QUARTIC GAUGE BOSON COUPLINGS AND TREE UNITARITY IN THE SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_N$ MODELS

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Abstract

The quartic gauge boson couplings in the SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_N$ models are presented. We find that the couplings of four different gauge bosons may have unusual Lorentz structure and the couplings satisfy the tree unitarity requirement at high energy limit.

1 Introduction

Although the standard model (SM) of electroweak interactions has been verified to great precision in the recent years at LEP, SLC and other places, there remain a few unanswered questions concerning the generation structure of quarks and leptons. In particular the question of the number of generations remains open and few progress has been made towards the understanding of the interrelation between generations. Amongst the possible extensions beyond the SM, the models based on the SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_N$ (3-3-1) gauge group \cite{1,2,3,4,5} are interesting from this point of view. They have the following intriguing features: Firstly, the models are anomaly free only if the number of generations $N$ is a multiple of three. If further one adds the condition of QCD asymptotic freedom, which is valid only if the number of generations of quarks is to be less than five, it follows that $N$ is equal to 3. The second characteristic of these models is that one generation of quarks is treated differently from two others. This could lead to a natural explanation for the unbalancing heavy top quarks, deviations of $A_b$ from the SM prediction, ...

In the SM, electroweak gauge bosons are introduced to preserve the local SU(2)$_L$ $\otimes$ U(1)$_Y$ symmetry. As a result, there is a universality among the couplings of the fermions to the gauge bosons, the three gauge bosons, and the four gauge bosons. This universality forms the basis of the success of the SM. So far the fermion-gauge-boson couplings were tested precisely at various colliders, however the direct measurement of the self-couplings of the gauge bosons is not precise enough to test the SM at loop level. The measurements performed at LEP1 have provided us with an extremely accurate knowledge of the parameters of the $Z$ gauge boson: its mass, partial widths, and total width. There even is first evidence that the contributions of gauge-boson loops to the gauge-boson self-energies are indeed required \cite{6}. Thus, an indirect confirmation of the existence of the trilinear gauge couplings (TGC’s) has been obtained. Deviation of non-abelian couplings from expectation would signal new physics. In addition, tests of the trilinear

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couplings aim at checking the non-abelian gauge structure, while quartic ones will provide important information on the nature of spontaneous symmetry breaking. In [7] the TGCs in the minimal model and in the 3 - 3 - 1 model with right-handed neutrinos have been presented. The TGCs and quartic gauge couplings (QGCs) in the minimal 3 - 3 - 1 model in a $U_e(1)$ covariant gauge were used in consideration of the static electromagnetic properties of the $W$ boson [8].

In this paper we present a complete set of the QGCs in two main versions of the 3 - 3 - 1 models. We will show that the tree unitarity requirement will be satisfied in all scatterings of longitudinal components of the vector gauge bosons.

## 2 Quartic gauge boson couplings in the 3 - 3 - 1 models

We outline two kinds of 3 – 3 – 1 models: the minimal proposed by Pisano, Pleitez and Frampton [2, 3], and the model with right-handed neutrinos [4].

### A. The minimal 3 – 3 – 1 model

The model treats the leptons as the SU(3)$_L$ antitriplet [3, 12]

$$f_L^a = \begin{pmatrix} e_L^a \\ -\nu_L^a \\ (e^c)^a_L \end{pmatrix} \sim (1, 3^*, 0); \ a = 1, 2, 3. \quad (1)$$

Two of the three quark generations transform as triplets and the third generation is treated differently - in antitriplet:

$$Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \\ D_{iL} \end{pmatrix} \sim (3, 3, -1/3), \quad (2)$$

$$u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -4/3), \ i = 1, 2,$$

$$Q_{3L} = \begin{pmatrix} d_{3L} \\ -u_{3L} \\ T_{3L} \end{pmatrix} \sim (3, 3^*, 2/3), \quad (3)$$

$$u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_{3R} \sim (3, 1, 5/3).$$

At the nine gauge bosons $W^a (a = 1, 2, ..., 8)$ and $B$ of SU(3)$_L$ and U(1)$_N$, four are light: photon $A$, $Z$ and $W^\pm$. The remaining five are new gauge bosons $Z'$, $Y^\pm$ and doubly charged bilepton $X^{\pm\pm}$. They are expressed in terms of $W^a$ and $B$ as [11]

$$\sqrt{2} \ W^+_\mu = W^1_{\mu} - i W^2_{\mu}, \quad \sqrt{2} \ Y^+_\mu = W^6_{\mu} - i W^7_{\mu},$$

$$\sqrt{2} \ X^{++}_\mu = W^4_{\mu} - i W^5_{\mu}. \quad (4)$$

In addition to these, neutral gauge bosons are photon, $Z$ and $Z'$ [12]:

$$A_\mu = s_W W^3_\mu + c_W \left( \sqrt{3} \ t_W W^8_\mu + \sqrt{1 - 3 \ t^2_W} \ B_\mu \right),$$

$$Z_\mu = c_W W^3_\mu - s_W \left( \sqrt{3} \ t_W W^8_\mu + \sqrt{1 - 3 \ t^2_W} \ B_\mu \right),$$

$$Z'_\mu = -\sqrt{1 - 3 \ t^2_W} \ W^8_\mu + \sqrt{3} \ t_W B_\mu. \quad (5)$$

†For recent proposed 3 -3 -1 models see [9]
‡The leptons may be assigned a triplet as in [2], however two models are mathematically identical.
where we denoted $s_W \equiv \sin \theta_W$, $c_W \equiv \cos \theta_W$, $t_W \equiv \tan \theta_W$.

The physical states are mixtures of $Z$ and $Z'$:

\[
Z_1 = Z \cos \phi - Z' \sin \phi, \\
Z_2 = Z \sin \phi + Z' \cos \phi,
\]

where $\phi$ is a mixing angle.

The mixing angle has to be very small \[12\] $-1.6 \times 10^{-2} \leq \phi \leq 7 \times 10^{-4}$, so that, we can safely neglect the mixing. It is interesting to note that in this model $\sin^2 \theta_W(m_{Z_2})$ should be less than 1/4, and it leads to $m_{Z_2} \leq 3.1 \text{ TeV}$. Moreover, from the muon decay experiment \[14\], $m_Y$ is found to be at least 230 GeV at 90% CL. The spontaneous symmetry breaking yields a splitting on the bileptons masses \[10\]

\[
|M_X^2 - M_Y^2| \leq 3 m_W^2.
\]

The quartic couplings arise from

\[
\mathcal{L}_{QGC} = \frac{g^2}{4} f^{abc} f_{ade} W_{b\mu} W_{c\nu} W^{d\mu} W^{e\nu}.
\]  

Expressing $W^a$ ($a = 1, 2, ..., 8$) in terms of physical fields thanks to Eqs \[4\] and \[5\], after straightforward but tedious calculation we get

\[
\frac{1}{g^2} \mathcal{L}_{QGC}^{\text{min}} = \frac{1}{2} \left( W^+.W^-W^+.W^- - W^+.W^+W^-.W^- \right) + \frac{1}{2} \left( Y^+.Y^-Y^+.Y^- - Y^+.Y^+Y^-.Y^- \right) \\
+ \frac{1}{2} \left( X^{++}.X^{--}X^{++}.X^{--} - X^{++}.X^{++}X^{--}.X^{--} \right) \\
+ \frac{1}{2} \left( W^+.W^-Y^+.Y^- + W^+.Y^+W^-Y^- - 2W^+.Y^-W^-Y^+ \right) \\
+ \frac{1}{2} \left( W^+.W^-X^{++}.X^{--} + W^+.X^{--}W^-X^{++} - 2W^+.X^{++}W^-X^{--} \right) \\
+ \frac{1}{2} \left( Y^+.Y^-X^{++}.X^{--} + Y^+.X^{--}Y^-X^{++} - 2Y^+.X^{++}Y^-X^{--} \right) \\
- s_W^2 \left[ (A.W^+.A.W^- - A.AW^+.W^-) + (A.Y^+.A.Y^- - A.AY^+.Y^-) \right] \\
- 4s_W^2 \left( A.X^{++}.A.X^{--} - A.AX^{++}.X^{--} \right) - c_W^2 \left( ZW^+ZW^- - ZZW^+.W^- \right) \\
- \frac{(c_W - 3s_W t_W)^2}{4} \left[ (Z.Y^+.Z.Y^- - Z.ZY^+.Y^-) + (Z.X^{++}Z.X^{--} - Z.ZX^{++}.X^{--}) \right] \\
- \frac{3}{4} \left( 1 - 3t_W^2 \right) \left[ (Z'.Y^+.Z'.Y^- - Z'.Z'^+Y^-) + (Z'.X^{++}Z'.X^{--} - Z'.Z'X^{++}.X^{--}) \right] \\
- c_W s_W \left( A.W^+.Z.W^- + A.W^-Z.W^+ - 2A.ZW^+.W^- \right) \\
+ \frac{1}{2} s_W (c_W + 3s_W t_W) \left( A.Y^+.Y^- + A.Y^-Z.Y^+ - 2A.ZY^+.Y^- \right) \\
- s_W (c_W - 3s_W t_W) \left( A.X^{++}.Z.X^{--} + A.X^{--}Z.X^{++} - 2A.ZX^{++}.X^{--} \right) \\
+ \frac{1}{2} s_W \sqrt{3(1 - 3t_W^2)} \left( A.Y^+.Z'.Y^- + A.Y^-Z'.Y^+ - 2A.Z'Y^+.Y^- \right) \\
+ s_W \sqrt{3(1 - 3t_W^2)} \left( A.X^{++}Z'.X^{--} + A.X^{--}Z.X^{++} - 2A.Z'X^{++}.X^{--} \right) \\
- \frac{1}{4} (c_W + 3s_W t_W) \sqrt{3(1 - 3t_W^2)} \left( Z.Y^+.Z'.Y^- + Z.Y^-Z'.Y^+ - 2Z.Z'Y^+.Y^- \right)
\]
\( + \frac{1}{4} (C_W - 3 s_W t_W) \sqrt{3(1 - 3 t_W^2)} \left( Z \cdot X^{++} Z', X^{--} + Z \cdot X^{--} Z', X^{++} - 2 Z \cdot Z', X^{++}, X^{--} \right) \)

\( + \frac{1}{4} \sqrt{6(1 - 3 t_W^2)} \left( Z', Y^+ W^+, X^{--} + Z', X^{--} W^+, Y^+ - 2 Z', W^+ Y^+, X^{--} \right) \)

\( + \frac{3 s_W}{\sqrt{2}} (A \cdot W^+ Y^+, X^{--} - A \cdot Y^+ W^+, X^{--}) \)

\( + \frac{3}{2 \sqrt{2}} \left[ s_W t_W \left( Z \cdot Y^+ W^+, X^{--} + Z \cdot X^{--} W^+, Y^+ - 2 Z \cdot W^+ Y^+, X^{--} \right) \right] + h.c \) (8)

where the following notation was used: \( X \cdot Y \equiv X_\mu Y^\mu \).

The vertices in this model are listed in Table 1. In our assumption, all charged boson lines are taken to be entering into the vertices. We remind that in the SM the QGCs contain two parts: the first is coupling strenght \( \propto g^2 \) and the second is common denoted by \( S_{\mu \nu, \rho \lambda} \) in the Cheng \& Li textbook [13].

Table 1: Quartic couplings in the minimal 3 \(-\) 3 \(-\) 1 model

| Vertex | coupling constant / \( g^2 \) |
|--------|--------------------------------|
| \( W^+ \mu \nu, W^- \alpha \beta \) | \( S_{\mu \nu, \alpha \beta} \) |
| \( Y^+ \mu \nu, Y^- \alpha \beta \) | \( S_{\mu \nu, \alpha \beta} \) |
| \( X^+ \mu \nu, X^- \alpha \beta \) | \( S_{\mu \nu, \alpha \beta} \) |
| \( W^+ \mu \nu, W^- \alpha \beta \) | \( S_{\mu \nu, \alpha \beta} \) |
| \( Y^+ \mu \nu, Y^- \alpha \beta \) | \( S_{\mu \nu, \alpha \beta} \) |
| \( X^+ \mu \nu, X^- \alpha \beta \) | \( S_{\mu \nu, \alpha \beta} \) |
| \( Z \cdot Y^+ W^+, X^{--} \) | \( -s_W^2 S_{\mu \nu, \alpha \beta} \) |
| \( Z \cdot X^{++} X^{--} \) | \( -s_W^2 S_{\mu \nu, \alpha \beta} \) |
| \( Z \cdot Z', W^+, X^{--} \) | \( -c_W s_W S_{\mu \nu, \alpha \beta} \) |
| \( Z \cdot Z', Y^+ X^{--} \) | \( s_W (c_W + 3 s_W t_W) S_{\mu \nu, \alpha \beta} / 2 \) |
| \( Z \cdot Z', Y^+ X^{--} \) | \( -s_W (c_W - 3 s_W t_W) S_{\mu \nu, \alpha \beta} / 2 \) |
| \( Z \cdot Z', Y^+ X^{--} \) | \( s_W \sqrt{(3 - 9 t_W^2)} S_{\mu \nu, \alpha \beta} / 2 \) |
| \( Z \cdot Z', Y^+ X^{--} \) | \( s_W \sqrt{(3 - 9 t_W^2)} S_{\mu \nu, \alpha \beta} / 2 \) |
| \( Z \cdot Z', Y^+ X^{--} \) | \( (c_W - 3 s_W t_W) \sqrt{(3 - 9 t_W^2)} S_{\mu \nu, \alpha \beta} / 2 \) |
| \( Z \cdot Z', Y^+ X^{--} \) | \( (c_W - 3 s_W t_W) \sqrt{(3 - 9 t_W^2)} S_{\mu \nu, \alpha \beta} / 2 \) |
| \( Z \cdot Z', Y^+ X^{--} \) | \( \sqrt{6(1 - 3 t_W^2)} S_{\mu \nu, \alpha \beta} / 2 \) |
| \( Z \cdot Z', Y^+ X^{--} \) | \( 3 s_W V_{\mu \nu, \alpha \beta} \) |
| \( Z \cdot Z', Y^+ X^{--} \) | \( 3 (s_W t_W S_{\mu \nu, \alpha \beta} + c_W U_{\mu \nu, \alpha \beta}) / (2 \sqrt{2}) \) |
Here the following notations were used

\[ S_{\mu\nu,\alpha\beta} \equiv g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - 2g_{\mu\nu} g_{\alpha\beta}, \]
\[ V_{\mu\nu,\alpha\beta} \equiv g_{\mu\nu} g_{\alpha\beta} - g_{\mu\alpha} g_{\nu\beta}, \]
\[ U_{\mu\beta,\nu\alpha} \equiv g_{\mu\beta} g_{\nu\alpha} - g_{\mu\alpha} g_{\nu\beta}. \]

(9)

From Table 1, we see that the two last vertices \( \gamma W Y X \) and \( Z W Y X \) are not proportional to the usual \( S_{\mu\alpha,\nu\beta} \), that is why we call it unusual Lorentz structure. \( S_{\mu\alpha,\nu\beta} \) is symmetric in permutation of \( \mu \) and \( \alpha \), and in permutation of \( \nu \) and \( \beta \), so the usual quartic gauge boson vertices (exclusive of two mentioned ones) are symmetric in permutation of two particles with the same electric charges.

B. The model with right-handed neutrinos

In this model, leptons are in a triplet:

\[ f^a_L = \begin{pmatrix} \nu^a_L \\ e^a_L \\ (\nu^c_L)^a \end{pmatrix} \sim (1, 3, -1/3), e^a_R \sim (1, 1, -1). \]

(10)

The first two generations of quarks are in antitriplets while the third one is in a triplet:

\[ Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ D_{iL} \end{pmatrix} \sim (3, 3^*, 0), \]
\[ u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -1/3), i = 1, 2, \]
\[ Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 1/3), \]
\[ u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3). \]

The doubly charged bileptons of the minimal model are replaced here by complex neutral ones:

\[ \sqrt{2} W^+_\mu = W^1_\mu - iW^2_\mu, \sqrt{2} Y^-_\mu = W^6_\mu - iW^7_\mu, \]
\[ \sqrt{2} X^o_\mu = W^4_\mu - iW^5_\mu. \]

(13)

For a shorthand notation, hereafter we will use \( X^o \equiv X \).

The physical neutral gauge bosons are again related to \( Z, Z' \) through the mixing angle \( \phi \). Together with the photon, these are defined as follows [3]:

\[ A_\mu = s_W W^3_\mu + c_W \left( -\frac{t_W}{\sqrt{3}} W^8_\mu + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \]
\[ Z_\mu = c_W W^3_\mu - s_W \left( -\frac{t_W}{\sqrt{3}} W^8_\mu + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \]
\[ Z'_\mu = \sqrt{1 - \frac{t_W^2}{3}} W^8_\mu + \frac{t_W}{\sqrt{3}} B_\mu. \]

(14)

The symmetry-breaking hierarchy gives us splitting on the bileptons masses [11]

\[ |M^2_X - M^2_Y| \leq m^2_W. \]

(15)
Therefore in the future studies it is acceptable to put \( M_X \simeq M_Y \).

The constraint on the \( Z \sim Z' \) mixing based on the \( Z \) decay, is given \([3]\): \(-2.8 \times 10^{-3} \leq \phi \leq 1.8 \times 10^{-4}\), and in this model we have not only a limit for \( \sin^2 \theta_W \) but also the upper limit for new gauge bosons.

From neutrino-electron scattering one gets a lower limit for \( M_{Z_1} \) in the range of 400 GeV, and the muon decay data \([4]\) gives a lower bound for \( Y \) bosons: 230 GeV (90 % CL). The symmetry-breaking hierarchy gives us a bilepton mass splitting: \( m_Y \simeq m_X \) (at least at the tree level) \([1]\)

\[
|M_X^2 - M_Y^2| \leq m_W^2. \tag{16}
\]

The similar cumbersome calculation gives the QGCs in this model:

\[
\frac{1}{g^2} \mathcal{L}^\text{rhb}_{\text{QGC}} = \frac{1}{2} \left[ (W^+.W^-W^+.W^- - W^+.W^-W^-W^-) + (Y^+.Y^-Y^+.Y^- - Y^+.Y^-Y^-Y^-) \right] + (X.X^*X.X^* - X.X.X.X^*) + 
\]

\[
(Y^+.Y^-X.X^* + Y^+.X.Y^-X^* - 2Y^+.X^*Y^-X) + 
\]

\[
(W^+.W^-X.X^* + W^+.X.W^-X^* - 2W^+.X^*W^-X)] - \]

\[
s_W^2 \left[ (A.W^+.A.W^- - A.AW^+.W^-) + (A.Y^+.A.Y^- - A.AY^+.Y^-) \right] - 
\]

\[
c_W^2 \left[ (Z.W^+.Z.W^- - Z.Z.W^+.W^-) - \frac{\cos^2 2\theta_W}{4c_W^2} (Z.Y^+.Z.Y^- - Z.Z.Y^+.Y^-) \right] - \]

\[
\frac{1}{4c_W^2} (Z.X^*X.Z - Z.Z.X.X^*) + \frac{3 - t_W^2}{4} \left[ (Z'.Y^+Z'.Y^- - Z'.Z^Y+.Y^-) \right] + 
\]

\[
(Z'.X.Z'.X^* - Z'.Z'.X.X^*) - 
\]

\[
c_W s_W \left[ (A.W^-.Z.W^+ + A.W^+.Z.W^- - 2A.ZW^+.W^-) \right] - 
\]

\[
t_W^2 \cos 2\theta_W \left[ A.Y^-.Z.Y^+ + A.Y^+.Z.Y^- - 2A.ZY^+.Y^- \right] + 
\]

\[
s_W^2 \frac{\sqrt{3 - t_W^2}}{2} \left[ A.Y^-.Z'.Y^+ + A.Y^+.Z'.Y^- - 2A.Z^Y+.Y^- \right] - 
\]

\[
\frac{\cos 2\theta_W}{4c_W} \sqrt{3 - t_W^2} (Z.Y^-.Z'.Y^+ + Z.Y^+.Z'.Y^- - 2Z.Z^Y+.Y^-) + 
\]

\[
\frac{\sqrt{3 - t_W^2}}{4c_W} (Z.X^*Z'.X + Z.X.Z'.X^* - 2Z.Z^X.X^*) - 
\]

\[
s_W \frac{\sqrt{2}}{2} \left[ A.Y^-.X^+.W^+ + A.W^+.X^+.Y^- - 2A.X^*W^+.Y^- \right] - 
\]

\[
\sqrt{\frac{3 - t_W^2}{8}} (Z'.X.W^+.Y^- + Z'.Y^-.X^+.W^+ - 2Z'.W^+X^*.Y^-) - 
\]

\[
s_W t_W \frac{2\sqrt{2}}{2} \left[ Z.X^*W^+.Y^- + Z.Y^-.W^+.X^* - 2Z.W^+X^*.Y^- \right] + 
\]

\[
\frac{c_W}{2\sqrt{2}} \left( Z.Y^-.W^+.X^* - Z.X^*W^+.Y^- \right) + h.c \tag{17}
\]

As before, the QGCs in the considered model are presented in Table 2.
Table 2: Quartic couplings in the 3 – 3 – 1 model with right-handed neutrinos

| Vertex                                                                 | coupling constant/g^2 |
|-----------------------------------------------------------------------|-----------------------|
| $W_+ W_+ W_+ W_-$                                                   | $S_{\mu\alpha,\beta}$ |
| $Y_+ Y_+ Y_+ Y_-$                                                   | $S_{\mu\alpha,\beta}$ |
| $X_\mu X_\nu X_\alpha X_\beta$                                     | $S_{\mu\alpha,\beta}$ |
| $W_\mu W_\nu Y_\alpha Y_\beta$                                     | $S_{\mu\alpha,\beta}$/2 |
| $W_\mu W_\nu X_\alpha X_\beta$                                     | $S_{\mu\alpha,\beta}$/2 |
| $Y_\mu Y_\nu X_\alpha X_\beta$                                     | $S_{\mu\beta,\nu\alpha}$/2 |
| $\gamma_\mu \gamma_\nu W_\alpha W_\beta$                          | $-s_W^2 S_{\mu\alpha,\beta}$ |
| $\gamma_\mu \gamma_\nu Y_\alpha Y_\beta$                           | $-s_W^2 S_{\mu\alpha,\beta}$ |
| $Z_\mu Z_\nu W_\alpha W_\beta$                                      | $-c_W^2 S_{\mu\alpha,\beta}$ |
| $Z_\mu Z_\nu Y_\alpha Y_\beta$                                      | $-\cos^2 2\theta W S_{\mu\alpha,\beta}/(4c_W^2)$ |
| $Z_\mu Z_\nu X_\alpha X_\beta$                                      | $-S_{\mu\alpha,\beta}/(4c_W^2)$ |
| $Z_\mu Z_\nu Y_\alpha Y_\beta$                                      | $(3 - t_W^2) S_{\mu\alpha,\beta}/4$ |
| $Z_\mu Z_\nu X_\alpha X_\beta$                                      | $(3 - t_W^2) S_{\mu\alpha,\beta}/4$ |
| $\gamma_\mu \gamma_\nu X_\alpha X_\beta$                          | $-t_W \cos 2\theta W S_{\mu\alpha,\beta}/2$ |
| $\gamma_\mu \gamma_\nu Y_\alpha Y_\beta$                           | $s_W \sqrt{3 - t_W^2} S_{\mu\alpha,\beta}/2$ |
| $Z_\mu Z_\nu Y_\alpha Y_\beta$                                      | $-\cos 2\theta W \sqrt{3 - t_W^2} S_{\mu\alpha,\beta}/(4c_W)$ |
| $Z_\mu Z_\nu X_\alpha X_\beta$                                      | $\sqrt{3 - t_W^2} S_{\mu\alpha,\beta}/(4c_W)$ |
| $Z_\mu Z_\nu Y_\alpha Y_\beta$                                      | $-\sqrt{(3 - t_W^2) S_{\mu\alpha,\beta}/(2\sqrt{2})}$ |
| $Z_\mu Z_\nu X_\alpha X_\beta$                                      | $-s_W S_{\mu\alpha,\beta}/\sqrt{2}$ |
| $Z_\mu Y_\alpha Y_\alpha Y_\beta$                                   | $-s_W t_W S_{\mu\alpha,\beta} - c_W V_{\mu\beta\alpha}/(2\sqrt{2})$ |

Our next step is to show that the vertices given here justify the unitarity requirement. For this purpose we show that the cancellation over quartic divergences in high energy scattering of gauge bosons.

3 Quartic divergence cancellation in high energy limit

In this section we show that the above presented QGCs satisfy tree unitarity requirement. For our purpose we consider processes in which two gauge bosons are in the initial and in the final state

$$V(p_1, m_1) + V(p_2, m_2) \rightarrow V(k_1, m_3) + V(k_2, m_4),$$

(18)

where momentum and mass of the corresponding particle are put in the bracket. Here $V$ stands for gauge bosons in these models: $\gamma, Z, W, X, Y$. As usual we denote the Mandelstam variables

$$s = (p_1 + p_2)^2, t = (p_1 - k_1)^2, u = (p_1 - k_2)^2$$

(19)

which satisfy the relation

$$s + t + u = \sum_{i=1}^{4} m_i^2.$$  

(20)
In the center of mass frame in which \( \mathbf{p}_1 = -\mathbf{p}_2 \equiv \mathbf{p}, \mathbf{k}_1 = -\mathbf{k}_2 \equiv \mathbf{k} \), the differential cross section is given by

\[
\left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{|M|^2 |\mathbf{k}| S}{64\pi^2 s |\mathbf{p}|}.
\]  

(21)

Here \( M \) is shorthand notation for the invariant amplitude \( \langle f||M||i \rangle \) and \( S = \prod_a 1/l_a! \) where \( l_a \) is the number of identical particles of type \( a \) in the final state. From (21) it is clear that in the high energy limit \( s \gg m_i^2 \) the unitarity requests \( M \sim \mathcal{O}(1) \). In other words, the amplitude \( M \) cannot contain the terms proportional to \( \frac{s}{m_i^2} \) or \( \frac{s^2}{m_i^4} \). It is known that for four gauge boson scattering the terms \( \propto s^2 \) arise only from purely gauge boson contribution (from Feynman diagrams (1a)–(1d) in figure 1), while the terms \( \propto s \) arise from both gauge boson contribution and the Higgs and would-be pseudo-Goldstone boson ones (Fig. 1e – 1g). The massless vector bosons have only two components, while the massive ones have three: two transverse and one longitudinal components. The longitudinal component plays a special role: namely in the high energy limit, they give the main contributions. For this reason the longitudinal components are usually used in estimation of high energy behaviour of scattering amplitudes. In this paper, therefore we shall consider two cases.

### 3.1 Scattering of massive gauge bosons

In the high energy limit, the component of vector of longitudinal polarization is given

\[
\epsilon^\mu_L(k) = \frac{k^\mu}{m} + \mathcal{O}\left(\frac{m}{k_0}\right)
\]  

(22)

Since the component of \( k^\mu \) are growing as \( |\mathbf{k}| \), the “bad” behaviour of the amplitude concerns with longitudinal vector bosons. For this reason hereafter we are only working with \( \epsilon^\mu_L(k) \).

We have checked that tree unitarity is satisfied for all possible processes such like in (15). For illustration we consider the following processes

\[
Z_L(p_1, m_Z) + W_L^+(p_2, m_W) \rightarrow X_{L}^{++}(k_1, m_X) + Y_{L}^-(k_2, m_Y)
\]  

(23)

in the minimal model and

\[
Z_L(p_1, m_Z) + W_L^+(p_2, m_W) \rightarrow X_{L}^0(k_1, m_X) + Y_{L}^+(k_2, m_Y)
\]  

(24)

in the model with right-handed neutrinos. The subscript \( L \) added to the fields indicates the longitudinal component of the vector bosons.

Now we turn to the process (23). At the tree level the Feynman diagrams are depicted in Figure 1.

[Diagram a: \( Z_L(p_1) \rightarrow X_{L}^{++}(k_1) \) with \( W_L^+(p_2) \rightarrow Y_{L}^-(k_2) \)]

[Diagram b: \( Z_L(p_1) \rightarrow X_{L}^+(k_1) \) with \( W_L^+(p_2) \rightarrow Y_{L}^-(k_2) \)]
Fig. 1  Tree-level diagrams for the process $Z_L W_L^+ \rightarrow X_{L+}^+ Y_{L-}^-$. Here the wave line represents gauge boson and the dashed line - pseudo-Goldstone boson associated with the gauge boson.

Using the Feynman-t’Hooft gauge, the contributions from Figs. 1a, 1b, 1c and 1d are given, respectively

\[ M_{1a} = g_{WWZ} g_{WXY} \frac{(u - t)s}{4m_Z m_W m_Y m_X} + \mathcal{O} \left( \frac{s}{m_i^2} \right) \]  \hspace{1cm} (25)

\[ M_{1b} = g_{WXZ} g_{WXY} \frac{(u - s)t}{4m_Z m_W m_Y m_X} + \mathcal{O} \left( \frac{s}{m_i^2} \right) \]  \hspace{1cm} (26)

\[ M_{1c} = g_{XYZ} g_{WXY} \frac{(t - s)u}{4m_Z m_W m_Y m_X} + \mathcal{O} \left( \frac{s}{m_i^2} \right) \]  \hspace{1cm} (27)

\[ M_{1d} = \frac{g_{ZWXY}}{4m_Z m_W m_Y m_X} \left[ 3s_W^2 (2s^2 - u^2 - t^2) + 3c_W^2 (u^2 - t^2) \right] + \mathcal{O} \left( \frac{s^2}{m_i^2} \right) \]  \hspace{1cm} (28)

With the TGCs given in [7] (see Appendix, Table 3) and the QGCs given in this work we see that the quartic divergences are indeed vanished.

For process (24), the Feynman diagrams are depicted in Fig.2
For the process involved photon, the above manipulation are not applicable. Let us consider in detail the following process:

\[
\gamma(p_1, 0) + W^+_L(p_2, m_W) \rightarrow X^{++}_L(k_1, m_X) + Y^-_L(k_2, m_Y)
\]  

(29)

The Feynman diagrams for the above process are shown in Fig.3

---

3.2 Processes with massless photon

In exactly the same way, using TGCs in Table 4, it is elementary exercise to show that the quartic divergences are cancelled.
+ three diagrams with inserting pseudo-Goldstone bosons

Fig. 3 Tree-level diagrams for the process $AW_L^+ \rightarrow X^+_L Y^-_L$.

In the Feynman-t’Hooft gauge, the diagrams in Figs. 3a, 3b, 3c give the following contributions

\begin{align*}
M_{3a} &= g_{WW} g_{WX} \frac{s [k_1 \cdot \epsilon(p_1) - k_2 \cdot \epsilon(p_1)]}{2 m_W m_Y m_X} + \mathcal{O}(1) \\
M_{3b} &= -g_{XX} g_{XY} \frac{t [k_2 \cdot \epsilon(p_1) + p_2 \cdot \epsilon(p_1)]}{2 m_W m_Y m_X} + \mathcal{O}(1) \\
M_{3c} &= -g_{YY} g_{XY} \frac{u [k_1 \cdot \epsilon(p_1) + p_2 \cdot \epsilon(p_1)]}{2 m_W m_Y m_X} + \mathcal{O}(1) \\
M_{3d} &= g_{XY} \frac{[uk_2 \cdot \epsilon(p_1) + sp_2 \cdot \epsilon(p_1)]}{2 m_W m_Y m_X} + \mathcal{O}(1). \tag{30}
\end{align*}

Noting that $t = -(s + u)$ and using momentum conservation we see that the leading divergence is cancelled

$$M_{3a} + M_{3b} + M_{3c} + M_{3d} = 0.$$ 

With the above manipulation one can check that vertices of gauge boson self-interaction satisfy unitarity requirement at the tree level.

4 Conclusions

In this paper we have presented a complete set of quartic gauge boson couplings in both 3-3-1 models. We have shown there exist four-different gauge boson couplings with unusual Lorentz structure. By consideration of scattering of longitudinal vector bosons and as well as the massless photons we have deduced that at the tree level the quartic divergences are cancelled and then the unitarity is satisfied. It is worth to mention that QGCs which contain the interactions between the SM gauge bosons and the bileptons in the minimal 3-3-1 model were presented in [8], and their results are consistent with ours. Other couplings were not presented there. We hope that the results in this paper are useful for anyone interested in studying processes involving these couplings.

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Appendix

In this appendix we rewrite trilinear gauge boson couplings (TGCs) in the 3-3-1 models [8].
Table 3: Trilinear couplings in the minimal 3 – 3 – 1 model.

| Vertex | coupling constant/e |
|--------|---------------------|
| $\gamma W^+W^-$ | 1 |
| $ZW^+W^-$ | $1/t_W$ |
| $\gamma Y^+Y^-$ | 1 |
| $ZY^+Y^-$ | $-(1 + 2s_W^2)/\sin 2\theta_W$ |
| $\gamma X^{++}X^{--}$ | $-2$ |
| $ZX^{++}X^{--}$ | $(1 - 4s_W^2)/\sin 2\theta_W$ |
| $Z'^+Y^-$ | $-\sqrt{3(1 - t_W^2)/(2s_W)}$ |
| $Z'^X^{++}X^{--}$ | $-\sqrt{3(1 - t_W^2)/(2s_W)}$ |
| $X^{--}Y^+W^+$ | $1/(\sqrt{2} s_W)$ |
| $X^{++}W^--Y^-$ | $1/(\sqrt{2} s_W)$ |

Table 4: Trilinear couplings in the 3 – 3 – 1 model with RH neutrinos.

| Vertex | coupling constant/e |
|--------|---------------------|
| $\gamma W^+W^-$ | 1 |
| $ZW^+W^-$ | $1/t_W$ |
| $\gamma Y^+Y^-$ | 1 |
| $ZY^+Y^-$ | $1/\tan 2\theta_W$ |
| $ZXX^*$ | $-1/\sin 2\theta_W$ |
| $Z'^+Y^-$ | $-\sqrt{3(1 - t_W^2)/(2s_W)}$ |
| $Z'^X^{++}X^{--}$ | $-\sqrt{3(1 - t_W^2)/(2s_W)}$ |
| $XW^--Y^+$ | $1/(\sqrt{2} s_W)$ |
| $X^*Y^-W^+$ | $1/(\sqrt{2} s_W)$ |

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