Exact description of D-branes in K-matrix theory *)

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Abstract

We summarize how to describe D-branes in a matrix theory based on unstable D-instantons, which we call K-matrix theory, and explicitly show that D-branes can be constructed as bound states of infinitely many unstable D-instantons. We examine the fluctuations around Dp-brane solutions in the matrix theory and show that they correctly reproduce fields on the Dp-brane world-volume. Plugging them into the action of the matrix theory, we precisely obtain the Dp-brane action as the effective action of the fluctuations.

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1. Introduction

In this paper, we consider matrix theories based on unstable D-brane systems in type II string theory. There are many choices of the unstable D-brane systems. The lowest dimensional ones are non-BPS D-instanton system in type IIA string theory or D-instanton - anti D-instanton system in type IIB string theory. If one wants to avoid the formal Wick rotation to obtain the D-instantons, one could start with D0-D0 system in type IIA string theory or non-BPS D0-brane system in type IIB string theory. We call this kind of matrix theory as K-matrix theory.

There are many reasons we think K-matrix theory is interesting. First of all, we can construct any D-brane configurations, including BPS D-branes, non-BPS D-branes, D-brane - anti D-brane pairs, commutative as well as non-commutative D-branes, etc., as solutions in the matrix theory. This is in contrast to the usual approach in string theory, in which D-branes are introduced by adding some open string sectors by hand and one has to change the setup depending on the brane configurations. It may also be possible to construct these D-brane configurations as solitonic solutions in open string field theory associated with space-time filling unstable D-brane systems, such as non-BPS D9-branes in type IIA and D9-D9 system in type IIB or type I string theory. But, K-matrix theory provides much simpler formulation than these, since the world-volume is just a point.

K-matrix theory is supposed to be a matrix formulation of string theory such that creation and annihilation of unstable D-branes are incorporated. The creation and annihilation of D-branes is represented by the value of tachyon, which is one of the matrix variables in K-matrix theory. Roughly speaking, D-branes are created when the tachyon is around the top of its potential, while they are annihilated if the tachyon is at the bottom of the potential. This tachyon is the very ingredient which makes the theory quite different from other matrix theories, such as Refs. [2], [3], and plays an essential role in the construction of D-branes.

We set the size of the matrix variables to be infinity from the beginning. Thus, the matrix variables are considered as operators acting on an infinite dimensional Hilbert space. Therefore, the geometry of the D-brane world-volume is represented in terms of operator algebras. Remarkably, it turned out that each D-brane configuration in K-matrix theory corresponds to an object which is well-known among mathematicians. As argued in Ref. [4], D-brane configurations are given as (a limit of) spectral triples, which are analytic description of Riemannian manifold [5]. Furthermore, the classification of D-brane is naturally given by a group called analytic K-homology. A physical interpretation of the K-homology group in terms of K-matrix theory has been given in Refs. [4], [6]. These results are considered to be dual to the K-theory classification of D-brane charges based on a physical interpretation of
the topological K-theory groups in terms of space-time filling unstable D-brane.

In this paper, we wish to explain more explicit description of D-branes following our recent paper Ref. 1. We adopt the boundary string field theory action of the unstable D-instantons as the action of K-matrix theory. The exact solutions for this action representing D-branes are given in Ref. 9. We will consider the fluctuations around the D-brane solution and show that the effective action of the fluctuations precisely agrees with the D-brane action. The argument is also applicable to the construction of D-branes as solitons in space-time filling unstable D-brane systems, which enables us to generalize the results given in Refs. 10, 11, 12. Our derivation is surprisingly simple and much more powerful. It is powerful enough to show that not only the D-brane tension, but also full effective action is precisely reproduced without making any approximations.

§2. Boundary state formulation of BSFT action

In this section, we quickly recapitulate some of the ingredients in boundary string field theory (BSFT) action for D-branes in type II superstring theory, which is needed in the following discussion. The BSFT action for the unstable D-brane system in superstring was proposed in Refs. 10 as a disk partition function with boundary interaction, generalizing the usual definition of the low energy effective action for massless fields on a BPS D-brane, i.e. Dirac-Born-Infeld action including its derivative corrections. It can be expressed in terms of closed string variables as the overlap between the vacuum state and a boundary state with boundary interaction: 

\[ S^{Dp}(A_\alpha, \Phi^i, T, \cdots) = \frac{2\pi}{g_s} \langle 0 | e^{-S_b(A_\alpha, \Phi^i, T, \cdots)} | Bp; + \rangle_{NS}. \]  

(2.1)

Here \( | Bp; + \rangle_{NS} \) is the NS-NS sector boundary state representing the Dp-brane world-volume and the open string fields \( (A_\alpha, \Phi^i, T, \cdots) \) are turned on through the boundary interaction \( S_b \). Let us next explain these objects in the following. The boundary state \( | Bp; \pm \rangle_{NS} \) used in (2.1) is defined by

\[ | Bp; \pm \rangle_{NS} = \int [d\bar{x}^\alpha] | x^\alpha, \bar{x}^\beta = 0; \pm \rangle_{NS}, \]  

(2.2)

where the superscript \( \alpha = 0, \ldots, p \) and \( i = p + 1, \ldots, 9 \) represent the directions tangent and transverse to the Dp-brane world-volume, respectively. Here we have introduced the boundary superfields \( \bar{x}^\alpha(\bar{\sigma}) = x^\mu(\sigma) + i\theta \bar{\psi}^\mu(\sigma) \) \( (\mu = 0, \ldots, 9) \), where \( x^\mu(\sigma) \) and \( \bar{\psi}^\mu(\sigma) \) are bosonic and fermionic functions defined on the boundary of the disk parametrized by \( \bar{\sigma} \).
0 ≤ σ ≤ 2π and \( \tilde{\sigma} = (\sigma, \theta) \) denotes the boundary supercoordinate. The integral \( \int [d\mathbf{x}^\alpha] \) means the path integral with respect to \( x^\alpha(\sigma) \) and \( \psi^\alpha(\sigma) \). The state \( |x^\mu; \pm\rangle_{NS} \) is the coherent state satisfying

\[
X^\mu(\tilde{\sigma}) |x^\mu; \pm\rangle_{NS} = x^\mu(\tilde{\sigma}) |x^\mu; \pm\rangle_{NS}, \tag{2.3}
\]

where \( X^\mu(\tilde{\sigma}) = X^\mu(\sigma) + i\theta \Psi^\mu_\pm(\sigma) \) is a superfield operator whose components \( X^\mu(\sigma) = \tilde{x}^\mu_0 + i \sum_{m \neq 0} (\tilde{a}^m_\mu e^{-im\sigma} + \tilde{a}^m_\mu e^{im\sigma}) \) and \( \Psi^\mu_\pm(\sigma) = \sum_{\nu} (\tilde{\psi}^\nu_\pm e^{-i\nu\sigma} \pm i\tilde{\psi}^\nu_\pm e^{i\nu\sigma}) \) are closed string operators in NS-R formulation evaluated at the boundary of the disk. We use NS-NS sector boundary interaction in (2.1) and hence the subscript \( r \) of \( \Psi^\mu_r \) and \( \tilde{\psi}^\nu_r \) runs half odd integers. We often omit the subscript \( \pm \) of the fermion \( \Psi^\mu_\pm \) in the following. The path integral in the right hand side of (2.2) represents summing over all possible configurations of the boundary of the string world-sheet attached on the Dp-brane world-volume.

It is also useful to introduce a momentum superfield operator \( P_\mu(\tilde{\sigma}) = \theta P_\mu(\sigma) + i\Pi_\mu(\sigma) \), where \( P_\mu(\sigma) \) and \( \Pi_\mu(\sigma) \) are the conjugate momenta of \( X^\mu(\sigma) \) and \( \psi^\mu(\sigma) \), respectively, and write the coherent state as

\[
|x^\mu; \pm\rangle_{NS} = e^{-i\int d\tilde{\sigma} P_\mu x^\mu} |x^\mu = 0; \pm\rangle_{NS}, \tag{2.4}
\]

where \( d\tilde{\sigma} = d\sigma d\theta \). From this expression, we can see that the boundary state (2.2) for a Dp-brane is related to that for a D(-1)-brane \( |B(-1); \pm\rangle_{NS} = |x^\mu = 0; \pm\rangle_{NS} \) by

\[
|Bp; \pm\rangle_{NS} = \int [d\mathbf{x}^\alpha] e^{-i\int d\tilde{\sigma} P_\alpha x^\alpha} |B(-1); \pm\rangle_{NS}. \tag{2.5}
\]

The fields on the D-brane couple to the boundary of the string world-sheet, and the disk partition function changes its value when the fields on the D-brane are turned on. The coupling is represented by the boundary interaction \( S_b \) in (2.1). The boundary interaction for the gauge field \( A_\alpha(x) \) on the D-brane is well-known and given as a supersymmetric generalization of Wilson loop operator:

\[
e^{-S_b(A_\alpha)} = \text{Tr} \hat{P} \exp \left( - \int d\tilde{\sigma} A_\alpha(X) D X^\alpha \right), \tag{2.6}
\]

where the covariant derivative is defined as \( D = \partial_0 + \theta \partial_\sigma \). Here \( \hat{P} \) denotes the supersymmetric generalization of the path ordered product, which is defined as

\[
\hat{P} \exp \left( \int d\tilde{\sigma} \mathbf{M}(\tilde{\sigma}) \right) = \sum_{n=0}^{\infty} \frac{(-1)^{n(n-1)}}{2} \int d\tilde{\sigma}_1 \cdots d\tilde{\sigma}_n \theta(\tilde{\sigma}_{12}) \theta(\tilde{\sigma}_{23}) \cdots \theta(\tilde{\sigma}_{n-1\,\,n}) \mathbf{M}(\tilde{\sigma}_1) \cdots \mathbf{M}(\tilde{\sigma}_n), \tag{2.7}
\]
where \( \hat{\sigma}_{ab} = \sigma_a - \sigma_b - \theta_a \theta_b \) and \( \Theta \) is a step function. When the gauge field is abelian and the field strength \( F_{a\beta} \) is constant, we can explicitly calculate the expression (2.1) with the boundary interaction (2.4) and obtain the DBI action.

On the Dp-brane with \( p < 9 \), we also have massless scalar fields \( \phi^i(x) \) which represent the transverse position of the Dp-brane world-volume. Since the vertex operators correspond to the scalar fields are given by the momentum operator \( P_i \), the boundary interaction including the scalar fields will become

\[
e^{-S_b(A, \phi^i)} = \text{Tr} \hat{P} \exp \left\{ -\int d\hat{\sigma} \left( A_a(X)DX^a + i\phi^i(X)P_i \right) \right\}. \tag{2.8}\]

In fact, we can see from (2.4) that turning on the scalar fields \( \phi^i \) in the boundary interaction (2.8) corresponds to shifting the D-brane world-volume in the transverse directions. In this paper, we will not turn on fermion fields and massive fields on the Dp-brane. Thus, (2.8) contains all what we need to describe BPS Dp-branes.

It is not difficult to include tachyon fields to obtain the boundary interaction corresponding to Dp-Dp systems or non-BPS Dp-branes.\(^{16, 10, 11, 12}\) First, we introduce a matrix consists of a condensate of the open string vertices in the superfield notation

\[
M = \left( \begin{array}{ccc}
-A_a(X)DX^a - i\phi^i(X)P_i & T(X) \\
T(X) & -\tilde{A}_a(X)DX^a - i\tilde{\phi}^i(X)P_i
\end{array} \right). \tag{2.9}\]

Here \( A_a, \tilde{A}_a, \phi^i, \tilde{\phi}^i \) and \( T \) are the fields on the D-brane. For Dp-\( \overline{D}p \) system, these fields are independent, and \( A_a \) and \( \phi^i \) represent the gauge field and scalar fields on the Dp-brane, \( \tilde{A}_\mu \) and \( \tilde{\phi}^i \) are those on the \( \overline{D}p \)-brane and \( T \) is the tachyon field which is created by the open string stretched between the Dp-brane and the \( \overline{D}p \)-brane. For non-BPS Dp-brane, we have constraints \( A_a = \tilde{A}_a, \phi^i = \tilde{\phi}^i \) and \( T^\dagger = T \).

Then the boundary interaction (for NS-NS sector) is given by

\[
e^{-S_b} = \kappa \text{Tr} \hat{P} e^{\int d\hat{\sigma} M(\hat{\sigma})} \tag{2.10}\]

where \( M \) is given as (2.9). The normalization constant \( \kappa \) is \( \kappa = 1 \) for Dp-\( \overline{D}p \) systems and \( \kappa = 1/\sqrt{2} \) for non-BPS D-branes.

It is often useful to rewrite the boundary interaction (2.10) in path integral formulation. Let us suppose that the number of the Dp-branes and \( \overline{D}p \)-branes are both \( 2^{m-1} \) and the matrix \( M \) in (2.9) can be expanded by \( SO(2m) \) gamma matrices \( \Gamma^I = \left( \gamma_I \gamma^I \right) (I = 1, \ldots, 2m) \) as

\[
M = \sum_{k=0}^{2m} \mathcal{M}^{I_1 \cdots I_k} \Gamma^{I_1 \cdots I_k}, \tag{2.11}\]
where $\Gamma^{I_1\cdots I_k}$ denote the skew-symmetric products of the gamma matrices and $M^{I_1\cdots I_k}$ are the coefficients. In this case, the boundary interaction becomes

$$e^{-S_b} = \int [d\Gamma] \exp\left\{ \int d\hat{\sigma} \left( \frac{1}{4} \Gamma^I D \Gamma^I + \sum_{k=0}^{2m} M^{I_1\cdots I_k} \Gamma^{I_1} \cdots \Gamma^{I_k} \right) \right\}.$$  

(2.12)

where $\Gamma^I(\hat{\sigma}) = \eta^I(\sigma) + \theta F^I(\sigma)$ are real fermionic superfields. Note that we impose anti-periodic boundary condition for $\eta^I(\sigma)$ for the NS-NS sector boundary interaction. This formula is obtained from (2.10) by replacing the gamma matrices $\Gamma^I$ in the gamma matrix expansion (2.11) with the superfields $\Gamma^I$ and arranging the kinetic term $\frac{1}{4} \Gamma^I D \Gamma^I$ for them.

For example, for the case with a single non-BPS $D_p$-brane, the matrix $M$ in (2.9) is expanded by $\sigma^1$ and the boundary interaction (2.12) will become

$$e^{-S_b(A_{\alpha}, \Phi^i, T)} = \int [d\Gamma] \exp\left\{ \int d\hat{\sigma} \left( \frac{1}{4} \Gamma D \Gamma - A_{\alpha}(X) DX^\alpha - i\Phi^i(X) P_i + T(X) \Gamma \right) \right\}.$$  

(2.13)

The equivalence of (2.10) and (2.12) follows from the equivalence between the operator formulation and the path integral formulation of boundary supersymmetric quantum mechanics. See appendix A of Ref. [1] for a formal proof. The essential point is as follows. The kinetic term of $\Gamma^I$ in (2.12) implies the canonical anti-commutation relation

$$\{ \eta^I, \eta^J \} = 2\delta^{IJ} \tag{2.14}$$

in the operator formulation, and hence $\eta^I$ play the same role as the gamma matrices $\Gamma^I$. In order to obtain a supersymmetric formula, it is natural to combine the boundary fermion $\eta^I(\sigma)$ with the bosonic auxiliary field $F^I(\sigma)$ into the superfield $\Gamma^I(\hat{\sigma})$. This is the reason that we replace $\Gamma^I$ with $\Gamma^I$ when we rewrite (2.10) to (2.12).

§ 3. D-brane configurations in K-matrix theory

In this section, we consider the construction of D-branes in K-matrix theory. We will use the matrix theory based on the non-BPS D-instantons in type IIA string theory for simplicity. The generalization to other unstable D-brane systems is straightforward.

3.1. D-brane solutions in K-matrix theory

Let us consider the matrix theory based on non-BPS D-instantons in type IIA string theory. The field contents and the action is obtained as the dimensional reduction of non-BPS D9-brane system. The system of $N$ non-BPS D9-branes is a ten dimensional $U(N)$

*) Note however that the supersymmetry is explicitly broken by the anti-periodic boundary condition of the boundary fermions in the NS-NS sector.
gauge theory with a tachyon field. In this paper, we only consider tachyonic and massless bosonic fields, that is, the tachyon field $T$ and the gauge field $A_\mu$, both of which transform as the adjoint representation under the gauge transformation. The dimensional reduction of these fields gives the tachyon $T$ and ten scalars $\Phi^\mu (\mu = 0, 1, \ldots, 9)$, which are $N \times N$ hermitian matrix variables in the matrix theory. We take $N$ to be infinity and regard these matrices as operators acting on an infinite dimensional Hilbert space. For the action of this system, we adopt the BSFT action (2.1)

$$S^{D(-1)}(\Phi^\mu, T) = \frac{2\pi}{g_s} \langle 0 | e^{-S_0(\Phi^\mu, T)} | B(-1); + \rangle_{NS}, \quad (3.1)$$

where the boundary interaction is now given by (2.10) with

$$M = \begin{pmatrix} -i\Phi^\mu P_\mu & T \\ T & -i\Phi^\mu P_\mu \end{pmatrix}, \quad (3.2)$$

A configuration representing a D$p$-brane extended along $x^0, \ldots, x^p$-directions is given by

$$T = u \sum_{\alpha=0}^p \hat{p}_\alpha \gamma^\alpha \quad (3.3)$$

$$\Phi^\alpha = \hat{x}^\alpha \quad (\alpha = 0, \ldots, p), \quad \Phi^i = 0 \quad (i = p + 1, \ldots, 9), \quad (3.4)$$

where $\hat{x}^\alpha$ and $\hat{p}_\alpha$ are operators on a Hilbert space $\mathcal{H}$ satisfying

$$[\hat{x}^\alpha, \hat{p}_\beta] = i\delta^\alpha_\beta. \quad (3.5)$$

and $\gamma^\alpha$ are hermitian gamma matrices. $T$ and $\Phi^\mu$ are operators acting on a Hilbert space $\mathcal{H} \otimes S$, where $S$ is the spinor space on which the gamma matrices $\gamma^\alpha$ are represented. $u$ is a real parameter and this configuration becomes an exact solution in the limit $u \to \infty$.

It will soon become clear in section 4 that this configuration becomes an exact solution representing a D$p$-brane (BPS D$p$-brane or non-BPS D$p$-brane for even or odd $p$, respectively). Before moving to the discussion using the action, let us explain some of the properties that we can immediately see from the configuration (3.3).

Recall that the eigen values of $\Phi^\mu$ represent the position of the non-BPS D-instantons. Since the spectrum of the operator $\hat{x}^\alpha$ spans the real axis, we can see from (3.4) that this configuration represent a $p+1$ dimensional object, which is interpreted as the D$p$-brane world-volume. In particular, the eigen value $x^\alpha$ of the operator $\hat{x}^\alpha$ is interpreted as a coordinate on the D$p$-brane world-volume.

This matrix theory has a huge gauge symmetry:

$$\Phi^\mu \to U\Phi^\mu U^{-1}, \quad T \to UTU^{-1}, \quad (3.6)$$
where \( U \) is a unitary transformation of the Chan-Paton Hilbert space on which the operators \( \Phi^\mu \) and \( T \) are defined. The configuration \((3.3,3.4)\) breaks most of the symmetry, but the unitary transformations of the form

\[
U = h \mathbf{1}, \quad (3.7)
\]

where \( \mathbf{1} \) is the identity operator on the Hilbert space and \( h \) is the \( U(1) \) phase factor, are left unbroken. This unbroken \( U(1) \) subgroup is interpreted as the global gauge symmetry of the Dp-brane. If we repeat the same argument for a configuration representing \( N \) Dp-branes, which can be obtained by piling \( N \) copies of a single Dp-brane configuration \((3.3,3.4)\), we obtain \( U(N) \) as the unbroken subgroup. This is consistent with the global gauge symmetry of \( N \) Dp-branes.

3.2. Fluctuations around the D-brane solution

Let us next consider the fluctuations around the Dp-brane solution \((3.3,3.4)\). Here we mainly consider the cases with even \( p \), in which \((3.3,3.4)\) represents a BPS Dp-brane, for simplicity. General fluctuations will be given by

\[
\begin{align*}
T &= u \sum_{\alpha=0}^{p} \hat{p}_\alpha \gamma^\alpha + \delta T, \\
\Phi^\alpha &= \hat{x}^\alpha + \delta \Phi^\alpha \quad (\alpha = 0, \ldots, p), \\
\Phi^i &= \delta \Phi^i \quad (i = p+1, \ldots, 9). \quad (3.8-3.10)
\end{align*}
\]

Fields on the Dp-brane are embedded in the fluctuations \( \delta T \) and \( \delta \Phi^\mu \). In principle, we have to deal with all the possible fluctuations, but there is a shortcut to extract the components corresponding to massless fields on the Dp-brane. The key observation is that the massless fluctuations can be interpreted as Nambu-Goldston modes associated with the symmetry of the system.

For example, the matrix theory has a translational symmetry:

\[
\Phi^i \rightarrow \Phi^i + \phi^i \mathbf{1}, \quad (3.11)
\]

where \( \phi^i \) is a real number. This symmetry is spontaneously broken in the configuration \((3.3,3.4)\). Then, the NG mode \( \delta \Phi^i = \phi^i(\hat{x}) \) corresponds to a massless scalar field on the Dp-brane. Here we allow \( \hat{x}^\alpha \) dependence for \( \phi^i \), since we know the eigen value of \( \hat{x}^\alpha \) represent a coordinate of the Dp-brane world-volume. Note that similar shift \((3.11)\) for \( \Phi^\alpha \) can be absorbed by the gauge transformation \((3.6)\) and the corresponding fluctuation \( \delta \Phi^\alpha = \phi^\alpha(\hat{x}) \) does not represent a physical field. In fact, the gauge transformation \((3.6)\) with \( U = \exp(i\{\hat{p}_\alpha, \lambda^\alpha(\hat{x})\}/2) \) implies

\[
U \hat{x}^\alpha U^{-1} = \hat{x}^\alpha + \lambda^\alpha(\hat{x}) + \mathcal{O}(\lambda^2) \quad (3.12)
\]
and we can always set $\phi^\alpha = 0$ by choosing $\lambda^{\alpha}$ appropriately. This represents the reparametrization invariance of the $Dp$-brane world-volume. Setting $\phi^\alpha = 0$ corresponds to choosing the static gauge on the $Dp$-brane. It is amusing that both the reparametrization of the world-volume coordinates and the gauge transformation of the $Dp$-brane are originated in the gauge transformation of the matrix theory.

Next, let us consider the fluctuation corresponding to the gauge field on the $Dp$-brane. The gauge field is associated with the local gauge symmetry on the $Dp$-brane world-volume. The local gauge symmetry is obtained by allowing $\hat{x}^\alpha$ dependence for the $U(1)$ factor $h$ in (3.7). This symmetry is spontaneously broken, as we can see that the tachyon operator in (3.3) doesn’t commute with $h(\hat{x})$: *\)

$$h(\hat{x}) T h(\hat{x})^{-1} = T - i u h(\hat{x}) \partial_\alpha h(\hat{x})^{-1} \gamma^\alpha.$$  \hspace{1cm} (3.13)

The gauge field can be considered as a NG mode for the spontaneously broken ‘local’ gauge symmetry with $h(x) \partial_\alpha h(x)^{-1} = \text{constant}$, and hence it corresponds to the fluctuation of the form $\delta T = -iu A_\alpha(\hat{x}) \gamma^\alpha$. It is now clear that the second term in the right hand side of (3.13) can be absorbed in the fluctuation $A_\alpha(\hat{x})$ provided it transforms as

$$A_\alpha(\hat{x}) \rightarrow A_\alpha(\hat{x}) + h(\hat{x}) \partial_\alpha h(\hat{x})^{-1}$$  \hspace{1cm} (3.14)

under the gauge transformation, which guarantees the local gauge invariance of the $Dp$-brane world-volume theory.

Including these fluctuations, the $Dp$-brane configuration becomes

$$T = u \sum_{\alpha=0}^{p} (\tilde{p}_\alpha - i A_\alpha(\hat{x})) \gamma^\alpha,$$  \hspace{1cm} (3.15)

$$\Phi^\alpha = \hat{x}^\alpha \hspace{1cm} (\alpha = 0, \ldots, p),$$  \hspace{1cm} (3.16)

$$\Phi^i = \phi^i(\hat{x}) \hspace{1cm} (i = p + 1, \ldots, 9).$$  \hspace{1cm} (3.17)

This is the configuration representing a BPS $Dp$-brane with massless bosonic fields turned on.

When $p$ is odd, the solution (3.3) represents a non-BPS $Dp$-brane, and hence, we expect a fluctuation corresponding to the tachyon field on it. Actually, we can find the tachyonic fluctuation in the tachyon operator as

$$T = u \sum_{\alpha=0}^{p} (\tilde{p}_\alpha - i A_\alpha(\hat{x})) \gamma^\alpha + t(\hat{x}) \gamma_*, \hspace{1cm} (3.18)$$

\* Of course, this doesn’t mean the theory is in Higgs phase, since the global gauge symmetry with $h(x) = \text{constant}$ is unbroken. Note also that our argument here is limited to classical theory.
where \( t(x) \) is a real scalar function and \( \gamma_* \) is the chirality operator satisfying
\[
\gamma_*^2 = 1, \quad \{ \gamma^\alpha, \gamma_* \} = 0 \quad (\alpha = 0, \ldots, p).
\] (3-19)
Note that such \( \gamma_* \) exists only when \( p \) is odd. The fluctuation \( t(\hat{\sigma}) \) in (3-18) corresponds to the tachyon field on the non-BPS \( D_p \)-brane. To understand this quickly, consider the case with \( A_\alpha(x) = 0 \) and \( t(x) = t \) (constant), for simplicity. Then, using (3-19), we obtain
\[
T^2 = u^2 \hat{p}^2_\alpha + t^2,
\] (3-20)
from which it immediately follows that the field \( t \) has a negative mass squared mass term in the effective action, which is inherited from that for \( T \). This can be more explicitly seen by plugging the configuration (3-18) into the action (3-1) as we will explain in the next section.

One might be wondering whether or not more (unwanted) fields are introduced when we turn on general fluctuations (3-8)–(3-10). As explained in Ref. 1, the fluctuations introduced in (3-15)–(3-17) and (3-18) are essentially the only relevant ones in the limit \( u \to \infty \). The others will disappear in the effective action when we take the limit \( u \to \infty \). We will come back to this point in the next section.

§4. \( D_p \)-brane action from K-matrix Theory

4.1. Derivation of \( D_p \)-brane action

The effective action of the fluctuations around the \( D_p \)-brane solution is obtained by plugging the configuration (3-15)–(3-17) or (3-18) into the action (3-1) of K-matrix theory. Here we will demonstrate this for the configuration (3-15)–(3-17) representing a BPS \( D_p \)-brane. Inserting (3-15)–(3-17) into (3-2), we obtain
\[
M = -i\tilde{\Gamma}^\alpha \mathcal{P}_\alpha - i\phi_i(\hat{\sigma}) \mathcal{P}_i + u(\hat{\rho}_\alpha - iA_\alpha(\hat{\sigma})) \Gamma^\alpha,
\] (4-1)
where \( \Gamma^\alpha = \begin{pmatrix} \gamma^\alpha \\ \gamma^\alpha \end{pmatrix} \). The boundary interaction is given as (2-10) with this \( M \) in the operator formulation. The key step is to rewrite this boundary interaction in path integral formulation. Using the same argument to obtain (2-12), the boundary interaction is now rewritten as
\[
e^{-S_{b(\phi^a,T)}} = \int [d\Gamma^\alpha][dx^\alpha][dp_\alpha] \exp \left\{ \int d\hat{\sigma} \left( \frac{1}{4} \Gamma^\alpha D\Gamma^\alpha + i\mathcal{P}_\alpha Dx^\alpha - i\tilde{\Gamma}^\alpha \mathcal{P}_\alpha - i\phi_i(x) \mathcal{P}_i + u(\hat{\rho}_\alpha - iA_\alpha(x)) \Gamma^\alpha \right) \right\},
\] (4-2)
where \( x^\alpha(\hat{\sigma}) = x^\alpha(\sigma) + i\theta \psi^\alpha(\sigma) \) and \( p_\alpha(\hat{\sigma}) = p_\alpha(\sigma) + i\theta \pi_\alpha(\sigma) \) are boundary superfields. To obtain (4-2), we replaced the operators \( \tilde{x}^\alpha \), \( \hat{\rho}_\alpha \) and \( \Gamma^\alpha \) with the superfields \( x^\alpha \), \( p_\alpha \) and
\[ \Gamma^\alpha \] respectively, and arranged the kinetic terms \( \frac{1}{4} \Gamma^\alpha D \Gamma^\alpha \) and \( i p_\alpha D x^\alpha \) to realize the (anti-) commutation relations (2.14) and (3.5), respectively. The equivalence of (2.10) and (4.2) again follows from the equivalence between the operator formulation and the path integral formulation of boundary supersymmetric quantum mechanics. The formal derivation of this can be found in the appendix A of Ref. [1].

Now we are ready to derive the \( D_p \)-brane action. It is surprisingly simple. First, note that we can perform the path integral over \( p_\alpha \), which implies a delta functional imposing the condition
\[ i D x^\alpha + u \Gamma^\alpha = 0. \] (4.3)
Integrating with respect to \( \Gamma^\alpha \), we obtain
\[ e^{- S_b(\phi^\mu; T)} = \int [dx^\alpha] \exp \left\{ - \int d\bar{\sigma} \left( \frac{1}{4 u^2} D x^\alpha D^2 x^\alpha + i x^\alpha P_\alpha + i \phi^i(x) P_i + A_\alpha(x) D x^\alpha \right) \right\}. \] (4.4)
The first term \( \frac{1}{4 u^2} D x^\alpha D^2 x^\alpha \) in the integrand of (4.4) drops in the limit \( u \to \infty \). Then, using (2.2)–(2.5), we have
\[ e^{- S_b(\phi^\mu; T)} \mid B(-1); \pm \rangle_{NS} \to e^{- S_b(A_\alpha, \phi^i)} \mid Bp; \pm \rangle_{NS}, \quad (u \to \infty), \] (4.5)
where
\[ e^{- S_b(A_\alpha, \phi^i)} = \exp \left\{ - \int d\bar{\sigma} \left( i \phi^i(X) P_i + A_\alpha(X) D X^\alpha \right) \right\}. \] (4.6)
The boundary interaction (4.6) is nothing but that for a \( D_p \)-brane given in (2.8)! In particular, if we apply this result to the action of the form (3.1), we precisely obtain the BSFT action for the \( D_p \)-brane (2.1):
\[ S^{D(-1)}(\phi^\mu; T) \to S^{Dp}(A_\alpha, \phi^i), \quad (u \to \infty). \] (4.7)
Thus, we correctly reproduced the \( D_p \)-brane action from K-matrix theory.

4.2. Some comments

Here we make several comments on the derivation of \( D_p \)-brane action given in the previous subsection.

- **On the \( u \to \infty \) limit**

To be more precise, we should regularize the path integral before taking \( u \to \infty \) limit. If we keep \( u \) finite, we obtain
\[ e^{- S_b(\phi^\mu; T)} \mid B(-1); \pm \rangle_{NS} \]
Here the path integral with respect to $x^\alpha$ only involves gaussian integral and it can be performed by using zeta-function regularization. The result justifies the naive $u \to \infty$ limit we took in (4.5). If we turn off the fluctuations $A_\alpha$ and $\phi^i$, the gaussian path integral with zeta function regularization implies

$$S^{D(-1)}(\Phi^\mu, T) = \frac{2\pi}{g_s} \int [dx^\alpha] e^{-\int d\sigma \frac{1}{4\sigma} Dx^\alpha D^2x^\alpha} \langle 0 \mid x^\alpha, x^i = 0; + \rangle_{\text{NS}}, \quad (4.9)$$

where $V \equiv \int d^{p+1}x$ is the volume of the Dp-brane world-volume, $T_p \equiv S^{Dp}|_{A_\alpha=\phi^i=0}/V$ is the Dp-brane tension and

$$K(z) = \sqrt{\frac{z}{4\pi}} \frac{\Gamma(z)^2}{\Gamma(2z)}. \quad (4.10)$$

This is the function which appeared in Ref. [10], in which it was shown that the tension of D-branes constructed via tachyon condensation in supersymmetric BSFT precisely agrees with the correct value. In their argument, the fact that $K(u^2)$ becomes 1 in the $u \to \infty$ limit was crucial to obtain the correct tension. Our argument provides an easy way to understand this fact. Once we accept the naive $u \to \infty$ limit (4.5), there is nothing mysterious and we can directly come to the same conclusion without performing detailed calculation.

**Generalization to non-BPS D-branes**

It is straightforward to generalize the above argument to include the tachyon fluctuation as (3.18) for the $p = \text{odd}$ case with a non-BPS Dp-brane. When we insert (3.18) in (3.2), (4.1) is replaced with

$$M = -i\hat{x}^\alpha P_{\alpha} - i\hat{\phi}^i(\hat{x}) P_i + u(\hat{\rho}_{\alpha} - i A_\alpha(\hat{x})) \Gamma^\alpha + t(\hat{x}) \Gamma_*, \quad (4.11)$$

where $\Gamma_* = \left( \begin{array}{c} \gamma^* \\ \Gamma^* \end{array} \right)$, and the boundary interaction becomes

$$e^{-S_b(\Phi^\mu, T)} = \int [d\Gamma^I][dx^\alpha][dp_{\alpha}] \exp \left\{ \int d\sigma \left( \frac{1}{4} \Gamma^I D\Gamma^I + i p_{\alpha} Dx^\alpha ight. \
- i x^\alpha P_{\alpha} - i\hat{\phi}^i(x) P_i + u(p_{\alpha} - i A_\alpha(x)) \Gamma^\alpha + t(x) \Gamma_* \right) \right\}, \quad (4.12)$$

where $\Gamma^I = (\Gamma^\alpha, \Gamma_*)$. The same argument as in section 4.1 implies

$$e^{-S_b(\Phi^\mu, T)} \mid B(-1); \pm \rangle_{\text{NS}} \to e^{-S_b(A_\alpha, \phi^i, t)} \mid Bp; \pm \rangle_{\text{NS}}, \quad \left( u \to \infty \right) \quad (4.13)$$
with

\[ e^{-S_b(A,\phi, t)} = \int [d\Gamma_*] \exp \left\{ \int d\hat{\sigma} \left( \frac{1}{4} \Gamma_* D\Gamma_* - i \phi^i(X) P_i - A_\alpha(X) D\Gamma^\alpha + t(X) \Gamma_* \right) \right\} . \]

(4.14)

As expected, (4.14) is nothing but the boundary interaction for a non-BPS D-brane given in (2.13).

It is also easy to generalize this argument to the construction of multiple D-branes, D-brane - anti D-brane systems, non-commutative D-branes and so on.

- More general fluctuations

In (4.1), we only considered the fluctuations up to linear order with respect to the gamma matrices. We could turn on more general fluctuations that induce higher order terms in the gamma matrix expansion (2.11). However, since (4.3) implies that \( \Gamma^\alpha \) is a variable of order \( O(u^{-1}) \), the higher order terms will vanish in the \( u \to \infty \) limit. *

- Coupling to closed string fields

In (4.5) or (4.13) we reproduced the Dp-brane boundary states with boundary interaction. The boundary states carry much more information than the effective action (2.1). In fact, coupling of the Dp-brane to closed string field \( \varphi \) can be expressed as \( \langle \varphi | B_p \rangle \). Therefore, our argument in the previous subsection shows that not only the effective action of the Dp-brane, but also its coupling to every closed string field is precisely reproduced from the configuration (3.15) - (3.18) in K-matrix theory. One of the interesting applications of this fact will be given in the next section.

- Application to the D-brane descent relations

Our method can also be applied to the construction of D-branes via tachyon condensation in the higher dimensional unstable D-brane systems. For example, a BPS Dp-brane configuration in non-BPS D9-branes in type IIA string theory is given by (4.10)

\[ T(x^\mu) = u \sum_{i=p+1}^{9} (x^i - \phi^i(x^\alpha)) \gamma^i, \]

\[ A_\alpha(x^\mu) = A_\alpha(x^\alpha) \quad (\alpha = 0, \ldots, p), \quad A_i(x^\mu) = 0 \quad (i = p + 1, \ldots, 9). \]

(4.15)\n
(4.16)

Here \( T(x^\mu) \) and \( A_\mu(x^\mu) \) are the tachyon and gauge fields on the non-BPS D9-brane, and \( \phi^i(x^\alpha) \) and \( A_\alpha(x^\alpha) \) are the fluctuations corresponding to scalar and gauge fields on the Dp- 

* Here we assumed the explicit \( u \) dependence of the fluctuations is \( \delta T \sim O(u^1) \) and \( \delta \phi \sim O(u^0) \) so that the fluctuations do not grow faster than the Dp-brane solution itself in the \( u \to \infty \) limit.
brane. Then, using (2.2)–(2.4), we have

\[ e^{-S_{b}} | B9; \pm \rangle_{NS} = \int [dx^{i}] [d\Gamma^{i}] \exp \left\{ \int d\hat{\sigma} \left( \frac{1}{4} \Gamma^{i} D\Gamma^{i} + u(x^{i} - \phi^{i}(x^{\alpha}))\Gamma^{i} - A_{\alpha}(x^{\alpha}) D x^{\alpha} - i x^{\mu} P_{\mu} \right) \right\} | x^{\mu} = 0; \pm \rangle_{NS} \]  

(4.17)

\[ \to \int [dx^{i}] \delta (x^{i} - \phi^{i}(x^{\alpha})) \times \exp \left\{ - \int d\hat{\sigma} \left( A_{\alpha}(x^{\alpha}) D x^{\alpha} + i x^{\mu} P_{\mu} \right) \right\} | x^{\mu} = 0; \pm \rangle_{NS} \]  

(4.18)

\[ = \exp \left\{ - \int d\hat{\sigma} \left( A_{\alpha}(X^{\alpha}) D X^{\alpha} + i \phi^{i}(X^{\alpha}) P_{i} \right) \right\} | Bp; \pm \rangle_{NS} \]  

(4.19)

This time, we performed $\Gamma^{i}$ integral and took $u \to \infty$ limit in (4.18). In (4.19), we again correctly reproduced the D$p$-brane boundary state with the boundary interaction.

- **Generalization to type I string theory**

Though we do not have enough space to explain in detail, the whole story can also be elegantly applied to type I string theory. The gauge group and the representation of tachyon and scalar fields are much more complicated than those in type II string theory as listed in table I. In this table, $p = 1, 5, 9$ are BPS D$p$-branes while the others are non-BPS D$p$-branes.

| $p$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|---|---|---|---|---|---|---|---|---|---|
| Gauge | $U$ | $O$ | $O$ | $O$ | $U$ | $Sp$ | $Sp$ | $Sp$ | $U$ | $O$ | $O$ |
| Tachyon | | | | | | | | | | | |
| Scalar | adj. | | | | | | | | | |

As we can see from the table, K-matrix theory based on the D-instantons ($p = -1$ on the table) is similar to that based on type IIA non-BPS D-instantons, but now the tachyon is in the anti-symmetric representation. We can construct D-branes from this matrix theory in an analogous way as above. Examining the fluctuations around the D-brane solutions, we correctly recover all the ingredients listed in the table. See Refs. 6, 11 for details.

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*) A handy way to obtain (4.18) is to rescale $u \Gamma^{i} \to \Gamma^{i}$ and drop the kinetic term of $\Gamma^{i}$ by taking the limit $u \to \infty$ before performing the $\Gamma^{i}$ integral.
§5. CS-term and index theorem

So far, we have considered NS-NS sector boundary states. But, our argument is also applicable to the R-R sector as well. In particular, we can also precisely reproduce CS-term of the Dp-brane from the CS-term of K-matrix theory. The CS-term of K-matrix theory is given as

$$S_{CS}^{D(-1)}(C, \Phi^\mu, T) = \langle C \mid e^{-S_b(\Phi^\mu, T)} B(-1); + \rangle_{RR},$$

(5.1)

where $\langle \mid$ represents a closed string state corresponding to the massless RR fields. Here the boundary interaction $e^{-S_b}$ in the R-R sector is given by replacing the trace ‘Tr’ with supertrace ‘Str’ in the previous expression (2.10). If we insert the Dp-brane configuration (3.15)–(3.17) in (5.1), using the analogous relation as (4.5) for the R-R sector boundary states, we obtain

$$S_{CS}^{D(-1)}(C, \Phi^\mu, T) \to \langle C \mid e^{-S_b(A^\mu, \phi^i)} | Bp; + \rangle_{RR} = S_{CS}^{D_p}(C, A^\alpha, \phi^i)$$

(5.2)
in the $u \to \infty$ limit.

As an interesting corollary of this, we naturally obtain a physical derivation of index theorem.

Here we consider type IIB string theory and take D-instanton - anti D-instanton system as the starting point. The CS-term (5.1) can be written more explicitly as follows. First we introduce fermionic operators $\psi_\mu^{\pm} \equiv \frac{1}{\sqrt{2}}(\psi_0^\mu \pm i\tilde{\psi}_0^\mu)$ satisfying

$$\{\psi_\mu^{\pm}, \psi_\nu^{\pm}\} = 0, \quad \{\psi_\mu^{\pm}, \psi_\nu^{\mp}\} = \delta^{\mu\nu}. \quad (5.3)$$

and the Fock vacuum $\langle - |$ and $| + \rangle$ annihilated by $\psi_-^\mu$ and $\psi_+^\mu$, respectively, with $\langle - \mid + \rangle = 1$. Then, the CS-term is

$$S_{CS}^{D(-1)} = \int d^{10} k \langle - \mid C(k^\mu, \psi_+^\mu) \text{Str} \left(e^{-ik_n\tilde{\Phi}^\mu + 2\pi\hat{\Phi}}\right) \mid + \rangle, \quad (5.4)$$

where

$$C(x^\mu; \psi_+^\mu) = \sum_{n:\text{even}} C_{\mu_1\cdots\mu_n}(x^\mu) \psi_+^{\mu_1} \cdots \psi_+^{\mu_n} = \int d^{10} k e^{-ik_n x^\mu} C(k^\mu, \psi_+^\mu)$$

(5.5)
is a formal sum over the massless RR $n$-form fields $C_{\mu_1\cdots\mu_n}(x)$ and

$$\tilde{\Phi}^\mu = \left( \begin{array}{c} \Phi^\mu \\ \tilde{\Phi}^\mu \end{array} \right), \quad (5.6)$$

$$\hat{\Phi} = \left( \begin{array}{cc} -TT^\dagger + \frac{1}{8\pi^2} [\Phi^\mu, \Phi^\nu] \psi_-^\mu \psi_-^\nu & -\frac{i}{8\pi^2} \tilde{\Phi}^\mu T - T \tilde{\Phi}^\mu \psi_-^\mu \\ -\frac{i}{8\pi^2} (\tilde{\Phi}^\mu T^\dagger - T^\dagger \tilde{\Phi}^\mu) \psi_-^\mu & -T^\dagger T + \frac{1}{8\pi^2} [\tilde{\Phi}^\mu, \tilde{\Phi}^\nu] \psi_-^\mu \psi_-^\nu \end{array} \right).$$

(5.7)

\(^*) In this section, we ignore possible $\alpha'$ corrections which do not contribute to the topological invariants that we are focusing on.
This expression can be obtained by either directly manipulating (5.1) or using T-duality
relations from the CS-term of the D9-\overline{D9} system given in Refs. [18], [11], [12].

Let us assume that the RR-field is constant (i.e. $k_{\mu} = 0$) and all the components except
for the zero-form part $C_0$ are zero. Then the CS-term takes very simple form, since $\Phi_{\mu}$ do
not contribute to it in this case. Actually, it becomes the index of the tachyon operator,

$$S_{CS}^{D(-1)} = C_0 \text{Str} \left( e^{-2\pi Q^2} \right) = C_0 \text{Tr} \left( \sigma^3 e^{-2\pi Q^2} \right) = C_0 \left( \dim \text{Ker} TT^\dagger - \dim \text{Ker} T^\dagger T \right) = C_0 \text{Index } Q,$$

(5.8)

where $Q \equiv \left( T^T \right)$. Since the coupling of the RR 0-form field represents D-instanton charge,
we conclude that the index of the tachyon operator is interpreted as the D-instanton charge.
This can also be seen from the fact that $\dim \text{Ker} TT^\dagger$ and $\dim \text{Ker} T^\dagger T$ correspond to the
number of D-instantons and anti D-instantons which are not annihilated by the tachyon
condensation, respectively.

Now consider the D-instanton charge in the presence of BPS D$p$-branes ($p = \text{odd}$). The
tachyon configuration representing D$p$-branes in type IIB K-matrix theory is also given as
(3.15), though $\gamma^\alpha$ are no longer hermitian matrices, and we have

$$Q = u \sum_{\alpha=0}^{p} (\hat{\rho}_\alpha - i A_{\alpha}(\hat{x})) \Gamma^\alpha \equiv -iu D,$$

(5.9)

where $\Gamma^\alpha = \left( \gamma_\alpha \gamma^\alpha \right)$ ($\alpha = 0, \ldots, p$) are SO($p + 1$) gamma matrices. Note that $\sigma^3$ in
(5.8) can be regarded as the chirality operator. Therefore, from the above argument, the
D-instanton charge in the presence of the BPS D$p$-branes is just the index of the usual Dirac
operator $\slashed{D}$ on the world-volume of the D$p$-branes.

On the other hand, as we have seen in (5.2), the CS-term of D-instanton - anti D-instanton
system with the D$p$-brane configuration is equal to the CS-term of D$p$-brane. In our case
with $C = C_0 = \text{constant}$, the CS-term of D$p$-brane $S_{CS}^{Dp}$ is given as

$$S_{CS}^{Dp} = C_0 \int_{Dp} \text{tr } e^{F/2\pi}.$$

(5.10)

Comparing this with (5.8), we obtain

$$\text{Index } (-i \slashed{D}) = \int_{Dp} \text{tr } e^{F/2\pi}.$$

(5.11)

This is nothing but the Atiyah-Singer index theorem[19]. It is quite interesting that we
have naturally reached this result considering D-brane physics. The physical interpretation
is now clear. The Dirac operator is the tachyon operator which represent the D$p$-brane in
the D-instanton - anti D-instanton system and its index gives the D-instanton charge. The
D-instanton can also be constructed as the instanton configuration in the gauge theory on
the Dp-brane world-volume and the instanton number is given by the Chern number of the
gauge bundle. And the two descriptions actually agree as expected.

§6. Conclusions

In this paper, we have explained how to describe D-branes in K-matrix theory. We
examined the fluctuations around the Dp-brane solution and showed that they correctly
reproduce the fields on the Dp-brane. Plugging the Dp-brane configurations into the action
of K-matrix theory, we precisely obtained the Dp-brane action as the effective action of the
fluctuations. Our method can easily be applied to construction of D-branes in a wide variety
of unstable D-brane systems and provides a simple derivation of D-brane ascent/descent
relations.

In K-matrix theory, D-branes are described in terms of the operators $T$ and $\Phi^\mu$ acting
on a Hilbert space. We translated this analytic description to a geometric description of the
D-branes given in terms of a gauge theory on the world-volume manifold of the D-branes.
The equivalence of the analytic and geometric descriptions of the D-branes follows from
the equivalence between path integral and operator formulation of the boundary quantum
mechanics. As a corollary, the Atiyah-Singer index theorem is naturally obtained by looking
at the coupling to RR-fields.

It is now clear that any D-branes can be constructed as a kind of bound states of unstable
D-instantons. Therefore, we can in principle study anything about D-branes by studying
K-matrix theory, just as some particle physicists sometimes say “the theory of elementary
particles is supposed to be a theory of everything, since everything in our world is made by
the elementary particles”. As we have shown in this paper, we precisely recover D-brane
action from K-matrix theory. Thus, at least, any analysis based on the D-brane action can
be recovered in the context of K-matrix theory.

Note however that our consideration so far is limited within the classical (disk) level.
The next important step would be to consider the quantum effects in K-matrix theory.

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