NEUTRINO PHENOMENOLOGY FROM UNCONVENTIONAL E\(_6\) MODELS\(^\dagger\)

ENRICO NARDI

Randall Laboratory of Physics, University of Michigan
Ann Arbor, MI 48109-1120, U.S.A.

ABSTRACT

Superstring derived E\(_6\) models can accommodate small neutrino masses if a discrete symmetry is imposed which forbids tree level Dirac neutrino masses but allows for radiative mass generation. The only possible symmetries of this kind are known to be generation dependent. Thus we explore the possibility that, as a consequence of such a symmetry, the three sets of light states in each generation do not have the same assignments with respect to the 27 of E\(_6\), implying that the gauge interactions under the additional U(1)\(_\prime\) factors are non universal. Models realising such a scenario are viable, and by requiring the number of light neutral states to be minimal, an almost unique pattern of neutrino masses and mixings arises. We briefly discuss a model in which, with a natural choice of the parameters, \(m_{\nu_\tau} \sim 0.1-10\text{ eV}\) is generated at one loop, \(m_{\nu_\mu} \sim 10^{-3}\text{ eV}\) is generated at two loops and \(\nu_e\) remains massless.

\(^\dagger\) Talk given at the “International School on Cosmological Dark Matter”, Valencia, Spain, October 4-8, 1993.

E-mail: nardi@umiphys.bitnet

UM-TH 94–01

January 1994
1. Introduction

It is generally believed that neutrinos possess very small but non-vanishing masses. While there is no fundamental reason for the neutrinos to be exactly massless, small $\nu$ masses are needed in any particle physics explanation of the solar neutrino problem, and at the same time they imply several interesting phenomenological consequences. A very attractive way of generating naturally small neutrino masses is through the use of the see-saw mechanism\(^1\). In $E_6$ supersymmetric Grand Unified Theories (GUTs)\(^2\), as derived from superstring theories, the see-saw mechanism cannot be easily implemented since the Higgs representation necessary to generate a large Majorana mass for the right-handed neutrinos is absent. However, even in the absence of Majorana terms, small masses can be generated through radiative corrections in models in which at the lowest order $m_\nu = 0$. As was pointed out by Campbell et. al.\(^3\) and Masiero et al.\(^4\), $E_6$ GUTs do offer the possibility of implementing this second mechanism.

The fermion content of models based on $E_6$ is enlarged with respect to the Standard Model (SM). In fact two additional lepton $SU(2)$-doublets, two $SU(2)$-singlet neutral states and two color-triplet $SU(2)$-singlet $d$-type quarks are present in the fundamental representation of the group. In order to forbid neutrino masses at the tree level an appropriate discrete symmetry has to be imposed on the super-potential of the model. Branco and Geng\(^5\) have shown that no generation-blind symmetry exists that forbids non vanishing neutrino masses at the tree level, and at the same time allows for the radiative generation of the masses at one loop. As a result, in order to implement this mechanism a symmetry that does not act in the same way on the three generations is needed.

It was recently pointed out\(^6\) that once we chose to build a model based on a symmetry that does distinguish among the different generations, there is no reason in principle to expect that this symmetry will result in a set of light fermions (i.e. the known states) that will exactly replicate throughout the three generations. To state this idea more clearly, we wish to suggest the possibility that what we call $\nu_\tau$ is actually assigned to an $SU(2)$ doublet which has a different embedding in $E_6$ with respect to the doublet that contains what we call $\nu_e$. In the following we will denote this kind of non-standard embeddings as ‘unconventional assignments’ (UA). As a consequence the two neutrinos will have different $E_6$ gauge interactions. Obviously, experimentally we know that the $SU(2) \times U(1)$ interactions of the fermions do respect universality with a high degree of precision, however, in the class of models that we want to investigate one or two additional $U(1)'$ abelian factors are always present, implying additional massive neutral gauge bosons possibly at energies $O(\text{TeV})$ or less. The possibility that the $U(1)'$ interactions of the known fermions could violate universality then is indeed still phenomenologically viable. In Section 2 we will briefly outline a scenario that realises this idea. A more complete description of the theoretical framework can be found in Ref.\(^6\). In Section 3 we will concentrate
on the neutrino phenomenology, and we will describe the pattern of masses and mixings that is predicted by our scheme, and in Section 4 we will draw the conclusions. We believe that the unconventional scenario that we are going to analyse here could be interesting in itself, since it is not a priori obvious that models in which the ‘low’ energy gauge interactions of the known fermions are not universal can be consistently constructed. However, it turns out that beyond being viable, these models also lead to an interesting phenomenology, especially in the neutrino sector, and as well imply some rather unusual consequences. For example a few peculiar effects in the propagation of the neutrinos through matter could arise, and have been discussed in Ref. [6]. Since neutrinos come in doubles with L-handed charged leptons, UA for the $\nu^i$’s also imply that the neutral current interactions for $e_L$, $\mu_L$ and $\tau_L$ can be different. This will result in deviations from unity for the rate of production of different lepton flavors, and represents the most clean signature of the UA models. Such a signature could be most easily detected in $e^+e^-$ annihilation at high c.m. energy, as for example at LEP II and at a 500 GeV Next Linear Collider. However, in order to identify completely the correct pattern of UA, the measurement of a large set of quantities is needed. A thorough analysis of the effect of UA on various cross sections and asymmetries, aiming to study the possibility of identifying unequivocally the different possible embeddings, can be found in Ref. [8].

2. Unconventional Assignments in $E_6$ models.

In $E_6$ grand unified theories, as many as two new neutral gauge bosons can be present, corresponding to the two additional Cartan generators that are not present in the SM gauge group. Here we will consider the embedding of the SM gauge group

$$\mathcal{G}_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

in $E_6$ through the maximal subalgebras chain:

$$E_6 \rightarrow U(1)_\psi \times SO(10)$$

$$\downarrow \quad U(1)_\chi \times SU(5)$$

$$\downarrow \quad \mathcal{G}_{SM}. \quad (2.2)$$

In general the two additional neutral gauge boson will correspond to some linear combinations of the $U_\chi(1)$ and $U_\psi(1)$ generators that we will parametrize in term of an angle $\theta$ as

$$Z'_\theta = Z_\psi \cos \theta - Z_\chi \sin \theta$$

$$Z''_\theta = Z_\psi \sin \theta + Z_\chi \cos \theta. \quad (2.3)$$

The angle $\theta$ is a model dependent parameter whose value is determined by the details of the breaking of the gauge symmetry. In the following we will denote the lightest of the two new gauge bosons as $Z_\theta$. 
In the kind of models we are considering here, each generation of matter fields belong to one fundamental \(27\) representation of the group. The \(27\) branches to the \(1 + 10 + 16\) representations of \(SO(10)\). The known particles of the three generations, together with an \(SU(2)\) singlet neutrino \(\nu^c\), are usually assigned to the \(16\) of \(SO(10)\), that in turn branches to \(1_{16} + 5_{16} + 10_{16}\) of \(SU(5)\). Giving in parenthesis the Abelian charges \(Q_\psi\) and \(Q_\chi\) for the different \(SU(5)\) multiplets, we have

\[
\begin{align*}
\mathbf{[16]} & \quad (1c_\psi)(-5c_\chi) = [\nu^c] \\
\mathbf{[5_{16}]} & \quad (1c_\psi)(3c_\chi) = [L = \begin{pmatrix} \nu^c \\ c \end{pmatrix}, d^c] \\
\mathbf{[10_{16}]} & \quad (1c_\psi)(1c_\chi) = [Q = \begin{pmatrix} u^c \\ d^c \end{pmatrix}, u^c, e^c]
\end{align*}
\tag{2.4}
\]

The \(10\) of \(SO(10)\) that branches to \(5_{10} + \bar{5}_{10}\) of \(SU(5)\) contains the fields

\[
\begin{align*}
\mathbf{[5_{10}]} & \quad (-2c_\psi)(-2c_\chi) = [H = \begin{pmatrix} N \\ E \end{pmatrix}, h^c] \\
\mathbf{[\bar{5}_{10}]} & \quad (-2c_\psi)(2c_\chi) = [H^c = \begin{pmatrix} E^c \\ N^c \end{pmatrix}, h^c].
\end{align*}
\tag{2.5}
\]

Finally the singlet \(1\) of \(SO(10)\) corresponds to

\[
\begin{align*}
\mathbf{[1]} & \quad (4c_\psi)(0c_\chi) = [S^c].
\end{align*}
\tag{2.6}
\]

According to the normalization \(\sum_{f=1}^{27}(Q^f_{\psi,\chi})^2 = \sum_{f=1}^{27}(\frac{3}{2}Y^f)^2 = 5\), in (2.4)-(2.6) we have respectively \(c_\psi = \frac{1}{6}\sqrt{\frac{5}{2}}\) and \(c_\chi = \frac{1}{8}\sqrt{\frac{2}{3}}\). Matter fields will couple for example to the \(Z'\) boson through the charge

\[
Q_\theta = Q_\psi \cos \theta - Q_\chi \sin \theta.
\tag{2.7}
\]

The most general renormalizable superpotential arising from the coupling of the three \(27\)'s in (2.4)-(2.6) and invariant under the low energy gauge group (2.2) is

\[
W = W_1 + W_2 + W_3 + W_4
\]

where

\[
\begin{align*}
W_1 & = \lambda^{(1)} H^c Q u^c + \lambda^{(2)} H Q d^c + \lambda^{(3)} H L e^c + \lambda^{(4)} S^c h h^c \\
W_2 & = \lambda^{(5)} h u^c e^c + \lambda^{(6)} L Q h^c + \lambda^{(7)} \nu^c h d^c \\
W_3 & = \lambda^{(8)} h Q Q + \lambda^{(9)} h^c u^c d^c \\
W_4 & = \lambda^{(10)} H^c L e^c + \lambda^{(11)} H^c H S^c.
\end{align*}
\tag{2.8}
\]

The Yukawa couplings in (2.8) are three index tensors in generation space, e.g. \(\lambda^{(1)} H^c Q u^c = \lambda^{(i,j,k)} H^c_{ij} Q_{jk} u^c_k\) with \(i, j, k = 1, 2, 3\) generation indices, and in general they are not constrained by the \(E_6\) Clebsch-Gordan relations.\(^{10}\) As it stands the model is not phenomenologically viable, since the simultaneous presence of \(W_2\) and \(W_3\) induces fast proton decay, and at the same time the presence of \(W_4\) would produce
(too large) tree level Dirac masses for all the neutral states. Both these problems can be cured by imposing on the superpotential (2.8) a discrete symmetry. Such a symmetry must be generation dependent if we want to leave open the possibility of having small neutrino masses generated by loop effects.5

3. Neutrino Masses in the Unconventional Schemes.

As it is clear from the second lines in (2.4) and (2.5), there is an ambiguity in assigning the known states to the 27 representation, since under the SM gauge group the 5_{10} in the 10 of SO(10) has the same field content as the 5_{16} in the 16. The same ambiguity is also present for the two \( g_{SM} \) singlets, namely 1_1 and 1_{16}. Then, as a starting point for investigating \( E_6 \) models with UA, we will assume that what we call “\( \nu_\tau \)" is in fact the \( N_3 \) weak doublet neutral state belonging to the \( \bar{5}_{10} \), while \( \nu_e \) and \( \nu_\mu \) are still assigned as usual to the \( \bar{5}_{16} \). We will henceforth use quotation marks to denote the known states with their conventional labels, since they might not correspond to the entries in (2.4)-(2.6). Labels not enclosed within quotation marks will always refer to the fields listed in these equations. Then, referring to the 10 and 16 of SO(10), our starting assumption for the assignments of the three \( SU(2) \) doublet light neutrinos reads:

\[
\begin{align*}
\text{“} \nu_\alpha \text{”} & \in L_\alpha \in 16, \quad \alpha = 1, 2 \\
\text{“} \nu_\tau \text{”} & \in H_3 \in 10.
\end{align*}
\tag{3.1}
\]

In order to realise this scenario we first have to require that the tree level masses for \( \nu_\alpha \) and \( N_3 \) vanish. This can be achieved by setting in \( W_4 \)

\[
\lambda^{(10)}_{i(1, 2)} (H^c_i L_\alpha \nu^c_j) = 0 \quad \text{and} \quad \lambda^{(11)}_{i(1, 2)} (H^c_i H_3 \nu^c_j) = 0.
\tag{3.2}
\]

For the sake of clarity we have enclosed inside \langle brackets\rangle the indices labeling the particular vacuum expectation values which are relevant for the actual discussion. From the LEP measurement of the number of weak-doublet neutrinos we know that all the remaining \( SU(2) \) doublet neutral states \( N_\alpha, \nu_3 \) and \( N^c_3 \) must be heavy ( \( \gtrsim 50 \text{ GeV} \)). This in turn implies that the following terms must be non-vanishing:

\[
\lambda^{(10)}_{i(1, 2)} (H^c_i L_3 \nu^c_\beta) \neq 0, \quad \lambda^{(11)}_{i(1, 2)} (H^c_i H_\alpha \nu^c_\beta) \neq 0.
\tag{3.3}
\]

Now, in order to allow for radiatively generated Dirac masses, we need massless R-handed neutrinos as well. For the sake of simplicity, we will require a minimum number of light neutral \( SU(2) \) singlets. In (3.3) we have already assumed that the couplings involving \( \nu^c_5 \) and \( S^c_5 \) are forbidden, thus preventing their fermionic component from acquiring a mass at tree level, so that at the lowest order three \( SU(2) \)-doublet and two \( SU(2) \)-singlet neutral states are massless, namely \( \nu_\alpha \) (\( \alpha = 1, 2 \)), \( N_3 \) and \( \nu^c_5, S^c_5 \). Dirac masses for these states can be induced by loops involving quarks, through the Yukawa couplings appearing in \( W_3 \) in the superpotential (2.8).
As is discussed in detail in Ref. [6] a set of one loop diagrams analogous to the one depicted in Fig. 1 will generate Dirac mass terms connecting the three L-handed neutrino with the singlet $\nu_c^3$, while at this order $S^c_3$ remains decoupled. However, additional diagrams are generated at the two loop level through diagrams similar to the one depicted in Fig. 2, and they induce additional mass terms for $S^c_3$ as well.

The final form of the mass matrix reads

$$\begin{pmatrix} \nu_1 & \nu_2 & N_3 \end{pmatrix} \cdot \mathcal{M} \cdot \begin{pmatrix} 0 \\ S^c_3 \\ \nu^c_3 \end{pmatrix},$$

$$\mathcal{M} = \begin{pmatrix} 0 & b_1 & a_1 \\ 0 & b_2 & a_2 \\ 0 & 0 & a_3 \end{pmatrix}$$

and is unique for the minimal scheme we have chosen. The entries $a_1$, $a_2$ and $a_3$ are generated at one loop, and with a reasonable choice of the parameters entering the diagram in Fig. 1 they can be estimated to fall naturally in the range $0.1 - 10$ eV. The entries $b_1$ and $b_2$ are generated at two loop, thus acquiring a typical suppression factor of order $10^{-3}$ with respect to the one loop masses. Once the mass matrix is diagonalized, a very interesting pattern of mixings and masses for the eigenstates
We end up with a massive neutrino $n_3$, mainly “$\nu_\tau$”, with a mass that can naturally fall in the range $\sim 0.1 - 10 \text{eV}$. The lower value is interesting for “$\nu_\mu$”-“$\nu_\tau$” atmospheric neutrino oscillation. On the other hand, due to the fact that $n_3$ is cosmologically stable, if its mass were close to the upper value, it could provide an interesting candidate for the hot component of the dark matter (DM). A second neutrino $n_2$, mainly “$\nu_\mu$”, acquires a much smaller mass ($\sim 10^{-3}$) at the two loop level, and can be relevant for matter enhanced “$\nu_e$”-“$\nu_\mu$” oscillations in the sun. Finally, due to the absence in our minimal scheme of a third helicity partner for the L-handed neutrinos, $n_1$ remains massless. We stress that such a hierarchy of masses (and a corresponding hierarchy of mixings) arises naturally as a direct consequence of the UA, and reflects the fact that being the neutrinos embedded in a different way in the gauge group, for one species a Dirac mass is generated at one-loop, while for the other two species the corresponding diagrams are forbidden, and only at the two loop level can a mass arise.

4. Conclusions.

In conclusion we have described the possibility of constructing consistent models in which the known neutrinos of the three different generations do not have the same gauge interactions under possible additional $U(1)'$ factors. We have carried out our analysis in the frame of the superstring-inspired $E_6$ models, taking as a guideline the requirement of having interesting neutrino phenomenology with naturally small radiatively generated Dirac masses. Models based on this scheme are indeed viable and can be realised by imposing a family-non-blind discrete symmetry on the superpotential. We have briefly described a minimal model, in which only two additional light $SU(2)$ singlet neutrinos are present, thus leaving one doublet neutrino massless. Clearly other models based on the same scenario but with a more rich structure in the neutrino sector can also be constructed. We have stressed that values of the neutrino masses in ranges interesting for explaining the solar and the atmospheric neutrino anomalies, or possibly for providing a hot DM component, can be obtained with a natural choice of the parameters. A hierarchy of masses and mixing is naturally generated as a consequence of the choice of UA assignments. We stress that, as it has been recently discussed, signals of models predicting UA could be detected at future $e^+e^-$ colliders by measuring ratios of cross sections for lepton productions. Also the correct pattern of embedding of the leptons into the gauge group could be identified through a large set of experimental observables including asymmetry measurements.
5. Acknowledgements

It is a pleasure to thank J.W.F Valle, A. Perez, D. Tommasini and F. Campos for inviting me at the International School on Cosmological Dark Matter and for making my stay in Valencia very pleasant.

6. References

1. M. Gell-Mann, P. Ramond, and R. Slansky, in 
Supergravity, F. van Nieuwenhuizen and D. Freedman eds., (North Holland, Amsterdam, 1979) p. 315; T. Yanagida, Proc. of the Workshop on Unified Theory and Baryon Number of the Universe, KEK, Japan, 1979.

2. For a review see J.L. Hewett and T.G. Rizzo, Phys. Rep. 183, 195 (1989).

3. B.A. Campbell et.al., Int. Jour. Mod. Phys. A2, 831 (1987).

4. A. Masiero et.al., Phys. Rev. Lett. 57, 663 (1986).

5. G.C. Branco and C.Q. Geng, Phys. Rev. Lett. 58, 969 (1986).

6. E. Nardi, Phys. Rev. D48, 3277 (1993).

7. E. Nardi, Report UM-TH-93-19, to appear on Phys. Rev. D.

8. E. Nardi and T.G. Rizzo, Report SLAC-PUB–6422 and UM-TH-93-29.

9. D. Gross, J. Harvey, E. Martinec and R. Rhom, Phys. Rev. Lett. 54, 502 (1985); Nucl. Phys. B256, 253 (1985); ibid. B 267 (1986) 75; P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258, 46 (1985); E. Witten, Phys. Lett. B149, 351 (1984).

10. E. Witten, Nucl. Phys. B258, 75 (1985).

11. L. Wolfenstein, Phys. Rev. D17, 2369 1978; D20, 2634 (1979); S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. 42, 1441 (1985); Nuo. Cim. 9C, 17 (1986).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9401275v1