Negative Parity $N^*$ Resonances in an Extended GBE

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Abstract

In this paper, we calculate the masses and mixing angles of $L = 1$ negative parity $N^*$ resonances in an extended GBE (Goldstone-boson-exchange model) with harmonic-oscillator wave functions. By using those mixing angles, we get their photoproduction amplitudes, and compare them with experimental data and the results of OPE (one-pion-exchange model), OPsE (only pseudoscalar meson exchange model), and OGE (one-gluon-exchange model). We find that the extended GBE gives right internal wave functions. It is essential to extend GBE to include not only pseudoscalar meson exchanges but also vector meson exchanges.

1 Introduction

Which is the interaction between quarks mediated by, gluons or mesons? It is an old problem. In one form or another, it has been used in a wide variety of models for the last two decades. In some models, such as, QCD-inspired Isgur-Karl model ¹¹ and flux-tube model ², which make success in explaining the spectra of baryons and mesons and their decay amplitudes, all spin dependencies are assumed to arise from gluon exchange. On the other hand, other models, such as cloudy bag model ³ and chiral quark model ⁴, assume the short-distance force between quarks is mediated by the exchange of nearly massless Goldstone bosons generated by the spontaneous breaking of chiral symmetry. It is argued that replacement of gluon exchange by meson exchange solves some problems of the quark model for baryons, such as spin-orbit problem, and gives superior description of baryon spectrum.

However, in 2000, Isgur published his critique of review by Glozman and Riska in which it is proposed that baryon spectroscopy can be described by OPE without the
standard OGE forces of Refs. [1] and [7]. In the critique, it is said that baryon internal wave functions are wrong in OPE. In fact, in a complex system like the baryon resonance, predicting the spectrum of the states is not a very stringent test of a model. The prototypical example is the case of the two $N_{1/2}^*$ states. Among models which perfectly describe the baryon spectrum, there is still a composition of these states since all values of $\theta_{1/2}^*$ from 0 to $\pi$ correspond to distinct states. OPE predicts $\theta_{1/2}^* = \pm 13^\circ$ and $\theta_{3/2}^* = \pm 8^\circ$. Such a $\theta_{1/2}^*$ will have almost no impact on explaining the anomalously large $N\eta$ branching ratio of the $N^*(1535)_{1/2}^-$ and the anomalously small $N\eta$ branching ratio of the $N^*(1650)_{1/2}^-$. In our previous calculation [8], by using the mixing angles of Chizma and Karl [9], we find OPE fails to explain the photo- and electro- production amplitudes for negative parity $N^*$ resonances.

In the argument of Glozman [10], it’s said that only a $\pi$-exchange tensor force was used for an estimate in Ref. [6]. Within GBE picture there are two sources for tensor force: $\pi$-like exchange and $\rho$-like exchange mechanisms. For quark separations smaller than $\approx 0.6$ fm, which is the scale relevant for baryon structure, the $\rho$-like, e.g. two-pion interaction, is dominant over the one-pion exchange interaction. And the spin-orbit component of linear scalar confining interaction in the P-shell multiplets can be overwhelmed by the spin-orbit component of $\rho$-like exchange[11]. This cancellation was requested by Isgur in his critique paper[5]. Both of these exchanges supply a spin-spin force with the same sign, while their tensor force components have opposite signs. So it is suggested that addition of $\rho$-like tensor potential will improve the results[10, 11]. So far, this improvement remains to be demonstrated. Recently, Wagenbrunn et al. [12] extended GBE from including only pseudoscalar mesons[13] to including all scalar, pseudoscalar and vector mesons, and including both spin-spin and tensor components. The extended GBE allows for an accurate description of all light and strange baryon spectra. In this paper we want to check whether the extended GBE can gives right internal wave functions of $L = 1$ $N^*$ resonances through an explicit calculation of mixing angles and photoproduction amplitudes.

In the following section, the eigenstates and eigenvalues of $L = 1$ $N^*$ resonances will be given. Then the numerical results of masses, mixing angles and photoproduction amplitudes will be shown in section 3. Summary and discussion are given in the last section.

2 Eigenstates and eigenvalues of $L = 1$ $N^*$ resonances

In the harmonic-oscillator model, we assume that the Hamiltonian of three quark system is of the form [1]:

$$H = \sum_i m_i + H_1 + H_{hyp}, \quad (1)$$
where

\[ H_1 = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} V_{\text{conf}}^{ij}, \]

and

\[ V_{\text{conf}}^{ij} = \frac{1}{2} Kr_{ij}^2 + U(r_{ij}). \]

In Eqs. (1)-(3), \( p_i, m_i \) are momentum and mass of the quark \( i \), and \( V_{\text{conf}} \) is a confining potential which is assumed to be a flavor-independent function of the relative quark separation. In our calculation, we employ following form for the confinement[12]:

\[ V_{\text{conf}}^{ij} = V_0 + C r_0 (1 - e^{-r_{ij}/r_0}) \sim \begin{cases} V_0 + C r_{ij}, & r_{ij} \ll r_0, \\ V_0 + C r_0 = \text{const}, & r_{ij} \gg r_0. \end{cases} \]

It is practically of linear form in the inner region and becomes a constant outside. The constant \( V_0 \) is needed to shift the ground-state energy to the nucleon mass of 939 MeV. \( C \) represents the strength of the linear term for \( r_{ij} \gg r_0 \). It’s necessary to point out that, when we calculate mixing angles, we don’t need the explicit form of confining potential. This point of view will be clearly seen later.

\( H_{\text{hyp}} \) in Eq. (1) is the hyperfine interaction. The extended GBE assumes pseudoscalar + vector + scalar exchanges. Then chiral interaction reads[12]:

\[
H_{\text{hyp}} = V^{ps}(ij) + V^{v}(ij) + V^{s}(ij) \\
= \sum_{a=1}^{3} [ V_\gamma(ij) + V_\rho(ij) ] \Lambda_a^i \Lambda_a^j + \sum_{a=4}^{7} [ V_K(ij) + V_{K^*}(ij) ] \Lambda_a^i \Lambda_a^j \\
+ [ V_\eta(ij) + V_{\omega}(ij) ] \Lambda_8^i \Lambda_8^j + \frac{2}{3} [ V_{\eta'}(ij) + V_{\omega'}(ij) ] + V_\sigma(ij),
\]

with \( \Lambda_a^i \) the Gell-Mann flavor matrices. The pseudoscalar meson nonet (\( \gamma = \pi, K, \eta, \eta' \)) comes with spin-spin and tensor forces

\[ V_\gamma(ij) = V^{SS}_\gamma(\vec{r}_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j + V^{T}_\gamma(\vec{r}_{ij}) [ 3 (\vec{r}_{ij} \cdot \vec{\sigma}_i) (\vec{r}_{ij} \cdot \vec{\sigma}_j) - \vec{\sigma}_i \cdot \vec{\sigma}_j ] , \]

where the spatial parts have the forms

\[
V^{SS}_\gamma(\vec{r}_{ij}) = \frac{g^2}{4\pi} \frac{1}{12m_im_j} \left[ \mu_\gamma^2 e^{-\mu_\gamma r} \left( \frac{2}{r} - \frac{3}{\mu_\gamma^2} \right) - \mu_\gamma^2 + \frac{\Lambda_\gamma}{2} \left( \frac{\Lambda_\gamma^2 - \mu_\gamma^2}{2} \right) e^{-\Lambda_\gamma r} \right] , \]

and

\[
V^{T}_\gamma(\vec{r}_{ij}) = \frac{g^2}{4\pi} \frac{1}{12m_im_j} \left[ \mu_\gamma^2 \left( 1 + \frac{3}{\mu_\gamma r} + \frac{3}{\mu_\gamma^2 r^2} \right) e^{-\mu_\gamma r} \right. \\
- \Lambda_\gamma^2 \left. \left( 1 + \frac{3}{\Lambda_\gamma r} + \frac{3}{\Lambda_\gamma^2 r^2} \right) e^{-\Lambda_\gamma r} \right] - \frac{\Lambda_\gamma^2}{2} \left( \frac{\Lambda_\gamma^2 - \mu_\gamma^2}{2} \right) \left( 1 + \Lambda_\gamma r \right) e^{-\Lambda_\gamma r} \right] .
\]
The vector meson nonet \((\gamma = \rho, K^*, \omega_8, \omega_0)\) produces central, spin-spin, tensor, and spin-orbit components.

\[
V_\gamma(ij) = V_\gamma^C(\vec{r}_{ij}) + V_\gamma^{SS}(\vec{r}_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j + V_\gamma^T(\vec{r}_{ij}) \left[ 3 (\vec{\tau}_{ij} \cdot \vec{\sigma}_i)(\vec{\tau}_{ij} \cdot \vec{\sigma}_j) - \vec{\sigma}_i \cdot \vec{\sigma}_j \right] + V_\sigma^{LS}(\vec{r}_{ij}) \vec{L}_{ij} \cdot \vec{S}_{ij} .
\] (9)

The spin-spin and tensor components of the vector meson nonet have the same spatial forms as for the pseudoscalar case above but with the coupling constants replaced in the following way:

\[
\text{spin-spin : } \frac{g_\gamma^2}{4\pi} \to 2\left(\frac{g_\gamma^V + g_\gamma^T}{4\pi}\right)^2
\] (10)

\[
\text{tensor : } \frac{g_\gamma^2}{4\pi} \to \left(\frac{g_\gamma^V + g_\gamma^T}{4\pi}\right)^2
\] (11)

The scalar singlet meson \((\gamma = \sigma)\) comes with only central and spin-orbit forces

\[
V(ij) = V_\gamma^C(\vec{r}_{ij}) + V_\sigma^{LS}(\vec{r}_{ij}) \vec{L}_{ij} \cdot \vec{S}_{ij} ,
\] (12)

where the central component has the spatial dependence

\[
V_\gamma^C(\vec{r}_{ij}) = -\frac{g_\gamma^2}{4\pi} \left[ \frac{e^{-\mu_\gamma r}}{r} - \left(1 + \frac{\Lambda_\gamma^2}{2\Lambda_\gamma^2} \right) \frac{e^{-\Lambda_\gamma r}}{r} \right] .
\] (13)

All formulae above correspond to a monopole-type parametrization of the meson-quark interaction vertices, i.e.

\[
F(\vec{q}^2) = \frac{\Lambda_\gamma^2 - \mu_\gamma^2}{\Lambda_\gamma^2 + \vec{q}^2} ,
\] (14)

where \(\vec{q}\) is the 3-momentum transferred by the exchanged meson \(\gamma\).

In the equations above, \(\vec{\tau}_i\) is Pauli matrix of the quark \(i\), and \(\mu_\gamma, g_\gamma\) are the mass and meson-quark coupling constant of the exchange meson \(\gamma\). The values of Cut-offs \(\Lambda_\gamma\) follow the linear scaling laws\[12\]:

\[
\Lambda_\gamma = \Lambda_\pi + \kappa (\mu_\gamma - \mu_\pi) \quad \text{for pseudoscalar mesons} ,
\] (15)

\[
\Lambda_\gamma = \Lambda_\rho + \kappa (\mu_\gamma - \mu_\rho) \quad \text{for vector and scalar mesons} .
\] (16)

In our calculation, we’ll leave out all spin-orbit forces and the central components from the vector meson exchanges as Ref. [12]. This is further supported on more theoretical grounds by a study of the two-pion exchange mechanism between constituent quarks in Ref. [11].

We take \(U + H_{hyp}\) as perturbation, then the main part of Hamiltonian is harmonic potential. In fact, the main part of Hamiltonian has \(SU(6) \otimes O(3)\) symmetry for the spin-flavor and orbital excitations. This symmetry is broken by introducing the perturbation
including particle exchange potential. Hence we can say that the admixture origins from symmetry breaking.

By using the main part of Hamiltonian, we can obtain the standard wave functions of Isgur-Karl model \[1\]:

\[
S = 3/2 : \quad \Psi(4P) = \frac{1}{\sqrt{2}} \chi^\ast \{\psi^\lambda \phi^\lambda + \psi^\rho \phi^\rho\},
\]

\[
S = 1/2 : \quad \Psi(2P) = \frac{1}{2} \{\chi^\lambda \psi^\rho \phi^\rho + \chi^\rho \psi^\lambda \phi^\lambda + \chi^\rho \psi^\lambda \phi^\lambda - \chi^\lambda \psi^\rho \phi^\rho\}.
\]

The explicit expressions of the physical states for the four \(L=1\) negative-parity resonances \(2\) and \(2\) states each at \(J=3/2\) and \(J=1\) to give the total angular momentum \(|L+S| \geq J \geq |L-S|\). As a result there are two states each at \(J=1/2\) and \(J=3/2\), namely spin doublet and spin quartet: \(2P_{1/2}, 4P_{1/2}\) and \(2P_{3/2}, 4P_{3/2}\). The physical eigenstates are linear combinations of these two states, and can be obtained by diagonalizing the Hamiltonian in this space of states. For example, the \(J^P = 3/2^-\) states are eigenstates of the matrix:

\[
\begin{pmatrix}
   \langle 4P_{3/2}|H|4P_{3/2}\rangle & \langle 4P_{3/2}|H|2P_{3/2}\rangle \\
   \langle 2P_{3/2}|H|4P_{3/2}\rangle & \langle 2P_{3/2}|H|2P_{3/2}\rangle
\end{pmatrix}.
\]

The explicit expressions of the physical states for the four \(L=1\) negative-parity resonances are:

\[
|N1700\rangle = \cos \theta_{3/2^-}|4P_{3/2}\rangle + \sin \theta_{3/2^-}|2P_{3/2}\rangle, \quad |N1520\rangle = -\sin \theta_{3/2^-}|4P_{3/2}\rangle + \cos \theta_{3/2^-}|2P_{3/2}\rangle, \\
|N1650\rangle = \cos \theta_{1/2^-}|4P_{1/2}\rangle + \sin \theta_{1/2^-}|2P_{1/2}\rangle, \quad |N1535\rangle = -\sin \theta_{1/2^-}|4P_{1/2}\rangle + \cos \theta_{1/2^-}|2P_{1/2}\rangle.
\]

where \(\theta_{1/2^-}, \theta_{3/2^-}\) are mixing angles, which are defined as Isgur and Karl \([14]\).

It should be mentioned that matrix elements of spin- and flavor-independent parts of \(H\) in diagonal are equal to each other. And the other two matrix elements in off-diagonal equal to zero. So mixing angles are independent of the confining potential and the scalar meson exchange interaction.

### 3 Masses and Amplitudes of photoproduction

With the formulae above, we first calculate the masses, \(i.e.\) physical eigenvalues, of \(L = 1\) \(N^\ast\) resonances. In our calculation, we adopt the parameters of Wagenbrunn \textit{et al.} \([12]\). Predetermined parameters are presented in Table 1.

| \(m_u = m_d\) | 340MeV | \(\mu_\pi\) | 139MeV | \(\mu_\eta\) | 547MeV | \(\mu_\eta'\) | 958MeV |
| \(\mu_\rho\) | 770MeV | \(\mu_\omega\) | 869MeV | \(\mu_\omega_0\) | 947MeV | \(\mu_\sigma\) | 680MeV |
| \(g_{ps}^2/4\pi\) | 0.67 \((g_{V,8}^V + g_{T,8}^T)^2/4\pi\) | 1.31 \((g_{V,8}^V + g_{T,8}^T)^2/4\pi\) | 1.31 \(g_{\pi}^2/4\pi\) | 0.67 |
| \(C_0\) | 2.53fm\(^{-2}\) | \(r_0\) | 7fm | \(\alpha\) | 0.41GeV |
The free parameters in this work are $\kappa$, $\Lambda_\pi$, and $\Lambda_\rho$. Here we fix $\kappa$ first, then search the best fitted masses of the five $L = 1$ $N^*$ resonances in the regions of, $0.4 GeV \leq \Lambda_\pi \leq 1.4 GeV$, and $0.4 GeV \leq \Lambda_\rho \leq 1.4 GeV$. The constant $V_0$ is determined by normalizing the ground-state energy to the nucleon mass of 939 MeV. We list our results in Table 2.

| $\kappa$ | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | exp. |
|----------|-----|-----|-----|-----|-----|------|
| $\Lambda_\pi^{fitted}$ (GeV) | 0.55 | 0.55 | 0.55 | 0.65 | 0.65 | -- |
| $\Lambda_\rho^{fitted}$ (GeV) | 1.3 | 1.2 | 1.1 | 0.95 | 0.85 | -- |
| $V_0$ (MeV) | -162 | -165 | -167 | -168 | -169 | -- |
| $D_{13}(1520)$ (MeV) | 1561 | 1558 | 1554 | 1562 | 1561 | 1515 $\sim$ 1530 |
| $D_{13}(1700)$ (MeV) | 1705 | 1701 | 1700 | 1704 | 1706 | 1650 $\sim$ 1750 |
| $S_{11}(1535)$ (MeV) | 1547 | 1534 | 1527 | 1543 | 1539 | 1520 $\sim$ 1555 |
| $S_{11}(1650)$ (MeV) | 1640 | 1644 | 1641 | 1649 | 1647 | 1640 $\sim$ 1680 |
| $D_{15}(1675)$ (MeV) | 1659 | 1653 | 1648 | 1662 | 1660 | 1670 $\sim$ 1685 |

In our calculation, the sum of the confining potential and hyperfine interaction is smaller than 200 MeV, while the main part is about 1900 MeV. So perturbation method is reasonable. Though we use the harmonic-oscillator wave functions, not the wave functions of Wagenbrunn et al. obtained by solving the Schrödinger equation with the stochastic variational method, the predicted masses of five resonances agree with experimental values quite well.

From the parameters of Table 1, we get the regions of $\Lambda_\pi$, and $\Lambda_\rho$ for the best mass spectrum. They are $0.5 GeV \sim 0.8 GeV$ for $\Lambda_\pi$, $0.8 GeV \sim 1.3 GeV$ for $\Lambda_\rho$, respectively. The fitted values of Wagenbrunn et al. are in those regions too. Thus we can calculate the maximum and minimum of mixing angles of the extended GBE for photoproduction of the first four $N^*$ resonances in those regions shown in Table 2. Then by using Eq. (20), we can get the physical wave functions of those resonances. Since mixing angles are independent of the confining potential and of the scalar meson exchange interaction, we don’t need the parameters, $C_0$, $r_0$, $g_\sigma^2/4\pi$, and $\mu_\sigma$ any longer.

To calculate the electromagnetic transition amplitudes, we use the electromagnetic interaction of Close and Li [16] which can be derived from B-S equation [17]. It avoids the explicit appearance of the binding potential through the method of McClary and Byers [18]. The explicit form is:

$$H^{em} = \sum_{i=1}^{3} H_i = \sum_{i=1}^{3} \{-e_i r_i \cdot E_i + i \frac{e_i}{2m^*} (p_i \cdot k_i - r_i \cdot A_i + r_i \cdot A_i p_i \cdot k_i) - \mu_i \sigma_i \cdot B_i$$

$$- \frac{1}{2m^*} (2\mu_i - \frac{e_i}{2m^*}) \frac{\sigma_i}{2} \cdot [E_i \times p_i - p_i \times E_i] \}$$

6
\[ + \sum_{i<j} \frac{1}{2M_T m^*} (\sigma_i - \sigma_j) \cdot [e_j E_j \times p_i - e_i E_i \times p_i] , \quad (21) \]

where we keep to \( O (1/m^2) \), and use long wave approximation. \( E_i \) and \( B_i \) are the electromagnetic fields, \( e_i \), \( \sigma_i \), \( \mu_i \) are the charge, spin, and magnetic moment of quark \( i \). \( M_T \) is recoil mass. \( m^*_i \) is the effective quark mass including the effect of long-range scalar simple harmonic potential, but it is independent on the exchange potential. So \( m^*_i \), or \( \mu_i \), in different models can be treated as the same free parameters.

By insertion of the usual radiation field for the absorption of a photon into Eq. (21), and by integrating over the baryon center-of-mass coordinate, we obtain the transverse photoexcited value over flavor spin and spatial coordinates [19]

\[ A_N^\lambda = \sum_{i=1}^{3} \langle X; J\lambda | H_i | N; 1/2 \lambda - 1 \rangle . \quad (22) \]

Here the initial photon has a momentum \( k||\hat{z} \). A simple procedure, that of transforming the wave function to a basis which has redefined Jacobi coordinates, allows the calculation of the matrix elements of the \( H_1 \) and \( H_2 \) operators to proceed in an exactly similar way to that of the operator \( H_3 \). Calculation of the matrix elements of \( H_3 \) avoids complicated functions of the relative coordinates in the "recoil" exponential.

By using physical wave functions and by using Hamiltonian in Eq. (21), we calculate the amplitudes of photoexitation from the ground state \( N(p,n) \) to the resonance \( X \) by Eq. (22) in Breit-frame. In the calculation, we follows the convention of Koniuk and Isgur [20]. For the photocouplings of the states made up of light quarks, and the states which are not highly exited, it should be a reasonable approximation to treat the quark kinetic mass \( m^*_i \) as a constant effective mass, \( m^* \). The recoil mass is kept at \( M_T = 3m^* \) as Ref. [19]. The origin of the effect mass of quark here is different from that of \( m_{u,s} \) above. So we adopt \( m^* = 437 \text{MeV} \), which is different from \( m_{u,s} = 340 \text{MeV} \). (In fact, the amplitudes aren’t sensitive to the values of \( M_T \) and \( m^* \)).

A useful measure of the quality of the fit is to form a \( \chi^2 \) statistic in the usual way. Introducing a "theoretical error" [21] avoids overemphasis in the fitting procedure of a few very well-measured photocouplings.

First, we calculate amplitudes for photoproduction of four states and \( \chi^2 \) of those amplitudes at \( \kappa = 1.2 \), which is the fitted value of \( \kappa \) in Ref. [12]. With \( \kappa = 1.2 \), the scopes of mixing angles in regions of \( \Lambda_\pi : 0.5 \text{GeV} \sim 0.8 \text{GeV} \) and \( \Lambda_\rho : 0.8 \text{GeV} \sim 1.3 \text{GeV} \) are:

\[
OPsE : \quad 22^\circ \leq \theta_{1/2}^\circ \leq 24^\circ, \quad -42^\circ \leq \theta_{3/2}^\circ \leq -35^\circ; \\
GBE : \quad -38^\circ \leq \theta_{1/2}^\circ \leq -23^\circ, \quad 5^\circ \leq \theta_{3/2}^\circ \leq 8^\circ. \quad (23)
\]

In Table 3, we give results for non-admixture (NA), for OPsE, and for GBE. We also list the experimental values in last column.
Table 3: Breit-frame photoproduction amplitudes at $\kappa = 1.2$ using wave functions of no-admixture (NA), of OPsE, and of extended GBE. Here $\alpha = 0.41GeV$, $m^* = 0.437GeV$, $g = 1.3$, $M_T = 3m^*$. Amplitudes are in units of $10^{-3}GeV^{1/2}$; a factor of $+i$ is suppressed for all amplitudes. Experimental values are from PDG [15].

| state       | $A_{\frac{1}{2}}^{p,n}$ | $A_{\frac{3}{2}}^{p,n}$ | $\chi_{\text{NA}}^2$ | $\chi_{\text{OPsE}}^2$ | $\chi_{\text{GBE}}^2$ | Expt. |
|-------------|--------------------------|--------------------------|-----------------------|------------------------|------------------------|-------|
| $N_{\frac{3}{2}}^{+}(1700)$ | $-21$ 0.0  | 23 $\sim$ 29  | 3.0 $\sim$ 3.9  | -22 $\sim$ -20  | 0.0 $\sim$ 0.0  | -18 $\pm$ 13 |
| $A_{\frac{1}{2}}^{p,n}$ | 19 0.1  | 21 $\sim$ 23  | 0.2 $\sim$ 0.2  | 5 $\sim$ 7  | 0.0 $\sim$ 0.0  | 0 $\pm$ 50 |
| $A_{\frac{3}{2}}^{p,n}$ | -36 1.3  | -105 $\sim$ -95 | 9.1 $\sim$ 11.1 | -13 $\sim$ -8 | 0.1 $\sim$ 0.2  | -1 $\pm$ 24 |
| $A_{\frac{5}{2}}^{p,n}$ | -14 0.1  | 40 $\sim$ 55  | 0.8 $\sim$ 1.4  | -51 $\sim$ -44 | 0.7 $\sim$ 1.0  | -3 $\pm$ 44 |
| $N_{\frac{3}{2}}^{-}(1520)$ | $-23$ 0.0  | -36 $\sim$ -35 | 0.3 $\sim$ 0.3  | -21 $\sim$ -20 | 0.0 $\sim$ 0.0  | -24 $\pm$ 9 |
| $A_{\frac{1}{2}}^{p,n}$ | -38 1.0  | -19 $\sim$ -23 | 2.7 $\sim$ 3.3  | -38 $\sim$ -39 | 0.8 $\sim$ 0.9  | -59 $\pm$ 9 |
| $A_{\frac{3}{2}}^{p,n}$ | 139 1.8  | 86 $\sim$ 101 | 9.9 $\sim$ 15.1 | 143 $\sim$ 144 | 1.1 $\sim$ 1.2  | 166 $\pm$ 5 |
| $A_{\frac{5}{2}}^{p,n}$ | -125 0.4  | -117 $\sim$ -110 | 0.9 $\sim$ 1.6  | -127 $\sim$ -128 | 0.2 $\sim$ 0.3  | -139 $\pm$ 11 |
| $N_{\frac{1}{2}}^{-}(1650)$ | $-19$ 1.8  | -43 $\sim$ -47 | 14.0 $\sim$ 15.2 | 70 $\sim$ 100 | 0.4 $\sim$ 3.4  | 53 $\pm$ 16 |
| $A_{\frac{1}{2}}^{p,n}$ | -1 0.2  | 46 $\sim$ 49  | 4.4 $\sim$ 4.9  | -49 $\sim$ -30 | 0.3 $\sim$ 1.4  | -15 $\pm$ 21 |
| $A_{\frac{3}{2}}^{p,n}$ | 109 0.3  | 131 $\sim$ 133 | 1.3 $\sim$ 1.4  | 99 $\sim$ 120 | 0.0 $\sim$ 0.7  | 90 $\pm$ 30 |
| $A_{\frac{5}{2}}^{p,n}$ | -82 1.1  | -91 $\sim$ -90 | 1.7 $\sim$ 1.8  | -83 $\sim$ -95 | 1.2 $\sim$ 2.1  | -46 $\pm$ 27 |
| $\sum \chi^2$ | -- 8.1  | -- 50 $\sim$ 59 | -- 7.1 $\sim$ 9.2 | --  | --  | --  |

In the first two columns of Table 3, the amplitudes without admixture and $\chi^2$ of those amplitudes are displayed. We can see that the amplitudes of many states have already agreed with experimental data well. The other noteworthy information we can get from the first two columns is that the difference between $A_{\frac{1}{2}}^{p,n}$ for $N(1650)$ and $A_{\frac{1}{2}}^{p,n}$ for $N(1535)$, and the difference between $A_{\frac{3}{2}}^{p,n}$ for $N(1700)$ and $A_{\frac{3}{2}}^{p,n}$ for $N(1520)$, are too large. So the admixture should not be very large. Otherwise the results which have agreed with experiment will be destroyed. The third and forth columns in Table 3 give the results of OPsE, where $\chi^2$ of most amplitudes increase. $\chi^2$ of $A_{\frac{3}{2}}^{p,n}$ for $N(1700)$, $A_{\frac{1}{2}}^{p,n}$ for $N(1650)$, or $A_{\frac{3}{2}}^{p,n}$ for $N(1520)$, is even about 10. All those amplitudes with large $\chi^2$ are obtained by mixing two amplitudes between which there is a large difference. The sum of $\chi^2$ for twelve amplitudes also increases from 8.0 to about 55. The fifth and sixth columns present results of GBE. Almost all amplitudes keep the agreement with experiment. There is no obvious destruction like OPsE. The sum of $\chi^2$ keeps at about 8.

To make our conclusion more clear, in Tables 4 and 5, we list the scopes of mixing angles and sums of $\chi^2$ for four resonances with OPsE and the extended GBE at $\kappa = 0.6$, 0.8, 1.0, 1.2, 1.4, respectively, and compare the results of the extended GBE with OPE and OGE. In the Tables 5, the mixing angles for OGE and OPE are from Ref. [14] and
Ref. [9], respectively. "Experimental" values of mixing angles in the last column are from Ref. [22].

Table 4: Scopes of mixing angles and sums of $\chi^2$ for twelve amplitudes for photoproduction of four $L = 1 N^*$ resonances for OPsE in regions of $\Lambda_\pi$: 0.5GeV~0.8GeV and $\Lambda_\rho$: 0.8GeV~1.3GeV. Parameters and conventions as Table 3.

| $\kappa$ | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 |
|----------|-----|-----|-----|-----|-----|
| $\theta_{1/2}$ ($^\circ$) | 23 ~ 24 | 23 ~ 24 | 23 ~ 24 | 22 ~ 24 | 22 ~ 23 |
| $\theta_{3/2}$ ($^\circ$) | -44 ~ -40 | -44 ~ -39 | -43 ~ -37 | -42 ~ -35 | -42 ~ -34 |
| $\chi^2$ | 56 ~ 62 | 56 ~ 62 | 53 ~ 60 | 50 ~ 59 | 48 ~ 58 |

Table 5: Scopes of mixing angles and sums of $\chi^2$ for twelve amplitudes for photoproduction of four $L = 1 N^*$ resonances for the extended GBE in regions of $\Lambda_\pi$: 0.5GeV~0.8GeV and $\Lambda_\rho$: 0.8GeV~1.3GeV. Parameters and conventions as Table 3.

| $\kappa$ | 0.8 | 1.0 | 1.2 | 1.4 | OPE | OGE | Exp. |
|----------|-----|-----|-----|-----|-----|-----|-----|
| $\theta_{1/2}$ ($^\circ$) | -34 ~ -15 | -36 ~ -20 | -38 ~ -23 | -39 ~ -26 | 25.5 | -32 | -32 |
| $\theta_{3/2}$ ($^\circ$) | 3 ~ 7 | 4 ~ 7 | 5 ~ 8 | 5 ~ 8 | -52.7 | 6 | 10 |
| $\chi^2$ | 7 ~ 9 | 7 ~ 9 | 7 ~ 9 | 7 ~ 10 | 80 | 11 | 10 |

To analyze our results, we can see that the mixing angles are not sensitive to $\kappa$. As a result, sum of $\chi^2$ keeps at about 60 for OPsE and at about 8 for the extended GBE, respectively. The mixing angles of OPsE which involves all pseudoscalar meson exchanges are similar to those of OPE. Only $\theta_{3/2}$ has a small improvement. The sum of $\chi^2$ is still about 60, which is larger than that of OGE obviously. However, after addition of vector mesons, a significant improvement is made. The mixing angles of the extended GBE are close to those of OGE and "experimental" values. The sum of $\chi^2$ of the extended GBE remains about 8. With those mixing angles, $N\eta$ branching ratio can be explained. So we can say the extended GBE can give right internal wave functions requested by Isgur.

4 Summary and discussion

We know that it is hard to judge which model is better only though spectrum. For example, with the same Hamiltonian but different wave functions, Ref. [13] gave quite different spectrum. In addition, the hyperfine interaction is quite small compared with the main part of the mass. Moreover, there are many free parameters and many kinds of confining potential. In fact, the difference of confining potentials can be regarded as a
hidden free parameter. The difference of hyperfine interactions may be veiled by adopting different parameter values. So predicting the spectrum is not enough to check the small differences between different models, especially of the hyperfine interactions. However, when we calculate the mixing angles of states with same $N$ and $L$ as this paper, the mixing angles are independent of all spin- and flavor-independent components. We find that the effect of the hyperfine interaction is dominant. In our calculation, we can see the mixing angles are more sensitive to hyperfine interaction than the spectra are. Hence it is necessary to check different models through mixing angles. It can be regarded as an important way to check the different hyperfine interactions, especially the tensor part. Though the extended GBE model yields an improved description of the light and strange baryon spectra only with detailed hyperfine splittings, the addition of vector mesons is essential to get good mixing angles and the amplitudes for photoproduction. After addition of vector mesons, the problem about baryon internal wave functions of Isgur is solved in this extended GBE.

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