Compilation of load spectrum for NC machine tool’s feed system based on the mixture weibull distribution

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Abstract. The load spectrum of the feed system for the numerical control (NC) machine tool is the objective basis of the reliability design and life prediction. So, a compilation method of load spectrum is proposed by using the mixture Weibull distribution (MWD). First, the load distribution in each feed direction is fitted by using the MWD. Among them, the Bayesian information criterion (BIC) determines the number of mixture components, the parameters of MWD are estimated by using the improved expectation maximization (IEM) algorithm, where the initial values of parameters are determined by the hidden Markov model (HMM). Second, the correlation between the loads in different feed directions is analyzed, and the joint distribution function (JDF) is established. Finally, the program loading spectrum is compiled by using the ratio coefficient. In this paper, the relative feed speed spectrum in the X-direction and Z-direction of NC lathes are established, case analysis results show that the relative feed speed distributions that use the proposed method have high fitting precision, and the load spectrum are compiled, which will be able to apply the reliability test, accelerated test and life prediction. Thus, the study of load spectrum has important theoretical significance and engineering application value.

1. Introduction

Numerical control (NC) machine tools is a typical repairable system, its reliability level represents the level of development in the manufacturing industry [1]. The feed system is a typical key functional part of NC machine tools, and its reliability level directly affects the performance index of NC machine tools. The inherent reliability of the product is determined by the design. Reliability design of NC machine tools and its key functional components is one of the main ways to improve the reliability level of NC machine tools. However, the load spectrum of NC machine tool is an objective basis for reliability design. The reliability test of key functional components for NC machine tools is the main technical means to obtain product failure data, perform failure analysis and establish the model of reliability [2], the load spectrum is also the precondition to simulate actual conditions for the reliability bench test and accelerated test of functional components, and feed speed load is the factor that has the greatest effect on surface roughness [3-4]. Thus, it has important theoretical significance and engineering application value to study the scientific and reasonable compilation method of load spectrum for NC machine tool’s feed system.

At present, certain research results about the compilation method of load spectrum have been obtained in the fields of aerospace, automobiles, and rail vehicles. Yan C systematically studied the key technologies and methods in the process of load spectrum compiled for the aircraft, and proposed
the principle of compiling the high-confidence median random fatigue load spectrum [5]. Liu Y proposed a non-parametric load extrapolation method based on multi-criterion, and compiled load spectrum of the power splitter axle gear [6]. Grubisic put forward the load spectrum test, which mainly include the structure of the vehicle, the road conditions, and different driver operations, and a test scheme of the corresponding load spectrum has been formulated [7]. Li Q adopted the compilation method of median load spectrum to determine the minimum test times and typical test data of high-confidence load spectrum compiled for trains, and compiled an effective load spectrum [8]. In addition, key components of aerospace, automobile, and rail trains have also been equipped with appropriate load measurement and compilation methods. However, no systematic research has been conducted on the compilation method of the load spectrum of NC Machine tool’s feed system.

Load spectrum is the load time process of the product or component. The extrapolation of the load spectrum is one of the most critical techniques in the compilation process of load spectrum. Extrapolation of the load spectrum can obtain the load that may occur throughout the product life cycle, especially the extreme load. Xiong JJ established a joint distribution function (JDF) for the mean and amplitude of the load for aircraft structural loads to apply parameter extrapolation [9]. Socie D F combined with quaternion extrapolation techniques to acquire a life-cycle load spectrum that takes into account the impact of different drivers on the road loads, and the extrapolation method of nonparametric extrapolated rainflow based on kernel density estimation is established [10].

Mixture distribution models are widely studied in the area of load spectrum [11-12]. MWD have many parameters that are difficult to estimate [13-14]. The EM algorithm is a method for estimating unknown parameters in a MWD based on the number of mixture components [15]. The basic principle of the EM algorithm is the combination of iterative method and maximum likelihood method, but its initial value is difficult to choose. To solve this problem, Wang J proposed an improved EM algorithm, the Bayesian classification method is used to determine the initial value of the EM algorithm [16].

In order to better carry out reliability test and design of the NC machine tool’s feed system, this paper used the MWD to establish the distribution function of relative feed speed. The BIC was used to determine the number of mixture components, the values of initial parameters for the IEM algorithm were calculated by using the hidden Markov model (HMM). The JDF of the relative feed speed in the X-direction and Z-direction for the NC lathe was established through the correlation analysis. Finally, the ratio coefficient was used to compile the eight-level spectrum of relative feed speed in the X-direction and Z-direction for NC machine tool’s feed systems. Refer to figure 1 for detailed steps.

![Figure 1. The flow chart of load spectrum compiled for NC machine tool’s feed system.](image-url)
2. Compilation of load spectrum for NC machine tool’s feed system

2.1. Establishment of MWD

The MWD is composed of many Weibull distributions with a certain proportion. The probability density function of the MWD is defined as:

\[
f(x) = p_1f_1(x) + \ldots + p_if_i(x) + \ldots + p_nf_n(x) \quad i = 1, 2, \ldots, n
\]  

(1)

where \(f_i(x) = (\beta_i/\eta_i)(x/\eta_i)^{\beta_i-1} \exp(-x/\eta_i^{\beta_i})\), the weight coefficients are set as \(p_i\). \(\beta_i\) is the shape parameter of Weibull distribution, and \(\eta_i\) denotes the scale parameter of Weibull distribution.

2.2. IEM Algorithm

2.2.1. Determining the initial value of the EM algorithm. In order to avoid the EM algorithm falling into the local optimal solution, this paper proposes a method combining K-means++ cluster analysis and HMM to determine the initial value of the parameter, which can fully reflect the influence of the original data on the parameters of Weibull distribution. The calculation steps are as follows:

Step 1: The original data of the load was classified using the K-means++ cluster analysis method.

Step 2: Each type of data is given as a sequence of observations. The Weibull distribution is used as the initial model of the HMM. The Viterbi algorithm is used to obtain the optimal state sequence. When the output probability of HMM is the largest, \((\beta_j, \eta_j)\), the distribution parameter of the \(j\)-th sample is estimated by the maximum likelihood method.

Step 3: According to the number of data in each sample, the weight coefficients are determined. These parameter values are used as initial values of the EM algorithm.

2.2.2. EM Algorithm. The EM algorithm estimates the unknown parameters iteratively using the maximum likelihood method under the number of mixture components for the MWD is known. Traditional EM algorithm mainly focuses on Gaussian mixture distribution [17]. Its iterative steps for MWD are as follows:

Step 1: these parameter values are initialized as \(p_1, p_2, \ldots, p_n, \eta_1, \eta_2, \ldots, \eta_n, \beta_1, \beta_2, \ldots, \beta_n\).

Step 2: the initial parameter values of the Weibull distribution are converted into the values of the normal distribution. The equation is as follows:

\[
\begin{align*}
\mu_j &= \mu_j + \eta_j \left(1 + \frac{1}{\beta_j} \right) \\
\sigma_j^2 &= \eta_j^2 \left[\Gamma\left(1 + \frac{2}{\beta_j}\right) - \Gamma^2\left(1 + \frac{1}{\beta_j}\right)\right]
\end{align*}
\]  

(2)

Step 3: the responsiveness of the iteration is calculated as:

\[
\gamma_j = \frac{p_jf(x_i)}{\sum_{i=1}^{N} p_i f(x_i)}
\]  

(3)

where \(i\) is the number of sample data, \(i = 1, 2, \ldots, N\), \(j\) is the number of sample classification, \(j = 1, 2, \ldots, n\).

Step 4: the normal distribution parameter is iterated as:
Step 5: the parameter values of normally distributed are converted into parameter values for Weibull distribution. Since there is an overriding equation in the above equation, this direct calculation will affect the speed of the iteration. By using the newton difference method to approximate the substitution, the operation speed can be improved.

Step 6: the maximum likelihood function is calculated as follows:

\[ L_{\text{max}} = \sum_{i=1}^{n} \ln \sum_{j=1}^{N} p_i f(x_j) \]  

(5)

Step 7: repeat Step 3 to Step 6 until the maximum likelihood function value reaches a fixed value, or reaches the specified number of iteration steps. Finally, the parameter values of each Weibull distribution and the corresponding weight coefficients are output.

2.3. Determining the number of mixture components

The MWD is composed of many Weibull distribution with a certain proportion. It approximates complex models by increasing the number of mixture components and adjusting the weight coefficients. It is important to determine the number of mixture components \([18]\). In order to determine the number of optimal mixture components, the BIC is used. The number of mixture components corresponding to the maximum value of BIC is the optimal number.

The minimum number of mixture components is determined by the K-S test. The maximum deviation of the cumulative probability density values of the empirical function and the fitting function is calculated. When the maximum deviation is within a given range, the minimum number of mixture components can be determined.

The number of mixture components is gradually increased, and the likelihood function values are solved. The BIC is then used to determine the number of optimal mixture components. The BIC is expressed as \([19]\):

\[ BIC = -2\max(\ln L) + m \ln(n) \]  

(6)

where \(m\) is the number of estimated parameters, \(n\) is the number of all observations, and \(\max(\ln L)\) is the maximized log-likelihood.

2.4. Analysis of correlation

Before establishing the JDF, the load correlation in each feed direction were analyzed to avoid the decrease of fitting accuracy because of seeing them as independent. Pearson coefficient, Kendall coefficient and Spearman coefficient are currently used. Among them, Pearson’s linear correlation coefficient is a statistic used to reflect the linear correlation of two variables. When Pearson's linear correlation coefficient is 0, its means that there is no linear correlation between these two variables. The Kendall rank correlation coefficient is applicable to the case where the two categorical variables are ordered, and the Spearman rank correlation coefficient uses the rank size of the two variables for linear correlation analysis. Therefore, this paper selects the Kendall rank correlation coefficient and the Spearman rank correlation coefficient to carry out the load correlation in each feed direction.
2.5. Compilation of program loading spectrum
The load spectrum is usually transformed into a program loading spectrum for reliability tests and fatigue life prediction in the practical application. First of all, we need to determine the extreme load of the feed system and then divide the grade interval. According to the traditional method, the measured extreme load is amplified by 1.15 times as the load extreme extrapolated [20].

In order to reduce the impact of loading frequency on the test results, the load is divided into eight levels according to the ratio coefficient (1, 0.95, 0.85, 0.725, 0.575, 0.425, 0.275, 0.125) [21].

When the correlation between the load of each feed directions for the feed system is strong, a two-dimensional program loading spectrum of the feed system can be obtained by equation (7):

\[
n_{ij} = N \left( \int_{R_i}^{R_{i+1}} \int_{R_j}^{R_{j+1}} f(x, z) \, dx \, dz \right)
\]  \hspace{1cm} (7)

where \( R_i \), \( R_{i+1} \) are the upper and lower limits of the X-direction load for the feed system; \( M_i \), \( M_{i+1} \) are the upper and lower limits of the Z-direction load for the feed system; \( f(x, z) \) is the JDF of the feed system load.

When the load correlation of each feed direction is weak, the one-dimensional program loading spectrum of the feed system can be obtained. In the reliability test, the two-dimensional program loading spectrum of the feed system is converted into a one-dimensional program loading spectrum by equation (8):

\[
M_i = \frac{\sum_{j=1}^{8} M_i n_{ij}}{\sum_{j=1}^{8} n_{ij}}
\]  \hspace{1cm} (8)

where \( M_i \) is the relative feed speed in the Z-direction for the \( j \)-th of the X-direction; \( n_{ij} \) is the relative frequency in the Z-direction for the \( i \)-th stage and the corresponding frequency of the X-direction.

3. Case analysis
In the previous period, loading data for 28 sets of NC lathes were collected in the user field tracing test. These included different machining methods such as the turning, boring, drilling, facing and breaking. This article will use the data to compile the program loading spectrum of relative feed speed for the NC lathe’s feed system in the X-direction and Z-direction.

Firstly, the ratio of the relative feed speed and the processing time are calculated. So, the relative feed rate histograms are plotted, as shown in figure 2 and figure 3. The relative speed \( V_r \) is defined as:

\[
V_r = \frac{V}{V_{\text{max}}}
\]  \hspace{1cm} (9)

where \( V \) is the feed speed, and \( V_{\text{max}} \) is the maximum feed speed of feed systems.

Secondly, the MWD is established by using equation (1). When \( n \), the number of mixture components is 3 by equation (6), the load distribution of relative feed speed in the X-direction and Z-direction can be better fitted, the IEM algorithm is used to estimate the parameters. The MWD functions \( f(x) \) and \( f(z) \) are obtained.

From figure 2 and figure 3, it can be found that when the number of mixture components is too less, the over fitting will appear, and when the mixture components are too more, excessive fitting and peak will appear. When \( n = 3 \), the load distributions of relative feed speed in the X-direction and Z-direction can be better fitted. The estimated values of the MWD parameters in the X-direction and the Z-direction are shown in table 1.

### Table 1. The parameters of MWD for the feed system.

|          | X-direction | Z-direction |
|----------|-------------|-------------|
| \( p_i \)| 0.2549      | 0.2563      |
| \( \eta_i \)| 262.8853   | 171.6587    |
| \( \beta_i \)| 6.6206     | 2.8174      |
| \( p_i \)| 0.5811      | 0.6611      |
| \( \eta_i \)| 54.3419    | 28.6667     |
| \( \beta_i \)| 1.4162     | 1.3161      |
| \( p_i \)| 0.1641      | 0.0826      |
| \( \eta_i \)| 408.0156   | 157.4061    |
| \( \beta_i \)| 5.8199     | 8.3221      |
Thirdly, the JDF is established through the correlation analysis. The Kendall rank correlation coefficient is $\tau = 0.0123$ and the Spearman rank correlation coefficient $\rho = 0.0325$ between the relative feed speed in the X-direction and Z-direction of the feed system were calculated separately. So, the load correlation between the X-direction and Z-direction of the feed system can be ignored, because of the turning process, many processing methods are one-way feeding. For example, the Z-direction feed is used for cylindrical turning and boring, and the X-direction feed is used for the face turning and groove turning, it is rare to use X-direction machining in conjunction with the Z-direction unless machining in cone turning and non-cylindrical turning is performed. Therefore, the joint distribution of the relative feed speeds for the feed systems in the X-direction and Z-direction is expressed as $f(x, z) = f(x)f(z)$. After the extreme load is increased by 1.15 times, the extreme feed speed range in different feed directions is obtained, as shown in table 2.

![Figure 2. The load distribution of X-direction.](image)

![Figure 3. The load distribution of Z-direction.](image)

| Table 2. The ranges of relative feed speed for X-direction and Z-direction feed systems. |
|-----------------------------------------------|-----------------------------|
| Feed system | X-direction | Z-direction |
|--------------|-------------|-------------|
| Relative feed speed | [0-0.92] | [0-0.83] |

Finally, the program loading spectrum of the relative feed speed is established. The one-dimensional program loading spectrum of the relative feed speed in the X-direction and Z-direction of the NC lathe’s feed system can be obtained by equation (8), as shown in table 3 and table 4.

| Table 3. The one-dimensional program loading spectrum of relative feed speed in the X-direction. |
|-----------------------------------------------|-----------------------------|
| Level | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Relative feed speed | 0.115 | 0.253 | 0.391 | 0.529 | 0.667 | 0.782 | 0.875 | 0.920 |
| Frequency (million times) | 668.6 | 372.5 | 437.5 | 176.8 | 83.8 | 16.8 | 3.4 | 0.2 |

| Table 4. The one-dimensional program loading spectrum of relative feed speed in the Z-direction. |
|-----------------------------------------------|-----------------------------|
| Level | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Relative feed speed | 0.104 | 0.228 | 0.394 | 0.436 | 0.602 | 0.705 | 0.788 | 0.830 |
| Frequency (million times) | 456.3 | 237.2 | 319.6 | 146.7 | 66.4 | 5.8 | 1.4 | 0.04 |

4. Conclusions

The load spectrum is the basic data to simulate actual conditions for the produce reliability test. By studying the distribution of load for NC machine tool’s feed system, a compilation method of load spectrum for NC machine tool’s feed system based on the MWD is proposed.

Based on the data of the 28 NC lathes in the machine tool user filed tracking test, the histograms of relative feed speed in the X-direction and Z-direction are plotted. The JDF $f(x, z)$ is determined.
through correlation analysis, and the spectrum of relative feed speed for the feed system is programmed. At the same time, the one-dimensional, eight-level program loading spectrums of the relative feed speed in the X-direction and Z-direction of the NC lathe’s feed system are established. It provides a theoretical basis to carry out the reliability test, acceleration test, life prediction and reliability design for the feed system of NC lathe, which has important theoretical significance and applied value of engineer.

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