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A two-dimensional electromagnetic vibration energy harvester with variable stiffness

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1. Introduction

Harvesting energy to power sensors for both wearable and Internet-of-Things (IoT) devices is a topic with a steep increasing interest [1–3]. Such energy harvesters can replace batteries in sensor devices and thereby save both the time and cost associated with replacing the batteries when these wear out, at the same time reducing the environmental issues with using batteries in the first place [4].

Energy can be harvested from the environment through a number of different sources. Here vibration energy harvesting is considered since it has the benefit of working both indoors and outdoors and in both cold and warm environments. It is also ideally suited for wearable devices, where the vibrations from human motion can directly be harvested [5]. Harvesting vibrational energy can be done through three approaches, namely piezoelectric, electrostatic and electromagnetic techniques. This study focuses on the latter type of harvester, as there is great flexibility in the design of such devices [6].

Electromagnetic vibration energy harvesters (EMVEH) are typically designed to fit their exact application, i.e. the harvester is tuned to resonate at a very specific frequency already in the design phase. This limits the power that can be harvested to a narrow range around exactly this resonant frequency [7,8]. A conventional EMVEH consists of three co-axial magnets arranged with opposing polarities, such that the middle magnet is floating and free to move relative to a set of coils [9–12] although other types of designs exists [13–16]. The power produced by a centimeter-sized device is typically in the μW-mW range [17]. EMVEHs are usually designed to vibrate only along one direction [17] with the inherent limitations of device design and functionality that this results in [18].

To overcome these limitations EMVEHs that can vibrate in multiple directions have been considered. Recently, a novel elliptically shaped EMVEH for two-dimensional motion was studied, which allowed for a wider bandwidth than traditional 1D harvesters [19]. This 2D-EMVEH consisted of a free-to-move ring-shaped permanent magnet with radial magnetization and ball-bearings on the top and bottom, a fixed system of square magnets placed inside an elliptically shaped holder, and coils located above and below the harvester. Unlike 1-D harvesters, the resulting output power showed two resonant frequencies at which the harvester generated around 1.5 mW. However, this device did not have any variable properties and no investigation was done on how to optimize power production, but the device merely experimentally showed that 2D motion can be harvested. A somewhat similar structure as discussed above, i.e. where the moving part is the middle magnet and also with coils placed on the outside of the design has been considered previously [20] but here the configuration was used only to decrease friction and not for scavenging energy from two-dimensional vibrations. Likewise, Ref. [21] considered a spherical permanent magnet...
inside a capsule which is wound with two coils. Due to the lack of a repulsive force, this prototype needs stoppers on the inner walls of the capsule so that the spherical magnet does not break. Finally, Ref. [22] considered how an eccentric mass brings about an oscillatory motion which, under certain conditions, results in a rotary 2D motion.

The above discussion shows that there is a clear knowledge gap in literature on the dynamics of two-dimensional EMVEHs, especially concerning the optimal harvester design. The influence of the critical system properties, such as magnetic stiffness and electromagnetic damping, on the behavior and output power of the harvesters is not known and existing designs in literature have only explored fixed designs with no variation of the system properties [19–22]. Therefore an investigation on the optimal harvester design with respect to the properties of the system that can be controlled after the design phase of the harvester is clearly warranted. The shape of the harvester cannot be altered once the harvester is produced, but the magnetic stiffness and the electromagnetic damping can. Here we investigate a multi-purpose device where the magnetic stiffness in the device can be altered at will. The hypothesis is that instead of modifying the geometry of the device to adapt it to the vibrational source, the magnetic stiffness in the device can instead be altered to change the resonant frequencies. This will be much more practical for applications as a single device can be used for multiple applications. In the design proposed here, changing the magnetic stiffness is simply done by adding or removing magnets in the outer confining structure of the harvester, thereby changing the magnetic stiffness independently in different directions, and through this obtaining a potentially broadband harvester. We quantify the behavior of such device for a total of nine different symmetric and asymmetric configurations of magnets to understand how to optimize a prototype to a given vibration source.

2. Device

In order to realize a harvester where the magnetic stiffness can be easily controlled, this needs to be highly modular. Hereby, a two-dimensional ring-shaped electromagnetic vibration energy harvester that consists of the three parts depicted in Fig. 1 is studied. The first part is a free-to-move three-cylinder magnetic structure with bearings on top and bottom. The second part encapsulates the moving structure, and consists of a ring holder with 24 holes placed in a circle on the top and bottom faces symmetrically, i.e. with 48 holes in total, where one or two disk-shaped magnets can be placed in each hole. The third part is a pair of coils placed on the top and bottom of the harvester. The dimensions and component specifications of the individual parts are listed in Table 1. A photo of the actual device is shown in Fig. 3.

The disk-shaped magnets placed in the holes of the ring holder are tilted 45°, so that they create a magnetic flux line distribution as illustrated in Fig. 2. This brings about a spring-like restoring force that pushes back the moving structure when it is affected by an external vibration source, resulting in a variable magnetic flux seen by the fixed coils, inducing an EMF in these. The magnetic stiffness of the device can be changed by changing the number of disk-shaped magnets in each hole, which can contain one or two disk-shaped magnets. The first magnet in each hole is glued in place. Inserting a second magnet is easy, as the magnetic force pulls it inwards. Removing it can also be done fairly easily as the slated holes allow for a prying tool to be inserted between the magnets in each hole, so that they can be separated and one of them removed. When two magnets are in a hole, they simply act as a twice-as-long magnet compared to a single magnet.

To ensure that the free-to-move structure of three cylindrical magnets does not flip over in accordance with Earnshaw’s theorem, two plates are placed just below and above this to confine the movement plane to two dimensions. Additionally, polymer ball transfer bearings (BB-S05-B180-POM from Igus) are placed on the free-to-move cylindrical structure to help decrease any friction between the moving structure and the plates.

Two 300-turn coils are glued to the top and bottom plates of the harvester, and their inner diameter is similar to the outer diameter of the free-to-move structure of the cylindrical magnets as can be seen in Fig. 3. The two coils are connected in series to get the sum of the two EMF signals, and the connected load matches with the internal resistance of the coils to attain maximum transfer power [23]. The low impedance due to the internal inductance of the coils is neglected. This

| Component Parameter | Quantity |
|---------------------|----------|
| Amount              | Varying  |
| Diameter            | 10 mm    |
| Height              | 5 mm     |
| Grade               | NdFeB, N52 |
| Amount              | 3        |
| Diameter            | 10 mm    |
| Height              | 20 mm    |
| Grade               | NdFeB, N45 |
| Inner diameter      | 27 mm    |
| Outer diameter      | 40 mm    |
| Height              | 2.5 mm   |
| Number of turns     | 300      |
| Wire gauge          | 0.202 mm |
| Resistance          | 16.5 Ω   |
| Inductance          | 3.77 mH  |
| Inner diameter      | 105 mm   |
| Outer diameter      | 154 mm   |
| Height              | 30 mm    |
is justified as e.g. at 10 Hz the impedance is 0.24 \Omega which is much lower than the internal resistance of the coils.

2.1. Physics of the 2D device

The physics and equation of motion of the 2D harvester is identical to that of the well-known 1D electromagnetic harvesters except that the motion is in two dimensions. A 1D electromagnetic harvester has the following equations of motion [8]:

\[
\begin{align*}
\sum_{n'} L_{n,n'} \dot{I}_{n'} + & R_a I_a + \frac{d\phi_n}{dz_a} = 0 \\
L_a I_a + & \sum_{n'} L_{n,a'} I_{a'} + R_a I_a \frac{d\phi_n}{dz_a} = 0 
\end{align*}
\]

where \( z \) is the position variable, \( m_{\text{free}} \) is the mass of the free-to-move magnet, \( F_{\text{coil}} \) is the force due to the interaction of the free-to-move magnet with the coil windings [12,24], \( F_{\text{mag}} \) is the force due to the interaction of the free-to-move magnet with the fixed magnets, \( F_r \) is the gravitational force and \( F_{\text{frict}} = -c z \) is a friction force. The second equation, which determines the electrical condition of a coil winding is stated for the \( n \)th coil winding but an equation exists for every independent coil winding. For the \( n \)th coil winding a current \( I_a \) is running, and where \( L_a \) is the self-inductance of each coil, \( L_{a,a'} \) is the mutual inductance between the \( n_{a'\text{th}} \) and \( n_{a\text{th}} \) coils, \( R_a \) is the resistance connected to each coil winding and \( \phi_n \) is the magnetic flux density

through the coil winding. If the coil windings are connected in series, the system of equations is simplified as only one current and one resistance is present.

For the harvester considered in this work, the free-to-move structure can move in two dimensions, i.e. in both the \( x \) and \( y \) directions. Thus the magnetic force depends on both the \( x \) and \( y \) coordinates and the coil winding force depend on the \( x \) and \( y \) coordinates as well as the velocities \( v_x \) and \( v_y \). For the flux through a coil winding the position variable is the planar distance between the free-to-move structure and the coil winding, \( r \). This planar distance is given as:

\[
r = \sqrt{x^2_{\text{rel}} + y^2_{\text{rel}}}
\]

Specifically, it shows the "F"-case described in Table 3.

The equations of motion for the 2D EMVEH is:

\[
m_{\text{free}} \ddot{x} = F_{\text{coil},x}(x,y,I_a) + F_{\text{mag},x}(x) + F_{r,x} + F_{\text{frict},x} \quad (8) \\
m_{\text{free}} \ddot{y} = F_{\text{coil},y}(x,y,I_a) + F_{\text{mag},y}(x,y) + F_{r,y} + F_{\text{frict},y} \quad (9)
\]

and

\[
L_a I_a + \sum_{n'} L_{n,a'} I_{a'} + R_a I_a + v_{\parallel,\text{dir}} \parallel \sum_{n} \frac{d\phi_n}{dz_a} = 0 \quad (10)
\]

Again there is a differential equation for each independent coil winding but only the \( x \) equation is given. Here \( v_{\parallel,\text{dir}} \) is the velocity parallel to the planar position vector \( r \) connecting the free-to-move structure and the coil winding. The direction \( v_{\parallel,\text{dir}} \) indicates if the free-to-move structure is moving towards or away from the coil winding.

The parallel component of the velocity in the \( x \)- and \( y \)-directions are thus

\[
\begin{pmatrix}
\dot{v}_{\parallel,x} \\
\dot{v}_{\parallel,y}
\end{pmatrix} = \frac{v_{x,\text{rel}} + v_{y,\text{rel}}}{\sqrt{x^2_{\text{rel}} + y^2_{\text{rel}}}} \begin{pmatrix} x_{\text{rel}} \\ y_{\text{rel}} \end{pmatrix}
\]

And the direction of the velocity of the free-to-move structure towards or away from the coil winding as given by \( v_{\parallel,\text{dir}} \) is

\[
v_{\parallel,\text{dir}} = \frac{v_{x,\text{rel}} x_{\text{rel}} + v_{y,\text{rel}} y_{\text{rel}}}{\sqrt{x^2_{\text{rel}} + y^2_{\text{rel}}}} = \text{sign} \left( v_{x,\text{rel}} x_{\text{rel}} + v_{y,\text{rel}} y_{\text{rel}} \right)
\]

This will be equal to \( +1 \) when the magnet is moving towards the coil winding and \( -1 \) when it is moving away from the coil winding.

The magnetic force can be found using e.g. a finite element framework for a specific device geometry. The coil winding force must consider the relative distance as mentioned above, i.e. the coil winding force on the free-to-move structure from the \( n \)th coil winding is given by

\[
F_{\text{coil},n}(x,y,I_n) = I_n \frac{d\phi_n}{dz_n} \frac{x_{\text{rel},n}}{\sqrt{x^2_{\text{rel},n} + y^2_{\text{rel},n}}}
\]
\[
F_{\text{out}}(x,y) = I_a \frac{dy}{dx} - \frac{V_{\text{rel}}}{\sqrt{x_{\text{rel}}^2 + y_{\text{rel}}^2}}
\]
and the total force is just the sum from all coil windings.

3. Experimental setup

To examine how the 2D EMVEH behaves for different magnetic stiffnesses, i.e., different amount of disk-shaped magnets in the ring holder, the device must be experimentally characterized. This was done by using the experimental setup depicted in Fig. 3, where a two-dimensional shaker (SRS-004-006-004-003-XY from H2W Technologies) was employed to generate the vibration source. This shaker has two platforms that move perpendicularly to each other, giving as a result a two-dimensional motion that follows Lissajous patterns, i.e. where the movement in the two directions are independently given by a sinusoidal curve. The induced EMF signals of the coils were measured by a USB-6351 data acquisition card from National Instruments with a sample frequency of 1 kHz.

The number of disk magnets in the ring holder determines the strength of the spring-like restoring force, i.e. the magnetic stiffness of the system. To examine and understand the movement and power production of the device with different magnetic stiffness, a set of nine different magnet distributions were tested. These are described in detail in the following. In order to ensure that there is a minimum magnetic stiffness (and therefore restoring force) in the device, each hole in the ring holder has a minimum of one disk-shaped magnet inserted at all times. The top and bottom configurations of disk-shaped magnet in the ring holder are always filled identically.

In all experiments the displacement amplitude of each platform of the XY-shaker was 2 mm, the bottom platform of the XY shaker is aligned with the y-axis of Fig. 3 and its motion has a constant frequency of 4 Hz. The top platform is aligned with the x-axis and its motion frequency was varied from 1 Hz to 10 Hz in steps of 0.25 Hz, and in steps 0.05 Hz around the resonance frequency, starting from rest for each frequency. These frequencies are chosen because the dominant step frequency of human walking falls at around 2–3 Hz [5]. However, due to the inherently inverse relationship between resonant frequency and harvester size, obtaining a resonant frequency in the 2–3 Hz range would require an obtrusively large harvester, excluding it from wearable electronics applications. For this reason, wearable energy harvesters are designed to have a resonant frequency matching one of the higher harmonics of the dominant step frequencies, around 6–8 Hz [5]. Energy from human motion has previously been harvested by electromagnetic energy harvesters [10,25–31]. However, this just is one of many potential applications, including vibrating structures, animal movement or transport applications. Especially applications where the moving object to which the harvester is attached has a potential two-dimensional motion would be beneficial. This could for example be around the collar of an animal, where the movement is both up and down due to the animal gait and left right due to head movement sideways.

4. Results

In each experiment the combined voltage in the coils was recorded for a period of 3 s after the system reached steady state. Fig. 4 shows the output voltage as function of time for an experiment with frequencies of 7.7 Hz and 4 Hz on the x- and y-axes respectively and amplitude of 2 mm on both axes.

4.1. Symmetric magnetic stiffness

First, increasing the magnetic stiffness symmetrically is explored, i.e. in such a way that the ring-shaped holder is as magnetically symmetric as possible in the xy-plane. Hence, four different arrangements of magnets in the ring holder are considered, as illustrated in Table 2. The magnetic stiffness is increased by placing more disk-shaped magnets in the ring holder successively. The power as measured experimentally for the different configurations are shown in Fig. 5. It is immediately apparent that for case "a" there is a low output power, which is because the amount of magnets is not enough to create a considerable restoring force which can manage to push back the moving structure. As a consequence, the moving structure has almost no relative motion, hence no voltage is induced in the coils. As the number of magnets in the ring holder is increased successively in cases "b", "c" and "d", the magnetic stiffness is increased and thus the moving structure experiences enough restoring force to push it back and forward inducing an EMF in the coils and hence producing output power. It is also clear that the higher the restoring force is, the larger the resonance frequency will be, as could also be expected from the physics of simple harmonic motion. The peak frequency is at 7.3 Hz, 7.7 Hz and 8.2 Hz for the cases "b", "c" and "d", respectively. Normalizing to the "b" peak frequency this gives a ratio of peak frequencies of 1.0:1.055:1.123.

To estimate the magnetic stiffness that each configuration of magnets in Table 2 gives rise to, we use a magnetostatic model in the finite element software Comsol, and calculate the magnetic force in the three cases "b", "c" and "d" on the cylindrical magnets in the moving structure as this is brought closer to the ring holder. The force is calculated from Maxwell’s stress tensor. This restoring force as function of displacement, \( r \), was then fitted with Hooks law, \( F_m = -kr \), in the central part of the system where the force is linear [8]. The determined values for the spring constant, i.e. the magnetic stiffness, \( k \), are 0.16 N/mm, 0.18 N/mm and 0.22 N/mm. The restoring force is shown in Fig. 6 as well as the linear fits. For a simple harmonic oscillator, the frequency scales as \( \sqrt{k} \), and thus the frequency ratios, if the system had been a simple harmonic oscillator, would be 1.0:1.061:1.173, which are close to the observed ratios. The discrepancy is caused by the fact that the experimental system is a damped driven nonlinear oscillator.
4.2. Two-pole asymmetric magnetic stiffness

Having explored a symmetric increase in magnetic stiffness, the dynamics of the device in asymmetric cases is explored. The disk-shaped magnets are placed such that the magnetic restoring force becomes asymmetric in either the $x$- or $y$-direction, as described in Table 3. For example, for case “e” the magnetic stiffness in one direction will be identical to that of case “a”, while it will be identical to that of case “d” in the orthogonal direction. The produced power as function of frequency is shown in Fig. 7. It is seen that by adding disk-shaped magnets asymmetrically as described in the “e”-case, the harvester reaches a maximum output power twice as high as that in the previous symmetric case. This is because the resulting magnetic stiffness comes from the concentrated magnetic flux in the $x$-direction, bringing about a stronger magnet–magnet interaction hence more generated power. However, this implies a higher resonance frequency as seen in Fig. 7. On the contrary, the “f”-case produces almost no power due to the lack of restoring force in the direction where the frequency sweep is performed. Finally the “g”-case shows that by only concentrating the magnetic flux in one side, the resulting power and resonance frequency are comparable to the ones attained in the symmetric magnetic stiffness analysis.

4.3. Three-pole asymmetric magnetic stiffness

Having seen that a two-pole asymmetric device can produce a substantial increase in power production, a further increase in pole number could result in an even higher power production. Therefore, the disk-shaped magnets in the ring holder are placed in a triangular pole distributions as detailed in Table 4. The results from the experimental characterization of the device are shown in Fig. 8. It can be seen that increasing the magnetic stiffness, i.e. going from the “h”-case to the “i”-case, slightly increases the resonance frequency as was also seen for the symmetric cases. However, here the output power also increases noticeably, which was not the case for the symmetric devices. Overall the produced power is lower than for the two-pole case and is comparable to the symmetric cases. This is most likely because these devices have a higher order symmetry (three-pole) than the vibrational directions (two directions), which causes them to behave as if they were fully symmetric devices.
4.4. Electromagnetic damping

Besides the magnetic stiffness in the harvester, the electromagnetic damping provided by the coils will also affect the movement of the free magnet and therefore the power output of the device. To investigate this effect, the “e”-case harvester is taken which was able to scavenge the highest power compared to the other configurations. For this system two different coil windings are employed, with different dimensions and different number of turns. Table 5 shows the characteristics of two new coils. Both use the original “e”-case coils as reference, see Table 1, but one coil has an increased diameter, whereas the other has an increased thickness. For both coils the number of turns is increased to 500 turns to accommodate the larger dimensions. This also increases the total wire length and increases the internal electrical resistance to 60 Ω and 68 Ω respectively. The load resistance in the experiments was changed according, i.e. to match the coil internal resistance. Performing the same vibrational experiments as described previously, i.e. a frequency sweep in one direction while maintaining a 4 Hz vibration in the other direction, the power produced by the harvester with the different coils mounted was measured. The results are shown in Fig. 9 where it can be seen that the different coils modify the state of the system, attaining a different resonance frequency as by modifying the magnetic stiffness. Moreover, it is seen that the wider coil clearly produces more power compared to the initial and thicker coils. The reason for this is that the thicker coil brings about a larger air-gap between the moving magnet and the coil-loops at one end of the coil. Consequently, lower voltage is induced on those coil-loops, and its higher internal resistance limits the electrical current, given as a result a lower output power compared with the other two coil windings.

4.5. Two dimensional frequency sweep

Until now, all experiments have been performed with a varying frequency in one direction and a 4 Hz vibration in the other direction. To explore the influence of the previously fixed 4 Hz frequency, the best performing device, i.e. “e”-case with a wider coil, was experimentally characterized with a frequency sweep study in both directions. The experimental procedure remains the same as previously described, i.e. for each pair of frequencies the system is started from rest and the steady state properties of the system is measured. The results are shown in Fig. 10 where it can be seen that the harvester has a fairly broad band with a width of around 6 Hz in the y-axis frequency centered on 3 Hz.

---

**Table 5**

| Coils   | Parameter       | Quantity   |
|---------|-----------------|------------|
| Wider   | Inner diameter  | 27 mm      |
|         | Outer diameter  | 51 mm      |
|         | Height          | 2.5 mm     |
|         | Number of turns | 500        |
|         | Wire gauge      | 0.202 mm   |
|         | Resistance      | 60 Ω       |
|         | Inductance      | 11.5 mH    |
| Thicker | Inner diameter  | 27 mm      |
|         | Outer diameter  | 40 mm      |
|         | Height          | 5 mm       |
|         | Number of turns | 500        |
|         | Wire gauge      | 0.202 mm   |
|         | Resistance      | 68 Ω       |
|         | Inductance      | 10.3 mH    |

---

Fig. 7. The power as function of frequency in the x-direction for the device configurations given in Table 3. The y-direction has a fixed frequency of 4 Hz.

Fig. 8. The power as function of frequency in the x-direction for the device configurations given in Table 4. The y-direction has a fixed frequency of 4 Hz.

Fig. 9. Power as function of frequency for configuration “e” with three different coils.

Fig. 10. Power as function of frequency in two directions for configuration “e” with two different coils.
increases to twice as large a value as the symmetric configurations. In harvester resonance frequency increases slightly but the output power of magnets on one motion axis, making the harvester asymmetric, the frequency of the harvester. Furthermore, by concentrating the number structure, the stronger restoring force and the higher the resonance frequency reached. The 2D shaker used in this work did not allow for this frequency, a different energy state with a different power might be through a different frequency history to reach the operated frequency, frequency tested is started from rest. If the harvester had been taken 36. For the harvester characterized in this work, the harvester for each system has multiple stable solutions in the device phase space [33–18,32,33]. This hysteresis behavior is a consequence of the fact that this work has a broader frequency span in the y-axis, which is likely due to the low magnetic stiffness in this direction for the particular harvester investigated here, i.e. the “e”-case.

4.6. Discussion

The above experiments have revealed a number of facts that is of interest when designing future harvesters. The most interesting of these is that breaking the symmetry of the restoring force internally in the harvester means that the free-to-move structure can move in a way that generates a factor of two more power for the size of device considered here. Furthermore, not only is the power increased, the frequency range at which the harvester generates useful power is also significantly increased.

One thing that has not been explored in this work is the hysteresis behavior, which electromagnetic harvesters are known to display [9, 18,32,33]. This hysteresis behavior is a consequence of the fact that the systems has multiple stable solutions in the device phase space [33–36]. For the harvester characterized in this work, the harvester for each frequency tested is started from rest. If the harvester had been taken through a different frequency history to reach the operated frequency, e.g. by reaching the operating frequency by decreasing from a higher frequency, a different energy state with a different power might be reached. The 2D shaker used in this work did not allow for this experiment, so this should be explored in a future study.

5. Conclusion

In this work a novel two-dimensional electromagnetic vibration energy harvester has been introduced and experimentally characterized by subjecting it to two-dimensional vibrations. The magnetic stiffness in the harvester has been varied by distributing the permanent magnets in the ring structure of the harvester in nine different configurations. Additionally, the electromagnetic damping created by the magnet-coil interaction has been compared for three different coil windings. The experimental results showed that the more magnets in the ring structure, the stronger restoring force and the higher the resonance frequency of the harvester. Furthermore, by concentrating the number of magnets on one motion axis, making the harvester asymmetric, theharvester resonance frequency increases slightly but the output power increases to twice as large a value as the symmetric configurations. In addition, it was shown that choosing the proper coil dimensions can lead to a larger power, but that this will also slightly increase resonance frequency of the harvester. The largest power output of the harvester was 42 mW, representing a good alternative as energy supply for the booming market of low-power devices.

CRediT authorship contribution statement

Carlos Imbaquingo: Writing – original draft, Software, Experimental work. Christian Bahl: Conceptualization, Writing. Andrea R. Insinga: Conceptualization, Writing. Rasmus Bjork: Conceptualization, Data curation, Software, Writing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data presented in this work is directly available from Ref. [37].

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