Power output and power coefficient calculations of a small HAWT with tubercles using Blade Element Momentum Theory

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Abstract. A 0.6 m diameter, three-bladed, untwisted, fixed-pitch, and small horizontal axis wind turbine with tubercles was analysed using the Blade Element Momentum (BEM) Theory. Calculation frameworks based from the Original BEM Theory, Wilson-Walker Method, Glauert’s Empirical Formula, and Buhl’s Theory were used in the prediction of the power output and power coefficient. Numerical results from the calculation frameworks were compared to the experimental data taken from the literature. The power output predicted by the Buhl’s Theory was the closest to the actual results. In the case of the power coefficient, the Wilson-Walker Method produced an almost equal prediction at $\lambda=4.06$, showing only a percentage difference of 0.47%. Results also showed that the power coefficient predicted by the four calculation frameworks were more accurate at higher tip speed ratios.

1. Introduction

Wind turbine technology had been improving gradually and remarkable advances in the wind turbine design had been achieved by a handful of researchers. In contrast to large horizontal axis wind turbines (HAWT) which provided reliable, cost-effective, and competitive power, small HAWT were required to produce power without necessarily the best of wind conditions [1-2]. Despite its growth and popularity, small HAWT had not been studied as thoroughly as their large counterparts.

One of the most noteworthy of all the remarkable advances on wind turbine technology was the correlation of turbine blades with the flippers of a humpback whale. The humpback whale (Megaptera novaeangliae), as seen on Figure 1, was incomparable among the large baleen whales because of its ability to undertake aquatic manoeuvres to catch prey despite its large size. Humpback whales utilized extremely mobile, wing-like flippers with rounded scalloped bumps called tubercles. Turbine blades with tubercles performed better than a straight blade by providing constant power output under the stall conditions, an important factor for sites with harsh atmospheric conditions and unsteady wind [3]. Addition of tubercles could even increase power generated, prevent loss of lift, and decrease noise [4-6]. Past studies have proven the aerodynamic promise of the tubercles. But despite its potential, wind turbines blades integrated with tubercles is not yet commercially available.

The most common tool in evaluating the aerodynamic performance of a HAWT is the Blade Element Momentum (BEM) Theory. BEM Theory is one of the oldest and simplest yet most effective methods in analyzing the loads of a HAWT. Despite its simplicity, the BEM theory provides relatively accurate results. For the past few years, researchers and wind turbine designers have optimized and
modified the BEM theory into various windmill brake state models like the Wilson-Walker method, Glauert’s empirical formula, and Buhl’s theory which incorporate various corrections to further increase the precision in the prediction.

The related literature on BEM theory analysis of small wind turbines with tubercles is rather limited, if not inexistent. Wind turbine researchers had been studying the BEM theory since its conception and marine biologist had been fascinated by the aerodynamic promise of the tubercles of humpback whales, but the combination of those fields produced a novel concept that was not yet thoroughly explored in a lot of studies. This paper investigated a small horizontal axis wind turbine with tubercles by analyzing its aerodynamic performance using the calculation framework based from the original BEM Theory and its three corrected models (Wilson-Walker method, Glauert’s empirical formula, and Buhl’s theory).

2. Materials and Methods

This paper adapted the blade design and experimental results from the study of Alsultan [8]. The turbine has a 0.6 m diameter, three blades, no twist, and fixed pitch at 0°. The design originated first from a straight blade. The chords lengths were manipulated throughout the blade span to transform the smooth edge into sinusoidal protuberances, resembling the tubercles of a humpback whale. Each cross section of the blade was a NACA4412 airfoil, one of the most widely used airfoil profile for the wind turbine applications. The initial parameters and geometrical inputs were entered in the code in Microsoft Excel to calculate the power output and coefficient using the calculation frameworks based from the Original BEM Theory, Wilson-Walker method, Glauert’s empirical formula, and Buhl’s theory. These particular methods were also used by various authors in the aerodynamic analysis of different HAWT [9–11]. The results were then compared to the actual results from the wind tunnel experiment done on the same blade from the literature.

3. Blade Element Momentum Theory

BEM Theory was a relatively straightforward method for aerodynamic analysis of horizontal axis wind turbines. The theory of the BEM combines the method of the one dimensional momentum theory and the two dimensional blade element theory.

Taking the concept of conservation of angular momentum in an annular stream tube, the thrust and torque on the annular element gives

\[ dF_{\text{thrust}} = 4a\pi\rho(1 - a)V^2rdr \]  

(1)

\[ dT = 4a'(1 - a)\rhoV\Omega\pi r^3 dr \]  

(2)
Meanwhile, the thrust and torque evolved by the elemental blade length positioned at a radius of “r” in the blade element theory, while multiplying it with the number of blades B, are given by

\[ dF_{\text{thrust}} = \frac{1}{2} \rho c W^2 (C_L \cos \phi + C_D \sin \phi) dr \] (3)

\[ dT = \frac{1}{2} \rho c W^2 (C_L \sin \phi - C_D \cos \phi) r dr \] (4)

where \( \rho \), \( c \), \( W \), \( C_L \), \( C_D \), and \( \phi \) are the density, chord length, resultant velocity, lift coefficient, drag coefficient, and flow angle respectively. Power output can be solved by applying the trapezoidal rule of integration to determine the overall torque. The equation for predicted power output was,

\[ P = T \Omega \] (5)

where \( T \) was the total torque for the entire blade length and \( \Omega \) was the angular speed of the rotor. If the unit for torque and angular speed are Newton-meter and radian per second, respectively, then the power output could be expressed as Watts.

4. Original BEM Theory

Combining equations (1) and (3) to eliminate the elemental thrust force, the new axial induction factor gives

\[ a = \frac{1}{\left[ \frac{4 \sin^2 \phi}{\sigma_r C_n} + 1 \right]} \] (7)

\[ C_n = C_L \cos \phi + C_D \sin \phi \] (8)

\[ \sigma_r = \frac{B c}{2 \pi r} \] (9)

where \( C_n \) is the normal force coefficient and \( \sigma_r \) was the local solidity factor. Similarly, combining equations (2) and (4) to cancel out the elemental torque, the new tangential induction factor becomes

\[ a' = \frac{1}{\left[ \frac{4 \sin \phi \cos \phi}{\sigma_r C_t} - 1 \right]} \] (10)

\[ C_t = C_L \sin \phi - C_D \cos \phi \] (11)

where \( C_t \) was the tangential force coefficient. This paper utilizes the iterative calculation feature in Microsoft Excel which eases the process of solving necessary variables for the analysis of the performance of the wind turbine. This study uses calculation frameworks to visualize the logical structure in solving the power output and power coefficient [12]. Figure 2 describes the logical structure needed in calculation.
Figure 2. Calculation framework based from the Original BEM Theory

Figure 3. Calculation framework based from the Wilson-Walker Method

Figure 4. Calculation framework based from the Glauert’s empirical formula

Figure 5. Calculation framework based from the Buhl’s Theory
5. Brake state models
The original BEM Theory had no correction factors integrated in its formula. Many corrections had been proposed to increase the accuracy of the calculations. One of the factors that could be incorporated on the formulation was the Prandtl’s tip loss factor (F).

\[ F = \frac{2}{\pi} \cos^{-1}\left( e^{-\frac{3}{2}r \sin \theta} \right) \]  

(12)

With \( F \) defined, the axial and tangential induction factors became

\[ a = \frac{1}{4 F \sin^2 \phi \sigma_r C_n + 1} \]  
(13)

\[ a' = \frac{1}{\left(\frac{4F \sin \phi \cos \phi}{\sigma_r C_t} - 1\right)} \]  
(14)

5.1. Wilson-Walker Method
In 1984, Wilson [13] reported the Wilson-Walker Method, which incorporated the Spera’s correction for the thrust coefficient based on the axial induction factor. If \( a \) became greater than the critical axial induction factor value, \( a_c \), which was 0.2, the following equations could be used,

\[ a = \frac{1}{2} \left\{ 2 + K(1 - 2a_c) - \sqrt{[2 + K(1 - 2a_c)]^2 + 4[Ka_c^2 - 1]} \right\} \]  
and \( K = \frac{4F \sin^2 \phi}{\sigma_r C_n} \)  
(15)

The step by step procedure of the Wilson-Walker method for each element of the blade can be summarized by Figure 3.

5.2. Glauert’s Empirical Formula
Glauert reported in 1926 that experimental results showing the thrust coefficients to be invalid if the axial induction factor exceeds approximately 0.4. Hence, Glauert came up with an empirical formula which addresses the validity of the numerical solution.

\[ a = \frac{0.143 + \sqrt{0.0203 - 0.6427(0.889 - C_T)}}{F} \]  
and \( C_T = \frac{F \sigma_r (1 - a)^2 C_n}{\sin^2 \phi} \)  
(16)

The procedure in Figure 4 describes the flow needed in calculation.

5.3. Buhl’s Theory
Buhl [13] derived a new equation for the thrust coefficient that solved the numerical instability commonly experienced in solving the BEM Theory iteratively.

\[ a = \frac{18\sigma C_n + 36F^2 \sin^2 \phi - 40F \sin^2 \phi - 6\sqrt{18F \sigma_r C_n \sin^2 \phi + 36F^4 \sin^4 \phi - 48F^3 \sin^4 \phi}}{2(9\sigma_r C_n - 50F \sin^2 \phi + 36F^2 \sin^2 \phi)} \]  
(17)

The procedure in applying the Buhl’s Theory for each element of the blade was described in Figure 5.
6. Results and Discussion
In this study, a small HAWT with tubercles was analyzed by using the original BEM Theory and other three brake state models (Wilson-Walker Method, Glauert’s Empirical Formula, and Buhl’s Theory). The numerical results from the four calculation frameworks were compared with the experimental results from the literature.

Figure 6 compared the numerical results of the original and corrected models of BEM theory to the experimental data taken from the wind tunnel test from the literature. The small HAWT with tubercles produced relatively small power outputs. Evidently, the four versions of the BEM Theory had underpredicted the power output from low to midrange wind velocities \((4.7824 \text{ m/s} \leq v \leq 6.2769 \text{ m/s})\). The predictions then became closer to the actual results as the wind velocity increased to 7.1736 m/s. But as the velocities were further increased, overprediction of the power output results started to become noticeable and the deviations of the predictions from the four models of BEM Theory to the actual results also started to increase. As can be seen from the graph above, the numerical results from the original BEM Theory were the most dissimilar as compared to the other three brake state models. Results from the original theory started to drift away with the increase in velocity, while the other three models stayed close with each other. This particular difference was expected since the original BEM Theory did not include factors and corrections, especially the Prandtl’s tip loss factor, in the calculation of the axial and tangential induction factor. The original version of the BEM Theory includes assumptions of ideal flow, which could be the main reason of the power overprediction at larger wind velocities.

Figure 7. Percentage differences of the power output predicted by BEM Theory to the actual wind tunnel data
To properly visualize the accuracy of the presented methods, the percentage differences were also computed, as can be seen on Figure 7. Since the study was dealing with small HAWT and small power outputs, small differences in the predictions could reflect greatly on the computed percentage errors. As seen from the graph above, the largest values of percentage error for the four methods occurred at the starting speed. But those percentage errors diminished as the velocity increased. When the velocity reached a value of 7.47 m/s, the methods integrated with the Glauert’s Empirical Formula and Buhl’s Theory registered their lowest percentage difference of 4.21% and 1.56% respectively. A 1.17% error was the lowest value for the Wilson-Walker Method at a speed of 7.17 m/s. For the original BEM Theory, a percentage difference as low as 2.88% was calculated at 6.88 m/s. As those four methods reached their lowest values, the percentage differences started to increase hugely as the wind speed were also increased. This paper used the mean absolute percentage difference (MAPD) as a statistical tool in selecting the framework that provided the closest prediction of power output. Among the four calculation methods, the Buhl’s Theory recorded the lowest MAPD of 17.21%, followed by the Glauert’s Empirical Formula with an average of 17.34%. The original BEM Theory had the highest MAPD of 21.40%.

Figure 8. Power coefficient predicted by BEM Theory as compared with actual wind tunnel data

Figure 8 showed the comparison of the numerical results from the four calculation frameworks with the experimental results from the wind tunnel test from the literature. At $\lambda=3.58$, 3.69, and 3.86, the four methods had underpredicted the power coefficient and had a noticeable significant difference to the actual wind tunnel data. But with the remaining tip speed ratios ($\lambda=3.94$, 4.06), the predictions of the four versions of the BEM Theory became closer to the actual data. The Wilson-Walker Method even predicted an almost equal value of the power coefficient ($c_p=0.124$) with the actual experimental data ($c_p=0.123$).

Figure 9. Percentage differences of the power coefficient predicted by BEM Theory to the actual wind tunnel data
The Wilson-Walker Method provided the closest prediction of the power coefficient at $\lambda=4.06$, showing only a percentage difference equivalent to 0.47%, but it also had the highest calculated percentage difference amounting to 51.06% ($\lambda=3.69$). The Original BEM Theory, Glauert’s Empirical Formula, and Buhl’s Theory had their lowest percentage differences at $\lambda=3.94$, with values of 3.79%, 8.56%, and 10.78% respectively, and their highest percentage differences at $\lambda=3.69$, with values amounting to 46.48%, 37.92%, and 40.66% respectively. Unlike the power output values which had thirteen actual values, there were only five power coefficient values from the wind tunnel experiment to be compared to the numerical results. This means that determining the MAPD were not as accurate as that of the power output.

7. Conclusion
The most common tool in evaluating the aerodynamic performance of a wind turbine is the Blade Element Momentum (BEM) Theory. This study focused on a small horizontal axis wind turbine with tubercles. Calculation frameworks based from the Original BEM Theory, Wilson-Walker Method, Glauert’s Empirical Formula, and Buhl’s Theory were used in the prediction of the power output and power coefficient. Numerical results from the calculation frameworks were compared to the actual wind tunnel experimental data taken from the literature. Since the study was dealing with small HAWT with tubercles, the power outputs calculated were relatively small. Because of that, slight differences in the predictions could reflect greatly on the computed percentage differences. The power output predicted by the Buhl’s Theory was the closest to the actual results, with a mean absolute percentage difference (MAPD) of 17.21%, followed by the Glauert’s Empirical Formula with a MAPD of 17.34%. In the case of the power coefficient, the Wilson-Walker Method produced an almost equal prediction at $\lambda=4.06$, showing only a percentage difference of 0.47%. Results also showed that the power coefficient predicted were more accurate at higher tip speed ratios. These findings proved that power output and power coefficient prediction were more accurate using the brake state models rather than the original BEM Theory, because these methods include formulas that addressed the instability in the calculation when the axial induction factor exceeded a certain value. Also, these brake state models integrated some corrections like the Prandtl’s tip loss factor.

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