Research on Durability Evaluation Method of Bridge Slings Based on Set Pair Analysis

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Abstract: The service durability of bridge slings is directly related to the safety of the entire bridge and has become one of the important indicators for measuring the performance of bridge structures. How to make a more accurate assessment of bridge slings is a problem that bridge management and maintenance departments must consider. Based on the investigation of the durability damage of the bridge sling structure in China, this paper establishes a comprehensive evaluation system for the durability of the bridge sling structure, divides the evaluation criteria, and proposes the durability of the sling based on set pair analysis Sex evaluation methods. On the basis of calculating the weight of the durability evaluation index by the analytic hierarchy process, the durability level of the bridge sling structure is accurately quantified through the set pair analysis operation. The example application shows that the method proposed in this paper has a good engineering application prospect.

1. Introduction
The pulling sling of modern suspension bridges, cable-stayed bridges and suspender arch bridges basically use high-strength steel wire or steel strand. Because the pulling sling is in a high stress state for a long time in the operation process, it is relatively sensitive to external damage. In the design, the life expectancy of the pulling sling is overestimated, and the maintenance is not timely in the use process, or the car accident, man-made damage, etc., resulting in the structural damage of the pulling sling. In serious cases, the pulling sling breaks, causing the bridge to collapse. In addition to strengthening management measures such as design, construction and maintenance, it is also one of the effective methods to timely grasp the damage situation of the pulling sling and accurately evaluate its durability level.

In order to evaluate the durability of the pulling sling comprehensively and accurately, this paper determines the evaluation system including the cable body protection, cable body and anchorage area, divides the evaluation criteria into five levels, and uses the set pair analysis method to evaluate the durability level of the sling structure.

2. Durability evaluation System and evaluation Standard of Bridge Pulling Sling Structure
The structure of pulling sling mainly consists of PE protection, cable body and anchorage zone. Generally speaking, the anchorage has a larger volume and is not very sensitive to corrosion. Even if serious corrosion occurs, its section loss is very small and has little impact on the bearing capacity.
Through the investigation of many pulling sling bridges at home and abroad, it is found that the main reason for the decrease of the load-bearing capacity of the pulling sling bridge is the damage of the protection system, which leads to the water entering into the internal part of the pulling sling bridge and the corrosion of the pulling sling bridge.

The outer sheath of pulling sling is the common protective form of modern pulling sling. The raw material of PE outer sheath is high-density polyethylene. Different manufacturers use different raw materials and formulations of outer sheath, so it also has different performance indicators. The ideal situation is that the PE sheath of the pull sling is suitable for the environment it will serve. The industrial polyethylene material is difficult to adapt to the special service environment, and the outer sheath often occurs diseases of cracks and breaks in advance. After the degradation of the outer sheath leads to the gradual cracking and peeling of the filler, the corresponding cable body steel wire will also be directly exposed to the service environment, which is prone to occurring diseases of corrosion and fracture under the action of corrosive medium and alternating load in the environment.

The cable body steel wire or steel strand of the pulling sling is from the manufacturing plant to the construction site. In order to facilitate transportation, the pulling sling is generally in the form of coiling, and coiling needs to pre twist the steel wire strand at a certain angle. After the installation and service of the pulling sling, through the stress process, the twisted steel wire strand will have a process of untwisting, but the outer sheath will not twist with the steel wire strand. The test results show that when the twist angle of the sling exceeds 40, after the stress process, the untwisted steel wire will also lead to the cracking of the outer sheath [1].

The damage of the outer sheath makes the cable body exposed to the service environment directly. The corrosive ions such as Cl\(^-\), SO\(_4\)\(^{2-}\) in the environment will adhere to the surface of the cable body steel wire or steel strand, produce chemical reaction on the surface of the cable body steel wire, make the protective layer such as the galvanized layer of the cable body steel wire lose its protective effect, and then cause the corrosion of the steel wire, resulting in the reduction of the effective section of the steel wire. Under the action of alternating stress coupling, the corrosion is accelerated, the plasticity of cable body steel wire is reduced, the brittleness is enhanced, and brittle fracture occurs.

The corrosion of the anchor head and the water accumulation in the lower anchorage area are also the most common diseases of the pulling sling. A lot of cable conduits for pulling slings are not sealed, the water in the conduit is accumulated, the built-in shock absorber is damaged, the filling materials in the conduit such as foaming polyurethane are aged and cracked, or the conduit and PE sheath are not firmly bonded, resulting in gaps or even peeling off. The pulling sling vibrates with the load, and the water will leak into the lower anchorage zone along with these cracks, which will rust the threads, anchor plates and even the pier hair of the steel wire [1].

To sum up, the durability evaluation system of pulling sling structure is composed of 7 indexes: the cracking and scratch of PE sheath, the corrosion and damage of cable body steel wire, the damage of anchorage, conduit and damping device.

![Comprehensive Evaluation System of Bridge Suspension Cable Structure Durability](image)
The damage classification of pulling sling durability evaluation index is based on the disease degree classification of the pulling sling according to the Standards for Technical Condition Evaluation of Highway Bridges (JTG/TH21-2011) and Specification for Inspection and Evaluation of durability of Highway Bridges (draft for approval). Combined with practical experience summary, the damage and deterioration of pulling sling durability evaluation index is divided into 5 levels, and each level has specific representation phenomenon.

3. Durability Evaluation Method of Bridge Pulling Sling Based on Set Pair Analysis

3.1 Basic Concepts of Set Pair Analysis

Set pair is a basic unit composed of two sets which are related to each other. It is also the most basic concept in set pair analysis and connection mathematics. It was formally put forward by Zhao Keqin in 1989[2]. Set pair analysis (SPA) is a systematic and mathematical analysis on the certainty and uncertainty of two sets in a set pair and the interaction between certainty and uncertainty under a certain problem background. This analysis is generally carried out by establishing the connection number of the two sets in discuss [3].

Set pair refers to a pair of two sets with a certain relationship, which can be represented by \( W = \{A, B\} \). The essence of set pair analysis is to analyze the connection and transformation of things from the same, different and opposite aspects. Among them, the calculation formulas of the two sets’ same, different and opposite connection degrees \( \mu \) based on the background of the problem is as follows:

\[
\mu = a + b_i + c_i = \left(\frac{N_i}{N}\right) + \left(\frac{N_j}{N}\right) + \left(\frac{N_k}{N}\right)
\]

(1)

In the above formula, \( N, N_1, N_2, N_3 \) respectively represent the total characteristic number, common characteristic number, neither common nor opposite characteristic number and mutual opposite characteristic number of the two sets in the set pair, among which \( N = N_1 + N_2 + N_3 \), \( a + b + c = 1 \) is also required. \( a, b, c \) are called the identity degree, difference degree and opposite degree of the set under the specified problem background respectively, reflecting the positive, negative and uncertain trend of each set in the set pair. \( i \) is the coefficient of differences, the value range is \([-1, 1]\), \( j \) is the coefficient of opposites, and \( j = -1 \) is specified. This quantitative description of certainty and uncertainty reflects uncertainty from three aspects of the same (identity), different (difference) and opposite (opposite), so it is also called ternary connection number [4].

In the system comprehensive evaluation, sometimes it is necessary to divide the evaluation level of the domain into four, five or more levels. In this case, multiple connection numbers can be used, and their general form is

\[
\mu = a + b_1i_1 + b_2i_2 + \cdots + b_ni_n + cj
\]

(2)

In the above formula, the requirements are: \( a, b_1, \cdots, b_n, c \in [0, 1] \), and \( a + b_1 + \cdots + b_n + c = 1 \). \( i_1, i_2, \cdots, i_n \) is the coefficient of difference, which is taken value in \([-1, 1]\) according to different situations, \( j = -1 \). Generally, when \( n = k, k + 2 \) is called the coupling coefficient, which is then called the multivariate connection number, and when \( n \geq 2 \), it is called the multiple connection number.

3.2 Comprehensive Evaluation Steps Based on Set Pair Analysis

(1) Determine evaluation index and evaluation level

If there are \( Q \) objects \( x_1, x_2, \cdots, x_Q \) to be evaluated to form a space \( X = \{x_1, x_2, \cdots, x_Q\} \), each index that represents the attributes of the objects to be evaluated forms an index set \( I = \{I_1, I_2, \cdots, I_m\} \), and the evaluation standard level set is \( V = \{v_1, v_2, \cdots, v_n\} \). Among them, \( v_1, v_2, \cdots, v_n \) constitutes an orderly division class of attributes, and \( v_1 < v_2 < \cdots < v_n \). The evaluation standards of each index are known and can be written in the form of evaluation standard matrix
In the evaluation matrix, it needs to meet: \( a_{p1} < a_{p2} < \cdots < a_{pm} \) or \( a_{p1} > a_{p2} > \cdots > a_{pm} \).

According to the measured value \( x_0 \) of each index of the object \( x_0 = (x_{p1}, x_{p2}, \ldots, x_{pn}) \) to be evaluated, which evaluation category the object belongs to is determined.

(2) The determination of comprehensive evaluation \( n \) variable connection number of each index[5] Define comprehensive evaluation \( n \) variable connection number of evaluation object \( x_0 \) index \( I_p \) is:

\[
\mu_p = r_{p1} + r_{p2}t_1 + r_{p3}t_2 + \cdots + r_{p(n-1)}t_{n-2} + r_{pn}t_n
\]

In the above formula, \( r_{pn} \in [0,1] (1 \leq p \leq m, 1 \leq l \leq n) \) is the connection degree component of evaluation index \( I_p \) relative to \( V_l \) levels.

If the measured value of the index \( I_p \) is \( t_p \), \( a_{p1} < a_{p2} < \cdots < a_{pm} \), then

1) When \( t_p \leq a_{p1} \),

\[
\mu_p = 1 + 0t_1 + 0t_2 + \cdots + 0t_{n-2} + 0t_n
\]

2) When \( a_{p1} < t_p < a_{p2} \),

\[
\mu_p = \frac{r_{p1} - a_{p2}}{a_{p1} - a_{p2}} t_1 + \frac{r_{p2} - a_{p3}}{a_{p2} - a_{p3}} t_2 + \cdots + \frac{r_{pn-1} - a_{pm}}{a_{pn-1} - a_{pm}} t_{n-2} + 0t_n
\]

3) When \( a_{p2} \leq t_p \leq a_{p(s+1)}(s=2,3,\ldots, n-2) \),

\[
\mu_p = 0 + \cdots + \frac{r_{p1} - a_{p(s+1)}}{a_{p1} - a_{p(s+1)}} t_1 + \frac{r_{p2} - a_{p3}}{a_{p2} - a_{p3}} t_2 + \cdots + \frac{r_{pn-1} - a_{pm}}{a_{pn-1} - a_{pm}} t_{n-2} + 0t_n
\]

4) When \( a_{p(s+1)} < t_p \leq a_{pm} \),

\[
\mu_p = 0 + \cdots + 0t_{n-3} + \frac{r_{p1} - a_{pn}}{a_{p1} - a_{pn}} t_{n-3} + \frac{r_{p2} - a_{p3}}{a_{p2} - a_{p3}} t_{n-2} + \frac{r_{pn-1} - a_{pm}}{a_{pn-1} - a_{pm}} t_n + j
\]

5) When \( a_{pm} \leq t_p \),

\[
\mu_p = 0 + 0t_1 + 0t_2 + \cdots + 0t_{n-2} + 1j
\]

In above-mentioned five cases, \( r_{pl} \) satisfies \( \sum_{l=1}^{n} r_{pl} = 1 \)

(3) Determine the \( n \) variable connection number of the total index

Similarly, comprehensive evaluation \( n \) variable connection number of the total index can be defined as

\[
\mu = r_1 + r_2t_1 + r_3t_2 + \cdots + r_{n-2}t_{n-2} + r_nj
\]

In the above formula, \( r_1 = \sum_{p=1}^{m} w_{p,pl} \), \((1 \leq p \leq m, 1 \leq l \leq n)\), \( w_p \) is the weight of evaluation index \( I_p \) in the index system. If it meets the requirements of \( \sum_{p=1}^{m} w_p = 1 \), \( w_p \) can be determined by AHP [6] according to the durability evaluation system of pulling sling given in Figure 1.

(4) Comprehensive evaluation
According to the comprehensive evaluation $n$ variable connection number of the object to be evaluated, we can get the extent of each level of the object to be evaluated. Because the evaluation level is in order, the confidence recognition criteria can be used to identify the evaluation results.

Let $k_o = \min \left\{ \sum_{i=1}^{n} a_i \lambda \leq \lambda \leq n \right\}$, usually take $\lambda = 0.8$, the evaluation level of the evaluation object is $k_o$.

### 4. Application Instance

A bridge in Southeast China has a total length of 390.52m. The main bridge is a half-through arch bridge, which was completed and opened to traffic in 2004. The main bridge is a half-through suspender arch bridge with a span of 64m and a rise-span ratio of 1:4.00. There are 18 suspenders (9 on one side) of the arch bridge with a suspender spacing of 5m. The suspender cable body is 55 $\varphi 7$ double-layer galvanized prestressed high-strength parallel steel wires with standard tensile strength of 1570mpa and elastic modulus of $E=2.0 \times 10^5$MPa. The parallel steel wires adopt double-layer galvanized and black PE protection. The two ends are anchored by special anchorage devices for suspenders. The current situation picture of the main bridge is shown in Figure 2.

![Real bridge pictures](image)

Figure 2. Real bridge pictures

The total weight value of each hierarchy evaluation index is calculated by AHP [6], as shown in the following table

| Level   | A1 PE Sheath | A2 Cable | A3 Anchoring Zone | Total Weight Value $\sum_{j=1}^{n} a_j b_i$ |
|---------|--------------|----------|------------------|----------------------------------|
| $A_{11}$ Crack | 0.104        | 0.669    | 0.227            | 0.084                            |
| $A_{12}$ Scratch | 0.807        |          |                  | 0.020                            |
| $A_{21}$ Damage | 0.193        | 0.273    |                  | 0.183                            |
| $A_{22}$ Rust | 0.727        |          |                  | 0.486                            |
| $A_{31}$ Anchor Damage |          | 0.600    |                  | 0.136                            |
| $A_{32}$ Catheter Injury |          | 0.270    |                  | 0.061                            |
| $A_{33}$ Damping Device Damage |          | 0.130    |                  | 0.030                            |

The total hierarchy weight of the durability damage evaluation index of the pulling sling is:

$$W = (0.084, 0.020, 0.183, 0.486, 0.136, 0.061, 0.030)$$

The state evaluation of each index is evaluated according to the percentage system. The values of I, II, III, IV, V, etc. five level standard segmentation points are 90, 75, 60 and 50 respectively, corresponding to five damage levels of pulling sling evaluation index.
Table 2 Evaluation index and grade standard of durability of pull rope

| Index system and grade standard division of durability evaluation of sling structure | Level 1 indicator (weight) | Level 2 indicator (weight) | Standard Split Points for Each Level | Actual Value |
|---|---|---|---|---|
| PE Sheath (0.104) | Crack (0.807) | 90 | 75 | 60 | 50 | 92 |
| | Scratch (0.193) | 90 | 75 | 60 | 50 | 85 |
| Cable (0.669) | Damage (0.273) | 90 | 75 | 60 | 50 | 95 |
| | Rust (0.727) | 90 | 75 | 60 | 50 | 92 |
| Anchoring Zone (0.227) | Anchor Damage (0.600) | 90 | 75 | 60 | 50 | 70 |
| | Catheter Injury (0.270) | 90 | 75 | 60 | 50 | 81 |
| | Damping Device Damage (0.130) | 90 | 75 | 60 | 50 | 83 |

According to the comprehensive evaluation steps given in the paper, the 4 variable connection number of each index is calculated as follows:

1. **4 variable connection number of two levels index:**
   \[ \mu_1 = 1 + 0i_1 + 0i_2 + 0j \]
   \[ \mu_2 = \frac{10}{15} + \frac{5}{15} i_1 + 0i_2 + 0j = 0.667 + 0.333i_1 + 0i_2 + 0j \]
   \[ \mu_3 = 1 + 0i_1 + 0i_2 + 0j \]
   \[ \mu_4 = \frac{10}{15} + \frac{5}{15} i_1 + 0i_2 + 0j = 0 + 0.667i_1 + 0i_2 + 0j \]
   \[ \mu_5 = \frac{6}{15} + \frac{9}{15} i_1 + 0i_2 + 0j = 0.4 + 0.6i_1 + 0i_2 + 0j \]
   \[ \mu_6 = \frac{8}{15} + \frac{7}{15} i_1 + 0i_2 + 0j = 0.533 + 0.467i_1 + 0i_2 + 0j \]

2. **4 variable connection number of one level index:**
   \[ \mu_1 = 0.936 + 0.064i_1 \]
   \[ \mu_2 = 1 \]
   \[ \mu_3 = 0.177 + 0.423i_1 + 0.4i_2 \]

3. **4 variable connection number of total indexes:**
   \[ \mu = 0.807 + 0.103i_1 + 0.09i_2 + 0j \]

According to the above formula, the evaluation object belongs to the connection degree of five level standard segmentation points, using the confidence identification criteria, if take \( \lambda = 0.8 \), \( 0.807 > 0.8 \), the durability level of the pulling sling can be "excellent". According to the actual situation, the pulling sling of the bridge does not need to be repaired or replaced in a large range, but the daily maintenance work should be strengthened.

5. Conclusion

1. The weight value of each index is obtained by AHP, and then the durability level of the bridge pulling sling structure is compared with \( \lambda = 0.8 \) by set pair analysis. Compared with the maximum membership principle adopted by the traditional evaluation method, the durability level of the pulling sling structure is more accurately quantified.

2. There are many differences in the structure of the pulling sling used in different regions and different bridge types, so the selection of the durability index of the pulling sling should be targeted and more detailed. There will be more accurate calculation methods for the corresponding weight calculation, durability level quantification and other aspects, which still need further study.
(3) It is verified by engineering instance that this method can effectively integrate the objective limit value of each index and the subjective experience of experts, evaluate the durability of the pulling sling more accurately, and the evaluation results are well adapted to the current specifications.

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