On the precision of assembly of complicated structures

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Abstract. The problems of forecasting the precision of assembly of complicated structures assembled from rods according to a given program (farms, platforms) have been considered. Because of assembly operational margins and random scatter of the lengths of the rods, the errors occur in position of the structure and its nodes. It is necessary to find out the probabilistic characteristics of errors and to assess the precision of assembly. For each model structure, two assembly ways have been studied: a «hard» assembly and «spring assembly». Under «hard assembly», the assembled parts while the subsequent assembly are not deformed, and master links allow adjustment of their length. The formulas for calculating the deviation of the steady-state length of (malformed) member, of which the structure is fitted, from the statistically distributed length of the non-deformed rod are given. Multiple implementations of the assembly process of various structures on a computer have been obtained and from these data the probability characteristics of the total error have been derived. The proposed method can be used in the construction industry when assembling complicated structures and allows to obtain sufficiently precise data, eliminating errors in the position of the building structure.

1. Introduction

It is known that the actual shape and position of the nodes of hinge-rod envelopment, and the actual erection stress-strain state (ESSS) is different from the state defined by the project. Therefore, the issues connected with the accounting of imperfections in structures at their design are current. It is known that the presence of imperfections usually leads to a lot of the unanticipated labor and financial costs usually not included in the cost of a construction project. The accumulation of imperfections can lead to the covering accident.

Definition of the ESSS of long-span hinge-rod metal coverings considering the accuracy of manufacturing and assembly sequence at the structural analysis is an important scientific and technical problem. Array hinge-rod structures are similar to the arrangement of the crystal lattice of the metal [1]. Allowance for imperfections, as a rule, is performed by the method of statistical tests. To determine the accounting of deviations at the design stage, a method of calculating the accuracy of long-span hinged-rod metal coatings has been developed. In order to prevent the deviations, the appropriate design solutions of shells has been developed. Such a comprehensive approach to the design of coatings allows increasing their bearing capacity.

In the theses of A.M. Yugov and A.B. Bondarev the magnitude of the imperfections and the ESSS of hinge-rod metal envelopments of coverings have been studied. As the result of these studies, the computer program and methods for calculating accuracy, and method of determining the ESSS have been developed. [2-6].
For the calculation of accuracy of metal structures, V.A. Savelyev together with E.V. Lebed have developed a method for determination of geometrical imperfections of spatial metal structures in a probabilistic setting as a numerical simulation on the computer of the process of its assembly and erection using the Monte-Carlo method. [7-11]

Currently, E.V. Lebed investigates the imperfections in the metal dome roofs, caused by inaccuracies in manufacturing and erection, [12, 13]. O.I. Efimov in his studies [14] have analyzed the work of space frames with a nodal run-outs and nonrigid nodes. As a result of studies, O.I. Efimov has developed the methods for calculation of structures with such imperfections.

D. M. Husannov and I. L. Kuznetsov [15,16] have conducted the analysis of imperfections. Affecting the operation of lightweight arched structures based on the results of in-situ surveys of the operated buildings. The estimation of the real condition of the arched buildings according to the results of in-situ surveys is given, the method for definition of allowances for production and installation of lightweight arched buildings is developed, recommendations for improving the design, production and installation of arch buildings are given. These problems have been considered without involvement of contemporary methods of computer modelling. Erection of neglected and insufficiently tested and risky constructive forms can lead to accident of covering (Water Park in Yasenevo, Baumsansky Market). One of the reasons for the accident was the version that the actual shape of the dome was significantly different from the design one. The installation was conducted by a hinged way from the bottom-up and the errors of installation, accumulating, reduced the already small curvature of the top part of a dome, as a result the top part of a dome worked not as a shell, but as a plate, some compressed rods lost stability. To increase the reliability of the structure, in individual theses it is recommended to "control their behavior" or to create so-called adaptable systems. [17]

2. Problem specification and decision

The slab consisting of hexagonal cells was considered. The cells are assembled from rods which lengths have a spread with a given mean-root-square deviation $\sigma$. The problem of estimation of deviations of the slab elements from calculated coordinates is put. The method of assembling simulation by means of a computer is applied. A single cell assembly program has been created. The variant of "rigid assembly" has been considered, when the collected sections at the further assembly are not deformed, and the closed segments allow adjustment from lengths. There are two such segments in each cell. Repeated implementation of the assembling process of the computer at different $\sigma$ have been conducted. On the basis of static processing of the received data, the estimations of values of residual errors for closing segments, and also for deviation of tops of a cell from a calculation position are given. If we know these values for a single cell, we can predict the errors in the configuration of the slab of any contour and size, in particular the deviation of the slab foundation from the planes. This can be done either analytically, using formulas of the error theory, or by modeling the assembly of the whole slab or its parts.

A class of structures consisting of the same repeating units (pyramids, tetrahedra, cubes, cells, etc.) is considered. The position and configuration of each unit are given by the coordinates of its vertices, that is, the points at which adjacent members are attached. The position of the first unit is determined up to its shift and rotation (as a whole), therefore six parameters-coordinates of its vertices are given randomly (see below). The positions of the vertices of subsequent units are derived in the process of assembly. It is assumed that they are completely determined by the lengths of the members in the structure (this corresponds to the natural condition of structural stability). Units are collected sequentially according to a given program (project), so that some members and vertices of the already assembled units are elements of the next unit to be assembled. Further we will consider the option of elastic assembly. In addition, in connection with the task of geodetic triangulation, calculations were made which are of interest for the accuracy of the assembly.

Due to operational margins in the assembly, random scatter in the length of the rods, «wiggle rooms» in the mounts, errors occur in the position of the structure and its units. The random deviations in the lengths (and consequently also in the directions) of the separate members determine the total
operational margins in the position and shape of the entire structure. Statistics of the scatter of rod lengths of different nominal sizes can be obtained by elementary measurements even before assembly. If the operational margins occur when the members are attached, then such measurements should be made on a series of separate assembled units (not necessarily combined into a single structure). Usually errors in rod lengths have an abridged normal distribution. In a number of cases, the distribution and statistical characteristics of these errors can be obtained from technological considerations, taking into account, for example, the nature of the member links at the vertices of the units. This problem was considered in the thesis of A.L. Garkavi, V.A. Shmatkov and G.E. Granovsky «Mathematical Methods for Calculating the Reliability of Construction Equipment». [18] In this thesis, mathematical methods are considered in the design of complicated systems with a large number of mutually-related elements. Such systems include contemporary construction machines and their complexes. The development and sophistication of technical systems requires the creation of a special mathematical theory of reliability which purpose is to develop methods for quantifying and calculating the reliability of technical devices. The calculation and forecast of the reliability of devices is of a probabilistic nature. This problem was also considered in the thesis of M.L. Kagan «Probabilistic Methods for Calculation and Processing of Information in the Construction Industry». This article is a continuation of the thesis of A.L. Garkavi, V.A. Shmatkov and G.E. Granovsky in which the task is to forecast the resulting deviations of the assembled structure from the calculated structure according to statistical data on the scatter of the lengths of the members. [19]

The problem of forecasting the summarizing deviations of the assembled structure from the calculated structure can be solved by simulating the process of assembly on a computer. Repetition of the same nodes will make it possible to organize in the corresponding program the cycles reproducing the assembly of units. Each cycle is an algorithm for solving a system of equations, as a result of which the coordinates of the vertices of the unit are obtained. As initial conditions in the algorithm, the coordinates of the vertices of adjacent already «assembled» units are introduced. As the parameters of the system, random lengths of rods are also introduced, of which the unit is assembled. The source of random deviations of lengths is a sensor or a table of random numbers.

Depending on the shape of the structure and the process technology, two assembly ways are possible: «hard» assembly – assembled nodes and units are not deformed during further assembly; «string» assembly – assembled nodes and units can be deformed during further assembly and their final configuration is fixed only after the assembly is completed.

Let $n$ – the number of members in the configuration; $m$ – the number of vertices of all units:

$$M_1(x_1, y_1, z_1), ..., M_m(x_m, y_m, z_m)$$  \hspace{1cm} (1)

As indicated, six coordinates are given randomly, for example:

$$x_1 = y_1 = z_1 = y_2 = z_2 = z_3 = 0$$  \hspace{1cm} (2)

The total number of coordinates to be determined is $N = 3m - 6$. For $n < N$, the structure is unstable: changes in the direction of the members are possible while preserving their lengths. For $n = N$, the structure, as a rule, (with a rational design and assembly order) can be assembled with observance of its stability from any rods, the random lengths of which differ little from the design ones.

For $n > N$ the structure, then for hard assembly some members (by the number $n^* = n - N$) must have a forced length, i.e. it should be possible to fit the length of these members.

For $n = N$, fitting the members is unnecessary and it is sufficient to consider the «hard assembly» way. However, for $n < N$, the «string assembly» way in many cases more adequately describes the real technology than the «hard assembly» way.

Let us dwell in more detail on the «string assembly» way. In this way, variations in the lengths of the members (due to their compression, extension, bending, deformation of the links, etc.) are allowed
during assembly. All members are assumed to be equal in rights: there are no members subject to special fit.

We introduce the following designations:

\[ \{ M_i \}, i = 1, \ldots, m \] – the vertices of the units from which the structure consists;

\[ (x_i^0, y_i^0, z_i^0) \] – the design (calculated) coordinates of the point \( M_i \);

\( (x_i, y_i, z_i) \) – the established coordinates of \( M_i \) at the completion of the assembly;

\( \Delta x_i = x_i - x_i^0, (\Delta y_i, \Delta z_i) \) – the deviations of the established coordinates of the vertex \( M_i \) from the design ones;

\[ l_i^0 = \sqrt{(x_i^0 - x_j^0)^2 + (y_i^0 - y_j^0)^2 + (z_i^0 - z_j^0)^2} \] (3)

\( l_i^0 \) – design length of the member \( M_i M_j \);

\( \bar{l}_i \) – the actual (random) length of the non-deformed rod (before assembly) used as a member \( M_i M_j \);

\[ l_i = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \] (4)

\( l_i \) – the established length of the deformed member at the completion of the assembly;

\( \delta_y = \bar{l}_i - l_i^0 \) – deviation of the random length of the rod from the design length of the member \( M_i M_j \);

\( \Delta l_i = l_i - \bar{l}_i \) – deviation of the established length of the (deformed) member from the random length of the non-deformed rod.

\[ \Delta l_i = l_i - \bar{l}_i = (l_i - l_i^0) - (\bar{l}_i - l_i^0) = (l_i - l_i^0) - \delta_y \] (5)

The deviations \( \Delta x_i = x_i - x_i^0, \Delta y_i, \Delta z_i, \Delta x_j, \Delta y_j, \Delta z_j \) are naturally considered small. Expanding in accordance with (4) the function \( l_i = l_i \left( x_i, x_j, y_i, y_j, z_i, z_j \right) \) near the point \( \left( x_i^0, x_j^0, y_i^0, y_j^0, z_i^0, z_j^0 \right) \) in powers of \( \Delta x_i, \Delta x_j, \Delta y_i, \Delta y_j, \Delta z_i, \Delta z_j \) and leaving only the linear terms, we obtain from (5) taking into account (3)

\[ \Delta l_i = \left[ \frac{(x_i^0 - x_j^0)(\Delta x_i - \Delta x_j)}{l_i^0} + \frac{(y_i^0 - y_j^0)(\Delta y_i - \Delta y_j)}{l_i^0} + \frac{(z_i^0 - z_j^0)(\Delta z_i - \Delta z_j)}{l_i^0} \right] + l_i^0 - l_i^0 - \delta_y = Z_i - \delta_y \] (6)

Where \( Z_i \) – the sum in square brackets.

So, \( \Delta l_i = Z_i - \delta_y \)

The structure is a string conservative system. For such a system, its steady position is characterized by a minimum of its potential energy \( P \). Assume that the deformation force \( f_{ij} \), acts on each member in a steady state, being proportional to Hooke's law \( \Delta l_i \); \( f_{ij} = C \cdot \Delta l_i \), where \( C \) – the stiffness factor.

Then \( P \) is equal to the total work of all forces \( f_{ij} \) upon transition of the system to a state with \( P = 0 \) (that is, with zero deformations). In this way, to

\[ P = \sum_{(i,j)}^{\Delta l_i} \int c dt = \frac{C}{2} \sum_{(i,j)} \left( \Delta l_i \right)^2 \] (7)

and the steady state the minimum of the following expression corresponds
\[
\frac{2}{c} P = \sum_{(i,j)} (\Delta l_{ij})^2 = \sum_{(i,j)} (Z_{ij} - \delta_{ij})^2
\]  
\hspace{1cm} (8)

over all \((\Delta x_i, \Delta y_i, \Delta z_i, i = 1, \ldots, m)\). Hence, the unknowns \((\Delta x_i, \Delta y_i, \Delta z_i, i = 1, \ldots, m)\) should be derived by the least square method as an approximate solution of the incompatible system of conditional equations

\[
Z_{ij} = \delta_{ij}
\]  
\hspace{1cm} (9)

(pairs \((i, j)\) run through all the members \(M_i M_j\) of the structure).

3. Main Results.
This problem can be solved on a computer using the standard program “The Solution of a Linear Problem by the Least Square Method”. The following initial data are introduced into the closed routine:

Calculated coordinates \(\{x_i^0, y_i^0, z_i^0\}\).

Random variables \(\{\delta_{ij}\}\) obtained with the help of the random normal numbers \(\{a\}\) introduced into the memory (with zero mean and unit variance), \(\delta_{ij}\) are related to \(a\) by the reduction transformation:

\[
\delta_{ij} = \begin{cases} 
\sigma_{ij} \cdot a; |a| \leq 2 \\
2\sigma_{ij}; |a| > 2 \\
-2\sigma_{ij}; |a| < -2
\end{cases}
\]  
\hspace{1cm} (10)

Where \(\sigma_{ij}\) – the mean-square deviation of \(\sigma_{ij}\).

The «hard assembly» and «spring assembly» ways are in some sense «extreme»: for the «hard assembly» way, the effect of subsequent assembly is excluded, for the «spring assembly» way, this effect is fully taken into account. A real assembly process may also correspond to some intermediate way, taking into account the attenuation of the links between the nodes as they are removed from each other. Therefore, the way «spring assembly» can be applied not to the entire structure, but in succession to its parts, in particular to its units.

From the obtained data, the probabilistic characteristics of the total errors are derived. As a measure \(M\) of the deviation of the assembled structure from the calculated one, it is natural to take the maximum of the mean-square deviations of the points of the assembled structure from their calculated position. However, it should be taken into account that, since the assembled structure allows shift and rotation, the assembly precision should be characterized by the value \(M^*\) of this maximum at the optimal position and orientation of the assembled structure. For structures consisting of 30–50 members of the same type, the \(M^*\) value for the «spring assembly» way is approximately 1.5 times smaller than for the «hard assembly» way, and lies between 1.5 \(\sigma\) and 2.5 \(\sigma\), where \(\sigma\) – mean-square deviation of the lengths of structure members.

The assembly of a long flat beam was simulated on the computer. Calculations were carried out upon the exact approximate method. Estimates of beam deviations from the horizontal position and from the straight line have been obtained. The estimates of both methods were comparable. The analysis of experimental data was also carried out, and empirical formulas for beam deviations have been derived. The experiment was also prepared to determine the influence of temperature difference on the design distortion due to thermal expansion of the part of the structure nodes. It is also planned to study the effect of assembly nodes sequence on possible errors.

The review of studies in the part of calculation of accuracy and accounting of deviations on erection strain-stress state of building structures is carried out. In order to increase the reliability of the
long-span coating, it is recommended to develop constructive measures aimed at «managing deviations». «Managing deviations» means such a way of erection at which the position of elements and nodes of a covering will be adjusted, that allows to eliminate deviations, and to reduce values of installation efforts up to minor level.

References
[1] Makowski Z S 2002 Development of jointing systems for modular prefabricated steel spacestructures. Proceedings of the international symposium. Warsaw: Poland. p.17-41.
[2] Bondarev A B 2016 The Method of Determination of Mounting Stress-Strain State-Span Hinge Rod Metal Coatings. Metal Constructions. 22(2), p.67-82.
[3] Bondarev A B, Yugov A M 2016 The method of calculating the accuracy of large-span metal rod systems. Magazine of Civil Engineering, 1(61), p.60-73.
[4] Bondarev A B, Yugov A M 2015 Evaluation of installation efforts in metal coatings, allowing for assembly process. Magazine of Civil Engineering. 4(56), p.28-37.
[5] Bondarev A B 2015 Deviations in assembly hinged-rod metal coating. Construction of Unique Buildings and Structures. 3(30), p.98-110.
[6] Bondarev A B, Yugov A M 2015 The Method of Generating Large-Span Rod Systems with the Manufacturer Defect and Assembly Sequence. Procedia Engineering, 117, p.953-963
[7] Lebed E V, Shebalina O V 1992 Analysis of distortions of the geometric shape in the assembly of composite metal structures. Industrial Construction, 5, p.23-24
[8] Lebed E V, Eterevsky V A 2012 Analysis of initial stresses in a sectorial-lattice dome during installation as an assembled structure in comparison with a starlattice dome. Bulletin of Peoples Friendship University of Russia. Series: Engineering Researches, 4, p.91-98.
[9] Lebed E V, Grigoryan A A 2015 Influence of assembly analytical models of the ribs of a double-layer metal dome on the initial forces in case of elimination of imperfections. Proceedings of Moscow State University of Civil Engineering, 8, p.66-79
[10] Lebed E V, Grigoryan A A 2016 Examination of initial efforts in two-zone metal dome while layer eliminating mounting errors of annular. Proceedings of Moscow State University of Civil Engineering, 4, p.36-51.
[11] Lebed E V, Shebalina O V 1993 Calculation of the Accuracy of Composite Structures Assembling. Industrial and Civil Engineering. 9, p. 27-28.
[12] Lebed E V 1988 Forecasting of errors of large-span metal dome construction based on geometric modeling of their installation: The thesis submitted for the Scientific Degree on competition of Candidate of Engineering. Moscow. p.171
[13] Savelev V A, Lebed E V, Shebalina O V 1991 Mathematical Modeling of Spatial Structures Installation. Industrial Construction., 1, p.18-20.
[14] Efimov O I 1982 Impact of compliant connections and nodal eccentricity on the work of the structural units: The thesis submitted for the Scientific Degree on competition of Candidate of Engineering. Kazan. p.152
[15] Kuznetsov I L, 1995 Stripped out construction arch buildings (Reserch, development, implementation) The thesis for the degree of Doctor of Technical Sciences, Kazan. p.426
[16] Khusainov D M 1996 Improving the quality of manufacturing design and installation of lightweight frame arch buildings, The dissertation for the degree of Candidate of Technical Sciences Kazan, p.252
[17] Kim H M, Doiron H H 1992, On-orbit modal identification of large space structures. Sound and Vibration, 26(6), p. 24-30
[18] Garkavi A L, Shmatkov V A, and Granovsky G E 1988 Mathematical Methods for Calculating the Reliability of Construction Equipment, 53 – Moscow: MCEI-MSUCE
[19] Kagan M L 1985 Probabilistic Methods for Calculation and Processing of Information in the Construction Industry, 132 – Moscow: MCEI-MSUCE