Performance evaluation of Nakagami-\(m\) fading with impulsive noise

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Abstract

The main motivation for considering noise to be Gaussian is the central limit theorem (CLT), which accounts for the perturbations that are additive in nature. However, a communication link may be severely affected due to the presence of potential non-Gaussian sources of noise. This paper considers an important class of non-Gaussian noise known as symmetric alpha-stable (\(S\alpha S\)) noise. To this end, using binary phase-shift keying (BPSK) modulation, the bit-error rate (BER) performance of a communication link subjected to Nakagami-\(m\) fading and \(S\alpha S\) noise is investigated by employing three approaches: exact, asymptotic and approximate. A closed-form expression for the probability of error over Nakagami-\(m\) fading subjected to bi-parameter Cauchy–Gaussian mixture noise (BCGM) model is obtained. The effect of fading parameter (\(m\)) and impulsive index (\(\alpha\)) on the BER is analyzed for different settings. The derived results corroborate with Monte Carlo simulations.

1 | INTRODUCTION

1.1 | Motivation

Fading and noise are two key phenomena that describe the statistical behaviour of a wireless channel. In case of fading, multiple copies of signal arrive at the receiver with different delay and phase, resulting in signal degradation. The noise which may result from man-made or natural phenomenon also affects the signal reception. For proper estimation and demodulation of signals, the behaviour of fading and noise should be properly accounted for [1]. As far as modelling of noise is concerned, Gaussian distribution has been extensively used in the technical literature. The main motivation for considering noise to be Gaussian is the central limit theorem (CLT), which accounts for the perturbations that are additive in nature. Moreover, due to simple analytical tractability of Gaussian noise, numerous tools and techniques have been proposed to evaluate the performance of a communication link subjected to Gaussian noise [2, 3]. However, in many practical scenarios (like urban and industrial scenario), a communication link is severely affected due to the presence of potential non-Gaussian sources of noise. In such scenarios, the Gaussian noise model may not be accurate [4].

Recent works on 5G [5], industrial millimetre-wave communication [5] and ultra-dense cellular networks [6] have taken into account the non-Gaussian nature of noise. In order to capture the non-Gaussian nature of noise, various non-Gaussian noise models have been reported in the literature [7, 8]. In this paper, an important class of non-Gaussian noise known as symmetric alpha-stable (\(S\alpha S\)) noise is considered. \(S\alpha S\) noise is a class of heavy-tailed stable distributions. It is a more general representation of additive white Gaussian noise (AWGN) since it includes Gaussian noise as a special case and satisfies generalized central limit theorem (GCLT) [9]. Furthermore, there are several reports that suggest that \(S\alpha S\) noise can effectively imitate a number of impulsive noise processes such as atmospheric noise [10], underwater acoustic noise [11] and noise in power line communications (PLC) [12]. Moreover, the statistical fitting reveals that electromagnetic interference (EMI) due to household appliances on the asymmetric digital subscriber line (ADSL) is well modelled by \(S\alpha S\) distribution [13]. In [14], \(S\alpha S\) distribution is used to model multiple-access interference (MAI) in a wireless ad hoc network. Based on the above reports, it can be concluded that \(S\alpha S\) noise is a more general and realistic representation of noise than the classical AWGN. Hence, by the introduction of such a noise model into the communication...
systems, a practical insight of a typical communication link can be acquired.

The other channel impairment considered in this paper is Nakagami-$m$ fading [1]. The main motivation for selecting this fading model is:

(a) It represents small-scale channel variations effectively.
(b) It includes Rayleigh fading and one-sided Gaussian distribution as special cases.
(c) To the best of our knowledge, the bit-error rate (BER) performance of Nakagami-$m$ fading with $\alpha S$ noise is not explored much and thus is the topic of research in this article.

In a practical wireless scenario, both fading and impulsive noise may affect the communication link at the same time and independently. The combined effect of fading and impulsive noise can be used to study the behaviour of the industrial wireless sensor network (IWSN). In industrial applications, various monitoring and control operations demand low BER. Proper modelling of the channel statistics and impulsive noise in an IWSN, the re-transmission rate can be reduced, which will subsequently reduce energy consumption and end-to-end latency.

In an industrial environment, the movement of large objects and highly reflective metal surfaces lead to multipath propagation that can be effectively captured by Nakagami-$m$ fading distribution [15, 16]. Another major challenge in an industrial environment is the impulsive noise generated by different machines. One of the important class of non-Gaussian noise model that effectively captures the impulsive noise in an IWSN scenario is the $\alpha S$ noise [17–19]. Due to frequent movement of machines (like cranes, forklifts etc.) and the impulsive noise generated by various machines operating in an IWSN scenario, the communication link under consideration may be subjected to the low, moderate and high intensity of fading and impulsive noise. Hence, the reliability of such a communication link varies with time [20]. In this work, the variation in channel dynamics is captured by the fading parameter ($m$) while the intensity of non-Gaussian noise is captured by the impulsive index ($\alpha$). The communication link under consideration is simulated for different values of $m$ and $\alpha$ to imitate an IWSN scenario that may be subjected to the low, moderate and high intensity of fading and impulsive noise.

Recently, the BER performance of various fading channels with different types of impulsive noise has been studied. In [21], the BER performance of BPSK over generalized Laplacian (GL) noise is evaluated. The error performance of the square $M$-ary quadrature amplitude modulation (MQAM) scheme subjected to different fading channels and multilevel double-gated additive white Gaussian noise (G$^2$AWGN) is addressed in [22, 23]. However, the BER analysis of Nakagami-$m$ fading and $\alpha S$ noise has not been addressed in the technical literature and thus is the topic of research in this article.

In the next subsection, a brief introduction of impulsive noise and its models is presented.

### 1.2 Impulsive noise and its models

In order to take into account the non-Gaussian nature of noise, various statistical models of noise have been reported in the literature. These include generalized Gaussian noise (GGN), Gaussian mixture noise (GMN) [24], Middleton Class A noise [25, 26] and Bernoulli Gaussian noise model [27]. Mathematically, impulsive noise can be represented as [8]

$$n(t) = \sum_k A_k P_{wk}(t - \tau_k),$$

where the amplitude $A_k$, pulse width $P_{wk}$ and inter-arrival time $\tau_k$ are random in nature, whose distribution is not a priori known. A simple model of impulsive noise that displays the aforementioned random parameters is shown in Figure 1.

### 1.3 Contribution of the work

The main contributions of this work are summarized as follows:

- BER analysis of BPSK modulation scheme subjected to $\alpha S$ noise is presented.
- BER analysis of Nakagami-$m$ fading channel with $\alpha S$ noise is presented using exact, asymptotic and approximate analysis.
- Closed-form expression of BER for Nakagami-$m$ fading with bi-parameter Cauchy–Gaussian mixture (BCGM) noise model is obtained.
- A comparison of asymptotic and approximate analysis with exact analysis is also presented.

The rest of the paper is organized as follows: In Section 2, the system model, $\alpha S$ noise and its approximate models are presented. In section 3, the Geometric signal-to-noise ratio (SNR) and the BER performance of BPSK modulation scheme with $\alpha S$ noise is presented. The performance of Nakagami-$m$ fading subjected to $\alpha S$ noise using different approaches is given in Section 4. Simulation results and useful concluding remarks are included in Sections 5 and 6, respectively.

### 2 SYSTEM MODEL

Mathematically, the received signal contaminated with $\alpha S$ noise can be expressed as

$$r[n] = bx[n] + z[n],$$

where $n$ is the time index. The transmitted symbol $x[n]$ is modulated with BPSK and Gray coding [28, 29]. $z[n]$ is the complex $\alpha S$ noise whose real and imaginary components are independent and identically distributed (i.i.d). $b$ represents the Nakagami-$m$ fading coefficient with slowly varying flat fading characteristics. The probability density function (PDF) of
Nakagami-\(m\) fading \cite{1} in terms of instantaneous SNR (\(\omega\)) can be expressed as

\[
p(\omega; \Omega) = \frac{m^m \omega^{m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\omega}{\Omega}\right),
\]

where \(\Omega = \frac{E_b}{N_0}\) and \(\Gamma(\cdot)\) is the Gamma function \cite{30, equation (8.310.1)}. The amount of fading (AF), which is defined as the measure of severity of fading, for Nakagami-\(m\) fading channel is given as

\[
AF = \frac{1}{m},
\]

where \(m\) is the Nakagami-\(m\) fading parameter that ranges from \(1/2\) to \(\infty\). For \(m = 1/2\) and \(m = 1\), (3) corresponds to one-sided Gaussian and Rayleigh distribution respectively.

### 2.1 S\(\alpha\)S noise and its approximate models

The PDF of S\(\alpha\)S noise is given by \cite{31}

\[
f_\alpha(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(j\mu t - \gamma^2 |t|^\alpha \right) \exp\left(-j|z|t\right) dt,
\]

where random variable \(z\) follows S\(\alpha\)S distribution. The physical meaning of various parameters is given in Table 1. The PDF of S\(\alpha\)S noise for different values of \(\alpha\) is shown in Figure 2. From this figure, it is clear that impulsive behaviour of S\(\alpha\)S noise increases with decrease in \(\alpha\), where \(\alpha \in (0, 2]\). The exaggerated view shown in Figure 2 signifies the heavy-tailed behaviour of S\(\alpha\)S noise, i.e. the tail of the PDF of S\(\alpha\)S noise decreases gradually. Equation (5) does not have any closed-form expression except for \(\alpha = 1\) and \(\alpha = 2\). When \(\alpha = 1\) and \(\alpha = 2\), S\(\alpha\)S distribution corresponds to Cauchy and Gaussian distribution, respectively.

### 2.2 Approximate model: Bi-parameter Cauchy–Gaussian mixture noise model

Equation (5) does not have a closed-form expression except for \(\alpha = 1\) and \(\alpha = 2\), hence Equation (5) cannot be used for real-time applications due to intensive computations. Although the power series expansion of S\(\alpha\)S distribution given in \cite{32} fits the S\(\alpha\)S curve around the origin and tail, the intermediate portion is not traced accurately. Furthermore, the Gaussian mixture model \cite{33} does not fit the S\(\alpha\)S model with a small number of components (\(N\)). Hence a tri-parameter (\(\epsilon, \sigma^2, \gamma\)) noise model, known as Cauchy–Gaussian mixture (CGM), was proposed to model the S\(\alpha\)S noise. The PDF of such a noise model is given as \cite{34}

\[
f_{\text{CGM}}(w) = (1 - \epsilon) f_G(w) + \epsilon f_C(w)
\]

\[
= \frac{(1 - \epsilon)}{2\sqrt{\pi \sigma^2}} \exp\left(-\frac{w^2}{2\sigma^2}\right) + \frac{\epsilon \gamma}{\pi \left(w^2 + \gamma^2\right)},
\]

where \(\epsilon \in [0, 1]\) is the mixture ratio, \(\sigma^2\) is the Gaussian variance and \(\gamma\) is the Cauchy parameter. The estimation of these three parameters increases the computational complexity. In order to reduce this computational complexity, a two-parameter (\(\epsilon, \gamma\)) noise model, known as bi-parameter Cauchy–Gaussian mixture (BCGM), was proposed in \cite{35}. Moreover, the KL divergence of BCGM model and S\(\alpha\)S distribution is less than CGM model and S\(\alpha\)S distribution. Hence, the BCGM noise model is a more...
appropriate representation of SaS noise. However, this noise model is valid for $\alpha \in [1, 2]$. The PDF of BCGM noise model is given as [35]

$$f_{BCGM}(w) = (1 - \varepsilon) f_C(w) + \varepsilon f_C(w)$$

$$= (1 - \varepsilon) \frac{1}{2\sqrt{\pi\gamma^2}} \exp\left(-\frac{w^2}{4\gamma^2}\right) + \frac{\varepsilon \gamma}{\pi (w^2 + \gamma^2)}, \quad (7)$$

where $\varepsilon \in [0, 1]$ is the mixture ratio, $\gamma$ is the dispersion parameter of SaS distribution. For $\varepsilon = 1$, Equation (7) reduces to Cauchy distribution and it results in Gaussian distribution for $\varepsilon = 0$. Various efforts have been made to establish a relationship between $\varepsilon$ and $\alpha$. McCulloch proposed a linear relation between $\varepsilon$ and $\alpha$, which is given as [36]

$$\varepsilon = 2 - \alpha. \quad (8)$$

Logarithm moment (ML) and fractional lower order moment (FLOM) are also used in the literature to establish the relation between $\varepsilon$ and $\alpha$, as given in (9) and (10), respectively [37]:

$$\varepsilon = \frac{4 - \alpha^2}{3\alpha^2} \quad (9)$$

$$\varepsilon = \frac{2\Gamma(-p/\alpha) - \alpha\Gamma(-p/2)}{2\alpha\Gamma(-p) - \alpha\Gamma(-p/2)}, \quad (10)$$

where $\Gamma(\cdot)$ is the Gamma function and $p$ is the fractional order moment where $p < \alpha$. Among all the three relations of $\alpha$ and $\varepsilon$ given above, Equation (10) gives the best fit when compared to maximum likelihood (ML) estimate as the benchmark [38]. Hence, Equation (10) has been employed in our analysis.

### 3 GEOMETRIC SNR

Since the second-order moment of SaS is infinite, the traditional definition of SNR is meaningless. This problem is solved by defining geometrical signal-to-noise ratio (GSNR), which is based on zero-order statistics (ZOS). For a logarithmic order random variable $Y$, the geometric power is defined as [39]

$$S_\varepsilon = S(Y) = e^{E[\log|Y|]}, \quad (11)$$

where $E(\cdot)$ denotes expectation operator. The geometrical power of SaS variable is defined as [40]

$$S_\varepsilon = \frac{\gamma (C_{\varepsilon} \gamma^{-1/2})}{C_{\varepsilon}}, \quad (12)$$

where $C_\gamma \approx 1.78$ is the exponential of the Euler’s constant. The geometrical signal-to-noise ratio ($SNRG$) for a signal which is amplitude modulated in the presence of additive noise is given as [39]

$$SNRG = \frac{1}{2C_{\varepsilon}} \left(\frac{A}{\sqrt{\gamma}}\right)^2, \quad (13)$$

where $A$ is the amplitude of the modulated signal. The normalization constant ($1/C_\gamma$) ensures that $SNRG$ corresponds to Gaussian noise when $\alpha = 2$. For a coded system with BPSK modulation, the $SNRG$ and code rate $R_c$ is given as

$$E_b/N_0 = \frac{SNRG}{2R_c} = \frac{1}{4R_c C_{\varepsilon}} \left(\frac{A}{\sqrt{\gamma}}\right)^2. \quad (14)$$

The $Q$-function for a channel with AWGN is given as [41]

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_{y}^{\infty} \exp\left(-\frac{x^2}{2}\right)dx. \quad (15)$$

The $Q$-function for SaS noise is defined as

$$Q_{\alpha}(\delta) = \int_{\delta}^{\infty} f_\alpha(\tau; 0, 1) d\tau \quad (16)$$

where $f_\alpha(\tau; 0, 1)$ is a standard SaS distribution with $\delta = 0$ and $\gamma = 1$. In [42], a consistent mapping has been established between $Q(y)$ and $Q_{\alpha}(\delta)$ for all values of $\alpha$, i.e. $\alpha \in (0, 2]$. Such a mapping is given as

$$Q(y) \rightarrow Q_{\alpha}(\sqrt{2C_\gamma^{-1}}) \quad (17)$$

Based on this mapping, the probability of error of various modulation schemes in Gaussian and SaS noise is given in Table 2. For $\alpha = 2$, the above mapping reduces to

$$Q(y) = Q_{\alpha}(\sqrt{2}) \quad (18)$$

Using the above mapping, the probability of error for AWGN and SaS noise for BPSK modulation can be expressed as

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q_{\alpha}\left(\sqrt{\frac{2E_b}{N_0}}\right). \quad (19)$$

Figure 3 shows the BER of BPSK modulation scheme in the presence of SaS noise. From this figure, it is clear that BER increases significantly as $\alpha$ approaches 0.
4 Performance evaluation of Nakagami-\(m\) fading with impulsive noise

Based on the analytical tractability, three approaches have been employed to evaluate the BER performance of Nakagami-\(m\) fading with \(S\alpha S\) noise. These are:

- Exact analysis.
- Asymptotic analysis.
- Approximate analysis.

4.1 Exact analysis

The probability of error for a coherent modulation scheme in \(S\alpha S\) noise can be expressed as

\[
P_{\text{ber}} = Q_{\alpha}(\sqrt{A}) \quad (20)
\]

where the constant \(A = 4C_{\alpha}^{\frac{\alpha-1}{\alpha}}\). Using BPSK modulation, the average probability of error over Nakagami-\(m\) fading channel with \(S\alpha S\) noise \((P_{\text{e}}^N)\) can be expressed as

\[
P_{\text{e}}^N = \int_{0}^{\infty} Q_{\alpha}(\sqrt{4C_{\alpha}^{\frac{\alpha-1}{\alpha}}} \omega) \rho(\omega; \Omega) d\omega \quad (21)
\]

where \(Q_{\alpha}(x)\) is defined in (16) and \(\rho(\omega; \Omega)\) is the PDF of Nakagami-\(m\) fading channel. The \(Q_{\alpha}(y)\) can be expressed as [31, 43]

\[
Q_{\alpha}(y) = \frac{1}{2} \tan^{-1}\left(\frac{y}{\pi}\right)
\]

when \(\alpha \neq 1\),

\[
Q_{\alpha}(y) = c_1 + \frac{\text{sign}(\alpha - 1)}{\pi} \int_{0}^{\frac{\pi}{2}} \exp\left(-\frac{y}{\pi} V(\theta; \alpha)\right) d\theta,
\]

where

\[
c_1 = \begin{cases} 
1, & \alpha < 1 \\
0, & \alpha > 1 
\end{cases}
\]

and

\[
V(\theta; \alpha) = \frac{\cos \theta}{\sin \alpha \theta} + \frac{\alpha - 1}{\alpha - 1} \frac{\cos(\alpha - 1)\theta}{\cos \theta}.
\]

Special Case: For \(\alpha = 2\), Equation (22) reduces to Gaussian noise as shown below:

\[
Q_{\alpha=2}(y) = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} \exp\left(-\frac{y^2}{4 \sin^2 \theta}\right) d\theta
\]

\[
= Q\left(\frac{y}{\sqrt{2}}\right) = \frac{1}{2} \text{erfc}\left(\frac{y}{\sqrt{2}}\right).
\]

Equation (21) cannot be expressed in a closed form. However, the effect of impulsive index (\(\alpha\)) and fading parameter (\(m\)) on the exact BER is shown in Figures 4–7. From these figures following conclusions can be drawn:

(i) As \(\alpha\) approaches 0, BER increases significantly. This is attributed to the fact that as \(\alpha\) approaches 0, the impulsive behaviour of the \(S\alpha S\) noise increases.
(ii) An increase in the value of $m$ leads to a decrease in BER as the severity of fading decreases.

(iii) As shown in Figure 7, changing the amount of fading $(AF = 1/m)$ for a fixed value of impulsive index ($\alpha = 1$) does not make much effect on the BER. This implies that effect of impulsiveness is more significant than the severity of fading.

### 4.2 Asymptotic analysis

Equation (21) involves a double integral and to the best of our knowledge no closed-form solution is available in the literature. However, using the asymptotic approach, this double integral can be reduced to a single integral as explained below.

The general expression for the probability of error in Nakagami-$m$ fading channel subjected to $\mathcal{S\alpha S}$ can be expressed as

$$P_b \rightarrow \frac{C_{\alpha}}{\Gamma(m)} \frac{m^w}{\Omega^w} \int_0^{\infty} \frac{\omega^m}{\Gamma(m)} \exp \left[ -\frac{m^w \omega}{\Omega^w} \right] d\omega,$$  

(26)

where the constant $A = 4C_{\alpha}^{-\alpha}$, $\Gamma(m) = \alpha^m \int_0^{\infty} t^{m-1} e^{-\alpha t} dt$ [30]. The asymptotic property of $\mathcal{S\alpha S}$ random variable $X$, is given as [9]

$$\lim_{x \to \infty} P(X > x) = \frac{C_{\alpha} x^{-\alpha}}{C_{\alpha}},$$  

(27)

where

$$C_{\alpha} = \frac{1}{\pi} \Gamma(\alpha) \sin(\pi \alpha / 2).$$  

(28)

For a standard $\mathcal{S\alpha S}$ random variable, (27) can be expressed as

$$\lim_{x \to \infty} Q_{\alpha}(x) = \frac{C_{\alpha}}{x^\alpha}.$$  

(29)

Using (29) in (26), we get

$$P_b \rightarrow \frac{m^w}{\Omega^w} \int_0^{\infty} \frac{\omega^m}{\Gamma(m)} \exp \left[ -\frac{m^w \omega}{\Omega^w} \right] d\omega.$$  

(30)

Capitalizing on Gamma function, the closed-form expression of (30) can be expressed as

$$P_b \rightarrow \frac{C_{\alpha}}{\Gamma(m)} \left( \frac{m^w}{\Omega^w A} \right)^{\alpha/2} \Gamma(m - \alpha/2).$$  

(31)
The operation $\rightarrow$ denotes asymptotic behaviour and the Gamma function is defined as $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ [30]. The special cases of (31) are given below:

1. For $m = 1$, (31) corresponds to Rayleigh fading distribution. It is given as

$$p^N_b \rightarrow \frac{C_\alpha}{(\Omega A)^{\alpha/2}} \Gamma(1 - \alpha/2).$$ (32)

2. For $m = 1/2$, (31) corresponds to one sided Gaussian distribution. It is given as

$$p^N_b \rightarrow \frac{C_\alpha}{\Gamma(1/2)(\sqrt{2\Omega A})^{\alpha/2}} \Gamma(1/2 - \alpha/2).$$ (33)

The effect of impulsive index ($\alpha$) and fading parameter ($m$) on the asymptotic BER is shown in Figure 8. From this figure, it is clear that for a fixed fading parameter ($m$), BER increases with decrease in $\alpha$, where $\alpha \in (0, 2]$.

### 4.3 Approximate analysis

The PDF of SâS noise given in (7) involves intensive computations, hence to reduce such computations, an approximate model of SâS noise, known as BCGM noise model, is employed. The probability of error over Nakagami-$m$ fading under the influence of BCGM noise model can be expressed as

$$P^N_{b,BCGM} = \int_0^\infty \int_0^\infty \mathcal{B}_{BCGM}(w) \frac{\omega^{m-1}m^m}{\Omega^m \Gamma(m)} \right) dw d\omega.$$ (34)

where $\mathcal{B}_{BCGM}$ is the PDF of the standard BCGM noise model. Therefore, (34) can be expressed as

$$P^N_{b,BCGM} = \int_0^\infty \int_0^\infty \frac{(1 - \varepsilon)}{2\sqrt{\omega}} \exp\left(-\frac{\omega^2}{4}\right)$$

$$+ \frac{\varepsilon}{\pi (\omega^2 + 1)} \left\{ \frac{\omega^{m-1}m^m}{\Omega^m \Gamma(m)} \exp\left(-\frac{\omega^2}{\Omega}\right) \right\} dw d\omega.$$ (35)

As shown in the appendix, the closed form of (35) can be expressed as

$$P^N_{b,BCGM} \approx \frac{m^m (1 - \varepsilon)}{2\omega^m} \left( \frac{A}{2} + \frac{m}{\Omega} \right)^{-m}$$

$$+ \frac{\varepsilon}{\pi} \left[ \frac{m^m \varepsilon a_1}{\Omega^m \pi} \left( \frac{A}{a_2} + \frac{m}{\Omega} \right)^{-m} - \frac{\varepsilon a_3}{\pi} \right].$$ (36)

The effect of impulsive index ($\alpha$) and fading parameter ($m$) on the approximate BER as shown in Figure 9–10 is explained in the next section.

### 5 Simulation results

In this section, the effect of the impulsive index ($\alpha$) and the fading parameter ($m$) on the BER is studied. As shown in Figure 3, when $\alpha$ is slightly perturbed from 2 to 1.99, the SNR penalty to maintain the same BER ($10^{-5}$) is $\approx 23.5$ dB. For a BER of the order of $10^{-3}$, it is observed from Figure 4 that as $\alpha$ is perturbed from Gaussian noise ($\alpha = 2$) to SâS noise ($\alpha = 1.5$), the SNR penalty with fading parameter $m = 2$ is $\approx 12.5$ dB. This SNR penalty increases further as $\alpha$ attains lower values (close to 0).

Moreover, it is observed that for higher values of $m$, the amount of fading ($AF = 1/m$) decreases, which subsequently improves the BER performance. On the other hand, for a fixed value of $\alpha = 1$, the BER variation due to fading parameter ($m$) is shown in Figure 5. It is observed that BER variation due to impulsive index ($\alpha$) is much pronounced than fading parameter ($m$). Thus, our results imitate a realistic communication link, which may be subjected to low, moderate and high intensity of fading and impulsive noise. The asymptotic and approximate BER curves given in Figures 6–7 follow the same behaviour as explained above. However, asymptotic and approximate BER analysis involve lesser computations as compared to the exact analysis.

For $\alpha = 2$, the exact and approximate BER curves shown in Figure 6 follow each other closely. There is a small deviation between exact and approximate BER curves for $\alpha = 1$. Such a deviation between the exact and approximate BER curves arises because (a) bi-parameter Cauchy noise model is an approximate representation of SâS noise and (b) this deviation also depends upon the mixture ratio ($\varepsilon$). However, as shown in Figure 7, the exact and asymptotic BER curves follow each other closely in the high SNR regime. By exploiting the mapping of Nakagami-$m$ fading parameter with the fading parameters $q$ and $n$, the performance of Nakagami-$q$ and Nakagami-$n$ fading can be also analyzed for different values of $m$, where $m \geq \frac{1}{2}$.

### 5.1 Impact of different noise models on the system performance

In technical literature, various fading and impulsive noise models have been studied to evaluate the performance of an IWSN link. In [44], an IWSN link is modelled with generalized-K fading and Middleton's class A noise. The effect of the impulsive index ($A$) and the fading parameter on the BER is studied. With $A = 10^{-3}$, multi-path fading parameter ($m = 2.5$), shadowing parameter ($m = 2$) and average SNR ($\vartheta \approx 22$ dB), the corresponding BER of the order of $10^{-3}$ is achieved. The same order of BER is achieved in our work with $m = 2$, average SNR ($\vartheta \approx 27$ dB) and $\alpha = 1.5$ as shown in Figure 4. In
employing M-QAM modulation, the performance of an IWSN scenario with Nakagami-\(m\) fading and double-gated additive white Gaussian noise (G2AWGN) is evaluated. With \(m = 2\),
signal-to-impulsive noise ratio (\(\delta_i = 20\) dB) and background
noise (\(\delta_b \approx 22.5\) dB), the BER of the order of 10^{-3} is attained. Further, it is shown that BER decreases as the fading parameter (\(m\)) attains higher values. In [45], the performance of an IWSN with multi-hop relaying was evaluated over Nakagami-\(m\) fading channel and Middleton’s class \(A\) noise model. With
\(m = 1, A = 10^{-3}\) and average 3\(\text{SNR per link (}\gamma \approx 8\text{ dB})\), approximately an end-to-end average BER of the order of 10^{-1} was obtained.

6 **CONCLUSION**

In this paper, the effect of impulsive noise and Nakagami-\(m\) fading on the BER performance of a communication link is studied using three approaches: exact, approximate and asymptotic. Closed-form expressions of BER are obtained using approximate and asymptotic analysis. The derived expressions can be used to study the BER performance of other fading models (like Rayleigh, Nakagami-\(g\) and Nakagami-\(m\)) available in the technical literature. The derived results are validated through Monte Carlo simulations. The results reveal that BER increases significantly as impulsive index (\(\alpha\)) approaches to 0. From the interplay between the impulsive index (\(\alpha\)) and fading parameter (\(m\)), it is concluded that both impulsive index (\(\alpha\)) and fading parameter (\(m\)) dictate the BER performance of a communication link.

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and

\[ I_2 = \int_0^\infty \int_0^\infty \frac{\varepsilon}{\pi (n^2 + 1)} \times \frac{\omega^{n-1} m^m}{\Omega^n \Gamma(m)} \exp \left(-\frac{m\omega}{\Omega}\right) d\omega d\varepsilon. \]  

(A.3)

On substituting \( w = \sqrt{2t} \) in \((A.2)\), we get

\[ I_1 = \frac{m^m(1 - \varepsilon)}{\Omega^n \Gamma(m)} \int_0^\infty \int_0^\infty \frac{1}{\sqrt{\omega^2/2}} \exp \left(-\frac{\omega^2}{2} - \varepsilon \right) dt \times \omega^{n-1} \exp \left(-\frac{m\omega}{\Omega}\right) d\omega. \]  

(A.4)

Since \( Q(y) \) is defined as \( \int_y^\infty \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2}) \, dx \) [41], Equation \((A.4)\) can be expressed as

\[ I_1 = \frac{m^m(1 - \varepsilon)}{\Omega^n \Gamma(m)} \int_0^\infty \int_0^\infty Q(\sqrt{-\omega^2/2}) \times \omega^{n-1} \exp \left(-\frac{m\omega}{\Omega}\right) d\omega dt. \]  

(A.5)

The upper Chernoff bound \( Q(y) \) is given as \[46\]

\[ Q(y) \approx \frac{1}{2} e^{-y^2} \cdot y \geq 0. \]  

(A.6)

Using Chernoff bound, \((A.5)\) can be expressed as

\[ I_1 \approx \frac{m^m(1 - \varepsilon)}{2 \Omega^n \Gamma(m)} \int_0^\infty \omega^{n-1} \exp \left(-\omega \left(\frac{A}{2} + \frac{m}{\Omega}\right)\right) d\omega. \]  

(A.7)

Using Gamma function \[30\], \( \Gamma(m) = \alpha^m \int_0^\infty (\alpha^{m-1}) e^{-\alpha} \, d\alpha \), \((A.7)\) can be expressed as

\[ I_1 = \frac{m^m(1 - \varepsilon)}{2 \Omega^n} \left(\frac{A}{2} + \frac{m}{\Omega}\right)^{-m}. \]  

(A.8)

From \((A.3)\), we have

\[ I_2 = \int_0^\infty \int_0^\infty \frac{\varepsilon}{\pi (n^2 + 1)} \times \frac{\omega^{n-1} m^m}{\Omega^n \Gamma(m)} \exp \left(-\frac{m\omega}{\Omega}\right) d\omega d\varepsilon. \]  

(A.9)

By performing simple mathematical manipulation, the above equation can be expressed as

\[ I_2 = \frac{\varepsilon}{2} \frac{m^m e}{\Omega^n \Gamma(m) \pi} \int_0^\infty \tan^{-1}\left(\sqrt{\alpha}\right) \, d\alpha. \]
\[ x \omega^{-1} \exp \left[ -\frac{m \omega}{\Omega} \right] \, d\omega. \quad (A.10) \]

Now \( I_2 \) can be expressed as

\[ I_2 = \frac{\epsilon}{2} - I_3, \quad (A.11) \]

where

\[ I_3 = \frac{m^\prime \epsilon}{\Omega \pi \Gamma(m) \pi} \int_0^\infty \tan^{-1} \left( \sqrt{A \omega} \right) \times \omega^{-1} \exp \left[ -\frac{m \omega}{\Omega} \right] \, d\omega. \quad (A.12) \]

Using curve fitting, \( \tan^{-1} \left( \sqrt{x} \right) \) can be approximated as

\[ \tan^{-1} \left( \sqrt{x} \right) \approx a_1 \exp \left( -\frac{x}{a_2} \right) + a_3, \quad (A.13) \]

where \( a_1 = -1.48525, a_2 = 4.61577 \) and \( a_3 = 1.51474 \). Therefore, \((A.12)\) can be expressed as

\[ I_3 \approx \frac{m^\prime \epsilon}{\Omega \pi \Gamma(m) \pi} \int_0^\infty \left\{ a_1 \exp \left( -\frac{A \omega}{a_2} \right) + a_3 \right\} \times \omega^{-1} \exp \left[ -\frac{m \omega}{\Omega} \right] \, d\omega. \quad (A.14) \]

Employing Gamma function, the above equation can be expressed as

\[ I_3 \approx \frac{m^\prime \epsilon}{\Omega \pi \Gamma(m) \pi} \left\{ \frac{A}{a_2} + \frac{m}{\Omega} \right\}^{-m} + \frac{\epsilon a_3}{\pi}. \quad (A.15) \]

On substituting \( I_3 \) in \((A.11)\), we get

\[ I_2 \approx \frac{\epsilon}{2} - \frac{m^\prime \epsilon}{\Omega \pi \Gamma(m) \pi} \left\{ \frac{A}{a_2} + \frac{m}{\Omega} \right\}^{-m} - \frac{\epsilon a_3}{\pi}. \quad (A.16) \]

On substituting \((A.8)\) and \((A.16)\) in \((A.1)\), we get

\[ p_{\nu,BCGM}^V \approx \frac{m^\prime}{2\Omega \pi} (1 - \epsilon) \left( \frac{A}{2} + \frac{m}{\Omega} \right)^{-m} \]

\[ + \frac{\epsilon}{2} - \frac{m^\prime \epsilon}{\Omega \pi} \left( \frac{1}{a_2} + \frac{m}{\Omega} \right)^{-m} - \frac{\epsilon a_3}{\pi}. \quad (A.17) \]