Improved Colorization Algorithm Using Lp Norm Minimization

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Abstract

We propose an algorithm to improve the colorization algorithm using the Lp-norm minimization approach. The proposed algorithm minimizes the Lp-norm of differences between neighboring pixels of a given luminance image to appropriately colorize a chrominance image. Since the Lp-norm minimization problem is difficult to solve, we apply the iterative reweighted least squares (IRLS) algorithm. Numerical examples show the effectiveness of the proposed method.

1. Introduction

This paper deals with a colorization problem, which is to recover a color image from a luminance image using a small number of color pixels. In order to solve this problem, various colorization algorithms have been proposed [1]-[5]. The colorization algorithm proposed in [1] is a well-known algorithm and gives high-quality color images from appropriately given color pixels. In [5], the authors proposed a colorization algorithm that achieves appropriate colorization even if the number of given color pixels is small.

A lot of colorization algorithms including those in [1,5] assume that the difference between the chrominance values of neighboring pixels is similar to the difference between the corresponding luminance values. However, in general images, luminance values change more than chrominance values. Figure 1 shows a luminance and two chrominance images of the Lena image, and we can see that the chrominance images have less variation than the luminance image.

In this paper, we propose an algorithm to improve the colorization performance of existing colorization algorithms by minimizing the redundant changes in a given luminance image. In order to achieve this, the Lp-norm minimization technique is applied to optimize a given luminance image, and the optimized luminance image is used to colorize the chrominance images. Figure 2 shows the outline of the proposed algorithm. Numerical examples show that the proposed algorithm improves the reconstruction quality of existing colorization algorithms.

Figure 1: Original image: (a) Luminance image, (b) Chrominance image (Cb) and (c) Chrominance image (Cr) of Lena

Figure 2: Outline of the proposed colorization algorithm
2. Proposed Method

This paper deals with the colorization problem of a luminance image with \( m \times n \) pixels, which is denoted as a vector. Then \( y \in R^{mn} \) denotes a given luminance image.

Define \( U \in R^{(m-1)\times mn} \), \( V \in R^{mn(2n-1)\times mn} \), \( U \in R^{(m-1)\times mn} \) and \( D \in R^{(2mn-m-n)\times mn} \) as

\[
U_{i,j} = \begin{cases} 
1, & \text{if } i = j \\
-1, & \text{if } i + 1 = j \\
0, & \text{otherwise}
\end{cases} \quad V_{i,j} = \begin{cases} 
1, & \text{if } i = j \\
-1, & \text{if } i + m = j \\
0, & \text{otherwise}
\end{cases}
\]

\( \bar{U} = \text{diag}(U, \ldots, U) \) and \( D = [U^T \ V^T]^T \), where \( \text{diag}(A_1, \ldots, A_m) \) denotes a block diagonal matrix consisting of \( A_1, \ldots, A_m \). The given vector \( Dy \) denotes the differences between the neighboring pixels of a whole image. Although many colorization algorithms assume that the change in chrominance values between neighboring pixels is similar to the change in luminance values between neighboring pixels, general chrominance images are smoother than the corresponding luminance images except in the edge regions. In order to recover precise chrominance images, we propose a method to smooth luminance images.

To minimize the difference between neighboring pixels except in the edge regions, the following Lp-norm minimization problem is considered:

\[
\text{Minimize} \quad \| x - y \|^2_2 + \lambda \| Dx \|_p \tag{1}
\]

where \( \| \cdot \|_p \) denotes the Lp norm of a vector. \( \lambda > 0 \) and \( p \geq 0 \) are given constants. Note that this problem with \( p = 1 \) corresponds to the total variation (TV) regularization problem, whose solution gives smooth images with maintaining sharp edges by making \( D x \) sparse. In the theory of compressed sensing [6], Lp-norm minimization with \( 0 \leq p \leq 1 \) gives a sparse solution, and therefore we consider the case of \( 0 \leq p \leq 1 \). The best value of \( p \) depends on the choice of colorization algorithm, and we determine it not theoretically but practically.

Since problem (1) with \( p < 1 \) is non-convex and difficult to solve exactly, we apply the iterative reweighted least squares (IRLS) algorithm [7, 8]. This algorithm gives an approximate solution to the Lp-norm minimization problem by solving the weighted least-squares solution and updating the weights iteratively.

Applying the IRLS algorithm to Eq. (1), the \( k \)th solution is obtained as

\[
x^{(k)} = \arg \min_{x \in R^{mn}} \| x - y \|^2_2 + \lambda \| W^{(k-1)} D x \|_2^2 \tag{2}
\]

that is,

\[
x^{(k)} = (I + \lambda D^T W^{(k-1)} D)^{-1} y \tag{3}
\]

where \( A^\dagger \) denotes the pseudoinverse of matrix \( A \). \( W^{(k)} \) is a diagonal matrix whose diagonal elements are updated as

\[
W^{(k)}_{i,i} = (\| D x^{(k)} \|_i + \epsilon)^{p/2-1} \quad (4)
\]

where \( \epsilon > 0 \) is a given constant. We propose an algorithm to improve the existing colorization algorithm, as shown in Algorithm 1, where \( T \) denotes the number of iterations of the IRLS algorithm.

Algorithm 1 Proposed algorithm

Require: \( y, \lambda, p, \epsilon \) and \( T \)
set \( k = 1 \)
set \( W^{(0)} \) as the identity matrices
repeat
    calculate \( x^{(k)} \) using Eq. (3)
    update \( W^{(k)} \) using Eq. (4)
    \( k \leftarrow k + 1 \)
until \( k = T \)
colorize an image using the optimized luminance image \( x^{(T)} \) by existing colorization algorithm

Ensure: color image

Figure 3: Test images: (a) Lena (512 \times 512), (b) Peppers (512 \times 512), (c) Mandrill (256 \times 256) and (d) Parrots (256 \times 256)

3. Numerical Examples

This section shows numerical examples to demonstrate the effectiveness of the proposed algorithm. In each example, a color image is recovered from its luminance image with color
information of only 0.1% of the pixels in a whole image, which are uniformly selected randomly. In all examples, we use \( \varepsilon = 10^{-5}, T = 10 \) and the test images shown in Fig. 3. Because the best values of \( \lambda \) and \( p \) depend on the colorization algorithm, we examine all combinations of \( \lambda \in \{1, 10, 100, 1000\} \) and \( p \in [0, 1] \) with intervals of 0.1 and select the values maximizing the peak signal to noise ratio (PSNR) of the recovered Lena image for each colorization algorithm.

First we use the colorization algorithm in [1] as the existing colorization algorithm and examine the reconstruction performance of the proposed algorithm. The proposed algorithm uses the parameters \( \lambda = 100 \) and \( p = 0.2 \). Table 1 shows the PSNR of these algorithms. The proposed algorithm gives larger PSNR values than the algorithm proposed in [1] except for the Cb of the Peppers image. Figure 4 shows the given luminance image, the optimized luminance image and colorization results. Figure 5 shows the recovery error, which is calculated as the average of the errors at each pixel.
Table 1: PSNR [dB] comparison of the proposed algorithm with the colorization algorithm in [1]

|        | Lena | Peppers | Mandrill | Parrots |
|--------|------|---------|----------|---------|
|        | Cb   | Cr      | Cb       | Cr      |
| Algorithm proposed in [1] | 30.9113 | 31.1157 | 27.4916  | 24.3527 |
| Proposed algorithm using [1]| 32.2260 | 31.4453 | 27.3132  | 24.8235 |

Table 2: PSNR [dB] comparison of the proposed algorithm with the colorization algorithm in [5]

|        | Lena | Peppers | Mandrill | Parrots |
|--------|------|---------|----------|---------|
|        | Cb   | Cr      | Cb       | Cr      |
| Algorithm proposed in [5] | 31.8462 | 31.3881 | 27.0119  | 25.1542 |
| Proposed algorithm using [5]| 33.2803 | 32.3387 | 27.5524  | 25.7235 |

in Cb and Cr. We can see that the proposed algorithm has less recovery error and that the color of the whole image of (c) in Fig. 4 is faded.

Next we adopt the colorization algorithm proposed in [5] as the existing colorization algorithm and examine the reconstruction performance. The proposed algorithm uses the parameters $\lambda = 10$ and $p = 0.8$. Table 2 shows the PSNR of these algorithms. Except for the Cr of the Mandrill image, the proposed algorithm achieves larger PSNR values than the algorithm proposed in [5]. Figure 6 shows the given luminance image, the optimized luminance image and the colorization results, and the recovery error is shown in Fig. 7. As can be seen, the proposed algorithm has less recovery error, and some parts of the image of (c) have a faded color, for example, Lena’s hair color is mixed with her skin color.

4. Conclusion

We propose a new algorithm to improve the performance of existing colorization algorithms by applying $L_p$-norm minimization to a given luminance image. The proposed algorithm minimizes the $L_p$ norm of differences between neighboring pixels of a given luminance image to appropriately colorize a chrominance image, and the IRLS algorithm is applied to solve the $L_p$-norm minimization problem. Numerical examples show that the proposed method enhances the colorization quality of existing algorithms [1, 5]. A method to determine appropriate parameters is planned as future work.

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