A method for system identification of a structure supported by nonlinear springs using evolutionary computing

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Abstract. A mechanical structure supported by nonlinear springs subjected to an external load is considered. If all mechanical parameters of the system were known, the displacement of the system subjected to this load could be easily calculated. If not all of the parameters are known, but the load and the displacement are measured at one location, an inverse problem exists. In the presented problem the nonlinear springs are unknown and have to be determined. At first glance a problem needs to be solved, which is underdetermined due to the number of unknown variables. However, evolutionary computing can be applied to solve this inverse, nonlinear and multimodal problem. Sometimes a prior knowledge exists on certain system properties, which is difficult to implement into analytical or numerical solver. This knowledge can play a decisive role in identifying the system properties and it can be easily included as boundary condition when applying evolutionary algorithm. This article examines how and under what conditions the spring resistances can be identified. The procedure is exemplified at a mechanical system of a pile foundation.

1. Introduction

The pile foundation with predominantly axially loaded driven piles is currently one of the most used foundations for offshore wind turbines in Germany (see fig. 1). The current state of scientific knowledge suggests that this type of foundation might be at risk due to cyclic loads caused by wind and waves. As part of on-going research projects, the Federal Institute for Materials Research and Testing is currently studying the behavior of cyclic axially loaded piles and developing an appropriate monitoring method for this type of foundation.

An axially loaded pile can be modeled numerically with the FE method as shown in Figure 2. The pile is considered there as a linear elastic body and the soil-pile interaction is represented by the so-called t-z springs (springs with a non-linear stiffness characteristic). According to [1] for the calculation of the displacements of the piles founded in sand a bilinear load vs. displacement function can be adopted for the t-z springs. The quality of this assumption is not of interest for the general approach and it is used only as an example for the studies described below. The crucial thing, however, is that the definition of bilinear springs with only one unknown (the maximal resistance t of...
the bilinear spring) is fixed, since the yield limit for displacements \( z_{pl} \) is assumed as constant and equal to 2.5 mm.

Measurements below the ground surface, and especially in the offshore environment, are extremely costly and time-consuming. Therefore, it is important to reduce the use of measurement to an absolute minimum. On the other hand, the measurements should be sufficient to obtain the system identification. This is a classical model-update problem where a parameter identification has to be performed. Based on mechanical considerations of the structure it can be shown that the measurement of the pile forces and of the displacements at the pile head could meet these two opposed requirements, and so despite a high number of unknown parameters (in this case, the number of springs).

This article proves the thesis above and provides an approach for the system identification of such a model using multi population evolution strategies [2]. Moreover, it also presents possibilities and limitations of this approach. This strategy permits the solution of problems that can be replaced by similar FE models.

2. Solution strategy
The problem of the system identification can be considered as classic black box model. The pile and the pile-soil interaction represent the unknown system. The pile loads are the input parameters in our unknown system. The system reacts to it and its output values are the displacements at the pile head. As mentioned before, the black box can be replaced by a FE model with nonlinear springs. The maximal resistances of these springs are the free parameters of the model that have to be determined (fig. 2). The number of free parameters depends consequently on the FEM discretization. Therefore the specific task to be performed is the determination of the resistance of the non-linear springs, so that the best possible match between the measured and the calculated load displacement curves at the pile head can be achieved.

A system can be referred to as identified if the substitute model shows the same reaction caused by the same action. However, the solution space can be multimodal. In this case, it means that a complete match can be achieved between the calculated and the measured progress of the load displacement curve for different sets of spring load capacities. A different possible method for providing the correlation between the problem variables and the problem solution might be the Monte Carlo simulation. This approach was discussed in [3]. The disadvantage of this method is that the representation of the dependencies between the input and the output parameters is difficult.
3. Example

A FE model, as shown in figure 2, with a given set of 35 springs is used here as target system to be identified. The full load displacement curve of the system is calculated for the given spring resistance values (as can be seen in figure 5 and $z_{pl}=2.5$ mm (figure 2). These values are the target values, which the method has to find by optimization with evolutionary computing out of the given load-displacement curve. Free parameters in this optimization problem are the 35 values for the spring resistances and the one value of the plastic threshold $z_{pl}$. In order to avoid the possibility that the algorithm finds always the same local optimum, different arbitrary start values can be used (fig. 5).

The solution to the problem is multimodal. However, we can use a prior knowledge to constrain the solution space. In this case, the following boundary conditions can be specified:

1. For each spring the maximum resistance can be constrained within an expected range based on a preliminary ground investigation for example
2. The ground constitutes a continuum. So for homogeneous soils, the difference between two consecutive springs should in general not be arbitrary. Based on this assumption, one may constrain it to 30% and disregard any solution outside this range of variation
The difference between two solution results without using both boundary conditions can be very significant. In figure 6 the results of two optimizations using both boundary conditions and in figure 7 only with the first boundary condition are shown. In the latter case the difference between two solution results is very large, while in former case it is relatively small. The results are not unimodal, but they can be adopted as a good engineering solution or as a good approximation.

Figure 6: Results of the parameter identification by using both of the boundary conditions \( R = \sum r_i \)

Figure 7: Results of the parameter identification by using only the first boundary condition \( R = \sum r_i \)

4. Summary and Outlook
This article shows that the identification of such static systems with non-linear springs is possible. The most important requirements for the application to real cases are:

- To provide a mathematical description of the spring characteristic as precisely as possible, since the number of free parameters is limited.
- High requirements on the measurement of the observation variables, as small measurement errors can lead to large scattering of the results by the inverse parameter identification.

As a summary, the present study has investigated the conditions to be met by the definition of a spring curve (its constitutive law) and how it should be developed in order to present a well-conditioned optimization problem for parameter identification.

References

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