Thermo-optical properties of silicon nitride Mach-Zehnder interferometer for the on-chip quantum random number generator

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Abstract. Here we study the thermo-optical properties of an on-chip silicon nitride Mach-Zehnder interferometer (MZI). The spectral shift of the MZI is associated with a change in the chip temperature. This can be explained by a change in the splitting ratio of the directional couplers, as well as a significant change in phase difference between waveguide arms. We experimentally found a phase shift of $2\pi$ when heated by $1.67\, ^\circ\text{C}$ and changes in resonant wavelength at different temperatures ($d\lambda/dT$) equal $12.0\, \text{pm}/^\circ\text{C}$, theoretically obtained a formula for an arbitrary splitting ratio of the directional couplers in an MZI, and determined the temperature stability required to the device operation inside a quantum cryptography system.

1. Introduction
High thermo-optical (TO) stability of photonic integrated circuits (PICs) is required in most modern applications, such as quantum key distribution and quantum random number generators, where passive elements, such as Mach-Zehnder interferometers (MZIs), are used to transfer phase-encoded signals into amplitude modulation [1]. In fact, MZIs generally require accurate resonant wavelengths; therefore, the temperature dependence of the effective refractive index $n$ should be taken into account. Inasmuch as the temperature coefficient of the Si$_3$N$_4$ refractive index ($2.45 \times 10^{-5} \, ^\circ\text{K}^{-1}$ [2]) is much smaller than in silicon on an insulator ($1.8 \times 10^{-4} \, ^\circ\text{K}^{-1}$ [3]), the former is less sensitive to temperature fluctuations. However, due to the large difference between the arms in our Si$_3$N$_4$ interferometers [4], even a small change in temperature can significantly affect the operation of the device due to the shift in the optical spectrum. For this reason, the use of such elements in PICs requires thermal control, which is usually solved by packaging. But, for example, maintaining the laser wavelength in high-throughput multiplexing systems requires maintaining the temperature within $\pm 0.1\, ^\circ\text{C}$ [5]. Here, we study the temperature dependence of the optical spectrum, visibility at wavelength, change resonance wavelength and the phase shift in the MZI.
2. Theory of a Mach-Zehnder Interferometer

In general case, an interferometer is an optical device that splits a wave into two parts and creates a phase difference between them by passing different lengths of optical paths, and then aligns them and measures their superposition. Figure 1 shows a free space Mach-Zehnder interferometer, which consists of two beam splitters that serve to split and combine waves, and two mirrors that set the optical path lengths of the interferometer arms.

![Mach-Zehnder Interferometer Diagram](image)

**Figure 1.** (a) Free space Mach-Zehnder Interferometer; (b) Spectral intensities at the two outputs of the interferometer at transmission coefficients of beam splitters equal to 0.9.

Let the transmission and reflection coefficients of the beam splitters be equal to \( \alpha \) and \( \beta \), respectively. In case of low absorption and scattering losses we get \( \alpha + \beta \approx 1 \). The wave with amplitude \( E_0 \) splits at the first beam splitter into \( E_0 \sqrt{\alpha_1} \) and \( E_0 \sqrt{\beta_1} \), then these two waves are redirected to the second beam splitter. Due to different optical paths, two waves come to the second beam splitter with a phase difference \( \Delta \phi \); moreover, additional losses may occur in one of the arms, which may be introduced via a transmission coefficient \( \xi \). In fact, when the length of one of the arms is much larger than the other, as in our case, the losses in one arm are greater than in the other. Thus, two waves fall on the second beam splitter \( \sqrt{\alpha_1} \) and \( \sqrt{\beta_1} \xi \). As a result, we get four waves after the second beam splitter: \( E_0 \sqrt{\alpha_1} \xi \beta_2 \) and \( E_0 \sqrt{\beta_1} \xi \alpha_2 \cdot e^{i\Delta \phi} \), which interfere in one port, and \( E_0 \sqrt{\beta_1} \xi \beta_2 \cdot e^{i\Delta \phi} \) and \( E_0 \sqrt{\alpha_1} \alpha_2 \) in the other. Using the equation (1):

\[
I = E \cdot E^* = I_1 + I_2 + 2 \sqrt{I_1 \cdot I_2} \cdot \cos(\Delta \phi),
\]

where \( I_1 \) and \( I_2 \) the intensities of the interfering waves, we find the intensities of the optical radiation at the output ports:

\[
I_{P_1}(\Delta \phi) = I_0(\alpha_1 \beta_2 + \beta_1 \xi \alpha_2 + 2 \sqrt{\alpha_1 \alpha_2 \beta_1 \beta_2} \cdot \xi \cdot \cos(\Delta \phi))
\]

\[
I_{P_2}(\Delta \phi) = I_0(\alpha_1 \alpha_2 + \beta_1 \xi \beta_2 + 2 \sqrt{\alpha_1 \alpha_2 \beta_1 \beta_2} \cdot \xi \cdot \cos(\Delta \phi + \pi)).
\]

Further, using these expressions, the interference visibility colour maps were built, which is defined as:

\[
V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \frac{I_p(2\pi) - I_p(\pi)}{I_p(2\pi) + I_p(\pi)},
\]
where \( I_{\text{max}} \), \( I_{\text{min}} \), are the intensities at the output of the interferometer in the case when the interfering waves arrived in-phase and antiphase, respectively. The maximum visibility of interference is achieved when the waves that interfere are equal in intensity to each other.

### 2.1. Lossless example

Consider the case when the losses in the interferometer arms are the same (\( \xi = 1 \)). Figure 2 shows the visibility maps for the interferometer outputs. We are interested in two special cases: 1) the beam splitters in the Mach-Zehnder interferometer are the same, which leads to the equality of the coefficients \( \alpha \) and \( \beta \), and 2) the coupling ratio of one of the splitters is 0.5.

![Figure 2](image)

**Figure 2.** (a) Dependence of the visibility on the splitting ratios of the first port without taking into account losses. (b) Dependence of the visibility on the splitting ratios for the second port without taking into account losses.

In case of identical beam splitters, the visibility takes the highest values at the output of the first port. At the output of the second port, the visibility has a maximum only at transmission coefficients close to 0.5. At the same time, with significant deviations of the transmittance \( \alpha \) from 0.5, the visibility drops sharply.

To understand why this happens, consider, for example, the case in which the transmittance \( \alpha \) is close to unity. The initial radiation \( I_0 \) is split into four waves. The most intense of them, which passed the two beam splitters, enters port 2. The wave with the lowest amplitude, which is reflected twice, also enters port 2. The two remaining waves once reflected and passed once through the beam splitter, are identical in intensity and enter port 1. All this leads to the fact that in the case of identical beam splitters and an arbitrary coupling ratio, the radiation intensity in port 1 significantly exceeds the intensity in the second port, while the radiation visibility, on the contrary, is maximal at the output of the second port.

If one beam splitter divides the radiation in half, there are no differences in the intensity and visibility of the interference at the output from the two ports. The outputs differ only in that the interference patterns are shifted relative \( \pi \) to each other (Figure. 1 (b)).

### 2.2. Example with losses

Let us consider the case when the losses in the arm, into which the light enters after reflection at the first beam splitter, are much higher than in the other. In this case, \( \xi \) is different from unity. Figure 3 shows the dependence of the visibility on the transmittance of the beam splitters \( \alpha_1 \) and \( \alpha_4 \) at \( \xi = 0.01 \).
Figure 3. (a) Dependence of the visibility on the splitting ratios of the first port, taking into account the loss. (b) Dependence of the visibility on the splitting ratios for the second port, taking into account the loss.

In the case of identical beam splitters, the visibility takes the highest values at the output of the second port at low values of the transmittance. In the case when it is less than 0.5, most of the radiation enters the arm with large losses. If you choose such that it will satisfy the condition \( \alpha^2 = \beta^2 \xi \) then the maximum visibility will be observed at the output of the second port, since the interfering waves will be equal in intensity. The visibility of the interference in port 1 is not affected by the values of the division factors, as in the case when \( \xi = 1 \). When \( \xi \) decreases, the visibility of the interference also decreases, since one of the waves loses its intensity, and the intensity of the second wave is not affected by \( \xi \), and the difference between the intensities of the interfering waves increases.

Since in our work along delay line is used in one of the arms, \( \xi \) differs significantly from one. The loss in the delay line was estimated to be about 20 dB \( (\xi = 0.01) \). Therefore, to get the highest visibility, the second port was chosen for the operation. In addition, the values of the transmittance of the beam splitters are optimal in the range from 0.05 to 0.15.

3. Device fabrication and experimental setup

To fabricate our device, we use a commercially available 525 µm silicon (Si) wafer with 2.6 µm thick buried silicon oxide (SiO\(_2\)) and over a 450 nm silicon nitride (Si\(_3\)N\(_4\)) layer. To form the geometry of the waveguide, we used single-stage electron-beam lithography with a high-contrast positive resist ZEP 520A and dry reactive-ion-etching (RIE) in a CHF\(_3\)-Ar mixture.

We fabricated a 2D array of the MZIs with a delay line, with various parameters of directional couplers interaction lengths in a range of 60 µm to 100 µm and gaps from 0.6 µm to 1.15 µm, the single device is shown in Figure 4 (a). The measurement transmission spectra of a 2D array with gap = 0.85 µm and different interaction length shown in Figure 5 (b).

In the second step, we characterized transmission spectra of on-chip MZI with different temperature substrate. For that, we used the experimental setup shown in Figure 5 (a).
Figure 4. (a) Optical micrograph of the on-chip MZI; (b) The normalized transmission spectra of MZI without coupling losses with temperature 25 °C and 30 °C; (c) Enlarged normalized transmission spectrum of MZI with temperature 25°C and 30°C.

Figure 5. (a) Schematic view of the experimental setup for MZI transmission measurements with different temperature of the substrate; (b) The measured transmission spectra of MZIs with constant gap = 0.85 µm and different interaction length.

Light from a tunable laser source through polarization controller coupled into the chip using a fibre array separated by with distance of 250 µm SMF-28 optical fibres as well. At the output fibre by a low-noise photodetector and amplifier and recorded by Ni-DAQ system. Chip was situated on rotation stage with x,y,z-motors. The change in the temperature of the stage in the range from 25 °C to 30 °C occurred due to the Joule heating of the resistor, which in turn heated the stage on which the sample was located. The stage temperature was controlled using a PID controller.

We characterized the transmission of MZI in the wavelength range 1510-1620 nm at a wavelength of 1552.85 nm. After the temperature normalization, the transmission spectrum of the MZI was measured. Figure 4 (a) shows the measured normalized transmission spectra of the MZI without coupling losses for two different temperatures (T), equal to 25 °C and 30 °C. In Figure 4 (c) we can see a zoomed image of optical transmission, and the maximum (P_{max}) and the minimum (P_T) optical power at a wavelength of 1552.82 nm for two different temperatures. In the final, we calculated visibility (ν_2) at wavelength use equation ν_2 = P_{max} - P_T/P_{max} + P_T. The results are shown in Figure 6 (a).
Figure 6. (a) Visibility at a wavelength = 1552.82 nm vs temperature; (b) Measured maximum optical power (black) and optical power at a wavelength = 1552.82 nm (red); (c) Wavelength in the minimum vs temperature.

Based on the measurement data (Figure 6 (c)) we found changes in resonant wavelength at different temperatures \((\Delta \lambda/\Delta T)\) equal 12.0 pm/°C as well as a phase shift by \(2\pi\) with a change in the substrate temperature by 1.67 °C. We also theoretically obtained a formula for an arbitrary division ratio of directional couplers in an interferometer and determined the temperature stability required to operate inside a quantum cryptography system.

4. Conclusion
In this work, we theoretically study the visibility of interference at splitting ratio in Mach-Zehnder interferometer and demonstrate visibility without and with losses in one of the interferometer arm. To study the TO effect, we change the temperature of the substrate in the range from 25 °C to 30 °C. Of the measured transmission spectra in this range we calculated the visibility of interference at wavelength 1552.85 nm. We have demonstrated a change in visibility and saw the change in resonance wavelengths by 12.0 pm/°C, as well as the phase shift by \(2\pi\) with a change in the substrate temperature by 1.67 °C. In particular, this will allow selecting the optimal temperature for work such device.

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