The spreading of a liquid on a solid is known to be strongly influenced by chemical inhomogeneities and roughness of the solid surface \[\gamma_0\]. Different situations arise for the motion of the co-existence line between a solid, a liquid and a gas phase (called the triple line or the contact line) depending on whether the liquid completely wets the solid surface \[\gamma_1\] or incompletely wets the solid \[\gamma_2\]. In the case of incomplete wetting, which is mainly determined by the ratio between the liquid-vapor surface energy, and the strength of the pinning potential \[\gamma_{LG}\] of the liquid-vapor interface. The case of a Gaussian pinning force is solved by a geometrical construction. We show that the hysteresis then has an additional dependence on the gravitational force determined by the weight of the liquid meniscus \[\rho g V\] along the fiber. The volume \[V\] of the liquid meniscus is given by an integration over Eq. (2) as,

\[A_p^\prime \Delta y \delta(x) \delta(y) + A_p^0 \delta(x)\]

where \[\gamma_{LG}\] is the liquid-gas surface energy, \[A_p\] the strength of the pinning potential of the fiber defect and \[A_p^0\] the strength of the pinning potential of the plate \[\gamma_{LG}\]. \[\rho\] is the mass density of the liquid, \[g\] the gravity constant and \[\Delta y\] the width of the fiber of defects. Solving Eq. (1) in the limit \(\nabla h^2 \ll 1\) gives:

\[h(x, y) = \frac{A_p^\prime \Delta y}{2\pi \gamma_{LG}} K_0(\frac{\sqrt{2} \sqrt{x^2 + y^2}}{l_c}) + \frac{A_p^0 l_c}{2\sqrt{2} \gamma_{LG}} \exp(-\frac{\sqrt{2} x}{l_c})\]

with \(l_c = \sqrt{\frac{2\gamma_{LG}}{\rho g}}\) the capillary length and \(K_0\) the zeroth-order modified Bessel function.

We consider first the case without plate-liquid interaction, \(A_p^0 = 0\). In this case the problem is rotational symmetric around the axis of the fiber. The force per unit length (for a given width \(\Delta y\)) needed to lift the liquid the distance \(h(0, \Delta y)\) along the fiber is determined by Eq. (2) as,

\[A_p^\prime = \frac{h(0, \Delta y) 2\pi \gamma_{LG}}{\Delta y K_0(\frac{\Delta y \sqrt{2}}{l_c})}\]

Dispite the fact that Eq. (3) is obtained for a fiber of width \(\Delta y\), the equation also describes the case of a defect of width \(\Delta y\) and zero extension in the \(z\)-direction. This is because the part of the fiber which is above/below the triple line does not enter in the force balance of the triple line described by \(h(x, y)\). The pinning force is in balance with the gravitational force determined by the weight of the liquid meniscus \(F_g = \rho g V\). The volume \(V\) of the liquid meniscus is given by an integration over Eq. (2) with the assumed \(A_p\) from Eq. (3). One finds...
where we introduced the effective elasticity constant $k$. Assuming the pinning force is Gaussian, with it’s center $z_d$ above $z = 0$,

$$F_p(z) = A_p \Delta y \exp \left(-[(z - z_d)/R]^2\right),$$

one can like [1] determine the critical amplitude $A_p^c$ for onset of hysteresis by equating the maximum slope of $F_p(z)$ to the slope of $F_y(z)$:

$$A_p^c = \frac{\sqrt{2\pi \gamma_{LG} R}}{\Delta y K_0(\sqrt{2\pi \gamma_{LG} R})}$$

Experimentally, hysteresis is defined as the difference between advancing and receding work done on the contact line. We define accordingly the hysteresis $H$ by,

$$H = \int_{-\infty}^{+\infty} F_p(z_d)dz_d + \int_{+\infty}^{+\infty} F_p(z_d)dz_d$$

where $z_d(z_d)$ describes the dependence of the position of the trip line as a function of the position of the defect $z_d$. Notice $H$ can be different from 0 since $F_p$ is not necessarily a single valued function of $z_d$. This is due to discontinuities in the function $z_d(z_d)$ when $A_p > A_p^c$. As depicted in Fig. 1b, $z_d(z_d)$ undergoes a jump from position $z_1$ to $z_2$ for decreasing $z_d$ and a jump from $z_3$ to $z_4$ for increasing $z_d$. The positions $z_1, z_3$ are determined by the condition $\frac{dF_p(z)}{dz} = k$ and the positions $z_2, z_4$ are determined by the condition $kz = F_p(z)$. The integrals in Eq. (7) can be transformed into integrals over $z_d$ by use of the Jacobian $dF_p/dz_d = (k - dF_p/dz_d)/k$ [2] and we can express the hysteresis in terms of the jump positions $z_n$, $n = 1, \ldots, 4$,

$$H = \sum_{n=1}^{4} (-1)^{n+1} \frac{1}{2k} F_p^2(z_n) + \int_{z_3}^{z_2} F_p(z)dz - \int_{z_4}^{z_1} F_p(z)dz.$$

In this case Eq. (8) leads to a hysteresis given by

$$H(\gamma, \rho g, \Delta y, R) = \frac{A_p^{c} \Delta y (1 + \frac{3R^2C}{4} + \frac{C^2R^3}{2})}{2} - \frac{3C^3R^4}{32} + \frac{R^4}{12}(C^2 + 4R^2 - 4C/R)^{3/2}$$

where $C \equiv k/(A_p \Delta y)$. The effect of the gravitational field enters through $k$ in $C$. Since the zero order Bessel function $K_0(x)$ depends logarithmically upon $x$ for small $x$ we conclude that when $\Delta y \ll l_c$ the hysteresis only depends logarithmically upon the acceleration of gravity, i.e., a very weak dependence. In this case the extension of the defect $(\Delta y, R)$ becomes the relevant length scale. Notice that this is in strong contrast to the common belief that the capillary length is the determining length scale in contact angle hysteresis. Contrary, when the defect is spatially extended (e.g. in the case of a wall with $A_p^{c} \neq 0$), we show below that the hysteresis depends on powers of $l_c$, i.e. a strong dependence of gravity.

Like the Gaussian case we have plotted the hysteresis for the parabolic force in Fig. 2 versus $\frac{\Delta y}{R}$. One notices that the two different pinning forces give rise to same the $1/k$ divergence for a weak elastic constant $\frac{1}{k} \rightarrow 0$, since in this limit the hysteresis is determined by the jump solution $z_3$, which coincide for the two different shapes of pinning force. On the other hand for $\frac{1}{k} \rightarrow 1$ all jump solutions $z_n$ become relevant in determining the hysteresis, and the specific form of the pinning force matters.

In order to illustrate our findings for a typical set of experimental values, we have plotted in Fig. 3 the hysteresis versus gravity $\rho g$ for different values of defect interaction range $R$. We have used a $\gamma_{LG}$ corresponding to water-air, and assumed a contact angle hysteresis of the defect of $20^0$. We have furthermore assumed a fixed given aspect ratio of the defect given by $R/\Delta y = 0.1$. The curves $(e) - (h)$ illustrate the very weak dependence of hysteresis on gravity when one is not in a parameter regime close to onset of hysteresis $(\rho g \approx 1000g/(cm^2s^2)$ corresponds to the density of water). For comparison note that a change in $R$ by one decade makes a change in $H$ by two decades. On the other hand when $l_c \approx \Delta y$ a pronounced dependence of gravity happens as illustrated by $(a) - (d)$. For a fixed $\rho g$ one notices from the curves $(b)$ and $(d)$ (or from Eq. (10)) that $H$ goes through a maximum as a function of $R$. The same trend happens for the Gaussian pinning force as can be seen from the curves $(a)$ and $(c)$.

Let us finally turn our attention to the general case with $A_p^{c} \neq 0$. We can repeat the procedure above. The gravitational restoring force for system of a plate with a fiber is given by

$$F_y = \frac{2\pi \gamma_{LG}}{K_0(\Delta y R)} h_0(0, \Delta y) - \frac{A_{p^{c}}^{\prime} \Delta y}{\sqrt{2K_0(\Delta y R)}} + L_y A_{p^{c}}^{\prime}$$

$L_y$ is the length of the system in the $y$ direction. The last
term on the right hand side of Eq. (11) is the contribution from the solid plate alone, whereas the negative term enters because the gravitational force on the fiber is now changed due to the additional capillary rise of the plate. The hysteresis in the general case with a plate plus fiber can be found analytically in the case of a pinning force given by Eq. (9):

\[
H(\gamma, \rho g, \Delta y, R) = A_p' \Delta y \left[ \frac{1}{C} \left( 1 - C_1 \right) - C R^2 \left( \frac{3}{4} + \frac{C_1}{4} \right) - \frac{3C^3 R^4}{32} \right] + \frac{R^4}{12} \left[ (C^2 + 4/R^2 + 4C_1/R^2 - 4C/R)^{3/2} - \left( \frac{4C_1}{R^2} \right)^{3/2} \right] \tag{12}
\]

where \( C_1 \equiv A_p' l_c \pi / [\sqrt{2} K_0(\Delta y) A_p' \Delta y] \). The linear dependence of \( C_1 \) on the capillary length \( l_c \) leads to a leading dependence on gravity in the form \((\rho g)^{-1/2}\).

**Conclusion:** We have studied the case of contact angle hysteresis, when a solid defect with a Gaussian pinning force is quasi-statically pulled out and pushed into a liquid, in the presence of gravity. Expanding the Gaussian pinning force to second order and assuming slow variations in the height of the liquid vapour interface, we have derived an analytical expression for the contact angle hysteresis. In strong contrast to the common belief, we have shown that the extension of the defect, and not the capillary length, is the determining length scale, when the extension of the defect is much smaller than the capillary length.

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FIG. 1. Fig. 1a

FIG. 1. Fig. 1b. $F_p, F_g$ versus $z$
FIG. 2. Hysteresis $H$ versus $\frac{k}{k^*}$ determined for a Gaussian defect force using Eq. (8) (thick solid line) and for a truncated parabolic defect force using Eq. (10) (thin solid line). Inset shows jump positions $z_n$ versus $\frac{k}{k^*}$.

FIG. 2. Inset to Fig. 2
FIG. 3. Hysteresis $H$ versus $\rho g$ for different values of interaction range $R$ determined for a Gaussian defect force using Eq. (8) (thick solid lines) and for a truncated parabolic defect force using Eq. (10) (thin solid lines). We have used $\gamma_{LG} = 73 \text{dyn/cm}$ corresponding to water, and a typical experimental value, $20^\circ$, for contact angle hysteresis of the defect, giving $A_p' = 69 \text{dyn/cm}$.

We have used a fixed aspect ratio of the defect $R/\Delta y = 0.1$ (a-b) $R = 0.02 \text{ cm}$, (c-d) $R = 0.01 \text{ cm}$, (e-f) $R = 0.001 \text{ cm}$, (g-h) $R = 0.0001 \text{ cm}$.
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