The Milky Way rotation curve in Horava–Lifshitz theory

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Accepted 2010 March 8. Received 2010 March 5; in original form 2010 January 28

ABSTRACT

The Horava–Lifshitz (HL) theory has recently attracted a lot of interest as a viable solution to some quantum gravity related problems and the presence of an effective cosmological constant able to drive the cosmic speed up. We show here that, in the weak field limit, the HL proposal leads to a modification of the gravitational potential because of two additive terms (scaling, respectively, as $r^2$ and $r^{-2}$) to the Newtonian $1/r$ potential. We then derive a general expression to compute the rotation curve of an extended system under the assumption that the mass density only depends on the cylindrical coordinates $(R, z)$ showing that the HL modification induces a dependence of the circular velocity on the mass function which is a new feature of the theory. As a first exploratory analysis, we then try fitting the Milky Way rotation curve using its visible components only in order to see whether the HL modified potential can be an alternative to the dark matter framework. This turns out not to be the case so that we argue that dark matter is still needed, but the amount of dark matter and the dark halo density profile have to be revised according to the new HL potential.

Key words: gravitation – Galaxy: kinematics and dynamics – dark matter.

1 INTRODUCTION

Inspired by the Lifshitz theory in condensed matter physics, Horava has recently proposed a new theory of gravity based on an anisotropic scaling of space and time in the UV limit. Usually referred to as Horava–Lifshitz (hereafter, HL) theory, the HL proposal shows a reduced invariance, dubbed foliation preserving diffeomorphism invariance, which, however, reduces to the standard one in the IR limit where general relativity is recovered. As an attractive feature, the HL theory turns out to be power counting renormalizable which has motivated the great interest in investigating with great detail its theoretical and cosmological aspects. It is worth remembering that, in its original formulation, two conditions were imposed in order to drive the choice of the field action. First, the projectability condition was supposed to hold true. Since time plays a fundamental role from the very beginning, it was assumed that the lapse function (defined when working in the Arnowitt–Deser–Misner formulation of gravity) has to be a projectable function on the space–time foliation, that is to say a function of time only. Secondly, in order to reduce the number of independent terms entering the action, the principle of detailed balance was used. Unfortunately, it soon became clear that this second condition leads to problems in the low energy limit (Lu, Jianwei & Pope 2009), thus motivating the search for the modification of the original HL theory where the action breaks the detailed balance condition either softly (Kehagias & Sfetsos 2009; Kiristis & Kofinas 2009; Lee, Kim & Myung 2009; Capasso & Polychronakos 2010; Kiristis & Kofinas 2010) or not. In particular, Sotiriou, Visser & Weinfturner (2009a,b) have worked out a modified HL theory with no detailed balance condition imposing that only parity preserving operators enter the potential.

Although the discussion about the foundations and the possible conceptual and phenomenological problems of the HL theory and its modified versions is still open, it is nevertheless worth systematically investigating its consequences at every scale. In particular, it is interesting to study its static spherically symmetric solutions since one can thus derive the gravitational potential generated from a point mass source. Recently, this problem has been addressed by Tang & Chen (2010, hereafter TC10) for the HL theory with the projectability condition and no detailed balance. They argue that only the Minkowski or de Sitter space–time are solutions, but we will show here that this is actually not the case. As a consequence, we find that the gravitational potential generated by a point mass differs

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from the Newtonian one because of the presence of additional terms depending on the HL coupling parameters. Such terms have to be taken into account when computing the potential generated by an extended system, such as a galaxy. Therefore, we work out a general formalism to estimate the rotation curve (i.e. the circular velocity $v_c$ as a function of the distance $R$ from the centre) for an extended source showing that the HL theory can boost the $v_c(R)$ with respect to the Newtonian value. Motivated by this consideration, we then try to fit the Milky Way rotation curve using visible matter only to both constrain the HL parameters and investigating whether it can work as an effective dark matter component.

The plan of the paper is as follows. In Section 2, we look for static spherically symmetric solutions for the HL theory with projectability condition and show that there is indeed a new solution leading to a modified gravitational potential. A general formalism to compute the rotation curve for an extended system is presented in Section 3 and then used in Section 4 to work out the predicted rotation curve for the Milky Way. Here, we also present the data and the results of fitting them with our modified potential and no dark matter. Conclusions are finally given in Section 5.

2 THE POINT MASS POTENTIAL

The weak field limit of the HL theory as proposed by Sotiriou, Visser and Weinfurtner has been yet discussed by TC10 so that here we will only summarize the main steps and stress where our work differs from their one. As stressed in TC10, we start from the observation that not all the static spherically symmetric solutions found for (modifications of) HL theory preserve the projectability condition. In fact, if one assumes a metric

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{g(r)} + r^2d\Omega^2,$$  

(1)

with $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ and use the coordinate transformation $dr = \rho \theta_{\rho} - [\{(1 - N^2)\}^{1/2}/N^2]d\rho$, one can show that a necessary condition for the projectability condition to hold is that $g = N^2$. The line element therefore reads

$$ds^2 = -N^2(t)dt^2 + \frac{(dr + N' dr)(dr + N' dr)}{f} + r^2d\Omega^2.$$  

(2)

The equations of motion for such a metric are given in TC10 and will not be repeated here for the sake of shortness. The authors then argue that in the IR limit the $f$ function is constrained to be 1 thus ending up with the usual Schwarzschild–de Sitter metric as the only solution. We here show that this is actually not the case. To this aim, we first note that the Euler–Lagrange equation for $N_r = N'/f$ in the IR limit reduces to

$$\frac{f'}{f} \frac{N_r}{r} = 0$$  

(3)

which is solved by either $N_r = 0$ or $f = \text{const}$. The first choice gives Minkowski back when choosing $g_0 = 0$ and $f = 1$, or simply reduces to Minkowski by means of a redefinition of the radial coordinate. The second choice therefore seems more interesting. In such a case, the equations obtained by varying with respect to $f$ and $N_r$ reduce to

$$\frac{dN_r^2}{dr} + \frac{N_r^2}{r} + \frac{N^2(t)}{2f^2} v_1(r) = 0,$$  

(4)

$$\int_0^\infty \left[ \frac{dN_r^2}{dr} + \frac{N_r^2}{r} + \frac{N^2(t)}{2f^2} v_2(r) \right] r^3 dr = 0,$$  

(5)

with

$$v_1(r) = -g_0 \xi^6 r + \frac{2\xi^4(1 - f)}{r} + \frac{2\xi^2(1 - f)}{r^3} [2g_2(1 + 7f) + g_4(1 + 5f)]$$

$$+ \frac{2(1 - f)^2}{r^5} [4g_4(1 + 23f) + 2g_2(1 + 17f) + g_4(1 + 14f)] + \frac{8f(1 - f)}{r^5} [2g_7(1 + 7f) + g_7(1 - 4f)].$$  

(6)

$$v_2(r) = -g_0 \xi^6 r + \frac{2\xi^4(1 - f)}{r} + \frac{2\xi^2(1 - f)}{r^3} (2g_2 + 4g_4)$$

$$+ \frac{2(1 - f)^2}{r^5} (4g_4 + 2g_6 + g_8) + \frac{8f(1 - f)^2}{r^5} (g_7 + g_8).$$  

(7)

For $f = 1$, we get $v_1(r) = v_2(r) = -g_0 \xi^6 r$ so that equations (4) and (5) are equal; hence, the solution for $N_r(r)$ is the same. However, differently from what stated in TC10, this is not the only possibility. Indeed, in order to have the same $N_r(r)$ solving both equations (4) and (5), one must have $v_1(r) = v_2(r)$ which is possible by equating the coefficients of the terms with equal orders in $r$ in the two functions. Comparing $v_1(r)$ and $v_2(r)$, one has just to equate the coefficients of the terms in $r^{-3}$ and $r^{-5}$, thus obtaining the following two equations:

$$(1 - f)[2g_2(1 + 7f) + g_4(1 + 5f)] = (1 - f)^2 (2g_2 + 4g_4),$$  

(8)

$$2(1 - f)^2 [4g_4(1 + 23f) + 2g_2(1 + 17f) + g_4(1 + 14f)]$$

$$+ 8f(1 - f) [2g_7(1 + 7f) + g_7(1 - 4f)]$$

$$= 2(1 - f)^3 (4g_4 + 2g_6 + g_8) + 8f(1 - f)^2 (g_7 + g_8).$$  

(9)

which allows us to set two of the quantities $(g_1, \ldots, g_8, f)$ as a function of the others. Provided this condition has been satisfied, the solution of equation (4) automatically solves also the integral condition (5), so that we can limit our attention only to equation (4). This is a linear first-order equation in $N_r^2(r)$ which can be analytically solved giving

$$N_r(r) = \pm \sqrt{\frac{M}{r^2}} - \frac{N^2(t)}{2f^2} \left( \frac{A}{r^2} + B - C \frac{D}{r^2} \right),$$  

(10)

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where $\mathcal{M}$ is an integration constant and we have defined

$$
\begin{align*}
A &= -g_0 \xi^6 \\
B &= 2 \xi^5 (1 - f) \\
C &= 2 \xi^2 (1 - f) [2 \xi_3 (1 + 7 f) + \eta_3 (1 + 5 f)] \\
D &= 2 (1 - f)^2 [4 \xi_4 (1 + 23 f) + 2 \xi_4 (1 + 17 f) \\
&+ \eta_8 (1 + 14 f)] + (1 - f) f [2 \xi_3 (1 + 7 f) \\
&+ \eta_8 (1 - 4 f)]
\end{align*}
$$

In order to derive the gravitational potential, we have first to rewrite the line element in the usual Schwartzschild-like form, i.e.

$$\mathrm{d}s^2 = -g_{00}(r)\mathrm{d}t^2 + g_{rr}(r)\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2$$

and then use the general relations

$$g_{00}(r) = N^2 - f N^2, \quad g_{rr}(r) = \frac{N^2}{f (N^2 - f N^2)}.$$ 

Fixing $N = 1$ (which is always possible by a rescaling of the $t$ coordinate), we then get

$$g_{00}(r) = 1 - \frac{f \mathcal{M}}{r} + \frac{1}{2 f} \left( \frac{A}{3} r^2 + B - \frac{C}{r^2} - \frac{D}{3 r^2} \right),$$

$$g_{rr}(r) = 1 / f g_{00}(r).$$

Since the gravitational potential generated by a point-like mass particle may be easily recovered from the usual relation $g_{00}(r) = 1 + 2 \Phi(r)$, we then find

$$\Phi(r) = -\frac{f \mathcal{M}}{2 r} + \frac{1}{4 f} \left( \frac{A}{3} r^2 + B - \frac{C}{r^2} - \frac{D}{3 r^2} \right).$$

Note that, up to now, the two integration constants $f$ and $\mathcal{M}$ are still undefined. However, in order to recover the usual Newtonian potential in the GR limit (i.e. for $A = B = C = D = 0$), we must set $f \mathcal{M} = 2 G m$, with $m$ being the mass of the gravitational field source. It is worth noting that, for $m = 0$, the potential (14) leads to unphysical divergences in the metric because of the terms in $C/r^2$ and $D/3r^4$. In order to avoid this problem, we must impose that both $C$ and $D$ vanish for $m = 0$. The first trivial choice is to impose $C = D = 0$ identically so that one recovers the usual Schwartzschild–de Sitter case. As a more attractive possibility, one can postulate a dependence of the metric coefficient $f$ on the mass which is not possible. As such, we consider them as algebraic equations for $f$ so that, in order to be fulfilled, we must equate the terms with equal powers of $f$ on the two sides. We then obtain

$$\begin{align*}
g_2 &= g_3 = 0 \\
g_4 &= -(36 g_3 + 15 g_6 + 4 g_7)/96 \\
g_8 &= (16/3) g_7
\end{align*}$$

which implies $C = 0$ and

$$D = \frac{(1 - f)^2}{12} [12 (1 - f) g_5 - 9 (1 - f) g_6 + 4 (1 - 153 f) g_7].$$

With these conditions, we can finally write the gravitational potential generated by a points mass $m$ as

$$\Phi(r) = \Phi_N(r) + \Phi_{\text{IL}}(r),$$

where $\Phi_N(r) = -Gm/r$ is the Newtonian potential and

$$\Phi_{\text{IL}}(r) = \frac{A}{12 f r^2} + \frac{B}{4 f} - \frac{D}{12 f r^2}$$

is the correction due to HL theory. It is worth noting that, while the second term is simply a constant having no impact on the dynamics, the other two terms have a simple relation with previous results in literature. Indeed, the first one plays the role of a cosmological constant thus remembering the Schwartzschild–de Sitter solution already found in TC10. On the other hand, the third term has the same asymptotical behaviour of the corrections to the Newtonian potential in the Kehagias–Sfetsos (2009, hereafter KS) static spherically symmetric solution of HL gravity. Iorio & Ruggiero (2009) have shown that such corrections are proportional to $m^2 r^{-4}$, thus suggesting that $D$ should actually be a function of the mass $m$ generating the gravitational field, a point which we will come back to later.

The corrective term may be conveniently written as

$$\Phi_{\text{IL}}(r) = -\frac{G M_0}{r_s} \left[ -\left( \frac{\eta}{\eta_A} \right)^2 - \frac{B_r}{4 f G M_0} + \left( \frac{\eta}{\eta_D} \right)^{-4} \right]$$

where $M_0$ being the Sun mass and $r_s$ an arbitrary chosen reference radius introduced to define the dimensionless quantity $\eta = r/r_s$. In equation (17), we have finally defined the scaling radii:

$$\begin{align*}
\eta_A &= \left( \frac{12 f G M_0}{A r_s} \right)^{1/2} \\
\eta_D &= \left( \frac{D r_s^{-3}}{12 f G M_0} \right)^{1/4}
\end{align*}$$

which can be used as model parameters instead of the $(A, D)$ coefficients. Some caveats are in order here. First, note that, in order to preserve the interpretation of $(r_s, r_D) = r_s \times (\eta_A, \eta_D)$ as physical radii, one has to postulate that the $(A, D)$ parameters are positive quantities, thus narrowing the space of the parameters $(g_0, g_5, g_6, g_7, f)$ entering $(A, D)$. Should $A$ or $D$ be negative, we could none the less define $(r_s, r_D)$ as above and accordingly change the sign of the corresponding term in the potential. For definiteness, both $A$ and $D$ are positive so that the potential is given by equation (17) without any change in the sign. We then remember that, in our scheme, $f$ is a dimensionless function of $\mu = m/M_0$, but its functional dependence cannot be obtained in any way. The only constraint we have is $f(\mu = 0) = 1$ so that $D = 0$, and we recover the Schwartzschild–de Sitter solution in accordance with the fact that $g_0$ (and hence $A$) plays the role of a cosmological constant term. In order to parametrize our ignorance, we can redefine the above scaling radii as

$$\begin{align*}
\eta_A &= \left( \frac{12 f G M_0}{A r_s} \right)^{1/2} \left[ f(\mu) \right]^{1/2} \\
\eta_D &= \left( \frac{D r_s^{-3}}{12 f G M_0} \right)^{1/4} \left[ D(\mu) f(\mu) \right]^{1/4}
\end{align*}$$

where quantities labelled with $\circ$ are evaluated for $\mu = 1$. It is worth wondering whether some hint on the functional expression of $f(\mu)$ can be retrieved. Up to now, we have postulated $f = f(\mu)$ in order to get a mathematically viable solution other than the Schwartzschild–de Sitter one. It is, however, worth noticing that

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1 Note that, following TC10, we have used units in which $Z = 1$ with $Z$ a dimensional parameter of the HL theory. In order to have $(A, B, C, D)$ expressed in the more common $c = 1$ units, one has simply to multiply each term in equation (11) by $\xi^{-4}$ with $\xi$ having, in these units, the dimension of a length.
a dependence of \( f \) on the mass may also be physically motivated. Although the HL theory is obtained by modifying general relativity, it is still true that the properties of the space–time are determined by the source of the gravitational field. As such, one can expect that the corrective HL term to the Newtonian potential is still related to the only property characterizing the field, i.e. the source mass \( m \).

Since the \((A, B, D)\) coefficients in equation (16) are related to the HL couplings (and hence are universal quantities), the only way to introduce a dependence of the solution on the source properties is to postulate \( f = f(\mu) \). Although qualitative, this discussion shows that our mathematical assumption is actually deeply related to a physical motivation.

Finally, we note that the second term in equation (17) simply adds a constant to the potential which has no effect in any situation of interest so that we will henceforth neglect this term. Note that this by no way means that \( B \) can be set to zero. Indeed, \( B = 0 \) means \( f = 1 \) so that also \( D \) vanish and we go back to the Schwarzschild–de Sitter solution, while we are here interested in the more general case. We therefore assume \( B \neq 0 \), but nevertheless neglect its contribution hereafter because drops off from the derivation of the quantities we are interested in.

3 THE ROTATION CURVE

The modified gravitational potential derived above deviates from the Newtonian one because of the additive terms in equation (17). Depending on the values of the scaling radii \((r_A, r_D)\), we can have different situations. As a general remark, we note that, for \( \eta \ll 1 \), the last term in equation (17) increases the potential with respect to the Newtonian one. In contrast, for \( \eta \gg 1 \), the first term boosts the potential making it deviating from the Keplerian fall off. When considering the rotation curve, \( v_c(r) = r d\Phi/dr \), we thus get a circular velocity which may significantly differ from the Newtonian one being larger than the classical value both in the inner and outer regions. It is therefore worth wondering whether such deviations may help in fitting spiral galaxies rotation curves without the need of other mass than the visible one. That is to say, we are here interested in investigating whether the HL modifications to the potential can also play the role of an effective dark halo. As a preliminary remark, we stress that a similar analysis could in principle be made also for the KS solution. However, in that case, the corrections fade away as \( r^{-2} \) and there is no cosmological constant term. It is therefore expected that the correction to the rotation curve in the outer regions is negligible so that we argue that the KS solution cannot play the role of an effective dark halo.

To this end, we have first to derive an expression for the rotation curve of an extended system generalizing the procedure adopted in the Newtonian gravity framework. In that case, the circular velocity in the equatorial plane is given by \( v^2_c(R) = R d\Phi/dR \), with \( \Phi \) being the total gravitational potential. Thanks to the superposition principle and the linearity of the point mass potential on the mass \( m \), the latter is computed by adding the contribution from infinitesimally small mass elements and then transforming the sum into an integral over the mass distribution. Such a simple procedure cannot be applied in the HL case since the corrective term \( \Phi_D \) does depend on \( m \) in a way that we do not explicitly know so that a non-linear dependence cannot be excluded a priori. In order to overcome this difficulty, we have developed an alternative procedure which can be actually applied to any kind of potential provided some general conditions hold.

As a starting point, let us denote by \( F_p(m, r) \) the gravitational force (per unit of test mass particle) generated by a point mass \( m \). Whatever is the dependence of \( F_p \) on \( m \), it is always true that the total force due to \( N \) particles of mass \( m \) is the sum of the single forces. As such, taking the continuum limit, we can estimate the (magnitude of the) total force as

\[
F(r) = \int n(m, r') F_p(m, |r - r'|) \, dm \, dV,
\]

where the integral is over the full mass range and volume and \( n(m, r) \) is the star mass function (hereafter MF), i.e. the number of stars in the volume element \( dV \) with mass between \( m \) and \( m + dm \). Because of its definition, we have

\[
\int_{m_{\text{min}}}^{m_{\text{max}}} n(m, r) \, dm = \rho(r),
\]

with \( \rho(r) \) being the mass density. As usual in literature, we will adopt the factorization hypothesis, thus writing \( n(m, r) = \psi(m) \bar{\rho}(r) \), with \( \psi(m) \) being the local\(^2\) MF and \( \bar{\rho} = \rho(r)/\rho_0 \) with \( \rho_0 = \rho(R_0) \). Defining \( \mu = m/M_\odot \), we have the following normalization condition for the local MF:

\[
\int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \mu \psi(\mu) \, d\mu = \rho_0/M_\odot
\]

so that henceforth we use as local MF the quantity \( N_1 \psi(\mu) \) with

\[
N_1 = \frac{\rho_0}{M_\odot} \left\{ \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \mu \psi(\mu) \, d\mu \right\}^{-1}.
\]

Let us now assume that the point mass gravitational force may be factorized as

\[
F_p(\mu, r) = \frac{GM_\odot}{r_s} f_\mu(\mu) f_r(\eta),
\]

with \( f_\mu \) and \( f_r(\eta) \) being the dimensionless functions depending on the particular form of the point mass gravitational potential \( \Phi_p \). Remembering that \( F_p = -\nabla \Phi_p \), it is only a matter of algebra to show that it is

\[
f_\mu(\mu) = \mu, \quad f_r(\eta) = 1/\eta^2,
\]

for the Newtonian potential. For the HL corrective potential, we can split it as the sum of two terms, i.e.

\[
\Phi_{\text{HL}}(r, r_A, r_D) = \Phi_A(r, r_A) + \Phi_D(r, r_D),
\]

and then obtain

\[
f_\mu = \left( \frac{\eta_{\text{A}}}{} \right) \left( \frac{\eta}{\eta_{\text{A}}} \right)^A, \quad f_r = \left( \frac{\eta_{\text{D}}}{} \right)^{-5}.
\]

for the \( \Phi_A \) and

\[
f_\mu = \left( \frac{\eta_{\text{D}}}{} \right) \left( \frac{\eta_{\text{D}}}{} \right)^{-5}, \quad f_r = \frac{D(\mu)}{D(\mu)} f_\mu(\mu),
\]

for the \( \Phi_D \) term. We now use cylindrical coordinates \((R, \theta, z)\) and the corresponding dimensionless variables \((\eta, \theta, \zeta)\) (with \( \zeta = z/r_s \)) and rely on the factorization hypotheses for both the MF and the point mass force to finally get

\[
F(r) = G \rho_0 r_s \times \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \mu \psi(\mu) \, d\mu \times \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \mu \psi(\mu) \, d\mu 
\]

\[
\times \int_0^\infty \eta' \, d\eta' \int_{-\infty}^{\infty} \, d\zeta' \int_0^{\eta'} f_A(\Delta \rho)(\eta', \theta', \zeta') \, d\theta' \int_0^{\eta'} f_A(\Delta \rho)(\eta', \theta', \zeta') \, d\theta'.
\]

\(^2\) Note that the term local refers to the Solar neighborhood when the galaxy is the Milky Way. For external galaxies, by local, we mean the MF in the neighborhood of a suitably chosen reference radius \( R_0 \).
with the shorthand notation
\[ \Delta = [\eta^2 + \eta'^2 - 2\eta\eta' \cos(\theta - \theta') + (\zeta - \zeta')^2]^{1/2}. \]

The circular velocity in the equatorial plane along the major axis (which is the quantity typically measured for spiral galaxies) will be simply
\[ v^2(R) = R\dot{F}(R, \theta = z = 0). \]
Since we will be interested in axisymmetric systems, we can set \( \dot{\rho} = \rho(\eta, \xi) \). Moreover, our systems will be spiral galaxies, hence made out of a spheroidal bulge and a circular disc, so that a convenient choice for the scaling radius \( r_o \) will be the disc scalelength \( R_d \). Under these assumptions, the rotation curve may then be evaluated as
\[ v_c^2(R) = G\rho_0 R_d^2 \eta \times \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} f_0(\mu) \psi(\mu) d\mu \times \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \mu \psi(\mu) d\mu \times \int_0^\infty \eta' d\eta' \int_{-\infty}^\infty \rho(\eta', \xi') d\xi' \int_0^\eta f_1(\Delta_0) d\eta' \]
with
\[ \Delta_0 = \Delta(\theta = \zeta = 0) = [\eta^2 + \eta'^2 - 2\eta\eta' \cos \theta' + \zeta^2]^{1/2}. \]

It is worth noting that, except in very particular cases (e.g. the Newtonian potential for a spherically symmetric mass distribution), the integrals over the coordinates in equation (25) have to be evaluated numerically. For computational reasons, it is useful to exchange the order of integration and resort to the logarithmic variables \( \lambda = \log \eta \) and \( \omega = \log \xi \) so that we get the following equivalent expression for the rotation curve:
\[ v_c^2(R) = G\rho_0 R_d^2 (\ln 10)^2 \delta \log(\lambda) \times \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} f_0(\mu) \psi(\mu) d\mu \times \int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \mu \psi(\mu) d\mu \times \int_{-1}^1 \frac{\hat{R}(\lambda, \xi)}{\sqrt{1 - \xi^2}} d\xi \]
with \( \delta = 10^\nu \) and
\[ R = \int_{-\infty}^{\infty} \delta(2\lambda') d\lambda' \int_{-\infty}^{\infty} f_1(\Delta_0) \rho(\lambda', \omega') d\omega' d\lambda'. \]

Equations (25) and (27) are fully general and can be used to compute the rotation curve provided the expressions for \( f_0(\mu) \) and \( f_1(\eta) \) are given. As a consistency check, it is easy to show that, for the Newtonian potential, the term depending on the MF is identically zero so that equation (25) reduces to a simple rewriting of the standard result. For the HL term, we get a dependence on the MF through the multiplicative term on the second row of equation (25). It is worth stressing that the MF here only plays the role of scaling the surface brightness profile due to the two visible components, namely the bulge and the disc. What we need to evaluate in the rotation curve is, however, the mass density so that, even assuming simple functional forms for the bulge and the disc, we still do not have any knowledge of the bulge and disc mass-to-light ratios (M/L), thus adding more parameters (and hence severe degeneracies) to be determined. As a first preliminary test, we therefore limit our attention to the Milky Way (hereafter MW) only. Our position within it allows us to determine the local MF (e.g. by star counts or converting the observed luminosity function into an MF through an empirically determined M/L ratio), thus reducing the uncertainties of the problem. Moreover, we also have direct determinations of the galactic parameters of interest so that the only unknown quantities are the HL parameters \( (r_A, r_D) \), thus strongly reducing the possibility of degeneracies. In the following, we first describe the MW mass models and the data on the rotation curve and then present the fitting procedure and the results.

4.1 The mass models

It is common to describe a spiral galaxies as the sum of a visible component (made out of stars and gas) embedded in a dark halo (mainly populated by cold dark matter particles). Lacking any definitive laboratory evidence for DM, the only reason why one has to include it in Galaxy modelling is to fit the rotation curve data. Since this evidence implicitly assumes that the Newtonian potential theory is the correct one, there is actually no compelling reason why one should add a priori a dark halo to the visible components in a modified gravity framework as the current HL theory. We therefore model the MW as made out of visible matter only distributed in a spheroidal bulge\(^3\) and a thick disc.

We follow Dehnen & Binney (1998, hereafter DB98) describing the bulge as a truncated power-law model, i.e. the scaled mass density reads
\[ \rho_b(R, z) = \left( \frac{\rho}{R_b} \right)^{-\nu} \left[ 1 + \frac{R}{R_b} \right]^{-\beta} \exp \left( -\frac{\rho^2}{R_b^2} \right), \]

\(^3\) Actually, there are different evidences that the bulge has a tri-axial structure. We nevertheless use a less detailed spheroidal model since the bulge contribute to the dynamics over the range probed by the data is much smaller than the disc one, independently of the gravitational theory adopted. Such a simplification allows us to use equation (25) without introducing any significative bias.
with $q^2 = R^2 + z^2/r^2$. Fitting the model to the infrared photometric Cosmic Background Explorer/Diffuse Infrared Background Experiment data yields values for the model parameters, namely

$$\beta = y = 1.8, \quad q = 0.6, \quad R_b = 1 \text{ kpc}, \quad R_t = 1.9 \text{ kpc}.$$  

The reference density is not determined from the photometry, but may be related to the total bulge mass $M_b$ as

$$\rho_{0,b} = \frac{M_b}{2\pi \int_0^\infty R \, dR \int_{-\infty}^\infty \rho(R, z) \, dz}.$$  

Dwek et al. (1995) have found $M_b = (1.3 \pm 0.3) \times 10^{10} \, M_\odot$ for the bulge mass so that we set $M_b$ to its central value neglecting the measurement uncertainty. We have then to set the local MF for the bulge. Zoccali et al. (2000) have determined it from the luminosity function of lower main-sequence stars finding $\psi_b(\mu) \propto \mu^{-\beta}$, with $\beta = -1.33 \pm 0.07$ for stars in the mass range $(0.15, 1.0) \, M_\odot$. We extend it to the brown dwarfs region and neglect the measurement uncertainty thus setting $(\beta, \mu_{\min}, \mu_{\max}) = (-1.33, 0.03, 1.0)$ as our bulge MF parameters.

While important in the inner regions, the bulge only plays a minor role in determining the circular velocity over the regions probed by the data. The dynamics is here dominated by the disc component that we model as a double exponential. The mass density then reads

$$\rho_d(R, z) = \exp\left(-\frac{R}{R_d}\right) \exp\left(-\frac{|z|}{z_d}\right),$$

where we follow DB98 fixing:

$$R_d = \kappa_d R_0, \quad z_d = 0.18 \text{ kpc},$$

with $\kappa_d = 0.30 \pm 0.05$ being a scaling parameter and $R_0$ being the Sun distance from the MW centre. We then follow Cardone & Sereno (2005) adopting $\kappa_d = 0.30$ and $R_0 = 8.5$ kpc as fiducial parameters. The disc reference density is related to the disc total mass as

$$\rho_{0,d} = \frac{M_d}{4\pi R_d^2 z_d},$$

while the disc mass is estimated as

$$M_d = 2\pi R_d^2 \Sigma_\odot \exp(R_0/R_d),$$

with $\Sigma_\odot = 48 \pm 8 \, M_\odot \text{ pc}^{-2}$ (Kuijken & Gilmore 1989) being the disc surface density at the Sun position. A caveat is in order here. The measured value of $\Sigma_\odot$ refers to the total mass, both stars and gas. In principle, we should separate the two components computing the total disc mass using $\Sigma_\star$ instead of $\Sigma_\odot$, with $\Sigma_\star = \Sigma_\odot - \Sigma_{\text{ISM}}$ and $\Sigma_{\text{ISM}} = 14.5 \, M_\odot \text{ pc}^{-2}$ (Olling & Merrifield 2001) being the gas surface density. We should then add a further disc-like component for the gas with its density profile. As a reasonable approximation, we can, however, assume that the gas follows the same density profile as the stellar disc with the same scalelength radius so that the total disc mass is indeed given by equation (33).

As a final ingredient, we need the local disc MF. This is determined quite accurately thanks to our position in the disc plane. We refer the reader to Chabrier (2003) and references therein for a detailed discussion of this issue motivating our choice of adopting the Kroupa MF; i.e. a power-law MF with slope $\beta$ changing with the mass range as

$$\beta = \begin{cases} 0.3 & 0.01 \leq \mu \leq 0.08 \\ 1.3 & 0.08 \leq \mu \leq 0.50 \\ 2.3 & 0.50 \leq \mu \leq 1.0 \\ 4.5 & 1.0 \leq \mu \leq 120.0. \end{cases}$$

The four different ranges contribute to the local mass density according to the following percentages:

$$(f_1, f_2, f_3, f_4) = (7.39, 48.21, 29.22, 15.18) \text{ per cent}$$

so that we can compute the multiplicative term entering the rotation curve for the HL term.

### 4.2 The data

Although not ideal targets for our test because of the poor knowledge of their parameter and MF, external galaxies are better suited for the measurement of precise and extended rotation curves. In contrast, our position in the MW disc equatorial plane makes it difficult to observationally determine this quantity. Indeed, one has to characterize the full line-of-sight velocity distribution in order to correct the observed velocity for asymmetric drift and projection effects. Moreover, the distance to the tracer should be known with great accuracy not to bias in a dangerous way the estimated circular velocity. Notwithstanding these difficulties, the rotation curve in the outer regions has been measured relying on Cepheids and H II regions. We follow DB98 to estimate the circular velocity from the Cepheids data of Pont et al. (1997) and the H II molecular clouds sample of Brand & Blitz (1993) using their same selection criteria. This data set probes the radial range $8.2 \leq R \leq 18.94 \text{ kpc}$ and is affected by a large scatter due to both measurement errors and the inhomogeneity of the data. In order to smooth the data without introducing any bias or spurious correlations, we use the local regression method (Loader 1999). Originally, proposed by Cleveland (1979) and further developed by Cleveland and Devlin (1988), the local regression technique combines much of the simplicity of linear least-squares regression with the flexibility of non-linear regression. The basic idea relies on fitting simple models to localized subsets of the data to build up a function that describes the deterministic part of the variation in the data, point by point. Actually, one is not required to specify a global function of any form to fit a model to the data so that there is no ambiguity in the choice of the interpolating function. Indeed, at each point, a low degree polynomial is fit to a subset of the data containing only those points which are nearest to the point whose response is being estimated. The polynomial is fit using weighted least squares with a weight function which quickly decreases with the distance from the point where the model has to be recovered. We hence use this method\(^4\) to smooth the sample points and cut data with $R > 14 \text{ kpc}$ since the local regression method becomes unreliable for these points because of the sparseness of the sample in this region. We finally give it away points with $\text{S/N} \leq (\text{S/N})_{\text{min}}$ where the threshold signal-to-noise ratio has been set to $\text{S/N}_{\text{min}} \simeq 8$ for reasons explained above. We will refer to this sample as the DB data in the following.

Another possible tracer of the velocity field in the outer regions is represented by the blue horizontal branch (BHB) stars. Photometry and spectra from the Sloan Digital Sky Survey (SDSS) for a sample

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\(^4\) See, e.g. Capozziello, Cardone & Troisi 2007 for a step-by-step description.
of 2466 BHB stars allow us to determine the rotation curve up to
~60 kpc as described in Xue et al. (2008, hereafter X08). In order
to extract \( v_c(R) \) taking care of all the possible projection effects
and systematic biases (due to, e.g. the survey mask and targeting
algorithm), X08 have relied on mock matched observations based
on two different cosmological simulations of MW-like galaxies. As
such, there are two different data sets labelled as \( N \) and \( S \) depending
on the simulation adopted. We here use only the \( N \) data set since it is
in better agreement with the DB sample over the range of overlap.
Note that this sample (referred to hereafter as the SDSS data set) is
made out of only 10 points obtained by binning the 2466 BHB stars
in almost equally spaced radial bins covering the range (7.5, 55) kpc.
The \( S/N \) is quite high with a median value \( (S/N)_{\text{med}} \approx 8 \) so that
we set \( (S/N)_{\text{min}} = (S/N)_{\text{med}} \) in order to give the same statistical
weight to both data sets. As a final remark, we note that, while
the DB data probe only the outer disc being \( R/R_0 \approx 3.2-5.4 \), the
SDSS sample extends mainly in the halo region most of the data
having \( R/R_0 > 4 \) and extending up to \( R/R_0 \approx 20 \). As such, the
two samples nicely complement each other and allow us to check
whether the outer rotation curve may be reproduced without any
dark matter contribution and, at the same time, still preserving the
agreement with the data in the disc region.

4.3 Fitting procedure

In order to constrain the HL model parameters, we employ a stan-
dard Bayesian approach first defining the likelihood function as

\[
\mathcal{L}(p) \propto \exp \left[ -\frac{\chi^2(p)}{2} \right] = \exp \left\{ -\frac{1}{2} \sum_{i} \left[ \frac{v_{\text{obs}}^i(R) - v_{\text{th}}^i(R)}{\epsilon_i} \right]^2 \right\}, \tag{34}
\]

where \( p = (\log \eta_A, \log \eta_C, \log \eta_D) \) are the HL model parameters, \( v_{\text{obs}}^i(R) \) and \( v_{\text{th}}^i(R) \) are the observed (with a measurement error \( \epsilon_i \))
and theoretically predicted circular velocities at the radius \( R_i \) of the \( i \)th point and the sum runs over the 101 DB and
10 SDSS data points.

The best fit is obtained by maximizing the likelihood \( \mathcal{L}(p) \), but
it is worth stressing that, according to the Bayesian philosophy,
the best estimate of the parameter \( p_i \) is not the best-fitting one. In
contrast, one has to marginalize over the remaining parameters
and look at the shape of the marginalized likelihood function defined as

\[
\mathcal{L}(p_i) \propto \int \mathcal{L}(p) dp_1 \ldots dp_{i-1} dp_{i+1} \ldots dp_n,
\]

with \( n \) being the total number of parameters. Actually, what we do
is running a Monte Carlo Markov Chain code to efficiently explore
the three-dimensional parameter space \( (\log \eta_A, \log \eta_C, \log \eta_D) \)
and use of the histogram of the values for the parameter \( p_i \) to estimate
the mean, the median and the 68 and 95 per cent confidence ranges.
Note that, because of degeneracies among the model parameters,
the best-fitting parameters \( p_{\text{ML}} \) may also differ from the maximum
likelihood ones \( p_{\text{ML}} \), i.e. the set obtained by maximizing each of
the marginalized likelihood functions.

5 We use logarithmic units in order to explore a wider range. Note also that,
rigorously speaking, equation (34) is not correct since the SDSS data points
are somewhat correlated being obtained by a binning procedure. However,
then the bin spacing is quite large, it is likely that the correlation matrix (not
available) is close to diagonal so that equation (34) is essentially correct.

Figure 1. Best-fitting HL model rotation curve superimposed to the DB and
SDSS data.

4.4 Results

As a preliminary discussion, clearly it is worth stating which are
the parameters we can constrain. Equation (25) shows that, in or-
der to compute the HL contributions to the rotation curve, namely
\( v_A^2(R) \) and \( v_D^2(R) \), we must know the functional expression of \( f(\mu) \)
entering \( f_a(\mu) \) through the scaling radii \( (R_A, R_D) \). Since we do not
know this function, we cannot separately constrain the parameters
\( (R_A, R_D) \). It is, however, easy to show that

\[
\kappa_A v_A^2(R, \eta_{A,0}) = v_A^2(R, \eta_{A,1}),
\]

\[
\kappa_D v_D^2(R, \eta_{B,0}) = v_D^2(R, \eta_{D,1}),
\]

where we have defined

\[
\kappa_A = \frac{\int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \left[ f(\mu) / f_C \right]^{-1} \psi(\mu) d\mu}{\int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \psi(\mu) d\mu},
\]

\[
\kappa_D = \frac{\int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \left[ D(\mu) / D_C \right] \left[ f(\mu) / f_C \right]^{-1} \psi(\mu) d\mu}{\int_{\mu_{\text{min}}}^{\mu_{\text{max}}} \psi(\mu) d\mu},
\]

while \( \eta_A = r_A / R_{\text{eff}} \) and \( \eta_D = r_D / R_{\text{eff}} \) are considered as a constant
parameter not depending on \( \mu \), and we assume that \( v_A^2(R, \eta_{A,0}) \)
and \( v_D^2(R, \eta_{D,0}) \) are evaluated setting \( f_{\mu} = \mu \) in equation (25). Our fit will
then give constraints on \( (\eta_A, \eta_D) \), while those on \( (\eta_{A,0}, \eta_{D,0}) \) could
be derived provided a theoretically motivated functional expression
for \( f(\mu) \) is given.

Skipping to logarithmic units to investigate a larger range, we run
a single chain with 150 000 points reduced to \~10 000 after cutting
out the initial burn in phase and thinning the chain (with a step of
10) to avoid spurious correlations. The best-fitting point turns out
to be

\[
(\log \eta_A, \log \eta_D) = (1.051, -0.060)
\]
giving \( \chi^2 / \text{dof} = 523.56 / 109 = 4.80 \). Needless to say, such a high
reduced \( \chi^2 \) is a strong evidence against the HL model with no dark
matter. A simple look at Fig. 1 shows that the large \( \chi^2 \) is due mainly
to the SDSS data. Indeed, it is \( \chi^2_{\text{SDSS}} / \text{dof} = 235.91 / 99 = 2.38 \) and
\( \chi^2_{\text{SDSS}} / \text{dof} = 287.65 / 8 = 35.96 \) for the DB and SDSS samples,
respectively. Although the model is clearly at odds with the data,
we give below, for completeness, the constraints on the parameters from the analysis of their histograms along the chain. For log $\eta_A$ we get

$$\langle \log \eta_A \rangle = 1.049, \quad \langle \log \eta_A \rangle_{\text{med}} = 1.050,$$

68 per cent: (1.040, 1.059), 95 per cent: (1.028, 1.066),

while it is

$$\langle \log \eta_D \rangle = -0.061, \quad \langle \log \eta_D \rangle_{\text{med}} = -0.060,$$

68 per cent: (−0.064, −0.056), 95 per cent: (−0.068, −0.054),

for log $\eta_D$. Converting median values in linear units, we get

$$\langle r_A, r_D \rangle = (11.2, 0.87) \text{ kpc},$$

which allows us to make some qualitative interpretation of the results. Considering the form of the potential for the point mass case, we see that only the first term can actually boost the stellar contribution to the rotation curve. This term essentially translates into a contribution to $v_r(R)$ which linearly increases with $R/r_A$ so that $r_A$ must be small in order to fit the data in the DB probed region without dark matter. On the other hand, a too small value may lead to a $v_r(R)$ soon diverging thus spoiling down the agreement with the SDSS data. The value $r_A = 11.2$ kpc we find is the result of this compromise, but it is nevertheless unable to reconcile the model with the SDSS data. A similar compromise drives the fit in the estimates of log $\eta_D$. Because of the scaling with $r^{-4}$ in the point mass potential, the term entering $r_D$ may contribute to the outer region circular velocity only if $r_D$ is quite large. But, in such a case, $R/r_D$ will be much smaller than 1 in the inner regions, thus making the contribution of this term to overcome by orders of magnitude the Newtonian one in the regions where this latter is yet able to fit the data. As a consequence, $r_D$ must be small so that it does not contribute enough to boost $v_r$ with respect to the Newtonian value hence motivating the need for a not too large $r_A$. Indeed, we find an almost linear correlation between log $\eta_A$ and log $\eta_D$ along the chain motivated by the fact that the larger is $r_D$, the larger is the contribution to $v_r(R)$ of the $r^{-4}$ term of the potential and hence the smaller is the need for the $r^{-2}$ term which translates in a larger $r_A$.

5 CONCLUSIONS

Initially motivated by its attractive features from the point of view of quantum gravity, the HL proposal has soon become one of the most investigated theories of gravity. We have here complemented recent works on its cosmological consequences by addressing its impact on the gravitational potential. In contrast to the claim in TC10, we have demonstrated that static spherically symmetric solutions other than the Schwarzschild–de Sitter one exist. As a consequence, we have found a modified gravitational potential made out of the Newtonian $1/r$ one corrected by the addition of three further terms scaling as $r^2$, which corresponds to the effect of a cosmological constant, and a quickly decreasing term, proportional to $1/r^4$. The importance of these two terms is parametrized in terms of two conveniently defined scaling radii, namely $(r_A, r_D)$, related to the couplings entering the HL Lagrangian. In order to consider astrophysically interesting situations, we have then developed a general formalism to compute the circular velocity curve provided the mass density and the mass function of the system are given. As an application, we have then evaluated the Milky Way rotation curve using as only source of the gravitational field a spheroidal truncated power-law bulge and a double exponential disc. It turns out that the modified rotation curve is unable to fit the data, thus demonstrating that the HL theory cannot play the role of an alternative solution to the missing mass problem.

It is worth noting that Mukohyama (2009) has shown that classical solutions to the infrared (IR) limit of the HL theory can mimic general relativity plus cold dark matter so that one can argue that it should be possible for the HL theory to provide an effective dark matter halo in apparent contradiction with our finding. Actually, this is not the case. First, Mukohyama only refers to the HL IR limit so that the Lagrangian he has considered does not include any potential term, while we here explicitly include this term through the coupling parameters $g_i$. Secondly, the cold dark matter term comes out as a consequence of the global Hamiltonian constraint being less restrictive than the local one typically imposed on general relativity. In our static spherically symmetric metric, the global Hamiltonian constraint reduces to equation (5). In order to find our solution, we then impose that the integrand in equation (4) identically vanishes so that we are actually converting the global constraint in a local one, thus going back to a situation similar to the general relativity case. As a consequence, we argue that the possibility to get a matter term as an integration constant is lost in our approach.

The next step in our analysis of the HL theory on galactic scale should naturally be the inclusion of a dark halo. There are, however, some subtle issues making such a logical step forward not so easy to address. First, needless to say, there are few hints on which the dark halo mass density profile should be. All the models used in literature are motivated by the outcome of numerical simulations, such as, e.g. the popular Navarro–Frenk–White (Navarro, Frenk & White 1997) and Einasto (Cardone, Piedipalumbo & Tortora 2005; Einasto 1965), or evidences from rotation curve fitting, such as the isothermal sphere (Binney & Tremaine 1987) and the Burkert (Burkert 1995; Burkert & Salucci 2000; Boselli & Salucci 2001) profile. All the previous works implicitly assume the validity of the Newtonian potential so that they are no more valid in a modified framework as the one we are using here. As a consequence, we have therefore to explore a wide class of density profile able to mimic most of the models in literature to finally select the most empirically motivated one. Moreover, as equation (25) shows, we also need to know the dark matter mass function which is completely unknown. Should the dark matter be composed of point-like particles all having the same mass, we can adopt a Dirac $\delta$ leaving the mass of the particle as an unknown or setting it according to some particle physics model. As a final consequence, the fit to the rotation curve should determine the three HL scaling parameters $(r_A, r_D)$ and $N_{\text{DM}}$ halo parameters so that severe degeneracies among these $N_{\text{DM}}$ quantities may take place. In order to reduce them, a better way should be to consider external galaxies rotation curves assuming that the MF is the same as the MW one. In such a way, we could take advantage of the data probing the full radial range and not only the outer disc as done here with the MW sample. Moreover, such a test can also probe the universality of the scaling radii $(r_A, r_D)$, thus providing a further mandatory test of the HL model.

As a final remark, we want to stress that, should such an analysis be successful, one has still to address a different issue. Let us suppose that we have indeed well fitted the rotation curves of a large sample of spiral galaxies, thus determining a halo model and the values of $(r_A, r_D)$. One could then use equations (11) and (18) to infer constraints on the HL coupling parameters $(g_i \ldots, g_8)$. Although we have thus five constraints (the two radii plus the two conditions imposed in the derivation of the potential) so being unable to determine all the eight quantities $(g_0 \ldots, g_8)$, one could nevertheless try to see whether the allowed region of the parameter space is consistent with what is inferred from cosmological
analyses (Dutta & Saridakis 2010) or Solar Sytem tests (see e.g. Iorio & Ruggiero 2009 and references therein). It is worth noting that such an attempt should give a consistent picture of the HL framework on all physical scales thus allowing to draw a deeper insight into its viability.

ACKNOWLEDGMENTS
We warmly thank Mauro Sereno for helpful comments on a preliminary version of this manuscript. MC is supported by Regione Piemonte and Università degli Studi di Torino.

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