Calculating the Vertical Displacement using the Method of Least
Squares Adjustment and 3D Objects Fitting

Nagham A. Abdulateef 1, Yousif H. Khalaf 1 and M Al-Bakri 1
1Department of Surveying. College of Engineering. University of Baghdad. Iraq

Abstract. In this research, the Iraqi flagpole in Baghdad University, which is the longest flagpole in Baghdad, with a height of 75m, was monitored. According to the importance of this structure, the calculation of the displacement (vertical deviation) in the structure monitored using the Total Station device, where several observation were taken at different times for a period of two years the monitoring started from November 2016 until May 2017, at a rate four observations for one year. And the observation processed using the least square method, and the fitting of circles, then the data was processed and the deviation was calculated using Matlab program with calculating the values of corrections, where the mathematical laws have been programmed in a format that suits the program, observation have been entered and correction were made on them, calculating corrected values and the amount of error between the observed and calculated values. The deviation was between (0.720 to 0.759)m during the observation period. The Auto CAD program and the 3D MAX program used to product the two-dimensional (2D) and (3D) Models of the structure.

1. Introduction
The preservation of cultural heritage objects requires regular monitoring and rehabilitation Work [1]. Baghdad University is the largest Iraqi university, which is located in the center of the Iraqi capital Baghdad. It was founded and financed by the Iraqi government in the late 1950s. The University of Baghdad has proved since its founding that it is the main and effective pillar of building, progression, and development in this country. This was clear through its contribution to all schools, institutes, and colleges of Iraq with professors, specialists, and educators, who have led the scientific process for many years. Therefore, it is entitled to be proud of what this university has achieved, which enriched the scientific and academic research committees and has played an outstanding role in every field of construction and development. The flagpole of the University of Baghdad is considered the highest flag in Iraq. The height of the Iraqi flag is more than 75 meters. It is placed near the main gate of the university. The weight of the flag and weather conditions may affect the alignment of the flagpole. Thus, this research investigates the pole that may happen through different periods of time. Technically the curve fitting technique is used to compute this which is one of the geodetic techniques in monitoring this technique depends on measuring the ground coordinates of selected points on the pole and baselines by using a total station (figure 1a). Then the coordinates of the selected point are used to compute the radius and center at different heights the old technique represented measuring tangential angles using theodolite instrument (figure 1b) [2]. Cylinder fitting there are different methods of least-squares fitting of a cylindrical shape.
2. Methodology

With the great development of the specifications of computers and languages, the development of the science of correction of measurements and its use in solving problems of geodetic network correction is possible and easy. Therefore, we used the calculation of the correction values on the Matlab program, where we programmed the mathematical laws in a format suitable for the program. The observations were entered and corrected, the corrected values were calculated and the error between the observed and computed values.

Where our project aims to calculate the slope of the Iraqi flag in the gate of the University of Baghdad and produce a 3D model of the Flagpole and calculate the coordinates according to the following steps:

- Selecting baselines.
- Measuring local coordinates from reference points (OCC) at Baghdad University by using the total station.
- Observing the points were distributed on the pole in the form of circles from the predetermined lines. The location of these circuits was fixed at a rate of six circles from the bottom of the base to the highest point of the pole.
- Monitoring the selecting points at a rate of four periodic times and between the monitoring, the results of the four observations in tables showing the time of observation.
- Using Matlab to create a special program to calculate the vertical amount of the flagpole.
- Finally, the flagpole was modeled in a 3D model using the AutoCAD program.

3. Least squares method

In surveying, observations must often satisfy established numerical relationships known as geometric constraints [3]. It is a set of equations in which there are two or more unknowns. The "least square" means that the comprehensive solution reduces the sum of error boxes made in the results of each equation. It is modifying the data or the final results due to the presence of random errors through observations which
increases the accuracy final results. The least-squares method was introduced and applied in many scientific and engineering fields. It was developed with the advent of computers and the development of matrices algebra and linking them with statistical concepts. In this method, a general mathematical model should be developed for the issue that will bring together both observations (n) and unknowns (u) and the relationship between them. It is important to know that there are always the fewest independent observations needed to solve the issue (no). If the value of (n) bigger than (no), it means a redundancy (r) theoretically, solving a mathematical issue by measuring a certain set of equations and these measured quantities should be sufficient to solve the remaining unknowns in the equation. In surveying matters, observations are often redundant, especially in geodesic networks and photogrammetry [4, 5].

The adjustment of independent observations which have equal variance least-squares adjustments can be divided into three categories:

- The observation equation.
- The condition equation.
- General or combined method.

The development of the general least square method is as follows:

\[ \phi = v^w - 2k(Av + B\Delta - f) \]  \hspace{1cm} (1)

\[ (B^wB)\Delta = (B^wf) \]  \hspace{1cm} (2)

Or

\[ [B'(AQ_A)^I]B\Delta = [B'(AQ_A)^I]f \]  \hspace{1cm} (3)

\[ NA = t \]  \hspace{1cm} (4)

Where:

\[ N = B^wB = B'(AQ_A)^I \]  \hspace{1cm} (5)

\[ t = B^w f = B'(AQ_A)^I f \]  \hspace{1cm} (6)

The vector \( \Delta \) may be obtained from equation, by direct inversion such that,[ 6].

\[ \Delta = N^t \]  \hspace{1cm} (7)

3.1 Fitting

The installation of lines, curves or circles for a set of points is an old problem that has often been driven by a large amount of repetition in various fields[7]. Generally, the installation of engineering features to give 2D/3D is required in various fields of science and engineering such as astronomy, physics, biology, quality control, image processing, metrics etc.

3.2 Curve Fitting

There are types of curves such as conical sections (circular, ellipse, Parabola) or trigonometric functions (e.g sine and cosine) used in some cases, for example, trajectories of objects under the influence of gravity follow an equivalent path, When air resistance is ignored. Thus, matching the data points of its path to its parabola curve can be said to try to find a curve that reduces the vertical (y-axis) which usually means trying to reduce the perpendicular distance with the curve (such least squares)[8]. Where the style of least squares is a way to bend the fitting, which is very popular for a long time. In addition, the style of least squares reduces the square of the error between the original data and the values predicted by the equation, but this technique may not be the most statistically effective method of fitting the function of a set of Data, it has the advantage of being twice as relatively simple (in terms of computing power required) and is well
understood. If the data point is significantly different than on the use of the training method and its sensitivity to the extreme values in it can be widely distorted from the majority of data, the more reasonable it is before taking the recognized results. Regression results. For this reason, data should always be studied.

![Figure 2. Ellipse Fitting](image)

### 3.3 Circle Fitting

Fit is the best alignment of the circle for a set of points in 2D. The issue of circle alignment may arise in some surveying or engineering applications, where follow the same method followed with the issue of adopting a straight line and curves using the method of less squares. In the general model as mentioned earlier, the coordinates of the points are considered to be meteorological in the matter, while the transactions of the circle (coordinates of the center and its radius) are considered to be the unknowns. For example, a set of (x,y) points that are close but do not accurately approach the circle. Then there is a center and radius point that represents the best circle that matches the point. By example: the gradient and intercept of the y-axis, the center ellipse, the main and secondary axes, and the rotation can be obtained as it is considered the projection of the circle and is better for the composition of the shapes with uses in the field of engineering, space, and computational engineering [9]. The best solution for restoring the circle or oval shape to be close to accuracy is by using the least-squares method shown in figure (2).

![Figure 3. Circle fitting](image)
After obtaining the coordinates of the center and base of the flagpole, the slope and direction can be calculated based on the mathematical equations below:

\[(x - x_o)^2 + (y - y_o)^2 - r_o = 0\]  
…………………………………………..(8)

\[((x_i + vxi) - (x_o))^2 + ((yi + vyi) - (y_o))^2 - r_o^2 = 0\]  
…………………………………………..(9)

The observation equations, in the general, will be as follows

\[A\nu + B\Delta = 0\]  
…………………………………………..(10)

The value of the derivatives that make up the two matrix elements in the first line will be as follows.[6]

\[\frac{\partial f_1}{\partial x_1} = 2(x_1 - x_o^e)\]  
…………………………………………..(11)

\[\frac{\partial f_1}{\partial y_1} = 2(y_1 - y_o^e)\]  
…………………………………………..(12)

\[\frac{\partial f_1}{\partial x_e} = -2(x_1 - x_o^e)\]  
…………………………………………..(13)

\[\frac{\partial f_1}{\partial y_e} = -2(y_1 - y_o^e)\]  
…………………………………………..(14)

\[\frac{\partial f_1}{\partial r} = -2r_o\]  
…………………………………………..(15)

4. The Softwares

4.1 Matlab
Matlab is an enormous software that is used in advanced mathematical and engineering applications. It analyzes and represents data according to its database, for example, the software takes down the work of calculus as well as the high-grade differential equations which are very complex. It also can achieve partial details and performs partial break-ups easily and easily, which requires a great time in traditional ways. In practical terms, the software can work in all engineering used for math computations, modeling and simulations, data analysis and processing, visualization and graphics, and algorithms development engineering applications [10]. With the great development of computer software and programming languages, the science of adjustment computation has evolved, and it becomes easy to solve geodetic networks. Therefore, in this research Matlab has been used to calculate correction values thorough programming the mathematical models, introducing estimation values, calculating the corrected values, and the number of errors between the observed and calculated values [11].

4.2 AutoCAD and 3DS MAX
The AutoCAD software can allow users to navigate from imagine design to its implementation, in addition to making adjustments to get the desired design. The user can create and work on a 3D design in an easy and simplified way. The beginning is an imaginary design for the two-dimensional project and then it can be developed into a 3D design. The basics of drawing and editing orders are fixed with the difference in the interface and some additional commands. This software can count quantities and extracting information easily, and making typical files to save time and effort, drawings accurately, as well as the software is considered a mirror of ideas and imagination. It can increase the abilities of creators and designers and showing their talent in the art of design achieved through 3D [12]. 3DS MAX It is one of the most powerful 3D engineering design software, created by Autodesk. The software is based on creating an environment to work on a wide land and create images as imagined by the designer and move
them from three-dimensional perspectives such as building or creating cars or any image that occurs in the imagination of the designer and then adjusted by the tools available in the software. The material is added to give the images a natural impression and not geometric design. Then the designer puts some external effects such as natural or industrial lighting and other effects to match natural design [13].

5. The Study Area
In this research, the flagpole Figure (3) was monitored for two years. It was divided into circular rings and the field surveying was performed through the use of a total station device. The monitoring was achieved at the rate of four observations per year using correction equations (fitting of circles) and the least-squares method. A modern-generation total station device was used to distribute control points by closed traverse method, in order to monitor all the mast hulls and assess the accuracy of the work. A special strategy has been developed to ensure that accurate results are obtained for monitoring purposes during the period of implementation of the research within the specifications of accuracy that meet the requirements of the search. The control points were distributed around the flagpole which can make measurements of points elected on the body of the pole. Therefore, each group of points can monitor a whole ring in a circular shape. These measurements are subsequently corrected based on the theory of the least-squares adapted to fitting. Finally, the flagpole was produced as a 3D model using the Front view of AutoCAD and 3D MAX software as shown in the figure (4).

Figure 3. (a) The flagpole with flag (b) The flagpole without flag
 Depending on the equations of circle fitting and the least-squares equation the radius of the circular rings, the center of each ring and at different elevations are computed in different times the results are listed in the table (1) and table (4).

**Table 1.** The calculation of the vertical deviation of the middle flagpole at 2016

| The vertical of the flagpole (m) | Tilt direction | The radius of flag middle top (m) | The base flagpole radius (m) | Height of flagpole (m) | Monitoring time |
|----------------------------------|----------------|---------------------------------|----------------------------|-----------------------|-----------------|
| 0.120                            | 34°44'23"      | 0.420                           | 0.615                     | 35.500                | September       |
| 0.150                            | 34°44'32"      | 0.420                           | 0.615                     | 35.500                | December        |
| 0.140                            | 34°44'29"      | 0.420                           | 0.615                     | 35.500                | March           |
| 0.120                            | 34°44'24"      | 0.420                           | 0.615                     | 35.500                | June            |
| 0.100                            | 34°44'20"      | 0.420                           | 0.615                     | 35.500                | August without the flag |

In tables no(2,3) it shows the changes in the amount of the vertical deviation calculated from the lower base of the flagpole to the center within a height of 35.5m. In different months of the year. It was found that the weight of the flag affects the increase in the amount of deviation so that the crawl is less without the flag. It was of the wind also affected by increased crawling.
### Table 2. The calculation of the vertical deviation of the flagpole at 2016

| The vertical of the flagpole (m) | Tilt direction | The radius of flagpole top (m) | The base flagpole radius (m) | Height of flagpole (m) | Monitoring time |
|----------------------------------|----------------|-------------------------------|-----------------------------|-----------------------|-----------------|
| 0.720                            | 34°44’23"      | 0.138                         | 0.615                       | 70.650                | September       |
| 0.756                            | 34°44’32"      | 0.138                         | 0.615                       | 70.650                | December        |
| 0.742                            | 34°44’29"      | 0.138                         | 0.615                       | 70.650                | March           |
| 0.725                            | 34°44’24"      | 0.138                         | 0.615                       | 70.650                | August without the flag |
| 0.710                            | 34°44’20"      | 0.138                         | 0.615                       | 70.650                |                 |

### Table 3. The calculation of the vertical deviation of the middle flagpole at 2017

| The vertical of the flagpole (m) | Tilt direction | The radius of flag middle top (m) | The base flagpole radius (m) | Height of flagpole (m) | Monitoring time |
|----------------------------------|----------------|----------------------------------|-----------------------------|-----------------------|-----------------|
| 0.130                            | 34°44’24"      | 0.420                            | 0.615                       | 35.500                | September       |
| 0.140                            | 34°44’27"      | 0.420                            | 0.615                       | 35.500                | December        |
| 0.140                            | 34°44’26"      | 0.420                            | 0.615                       | 35.500                | March           |
| 0.130                            | 34°44’24"      | 0.420                            | 0.615                       | 35.500                | June            |
| 0.110                            | 34°44’20"      | 0.420                            | 0.615                       | 35.500                | August without the flag |

### Table 4. The calculation of the vertical deviation of the flagpole at 2017

| The vertical of the flagpole (m) | Tilt direction | The radius of flagpole top (m) | The base flagpole radius (m) | Height of flagpole (m) | Monitoring time |
|----------------------------------|----------------|-------------------------------|-----------------------------|-----------------------|-----------------|
| 0.725                            | 34°44’24"      | 0.138                         | 0.615                       | 70.650                | September       |
| 0.759                            | 34°44’27"      | 0.138                         | 0.615                       | 70.650                | December        |
| 0.745                            | 34°44’26"      | 0.138                         | 0.615                       | 70.650                | March           |
| 0.732                            | 34°44’24"      | 0.138                         | 0.615                       | 70.650                | June            |
| 0.713                            | 34°44’20"      | 0.138                         | 0.615                       | 70.650                | August without the flag |

### 6. Conclusions

Tables (1,3) show the calculated vertical deviation from the base to the middle of the pole height. It was found that the deviation is less than the top and more stable during the observation period because the pole radius has a role that is measured by the weight of the pole and the effect of the wind speed opposite the top. The first observation was in September 2016 using the total station, where the amount of deviation was (0.720 m) which is considered a significant value for the flagpole did not have a long time. The amount of deviation from September to December, approximately three months, was 0.720m to 0.756m as in Table 2. This is probably because of the diameter of the upper part of the flagpole, which is about 14 cm which is not suitable with the height of the flagpole. The amount of deviation during the month of March...
to June, approximately three months, was found (0.742m to 0.725m) as in table 2. The observation in August was without the flag. This month was chosen in order to be a clear sky and the wind is relatively low. It was found that the amount of deviation is (0.710m) as in table 2 and this represents the real deviation of the flagpole for 2016. The observations during the period of September and December 2017 found that the deviation value was (0.725m to 0.759m). It can be noted that the deviation increased with the presence of the Iraqi flag. Observations in March and June were as follows (0.745m and 0.732m) as in Table 4. These results are close to the results of the same months in 2016. This indicates the stability of the deviation trend of the flagpole during this period. The observations in August 2017 were without the presence of the flag. It was found that the amount of crawl was (0.713m) as in table 4. It is a clear indication of the stability of the deviation during the monitoring period from 2016 to 2017. This may be due to a defect during the installation of the flagpole, and heterogeneity of its diameter from the lower base with a radius of (0.62cm) to top which is about (0.14cm).

7. Recommendations
- Continuity in monitoring the flagpole in order to determine whether the deviation is fixed or an increase through long periods of years.
- It is suggested to use other techniques, in addition to the total station, such as terrestrial laser scanner for observations.
- It is recommended to investigate and study the suitability of flagpole material with its height.

Reference
[1] Beshr A A 2014 Structural Data Analysis for Monitoring the Deformation of Oil Storage Tanks Using Geodetic Techniques J. Surv. Eng 140(1).
[2] Bashar S, Nagham A, Yousif H 2019 Out of Plumb Assessment for Cylindrical-Like Minaret Structures Using Geometric Primitives Fitting ISPRS Int. J. Geo-Inf. 8, 64.
[3] Forbes B A Least-Squares Best-Fit Geometric Elements; Report Number: NPL Report DITC 140/89; National.
[4] Ghilani C D and Wolf P R 1989 Adjustment Computations Spatial Data Analysis, 4th ed.; John Wiley & Sons, Inc. :Hoboken, NJ, USA, 2006; p. 398. Physical Laboratory: London, UK, 1989.
[5] Hume and Rainsford F 1979 Surveying Adjustment and least Squares, 3rd edition, London.
[6] Yousif H KH 2009 Displacement Computation of Mosul dam by using Free Geodetic Network Adjustment, M.Sc. thesis, college of Engineering, University of Baghdad, Department of Surveying Engineering.
[7] Mikhail E M 2000 Observations and Least Squares 3rd edition, Donnelly publisher, New York.
[8] Panyam M, Kurfessa T, Tucker T 2008 Least Squares Fitting of Analytic Primitives on a GPU J. Manuf. Syst 27 130–135.
[9] Sandra Lach Arllinghaus, PHB 1994 paractical handbook of curve fitting. CRC Press.
[10] Yousif H, Nagham A 2018 Orthophoto Production from Aerial Photograph by using Matlab and GIS International Journal of Civil Engineering and Technology (IJCIET) 9(9).
[11] Abulateef N A 2009 Accuracy Evaluation of Digital Close-Range Photogrammetry by Free Adjustment Method, MSc. Thesis, College of Engineering, Baghdad University.
[12] Kersten T and Mass H G 1995 Photogrammetric 3-D point determination for dam monitoring. Optical 3-D Measurement III, Eds. Gruen / Kahmen, Wichmann Verlag, 161-168
[13] Zamirroshin A 2006 Designing Software for 3D Object Modeling Using Digital Close-Range Photogrammetry.