Electronic transport of bilayer graphene with asymmetry line defects

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In this paper, we study the quantum properties of a bilayer graphene with (asymmetry) line defects. The localized states are found around the line defects. Thus, the line defects on one certain layer of the bilayer graphene can lead to an electric transport channel. By adding a bias potential along the direction of the line defects, we calculate the electric conductivity of bilayer graphene with line defects using Landauer-Büttiker theory, and show that the channel affects the electric conductivity remarkably by comparing the results with those in a perfect bilayer graphene. This one-dimensional line electric channel has the potential to be applied in the nanotechnology engineering.

I. INTRODUCTION

Recently, graphene, a two-dimensional Dirac material[1–7] becomes an attractive research field due to its exotic quantum properties, such as the unconventional quantum Hall effect, the Klein paradox [8], the Josephson effect [9], and the n-p junction [10]. Bilayer graphenes are weakly-coupled two single-layer graphene by interlayer carbon hopping, typically arranged in the Bernal (AB) stacking arrangement. In addition to the interesting underlying physics properties, bilayer graphenes have the potential electronics applications, owing to the possibility of controlling both the carrier density and energy band gap through doping or gating [11–14]. A dual-gated structure allows electrical and independent control of the perpendicular electric field and the carrier density [15–17]. Intrinsic bilayer graphene has no band gap between its conduction and valence bands and the low-energy dispersion is quadratic with massless chiral quasiparticles [18, 19, 21]. This is in contrast to what is observed in the monolayer which has a linear dispersion with massless quasiparticles.

In this paper, we consider a bilayer graphene with asymmetry line defects and study the defect states and the electric conductivity using the Landauer-Büttiker theory. This nontrivial physics properties of the defect states may be applied to a new type of devices based on the bilayer graphene.

The paper is organized as follows: Firstly, we introduce the tight-binding Hamiltonian of the bilayer graphene. Secondly, we calculate the energy structure of bilayer graphene with line defects. Next, we show the effects of the line defects on the electric conductance, including the effects of defect-line number and the vertical electric field. Finally, summary and conclusions are provided.

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energy difference between the two layers is parameterized by
potentials), the inversion symmetry is broken. Hence, the
mass gap opens in the low energy spectrum [23–25].

A When we assume that the upper and lower layers are at
conduction bands. If the inversion symmetry is broken,
exists the degeneracy of the highest valence and lowest
energy spectrum near $V / \gamma = 16$ eV, [20].

Here, $a^\dagger_l,i$ ($b^\dagger_l,i$) is the annihilation operator at sublattice $A_l$ ($B_l$) at site $\mathbf{R}_i$, where $l = 1$ represents the upper
layer and $l = 2$ represents the lower layer of the bilayer
material. The first term in the Hamiltonian describes intralayer nearest-neighbor coupling inside the layer, the
second term describes the interlayer coupling between the
sublattice of the lower layer $B_2$ and the sublattice of the
upper layer $A_1$ and here we set $\gamma_0 = 3.16$ eV, $\gamma_1 = 0.39$
eV [21].

Due to spacial translation symmetry, intrinsic bilayer
graphene can be described in momentum space and the
number of sites in a primitive cell is four. Owing to
the inversion symmetry in neutral bilayer graphene, there
exists the degeneracy of the highest valence and lowest
conduction bands. If the inversion symmetry is broken,
mass gap opens in the low energy spectrum [22, 23].

When we assume that the upper and lower layers are at
different electrostatic potentials $V$ (normally called bias
potentials), the inversion symmetry is broken. Hence, the
energy difference between the two layers is parameterized
by the energy $V$. Then, the Hamiltonian becomes

$$
H = H_0 + HV
$$

$$
= - \gamma_0 \sum_{l, (i,j)} \left( a^\dagger_{l,i} b_{l,j} + h.c. \right) - \gamma_1 \sum_i \left( a^\dagger_{1,i} b_{2,i} + h.c. \right)
+ \sum_{l, (i,j)} (V_{l,i} a^\dagger_{l,i} a_{l,i} + V_{l,j} b^\dagger_{l,j} b_{l,j}),
$$

(2)

where $HV$ is the external electrostatic potential. Here, we
consider a staggered energy potential for bilayer graphene
with $V_{1,i} = V$ on the upper layer and $V_{2,i} = -V$ on the
lower layer, where $V \geq 0$ is a controllable constant
quantity.

III. LOCALIZED STATES INDUCED BY LINE
DEFECTS

In this paper, we assume that a line-lattice defect lies
on the upper layer, as represented by the highlighted red
line marked in Fig.1(b), which is a zigzag lattice chain.

To study the line defects on bilayer graphene, we simu-
late lattice defects on bilayer graphene by the following
method: for a target lattice site $i$ on line defects, we set
all the coupling hopping parameters connected to the site
$i$ to 0 and the local on-site energy potential $V_i$ to be a
relatively larger value. To study the energy spectrum we
assume the bilayer graphene is semi-infinite, that is, the
system is finite along the $y$-direction and infinite along the
$x$-direction. Thus, the line defect is an infinite zigzag
chain along the $x$-direction. To study the localized states
induced by line defects, we use

the period boundary conditions (PBC) along the $y$-
direction which makes the system a nanotube. By per-
foming Fourier analysis and diagonalization, we may ob-

FIG. 2: (Colour online) Energy spectra and localized states with potential $V / \gamma_0 = 0.00$. The spectra of a sin-
gle particle in the zigzag bilayer graphene nanoribbons with
a line defect located at $n_{yy} = 120$ on the upper layer is shown
in Fig.1(b), which has PBC along the $y$-direction. (a) En-
ergy spectrum near $E(k_x) = 0$, where the red and green lines
denote the defect states of the graphene with a line-defect.
The blue dot indicates an energy state with $E(k_x) = 0$. (b)
shows the particle-distribution of the zero-energy energy state
indicated by the blue dot in (a) on the upper layer (blue solid
line) and lower layer(red solid line), respectively.
tain the eigenvalues (or energy spectrum) $E(k_z)$ and the corresponding eigenstates. The effects of the line defects and vertical electric potential are shown in Fig.2, Fig.3, Fig.4, respectively. For the case of $V > 0$, there exists bulk energy gap, and the value of the gap becomes larger with an increase of $V$. In particular, a line defect leads to localized states (or defect states).

We consider a line defect on the upper layer and set the total number of zigzag chain (parallel to the red zigzag line in Fig.1(b)) along $y$-direction to be $N_y = 240$. When $V = 0.00$ there exist defect states (marked by the red solid line and green solid line in Fig.2(a)) and zero-energy states, which has a flat band. For this case, the wave function of zero-energy state is localized around the defects (Fig.2(b)). The length-scale of the localized states along $x$-direction is $\xi \sim 1/\Delta_f$ where $\Delta_f$ is the energy gap of bulk system. For the case of $V > 0$, we also have the localized states around line defects. For this case, a bulk gap appears and the defect states becomes dispersive. The probability density of zero-energy wave function $|\psi_E=0(y)|^2$ becomes extended on both upper layer and lower layer of the bilayer graphene, and the maximum value of $|\psi_E=0(y)|^2$ is on the corresponding positions on lower layer.

Furthermore, we can also study the cases with several parallel line defects. For two parallel line defects on the upper layer, there exists localized defect states which are located around corresponding line defects, and the electric field induces a finite bulk energy gap (Fig.4).

IV. TRANSPORT PROPERTIES OF A BILAYER GRAPHENE WITH ASYMMETRY LINE DEFECTS

Bilayer graphene exhibits nontrivial transport properties owing to its unusual band structure where the conduction and valence bands touch with quadratic dispersion. One of the earliest theoretical papers studying the conductivity through AB-stacked bilayer graphene was Ref. 26 where it was assumed that the band structure of the bilayer is described by two bands closest to the Dirac point energy. Transport properties and the nature of conductivity near the Dirac point were probed experimentally 1, 27 and investigated theoretically 28.

We use the well-known Landauer-Büttiker equation to obtain the electric conductivity. To show the effect of the line defect on the electric conductance, we separate the original system into three parts: left lead, right lead and the center device area, as shown in the schematic diagram in Fig.5. The left and right leads are semi-infinite and the center device area has finite size. The number of primitive cells along the $x$-direction is $N_x$; the number of zigzag chain along the $y$-direction is $N_y$. Here, it is a zigzag edge along the $x$-direction and an armchair edge along the $y$-direction. Hence, for the lead-center-left bilayer graphene system, the zero-temperature conductance is calculated using the Landauer-Büttiker formalism 29:

$$G = \frac{2e^2}{h} \sum_{\alpha,\beta} \langle T \rangle_{\beta\alpha},$$

where $\langle T \rangle_{\beta\alpha}$ is the transmission from the incoming state $\alpha$ in the left lead to the outgoing state $\beta$ in the right lead. The terminal electric conductance can be written as $G = \frac{2e^2}{h}T$, and the transmission probability $T$ through it is

$$T = Tr[\Gamma_L G_C^r \Gamma_R G_C],$$

where $\Gamma_L$ ($\Gamma_R$) is the left (right) coupling matrix, and $G_C^r$ ($G_C$) is the retarded (advanced) Green’s function of the center part of the system.

The single particle Green’s function of the center devise area $G_c$ is defined as

$$G_c = [(E + i\eta)I - H_c - \Sigma_L - \Sigma_R]^{-1}$$

where $E$ is the energy of the particle, $\eta$ is a positive infinitely-small real number, $I$ represents the identity matrix, and $H_c$ is the Hamiltonian of the center devise area.

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**FIG. 4:** (Colour online) **Energy spectra and localized states when** $V/\gamma_0 = 0.08$. The wave function in (b) corresponds to the quantum state of the energy indicated by the blue dot in (a). Here are two parallel line defects at $n_y = 80$ and $n_y = 160$ on the upper layer.

**FIG. 5:** (Color online) The illustration of the system to calculate the electric conductance, which consist of center device area (denoted by 0 with green area) and left/right leads (denoted by other integer numbers with yellow area). The structure of center can be various, but here we focus on the bilayer graphene with line defects shown in Fig.1(b).
To compare with the electrical transport effects of the line defects, the electric conductance of perfect bilayer graphene devices is also given (Fig.6). We use PBC along the $y$-direction, that is, the center devices area is a bilayer graphene nanotube. When $V = 0$, the value of $G$ near $E = 0$ is $2(\frac{2e^2}{h})$, which is double to the value of one layer graphene $\frac{2e^2}{h}$ owing to the degeneracy of the highest valence and lowest conduction bands. When $V > 0$, the value of $G$ at $E = 0$ approaches zero, which indicates that the system generates a bulk energy gap, and the larger $V$ results in a larger width of gap. However, the electric conductance shows additional features induced by the line defects as shown in Fig.7. When $V = 0$ the value of $G$ near $E = 0$ is about $\frac{2e^2}{h}$. When $V > 0$ there is $G(E) \neq G(-E)$, and from the contrast of pure bilayer graphene and symmetric defects it is clear that this is the result of the particle-hole symmetry breaking by the asymmetric defects [30,31].

Now, the degeneracy of the zero energy is left and the energy spectrum becomes asymmetry. Hence, the conductance can be controlled by the vertical electric potential and the line defects in this case. As a result, this one-dimensional line electric channel has the potential to be applied in the nanotechnology engineering.

V. CONCLUSION

In this paper, we studied the physics properties of a bilayer graphene with line defects, including the defect-induced localized states and the electric conductivity. We found that the line defect on a certain layer of the bilayer graphene leads to an electric channel. When $V = 0$, the localized states on single layer has a flat band with zero energy. When $V > 0$ the system becomes gapped and the localized modes have the distribution on both layers. The additional conductance $G$ from line defect is obtained. This effect from line defects may be applied in electronic devices based on bilayer graphene.

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