Role of compressive viscosity and thermal conductivity on the damping of slow waves in the coronal loops with and without heating cooling imbalance

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Abstract In the present paper, we derive a new dispersion relation for the slow magnetoacoustic waves invoking the effect of thermal conductivity, compressive viscosity, radiation and unknown heating term along with the consideration of heating cooling imbalance from linearized MHD equations. We solve the general dispersion relation to understand the role of compressive viscosity and thermal conductivity in damping of the slow waves in the coronal loops with and without heating cooling imbalance. We have analyzed the wave damping for the range of loop length $L=50$-$500$ Mm, temperature $T=5$-$30$ MK, and density $\rho=10^{-11}$-$10^{-9}$ kg m$^{-3}$. It was found that the inclusion of compressive viscosity along with the thermal conductivity significantly enhances the damping of the fundamental mode oscillations in shorter (e.g., $L=50$ Mm) and super-hot ($T >10$ MK) loops. However, the role of the viscosity in damping is insignificant in longer (e.g., $L=500$ Mm) and hot loops ($T \leq10$ MK) where, instead, thermal conductivity along with the presence of heating cooling imbalance plays a dominant role. For the shorter loops at the super-hot regime of the temperature, the increment in the loop density substantially enhances damping of the fundamental modes due to the thermal conductivity when the viscosity is absent, however, when the compressive viscosity is added the increase in density substantially weakens the damping. Thermal conductivity alone is found to play a dominant role in longer loops at the lower temperatures ($T \leq10$ MK), while compressive viscosity dominates in damping at super-hot temperatures ($T >10$ MK) in shorter loops. The predicted scaling law between damping time ($\tau$) and wave period ($P$) is
found to better match to the observed SUMER oscillations when the heating cooling imbalance is taken into account in addition to the thermal conductivity and compressive viscosity for the damping of the fundamental slow mode oscillations.

**Keywords:** Flares, Dynamics; oscillations and waves, MHD; Magnetic fields, Corona

## 1. Introduction

The Doppler shift oscillations in hot coronal loops are first directly observed by the SUMER spectrograph onboard Solar and Heliospheric Observatory (SoHO) (Wang et al., 2002, 2003a). These observations are found to be associated with the fundamental mode of the slow magnetoacoustic oscillations exhibiting an efficient damping (Ofman and Wang, 2002). Wang et al. (2003b) have detected such slow-mode oscillations in numerous hot coronal loops, and using a large enough statistics established their physical properties consistently, e.g., the phase speed derived from observed period and loop length matches approximately to the local sound speed; the intensity fluctuation lags the Doppler shift by 1/4 period. It was also found that the observed scaling of the damping time with the wave period matches to the predicted scaling for slow waves when the damping effects due to thermal conduction and compressive viscosity are considered. The numerical modeling by Taroyan and Bradshaw (2008) showed that such oscillations are also expected to be detected in the normal coronal loops maintained at 1-2 MK temperature, and later this supposition was confirmed with observations from EUV Imaging Spectrometer (EIS) onboard Hinode (Mariska et al., 2008; Erdélyi and Taroyan, 2008; Srivastava and Dwivedi, 2010). Damped slow-mode oscillations are also suggested to be produced in stellar flaring loops because some quasi-periodic pulsations (QPPs) detected in stellar flares show many features similar to those observed in solar flares (Mitra-Kraev et al., 2005; Srivastava, Lalitha and Pandey, 2013; Cho et al., 2016).

The SUMER and Yohkoh/SXT observations suggested that such loop oscillations may be associated with an impulsive deposition of the heat at its footpoint due to the localized transient events (e.g., micro-flares) that may lead a perturbations both in velocity and density within the loop (Wang et al., 2003). While impulsive heating is proposed as a primary exciter of the slow magnetoacoustic oscillations in solar loops (Wang et al., 2005; Patourakos and Klimchuk, 2006; Taroyan et al., 2007), there are several other mechanisms that may also trigger it, e.g., pressure or velocity pulses, kink instability, etc (e.g., Selwa, Murawski and Solanki, 2005; Selwa, Ofman and Murawski, 2007; Haynes, Arber and Verwichte, 2008, and references cited there). Additionally, the pressure pulse and flows may also inevitably be associated with a response of the impulsive heating. Since such loops may cool after the transient energy release and heating, therefore, the thermal conduction is termed initially as a viable dissipation mechanism for interpreting the observed damping of the slow magnetoacoustic oscillations (Ofman and Wang, 2002). The effect of dissipative
agents, e.g., thermal conduction, compressive viscosity, and radiation have been studied on the slow magnetoacoustic oscillations in a greater detail by Pandey and Dwivedi (2006). They have found that by varying density from $10^8$ to $10^{10}$ cm$^{-3}$ at a fixed temperature in the range 6-10 MK as observed by SUMER, strong damping occurs at lower density and weak damping occurs at higher density. It was also noted that the effect of optically thin radiation provides some additional dissipation apart from thermal conductivity and viscosity in weak-damped oscillations. Mendoza-Briceño, Erdélyi and Sigalotti (2004) have pointed out the effect of stratification on the wave damping, and concluded that the dissipation rates of slow waves by thermal conduction and compressive viscosity are enhanced by the nonlinear effect caused by the gravitational stratification. Later, Sigalotti, Mendoza-Briceño and Luna-Cardozo (2007) argued that thermal conduction alone cannot produce the strong damping as observed in the SUMER oscillations, while the inclusion of compressive viscosity is required.

Bradshaw and Erdélyi (2008) have reported the effect of the radiative emission arising from a non-equilibrium ionization on the damping of the slow magnetoacoustic oscillations in the loops, and inferred that this loss may reduce the damping timescale by up to 10% than that typically observed by SUMER. Haynes, Arber and Verwichte (2008) have studied the fact that even in the absence of thermal conduction, the large-amplitude slow-mode oscillations are getting damped strongly by the dissipation of the shocks. Verwichte et al. (2008) have further found that in the presence of thermal conduction, shock dissipation at large amplitudes enhances the damping rate by 50% higher than the rate achieved during the presence of only the thermal conduction enabled dissipation. Erdélyi, Luna-Cardozo and Mendoza-Briceño (2008) have reported the damping scenario of standing slow waves in nonisothermal, hot, gravitationally stratified coronal loops, and established the physical fact that the decay time of waves decreases with the increase of the initial temperature. Al-Ghafri and Erdélyi (2013) have found that although the background plasma is cooling, thermal conduction is still found to cause a strong damping for the slow magnetoacoustic oscillations in hot coronal loops. However, they consider only the effect of thermal conduction that has weak damping effect at very higher temperature. Therefore, when the loop cools from superhot to hot regime close to a maximum damping temperature, then damping due to conduction increases with the cooling (De Moortel and Hood, 2003). Kumar, Nakariakov and Moon (2016) have presented a theoretical model of the standing slow magnetoacoustic mode and found that these modes are highly sensitive to the radiative cooling and choice of the heating function also.

It was found that the thermal conductivity is nearly suppressed and compressive viscosity is enhanced by more than an order of magnitude in the very hot loops, and affecting the dissipation of the various harmonics of the slow magnetoacoustic oscillations (Wang et al., 2013, 2018). Many previous studies on the dynamics of compressive MHD wave modes in coronal loops have described the significance of the thermal equilibrium along with the mechanical equilibrium in the wave guiding magnetic structures (e.g., Kumar, Nakariakov and Moon, 2016; Nakariakov et al., 2017). Various representative theories, on the physical processes that balance the internal energy losses to maintain thermal
equilibrium, have been given to explain the unknown mechanism underlying the observed coronal heating. For a long time, it has been termed as the coronal heating problem (e.g., Parnell and De Moortel, 2012). Although it is established that a lot of development has been made to understand different heating and cooling mechanisms in coronal loops, however, no explanation has proven to be accepted preferably. Since the specific heating mechanism is unknown, therefore, many previous theoretical models of MHD waves in coronal loops have taken into account the heating function to depend upon the loop parameters, e.g., temperature, density, and magnetic field, and static or time-dependent heating function, more specifically in the form of power law functions (e.g., Rosner, Tucker, and Vaiana, 1978; Reale, 2014; Kolotkov, Nakariakov and Zavershinskii, 2019, and references cited there). Compressive and longitudinal MHD wave modes change the local thermal equilibrium by locally perturbing the background equilibrium quantities such as density, temperature, pressure which ultimately leads to a heating cooling imbalance. This physical scenario is inferred as the imbalance between the equilibrium balance of radiative cooling losses and unknown coronal heating. This affects the slow wave by modifying the dispersion relation and can attribute to the suppressed or enhanced damping of the waves. Many previous theoretical studies have been concentrated on the effects of this imbalance in the limit of weak non-adiabacity. However more recent work by Kolotkov, Nakariakov and Zavershinskii (2019) have removed this approximation by taking into account the non-adiabatic terms to be arbitrarily large, and we have followed a similar procedure in our present model.

As we mentioned above, significance of heating and cooling imbalance and its effect on the properties of slow magnetoacoustic oscillations is recently studied by Kolotkov, Nakariakov and Zavershinskii (2019). They have studied the damping of standing slow magnetoacoustic oscillations in the solar coronal loops by taking into account the field-aligned thermal conductivity and a wave-induced misbalance between radiative cooling and some unspecified heating rates, and found that the slow wave dynamics is highly sensitive to the characteristic timescales of the thermal misbalance. Wang and Ofman (2019) have recently shown the significance of suppressed thermal conductivity and enhanced effect of the compressive viscosity that determine damping properties of the magneto-acoustic oscillations in hot loops at ~10 MK. In the present paper, we have derived a new dispersion relation taking into account the compressive viscosity, thermal conduction, and radiative cooling as dissipative mechanisms, and an appropriate heating function in order to heat the coronal loop plasma. We consider the coronal loops for a wide range of length (50-500 Mm), temperature (5-30 MK), and density ($10^{-11}$–$10^{-9}$ kg m$^{-3}$) with and without heating cooling misbalance to understand the evolutionary and dissipative properties of the slow magnetoacoustic oscillations. We refer to hot loops as the loops hosting the SUMER oscillations with T=5-15 MK (including the Doppler shift oscillations observed with Yohkoh/BCS and longitudinal oscillations observed with SDO/AIA) and Super-hot loops with T=20-30 MK is referred to the RHESSI detected oscillations or QPPs in flares by Cho et al. (2016). Moreover, we have chosen the damped oscillatory regime of the loop’s magnetoacoustic oscillations both with and without heating cooling imbalance, and also perform the detailed analytical study of the effect of
compressive viscosity and thermal conductivity on the damping of slow waves. We have also compared their individual roles in the damping of slow waves under the consideration of heating cooling imbalance. We also obtain new scaling relations between damping-time ($\tau$) and wave-period ($P$) from the results of these studies, and compare the theoretical results and various scaling laws with the observed damped SUMER oscillations. In Sect.2, we present the basic model and dispersion relation. The numerical solution and related results are described in Sect. 3. Last section depicts the discussion and conclusions.

2. Analytical Model and New Dispersion Relation

The description of the model of slow magnetoacoustic oscillations in the hot and denser coronal loop is given below. It depicts the properties of slow waves in the viscous, thermal conductive, and radiative plasma with a certain heating, as well as with and without considering the effect of heating cooling imbalance.

2.1. Basic MHD Equations

We consider the effects of thermal conductivity, imbalance of radiative cooling and unknown coronal heating, dissipative viscous forces, and heating in our model. For the generalization of the results we compare the model results with and without heating-cooling imbalance also. To exclude the heating cooling imbalance, we consider the constant heating term in our model. To the best of our knowledge, this is the first effort here to perform such a detailed analytical calculation. Apart from these effects, we have the infinite magnetic field approximation under which the perturbations are confined along the rigid magnetic field lines. We derive a new most general dispersion relation for the slow magnetoacoustic waves in the non-ideal coronal loop plasma. The governing magnetohydrodynamic equations in 1-D are given as follows:

Mass Equation-

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial z} = 0 \quad (1)$$

Momentum Equation-

$$\rho \left( \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial z} \right) + \frac{\partial p}{\partial z} - \frac{4}{3} \frac{\partial}{\partial z} \left( \eta_0 \frac{\partial V}{\partial z} \right) = 0 \quad (2)$$

Here $\eta_0$ is the coefficient of compressive viscosity.

Energy Equation-

$$C_v \left( \frac{\partial T}{\partial t} + V \frac{\partial T}{\partial z} \right) - \left( \frac{k_B T}{\rho m} \right) \left( \frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial z} \right) =$$

$$- Q(\rho, T) + \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho \frac{\partial T}{\partial z} \right) + \frac{4 \mu}{3} \left( \frac{\partial V}{\partial z} \right)^2 \quad (3)$$
Here $C_v$ is defined as

$$C_v = \frac{k_B}{m(\gamma - 1)}$$

(4)

and $Q(\rho, T)$ is composed of two functions

$$Q(\rho, T) = L(\rho, T) - H(\rho, T)$$

(5)

$$L(\rho, T) = \chi \rho T^\alpha$$

(radiative cooling function)

(6)

$$H(\rho, T) = h \rho^a T^b$$

(unknown heating function),

(7)

where $a, b$ are power index factors and $h, \chi, \alpha$ are respectively the unknown heating coefficient and coefficients to radiative cooling term while $\kappa$ is thermal conductivity.

Gas Equation-

$$p = \rho k_B T \frac{m}{m_{p}}$$

(8)

where $k_B$ is the Boltzmann constant and $m$ is the mean particle mass equal to $0.6 m_p$ where $m_p$ is the mass of proton.

Further we take the temperature dependence of viscosity as (Braginskii, 1965)

$$\eta_0 = 10^{-17} T^{5/2} \text{ kg m}^{-1} \text{ s}^{-1}.$$  

(9)

We also take the temperature dependence of thermal conductivity as (Braginskii, 1965)

$$\kappa = 9 \times 10^{-12} T^{5/2} \text{ W m}^{-1} \text{ K}^{-1}.$$  

(10)

2.2. Linearised MHD Equations

In order to study the dynamics of slow magnetoacoustic waves, we consider linear perturbations in the basic state of plasma. In equilibrium, the plasma is isothermal with a constant uniform density and pressure throughout. It is also assumed to be stationary having zero velocity field and the gravitational effects are entirely ignored in the analysis.

$$p = p_0 + p_1$$  

(Pressure)

(11)

$$\rho = \rho_0 + \rho_1$$  

(Density)

(12)

$$T = T_0 + T_1$$  

(Temperature)

(13)

$$V = V_1$$  

(Velocity Field)

(14)
Further using these linear perturbations, we linearize the MHD equations as below

**Linearized Mass equation**

\[
\frac{\partial \rho_1}{\partial t} + \rho_0 \left( \frac{\partial V_1}{\partial z} \right) = 0
\]  

(15)

**Linearized Momentum equation**

\[
\rho_0 \left( \frac{\partial V_1}{\partial t} \right) + \frac{\partial p_1}{\partial z} = \left( \frac{4\eta_0}{3} \right) \left( \frac{\partial^2 V_1}{\partial z^2} \right)
\]

(16)

**Linearized Energy equation**

\[
\frac{\partial T_1}{\partial t} - \left( \frac{(\gamma - 1)T_0}{\rho_0} \right) \left( \frac{\partial \rho_1}{\partial t} \right) = \left( \frac{\kappa \rho_0 C_v}{\tau_2} \right) \left( \frac{\partial^2 T}{\partial z^2} \right) - \left( \frac{T_1}{\tau_2} \right) - \left( \frac{1}{\tau_1} \right) \left( \frac{T_0}{\rho_0} \right) \rho_1
\]

(17)

Here \( \tau_1 \) and \( \tau_2 \) are the corresponding characteristic time scales of the heating cooling mechanisms involved. They are defined as below by Kolotkov, Nakariakov and Zavershinskii (2019): 

\[
\tau_2 = \frac{C_v}{(\partial Q/\partial T)}
\]

\[
\tau_1 = \frac{\gamma C_v}{(\partial Q/\partial T - (\rho_0/T_0)(\partial Q/\partial \rho))}
\]

**Ideal gas equation**

\[
\frac{p_1}{p_0} = \frac{T_1}{T_0} + \frac{\rho_1}{\rho_0}
\]

(18)

### 2.3. Fourier analysis and new dispersion relation

We substitute the Fourier solutions of the form 

\[
F = \hat{F} e^{i(kz - \omega t)}
\]

(19)

to obtain our dispersion relation

\[
\omega^3 + A\omega^2 + B\omega + C = 0,
\]

(20)

where

\[
A = i \left( \frac{4\eta_0 k^2}{3\rho_0} + \frac{\kappa k^2}{\rho_0 C_v} + \frac{1}{\tau_2} \right)
\]
3. Numerical solutions of the dispersion relation

In order to solve our dispersion relation (Equation 20), we consider the following form of unknown heating function

$$H(\rho, T) = h\rho^{-1/2}T^{-3}$$  

where the power index factors $a, b$ are chosen as -0.5 and -3 respectively, which is associated with determining a good quality factor of the damped oscillatory slow magnetoacoustic oscillations during heating cooling imbalance (Kolotkov, Nakariakov and Zavershinskii, 2019) and $h$ is determined by the equilibrium condition initially i.e $Q(\rho_0, T_0) = 0$. Throughout the paper we shall use this form of heating function whenever heating cooling imbalance is to be considered, otherwise $H(\rho, T) = \text{constant}$ is assumed.

Many previous studies have investigated the damping of slow waves by assuming the unspecific heating term as constant ($H = \text{constant}$), which infers that the heating only plays a role in balancing the optically thin radiation loss in the equilibrium but does not contribute to the evolution of waves (Pandey and Dwivedi, 2006; Sigalotti, Mendoza-Briceño and Luna-Cardozo, 2007). These studies have demonstrated that the effect of radiation on slow wave damping in the hot plasma is weak or negligible. However, when considering the heating as a function of density and temperature, the waves will, in turn, cause variations of heating, leading to the so-called heating cooling imbalance that can significantly change the behavior of the wave evolution (Kumar, Nakariakov and Moon, 2016; Nakariakov et al., 2017; Kolotkov, Nakariakov and Zavershinskii, 2019).

We have used the CHIANTI atomic database v. 9.0.1 (Dere et al., 1997) in determining the specific values of $\chi$ and $\alpha$ at different temperature and density for the radiative cooling. Using this we obtain the values of $\tau_1$ and $\tau_2$ for different temperatures at density of $10^{-11}kgm^{-3}$. In the hot regime of the temperature ($T \leq 10$ MK), for $T=5.0, 6.3, 8.9, 10$ MK, $\tau_1$ is estimated as 36, 37, 65, 100 mins, while $\tau_2$ as 10, 12, 19, 22 mins respectively. In the super hot regime of the temperature ($T > 10$ MK), for $T=20, 30$ MK, $\tau_1$ is estimated as 308, 566 mins, while $\tau_2$ as 123, 226 mins respectively. The values of $\tau_1$ and $\tau_2$ are reduced respectively by 10, and 100 times at each given temperature for the loops of density $10^{-10}kgm^{-3}$ and $10^{-9}kgm^{-3}$.

We solve our dispersion relation numerically using Wolfram Mathematica environment (2016) for solution of standing wave form, i.e, where the wave number $k$ is real and frequency $\omega$ is complex $\omega_R + i\omega_I$. Further we solve our dispersion relation for a range of temperature, density (normal and over-dense postflare...
loops), and loop lengths which also include the values for observed temperature and loop length of SUMER oscillations (e.g., Wang et al., 2003b; Wang, 2011, and references cited there).

4. Theoretical Results

In the sub-section 4.1 of the theoretical results, we analyse the effect of viscosity and thermal conductivity in the damping of slow waves in coronal loops by considering heating cooling imbalance as well as without it (i.e., at H=const heating rate). Thermal conductivity and radiative cooling is always present as the damping mechanism in these analyses. However, we switch on and off the effect of compressive viscosity both in the case of heating cooling imbalance and without it in order to understand its effects w.r.t. thermal conductivity on the damping of slow magnetoacoustic oscillations. Role of loop density in the damping of slow waves in coronal loop of length $L=500 \text{ Mm}$ within hot regime of temperature ($T \leq 10 \text{ MK}$) is discussed in sub-section 4.2. In sub-section 4.3, we study the role of loop density in the damping of slow waves in coronal loop of length $L=50 \text{ Mm}$ within super hot regime of temperature ($T > 10 \text{ MK}$). In the sub-section 4.4, we compare the individual role of viscosity and thermal conductivity on the damping of slow waves in the coronal loops in presence of heating cooling imbalance. Thereafter, in the light of detailed analytical results, we made new scaling laws between damping time ($\tau$) and wave period ($P$) of the fundamental mode of slow magnetoacoustic oscillations and compare them with the SUMER oscillations in sub-section 4.5.

4.1. Effect of viscosity and thermal conductivity on the damping of slow waves in coronal loops with and without heating cooling imbalance

We have solved a new dispersion relation (equation 18) in order to understand the effect of compressive viscosity and thermal conductivity at different sets of the physical parameters, e.g., loop length, temperature, and density. The main objective of the present work is to understand the evolution and damping of the fundamental mode of the slow magnetoacoustic oscillations, i.e, $k = \pi/L$, where $L$ is the Loop length and $k$ is the wave number. However, the most general solution of the dispersion relation that depends on the dimensionless wave number $K$, where it is defined as $K = kL/\pi$ ($K=1$ corresponds to the fundamental mode (or first harmonics), $K=2, 3, 4$ correspond to the second, third, fourth harmonics respectively). We study the damping of slow magnetoacoustic oscillations with and without heating cooling imbalance in our analysis due to linear perturbation of the plasma medium. This is a very detailed parametric study by considering the variation of the parameters in a wide range, e.g. loop length ($L=50-500 \text{ Mm}$), temperature ($T=5-30 \text{ MK}$), and density ($\rho=10^{-11}-10^{-9} \text{ kg m}^{-3}$) in order to understand the evolution and damping of slow magnetoacoustic oscillations. We define two regimes of the loops based on
their temperature, i.e., (i) hot loops with $T \leq 10$ MK; (ii) super-hot loops with $T > 10$ MK. Later, for the loop lengths of $L=50$ Mm (shortest) and 500 Mm (longest), we perform the similar parametric studies at three different densities of the coronal loops, e.g., normal ($\rho=10^{-11}$ kg m$^{-3}$), and over-dense ($\rho=10^{-10}$-$10^{-9}$ kg m$^{-3}$) hot loops that are also maintained at wide range of the temperature (5-30 MK).

From the following analysis of the results shown in Figures 1-5, we find that the effect of heating cooling imbalance on damping is more remarkable for the loops of lower temperatures but greater lengths. Panels in figures 1-3 show the variation of $\omega_I$ with dimensionless wave number $K$ at a fixed loop-lengths (50, 180, 500 Mm) for different temperatures from 5 MK to 30 MK. In each panel, the red and yellow curves are the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating cooling imbalance is present. The blue-dotted and green-dotted curves are the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating cooling imbalance is not present. It should be noted that the thermal conductivity is by default present in these analytical calculations. Therefore, when we consider an absence of compressive viscosity, this means that in background the thermal conductivity is present as a natural damping mechanism. When we include the compressive viscosity, its effect will be added to the presence of the thermal conductivity. These estimations and related analyses are performed both in case of heating cooling imbalance and without it. Therefore, in principle, the red curve shows the joint effect of both thermal conductivity and compressive viscosity during heating cooling imbalance, while yellow exhibits only the presence of thermal conductivity with heating cooling imbalance. Similarly blue-dotted curve shows the joint effect of both thermal conductivity and compressive viscosity during the absence of heating cooling imbalance, while green-dotted curve includes only the effect of thermal conductivity without heating cooling imbalance.

In the next two sub-sections of 4.1, we will provide the detailed results and their analyses/interpretations about the effect of thermal conductivity and viscosity on the fundamental modes, and higher order harmonics of the slow waves respectively.

4.1.1. Effect of thermal conductivity and viscosity on the damping of the fundamental modes

In figure 1, we present the variation of $\omega_I$ w.r.t. $K$ for the fundamental modes in the loop of length $L=50$ Mm. We present this variation at $T=5.0, 6.3, 8.9, 10, 20$, and 30 MK. In the super hot regime at $T=20$, and 30 MK, for the fundamental mode ($K=1.0$), the value of $\omega_I$ on red and blue-dotted curves lowers down significantly compared to the same on yellow and green-dotted curve (cf., bottom-left and right panels in figure 1). This shows that the compressive viscosity causes an enhanced damping of the fundamental mode in a super hot regime of shorter loops, when it is added in the presence of thermal conductivity. The red and blue-dotted (or yellow and green-dotted) curves coincide with each other (cf., bottom-left and right panels in figure 1), which depicts that heating
Figure 1. Panels above show the variation of $\omega_I$ with dimensionless wave number $K$ at a fixed loop-length of 50 Mm and for different temperatures from 5 MK to 30 MK. In each panel, the red and yellow curves correspond to the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating-cooling imbalance is present. The blue-dotted and green-dotted curves represent the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating-cooling imbalance is not present. Thermal conductivity is always present as a damping mechanism in these analyses.
Figure 2. Panels above show the variation of $\omega_I$ with dimensionless wave number $K$ at a fixed loop-length of 180 Mm and for different temperatures from 5 MK to 30 MK. In each panel, the red and yellow curves correspond to the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating-cooling imbalance is present. The blue-dotted and green-dotted curves represent the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating-cooling imbalance is not present. Thermal conductivity is always present as a damping mechanism in these analyses.
cooling imbalance has no significant effects on the damping of the fundamental modes in the super hot regime of shorter loops \((T > 10 \text{ MK})\). In conclusion, figure 1 demonstrates that the compressive viscosity clearly enhances the damping of the fundamental modes in shorter loops \((L=50 \text{ Mm})\) at super-hot regime of \(T > 10 \text{ MK}\) in the presence of thermal conductivity. At hot regime of \(T \leq 10 \text{ MK}\) at \(T=8.9\) and \(10 \text{ MK}\) (cf., middle-left and right panels in figure 1), the compressive viscosity still plays certain roles in the damping of the fundamental mode of slow waves along with the thermal conductivity, but not as effective as we have already seen in the super hot regime. Also, the effect of heating cooling imbalance in this condition is still negligible. As the temperature goes down to \(T=6.3 \text{ MK}\) and then further at \(T=5.0 \text{ MK}\) (cf., top-left and right panels in figure 1), for the fundamental mode \((K=1.0)\), the value of \(\omega_I\) on red, blue-dotted, yellow, and green-dotted curves have almost same values. This indicates that the effects of both the heating cooling imbalance and the viscosity on damping of the fundamental mode are insignificant compared to the thermal conductivity.

In figure 2, we present the variation of \(\omega_I\) w.r.t. \(K\) in the case of \(L=180 \text{ Mm}\) and \(T=5.0, 6.3, 8.9, 10, 20,\) and \(30 \text{ MK}\). In the super hot regime at \(T=20,\) and \(30 \text{ MK}\), for the fundamental mode \((K=1.0)\), the value of \(\omega_I\) on red and blue-dotted curves lowers down compared to that on yellow and green-dotted curve (cf., bottom-left and right panels in figure 2). This shows that the compressive viscosity along with thermal conductivity cause an enhanced damping of the fundamental mode in a super hot regime in the loop of intermediate length \((L=180 \text{ Mm})\). It is also seen that the red and blue-dotted (or yellow and green-dotted) curves coincide with each other, which refers that heating cooling imbalance has little effects on the damping of the fundamental mode oscillations in super hot regime loops even with the intermediate length. In conclusion, the compressive viscosity has a definite role along with the presence of thermal conductivity in damping of the fundamental oscillations in intermediate loops (e.g., \(L=180 \text{ Mm}\)) at super-hot regime \(T > 10 \text{ MK}\). However, this physical effect is comparatively weaker compared to that in the case of shorter loops (cf., compare the bottom panels of figure 1 and 2). In the hot regime at \(T=8.9\) and \(10 \text{ MK}\) (cf., middle-left and right panels in figure 2), for the fundamental mode \((K=1.0)\), the value of \(\omega_I\) on red/blue-dotted curve has smaller lowering in its value compared to the yellow/green-dotted curve. Moreover, red and blue-dotted curves coincide with each other, and same is true for the yellow and green-dotted one. These results infer that the heating cooling imbalance affects little the damping of the fundamental mode oscillations. This also indicates that the damping caused by the viscosity is insignificant compared to that caused by thermal conductivity. The effect of the compressive viscosity is nearly minimal at these temperatures. At \(T=5.0\) and \(6.3 \text{ MK}\) (cf., top-left and right panels in figure 2), for the fundamental mode, the value of \(\omega_I\) on red/yellow curve has lower values compared to the one on blue/green-dotted curve. Moreover, the red curve coincides with the yellow, and the same is true for green and blue-dotted curves. The physical scenario for the damping of the fundamental mode in the loops of intermediate length (e.g., \(L=180 \text{ Mm}\)) at these lower temperatures now describe that (i) the heating cooling imbalance enhances damping of the fundamental mode, (ii) the
compressive viscosity has almost no effect on the damping, therefore, thermal conductivity dominates.

In figure 3, we present the variation of $\omega_I$ with $K$ in the case with $L=500$ Mm and $T=5.0, 6.3, 8.9, 10, 20, and 30$ MK. At the highest temperature of 30 MK in the super hot regime, for the fundamental mode ($K=1.0$), the value of $\omega_I$ on red and blue-dotted curves lowers down compared to that on yellow and green-dotted curve (cf., bottom-right panel in figure 3). This shows that the compressive viscosity along with thermal conductivity causes an enhanced damping of the fundamental mode at this highest temperature in the longest loop considered in our analysis ($L=500$ Mm). It is also seen that the red and blue-dotted (or yellow and green-dotted) curves coincide with each other at 30 MK, which refers that heating cooling imbalance has no distinct effects on the damping of the fundamental mode at this highest temperature compared to the case when it was absent. At $T=20$ MK (cf., bottom-left panel in figure 3), for the fundamental mode ($K=1.0$), the value of $\omega_I$ on red, blue-dotted, yellow, and green-dotted curves have almost same values. The heating cooling imbalance, therefore, does not cause any enhanced damping of the fundamental mode compared to the case when it was absent. Moreover, the damping effect of compressive viscosity is not important compared to that by thermal conductivity even at this higher temperature. In the hot regime at $T=5-10$ MK, for the fundamental mode ($K=1.0$), $\omega_I$ on red/yellow curve has lower values compared to the case on blue/green-dotted curve (cf., top-left and right, middle-left and right in figure 3). Further, the red curve coincides with the yellow, and the same is true for green and blue-dotted curves. This suggests that the heating cooling imbalance causes more damping of the fundamental mode compared to the case when it was not present. In addition, this indicates that the compressive viscosity has insignificant effect on the damping of the fundamental mode, therefore, the thermal conductivity dominates. Moreover, as the temperature goes towards the lowest value the heating cooling imbalance causes larger damping compared to the case without it.

4.1.2. Effect of thermal conductivity and viscosity on the damping of the higher order harmonics

Figure 1 also shows that in the case of shorter loops ($L=50$ Mm) at super-hot regime ($T > 10$ MK) the compressive viscosity dominates in damping and its effect increases with the harmonic number, whereas heating cooling imbalance has little effects on the damping and nearly independent of the harmonic number.

Figure 2 shows that in the case of intermediate loops (e.g., $L=180$ Mm), the behavior of the compressive viscosity and heating cooling imbalance on damping of higher harmonics is similar to that for shorter loops in super hot regime. While at the lowest temperature of $T=5$ MK, the thermal conduction dominates in damping and its effect increases with the harmonic number, the heating cooling imbalance plays some role in damping and its dependence on the harmonic number is weak.

Figure 3 shows that in super hot regime, the longest chosen loop ($L= 500$ Mm) has the similar damping properties for higher harmonics as the intermediate
loops. While at the lowest temperature of $T=5 \text{ MK}$, thermal conduction still dominates in damping and its effect increases with the harmonic number. Compared to the intermediate loops, the longest loop shows the significant effect of heating cooling imbalance on damping, and its effect appears to be independent of the harmonic number.

In the next sub-section of 4.1, we present a brief summary of the results obtained in 4.1.1 and 4.1.2.

4.1.3. Brief summary

Note that in sub-sections 4.1.1 and 4.1.2, we have not compared the individual roles of thermal conduction and viscosity in the damping of the fundamental modes and higher order harmonics of slow waves, which will be done in the next sub-section 4.2. In summary, from Figures 1-3, we find that (i) the viscosity causes the significant and dominant damping of slow modes in the super-hot regime of shorter loops with the damping rate even higher for higher harmonics (cf., bottom panels of figure 1), while the heating cooling imbalance has little effect on damping of slow modes in this condition; (ii) The heating cooling imbalance causes the significantly enhanced damping of slow modes in the less hot and longer loops with the effects that weakly depend on the frequency (or harmonic number; cf., top panels of figure 3), while the viscosity plays nearly no role or very small role in damping in this condition. It is worth noting that the y-axes on figures 1-3 have varying scales of $\omega_I$, therefore, describe significantly different decay rates in the different cases.

Many previous studies on the damping of standing slow MHD waves in coronal loops have defined dimensionless parameters in order to quantify the damping effect of thermal conductivity, viscosity and radiative losses/gains (De Moortel and Hood, 2003, 2004; Sigalotti, Mendoza-Briceño and Luna-Cardozo, 2007). These parameters are defined as below

\[
\epsilon = \frac{\eta_0}{\rho_0 L c_s} \quad \text{(Viscous ratio)} \quad (22)
\]

\[
d = \frac{(\gamma - 1) \kappa \rho_0 T_0 c_s}{\gamma^2 \rho_0^2 L} \quad \text{(Thermal ratio)} \quad (23)
\]

\[
r = \frac{(\gamma - 1) L p_0 L (\rho_0, T_0)}{\gamma \rho_0 c_s} \quad \text{(Radiative ratio)} \quad (24)
\]

Here $c_s = \left(\frac{2 k T_0}{m}\right)^{1/2}$.

The damping rate of the standing waves have been analytically studied in terms of these dimensionless ratios and it was found that it is a monotonically increasing function of $\epsilon$ (Sigalotti, Mendoza-Briceño and Luna-Cardozo, 2007) whereas it increases up to a peak maximum value and then reduces with increasing $d$ ratio (De Moortel and Hood, 2003). In the regime for hot (5-10 MK) and longer loops (500Mm) with normal density, both $d$ and $\epsilon$ are small ($\ll 1$), this is the condition for so called the weak dissipation approximation, in this case, $\frac{d^2}{\epsilon}$ is a constant and
\[ d \gg 1, \] so thermal conduction dominates in damping. In the regime of super-hot (20-30MK) and short loops (50Mm) with normal density both the thermal ratio \( d \propto \frac{T^2}{\rho L} \) and viscous ratio \( \epsilon \propto \frac{T^2}{\rho L} \) are large \((\gg 1)\). The different behaviour of damping rate depending on \( d \) and \( \epsilon \) implies that when both \( d \) and \( \epsilon \) are \( \gg 1 \), the viscous damping is dominant.

In the next sub-section of 4.1, we analyse the velocity oscillations of the fundamental mode in the hot regime of the temperature \( T \leq 10 \) MK.

4.1.4. Analyses of velocity oscillations of the fundamental mode in the hot regime of the temperature \( T \leq 10 \) MK

We analyse the time evolution of damped slow-mode oscillations in a similar manner as done by Wang et al. (2003a) and consider the velocity perturbations as

\[
V(z, t) = V_0 \sin(kz) \cos(\omega_R t) e^{-|\omega_I|t}
\]

where \( k \) and \( V_0 \) are the wave number and amplitude of velocity oscillations, respectively. We calculate the time variation of velocity oscillations using our numerical solutions of dispersion relation in different cases.

In figure 4, the time variation of \( V \) at the chosen position of \( z = L/2 \) for the damped fundamental mode oscillations in the loops of \( L=50 \) Mm (left column), 180 Mm (middle column), and 500 Mm (right column) are plotted respectively at \( T=5.0, 6.3, 8.9 \) MK. Since we estimate normalized temporal variations of \( V \), and \( \sin(kz) \) varies between \( \pm 1 \), therefore, any of the arbitrary choice of \( z \) will not change the shape of the velocity oscillations. In each panel, the yellow (red) solid curve shows the temporal variation of the oscillations without (with) the effect of compressive viscosity under the consideration of heating cooling imbalance and presence of thermal conductivity as a default damping mechanism. The dotted-green (dotted-blue) curve shows the temporal variation of the oscillations without (with) the effect of compressive viscosity without the consideration of heating cooling imbalance and presence of thermal conductivity as a default damping mechanism.

It is clear that the velocity oscillations decay quickly in the shorter loop of length 50 Mm at \( T=5.0-8.9 \) MK. However, all the curves are merged together (cf., left-column in figure 4). Although, the case shown in left-bottom panel \((L=50 \text{ MK and } T=8.9)\) indicates the weak effect of viscous damping. Firstly this implies no difference in the damping between the cases with and without consideration of heating cooling imbalance. In both the cases, the thermal conductivity is always switched on as a default damping mechanism. Therefore, it is clear that the fundamental mode oscillations are damped dominantly due to thermal conductivity in case of \( L=50 \) Mm loop, and both the heating cooling imbalance and viscosity cause no enhancement of the damping. The top and middle panels in the left-column in figure 4 for \( T=5.0 \) and 6.3 MK clearly demonstrate such cases where thermal conduction dominates in damping, while both the viscosity and heating cooling imbalance have nearly no effects on the damping.

The velocity oscillations also show decay in the loop of length 180 Mm at \( T=5.0-8.9 \) MK. At \( T=5 \) MK, the heating cooling imbalance causes reduction in
Figure 3. Panels above show the variation of $\omega_I$ with dimensionless wave number $K$ at a fixed loop-length of 500 Mm and for different temperatures from 5 MK to 30 MK. In each panel, the red and yellow curves correspond to the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating-cooling imbalance is present. The blue-dotted and green-dotted curves represent the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating-cooling imbalance is not present. Thermal conductivity is always present as a damping mechanism in these analyses.
Figure 4. Variation of $V$ with time for the fundamental slow magnetoacoustic oscillations at $z = \frac{L}{2}$ in the loops of length $L = 50$ Mm (left column), 180 Mm (middle column), and 500 Mm (right column) at $T = 5.0, 6.3, 8.9$ MK. The yellow (red) curve represents the oscillations without (with) the effect of viscosity under heating cooling imbalance. The dotted-green (dotted-blue) curve represents the oscillations without (with) the effect of viscosity in the absence of heating cooling imbalance. Thermal conductivity is always present as a damping mechanism in these analyses.

the velocity amplitude (red/yellow curve) and thus the enhanced damping compared to the case in the absence of imbalance (dotted-green/dotted-blue curve; cf., top-panel in the middle-column). The red and yellow curves superimpose on each other, which tells that the compressive viscosity has insignificant effect on enhancing the damping in the presence of heating cooling imbalance, and its damping is mainly caused by thermal conductivity. Similarly the dotted green and blue one also superimposed with each other indicating that the effect of viscous damping is negligible compared to the damping due to thermal conductivity in the absence of heating cooling imbalance. As temperature reaches to
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![Graphs showing velocity vs time for different loop lengths and temperatures](image)

Figure 5. Variation of $V$ with time for the fundamental slow magnetoacoustic oscillations at $z = \frac{L}{2}$ in the loops of length $L=50$ Mm (left column), 180 Mm (middle column), and 500 Mm (right column) at $T=10, 20, 30$ MK. The yellow (red) curve represents the oscillations without (with) the effect of viscosity under heating cooling imbalance. The dotted-green (dotted-blue) curve represents the oscillations without (with) the effect of viscosity in the absence of heating cooling imbalance. Thermal conductivity is always present as a damping mechanism in these analyses.

6.3 MK, the enhanced damping is again observed in the presence of heating cooling imbalance, but its effect decreases as there is only comparatively smaller reduction in the velocity amplitude compared to the one observed at $T=5.0$ MK (cf., middle-panel in the middle-column). At $T=8.9$ MK, still the heating cooling imbalance has a little higher effect on the damping of the fundamental mode oscillations compared to the one without it. However, the reduction in the velocity amplitude oscillations is even less compared to the one seen at lower temperatures of 6.3 and 5.0 MK (cf., bottom-panel in the middle-column). In conclusion, this scenario indicates that for intermediate loop length (e.g., $L=180$
Mm here), the damping is mainly due to the presence of thermal conductivity and the effect of heating cooling imbalance on damping is evident mostly at the hot regime of the temperature \( T \leq 10 \) MK. Heating cooling imbalance causes more damping when we go towards more lower temperature in the longer loops.

For the loop of length \( L=500 \) Mm maintained at \( T=5.0 \) MK, the heating cooling imbalance enhances the damping of the fundamental mode oscillations very significantly. The velocity oscillation is reduced significantly in magnitude (cf., top-panel in the right-column). When temperature increases to 6.3 MK, still the heating cooling imbalance strongly affects the damping of the fundamental mode as the velocity oscillations are still reduced significantly (cf., middle-panel in the right-column). In case of \( T=8.9 \) MK, the same scenario holds with the reduction of the velocity oscillations in the presence of heating cooling imbalance (cf., bottom-panel in the right-column). In this case also compressive viscosity has no appreciable role in damping of the fundamental modes compared to the thermal conductivity, and even the presence of heating cooling imbalance enhances the dissipation caused by the thermal conductivity. In conclusion, in the hot regime \( T \leq 10 \) MK, the compressive viscosity has insignificant role in enhancing damping of the fundamental mode oscillations. The thermal conductivity plays an appreciable role in damping though, and its effect is enhanced by the presence of heating cooling imbalance especially in the longer loops of lower temperature. The case at \( T=5.0 \) MK in this longest loop is showing the most significant effect for heating cooling imbalance in damping while no effect due to viscosity. In the hot temperature regime, especially in the longer loops, there is a phase-shift in the velocity oscillations (e.g. top-right panel of figure 4). The detailed study on the physical cause of this is out of the scope of present paper, and will be taken-up extensively in future studies.

In the next sub-section of 4.1, we analyse the velocity oscillations of the fundamental mode in the super hot regime \( T >10 \) MK.

4.1.5. Analyses of velocity oscillations of the fundamental mode in the super hot regime of the temperature \( T >10 \) MK

In figure 5, the time variation of \( V \) at \( z = \frac{L}{2} \) for the fundamental mode oscillations in the loops of \( L=50 \) Mm (left column), 180 Mm (middle column), and 500 Mm (right column) are plotted respectively at \( T=10, 20, 30 \) MK for each loop. In each panel, the yellow (red) solid curve shows the temporal variation of the oscillations without (with) the effect of compressive viscosity under the consideration of heating cooling imbalance and presence of thermal conductivity as a default damping mechanism present. The dotted-green (dotted-blue) curve shows the temporal variation of the oscillations without (with) the effect of compressive viscosity without the consideration of heating cooling imbalance.

For the loop of length 50 Mm, at the temperature 10 MK (cf., top panel in left-column), the velocity curves without viscosity in both the cases with heating cooling imbalance (yellow) and without it (dotted-green curve) are superimposed with each other and have the higher amplitude of oscillations. On the other hand, the velocity curves with viscosity in both the cases with heating cooling imbalance (red) and without it (dotted-blue curve) are also almost superimposed
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each other and have the lower amplitude of oscillations. This scenario is also true for the higher temperatures of 20 and 30 MK, in general. It describes that the heating cooling imbalance or constant heating rate without imbalance plays no role in the damping of fundamental mode oscillations in the shorter loops. On the other hand in the super hot regime, it is obvious that the inclusion of viscosity (red/blue-dotted curve) significantly enhance the damping of the fundamental modes compared to the case with the thermal conductivity alone (yellow/green-dotted curve). At $T=10$ MK, the velocity oscillations are damped (cf., figure 5, top panel in left-column) due to the effects of both compressive viscosity and thermal conductivity. It can be seen that when the temperature increases to 20 MK for the loop of $L=50$ Mm, the damping effect by the viscosity becomes even more prominent compared to the case with thermal conduction alone (cf., figure 5, middle panel in left-column). We find that the velocity oscillations show the most serious damping (only one cycle of the oscillations is visible) at $T=30$ MK for the shorter loop of $L=50$ Mm in the case when the viscosity is included, while the damping of oscillations in the case with thermal conduction alone become weaker with increase in temperature (cf., figure 5, bottom panel in left-column). The cases with $L=50$ Mm and $T=20$ and 30 MK have the damping dominated by viscosity because the cases without viscosity (or with thermal conduction alone) shows only a weakly damping. While the effect of heating cooling imbalance in this condition is negligible.

For the intermediate loop length $L=180$ Mm, the red/yellow as well as blue/green-dotted curves are almost superimposed with each other at 10 MK temperature. This behaviour infers that the damping of the fundamental mode oscillations is mostly done by the presence of thermal conductivity, and the presence of heating cooling imbalance and compressive viscosity do not enhance the amount of damping (cf., figure 5, middle panel in middle-column). At $T=20$ MK, the velocity oscillations are seen damped due to the joint effect of viscosity and conductivity while the imbalance plays no role in damping in the case of 180 Mm loop (cf., figure 5, middle panel in middle-column). At $T=30$ MK, the velocity oscillations are damped significantly largely due to the joint effect of viscosity and conductivity while the imbalance plays no role in damping in the case of 180 Mm loop (cf., figure 5, bottom panel in middle-column).

For the highest considered loop length $L=500$ Mm at 10 MK temperature (cf., figure 5, top panel in right-column), heating cooling imbalance dominates along with thermal conductivity (red/yellow curve) in damping of the fundamental modes. The effect of the compressive viscosity is not likely the cause enhancing the damping of the fundamental modes at 10 MK as indicated by the feature of the almost superimposed yellow and red curves. At $T=20$ MK, the red/yellow and blue/green-dotted curves are almost superimposed with each other, which suggests that the damping of the fundamental mode slow magnetoacoustic oscillations is mostly due to the presence of thermal conductivity while both heating cooling imbalance and compressive viscosity do not play a role in damping (cf., figure 5, middle panel in right-column). At $T=30$ MK, the damping effect of the compressive viscosity along with the thermal conductivity only slightly takes over that of thermal conductivity, but the both cases attenuates the velocity
oscillations significantly in the case of 500 Mm loop, while the effect of heating cooling imbalance on damping is negligible (cf., figure 5, bottom panel in right-column). This implies that thermal conduction is dominant in damping. In conclusion, the super-hot loops have quite appreciable velocity damping of the fundamental modes under the joint effects of compressive viscosity and thermal conductivity especially in the shorter loops, where the viscous damping dominates over thermal conduction damping, while the effect of heating cooling imbalance is negligible. Although in the longest loop ($L=500$ Mm), once we go towards $10$ MK temperature the effect of heating cooling imbalance along with the thermal conductivity causes dominant effect on the damping. While, towards highest temperature of $30$ MK, as usual the joint effect of compressive viscosity and thermal conductivity causes the enhanced damping. It should be noted that all these estimations are made for the appropriate density values of the normal coronal loops, i.e., typically of the order of $\rho=10^{-11}$ kg m$^{-3}$.

In the next sub-section of 4.1, we present a brief summary of the results obtained in 4.1.4 and 4.1.5.

4.1.6. Brief Summary

In summary, there are mainly three different cases on the damping of slow modes by thermal conduction, viscosity and heating cooling imbalance which show the dominant role in different conditions: (i) the viscosity dominates in damping in super-hot shorter loops (cf., bottom-left panel of figure 5) while the heating cooling imbalance plays nearly no role in damping in this condition; (ii) The slow modes are damped mainly due to the joint effects of thermal conduction and the heating cooling imbalance for the less hot loops with a longer length (cf., top-right panel of figure 4) while the viscous effect is negligible in this condition; (iii) the thermal conduction dominates in damping while the effects of both viscosity and heating cooling imbalance are negligible (cf., the cases with $L=50$ and $T=5.0$ and $6.3$ MK in upper two panels in left column of figure 4, the cases with $L=180$ and $T=8.9$ and $10$ MK in bottom-middle panel of figure 4 and top-middle panel of figure 5, and the case with $L=500$ Mm, and $T=20$ MK in middle-right panel of figure 5).

Upto this point we have analyzed and established comprehensively the damping scenario of the slow waves in the presence of thermal conductivity and compressive viscosity with and without heating cooling imbalance for a wide range of loop length and temperature. In the following sections, we will analyze the role of loop density in the damping of slow waves in two extreme cases: one for the longest loop of the length $500$ Mm in the hot regime $T \leq 10$ MK (section 4.2), and another in the shortest loop of the length $50$ Mm in the super-hot regime $T > 10$ MK (section 4.3).

4.2. Role of loop density on the damping of slow waves in coronal loop of length $L=500$ Mm within hot regime of temperature ($T \leq 10$ MK)

In previous section, we have found that the thermal conductivity along with heating cooling imbalance causes the efficient damping of the fundamental slow
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Figure 6. Variation of $\omega_i$ with non-dimensional wave number $K$ at 5.0, 6.3, 8.9 MK temperatures in a loop of length $L=500$ Mm. The columns from left to right show the loops with densities $10^{-11}$ kg m$^{-3}$, $10^{-10}$ kg m$^{-3}$, and $10^{-9}$ kg m$^{-3}$ respectively. In each panel, the red and yellow curves correspond to the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating-cooling imbalance is present. The blue-dotted and green-dotted curves represent the same meaning as the red and yellow curves but in the case when the heating-cooling imbalance is not present. Thermal conductivity is always present as a damping mechanism in these analyses.

mode oscillations in the hot regime ($T \leq 10$ MK) of the longer loops, while compressive viscosity does not lead any significant enhancement of the damping on this oscillation. Keeping this scenario in view, in this sub-section we present some additional analyses about the role of loop density in damping of slow waves in coronal loop of length $L=500$ Mm within the hot regime.
Figure 7. Left panels show the temporal variation of velocity $V$ for the fundamental mode of slow magnetoacoustic oscillations at $z = \frac{L}{2}$ for a loop of length $L=500$ Mm at the densities $\rho = 10^{-11}$ (red), $10^{-10}$ (blue-dotted), $10^{-9}$ (green-dotted) kg m$^{-3}$ at $T = 5.0$, $6.3$, $8.9$ MK when heating cooling imbalance is present. Right-column: Similar panels as shown in left-column but for the case without heating cooling imbalance.
In figure 6, the variation of $\omega_I$ with non-dimensional wave number $K$ is presented at $T=5.0$, 6.3, 8.9 MK in the loop with densities ($\rho$) $10^{-11}$, $10^{-10}$, and $10^{-9}$ kg m$^{-3}$. These panels collectively show that in the hot regime at each temperature values of 5.0, 6.3, and 8.9 MK, the $\omega_I$ values for the fundamental mode do not show any appreciable lowering (red/yellow curves when they are compared each other) as the density increases. This depicts that the damping rate due to heating cooling imbalance does not affect much in more bulky loops (e.g., post-flare loop arcades) compared to the loops with normal coronal density. At a given temperature, the value of $\omega_I$ on red-yellow curve only slightly decreased with the increase in density for the fundamental mode, which depicts that the increase in density slightly suppress the damping due to thermal conduction and heating cooling imbalance. The red and yellow curves are superimposed with each other, and similar scenario is true for the green-dotted and blue-dotted curves. This implies that the compressive viscosity does not add any enhancement in the damping of the fundamental modes, while the damping is mostly performed by the thermal conductivity. The similar physical scenario is also evident for the damping of the higher order harmonics in the longest considered loop of length 500 Mm. The damping effects caused by the heating cooling imbalance and thermal conductivity on the higher order harmonics (e.g., second harmonics with $K=2.0$) do not change appreciably with the increase of the loop density. However, the curves of $\omega_I$ become flattening with increase of the harmonic number, showing a behavior distinctly different from those when thermal conduction dominates in damping (compare the right columns of figure 6 and figure 8). This may suggest that the role of optically thin radiation becomes important in damping of higher harmonic slow modes in the over-dense loops (De Moortel and Hood, 2004; Pandey and Dwivedi, 2006). The detailed analysis on the effects of radiation has beyond the scope of this study. In the longest loop maintained at normal coronal density at a temperature 6.3-8.9 MK, some weak damping effect caused by the viscosity on the higher mode harmonics can been seen (cf., middle and bottom panels in the left-column in figure 6). In all the panels of figure 6, the values of $\omega_I$ have significantly lower values on the red/yellow curves compared to the one on blue/green-dotted curves. This implies that overall the heating cooling imbalance has larger effects on the damping of slow waves compared to the one without it.

In summary, in the hot ($T \leq 10$ MK) longer loops, both thermal conduction and heating cooling imbalance play significant roles in damping of fundamental mode oscillations in the loop of different densities, however, the damping rate due to the heating cooling imbalance only decreases a little as the loop density...
In addition, the damping effect of radiation may become more important than that of thermal conduction for higher harmonics in the over-dense long loops, which needs to be investigated further in the future.

4.2.1. In coronal loop of length \( L = 50 \text{ Mm} \) within super hot regime of temperature \((T > 10 \text{ MK})\)

As we have detected the significant role of the compressive viscosity and thermal conductivity jointly for the shorter loops (e.g., \( L = 50 \text{ Mm} \)) maintained at typical coronal loop density and at the super hot regime of the temperature \((T > 10 \text{ MK})\). Therefore, we examine in the next the effect of density variations on the properties of the slow magnetoacoustic oscillations in a shorter loop \((L = 50 \text{ Mm})\) for different temperatures ranging between 10-30 MK. It should be noted that addition of the compressive viscosity along with the thermal conductivity has significant effects on the damping of the fundamental mode for all considered temperatures ranging between 5-30 MK (figure 1) compared to the case by thermal conductivity alone, however, the contribution of compressive viscosity becomes significant only at the very higher temperatures, e.g., 10-30 MK (figures 1). We divide such loops into two categories based on their density values, namely, the normal density loops \((\rho = 10^{-11} \text{ kg m}^{-3})\), and over-dense flaring loops \((\rho = 10^{-10}-10^{-9} \text{ kg m}^{-3})\) (Aschwanden, 2004), and analyze the variation of \( \omega_I \) with \( K \) for the super hot regime \( T \geq 10 \text{ MK} \). The results of this analysis are presented in figure 8.

In the top row of figure 8, at \( T = 10 \text{ MK} \) and \( \rho = 10^{-11} \text{ kg m}^{-3} \) (normal dense loops), for the fundamental mode, the value of \( \omega_I \) on red/blue-dotted curve lowers down compared to the same on yellow/green dotted curve. Like the previous explanation, it infers that when viscosity is added to the thermal conductivity, it enhanced the damping (cf., top-left panel in the first row of figure 8). Since red and blue-dotted curves is coincident with each other this means heating cooling imbalance does not have significant role in enhancing the damping in the present case. The similar physical scenario also works for the higher order harmonics. However, when density is increased by an order of magnitude (mildly over-dense loops), i.e., \( 10^{-10} \text{ kg m}^{-3} \) (cf., top-middle panel in the first row of figure 8), for the fundamental mode, the red, yellow, green-dotted, blue-dotted curves are almost all coincident. This describes the fact that even at higher temperature, if the density of the shorter loop is higher, the inclusion of compressive viscosity does not cause more damping than the case with the thermal conductivity alone (yellow/green dotted curves). Moreover, the effect of heating cooling imbalance on damping in this condition is almost negligible. However, when examining the higher order harmonics (e.g., \( K = 2.0 \)), we find that viscosity still has some small effects in enhancing the damping along with the thermal conductivity (red/blue-dotted curves still lower down) compared to the one caused by the thermal conductivity alone (yellow/green-dotted curves). When density is increased by one more order (over-dense loops), i.e., \( 10^{-9} \text{ kg m}^{-3} \) (cf., top-right panel in the first row of figure 8), then for the fundamental mode, the red and yellow (green-dotted and blue) curves coincide with each other and the values of \( \omega_I \) on red/yellow curve lower down compared to the
Figure 8. The variation of $\omega_i$ with non-dimensional wave number $K$ at 10, 20, 30 MK temperatures in a loop of length $L=50$ Mm. The left-most column shows the variations for the loop with normal coronal density $\rho=10^{-11}$ kg m$^{-3}$. The middle and right-most columns represent the estimations for the bulky loops with higher densities $\rho=10^{-10}$ kg m$^{-3}$ and $\rho=10^{-9}$ kg m$^{-3}$ respectively. In each panel, the red and yellow curves correspond to the solution of the dispersion relation respectively with and without the effect of compressive viscosity when the heating-cooling imbalance is present. The blue-dotted and green-dotted curves have the same meaning as the red and yellow curves but for the case when the heating-cooling imbalance is not present. Thermal conductivity is always present as a damping mechanism in these analyses.
Figure 9. Left-column: Temporal variation of the velocity amplitude $V$ of the fundamental mode of slow magnetoacoustic oscillations at $z = \frac{d}{2}$ for a loop of length $L=50$ Mm at the densities $\rho=10^{-11}$ (red), $10^{-10}$ (blue-dotted), $10^{-9}$ (green-dotted) kg m$^{-3}$ at $T=10$ MK (top), 20 MK (middle), and 30 MK (bottom) when heating cooling imbalance is present. Right-column: Similar panels as shown in left-column but without heating cooling imbalance.
same on green/blue-dotted curves. Now, the inclusion of compressive viscosity does not cause any appreciable enhancement of the damping compared to the one already caused by thermal conductivity. Therefore, thermal conductivity remains dominating damping mechanism while viscosity does not lead any significant role in the bulky loops (e.g., post flare loops) even if they are shorter and maintained at high temperature. Moreover, it is noticed that the heating cooling imbalance enhances slightly more damping compared to the one without its presence. For the higher order harmonics, the compressive viscosity still plays some roles in slightly enhancing the damping. At the temperatures $T=20$ MK (cf., middle row in figure 8) and $30$ MK (cf., bottom row in figure 8), it is seen the similar trend that as the density increases, the effect of the compressive viscosity diminishes. However, it still remains effective for the second harmonics. In conclusion, even for the shorter and super hot loops, the damping effect of compressive viscosity may be weak or negligible if the loop density is high enough.

For post flare bulky loops, since the thermal ratio $d \gg 1$ for super-hot and short loops, therefore, the thermal conduction damping is weak but with increase in density, $d$ decreases since $d \propto 1/\rho_0$ and comes close to the value for the peak damping rate (De Moortel and Hood, 2003) causing the increase in damping. When the viscosity is added, which is a dominant damping mechanism in the regime, the increase in density reduces viscous ratio ($\epsilon \propto 1/\rho$) thus reducing the damping effect by the viscosity. Note that the scales for Y-axis between the left and right panels are distinctly different in figure 8.

In contrast, both the thermal ratio ($d$) and viscous ratio ($\epsilon$) are $\ll 1$ for hot (5-10 MK) and longer loops (500Mm) and thermal conduction is a dominant damping mechanism while the damping due to viscosity is weak. As the value of density is increased, both $\epsilon$ and $d$ decrease even further and thus damping rate due to thermal conductivity is reduced for over dense loops (cf., figure 6) while the addition of viscosity does not make significant difference in the overall damping for normal as well as post flare bulky loops.

Figure 9 shows the temporal variation of the velocity $V$ of the fundamental mode for a loop of length $L=50$ Mm with the densities $\rho=10^{-11}$ (red), $10^{-10}$ (blue-dotted), $10^{-9}$ (green-dotted) kg m$^{-3}$ at different temperatures in the cases with and without the heating cooling imbalance. It is seen that left and right columns are almost same, implying that in the super-hot regime the damping of the velocity amplitude oscillations are less influenced by heating cooling imbalance. In left (or right) column, when seen from top-panel (at 10 MK) to the bottom-panel (at 30 MK), we notice that the velocity amplitude oscillations are largely damped in the loop of density $10^{-11}$ kg m$^{-3}$ (red curve) under the inclusion of compressive viscosity with the the thermal conductivity, while it is less damped in the loop of densities $10^{-10}$ and $10^{-9}$ kg m$^{-3}$ (blue-dotted and green-dotted curves). The damping rate of velocity oscillations in normal, mild, and over-dense loops increasing with the increment in temperature can be seen clearly from these plots.
Figure 10. Panels above show the variation of $\omega_I$ with dimensionless wave number $K$ at a fixed loop-length of 50 Mm and for different temperatures from 5 MK to 30 MK. In each panel, the cyan and yellow curves correspond to the solution of the dispersion relation respectively for compressive viscosity and thermal conductivity when the heating-cooling imbalance is present.
Figure 11. Panels above show the variation of $\omega_I$ with dimensionless wave number $K$ at a fixed loop-length of 180 Mm and for different temperatures from 5 MK to 30 MK. In each panel, the cyan and yellow curves correspond to the solution of the dispersion relation respectively for compressive viscosity and thermal conductivity when the heating-cooling imbalance is present.
Figure 12. Panels above show the variation of $\omega_I$ with dimensionless wave number $K$ at a fixed loop-length of 500 Mm and for different temperatures from 5 MK to 30 MK. In each panel, the cyan and yellow curves correspond to the solution of the dispersion relation respectively for compressive viscosity and thermal conductivity when the heating-cooling imbalance is present.
4.3. Comparison of the role of thermal conductivity and viscosity in damping of slow waves in coronal loops with heating cooling imbalance

In the previous sub-sections, we have done a comprehensive analyses on the damping of slow magnetoacoustic oscillations. We find that at hot regime of the temperature \( T \leq 10 \text{ MK} \), the thermal conductivity along with heating cooling imbalance plays significant role in enhancing the damping of the fundamental mode. While, in the super hot regime of the temperature \( T > 10 \text{ MK} \), the inclusion of the compressive viscosity along with thermal conductivity causes the enhanced wave damping. Therefore, in the present sub-section, we estimate and analyze the individual roles of compressive viscosity and thermal conductivity in the wave damping under the effect of heating cooling imbalance.

In figure 10, we find that the damping of fundamental mode is much higher due to thermal conductivity (yellow) compared to the compressive viscosity (cyan) in the hot regime \( T = 5-10 \text{ MK} \) in the shortest considered loop of length 50 Mm. However, their role is getting reversed in the super hot regime \( T > 10 \text{ MK} \) as the compressive viscosity dominates in the damping of the fundamental mode at 20 and 30 MK temperatures. This result also supports the conclusions of Mendoza-Briceño, Erdélyi and Sigalotti (2004), Sigalotti, Mendoza-Briceño and Luna-Cardozo (2007), and Abedini, Safari and Nasiri (2012). The similar physical scenario is also true for the higher order harmonics with \( K \geq 2.0 \). In figure 11, for the loop of length 180 Mm, the thermal conductivity dominates over the temperature range of 5-20 MK for the dissipation of the fundamental mode oscillations. This result also supports the conclusion derived in Ofman and Wang (2002) that the thermal conductivity is a dominant damping mechanism in typical hot coronal loops. The compressive viscosity begins to play some roles in damping when at 30 MK, and its effect is slightly stronger than that of thermal conduction. At \( T > 20 \text{ MK} \), the damping of higher harmonics is dominated by compressive viscosity, while at \( T < 20 \text{ MK} \), it is dominated by thermal conduction. In figure 12, for the longest loop of the length 500 Mm, the thermal conductivity at all temperatures dominates over the compressive viscosity in damping of the fundamental mode.

4.4. Comparison of various scaling laws between \( \tau \) and \( P \)

Figure 13 displays the damping time vs period for all kinds of theoretically estimations. Filled red circles are the measurements related to the theoretically estimated \( \tau \) and \( P \) for the damped fundamental mode when we consider thermal conductivity, compressive viscosity, and heating cooling imbalance. Cyan circles are the theoretical data points when we consider only compressive viscosity and heating cooling imbalance. While, pink circles are the data points when we consider only thermal conductivity and heating cooling imbalance. Blue rectangles are the points under the effect of compressive viscosity and thermal conductivity without heating cooling imbalance. At the end, filled black rectangles are the data related to the observed SUMER oscillations (Wang et al., 2003) that are overplotted on figure 13. Various dashed-dot yellow lines show the fittings.
Figure 13. Variation of damping time ($\tau$) vs period ($P$): (i) filled-red circles (with thermal conductivity+compressive viscosity+heating cooling imbalance); (ii) cyan circles (only with compressive viscosity+heating cooling imbalance); (iii) pink circles (only with thermal conductivity+heating cooling imbalance); (iv) blue rectangles (without imbalance+compressive viscosity+thermal conductivity); (v) filled-black rectangles (observed SUMER oscillations). Various dashed-dot yellow lines show the fittings on (i)-(iii) theoretical data points. Lower dark green-dashed line is $\tau = P$ line.

Theoretical values with constant heating
Theoretical values with H/C Imbalance
SUMER Observations
Theoretical values without Thermal conductivity
Theoretical values without viscosity

on various sets of theoretical data points while lower dark green-dashed line is $\tau = P$ line. It should be noted that we have taken all the estimated data ($\tau$, $P$) corresponding to the fundamental mode oscillations derived from coronal loops with the normal density $10^{-11}$ kg m$^{-3}$.

These damped oscillations and related $\tau$ and $P$ are being studied and plotted in Figure 13 without bothering any information about loop-length and temperature. Keep in mind that SUMER oscillations were measured in the variety of loops with different length and temperature (Wang, 2011). It is clear that the observed data points (black rectangles) from SUMER oscillations match quite closely with the theoretically estimated data of the slow-mode oscillations when we consider thermal conductivity, compressive viscosity, and heating cooling imbalance (red circles). However, the close inspection of Figure 13 provides some more interesting scientific facts. For the period $P \leq 25$ min, the almost linear trend is observed between $\tau$ and $P$ with a scaling $\tau \propto P^{1.05}$. Note that dark green-dotted line is $\tau = P$ line in Figure 13. These first 14 theoretically estimated data points are basically associated with the oscillations for the shorter loops, and they are closely matching to the observed SUMER oscillations (black rectangles) which were shown to follow a scaling law of $\tau \propto P^{0.96}$ as per the improved measurements taking into account the flow effects (Wang et al., 2005).
and also the more recent oscillations detected by RHESSI (Cho et al., 2016) have shown a similar scaling law for hotter and shorter loops. For $P \geq 25$ min, the next 10 theoretically estimated data points are more scattered and they are basically related to the various oscillations detected in the longer loops. The lower dot-dashed yellow line that fits the red filled cycles shown in Figure 13 is indeed composed of two power-law scalings. For the period $P \leq 25$ min the scaling between $\tau$ and $P$ is $\tau \propto P^{1.05}$. While beyond this period, it is found to be $\tau \propto P^{0.95}$. A break point is detected at $P = 25$ min, and the next linear trend/fit of the data has lesser slope with a scaling $\tau \propto P^{0.95}$ close to the one estimated by Mariska et al. (2008) and Nakariakov et al. (2019). It should be noted that this break point might occur due to the fact that the heating cooling imbalance becomes effective in the loops of longer length and help to account for the observed excessive damping. Therefore, the analytical data and corresponding scaling law falls more towards the observations. The data points (blue rectangles) estimated under the consideration of constant heating rate (i.e., without heating cooling imbalance), has a scaling law (the middle yellow-dotted line in figure 13) that is expressed by $\tau \propto P^{1.12}$. This is slightly deviated from the SUMER observations, and damping is underestimated when heating cooling imbalance is not considered. The another set of data points (cyan circles) estimated under the consideration of only compressive viscosity (i.e., without considering thermal conductivity), has a scaling law (upper yellow-dashed line on these points in figure 13 especially below $P=35$ min) that is expressed by $\tau \propto P^{1.55}$. This is far beyond the scaling related to the SUMER observations, and damping is heavily underestimated in the case when thermal conduction is not considered. Note that in the period regime of $P > 35$ min, which is mostly related to the longer loops, the heating cooling imbalance significantly enhances the damping and can cause the deviation of some data points (cyan circles) towards $\tau = P$ line. In conclusion, the consideration of joint effect of thermal conductivity, compressive viscosity, and heating cooling imbalance predicts a theoretical scaling law in between $\tau$ and $P$ better matching to the SUMER observations. Even if we remove the effect of viscosity, and only consider thermal conductivity and heating cooling imbalance, most of the data point in figure 13 (empty pink circles) still fall close to the filled red circles except a few of the data points related to the shorter loops that are associated with the lowest periods. This suggests that thermal conductivity along with heating cooling imbalance may play a vital role in explaining the strongly damped slow-mode oscillations observed with SUMER for most of the events, while the role of compressive viscosity may be essential in interpreting damping of short loops in super hot temperature regime such as in postflare loops in solar and stellar flares (Cho et al., 2016).

5. Discussion and Conclusions

To the best of our knowledge, the present paper firstly provides a comprehensive overview on the physical scenario of the damping of fundamental mode of slow magnetoacoustic oscillations and its higher harmonics in the coronal loops with diverse length (50-500 Mm), temperature (5-30 MK), and density ($10^{-11}$-$10^{-9}$
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Table 1. Summary of the dominant damping mechanisms of the fundamental slow modes in the various regimes of loop parameters considered throughout our study. T.C : Thermal conductivity, H/C : heating cooling imbalance, Vis : Viscosity

| Loop Length       | Short Loops (50Mm) | Long Loops (500Mm) |
|-------------------|--------------------|--------------------|
|                   | Normal Loops       | Bulky Loops        | Normal loops | Bulky Loops |
| Hot Loops (5-10MK)| T.C                | T.C                | T.C + H/C    | T.C + H/C   |
| Super-Hot Loops (20-30MK) | Viscosity | T.C                | T.C + Vis.   | T.C         |

kg m⁻³) under the consideration of thermal conductivity, compressive viscosity, and heating cooling imbalance. In the present work, the temperature range chosen for the theoretical analysis is based on the observations. The similar damped harmonic oscillations have been found with RHESSI observations that showed average periods of 0.9 min and average decay times of 1.5 min, much short compared to those of SUMER oscillations (Cho et al., 2016), while their oscillation quality (i.e., ratio of damping time to period) is similar. Therefore, the assumption of quasi-steady equilibrium may still hold if the wave period is much smaller than the cooling timescale at T=20-30 MK. The quasi-static assumption here has two-fold meanings: 1) linear theory of different damping mechanisms is applicable on a time scale comparable to the wave period corresponding to the "equilibrium" background condition (T₀, ρ₀, and L) at a certain instant; 2) the WKB approximation, i.e. \( \frac{d^2}{dT^2} \ll 1 \), the changing of wave period is very slow during a certain period (e.g. the decay time considered). This allows us to analyse the decay of relative perturbations, \( \frac{dT}{T₀} \) and \( \frac{dρ}{ρ₀} \) where T₀ and ρ₀ are the averaged parameters over this period. For example, Sharykin and Kosovichev (2015) showed the observations of super-hot flare loop with GOES that can reach T>30 MK for a few minutes. Their damping behaviour can be understood based on our parametric study in the regime of super-hot short loops. It is known that the RHESSI energy band (3-25 keV) used to detect the oscillations (or QPPs) is associated with super-hot loops with T=20-30 MK (e.g., Caspi, Krucker, and Lin, 2014; Ryan et al., 2014, and references cited there). The QPPs are commonly detected in solar flares (e.g., Caspi, Krucker, and Lin, 2014; Ryan et al., 2014; Sharykin and Kosovichev, 2015) which most likely correspond to short loops, however, parametric studies of long loops with super-hot temperatures may help understand the case of stellar flares which have a wide length scale range (e.g., Mitra-Kraev et al., 2005; Srivastava, Lalitha and Pandey, 2013), therefore, T=20-30 MK is well relevant to the solar atmosphere and related dynamics in the confined heated loops. On the other hand, as we know the slow-mode oscillations in cooler loops (1 MK<T<5 MK), particularly related to impulsive heating, are rarely observed compared to the temperature range considered in this study.

In general, the role of heating cooling imbalance highly depends on the form of heating function assumed which is however unknown, so one can only say this
Damping of Slow Waves in coronal loops result is correct based on our assumption (i.e., \( a = -\frac{1}{2}, b = -3 \)), which suits to the damped oscillatory regime of the slow modes as described by Kolotkov, Nakariakov and Zavershinskii (2013). On the other hand, as the result in figure 13 suggested, the inclusion of heating cooling imbalance can produce the theoretical prediction better matching to the observations. Moreover, the results are compared with the case that no heating cooling imbalance is present, or heating rate is assumed to be constant. The present model also adds in background the radiative cooling term. However, the radiative effect is insignificant compared to other dissipation mechanisms, we did not study its effect separately (Abedini, Safari and Nasiri, 2012). In the present work, we found that the compressive viscosity along with the thermal conductivity causes strong damping of the fundamental mode oscillations in the shorter (e.g., \( L = 50 \text{ Mm} \)) and super-hot (\( T > 10 \text{ MK} \)) loops, and the compressive viscosity plays the dominant role in this regime. However, the effect of viscosity is insignificant in the damping of these modes in longer (e.g., \( L = 500 \text{ Mm} \)) and hot loops (\( T \leq 10 \text{ MK} \)), instead thermal conductivity along with the presence of heating cooling imbalance plays an important role in this condition. Moreover, for the longer loops at the hot regime of the temperature, the increase in density slightly decreases the damping due to thermal conduction and heating cooling imbalance (cf., figure 6 and left-bottom panels of figure 7). However, for the shorter loops at the super-hot regime of the temperature, the increment in the loop density substantially enhances damping of the fundamental modes due to the thermal conductivity when the viscosity is absent (cf., bottom panels of Figure 8). Individual role of thermal conductivity is found to be dominant in longer loops at the lower temperatures (\( T \leq 10 \text{ MK} \)), while compressive viscosity dominates in damping at super-hot temperatures (\( T > 10 \text{ MK} \)) in the shorter loops only. We have summarised our results for the fundamental mode in the various physical conditions of loops in Table 1. The scaling law between \( \tau \) and \( P \) obtained by fitting the theoretical data is found to be closer to the observed SUMER oscillations when we add the effect of heating cooling imbalance to the case with thermal conduction and viscosity for the damping of fundamental slow mode oscillations.

Pandey and Dwivedi (2006) have reported that by varying loop density from \( 2 \times 10^8 \) to \( 2 \times 10^{10} \text{ cm}^{-3} \) (or \( \rho = 2 \times 10^{-13} \) to \( 2 \times 10^{-11} \text{ kg m}^{-3} \)) at a fixed temperature in the temperature range 6-10 MK (we consider it as a hot regime of the temperature, i.e., \( T \leq 10 \text{ MK} \) in our present work), they get two sets of damping of fundamental slow mode oscillations, in which one was for \( \tau/P \sim 1 \) (dark-green dotted line in figure 13) related to the strong damping occurred at lower density \( (10^8 \text{ cm}^{-3}) \), while another was for \( \tau/P \geq 2 \) corresponding to a weak damping occurring at higher density \( (10^{10} \text{ cm}^{-3}) \). Note that \( \rho = 10^{-11} \text{ kg m}^{-3} \) corresponds to \( N_e = 5 \times 10^9 \text{ cm}^{-3} \) or \( \log_{10}(N_e) = 9.6 \), which gives \( \tau/P \sim 1.5 \) from figure 1 of Pandey and Dwivedi (2006), while all theoretical data in figure 13 of our paper are for \( \rho = 10^{-11} \text{ kg m}^{-3} \). Therefore, all the predicted data here are below \( \tau = 2P \) line as given in Pandey and Dwivedi (2006) that is consistent with their prediction. These results presented here are thus consistent with the findings of Pandey and Dwivedi (2006). In the present paper, a two-part power-law scaling \( \tau \propto P^{1.05} \) (at \( P \leq 25 \text{ min} \)) and \( \tau \propto P^{0.95} \) (at \( P \geq 25 \text{ min} \)) is achieved for the fundamental mode oscillations for the range of loop-length (50-500 Mm).
and temperature (5-30 MK) under the effect of thermal conductivity, compressive viscosity, and heating cooling imbalance, which is very close to the strong damping of $\tau/P \approx 1.0$ and also better satisfy the observed SUMER oscillations. Joint effect of thermal conductivity, compressive viscosity, along with heating cooling imbalance produces strong damping of the fundamental mode oscillations in the normal coronal loops itself, which matches well with the SUMER observations.

Sigalotti, Mendoza-Briceño and Luna-Cardozo (2007) have numerically studied the damped oscillations observed by SUMER in the linear and nonlinear regimes, and concluded that the damping times of the oscillations are mostly shaped by compressive viscosity rather than thermal conduction, and the damping due to optically thin radiation is negligible when considering a constant heating rate. They showed that thermal conduction alone results in slower damping of the density and velocity oscillations than that observed, while it is required to add the compressive viscosity so that the waves can be damped quickly enough to match the SUMER observations. However, their conclusion for viscosity dominating over thermal conduction in damping was deduced in the loops with very lower density on an order of $10^8$ cm$^{-3}$. However, here we find that in coronal loops of typical density ($\rho=10^{-11}$ kg m$^{-3}$ or $N_e=5 \times 10^{9}$ cm$^{-3}$), the effect of compressive viscosity on the damping of the slow waves is significant only in very short loops at super-hot temperature regime. Instead, thermal conductivity along with heating cooling imbalance dominates in damping of the slow waves. Moreover, to better match to the observed damped SUMER oscillations (figure 13), the joint effect of the thermal conductivity, compressive viscosity, and heating cooling imbalance is required (red circles; $\tau \propto P^{1.05}$ and $\tau \propto P^{0.95}$). On the other hand, if we consider the compressive viscosity alone (cyan circles), in the observed range of SUMER oscillations ($P \leq 35$ min), its variation ($\tau \propto P^{1.55}$) goes out of even the weak damping regime ($\tau \geq 2P$ line) as shown in figure 13. This infers that the compressive viscosity alone can not explain the observed damping of the SUMER oscillations as pointed by many previous reports (Sigalotti, Mendoza-Briceño and Luna-Cardozo, 2007; Abedini, Safari and Nasiri, 2012). If we exclude the effect of the compressive viscosity, and only consider the thermal conductivity supported by heating cooling imbalance also, then most of the data points in figure 13 (empty pink circles) lie very close to the filled red circles (thermal conductivity+viscosity+imbalance) except of a few points for the oscillations of very shorter periods associated with very short/super-hot loops. Therefore, we conclude that thermal conductivity along with heating cooling imbalance may be the dominant damping mechanism for interpreting the strongly-damped slow-mode oscillations observed with SUMER (filled black rectangles in figure 13).

There are also several attempts in the past also that investigated the damping of slow waves in nonisothermal, hot, gravitationally stratified coronal loops (e.g. Erdélyi, Luna-Cardozo and Mendoza-Briceño, 2008). In hot and super-hot regime, the non-uniformity in $T_0$ and $\rho_0$ along the loop is expected to be negligible because of highly efficient thermal conduction and very large density scale height. For example, the density scale height is $H=500$ Mm for $T=10$ MK plasma, which implies that the effect of gravitational stratification is very small on damping even for the longest loop of $L=500$ Mm that has a height $h=160$
Mm if the loop is semi-circular.

We would like to mention that the background heating function which is described as a function of density and temperature throughout our study has been taken as just a typical physical scenario developing on the model of Kolotkov, Nakariakov and Zavershinskii (2019). Many previous works in the area have studied different functional dependencies of this unknown heating function such as its dependence on magnetic field and loop length (e.g., Lionello et al., 2009; Nakariakov et al., 2017) as well as time (e.g., Taroyan et al., 2007; Reale, 2016) since the specific heating mechanism in solar corona is still unknown and remains an open research problem to date.

We conclude finally that although thermal conduction along with heating cooling imbalance may be the dominant damping mechanism for interpreting the strongly-damped slow-mode oscillations observed with SUMER, the role of compressive viscosity is essential to explain the damping of SUMER oscillations in coronal loops at short length and super-hot temperature regimes such as the slow modes excited in postflare loops in solar and stellar flares.

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