Pulsating combustion of solid particles in Helmholtz resonator type device

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Abstract. There are a number of works in which pulsating combustion of the wood particles in Helmholtz resonator type device was studied. The dependences of the frequency and amplitude of oscillations on the geometry of the device were determined. The aim of this work is to study the thermodynamic properties of pulsating combustion. P-V diagrams of pulsating combustion and the amplitude of oscillations of the heat release rate are determined. It is proved that the acoustic power generated in the combustion zone is the result of work performed on the gas during one oscillation cycle.

1. Introduction

It is known that gas oscillations has a number of practical applications [1-4]. There are a number of works in which self-excited acoustic oscillations in solid fuel combustors were studied. One of these devices includes the combustion chamber, the air supply pipe and the pipe for the output of the burned gases [5, 6]. A mathematical model of pulsating combustion was developed. The conditions of self-excitation of gas oscillations, the frequency and amplitude of steady-state oscillations depending on the geometrical parameters of the device were determined. The energy method [7, 8] was used, which is based on the analysis of comparison of produced acoustic energy with its losses. Currently, the acoustic properties of the device are determined. However, thermodynamic properties associated with fluctuations of heat release rate in the combustion zone have not been studied. The purpose of this work is the analysis of pulsating combustion as a thermodynamic process. The evaluation of the work performed in the combustion zone, the amplitude of the heat release rate fluctuations will be given.

2. Thermodynamic model of pulsating combustion

Suppose that at time t some mass of gas has a volume \( V_c \) equal to the volume of the combustion chamber (Fig. 1).
During the small $dt$ interval, the gas volume change is equal to:

$$dV(t) = Re(S_4 u_{4,0} - S_0 u'_{0,0})dt,$$

where $u'_{0,0}$ and $u_{4,0}$ are gas velocity pulsations at the combustion chamber inlet and outlet, respectively.

It is known [7] that the linearized equation of energy conservation in the combustion chamber has the form:

$$S_4 u_{4,0} - S_0 u'_{0,0} = \frac{q'}{\rho_0 c_0 T_0} - \frac{i \omega V_c p_c'}{\rho_0 c_0^2},$$

where $q'$ - fluctuations of heat release rate, $p_c' = p_c \exp(\text{i}\omega t)$ is the pressure pulsation in the combustion chamber, $\rho_0, T_0, c_0$ - average density, average temperature, specific heat, of sound speed the air flow, $\omega$ is the angular frequency.

Analysis of works [7, 8] shows the following

$$Req' = q_c \sin(\omega \tau_u - \omega \tau), \quad q_c = \frac{2 \rho_0 c_0 T_0 A_c}{p_c},$$

where $A_c$ is the acoustic energy generated in the combustion zone, $\tau_u$ is the time of delay of the heat release fluctuations rate relative to the air velocity fluctuations.

After substitution (2), (3) in (1) and integration we have:

$$V(t) = \frac{2A_c}{p_c} \int \sin(\omega \tau_u - \omega \tau) \, dt + \frac{\omega p_c V_c}{\rho_0 c_0^2} \int \sin \omega t \, dt + V_0(t).$$

The gas pressure in the combustion chamber is defined as $P(t) = P_a + Re p_c(t)$, where $P_a$ is the atmospheric pressure.

Calculations were carried out under the following conditions: the volume of the combustion chamber $V_c = 0.0011 \text{m}^3$; the pipe for the exit of the burned gases has a length of $1.06 \text{ m}$, a radius of $0.015 \text{ m}$; the air supply pipe has a radius of $0.02 \text{ m}$, a length of $L_0 = 1.2 \text{ m}$ and $1.6 \text{ m}$.

The table contains known data as well as $q$ computed in this paper.

Table. Acoustic and thermodynamic characteristics of pulsating combustion

| $l_0$, m | f, Hz | $p_c$, Pa | $\omega \tau_u$ | $A_c$, W | $q_c$, W |
|---------|-------|-----------|----------------|----------|-----------|
| 1.2     | 64    | 1684      | 1.308          | 0.348    | 149       |
The pressure and volume changes during one cycle of oscillations are shown in Fig. 2.

![Figure 2. PV diagram of a pulsating combustion, $l_0 = 1, 2 \text{ m}$ (curve 1), $l_0 = 3, 6 \text{ m}$ (curve 2).]

Now we provide a qualitative analysis of the diagrams (Fig. 3).

![Figure 3. PV-diagram in General.]

At the initial time $t = 0$ we have $P_e = P_a + p'_c$, $q'(t) = q_c \sin(\omega \tau - \omega t)$, $V = V_c$, i.e. the gas pressure is maximum, the volume is equal to the volume of the combustion chamber (point 0). Then the gas expands, i.e. the pressure decreases, the volume increases. When $\omega \tau = \pi/2$, we have $p'_c = 0$, $p_e = P_a$ (point 1). Then the gas continues to expand and reaches the maximum volume $V_2$ at $\omega \tau = \pi$ (point 2). At this point, the gas pressure becomes the minimum $P_a - p_c$. After that, the gas begins to compress. When $\omega \tau = 3\pi/2$ we have $p'_c = 0$ and the gas pressure is atmospheric (point 3). Then the gas continues to compress. The cycle ends when $\omega \tau = 2\pi$ and the gas returns to its initial state.
Thus, during one cycle, that is, during the period of gas oscillations a positive work is performed over the gas in the combustion zone. These work numerically equal to the area of the shaded figure.

3. Conclusion
Thus, P-V diagrams of pulsating combustion of wood particles the Helmholtz resonator type device are constructed. These diagrams showed that in combustion chamber during the period of gas oscillations positive work is done. As a result, the gas acquires acoustic energy necessary for excitation of gas oscillations.

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