Semiquantum private comparison without quantum measurements from the classical user

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Abstract: In this paper, we propose a novel semiquantum private comparison (SQPC) protocol based on Bell states, which enables one quantum user and one classical user to compare the equality of their private inputs with the help of a semi-honest quantum third party (TP). TP is assumed to be semi-honest in the sense that she may take all possible attacks to steal the private inputs except conspiring with anyone. The correctness analysis and the security analysis validate that our protocol can draw the comparison results correctly and securely, respectively. Besides, our protocol doesn’t need to implement quantum entanglement swapping, only needs TP to perform Bell basis measurement, and doesn’t require the classical user to conduct quantum measurements. Moreover, our protocol can take advantage over previous SQPC protocols based on Bell states in qubit efficiency.

Keywords: Semiquantum private comparison; semi-honest third party; Bell state; quantum measurement

1 Introduction

In the year of 1984, Bennett and Brassard [1] proposed the first quantum cryptography protocol, namely the famous BB84 quantum key distribution (QKD) protocol. Hereafter, quantum cryptography has been developed rapidly. In the year of 2009, Yang and Wen [2] proposed the novel concept named as quantum private comparison (QPC), which can compare the equality of two users’ private information without disclosing them through the quantum means. Subsequently, scholars put forward a series of QPC protocols by using different quantum resources, such as single particles [3,4], Bell states [5-8], GHZ states [9,10], χ-type entangled states [11], five-qubit entangled states [12], six-qubit entangled states [13], etc.

In the years of 2007 and 2009, Boyer et al. [14,15] respectively proposed two pioneering semiquantum key distribution (SQKD) protocols by using polarized single photons to claim the birth of semiquantum cryptography. Later, Zou et al. [16] proposed a novel SQKD protocol to release the classical user from quantum measurements; Ye et al. [17,18] put forward two novel SQKD protocols with single photons in both polarization and spatial-mode degrees of freedom. According to the two works of Refs. [14,15], in the realm of semiquantum cryptography, it is generally believed that the classical user is limited to the following operations: (a) transmitting the qubits without disturbance; (b) measuring the qubits with the $Z$ basis $|0⟩,|1⟩$; (c) producing the qubits in the $Z$ basis; and (d) scrambling the qubits with delay lines. In the year of 2016, Chou et al. [19] proposed the first semiquantum private comparison (SQPC) scheme based on Bell states and quantum entanglement swapping. Since then, scholars have proposed numerous SQPC protocols with different quantum states. For example, Refs. [20-24] utilized Bell states to propose different SQPC protocols; Ref. [25] put forward a measure-resend SQPC protocol by using two-particle product states; Refs. [26,27] proposed different SQPC protocols by using single photons; Ref. [28] put forward a SQPC scheme based on Greenberger-Horne-Zeilinger (GHZ) class states; Ref. [29] suggested a novel SQPC scheme based on entanglement swapping of
four-particle cluster state and Bell state; Ref.[30] adopted \(d\) -dimensional Bell states to design the first SQPC protocol which can compare the size relationship of two classical users’ private inputs; Ref.[31] proposed a novel SQPC protocol of size relationship by using \(d\) -level single-particle states. Each of the above SQPC protocols in Refs.[19-31] compares the equality of two classical users’ private inputs. As far as we know, at present, there is no SQPC protocol which can compare the equality of one quantum user’s private input and one classical user’ private input.

Based on the above analysis, in this paper, we propose a novel SQPC protocol based on Bell states, which can correctly and securely compare the equality of one quantum user’s private input and one classical user’ private input with the help of a semi-honest quantum third party (TP). Besides, our protocol doesn’t need quantum entanglement swapping, only needs TP to perform Bell basis measurement, and doesn’t require the classical user to conduct quantum measurements. Moreover, the qubit efficiency of our protocol exceeds that of each of previous SQPC protocols based on Bell states [19-24].

### 2 Protocol description

Suppose that Alice is the user equipped with unlimited quantum capabilities, while Bob is the user only having limited quantum capabilities. Alice and Bob hold private inputs \(X = (x_1, x_2, \ldots, x_n)\) and \(Y = (y_1, y_2, \ldots, y_n)\), respectively, where \(x_i, y_i \in \{0,1\}, i = 1, 2, \ldots, n\). They want to compare the equality of their private inputs with the help of a semi-honest TP, who is assumed to have unlimited quantum capabilities. Relying on Ref.[32], “semi-honest” means that TP may take all possible attacks to obtain two users’ private inputs except conspiring with anyone. The specific steps of the proposed SQPC protocol are as follows, which are further shown in Fig.1 for clarity.

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**Fig.1** The flowchart of the proposed SQPC protocol

**Step 1:** Alice (Bob) generates two \(n\) -bit random number sequence \(R_{AI} = (r_{A1}^1, r_{A1}^2, \ldots, r_{A1}^n)\) (\(R_{BI} = (r_{B1}^1, r_{B1}^2, \ldots, r_{B1}^n)\)) and \(R_{A2} = (r_{A2}^1, r_{A2}^2, \ldots, r_{A2}^n)\) (\(R_{B2} = (r_{B2}^1, r_{B2}^2, \ldots, r_{B2}^n)\)) by applying a random number generator, where \(r_{A1}^i, r_{A2}^i \in \{0,1\}\) and \(i = 1, 2, \ldots, n\). Thereafter, Alice (Bob) encrypts \(X (Y)\) with \(R_{AI} (R_{BI})\) and obtains \(X' = (x_1 \oplus r_{A1}^1, x_2 \oplus r_{A1}^2, \ldots, x_n \oplus r_{A1}^n)\) (\(Y' = (y_1 \oplus r_{B1}^1, y_2 \oplus r_{B1}^2, \ldots, y_n \oplus r_{B1}^n)\) except conspiring with anyone. The specific steps of the proposed SQPC protocol are as follows, which are further shown in Fig.1 for clarity.

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2. Prepare Bell states \(\{\varphi^+, \varphi^-\}\);  
4. Perform Bell basis measurements on \(S_x^e\) and \(S_y^e\), and obtain \(c^e\);  
6. Calculate \(m_e\) and obtain comparison result.

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2. Send \(S_x^e\);  
4. Check the transmission security of \(S_x^e\);  
6. Send \(c^e\);

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2. Send \(S_y^e\);  
4. Check the transmission security of \(S_y^e\);  
6. Send \(c^e\);

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1. Generate \(r_{A1}^i\) and \(r_{A2}^i\), encrypt \(x_i\), and share \(k_{AI}^i\) and \(k_{AB}^i\) with Bob;  
3. Perform SIFT or CTRL on \(S_x^e\);  
5. Obtain \(c^e\);

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1. Generate \(r_{B1}^i\) and \(r_{B2}^i\), encrypt \(y_i\), and share \(k_{AI}^i\) and \(k_{AB}^i\) with Alice;  
3. Perform SIFT or CTRL on \(S_y^e\);  
5. Obtain \(c^e\);

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6. Send \(r_{A1}^i \oplus k_{AI}^i\);  
6. Send \(r_{B1}^i \oplus k_{AI}^i\).

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Alice  
Bob
\( Y = \{ y_1, y_2, \ldots, y_n \} = \{ (y_1 \oplus r_{1i}^1), (y_2 \oplus r_{2i}^1), \ldots, (y_n \oplus r_{ni}^1) \} \), where \( \oplus \) is the bitwise XOR operation.

Besides, Alice agrees on with Bob in advance to place \( R_{A2} \) and \( R_{B2} \) in the same positions of \( X' \) and \( Y' \), so as to form new sequences \( A = \{ a_1, a_2, \ldots, a_{2n} \} \) and \( B = \{ b_1, b_2, \ldots, b_{2n} \} \), respectively. For example, let \( A = \{ x_1, x_2, \ldots, x_n, r_{12}, r_{22}, \ldots, r_{nn} \} \) and \( B = \{ y_1, y_2, \ldots, y_n, r_{12}, r_{22}, \ldots, r_{nn} \} \). Moreover, Alice and Bob share a \( n \)-bit private key \( K_{AB} = \{ k_{AB}^1, k_{AB}^2, \ldots, k_{AB}^n \} \) via the SQKD protocol in Ref.\[16\] beforehand, where \( k_{AB}^i \in \{0, 1\} \) and \( i = 1, 2, \ldots, n \). According to \( K_{AB} \), Alice and Bob produce another \( 4n \)-bit private key
\[
K_{AB} = \{ k_{AB}^1, k_{AB}^2, \ldots, k_{AB}^n \} = \{ k_{AB}^1, k_{AB}^2, \ldots, k_{AB}^n \}.
\]

Step 2: TP prepares \( 4n \) Bell states, each of which is randomly selected from one of the four states \( \{ |\phi^-\rangle, |\phi^-\rangle, |\psi^-\rangle, |\psi^-\rangle \} \), where \( |\phi^+\rangle = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle) \) and \( |\psi^+\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle) \). Then, she splits all first particles and all second particles into sequences \( S_A \) and \( S_B \), respectively. Finally, TP sends the particles of \( S_A \) (\( S_B \)) to Alice (Bob) one by one. Take notes that after TP sends the first particle to Alice (Bob), she sends a particle only after receiving the prior one.

Step 3: After receiving \( s_A' \) (\( s_B' \)) from TP, Alice (Bob) performs the corresponding operation on it according to \( k_{AB}^i \), where \( s_A' \) (\( s_B' \)) is the \( j \)th particle of \( S_A \) (\( S_B \)), \( j = 1, 2, \ldots, 4n \). That is, when \( k_{AB}^i = 0 \), Alice (Bob) performs the SIFT operation on it; and when \( k_{AB}^i = 1 \), Alice (Bob) performs the CTRL operation on it. Here, the CTRL operation means to directly return the received particle to TP, while the SIFT operation means to perform the corresponding unitary operation on the received particle and then send the resulted particle to TP. The rule of performing the unitary operation on the \( l \)th SIFT particle is that: when \( a_i = 0 \) (\( b_i = 0 \)), Alice (Bob) performs \( I = |0\rangle \langle 0| + |1\rangle \langle 1| \) on the \( l \)th SIFT particle; and when \( a_i = 1 \) (\( b_i = 1 \)), Alice (Bob) performs \( \pi = |0\rangle \langle 1| + |1\rangle \langle 0| \) on the \( l \)th SIFT particle. Here, \( l = 1, 2, \ldots, 2n \). For convenience, \( S_A \) (\( S_B \)) after Alice’s (Bob’s) operations is represented by \( S_A' \) (\( S_B' \)).

Step 4: After TP obtains the particles returned from Alice and Bob, she measures the particles of the same positions in \( S_A' \) and \( S_B' \) with Bell basis and writes down the corresponding measurement results. Alice and Bob tell TP the positions where they chose the CTRL operations. TP derives a bit sequence \( C = \{ c_1, c_2, \ldots, c_{2n} \} \) from her Bell basis measurement results on the SIFT particles of the same positions in \( S_A' \) and \( S_B' \); when her measurement result is same as the initial prepared state, TP sets \( c_i = 0 \); otherwise, she sets \( c_i = 1 \). Here, \( l = 1, 2, \ldots, 2n \). For clarity, the relationships among...
different parameters corresponding to SIFT particles of the same positions in $S_A$ and $S_B$ are summarized in Table 1.

Table 1  Relationships among different parameters corresponding to SIFT particles of the same positions in $S_A$ and $S_B$

| Initial prepared states | $a_j$ | $b_j$ | Alice’s unitary operation | Bob’s unitary operation | TP’s measurement result | $c_i$ |
|-------------------------|-------|-------|----------------------------|-------------------------|-------------------------|-------|
| $|\phi^+\rangle$        | 0     | 0     | $I$                        | $I$                     | $|\phi^+\rangle$          | 0     |
| $|\phi^+\rangle$        | 0     | 1     | $I$                        | $\sigma$               | $|\psi^+\rangle$          | 1     |
| $|\phi^+\rangle$        | 1     | 0     | $\sigma$                   | $I$                     | $|\psi^+\rangle$          | 1     |
| $|\phi^+\rangle$        | 1     | 1     | $\sigma$                   | $\sigma$               | $|\phi^+\rangle$          | 0     |
| $|\phi^-\rangle$        | 0     | 0     | $I$                        | $I$                     | $|\phi^-\rangle$          | 0     |
| $|\phi^-\rangle$        | 0     | 1     | $I$                        | $\sigma$               | $|\psi^-\rangle$          | 1     |
| $|\phi^-\rangle$        | 1     | 0     | $\sigma$                   | $I$                     | $|\psi^-\rangle$          | 1     |
| $|\phi^-\rangle$        | 1     | 1     | $\sigma$                   | $\sigma$               | $|\phi^-\rangle$          | 0     |
| $|\psi^+\rangle$        | 0     | 0     | $I$                        | $I$                     | $|\psi^+\rangle$          | 0     |
| $|\psi^+\rangle$        | 0     | 1     | $I$                        | $\sigma$               | $|\phi^+\rangle$          | 1     |
| $|\psi^+\rangle$        | 1     | 0     | $\sigma$                   | $I$                     | $|\phi^+\rangle$          | 1     |
| $|\psi^+\rangle$        | 1     | 1     | $\sigma$                   | $\sigma$               | $|\psi^+\rangle$          | 0     |
| $|\psi^-\rangle$        | 0     | 0     | $I$                        | $I$                     | $|\psi^-\rangle$          | 0     |
| $|\psi^-\rangle$        | 0     | 1     | $I$                        | $\sigma$               | $|\phi^-\rangle$          | 1     |
| $|\psi^-\rangle$        | 1     | 0     | $\sigma$                   | $I$                     | $|\phi^-\rangle$          | 1     |
| $|\psi^-\rangle$        | 1     | 1     | $\sigma$                   | $\sigma$               | $|\psi^-\rangle$          | 0     |

In order to check the transmission security of CTRL particles, for the positions where Alice and Bob chose the CTRL operations, TP checks whether her measurement results on the CTRL particles of the same positions in $S_A$ and $S_B$ are same as the initial states prepared by herself. If all
results are positive, the protocol will be continued; otherwise, the protocol will be halted.

In order to check the transmission security of SIFT particles, Alice and Bob pick out the particles on which they performed the unitary operations according to $R_{A2}$ and $R_{B2}$, respectively. Then, Alice and Bob require TP to publish the initial prepared states of these chosen positions and the corresponding measurement results, while TP requires Alice and Bob to publish the values of $R_{A2}$ and $R_{B2}$, respectively. Then, Alice, Bob and TP check whether the initial prepared states of these chosen positions, TP’s corresponding measurement results and Alice and Bob’s corresponding unitary operations are correctly related or not. If all results are positive, the protocol will be continued; otherwise, the protocol will be halted.

Step 5: TP drops out the bits in $C$ corresponding to $R_{A2}$ and $R_{B2}$, and obtains the new bit sequence $C' = \{c_1, c_2, \ldots, c_n\}$. Note that the bits in $C$ are corresponding to $X$ and $Y$.

Step 6: For $i = 1, 2, \ldots, n$ : TP sends $c_i$ to Alice and Bob. Then, Alice (Bob) informs Bob (Alice) of the value of $r_{Ai} \oplus k'_{Ai}$ ($r_{Bi} \oplus k'_{Bi}$). Afterward, Alice (Bob) calculates

$$m_i = (r_{Ai}' \oplus k'_{Ai}) \oplus (r_{Bi}' \oplus k'_{Bi}) \oplus c_i.$$  

Alice and Bob finish the protocol and conclude that $X \neq Y$ as long as they find out $m_i \neq 0$; otherwise, they set $i = i + 1$ and repeat this step.

If they find out $m_i = 0$ for $i = 1, 2, \ldots, n$ in the end, they will conclude that $X = Y$.

Until now, it has finished the description of the proposed SQPC protocol. Obviously, in this protocol, besides being in charge of preparing Bell states as the initial quantum resource, TP is required to implement Bell basis measurements. As for Bob, he needs to send the particles via the quantum channel, prepare fresh particles in the $Z$ basis, reorder particles via different delay lines and perform the two unitary operations $I$ and $\sigma$. To perform $I$ on one particle means to do nothing on it. According to Ref.[33], the unitary operation $\sigma$ can be regarded to be classical, so Bob is indeed a classical user. Moreover, Bob is exempted from quantum measurements in this protocol.

3 Correctness analysis

In Step 4, TP produces $C$ from her Bell basis measurement results on the SIFT particles of the same positions in $S'_A$ and $S'_B$. According to Table 1, we can clearly obtain that $c_i = x_i \oplus y_i$, where $i = 1, 2, \ldots, n$. Besides, we can know $x_i = x_i \oplus r_{Ai}'$ and $y_i = y_i \oplus r_{Bi}'$ from Step 1. Hence, we have

$$m_i = (r_{Ai}' \oplus k'_{Ai}) \oplus (r_{Bi}' \oplus k'_{Bi}) \oplus c_i = r_{Ai}' \oplus r_{Bi}' \oplus c_i.$$
\[ \begin{align*}
&= r_{ai} \oplus r_{bi} \oplus (x_i \oplus y_i) \\
&= r_{ai} \oplus r_{bi} \oplus (x_i \oplus r_{ai}) \oplus (y_i \oplus r_{bi}) \\
&= x_i \oplus y_i .
\end{align*} \]

According to Eq.(1), \( m_i = 0 \) means that \( x_i = y_i \). It can be drawn the conclusion now that the output of the proposed protocol is correct.

4 Security analysis

In this section, we analyze the security of the proposed protocol and verify that the proposed protocol can resist the attacks from an external eavesdropper and internal participants.

4.1 Outside attacks

An external eavesdropper, Eve, wants to obtain something useful about users’ private inputs by launching various famous attacks, such as the Trojan horse attacks, the intercept-resend attack, the measure-resend attack, etc. However, Eve will be inevitably detected.

(1) The Trojan horse attacks

Because the particles of \( S_A( S_B ) \) are transmitted from TP to Alice (Bob) and back to TP, we need to think over the Trojan horse attacks from Eve, mainly including the invisible photon eavesdropping attack [34] and the delay-photon Trojan horse attack [35,36]. For resisting the invisible photon eavesdropping attack, a wavelength filter can be added in front of Alice’s (Bob’s) device to erase the illegitimate photon signal [36,37]. Moreover, the method of preventing the delay-photon Trojan horse attack is that Alice (Bob) uses a photon number splitter (PNS: 50/50) to divide each sample signal into two parts and evaluates the multiphoton rate after measuring the resulted signals in the right measuring bases [36,37].

(2) The intercept-resend attack

In Step 1, TP sends the particles of \( S_A( S_B ) \) to Alice (Bob). Eve intercepts the particles sent from TP to Alice (Bob), and sends Alice (Bob) the false ones she prepared in advance in the \( Z \) basis; after Alice’s (Bob’s) operations, Eve measures the particles sent out from Alice (Bob) with the \( Z \) basis, in hoping of getting something useful about Alice’s (Bob’s) private input, and sends the resulted states to TP. When Alice and Bob tell TP the positions where they chose the CTRL operations, Eve may hear of them. In this way, Eve may know the positions where Alice and Bob chose the SIFT operations. Through this intercept-resend attack, Eve can decode out \( A \) ( \( B \) ) from the initial prepared states of the fake particles corresponding to Alice’s (Bob’s) SIFT operations and her measurement results on the corresponding particles sent out from Alice (Bob) before being detected. However, she still cannot obtain \( X \) ( \( Y \) ) before being detected, because Eve has no knowledge about \( R_{ai} \) ( \( R_{bi} \) ) and the position of \( X' \) ( \( Y' \) ) in \( A \) ( \( B \) ).

In the following, we validate that this kind of attack from Eve can be discovered by Alice, Bob and TP inevitably. For example, assume that an initial pair of particles prepared by TP is \( \{ \phi^+ \} \)
and that the two fake particles prepared by Eve are both in the state of $|0\rangle$. After Alice and Bob perform their operations, TP executes Bell basis measurement on this pair of returned particles. When both Alice and Bob choose the CTRL operations, TP’s measurement result is randomly in the state of $|\phi^+\rangle$ or $|\phi^-\rangle$; as a result, Eve is detected with the probability of $\frac{1}{2}$, as TP’s measurement result should be $|\phi^+\rangle$ when no attack happens. When both Alice and Bob choose the SIFT operations, Eve is also detected with the probability of $\frac{1}{2}$ when this pair of particles is chosen for security check.

(3) The measure-resend attack

In the process of TP transmitting the particles of $S_A$ ($S_B$) to Alice (Bob), Eve intercepts them, measures them with the Z basis and sends the resulted states to Alice (Bob); after Alice’s (Bob’s) operations, Eve transmits the resulted states to TP after measuring the particles sent out from Alice (Bob) with the Z basis. Eve may know the positions where Alice and Bob chose the SIFT operations after they tell TP the positions where they chose the CTRL operations. Through this measure-resend attack, Eve can deduce $A$ ($B$) from her measurement results on the SIFT particles from TP to Alice (Bob) and back to TP before being detected. Unfortunately, Eve still cannot know $X$ ($Y$) before being detected, due to lack of $R_{ai}$ ($R_{bi}$) and the position of $X$ ($Y$) in $A$ ($B$).

In addition, Alice, Bob and TP can successfully detect Eve’s this kind of attack. For example, assume that an initial pair of particles prepared by TP is $|\phi^+\rangle$. After Eve intercepts these two particles and measures them with the Z basis, they are collapsed randomly into $|00\rangle$ or $|11\rangle$. Without loss of generality, assume that they are collapsed into $|00\rangle$. After Alice and Bob perform their operations, TP performs Bell basis measurement on this pair of returned particles. When both Alice and Bob choose the CTRL operations, TP obtains $|\phi^+\rangle$ or $|\phi^-\rangle$ with the same probability; hence, Alice, Bob and TP can detect Eve with the probability of $\frac{1}{2}$, as TP’s measurement result should be $|\phi^+\rangle$ when no attack happens. When both Alice and Bob choose the SIFT operations, Alice, Bob and TP can also detect Eve with the probability of $\frac{1}{2}$ when this pair of particles is chosen for security check.

4.2 Participant attacks

In 2007, Gao et al. [38] reminded that participant attacks must be given more concerns to, due to their strong powers. Therefore, in the proposed protocol, we need to pay special attention to
the attacks launched by Alice, Bob or TP.

(1) The participant attack from Alice or Bob

In the proposed protocol, Alice possesses unlimited quantum capabilities, while Bob only has limited quantum capabilities. Hence, it can be thought that Alice is more powerful than Bob. In this regard, here analyzes the participant attack from Alice first.

Firstly, we analyze the intercept-resend attack from Alice. Alice introduces no disturbances on the particles of $S_B$ sent out from TP corresponding to the CTRL operations, but intercepts the particles of $S_B$ sent out from TP corresponding to the SIFT operations and uses the fake particles prepared by her in the Z basis beforehand to replace them. After Bob’s operations, Alice introduces no disturbances on the particles sent out from Bob corresponding to the CTRL operations, but intercepts the particles sent out from Bob corresponding to the SIFT operations, uses the Z basis to measure them and sends the resulted states to TP. In this way, Alice can easily deduce $B$ from the initial prepared states of the fake particles and her measurement results on the corresponding particles sent out from Bob before being detected. However, although Alice knows the positions of $Y$ in $B$, she still cannot obtain $Y$ before being detected, because she has no access to $R_{B_1}$ at the moment.

Apparently, the above intercept-resend attack from Alice introduces no error on the CTRL particles. In the following, we validate that when launching the above intercept-resend attack, Alice leaves her trace on the SIFT particles so that her attack behavior can be discovered by Bob and TP. For example, assume that an initial pair of particles corresponding to Alice and Bob’s SIFT operations is prepared by TP in the state of $\ket{\Phi^+}$ and that one fake particle prepared by Alice is in the state of $\ket{0}$. Alice performs the above intercept-resend attack on the particle from TP to Bob and back to TP. TP executes Bell basis measurement on this pair of returned particles. When both Alice and Bob choose the SIFT operations, TP’s measurement result is randomly in the state of $\ket{\Phi^+}, \ket{\Psi^-}, \ket{\Psi^+}$ or $\ket{\Psi^-}$; as a result, Alice’s attack behavior is detected with the probability of $\frac{3}{4}$ when this pair of particles is chosen for security check.

Secondly, we analyze the measure-resend attack from Alice. Alice introduces no disturbances on the particles of $S_B$ sent out from TP corresponding to the CTRL operations, but intercepts the particles of $S_B$ sent out from TP corresponding to the SIFT operations, uses the Z basis to measure them and sends the resulted states to Bob. After Bob’s operations, Alice introduces no disturbances on the particles sent out from Bob corresponding to the CTRL operations, but intercepts the particles sent out from Bob corresponding to the SIFT operations, uses the Z basis to measure them and sends the resulted states to TP. In this way, Alice can easily deduce $B$ from her
measurement results on the particles of $S_B$ sent out from TP corresponding to the SIFT operations and the corresponding particles sent out from Bob before being detected. Unfortunately, although Alice is aware of the positions of $Y$ in $B$, she still has no way to get $Y$ before being detected, due to lack of $R_n$ at the moment.

Obviously, no error is induced by the above measure-resend attack from Alice on the CTRL particles. Then, we validate that Alice’s above measure-resend attack disturbs the SIFT particles so that Bob and TP can detect her attack behavior. For instance, suppose that TP prepares an initial pair of particles corresponding to Alice and Bob’s SIFT operations in the state of $|\phi^+\rangle$. Alice imposes the above measure-resend attack on the particle from TP to Bob and back to TP. Without loss of generality, assume that after Alice measures the particle from TP to Bob with the $Z$ basis, this pair of particles is collapsed into $|0\rangle|0\rangle$. TP performs Bell basis measurement on this pair of returned particles. When both Alice and Bob choose the SIFT operations, Alice’s attack behavior is discovered with the probability of $\frac{1}{2}$ when this pair of particles is chosen for security check.

It is easy to know after similar analysis that, through his attack behaviors, the dishonest user Bob cannot know $X$ either before being detected, due to lack of $R_n$ at the moment, although he knows the positions of $X$ in $A$; and moreover, his attack behavior can be detected inevitably by Alice and TP.

(2) The participant attack from TP

In the proposed protocol, TP is supposed to be semi-honest, that is, she may try all possible means to get two users’ private inputs but cannot be allowed to conspire with anyone. TP may adopt the following attack strategy: TP prepares $8n$ single particles in the $Z$ basis instead of $4n$ Bell states in Step 2, picks out half of these single particles to form $S_A$, and makes the remaining half particles compose $S_B$; afterward, TP sends the particles of $S_A$ ($S_B$) to Alice (Bob) one by one. After Alice’s (Bob’s) operations, when Alice and Bob tell TP the positions where they chose the CTRL operations, TP uses the $Z$ basis to measure the received particles whose positions Alice and Bob chose the SIFT operations. As a result, TP can decode out $A$ ($B$) from the initial prepared states of the SIFT particles in $S_A$ ($S_B$) and her measurement results on the corresponding particles in $S_A$ ($S_B$). In order not to be detected by Alice and Bob when checking the transmission security of CTRL particles, TP always announces Alice and Bob that her Bell basis measurement results on the CTRL particles of the same positions in $S_A$ and $S_B$ are identical to the initial Bell states prepared by herself; and in order not to be detected by Alice and Bob when checking the transmission security of SIFT particles, TP always publishes the initial prepared Bell states of the chosen positions and the corresponding Bell basis measurement results, which are correctly related to
Alice’s and Bob’s corresponding unitary operations. In this way, TP’s attack behavior can escape from being detected. In Step 6, TP may hear the value of $r_{ai}' \otimes k_{ab}'$ $(r_{ai}' \otimes k_{ab}')$ from Alice (Bob). However, due to lack of $k_{ab}'$, TP still cannot decode out $r_{ai}'$ $(r_{ai}')$. As a result, TP cannot deduce $x_i$ $(y_i)$ which is known by her ahead.

In Step 6, once Alice and Bob halt the communication in the middle, TP automatically knows that $X \neq Y$. The probability that this case occurs is $1 - \left(\frac{1}{2}\right)^{n-1}$. Or if Alice and Bob do not finish the communication until the last bits of $X$ and $Y$ are compared, TP has to guess the comparison result of $X$ and $Y$ randomly. The probability that this case happens is $\left(\frac{1}{2}\right)^{n-1}$. To sum up, the probability that TP correctly obtains the comparison result of $X$ and $Y$ (i.e., $X \neq Y$) is $1 - \left(\frac{1}{2}\right)^{n-1}$.

### 5 Discussions and conclusions

In this part, we compare the proposed protocol with previous SQPC protocols based on Bell states in Refs.[19-24]. The specific comparison results are shown in Table 2. Referring to Ref.[39], we define the qubit efficiency as $\eta = \frac{\lambda_x}{\lambda_q + \lambda_c}$, where $\lambda_x$, $\lambda_q$, and $\lambda_c$ represent the length of compared private bits, the number of consumed qubits and the number of classical bits for the classical communication, respectively. Here, we take no account of the classical resources consumed in the eavesdropping detection processes.

In the proposed protocol, the length of $X$ or $Y$ is $n$ bits, so it has $\lambda_x = n$. TP needs to prepare $4n$ initial Bell states; moreover, Alice and Bob share $K_{ab}$ via the SQKD protocol in Ref.[16] beforehand, which requires Alice to generate $N = 4n(1 + \delta)$ qubits randomly in $\{|0\}, \{|1\}, \{|+\}, \{|-\} \}$ and Bob to prepare $M$ qubits randomly in the $Z$ basis, where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ and $M \geq N$. Hence, it has $\lambda_q = 4n \times 2 + N + M = 8n + N + M$. In addition, considering the case that $x_i = y_i$ for $i = 1, 2, \ldots, n - 1$, TP need send $c_i$ to Alice and Bob, while Alice (Bob) need tell Bob (Alice) the value of $r_{ai}' \otimes k_{ab}'$ $(r_{ai}' \otimes k_{ab}')$, where $i = 1, 2, \ldots, n$. As a result, it has $\lambda_c = n \times 3 = 3n$. Consequently, the qubit efficiency of the proposed protocol is $\eta = \frac{n}{8n + N + M + 3n} \leq \frac{1}{19}$. Using the same method, we calculated the qubit efficiency of each of the protocols in Refs.[19-24].

According to Table 2, compared with the SQPC protocols of Refs.[19-24], the proposed
SQPC protocol has the following merits: (1) it can be used to compare the equality of one quantum user’s private input and one classical user’s private input, but each of Refs.[19-24] is suitable for two classical users to compare the equality of their private inputs; (2) it only requires TP to carry out Bell basis measurements, but each of the protocols of Refs.[19,20,22,23] requires TP to perform both Bell basis measurements and Z basis measurements; (3) it releases the classical user from quantum measurements, but each of the protocols of Refs.[19-24] requires the classical users to perform quantum measurements; (4) it doesn’t need quantum entanglement swapping, but the protocol of Ref.[19] requires quantum entanglement swapping; (5) its qubit efficiency can be larger than that of each of the protocols in Refs.[19-24].

In conclusion, in this paper, we propose a novel SQPC protocol based on Bell states, which is suitable for one quantum user and one classical user to compare the equality of their private inputs with the help of a semi-honest TP. Detailed security analysis shows that our protocol can resist a variety of outside and participant attacks. Our protocol only requires TP to perform Bell basis measurements, releases the classical user from quantum measurements and doesn’t need quantum entanglement swapping. Moreover, the qubit efficiency of our protocol can be improved, compared with previous SQPC protocols based on Bell states in Refs.[19-24].

Table 2  Comparison results of our SQPC protocol and previous SQPC protocols based on Bell states

| Function | The protocol of Ref.[19] | The protocol of Ref.[20] | The second protocol of Ref.[21] | The protocol of Ref.[22] | The protocol of Ref.[23] | The protocol of Ref.[24] | Our protocol |
|----------|--------------------------|--------------------------|---------------------------------|--------------------------|--------------------------|--------------------------|--------------|
| Type of TP | Semi-honest | Semi-honest | Semi-honest | Semi-honest | Semi-honest | Almost-honest | Semi-honest |
| Feature | Measure | Measure | Measure | Discard | Measure | Measure | Unitary |
|         | -resend | -resend | Randomization | -resend | -discard | -resend | operation- |
|         |         |         | -resend |         | -resend |         | resend |


| TP's measurement operation and $z$ basis测量 | Bell basis测量 | Bell basis测量 | Bell basis测量 | Bell basis测量 | Bell basis测量 | Bell basis测量 |
|---------------------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $m$ basis测量和$z$ basis测量 | $z$ basis测量 | $z$ basis测量 | $z$ basis测量 | $z$ basis测量 | $z$ basis测量 | $z$ basis测量 |
| The classical $z$ basis测量 | $z$ basis测量 | $z$ basis测量 | $z$ basis测量 | $z$ basis测量 | $z$ basis测量 | No |
| user's quantum measurement | measurements | measurements | measurements | measurements | measurements | measurements |
| Usage of SQKD or SQKA | No | Yes | Yes | Yes | Yes | Yes | Yes |
| Usage of quantum entanglement swapping | Yes | No | No | No | No | No | No |
| Usage of unitary operations | No | No | No | No | No | No | Yes |
| Usage of delay lines | No | Yes | Yes | No | No | No | Yes |
| Qubit efficiency | $\frac{1}{82}$ | $\frac{1}{60}$ | $\frac{1}{32}$ | $\frac{1}{48}$ | $\frac{1}{36}$ | $\frac{1}{58}$ | $\leq \frac{1}{19}$ |

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