CMB Distortions from Superconducting Cosmic Strings

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Abstract

We reconsider the effect of electromagnetic radiation from superconducting strings on cosmic microwave background (CMB) $\mu$- and $\gamma$-distortions and derive present (COBE-FIRAS) and future (PIXIE) constraints on the string tension, $\mu_s$, and electric current, $I$. We show that absence of distortions of the CMB in PIXIE will impose strong constraints on $\mu_s$ and $I$, leaving the possibility of light strings ($G\mu_s \lesssim 10^{-18}$) or relatively weak currents ($I \lesssim 10$ TeV).

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I. INTRODUCTION

Phase transitions are key milestones in the thermal history of the universe. As the universe evolves and cools, fundamental symmetries are spontaneously broken and cosmological phase transitions occur. Therefore, the observation of phase-transition remnants can give us direct access to high energy particle physics and the very early universe.

Superconducting cosmic strings are one of a variety of topological defects that could be produced at phase transitions in the early universe [1, 2]. As superconducting strings move through the cosmic magnetized plasma, they can develop and carry large currents, and oscillating loops of superconducting strings will emit copious amounts of electromagnetic radiation and particles mostly as bursts [3, 4]. This led to the idea that superconducting strings may be a candidate for the engine driving observed gamma ray bursts [5–7], sources of cosmic ray bursts [8] and radio transients [9, 10].

In this paper we focus on the role of superconducting cosmic strings as sources that inject energy in the cosmic medium and cause spectral distortions of the cosmic microwave background (CMB). The measurement of the CMB spectral distortion is a good probe of the thermal history of the universe and has been studied analytically and numerically in Refs. [11–16]. In the early universe \( z > 10^6 \) where \( z \) is the cosmic redshift), double Compton and Compton scatterings are very efficient, and any energy that is injected in photons into the cosmic medium is thermalized, and the cosmic radiation spectrum remains that of a blackbody. However, the expansion of the universe makes these scatterings less efficient with time and energy injected at epochs with \( z < 10^6 \) produces CMB spectral distortions. That is, the spectrum departs from a blackbody spectrum. Such distortions are commonly described by two parameters: the \( \mu \) (chemical potential) distortion parameter, and the Compton \( y \)-parameter. Current constraints on these parameters have been obtained by COBE FIRAS and are: \( |\mu| < 9 \times 10^{-5} \) and \( y < 1.5 \times 10^{-5} \) [17, 18]. The recently proposed future space mission called PIXIE has the potential to give dramatically tighter constraints on both types of distortion, \( |\mu| \sim 5 \times 10^{-8} \) and \( y \sim 10^{-8} \) at the 5 \( \sigma \) level [19].

There are several more conventional reasons for expecting CMB distortions. The diffusion of density fluctuations before recombination, known as Silk damping [20], is an energy injection source which produces CMB distortions [21, 23] at the level of \( \mu \sim 8 \times 10^{-9} \) [24, 25]. Other energy injection sources include massive unstable relic particles which decay during the thermalization epoch [26], dissipation of primordial magnetic fields during the recombination epoch [27], and Hawking radiation from primordial black holes [28]. An observation of CMB distortions will not by itself definitively point to a particular injection source, though upper limits on the distortions can be used to place constraints on models.

We evaluate both \( \mu \)- and \( y \)-distortions due to energy injected from superconducting string loops. The injected energy depends on the current, \( I \), carried by the strings, and on the string tension, \( \mu_s \). In general, the current arises due to the interaction of strings with ambient magnetic fields and need not be constant along a string, and may also vary among different parts of the string network. However, we shall simplify our analysis by assuming the same constant current along all strings in the network. This simplification is expected to be accurate in the presence of primordial magnetic fields so that there is sufficient time for the current to build up and saturate at its maximum possible value.

In this paper, we will obtain constraints in the two dimensional parameter space given by the electric current on superconducting cosmic string loops and the string tension (\( I - G\mu_s \) plane) due to present limits on CMB distortions. Early analyses placed a constraint on the
fraction of electromagnetic to gravitational radiation from strings \[3, 29, 30\]. However, since both the electromagnetic and gravitational power depend on the string tension, as described in Sec. II, those results cannot be directly used to produce a constraint plot in the $I - G\mu_s$ plane. Our analysis also differs from earlier work in details of the string network, and we are able to forecast constraints from future observation missions such as PIXIE.

The organization of this paper is as follows. In Sec. II we discuss the cosmic string network properties and number density of loops, and then, derive the rate of electromagnetic energy density emitted from cusps of superconducting cosmic string loops. In Sec. III we calculate the spectral distortions of the CMB parametrized by chemical potential $\mu$ and Compton $y$-parameter due to cosmic strings, and obtain the corresponding constraints from COBE and PIXIE. Finally, in Sec. IV we summarize our findings.

Throughout this paper, we use parameters for a flat ΛCDM model: $h = 0.7 \ (H_0 = h \times 100 \ \text{km/s/Mpc}), \Omega_b = 0.05 \ \text{and} \ \Omega_m = 0.26$. Note also that $1 + z = \sqrt{u/t}$ in the radiation dominant epoch and $1 + z = (1 + z_{eq})(t_{eq}/t)^{2/3}$ in the matter dominant epoch, where $t' = (2\sqrt{\Omega_r}H_0)^{-1}$ and $z_{eq} = \Omega_m/\Omega_r$ with $h^2\Omega_r = 4.18 \times 10^{-5}$. We also adopt natural units, $\hbar = c = 1$, and set the Boltzman constant to unity, $k_B = 1$.

## II. STRING NETWORK AND RADIATION

A superconducting string loop emits electromagnetic radiation at frequency harmonics defined by its inverse length. The emitted power is dominated by the highest frequency and is cut off by the finite thickness of the string. The total power emitted in photons from loops with cusps is

$$P_\gamma = \Gamma_\gamma I \sqrt{\mu_s},$$

(1)

where $I$ is the current on the string (assumed constant), $\mu_s$ is the string tension, and $\Gamma_\gamma \sim 10$ is a numerical coefficient that depends on the shape of the loop. The string network also contains a similar number of loops without cusps which emit much less power,

$$P \sim I^2,$$

in electromagnetic radiation than loops with cusps. Therefore, the contribution of cuspless loops can be ignored.

The loop also emits gravitational radiation with power \[31\]

$$P_g = \Gamma_g G\mu_s^2,$$

(2)

where $\Gamma_g \sim 100$. Therefore, for every $\mu_s$, there is a critical current

$$I_* = \frac{\Gamma_g G\mu_s^{3/2}}{\Gamma_\gamma},$$

(3)

and for $I > I_*$ electromagnetic radiation dominates and determines the lifetime of the loop, while for $I < I_*$ gravitational losses are more important. Hence, we can write the lifetime of a string loop of length $L$ as

$$\tau = \frac{L}{\Gamma G\mu_s},$$

(4)

where

$$\Gamma = \Gamma_g, \quad I < I_*,$$

$$\Gamma = \frac{\Gamma_\gamma I}{G\mu_s^{3/2}} = \Gamma_g \frac{I}{I_*}, \quad I > I_*.$$

(5)
If a loop is born with length \( L_i \), its length changes with time as
\[
L = L_i - \Gamma G \mu_s (t - t_i). \tag{6}
\]
Assuming slow decay, we take \( t \gg t_i \) and hence
\[
L_i \approx L + \Gamma G \mu_s t. \tag{7}
\]

Analytical studies [32–36] and simulations [37–45] all yield a consistent picture for large cosmic string loops but differing results for small loops. The results can be summarized during the cosmological radiation dominated epoch \((t < t_{eq})\), by giving the number density of loops of length between \( L_i \) and \( L_i + dL_i \),

\[
\frac{dn(L_i, t)}{L_i} = \kappa \frac{dL_i}{t^{1-p} L_i^p}.
\tag{8}
\]

The overall normalization factor, \( \kappa \), will be assumed to be \( \sim 1 \). In our calculation, we shall take the exponent \( p = 2.5 \), though somewhat different values are suggested in other studies, e.g., \( p = 2.6 \) and \( \kappa \sim 0.1 \) in [36]. Our final constraints are not affected significantly by such a slight increase in the value of \( p \), especially because the increased effect from small loops is compensated by the smaller value of \( \kappa \).

Inserting Eq. (7) in (8) gives
\[
\frac{dn(L, t)}{L} = \kappa \frac{d}{t^{3/2}(L + \Gamma G \mu_s t)^{5/2}}, \quad t < t_{eq}, \tag{9}
\]
as the number density of loops of length \( L \) at cosmic time \( t \).

Similarly, during the cosmological matter dominated epoch the number density of loops is
\[
\frac{dn(L, t)}{L} = \frac{\kappa C_L dL}{t^2 (L + \Gamma G \mu_s t)^2}, \quad t > t_{eq}, \tag{10}
\]
where
\[
C_L \equiv 1 + \sqrt{\frac{t_{eq}}{L + \Gamma G \mu_s t}}. \tag{11}
\]
The second term takes into account the loops from the radiation dominated epoch that survive into the matter dominated epoch.

The energy injection rate into photons from cosmic strings is found by multiplying Eq. (1) by the number density of loops in Eq. (10), and integrating over loop length
\[
\frac{dQ}{dt} = \Gamma \gamma I \sqrt{\mu_s} \int_0^\infty dn(L, t).
\tag{12}
\]

We can write \( dQ/dt \) in the radiation and the matter dominated epochs as
\[
\frac{dQ}{dt} = \frac{2\kappa \Gamma \gamma I \sqrt{\mu_s}}{3(\Gamma G \mu_s)^{3/2} t^3}, \quad t < t_{eq}, \tag{13}
\]
and
\[
\frac{dQ}{dt} \approx \frac{2\kappa \Gamma \gamma I \sqrt{\mu_s}}{3(\Gamma G \mu_s)^{3/2} t^3} \left[ 1 + \frac{2}{3} \sqrt{\frac{t_{eq}}{\Gamma G \mu_s t}} \right], \quad t > t_{eq}, \tag{14}
\]
\[
\approx \frac{2\kappa \Gamma \gamma I \sqrt{\mu_s}}{3(\Gamma G \mu_s)^{3/2} t^3} \sqrt{\frac{t_{eq}}{t}},
\]
where in the second line, we have restricted attention to strings such that $\Gamma G_{\mu s} t_0 \ll t_{eq}$ — $t_0$ being the present cosmological epoch — a condition that is satisfied in a large part of the allowed range of string parameters.

III. CMB DISTORTIONS DUE TO COSMIC STRINGS

In the early universe ($z > 10^6$ where $z$ denotes cosmic redshift), we expect that energy injected into the cosmological medium will be thermalized by photon-electron interactions, i.e., by Compton and double Compton scatterings. As a result, the photon distribution in the early universe maintains its blackbody spectrum. However, the energy injection from cosmic strings mainly consists of very high energy photons ($\omega \sim \sqrt{\mu} \gg m_e$) and the optical depth of such high energy photons for Compton and double Compton scatterings is not high, because the scattering cross-sections are suppressed by the photon energy. Then, thermalization proceeds in two steps. First, the high energy photons lose their energy quickly via photon-photon scattering or photo pair production (see Ref. [46]). Once the energy of the photons is reduced by these processes, they can be thermalized by Compton and double Compton scatterings and the photons again achieve a blackbody spectrum.

At lower redshifts ($z < 10^6$), Compton and double Compton scatterings decouple and the injected photons can no longer be thermalized efficiently. Accordingly, energy injection produces distortions in the blackbody spectrum of the CMB.

First, the decoupling of double Compton scattering takes place at $z \sim 10^6$. As a consequence, photon number is conserved for $z < 10^6$, and only the energy among the photons can be re-distributed. This is insufficient to establish a blackbody spectrum for the photons. However, the injected photons are still thermalized by Compton scattering, and the CMB spectrum in this thermal equilibrium state is described by the Bose-Einstein distribution with a chemical potential $\mu$.

Thermalization due to Compton scattering also becomes inefficient at $z \sim 10^5$, when the time scale of the Compton scattering process becomes longer than the Hubble time. The energy injection after Compton decoupling produces a distortion which is parameterized by the Compton $y$-parameter.
\[ \frac{d\mu}{dt} = -\mu \left( \frac{t_{DC}(z)}{t_{DC}} \right) + \frac{1.4 dQ}{\rho_\gamma dt}. \]  

Here \( \rho_\gamma \) is the photon energy density, \( t_{DC} \) is the time scale for double Compton scattering

\[ t_{DC} = 2.06 \times 10^{33} \left( 1 - \frac{Y_p}{2} \right)^{-1} (\Omega_b h^2)^{-1} z^{-9/2} \text{ s}, \]

where \( Y_p \) is the primordial helium mass fraction.

As explained above, the \( \mu \)-distortion is only produced in a redshift range \( z_1 \approx 10^6 \) to \( z_2 \approx 10^5 \), when double Compton scattering is inefficient, but Compton scattering is still operative. Then, the solution to Eq. (15) is

\[ \mu = 1.4 \int_{t(z_1)}^{t(z_2)} dt \frac{dQ}{\rho_\gamma} \exp\left[-\frac{(z(t)/z_{DC})^{5/2}}{2} \right] = 1.4 \int_{z_1}^{z_2} dz \frac{dQ}{\rho_\gamma} \exp\left[-\frac{(z/z_{DC})^{5/2}}{2} \right]. \]

Performing the integration in Eq. (17) with Eq (13), we find

\[ \mu = 4.6 \times 10^{-6} \left( \frac{I/\Gamma G \mu_s}{10^{11} \text{ GeV}} \right), \]

which is plotted in Fig. 2 (the result depends only very weakly on the integration limits \( z_1 \) and \( z_2 \)). According to the COBE constraint [17, 18], we obtain

\[ \frac{I}{\Gamma G \mu_s} < 1.95 \times 10^{12} \text{ GeV \ , (COBE)}, \]

FIG. 2: The \( \mu \)-distortion as a function of \( I/\Gamma G \mu_s \). The dotted and dashed lines represent COBE FIRAS limit and the detection limit by PIXIE in its current design.

**A. \( \mu \)-distortion**

The time evolution of the \( \mu \)-distortion of the CMB spectrum due to energy injection is given by [16]

\[ \frac{d\mu}{dt} = -\mu \left( \frac{t_{DC}(z)}{t_{DC}} \right) + \frac{1.4 dQ}{\rho_\gamma dt}. \]
FIG. 3: The constraint from $\mu$-distortion on $I-G_{\mu s}$ plane. The dark shaded area is ruled out by COBE constraint on $\mu$-distortion. If there is no detection of $\mu$-distortion by PIXIE, the lightly shaded region within the thick dashed line will be ruled out. The region above the thin dashed line is where gravitational radiation dominates over electromagnetic radiation, i.e., $I < I_*$ (see Eq. (3)). The hatched region is excluded because $I > \sqrt{\mu_s}$, and exceeds the saturation value of the current on superconducting strings. Also, millisecond pulsar observations constrain $G_{\mu s} \lesssim 10^{-7}$ as shown by the dot-dashed line.

and the predicted constraint from PIXIE is more severe \cite{19},

$$\frac{I}{G_{\mu s}} < 1.08 \times 10^9 \text{ GeV}, \quad \text{(PIXIE).} \tag{21}$$

We plot these constraints in the $I-G_{\mu s}$ plane in Fig. 3.

In Fig. 3, the region above the short-dashed line is where gravitational radiation losses dominate the electromagnetic radiation, i.e., $I < I_*$, where $I_*$ is defined in Eq. (3). In this region, $\Gamma \sim 100$ from Eq. \label{Eq:Gamma}. Then, Eqs. (20) and (21) give the constraints,

$$\frac{I}{G_{\mu s}} < 1.95 \times 10^{14} \text{ GeV}, \quad \text{(COBE),} \tag{22}$$

$$\frac{I}{G_{\mu s}} < 1.08 \times 10^{11} \text{ GeV}, \quad \text{(PIXIE).} \tag{23}$$

In the region below the short-dashed line in Fig. 3 electromagnetic radiation dominates over gravitational radiation. As a result, in this region, the constraints from $\mu$-distortion represented by Eqs. (20) and (21) can be written using Eq. \label{Eq:Gamma},

$$G_{\mu s} < 2.5 \times 10^{-12}, \quad \text{(COBE),} \tag{24}$$

$$G_{\mu s} < 7.9 \times 10^{-19}, \quad \text{(PIXIE).} \tag{25}$$

Note that the constraint is independent of the current provided $I > I_*$ holds.
B. Compton $y$-distortion

For $z < 10^5$, the injected energy is no longer thermalized by Compton scattering. Instead, the injected energy heats up electrons, which then scatter the CMB photons by the inverse Compton process, leading to $y$-distortions of the CMB.

The Compton $y$-parameter is given by \[ y = \int_{t_{\text{freeze}}}^{t_0} dt \frac{T_e - T}{m_e} n_e \sigma_T, \] (26)

where $T_e$ is the electron temperature, $T$ is the temperature of the cosmic background radiation, $n_e$ is the number density of free electrons, $\sigma_T$ is the Thomson scattering cross section and $t_0$ is the present time. The time $t_{\text{freeze}}$ represents the freeze out time of thermalization, which we set it to be $z \sim 10^5$.

The evolution of the electron temperature $T_e$ with injected photon energy is written as

\[ n_e \frac{dT_e}{dt} = \frac{n_e \sigma_T}{3} \int \frac{\omega - 4T_e}{m_e} \omega f_\omega d\omega - \frac{4}{3} \frac{n_e \sigma_T}{m_e} \rho_\gamma (T_e - T) - 2 \frac{\dot{a}}{a} T_e n_e, \] (27)

where $f_\omega$ is the spectrum of photons injected by time $t$ — in other words, $f_\omega$ is the spectrum of all photons minus the spectrum of blackbody photons. The first term on the right hand side (rhs) of Eq. (27) describes Compton heating of electrons by injected photons; the second term describes the Compton cooling of electrons by photons; the third describes cooling due to cosmic expansion.

The evolution of the spectrum $f_\omega$ is obtained from the equation,

\[ \frac{\partial f_\omega}{\partial t} = \frac{\omega - 4T_e}{m_e} \omega \frac{\partial f_\omega}{\partial \omega} n_e \sigma_T + \frac{2\omega - 4T_e}{m_e} f_\omega n_e \sigma_T + \frac{\dot{a}}{a} \omega \frac{\partial f_\omega}{\partial \omega} - 2 \frac{\dot{a}}{a} f_\omega + \delta f_\omega, \] (28)

where $\delta f_\omega$ is the injected number of photons with frequency $\omega$ per unit time. The first two terms on the rhs of Eq. (28) describe Compton cooling of injected photons, and the third and forth terms describe cooling due to Hubble expansion.

In order to analytically evaluate the electron temperature, we note that the last term in Eq. (27) is suppressed by the inverse cosmic time, which is much larger than the microphysical time involved in Compton processes. So we ignore the Hubble expansion term and assume the quasi-steady state condition: $dT_e/dt = 0$. Then, the electron temperature is

\[ T_e - T = \frac{m_e}{4\rho_\gamma} \int_0^\infty d\omega \frac{\omega - 4T_e}{m_e} \omega f_\omega. \] (29)

The term on the rhs can be found by integrating Eq. (28) over the photon energy

\[ \frac{\partial}{\partial t} \int_0^\infty d\omega f_\omega = - \int_0^\infty d\omega \frac{\omega - 4T_e}{m_e} \omega f_\omega n_e \sigma_T + \frac{dQ}{dt}, \] (30)

where we ignore cosmic expansion again because the time scale of Thomson scattering is much shorter than the cosmological time. Also, the total injected energy rate by cosmic strings is

\[ \frac{dQ}{dt} = \int d\omega \omega \delta f_\omega. \] (31)
The first term on the rhs of Eq. (30) describes the energy loss rate from the photons due to Compton cooling. We express this term as $\partial E_\gamma / \partial t$. Note that $f_\omega$ is the spectral distribution of photons minus the blackbody distribution, and $E_\gamma$ is also the energy of photons that are in the spectral deviation from blackbody.

Then, from Eq. (29), we can rewrite the electron temperature in terms of $E_{\text{loss}}$ as

$$T_e - T = \frac{m_e}{4 \rho \gamma n_e \sigma T} \left[ \frac{dQ}{dt} - \frac{\partial E_\gamma}{\partial t} \right].$$

We will now argue that the rate of change of $E_\gamma$ is of order the Hubble expansion rate and can be ignored in our quasi-steady state treatment. Basically, the idea is that the energy of high frequency photons injected by strings is transferred very efficiently to electrons by Compton scattering. A photon with frequency $E_\gamma$ loses energy $E_\gamma (E_\gamma - 4T_e)/m_e$ per Compton scattering. Hence, the energy loss of a photon with high initial energy $E_{\gamma 0} \gg T_e$ within a Hubble time is approximated as

$$\delta E_\gamma \approx \frac{E^2_{\gamma 0} n_e \sigma T}{m_e H}.$$

Fig. 4 shows that $\delta E_\gamma \gg E_{\gamma 0}$ at $z = 10^6$ and $10^3$ for high energy photons and it is clear that the injected photon energy is fully transferred into electrons well within a Hubble time. So, the energy $E_\gamma$ only varies on a cosmological time scale, and its time derivative can be ignored in the quasi-steady state approximation. Therefore, we can drop the last term in Eq. (32) and obtain

$$T_e - T \approx \frac{m_e}{4 \rho \gamma n_e \sigma T} \frac{dQ}{dt},$$

which, from Eq. (26), leads to

$$y = \frac{1}{4} \int_{t(z_{\text{rec}})}^{t(z_{\text{freeze}})} dt \frac{1}{\rho_r} \frac{dQ}{dt},$$

FIG. 4: The energy loss from photons in a Hubble time due to Compton scattering as a function of injected photon energy. The curves are for $z = 10^6$ and $10^3$. 

![Graph showing energy loss from photons in a Hubble time due to Compton scattering as a function of injected photon energy. The curves are for $z = 10^6$ and $10^3$.](image-url)
where the upper bound of the integration $t(z_{\text{rec}})$ is the recombination epoch, which is introduced since the injected energy does not transfer into the background electrons once the optical depth becomes very low after recombination.

We now calculate the integral in Eq. (35) with Eq. (13) and Eq. (14), and plot the result in Fig. 5. The fit is

$$y = 9.53 \times 10^{-7} \left( \frac{I/G_{\mu_s}}{10^{11} \text{GeV}} \right).$$

The corresponding constraint from COBE yields

$$\frac{I}{G_{\mu_s}} < 1.57 \times 10^{12} \text{ GeV}, \quad \text{(COBE)}$$

and PIXIE will be able to constrain up to

$$\frac{I}{G_{\mu_s}} < 1.09 \times 10^9 \text{ GeV}, \quad \text{(PIXIE)}.$$  

These constraints are very similar to those obtained from $\mu$-distortion in Eqs. (22) and (23).

IV. CONCLUSIONS

We studied the effect of electromagnetic radiation from superconducting cosmic string loops on CMB spectral distortions, and obtained constraints on the parameter space of string tension $G_{\mu_s}$ and the current $I$. Earlier studies by Refs. [29, 30] to constrain superconducting cosmic string parameters from CMB spectral distortions assumed that the power going into electromagnetic radiation, $P_\gamma$, is a $\mu_s$ independent, small, constant fraction of the power going into gravitational radiation, $P_g$, and hence, the loop lifetime is determined by gravitational radiation. In Sec. [11] we explained that the electromagnetic power depends on the current in the string [see Eq. (1)], hence, it is not simply a fraction of $P_g$. Besides, since $P_\gamma$ depends on the current, $I$, at some value of the current given by Eq. (3), the
electromagnetic radiation becomes the dominant energy loss mechanism, and the lifetime of the loops is determined by $P_\gamma$.

We made some simplifying assumptions in this paper. First of all, we assumed that the cosmic network characteristics for the superconducting strings is the same as ordinary ones with no current, as always assumed in cosmic string simulations. We do not think that the effect of the current will be very significant in the accuracy of our order of magnitude estimates. Another simplifying assumption was that cosmic string cusps produce homogeneous CMB distortions. Therefore, we assumed that the beamed radiation from cusps are quickly isotropized since we focus on very early epochs $z < z_{rec} \sim 1100$, where the injected photons quickly thermalize [46]. On the other hand, if the radiation from cusps is not isotropized efficiently, the CMB distortions will depend on the direction of observation.

In Sec. III we showed that both $\mu$- and $y$-distortions give comparable constraints on the parameter space. COBE-FIRAS measurement of no spectral distortion of the CMB places upper bounds on the distortion parameters, $|\mu| < 9 \times 10^{-5}$ and $y < 1.5 \times 10^{-5}$ [17, 18]. On the other hand, the proposed future space mission PIXIE can constrain them up to, $|\mu| \sim 5 \times 10^{-8}$ and $y \sim 10^{-8}$ at the $5 \sigma$ level [19]. The corresponding constraints from COBE and PIXIE on string parameters for $\mu$ distortion are relatively given by

$$\frac{I}{G\mu_s} < 1.95 \times 10^{14} \text{ GeV}, \quad \text{(COBE)},$$

$$\frac{I}{G\mu_s} < 1.08 \times 10^{11} \text{ GeV}, \quad \text{(PIXIE)},$$

where the loop lifetime is determined by gravitational energy losses, $I < I_*$. In the opposite regime, $I > I_*$, we obtained

$$G\mu_s < 2.5 \times 10^{-12}, \quad \text{(COBE)},$$

$$G\mu_s < 7.9 \times 10^{-19}, \quad \text{(PIXIE)}.$$

These constraints are summarized in Fig. 3. We have also calculated the CMB $y$—distortion due to superconducting strings. These lead to constraints that are similar in magnitude to those from the $\mu$—distortion and are shown in Fig. 5.

If PIXIE does not detect suitable distortions, only light superconducting strings with modest currents (up to $\sim 10^8$ GeV), or somewhat heavier strings but with small currents ($\lesssim 10^4$ GeV) will be allowed. Of course, there is a possibility that CMB distortions will be detected, in which case, one needs to look at other distinguishing signatures from superconducting cosmic strings such as neutrino bursts [8] and radio transients [9, 10].

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[1] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 1994).
[2] E. Witten, Nucl. Phys. B249, 557 (1985).
[3] J. P. Ostriker, A. C. Thompson, and E. Witten, Phys. Lett. B180, 231 (1986).
[4] A. Vilenkin and T. Vachaspati, Phys. Rev. Lett. 58, 1041 (1987).
[5] A. Babul, B. Paczynski, and D. Spergel, Astrophys. J. L. 316, L49 (1987).
[6] B. Paczynski, Astrophys. J. 335, 525 (1988).
[7] V. Berezinsky, B. Hnatyk, and A. Vilenkin, Phys. Rev. D64, 043004 (2001), astro-ph/0102366.
[8] V. Berezinsky, K.D. Olum, E. Sabancilar and A. Vilenkin, Phys. Rev. D80, 023014 (2009) [arXiv: 0901.0527].
[9] T. Vachaspati, Phys. Rev. Lett. 101, 141301 (2008), 0802.0711.
[10] Y.-F. Cai, E. Sabancilar, and T. Vachaspati (2011), 1110.1631.
[11] R. Sunyaev and Y. Zeldovich, Ann.Rev.Astron.Astrophys. 18, 537 (1980).
[12] L. Danese and G. de Zotti, Nuovo Cimento Rivista Serie 7, 277 (1977).
[13] L. Danese and G. de Zotti, astron. astrophys. 107, 39 (1982).
[14] C. Burigana, L. Danese, and G. de Zotti, Astron. Astrophys. 246, 49 (1991).
[15] C. Burigana, L. Danese, and G. de Zotti, Astrophys. J. 379, 1 (1991).
[16] W. Hu and J. Silk, Phys. Rev. D48, 485 (1993).
[17] J. C. Mather et al., Astrophys. J. 420, 439 (1994).
[18] D. Fixsen, E. Cheng, J. Gales, J. C. Mather, R. Shafer, et al., Astrophys.J. 473, 576 (1996), astro-ph/9605054.
[19] A. Kogut, D. Fixsen, D. Chuss, J. Dotson, E. Dwek, et al., JCAP 1107, 025 (2011), 1105.2044.
[20] J. Silk, Astrophys. J. 151, 459 (1968).
[21] J. D. Barrow and P. Coles, MNRAS 248, 52 (1991).
[22] R. A. Daly, Astrophys. J. 371, 14 (1991).
[23] W. Hu, D. Scott, and J. Silk, Astrophys.J. 430, L5 (1994), astro-ph/9402045.
[24] J. Chluba and R. Sunyaev (2011), 1109.6552.
[25] R. Khatri, R. A. Sunyaev, and J. Chluba (2011), 1110.0475.
[26] W. Hu and J. Silk, Phys. Rev. Lett. 70, 2661 (1993).
[27] K. Jedamzik, V. Katalinic, and A. V. Olinto, Phys. Rev. Lett. 85, 700 (2000), astro-ph/9911100.
[28] H. Tashiro and N. Sugiyama, Phys.Rev. D78, 023004 (2008), 0801.3172.
[29] N.G. Sanchez, M. Signore, Phys.Lett. B219, 413 (1989).
[30] N.G. Sanchez, M. Signore, Phys.Lett. B241, 332 (1990).
[31] T. Vachaspati and A. Vilenkin, Phys. Rev. D 31, 3052 (1985).
[32] J. V. Rocha, “Scaling solution for small cosmic string loops”, Phys. Rev. Lett. 100, 071601 (2008) [arXiv: 0709.3284].
[33] J. Polchinski and J. V. Rocha, Phys. Rev. D 75, 123503 (2007).
[34] F. Dubath, J. Polchinski and J.V. Rocha, Phys. Rev. D 77, 123528 (2008).
[35] V. Vanchurin, Phys. Rev. D 83, 103525 (2011) [arXiv: 1103.1593].
[36] L. Lorenz, C. Ringeval and M. Sakellariadou, “Cosmic string loop distribution on all length scales and at any redshift”, JCAP 1010, 003 (2010) [arXiv:1006.0931].
[37] D.P. Bennett and F.R. Bouchet, Phys. Rev. D 41, 2408 (1990).
[38] B. Allen and E.P.S. Shellard, Phys. Rev. Lett. 64, 119 (1990).
[39] G.R. Vincent, M. Hindmarsh and M. Sakellariadou, Phys. Rev. D 56, 637 (1997).
[40] C.J.A.P. Martins and E.P.S. Shellard, Phys. Rev. D 73, 043515 (2006).
[41] C. Ringeval, M. Sakellariadou and F. Bouchet, JCAP 0702, 023 (2007).
[42] V. Vanchurin, K. D. Olum and A. Vilenkin, Phys. Rev. D 74, 063527 (2006).
[43] K. D. Olum and V. Vanchurin, Phys. Rev. D 75, 063521 (2007).
[44] J.J. Blanco-Pillado, K. D. Olum and B. Shlaer, Journal of Computational Physics 231 98 (2012).
[45] J.J. Blanco-Pillado, K. D. Olum and B. Shlaer, Phys. Rev. D 83, 083514 (2011) [arXiv: astro-ph.CO/1101.5173].
[46] A. A. Zdziarski and R. Svensson, Astrophys. J. 344, 551 (1989).