Optimisation of the FE Model Based on the No-Load Test Measurement for Estimating Electromagnetic Parameters of an Induction Motor Equivalent Circuit Including the Rotor Deep-Bar Effect

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Abstract: The various measurement procedures for determination of electromagnetic parameters for the induction motor (IM) equivalent circuits including the rotor deep-bar effect were proposed in the literature. One of them is the procedure based on the load curve test (LCT). Since the execution of the LCT can pose some difficulties, especially in industrial conditions, as an alternative, the finite element method (FEM) can be employed to simulate the IM operation under the LCT. In this work we developed the optimisation technique for the finite element (FE) model. This technique is performed with the use of the stator current space-vector components which determine the IM input active and reactive power consumption during no-load operation. Relying on the LCT simulation carried out with the optimised FE model the inductance frequency characteristic can be determined and then used as the reference characteristic in the electromagnetic parameter estimation for the IM equivalent circuit including the rotor deep-bar effect. The presented research results demonstrate proper conformity between the inductance frequency characteristics obtained from the LCT performed experimentally and determined by means of the optimised FE model. Satisfactory conformity is also achieved in the case of the torque-versus-slip frequency curves acquired from the measurement and calculated by the IM space-vector model with estimated electromagnetic parameters. All of this validates the effectiveness of the proposed technique for the FE-model optimisation and the usefulness of the presented approach using the FEM in the electromagnetic parameter estimation for the IM equivalent circuit including the rotor deep-bar effect.

Keywords: induction motors; equivalent circuits; finite element analysis; deep-bar effect; optimisation

1. Introduction

Numerical methods such as the finite element method (FEM) are effective tools for analysing and modelling electrical machines. These methods enable investigations of machine static and dynamic states while accounting for machine geometry, material properties of its components and also phenomena such as magnetic saturation of stator and rotor cores and the skin effect in rotor cage bars of induction motors (IMs). However, these methods are time consuming and complex. In fact, this eliminates their use in the applications where the rate of obtaining information on the current machine state plays a significant role, e.g., control of high-performance electrical machine drives. The control schemes are developed based on the space-vector models which are characterised by much lower computational complexity compared to the finite element (FE) models, and thus their microprocessor implementation does not pose too many difficulties.

The major problem related to the use of the IM space-vector models in the field oriented control (FOC) schemes is the variability of IM electromagnetic parameters. This variability
results from the changes of motor winding temperature, stator and rotor core saturation, as well as the rotor deep-bar effect [1]. The lack of capability of proper representation of the variability of IM electromagnetic parameters by the space-vector model with constant parameters results in inaccurate orientation of the control space vectors. This, in turn, is associated with precision deterioration of decoupling effectiveness of electromagnetic torque and rotor flux amplitude control paths, thus deteriorating the overall performance of the FOC [2,3].

Development of the IM space-vector model that would include the electromagnetic parameter variability of the real IM has been a subject investigated in many research institutions around the world [4–20]. In the literature, i.a., [4,5], the control schemes of IM drives were presented which were based on the space-vector model extended with algorithms tracking the variability of selected IM electromagnetic parameters. These algorithms constitute the additional estimators in the IM control scheme to the one responsible for determination of the adopted coordinate system phase position, and so the orientation of the control space vectors. Despite the efficiency of these estimators, their use increases the computational complexity of the control scheme. Alternatively, the variability of selected IM electromagnetic parameters can be represented by model parameters varying as a function of changes in specified physical parameters. In the work [6] the inverse magnetising inductance and leakage inductances of the T-type IM equivalent circuit were expressed as a function of the magnetising flux. A different approach was reported in another work [7] where the stator and slot-bridge leakage inductances of the $\Gamma$-type IM equivalent circuit were defined as a function of the stator flux and the slot-bridge leakage flux, respectively. In other studies [8,9] the rotor resistance variability resulting from the deep-bar effect was conditioned by slip frequency changes. In a number of studies, the variability of IM electromagnetic parameters related to the rotor deep-bar effect was modelled by the IM space-vector models with the rotor represented by the cascade connection of two-terminal-pair networks [7,10–21]. Relying on such a model, the airgap-flux-oriented control [20,22] and pseudorotor-flux-oriented control [23] schemes were developed. In the work [24] the presented theoretical basis led to the formulation of the rotor flux space-vector estimator based on the IM space-vector model with the rotor two-terminal network representation. This allows the implementation of the rotor-flux-oriented control scheme with the use of such a model.

Electromagnetic parameters of the IM equivalent circuit including the rotor deep-bar effect can be determined through an estimation process where the approximation accuracy of the investigated IM reference characteristic by the characteristic resulting from the adopted mathematical model is put to use as the parameter selection criterion. Numerous experimental procedures for determination of the IM reference characteristics were proposed in the literature. In [10] the reference characteristic of the tested IM constituted the stator current response to the DC step voltage disturbance applied across stator terminals at standstill. In another study, [11], the characteristics of the instantaneous input impedances of the tested IMs were considered as the reference. These characteristics were determined based on the measurement of stator voltages, currents and shaft angular velocity during a direct startup.

In many studies the electromagnetic parameter estimation of the space-vector IM model with the rotor two-terminal-pair network representation was conducted based on IM frequency characteristics. The use of these types of reference characteristics is particularly justified in the case of the estimation of the IM space-vector model parameters representing the variability of IM electromagnetic parameters conditioned by slip frequency changes. The rotor-cage impedance frequency characteristic was used in [7]. This characteristic was determined based on a series of locked-rotor tests where the stator frequency varied in the considered range with maintaining constant magnitude of the stator current. In other works [12,13] the IM impedance frequency characteristics obtained from the phase-to-phase standstill variable frequency test were adopted as the reference. Another approach for determination of the reference frequency characteristic was reported in [14]. In this
work the electromagnetic parameter estimation was conducted based on the inductance frequency characteristic which was determined using the measurement data derived from the load curve test (LCT).

Realisation of the above-mentioned measurement procedures for determination of the IM reference characteristics can cause some problems, especially in the industrial conditions. In this case, electromagnetic parameters of the IM space-vector models can be calculated or estimated with the use of analytical or numerical methods.

The papers [15,16] present analytical investigations where the rotor bar was split into a finite number of sections such that the current density was approximately uniform along the depth of each section. This allowed for calculation of electromagnetic parameters for each section based on its geometry and material properties. Then, these parameters were included in the adopted equivalent circuits with the rotor two-terminal-pair network representation.

Analytical methods can also be used to determine the IM reference frequency characteristics for the estimation of electromagnetic parameters of the IM equivalent circuit including the rotor deep-bar effect. In [17] the electromagnetic field analysis was carried out based on the IM multi-layer model and as a result the Laplace-domain inductance was formulated. This inductance, as an IM transfer function, can be represented in the form of frequency characteristics. In another study [18] the rotor resistance and self-inductance variability were calculated from the Poynting flux after solving the electromagnetic field problem formulated for the simplified rotor bar model. Subsequently, the obtained frequency characteristics were considered as a reference in the electromagnetic parameter estimation for the rotor equivalent circuit composed of cascade connected Π-networks.

A different analytical investigation was reported in [19] where the electromagnetic field problem was formulated for the multiple-cage rotor bar model and as a result the expressions for the self-impedance of each conductor as well as the mutual inductances between conductors were formulated. Based on these expressions the frequency characteristics of rotor resistance and inductance can be determined.

Analytical methods are characterised by certain constraints, i.e., the simplified geometry of motor and rotor cage bar models, isotropic material properties and a model feeding usually reduced to the current fundamental harmonic. These all affect the accuracy of electromagnetic field calculations and so the electromagnetic parameter estimation for the IM space-vector models. The FEM is devoid of such constraints and allows one to model the exact IM geometry and material properties of its individual components without limiting the model feeding to the current fundamental harmonic. For this reason, the FEM is an attractive alternative to the measurement procedures for the reference characteristic determination in the electromagnetic parameter estimation of the IM space-vector models. In the work [20], the FEM was used to determine the variability of the rotor bar impedance as a function of slip frequency and subsequently the obtained characteristic was employed in the electromagnetic parameter estimation. In another paper [21], the FEM served for simulation of the impulse response test of the investigated IM, the results of which were used for the electromagnetic parameter estimation of the IM space-vector model with the rotor represented by two parallel connected two-terminals.

In this paper we are proposing an effective optimisation technique for the IM FE model based on the measurement of stator voltages and currents and shaft speed conducted during IM no-load operation. The overall goal is to develop a procedure for the electromagnetic parameter estimation of the IM space-vector models including the rotor deep-bar effect which constitutes an alternative for the procedure based on the LCT conducted experimentally, since its practical implementation can cause some difficulties. The main contributions to the state of the art presented in this paper are the following:

- The methodology based on the LCT simulation performed by the FEM for determination of the reference frequency characteristic, intended for the electromagnetic parameter estimation of the IM space-vector models, where the measurement conducted under the IM no-load operation is only required;
• The theoretical background supported by the results of a number of FEM simulations indicating the impact of adjustments of the FE-model selected parameters (e.g., geometric dimensions, material properties of model components or damping coefficient) on the individual stator current space-vector components and the slip frequency; and

• The validation of the proposed optimisation technique for the IM FE model and the procedure for the electromagnetic parameter estimation of the IM space-vector model through experiments performed on an industrial IM.

The structure of the paper consists of six sections. The IM space-vector model including the rotor deep-bar effect is concisely described in Section 2. This section also includes the procedure for the electromagnetic parameter estimation for such a model based on the reference frequency characteristic derived from the LCT carried out experimentally. Section 3 discusses the theoretical background which demonstrates the influence of the FE-model selected parameter modifications on the individual stator current space-vector components and the slip frequency. Section 4 includes the results of IM operation simulations conducted by the FEM which prove the theoretical considerations from Section 3. Section 5 describes the optimisation process of the FE model and confirms that the use of the optimised FE model enables accurate modelling of the actual IM operation. This translates into the reliable estimation of electromagnetic parameters for the IM equivalent circuit including the rotor deep-bar effect. The conclusions are collected in Section 6.

The paper also includes four appendices: Appendix A presents the rated values of the investigated IM corresponding to the adopted operating point, the cross-section dimensions and the material properties of the IM individual components. In Appendix B, the IM modelling procedure employing the FEM is concisely described. Then, a short description of the laboratory test bench is included in Appendix C. The evaluation function and the GA settings employed in the presented studies are given in Appendix D.

2. Estimation of IM Equivalent Circuit Electromagnetic Parameters Based on the Inductance Frequency Characteristic

2.1. The Induction Motor Mathematical Model Including Deep-Bar Effect

The literature presents analytical considerations leading to an approximate representation of the electromagnetic parameters variability resulted from the rotor deep-bar effect by means of the IM space-vector mathematical models with the rotor represented by the cascade connection of two-terminal-pair networks. Electromagnetic parameters of such a model can be determined based on the IM Laplace-domain inductance described by the equation:

$$\begin{align*}
L_1(p) &= L_{\sigma 1} + L_{1 \delta}(p), \\
&= L_{1 \delta}(p) + Z_{ab}(p) = \frac{\psi_1^{(r)}(p)}{I_1^{(r)}(p)} = \frac{Z_{ab}(p)}{p}.
\end{align*}$$

(1)

where, $\psi_1^{(r)}(p)$, $I_1^{(r)}(p)$ are the Laplace transforms of the stator flux and current space vectors, respectively, $p$ is a complex frequency, $Z_{ab}(p)$ denotes the Laplace-domain impedance between the terminals “a” and “b” of the IM equivalent circuits presented in Figure 1 and the superscript (r) denotes space vectors expressed in an orthogonal coordinate system rotating at the shaft angular velocity $\omega_{sh}$.

The Laplace-domain inductance $L_{1 \delta}(p)$ is a series connection of the stator leakage inductance $L_{\sigma 1}$ and the Laplace-domain inductance associated with the airgap flux $L_{1 \delta}(p)$:

$$L_1(p) = L_{\sigma 1} + L_{1 \delta}(p).$$

(2)

The formula for the Laplace-domain inductance $L_{1 \delta}(p)$ can be derived as a result of a solution of Maxwell’s differential system of equations formulated on, e.g., the IM multi-layer model as in [17]. The inductance $L_{1 \delta}(p)$ is a certain type of the IM equivalent circuits presented in Figure 1 and the superscript (r) denotes space vectors expressed in an orthogonal coordinate system rotating at the shaft angular velocity $\omega_{sh}$.
rotor representation in the form of the parallel connected two-terminal network with $R_{2(n)}$, $L_{\alpha 2(n)}$ parameters (Figure 1b) which are determined analytically in that case.

$$\frac{1}{pL_{1s}(p)} \cong \frac{1}{pL_{\mu}} + \sum_{n=1}^{N} \frac{1}{R_{2(n)} + pL_{\alpha 2(n)}}, \quad (3)$$

$$\frac{1}{Z_2(p)} \cong \sum_{n=1}^{N} \frac{1}{R_{2(n)} + pL_{\alpha 2(n)}}.$$  

where, $L_{\mu} = L_{1s}(p = 0)$ is the magnetising inductance, $Z_2(p)$ stands for the Laplace-domain rotor impedance and $N$ indicates the number of two-terminal-pair networks connected in cascade.

![Image of IM equivalent circuits](image-url)

**Figure 1.** The IM equivalent circuits expressed in the Laplace-domain: (a) the IM equivalent circuit with rotor resistance and leakage inductance modelled by parameters varying as a function of slip frequency, (b) the approximation of the rotor electromagnetic parameter variability by the parallel connected two-terminal network in the rotor equivalent circuit.

Alternatively, while reducing the consideration from the Laplace-domain to the frequency-domain through the assumption $L_{1s}(p = j\omega_2)$, where $\omega_2$ stands for slip frequency and $j = \sqrt{-1}$ denotes the imaginary unit, this inductance can be applied as the reference frequency characteristic in the estimation of $R_{2(n)}, L_{\alpha 2(n)}$ parameters. The number $N$ of two-terminals required in the rotor equivalent circuit (Figure 1b) is related to the considered range of slip frequency $\omega_2$, since the wider range of slip frequency is of interest as the greater order of the rotor equivalent circuit is essential for accurate approximation of the reference frequency characteristic $L_{1s}(p = j\omega_2)$.

The procedures of the electromagnetic parameter estimation for the IM space-vector models where the reference characteristics are determined based on experimental tests or numerical calculations do not a priori impose the configuration of electromagnetic parameter connections in individual two-terminal-pair networks of the rotor equivalent circuit, as in the case with the above-described analytical procedure shown in Equations (3) and (4). The selection of the rotor equivalent circuit structure is a compromise between the possible accuracy of reference characteristic approximation and the computational complexity of the IM space-vector model [16]. Subsequently, electromagnetic parameters of individual two-terminal-pair networks in the rotor equivalent circuit, as well as their number $N$, ensuring the required accuracy of reference characteristic approximation are searched empirically. In this manner, similar to the analytical approach, the variability of IM electromagnetic parameters resulting from the rotor deep-bar effect is represented by means of the IM space-vector model with the rotor two-terminal-pair network representation.

### 2.2. Determination of the Inductance Frequency Characteristic

The work [14] presents the estimation procedure of electromagnetic parameters for the IM equivalent circuit including the rotor deep-bar effect where the IM inductance frequency characteristic $L_2(\omega_2)$ determined based on measurement data obtained from the LCT was adopted as the reference characteristic. According to the LCT, the measurement of stator
active power, RMS voltages and currents and the shaft angular velocity is conducted at the rated input voltage and frequency while changing the IM load in the range (0.25–1.25) of the rated load [25]. Relying on the measurement data, the IM input impedance-versus-slip frequency characteristic $Z_1(\omega_2)$ can be calculated by Equations (5) and (6) from which the inductance frequency characteristic $L_1(\omega_2)$ is then determined through Equation (7) [14].

The stator resistance $R_1$ appearing in Equation (7) is obtained from the DC line-to-line resistance measurement [25].

$$Z_1(\omega_2) = \frac{|U_{1ph}|}{|I_{1ph}(\omega_2)|} \left( \cos(\phi_1(\omega_2)) + j\sqrt{1 - \cos(\phi_1(\omega_2))^2} \right), \quad (5)$$

$$\cos(\phi_1(\omega_2)) = \frac{P_1(\omega_2)}{3|U_{1ph}| |I_{1ph}(\omega_2)|}, \quad (6)$$

$$L_1(\omega_2) = \frac{Z_{ab}(\omega_2)}{j\omega_1} = \frac{Z_1(\omega_2) - R_1}{j\omega_1}. \quad (7)$$

where, $Z_1(\omega_2)$ is the IM input impedance expressed as a function of slip frequency, $\omega_1$ represents the stator voltage angular frequency, $|U_{1ph}|$ and $|I_{1ph}(\omega_2)|$ constitute the measured root mean squared (RMS) values of stator phase voltages and currents, respectively, and $P_1(\omega_2)$ and $\cos(\phi_1(\omega_2))$ are the measured stator power and calculated stator power factor, respectively.

Approximation of the inductance frequency characteristic $L_1(\omega_2)$ by means of the frequency-domain inductance $L_1(\sigma = j\omega_2)$ resulting from the adopted IM space-vector model enables the estimation of its electromagnetic parameters. Expressing the IM inductance $L_1(\sigma = j\omega_2)$ with Equations (2) and (3) leads to the IM equivalent circuit with the rotor two-terminal network representation presented in Figure 1b. In this approach, the following model parameters $L_{1\sigma}$, $L_{1\mu}$, $R_{2(n)}$, $L_{a2(n)}$ and $N$ are determined through the estimation process which can be considered as a minimisation issue of the evaluation function defined as the sum of the mean squared errors of the reference characteristic approximation [14,24].

2.3. Determination of the Torque-versus-Slip Frequency Curve

The measurement data obtained under the LCT can be used to determine the torque-versus-slip frequency curve in line with Equations (8) and (9):

$$T_{em}(\omega_2) = 3p_p \text{Im}\left( \Psi_{1ph}^* L_{1ph}(\omega_2) \right) \quad (8)$$

$$\Psi_{1ph}(\omega_2) = \frac{1}{j\omega_1} \left( \frac{U_{1ph}}{R_1 L_{1ph}(\omega_2)} - 1 \right) \quad (9)$$

where, $U_{1ph}$, $I_{1ph}$ and $\Psi_{1ph}$ are the stator voltage, current and flux complex RMS values, respectively; $p_p$—stands for the pole pairs, $^*$ denotes the complex conjugate.

Both the inductance frequency characteristic and the torque-versus-slip frequency curve were employed in the presented studies for verification of the proposed approach for FE-model optimisation.

3. Optimisation of the FE Model Based on the No-Load Test Measurement

The realisation of the LCT entails the need to apply a braking mechanism for controlling of the tested IM load which may pose difficulties in industrial conditions, especially in relation to high-power machines. Alternatively, the LCT can be simulated with the use of the FEM. This method enables analysis and modelling of the IM operation under steady and transient states with accuracy dependant on the FE-model fidelity to the real motor.

For the FE-model construction, knowledge of geometric dimensions and material properties of individual IM components is required. Based on this information, iron and
winding losses and the reactive power required for magnetic flux generation are modelled. This knowledge is complemented by the information on friction and windage losses. The geometric dimensions with the assumed tolerance and the material properties of machine components are included in the manufacturer’s catalogues, while the information on friction and windage losses is usually acquired through the experimental test. Incorrect geometric dimensions of the FE model, inaccurately defined material properties of its components or wrongly determined friction and windage losses bring errors in modelling of considered IM operation under the LCT simulation. This, in turn, has an impact on correctness of the inductance frequency characteristic determination and, consequently, estimated electromagnetic parameters for the IM space-vector model.

Figure 2 presents the stator voltage and current space vectors under various IM load conditions. These space vectors are expressed in the coordinate system (x-y) that rotates synchronously with the stator voltage (SV) space vector. When the SV space vector is oriented towards the x-axis of the adopted coordinate system \( U_{1y} \), the x-axis and y-axis components of the stator current (SC) space vector determine the total input active and reactive power of the considered IM, respectively. In such a coordinate system, the changes of the motor load involve the linear changes of the x-axis component of the SC space vector \( I_{1x} \). At the same time, the y-axis component of the SC space vector \( I_{1y} \), determining the IM demand for the reactive power for magnetic flux generation, is maintained at approximately the same level with changes of the IM load within its rated range. Thus, with IM load changes, the SC space vector moves along a straight line parallel to the x-axis of the adopted coordinate system and shifted against the x-axis by amplitude of the y-axis component of the SC space vector (Figure 2).

![Figure 2](image-url)

**Figure 2.** The stator voltage and current space vectors under various IM load conditions in the coordinate system oriented with respect to the stator voltage space vector.

Under no-load operation when the load torque is equal to zero the x-axis component of the SC space vector determines the input active power consumed by the machine to cover the power losses including iron, winding, friction and windage losses. Therefore, the knowledge of the x-axis component of the SC space vector supplemented by the information on the slip frequency of the considered IM allows one to optimise the FE model in terms of the generated power losses by adjusting the damping coefficient or conductivity of the rotor cage or machine cores. Whereas the knowledge on the y-axis component of the SC space vector enables optimisation of the FE model with respect to reactive power consumption by adapting the magnetic properties of machine cores or the airgap thickness.

Concluding the above discussion, the measurement of the stator instantaneous voltages and currents carried out under no-load operation and complemented by the no-load shaft angular velocity measurement provides sufficient information on how to optimise the
FE model. In cases where the tested IM is supplied directly from the AC line, the complex RMS values of the stator voltage and current can be used in place of the SV and SC space vectors since FE-model optimisation can be conveniently conducted on the data measured during the IM steady state operation. The optimised FE model can then be employed for simulation of the IM operation under the LCT. Based on the data derived from the LCT simulation, the IM inductance frequency characteristic $L_1(\omega_2)$ is determined which can be subsequently used as the reference characteristic in the electromagnetic parameter estimation for the IM space-vector model with the rotor two-terminal network representation (Figure 1b).

4. Impact of Motor FE-Model Parameters on SC Space-Vector Components and Slip Frequency

The presented studies were conducted to the squirrel-cage IM of type Sg 132S-4. The IM rated values corresponding to the adopted operating points are listed in Table A1 included in Appendix A. The data from the manufacturers’ catalogues involving dimensions of the tested IM cross-section (Figure A1a), the stator and rotor slot cross-sections (Figure A1b) and the material properties of IM individual components are also presented in Appendix A.

The numerous simulations of the IM operation were conducted by means of the FEM where one of the FE-model parameters: the damping coefficient, the airgap thickness or selected material properties of model components was modified in the specified range, whilst the other parameters were unaltered and consistent with the data from manufacturers’ catalogues. This was aimed to visualise the impact of adjustments of the FE-model selected parameters on the SC space-vector components and the slip frequency. The concise description of the IM modelling procedure with the use of the ANSYS Maxwell software is included in Appendix B. There is also shown the 2D FE mesh (Figure A2) employed in the presented studies.

In these studies, the direct-on-line starting of the tested IM under no-load operation was simulated. After starting, under the IM steady state operation the $x$-axis and $y$-axis components of the SC space vector were determined on the basis of the stator instantaneous voltages and currents returned by the FE model. Additionally, the relative differences between the SC space vector components or the slip frequency obtained from the FE model with the initially adopted catalogue parameters (the initial FE model) and the FE model with modified parameters were taken into account:

$$\Delta x_{(k)} = \frac{x_{(k)} - x_{(\text{int.})}}{x_{(\text{int.})}} \times 100\% , \quad (10)$$

where: $x_{(\text{int.})}$, $x_{(k)}$ are the $x$-axis or $y$-axis components of the SC space vector or the slip frequency acquired by the FE model with the initial parameters and by the FE model with $k$-th modification of its selected parameter, respectively.

In the first instance, the FE model with the initial parameters was used for simulation of the direct-on-line starting of the tested IM under no-load operation. The damping coefficient, specifying the IM friction and windage losses, was assumed at 0.010 (N·m·s/rad) which corresponded to about 10 (%) of the IM rated torque. The $x$-axis and $y$-axis components of the SC space vector and the slip frequency determined on the basis of this simulation, employed as the reference values in the present considerations, are listed in Table 1. The reference values are also included in Tables 2 and 3 to facilitate the visualisation of the impact of modifications of the FE-model selected parameters on the SC space-vector components and the slip frequency.
Table 1. The x-axis and y-axis components of the SC space vector and the slip frequency at the modification of the FE-model damping coefficient.

| Quantity | Damping Factor (N m s/rad) |
|----------|----------------------------|
|          | 0.005 | 0.010 (int. FE Model) | 0.015 | 0.020 | 0.025 |
| $I_{1x}^{(U_1)}$ (A) | 0.2476 | 0.5378 | 0.8373 | 1.0719 | 1.3137 |
| $\Delta I_{1x}^{(U_1)}$ (%) | -53.9583 | - | 55.6824 | 99.3030 | 144.2574 |
| $I_{1y}^{(U_1)}$ (A) | -1.3496 | -1.3537 | -1.3675 | -1.3880 | -1.4134 |
| $\Delta I_{1y}^{(U_1)}$ (%) | -0.3053 | 0 | 1.0158 | 2.5393 | 4.4107 |
| $\omega_2$ (Hz) | 0.0542 | 0.1316 | 0.2188 | 0.2839 | 0.3454 |
| $\Delta \omega_2$ (Hz%) | -53.9583 | -26.5682 | 99.3030 | 144.2574 | 201.4807 |

Table 2. The x-axis and y-axis components of the SC space vector and the slip frequency at the modification of the stator and rotor B-H curve slope of the FE model.

| Quantity | B-H Curves |
|----------|------------|
|          | B-H Curve 1 | M600-50A (int. FE Model) | B-H Curve 2 |
| $I_{1x}^{(U_1)}$ (A) | 0.5392 | 0.5378 | 0.5362 |
| $\Delta I_{1x}^{(U_1)}$ (%) | 0.2603 | - | -0.2975 |
| $I_{1y}^{(U_1)}$ (A) | -1.3188 | -1.3537 | -1.3880 |
| $\Delta I_{1y}^{(U_1)}$ (%) | -2.5781 | - | 2.5337 |
| $\omega_2$ (Hz) | 0.1324 | 0.1316 | 0.1313 |
| $\Delta \omega_2$ (%) | 0.6079 | - | -0.2279 |

Table 3. The x-axis and y-axis components of the SC space vector and the slip frequency at the modification of the airgap thickness of the FE model.

| Quantity | Airgap Thickness (mm) |
|----------|-----------------------|
|          | 0.30 | 0.35 (int. FE Model) | 0.40 | 0.50 | 0.60 |
| $I_{1x}^{(U_1)}$ (A) | 0.5519 | 0.5378 | 0.5218 | 0.5535 | 0.5448 |
| $\Delta I_{1x}^{(U_1)}$ (%) | 2.6218 | - | -2.9751 | 2.9139 | 1.3016 |
| $I_{1y}^{(U_1)}$ (A) | -1.2023 | -1.3537 | -1.5061 | -1.8122 | -2.1080 |
| $\Delta I_{1y}^{(U_1)}$ (%) | -11.1842 | - | 11.2580 | 33.8701 | 55.7214 |
| $\omega_2$ (Hz) | 0.1318 | 0.1316 | 0.1300 | 0.1317 | 0.1299 |
| $\Delta \omega_2$ (%) | 0.1452 | - | -1.1052 | 0.0436 | -1.3084 |

In the second step, the impact of adjustments of the FE-model damping coefficient was examined. In the presented studies, the damping coefficient was modified within the relatively wide interval of (0.005 N·m·s/rad to 0.025 N·m·s/rad). The x-axis and y-axis components of the SC space vector and the slip frequency obtained from conducted simulations are included in Table 1. The relative differences between the SC space-vector components and the slip frequency obtained from the FE model with the initial parameters and the FE model with the modified damping coefficient are also listed in Table 1.

According to the assumptions, the damping coefficient modification has an impact on the x-axis component of the SC space vector and the slip frequency while having a slight influence on the y-axis component of the SC space vector. It is worth pointing out that the approximately linear increments of the x-axis component of the SC space vector and the slip frequency as a function of the damping coefficient changes are observed.
In the next step, the impact of the B-H curve slope of the stator and rotor cores on the SC space-vector components and the slip frequency was examined. Figure 3 presents the B-H curve of the M600-50A electrical steel. This figure also includes the two additional B-H curves formed for the purpose of the presented analysis through increasing (B-H curve 1) and decreasing (B-H curve 2) the slope of the M600-50A B-H curve on average by 25 (%). The adjustment of the M600-50A B-H curve slope in the analysed range had a negligible effect on the x-axis component of the SC space vector and the slip frequency.

Figure 3. The B-H curve of the M600-50A electrical steel and its modifications formed through increasing (B-H curve 1) and decreasing (B-H curve 2) the slope of the M600-50A B-H curve.

Table 2 presents the x-axis and y-axis components of the SC space vector and the slip frequency achieved from the FE model with the three considered B-H curves of the stator and rotor cores. A modification of the B-H curve slope influences the IM magnetising current, i.e., the y-axis component of the SC space vector which is found to be in line with expectations. The linear relationship between the B-H curve slope and the y-axis component of the SC space vector is observed—this component changes approximately by 2.5 (%) with slope modifications of the M600-50A B-H curve on average by 25 (%). The adjustment of the M600-50A B-H curve slope in the analysed range had a negligible effect on the x-axis component of the SC space vector and the slip frequency.

In the subsequent stage, the impact of the IM airgap thickness on the SC space-vector components and the slip frequency was analysed. Under these studies, the IM airgap thickness was altered in the interval (0.30 mm–0.60 mm) through changing the rotor diameter. The airgap thickness consistent with the data from the manufacturer catalogue is equal to 0.35 (mm) (Figure A1). The x-axis and y-axis components of the SC space vector and the slip frequency obtained from the FE model with the airgap thickness altered within the above-mentioned interval are included in Table 3.

As expected, the linear relationship between the IM airgap thickness and the y-axis component of the SC space vector is registered. Slight changes in the x-axis component of the SC space vector are also observed. However, due to their non-systematic nature, these few-percent fluctuations of the x-axis component of the SC space vector should be related to the accuracy of numerical methods rather than looking for their relationship with a modification of the airgap thickness. As it results from Table 3, the change in the IM airgap thickness had a negligible effect on the slip frequency.

The adjustment of the rotor-cage conductivity had no impact on the x-axis and y-axis components of the SC space vector. However, it influenced the slip frequency. This means that with some modifications of the rotor-cage conductivity the machine active and reactive power consumption remains unaltered, but the distribution of the power transmitted to the rotor into torque generation and rotor winding losses is affected.

5. Verification of the Proposed Approach for FE-Model Optimisation

The proposed approach for FE-model optimisation was verified based on the measurement data derived from the LCT performed experimentally. The description and
schematic diagram (Figure A3) of the laboratory test bench are included in Appendix C. Before starting the measurement conducted under the LCT, the tested IM operated with the given load torque to stabilise its temperature. This was carried out to limit the impact of the stator and rotor resistance changes on the measurement data employed in the verification studies. Afterwards, the measurement of the stator instantaneous voltages and currents, the stator winding temperature and the shaft angular velocity was realised at steady state operation for each IM load conditions adopted under the LCT.

The data from measurement performed under IM no-load operation served for optimisation of the FE model, whilst the data acquired in the whole considered range of the slip frequency were used in verification of the optimised FE model. Based on the conducted measurement the SV and SC space vectors were formed and the input IM impedance-versus-slip frequency characteristic was calculated as their quotient. Afterwards, the inductance frequency characteristic was determined following Equation (7). The modulus and argument of this characteristic are presented in Figure 4.

![Figure 4](image_url)

**Figure 4.** The inductance frequency characteristics of the tested IM determined based on the data derived from the LCT performed experimentally and with the use of the FEM: (a) modulus and (b) argument.

### 5.1. Optimisation Process of the FE Model

The $x$-axis and $y$-axis components of the SC space vector, calculated based on the measurement data acquired under IM no-load operation, along with the no-load slip frequency, are listed in Table 4. These values constitute the reference quantities in the optimisation process. As a comparison, the SC space-vector components and the slip frequency obtained from the simulation of the IM no-load operation conducted using the FE model with the catalogue parameters are also presented in Table 4.

| Quantity | Measurement | Initial FE Model | Optimised FE Model |
|----------|-------------|------------------|--------------------|
| $I^{(U)}_x$ (A) | 0.5674 | 0.5378 | 0.5540 |
| $I^{(U)}_y$ (A) | $-2.0318$ | $-1.3537$ | $-1.9585$ |
| $\omega_2$ (Hz) | 0.1186 | 0.1316 | 0.1362 |

The study results presented in Section 4 demonstrate the approximate linearity between the specified FE-model parameters and the $x$-axis or $y$-axis components of the SC space vector. For this reason, the results of the FE analysis conducted for two different
settings of the selected FE-model parameter enable estimating the variability of the x-axis or y-axis components of the SC space vector as a function of the studied parameter. This property can be conveniently used in determining the FE-model parameters which provide the requested conformity between the SC space-vector components obtained from the measurement and the optimised FE model.

The increase of the y-axis component of the SC space vector was achieved through the increment of the FE-model airgap thickness from 0.35 (mm) to 0.50 (mm). It allowed the reduction of the relative error between the SC space-vector y-axis components obtained by the measurement and the FE models from about 33 (%) for the initial FE model to 3.6 (%) for the optimised FE model. Additionally, in order to increase the x-axis component of the SC space vector, the FE-model damping coefficient was also slightly modified. Since the increment of the FE-model damping coefficient is associated with an increase in the slip frequency (Table 1), which was not desired in the case under consideration, the damping coefficient was boosted only to 0.011 (N·m·s/rad). The decrease in the FE-model slip frequency could be achieved through increasing its rotor cage conductivity, and thus reducing the rotor winding losses. In this case, the absolute error between the no-load slip frequencies derived from the measurement and the simulation conducted with the use of the optimised FE model was approximately equal to 0.02 (Hz). The corrections of the FE-model slip frequency at the level of 0.02 (Hz) turned out to be problematic due to the limited calculation accuracy inherent for numerical methods. For this reason, the FE model of the tested IM was not subjected to further corrections. The SC space-vector components and the slip frequency derived from the simulation of the IM no-load operation performed with the use of the optimised FE model are included in Table 4.

The optimised FE model was subsequently employed for the LCT simulation. The simulation was performed at the following values of the IM load torque $T_L = (0.0, 2.5, 5.0, 7.5, 10, 12.5, 15.0, 17.5, 20.0)$ (N·m). These values were adopted based on the electromagnetic torque values determined from the LCT conducted experimentally (Figure 7), minus friction and windage losses which were assumed to be approximately 10 (%) of the IM rated torque. The stator instantaneous voltages and currents and the shaft angular velocity obtained from the LCT simulation were used to determine the inductance frequency characteristic. This characteristic is presented in Figure 4 together with the characteristic obtained as a result of the LCT performed experimentally. For comparison, the LCT simulation was also carried out using the initial FE model. The inductance frequency characteristic determined on the data derived from this simulation is also included in Figure 4.

The optimisation of the FE model according to the presented approach allowed for much more accurate modelling of the tested IM operation under the LCT in comparison to the initial FE model. A satisfying conformity between the inductance frequency characteristics obtained from the measurement and the simulation conducted using the optimised FE model was achieved. This was not attained with the use of the initial FE model.

5.2. Validation of the Electromagnetic Parameter Estimation for the IM Space-Vector Model

In the next stage, the verification of the proposed approach for FE-model optimisation was based on the reliability evaluation of the IM space-vector model in which electromagnetic parameters were estimated using the inductance frequency characteristics determined from the LCT performed experimentally and by means of the optimised FE model. These characteristics were employed as the reference characteristics in the estimation process which was performed with the genetic algorithm (GA) according to the procedures described in [14,24]. The evaluation function and a brief description of the GA settings employed in the presented studies are included in Appendix D. As in the works [14,24], the criterion adopted in the estimation process of electromagnetic parameters for the IM equivalent circuit including the rotor deep-bar effect assumed the approximation of the reference characteristic modulus with an error not exceeding 2 (%) in the considered range of slip frequency. At the same time, a possible minimum approximation error of the reference characteristic argument and a minimum number $N$ of two-terminals in the
rotor equivalent circuit were required under this criterion. The approximation of the reference characteristics with the adopted permissible modulus error was achieved by the IM space-vector model with two \((N = 2)\) parallel connected two-terminals in the rotor network representation (Figures 5 and 6). In parallel, the approximation errors of the reference characteristic argument were at the acceptable level, not exceeding \(\pm 1.72\) (deg) and \(\pm 1.4\) (deg) for the characteristics obtained from measurement and simulation, respectively.

![Figure 5](image1.png)  
**Figure 5.** The inductance frequency characteristic determined based on the data derived from the LCT performed experimentally and its approximation by means of the IM mathematical model: (a) modulus and (b) argument.

![Figure 6](image2.png)  
**Figure 6.** The inductance frequency characteristic determined based on the data derived from the simulation of the LCT and its approximation by means of the IM mathematical model: (a) modulus and (b) argument.

The electromagnetic parameters estimated based on the inductance frequency characteristics derived from the LCT performed experimentally and with the use of the optimised FE model were referred to as Set 1 and Set 2, respectively. These parameters are summarised in Table 5.

The space-vector model with the electromagnetic parameters Set 1 and Set 2 was employed to calculate the torque-versus-slip frequency curves. These curves are set together with the torque-versus-slip frequency curve determined according to Equations (8) and (9) based on the data obtained from the LCT realised experimentally (Figure 7a). Additionally, Figure 7b presents the relative errors between the torque-versus-slip frequency curve derived from the experimental realisation of the LCT and the curves...
calculated by the IM space-vector model with two parallel connected two-terminals in the rotor network representation.

**Table 5.** Electromagnetic parameters of the IM equivalent circuit with two \((N = 2)\) parallel connected two-terminals in the rotor network representation. The resistances correspond to the reference winding temperature of 25 °C.

| Electromagnetic Parameters | \(L_{a1}\) (H) | \(L_\mu\) (H) | \(R_{2(1)}\) (Ω) | \(L_{a2(1)}\) (H) | \(R_{2(2)}\) (Ω) | \(L_{a2(2)}\) (H) |
|----------------------------|----------------|----------------|-------------------|-------------------|-------------------|-------------------|
| Measurement (Set 1)        | 0.0181         | 0.4963         | 2.1683            | 0.0069            | 5.7543            | 0.2146            |
| Optimised FE model (Set 2) | 0.0185         | 0.5074         | 2.6957            | 0.0102            | 3.6622            | 0.1115            |

**Figure 7.** The torque-versus-slip frequency curves determined based on the data derived from the LCT performed experimentally and by means of the IM mathematical model: (a) torque-versus-slip frequency curves and (b) relative errors.

Both the torque-versus-slip frequency curves, calculated by the IM space-vector model with electromagnetic parameters Set 1 and Set 2, are in good agreement with the torque-versus-slip frequency curve derived from the measurements conducted under the LCT. The relative errors between the torque-versus-slip frequency curve determined from the experimental realisation of the LCT and the curves calculated by the IM space-vector model with electromagnetic parameters Set 1 and Set 2 are not significant in the whole considered range of slip frequency and, apart from the slip frequency corresponding to IM no-load operation, they do not exceed ±3.3 (%) and ±5.1 (%), respectively.

The relative error of 8 (%) between the torque-versus-slip frequency curve derived from the measurement under the LCT and the curve obtained by the IM space-vector model with electromagnetic parameters Set 1 results from limited approximation precision of the reference inductance frequency characteristic—also derived from the measurement under the LCT—by this model at the slip frequency corresponding to the IM no-load operation. The absolute approximation error of the reference characteristic argument is equal to about −1.72 (deg) at the no-load slip frequency and is the highest in the considered range of slip frequency.

The relative errors introduced by the IM space-vector model with the electromagnetic parameters Set 2 are predominantly related to limited conformity between the inductance frequency characteristics obtained in the result of the LCT performed experimentally and through the simulation conducted with the optimised FE model.
6. Conclusions

The paper presents an effective optimisation technique for the FE model which is intended for determination of the reference frequency characteristic employed in the electromagnetic parameter estimation of the IM equivalent circuit including the rotor deep-bar effect. The crucial advantage of the developed technique is the uncomplicated method for the acquisition of the reference quantities in FE-model optimisation since the measurement of the stator voltages and currents and the shaft angular velocity does not require advanced measuring equipment. Moreover, the IM no-load test does not pose too many difficulties in industrial conditions.

The inductance frequency characteristic derived from the LCT simulation conducted by the optimised FE model is in good conformity with the characteristic determined based on the data acquired from the experimental realisation of the LCT. Equally good conformity is achieved between the torque-versus-slip frequency curves obtained as a result of the LCT performed experimentally and calculated by the IM space-vector model in which electromagnetic parameters were estimated with the use of the inductance frequency characteristic sourced by means of the optimised FE model.

Concluding, the use of the FE model optimised in accordance with the developed technique in the estimation of electromagnetic parameters for the IM space-vector models constitutes an attractive alternative to the measurement procedures defined in the world literature, particularly in industrial conditions where the realisation of these procedures can cause some difficulties.

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**Appendix A**

**Table A1.** The rated values of the squirrel-cage IM of type Sg 132S-4 corresponding to the adopted operating points.

| Rating       | Unit Description       | Unit Value |
|--------------|------------------------|------------|
| Output power | kilowatt (kW)          | 2.358      |
| Stator voltage| volt (V)               | 400 (wye)  |
| Stator frequency| hertz (Hz)           | 50         |
| Stator current| ampere (A)             | 4.536      |
| Torque       | newton meter (N·m)     | 15.53      |
| Rotational speed| revolution per minute (r/m) | 1450 |
| Power factor | -                      | 0.8819     |
| Efficiency   | -                      | 0.8525     |
| Stator flux  | weber (Wb)             | 0.973      |
The B-H curve of the M600-50A electrical steel is shown in Figure 3 in the main text of the paper. The other material parameters of the FE-model individual components were adopted as follows:

- Conductivity of the stator and rotor cores \( \sigma_{Fe} = 3.125 \times 10^6 (\Omega \cdot m)^{-1} \);
- Conductivity of the rotor cage \( \sigma_{Al} = 36 \times 10^6 (\Omega \cdot m)^{-1} \); and
- Conductivity and relative permeability of the rotor shaft \( \sigma_{sh} = 10.3 \times 10^6 (\Omega \cdot m)^{-1}, \mu_{sh} = 4000 (\cdot) \).

The stator resistance, determined based on the DC line-to-line resistance measurement is equal to \( R_1 = 2.9597 (\Omega) \) at the temperature of 25 \( ^\circ \text{C} \).

**Figure A1.** The cross-section dimensions: (a) the squirrel-cage IM of type Sg 132S-4 and (b) stator and rotor slots of the squirrel-cage IM of type Sg 132S-4.

**Appendix B**

Simulations of the tested IM operation were performed with the use of the ANSYS Maxwell software. The Maxwell Transient Solver was employed for accurate modelling of the IM operation conditions under the LCT, since it enables modelling of the IM operation while taking account of rotor motion. The 2D FE mesh (Figure A2) was generated by means of the TAU flex meshing algorithm. The length of the IM and the skew of rotor slots were included in the FE model by making use of 2.5D modelling. The direct-on-line starting of the investigated IM under specified load torque was simulated. The stator instantaneous voltages and currents and the shaft angular velocity returned by the FE model were subsequently exported to the Matlab environment for further processing.
Appendix C

The investigated IM was powered by the sinusoidal voltage generated by the programmable AC source AMETEK Model: 3001iX. The 4Q thyristor converter Parker DC590P powering the separately excited DC machine of type PCMb 54b enabled controlling the load torque of the tested IM under the LCT. The National Instruments USB-6255 high-resolution, multifunction I/O device served for data acquisition. The configuration of the I/O device and measurement data acquisition were performed with the use of the National Instruments LabView software. Processing of the measurement data and visualisation of the research results were conducted in the Matlab environment.

Figure A3. The schematic diagram of the laboratory test bench.
Appendix D

The electromagnetic parameter estimation for the IM equivalent circuit including the rotor deep-bar effect was conducted with the use of the Genetic Algorithms for Optimisation Toolbox in the Matlab environment. In the presented studies, the following evaluation function was adopted [14,24]:

\[
F(|L_1(\omega_1)|, \angle L_1(\omega_2)) = \sum_{\omega_{2\min}}^{\omega_{2\max}} k_{\text{mod}} \left( \frac{|L_1(\omega_1)| - |L_1(p=j\omega_2)|}{|L_1(\omega_2)|} \right)^2 + \sum_{\omega_{2\min}}^{\omega_{2\max}} k_{\text{arg}} (\angle L_1(\omega_2) - \angle L_1(p=j\omega_2))^2
\]

where, \(|L_1(\omega_1)|, \angle L_1(\omega_2)|\) constitute the modulus and argument of the reference frequency characteristic, respectively, \(|L_1(p=j\omega_2)|, \angle L_1(p=j\omega_2)|\) indicate the modulus and argument of the frequency-domain inductance, respectively, \(\omega_{2\min}\) and \(\omega_{2\max}\) are the lower and upper limit of the considered slip frequency range and \(k_{\text{mod}}, k_{\text{arg}}\) represent the weighting factors of the evaluation function components.

The search space bounds for individual electromagnetic parameters of the IM equivalent circuit including the rotor deep-bar effect were defined based on parameters of the T-type equivalent circuit. The electromagnetic parameters of the T-type equivalent circuit were determined according to the procedure reported in the standard [25] relying on the measurement data from the LCT conducted experimentally.

Satisfactory conformity between the reference frequency characteristics and the approximating ones was achieved with the use of the ranking selection function and the following genetic operators: the heuristic crossover and the multi-non-uniform mutation.

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