RaLL: End-to-end Radar Localization on Lidar Map Using Differentiable Measurement Model

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Abstract—Radar sensor provides lighting and weather invariant sensing, which is naturally suitable for long-term localization in outdoor scenes. A basic technique is to localize the robot via GPS/INS in outdoor scenes, which is infeasible in GPS-denied environments. Localization can also be accumulated by odometry data, while it is impracticable when the drift error becomes significant. Therefore, localization on pre-built map is indispensable, which tracks the robot pose with bounded uncertainty.

Recently many visual and lidar based approaches have been widely proposed in the field of robotics, but there still exist some difficulties in real applications. For instance, visual localization on visual map [1], [2] and lidar map [3] are still challenging when appearance changes significantly, like weather or season varies. For lidar localization, the technique still challenging when appearance changes significantly, like foggy and snowy, the lidar range measurements are very noisy, calling for the aid of other sensors.

To build a robust localization system, we consider radar is a good choice, which is naturally lighting and weather invariant. Therefore, radar odometry becomes a research focus recently [8]–[12], and radar simultaneous localization and mapping [13]. A general idea is to build a map using radar, then localize the robot by aligning the radar measurements against the radar map. Considering that the large scale lidar map is available, we argue that repeating the whole mapping process for radar is extremely time and manual labor consuming, also bringing extra calibration between radar map and lidar map [14].

In this paper, we propose an end-to-end learning system to achieve Radar Localization directly on Lidar map (RaLL), as shown in Figure 1. There are two challenges for RaLL. First, the common feature space of the two sensor modals is not explicitly supervised. We propose a network to learn the feature shared embeddings from the pose supervision by introducing a differentiable pose estimator to back-propagate the gradient. Second, the measurement uncertainty is not explicitly supervised, preventing the probabilistic fusion between measurement and motion. We apply the learned model above as a measurement model in a pose tracking Kalman filter (KF), enforcing the measurement uncertainty to be compatible to Gaussian fusion. By modeling the whole localization process...
as a sequential Gaussian distribution estimator, we can supervise the measurement uncertainty via maximizing likelihood. To evaluate the system performance, we utilize RobotCar dataset [15], [16] for benchmark. Besides, the MulRan dataset [17] is employed for testing the generalization of RaLL. In summary, the contributions of this paper are as follows:

- A deep neural network architecture is proposed to learn the cross-modal shared feature embedding by backpropagating gradient from the pose supervision, which leverages the lidar map for radar localization.
- With the learned network being a differentiable measurement model, a Kalman filter is proposed to generate fused estimation, which is learned in an end-to-end way for improve the accuracy.
- We conduct the model training and testing using RobotCar driving dataset (UK), and achieve similar experimental performance in Mulran dataset (South Korea), demonstrating good generalization of RaLL.

The rest of this paper is organized as follows: Section II reviews the related topics in recent years. The whole system is introduced in Section III. Section IV reports the experimental results on two datasets. We conclude a brief overview on our system and a future outlook in Section V.

II. RELATED WORK

Localization using radar sensor. Radar sensor has been applied for various perception tasks widely in autonomous driving area, mainly focused on dynamic objects detection on roads. Some researchers achieved indoor positioning using low-cost radar sensor, including localization on CAD model [18] or laser map [19]. But in challenging outdoor scenarios, precise radar localization is blocked by diverse types of sensor noises and Doppler velocity, therefore, most of studies are based on data pre-processing and sensor modeling [20]–[23].

Recently the Navtex Frequency Modulated Continuous Wave (FMCW) radar sensor brings less Doppler effect, higher resolution and 360°-view in data collection [13], [15]–[17]. The development of this sensor technique results in new applications for real-time object detection and mapping. Various types of noises exist in radar data, while the lidar map is much cleaner. Second, there are occlusions in current view images. At time \( t \), one scan \( R_t^{H \times W} \) is generated from the radar, and an initial estimation of the pose is \( x_t \in SE(2) \). For the prior 2D laser map \( M_t^{H \times W} \), we crop it to \( M_t^{H \times W} \) at pose \( x_t \). With the input \( R_t \) and \( M_t \), a neural network \( F \) is designed to estimate the offset \( \Delta x = {\Delta x, \Delta y, \Delta \theta} \) between \( x_t \) and the ground truth pose \( x_t^* \). Therefore, the ground truth offset \( \Delta x^* \) is the supervision for training \( F \).

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We establish a two-stream neural network for radar and lidar, shown in Figure 2. The whole network consists of three U-Net architectures [33], denoted as \( F_m, F_r \) and \( F_l \). To suppress the noise in radar, we apply a masking network \( F_m \) to learn a noise mask as in [11]. The sigmoid activation in the final layer constrains the masking output range to \([0, 1]\), then a de-noised representation \( R_t^{m} \) is obtained by element-wise

GAN was used to transfer the radar to fake lidar data, and then Monte Carlo localization was applied for pose tracking.

Deep learning for localization. With the advances of deep learning, some geometric or theoretical problems of robot localization were solved by data-driven [29], [30]. A direct way is to localize the sensor by end-to-end learning without geometric information [31], which requires large training data to avoid overfitting. Some researchers [32] established feature correspondences by deep learning and used traditional solvers for pose estimation, while these solvers are indifferentiable for end-to-end learning.

This paper is inspired by several recent deep learning based approaches [5], [6], [11]. In these research works, localization is performed on same sensor modalities. While in this paper, we model the similarity measure and pose regression on the different modalities via deep neural networks. We also train and test the networks with the sequential data, thus building an end-to-end localization system in time domain.

III. METHODS

The framework of RaLL is a differentiable Kalman filter embedded with a neural network based measurement model, which is introduced in Section III-C. The network architecture of the measurement model consists of two stages. The first stage embeds the radar scan and lidar map into a common feature space, which is introduced in Section III-A. The second stage evaluates the similarity of the features extracted from two branches above, and yield the final estimation, which is introduced in Section III-B. Finally, we show the training strategy for RaLL in Section III-D.

A. Comparable Feature Embeddings

We represent both radar scan and lidar map as bird-eye view images. At time \( t \), one scan \( R_t^{H \times W} \) is generated from the radar, and an initial estimation of the pose is \( x_t \). For the prior 2D laser map \( M_t^{H \times W} \), we crop it to \( M_t^{H \times W} \) at pose \( x_t \). With the input \( R_t \) and \( M_t \), a neural network \( F \) is designed to estimate the offset \( \Delta x = {\Delta x, \Delta y, \Delta \theta} \) between \( x_t \) and the ground truth pose \( x_t^* \). Therefore, the ground truth offset \( \Delta x^* \) is the supervision for training \( F \).

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multiplication $R_i^m = R_i \circ \mathcal{F}_m(R_i)$. We notice that the filtered $R_i^m$ is much clearer as shown in Fig. 3. Then we learn the feature embedding which is comparable between the de-noised radar scan and the lidar map: $\mathcal{F}_r(R_i^m) \rightarrow E'_i, \mathcal{F}_l(M_i) \rightarrow E'_l$. We set the final activation function as ReLU in $\mathcal{F}_r$ and $\mathcal{F}_l$ for feature extraction. Figure 3 demonstrates several learned deep embeddings. It is clear that the features salient in both modalities, are directly comparable in the feature embedding space, such as buildings facade etc.

**B. Similarity based Offset Estimation**

Given the initial pose $x_i$, the pose offset should lie in a limited interval, denoted as $-\Delta x_m \leq \Delta x \leq \Delta x_m, \Delta x_m > 0$, the same for $\Delta y$ and $\Delta \theta$ in the range $[-\Delta y_m, \Delta y_m]$ and $[-\Delta \theta_m, \Delta \theta_m]$. Then we divide the 3D solution space for the offset into grids with certain resolutions $\Delta x, \Delta y, \Delta \theta$, and there are $n_{xym} = n_x \times n_y \times n_{\theta}$ candidates in total, e.g. $n_x = \frac{\Delta x_m - (-\Delta x_m)}{\Delta x} + 1$. Hence each candidate in the solution space is denoted as $\Delta x_{ijk} = \{\Delta x_i, \Delta y_j, \Delta \theta_k\}$, where $1 \leq i \leq n_x, 1 \leq j \leq n_y, 1 \leq k \leq n_{\theta}$. As a result, the absolute pose of each candidate is calculated by transform the initial pose $x_i$ using the offset $\Delta x_{ijk}$ as follow:

$$x_i \oplus \Delta x_{ijk} = \left[ \begin{array}{c} x_i + \cos(\theta_i + \Delta \theta_k) \Delta x_i - \sin(\theta_i + \Delta \theta_k) \Delta y_j \\ y_j + \sin(\theta_i + \Delta \theta_k) \Delta x_i + \cos(\theta_i + \Delta \theta_k) \Delta y_j \\
\theta_i + \Delta \theta_k \end{array} \right]$$

(1)

At the level of feature embeddings, we rotate and translate the map embedding $E'_l$ using $n_{xym}$ candidates, resulting in $n_{xym}$ map blocks $E'_{l,ijk}$. When feature embedding is learned, the feature should be comparable by the subtraction $(E'_l - E'_{l,ijk})$. Repeating the subtraction for each candidate, we form a difference tensor as $\Delta E$ with the size $n_{xym} \times H \times W$.

Considering the occlusion, we further divide $\Delta E$ into $k \times k$ small patches as shown in Fig. 2. For each patch, we utilize a patch network $\mathcal{F}_p$ to derive a patch based difference. Finally, we average the $k^2$ patch based differences and obtain a difference score between the radar scan feature $E'_r$ and each candidate $E'_{r,ijk}$, leading to a difference vector with a length of $n_{xym}$.

Essentially, if a candidate pose, say $(x_i \oplus \Delta x_{ijk})$ is close to the ground truth pose $x^*_i$, $E'_r$ should be similar with $E'_{r,ijk}$, hence leading to a low difference at the $ijk$th position of the difference vector. As the lower the difference is, the closer the candidate and the ground truth lie, we take a softmax module to transform and normalize the difference vector to form a probability distribution. Then, we reshape the distribution into a 3-dimensional cost volume $V(\Delta x_{ijk})$. In summary, from the current radar scan and the lidar map, the proposed network produces a probability distribution of the pose offset to be estimated, states as follow:

$$\mathcal{F}_{m,r,l,p}(R_i, M, x_i, \Delta x_{ijk}) \rightarrow V(\Delta x_{ijk})$$

(2)

We then sum over the values along each axis to derive the marginal distribution:

$$\begin{align*}
P_x(\Delta x_i) &= \sum_{\Delta y_j, \Delta \theta_k} V(\Delta x_{ijk}) \\
P_y(\Delta y_j) &= \sum_{\Delta x_i, \Delta \theta_k} V(\Delta x_{ijk}) \\
P_{\theta}(\Delta \theta_k) &= \sum_{\Delta x_i, \Delta y_j} V(\Delta x_{ijk})
\end{align*}$$

(3)

Upon the marginal distribution, we design two losses to train the network: $\mathcal{L}_1 + \mathcal{L}_2$. When the estimation problem is regarded as three separated classification problems of $\{\Delta x_i, \Delta y_j, \Delta \theta_k\}$, a cross entropy loss can be formed as follows:

$$\begin{align*}
\mathcal{L}_1 &= - \sum_{\Delta x_i} C_x \log(P_x) - \sum_{\Delta y_j} C_y \log(P_y) - \sum_{\Delta \theta_k} C_{\theta} \log(P_{\theta})
\end{align*}$$

(4)

where $C_x$, $C_y$ and $C_{\theta}$ are one hot encodings of the ground truth, which is the supervision. On the other hand, as a regression problem, we utilize the marginal expectations of the three marginal distributions as the estimators:

$$\begin{align*}
\Delta \hat{x} &= \sum_{\Delta x_i} P_x(\Delta x_i) \Delta x_i \\
\Delta \hat{y} &= \sum_{\Delta y_j} P_y(\Delta y_j) \Delta y_j \\
\Delta \hat{\theta} &= \sum_{\Delta \theta_k} P_{\theta}(\Delta \theta_k) \Delta \theta_k
\end{align*}$$

(5)

and then the second loss is constructed by the squared error between the estimation $\Delta \hat{x}$ and ground truth $\Delta x^*$ as follows:

$$\mathcal{L}_2 = (\Delta \hat{x} - \Delta x^*)^2 + (\Delta \hat{y} - \Delta y^*)^2 + \alpha \cdot (\Delta \hat{\theta} - \Delta \theta^*)^2$$

(6)
where $\alpha$ is a constant value to balance the translation metric (m) and rotation angle metric (°).

C. Differentiable Kalman Filter

Kalman filter is a general technique to fuse the sensor data in sequential. It consists of two steps, the prediction step using motion model and then followed by the updating step using measurement model, formulating a Bayesian recursive estimator with both models having Gaussian distributions.

Firstly, we propose an iterative closest point (ICP) based radar odometry as the motion model, which estimates the relative pose between two timestamps i.e. $u_{t−1}$ from $R_{t−1}$ to $R_t$. Specifically, radar points with high intensities are extracted from the raw data, and an intensity threshold is used to select salient points. Then ICP [34] is performed on these filtered points to estimate the pose. Based on this radar odometry, the prediction step of is formulated as follows:

$$x_t = f(x_{t−1}, u_{t−1})$$
$$\Sigma_t = F_t \Sigma_{t−1} F^T_t + \Sigma_m$$

(7)

where $f(\cdot)$ accumulates the ego-motion on the previous estimated pose, and $F_t$ is the Jacobian of $f(\cdot)$. The covariance is also propagated and inserted with the odometry covariance $\Sigma_m$, which is a pre-defined coefficient.

In the update step, we utilize the networks $F_m, r, l, p$ as the measurement model. We estimate the offset $\Delta x_t$ at the predicted location $x_t$, then a global observation is generated by applying $z_t = x_t \oplus \Delta x_t$. With this GPS-like observation, we can derive the measurement model. We also calculate the observation covariance $\Sigma_o$, according to the probability distributions $P_x, P_y$ and $P_0$ derived above. Overall, the update step is formulated as follows:

$$K = \Sigma_t (\Sigma_t + \Sigma_o)^{-1}$$
$$\hat{x}_t = x_t + K(z_t - x_t)$$
$$\Sigma_t = (I - K) \Sigma_t$$

(8)

The network aided KF system is presented in Figure. 4. As both models are differentiable, the filter can be regarded as a recurrent network, which means that the measurement model can be trained by back-propagating gradients from future steps.

At the probabilistic perspective, KF generates the Gaussian posterior of the 3-dimensional pose in sequential, which is $\mathcal{N}(\hat{x}_t, \Sigma_t)$. Therefore, with the sequential ground truth pose $\{x^*_t\}$ available, we apply the maximum likelihood to tuning the network parameters as follows

$$\text{maximize } \mathcal{N}(x^*_t; \hat{x}_t, \Sigma_t)$$
$$= \text{maximize } \frac{1}{\sqrt{(2\pi)^3 \det(\Sigma_t)}} \cdot \exp(-\frac{1}{2}(\hat{x}_t - x^*_t)^T \Sigma_t^{-1}(\hat{x}_t - x^*_t))$$

(9)

and we apply negative log-likelihood to formulate the minimization as

$$\text{minimize } \frac{1}{2} \log((2\pi)^3 \det(\Sigma_t)) + \frac{1}{2}(\hat{x}_t - x^*_t)^T \Sigma_t^{-1}(\hat{x}_t - x^*_t)$$

(10)

where $\det(\cdot)$ is the determinant of matrix. Based on the analysis, by discarding the constant terms, we can simply derive the third loss $L_3$ for end-to-end training of the sequential localization as

$$L_3 = \frac{1}{k} \sum_{t-k}^{t} (\hat{x}_t - x^*_t)^T \Sigma_t^{-1}(\hat{x}_t - x^*_t) + \beta \cdot \det(\Sigma_t)$$

(11)

where $\beta$ is a constant balance factor, and $k$ denotes the length of sequence for training. This loss forms a balance between the likelihood and a lower uncertainty in the posterior.

D. Implementation and Training Strategy

We implement the proposed system using Python and PyTorch. The whole network is trained on a single Nvidia Titan X GPU. As for the motion model in the KF, the ICP based radar odometry is implemented by using libpointmatcher [35].
We first train the measurement model $F_{\mathcal{L}_1, \mathcal{L}_2}$ using the single step loss $\mathcal{L}_1 + \mathcal{L}_2$. The training and test data is augmented by randomly sampling the initial poses near the ground truth poses. The sensitivity analysis and comparisons are conducted in Section IV-B and IV-D by using $F_{\mathcal{L}_1, \mathcal{L}_2}$. Furthermore, the pre-trained network is then trained with the sequential loss $\mathcal{L}_3$ in an end-to-end manner, denoted as $F_{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3}$. The input data for training $F_{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3}$ is sequences of temporal radar scans and corresponding ground truth poses. The pose tracking tests with the trained $F_{\mathcal{L}_1, \mathcal{L}_2}$ and $F_{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3}$ are presented in Section IV-C.

IV. EXPERIMENTS

In Section IV-A, we first introduce the datasets for evaluation, then followed an ablation study in Section IV-B. The performance of pose tracking and localization are presented in Section IV-C and IV-D, with comparisons to recent methods.

A. Datasets

We conduct the experiments on the public autonomous driving datasets, Oxford Radar RobotCar (RobotCar)\(^2\) [15], [16] in UK and Multimodal Range Dataset (MulRan)\(^3\) [17] in South Korea. Both of these datasets include ground truth poses, extrinsic calibration, raw lidar and radar data. The sensors types and locations on vehicles are different in the two datasets, thus validating the generalization of the proposed method indirectly.

Table I and Table II present the details of the sequences in these two datasets. For RobotCar dataset, we select six sequences at different time, and Oxford-01 is used for mapping and network training. To validate RaLL in this paper, we follow the path split in [27], shown in Figure 6(a). As for MulRan dataset, sessions at DCC, KAIST and Riverside are used for evaluation. We build the laser maps using DCC-01, KAIST-02 and Riverside-02, and generalize all sequences directly using the learned model in RobotCar. This generalization strategy is performed in all the following experimental sections IV-B, IV-C and IV-D.

\(^2\)https://oxford-robotics-institute.github.io/radar-robotcar-dataset

\(^3\)https://sites.google.com/view/mulran-pr

B. Ablation Study on Various Configurations

First of all, we employ the sensitivity analysis on the input range images, including the scan size $H$, $W$ and resolution $r$. Input radar scans and lidar maps share four types of image settings, as shown in Figure 5(a); and then four networks are trained with these different inputs. The parameter $k$ varies when $H = W = 256$ or 512 relatively, thus guaranteeing the invariance of input patch sizes for patch networks. The ablation study is manual labor consuming for training networks, but we consider it is worthy to explore an appropriate solution for radar localization.

We set the offset in the limit range as follows: $\Delta x_m = \Delta y_m = 2m$, and $\Delta \theta_m = 2^\circ$, and also set the resolution $\delta x = \delta y = 2m$, $\delta \theta = 2^\circ$ in solution space, thus the cost volume is divided to $n_{xyz\theta} = 3 \times 3 \times 3$. Samples are generated randomly with known offsets for training and evaluation. In RobotCar dataset, we collect more than 1000 samples in the test path of Oxford-02 and Oxford-03; and more than 3000 samples in DCC-02 and KAIST-02 of MulRan dataset. The estimated offset is inferred by the trained networks $F_{\mathcal{L}_1, \mathcal{L}_2}$. We calculate the mean errors of the estimated offsets alongside the three dimensions $x, y, \theta$ in robot coordinate.

The evaluation results are shown in Figure 5(a). It is obvious that the neural networks performs best than the others when $H = W = 512$ and $r = 0.25m/pixel$. And the network also works well with $H = W = 256$ and $r = 0.5m/pixel$. In summary, conclusions can be drawn from these comparisons: the higher range resolution is, or the longer detection range is, the more precise estimation will be achieved. But due to the constrained resources, it is a tradeoff between the image sizes and computing efficiency.

Based on the ablation analysis above, the best two con-
(a) $\Delta x_m = \Delta y_m = 2m$, $\Delta \theta_m = 2^\circ$

(b) $\Delta x_m = \Delta y_m = 6m$, $\Delta \theta_m = 6^\circ$

Fig. 5. We conduct the sensitivity experiments with various configurations, and conclusions can be drawn after comparisons and analysis.

### TABLE III

| Dataset | Sequence | Fake-Lidar [28] | Radar Odometry | RaLL ($\mathcal{F}_{\ell_1+\ell_2}$) | RaLL ($\mathcal{F}_{\ell_1+\ell_2+\ell_3}$) |
|---------|----------|----------------|----------------|----------------------------------|----------------------------------|
|         |          | Trans.(m) Rot.($^\circ$) | Trans.(m) Rot.($^\circ$) | Trans.(m) Rot.($^\circ$) | Trans.(m) Rot.($^\circ$) |
| RobotCar| Oxford-02| 8.46 5.43 | 263.27 26.93 | 1.42 1.53 | 0.98 1.45 |
|         | Oxford-03| 6.93 2.46 | 229.95 17.04 | 1.65 1.66 | 1.14 1.62 |
|         | Oxford-04| 9.12 4.47 | 131.66 11.16 | 2.18 2.00 | 1.71 1.93 |
|         | Oxford-05| - - | 439.23 44.14 | 1.47 1.57 | 1.11 1.48 |
|         | Oxford-06| 14.10 4.25 | 333.04 19.45 | 1.52 1.57 | 1.14 1.52 |
| MulRan  | DCC-01   | 2.86 2.46 | 218.47 51.41 | 2.90 2.01 | 2.11 1.97 |
|         | DCC-02   | 6.93 2.46 | 229.95 17.04 | 1.65 1.66 | 1.14 1.62 |
|         | DCC-03   | 9.12 4.47 | 131.66 11.16 | 2.18 2.00 | 1.71 1.93 |
|         | KAIST-01 | - - | 439.23 44.14 | 1.47 1.57 | 1.11 1.48 |
|         | KAIST-02 | - - | 439.23 44.14 | 1.47 1.57 | 1.11 1.48 |
|         | KAIST-03 | - - | 439.23 44.14 | 1.47 1.57 | 1.11 1.48 |
|         | Riverside-01 | - - | 439.23 44.14 | 1.47 1.57 | 1.11 1.48 |
|         | Riverside-02 | - - | 439.23 44.14 | 1.47 1.57 | 1.11 1.48 |

Fig. 6. (a) We split the data in RobotCar dataset for training and evaluation, and the overlapping area is removed. (b) The pose tracking on Oxford-02, including a zoomed view at $\star$ location. (c) The learned model is also generalized to new environments in MulRan dataset.

Conclusions can be drawn after comparisons and analysis.

Fig. 5. We conduct the sensitivity experiments with various configurations, and conclusions can be drawn after comparisons and analysis.

The comparison between $\alpha$ and $\beta$ when $r = 0.25m/pixel$ shows that higher resolution in offset space may not bring more precise estimation. We think it is because of the limited neurons in the network, which is hard to learn the distribution.
of a large number of possibilities.

From these extensive ablation experiments, we also notice that the error in $x$ is often larger than $y$, which are longitudinal and lateral axes for automotive vehicles relatively. We consider this result is interpretable due to the road environments in the two autonomous driving datasets. Most of the roads are straight for vehicles, and are more distinguishable along the lateral direction, especially when there are few features on the road sides. Therefore, it is more difficult to estimate the $\Delta x$ than $\Delta y$ in this paper.

### C. Pose Tracking Evaluation

For the experiments in Section IV-C and IV-D, we select the best configuration in IV-B: $H = W = 512$, $r = 0.25m/pixel$, and the volume is set as $n_{xy\theta} = 7^3$ for estimation in $\Delta x_m = \Delta y_m = 6m$, $\Delta \theta_m = 6^\circ$.

The pose tracking is performed on the whole path of the test sequences. To compare the performance, we accumulate the proposed radar odometry as a comparison, and this odometric localization results in large errors. The direct radar odometry PhaRaO in [12] was performed on DCC-02 and Riverside-02, and we also compare our method on these two sequences.

### D. Localization with the Large Offsets

Besides the pose tracking task, we also perform the localization with large initial offset by using the proposed networks directly. This experiment is compared to the methods in [26], [27]. For a fair comparison, we expand the solution space to...
$\Delta x_m = \Delta y_m = 18m$, $\Delta \theta_m = 18^\circ$, which is close to the experiments in [26], [27], and is also a very large offset on pixel levels in this paper. More than 200 and 500 samples are generated in the test path of Oxford-02 and KAIST-02.

However, our proposed network is designed for limited offsets originally, and not applicable to large offsets. To achieve this goal, we divide the entire space to several sub-spaces, illustrated in Figure 8. And initial guesses are raised in the center of these sub-spaces as initial offsets ($\Delta x$). Then the proposed network is applied at each initial offset, and we can obtain the estimated $\{\Delta x_s\}$ in sub-spaces. While in the entire space, all the potential solutions are $\{\Delta x \pm \Delta x_s\}$, and the problem is then transformed to find the best solution in this answer set. In Section III-B, the coarse similarity is measured by subtraction directly, and here we calculate this measurement for selection. Specifically, for every pair of embeddings, the individual similarity or difference is formulated as $||E_r^t - E_t^{t+j,k}||^2$. Finally, we regard the offset with minimum difference as the estimated offset.

The evaluation results are presented in Table V, and we exchange $x$ and $y$ in [26], [27] because of the different representations in methodologies. Our proposed method performs better overall the performance, compared to another two deep learning based methods. This experiment indicates that our network not only handles long-term pose tracking, but also the localization with large offsets.

V. CONCLUSION

An end-to-end method RaLL is proposed in this paper, which can localize a rotating radar sensor on a prior laser map. We demonstrate the effectiveness of the proposed differentiable localization system in multi-session multi-scene datasets. In the future, we consider that a learned network is desired to replace the traditional filter method in the back-end, and the whole system can be data-driven completely.

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