Non-relativistic D3-brane in the presence of higher derivative corrections

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Abstract

Using $\alpha'^3$ terms of type IIB supergravity action we study higher order corrections to the non-relativistic non-extremal D3-brane. Utilizing the corrected solution we evaluate corrections to temperature, entropy and shear viscosity. We also compute the $\eta/s$ ratio which although within the range of validity of the supergravity approximation and in the lowest order of the correction the universal bound is respected, there is a possibility for a violation of the bound when higher terms in the expansion are taken into account.
1 Introduction

AdS/CFT correspondence [1] has been found to be a powerful tool for studying strongly coupled field theories in terms of weakly coupled gravities. For instance it has provided us a framework to study thermodynamic and hydrodynamic properties of certain strongly coupled gauge theories which have gravity dual (see for example [2]). In particular in this context the AdS/CFT correspondence has been used to evaluate the ratio of shear viscosity to the entropy density (see for example [3–12]).

For example, for $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in 4D at finite temperature whose gravity dual is given in terms of the near horizon of the non-extremal D3-brane it was shown in [13] that at large ’t Hooft coupling and for large number of colors (large $N$) the gravity calculations give $\frac{\eta}{s} = \frac{1}{4\pi}$, which is compatible with the RHIC data [14]. In fact motivated by gravity calculations it was proposed [3] that the $\frac{\eta}{s}$ is bounded from below

$$\frac{\eta}{s} \geq \frac{1}{4\pi}. \quad (1.1)$$

Further investigations have confirmed this, rather universal, behavior for broader examples [15]. We note, however, that since in the gravity side we have higher derivative corrections to the tree level action which has been used to reach to the above conclusion, one may wonder if these corrections would spoil the bound. Indeed this point has been studied in several papers including [16–19]) where it was shown that there is a possibility for the bound to be violated when higher order corrections are taken into account.

In this article we would like to extend the above considerations for those non-relativistic field theories whose gravity duals are given in [20, 21]. The corresponding gravity solutions may be embedded in ten dimensional type II supergravities. Indeed starting from brane solutions in type II supergravities and using a Null Melvin Twist [22, 23] one can find new solutions (non-relativistic branes) which provide gravity duals for non-relativistic field theories [24–26]. Following the relativistic case one would expect that heating up the non-relativistic field theory corresponds to adding a black hole solution to the bulk [27–30].

Having had the gravity description of the non-relativistic field theory one may proceed to study thermodynamic and hydrodynamic properties of the theory. Indeed at leading order this has been done in [31, 32] where it was shown that the ratio of the shear viscosity to entropy density is the same as that in the relativistic case; $\frac{\eta}{s} = 1/4\pi$. It is then natural to pose the question whether there is a lower bound for this ratio in the non-relativistic case as well. To explore this question we will study the effects of higher order corrections to this ratio in the context of AdS/NRCFT.

To be specific we will consider non-relativistic non-extremal D3-brane in type IIB string theory which can be used to study thermodynamic and hydrodynamic properties of the dual three dimensional non-relativistic field theory. In leading order the gravity solution can be obtained from D3-brane solution by making use of the Null Melvin Twist. The resultant background is still a solution to the leading order action of type IIB supergravity. Now the task is to find the corrections to the solution when $\alpha'^3$ terms are added to the action. This can
be used to study the effects of higher order terms to the \( \eta/s \) ratio \(^1\).

We note, however, that in general it is difficult to find a closed form for the corrected solution. Nevertheless one can solve the equations of motion in the presence of higher derivative terms perturbatively. In fact there are two parameters that control the perturbation. The first one, denoted by \( \gamma \), controls the higher order terms in the action. The other, denoted by \( \lambda \), parametrizes the deviation of the geometry from that in the relativistic case \(^2\). Taking these two as the expansion parameters one finds the following schematic expansion for a typical quantity \( Q \)

\[
Q = Q_0 \left[ 1 + \left( c_0^0 + c_1^1 (\lambda^2 T) + c_1^2 (\lambda^2 T)^2 + \cdots \right) \gamma + \cdots \right], \tag{1.2}
\]

where \( c_i^j \) are some numerical factors. In particular for \( \eta/s \) ratio at first subleading order we find

\[
\eta/s = \frac{1}{4\pi} \left[ 1 + \left( 120 - 183.31 \pi^2 (\lambda^2 T)^2 + O((\lambda^2 T)^4) \right) \right], \tag{1.3}
\]

which has a potential to violate the bound. We will back to this point later.

The paper is organized as follows. In the next section we will review the tree level non-relativistic non-extremal D3-brane to fix our notation. Then we will find higher order corrections to the solution when the \( \alpha'^3 \) terms are added. In section three we will study the thermodynamic and hydrodynamic properties of the dual non-relativistic theory in the presence of higher derivative terms. In particular we will find the corrections to the temperature, entropy and shear viscosity. The last section is devoted to conclusions where we also give comments on the \( \eta/s \) ratio.

## 2 Higher derivative corrections

In this section we will study higher derivative corrections to the non-relativistic non-extremal D3-brane. To proceed we will first fix our notation by reviewing the tree level action of type IIB supergravity and its non-extremal D3-brane solution. When only dilaton, metric, B-field and RR-five form are non-zero the tree level action of type IIB supergravity in the string frame is given by

\[
I_0 = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} L_0 = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ e^{-2\Phi} \left( R + 4 \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2 \cdot 3!} H^2 - \frac{1}{4 \cdot 5!} F_{(5)}^2 \right) \right\}. \tag{2.1}
\]

The near horizon limit of the non-extremal D3-brane solution may be written as follows

\[
ds^2 = r^2 \left( - f_0^2 (r) dt^2 + dy^2 + dx_1^2 + dx_2^2 + \frac{dr^2}{f_0^2 (r) r^2} + (d\chi + A)^2 + ds_{CP^2}^2 \right), \quad e^{-2\Phi} = 1,
\]

\(^1\)Higher order corrections to the non-relativistic background have recently been studied in \([33]\) where the higher derivative terms were given by Gauss-Bonnet action. It was then shown that the higher order terms may correct the power exponent and as a result the \( \eta/s \) bound may be violated.

\(^2\)To be precise the relevant expansion parameter is \( \lambda^2 T \) where \( T \) is the temperature.
\[ F_{(5)} = dC_{(4)} = 2(1 + \ast) d\chi \wedge J \wedge J, \quad dA = 2J, \quad Vol(CP^2) = \frac{1}{2} J \wedge J. \] (2.2)

Here \( f_0^2(r) = 1 - r_0^4/r^4 \) and

\[ A = \frac{1}{2} \sin^2 \mu(d\psi + \cos \theta d\phi), \quad ds_{CP^2}^2 = d\mu^2 + \frac{1}{4} \sin^2 \mu(\sigma_1^2 + \sigma_2^2 + \cos^2 \mu \sigma_3^2), \] (2.3)

where \( \sigma_i \) are the \( SU(2) \) left invariant one-form

\[ \sigma_1 = \cos \psi d\theta + \sin \theta \sin \psi d\phi, \quad \sigma_2 = -\sin \psi d\theta + \sin \theta \cos \psi d\phi, \quad \sigma_3 = d\psi + \cos \theta d\phi. \] (2.4)

Using the Null Melvin Twist one can map the solution (2.2) to the non-relativistic non-extremal D3-brane which is still a solution of the tree level type IIB supergravity action. The obtained solution is [25]

\[ ds^2 = \frac{r^2 f_0^2(r)}{k(r)} \left( -(1 + r^2 \lambda^2) dt^2 + \frac{1 - r^2 \lambda^2 f_0^2(r)}{f_0^2(r)} dy^2 - 2r^2 \lambda^2 dt dy \right) + r^2 (dx_1^2 + dx_2^2) + \frac{dr^2}{f_0^2(r) r^2} (d\chi + A)^2 + ds_{CP^2}^2, \]

\[ B_{(2)} = \frac{r^2 \lambda}{k(r)} (d\chi + A) \wedge (f_0(r) dt + dy), \quad e^{2\Phi} = \frac{1}{k(r)}, \] (2.5)

where \( k(r) = 1 + r^2 \lambda^2 (1 - f_0^2(r)) \) and the RR-five form remains unchanged.

Now the aim is to study the effects of higher derivative terms to the non-relativistic non-extremal D3-brane solution. Here we restrict ourselves to corrections from gravity side. There are other efforts have been done to include other correction terms, for example see [17]. The higher derivative terms which we are interested in are the \( \alpha'^3 \) corrections to type IIB supergravity action. In string frame they are given by

\[ I_1 = \frac{1}{2 \kappa_1^2} \int d^{10} x \sqrt{-g} \gamma L_1 = \frac{1}{2 \kappa_1^2} \int d^{10} x \sqrt{-g} \gamma e^{-2\phi} W, \] (2.6)

where \( \gamma = \frac{1}{8} \zeta(3)(\alpha')^3 \) and \( W \) can be written in terms of the Weyl tensors

\[ W = C^{hmnk} C_{pqmn} C_h^{rsp} C_{rsk}^q + \frac{1}{2} C^{hkmn} C_{pqmn} C_h^{rsp} C_{rsk}^q. \] (2.7)

In principle one needs to solve the equations of motion coming from the action \( I_0 + I_1 \). Of course, in general, it is not an easy task to solve the resultant equations. Nevertheless we may start from an ansatz and try to solve the equations of motion for the parameters of the ansatz. We note, however, that in general the obtained equations cannot be solved exactly, though they may be solved perturbatively.

For making our ansatz, we start from the same general ansatz as [34] for relativistic non-extremal D3-branes. Then we do all the steps for Null Melving Twisting of \( D_p \)-branes noted
in [25] for this general background. Finally we find the following ansatz for the non-relativistic non-extremal D3-branes

$$ds^2 = \frac{r^2 f^2(r)}{k(r)} \left( -(1 + r^2 \lambda^2 s^2(r))dt^2 + \frac{1 - r^2 \lambda^2 s^2(r) f^2(r)}{f^2(r)} dy^2 - 2 r^2 \lambda^2 s^2(r) dtdy \right)$$

$$+ \frac{r^2 (dx_1^2 + dx_2^2) + \frac{h^2(r)}{r^2} dr^2 + \frac{s^2(r)}{k(r)} (d\chi + A)^2 + s^2(r) ds_{CP^2}}{k(r)} ,$$

$$F(5) = dC(4) = 2(1 + \ast) d\chi \wedge J \wedge J , \quad e^{2\phi} = \frac{e^{2v(r)}}{k(r)} ,$$

$$B(2) = \frac{r^2 \lambda s^2(r)}{k(r)} (d\chi + A) \wedge (f(r) dt + dy) ,$$

(2.8)

for arbitrary functions $f(r), h(r), s(r)$ and $v(r)$. Moreover

$$k(r) = 1 + r^2 \lambda^2 s^2(r) (1 - f^2(r)) .$$

(2.9)

Note that in the limit of $\lambda \to 0$ the above ansatz reduces to that in the relativistic case. We will also assume that $f(r)$ and $h(r)$ have zero at $r = \pm r_0$ and the extremal limit is obtained at $r_0 \to 0$.

Using the above ansatz the equations of motion of the total action $I_0 + I_1$ read

$$\frac{\partial L_0}{\partial X_i} - d \frac{d \psi}{dr} \frac{\partial L_0}{\partial \psi_i} + \frac{d^2 \partial L_0}{dr^2 \partial \psi_i} = -\gamma (\frac{\partial L_1}{\partial X_i} - \frac{d \partial L_1}{dr} \frac{\partial \psi_i}{\partial X_i} + \frac{d^2 \partial L_1}{dr^2 \partial \psi_i} ) ,$$

(2.10)

where prime denotes the derivative with respect to $r$ and $X_i = \{ f(r), h(r), s(r), v(r) \}$. To proceed we choose the following perturbed ansatz over the tree level solution

$$f(r) = (1 - \frac{r_0^4}{r^4})^{\frac{1}{2}} (1 + \gamma F(r)) , \quad h(r) = (1 - \frac{r_0^4}{r^4})^{-\frac{1}{2}} (1 + \gamma H(r)) ,$$

$$s(r) = 1 + \gamma S(r) , \quad v(r) = \gamma V(r) .$$

(2.11)

It is straightforward, though messy, to plug the ansatz to the equations of motion to find the following differential equations for the perturbed functions $\{ F(r), H(r), S(r), V(r) \}$. Setting $\rho = \frac{r}{r_0}$ and $\lambda = \lambda r_0$ one gets

$$(\rho^4 - 1) (\rho S'' - \frac{2}{5} \rho V'' - \frac{3}{5} H') + 4 (\rho^4 - \frac{1}{2}) (S' - \frac{2}{5} V') - 4 \rho^3 (S - \frac{2}{5} V + \frac{3}{5} H)$$

$$= \frac{F_1(\rho; \lambda)}{\rho^{13} (\rho^2 + \lambda^2)^9} ,$$

(2.12)

$$\rho (\rho^4 - 1) (F'' + 4 S'' - 2 V'') + 10 (\rho^4 - \frac{1}{2}) (S' - 2 V') - 4 (\rho^4 - \frac{1}{2}) H'$$

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\[ \rho^3 (6 \rho F' + 52S - 8V - 20H) = \frac{F_2(\rho; \tilde{\lambda})}{\rho^{13}(\rho^2 + \tilde{\lambda}^2)^9}, \]  
(2.13)

\[ (\rho^4 - 1) F' + \frac{4}{3}(\rho^4 - \frac{1}{2})(5S' - 2V') - 4\rho^3 (H + \frac{5}{3}S - \frac{2}{3}V) = \frac{F_3(\rho; \tilde{\lambda})}{\rho^{13}(\rho^2 + \tilde{\lambda}^2)^8}, \]  
(2.14)

\[ \rho (\rho^4 - 1) (F'' + 5S'' - 2V'') + 5(\rho^4 - \frac{1}{2}) (5S' - 2V') - 4(\rho^4 - \frac{1}{2}) H' \]
\[ + \rho^3 (6\rho F' - 20H + 20S) = \frac{F_4(\rho; \tilde{\lambda})}{\rho^{13}(\rho^2 + \tilde{\lambda}^2)^8}. \]  
(2.15)

Here \( F_i \) are functions of \( \rho \) whose explicit forms are given in appendix A. Now the task is to perturbatively solve the above equations with given boundary conditions. First of all we would like to have a solution whose asymptotic is the same as that for the non-relativistic non-extremal D3-brane. Moreover we require that the solution to be finite at the horizon.

For our purpose we just need to solve the equations up to \( O(\tilde{\lambda}^2) \). The results are

\[ F(\rho) = \frac{640 - 1375\rho^4 - 1465\rho^8}{32\rho^{12}} + \left( \frac{11978}{15} (\rho^4 - \frac{2837}{5989} \ln(1 + \frac{1}{\rho^2}) - \frac{6304}{15} \ln(2) \right) \tilde{\lambda}^2. \]  
(2.16)

\[ H(\rho) = \frac{2480\rho^8 + 1895\rho^4 - 10000}{64\rho^{12}} + \left( \frac{5989}{5} \rho^8 - \frac{9036}{5} \rho^4 + \frac{2837}{15} \right) \ln(1 + \frac{1}{\rho^2}) \]
\[ + \frac{6304}{15} \ln(2) - \frac{1657358}{1575} \rho^{14} - \frac{18777}{28} \rho^{10} + \frac{45493}{75} \rho^6 + \frac{607097}{6300} \rho^2 + \frac{24803}{75} \frac{1}{\rho^2} \]
\[ + \frac{21119}{15} \rho^2 + \frac{11978}{25} \rho^4 - \frac{5989}{5} \rho^6 \right) \tilde{\lambda}^2. \]  
(2.17)

\[ S(\rho) = -\frac{15 + 90\rho^4}{64\rho^8} + \left( \frac{5989}{25} \rho^4 - \frac{3257}{25} \right) \ln(1 + \frac{1}{\rho^2}) - \frac{5989}{25} \rho^2 + \frac{3782}{75} \frac{1}{\rho^2} - 1532 \frac{1}{375} \rho^6 \]
\[ + \frac{184273}{10500} \rho^{10} - \frac{53}{20} \rho^{14} \] \( \tilde{\lambda}^2. \)  
(2.18)

\[ V(\rho) = -\frac{30 + 45\rho^4 + 90\rho^8}{16\rho^{12}} - \frac{1}{175} \left( 83846 + 7350 \ln(1 + \frac{1}{\rho^2}) - 7350 \frac{1}{\rho^2} - 2450 \frac{1}{\rho^6} \right) \]  

\[ ^3 \text{Note that using the expression of the temperature, (3.2), at leading order one has } \tilde{\lambda} = \lambda r_0 = \lambda^2 T. \]

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\[ - 16170 \frac{1}{\rho^{10}} + 3375 \frac{1}{\rho^{14}} \lambda^2. \]  
\[ (2.19) \]

These are all information we need to evaluate the corrections to the temperature, entropy and shear viscosity which we will do in the next section. It is worth noting that the above solution, after going to Einstein frame and using a field redefinition, reduces to the known results in [34] for \( \lambda = 0 \).

### 3 Entropy and Viscosity

In this section we will study thermodynamic and hydrodynamic properties of the finite temperature non-relativistic three dimensional field theory whose gravity dual is given in terms of the non-relativistic non-extremal D3-brane. In fact, by making use of the results of the previous section, we will be able to study the effects of the higher derivative corrections to the entropy and the shear viscosity. To proceed let us start from the thermodynamical quantities.

#### Temperature

Since in the presence of the higher derivative terms the solution gets correction, one expects that the temperature will also be corrected. To find the temperature one may follow the standard route. Namely we will have to identify the Euclidean time in the corrected geometry in such a way that the corrected metric will be free of conical singularity. Consider \( \zeta^\mu \) as the null generator of the horizon, the surface gravity is defined by

\[ \kappa^2 = -\frac{1}{2} \nabla^\mu \zeta^\nu \nabla_\mu \zeta_\nu, \]
\[ (3.1) \]

and the temperature is related to surface gravity by \( T = \frac{\kappa}{2\pi} \). As indicated in [27] the Killing generator of the horizon is given by \( \zeta = \frac{1}{\lambda} \frac{\partial}{\partial t} \) which differs from the relativistic case by a prefactor. Knowing these, to first order of \( \gamma \) and \( \tilde{\lambda}^2 \) one finds

\[ T = \left[ (1 + \frac{1225}{64} \gamma) + \left( \frac{3152}{15} \ln(2) - \frac{206806}{1575} \right) \gamma \tilde{\lambda}^2 \right] \frac{r_0}{\pi \lambda}. \]
\[ (3.2) \]

Note that in the limit of \( \gamma \to 0 \) this reduces to known leading order expression [27]. On the other hand for \( \tilde{\lambda} \to 0 \) and \( \lambda T \to T \) we recover the relativistic result as well [5, 34].

#### Entropy

Entropy is one of the interesting quantities usually people study in this context. To compute the entropy one may use the Euclidean method by which we need to evaluate the value of the
action on our solution. We note, however, that in general the action diverges and needs to be regularized by subtracting the value of the action evaluated on a reference background. To do this we will have to be careful about how to define the cut off. Although this method is found useful for geometries which are asymptotically AdS, in general for geometries with arbitrary asymptotic, this procedure is not easy to be applied. Therefore it will be useful if one can use another method.

Indeed there is an alternative way to compute the entropy via Wald’s formula for the entropy \([35–37]\). When the Lagrangian does not contain the covariant derivative of the curvature the Wald formula for the entropy reduces to the following simple form \([38]\)

\[
S_{BH} = \int_H dx^H \sqrt{g^H} \frac{\partial L}{\partial R_{\mu\nu\lambda\rho}} g^\perp_{\mu\lambda} g^\perp_{\nu\rho},
\]

(3.3)

where \(L\) is the total Lagrangian and the integration is over the horizon. \(g^\perp\) is the orthogonal metric to the horizon which can be obtained in terms of the normal vectors to the horizon as follows

\[
g^\perp_{\mu\nu} = (N_t)_\mu(N_t)_\nu + (N_r)_\mu(N_r)_\nu.
\]

(3.4)

In our case setting \(L = \frac{4\pi}{2\kappa_{10}^2} (\mathcal{L}_0 + \gamma \mathcal{L}_1)\) and taking into account that

\[
N_t = \sqrt{g_{tt}}(1, \frac{g_{ty}}{g_{tt}}, 0, 0, 0, \bar{0}), \quad N_r = (0, 0, 0, 0, \sqrt{g_{rr}}, \bar{0}),
\]

(3.5)

the Wald entropy (3.3) reads (for more technical details see e.g. \([39]\))

\[
S_{BH} = \frac{8\pi}{2\kappa_{10}^2} \int_H dx^H \sqrt{g^H} \left\{ \frac{\partial L}{\partial R_{trtr}} g^\perp_{tt} g^\perp_{rr} + \frac{\partial L}{\partial R_{gyry}} g^\perp_{yy} g^\perp_{rr} + 2 \frac{\partial L}{\partial R_{tgyr}} g^\perp_{ty} g^\perp_{rr} \right\}.
\]

(3.6)

Using the corrected geometry presented in the previous section it is easy to compute each term in the above expression for the entropy. Altogether up to \(O(\bar{\lambda}^2)\) one arrives at

\[
S_{BH} = \frac{2\pi^2 R V_2 V_{CP} V_1}{2\kappa_{10}^2} r_0^3 \left( 4 + \frac{4635}{16} \gamma + \left( \frac{12608}{5} \ln(2) - \frac{1356424}{525} \right) \gamma \bar{\lambda}^2 \right),
\]

(3.7)

\[
= \frac{2\pi^2 R V_2 V_{CP} V_1}{2\kappa_{10}^2} r_0^3 \left( 4 + 289.68 \gamma \right) - 835.82 \gamma \bar{\lambda}^2.
\]

Here \(R\) is the radius of compact direction \(y\), \(V_1\) is the volume of the \(\chi\) coordinate and \(V_2\) is the volume of \(x_1\) and \(x_2\) directions. Using the expression for the temperature (3.2) we can eliminate \(r_0\) to express the entropy in terms of the temperature

\[
S_{BH} = \frac{8\pi^5 R V_2 V_{CP} V_1}{2\kappa_{10}^2} (\lambda T)^3 \left[ 1 + \left( 15 - 252\pi^2(\lambda^2 T)^2 \right) \gamma \right].
\]

(3.8)
Shear viscosity

The shear viscosity of a translation invariant theory at the hydrodynamic limit can be evaluated by making use of the Kubo formula \[40\]

\[
\eta = - \lim_{\omega \to 0} \frac{1}{\omega} \Im G_{x_1 x_2, x_1 x_2}^R(\omega, k = 0),
\]

(3.9)

where \(G_{x_1 x_2, x_1 x_2}^R\) is the retarded Green’s function of the component \(T_{x_1 x_2}\) of the energy momentum tensor. To compute the retarded Green’s function we utilize the AdS/CFT dictionary \[40, 41\], by which the retarded Green’s function can be related to the on-shell bulk action

\[
G_{x_1 x_2, x_1 x_2}^R(\omega, k, r) = \lim_{r \to \infty} 2F(\omega, k, r).
\]

(3.10)

The shear viscosity in the non-relativistic field theory has been evaluated at leading order in \[32\] where it was shown that the ratio of the shear viscosity to the entropy density saturates the universal bound

\[
\frac{\eta}{s} = \frac{1}{4\pi}.
\]

(3.11)

Following \[5\] we would like to study the effects of higher derivative corrections to the value of the shear viscosity. In order to evaluate the shear viscosity we consider a perturbation to the metric as \(h_{x_1 x_2} = r^2 \varphi(t, r, x_1, x_2)\) which can be Fourier transformed along \(t\) and \(x_i\)’s directions leading to

\[
\varphi(t, r, y, x_1, x_2) = \int d\omega d^3k (2\pi)^4 e^{-i\omega t + ik \cdot x} \varphi_{\omega, k}(r).
\]

(3.12)

Since in the Kubo formula, the spatial momentum is zero, we restrict our calculations to \(k = 0\). On the other hand since the energy momentum tensor has zero particle number the perturbation must be \(y\)-independent. Actually it is easy to see that the relevant perturbation of the metric decouples from other perturbations and in fact it obeys the equation of motion of a minimally coupled massless scalar \[16\] which in our case we have

\[
A \varphi''_{\omega} + C \varphi'_{\omega} + 2D \varphi_{\omega} - \frac{d}{dr} (F \varphi''_{\omega} + 2B \varphi'_{\omega} + C \varphi_{\omega}) + \frac{d^2}{dr^2} (2E \varphi''_{\omega} + F \varphi'_{\omega} + A \varphi_{\omega}) = 0.
\]

(3.13)

The coefficients \(A, ..., F\) are given in the appendix B. We note, however, that in general it is difficult to find an explicit solution for the above equation. Nevertheless we may solve it perturbatively. Changing the variables as before, \(\rho = \frac{d}{r_0}\) and \(\bar{\lambda} = \lambda r_0\), and expanding the above equation up to \(O(\bar{\lambda}^3)\), one finds

\[
\rho(\rho^4 - 1)^2 \frac{d^2 \varphi_{\omega}}{dp^2} + (5\rho^8 - 6\rho^4 + 1) \frac{d \varphi_{\omega}}{dp} - 4\rho^3 \Omega^2 (\bar{\lambda}^2 \rho^4 - \rho^2 - \bar{\lambda}^2) \varphi_{\omega}
\]

\[
+ \frac{1}{\rho^{14}(\rho^4 - 1)} \left( C_1 \frac{d^4 \varphi_{\omega}}{dp^4} + C_2 \frac{d^3 \varphi_{\omega}}{dp^3} + C_3 \frac{d^2 \varphi_{\omega}}{dp^2} + C_4 \frac{d \varphi_{\omega}}{dp} + C_5 \varphi_{\omega} \right) = 0,
\]

(3.14)
where the coefficients $C_i$ are given in appendix C. Next we set
\[ \varphi_\omega(\rho) = (1 - \frac{1}{\rho^2})^\Delta G_\omega(\rho), \]
where $G_\omega(\rho)$ is a regular function at horizon and $\Delta$ is a constant to be determined. Setting $\omega = 2r_0w$, for a solution normalized to one at $\rho \to \infty$ up to the order of $O(w^2)$ we have
\[ \varphi_w(\rho) = (1 - \frac{1}{\rho^2})^\Delta(w) \left\{ 1 + \frac{1}{15} \left( 120 \ln(1 + \frac{1}{\rho^2}) - \frac{120}{\rho^2} + \frac{155}{\rho^4} + \frac{67}{\rho^6} \right) + \Delta(w) \left( \ln(1 + \frac{1}{\rho^2}) - \frac{\gamma}{16} \left( \frac{3385}{\rho^4} + \frac{2335}{\rho^8} + \frac{768}{\rho^{12}} \right) + \gamma \bar{\lambda}^2 W(\rho) \right\}. \]

Here $W(\rho)$ is an unknown function satisfying the following differential equation
\[ \rho^{16}(\rho^4 - 1)^3 \frac{d^2 W}{d\rho^2} - (\rho^{15} - 7 \rho^{19} + 11 \rho^{23} - 5 \rho^{27}) \frac{dW}{d\rho} - \frac{201728}{15} \rho^{18} \ln(1 + \rho^2) \]
\[ + \frac{1}{3} \left( 96 \rho^{24} - 660 \rho^{20} + 1344 \rho^{16} + 248 \rho^{12} - 2368 \rho^8 + 1340 \rho^4 \right) \ln(1 + \frac{1}{\rho^2}) \]
\[ - 32 \left( \rho^{22} - 29 \rho^{20} + \frac{7278127}{12600} \rho^8 - \frac{9859}{15} \rho^{16} - \frac{827}{120} \rho^{14} - \frac{184637}{360} \rho^{12} - \frac{431}{120} \rho^{10} \right) \rho^4 \]
\[ + \frac{12381}{200} \rho^8 + \frac{201}{40} \rho^6 + \frac{96023}{56} \rho^4 - \frac{9205}{8} + \frac{12608}{15} \rho^{18} \ln(4\rho) = 0. \]

Using Maple one can solve the above equation which in turn can be used to find the solution $\varphi_w$. On the other hand the constant $\Delta$ can be fixed by the regularity condition on $G_\omega(\rho)$. Following [19] we note that in order to have the regularity up to order $\gamma$, we need to check the regularity condition up to order of $O(w^2)$. Doing so one finds
\[ \Delta(w) = -\frac{2ir_0w}{4\pi T} = -\frac{1}{2\lambda} \left[ 1 + \left( -\frac{3152}{15} \ln(2) r_0^2 \lambda^2 + \frac{206806}{1575} r_0^2 \lambda^2 - \frac{1225}{64} \right) \right]. \]

Using the equation of motion (3.13) one finds the quadratic part of the on-shell action of the bulk theory as follows (for more detail see [5])
\[ \mathcal{F}(\omega, 0, r) = -\frac{V_{CP}^2 V_1}{2\kappa_{10}^2} \left[ \left( B - A - \frac{F'}{2} + 2p_0 E \right) \varphi_\omega \varphi_\omega - \frac{1}{2} (C - A') \varphi_\omega \varphi_\omega + E p_1 \varphi'_\omega \varphi'_\omega \right. \]
\[ \left. - E' \varphi''_\omega \varphi'_\omega + E \varphi''_\omega \varphi'_\omega - E' \varphi''_\omega \varphi'_\omega \right]. \]

Putting everything together we get
\[ G^R_{x_1x_2x_1x_2}(w_0, 0) = \lim_{\rho \to \infty} 2\mathcal{F}(w_0, 0, \rho) = \lim_{\rho \to \infty} -\frac{2V_{CP}^2 V_1}{2\kappa_{10}^2} r_0^4 \left( 2 \rho^2 \bar{\lambda}^2 - 1 + iw \right) + \rho^4. \]
Finally the shear viscosity becomes
\[
\eta = \lim_{\omega \to 0} \frac{2V_{CP^2}V_1w}{2\kappa_{10}^2\omega} \left[ 1 + \frac{12315}{64} \gamma + \left( \frac{3232}{5} \ln(2) - \frac{441166}{525} \right) \frac{\gamma}{\lambda^2} \right].
\] (3.21)

Using the expression for the temperature (3.2) we can eliminate \( r_0 \) to write the shear viscosity in terms of the temperature as follows
\[
\eta = \frac{\pi^3V_{CP^2}V_1}{2\kappa_{10}^2}(\lambda T)^3 \left[ 1 + \left( 135 - 435.31\pi^2(\lambda^2T)^2 \right) \frac{\gamma}{\lambda^2} \right].
\] (3.22)

4 Conclusions

In this paper we have studied the non-relativistic non-extremal D3-brane in the presence of higher order corrections given by \( \alpha'^3 \) terms in type IIB supergravity. By making use of the corrected solution we have computed the effects of the higher order corrections to the temperature, entropy and shear viscosity. Having computed the corrected entropy and shear viscosity it is natural to obtain the ratio of the shear viscosity to entropy density. To find it, we first need the entropy density which turns out to be
\[
s = \frac{4\pi^4V_{CP^2}V_1}{2\kappa_{10}^2}(\lambda T)^3 \left[ 1 + \left( 15 - 252\pi^2(\lambda^2T)^2 \right) \frac{\gamma}{\lambda^2} \right].
\] (4.1)

Using the expression for shear viscosity one finds
\[
\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 + 120\gamma + \left( 16\ln(2) - \frac{972}{5} \right) \pi^2(\lambda^2T)^2 \right]
\] (4.2)

which presents the correction to the tree level bound \( 1/4\pi \). Note that in the limit of \( \lambda^2T = 0 \) we recover the result of the relativistic field theory [19]. When \( \gamma = 0 \) we get \( \eta/s = 1/4\pi \) which is independent of \( \lambda \) showing that at tree level as far as the ratio is concerned both relativistic and non-relativistic cases lead to a universal result [32].

It is now important to see whether this correction can violate the lower bound of the \( \eta/s \) ratio. This depends on the sign of the \( \gamma \) term, \( 120 - 183.31\pi^2(\lambda^2T)^2 \). In fact for \( \lambda^2T > 0.257 \) the coefficient of \( \gamma \) is negative leading to a violation of the bound. On the other hand for \( \lambda^2T < 0.257 \) the higher order corrections respect the bound. Therefore it is crucial to estimate \( \lambda^2T \) as precise as we can. To find a bound on the value of the corrections we note that the supergravity approximation is applicable when the curvature is small\(^4\) which in our notation

\(^4\)In our notation we have set the radius of the space time to one.
it means \(|R| = 20 r_0^2 \lambda^2 \ll 1\), in other words, \(\lambda^2 T \ll 0.07\). Therefore within the supergravity approximation and to the order of \(O(\lambda^4 T^2)\) the bound is not violated. On the other hand if one considers the effects of higher orders, \(e.g.\) \(O(\lambda^8 T^4)\), since the corrections may decrease the lower bound of 0.257, there could be a possibility for violation of the bound in a narrow window where \(\lambda^2 T \ll 0.07\). It would be interesting to explore this point better.

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Appendix A

The \(F_1(\rho; \tilde{\lambda})\) functions on the right hand side of equations of motion in (2.12) to (2.15) are given by

\[
F_1 = \frac{1296}{5} \left[ \tilde{\lambda}^4 \rho^{34} + \frac{91273}{15522} \tilde{\lambda}^6 \rho^{32} + \left( \frac{2003147}{93312} \tilde{\lambda}^4 - \frac{101}{108} \right) \tilde{\lambda}^4 \rho^{30} + \frac{9179675}{279936} \tilde{\lambda}^8 + \frac{19861}{4374} \tilde{\lambda}^4 \right]
\]

\[
- \frac{485}{648} \tilde{\lambda}^2 \rho^{28} + \left( \frac{2742697}{139968} \tilde{\lambda}^8 - \frac{17461043}{223988} \tilde{\lambda}^4 + \frac{8569}{1944} \tilde{\lambda}^4 \rho^{26} + \frac{3091}{972} \tilde{\lambda}^{12} - \frac{83656099}{7121664} \tilde{\lambda}^8 \right)
\]

\[
- \frac{72409}{148368} \tilde{\lambda}^4 + \frac{4827}{6182} \tilde{\lambda}^2 \rho^{24} + \frac{5545}{648} (\tilde{\lambda}^{16} - \frac{116087}{9962} \tilde{\lambda}^{12} + \frac{153073}{95816} \tilde{\lambda}^8 - \frac{193517}{130308} \tilde{\lambda}^4)
\]

\[
+ \frac{432}{1019} \rho^{22} + \frac{206}{81} \tilde{\lambda}^{16} + \frac{132625}{59328} \tilde{\lambda}^{12} - \frac{388271}{474624} \tilde{\lambda}^8 - \frac{2123}{1648} \tilde{\lambda}^4 - \frac{407}{1648} \tilde{\lambda}^{20} - \frac{1463}{1944} \tilde{\lambda}^{16}
\]

\[
- \frac{1229863}{19152} \tilde{\lambda}^{12} + \frac{5417857}{93632} \tilde{\lambda}^8 - \frac{309279}{5852} \tilde{\lambda}^{14} + \frac{405}{77} \rho^{18} + \frac{407}{432} (\tilde{\lambda}^{16} - \frac{471325}{43956} \tilde{\lambda}^{12})
\]

\[
- \frac{4332419}{1054944} \tilde{\lambda}^8 - \frac{1694084}{32967} \tilde{\lambda}^4 + \frac{13396}{1221} \tilde{\lambda}^{16} + \frac{12197}{5832} (\tilde{\lambda}^{12} - \frac{44747}{73182} \tilde{\lambda}^8 + \frac{49288231}{1179012} \tilde{\lambda}^4)
\]

\[
- \frac{1601631}{97576} \tilde{\lambda}^{14} + \frac{137}{324} (\tilde{\lambda}^{12} + \frac{122791}{13152} \tilde{\lambda}^8 + \frac{44678345}{236736} \tilde{\lambda}^4 - \frac{411869}{3288} \tilde{\lambda}^6 \rho^{12} - \frac{20891}{8748} \tilde{\lambda}^8)
\]

\[
+ \frac{14499791}{668512} \tilde{\lambda}^{14} - \frac{141442003}{5348096} \tilde{\lambda}^8 \rho^{10} + \frac{15707}{34992} (\tilde{\lambda}^8 - \frac{784154}{15707} \tilde{\lambda}^4 - \frac{121617627}{1005248} \tilde{\lambda}^{10} \rho^8)
\]

\[
+ \frac{409103}{69984} (\tilde{\lambda}^4 - \frac{4562125}{818206} \tilde{\lambda}^{12} \rho^6 + \frac{901}{1296} (\tilde{\lambda}^4 - \frac{3603757}{194616} \tilde{\lambda}^{14} \rho^4 - \frac{46409}{15552} \tilde{\lambda}^{16} \rho^2 - \frac{42715}{139968} \tilde{\lambda}^{18})
\]

\[
F_2 = - \frac{2876}{5} \left[ \tilde{\lambda}^4 \rho^{30} - \frac{933871}{621216} \tilde{\lambda}^4 - \frac{10}{719} \tilde{\lambda}^2 \rho^{28} - \left( \frac{12450601}{4969728} \tilde{\lambda}^4 - \frac{365}{719} \tilde{\lambda}^4 \rho^{26} + \frac{57784183}{4969728} \tilde{\lambda}^8 \right)
\]

\[
+ \frac{38395}{6471} \tilde{\lambda}^4 - \frac{24265}{8628} \tilde{\lambda}^2 \rho^{24} + \left( \frac{5721773}{207072} \tilde{\lambda}^8 - \frac{287905}{138048} \tilde{\lambda}^4 - \frac{230515}{34512} \tilde{\lambda}^4 \rho^{22} + \frac{108895}{5752} (\tilde{\lambda}^{12}
\right)]
\[
\begin{align*}
F_3 &= -\frac{2228}{3} \left( \lambda^4 \rho^{28} + \frac{1751323}{481248} \lambda^4 - \frac{10}{557} \lambda^2 \rho^{26} + \frac{2201797}{3849984} \lambda^4 + \frac{403}{1671} \lambda^4 \rho^{24} + \frac{988465}{160416} \lambda^8 \right) \\
&\quad + \frac{92237}{80208} \lambda^4 - \frac{1231}{6684} \lambda^2 \rho^{22} + \frac{80353}{13368} \left( \lambda^8 + \frac{1926317}{5785416} \lambda^4 - \frac{20513}{160706} \lambda^4 \rho^{20} + \frac{8617}{2228} \lambda^8 \right) \\
&\quad + \frac{2031685}{1861272} \lambda^8 - \frac{91487}{155106} \lambda^4 + \frac{2362}{25851} \lambda^2 \rho^{18} + \frac{1587}{6348} \left( \lambda^8 + \frac{55925}{16928} \lambda^8 \rho^{12} - \frac{76121}{16928} \lambda^8 \right) \\
&\quad + \frac{25091}{9522} \lambda^8 - \frac{270}{529} \lambda^8 \rho^{16} + \frac{5161}{1671} \left( \lambda^8 - \frac{55141}{495456} \lambda^4 + \frac{663257}{743184} \lambda^4 - \frac{7575}{20644} \lambda^2 \rho^{14} + \frac{463}{557} \lambda^8 \right) \\
&\quad + \frac{4315}{44448} \lambda^8 + \frac{3655517}{800064} \lambda^4 - \frac{25291}{7408} \lambda^4 \rho^{12} + \frac{5821}{13368} \left( \lambda^8 + \frac{125535}{23284} \lambda^4 - \frac{223471}{23284} \lambda^8 \right) \\
&\quad + \frac{347}{2228} \lambda^8 + \frac{99827}{18738} \lambda^4 - \frac{16771699}{599616} \lambda^8 \rho^{8} + \frac{18701}{120312} \left( \lambda^8 + \frac{376574}{18701} \lambda^8 \rho^{6} + \frac{301}{20052} \lambda^4 \right) \\
&\quad - \frac{78325}{53472} \lambda^8 \rho^{8} - \frac{24203}{60156} \lambda^8 \rho^{8} - \frac{11795}{20624} \lambda^16 \right). \\
F_4 &= 324 \left( \lambda^4 \rho^{28} + \frac{1027}{324} \lambda^6 \rho^{26} + \frac{1893883}{559782} \lambda^4 + \frac{17}{27} \lambda^2 \rho^{22} + \frac{287}{1458} \lambda^4 \rho^{24} + \frac{3103}{1296} \lambda^4 - \frac{20}{27} \lambda^2 \rho^{22} \\
&\quad + \frac{2479}{648} \lambda^8 + \frac{121927}{535464} \lambda^4 - \frac{7135}{14874} \lambda^4 \rho^{20} + \frac{1775}{324} \left( \lambda^8 + \frac{6347}{21300} \lambda^4 - \frac{125927}{383400} \lambda^8 - \frac{28}{355} \lambda^2 \rho^{18} \\
&\quad + \frac{295}{108} \lambda^8 + \frac{401}{1180} \lambda^8 \rho^{12} - \frac{621}{50976} \lambda^8 + \frac{4199}{5310} \lambda^8 \rho^{16} + \frac{1139}{324} \left( \lambda^8 + \frac{2645}{4824} \lambda^8 - \frac{14053}{13668} \lambda^4 \\
&\quad + \frac{300}{1139} \lambda^2 \rho^{14} + \frac{61}{81} \lambda^12 + \frac{104215}{17568} \lambda^8 - \frac{520673}{976} \lambda^8 \rho^{12} + \frac{1063}{486} \lambda^8 \rho^{16} - \frac{30295}{5104} \lambda^4 \right) \right) \\
&\quad + \frac{11235}{8504} \lambda^6 \rho^{10} + \frac{137}{324} \lambda^8 + \frac{211}{2466} \lambda^4 + \frac{1529700}{236736} \lambda^8 \rho^{8} + \frac{1921}{17496} \lambda^4 + \frac{129061}{7684} \lambda^8 \rho^{6} \\
&\quad + \frac{10478339}{23521320} \lambda^8 - \frac{775829}{980055} \lambda^4 + \frac{61664}{32685} \lambda^2 \rho^{20} + \frac{10465}{1438} \left( \lambda^8 - \frac{582367}{347760} \lambda^8 + \frac{7134823}{6027840} \lambda^8 \right)
\right)
\end{align*}
\]
\[ A = \frac{79}{2916} (\tilde{\lambda}^4 + \frac{19495}{632} \tilde{\lambda}^{12} \rho^4 + \frac{3937}{17496} \tilde{\lambda}^{14} \rho^2 + \frac{955}{34992} \tilde{\lambda}^{16}) \]

Appendix B

\[ B = \frac{3}{2} (r^4 - r_0^4) r + \frac{r_0^2 \gamma}{16800 r_{13}^2} \left( 12073824 \lambda^2 r_{18}^4 + 21181440 \lambda^2 r_{18}^4 \ln(1 + \frac{r_0^2}{r^2}) \right) \]

\[ C = \frac{4}{r^2} \left( 3 r^6 + \lambda^2 r_0^4 r^4 - r^2 r_0^4 - \lambda^2 r_0^8 \right) + \frac{r_0^2 \gamma}{12600 r_{14}^2} \left( 2299500 r_{14}^2 r_0^6 - 37910250 r_{14}^2 r_0^6 \right) \]
\[ D = \frac{1}{2(r^4 - r_0^4)r^3} \left( (-8 + \lambda^2 \omega^2)r^{10} - (8\lambda^2 r_0^4 + \omega^2)r^8 + (8 - \lambda^2 \omega^2)r_0^4 r^{10} + 8\lambda^2 r_0^{12} \right) \]

\[ + \frac{\gamma}{100800r^{15}(r^4 - r_0^4)^2 r_0^2} \left( 120738240\lambda^2 r_0^{18}(-\frac{32782}{17967}r_0^4 \omega^2 r^6 - \frac{100864}{17967}r_0^8 r^4 + \omega^2 r^{10} \right) \]

\[ + \frac{2837}{5989} r_0^8 \omega^2 r^2 + \frac{50432}{17967} r_0^4 r^8 + \frac{50432}{17967} r_0^{12} \ln(1 + \frac{r_0^2}{r^2}) + 42362880\lambda^2 r_0^8 r^{20} \ln(2) \omega^2 \]

\[ + 193181184\lambda^2 r_0^2 (-\frac{5}{8} \omega^2 + r_0^2)r^{26} + (-301985040\lambda^2 r_0^6 + 72442944\lambda^2 r_0^4 \omega^2)r^{24} \]

\[ + (-216910848\lambda^2 r_0^8 + 180332460\lambda^2 r_0^6 \omega^2)r^{22} + (588652400\lambda^2 r_0^{10} \]

\[ - 151400000\lambda^2 r_0^8 \omega^2 + (4473000\omega^2 - 756000\lambda^2 \omega^4)r_0^6 )r^{20} + (23729664\lambda^2 r_0^{12} \]

\[ + (-4914000 + 9837212\lambda^2 \omega^2)r_0^{10})r^{18} + (1366241912\lambda^2 r_0^{14} + (-590625\omega^2 \]

\[ - 1314600\lambda^2 \omega^4)r_0^{10})r^{16} + ((-45108000 + 9495348\lambda^2 \omega^2)r_0^{14} + 466200r_0^{10} \omega^4)r^{14} \]

\[ + (-7974974584\lambda^2 r_0^{18} + (-331800\lambda^2 \omega^4 - 15600375\omega^2)r_0^{14})r^{12} + (48816276\lambda^2 \omega^2 \]

\[ + 159390000)r_0^{18} r^{10} + (6879600r_0^{18} \omega^2 + 12897439512\lambda^2 r_0^{22})r^8 + (-163800000 \]

\[ - 29432400\lambda^2 \omega^2)r_0^{22} r^6 - 8635726200\lambda^2 r_0^{26} r^4 + 54432000r_0^{26} r^2 + 2060352000\lambda^2 r_0^{30} \]

\[ E = -\frac{\gamma r_0^4}{24r^{11}} \left( 204\lambda^2 r^{16} - 317\lambda^2 r_0^4 r^{12} - 111r_0^4 r^{10} + 299\lambda^2 r_0^8 r^8 + 222r_0^8 r^6 \right) \]

\[ - 463\lambda^2 r_0^{12} r^{4} - 111r_0^{12} r^2 + 277\lambda^2 r_0^{16} \right) \]

\[ F = -\frac{\gamma r_0^4}{12r^{12}} \left( 708\lambda^2 r^{16} - 753\lambda^2 r_0^4 r^{12} - 267r_0^4 r^{10} + 805\lambda^2 r_0^8 r^8 + 378r_0^8 r^6 \right) \]

\[ - 1047\lambda^2 r_0^{12} r^{4} - 111r_0^{12} r^2 + 287\lambda^2 r_0^{16} \right) \]

**Appendix C**

Introducing the \( \omega = 2r_0 \omega \) the coefficients in equation of motion are

\[ C_1 = -17 (\rho^4 - 1)^4 \rho^3 \left( \frac{91}{204} \rho^4 + \frac{277}{204} - \frac{37}{68} \rho^2 \right) \]

\[ C_2 = -\frac{1}{6} (\rho^4 - 1)^3 \rho^2 \left( 1020 \lambda^2 \rho^4 + 703 \lambda^2 \rho^8 + 111 \rho^6 - 194 \lambda^2 \rho^4 - 999 \rho^2 + 3047 \lambda^2 \right) \]
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