Effect of the length-diameter ratio on the initial fragment velocity of cylindrical casing

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Abstract. The initial velocity of fragment from cylindrical casing, which detonates at one end along the central of the casing, is the key issue in the field of explosion technology and its protecting. Most of the formula available can predict the initial velocity or the velocity of fragments at middle part of the cylindrical casing with greater length-diameter ratio (L/d>2). However, when the length-diameter ratio is less than two, the initial velocity of the cylindrical casing filled with explosives will have a big difference. In the present work, a numerical simulation model acknowledged by X-ray radiography experiments was used to determine the influence of the length-diameter ratio. The formula was built on top of the Gurney formula and made use of a correctional function to account for the effect of the length-diameter ratio. The formula was further acknowledged with the established numerical simulation model. The results indicate that the calculation formula can accurately predict the initial fragment velocity with different length-diameter ratio (L/d≤ 2).

1. Introduction
The cylindrical casing filled up with explosive is a usual used structure in the design of warhead. There are a large amount of numerical simulation and experiments about cylindrical casing. It is critical to calculate the initial velocities for design in explosion techniques and protective devices.

The fragmentation is a very interest dynamic issue to the researchers who focus on explosion drive and protective applications. During the past decades, there are many formulas calculating the initial fragment velocity which have been covered. Assuming the velocity of detonation products is linearly distributed at any time and the shell is infinite in length, Gurney [1] proposed a typical formula to calculate the initial velocity, and it is shown as:

\[ v_0 = \sqrt{2E} \cdot (1/ \beta + 0.5)^{-1/2} \] (1)

where \( \sqrt{2E} \) is Gurney constant of explosive, \( v_0 \) is Gurney velocity, and \( \beta \) is the mass ratio of explosive to casing(\( \beta=C/M \)), C and M is the mass of explosive and cylindrical casing. Gurney found that the initial velocity of fragment was only related to the mass ratio and explosive energy. However, the influence of length-diameter ratio was not taken into account. It performs well when the length-diameter ratio is greater than 2, but when the ratio is less than 2, there will be a large deviation. Charron [2] found that Gurney’s method is not suitable well for allocations where length-diameter ratio is no more than 2, even if local C/M’s were took into account. According to a lot of experiments, There is a conclusion that the range of the initial fragment velocity would change about 1.7AL/r₀√C/M because of changes in the...
casing length, which A is cross-sectional area, L is the length of the cylindrical casing, and \( r_0 \) is the initial radius of the casing. According to conservation of energy, a formula was proposed to count the initial velocity of fragment, and it is shown as:

\[
v_0 = \frac{D_e}{2} \left( \frac{Q_v}{\alpha_0} \cdot \frac{1}{2} \right)^{1/2}
\]

where \( D_e \) is the explosive detonation velocity, \( Q_v \) is the heat of detonation of the charge, \( \alpha_0 \) is the loading coefficients. Considering the motion equation of the cylindrical casing, an equation to calculate the initial velocity was proposed [3], which can be expressed as:

\[
v_0 = \frac{D_e}{2} \cdot \left\{ \frac{\beta}{2 + \beta} \left[ 1 - \left( \frac{r}{r_0} \right)^4 \right] \right\}^{1/2}
\]

where \( r \) is the expanding radius in one moment. The equations mentioned above are put forward considering that the cylindrical casing is infinite in length, However, according to experimental data of some articles [4], their calculated results are inconsistent with them. The initial velocity is less than that calculated by the equations mentioned above for small length-diameter ratio. The kinetic energy (\( E_k \)) is one of the important scales to measure the fragment killing power. \( E_k = \frac{1}{2} m v^2 \), so it is very important to predict the velocity as accurately as possible for the warhead structure design. Because the length of the warhead is always limited, especially when the length-diameter ratio is less than 2, such as grenades, tandem warhead, etc. Thus, a formula predicting the initial velocity of cylindrical casing with different length-diameter ratio (\( L/d \leq 2 \)) is in great request.

2. Validation of simulation model
The SPH method was used to investigate the fragmentation process in the present work [5], which can be used to simulated material when there was large deformation, and it is widely used in the explosive field. To acquire a precise and reliable simulation model, Feng’s work initiated at one end [6,7] was used to calibrate the model.

2.1. Characterization of the experiment: cylindrical metal casing initiated at one end
The experiments were carried out to figure out the axial distribution of initial velocity of the casing, which was filled up with composition B (COMP-B). AISI 1045 steel was used as the casing, which is widely used in engineering. The length of casing and charge (L) is 77.3 mm, the exterior and interior diameter of casing and charge are 29.68 mm and 23.60 mm, and the density of casing and COMP-B are 7.85 g cm\(^{-3}\) and 1.717 g cm\(^{-3}\).

2.2. Simulation model of Feng’s trial
Due to the symmetry of experiment devices, it only need model a quarter of them (Figure 1). To obtain the most accurate output possible, a great number of evenly distributed particles are needed. Therefore, particles need to be as small as possible. But smaller size requires more computer resources. Li [8] discovered that 0.4 mm is precise enough in the SPH model. The 0.4 mm particle radius was used in our present work, which showed that the results of simulation results are consistent with the experiment.
2.3. Material model
In our present work, the Johnson and Cook (JC) model (equation (4)) is employed to simulated the material properties of the AISI 1045 steel [9]. The model can be expressed as:

$$\sigma = \left( A + B e_{ep}^n \right) \left( 1 + C \ln \& \right) \left( 1 - T^m \right)$$

(4)

where A, B, C, n and m are the constants of the material, which are 507, 320, 0.064, 0.28, and 1.06 for the casing respectively[10]. Failure model is applied to model the fragmentation [11-13]. As a result, the material involves weak points where the failure occurred [14]. The failure model [13] would be described as:

$$dp = (1 - p)Ce^{\gamma} d\varepsilon$$

(5)

where \(\gamma\) and \(C\) are constants, \((1-p)\) is the probability that there is not rupture when the strain values is less than \(\varepsilon\). \(\gamma\) can be expressed as:

$$\gamma \approx 160 \frac{\sigma_2}{\sigma_f \left(1 + \varepsilon_f\right)}$$

(6)

where \(\sigma_2\) is the proportionality coefficient of the law, \(\sigma_f\) is the true stress at the fracture and \(\varepsilon_f\) are the plastic strain at fracture. According to Li’s stand tensile tests [8], the value of \(\gamma\) is 53.8.

The material properties of COMP-B are simulated using the JWL equation of state (EOS), which can be shown as:

$$P_b = C_1 \left( 1 - \frac{\omega}{r_1^v} \right) e^{-\omega} + C_2 \left( 1 - \frac{\omega}{r_2^v} \right) e^{-\omega} + \frac{\omega E}{v}$$

(7)

where \(P_b\) is the blast pressure, \(v\) is the initial relative volume, \(E\) is the internal energy every initial volume, \(\omega r_1, r_2, C_1,\) and \(C_2\) are the constants of the material. The parameters of the material are shown in Table [15].

| Material | \(\rho_0/g \cdot cm^{-3}\) | D/m \cdot s^{-1} | \(P_c/GPa\) | \(E_0/KJ/m^3\) | \(C_1/GPa\) | \(C_2/GPa\) | \(R_1\) | \(R_2\) | \(\omega\) |
|----------|-----------------|----------------|---------|--------------|-------------|-------------|-------|-------|-------|
| COMP-B   | 1.717           | 7980           | 29      | \(8.5 \times 10^6\) | 542         | 7.68        | 4.2   | 1.1   | 0.24  |

2.4. Simulation results
The experimental photograph and the numerical image of the fragment outline at the almost same moment (52.6 \(\mu s\) and 50 \(\mu s\)) are shown in Figure 2. From Figure 3, it shows up clearly that the fragment...
distributions in simulation and trial are in good agreement, which shows that the simulation model is capable to provide responsible predictions and the size of the particles are adequate to provide relatively accurate prediction.

![Image](image1.png)

**Figure 2.** The X-ray image (52.6 μs) (left) and numerical simulation image (50 μs) (right) of a cylindrical casing.

![Image](image2.png)

**Figure 3.** The congruency between the experiment and numerical simulation of fragment velocity.

3. The influence formula of the length-diameter ratio

3.1. Theoretical analysis

In cases where length-diameter ratio is small, the rarefaction waves get to farther inside the casing and the pressure would decrease over a wider axial distance. Hence, the influence formula of the length-diameter ratio can be modelled by multiplying a correctional function with $\beta$, which is shown as:

$$v(a) = \sqrt{2E \left(1/(\beta \cdot f(a)) + 0.5\right)^{1/2}}$$  \hspace{1cm} (8)

where $a$ is the length-diameter ratio of a cylindrical casing.

According to equation (8), in contrast to long cylindrical casing ($a > 2$), which the initial fragment velocity is equal to the velocity calculated by Gurney formula, the cylindrical casing with small length-diameter ratio ($a \leq 2$) has an initial velocity gain, which can be shown as:

$$g(a) = \frac{v(a)}{v(0)} = \left(\frac{1/(\beta \cdot f(a)) + 0.5}{1/\beta + 0.5}\right)^{1/2}$$  \hspace{1cm} (9)

Where $v(0)$ is the initial velocity calculated by Gurney formula, $v(a)$ is the initial velocity of a cylindrical casing with a specific length-diameter ratio.
3.2. Numerical simulation
The acknowledged model above was used to simulate the effect of the length-diameter ratio on the initial velocity (equation (6)). Table lists the eight cylindrical casings with different \( a \). Figure shows the fragment velocity of the casing with different \( a \), which means that the length-diameter ratio has a significant influence on the initial fragment velocity, especially when the length-diameter ratio is less than 2. The biggest fragment velocity of each cylindrical casing is treated as the initial velocity.

**Table 2.** Parameters and results of eight cylindrical casings modelled with various \( a \).

| No. | Length of Casing (L, mm) | Exterior diameter of charge (D, mm) | Interior diameter of casing (d, mm) | \( a \) | \( \beta \) | Fragment velocity \( v(\alpha) \) (m/s) | Velocity gain \( g(\alpha) \) |
|-----|-------------------------|----------------------------------|-----------------------------------|------|------|-------------------------|------------------|
| 1   | 7.5                     | 30                               | 24                                | 0.3125 | 0.39 | 536.923                  | 0.34822 |
| 2   | 15                      | 30                               | 24                                | 0.625  |      | 881.017                  | 0.57135 |
| 3   | 22.5                    | 30                               | 24                                | 0.9375 |      | 1093.09                  | 0.70888 |
| 4   | 30                      | 30                               | 24                                | 1.25   |      | 1230.86                  | 0.79822 |
| 5   | 37.5                    | 30                               | 24                                | 1.5625 |      | 1306.12                  | 0.84703 |
| 6   | 45                      | 30                               | 24                                | 1.875  |      | 1392.52                  | 0.90306 |
| 7   | 52.5                    | 30                               | 24                                | 2.1875 |      | 1466.32                  | 0.95092 |
| 8   | 60                      | 30                               | 24                                | 2.5    |      | 1501.72                  | 0.97388 |

**Figure 4.** The velocity of gauge points with different \( a \).

3.3. Simulation results
Figure is the front elevation of the outline distribution in the numerical simulations. The distributions of fragment show that when the casing expand to a certain extent, fragment velocity becomes stable [16].

**Figure 5.** Front view of the outline distribution of cylindrical casings with various \( a \).

Table shows the initial velocity gain of the eight cylindrical casing, which is drew against the length-diameter ratio in Figure. The initial velocity gain of the casing increases as the length-diameter ratio of the cylindrical casing gets bigger, and it gets close to 1 when \( a \) approaches 2. Therefore, the influence
of the length-diameter ratio cannot be neglected.

It found that the modification function $f(a)$ which is an exponential form can accurately depict the influence of the length-diameter ratio. Therefore, considering that the initial velocity gain of the eight cylindrical casings in Figure 6, the modification function $f(a)$ that we can get by data fitting is shown as:

$$f(a) = ke^{a^t} + b$$

(10)

where $k$, $t$ and $b$ are the constant. When subrogating equation (10) into equation (8), the initial velocity with different length-diameter ratio of the cylindrical casing is shown as:

$$v(a) = \sqrt{\frac{1}{2E}} \left( \frac{1}{\beta \cdot (ke^{a^t} + b) + 0.5} \right)^{-1/2}$$

(11)

The initial velocity gain could also be shown as equation (12) rather than equation (9).

$$g(a) = \frac{v(a)}{v(0)} = \left( \frac{1}{\beta + 0.5} \right)^{-1/2}$$

(12)

The results of simulation of the eight cylindrical casings (Figure 6) is fitted by using least square method, the fitting result gives $k=-1.353$, $t=1.606$ and $b=1.212$ ($R^2=0.998$).

![Figure 6. Simulated initial velocity gain of cylindrical casings with various $a$.](image1)

![Figure 7. The initial fragment velocity of cylindrical casings calculated by simulation and equation (11) with another different $a$.](image2)

4. Validation of the proposed formula
The acknowledged numerical model needs to be employed to identify whether equation (11) can precisely predict the initial velocity with different length-diameter ratio. Cylindrical casings with another different $a$ are calculated in both calculations with equation (11) and numerical simulation. The initial fragment velocities in numerical simulation and equation (11) are shown in Figure 7. It can be seen that the calculation results are almost the same with that calculated by equation (11).

5. Conclusion
According to the numerical simulation and calculation process above, we dedicate to the initial velocities of cylindrical casings with different length-diameter ratios. It obvious that the effect of the length-diameter ratio on the initial velocity should not be neglected. In our present work, there are following
conclusions could be drawn:
- A simulation model is put forward, which is acknowledged by experimental data and X-ray images, and it can be applied to model fragmentation of cylindrical casing.
- A novel formula is proposed to describe the effect of the length-diameter ratio on the initial velocity of a cylindrical casing. The formula can predict the initial velocity initiated at the near end along the central of the casing with different length-diameter ratios ($a \leq 2$), and it is further verified by the validated simulation model, which shows satisfying results.

Acknowledgments
The authors wish to thank to Zhang Bo who provides simulation support, Feng Shun-shan offering the experimental data, and other seniors in our group.

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