Minimal Flavor Violation with Axion-like Particles

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Abstract

We revisit the flavor-changing processes involving an axion-like particle (ALP) in the context of generic ALP effective lagrangian with a discussion of possible UV completions providing the origin of the relevant bare ALP couplings. We focus on the minimal scenario that ALP has flavor-conserving couplings at tree level, and the leading flavor-changing couplings arise from the loops involving the Yukawa couplings of the Standard Model fermions. We note that such radiatively generated flavor-changing ALP couplings can be easily suppressed in field theoretic ALP models with sensible UV completion. We discuss also the implication of our result for string theoretic ALP originating from higher-dimensional $p$-form gauge fields, for instance for ALP in large volume string compactification scenario.
I. INTRODUCTION

Axion-like particle (ALP) is a compelling candidate for physics beyond the standard model (BSM) in the intensity frontier searching for a light particle with feeble interactions to the standard model (SM) particles. Indeed ALP is ubiquitous in many well-motivated BSM scenarios, including the QCD axion introduced to solve the strong CP problem [1, 2], string theories [3–5], and the cosmological relaxation of the weak scale [6].

Among the various experimental searches for ALP having a mass below few GeV, one of the most sensitive probe is the flavor-changing processes. Even when the ALP has flavor-conserving couplings to the SM fermions at tree level, there can be radiatively induced flavor-changing couplings which may yet provide a meaningful constraint on the model [7–9]. In particular, if the ALP has a proper form of tree-level couplings to the SM fermions and/or to the Higgs fields, radiative flavor violation can arise at one-loop, with logarithmically divergent effective couplings proportional to $y^\dagger y \ln \Lambda$, where $y$ denotes the fermion Yukawa couplings and $\Lambda$ is the cutoff scale of the ALP effective theory. Such radiatively induced flavor violations have been studied before [7, 8], leading to a rather strong phenomenological constraint on the model parameters. However, these studies are based on the ALP effective interactions which are not manifestly invariant under the electroweak gauge symmetry, while the logarithmic divergence indicates that the dominant contribution comes from the high scales where the electroweak gauge symmetry is restored. This makes the implication of the previous results [7, 8] less clear.

In this paper, we revisit the radiatively induced flavor-changing ALP couplings in the context of manifestly gauge invariant effective lagrangian, and examine their implications with a discussion of the possible UV completion providing the origin of the relevant bare ALP couplings. It is noted that the most dangerous flavor-changing ALP couplings to down-type quarks can be naturally suppressed in field theoretic ALP models with sensible UV completion, in which the ALP originates from the phase degrees of complex scalar fields $X$ charged under the Peccei-Quinn (PQ) symmetry in a UV theory with cutoff scale significantly higher than the PQ scale $f_a \sim \langle X \rangle$. The reason is that bare
ALP couplings at $f_a$ are constrained by the condition that the UV theory allows the top quark Yukawa coupling of order unity, which results in a suppression by $1/\tan^2 \beta$ of the radiative correction to flavor-changing ALP couplings to down-type quarks, where $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$ for the Higgs doublet $H_2$ responsible for the up-type quark masses and the additional Higgs doublet $H_1$ introduced to accommodate the DFSZ-type ALP in the discussion [2]. As a consequence, the flavor constraints on field theoretic ALP become significantly weaker than the estimation of [7, 8] in the large $\tan \beta$ limit. Our analysis captures also the result of [9], which examined the flavor-changing ALP couplings in a model in which the ALP couples to the SM fields through the Higgs bilinear term $H_1 H_2$ in the scalar potential of two Higgs doublet model (2HDM).

We also discuss the implication of our results for string theoretic ALP in large volume scenario of string compactification [10], in which the relevant ALP originates from higher dimensional $p$-form gauge field with a relatively low decay constant in phenomenologically interesting range. It is noticed that for a given value of $f_a$ set by the couplings to gauge fields, flavor-conserving tree level couplings of string theoretic ALP to matter fermions are smaller than those of field theoretic ALP by a factor of $O(1/16\pi^2)$. This distinctive feature of stringy ALP makes the flavor constraints weaker than the naive expectation, independently of the size of $\tan \beta$.

The organization of this paper is as follows. In the next section, we discuss the radiatively induced flavor-changing ALP couplings in the context of generic ALP effective lagrangian constrained just by the ALP shift symmetry and the SM gauge invariance. We consider first the case that the effective theory below the ALP decay constant is non-supersymmetric, but possibly with additional Higgs doublets, and then discuss the supersymmetric case also. In Sec. [II], we discuss the possible UV completion of ALP models, particular the UV origin of the relevant bare ALP couplings. We consider two different possibilities, a field theoretic ALP originating from the phase of PQ-charged complex scalar fields whose vacuum values break the PQ symmetry spontaneously, and a string theoretic ALP originating from $p$-form gauge field in string theory. In Sec. [IV], we examine the ALP parameter region allowed by phenomenological constraints, including
those from the flavor-changing ALP processes, for some specific UV models discussed in Sec. III. Sec. V is the conclusion.

II. RADIATIVELY INDUCED FLAVOR-CHANGING ALP COUPLINGS

In this section, we discuss radiatively induced flavor-changing couplings of an axion-like particle in the context of generic effective lagrangian defined at scales above the weak scale, but below the ALP decay constant $f_a$. We will use the Georgi-Kaplan-Randall (GKR) field basis \cite{11, 12}, in which only the ALP \( a \) experiences a constant shift, while all other low energy fields \( \Phi \) are invariant under the non-linear PQ symmetry:

$$U(1)_{PQ} : a \to a + \text{constant}, \quad \Phi \to \Phi.$$  \hspace{1cm} (1)

Note that one can always take such a field basis with an appropriate ALP-dependent field redefinition of the form \( \Phi \to e^{iq_\Phi a/f_a} \Phi \), where \( q_\Phi \) is the PQ charge carried by \( \Phi \) in the original field basis.

In the GKR basis, PQ-invariant ALP interactions at scales below \( f_a \), which are relevant for our subsequent discussion, can be generally written as

$$\mathcal{L}_{\text{inv}} = \frac{\partial \mu}{f_a} \left[ \sum_i (c_{\psi})_{ij} \bar{\psi}_i \gamma^\mu \psi_j + \sum_\alpha c_{H_\alpha} H_\alpha^\dagger i \sigma^\mu \sigma^\nu D^\nu H_\alpha \right],$$  \hspace{1cm} (2)

where \( \psi_i = \{Q_i, u^c_i, d^c_i, L_i, e^c_i\} \) \((i = 1, 2, 3)\) stands for the 3 generations of the left-handed quarks and leptons, and \( H_\alpha \) \((\alpha = 1, 2)\) denote the Higgs doublets with the following Yukawa couplings at scales just below \( f_a \):

$$\mathcal{L}_{\text{Yukawa}} = (\tilde{y}_u)_{ij} u^c_i Q_j H_2 + (\tilde{y}_d)_{ij} d^c_i Q_j H_d + (\tilde{y}_e)_{ij} e^c_i L_j H_e,$$  \hspace{1cm} (3)

\footnote{Here for simplicity we assume the CP invariance, and ignore the terms such as \( \partial_\mu a H^T_\alpha i \sigma_2 D^\nu H_\beta \) \((\alpha \neq \beta)\) which are assumed to be small in order to forbid the tree level flavor changing neutral current in two (or more) Higgs doublet models. As we consider the effective theory at scales well above the weak scale, the electroweak gauge symmetry is linearly realized in this ALP effective lagrangian. For a discussion of ALP couplings with non-linearly realized electroweak gauge symmetry, see \cite{13}.}
where each of $H_d$ and $H_e$ can be identified as either $H_1$ or $i\sigma_2 H_2^*$, depending upon the model under consideration. Making an appropriate ALP-dependent phase rotation of $\psi$ and $H_\alpha$, together with a proper redefinition of the PQ symmetry, one may choose a specific form of GKR basis for which some of the ALP couplings $(c_\psi, c_{H_\alpha})$ are vanishing. However, as we are interested in the UV origin of the above ALP couplings, which will be discussed in the next section, here we take more general field basis which allows a straightforward matching to the UV completion. On the other hand, we limit the discussion to the models with 2HDM, except for the type III 2HDM which can give rise to a tree level flavor changing neutral current (FCNC). It is straightforward to extend the discussion to models with more Higgs doublets or to the SM without $H_1$, in which $H_d = H_e = i\sigma_2 H_2^*$.

As $U(1)_{PQ}$ is an approximate symmetry, there can be PQ-breaking ALP interactions also, particularly the non-derivative couplings to gauge fields and the scalar potential providing a nonzero ALP mass:

$$\Delta L_{br} = \frac{a}{f_a} \sum_A C_A \frac{g_A^2}{32\pi^2} F^{\mu\nu}_A \tilde{F}^{\mu\nu}_A - \frac{1}{2} m_a^2 a^2 + \ldots,$$

where $F^{\mu\nu}_A$ $(A = 3, 2, 1)$ denote the canonically normalized gauge field strength of the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. Here we assume that the ALP mass is determined by some unspecified UV physics other than the QCD anomaly, which results in

$$m_a \gg f_\pi m_\pi / f_a.$$  

Such ALP mass allows that $f_a$ is small enough to give rise to sizable flavor-changing ALP couplings in the range of phenomenological interest.

We are interested in the case that ALP has flavor-universal couplings to the SM fermions at tree level, so that the $3 \times 3$ ALP coupling matrix $c_\psi$ takes the flavor-universal form at the cutoff scale $\Lambda_a$ of the ALP effective lagrangian (2):

$$(c_\psi)_{ij}(\mu = \Lambda_a) = c_\psi \delta_{ij} \quad (\psi = \{Q, u^c, d^c, L, e^c\}).$$

For field theoretic ALP originating from the phase of PQ charged complex scalar fields, $\Lambda_a$ can be identified as the scale where the PQ symmetry is spontaneously broken, i.e.

$$\Lambda_a \sim f_a.$$
On the other hand, for string theoretic ALP from higher-dimensional $p$-form gauge field, one finds \[ \Lambda_a \sim M_{st} \sim \frac{8\pi^2}{g^2} f_a, \]
where $M_{st}$ is the string scale and the additional factor $8\pi^2/g^2$ originates from the convention to define the ALP decay constant through the ALP interaction to gauge fields in [1], while assuming $C_A = \mathcal{O}(1)$.

Even when $c_\psi$ are flavor-universal at $\Lambda_a$, non-universal piece can be generated by radiative corrections at lower scales. The leading part of those radiative corrections can be captured by the following form of one-loop renormalization group (RG) equations, which can be determined up to an overall coefficient $\xi$ by the covariance under the $SU(3)$ flavor rotations of $\psi$ and the ALP-dependent field redefinitions $\psi \rightarrow e^{ix_a/J_a} \psi$, $H_\alpha \rightarrow e^{ix_a/J_a} H_\alpha$:

\[
\begin{align*}
\frac{d c_Q}{d \ln \mu} &= \frac{\xi}{32\pi^2} \left( c_Q (\tilde{y}_u^\dagger \tilde{u}^\dagger \tilde{y}_d^\dagger \tilde{d}) + \tilde{y}_u^\dagger c_{Q}^T \tilde{u}^\dagger \tilde{d} \tilde{y}_d^\dagger \tilde{y}_u^\dagger c_{Q}^T \tilde{y}_d^\dagger \tilde{y}_u^\dagger + c_{H_2} \tilde{y}_u^\dagger \tilde{y}_u^\dagger c_{H_2} \tilde{y}_d^\dagger \tilde{y}_d^\dagger + h.c. \right), \\
\frac{d c_{u^c}}{d \ln \mu} &= \frac{\xi}{16\pi^2} \left( \tilde{y}_u^c \tilde{y}_u^\dagger + c_{u^c}^T \tilde{y}_u^\dagger \tilde{y}_d^\dagger c_{H_2} \tilde{y}_u^\dagger \tilde{y}_d^\dagger + h.c. \right), \\
\frac{d c_{d^c}}{d \ln \mu} &= \frac{\xi}{16\pi^2} \left( \tilde{y}_d^c \tilde{y}_u^\dagger + c_{d^c}^T \tilde{y}_d^\dagger \tilde{y}_d^\dagger c_{H_2} \tilde{y}_d^\dagger \tilde{y}_d^\dagger + h.c. \right), \\
\frac{d c_L}{d \ln \mu} &= \frac{\xi}{32\pi^2} \left( c_L \tilde{y}_e^\dagger \tilde{y}_e + \tilde{y}_e^\dagger c_L^T \tilde{y}_e + c_{H_2} \tilde{y}_e^\dagger \tilde{y}_e + h.c. \right), \\
\frac{d c_{e^c}}{d \ln \mu} &= \frac{\xi}{16\pi^2} \left( \tilde{y}_e^c \tilde{y}_e^\dagger + c_{e^c}^T \tilde{y}_e^\dagger \tilde{y}_e + c_{H_2} \tilde{y}_e^\dagger \tilde{y}_e + h.c. \right),
\end{align*}
\]

where we include only the Yukawa-dependent parts which can generate flavor-changing ALP couplings at low energy scales. In the following, we will use the above RG equation at the leading log approximation to derive the ALP-fermion coupling $c_\psi$ around the weak scale. Note that the radiatively generated flavor-changing ALP couplings are produced dominantly by the loops involving the top quark and the (Goldstone-mode) Higgs fields, which would be encoded in the RG running from $f_a$ down to the weak scale. At any rate, for non-supersymmetric ALP model, one easily finds from the diagrams in Fig. 1 that the RG coefficient $\xi$ is given by

\[ (\xi)_{\text{non-SUSY}} = 1. \]
In supersymmetric (SUSY) ALP models, there can be additional diagrams involving the superpartner particles, which would contribute to the RG coefficient $\xi$ in (7). One the other hand, in SUSY models there is a simple connection between the beta function of ALP coupling and the anomalous dimension of chiral matter field [14], with which one can easily compute the RG coefficient $\xi$. To see this, we first note that in SUSY model, the ALP interaction (2) can be encoded in the following superfield interactions

$$\int d^4\theta (c_{\Phi})_{IJ} \frac{(A + A^*)}{f_a} \Phi^I \Phi^*_J$$

where $\Phi_I$ denote the chiral superfields including the quark and lepton superfields, as well as the Higgs doublet superfields in SUSY models, and $A$ is the ALP superfield which contains the saxion ($s$) and the axino ($\tilde{a}$) as

$$A = (s + ia) + \sqrt{2}\theta\tilde{a} + \theta^2 F^A.$$

To proceed, it is enough to consider a toy model involving the ALP superfield and a single chiral matter superfield $\Phi$, with the following effective lagrangian

$$\int d^4\theta Z_{\Phi} \Phi^* \Phi + \int d^2\theta \frac{1}{3} \lambda_{\Phi} \Phi^3 + h.c,$$

where

$$Z_{\Phi} = Z_0 \left( 1 + c_{\Phi} \frac{(A + A^*)}{f_a} \right),$$

and $Z_0$ and $\lambda_{\Phi}$ are constants. One then finds

$$c_{\Phi} = f_a \left. \frac{\partial \ln Z_{\Phi}}{\partial A} \right|_{A=0},$$
and therefore
\[
\frac{d c_\phi}{d \ln \mu} = f_a \frac{\partial}{\partial A} \left( \frac{d \ln Z_\phi}{d \ln \mu} \right) \bigg|_{A=0}.
\] (13)

On the other hand, \(d \ln Z_\phi / d \ln \mu\) corresponds to the superspace anomalous dimension, whose one-loop expression is given by
\[
\frac{d \ln Z_\phi}{d \ln \mu} = -\frac{1}{8\pi^2} \frac{\lambda_\phi^* \lambda_\phi}{Z_\phi Z_\phi^*}.
\] (14)

It is straightforward to generalize this observation to the ALP couplings to the MSSM chiral superfields, from which we find that the RG coefficient \(\xi\) in SUSY ALP model is given by
\[
(\xi)_{\text{SUSY}} = 2,
\] (15)
with \(c_{H_d} = c_{H_e} = c_{H_1}\). Note that here we consider only the minimal radiative flavor violation induced by the Yukawa couplings of the SM fermions, while ignoring other sources of flavor violation which might exist in SUSY models.

The BSM degrees of freedom in our ALP model, i.e. additional Higgs doublet and/or the superpartners, might have a mass well above the weak scale. In such case, we should integrate out those BSM particles to derive the ALP couplings at the weak scale. For simplicity, we assume that all BSM particles have a similar mass \(m_{\text{BSM}}\) which would correspond to the charged Higgs boson mass in the 2HDM, \(m_{\text{BSM}} = m_{H^\pm}\), or the superpartner masses in SUSY ALP models, \(m_{\text{BSM}} = m_{\text{SUSY}}\).

In the process to integrate out the BSM particles at \(m_{\text{BSM}} \gg m_W\), the only matching condition relevant for low energy ALP couplings in our approximation is those for the Higgs doublets, which are given by
\[
H_1 = H^* \cos \beta, \quad H_2 = H \sin \beta,
\] (16)
where \(H\) corresponds to the SM Higgs doublet. Then the PQ invariant ALP couplings at scales below \(m_{\text{BSM}}\) are given by
\[
\mathcal{L}_{\text{inv}} = \frac{\partial \mu}{f_a} \left[ \sum_{\psi_\psi} (c_\psi)_{ij} \bar{\psi}_i \gamma^\mu \psi_j + c_H H^i D^\mu H^i \right],
\] (17)
with the matching condition

\[ c_H(\mu = m_{BSM}) = c_{H_2} \sin^2 \beta - c_{H_1} \cos^2 \beta , \]  

(18)

and the SM Yukawa couplings

\[ \mathcal{L}_{Yukawa} = (y_u)_{ij} u_i c_j Q_j H + (y_d)_{ij} d_i c_j Q_j H^* + (y_e)_{ij} e_i c_j L_j H^* , \]  

(19)

where

\[ y_u = \tilde{y}_u \sin \beta, \quad y_d = \tilde{y}_d \cos \beta \text{ or } \tilde{y}_d \sin \beta, \quad y_e = \tilde{y}_e \cos \beta \text{ or } \tilde{y}_e \sin \beta , \]  

(20)

where the matching conditions for \( y_d \) and \( y_e \) depend on the type of 2HDM under consideration. The relevant RG evolution of ALP couplings from \( m_{BSM} \) to the weak scale are given by

\[
\frac{d c_Q}{d \ln \mu} = \frac{1}{32 \pi^2} \left( c_Q (y_u^\dagger y_u + y_d^\dagger y_d) + y_u^\dagger c_{uQ}^T y_u + y_d^\dagger c_{dQ}^T y_d \\
+ c_H (y_u^\dagger y_u - y_d^\dagger y_d) + h.c \right),
\]

\[
\frac{d c_{uQ}^T}{d \ln \mu} = \frac{1}{16 \pi^2} \left( y_u c_Q y_u^\dagger + c_{uQ}^T y_u y_u^\dagger + c_H y_u y_u^\dagger + h.c \right),
\]

\[
\frac{d c_{dQ}^T}{d \ln \mu} = \frac{1}{16 \pi^2} \left( y_d c_Q y_d^\dagger + c_{dQ}^T y_d y_d^\dagger - c_H y_d y_d^\dagger + h.c \right),
\]

\[
\frac{d c_L}{d \ln \mu} = \frac{1}{32 \pi^2} \left( c_L y_e^\dagger y_e + y_e^\dagger c_{eL}^T y_e - c_H y_e y_e^\dagger + h.c \right),
\]

\[
\frac{d c_{eL}^T}{d \ln \mu} = \frac{1}{16 \pi^2} \left( y_e c_L y_e^\dagger + c_{eL}^T y_e y_e^\dagger - c_H y_e y_e^\dagger + h.c \right). \]  

(21)

The RG induced non-universal elements of \( c_\psi \) will lead to flavor-changing ALP interactions at low energy scales after rotating to the fermion mass eigenbasis. The dominant experimental constraints on flavor-changing ALP interactions come from the down-type quark processes. In the mass eigenbasis, the ALP couplings to the left-handed down-type quarks are given by

\[
(c_{ij}^d \frac{\partial \mu a}{f_a} \bar{d}_i \gamma^\mu d_j) \rightarrow -ie_{ij}^d \frac{a}{f_a} \bar{d}_i \left( m_{d_i} P_L - m_{d_j} P_R \right) d_j , \]  

(22)
where

$$c_{ij}^d = (U_{dL}^\dagger c_Q U_{dL})_{ij},$$

(23)

and $d_{Li}$ ($d_i$) denote the left-handed (Dirac) down-type quark fields in the mass eigenbasis, which is obtained by the unitary rotation $d_L \rightarrow U_{dL}d_L$. Here we used the equations of motion of the fermion fields to get the last expression. Applying the one-loop RG equations (7) and (21), we find

$$c_{ij}^d = -\frac{\xi}{16\pi^2} (c_Q + c_{u^c} + c_{H_2}) \left( V_{\text{CKM}}^\dagger \tilde{y}_u^D \tilde{y}_u^D V_{\text{CKM}} \right)_{ij} \ln \left( \frac{\Lambda_a}{m_{\text{BSM}}} \right)$$

$$- \frac{1}{16\pi^2} (c_Q + c_{u^c} + c_H) \left( V_{\text{CKM}}^\dagger y_u^D y_u^D V_{\text{CKM}} \right)_{ij} \ln \left( \frac{m_{\text{BSM}}}{\mu} \right) + \ldots,$$

$$\approx -\frac{m_t^2}{16\pi^2 v^2} (V_{\text{CKM}}^\dagger)^3_i (V_{\text{CKM}})^3_j \left[ \frac{\xi}{\sin^2 \beta} (c_Q + c_{u^c} + c_{H_2}) \ln \left( \frac{\Lambda_a}{m_{\text{BSM}}} \right) \right. $$

$$+ \left. (c_Q + c_{u^c} + c_{H_2} - (c_{H_1} + c_{H_2}) \cos^2 \beta) \ln \left( \frac{m_{\text{BSM}}}{m_t} \right) \right] + \ldots,$$

(24)

where $y_{\psi}^D$ denotes the diagonalized Yukawa matrices in the CKM basis, $v = 174$ GeV, and the ellipses stand for the irrelevant flavor-diagonal parts. Note that the down-type Yukawa couplings do not give rise to a flavor-violating coupling of the down-type quarks at one-loop approximation due to the GIM mechanism. Likewise, the other ALP couplings $c_{\psi} (\psi = u^c, d^c, e^c, L)$ in the one-loop approximation are diagonalized in the CKM basis as long as the flavor-universal condition (6) is satisfied at the scale $f_a$, so they do not generate a flavor violation at one-loop.\(^2\)

From (24), we find that in the large $\tan \beta$ limit, the flavor-changing processes like $b \rightarrow s + a$ or $s \rightarrow d + a$ can happen with a sizable rate if $c_Q + c_{u^c} + c_{H_2}$ has a non-zero value. In case that $c_Q + c_{u^c} + c_{H_2} = 0$, the next leading order contribution arises from a non-zero value of $c_{H_1} + c_{H_2}$, multiplied by an additional suppression factor $1/\tan^2 \beta$.\(^2\)

\(^2\) If we include the right-handed neutrinos with proper Yukawa couplings, the one-loop corrected ALP-lepton coupling $c_L$ includes flavor-changing piece proportional to the square of the right-handed neutrino Yukawa couplings. However, such lepton-flavor-changing ALP couplings can be safely ignored because either the right-handed neutrinos are superheavy or the neutrino Yukawa couplings are negligibly small in order to be compatible with the observed small neutrino masses.
In the next section, we will discuss the implication of this point in terms of the possible UV completion of the ALP effective coupling \( (2) \). Especially, we will see that this implies a suppression of the flavor-changing ALP couplings to the down-type quarks for field theoretic ALP with a sensible UV completion.

We emphasize that the above expression \( (24) \) of the ALP coupling to the down-type quarks is independent of the type of 2HDM under consideration, as far as the SM-like Higgs in the decoupling limit, i.e. \( H_2 \) in our convention, couples only to the up-type quark sector, which is the case for all 2HDMs not involving FCNC at tree-level. Furthermore, even in models involving more Higgs doublets beyond the 2HDMs, the suppression factor \( 1/\tan^2 \beta \) should generically appear. This can be shown by considering the corresponding generalization of the matching condition \( (18) \) for multiple Higgs doublet models. If we define \( \sin \beta \equiv \langle H_2 \rangle / \langle H \rangle \) (i.e. the ratio of the vacuum value of the Higgs of the up-type quark sector to the SM Higgs vacuum value), the matching condition is generalized to

\[
\begin{align*}
    c_H(\mu = m_{\text{BSM}}) &= c_{H_2} \sin^2 \beta + \cos^2 \beta \left( \sum_{\alpha \neq 2} 2Y_{H_\alpha} c_{H_\alpha} \frac{v_{\alpha}^2}{\sum_{\beta \neq 2} v_{\beta}^2} \right),
\end{align*}
\]

where \( v_{\alpha} \equiv \langle H_\alpha \rangle \), and \( Y_{H_\alpha} \) denotes the \( U(1)_Y \) hypercharge of \( H_\alpha \) which should be either \( 1/2 \) or \( -1/2 \) to preserve the electromagnetic \( U(1)_{\text{EM}} \) symmetry. Then

\[
\begin{align*}
    c_Q + c_{u^c} + c_H &= c_Q + c_{u^c} + c_{H_2} - \left[ c_{H_2} - \left( \sum_{\alpha \neq 2} 2Y_{H_\alpha} c_{H_\alpha} \frac{v_{\alpha}^2}{\sum_{\beta \neq 2} v_{\beta}^2} \right) \right] \cos^2 \beta.
\end{align*}
\]

Therefore, if \( c_Q + c_{u^c} + c_{H_2} = 0 \), the dominant term in \( (24) \) is still accompanied by \( 1/\tan^2 \beta \).

We also comment that inclusion of singlet fields or \( SU(2)_L \)-triplet fields etc in the Higgs sector contributes to the flavor violation only by higher dimensional operators and does not change our results at leading order.

Flavor-changing ALP couplings to the up-type quarks can be similarly derived from the one-loop corrected \( c_Q \). Contrary to the case of down-type quarks, the couplings to the up-type quarks depend on the type of 2HDM under consideration. In the mass eigenbasis, the resultant ALP couplings turn out to be

\[
- i \frac{a}{f_a} e^{\alpha} c^{u}_{ij} \bar{u}_i \left( m_{u_i} P_L - m_{u_j} P_R \right) u_j
\]

\( \text{(27)} \)
with

\begin{align*}
c^{v}_{ij} &= -\frac{\xi}{16\pi^2} (c_Q + c_d + c_{H_d}) \left( V_{\text{CKM}} \tilde{y}_d D^i \tilde{y}_d V_{\text{CKM}}^\dagger \right)_{ij} \ln \left( \frac{\Lambda_{a}}{m_{\text{BSM}}} \right) \\
&\quad - \frac{1}{16\pi^2} (c_Q + c_d - c_{H}) \left( V_{\text{CKM}} y_d D^i y_d V_{\text{CKM}}^\dagger \right)_{ij} \ln \left( \frac{m_{\text{BSM}}}{\mu} \right) + \ldots,
\end{align*}

\approx -\frac{m_b^2}{16\pi^2 v^2} (V_{\text{CKM}})_{ij} (V_{\text{CKM}})^*_j \cdot j^3 \left[ \xi (c_Q + c_d + c_{H_d}) \left\{ 1/\cos^2 \beta \ , 1/\sin^2 \beta \right\} \ln \left( \frac{\Lambda_{a}}{m_{\text{BSM}}} \right) \\
&\quad + \left( c_Q + c_d + c_{H_d} - (c_{H_1} + c_{H_2}) \left\{ \sin^2 \beta \ , -\cos^2 \beta \right\} \ln \left( \frac{m_{\text{BSM}}}{m_{\text{W}}} \right) \right) + \ldots, \quad (28)
\end{align*}

where the upper entry of the column applies to the SUSY, type II and type Y 2HDMs with \( c_{H_d} = c_{H_1} \), while the lower entry corresponds to the type I and type X 2HDMs with \( c_{H_d} = -c_{H_2} \), and the ellipses denote the flavor-diagonal part. Here we see that flavor-changing ALP couplings to the up-type quarks arise from the down-type Yukawa couplings, while the up-type Yukawa couplings generate only a flavor-conserving piece due to the GIM mechanism, and therefore the resultant couplings are suppressed by small \( m_b^2/m_t^2 \) compared to the couplings to the down-type quarks. Moreover, the experimental sensitivity of the up-type quark sector to a new physics involving FCNC process is known to be rather weak as it is screened by the QCD long distance effect \[15\]. Yet, in certain models such as SUSY, type II and type Y 2HDMs, the couplings are multiplied by \( \tan^2 \beta \), and therefore can be sizable in the large \( \tan \beta \) limit. This is because the \( b \)-quark Yukawa coupling is enhanced by \( \tan \beta \) at scales above the BSM scale \( m_{\text{BSM}} \). As a result, depending on the type of 2HDM under consideration, the flavor-changing processes of the up-type quarks might impose a meaningful constraint on the ALP decay constant \( f_a \). In the next section, we will address this point with a discussion of possible UV completion of ALP models.

### III. IMPLICATION FOR UV COMPLETED ALP MODELS

In this section, we discuss possible UV completion of ALP models to examine the implication of radiatively induced flavor-changing ALP interactions. As for the UV origin
of ALP, there are two possibilities. ALP might originate from the phase of PQ-charged complex scalar fields whose vacuum values break the PQ symmetry spontaneously, which we call field theoretic ALP, or from higher dimensional $p$-form gauge fields in UV theory with extra spacial dimension, which we call string theoretic ALP. For both type of ALPs, there exist a scalar partner in the UV theory, i.e. the radial mode of PQ-breaking complex scalar field for field theoretic ALP and the modulus partner of string theoretic ALP, whose vacuum value determines the ALP decay constant $f_a$. As we will see, the ALP couplings to the SM fermions and the Higgs doublets, which are of our primary concern, have a definite connection to the couplings of the scalar partner in the Yukawa sector and the Higgs potential.

A. Field theoretic ALP

Let us first consider an ALP originating from the phase of PQ-charged complex scalar fields. For simplicity, we assume that the ALP corresponds mostly to the phase of a single complex scalar field $X$ with PQ charge $q_X = -1$:

$$X = \frac{1}{\sqrt{2}} \rho e^{ia/f_a},$$

(29)

where the vacuum value of the radial mode, $\langle \rho \rangle = f_a$, can be identified as the ALP decay constant in low energy effective theory. Generically this PQ-charged $X$ can couple to the Yukawa sector and the Higgs potential as

$$\left( \frac{X}{M_*} \right)^{q_u + q_{Q_j} + q_{H_2}} (\lambda_{u})_{ij} u_i^c Q_j H_2 + \left( \frac{X}{M_*} \right)^{q_d + q_{Q_j} + q_{H_d}} (\lambda_{d})_{ij} d_i^c Q_j H_d$$

$$+ \left( \frac{X}{M_*} \right)^{q_e + q_{L_j} + q_{H_e}} (\lambda_{e})_{ij} e_i^c L_j H_e + b_0 \left( \frac{X}{M_*} \right)^{q_{H_1} + q_{H_2}} H_1 H_2 + \text{h.c.},$$

(30)

where $q_I$ denote the PQ charge of the corresponding field $\Phi_I$, $M_*$ is the cut-off scale of the above effective interactions, which should be bigger than $f_a$ for consistency, and $b_0$ is a parameter with mass-dimension two. Again we remark that each of $H_d$ and $H_e$ corresponds to either $H_1$ or $i \sigma_2 H_2^*$ depending on the type of 2HDM under consideration.
After replacing $X$ with its vacuum value,

$$\langle X \rangle = \frac{1}{\sqrt{2}} f_a e^{ia/f_a},$$  \hspace{1cm} (31)$$

the UV Yukawa couplings in (30) can be matched to the effective theory Yukawa couplings in (3) in the GKR field basis, with an ALP-dependent field redefinition

$$\Phi_I \rightarrow e^{-i\bar{q}Ia/f_a} \Phi_I \quad (\Phi_I = \psi_i, H_{1,2}),$$  \hspace{1cm} (32)$$

which results in the following matching condition\(^3\) for the ALP couplings at the scale $f_a$:

$$(c_\Phi)_{IJ}(\mu = f_a) = q_{IJ} \delta_{IJ}. \hspace{1cm} (33)$$

It is an interesting possibility that the PQ charges are flavor-non-universal in such a way that the observed hierarchical masses and mixing angles of charge fermion originate from the PQ-breaking spurion factor $(X/M_*)^q_{\psi_i} + q_{\psi_j} + q_{H} \hspace{1cm} [16, 17].$ However, in such case ALP has flavor-changing couplings at tree-level, and the radiative corrections discussed in the previous section give only a small subleading correction to the tree level result.

If the PQ charges of the SM fermions are flavor-universal, i.e.

$$q_{\psi_i} = q_\psi \quad (\psi = Q, u^c, d^c, L, e^c), \hspace{1cm} (34)$$

then there is no flavor-changing ALP coupling at tree level, and the one-loop radiative corrections discussed in the previous section might provide the dominant source of flavor violating ALP processes at low energy scales. After the spontaneous breaking of PQ symmetry, the fermion Yukawa couplings and the coefficient of the Higgs bilinear term are given by

$$(\tilde{y}_\psi)_{ij} = \left( \frac{f_a}{M_*} \right)^{n_\psi} (\lambda_\psi)_{ij} \quad (\psi = u, d, e), \hspace{1cm} (35)$$

$$b = \left( \frac{f_a}{M_*} \right)^{n_H} b_0, \hspace{1cm} (36)$$

\(^3\) Note that there can be a small correction of $\mathcal{O}(f_a^2/M_*^2)$ to this matching condition due to the higher-dimensional operators such as $X^* \partial_{\mu} X \bar{\psi} \gamma^\mu \psi$, which will be ignored in the following discussion.
where the non-negative integer $n_\psi$ and $n_H$ are given by

\[
\begin{align*}
    n_u &= qQ + q_{u^c} + q_{H_2} = cQ + c_{u^c} + c_{H_2}, \\
    n_d &= qQ + q_{d^c} + q_{H_d} = cQ + c_{d^c} + c_{H_d}, \\
    n_e &= qL + q_{e^c} + q_{H_e} = cL + c_{e^c} + c_{H_e}, \\
    n_H &= q_{H_1} + q_{H_2} = c_{H_1} + c_{H_2}.
\end{align*}
\]

One then finds from (24) and (28) that $n_{u,d}$ correspond to the coefficients of RG running generating the flavor-changing ALP couplings starting from the ALP scale $f_a$. In other words, a nonzero value of $n_{u,d}$ can be identified as the dominant source of flavor-changing ALP couplings, which would be enhanced by the large logarithmic factor $\ln(f_a/m_{t,W})$.

Also one finds that $n_H$ corresponds to the RG running coefficient generating flavor-changing ALP couplings starting from the BSM scale $m_{BSM}$ to the weak scale.

Obviously it is not possible to get the correct top quark Yukawa coupling with nonzero $n_u$, while satisfying the perturbativity bound $\lambda_t \lesssim \mathcal{O}(1)$, unless the cutoff scale $M_*$ is comparable to $f_a$. Although $M_*$ can be determined only by the next step of UV completion, which is beyond the scope of this work, there is neither theoretical nor phenomenological motivation for $M_* \sim f_a$. Rather, the spontaneous breaking of PQ symmetry should be interpreted as an IR phenomenon even within the present level of UV completion, which means that it is implicitly assumed that the cutoff scale $M_* \gg f_a$. As we will see in the next section, the radiatively generated flavor-changing ALP couplings discussed in the previous section can be sizable enough to be phenomenologically relevant, only when $f_a \lesssim 10^7$ GeV. For such low PQ scale, the cutoff scale $M_*$ of the present level of UV completion involving the effective interaction (30) is likely to be much higher than $f_a$, for instance at least by one order of magnitude. This implies that

\[ n_u = cQ + c_{u^c} + c_{H_2} = 0 \]  

for generic field theoretic ALP which has a sensible UV completion. Then the flavor-changing ALP couplings to the down-type quarks start to be radiatively generated only from the scale $m_{BSM}$, with a further suppression by $1/\tan^2 \beta$ (see eq. (24)). In fact, the
RG-induced flavor-changing ALP couplings generated at scales below $m_{BSM}$ correspond to the leading piece of the finite result calculated in [9] for a specific UV-completed ALP model in which the ALP couples to the SM sector only through the Higgs bilinear term $H_1H_2$, which amounts to the case with $n_H \neq 0$ and $n_u = n_d = n_e = 0$ in our terminology. Our discussion suggests that the suppression by $1/\tan^2 \beta$ of the one-loop flavor violation is rather generic, and applies for a wide class of field theoretic ALP models beyond the specific example discussed in [9].

Since the major constraint on the ALP decay constant $f_a$ comes from the down-type quark sector, the above observation suggests that the constraints on the ALP models with $n_u = 0$ will be significantly weaker than the previous results which have been obtained based on a simple ansatz for the tree level ALP couplings [7, 8], which is in fact hard to be realized within a sensible field theoretic UV completion. For instance, the Yukawa-like ALP couplings\(^4\) assumed in [8] correspond to the case of $n_u = n_d \neq 0$, which can not be achieved from field theoretic UV completion with a cutoff scale $M_\ast$ significantly higher than $f_a$. The universal ALP couplings assumed in [7] correspond to the case of $q_Q = q_u^c = q_d^c \neq 0$ and $q_{H_1} = q_{H_2} = 0$, which again can not be achieved from sensible field theoretic UV completion.

Given that $n_u = 0$ for field theoretic ALP models with sensible UV completion, and as a result the one-loop flavor-changing ALP couplings to the down-type quarks are suppressed by $1/\tan^2 \beta$, higher loop effects might be even more important than the one-loop contribution if $\tan \beta$ is large enough. Recently, it was pointed out that the following ALP coupling to the $W$-bosons,

$$C_{aWW} \frac{a}{f_a} \frac{g_\ast^2}{32\pi^2} W \bar{W},$$

(39)

\(^4\) If non-derivative ALP couplings are used for the calculation as in [8], one must take into account the additional couplings $i(\tilde{y}_u)_{ij} \frac{a}{f_a} u_R^c d_{Lj} H_2^+ + i(\tilde{y}_d)_{ij} \frac{a}{f_a} d_R^c u_{Lj} H_d^-$ in order to maintain the gauge invariance. Similarly, if one considers an ALP coupling to quark axial vector current as in [7], the associated coupling to vector current must be included for the gauge invariance. This additional vector current coupling can be rotated to the couplings $i(\tilde{y}_u)_{ij} \frac{a}{f_a} u_R^c d_{Lj} H_2^+ + i(\tilde{y}_d)_{ij} \frac{a}{f_a} d_R^c u_{Lj} H_d^-$ by an appropriate ALP-dependent redefinition of the quark fields.
which might exist as a part of (4), can generate flavor-changing ALP couplings to the
down-type quarks [18]. The resulting flavor-changing ALP couplings are essentially two-
loop effects as the above ALP coupling to the $W$-bosons is generated by the one-loop
threshold of PQ-charged heavy particles in field theoretic ALP models. Combining the
results of [18] with ours, we find that the effectively two-loop ALP couplings induced by
(39) dominate over our one-loop contribution if $n_u = 0$ and $\tan \beta$ is large as
\[
\tan \beta \gtrsim 17 \times \sqrt{n_H} \left[ \frac{3}{C_{aWW}} \right]^\frac{1}{2} \left[ \frac{\ln(m_{H^\pm}/m_t)}{2} \right]^\frac{1}{2}.
\] (40)
We also remark that any new physics effect which contributes to the ALP-Higgs derivative
coupling as in [19] can have an important consequence on flavor violating ALP couplings
to the down-type quarks as can be seen from (24).

Finally, let us comment on the flavor violation in the up-type quark sector for field
theoretic ALP. For certain class of UV models including the type-II, type Y 2HDMs and
SUSY, the bottom Yukawa coupling is enhanced by $\tan \beta$ compared to the SM. One may
then expect a sizable amplitude for up-type quark FCNC process for models with $n_d \neq 0$
and large $\tan \beta$ (see (28) and (37)). However such scenario is constrained by the following
matching condition from (36):
\[
\frac{m_b}{v} \cos \beta = \left( \frac{f_a}{M_*} \right)^{n_d} \lambda_b.
\] (41)
Again, for a cutoff scale $M_*$ significantly higher than $f_a$, e.g. by one order of magnitude,
the perturbativity bound $\lambda_b \lesssim \mathcal{O}(1)$ requires $n_d = 0$ for $\tan \beta \gtrsim 10$. On the other hand, in
order for the up-quark sector to compete with the down-quark sector, we need $\tan \beta \gtrsim 20$.
This means that for field theoretic ALP the up-type quark sector is less sensitive to the
ALP-involving flavor violation than the down-type quark sector over the most of the ALP
parameter region provided by sensible UV completion.

B. String theoretic ALP

So far, we have discussed field theoretic UV completion in which the ALP originates
from the phase of PQ-charged complex scalar fields. In such models, the spontaneous
breaking of PQ symmetry should be interpreted as an IR phenomenon in the context of a proper UV completion with the cutoff scale $M_\ast \gg f_a$, which then implies $n_u = 0$. There exists in fact a totally different, but equally attractive UV completion. ALP might originate from higher-dimensional gauge fields in higher-dimensional theory with an ALP decay constant $f_a$ which has a direct connection to the fundamental scale such as the string scale or the compactification scale [4, 20, 21]. The best-motivated example is string theoretic ALP originating from $p$-form gauge field [4] as
\begin{equation}
C_{[m_1 m_2 \ldots m_p]} = \sum_\alpha a_\alpha(x) \omega^\alpha_{[m_1 m_2 \ldots m_p]},
\end{equation}
where $\omega^\alpha$ are harmonic $p$-form on the compact internal space. Typically such ALP arises in SUSY-preserving compactification with a modulus partner $\tau_\alpha$ describing the volume of $p$-cycle dual to $\omega^\alpha$, and forms a chiral superfield as
\begin{equation}
T_\alpha = \frac{\tau_\alpha + ia_\alpha}{\sqrt{2}},
\end{equation}
where we omitted the fermionic and auxiliary $F$-components. The effective theory just below the compactification scale is described by 4D $N = 1$ supergravity model with a Kähler potential
\begin{equation}
K = K_0(T_\alpha + T_\alpha^*) + Z_{IJ}(T_\alpha + T_\alpha^*)\Phi_I^*\Phi_J,
\end{equation}
where $\Phi_I$ denote the gauge-charged chiral matter superfields. The effective theory is controlled by approximate non-linear PQ symmetries under which
\begin{equation}
a_\alpha \rightarrow a_\alpha + \text{constant},
\end{equation}
which are the low energy remnant of the higher-dimensional gauge transformation:
\begin{equation}
\delta C_{[m_1 m_2 \ldots m_p]} = \partial_{[m_1} \Lambda_{m_2 \ldots m_p]}.
\end{equation}
Note that the above non-linear PQ symmetries are defined in the GKR field basis [11], so that $\Phi_I$ are invariant under the PQ symmetries.
With the Kähler potential (44), the ALP effective lagrangian at the scale just below the string scale is given by
\[
L_{\text{eff}} = -\frac{1}{2} (\partial_\alpha \partial_\beta K_0) \partial_\mu a_\alpha \partial_\mu a_\beta - Z_{IJ} (D_\mu \phi^*_I D^\mu \phi_J - i \bar{\psi}_I \Slash{D} \psi_J)
 \]
\[
- \frac{\partial_\mu a_\alpha}{\sqrt{2}} \left[ \left( \frac{\partial Z_{IJ}}{\partial T_\alpha} \right) \phi^*_I \overset{\leftrightarrow}{D}^\mu \phi_J + \left( \frac{\partial Z_{IJ}}{\partial T_\alpha} - \frac{Z_{IJ}}{2} \frac{\partial K_0}{\partial T_\alpha} \right) \bar{\psi}_I \sigma_\mu \psi_J \right],
\] (47)
where we set the reduced Planck scale \( M_P = 1/\sqrt{8\pi G_N} = 1 \). The above lagrangian can be rewritten in terms of the canonically normalized ALP and the matter fermions and sfermions as
\[
L_{\text{eff}} = -\frac{1}{2} \partial_\mu a_p \partial_\mu a_p - D_\mu \phi^*_M D^\mu \phi_M + i \bar{\psi}_M \Slash{D} \psi_M
 \]
\[
- \frac{\partial_\mu a_p}{\sqrt{2}} \left[ c_{pMN} \phi^*_M \overset{\leftrightarrow}{D}^\mu \phi_N + \left( c_{pMN} - \frac{1}{2} c_p \delta_{MN} \right) \bar{\psi}_M \sigma_\mu \psi_N \right],
\] (48)
where
\[
c_{pMN} = \Omega^a_{\alpha p} \Omega^f_{IM} \Omega^f_{JN} \frac{\partial Z_{IJ}}{\partial T_\alpha}, \quad c_p = \Omega^a_{\alpha p} \frac{\partial K_0}{\partial T_\alpha}
\] (49)
for the field redefinition matrices
\[
\Omega^a_{\alpha M} \Omega^a_{\beta N} (M_P^2 \partial_\alpha \partial_\beta K_0) = \delta_{MN}, \quad \Omega^f_{IM} \Omega^f_{JN} Z_{IJ} = \delta_{MN}.
\] (50)

Unless the compactification involves a large internal space volume or an exponential warp factor, the stringy ALP decay constant is generically near \( M_P/8\pi^2 \sim 10^{16} \) GeV [3, 4]. In such case, the flavor-changing ALP interactions would be too weak to be phenomenologically relevant. On the other hand, in models with a large internal volume or warp factor, the resulting ALP scale can be lower than \( M_P/8\pi^2 \) by many orders of magnitude, even might be around the TeV scale [10, 20–23]. In the following, we consider one such example, the stringy ALP in the large volume scenario (LVS) proposed in [10].

For simplicity, we consider the minimal LVS with two ALPs and their modulus partners:
\[
T_1 = \frac{\tau_1 + i a_1}{\sqrt{2}}, \quad T_2 = \frac{\tau_2 + i a_2}{\sqrt{2}},
\] (51)
where \( \tau_1 \) corresponds to the volume of big 4-cycle \( C_b \), which is connected to the bulk volume of the 6-dimensional internal space as \( V \sim \tau_1^{3/2} \), while \( \tau_2 \) is the volume of small
4-cycle $C_s$ supporting a hidden non-perturbative dynamics, as well as the visible sector. Following [10], we assume $\tau_1$ is stabilized at an exponentially large value as

$$\frac{1}{\tau_1^{3/2}} \sim e^{-a\tau_2},$$

(52)

where $e^{-a\tau_2}$ parametrizes the strength of hidden non-perturbative dynamics with $a\tau_2 = O(\pi^2/g_{\text{GUT}}^2)$, which competes with the stringy $\alpha'$ corrections of $O(1/\tau_1^{3/2})$ to stabilize $\tau_1$ at an exponentially large vacuum value.

To be specific, let us consider the Kähler potential in the limit $\tau_1 \gg \tau_2 \gtrsim 1$, which is given by [10]

$$K = -3 \ln(T_1 + T_1^*) + \frac{(T_2 + T_2^*)^{3/2}}{(T_1 + T_1^*)^{3/2}} + \frac{(T_2 + T_2^*)^{\omega_N}}{(T_1 + T_1^*)^{\omega_N}} \Phi_N^* \Phi_N,$$

(53)

where the modular weights $\omega_N$ of gauge charged matter superfields $\Phi_N$ are rational numbers. The holomorphic gauge kinetic function of the model for the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ takes the form:

$$F_A = k_A T_2 \quad (A = 3, 2, 1),$$

(54)

where $k_A$ are rational numbers of order unity, and the visible sector Yukawa couplings in the superpotential are given by

$$\Delta W = \frac{1}{6} \lambda_{LMN} \Phi_L \Phi_M \Phi_N,$$

(55)

where $\lambda_{LMN}$ are independent of $T_i$ ($i = 1, 2$) due to the ALP shift symmetries. As we will see, the ALP $a_1$ associated with the big cycle has a decay constant near $M_P$, while the small-cycle ALP $a_2$ can have a much lower decay constant in phenomenologically interesting range.

Following the usual convention for ALP couplings, let us define the decay constant of the canonically normalized $a_2$ though its coupling to the gauge fields. For the Kähler potential and the gauge kinetic function given by (53) and (54), we find

$$\frac{1}{2} \partial_a a_2 \partial^a a_2 - \frac{1}{4g_A^2} F_A^{\mu \nu} \tilde{F}_A^{\mu \nu} - \frac{1}{32\pi^2} \frac{a_2}{f_a} F_A^{\mu \nu} \tilde{F}_A^{\mu \nu},$$

(56)
where
\[
\frac{1}{g^2_A} = k_A \frac{\tau_2}{\sqrt{2}},
\] (57)
and
\[
f_a = \frac{\sqrt{3}}{2\tau_2^{1/4}} \frac{1}{M_P} \frac{M_P}{8\pi^2} \sim e^{-a\tau_2/2} \frac{M_P}{8\pi^2}.
\] (58)

Here we used \(\tau_2 = \mathcal{O}(1/g^2_{\text{GUT}})\) and the large volume condition [52] for the last expression of \(f_a\). In the canonically normalized field basis, we find also the following physical Yukawa couplings and the ALP couplings to the matter fields:

\[
\mathcal{L}_{\text{Yukawa}} = \frac{1}{2} y_{LMN} \phi_L \bar{\psi}_M \psi_N,
\]
\[
\mathcal{L}_{\text{inv}} = \frac{\partial_\mu a_1}{\sqrt{6} M_P} \left( \phi_N^* i D^\mu \phi_N - \frac{1}{2} \bar{\psi}_N \bar{\sigma}^\mu \psi_N \right) - c_N \frac{\partial_\mu a_2}{f_a} \left( \phi_N^* i D^\mu \phi_N + \bar{\psi}_N \gamma^\mu \psi_N \right),
\] (59)

where
\[
y_{LMN} = \frac{\lambda_{LMN}}{(\sqrt{2}\tau_2)(\omega_L + \omega_M + \omega_N)/2},
\]
\[
c_N = \frac{\sqrt{2}}{16\pi^2 \tau_2 \omega_N}.
\] (60)

Note that the above couplings are defined at scales around the string scale which is related to the ALP scale as [10]
\[
M_{st} \sim \frac{M_P}{\tau_1^{3/4}} \sim 8\pi^2 f_a.
\] (61)

Although the big-cycle ALP \(a_1\) has a too large decay constant to give any observable consequence in the laboratory experiments, the small-cycle ALP \(a_2\) can have a decay constant in the phenomenologically interesting range, if \(\tau_1\) has an exponentially large vacuum value as \(\tau_1^{3/4} \sim e^{a\tau_2/2}\) with \(a\tau_2 \gg 1\) [10, 22, 23]. Yet, the pattern of the couplings of \(a_2\) is determined by the matter modular weights \(\omega_N\). It has been noticed in [24] that these modular weights can be determined by the behavior of the physical Yukawa couplings under the rescaling of the metric on the small-cycle \(C_s\), which results in flavor-universal \(\omega_N\) in the range [0, 1].
Here are the values of modular weights in some interesting cases. One possible scenario (Case 1) is that matter zero modes live on the 4-cycle $C_s$, with four dimensional triple intersections for Yukawa couplings, yielding

$$\omega_N = 1/3.$$  

Another possible scenario (Case 2) is that matter zero modes are confined on two dimensional curves in $C_s$, with point-like triple intersection for Yukawa couplings, which gives

$$\omega_N = 1/2.$$  

The final example (Case 3) we can consider is that $Q, u^c$ and $H_2$ are confined on a singular point, while $H_1$ and/or $d^c$ can propagate over two or four dimensional surface in $C_2$, which gives

$$\omega_Q = \omega_{u^c} = \omega_{H_2} = 0, \quad \omega_{d^c} + \omega_{H_1} > 0.$$  

Note that $\tau_2 = \mathcal{O}(\frac{1}{g_{GUT}^2})$, and therefore contrary to the case of field theoretic ALP, all of these examples can be compatible with the perturbativity constraint $\lambda_{LMN} \lesssim \mathcal{O}(1)$, while giving the correct top quark Yukawa coupling $y_t = \mathcal{O}(1)$.

For the Cases 1 and 2, the model predicts that $n_u$ is nonzero as

$$n_u = c_Q + c_{u^c} + c_{H_2} = \frac{\sqrt{2}}{16\pi^2 \tau_2} (\omega_Q + \omega_{u^c} + \omega_{H_2}) = \mathcal{O} \left( \frac{1}{16\pi^2} \right).$$ (62)

Although being the order of $10^{-2}$, a nonzero $n_u$ still can yield relatively strong flavor-changing ALP couplings to the down-type quarks due to the large logarithmic factor $\xi \ln(\Lambda_a/m_t) \approx 2 \ln(8\pi^2 f_a/m_t)$ (see eq. (24)). For the Case 3, $n_u = 0$ and therefore the resulting flavor-changing ALP couplings to the down-type quarks are further suppressed by $1/\tan^2 \beta$. However, in this case we have a nonzero $n_d$ as

$$n_d = c_Q + c_{d^c} + c_{H_1} = \frac{\sqrt{2}}{16\pi^2 \tau_2} (\omega_Q + \omega_{d^c} + \omega_{H_1}) = \mathcal{O} \left( \frac{1}{16\pi^2} \right),$$ (63)

and then the resulting ALP couplings to the up-type quarks might provide a meaningful constraint on the model if $\tan \beta$ is large enough. In the next section, we will give a detailed analysis of the phenomenological constraints on the string theoretic ALP decay constant for the Cases 1 and 3.
IV. CONSTRAINTS ON THE ALP DECAY CONSTANT

In this section, we examine the experimental constraints on the ALP decay constant $f_a$ from flavor-changing processes, while taking into account the properties of ALP inferred from the possible UV completions. As we will see, the FCNC processes of down-type quarks provide a dominant constraint on $f_a$, because flavor-changing ALP couplings to the up-type quarks are suppressed by the relatively small bottom Yukawa coupling as discussed in Sec. II and have weaker experimental sensitivity due to the long distance QCD effect [15].

According to Eqs. (24) and (37), flavor-changing ALP couplings to the down-type quarks in the fermion mass eigenbasis are given by

$$\frac{\partial a}{f_a} c^d_{ij} \bar{d}_i \gamma^\mu P_L d_j,$$

where

$$c^d_{ij} \approx \frac{m_t^2}{16\pi^2 v^2} (V_{CKM})_{3i}^*(V_{CKM})_{3j} \left[ \xi_n u \ln \left( \frac{\Lambda_a}{m_{BSM}} \right) + \left( n_u - \frac{n_H}{\tan^2 \beta} \right) \ln \left( \frac{m_{BSM}}{m_t} \right) \right].$$

Here $\Lambda_a \sim f_a$ for field theoretic ALP, while $\Lambda_a \sim 8\pi^2 f_a$ for string theoretic ALP, $\xi = 1 \ (2)$ for non-SUSY (SUSY) model, and the BSM scale $m_{BSM}$ corresponds to the charged Higgs boson mass in 2HDMs, which is also taken to be the superpartner mass scale for SUSY models. These ALP couplings give rise to the rare meson decays such as $B \to K^{(*)} a$ and $K \to \pi a$. We use the hadronic matrix elements using the light-cone QCD sum rules for the $B$ or $K$ meson transitions [7, 25, 26], yielding

$$\Gamma (B \to Ka) = \frac{m_B^3}{64\pi} \frac{|c^{d}_{sb}|^2}{f_a^2} \left( 1 - \frac{m_K^2}{m_B^2} \right)^2 \mathcal{F}^2_K \left( \frac{m_B^2}{m_K^2} \right) \lambda^{1/2}_{BKa},$$

$$\Gamma (B \to K^{*}a) = \frac{m_B^3}{64\pi} \frac{|c^{d}_{sb}|^2}{f_a^2} \mathcal{F}^2_K \left( \frac{m_B^2}{m_K^2} \right) \lambda^{3/2}_{BK^{*}a},$$

$$\Gamma (K^{+} \to \pi^{+} a) = \frac{m_K^3}{64\pi} \frac{|c^{d}_{ds}|^2}{f_a^2} \left( 1 - \frac{m_{\pi}^2}{m_K^2} \right)^2 \lambda^{1/2}_{K\pi a},$$

$$\Gamma (K_{L} \to \pi a) = \frac{m_{K_{L}}^3}{64\pi} \frac{|\text{Im}(c^{d}_{ds})|^2}{f_a^2} \left( 1 - \frac{m_{\pi}^2}{m_{K_{L}}^2} \right)^2 \lambda^{1/2}_{K_{L}\pi a}.$$
where
\[ \lambda_{xyz} \equiv \left( 1 - \frac{(m_y + m_z)^2}{m_x^2} \right) \left( 1 - \frac{(m_y - m_z)^2}{m_x^2} \right), \]  
(70)
and the form factors for the $B$ meson transition are given by [27, 28]
\[ F_K \left( m_a^2 \right) = \frac{0.33}{1 - m_a^2/(38 \text{GeV}^2)}, \]  
(71)
\[ F_{K^*} \left( m_a^2 \right) = \frac{1.35}{1 - m_a^2/(28 \text{GeV}^2)} - \frac{0.98}{1 - m_a^2/(37 \text{GeV}^2)}, \]  
(72)
while the form factors for the $K$ meson transition are taken to be unity.

We also have flavor-changing ALP couplings to the up-type quarks,
\[ \frac{\partial_{\mu} a}{f_a} \bar{c}_i \gamma\mu P_L c_j, \]  
(73)
where the coupling coefficients $c_{ij}^u$ are given in Eq. [28] and [37]:
\[ c_{ij}^u \approx - \frac{m_b^2}{16\pi^2v^2} (V_{\text{CKM}})_{i3}(V_{\text{CKM}})_{j3}^* \times \left[ \frac{\tan^2\beta}{n_d - n_H} \left\{ \frac{\Lambda_a}{m_{BSM}} + \left( 1 - \frac{1}{\tan^2\beta} \right) \ln \left( \frac{m_{BSM}}{m_W} \right) \right\} \ln \left( \frac{m_{BSM}}{m_W} \right) \right], \]
where the upper entry of the column is for the 2HDMs with $H_d = H_1$, while the lower entry corresponds to the other models with $H_d = i\sigma_2 H_2^*$. The most stringent constraint on the above ALP couplings comes from the rare charm meson decay $D^+ \to \pi^+ + a$ whose width is given by [29]
\[ \Gamma \left( D^+ \to \pi^+ a \right) = \frac{m_{D^+}^3}{64\pi} \frac{|c_{cu}^u|^2}{f_a^2} \left( 1 - \frac{m_{\pi^+}^2}{m_{D^+}^2} \right)^2 \lambda_{D^+ \pi^+ a}^{1/2} \mathcal{F}_{D^+} \left( m_a^2 \right), \]  
(74)
where
\[ \mathcal{F}_{D^+} \left( m_a^2 \right) = \frac{0.67}{1 - m_a^2/(4.58 \text{GeV}^2)}. \]  
(75)

The ALP produced by the rare meson decays subsequently decays into lighter SM particles with the branching ratio determined by the flavor-conserving ALP couplings. In Appendix A we provide a summary of the low energy ALP couplings relevant for the ALP.
decays. Given the rare meson decay width to the final state involving ALP, and also the subsequent ALP decay branching ratios, one can predict an excess in each specific rare meson decay channel over the background. For instance, if the ALP decays mainly into leptons as $a \to e^+ e^-$ or $\mu^+ \mu^-$, the experimental upper bound on the branching fraction of the leptonic rare meson decay $B^+ \to K^+ l^+ l^-$ puts an upper limit on the rare meson decay width $B^+ \to K^+ a$ times the branching ratio $a \to l^+ l^-$, providing a lower bound on the ALP decay constant $f_a$ for given values of the other model parameters. If the ALP decay width is so small that the ALP escapes the detector before it decays, the event will be identified as an invisible decay, constrained by the channel $K^+ \to \pi^+ + inv$, for example. In Appendix B, we provide a short description of the various experimental channels which are relevant for our study.

In Fig. 2, we show the excluded range of the ALP decay constant $f_a$ in terms of the ALP mass $m_a$ for field theoretic ALP with sensible UV completion, which has $n_u = 0$ as discussed in the previous section. Although this is about a specific benchmark model, i.e. non-supersymmetric ALP model with the type II 2HD Yukawa sector, similar results are obtained also for other type of 2HD models or SUSY models. The left panel corresponds to the case with $n_H = 1$ and a moderate value of $\tan \beta$, in which the one-loop induced flavor-violating ALP couplings in (65) provide the dominant source of constraints. The plot shows that the bound is more than an order of magnitude weaker than the results of [8]. Only for ALP mass above the two muon threshold $m_a > 2m_{\mu}$, the bound is similar to the previous results found in [7, 8], since the experimental upper limits on $\text{Br} (B \to K + a(\mu^+ \mu^-))$ have been significantly improved recently [30, 31]. This overall weaker bound is due to that the condition of sensible UV completion requires $n_u = 0$, and as a result the radiative correction to generate flavor-changing ALP couplings starts to operate from the BSM scale, which is the charged Higgs mass in our benchmark example, with a suppression by $1/\tan^2 \beta$ (see Eq. (65)). Note that yet a sizable fraction

\[5\] Although our ALP model involves BSM physics at scales above $m_{BSM}$, we assume that the BSM scale is high enough, e.g. heavier than 1 TeV, so that the background event rates are essentially same as those for the SM.
FIG. 2: Parameter region excluded by the FCNC constraints for field theoretic ALP \((n_u = 0\) and \(\Lambda_a \sim f_a\)). The left panel is for the case with a moderate \(\tan \beta = 5\), in which the one-loop induced ALP couplings in (65) provide the dominant source of flavor violation. The right panel is for the case with a larger \(\tan \beta = 30\) satisfying (40), in which the effective two-loop effects associated with the ALP coupling (39) to the W-bosons, which was discussed in [18], provide the dominant constraints. Here we consider the type II 2HDM with \(m_{H^\pm} = 1\) TeV as a benchmark model. The results do not change much for other type of 2HDMs and SUSY models. Gray parts correspond to the parameter region excluded by the conventional astrophysical considerations (SN1987 + Red giant evolution).

of the parameter space for \(m_a \gtrsim \mathcal{O}(0.1)\) MeV, which would be allowed by astrophysical constraints, is excluded by the FCNC constraints on the radiatively generated flavor-changing ALP couplings. If we take an even lower value of \(\tan \beta\) around 1, the overall flavor constraints get severer by an order of magnitude, approaching to the previous results in [8] except the mass region \(m_a > 2m_\mu\) where the experimental sensitivity has been upgraded. This limit corresponds to the strongest flavor bound on field theoretic ALP. However, since such a small \(\tan \beta\) would have a problem with the perturbativity
bound on the top Yukawa coupling, theoretically more sensible bound is expected to be weaker being similar to the left panel of Fig. 2 within order one uncertainty.

For large tan β satisfying the condition \( (40) \), or for the case with \( n_H = 0 \), the effective two-loop contribution associated with the ALP coupling \( (39) \) to the W-bosons becomes dominant over the one-loop contribution of \( (65) \). The flavor constraints in such situation were discussed in [18] under the assumption that ALP does not have a tree level coupling to the charged leptons, so decays mostly into photons, which would be the case for the KSVZ-type ALP model [2]. Here we are concerned with the DFSZ-type ALP having nonzero tree level coupling \( c_\psi \) to the SM fermions, but with \( n_u = c_Q + c_{uc} + c_{H_2} = 0 \) for field theoretic ALP models. As a result, in our case the ALP decays mainly into lepton pair, and we depict the resulting constraints in the right panel of Fig. 2. We see that still a sizable fraction of parameter space allowed by other constraints is excluded by the flavor constraints. Notice that this corresponds to the weakest flavor bound for the (DFSZ-type) field theoretic ALP with non-vanishing \( C_{aWW} \) coupling \( (39) \). Since it is dominated by the effective two-loop contribution, it does not depend on tan β as long as tan β is large enough to satisfy the condition \( (40) \). If \( C_{aWW} = 0 \), the bound can be even weaker dominated by the one-loop contribution suppressed by \( 1/\tan^2 \beta \). In this case, we find that the lower bound on \( f_a \) becomes around TeV scale if \( \tan \beta > 60 \) for \( m_a < 1 \) MeV or \( m_a \gtrsim 100 \) MeV. For \( 1 \) MeV \( < m_a \lesssim 100 \) MeV, the dominant constraint comes from the CHARM beam dump experiment, which shows a rather insensitive dependence on tan β. For this region, the resultant lower bound on \( f_a \) is larger than 10 TeV unless \( \tan \beta \gtrsim 100 \).

In Fig. 3 we show the excluded parameter region for string theoretic ALP in the LVS scenario. In the plot, we examine the case of universal modular weights \( \omega_N = 1/3 \) with \( \tau_2 = \sqrt{2} \), giving

\[
\begin{align*}
n_u &= c_Q + c_{uc} + c_{H_2} = \frac{\sqrt{2}}{16\pi^2\tau_2} (\omega_Q + \omega_{uc} + \omega_{H_2}) = \frac{1}{16\pi^2}, \\
n_d &= c_Q + c_{dc} + c_{H_1} = \frac{\sqrt{2}}{16\pi^2\tau_2} (\omega_Q + \omega_{dc} + \omega_{H_1}) = \frac{1}{16\pi^2}, \\
n_H &= c_{H_1} + c_{H_2} = \frac{\sqrt{2}}{16\pi^2\tau_2} (\omega_{H_1} + \omega_{H_2}) = \frac{1}{24\pi^2}.
\end{align*}
\]
FIG. 3: Excluded parameter region for string theoretic ALP in the LVS scenario with universal modular weights \( (n_u = n_d = 1/16\pi^2, n_H = 1/24\pi^2, \Lambda_a \sim 8\pi^2 f_a) \). Gray parts correspond to the parameter region excluded by the conventional astrophysical considerations (SN1987 + Red giant evolution).

This is similar to the Yukawa-like coupling ansatz of \([7, 8]\), but with additional suppression factor of \(1/16\pi^2\), which is due to our convention\(^6\) to define \( f_a \) in terms of the ALP couplings to gauge fields:

\[
\frac{a}{f_a} \frac{g_A^2}{32\pi^2} F^{\mu\nu}_A \tilde{F}^{\mu\nu}_A.
\]

However the resultant bound on \( f_a \) turns out to be only an order of magnitude weaker than the results of \([7, 8]\), rather than two orders of magnitude expected from the factor \(1/16\pi^2\). This is mostly due to the logarithmic factor \( \xi \ln(\Lambda_a/m_t) \simeq 2 \ln(8\pi^2 f_a/m_t) \) for the down-type quark flavor violation with non-zero \( n_u \) as can be seen in \([65]\), which provides nearly an order of magnitude enhancement in our case. Note that in \([7, 8]\) \( \Lambda_a \) is taken to be around 1 TeV, and as a result the corresponding logarithmic factor is of order unity.

\(^6\) Note that in this convention, \( c_\psi = \mathcal{O}(1) \) for the DFSZ-type ALP, \( c_\psi = \mathcal{O}(\ln(f_a/\mu)/(16\pi^2)^2) \) for the KSVZ-type ALP, and \( c_\psi = \mathcal{O}(1/16\pi^2) \) for string theoretic ALP.
Since the major constraint comes from the flavor-changing ALP couplings to the down-type quarks induced by non-zero $n_u$, the bound on $f_a$ does not depend on $\tan\beta$ and the charged Higgs mass $m_{H^\pm}$.

In case that the matter modular weights give $n_u = 0$ and $n_d \neq 0$, e.g. the Case 3 described in the previous section, the flavor constraints from the up-type quark sector might be important if $\tan\beta$ is large enough. We examined this issue also, and find that for ALP mass $m_a > 100$ MeV, the flavor constraints from rare charm decay (with leptonic decay channel) provide a stronger bound on $f_a$ than the down-quark sector only for a very large $\tan\beta > 70$, which would constrain the ALP decay constant as $f_a \gtrsim 1$ TeV. For smaller $\tan\beta$, it turns out that the effective two loop flavor violation in the down-quark sector \[18\] arising from the ALP coupling \[39\] to the W-bosons provides a stronger constraint than the up-quark sector. However it should be noted that for a stringy ALP with $C_{aWW} = 0$, the up-type quark sector can be the dominant source of flavor constraints once $\tan\beta \gtrsim 20$.

V. CONCLUSION

In this paper, we examined the radiatively induced flavor-changing ALP couplings in the context of manifestly gauge-invariant effective lagrangian, while taking into account the UV origin of the relevant bare ALP couplings. We focus on the minimal scenario that ALP has only flavor-conserving couplings at tree level, and the dominant flavor-violating couplings are induced at one loop order due to the SM Yukawa couplings. As for the UV origin of ALP, we consider two possibilities: (i) field theoretic ALP originating from the phase degrees of PQ charged complex scalar fields in a UV theory with linearly realized PQ symmetry, and (ii) string theoretic ALP originating from higher dimensional $p$-form gauge fields in compactified string theory with relatively low string scale.

For field theoretic ALP, the bare ALP parameter $n_u = c_Q + c_{u^c} + c_{H^2}$, which is responsible for radiative generation of the most dangerous flavor-changing ALP couplings, is required to be vanishing in order for the underlying UV theory to admit the top quark
Yukawa coupling of order unity. As can be noticed easily from the expression (65), this results in a suppression of the flavor-changing ALP couplings to down-type quarks, which is particularly efficient in the large $\tan \beta$ limit. Then, depending upon the value of $\tan \beta$, the experimental lower bound on $f_a$ for field theoretic ALP can be significantly relieved compared to the previous estimation \[7, 8\], which was based on the simple ansatz for ALP couplings that would not be realized in a sensible UV theory.

We examined also the flavor constraints on string theoretic ALP in large volume scenario of string compactification \[10\], in which some of the ALPs can have a low decay constant in phenomenologically interesting range. One of the distinctive features of such string theoretic ALP is that $c_\psi = \mathcal{O}(1/16\pi^2)$ for $f_a$ defined through the ALP couplings to gauge fields under the assumption $C_A = \mathcal{O}(1)$. (See Eqs. (2) and (4) for our notations.) Note that $c_\psi = \mathcal{O}(1)$ for DFSZ-type field theoretic ALP in the same convention \[2\]. Even with $c_\psi = \mathcal{O}(10^{-2})$, the resulting flavor constraints can be stronger than those for field theoretic ALP, in particular when $\tan \beta \gg 1$. This is because $n_u$ for string theoretic ALP is generically non-vanishing, although small as $\mathcal{O}(10^{-2})$, and therefore the flavor-violating radiative corrections are enhanced by the large logarithmic factor $\ln(\Lambda_a/m_t) \sim \ln(8\pi^2 f_a/m_t)$ without a suppression by $1/\tan^2 \beta$.

VI. ACKNOWLEDGEMENT

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Appendix A: Low energy effective ALP couplings

In this appendix, we briefly summarize the flavor-conserving low energy couplings which are relevant for the decays of axion-like particles (ALPs) which were produced by flavor-changing rare meson decays. General flavor and CP conserving effective interactions of an ALP with the SM particles at scales just above the weak scale are given by

\[ \frac{\partial \mu a}{f_a} \left( c_Q \bar{Q} \gamma^\mu Q + c_u \bar{u}_R \gamma^\mu u_R + c_d \bar{d}_R \gamma^\mu d_R + c_e \bar{e}_R \gamma^\mu e_R + c_L \bar{L} \gamma^\mu L + c_c \bar{c}_R \gamma^\mu c_R + c_H H^\dagger i \tilde{D}^\mu H \right) \]

\[ - \frac{\bar{a} f_a}{2} \left( C_{agg} \frac{g_3^2}{32\pi^2} G\tilde{G} + C_{aWW} \frac{g_2^2}{32\pi^2} W\tilde{W} + C_{aBB} \frac{g_1^2}{32\pi^2} B\tilde{B} \right) - \frac{1}{2} \hat{m}_a^2 a^2 + \ldots, \]  

(A1)

where \( \hat{m}_a \) denotes the ALP mass not including the contribution from the ALP coupling to the gluon anomaly \( G\tilde{G} \). After the electroweak symmetry breaking, the ALP-Higgs coupling induces a mixing between the ALP and Z-boson. Integrating out the Z-boson then gives a threshold correction to the ALP-fermion couplings at scales below the Z-boson mass. This threshold correction can be taken into account by making the following ALP-dependent \( U(1) \) rotation [11],

\[ H \rightarrow \exp \left( i c_H \frac{a}{f_a} \right) H, \]

\[ \psi \rightarrow \exp \left( i 2 c_H Y_\psi \frac{a}{f_a} \right) \psi, \]  

(A2)

where \( Y_\psi \) denotes the \( U(1)_Y \) hypercharge of the fermion \( \psi \). This hypercharge-proportional rotation enables us to remove the ALP-Higgs coupling without affecting the ALP couplings to gauge bosons. Then, after integrating out the massive weak gauge bosons and the top quark, one finds the relevant ALP interactions given by

\[ \frac{\partial \mu a}{f_a} \left[ \sum_{u_i = u, c} A_u \bar{u}_i \gamma^\mu \gamma^5 u_i + \sum_{d_i = d, s, b} A_d \bar{d}_i \gamma^\mu \gamma^5 d_i + \sum_{l = e, \mu, \tau} A_l \bar{c}_i \gamma^\mu \gamma^5 l \right] \]

\[ - \frac{\bar{a} f_a}{2} \left( C_{agg} \frac{g_3^2}{32\pi^2} G\tilde{G} + C_{a\gamma\gamma} \frac{e^2}{32\pi^2} F\tilde{F} \right), \]  

(A3)
where now all fermions are written as the Dirac fermions, and
\[
A_u = -\frac{1}{2}(c_Q + c_u e + c_H),
\]
\[
A_d = -\frac{1}{2}(c_Q + c_d e - c_H),
\]
\[
A_l = -\frac{1}{2}(c_L + c_e e - c_H),
\]
\[
C_{a\gamma\gamma} = C_{aWW} + C_{aBB}.\]  

(A4)

At the lower scale below the charm quark mass, but above the QCD scale \(\Lambda_{\text{QCD}}\), the relevant ALP interactions are further reduced to
\[
\frac{\partial}{\partial a} \left[ \bar{q} \gamma^\mu (X_q \gamma^5 q + \bar{l} \gamma^\mu A_l \gamma^5 l) - \frac{a}{f_a} \left( C_{a\gamma\gamma} \frac{g_3^2}{32\pi^2} G \tilde{G} + C_{a\gamma\gamma} \frac{e^2}{32\pi^2} F \tilde{F} \right) \right], \tag{A5}
\]
where \(q \equiv (u, d, s)\), \(l \equiv (e, \mu)\), and \(X_q \equiv \text{diag}(A_u, A_d, A_d)\). Below the QCD scale, one should apply the chiral perturbation theory to describe the ALP interactions with mesons and baryons. For convenience, we first eliminate the term \(a G \tilde{G}\) by the following quark field rotation\(^7\),
\[
q \rightarrow \exp \left[ \frac{i}{f_a} a q A \gamma^5 \right] q, \tag{A6}
\]
where
\[
\text{Tr} [qA] = \frac{C_{a\gamma\gamma}}{2}. \tag{A7}
\]
Then the effective lagrangian becomes
\[
\frac{\partial}{\partial a} \left[ \bar{q} \gamma^\mu (X_q - q A) \gamma^5 q + \bar{l} \gamma^\mu A_l \gamma^5 l \right] - \frac{a}{f_a} \left[ 2i \frac{a}{f_a} q A \gamma^5 \right] q \tag{A8}
\]
\[
- \frac{a}{f_a} \left( C_{a\gamma\gamma} - 12 \text{Tr} [qA Q_k^2] \right) \frac{e^2}{32\pi^2} F \tilde{F},
\]
\(^7\) For heavy ALP with a mass around the \(\eta'\) meson mass (\(\sim 1\) GeV), this chiral rotation is no longer more convenient for calculation since the mixing between the ALP and \(\eta'\) becomes important. Nevertheless, we keep this approach, while keeping all kinetic or mass mixing terms in the following calculation, which would guarantee that the final results are independent of the used field basis.
where \( M_q \) denotes the light quark mass matrix. According to the chiral perturbation theory, the above ALP-quark couplings are matched to
\[
\frac{\partial_{\mu} a}{f_a} \sum_b j^\mu_{A b} \text{Tr} \left[ \lambda_b \left( X_q - q_A \right) \right]
+ \frac{1}{2} f^2_{\pi \mu} \left( -i \frac{\alpha}{f_a} \text{Tr} \left[ \left\{ M_q, q_A \right\} \Sigma \right] - \frac{1}{2} \left( \frac{\alpha}{f_a} \right)^2 \text{Tr} \left[ \left\{ \left\{ M_q, q_A \right\} , q_A \right\} \Sigma \right] + \text{h.c} + \ldots \right),
\]
where \( \mu_{\pi} \equiv m_{\pi_0}^2 / (m_u + m_d) \) and \( \lambda_a \ (a = 1, 2, \ldots, 8) \) are the Gell-Mann matrices. Here the \( U(3) \)-valued \( \Sigma = \exp i \left( \pi a \lambda_a / f_\pi + 2 \pi_9 / \sqrt{6} f_9 \right) \) parametrizes the pseudo-scalar mesons \( f_9 \simeq f_\pi \simeq 93 \text{ MeV} \), and the axial vector currents of meson fields are given by
\[
j^\mu_{A b} = i f^2_{\pi \mu} \text{Tr} \left[ \lambda_b \left( 1 - \delta_{b0} \right) \left( \Sigma D^\mu \Sigma^\dagger - \Sigma^\dagger D^\mu \Sigma \right) + \delta_{b0} f_9 \partial^\mu \pi_9 \right] = f_\pi \partial^\mu \pi_9 \left( 1 - \delta_{b0} \right) + \delta_{b3} f_9 \partial^\mu \pi_9 + \delta_{b3} \left( \pi^0 \partial^\mu \pi^+ + \pi^0 \partial^\mu \pi^- - 2 \pi^+ \partial^\mu \pi^- \right) + \ldots,
\]
where \( \pi^0 \equiv \pi_3 \) and \( \pi^\pm \equiv (\pi_1 \mp i \pi_2) / \sqrt{2} \).

One can choose \( q_A \) as
\[
q_A = \frac{C_{agg}}{2} \frac{M_q^{-1}}{\text{Tr} \left[ M_q^{-1} \right]},
\]
and then the ALP mass mixing with the meson octet \( \pi_a \) disappears. Then the mass-square matrix of \( \left( \pi_3, \pi_8, \pi_9, a \right) \) in this basis is given by
\[
\mu_\pi = \begin{pmatrix}
  m_u + m_d & \frac{m_u-m_d}{\sqrt{3}} & \frac{2f_\pi m_u-m_d}{\sqrt{6}} & 0 \\
  \frac{m_u-m_d}{\sqrt{3}} & m_u + m_d + 4m_s & \frac{2f_\pi m_u+m_d-2m_s}{\sqrt{6}} & 0 \\
  \frac{2f_\pi m_u-m_d}{\sqrt{6}} & \frac{2f_\pi m_u+m_d-2m_s}{\sqrt{6}} & \frac{3f_\pi^2}{f_\pi^2} x m_s + \frac{2f_\pi^2 m_u+m_d+m_s}{3} - \frac{\sqrt{3} f_\pi^2}{f_\pi} C_{agg} f_\pi f_9 \left( \frac{1}{\text{Tr} M_q} \right) + \frac{1}{\mu_i} \tilde{m}_a^2 \\
  0 & 0 & -\frac{\sqrt{3} f_\pi^2}{f_\pi} C_{agg} f_\pi f_9 \left( \frac{1}{\text{Tr} M_q} \right) + \frac{1}{\mu_i} \tilde{m}_a^2 \\
\end{pmatrix},
\]
where \( x \approx 1.68 \) for the \( \eta-\eta' \) mixing angle \( \theta_{\eta\eta'} \approx -11.4^\circ \) \cite{[32]}, which is defined by
\[
\begin{pmatrix}
  \eta \\
  \eta'
\end{pmatrix} = \begin{pmatrix}
  \cos \theta_{\eta\eta'} & -\sin \theta_{\eta\eta'} \\
  \sin \theta_{\eta\eta'} & \cos \theta_{\eta\eta'}
\end{pmatrix} \begin{pmatrix}
  \pi_3 \\
  \pi_8 \\
  \pi_9
\end{pmatrix}.
\]
We also have the following ALP-meson kinetic mixings
\[
\partial_\mu a \partial^\mu \pi_3 \cdot \frac{f_\pi}{f_a} \kappa_3 + \partial_\mu a \partial^\mu \pi_8 \cdot \frac{f_\pi}{f_a} \kappa_8 + \partial_\mu a \partial^\mu \pi_9 \cdot \frac{f_\eta'}{f_a} \kappa_9,
\]
where
\[ \kappa_3 = A_u - A_d - \frac{C_{agg}}{2} \frac{m_u^{-1} - m_d^{-1}}{m_u^{-1} + m_d^{-1} + m_s^{-1}}, \]
\[ \kappa_8 = \frac{A_u - A_d}{\sqrt{3}} - \frac{C_{agg}}{2\sqrt{3}} \frac{m_u^{-1} + m_d^{-1} - 2m_s^{-1}}{m_u^{-1} + m_d^{-1} + m_s^{-1}}, \]
\[ \kappa_9 = \frac{2(A_u + 2A_d)}{\sqrt{6}} - \frac{C_{agg}}{\sqrt{6}}. \]

After diagonalizing the kinetic and mass terms, we find the relevant low energy couplings of the canonically normalized mass eigenstate ALP are given by
\[ \frac{\partial \mu_a}{f_a} \left[ A_l \bar{\gamma}^\mu \gamma^5 l + \frac{C_{a\pi}}{f_\pi} \left( \pi^0 \pi^- \partial^\mu \pi^+ + \pi^0 \pi^+ \partial^\mu \pi^- - 2\pi^+ \pi^- \partial^\mu \pi^0 \right) \right] - \frac{e^2}{32\pi^2} \bar{C}_{a\gamma\gamma} \frac{a}{f_a} F \tilde{F} \]
where
\[ \bar{C}_{a\gamma\gamma} \simeq C_{a\gamma\gamma} - 12 \text{Tr} \left[ q_A Q_E^2 \right] - 2\kappa_3 \frac{m_a^2}{m_\pi^2 - m_a^2} - 1.3\kappa_9 \frac{m_a^2}{m_\eta^2 - m_a^2} - 2.9\kappa_{\eta'} \frac{m_{a'}^2}{m_{\eta'}^2 - m_{a'}^2}, \]
\[ C_{a\pi} = \frac{2}{3} \text{Tr} \left[ \lambda_3 (X_q - q_A) \right], \quad (A12) \]
for
\[ \kappa_\eta = \kappa_8 \cos \theta_{\eta\eta'} - \kappa_9 \sin \theta_{\eta\eta'}, \]
\[ \kappa_{\eta'} = \kappa_8 \sin \theta_{\eta\eta'} + \kappa_9 \cos \theta_{\eta\eta'}. \]

and \( q_A \) given by \( (A10) \).

Appendix B: Summary of experimental constraints

Here we describe the experimental constraints coming from the various rare meson decay channels used in this paper. We are basically summarizing the results of Ref. \([8]\) with some updates.

First, let us discuss the semi-invisible decay channels. If the decay length of ALP, i.e. \( l_d \equiv |\vec{p}_a|/m_a \Gamma_a \), where \( \vec{p}_a \) and \( \Gamma_a \) denote the ALP momentum in the laboratory frame and the total decay width, respectively, is much larger than the detector size, the ALP leaves no trace inside the detector. In such case, the event is to be interpreted as an invisible
decay mode like $B \to K \bar{\nu}\nu$ or $K \to \pi \bar{\nu}\nu$. The rare $K$ decay modes $K \to \pi + inv$ have been measured by the E949 and E787 collaborations \cite{33}. The combined results at the 68% confidence level (CL) give

$$\text{Br} (K^+ \to \pi^+ + inv) \simeq \left\{ \begin{array}{ll}
1.73_{-1.05}^{+1.15} \times 10^{-10} , & m_a = [0 - 110][150 - 260] \, \text{MeV} \\
5.6 \times 10^{-8} , & m_a \approx m_\pi
\end{array} \right. \quad (B1)$$

where the second result for $m_a \approx m_\pi$ is from the E949 90% CL upper limit \cite{34} on $\text{Br}(\pi^0 \to \nu\nu) < 2.7 \times 10^{-7}$. Here, we take the detector size as 4 m. In the near future, the proposed NA62 \cite{35} experiment will reach a sensitivity of $O(10^{-12})$ \cite{36}. For the rare $B$ invisible decays, the BaBar measurement \cite{37} gives the 90% CL upper limits

$$\text{Br} (B \to K + inv) < 3.2 \times 10^{-5},$$

$$\text{Br} (B \to K^* + inv) < 7.9 \times 10^{-5} \quad (B2)$$

for the ALP mass $m_a = 0 - 4700$ MeV.

Next we discuss the leptonic decay channels where the vertex resolution of detectors should be taken into account. If the ALP decay length is larger than the resolution, the event will be discarded. Therefore, when estimating an ALP branching ratio, one should multiply it by the probability that ALP decays within the resolution length in order to get an actual number of events to be taken by the detector. The decay mode of $K^\pm \to \pi^\pm + l^+l^-$ have been measured by the NA48/2 \cite{38,39} (with a vertex resolution $\sim 1\text{cm}$), which results in

$$\text{Br} (K^\pm \to \pi^\pm + e^+e^-) = (3.11 \pm 0.12) \times 10^{-7} \quad (m_a = 140 - 350 \, \text{MeV}),$$

$$\text{Br} (K^\pm \to \pi^\pm + \mu^+\mu^-) = (9.62 \pm 0.25) \times 10^{-8} \quad (m_a = 210 - 350 \, \text{MeV}), \quad (B3)$$

where the ALP mass range relevant for each branching ratio is specified also. The decay mode of $K_L \to \pi^0 + l^+l^-$ have been measured by the KTeV/E799 \cite{40,41} (with a vertex resolution $\sim 0.4\text{cm}$) and the resulting 90% CL upper limits on the branching ratios are given as

$$\text{Br} (K_L \to \pi^0 + e^+e^-) < 2.8 \times 10^{-10} \quad (m_a = 140 - 350 \, \text{MeV}),$$

$$\text{Br} (K_L \to \pi^0 + \mu^+\mu^-) < 3.8 \times 10^{-10} \quad (m_a = 210 - 350 \, \text{MeV}). \quad (B4)$$
As for the decay mode $B \to K^{(*)} + l^+l^-$, the current world average combined result on the branching ratio on $B^+ \to K^+ + l^+l^-$ is given as [32]

$$\text{Br} \left( B^+ \to K^+ + l^+l^- \right) = (4.51 \pm 0.23) \times 10^{-7} \quad (m_a = 220 - 4690 \text{ MeV}). \quad (B5)$$

This is in good agreement with the recent result on $B^+ \to K^+ + \mu^+\mu^-$ from the LHCb experiment [42]. We take the vertex resolution factor of the LHCb as 0.5cm [8]. Furthermore, we use the recent analyses of the LHCb collaboration on $B^0 \to K^{*0} + a (\mu^+\mu^-)$ [30] and $B^+ \to K^+ + a (\mu^+\mu^-)$ [31], which turn out to be able to put much stronger constraints depending on ALP mass and lifetime by up to $10^{-10}$ order of upper limit on the branching fraction. For the dimuon invariant mass near the masses of $J/\psi$ and $\psi(2S)$, the long-distance effect from the charmonium resonances becomes dominant and normally screens a short-distance BSM contribution, so that one cannot simply use the above value to constrain the ALP physics [30, 31, 42]. Yet, the branching ratio of

$$\text{Br} \left( B^+ \to K^+ + l^+l^- \right) < 6.0 \times 10^{-5} \quad (m_a = 2950 - 3180 \text{ MeV}),$$
$$\text{Br} \left( B^+ \to K^+ + l^+l^- \right) < 4.9 \times 10^{-6} \quad (m_a = 3590 - 3770 \text{ MeV}). \quad (B6)$$

Let us now discuss the photon decay channels. The decay mode of $K_L \to \pi^0 + \gamma\gamma$ have been measured by the KTeV [43], which results in

$$\text{Br} \left( K_L \to \pi^0 + \gamma\gamma \right) = (1.29 \pm 0.03_{\text{stat}} \pm 0.05_{\text{sys}}) \times 10^{-6}.$$ \quad (B7)

For $m_a \sim m_\pi$, the SM background from $K_L \to \pi^0\pi^0$ reduces the sensitivity. As in the case of rare $B$ meson leptonic decays, the branching ratio of $K_L \to \pi^0 + a$ with $a \to \gamma\gamma$ should not exceed $\text{Br}(K_L \to \pi^0 + J/\psi \to K^+ + l^+l^-)$ and $\text{Br}(B^+ \to K^+ + \psi(2S) \to K^+ + l^+l^-)$ [32], and therefore

$$\text{Br} \left( K_L \to \pi^0 + l^+l^- \right) < 6.0 \times 10^{-5} \quad (m_a = 2950 - 3180 \text{ MeV}),$$
$$\text{Br} \left( K_L \to \pi^0 + l^+l^- \right) < 4.9 \times 10^{-6} \quad (m_a = 3590 - 3770 \text{ MeV}). \quad (B6)$$

The rare $B$ decay mode $B \to K + \gamma\gamma$ has been measured previously by the B-factories (BaBar [45] and Belle [46] with a relatively large vertex resolution $\sim 30$ cm), but only for
the diphoton invariant mass $m_{\gamma\gamma} \sim m_{\pi}$. Since the measured branching fraction is order of $10^{-5} \sim 10^{-6}$, we choose a conservative upper limit of $10^{-6}$ for the ALP decay mode.

As for the flavor constraints coming from the up-type quarks, the rare charm meson decay can be relevant. In spite of the long distance QCD effect screening short distance physics, the process $D^+ \rightarrow \pi^+\mu^+\mu^-$ with dimuon invariant mass which is potentially sensitive to short distance BSM physics has been measured by the LHCb, yielding the following 90% CL upper limit on the branching ratio:

$$\text{Br} \left( D^+ \rightarrow \pi^+\mu^+\mu^- \right) \simeq \begin{cases} 
2.0 \times 10^{-8} & (m_a = [250 - 525] \text{ MeV}), \\
2.6 \times 10^{-8} & (m_a = [1250 - 1700] \text{ MeV}), \\
7.3 \times 10^{-8} & (\text{total}).
\end{cases} \quad \text{(B9)}$$

Finally, the beam dump experiment searching for long-lived light particle can also constrain the ALP FCNC processes \[47, 48\]. It turns out that presently the CHARM experiment \[49\] using the proton-proton beam collision gives the most stringent constraint. The total number of produced ALPs can be estimated by the following ratio to the pion production cross section \[47, 50\]:

$$N_a \approx \left( 2.9 \times 10^{17} \right) \cdot \frac{\sigma_a}{\sigma_{\pi}}, \quad \text{(B10)}$$

where

$$\frac{\sigma_a}{\sigma_{\pi}} \approx 3 \cdot \left( \frac{1}{14} \text{Br} \left( K^+ \rightarrow \pi^+ + a \right) + \frac{1}{28} \text{Br} \left( K_L \rightarrow \pi^0 + a \right) + 3 \cdot 10^{-8} \text{Br} \left( B \rightarrow X + a \right) \right).$$

Since the detector is 35m long and 480m away from the target, the number of the signals from ALP decays is estimated as

$$N_d \approx N_a \cdot \text{Br} \left( a \rightarrow \gamma\gamma, ee, \mu\mu \right) \cdot \left[ \exp \left( -\Gamma_a \frac{480\text{m}}{\gamma} \right) - \exp \left( -\Gamma_a \frac{515\text{m}}{\gamma} \right) \right], \quad \text{(B11)}$$

with $\gamma \simeq 25 \text{ GeV}/m_a$. From that there is no signal from CHARM experiment, one then finds the 90% CL bound \[50\]

$$N_d < 2.3. \quad \text{(B12)}$$
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