A surrogate-based reliability analysis method of the motion of large flexible space structures
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Abstract:
Satellites and their instruments are subject to the motion stability throughout their lifetimes. The reliability of the large flexible space structures (LFSS) is particularly important for the motion stability of satellites and their instruments. In this paper, the reliability analysis of large flexible space structures is conducted based on Bayesian support vector regression (SVR). The kinematic model of a typical large flexible space structure is first established. Based on the kinematic model, the surrogate model of the motion of the large flexible space structure is then developed to further reduce the computational cost. Finally, the reliability analysis is conducted using the surrogate model. The proposed method shows high accuracy and efficiency for the reliability assessments of the typical large flexible space structure and can be further developed for other LFSS.

1. Introduction
The large flexible space structures (LFSS) are widely used in construction of various projects to cover large open areas. The most common applications of the LFSS are payload support structure, solar array, and deployable antenna as shown in Fig. 1.
Under the consideration of the complex environment of the outer space, the reliability of the large flexible space structures is crucial. Therefore, in order to estimate and ensure the safety of the large flexible space structures, it is necessary to consider the effects of uncertainties. In this context, numerous studies have been conducted so far by many researchers, considering the effects of some of these random variables in their investigations. Yang et al. [1] proposed the health monitoring method of the deployable antenna module in space solar power satellites. Bai et al. [2] studies the importance sampling-based reliability analysis method of the large complex structure and flexible systems. Mousavi et al [3] investigated the effects of applying different buckling modes on the sensitivity analysis of the space structures. All of the above works investigated the reliability and safety of LFSS by using the direct strategy, which inevitably requires large computational cost on the response of the LFSS. However, the LFSS usually contains various components and is difficult to test due to the particularity of the LFSS. Therefore, sufficient data or experiments may not be available. Another way to gain more information about the LFSS is to use the simulation. However, for reliability analysis, the tedious simulation is computationally expensive.

Fortunately, in reliability analysis, the surrogate models are well developed in recent years. Surrogate models can establish a full relationship between the inputs and the outputs of the structures or systems, which requires only very few experiments of the structures. Kriging surrogate model is the most popular among the surrogate methods. [4-8]. Nevertheless, for problems with higher dimensions (where the case is common in LFSS), the Kriging requires large number of samples to guarantee the accuracy. Consequently, Kriging models can be unstable when it deals with the high dimensional problems. Compared with Kriging model, a regression-based instead of
interpolation-based method called support vector regression (SVR) has not been widely used in the reliability analysis. The reason for the few applications of SVR is the point-wise prediction requirement.

To solve the problems above, a Bayesian SVR which has successfully applied in global approximation [9] is used to conduct the reliability analysis, where the new adaptive update strategy is equipped with the Bayesian SVR to train a more accurate classifier instead of an approximator. Due to the regression-based feature, the Bayesian SVR performs better than Kriging in three or higher dimensional problems. The rest of this paper is organized as follows. In section 2, the basic statement of the motion reliability of LFSS is introduced. Then, the general framework for solving the motion reliability problems is proposed in section 3 and an example of a typical LFSS is shown simultaneously to further detail the process. In section 4, the results of the shown example are presented and compared with other widely used reliability analysis methods. Finally, the conclusions are presented in the end.

2. Problem statement

In this work, we focus on the motion of the LFSS, which is a response that is time-variant and uncertain due to the time-varying working conditions and stochastic errors from manufacturing and assembling. The motion reliability of the LFSS during a given period of time $[t_s, t_e]$ is the probability that the LFSS has not failed during the movement before time $t_e$. In general, the typical expression of the time-variant reliability of the LFSS is defined as

$$R(t_0, t_e) = \text{Pr}\{g(X, Y(t), t) > 0, \forall t \in [t_0, t_e]\}$$  \hspace{1cm} (1)$$

where $g(\cdot)$ is the limit state function for the LFSS, usually the motion feature of the LFSS. $X$ denotes the vector of the random variables, $Y(t)$ denotes the vector of stochastic process and $t$ is the temporal parameter. Since the response is a function of random variables and stochastic process, it is also a random field. The reliability assessments of this kind of problem are much more difficult.
3. The principle of motion reliability assessments of the LFSS

In this section, a general framework for the solution to the motion reliability of LFSS is proposed and a typical LFSS will be illustrated to clarify the process. The deployable structure of the solar array is a typical type of LFSS and is used almost in any kind of solar array. A deployable structure consists of support beams and hinge units. The solar cell blanket for power generation is unfolded in each deployable structure and the task of the LFSS in the solar cell blanket is to unfold as shown in Fig. 2. The LFSS is supposed to unfold as the desired trajectory. However, due to the stochastic errors from manufacturing and assembling, the LFSS may not span correctly. The motion reliability assessments of the LFSS focus on the evaluation of the deviation of the motion, which mainly reflects the operating status of the LFSS.

![Diagram of LFSS in solar cell blanket](image)

Fig. 2 The LFSS in solar cell blanket

3.1 Kinematic model under uncertainties of a typical LFSS

For the convenience of analyzing the problem, we take the side view of the single unfolding mechanism module to analyze the motion of the LFSS. As Fig.3 shows, the connecting frame $OA$ rotates around the node $O$ and the windsurfing beams $AB$ and $BC$ also moves. We concern about the deviation of tip trajectory, which is the trajectory of the node $C$ in Fig. 3.
To establish the kinematic model, a Cartesian coordinate system is established with the origin $O$. The angle between the connecting frame and negative $y$-axis is $\theta(t)$. Then the tip trajectory $CT$ can be calculated according to the angle and corresponding length of the frame and beam, which is

$$CT_x = r \sin \theta + l_1 \sin \theta + l_2 \sin \theta$$
$$CT_y = l_1 \cos \theta - l_2 \cos \theta - r \cos \theta$$
$$CT = \sqrt{CT_x^2 + CT_y^2}$$

(2)

The desired trajectory $CT^*$ is set according to the standard lengths of $OA$ and $AB$, which is obtained by the simulation. However, due to the stochastic errors from manufacturing and assembling, the practical length of the connecting frame and windsurfing beams are not the standard lengths, which may cause the deviation of the tip trajectory. If the deviation of the tip trajectory is larger than the threshold, the module cannot finish the unfolding task. Therefore, by denoting $l_1, l_2, r, \theta$ as $X_1, X_2, X_3, t$, the limit state function can be concluded as

$$g(X, t) = g(X_1, X_2, X_3, t) = \varepsilon - \left|CT - CT^*\right|$$

(3)

where $\varepsilon$ is the allowable deviation.

### 3.2 Surrogate model of the motion of the LFSS

As mentioned above, the computational cost of $CT^*$ is expensive. For many other more complicated LFSS, the computational cost will be higher. Therefore, it is necessary to reduce the evaluation of the real limit state function. In many existing works [4-8], the surrogate model is widely used to reduce the computational cost, especially the Kriging model. Due to the instability of the Kriging model and many
LFSS contain many variables, the Bayesian support vector regression [10] which performs better in higher dimensional problems is used to establish the surrogate model of LFSS.

Given a set of training sample set \( \{X, Y\} \), where \( X = \{x_1, \ldots, x_N\}^T \) are the input samples and \( Y = \{y_1, \ldots, y_N\}^T \) are the corresponding response quantity of interest, the training sample set can be expressed as
\[
y_i = \tilde{g}(x_i) + \delta_i
\]
where \( \delta_i, i = 1, 2, \ldots, N \) are independent identically distributed random noises and \( \tilde{g}(x) \) is the SVR regression model. By using the Gaussian process assumption, the joint distribution of \( \tilde{g}(x^*) \) at an unexploited point \( x^* \) can be predicted as
\[
\tilde{g}(x^*) \sim N(\tilde{\mu}(x^*), \tilde{\sigma}^2(x^*))
\]
where \( \tilde{\mu}(x^*) \) and \( \tilde{\sigma}^2(x^*) \) are the predicted mean and variance at \( x^* \) according to the established Bayesian SVR model. More details about the Bayesian SVR can be found in Refs. [9-10].

To establish the initial surrogate model of the LFSS, \( N_{\text{test}} \) testing samples \( x_{\text{test}} = [(X^1, t^1), (X^2, t^2), \ldots, (X^{N_{\text{test}}}, t^{N_{\text{test}}})] \) are generated with corresponding marginal probability distributions and \( t \) is treated as a uniformly distributed variable. \( N_{\text{train}} \) initial training samples are randomly selected from the testing sample pool. Then, the \( N_{\text{train}} \) initial training samples and their corresponding response \( g \) defined in Eq. (3) are used to establish the initial model. After establishing the initial surrogate model, an active learning algorithm is presented to update the training sample pool. The new sample is selected by
\[
x_{\text{new}} = \arg \max_{x_{\text{new}}} \left\{ \text{sign}(\tilde{\mu}(x))\tilde{\mu}(x)\Phi\left(\frac{\text{sign}(\tilde{\mu}(x))\tilde{\mu}(x)}{\tilde{\sigma}(x)}\right) + \tilde{\sigma}(x)\phi\left(\frac{\tilde{\mu}(x)}{\tilde{\sigma}(x)}\right) \right\}
\]
which represents the maximal expected risk of misclassification of a testing sample, where \( \text{sign}(\cdot) \) is the sign function, \( \Phi(\cdot) \) and \( \phi(\cdot) \) are CDF and PDF of normal distribution respectively. And the new training sample in Eq. (5) and its response
\( g(x_{\text{new}}) \) will be added to the training sample pool. Then the new training sample pool will be used to update the surrogate model. The stopping criteria is given as

\[
\Phi \left( \frac{\mu(x_{\text{new}})}{\sigma(x_{\text{new}})} \right) \geq \Phi(2)
\]

which represents the probability of misclassification of the potential new sample. According to the proposed method for enriching samples and stopping criteria, the surrogate model of the LFSS can be accurately established.

### 3.3 Motion reliability assessments of the LFSS based on the surrogate model

The established surrogate model of LFSS can accurately classify the safety and failure sample without the call of the original limit state function, which is extremely computational-friendly for the LFSS. By using the established surrogate model of LFSS, the MCS can be equipped to conduct the reliability analysis. The reliability can be estimated by MCS as

\[
R \approx \frac{1}{N\text{MCS}} \sum_{i=1}^{N\text{MCS}} I(\min_{t} \tilde{\mu}([X_{i}, t]), j = 1,2,...N)
\]

where \( X_{i}, i = 1,2,...N\text{MCS} \) are \( N\text{MCS} \) random samples generated with corresponding marginal probability distributions and \( \tilde{\mu}(\bullet) \) is the predicted mean value of the corresponding inputs according to the established surrogate model. It should be noted that the surrogate model established above contains the temporal parameter. Therefore, the temporal parameter is discretized as \( h \) instants here to simplify the calculation. \( I(\bullet) \) is the indicator function which is defined as

\[
I(\min_{t} \tilde{\mu}([X, t])) = \begin{cases} 0, & \min_{t} \tilde{\mu}([X, t]) \leq 0 \\ 1, & \min_{t} \tilde{\mu}([X, t]) > 0 \end{cases}
\]

By using the MCS and established surrogate model, the reliability analysis of the LFSS requires no extra computational resources on the original limit state function.

The general framework of the motion reliability analysis of LFSS can be concluded as:

Step 1: Establish the limit state function of the motion of LFSS as shown in section 3.1.
Step 2: Mapping the relationships between the inputs and outputs of the motion of LFSS by Bayesian SVR proposed in section 3.2.

Step 3: Combining MCS with the established surrogate model in Step 2 to conduct the reliability analysis.

4. Results

The basic information of the inputs is shown in Table 1. Also, the component \( CT^* \) in Eq. (3) is obtained from the simulation, where \( t \in [0, \frac{\pi}{2}] \).

| Input variable | Mean | Standard deviation | Distribution type |
|----------------|------|--------------------|-------------------|
| \( X_1 \)      | 1    | 0.01               | Lognormal         |
| \( X_2 \)      | 2.5  | 0.025              | Normal            |
| \( X_3 \)      | 2.5  | 0.025              | Normal            |

The final results of the reliability analysis of the LFSS are shown in Table 2. Additionally, reliability analysis results at the end time instant \( t_e = \frac{\pi}{2} \) by the widely used extreme moment method [11], Kriging-based method and the accurate directly MCS are provided in Table 1. The MCS requires \( 10^6 \times 10^2 = 10^8 \) function calls, which is computationally expensive and its result is used as a reference. The extreme value moment requires less computational resources with only \( 25 \times 20 = 500 \) function calls but it cannot give an accurate result due to the large skewness. The proposed method uses the least computational resources with 76 function calls and also provides an accurate enough result.

| Method                        | Reliability | \( err \) (%) | Function calls |
|-------------------------------|-------------|---------------|----------------|
| MCS                           | 0.9495      | -             | \( 1 \times 10^8 \) |
| Extreme moments method        | 0.9005      | 5.1606        | 500            |
| Kriging-based method          | 0.9445      | 0.5266        | 103            |
| Proposed method               | 0.9470      | 0.2633        | 76             |

5. Conclusions
In this work, we propose a general framework and solution for motion reliability of LFSS and use a typical LFSS to show the feasibility of the framework. Considering the complex structures and the time-consuming simulations are common in the reliability analysis of LFSS, the proposed framework uses the Bayesian SVR to establish the surrogate model of the motion of the LFSS. Then, by a novel active learning strategy and stopping criteria, the approximator of the usual Bayesian SVR is further developed as a classifier, which is more powerful in reliability analysis. Finally, by equipping the MCS with the established surrogate model, the motion reliability analysis can be easy to conduct and it uses much less computational resources than the traditional methods. Owing to the universality of the surrogate model, the proposed framework is suitable for any type of motion equations. It should be noted that although the proposed method performs better than the Kriging-based method in higher dimensional problems, the curse of dimensionality cannot be fully avoided. Therefore, the method for extremely high-dimensional problems should be further investigated.

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