NON-DIAGONAL PARTON DISTRIBUTION
FUNCTIONS

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Abstract
We show why the time ordering in the definition of non-diagonal parton distributions can be omitted and discuss the physics implications.

1 The physics context

The idea that the gluon distribution in the proton is relevant to diffractive photon-proton reactions [1] has been very fruitful and now been applied to a variety of processes: diffractive production of vector mesons, of a real photon or $Z$, of dijets and of open charm. It can be generalised to the non-diffractive region, where gluon exchange is no longer dominant and the quark distribution is probed [2, 3]. Then one can also exchange quantum numbers and consider final states such as $p\pi^0$ or $n\pi^+$. In all cases one exchanges a quark-antiquark or a gluon pair, attached to a parton distribution $S$ on the proton and to a hard scattering $H$ on the photon side as shown in Fig. 1; it is required that the photon virtuality or the final state provide a hard scale while $t$ is small. For certain processes this factorisation has been derived in QCD [3].

To make the transition from $\gamma^*$ to $X$ possible the hadron momenta on either side of the blob $S$ must be different, in contrast to usual parton distributions. We thus have new

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Figure 1: The factorisation of $\gamma^* + p \rightarrow X + p$ into a non-diagonal parton distribution $S$ and a hard scattering $H$. If $X$ is a meson, $H$ includes a distribution amplitude describing the formation of the meson from its valence quarks. The partons have momenta $k$ and $k'$ with plus-components $k^+ = x_1 p^+$ and $k'^+ = x_2 p^+$. 

quantities, non-diagonal parton distributions, which depend on $t$ and on the momentum fractions $x_1$ and $x_2$ of the partons. Their difference $x_1 - x_2$ is fixed to be $\xi = [(p - p') \cdot q]/[p \cdot q]$ by kinematics.

2 Time ordering: why we want to drop it . . .

Taking for $X$ in Fig. 1 the incoming $\gamma^*$ and cutting in the $s$-channel we obtain the diagram for the inclusive $\gamma^* p$ cross section via the optical theorem. The blob $S$, described now by usual parton densities, is then cut, whereas the non-diagonal parton distributions appear in the diagram of an amplitude where $S$ is not cut. On a formal level ordinary quark or gluon distributions are Fourier transformed matrix elements of products $\bar{\psi}_\beta(0)\psi_\alpha(z)|_{z^2=0}$ or $G_{\mu\nu}(0)G_{\rho\sigma}(z)|_{z^2=0}$ of field operators, whereas the non-diagonal ones appear with time ordered products.

It can however be shown that, because the separation $z$ of the fields is lightlike, one can leave out the time ordering, in other words it is the same whether one cuts the blob $S$ or not [4]. Before giving the idea of the proof let us point out why this property is so important:

— As non-diagonal distributions can be expressed without time ordering they become equal to the usual parton distributions in the limit $p = p'$.

— If the blob $S$ is cut one can show that, apart from convention dependent phases, the corresponding parton distributions are real valued due to time reversal invariance.

— Since the blob $S$ is “already cut” the real and imaginary parts of the $\gamma^* p$ amplitude in Fig. 1 are obtained just from the real and imaginary parts of the hard scattering $H$, convoluted with the same soft quantity. This is needed to obtain a simple dispersion relation for the amplitude [3].

— While the momentum fractions $x_1, x_2$ can a priori range from $-\infty$ to $+\infty$ in the loop integral of Fig. 1, they are restricted to a finite interval because the lower blob is effectively cut. This is crucial for a parton interpretation: no parton in a hadron can be faster than the hadron itself.

— An ordinary parton distribution can be interpreted as the squared amplitude for the
emission of a specified parton, summed over all configurations of spectator partons, which go across the cut of the blob $S$. Since non-diagonal distributions are cut as well they have a similar interpretation. Because $x_1$ and $x_2$ are different they involve however the interference of two different amplitudes. In this sense they contain essentially new information on the proton structure.

The question of time ordering is also relevant for higher twist parton distributions, i.e. soft blobs with more than two parton lines. For the diagonal case, $p = p'$, Jaffe has shown that time ordering can be dropped in leading and nonleading twist distributions [5].

3 . . . and why we are allowed to

The idea of our proof is to use the analytic properties of scattering amplitudes, following Landshoff and Polkinghorne [3]. Let us work in the collision c.m. with the 3-axis along $p$ and introduce for any vector $v$ its light cone coordinates $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and its transverse part $v_T = (v^1, v^2)$. The parton distributions can be expressed in terms of

$$\int dk^- d^2k_T \mathcal{A}\big|_{k^+ = x_1 p^+},$$

where $\mathcal{A}$ is the proton-parton scattering amplitude depicted by the blob $S$, given by $\mathcal{A} = \int d^4z \, e^{i k^- z} \langle p' | T \bar{\psi}_\beta(0) \psi_\alpha(z) | p \rangle$ if for definiteness we take quark distributions. In position space the integral over $k^-$ corresponds to setting $z^+ = 0$ in the matrix element, and the combined integral over $k^-$ and $k_T$ puts $z$ on the light cone. It turns out that the integration over $k^-$ is enough for our purpose (we may thus also consider $k_T$-unintegrated distributions).

According to the usual ideas on analyticity the amplitude $\mathcal{A}$ has singularities (poles and branch cuts) in the parton virtualities $k^2, k'^2$ and in the Mandelstam variables $s = (-k+p)^2, u = (k+p')^2$; in the standard convention they lie slightly below the real axis of the variable concerned. Expressing each variable in terms of the components of $p, p'$ and $k$ we can map these singularities onto the complex $k^-$-plane. We find that their location is controlled by the fractions $x_1$ and $x_2 = x_1 - \xi$ and have several cases:

— If $x_1 > 1$ or $x_1 < \xi - 1$ then all singularities lie on the same side of the real $k^-$-axis. Thus we can close the integration contour by an infinite semicircle in the other half-plane without encircling any singularity, and obtain a zero integral. Under the assumption that $\mathcal{A}$ vanishes fast enough as $|k^-| \to \infty$ at fixed $k^+$ and $k_T$ the semicircle does not contribute and the integral along the real $k^-$-axis is zero itself. We thus find that our distributions are only nonzero in the interval $\xi - 1 < x_1 < 1$.

— For $\xi < x_1 < 1$ we find the singularities in $s$ above and all others below the real $k^-$-axis. Closing the contour in the upper half-plane our integral is given by the discontinuity of $\mathcal{A}$ across the $s$-cut, which according to the Cutkosky rules corresponds to cutting the soft amplitude in the $s$-channel, as shown in Fig. 2(a). This cut amplitude can be written in terms of $\langle p' | \bar{\psi}_\beta(0) | I \rangle \langle I | \psi_\alpha(z) | p \rangle$ summed over all intermediate states $I$; using the closure property we find just the product $\bar{\psi}_\beta(0)\psi_\alpha(z)$ without time ordering as we wanted.
Figure 2: Different ways to cut the proton-parton amplitude $\mathcal{A}$: (a) in the $s$-channel, (b) in the parton virtuality $k'^2$. $I$ and $I'$ denote the intermediate states.

— For $\xi - 1 < x_1 < 0$ we can pick up in a similar way the $u$-channel cut of the soft amplitude and obtain a matrix element of $-\bar{\psi}_\alpha(z)\psi_\beta(0)$. While for $\xi < x_1 < 1$ we had the emission of a quark with momentum fraction $x_1$ and its reabsorption with fraction $x_2$ we now have emission of an antiquark with fraction $-x_2$ and its reabsorption with fraction $-x_1$.

— The region $0 < x_1 < \xi$ has the particularity that $x_1$ is positive but $x_2$ negative; in a parton picture one has now the transition from a proton $p$ to a proton $p'$ and a quark-antiquark pair. In the $k^-$-plane the singularities in $s$ and $k'^2$ lie above, those in $u$ and $k^2$ below the real axis. If we chose to pick up the former two we must cut the soft amplitude in the $s$-channel as before, but we must also cut in $k'^2$, as shown in Fig. 2 (b). Altogether we find precisely the sum over intermediate states $I$ and $I'$ needed to go from a time ordered product of quark fields to an ordinary one.

In summary, we can show under fairly general assumptions on the amplitude $\mathcal{A}$ that in parton distributions the time ordering of fields is irrelevant because the component $z^+$ of their separation is zero, in other words because parton distributions are integrated over the minus component of the parton momentum.

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