Evaluation of damage accumulation zone in the vicinity of the crack tip: FEM analysis via UMAT procedure

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Abstract. Nowadays the damage and failure of the material are of critical interest to designers of engineering structures, especially in aircraft and spacecraft industry. As there is a lack of methods for accurate failure prediction and damage analysis. Especially for early stages of the design process, a quick way of estimating material behavior is needed. There is a need for a material model coupled with a damage evolution law. In the paper a damage model, based on continuum damage mechanics, is presented. The material law is implemented computationally as a user defined subroutine (UMAT) in a commercially available FEM package Simulia Abaqus. The active damage accumulation zones in the vicinity of the crack tip for a wide class of cracked specimens are analyzed. Distribution of each damage component is given.

1. Introduction. Description of damage accumulation processes in solids

Fracture mechanics is the field of solid mechanics that deals with the behavior of cracked bodies subjected to stresses and strains. These can arise from primary applied loads or secondary self-equilibrating stress fields (e.g., residual stresses). For engineering materials, such as metals, there are two primary modes of fracture: brittle and ductile. The fracture process zone is the region around the crack tip where dislocation motions, material damage occur. It is a region of nonlinear deformation. Different theories have been advanced to describe the fracture process in order to develop predictive capabilities: Linear Elastic Fracture Mechanics, Cohesive zone models, Nonlinear Fracture Mechanics, etc. [1–2].

Kachanov L.M. and Rabotnov Y.N. in their pioneer works [3–4] proposed the continuity and damage parameters for the mathematical description of damage accumulation processes. Since then the set of damage evolution equations has been considerably developed [2,5–7] and many works describe mutual influence of stress-strain state and damage evolution near-stress concentrators [8–22].

The directions of continuum damage mechanics development can be classified. First of all, different damage evolution equations have been proposed which take into account different aspects of damage accumulation [2]. Then several models for anisotropic damaged materials have been suggested [23–29]. Finally, one can note that many asymptotic solutions to the coupled elasticity-damage, plasticity damage and creep damage problems have been obtained [2,8–24,26,27]. To elucidate the effect of damage in structures working in real conditions it is necessary to apply finite element method for cracked bodies under complex loading [30–32].

Damage accumulation process in solids can be described by scalar or tensor damage parameter.

In the simplest case damage is presented as scalar value $1 \geq \psi \geq 0$ [3]. In initial state, when structure is undamaged $\psi = 1$, and eventually function $\psi$ decreases. It is possible to interpret function $\psi$ as discontinuity.
In 1959 Rabotnov Y. N. established function \( \omega > 0 \), zero meaning that the material is undamaged and one meaning that the material is fully damaged [4]. It is essential to interpret function \( \omega \) as damage and then the equation \( \psi = 1 - \omega \) is true.

The damage parameter \( \omega \) represents the relative cross-sectional area of the specimen occupied with cracks. Damage rate \( \partial \omega / \partial t \) depends on stress and \( \omega \). This assumption let us consider \( \omega \) as one of the structural parameters of the material model.

The simplest hypothesis assumes that \( \partial \omega / \partial t \) is a power function of the relation \( \sigma / (1 - \omega) \). This relation is explained as medium stress on cross-sectional area, free from cracks. Constitutive equations of the material are based on steady-state creep theory power law of Baily-Norton:

\[
\frac{\partial \omega}{\partial t} = \frac{(3/2) B(\sigma / \psi)^{m-1} s_{ij}}{\psi},
\]

where \( \psi \) is continuity parameter, evolving according to the power law of damage accumulation:

\[
\frac{d\psi}{dt} = -A(\sigma / \psi)^{m}.
\]

Nowadays the damage and failure of the material are of particular interest to designers of engineering structures. While composites applications in the aircraft and spacecraft industry are rapidly increasing, there is a lack of accurate failure prediction and progressive damage analysis. The efficient design of a composite structure depends on developing accurate analytical and numerical material models. Critical to this advancement is a thorough understanding of damage mechanisms and their interactions [27]. Especially for early stages of the design, a quicker way of estimating complicated material behavior is needed. Engineers must be able to predict the strength of the future structure element and design on the whole.

However, the numerical modeling of these materials poses several challenges. There is a need for a material model coupled with a damage evolution law. In the paper a damage model, based on continuum damage mechanics, is presented. The material law is implemented computationally as a user defined subroutine (UMAT) in a commercially available FEM package SimuliaAbaqus. The active damage accumulation zones in the vicinity of the crack tip are analyzed.

2. Material model based on the damage tensor

In recent work we use the defined in [24] damaged material model, based on continuum damage mechanics.

The intact undamaged material was considered to be linear, elastic, isotropic. The stiffness degradation arising from the damage accumulation was simulated by adding a damage tensor into the constitutive equation initially proposed in [8]:

\[
\sigma_{ij} = \left[ K_{ijkl}^e(T) + K_{ijkl}^d(T) \right] \times \left( \epsilon_{ijkl} - \epsilon_{ijkl}^d \right),
\]

where \( \sigma_{ij} \) and \( \epsilon_{ijkl} \) are the stress and strain tensor components, \( K_{ijkl}^e(T) \) is the temperature-dependent fourth order stiffness tensor representing the undamaged isotropic material, \( K_{ijkl}^d(T) \) is the temperature-dependent fourth order stiffness tensor representing the added influence of damage, \( \epsilon_{ijkl}^d \) are the thermal strain tensor components.

The components of the stiffness tensors are given by:

\[
K_{ijkl}^e = \lambda(T) \delta_{ij} \delta_{kl} + \mu(T) \left( \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right),
\]

\[
K_{ijkl}^d = C_1(T) \left( \delta_{ij} D_{kl} + \delta_{il} D_{kj} \right) + C_2(T) \left( \delta_{ik} D_{lj} + \delta_{ij} D_{lk} \right),
\]

where \( \lambda(T) \) and \( \mu(T) \) are Lame’s constants, \( D_{ij} \) are the damage parameters, \( C_1(T) \) and \( C_2(T) \) are constants of the material [8].

We are interested in analysis of active damage accumulation zone. The components of the damage tensor \( D_{ij} \) are functions of the stress state, \( 0 \leq D_{ij} \leq 1 \).
The original damage model \[8\] took into account the effect of the shear stresses on the damage evolution, only the diagonal terms of the damage tensor, so the damage parameters due to tensile principal stresses, were accounted for in the paper \[24\]. Their values follow a linear evolution law:

\[
D_{ii} = \begin{cases} 
0 & \sigma_i \leq \sigma_{th} \\
\frac{\sigma_i - \sigma_{th}(T)}{\sigma_i(T) - \sigma_{th}(T)} & \sigma_{th} < \sigma_i < \sigma_c \\
1 & \sigma_i \geq \sigma_c 
\end{cases},
\]

where \(i = 1, 2, 3\), \(\sigma_i\) is the \(i\)th principal tensile stress, \(\sigma_{th}\) is the temperature-dependent threshold stress under which no damage occurs, \(\sigma_c\) is the temperature-dependent critical stress above which the material is fully damaged \[24\].

The damage tensor will thus be naturally defined as follows. From equations (3)–(5) and neglecting the non-diagonal terms of the damage tensor, the constitutive equations of the material are \[12, 24–25\]:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 \\
\end{bmatrix} = \begin{bmatrix}
\lambda + 2\mu + 2D_{11}(C_1 + C_2) \\
\lambda + C_1(D_1 + D_2) + 2\mu + 2D_{11}(C_1 + C_2) + D_{11} \lambda \\
\lambda + C_2(D_1 + D_2) + 2\mu + 2D_{11}(C_1 + C_2) + D_{22} \lambda \\
0 \\
0 \\
0 \\
\end{bmatrix} \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6 \\
\end{bmatrix}
\]

The values of \(C_1\) and \(C_2\) are calculated by stating that the stresses \(\sigma_{11}\) and \(\sigma_{22}\) drop to zero when \(D_{11} = D_{22} = 1.0\), for equi-biaxial tension in directions 1 and 2 i.e., for \(D_{11} = 0\). For such an equi-biaxial tension test, \(\varepsilon_{11} = \varepsilon_{22} = \varepsilon\):

\[
\begin{align*}
\sigma_{11} = \sigma_{22} &= 0 = (2(\lambda + \mu) + 2(2C_1 + C_2))\varepsilon + (\lambda + C_1)\varepsilon_{33} \\
\sigma_{33} &= 0 = (2(\lambda + C_1))\varepsilon + (\lambda + 2\mu)\varepsilon_{33}
\end{align*}
\]

Equations (8) and (9) must be equivalent. Then \(C_1 = 2\mu\) and \(C_2 = -3\mu\) \[12, 24–25\].

In the following calculations these values of material constants are used.

3. Finite element modelling and calculations

Abaqus provides an extensive number of user subroutines that allow us adapting Abaqus to particular problem requirements. User subroutines provide an extremely powerful and flexible tool for analysis \[23\].

3.1. User subroutine UMAT in Simulia Abaqus

To define any complex, constitutive models for materials that cannot be modeled with the available Abaqus standard material models we use subroutine UMAT (User material). The material model developer should be concerned only with the development of the material model and not the development and maintenance of the software \[33\].

Solution-dependent state variables (SDVs) are values that can be defined to evolve with the solution of an analysis. An example of a solution-dependent state variable for UMAT subroutine in this particular work is damage parameter.

3.2. The analysis of active damage zone in structures

The objective of this work is the analysis of active damage zone in the vicinity of the crack tip in various types of cracked specimens. The models are implemented in Simulia Abaqus using UMAT subroutine.

3
3.2.1. Damage distribution in a plate with central crack

The elaborated UMAT subroutine was applied to the simplest possible model – a 2D elastic plate with central crack. Additional complexity was not included in the model in order to compare problem results with standard Abaqus material model. The plate is subjected to uniaxial loading in direction 1.

The mesh example near the crack tips is shown in Figure 1. For mesh convergence in a small-strain analysis, the singularity at the crack tip must be considered. As it is seen from Figure 1 the entire 2D part of the plate is filled with a quad or quad-dominated mesh. At the crack tips, a ring of triangles are inserted along with concentric layers of structured quads. All triangles in the contour domains must be represented as degenerated quads [1].

![Figure 1. Mesh example for a plate with central crack and two crack tips with singular element mesh.](image)

FEM analysis of the plate showed that tension of the damaged structure and undamaged structure differs (Figures 2 – 5). Equivalent stresses in the plate constituted of standard Abaqus elastic material without using UMAT are considerably higher: \( \sigma_e = 6.890 \text{ GPa} \) compared to \( \sigma_e = 5.396 \text{ GPa} \) in the damaged plate (Figure 2). The same happens with the values of stress-field components of the plate: \( \sigma_{11} = 4.070 \text{ GPa} \) for standard material model and \( \sigma_{11} = 3.654 \text{ GPa} \) using UMAT (Figure 3); \( \sigma_{12} = 2.803 \text{ GPa} \) for standard material model and \( \sigma_{12} = 2.602 \text{ GPa} \) using UMAT (Figure 4); \( \sigma_{22} = 7.130 \text{ GPa} \) for standard material model and \( \sigma_{22} = 6.707 \text{ GPa} \) using UMAT (Figure 5).

![Figure 2. Equivalent stress in the model: undamaged material (left), damaged material (right).](image)

![Figure 3. Stress component \( \sigma_{11} \): undamaged material (left), damaged material (right).](image)
Distributions for damage tensor components of the cracked plate were also obtained and shown in Figure 6. Thus damage accumulation effects on the stress-strain state in the vicinity of the crack tip.
3.2.2. Damage distribution in a semi disk with vertical crack
The other problem considered is a semicircular bend (SCB) specimen. It is a 2D elastic semidisk containing an inclined edge crack under 3-point loading configuration. The obtained results of damage distribution are shown in Figure 7.

![Damage distribution in a semi disk with vertical crack](image)

Figure 7. Distribution of damage tensor components: a – for $D_{11}$, b – for $D_{22}$, c – for $D_{33}$.

3.2.3. Damage distribution in a plate with central hole
As the third specimen, an elastic 2D square plate with central circular hole was regarded. Plate was subjected to two types of loading: equi-biaxial and uniaxial. Distributions for damage tensor components of the plate with stress-concentrator were also obtained and shown in Figures 8-9.
4. Conclusion
The paper presents numerical solutions obtained by FEM and the user procedure UMAT for a wide class of cracked specimens. Plate with the central circular hole under biaxial and uniaxial loading, semi-disk with vertical notch, plate with the central crack in a damaged material are considered. The geometry of active damage accumulation zones in the vicinity of the notch and cracks are studied. Distribution of each damage component is given.

5. References
[1] Modeling Fracture and Failure with Abaqus Dassault Systems Educational Course
[2] Murakami S 2012 Continuum Damage Mechanics A Continuum Mechanics Approach to the Analysis of Damage and Fracture (Dordrecht: Springer)
[3] Kachanov L M 1958 Ovremeni razrushenija v usloviyah polzuchesty Izv AN SSSR OTN 8 26-31
[4] Rabotnov J N 1959 Omechanizme dlitel’nogo razrushenia Voprosi prochnosti materialov i konstrukcij AN SSSR OTN 5-7
[5] Voyiadjis G Z and Kattan PI 2012 Damage Mechanics with Finite Elements: Practical Applications with Computer Tools (Berlin: Springer)
[6] Altenbach H and Sadowski T 2015 Failure and Damage Analyses of Advanced Materials (Berlin: Springer)
[7] Riedel H 1987 Fracture at High Temperature (Berlin: Springer)
[8] Sun X and Khaleel M 2004 Modeling of glass fracture damage using continuum damage mechanics – static spherical indentation Int. J. of Damage Mechanics 13(3) 263-285
[9] Stepanova L V and Fedina M E 2005 Asymptotic behavior of the far stress field in the problem of crack growth in a damaged medium under creep conditions Journal of Applied Mechanics and Technical Physics 46(4) 570–580
[10] Wen Z X, Hou N X and Yue Z F 2009 Creep damage and crack initiation behaviour of nickel-base single crystalline superalloys compact tension specimen with a void ahead of crack tip Materials Science and Engineering A 510-511 284-288
[11] Belnoue J P, Garnham B, Bache M and Korsunsky A M 2010 The use of coupled nonlocal damage-plasticity to predict crack growth in ductile metal plates Engineering Fracture Mechanics 771 721-729
[12] Doquet V, Ben Ali N, Constantinescu A and Boutillon X 2013 Fracture of a borosilicate glass under triaxial tension Mechanics of Materials 57 15-29
[13] Cousigné O, Moncayo D, Coutellier D, Camanho P, Naceur H and Hampel S 2013 Development of a new nonlinear numerical material model for woven composite materials accounting for permanent deformation and damage Composite Structures 106 601-614
[14] Stepanova L V and Igonin S A 2014 Perturbation method for solving the nonlinear eigenvalue problem arising from fatigue crack growth problem in a damaged medium Appl. Math. Modeling 38 3436-3455
[15] Stepanova L V and Igonin S A 2015 Asymptotics of the near-crack-tip stress field of a growing fatigue crack in damaged materials: Numerical experiment and analytical solution Numerical Analysis and Applications 8(2) 168–181
[16] Stepanova L V and Yakovleva E M 2015 Asymptotic stress field in the vicinity of a mixed-mode crack under plane stress conditions for a power-law hardening material Journal of Mechanics of Materials and Structures 10(3) 367-393
[17] Stepanova L and Roslyakov P 2016 Complete Williams asymptotic expansion of the stress field near the crack tip: Analytical solutions, interference-optic methods and numerical experiments AIP Conf. Proceedings 1785 030029
[18] Stepanova L and Yakovleva E 2016 Asymptotic stress field in the vicinity of the mixed-mode crack in damaged materials under creep conditions Procedia Structural Integrity 2 793-800
[19] Stepanova L V and Yakovleva E M 2016 Stress-strain state near the crack tip under mixed-mode loading: Asymptotic approach and numerical solutions of nonlinear eigenvalue problems AIP Conference Proceedings 1785 030030
[20] Stepanova L V, Dolgikh V S and Turkova V A 2017 Digital photoelasticity for calculating coefficients of the Williams series expansion in plate with two collinear cracks under mixed mode loading CEUR Workshop Proceedings 1904 200-208
[21] Chen Y, Cui Y and Gong W 2017 Crack propagation calculations for optical fibers under static bending and tensile loads using continuum damage mechanics Sensors 17 2633
[22] Gao Z, Zhang L and Yu W 2018 A nonlocal continuum damage model for brittle fracture Engineering Fracture Mechanics 189 481-500
[23] Fengxia Ouyang 2005 Abaqus implementation of creep failure in polymer matrix composites with transverse isotropy (master thesis)
[24] Dubé M, Doquet V, Constantinescu A, George D, Rémond Y and Ahzi S 2010 Modeling of thermal shock-induced damage in a borosilicate glass Mechanics of Materials 42 863-872
[25] Doquet V, Ben Ali N, Chabert E and Bouyer F 2015 Experimental and numerical study of crack healing in a nuclear glass Mechanics of Materials 80 145-162
[26] Lee C-S, Kim J-H, Kim S-K, Ryu D-M and Lee J-M 2015 Initial and progressive failure analyses for composite laminates using Puck failure criterion and damage-coupled finite element method Composite Structures 121 406-419
[27] Salavatian M and Smith L V 2015 An investigation of matrix damage in composite laminates using continuum damage mechanics Composite Structures 131 565-573
[28] Hazar S, Zaki W, Moumni Z and Anlas G 2015 Modeling of steady-state crack growth in shape memory alloys using a stationary method International Journal of Plasticity 67 26-38
[29] Jun L, Guiqiong J, Bo W, Liang L and Chengpeng Y 2014 Damage characteristics and constitutive modeling of the 2D C/ SiC composite: Part II – Material model and numerical implementation Chinese Journal of Aeronautics 24
[30] Turkova V A, Stepanova L V 2016 Finite element analysis of biaxial cyclic tensile loading of elasto-plastic plate with central elliptical hole PNRPU Mechanics Bulletin 3 207-221
[31] Kim J-H, Lee C-S, Kim M-H, Ryu D-M and Lee J-M 2013 Prestrain-dependent viscoplastic damage model for austenitic stainless steel and implementation to ABAQUS user-defined material subroutine Computational Materials Science 67 273-281
[32] Lua J, Beckera A, Suna W and Tannerb D 2014 Simulation of cyclic plastic behavior of 304L steel using the crystal plasticity finite element method Procedia Materials Science 3 135-140
[33] Writing User Subroutines with ABAQUS Dassault Systems Lectures

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