Abstract

Two spatial equilibria, agglomeration and dispersion, in a continuous space core-periphery model are examined to discuss which equilibrium is socially preferred. It is shown that when transport cost is lower than a critical value, the agglomeration equilibrium is preferable in the sense of Scitovszky, while when the transport cost is above the critical value, the two equilibria can not be ordered in the sense of Scitovszky.

Keywords: compensation principle; continuous racetrack economy; core-periphery model; economic agglomeration; new economic geography; self-organization; transport cost; welfare economics

JEL classification: R12, R40, D60

1 Introduction

Charlot et al. (2006) discuss the welfare economic properties of agglomeration and dispersion equilibrium in the two regional core-periphery model, which has been introduced by Krugman (1991). One of their main results is that the agglomeration can be preferred over dispersion in the sense of Scitovszky (Scitovszky, 1941) if the transport cost is sufficiently low.

In this paper, we apply the method devised by Charlot et al. (2006) to the core-periphery model in a continuous periodic space and compare the agglomeration and dispersion equilibrium. Similar to Charlot et al. (2006), we prove that in a periodic continuous space, agglomeration is also preferred in the sense of Scitovszky under sufficiently low transport cost.

Let us describe the model setup we use. The geographical space is assumed to be a circle $S$ of which radius is $r \geq 1$. The industry consists of two sectors: manufacturing under the monopolistic competition and agriculture under the perfect competition. The manufacturing produces various differentiated goods with the increasing returns, while the agriculture produces one variety of homogeneous good with the constant returns. The supply of...
each variety of the manufacturing goods is normalized to be $\mu \in (0, 1)$ as in \cite{Fujita et al., 2001}, Chapter 4, (4.33)). There are two different types of workers for each sector. Manufacturing workers whose total population is $\mu \in (0, 1)$ are assumed to move around $S$ in search of higher real wages, while agricultural workers whose total population is $1 - \mu$ are assumed to be unable to move. The transportation of the manufacturing goods incurs the iceberg transport cost. That is, to deliver one unit of any one variety of the manufacturing goods, $T(x, y) \geq 1$ units of it must be shipped from $x \in S$ to $y \in S$. Meanwhile, the transportation of the agricultural good does not incur any transport cost.

The model which we use is described as follows.\footnote{The fifth equation of \cite{1} represents the movement of the manufacturing workers, and $v > 0$ denotes the speed of adjustment. However, since comparative statics is the main subject, the differential equation is not explicitly considered in this paper.}

\[
\begin{align*}
Y(t, x) &= \mu \lambda(t, x) w(t, x) + (1 - \mu) \phi(x), \\
w(t, x) &= \left[ \int_S Y(t, y) G(t, y)^{\sigma - 1} e^{-(\sigma - 1)\tau |x - y|} dy \right]^{\frac{1}{\sigma}}, \\
G(t, x) &= \left[ \int_S \lambda(t, y) w(t, y)^{1 - \sigma} e^{-(\sigma - 1)\tau |x - y|} dy \right]^{\frac{1}{1 - \sigma}}, \\
\omega(t, x) &= w(t, x) G(t, x)^{-\mu}, \\
\frac{\partial \lambda}{\partial t}(t, x) &= v \left[ \omega(t, x) - \int_S \omega(t, y) \lambda(t, y) dy \right] \lambda(t, x),
\end{align*}
\]

with an initial condition $\lambda(0, x) = \lambda_0(x)$. For any function $f$ on $S$, we denote the integration of $f$ over any subset $\Sigma \subset S$ by $\int_\Sigma f(x) dx$, where $dx$ is a line element.\footnote{Names of variables on $S$ can be $x, y, z,$ and so on, as the case may be. The functions $Y(t, x), w(t, x), G(t, x)$ and $\omega(t, x)$ represent the income, manufacturing nominal wage, manufacturing price index, and manufacturing real wage at time $t \geq 0$ in region $x \in S$, respectively. Each of the nominal wage of the agricultural workers and the price of the agricultural good is assumed to be one. The manufacturing population density is given by $\mu \lambda(t, x)$ at $t \geq 0$ in $x \in S$, and the function $\lambda$ must satisfy $\int_S \lambda(t, x) dx = 1, \forall t \geq 0$. Similarly, the agricultural population density is given by $(1 - \mu) \phi(x)$ in $x \in S$, and the function $\phi$ must satisfy $\int_S \phi(x) dx = 1$. The iceberg transportation is assumed by $T(x, y) = e^{\tau |x - y|}$, where $\tau > 0$. Here, $|x - y|$ denotes the shorter distance between $x, y \in S$. The parameter $\sigma > 1$ denotes the consumer’s preference for manufacturing variety; the closer $\sigma$ is to one, the stronger the degree to which consumers prefer diversity of the manufacturing goods. Since the parameters $\tau$ and $\sigma$ often appear in the form $\tau(\sigma - 1)$, it is convenient to introduce}

\[
\alpha := \tau(\sigma - 1) > 0
\]
in the following.

Any \( x \in S \) corresponds one-to-one to a certain \( \theta \in [−\pi, \pi) \) as \( x = x(\theta) \), so it is convenient for specific calculations to use \( \theta \) as the coordinates put into \( S \). Then, when \( x = x(\theta) \) and \( y = y(\tilde{\theta}) \), the shorter distance \( |x−y| \) between them is computed by \( \min \left\{ r|\theta−\tilde{\theta}|_{\text{abs}}, 2\pi r−r|\theta−\tilde{\theta}|_{\text{abs}} \right\} \) where \( |\cdot|_{\text{abs}} \) denotes the absolute value.

Let us refer to related works. The cpre-periphery model in a periodic continuous space was initially introduced by Fujita et al. (2001, Chapter 6). Ohtake and Yagi (2022) review the behavior of solutions of (1) and discuss its economic implications. Tabata et al. (2013) consider the core-periphery model in a \( n \)-dimensional continuous space, and study it analytically. Pflüger and Südekum (2008) investigate the welfare-economic properties of a NEG model with a quasi-linear utility function.

2 Equilibrium and welfare

In this section, we first give precise definitions for the two equilibria. We then discuss whether compensation is possible in each equilibrium, i.e., whether each equilibrium is potentially Pareto-dominant over the other.

2.1 Agglomeration equilibrium

We first consider the equilibrium in which all the manufacturing workers concentrate in \( x^* \in S \). We call this equilibrium (A) for short. It can be expressed as

\[
\lambda(x) = \delta^S(x-x^*)
\]

by using the delta function on \( S \). Here, the delta function on \( S \) is a linear functional satisfying

\[
\int_S \delta^S(x-x^*)dx = 1,
\]

\[
\int_S \delta^S(x-x^*)f(x)dx = f(x^*),
\]

where \( f \) is any continuous function on \( S \). By substituting (2) into the first three equations of (1), one can see that

\[
w(x^*) = G(x^*) = 1,
\]

\[
G(x) = e^{r|x-x^*|}, \ x \in S.
\]

Then, from the fourth equation of (1), the manufacturing real wage in \( x^* \) is given by

\[
\omega(x^*) = 1.
\]
2.2 Dispersion equilibrium

We next consider the equilibrium in which the manufacturing workers are evenly distributed throughout $S$, and the nominal wages and the price indices of all regions are homogeneous. We call this equilibrium (D) for short. By substituting

$$
\lambda(x) \equiv \lambda = \frac{1}{2\pi r}, \quad \forall x \in S,
$$
$$
\overline{w} \equiv \overline{w}, \quad \forall x \in S,
$$
$$
G(x) \equiv \overline{G}, \quad \forall x \in S.
$$

into the first three equations of (1), one can obtain

$$
\overline{w} = 1,
$$
$$
\overline{G} = \left[1 - e^{-\alpha r \pi} \frac{\mu}{\sigma} \right] \frac{1}{\pi r}. \quad (6)
$$

Then, from the fourth equation of (1), the manufacturing real wage also becomes homogeneous as

$$
\overline{w} = \left[1 - e^{-\alpha r \pi} \frac{\mu}{\sigma} \right] \frac{1}{\pi r}. \quad (7)
$$

2.3 Comparison of welfare levels

2.3.1 Welfare levels at (A)

Let $\omega^A$, $\psi^A(x)$ denote levels of welfare in (A) of the manufacturing workers and the agricultural workers in $x \in S$, respectively. The level of welfare is measured by real wages of workers. Therefore, from (5) $\omega^A = 1$, and from (4) $\psi^A(x) = e^{-\mu \tau |x-x^*|}$. Note that the agricultural workers at $x^*$ enjoy the same level of welfare as the manufacturing workers because $\psi^A(x^*) = 1 = \omega^A$. The results are summarized in Table 1.

| Type of workers | Welfare      |
|-----------------|-------------|
| Manufacturing workers | 1           |
| Agricultural workers at $x^*$ | 1           |
| Agricultural workers at $x$ | $e^{-\mu \tau |x-x^*|}$ |

Table 1: Levels of welfare at (A)

2.3.2 Welfare levels at (D)

Let $\omega^D$, $\psi^D$ denote levels of welfare in (D), which is measured by the real wages, of the manufacturing workers and the agricultural workers, respectively. From (7), $\omega^D = \left[1 - e^{-\alpha r \pi} \frac{\mu}{\sigma} \right] \frac{1}{\pi r}$, and from (6), $\psi^D = \left[1 - e^{-\alpha r \pi} \frac{\mu}{\sigma} \right] \frac{1}{\pi r}$. 

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Note that all the agricultural workers enjoy the same level of welfare as the manufacturing workers. The results are summarized in Table 2.

| Type of workers | Welfare |
|-----------------|---------|
| Manufacturing workers | $\left[ \frac{1-e^{-\alpha r\pi}}{\alpha r\pi}, \frac{1}{\alpha r}\right]$ |
| Agricultural workers | $\left[ \frac{1-e^{-\alpha r\pi}}{\alpha r\pi}, \frac{1}{\alpha r}\right]$ |

Table 2: Levels of welfare at (D)

**Theorem 1.** *The real wage of the workers in (D) is lower than 1:*

$$\omega^D (= \psi^D) < 1.$$ 

See Appendix for the proof.

Therefore, it is immediate that

$$\omega^D < 1 = \omega^A,$$

$$\psi^D < 1 = \psi^A(x^*).$$

This means that all the manufacturing workers and the agricultural workers in $x^* \in S$ prefer (A) over (D). Meanwhile, as for the agricultural workers in $S \setminus \{x^*\}$, there exists a neighbor of $x^*$ denoted by $\Gamma \subset S$ such that

$$\psi^A(x) \geq \psi^D, \forall x \in \Gamma,$$

and

$$\psi^A(x) < \psi^D, \forall x \notin \Gamma.$$ 

See Figure 1 for the situation.

![Figure 1: Sketch of the graph of $\psi^A$ and $\psi^D$](image)

This means that on the one hand the agricultural workers in $x \in \Gamma$ prefer (A) over (D), on the other hand, the agricultural workers in $x \notin \Gamma$ prefer (D) over (A).

In the following, we set $x^* = x(0), 0 \in [-\pi, \pi]$ without loss of generality. Then, $\Gamma \subset S$ corresponds to an interval $[-\Delta, \Delta] \subset [-\pi, \pi]$ where $\Delta > 0$. These notations are essential when computing specific integrals.
2.4 Compensation scheme

2.4.1 Determination of nominal compensation

Under (A), assume that the manufacturing workers and the agricultural workers in \( \Gamma \) compensate to the agricultural workers in \( S \setminus \Gamma \). Let \( C_A(x - x^*) \) denote the compensation received per agricultural workers in \( S \setminus \Gamma \). Then, their welfare level after receiving the compensation is given by

\[
(1 + C_A(x - x^*)) e^{-\mu \tau |x - x^*|},
\]

and it must be equal to their welfare level under (D):

\[
\psi_D = \left[ 1 - e^{-\alpha r \pi} \right]^\mu \sigma^{-1}.
\]

Therefore, the compensation is calculated as

\[
C_A(x - x^*) = \psi_D e^{\mu \tau |x - x^*|} - 1,
\]

and the total compensation is calculated by

\[
\int_{S \setminus \Gamma} C_A(x - x^*)(1 - \mu) \phi dx = \int_{S \setminus \Gamma} \left\{ \psi_D e^{\mu \tau |x - x^*|} - 1 \right\} (1 - \mu) \phi dx \tag{8}
\]

The manufacturing workers and agricultural workers in \( \Gamma \) pay \( T_A^* \) and \( T_A(x - x^*) \) per person, respectively, to compensate the agricultural workers in \( S \setminus \Gamma \). Since the total payment is equal to the total compensation,

\[
\mu T_A^* + (1 - \mu) \phi \int_{\Gamma} T_A(x - x^*) dx = (1 - \mu) \phi \int_{S \setminus \Gamma} C_A(x - x^*) dx \tag{9}
\]

holds.

2.4.2 Balance of supply and demand.

The demand from region \( x \) for one variety of manufacturing goods produced in region \( x^* \) is given by

\[
\mu Y(x) \left( p(x^*) e^{\tau |x - x^*|} \right)^{1-\sigma} G(x)^{\sigma-1}, \tag{10}
\]

where \( p(x) \) denotes the f.o.b price \(^{[1]}\) of the manufacturing goods produced in \( x \in S \). Since \( p(x^*) = w(x^*) \) from normalization in \(^{[2]}\), and we now have (3), so (10) becomes

\[
\mu Y(x) e^{-\alpha r \pi |x - x^*|} G(x)^{\sigma-1}.
\]

\(^{[1]}\) Fujita et al. (2001, p.50, (4.16))
Under the assumption of the iceberg transport cost, $e^{r|x-y|}$ times this amount have to be shipped to meet this demand. Thus, the aggregate demand for one variety of manufacturing goods produced in region $x^*$ is given by

$$q(x^*) = \int_S \mu Y(x) G(x)^{\sigma-1} e^{-\alpha|x-x^*|} dx.$$  \hspace{1cm} (11)

The income after the compensation is made under (A) is given by

$$Y(x) = \begin{cases} \mu \delta^S (x - x^*) (1 - T_A^*) \\ + (1 - \mu) \overline{\delta} (1 - T_A(x - x^*)) \end{cases}, \quad \forall x \in \Gamma, \hspace{1cm} (12)$$

$$\begin{cases} (1 - \mu) \overline{\phi} (1 + C_A(x - x^*)) \end{cases}, \quad \forall x \in S \setminus \Gamma.$$

Substituting (12) into (11) yields

$$q(x^*) = \mu - \mu^2 T_A^* - \mu(1 - \mu) \overline{\phi} \int_{\Gamma} T_A(y - x^*) dy + \mu(1 - \mu) \overline{\phi} \int_{S \setminus \Gamma} C_A(y - x^*) dy.$$

Applying (12) to this gives

$$q(x^*) = \mu$$

which means the demand for a variety of the manufacturing goods is equal to its supply $\mu$. Thus, it is possible to implement such compensation without disturbing the equilibrium price.

### 2.4.3 Condition under which agglomeration is preferred

For (A) to be preferred, the following two conditions must be satisfied.

1. The welfare level of the manufacturing workers in (A) must be higher than that in (D) even after compensation, i.e.,

$$1 - T_A^* > \psi^D.$$  \hspace{1cm} (13)

2. The welfare level of the agricultural workers in $\Gamma$ after compensation must be higher than or equal to that in (D), i.e.,

$$(1 - T_A(x - x^*)) e^{-\mu r|x-x^*|} \geq \psi^D, \quad \forall x \in \Gamma.$$  \hspace{1cm} (14)

In the following, we consider a compensation scheme in which the agricultural workers in $\Gamma$ exhaust all the surplus generated by the agglomeration. In this case, (14) holds as

$$T_A(x - x^*) = 1 - \psi^D e^{\mu r|x-x^*|}.$$  \hspace{1cm} (15)
Considering (15), one can find the total payment by the manufacturing workers and the agricultural workers in $\Gamma$ becomes

$$\mu T_A^* + (1 - \mu)\bar{\phi} \int_{\Gamma} \left(1 - \psi^D e^{\mu |x - x^*|}\right) dx$$

(16)

Since (16) equals to the total compensation (8), we see

$$T_A^* = \frac{1 - \mu}{\mu} \left[\psi^D \cdot \bar{\phi} \int_{S} e^{\mu |x - x^*|} dx - 1\right].$$

We can show (13), i.e., $F(\tau) := 1 - T_A^* - \psi^D > 0$ holds for sufficiently small values of $\tau > 0$. See Appendix for the proof.

**Theorem 2.** There exists $\tau_K > 0$ such that $F(\tau) > 0$ for any $\tau \in (0, \tau_K)$.

Therefore, we can claim that (A) is preferred over (D) in the sense of Kaldor [Kaldor, 1939] if and only if $\tau < \tau_K$.

### 2.4.4 Compensation under (D) disturbs equilibrium price

Contrary to the previous discussion, consider compensation under (D) from the agricultural workers in $S \setminus \Gamma$, who gains more at (D) than at (A), to the manufacturing workers and the agricultural workers in $\Gamma$, who prefer (A).

In this case, let $T_D(x - x^*)$ be the amount paid by one agricultural worker in $S \setminus \Gamma$. This amount generally depends on the distance $x - x^*$ from the potential city in $x^*$, since those who live farther away from $x^*$ will suffer greater losses from the agglomeration and will therefore be prepared to pay more to prevent it.

Meanwhile, let $C_D^*$ and $C_D(x - x^*)$ be the amount of compensation received by a manufacturing worker, and an agricultural worker in $\Gamma$, respectively. The latter generally depends on the distance $x - x^*$ from the potential city in $x^*$, since the closer a player lives to $x^*$, the greater the gain he enjoys under (A), and he will not be satisfied unless he receives a larger compensation.

The income distribution after compensation under (D) is given by

$$Y(x) = \begin{cases} \mu \lambda (1 + C_D^*) + (1 - \mu) \bar{\phi} (1 + C_D(x - x^*)) , & \forall x \in \Gamma , \\ \mu \lambda (1 + C_D^*) + (1 - \mu) \bar{\phi} (1 - T_D(x - x^*)) , & \forall x \in S \setminus \Gamma. \end{cases}$$

(17)

The total payment and the total compensation must be equal, so

$$(1 - \mu)\bar{\phi} \int_{S \setminus \Gamma} T_D(x - x^*) dx = \mu C_D^* + (1 - \mu)\bar{\phi} \int_{\Gamma} C_D(x - x^*) dx$$

(18)

holds.
Same as (11), the aggregate demand for one variety of manufacturing goods produced in region $x$ is given by

$$q(x) = \int_S \mu Y(y) G(y)^{\sigma-1} e^{-|y-x|} dy. \quad (19)$$

Using the fact that

$$\overline{G^{\sigma-1}} \int_S e^{-\alpha|x-y|} dy = 2\pi r$$

and substituting (17) into (19) gives

$$q(x) = \mu + \mu^2 C_D^* + \mu (1 - \mu) \phi \int_{\Gamma} C_D(y - x^*) e^{-\alpha|x-y|} dy \quad (20)$$

Let us especially consider $q(x^*)$. Then, the integral terms in (20) are estimated as

$$\int_{\Gamma} C_D(y - x^*) e^{-\alpha|x^*-y|} dy > e^{-\alpha r \Delta} \int_{\Gamma} C_D(y - x^*) dy \quad (21)$$

and

$$\int_{S\backslash \Gamma} T_D(y - x^*) e^{-\alpha|x^*-y|} dy < e^{-\alpha r \Delta} \int_{S\backslash \Gamma} T_D(y - x^*) dy, \quad (22)$$

and note that

$$\overline{G^{\sigma-1}} = \frac{\alpha \pi r}{1 - e^{-\alpha r \Delta}} > 1. \quad (23)$$

These estimations (21)-(23) with (20) give

$$q(x^*) > \mu + \mu^2 C_D^* + \mu (1 - \mu) \phi e^{-\alpha r \Delta} \int_{\Gamma} C_D(y - x^*) dy \quad (24)$$

Finally, applying (18) to (24), we obtain

$$q(x^*) = \mu + (1 - e^{-\alpha r \Delta}) \mu^2 C_D^* > \mu$$

which means excess demand for varieties of the manufacturing goods produced in $x^*$ occurs. Furthermore, since $q(x)$ is continuous as for $x$, there exists a neighborhood of $x^*$ such that the excess demand $q(x) > \mu$ occurs in any $x$ in the neighborhood.

This result shows that such an compensation scheme is not feasible, since it cannot be implemented without disturbing the prices in (D). Therefore, (A) is preferred over (D) in the sense of Hicks (Hicks, 1939) under any values of $\tau > 0$. 

9
3 Conclusion

Based on the above discussion, we can confirm as follows that the main result of Charlot et al. (2006) holds for a periodic continuous space model. Subsection 2.4.4 shows that (A) is preferred over (D) in the sense of Hicks for any value of \( \tau > 0 \). Then, Theorem 2 claims that if \( \tau < \tau_K \), then (A) is preferred over (D), therefore, when \( \tau < \tau_K \), (A) is preferred over (D) in the sense of Scitovszky. On the other hand, when \( \tau \geq \tau_K \), there is no equilibrium that is preferred in the sense of both Hicks and Kaldor, and thus, in the sense of Scitovszky, the preferability is undetermined.

4 Appendix

4.1 Proof for Theorem 1

Let \( X := \alpha r \pi > 0 \). When \( X \geq 1 \), it is obvious that

\[
\frac{1 - e^{-X}}{X} < 1. \quad (25)
\]

When \( X < 1 \), by Maclaurin expansion of \( e^X \), it is easy to see that (25) is equivalent to

\[
\sum_{k=2}^{\infty} \left\{ \frac{X^{k+1}}{(k+1)!} - \frac{X^k}{k!} \right\} < 0
\]

This is true from the assumption \( X < 1 \).

4.2 Proof for Theorem 2

Tedious calculations yield the following Lemmas 1-2.

Lemma 1. For any \( \tau > 0 \), the function \( \psi_D = \psi_D(\tau) \) given by

\[
\psi_D = \left[ 1 - e^{-\alpha r \pi} \right] \frac{\tau^\alpha}{\alpha r \pi}, \quad \alpha := (\sigma - 1)\tau
\]

satisfies

\[
\lim_{\tau \to 0} \psi_D = 1,
\]

\[
\lim_{\tau \to \infty} \psi_D = 0,
\]

\[
\frac{d\psi_D}{d\tau} < 0,
\]

\[
\lim_{\tau \to 0} \frac{d\psi_D}{d\tau} = -\frac{\mu r \pi}{2},
\]

\[
\lim_{\tau \to \infty} \frac{d\psi_D}{d\tau} = 0.
\]
Lemma 2. For any $\tau > 0$, the function $\chi = \chi(\tau)$ given by
\[
\chi := \phi \int_S e^{\mu r |x-x^*|}dx = \frac{e^{\mu r \tau \pi} - 1}{\mu \tau \pi}
\]
satisfies
\[
\lim_{\tau \to 0} \chi = 1, \\
\lim_{\tau \to \infty} \chi = \infty, \\
\frac{d\chi}{d\tau} > 0, \\
\lim_{\tau \to 0} \frac{d\chi}{d\tau} = \frac{\mu r \pi}{2}, \\
\lim_{\tau \to \infty} \frac{d\chi}{d\tau} = \infty.
\]

Lemmas 3-4 immediately follow from Lemmas 1-2.

Lemma 3. For a sufficiently small $\tau > 0$,
\[
\frac{dF}{d\tau}(\tau) > 0.
\]

Lemma 4. For a sufficiently large $\tau > 0$,
\[
\frac{dF}{d\tau}(\tau) < 0.
\]

A bit of technical discussion is required to prove Lemma 5.

Lemma 5. For any $\tau > 0$,
\[
\frac{d^2 F}{d\tau^2}(\tau) < 0.
\]

Proof. Let $f'$ denote the first derivative of any function $f$. To estimate the sign of the second derivative $f''$ of $f$, it is convenient to use the fact that
\[
(ln f)'' = \frac{f'' f - (f')^2}{f^2}.
\]

It follows that if $(ln f)'' > 0$, then $f'' f > (f')^2$ holds. Therefore, if $f > 0$, then we have $f'' > 0$.

The second derivative of $F$ is
\[
F'' = -\frac{1 - \frac{\mu}{\mu}}{\mu} (\psi^D \chi)' - \psi^D''.
\]

Tedious calculation shows that $(ln \psi^D \chi)'' > 0$. Since $\psi^D \chi > 0$, we have
\[
(\psi^D \chi)'' > 0.
\]
Another tedious calculation gives $(\ln \psi_D)'' > 0$, and we have
\[ \psi_D'' > 0 \quad (28) \]
by the same manner. As a result, by considering (27) and (28) together with (26), we get $F'' < 0$. \hfill \square

It is a simple matter to see Lemma 6 holds.

**Lemma 6.**

1. \[ \lim_{\tau \to 0} F(\tau) = 0, \]

2. \[ \lim_{\tau \to \infty} F(\tau) < 0. \]

Then Theorem 2 obviously follows from Lemmas 3-6. Figure 2 shows the sketch of the graph of $F(\tau)$.

![Figure 2: Sketch of the graph of $F(\tau)$](image)

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\[ ^4 \text{It does not plot the numerically exact values of the function.} \]
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