Spin–orbit interaction in the magnetization of two-dimensional electron systems

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We review recent experimental and theoretical work on the quantum oscillations of the magnetization $M$, i.e., the de Haas–van Alphen (dHvA) effect, in two-dimensional electron systems (2DESs) with spin–orbit interaction (SOI). We focus first on a theoretical modeling by numerically solving the Hamiltonian including the Rashba (R) and Dresselhaus (D) SOI and the Zeeman term in an arbitrarily tilted magnetic field $B$. We second present experimental data on the SOI-modified quantum oscillations of $M(B)$ in 2DESs formed in the InGaAs/InP and AlGaAs/GaAs material systems for different tilt angles between the 2DES normal and the direction of $B$. We find pronounced beating patterns in InGaAs/InP that are described quantitatively by assuming a dominant R-SOI except for a distinct frequency anomaly in $M$ present in nearly perpendicular $B$. In AlGaAs/GaAs, beating patterns occur at large tilt angles. Here, anomalies in the dHvA wave form occur. The findings demonstrate that the understanding of the ground state energy of a 2DES is incomplete when SOI is present. Finally, we predict that the amplitude and anisotropy of specific dHvA oscillations with respect to the in-plane magnetic field component allow one to quantify the magnitude and relative signs of both R-SOI and D-SOI when simultaneously present.

1 Introduction

The spin–orbit interaction (SOI) plays a central role in spintronics [1–4] due to a number of reasons. On the one hand, control of the spin degree of freedom of charge carriers in semiconductors by electrical means is desired. On the other hand, unwanted spin decoherence effects are intimately linked to the SOI. In two-dimensional electrons systems (2DESSs), spin control can be achieved by means of the Rashba spin–orbit interaction (R-SOI) [5] that arises due to the artificially tailored structure inversion asymmetry. Tuning of the R-SOI by means of field-effect electrodes is at the heart of the ballistic spin transistor proposed by Datta and Das [6] and has been first demonstrated experimentally by analyzing the beating patterns in Shubnikov–de Haas (SdH) oscillations.
occuring in the longitudinal magneto-resistivity \( \rho_{//}(B) \) of 2DESs [7–9]. Such beating patterns can arise due to two unequally spaced sets of Landau levels (LLs) caused by the hybridization of the SOI-induced zero-field spin splitting and the field-dependent Zeeman splitting in a transverse magnetic field \( B \). However, the origin of beating patterns observed in magnetotransport measurements has been discussed controversially in the literature [10, 11].

In 1984, Bychkov and Rashba [5] proposed in their pioneering paper to measure the magnetic susceptibility and the de Haas–van Alphen (dHvA) effect to observe and quantify the spin splitting induced by R-SOI in asymmetric heterostructures. The magnetization \( M = -\partial U/\partial B \) is a thermodynamic state function that at low temperature \( T \to 0 \) reflects the variation of the ground state energy \( U \) with magnetic field. In particular, the authors predicted a beating pattern in the quantum oscillations of \( M \). Thermodynamic quantities like \( M \) are powerful to study the evolution of electronic states in a magnetic field since they allow for an explicitly quantitative analysis and provide fundamental insight without further assumptions [12, 13]. Since the seminal work of Bychkov and Rashba, numerous theoretical papers have discussed the beating patterns in dHvA traces and the information they yield on the SOI [14–17]. Experimental data, however, have not been available for a long time due to the challenging magnetometry [18, 19].

In addition to R-SOI, also the Dresselhaus type spin–orbit interaction (D-SOI) arising from the bulk inversion asymmetry of the host crystal is relevant. This was pointed out, e.g., in the proposal of a robust nonballastic spin transistor by Schliemann et al. [20], which requires the tuning of R-SOI and D-SOI to equal strength. However, it is challenging to experimentally determine the relative contributions of Rashba and Dresselhaus terms to the SOI. In the analysis of experimental beatings aiming to obtain the R-SOI value, the D-SOI contribution is normally neglected [7–9, 21–24]. This is owed to the fact that the straightforward analysis of beating patterns in perpendicular magnetic fields alone does not yield unambiguous results when both R-SOI and D-SOI play a comparable role. Further, the weak antilocalization phenomenon in \( \rho \) near \( B = 0 \) can be used to determine SOI constants. Here, different model have been developed which help to evaluate experimental data only in specific parameter regimes [25–30]. The spin–Galvanic effect [31] was employed to extract the ratio and relative sign, but not the absolute values of R-SOI and D-SOI constants. R-SOI and D-SOI strength were also inferred from time-resolved Faraday rotation experiments [32] and from simultaneous measurements of the in-plane electron g-factor and the spin relaxation rate monitored by spin-quantum beat spectroscopy [33]. Fal’ko [34] considered cyclotron and electric-dipole spin resonance theoretically in tilted magnetic fields. The author provided an approximate analytical formula to determine both SOI contributions but left the exact diagonalization outside the scope of the article.

In this feature article, we review recent experimental and theoretical work addressing the quantum oscillatory magnetization \( M(B) \) in 2DESs subject to SOI. We first introduce a theoretical model that describes \( M(B) \) by numerically solving the Hamiltonian including R-SOI and D-SOI and the Zeeman term in an arbitrarily tilted magnetic field. We then discuss experimental data \( M(B) \) exhibiting SOI signatures obtained on an InGaAs/InP asymmetric quantum well (QW) and an AlGaAs/GaAs heterojunction for different tilt angles between the 2DES normal and the direction of \( B \) [35, 36]. We find pronounced beating patterns in InGaAs/InP already in nearly perpendicular magnetic fields. The data are analyzed using the theoretical framework and compared to the results of magnetotransport experiments. The experimental data of the InGaAs/InP samples exhibit a clear anomaly in the frequency of the magnetic quantum oscillations at small tilt angles between the 2DES normal and the direction of \( B \). SdH oscillations do not exhibit the anomaly. We show that the anomaly is unexpected and in contrast to the established understanding. In AlGaAs/GaAs beating patterns are not resolved for small tilt angles but occur at large tilt angles, where the beating patterns are shifted to higher perpendicular magnetic field values by the interplay of SOI and Zeeman effect. Here, an anomalous dHvA wave form is observed that is not explained by existing theories. The results show that the thermodynamic properties of 2DESs subject to SOI are not yet fully understood. Finally, we discuss that the quantum oscillatory magnetization \( M \) can be used to extract both R-SOI and D-SOI constants as well as their relative sign. The approach relies on measuring the amplitudes of specific magnetization oscillations corresponding to SOI-induced level anticrossings in tilted fields near the quantum limit, i.e., in a regime where beating patterns are absent.

The article is organized as follows. We present the theory in Section 2 and describe the experimental details in Section 3. Experimental data on \( M \) at low tilt angles are presented in Section 4. In Section 5, data obtained at large tilt angles between magnetic field and sample normal are discussed. In Section 6, we propose a magnetization experiment to quantitatively extract both SOI constants when simultaneously present. Conclusions are drawn in Section 7.

### 2 Theory

The equilibrium magnetization of an electron system is a thermodynamic state function defined as \( M = -\partial F/\partial B|_{T,n} \), where \( F = U - TS \) is the free energy of the system (with the ground state energy \( U \) and entropy \( S \)) and \( n \) is the sheet electron density. At \( T \to 0 \) this definition reduces to \( M = -\partial U/\partial B|_{T,n} \), making the magnetization an experimentally observable quantity that can directly be linked to the energy eigenvalues of a model Hamiltonian \( H \) without considering the quantum-statistical treatment of level occupation. In the following, we describe the exact numerical calculation of the quantum oscillatory magnetization \( M(B) \), i.e., the dHvA effect of III–V semiconductor 2DESs including R-SOI, D-SOI, and the Zeeman effect in arbitrarily tilted magnetic fields.

#### 2.1 Model Hamiltonian

In III–V semiconductor 2DESs with zinc blende structure grown in the [001] dire
tion, both R-SOI and D-SOI can play a role when the
confining potential in the growth direction is asymmetric.
We consider only the lowest subband of the quantum well.
The Hamiltonian of the problem can be written as \( H = H_0 + H_{SO} + H_{SO}^2 \). The spin–orbit parts of the Hamiltonian read [37]

\[
H_{SO}^k = \frac{\alpha_k}{\hbar} \left( \sigma \cdot \pi - \sigma \cdot \pi_z \right) + \frac{\beta_k}{\hbar} \left( \sigma \cdot \pi_z - \sigma \cdot \pi_x \right),
\]

\[
H_{SO}^3 = \frac{\gamma_0}{\hbar} \left( \sigma \cdot \pi^2 - \sigma \cdot \pi_z \right).
\]

Here, \( \alpha_k \) is the R-SOI coupling strength, and \( \gamma_0, \beta_0 \) are the coupling parameters for the D-SOI that are cubic in the in-
plane wave vector \( k \) and linear in \( k \), respectively. The \( k \)-linear term arises from the \( k^3 \) bulk D-SOI due to the confinement in the growth direction (\( z \)-direction in our notation). \( \gamma_0 \) and \( \beta_0 \) are connected by \( \beta_0 = \gamma_0 (k^2) \), where \( (k^2) \) is the averaged squared wave vector in the growth direction. Further, \( \pi_{x, y, z} = \pi_{x, y, z} \), where \( \pi_{x, y, z} = -i \hbar \partial / \partial (x, y) \) are the components of the in-plane momentum operator, \( A_{x, y} \) are the components of the vector potential, and \( e \) is the electron charge. \( H_0 = \pi^2/2m^* + 1/2g^* \mu_B \pi \times \mathbf{B} \), \( \pi = (\pi_x, \pi_y, \pi_z) \) includes the Zeeman term with the effective g-factor \( g^* \), the Bohr magneton \( \mu_B = e \hbar / 2m^* \), and the vector of the Pauli matrices \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \).

Here, \( m^* \) denotes the free electron mass whereas \( m^* \) is the effective mass. For an arbitrary direction of the external magnetic field \( \mathbf{B} = B \mathbf{B} = B \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta \) no analytical solution for \( H \) is known. Here, \( \theta \) is the tilt angle of \( \mathbf{B} \) with respect to the 2DES normal \( \mathbf{n} \) and \( \phi \) is the azimuthal angle with respect to the [100] direction. For a numerical diagonalization, we use the matrix elements of \( H \) between the well-known Landau eigenstates \( | n, \pm \rangle \) of \( H_0 \) with eigenenergies \( E_n^\pm = (n (1 + 1/2)) \hbar \omega_c \pm (1/2)g^* \mu_B B \). Here, \( n = 0, 1, 2, \ldots \) is the Landau quantum number and \( \omega_c = eB/m^* \) is the cyclotron frequency with \( B_c = B \cos \theta \) denoting the magnetic field component parallel to \( \mathbf{n} \). The corresponding matrix elements have been first calculated by Das et al. [38]. The resulting matrix is diagonalized numerically by truncating the matrix dimensions, while including a sufficient number of levels to achieve convergence in all eigenvalues \( E_n^\pm \) used for the calculation of the 2DES properties.

### 2.2 Beating patterns in quantum oscillations

The quantum oscillations of the Fermi energy \( E_F \) are determined as a function of \( B \) using

\[
n = \int_{-\hbar \omega_c}^{\hbar \omega_c} \sum \frac{g(E - E_n)}{dE}.
\]

Here, \( N_n = eB/\hbar \) is the (spin-resolved) LL degeneracy per unit area and \( g(E) \) is an appropriate distribution function accounting for the level broadening. \( g(E) \) will be taken as a Gaussian distribution with broadening parameter \( \Gamma \) in the following [39–41]. \( E_F(B) \) exhibits a clear beating pattern as shown in Fig. 1a for realistic parameters reported by Guzenko et al. [42] for a 10 nm wide InGaAs/InP quantum well. \( E_F(B) \) for \( \Gamma = 0.3 \) meV (\( \Gamma = 0 \)) is shown as dark line (light line). This pattern is a consequence of the nonlinear LL dispersion induced by the SOI, leading to small \( \Delta E \) levels at \( \Delta E \). Both \( E_F(B) \) and \( M(B) \) exhibit pronounced beating patterns in low magnetic fields. The beatings arise due to the nonlinear LL dispersion induced by the SOI, leading to small \( \Delta E \) levels at \( \Delta E \). This pattern is a consequence of the nonlinear LL dispersion induced by the SOI, leading to small \( \Delta E \) levels at \( \Delta E \).
Following Refs. [39, 44, 45], the jumps in the field values, where the LLs are evenly spaced. We emphasize that this condition is fulfilled when the total spin splitting at \( E_F \) is denoted by \( \delta \) in the following, with \( p = 0, 1, 2, \ldots \) [43]. We show \( \delta(B_\perp) \) in Fig. 1c. The crossing points with \( (p + 1/2)\hbar\omega_0 \) mark the node positions in the quantum oscillations as depicted by the dashed vertical lines.

The beating in \( E_F \) manifests itself in many physical quantities. In this article, we focus on the equilibrium magnetization \( M \) as a thermodynamic state function that can be obtained from the ground state energy \( U \) with the following formula:

\[
U = \int_{-\infty}^{E_F} N_{\pm} \sum_{n, \pm} g \left( E - e_n^\pm \right) EdE,
\]

(4)

where \( N \) is the number of charge carriers in the system. The way it can be understood that the quantum oscillations of \( M \) are similar to the oscillations of \( E_F \), when divided by \( B_\perp \). Oscillations in both \( E_F \) and \( M \) are periodic in \( 1/B_\perp \), with sharp jumps occurring at integer values of the filling factor \( v = n/(eB_\perp/h) \). Their envelopes exhibit minima (nodes) at the field values, where the LLs are evenly spaced. We emphasize here that the fast oscillations in \( M \) are strictly periodic in \( 1/B_\perp \), since their period is only determined by the LL degeneracy and the carrier density via \( \Delta(1/B_\perp) = e/(\hbar n_\perp) \).

Calculations of the longitudinal resistivity \( \rho_{xx} \) are performed following the approach in Refs. [7, 46] using the \( e_n^\pm \) determined as described above.

3 Experimental details Magnetization measurements are performed using torque magnetometry in the form of highly sensitive micro-mechanical cantilever magnetometers (MCMs) as shown in Fig. 2. The magnetometers measure the anisotropic magnetization \( M \) of a 2DES via the torque

\[
\tau = M \times B
\]

acting in an external magnetic field \( B \). We note that the torque acting on \( M \) at a fixed value of the magnetic field component \( B_\perp \) perpendicular to the 2DES increases as \( \tan \theta \). The sensitivity of the setup towards oscillations in \( M \) thus increases with increasing tilt angle. The sensor of Fig. 2 is prepared from an undoped AlGaAs/GaAs heterostructure grown by molecular beam epitaxy. A thin-film coil contacted by wire bonds is surrounding the 2DES on the sensor. By passing a current through the coil a well-defined magnetic moment \( M \) is generated and used for the accurate calibration of the MCM. Further leads can be used for transport measurements. The scale bar in the image is 1 mm. The inset shows a schematic side view. The magnetic field \( B \) is tilted with respect to the 2DES normal \( n \). The deflection of the flexible cantilever beam due to the torque \( \tau \) is detected either capacitively or by optical interferometry. For the former, the cantilever backside is metallized and placed parallel to a lithographically defined metallic ground plate on the substrate at a distance \( d \), forming a plate capacitor with capacitance \( C \). For deflections \( \Delta d \ll d \), the capacitance change is a linear measure of the torque. For the optical readout, the cleaved edge of an optical fiber forms a cavity with an Au-coated mirror pad on the cantilever surface. The cantilever deflection is detected by feeding the interference signal into a feedback loop that keeps the fiber-to-cantilever distance constant via adjustment of the fiber position using a piezo-tube.

The magnetization is extracted from \( M = K \Delta(C, V)/B_\perp \sin \theta \), where \( K \) is a calibration constant and \( \Delta(C, V) \) is the change in capacitance or piezo voltage, respectively. The oscillatory part of \( M \) is extracted from the raw data by fitting and subtracting a low-order polynomial in \( 1/B_\perp \) [41, 45, 47, 48]. MCMs were operated either in vacuum loading \(^3\)He cryostats or in a \(^3\)He-\(^4\)He dilution refrigerator equipped with superconducting magnets.

4 Magnetization and magnetotransport at low tilt angles Measurements were performed on an AlGaAs/GaAs heterojunction and on InGaAs/InP quantum wells. We concentrate on two specific samples showing a
small LL broadening in the following. Results on further samples from the same wafer and from different wafers were consistent with the results presented here. We denote the samples as GaAs#1 and InGaAs#2 in the following.

Sample GaAs#1 consists of 2DES that is formed at the heterojunction of Al$_{0.33}$Ga$_{0.67}$As and GaAs layers. A 40 nm spacer separates the 72-mm-thick Si-doping layer in the AlGaAs barrier from the 2DES channel. The sample was grown by molecular beam epitaxy on a [001] GaAs wafer [35]. Measurements on the sample with an area of ~1.2 mm$^2$ were performed after brief illumination with a red light emitting diode. Carrier densities after illumination varied between $n_e = 3.2 \times 10^{15}$ m$^{-2}$ and $n_e = 3.3 \times 10^{15}$ m$^{-2}$ in different cooling cycles. The zero-field mobility measured on a reference sample from the same wafer was $\mu = 900$ m$^2$ V$^{-1}$ s$^{-1}$ at $T = 0.3$ K. The $M(B)$ data for small tilt angles $\theta$ were discussed in Ref. [49]. Further details on the data at large tilt angles are found in Ref. [35]. Electrons occupied only the lowest subband in this heterojunction. The effective mass was experimentally determined as $m^* = 0.065 m_e$ [41].

Sample InGaAs#2 consists of an inverted asymmetric 10 nm wide Ga$_{0.23}$In$_{0.77}$As QW embedded between a 150 nm Ga$_{0.47}$In$_{0.53}$As top barrier and an InP bottom barrier. A 20 nm spacer separates the 10-nm-thick Si-doping layer located in the bottom barrier from the QW. The sample was grown by metal organic vapor phase epitaxy on a [001] InP wafer [36]. All measurements were performed after brief illumination with a blue light emitting diode on the sample piece with an area of ~1.2 mm$^2$. A carrier density $n_e = 8.7 \times 10^{15}$ m$^{-2}$ and zero-field mobility $\mu = 37$ m$^2$ V$^{-1}$ s$^{-1}$ were measured at $T = 0.3$ K. Again, the zero-field mobility was determined on a reference sample from the same wafer. The effective mass was determined to be $m^* = 0.037 m_e$ [50].

We show experimental data for sample GaAs#1 for tilt angles $\theta$ between 15° and 60° taken at 30 mK in Fig. 3a. The data are displayed versus $1/B_\perp$. The magnetization exhibits sawtooth-like oscillations characteristic of the dHvA effect. The magnetization exhibits sharp jumps at even integer filling factors $\nu = n_e/(eB/h)$, i.e., at magnetic field positions where the Fermi energy jumps between adjacent LLs as the magnetic field is varied. Additional oscillations with small amplitude become visible at odd integer $\nu$ for field strengths $1/B_\perp \lesssim 1$ T$^{-1}$ as highlighted in the inset. These oscillations are due to the spin splitting of LLs. We find that these oscillations are more pronounced than expected from the Zeeman energy of noninteracting electrons. Such an enhancement is well known in high-mobility GaAs 2DES, where electron-electron interaction effects are important [45, 51]. In Ref. [41], these oscillations have been attributed to exchange enhancement of the $g$-factor, leading to effective $g$-factors of $|g^{*}| \approx 3$ at $\nu = 15$ and $|g^{*}| \approx 7$ in the quantum limit at $\nu = 1$ in this specific sample. The high field behavior of the Zeeman splitting $\Delta E_Z$ was found to be described by $\Delta E_Z = |g| \mu_B B + E_{ex} \approx g^{*} \mu_B B$, where $|g| = 0.44$ is the band structure $g$-factor and $E_{ex} = 0.33/(E_C v)$. Here, we introduced the Coulomb exchange energy $E_C = e^2/(4\pi\epsilon\varepsilon_0 a_B)$ with the magnetic length $l_B = (\hbar/eB)^{1/2}$.

Beating patterns as considered in Section 2.2 are not resolved in this angular regime. This can be due to two different scenarios: First, the R-SOI and D-SOI coupling constants in this heterojunction could be so small that the condition for the last beat node $\delta = (1/2)n_0$ occurs at such small values of $B_\perp$, that they are not resolved due to the level broadening and the experimental resolution. Second, R-SOI and D-SOI are not necessarily small, but of similar strength, such that beating patterns are suppressed in perpendicular magnetic

Figure 3 Magnetization $M(1/B_\perp)$ measured for different tilt angles $\theta$ for (a) sample GaAs#1 and (b) sample InGaAs#2. Both datasets are taken at $T = 30$ mK and the curves are offset in vertical direction for clarity. The oscillation amplitudes increase monotonically with increasing $B_\perp$ in (a). In high magnetic fields, additional oscillations due to the spin splitting become visible at odd $\nu$ (inset). Inset curve was taken at 15° and $T = 1.6$ K. In contrast to (a), the magnetization curves in (b) exhibit a clear beating pattern. The dashed vertical lines in (b) indicate the position of the beat nodes. (c) Longitudinal resistivity $\rho_{xx}$ for the same InGaAs/InP QW as in (b), measured on a Hall-bar sample prepared from the same wafer. A beating pattern is also present in $\rho_{xx}$, with the beat nodes at consistent positions if one accounts for a slight difference in carrier density of $\approx 2\%$. Data in (a) taken from [35], data in (b) and (c) taken from [36].
fields (see discussion in Section 6 and [52]). In both cases, beating patterns may however occur in strongly tilted magnetic fields due to the interplay of R-SOI and D-SOI with the enlarged Zeeman energy. This situation will be discussed in Section 5.

In Fig. 3b, we show $M(1/B_\perp)$ for sample InGaAs#2 for the same tilt angle and magnetic field regime as for sample GaAs#1 in Fig. 3a. A pronounced beating pattern is observed in the dHvA oscillations as first predicted by Bychkov and Rashba [5]. At $\theta = 15^\circ$, the last beat node is found at $1/B_\perp \simeq 1.6$. For larger $\theta$, the second beat node is resolved at $1/B_\perp \simeq 3.3$ due to the stronger torque signal.

Before we analyze the magnetization data in Fig. 3b in more detail, it is instructive to consider the SdH oscillations in $\rho_{xx}(1/B_\perp)$ measured on a reference sample from the same wafer as displayed in Fig. 3c. Note that in the $\rho_{xx}$ traces, the minima are known to mark the positions of integer $v$. The $\rho_{xx}$ traces exhibit the well-known beating pattern attributed to SOI as well. Comparing Fig. 3b and c, we find that the beat node positions in $M$ and $\rho_{xx}$ are at corresponding field positions if one takes into account the small difference in carrier density of about 2%. Thus, at first sight, the overall behavior of $M$ and $\rho_{xx}$ seems to be consistent. This is, however, not the case if one considers the periodicity of the oscillations in $1/B_\perp$. We will discuss the anomalous behavior of the oscillation frequency that we found for $M$ in Section 4.1.

We first focus on the beating patterns in $M$ and $\rho_{xx}$ as described by their beat nodes and envelopes. We analyze them in the framework of the theory introduced in Section 2. For this, we replotted the experimental data for $\rho_{xx}$ at $\theta = 15.2^\circ$ and $M$ at $\theta = 60.2^\circ$ as the uppermost curves in Fig. 4a and c, respectively. We now show that the experimental results are well described by assuming dominant R-SOI, with a small admixture of D-SOI. To illustrate this, we plot the results of calculations assuming only $\delta_R \neq 0$ and $\alpha_R, \gamma_R = 0$ as the center curves in Fig. 4a and c. Here, the size of the zero-field Dresselhaus spin-splitting $\Delta_D = 2\beta_{\text{D}} k_F = 2.4$ meV was chosen to match the position of the last beat node in the experiment, indicated by the black vertical line. Here, we have introduced the Fermi wave vector $k_F = \sqrt{2\pi n}$. We find that the position of the second last beat node cannot be modeled with this assumptions (dashed vertical line). The lowermost curves in Fig. 4a and c represent calculations with only $\alpha_R \neq 0$. Again, $\alpha_R$ was adjusted to match the position of the last beat node. With this assumption, the position of the second last node is well modeled (dotted vertical line) with only a slight deviation attributed to a remaining admixture of the D-SOI. Calculations assuming R-SOI and D-SOI simultaneously (not shown) reveal that the coupling strength of the $k$-linear D-SOI is $\lesssim 20\%$ of the R-SOI term, while the $k^3$ D-SOI can be neglected. The lowermost curves correspond to $\alpha_R = 4.5 \times 10^{-12}$ eV m giving $\Delta_R = 2\alpha_R k_F = 2.1$ meV. The physical origin of the different relative beat node positions for the R-SOI and D-SOI case is illustrated in Figs. 4b and d for $\theta = 15.2^\circ$ and $\theta = 60.2^\circ$, respectively. Here, the total spin splitting $\delta$ stemming in general from the hybridization of R-SOI, D-SOI and Zeeman terms is plotted versus $1/B_\perp$.

![Figure 4](image)

**Figure 4** (a) $\rho_{xx}(1/B_\perp)$ at $\theta = 15.2^\circ$ taken from the experiment (uppermost curve) and from calculations assuming pure D-SOI (center curve) and pure R-SOI (lowermost curve). Note that the calculations have been scaled in amplitude to match the experiment. For the calculations, the D-SOI (R-SOI) strength has been adjusted to match the position of the last beat node observed in the experiment (black vertical line), yielding $\Delta_D = 2.4$ meV and $\Delta_R = 2.1$ meV, respectively. (b) Total spin splitting $\delta(1/B_\perp)$ (solid line) for R-SOI (D-SOI: dashed line) as in (a). $\delta$ starts from the corresponding zero-field splitting at $1/B_\perp \to \infty$, then decreases and finally approaches the Zeeman splitting in the high-field limit. The crossing points with odd half-integer values of $\hbar \omega_c$ correspond to the beat nodes in (a). (c) $M(1/B_\perp)$ at $\theta = 60.2^\circ$. The beat node positions and analysis are consistent with (a). The magnetization per electron is calculated in absolute units of $\mu_B$. The splitting assuming D-SOI does not approach zero in the intermediate field regime any more due to the SOI-induced avoided crossing of levels in tilted magnetic fields. Experimental data in (a) and (c) taken from [36].

The behavior of $\delta$ for $\Delta_R = 2.1$ meV is shown as solid line. Starting from the zero-field value of 2.1 meV at $1/B_\perp \to \infty$, the total spin splitting first decreases since the energy separation of the different LLs coupled by the SOI increases with magnetic field, making the coupling less efficient. At the same time, the Zeeman energy (dotted lines) increases with magnetic field, dominating the behavior of the total spin splitting at high magnetic fields. The crossing points of $\delta$ with odd half-integer values of $\hbar \omega_c$ (dash-dotted lines) yield the positions of the beat nodes, as discussed in Section 2.2. The dashed line shows $\delta$ for pure $k$-linear D-SOI. Again starting from the zero-field value the hybridized spin splitting reaches zero at an intermediate field value, before it approaches the Zeeman splitting from below in the high field limit. The signs of the D-SOI and the Zeeman term are opposite, leading
to a vanishing splitting at an intermediate field value for perpendicular magnetic field. The R-SOI has the same sign compared to the Zeeman term, thus avoiding $\delta = 0$. Due to this difference, the slopes of $\delta (1/B_n)$ are different for R-SOI and D-SOI. When we now match the crossing points with $1/2\hbar \omega_c$, the crossing points with $3/2\hbar \omega_c$ occur at different field values. This allows us to distinguish the two cases.

The same discussion applies qualitatively to the case of $\theta = 60.2^\circ$ in Fig. 4d with the difference that with increasing tilt angle the spin splitting never reaches zero also in the D-SOI case. Instead, avoided level crossings occur in tilted fields, as has first been pointed out in [53] for the case of R-SOI. We note that the behavior of $\delta$ depends on the relative sign of the appropriate matrix elements. This means in particular that the beating pattern for a given $\Delta_n = \Delta$ and negative sign of the $g$-factor is exactly the same as for $\Delta_0 = -\Delta$ and positive sign of the $g$-factor. However, for systems with large SOI as the InGaAs/InP QW considered here, the $g$-factor can safely be assumed to be negative [14], such that no ambiguity arises from this.

At even higher tilt angles, the last beat nodes start to move to higher $B_\perp$ when the Zeeman splitting approaches $\sfrac{1}{2}\hbar \omega_c$ from below. For $g^*\mu_B B > (1/2)\hbar \omega_c$, the node vanishes and the process is repeated with the second last beat node. This behavior has been discussed in Refs. [21, 22, 38] for $\rho_{\alpha\nu}$. The characteristics of $M$ in sample InGaAs#2 at high tilt angles will not be discussed here. Instead, we focus on the anomalies discovered in the dHvA frequency of sample InGaAs#2 in the regime of low tilt angles in Section 4.1 and on the anomalies in the dHvA waveform in sample GaAs#1 occurring at high tilt angles in Section 5.

### 4.1 Frequency anomaly in the magnetization

The frequency of the dHvA oscillations as a function of $1/B_\perp$ displayed in Fig. 3b exhibits a distinct anomaly. In order to highlight this frequency anomaly in $M$ at $\theta = 15.2^\circ$, we plot integer $\nu$ versus $1/B_\perp$ that were determined separately for each oscillation in Fig. 5a as open symbols. We find that the field positions of integer $\nu$ follow two different slopes after and before the last beat node. This is highlighted by linear fits to the data after (before) the last beat node shown as dashed (dashed-dotted) lines. The solid line corresponds to the sum frequency extracted from a fast Fourier transform (FFT) of the data over the full field regime (not shown). The sum frequency is found to be irrelevant to describe the experimental data $M(B)$. This finding is unexpected and in contrast to the transport data shown in Fig. 5b as crosses. For $\rho_{\alpha\nu}$, the positions of integer $\nu$ strictly follow the slope corresponding to the FFT sum frequency (solid line). Here, the slope does not change in the vicinity of the last beat node. The linear fits from (a) are depicted for reference. The slope of $\nu$ versus $1/B_\perp$ has been interpreted as $(h/e)\eta_\nu$ for decades, thereby yielding the sheet carrier density $n_\nu$ of the 2DES. Following this well-established method, we now define the parameter $n^* = (eB_\perp/\hbar)\nu$ and evaluate the parameter for both magnetization and magnetotransport data. The results are shown in Fig. 5c. For $\rho_{\alpha\nu}$, $n^*$ is found to be constant within the experimental uncertainty for all tilt angles (crosses). By analyzing $M$ in the same way, $n^*$ is found to jump near the last beat node at $\theta = 15.2^\circ$ in Fig. 5c (uppermost graph). This jump is larger than the experimental error bar derived as follows: For $\rho_{\alpha\nu}$, it is well known that $n^* = n_\nu$ = const. for all fields $B$ and angles $\theta$. The variation in the $n^*$ data from $\rho_{\alpha\nu}$ displayed in Fig. 5c thus reflects the error bar of the $n_\nu^*$ evaluation. The remaining angular variation of $n^*$ is attributed to the

---

2The jump is found in $M$ of the 2DES with SOI after and before (not shown) illumination.
small experimental uncertainty in $\theta$. The maximum peak-to-peak error amounts to $\delta n_s^* = (8.68 - 8.52) \times 10^{15} \text{m}^{-2} = 0.18 \times 10^{15} \text{m}^{-2}$, i.e., 2.1%. At $\theta = 15.2^\circ$, the jump in $n_s^*$ extracted from $M(B)$ amounts to 5.2%, being far larger than the maximum error in $n_s^*$. We have analyzed this striking discrepancy between the transport and magnetization data towards higher tilt angles. The jump in $n_s^*$ present in $M$ in nearly perpendicular magnetic field decreases systematically with increasing angle, whereas for $\rho_{xx}$, $n_s^* \approx \text{const.}$ at all tilt angles. We find a constant $n_s^*$ for $M$ only at large tilt angles $\theta \geq 60.2^\circ$ (lowest graph). Here, $n_s^*$ is consistent with a constant (field-independent) $n_s$. The $\rho_{xx}$ data have been obtained on a separate 2DES sample from the same wafer in a separate cooling cycle and illumination step. This accounts for the small difference in $n_s^*$ for $\rho_{xx}$ and $M$ visible at $\theta = 60.2^\circ$.

The thermodynamic ground state property $M(B)$ thus behaves strikingly different from $\rho_{xx}(B)$. We have ruled out experimental artifacts by repeating measurements on different samples in two setups and several cooling cycles. All data sets are consistent, suggesting that there is a fundamental difference in the physics underlying the beating patterns in $\rho_{xx}$ and $M$.

In the following, we demonstrate that the frequency anomaly in $M$ is completely unexpected also from the theoretical side and discuss possible origins. For this, we use the model calculations developed in Section 2 and the R-SOI coupling strength $\alpha_R = 4.5 \times 10^{-12} \text{eVm}$ determined above. We plot the results of the model calculations (assuming R-SOI with $\alpha_R = 4.5 \times 10^{-12} \text{eVm}$ as detailed above) at $\theta = 15.2^\circ$ and $\theta = 60.2^\circ$ as light (red) lines in Fig. 6a and b, respectively. In the calculation, the oscillations occur always at integer $v$ since only the total carrier density $n$, and the LL degeneracy $eB_\perp / h$ enter in the position of integer $v$ [54]. We plot the corresponding experimental data at $\theta = 15.2^\circ$ and $\theta = 60.2^\circ$ as black lines in Fig. 6a and b, respectively. The insets show blow-ups of the regions above and below the last beat node. In (a), the carrier density in the calculations was adjusted to match the $n_s^*$ in the high-field region after the last beat node (left inset). The inset highlights the discrepancy between the constant frequency in the calculation and the frequency anomaly in the experimental data present for $\theta = 15.2^\circ$.

In the bottom-left inset, the experimental oscillation follows the calculated trace. In the bottom-right inset, however, the experimental data are found to oscillate at a higher frequency if compared to the calculation. The insets in (b) highlight the excellent and quantitative reproduction of the experimental oscillation amplitude and frequency for the full field regime by the theory at $\theta = 60.2^\circ$, where the anomaly is absent.

The state-of-the-art theoretical approach presented in Section 2 based on the current understanding of SOI is able to reproduce even fine details in the beating patterns of $M(B)$ in sample InGaAs#2. Furthermore, a quantitative match between experiment and theory is achieved concerning the absolute amplitudes. The only experimental observation that is not captured by the model is the frequency anomaly occurring at tilt angles $\theta < 60^\circ$. Since the magnetization $M$ and the model Hamiltonian are linked in a straightforward way, our finding of the anomaly raises the fundamental question whether the current picture of SOI-induced beating patterns in magnetic-field-dependent quantum oscillations is comprehensive. The physics underlying the frequency anomaly and its evolution with the tilt angle is unknown at present. In the following, we speculate about the possible origin.

The data on the frequency anomaly seem to suggest intuitively that quantum oscillations of $M$ from one carrier subset are seen in the low field regime before the last beat node, whereas in the high field regime oscillations of another carrier subset with a higher density are seen. One might get the idea to associate the two frequencies (apparent densities $n_s^*$) with the two spin subsets present in the 2DES. This is, however, not reasonable since the difference in $n_s^*$ before and after
the node vanishes with increasing tilt angle, while, strikingly, the envelope of the beating pattern itself remains unaffected. How can two different apparent densities \( n^*_s \) appear in the two regimes? It has been put forward that the quantum relaxation times might differ between the two unequally populated spin subsets \([22, 55]\). This might potentially lead to a preferential observation of only one subset in a given field regime. We have modeled \( M \) including a subset-dependent relaxation time and find that this cannot account for our experimental observations.

A rather fundamental difference between \( M \) and \( \rho_{xx} \) is that the energy of all LLs including those well below \( E_F \) enter in \( M \) through the ground state energy \( U \). In contrast, the resistance probes the density of states at \( E_F \), i.e., it does not monitor lower lying energy levels. Thus, an interaction-induced renormalization \([56]\) of LL energies below \( E_F \) beyond the straightforward Hartree–Fock theory \([57, 58]\) could lead to differences between \( M \) and \( \rho_{xx} \). An angular dependence might be introduced by the interplay with the SOI-induced anticrossing of levels that depends strongly on the tilt angle.

### 5 Magnetization at high tilt angles

In this section we consider beating patterns at high tilt angles, focussing on sample GaAs#1 where no beating patterns have been detected in the regime of low tilt angles (see Fig. 3a). We show experimental data \( M(1/B_\perp) \) at different large tilt angles \( \theta \) in Fig. 7. The following striking observations are made: First, at \( \theta = 76.5^\circ \), the node of a beating is located at \( 1/B_\perp \simeq 3 \times 10^{-1} \). With increasing tilt angle the beat node systematically moves towards higher perpendicular magnetic field positions. At tilt angles \( \theta > 8^\circ \), a second node enters from the low field side in Fig. 7. Second, the maximum dHvA amplitudes in the traces decrease strongly with increasing \( \theta \). Third, the waveform of the dHvA oscillations changes systematically when approaching a beat node.

In comparison to sample InGaAs#2, an analysis using the SOI theory from Section 2 is more challenging here, because (i) no beating patterns were found in nearly perpendicular magnetic fields and (ii) the experimental results do not lend themselves to a straightforward modeling in terms of one dominant SOI contribution and the experimentally determined values for \( m^* \) and \( g^* \). This is at least partly owed to the fact that the Zeeman splitting is strongly exchange-enhanced in this high-mobility GaAs 2DES as discussed above. At the same time the beating patterns at such large tilt angles, where the node positions in \( B_\perp \) change with \( \theta \) are strongly sensitive to the value of \( g^* \). The exchange-enhancement of the \( g \)-factor in sample GaAs#1 was discussed in Section 4 and could be evaluated up to \( \nu = 15 \). \( g^* \) values for the low \( B_\perp \) regime addressed here could not be determined, leaving an uncertainty in the relevant value between \( |g^*| \sim 0.4 \) to \( \sim 3 \) with a possible field dependence.

We have presented a detailed discussion that rules out sample inhomogeneities, misalignments, higher subband occupation and magnetointersubband scattering as possible origin of the beatings in Ref. \([35]\). The effect recently observed by Hatke et al. \([59]\) can also be ruled out as the origin of the beating due to the contrasting tilt angle dependence of the beat nodes: In their work, the beatings in wide GaAs QWs were found at high tilt angles with the node positions moving to smaller field values with increasing \( \theta \). The authors attributed this to an in-plane field-induced increase of the effective carrier mass. In contrast, we find that the beat nodes move to larger values of \( B_\perp \) with increasing \( \theta \). Armed with the SOI theory introduced in Section 2 we can attribute this characteristics to the tilt-induced enlargement of the Zeeman term that enters in the total spin splitting \( \delta \): The total spin splitting \( \delta \) is the result of a nontrivial hybridization of the Zeeman splitting and the SOI spin splitting, with the former dominating in high magnetic field values \( B \) and the latter dominating for \( B \to 0 \). Tilting the field now increases the Zeeman splitting at a given perpendicular magnetic field. The last beat node thus starts moving to larger \( B_\perp \) when the Zeeman splitting approaches \((1/2)\hbar \omega_\perp \).

This picture is considerably complicated when both R-SOI and D-SOI are large, leading to a complicated anticrossing behavior of the LLs that gives rise to anomalous beatings \([14, 52]\). We demonstrated in Ref. \([35]\), that the evolution of the beat node positions with angle could be roughly reproduced assuming dominant R-SOI and \( g^* = -1.1 \).
However, the calculated trace did not model the observed functional form as depicted by the solid line in the inset of Fig. 7. More data, in particular for different directions of the in-plane magnetic field component would be needed for a thorough analysis. We describe an experimental procedure capable of separating R-SOI and D-SOI reliably from a magnetization trace for magnetic quantum oscillations presented in Fig. 7. For this, we first numerically solve the model Hamiltonian $H = H_0 + H_{SO}$ defined in Section 2.1 including the $k$-linear R-SOI and D-SOI terms and the Zeeman term in an in particular doubly tilted magnetic field, i.e., explicitly considering the dependence of the energy spectrum on the angles $\theta$ and $\phi$ defined in Section 2.1. The magnetization $M(B)$ is then calculated for different directions of the in-plane magnetic field component. The amplitudes of specific magnetic quantum oscillations and in particular their anisotropy in the 2DES plane are found to quantify the magnitude of both R-SOI and D-SOI constants as well as their relative sign. We use realistic sample parameters in the calculations and provide guidelines for experimental investigations using highly sensitive magnetometry techniques as introduced in Section 3.

Armed with the theoretical framework introduced in Section 2, the numerical calculation of the quantum oscillatory magnetization based on the Hamiltonian first written out by Das et al. [38] is straightforward. However, in order to get analytical insight highlighting the physics underlying this proposal on the one hand and useful approximations for fitting experimental data on the other hand, we choose to reexpress the Hamiltonian following Refs. [34, 65]. For this, the spin quantization axis is chosen along the direction of the oscillations is expected for all dHvA oscillations in a system with fixed electron density (canonical ensemble). We find however, that the steep flank moves from the high-field side to the low field side of the dHvA oscillations as highlighted in Fig. 8b for $\theta = 85^\circ$. In the center of a beat window between the two nodes the wave form of the dHvA signal is triangular. This is unexpected, but is observed reproducibly in the beating patterns of sample GaAs#1 at different $\theta$. We thus find an anomaly of the dHvA waveform in sample GaAs#1 at high tilt angles. We point out that this is not accompanied by a frequency anomaly as in sample InGaAs#2 at low $\theta$, since the positions of the dHvA flanks are strictly periodic in $1/B_z$. The 180° phase shift at the nodes is consistent with the theory.

Speculating about the possible origin of the wave form anomaly it is worth noting that theoretically, the abrupt jumps are expected on the high-field side of the oscillations when calculating $M$ for a canonical ensemble, i.e., a system with fixed $n_i$ and oscillating $E_F(B)$ as has been done in Section 2. However, for a grand canonical ensemble with fixed $E_F$ due to contact to a reservoir, the jumps are predicted to be on the low-field side [60–62]. Such a case has been observed experimentally in Ref. [63]. There is no obvious reason, why a field-dependent change in the thermodynamic boundary conditions should occur in sample GaAs#1 that has no electrical contacts. The complex behavior shown in Fig. 8 remains an open question.

6 Method for the simultaneous determination of R-SOI and D-SOI constants We now present an alternative method for the quantitative determination of R-SOI and D-SOI coupling constants in 2DESs using the magnetization [64]. For this, we first numerically solve the model Hamiltonian $H = H_0 + H_{SO}$ defined in Section 2.1 including the $k$-linear R-SOI and D-SOI terms and the Zeeman term in an in particular doubly tilted magnetic field, i.e., explicitly considering the dependence of the energy spectrum on the angles $\theta$ and $\phi$ defined in Section 2.1. The magnetization $M(B)$ is then calculated for different directions of the in-plane magnetic field component. The amplitudes of specific magnetic quantum oscillations and in particular their anisotropy in the 2DES plane are found to quantify the magnitude of both R-SOI and D-SOI constants as well as their relative sign. We use realistic sample parameters in the calculations and provide guidelines for experimental investigations using highly sensitive magnetometry techniques as introduced in Section 3.

Armed with the theoretical framework introduced in Section 2, the numerical calculation of the quantum oscillatory magnetization based on the Hamiltonian first written out by Das et al. [38] is straightforward. However, in order to get analytical insight highlighting the physics underlying this proposal on the one hand and useful approximations for fitting experimental data on the other hand, we choose to reexpress the Hamiltonian following Refs. [34, 65]. For this, the spin quantization axis is chosen along the direction of the
magnetic field $B$, leading to the rotation

$$
\sigma_+ \rightarrow \sigma_+ \cos \theta \cos \phi - \sigma_- \sin \phi + \sigma_\uparrow \sin \theta \cos \phi, \\
\sigma_- \rightarrow \sigma_- \cos \theta \sin \phi - \sigma_\uparrow \cos \phi + \sigma_\downarrow \sin \phi, \\
\sigma_\uparrow \rightarrow -\sigma_\downarrow \sin \theta + \sigma_\uparrow \cos \theta.
$$

Equation (7) yields for $H_{SO}^k$ [65]

$$
H_{SO}^k = \frac{\pi}{2} \big[ \sigma_+ (\eta_+ + \eta_- \cos \theta) - \sigma_- (\eta_+ - \eta_- \cos \theta) \\
+ \sigma_\uparrow \eta_- \sin \theta \big] + H.c.,
$$

where we used $\sigma_\pm = (\sigma_x \pm i \sigma_y)/2$, $\pi_\pm = \pi_x \pm i \pi_y$, and the convenient notation $\eta_\pm = \beta_0 e^{i \phi} \pm i \gamma_0 e^{-i \phi}$ introduced in [65]. The corresponding nonvanishing matrix elements can be written as

$$
\langle n, \pm | H_0 | n, \pm \rangle = \hbar \omega_c \left( n + \frac{1}{2} \right) \pm \frac{1}{2} \frac{\hbar^* \mu_B}{2} B,
$$

$$
\langle n+1, \pm | H_{SO} | n, \mp \rangle = \pm i \sqrt{\frac{n+1}{2 \hbar^*}} (\eta_+ (\phi) \pm \eta_- (\phi) \cos \theta),
$$

$$
\langle n+1, \pm | H_{SO} | n, \pm \rangle = \pm i \sqrt{\frac{n+1}{2 \hbar^*}} \eta_- (\phi) \sin \theta,
$$

and their Hermitian conjugates.

In the following, we focus on specific tilted field situations where levels are brought to coincidence in a system without SOI, and show that these level coincidences are avoided by an SOI-induced level repulsion in the case of finite SOI. In a system without SOI in the interplay of the Landau quantization energy $(n + 1/2) \hbar \theta c B / m^*$ (which depends on $B_{\perp}$) and Zeeman energy $g^* \mu_B B$ (which depends on the total $B = B_{\perp}/ \cos \theta$) leads to a degeneracy of spin-split LLs whenever $l \cos \theta_c = (g^*/2)(m^*/m_0)$ with $l = 1, 2, 3, \ldots$ Two levels with different Landau and spin indices thus cross as a function of $B/B_{\perp} = 1/ \cos \theta$. Such a crossing between $(n = 1, -1)$ and $(n = 2, +1)$ LLs is shown by the dashed lines in Fig. 9a. Here, the same InGaAs/InP QW parameters as in Fig. 1 were used. The condition for the crossing [66, 67] – commonly named coincidence condition – has been used extensively in experiments to measure $|g^*/m^*|$ [67, 68]. When SOI is present, the situation changes: An anticrossing gap $\Delta_{SO}$ is opened at the critical angle $\theta_c$ due to the resonant coupling induced by the SOI matrix elements (solid lines). The gap $\Delta E$ between the levels $(1, -)$ and $(2, +)$ at integer $\nu = 4$ shown in Fig. 9a has a finite value $\Delta_{AC}$ at $\theta_c$ (Fig. 9b) (although $\Delta E = 0$ would be expected without SOI). At this angle $\theta_c$, the SOI-induced mixing of levels is strong. To first order, the two levels are coupled by the matrix elements

$$
\langle n+1, \pm | H_0 | n, \mp \rangle \quad \text{at} \quad \theta_c.
$$

The anticrossing gap is thus

$$
\Delta_{AC} \approx \frac{\sqrt{2(n+1)}}{\hbar^* B_{\perp}} \times \\
\sqrt{\frac{\beta_0}{B_{\perp}^2}} \left( 1 - \frac{g^*}{|g^*|} b_1 \right) \left[ 4 \alpha_R \beta_0 b_1 + \alpha_R^2 \left( 1 + \frac{g^*}{|g^*|} b_1 \right) \right].
$$

Equation (12) is consistent with the results of [34, 65]. $\Delta_{AC}$ is anisotropic with respect to the direction of the in-plane magnetic field component when both $\alpha_R$ and $\beta_0$ are $\neq 0$. Following Eq. (5), the peak-to-peak amplitude of the corresponding magnetization oscillation as depicted in Fig. 9c and d is given by

$$
\Delta M_{AC}(\phi) \approx \frac{1}{B_{\perp}} \sqrt{\Delta^2 + \Delta_D^2 \cos^2 \theta_c}, \quad \frac{\Delta_D \Delta_{2D}}{2} \sin^2 \theta_c \sin 2\phi.
$$
This equation holds for negative sign of \( g^* \), which is generally the case in 2DESs with small band gap and large SOI [69]. For positive \( g^* \), \( \Delta_2^R \) and \( \Delta_2^D \) have to be exchanged.

For the calculations we choose a material system where R-SOI and D-SOI are large and might be tuned to comparable strength. Numerical results are shown in Fig. 10 for a 7 nm InSb QW with parameters \( m^* = 0.014 m_e \), \( g^* = -51 \), and \( \beta_D = 3.22 \times 10^{-12} \text{eV m}\). \( \Delta_2^R \) is varied as shown in the legend. These parameters lead to \( \gamma_0 = 69.08^\circ \). We find that the presence of, both, R-SOI and D-SOI provokes a pronounced anisotropy of \( \Delta M_{AC} \) with respect to the direction of the in-plane magnetic field component. Dashed lines in Fig. 10 denote the approximate analytical results. The analytical approximation is valid in the regime \( \Delta M_{AC} B_L \ll \hbar \omega_C \). For the discussion we have chosen filling factor \( v = 4 \), which occurs at \( B_L \approx 2.59 \text{T} \) leading to \( B \approx 7.25 \text{T} \) at the critical angle, which is easily accessible in experiments [41]. With the R-SOI strength approaching the D-SOI value from below, a pronounced anisotropy of the oscillation amplitude develops. This anisotropic \( \Delta M_{AC} \) contains the information on the absolute values of \( \alpha_R \) and \( \beta_D \) as well as their relative sign as will be detailed in the following: The strength of the anisotropy depends on \( \alpha_R/\beta_D = \Delta_R/\Delta_D \). The minimum amplitude occurs at \( \phi = -45^\circ \) and the maximum amplitude at \( \phi = 45^\circ \). The orientation of the double-loop figure depends on the relative sign of \( \alpha_R \) and \( \beta_D \) (bottom right inset of Fig. 10): For \( \alpha_R/\beta_D = -2.1 \), the position of minima and maxima occur at \( \phi = 45^\circ \) and \( \phi = -45^\circ \), respectively. Note that the definition of the sign of \( \beta_D \) varies in the literature [20, 31, 34, 38, 52, 65].

The case \( \Delta_R = \Delta_D = \Delta \) is special due to its relevance for spintronics [20] and because beating patterns do not exist in perpendicular magnetic fields. Here, Eq. (13) reduces to \( \Delta M_{AC} \approx (\Delta/B_L) \sqrt{1 + (1/2) \sin^2 \theta_c} \sin 2 \phi \) and the value of \( \Delta \) and the relative sign of the contributions can be extracted.

Starting values \( \Delta_R \) and \( \Delta_D \) for the exact modeling can be extracted from the maximal and minimal oscillation amplitudes in experimental data occurring at \( \phi = \pm 45^\circ \) using the analytical approximation

\[
\Delta M_{AC}^C(\phi = \pm 45^\circ) \approx \frac{\Delta_R \sin^2(\theta_c/2) \pm \Delta_D \cos^2(\theta_c/2)}{B_L}\]

(14)

Here, we assume \( \Delta_R/\Delta_D > 0 \) and \( g^* < 0 \). For \( \Delta_R/\Delta_D < 0 \) Eq. (14) applies to \( \phi = \mp 45^\circ \). Note that for \( \phi = -45^\circ \) we get \( \Delta M_{AC} = 0 \) for \( \alpha_R/\beta_D = 1/\tan^2 \theta_c/2 \), i.e., the anticrossing gap vanishes to first order for this specific in-plane field direction. For the InSb QW considered here this condition is true for \( \alpha_R/\beta_D \approx 2.1 \). Equation (14) yields in general four solutions for \( \Delta_R \), \( \Delta_D \). Acknowledging that we can only determine the relative sign of \( \Delta_R \) and \( \Delta_D \) but not the absolute sign of both we end up with two solutions

\[
\Delta_R \approx \frac{(\Delta M^+ \pm \Delta M^-) B_L}{2 \sin^2(\theta_c/2)},
\]

\[
\Delta_D \approx \frac{(\Delta M^+ \mp \Delta M^-) B_L}{2 \cos^2(\theta_c/2)}.
\]

(15)

These reduce to one for \( \Delta_R/\Delta_D = 1/\tan^2(\theta_c/2) \), since here \( \Delta M^* = 0 \). We point out that for the general case of two non-degenerate solutions, the correct solution can be determined by comparing the numerically calculated magnetization with the experiment: the two solutions are distinguished by the beating patterns in the low-field regime. To substantiate this, we show numerically calculated magnetization traces in Fig. 11 for the case \( \Delta_R = 8.07 \text{meV} \) and the corresponding second solution of Eq. (15) given by \( \Delta_D = 17.03 \text{meV} \) and \( \Delta_D = 3.82 \text{meV} \). The beating patterns at low fields differ as shown in Fig. 11a, although the behavior of \( M \) at large \( B_L \) shown in (b) is almost identical for both cases. In this way, the full numerical calculations producing the traces in Fig. 11a allow one to distinguish the two solutions of Eq. (15). Note that the same procedure applies when only one term of \( \Delta_R \) and \( \Delta_D \) is finite, and it is not known a priori which on it is. In such a case \( \Delta M_{AC}(\phi = \pm 45^\circ) \) is isotropic, and the two possible solutions are \( \Delta_R \approx MB_L/\sin^2(\theta_c/2) \) and \( \Delta_D \approx MB_L/\cos^2(\theta_c/2) \). Again, the applicable solution can be found from the different low-field beating patterns.
The analysis of the anisotropy of $\Delta M_{AC}$ based on Eq. (15) can be used as a starting point for the exact diagonalization. The full numerical treatment then provides the definite values of $\alpha_R$ and $\beta_D$ by one-to-one comparison with the experimental data over a larger field regime.

Including the $k^3$ D-SOI using Eq. (2) into the numerical consideration is straightforward. We consider a 30 nm wide In$_{0.85}$Al$_{0.15}$Sb/InSb/In$_{0.9}$Al$_{0.1}$Sb asymmetric quantum well with $n_s = 2.5 \times 10^{15} \text{ m}^{-2}$ and $\alpha_R = 5.1 \times 10^{-12} \text{ eV m}$, $\beta_D = 3.2 \times 10^{-12} \text{ eV m}$ and $\gamma_D = 4.5 \times 10^{-28} \text{ eV m}^3$ calculated in the work of Gilbertson et al. [71] in Fig. 12. Solid squares represent the values including the $k^3$ D-SOI terms from Eq. (2). For comparison, we show the result where $k^3$ D-SOI is neglected as open circles. The difference shows that the $k^3$ D-SOI can become important in wide QWs. However, the angular dependence is qualitatively the same. This is owed to the fact that the Hamiltonian in Eq. (2) gives rise to two non-vanishing matrix elements. One couples different spin levels that differ in the Landau index by 3. A calculation of the anticrossing gap neglecting this matrix element is shown as dashed line in Fig. 12, highlighting that this matrix element can be safely neglected. The other matrix element couples exactly the same levels as the only non-vanishing $k$-linear Dresselhaus matrix element. It has opposite sign compared to the $k$-linear element, leading to a decreased value of this entry. Fitting Eq. (13) to experimental data thus yields useful information also in the limit of wide quantum wells, when $\Delta_D = 2k_\theta (\beta_D - \gamma_D k_\theta^2 / 4)$ is substituted for $\Delta_D$. Information about the relative strength of $\beta_D$ and $\gamma_D$ could be obtained by variation of $k_\theta = \sqrt{2\pi n_s}$ via the carrier density.

In an experiment, $\theta_i$ is determined from the minimum of $\Delta M(\theta) = \Delta M_{AC}$ for each $\phi$. Two measurements at $\phi = \pm 45^\circ$ are in principle sufficient to determine both SOI constants and their relative signs.

### 7 Conclusions

In conclusion, we have presented both experimental and theoretical studies on the influence of SOI on the quantum oscillatory magnetization, i.e., the dHvA effect. We have shown that micromechanical cantilever torque magnetometry is powerful to experimentally explore the quantum oscillations of the magnetization when SOI is present. We have developed an extensive theoretical description directly linking the Hamiltonian including R-SOI, D-SOI, and the Zeeman field in arbitrarily tilted magnetic fields to the magnetization as an observable ground state property. In particular, comparing experiment and theory we found an unexpected frequency anomaly in the magnetization at low tilt angles and an anomalous dHvA wave form at large
tilt angles when SOI was present. Both cannot be explained in the framework of the well-accepted quantum mechanical description presented here. This is especially intriguing since $M$ is a thermodynamic state function that depends in a straightforward way on the eigenvalues of the model Hamiltonian yielding the Landau level energies and the ground state energy $U$. The analysis of beating patterns in quantum oscillations to extract SOI-induced properties plays an important role in spintronics. Our results on beating patterns in the quantum oscillations of the magnetization shed new light on the underlying physics, deepening the understanding of the interplay of SOI with quantum oscillations on the one hand, but raising new questions about the completeness of existing descriptions on the other hand. Finally, we have shown that the values of both R-SOI and D-SOI coupling constants and their relative signs can be determined from the anisotropy of magnetic oscillations in high magnetic fields, where beating patterns are absent. Experiments addressing the magnetization in this regime could thus lift ambiguities that exist in the analysis of beating patterns and promote the understanding of SOI in 2DESs based on thermodynamic ground state properties.

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