Constraints on Braneworld Gravity Models from a Kinematic Limit on the Age of the Black Hole XTE J1118+480

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In braneworld gravity models with a finite AdS curvature in the extra dimension, the AdS/CFT correspondence leads to a prediction for the lifetime of astrophysical black holes that is significantly smaller than the Hubble time, for asymptotic curvatures that are consistent with current experiments. Using the recent measurements of the position, three-dimensional spatial velocity, and mass of the black hole XTE J1118+480, I calculate a lower limit on its kinematic age of $\geq 11$ Myr (95% confidence). This translates into an upper limit for the asymptotic AdS curvature in the extra dimensions of $< 0.08$ mm, which significantly improves the limit obtained by table-top experiments of sub-mm gravity.

The existence of large extra dimensions promises to offer a solution to the hierarchy problem in physics. If the standard model is restricted to act only on a (3+1)-dimensional brane, whereas gravity is allowed to propagate in the higher-dimensional bulk, the effective Planck scale in the four-dimensional spacetime can be made significantly larger than the electroweak scale, matching the experimental requirements.

In a braneworld model, even though it is only gravity that feels the presence of the extra dimensions, there are still a number of detectable effects on our (3+1) brane that can be used in constraining the properties of the bulk. The most direct implication of braneworld gravity models with large extra dimensions is the deviation of the inverse-square law for gravity at small scales. Torsion-balance experiments have been used successfully in rejecting the possibility of deviations of order unity from Newtonian gravity at scales as small as $\sim 0.2$ mm.

Significantly tighter bounds on the higher-dimensional Planck scale can be obtained by comparing model predictions to a number of astrophysical phenomena. Unfortunately, such bounds typically depend on the particular interpretation of astrophysical observations and suffer from large systematic errors. However, if a compelling case can be made for any of these phenomena, a very tight bound on braneworld models can be achieved.

Among the astrophysical tests, the one that requires the least amount of information is related to the evaporation of black holes. Within general relativity, the evaporation timescale of a black hole of any astrophysical mass ($\sim 1 - 10^9 M_\odot$) is much longer than the age of the universe. However, application of the AdS/CFT duality in braneworld gravity models that are asymptotically AdS (such as the Randall-Sundrum models) shows that this timescale can be significantly reduced, if the higher-dimensional Planck scale is not much larger than the electroweak energy scale. In particular, the evaporation time is given by

$$\tau \simeq 1.2 \times 10^2 \left( \frac{M}{M_\odot} \right)^3 \left( \frac{L}{1 \text{ mm}} \right)^{-2} \text{ yr},$$

where $M$ is the mass of the black hole and $L$ is the asymptotic AdS radius of curvature of the bulk.

Because of the no-hair theorem, this theoretical prediction depends only on assumptions regarding the braneworld model and in particular on the validity and implementation of the AdS/CFT correspondence. The basis of the calculation discussed above has been recently challenged, where it was argued that the strongly coupled nature of the holographic conjecture allows for time-independent, i.e., non evaporating, black-hole solutions. This issue will, of course, be resolved when complete black-hole solutions in braneworld gravity models are derived. However, even in the absence of such solutions, heuristic arguments based on similarities with black-string solutions and the thermodynamic properties of black holes indicate that static solutions will be unstable, at a timescale given by equation (1).

The lifetime given by equation (1) was calculated for black holes in vacuum, and therefore, its application to astrophysical black holes may not be justified. However, it is important to note that the fast “evaporation” of the black holes described in [4] is in fact a classical phenomenon and is simply equivalent to the radiation of a large number of modes in the bulk, which is devoid of matter, by construction. Matter external to the black hole on the brane will affect this result only if its gravitational properties of the latter in the bulk.

This is never the case in systems with stellar-mass black holes, for which the fraction of mass in the accretion flow is

$$\frac{M_{\text{acc}}}{M} \simeq 5 \times 10^{-10} \left( \frac{\epsilon}{0.1} \right) \left( \frac{R_{\text{acc}}}{1 \text{ AU}} \right)^{3/2} \left( \frac{M}{10 M_\odot} \right)^{-1/2}.$$

Here I assumed a free-falling accretion flow of radial extent $R_{\text{acc}}$, accreting at the Eddington critical rate, with a radiation efficiency $\epsilon$. Clearly, the presence of matter on the brane external to the black hole will not affect the gravitational properties of the latter in the bulk.

A final concern arises by the rapid accretion of matter by astrophysical black holes, which may overwhelm the rate of evaporation. For a black hole accreting at the
Eddington critical rate, the rate of evaporation is larger than the rate of accretion, as long as the mass of the black hole is \(M\):

\[
M \leq 50.3 \left(\frac{\epsilon}{0.1}\right)^{1/3} \left(\frac{L}{1 \text{ mm}}\right)^{2/3}.
\]

For a 10\(M_\odot\) black hole, the rate of accretion of matter is \(\sim 125\) slower than the rate of evaporation and can, therefore, be neglected.

The prediction given by equation (1) does not depend on unknowns that usually hamper other astrophysical tests. In fact, using it to place a constraint on braneworld gravity models requires only a firm lower limit on the age of an astrophysical black hole. In this Letter, I use the results of recent observations of the black hole XTE J1118+480 to place a lower limit on its kinematic age and a lower bound on the asymptotic AdS radius of braneworld gravity models [6].

XTE J1118+480 is a compact object in a binary system that lies away from the galactic plane [7]. The high (\(\geq 6.4\)\(M_\odot\)) measured mass function of the binary secures the identification of the compact object as a black hole [7]. Its position away from the galactic plane, as well as its large inferred spatial velocity, had led to the original suggestion that XTE J1118+480 is a halo object, probably as old as several Gyr [8]. However, the recent detection of high-Z elements in the optical spectrum of the source is inconsistent with the hypothesis that it was formed from the direct collapse of an old halo star [8]. On the other hand, the metallicity, position, and spatial velocity of this binary system requires a formation mechanism according to which it was produced in the Galactic disk and was ejected from the disk by the asymmetric explosion that formed the black hole [8, 10]. Even though this formation history makes the black hole significantly younger than estimated earlier, it also allows for an accurate bound to be placed on its kinematic age.

Given the current position and 3D velocity of the source, it is possible to integrate backwards its trajectory in the galactic potential to infer the times at which it crossed the galactic plane. Any of these crossings represents a plausible time at which the black hole was formed. However, the last time at which the black hole emerged from the galactic plane provides a lower limit on its kinematic age. This approach has been used successfully in the past to show that, in any of the recent crossing of the galactic plane, the velocity imparted to the natal black-hole binary is consistent with the kick velocities measured for other compact objects [10]. In this paper, I am only concerned with the time elapsed since the last galactic crossing and not with the properties of the binary star before or after the formation of the black hole.

There are four observable quantities that were recently measured accurately for XTE J1118+480 and allow for the reconstruction of its evaporation history (see Table I).

### Table I: Observed kinematic properties of XTE J1118+480

| Measurement                  | Uncertainty          |
|------------------------------|----------------------|
| Proper Motion, \(\mu\)       | 18.3 mas yr\(^{-1}\) ±1.6 mas yr\(^{-1}\) |
| Position Angle, \(\theta\)   | 246° ±6°             |
| Radial Velocity, \(V_r\)     | 10 km s\(^{-1}\) ±35 km s\(^{-1}\) |
| Distance, \(D\)              | 1.72 kpc ±0.1 kpc    |
| Mass, \(M\)                 | 8.53 \(M_\odot\) ±0.6 \(M_\odot\) |

First, in order of decreasing accuracy, are VLBA observations of the radio counterpart to the source showed a measurable proper motion of \(\mu = 18.3 ± 1.6\) mas yr\(^{-1}\) along a position angle of \(\theta = 246 ± 6°\). Given a distance to the source, these numbers specify the magnitude and direction of the source velocity on a plane perpendicular to our line of sight.

Second, models of the orbital variation of the spectral lines observed from the source led to a measurement of its radial velocity [7]. In the particular case of XTE J1118+480, the amplitude of orbital variations of the radial velocity (\(\sim 500\) km s\(^{-1}\)) is much larger than the reported systemic radial velocity of the binary (\(\sim\)a few tens of km s\(^{-1}\)), which is also comparable to the measurement uncertainties (\(\sim 20\) km s\(^{-1}\)) [7]. This results in an uncertain measurement, with the values reported ranging from \(-15±10\) km s\(^{-1}\) to \(+26±17\) km s\(^{-1}\) [7]. Because of this large systematic uncertainty, I will assume that the radial velocity is in the range \((-25, +45)\) km s\(^{-1}\), which I will write approximately as \(10 ± 35\) km s\(^{-1}\).

Finally, models of the optical/IR lightcurve of the source together with a characterization of the spectrum of the companion star led to a measurement of both the distance, \(D\), to the source and of the masses of the companion star and the black hole [11]. Uncertainties related to the contamination of the optical lightcurve by the accretion disk [7] and to the spectral characterization of the companion star [11] typically limit the accuracy of the measurements. Here I use the rather optimistic values quoted in reference [12] rather than the more conservative estimates of reference [7]. Having made the opposite choice would have affected only marginally the final result, the accuracy of which is mostly determined by the uncertainty in the radial velocity measurement.

A final ingredient in the calculation is the model for the galactic potential in the vicinity of the sun. Because of the large number of integrations needed in order to estimate the uncertainty of the results (see below), I use here the simpler Paczynski potential for the Galaxy [13]. Changing the model parameters of the potential within the range allowed by more recent studies of the galactic kinematics in the solar vicinity [14] affects the results by an amount smaller than the uncertainty introduced by the systematics in the radial velocity measurement. Given the model of the galactic potential and the values
for the observable properties of XTE J1118+480, I calculated the current 3D spatial velocity of the source according to references $[8,15]$ and performed the calculation of the galactic crossing times according to reference $[13]$. For the average values of the current kinematic parameters of XTE J1118+480 shown in Table 1, the time elapsed since the last crossing of the galactic plane is $\approx 18$ Myr. For a 8.5$M_\odot$ black hole, this corresponds to a limit on the asymptotic AdS curvature of the extra dimensions (see eq. (1)) of $L \leq 0.06$ mm, which is tighter than the $L \leq 0.2$ mm bound obtained by current tabletop experiments of sub-mm gravity [2]. However, because of the non-negligible uncertainties in the measurement of the kinematic properties of XTE J1118+480, and especially of its radial velocity, I will now estimate the formal uncertainty on this limit.

I will assume that the measurements of the various kinematic properties are independent of each other and assign to each one a Gaussian probability distribution with a mean and standard deviation equal to the values shown in Table 1. The measurement of the radial velocity is indeed independent of the measurement of the distance and they are both independent of the measurements related to the proper motion. On the other hand, the measurements of the proper motion and of the corresponding position angle are correlated to some level. However, because the uncertainties in these two parameters are much smaller than the uncertainty in the radial velocity, it is safe to neglect any correlated errors in them.

The assumption of Gaussian uncertainties is also easily justified for the measurements of the proper motion and position angle, since they are both dominated by statistical errors. However, the uncertainties quoted in Table 1 for the radial velocity and distance are dominated by systematic errors and rather represent a range of allowed values. Making the assumption that these uncertainties are also Gaussian leads to a more conservative calculation, since the exponential wings of the Gaussian distribution function allow, in principle, for a wider range of values for the measured quantities.

Given the assumptions discussed above, I can write the probability that the current kinematic state of XTE J1118+480 is described by a proper motion between $(\mu, \mu + d\mu)$, a position angle between $(\theta, \theta + d\theta)$, a radial velocity between $(V_r, V_r + dV_r)$, and a distance between $(D, D + dD)$ as

$$dP(\mu, \theta, V_r, D) = \int F_\mu(\mu) d\mu F_\theta(\theta) d\theta F_{V_r}(V_r) dV_r F_D(D) dD ,$$  

where $F_\mu(\mu)$, $F_\theta(\theta)$, $F_{V_r}(V_r)$, and $F_D(D)$ are the probability distribution functions for the mean proper motion, position angle, radial velocity, and distance, respectively. The probability distribution over each individual quantity is a Gaussian. For each set of values for the kinematic properties, I can now use the numerical method described above in order to calculate the time $t_c$ that has elapsed since the last crossing of the galactic plane, which

FIG. 1: The probability distribution over time elapsed since the last crossing through the galactic plane of the black hole XTE J1118+480. The median time is $\approx 24$ Myr and the (95%-probability) lower limit is $\approx 11$ Myr.
FIG. 2: The curved lines show the constraints on the asymptotic AdS curvature $L$ in the extra dimension placed by (short dashed line) the most probable time since the last galactic crossing of XTE J1118+480 and (long dashed line) the lower limit (at 95% confidence) on that time. The horizontal straight lines show the uncertainty in the dynamical mass measurement of the black hole. The intersection of the two gives the limit imposed by the kinematic age of the black hole of $L > 0.08$ mm.

source position with VLBA during a subsequent outburst will also reduce significantly the uncertainty in the proper motion. The best case scenario would be the detection of a black hole outside the galactic plane, traveling with a large velocity towards the galactic plane. As an example, for a $4M_\odot$ black hole, at the same position as XTE J1118+480, traveling with a velocity of 100 km s$^{-1}$ towards the galactic plane, the minimum kinematic age would be $\approx 205$ Myr and the upper bound on the asymptotic AdS curvature of the extra dimensions would be $L < 0.006$ mm. This represents the most stringent constraint that can be achieved with the method discussed here.

Note added: After submission of this paper, Kapner et al. reported an improved limit on the length scale in the extra dimensions of $\leq 50\mu$m, which is comparable to the one obtained here.

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