ρN resonance dynamics in the proton nucleus reaction

Swapan Das
Nuclear Physics Division, Bhabha Atomic Research Centre,
Mumbai-400 085, India

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Abstract

The strong coupling of rho meson to the nucleon produces $s$ and $p$ wave rho meson nucleon ($\rho N$) resonances. In a nucleus, the $\rho N$ resonance-hole polarization generates the optical potential or self-energy for the rho meson. The scattering of rho meson due to this potential provides valuable informations about the $\rho N$ resonance dynamics in a nucleus. To investigate it, we use this potential to calculate the mass distribution spectrum for the $\rho$ meson produced coherently in the proton nucleus reaction. The cross sections arising due to $s$ and $p$ wave $\rho N$ resonances have been presented. The coherent and incoherent contributions to the cross sections due to these resonances are compared. In addition, the calculated results due to non-relativistic and relativistic $\rho$ meson self-energy are illustrated.

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1 INTRODUCTION

The rho meson nucleon resonances arise because of the strong coupling of this meson to the nucleon. The importance of the hadronic resonances in the particle production phenomena is undoubtedly understood from long ago. For example, the pion production data in the nuclear reaction below 1 GeV had been interpreted successfully by the formation of the $\Delta(1232)$ resonance in the intermediate state [1]. Therefore, the dynamics of the $\rho N$ resonances can have intimate relationship with the rho meson production in the nuclear reaction. The sub-threshold rho meson emission is well described by the $N(1520)$ resonance production in the intermediate state [2]. At higher energy, the importance of $N(1720)$ resonance has been discussed [3] in context to the $\rho$ meson production in the
The $\rho N$ resonances were also found significant to investigate the modification of the rho meson in the nuclear medium. Sometime back, Kondratyuk et al., [4] estimated the rho meson nucleon scattering amplitude, i.e., $f_{\rho N}$, due to the formation of $\rho N$ resonances in the intermediate state. Using this amplitude, they have shown the $\rho$ meson mass in a nucleus is below 770 MeV in the static limit. This agrees with the result of scaling hypothesis formulated by Hatsuda and Lee [5]. Friman et al., [6] calculated the $\rho$ meson self-energy arising due to $p$ wave $\rho N$ scattering, via $N(1720)$ and $\Delta(1905)$ resonances. According to their calculation, this self-energy (which appears in the $\rho$ meson spectral function) reduces the mass of the $\rho$ meson at high baryon density. Peters et al., [7] extended this calculation by incorporating contributions from all four starred $s$ and $p$ wave $\rho N$ resonances. As shown by them, the $s$ wave $\rho N$ resonances (specifically, $N(1520)$) have significant influence on the $\rho$ meson spectral function. The $\rho$ meson self-energy due to $s$ wave $\rho N$ resonance-hole polarization shows an important feature, as it depends upon both energy and momentum of the rho meson. Therefore, the self-energy due to $s$ wave $\rho N$ resonance can contribute to the $\rho$ meson spectral function in the static limit. This is unlike to that occurring due to $p$ wave $\rho N$ scattering, where the rho meson self-energy depends only on its momentum.

The self-energy or optical potential is inevitable to describe the elastic scattering of a particle by the nucleus. Using it, we studied various aspects for the coherent (elastic) scattering of $\rho^0$ meson in the proton nucleus reaction [8]. In one part of this study, we investigated the sensitivity of the coherent $\rho^0$ meson mass distribution on its optical potential formulated by various authors [4, 5, 6, 9] including Peters et al., [7]. The later authors, as mentioned above, have shown non-relativistically that various $\rho N$ resonances can generate the self-energy for the rho meson in a nucleus. Due to lack of scope, we could not show in our earlier work [8] the distinct contribution arising from each $\rho N$ resonance to the $\rho^0$ meson production cross section. Therefore, those are illustrated in the present work. In addition, we extend our calculation to accommodate the relativistic evaluation of the $\rho$ meson self-energy due to these resonances and discuss how the $\rho N$ resonance behave relativistically in the above reaction. We compare the non-relativistic and relativistic optical potentials for the $\rho^0$ meson arising due to each $\rho N$ resonance-hole polarization in the nucleus. We also present the results for the coherent and incoherent contributions of these resonances to the overall cross sections.

2 FORMALISM

The coherent $\rho^0$ meson production in the proton nucleus reaction describes the elastic scattering of the virtual rho meson (emitted by the beam proton) to its real state. Symbolically, this reaction is used to express as $p + A(\text{g.s.}) \rightarrow p' + \rho^0 + A(\text{g.s.})$. As we worked out earlier [8], the double differential cross section for the coherent $\rho$ meson mass
distribution in the \((p, p')\) reaction on a nucleus is given by
\[
\frac{d\sigma}{dmd\Omega_{p'}} = \int \int d\Omega_{p}dE_{p'} [KF] S(m^2) < |T_{fi}|^2 > ,
\tag{1}
\]
where \([KF]\) is the kinematical factor associated with this reaction. The expression for it is
\[
[KF] = \frac{m_p^2 E_{p'}}{(2\pi)^5 k_{p'}k_{p}^2 m} \frac{k_{p'}k_{p}^2 m}{k_{p'}|k_{p}(E_{p'} - E_{p'}) - (k_{p} - k_{p'}) \cdot k_{p}E_{p}|}.
\tag{2}
\]

The factor \(S(m^2)\) in Eq. (1) denotes the mass distribution function for the rho meson of mass \(m\) in the free space. It is well addressed by the Breit-Wigner form [10]:
\[
S(m^2) = \frac{1}{\pi} \frac{m_p \Gamma_{\rho}(m^2)}{[(m^2 - m_p^2)^2 + m_p^2 \Gamma_{\rho}(m^2)]^2},
\tag{3}
\]
with \(m_{\rho}(\approx 770\text{ MeV})\) being the resonance mass of the \(\rho\) meson in the free state. \(\Gamma_{\rho}(m^2)\) represents the total free space decay width for the \(\rho\) meson of mass \(m\), arising dominantly (\(\sim 100\%\)) due to \(\rho^0 \rightarrow \pi^+\pi^-\). Therefore, \(\Gamma_{\rho}(m^2)\) can be written as
\[
\Gamma_{\rho}(m^2) \approx \Gamma(m_{\rho}^2)_{\rho \rightarrow \pi \pi} \left(\frac{m_{\rho}}{m}\right) \left[\frac{k(m^2)}{k(m_{\rho}^2)}\right]^{3/2} \Theta(m^2 - 4m_{\pi}^2),
\tag{4}
\]
with \(\Gamma(m_{\rho}^2)_{\rho \rightarrow \pi \pi} \approx 150\text{ MeV}\). \(k(m^2)\) is the pion momentum in the rest frame of decaying \(\rho^0\) meson of mass \(m\). This expression intrinsically implies the detection of rho meson through its decay products \(\pi^+\pi^-\) in the final state. We have not considered here the distortion due to pion nucleus scattering since the effect of it (in this kind of reaction) has been found insignificant [11].

\(T_{fi}\) in Eq. (1) is the \(T\)-matrix for the coherent rho meson production in the proton nucleus reaction. The annular bracket around \(|T_{fi}|^2\) represents the average and summation over spins and polarization of the initial and final states respectively. \(T_{fi}\) is given by
\[
T_{fi} = \int \int d\mathbf{r}'d\mathbf{r} \chi^{(-)*}(\mathbf{k}_p, \mathbf{r}') \Pi_{\rho}(\mathbf{r}' - \mathbf{r}) G_{\rho}(\mathbf{r}' - \mathbf{r}) \Psi_{\rho}(\mathbf{q}, \mathbf{r}).
\tag{5}
\]

In this equation, \(G_{\rho}(\mathbf{r}' - \mathbf{r})\) describes the propagation of the virtual \(\rho^0\) meson from \(\mathbf{r}\) to \(\mathbf{r}'\), where the elastic scattering of this meson (to its real state) is taken place due to the \(\rho\) meson nucleus optical potential \(V_{O\rho}(\mathbf{r}')\). This potential appears in the above equation through the \(\rho\) meson self-energy, i.e., \(\Pi_{\rho}(\mathbf{r}') = 2q_0 V_{O\rho}(\mathbf{r}')\). Here, \(q_0(= E_p - E_{p'})\) is the energy of the virtual rho meson. \(\Pi_{\rho}(\mathbf{r}')\) will be elaborated in the next section.

The symbol \(\Psi_{\rho}(\mathbf{q}, \mathbf{r})\) in Eq. (5) stands for the virtual rho meson production amplitude in the \((p, p')\) reaction. The expression for it is
\[
\Psi_{\rho}(\mathbf{k}_p', \mathbf{k}_p, \mathbf{r}) = \chi^{(-)*}(\mathbf{k}_p', \mathbf{r}) \Gamma_{\rho NN} \chi^{(+)}(\mathbf{k}_p, \mathbf{r}),
\tag{6}
\]
where \(\Gamma_{\rho NN}\) denotes the vertex function at the \(\rho^0 pp'\) vertex. It is governed by the \(\rho NN\) Lagrangian as described in Ref. [8]. \(\chi\)s represent the distorted wave functions for protons.
in the continuum. In the energy region of $\rho$ meson production, the distortion arising due to proton nucleus scattering is purely absorptive. Therefore, the distorted wave functions for protons can be approximated by their plane waves multiplied by the attenuation factor $A_F$ for them [12], i.e.,

$$\chi^{(-)}(k_p', r)\chi^{(+)}(k_p, r) \approx A_F e^{i\mathbf{q} \cdot \mathbf{r}}, \quad \mathbf{q} = \mathbf{k}_p - \mathbf{k}_p'.$$

This equation shows $\mathbf{q}$ is the momentum of the virtual rho meson emitted at the $\rho NN$ vertex.

The $\rho^0$ meson in the final state can emit in all directions. Therefore, the partial wave expansion is preferred to express its wave function $\chi^{(-)}(k_\rho, r')$ appearing in Eq. (5), i.e.,

$$\chi^{(-)}(k_\rho, r) = \frac{4\pi}{k_\rho r} \sum_{lm} (-i)^l u_l(k_\rho r) Y_{lm}^\ast(\mathbf{r}) Y_{lm}(\mathbf{r}).$$

The radial part of this wave function, i.e., $u_l(k_\rho r)$, is generated by solving the relativistic wave (Schrödinger) equation using the $\rho$ meson optical potential $V_{Op}(r)$, described later.

### 3 RESULTS AND DISCUSSION

The attenuation factor $A_F$ in Eq. (7), which accounts the distortion for protons, can be estimated by using the following equation [12]:

$$A_F = \frac{1}{A} \int dB T(b) \exp[-Im\delta(b)],$$

where $A$ is the mass number of the nucleus. $T(b) = \int_{-\infty}^{+\infty} \varrho(b, z) dz$ represents the thickness function for the nucleus at the impact parameter $b$. $\varrho(b, z)$ describes the density distribution for the nucleus, given in Eq. (12) for $^{12}\text{C}$ nucleus. $\delta(b)$ denotes the total phase shift function for the proton nucleus scattering, i.e., $\delta(b) = \delta_p(b) + \delta_{p'}(b)$. The form for it is given by

$$\delta_p(b) = -\frac{1}{v_p} \int_0^{\infty} V_{Op}(b, z) dz,$$

where $v_p$ is the velocity of the proton. $V_{Op}(b, z)$ denotes the optical potential for the proton nucleus scattering.

The imaginary part of the proton nucleus optical potential $V_{Op}(b, z)$ is required to evaluate the attenuation factor $A_F$ for protons (see Eqs. (9) and (10)). Using the high energy ansatz, i.e., “$t\varrho$” approximation, the imaginary part of $V_{Op}(b, z)$ can be written as

$$ImV_{Op}(r) = -\frac{1}{2} v \sigma^{pN}_t \varrho(r).$$

$v$ is the relative velocity in the proton nucleon cm system. $\sigma^{pN}_t$ denotes the total cross section for the proton nucleon scattering. The energy dependent values for it is taken
from the measurements [13, 14]. \( \varrho(\mathbf{r}) \) describes the spatial shape of the optical potential \( V_0(\mathbf{r}) \). It is usually approximated by the density distribution of the nucleus. The form of \( \varrho(\mathbf{r}) \) for the \(^{12}\text{C} \) nucleus, as extracted from the electron scattering data [15], is given by

\[
\varrho(\mathbf{r}) = \varrho_0[1 + w(r/c)^2]e^{-(r/c)^2}; \quad w = 1.247, \; c = 1.649 \text{ fm}.
\]

This density is normalized to the mass numbers of the nucleus.

As mentioned earlier, we look for the contribution of the individual \( \rho N \) resonance to the cross section for the coherent rho meson production in the proton nucleus reaction. Upto \( \sim 1.9 \) GeV, the existence of these resonances (which emit \( \rho \) meson) is confirmed [14]. The formation of these resonances in a nucleus generates the \( \rho N \) resonance-hole polarizations in it. The interaction energy associated with these polarizations is referred as the self-energy or optical potential for the rho meson. This potential can modify the \( \rho \) meson propagation through the nucleus. Therefore, the \( \rho \) meson production process in the nuclear reaction can be thought as a potential tool to investigate the \( \rho N \) resonance dynamics in a nucleus.

To disentangle the above issue explicitly, we use the rho meson self-energy (or optical potential) evaluated by Peter et al., [7]. According to them, the \( \rho \) meson self-energy due to \( s \) wave (negative parity) \( \rho N \) resonance is given by

\[
\Pi_{(s)}^{\mu\nu}(k_0, \mathbf{k}) = -(k_0^2 P_T^{\mu\nu} - k^2 P_{L}^{\mu\nu}) \left( \frac{f_{RN\rho}}{m_\rho} \right)^2 F'(k^2) S_\Sigma \beta(k_0, \mathbf{k}),
\]

with \( k^2 = k_0^2 - \mathbf{k}^2 \). \( k_0 \) and \( \mathbf{k} \) are the energy and momentum respectively of the rho meson. \( F'(k^2) \) is the monopole form factor associated with the \( \rho \) meson self-energy: \( F'(k^2) = \frac{\Lambda'^2}{\Lambda'^2 + k^2} \) with \( \Lambda' = 1.5 \) GeV [7]. \( P_T^{\mu\nu} \) and \( P_L^{\mu\nu} \) denote the respective longitudinal and transverse projection operators for the self-energy. They are normalized as \( P_L^{\mu\nu} P_{\mu\nu} = 1 \) and \( P_T^{\mu\nu} P_{\mu\nu} = 2 \). The longitudinal and transverse self-energies can be obtained by using following relations [7]:

\[
\Pi_L^{(s)}(k_0, \mathbf{k}) = -P_T^{\mu\nu} \Pi_{\mu\nu}(k_0, \mathbf{k}); \quad \Pi_T^{(s)}(k_0, \mathbf{k}) = -\frac{1}{2} P_T^{\mu\nu} \Pi_{\mu\nu}(k_0, \mathbf{k}).
\]

The average \( \rho \) meson self-energy due to \( s \) wave \( \rho N \) resonances is given by \( \Pi^{(s)}(k_0, \mathbf{k}) = \frac{1}{3}[2\Pi_L^{(s)}(k_0, \mathbf{k}) + \Pi_T^{(s)}(k_0, \mathbf{k})] \). On the other hand, the \( \rho \) meson self-energy due to \( p \) wave (positive parity) \( \rho N \) resonance is purely transverse. The expression for it [7] is

\[
\Pi^{(p)}(k_0, \mathbf{k}) \equiv \Pi_T^{(p)}(k_0, \mathbf{k}) = k^2 \left( \frac{f_{RN\rho}}{m_\rho} \right)^2 F'(k^2) S_\Sigma \beta(k_0, \mathbf{k}).
\]

In Eqs. (13) and (15), \( f_{RN\rho} \) denotes the constant for the rho meson-nucleon coupling to the resonance \( R \), and \( S_\Sigma \) represents the associated spin-isospin transition factor. As shown by Peters et al. [7], there are four \( s \)-wave \( \rho N \) resonances, namely \( N(1520), \Delta(1620), N(1650), \Delta(1700) \), and five \( p \)-wave \( \rho N \) states \( N(940), \Delta(1232), N(1680), N(1720) \) and
\( \Delta(1905) \). Values of \( f_{RN\rho} \) and \( S_\Sigma \) for these resonances, as estimated by Peters et al. [7], are listed in Table 1. For \( \Delta(1232) \) resonance, the Landau-Migdal parameter \( g \) is introduced to account for the short-range correlation:

\[
\Pi^{(p)}(k_0, k) \rightarrow \Pi^{(p)} \left[ 1 - g \frac{\Pi^{(p)}}{k_0} \right],
\]

with \( g = 0.5 \). The inclusion of Landau-Migdal parameter for other resonances was not recommended [7]. The resonance \( N(1440) \) was not incorporated in this calculation, since its width decaying to the \( \rho N \) channel is insignificant (8 < %).

The Linhard function \( \beta(k_0, k) \) appearing in Eqs. (13) and (15) is given by

\[
\beta(k_0, k) = -\int_0^{p_F} \frac{d\mathbf{p}}{(2\pi)^3} \left[ \frac{1}{k_0 - E_N(\mathbf{p}) + E_R(\mathbf{p} + k)} + \frac{1}{-k_0 - E_N(\mathbf{p}) + E_R(\mathbf{p} - k)} \right].
\]

This integration eventually means the summation over all nucleons, i.e., \( \Sigma_i \rightarrow 4V \int_0^{p_F} \frac{d\mathbf{p}}{(2\pi)^3} \).

\( E_N(Q) \) and \( E_R(Q) \) represent energies for the nucleon \( N \) and resonance \( R \) respectively. They are defined as

\[
E_N(Q) = \sqrt{m_N^2 + Q^2}; \quad E_R(Q) = \sqrt{m_R^2 + Q^2} - \frac{i}{2} \Gamma_R,
\]

where \( \Gamma_R \) denotes the width of the resonance \( R \). It is composed of the pion-nucleon and rho meson-nucleon decay widths, i.e., \( \Gamma_R = \Gamma_{R \to N\pi} + \Gamma_{R \to N\rho} \), elaborated in Ref. [7].

In the non-relativistic calculation done by Peters et al. [7], the \( \rho \) meson self-energies (given in Eqs. (13) - (15)) were evaluated in the nucleon rest frame except the coupling constant \( f_{RN\rho} \) appearing in it. This constant was calculated in the resonance rest frame. In the nucleon rest frame, the Linhard function \( \beta(k_0, k) \) in Eq. (16) is reduced to

\[
\beta(k_0, k) = \frac{1}{2} \frac{E_R(k) - m_N}{k_0^2 - [E_R(k) - m_N]^2}.
\]

In this case, \( k \) is the four momentum of the \( \rho \) meson in the nucleon rest frame. In fact, this is the expression for \( \beta(k_0, k) \) normally used in the nuclear matter calculation in the limit of low nuclear density \( \rho \) or high momentum \( |k| \gg p_F \). The rho meson optical potential obtained from its self-energy, i.e., \( V_{\rho p} = \frac{1}{2k_0} \Pi_\rho \), is transformed in the proton nucleus cm system using proper kinematics. The radial density distribution \( \rho \) for the \( ^{12}C \) nucleus is expressed in the Eq. (12).

Using the \( \rho \) meson self-energy described above, we calculate the mass distribution spectra for the coherent rho meson produced in the \( (p, p') \) reaction on \( ^{12}C \) nucleus. The beam energy for this reaction is taken equal to 3.5 GeV, and the ejectile \( p' \) emission angle is taken fixed at \( ^{1}0^\circ \). In Fig. 1, the cross sections arising due to \( s \) wave \( \rho N \) resonances are presented, and those due to \( p \) wave \( \rho N \) resonances are shown in Fig. 2. It is noticeable in these figures that the cross sections due to \( p \) wave \( \rho N \) resonances are much larger than those due to the \( s \) wave \( \rho N \) resonances. In addition, these figures also show that all \( s \) wave or \( p \) wave \( \rho N \) resonances do not contribute identically to the cross section. It occurs since the cross section of this reaction depends upon various quantities, such
as coupling constant \( f_{\rho N} \), spin-isospin factor \( S_\Sigma \), hadron parameters for the resonance, etc. These quantities, as mentioned earlier, are listed in Table-1 for all resonances. Remarkably, the magnitude of the cross section strongly depends on the resonance(\( R \))-rho meson(\( \rho \))-nucleon(\( N \)) coupling constant \( f_{\rho N} \). As shown in Table-1, \( f_{\rho N} \) for the \( p \) wave \( \rho N \) resonances are relatively larger. They lie within the range 6.3 to 15.3, whereas \( f_{\rho N} \) for the \( s \) wave \( \rho N \) resonances are within the range of 0.9 to 7.0. Therefore, the cross section due to \( p \) wave \( \rho N \) resonance is expected to be larger than that due to \( s \) wave \( \rho N \) resonance.

Fig. 1 shows that the \( \rho^0 \) meson production cross section due to \( N(1520) \) resonance (solid curve (a)) is the largest compared to those originating due to other \( s \) wave \( \rho N \) resonances. This resonance, as compared in Table-1, has the largest \( \rho N \) coupling constant, i.e., \( f_{\rho N} = 7 \). The spin-isospin factor \( S_\Sigma \) for it (equal to 8/3) is also quite significant. The peak cross section due to this resonance is found equal to 5.64 \( \mu b/(GeV \ sr) \), and it appears at 720 MeV in the \( \rho \) meson mass distribution spectrum. The second largest cross section distribution for the rho meson production, presented by the dot dot dashed curve (b), arises due to \( \Delta(1700) \) resonance. This curve shows that the cross section at the peak is equal to 1.15 \( \mu b/(GeV \ sr) \), and the peak position appears at the \( \rho \) meson mass equal to 710 MeV. For this resonance, values for \( f_{\rho N} \) and \( S_\Sigma \) are equal to 5 and 16/9 respectively. The \( \rho \) meson production cross section due to \( \Delta(1620) \) resonance (dashed curve (c)) is 0.37 \( \mu b/(GeV \ sr) \) at the peak, appearing at the \( \rho \) meson mass equal to 710 MeV. \( f_{\rho N} \) and \( S_\Sigma \) for \( N(1620) \) are equal to 2.5 and 8/3 respectively. The \( \rho \) meson production cross section due to \( N(1650) \) resonance (dotted curve (d), not properly visible in Fig. 1) is very small. It happens due to its weak coupling to the \( \rho \) meson and nucleon, i.e., \( f_{\rho N} = 0 \). However, \( S_\Sigma \) for \( N(1650) \) is significant (equal to 4).

The rho meson mass distribution spectra due to \( p \) wave \( \rho N \) resonances, as mentioned earlier, are presented in Fig. 2. As shown in this figure, the maximum cross section arises due to \( \Delta(1905) \) resonance (see dashed curve (a)). For this resonance, \( f_{\rho N} \) and \( S_\Sigma \) are equal to 12.2 and 4/5 respectively. The cross section at the peak is equal to 36.63 \( \mu b/(GeV \ sr) \), and it appears at the \( \rho \) meson mass equal to 640 MeV. The dot dot dashed curve (b) represents the second largest cross section distribution originating due to \( \Delta(1232) \) isobar. This curve shows its peak at the \( \rho \) meson mass equal to 720 MeV, and the cross section at the peak is equal to 35.17 \( \mu b/(GeV \ sr) \). The constant \( f_{\rho N} \) for the \( \Delta(1232) \) coupling to the \( \rho \) meson and nucleon is quite large. The magnitudes for \( f_{\rho N} \) and \( S_\Sigma \) for this resonance are equal to 15.3 and 16/9 respectively. It must be mentioned that \( \Delta(1232) \) does not have measurable branching ratio into two pions. Therefore, the value of the \( \Delta N \rho \) coupling constant, i.e., \( f_{\Delta N \rho} = f_{\Delta N \rho} \), can’t be determined from the measured decay width. Since the magnitude of \( f_{\Delta N \rho} \) is uncertain, the cross section arising due to \( \Delta(1232) \) resonance can be interpreted only qualitatively. We have taken the above value of \( f_{\Delta N \rho} \) since it is used by Peters et al., [7] to calculated the rho meson self-energy.

The \( \rho^0 \) meson production cross section due to \( N(1720) \) resonance (shown by the solid curve (c)) in Fig. 2 is also significant. \( f_{\rho N} \) and \( S_\Sigma \) for this resonance are 7.8 and 8/3 respectively. The calculated spectrum due to this resonance shows a peak at the \( \rho \)
meson mass equal to 720 MeV, and the cross section at the peak is 31 µb/(GeV sr). The \( \rho^0 \) meson production cross section due to nucleon \( N(940) \) is presented by the dot dashed curve (d) in Fig. 2. The \( \rho NN \) coupling constant \( f_{NN\rho} \) and \( S_\Sigma \) are taken equal to 7.7 and 4 respectively. This curve shows that the cross section at the peak is equal to 18.99 µb/(GeV sr) and the peak position appears at the \( \rho \) meson mass equal to 720 MeV. The cross section due to \( N(1680) \) resonance (dotted curve (e)) is least compared to those due to other \( p \) wave \( \rho N \) resonances. For this resonance, \( f_{NN\rho} \) and \( S_\Sigma \) are equal to 6.3 and 6/5 respectively.

The coherent and incoherent addition of the cross sections due to all \( (s \ and \ p \ wave) \) \( \rho N \) resonances have been compared in Fig. 3. The incoherent addition of the cross sections due to all \( \rho N \) resonances (dot dashed curve) shows the cross section at the peak is equal to \( \sim 0.126 \, \text{mb/(GeV sr)} \) and it appears at the \( \rho \) meson mass equal to 710 MeV. The peak cross section due to coherent contribution of all \( \rho N \) resonances (solid curve) appears at 730 MeV in the \( \rho \) meson mass distribution spectrum, and its value is equal to 1.35 mb/(GeV sr). Therefore, the coherent addition increases the cross section drastically. It enhances the value of the cross section at the peak by a factor about 10.7 over that due to incoherent contribution. The peak position due to later appears 20 MeV higher in the \( \rho \) meson mass distribution spectrum.

The calculated results presented in Figs. 1–3 are based on the choice of the \( \rho \) meson self-energies (given in Eqs (13) - (18)) evaluated by Peters et al., [7]. As mentioned earlier, this is a preferable choice in the nuclear matter calculation. However, this choice had been criticized by Post et al. [16], since it leads serious uncertainty in the non-relativistic calculation. According to them, the \( \rho \) meson self-energies in Eqs. (13) and (15) also should be calculated in the resonance rest frame. This approach gives very reasonable agreement with the relativistic calculation for the rho meson self-energy [16]. Therefore, the calculated cross section using this approach can be considered as the relativistic results. Since the spectra appearing in Figs. 1–3 are due to the non-relativistic calculation for the \( \rho \) meson self-energy, we refer them henceforth as non-relativistic results. Another advantage of the resonance rest frame calculation is that the complicated relativistic calculation for the rho meson self-energy due to spin-\( \frac{5}{2} \) \( \rho N \) resonance can be avoided. In the resonance rest frame, the form for the Linhard function \( \beta(k_0,k) \) in Eq. (16) can be simplified to

\[
\beta(k_0, k) = \frac{1}{2} \phi \frac{m_R - i \frac{1}{2} \Gamma_R - E_N(k)}{k_0^2 - [m_R - i \frac{1}{2} \Gamma_R - E_N(k)]^2}, \tag{19}
\]

where \( k \) is the four momentum of the \( \rho^0 \) meson in the resonance rest frame. All other quantities have been defined earlier.

To explore the relativistic effect on the cross section, we recalculate the \( \rho \) meson mass distribution spectra in the \( (p, p') \) reaction using the relativistic \( \rho \) meson nucleus optical potential illustrated above. The cross sections originating due to \( s \) wave \( \rho N \) resonances are shown in Fig. 4. This figure looks qualitatively similar to the non-relativistic results presented in Fig. 1. It is noticeable that magnitudes of the cross sections in the relativistic
calculation have gone up significantly compared to those obtained in the non-relativistic case. Specifically, the enhancement in the cross section is found more than 3 times larger for the $N(1520)$ and $\Delta(1700)$ resonances. This enhancement occurs due to the increase in the rho meson potential, listed in Table-2, in the relativistic calculation.

The cross section due to $N(1520)$ resonance (solid curve (a) in Fig. 4) is distinctly largest in the relativistic case also. The peak in the calculated $\rho$ meson mass distribution spectrum due to this resonance appears at 730 MeV, and the cross section at the peak is equal to $20.04 \mu b/(GeV sr)$. The second largest cross section for the rho meson mass distribution arises due to the $\Delta(1700)$ resonances (dot dot dashed curve (b)). The cross section at the peak is equal to $3.82 \mu b/(GeV sr)$, and the peak appears at the rho meson mass equal to 730 MeV. The dashed curve (c) represents the mass distribution spectrum for the rho meson due to the $\Delta(1620)$ resonance. The peak cross section for this distribution is equal to $0.70 \mu b/(GeV sr)$, and it appears at the $\rho$ meson mass equal to 720 MeV. Similar to the non-relativistic case, the cross section due to $N(1650)$ resonance (dotted curve (d), not prominently seen in Fig. 4) is insignificant compared to those due to other $s$ wave $\rho N$ resonances.

The calculated (relativistic) rho meson mass distribution spectra due to $p$ wave $\rho N$ resonances are shown in Fig. 5. Unlike the case of $s$ wave $\rho N$ resonances, these spectra are not qualitatively similar to the non-relativistic results presented in Fig. 2. The $\rho$ meson mass distribution spectra in Fig. 5 show that the largest cross section arises due to $\Delta(1232)$ resonance. The second largest cross section originates due to the $\Delta(1905)$ resonance. It is exactly opposite to the non-relativistic results (see in Fig. 2). We again mention that the cross section due to $\Delta(1232)$ resonance can be described only qualitatively, since uncertainty exists in determining the $\Delta N \rho$ coupling constant $f_{\Delta N \rho}$. We do not incorporate the cross section due to $N(940)$ in Fig. 5, since the relativistic calculation for it, as mentioned by Post et al. [16], leads huge over estimation of the elementary rho production (e.g., $\gamma N \rightarrow \rho N$ and $\pi N \rightarrow \rho N$) data. It is remarkable that the relativistic calculation for the $p$ wave $\rho N$ resonance gives on an average about 67% less cross section compared to that due to non-relativistic calculation. This reduction occurs since the relativistic optical potential due to $p$ wave $\rho N$ resonance-hole polarization, as presented in Table-2, is less than that due to non-relativistic calculation. This is in contrast to the $s$ wave $\rho N$ resonance case.

The cross section distribution due to the $\Delta(1905)$ resonance is described by the dashed curve (a) in Fig. 5. It shows that the peak cross section is equal to $20.92 \mu b/(GeV sr)$, and the peak position appears at 650 MeV in the $\rho$ meson mass distribution spectrum. The dot dot dashed curve (b) represents the $\rho$ meson mass distribution spectrum arising due to the $\Delta(1232)$ resonance. This curve shows that the calculated cross section at the peak is equal to $26.84 \mu b/(GeV sr)$, and the peak appears at the rho meson mass equal to 720 MeV. The solid curve (c) illustrates the $\rho$ meson mass distribution spectrum due to the $N(1720)$ resonance. The cross section due to it is equal to $19.68 \mu b/(GeV sr)$ at the peak, and it appears at the rho meson mass equal to 720 MeV. The cross section due to $N(1680)$ resonance (dotted curve (d)) is the smallest amongst those due to other $p$
wave $\rho N$ resonances. To be mentioned, the non-relativistic calculation corroborates this finding qualitatively.

The calculated relativistic results for the coherent and incoherent contributions to the cross section arising from all ($s$ and $p$ wave) resonances are compared in Fig. 6. The calculated cross sections in this case show features qualitatively similar to the non-relativistic results presented in Fig. 3. The incoherent addition of the cross sections due to all $\rho N$ resonances gives smaller cross section (dot dashed curve). It is equal to 0.11 mb/(GeV sr) at the peak, appearing at the $\rho$ meson mass equal to 720 MeV. Whereas, the coherent contribution of all $\rho N$ resonances (solid curve) enhances the cross section by a factor about 7.82 to 0.86 mb/(GeV sr). The peak position in the later case appears at 730 MeV in the $\rho$ meson mass distribution spectrum.

4 CONCLUSIONS

The $\rho$ meson nucleon resonance dynamics in the nucleus have been studied through the coherent $\rho^0$ meson production process in the proton nucleus reaction. This study shows that the cross section strongly depends upon the $\rho$ meson-nucleon-resonance coupling constant. It also depends on various other quantities, such as the spin-isospin transition factor and the hadron parameters for the resonance .... etc. The cross sections arising due to $p$ wave $\rho N$ resonances are larger than those due to $s$ wave $\rho N$ resonances. Amongst the $s$ wave resonances, the largest cross section originates distinctly due to $N(1520)$ resonance. The relativistic results are qualitatively similar to the non-relativistic results in the case of $s$ wave $\rho N$ resonances. The relativistic calculation for these resonances gives larger cross sections. These features are opposite in the case of $p$ wave $\rho N$ resonances dynamics. The coherent contribution of all resonances enhances the cross section drastically over that due to incoherent contribution.

5 ACKNOWLEDGEMENTS

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Table 1: Values for $\rho N$ resonances’ parameters taken from the Ref. [7]. $L_{2I,2J}$ is the spectroscopic symbol for the pion-nucleon resonance state. $S_\Sigma$ denotes the spin-isospin transition factor. All other symbols carry their usual meanings.

| $R(m_R)L_{2I,2J}$ | $I(J^P)$ | $l_\rho$ | $\Gamma_\pi$ | $\Gamma_\rho$ | $f_{RN\rho}$ | $S_\Sigma$ |
|---------------------|----------|----------|--------------|--------------|--------------|-----------|
| $N(1520)D_{13}$    | $\frac{1}{2}$($\frac{3}{2}^-$) | 0        | 95           | 25           | 7.0          | 8/3       |
| $\Delta(1620)S_{31}$ | $\frac{1}{2}$($\frac{1}{2}^-$) | 0        | 130          | 20           | 2.5          | 8/3       |
| $N(1650)S_{11}$    | $\frac{1}{2}$($\frac{1}{2}^-$) | 0        | 135          | 15           | 0.9          | 4         |
| $\Delta(1700)D_{33}$ | $\frac{1}{2}$($\frac{3}{2}^-$) | 0        | 180          | 120          | 5.0          | 16/9      |
| $N(940)$ -          | $\frac{1}{2}$($\frac{1}{2}^+$) | 1        | 0            | 0            | 7.7          | 4         |
| $\Delta(1232)P_{33}$ | $\frac{1}{2}$($\frac{3}{2}^+$) | 1        | 120          | 0            | 15.3         | 16/9      |
| $N(1680)F_{15}$    | $\frac{1}{2}$($\frac{3}{2}^+$) | 1        | 118          | 12           | 6.3          | 6/5       |
| $N(1720)F_{13}$    | $\frac{1}{2}$($\frac{1}{2}^+$) | 1        | 50           | 100          | 7.8          | 8/3       |
| $\Delta(1905)F_{35}$ | $\frac{1}{2}$($\frac{3}{2}^+$) | 1        | 140          | 210          | 12.2         | 4/5       |

Table 2: Optical potentials for the $\rho$ meson due to $\rho N$ resonance-hole polarizations in the $^{12}$C nucleus. Potentials are tabulated below for the $\rho$ meson kinetic energy $T_\rho = 1227.25$ MeV. This value corresponds to $m_\rho = 770$ MeV and $T_p = 1500$ MeV.

| $\rho N$ partial wave $(l_\rho)$ | $N/\Delta$ | $V_{O\rho}$ (MeV) (Non-relativistic) | $V_{O\rho}$ (MeV) (Relativistic) |
|----------------------------------|-------------|-------------------------------------|----------------------------------|
| $l_\rho = 0$ (s wave)            |             |                                     |                                  |
| $N(1520)$                        | -1.94, -11.12 | -2.24, -23.95                        |
| $\Delta(1620)$                   | 1.24, -2.66  | 0.99, -4.14                          |
| $N(1650)$                        | 0.56, -0.52  | 0.51, -0.65                          |
| $\Delta(1700)$                   | -0.38, -4.88 | -1.20, -10.01                        |
| $l_\rho = 1$ (p wave)            |             |                                     |                                  |
| $N(940)$                         | 34.46, -     | -                                   |
| $\Delta(1232)$                   | -8.57, -27.81| -1.40, -28.13                        |
| $N(1680)$                        | -0.99, -4.72 | -0.69, -4.89                         |
| $N(1720)$                        | 14.11, -33.92| 6.53, -25.61                         |
| $\Delta(1905)$                   | 1.17, -22.74 | -0.96, -20.01                        |
Figure Captions

1. The $\rho$ meson mass distribution spectra due to $s$ wave $\rho N$ resonances have been compared. The non-relativistic $\rho$ meson self-energy is used in this calculation (see text).

2. The $\rho$ meson mass distribution spectra due to $p$ wave $\rho N$ resonances have been compared. The non-relativistic $\rho$ meson self-energy is used in this calculation (see text).

3. The coherent and incoherent contributions to the cross section arising from all $\rho N$ resonances are compared. The non-relativistic $\rho$ meson self-energy is used in this calculation (see text).

4. The $\rho$ meson mass distribution spectra due to $s$ wave $\rho N$ resonances have been compared. The relativistic $\rho$ meson self-energy is used in this calculation (see text).

5. The rho meson mass distribution spectra due to $p$ wave $\rho N$ resonances have been compared. The relativistic $\rho$ meson self-energy is used in this calculation (see text).

6. The coherent and incoherent contributions to the cross section arising from all $\rho N$ resonances are compared. The relativistic $\rho$ meson self-energy is used in this calculation (see text).
$^{12}\text{C}(p,p'\rho^0)^{12}\text{C}$

$T_p = 3.5 \text{ GeV}$

$\theta_{p'} = 1^0$

$s$-wave $pN$

resonances:

(1) $N(1520)$

(2) $\Delta(1700)$

(3) $\Delta(1620)$

(4) $N(1650)$

Fig. 1

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$^{12}\text{C}(p,p'p^0)^{12}\text{C}$

$T_p = 3.5$ GeV

$\theta_p = 1^0$

*p-wave $\rho N$ resonances:*

(1) $\Delta(1905)$
(2) $\Delta(1232)$
(3) $N(1720)$
(4) $N(940)$
(5) $N(1680)$

Fig.2
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$d^2\sigma/dm d\Omega_{\rho'}$

$1^2C(p,p'/\rho^0)^{12}C$

$T_p = 3.5$ GeV

$\theta_{\rho'} = 1^0$

Coherent contribution from all $\rho N$ resonances

Incoherent contribution from all $\rho N$ resonances

Fig. 3

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$^{12}\text{C}(p,p'\rho^0)^{12}\text{C}$

$T_p=3.5\text{ GeV}$

$\theta_p=1^0$

$s$ wave $\rho N$

resonances:

$(1) N(1520)$

$(2) \Delta(1700)$

$(3) \Delta(1620)$

$(4) N(1650)$

---

**Fig. 4**

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$^{12}\text{C}(p,p'p^0)^{12}\text{C}$

$T_p = 3.5 \text{ GeV}$

$\theta_p = 1^0$

$p$ wave $\rho N$

resonances:

(1) $\Delta(1905)$

(2) $\Delta(1232)$

(3) $N(1720)$

(4) $N(1680)$

Fig. 5

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$^{12}\text{C}(p,p'\rho^0)^{12}\text{C}$

$T_p = 3.5$ GeV

$\theta_{p} = 1^0$

Coherent contribution from all $\rho N$ resonances

Incoherent contribution from all $\rho N$ resonances

Fig. 6
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