Simulation of Piezoelectric Energy Harvester Based on the Vortex Flow

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Abstract. In this article, numerical research on the fluid-structure interaction between the flexible piezoelectric energy harvester (FPEH) and the Von Karman vortex street forming behind a bluff body is carried out to optimize the oscillation of FPEH to obtain more electrical energy. Using ANSYS Workbench platform, the simulation is performed. The numerical results show that the maximal deformation of the PEH is 1.7428 mm, meanwhile the maximal voltage is 4.6144 V. Besides, these numerical results generated by the ANSYS simulation are in good agreement with the experimental results.

1. Introduction
With the increasing consumption of fossil energy in the industrial production, the energy crisis has been placed in front of the world. In recent years, people have developed a variety of renewable energy to replace the traditional fossil energy, such as solar energy, wind energy, wave energy and so on. Among them, ocean wave energy has been focused more attention because of its huge energy reserves and environmental pollution-free features. Piezoelectric materials are well known for their ability to generate electrical charge when they are deformed. With this feature, piezoelectric flags have been placed in the fluid flow using as interested candidates [1-2]. Periodic deformation of the piezoelectric flags leads a periodic charge transfer between the electrodes of piezoelectric patches positioned on the surface of the piezoelectric flags as they exhibit spontaneous self-sustained flapping when the surrounding flow exceed a critical velocity. There are several literatures about the response of flexible piezoelectric energy harvester (FPEH) placed in a cross-flow and obtained the maximal strain energy and output power. To maximize the amount of power generated by the eel, many factors have been considered in the investigation, such as the thickness and stiffness of the eel materials, the eel length, the bluff body width and the spacing between the body and eel head [3-4]. The feasibility and performance of harvesting the electrical charge by a resistive circuit was recently studied, and it was shown that both the stability of the system and the fluttering dynamics are influenced by electrical coupling [5-6]. The energy harvesting eel is a device that uses piezoelectric polymer to convert energy from fluid flow into electricity power, which can harvest energy in both ocean and river. In this research, interaction between FPEH and the vortex behind a bluff body is studied.

2. Basic Principle of Analysis Model
Under the condition that the fluid model is a continuous model, the law of conservation of mass in fluid dynamics refers to the continuity equation, and the velocity and density of matter in the model will be the function of time and space coordinates. The incompressible viscous fluid is considered in modeling. The governing equations, which are continuous equations, are [6]:

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\[ \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{w}}{\partial z} + \frac{\partial \rho}{\partial t} = 0 \]  

(1)

Where \( \tilde{u} \), \( \tilde{v} \), and \( \tilde{w} \) are speed vectors, \( \rho \) is a constant for the incompressible viscous fluids. The governing equations for the FSI model are [7]:

\[ \rho \frac{\partial \tilde{u}_{\text{fluid}}}{\partial t} + (\tilde{u}_{\text{fluid}} \cdot \nabla) \tilde{u}_{\text{fluid}} = \nabla \cdot \left[ -pI + \mu \left( \nabla \tilde{u}_{\text{fluid}} + (\nabla \tilde{u}_{\text{fluid}})^T \right) \right] + \tilde{F} \]

(2)

\[ \frac{\partial^2 \tilde{u}_{\text{solid}}}{\partial t^2} - \nabla \sigma = \rho \tilde{F} \]

(3)

Where \( \tilde{u}_{\text{fluid}} \) and \( \tilde{u}_{\text{solid}} \) represent the velocity of the fluid flow and the solid mechanics respectively; \( \tilde{F} \) is the volume force field, \( \sigma \) is the Cauchy stress tensor which is related to the solid material being used for the FPEH.

In this paper, the \( k-\varepsilon \) turbulence model provided in ANSYS Fluent is used. In the standard \( k-\varepsilon \) model, \( \varepsilon \) is the turbulent dissipation factor. And the corresponding transport equation is:

\[ \frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ (\mu + \frac{\mu_t}{\sigma_k}) \frac{\partial k}{\partial x_j} \right] + G_k + G_b - \rho \varepsilon - Y_M + S_k \]

(4)

Where \( G_k \) represents the turbulent kinetic energy caused by average velocity gradient; \( G_b \) represents the turbulent kinetic energy caused by buoyancy; \( Y_M \) represents effects of pulsating expansion on total turbulent kinetic energy in compressible turbulence.

3. Piezoelectric Equation

The piezoelectric equation is an expression that describes the relationship between the mechanical and electrical quantities of piezoelectric materials. This kind of piezoelectric equation is used in the ANSYS coupling solutions for the analysis about piezoelectric materials. In linear piezoelectricity, the equations of elasticity are coupled to the charge equation of electrostatics by means of piezoelectric constants [8]:

\[ \{S\} = \left[ s^{E} \right] \cdot \{T\} + \{d\} \cdot \{E\} \]

(5)

\[ \{D\} = \left[ d^{T} \right] \cdot \{T\} + \left[ e^{T} \right] \cdot \{E\} \]

(6)

Where \( \{S\} \) is the elastic strain vector; \( \{T\} \) is the stress vector \((N/m^2)\); \( \{D\} \) is the electric flux density vector \((C/m^2)\); \( \{E\} \) is the electric field intensity vector \((V/m)\); \( [s^{E}] \) is the compliance matrix evaluated at constant electric field, i.e. short circuit; \( [d] \) and \( [d^{T}] \) are the piezoelectric strain coefficient matrix \((C/N)\), and the superscript \( \{t\} \) is the matrix transpose.

4. Case Simulation

When fluid flows from the left inlet side and passes through the bluff body, the Von Karman vortex street is generated, which takes on the repeating pattern of swirling vortices. The driving force coming from the shedding vortex makes the FPEH vibrate continuously. In this study, in order to perform the numerical simulation of fluid structure interaction, a three-dimensional model has been set up by ANSYS (shown in figure 1), which includes the fluid flow domain, a circular cylinder acting as a bluff body, and a FPEH. \( D \) is the cylinder diameter and used as the characteristic length, so the size of fluid domain can be described as \( 20D \times 10D \times 7.5D \) in length, width and height respectively. The FPEH is fixed on the outlet side of the cylinder, and its length, width and height are 200 mm, 0.1 mm and 20 mm respectively. Detailed geometry and material parameters for the model simulation is given in table...
1. And the local meshing of both blunt body and FPEH is given in figure 2. The volume meshing method is adopted for the FPEH and the bluff body, and their minimum element sizes are 0.0005m and 0.01m respectively. Obviously, the grid density of the FPEH is more intensive than that of the blunt body.

**Figure 1.** Computational domain.  
**Figure 2.** Mesh of partial enlargement.

| Table 1 Parameters for the model simulation. |
|---------------------------------------------|
| $L$  | $W$  | $H$  | $l$  | $w$  | $h$  | $D$  | $\rho_{\text{solid}}$ | $\mu_{\text{fluid}}$ | $E$  | $\nu$ |
| [mm] | [mm] | [mm] | [mm] | [mm] | [mm] | [mm] | [kg/m$^3$]   | [kg/ms]   | [GPa] | 0.3   |
| 800  | 400  | 300  | 200  | 0.1  | 20   | 40   | 1770         | 1.420     | 2.5   | 0.3   |

There are many kinds of flexible piezoelectric materials. In this study, PVDF is used, and its detailed material parameters for the model simulations are given in table 2.

**Table 2** Details of flexible piezoelectric materials.

| Symbol | Parameter                              | PVDF | Units                        |
|--------|----------------------------------------|------|------------------------------|
| $d_{31}$ | Piezo Strain Constant                  | 23   | $[(10^{-12})\text{C/N}]$    |
| $d_{33}$ | -33                                    |      |                              |
| $g_{31}$ | Piezo Stress Constant                  | 216  | $[(10^{-3})\text{Vm/N}]$    |
| $g_{33}$ | -330                                   |      |                              |
| $k_{31}$ | Electromechanical Coupling Factor      | 12%  |                              |
| $k_{t}$  | -                                      | 14%  |                              |
| $C$     | Capacitance                            | 380  | $[\text{pF/cm}^2, @1\text{kHz}]$ |
| $V_0$   | Speed of Sound                         | 2.2  | $[10^3\text{m/s in thickness direction}]$ |

5. Results and Discussion
Before simulation, the voltage boundary on one side of the FPEH is set to 0 V. The uniform flow is used for the motion of the fluid, and the speed of fluid is set as 0.3 m/s. After calculation, the cloud maps of the generated voltage, the deformation and the pressure of the piezoelectric material are given in figure 3, figure 4 and figure 5 respectively. And the maximal deformation, maximal and minimal voltages of the FEPH at different times are given in table 3. Because of the influence of the shedding vortex, the voltage generated by the FPEH mainly concentrates in the fixed end of the piezoelectric sheet (shown in figure 3), and the maximal voltage is 4.6144V. The maximal deformation is
1.7428 mm and occurs at the free end of the FPEH. For comparison, an actual FPEH with same dimensions was tested in a water tank. The maximal output voltage was measured as 4.78 volts under open-circuit condition with an oscilloscope. It can be found that the values of simulated voltage and experimental voltage are very close.

6. Conclusion
In this article, based on the phenomenon of Von Karman vortex street, the driving force originating from the shedding vortex acts on the FPEH and makes it vibrate continuously. Results indicate that the simulation result of power generation of FPEH is in good agreement with that of the actual measurement. The validity of the ANSYS software in the analysis of this kind of problem about fluid-structure interaction has been proved.
Table 3 Max deformation, max and min voltage at different time.

| Time[s] | Max deformation [mm] | Max voltage [V] | Min voltage [V] |
|---------|---------------------|-----------------|-----------------|
| 0.4     | 0.438               | 3.756           | -2.044          |
| 0.8     | 1.682               | 3.785           | -3.963          |
| 1.2     | 0.602               | 1.871           | -1.605          |
| 1.6     | 0.174               | 3.441           | -4.064          |
| 2.0     | 0.214               | 3.521           | -4.526          |
| 2.4     | 0.238               | 3.584           | -4.252          |
| 2.8     | 0.215               | 3.414           | -4.177          |
| 3.2     | 0.162               | 3.542           | -4.255          |
| 3.6     | 0.242               | 3.678           | -4.433          |
| 4.0     | 0.249               | 3.461           | -4.313          |
| 4.4     | 0.152               | 3.477           | -4.153          |
| 4.8     | 0.237               | 3.708           | -4.423          |
| 5.2     | 0.269               | 3.517           | -4.404          |
| 5.6     | 0.179               | 3.532           | -4.306          |
| 6.0     | 0.233               | 3.792           | -4.614          |

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8. References
[1] Williamson C H K and Govardhan R A brief review of recent results in vortex-induced vibrations 2008 J. Wind Eng. Ind. Aerod. 96 713
[2] Taylor G W, Burns J R, Kammann S A, Powers W B and Welsh T R The energy harvesting eel: a small subsurface ocean/river power generator 2002 IEEE J. Oceanic Eng. 26 539
[3] Calio R, Rongala U B, Camboni D, Milazzo M, Stefanini C, Petris G and Oddo C M Piezoelectric energy harvesting solutions 2014 Sensors 14 47
[4] Doare O and Michelin S Piezoelectric coupling in energy-harvesting fluttering flexible plates: linear stability analysis and conversion efficiency 2011 J. Fluid. Struct., 27 1357
[5] Allen J J and Smits A J Energy harvesting eel 2001 J. Fluid. Struct. 15 629
[6] COMSOL A B 2010 COMSOL Multiphysics User’s Guide
[7] Sin V K, Deng W Y and Xiao W H Study of motion of flexible eel from the wake of bluff body in a cross flow 2012 Int. Conf. Numerical Analysis and Applied Mathematics 1479 161
[8] Zeng C X and Jiang L The finite element analysis of piezoelectric transducer based on ANSYS 2011 Instrum. Technol. 6 31