FINE-GRAINED PRIVATE KNOWLEDGE DISTILLATION

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ABSTRACT

Knowledge distillation has emerged as a scalable and effective way for privacy-preserving machine learning. One remaining drawback is that it consumes privacy in a client-level manner. In order to attain fine-grained privacy accountant and improve utility, this work proposes a model-free reverse \(k\)-NN labeling method towards record-level private knowledge distillation, where each private record is employed for labeling at most \(k\) queries. Theoretically, we provide bounds of labeling error rate under the centralized/local model of differential privacy. Experimentally, we demonstrate that it achieves new state-of-the-art accuracy in MNIST/SVHN/CIFAR-10 dataset with one order of magnitude lower of privacy loss.

Index Terms— Differential Privacy, Federated Learning, Knowledge Distillation

1. INTRODUCTION

Federated learning benefits from data across multiple individuals or organizations. However, data privacy has been a critical issue during collaboration, especially under increasingly rigid privacy laws, such as Data Security Law of the PRC and General Data Protection Regulation in the Europe Union. In contrast to transmitting raw data among clients, the seminal work of federated learning [1] proposes to share gradients. Subsequent works [2, 3] further impose rigorous protections (e.g., differential privacy [4]) on the gradients.

Since iteratively transmitting gradients is inefficient, researchers [5, 6] begin to employ the paradigm of knowledge distillation [7]. Federated clients are asked to label public-available data with locally-trained models (see the top of Figure 1), meanwhile preserving the privacy of clients’ local records. Because labels have much lower dimensionality than gradients, federated knowledge distillation has become a communication & privacy efficient and thus prevalent way to federated deep learning [8, 9]. One line of these studies inject Laplace/Gaussian random noise for preserving centralized differential privacy on aggregated labels from teacher models [6, 10]; another line of studies sanitize the pseudo label predicted by teacher model locally [11, 8] to enable local differential privacy.

Despite many advantages over sharing gradients, the current knowledge distillation paradigm is still suffering from a critical drawback on privacy accountant. Instead of accounting privacy loss at the record level as the gradient-sharing paradigm, the current knowledge distillation paradigm summarizes local records to a privacy-sensitive local model. As a result, it inevitably reckons in each private record’s contribution to the answers (i.e., client-level privacy).

In contrast, we propose the reverse \(k\)-NN labeling method\(^1\) to limit the single record’s maximum impact on the answers to multiple queries (i.e., record-level privacy). Besides, instead of relying on a locally-trained model that needs hundreds of training records, we utilize the unsupervised learning to generate public representation space where the private records are connected with public queries. Accordingly, our method is naturally immune to data Non-I.I.D. settings. As demonstrated in Figure 1, every private record is associated with \(k\)-nearest neighboring query samples, hence the privacy loss scales with only \(k\) instead of the number of total queries. The contributions of this paper are as follow:

(a) We initialize the study of federated knowledge distillation with record-level privacy preservation, and propose the model-free reverse \(k\)-NN query labeling method. (b) We for-

\(^1\)Source Code: https://github.com/liyuntong9/rknn
mulate the reverse k-NN labeling as Bucketized Sparse Vector Summation problem, and provide concrete mechanisms & theoretical guarantees for the problem under centralized/local differential privacy. (c) Through extensive experiments in centralized/local privacy setting, our method achieves a significant accuracy boost with one magnitude lower of privacy consumption when compared to existing approaches.

2. PROPOSED APPROACH

2.1. Background

Federated knowledge distillation. Every data record \((x, y)\) is sampled from a Cartesian domain \(X \times Y\), where the sample \(x\) might be a tabular image, an image, and etc. The class label \(y\) can be a binary value (i.e. \(Y = \{0, 1\}\)) or categorical value (e.g., \(Y = \{0, 1, \ldots, 9\}\)). The label \(y\) is often represented as vector form for the convenience of calculation. Assume that the client \(i\) possesses \(m_i\) records, let \(D^i = \{(x^i_1, y^i_1), \ldots, (x^i_{m_i}, y^i_{m_i})\}\) denote these records, let \(D_{\text{priv}} = \bigcup_{i=1}^{n} D^i\) denote the union of all local datasets, and let \(D_{\text{pub}} = \{(x_1,?), \ldots, (x_{m_{\text{pub}}},?)\}\) denote the public unla- beled dataset possessed by the federated server. The primary goal of federated knowledge distillation is then labeling \(D_{\text{pub}}\) with the knowledge from \(D_{\text{priv}}\).

Centralized differential privacy. Let \(Z\) denote the output domain. A randomized mechanism \(K\) satisfies \((\epsilon, \delta)\)-differential privacy [4] (DP) if for any neighboring datasets \(D, D’\) and any outputs \(z \subseteq Z\) it holds that \(P[K(D) \in z] \leq \exp(\epsilon) \cdot P[K(D’) \in z] + \delta\).

Local differential privacy. Assume that each client holds a dataset \(D\) with only one record. A randomized mechanism \(K\) satisfies local \(\epsilon\)-differential privacy [12] (local DP) if for any data pair \(D, D’\) \(\in X \times Y\) and any output \(z \in Z\) it holds that \(P[K(D) = z] \leq \exp(\epsilon) \cdot P[K(D’) = z]\).

2.2. Reverse k-NN Labeling

In this subsection, we provide the detailed description of reverse k-NN labeling in federated learning. The whole algorithm is illustrated in Algorithm 1.

Learning to represent: Since raw pixels are unstable w.r.t. semantic labels, we measure sample distance by their latent representations. One can use pre-trained representation models or train an unsupervised representation model from scratch with public-available \(D_{\text{pub}}\).

Selecting queries: Labeling all samples in \(D_{\text{pub}}\) is privately expensive. Following current approaches [6, 10], we select representative samples from \(D_{\text{pub}}\). At the first iteration, we cluster \(D_{\text{pub}}\) into \(s\) groups in the representation space, and treat cluster centers \(Q = \{q_1, q_2, \ldots, q_s\}\) as query samples. For later iterations, samples are selected adaptively w.r.t. uncertainty of the current model \(M_s\).

Local labeling: Given queries \(Q = \{q_1, q_2, \ldots, q_s\}\) and public representation model, every local record \((x^i_j, y^i_j)\) is connected to \(k\)-nearest query samples in the representation space. Let \(N^i_j \subseteq [1:s]\) denote the set of query indices that are \(k\)-nearest neighbors of \(x^i_j\). Then the labeling answer from client \(i\) is \(A^i = \{a^i_1, a^i_2, \ldots, a^i_s\}\), where \(a^i_j = \sum_{l=1}^{m_i} \mathbb{1}[l \in N^i_j] y^l_j \in \mathbb{R}^{|Y|}\) for each \(l \in [1:s]\).

Algorithm 1 Fine-grained Private Knowledge Distillation

**Input:** \(n\) clients, private datasets \(D^n_1, \ldots, D^n_n\), unlabeled public dataset \(D_{\text{pub}}\).

**Parameter:** number of iterations \(T\), number of nearest neighbors \(k\), number of query samples \(s\), privacy budget \(\epsilon\).

**Output:** student model \(M_s\) that satisfies \(\epsilon\)-differential privacy

1. for \(t=1, \ldots, T\) do
   2. select \(s\) query samples \(D_{\text{query}}\) from \(D_{\text{pub}}\)
   3. // Client side
   4. for \(i=1, \ldots, n\) do
   5.    connect each local record \((x^i_j, y^i_j) \in D^i\) to \(k\)-nearest queries in \(D_{\text{query}}\)
   6.    find \(k\)-nearest neighbors \(N^i_j \subseteq [1:s]\) of \(x^i_j\)
   7.    represent the labeling answer on each query \(l \in [1:s]\) as \(a^i_j = \sum_{l \in [1:s]} \mathbb{1}[l \in N^i_j] y^l_j \in \mathbb{R}^{|Y|}\)
   8.    send all labeling answers \(A^i = \{a^i_1, a^i_2, \ldots, a^i_s\}\)
   9. end for
10. // Sever side
11. aggregate the label counts \(a_i = \sum_{i=1}^{n} a^i_j \in \mathbb{R}^{|Y|}\) from all clients for each \(l \in [1:s]\)
12. ensemble \(A = \{a_1, a_2, \ldots, a_s\}\) and add noise to \(A\) by \(\epsilon\)-differential privacy
13. derive labels \(\{\hat{y}_i\}_{i=1}^{s}\) from noisy label counts \(A\)
14. train student model \(M_s\) on \(D_{\text{query}}\) with labels \(\{\hat{y}_i\}_{i=1}^{s}\)
15. end for
16. return \(M_s\)
2.3. Centralized Private Mechanisms

In this subsection, we reformulate the reverse k-NN labeling as Bucketized Sparse Vector Summation (BSVS). Then we present centralized DP mechanisms and provide corresponding labeling error bounds.

Definition 1 (Bucketized Sparse Vector Summation). In the BSVS problem, each datum corresponds to a set $T_j \subseteq T$ of $k$ buckets and a sparse vector $y_j \in \{0, 1\}^{|Y|}$ and $|y_j| = r$. The goal is to determine, for a given bucket $t \in T$, the vector sum of $t$, which is $a_t := \sum_{j=1}^{n} m_j y_j[t \in T_j]$. An approximate oracle $\tilde{a}$ is said to be $(\eta, \beta)$-accurate at bucket $t$ if we have $|a_t - \tilde{a}_t|_\infty < \eta$ with probability $1 - \beta$.

In the above reformulation, the number of all buckets is equal to the number of query samples: $|T| = s$. Note that in conventional multi-class classification, we have $r \equiv 1$.

When centralized $(\epsilon, 0)$-DP is imposed on the BSVS problem, we employ the classical Laplace mechanism for privacy preservation. Apparently, the maximum possible changing magnitude of privacy preservation. Therefore, we employ the classical Laplace mechanism for privacy preservation. The maximum possible changing magnitude of privacy preservation. Therefore, we employ the classical Laplace mechanism for privacy preservation. Therefore, we employ the classical Laplace mechanism for privacy preservation.

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Proposition 1. There is an $\left(\frac{2k \cdot r \cdot \log(1/\beta)}{\epsilon}, \beta\right)$-accurate centralized $\epsilon$-DP algorithm for the BSVS problem.

For the $t$-th query/bucket, we define the non-private count gap between the true label $y^t \in \{1 : |Y|\}$ and false labels as $Gap_t = a_t(y^t) - \max_{c \in \{1: |Y|\}} a_t(c)$. Then we have the following conclusion on the private labeling accuracy w.r.t. the accuracy of the BSVS problem:

Remark 1. If $Gap_t \geq 2\eta$ and the private algorithm is $(\eta, \beta)$-accurate, then with probability $1 - \beta$, the estimated hard labeling result is accurate (equals to the true label $y^t$).

2.4. Locally Private Mechanisms

Considering the most stringent case of imposing local DP on every client who holds only one record (i.e., $m_i \equiv 1$), every client $i$ now sanitize the labeling answer $A^i = [a^i_1, a^i_2, ..., a^i_{|Y|}]$ independently. We employ the randomized response mechanism [12] for local differential privacy, which randomly flips every binary value in $A^i$ with probability $e^{\epsilon/(2kr)}$. We show randomized response is $O(\sqrt{\frac{ckr \cdot \log(1/\beta)}{\epsilon^2}})$-accurate (in Theorem 1).

Theorem 1. The local $\epsilon$-DP randomized response mechanism is an $\left(\frac{e^{\epsilon/(2kr)} + 1}{e^{\epsilon/(2kr)} + 1}, \beta\right)$-accurate algorithm for the BSVS problem when $\epsilon = O(1)$.

Proof. For a binary value $b$ flipped with probability $\frac{\epsilon}{e^{\epsilon/(2kr)} + 1}$, the unbiased estimation given the observation $b'$ is $\hat{b} = \frac{b' - 1/e^{\epsilon/(2kr)} + 1}{e^{\epsilon/(2kr)} - 1/e^{\epsilon/(2kr)} + 1}$. The total count of observed ones is a summation of $n$ Bernoulli variables with a success rate of either $1/e^{\epsilon/(2kr)}$ or $1/e^{\epsilon/(2kr)} + 1$. Let $u$ denote the estimation bias of one element in $\tilde{a}_t$, and we have $P[|u| > \eta \cdot e^{\epsilon/(2kr)} + 1] \leq \exp(-\eta^2 e^{\epsilon/(2kr)}/3n)$. Therefore, with probability of $1 - \beta$, we have $|a_t - \tilde{a}_t|_\infty \leq \frac{e^{\epsilon/(2kr)} + 1}{e^{\epsilon/(2kr)} + 1} \sqrt{\frac{3n \log(|Y|/\beta)}{\epsilon^2}}$.

Additionally, we can adopt an optimal sparse vector summation oracle (Collision Mechanism) in the high privacy regime [15] for the BSVS problem and achieve an error rate of $\Theta(\sqrt{\frac{n}{\epsilon}})$.

3. EXPERIMENTS

3.1. Datasets, Networks and Performance Metric

To validate the proposed private knowledge distillation algorithm, we conduct extensive experiments on real-world datasets: MNIST2 that contains 70,000 gray-scale images of size $28 \times 28$ and 10 categories; SVHN3 that contains 630,420 digit images of size $32 \times 32$ and 10 categories; CIFAR-10 [16] that contains 60,000 images of size $32 \times 32$ and 10 categories.

Following common settings in the literature, for the MNIST/SVHN dataset, the public data $D_{pub}$ are 5,000/26,000 samples from the test dataset, the remaining 5,000/1,000 test samples are used for evaluating the performance of the student classifier, and the training dataset (together with the extended data in SVHN) is used as the private data $D_{priv}$. For the CIFAR-10 dataset, the public data $D_{pub}$ are 30,000 samples from the training set, the 1,000 samples from the test dataset are used for evaluation, and other 29,000 samples are used as the private data $D_{priv}$.

For the MNIST dataset, the architecture of the student classifier is from [17], and the DTI [18] is employed for general-purpose representation & clustering on $D_{pub}$ (denoted as [general]). For the SVHN dataset, the architecture of the student classifier is Mixmatch [19], and the histogram of oriented gradients (HOG) [20] and k-means++[21] are employed for general-purpose representation & clustering on $D_{pub}$. For the CIFAR-10 dataset, the network architecture is DenseNet121 [22], and the SimCLR [23] and k-means++ used for representation learning & clustering on $D_{pub}$.

Two accuracy indications are employed for measuring the performances. One is the accuracy of the private label answering (Acc$_{pl}$), the other is the test accuracy of the privately learned classifier (Acc$_{pc}$). As we use unsupervised clustering for query selection at iteration 1, here the Acc$_{pl}$ is the number of public samples receiving correct labels divided by $|D_{pub}|$.  

\footnotesize
\begin{itemize}
\item [2]http://yann.lecun.com/exdb/mnist
\item [3]http://ufldl.stanford.edu/housenumbers
\end{itemize}
3.2. Varying Number of Clusters and Nearest Neighbors

We here explore the choice of $s$ and $k$ of our method in Figures 2. The purity of clusters (w.r.t. class labels) upper bounds the Acc$_{cl}$. Increasing the number of clusters $s$ can roughly increase purity, but reduce the number of local records associated with one query. Besides there is no noticeable difference between choosing the number of nearest neighbors $k$ at 1, 2, 3 or 4. Theoretically, as $k$ gets larger, the label count of each query grows with $k$, but the count gap grows sublinearly with $k$ and the standard deviation of the privacy noise grows with $k$. The $k$ here in $[1, 2, 3, 4]$ are all small, thus the sublinearity is negligible and the noises hardly overwhelm count gaps.

3.3. Comparison with Existing Approaches and Local DP

The competitive approaches include SOTA private knowledge distillation methods by adding Laplace noise (LMAX) or Gaussian noise (GNMAX) in [5, 6], private $k$-NN [13] and the noisy SGD methods in [3]. Given the representation model trained on the $D_{pur}$, and the labeled samples in $D_{priv}$, one may simply train a prediction head with noisy SGD [24] as the classifier. We denote this straight-forward approach as Layer-1/Layer-2 noisy SGD (with one/two prediction layer(s)).

In Table 1, we compare our method with reported results of existing approaches under the same settings. The hyper-parameter of our method is set to $T = 1$ and $k = 1$. When employing general-purpose unsupervised representation learning and clustering, our method achieves better accuracy with an order magnitude smaller privacy compared to LMAX, GNMAX, noisy SGD and private $k$-NN. Specifically, if we employ end-to-end unsupervised clustering [18, 25] (denoted as [end2end]), we are able to achieve average accuracy of 86.1% for CIFAR-10, and 99.1% for MNIST with centralized privacy budget $\epsilon = 0.01$. Note that when equipped with more relaxed DP and tighter privacy accountant [26], our experiment results will be better.

For the local DP, we present results in Figure 2 on MNIST/CIFAR-10 dataset. Since noises due to local DP easily dominate Gap, we fix hyper-parameters at $s = |Y| = 10$ and $k = 1$, and employ end-to-end unsupervised clustering on MNIST with DTI [18] and on CIFAR-10 with SCAN [25]. It is observed that the Collision mechanism [15] achieves test accuracy of 98.5% for MNIST and 78.2% for CIFAR-10 with privacy budget $\epsilon = 0.4$. To the best of our knowledge, it is the first time locally private deep learning provides meaningful privacy/accuracy trade-offs.

4. CONCLUSION

This work proposes a fine-grained private knowledge distillation method (i.e., reverse $k$-NN labeling). Theoretically, we provide concrete differentially private mechanisms that are guaranteed for labeling accuracy. Experimentally, our solution improves significantly upon existing methods.

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**Table 1.** Test accuracy & privacy consumption comparison of centralized differentially private methods.

| Dataset | Methods | $\#$Queries | $\epsilon$ | Test Acc. | Label Acc. | Non-priv Acc. |
|---------|---------|-------------|------------|-----------|------------|---------------|
| MNIST   | LNMAX   | 1000        | 8.03       | 98.1%     | 99.2%      |               |
|         | GNMAX   | 286         | 1.97       | 98.5%     |            |               |
|         | Private $k$-NN | 735 | 0.47 | 98.8% |            |               |
|         | Ours [general] | 40 | 0.1 | 99.1% | 99.2% |               |
|         | Ours [general] | 40 | 0.04 | 98.6% | 97.7% |               |
|         | Ours [end2end] | 40 | 0.003 | 98.2% | 97.3% | 98.7% |
| SVHN    | LNMAX   | 1000        | 8.19       | 90.1%     |            |               |
|         | GNMAX   | 3098        | 4.96       | 91.6%     |            |               |
|         | Private $k$-NN | 2939 | 0.48 | 91.6% |            |               |
|         | Noisy SGD [24] | 40 | 4.0 | 76.0% | 84.4% |               |
|         | Ours [general] | 500 | 0.1 | 99.6% |            |               |
|         | Ours [general] | 500 | 0.04 | 95.3% |            |               |
| CIFAR-10| LNMAX   | 3877        | 2.92       | 70.8%     |            | 80.5%         |
|         | GNMAX   | 500         | 1.0        | 73.7%     |            |               |
|         | Private $k$-NN | 3877 | 1.0 | 73.7% |            |               |
|         | Finetuning Noisy SGD [3] | 500 | 0.29 | 79.4% | 87.9% | 82.8% |
|         | Layer-1 Noisy SGD [24] | 40 | 0.80 | 80.5% | 77.7% |               |
|         | Layer-2 Noisy SGD [24] | 40 | 0.86 | 98.9% |            |               |
|         | Ours [general] | 500 | 1.0 | 82.1% | 77.1% | 82.8% |
|         | Ours [general] | 500 | 0.29 | 79.4% | 87.9% | 82.8% |
|         | Ours [end2end] | 500 | 0.005 | 86.1% | 85.7% | 86.2% |

Fig. 2. Results of varying number of clusters/nearest neighbors on MNIST/SVHN/CIFAR-10 dataset (a, b, c) and results of local differential privacy on MNIST/CIFAR-10 dataset (d).
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