THz amplification via a Raman scattering

S. Son
169 Snowden Lane, Princeton, NJ, 08540

A THz light amplifier and the corresponding methods are proposed, based on the Raman interaction between a THz light and visible light lasers. Two lasers, with their frequencies differing by that of a Langmuir wave, counter-propagate inside a background plasma, exciting a rather strong Langmuir wave. The THz light amplification occurs through the non-resonant Raman scattering between the THz light and one of the lasers in the presence of the mentioned Langmuir wave. It is normally the case that the non-resonant scattering does not exchange the energy between E & M fields but the author demonstrates that it is possible under the pre-exciting Langmuir wave. In the presence of a pre-existing Langmuir wave, the ponderomotive density perturbation between the THz light and the visible-light laser is induced with the phase suitable for transferring the energy from the visible light laser to the THz light. The condition for achieving the favorable phase and the amplification strength, when the condition is met, is estimated as an mathematical formula. The gain is as high as or even higher than 100 per centimeter.

PACS numbers: 42.55.Vc, 42.65.Dr, 42.65.Ky, 52.38.-r, 52.35.Hr

I. INTRODUCTION

For a few decades, there have been growing interests for a commercially-viable THz light source, as it would be critical in the fusion plasma diagnostic, the molecular spectroscopy, the tele-communication and many others [1,2]. Numerous THz light sources have been invented [3-22] and in particular, the great advance has been made in the laser-based technology [2]. However, THz light source is not still intense enough for many applications and the current inability to produce intense THz light is referred to as the “THz Gap” [1]. In addition, current light sources often require expensive strong magnets and accelerators, or often need to be operated in extremely low-temperature. Significant progresses, in enhancing the intensity (power), reducing the cost (size) or increasing the efficiency, are necessary for commercial applications of the THz light. If a comparison is made between the progress in the visible light laser and the THz light laser, one obvious missing ingredient in the THz light is an amplifier, wherein a small signal gets amplified to an intense one. If an intense amplifier is available, many obstacles for the THz applications could be overcome. One such amplifier is proposed in this paper based on the Raman scattering.

In the backward Raman scattering (BRS) between two lasers, the ponderomotive force between these lasers excite a Langmuir wave, which is phase-locked to the laser ponderomotive interaction by a quarter cycle. Due to this phase lock, the beating current of this Langmuir density and the laser quiver transfers the energy from the higher frequency laser to the lower frequency laser. It is a tempting idea that the Raman scattering could rotate the energy from the visible light laser to the THz light in a similar fashion. When the THz light interacts with the visible light laser, however, their beating ponderomotive force cannot be resonant, due to their big frequency difference. If it is not resonant, the phase of the excited density is in sync with the ponderomotive force, resulting no energy transfer between the laser and the THz light. Because of this consideration, the non-resonant Raman scattering between a laser and THz light is not seemingly suitable for the THz light amplification.

The main idea of this paper is to circumvent this problem utilizing the pre-existing background Langmuir wave; Two-counter propagating lasers excite a Langmuir wave and the secondary interaction of this Langmuir wave and ponderomotive force between the lasers and the THz light creates a density perturbation with the desirable phase. The condition for achieving the mentioned favorable phase and the amplification strength, when the condition is met, is estimated. The amplification strength is very strong, reaching the gain per-length as high as or even higher than 100 per centimeter.

II. RAMAN SCATTERING AND THE ENERGY TRANSFER BETWEEN THE LASERS

Let us briefly revisit the backward Raman scattering physics. Consider two lasers with the frequency and the secondary interaction of this Langmuir wave and ponderomotive force between the lasers and the THz light creates a density perturbation with the desirable phase. The condition for achieving the mentioned favorable phase and the amplification strength, when the condition is met, is estimated. The amplification strength is very strong, reaching the gain per-length as high as or even higher than 100 per centimeter.

\[
\frac{\partial \delta n_e}{\partial t} = -\nabla \cdot (\delta n_e \mathbf{v}),
\]
\[
\frac{m_e \mathbf{v}}{dt} = e \left( \nabla \phi - \frac{\mathbf{v}}{c} \times \mathbf{B} \right),
\]
Combining the above equations with the Poisson equation $\nabla^2 \phi = -4\pi \delta n_e$, the density response is governed by

$$ (\frac{\partial^2}{\partial t^2} + \omega_{pe}^2) \delta n_e = \frac{e n_0}{m_e c} \nabla \cdot (\mathbf{v} \times \mathbf{B}). $$

$$ = -n_0 (ck_1 + ck_2)^2 a_1^* a_2, \quad (1) $$

where $\omega_{pe}^2 = 4\pi n_0 e^2/m_e$ is the plasma Langmuir wave frequency, $n_0$ is the background electron density, all physical quantities is expressed as $b(z, t) = b_0 \exp(i\omega t - k z) + b^* \exp(-i\omega t + k z)$, $a_{1,2} = e E_i/m\omega_{1,2}c$ is the laser quiver velocity normalized by the velocity of the light and $E_i$ is the electric field of the laser $i$.

The density perturbation from the ponderomotive force of any one pair of the lasers (1, 2) can be estimated from Eq. (1). For an example, the density perturbation from 1, 2 with the frequency $\omega_{pe} \neq \omega_2 - \omega_1$ is given

$$ \delta n(\omega_2 - \omega_1, k_1 + k_2) = -n_0 \frac{(ck_1 + ck_2)^2}{(\omega_1 - \omega_2)^2 - \omega_{pe}^2} a_1^* a_2 $$

$$ = -n_0 C_B a_1^* a_2, \quad (2) $$

where $C_B = (ck_2 + ck_1)^2/(\omega_1 - \omega_2)^2 - \omega_{pe}^2 \gg 1$ is a positive real constant, because $\omega_{1,2} \gg \omega_{pe}$. The lasers will respond to the density perturbation in Eq. (1) by $\delta n$.

FIG. 1: The schematic diagram about the propagation direction of the BRS pump, THz pulse, the electron beam and the plasmon.

$$ \left( \frac{d^2}{dt^2} + \omega_{pe}^2 \right) a = A \cos(\omega t). \quad (6) $$

If $\omega_0 \neq \omega$ ($\omega_0 = \omega$), then the particular solution is $a = A/\omega_0 \sin(\omega t)$ ($a = (A/\omega)t \sin(\omega t)$). This phase change from the cosine function to the sine function is what it needs to happen for the energy channeling between the lasers.

It would be desirable if a energy channeling between the THz light and one laser is possible by the above physics of Raman scattering. However, in the plasma where the THz light can propagate, $\omega_1 \gg \omega_{pe}$ and $\omega_2 \sim \omega_{pe}$, wherein $\omega_3$ is the frequency of the THz light.

### III. THZ LIGHT AMPLIFICATION

Consider two lasers with the frequency $\omega_1$ and $\omega_2$ counter-propagating in the z-direction, where $\omega_2 - \omega_1 = \omega_{pe}$. The density perturbation $\delta n$ will respond to the beating ponderomotive force as given in Eq. (6). As the density excited reaches a considerable level, the assumption that $\delta n \ll n_0$ might not be valid any more, in which situation the zeroth order density would be given as

$$ n = n_0 + \delta n = n_0 + n_L(\omega_{pe}, k_1 + k_2) \equiv n_0 + iC_B a_1^* a_2.$$ 

Let me assume a THz light with the frequency $\omega_3$ and the wave vector $k_3$ is moving in the same direction with $\omega_1$ laser. See Fig. (1) for the directions of lasers and THz light. The density response of the plasma to the ponderomotive interaction between the THz light and two lasers are what we want to estimate. The expansion of the continuity equation is given as

$$ \left( \frac{\partial^2}{\partial t^2} + \omega_{pe}^2 \right) \delta n_e = -\nabla \cdot (\frac{\partial n_L}{\partial t} \mathbf{v}) - \nabla \cdot ((n_0 + n_L) \frac{\partial \mathbf{v}}{\partial t})) $$

$$ \cong -(n_0 + n_L) \nabla \cdot (\frac{\partial \mathbf{v}}{\partial t}) - (\nabla n_L) \cdot (\frac{\partial \mathbf{v}}{\partial t}). \quad (7)$$
The term involving $(\partial n_L/\partial t)$ in Eq. (7) is smaller than other terms by $\omega_{pe}/\omega_{1,2}$, which will be ignored. Then, there will be three remaining terms on the right side of Eq. (7). The first term involving $n_0$ is the usual term of the density response to the ponderomotive force without the presence of the Langmuir wave. The second term with $n_L$ and the third term with $\nabla(n_L)$ is what we will compute now.

In the above equation, the velocity $\mathbf{v}$ is excited by the ponderomotive force between a laser and one laser. There are four ponderomotive interaction $(a_3a_1, a_3a_2, a_3a_3^*)$, which could interact with $n_L$ or $n_L^*$. Then, there are eight density perturbations, $(n_La_3a_1, n_La_3a_2, n_La_3a_3, n_La_3a_3^*, n_L^*a_3a_1, n_L^*a_3a_2, n_L^*a_3a_3, n_L^*a_3a_3^*)$. Those eight density perturbations beat with the laser quiver $(a_1, a_1^*, a_2, a_2^*)$. Therefore, there are 32 combinations. In the end, the author’s interest is how the THz light $a_3$ evolve and, therefore, the whole beating must have the total momentum $k_3$ and the frequency $\omega_3$. There are four such combinations $(n_L[a_3a_3^*]a_2, n_L[a_3^*a_3]a_1, n_L^*[a_3a_3^*]a_2, n_L^*[a_3^*a_3]a_1)$, where $n_L[a_3a_3^*]$ for an example, represents a density perturbation, by the beating of a Langmuir wave $n_L$ and the density perturbation to the ponderomotive interaction by $a_1a_3$, making a beat current with the laser quiver $a_2^*$.

With the above consideration in mind, by computing all four terms, the relevant density response for the second term in Eq. (7) is given as

$$\delta n_1 = +i n_0 \left(-A_1|a_1|^2a_2 + A_2|a_2|^2a_1^*\right) a_3$$

$$= +i n_0 \left(-A_1|a_1|^2a_2^* + A_4|a_2|^2a_1^*\right) a_3,$$  

where $A_1 = (ck_1 + c_3^2)/(\omega_2 + \omega_3)^2$ represents the process of the ponderomotive force by $a_3a_1$ and producing the THz current by beating with the $a_2^*$ quiver and $n_L$, $A_2 = (ck_2 + c_3^2)/(\omega_2 + \omega_3)^2$ represents the ponderomotive force being excited by $a_3a_2^*$, producing the THz current with the $a_1$ quiver and $n_L^*$, $A_3 = (ck_1 - c_3^2)/(\omega_2 - \omega_3)^2$ represents the ponderomotive force by $a_3^*a_1^*$, producing the THz current by beating with the $a_2$ quiver and $n_L^*$ and $A_4 = (ck_2 - c_3^2)/(\omega_2 + \omega_3)^2$ represents the ponderomotive force by $a_3a_3^*$ producing the THz current by beating with the $a_1^*$ quiver and $n_L^*$. The relevant density response for the third term is given as

$$\delta n_2 = +i n_0 \left(B_1|a_1|^2a_2 - B_2|a_2|^2a_1^*\right)$$

$$= +i n_0 \left(B_3|a_1|^2a_2^* - B_4|a_2|^2a_1^*\right) a_3,$$  

where $B_1 = C(c_1 + c_3^2)(ck_1 + c_3^2)/(\omega_2 + \omega_3)^2$ represents the ponderomotive force by $a_3a_1$, producing the THz current by beating with the $a_2^*$ quiver and $n_L$, $B_2 = C(c_2 + c_3^2)(ck_2 + c_3^2)/(\omega_2 - \omega_3)^2$ represents the ponderomotive force by $a_3a_2^*$ producing the THz current by beating with the $a_1$ quiver and $n_L$, $B_3 = C(c_1 - c_3^2)(ck_1 + c_3^2)/(\omega_2 - \omega_3)^2$ represents the ponderomotive force by $a_3^*a_1^*$ producing the THz current by beating with the $a_2$ quiver and $n_L^*$ and $B_4 = C(c_2 - c_3^2)(ck_1 + c_3^2)/(\omega_1 + \omega_3)^2$ represents the ponderomotive force by $a_3a_2$, producing the THz current by beating with the $a_1^*$ quiver and $n_L^*$. Combining Eq. (8) and Eq. (9), we obtain the total density response if we replace $A_{1,2,3,4}$ by $D_{1,2,3,4} = A_{1,2,3,4} - B_{1,2,3,4}$. Combining Eq. (8) and Eq. (9) with Eq. (3) or Eq. (4), we obtains

$$L_3a_3 = \frac{\omega_{pe}^2}{2\omega_3} \left(\Gamma C_B |a_1|^2|a_2|^2\right) a_3$$

$$= \frac{\omega_{pe}^2 n_L}{2\omega_3} (\Gamma |a_1||a_2|) a_3,$$  

where $\Gamma = D_3 + D_4 - D_1 - D_2$ and $|n_L| \equiv C_B |a_1^*a_2|$. The equation (10) is the major result of this paper. As an example, consider $\omega_1 = 5\omega_3$ and $\omega_2 = 6\omega_3$. Then, $A_1 = 0.73, A_2 = 3.06, A_3 = 0.64, A_4 = 0.69, B_1 = 1.35, B_2 = 4.81, B_3 = 1.76, B_4 = 1.53$ and $\Gamma = 0.4$, which is positive. There will be an amplification. As another example, consider $\omega_1 = 10\omega_3$ and $\omega_2 = 11\omega_3$. Then, $\Gamma = 0.1$. As another example, consider $\omega_1 = 15\omega_3$ and $\omega_2 = 16\omega_3$. $\Gamma = 0.05$. The more frequency difference between the THz light and the lasers, the efficiency for the amplification becomes weaker. If $\omega_1 < \omega_3$, then $\Gamma$ will be negative and then there will be decay. It can be concluded that the excitation of the Langmuir wave by the counter-propagating lasers could amplify the THz light if the THz is co-propagate with the lower frequency laser.

For the example with the specific parameter, consider a plasma with $n_0 = 10^{17}$/cm$^3$, $n_L/n_0 = 0.3$, a 6 THz light and two lasers with 10 $\mu$m and 9.0 $\mu$m. Then, it can be estimated as $\Gamma \equiv 1.5 \times 10^{11}/\sqrt{T_{16}P_{16}}$ sec, where $T_{16}$ ($P_{16}$) is the intensity of $a_1$ ($a_2$) laser normalized by $10^{16}$ W/cm$^2$. For the relativistic intensity, the gain-per-length could reach 10 per centimeter. As another example, consider a plasma with $n_0 = 4 \times 10^{17}$/cm$^3$, $n_L/n_0 = 0.3$, a 10 THz light and two lasers with 10 $\mu$m and 8.5 $\mu$m. Then, it can be estimated as $\Gamma \equiv 3 \times 10^{12}/\sqrt{T_{16}P_{16}}$ sec. For the relativistic intensity, the gain-per-length could reach 100 per centimeter.

To summarize, in this paper, a new amplifying mechanism of the THz light is proposed based on the Raman scattering. The desirable phase-locking is provided by the pre-existing Langmuir wave excited by the BRS. As the plasma density responds to the ponderomotive interaction between the THz light and the lasers, the lasers could give away energy into the THz light via the non-resonant Raman scattering. The THz light needs to be propagating with the low-frequency laser. The gain per length could reach 100 per centimeter.

If the THz light is propagating with the low-frequency laser, it is also beneficial. As the Langmuir wave is excited by the BRS between lasers, the energy will be also transferred from the higher frequency laser to the lower one. The lower frequency laser will be amplified as it
extract the energy from the fresh higher frequency laser because the lasers are counter-propagating. The THz light will also benefit from this amplified lower frequency laser because it is co-propagating and experiencing more intense laser.