The superluminal motion of the jet launched in GW170817, the Hubble constant, and critical tests of gamma ray bursts theory

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Introduction. A 3σ level discrepancy exists between the measured local value of the Hubble constant provided by standard candle Type Ia supernovae, \( H_0 = 73.24 \pm 1.74 \) km/s/Mpc, and the cosmic value obtained from the Planck measurements of the microwave background radiation assuming the standard ΛCDM cosmology, \( H_0 = 67.74 \pm 0.46 \) km/s/Mpc. The origin of this discrepancy is still unknown. An independent method of measuring the local value of \( H_0 \) from gravitational wave (GW) sources accompanied by emission of electromagnetic waves (EMW) was suggested before and applied recently to obtain the value \( H_0 = 74.3^{+8}_{-6} \) km/s/Mpc from the GW emission from the nearby neutron stars merger (NSM) event GW170817 followed by the short gamma ray burst SHB170817A. Moreover, in a recent paper it was claimed that the VLBA and VLBI radio observations of the mean apparent superluminal velocity of the jet in the short hard burst SHB170817A improves the measurement of the local Hubble constant based on the gravitational waves emission from the NSM GW170817 event and the electromagnetic localization of the counterpart short hard gamma ray burst SHB170817A. This claimed improvement, however, was based on a debatable model, which involved many free adjustable parameters and a jet direction which does not necessarily coincide with the axis of the orbit of the merging neutron stars. A coincidence between the axis of the orbital motion and the direction of the emitted highly relativistic jet is observed in quasars, blazars and microquasars.

Below, we present evidence from the combined GW170817/SHB170817A event and the measured apparent superluminal velocity of the far off-axis jet launched in this event, which seems to support the local value of the Hubble constant obtained from standard candle supernovae of type Ia, but is different, roughly by 3σ from the cosmic value measured by Planck, assuming the standard ΛCDM cosmology. We also show that the measured superluminal motion of the jet allows critical tests of the assumed production mechanism of SHBs in general and of SHB170817A in particular.

Apparent Superluminal Motion Evidence. The apparent velocity in the plane of the sky of a highly relativistic compact (unresolved) source moving at a small redshift \((1+z) ≈ 1\) with a constant bulk motion Lorentz factor \(\gamma \gg 1\) (i.e., \(\beta = \sqrt{1 - 1/\gamma^2} ≈ (1 - 1/2\gamma^2) \approx 1\)), and viewed from an angle \(\theta\) relative to its direction of motion, is given by

\[
V_{app} = \frac{\beta c \sin \theta}{(1+z)(1-\beta \cos \theta)} \approx \frac{c \sin \theta}{1-\cos \theta} ,
\]

which depends only on \(\theta\). In that case, the angular distance to the afterglow source satisfies

\[
D_A \approx V_{app} \Delta t / \Delta \theta_s \approx c z / H_0
\]

where \(\Delta \theta_s\) is the change in its angular location during the time \(\Delta t\). In the case of SHB170817A in NGC 4993 at redshift \(z = 0.009783\), the angular location of the source in the plane of the sky has changed during \(\Delta t = 155\) day (between day 75 and day 230) by \(\Delta \theta_s = 2.68 \pm 0.3\) mas. Thus, assuming the local value \(H_0 = 73.24 \pm 1.74\) km/s/Mpc, eq.(2) yields \(V_{app} \approx (4.32 \pm 0.43)\) c and consequently \(\theta \approx 28 \pm 2\) deg, which follows from eq.(1). This value of \(\theta\) is in agreement with the value \(5 \pm 4\) deg, which was obtained from GW170817 and its electromagnetic location assuming the local value of \(H_0\) obtained from Type Ia supernovae. Eq.(2) yields \(V_{app} \approx (3.42 \pm 0.43)\) c, and consequently \(\theta \approx 26 \pm 2\) deg, which follows from eq.(1) for the Planck cosmic value \(H_0 = 67.74 \pm 0.46\) km/s/Mpc. It is however in tension with \(\theta = 18 \pm 4\) deg, which was obtained from GW170817 and its electromagnetic location assuming the Planck cosmic value of \(H_0\) obtained from the MBR.

Note that if \(\gamma(t)\) is time dependent, then

\[
V_{app} = \beta \gamma \delta c \sin \theta / (1+z)
\]

where \(\delta = 1/\gamma (1 - \beta \cos \theta)\) is the Doppler factor of the source. Thus, \(V_{app}(t)\) generally depends on both \(\theta\) and \(\gamma\) which are generally unknown. But, as long as \(\gamma^2 \gg 1\), \(\delta^2 \ll 1\), and \(\gamma^2 \theta^2 \gg 1\), \(V_{app}\) remains constant in time and satisfies eq.(1).
A constant superluminal velocity $V_{\text{app}} \approx c \sin \theta / (1 - \cos \theta)$ was predicted by the cannonball (CB) model of GRBs for far-off axis (low luminosity) GRBs within a face-on spiral galaxies\textsuperscript{12}, where $\gamma^2 \theta^2 \gg 1$ is satisfied within the disk and remains so after escape into the galactic halo. Both GRB980425\textsuperscript{13} and SHB170817A\textsuperscript{14} took place in such locations. The deceleration of a CB stops practically when the CB moves out from the galactic disk into the low density galactic halo\textsuperscript{12,15}.

Note in particular that the shape of the prompt emission and of the afterglow, in the CB model, depends on the product $\gamma(0) \theta$ and not on the individual values of $\gamma(0)$ and $\theta$. This degeneracy has now been removed, for the first time, by the measured\textsuperscript{9} apparent superluminal motion of the jet in SHB170817A, which yields $\theta = 28$ deg. As shown below, it indicates that the glory around NSMs has its peak energy in the X-ray band. This is in contrast to the eV peak energy of the optical glory in SN-GRBs, which, in the past, was wrongly assumed\textsuperscript{15} also for SHBs.

Falsifiable tests of the production mechanism of SHBs. In the CB model\textsuperscript{16} the peak energy of the prompt emission produced by ICS of glory photons with a bremsstrahlung (or a cutoff power-law) spectrum, $\varepsilon (dn/d\varepsilon) \propto \exp(-\varepsilon/\varepsilon_p)$, is given by

$$ (1+z) E_p = \gamma \delta \varepsilon_p. \quad (4) $$

In low luminosity SHBs, i.e., SHBs viewed far off-axis, $\gamma^2 \theta^2 \gg 1$ and $\beta = \sqrt{1 - 1/\gamma^2} \approx 1 - 1/2 \gamma^2 \approx 1.$ Thus,

$$ (1+z) E_p \approx \varepsilon_p / (1 - \cos \theta). \quad (5) $$

Hence, the observed\textsuperscript{17} (T90) $E_p \approx 86 \pm 19$ keV and $\theta \approx 28$ deg yield an X-ray glory with $\varepsilon_p \approx 10.2 \pm 2.2$ keV.

The viewing angle extracted from the observed superluminal motion of the jet in SHB170817A and the rise time $\Delta \approx 0.58$ s to peak value of its prompt emission pulse yield a glory radius

$$ R_g \approx \frac{\gamma \delta c \Delta}{(1+z)} \approx \frac{c \Delta}{(1+z)(1 - \cos \theta)} \approx 1.5 \times 10^{11} \text{ cm}, \quad (6) $$

Note also that the escape by diffusion of a 10.2 $\pm$ 2.2 keV glory/thermal light through the merger ejecta surrounding the newly born neutron star may explain the origin of the second prompt emission pulse in SHB170817A with $\sim 10.3 \pm 1.5$ keV grey body-like spectrum\textsuperscript{17}.

Moreover, assuming that the glory and the CB Lorentz factor in SHBs are approximately standard candles, then the observed mean rise time $\langle \Delta \rangle \approx 25$ ms of individual pulses in ordinary SHBs (where $\theta \approx 1/\gamma$, i.e., $\delta \approx \gamma$) and (eq.6) yields

$$ \gamma^2 \approx \frac{R_g (1+z)}{\Delta c} \approx 202, \quad (7) $$

i.e., $\gamma \approx 14.2$, both in SHB170817A and ordinary SHBs. This value is roughly a factor 30 smaller than the mean value of $\gamma(0)$ of CBs launched in ordinary SN-GRBs\textsuperscript{16}.

The isotropic equivalent energy of GRBs, in the CB model, satisfies\textsuperscript{16} $E_{\text{iso}} \propto \varepsilon_p \gamma^3$. Thus, the CB model predicts that ordinary SHBs and SN-GRBs satisfy

$$ \langle E_{\text{iso}}(SHB) \rangle \sim 10^{-2} \langle E_{\text{iso}}(SN-GRB) \rangle \sim 10^{51} \text{ erg}, \quad (8) $$

and the far off-axis SHB170817A satisfies

$$ \frac{\langle E_{\text{iso}}(170817A) \rangle}{\langle E_{\text{iso}}(SHB) \rangle} \approx \frac{1}{\gamma^6 (1 - \cos \theta)^3} \approx 7.5 \times 10^{-5}, \quad (9) $$

i.e., an $E_{\text{iso}}(170817A) \sim 7.5 \times 10^{46}$ erg, compared to its observed value\textsuperscript{17}, $E_{\text{iso}}(170817A) \approx (5.6 \pm 1.1) \times 10^{46}$ erg.

Note also that the peak energy of the far off-axis SHB170817A, as given by eq.(4), is related to the mean peak energy of ordinary SHBs by

$$ E_p(170817A) \approx \langle (1+z) E_p(SHB) \rangle / \gamma^2 / (1 - \cos \theta) \quad (10) $$

where $\langle E_p(SHB) \rangle \approx 650^{+17}$ keV. Assuming that the evolution function of SHBs is the same as that of long GRBs, (which is the case if SHBs are produced by NSM of compact binaries in core fission during core collapse supernovae explosions of fast rotating massive stars\textsuperscript{15}), then $(1+z) \approx 3$. Thus, for $\gamma^2 \approx 202$ as given by eq.(7), and $\theta \approx 28$ deg, the right hand side of eq.(10) yields 82 keV, in agreement with the observed\textsuperscript{17} $E_p = 86 \pm 19$ keV (T90 = 2.1 s).

Pulse Shape. The observed pulse-shape produced by ICS of glory photons with an exponentially cut off power law (CPL) spectrum, $dn_0/d\varepsilon \propto e^{-\alpha \varepsilon} \exp(-\varepsilon/\varepsilon_p)$ at redshift $z$, by a CB is given approximately\textsuperscript{15} by

$$ E \frac{d^2 N_z}{d\varepsilon dt} \propto \frac{t^2}{(t^2 + \Delta^2)^2} E_{1 - \alpha} \exp(-E/E_p(t)), \quad (11) $$

where $\Delta$ is approximately the peak time of the pulse in the observer frame, which occurs when the CB becomes transparent to radiation, and $E_p \approx E_p(t=\Delta)$. In eq.(11), the early temporal rise like $t^2$ is produced by its increasing cross section, $\pi R^2_{\text{BP}} \propto t^2$, of the fast expanding CB when it is still opaque to radiation. When the CB becomes transparent to radiation due to its fast expansion, its effective cross section for ICS becomes constant. Then, and the density of the ambient photons, which for a distance $r = \gamma \delta c t / (1 + z) > R$ decreases like $n_g(r) \approx n_g(0) (R/r)^2 \propto t^{-2}$, produce the temporal decline like $t^{-2}$. If CBs are launched along the axis of a glory of torus-like pulsar wind nebula (PWN), or of an accretion disk with a radius $R$, then glory photons that intercept a CB at a distance $r$ from the center intercept it at a lab angle $\theta_{\text{int}}$, which satisfies $\cos \theta_{\text{int}} = -r/\sqrt{r^2 + R^2}$. It yields a $t$-dependent peak energy,

$$ E_p(t) = E_p(0)(1 - t/\sqrt{t^2 + \tau^2}), \quad (12) $$

with $\tau = R(1+z)/\gamma \delta c \approx 0.59$ s and $E_p \approx E_p(t=\Delta)$, where $\Delta$ is approximately the peak time of the pulse. For $\alpha$ not very different from 1, integration of $d^2 N(E,t)/dE dt$ from $E = E_m$ upwards yields

$$ N(t, E > E_m) \propto \frac{t^2}{(t^2 + \Delta^2)^2} \exp(-E_m/E_p(t)). \quad (13) $$
A best fit of eq.(13) to the observed pulse shape\textsuperscript{17} for \(E_m = 50\) keV, shown in Figure 1, returned \(\Delta = 0.58\) s, \(\tau = 0.65\) s, and \(E_p(\Delta) \approx 320\) keV. The best fit value of \(\Delta\) yields \(E_p \approx E_p(\Delta) = 94\) keV compared to the observed\textsuperscript{17} \((T90) E_p = 86 \pm 19\) keV.

**Early-time Afterglow.** The X-ray afterglow of ordinary SHBs are well explained by PWN emission powered by the rotational energy loss through magnetic dipole radiation (MDR), relativistic winds and high energy particles of the newly born millisecond pulsars (MSPs) in NSMs In a steady state, the X-ray luminosity powered by the spin-down of the MSP has the universal form\textsuperscript{18}

\[
L(t) = L(0) 1 / (1 + t/t_b)^2, \tag{14}
\]

where \(t_b = P/2 \dot{P}\) and \(P\) being the period of the newly born pulsar at birth. A best fit of eq.(14) to the bolometric light curve\textsuperscript{19} of SHB170817A shown in Figure 2 yields \(L(0) = 2.27 \times 10^{42}\) erg/s, \(t_b = 1.15\) d, with an entirely satisfactory \(\chi^2/\text{dof} = 1.04\).

**The Synchrotron Afterglow.** In the CB model, the electrons that enter the CB with a Lorentz factor \(\gamma(t)\) in its rest frame are Fermi accelerated there, and cool by emission of synchrotron radiation - an isotropic afterglow in the CBs rest frame. As for the rest of the CBs radiations, the emitted photons are beamed into a narrow cone along the CBs direction of motion, their arrival times are aberrated, and their energies boosted by the Doppler factor \(\delta(t)\) and redshifted by the cosmic expansion. The observed spectral energy density of the unabsorbed synchrotron afterglow produced by a CB has the form (e.g., eq.(1) in Ref. [20]),

\[
F_\nu \propto n^{(1+\beta_e)/2} \left[ \gamma(t) \right]^{3\beta_e - 1} \left[ \delta(t) \right]^{\beta_e - 1} \nu^{-\beta_e}, \tag{15}
\]

where \(n(t)\) is the density of the medium encountered by the CB at time \(t\), and \(\beta_e\) is the spectral index of the emitted radiation at a frequency \(\nu\). For a constant density, the deceleration of the CB yields a late time \(\gamma(t) \propto t^{-1/4}\) and \(\delta(t) = 1/\gamma(t) (1 - \cos \theta) \propto t^{1/4} / (1 - \cos \theta)\). Consequently, the apparent superluminal velocity of a CB in a constant low-density environment, stays constant, as long as \(\beta \approx 1\), while its late-time afterglow luminosity increases like \(t^{(1-\beta_e)/2}\). When the CB exits the disk into the halo, it turns into a fast decay \(\propto [n(r)]^{(1+\beta_e)/2}\). Approximating the disk density perpendicular to the disk by \(n(z) = n(0)/(1 + e^{z/d})\) where \(d\) is the surface depth of the disk, the lightcurve of the afterglow of SHB170817A can be approximated by,

\[
F_\nu(t) \propto \frac{(t/t_e)^{1-\beta_e/2} \nu^{-\beta_e}}{[1 + e^{(z-w)/(1+\beta_e)]^{1+\beta_e}/2}} \tag{16}
\]

where \(t_e\) is roughly the escape time of the CB from the galactic disk into the halo after its launch, and \(w\) is the crossing time of the surface of the disk. Such
a behavior of the late-time afterglow may have been observed\textsuperscript{21} in GRB09042515. It is compared in Figures 3,4 to the observed late-time X-ray\textsuperscript{22} and radio\textsuperscript{23} afterglows of SHB170817A.

Conclusions. The first successful measurement of the apparent superluminal motion of the highly relativistic jets, which produce the prompt emission and the beamed afterglow of GRBs and SHBs, finally took place\textsuperscript{9} two decades after the discovery of the afterglow of GRBs at lower frequencies\textsuperscript{24}. This recent achievement not only is an important contribution towards resolving the long standing debate of what is the production mechanism of SHBs and their afterglows, it also appears to confirm a fundamental physical difference between the local and the cosmic values of the Hubble constant. Few more Ligo virgo detections of nearby neutron stars merger followed by far-off axis SHBs with accurate VLBA and VLBI measurement of the superluminal motion of their jets will be required to confirm that beyond doubt.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4.png}
\caption{Left: The measured\textsuperscript{23} lightcurve of the 3 GHz radio afterglow of SHB170817A and the lightcurve predicted by the CB mode $\beta_r = 0.56$, $t_e = 245.6$ d and $w = 63.4$ d.}
\end{figure}

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