Research Article

Chebyshev Neural Network-Based Adaptive Nonsingular Terminal Sliding Mode Control for Hypersonic Vehicles

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Abstract

This paper presents an adaptive nonsingular terminal sliding mode control approach for the attitude control of a hypersonic vehicle with parameter uncertainties and external disturbances based on Chebyshev neural networks (CNNs). First, a new nonsingular terminal sliding surface is proposed for a general uncertain nonlinear system. Then, a nonsingular sliding mode control is designed to achieve finite-time tracking control. Furthermore, to relax the requirement for the upper bound of the lumped uncertainty including parameter uncertainties and external disturbances, a CNN is used to estimate the lumped uncertainty. The network weights are updated by the adaptive law derived from the Lyapunov theorem. Meanwhile, a low-pass filter-based modification is added into the adaptive law to achieve fast and low-frequency adaptation when using high-gain learning rates. Finally, the proposed approach is applied to the attitude control of the hypersonic vehicle and simulation results illustrate its effectiveness.

1. Introduction

In recent years, hypersonic vehicles have received much attention because they are viewed as a cost-effective and promising method for access to space. The dynamics of hypersonic vehicles is severely nonlinear, fast time-varying, highly uncertain, and strongly coupled. Moreover, hypersonic vehicles often suffer unexpected disturbances [1, 2]. Therefore, it is a great challenge to design a robust fight control system for hypersonic vehicles.

So far, many advanced control methods have been used to design control systems for hypersonic vehicles, such as dynamic inversion [3–5], backstepping control [6–9], and fuzzy control [10–13]. In particular, sliding mode control (SMC) is well known for its robustness against parameter uncertainties and external disturbances [14]. However, the conventional linear SMC can only ensure the asymptotic stability. To achieve finite-time convergence, terminal sliding mode control (TSMC) techniques have been proposed by introducing a nonlinear term into the sliding surface design [15, 16]. Unfortunately, TSMC suffers from the singularity problem. To deal with this drawback, nonsingular TSMC (NTSMC) techniques have been developed [17–19]. Compared with conventional linear SMC, TSMC and NTSMC offer some superior properties such as faster convergence and higher control precision and are appropriate for flight control design. In [20], an adaptive nonsingular terminal sliding mode control scheme was proposed for the attitude control of the hypersonic vehicle. Mu et al. proposed a fast terminal sliding mode control approach for an air-breathing hypersonic vehicle [21].

Even though TSMC and NTSMC have the above-mentioned advantages, they need the prior knowledge of the upper bound of the uncertainties and external disturbances, which is very hard to know in practice. To tackle this issue, neural networks have been widely used in flight control design. In [22], a second-order nonsingular terminal sliding mode control approach was developed for a hypersonic vehicle by using recurrent neural networks to approximate the parameter uncertainties and external disturbances. Zhu et al.
[23] proposed a robust adaptive control strategy for a hypersonic flight vehicle subject to stochastic disturbances and dynamic uncertainties based on neural network approximation. In [24], radial basis function neural networks are utilized to estimate the unknown additive faults and external disturbances of the hypersonic vehicle. Chebyshev neural network (CNN) is a kind of functional link network whose input is generated based on a subset of orthogonal Chebyshev polynomials. It has been shown that CNN has some important advantages, such as simplicity of its structure, fast learning speed, low computational complexity, and powerful approximation abilities [25, 26]. Due to these advantages, CNN has been successfully applied in the control of bilateral teleportation systems [27], attitude control of spacecrafts [28], online system identification [29], and so on.

Inspired by the above analysis, this paper proposes an adaptive nonsingular terminal sliding mode control based on CNN for the attitude control of the hypersonic vehicle under parameter uncertainties and external disturbances. First, a new nonsingular terminal sliding surface is introduced and its convergence time is calculated analytically. Then, a nonsingular terminal sliding mode control law is developed to drive the tracking error to converge to zero in finite time. Furthermore, to relax the requirement of the upper bound of the lumped uncertainty, a CNN is employed to approximate the lumped uncertainty and network weights are updated online. Besides, to achieve fast and low-frequency adaptation for network weights through a high-gain learning rate, a low-pass filter-based modification is applied to the adaptive law of network weights to filter out the high-frequency component. Finally, the proposed control approach is applied to the hypersonic vehicle attitude control. The main contributions of this paper are as follows: (1) A new nonsingular terminal sliding surface is proposed. (2) The proposed control approach does not depend on any prior knowledge of the upper bound of the lumped uncertainty. (3) A low-pass filter-based update law for network weights is proposed so that fast and low-frequency adaptation can be achieved through high-gain learning rates with guaranteed system stability.

The remainder of this paper is organized as follows: Section 2 presents the problem formulation. The design of the adaptive nonsingular terminal sliding mode control and the stability analysis are introduced in Section 3. Simulation results for the attitude control of the hypersonic vehicle are provided in Section 4. Finally, some conclusions are drawn in Section 5.

### 2. Problem Formulation

The considered attitude dynamic equations of a hypersonic vehicle are given as [20, 30] follows:

\[
\dot{\Omega} = f_j(\Omega) + g_j(\Omega)\omega + \Delta_j(\Omega),
\]

\[
\dot{\omega} = f_j(\omega) + g_j(\omega)\omega + \Delta_j(\omega, t),
\]

where \(\Omega = [\alpha, \beta, \gamma]^T\) is the attitude angle vector composed of angle of attack, sideslip angle, and bank angle. \(\omega = [p, q, r]^T\) is the angular rate vector consisting of roll rate, pitch rate, and yaw rate. \(M_e = g_{j_3}\delta\) is a control moment vector, where \(\delta = [\delta_e, \delta_a, \delta_r]^T\) is the control surface deflection composed of elevator, aileron, and rudder, and \(g_{j_3}\) is the control allocation matrix. \(g_s\) and \(g_f\) are the invertible system matrices, and \(f_e, g_e, f_f, g_f\) are the state function vectors. The concrete expressions for \(g_{j_3}, f_f, f_e, g_s,\) and \(g_f\) can be found in [20, 30]. \(\Delta_j(\Omega) = \Delta_j(f_j(\Omega))\) and \(\Delta_j(\omega, t) = \Delta_j(f_j(\omega) + d_j(t))\) are the lumped uncertainties, where \(\Delta_j(f_j(\omega) + d_j(t))\) stand for the system uncertainties and \(d_j(t)\) represents the external disturbance.

According to the singular perturbation theory, the aforementioned equations can be divided into fast-loop and slow-loop, respectively. The slow-loop and fast-loop control systems can be designed separately.

The main goal of this paper is to design the adaptive nonsingular terminal sliding mode control law, such that attitude angle \(\Omega\) can track the given command \(\Omega_s\) stably and robustly in the presence of parameter uncertainties and external disturbances.

To proceed the control design for the hypersonic vehicle, the following assumptions and lemmas are required.

**Assumption 1.** All system states are available.

**Assumption 2.** The lumped uncertainties \(\Delta_j\) and \(\Delta_f\) are bounded, i.e., \(\|\Delta_j\| \leq \rho_j\) and \(\|\Delta_f\| \leq \rho_f\), where \(\rho_j\) and \(\rho_f\) are unknown positive constants.

**Lemma 1** (see [31]). Consider the following system:

\[
\dot{x} = f(x),
\]

\[
f(0) = 0, \quad x \in \mathbb{R}^n,
\]

where \(f: D \rightarrow \mathbb{R}^n\) is continuous on an open neighborhood \(D\) of the origin. Suppose there exist a continuous positive-definite function \(V(x)\), real numbers \(c > 0\), and \(0 < \xi < 1\), such that

\[
\dot{V}(x) + cV^\xi(x) \leq 0.
\]

Then, the origin of system (3) is a locally finite-time stable equilibrium, and the settling time, depending on the initial state \(x(0) = x_0\), satisfies \(T(x_0) \leq (V(x_0))^{1-\xi}/(c - \xi)\). In addition, if \(D = \mathbb{R}^n\) and \(V(x)\) is also radially unbounded, then the origin of system (3) is a globally finite-time stable equilibrium.

### 3. Control Design

Equations (1) and (2) can be described as the following general uncertain MIMO nonlinear system:

\[
\begin{align*}
\dot{x} &= f(x) + \Delta f(x) + g(x)u + d(t), \\
y &= x,
\end{align*}
\]

where \(x \in \mathbb{R}^n\) is the system state vector, \(u \in \mathbb{R}^m\) is the control input vector, \(y \in \mathbb{R}^p\) is the system output vector, \(f(x) \in \mathbb{R}^n\) is the system state function vector, \(g(x) \in \mathbb{R}^{m \times n}\) is the system control gain matrix, \(\Delta f(x)\) is the system uncertainty, and \(d(t)\) is the external disturbance.
Define
\[ \Delta(x, t) = \Delta f(x) + d(t), \]  
where \( \Delta(t, x) \) represents the lumped uncertainty and it is assumed that \( \|\Delta(t, x)\| \leq \rho \), where \( \rho \) is an unknown positive constant.

In this section, the control objective is to design a control law so that the output \( y \) tracks a desired reference trajectory \( x_c \) in finite time.

### 3.1. Sliding Surface Design

Define the tracking error as \( e = x - x_c \). Then, a new nonsingular terminal sliding surface is designed as
\[ \sigma = e(t) + l \int_0^t \Delta(e(\tau)) \text{sign}^{1-y}(e(\tau)) d\tau, \]  
where \( l = [l_1, l_2, \ldots, l_n]^T \), \( \Delta(e) = \text{diag}(\exp(k|e|^{\tau}), \exp(k|e|^{\tau}), \ldots, \exp(k|e|^{\tau})) \), and \( \text{sign}^{1-y}(e) = [|e|^{1-y} \text{sign}(e_1), |e|^{1-y} \text{sign}(e_2), \ldots, |e|^{1-y} \text{sign}(e_n)]^T \). \( k > 0, \ 0 < y < 1, \) and \( l > 0 \) are constants.

When the system reaches the sliding mode, it satisfies the following equations:
\[ \sigma_i = 0, \]  
\[ \dot{\sigma}_i = 0, \quad i = 1, 2, \ldots, n \]  
Consequently, the following sliding mode dynamics is obtained as
\[ \dot{e}_i = \dot{e}_i + l \exp(k|e|^{\tau}) |e|^{1-y} \text{sign}(e), \]  
Equation (10) can be rewritten as
\[ \frac{de_i}{dt} = -l \exp(k|e|^{\tau}) |e|^{1-y} \text{sign}(e). \]  
Let \( e_{i0} \) be the initial value of \( e_i \). Integrating (12) from \( e_{i0} \) to \( e_i = 0 \) yields
\[ t_{oi} = \int_{e_{i0}}^0 \frac{de_i}{l \exp(k|e|^{\tau}) |e|^{1-y} \text{sign}(e)}. \]

Therefore, for any given initial state \( x(0) \), the tracking error \( e_i (i = 1, 2, \ldots, n) \) can converge to zero in finite time.

**Remark 1.** In [32], a nonsingular terminal sliding surface is proposed as follows:
\[ s_i = ce_i(t) + \int_0^t |e_i(\tau)| \text{sign}(e_i(\tau)) d\tau, \quad i = 1, 2, \ldots, n, \]  
where \( c > 0 \) and \( 0 < a < 1 \) are constants. The convergence time of the tracking error \( e_i \) from the initial error state \( e_i(0) \neq 0 \) to zero in the sliding mode \( s_i = 0 \) is given as
\[ t_{si} = \frac{|e_i(0)|^{1-a}}{(1-a)(1/c)}. \]  
Compared with the sliding surface (14), a nonlinear exponential term is added in the proposed sliding surface. Consequently, a faster convergence rate of the proposed sliding surface can be achieved by tuning the parameter \( k \).

### 3.2. Nonsingular Terminal Sliding Mode Control Design

After the sliding surface is defined, the next step is to design a sliding mode control law to drive the system trajectories onto the sliding surface in finite time. A nonsingular terminal sliding mode control law is proposed as
\[ u = g^{-1} \left[ -f + (l\Delta(e) \text{sign}^{1-y}(e) - \rho \text{sign}(\sigma)) \right]. \]  
Combining (5), (9), and (16) yields
\[ \dot{\sigma} = \Delta - \rho \text{sign}(\sigma). \]  
Furthermore, choose a Lyapunov function as
\[ V = \frac{1}{2} \sigma^T \sigma. \]
Differentiating (18) with respect to time and using (17) yield
\[ \dot{V} = \sigma^T (\Delta - \rho \text{sign}(\sigma)) \leq - (\rho - \|\Delta\|) \|\sigma\| \leq - \overline{p} \|\sigma\| \leq -\sqrt{2pV^{1/2}}, \]  
where \( \overline{p} = \rho - \|\Delta\| \).
According to Lemma 1, the tracking error \( e \) will converge to the sliding surface \( \sigma = 0 \) in a finite time \( T_1 \leq \sqrt{2V^{1/2}} \).

### 3.3. Adaptive Nonsingular Terminal Sliding Mode Control Design

Since the uncertainty bound \( \rho \) is usually difficult to measure in practical applications, a large control gain \( \rho \) is often chosen to achieve system robustness and ensure system stability. Unfortunately, it will result in a severe chattering phenomenon. To relax the requirement of previous determination of the uncertainty bound, a CNN is used to estimate the lumped uncertainty online.
CNN is a functional link neural network based on Chebyshev polynomials, which are a set of orthogonal polynomials derived from the solution of the Chebyshev differential equation. The Chebyshev polynomials can be obtained by the following recursive formula:

\[ T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x), \quad T_0(x) = 1, \]

where \( x \in \mathbb{R} \) and \( T_1(x) \) have several definitions, such as \( x, 2x, \ldots, 2x - 1 \) and \( 2x + 1 \). Here, \( T_1(x) \) is chosen as \( x \). Then, an enhanced pattern using the Chebyshev polynomials for a vector \( X = [x_1, x_2, \ldots, x_m]^T \in \mathbb{R}^m \) is given by

\[ \psi(X) = [1, T_1(x_1), \ldots, T_N(x_1), \ldots, T_1(x_m), \ldots, T_N(x_m)]^T, \]

(21)

where \( T_i(x_j) (i = 1, \ldots, N, j = 1, \ldots, m) \) represents a Chebyshev polynomial, \( N \) is the order of Chebyshev polynomials, and \( \psi(X) \) is the Chebyshev polynomial basis function.

Thus, the CNN output can be expressed as

\[ y = W^T \psi(X), \]

(22)

where \( W \) is the weight matrix.

Based on the approximation property of CNN, the lumped uncertainty \( \Delta (x, t) \) can be approximated by CNN as

\[ \Delta = W^* T \psi(X_m) + \varepsilon, \]

(23)

where \( W^* \) is the optimal weight matrix and \( X_m = [e^T, e^T]^T \) is the input. \( \varepsilon \) is the CNN approximation error and satisfies \( \| \varepsilon \| \leq \varepsilon_N \) with \( \varepsilon_N \) being an unknown positive constant.

Then, an estimation of \( \Delta \) is given as

\[ \hat{\Delta} = \hat{W}^T \psi, \]

(24)

where \( \hat{W} \) is the estimate of \( W^* \).

According to the Lyapunov stability theory, a standard update law for \( \hat{W} \) is derived as

\[ \hat{\dot{W}} = \xi \psi \sigma^T, \]

(25)

where \( \xi \) is the learning rate.

In order to speed up the CNN parameter convergence, a high-gain learning rate is necessary to achieve fast adaptation. However, this can cause high-frequency chattering in control response and may lead to system instability. Therefore, a low-pass filter weight estimate of \( \hat{W} \) is designed as [33]:

\[ \hat{W}_L = a(\hat{W} - \hat{W}_L), \]

(26)

\[ \hat{W}_L(0) = \hat{W}(0), \]

where \( a > 0 \) is a filter time constant.

The adaptive law of \( \hat{W} \) is redesigned as follows:

\[ \hat{\dot{W}} = [\xi \psi \sigma^T - \xi \kappa (\hat{W} - \hat{W}_L)] \]

(27)

where \( \kappa \) is a positive constant.

Define the estimation error as

\[ \Delta - \hat{\Delta} = W^* T \psi + \varepsilon - \hat{W}^T \psi = \hat{W} \psi + \varepsilon, \]

(28)

where \( \hat{W} = W^* - \hat{W} \).

Then, an adaptive nonsingular terminal sliding mode control law is designed as

\[ u = g^{-1}[-f + \hat{x}_e - l \alpha (e) \sigma^{1-\gamma} (e) - \hat{\Delta} - (E + \eta) \text{sign}(\sigma)], \]

(29)

where \( E > \varepsilon_N \) and \( \eta \) is a positive constant.

Using equations (9) and (29), the closed-loop sliding mode dynamics is obtained as follows:

\[ \dot{\sigma} = \hat{W}^T \psi + \varepsilon - (E + \eta) \text{sign}(\sigma). \]

(30)

**Theorem 1.** Consider the nonlinear system (5), if the control law is designed as (29) and parameter adaptive laws are
designed as (26) and (27), then, all signals of the closed-loop system are bounded. Moreover, if the design parameter $E$ satisfies $E > \|W\psi + \epsilon_N\|$, the sliding variable $\sigma$ will converge to zero in finite time.

**Proof.** Define a Lyapunov function candidate as
\[
V_1 = \frac{1}{2} \sigma^T \sigma + \frac{1}{2 \kappa} \text{tr}\left( W^T \hat{W} \right) + \frac{\kappa}{2 \alpha} \text{tr}\left( W_L^T \hat{W}_L \right),
\] (31)
where $\hat{W}_L = W^* - \hat{W}_L$

Taking the time derivative of the Lyapunov function (31) and using (30), we have
\[
V_1 = \sigma^T \dot{\sigma} + \frac{1}{\xi} \text{tr}\left( W^T \dot{W} \right) + \frac{\kappa}{\alpha} \text{tr}\left( W_L^T \dot{W}_L \right)
= \sigma^T \left( W^T \psi + \epsilon - (E + \eta) \text{sign}(\sigma) \right) - \frac{1}{\xi} \text{tr}\left( W^T \dot{W} \right)
- \frac{\kappa}{\alpha} \text{tr}\left( W_L^T \dot{W}_L \right)
= \sigma^T W^T \psi + \sigma^T \epsilon - \sigma^T (E + \eta) \text{sign}(\sigma) - \frac{1}{\xi} \text{tr}\left( W^T \dot{W} \right)
- \frac{\kappa}{\alpha} \text{tr}\left( W_L^T \dot{W}_L \right).
\] (32)

Substituting the adaptive laws (26) and (27) into (32) yields
\[
V_1 = \sigma^T W^T \psi + \sigma^T \epsilon - \sigma^T (E + \eta) \text{sign}(\sigma) - \text{tr}\left( W^T \psi \sigma^T \right)
+ \kappa \text{tr}\left( W^T \dot{W} \right) - \kappa \text{tr}\left( W_L^T \dot{W}_L \right) - \kappa \text{tr}\left( W_L^T \dot{W}_L \right)
= \sigma^T \epsilon - \sigma^T (E + \eta) \text{sign}(\sigma) + \kappa \text{tr}\left( W^T \left( \dot{W} - \dot{W}_L \right) \right)
- \kappa \text{tr}\left( W_L^T \left( \dot{W} - \dot{W}_L \right) \right)
= \sigma^T \epsilon - \sigma^T (E + \eta) \text{sign}(\sigma) - \kappa \text{tr}\left( (W_L - \dot{W})^T (\dot{W} - \dot{W}_L) \right)
= \sigma^T \epsilon - \sigma^T (E + \eta) \text{sign}(\sigma) - \kappa \text{tr}\left( (W - \dot{W}_L)^T (\dot{W} - \dot{W}_L) \right).
\] (33)

Using the fact $\text{tr}\left( (\dot{W} - \dot{W}_L)^T (\dot{W} - \dot{W}_L) \right) \geq 0$, one can obtain
\[
\dot{V}_1 \leq \sigma^T \epsilon - \sigma^T (E + \eta) \text{sign}(\sigma)
\leq -(E + \eta - \epsilon_N) ||\sigma||
\leq -\eta ||\sigma||.
\] (34)

Thus, the stability of the closed-loop system can be guaranteed, and it can be concluded that $\sigma, W, \dot{W}_L, \epsilon, \dot{\epsilon}$ are bounded. Then, it is easy to obtain that $W^T \psi$ is also bounded.

Next, it will be proven that the sliding variable $\sigma$ can converge to zero in finite time. Choose a Lyapunov function as
\[
V_2 = \frac{1}{2} \sigma^T \sigma.
\] (35)

Differentiating (35) with respect to time and substituting (30) into the derivative of (35), one obtains
\[
\dot{V}_2 = \sigma^T W^T \psi + \sigma^T \epsilon - \sigma^T (E + \eta) \text{sign}(\sigma)
\leq -\eta ||\sigma|| - \left( E - \|W^T \psi\| - \epsilon_N \right) ||\sigma||.
\] (36)

Since $E - \|W^T \psi\| - \epsilon_N \leq 0$ and $\eta > 0$, it can be obtained that
\[
\dot{V}_2 \leq -\eta ||\sigma||.
\] (37)

Therefore, according to Lemma 1, it can be concluded that the sliding variable $\sigma$ can converge to zero in a finite time $T_2 \leq \sqrt{2V_2^{1/2}(\sigma_0)/\eta}$. Then, the tracking error $e$ will converge to zero within a finite time.

**Remark 2.** As $\hat{W}$ and $\epsilon_N$ are actually unknown, $E$ is acquired by the trial and error method in general.

**Remark 3.** Since the discontinuous function sign involved in (29) may cause the chattering phenomenon, a hyperbolic tangent function $\tanh(\sigma/h)$ is used to replace the sign function, where $h$ is the thickness of the boundary layer.

The parameter design guideline is presented as follows:

(1) From (13), it can be concluded that the parameters $k$ and $l$ determine the convergence time $t_{0i}$. Large $k$ and $l$ will result in a shorter convergence time. However, too large $k$ and $l$ may substantially increase the magnitude of control input (29). Therefore, $k$ and $l$ should be properly chosen to avoid excessive control input, considering the practical implementation conditions. The parameter $\gamma (0 < \gamma < 1)$ should satisfy the constraint $0 < \gamma < 0.5$, so as to avoid causing chattering in the control input.

(2) The parameter $\xi$ is the adaptation gain as shown in (27). A large $\xi$ should be chosen to achieve fast adaptation for the network weight. The parameter $a$ needs to be fine-tuned such that the high-frequency content of network weight can be cut off. The parameter $\kappa$ is a small constant, and its range is usually set as $0 < \kappa \leq 0.1$.

(3) The parameter $\eta$ affects the reaching time of the sliding surface (7), which can be gradually increased from zero until the tracking performance is satisfactory.

(4) The boundary-layer thickness $h$ should be kept small to ensure a tradeoff between control performance and chattering attenuation.

### 4. Simulation Verification

In this section, the proposed approach is applied to the attitude control of the hypersonic vehicle and simulation...
results are presented to validate the effectiveness. The initial values of the states are set as $V_0 = 2500 \text{ m/s}$, $H_0 = 30 \text{ km}$, $\rho_0 = q_0 = r_0 = 0 \text{ rad/s}$, $\alpha_0 = 1^\circ$, $\beta_0 = 0^\circ$, and $\mu_0 = 0^\circ$.

The command signals are chosen as $\alpha_c = 4^\circ$, $\beta_c = 0^\circ$, and $\mu_c = 3^\circ$. Moreover, the command signals pass three same command filters, respectively, to obtain a smooth attitude trajectory. The command filter is given by

$$\frac{\Omega_r}{\Omega_c} = \frac{2}{s + 2}$$

(38)

where $\Omega_c = [\alpha_c, \beta_c, \mu_c]^T$. $\Omega_r$ is the command filter output.

Suppose that there are $+30\%$ and $-30\%$ uncertainties in the aerodynamic coefficients and aerodynamic moment coefficients, respectively. Besides, the time-varying disturbance moment imposed on the fast loop is given by

$$d_f(t) = 10^5 \left[ \sin(3t) + 0.5, \sin(8t) + 0.1, \sin(5t) \right]^T \text{Nm}$$

(39)

The parameters of the proposed control law are chosen as follows:

$$k_s = 2, k_f = 2, l_s = 3, l_f = 1, \gamma_s = 0.2, \gamma_f = 0.2, \xi_s = 100, \xi_f = 1000, \kappa_s = 0.1, \kappa_f = 0.1, \alpha_s = 5, \alpha_f = 5, E_s + \eta_s = 0.8, E_f + \eta_f = 2, h_s = 0.01, h_f = 0.01$$

(40)

where the parameters with the subscript “s” represents the slow-loop controller parameters and the parameters with the subscript “f” represents the fast-loop controller parameters.
To compare the control performances of the proposed control approach, a conventional sliding mode control (CSMC) method with a linear sliding surface $S = e(t) + \lambda \int_0^t e(\tau) d\tau$ [24] is adopted to design the attitude control system for the hypersonic vehicle, as follows:

$$\omega_c = -g_\varepsilon[ f_\varepsilon - \dot{x}_\varepsilon + \lambda_\varepsilon e_\varepsilon + \Gamma_\varepsilon \tanh(S_\varepsilon/\nu)],$$  

(41)

$$M_c = -g_f[ f_f - \omega_f + \lambda_f e_f + \Gamma_f \tanh(S_f/\nu)],$$  

(42)

where $\nu$ is the thickness of the boundary layer around the sliding surface.

The parameters of the CSMC system (41) and (42) are selected as $\lambda_\varepsilon = \lambda_f = 8$, $\Gamma_\varepsilon = \Gamma_f = 50$, and $\nu = 0.01$. The simulation results are shown in Figures 2–5.

From Figure 2, it can be seen that under the effects of the parameter uncertainties and time-varying external disturbances, the tracking performance of CSMC is very poor. By contrast, the proposed approach shows better tracking performance with faster convergence speed and higher precision. Figure 3 shows the control inputs generated by the two approaches. It is observed that high adaptive gains used in the proposed approach never lead to high-frequency oscillations appearing in the control input signal. The time responses of sliding surfaces $\sigma_\varepsilon$ and $\sigma_f$ are illustrated in Figure 4. It can be seen that sliding variables $\sigma_\varepsilon$ and $\sigma_f$ converge to $|\sigma_\varepsilon| < 2 \times 10^{-3}$ and $|\sigma_f| < 2 \times 10^{-3}$, $i = 1, 2, 3$, in finite time, respectively, and all reaching times are less than 1s. Figure 5 shows the CNN output $\hat{\Delta}_{\varepsilon}$ and $\hat{\Delta}_f$. Therefore, it is concluded that the proposed approach is effective.
5. Conclusion

This paper presents a novel adaptive nonsingular terminal sliding mode control approach for the attitude control of hypersonic vehicles under parameter uncertainties and external disturbances. CNNs are used to approximate the lumped disturbance, and the adaptive laws with filter-based modification term for the CNN weights are derived to improve the transient performance. The stability and finite-time convergence of the closed-loop system can be guaranteed by the Lyapunov stability theory. Simulation results in the hypersonic vehicle attitude system verify the effectiveness and robustness of the proposed approach.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.
Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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