Mechanical challenges of electrical transmission lines inspection robot

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Abstract. In this article, the dynamics of electrical transmission lines inspection robot while moving on the line has been investigated and mathematical modeling issues of its movement along the conductors have been considered. In order to achieve dynamical stability during the movement of inspection robot on the electrical line variation methods for problems of stretched string have been considered and solved. However, line inspection robot operation might be subject to failures because of conductor vibration. In this article, the robot-inspector considered as moving load and pendulum. As a result of even steady motion of inspection robot on the line, saw-tooth oscillations have been observed in the vertical plane that causes parametric oscillations in the perpendicular plane. In order to avoid dangerous oscillation, dynamic vibration absorber has been designed. The behavior of electrical line inspection robot motion through transmission lines has been studied.

1. Introduction
Nowadays electrical energy plays an inseparable role in human life. Any damage and disruption during the transmission of the electrical energy lines from the power plants to the end users (cities, industries, etc.) may affect human life. To prevent any problems in the mentioned areas, inspection and maintenance of electrical transmission lines are necessary.

On the other hand, an inspection of electrical transmission lines performed by human forces would be faced with danger. To check the condition of the power lines (high voltage electrical transmission lines), robot inspectors are created and used in some countries [1]. Figure 1 shows one of proposed robot inspector [2], capable not only to move along the wire and conduct diagnostics but also able to pass through obstacles (dampers, clamps, warning balls located on electrical transmission lines). Operating experience in some countries [3-7] shows that when DAM moves along the wire, intense dangerous vibrations can occur. To understand the essence of these oscillations, their quantitative evaluation, prevention of accidents and ensuring personnel safety, it is necessary to perform mathematical modeling of the robot inspector motion along the conductor [8-10].

In this article a mathematical model developed to study the behavior of the conductor as a stretched string with a moving load and also a dynamic vibration absorber system has been designed in order to avoid dangerous oscillation in the horizontal plane of motion.
2. Saw-tooth vibration of the string

In this section, the dynamics of the conductor with a moving load is considered (figure 2). This is the simplest model in which the conductor is considered as a stretched string, and the robot inspector is considered as a moving load.

\[
Tu'' + f(x,t) = \rho \ddot{u}; \quad x = 0, l; \quad u = 0; \quad t = 0; \quad u = 0, \dot{u} = 0
\]

(1)

where \( T \) is the string tension force, \( f \) is the linear load (per unit length), \( \rho \) is the density, \( l \) is the string length; prime and dot mean differentiation by coordinate \( x \) and time \( t \).

Under load \( F(t) \), the force concentrated at a point \( x=\xi \) will be \( f(x,t) = F(t)\delta(x-\xi(t)) \) (with a delta function). The loading point can move according to an arbitrary law \( \xi(t) \) with the condition of \( 0 < t < t_1, \xi(0) = 0, \xi(t_1) = l \).

The solution to the problem (1) can be constructed by the method of eigenfunctions [11]:

\[
\varphi_n = \sqrt{\frac{2}{l}} \sin \lambda_n x, \quad \lambda_n = \frac{n\pi}{l},
\]

\[
\int_0^l \varphi_n \varphi_k dx = \delta_{n k}
\]

(2)

Multiplying both sides of equation (1) by \( \varphi_n \) and integrating, we obtain ordinary differential equations (ODE) for \( u_n \):

\[
\ddot{u}_n + \omega_n^2 u_n = f_n(t)/\rho, \quad \omega_n = \lambda_n c, \quad c = \sqrt{T/\rho}
\]

(3)
These ODEs are solved by the Duhamel integral. Under zero initial conditions
\[ u_n(t) = \frac{1}{\rho \omega_n} \int_0^t f_n(\tau) \sin \omega_n (t - \tau) d\tau. \]
Substituting in (4), we obtain the solution of problem (1).

With a concentrated load we have \[ f_n(t) = \frac{2}{l} F(t) \sin \lambda_n \xi(t). \]
If the functions \( F(t), \xi(t) \) are arbitrary, then Mathcad [12] can be used to determine the definition \( u_n(t) \). But with \( F(t) = \text{constant} \) and \( \xi = vt \) (the load moves at a constant velocity \( v \)) we get
\[ u_n = \frac{F}{\rho \omega_n} \sqrt{\frac{2}{l}} \int_0^t \sin \lambda_n \nu \tau \sin \omega_n (t - \tau) d\tau = \frac{F}{\lambda_n (v^2 - c^2)} \sqrt{\frac{2}{T \rho l}} (v \sin \lambda_n ct - c \sin \lambda_n vt) \quad (4) \]

Calculation using the equations is done by Mathcad [12]. Figure 3 illustrates the deflection (a) and acceleration (b) of the conductor using the following parameters: \( T=10 \text{ kN}, F=1 \text{ kN}, v=2 \text{ m/s}, \rho=5 \text{ kg/m}, l=200 \text{ m}. \) As shown in figure 3(a) saw-tooth oscillations appear in the vertical plane although the motion of transferring load is steady. Acceleration showed in figure 3(b) cause inertial load on the construction of the robot.

![Figure 3](image_url)

**Figure 3.** (a) Deflection and (b) acceleration of the conductor.
3. Design of dynamic vibration absorber for a pendulum with a movable suspension base

The consequence of these saw-tooth oscillations that may cause dangerous oscillations of the robot as a pendulum with a movable suspension base has been studied in [8]. In that case, although it is not the classical parametric resonance, the growth of fluctuations can be seen however limited only by the time of the process. This example illustrates that dangerous oscillations with increasing amplitude are possible.

In this article, to prevent dangerous oscillation in the horizontal plane, dynamic vibration absorber (DVA) has been designed. The performance of DVA using in inspection robot motion through transmission lines has been investigated.

The concept of the dynamic vibration absorber DVA for reducing pendulum sway motion with a suspension base is shown in figure 4. The pendulum mass (robot mass) is denoted by $m_1$, the deflection angle is designated by $\theta$ measured in the $z$-$y$ plane from the vertical axis. The position of the center of mass of the pendulum is indicated by $r_1$. The tuned mass $m_2$ locates in the static position is shown by $r_2$. $k$ and $c$ are the spring stiffness and the damping coefficient of the DVA system. Function $u(t)$ describes the base motion in the vertical direction while the motion of the tuned mass of the DVA system describes by $d(t)$.

![Figure 4. Pendulum with a movable suspension base using dynamic vibration absorber.](image)

Consider the coordinate system as shown in figure 4, the positions of the main system mass $(z_1, y_1)$ and the DVA mass $(z_2, y_2)$ and the velocities of both masses are:

$$
\begin{align*}
  z_1 &= r_1 \sin(\theta),
  y_1 &= r_1 \cos(\theta) + u(t),
  z_2 &= (r_2 - d) \sin(\theta),
  y_2 &= (r_2 - d) \cos(\theta) + u(t),
  \dot{z}_1 &= r_1 \theta \cos(\theta),
  \dot{y}_1 &= -r_1 \theta \sin(\theta) + \dot{u}(t),
  \ddot{z}_2 &= (r_2 - d) \ddot{\theta} \cos(\theta) - \dot{\theta} \sin(\theta) - \dot{d} \cos(\theta) + \dot{u}(t).
\end{align*}
$$

The twice of the kinetic energy using equation (5) can be written as:

$$
\begin{align*}
  2K &= m_1(\dot{z}_1^2 + \dot{y}_1^2) + m_2(\dot{z}_2^2 + \dot{y}_2^2) \\
  2K &= m_1 \left( r_1^2 \ddot{\theta}^2 + \dot{u}^2 - 2r_1 \dot{u} \dot{\theta} \sin(\theta) \right) + \\
  ... + m_2 \left( (r_2 - d)^2 + \dot{u}^2 + \dot{d}^2 - 2u \dot{d} \cos(\theta) - 2(r_2 - d) u \dot{\theta} \sin(\theta) \right).
\end{align*}
$$

The potential energy is given by:
\[ U = \frac{1}{2} kd^2 + g (1 - \cos \theta)(m_1 r_1 + m_2 (r_2 - d)). \] (7)

After completing all the necessary actions in accordance with the Lagrange equation (second order), knowing the general force in \( d \) direction (general coordinate) which is \( Q_d = -c \dot{d} \), one can obtain the system of equations for general coordinates \( \theta \) and \( d \):

\[
\frac{d}{dt} \left( \frac{\partial K}{\partial \dot{q}_i} \right) - \left( \frac{\partial K}{\partial q_i} \right) + \frac{\partial U}{\partial q_i} = Q_i \Rightarrow \\
\ddot{\theta}[m_1 r_1^2 + m_2 (r_2 - d)^2] - 2m_2 \dot{\theta} \dot{d} (r_2 - d) + (g - \ddot{u})(m_1 r_1 + m_2 (r_2 - d))\sin(\theta) = 0, \tag{8}
\]

\[
m_2 \dddot{d} + cd + kd + m_2 \ddot{\theta}^2 (r_2 - d)^2 = m_2 \cos(\theta)\ddot{u} + m_2 g(1 - \cos(\theta)).
\]

Study of this character can be done by mathematical modeling: by setting \( \theta(0) = 0.1, \dot{\theta}(0) = 0 \) and \( a(t) \) as shown in figure 3 (b), numerical solution of (8) been obtained. Figure 5 shows the result using the following parameters: \( m_1 = 100 \text{ kg}, m_2 = 5 \text{ kg}, r_1 = 0.3 \text{ m}, r_2 = 0.5 \text{ m}, k = 0.1 \text{ N/m}, c = 25 \text{ N.s/m}. \)

![Figure 5. Performance analysis of DVA used in a pendulum with a movable suspension.](image)

It can be seen that the amplitude of fluctuations decreased by the time of the process. This example shows the possibility of usage of DVA in electrical transmission inspection robots.

4. Conclusion
The dynamical behavior of the electrical transmission lines inspection robot motion traveling through the line has been studied. As a result of mathematical modeling, it is shown that even by steady motion of the robot on the line saw-tooth vibration appears in the vertical plane. This oscillation can cause parametric type vibrations in the perpendicular plane. In order to achieve dynamical stability during the movement of inspection robot on the electrical line and to avoid dangerous oscillation in the horizontal plane, dynamic vibration absorber has been designed and investigated. The result shows the possibility of usage of DVA in electrical transmission inspection robots. Although, practical results can be useful for engineers to select proper parameters and speed during the design stage.

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