Enhanced output entanglement with reservoir engineering

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I. INTRODUCTION

Cavity optomechanics [1] exploring the interaction between macroscopic mechanical resonators and light fields, has received increasing attention for the potential to detect tiny mass, force and displacement [2–5]. The common optomechanical cavity contains one end mirror being a macroscopic mechanical oscillator or a vibrating membrane [6–11]. In these optomechanical systems, the motion of mechanical oscillator can be effected by the radiation pressure of cavity field, and this interaction can generate various quantum phenomena. Such as ground-state cooling of mechanical modes [12–17], electromagnetically induced transparency and normal mode splitting [18–22], nonlinear interaction effects [23–26] and quantum state transfer between photons with vastly differing wavelengths [27–30].

Entanglement is the characteristic element of quantum theory because it is responsible for nonlocal correlations between observables and an essential ingredient in most applications in quantum information. For these reasons, there are a number of theoretical and experimental works on entanglement between macroscopic objects such as, between atomic ensembles [31–34] and between superconducting qubits [35–36]. Recently, quantum entanglement in cavity optomechanics has received increasing attention for the potential to use the interaction to generate various entanglement between subsystems. For example, quantum entanglement between mechanical resonators [37–40], between different optical modes [41–44], and between mechanical resonators and light modes [45–47] have been studied theoretically and the entanglement between mechanical motion and microwave fields has been demonstrated in a recent experiment [48].

Here, we consider a three-mode optomechanical system in which two cavities are coupled to a common mechanical resonator (see Fig. 1). This setup has been realized in several recent experiments [49–51]. Because in such a system the parametric-amplifier interaction and the beam-splitter interaction can entangle the two intracavity modes, the output cavity ones are also entangle with each other. In previous works [50, 52], the entanglement of two output optical fields with their center frequencies same as the resonant frequencies of the cavities has been studied. In Ref. [50], the entanglement between the two output fields is enhanced obviously via reservoir engineering [62, 63]: cooling the Bogoliubov mode through enhancing mechanical decay results in large entanglement between the two target output fields. But these output entanglement in Ref. [50, 52] will be largely limited by the bandwidth of filter function, and the optimal time delay in Ref. [50] between the two output fields only suitable for the case of little bandwidth of filter function.

In this paper, we first study the effect of filter bandwidth on the output entanglement between the two optical fields without time delay. We find the bandwidth will strongly suppress the output entanglement, specifically as the center frequency of the output fields in the vicinity of resonant frequency. While the output entanglement will become prosperous if the center frequency of output fields departs from the resonant frequency. We will see that the physics behind this phenomenon is the reservoir engineering mechanism because shifting the center frequency can cool the temperature of the system. We obtain all the approximate analytical expressions of the output entanglement in various case, and from which we give the corresponding optimal center frequencies making the entanglement maximum. Finally, we study the effect of the time delay between the two output fields on the output entanglement according to the reservoir engineering mechanism, from which we obtain the approximate analytical expression of the optimal time delay for the case of large filter bandwidth. We think the results of this paper may be used for reference to experimental and theoretical physicists who work on entanglement or quantum information processing.

The rest of this paper is organized as follows. In Section II, we introduce the three-mode optomechanical model with a corresponding equivalent model, and the definition of canonical mode operators of the two output.
optical fields. In Section III, we study the entanglement between the two output optical fields by shifting the center frequency of filter function from resonant frequency. And we study the effects of time delay on the output entanglement. Finally, the conclusions are given in the Section IV.

II. SYSTEM AND AN EQUIVALENT MODEL

We consider a three-mode optomechanical system in which two cavities are coupled a common mechanical resonator (see Fig. 1).

The standard optomechanical Hamiltonian

\[
H = \omega_m \hat{b}^\dagger \hat{b} + \sum_{i=1,2} [\omega_i \hat{a}_i^\dagger \hat{a}_i + g_i (\hat{b} + \hat{b}^\dagger) \hat{a}_i^\dagger \hat{a}_i] \tag{1}
\]

governs the system’s dynamics, where \(\hat{a}_i\) is the annihilation operator for cavity \(i\) with frequency \(\omega_i\) and damping rate \(\kappa_i\), \(\hat{b}\) is the annihilation operator for mechanics resonator with frequency \(\omega_m\) and damping rate \(\gamma\), and \(g_i\) is the optomechanical coupling strength. In order to generate the steady entanglement between the two output fields, we drive cavity 1 (2) at the red (blue) sideband with respect to mechanical resonator: \(\omega_{d1} = \omega_1 - \omega_m\) and \(\omega_{d2} = \omega_2 + \omega_m\). If we work in a rotating frame with respect to the free Hamiltonian, following the standard linearization procedure, and make the rotating-wave approximation (in this paper, we focus on the resolved-sideband regime \(\omega_m \gg \kappa_1, \kappa_2\)), hence, the Hamiltonian of the system can be written as

\[
\hat{H}_{int} = G_1 \hat{b}^\dagger \hat{d}_1 + G_2 \hat{b} \hat{d}_2 + H.c. \tag{2}
\]

Here, \(\hat{d}_i = \hat{a}_i - \hat{a}_i^\dagger\), \(\hat{a}_i\) being the classical cavity amplitude. \(G_i\) is the effective coupling strength. The combined swapping and entangling interactions in \(\hat{H}_{int}\) lead to a net entangling interaction between the two intracavity modes as discussed in \([47]\).

Based on Eq. (2), the dynamics of the system is described by the following quantum Langevin equations for relevant annihilation operators of mechanical and optical modes

\[
\begin{align*}
\frac{d}{dt} \hat{b} &= -\frac{\gamma}{2}\hat{b} - i(G_1 \hat{d}_1 + G_2 \hat{d}_2) - \sqrt{\gamma} \hat{b}^\dagger, \\
\frac{d}{dt} \hat{d}_1 &= -\frac{\kappa_1}{2} \hat{d}_1 - iG_1 \hat{b} - \sqrt{\kappa_1} \hat{d}_1^\dagger, \\
\frac{d}{dt} \hat{d}_2 &= -\frac{\kappa_2}{2} \hat{d}_2 + iG_2 \hat{b} - \sqrt{\kappa_2} \hat{d}_2^\dagger, \tag{3}
\end{align*}
\]

In Eq. (3), \(\hat{b}^\dagger\), \(\hat{d}_i^\dagger\) are the input noise operators of mechanical resonator and cavity \(i\)\((i = 1, 2)\), whose correlation functions are \(\langle \hat{b}^\dagger(t)\hat{b}^\dagger(t') \rangle = N_m \delta(t - t')\) and \(\langle \hat{d}_i^\dagger(t)\hat{d}_i^\dagger(t') \rangle = N_i \delta(t - t')\) respectively. Here, \(N_m\) and \(N_i\) are the average thermal populations of mechanical mode and cavity \(i\), respectively. In the following discussion, we mainly concentrate on how the effects of the center frequency departing from the resonance, the bandwidth of filter function on the entanglement, so we assume these average thermal populations are zero (zero temperature). According to the Routh-Hurwitz stability conditions \([64]\) and we focus on the regime of strong cooperativities \(C_i \equiv 4G_i^2/(\gamma \kappa_i) \gg 1\) and \(\kappa_i \gg \gamma\) in this paper, the stability condition of our system can be obtained as \(G_1^2/G_2^2 > \max(\kappa_1/\kappa_2, \kappa_2/\kappa_1)\) for \(\kappa_1 \neq \kappa_2\), and the system is always stable if \(\kappa_1 = \kappa_2\) and \(G_1 \leq G_2 \leq G_1 \sqrt{2}\) \([47, 50]\).

For simplicity, we adopt a rectangle filter with a bandwidth \(\sigma\) centered about the frequency \(\omega\) to generate the output temporal modes. Then, the canonical mode operators of the two output fields can be described as

\[
\hat{D}_i^{\text{out}}[\omega, \sigma, \tau_i] = \frac{1}{\sqrt{\sigma}} \int_{-\infty}^{\infty} d\omega' e^{-i\omega' \tau_i} \hat{d}_i^\dagger[\omega']. \tag{4}
\]

Here, \(\omega_{\pm} = \omega \pm \frac{\sigma}{2}\), and \(\tau_i\) is the absolute time at which the wavepacket of interest is emitted from cavity \(i\). The frequency-resolved output modes \(\hat{d}_i^{\text{out}}[\omega] \equiv \int d\omega' e^{i\omega' t} \hat{d}_i^\dagger[\omega]/\sqrt{2\pi}\) which can be obtained straightforwardly from the system Langevin equations and input-output relations \([65]\). And we use the logarithmic negativity \([66, 67]\) to quantify the entanglement between the two output cavity modes \(\hat{D}_1^{\text{out}}[\omega, \sigma, \tau_1]\) and \(\hat{D}_2^{\text{out}}[-\omega, \sigma, \tau_2]\). Without loss of generality, we set \(\tau_2 = 0\), and we write \(\hat{D}_i^{\text{out}}[\omega, \sigma, \tau_i] \equiv \hat{D}_i\) for simplicity in the following.

It can be proved that our system can be mapped to a two-mode squeezed thermal state \([50]\)

\[
\hat{\rho}_{12} = \hat{S}_{12}(R_{12})[\hat{b}_1(\tilde{n}_1) \otimes \hat{b}_2(\tilde{n}_2)]\hat{S}_{12}^\dagger(R_{12}) \tag{5}
\]

Here,

\[
\hat{S}_{12}(R_{12}) = \exp[R_{12} \hat{D}_1 \hat{D}_2 - H.c.] \tag{6}
\]

is the two-mode squeeze operator, with \(R_{12}\) being the squeezing parameter, and \(\rho_i^{\text{th}}(\tilde{n}_i)\) describes a single-mode thermal state with average population \(\tilde{n}_i\). Hence, the output fields are thus completely characterized by just
three parameters: $\bar{n}_1$, $\bar{n}_2$, $R_{12}$. The relationship between the two-mode squeezed thermal state and our system can be obtained as follows

$$\bar{n}_1 = \frac{\langle \hat{D}_1^\dagger \hat{D}_1 \rangle - \langle \hat{D}_2^\dagger \hat{D}_2 \rangle - 1 + \sqrt{A^2 - 4|\langle \hat{D}_1^\dagger \hat{D}_2 \rangle|^2}}{2},$$

$$\bar{n}_2 = \frac{\langle \hat{D}_2^\dagger \hat{D}_2 \rangle - \langle \hat{D}_1^\dagger \hat{D}_1 \rangle - 1 + \sqrt{A^2 - 4|\langle \hat{D}_1^\dagger \hat{D}_2 \rangle|^2}}{2},$$

$$R_{12} = \frac{1}{2} \arctan \left( \frac{2|\langle \hat{D}_1^\dagger \hat{D}_2 \rangle|}{A} \right),$$

(7)

here, $\langle \hat{D}_1^\dagger \hat{D}_1 \rangle$, $\langle \hat{D}_1^\dagger \hat{D}_2 \rangle$, $\langle \hat{D}_2^\dagger \hat{D}_2 \rangle$ are the correlators of the output cavity modes, which can be obtained by Langevin equations Eq. (3) and input-output relation, and $A = \langle \hat{D}_1^\dagger \hat{D}_1 \rangle + \langle \hat{D}_2^\dagger \hat{D}_2 \rangle + 1$. According to Eq. (5) and Eq. (6), the output entanglement $E_n$ of this two-mode squeezed thermal state (if $E_n \geq 0$) can be simply given by

$$E_n = -\ln(n_R - \sqrt{\bar{n}_R^2 - (1 + 2\bar{n}_1)(1 + 2\bar{n}_2)}),$$

(8)

with $n_R = (\bar{n}_1 + \bar{n}_2 + 1) \cosh 2R_{12}$. It can be seen from Eq. (8) that the entanglement will increase with the increase of the squeezing parameter $R_{12}$, while decrease with the increase of the average populations $\bar{n}_1$, $\bar{n}_2$. In the following, it can be seen that shifting the center frequency of filter function from the resonance can evidently cool the temperature of the system (decrease the average populations $\bar{n}_1$, $\bar{n}_2$).

### III. CAVITY OUTPUT ENTANGLEMENT

For simplicity, we set equal cavity damping rate $\kappa_1 = \kappa_2 = \kappa$, equal coupling $G_1 = G_2 = G$, and $\gamma \ll \sigma, \kappa, G$ in the following. We discuss the output entanglement on two cases: shifting the filter center frequency $\omega$ from the resonant frequency (the resonant frequency is zero in the rotating frame) under the condition of small bandwidth ($\sigma \ll \kappa$), and large bandwidth ($\sigma = \kappa$) respectively.

#### A. Small bandwidth

In this section we discuss the effects of small bandwidth $\sigma$ ($\sigma \ll \kappa$) on the entanglement between the two output fields. If we shift the filter center frequency $\omega$ to satisfy $0 \leq \omega \leq \frac{\kappa}{2}$ (in the rotating frame), the approximate analytical expression of the output entanglement can be written as

$$E_n \approx \frac{\pi \gamma}{2\sigma}.$$  

(9)

It can be seen from Eq. (9) that the entanglement between output fields is not related to the filter center frequency $\omega$ and the coupling strength $G$. And increasing the mechanical decay rate $\gamma$ can enhance the output entanglement in the vicinity of resonant frequency $\omega = 0$ just as what the author did in Ref. [50], which is the reservoir engineering mechanism because increasing mechanical decay rate $\gamma$ can cool the Bogoliubov mode [50]. If the mechanical damping rate $\gamma$ satisfies $\gamma \ll \sigma$, the entanglement will almost equal to zero. It can also be seen from Eq. (9) that the output entanglement can be largely suppressed by increasing the filter bandwidth $\sigma$.

If the center frequency $\omega$ satisfies $\frac{\kappa}{\sigma} < \omega < \frac{\kappa}{2}$, and the coupling strength $G$ is weak coupling ($G < \kappa$), the analytical expression of the entanglement can be simplified to

$$E_n \approx -\ln \left( \frac{2G^4\sigma^2 + 3\kappa^2\omega^4}{3\omega^2(64G^4 + \sqrt{2}\kappa^2\omega^2)} \right).$$

(10)

The entanglement is plotted in Fig. 2(a) with parameters $\gamma = 1, \sigma = 10, \kappa = 10^5, G = \kappa/10$. The black-solid line is numerical result according to logarithmic negativity, while the red-dashed line is plotted according to simplified analytical expression Eq. (10). The entanglement...
is not monotonic with the change of center frequency \( \omega \), and will reach a maximum as the optimal center frequency satisfy \( \omega_{opt} \approx 6^{1/4}G(\sigma/\kappa)^{1/2} \). The entanglement will appear a peak value at resonant frequency \( (\omega = 0) \) for the case \( \sigma = 0 \) \[50\], but the peak will emerge at some a center frequency \( \omega \) for the case \( \sigma \neq 0 \). We can give a clear reason for this phenomenon from Fig. 2(b) in which the squeezing parameter \( R_{12} \) (red-dashed line), the thermal populations \( \bar{n}_1/10^7 \) (blue-dotted line), \( \bar{n}_2/10^7 \) (black-solid line) vs the normalized center frequency \( \omega/\sigma \) are plotted. It can be seen from Fig. 2(b) the two thermal populations \( \bar{n}_1, \bar{n}_2 \) are very large (the temperature of the equivalent two-mode squeezing thermal state is very high) for \( \omega < \sigma/2 \), then the entanglement is almost zero. But if the center frequency \( \omega \) become larger (\( \omega > \sigma/2 \)), the two thermal populations \( \bar{n}_1, \bar{n}_2 \) will decrease rapidly while the squeezing parameter \( R_{12} \) decrease very slowly. Hence, the entanglement become larger with the increase of center frequency \( \omega \) until the highest point. As a result, the optimal center frequency \( \omega_{opt} \) at which the entanglement reaches a maximum must be greater than \( \sigma/2 \).

If the coupling strength \( G \) is strong coupling \( (G > \kappa) \), and the filter center frequency \( \omega \) still satisfies \( \frac{\kappa}{2} < \omega < \frac{\sigma}{2} \), the analytical expression of the entanglement can be simplified to

\[
E_n \approx -\frac{1}{2} \ln \left[ \frac{G^8 \sigma^4 + G^4 \sigma^2 \omega^4 \kappa^2 + 2 \omega^2 \kappa^2}{144 G^8 \omega^4} \right],
\]

which reaches a maximum as the optimal center frequency satisfy \( \omega_{opt} \approx (G^2 \sigma^4/3\kappa^2)^{1/5} \). The entanglement is plotted in Fig. 3(a) with parameters \( \gamma = 1, \sigma = 10, \kappa = 10^5, G = 10\kappa \). The black-solid line is numerical result according to logarithmic negativity, while the red-dashed line is plotted according to simplified analytical expression Eq. (11). It can be seen from Fig. 2, Fig. 3 that the curves of entanglement plotted by simplified analytical expressions fits the numerical results very well, the squeezing parameter \( R_{12} \) of strong coupling is larger than the case of weak coupling, and the two thermal populations \( \bar{n}_1, \bar{n}_2 \) of strong coupling will also decrease rapidly as the center frequency \( \omega > \sigma/2 \) just as the case of weak coupling. That is the reason why the entanglement of strong coupling will be larger than the one of weak coupling.

According to the above analysis that the optimal center frequency \( \omega_{opt} \) must be greater than \( \sigma/2 \), hence \( \omega_{opt} \) will be far away from the resonant frequency \( \omega (\omega = 0) \) if \( \sigma \) is very large. We will discuss the case \( \sigma = \kappa \) in the following.

\[7\]

### B. Large bandwidth

For \( G < \kappa \) and large \( \sigma \), such as \( G = \kappa/10 \) and \( \sigma = \kappa \), the entanglement will be very small. Hence, in this section, we just discuss the entanglement of strong coupling \( G > \kappa \) with the bandwidth \( \sigma = \kappa \). Because of \( \sigma = \kappa \approx \gamma \), the entanglement almost be zero when \( 0 \leq \omega \leq \frac{\sigma}{2} \) according to Eq. (9). The analytical expression of the entanglement can be simplified to

\[
E_n \approx \ln \left[ \frac{\sqrt{2}(3G^4 \kappa^2 (\omega^2 + \frac{3\gamma^2}{4}) + G^2 \kappa^2 \omega^4 + \omega^8)}{3G^4 \kappa^4 + 2G^2 \omega^2 \kappa^4 + \omega^8} \right]
\]

for \( \frac{\sigma}{2} \leq \omega \leq \frac{7 \kappa}{2} \). And the optimal center frequency \( \omega_{opt} \approx \sqrt{G\kappa} \). In Fig. 4(a), we plot the entanglement vs center frequency \( \omega/\kappa \) according to the analytical expression Eq. (9), Eq. (12) (black-solid line) and the numerical result according to the logarithmic negativity (green-dashed-dotted line) under the parameters: \( \gamma = 1, \sigma = \kappa = 10^5, G = 10\kappa \). It can be seen from Fig. 4(a) that there still is large entanglement even with large bandwidth (\( \sigma = \kappa \)). This because shifting center frequency can effectively cool the two thermal populations \( \bar{n}_1, \bar{n}_2 \) via reservoir engineering as above. And the tendencies of the two thermal populations \( \bar{n}_1, \bar{n}_2 \) and the
squeezing parameter $R_{12}$ are almost the same as the previous cases in Fig. 2(b), Fig. 3(b), we don’t discuss them any more.

As the above analysis, large bandwidth $\sigma$ must strongly influence the entanglement of the two output fields. According to the definition of the canonical mode operators $D_{\omega}$ (see Eq. (4)), the correlator of the output cavity modes $\langle \hat{D}_{1} \hat{D}_{2} \rangle$ is connected with time delay $\tau$, while the other two correlators $\langle \hat{D}_{1}^{\dagger} \hat{D}_{1} \rangle$, $\langle \hat{D}_{2}^{\dagger} \hat{D}_{2} \rangle$ are not. The expression $\langle \hat{D}_{1} \hat{D}_{2} \rangle$ can be written explicitly as

$$\langle \hat{D}_{1} \hat{D}_{2} \rangle = \int_{-\infty}^{\infty} e^{-i\tau\Omega} \frac{8G^{2}\kappa + (\gamma + 2i\Omega)(\kappa^{2} + 4\Omega^{2})}{-(\gamma^{2} + 4\Omega^{2})(\kappa^{2} + 4\Omega^{2})^{2}/(8G^{2}\kappa)} d\Omega.$$  

(13)

The effect of time delay $\tau$ on entanglement $E_n$ can be seen easily form the equivalent two-mode squeezing thermal state. From Eq. (7), we can see that the two-mode squeezing parameters $\bar{n}_{1}$, $\bar{n}_{2}$, and $R_{12}$ are affected by time delay $\tau$ just through the correlator $\langle \hat{D}_{1} \hat{D}_{2} \rangle$. More specifically, $\bar{n}_{1}$, $\bar{n}_{2}$ will decrease and $R_{12}$ will increase if the modulus $|\langle \hat{D}_{1} \hat{D}_{2} \rangle|$ becomes large as other parameters fixed except for time delay $\tau$. Hence, we can assert categorically that the output entanglement $E_n$ will increase with the increasing of the modulus of the correlator $\langle \hat{D}_{1} \hat{D}_{2} \rangle$. The optimal time delay $\tau_{opt}$ is the delay which makes the $|\langle \hat{D}_{1} \hat{D}_{2} \rangle|$ reach a maximum. After obtaining the approximate analytical expression about $|\langle \hat{D}_{1} \hat{D}_{2} \rangle|$ and making some corrections, we find the optimal time delay is

$$\tau_{opt} \approx \begin{cases} \frac{3G^{2}\kappa\omega^{2}}{G^{2}\kappa^{2} + \omega^{6}} , & \omega \geq \frac{\kappa}{2} , \\ \frac{3\kappa}{\pi(2G^{2}\kappa)} , & 0 \leq \omega < \frac{\kappa}{2} . \end{cases} \tag{14}$$

We plot the output entanglement $E_n$ with optimal time delay $\tau_{opt}$ (red-solid line) based on Eq. (14), and that with numerical optimal time delay which makes the entanglement $E_n$ reach a maximum (blue-dashed line) in Fig. 4(a) and the corresponding time delays are plotted in Fig. 4(b) with the parameters: $\gamma = 1, \sigma = \kappa = 10^{\delta}, G = 10\kappa$, and they all fit very well. It can be seen from Fig. 4(a) that the time delay $\tau$ strongly affects the entanglement $E_n$ as long as the center frequency $\omega$ is not big enough compared with bandwidth $\sigma$, while has no effect on the entanglement $E_n$ as $\omega \gg \sigma$. The reason is that the effect of fixing $\sigma$ and increasing $\omega$ is equivalent to that of fixing $\omega$ and decreasing $\sigma$. And the time delay $\tau$ has no effect on entanglement for the case of $\sigma \to 0$, which can be seen according to Eq. (13) that the factor $e^{-i\tau\Omega}$ can be extracted out of the integration for small bandwidth $\sigma$ with the result that the modulus $|\langle \hat{D}_{1} \hat{D}_{2} \rangle|$ will be not related to $\tau$. The steep entanglement in the vicinity $\omega = \sigma/2$ is because of the special rectangle filter and reaches a local minimum ($E_{n\min} \approx 1.68$) at $\omega = \sigma/2$ according to the numerical result.

**IV. CONCLUSIONS**

In summary, we have studied theoretically the output entanglement between two output cavity fields via reservoir engineering by shifting the center frequency of the causal filter function away from the resonance ($\omega = 0$ in the rotating frame) in a three-mode cavity optomechanical system. We find that the nonzero bandwidth $\sigma$ can largely suppress the entanglement $E_n$, specifically in the vicinity of resonant frequency $E_n \sim 1/\sigma$. While the output entanglement will become prosperous, if we shift the center frequency of output fields away from the resonant frequency. This is because shifting center frequency can effectively cool the two-mode squeezing thermal state which is equivalent to our model. We obtain all the approximate analytical expressions of the output.
entanglement, and from which we give the corresponding optimal center frequencies $\omega_{\text{opt}}$. In addition, we find the time delay $\tau$ between the two output optical fields can evidently effect the output entanglement. And we obtain the analytical expression of the optimal time delay $\tau_{\text{opt}}$ in the case of large filter bandwidth ($\sigma = \kappa$). Our results can also be applied to other parametrically coupled three-mode bosonic systems, and may be useful to experimentalists to obtain large entanglement.

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