Study on Mathematical Stress-strain Model of Frozen-thawed Soil Based on Structure Mechanism

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Abstract

The stress-strain behavior of frozen-thawed soil is a main issue of the frozen soil mechanics. In this paper, the change rules of soft soil shear strength, friction angle, cohesive force and other mechanical parameters were studied. After frozen-thawed cycles, the friction angle of soft soil increases with lower cohesive force, the constitutive property gradually weakened with the nature of transformation from cohesive soil to cohesionless soil. Mathematical characteristics for shear stress-strain curve of frozen-thawed soil are analyzed. Structure mechanism index is presented to indicate the relation between stress-strain curve and soil structure mechanism. The result of this paper is helpful to the numerical analysis and practical engineering design of frozen-thawed soil foundations.

1. Introduction

At present, along with China’s rapid economic development, the investment to infrastructure are continually increased, roads, railways, major airports, pipelines, oil storage tanks, urban high-rise buildings and other projects in northwest and northeast China are gradually increased too. The Qinghai-Tibet railway, the Qinghai-Tibet highway and the Sino-Russian oil pipeline represented in these areas due to the special geological conditions in which seasonally frozen ground are widely distributed. What’s more, the projects of the subway, river-crossing tunnel and deep excavation in soft soil area along the coast, freezing methods are always applied in the construction. Nanjing, Shanghai and Hangzhou are as the main representative of subway construction. The soil are usually required to go through different times of the frozen-thawed cycles, the engineering mechanical properties are changed a lot after frozen-thawed cycles: the strength is better in the beginning of construction, but after a few times
of frozen-thawed cycles the strength decreased significantly which resulting in the nonuniform settlement is too large and the stability reduces.

The stability of soft soil after freezing and thawing (hereinafter referred to as "frozen-thawed soil") will directly determine the stability of built structures and buildings on the foundation and its safety. Therefore, the study on the engineering mechanical property of frozen-thawed soil and the establishment of an appropriate constitutive model is very important and has become a common concern on the related engineering fields. The core problem of constitutive model of frozen-thawed soil is the features of the deviatoric stress (hereinafter referred to as "stress", \( q = \sigma_1 - \sigma_3 \)) and axial strain (hereinafter referred to as "strain", \( \varepsilon \)) under triaxial condition. Most of the current researches are limited to theoretical analysis under many hypothesis or rough empirical guess; there is still a large gap between the research and engineering needs. In recent years, many important projects at home and abroad both in the construction and use, as a result of the lack of adequate knowledge of its mechanical properties of frozen-thawed soil and reasonable understanding of engineering measures resulting in serious accidents and casualties. These accidents and disasters call attention to us that we shall strengthen the understanding on mechanical properties of frozen-thawed soil especially the stress-strain behavior which is also demanding prompt solution in engineering field.

In this paper, we take soft soil foundation as the object to analyze its stress-strain behavior under different freezing and thawing times, different confining pressure and different stress path, and explore stress-strain developmental mechanism of frozen-thawed soil under different states; analyze its structural characteristics and stress-strain model, which is rich in academic content and with significant engineering applications.

2. Laboratory Test of Frozen-thawed Soil

Many domestic and foreign scholars have done many in-depth studies on the stress-strain behavior of frozen-thawed soil (including the soft soil which is not frozen, we take it as the soft soil which experiences zero time frozen-thawed process).

Liu Jiankun, Vassallo, Wang Dayan, Lerouei, Kelln and other scholars concluded from the test that the frozen-thawed soil’s stress-strain curves are generally divided into harden curves and the soften curves. In general, the normal consolidated clay, loose sand and medium sand, frozen-thawed soil, the stress-strain curve obtained from the triaxial test is harden curve; overconsolidated clay and dense sand, the stress-strain curve obtained from the triaxial test is soften curve; while for the strong structural soil, its stress-strain curves is integrating the harden and soften curve. Feng Yong, Qi Jilin and Ma Wei, Liu Jiankun, Zhang Erqi, Zeng Junjun, Chai and other scholars’ researches show that: for the compacted silt and strong structural frozen-thawed soft clay, the nature of stress-strain curves is related to the consolidation pressure with a particular individual yield stress existed. When the consolidation pressure is higher than the yield stress, the stress strain curve is a harden curve; when the consolidation pressure is lower than the yield stress, the stress-strain curve is a soften curve. Dr. Wang Dayan has done experimental analysis on the physical and mechanical properties of silty clay under frozen-thawed cycles along the Qinghai-Tibet Railway and concluded that the frozen-thawed cycles would not change the form of stress-strain curves but only to change its elastic constants and fracture strength; Dr. Qi Jilin thought that the soil’s stress-strain behavior under frozen-thawed cycles was affected by the test conditions (consolidation, triaxial, etc.) which caused the different mechanical behavior. Mao Ran, Yu Linlin, Wang Yongzhong, Dong Xiaohong and other scholars have done many tests of the effects of frozen-thawed cycles on shear behavior on China’s different types of permafrost soil: middle and south area’s frozen soil, the northeast saturated silty clay, south silty clay, northwest loess, such as different types of loess was freezing and thawing on in-depth analyzed the change rules of the shear strength, friction angle, cohesion and other mechanical parameters. The test results showed that the soils’ structure has changed a
lot through frozen-thawed cycles. Lerouei thought that the original clay in undrained tests, the peak stress disappeared after frozen-thawed cycles.

WANG carried out some tests through two group clayey soils with different water contents under quick-soaking, slow-soaking, freezing-thawing, different stresses. Following studies were performed on relationship between mechanical strength and microstructure dynamic environmental energy field of clayey soil, interrelation between microstructure dynamic environmental energy field, changing of external environment and medium environmental intrinsic of clayey soil. Their results showed that the water soaking is vital to microstructure, its influence on shear strength is higher than it on deformation properties of clayey soil obviously; when number of blows of more wet and vibrated soil increased, the damage of initial construction increased, but the later strength of construction increased for a little. Analysis indicates that the freezing-thawing is most important to failure of microstructure of soil; the second is slow-soaking.

We conducted some triaxial test on frozen soft soil at State Key Laboratory of Frozen Soil Engineering, CAS, Lanzhou, China. The soil is water content 23% with grey color, from Zhejiang province, China. The specimen is 125mm height with 61.8 mm diameter. Frozen and test temperatures are both -8 °C. One tested typical stress-strain curve with 0.45Mpa confining pressure is shown as in Figure 1. The experiment result showed that make big difference to initial tangent modulus of normal soil, frozen soil and frozen-thawed soil.

Through preliminary tests of frozen-thawed soil we have found that its stress-strain curves gradually changed along with the number of frozen-thawed cycles, that was the peak stress decreased until it disappeared. Therefore, the urgent need to establish a good mathematical model to describe the complex stress-strain relationship of frozen-thawed soil is the key problem to establish the basis for its constitutive model and also the prerequisite of numerical simulation.

![Stress-strain Curve of Frozen Soft Soil](image)

**3. Traditional Stress-strain Model**
3.1 Harden Type Model

Duncan analyzed based on the statistical data according to the test, first proposed to use the two parameter hyperbolic model to simulate the harden stress-strain curves, the expression is showed as follows:

\[ q = f(\varepsilon) = \frac{ab\varepsilon}{a + b\varepsilon} \]  

(1)

Two parameters of the model are respectively related to the initial tangent modulus \( G(0) \) and the ultimate stress \( q_u \) (see Figure 2 and Equation (2)) which has a more clear physical meaning. With simple curve form, it is widely applied and popularized in the soil and structural soil and other aspects.

![Fig. 2 Harden Model of Stress-strain Curve of Frozen-thawed Soils](image)

\[ \begin{cases} 
G(0) = \frac{df}{d\varepsilon} \bigg|_{\varepsilon=0} = b \\
q_u = f(\varepsilon) = a 
\end{cases} \]  

(2)

According to the permafrost-related test, Dr. Wang Dayan, Professor Yang Ping and our project team proposed that we could use two-parameter exponential model to simulate the stress-strain curve of related soft soils, its expression can be written as follows:

\[ q = F(\varepsilon) = A(1 - e^{-B\varepsilon}) \]  

(3)

Two undetermined parameters \( A \) and \( B \) of the model are also related to \( G(0) \) and \( q_u \). This model has the advantages of a hyperbolic model and has been popularized.

\[ \begin{cases} 
G(0) = \frac{dF}{d\varepsilon} \bigg|_{\varepsilon=0} = AB \\
q_u = F(\varepsilon) = A 
\end{cases} \]  

(4)

However, the above two stress-strain models only focus on the approximate fitting of the test values and shape of the curve with insufficient theoretical basis which can only approximately describe the stress-strain behavior and is not a very good description of the mechanism of its development process. Our preliminary research and related test data indicate that the two parameter hyperbolic model and exponential model can not well simulate the harden stress-strain curve of frozen-thawed soil. There are scissors-type error between the fitting data and the measured data and sometimes this error is very large. Although a limited set of test results can still fit the data points, but in fact the essential characteristic of the curve is not fully consistent with the experimental data. The root cause of this error is the limitations
of the model; we can introduce the concept of the function’s “stress factor” to note this issue. The initial tangent modulus \( G(0) \), the ultimate stress \( q_u \), stress factor \( \varepsilon^* \) are the three basic characteristics of harden model which form a mathematical equation of the harden stress-strain model. In theory, they should be independent of each other, but we found that in the hyperbolic model and exponential model they are not independent, and this was just the root cause of the larger fitting error.

We can study further resource of traditional model restrictions form relation between stress-strain curve and soil structure mechanism. Let \( \varepsilon_X \) denote structure mechanism index (SMI), where \( x \) means the ratio of current strain concerned stress to final stress ranging from 0 to 1.

Take \( x = 0.2, 0.5 \) and \( 0.9 \) as example, as to hyperbolic model, we can get:

\[
\begin{align*}
\varepsilon_{t,0.2} & = 0.25a / b \\
\varepsilon_{t,0.5} & = a / b \\
\varepsilon_{t,0.9} & = 9a / b
\end{align*}
\]  

(5)

As to exponential model, \( \varepsilon_X \) can be expressed as

\[
\begin{align*}
\varepsilon_{E,0.2} & = (\ln 1.25) / B \\
\varepsilon_{E,0.5} & = (\ln 2) / B \\
\varepsilon_{E,0.9} & = (\ln 10) / B
\end{align*}
\]  

(6)

It is obviously that three \( \varepsilon_X \) of hyperbolic model are absolutely bigger than that of exponential model, which show that hyperbolic model is good for freezing-thawing soil with lower structure property and exponential model is good for soil with higher structure property.

Take strain difference of \( \varepsilon_X \) into accounted, namely:

\[
\begin{align*}
\alpha & = \varepsilon_{0.5} - \varepsilon_{0.2} \\
\beta & = \varepsilon_{0.9} - \varepsilon_{0.5}
\end{align*}
\]  

(7)

For hyperbolic model, we can deduce:

\[
\begin{align*}
\alpha_f & = 0.75a / b \\
\beta_f & = 8a / b
\end{align*}
\]  

(8)

For exponential model, we can obtain:

\[
\begin{align*}
\alpha_F & = 0.47 / B \\
\beta_F & = 1.61 / B
\end{align*}
\]  

(9)

Combining (8) and (9) gives raise that:

\[ \alpha_f < \alpha_F; \beta_f < \beta_F \]  

(10)

Equation (10) demonstrates that strain differences of hyperbolic model are bigger than that of exponential model with same stress differences, respectively, which signify once more that exponential model is good for soil with higher structure property.

3.2 Soften Type Model

The stress-strain curve of frozen-thawed soil is sometimes showed as a softening type, the mathematical nature of the model curve is much more complicated than the harden curve, involving the 5 basic features: the initial modulus \( G(0) \), the stress factor \( \varepsilon^* \), the peak stress \( q_p \), inflexion stress \( q_i \), and ultimate stress \( q_u \), and these 5 characteristics form the soften curve’s mathematic property set which is
shown in Figure 3. Here the max value of the deviatoric stress in the definition of the stress factor corresponding to the peak deviator stresses $q_p$.

Currently, many researchers conducted a lot of research work on the simulation of such a curve and achieved fruitful results. Professor Zhang Erqi, Academician Shen Zhujiang proposed a different soften curve model with 4 parameters, the model uses 4 undetermined parameters which can not be independently describe the mathematical equation with 5 characteristics; and in the sub-curve model, the model curve is discontinued which cause the turning point is difficult to determine, thus it is not conducive to engineering applications. Guo Ruiping, Zeng Junjun, Zhang Hongbo respectively proposed the soften stress-strain model of the soil from heat engineering and damage, and all of the tests had obtained experimental data’s preliminary validation. Guo Xiaoxia and Professor Chi Shichun, Professor Gutierrez and others also made preliminary study on the stress-strain model of structural soil from the perspective of the energy dissipation. Although these models have a better application, but only from the shape similarity of the curve to simulate the soften stress-strain curve, the mathematical characteristic equation is not independent with a big fit simulation error.

4. Some Existing Problems

In summary, the problems existed in the current research of stress-strain model of frozen-thawed soil primarily the following several aspects:

(1) We only roughly through the apparent phenomenon data from the test to determine the of stress-strain curves of frozen-thawed soil is soften model when the confining pressure is small, harden model when the confining pressure is large, relatively lack of quantitative study of the freezing and thawing and confining pressure, and also lack of the study on the relation between stress-strain curves form and soil state parameter.

(2) Few studies on the structural changes which are the nature of the frozen-thawed soil’s stress-strain change. The stress-strain models are mostly established based on mathematical models and experimental data point curve shape, which remain in the experience stage to some extent only be characterized by the appearance of the state and can not well describe the stress-strain development of soft soil movement, leading to the model’s mathematical characteristic equation is not independent thereby resulting in large errors.
(3) We should combine the macroscopic energy dissipation principle and the microscopic mathematical differential equations to build a stress-strain model. Less interdisciplinary study can lead to the established model with great limitations. The mathematical model which can reflect both soften and harden nature of the stress-strain model has been reported rarely.

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