Non factorizable effects in nonleptonic $B$ decays to charmonium

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We discuss the validity of factorization for exclusive two-body $B$ to charmonium transitions. In particular, we consider the role of non factorizable corrections in selected two-body modes.

1 Introduction

Nonleptonic two-body $B$ decays play a crucial role in the study of CP violation and in the determination of fundamental Standard Model parameters, such as the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements or the angles of the unitarity triangle. However, it is very difficult to deal with strong interaction effects for these purely hadronic processes. One of the oldest prescriptions to compute nonleptonic transitions is the factorization ansatz, of which various formulations have been proposed. More recent developments exploit the presence of a large parameter, i.e. the $b$ quark mass $m_b$. Since $m_b$ is much larger than the QCD scale $\Lambda_{QCD}$, it is possible to evaluate the relevant hadronic matrix elements as an expansion in the strong coupling constant $\alpha_s(m_b)$ and in the ratio $\Lambda_{QCD}/m_b$. Several approaches are based on such an expansion, such as QCD factorization [1] or perturbative QCD [2]. Another possibility is to combine the heavy quark expansion with non perturbative approaches such as QCD sum rules [3] or to exploit the large energy release to the final state in selected exclusive $B$ decays [4]. The realm of applicability of each method should be assessed through the comparison with experimental data to theoretical predictions based on factorization. In this case, such a comparison shows the existence of sizable violations. After a brief review of the factorization approach, we analyse specific decay modes for which the factorized amplitudes vanish; nevertheless, they have been observed experimentally.

2 $B$ to charmonium transitions: factorization versus experimental data

Let us consider a generic nonleptonic two-body $B$ decay: $B \to M_1M_2$. The effective hamiltonian describing this process is obtained integrating out the W field and can be written as: $H_{eff} = \sum_i c_i(\mu) O_i$, where $c_i(\mu)$ are short distance coefficients and $O_i$ local operators. As a consequence, the relative amplitude reads as:

$$A(B \to M_1M_2) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i c_i(\mu) \langle M_1M_2|O_i(\mu)|B \rangle$$

(1)

where $\lambda_i$ represents the product of the two elements of the Cabibbo-Kobayashi-Maskawa matrix (CKM) involved in the considered transition. Long distance effects are encoded in the matrix elements of the operators $O_i$ and hence represent the non perturbative ingredient of the calculation. In order to determine such matrix elements, various approaches have been proposed. We shall not review all of them here; instead, we only consider the factorization approach (in its various formulations) with particular reference to final products consisting of a kaon and a charmonium state.

The effective weak hamiltonian governing the process $B^- \to K^- + \bar{c}c$ is:

$$H_W = \frac{G_F}{\sqrt{2}} \left( V_{cb} V_{cs}^* \left[ c_1(\mu) O_1 + c_2(\mu) O_2 \right] - V_{ub} V_{us}^{*} \sum_{i=3}^{10} c_i(\mu) O_i \right)$$

(2)

where $O_1 = (\bar{c}b)v_{-A}(\bar{c}c)v_{-A}$ and $O_2 = (\bar{c}b)v_{-A}(\bar{c}c)v_{-A}$ are current-current operators, and $O_3 - O_{10}$ are QCD and electroweak penguin operators [5]. Let us consider the mode $B^- \to K^- \chi_{c0}$, where $\chi_{c0}$ is the $0^{++}$ state of charmonium. Naive factorization [6] amounts to factorize the currents appearing in the $O_i$ and computing the relevant amplitude inserting the vacuum in all possible ways. As a consequence, one has:

$$A_{fact}(B^- \to K^- \chi_{c0}) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \hat{c} \langle K^-|\bar{c}b)v_{-A}|B^-\rangle \times \langle \chi_{c0}(\bar{c}c)v_{-A}|0 \rangle ,$$

(3)

where $\hat{c}$ is a combination of Wilson coefficients. Since the $\chi_{c0}$ is a scalar $\bar{c}c$ particle, the latter matrix element in (3) vanishes and therefore $A_{fact} = 0$. Nevertheless, both Belle...
and BaBar Collaborations have reported observation of this decay mode, giving:

\[ B(B^- \to K^- \chi_{c0}) = (6.0_{-2.1}^{+2.1} \times 1.1) \times 10^{-4} \quad (7) \]

\[ B(B^- \to K^- \chi_{c0}) = (2.4 \pm 0.7) \times 10^{-4} \quad (8) \]

Such results clearly indicate that factorization is indeed violated in the considered mode. Furthermore, Belle Collaboration also provides the ratio:

\[ \frac{B(B^- \to K^- \chi_{c0})}{B(B^- \to K^- J/\psi)} = (0.60^{+0.21}_{-0.18} \pm 0.05 \pm 0.08) \quad (6) \]

showing that the mode \( B^- \to K^- \chi_{c0} \) proceeds with a rate comparable to that of \( B^- \to K^- J/\psi \).

A possible improvement of the naive factorization approach is represented by the so called generalized factorization [9], in which the Wilson coefficients in the factorized amplitude are treated as free parameters. If data are available for some process, they can be fitted and used as input into other similar modes. However, it still holds that \( A(B^- \to K^- \chi_{c0}) = 0 \) in generalized factorization.

It is interesting to consider a decay mode allowed in this approximation, such as \( B^- \to K^- J/\psi \). The factorized amplitude reads:

\[ A_{\text{fact}}(B^- \to K^- J/\psi) = 2 \frac{G_F}{\sqrt{2}} V_{cb} V_{cN} a_2 f_0 M_\phi \times \]

\[ \times F_{MK}^2 (M_\phi) (\epsilon \cdot q), \]

where \( a_2 = c_2 + c_1/N_c \), \( f_0 \) is the \( J/\psi \) decay constant, \( q \) the kaon momentum and \( F_{MK}^2 \) one of the form factors parameterizing the matrix element \( \langle K^- | \bar{s} \gamma \mu b | B^- \rangle \).

Using the form factor \( F_{MK}^2 \) computed in [10] and \( B(B^- \to K^- J/\psi) = (1.01 \pm 0.05) \times 10^{-3} \) [11], one obtains: \( a_2^{eff} = 0.38 \pm 0.05 \). Scanning several form factor models, the result would vary in the range: \( a_2^{eff} = 0.2 - 0.4 \). The obtained effective value of \( a_2 \) should be compared to the QCD calculation, which gives \( a_2^{NLO} (\mu = m_b) = 0.163 (0.126) \), in the naive dimensional regularization (’tHooft and Veltman) scheme [5]. This comparison shows that non factorizable effects are sizable also in the case of \( B^- \to K^- J/\psi \).

A QCD improved factorization approach has been proposed for \( B^- \) decays, exploiting the large value of \( m_b \) [11]. The approach holds for \( B \to M_1 M_2 \) non leptonic decays when \( M_2 \) is light, \( M_1 \) being the meson picking up the spectator quark in the decay. In this case, it has been shown that non factorizable corrections are dominated by hard gluon exchanges, while soft effects are confined to the \((BM_1)\) system. Naive factorization is recovered at the leading order in \( \alpha_s \) and \( \Lambda_{QCD}/m_b \). The approach does not hold when \( M_2 \) (the emitted meson) is heavy, since a large overlap is expected between \( M_2 \) and the \((BM_1)\) system. An exception is represented by the emission of a quarkonium state, since, in the heavy quark limit, its transverse size becomes small.

An analysis performed for \( B^- \to K^- \chi_{c0} \) shows that infrared divergences in the final result do not allow to apply this method to such a process [12]. As for \( B^- \to K^- J/\psi \), although the cancellation of infrared divergences has been proven at the leading order in the \( 1/m_b^2 \) expansion, the experimental data are not reproduced [13]. In the following section, we discuss the possibility that rescattering diagrams, taking contribution from intermediate charmed mesons, could play a role in the considered modes.

3 Role of rescattering processes

The decay \( B^- \to K^- \chi_{c0} \) can be obtained by rescattering of charmed intermediate states, as shown in fig. 1 [14]. The decay is still induced by the transition \( b \to s c \bar{c} \) and the relevant CKM structure is the same as for the direct transition.

A first analysis of rescattering diagrams of the kind shown in fig. 1is reported in [14] and briefly summarized below.

The computation involves the weak matrix elements governing the transitions \( B \to D^{(*)}_{s,J} D^{(*)}_{c,J} \) and the strong couplings between the charmed states \( D^{(*)}_{s,J} D^{(*)}_{c,J} \) and the kaon and the \( \chi_{c0} \). There is experimental evidence that the calculation of the amplitude by factorization reproduces the main features of the \( B \to D^{(*)}_{s,J} D^{(*)}_{c,J} \) decay modes [15]. Therefore, neglecting the contribution of penguin operators in (1), we can write:

\[ \langle D^{(*)}_{s,J} D^{(*)}_{c,J} | H_W | B^- \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{cN} a_1 \langle D^{(*)}_{s,J} | (V - A)^I | B^- \rangle \]

\[ \langle D^{(*)}_{s,J} | (V - A)^I | B^- \rangle \]

\[ (D^{(*)}_{s,J} | (V - A)^I | B^- \rangle \]

where \( a_1 = c_1 + c_2/N_c \). In the heavy quark effective theory, the matrix element \( (D^{(*)}_{s,J} | (V - A)^I | B^- \rangle \) can be expanded in terms of a single form factor, the Isgur-Wise function \( \xi \) [8].

Figure 1. Typical rescattering diagram contributing to \( B^- \to K^- \chi_{c0} \).
while \( (D_s^- (p, e) (V - A)_\mu |0) = f_{D_s} M_{D_s} e^\mu \), where \( f_{D_s} \) is the \( D_s^- \) leptonic constant and \( e \) its polarization vector; the analogous matrix element for \( D_s \) involves the corresponding constant \( f_{D_s} \).

Other hadronic quantities involved in the calculations are the strong couplings of a pair of charmed mesons to the kaon and to the \( \chi_{c0} \).

The \( D_s^- (V^-) K \) couplings, in the soft \( p_K \to 0 \) limit, can be related to a single effective constant \( g \), as it turns out considering the effective QCD Lagrangian describing the strong interactions between the heavy mesons and the octet of the light pseudoscalar mesons [17]. \( L_I = i g \, Tr[H_\alpha \gamma_\mu A_\alpha^\mu H_\alpha] \), with \( A_\mu H_\alpha = \frac{i}{2} \left( \xi^\alpha \partial_\mu \xi - \xi \partial_\mu \xi^\alpha \right) H_\alpha \) and \( \xi = e^{i \frac{m}{m_0}} \). The matrix \( M \) contains the fields of the octet of the light pseudoscalar mesons. For example, the \( D_s^- D K \) coupling, defined through the matrix element

\[
< D_s^0 (p) K^- (q) | D_s^- (p + q, e) > = g_{D_s^- D K^-} (e \cdot q) \quad (9)
\]

is related to the effective coupling \( g \) through the relation:

\[
g_{D_s^- D K^-} = -2 \sqrt{m_0 m_{D_s}} \frac{g}{m_K} \quad (10)
\]

As for the coupling of the \( \chi_{c0} \) state to a pair of \( D_s \) mesons, defined by the matrix element

\[
< D_s^0 (p_1) D_s^0 (p_2) | \chi_{c0} (p) > = g_{D_s^0 D_s^0 \chi_{c0}} \quad (11)
\]

an estimate can be made by considering the \( D_s \) matrix element of the scalar \( \bar{c}c \) current: \( < D_s (v') | \bar{c}c | D_s (v) > \), assuming the dominance of the nearest resonance, i.e. the scalar \( \bar{c}c \) state, in the \( (v - v')^2 \) channel and using the normalization of the Isgur-Wise form factor at the zero-recoil point \( v = v' \). This allows us to express \( g_{D_s^0 D_s^0 \chi_{c0}} \) in terms of the constant \( f_{\chi_{c0}} \) that parameterizes the matrix element \( <0 | \bar{c}c | \chi_{c0} (q) > = f_{\chi_{c0}} m_{\chi_{c0}} \). One obtains:

\[
g_{D_s^0 D_s^0 \chi_{c0}} = -2 \frac{m_{D_s} m_{\chi_{c0}}}{f_{\chi_{c0}}} \quad (12)
\]

The method can also be applied to \( g_{D_s^- D_s^0 \chi_{c0}} \).

However, the determinations of the couplings described above do not account for the off-shell effect of the exchanged \( D_s \) and \( D_s^0 \) particles, the virtuality of which can be large. As discussed in the literature, a method to account for such effect relies on the introduction of form factors \( g_1 (t) = g_0 F_1 (t) \), with \( g_0 \) the corresponding on-shell couplings [21]. A simple pole representation for \( F_1 (t) \) is:

\[
F_1 (t) = \frac{\Lambda^2 - m_{\chi_{c0}}^2}{\Lambda^2 - t}, \quad \text{consistent with QCD counting rules} \quad [18].
\]

The parameters in the form factor represent a source of uncertainty in our analysis. In the evaluation of the diagrams, we compute at first the absorptive part, and then derive the real part through a dispersive representation.

Before turning to the numerical analysis, it is worth considering rescattering contributions of intermediate charm mesons to the decay mode \( B^- \to K^- J/\psi \). The hadronic information for determining rescattering amplitudes are the same as for \( B^- \to K^- \chi_{c0} \), with the only difference in the strong \( D_s^- (V^-) J/\psi \) couplings that can be expressed in terms of the parameter \( f_{J/\psi} \), using the same vector meson dominance method applied to derive eq. [12].

We have now to fix the values of the various hadronic parameters. The Wilson coefficient \( a_1 \), common to all the amplitudes, can be put to \( a_1 = 1.0 \) as obtained by the analysis of exclusive \( B \to D_s^- (V^-) \) transitions. Moreover, we use \( f_{D_s} = 240 \text{ MeV} \), in the range quoted by the Particle Data Group [11]. and \( f_{D_s} = f_{D_s} \) consistently with our approach that exploits the large \( m_0 \) limit. For the Isgur-Wise universal form factor \( \xi \), the expression \( \xi (y) = \left( \frac{2}{y + 1} \right)^2 \) is compatible with the current results from the semileptonic \( B \to D^0 \) decays.

A discussion is needed about the \( D_s^- (V^-) K \) vertices. For the effective coupling \( g \) one can use the CLEO result \( g = 0.59 \pm 0.01 \pm 0.07 \) obtained by the measurement of the \( D^* \) width [19]. Several estimates of \( g \) have appeared in the literature; in particular, potential models give values close to one [20], while other determinations point towards lower values [21]. The discrepancy may be attributed to relativistic effects [22]. The value obtained by CLEO is in the upper side of the theoretical calculations [3][23]. We choose to be conservative, and vary this parameter in the range: \( 0.35 \leq g \leq 0.65 \) that encompasses the largest part of the predictions.

In [12] we use \( f_{K^0} = 510 \pm 40 \text{ MeV} \) obtained by a standard two-point QCD sum rule analysis [13]. As for the couplings \( D_s^- (V^-) J/\psi \), expressions analogous to [12] involve \( f_{J/\psi} \), for which we use the experimental measurement. Assuming that the amplitude relative to \( B^- \to K^- J/\psi \) deviates from the factorized result because of the contribution of the rescattering term: \( \bar{A}_{\text{fact}} = \bar{A}_{\text{fact}} + \bar{A}_{\text{resc}} \), one can constrain the values of \( \Lambda \), for the calculation of \( B (B^- \to K^- \chi_{c0}) \). The result is \( B (B^- \to K^- \chi_{c0}) = (1.1 - 3.5) \times 10^{-4} \), to be compared to [45].

Two conclusions can be drawn from the present study: i) rescattering amplitudes are sizable in \( B^- \to K^- \chi_{c0} \) and in \( B^- \to K^- J/\psi \); ii) they might explain the large branching ratio observed for \( B^- \to K^- \chi_{c0} \).

The calculation could be improved in several points: considering additional intermediate states\(^1\), using improved

\(^1\)The contribution of orbital and radial excitations of the intermediate charmed mesons is expected to be suppressed by smaller values of the leptonic constants and of the strong couplings.
values for the input parameters or inserting new experimental data. A more refined analysis is indeed in progress.

It is also interesting to estimate rescattering effects in other channels which are also forbidden in the factorization approximation, such as \( B^{-} \to K^{-} \chi_{0} \) or \( B^{-} \to K^{-} h_{c} \), \( h_{c} \) being the 2\(^{+}\) and the 1\(^{+}\) charmonium states, respectively. In particular, the latter decay mode would also be interesting per se, since the \( h_{c} \) has been observed in \( p \bar{p} \) annihilation, but it is not an established particle yet [21]. A preliminary result gives \( \mathcal{B}(B^{-} \to K^{-} h_{c}) \approx (1 - 3) \times 10^{-4} \) [24] to be compared to the prediction for the inclusive mode \( \mathcal{B}(B^{-} \to X h_{c}) \approx (0.13 - 0.14)\% \) [25].

It has been suggested that a possible decay chain to observe this mode could be: \( B^{-} \to K^{-} h_{c} \to \eta_{c} \gamma_{c} \) with \( \eta_{c} \to K K \pi \) or \( \eta_{c} \to \eta \pi \pi \), and \( \mathcal{B}(h_{c} \to \eta_{c} \gamma_{c}) \approx 0.5 \pm 0.1 \) [26]. Using such a prediction, our result would give: \( \mathcal{B}(B^{-} \to K^{-} \eta_{c} \gamma_{c} \to K(K K \pi \gamma_{c})) \approx (2.5 - 7.5) \times 10^{-6} \), suggesting that this mode could be accessible at the currently operating experimental facilities.

### 4 Conclusions

Present experimental results on two-body \( B \) to charmonium transitions show large non factorizable contributions. We suggested that such contributions can be interpreted by rescattering of intermediate charmed resonances. The numerical evaluation of such effects for \( B^{-} \to K^{-} \chi_{0} \) shows some agreement with experimental data. Contributions of similar sizes are expected for \( B^{-} \to K^{-} h_{c} \). Predictions for other decay modes could confirm this picture.

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