Vibrations occur in many areas ranging from heavy-duty industry over micro sensors and production machines to appliances in the private household. A key concept for limiting such vibrations is vibration control that avoids or at least minimize unintentional product behavior or negative effects on the health and safety of humans and animals. A concept for attacking vibrations in mechanical systems is outlined based on time-periodic modulation of a physical parameter in the system. This concept can be applied in parallel to existing controls and allows the artificial increase of damping transient dynamics.

**Keywords**
vibration control, parametric anti-resonance

**1. Introduction**

Vibrations occur in many areas ranging from heavy-duty industry over micro sensors and production machines to appliances in the private household. A key concept for limiting such vibrations is vibration control that avoids or at least minimize unintentional product behavior or negative effects on the health and safety of humans and animals, see (ISO 5349-1). For example, the whole-body vibrations in agricultural track-laying tractors may affect the operator’s health and work efficiency [1-2]. Another example is the generation of electrical energy in heavy-duty power plants [3]. The efficiency with which such power plants translate gas/oil to electrical energy depends strongly on the gap between blades and casing. This gap needs to be large enough for allowing transient operation of the power plant due to steadily changing power needs but small enough for ensuring a cost-effective and efficient operation. A further example are wind turbine blades in the so-called green industry [3-4]. The trend towards longer blades makes wind turbines prone to wind-induced vibrations that may limit the turning speed or reduce the lifetime of the blade material. A modern concept for attacking vibrations in mechanical systems is outlined that can be used in parallel to existing controls and allows the artificial increase of damping transient dynamics [4].

Introducing a time-periodic modulation of a system parameter may trigger an energy transfer between vibration modes [5]. If this energy transfer induces a vibration reduction the associated modulation frequency is called parametric anti-resonance frequency. For a system with multiple natural frequencies, the simplest parametric anti-resonance may occur at the sum or difference between two of the many natural frequencies

\[ \nu = \frac{|\omega_i \mp \omega_j|}{n} \quad i, j, n = 1, 2, \ldots \]  

The main benefit is outlined in Fig. 1 for the example of a linear, unstable system. The original system possess some unstable and stable poles. Additional poles may be located on the left half-plane. Introducing a time-periodic modulation of a physical parameter in the system, for example modulating the original stiffness \( k \), leads to the time-periodic stiffness \( k(t) \). Tuning the modulation close to a parametric anti-resonance frequency induces a modal interaction between the original system modes and allows for an energy transfer between these two
vibration states of the system. This is a selective, externally triggered energy transfer. Translated to the complex plane in Fig. 1, the original system poles move with respect to the real parts towards a fix point, which eventually results in a stable system. The concept goes back to the pioneering idea of Tondl (1978) [6] and was verified theoretically and experimentally for different test rigs with flexible structures [7].

Figure 1. Movement of the system poles at induced modal interaction: The original system poles of the unstable system with constant stiffness k (left) move with respect to the real parts towards a fix point if an intentional modal interaction is triggered by k(t) which results in a stable system at parametric anti-resonance (right).

**Potential applications**

The strength of this concept is that it is applicable in parallel to already existing control concept. Some potential areas for different mechanical systems are outlined in the following. The first experimental verification was performed by Tondl using an analog electronic circuit. This work motivated analytical, numerical and experimental work on systems utilizing a parametric anti-resonance [7-8]. A test rig of flexible structure is shown exemplary in Fig. 2: an uniaxial electromagnetic actuator used as a time-modulated support of a flexible cantilever. The effective damping observed during transient dynamics of the cantilever are collected at different modulation frequencies ν. It is clearly shown that if the modulation frequency hits a parametric resonance frequency, the effective damping is decreased and the system response may become unstable. However, tuning the modulation frequency ν close to a parametric anti-resonance frequency as defined in eq. (1), triggers a significant increase of the effective damping. A comparison between the transient vibrations for the nominal system and at activated time-periodic modulation of the additional support shows the increase in effective damping.

Figure 2. Uniaxial electromagnetic actuator [7]: (left) Cantilever with additional electromagnetic mount, (right, top) Effective damping at a parametric excitation amplitude of ε = 0.8, (right, bottom) Comparison of transient vibrations for the nominal system and at activated parametric anti-resonance.
Since then, the concept was tested for different applications in structural dynamics at macroscale, e.g. rotordynamics and microscale, e.g. micro-electromechanical systems (MEMS). A milling machine may experience vibrations, a kind of self-excited vibration mechanism, that occur above a certain cutting depth. A typical stability map is shown in Fig. 3. Employing a magnetically supported spindle for the milling operation [1] [8] allows for an additional time-periodic modulation of the bearing stiffness. Tuning the frequency of this modulation properly generates a dense region of stable milling operation even for large values of the cutting depth.

An efficient implementation of time-periodic modulation in MEMS is shown in Fig. 5. A sensor beam driven at a forced excitation during the time interval $t_d$ allows for a desired measurement. After the successful measurement, the sensor beam vibrations need to be mitigated quickly for getting ready for the next measurement cycle. A parametric anti-resonance triggers an energy transfer between the first mode of the sensor beam and the first mode of the auxiliary beam [9-10]. The transient time histories are depicted in Fig. 4 on the top right. The energy transferred to the auxiliary beam is eventually dissipated by structural damping.

Vibrations play also an important role in the power conversion industry. Rotors in this industry are typically supported by fluid-film bearings that may show instability (whirl) above a critical speed, see Fig. 5 [12]. Introducing an adjustable bearing enables the implementation of a time-periodic modulation and
the chance of mitigating vibrations by a parametric anti-resonance. The vibration mitigation was confirmed theoretically in [13-14] and experimentally in [14]. A real rotor was analyzed [14-15] and shows a significant increase of the onset speed of rotor whirl. It is also possible to perform high speed balancing of a rotor without turning the rotor at high speed [15].

The concept of parametric anti-resonance is most beneficial for systems with low structural damping. A minimum requirement is that the system possesses at least two vibration modes. If the original system has only one vibration mode, an additional subsystem must be attached. It should be noted that the concept allows for the reduction, and sometimes even suppression of transient vibrations. Other potential applications that should be investigated in future are the transient vibration of long wind turbines blades [16-17], the drivetrain of cars and wind turbines in Fig. 6 [18] or the seat dynamics of a farming vehicle [19-20].

Figure 5. Rotordynamic systems at parametric anti-resonance: (left, top) active deformation of the bearing shell [13], (right, top) experimental test rig of a fluid-film bearing with moveable bearing half shell [14], (bottom) typical rotor for power generation up to 12 MW and the corresponding stability map showing a significant increase of the whirl speed [15]

Figure 6. Drivetrain of a wind turbine: (left) assembly, (right) simplified mechanical model [18]
Conclusions

The concept of parametric anti-resonance is applicable in parallel to existing control strategies and allows for suppressing or at least damping of transient vibrations. Several proven examples and future directions are outlined briefly. Time-periodic systems have been analyzed since decades due to their severe vibration response at parametric excitation. On the contrary, the concept of parametric anti-resonance at macroscale systems was understood recently and enable a structured and efficient implementation in real engineering applications.

References

[1] Dohnal, F. (2012a), A contribution to the mitigation of transient vibrations: Parametric anti-resonance: Theory, experiment and interpretation, Habilitation thesis, Technical University Darmstadt, Germany.
[2] Altintas, Y. (2012), Manufacturing automation: metal cutting mechancis, machine tool vibrations, and CNC design, Cambridge University Press.
[3] Becker, K. (2019), Dynamisches Verhalten hydrodynamisch gelagerter Rotoren unter Berücksichtigung veränderlicher Lagergeometrien (in German), PhD thesis, Karlsruher Institut für Technologie (KIT).
[4] Dohnal, F., Hörttagel, W. and Zamojski, M. (2019), Numerical and analytical investigation of chatter suppression by parametric excitation, Proceedings of the 15th International Conference Dynamical Systems – Theory and Applications, DSTA, Łódź, Poland, 8 pages.
[5] ISO 5349-1, International Standard, Mechanical vibration and shock, Human exposure to mechanical vibration and shock.
[6] Li, C., Zhou, Y., Lim, T.C. and Sun, G. (2016), Dynamic responses of a wind turbine drivetrain under turbulent wind and voltage disturbance conditions, Advances in Mechanical Engineering, Vol. 8(5), pp. 1–12.
[7] Chasalevris, A. and Dohnal, F. (2016) Improving stability and operation of turbine rotors using adjustable journal bearings, Tribology International, Vol. 104, pp. 369–382.
[8] Al-Hadad, M., McKee, K.K. and Howard, I. (2019), Vibration characteristic responses due to transient mass loading on wind turbine blades, Engineering Failure Analysis, Vol. 102, pp. 187–202.
[9] Dohnal, F. (2012b), Experimental studies on damping by parametric excitation using electromagnets, Proceedings of the Institution of Mechanical Engineering, Part C: Journal of Mechanical Engineering Science, Vol. 226, pp. 2015–2027.
[10] Abele, E., Dohnal, F., Feulner, M., Sielaff, T. and Daume, C. (2018), Numerical investigation of chatter suppression via parametric anti-resonance in a motorized spindle unit during milling, Production Engineering, Vol. 12, pp. 309–317.
[11] Mendes, R.U. and Dohnal, F. (2019), Tuning of parametric excitation for rotor balancing, Journal of Physics: Conference Series, Vol. 1264, 012036, 11 pages.
[12] Mohanta, R.K., Chelliah, R.C., Allamsetty, S., Akula, A. Ghosh, R. (2017), Sources of vibration and their treatment in hydro power stations – A review, Engineering Science and Technology, Vol. 20, pp. 637–648.
[13] Ramírez Barrios, M., Dohnal, F. and Collado, J. (2019), Enhanced vibration decay in high-Q resonators by confined of parametric excitation, submitted to Archive of Applied Mechanics.
[14] Tondl, A. (1978). On the Interaction between self-excited and parametric vibrations, Monographs and Memoranda, Vol. 25, Prague: National Research Institute for Machine Design.
[15] Tondl, A. and Dohnal, F. (2013), Using time-periodicity for inducing energy transfer between vibration Modes, Proceedings of ASME 9th International Conference on Multibody Systems, Nonlinear Dynamics, and Control, USA, DETC2013-12936, 10 pages.
[16] Cutini, M., Brambilla, M. and Bisaglia, C. (2017), Whole-body vibration in farming: Background document for creating a simplified procedure to determine agricultural tractor vibration comfort, Agriculture, Vol. 7(84), 20 pages.
[17] Vallone, M., Bono, F., Quendler, E., Febo, P. and Catania, P. (2016), Risk exposure to vibration and noise in the use of agricultural track-laying tractors, Annals of Agricultural and Environmental Medicine, Vol. 23, pp. 591–597.
[18] Yang, J., Mu, A. and Li, N. (2019), Dynamical analysis and stabilization of wind turbine drivetrain via adaptive fixed-time terminal sliding mode controller, Mathematical Problems in Engineering, Vol. 2019, 8982028, 14 pages.
[19] Staino, A. and Basu, B. (2015), Emerging trends in vibration control of wind turbines: A focus on a dual control strategy, Philosophical Transactions of the Royal Society A, Mathematical, Physical and Engineering Sciences, Vol. 373(2035), 16 pages.
[20] Pfau, B. and Markert, R. (2017), A two-lobe journal bearing with adjustable gap geometry for vibration reduction of flexible rotors, Technische Mechanik, Vol. 37, pp. 109–119.