UV complete composite Higgs models

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Outline

• Model-building framework and viable models
• Predictions for Dark Matter
• Phenomenology
• Conclusions
Framework

In SM, all observed global symmetries (B and L) are understood as accidental symmetries of the renormalizable Lagrangian. This leads to the proton stability

We need at least one more stable particle to explain DM ...
let’s assume DM stability is due to new accidental symmetries

- We take SM with elementary Higgs and add NF new “hyperquarks” \( \Psi \) charged under new “hypercolor” interactions

- We assume that “hypercolor” confines and hyperquarks condensate is formed \( \sim \) TeV scale

- We also assume that hyperquarks lie in a real representation under the SM so that their condensate does not break EW

\[
\mathcal{L} = \mathcal{L}_{SM} + \bar{\Psi}_i(i\not\!\!\!d - m_i)\Psi_i - \frac{g_{\mu\nu}^2}{4g_{TC}^2} + \frac{\theta_{TC}}{32\pi^2} G_{\mu\nu}^A \tilde{G}_{\mu\nu}^A + [H \bar{\Psi}_i(y_{ij}^L P_L + y_{ij}^R P_R)\Psi_j + h.c.]
\]

\[\mathcal{L} \supset |D_\mu H|^2 - \lambda (H^\dagger H)^2 + m^2 H^\dagger H\]
SM Higgs
(models with Higgs coupling)

The models will always contain “half-composite” 2HDM sector (due to elementary and composite doublets).

Depending on the mixing induced by Yukawa (y), the 125 GeV Higgs can be mainly elementary or composite.
What do we gain?

- Natural DM candidates (hyperbaryons and hyperpions) to be probed in the next round of DM experiments
- Each model predicts concrete set of hypermesons to be probed at LHC 13
- Deviations in the Higgs couplings and EDMs
- Automatic MFV to avoid all flavor bounds (since SM quarks couple only to the elementary Higgs)
- Naturalness is solved via relaxion mechanism or by hypothesis of scale invariance
Our model-building rules

- We study SU(N) and SO(N) “hypercolor” gauge theories with fermionic hyperquarks in the fundamental reps

- Under SM, hyperquark reps are embeddable in unified SU(5) multiplets

| SU(5) | SU(3)_c | SU(2)_L | U(1)_Y | charge | name | Δb_3 | Δb_2 | Δb_Y |
|-------|---------|---------|--------|--------|------|------|------|------|
| 1     | 1       | 1       | 0      | 0      | N    | 0    | 0    | 0    |
| 5     | 3       | 1       | 1/3    | 1/3    | D    | 1/3  | 0    | 2/9  |
|       | 1       | 2       | -1/2   | 0, -1  | L    | 0    | 1/3  | 1/3  |
| 10    | 3       | 1       | -2/3   | -2/3   | U    | 1/3  | 0    | 8/9  |
|       | 1       | 1       | 1      | 1      | E    | 0    | 0    | 2/3  |
|       | 3       | 2       | 1/6    | 2/3, -1/3 | Q | 2/3 | 1 | 1/9 |
| 15    | 3       | 2       | 1/6    | 2/3, -1/3 | Q | 2/3 | 1 | 1/9 |
|       | 1       | 3       | 1      | 0, 1, 2 | T    | 0    | 4/3  | 2    |
|       | 6       | 1       | -2/3   | -2/3   | S    | 5/3  | 0    | 8/9  |
| 24    | 1       | 3       | 0      | -1, 0, 1 | V    | 0    | 4/3  | 0    |
|       | 8       | 1       | 0      | 0      | G    | 2    | 0    | 0    |
|       | 3       | 2       | 5/6    | 4/3, 1/3 | X | 2/3 | 1 | 25/9 |
|       | 1       | 1       | 0      | 0      | N    | 0    | 0    | 0    |

“Species”

\[
R \equiv R_N \oplus \bar{R}_N,
\]

\[
\langle \psi_R^{(N)} \psi_{\bar{R}}^{(\bar{N})} \rangle \neq 0
\]

- Demand that HC gauge group is asymptotically free and SM gauge couplings do not develop Landau poles below Planck scale

* Sp(N) models don’t have stable baryons
Accidental symmetries

1) U(1) hyperbaryon number

Leads to stable HyperBaryons (HB)

2) “Species” number

The NF hyperflavors organize themselves into S “species”

Leads to stable hyper-pions made of different species

Example: in QCD + QED \[\Psi_1, \Psi_2, ..., \Psi_{NF}\] would be stable

3) G-parity

Modified version of the charge conjugation

Even (odd) weak isospin hyperpions are even (odd) under G-parity

Leads to lightest odd weak isospin hyperpions stable

Example: \[\pi^0\] would be stable

Bai, Hill ’10
Breaking of accidental symmetries

The above symmetries can be violated by various effects

- **Yukawa interactions**, if allowed, break “species symmetry” and G-parity
  \[ \bar{\Psi}_I H \Psi_J \]

- **Dim-5 operators** break “species” number and G-parity:
  \[ \frac{1}{M} \bar{\Psi} \Psi HH, \quad \frac{1}{M} \bar{\Psi} \sigma^{\mu\nu} \Psi B_{\mu\nu} \]

- **U(1) hyperbaryon and “species” symmetry** can be broken by **dim-6 operators**:
  \[ \tau_B \sim \frac{8\pi M^4}{m_B^5} \sim \left( \frac{M}{10^{16} \text{ GeV}} \right) \times \left( \frac{10^5 \text{ GeV}}{m_B} \right) \times 10^{10} \text{ years} \]

Within EFT hyperbaryons (HB) are more likely to be cosmologically stable
SU(N) composite DM models
Dynamics is QCD-like:

\[ SU(N_F)_L \otimes SU(N_F)_R \rightarrow SU(N_F)_V \rightarrow N_F^2 - 1 \text{ hyperpions} \]

We assume the standard large-N scaling:

\[ \Lambda_{HC} \sim \frac{4\pi}{\sqrt{N}} f \quad m_{HB} \sim N \Lambda_{HC} \]

Model has viable DM candidates if all stable particles have zero charge, hypercharge and QCD color.

DM should belong to the multiplets with integer weak isospin \( J=0,1,2,.. \)
Hyperpions in SU(N) models

Hyperpions belong to the adjoint reps and decompose under SM as:

$$\bar{\Psi} \Psi \text{ states: } \text{Adj}_{SU(N_F)} = \left[ \sum_{i=1}^{N_S} R_i \right] \otimes \left[ \sum_{i=1}^{N_S} \bar{R}_i \right] \oplus 1$$

Charged pions acquire positive mass.

$$m_\pi^2 = \frac{3g_i^2}{(4\pi)^2} C_2(\pi) m_\rho^2 + m_\Psi f$$

After electro-weak symmetry breaking multiplets further split. Neutral component is the lightest. For triplets:

$$m^+ - m^0 = 166 \text{ MeV}$$

Hyperpions may be stable due to “species” symmetry or G-parity
HyperBaryons in SU(N) models

Hypercolor (HC) singlets constructed with N hyperquarks.

Fermions (scalars) for odd (even) N

| Lightest HB w.f. | = | HC x spatial x spin x flavour |
|------------------|---|-------------------------------|
| antisymm         |   | symmetric (s-wave)            |
| (Fermi statistics)| ▲| has to be symmetric           |

Spin x flavor:

- \( N=3 \) (spin=1/2), QCD octet (p, n, Σ, Ξ, Λ)
- \( N=4 \) (spin=0)
- \( N=5 \) (spin=1/2)

Heavier HB:

- \( N=3 \) (spin=3/2), QCD decuplet
- \( N=4 \) (spin=1,2)
- \( N=5 \) (spin=3/2, 5/2)
Final spectrum in SU(N) models

$\Lambda_{TC}$

$\Delta m = \alpha_2 Q^2 m_W \sin^2 \theta_W \frac{\theta_W}{2}$

$\sim 100$ MeV
Viable renormalizable SU(N) models

We scan over combination of HC quarks and impose constraints to obtain viable DM candidates (multiplet with integer weak isospin)

| SU(N) techni-color. | Yukawa couplings | Allowed N | Techni-pions | Techni-baryons | under |
|---------------------|------------------|-----------|--------------|---------------|-------|
| N<sub>TF</sub> = 3  | Ψ = V            | 0         | 3            | 3             | VVV = 3 |
|                     | Ψ = N ⊕ L       | 1         | 3,.., 14     | unstable      | N<sub>N</sub>* = 1 |
| N<sub>TF</sub> = 4  | Ψ = V ⊕ N       | 0         | 3            | 3 x 3         | VVV, VNN = 3, VVN = 1 |
|                     | Ψ = N ⊕ L ⊕ Ė    | 2         | 3, 4, 5      | unstable      | N<sub>N</sub>* = 1 |
| N<sub>TF</sub> = 5  | Ψ = V ⊕ L       | 1         | 3            | unstable      | VVV = 3 |
|                     | Ψ = N ⊕ L ⊕ L̄  | 2         | 3            | unstable      | NLL = 1 |
|                     | =               | 2         | 4            | unstable      | N<sub>N</sub>LL, LLL = 1 |
| N<sub>TF</sub> = 6  | Ψ = V ⊕ L ⊕ N   | 2         | 3            | unstable      | VVV, VNN = 3, VVN = 1 |
|                     | Ψ = V ⊕ L ⊕ Ė    | 2         | 3            | unstable      | VVV = 3 |
|                     | =               | 3         | 3            | unstable      | NLL, LLL = 1 |
|                     | =               | 3         | 4            | unstable      | N<sub>N</sub>LL, LLL, NLL = 1 |
| N<sub>TF</sub> = 7  | Ψ = L ⊕ L̄ ⊕ E ⊕ Ė ⊕ N | 4 | 3      | unstable      | VVV, VNN = 3, VVN = 1 |
|                     | Ψ = N ⊕ L ⊕ Ė ⊕ V | 3         | 3            | unstable      | VVV, VNN = 3, VVN = 1 |
| N<sub>TF</sub> = 9  | Ψ = Q ⊕ Ė̃       | 1         | 3            | unstable      | QQĚ = 1 |
|                     | Ψ = Q ⊕ Ė̃ ⊕ U   | 2         | 3            | unstable      | QQĚ̃, Ė̃DU = 1 |
Exemplary SU(N) model

1) $SU(N)_{HC}$ model with $\Psi = V$

- One specie of hyperquark in the adjoint of SU(2) so that $N_F=3$
- No Yukawa with the Higgs is allowed (because $3 \otimes 3 \otimes 2$ contains no singlets)
- If $N>3$, the SU(2) coupling becomes non-perturbative below the Planck scale
- HB and $H\pi$ lie in 8 of hyper-flavor SU(3): $8 = 3_0 \oplus 5_0$ under $SU(2)_L \otimes U(1)_Y$
- The $H\pi$ triplet is stable because of G-parity ($J=1$ odd) and the HB triplet is stable because of HB number
Dark Matter (WIMP)

Hyperpion DM: behave as minimal DM

Let's concentrate on....

HyperBaryon DM

Cirelli, Fornengo, Strumia '05
Crucially depends on the HBaryon mass:

\[
M_{DM} \approx \begin{cases} 
100 \text{ TeV} & \text{if DM is a thermal relic,} \\
3 \text{ TeV} & \text{if DM is a complex state with a TCB asymmetry}
\end{cases}
\]

Relic abundance determined by non-relativistic annihilation xsec of HB into hyperpions rescaling the measured QCD pp xsec

\[
\langle \sigma^{ANN} v \rangle \sim \frac{4\pi}{m_B^2}
\]

THERMAL ABUNDANCE

\[
m_B \sim 50 - 100 \text{ TeV}
\]

\[
(m_B \sim N\Lambda_{HC} \Rightarrow \Lambda_{HC} \sim 10 \text{ TeV})
\]
Direct detection of HBaryon DM

Weak interactions lead to the too small direct detection xsec for 100 TeV DM

Main hope for direct detection of the fermionic DM is the dipole interactions with the photon:

$$\bar{\Psi} \gamma_{\mu\nu} (\mu_M + i d_E \gamma_5) \Psi \frac{F_{\mu\nu}}{2}$$

$$\mu_M = \frac{e g_M}{2 M_{\text{DM}}}$$

$$d_E = \frac{e g_E}{2 M_{\text{DM}}}$$

$$\frac{d\sigma}{dE_R} \approx \frac{e^2 Z^2}{4\pi E_R} \left( \mu_M^2 + \frac{d_E^2}{v^2} \right)$$

In models with QCD-colored hyperquarks we also have chromo-dipole moments
Additional effects in theories with Yukawa coupling
Add lepton doublet $L$ and singlet $N$ in the fundamental of new $QCD'$

$$\mathcal{L}_M = m_L L L^c + m_N N N^c + yH L N^c + \tilde{y} H^\dagger L^c N + h.c.$$

CP phase : \[ \text{Im}(m_L m_N y^* \tilde{y}^*) \]

After $\chi_{SB}$, octet of $SU(3)$ GB decompose under EW as:

$$8 = 3_0 \oplus 2_{\pm 1/2} \oplus 1_0$$

$$\Pi = \begin{pmatrix} \pi_3^0 / \sqrt{2} + \eta / \sqrt{6} & \pi_3^+ & K_2^+ \\ \pi_3^- & -\pi_3^0 / \sqrt{2} + \eta / \sqrt{6} & K_2^0 \\ K_2^- & \bar{K}_2^0 & -2\eta / \sqrt{6} \end{pmatrix} + \frac{\eta'}{\sqrt{3}} \mathbb{1}_3.$$
Low energy effective theory of hyperpions

Yukawas and explicit masses

\[ \mathcal{L} = \frac{f^2}{4} \text{Tr}[D_\mu UD^\mu U^\dagger] + (g_\rho f^3 \text{Tr}[MU] + h.c) + \frac{f^2}{16N} \left[ \ln(\det U) - \ln(\det U^\dagger) \right]^2 \]

Anomaly with SM vectors

1-loop gauge contribution

\[ M = \begin{pmatrix} m_L & 0 & yh^+ \\ 0 & m_L & yh^0 \\ yh^- & yh^0 & m_N \end{pmatrix} \quad \text{and} \quad U \equiv e^{i\sqrt{2}\Pi/f_\pi} \]
Electron EDM

CP phase: \( \text{Im}(m_L m_N y^* \tilde{y}^*) \)

Heavy fermions

Light fermions

Integrating out \( \eta, \pi_3 \):

\[
L_{\text{EDM}}^\text{eff} \subset \frac{e^2 N}{48\pi^2} \frac{\text{Im}(y\tilde{y})(3m_\eta^2 - 2m_{\pi_3}^2)m_\rho^2}{m_{\pi_3}^2 m_\eta^2 m_{K_2}^2} F \tilde{F} h^0 h^0
\]

\[
d_e \approx 10^{-27} \text{ e cm} \times \text{Im}[y\tilde{y}] \times \frac{N}{3} \times \left( \frac{\text{TeV}}{m_{\pi_3,\eta}} \right)^4 \times \left( \frac{m_\rho}{\text{TeV}} \right)^2
\]
LHC phenomenology
and other predictions
LHC Phenomenology and Constraints

Very weak bounds:

- Automatic MFV
- Precision tests ok
- LHC: $m_\rho > 1 - 2\,\text{TeV}$

Interesting phenomenology:

- Plausible at LHC13
- Automatic dark matter candidates
- Simple UV models
Collider Signatures

Vector resonances with SM quantum numbers predicted

\[ q \xrightarrow{g_{SM}^2} \rho \]

Decay to hidden pions and back to SM gauge bosons,

Pions can also be stable or long lived.
Gravitational waves (GW)

SU($N$) confining theories with $N_F$ massless flavours give rise to a 1st order P.T. for

\[ 3 \leq N_F \leq 4N \quad \text{and} \quad N > 3 \]

P.T. occurs at:

\[ T \sim \Lambda_{TC} \sim \mathcal{O}(10 \text{ TeV}) \]

Peak frequency of the GW signal:

\[ f_{\text{peak}} = 3.3 \times 10^{-3} \text{ Hz} \times \left( \frac{T}{10 \text{ TeV}} \right) \times \left( \frac{\beta}{10H} \right) \]

Amplitude of the GW signal:

\[ h^2 \Omega_{GW} \sim 10^{-9} \]

P. Schwaller 15’
Unification of the SM gauge couplings

Incomplete SU(5) multiplets modify SM running

\[ \frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} + \frac{b_i^{SM}}{2\pi} \log \frac{M_{GUT}}{M_Z} + \frac{\Delta b_i}{2\pi} \log \frac{M_X}{\Lambda_{TC}} + \frac{\Delta b}{2\pi} \log \frac{M_{GUT}}{M_X} \]

Examples:

| SU(N) techni-color. | Yukawa | Allowed N | Techni-pions | Techni-baryons | under |
|---------------------|--------|-----------|--------------|----------------|-------|
| Techni-quarks       |        |           |              |                |       |
| \( N_{TF} = 9 \)    |        |           |              |                | SU(9)_{TF} |
| \( \Psi = Q \oplus D \) | 1 3 | unstable | Q\(Q\bar{Q}D = 1 \) | SU(2)_{L} |

\[ \alpha_{GUT} \approx 0.06, \quad M_{GUT} \approx 2 \times 10^{17} \text{ GeV}, \]
\[ M_X \approx 2 \times 10^{11} \text{ GeV} \times \frac{\Lambda_{HC}}{100 \text{ TeV}} \]

\[ \alpha_{GUT} \approx 0.065, \quad M_{GUT} \approx 3 \times 10^{14} \text{ GeV}, \]
\[ M_X \approx 4 \times 10^7 \text{ GeV} \times \frac{\Lambda_{HC}}{100 \text{ TeV}} \]

\[ \Lambda_{HC} = 100 \text{ TeV} \quad M_X \approx 2 \times 10^{11} \text{ GeV} \]
What about naturalness?
Relaxion mechanism

Minimal model: \( \text{SM} + \text{QCD axion} + \text{inflaton} \)

\[
L = (-M^2 + g\phi)|h|^2 + gM^2\phi + \frac{\phi}{f} \tilde{G}'_{\mu\nu}G'^{\mu\nu}
\]

- Soft-breaking of shift symmetry (via coupling to Higgs)
- Large (non-compact) axion field excursions

How it works?

- During inflation axion slow-rolls and scans Higgs mass
- Once mass gets negative, Higgs obtains a vev
- Axion potential barriers (linear in the vev) grow and stop scanning

\[
m_{\pi}^2 \sim m_q f_\pi \sim y_q <h> f_\pi \quad \Rightarrow \quad y_q f_\pi^3 < h > \cos \frac{\phi}{f}
\]
Relaxion mechanism

Minimal model: $\text{SM + QCD axion + inflaton}$

$$L = (-M^2 + g\phi)|h|^2 + gM^2\phi + f^2_\pi m^2_\pi \cos \frac{\phi}{f}$$

- Soft-breaking of shift symmetry (via coupling to Higgs)
- Large (non-compact) axion field excursions

How it works?

- During inflation axion slow-rolls and scans Higgs mass
- Once mass gets negative, Higgs obtains a vev
- Axion potential barriers (linear in the vev) grow and stop scanning

$$m^2_\pi \sim m_q f_\pi \sim y_q < h > f_\pi \quad \Rightarrow \quad y_q f^3_\pi < h > \cos \frac{\phi}{f}$$
Relaxion mechanism

Rolling stops when slopes match:

\[ gM^2 \sim \frac{m^2 \pi f^2}{f} \]

slow-roll

\[ \langle h \rangle \neq 0 \]

\[ \langle h \rangle = 0 \]

Slope shifts minima by \( O(f) \) which leads back to strong CP problem

Axion is oscillations around minima
Solution: barriers for axion arise from a new strong group (QCD')

\[ \frac{\phi}{f} \tilde{G}'_{\mu\nu} G''_{\mu\nu} \]

and this is precisely our framework

\[ \mathcal{L} = \mathcal{L}_{SM} + \bar{\Psi}_i (i\mathcal{D} - m_i) \Psi_i - \frac{G_{A2}^{\mu\nu}}{4g_{TC}^2} + \frac{\theta_{TC} G_A^A \tilde{G}_A^A}{32\pi^2} + [H \bar{\Psi}_i (y_{ij}^L P_L + y_{ij}^R P_R) \Psi_j + h.c.] \]

Compared to original paper, our vector-like fermions are lighter than confinement scale leading to parametric enhancement of the cutoff

Scales to be tested at the LHC 13:

\[ m_{K^2} \sim f_\pi \sim 500 \text{ GeV} \text{ and } m_\rho \sim 5 \text{ TeV} \]
In conclusions...

- We discussed electroweak-preserving strong sector
- We showed that these theories are consistent with all present bounds and naturally feature DM candidates to be probed in the next round of DM experiments
- Each model predicts concrete set of hyperpions to be probed at LHC 13 and some models allow for unification of SM gauge couplings
- Among other predictions are gravity waves and electron EDM which are also within the reach of the upcoming experiments
Back up slides
Expand around the origin of fields space to cubic order:

\[ \mathcal{L}_m = g_\rho f_\pi^3 Tr[MU] + h.c. + \frac{3g_\rho^2 g_\rho^4 f_\pi^4}{2(4\pi)^2} \sum_{i=1}^{3} Tr[UT^i U^\dagger T^i] \]

\[ \approx \text{mass terms} \]

+ mixing and trilinear

\[ i\sqrt{2}g_\rho f_\pi^2 BK^\dagger H - \frac{g_\rho}{\sqrt{2}} A f_\pi \left( K^\dagger \sigma^a \pi_3^a - \frac{\eta K^\dagger}{\sqrt{3}} \right) H + h.c. \]

+ \[ \eta \text{-tadpole} \]

\[ A \equiv (y + \tilde{y}^*) \quad B \equiv (y - \tilde{y}^*) \]
Direct detection of real HB DM

In most of SO(N) models there is Yukawa interaction with the Higgs and therefore, after EWSB, HB DM candidates with $Y=0$ mix with $Y\neq 0$ HB

Example:

| SO(N) techni-color. | Yukawa couplings | Allowed $N$ | Techni-pions | Techni-baryons | under |
|---------------------|------------------|------------|--------------|----------------|-------|
| Techni-quarks       |                  |            |              |                |       |
| $N_{TF} = 5$        |                  |            | 14           | 5, 1...        |       |
| $\Psi = L \oplus N$ | 1                | 3, 4, .., 14 | unstable     | $LLN = 1$,     |       |

Axial coupling to $Z$:

$$Axial\,\,coupling\,\,to\,\,Z:\quad g_A \frac{g_2}{\cos \theta_W} \frac{\bar{X} \gamma_{\mu} \gamma_5 X}{2}$$

The resulting lightest HB is a Majorana fermion for $N$-odd and real scalar for $N$-even.

Majorana fermion can neither have vector coupling to $Z$ nor dipole moments.

spin-dependent xsec with nuclei
Direct detection of real HB DM

Using the present LUX bound: \( \sigma_{nSD}^{n} < 1.7 \times 10^{-39} \frac{M_{DM}}{\text{TeV}} \)

\[ |g_A| < 1.2 \frac{M_{DM}}{\text{TeV}} \]
Exemplary SO(N) model

- One specie of hyperquark in the adjoint of SU(2) so that NF=3
- No Yukawa with the Higgs is allowed (because $3 \otimes 3 \otimes 2$ contains no singlets)
- If $N>7$, the SU(2) coupling becomes non-perturbative below the Planck scale
- $H\pi$ are unstable and lie in 5 SU(2)
- HB: for $N=3$ is a fermion triplet while for $N=4$ is a scalar singlet

$SO(N)_{HC}$ model with $\Psi = V$

| SO(N) techni-color. | Yukawa couplings | Allowed $N$ | Techni-pions | Techni-baryons | under |
|---------------------|------------------|------------|--------------|----------------|-------|
| Techni-quarks       |                  |            |              |                |       |
| $N_{TF} = 3$        |                  | 5          | 3, 1, ... for $N = 3, 4, ...$ | $V^N = 3, 1, ...$ | $SO(3)_{TF}$ |
| $\Psi = V$          | 0                | 3, 4, ..., 7| unstable     |                | $SU(2)_L$ |
Viable renormalizable $SO(N)$ models

Again, scan over combination of HC quarks and impose constraints to obtain viable DM candidates

| $SO(N)$ techni-color. | Yukawa | Allowed $N$ | Techni-pions | Techni-baryons | under |
|----------------------|--------|-------------|--------------|----------------|-------|
|                      |        |             |              |                |       |
| $N_{TF} = 3$         |        |             |              |                |       |
| $\Psi = V$           | 0      | 3, 4, .., 7 | unstable     | $V^N = 3, 1, ...$ | $SO(3)_{TF}$, $SU(2)_L$ |
| $N_{TF} = 4$         |        |             |              |                |       |
| $\Psi = N \oplus V$  | 0      | 3, 4, .., 7 | 3            | $VV(N = 1, V(VV + NN) = 3,$ | $SO(4)_{TF}$, $SU(2)_L$ |
|                      |        |             |              | $VV(VV + NN) = 1, ...$ |       |
| $N_{TF} = 5$         |        |             |              |                |       |
| $\Psi = L \oplus N$  | 1      | 3, 4, .., 14| unstable     | $L\bar{L}NN = 1,$ | $SO(5)_{TF}$, $SU(2)_L$, $SU(2)_L$ |
|                      |        |             |              | $L\bar{L}(L\bar{L} + NN) = 1, ...$ |       |
| $N_{TF} = 7$         |        |             |              |                |       |
| $\Psi = L \oplus V$  | 1      | 4           | unstable     | $(L\bar{L} + VV)^2 = 1$ | $SO(7)_{TF}$, $SU(2)_L$ |
| $\Psi = L \oplus E \oplus N$ | 2      | 4, 5        | unstable     | $(E\bar{E} + L\bar{L})^2 + NN(L\bar{L} + E\bar{E}) = 1$ | $SU(2)_L$, $SU(2)_L$ |
| $N_{TF} = 8$         |        |             |              |                |       |
| $\Psi = G$           | 0      | 4           | unstable     | $GGGG = 1$ | $SU(2)_L$ |
| $\Psi = L \oplus N \oplus V$ | 2      | 4           | unstable     | $(L\bar{L} + VV)^2 + NN(L\bar{L} + VV) = 1$ | $SU(2)_L$ |
| $N_{TF} = 9$         |        |             |              |                |       |
| $\Psi = L \oplus E \oplus V$ | 2      | 4           | unstable     | $(E\bar{E} + L\bar{L} + VV)^2 = 1$ | $SU(2)_L$ |
| $N_{TF} = 10$        |        |             |              |                |       |
| $\Psi = L \oplus E \oplus V \oplus N$ | 3      | 4           | unstable     | as $L \oplus E \oplus V + NN(L\bar{L} + E\bar{E} + VV) = 1$ | $SU(2)_L$ |
Vectorial hyperquarks $\Psi$ are defined as

$$\Psi \equiv \begin{cases} \ C_N \oplus \bar{C}_N & \text{for complex SM representations } C \in \{E, L, D, U, Q, S, T, X\} \\ R_N & \text{for real SM representations } R \in \{N, V, G\} \end{cases}$$

Symmetry breaking pattern is:

$$SU(N_F) \to SO(N_F) \otimes Z_2$$

$$\langle C_N \bar{C}_N \rangle = 2 \langle R_N R_N \rangle \sim 4\pi \Lambda_{HC}^3$$

$$N_F(N_F + 1)/2 - 1 \text{ hyperpions in } \square \text{ of } SO(N_F)$$

$$HB = \text{anti} - HB$$

Two HB can annihilate into hyperpions (HB stability follows from the $Z_2$ symmetry)
Hyperbaryons in SO(N) models

Start from the SU(N_F) HB and decompose under SO(N_F)

\[
N = 3 : \quad (\begin{\array}{c} \bullet \\ \text{SU}(N_F) \end{\array}) = (\begin{\array}{c} \bullet \\ \text{SO}(N_F) \end{\array})
\]

\[
N = 4 : \quad (\begin{\array}{c} \bullet \\ \text{SU}(N_F) \end{\array}) = (\begin{\array}{c} \bullet \oplus \bullet \oplus 1 \\ \text{SO}(N_F) \end{\array})
\]

\[
N = 5 : \quad (\begin{\array}{c} \bullet \\ \text{SU}(N_F) \end{\array}) = (\begin{\array}{c} \bullet \oplus \bullet \oplus \bullet \oplus \bullet \oplus \bullet \\ \text{SO}(N_F) \end{\array})
\]

Example: QCD “eightfold way” splits spin-1/2 HB

\[
8 = (\begin{\array}{c} \bullet \\ \text{SU}(3) \end{\array}) = (\begin{\array}{c} \bullet \oplus \bullet \\ \text{SO}(3) \end{\array}) = 5 \oplus 3
\]

similarly for the heavier spin-3/2 HB:

\[
10 = (\begin{\array}{c} \bullet \bullet \\ \text{SU}(3) \end{\array}) = (\begin{\array}{c} \bullet \bullet \oplus \bullet \\ \text{SO}(3) \end{\array}) = 7 \oplus 3
\]
HyperBaryon EDM

- HC CP phase leads to EDM for HBaryons

\[ \mathcal{L}_{BB\Pi,\theta} = -\frac{2\sqrt{2}a}{3f} (\theta_{TC} - 2\phi_L - \phi_E) \left( b_1 \text{Tr}[\bar{B}\Pi B] + b_2 \text{Tr}[\bar{B}B\Pi] \right) + \ldots, \]

\[ \mathcal{L}_{BB\Pi} = -\frac{D + F}{\sqrt{2}f} \text{Tr}[\bar{B}\gamma^\mu\gamma_5(D_\mu\Pi)B] - \frac{D - F}{\sqrt{2}f} \text{Tr}[\bar{B}\gamma^\mu\gamma_5B(D_\mu\Pi)] + \ldots, \]

\[ d_E = \frac{eg_E}{2M_{DM}}. \]

\[ g_E^{B_1} \approx -0.15 \frac{m^2_{\pi_2}}{f^2} \log \frac{m^2_B}{m^2_{\pi}} \times \theta_{TC}. \]