On liquid crystal diffractive optical elements utilizing inhomogeneous alignment

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Abstract: Formation of a desired liquid crystal (LC) director distribution by the use of inhomogeneous anchoring and pre-tilt angle for electrically controlled diffractive optical elements (DOE) is studied. Such LC DOE can have high periodicity and diffraction efficiency. At the same time they are free of constructive regularities, e.g. a periodic arrangement of the electrodes or thickness deviations, which have undesired impact on diffractive characteristics of LC DOE of other types. We focus on evaluation of potential functional abilities of LC DOE with inhomogeneous alignment. The reasons causing restriction of the LC DOE diffraction efficiency and periodicity are considered. Approaches for improvement of characteristics of the LC DOE are discussed.

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1. Introduction

Electrically controllable optical elements such as lenses with variable focal distance, dynamic diffraction gratings and tunable prisms are highly desirable for numerous applications. There are devices for telecommunication, machine vision, displays, data storage, measurement equipment, military systems, energy-related applications, etc. Diffractive optical elements (DOE) offer more degrees of freedom in performance in comparison with traditional refractive elements [1,2]. They are also thin and can be implemented into micron-scaled optical systems.

Due to their electrical and optical anisotropy coupled with fluidity, liquid crystal (LC) constitutes a materials class among the advanced materials suited for tunable DOE [1,2]. From an optical point of view, a nematic LC having a spatial distribution of the LC molecules (director) can be considered as a medium with a refractive index varying in space. Such a
medium is known as gradient index (GRIN) medium [2]. A change of the electric field causes a change of the refractive index distribution. This leads to a change of phase retardation and the direction of light propagation through the medium.

Among the attractive features of LC DOE are non-mechanical tunability, low driving voltage, relatively low cost, and small sizes. In fact, they can be much smaller and have less weight than conventional glass and plastic analogues. In addition, LC DOE can be integrated into others optical components and compact systems [2].

Several types of LC optical elements have been proposed [1–19]. The needed spatial distribution of the LC director is achieved in the majority cases by means of an inhomogeneous electric field [1–12] or with a thickness variations [1, 2, 13, 14]. Adaptation of such approaches for micron-scaled systems poses technological difficulties due to that electrodes are required or due to geometrical non-uniformities. Moreover, the methods utilizing a non-uniform field distribution generated by the patterned structure of the electrodes are associated with additional complexities of the multiplexing driving. The periodicity of the electrodes or geometrical non-uniformities causes undesired diffraction. The fringe field effect [10, 11] also contributes to the restriction of the characteristics of DOE.

Implementation of inhomogeneous alignment, in particular inhomogeneous anchoring [15,16] or the pre-tilt angle variation [17–19], makes possible to achieve a desired formation of the LC director distribution under a uniform electric field. The corresponding optical element can be controlled through a pair of continuous electrodes and be free of the drawbacks mentioned above. Photoalignment techniques [20] enable performance of the alignment non-uniformities with a resolution of order of the light wavelength. This fact, as well as the reports about possibilities to alter alignment conditions with polarized light in already fabricated LC devices [21,22], makes the approach based on inhomogeneous alignment especially attractive for DOE.

Despite the apparent simplicity of the methods utilizing inhomogeneous alignment and several projects devoted to this subject [15–19], a set of questions related to the potential of the corresponding LC DOE remains unclear. For example, there is no clear evidence about the extreme characteristics that can be achieved by such kind of LC DOE. A clarification regarding optimal design parameters, values of the driving voltages, the ranges of anchoring energy and the pre-tilt angle variations are also required. Merits and limitations of each of two methods utilizing alignment inhomogeneities i.e. the pre-tilt angle and anchoring energy variations should be investigated. In this work, we make an attempt to clarify the questions mentioned above. The goal is to investigate the factors that play a key role in limitations of formation of a desired LC director distribution caused by inhomogeneous anchoring and pre-tilt angle variations.

2. Definition of the problem

The orientation of an LC director is determined by an external electric (or magnetic) field, the boundary conditions, and the elastic and dielectric (magnetic) features of the LC material. A certain alignment inhomogeneity can lead to a spatial distribution of the LC director which, through variation of the refractive index, forms a required optical element as shown in Fig. 1. Examples of such elements are a prism (Fig. 1), a Fresnel’s lens, a diffractive grating with a specific phase profile, etc. A change of the electric field causes a change of the LC director orientation and as a result the properties of the optical element are changed also.
Fig. 1. Fragment of LC DOE based on non-uniform alignment.

In order to evaluate possible operating modes of LC DOE based on inhomogeneous alignment, we will consider how anchoring and pre-tilt angle exert action on the LC director distribution under an electric field. Figure 2 demonstrates the dependences of the parameter \( \eta \) defined as \( \frac{n_e - n_{\text{eff}}}{n_e - n_o} \) versus the reduced voltage \( \frac{V}{V_c} \) for different anchoring energies (Fig. 2(a)) and pre-tilt angles (Fig. 2(b)). Here \( n_o \) and \( n_e \) are principal refractive indexes of the LC material, \( n_{\text{eff}} \) is the effective refractive index for the extraordinary wave: \( n_{\text{eff}} = \frac{1}{L} \int n^* dl \), where \( L \) is the length of the trajectory of the light ray inside the LC medium and

\[
n^* = \frac{n_e n_o}{\sqrt{n_o^2 + (n_e^2 - n_o^2)(\vec{N}(l) \cdot \vec{k}(l))^2}}.
\]

In the expression for \( n^* \), \( \vec{N} \) is the unit vector describing the LC director orientation, and \( \vec{k} \) is the light propagation vector. \( V_c \) is the threshold voltage of the LC with strong anchoring and zero pre-tilt angle and is given by

\[
V_c = \frac{\pi K_{11}}{\varepsilon_0 \Delta \varepsilon},
\]

where \( K_{11} \) is the splay elastic constant, \( \Delta \varepsilon \) is the static dielectric anisotropy of the LC material and \( \varepsilon_0 \) the permittivity of free space. The calculations were done for the LC material E44 (from Merck) that have the following principal physical constants \( K_{11} = 1.55 \times 10^{-11} \text{N}, K_{33} = 2.8 \times 10^{-11} \text{N}, \Delta \varepsilon = 16.8, n_e = 1.7859, n_o = 1.5278 \). The cell gap is 5\( \mu \)m. In the calculations with inhomogeneous anchoring (Fig. 2(a)) the pre-tilt angle is \( 2^\circ \). The curves of Fig. 2(b) were obtained at a strong anchoring energy equal to \( 10^{-3} \text{J/m}^2 \). The refractive indices correspond to wavelength 588 nm.
The parameter $\eta$ can also be considered to represent the normalized phase delay $\phi$ for the outgoing light wave. The relationship between $\phi$ and $\eta$ is expressed as

$$\phi = \eta \frac{2\pi}{\lambda} d \Delta n,$$

where $\lambda$ is the wavelength of the light, $\Delta n = n_e - n_o$. The phase delay $\phi$ reaches its maximum when $\eta = 1$ and its minimum when $\eta = 0$. This fact justifies the intention to present the further results in terms of $\eta$.

From the curves in Fig. 2(a) follow that an LC cell with inhomogeneous anchoring can have uniform structure at no voltage applied. By applying a voltage, the forces balance occurs at new orientations of the LC directors. The reorientation (deformation) is higher if the anchoring energy is lower. When the applied voltage takes a value between the threshold voltage corresponding to the strong anchoring ($V_{st}$) and the saturation voltage corresponding to the weak anchoring ($V_{sw}$), the LC director orientation depends critically on the anchoring energy. In other words, a variation of the anchoring energy causes a variation of the outgoing light phase profile (Fig. 1).

The curves presented in Fig. 2(b) indicate that in the case of pre-tilt angle variations, the situation is opposite – a distribution of the pre-tilt angle forms a phase profile of the LC slab at no voltage. For sufficiently high voltage, the LC structure transforms into the homeotropic state.

There are two main requirements for a DOE: 1) the optical retardation within a diffraction zone has to be changed between 0 and $\lambda$, and 2) the optical retardation must have a stepwise change over $\lambda$ through the border between two adjacent diffraction zones. The first requirement defines the minimum thickness of the LC layer: $d = \frac{\lambda}{\Delta n}$ on condition that $n_{eff}$ varies between $n_o$ and $n_e$ which is equivalent to $0 \leq \eta \leq 1$.

The degree of the sophistication of a DOE depends on the width of the transition region between two adjacent diffraction zones. The sharper the phase profile is in the transition region, i.e. the narrower this region, the higher the diffraction efficiency will be. The stepwise retardation can in principle be achieved with a disclination. However, a controllable disclination is not easy to realize in practice. Another solution involves continuum transition between two adjacent zones with different LC director orientations. The transition in this case is not strictly stepwise. It will depend on the elastic properties of the LC and the external forces originating from an electric field and the boundary conditions. In order to obtain a picture about the width of the transient area, also known as the fly-back zone [10], computer
simulation is implemented. By doing so, we are able to determine the maximum diffraction efficiency and minimum period of LC DOE.

To be more specific, we have studied DOE for the LC material E44 from Merck at wavelength 588nm. It is a widely used material and has a relatively large birefringence. However, it should be noted that this material is far not the best one for LC DOE in general. A series of materials has been reported with much higher birefringence [23, 24]. On the other hand, the maximum phase difference depends on the ratio \( \frac{\Delta n}{\lambda} \). Many tunable DOE are requested for optical communication at \( \lambda = 1550 \text{ nm} \). The ratio \( \frac{\Delta n}{\lambda} \) for \( \Delta n \) close to 0.7 [23, 24] and \( \lambda = 1550 \text{ nm} \) is approximately the same as for \( \Delta n = 0.26 \) and \( \lambda = 588 \text{ nm} \) to be considered in the paper. Actually, the results obtained in our work can be used for inferring about LC DOE utilizing another LC materials and can be applied at other wavelengths.

3. Simulation approach

Simulation of electro-optical properties of LC DOE includes three interrelated parts: determination of the LC director distribution, computation of the light propagation through this distribution in the near-field case, and solving the far-field problem. The LC director distribution is formed as a balance of three main forces originated from the elastic strain, the interaction with applied field, and the contact with the surface. The basic assumption for finding the balance is that the system tends to minimize its free energy. The elastic free energy density of untwisted LC is defined by the Frank-Oseen equation [25] as

\[
\frac{f_{el}}{2} = K_{11}(\hat{\n} \cdot \n)^2 + K_{33} [\hat{n} \times \hat{\n} \times \hat{n}]^2 ,
\]

(2)

where \( \n \) is the unit vector describing the orientation of the LC director.

The energy density of the dielectric interaction with the external electrical field \( \hat{E} \) is

\[
f_{d} = \frac{1}{2} \hat{\varepsilon} \cdot \hat{E} \cdot \hat{E} ,
\]

(3)

where \( \hat{\varepsilon} \) denotes the dielectric tensor of the LC material.

The anchoring energy density consists of two terms \( f_{s}^{top} \) and \( f_{s}^{bottom} \) representing the anchoring energy density on the top and bottom boundaries, respectively. From the Rapini-Papoular model and for an LC with symmetric boundary conditions, \( f_{s}^{top} \) and \( f_{s}^{bottom} \) are [26]

\[
f_{s}^{bottom} = f_{s}^{top} = \frac{1}{2} W \sin^2 (\alpha - \alpha_o) ,
\]

(4)

where \( W \) is the anchoring strength coefficient, \( \alpha \) the tilt angle at the boundary and \( \alpha_o \) is the angle between the boundary and the easy axis of the LC.

Following the mathematics introduced by Berreman [27] and applying the finite element method [28, 29], one can find the minimum of the total free energy and as a result the equilibrium state.

Several approaches with different degrees of approximations can be applied for solving the near field problem. The simplest way is to adapt a one-dimensional (1D) optical simulation method such as Jones calculus [30] or the Berreman matrix method [31] for finding the outgoing field in terms of the phase profile. However, a 1D simulation method yields an inaccurate result for a structure with two-dimensional (2D) non-uniformity [32]. The exact solution for 2D non-uniformity can be found by solving Maxwell’s equations. The disadvantage of such a way is a long computation time. As a compromise between the rigor
but slow method involving direct solution of Maxwell’s equations and inaccurate but fast 1D
technique, the eikonal approximation [33, 34] can be utilized. In this case, the trajectory of
the light ray is found from the equation
\[
\frac{d}{ds} \left( \hat{n}(\vec{r}) \frac{d\vec{r}}{ds} \right) = \hat{v} \hat{n}(\vec{r}).
\]  
(5)

where \( \vec{r} \) is the position vector on a point on the ray, \( ds \) is an element of the arc length along
the ray. In general, the direction of the Poynting vector of the extraordinary wave does not
coincide with the wave vector. The angle \( \delta \) between them (the dispersion angle) in the plane
containing the crystal axis is given by
\[
\tan \delta = \frac{(n_e^2 - n_o^2) \left( 1 - (\hat{n} \hat{k})^2 \right)}{n_o^2 + (\hat{n} \hat{k})^2 (n_e^2 - n_o^2)}.
\]  
(6)

To trace the ray, it is necessary to apply an algorithm utilizing iterations of the Eq. (5) for the
wavevector \( \vec{k} = \frac{d\vec{r}}{ds} \) with the following determining the direction of the Poynting vector
according Eq. (6).

For far field calculations, the Fraunhofer diffraction integral [34] can be utilized. In the
case of normal incidence, the complex amplitude of the light wave observed at an angle \( \beta \) is
found as
\[
U_o(\alpha) = C \int_{-\lambda/2}^{\lambda/2} \psi(x) e^{-ikx \sin \beta} dx.
\]  
(7)

Here \( C \) is a constant, \( \lambda \) is the length of the LC structure under consideration, \( \psi \) is the solution
of the near field problem. The amplitude of the light wave for a periodic structure with the
period \( \lambda \) is expressed as
\[
U(\alpha) = U_o(\alpha) \frac{\sin N\sigma}{\sin \sigma} e^{-i(N-1)\sigma},
\]  
(8)

where \( N \) is the number of the periods, \( \sigma = \frac{k\lambda \sin \alpha}{2} \).

4. Results and discussions

4.1 Inhomogeneous anchoring energy

The simplest way for evaluation of the range of the anchoring energy variation is to utilize the
formula derived by us earlier [15]:
\[
\delta_{max} = \pi K_{11} \Delta n \rho \left( \frac{1}{W_2} - \frac{1}{W_1} \right).
\]  
(9)

Here \( \delta_{max} \) is the maximum retardation that can be obtained in an LC cell due to
inhomogeneous anchoring, \( \rho = \frac{\eta_2 - \eta_1}{\xi_2 - \xi_1} \), where \( \xi_1 = \frac{\pi K_{11}}{d W_1} \) [15, 35], sub-indexes 1 and 2 refer
to the parameters related to the areas with the strongest (\( W_1 \)) and weakest (\( W_2 \)) anchoring
energies, respectively. Typically, \( \rho \) has values between 1 and 1.5 at an appropriate LC
thickness and applied voltage [15]. If we assume that \( W_2 \) is much smaller than \( W_1 \) Eq. (9) gives

\[
W_2 = \frac{\pi K_{11} \Delta n \rho}{\delta_{\text{max}}}.
\]  

Substitution of the principal physical constants of the LC material E44 gives \( W_2 \approx 3.2 \times 10^{-5} \text{ J/m}^2 \) for \( \delta_{\text{max}} = 588\text{nm} \) (\( \lambda \)), and \( W_2 \approx 6.4 \times 10^{-5} \text{ J/m}^2 \) for \( \delta_{\text{max}} = 289\text{nm} \) (\( \lambda/2 \)). Such values of the anchoring energy are weak. In other types of LC devices, e.g. displays, \( W \) usually ranges between \( 10^{-3} \text{ J/m}^2 \) and \( 10^{-4} \text{ J/m}^2 \) which is suitable for \( W_1 \). However, it is not easy to reach controllable broadband variations of anchoring energy [36–38]. According to literature results [36–38], polar anchoring energy was controlled within relatively narrow range.

Assuming that the ratio between \( W_1 \) and \( W_2 \) is \( m \), Eq. (9) can be transformed as

\[
W_2 = \frac{\pi K_{11} \Delta n \rho (m-1)}{\delta_{\text{max}} m}.
\]  

The results obtained for \( W_2 \) at different \( m \) and \( \delta_{\text{max}} \) are summarized in Table 1. From these data is seen that \( W_2 \) varies slightly when \( m \) changes drastically. For example, increasing \( m \) from 1.5 to 10 leads to increasing \( W_2 \) from \( 1.1 \times 10^{-5} \text{ J/m}^2 \) to \( 2.9 \times 10^{-5} \text{ J/m}^2 \) for \( \delta_{\text{max}} = 588\text{nm} \).

It should be noted that neither the LC thickness \( d \) nor the applied voltage \( V \) are explicitly included in Eqs. (9)–(11). Influence of these parameters has an impact through the constant \( \rho \) that depends on \( d \) and \( V \) [15]. To obtain a view about optimal values of \( d \) and \( V \), we have calculated optical retardation \( \delta_{\text{max}} \) as a function of the reduced voltage \( V/V_c \) for different \( d \). The obtained results are presented in Fig. 3. The calculations were carried out for \( W_1 = 1 \times 10^{-4} \text{ J/m}^2 \), \( W_2 = 2 \times 10^{-5} \text{ J/m}^2 \) with other parameters as the above.

| \( m \) | \( W_2 \) for \( \delta_{\text{max}} = 588\text{nm} \) (\( \lambda \)) | \( W_2 \) for \( \delta_{\text{max}} = 289\text{nm} \) (\( \lambda/2 \)) |
|---|---|---|
| 1.5 | \( 1.1 \times 10^{-5} \text{ J/m}^2 \) | \( 2.2 \times 10^{-5} \text{ J/m}^2 \) |
| 2 | \( 1.6 \times 10^{-5} \text{ J/m}^2 \) | \( 3.2 \times 10^{-5} \text{ J/m}^2 \) |
| 3 | \( 2.2 \times 10^{-5} \text{ J/m}^2 \) | \( 4.4 \times 10^{-5} \text{ J/m}^2 \) |
| 5 | \( 2.6 \times 10^{-5} \text{ J/m}^2 \) | \( 5.2 \times 10^{-5} \text{ J/m}^2 \) |
| 10 | \( 2.9 \times 10^{-5} \text{ J/m}^2 \) | \( 5.8 \times 10^{-5} \text{ J/m}^2 \) |
The preference in decision regarding the appropriate thickness $d$ should be given to the lowest values, at which the required retardation is achieved. The reason is that the volume energy of the LC decreases when the thickness decreases. Hence, the surface energy has more influence on the director deformations at small thicknesses of the LC layer. Moreover, the transition processes in LC are faster or, in other words, the switching times are shorter at lower $d$. From Fig. 3, it is possible to conclude that the optimal $d$ of a saw-tooth grating or a Fresnel lens is around $3 \, \mu m$, when the driving voltage is close to $V_c (1.01V)$. For a diffraction grating with $\delta_{max} = \lambda/2$, the LC thickness can be less than $2 \, \mu m$ and the driving voltage can be around $0.5V_c (0.5V)$.

Equations (9)-(11) can be applied also for evaluation of distribution of anchoring energy as a function of the spatial coordinate $x$. The local energy $W(x)$ and the required retardation $\delta(x)$ are then replacing $W_1$ and $\delta_{max}$. As an example, we have calculated the distributions of the anchoring energy within the period $A$ of the saw-tooth diffraction grating (Fig. 1) of the LC thickness $3 \, \mu m$ when $W_f = 1\cdot10^{-4} \, J/m^2$ and $W_f = 5\cdot10^{-5} \, J/m^2$. The driving voltage is $V_c$. The obtained results are shown in Fig. 4. The solid curves are the outcome of the computer simulation based on Eqs. (2)-(8). The dash curves were obtained according to Eq. (9) for $\rho = 1.2$ determined from the curve for $V = V_c$ of Fig. 1 from [15].

The fly-back zone in a DOE based on inhomogeneous alignment and the corresponding restrictions were studied by us in [15].
4.2 Inhomogeneous pre-tilt angle

In contrast to the approach based on inhomogeneous anchoring energy, the method utilizing inhomogeneous pre-tilt angles enables generation of a larger phase difference at the expense of a larger LC layer thickness. Another advantage is the fact that in practice it is easier to realize a pre-tilt angle variation in a wide range [39–41] than an anchoring energy variation.

As was mentioned above, the performance of LC DOE is restricted mainly by the fly-back zone due to the LC elasticity. In order to analyze the width of the fly-back zone and its relationship to the pre-tilt angles allocation, we investigate the response of the LC phase profile on the “slit”-like pre-tilt angle allocation in the alignment layer. A portion of the surface with the length $L$ has a vertical alignment (the pre-tilt angle equals 90°), whereas the pre-tilt angle of the rest area is 2°. The results for $\eta$ as a function of $X/d$ for different values of $L$ with respect to the thickness of the LC layer $d$ are plotted in Fig. 5. $L$ equal to 0.25$d$, 0.5$d$, $d$, and 1.5$d$ is considered. From the presented results is seen that the variation of $\eta$ as well as the features of the transition zone depends on the ratio $L/d$. The maximum peak-to-peak value is achieved for $L > d$.

If the period of a DOE structure is much larger than the thickness $d$, the distribution of the pre-tilt angle $\alpha(x)$ along the spatial coordinate $x$ can be obtained as

$$\sin^2(\alpha(x)) = \frac{n_0^2}{n_r^2 - n_o^2} \left( \frac{n_r d}{n_r d + \delta(x)} \right)^2 - 1,$$

(12)

where $\delta(x)$ is the required retardation as a function of $x$. 

Fig. 4. Distributions of the anchoring energy within a period in a saw-tooth diffraction grating. The solid curves are results of the computer simulation, the dash curves are obtained according to Eq. (8).
A saw-tooth diffraction gratings of different periods $\Lambda$ were considered in order to investigate influence of a high periodicity of the DOE on the perfection of the required phase profile and, as a result, to obtain knowledge about possible minimum resolutions of the periodic structure of the DOE utilizing non-uniform pre-tilt angle distribution. Figure 6 shows dependences of $\eta$ versus the ratio $\frac{X}{d}$ for $\Lambda = 2d$, $4d$, $8d$, $20d$ and $100d$. The pre-tilt angle in the considered cases satisfies to Eq. (12). The calculated diffraction efficiencies for the first and zero diffraction orders for the profiles shown in Fig. 6 are summarized in Table 2. From the presented results follows that the diffraction grating of period around $100d$ has characteristics close to the ideal case. However, if $d = 2.3 \mu m$ which gives the maximum phase difference $2\pi$ for the LC material E44 at $\lambda = 588$ nm, the grating period $\Lambda$ will be $230 \mu m$. The diffraction angle $\beta$ defined as $\sin \beta = \frac{\lambda}{\Lambda}$ will in this case be around $0.15^\circ$. If the grating period $\Lambda$ is equal to $8d$, the diffraction angle $\beta$ reaches $2^\circ$. 

Fig. 5. $\eta$ as a function of $X/d$ for different wideness of the "slit".

Fig. 6. $\eta$ as a function of $X/\Lambda$ for different values of $\Lambda$. 

Table 2. Diffraction Efficiency of the Grating with the Phase Profiles Shown in Fig. 6

| Period | 0 order | 1st order |
|--------|---------|-----------|
| 2d     | 12.7%   | 39.4%     |
| 4d     | 8.6%    | 54.1%     |
| 8d     | 3.7%    | 75.6%     |
| 20d    | 1.1%    | 93.2%     |
| 100d   | 0.04%   | 99.9%     |

High diffraction angles can be achieved with a sinusoidal phase grating. One of the appropriated modes of the switchable sinusoidal grating can be a retardation modulation between 0 and 0.38λ. For such a sinusoidal grating the diffraction efficiency at the 0 diffraction order is 0, at the 1st diffraction order is 27.5% and at the 2nd diffraction order is 18.5%. Figure 7 shows differences between maximum and minimum values of $\eta$ ($\Delta \eta$) which can be obtained in a sinusoidal diffraction grating of period $\Lambda$. A half of the period of the grating has the pre-tilt angle 2° and the other half of the period has the pre-tilt angle 90°. The points in the graph are the results of the calculations of $\Delta \eta$ for different values of $\Lambda/d$. The dependence $\Delta \eta(\Lambda/d)$, which is described by a linear function ($\Delta \eta = a \frac{\Lambda}{d} + b$), enables evaluation of the minimum value of $\Lambda$ for retardation $d\Delta n \Delta \eta = 0.38 \lambda$. By expressing $\Lambda$ as a function of $d$, we get

$$\Lambda = \frac{1}{a} \left( \frac{0.38 \lambda}{\Delta n} - bd \right).$$

(13)

By substituting the coefficients $a$ and $b$ ($a = 0.446$, $b = 0.026$) in Eq. (13), it is possible to find the range of $\Lambda$ variation. The upper limit of $\Lambda$ is 1.87 μm ($d = 0.86$ μm) and the lower limit is 0 ($d \geq 33.1$ μm). It should be noted that Eq. (13) is obtained with the assumption that the anchor energy is very strong and that the orientation of the LC molecules near the surface is determined only by the surface, i.e. the orientations of the LC molecules near the surfaces are strictly 0° or 90°. However, in reality when $\Lambda$ is too small, the orientation of the molecules is also influenced by the near-lying parts on the surface. For low values of $\Lambda$, of the order of a few hundreds of nanometers, Eq. (13) becomes incorrect.

To transform the LC into a homeotropic state and to remove the periodical phase profile (“off” state), a high voltage must be applied. The voltage has to be sufficiently high for reorienting the LC molecules bordering with the alignment layers. Otherwise, the residual birefringence of the LC forms the phase profile of the outgoing light wave in the same
manner as in the case of no voltage. As a result, the diffraction efficiency of the 0 diffraction order will not be zero. On the other hand, the voltage should not be as high as the electrical breakdown voltage of the LC cell.

If the anchoring energy is strong \((10^{-3} J/m^2)\), the residual phase difference \(\Delta \varphi\) under high voltage (25V) may be up to 10% from the initial phase difference at no voltage applied. The residual phase grating decreases the ratio between intensities “on” and “off” states. The effect of residual phase grating can be dramatically decreased, due to decreasing the anchoring energy. Figure 8 illustrates the residual profile of the saw-tooth diffraction grating having different values of anchoring energy when 25V is applied. The calculations are made for \(d = 2.3\mu m\) and \(\Lambda = 25\mu m\). It is seen that the residual birefringence is slightly more that 1% for \(W = 2.5 \cdot 10^{-4} J/m^2\) and disappears completely for \(W = 2 \cdot 10^{-3} J/m^2\). On further decreasing anchoring energy, it is possible to reduce the voltage. Thus, the saturation voltage for \(W = 1 \cdot 10^{-4} J/m^2\) is around 7V.

![Graph showing \(\eta\) versus \(X/\Lambda\) in a saw-tooth diffraction grating with 25V applied for different anchoring energies.](image)

**5. Conclusions**

Potential abilities of LC DOE based on inhomogeneous alignment are studied. Formation of a desired LC director distribution due to inhomogeneous anchoring and pre-tilt angles are considered. These two approaches enables one to design a LC DOE controlled by continues electrodes, i.e. there are no constructive periodicities in the structure of the LC DOE. An advantage of utilizing inhomogeneous anchoring is low switching voltage which can be less 1V. LC DOE with non-uniform anchoring is optically homogeneous at no voltage, whereas the non-uniform pre-tilt angles distribution has the LC distribution endowing the LC medium with the properties of DOE.

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