Signals of Unconventional $E_6$ Models
at $e^+ e^-$ Colliders.

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Abstract

Generation dependent discrete symmetries often appear in models derived from superstring theories. In particular, in the framework of $E_6$ models the presence of such symmetries is required in order to allow for the radiative generation of naturally small neutrino masses. Recently it was shown that by imposing suitable generation dependent discrete symmetries, a class of models can be consistently constructed in which the three sets of known fermions in each generation do not have the same assignments with respect to the $27$ representation of $E_6$. In this scenario, the different embedding in the gauge group of the three generations implies in particular that the known charged leptons couple in a non–universal way to the new neutral gauge bosons ($Z_\beta$) present in these models. We exploit this fact to study the signature of this class of models at present and future $e^+e^-$ colliders. We show that some signals of deviation from lepton universality as well as some other discrepancies with the standard model predictions which have been observed at the TRISTAN collider in the production rate of $\mu$ and $\tau$, can be accounted for if the $Z_\beta$ mass is not much heavier than 300 GeV. We also study the discovery limits for lepton universality violation of this type at LEP-2 and at the 500 GeV $e^+e^-$ Next Linear Collider (NLC). We show that models predicting unconventional assignments for the leptons will give an unmistakable signature, when the $Z_\beta$ mass is as heavy as $\sim 800$ GeV (LEP-2) and $\sim 2$ TeV (NLC).

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I. INTRODUCTION

It was recently shown [1] that in the framework of superstring derived $E_6$ models, it is possible to implement an unconventional scenario in which some of the known fermions of the three families are embedded in the fundamental $27$ representation of the group in a generation dependent way, meaning that their gauge quantum numbers do not replicate throughout the three generations. It was also argued [1] that if $E_6$ models are required to allow for small neutrino masses (as is needed for any particle physics solution of the solar neutrino problem, and/or for explaining the atmospheric $\nu_\mu$ deficit via $\nu_\mu$ oscillations) the Unconventional Assignments (UA) scenario should be considered as a natural alternative to the standard schemes. In fact, in the framework of superstring inspired $E_6$ theories the most natural way for generating small neutrino masses is through radiative corrections [2,3] since in these models the Higgs representation necessary to implement a see-saw mechanism [4] is absent.\footnote{See however Ref. [5] for a discussion of a see-saw mechanism induced by gravitational effects.} In order to implement the generation of $\nu$ masses at the loop level, a suitable discrete symmetry must be imposed on the superpotential to insure that at the lowest order $m_\nu = 0$. Branco and Geng [6] have shown that no generation-blind symmetry exists that forbids non–vanishing neutrino masses at the tree level and at the same time allows for their radiative generation. As a result, in order to implement in a consistent way the generation of $\nu$ masses via loop diagrams, a symmetry that does not act in the same way on the three generations is needed.
The main motivation for investigating the UA schemes stems from the observation that once we chose to build a model based on a symmetry that distinguishes among the different generations, there is no reason in principle to expect that this symmetry will result in a set of light fermions (i.e. the known states) that will exactly replicate throughout the three generations [1]. This implies the possibility that the known states belonging to different generations could have different $E_6$ gauge interactions. Of course, experimentally we know that the $SU(2) \times U(1)$ gauge interactions of the known fermions do respect universality with a high degree of precision. However, since the Standard Model (SM) gauge group is rank 4, while $E_6$ is rank 6, as many as two additional massive neutral gauge bosons ($Z_β$) can be present, possibly with $M_β \sim 1$ TeV or less, and the possibility that the additional $U_β(1)$ interactions could violate universality is still phenomenologically viable.

Since the fundamental representation of $E_6$ is 27 dimensional, the fermion content of models based on this group is enlarged with respect to the SM. In fact, in addition to the standard fermions, two additional leptonic $SU(2)$-doublets, two $SU(2)$-singlet neutral states and two color-triplet $SU(2)$-singlet $d$-type quarks are present. UA models are realised by identifying in a generation dependent way some of the known doublets of lepton and/or $d$-type singlet quarks with the additional fermion multiplets [1]. Models based on the UA scenario can be implemented without conflicting with phenomenological or theoretical constraints. For example the model described in Ref. [1] was shown to be consistent with a large number of experimental constraints, ranging from the direct and cosmological limits on the neutrino masses, to the stringent limits on flavor changing neutral currents (FCNCs). In this model the left (L) handed lepton doublet \( \left(\nu_τ\right)_L \) and the right (R) handed quark singlet \( b_R \) of the third generation are assigned to $SU(2)$ multiplets having a different embedding in $E_6$ with respect to the corresponding states of the first two generations. The non standard phenomenology resulting from this model implies in particular that the \( \nu_τ \) neutrinos have different neutral current (NC) interactions than to \( \nu_e \) and \( \nu_μ \).
In Sec. 2 we will briefly outline the main features of the E\(6\) models based on the UA scenario, and establish our conventions and notations. A more complete discussion of the theoretical framework can be found in Ref. [1]. A very clean signature for the UA models would be the detection of deviations from universality in neutral current (NC) processes. Due to the clean experimental environment of \(e^+e^-\) annihilation, such a signature could be more easily detected at \(e^+e^-\) colliders rather than at hadron colliders. In Sec. 3 we will investigate the phenomenology of UA models at the present and future \(e^+e^-\) machines. Since the UA for the known fermions would result in a violation of universality only in the fermion couplings to the new \(Z_\beta\) bosons without affecting the couplings to the standard \(Z_0\), the large amount of data collected at the \(Z_0\) resonance by the Large Electron Positron (LEP) collaborations are not effective for the search of these kind of effects. In fact, the contribution of \(Z_\beta-Z_0\) interference to the various cross sections and asymmetries measured at LEP-1 vanishes at the peak, and the contribution of pure \(Z_\beta\) exchange is also vanishingly small. Some effects could still be detected if the \(Z_0\) had a sizeable mixing with \(Z_\beta\), however the existing bounds on the \(Z_0-Z_\beta\) mixing angle are extremely tight [7], so that we will disregard this possibility throughout this paper.

Among presently operating colliders, the one best suited to reveal the kind of effects we are looking for is the TRISTAN collider at the KEK laboratories, which is collecting data at about 60 GeV c.m. energy. It is intriguing, though certainly not compelling, that a few discrepancies between the TRISTAN data on the total hadronic and leptonic cross sections and the SM predictions do exist [8], and they point towards the existence of a \(Z_\beta\) at rather low energies \(M_\beta \lesssim 300\) GeV [9]. At the same time the data on the rate of production of \(\tau\) and \(\mu\) leptons do show a signal of violation of universality (at the level of 1.6 standard deviations) which could not be explained by conventional extended gauge models. However, as we will show, these data can be well accounted for in the framework of models with UA. We will complete our discussion by analyzing the discovery limits for UA models at LEP II and at the 500 GeV Next Linear \(e^+e^-\) Collider (NLC). Finally,
in Sec. 4 we will summarize our results and draw the conclusions.

II. UNCONVENTIONAL ASSIGNMENTS
IN $E_6$ MODELS

In $E_6$ grand unified theories, matter fields belong to the fundamental $27$ representation of the group. $E_6$ contains $SO(10) \times U_\psi(1)$ as a maximal subalgebra, and the $27$ branches to the $1 + 10 + 16$ representation of $SO(10)$. In turn $SU(10)$ contains $SU(5) \times U_\chi(1)$. The $SO(10)$, $SU(5)$, $U_\psi(1)$ and $U_\chi(1)$ assignments for the fermions in the $27$ representation are listed in Table I. Usually the known particles of the three generations are assigned to the $16$ of $SO(10)$ that also contains a $SU(2)$ singlet neutrino $\nu^c$

$$[16]_i = \begin{bmatrix} u \\ d \end{bmatrix}, \ u^c, \ e^c, \ d^c, \ L \equiv \begin{bmatrix} \nu \\ e \end{bmatrix}, \ \nu^c \end{bmatrix}_i \quad i = 1, 2, 3. \quad (2.1)$$

The $10$ and the $1$ of $SO(10)$ contain the new fields

$$[10]_i = \begin{bmatrix} H^c \equiv \begin{bmatrix} E^c \\ N^c \end{bmatrix}, \ h, \ H \equiv \begin{bmatrix} N \\ E \end{bmatrix}, \ h^c \end{bmatrix}_i$$

$$[1]_i = [S^c]_i \quad i = 1, 2, 3. \quad (2.2)$$

As it is clear from Table I, there is an ambiguity in assigning the known states to the $27$ representation, since under the SM gauge group

$$G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$$

the $\bar{5}_{(10)}$ in the $10$ of $SO(10)$ has the same field content as the $\bar{5}_{(16)}$ in the $16$. The same ambiguity is also present for the two $G_{SM}$ singlets, namely $1_1$ and $1_{16}$. In the present paper we will concentrate on the consequences of having different assignments for the known L-handed leptons. Since these leptons might not correspond to the entries as listed in Table I, we use quotation marks to denote
TABLE I. $SO(10)$, $U_\psi(1)$, $SU(5)$ and $U_\chi(1)$ assignments for the left-handed fermions of the $27$ fundamental representation of $E_6$. The $SU(2)$ doublets $H^c$, $H$, $L$ and $Q$ are explicitly written in components. The Abelian charges $Q_\psi$ and $Q_\chi$ can be obtained from the quantum numbers in the brackets by dividing by $c_\psi = 6\sqrt{2}/5$ and $c_\chi = 6\sqrt{2}/3$, respectively. The charges are normalized to the hypercharge according to: 

$$
\sum_{f=1}^{27} (Q_f^\psi, Q_f^\chi)^2 = \sum_{f=1}^{27} (\frac{1}{2}Y_f)^2 = 5.
$$

|       | $S^c$ | $(E_{N^c}^c)$ | $h$ | $(\tilde{N})_{E}^c$ | $h^c$ | $\nu^c$ | $(\bar{\nu})_{E}^c$ | $d^c$ | $e^c$ | $(u)_{d}^c$ |
|-------|-------|---------------|-----|-------------------|--------|---------|-------------------|------|------|----------|
| $SO(10)$ ($c_\psi Q_\psi$) | 1 (4) | 10 (−2) | | | | | | | | |
| $SU(5)$ ($c_\chi Q_\chi$) | 1 (0) | 5 (2) | 5 (−2) | 1 (−5) | 5 (3) | 10 (−1) | | | | |

the known states with their conventional labels, while labels not enclosed within quotation marks will always refer to the fields listed in the table.

In the models under investigation as many as two additional neutral gauge bosons can be present, corresponding for example, to some linear combinations of the $U_\chi(1)$ and $U_\psi(1)$ generators. The interaction of the fermions in the $\bar{5}$ of $SU(5)$ with these gauge bosons will depend on the specific assignments to the $16$ or to the $10$ of $SO(10)$. The two additional neutral gauge bosons are usually parametrized as

$$
Z_\psi' = Z_\psi \sin \beta + Z_\chi \cos \beta
$$

$$
Z_\beta'' = Z_\psi \cos \beta - Z_\chi \sin \beta,
$$

where $\beta$ is a model dependent parameter. In the following we will denote the lightest of the two new gauge bosons as $Z_\beta$. In the presence of a ‘light’ $Z_\beta$ different assignments will lead to a different phenomenology. In contrast, in the limit $M_\beta \rightarrow \infty$ the choice of the assignment is irrelevant as long as we are only concerned with the gauge interactions. However, even in this limit the requirement of $U_\psi,\chi(1)$ gauge invariance for the superpotential, together with the phenomenolog-
ical constraints on the absence of FCNCs in the Higgs sector, strongly constrain the structure of the viable models [1].

A model realizing an UA scenario, in which we call "$\tau_L$" corresponds to the charged component of the $H_3$ weak doublet belonging to $\bar{5}_{10}$, while the "$e_L$" and the "$\mu_L$" leptons are as usual assigned to the $\bar{5}_{16}$, was recently proposed in Ref. [1]. This model is realized by imposing on the superpotential a particular family-non-blind $Z_2 \times Z_3$ discrete symmetry. As a result of such a symmetry, the masses of the known (light) chiral leptons are generated by vacuum expectation values (VEVs) of Higgs doublets, through the terms $m_\tau E_3 L e_3 R$ (with $m_\tau \sim \langle \bar{L}_3 \rangle_0$) and $m_{\alpha \beta} e_\alpha L e_\beta R$ (with $m_{\alpha \beta} \sim \langle \bar{H}_2 \rangle_0$ and $\alpha, \beta = 1, 2$). The remaining charged leptons $e_3 L, E_3 R, E_\alpha L, E_\alpha R$ are vectorlike, and acquire large masses from VEVs of Higgs singlets.

As it was argued in Ref. [1], several interesting features of this model are peculiar to the UA schemes in general, independently of this particular realization. For example, in contrast to the conventional $E_6$ models [2], in the UA schemes rank 6 models are not disfavoured with respect to rank 5, so that the general parametrization of the two additional gauge bosons given in (2.3) is well motivated. We stress that other assignments, leading to models with a structure similar to the model proposed in Ref. [1], but implying a different phenomenology, can be easily obtained by means of some different discrete symmetries.

If some of the $\nu^c$ and/or $S^c$ $SU(2)$ neutral singlets are massless or are very light ($m_s \lesssim 1$ MeV), cosmological arguments suggest that the $Z_3$ bosons should be heavier than about $\sim 1-2$ TeV [10], thus excluding the possibility of detecting any signal at TRISTAN and LEP-2. In fact, though singlet under $G_{\text{SM}}$, these states do have $U(1)_\beta$ interactions. Then, not to conflict with the limit of 3.6 relativistic neutrinos in thermal equilibrium at the time of nucleosynthesis [11] (which can be derived from the data on the light element abundances) we have to require this interaction to be weak enough to allow for the decoupling of the light $G_{\text{SM}}$-singlets at a sufficiently early time (for example before the QCD phase transition) so that their number density can be safely diluted. Requiring the $U_\beta(1)$ interaction to be
“superweak” results in the quoted lower bound on the $Z_\beta$ mass [10]. We would like to mention that there are two models corresponding to the particular values of the angle $\beta$ in (2.3) ($\tan 2\beta = -\sqrt{\frac{15}{7}}$, 0) in which the nucleosynthesis constraints on $M_\beta$ can be evaded even in the presence of light $SU(2)$ singlets [1]. In fact, for these two values of $\beta$, respectively the $\nu^c$ and the $S^c$ degrees of freedom decouple from the lightest $Z_\beta$, behaving as ‘effective singlets’ with respect to all the ‘light’ gauge bosons. Then they could play the role of the helicity partners of the standard neutrinos, allowing in particular for non zero neutrino Dirac masses, while at the same time their gauge interactions would not be effective to keep them in thermal equilibrium in the early Universe.

However, for the sake of generality, in the present analysis we will assume that all the $\nu^c$ and $S^c$ $G_{\text{SM}}$-singlets are heavy ($m_s \gg 1$ MeV). In this case, independently of the value of $\beta$, the nucleosynthesis constraints on $M_\beta$ are evaded. Therefore the $Z_\beta$ boson could be as light as allowed by the present limits from direct searches at colliders [12] and from the analysis of $Z_\beta$ indirect effects [7], resulting in both cases in $M_\beta \gtrsim 200 - 300$ GeV. As we will see, in the UA schemes the presence of a $Z_\beta$ with a mass in this range can give rise to lepton universality violating effects that could be detected at the colliders presently in operation.

III. SIGNALS OF UNCONVENTIONAL ASSIGNMENTS AT $e^+ e^-$ COLLIDERS

In the presence of additional neutral gauge bosons, the lowest order cross section\(^2\) for the process $e^+ e^- \rightarrow l^+ l^-$ with $l \neq e$, is

$$\sigma(s) = \frac{4 \pi \alpha^2}{3} \sum_{m,n=0}^{N} C(m, n) \chi_m(s) \chi^*_n(s) \tag{3.1}$$

\(^2\) In the numerical computations we have taken into account the leading one-loop corrections by using an improved Born approximation [13].
\[
C(m, n) = [v_m(e)v_n^*(e) + a_m(e)a_n^*(e)] \cdot [v_m(l)v_n^*(l) + a_m(l)a_n^*(l)]
\]

(3.2)

\[
\chi_m(s) = \frac{g_m^2}{4\pi\alpha s} \frac{s}{s - M_m^2 - iM_m\Gamma_m}.
\]

(3.3)

We will henceforth assume that one of the two new bosons in (2.3) is heavy enough so that its effects on the low energy physics are negligible. Then \(m, n = 0, 1, 2\) correspond respectively to the \(\gamma\), \(Z_0\) and \(Z_\beta\) amplitudes. The couplings in (3.2) and (3.3) are

\[
g_0 = e \quad v_0(\ell) = Q_{em}^\ell \quad a_0(\ell) = 0
\]

\[
g_1 = (\sqrt{2}G_\mu M_Z^2)^{1/2} \quad v_1(\ell) = T_{3L}^\ell - 2Q_{em}^\ell s_w^2 \quad a_1(\ell) = T_{3L}^\ell
\]

\[
g_2 = s_w g_1 \quad v_2(\ell) = Q_{\beta}^\ell - Q_{\chi_\beta}^c \quad a_2(\ell) = Q_{\beta}^\ell + Q_{\chi_\beta}^c \quad \ell = e, \mu, \tau
\]

(3.4)

where \(Q_{em}^\ell = -1\) is the electric charge of the leptons, \(T_{3L}^\ell = -\frac{1}{2}\) is the third component of the weak isospin, \(s_w \equiv \sin \theta_w\) with \(\theta_w\) the weak mixing angle, \(Q_{\beta}^\ell, Q_{\chi}^c = Q_{w}^\ell, c\sin \beta + Q_{x}^\ell, c\cos \beta\) is the lepton coupling to the \(Z_\beta\) boson in (2.3). The new charges \(Q_{w}^\ell, c, Q_{x}^\ell, c\) are given in Table I, and are normalized to the hypercharge generator \(Y/2\). In addition, in (3.4) we have assumed for the abelian coupling, \(g_2\), a renormalization group evolution down to the electroweak scale similar to that of the hypercharge coupling \(g_Y \simeq s_w g_1\).

In (3.4) the vector and axial-vector couplings, \(v_{0,1}(\ell)\) and \(a_{0,1}(\ell)\), do not depend on the specific assignments for the leptons, and are unmodified with respect to the SM. In contrast, \(v_2(\ell)\) and \(a_2(\ell)\) do depend on the particular assignments of the \(\ell\) lepton. With the notations given in (2.1) and (2.2), and referring to the 16 and to the 10 representations of \(SO(10)\), the possible assignments for the L-handed \(\ell_1 = "e_L"\), \(\ell_2 = "\mu_L"\), \(\ell_3 = "\tau_L"\) charged leptons are

\[
"\ell_i^+" \in L_i \in 16
\]

or

(3.5)

\[
"\ell_i^-" \in H_i \in 10 \quad i = 1, 2, 3.
\]
With these assignments three different cross sections for the process $e^+e^- \rightarrow l^+l^-$ ($l \neq e$) are possible. They are $\sigma_{16 \rightarrow 16}$, $\sigma_{16 \rightarrow 10} = \sigma_{10 \rightarrow 16}$ and $\sigma_{10 \rightarrow 10}$, where the subscripts refer to the specific embedding of the L-handed components of the initial $e^-$ and final $l^-$ states in the 16 or in the 10 of $SO(10)$. In the following we will give results for the quantities $R_{16 \rightarrow 16}$, $R_{16 \rightarrow 10}$ and $R_{10 \rightarrow 10}$ corresponding to the different cross sections normalized to the point-like QED cross section for muon pair production.

In Fig. 1 we compare the theoretical values for $R_{16 \rightarrow 16}$, $R_{16 \rightarrow 10}$ and $R_{10 \rightarrow 10}$ at $\sqrt{s} = 58$ GeV c.m. energy, with the SM prediction $R_{ll}^{SM} = 1.053$ (heavy solid line), and with the TRISTAN experimental data $R_{\tau\tau} = 1.026 \pm 0.037$ and $R_{\mu\mu} = 0.982 \pm 0.036$. These figures have been obtained by combining the results of the AMY, TOPAZ and VENUS collaborations given in Ref. [8]. In deriving the averages we have assigned a common systematic error of $\pm 0.030$ for the uncertainty in the luminosity. Fig. 1 shows that the measured values of $R_{\tau\tau}$ and $R_{\mu\mu}$ are both lower than the SM prediction. However, while the value of $R_{\tau\tau}$ is consistent with the SM within one standard deviation, $R_{\mu\mu}$ is about two standard deviations off the expected value. The shaded areas show the predictions for the three ratios $R_{16 \rightarrow 16}$, $R_{16 \rightarrow 10}$ and $R_{10 \rightarrow 10}$ for a $Z_\beta$ mass ranging between 200 GeV and 300 GeV. This range coincides with the range of the lower bounds on $M_\beta$ obtained in the framework of conventional $E_6$ models. For example, for the models usually referred to as $\psi$, $\eta$ and $\chi$ models which correspond to the particular values $\sin \beta = -\sqrt{5}/8$, 0, 1 [14], the most conservative direct bounds are respectively 200, 230 and 280 GeV. These bounds have been obtained at hadron colliders from the limits on the process $p\bar{p} \rightarrow Z_\beta \rightarrow l^+l^-$, by assuming $Z_\beta$ decay to all allowed fermions and supersymmetric fermions [12]. Other indirect limits have been obtained from high precision electroweak data by analysing the $Z_\beta$ indirect effects on NC observables. The indirect bounds also suggest $M_\beta \gtrsim 200$ GeV for all the values of the parameter $\beta$ [7]. Clearly the limits derived from analyses based on the conventional scheme cannot be straightforwardly applied to the $Z_\beta$ of UA models, since in the present case a large number of fermion couplings could be different. However, we have no
FIG. 1. The TRISTAN total cross sections for $\mu$ pair production ($R_{\mu\mu}$) and $\tau$ pair production ($R_{\tau\tau}$), normalized to the point-like QED cross section, compared to the standard model prediction $R_{ll}^{SM}$. The dotted and dashed lines correspond to the one standard deviation experimental errors for $R_{\mu\mu}$ and $R_{\tau\tau}$ respectively. The shaded areas, enclosed between the solid lines ($M_{\beta} = 200$ GeV) and the dot-dashed lines ($M_{\beta} = 300$ GeV) depict the predictions for lepton pair production at the TRISTAN c.m. energy $\sqrt{s} = 58$ GeV in the unconventional assignments $E_6$ models. The results are given for a general $Z_{\beta}$ from $E_6$, as a function of $\sin \beta$. $R_{16 \rightarrow 16}$ refers to the case in which the L-components of both the initial and final leptons are assigned to the 16 representation of $SO(10)$, and similarly for $R_{16 \rightarrow 10}$ and $R_{10 \rightarrow 10}$.

reason to expect that by assuming UA the bounds could be greatly strengthened or relaxed, and hence we will take the quoted limits as a reasonable guess for the lower bounds on $M_{\beta}$ also in the UA schemes.

From Fig. 1, it is apparent that in the presence of a light $Z_{\beta}$, the experimental data on $R_{\mu\mu}$ would be better accounted for by either assigning both “$e_L$”
and "µL" to the 16 representation of SO(10) (as in conventional E6 models), and for values of sin β centered around zero, or by assigning both these leptons to the 10 and for sin β close to unity. At the same time, for any choice of the assignments and for any value of β the various R are in good agreement with the experimental value of $R_{\tau\tau}$. Only a small region in the vicinity of sin β $\sim 0.6$ is slightly disfavoured if the assignments "e_L" $\in$ 16 and "τ_L" $\in$ 10 are chosen.

One of the most spectacular signals of UA models would be a deviation of the ratio $\rho_{\mu/\tau} \equiv \sigma(e^+ e^- \to \mu^+ \mu^-)/\sigma(e^+ e^- \to \tau^+ \tau^-)$ from unity. Since many systematic errors, take for example the uncertainties in the luminosity measurements, cancel in this ratio, the experimental error is statistically dominated, implying a very transparent significance for such a measurement. At the $Z_0$ resonance, the measured value for this observable $\rho_{\mu/\tau} = 0.998 \pm 0.006$ [15], in striking agreement with µ–τ universality. Undoubtedly it would be difficult to accommodate any large deviation from unity at $\sqrt{s} \neq M_Z$ by means of some mechanism different from the one discussed here. For this reasons we believe that if a value $\rho_{\mu/\tau} \neq 1$ is measured off $Z_0$ resonance, this would represent a very clean and almost unmistakable signature of the UA models.

According to the assignments in (3.5), and without referring to any specific lepton flavor, we can write two expressions for this ratio which deviate from unity: $\rho_{16} \equiv \sigma_{16 \to 16}/\sigma_{16 \to 10}$ and $\rho_{10} \equiv \sigma_{10 \to 10}/\sigma_{10 \to 16}$, where the subscripts label the assignment for the L-electron in the initial state. Fig. 2 shows the two ratios $\rho_{16}$ and $\rho_{10}$ compared to the combined TRISTAN measurement $\rho_{\mu/\tau} = 0.957 \pm 0.027$. This value is about 1.6 standard deviation off the value of unity predicted by any model which assumes lepton universality. Again it is apparent that the experimental data can be better accounted for by taking

\[ "e_L", "\mu_L" \in 16, \quad "\tau_L" \in 10 \quad \text{and} \quad -0.5 \lesssim \sin \beta \lesssim 0.8 \quad (3.6) \]

or

\[ "e_L", "\mu_L" \in 10, \quad "\tau_L" \in 16 \quad \text{and} \quad \sin \beta \gtrsim 0.4 \quad (3.7) \]

and assuming $M_\beta \lesssim 300$ GeV. We note that the set of assignments in (3.6) coincides with the assignments in the model discussed in Ref. [1].
FIG. 2. The TRISTAN result for the ratio of $\mu$ to $\tau$ pair productions $(\sigma_{\mu\mu}/\sigma_{\tau\tau})_{\text{EXP}}$ compared to the standard model prediction $(\sigma_{\mu\mu}/\sigma_{\tau\tau})_{\text{SM}} = 1$. The dotted lines correspond to the one standard deviation experimental error. The shaded areas, enclosed between the solid lines ($M_\beta = 200$ GeV) and the dot-dashed lines ($M_\beta = 300$ GeV) depict the predictions for the ratio of pair productions of two different lepton flavors at $\sqrt{s} = 58$ GeV, in the unconventional assignments $E_6$ models. The results are given for a general $Z_\beta$ from $E_6$, as a function of $\sin \beta$. $\sigma_{16\rightarrow16}$ refers to the case in which the L-components of both the initial and final leptons are assigned to the 16 representation of $SO(10)$, and similarly for $\sigma_{16\rightarrow10} = \sigma_{10\rightarrow16}$ and $\sigma_{10\rightarrow10}$.

For the particular value $\sin \beta = -\sqrt{\frac{5}{8}}$, which corresponds to the rank 5 $\eta$ model [14], the cross section (3.1) is invariant with respect to the different choices of the assignments. This is apparent from Fig. 1, and in particular Fig. 2 shows that in this case lepton universality is preserved. This follows from the fact that for
this value of $\beta$, the $Q_\eta$ charges for the the $\bar{5}_{(10)}$ and for the $\bar{5}_{(16)}$ are equal [14,16] (this is true also for $Q_\eta(1_{(3)})$ and $Q_\eta(1_{(16)})$) implying that for all the leptons the couplings to the $Z_\eta$ are the same independently of the UA.

![Graph showing predictions for the ratio of pair productions of two different lepton flavors in the unconventional assignments E$_6$ models at the LEP-2 c.m. energy $\sqrt{s} = 190$ GeV (shaded areas), compared to the standard model prediction (thick solid line). The dotted lines depict the expected one standard deviation experimental error, based on an integrated luminosity of 500 pb$^{-1}$. The solid lines enclosing the shaded areas correspond to $M_\beta = 600$ GeV while the dot-dashed lines correspond to $M_\beta = 800$ GeV. The results are given for a general $Z_\beta$ from E$_6$, as a function of $\sin \beta$.]

In Fig. 3 we depict the theoretical values of $\rho_{16}$ and $\rho_{10}$ for $M_\beta$ in the range 600 – 800 GeV, and for $\sqrt{s} = 190$ GeV, corresponding to the c.m. energy at LEP-2.
The dotted lines depict the one standard deviation statistical error achievable with 500 pb$^{-1}$ of integrated luminosity corresponding to one year run [17] ($\sim 3 \times 10^3$ leptonic events per flavor). It is apparent that the signature of UA models could be easily recognized for $Z_\beta$ bosons corresponding to most of the $\sin \beta$ positive values.

Fig. 4 depicts the ratios $\rho_{16}$ and $\rho_{10}$ for $M_\beta$ in the range 1500 – 2200 GeV, and for $\sqrt{s} = 500$ GeV, corresponding to the NLC c.m. energy. The one standard deviation error corresponds to a statistics of $\sim 8.6 \times 10^3$ leptonic events per flavor, based on an integrated luminosity of 20 fb$^{-1}$ (one year run) and taking into account the efficiency for a cut to suppress the two-photon background [18]. The violation of
lepton universality, intrinsic of the UA models, would produce striking effects for a $Z_\beta$ as heavy as $\sim 2$ TeV and for values of $\sin \beta$ not too close to the $\eta$ model.

We have seen that by measuring the various quantities $R_{ll}$ and $\rho_{l/l'}$, it will be easy to detect the effects of the UA. However, from Figs. 1–4 it is also apparent that these observables alone would not be sufficient to determine the exact pattern of lepton assignments, since different assignments can account for the same set of experimental data by means of a different choice of the $\beta$ parameter. Working out a procedure for identifying univocally the correct pattern of assignments is beyond the scope of the present analysis, however, we believe that once signals of UA are detected in the leptonic cross sections, a measurement of the various leptonic asymmetries would be quite effective to achieve this result. Also, we would like to point out that if some mechanism resulting in generation dependent assignments for the lepton doublets is effective, quite naturally the same mechanism would imply UA for the $d$-type $SU(2)$ singlet quarks as well. For example in the model discussed in Ref. [1] the assignments “$e_L$”, “$\mu_L$” $\in 16$ and “$\tau_L$” $\in 10$ did imply, for the consistency of the model, the UA “$d^c_L$”, “$s^c_L$” $\in 10$ and “$b^c_L$” $\in 16$ in the quark sector. Clearly, due to the experimental difficulties in tagging the quark flavors, identifying UA for the $d$-quarks would be a much harder task.

IV. CONCLUSIONS

In conclusion we have examined the possibility of detecting with the present and future $e^+e^-$ colliders, signals of models predicting UA for the charged leptons. We have shown that a class of models based on the gauge group $E_6$, in which the known $SU(2)$ lepton doublets are embedded in the fundamental representation of the group in a generation dependent way, would result in a unique type of violation of lepton universality, which is induced by the exchange of new $Z_\beta$ bosons.
In agreement with LEP-1 data, no observable effects are predicted at the $Z_0$ resonance, however, some signals could be detected off $Z_0$ resonance. For example, we have shown that a few anomalies in the production rate of leptons, as well a hint of violation of $\mu-\tau$ universality which have been observed at the TRISTAN collider, could be well accounted for in the UA scenarios. As we have discussed, though these anomalies are not statistically compelling, it will not be easy to find an alternative particle physics mechanism that could simultaneously account for the LEP-1 and the TRISTAN observations. However, the mechanism proposed here would be effective only if the $Z_\beta$ mass is not much heavier than $\sim 300$ GeV. Though this value is still consistent with the direct limits from the $p\bar{p}$ collider [12], in the near future the data obtainable at Tevatron will be able to probe or rule out such an explanation of the leptonic cross section anomalies [19].

We have also discussed the discovery potential for this class of models at LEP-2, operating at 190 GeV c.m. energy, and at the NLC, operating at 500 GeV c.m. energy. We have shown that at these future colliders, striking effects of lepton universality violation resulting from the various UA, could be observed up to $M_\beta \sim 800$ GeV (LEP-2) and $M_\beta \sim 2200$ GeV (NLC) for most of the values of the model dependent parameter $\beta$.

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References

[1] E. Nardi, Report No. UM-TH-93-09 to be published on Phys. Rev. D48, n.7 (1 October 1993).

[2] B.A. Campbell et.al., Int. J. Mod. Phys. A 2, 831 (1987).

[3] A. Masiero, D.V. Nanopoulos and A.I. Sanda, Phys. Rev. Lett. 57, 663 (1986).

[4] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, Proceedings of the Workshop, Stony Brook, New York, 1979, edited by F. van Nieuwenhuizen and D. Freedman, (North Holland, Amsterdam, 1979) p. 315;
T. Yanagida, Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe, Tsukuba, Japan, 1979, edited by A. Sawada and A. Sugamoto, (KEK Report No. 79-18, Tsukuba, 1979).

[5] S. Nandi and U. Sarkar, Phys. Rev. Lett. 56, (1986) 564;
M. Cvetič and P. Langacker, Phys. Rev. D46, (1992) R2759.

[6] G.C. Branco and C.Q. Geng, Phys. Rev. Lett. 58, 969 (1986).

[7] E. Nardi, E. Roulet and D. Tommasini, Phys. Rev. D46, 3040 (1992);
P. Langacker and M. Luo, ibid. 45 278 (1992);
F. del Aguila, W. Hollik, J.M. Moreno and M. Quirós, Nucl. Phys. B372, 3 (1992);
J. Layssac, F.M. Renard and C. Verzegnassi, Z. Phys. C53, 97 (1992);
M.C. Gonzalez García and J.W.F. Valle; Phys. Lett. B259, 365 (1991);
G. Altarelli et.al., ibid. 263 459 (1991).

[8] AMY Collaboration, T. Kumita; TOPAZ Collaboration, B.L. Howell;
VENUS Collaboration, T. Sumiyoshi; in Proceedings of the Workshop on TRISTAN Physics at High Luminosities, KEK, Tsukuba, Japan, 1992, edited by M. Yamauchi (KEK Proceedings No. 93-2);
see also A. Maki, in Colmar 1991, Proceedings of the 11th International
Conference on Physics in Collision Colmar, France, 1991 (KEK Report No. 91-100).

[9] K. Hagiwara, R. Najima, M. Sakuda and T. Terunuma; Phys. Rev. D41, (1990) 815.

[10] M.C. Gonzales-Garcia and J.W. Valle, Phys. Lett. B240, 163 (1990); J. L Lopez and D.V. Nanopoulos, ibid. 241 392 (1990); A.E. Faraggi and D.V. Nanopoulos, Mod. Phys. Lett. A6 61 (1991).

[11] T. Walker, in Texas/PASCOS 92: Relativistic Astrophysics and Particle Cosmology, Proceedings of the 16th Texas Symposium on Relativistic Astrophysics and 3rd Particles, Strings, and Cosmology Symposium, Berkeley, CA, 1992, edited by C.W. Akerlof and M.A. Srednicki (The New York Academy of Sciences, vol. 688, New York, 1993), p. 745.

[12] CDF Collaboration, F. Abe et.al., Phys. Rev. Lett. 67, 2609 (1991); ibid. 68, 1463 (1992).

[13] M. Consoli and W. Hollik, in Z physics at LEP 1 Vol. 1, edited by G. Altarelli et.al., CERN Report No. 89–08, p. 7; G. Burgers and F. Jegerlehner, ibid. , p. 55; G. Burgers and W. Hollik, in Polarization at LEP Vol. 1, edited by G. Alexander et.al., CERN Report No. 88–06, p. 136; D.C. Kennedy and B.W. Lynn, Nucl. Phys. B322, (1989) 1.

[14] See J.L. Hewett and T.G. Rizzo, Phys. Rep. 183, 195 (1989) and references therein.

[15] See A. Gurtu, invited talk given at the 10th DAE Symposium on High Energy Physics, Bombay, India, 1992, Report No. TIFR/EHEP 93-1 (unpublished).

[16] E. Nardi, Phys. Rev. D48, 1240 (1993).

[17] D. H. Perkins, in Proceedings of the ECFA Workshop on LEP 200, Aachen,
Germany, 1986, edited by A. Böhm and W. Hoogland (CERN Report No. 87–08, p.1).

[18] A. Djouadi et al., in *Proceedings of the Workshop on $e^+e^-$ Collisions at 500 GeV*, Munich, Annecy, Hamburg, 1991, edited by P.M. Zerwas (DESY Report No. 92-123B, p. 491).

[19] F. del Aguila, M. Cvetič and P. Langacker, in *Proceedings of the Workshop on Physics and Experiments with Linear $e^+e^-$ colliders*, Waikoloa, HI, 1993, edited by F. Harris, Report No. UPR-0583-T (Unpublished).