Kaon Condensates, Nuclear Symmetry Energy and Cooling of Neutron Stars

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Abstract

The cooling of neutron stars by URCA processes in the kaon-condensed neutron star matter for various forms of nuclear symmetry energy is investigated. The kaon-nucleon interactions are described by a chiral lagrangian. Nuclear matter energy is parametrized in terms of the isoscalar contribution and the nuclear symmetry energy in the isovector sector. High density behaviour of nuclear symmetry energy plays an essential role in determining the composition of the kaon-condensed neutron star matter which in turn affects the cooling properties. We find that the symmetry energy which decreases at higher densities makes the kaon-condensed neutron star matter fully protonized. This effect inhibits strongly direct URCA processes resulting in slower cooling of neutron stars as only kaon-induced URCA cycles are present. In contrast, for increasing symmetry energy direct URCA processes are allowed in the almost whole density range where the kaon condensation exists.

Key words: dense matter, nuclear symmetry energy, kaon condensation, neutron star cooling
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1 Introduction

The possibility of the kaon condensation in dense nuclear matter, proposed by Kaplan and Nelson [1], is recently a subject of intensive theoretical and experimental research [2,3]. Such a kaon-condensed phase of dense matter is...
of direct astrophysical relevance as it would form in neutron stars and could affect their properties. In this paper we study how sensitive is the cooling of neutron stars to the presence of the kaon condensates. A particularly important problem in this regard, which we address here, is to assess the influence of the uncertainty of the high density behaviour of nuclear symmetry energy on the charged kaon condensation itself. Other strange hadrons, like neutral kaons and hyperons could also appear in neutron star matter. Their inclusion here, however, would blur the role of the nuclear symmetry energy as it would introduce additional uncertainty due to poorly known nucleon-hyperon interactions. The analysis reported here will be extended to account for hyperons in a forthcoming paper. Model calculations show that hyperons are likely to influence the results concerning kaon condensation [4].

Among theoretical approaches we may distinguish those based on the chiral theory [1] and the ones using the meson exchange picture. The former ones have their roots in the spontaneously broken $SU(3) \times SU(3)$ symmetry. It gives a solid base to study kaon-nucleon interactions where kaons are treated as pseudo-Goldstone bosons. In the meson exchange picture, the kaon couplings to nucleons are realized indirectly through the exchange of vector and scalar mesons [5]. The predictive power of chiral theory is lost in such approaches. In order to study the kaon condensation in the neutron star matter a realistic model of nucleon-nucleon interactions should be considered.

The nucleon-nucleon part of the interaction Hamiltonian is usually taken from other theories of nuclear matter, such as e.g. many-body models with realistic potentials or relativistic mean field (RMF) theory. Realistic potential approach is firmly based on experiment. Two-nucleon potentials are fitted to the scattering data. Three-body potentials must be added to get agreement with the binding energy of light nuclei ($^3$H, $^4$He) and the saturation properties of nuclear matter. The energy of nuclear matter is usually derived in a variational approach [6] or by using other techniques as e.g. some version of the Brueckner theory.

The RMF approach starts from postulating the Lagrangian for nucleons and mesons which takes into account relevant internal symmetries (as e.g. isospin). The meson-nucleon interactions are assumed to be of a Yukawa type with coupling constants treated as free parameters which are fitted to the saturation point properties obtained from the mass formula and from breathing modes of giant resonances [7].

Unfortunately, up to now there is no consistent description of the ground state properties of nuclear matter in terms of the chiral theory with its free-space parameters. Recently some novel approaches were proposed. One of them starts from the chiral Lagrangian [8] but still treats its crucial parameters in the spirit of RMF model – fitting them to the saturation point properties. A
different idea based on the power counting scheme was presented in [9], but its extrapolation to higher densities seems not to be well justified.

In the next section we discuss the high density behaviour of the nuclear symmetry energy. One should stress that the symmetry energy governs the chemical composition of nuclear matter in the beta equilibrium and hence it determines the particle species present in neutron stars. In Sect.3 the chiral model kaon-nucleon interactions is derived. In Sect.4 the kaon condensate properties and the role of nuclear symmetry energy in a kaon-condensed matter are studied. In particular we assess how uncertainty of the behaviour of the symmetry energy at high densities influences conclusions regarding the formation and properties of the kaon-condensed neutron star matter. Implications for the cooling of neutron stars are investigated in Sect.5.

2 Nuclear symmetry energy

A common feature of the RMF models is that the nuclear symmetry energy, whose value at the saturation density is well known to be about 30 MeV, strongly increases with density of the nuclear matter. It is not the case for realistic potential models of nuclear matter in the variational approach [6], where the symmetry energy saturates and then bends over at higher densities. The high-density behaviour of symmetry energy is the worst known property of dense matter [10,12], with different nuclear models giving contradictory predictions, as in the case of the RMF theory and realistic potential models which provide incompatible predictions of the nuclear symmetry energy at high densities. Such divergent predictions are the source of a major uncertainty of high density properties of neutron star matter [11], including the formation of kaon condensates [16].

To assess the influence of the symmetry energy we explore various forms of its high density behaviour as provided by different models of dense nuclear matter. We use three types of the high density symmetry energy obtained from the realistic potential theory [6] and also the symmetry energy derived from the RMF model which are displayed in Fig.1. Some preliminary results are given in [16], where it was shown that low values of the symmetry energy may exclude the condensation. However, if the condensate is formed, the proton fraction goes very high, even to complete protonization of nuclear matter, $x = 1$. These results encourage us to explore the implications for cooling of neutron stars.

The nuclear symmetry energy is defined as

$$E_s(n) = \frac{1}{8} \frac{\partial^2 E}{\partial x^2}(n, \frac{1}{2}),$$

where $x$ is the proton fraction.
where \( E(n, x) \) is the energy per baryon as a function of the baryon number density, \( n \), and the proton fraction, \( x \). The nuclear symmetry energy plays an essential role in the neutron star matter when the beta equilibrium among all its components is achieved. From thermodynamic identities it follows that \( \partial E / \partial x = \mu_n - \mu_p \), where \( \mu_n, \mu_p \) are neutron and proton chemical potentials. The isospin symmetry allows us to expand \( E(n, x) \) in even powers of \( (x - \frac{1}{2}) \). The expansion up to the second order is enough, i.e. the term \( (x - \frac{1}{2})^4 \) seems to be negligible [13]. We may thus write with a sufficient accuracy

\[
4(1 - 2x)E_s = \mu_n - \mu_p. \tag{1}
\]

The beta equilibrium yields a sequence of equalities between chemical potentials for particles present in the system and hence the symmetry energy governs the chemical composition of the neutron star matter. For example, in the simplest case of \( n, p, e \) matter, we have \( 4(1 - 2x)E_s = \mu_n - \mu_p = \mu_e \). Taking into account the neutrality condition, \( k_e = k_p \), and neglecting the electron mass we obtain the proton fraction

\[
x = \frac{1}{2}(1 - \frac{k_p}{4E_s}). \tag{2}
\]

At the saturation density, where the value of symmetry energy is known to be \( E_s(n_0) = a_4 = 30 \text{ MeV} \) [14], the proton abundance is about 4%. As the proton Fermi momentum is slowly varying with density, \( k_p \sim n_p^{1/3} \), we conclude that if \( E_s \) increases faster than \( n^{1/3} \) then the neutron star matter becomes more and more symmetric at high densities: \( x \to \frac{1}{2} \), (muons, produced when \( k_e > m_\mu \), make this effect even stronger because protons have to neutralize additional negative charge carriers). This is the case of RMF models. In its basic version where the only isovector interaction is due to Yukawa coupling to the \( \rho \) meson, the symmetry energy takes the form [15]:

\[
E_s(n) = \frac{k_F^2}{6\sqrt{k_F^2 + m_e^2}} + \frac{g_\rho^2}{8m_\rho^2}n, \quad k_F = (\frac{3\pi^2}{2}n)^{1/3} \tag{3}
\]

The first term is purely kinematic, coming from the energy difference of proton and neutron Fermi seas. The second term comes just from interaction and is determined by the coupling constant. As we see, \( E_s \) grows slightly faster than linear with density and this is its typical behaviour in RMF theories\(^1\). In an opposite case, when the symmetry energy decreases with density, one expects the neutron star matter to be more neutron-rich, \( x \to 0 \), at high densities. This simple picture need not be always true if kaons appear. Although kaons,

\(^1\) In our earlier work [15] we tested if the inclusion of isovector-scalar component due to the \( \delta \) meson, could change the rapid growth of the symmetry energy.
similarly as electrons and muons, carry negative charge, their presence in the neutron star matter may have unexpected consequences.

3 Model for $K-N$ interactions

In our investigation we use the effective chiral theory proposed by Kaplan and Nelson [1] and extensively explored in subsequent works. The Lagrangian for kaons represents a nonlinear effective theory which is not renormalizable, but it makes a good starting point for chiral perturbation theory [17] (we use the same symbols as in [18], for details see Appendix A.1).

\[
\mathcal{L}_\chi = \frac{f^2}{4} \text{Tr} \partial_\mu U \partial^\mu U^\dagger + \text{Tr} B(i \gamma^\mu D_\mu - m_B)B + F \text{Tr} \bar{B} \gamma^\mu \gamma_5 [A_\mu, B] + D \text{Tr} \bar{B} \gamma^\mu \gamma_5 \{A_\mu, B\} + c \text{Tr} \mathcal{M}(U + U^\dagger) + a_1 \text{Tr} B(\xi \mathcal{M} \xi + \xi^\dagger \mathcal{M} \xi^\dagger)B + a_2 \text{Tr} \bar{B} B(\xi \mathcal{M} \xi + \xi^\dagger \mathcal{M} \xi^\dagger) + a_3 \text{Tr} \bar{B} B \text{Tr}(\xi \mathcal{M} \xi + \xi^\dagger \mathcal{M} \xi^\dagger).
\]

The Lagrangian consists of one part which is $SU(3) \times SU(3)$ symmetric (the first four terms in (4)), and the other part which breaks the chiral symmetry, where the breaking strength is determined by the parameters $a_1, a_2, a_3,$ and $c$, as well as by the quark mass matrix $\mathcal{M}$ which is

\[
\mathcal{M} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & m_s
\end{pmatrix},
\]

i.e. the light quarks are assumed to be massless. Parameters in the symmetric part are $f = 93$ MeV, $D = 0.81, F = 0.44$. In the symmetry-breaking sector $c$ represents the vacuum expectation value of the scalar quark density $\bar{q}q$ [19]. The Gell-Mann-Oakes-Renner relation states that $m_K^2 = 2cm_s/f^2$. Mass splittings for the strange baryons determine the values of $a_1m_s, a_2m_s$ and we use after [20] $a_1m_s = -67$ MeV and $a_2m_s = 134$ MeV. The last parameter $a_3m_s$ is subject to a large uncertainty. In principle it should be extracted from the kaon-nucleon scattering or kaonic atoms data [2] through the kaon-nucleon sigma term $\Sigma_{KN} = \frac{1}{2}m_s \langle N | \bar{u}u + \bar{s}s | N \rangle = -\left(\frac{1}{2}a_1 + a_2 + 2a_3\right)m_s$, but up to now its value has not been determined. To assess its influence on further results we adopt its values in the range $-310$ MeV $< a_3m_s < -134$ MeV which corresponds to a reasonable strangeness content of the proton in the range: $0.2 > \langle \bar{s}s \rangle_p/\langle \bar{u}u + \bar{d}d + \bar{s}s \rangle_p > 0$ [21]. The value of $a_3m_s$ appears to be important for the kaon condensation because the kaon-nucleon sigma term provides
the attractive interaction between kaons and nucleons, and makes the kaon
effective mass smaller in the nuclear medium. Negatively charged kaons con-
dense in the nucleon matter if their effective mass becomes smaller than the
electron chemical potential $\mu_e$. Assuming that the only relevant kaon-nucleon
interaction is in the s-wave, the expectation value of the kaon field $\langle K^- \rangle$ is
spatially uniform. According to the Baym theorem [22], its time dependence is

$$\langle K^- \rangle = \frac{f \theta}{\sqrt{2}} \exp(-\mu_K t),$$

where $\theta$ is an unknown amplitude of the condensate.

Formally, pure nucleon-nucleon interactions should also emerge from the chiral
theory as it is supposed to represent the low energy limit of QCD. However,
it was not derived up to now, so we use the chiral Lagrangian only to extract
the kaon-nucleon part of the interactions and we adopt the nucleon-nucleon
part from the realistic potential models of nuclear matter. This means that
the energy density of our problem is a sum of three contributions: the kaon-
nucleon interaction energy density (derived from $\mathcal{L}_\chi$ restricted only to kaon
and nucleon fields), the nucleon-nucleon energy density taken from [6] and the
energy density for leptons ($e, \mu$):

$$\varepsilon = \varepsilon_{KN} + \varepsilon_{NN} + \varepsilon_{lep}.$$  \hspace{1cm} (7)

The nucleon-nucleon contribution is

$$\varepsilon_{NN} = \frac{3}{5} E_F^{(0)} n_0 \left( \frac{n}{n_0} \right)^{5/3} + V(n) + mn + n(1-2x)^2 E_s(n),$$  \hspace{1cm} (8)

where

$$V(n) = nV_0(n),$$  \hspace{1cm} (9)

$$E_s(n) = \frac{1}{3} E_F^{(0)} \left( \frac{n}{n_0} \right)^{2/3} + V_2(n).$$  \hspace{1cm} (10)

The functions $V_0$ and $V_2$ depend on the parameterization of the nucleon-
nucleon potential used to fit the experimental data. We made use of the functions
given in [6] corresponding to UV14+UVII, AV14+UVII, UV14+TNI inter-
actions, where UV14 and AV14 are 14-operators two-body potentials from
Urbana and Argonne. The three-body potentials UVIII or TNI are important
to reproduce the saturation point properties by providing a proper contribu-
tion in the isoscalar sector. The UV14+TNI potentials reproduce best the
saturation density $n_0 = 0.16 \text{ fm}^3$ and the binding energy $w_0 = 16 \text{ MeV}$. One
Fig. 1. The nuclear symmetry energy for different realistic potential models: UV14+UVII, AV14+UVII, and UV14+TNI. The line labelled $E_1$ corresponds to the symmetry energy for the RMF model [18].

can notice that the three-body forces give different energies also in the isovector sector, i.e. the symmetry energy, as shown in Fig.1. For the same two-body potential, UV14, the three-body potentials UVII and TNI give different energies at higher densities, although both of them produce the same symmetry energy at saturation point $E_s(n_0) \approx 30$ MeV. In this figure, also the symmetry energy, typical for RMF theory, is shown. It is given by the formula [18]

$$E_s(n) = \frac{3}{5}(2^{2/3} - 1)E_F^{(0)}(u^{2/3} - F(u)) + E_s(n_0)F(u), \quad (11)$$

where $u = n/n_0$ and the interaction contribution $F(u)$ depends linearly on density, $F(u) = u$. The nuclear symmetry energies from realistic potential models differs from one another but only quantitatively, whereas the RMF prediction $E_1$ is qualitatively different - it never saturates at higher densities.

The energy density for leptons is determined in a standard way through their Fermi momenta

$$\varepsilon_{lep} = \frac{k_F^4}{4\pi^2} + m_\mu^4g(\sqrt{\eta(k_F^2 - m_\mu^2)/m_\mu}), \quad (12)$$

where the function $g(t)$ comes from integration over the Fermi sea of massive fermions:

$$g(t) = \frac{1}{8\pi^2}((2t^3 + t)\sqrt{1 + t^2} - \arcsinh t), \quad (13)$$
Fig. 2. Energy per particle for kaon-condensed neutron star matter with UV14+TNI (upper panel) and AV14+UVII (lower panel) interactions. The labels a, b and c correspond to the values of \( a_3 m_s \) from Table 1.

\[
\eta(x) = \begin{cases} 
  x &: x \geq 0 \\
  0 &: x \leq 0 
\end{cases}
\]

To derive the kaon-nucleon contribution, \( \varepsilon_{KN} \), in (7) we use the standard prescription to get the energy density from a given Lagrangian (Appendix A.1). Employing the Baym theorem we obtain finally

\[
\varepsilon_{KN} = f^2 \frac{\mu^2}{2} \sin^2 \theta + 2m_K^2 f^2 \sin^2 \frac{\theta}{2} + n(2a_1 x + 2a_2 + 4a_3) m_s \sin^2 \frac{\theta}{2}. \quad (14)
\]

4 Thermodynamics of the system

At zero temperature the energy density should be canonically a function of the number densities \( \varepsilon(n_p, n_n, n_K) \) (leptons may be disregarded without loss of generality). Instead of number densities we have \( n, x, \mu_K \) in the expressions (14),(8) entering \( \varepsilon \), which describe the thermodynamic state completely. The fourth, superfluous variable, \( \theta \), is a free microscopic parameter with respect to
which the energy is minimized when all densities are fixed,

\[
\left( \frac{\partial \varepsilon}{\partial \theta} \right)_n = 0.
\]  

(15)

This condition leads to the equation for \( \theta \) (Appendix A.2) which now may be expressed in terms of the equilibrium state determined by \( n, x, \mu_K \),

\[
\cos \theta = \frac{1}{f^2 \mu_k^2} \left( m_k^2 f^2 + \frac{1}{2} n (2a_1 x + 2a_2 + 4a_3) m_k - \frac{1}{2} \mu_k n (1 + x) \right).
\]  

(16)

The function \( \theta(n, x, \mu_K) \) plays the role of the order parameter in the analysis of the phase transition. The kaon condensate develops provided the above equation possesses a solution. If there is no solution, i.e. the right-hand side of eq.(16) does not fall in the range (-1,1), this means dense matter is allowed to contain only the normal nucleon phase. In the neutron star matter the free thermodynamic parameters \( n, x, \mu_K, \mu_e, \mu_\mu \) are subject to further constraints which come from the beta equilibrium and charge neutrality.

On the timescale relevant to cooling of neutron stars, the nucleon matter with a kaon condensate approaches the beta equilibrium through the following reactions:

\[
\begin{align*}
n &\leftrightarrow p + l + \nu_l \\
n &\leftrightarrow p + K^- \\
l &\leftrightarrow K^- + \nu_l , \quad l = e, \mu
\end{align*}
\]  

(17)

As neutrinos are supposed to leave freely, from the above reactions one derives corresponding relations among chemical potentials for all particles present in the system:

\[
\mu_K = \mu_e = \mu_\mu = \mu_n - \mu_p
\]  

(18)

The above sequence of equalities shows that there are two independent chemical potentials, let’s say \( \mu_e \) and \( \mu_n \). These chemical potentials represent two quantities, the electric charge and the baryon number, which are conserved in the beta-reactions (17). This means that the charge and the baryon number uniquely describe the thermodynamic state of our system. In further calculations we put

\[
\mu \equiv \mu_e ,
\]

and \( \mu \) will represent the chemical potential of all negatively charged particles. Chemical potentials for leptons are determined by their Fermi momenta, \( \mu_e = \)
$k_e$, and, $\mu_\mu = \sqrt{k_\mu^2 + m_\mu^2}$, and for nucleons they must be derived through a proper differentiation, $\mu_{p,n} = (\partial \varepsilon / \partial n_{p,n})_{n_i}$, i.e. with all remaining densities fixed:

$$\mu_{\bar{p}} = E^{(0)}_F \left( \frac{n}{n_0} \right)^{2/3} + m + V'(n) + n(1-2x)^2 E_s'(n) + (1-4x^2) E_s(n)$$

$$+ (2a_2 m_s + 4a_3 m_s - \mu_K) \sin^2 \frac{\theta}{2}, \quad (19)$$

$$\mu_p = E^{(0)}_F \left( \frac{n}{n_0} \right)^{2/3} + m + V'(n) + n(1-2x)^2 E_s'(n) - (1-2x)(3-2x) E_s(n)$$

$$+ (2a_1 m_s + 2a_2 m_s + 4a_3 m_s - 2\mu_K) \sin^2 \frac{\theta}{2} \sin \frac{\theta}{2}, \quad (20)$$

With these expressions for $\mu_p, \mu_p$, the beta equilibrium condition (18) leads to the equation

$$\mu = 4(1-2x) E_s(n) / \cos^2 \frac{\theta}{2} - 2a_1 m_s \tan^2 \frac{\theta}{2} \sin \frac{\theta}{2} \quad (21)$$

While the beta equilibrium condition is easily implemented, the assumption as to the charge neutrality needs further qualification. We must make distinction between local and global electric neutrality of the system. For locally neutral matter in any point densities of charged particles satisfy the condition

$$n_p - n_K - n_e - n_\mu = 0.$$ 

The only unknown quantity in the above formula is the negative kaon density $n_K$, which is obtained from the conserved electric current corresponding to the $U(1)$ symmetry of the kaon-nucleon Lagrangian

$$n_K = i(p_{K^+} - p_{K^-}),$$

where $p_{K^+}, p_{K^-}$ are the canonical momenta. After somewhat laborious calculations they may be derived directly from the Lagrangian (Appendix A.1). Making use of the Baym theorem we get the final expression for the charge density carried by kaons,

$$n_K = f^2 \mu \sin^2 \theta + n(1+x) \sin^2 \frac{\theta}{2}, \quad (22)$$

and the local charge neutrality condition becomes:

$$0 = f^2 \mu \sin^2 \theta + n(1+x) \sin^2 \frac{\theta}{2} + \frac{\mu^3}{3\pi^2} + \theta(\mu)\frac{(\mu^2 - m_\mu^2)^{3/2}}{3\pi^2} - nx. \quad (23)$$
When a phase transition takes place, the charge separation in two different phases preserving merely the neutrality of the matter as a whole, but not on a small scale, can occur, as it was shown in [23]. Then the equation (23) does not longer hold. The problem of mixed phase formation will be discussed elsewhere.

The equations (23, 21, 16) allow us to find the total energy density (7) as a function of the baryon density only, $\varepsilon(n)$. The onset of condensation occurs at a critical density $n_c$ where the function $\theta$ starts to deviate from zero and the energy density of the phase with the condensate becomes smaller than that of normal matter, Fig. 2. As it was shown in [16], the critical density, $n_c$, depends sensitively on the shape of the symmetry energy, $E_s(n)$, with the eventual exclusion of condensation if the strangeness is too small. The values of $n_c$ for different models of the symmetry energy, $E_s$, and some representative values of strangeness are presented in Table 1. Likewise, the other quantities,

| $a_3 m_s$ [MeV] | $E_1$ | UV14+UVII | AV14+UVII | UV14+TNI |
|-----------------|------|-----------|-----------|---------|
| a               | -310 | 2.4       | 2.6       | 2.7     | 2.8     |
| b               | -230 | 3.0       | 3.4       | 3.7     | 4.1     |
| c               | -134 | 4.2       | 5.0       | 5.9     | –       |

important for the structure of neutron stars, change radically with $E_s$. Paradoxically, low values of the symmetry energy (which usually reduce $x$) in the presence of the condensation lift the proton abundance even up to a complete protonization, $x = 1$. In the most extremal case of the UV14+TNI interactions with the minimal strangeness $a_3 m_s = -134$ MeV the kaon condensate does not appear.
tions the protonization sets in just after formation of the condensate. This is because kaons appear at a small value of the kaon chemical potential $\mu$, that means they may be easily produced as the chemical potential measures the cost of energy to add one particle to the system. The kaon chemical potential in the condensate is shown in Fig.3 for some models we study here.

With the presence of kaon condensate the overall chemical composition of the neutron star matter becomes somewhat exotic. This is shown in Fig.4 and 5. Let us consider first the case of AV14+UVII (UV14+UVII gives similar results but less pronounced). Kaons, as bosons, are produced in the zero momentum state unlike leptons which must be put on the top of the Fermi sea. The negative charge carried by kaons is so abundant that the kaons supersede electrons and muons in dense matter - the curves for $x$ and $x_K$ overlap (case (c) in Fig.4). When strangeness is high (case (a) in Fig.4) the kaons are produced at lower density, $x_K$ approaches the value $x_K = 1$ and even may exceed unity. If $x_K$ is so high, positive leptons ($x_{e+\mu} < 0$) have to appear to ensure charge neutrality. With further increase of density the situation changes again. As the proton fraction $x$ cannot be greater than 1, the equation (16) is no longer valid. Strictly speaking, the energy has no minimum for $x$ inside the allowed range, $x \in (0,1)$. So, it takes its minimal value at the edge of the range of $x$ which is just at $x = 1$. Then $\mu$ stops to decrease, positive leptons disappear and negative leptons may be restored. This fact is better seen in Fig.5. In the UV14+TNI case positive leptons appear as well but with much smaller amount because the effect of $x = 1$ constraint is very strong. And then, at higher density, the composition of matter does not depend on the strangeness value. The curves for $x_K$ and $x_{e+\mu}$ overlap for high $n$ (see Fig.5).

The above discussion clearly indicates the situation becomes complex. There is a common conviction that high proton fraction $x \gtrsim 0.15$ ensures very fast cooling of neutron stars by direct URCA processes [24]. This statement is true only for usual $n, p, e, \mu$ matter. Direct URCA cycles take place if Fermi momenta for protons, neutrons and leptons form a closed triangle. All the Fermi momenta may be expressed in terms of $x$. So, the triangle condition leads to the existence of the threshold value, $x_{URCA}$, which lies between 0.11 and 0.15, depending on the presence of muons. For nuclear matter with kaons we have an additional degree of freedom - $\theta$, which determines the kaon abundance altogether with $\mu$ (22). This is the reason why we cannot formulate any simple answer to the question if direct URCA processes are allowed above some value of $x$ or not. In our case, the triangle condition must be examined explicitly in the whole range of densities.
Fig. 4. The composition of the kaon-condensed neutron star matter for AV14+UVII potentials. Two families of curves indicated as (a) and (c) correspond to maximal and zero strangeness in Table 1, respectively. Bold lines represent the proton fraction $x$, thin lines $x_K$ and $x_{e+\mu}$. For comparison, dashed lines correspond the proton fraction $x$ calculated with the same function $V_0$ and the $E_1$ model of the nuclear symmetry symmetry energy.

Fig. 5. The composition of the kaon-condensed neutron star matter for UV14+TNI potentials. Line labels as in Fig.4. For zero strangeness (c) there is no condensation, so only (a) and (b) families are presented. At high density all particle fractions $x_i$ are independent of the strangeness and corresponding curves overlap for different families.

5 Cooling of matter with kaon condensate

When a kaon condensate is formed, besides the usual direct beta reactions there are also possible reactions which are specific to the kaon-condensed neutron star matter. The direct URCA cycles take place among quasi-particles $\tilde{p}, \bar{n}$.
\[ \tilde{n} \rightarrow \tilde{p} + l + \bar{\nu}_l \]

\[ \downarrow \]

\[ \tilde{p} + l \rightarrow \tilde{n} + \nu_l, \]

(24)

and we call them dURCA. The states \(|\tilde{p}\rangle, |\tilde{n}\rangle\) are linear combinations of pure proton and neutron state which depend on the condensate amplitude \(\theta\) [25]. As the quasi-particles have no definite charge one may write another admitted type of reactions

\[ \tilde{n} \rightarrow \tilde{n} + l + \bar{\nu}_l, \]

\[ \tilde{p} \rightarrow \tilde{p} + l + \bar{\nu}_l. \]

(25)

(26)

Microscopically, it means that the necessary charge is taken from the condensate, e.g. \(n + K \rightarrow n + l + \nu_l\) and thus they are called kaon induced URCA. For all particles taking part in the above reactions, the momentum conservation gives the following relations among their Fermi momenta:

\[ \text{kURCA : } 2k_{\tilde{n}} > k_l \text{ or } 2k_{\tilde{p}} > k_l \]

\[ \text{dURCA : } |k_{\tilde{p}} - k_{\tilde{n}}| < k_l < k_{\tilde{p}} + k_{\tilde{n}}. \]

(27)

(28)

The neutrino momentum, which is \(k_{\nu} \sim kT < 1\text{MeV}\), may be neglected as it is much smaller than momenta of the other particles. Likewise, kaons do not enter into the relation (28) as the condensation takes place in the s-wave and the kaon momenta do not contribute. Let’s focus on the direct URCA cycle. The three terms of the double inequality are presented for various models in Fig.6. When negative leptons are present, the right-hand side inequality in (28) is always fulfilled, because the local neutrality means that \(k_e \leq k_{\tilde{p}}\). It does not have to be true when positive leptons appear. In principle, although very unlikely, it is possible that \(k_e > k_{\tilde{n}}\) and for clarity we put the sum \(k_{\tilde{p}} + k_{\tilde{n}}\) on the plots as well. The main conclusion from the calculations shown in Fig.6 is that the exceeding by the proton fraction the threshold value \(x_{\text{URCA}}\) does not ensure the presence of the dURCA cycle. Too high values of \(x\) block this cycle due to small fraction of neutrons. Also, a non-typical behaviour of the chemical potential \(\mu\), which crosses zero at some density (Fig.3), suppresses dURCA in a narrow zone due to small lepton abundance. From the plots in Fig.6 the tendency of suppressing the dURCA process with smaller values of the symmetry energy can be inferred. However, the importance of this effect is not so great, because always one of the kaon-induced processes (with a proton or a neutron) operates as may be easily concluded from Fig.6. The sum \(k_{\tilde{p}} + k_{\tilde{n}}\) is always greater than \(k_e\) so, either \(2k_{\tilde{p}} > k_e\) or \(2k_{\tilde{n}} > k_e\) or both of them are fulfilled. In the thermal evolution of neutron stars we distinguish two cooling scenarios: fast – driven by direct URCA and slow – driven by modified URCA processes. Modified URCA always operate, as they are not limited by any kinematical constraints, however their emissivities are much
Fig. 6. The dURCA triangle condition for various nuclear models and different values of strangeness. The solid lines correspond to $k_\bar{p} + k_{\bar{n}}$, dashed lines to $k_e$ and dotted lines to $|k_\bar{p} - k_{\bar{n}}|$. The shaded areas represent regions where dURCA is allowed.
Table 2
Emissivities of the three fermion processes in the kaon-condensed neutron star matter.

| cycle | reaction | $I_\theta/I_{URCA}$ |
|-------|----------|---------------------|
| dURCA | $\bar{n} \rightarrow \bar{p} + l + \bar{\nu}_l$ | $\cos^2 (\theta/2)$ |
| kURCA | $\bar{n} \rightarrow \bar{n} + l + \bar{\nu}_l$ | $\frac{1}{4} \sin^2 \theta \tan^2 \theta_c$ |
|       | $\bar{p} \rightarrow \bar{p} + l + \bar{\nu}_l$ | $\sin^2 \theta \tan^2 \theta_c$ |

Fig. 7. The total neutrino emissivity in URCA processes for various nuclear models.

smaller, about 5 to 6 orders of magnitude less than the direct ones. So they provide much slower cooling. The kaon-induced URCA processes (like direct URCA) belong to the class of three-fermion processes but their emissivity is smaller because they proceed through strangeness-changing reactions. The rate of such reactions scales as $\sin \theta_c$, whereas direct URCA scales as $\cos \theta_c$, where $\theta_c$ is the Cabibbo angle. Both kinds of processes depend on the kaon abundance which is described by the condensate amplitude $\theta$ [26]. In Table 2 the ratio of emissivities of kaon-condensed neutron star matter to emissivity by direct URCA of normal matter with no condensate is presented. The emissivity finally depends on the baryon number density through $I_{URCA}$ and the condensate amplitude $\theta$. The total emissivity is a sum of all the contributions from direct and kaon-induced URCA cycles with electrons as well as muons which are present if $\mu_e > m_\mu$. In Fig.7 the total emissivity is shown.
6 Summary and discussion

We have shown that the formation and properties of the kaon-condensed phase of neutron star matter are quite sensitive to the high density behaviour of the nuclear symmetry energy, $E_s(n)$, which still remains the most poorly known property of dense matter. In particular, for $E_s$ decreasing to negative values the formation of the kaon condensate can be inhibited for the lowest absolute value of $a_3m_s$. This is in stark contrast to the results for monotonically increasing $E_s(n)$, when the condensation occurs for any value of $a_3m_s$. The properties of the kaon-condensed neutron star matter and the abundance of particle species are sensitive to the high density form of the nuclear symmetry energy, as shown in Figs.4-5. A surprising finding is that the direct URCA process is often not allowed in the kaon-condensed matter for decreasing $E_s(n)$.

The above results allow us to assess how great is the influence of nuclear force models on the total neutrino emissivity of kaon-condensed neutron star matter. In case of increasing symmetry energy as in the RMF model ($E_1$) dURCA operates almost always when kaon condensate is formed, except of a narrow zone around density where the chemical potential $\mu$ is zero. For realistic potential models dURCA is systematically suppressed to more narrow density range with decreasing symmetry energy. For an extreme case of the UV14+TNI interactions, the allowed zone shrinks eventually to a very narrow vicinity of some density close to the threshold value for condensation $n_c$. The kaon-induced URCA branch is always present but at the level of about 10 times smaller, as shown in Fig.7.

Another interesting question concerns the role of $\bar{K}^0$. As was shown in [27] neutral kaons are easily produced just after the onset of $K^-$ condensation. Then the kaon-condensed matter becomes even more effectively isospin-symmetrized than in the absence of $\bar{K}^0$. We would like to stress that this effect is sensitive to the form of the nuclear symmetry energy. The effect occurring for increasing $E_s(n)$ is not observed in the case of decreasing $E_s(n)$. High values of $x$ which we obtain in our model make the $\bar{K}^0$ condensation more difficult. The dispersion relations for anti-kaons are:

$$\omega_{K^-} = -\frac{n(1 + x)}{4f^2} + \left(\frac{n(1 + x)}{4f^2}\right)^2 + m_K^2 + (2a_1x + 2a_2 + 4a_3)m_sn\frac{n}{2f^2} \right)^{1/2} \tag{29}$$

$$\omega_{\bar{K}^0} = -\frac{n(2 - x)}{4f^2} + \left(\frac{n(2 - x)}{4f^2}\right)^2 + m_K^2 + (2a_1(1 - x) + 2a_2 + 4a_3)m_sn\frac{n}{2f^2} \right)^{1/2} \tag{30}$$

The first term in the expression (30) for $\omega_{\bar{K}^0}$, which is the leading term, includes positive contribution from $x$, so high values of $x$ make the slope of $\omega_{\bar{K}^0}$,
as a function of \( n \), more flat after the charged kaons appear. Fig. 8 presents this effect clearly. We leave more detailed discussion of the neutral kaon condensate for future work.

![Graph showing proton fraction and anti-kaons energy in dense matter](image)

Fig. 8. The proton fraction \( x \) and anti-kaons energy in dense matter. The leveling off of \( \omega_K \) for \( \bar{K}^0 \) corresponds to the onset of \( K^- \) condensation which drives the strong increase of the proton fraction shown in the upper panel.

Note added: After completion of this work dr. T. Muto brought to our attention his research [28] concerning kaon condensation and cooling of neutron stars.
A Calculation of thermodynamic quantities

A.1 Basic formulae

A matrix $U \in SU(3)$ is defined as $U = \exp(i\sqrt{2}M/f)$, where $M$ represents the pseudoscalar meson octet

\[
M = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\
K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta_8
\end{pmatrix}.
\]  

(A.1)

Matrix $\xi$ appearing in (4) is such that $\xi^2 = U$ and $B$ corresponds to the baryon octet

\[
B = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\
\Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\
\Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}} \Lambda
\end{pmatrix}.
\]  

(A.2)

When the only non-zero elements of the meson matrix $M$ are $K^+$ and $K^-$ fields, then $U$ has the form (below we use dimensionless meson fields $k^\pm = \sqrt{2}K^+/f$):

\[
U = \begin{pmatrix}
\cos \sqrt{k^-k^+} & 0 & \frac{i k^+ \sin(\sqrt{k^-k^+})}{\sqrt{k^-k^+}} \\
0 & 1 & 0 \\
\frac{i k^- \sin(\sqrt{k^-k^+})}{\sqrt{k^-k^+}} & 0 & \cos(\sqrt{k^-k^+})
\end{pmatrix}.
\]  

(A.3)

Hence the canonical momenta corresponding to the rescaled kaon fields $p_\pm = \frac{\partial \mathcal{L}}{\partial (\partial t k^\pm)}$ are

\[
p_+ = -i (\bar{n}n + 2\bar{p}p) \frac{\sin^2 \chi}{2\chi^2} k^- \\
+ \frac{f^2}{4} \left( \frac{k^+ \partial t k^- + k^- \partial t k^+}{k^+} + \frac{k^+ \partial t k^- - k^- \partial t k^+}{k^+} \frac{\sin^2 \chi}{\chi^2} \right)
\]  

(A.4)

\[
p_- = i (\bar{n}n + 2\bar{p}p) \frac{\sin^2 \chi}{2\chi^2} k^+
\]
where $\chi = \sqrt{k^+k^-}$. The energy density for the kaon-nucleon sector is given by the Hamiltonian $H_{KN} = p_+ \partial_t k^+ - p_- \partial_t k^- - \mathcal{L}_\chi$ and it reads

$$H_{KN} = \frac{f^2(k^+\partial_t k^- + k^-\partial_t k^+)^2}{8\chi^2} - \frac{f^2(k^+\partial_t k^- - k^-\partial_t k^+)^2 \sin^2 \chi}{8\chi^4} \quad (A.6)$$

$$\quad + \left(2f^2m_K^2 + (2a_2 + 4a_3)m_s \bar{n}n + (2a_1 + 2a_2 + 4a_3)m_s \bar{p}p\right) \sin^2 \frac{\chi}{2}$$

The charge density $n_K = i(p_+ k^+ - p_- k^-)$ becomes

$$n_K = (\bar{n}n + 2\bar{p}p) \sin^2 \frac{\chi}{2} + \frac{if^2}{2}(k^+\partial_\mu k^- - k^-\partial_\mu k^+) \frac{\sin^2 \chi}{\chi^2} \quad (A.7)$$

The Baym theorem for $k^\pm$ fields allows us to write them as $k^\pm = \theta \exp^{\pm i\mu_K t}$ and with this form we get the equation (14,22).

A.2 Minimum of energy

The derivative of $\varepsilon_{KN}$ with respect to $\theta$, keeping $n_i = n, x, n_K$ fixed, is

$$\left(\frac{\partial \varepsilon_{KN}}{\partial \theta}\right)_{n_i} = \frac{\partial \varepsilon_{KN}(n, x, \mu_K, \theta)}{\partial \theta} + \frac{\partial \varepsilon_{KN}(n, x, \mu_K, \theta)}{\partial \mu_K} \left(\frac{\partial \mu_K}{\partial \theta}\right)_{n_i} \quad (A.8)$$

It remains to find $\left(\frac{\partial \mu_K}{\partial \theta}\right)_{n_i}$. This may be done by taking derivative of both sides of (22)

$$0 = f^2 \sin^2 \theta \left(\frac{\partial \mu_K}{\partial \theta}\right)_{n_i} + 2f^2 \mu_K \sin \theta \cos \theta + \frac{1}{2}n(1 + x) \sin \theta$$

This approach is, of course, equivalent to minimization of a new thermodynamic potential $\tilde{\varepsilon} \equiv \varepsilon_{KN} - \mu n_K$ with respect to $\theta$ with $n, x, \mu$ fixed, as was done in [18].

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