Rate Allocation for Block-based Compressive Sensing

Quang Hong Nguyen¹, Khanh Quoc Dinh², Viet Anh Nguyen¹, Chien Van Trinh³, Byeungwoo Jeon⁴

Abstract

Compressive sensing (CS) has drawn much interest as a novel sampling technique that enables sparse signal to be sampled under the Nyquist-Shannon rate. By noting that the block-based CS can still keep spatial correlation in measurement domain, this paper proposes to adapt sampling rate of each block in frame according to its characteristic defined by edge information. Specifically, those blocks containing more edges are assigned more measurements utilizing block-wise correlation in measurement domain without knowledge about full sampling frame. For natural image, the proposed adaptive rate allocation shows considerable improvement compared with fixed subrate block-based CS in both terms of objective (up to 3.29 dB gain) and subjective qualities.

Keywords: Block-based Compressive Sensing, Adaptive Measurement Rate, Edge Detection.

¹ Sungkyunkwan University, College of Information & Communication Engineering
² Corresponding Author: bjeon@skku.edu

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II. Related works

1. Compressive Sensing

A finite-length signal $x \in \mathbb{R}^N$ is called a $k$-sparse if it has at most $k$ non-zero coefficients for some $k \leq N$. Compressive sensing (CS) [5] is an emerging framework which can reduce the sampling cost; it is proved that a $k$-sparse signal $x$ can be recovered from far fewer measurements $y \in \mathbb{R}^M$ (i.e., $M < N$) than the Nyquist/Shannon sampling rate where the measurement vector $y$ is a linear projection of the signal $x$ by a measurement (or sensing) matrix $\Phi$ as:

$$ y = \Phi x \quad (1) $$

The ratio of $M/N$ is called the sparsity (or measurement rate), which represents how much sub-sampling is done since $M < N$. Note that, the measurement matrix $\Phi$ needs to satisfy the restricted isometry property (RIP) [6]. Matrix $\Phi$ satisfies RIP of order-k if there exits a $\delta_k \in (0, 1)$ holding for all $k$-sparse vector $x$ such that:

$$ (1 - \delta_k) \|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \delta_k) \|x\|_2^2 \quad (2) $$

In fact, most of the natural signals are not exactly sparse, but can be sparsely represented in a proper transform domain, or compressible in some transform domains. By assumption that the signal $x$, through a sparse transform with its kernel $\Psi$, can be represented as $x = \Psi \theta$, where $\theta$ is the transform coefficient of $x$, and the signal $\theta$ is sparse (or at least compressible), the sparse vector $\theta$ of the signal $x$ can be recovered from the $M$ measurements, $y = \Phi x = \Phi \Psi \theta$ by $l_1$-minimization as following:

$$ \hat{\theta} = \min \| \theta \|_1 \text{ such that } y = \Phi \Psi \hat{\theta} \quad (3) $$

where the $l_1$-norm $\| \theta \|_1$ is number of non-zero vector elements of $\theta$. However, the $l_1$-minimization is a non-convex problem, which means potentially very difficult to solve, especially when the number of vector elements are large. Instead, CS solves (3) by $l_1$-minimization, which is a convex optimization problem, as:

$$ \hat{\theta} = \min \| \theta \|_1 \text{ such that } y = \Phi \Psi \hat{\theta} \quad (4) $$

Here, $\| \theta \|_1 = \sum_{i=1}^{N} |\theta_i|$ is the $l_1$-norm of $\theta$.

It is already known that CS can successfully recover a $k$-sparse signal under the condition that the number of acquired measurements $M$ should satisfy [7]:

$$ M = O(k \log \frac{N}{k}) \quad (5) $$

There are many algorithms to solve the CS reconstruction problem; for example, Matching Pursuit [8], Orthogonal Matching Pursuit [9], Multiple Candidate Matching Pursuit [10], SL0 [18], or TV/L1 [19] algorithms.

2. Block-based Compressive Sensing

In CS for image/video processing, the block-based CS (BICS) method has been proposed to reduce the complexity of system [11-13]. An input image is split into non-overlapping blocks denoted by $x^{(i)} \in \mathbb{R}^{B^2}$ where $i$ is a block index, $x^{(i)}$ is a column vector with $B^2$ coefficients, each of which is a pixel value inside the $i$-th $B \times B$ block. Then, blocks are compressively sampled into measurement domain as $y_i = \Phi_{B}x^{(i)}$ where $y_i \in \mathbb{R}^{M_B}$ is a column vector measurement and $\Phi_{B}$ is a $M_B \times B^2$ measurement matrix; in this case, subrate of a block is calculated as a ratio of $M_B/B^2$. The measurement matrix of a whole frame is a block-diagonal matrix as:

$$ \Phi = \begin{bmatrix} \Phi_{B_1} & & \\ & \ddots & \\ & & \Phi_{B_2} \end{bmatrix} $$

For recovery, [11] suggests a procedure that combines projected Landweber iteration with smoothing in the form of Wiener filtering. The overall process of BCS sampling and SPL reconstruction are called BCS-SPL. An improvement of BCS-SPL is also presented in [12].

III. Rate Allocation for Block-based Compressive Sensing

It is worth noting that BCS still keeps the block correlation in spatial domain also in measurement domain [13-15]. S. Mun et al. empirically confirmed the high correlation among measurements of BCS using Lena image, which showed average correlation even larger than 0.95 [13].

Additionally, characteristics of natural image is changed from region to region. Some regions can be smooth, while others can be complex with many objects. As a result, the sparsity (or compressibility) of image is also changed with different regions. For example, shown more clearly about this problem. From original Lena image, we extract two blocks with two different characteristics; the upper one locates in a smooth region and the lower one locates in a complex region with many edge information (see the edge image on the left side). And then, DCT transformation is applied for two those blocks. Obviously, the sorted in descending order of absolute DCT coefficients of upper one decrease much quickly than the lower one. That means the upper one (smooth block) is more compressible than the lower one (complex block). As a result, smooth block needs less measurements than complex block to achieve equally reconstructed performance.

Motivated by the correlation among BCS measurements, we detect edges in image using the measurement data only. That is, if the block measurement data of blocks at an initial subrate $s_0$ (i.e., same original subrate for all blocks) $Y = (y_1, y_2, ..., y_M)$ is block-averaged as:

$$ \bar{Y} = \frac{1}{M} \sum_{i=1}^{M} y_i $$

where $y_i$ is the measurement vector of a $i$-th block, and

![Image](image_url)
\( \hat{y} \) is a mean value of measurement vector \( y \). Then \( \hat{Y} \) is reshaped into a form of 2D image \( \hat{Y}_{id} = \text{reshape}(\hat{Y}) \), we can see a roughly estimated original image in a low resolution (i.e., \( \hat{Y}_{id} \) still has good spatial correlation among blocks in measurement domain). Subsequently, the reshaped image \( \hat{Y}_{id} \) is scaled up to original image size by a bicubic interpolation \[ \hat{f}_{yt} = \text{interp}(\hat{Y}_{id}) \] where \( \text{interp}(\cdot) \) is the bicubic interpolation operator. Interestingly, the interpolated image still clearly contains edges and boundaries of the original image, and therefore, a typical edge detection algorithm is expected to identify those edge pixels well.

Fig. 2 shows the whole process of edge detection in measurement domain. Clearly, the detected edge image shows edges quite well. Thus, it is possible to characterize blocks based on the edge information, for example, a smooth block will contain very few edge pixels while a complex block will contain many edge pixels. By this way, it is expected that the more complex (less compressible) the block is, the more edge pixels the block contains. Therefore, in this paper, we estimate the sparsity of \( i \)-th block proportionally with number of edge pixels in block as:

\[
\epsilon_i = \frac{\text{number of edge pixels in } i\text{-th block}}{\text{number of pixels in } i\text{-th block}}
\]

The bigger \( \epsilon_i \) means that block is less compressible. On the other hand, the number of required measurements for CS reconstruction depends much on the sparsity of block as shown in (5); hence, a sparser block needs fewer measurements [1]. Thus we should look for a way to sample each block with its proper subrate adaptively. In this paper, the rate allocation for each block is based on the edge image - blocks containing more edges (i.e., more complex) are sampled with a higher subrate than otherwise.

The whole proposed compressed sensing system is shown in Fig. 3; sparsity estimation is equivalent to edge detection process which determines the edge information in each block. The subrate of each block is assigned proportionally to sparsity of block as following:

\[
s_i = s_{i0} + \alpha \times \epsilon_i
\]

where \( s_i \) is an adaptively selected subrate for \( i\)-th block; \( s_{i0} \) is the minimum subrate of block. \( s_{i0} \) is firstly setup \( s_{i0} = r \times s_0 \). \( r \) is predefined parameter to determine how small the minimum subrate assigned for each block is, \( 0 < r < 1 \). The meaning of \( s_{i0} \) is to guarantee that the subrate assigned for each block is not so small compared with \( s_0 \) which can make unsteady or inaccurate in reconstruction and we may not get the improvement. Parameter \( \alpha \) is a control parameter (its value is calculated below).

The subrate of each block is adjusted so that the final subrate of whole image is not changed compared with \( s_0 \) such that:

\[
\sum_{i=1}^{n} s_i = L \times s_0
\]

where \( L \) is the number of blocks in image . From (8) and (9):

\[
\sum_{i=1}^{n} s_i = L \times s_{i0} + \alpha \sum_{i=1}^{n} \epsilon_i = L \times s_0
\]

\[
\Rightarrow \epsilon_i = \frac{(s_{i0} - s_0)}{L}
\]

The value of \( \alpha \) in (11) is then substituted into (8) to calculate subrate of each block. However, if the assigned subrate for a block exceed \( 1 \) (which is nonsense in CS system since number of measurements is not higher than signal length), \( s_i \) is adjusted by increasing \( r \) (note that \( r \) is still smaller than 1), and then subrate assigned for each block is re-calculated.

As a result, a smooth block may be assigned a smaller subrate than \( s_0 \) while a complex block is assigned a higher subrate than \( s_0 \). Finally, since smooth blocks are assigned subrate lower than \( s_0 \), we only take required number of measurement \( B^2 \times s_i \) from \( y \). Otherwise, those complex blocks are assigned with higher subrate than \( s_0 \), we have to sample more \( B^2 \times (s_i - s_0) \) measurements.

### IV. Experimental results

In this section, performance of the proposed method is presented using four 512x512 gray natural images, namely, Lena, Barbara, Boat, and Cameraman. At encoder, the input image is split into small non-overlapped blocks and...
then mapped into measurement domain by an i.i.d. random Gaussian. In our experimental results, we tested three subrates 0.2, 0.4, 0.6 with two block sizes of 8x8 and 16x16.

Performance of the proposed method is evacuated using three reconstructed algorithms: Iterative Support Detection (ISD) [17], Smoothed L0 Norm (SLO) [18], and total variation (TVL3) [19]. For edge detection, Canny edge detection algorithm is used considering its good performance compared with other edge detection algorithms [20].

Additionally, value of β/3 is assigned for the initiation of parameter r. More detail result of the experiments is shown in Table 1, Table 2, Table 3 and Fig. 4.

| Image | Subrate | ISD [17] | SLO [18] | TVL3 [19] |
|-------|---------|-----------|-----------|------------|
| Lena  | 0.2     | 25.66     | 25.81     | 25.61      |
|       | 0.4     | 30.83     | 31.72     | 30.89      |
|       | 0.6     | 35.05     | 36.22     | 35.43      |
|       |anny     | 22.92     | 23.07     | 22.98      |
|       | 0.4     | 27.52     | 28.04     | 27.61      |
|       | 0.8     | 31.86     | 32.43     | 31.92      |
| Boat  | 0.2     | 23.17     | 23.74     | 23.22      |
|       | 0.4     | 27.70     | 28.31     | 27.59      |
|       | 0.6     | 31.87     | 32.39     | 31.89      |
|       | 0.4     | 32.43     | 34.56     | 32.32      |

Table 1. Experimental results with various CS recovery methods for block size of 16x16 | PSNR in dB |

Table 2. Image Lena, Barbara, Boat, and Cameraman compare the performance of the proposed method for block size of 8x8 and 16x16. Clearly, the proposed method gains remarkably compared with the case of fixed subrate BCS. Those images with much detail, like Barbara, show PSNR improvement of 0.64 dB to 1.42 dB. With smooth images, like Cameraman, the improvement is up to 3.29 dB (subrate 0.2 with ISD algorithm).

Additionally, Table 2 shows the performance of CS recovery with block size of 16x16. Obviously, performance of the proposed method is better than the fixed subrate BCS; however, improvement of the proposed method with block size of 16x16 is lower than that with block size of 8x8. It is because the block size of 8x8 is better in keeping the block correlation and the spatial information is also preserved well in measurement domain than the block size of 16x16. The proposed adaptive subrate assignment for each block becomes more accurate leading to higher performance in CS recovery.

Moreover, Table 3 shows visual comparison between the fixed subrate BCS and the proposed method by SSIM index with block size 8x8; it is obvious that, in all cases, the proposed method presents better performance. For more clearly, Fig. 4 illustrates an example of cropped Lena image at subrate 0.2 and TVL3 algorithm. Fig. 4(c) shows better in visual quality (the fine detail becomes much clearer) compared with Fig. 4(b), especially in some complex regions which contains much edge information (subrate for those blocks in this region is high) leading to good reconstructed block both in subjective and objective qualities.

V. Conclusion

In this paper, an adaptive rate allocation for BCS of images is proposed. The characteristic of block is defined based on edge detection and then subrate of each block is assigned commensurately to how many edge pixels it contains in the edge image - blocks containing more edges are assigned more measurement data than others. The experimental results showed remarkable improvement of the proposed method compared with the fixed subrate BCS.
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Khánh Quoc Dinh
- 2013 년 박사: Hanoi University of Science and Technology (Vietnam)
- 2012 년 석사: 성균관대학교 전자전기정보공학부
- 2013 년 학사: 한국경찰청 정보통신경찰단
- ORCID: http://orcid.org/0000-0002-7141-5863

Phạm Văn Minh
- 2013 년 박사: Hanoi University of Science and Technology (Vietnam)
- 2012 년 석사: 성균관대학교 전자전기정보공학부
- ORCID: http://orcid.org/0000-0002-4734-8555

Do Thi Lan Quyen
- 2013 년 박사: Hanoi University of Science and Technology (Vietnam)
- 2012 년 석사: 성균관대학교 전자전기정보공학부
- ORCID: http://orcid.org/0000-0002-4734-8555

Nguyen Hong Quang
- 2015 년 박사: Hanoi University of Science and Technology (Vietnam)
- 2013 년 석사: 성균관대학교 전자전기정보공학부
- ORCID: http://orcid.org/0000-0002-5650-2881

Houbin Lan
- 2014 년 석사: 전자공학과
- 2013 년 학사: 성균관대학교 전자전기정보공학부
- ORCID: http://orcid.org/0000-0002-5650-2881