Mathematical and computer modeling of a planar mechanism with a certain number of links

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Abstract. The article describes a mathematical model of the functioning of a certain planar mechanism, consisting of a finite number of links, interconnected by various types of communication. A software application has been developed that allows simulating various assembly options for most mechanisms based on analytical calculations and computer analysis. The mathematical basis of the study is the numerical methods for solving systems of nonlinear equations and coordinate - vector method for describing the position of the links of the planar mechanism.

1. Introduction
Under a mechanism we mean a kinematic chain in which, for a given movement of one or more links relative to any of them, all the other links make clearly determined movements. The determination of the movement of the mechanism is achieved by the proper pairwise connection of its parts. The problem of determining the absolute and relative positions of the links of the mechanism for any number of links is relevant and allows determining the areas of the existence of a certain planar mechanism with different options for the connections between its links.

2. Problem statement
Any planar mechanism with two links can be described using nonlinear equations. As the complexity of the mechanism increases, it becomes necessary to solve systems of nonlinear equations with a large number of variables to find the position of the links at any time, speeds and accelerations of the links [1-2].

The main goal of the study is to develop a mathematical model of a certain planar mechanism based on the coordinate-vector method and create software in which all the necessary analytical calculations will be carried out with the main characteristics and positions of all links of the mechanism displayed on the screen, as well as the demonstration of the successful operation of the studied mechanism [3-4].

3. Theoretical basis for research
Let us consider a number of simplest problems on the work of a planar mechanism with known distances between its links (lij), and angles (φij).
Problem 1. Let two fixed points be given \(A(x_0; y_0), B(x_1; y_1)\) and distances \(AC= l_{02}\) and \(CB=l_{12}\) are fixed (figure 1). Point coordinates \(C(x_2; y_2)\) can be found from system (1).

\[
\begin{align*}
(x_2-x_0)^2 + (y_2-y_0)^2 - l_{02}^2 &= 0, \\
(x_2-x_1)^2 + (y_2-y_1)^2 - l_{12}^2 &= 0
\end{align*}
\]

(1)

Changing one of the distances \(l_{12}, l_{02}\) or both at the same time, each time we will get a working mechanism, if the conditions are true (2), otherwise the work of the mechanism is impossible.

![Figure 1. One rotational pair and the given lengths of links.](image)

Figure 1. One rotational pair and the given lengths of links.

Systems (1), (2) also allow defining maximum and minimum link lengths and the relationships between them, based on the study of the function of two variables on a constrained extremum.

\[
\begin{align*}
\sqrt{(x_2-x_0)^2 + (y_2-y_0)^2} + \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} &\leq \sqrt{(x_0-x_0)^2 + (y_0-y_0)^2}, \\
\sqrt{(x_2-x_0)^2 + (y_2-y_0)^2} + \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2} &\leq \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}, \\
\sqrt{(x_2-x_0)^2 + (y_2-y_0)^2} + \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} &\leq \sqrt{(x_0-x_0)^2 + (y_0-y_0)^2}
\end{align*}
\]

(2)

Problem 2. Let two fixed points be given \(A(x_0; y_0)\) and \(B(x_1; y_1)\) with a fixed distance \(AC\), axial angle \(Ox\) and a link \(CB\) (figure 2). Point coordinate value \(C(x_2; y_2)\) can be found from system (3).

\[
\begin{align*}
(x_2-x_0)^2 + (y_2-y_0)^2 - l_{02}^2 &= 0, \\
x_1 - x_0 - \cos \varphi_{12} \cdot \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} &= 0, \\
y_1 - y_0 - \sin \varphi_{12} \cdot \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} &= 0
\end{align*}
\]

(3)

Changing distance \(l_{02}\) or angle \(\varphi_{12}\) or both values at the same time, each time we will get a working mechanism, if the conditions are true (2), otherwise the work of the mechanism is impossible. Systems (2), (3) allow defining all the possible lengths of the mechanism links.

![Figure 2. One rotational pair, given length of link and angle.](image)

Figure 2. One rotational pair, given length of link and angle.
Problem 3. Let two fixed points be given $A(x_0; y_0)$ and $B(x_1; y_1)$, with fixed angles between axis Ox and links $AC$ and $CB$ (figure 3). Point coordinate value $C(x_2; y_2)$ can be found from system (4).

$$
\begin{align*}
&x_2 - x_0 - \cos \varphi_{02} \cdot \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = 0, \\
&x_1 - x_2 - \cos \varphi_{12} \cdot \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 0, \\
&y_2 - y_0 - \sin \varphi_{02} \cdot \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = 0, \\
&y_1 - y_2 - \sin \varphi_{12} \cdot \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 0.
\end{align*}
$$

(4)

Changing angles $\varphi_{02}$, $\varphi_{12}$, we will get a working mechanism, if the conditions are true (2), (4).

![Figure 3. One rotational pair and given angles.](image)

The theoretical basis of the mathematical model of the functioning of a certain planar mechanism, consisting of a finite number of links connected by the listed types of communication, is a combination of equations obtained in problems 1-3.

Problem 4. Let the coordinates of the support be given in the four-bar linkage $A(x_0; y_0)$ and distances $l_{01}, l_{12}, l_{03}, l_{32}$ and angles $\varphi_{01}, \varphi_{03}$ are fixed (figure 4).

![Figure 4. Four-bar linkage with one support.](image)

To state the point position $B(x_1; y_1)$, $C(x_2; y_2)$, $D(x_3; y_3)$, we will compose a system of equations (5).
\[
\begin{align*}
(x_1 - x_0)^2 + (y_1 - y_0)^2 - l_{10}^2 &= 0, \\
x_1 - x_0 - \cos \varphi_{10} \cdot \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} &= 0, \\
x_1 - x_0 - \sin \varphi_{10} \cdot \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} &= 0, \\
(x_2 - x_1)^2 + (y_2 - y_1)^2 - l_{12}^2 &= 0, \\
(x_3 - x_2)^2 + (y_3 - y_2)^2 - l_{13}^2 &= 0, \\
x_4 - x_3 - \cos \varphi_{30} \cdot \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2} &= 0, \\
x_4 - x_3 - \sin \varphi_{30} \cdot \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2} &= 0.
\end{align*}
\]

(5)

Changing the distances and angles between links with the help of systems (2), (5) one can find all the possible link lengths and consequently all types of the mechanism assembly.

4. Description of the software application

The software application has been developed in the programming environment Builder C++ using the OpenGL library [5-6]. The program allows simulating various assembly options for most planar mechanisms, consisting of any number of links and related communication conditions listed in problems 1-3. In the created application, the ability to input/output and change input data (the number of links, their lengths and angles) is organized, and protection against conflicting data is established. In the program the distribution and type checking of all elements is organized depending on the entered data; the output of information on the mechanism under study is organized for various input data and a graphical implementation of the obtained model of the planar mechanism is schematically performed [7].

Let us describe the algorithm of the software application. Based on the entered input data, the system of nonlinear equations of type (5) is composed, the number of variables and constants is determined, and the Jacobian of the system is compiled. The algorithm of the Newton method consists of several steps [8]. The equations of the system must be specified implicitly (6).

\[ F_i(X) = 0, \quad X = (x_1, x_2, \ldots, x_n), \quad i = 1, 2, \ldots, n. \]

(6)

The Jacobian matrix \( W(x) \) is composed of partial derivatives of functions (6) with respect to each variable. For problem 4, the Jacobian matrix has form (7).

\[
\begin{pmatrix}
2(x_1^0 - x_0) & 2(y_1^0 - y_0) & 0 & 0 & 0 & 0 \\
\frac{\cos \varphi_{10}(y_1^0 - y_0)}{\sqrt{(x_1^0 - x_0)^2 + (y_1^0 - y_0)^2}} & \frac{\cos \varphi_{10}(y_1^0 - y_0)}{\sqrt{(x_1^0 - x_0)^2 + (y_1^0 - y_0)^2}} & 0 & 0 & 0 & 0 \\
-2(x_1^0 - x_0) & 2(y_1^0 - y_0) & 2(x_2^0 - x_1) & 2(y_2^0 - y_1) & 2(x_3^0 - x_2) & 2(y_3^0 - y_2) \\
-2(y_1^0 - y_0) & -2(x_1^0 - x_0) & 2(y_1^0 - y_0) & 2(x_2^0 - x_1) & 2(y_2^0 - y_1) & 2(x_3^0 - x_2) \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\cos \varphi_{10}(x_3^0 - x_0)}{\sqrt{(x_3^0 - x_0)^2 + (y_3^0 - y_0)^2}} & \frac{\cos \varphi_{10}(x_3^0 - x_0)}{\sqrt{(x_3^0 - x_0)^2 + (y_3^0 - y_0)^2}} & 1 & \frac{\cos \varphi_{10}(x_3^0 - x_0)}{\sqrt{(x_3^0 - x_0)^2 + (y_3^0 - y_0)^2}} & 1 & \frac{\cos \varphi_{10}(x_3^0 - x_0)}{\sqrt{(x_3^0 - x_0)^2 + (y_3^0 - y_0)^2}}
\end{pmatrix}
\]

(7)

The Newton method assumes the inversion of the Jacobian matrix at each step, and has quadratic convergence near the root, so to search each next approximation we use formula (8).

\[
X^{(i+1)} = X^{(i)} - W^{-1}(X^{(i)})F(X^{(i)}).
\]

(8)

The program solves the obtained system of linear equations, and the results are used to construct a graphic image of one position of all the links in the mechanism. The process is over if two adjacent approximations coincide with a given degree of accuracy.
The program calculates not only the coordinates of variable points, but also their speeds, which are calculated on the basis of numerical differentiation of the obtained table function. The program contains a complete modular and cross-platform description of each element, which allows adding new types of connections between links and modeling more complex connections, developing more modern mechanisms.

5. Conclusions
The mathematical model is presented in the work and on its basis the software is developed for solving the problem of the positions of a planar mechanism with any number of links. Computer simulation allows establishing options for assembling the mechanism and the area of their existence, as well as calculates the necessary parameters of the mechanism. The developed application builds planar mechanisms and models their behavior based on theoretical calculations.

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