MODEL-INDEPENDENT PREDICTIONS OF
BIG BANG NUCLEOSYNTHESIS

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Abstract

We derive constraints from standard (with $N_\nu = 3$) BBN arising solely from two cosmologically produced nuclides, $^4$He and $^7$Li, from which the extrapolation to primordial abundances is straightforward. The abundances of D and $^3$He are at present only inferred from their solar and local interstellar medium values using models of galactic chemical evolution. However, our knowledge of chemical evolution suffers from large uncertainties, and so it is of use to take an approach which minimizes the reliance on chemical evolution in determining the consistency of BBN. Using only the data on $^4$He and $^7$Li, and reasonable estimates of possible systematic errors in their abundance determinations, concordance is found if the baryon-to-photon ratio lies in the range $1.4 < 10^{10} \eta < 3.8$ (95% CL). The $^4$He and $^7$Li abundances are also used to predict the primordial abundances D and $^3$He, which provides an initial condition for their chemical evolution.
The consistency of big bang nucleosynthesis (hereafter BBN) is of major importance to standard cosmology, as BBN tests the big bang at the earliest epoch accessible thus far [1]. While BBN theory is well-understood, determining its consistency with the data is difficult because the observables are not the primordial abundances themselves, but rather present-day or solar abundances. We have, at present, only local and recent measurements from which we must infer the primordial light element abundances. Such an extrapolation falls within the domain of chemical evolution, a discipline fraught with its own difficulties and uncertainties. In this paper, we will seek to minimize the use of chemical evolution in determining the viability of BBN, and indeed we will turn the problem around and offer BBN results as constraints on chemical evolution models.

For clarity, we explicitly distinguish the three features inherent to “BBN” and required in evaluating its consistency. (1) Theory, i.e., the calculation of light element abundances as a function of the baryon-to-photon ratio \( n_B/n_\gamma \equiv \eta \). We will consider here only the standard model of big bang nucleosynthesis with three massless neutrinos, \( N_\nu = 3 \); with \( N_\nu \) fixed, \( \eta \) is the only free parameter. The calculation proceeds from first principles and is well understood; its uncertainties are those due to nuclear cross sections measured in the lab and so admit the usual error analysis techniques. This has led to monte carlo calculations of the big bang abundances [2] - [5], all of which are in good agreement. (2) Data, i.e., the observed light element abundances in stars, or in galactic and extragalactic gas. The observations have varying degrees of statistical uncertainty, but the more serious problem is that the systematic uncertainties, while hard to determine, could well dominate the error in the data. (3) Chemical evolution, which accounts for galactic nucleosynthesis processing in the observed abundances and so allows extrapolation to the primordial abundances needed to test the theory. Beyond a set of general principles, chemical evolution is very uncertain in its details, and we are lacking as yet a first-principles theory, relying rather on a simplified (yet in some aspects, surprisingly successful) model, which itself has many uncertain parameters (see e.g. [6]). Notice that BBN is an empirically testable theory only with all three features, in particular, only with the introduction, at some level, of chemical evolution.

Given that the detailed predictions of chemical evolution have questionable reliability and accuracy, and also given the need to include chemical evolution in the evaluation of BBN,
several approaches suggest themselves. One is to try to find the most generic (and hopefully model-independent) predictions of chemical evolution, either by using the best available parameterizations and examining the range in which they may vary \([7] - [9]\), or by making a simple parameterization of the basic predictions as relevant to BBN \([10] - [12]\). Such an approach makes all four light element species available to test BBN, and so can provide a powerful test of the theory as the (one-parameter) predictions are strongly over-constrained. However, such an approach must always be subject to criticisms of the chemical evolution procedure used, particularly for elements for which the effects of evolution are large.

Another means of addressing the role of chemical evolution uncertainties in evaluating BBN—and the approach we will take—is to see what conclusions one can reach with a minimal reliance on chemical evolution. To do so, we first note that the different light nuclides have very different degrees of galactic evolution. Specifically, to infer the primordial abundances of \(^4\)He and \(^7\)Li involves only an appeal to the most general principles of chemical evolution (namely the increase of metal abundances with time) while to determine D and \(^3\)He evolution relies on calculations that are much more detailed (and thus much more model-dependent). Consequently, for the purposes of evaluating BBN, we will restrict ourselves to just the abundances of \(^4\)He and \(^7\)Li; note that these two abundances alone are enough to test the one-parameter, standard BBN theory. We will thus see if there is a concordant range in \(\eta\) for current best estimates of primordial \(^4\)He and \(^7\)Li, and indeed we find one. Then we set confidence limits on BBN predictions for primordial D and \(^3\)He, which then serve as inputs for chemical evolution models.

Our approach to obtaining bounds on \(\eta\) is clearly different from previous ones. In particular, the lower bound on \(\eta\) was first established using the combination of D + \(^3\)He in \([13]\). This argument, which is based on the fact \(^3\)He is not totally destroyed in stars, was used and/or improved upon in later work \([1, 11, 12]\). It has been argued \([11]\) that the lower bound on \(\eta\) is now large enough to indicate a potential inconsistency in BBN. Indeed, when \(^3\)He production in low mass stars is included \([14]\), (this inclusion is implied by recent observations of high \(^3\)He in planetary nebulae \([15]\) ) one is faced with an overproduction of \(^3\)He. As we state below, we will not make any speculations on the source of the problem concerning \(^3\)He here (see e.g. \([10]\)). It is clear however, that the assumed primordial abundances of D and
$^3$He will be strongly dependent on models of chemical and stellar evolution. It is for this reason, that we will perform our analysis independently of these two isotopes. Due to our analysis based only on $^4$He and $^7$Li, our conclusions regarding the consistency range for $\eta$ are also different than in previous work. We will find that a lower value for $\eta$ is preferred and that overlap with other work occurs at about the 2\,$\sigma$ level. In addition, our predicted range for primordial D/H is correspondingly higher than in previous work.

This straightforward analysis is complicated by the need to consider the presence of significant systematic errors in the $^4$He and $^7$Li abundances. While such errors are intrinsically hard to quantify, they are likely to be present and could in fact dominate the error. We will thus consider several possibilities for the size and distribution of the systematic errors, and their possible combinations for the two elements.

The light elements are observed in disparate astrophysical sites, from old stars to galactic as well as extragalactic gas. Questions of chemical evolution aside, the abundances themselves are difficult to determine to the needed accuracy. Systematic errors can arise in the procedure used to deduce an abundance from a line strength, and from the idealizations employed in modeling the sites themselves. In the following we briefly review the situation for each light nuclide, with attention to the possible systematic errors.

$^4$He:

The $^4$He abundance is best determined from observations of HeII $\rightarrow$ HeI recombination lines in extragalactic HII regions. There are extensive compilations of observed abundances of $^4$He, as well as the abundances of N, and O, in many different low metallicity HII regions [17]-[19]. The oxygen abundance in these regions ranges from one fifth to one fiftieth of the solar oxygen abundance. However because $^4$He is produced in stars along with oxygen, the primordial abundance of $^4$He can only be determined from an extrapolation of the data to zero metallicity. Fortunately there is data at low metallicity which lends confidence to such an extrapolation without the reliance of specific models of galactic chemical evolution other than the assumption that both oxygen (as well as nitrogen) and $^4$He increase with time due to their production in stars. In an extensive analysis, using the data of Pagel

\[3\]
and Skillman et al. [18], Olive and Steigman [20] derived a primordial abundance of $Y_p = 0.232 \pm 0.003$. With the inclusion of the recent data of Izatov et al. [19], the best estimate for the primordial $^4\text{He}$ mass fraction becomes [21]

$$Y_p = 0.234 \pm 0.003 \pm 0.005$$

where the first error in eq. (1) is purely statistical. The magnitude of the of the statistical uncertainty is dominated by the large number of extragalactic HII regions observed (over 50) while typical errors in any individual observation are of order 0.01. One of the best observed HII region (and the one with the lowest metallicity), I Zw 18, has an average $^4\text{He}$ abundance [17, 22] of 0.229 $\pm$ 0.004. There are in addition several sources of systematic uncertainties due to ionization corrections, collisional excitation, and the presence of neutral helium. The cumulative uncertainty from these effects has been estimated to be of order 0.005 in the $^4\text{He}$ mass fraction [17, 18, 20] though it may be somewhat higher [23, 1].

$^7\text{Li}$:

The primordial Li abundance is best determined from observations of old, extremely metal poor (population II) halo stars. These stars are observed to have a constant Li/H abundance (the “Spite plateau;” [24]) below a metallicity less than about 1/20 of solar, i.e., $[\text{Fe/H}] \lesssim -1.3$. Given that Fe must increase with time in the Galaxy, the constant Li abundance for all low metallicities indicates that the Li is primordial. Notice that this conclusion relies on chemical evolution only insofar as it is assumed that Fe increases with time in the early Galaxy.

The pop II Li abundance is normally assumed to measure the primordial Li abundance; and indeed Li has been measured in many such stars and so the average abundance can in principle be determined to a high precision. For a given method of converting the raw observations to an abundance, the statistical errors are small. For an individual measurement, a $^7\text{Li}$ abundance typically carries an uncertainty of 0.1 - 0.2 dex in $[\text{Li}]$ ($^7\text{Li}$ abundances are normally quoted as logarithmic quantities so that $[\text{Li}] = 12 + \log \text{Li/H}$). Again, due to the large number of observations the mean value can be determined to within 0.02 dex. We will
use the recent analysis of Molaro et al. [25] in our computations of BBN consistency. This leads to a mean value $[\text{Li}] = 2.21 \pm 0.02$ or

$$\frac{\text{Li}}{\text{H}} = (1.6 \pm 0.1) \times 10^{-10} \quad (2)$$

However, the overall accuracy of the observations suffers from two sources of systematic error. First, the abundance determinations all depend on the model one adopts for the stellar atmosphere; while different models (and different researchers!) get roughly the same answer for the same stars, some discrepancies still remain. Thus we allow for a systematic error of magnitude $\Delta \log_{10}(\text{Li}/\text{H}) = 0.10$ dex or $\Delta_1 = \Delta \text{Li}/\text{H} = +0.4 \times 10^{-10}$. A second and potentially more serious problem is that the Li may have been depleted over the long lifetimes of these stars, and it has been argued that rotational mixing could lead to very large depletions [26].

While such models are hard to exclude, we note that the observations (to include recent determinations of the $^7\text{Li}/^6\text{Li}$ ratio; [27], see also [28]) are well explained by non-rotational models [29] which do not give a significant depletion. However, to be conservative, we will examine the impact of allowing a Li depletion by a factor of 2. Indeed it is also possible that some of the observed $^7\text{Li}$ in halo stars is not primordial and was produced by cosmic-ray nucleosynthesis. While consistency of cosmic-ray nucleosynthesis with the observations of Be and B in the same halo stars restricts the amount of cosmic-ray produced lithium [30], we cannot be sure that some fraction of order 20% is not primordial. Thus we allow for a second systematic error in $^7\text{Li}$ which we take as $\Delta_2 = \Delta \text{Li}/\text{H} = +0.6 \times 10^{-10}$.

**$^3\text{He}$:**

We will not use $^3\text{He}$ to constrain BBN, but we will compare BBN predictions with the observed abundances to evaluate what chemical evolution models will have to do. The observational data is well reviewed elsewhere (for a recent discussion see [16]); the upshot is that solar abundances are

$$\left(\frac{^3\text{He}}{\text{H}}\right)_{\odot} = (2.6 \pm 0.6 \pm 1.4) \times 10^{-5} \quad (3)$$

$$\left(\frac{^3\text{He}}{\text{H}}\right)_{\odot} = (1.5 \pm 0.2 \pm 0.3) \times 10^{-5} \quad (4)$$
The interstellar medium (ISM) abundance for D is reported as \[ [31] \]

\[
\left( \frac{D}{H} \right)_{\text{ISM}} = \left( 1.6 \pm 0.09^{+0.05}_{-0.10} \right) \times 10^{-5}
\]

(5)

however the error bar may be misleading, as recently reported preliminary measurements of D in other lines of sight, with abundances as low as D/H \( \sim 0.5 \times 10^{-5} \) \[32\]. Such variations are surprising in light of chemical evolution predictions for the slow evolution of D, and further suggest that chemical evolution predictions for this element are to be viewed with caution. At any rate, D chemical evolution, while model-dependent in it details, has a clear general trend: D decreases with time as D is only destroyed in Galactic processing. Thus both the solar and the ISM abundances should be lower than the primordial abundance. The ISM \( ^{3}\text{He} \) abundances also show a large dispersion and are found in the range \( ^{3}\text{He}/H \approx 1 - 5 \times 10^{-5} \) \[33\].

While chemical evolution models can in principle account for large destruction factors for D/H over the age of the galaxy, the relative flatness of the \( ^{3}\text{He}/H \) evolution is very difficult to explain. Indeed, if we had confidence in the predictions of galactic chemical evolution one would be able to constrain the primordial D abundance \[1\] and ultimately the consistency of BBN \[11\]. Because stars in their pre-main-sequence stage convert D to \( ^{3}\text{He} \), a high primordial D abundance usually leads to an increasing \( ^{3}\text{He} \) over time and when \( ^{3}\text{He} \) production in low mass stars is included the problem becomes more acute \[14\]. In this case even dramatic changes to standard models of chemical evolution fail, suggesting that perhaps part of the problem lies in the stellar evolutionary predictions for \( ^{3}\text{He} \) \[16\]. However as we stated at the outset, here we will not make any assumptions regarding chemical (or stellar) evolution and use only \( ^{4}\text{He} \) and \( ^{7}\text{Li} \) to test for consistency and therefore make predictions regarding the primordial values of D and \( ^{3}\text{He} \).

Finally, a recent and very exciting development is the improvement of spectral resolution in Lyman-\( \alpha \) forest allows for the possibility to observe D at high-redshift QSO absorption line systems. A solid D abundance for such systems would be of the utmost interest, as these primitive systems have not suffered much evolution (though they do contain some metals) and so will show a D abundance much nearer to its primordial value. Indeed, such observations have already been reported, with initial published values being surprisingly
high, with $D/H \simeq 2 \times 10^{-4}$ \cite{34}. However, with the report of a much lower abundance in a
different line of sight \cite{35}, the situation has become confused. We feel that this technique is
too new to provide a basis for an evaluation of BBN, but clearly this method may come to
provide a strong and clean test of BBN.

There is one unknown parameter in the standard model of big bang nucleosynthesis, the
baryon to photon ratio, $\eta$. For a given value of $\eta$, the only real uncertainty in the calculation
of the light element abundances comes from the uncertainties in the nuclear (and weak)
interaction rates employed. Thus one can obtain a distribution of abundances at each value
of $\eta$ based on these uncertainties which we assume are Gaussian distributed. Here, we will
use\cite{the Monte Carlo results from Hata et al. \cite{5}}. We therefore have a likelihood distribution
(unnormalized) from the BBN calculation,

$$L_{BBN}(Y, Y_{BBN}) = e^{-(Y - Y_{BBN}(\eta))^2/2\sigma_1^2}$$ \hspace{1cm} (6)

where $Y_{BBN}(\eta)$ is the central value for the $^4\text{He}$ mass fraction produced in the big bang, and
$\sigma_1$ is the uncertainty in that value derived from the Monte Carlo calculations.

There is also a likelihood distribution based on the observations. In this case we have two
sources of errors as discussed above, a statistical uncertainty, $\sigma_2$ and a systematic uncertainty,
$\sigma_{sys}$. For the most part we will assume that the systematic error is described by a top hat
distribution \cite{5, 34}. The convolution of the top hat distribution and the Gaussian (to describe
the statistical errors in the observations) results in the difference of two error functions

$$L_O(Y, Y_O) = \text{erf} \left( \frac{Y - Y_O + \sigma_{sys}}{\sqrt{2}\sigma_2} \right) - \text{erf} \left( \frac{Y - Y_O - \sigma_{sys}}{\sqrt{2}\sigma_2} \right)$$ \hspace{1cm} (7)

where in this case, $Y_O$ is the observed (or observationally determined) value for the $^4\text{He}$
mass fraction. As there is some doubt as to how to treat the systematic uncertainty, we
have also derived the likelihood functions assuming that the systematic errors are Gaussian
distributed. In this case the convolution also leads to a Gaussian, with an error $\sigma^2 = \sigma_2^2 + \sigma_{sys}^2$.

Finally we have also simply shifted the mean value $Y_O$ by an amount $\pm \sigma_{sys}$. In this case

\footnote{We thank Dave Thomas for his invaluable assistance here.}
$L_O$ is also a Gaussian with spread $\sigma_2$. These functions were similarly derived for $^7\text{Li}$. The asymmetric systematic errors in the top hat-Gaussian convolution are easily incorporated: the error on the positive side is inserted in the right error function in (7) while the error on the negative side is inserted in the left error function.

For $^4\text{He}$ we constructed a total likelihood function for each value of $\eta_{10} \equiv 10^{10}\eta$, convolving for each the theoretical and observational distributions

$$L^{4\text{He}}_{\text{total}}(\eta) = \int dY L_{\text{BBN}}(Y, Y_{\text{BBN}}(\eta)) L_O(Y, Y_O)$$

(8)

An analogous calculation was performed for $^7\text{Li}$.

Of course, each observable can individually be reconciled with the one-parameter theory. However, when demanding that both observables be fit simultaneously, one tests the theory. To do this, one examines the product of the individual likelihoods, $L^{4\text{He}}_{\text{total}}(\eta)L^{7\text{Li}}_{\text{total}}(\eta)$; this gives a quantitative measure of the goodness of fit and of the spread in the allowed values in $\eta_{10}$ (if there are any).

We first examine the case that we feel combines the most “standard” of assumptions, namely: (1) $^4\text{He}$ takes the central value $Y_p^0 = 0.234$ as in eq. (1); (2) $^7\text{Li}$ is not depleted in Pop II stars, nor is produced in any great quantity by cosmic-rays, i.e., we ignore the second set of systematic errors $\Delta_2$; (3) $^4\text{He}$ systematics are at the level $\Delta Y_{\text{sys}} = 0.005$; (4) the systematic errors are given by a flat (“top hat”) distribution. With these assumptions, we have calculated the likelihoods for $^4\text{He}$ and $^7\text{Li}$; results appear in figure 1. The shapes of these curves are characteristic, with one peak for $^4\text{He}$, which rises monotonically with $\eta$, and two for $^7\text{Li}$, which goes through a minimum. In this case (and most others) the minimum theoretical $^7\text{Li}$ is somewhat below most of the observational values and so the sides of the minimum are favored, leading to the two peaks, i.e. for a given observational value of $^7\text{Li}$, there are two values for $\eta$ at which this may be achieved.

The combined likelihood, for fitting both elements simultaneously, is given by the product of the two functions in figure 1, and is shown in figure 2. From figure 2 it is clear that $^4\text{He}$ overlaps the lower $^7\text{Li}$ peak, and so one expects that there will be concordance, in an allowed range of $\eta$ given by the overlap region. This is what one finds in figure 2, which does show concordance, and gives an allowed (95% CL) range of $1.4 < \eta_{10} < 3.8$. As we will really only
be interested in the upper limit of this range we will from here on only quote the 95% CL upper limit as being the upper limit of the entire range. Note that the likelihood functions shown in figures 1 and 2 are not normalized to unity. The $\eta$ dependant normalization has however been included. Any further normalization would have no effect on the predicted range for $\eta$.

Thus, for this “standard” case, we find that the abundances of $^4$He and $^7$Li are consistent, and select an $\eta_{10}$ range which overlaps with (at the 95% CL) the longstanding favorite range around $\eta_{10} = 3$. Further, by finding concordance (in this case) using only $^4$He and $^7$Li, we deduce that if there is problem with BBN, it must arise from D and $^3$He and is thus tied to chemical evolution. The most model-independent conclusion is that standard BBN with $N_\nu = 3$ is not in jeopardy, but there may be problems with our detailed understanding of D and particularly $^3$He chemical evolution.\footnote{It is interesting to note that the central (and strongly) peaked value of $\eta_{10}$ determined from the combined $^4$He and $^7$Li likelihoods is at $\eta_{10} = 1.8$. The corresponding value of D/H is $1.8 \times 10^{-4}$ very close to the value of D/H in quasar absorbers in the published set of observations\cite{34}. It is not clear whether this is a coincidence or that we really have evidence that three of light element abundances point to the same value of $\eta_{10}$.}

When we vary some of the values or assumptions concerning systematic errors, our 95% CL range for $\eta$ is somewhat affected. These results are summarized in the table below. Had we used the central value of $^4$He as determined by Olive and Steigman\cite{20}, $Y_p = 0.232$ (all other assumptions held fixed) our 95% CL upper limit shifts down to $\eta_{10} < 3.3$. At this point one should take note at the sensitivity of the upper limit on $\eta$ to the $^4$He abundance.

As the size of the assumed systematic error for $^4$He is sometimes questioned\cite{23,4} we have run our likelihood test for $Y_p = 0.234 \pm 0.003 \pm 0.010$, i.e., we have doubled the assumed systematic error (still treated as a top hat). In this case there is a broad overlap between the likelihood functions for $^4$He and and both peaks for $^7$Li. There are now two peaks in the product of the distributions at $\eta_{10} = 1.8$ and $\eta_{10} = 3.6$ the 95% CL upper limit increases to

\footnote{In fact, this conclusion is firm only if the predicted $D_p$ satisfies the only unquestioned prediction of chemical evolution, namely that D decrease with time and so $D_p > D_\odot$. This is true for all of the low-$\eta$ regions we consider.}
$\eta_{10} < 4.5$.

Table 1: Limits on $\eta$

| $Y_p$       | $\Delta Y_{\text{sys}}$ | sys type | $^7\text{Li}_{\text{sys}}$ | $\eta_{10}^{\text{min}}$ | $\eta_{10}^{\text{max}}$ |
|-------------|------------------------|----------|-----------------------------|-------------------------|-------------------------|
| .234 ± .003 | .005                   | top hat  | $\Delta_1$                  | 1.4                     | 3.8                     |
| .232 ± .003 | .005                   | top hat  | $\Delta_1$                  | 1.3                     | 3.3                     |
| .234 ± .003 | .010                   | top hat  | $\Delta_1$                  | 1.4                     | 4.5                     |
| .234 ± .003 | .005                   | Gaussian | $\Delta_1$                  | 1.3                     | 4.4                     |
| .239 ± .003 | .000                   | shift    | $\Delta_1$                  | 1.7                     | 4.4                     |
| .234 ± .003 | .010                   | top hat  | $\Delta_1 + \Delta_2$      | 1.2                     | 3.8                     |

If we return to our standard values and treat the systematic errors as if they were Gaussian distributed, then there is again a broad overlap between the likelihood functions for $^4\text{He}$ and $^7\text{Li}$ though the two peaks (at $\eta_{10} = 1.8$ and $\eta_{10} = 3.3$) in the product distribution overlap. The 95% CL upper limit to $\eta$ is now $\eta_{10} < 4.4$. One can also imagine the systematic errors amounting to a shift to the central value by $\sigma_{\text{sys}}$. For $^4\text{He}$, we apply this shift upwards and take $Y_p = 0.239 \pm 0.003$. For $^7\text{Li}$, our results are sensitive to the direction we shift the central value. When a shift by $4 \times 10^{-11}$ is added to the $^7\text{Li}$ central value (making it $^7\text{Li}/H = 2.02 \times 10^{-10}$) the $^7\text{Li}$ peaks are split farther apart as this value of $^7\text{Li}/H$ is obtained in BBN calculations at either higher or lower values of $\eta$. There is now only a minimal overlap between the likelihood functions of $^4\text{He}$ (whose peak now sits between the two $^7\text{Li}$ peaks) and $^7\text{Li}$. The product of the likelihood functions now shows two separate peaks, however because of the poor agreement between the two elements we discard this possibility and do not include it in the table. In fact, this disagreement can be quantified by taking the product of the normalized likelihood functions. The (very) low value of the product relative to the other cases we consider would be such a signal. If instead we apply a downward shift of $3 \times 10^{-11}$ in $^7\text{Li}$, there is substantially more overlap and the 95% CL upper limit to $\eta$ is 3.9. When no shift is applied to $^7\text{Li}$, there is still a reasonable amount of overlap and there are still two peaks in the product distribution at $\eta_{10} = 2.0$ an 3.5, and the upper limit is now $\eta_{10} < 4.4$. It is this case that appears in the table.
Finally, we consider the possibility that the errors in $^7$Li are in fact larger than assumed above, due to either stellar depletion or cosmic-ray production of $^7$Li. In this case, as the systematic errors are treated as top hat distributions, the asymmetric uncertainties in $^7$Li effectively shift upward the central value (cf. eq. (7)). This causes the two peaks in the $^7$Li likelihood distribution to move apart, as we have seen above. The $^4$He likelihood distributions is now almost entirely under the low $\eta$ peak of the $^7$Li distribution. The result is an upper limit, $\eta_{10} < 3.8$. The peak of the product distribution is at $\eta_{10} = 1.7$. In general we found that, contrary to the naïve expectation, when the $^7$Li uncertainties increase substantially (when the effects of depletion are allowed for) the range for BBN consistency actually shrinks rather than expands.

Having found the allowed range of $\eta$, we now turn to the predictions for primordial D and $^3$He. Since D and $^3$He are monotonic functions of $\eta$, a prediction for $\eta$, based on $^4$He and $^7$Li, can be turned into a prediction for D and $^3$He. In figure 3 we show the abundances of D and $^3$He as a function of $\eta_{10}$ along with the one $\sigma$ uncertainty in the calculations from the Monte Carlo results of [3]. We also show by a set of rectangles the 68% (dashed) and 95% CL (dotted) ranges for D and $^3$He as given by our likelihood analysis above. The corresponding 95% CL ranges are $D/H = (5.5 - 27) \times 10^{-5}$ and $^3$He/H = $(1.4 - 2.7) \times 10^{-5}$. Again, any potential inconsistency with BBN must be related to D and $^3$He and thus from our more detailed understanding (or lack thereof) of chemical (or stellar) evolution.

We would like to stress that in essentially all of the cases we considered (with the possible exception of shifting up the observed $^4$He abundance while at the same time shifting up the $^7$Li abundance) we found a broad region of concordance between the predictions of BBN and the observations of $^4$He and $^7$Li. This region of concordance in what we deemed the most standard case allowed values of $\eta$ as are high as 3.8 at the 95% CL. We note that this region overlaps with the one found in [11, 12] but that the overlap is in the “2 $\sigma$” error bars on each side. More importantly, this upper limit as well as lower values of $\eta$ can easily accommodate the evolution of D [8, 14]. The main problem lies with $^3$He. Indeed, it was argued in [11] that a high value of $\eta$ corresponding to a relatively low value of D/H was necessary to account for the evolution of $^3$He. However, when the effects of $^3$He production in low mass stars is included [14, 57, 58] even the most successful models of galactic chemical evolution fail to
explain the observed abundances of $^3\text{He}$. Thus, as these models are not capable of explaining the $^3\text{He}$ abundances at high $\eta$, they can not be used as a constraint on BBN forbidding lower values of $\eta$. We believe that the values derived here must ultimately be incorporated in any model of chemical evolution.

Of all the light element abundances, $^4\text{He}$ and $^7\text{Li}$ are the least dependent on specific models of galactic chemical evolution. In using the best observationally determined values for these elements as well as simple assumptions concerning the treatment of systematic errors, we have found concordance in the one parameter theory of standard big bang nucleosynthesis with $N_\nu = 3$. Though our prediction for the primordial value for D/H is consistent with chemical evolution models we know of no models at present which can account for the evolution of $^3\text{He}$ as implied by the observations of $^3\text{He}$. We also know of no “standard” model which can account for the evolution of $^3\text{He}$ at higher values of $\eta$ (lower values of D/H) when the production of $^3\text{He}$ in low mass stars is included. Of course, if a firm value for primordial D/H can be established from the observations of quasar absorption systems, these predictions will tested.

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Figure Captions

1. Likelihood distribution for each of $^4$He and $^7$Li, shown as a function of $\eta$. The one-peak structure of the $^4$He curve corresponds to its monotonic increase with $\eta$, while the two-peaks for $^7$Li arise from its passing through a minimum.

2. Combined likelihood for simultaneously fitting $^4$He and $^7$Li, as a function of $\eta$.

3. D/H and $^3$He/H as a function of $\eta_{10}$ from BBN along with the one $\sigma$ uncertainty from Monte Carlo calculations $^3$. Also shown are the values (demarcated by rectangles) of D/H and $^3$He/H consistent with 68% (dashed) and 95% CL (dotted) likelihood values for $\eta_{10}$. 
$L_{\text{total}}^{\eta}$, $L_{\text{total}}^{\eta_1}$,

$\eta_{10}$, $4\text{He}$, $7\text{Li}$,
$L_{\text{total}}(\eta) \times ^7\text{Li}$
