Nonclassical microwave radiation from the dynamical Casimir effect

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We investigate quantum correlations in microwave radiation produced by the dynamical Casimir effect in a superconducting waveguide terminated and modulated by a superconducting quantum interference device. We apply nonclassicality tests and evaluate the entanglement for the predicted field states. For realistic circuit parameters, including thermal background noise, the results indicate that the produced radiation can be strictly nonclassical and can have a measurable amount of intermode entanglement. If measured experimentally, these nonclassicality indicators could give further evidence of the quantum nature of the dynamical Casimir radiation in these circuits.

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Vacuum fluctuations are fundamental in quantum mechanics, yet they have not so far played an active role in the rapidly advancing field of engineered quantum devices, e.g., for quantum information processing and communication. The main reason being that it has been notably difficult to observe dynamical consequences of the vacuum fluctuations [1], let alone use them for applications. The dynamical Casimir effect (DCE) [2, 3] is a vacuum amplification process that can produce pairs of photons from vacuum fluctuations by means of nonadiabatic changes in the mode structure of the quantum field, e.g., by a changing boundary condition [4, 5] or index of refraction [6, 7]. As such it could potentially be applied as a source of entangled microwave photons.

For decades the DCE eluded experimental demonstration, largely due to the challenging prerequisite of nonadiabatic changes in the mode structure with respect to the speed of light. However, using a varying boundary condition in a superconducting waveguide [8, 9], the experimental observation of the DCE was recently reported [10]. This experiment also demonstrated that the dynamical Casimir radiation exhibits the expected two-mode squeezing [9, 11–13], which is a consequence of a nonclassical pairwise photon-creation process.

The microwave radiation produced by the DCE in superconducting circuits therefore has high potential of being distinctly nonclassical. Whether the state of a quantum field can be considered nonclassical, or if it could be produced by a classical process, may be demarcated by evaluating certain carefully-designed inequalities [14] for the field observables (nonclassicality tests). In this paper, we apply such nonclassicality tests to show that the microwave radiation produced by the DCE in these superconducting circuits can be distinctly nonclassical, even when taking into account the background thermal noise [15, 16] and higher-order scattering processes. Using auxiliary quantum systems as detectors [17, 18] could be an alternative to directly measure the field quadratures, which could provide further opportunities to detect nonclassical correlations, e.g., on the single photon-pair level [19].

DCE in superconducting circuits.—Superconducting circuits are strikingly favorable for amplifying vacuum fluctuations because of their inherently low dissipation, which allows the vacuum state to be reached, and the in-situ tunability of an essential circuit element, namely the Josephson junction (JJ). A JJ is characterized by its Josephson energy, and by arranging two such junctions in a superconducting loop – a superconducting quantum interference device (SQUID) – an effective tunable JJ can be produced. The Josephson energy of the effective junction can be tuned by the applied magnetic flux through the SQUID-loop. This in-situ tunability can be used to produce waveguide circuits with tunable boundary conditions [20, 23], as employed in the DCE experiment in Ref. [10], and tunable index of refraction [24–26]. Tunable JJs are also essential in related DCE proposals based on circuit QED with tunable coupling [27].

The electromagnetic field confined by a superconducting waveguide, such as a coplanar or strip-line waveguide, can be described quantum mechanically in terms of the flux operator $\Phi(x, t)$. It is related to the voltage operator by $\Phi(x, t) = \int dt' V(x, t')$, and to the gauge-invariant superconducting phase operator $\varphi = 2\pi \Phi/\Phi_0$, where $\Phi_0 = h/2e$ is the magnetic flux quantum. The flux field in the transmission line obeys the massless, one-dimensional Klein-Gordon wave equation, $\partial_{xx} \Phi(x, t) - \omega^2 \partial_t \Phi(x, t) = 0$, which has independent left- and right-propagating components. Using this decomposition, the field can be written in the form

$$\Phi(x, t) = \sqrt{\frac{\hbar Z_0}{4\pi}} \int_{-\infty}^{\infty} \frac{d\omega}{\sqrt{\omega}} \times \left[ a(\omega) e^{-i(k_{\omega} x + \omega t)} + b(\omega) e^{-i(k_{\omega} x - \omega t)} \right], \quad (1)$$

where $a(\omega)$ and $b(\omega)$ are the annihilation operators for photons with frequency $\omega/2\pi > 0$ propagating to the right (incoming) and left (outgoing), respectively. Here we have used the notation $a(-\omega) = a^\dagger(\omega)$, and $k_{\omega} = \omega/c$.

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\( \omega/v \) is the wavenumber, \( v \) is the speed of light in the
guide, and \( Z_0 \) the characteristic impedance.

Using the previously discussed flux-tunable SQUID
termination of the waveguide, one can produce a tun-
able boundary condition (see also Refs. \[23 \, 29\]) for the
quantum field [Eq. (1)],

\[
\Phi(0, t) + L_{\text{eff}}(t) \partial_x \Phi(x, t)|_{x=0} = 0,
\]

(2)

can be characterized by an effective length \( L_{\text{eff}}(t) = (\Phi_0/2\pi)^2/(E_J(t)L_0) \), where \( L_0 \) is the characteristic
inductance per unit length of the waveguide and \( E_J(t) = E_J(\text{ext}(t)) \) is the flux-dependent effective Josephson
energy. To arrive at this boundary condition we have
neglected the capacitance of the SQUID and assumed
small phase fluctuations, which is justified for a large
SQUID plasma frequency \[3, 9\]. For sinusoidal modula-
tion with frequency \( \omega_d/2\pi \) and normalized amplitude \( \epsilon \),
\( E_J(t) = E_J^0[1 + \epsilon \sin \omega_dt] \), we obtain an effective length
modulation amplitude \( \delta L_{\text{eff}} = L_0^0 - L_{\text{eff}}(0) \).
A strong modulation (corresponding to an effective ve-
locity \( \nu_{\text{eff}} = \delta L_{\text{eff}}/\omega_d \)) that is a significant fraction of the
speed of light in the waveguide \( v \), results in nonadiabatic
changes in the mode structure of the quantum field, and
the emission of photons as described by the DCE.

The DCE can be analyzed using scattering theory that
describes how the time-dependent boundary condition,
or region of the waveguide with a time-dependent index
of refraction, mixes the otherwise independent left and
right propagating modes \[30\]. The superconducting
circuits considered here were analyzed using this method
in Refs. \[3, 9\], where the weak-modulation regime was
studied analytically using perturbation theory, and the
strong-modulation regime was studied using a higher-
order numerical method.

In the perturbative regime, the resulting output field is
correlated at modes with angular frequencies \( \omega \) and \( \omega_d - \omega \), i.e., symmetrically around half the driving frequency.
This intermode symmetry is emphasized when the output
field is written for two such correlated modes:

\[
b_\pm = -a_\pm - i \frac{\delta L_{\text{eff}}}{\nu} \sqrt{\omega + \omega - a_\pm^2},
\]

(3)

where we have introduced the short-hand notation
\( a_\pm = a(\omega_\pm) \) and \( b_\pm = b(\omega_\pm) \), and where \( \omega_\pm = \omega_d/2 \pm \delta \omega \) is the symmetric detuning. In
this perturbation calculation, the small parameter is
\( \delta L_{\text{eff}}/\sqrt{\omega + \omega - \nu} \approx \epsilon L_{\text{eff}}(0)\omega_d/2\nu \). Here, even if the input
field is in the vacuum state, \( \langle a_\pm^\dagger a_\pm \rangle = 0 \), the output
field Eq. (3) has a nonzero, symmetric photon flux
\( \langle b_\pm^\dagger b_\pm \rangle = (\delta L_{\text{eff}}/\nu)^2\omega_\pm \), i.e., the dynamical Casimir
radiation. Furthermore, the photons in the two modes
have bunching-like statistics, where the probability of si-
multaneously observing one photon in each mode is equal
to the probability of observing a photon in one of the
modes \( \langle b_\pm^\dagger b_\pm \rangle \approx \langle b_\pm^\dagger b_\pm \rangle \), i.e., they appear in pairs.

For finite temperatures, where thermal noise is present
in the input field, and for not so weak modulation, when
for example \( \delta L_{\text{eff}}/\sqrt{\omega + \omega} \nu \) no longer is a small parameter,
it is not obvious if or to what extent the above results
apply. In these cases there are both classical and non-
classical contributions to the photon flux in the output
field, and it becomes necessary to systematically compare
the relative importance of such contributions in order to
tell if the resulting output field remains nonclassical or
not. In the following, we carry out such an analysis using
nonclassicality tests and by evaluating the degree of
entanglement in the predicted output field.

**Nonclassicality tests.**—The theory of nonclassicality
tests has been well developed in quantum optics, and
here we briefly review the important results in the nota-
tion introduced above for superconducting waveguides.
We consider an operator \( \tilde{f} \) which is defined as a func-
tion of the creation and annihilation operators. For the
Hermitian operator \( \tilde{f} \) it can then be shown \[14\], using the Glauber-Sudarshan \( P \) function formalism, that any
classical state of the field satisfies

\[
\left\langle \tilde{f}^\dagger \tilde{f} \right\rangle \geq 0,
\]

(4)

where the condition for classicality that has been used is
that the \( P \) function must always be non-negative. The \( : \)
denotes normal ordering.

For the two-mode quadrature-squeezed states that the
DCE is known to produce, the natural definition of \( \tilde{f} \) is

\[
\tilde{f}_\theta = e^{i\theta} b_- + e^{-i\theta} b_+^\dagger + i(e^{i\theta} b_+ - e^{-i\theta} b_-^\dagger),
\]

(5)

where \( \theta \) is the angle that defines the principal squeezing
axis. With this definition of \( \tilde{f}_\theta \), a pure two-mode
squeezed state is known to violate the inequality \( \tilde{f} \),
see, e.g., Ref. \[14\] and references therein. This choice
of \( \tilde{f}_\theta \) is also suitable from an experimental point of view,
since \( \left\langle \tilde{f}_\theta^\dagger \tilde{f}_\theta \right\rangle \) can be evaluated from experimentally-
accessible quadrature correlations.

We now evaluate the quantum-classical indicator

\[
\left\langle \tilde{f}_\theta^\dagger \tilde{f}_\theta \right\rangle = \min_\theta \left\langle \tilde{f}_\theta^\dagger \tilde{f}_\theta \right\rangle
\]

for the field state produced by the DCE, and discuss the conditions under which this
nonclassicality test is violated. For weak driving, using
output field Eq. (3), and a thermal input field we obtain

\[
\left\langle \tilde{f}_\theta^\dagger \tilde{f}_\theta \right\rangle = 2(n_\text{th}^+ + n_\text{th}^-)
- 4 \cos 2\theta \frac{\delta L_{\text{eff}}}{\nu} \sqrt{\omega + \omega_-} (1 + n_\text{th}^+ + n_\text{th}^-),
\]

(6)

where \( n_\text{th}^\pm = \langle a_\pm^\dagger a_\pm \rangle = (\exp(\hbar \omega_\pm/k_B T) - 1)^{-1} \) is the
dynamical thermal photon flux of the input mode with frequency
\( \omega_\pm \). In this case, \( \left\langle \tilde{f}_\theta^\dagger \tilde{f}_\theta \right\rangle \) is minimized by taking \( \theta = 0 \),
and it is negative if \( (\delta L_{\text{eff}}/\nu) \sqrt{\omega + \omega_-} \geq (n_\text{th}^+ + n_\text{th}^-)/2 \), or,
equivalently, \( \epsilon \geq 2\nu/(L_{\text{eff}}\omega_d)(n_\text{th}^+ + n_\text{th}^-)/2 \). This indicates
that the field state in the form Eq. (3) is distinctly nonclassical for a vacuum input field, and potentially also for low-temperature thermal input fields.

To investigate whether the nonclassical characteristics of the DCE radiation remain for realistic input field temperatures and when the driving amplitude is increased beyond the perturbative regime, we also evaluate $\langle f_\theta^\dagger f_\theta \rangle$ by solving the scattering problem numerically. The results of this calculation are presented in Fig. 1(a), showing that for sufficiently large driving amplitude $\langle f_\theta^\dagger f_\theta \rangle < 0$ even at typical temperatures for superconducting circuits, and including higher-order scattering processes. We therefore conclude that nonclassical characteristics of the DCE radiation can be sufficiently robust to remain important in realistic experimental situations. Evaluating $\langle f_\theta^\dagger f_\theta \rangle$ from experimentally measured field quadratures therefore appears to be a viable method to conclusively demonstrate the quantum statistics of the dynamical Casimir radiation.

The nonclassicality test in terms of $\sigma_2$.—To further relate to the experimental demonstrations of the DCE, it is instructive to formulate the nonclassicality test in terms of the two-mode squeezing $\sigma_2$, which was measured in Ref. [10]. The two-mode squeezing is defined as $\sigma_2 = \langle (I_+^2 - Q_- Q_+) \rangle / \langle (I_+^2 + I_-^2 + Q_-^2 + Q_+^2) / 2 \rangle$, where $I_{\pm} = (\hbar \omega_{\pm} Z_0 / 8n_s)^{1/2} \left( e^{i\phi} b_{\pm} + e^{-i\phi} b_{\pm}^\dagger \right)$ and $Q_{\pm} = -i (\hbar \omega_{\pm} Z_0 / 8n_s)^{1/2} \left( e^{i\phi} b_{\pm} - e^{-i\phi} b_{\pm}^\dagger \right)$ are the voltage quadratures. Using this expression for $\sigma_2$, we can write the inequality $\langle f_\theta^\dagger f_\theta \rangle < 0$ as

$$\sigma_2 > \frac{2\sqrt{\omega_+ \omega_-} (n_+ + n_-)}{\omega_+ (2n_+ + 1) + \omega_- (2n_- + 1)},$$

where $n_\pm = \langle b_{\pm}^\dagger b_{\pm} \rangle$ is the photon flux (thermal and DCE) for the output mode with frequency $\omega_{\pm}$, and where we have taken $\theta = \phi + \pi/4$ to relate $\sigma_2$ and $\langle f_\theta^\dagger f_\theta \rangle$.

Equation (7) suggests that a non-zero two-mode squeezing does not necessarily imply that the field is a strictly nonclassical state [by the criterium of Eq. (4) and the current definition of the operator $\hat{f}$]. However, if the magnitude of the two-mode squeezing exceeds the right-hand side of Eq. (7), the field is guaranteed to be distinctively nonclassical (i.e., squeezed vacuum rather than a squeezed thermal state). Since the expectation values in the right-hand side of Eq. (7) can be measured experimentally, this could be a practical formulation for the experimental evaluation of the nonclassicality test.

Figure 1(b) shows the two-mode squeezing together with the boundary between the classical and quantum regimes, as defined by Eq. (7). With the parameters used in Fig. 1 the boundary corresponds to the squeezing $\sigma_2 \approx 0.04$. Experimental measurements [10] have demonstrated significantly larger squeezing for the dynamical Casimir radiation, but at the same time the measured photon flux was larger than in the current calculations due to the presence of low-$Q$ resonances in the transmission line. An increased photon flux increases the value of the boundary in Eq. (7) and makes the violation of the inequality more demanding. However, by reducing the driving strength to get a lower photon flux a violation of the nonclassicality test Eq. (7) should be achievable with an experimental setup like the one in Ref. [10], although increased measurement time and averaging may be necessary to obtain sufficient sensitivity.

Entanglement.—The two-mode squeezing and the nonclassicality tests discussed above demonstrate that the DCE radiation is nonclassical. The quantum nature of the radiation originates from the entanglement in individual pairs of photons. To quantify the entanglement between two entire modes with frequencies adding up to the driving frequency, we evaluate the logarithmic negativity $N$ [31], which is an entanglement measure for Gaussian states that is frequently used in quantum optics, and recently also in microwave circuits [32] and nanomechanical systems [33]. The logarithmic negativity is positive for entangled states, and it can be calculated from the covariance matrix $V_{\alpha \beta} = \tfrac{1}{2} (R_{\alpha} R_{\beta} + R_{\beta} R_{\alpha})$, where $R^T = (q_-, p_-, q_+, p_+)$ is a vector with the quadratures as elements: $q_\pm = (b_{\pm} + b_{\pm}^\dagger) / \sqrt{2}$ and $p_\pm = -i (b_{\pm} - b_{\pm}^\dagger) / \sqrt{2}$.

The covariance matrix can be evaluated both analytically and numerically, and also constructed from experimental quadrature measurements. The numerically calculated covariance matrix is shown in the
Fig. 2: (color online) The logarithmic negativity $N$ as a function of the normalized modulation amplitude $\epsilon$. The onset of nonzero $N$ is $\epsilon_0$. The parameters are the same as in Fig. 1. Inset: The covariance matrix for $\epsilon = 0.5$. The diagonal quadrature correlations correspond to the vacuum fluctuations and the photon flux due to the DCE and of thermal origin. The nonzero, off-diagonal elements correspond to the two-mode correlations produced by the DCE.

Fig. 3: (color online) The region of nonclassical radiation (blue), visualized using $-\langle \hat{f}\hat{f}^\dagger \rangle$ (left) and the logarithmic negativity $N$ (right), as a function of the temperature $T$ and the detuning $\delta\omega$, for $\epsilon = 0.15$ and other parameters as in Fig. 1. Although the nonclassical region is larger for $N$ than for $-\langle \hat{f}\hat{f}^\dagger \rangle$, $N$ is small in the region where $-\langle \hat{f}\hat{f}^\dagger \rangle$ is non-negative (white), and the regions where the measures violate classicality with a one-$\sigma$ confidence are quite similar.

We have theoretically investigated quantum correlations in the radiation produced by the DCE in a superconducting waveguide by evaluating nonclassicality tests and the logarithmic negativity. These measures indicate that the devices used in Ref. [10], should have access to regimes where the produced radiation is strictly nonclassical. We have formulated practical inequalities with experimentally obtainable observables that could be used to directly verify the quantum nature of the measured radiation in future DCE experiments. We also note that recently two-mode squeezed states have been generated in microwave circuits using other mechanisms, for example parametric amplification using the nonlinear response [24,25] or time-varying index of refraction [26] of SQUID arrays and JJs [22,30]. The nonclassicality tests discussed here could also be applied to analyze the radiation produced in these experiments. We believe that a demonstration of a nonclassicality violation in superconducting circuits, or other promising systems [37–40], could pave the way to the experimental exploration of the continuous production of entangled microwave photons by the DCE, and possible applications thereof in, for example, quantum information processing [41–43]. As such it could become a novel practical application of microwave quantum vacuum fluctuations.

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