Error analysis of mathematics students who are taught by using the book of mathematics learning strategy in solving pedagogical problems based on Polya’s four-step approach

Budi Halomoan Siregar, Izwita Dewi, Ade Andriani

Department of Mathematics Education, Faculty of Mathematics and Natural Science, State University of Medan

budi_or_fuzzy@yahoo.com

Abstract. The purpose of this study is to analyse the types of students’ errors and causes of them in solving of pedagogic problems. The type of this research is qualitative descriptive, conducted on 34 students of mathematics education in academic year 2017 to 2018. The data in this study is obtained through interviews and tests. Furthermore, the data is then analyzed through three stages: 1) data reduction, 2) data description, and 3) conclusions. The data is reduced by organizing and classifying them in order to obtain meaningful information. After reducing, then the data presented in a simple form of narrative, graphics, and tables to illustrate clearly the errors of students. Based on the information then drawn a conclusion. The results of this study indicate that the students made various errors: 1) they made a mistake in answering what being asked at the problem, because they misunderstood the problem, 2) they fail to plan the learning process based on constructivism, due to lack of understanding of how to design the learning, 3) they determine an inappropriate learning tool, because they did not understand what kind of learning tool is relevant to use.

Keyword: error analysis, problem solving, polya four-step approach

1. Introduction

Problem solving is an activity to overcome any problem by using concepts and rules, finally expected to get a solution to reach the goal (Dewi, et al 2017). Wilson (1993: 57) stated, "The problem solving has a special interest in learning mathematics. The main purpose of teaching and learning of mathematics is to develop the ability to solve complex mathematical problems". Furthermore, the approach of problem solving applied in this study is a Polya’s four-step approach to problem solving.

Pedagogic ability is a competency related to learning or education. This ability includes an understanding of the character, the level of development of psychology, educational concepts, teaching methods which are in accordance with student development, etc. Finally, it can be concluded that pedagogic competence is the ability of a teacher to plan the learning and apply learning strategies which in accordance with the development and character of learners.

There are several reasons why teachers should master pedagogic competence. Firstly, the teacher will understand easily the difficulties faced by the students when they have problems in understanding the subject matter, able to stimulate the development of students well, able to apply active learning,
able to apply various models of learning in accordance with the material and development of learners, able to build a conducive atmosphere when the learning process.

Pedagogics problems is related to education and how to educate. Furthermore, the ability to solve pedagogic problems is one of the most basic and important competencies to be mastered by teachers, in order to be able to educate them according to the rules, principles and norms.

On the other hand, as a researcher and lecturer in mathematics teaching and learning strategy of mathematics, has identified some student mistakes in solving pedagogic problems. These errors are known based on interviews and answer sheets on quiz questions. Based on these errors, it is necessary to analyze the types of errors and sources of cause. Then, the results of this analysis are used to improve the learning strategy, so that students do not make the same mistakes.

2. Theoretical Review
The type of this research is qualitative descriptive research in which aims to identify errors and causes in solving pedagogic problems. There are 34 students as subjects in this research, they have been learning about mathematics learning strategy in academic year 2017-2018. John C. Creswell (2009: 22) said that Qualitative research is a means for exploring and understanding the meaning of individuals or groups as a social or human problem. The process of research involves emerging questions and procedures, data typically collected in the participant's settings, data analysis inductively building from particulars to general themes, and the researcher making interpretations of the meaning of the data. The final written report has a flexible structure. Those who engage in this form of inquiry support a way of looking at research that honors an inductive style, a focus on the individual meaning, and the importance of rendering the complexity of a situation.

Data were obtained through a test and interviews. The test is given to obtain the kinds of errors in solving pedagogic problems. Before the tests were conducted, the questions were validated by three lecturers who were experts in the field of evaluation. Moreover, interview is an exchange of information through a question and answer process conducted by two people to obtain meaningful information in a particular topic (Esterberg in Sugiyono: 2012, 317).

After obtaining the data, then analyzed through three stages, namely: 1) data reduction, 2) displaying and explaining data and 3) drawing conclusions. Furthermore, the determination of the student's answer score is done by applying Polya’s four-step approach to problem-solving steps. Then, the test results are consulted on the scoring guidelines of problem solving test.

After reducing, then the data is presented in a simple form of narration, tables and diagrams. To determine the level of problem solving ability of each indicator then used the following formula: \[ NP_k = \frac{R_k}{SM_k} \times 100\% \] (Purwanto, 2009: 102). Where NP\(_k\): The percent value to be determined on k indicator, \( k = 1, 2, 3, 4 \). R\(_k\): score obtained by student on k-indicator, SM\(_k\): maximum score on k-indicator. After the score is obtained, then consulted on 5 categories, they are very high, high, medium, low, and very low.

The third stage is the process of concluding the results of student error analysis. This stage is done by analyzing the results obtained based on previously presented data.

3. Result And Discussion
After the student answer sheets are checked and reduced, then the data obtained in table 1.

| Value Interval (%) | Types of category | Percentage of student achievement |
|--------------------|-------------------|----------------------------------|
| 90-100             | Very High         | Indicator 1: 24% Indicator 2: 18% Indicator 3: 18% Indicator 4: 18% |
| 80-89              | High              | Indicator 1: 35% Indicator 2: 24% Indicator 3: 24% Indicator 4: 21% |
| 65-79              | Medium            | Indicator 1: 32% Indicator 2: 26% Indicator 3: 26% Indicator 4: 26% |
| 55-64              | Low               | Indicator 1: 9% Indicator 2: 32% Indicator 3: 32% Indicator 4: 35% |
| 0-54               | Very Low          | Indicator 1: 0% Indicator 2: 0% Indicator 3: 0% Indicator 4: 0% |
3.1. Error in Preparation: Understand The Problem
Based on table 1, there are some students who make mistakes in understanding of pedagogic problems with different error levels. 32% and 3% of students who have not been able to understand the problem well, in which consist of in the medium and low category, respectively. To find out the types and causes of the error, student answer 5 (M5) is shown as below.

a. Understanding of pedagogic problems
Based on the problem can be known:
- The material already taught is the concept of cylinder volume
- The following material to be taught is the concept of cone volume
- It is recommended to use learning tools and learning models
- The learning process to be implemented is based on constructivism.

The question is:
What is the relation of constructivism theory to the concept of cylinder and cone volume.

According to the answer sheet, it is known that M5 has been able to identify what are known on the problem. However, M5 has not been able to identify what are being asked on the problem well. To find out the error, the results of interviews between researchers and M5 are shown below.

Researcher: According to your answers, you have been able to identify information on the problem. However, your explanation is quite short at the point of what is being asked on the problem. Why?
Student 5: Yes sir, I think that explanation already represents what is asked on the problem.
Researcher: Are you sure of your answer?
Student 5: I am sure that my answer is correct
Researcher: Please re-read the problem, I give you time for 7 minutes.
Student: ok sir
Researcher: After reading it, how do you think?
Student: My answer is correct sir, but there is one thing that must be added.
Researcher: What is it?
Student: The problem instructs to describe the initial, core, and closing activities.
Researcher: Why did not you write down this point at that time?
Student: I did not identify this point at that time sir. I was not careful to understand the problem.
Researcher: According to your answer sheet, you wrote in the point of what beeing asket “The relationship between the constructivism theory and the material of the cylinder and cone. Are you sure of your answer?
Student 5. I'm sure sir. Etc....

According to interview, it can be seen that the student misunderstood at the point of what being asked on the problem. Furthermore, after the researcher asked to re-read the problem, the student was aware of his mistake and able to fix it by himself. This is because the student has not been careful to understand the problem during the test.

After analyzing all student answer sheets, it can be concluded that there is a uniform error. They make mistakes in identifying and understanding what is being asked on the problem, because they do not read the problem carefully.

3.2. Error in Thinking Time: devising a plan
According to table 1, it can be concluded that there are 26% and 11% of students in medium and low category in devising a plan. This data shows that there are still some students who are wrong in making plans. In order to know the types of errors, the following answer sheet shown below.

a. Devising a plan
- Design a student worksheet based on constructivism.
- Using the PBL Model
- Develop a plan of learning based on PBL syntax
- Students will be grouped, 3-4 students per group.
- Prepare cylinder and cone objects where both types of objects have the same base area and height.
- In the learning process, the teacher will show cones and cylinders, then explain the formulas of each object.
- Teacher will facilitate students in experimenting.

The results of interview between lecturer and student 9 shown as follows.

Researcher: After checking your answer sheet, the result is good at point b.
Student: Thank you sir.
Researcher: However, do you only need cylinders and cones in experimenting?
Student: You are right Sir, I just need those two learning tools
Researcher: How the use of cylinders and cones in the learning process that you will do.
Student: I will show the cylinders and cones in front of the class. It is expected that they will recognize the two objects well.
Researcher: In addition, what are the cylinders and cones for? And what do you mean by the answer at the last point?
Student: I will give cylinder and cone to each group. Then I will ask them to disassemble these objects, so they will know the shape of skin both of objects.
Researcher: Then when do you plan to teach the concept of cone volume?
Student: After recognizing the shape of cylinder and cone well, then I will explain the concept of cone volume, and then give some problems related to it. Etc....

Based on the answer sheet and the results of the interview, it can be seen that the learning tool used in the learning process is incomplete. This is due to that the student does not understand how to use the learning in finding the concept of cone volume.

After analyzing the answer sheets and interview, there are several types of errors found as shown in Table 2.

| Types of errors                      | Causes of the errors                                      |
|-------------------------------------|-----------------------------------------------------------|
| They make mistake to plan a constructivism-based learning | They did not understand how to design a constructivism-based learning |
| They determine an inappropriate learning tool | Students did not know well what learning tool is appropriate to use |
| They demonstrate improperly the use of learning tools | They did not understand how to use the tool in finding the concept of cone volume |

3.3. Error in Insight: carrying out the plan

Table 1 shows that there are 32% and 26% of students in low and medium categories in carrying out problem solving. This data shows that there are still many students who make mistakes in that step. To find out the type of errors and its causes, the following answer sheet and interviews are shown.

b. Carry out the plan
Initial activity
Distribute student worksheets and learning tools, then explain the competencies to be achieved.
Core activities
The teacher will explain material: “if there are any cylinders and cones with the same base and height, then: cylinder volume = 3xthe volume of the cone, so it can be concluded that the cone volume = \( \frac{1}{3} \) volume of the cylinder = \( \frac{1}{3} \pi \text{r}^2 \text{t} \)”

After the students understand the concept, students are then instructed to do the following experiment: Fill mung beans into the cone until full. Then pour the contents of the cone into the cylinder. This activity is repeated until the volume of the cylinder is full. Based on the experiment, students are expected to understand that the cone volume is equal to \( \frac{1}{3} \) the volume of the cylinder. So
the formula is obtained: \( \text{cone volume} = \frac{1}{3} \text{volume of the cylinder} = \frac{1}{3} \pi^2 t \)

Closing Activity

The teacher and the students will conclude the results of the experiment.

Here are the results of interviews between lecturer and student 21.

Researcher: According to your answer sheet, you plan that the teacher will explain the material before the experiment. Why?

Student 21. I think if i do it, the experiments process can be run smoothly.

Researcher: May you explain what are the principles of constructivism-based learning?

Students: In essence: knowledge must be constructed by the students themselves

Researchers: do you believe by explaining the material at the beginning of the activity can stimulate the students to construct the knowledge by themselves?

Student: ohh i am sorry Sir. I just realized that my answer is wrong.

Researcher: Why do you say like that? What is wrong exactly?

Student: teachers should stimulate students to construct their own knowledge. The experiment should be done at the beginning of the activity. With such that, it is expected that they will find the concept of cone volume. Etc....

Based on the answer sheet and interview, it can be seen that the design of core activities does not conform to the principles of constructivism-based learning. It is recommended that the students should conduct an experiment in the beginning activity, so that they will be able to construct the knowledge by themselves.

After analyzing the data, the types of errors and causes are shown below.

Table 3. The types of errors and causes in carrying out the plan

| Type of error                        | Cause of the error                                                                 |
|--------------------------------------|-----------------------------------------------------------------------------------|
| Not implementing constructivist-based learning | They did not understand well how to design the learning process based constructivism |
| The use of learning tools is irrelevant to find the concept | They did not understand well how to use the learning tools to find the concept of cone |
| They make mistake in designing of core activity. | Not using a relevant model, so the learning process is not organized according to the principles of constructivist-based learning |

3.4. Error in verification: looking back

Table 1 shows that 35% of students still make a lot of mistakes in verification process. Here is the answer sheet of student 30 (M30).

d. look back.

Initial activities, core activities, and the closing have been described at the planning step. These three activities have been designed to be interrelated and structured based on constructivism-based learning.

The interview between researcher with student 30 shown below.

Researcher: Why do you check on the beginning, core, and closing activities only?

Student: I think by describing these three sections can already represent the answer point.

Researcher: Do you think by the explanation that means you have already checked your answer?

Student: I doubt sir.....etc

Researchers: Do you need to ensure that the learning process has applied the constructivism based-learning?

Student: Yes i do, because this point asked on the question.

Researcher: What else is needed to be explained in verification?

Student: I think it is important to check whether to find the cone formula out has been associated with the concept of cylinder volume. Etc....
After analyzing the data, the following types of errors and causes are shown below.

**Table 4.** The types of errors and causes in looking back

| Type of error                                                                 | Cause of the error                                                |
|--------------------------------------------------------------------------------|-------------------------------------------------------------------|
| The answer is still very common, so it is still not check the main point of the problem. | Students do not understand the technique and procedures to look back the answers. |
| The student's answer in the step c is not synchronized with point d (verification) | Students can not be able to synchronize the answer point d with the answer point c. |
| They did not check their answer completely                                      | They did not verify the all answer                                  |

4. **Conclusion**

Based on result and discussion, the type of student errors in solving pedagogic problem based on Polya procedure were: 1) the student made a mistake to answer what being asked at the problem, this is because they misuderstood at the problem, 2) they made a mistake to plan the learning process based on constructivism, due to lack of understanding of how to design the learning, 3) they chose an inappropriate learning tool, because they did not know what kind of learning tool is relevant to use, 4) They were wrong in sorting and organizing the learning process, because they did not yet understand how to use the learning model appropriately.

Based on the above conclusions, the researchers recommend to: 1) to improve the students’ ability to design the active learning, 2) to increase students’ ability to determine the appropriate learning tools, 3) improving students' ability to apply a learning model.

**References**

[1] Arikunto, S.2010. *Prosedur Penelitian Suatu Pendekatan*. (Yogyakarta: RinekaCipta). P317
[2] Ario M. 2016. *Analisis kemampuan penalaran matematis siswa SMK setelah mengikuti pembelajaran berbasis masalah*. Jurnal Ilmiah Edu Research Vol.5 No.2. Dec 2016 P125
[3] Creswell, John C. 2009. *Research design: Qualitative, quantitative, and mixed methods approaches, 3rd ed.* (Los Angeles: SAGE Publications, Inc.)
[4] Dewi I, Andriani A, Halomoan B. 2017. *Strategi Belajar Matematika Berbasis HOTS*: (Medan).
[5] Polya G. 1973. *How To Solve it: A New Aspect of Mathematical Method*. (New Jersey, USA: Pricenton University Press).
[6] Purwanto, (2009), *Evaluasi Pengajaran*. (Bandung: Rosdakarya). P.102
[7] Suherman E.  2003. *Strategi Pembelajaran Matematika Kontemporer*. (Bandung: Jurusan Pendidikan Matematika UPI).
[8] Wilson. 1993. *Mathematical Problem Solving*. (New York: Macmilan Publishing Company).