Profile imitation algorithm research for the cam contour

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Abstract. In this paper, the characteristics and simulation accuracy of different simulation algorithms are studied for the cam profile. The triangular square approximation is improved, and a high-precision simulation algorithm is proposed. The improved simulation algorithm has achieved good results in practical applications. Comparative analysis using examples: comparing the accuracy of the three approximation methods of spline interpolation, unconstrained best square approximation, and best square approximation with first-order boundary. It is concluded that the optimal triangular square approximation of the first-order boundary can more accurately reflect the law of periodic motion, and is suitable for the state approximation and characteristics of periodic motion.

1. Analysis of common algorithms for copying cams.

The most commonly algorithm of copying cam is spline interpolation. We can also use finite Fourier approximation and best square approximation [1-4]. It is difficult to avoid the Ruge phenomenon of data distortion in the interpolation of higher-order algebraic polynomials and trigonometric polynomials, which is not suitable for cam copying. Interpolation is a function approximation which is equal to the known data everywhere. The finite Fourier approximation and the best square approximation are not all point by point function approximation. Because the known data has rounding error or measurement error, equality everywhere is not better than approximation. Square approximation is not conducive to kicking out the bad points of the data. However, the smooth order of spline interpolation is relatively low. Only equidistant value points of finite Fourier approximation can easily obtain the expansion coefficient. The best triangle square approximation can be used in the cam with periodic motion. We can study the advantages and disadvantages of different methods through in-depth and detailed comparative analysis.

In this paper, the general best trigonometric square approximation [5-7] and the cubic spline interpolation satisfying the first-order boundary are explored [8]. And the best trigonometric square approximation with the first-order boundary is studied in depth.

2. The best square approximation of trigonometric polynomials and improvement.

2.1. Non equidistant best trigonometric square approximation without boundary constraints.

Given a set of points in $[0, 2\pi]$: $0 \leq x_1 < x_2 < \ldots < x_n < 2\pi$, The value of function $f(x)$ at these points is known. The trigonometric polynomials are solved:

$$T(x) = a_0 + \sum_{k=1}^{n} (a_k \cos kx + a_{n+k} \sin kx)$$  \hspace{1cm} (1)

Sum of the squares of the deviations at the known points is the minimum:
2.2. Non equidistant optimal trigonometric square approximation with first order boundary constraints.

For $T(x)$, the first reciprocal is:

$$T(x) = \sum_{k=1}^{n} (-a_{k} \sin kx + a_{n+k} \cos kx)$$

(2)

When necessary, it is easy to get the second derivative as:

$$T''(x) = \sum_{k=1}^{n} (-a_{k} k^2 \cos kx + a_{n+k} k^2 \sin kx)$$

Now it is required that $T(x)$ not only has the smallest sum of squares of deviations from known points, but also has the smallest deviation from the specified value of the first derivative of the boundary. So that, $\Delta = \sum_{k=1}^{n} [T(x_k) - f(x_k)]^2 + [T'(x_k) - f'(x_k)]^2 + T''(x_m) - f''(x_m)]^2$ is the minimum.

Each coefficient meet the following equations:

$$\frac{1}{2} \frac{\partial \Delta}{\partial a_n} = \sum_{j=1}^{n} [T(x_j) - f(x_j)] \cos jx_n = 0$$

$$\frac{1}{2} \frac{\partial \Delta}{\partial a_{n+k}} = \sum_{j=1}^{n} [T(x_j) - f(x_j)] \sin jx_n = 0$$

$$\frac{1}{2} \frac{\partial \Delta}{\partial a_{n+k+1}} = \sum_{j=1}^{n} [T(x_j) - f(x_j)] \cos jx_n = 0$$

$$\frac{1}{2} \frac{\partial \Delta}{\partial a_{n+k+2}} = \sum_{j=1}^{n} [T(x_j) - f(x_j)] \sin jx_n = 0$$

After sorting out the above equation and writing it into the matrix, we can get:

$$BA = C$$

(3)

Among that, $A = (a_0, a_1, a_2, ..., a_n, a_{n+1}, a_{n+2}, ..., a_{2n})^T$

$B$ is a real symmetric matrix, and each element is calculated according to the following equations.

$$b_{ij} = b_{i,j} = \sum_{j=1}^{n} \cos kx_j \cos jx_i + k \sin kx_j \sin jx_i + k \sin kx_n \sin jx_n$$

$$b_{n+k,n+i} = b_{n+i,n+k} = \sum_{j=1}^{n} \sin kx_j \sin jx_i + k \cos kx_j \cos jx_i + k \cos kx_n \cos jx_n$$

$$b_{n+k+1,n+i} = b_{n+i,n+k+1} = \sum_{j=1}^{n} \cos kx_j \sin jx_i - k \cos kx_j \sin jx_i + k \cos kx_n \sin jx_n$$

$$i = 1, 2, ..., n$$

$$k = 1, 2, ..., n$$

$$c = (c_0, c_1, c_2, ..., c_n, c_{n+1}, c_{n+2}, ..., c_{2n})^T$$

$$c_0 = \sum_{j=1}^{n} y_j$$

$$c_i = \sum_{j=1}^{n} y_j \cos jx_i - f'(x_1) \sin jx_i - f'(x_n) \sin jx_n$$

$$c_{n+i} = \sum_{j=1}^{n} y_j \sin jx_i + f'(x_1) \cos jx_i + f'(x_n) \cos jx_n$$

$$i = 1, 2, ..., n$$

Equation (4) and (5) are known, and $a$ can be obtained by Gauss elimination method.
3. Example comparison

The intake and exhaust cams of a practical diesel engine are asymmetric cams with different turning angles. On the drawing, the values of one lift in front and two in back and one lift in the middle only prove the configuration relationship of the exhaust cams. In order to meet the needs of production, the intake cam can only be calculated and analyzed by numerical method. In order to investigate the accuracy of the approximation model, based on the calculated value of the accurate relationship of the exhaust cam, the spline interpolation, unconstrained best square approximation and best square approximation with first-order boundary are used as the similarity calculation with the drawing data. Within the effective lift range of 172.5 °C, the mean square deviation and the maximum of 346 lift, velocity, acceleration and profile coordinate values are examined respectively with a value of half degree Large absolute deviation, see Table 1.

| Compare with exact relation value | Spline interpolation | Approximation with first order boundary | Free boundary approximation | Data unit |
|----------------------------------|----------------------|----------------------------------------|-----------------------------|-----------|
|                                  | Option 1            | Option 2                               | Option 3                    | Data unit |
| Average lift equation           | 0.00 241            | 0.00 002 5                             | 0.00 798                    | mm        |
| Maximum lift deviation          | 0.01 789            | 0.00 006 9                             | 0.05 306                    | mm        |
| And corner                      | 58.0 19              | 145.0 0                               | 58.0 15.0                   | deg       |
| Mean square deviation of velocity | 0.07 682            | 0.00 110 3                             | 0.30 845                    | mm/r ad   |
| Maximum speed deviation         | 0.45 924            | 0.00 252 6                             | 1.62 983                    | mm/r ad   |
| And corner                      | 56.0 31.0            | 143.5 0                               | 54 0.5                      | deg       |
| Mean square deviation of acceleration | 3.72 852          | 0.7 845 5                             | 17.4 669                    | mm/r ad$^2$|
| Maximum acceleration deviation  | 29.2 068            | 2.4 787 9                             | 121.992                     | mm/r ad$^2$|
| And corner                      | 58.0 42.5            | 145.0 0                               | 0.0 0.0                     | deg       |
| Contour x mean square deviation | 0.01 900            | 0.0 019 4                             | 0.03 213                    | mm        |
| x maximum deviation             | 0.12 525            | 0.0 067 9                             | 0.22 960                    | mm        |
| And corner                      | 59.5 108.5           | 143.5 0                               | 61.0 52.0                   | deg       |
| Contour y mean                  | 0.01 0.010           | 0.00 0.00                             | 0.08 0.02                   | mm        |
Scheme 1 in the table is the interpolation and approximation based on the nonisometric lift inspection data of the drawing (one lift value for every 2 degrees in front and back 10 degrees, and one lift value for every 3 degrees in between), from which it can be seen that three methods are available. In particular, the trigonometric square approximation satisfying the first-order boundary is the best, the average fluctuation of the exact relationship is the smallest, and the displacement, velocity, acceleration and profile Monogram are closer to the exact values. The trigonometric square approximation of free boundary only has absolute deviation greater than spline deviation at some boundary points.

Scheme 2 in the table is the interpolation and approximation of the equidistant data (2.5 degrees lift value) calculated according to the precise relationship. At this time, the spline interpolation and the first-order boundary trigonometric square approximation are both ideal, while the free boundary trigonometric square approximation fluctuates greatly at the boundary point. For very smooth small equidistant data, spline interpolation is slightly better. However, neither the drawing data nor the measured data can be the ideal data.

Scheme 3 in the table is accurate to the thousandth place in scheme 2 in the table, and there is a secondary point at 15 degrees before and after, that is, only accurate to the tenth place, which obviously meets the requirements of the first-order boundary trigonometric square approximation, and basically meets the requirements of machining accuracy, and the other two methods are equally poor.

Scheme 3 is smoothed and then interpolated or approximated by rebound method, which improves the trigonometric square approximation satisfying the first-order boundary obviously.

4. Conclusion
The best trigonometric square approximation of the first-order boundary can more accurately reflect the periodic motion law, which is very suitable for the state approximation and characteristic research of periodic motion, and is a very ideal cam simulation and manufacturing calculation tool.

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