closure disrupted supply chains and reduced productivity.

Scientists in different disciplines such as science, technology, and engineering have shifted their interest, time, and energies to understand and combat this new enemy of humanity to pop out normal scenario of the countries. Many works to discover new and adequate vaccine are undergoing in many labs in distinct nations around the globe. Some outstanding outcomes were acquired as ventilators, and other items used to get some recovered patients in many nations.

The contributions from biologists and mathematicians have been playing a crucial role in forecasting and controlling the COVID-19 disease with their outstanding findings regularly by using mathematical models. The investigations from [5] revealed that the COVID-19 pandemic may be suppressed through a lockdown. However, lockdown measures can have a profound terrible effect on people, communities, and societies by way of bringing social and economic lifestyles to a near stop. Such measures disproportionately affect deprived organizations, inclusive of human beings in poverty, migrants, internally displaced people, and refugees, who most often live in overcrowded and beneath-resourced settings, and depend upon daily exertions for subsistence [6]. Quarantine is another measure taken into consideration as pretty effective countermeasures for the COVID-19 (Hou et al., [7]), but it increases the concern that the risks and advantages need to be taken into consideration in-domestic quarantine for confirmed cases, which can result in family case clusters if they transmit the virus to different individuals of the same family (Khan et al., [8]). The studies from Margraf et al. [9] and Lin Chen [10] suggested that the disease spread/impact is extensively less when governments need to interact in conduct amendment at the societal, community, and personal levels.

Numerous studies of epidemiological models of the disease COVID-19 were constructed based on dividing the populations into various compartments/classes highlighted the possibility of the disease transmission and the necessary steps to reduce its impact in populations through the analytical and stochastically analysis, some of which can be found from (Zhang [11], Bhadauria et al. [12], Li and Zhang [13], Higazy [14], Ramos et al. [15], Batistela et al. [16], and Paré et al. [17]). The incubation period of the disease and its recovery time are dealt by Zhu and Zhu [18] and Scheiner et al. [19] considering time delay epidemic models. Furthermore, control strategies play an important role to find effective strategies in controlling the disease transmission among the individuals which facilitate the policy makers to implement necessary steps in public sectors. Some investigations incorporating control strategies of which is better to carry out immediately to the community were explored in the studies of Ali et al. [20], Djaoue et al. [21], Kassa et al. [22], Mumba and Hugo [23], Mwalili et al. 2020, Ndaireu et al. 2020, Oluyori Adebayo, 2020, Pal et al. 2020, and C. Yang Wang 2020.

The organization of this paper is as follows. In Section 2, we formulate our stochastic model. Preliminaries are
explored in Section 3. In Section 4, we introduce the reproduction number of the deterministic model. Extinction and persistence in mean results are explored in Section 5 and Section 6, respectively. In Section 7, the theoretical results are illustrated with the support of numerical examples.

2. Model Formulation

As in [24], we consider a population divided into five classes (see, Figure 1): susceptible (S), exposed (E), quarantined (Q), hospitalized infected (I), and recovered (R) and presented by the following system

\[
\begin{align*}
\dot{S}(t) &= A - \beta(1 - \rho_1)(1 - \rho_2)SE + b_1Q - \theta S - pSM, \\
\dot{E}(t) &= \beta(1 - \rho_1)(1 - \rho_2)SE - (b_2 + \alpha + \theta + \delta)E, \\
\dot{Q}(t) &= b_2E - (b_1 + c + \theta)Q, \\
\dot{I}(t) &= \alpha E - cQ - \eta + \theta + \delta)I, \\
\dot{R}(t) &= \eta I + \delta E - \theta R + pSM,
\end{align*}
\]

(1)

with initial values \(S(0) \geq 0, E(0) \geq 0, Q(0) \geq 0, I(0) \geq 0,\) and \(R(0) \geq 0,\)

where \(S(t), E(t), Q(t), I(t),\) and \(R(t)\) denote the numbers of susceptible, exposed, quarantined, infective, and removed individuals at time \(t,\) respectively. The parameters of the above model have the following meaning: \(A\) represents the recruitment rate of the susceptible population, and \(\beta\) denotes the disease transmission rate. If the susceptible individuals regroup with exposed ones, the epidemic propagates exponentially. However, the parameters \(\rho_1(0 < \rho_1 < 1)\) represent the proportion of susceptible individuals with prudent precaution, \(\rho_2(0 < \rho_2 < 1)\) is the proportion of the exposed individuals with prudent precaution, and \((1 - \rho_1)S\) represents the proportion of \(S\) class due to the contact of \((1 - \rho_2)E\) portion of exposed individuals. The parameters \(\alpha\) and \(b_2\) signify, respectively, the portions in which the exposed individuals and quarantined individuals go to the infected class; \(\theta\) is the natural death rate in the population, and \(\delta\) represents the death rate due to the COVID-19. The terms \(cQ\) and \(b_1Q\) represent the proportion in which the quarantined individuals goes to the infected class and susceptible class, respectively. The parameters \(\eta\) and \(\theta\) represent the recovery rates for the hospitalized infected individuals and exposed individuals, respectively. \(M\) and \(p\) represent the government policy parameters [24].

The quantity \(\beta SE\) represents the incidence rate of the population that models the number of new infections recruited in the population per unit of time. In literature, most works used the bilinear incidence to model the new infection cases in their models. And others used the standards incidence rate presented by \(\beta SE/N\) (with \(N\) is the total population number). So, in many situations existing in the natural environment, the bilinear incidence is not the perfect incidence to describe them. Therefore, we use the saturated incidence rate developed by Capasso and Serio [25] when the number of the infected groups in the population is large, and this situation conducts leading to an increase slowly of the incidence rate.

On the other hand, natural systems can generally be affected by environmental noise such as white noise [26–28] and Lévy noise [29–31]. Then, random fluctuations affect the distribution of infectious diseases since this type of epidemic spreads randomly. Therefore, some authors have expressed the random effect in their deterministic models by employing the direct perturbation of parameter approach [32–35]. For example, El Koufi et al. in [26] have assumed that the stochastic perturbations are of white noise type and affect the transmission rate of disease. In addition, they showed the global existence and uniqueness of a positive solution to their stochastic model. Also, they established conditions for the extinction and the persistence of the epidemic in their model. In [36], the authors have supposed that the random perturbations are of white noise type, and that they are proportional to the variables of \(S, E, Q, I,\) and \(R.\)

However, natural and massive phenomena such as Covid-19, earthquakes, tsunamis, and volcanoes cannot be modeled by the stochastic differential equation because these phenomena cause to break the continuity of the solution and provoke jumps in the system. Consequently, including a jump process (Lévy process [30, 37, 38]) in a stochastic system may well model these phenomena.
This paper is aimed at studying the effect of environmental fluctuations on the model (1). Then, we suppose that the stochastic perturbation is of the white and Lévy noise type and proportional to the variable $S, E, Q, I$, and $R$. Therefore, we present the stochastic version of the deterministic model (1) by the following stochastic equation system:

\[
\begin{align*}
    dS(t) &= [A - \beta(1 - \rho_3)(1 - \rho_2)SE + b_1 Q - \theta S - pSM]dt + \sigma_1 SdW_1(t) + \int \pi_1(a)S(t - )\tilde{N}(dt, da), \\
    dE(t) &= [\beta(1 - \rho_3)(1 - \rho_2)SE - (b_2 + \alpha + \theta + \delta)E]dt + \sigma_2 EdW_2(t) + \int \pi_2(a)E(t - )\tilde{N}(dt, da), \\
    dQ(t) &= [b_2 E - (b_1 + c + \theta)Q]dt + \sigma_3 QdW_3(t) + \int \pi_3(a)Q(t - )\tilde{N}(dt, da), \\
    dI(t) &= [\alpha E - cQ - (\eta + \theta + \delta)I]dt + \sigma_4 IdW_4(t) + \int \pi_4(a)I(t - )\tilde{N}(dt, da), \\
    dR(t) &= [\eta I + \theta E - \theta R + pSM]dt + \sigma_5 RdW_5(t) + \int \pi_5(a)R(t - )\tilde{N}(dt, da),
\end{align*}
\]

where $W_i(t) (i = 1, 2, 3, 4, 5)$ are independent standard Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ with the filtration $(\mathcal{F}_t)_{t \geq 0}$ satisfying the usual conditions, and $\sigma_i (i = 1, 2, 3, 4, 5)$ represents the intensities of $W(t) (i = 1, 2, 3, 4, 5)$, where $S(t - ), E(t - ), Q(t - ), I(t - ),$ and $R(t - )$ are the left limit of $S(t), E(t), Q(t), I(t),$ and $R(t)$, respectively, $N(dt, da)$ is a Poisson counting measure with the stationary compensator $v(da), \tilde{N}(dt, da) = N(dt, da) - v(da)dt$, and $v$ is defined on measurable subset $\mathcal{A}$ of $[0, \infty)$, satisfying $v(\mathcal{A}) < \infty; \pi : \mathcal{A} \times \Omega \rightarrow \mathbb{R}$ and $i = 1, 2, 3, 4, 5$ are the effects of random jumps which are assumed to be bounded and continuously differentiable.

3. Preliminaries

Throughout this paper, we let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ be a complete probability space satisfying the usual conditions (i.e., it is increasing and right continuous, while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets).

A standard Brownian motion (Wiener process) is a stochastic process $\{B_t\}_{t \geq 0}$ indexed by positive real numbers $t$ and satisfying the following properties:

(i) $B(0) = 0$ with probability 1
(ii) $0 \leq s \leq t$ and $B(t) - B(s)$ are independent of $\mathcal{F}_s$
(iii) $0 \leq s \leq t$ and $B(t) - B(s) \sim \mathcal{N}(0, t - s)$

Let $\{p^* = p^*(t), t \geq 0\}$ represent the stationary $\mathcal{F}_t$-adapted and $\mathbb{R}^n$-valued Poisson point process. Then, for $K$ belongs to the Borel $\sigma$-field $\mathcal{B}(\mathbb{R}^n - 0)$, with $0 \notin K$, we define the Poisson counting measure $N$ associated with $p^*$ by

\[
N([0, t] \times K) := \# \{0 < s \leq t, p^* \in K\} = \sum_{i < t \in I^T} I_K(p^*(s)),
\]

where $\#$ is the cardinality of set $\{\}$, We pose $N(t, K) = N(0, t] \times K)$. It is known that there exists a $\sigma$-finite measure $\Pi$ such that

\[
E[N(t, K)] = \pi(K)t, \quad \mathbb{P}(N(t, K) = n) = \frac{e^{-\pi(K)}(\pi(K)t)^n}{n!},
\]

where $\pi$ denotes the Lévy measure (for more detail, see [39]). Therefore, by Doob-Meyer's decomposition theorem, there exists a unique compensated Poisson random measure $\tilde{N}(t, K)$ and a unique compensator process $\nu(K)t$ such that

\[
\tilde{N}(t, K) = N(t, K) - \nu(K)t.
\]

The differential operator $\mathcal{C}$ (see, [40]) is associated with the following stochastic differential equation with Lévy processes:

\[
dZ(t) = f(Z(t), t)dt + g(Z(t), t)dW(t) + \int \pi(h(Z(t), \alpha))\tilde{N}(dt, da)
\]

is presented by

\[
\mathcal{C}Z(t) = \frac{\partial Z(t - )}{\partial t} + \sum_{i=1}^n \frac{\partial Z(t - )}{\partial Z_i} f_i(Z(t) \\
+ \frac{1}{2} \sum_{i,j=1}^n \frac{\partial^2 Z(t - )}{\partial Z_i \partial Z_j} [g^T(Z(t))g(Z(t))]_{ij} \\
+ \int \{(Z(t - ) + h(Z(t - ), \alpha)) - Z(t - ) - \frac{\partial Z(t - )}{\partial Z} h(x(t - ), \alpha)\} \nu(da).
\]
Figure 2: Dynamical simulation of $S(t), E(t),$ and $Q(t)$ for the deterministic system (1) and stochastic system (2) with the parameter’s value presented in Table 1 and noise parameter values: $\sigma_1 = 0.6, \sigma_2 = 0.7, \sigma_3 = 0.6, \sigma_4 = 0.6, \pi_1(\alpha) = 0.01, \pi_2(\alpha) = 0.1, \pi_3(\alpha) = 0.04, \pi_4(\alpha) = 0.01, \text{ and } \pi_5(\alpha) = 0.01.$
If \( \mathcal{L} \) acts on a function \( \mathcal{P} \in \mathcal{C}^{1,2}(\mathbb{R}^n \times \mathbb{R}_+ ; \mathbb{R}_+) \), then

\[
\mathcal{L}(Z(t)) = \mathcal{P}_1(Z(t^-)) + \mathcal{P}_2(Z(t^-))f(Z(t^-), t) + \frac{1}{2} \text{trace} \left[ \mathcal{P}_{22}(Z(t^-))g(Z(t^-), t) \right] + \int \left[ \mathcal{P}(Z(t^-)) + h(Z(t^-), \alpha) - \mathcal{P}(Z(t^-)) \right] (Z(t^-), \alpha) \|v(t)\| dt,
\]

where \( \mathcal{P}_t = \partial \mathcal{P}/\partial t, \mathcal{P}_Z = (\partial \mathcal{P}/\partial Z_1), \ldots, (\partial \mathcal{P}/\partial Z_n) \), and \( \mathcal{P}_{ZZ} = (\partial^2 \mathcal{P}/\partial Z_i \partial Z_j)_{nn} \). Then, the generalized Itô’s formula (see, [39]) is defined by

\[
d\mathcal{P}(Z(t)) = \mathcal{L}(Z(t^-))dt + \mathcal{P}_2(Z(t^-))g(Z(t), t)dW(t)
+ \int \left[ \mathcal{P}(Z(t^-)) + h(Z(t^-), \alpha) - \mathcal{P}(Z(t^-)) \right] (Z(t^-), \alpha) \|v(t)\| dt.
\]

In this paper, we assume the following assumptions.

**Assumption 1.** For each \( N > 0 \), there exists \( C_N > 0 \) such that

\[
\int_{\mathbb{V}} |H_i(x, \alpha) - H_i(y, \alpha)|^2 v(ds) \leq C_N |x - y|^2,
\]

for \( i = 1, 2, 3, 4, 5 \), where

\[
\begin{align*}
H_1(z, \alpha) &= \pi_1(\alpha)z \quad \text{for } z = S(t^-), \\
H_2(z, \alpha) &= \pi_2(\alpha)z \quad \text{for } z = E(t^-), \\
H_3(z, \alpha) &= \pi_3(\alpha)z \quad \text{for } z = Q(t^-), \\
H_4(z, \alpha) &= \pi_4(\alpha)z \quad \text{for } z = I(t^-), \\
H_5(z, \alpha) &= \pi_5(\alpha)z \quad \text{for } z = R(t^-),
\end{align*}
\]

with \( |x| v(y) \leq N \).

**Assumption 2.** \( |\ln(1 + \pi_i(\alpha))| \leq K \), for \( \pi_i(\alpha) > -1, (i = 1, 2, 3, 4, 5) \) where \( K \) is positive constant.

**Theorem 3.** Let Assumptions 1 and 2 hold. For any initial condition \( (S(0), E(0), Q(0), I(0), R(0)) \in \mathbb{R}_+ \), there exists a unique positive solution \( (S(t), E(t), Q(t), I(t), R(t)) \in \mathbb{R}_+^5 \) for all \( t \geq 0 \).

To show that the solution of the system (2) is positive and global, we use the Lyapunov method (see, [41]). Then, using the same technique as in [33], we can prove the above theorem.

**Lemma 4.** (see [42]). Let \((S(t), E(t), Q(t), I(t), R(t))\) be the solution of model (2) with any initial value \((S(0), E(0), Q(0), I(0), R(0)) \in \mathbb{R}_+^5 \). Then,

\[
\lim_{t \to \infty} \frac{\int_S^t S(s) dB_1(s)}{t} = 0, \quad \lim_{t \to \infty} \frac{\int_S^t E(s) dB_2(s)}{t} = 0,
\]

where \( S(t) = S(t^-) + \int_{t^-}^t S(s) dB_1(s) \), \( E(t) = E(t^-) + \int_{t^-}^t E(s) dB_2(s) \).

**Lemma 5.** (see [42]). Let \( (S(t), E(t), Q(t), I(t), R(t)) \) be the solution of model (2) with any initial value \((S(0), E(0), Q(0), I(0), R(0)) \in \mathbb{R}_+^5 \). Then,

\[
\lim_{t \to \infty} \frac{S(t) + E(t) + Q(t) + I(t) + R(t) - 16}{t} = 0, \quad a.s.
\]

Moreover,

\[
\lim_{t \to \infty} \frac{S(t)}{t} = 0, \quad \lim_{t \to \infty} \frac{E(t)}{t} = 0, \quad \lim_{t \to \infty} \frac{Q(t)}{t} = 0,
\]

**Lemma 6.** (see [37]). Suppose that \( Y(t) \in \mathcal{C}(\Omega \times [0, \infty) ; \mathbb{R}_+) \), under Assumption 2, then

(a) If positive constants \( a_0 \) and \( T \) exist such that

\[
\ln Y(t) \leq a_0 \int_0^t Y(s) ds + \sum_{i=1}^n k_i B_i(t) + \sum_{i=1}^m q_i \int_0^t (1 + t)[u(t)] N(dt, da) \quad a.s.,
\]

for all \( t \geq T \), where \( a, k_i, q_i \) are constants, then

\[
\begin{align*}
Y(t) \leq & \frac{a}{a_0} \quad a.s. \quad \text{if } a \geq 0, \\
\lim_{t \to \infty} Y(t) = 0 \quad a.s. \quad \text{if } a < 0.
\end{align*}
\]

(b) If there exists three positive constants \( a, T \) and \( a_0 \) suchthat

\[
\lim_{t \to \infty} \frac{S(t) + E(t) + Q(t) + I(t) + R(t) - 16}{t} = 0, \quad a.s.
\]
The reproduction number of system (1) is given by (see, [43]). According to the next-generation approach, the reproduction number indicates the number, which coincides with the basic reproduction number, noted \( R_0 \) of the system (1).

\[
\Pi_s = \frac{\beta(1 - \rho_1)(1 - \rho_2)}{\theta + pM} - \frac{(b_2 + \alpha + \theta)^2}{2} - \int \pi_2(\alpha) - \log(1 + \pi_2(\alpha)) \nu(\alpha) \quad (19)
\]

\[
(\beta(1 - \rho_1)(1 - \rho_2))\left(\frac{A}{\theta + pM}\right) - (b_2 + \alpha + \theta)^2 - \int \pi_2(\alpha) - \log(1 + \pi_2(\alpha)) \nu(\alpha).
\]

**Theorem 7.** Let \((S(t), E(t), Q(t), I(t), R(t))\) be the solution of the system (2) with any initial value \((S(0), E(0), Q(0), I(0), R(0)) \in \mathbb{R}_+^5\). If \( \Pi_s < 0 \), then

\[
\limsup_{t \to \infty} \frac{\log E(t)}{t} \leq \Pi_s, \text{ a.s.} \quad (20)
\]

In other words, \( E(t) \) will go to zero exponentially a.s. Moreover,

\[
\lim_{t \to \infty} \frac{1}{t} \int_0^t S(s) ds = \frac{A}{\theta + pM}, \text{ a.s.} \quad (21)
\]

**Proof.** Applying the Itô’s formula to the function \( \log E(t) \), we obtain

\[
d \log E(t) = \left[ \frac{\beta(1 - \rho_1)(1 - \rho_2)}{\theta + pM}(b_2 + \alpha + \theta) - \frac{\sigma^2}{2} \right]
\]

\[
- \int \pi_2(\alpha) - \log(1 + \pi_2(\alpha)) \nu(\alpha) dt + \sigma_i dW_i(t) \quad (22)
\]

For more description, see [24].

**5. Extinction**

In this section, we investigate a condition for the extinction of the disease in model (2). For this, we introduce the following number, which coincides with the basic reproduction number, noted \( R_0 \) in epidemiology, the basic reproduction number, noted \( R_0 \), represents “the expected number of secondary cases produced by a single infection in a completely susceptible population” [43]. Also, the reproduction number indicates the dynamic behavior of the deterministic system:

(i) If the value of \( R_0 \) is less than one, then the disease dies out

(ii) If the value of \( R_0 \) is greater than one, then the epidemic persists

To calculate the number \( R_0 \), there exists many methods (see [43]). According to the next-generation approach, the reproduction number of system (1) is given by

\[
R_0 = \frac{A(1 - \rho_1)(1 - \rho_2)}{(\theta + pM)((b_2 - \alpha - \theta)^2 + \rho_1 \rho_2)}. \quad (18)
\]

For all \( t \geq T \), then \( Y(t) \to \infty \) a.s.
Integrating the above equality from 0 to t and dividing by t on both sides, we get

$$\log \frac{E(t)}{t} - \log \frac{E(0)}{t} = \beta(1 - \rho_1)(1 - \rho_2) \int_0^t S(s)ds - (b_2 + a + \theta + \theta) - \frac{\sigma^2}{2} - \int_0^t \pi_2(a)$$

$$- \log (1 + \pi_2(a)) \nu(da) + \frac{1}{t} \int_0^t \log (1 + \pi_2(a)) \tilde{N}(ds, da).$$

(23)

On the other hand, from the system (2), one can derive that

$$dS(t) + dE(t) + \frac{b_1}{b_1 + c + \theta} dQ(t) = A - (\theta + pM)S(t) - K, E(t) + \sigma(s) \frac{dW_2(t)}{t}$$

$$+ \int_0^t \frac{\pi_1(a)}{\theta} S(s)ds + \frac{b_1}{b_1 + c + \theta} \sigma(s) dW_2(t)$$

$$+ \int_0^t \pi_2(a) E(s)ds + \frac{b_1}{b_1 + c + \theta} \sigma(s) dW_2(t)$$

$$= A - (\theta + pM) t$$

where $K_1 = \alpha + \theta + b_1 - b_1 b_2 / b_1 + c + \theta$. Integrating from 0 to t on both sides of the equation (24) gives us

$$S(t) - S(0) + E(t) - E(0) + \frac{b_1}{b_1 + c + \theta} Q(t) - Q(0)$$

$$= A - (\theta + pM) - \int_0^t E(s)ds$$

$$+ \frac{1}{t} \int_0^t \pi_1(a) S(s) ds - \int_0^t \sigma_2 E(s) dW_2(s)$$

$$+ \frac{1}{t} \int_0^t \pi_2(a) E(s) ds - \int_0^t \sigma_3 Q(s) dW_3(s)$$

$$+ \frac{1}{t} \int_0^t \pi_3(a) Q(s) ds$$

$$= \frac{A}{\theta + pM} - \frac{K_1}{\theta + pM} t$$

$$+ \frac{1}{t} \int_0^t S(s) ds - \frac{1}{t} \int_0^t E(s)ds - \phi(t),$$

(25)

and then

$$\frac{1}{t} \int_0^t S(s) ds = \frac{A}{\theta + pM} - \frac{K_1}{\theta + pM} t - \frac{1}{t} \int_0^t E(s)ds - \phi(t),$$

(26)

where

$$\phi(t) = \frac{1}{\theta + pM} \left[ \frac{S(t) - S(0)}{t} + \frac{E(t) - E(0)}{t} - \frac{b_1}{b_1 + c + \theta} \frac{Q(t) - Q(0)}{t} \right]$$

$$- \frac{1}{t} \int_0^t \sigma_2 E(s) dW_2(s) - \frac{1}{t} \int_0^t \pi_1(a) S(s) ds$$

$$- \frac{1}{t} \int_0^t \pi_2(a) E(s) ds - \frac{1}{t} \int_0^t \pi_3(a) Q(s) ds$$

$$= (b_2 + a + \theta + \theta) [\mathcal{A}_0 - 1] - \frac{\sigma^2}{2} - \int_0^t \pi_2(a)$$

$$- \log (1 + \pi_2(a)) \nu(da)$$

$$= (b_2 + a + \theta + \theta) [\mathcal{A}_0 - 1] - \frac{\sigma^2}{2} - \int_0^t \pi_2(a)$$

$$- \log (1 + \pi_2(a)) \nu(da) < 0,$$

(35)

which implies that

$$\lim_{t \to \infty} E(t) = 0 \ a.s.$$
From \((26)\), we therefore have
\[
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} S(s)ds = \frac{A}{\theta + pM}.
\] (37)

Integrating the third equation of \((2)\) from 0 to \(t\) and dividing by \(t\) on both sides, we have
\[
\frac{Q(t)}{t} - \frac{Q(0)}{t} = b_{2} \frac{1}{t} \int_{0}^{t} E(s)ds - (b_{1} + c + \theta) \frac{1}{t} \int_{0}^{t} Q(s)ds + \sigma_{3} \frac{1}{t} \int_{0}^{t} Q(s)dW_{3}(s) + \frac{1}{t} \int_{0}^{t} \pi_{3}(\alpha)Q(s-)\tilde{N}(ds, d\alpha).
\] (38)

According to the above results, we conclude that
\[
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} Q(s)ds = 0 \quad a.s.
\] (39)

By the same idea, we can prove that
\[
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} I(s)ds = 0 \quad a.s. \\
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} R(s)ds = 0 \quad a.s.
\] (40)

This completes the proof.

**Remark 8.** According to Theorem 7, if \(\Pi_{i} < 0\), then the disease goes extinct in the system \((2)\). Thus, large environmental noises can make the disease extinct.

**6. Persistence**

**Theorem 9.** Let \((S(t), E(t), Q(t), I(t), R(t))\) be the solution of the system \((2)\) with any initial value \((S(0), E(0), Q(0), I(0), R(0)) \in \mathbb{R}_{+}^{5}\). If \(\Pi_{i} > 0\), then

\[
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} S(r)dr = \bar{S}^* \quad a.s.,
\]
\[
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} E(r)dr = \bar{E}^* \quad a.s.,
\]
\[
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} Q(r)dr = \bar{Q}^* \quad a.s.,
\]
\[
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} I(r)dr = \bar{I}^* \quad a.s.,
\]
\[
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} R(r)dr = \bar{R}^* \quad a.s.,
\] (41)

where \(\bar{S}^*, \bar{E}^*, \bar{Q}^*, \bar{I}^*, \text{ and } \bar{R}^*\) are positive constants.

**Proof.** Using the equation \((23)\), we have
\[
\frac{\log E(t)}{t} = \beta(1 - \rho_{1})(1 - \rho_{2}) + \frac{1}{t} \int_{0}^{t} S(s)ds - (b_{2} + \alpha + \theta)\theta + pM
\]
\[
- \frac{\sigma_{3}^2}{2} - \int_{\gamma} \pi_{2}(\alpha) - \log (1 + \pi_{2}(\alpha))v(\alpha) + \frac{1}{t} \int_{0}^{t} \pi_{2}(\alpha)ds + \frac{1}{t} \int_{0}^{t} \log (1 + \pi_{2}(\alpha))\tilde{N}(ds, d\alpha).
\] (42)

Then,
\[
\log E(t) = \frac{A\beta(1 - \rho_{1})(1 - \rho_{2})}{(\theta + pM)} - (b_{2} + \alpha + \theta) - \frac{\sigma_{3}^2}{2}
\]
\[
- \int_{\gamma} \pi_{2}(\alpha) - \log (1 + \pi_{2}(\alpha))v(\alpha) + \frac{1}{t} \int_{0}^{t} \pi_{2}(\alpha)ds + \frac{1}{t} \int_{0}^{t} \log (1 + \pi_{2}(\alpha))\tilde{N}(ds, d\alpha)
\]
\[
- \beta(1 - \rho_{1})(1 - \rho_{2})\phi(t).
\] (43)

According to Lemma 6, we have
\[
\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} E(s)ds = K_{1}\left[\frac{A\beta(1 - \rho_{1})(1 - \rho_{2})}{(\theta + pM)} - (b_{2} + \alpha + \theta)\right]
\]
\[
- \frac{\sigma_{3}^2}{2} - \left[\int_{\gamma} \pi_{2}(\alpha) - \log (1 + \pi_{2}(\alpha))v(\alpha)\right]
\]
\[
= K_{1}\left[(b_{2} + \alpha + \theta)\|\mathcal{R}_{0}\| - 1\right] - \frac{\sigma_{3}^2}{2}
\]
\[
- \int_{\gamma} \pi_{2}(\alpha) - \log (1 + \pi_{2}(\alpha))v(\alpha) > 0 \quad a.s.,
\] (44)

where \(\theta + pM/K_{1}\beta(1 - \rho_{1})(1 - \rho_{2})\).

From the system \((2)\), we have
\[
\frac{Q(t) - Q(0)}{t} = b_{2} \frac{1}{t} \int_{0}^{t} E(s)ds - (b_{1} + c + \theta) \frac{1}{t} \int_{0}^{t} Q(s)ds
\]
\[
+ \frac{1}{t} \int_{0}^{t} \sigma_{3}Q(s)dW_{3}(s) + \int_{\gamma} \pi_{3}(\alpha)Q(s-)\tilde{N}(ds, d\alpha).
\] (45)
Then, 

$$\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} Q(s) ds = \frac{b_{2}}{(b_{1} + c + \theta)} \Pi_{s} = Q^{*} \quad a.s. \quad (46)$$

By the same process, we can prove

$$\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} S(s) ds = S^{*} \quad a.s.,$$

$$\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} I(s) ds = I^{*} \quad a.s.,$$

$$\lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} R(s) ds = R^{*} \quad a.s. \quad (47)$$

7. Conclusion and Numerical Simulation

In this paper, we have proposed a stochastic COVID-19 epidemic model with Lévy noise. We have presented the model and its advantage compared with the corresponding deterministic and stochastic systems. Employing some stochastic calculus background, we have proved that the extinction and persistence of the COVID-19 epidemic are determined by the value of $\Pi_{s}$, namely, if $\Pi_{s} < 0$, the COVID-19 epidemic is extinct. Furthermore, if $\Pi_{s} > 0$, the COVID-19 pandemic persists. In the rest of this section, we present some numerical examples to confirm our theoretical results achieved. To simulate, we use the Milstein method presented in [44].

Example 10. Taking the parameter values described in Table 1 and the noise values: $\sigma_{1} = 0.6, \sigma_{2} = 0.7, \sigma_{3} = 0.6, \sigma_{4} = 0.6, \sigma_{5} = 0.6, \pi_{1}(a) = 0.01, \pi_{5}(a) = 0.1, \pi_{4}(a) = 0.04, \pi_{1}(a) = 0.01$, and $\pi_{3}(a) = 0.01$, we obtain that $R_{0} = 0.4358 < 1$ and $\Pi_{s} = 1.2258 < 0$. Then, by Theorem 7, $E(t)$ will tend to zero exponentially with probability one, and Figure 2 exposes this result. If we choose the parameters $\beta = 1.2, \rho_{1} = 0.2$, and $\rho_{2} = 0.3$ and the noise values: $\sigma_{1} = 0.2, \sigma_{2} = 0.2, \sigma_{3} = 0.1, \sigma_{4} = 0.1$, and $\sigma_{5} = 0.1$, then we obtain $\Pi_{s} > 0$. According to Theorem 9, the disease will persist in the population (see, Figure 3). From the discussion above, we deduce that the epidemic disappears from the population if the value of the noise is very large. And if it is weak, the pandemic persists.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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