production in $e^+e^-$ annihilation near the threshold revisited

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Production of $p\bar{p}$ and $n\bar{n}$ pairs in $e^+e^-$ annihilation near the threshold of the process is discussed with account for the new experimental data appeared recently. Since a significant part of these new data was obtained at energies noticeably exceeding the threshold, we also take into account the form factor describing the amplitude of $N\bar{N}$ pair production at small distances. The effective optical potential, which describes a sharp dependence of the $N\bar{N}$ production cross sections near the threshold, consists of the central potential for $S$ and $D$ waves and the tensor potential. These potentials differ for the states with isospin $I = 0$ and $I = 1$ of $N\bar{N}$ pair. The optical potential describes well $N\bar{N}$ scattering phases, the cross sections of $pp$ and $n\bar{n}$ production in $e^+e^-$ annihilation near the threshold, the electromagnetic form factors $G_K$ and $G_M$ for protons and neutrons, as well as the cross sections of the processes $e^+e^-\rightarrow 6\pi$ and $e^+e^-\rightarrow K^+K^-\pi^+\pi^-$. 

I. INTRODUCTION

A strong energy dependence of the cross sections of baryon-antibaryon and meson-antimeson pair production has been observed in many processes near the thresholds of the corresponding reactions. Some of these processes are $e^+e^-\rightarrow pp\ [1–8]$, $e^+e^-\rightarrow n\bar{n}\ [9–11]$, $e^+e^-\rightarrow \Lambda(\bar{\Lambda})\ [12–15]$, $e^+e^-\rightarrow BB\ [16]$, and $e^+e^-\rightarrow \phi\Lambda\bar{\Lambda}\ [17]$. This anomalous behavior can naturally be explained by small relative velocities of the produced particles. Therefore, they can interact strongly with each other for a sufficiently long time. As a result, the wave function of the produced pair changes significantly (the so-called final-state interaction). The idea on the final-state interaction as a source of anomalous energy dependence of the cross sections near the thresholds has been expressed in many papers [18–28]. However, the technical approaches used in these papers were different. It turned out that in almost all cases the anomalous behavior of the cross sections is successfully described by the final-state interaction.

Unfortunately, information on the potentials, which are responsible for the final-state interaction, is very limited. However, instead of trying to find these potentials from the first principles, one can use some effective potentials, which are described by a small number of parameters. These parameters are found by comparison of the predictions with a large amount of experimental data. Such an approach has justified itself in all known cases.

One of the most complicated processes for investigation is $N\bar{N}$ pair production in $e^+e^-$ annihilation near the threshold. To describe the process, it is necessary to take into account the central part of the potential for $S$ and $D$ waves and the tensor part of the potential. In addition, these potentials are different in the isoscalar and isovector channels. Another circumstance, that is necessary to take into account, is a large number of $N\bar{N}$ annihilation channels to mesons. As a result, instead of the usual real potentials, one has to use the so-called optical potentials containing the imaginary parts. Note that in a narrow region near the thresholds of $p\bar{p}$ and $n\bar{n}$ production, the Coulomb interaction of $p$ and $\bar{p}$ should also be taken into account as well as the proton and neutron mass difference.

The details of approach that allows one to solve the specified problem are given in our paper [24]. However, in that paper the parameters of the potentials and the corresponding predictions for various characteristics of the processes were based on the old experimental data on the production of $p\bar{p}$ and $n\bar{n}$ pairs. Moreover, a significant part of the uncertainty in the parameters of the model was related to a poor accuracy of the experimental data on the cross section of $n\bar{n}$ pair production. Recently, new data have appeared on $n\bar{n}$ pair production in $e^+e^-$ annihilation near the threshold [10, 11]. These data differ significantly from the previous ones and have a fairly high accuracy compared to the previous experiments. Therefore, it became necessary to perform a new analysis of the numerous experimental data within our model.

The approach in Ref. [24] was based on the assumption that the amplitude of a hadronic system production at small distances weakly depends on energy of the system near the threshold of the process. Therefore, in Ref. [24] this amplitude was considered as energy independent, and strong energy dependence of the cross section has appeared via the energy dependence of the wave function due to the final-state interaction. In order to use the new data obtained at energies significantly above the threshold (but in the non-relativistic approximation), in the present paper we

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introduce the phenomenological dipole form factor which describes the amplitude of a hadronic system production at small distances.

The aim of the present work is the analysis of $NN$ real and virtual pair production in $e^+e^-$ annihilation with the new experimental data taken into account. We show that our model, which contains a relatively small number of parameters, successfully describes the energy dependence of $NN$ scattering phases (see Ref. [29] and references therein), the energy dependence of the cross sections of $p\bar{p}$ and $n\bar{n}$ pair production near the threshold [1–11], the electromagnetic form factors $G_E$ and $G_M$ for protons and neutrons in the time-like region [1–5, 8], as well as the anomalous behavior of the cross sections of the processes $e^+e^- \rightarrow 6\pi$ [6, 30–32] and $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ [6, 33, 34].

II. DESCRIPTION OF THE MODEL

The wave function of the $NN$ system produced in $e^+e^-$ annihilation through one virtual photon contains four components, namely, $p\bar{p}$ pair in $S$ and $D$ waves and $n\bar{n}$ pair in $S$ and $D$ waves. It is necessary to take into account $p\bar{p}$ and $n\bar{n}$ pairs together in the wave function due to the charge-exchange processes $p\bar{p} \leftrightarrow n\bar{n}$. Contributions of $S$ and $D$ waves must be taken into account together due to a tensor potential, which, for the total angular momentum $J = 1$ and the total spin $s = 1$, leads to mixing of states with the orbital angular momenta $L = 0$ and $L = 2$. In the absence of the effects violating the isotopic invariance (the Coulomb $p\bar{p}$ interaction and the proton and neutron mass difference), the potential in the states with a certain isospin $I = 0$, 1 has the form

$$V^I = V^D_0(\delta_{L0}) + V^D_1(\delta_{L2}) + V^P_1(r)\left[6(s \cdot n)^2 - 4\right],$$

where $s$ is the spin operator of $NN$ pair ($s = 1$), $n = r/r$, and $r = r_N - r_N$. The potentials $V^D_0(R), V^D_1(R)$, and $V^P_1(R)$ correspond to interaction in the states with $L = 0$ and $L = 2$, as well as the tensor interaction. With account for the effects violating the isotopic invariance we have to solve not two independent systems for each isospin but one system of equations for the four-component wave function $\Psi$ (see Ref. [24] for more details)

$$[p^2_r + \mu V - \kappa^2] \Psi = 0,$$

$$\kappa^2 = \begin{pmatrix} k^2_S & 0 \\ 0 & k^2_n \end{pmatrix},$$

$$\mu = \frac{1}{2}(m_p + m_n), \quad k^2_p = \mu E, \quad k^2_n = \mu(E - 2\Delta), \quad \Delta = m_n - m_p,$$

where $\Psi^T$ denotes a transposition of $\Psi$, $(-p^2_r)$ is the radial part of the Laplace operator, $w^p(r)$, $w^n(r)$ are the radial wave functions of $p\bar{p}$ or $n\bar{n}$ pair with $L = 0$ and $L = 2$, respectively, $m_p$ and $m_n$ are the proton and neutron masses, $E$ is the energy of a system counted from the $p\bar{p}$ threshold, $\hbar = c = 1$. In Eq. (2), $V$ is the matrix $4 \times 4$ which accounts for the $p\bar{p}$ interaction and $n\bar{n}$ interaction as well as transitions $p\bar{p} \leftrightarrow n\bar{n}$. This matrix can be written in a block form as

$$V = \begin{pmatrix} V^{pp} & V^{pn} \\ V^{np} & V^{nn} \end{pmatrix},$$

where the matrix elements read

$$V^{pp} = \frac{1}{2}(U^1 + U^4) - \frac{\alpha}{r} I + U_{cf},$$

$$V^{nn} = \frac{1}{2}(U^1 + U^4) + U_{cf},$$

$$V^{pn} = \frac{1}{2}(U^0 - U^1),$$

$$U^I = \begin{pmatrix} V^S_I + V^D_I - 2\sqrt{2} V^I \bar{V}^I_D \\ -2\sqrt{2} V^I_D - 2 V^I_D \end{pmatrix},$$

$$U_{cf} = \frac{6}{\mu r^2} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

and $\alpha$ is the fine-structure constant and $I$ is the unit matrix $2 \times 2$.

The equation (2) has four linearly independent regular at $r \rightarrow 0$ solutions $\Psi_{iR}$ ($i = 1 \div 4$) with asymptotics at $r \rightarrow \infty$ given in [24]. The proton and neutron electromagnetic form factors are expressed in terms of the components
of these wave functions as follows

\[ G_M^p = \left\{ g_p u^p_{1R}(0) + g_n u^n_{1R}(0) + \frac{1}{\sqrt{2}} \left[ g_p u^p_{2R}(0) + g_n u^n_{2R}(0) \right] \right\} F_D(q), \]

\[ G_E^p = \frac{q}{2\mu} \left\{ g_p u^p_{1R}(0) + g_n u^n_{1R}(0) - \sqrt{2} \left[ g_p u^p_{2R}(0) + g_n u^n_{2R}(0) \right] \right\} F_D(q), \]

\[ G_M^n = \left\{ g_p u^p_{3R}(0) + g_n u^n_{3R}(0) + \frac{1}{\sqrt{2}} \left[ g_p u^p_{4R}(0) + g_n u^n_{4R}(0) \right] \right\} F_D(q), \]

\[ G_E^n = \frac{q}{2\mu} \left\{ g_p u^p_{3R}(0) + g_n u^n_{3R}(0) - \sqrt{2} \left[ g_p u^p_{4R}(0) + g_n u^n_{4R}(0) \right] \right\} F_D(q), \]

\[ F_D(q) = \frac{1}{\left( 1 - \frac{q^2}{q_0^2} \right)^2}, \quad q = 2\mu + E, \quad q_0 = 840 \text{ MeV}. \]  

Here \( F_D(q) \) is the phenomenological dipole form factor that takes into account the energy dependence of the amplitude of the hadronic system production at small distances, \( u^p_{1R}(0) \) and \( u^n_{1R}(0) \) are the energy-dependent components of the wave function at \( r = 0 \), \( g_p \) and \( g_n \) are energy-independent fitting parameters.

The cross sections of \( pp \) and \( n\bar{n} \) pair production, which we refer to as the elastic cross sections, have the form

\[ \sigma_{el}^p = \frac{4\pi k_p \alpha^2}{q^3} F_D^2(q) \left[ |g_p u^p_{1R}(0) + g_n u^n_{1R}(0)|^2 + |g_p u^p_{2R}(0) + g_n u^n_{2R}(0)|^2 \right], \]

\[ \sigma_{el}^n = \frac{4\pi k_n \alpha^2}{q^3} F_D^2(q) \left[ |g_p u^p_{3R}(0) + g_n u^n_{3R}(0)|^2 + |g_p u^p_{4R}(0) + g_n u^n_{4R}(0)|^2 \right]. \]  

In the absence of the final-state interaction, we have \( u^p_{1R}(0) = u^n_{1R}(0) = 1 \), and the rest \( u^p_{2R}(0) \) and \( u^n_{2R}(0) \) vanish. The functions \( u^p_{3R}(0) \) and \( u^n_{3R}(0) \) differ from zero due to the charge-exchange process, while nonzero values of \( u^p_{4R}(0) \), \( u^n_{4R}(0) \), and \( u^n_{4R}(0) \) are the consequence of the tensor forces. Note that \( |G_E^n/G_E^p| \) and \( |G_M^n/G_M^p| \) differ from unity solely due to the tensor forces. For \( E = 0 \) these ratios are equal to unity, since at the threshold the contribution of the \( D \) wave vanishes.

In addition to the strong energy dependence of the cross sections \( \sigma_{el}^p \) and \( \sigma_{el}^n \) near the threshold, a strong energy dependence reveals also in the cross sections of meson production in \( e^+e^- \) annihilation near the \( NN \) pair production threshold \([6, 30−34]\). Such a behavior is related to the production of virtual \( NN \) pair below and above the threshold with the subsequent annihilation of this pair into mesons. Since the probability of virtual \( NN \) pair production strongly depends on energy, then the probability of meson production through the intermediate \( NN \) state also strongly depends on energy. Meanwhile, the probability of meson production through other mechanisms has weak energy dependence near the \( NN \) threshold. To find the cross section \( \sigma_{tot} \) of meson production through \( NN \) intermediate state (the inelastic cross section) with a certain isospin \( I \), one can use the optical theorem. Due to this theorem, the cross sections \( \sigma_{tot} = \sigma_{el} + \sigma_{in} \) are expressed via the imaginary part of the Green’s function \( D(r, r'|E) \) of the Schrödinger equation:

\[ \sigma_{tot} = \frac{2\pi \alpha^2}{q^3} F_D^2(q) \text{Im} \left[ (G^I)^\dagger D(0,0|E) G^I \right], \]

\[ (G^0)^T = \frac{g_p + g_n}{2} \cdot (1, 0, 1, 0), \quad \quad (G^1)^T = \frac{g_p - g_n}{2} \cdot (1, 0, -1, 0). \]  

| \( U'_{S} \) | \( U'_{D} \) | \( U''_{S} \) | \( U''_{D} \) | \( U''_{S} \) | \( U''_{D} \) | \( U''_{F} \) |
|---|---|---|---|---|---|---|
| \( U_{1}(\text{MeV}) \) | −196 | 80.8 | −2.2 | −36.3 | 40.1 | 15.2 |
| \( W_{1}(\text{MeV}) \) | 167.3 | 225.4 | −2 | −16.4 | 217.2 | 1.5 |
| \( \alpha_{i}(\text{fm}) \) | 0.701 | 1.185 | 2.704 | 1.294 | 0.739 | 1.289 |

Table I. The parameters of the model.
The cross sections $\sigma^I_{el}$ have the form

$$\sigma^0_{el} = \frac{4\pi k_p \alpha^2}{q^3} F^2_D(q) \left| \frac{g_p + g_n}{2} \right|^2 \left[ |u^p_{1R}(0) + u^n_{1R}(0)|^2 + |u^p_{2R}(0) + u^n_{2R}(0)|^2 \right]$$

$$+ \frac{4\pi k_n \alpha^2}{q^3} F^2_D(q) \left| \frac{g_p + g_n}{2} \right|^2 \left[ |u^p_{1R}(0) + u^n_{1R}(0)|^2 + |u^p_{4R}(0) + u^n_{4R}(0)|^2 \right],$$

$$\sigma^1_{el} = \frac{4\pi k_p \alpha^2}{q^3} F^2_D(q) \left| \frac{g_p - g_n}{2} \right|^2 \left[ |u^p_{1R}(0) - u^n_{1R}(0)|^2 + |u^p_{2R}(0) - u^n_{2R}(0)|^2 \right]$$

$$+ \frac{4\pi k_n \alpha^2}{q^3} F^2_D(q) \left| \frac{g_p - g_n}{2} \right|^2 \left[ |u^p_{4R}(0) - u^n_{4R}(0)|^2 + |u^p_{4R}(0) - u^n_{4R}(0)|^2 \right].$$

The Green’s function satisfies the equation,

$$[p_r^2 + \mu \gamma - \gamma] \mathcal{D}(r, r'|E) = \frac{1}{r^2} \delta (r - r'),$$

and is expressed in terms of regular and irregular solutions of the Schrödinger equation (2) (see Ref. [24] for details).

### III. RESULTS AND DISCUSSION

The optical potentials $V(r)$ in Eq. (1) are expressed in terms of the potentials $\tilde{U}^0(r)$ and $\tilde{U}^1(r)$ associated with isoscalar and isovector exchange,

$$V(r) = \tilde{U}^0(r) + (\tau_1 \cdot \tau_2) \tilde{U}^1(r),$$

$$V(r) = \tilde{U}^0(r) + (\tau_1 \cdot \tau_2) \tilde{U}^1(r),$$

Figure 1. The predictions for the cross sections of $p\bar{p}$ scattering compared with the Nijmegen data [29].
where $\tau_{1,2}$ are isospin Pauli matrices for nucleon and antinucleon, respectively. Therefore, $V^i_{S,D,T}$ in Eq. (1) have the form

$$V^i_{S}(r) = \tilde{U}^0_{S}(r) + \tilde{U}^1_{S}(r), \quad V^0_{S}(r) = \tilde{U}^0_{S}(r) - 3\tilde{U}^1_{S}(r), \quad i = S, D, T. \quad (11)$$

In our model, we use the simplest parametrization of the potentials $\tilde{U}^i(r)$,

$$\tilde{U}^0_{S}(r) = (U^0_{S} - i W^0_{S}) \theta(a^S_0 - r), \quad \tilde{U}^1_{S}(r) = (U^1_{S} - i W^1_{S}) \theta(a^S_1 - r) + U^0_{T}(r) \theta(r - a^T_1), \quad i = S, D, T, \quad (12)$$

where $\theta(x)$ is the Heaviside function, $U^i_S$, $W^i_S$, $a^i_S$ are free real parameters fixed by fitting the experimental data, and $U^i_T(r)$ are the terms in the pion-exchange potential (see, e.g., [35]).

To fit the parameters of our model, we use the following experimental data: $NN$ scattering phases obtained by the Nijmegen group (see Ref. [29] and references therein), the cross sections of $p\bar{p}$ and $n\bar{n}$ production near the threshold [2–6, 10, 11], modules of electromagnetic form factors $|G_E^p|$ and $|G_M^p|$ [4], as well as the ratios $|G_E^N/G_M^N|$ [2–5, 8] and $|G_E^N/G_M^N|$ [11]. The resulting values of parameters are given in Table I. For these parameters we obtain $\chi^2/N_{df} = 98/85$, where $N_{df}$ is the number of degrees of freedom.

Fig. 1 shows a comparison of our predictions for partial cross sections of $p\bar{p}$ scattering with the results of partial wave analysis [29]. Fig. 2 shows the energy dependence of $p\bar{p}$ and $n\bar{n}$ pair production cross sections. Fig. 3 shows $|G_E^p|$ and $|G_M^p|$, as well as the ratios $|G_E^N/G_M^N|$ and $|G_E^N/G_M^N|$. Good agreement of the predictions with the available experimental data is seen everywhere.

As mentioned above, the optical theorem allows one to predict the contributions $\sigma_{in}^I$ to the cross sections of meson production in $e^+e^-$ annihilation associated with the $NN$ pairs in an intermediate state. In Fig. 4 the cross sections $\sigma_{tot}^I$, $\sigma_{el}^I$, and $\sigma_{in}^I$ are shown. It can be seen that in the channel with $I = 1$ there is a large dip in the cross section $\sigma_{in}^I$ at the threshold of real $NN$ pair production. At the same time, in the channel with $I = 0$ this dip is practically invisible.
A dip was found in the cross sections of the processes \( e^+ e^- \rightarrow 3 (\pi^+ \pi^-) \) [6, 30, 31], \( e^+ e^- \rightarrow 2 (\pi^+ \pi^- \pi^0) \) [30, 32], and \( e^+ e^- \rightarrow K^+ K^- \pi^+ \pi^- \) [6, 33, 34]. Since in our approach we cannot predict the cross sections in each channel, for comparison of our predictions with experimental data we use the following procedure. We assume that strong energy dependence of the cross sections for the production of mesons in each channel near the \( N\bar{N} \) threshold is related to a strong energy dependence of the amplitude of virtual \( N\bar{N} \) pair production in an intermediate state. We also suppose that the amplitudes of virtual \( N\bar{N} \) pair transitions to specific meson states weakly depend on energy near the threshold of \( N\bar{N} \) production. Evidently, other contributions to meson production cross sections, which are not related to \( N\bar{N} \) in an intermediate state, have also a weak energy dependence. Therefore, we approximate the cross section \( \sigma_{\text{mesons}} \) of meson production in a state with a certain isospin by the function

\[
\sigma_{\text{mesons}}^I = a \cdot \sigma_{\text{in}}^I + b \cdot E^2 + c \cdot E + d, \tag{13}
\]

where \( a, b, c \) and \( d \) are some fitting parameters, which depend on the specific final states.

The \( 6\pi \) final state has isospin \( I = 1 \) due to \( G \)-parity conservation. Comparison of our predictions for the \( 6\pi \) production cross section with the experimental data is shown in Fig. 5. For these processes the fit shows that we can set \( b = 0 \), and the remaining parameters are \( a = 0.14, c = 3.3 \cdot 10^{-3} \text{nb/MeV}, d = 0.84 \text{nb} \) for \( 3 (\pi^+ \pi^-) \) production and \( a = 0.4, c = 2 \cdot 10^{-3} \text{nb/MeV}, d = 3.8 \text{nb} \) for \( 2 (\pi^+ \pi^- \pi^0) \) case. It can be seen that there is good agreement between our predictions and experimental data.

Consider now the process \( e^+ e^- \rightarrow K^+ K^- \pi^+ \pi^- \). Unlike the \( 6\pi \) state, the state \( K^+ K^- \pi^+ \pi^- \) may be in both isospin states, \( I = 1 \) and \( I = 0 \). Since our calculations show that the cross section \( \sigma_{\text{in}}^I \) has no sharp energy dependence near the \( N\bar{N} \) threshold, then the contribution of state with \( I = 0 \) can be taken into account in the parameters \( b, c, \) and \( d \). Thus, we can compare the cross section of the process \( e^+ e^- \rightarrow K^+ K^- \pi^+ \pi^- \) with formula (13) for \( I = 1 \). The fitting parameters for this process are \( a = 0.11, b = -6.1 \cdot 10^{-3} \text{nb/MeV}^2, c = 1.7 \cdot 10^{-3} \text{nb/MeV}, d = 4.2 \text{nb} \). Comparison of our predictions with experimental data is also shown in Fig. 5. Again, there is good agreement of our predictions and experimental results.
Figure 4. The energy dependence of the cross sections $\sigma_{I_{\text{tot}}}^I$ (solid line), $\sigma_{I_{\text{el}}}^I$ (dashed line), and $\sigma_{I_{\text{in}}}^I$ (dotted line) for isospins $I = 0, 1$.

Figure 5. The energy dependence of the cross sections for the processes $e^+e^- \rightarrow 3(\pi^+\pi^-)$, $e^+e^- \rightarrow 2(\pi^+\pi^-\pi^0)$, and $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$. The experimental data are taken from Refs. [6, 30, 31], [30, 32], and [6, 33, 34], respectively.

IV. CONCLUSION

Using new experimental data on the production of $p\bar{p}$ and $n\bar{n}$ pairs in $e^+e^-$ annihilation, a simple model is suggested that successfully describes the cross sections of a few processes with production of real or virtual $NN$ pairs. These processes are $e^+e^- \rightarrow p\bar{p}$, $e^+e^- \rightarrow n\bar{n}$, $e^+e^- \rightarrow 6\pi$, and $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ near the $NN$ production threshold. Moreover, this model describes well the energy dependence of partial cross sections for nucleon-antinucleon scattering in states with $L = 0, 2$, $s = 1$ and $J = 1$, as well as the electromagnetic form factors of proton and neutron in the
time-like region. Since new experimental data were obtained at energies noticeably exceeding the \( N\bar{N} \) production threshold, an effective dipole form factor was introduced. It accounts for the energy dependence of the amplitude of real or virtual \( N\bar{N} \) pair production at small distances. Since the new data on \( n\bar{n} \) production have noticeably better accuracy compared to the previous ones, our predictions became more accurate. The analysis of meson production in different channels shows that the strong energy dependence of the meson production cross sections near the \( N\bar{N} \) threshold is related solely to a strong energy dependence of the amplitude of virtual \( N\bar{N} \) pair production in an intermediate state.

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