Gap formation around $\Omega_e/2$ and generation of low-band whistler waves by Landau-resonant electrons in the magnetosphere: Predictions from dispersion theory

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Key Points:
- Gap formation at half the electron cyclotron frequency is a significant feature in magnetospheric whistler wave emission
- The Doppler-shifted cyclotron mode of the Landau-resonant electron population causes mode splitting leaving behind the frequency gap
- Lower-band whistler waves of very oblique propagation are generated by Landau-resonant beam electrons

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Abstract: In this paper we show that two significant phenomena of magnetospheric chorus emission can be explained by the participation of beam-like electron structures, created by Landau-resonant interaction with growing oblique whistler waves. The first concerns the widely observed spectral gap near half the electron cyclotron frequency $\Omega_e$; the second is related to the observation of very obliquely propagating lower-band waves that cannot be directly generated by temperature anisotropy. Concerning the gap, kinetic dispersion theory reveals that interference of the beam-related cyclotron mode $\omega \approx \Omega_e - kV_b$ with the conventional whistler mode leads to mode splitting and the appearance of a ‘forbidden’ area in the $\omega$-$k$ space. Thereby the beam velocity $V_b$ appears as an essential parameter. It is directly related to the phase velocity of the most unstable whistler wave mode, which is close to $V_{Ae}/2$ for sufficiently hot electrons ($V_{Ae}$ is the electron Alfvén velocity). To clarify the second point, we show that Landau-resonant beams with $V_b < V_{Ae}/2$, which arise in cold plasmas from unstable upper-band waves, are able to generate lower-band whistler mode waves at very oblique propagation ($\theta \geq 60^\circ$). Our studies demonstrate the important role of Landau-resonant electrons in nonlinear whistler wave generation in the magnetosphere.

Keywords: important role play Landau-resonant electrons; modification of the electron distribution function; gap formation at half the electron cyclotron frequency

1. Introduction

It has been known for decades that whistler mode emission in the Earth’s magnetosphere often appears in a banded structure, called chorus waves. It is now widely accepted that two kinds of hot anisotropic electron populations, in addition to cold isotropic background electrons, are responsible for the excitation of chorus waves via cyclotron resonant interaction. One of the unique observational properties of the chorus waves is the appearance of a power gap in the time-frequency spectrum around half the electron cyclotron frequency ($\Omega_e/2$) by which a banded structure with a lower and upper band arises (Burtis and Helliwell, 1969; Tsurutani and Smith, 1974; Santolik et al., 2010; Fu XR et al., 2014; Gao XL et al., 2018, 2019). Considerable attention has been focused on this topic recently, stimulated by satellite observations of whistler activity (Cluster 2: Santolik et al., 2009, 2010; Cluster 4: Schriver et al., 2010; THEMIS: Min K et al., 2014, Gao XL et al., 2018, 2019; Van Allen Probe B: Agapitov et al., 2015). On the basis of simultaneous observations of electron velocity distributions and chorus waves, evidence is provided for the formation of a substantial suprathermal plateau in the parallel electron distribution (Min K et al. 2014; Agapitov et al., 2015; Li W et al., 2016). Omura et al. (2008, 2009) suggested that the gap results from nonlinear damping of obliquely propagating waves close to half the gyrofrequency through Landau resonance. This approach, in which the gradient of the background magnetic field is an essential ingredient, is supported by test particle simulations on wave-particle interaction via Landau resonance by Hsieh and Omura (2018). Most of the recent
theoretical efforts rely on two-dimensional PIC simulations of the whistler anisotropy instability. Liu KJ et al. (2011), Gary et al. (2000, 2011), and later Fu XR et al. (2014) postulated that the upper and the lower bands of the chorus emission can be generated independently by distinct anisotropic electron populations. Ratcliffe and Watt (2017) also used 2D PIC simulations to study this instability and concluded that the gap arises self-consistently due to subtle changes in the electron distribution produced by Landau resonant interaction of electrons with obliquely propagating whistler modes during their growth phase. However, definitive conclusions on the origin of the spectral gap and the statistically relevant presence of upper-band and lower-band whistler modes in the observations have not been drawn.

From the point of view of quasi-linear diffusion models, Landau resonant interaction shifts electrons around the resonant velocity down the parallel particle distribution function and supports the formation of plateau structures (Kennel and Engelmann, 1966; Swanson, 2003). For intense, nearly coherent chorus waves, this phenomenon may lead to Landau resonant electron trapping in the electric field of the most dominant whistler mode. Indeed, both simulations and observations provide evidence that the non-linear modifications of the parallel electron velocity distribution are associated with the fastest growing mode of the anisotropy instability. Trapped particles can be considered as a low-energy electron beam which subsequently undergoes relaxation to a plateau. Plateau formation was first seen by Schr"{u}ver et al. (2010) and Gary et al. (2011) in 2D PIC simulations of the whistler anisotropy instability in an electron population of bi-Maxwellian type; see also An X et al. (2019). Experimental verification of plateau formation by Landau-resonant interactions with chorus waves has been reported by Min K et al. (2014) and Agapitov et al. (2015) using THEMIS and Van Allen Probe B satellite measurements, respectively. In a recent investigation by Dokgo et al. (2018) using PIC simulations, the formation of an electron beam and subsequent broadening to a plateau, owing to interaction of an initial large amplitude oblique whistler wave with electrons, has been shown.

In the present work we adopt the beam character of the Landau-resonant electrons, although not clearly confirmed by observations, and apply kinetic dispersion theory to study the consequences of their presence for the whistler mode dispersion. Thereby we pay attention to the electron temperature of the anisotropic population (specifically, their parallel electron plasma beta $\beta_{\parallel}$). Dispersion theory reveals that the phase velocity $V_{\text{ph} \parallel}$ of the fastest growing anisotropy-driven mode is sensitive to $\beta_{\parallel}$. For plasmas with only one anisotropic electron population it varies as $V_{\text{ph} \parallel}/V_{\text{Be}} \sim 2.8\beta_{\parallel}^{1/2}$ in the range $0.001 \leq \beta_{\parallel} \leq 0.025$ and becomes $V_{\text{ph} \parallel}/V_{\text{Be}} \geq 0.5$ for $\beta_{\parallel} \geq 0.025$ ($V_{\text{Be}}$ is the electron Alfvén velocity), as described in Gary et al. (2011), Fu XR et al. (2014), Yue C et al. (2016), Fan K et al. (2019) and An X et al. 2017. Two interesting results have been found in our analysis. First, the widely discussed spectral gap near half the electron gyrofrequency $\Omega_e/2$ appears as a natural consequence of the weak electron beam formed by Landau resonant trapping. The associated stop band results from splitting of the whistler mode by the Doppler-shifted cyclotron mode $\omega-\Omega_e-kV_{\parallel}$ with $V_{\parallel}=V_{\text{ph} \parallel} \sim 0.5V_{\text{Be}}$ at $k/c\omega_e \sim 1$. This mode splitting creates a $\omega-k$-crossing area of forbidden wave propagation. The requirement that $V_{\text{ph} \parallel} \sim 0.5V_{\text{Be}}$ suggests that gap formation is restricted to occur only for electron populations in a certain temperature range. For a bi-Maxwellian electron distribution it results in the condition $\beta_{\parallel} \geq 0.025$. For plasmas with multiple populations, their densities come additionally into play. Secondly, very oblique lower band waves, which cannot be directly excited by temperature anisotropy, are another challenging signature of observed whistler wave spectra. Their origin can also be traced back to the beams of Landau-resonant electrons in relatively cold plasmas with phase velocities $V_{\text{ph} \parallel}$ in the range $V_{\text{ph} \parallel} \leq 0.2V_{\text{Be}}$. These lower-band waves are occasionally observed as a dominant spectral line (Min K et al. (2014); Mourenas et al. (2015); Li W et al. (2016)) or as a hidden secondary effect, as found, for example, by Li W et al. (2013) in the probability distribution of wave normal angles. A recent review of the distribution, origin, and roles of such waves is given by Artemyev et al. (2016).

In Section 2 we present a simple basic model for the gap formation by Landau-resonant electron beams. Section 3 recalls dispersion characteristics of whistler waves driven by temperature-anisotropy and in Section 4 we discuss the gap formation for anisotropy-driven whistler mode. In Section 5, we demonstrate the excitation of oblique (quasi-parallel) whistler waves by Landau-resonant beams of temperature-anisotropy origin. Section 6 presents the generation of lower-band, highly oblique whistler mode waves driven by Landau-resonant beams in cold plasmas ($\beta_{\parallel} < 0.025$), and a summary follows in Section 7.

2. Gap Formation Around $\Omega_e/2$, Preview

In this preliminary section we present the basic idea for a simple model explanation of the spectral gap in the magnetospheric whistler emission; our model includes two (counter-streaming) beams/plateaus in the parallel electron velocity distribution. These beams/plateaus should model the group of Landau-resonant electrons trapped in the electric field of the fastest growing obliquely propagating whistler wave mode. Their speed is assumed here as $0.5V_{\text{Be}}$. Two counter-streaming beams and related plateaus due to their possible relaxation, respectively, need be considered because of the symmetry of whistler mode generation by the anisotropy instability. First, the beam situation is considered. In an idealized picture with cold background and beam electrons, one can write $f(v) = n_0 \delta(v) + n_b (\delta(v - V_{\parallel}) + \delta(v + V_{\parallel}))$ with $n_0$ the background density, $n_b$ the beam density, and $V_{\parallel}$ the beam velocity. Using, for example, the parameters $n_0/n_b = 0.99$, $n_b/n_e = 0.01$ ($n_e = n_b + n_b$) and $V_{\parallel} = 0.5V_{\text{Be}}$, the parallel electron velocity distribution, including the beams, is sketched in the upper panel of Figure 1a. For cold electrons, the dispersion relation for parallel propagating whistler waves reduces to a simple algebraic relation (polynomial in $\omega(k)$ of sixth order) and can easily be solved numerically; see Sauer and Sydora (2010). The corresponding dispersion properties are displayed in the two lower left panels of Figure 1. The crucial feature is mode splitting of the whistler wave in the vicinity of $\omega \sim 0.5\Omega_e$ at $k/c\omega_e \sim 1$, resulting in a pronounced frequency gap caused by the beam which moves opposite to the whistler wave. In the range of positive wave numbers, the cyclotron-beam mode $\omega - \Omega_e-kV_{\parallel}$ with the negative slope is responsible for mode splitting and the associated gap formation.
Figure 1. Simple model of gap formation in the whistler frequency range around $\omega - \Omega_e/2$ by (a) counter-streaming beams and by (b) plateaus, respectively. The upper panels show the model of the parallel velocity distribution, containing (a) cold background electrons together with two counter-streaming weak beams of velocity $V_b = \pm 0.5 V_{ae}$ and density $n_b/n_e = 0.01$ and (b) a warm water-bag plasma with the modification by a plateau distribution on both sides. The beams and plateaus should model the modifications by Landau-resonant trapped electrons that arise from unstable obliquely propagating whistler waves. The lower panels display results of associated dispersion analysis (real part of normalized frequency $\omega/\Omega_e$ and phase velocity parallel to the magnetic field $V_{ph}/V_{ae}$ versus the normalized wave number $kc/\omega_e$). The mode splitting around $kc/\omega_e \sim 1$ ($\omega - 0.5\Omega_e$) arises due to the intersection of the common whistler mode with the cyclotron-beam mode $\omega - \Omega_e k V_b$, associated with the counter-streaming beam in (a) and the plateau component in (b), respectively.

The same happens in the opposite direction. Similar mode splitting in kinetic theory has already been described by Hashimoto and Matsumoto (1976). Two things should be noted from Figure 1: the location of the gap is essentially determined by the beam velocity $V_b$, and a gap around $\omega - 0.5\Omega_e$ will arise only for beam velocities $V_b \sim 0.5V_{ae}$. These conditions have implications for the gap formation in the case of anisotropy-driven whistler modes because of the spread in the phase velocity of the fastest growing waves due to different electron temperatures. For later considerations it is important to note that the plateau distribution shown in the upper panel of Figure 1b leads, with respect to the gap formation, to the same results.

The simple model of gap formation described above is supported by treating whistler emission in the framework of the so-called response theory (Sauer and Sydora, 2015; Sauer et al., 2019). This procedure describes the reaction of a plasma to an external current $j_{ext}$ by Fourier transforming in both space and time, including the entire response in the field vector $E$. The $\omega-k$ power spectrum of the transverse electric field intensity for the whistler mode generated by a sudden switch-on of an external current transverse to the ambient magnetic field. Figure 2b represents the transverse electric field intensity and results from integrating $E(\omega,k)$ over $k$. In both panels the spectral gap around $\Omega_e/2$ is clearly seen.

$$E(\omega,k) = \frac{M^*(\omega,k) j_{ext}(\omega,k)}{D(\omega,k)},$$

$M(\omega,k)$ is the dispersion tensor that which completely specifies the electromagnetic properties of the medium in the whistler frequency range; $M^*$ is its adjoint; $D(\omega,k) = \text{Det}(M) = 0$ is the dispersion relation of the electromagnetic waves in the medium (see Sauer and Sydora (2010)). Here, the parallel velocity distributions from the top panels of Figure 1 are taken into account. The colour plot of Figure 2a displays the $\omega-k$ power spectrum of the transverse electric field intensity for the whistler mode generated by a sudden switch-on of an external current transverse to the ambient magnetic field. Figure 2b represents the transverse electric field intensity and results from integrating $E(\omega,k)$ over $k$. In both panels the spectral gap around $\Omega_e/2$ is clearly seen.

3. Dispersion Characteristics of Temperature-Anisotropy Driven Whistler Waves

The simple model for the generation of the spectral gap in the whistler emission, presented in the previous section, demon-
strates that a gap around $\Omega_e/2$ can occur only when the beam velocity is close to half the electron Alfvén velocity. Since the beams are thought to be generated via Landau resonance with unstable whistler modes, for a gap to be formed it is necessary that sufficiently intense oblique modes can be excited that satisfy the condition $V_{ph}||/V_Ae \approx 0.5$. This condition implies restrictions on parameters of the electron plasma, especially its temperature. To check the situation, Figure 3 presents numerical solutions of the full electromagnetic linear kinetic dispersion relation of whistler wave modes excited by the anisotropy instability (Gary, 1993). As a first step, we do not discriminate between different electron populations and consider a continuum of anisotropic electrons (bi-Max-

Figure 2. Result of response theory using Equation (1). The colour plot (a) shows the amplitude (arbitrary units) of the transverse electric field of the parallel propagating whistler mode in an electron plasma with velocity distributions displayed in the top panels of Figure 1. For both cases one gets the same results. The waves are excited by a sudden switch-on of an external current transverse to the magnetic field. (b) Transverse electric field intensity $|E(\omega)|^2$ versus frequency (from integrating $E(\omega, k)$ over $k$). The gap around half the electron gyrofrequency is a clear signature.

Figure 3. Results of kinetic dispersion theory. Maximum growth rate, maximized over the wave number (top panels), as function of the propagation angle for three ranges of $\beta_{e||}$ choosing different temperature anisotropies $A = T_{e\perp}/T_{e||}−1$: (a) $A = 3.3$, (b) $A = 2.0$ and (c) $A = 1.0$. The lower panels display corresponding quantities such as parallel phase velocity $V_{ph}/V_A$, frequency $\omega/\Omega_e$ and wave number $kc/\Omega_e$. The shadowed area indicates the $\beta_{e||}$ range over which the sharp transition from oblique (0°–50°) to quasi-parallel propagation takes place. Here, $V_A = B_0/(\mu_0 n_0 m_e)^{1/2}$ is the electron Alfvén velocity. Different values of $A$ have been chosen for the three $\beta_{e||}$ intervals in order to limit the growth rate.
wellian distribution with larger perpendicular than parallel temperature), following the approach of Gary et al. (2000, 2011). Figure 3 shows how the maximum growth rate of the instability, maximized over the wave number and related parameters (propagation angle, phase velocity, frequency and wave number) varies with increasing electron beta. These results demonstrate the strong temperature influence and complete the findings of Gary et al. (2011). An especially interesting behavior is exhibited in the propagation angle for optimum growth, which abruptly changes at $\beta_{\parallel} \sim 0.025$, marked by the shadowed area. Whereas optimum wave excitation below this value happens for strongly oblique propagation ($\theta \geq 50^\circ$), a rapid transition to almost parallel propagation ($\theta \sim 0^\circ$) takes place above this value. The phase velocity continually increases with growing temperature from $V_{\text{ph}}/V_{\text{ke}} \sim 0.1$ at $\beta_{\parallel} = 0.001$ to $V_{\text{ph}}/V_{\text{ke}} \sim 0.5$ at the critical value $\beta_{\parallel} \sim 0.025$ and remains nearly constant above this value. Recently, very similar results using growth rate calculations have been obtained by Yue C et al. (2016) and Fan K et al. (2019). Moreover, these authors showed that Van Allen Probes observations of parallel and oblique chorus waves are globally in agreement with such theoretical results (see their Fig. 3).

To assess the extent to which Landau resonant electrons can be generated by obliquely propagating whistler waves, the rate of growth and related dispersion quantities have been calculated for a given electron temperature as a function of the propagation angle $\theta$. Figure 4 shows results for $\beta_{\parallel} = 0.07$. It should be noted that the growth rate up to $\theta \sim 30^\circ$ is diminished against its maximum value at parallel propagation only by a factor 2–3. Thus, the generation of Landau-resonant electrons under these conditions can be expected. In addition, the other conditions for gap formation are met. That means: phase velocity, frequency, and wave number are with $V_{\text{ph}}/V_{\text{ke}} \sim 0.5$, $\omega/\Omega_{e} \sim 0.5$, and $k c/\omega_{e} \sim 1$, respectively, in the required range.

4. Gap Formation for Anisotropy-Driven Whistler Mode Waves

The results of the preceding section’s dispersion analysis for bi-Maxwellian electron populations suggest that whistler wave modes generated by the anisotropy instability match to a large extent the conditions for gap formation given in the preview. The critical parameter is the parallel electron temperature, which in the case of a bi-Maxwellian distribution function should satisfy the condition $\beta_{\parallel} > 0.025$. In this section we study the feedback of the self-generated modifications of the parallel velocity distribution (Landau-resonant beams or plateaus) to the wave field of whistlers excited by the temperature anisotropy. As a first step, we consider bi-Maxwellian electron populations, for which the dispersion properties above have been presented. Sufficiently hot electron plasmas are assumed for which the whistler waves have maximum growth rate at parallel propagation and phase velocities $V_{\text{ph}}$ in a narrow range around $0.5 V_{\text{ke}}$. We introduce the distribution function, modified by beams or plateaus at $V_{0} \sim 0.5 V_{\text{ke}}$, into the kinetic dispersion relation and analyze the changes in the wave activity. The small beam-like components are modelled by shifted Maxwellians. Figure 5 presents results, comparing the dispersion characteristics of three velocity distributions. Panel (a) corresponds to a pure bi-Maxwellian distribution; the right panels (b) and (c) show cases with slightly modified distributions by counter-streaming Landau-resonant beams (panel (b)) or humps (panels (c)). Panel (a) represents the common signatures of unstable whistler waves in a plasma with $\beta_{\parallel} = 0.07$ (see Figure 3) in which maximum growth happens at parallel propagation. Panel (b) demonstrates the mode splitting owing to the beam mode, as described in Section 2, for a beam density of $n_{0}/n_{e} = 0.01$. Significant changes in the dispersion properties are clearly seen in panel (b). The mode splitting is accompanied by the generation of two separate frequency bands, above and below half the electron gyrofrequency $\Omega_{e}/2$, similar to the situation shown in Figure 1. In addition, the shape of the growth rate variation is split into two parts exhibiting separate maxima in the lower and upper frequency band. The corresponding wave number regions around $k c/\omega_{e} \sim 1$ are marked by different colours. As shown in Figure 5c, these signatures almost survive when the beam broadens to a plateau-like distribution.

The considerations so far are based on the assumption that the whistler instability is driven by a single electron distribution of bi-Maxwellian type. To approach more realistic conditions in the
magnetosphere, we have extended the analysis to distributions consisting of two populations, an isotropic cold one (index c) and an anisotropic hot one (index h). An example is shown on top in Figure 6a, with the parameters $n_c/n_h = 0.9$, $\beta_c = 0.001$; $n_h/n_c = 0.1$, $\beta_h = 0.025$. The corresponding anisotropy instability ($A_h = 1.6$) produces almost the same variation of the optimum growth rate with increasing propagation angle as shown in Figure 4. Thus, one can expect the generation of Landau-resonant electrons for the two-component velocity distribution as well. Similar to Figure 5, Figure 6 compares the dispersion characteristics of three types of composed electron velocity distributions. As before, small changes in the velocity range enclosing $V/V_{Ae} = 0.5$ in the form of beams (panels b) or plateau-like humps (panels c), again with $n_b/n_h = 0.01$, have been added to the parallel distribution function. As seen in Figure 6, the modifications of the whistler dispersion properties are almost the same as those shown in Figure 5 for a single anisotropic distribution. These include both the occurrence of a frequency gap around $\Omega_e/2$ and the splitting of the growth rate with separate maxima in the lower and upper frequency band near to the wave number $kc/\omega_e \sim 1$. As shown in Figure 6c, these signatures survive if the beam broadens to a plateau-like distribution. To sum up, the results of our dispersion analysis strongly suggest the key role of Landau-resonant electrons in explaining the widely discussed frequency gap around $\Omega_e/2$. Further discussion will follow in the summary section.

5. Beam Excitation of Quasi-Parallel Whistler Waves and Gap Formation

Self-generated Landau resonant beams due to oblique whistler waves may act on the dispersion of these waves in two ways. In the previous section, it has been shown that these beams are responsible for the $\Omega_e/2$ gap formation of parallel propagating waves that are excited by temperature anisotropy. In this case, the gap in the frequency spectrum is a consequence of mode coupling of the whistler wave of the main plasma with the beam mode $\omega - \Omega_e - kV_{cb}$, which belongs to the counter-streaming beam. For oblique propagation, there is a possibility, in addition, that the co-streaming beams (generating the beam mode $\omega = \Omega_e + kV_{cb}$) may excite whistler waves in a slightly modified frequency spectrum. Thus, a frequency gap at about $\Omega_e/2$ can be generated similar to...
the whistler wave excitation by temperature anisotropy. Again, phase velocity of the whistler waves of $V_{ph} \sim 0.5V_{Ae}$ and thus plasmas with $\beta_e > 0.025$ have to be assumed.

As before, dispersion analysis is used to investigate the action of Landau-resonant beams with respect to the excitation of whistler waves in the case of oblique propagation; no temperature anisotropy is assumed. Examples are shown in Figure 7 using the same panel arrangement as before. Panels (a) belong to a single electron plasma with $\beta_e = 0.05$. The assumed beam density is $n_b/n_e = 0.01$. Panels (b) are for a plasma consisting of two populations, a cold one with $\beta_e = 5 \times 10^{-4}$ and a hot one with $n_b/n_e = 0.15$ and $\beta_e = 0.02$. Here, a beam density of $n_b/n_e = 2 \times 10^{-3}$ was assumed. For both cases we clearly see that two separated regions of instability arise, separated by a gap at $\omega \sim \Omega_e/2$; thus both an upper and lower frequency band are formed. This effect can be summarized briefly as follows: The instability of obliquely propagating whistler waves is caused by the co-streaming beam; the arising frequency gap in turn is a consequence of the simultaneous presence of the counter-streaming beam, which leads to the mode splitting.

6. Lower-Band, Highly Oblique Whistler Waves Driven by Landau-Resonant Beams

In the previous sections our focus was directed to the frequency gap around $\Omega_e/2$ which could be explained by splitting of the whistler wave mode due to the presence of counter-streaming beams with velocities close to half of the Alfvén velocity. This condition for the beam velocity requires a sufficiently hot anisotropic electron population. In this section we demonstrate that beam-like electron distributions are not only responsible for whistler mode splitting, but may themselves excite highly oblique low-band whistler waves in the case of lower electron temperatures. In the case of cold electrons ($|\beta_{\parallel}| < 0.025$) the temperature anisotropy leads to the excitation of whistler waves at oblique propagation ($\theta \sim 50^\circ$) in the upper band ($\omega \sim 0.7\Omega_e$); see Figure 3. Their phase velocity varies as $V_{ph}/V_{Ae} \sim 2.8|\beta_{\parallel}|^{1/2}$. If we consider a plasma with $|\beta_{\parallel}| = 0.001$, for example, accordingly waves with $V_{ph}/V_{Ae} \sim 0.1$ are excited. That is, Landau resonant electrons can be generated at the same slow speed. As we will see, the associated beam modes with $\omega \sim \pm kV_{pe}$ can in turn excite very oblique whistler waves ($\theta \geq 50^\circ$) in the low-frequency band ($\omega < \Omega_e/2$). In
this way, a wave transmission from the upper to the lower frequency band takes place.

For the subsequent dispersion analysis of the described whistler transfer process we chose an electron plasma with $\beta_e = 0.0015$. According to the second panel of Figure 3, the strongest whistler mode waves are then generated at a propagation angle of $\theta \approx 50^\circ$ with a parallel phase velocity of $V_{ph,||} \approx 0.12$. Figures 8b and 8c show the dispersion properties for this case. Maximum growth rate appears at $k c / \omega_e \approx 8$. As a leftover from earlier PIC simulations (Sydora et al., 2007) using the same parameters as in the dispersion analysis, except a larger anisotropy of $A = 6$, Figure 8d gives indirect evidence for the presence of an electron beam by showing the associated beam mode. The Fourier transform $B_y(\omega,k)$ of the perpendicular magnetic field is displayed in this figure. As seen there, a beam mode $\omega - k v_b$ with $V_b \approx 0.12$ appears, indicated by the dash-dot line, which crosses the whistler mode at $k c / \omega_e \approx 8$. The presence of this beam mode is a clear hint at the generation of Landau-resonant electrons by the obliquely propagating unstable whistler wave.

We now proceed to the analysis of whistler mode generation by a Landau-resonant beam in the case of cold electrons. For this purpose, a reasonable beam density of $n_b / n_e = 5 \times 10^{-3}$ was chosen. The other parameters are the same as above: $\beta_e = 0.0015$, $V_b \approx 0.12$. Figure 9 compares the results of the dispersion analysis for three cases. The distribution functions of the different beam-plasma configurations are shown on top of Figure 9. Results of the corresponding kinetic dispersion analysis are plotted below: normalized frequency $\omega / \Omega_e$, parallel phase velocity $V_{ph,||} / V_{Ae}$ and growth rate $\gamma / \Omega_e$ respectively. An optimum propagation angle $\theta \approx 75^\circ$ was found. The maximum growth rate $\gamma / \Omega_e \approx 0.02$ occurs where the beam mode $\omega - k v_b$ intersects the whistler mode at $k c / \omega_e \approx 1$. The presence of this beam mode is a clear hint at the generation of Landau-resonant electrons by the obliquely propagating unstable whistler wave.

Figure 7. Excitation of oblique whistler waves ($\theta = 30^\circ$) and gap formation by the combined action of co- and counter-streaming electron beams with $|v_b/V_e| = 0.5$. There is no temperature anisotropy; $A = 0$. The four panels show the same quantities as before. (a) belongs to a single electron plasma with $\beta_e = 0.05$. The assumed beam density is $n_b / n_e = 0.01$. (b) is for a plasma consisting of two populations, a cold one with $\beta_e = 5 \times 10^{-4}$ and a hot one with $n_h / n_e = 0.15$ and $\beta_h = 0.02$. Here, a beam density of $n_b / n_e = 2 \times 10^{-3}$ was taken. The formation of both the lower and upper frequency bands around $\omega / \Omega_e \approx 0.5$ ($k c / \omega_e \approx 1$) is clearly indicated.
described by a water-bag distribution. Details of the dispersion analysis are described in Sauer and Sydor (2010, 2011). Aside from the absence of strong damping of the beam modes in the fluid model, there is a reasonable agreement between both approaches.

7. Discussion and Summary

The aim of our paper is the interpretation of frequency spectra observed at whistler chorus mode emission in the magnetosphere. The most distinctive spectral signature is a frequency gap around half the electron cyclotron frequency. Several models have been proposed to explain its origin (Omura et al., 2008, 2009; Bell et al., 2009; Liu et al., 2011; Fu et al., 2014), but a mechanism covering the various aspects of observed spectra has been elusive.

Two-dimensional PIC simulations of the anisotropic whistler instability (Ratcliffe and Watt, 2017) revealed that the gap develops self-consistently, accompanied by slight local modifications to the parallel electron velocity distribution generated by growing whistler wave modes. We demonstrate that kinetic dispersion theory of anisotropy-driven whistler modes provides sufficient information to establish a plausible picture for the gap formation. Thereby one may reason as follows: (1) A necessary condition for a gap to be formed near \( \Omega_e/2 \) is a sufficiently hot anisotropic electron population \( \beta_{\parallel} > 0.025 \) for a single bi-Maxwellian which generates maximum whistler instability at parallel propagation, see Figure 3. (2) These unstable whistler waves have phase velocities close to half the electron Alfvén velocity \( V_{Ae}/V_{pe} = 0.5 \). (3) Oblique whistler mode waves are simultaneously excited, the growth rates of which remain significant in a narrow cone of \( 20° – 30° \) around \( \vec{B}_0 \). (4) The parallel electric field component of the oblique whistler waves allows them to interact efficiently with electrons via Landau resonance to produce electron beams propagating in opposite directions. (5) The beam speed corresponds to the phase velocity of the strongest growing whistler mode, i.e., \( V_\beta/V_{pe} \sim \pm 0.5 \). (6) The Doppler-shifted cyclotron mode \( \omega - \Omega_e + kV_\beta \) associated with the beam in the opposite direction \( (V_\beta/V_{pe} = -0.5) \) interferes with the right-propagating whistler branch at \( kC/\omega_e \sim 1 \) to produce mode splitting, which creates an inadmissible \( \omega - k \) area and thus a frequency gap around \( \Omega_e/2 \). In this context, it should be mentioned that even parallel chorus waves become more oblique as they propagate from their source region near the equator along a magnetic field line of increasing magnetic strength, reaching wave normal angles of \( \theta \sim 20° – 30° \) at latitudes \( 4° – 7° \) in both observations and simulations (e.g., see Section 1.3 and Fig. 6 in Artemyev et al., 2016, and references therein). We believe that the observations of \( \Omega_e/2 \)-gaps in the magnetosphere (Burts and Helliwell, 1969; Tsurutani and Smith, 1974; Santolik et al., 2010, Fu et al., 2014; Gao et al., 2018, 2019) can be explained by our model. However, experimental evidence for the existence of Landau-resonant electrons in connection with the observation of gaps is still lacking. Recently, beam formation during the nonlinear evolution of large amplitude oblique whistler waves has been described by Dokgo et al. (2018) in 1D PIC simulations. Local subtle structuring or plateau formation of the electron velocity distribution that occurred in earlier PIC simulations (Gary et al., 2011; Schriver et al., 2010; Ratcliffe and Watt, 2017) and the detection of the beam mode in Figure 7d may only serve as indirect evidence.

Figure 8. (a)–(c): Dispersion of whistler waves driven by temperature-anisotropy \( A = 3.3 \) in a cold electron plasma \( (\beta_{\parallel} = 0.0015) \) at a propagation angle of \( \theta = 50° \). (a) parallel Maxwellian velocity distribution function; (b), (c) variation of frequency \( \omega/\Omega_e \) and growth rate \( \gamma/\Omega_e \) with the wave number \( kC/\omega_e \). Maximum instability is at \( kC/\omega_e \sim 8 \). The colour plot (d) is from PIC simulations from Sydora et al. (2007) up to a final time of \( \Omega_e t = 600 \) and displays the Fourier component of the transverse (to \( \vec{k} \) and \( \vec{B}_0 \)) magnetic field \( B_y(\omega/\Omega_e, \vec{k}) \). The dash-dot line indicates the beam mode due to Landau-resonant electrons \( (\omega - kV_\beta) \) with \( V_\beta - V_{\parallel||} \sim 0.12V_{Ae} \).
confirmation of beam generation by oblique whistler waves. In this context, the paper by Drake et al. (2015) should be mentioned. A more direct indication for the acceleration of electrons by wave-particle interaction is seen in the chorus wave modulation of Langmuir waves (Li JX et al., 2017, 2018).

In addition, there is another striking phenomenon that underlines the importance of Landau-resonant electrons. It is the observation of very oblique lower band whistler waves ($\theta \leq 75^\circ$, $\omega < 0.5 \Omega_e$) for the case of relatively cold electrons (Mourenas et al., 2015; Li W et al., 2016; Ma Q et al., 2017). Such waves, however, cannot directly be excited by temperature anisotropy, as shown in Figure 3a. This discrepancy can be overcome by including Landau-resonant electrons via the following nonlinear mechanism: In the case of cold electrons ($\beta_e \leq 0.01$), unstable upper band waves ($\omega \sim 0.7 \Omega_e$, $\theta \sim 50^\circ$) are produced by temperature anisotropy. The phase velocity of these waves is below $0.5 V_{pe}$ and reaches, e.g. for $\beta_e = 0.001$, a value of $V_{ph} \sim 0.1 V_{pe}$. The oblique propagation of these waves promotes the generation of Landau-resonant electrons with almost the same velocity. Dispersion analysis has shown that these beams, in turn, are able to excite very obliquely propagating whistler waves in the lower frequency band via the beam instability, i.e., waves with frequencies well below $0.5 \Omega_e$, close to the resonance frequency $\omega \sim \Omega_e \cos \theta$. In this way, Landau resonant electrons can act as mediator, transforming whistler waves from the upper into the lower frequency band. Corresponding observations are described in the papers of Mourenas et al. (2015), Li W et al. (2016) and Ma Q et al. (2017). In this context we should point to the following suggestion: whistler waves initially present in the upper band should be damped during the transfer of their energy to an electron beam and subsequently to lower band waves. Such a damping, combined with effects of propagation from their generation region, could explain why very oblique lower band chorus waves are often seen without upper-band waves. An example is shown in the right bottom panel of Figure 9, adapted from Li W et al. (2016). Finally, it should be added that the excitation of lower band waves by a Landau-resonant electron beam may possibly

![Figure 9](https://example.com/figure9.png)

**Figure 9.** Excitation of highly oblique lower-band whistler waves in a low-temperature plasma ($\beta_e = 0.0015$) by an electron beam with $n_b = 0.005 n_e$ and $V_b = 0.12 V_{pe}$. Optimum beam-instability appears at $\theta \sim 75^\circ$. The beam is assumed to be generated by Landau resonant interaction of electrons with upper-band anisotropy-driven whistler waves. From top to bottom: parallel electron distribution function, normalized frequency $\omega/\Omega_e$, parallel phase velocity $V_{ph}/V_{pe}$ and growth rate $\gamma/\Omega_e$, respectively, versus $kc/\omega_e$. (a) Beam and plasma have the same temperature; (b) Transition to a plateau by broadening of the beam; (c) Fluid approximation by using a cold plasma and a water-bag distribution for the beam.
lead to the excitation of higher harmonics, called lower-band cascade by Gao XL et al. (2016, 2018, 2019).

The generation of lower-band waves by electron beams raises the question of the extent to which this process can be suppressed by beam relaxation and plateau formation in the frequency range of the electron plasma waves. This problem has been discussed, e.g. in the paper of Mourenas et al. (2015), but it can in principle be ruled out based on the following simple consideration. The phase velocity of the most unstable whistler wave due to temperature anisotropy, and thus the velocity of the Landau resonant electrons, varies with $\beta_{\parallel e}$ as $V_{ph}/V_{ke} = V_b/V_{ke} \approx 2.8 \beta_{\parallel e}^{-1/2}$. Together with the relation $\beta_{\parallel e} = 2V_e^2/V_{ke}^2$, this gives $V_b/V_{ke} \approx 4$ ($V_b$ is the parallel thermal electron velocity). For such a low beam speed (in terms of $V_b$), the Langmuir instability is normally suppressed by Landau damping; that is, beam relaxation and plateau formation is based solely on the interaction of the beam with the low-band whistler waves. Thus, it can be assumed that the measured plateaus in the electron velocity distributions (e.g. Min K et al., 2014 and Agapitov et al., 2015) have evolved by beam-whistler interaction. Only in the case of larger beam densities may the simultaneous excitation of lower-band whistler waves and Langmuir waves, sometimes seen in the electric field and magnetic data of Van Allen Probes (Mourenas et al., 2015; Li JX et al., 2017), occur. Using high-resolution waveforms, Li JX et al. (2017) conclude that the chorus waves accelerate electrons via Landau resonance and generate a localized electron beam in the phase-space density by which Langmuir waves are excited and modulated. For an electron beam of given density and velocity, the simultaneous excitation of whistler and Langmuir waves has been modelled theoretically in a paper by Sauer and Sydora (2012) by means of 1D PIC simulations (see their Figure 6).

The suggestion that upper-band whistler waves are possibly damped by energy transfer to the electrons brings us back to the interpretation by Omura et al. (2008, 2009) and Hsieh and Omura (2018), mentioned in our Introduction, that the frequency gap at $\Omega_e/2$ is due to nonlinear wave suppression. Hereby, one has to consider that in sufficiently hot plasmas ($\beta_{\parallel e} > 0.025$) the frequencies and phase velocities of unstable quasi-parallel waves are re-
stricted to $\omega \sim \Omega/2$ and $V_{\text{wh}} \sim V_{\text{AE}}/2$, respectively. Although the energy transfer to electron beams may well lead to an amplitude reduction of the primary excited wave, one has to consider that the beams themselves may drive whistler waves in the same frequency range. Therefore, the effect of wave suppression alone cannot cause an $\Omega/2$ gap.

In conclusion, the schematic view in Figure 10 summarizes once again the proposed mechanisms for the generation of chorus whistler wave spectra. Generally, whistler waves that are driven by temperature anisotropy are considered. The kind of interaction chain that develops depends on the electron plasma beta. For a single electron population with bi-Maxwellian electron distribution, $\beta_e = 0.025$ is a critical value. The left part of the scheme represents the process that finally ends in the gap formation. An example is given in the left colour plot (adapted from Fu XR et al., 2014), showing the spectral variation of the electric field intensity. The right part of the scheme clarifies the processes leading to the generation of a whistler anisotropy instability as the source of banded chorus: Van Allen Probes observations and particle-in-cell simulations. J. Geophys. Res. Space Phys., 119(10), 8288–8298. https://doi.org/10.1002/2014JA020364

It remains to be seen to what extent the described processes of the nonlinear transformation of whistler waves can be confirmed by targeted PIC simulations or by more detailed plasma measurements. Lastly, we would like to draw attention to the comprehensive work of LI JX et al. (2019) which appeared during the review process of this paper. Their Van Allen Probe measurements and accompanying numerical simulations largely support our interpretation of the origin of the two-band whistler wave emission in the magnetosphere.

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This paper is theoretical and does not use external data.

References
Agapitov, O. V., Artemyev, A. V., Mourenas, D., Mozzer, F. S., and Krasnoselskikh, V. (2015). Nonlinear local parallel acceleration of electrons through Landau trapping by oblique whistler mode waves in the outer radiation belt. Geophys. Res. Lett., 42(23), 10140–10149. https://doi.org/10.1002/2015GL066887
An, X., Yue, C., Bortnik, J., Decyk, V., Li, W., and Thorne, R. M. (2017). On the parameter dependence of the whistler anisotropy instability. J. Geophys. Res. Space Phys., 122(2), 2001–2009. https://doi.org/10.1002/2017JA023895
An, X., Li, J. X., Bortnik, J., Decyk, V., Kletzing, C., and Hospodarsky, G. (2019). Unified view of nonlinear wave structures associated with whistler-mode chorus. Phys. Rev. Lett., 122(4), 045101. https://doi.org/10.1103/PhysRevLett.122.045101
Artemyev, A., Agapitov, O., Mourenas, D., Krasnoselskikh, V., Shastun, V., and Mozzer, F. (2016). Oblique whistler-mode waves in the Earth’s inner magnetosphere: Energy distribution, origins, and role in radiation belt dynamics. Space Sci. Rev., 200(1-4), 261–355. https://doi.org/10.1007/s11214-016-0252-5
Bell, T. F., Inan, U. S., Haque, N., and Pickert, J. S. (2009). Source regions of banded chorus. Geophys. Res. Lett., 36(11), L11101. https://doi.org/10.1029/2009GL037629
Burtis, W. J., and Helliwell, R. A. (1969). Banded chorus—A new type of VLF radiation observed in the magnetosphere by OGO 1 and OGO 3. J. Geophys. Res., 74(11), 3002–3010. https://doi.org/10.1029/JA074i011p03002
Dokgo, K., Min, K. W., Choi, C. R., Woo, M., Yoon, P. H., and Hwang, K. J. (2018). Nonlinear evolutions of large amplitude oblique whistler waves. Phys. Plasmas, 25(6), 062904. https://doi.org/10.1063/1.511775
Drake, J. F., Agapitov, O. V., and Mozer, F. S. (2015). The development of a bursty precipitation front with intense localized parallel electric fields driven by whistler waves. Geophys. Res. Lett., 42(8), 2563–2570. https://doi.org/10.1002/2015GL063528
Fan, K., Gao, X. L., Lu, Q. M., Guo, J., and Wang, S. (2019). The effects of thermal electrons on whistler mode waves excited by anisotropic hot electrons: linear theory and 2-D PIC simulations. J. Geophys. Res. Space Phys., 124(7), 5234–5245. https://doi.org/10.1029/2019JA026463
Fu, X. R., Cowee, M. M., Friedel, R. H., Funsten, H. O., Gary, S. P., Hospodarsky, G. B., Kletzing, C., Kurth, W., Larsen, B. A.,... Winske, D. (2014). Whistler anisotropy instabilities as the source of banded chorus: van Allen Probes observations and particle-in-cell simulations. J. Geophys. Res. Space Phys., 119(10), 8288–8298. https://doi.org/10.1002/2014JA020364
Gao, X. L., Lu, Q. M., Bortnik, J., Li, W., Chen, L. J., and Wang, S. (2016). Generation of multiband chorus by lower band cascade in the Earth’s magnetosphere. Geophys. Res. Lett., 43(6), 2343–2350. https://doi.org/10.1002/2016GL068313
Gao, X. L., Lu, Q. M., and Wang, S. (2018). Statistical results of multiband chorus by using THEMIS waveform data. J. Geophys. Res. Space Phys., 123(7), 5506–5515. https://doi.org/10.1029/2018JA025393
Gao, X. L., Chen, L. J., Li, W., Lu, Q. M., and Wang, S. (2019). Statistical results of the power gap between lower-band and upper-band chorus waves. Geophys. Res. Lett., 46(8), 4098–4105. https://doi.org/10.1029/2019GL082140
Gary, S. P. (1993). Theory of Space Plasma Microinstabilities. New York: Cambridge University Press.
Gary, S. P., Winske, D., and Hesse, M. (2000). Electron temperature anisotropy instabilities: Computer simulations. J. Geophys. Res. Space Phys., 105(A5), 10751–10759. https://doi.org/10.1029/1999JA000322
Gary, S. P., Liu, J., and Winske, D. (2011). Whistler anisotropy instability at low electron $\beta$: particle-in-cell simulations. Phys. Plasmas, 18(8), 082902. https://doi.org/10.1063/1.3610378
Hashimoto, K., and Matsumoto, H. (1976). Temperature anisotropy and beam type whistler instabilities. Phys. Fluids, 19(10), 1507–1512. https://doi.org/10.1063/1.861342
Hsieh, Y. K., and Omura, Y. (2018). Nonlinear damping of oblique whistler mode waves via Landau resonance. J. Geophys. Res. Space Phys., 123(9), 7462–7472. https://doi.org/10.1029/2018JA025848
Kennel, C. F., and Engelfmann, F. (1996). Velocity space diffusion from weak plasma turbulence in a magnetic field. Phys. Fluids, 9(12), 2377–2388. https://doi.org/10.1063/1.1761629
Li, J. X., Bortnik, J., An, X., Li, W., Thorne, R. M., Zhou, M., Kurth, W. S., Hospodarsky, G. B., Funsten, H. O., and Spence, H. E. (2017). Chorus wave modulation of Langmuir waves in the radiation belts. Geophys. Res. Lett., 44(23), 11171–11172. https://doi.org/10.1002/2017GL075877
Li, J. X., Bortnik, J., An, X., Li, W., Russell, C. T., Zhou, M., Berchem, J., Zhao, C., Wang, S.,... Burch, J. L. (2018). Local excitation of whistler mode waves and associated Langmuir waves at dayside reconnection regions. Geophys. Res. Lett., 45(17), 8793–8802. https://doi.org/10.1002/2018GL078287
Li, J. X., Bortnik, J., An, X., Li, W., Angelopoulos, V., Thorne, R. M., Russell, C. T., Ni, B. B., Shen, X. C.,... Baker, D. N. (2019). Origin of two-band chorus in the radiation belt of Earth. Nat. Commun., 10(1), 4672. https://doi.org/10.1038/s41467-019-12561-3
Li, W., Bortnik, J., Thorne, R. M., Cully, C. M., Chen, L., Angelopoulos, V., Nishimura, Y., Tao, J. B., Bonnell, J. W., and LeContel, O. (2013). Characteristics of the Poynting flux and wave normal vectors of whistler-mode waves observed on THEMIS. J. Geophys. Res. Space Phys., 118(4),...
1461–1471. https://doi.org/10.1029/2009JA015218
Li, W., Mourenas, D., Artemyev, A. V., Bortnik, J., Thorne, R. M., Kletzing, C. A., Kurth, W. S., Hospodarsky, G. B., Reeves, G. D., … Spence, H. E. (2016). Unraveling the excitation mechanisms of highly oblique lower band chorus waves. Geophys. Res. Lett., 43(17), 8867–8875. https://doi.org/10.1002/2016GL070386
Liu, K. J., Gary, S. P., and Winske, D. (2011). Excitation of banded whistler waves in the magnetosphere. Geophys. Res. Lett., 38(14), L14108. https://doi.org/10.1029/2011GL048375
Ma, Q., Artemyev, A. V., Mourenas, D., Li, W., Thorne, R. M., Kletzing, C. A., Kurth, W. S., Hospodarsky, G. B., Reeves, G. D., … Wygant, J. (2017). Very oblique whistler mode propagation in the radiation belts: Effects of hot plasma and Landau damping. Geophys. Res. Lett., 44(24), 12057–12066. https://doi.org/10.1002/2017GL075892
Min, K., Liu, K. J., and Li, W. (2014). Signatures of electron Landau resonant interactions with chorus waves from THEMIS observations. J. Geophys. Res. Space Phys., 119(7), 5551–5560. https://doi.org/10.1002/2014JA019903
Mourenas, D., Artemyev, A. V., Agapitov, O. V., Krasnoselskikh, V., and Mozer, F. S. (2015). Very oblique whistler generation by low-energy electron streams. J. Geophys. Res. Space Phys., 120(5), 3665–3683. https://doi.org/10.1002/2015JA021135
Omura, Y., Katoh, Y., and Summers, D. (2008). Theory and simulation of the generation of whistler-mode chorus. J. Geophys. Res. Space Phys., 113(A4), A04223. https://doi.org/10.1029/2007JA012622
Omura, Y., Hikishima, M., Katoh, Y., Summers, D., and Yagitani, S. (2009). Nonlinear mechanisms of lower-band and upper-band VLF chorus emissions in the magnetosphere. J. Geophys. Res. Space Phys., 114(A7), A07217. https://doi.org/10.1029/2009JA014206
Ratcliffe, H., and Watt, C. E. J. (2017). Self-consistent formation of a 0.5 cyclotron frequency gap in magnetospheric whistler mode waves. J. Geophys. Res. Space Physics, 122, 8166–8180. https://doi.org/10.1002/2017JA024399
Santolik, O., Garnett, D. A., Pickett, J. S., Chum, J., and Cornilleau-Wehrlin, N. (2009). Oblique Propagation of whistler mode waves in the chorus source region. J. Geophys. Res. Space Phys., 114(A12), A00F03. https://doi.org/10.1029/2009JA014586
Santolik, O., Garnett, D. A., Pickett, J. S., Grimald, S., Décreau, P. M. E., Parrot, M., Cornilleau-Wehrlin, N., El-Lemdani Mazouz, F., Schriver, D, … Fazakerley, A. (2010). Wave-particle interactions in the equatorial source region of Whistler-mode emissions. J. Geophys. Res. Space Phys., 115(A8), A00F16. https://doi.org/10.1029/2009JA015218
Sauer, K., and Sydora, R. D. (2010). Beam-excited whistler waves at oblique propagation with relation to STEREO radiation belt observations. Ann. Geophys., 28(6), 1317–1325. https://doi.org/10.5194/angeo-28-1317-2010
Sauer, K., and Sydora, R. D. (2011). Whistler-Langmuir oscillations and their relation to auroral hiss. Ann. Geophys., 29(10), 1739–1753. https://doi.org/10.5194/angeo-29-1739-2011
Sauer, K., and Sydora, R. D. (2012). Mode crossing effects at electron beam-plasma interaction and related phenomena. Plasma Phys. Control. Fusion, 54(12), 124045. https://doi.org/10.1088/0741-3335/54/12/124045
Sauer, K., and Sydora, R. D. (2015). Current-driven Langmuir oscillations and amplitude modulations—Another view on electron beam-plasma interaction. J. Geophys. Res. Space Physics, 119. https://doi.org/10.1002/2014JA020409
Sauer, K., Malaspina, D. M., Pulupa, M., and Salem, C. S. (2017). Parametric decay of current-driven Langmuir waves in plateau plasmas: Relevance to solar wind and foreshock events. J. Geophys. Res., 122. https://doi.org/10.1002/2017JA024258
Sauer, K., Baumgärtel, K., Sydora, R., and Winterhalter, D. (2019). Parametric decay of Beam-generated Langmuir waves and three-wave interaction in plateau plasmas: implications for type III radiation. J. Geophys. Res. Space Phys., 124(1), 68–89. https://doi.org/10.1029/2018JA025887
Schriver, D., Ashour-Abdalla, M., Coroniti, F. V., LeBoeuf, J. N., Decyk, V., Travniece, P., Santolik, O., Winningham, D., Pickett, J. S., … Fazakerley, A. N. (2010). Generation of whistler mode emissions in the inner magnetosphere: an event study. J. Geophys. Res. Space Phys., 115(A8), A00F17. https://doi.org/10.1029/2009JA014932
Swanson, D. G. (2003). Plasma Waves. Bristol and Philadelphia: Institute of Physics Publishing.
Sydora, R. D., Sauer, K., and Silin, I. (2007). Coherent whistler waves and Oscilliton formation: kinetic simulations. Geophys. Res. Lett., 34(22), L22105. https://doi.org/10.1029/2007GL031839
Tsurutani, B. T., and Smith, E. J. (1974). Postmidnight chorus: a substorm phenomenon. J. Geophys. Res., 79(1), 118–127. https://doi.org/10.1029/JA079i001p00118
Yue, C., An, X., Bortnik, J., Ma, Q. L., Li, W., Thorne, R. M., Reeves, G. D., Gkioulidou, M., Mitchell, D. G., Kletzing, C. A. (2016). The relationship between the macroscopic state of electrons and the properties of chorus waves observed by the Van Allen Probes. Geophys. Res. Lett., 43(15), 7804–7812. https://doi.org/10.1002/2016GL070084