Calculation of $f_B$ and the “Isgur-Wise Function” using a non-perturbatively improved fermion action

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We present a calculation of $f_B$ and of the form factors for the semi-leptonic decay $B \rightarrow D l \bar{\nu}$ in the quenched approximation to QCD. Results are generated on lattices at $\beta = 6.2$, using an $O(a)$-improved fermion action, with the clover coefficient determined non-perturbatively.

1. Introduction

A knowledge of the hadronic matrix elements for the decays of mesons containing heavy quarks is essential to the experimental determination of elements of the CKM matrix involving the heavy quarks. In this contribution, we concentrate on the hadronic matrix elements for the semi-leptonic decay $\bar{B} \rightarrow D l \bar{\nu}$, and also discuss $f_B$. The calculations of the $D \rightarrow K$ and $D \rightarrow \pi$ matrix elements are contained in the talk of Chris Maynard\cite{1}. An important element of this programme is the use of a “clover” fermion action with the clover coefficient, $c_{SW}$, determined non-perturbatively. Thus we expect a substantial reduction in discretisation errors compared to earlier calculations using the tree-level value of $c_{SW}$.

2. Simulation details

The calculation is performed in the quenched approximation on an ensemble of $216 \times 24^3 \times 48$ lattices at $\beta = 6.2$, generated using the Wilson gauge action. In the calculation of the semi-leptonic decay matrix element, we used four values of the final heavy-quark mass, corresponding to $\kappa_l = 0.1200, 0.1233, 0.1266$ and 0.1299, and two values of the initial heavy-quark mass, corresponding to $\kappa_h = 0.1200$ and 0.1266; the charm-quark mass corresponds roughly to $\kappa = 0.1233$. Further details of the calculation are contained in ref. \cite{2}.

In the non-perturbatively improved scheme, the axial vector and pseudoscalar currents mix:

$$A_R^{\mu} = Z_A (1 + b_A m_q) \times \left[ A_{latt}^{\mu} + c_A a \frac{1}{2} (\partial_{\mu}^{\ast} + \partial_{\mu}) P_{latt} \right]$$

where

$$m_q = \frac{1}{2} \left( \frac{1}{\kappa} - \frac{1}{\kappa_{crit}} \right)$$

We take the ALPHA Collaboration’s numerical values\cite{3} for the overall renormalisation factor, $Z_A$, and for $c_A$, and the one-loop calculation of $b_A$. Note that this form is modified for the case of a current constructed of non-degenerate quarks through the addition of a term linear in the difference of the quark masses, with coefficient $b'_A$; this coefficient not been computed.

The renormalisation of the vector current proceeds likewise:

$$V_R^{\mu} = Z_V (1 + b_V m_q) \times \left[ V_{latt}^{\mu} + c_V a \frac{1}{2} (\partial_{\nu}^{\ast} + \partial_{\nu}) \Sigma_{latt}^{\mu\nu} \right]$$

where $Z_V$, $b_V$ and $c_V$ are all known non-perturbatively.

3. Pseudoscalar decay constant

We obtain $f_P/f_\pi$ at each value of $\kappa_l$ and $\kappa_h$, where $P$ is the heavy-light pseudoscalar, and $f_\pi$ is the decay constant for a light meson constructed from degenerate quarks of mass corresponding to $\kappa_l$. We then extrapolate $\kappa_l$ to $\kappa_{crit}$ at fixed $\kappa_h$. Note that, whilst $Z_A$ cancels between the denominator and numerator, an effective relative renormalisation enters through $b_A$. 


Figure 1. The ratio $\phi(M_p)$ is shown at $\kappa_l = \kappa_{\text{crit}}$ for each value of $\kappa_h$. The solid line is a quadratic fit to the data, and the dashed line corresponds to the extrapolation to $M_B$.

To investigate the heavy-mass dependence, we construct the scaling quantity

$$\Phi \equiv \frac{\alpha_s(M_P)/\alpha_s(M_B)}{f_P/f_{\pi}} \sqrt{M_P},$$

(4)

where we take $N_f = 0$, and perform a quadratic fit in $1/M_P$, as shown in Figure 1. Setting $a^{-1} = 2.64 \text{ GeV}$, the value obtained from $m_\rho$, we find

$$f_D/f_{\pi} = 1.19 \pm \frac{3}{1},$$

$$f_B/f_{\pi} = 1.34 \pm \frac{4}{3},$$

(5)

where the quoted errors are purely statistical, and hence

$$f_D = 190 \pm \frac{5}{2} \text{ MeV},$$

$$f_B = 176 \pm \frac{5}{4} \text{ MeV}.$$  

(6)

4. Semi-leptonic decays and the Isgur-Wise function

The form factors for the semi-leptonic decay $\bar{\tau} \rightarrow P' l \nu$, where $P'$ is a heavy-light pseudoscalar meson, may be parametrised as

$$\frac{\langle P'(p')|\bar{Q}'\gamma_\mu Q|P(p)\rangle}{\sqrt{M_P M_{P'}}} = (v + v') h^+(\omega; m_Q, m_{Q'}) + (v - v') h^-(\omega; m_Q, m_{Q'}).$$

(7)

Here $v'$ is the four velocity of the initial (final) meson, and $\omega = v \cdot v'$. In the Heavy Quark Effective Theory (HQET), the form factors display an additional spin-flavour symmetry, and are related to a universal “Isgur-Wise” function $\xi(\omega)$:

$$h^i(\omega; m_Q, m_{Q'}) = \xi(\omega) \times$$

$$(\alpha^i + \beta^i(\omega; m_Q, m_{Q'}) + \gamma^i(\omega; m_Q, m_{Q'}))$$

(8)

where $\alpha^+ = 1$, $\alpha^- = 0$, and $\beta^i$ and $\gamma^i$ represent the radiative and power corrections respectively. Note that $\xi$ is normalised: $\xi(\omega = 1) = 1$.

For the case of degenerate transitions at zero momentum transfer, we can obtain a direct measurement of the effective renormalisation constant $Z_V^{\text{eff}} = Z_V(1 + b_V a m_Q)$.

Figure 2. $Z_V^{\text{eff}}$ is shown for two values of the light-quark mass and for two values of the heavy-quark mass. The dashed lines represent the predictions using the non-perturbative values of eqn. 3.
Figure 3. $h^+(\omega)$ is shown for fixed light-quark and initial heavy-quark masses.

and compare with the value using the non-

perturbative parameters of eqn. 3, as shown in

Figure 2. The agreement is striking, and perhaps

surprising since the NP prescription only removes

$O(a m_Q)$ errors.

In Figure 3, we show the form factor $h^+(\omega)$ for

fixed value of the light-quark mass, close to the

strange, for fixed initial heavy-quark mass, and

for four values of the final heavy-quark mass; as

expected, $h^-$ is very small. We compute $\beta^+$ using

Neubert’s short-distance expansion of the currents 3, and “define” the Isgur-Wise function through

$$\xi(\omega) = \frac{h^+(\omega)}{1 + \beta^+(\omega)}.$$  \hspace{1cm} (10)

The corrected form factor is shown in Figure 4, together with a one-parameter fit for $\omega < 1.2$
to

$$\xi(\omega) = \frac{2}{1 + \omega} \exp(-(2 \rho^2 - 1)(\omega - 1)/(\omega + 1)).$$

The use of the NP prescription has enabled a

far more satisfactory treatment of discretisation

errors than the calculation using the tree-level-

improved SW action 3. It remains to obtain a

quantitative estimate of the power corrections,

and a determination of the remaining form factors.

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