Determination of Ground Heat Exchangers Temperature Field in Geothermal Heat Pumps

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Abstract. For the heating and cooling supply of buildings and constructions geothermal heat pumps using low-potential ground energy are applied by means of ground exchangers. The process of heat transfer in a system of ground exchangers is a phenomenon of complex heat transfer. The paper presents a mathematical modeling of heat exchange processes, the temperature fields are built which are necessary for the determination of the ground array that ensures an adequate supply of low potential energy excluding the freezing of soil around the pipes in the ground heat exchangers and guaranteeing a reliable operation of geothermal heat pumps.

1. Introduction
Using of renewable energy sources is an effective way to improve energy efficiency and environmental ecology. Geothermal heat pumps use ground low-potential energy for heating and cooling of buildings and constructions, the important element of which is the collection system of low-potential energy representing ground heat exchangers. Structure analysis of ground heat exchangers and the practical implementation of such systems, followed by monitoring during the operation showed that vertical ground heat exchangers have the priority benefit. The study of heat transfer and temperature field formation is necessary to determine the ground array that ensures an adequate supply of low potential energy for reliable operation of the geothermal heat pumps, eliminating the soil freezing around ground heat exchanger pipes.

2. The processes of heat transfer in the system of collecting low-grade ground energy
Modeling of the tasks associated with finding the temperature field, is based on the solution of the differential equation of heat conduction. [1]

\[ c \rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) + Q_0(x, y, z, \tau, T) \]  

where \( T = T(x, y, z, \tau) \) is the temperature, °C; \( c\rho \) - volumetric heat capacity, \( J/(K \cdot m^3) \); \( \lambda \) - thermal conductivity, \( W/(m \cdot K) \); \( t \) - time sec.; \( Q_0(x, y, z, \tau, T) \) - specific performance of internal heat sources, W/m³.

In connection with diversity and some informative uncertainty of the factors affecting the thermal conductivity process, modeling of such systems requires the introduction of certain simplifications.
For making a closed mathematical model it is offered to produce a geometric transformation, i.e. conditionally "straightening" U-tube of the ground heat exchanger (Figure 1).

![Figure 1. Detail of low-grade energy collection system of the ground: left - in the form of a multi-layer cylinder; right - a "straightened" U-tube of the ground heat exchanger.](image)

For convenience of the heat transfer modeling, it is taken a planar element consisting of soil layer, filler and pipe wall that is in contact with the working fluid. It will carry out a study of temperature changes along the x-axis, but in order to accommodate these changes along the y-axis it is necessary perform separate heat transfer calculation for the working fluid.

Finding the temperature field in an environment where there may be convection, radiation and moisture transfer is a difficult task, but given the low value of the ground array temperature in the natural state [2-4], these factors can be omitted, and it should enter additional simplification - the soil is considered as a solid, continuous, isotropic medium, wherein the heat transfer occurs only by conduction of heat transmission capacity [5, 6]. A similar allowance is made for the borehole filler. It should be considered the effects of convection, radiation, moisture transfer, if the medium temperature is higher than +50 ° C. [5,6]

The impact of solar radiation and outside air temperature on the formation of the ground temperature condition in its natural state applies only to 10 meters depth, below, where the "neutral zone" begins, it is influenced only the radiogenic heat and it is observed temperature constancy of the ground array [2,3], for example, for Vladivostok equal to +8 °C [4].

Taken into account made simplifications, finding of the temperature field is carried out by one-dimensional heat equation (2) for the selected item:

\[
\begin{align*}
\rho_1 \cdot c_1 \cdot \frac{\partial T_1}{\partial t} = & \frac{\partial}{\partial x} \left( \lambda_1 \cdot \frac{\partial T_1}{\partial x} \right), 0 < x < x_{(N_1)}^*, \\
\rho_2 \cdot c_2 \cdot \frac{\partial T_2}{\partial t} = & \frac{\partial}{\partial x} \left( \lambda_2 \cdot \frac{\partial T_2}{\partial x} \right), x_{(N_1)}^* < x < x_{(N_2)}^*, \\
\rho_3 \cdot c_3 \cdot \frac{\partial T_3}{\partial t} = & \frac{\partial}{\partial x} \left( \lambda_3 \cdot \frac{\partial T_3}{\partial x} \right), x_{(N_2)}^* < x < L_{(n)} \\
\end{align*}
\]
The initial and boundary conditions can be written as follows:

$$t = 0: T = T_0, 0 \leq x \leq L$$

$$x = 0: -\lambda_1 \cdot \frac{\partial T}{\partial x} = q_0, t > 0$$

$$x = L: \lambda \cdot \frac{\partial T}{\partial x} = \alpha_f \cdot (T_3 - T_f), t > 0, \alpha_f > 0$$

$$T_1(t, x_{1,N}) = T_2(t, x_{2,N})$$

$$T_2(t, x_{2,N}) = T_3(t, x_{3,N})$$

$$-\lambda_1 \cdot \frac{\partial T_1}{\partial x} = -\lambda_2 \cdot \frac{\partial T_2}{\partial x}$$

$$-\lambda_2 \cdot \frac{\partial T_2}{\partial x} = -\lambda_3 \cdot \frac{\partial T_3}{\partial x}$$

where $T_0$ - temperature at the initial time, °C; $L$ - length of the multilayered wall, m; $\lambda$ - coefficient of thermal conductivity, W/(m²·K); $\alpha_f$ - heat transfer coefficient of the working fluid, W/(m²·K); $T_f$ - working fluid temperature, washing the right boundary, °C; index 1 corresponds to the ground array, 2 - filler (sand aqueous solution), 3 - the pipe wall.

On the left edge it is arisen the influx of radiogenic heat $q_0$ (the value of the amount of radiogenic heat may be taken as 0.1 W/m² [7]) and given ground temperature $T_{gr}$. The value temperature $T_{gr}$ in calculations is assumed equally throughout the depth of the borehole, according to [2-4,8]. At the boundary $x = 0$ are given the limit values of the second kind. On the right edge is defined working fluid temperature $T_f$, washing the pipe wall. At the initial time the working fluid temperature is equal to the temperature at the entrance to the ground heat exchanger, and the next time it is calculated. In this case, on the boundary $x = L$ are given boundary conditions of the third kind. At the boundaries, separating the interconnected layers of the wall, it is set the boundary conditions of the fourth kind.

To solve the one-dimensional differential heat equation it is used finite-difference method on the uniform grid. It is determine the temperature in the i-th node at a time value $t = t_n = n \cdot \tau$ as $T(x_i, t_n) = T^n_i$, here $\tau$ - integration step over temporal value, $n$ - step number at the time.

Due to approximation of the relevant partial finite differences it is obtained the following system of the linear algebraic equations:

$$\rho_1 \cdot c_1 \cdot \frac{T^{n+1}_{1,i} - T^n_{1,i}}{\tau} = \lambda_1 \cdot \left( \frac{T^{n+1}_{1,i+1} - 2 \cdot T^n_{1,i} + T^{n+1}_{1,i-1}}{\Delta x^2} \right), i = 1, 2, ..., N1 - 1, n \geq 0$$

$$\rho_2 \cdot c_2 \cdot \frac{T^{n+1}_{2,i} - T^n_{2,i}}{\tau} = \lambda_2 \cdot \left( \frac{T^{n+1}_{2,i+1} - 2 \cdot T^n_{2,i} + T^{n+1}_{2,i-1}}{\Delta x^2} \right), i = N1, ..., N2 - 1, n \geq 0$$

$$\rho_3 \cdot c_3 \cdot \frac{T^{n+1}_{3,i} - T^n_{3,i}}{\tau} = \lambda_3 \cdot \left( \frac{T^{n+1}_{3,i+1} - 2 \cdot T^n_{3,i} + T^{n+1}_{3,i-1}}{\Delta x^2} \right), i = N2, ..., N - 1, n \geq 0$$

It is used four-point implicit difference scheme, where to determine the temperature field at the new time level is necessary to solve the linear equations system (4).

Approximation of the differential problem (2), (3) the finite-difference (4) is provided with first order accuracy over time and the second one in coordinate. This implicit difference scheme is absolutely stable, i.e., the integration of the boundary problem (2), (3) may be performed with any step difference in time [9-12].
The resulting system can be solved using marching method, it is necessary (4) to make the most general form, thus obtaining the three-point differential equation of the second order.

When the temperatures at the new time level at each step in the spatial coordinate are determined, it is occurred calculation of changes of working fluid temperature along the y-axis. Knowing the entering fluid temperature into the heat exchanger pipe, and determining the heat flow from the solutions of equations (2), it is possible to calculate the temperature of the liquid in the next moment of time \( \tau \) on at the next step \( \Delta y \) coordinate.

Determination of the heat flow, which goes into heating the working fluid from the ground array:

\[
q = \pi \cdot d_w \cdot \alpha_f \cdot (T_w - T_{f1})
\]

(5)

where \( d_w \) - the inner diameter of the ground heat exchanger pipe, m; \( \alpha_f \) - heat-transfer coefficient on the boundary pipe wall and the working fluid, W/(m\(^2\)·K); \( T_w \) - the temperature inside the pipe wall of the ground heat exchanger is defining during the solutions process of the system (2) \( T_w = T_{N,n+1}, \ ^\circ C; \) \( T_{f1} \) - working fluid temperature, \(^\circ C; \)

The heat transfer coefficient is determined by the formula:

\[
\alpha_f = \frac{Nu \cdot \lambda_f}{d_w}
\]

(6)

where \( Nu \) - Nusselt number, which characterizes the heat transfer at the interface of the pipe wall and the working fluid, which can be determined by formulas from [13-17]; \( \lambda_f \) - thermal conductivity of the working fluid, W/(m·K).

Applying the heat balance equation for the working fluid, it is possible to calculate its temperature at the next instant in the next step, in the coordinate:

\[
T_{f2} = \frac{(q \cdot \tau \cdot \Delta y)}{M \cdot c} + T_{f1}
\]

(7)

where the expression \(- (q \cdot \tau \cdot \Delta y) = Q\) is like the total amount of the heat applied to the liquid in the time interval \( T \), W; \( \Delta y \) - spatial coordinate step \( 0 < y < N \), where \( N \) - the length of the ground heat exchanger pipe, m; \( M \) - the mass of the working fluid, kg; \( s \) - specific heat of the working fluid J/kg·K.

3. Evaluation of the mutual influence of ground heat exchanger pipes in the borehole

To determine the temperature field in the vicinity of the pipes inside the borehole, it used method by E. Shubin [19, 20] for calculation of the heat loss of several pipes laid in the ground.

In this method the mutual influence of the pipes takes into account nominal resistance defined by the formula:

\[
R_{com} = \frac{1}{2 \cdot \pi \cdot \lambda_{fil}} \cdot \ln \left[ 1 + \left( \frac{2 \cdot h_0}{b} \right)^2 \right]
\]

(8)

where \( \lambda_{fil} \) - borehole filler thermal conductivity, W/(m·K); \( h_0 \) - axis distance from the pipes to the x-axis, m; \( b \) - the distance between the axes of the pipes, m.

Having received fluid temperature along the entire ground heat exchanger length and the temperature of the borehole boundary, by simultaneously solving equations (2), (3), (5) and (7) it is possible to determine which heat flow is necessary for every pipe in their mutual influence on each other within the borehole while the borehole section is considered only at different depths. Diagram of a single cross-section is shown in figure 2.
The heat flow for the first pipe the working fluid is lowered into the borehole can be determined by the formula:

$$q_1 = \frac{(T_{f1} - T_0) \sum R_1 - (T_{f2} - T_0) \sum R_{con}}{\sum R_1 \cdot \sum R_2 - R_{con}^2}$$  \hspace{1cm} (9)$$

The heat flow to the second pipe the working fluid is raised from the borehole, can be determined by the formula:

$$q_2 = \frac{(T_{f2} - T_0) \sum R_1 - (T_{f1} - T_0) \sum R_{con}}{\sum R_1 \cdot \sum R_2 - R_{con}^2}$$  \hspace{1cm} (10)$$

where \(T_{f1}, T_{f2}\) - fluid temperature in the first and second pipes \(\circ C\); \(T_0\) - temperature at the boundary of the contact of the ground filler and ground array, of the considered section, \(\circ C\); \(R_{con}\) - conventional thermal resistance determined by the formula (9), m·K/W; \(\Sigma R_1, \Sigma R_2\) - total thermal resistance for the first and second pipes, m·K/W.

The temperature at any point of the borehole in accordance with Fig. 3 can be determined:

$$T = T_0 + \frac{q_1}{2 \cdot \pi \cdot \lambda_{fil}} \cdot \ln\left(\frac{x^2 + (y + h_0)^2}{x^2 + (y - h_0)^2}\right) + \frac{q_2}{2 \cdot \pi \cdot \lambda_{fil}} \cdot \ln\left(\frac{(x - b)^2 + (y + h_0)^2}{(x - b)^2 + (y - h_0)^2}\right)$$  \hspace{1cm} (11)$$

where \(x\) - the distance of the point from the vertical axis, m.; \(y\) - the distance from a point to a horizontal axis, m.

Evaluation of the mutual influence of pipes was produced for the two sections, the first one is a zone of maximum temperature difference between the ground heat exchanger pipes, and the second one is the same area of the coolant temperature in the ground heat exchanger pipes (figure 3).
4. Conclusion
As a result of calculations it is determined the temperature field in the two sections of the borehole and found that the mutual influence of the ground heat exchanger pipes has an impact only if descent pipe and riser pipe are located at a distance of less than 0.04 m, and at a greater distance such interaction is not observed, for this reason the ground heat exchanger pipes should be placed closer to the edges of the borehole. These statements are related to ground heat exchangers made of plastic pipes with an outer diameter of 0.032 m and a wall thickness of 0.003 m.

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