Analysis of BeiDou/Galileo combined pseudo-range positioning performance in urban environment

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Abstract. The observation conditions have a significant influence on the positioning performance. GNSS signals are easily blocked in urban environments, which leads to discontinuity and instability of position solution. With the construction of BeiDou and Galileo satellite navigation system in the final stage, multi-satellite fusion will be the future development trend of GNSSs. This paper introduces the mathematical fusion model of Galileo and BeiDou satellite navigation system, typical urban coverage conditions are simulated, and a detailed in the aspects of number of visible satellites, positioning error and PDOP values are analysed. The results show that the positioning accuracy of the integrated navigation system in the horizontal is within 3 m, the elevation is within 5 m, and the PDOP value are less than 1.5 in the open sky condition. In any coverage conditions, Galileo stand-alone system cannot guarantee the positioning continuity in the whole day, and positioning performance is the worst in the north coverage condition, unable to obtain the position solutions for more than 40% time in the whole day. In the north coverage condition, BeiDou has the best positioning performance, with a minimum of 4 visible satellites and 99% of PDOP values are less than 10.

1. Introduction
Global Navigation Satellite System (GNSS) can provide high-precision navigation, positioning and timing services, and is widely used in military, national economic construction, space technology and other fields [1-2]. However, the number of visible satellites, positioning accuracy and availability decreases significantly under complicated urban observation conditions, such as urban canyon, tunnel, GNSS positioning performance, mainly due to the blocking and reflection of GNSS signals by dense high-rise buildings. Under these conditions, the number of visible satellites of a GNSS stand-alone system can even be less than 4, which will make GNSS impossible to obtain the position solution [3-5].

BeiDou has started in 2019 to provide global navigation services, and Galileo is going full speed ahead to achieve full constellation by 2020 [6]. With the prosperity of GNSS technology, multi-system fusion positioning has improved satellite visibility in urban environments. The multi-mode navigation satellite system enables users to obtain more observation values and strength satellite spatial geometry, thus contributing to the improvement of positioning accuracy in the urban environment.

This paper aims to study the positioning advantage of Galileo/BeiDou system in the urban environment. The positioning performance of single and double systems is compared and analysed from the perspective of good and poor urban observation conditions. Finally, the influence of complex urban observation environment on the positioning performance of different system combinations is analysed.
2. Mathematical model

When the satellite clock error, ionospheric and tropospheric delay errors are compensated, the modified pseudo-range observation equation of a single satellite can be simplified as [3]

\[
\rho^m = \sqrt{(x - x^m)^2 + (y - y^m)^2 + (z - z^m)^2} + b_g + \varepsilon^m_p
\]  

\[
\rho^n = \sqrt{(x - x^n)^2 + (y - y^n)^2 + (z - z^n)^2} + b_B + \varepsilon^n_p
\]

Where \( \rho \) is the pseudo-distance measured; Subscripts \( G \) and \( B \) represent Galileo and BeiDou respectively. Superscript \( m \) and \( n \) represent Galileo and BeiDou satellite Numbers respectively. \((x, y, z)\) are user coordinates in the earth coordinate system; \((x^m, y^m, z^m)\) and \((x^n, y^n, z^n)\) represent the coordinates of Galileo and BeiDou satellites in the earth coordinate system respectively; \( b \) represents the receiver clock equivalent distance error; \( \varepsilon \) represents the pseudo-range measurement noise.

By Taylor expansion of equations (1) and (2) at the receiver approximate coordinates \((x_0, y_0, z_0)\), the pseudo-range positioning linear error equation is obtained [3]

\[
\begin{bmatrix}
1 \\ l^m \\ l^{m+1} \\ p^{m+n}
\end{bmatrix} = 
\begin{bmatrix}
\frac{x_0 - x^1}{\tilde{r}^1} & \frac{y_0 - y^1}{\tilde{r}^1} & \frac{z_0 - z^1}{\tilde{r}^1} & 1 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x_0 - x^m}{\tilde{r}^m} & \frac{y_0 - y^m}{\tilde{r}^m} & \frac{z_0 - z^m}{\tilde{r}^m} & 1 & 0 \\
\frac{x_0 - x^{m+1}}{\tilde{r}^{m+1}} & \frac{y_0 - y^{m+1}}{\tilde{r}^{m+1}} & \frac{z_0 - z^{m+1}}{\tilde{r}^{m+1}} & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x_0 - x^{m+n}}{\tilde{r}^{m+n}} & \frac{y_0 - y^{m+n}}{\tilde{r}^{m+n}} & \frac{z_0 - z^{m+n}}{\tilde{r}^{m+n}} & 0 & 1
\end{bmatrix} 
\begin{bmatrix}
\delta x \\ \delta y \\ \delta z \\ \delta b_G \\ \delta b_B
\end{bmatrix} + 
\begin{bmatrix}
\varepsilon^1_p \\ \vdots \\ \varepsilon^m_p \\ \varepsilon^{m+1}_p \\ \vdots \\ \varepsilon^{m+n}_p
\end{bmatrix}
\]  

\( (\Delta x, \Delta y, \Delta z) \) is the correction number of position coordinates solved by iteration; \( \tilde{r}^k (k = 1, 2, \cdots, m + n) \) for signal time satellite \( k \) to receiver the approximate location of the geometric distance; \( l^m = \rho^m - \tilde{r}^k \) as constant observation equation.

Equation (3) can be written as a matrix:

\[
L = AX + V
\]

where \( L \) is the observation vector, which is the difference between the corrected pseudo-distance and the pseudo-distance calculated by the approximation coordinates. \( X \) is the unknown parameter vector including three-dimensional position and receiver clock error, \( A \) is the n-by-4 design matrix, and the residual matrix is \( V \). If the weight matrix \( P \) of the observation vector \( L \) is considered, the least squares solution of the unknown parameter vector is [3]

\[
\bar{X} = (A^T PA)^{-1} A^T PL
\]

The weight coefficient matrix is:

\[
H = (A^T PA)^{-1} = 
\begin{bmatrix}
q_{11} & q_{12} & q_{13} & q_{14} \\
q_{21} & q_{22} & q_{23} & q_{24} \\
q_{31} & q_{32} & q_{33} & q_{34} \\
q_{41} & q_{42} & q_{43} & q_{44}
\end{bmatrix}
\]

Where \( PDOP = \sqrt{q_{11} + q_{22} + q_{33}} \) is an important parameter to evaluate the positioning accuracy.

3. Simulation and Results

3.1. Constellation status

In order to calculate the PDOP values and number of visible satellites in Berlin (52.52°N, 13.03°E) urban environment, the simulation time was set to 24h. The position parameters for BeiDou and Galileo were calculated by using Kepler orbital parameters, which can be referred to the reference [3]. The satellite elevation mask angle was set to 5°, and the sampling interval was 5minutes for 24h. The altitude was set to 10m. During the simulation period, 30 Galileo satellites and 35 BeiDou satellites
were employed. Figure 1 shows the ground traces of BeiDou GEO, IGSO and MEO satellites in this study. Ground traces of satellites are represented by yellow dots, blue and red curves for GEO, IGSO and MEO satellites, respectively.

Table 1. Half-sky coverage typical masking azimuth schemes/ Unit(°)

| Azimuth          | East   | South  | West   | North   |
|------------------|--------|--------|--------|---------|
| Remove Satellite | [0, 90]| [90, 180]| [-180, -90]| [-90, 0]|
|                  | [90, 180]| [-180, -90]| [-90, 0]  | [0, 90]  |

Four typical coverage schemes are simulated, namely, the east, south, west and north half sky is completely blocked, and the satellites in the blocked area are not visible. The azimuth sky plot can be viewed as shown in figure 2, and the invisible satellite is removed according to the scheme in table 1.

![Figure 1. Sub-satellite point track of BeiDou constellation](image1)

![Figure 2. Azimuth sky plot](image2)

![Figure 3. Number of visible satellites for Galileo only (top), BeiDou only (middle) and Galileo/BeiDou integrated system (bottom) under different azimuth coverage.](image3)

The number of visible satellites in typical coverage conditions and the open sky condition is shown in figure 3 and table 2. In the north coverage condition for Galileo system, it can be seen that the
number of visible satellites is between 3 and 8, with an average of 4.8. Moreover, the number of visible satellites is less than 5 in more than 40% of the time, and the positioning performance is not ideal. The number of satellites is less than 4 in a certain percentage of the time, which leading to the failure to solve. In the east, south, and west coverage conditions, the minimum number of visible satellites is 3, which cannot obtain the solution for about 2 hours to 5 hours. When the west and north are covered, BeiDou can obtain the position solution due to the minimum number of visible satellites is more than 4. When the east and south are blocked, the minimum number of visible satellites is only 2, so there are many epochs that cannot be solved the position. For the Galileo and BeiDou combined system, stand-alone system positioning performance in Europe can be further improved. In the four coverage conditions, the minimum and maximum number of visible satellites can achieve 19 and 30, respectively, which greatly improves the positioning performance in the urban environment.

Table 2. Mean values of Visible Satellites for all epochs in 24 hours

| Mode            | Open sky | Half open sky coverage |
|-----------------|----------|------------------------|
|                 |          | East      | South     | West     | North    |
| Galileo         | Mean     | 10.7      | 5.20      | 5.84     | 5.51     | 4.87     |
|                 | Max      | 14        | 7         | 10       | 8        | 8        |
|                 | Min      | 7         | 3         | 3        | 3        | 3        |
| BeiDou          | Mean     | 12.2      | 5.32      | 5.37     | 6.90     | 6.85     |
|                 | Max      | 16        | 8         | 10       | 10       | 10       |
|                 | Min      | 8         | 2         | 2        | 5        | 4        |
| Galileo/BeiDou  | Max      | 22.9      | 10.52     | 11.2     | 12.4     | 11.72    |
|                 | Min      | 19        | 8         | 7        | 9        | 8        |

Table 3 shows the PDOP values in different coverage conditions for satellite navigation system. According to the PDOP values, this paper divides into the three grades of excellent, good and poor, corresponding values are (1, 3), (3, 10) and (10, +∞). In the open sky condition, all PDOP values are in the range (1, 3), and the positioning performance is optimal. In the four coverage conditions, any single positioning system has a certain proportion of PDOP values in the range (10, +∞), which cannot meet the positioning requirements. For Galileo stand-alone system, in the north coverage condition, 40.98% of the PDOP values is in the range (10, +∞), and in the east and south coverage conditions for BeiDou stand-alone system are about 40% of PDOP values in the range (10, +∞).

Table 3. PDOP value in (1, 3), (3,10) and (10, +∞) of (unit: %)

| Value       | mode      | Open sky | Half-open sky coverage |
|-------------|-----------|----------|------------------------|
|             |           | East     | South     | West     | North    |
| 3>PDOP>1    | Galileo   | 100      | 12.50     | 3.82     | 25.35    | 6.94     |
|             | BeiDou    | 100      | 12.15     | 0.69     | 11.11    | 22.92    |
|             | Galileo/BeiDou | 100  | 62.85     | 40.28    | 72.57    | 93.06    |
| 10>PDOP>=3  | Galileo   | 0        | 69.44     | 76.39    | 62.85    | 52.08    |
|             | BeiDou    | 0        | 51.39     | 56.60    | 67.01    | 76.04    |
|             | Galileo/BeiDou | 0     | 37.15     | 58.68    | 27.43    | 6.94     |
| PDOP>=10 or unavailable | Galileo | 0        | 18.06     | 19.79    | 11.80    | 40.98    |
|             | BeiDou    | 0        | 36.46     | 42.71    | 21.82    | 1.04     |
|             | Galileo/BeiDou | 0     | 0        | 1.04     | 0        | 0        |

Table 4 shows the RMS statistics of combined positioning results in the open sky condition. The positioning errors of the combined system in the E, N and U directions are 1.031, 1.206 and 2.300 m, respectively. In the south coverage condition, the positioning performance is the worst, because 58.68%
of PDOP values are in the range (3,10). The position performance is optimal in the north coverage condition, with the 93.06% of PDOP values are in the range (1, 3). In the east and west coverage conditions, the performance of the positioning results is similar. Figure 4 and 5 show the positioning error and PDOP values for the three systems within 24 hours in the open sky condition, respectively. It can be seen that Galileo and BeiDou have similar horizontal and elevation positioning performance in this region, with the horizontal within 5 m and the elevation within 10 m. For the combined system, the positioning performance is greatly improved, with the horizontal within 3 m and the elevation within 5 m. The PDOP values are less than 2.5 for any stand-alone system and 1.5 for combined system.

Table 4. The position error RMS values for Galileo/BeiDou integration

| Direction | Open sky | Half open sky coverage |
|-----------|----------|------------------------|
|           |          | East       | South      | West       | North      |
| E         | 1.031    | 5.256      | 1.670      | 4.960      | 1.784      |
| N         | 1.206    | 2.352      | 7.562      | 2.162      | 4.206      |
| U         | 2.300    | 4.761      | 7.489      | 4.561      | 3.972      |

Figure 4. The position errors of E, N and U in the open-sky

Figure 5. PDOP values in the open sky

4. Conclusion
This paper introduces Galileo and BeiDou combined navigation mathematical models, the positioning performance of typical coverage environments are simulated and analysed, and following conclusions in the aspect of positioning accuracy, number of visible satellites and PDOP values can be obtained.

(1) In the open sky condition, the positioning accuracy of the integrated navigation system in the horizontal is within 3 m, the elevation is within 5 m, and the PDOP values are less than 1.5. In any coverage condition, the number of visible satellites of the combined navigation system are more than 4, and the PDOP value are less than 10, which have the optimal positioning performance.

(2) In any coverage condition, Galileo stand-alone system cannot guarantee the continuity of positioning in the whole day. In the north coverage condition, positioning performance is the worst, unable to solve the position for more than 40% time in the whole day. In the north coverage condition, BeiDou has the best positioning performance, with a minimum of 4 visible satellites and 99% of PDOP values are within 10.

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