Non-Supersymmetric Type I Strings with Zero Vacuum Energy

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Abstract

We study open descendants of non-supersymmetric type IIB asymmetric (freely acting) orbifolds with zero cosmological constant. A generic feature of these models is that supersymmetry remains unbroken on the brane at all mass levels, while it is broken in the bulk in a way that preserves Fermi-Bose degeneracy in both the massless and massive (closed string) spectrum. This property remains valid in the heterotic dual of the type II model but only for the massless excitations. A possible application of these constructions concerns scenarios of low-energy supersymmetry breaking with large dimensions.
1. Introduction

One of the longest outstanding problems in theoretical physics is to explain why the cosmological constant is extremely small and possibly vanishes after supersymmetry breaking. It was recently argued that there is a class of non-supersymmetric type II string compactifications whose vacuum energy vanishes to all orders in perturbation theory due to Fermi-Bose degeneracy \[1,2,3\]. These models are based on asymmetric orbifold constructions of non-abelian nature, where supersymmetry is broken in a special way, so that the vacuum amplitude can be shown to vanish up to genus-2 by explicit computation \[3\]. This vanishing was argued to persist in higher orders as well, while at the non-perturbative level a cosmological constant may be generated, although it will be exponentially suppressed in the weak coupling limit \[4\].

Since these constructions were made in the framework of type II string theories, their obvious disadvantage is the absence of non-abelian gauge symmetries at the perturbative level. It is then natural to ask whether they admit open descendants that support non-abelian gauge groups and chiral matter, without destroying their main property of vanishing vacuum energy \[5\]. A recent analysis has indeed shown that such a generalization is possible \[6\].

In this work we perform a systematic study of the open descendants in non-supersymmetric type IIB asymmetric orbifolds with zero cosmological constant, using the formalism of \[7,8\]. The models we study can be described as Scherk-Schwarz deformations of six-dimensional (6d) vacua with \(\mathcal{N} = 1\) supersymmetry that are asymmetric generalizations of the class studied in \[9\]. In particular, we derive the full partition function of the type I model based on the freely acting orbifold of \[4\]. A peculiar feature of the parent type IIB vacuum is that both \(R \to \infty\) and \(R \to 0\) limits lead to (6d) supersymmetric theories, with \(R\) the radius of the circle used to break supersymmetry. This is due to an exact T-duality symmetry present even after supersymmetry breaking. T-duality is of course broken in the type I theory that has a (6d) supersymmetric decompactification limit, while for \(R \to 0\) there are linear or logarithmic divergences, in the case of one or two dimensions, respectively. In the T-dual type I' picture, the divergences reflect the existence of non vanishing local tadpoles in the directions transverse to the D-branes \[10\].

\[\text{It would be interesting to study these models at special points of moduli space and in the presence of RR-backgrounds, where extra massless non-abelian gauge particles appear non-perturbatively. In section 5, we discuss this issue using heterotic–type II duality.}\]
These tadpoles remain non-vanishing for any configuration of D-branes, unlike the familiar situation for the SO(16) \( \otimes \) SO(16) model in nine dimensions \[11\].

Although broken in the closed string sector, supersymmetry remains unbroken on the D-branes. This is consistent with the results of \[9\], since using T-duality, an exact symmetry of the parent type IIB theory, one can always break supersymmetry along a direction transverse to the branes. Notice, however, that unlike the constructions of \[9\], where supersymmetry is present only for the massless excitations of the D-branes, in our case it is preserved in all mass levels. The same property appears to be present for the massless excitations in the gauge (non-perturbative) sector of the type of closed string models of \[4\], at enhanced symmetry points of their moduli space, as we argue by analysing the partition function on the heterotic side. We also study the open descendants of another class of non-supersymmetric type IIB vacua based on the free-fermionic construction \[12\]. These models are defined at particular points of moduli space, and thus they can not be continuously deformed to higher dimensional supersymmetric models. The open sector has a gauge group with reduced rank and, as in the previous models, unbroken supersymmetry at all mass levels.

The present paper is organized as follows. In section 2 we review the non-supersymmetric type II model of \[4\] with vanishing vacuum energy. In section 3 we derive its one-loop partition function and discuss its 6d limit. In section 4 we derive its open descendants that have \( \mathcal{N} = 2 \) unbroken supersymmetry to all mass levels, and give the full genus-1 vacuum amplitude, that receives contributions from the torus, the Klein bottle, the annulus and the Möbius strip. In section 5 we study the effects of supersymmetry breaking on the non-abelian gauge sector of the heterotic dual of the type II model of \[4\], by analysing the corresponding partition function. In section 6 we derive the open descendants of a different type IIB 4d model, with fixed values for all the internal radii, using the free-fermionic formulation. Finally, section 6 contains our concluding remarks.

2. The orbifold generators and their algebra

The asymmetric orbifold we will consider in this section is generated by the following two elements \[4\]:

\[
\begin{align*}
f &= \left[ (-1^4, 1^5), (0^4, v_L; \delta^4, v_R), (-)^{F_R} \right], \\
g &= \left[ (1^5; -1^4, 1), (\delta^4, w_L; 0^4, w_R), (-)^{F_L} \right].
\end{align*}
\] (2.1)
Here the first entry inside the square brackets denotes rotations, the second denotes shifts on the internal compactification lattice while the third corresponds to a genuine world-sheet symmetry. Moreover, a semicolon separates holomorphic and antiholomorphic coordinates. In order to implement an asymmetric $\mathbb{Z}_2$ rotation, the internal four-dimensional torus must split into a product of four circles with self-dual radius $R = \sqrt{\alpha'}$, such that the lattice factorizes into a holomorphic and an antiholomorphic part. $\delta$ is then a shift by $R/2$, as required by level matching and multi-loop modular invariance conditions [13]. No further constraints are imposed on the radius of the fifth coordinate, while $v_{L,R} = w_{R,L}$. The shifts $v_{L,R}$ in the fifth coordinate, $A_2$ shifts in the notation of [14], act as

$$X(z) \rightarrow X(z) + \frac{1}{2} \left( \frac{\alpha'}{R} + R \right),$$

$$\overline{X}(\bar{z}) \rightarrow \overline{X}(\bar{z}) - \frac{1}{2} \left( \frac{\alpha'}{R} - R \right),$$

and give a contribution

$$\frac{1}{8} \left( \frac{\alpha'}{R} + R \right)^2 - \frac{1}{8} \left( \frac{\alpha'}{R} - R \right)^2 = \frac{1}{2} \alpha',$$

to the level matching condition, thereby balancing the contribution of the shifts that act on $T^4$.

Due to the presence of $(-)^{F_{L,R}}$, the $g$ ($f$) generator projects out all the gravitini coming from the (anti-)holomorphic sector, and therefore the combined action of $f$ and $g$ breaks supersymmetry completely. The shifts give masses to the corresponding twisted sectors, ensuring that no massless gravitons originate from them. Although the model is non-supersymmetric, it has been argued in [1] that one-loop and higher order perturbative corrections to the cosmological constant vanish. There are, however, non-perturbative contributions originating from wrapped D-branes that can be studied perturbatively on the dual heterotic theory [4].

The generators in (2.1) satisfy the algebra

$$f \circ f = \left[ \left( 1^5 ; 1^5 \right), \left( 0^5 ; (2\delta)^4, 0 \right), 1 \right],$$

$$g \circ g = \left[ \left( 1^5 ; 1^5 \right), \left( (2\delta)^4, 0 ; 0^5 \right), 1 \right],$$

$$f \circ g = \left[ \left( -1^4, 1 ; -1^4, 1 \right), \left( -\delta^4, 0 ; \delta^4, 0 \right), (-)^{F_L+F_R} \right],$$

$$g \circ f = \left[ \left( -1^4, 1 ; -1^4, 1 \right), \left( \delta^4, 0 ; -\delta^4, 0 \right), (-)^{F_L+F_R} \right],$$
thus revealing the non-abelian nature of the orbifold group $S$. When restricted to the point group $\overline{\mathcal{P}}$, defined as the quotient of the space group $S$ by the generators of pure translations $[13]$, the orbifold group reduces to a simple abelian group, $Z_2 \otimes Z_2$, where the $Z_2$ factors are generated by $f$ and $g$. Then one can first mod out the theory by $\Lambda = \{f^2, g^2\}$, thereby defining a new compactification lattice, and then quotient by the point group $\mathcal{P} = Z_2 \otimes Z_2$.

In the next section we will follow this procedure. In fact, the model (2.1) is a freely acting asymmetric orbifold of $T^4/Z_2$ by $f$, and can also be interpreted as an asymmetric Scherk-Schwarz deformation by doubling the radius of the fifth coordinate $[16,9]$. Due to the asymmetric nature of the deformation that acts simultaneously on momentum and winding modes, the lattice contribution to the resulting model is invariant under T-duality. As a result, the standard Scherk-Schwarz breaking model is equivalent to the M-theory breaking one. An alternative approach, leading to the same result, would be to mod out directly by the space group $S$ $[3]$.

3. The torus partition function

The starting point in the construction of the orbifold described in the previous section is the toroidal compactification of the type II superstring on a five-dimensional lattice $\Gamma_{(5,5)} = [\Gamma_{SU(2)}]^4 \times [\Gamma_{(1,1)}(R)]$, where $\Gamma_{SU(2)}$ denotes the SU(2) lattice and $\Gamma_{(1,1)}(R)$ is the contribution of a single circle of radius $R$. In order to construct the open-descendants of this model we start from the type IIB superstring, that is invariant under the action of the world-sheet parity $\Omega$. Using the SO(8) characters $O_8$, $V_8$, $S_8$ and $C_8$, associated to the conjugacy classes of identity, vector, spinor and conjugate spinor representations, respectively, to represent the contribution of the world-sheet fermions, the partition function reads:

$$\mathcal{T}_0 = |V_8 - S_8|^2 \left[|\chi_1|^2 + |\chi_2|^2\right]^4 Z_{m,n}(q, \bar{q}) \, . \quad (3.1)$$

Here $\chi_1$ and $\chi_2$ indicate the two characters of SU(2) at level one, corresponding to the conjugacy classes of the singlet and the doublet representation, and

$$Z_{m,n}(q, \bar{q}) = \sum_{m,n \in \mathbb{Z}} q^\frac{\alpha'}{4}(m/R+nR/\alpha')^2 \frac{\bar{q}}{4}(m/R-nR/\alpha')^2$$

with $q = e^{2\pi i\tau}$.
Following the strategy outlined in the previous section, we mod out the toroidal amplitude \( (\mathcal{A}) \) by the generators of \( \Lambda \) that act as pure asymmetric lattice shifts. This results in a new toroidal compactification where the SU(2)\(^4\) lattice is turned into an SO(8) lattice:

\[
T_{\Lambda} = |V_8 - S_8|^2 \left[ |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2 \right] Z_{m,n}(q, \bar{q}).
\]

The presence of a non-trivial background for the NS \( \otimes \) NS antisymmetric tensor in the SO(8) lattice reduces the rank of the Chan-Paton (CP) gauge group by a factor equal to the rank of the antisymmetric tensor \([17,18,19,20]\). As we will see in the next section, this turns out to be crucial in order to get a consistent model.

Acting now with the point group \( \mathcal{P} \), one gets the following formal expression for the partition function

\[
\mathcal{T} = \frac{1}{4} \left[ \mathcal{T}_{(0,0)} + \mathcal{T}_{(0,f)} + \mathcal{T}_{(0,g)} + \mathcal{T}_{(0,fg)} + \mathcal{T}_{(f,0)} + \mathcal{T}_{(f,f)} + \mathcal{T}_{(g,0)} + \mathcal{T}_{(g,g)} + \mathcal{T}_{(fg,0)} + \mathcal{T}_{(fg,fg)} \right].
\]  \( (3.2) \)

Note the absence of the disconnected modular orbit generated by \( \mathcal{T}_{(f,g)} \). This fact has two different explanations in the space group or in the point group approaches of the orbifold. In the former case, it is due to the fact that the path integral receives contributions only from commuting pairs of spin structures \([13]\), whereas in the latter case, \( \mathcal{T}_{(f,g)} \) vanishes due to the simultaneous action of shifts and rotations \([16]\).

The amplitudes \( \mathcal{T}_{(a,b)} \) with \( a, b \in \{0, f, g, fg\} \) are given by

\[
\mathcal{T}_{(0,0)} = |V_4O_4 + O_4V_4 - S_4S_4 - C_4C_4|^2 \left[ |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2 \right] Z_{m,n},
\]

\[
\mathcal{T}_{(0,f)} = (V_4O_4 - O_4V_4 + S_4S_4 - C_4C_4)(\overline{V_4O_4} + \overline{O_4V_4} + \overline{S_4S_4} + \overline{C_4C_4}) \times \frac{\vartheta^2_3 \vartheta^2_4}{\eta^4} (\overline{O_4O_4} - \overline{V_4V_4}) (-)^{m+n} Z_{m,n},
\]

\[
\mathcal{T}_{(0,g)} = (V_4O_4 + O_4V_4 + S_4S_4 + C_4C_4)(\overline{V_4O_4} - \overline{O_4V_4} + \overline{S_4S_4} - \overline{C_4C_4}) \times \frac{\vartheta^2_3 \vartheta^2_4}{\eta^4} (\overline{O_4O_4} - \overline{V_4V_4}) (-)^{m+n} Z_{m,n},
\]

\[
\mathcal{T}_{(0,fg)} = |V_4O_4 - O_4V_4 - S_4S_4 + C_4C_4|^2 \left[ \frac{\vartheta^2_3 \vartheta^2_4}{\eta^4} \right]^2 Z_{m,n},
\]

\( (3.3) \)
in the untwisted sector, by
\[
\mathcal{T}_{(f,0)} = \frac{1}{2} (O_4 C_4 + V_4 S_4 - S_4 O_4 - C_4 V_4)(\overline{O}_4 \overline{C}_4 + \overline{V}_4 \overline{V}_4 - \overline{S}_4 \overline{C}_4 - \overline{C}_4 \overline{V}_4) \times \\
\times \frac{\partial_2^2 \partial_3^2}{\eta^4} [(O_4 + V_4)(S_4 + C_4) + (S_4 + C_4)(O_4 + V_4)] Z_{m+1/2,n+1/2},
\]
\[
\mathcal{T}_{(f,f)} = \frac{1}{2} (O_4 C_4 - V_4 S_4 - S_4 O_4 + C_4 V_4)(\overline{O}_4 \overline{C}_4 + \overline{V}_4 \overline{V}_4 + \overline{S}_4 \overline{C}_4 + \overline{C}_4 \overline{V}_4) \times \\
\times \frac{\partial_2^2 \partial_3^2}{\eta^4} [(O_4 - V_4)(S_4 + C_4) + (S_4 + C_4)(O_4 - V_4)] (-)^{m+n} Z_{m+1/2,n+1/2},
\]
(3.4)
in the \(f\)-twisted sector, by
\[
\mathcal{T}_{(g,0)} = \frac{1}{2} (O_4 O_4 + V_4 V_4 - S_4 C_4 - C_4 S_4)(\overline{O}_4 \overline{C}_4 + \overline{V}_4 \overline{V}_4 - \overline{S}_4 \overline{O}_4 - \overline{C}_4 \overline{V}_4) \times \\
\times [(O_4 + V_4)(S_4 + C_4) + (S_4 + C_4)(O_4 + V_4)] \frac{\partial_2^2 \partial_3^2}{\eta^4} Z_{m+1/2,n+1/2},
\]
\[
\mathcal{T}_{(g,g)} = \frac{1}{2} (O_4 O_4 + V_4 V_4 + S_4 C_4 + C_4 S_4)(\overline{O}_4 \overline{C}_4 - \overline{V}_4 \overline{V}_4 - \overline{S}_4 \overline{O}_4 + \overline{C}_4 \overline{V}_4) \times \\
\times [(O_4 - V_4)(S_4 + C_4) + (S_4 + C_4)(O_4 - V_4)] \frac{\partial_2^2 \partial_3^2}{\eta^4} (-)^{m+n} Z_{m+1/2,n+1/2},
\]
(3.5)
in the \(g\)-twisted sector, and by
\[
\mathcal{T}_{(f,g,0)} = \vert O_4 S_4 + V_4 C_4 - C_4 O_4 - S_4 V_4 \vert^2 \frac{\partial_2^2 \partial_3^2}{\eta^4} Z_{m,n},
\]
\[
\mathcal{T}_{(f,g,g)} = \vert O_4 S_4 - V_4 C_4 - C_4 O_4 + S_4 V_4 \vert^2 \frac{\partial_2^2 \partial_3^2}{\eta^4} Z_{m,n},
\]
(3.6)
in the \(fg\)-twisted sector. Here \(\vartheta_j\) are Jacobi theta functions, while \(\eta\) is the Dedekind eta-function. In order to implement the orbifold projection, we have broken the \(SO(8)\) characters into products of \(SO(4)\) characters, and we have used the \(S : \tau \to -1/\tau\) and \(T : \tau \to \tau + 1\) modular transformation matrices
\[
S_{SO(2n)} = \frac{1}{2} \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & i^{-n} & -i^{-n} \\
1 & -1 & -i^{-n} & i^{-n}
\end{pmatrix}, \quad T_{SO(2n)} = e^{-i\pi n/12} \text{diag} \left(1, -1, e^{i\pi n/4}, e^{i\pi n/4}\right).
\]
From expressions (3.3)-(3.6), one can check that the partition function vanishes identically in each sector.

Expanding the internal characters and the theta functions in powers of \(q\), and keeping only leading terms, we get the following massless contributions
\[
\mathcal{T}_{\text{untw}} \sim \vert V_4 O_4 \vert^2 + \vert S_4 S_4 \vert^2 - (O_4 V_4)(\overline{C}_4 \overline{C}_4) - (C_4 C_4)(\overline{O}_4 \overline{V}_4),
\]
\[
\mathcal{T}_{fg-tw} \sim 8 \vert O_4 S_4 - C_4 O_4 \vert^2,
\]
(3.7)
that translate into the following five-dimensional field content: \( \{ g_{\mu\nu}, 7A_\mu, 6\phi; 8\psi \} \) from the untwisted sector, and \( 8 \{ A_\mu, 5\phi; 2\psi \} \) from the \( fg \)-twisted sector. Here, \( g_{\mu\nu}, A_\mu, \phi \) and \( \psi \) denote the five dimensional graviton, vector, real scalar and Dirac fermion, respectively. The factor 8 in \( T_{fg-tw} \) counts the number of fixed points left invariant by the shifts.

Due to the asymmetric shift on the fifth coordinate, the torus amplitude is invariant under T-duality. As a result, the amplitude in the decompactification limit \( R \to \infty \) coincides with the one in the \( R \to 0 \) limit, aside from volume factors. The former case corresponds to the decompactification limit of type IIB on the supersymmetric \( T^4/Z_2 \) orbifold, with 21 tensor multiplets coupled to \( \mathcal{N} = (2, 0) \) supergravity in six dimensions. The latter case has a natural interpretation in terms of type IIA compactified on the same orbifold, with 6d \( \mathcal{N} = (1, 1) \) supergravity coupled to 20 vector multiplets. In order to have a consistent assignment of quantum numbers [9], in the \( R \to \infty \) (\( R \to 0 \)) limit it is required to double (halve) the radius. Consequently, the terms \( Z_{m,n} \) contribute with an additional factor of 2 that partially compensates the \( 1/4 \) factor in the partition function (3.2), yielding the expected multiplicity of states in the \( T^4/Z_2 \) orbifolds.

4. Open descendants

Following [7,8] the construction of open descendants resembles a \( Z_2 \) orbifold where the \( Z_2 \) symmetry is the world-sheet parity \( \Omega \). The “untwisted sector” of the parameter space orbifold consists of closed unoriented strings, whose contribution to the total partition function is

\[
\frac{1}{2} (T + K) ,
\]

where the Klein bottle amplitude

\[
K = \frac{1}{2} \left[ (V_4 O_4 + O_4 V_4 - S_4 S_4 - C_4 C_4) (O_8 + V_8 + S_8 + C_8) Z_m + \right.
\]

\[
+ (V_4 O_4 - O_4 V_4 - S_4 S_4 + C_4 C_4) \frac{\partial^2 \varphi^2}{\eta^4} (-)^m Z_m + \quad (4.1)
\]

\[
+ 2(O_4 S_4 + V_4 C_4 - C_4 O_4 - S_4 V_4) \frac{\partial^2 \varphi^2}{\eta^4} Z_m \right] ,
\]

completes the \( \Omega \) projection. In the following we will use the same conventions as in [9] for the lattice sums

\[
Z_{m+a} = \sum_{m \in \mathbb{Z}} q^{\frac{1}{2}[(m+a)/R]^2} , \quad \bar{Z}_{n+b} = \sum_{n \in \mathbb{Z}} q^{\frac{1}{4}[(n+b)R/2]^2} ,
\]
that correspond to the choice $\alpha' = 2$. At the massless level one finds
\[
\mathcal{K} \sim (V_4 O_4 - S_4 S_4) + 4 (O_4 S_4 - C_4 O_4),
\]
that symmetrizes correctly the torus amplitude (3.7). The factor 4 has to be interpreted as $n^+ - n^- = 6 - 2$, where $n^\pm$ is the number of fixed points with $\Omega = \pm 1$ \cite{19,20}. The resulting spectrum of massless excitations then results in the following five-dimensional fields: \{g_{\mu\nu}, 2A_\mu, 5\phi; 4\psi\} from the untwisted sector and \{2A_\mu, 26\phi; 8\psi\} from the $fg$-twisted sector. Since $\mathcal{K}$ is supersymmetric, the contribution of the Klein bottle amplitude to the 1-loop cosmological constant vanishes identically. Although the asymmetric nature of the orbifold is reflected in the presence of the signs in the lattice sums in (4.1), the open descendants only feel the left-right symmetric (supersymmetric) generator $fg$ \cite{21}.

In the transverse channel, the Klein bottle amplitude is
\[
\tilde{\mathcal{K}} = \frac{2^4}{2} R \left[ (V_4 O_4 + O_4 V_4 - S_4 S_4 - C_4 C_4) O_8 \tilde{Z}_{2n} + \right. \\
+ (V_4 O_4 - O_4 V_4 - S_4 S_4 + C_4 C_4) \frac{\eta_3^2 \eta_4^2}{\eta_4^4} \tilde{Z}_{2n} + \\
\left. + \frac{1}{2} (O_4 S_4 + V_4 C_4 - C_4 O_4 - S_4 V_4) \frac{\eta_2^2 \eta_3^2}{\eta_4^4} \tilde{Z}_{2n+1} \right],
\]
with a massless tadpole proportional to
\[
\tilde{\mathcal{K}} = 2^4 R (V_4 O_4 - S_4 S_4) + \text{massive}. \quad (4.2)
\]
There are no volume factors relative to $T^4$, because we are working at the SO(8) rational point. Something particularly interesting happens due to the presence of the SO(8) lattice, which ensures the consistency of the theory. To appreciate the meaning of eq. (4.2), let us recall that, in the presence of a quantized background for the NS $\otimes$ NS antisymmetric tensor $B_{IJ}$, the transverse Klein bottle amplitude associated to the $T^4/Z_2$ orbifold reads \cite{20}
\[
\tilde{\mathcal{K}} \sim \left( \sqrt{v} + \frac{2^{-r/2}}{\sqrt{v}} \right)^2 (V_4 O_4 - S_4 S_4) + \left( \sqrt{v} - \frac{2^{-r/2}}{\sqrt{v}} \right)^2 (O_4 V_4 - C_4 C_4). \quad (4.3)
\]
Here $v$ is the volume of the internal lattice and $r$ is the rank of $B_{IJ}$. For the SO(8) lattice $r = 2$ and $v = \frac{1}{2}$ \cite{22}, and the transverse Klein bottle amplitude is
\[
\tilde{\mathcal{K}} \sim (V_4 O_4 - S_4 S_4). \quad (4.4)
\]
Although both (4.3) and (4.4) are consistent with the associated torus amplitude

\[ T \sim |V_4 O_4 - S_4 S_4|^2 + |O_4 V_4 - C_4 C_4|^2 + \ldots , \]  

(4.5)

only (4.4) is compatible with (3.7). A similar phenomenon will take place in the open sector, along with an identification of Neumann and Dirichlet charges.

The “twisted sector” of the parameter space orbifold

\[ \frac{1}{2}(A + M) , \]

corresponds to the open unoriented sector of the spectrum and carries multiplicities associated to CP charges (D-branes) that live at the ends of the open strings. In five dimensions, there are two different open sectors that differ by a sign in the massive \( fg \)-twisted states. This ambiguity is related to the two possible identifications of Neumann and Dirichlet charges under the action of the \( f \) and \( g \) generators. The annulus and Möbius strip amplitudes are then given by

\[ A_{1,2} = \frac{1}{2} \left[ (M + \overline{M})^2 (V_4 O_4 + O_4 V_4 - S_4 S_4 - C_4 C_4) O_8 + \right. \]

\[ - (M - \overline{M})^2 (V_4 O_4 - O_4 V_4 - S_4 S_4 + C_4 C_4) \frac{\partial_3^2 \partial_4^2}{\eta^4} \right] Z_{2m} \]

\[ + \frac{2}{8} \left[ (M + \overline{M})^2 (O_4 S_4 + V_4 C_4 - C_4 O_4 - S_4 V_4) \frac{\partial_2^2 \partial_3^2}{\eta^4} \right. + \]

\[ \left. \begin{array}{c}
\pm (M - \overline{M})^2 (O_4 S_4 - V_4 C_4 - C_4 O_4 + S_4 V_4) \frac{\partial_2^2 \partial_4^2}{\eta^4} \end{array} \right] Z_{2m+1} , \]

(4.6)

and

\[ M_{1,2} = - \frac{1}{2} (M + \overline{M}) \left[ (\hat{O}_4 \hat{V}_4 - \hat{V}_4 \hat{O}_4 + \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4) \hat{O}_8 \right. \]

\[ + (\hat{V}_4 \hat{O}_4 + \hat{O}_4 \hat{V}_4 - \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4) \frac{\partial_3^2 \partial_4^2}{\eta^4} \right] \hat{Z}_{2m} + \]

\[ + \frac{2}{8} (M - \overline{M}) \left[ (\hat{O}_4 \hat{S}_4 - \hat{V}_4 \hat{C}_4 - \hat{C}_4 \hat{O}_4 + \hat{S}_4 \hat{V}_4) \frac{\partial_2^2 \partial_3^2}{\eta^4} \right. \]

\[ \left. \begin{array}{c}
\pm (\hat{O}_4 \hat{S}_4 + \hat{V}_4 \hat{C}_4 - \hat{C}_4 \hat{O}_4 - \hat{S}_4 \hat{V}_4) \partial_3^2 \partial_4^2 \eta^4 \end{array} \right] (-)^m \hat{Z}_{2m+1} , \]

(4.7)

where in the Möbius strip amplitude we have defined suitable hatted characters \( \hat{\xi} \). The parametrization of the annulus and Möbius strip amplitudes in terms of CP charges \( M \) and \( \overline{M} \) follows from the requirement of a consistent particle interpretation for the spectrum and
from the tadpole conditions, that lead also to a supersymmetric open unoriented sector. In the transverse channel, these amplitudes are

\[ \tilde{A}_{1,2} = \frac{2^{-4}}{2} R (M + \overline{M})^2 \left[ \left( V_4O_4 + O_4V_4 - S_4S_4 - C_4C_4 \right) (O_8 + V_8 + S_8 + C_8) \tilde{Z}_n + \right. \]

\[ + \left. \left( V_4O_4 - O_4V_4 - S_4S_4 + C_4C_4 \right) \frac{\partial_3^2 \partial_4^2}{\eta^4} (-)^n \tilde{Z}_n \right] + \]

\[ - \frac{2^{-3}}{8} R (M - \overline{M})^2 \left[ 4 \left( O_4S_4 + V_4C_4 - C_4O_4 - S_4V_4 \right) \frac{\partial_3^2 \partial_4^2}{\eta^4} \tilde{Z}_n + \right. \]

\[ \left. \pm 2 \left( O_4S_4 - V_4C_4 - C_4O_4 + S_4V_4 \right) \frac{\partial_3^2 \partial_4^2}{\eta^4} (-)^n \tilde{Z}_n \right] , \]

and

\[ \tilde{M}_{1,2} = - \frac{2}{3} R (M + \overline{M}) \left[ \left( \tilde{V}_4\tilde{O}_4 - \tilde{O}_4\tilde{V}_4 - \tilde{S}_4\tilde{S}_4 + \tilde{C}_4\tilde{C}_4 \right) \tilde{O}_8 + \right. \]

\[ + \left. \left( \tilde{V}_4\tilde{O}_4 + \tilde{O}_4\tilde{V}_4 - \tilde{S}_4\tilde{S}_4 - \tilde{C}_4\tilde{C}_4 \right) \frac{\partial_3^2 \partial_4^2}{\eta^4} \tilde{Z}_n + \right. \]

\[ \left. + \frac{4}{8} R (M - \overline{M}) \left[ \left( \tilde{O}_4\tilde{S}_4 - \tilde{V}_4\tilde{C}_4 - \tilde{C}_4\tilde{O}_4 + \tilde{S}_4\tilde{V}_4 \right) \frac{\partial_3^2 \partial_4^2}{\eta^4} + \right. \]

\[ \left. \left. + \left( \tilde{O}_4\tilde{S}_4 + \tilde{V}_4\tilde{C}_4 - \tilde{C}_4\tilde{O}_4 - \tilde{S}_4\tilde{V}_4 \right) \frac{\partial_3^2 \partial_4^2}{\eta^4} \right] (-)^n \tilde{Z}_{2n+1} \right] \]

that imply the following contributions to the massless tadpoles:

\[ \tilde{A}_{1,2} = 2^{-4} R (M + \overline{M})^2 (V_4O_4 - S_4S_4) - 2^{-2} R (M - \overline{M})^2 (O_4S_4 - C_4O_4) + \text{massive} , \]

and

\[ \tilde{M}_{1,2} = -2 R (M + \overline{M}) (\tilde{V}_4\tilde{O}_4 - \tilde{S}_4\tilde{S}_4) + \text{massive.} \]

Together with the contribution (4.2) from the transverse Klein bottle amplitude, tadpole conditions result in the following constraints on the CP charges:

\[ M + \overline{M} = 16 \quad M - \overline{M} = 0. \quad (4.8) \]

Keeping only the leading terms in the expansion of the direct amplitudes (4.6) and (4.7) yields

\[ A_1 \sim \left[ 2M\overline{M}(V_4O_4 - S_4S_4) + (M^2 + \overline{M}^2)(O_4V_4 - C_4C_4) \right] \mathcal{Z}_{2m} + \]

\[ + 2 \left[ 2M\overline{M}(O_4S_4 - C_4O_4) + (M^2 + \overline{M}^2)(V_4C_4 - S_4V_4) \right] \mathcal{Z}_{2m+1} , \]
\[ \mathcal{M}_1 \sim -(M + M)(\hat{O}_4\hat{V}_4 - \hat{C}_4\hat{C}_4)\hat{Z}_{2m} + 2(M - M)(\hat{V}_4\hat{C}_4 - \hat{S}_4\hat{V}_4)(-)^m\hat{Z}_{2m+1}, \]

and

\[ \mathcal{A}_2 \sim \left[ 2M\bar{M}(V_4O_4 - S_4S_4) + (M^2 + \bar{M}^2)(O_4V_4 - C_4C_4) \right] \hat{Z}_{2m} + \\
+ 2 \left[ (M^2 + \bar{M}^2)(O_4S_4 - C_4O_4) + 2M\bar{M}(V_4C_4 - S_4V_4) \right] \hat{Z}_{2m+1}, \]

\[ \mathcal{M}_2 \sim -(M + M)(\hat{O}_4\hat{V}_4 - \hat{C}_4\hat{C}_4)\hat{Z}_{2m} + 2(M - M)(\hat{O}_4\hat{S}_4 - \hat{C}_4\hat{V}_4)(-)^m\hat{Z}_{2m+1}. \]

The spectrum associated with the open unoriented sector can thus be arranged in $N = 2$ five-dimensional supersymmetric representations, and comprises a vector multiplet in the adjoint representation of $U(8)$ and a massless hypermultiplet in the representations $28 \oplus \overline{28}$ from the untwisted sector, as well as 2 full massive hypermultiplets in the adjoint representation for model 1 and in the $28 \oplus \overline{36}$ for model 2 from the twisted sector. The doubling of the number of hypermultiplets in the twisted sector is due to the presence of the antisymmetric tensor $[19,20]$, and to the fact that the lattice sum $\hat{Z}_{2m+1}$ contributes twice to each mass level. Since $\mathcal{A}$ and $\mathcal{M}$ are supersymmetric, the contribution of the open unoriented sector to the 1-loop cosmological constant vanishes identically.

As shown in [6], one can derive this open unoriented spectrum by modding out the supersymmetric $T^4/Z_2$ orbifold compactification [23,8,24] by the T-duality contained in $f$. This operation is not always allowed. In fact, for orbifold compactifications in the presence of a quantized antisymmetric tensor $B_{IJ}$, the transverse annulus amplitude involves the following terms [20]

\[ \tilde{\mathcal{A}} \sim \left[ 2^{r/2}(N + \overline{N})\sqrt{v} + \frac{(D + \overline{D})}{\sqrt{v}} \right]^2 (V_4O_4 - S_4S_4) + \\
+ \left[ 2^{r/2}(N + \overline{N})\sqrt{v} - \frac{(D + \overline{D})}{\sqrt{v}} \right]^2 (O_4V_4 - C_4C_4), \]

coming from the torus amplitude (4.5). In our case, it is crucial that the compactification four-torus is an SO(8) lattice, since at the SO(8) point

\[ \tilde{\mathcal{A}} \sim [(N + \overline{N}) + (D + \overline{D})]^2 (V_4O_4 - S_4S_4) + [(N + \overline{N}) - (D + \overline{D})]^2 (O_4V_4 - C_4C_4). \]

Thus, we can mod out by T-duality, identifying the Neumann and Dirichlet charges\(^3\), and obtain the amplitude

\[ \tilde{\mathcal{A}} \sim (N + \overline{N})^2(V_4O_4 - S_4S_4), \]

\(^2\) This corrects a mistake in [6] in the counting of multiplicities for the open sector.

\(^3\) In our conventions $N \equiv D$ ($N \equiv \overline{D}$) corresponds to model 1(2).
We now study the limiting behaviour for large and small radius. As $R \to 0$, inspection of the transverse amplitudes reveals that new tadpoles arise due to odd windings, that become massless. These new tadpoles receive a contribution also from the transverse Klein bottle amplitude. Trying to impose local tadpole cancellation, one then finds $M - \bar{M} \neq 0$, in contrast with (4.8). This incompatibility cannot be resolved by adding Wilson lines as in the case of toroidal and supersymmetric orbifold compactifications. Thus, for small $R$, the local non-vanishing tadpoles induce linear divergences in the 5d gauge theory on the branes [10].

The situation is different for $R \to \infty$. After doubling the radius, the sums over momenta with alternating signs now vanish, while the others give additional factors of two. In the closed unoriented sector this leads to a massless spectrum consisting of 5 tensor multiplets and 16 hypermultiplets coupled to $\mathcal{N} = (1,0)$ supergravity in six dimensions. The interpretation of the open unoriented sector is more subtle. After taking the limit $R \to \infty$

$$A \sim 2[2M\bar{M}(V_4O_4 - S_4S_4) + (M^2 + \bar{M}^2)(O_4V_4 - C_4C_4) + 2(M^2 + \bar{M}^2)(O_4S_4 - C_4O_4) + 2M\bar{M}(V_4C_4 - S_4V_4)],$$

and

$$\mathcal{M} \sim -2(M + \bar{M})(\hat{O}_4\hat{V}_4 - \hat{C}_4\hat{C}_4).$$

The factor of two reveals a doubling of degrees of freedom that reflects a doubling of CP charges, $M \to (N, D)$. Then, consistency of the model leads to the following amplitudes

$$A \sim (2N\bar{N} + 2D\bar{D})(V_4O_4 - S_4S_4) + (N^2 + \bar{N}^2 + D^2 + \bar{D}^2)(O_4V_4 - C_4C_4) + 2(2N\bar{D} + 2N\bar{D})(O_4S_4 - C_4O_4) + 2(2N\bar{D} + 2N\bar{D})(V_4C_4 - S_4V_4),$$

and

$$\mathcal{M} \sim -(N + \bar{N} + D + \bar{D})(O_4V_4 - C_4C_4).$$

All these results would be obtained directly starting from the limiting torus amplitude.

The massless spectrum consists of $\mathcal{N} = (1,0)$ vector multiplets in the adjoint of $U(8) \otimes U(8)$, with hypermultiplets in the representations $(28; 1) \oplus (28; 1) \oplus (1; 28) \oplus (1; 28) \oplus (8; \bar{8}) \oplus (\bar{8}; 8)$. This is precisely the spectrum associated to the $T^4/Z_2$ orbifold in the presence of a quantized $B_{IJ}$ [3, 19, 20]. The additional tensor multiplets take part in a generalized Green-Schwarz mechanism for the cancellation of the residual anomalies [25].
5. Supersymmetry breaking on the heterotic side

An interesting question is whether the open sector of the model discussed above remains supersymmetric at the non-perturbative level. A way to address this question would be to find a heterotic dual. Before breaking supersymmetry with the freely acting projection, the presence of a quantized $B_{IJ}$ in the $T^4/Z_2$ suggests that a possible heterotic dual should have reduced rank [18,26]. This is possible only for the type $I'$ description which is related to type IIA and has only one tensor multiplet. Moreover, local tadpole cancellation requires to separate the branes so that the resulting gauge group is $U(4)^2_{88} \otimes U(4)^2_{44}$. However, in the presence of supersymmetry breaking there is no perturbative heterotic dual, since the action of the freely acting projection identifies the D8 with the D4 branes on the type $I'$ side.

Nevertheless, one can address the question of supersymmetry breaking in the non-perturbative gauge sector of the type II model of [4], by analysing the partition function of its heterotic dual. The starting point of the construction is the compactification on the Narain lattice:

$$\Gamma_{(5,21)} = \Gamma_{E_8} \oplus \Gamma_{E_8} \oplus \Gamma_{SO(8)} \oplus \Gamma_{(1,1)}(R),$$

where the $\Gamma_{E_8}$ factors refer to the affine $\hat{E}_8$ algebra, $\Gamma_{SO(8)}$ denotes the four dimensional compactification torus at the $SO(8)$ symmetry enhancement point and $\Gamma_{(1,1)}(R)$ denotes the compactification on a circle of radius $R$. In terms of $SO(8)$ and $E_8$ characters, the partition function is then

$$Z_{\text{het}}^{(0)} = \frac{1}{\eta^4 \bar{\eta}^4} (V_8 - S_8) \chi_{E_8} \chi_{E_8} \left[ |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2 \right] Z_{m,n}.$$ 

The massless level contains the 5d $\mathcal{N} = 4$ supergravity multiplet coupled to vector multiplets with gauge group $E_8 \otimes E_8 \otimes SO(8)$.

The action of the non-supersymmetric orbifold generator $f_{\text{het}}$ on the heterotic side can be defined using heterotic–type IIA duality and the adiabatic argument [4]. It is given by

$$f_{\text{het}} = \left[ (1^5; -1^4, 1), (\delta^5; 0^4, \delta), S_2(E_8 \otimes E_8), (-)^F \right],$$

Strictly speaking, the type II model of [4] does not have points of enhanced non-abelian symmetry because of the presence of the NS $\otimes$ NS antisymmetric tensor in the SO(8) lattice. However, its heterotic dual can be deformed to a point of enhanced gauge symmetry that we discuss here.
with $\delta$ denoting the action of a $Z_2$ shift on the compactification lattice and $S_2$ the permutation of the two $\Gamma_{E_8}$ lattices \footnote{\textit{\textsuperscript{[ ]}}}. The resulting partition function is

$$
Z_{\text{het}} = \frac{1}{2} \eta^4(q) \eta^4(\bar{q}) \left[ (V_8 - S_8)(q) (\chi_{E_8} \chi_{E_8}) \bar{q} \right] \left[ |O_4 O_4 + V_4 V_4|^2 + |V_4 O_4 + O_4 V_4|^2 + |C_4 C_4 + S_4 S_4|^2 + |S_4 C_4 + C_4 S_4|^2 \right] Z_{m,n}(q, \bar{q}) +
+ (V_8 + S_8)(q) \chi_{E_8} \chi_{E_8} (q^2) \left[ O_4 O_4 - V_4 V_4 \right]^2 \left[ (-)^m Z_{m,n}(q, \bar{q}) \right] +
+ \frac{1}{4} (O_8 - C_8)(q) \chi_{E_8} \chi_{E_8} (\sqrt{q}) \left[ (O_4 + V_4)(S_4 + C_4) \right] +
+ (S_4 + C_4)(O_4 + V_4)^2 \left[ Z_{m,n+1/2}(q, \bar{q}) \right] +
+ \frac{1}{4} (O_8 + C_8)(q) \chi_{E_8} \chi_{E_8} (-\sqrt{q}) \left[ (O_4 - V_4)(S_4 + C_4) \right] +
+ \left( S_4 + C_4 \right)^2 (O_4 - V_4)^2 \left[ (-)^m Z_{m,n+1/2}(q, \bar{q}) \right].
$$

As a result of the shift on the circle, the twisted sector is massive, while the $q$-expansion of the untwisted sector

$$
(16 + 128q + \ldots)(q^{-1} + 252q + \ldots)(-)^m Z_{m,n},
$$

reveals a Fermi-Bose degeneracy for the massless states, because of the absence of the $q^0$ term. It is interesting to study the contributions of the various massless states to the vacuum energy. Expanding each term of the untwisted sector

$$
Z_{\text{het}}^{\text{untw}} \sim q^{-1/2} \bar{q}^{-1} (1 + 4\bar{q})(4 + 4q^{1/2} - 2(2 + 2)q^{1/2})(1 + (248 + 248)\bar{q}) \times
\times [(1 + 6\bar{q})(1 + 6q) + 4 \times 4\bar{q}]+
+ q^{-1/2} \bar{q}^{-1} (1 + 4\bar{q})(4 + 4q^{1/2} + 2(2 + 2)q^{1/2})(1 + (248 - 248)\bar{q}) \times
\times [(1 + 6\bar{q})(1 + 6q) - 4 \times 4\bar{q}],
$$

one can see that the contribution of the massless states to the vacuum energy vanishes for the following reasons. On the one hand, in the “gravitational sector”, whose massless excitations are $\{g_{\mu\nu}, B_{\mu\nu}, \phi, 4A_{\mu}\}, \{A_{\mu}, 4\phi\}$ and the spinors $\{\psi_\alpha, \bar{\psi}_\alpha\}$ in the adjoint and $(4, 4)$ representations of $\text{SO}(4) \otimes \text{SO}(4)$, respectively, the $f_{\text{het}}$ projection breaks effectively supersymmetry, while preserving Fermi-Bose degeneracy. On the other hand, the “gauge sector” is effectively supersymmetric. In fact, the projection gives a plus sign to the space-time bosons $V_8$ and to the diagonal combination of the two $E_8$ factors and a minus sign to the space-time fermions $S_8$ and to the antisymmetric combination of the gauge factors. As a result, a full $\mathcal{N} = 4$ vector supermultiplet in the adjoint of $E_8$ survives the orbifold
projection. This property, however, holds only for massless states. For instance, at the first mass level the decomposition of the product of two adjoint representations,

\[ 248 \otimes 248 = 1_s \oplus 248_a \oplus 3875_s \oplus 27000_s \oplus 30380_a , \]

reveals that bosons and fermions in the “gauge sector” appear in symmetric and antisymmetric representations, respectively, and thus do not fit any more into supermultiplets.

By heterotic–type II duality, one can then argue that supersymmetry remains unbroken for the massless non-abelian gauge sector that arises non-perturbatively at singular points of K3 from D2 branes wrapped around collapsing 2-cycles. This phenomenon is similar to the “M-theory breaking” of type I models with the direction of supersymmetry breaking transverse to the D-brane \[9\].

6. Free-fermions and open strings

An alternative approach to the construction of non-supersymmetric vacua with vanishing cosmological constant is based on the fermionic construction \[12\]. Using this approach, the authors of \[5\] constructed a series of non-supersymmetric models in \(D = 4\) with Fermi-Bose degeneracy at each mass level. They found two different classes of models related to asymmetric \(Z_2 \otimes Z_2\) orbifolds. In the first class the two \(Z_2\) twists break \(\mathcal{N} = (4, 4)\) supersymmetry to \(\mathcal{N} = (2, 0)\) and \(\mathcal{N} = (0, 2)\), respectively. In the second class supersymmetry is broken to \(\mathcal{N} = (2, 0)\) for the first \(Z_2\) and to \(\mathcal{N} = (0, 4)\) for the second one. Although the full \(Z_2 \otimes Z_2\) model is not supersymmetric, each projection is supersymmetric thus ensuring the vanishing of the cosmological constant. Only model I in \[5\] is left-right symmetric and can therefore be modded out by \(\Omega\) to construct open descendants.

The construction of the four dimensional \(Z_2 \otimes Z_2\) orbifold starts with the type IIB superstring compactified on the SO(12) lattice whose partition function is

\[ T = |V_8 - S_8|^2 \left[ |O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2 \right]. \]

Following \[12\], the \(Z_2 \otimes Z_2\) projection can be defined by adding the following sets of periodic fermions

\[ S = \{ \chi^I, \omega^I \}, \quad S' = \{ \chi^I, y^I \}, \quad \mathcal{S} = \{ \bar{\chi}^I, \bar{\omega}^I \}, \quad \mathcal{S}' = \{ \bar{\chi}^I, \bar{y}^I \}, \]
to those that define the SO(12) lattice, where $I = 1, \ldots, 4$ labels the coordinates of the internal $T^4$ on which the orbifold generators act non-trivially. In the orbifold language this translates into the generators:

$$f = \left[(-1^4, 1^2; 1^6), (0^6; s^4, 0^2), (-)^{F_R}\right]$$

$$g = \left[(1^6; -1^4, 1^2), (s^4, 0^2; 0^6), (-)^{F_L}\right],$$

where $s$ is a $Z_2$ shift on the internal bosons. As a result, the partition function of the orbifold model can be arranged into 64 characters. They can be generated from the identity

$$\chi_{0,1} = V_4O_4O_4O_4O_4 + O_4V_4O_4V_4V_4 - S_4S_4V_4O_4V_4 - C_4C_4V_4V_4O_4.$$ 

by applying a modular $S$ transformation. Besides $\chi_{0,1}$, the massless characters in the untwisted sector are:

$$\chi_{0,2} = V_4O_4V_4O_4V_4 + O_4V_4V_4O_4 - S_4S_4O_4O_4 - C_4C_4O_4V_4V_4,$$

$$\chi_{0,9} = V_4O_4O_4V_4V_4 + O_4V_4O_4O_4 - S_4S_4V_4V_4O_4 - C_4C_4V_4O_4V_4,$$

$$\chi_{0,10} = V_4O_4V_4V_4O_4 + O_4V_4V_4O_4V_4 - S_4S_4O_4V_4V_4 - C_4C_4O_4O_4O_4.$$ 

In the $f$-twisted sector, they are given by

$$\chi_{f,1} = O_4C_4O_4O_4C_4 + V_4V_4O_4V_4C_4 - C_4V_4V_4O_4C_4 - S_4O_4V_4V_4S_4,$$

$$\chi_{f,2} = O_4C_4V_4V_4S_4 + V_4S_4V_4O_4C_4 - C_4V_4V_4O_4C_4 - S_4O_4O_4O_4S_4,$$

$$\chi_{f,3} = O_4C_4V_4V_4C_4 + V_4S_4O_4V_4S_4 - C_4V_4V_4O_4S_4 - S_4O_4O_4O_4C_4,$$

$$\chi_{f,4} = O_4C_4O_4O_4C_4 + V_4S_4V_4O_4S_4 - C_4V_4V_4O_4S_4 - S_4O_4V_4V_4C_4,$$

in the $g$-twisted sector by

$$\chi_{g,1} = O_4O_4O_4C_4C_4 + V_4V_4O_4S_4S_4 - S_4C_4V_4C_4S_4 - C_4S_4V_4S_4C_4,$$

$$\chi_{g,2} = O_4O_4O_4S_4S_4 + V_4V_4O_4C_4C_4 - S_4C_4V_4C_4S_4 - C_4S_4V_4C_4S_4,$$

$$\chi_{g,3} = O_4O_4O_4S_4C_4 + V_4V_4O_4C_4S_4 - S_4C_4V_4C_4S_4 - C_4S_4V_4S_4S_4,$$

$$\chi_{g,4} = O_4O_4O_4C_4S_4 + V_4V_4O_4S_4C_4 - S_4C_4V_4S_4S_4 - C_4S_4V_4C_4C_4,$$

and, finally, in the $fg$-twisted sector by

$$\chi_{fg,1} = O_4S_4O_4C_4O_4 + V_4C_4O_4S_4V_4 - S_4V_4V_4S_4O_4 - C_4O_4V_4C_4V_4,$$

$$\chi_{fg,2} = O_4S_4V_4C_4V_4 + V_4C_4V_4S_4O_4 - S_4V_4O_4S_4V_4 - C_4O_4O_4C_4O_4,$$

$$\chi_{fg,9} = O_4S_4O_4S_4O_4 + V_4C_4O_4C_4V_4 - S_4V_4V_4C_4O_4 - C_4O_4V_4S_4V_4,$$

$$\chi_{fg,10} = O_4S_4V_4S_4V_4 + V_4C_4V_4C_4O_4 - S_4V_4O_4C_4V_4 - C_4O_4O_4S_4O_4.$$
These massless characters appear in the partition function as follows

$$\mathcal{T}_{m^2=0} = |\chi_{0,1}|^2 + |\chi_{0,2}|^2 + \chi_{0,9}\bar{\chi}_{0,10} + \chi_{0,10}\bar{\chi}_{0,9} +$$

$$+ (\chi_{f,1}\bar{\chi}_{g,1} + \chi_{f,2}\bar{\chi}_{g,2} + \chi_{f,3}\bar{\chi}_{g,3} + \chi_{f,4}\bar{\chi}_{g,4} + c.c.) +$$

$$+ |\chi_{fg,1}|^2 + |\chi_{fg,2}|^2 + \chi_{fg,9}\bar{\chi}_{fg,10} + \chi_{fg,10}\bar{\chi}_{fg,9}.$$ 

The 4-dimensional massless spectrum resulting from the above expression yields \{\(g_{\mu\nu}, B_{\mu\nu}, 8A_\mu, 13\phi, 16\psi, 32\phi; 16\psi\)\} for the untwisted sector, \{32\phi; 16\psi\} for the \(f\) and \(g\) twisted sectors and \{4A_\mu, 24\phi; 16\psi\} for the \(fg\)-twisted sector.

In order to construct the open descendants, one needs the characters that appear in the partition function symmetrically in the holomorphic and in the antiholomorphic sectors or combined with their conjugates. Since in our case all characters are self-conjugate the only relevant contribution to the torus partition function reads:

$$\mathcal{T}_{\text{diag}} = \sum_{\alpha \in \{0, fg\}} \sum_{\ell=1}^{8} |\chi_{\alpha,\ell}|^2,$$ 

(6.1)

with

$$\chi_{0,3} = V_4 O_4 V_4 O_4 V_4 + O_4 V_4 V_4 V_4 V_4 - S_4 S_4 O_4 O_4 V_4 - C_4 C_4 O_4 V_4 V_4,$$
$$\chi_{0,4} = V_4 O_4 O_4 O_4 V_4 + O_4 V_4 O_4 V_4 V_4 - S_4 S_4 V_4 O_4 V_4 - C_4 C_4 V_4 V_4 V_4,$$
$$\chi_{0,5} = V_4 O_4 S_4 C_4 S_4 + O_4 V_4 S_4 S_4 S_4 - S_4 S_4 C_4 S_4 C_4 - C_4 C_4 S_4 C_4,$$
$$\chi_{0,6} = V_4 O_4 C_4 C_4 S_4 + O_4 V_4 C_4 S_4 C_4 - S_4 S_4 C_4 C_4 S_4 - C_4 C_4 S_4 S_4 C_4,$$
$$\chi_{0,7} = V_4 O_4 C_4 C_4 C_4 + O_4 V_4 C_4 S_4 S_4 - S_4 S_4 S_4 C_4 S_4 - C_4 C_4 S_4 S_4 C_4,$$
$$\chi_{0,8} = V_4 O_4 S_4 C_4 S_4 + O_4 V_4 S_4 S_4 C_4 - S_4 S_4 C_4 C_4 S_4 - C_4 C_4 C_4 S_4 S_4$$

from the untwisted sector, and

$$\chi_{fg,3} = O_4 S_4 O_4 C_4 V_4 + V_4 C_4 O_4 S_4 O_4 - S_4 V_4 V_4 S_4 V_4 - C_4 O_4 V_4 C_4 O_4,$$
$$\chi_{fg,4} = O_4 S_4 V_4 C_4 O_4 + V_4 C_4 V_4 S_4 V_4 - S_4 V_4 O_4 S_4 O_4 - C_4 O_4 O_4 C_4 V_4,$$
$$\chi_{fg,5} = O_4 S_4 S_4 O_4 S_4 + V_4 C_4 S_4 V_4 C_4 - S_4 V_4 C_4 V_4 S_4 - C_4 O_4 C_4 O_4 C_4,$$
$$\chi_{fg,6} = O_4 S_4 C_4 O_4 C_4 + V_4 C_4 C_4 V_4 S_4 - S_4 V_4 S_4 V_4 C_4 - C_4 O_4 S_4 O_4 S_4,$$
$$\chi_{fg,7} = O_4 S_4 C_4 O_4 S_4 + V_4 C_4 C_4 V_4 C_4 - S_4 V_4 S_4 V_4 S_4 - C_4 O_4 S_4 O_4 C_4,$$
$$\chi_{fg,8} = O_4 S_4 S_4 O_4 C_4 + V_4 C_4 S_4 V_4 S_4 - S_4 V_4 C_4 V_4 C_4 - C_4 O_4 C_4 O_4 S_4,$$

5 SO(2n) characters are self-conjugate for even \(n\).
from the $fg$-twisted sector. The $f$ and $g$-twisted sectors do not contribute to the Klein bottle, annulus and Möbius strip amplitudes, because they give rise to non diagonal contributions in the partition function.

We can now proceed to construct the open descendants following [7,8]. The direct channel Klein bottle amplitude is:

$$\mathcal{K} = \sum_{\alpha \in \{0, fg\}} \sum_{\ell = 1}^{8} \chi_{\alpha, \ell}.$$ 

The massless excitations of the closed unoriented sector comprise the graviton, 4 abelian vectors, 54 scalars and 32 Dirac spinors. The Klein bottle amplitude has $\mathcal{N} = 2$ supersymmetry and therefore does not generate any cosmological constant. In the transverse channel

$$\tilde{\mathcal{K}} = 2^3 (\chi_{0,1} + \chi_{0,2} + \chi_{fg,3} + \chi_{fg,4}),$$

develops massless tadpoles, whose cancellation require the introduction of open strings.

In principle, one is free to introduce in the Klein bottle amplitude signs that are consistent with the crosscap constraint [27][28] (see also [29]). The choice which includes eight positive and eight negative signs results in a model without an open sector, similarly to what was found in [30]. This follows from the unitarity of the $S$ matrix, that implies that each character transforms into $\chi_{0,1}$ with a positive sign, so that the transverse Klein bottle does not contain the identity and has no IR divergences. The only possible solution is an open descendant without open strings.

Inspection of (6.1) allows for 16 different CP charges. The transverse channel annulus thus reads

$$\tilde{\mathcal{A}} = 2^{-3} \sum_{\alpha \in \{0, fg\}} \sum_{\ell = 1}^{8} B_{\alpha, \ell}^2 \chi_{\alpha, \ell},$$

while the transverse Möbius amplitude is

$$\tilde{\mathcal{M}} = 2 \left[ \epsilon_{0,1} B_{0,1} \tilde{\chi}_{0,1} + \epsilon_{0,2} B_{0,2} \tilde{\chi}_{0,2} + \epsilon_{fg,3} B_{fg,3} \tilde{\chi}_{fg,3} + \epsilon_{fg,4} B_{fg,4} \tilde{\chi}_{fg,4} \right],$$

where the boundary-to-boundary coefficients $B_{\alpha, \ell}$ are 16 orthogonal combinations of the CP multiplicities and $\epsilon_{\alpha, \ell}$ are signs. The solution of the inhomogeneous tadpole conditions

$$B_{0,1} = \sum_{\ell = 1}^{8} n_{0, \ell} + \sum_{\ell = 1}^{8} n_{fg, \ell} = -8 \epsilon_{0,1},$$

$$B_{0,2} = \sum_{\ell = 1}^{8} n_{0, \ell} - \sum_{\ell = 1}^{8} n_{fg, \ell} = -8 \epsilon_{0,2},$$
then requires that the $n_{fg}$ charges vanish identically and fixes the signs $\epsilon_{0,1} = -1 = \epsilon_{0,2}$ in the Möbius strip amplitude.

In the direct channel, the annulus and Möbius strip amplitudes can be cast into $\mathcal{N} = 2$ supersymmetric extended characters, thus ensuring the vanishing of the cosmological constant also in the open unoriented sector. Introducing a minimal set of charges, these amplitudes are

$$\mathcal{A} = 2n\mathbf{m}(\chi_{0,1} + \chi_{0,2} + \chi_{fg,3} + \chi_{fg,4}) + (n^2 + \mathbf{m}^2)(\chi_{0,3} + \chi_{0,4} + \chi_{fg,1} + \chi_{fg,2}),$$

and

$$\mathcal{M} = (n + \mathbf{m})(\tilde{\chi}_{0,3} + \tilde{\chi}_{0,4}) + (n - \mathbf{m})(\tilde{\chi}_{fg,1} + \tilde{\chi}_{fg,2}).$$

The corresponding massless excitations are an $\mathcal{N} = 2$ vector multiplet in the adjoint representation of $U(4)$ and one hypermultiplet in the $10 \oplus 6$ representations. The reduction of the rank of the CP gauge group is due to the presence of a rank four antisymmetric tensor in the definition of the $SO(12)$ lattice [17].

7. Conclusion

In this paper we studied open descendants of non-supersymmetric type IIB compactifications with zero cosmological constant. The construction of the parent closed string theory is based on a freely acting orbifold that resembles an asymmetric Scherk-Schwarz deformation. Whereas supersymmetry is broken in the bulk, the open sector remains supersymmetric at all mass levels. An interesting open question concerns the radiative corrections induced by the non-supersymmetric bulk in the supersymmetric open string spectrum. In particular, it would be interesting to understand the magnitude of the induced mass splittings, and whether these splittings preserve the property of the vanishing vacuum energy.

A possible application of the constructions discussed here is to models with a low string scale and supersymmetry breaking by large dimensions [31,32,21]. An immediate limitation, however, is that these constructions allow only one free internal dimension, since four of them are fixed at the fermionic point, while the fifth one determines the scale of supersymmetry breaking in the bulk. One possibility within this limitation is, for instance, to take the compactification scale of the fifth dimension close to the string scale at intermediate energies, in such a way that the (gravitationally) induced mass splittings
on the brane are of the order of a TeV. This may be possible by adjusting the size of the remaining (free) transverse dimension, that can be as large as a millimeter, implying a string scale as low as $10^8$ GeV. Furthermore, in order to avoid large (linearly divergent) corrections to the effective field theory, one should impose local tadpole cancellation, that leads to additional constraints in model building \[10\]. New possibilities, however, may arise if some of the four fixed internal radii are liberated in more general constructions.

In the context of type II theories with low string scale \[33\], it is also interesting to study the closed string models with vanishing vacuum energy at special points of moduli space, where non-perturbative gauge symmetries appear. In particular, one should understand the effects of supersymmetry breaking in the non-perturbative gauge sector of the theory. In this work, we argued that supersymmetry remains unbroken for the massless excitations to lowest order, by analysing the partition function of the heterotic dual. These models are particularly attractive since the induced non-perturbative cosmological constant will be exponentially suppressed in the weak coupling limit.

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