Thermal analysis based on heat transfer equation

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Abstract. When working in a high temperature environment, people need to wear special clothes to avoid burns. High-temperature special clothing can avoid burns, which greatly improves work safety. In order to design a special garment with high strength, heat insulation and low cost, we need to consider that the thickness of the clothes should not be too thick and increase the cost, and it should not be too thin to affect the function. So, we built a model through micro-element analysis to determine the thickness of the clothes. This allows the garment to reduce costs while ensuring insulation.

1. Introduction
When working in a high temperature environment, people need to wear special clothes to avoid burns. Therefore, there is an urgent need to design a special garment having high strength heat insulation. Special clothing is usually composed of three layers of fabric material, referred to as layers I, II and III, wherein layer I is in contact with the external environment, and there is a gap between layer III and the skin, and the gap is recorded as an IV layer. When considering the thickness of the clothes, we assume that layer I and layer III are fixed values, and layer II and layer IV can be changed, as shown in Table 1.

Table 1. Special clothing material parameters.

| Layer | Density(kg/m³) | Specific heat(J/kg °C) | Thermal conductivity(W/(m °C)) | Thickness(mm) |
|-------|----------------|------------------------|-------------------------------|---------------|
| I layer | 300 | 1377 | 0.082 | 0.6 |
| II layer | 862 | 2100 | 0.37 | 0.6-25 |
| III layer | 74.2 | 1726 | 0.045 | 3.6 |
| IV layer | 1.18 | 1005 | 0.028 | 0.6-6.4 |

To simplify our model, we assume that the thickness of the material is stable at different temperatures, and that heat is diffused according to the shortest path, and other heat radiation effects are not considered.

2. Simplified model
To solve the problem, we simplified the model again. We assumed that the ambient temperature is 65 °C and the thickness of the IV layer is 5.5mm. Then we can solve the optimal thickness of the II layer, and ensure that the outside temperature of the dummy skin does not exceed 47°C when working for 60 minutes, and the time exceeding 44°C does not exceed 5 minutes.

For the selection of the thickness of the II layer, we use the micro-element method to divide 0.6 - 25mm into 60 small parts for detailed performance calculation. According to the heat transfer equation and the investigation of the high temperature overalls, we can approximate the relationship between the thickness of each layer of clothing and the clothes:
In the formula, the thickness of layer II is \( T \) (mm); and \( L = 9.7 + T \) (mm).

Firstly, the average thickness of the layer II is between 0.6 and 25, and 60 micro-elements are selected into the model one by one, and the set of data closest to the human skin outside the IV layer (\( y_4 = 60 \)) is selected, and \( t = 60 \times 60 \text{s} \) is extracted. The data of 60*60 s time constitutes a matrix. After screening, it is found that the temperature outside the human skin does not exceed 47ºC when working for 60 minutes, and the time exceeding 44ºC does not exceed 5 minutes. Therefore, we further deepen the conditions to screen the data and find that the temperature of the first ten values of each row decreases rapidly, and then basically stabilizes.

Therefore, we take the temperature of the micro-element and the temperature of the IV-layer \( t = 60 \times 60 \text{s} \) as the independent variable and the dependent variable, and adopt the method of image fitting to fit the function (Figure 1).

\[
\begin{align*}
\alpha & \text{ is the thermal diffusivity; } k \text{ is the thermal conductivity; } c \text{ is the heat capacity; } \rho \text{ is the density. } T \text{ is the temperature at the position; } z \text{ is the shortest distance of the position for the outermost side of the IV layer; } t \text{ is the time; } L \text{ is the total value of the thickness of the four layers of I, II, III, IV, } y_i \text{ represents the i-th layer material.} \\
\end{align*}
\]

\[
\alpha = \frac{k}{c\rho} \\
T(0, t) = 75 \\
L = 15.2 \text{mm} \\
T(y_i, 0) = \begin{cases} 
75 - 1.98 \times 0.0006 \times y_1^2 & (1 \leq y_1 \leq 60) \\
70 - 2.04 \times 0.001 \times y_2^2 & (1 \leq y_2 \leq 60) \\
62 - 3.15 \times 0.0005 \times y_3^2 & (1 \leq y_3 \leq 60) \\
55 - 2.36 \times 0.004 \times y_4^2 & (1 \leq y_4 \leq 60)
\end{cases}
\]

Figure 1. Data curve fitting image.
Then, the derivative curve [1] is used to solve the change trend of the derivative curve. It can be obtained that when the 44th micro-element is selected, the derivative is less than -0.2, which can be approximated as the curve is close to the stationary state, which can be regarded as satisfying the characteristics of the clothing. The optimum thickness is 17.893.

3. Model complexity

Now let's return the problem to the beginning. We assume that when the ambient temperature is 80°C, the optimal thickness of layer II and layer IV is determined, ensuring that the outside temperature of the dummy's skin does not exceed 47°C when working for 30 minutes, and the time exceeding 44°C does not exceed 5 minutes.

First, we divide the Layer II and IV layers into 60 aliquots according to the micro-element method.

In information theory, entropy is a measure of uncertainty. The larger the amount of information, the smaller the uncertainty and the smaller the entropy; the smaller the amount of information, the greater the uncertainty and the greater the entropy. According to the characteristics of entropy, the randomness and disorder degree of an event can be judged by calculating the entropy value. The entropy value can also be used to judge the degree of dispersion of an index. The greater the degree of dispersion of the index, the influence of the index on the comprehensive evaluation. The larger (weight), the smaller its entropy value. Therefore, in order to evaluate the heat resistance of a certain thickness in this question, we introduce the evaluation method of the entropy method.

1) Select n samples, the four indicators are the total mass of professional clothing $x_1$, the total thickness of the clothing material $x_2$, the temperature of the last moment of the 60th micro-layer of the VI layer $t=30*60s$ (ie the outer temperature of the skin of the dummy) [2]) $x_3$ and the temperature average $x_4$ of the micro-layer from the 1st to the last second, then $x_{ij}$ is the value of the j-th index of the i-th sample ($i=1, 2, ..., n; j=1,2,3,4$).

2) Normalization of indicators: heterogeneity index homogenization. Since the units of measurement of the indicators are not uniform, they should be standardized before they are used to calculate the comprehensive indicators, that is, the absolute value of the indicators is converted into relative values, and $x_{ij}'=|x_{ij}|$. Thereby solving the homogenization problem of different index values. Moreover, since the positive indicator and the negative indicator value represent different meanings (the higher the positive indicator value is, the lower the negative indicator value is, the better). Therefore, for the high and low indicators, we use different algorithms for data standardization. The specific method is as follows:

Positive indicator:

$$x_{ij}' = \frac{x_{ij} - \min\{x_{ij}, \cdots, x_{nj}\}}{\max\{x_{ij}, \cdots, x_{nj}\} - \min\{x_{ij}, \cdots, x_{nj}\}}$$

(1)

Negative indicators:

$$x_{ij}' = \frac{\max\{x_{ij}, \cdots, x_{nj}\} - x_{ij}}{\max\{x_{ij}, \cdots, x_{nj}\} - \min\{x_{ij}, \cdots, x_{nj}\}}$$

(2)

Then $x_{ij}'$ is the value of the j-th indicator of the i-th sample ($i=1, 2, ..., n; j=1,2,3,4$). For the sake of convenience, the normalized data is still recorded as $x_{ij}$.

3) Calculate the proportion of the i-th sample under the j-th indicator to the indicator:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} X_{ij}} \quad i=1,2,3,\cdots,n \quad j=1,2,3,\cdots,n$$

(3)

4) Calculate the entropy of the j-th indicator:
\[ e_j = -k \sum_{i=1}^{n} p_j \ln(p_j) \] 

5) Calculate information entropy redundancy:
\[ d_j = 1 - e_j \] 

6) Calculate the weight of each indicator:
\[ w_j = \frac{d_j}{\sum_{j=1}^{m} d_j} \] 

7) Calculate the composite score for each sample:
\[ s_i = \sum_{j=1}^{m} w_j \cdot p_{ij} \]

**Table 2.** Weights corresponding to 4 indicators.

|                                | Weights |
|--------------------------------|---------|
| Total quality of professional clothing x₁(mm) | 0.283123 |
| Total thickness of clothing materials x₂(mm) | 0.224504 |
| The temperature at the last moment of the 60th micro-element of the VI layer is t=30*60s (the outside temperature of the dummy skin)x₃(ºC) | 0.224504 |
| The average temperature of the micro-layer from 1st to the last second x₄(ºC) | 0.267868 |

The formula for finally solving the comprehensive score is:
\[ y = 0.283123 \times x_1 + 0.224504 \times x_2 + 0.224504 \times x_3 + 0.267868 \times x_4 \] 

By comparing the composite scores of 60*60 cases, we get the following results:

**Table 3.** Optimal thickness related data calculated by entropy method.

|   |   | Mass/(kg) | Total thickness(mm) | Final temperature | Average temperature | Score(highest value) |
|---|---|-----------|---------------------|------------------|--------------------|---------------------|
| 2 | 33| 2.120905837 | 8.203333333       | 37.71268724      | 38.01000281        | 0.000914068         |

4. Conclusion

The second layer of the second layer, that is, the thickness of the second layer is 1.4133mm, and the thickness of the third layer of the IV layer, that is, the IV layer is 3.79mm, which is guaranteed when the ambient temperature is 80ºC, when working for 30 minutes. The outside temperature of the dummy’s skin does not exceed 47ºC, and the time exceeding 44ºC does not exceed the optimal thickness of 5 minutes.

References

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[2] Lu Linzhen, Xu Dinghua, Xu Yinghong. Prediction of skin burn degree using a three-layer thermal protective clothing heat transfer improved model[J]. Journal of Textile Research, 2018, (1): 111-118, 125.