unusual temperature dependent resistivity of a semiconductor quantum wire

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Abstract

We calculate the electronic resistivity of a GaAs-based semiconductor quantum wire in the presence of acoustic phonon scattering. We find that the usual Drude-Boltzmann transport theory leads to a low temperature activated behavior instead of the well-known Bloch-Grüneisen power law. Many-body electron-phonon renormalization, which is entirely negligible in higher dimensional systems, has a dramatic effect on the low temperature quantum wire transport properties as it qualitatively modifies the temperature dependence of the resistivity from the exponentially activated behavior to an approximate power law behavior at sufficiently low temperatures.

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Much recent interest has focused on electronic properties of semiconductor quantum wire structures, particularly in the extreme quantum limit where the electrons are essentially confined in one dimensional channels. One-dimensional quantum wire systems possess many unique properties. One of them is the suggested high mobility $[1]$ for electrons at low temperatures. Because of the severe phase-space restriction, the only possible low energy excitations of a one-dimensional electron gas (1DEG) have wavevectors of magnitude 0 or $2k_F$, where $k_F$ is the 1D Fermi wavevector. If one considers the low temperature momentum relaxation of a 1DEG, it has to occur through the transfer of momentum $2\hbar k_F$ from the electrons to phonons or impurities. In the case of electrons coupled to acoustic phonons, the situation addressed in this letter, which have a linear dispersion of the form $\omega_q = cq$, the resistive scattering events occur with a characteristic energy scale of $\hbar \omega_{2k_F} = 2\hbar ck_F$. One would expect that when $k_B T \ll \omega_{2k_F}$, where $T$ is the temperature and $k_B$ is the Boltzmann constant, the resistivity of an electron gas interacting with acoustic phonons would follow an exponential behavior $\exp(-\hbar \omega_{2k_F} / k_B T)$, simply because the low temperature scattering is entirely mediated by phonons of energy $\hbar \omega_{2k_F}$. Thus, in a 1DEG there is no conventional Bloch-Grüneisen temperature regime in the resistivity—all low temperature acoustic phonon scattering processes are exponentially activated in a 1DEG. The usual Drude-Boltzmann transport theory therefore leads to activated transport behavior at low temperature even in free carrier “metallic” quantum wires! This is in striking contrast to higher-dimensional systems. In this letter, we report our theoretical study on the resistivity of an interacting 1DEG in the presence of acoustic phonon scattering using the memory-function formalism. We show that the above Drude-Boltzmann conclusion of an exponentially activated resistivity due to phase-space restriction is in fact incorrect at sufficiently low temperatures. Phonon renormalization, which is inevitable in the presence of the electron-phonon interaction, enhances the resistive scattering rate at low temperatures and results in a power law temperature dependence of the resistivity at sufficiently low temperatures. This dramatic effect of phonon renormalization is shown to be unique to 1DEG with no corresponding higher dimensional analogy. Although there exists a substantial body of theoretical work $[2]$
on the transport properties of quantum wires, the phenomenon discussed in this letter has not been predicted before to the best of our knowledge.

In a coupled electron-acoustic phonon many-body system, the properties of the electrons and the phonons are mutually coupled through electron-phonon interaction. In addition to renormalizing the bare phonon frequency, the interaction also results in a finite lifetime for the phonons, i.e., a broadening of the phonon spectral function. This means that the renormalized frequency of the phonons interacting with the electrons is not a sharply defined quantity, but rather a resonance of finite width around the bare phonon frequency: there is a nonzero amplitude for the renormalized phonons to have arbitrary frequencies for a given wavevector. As a consequence, the resistive scattering of the 1DEG by the acoustic phonons, which is restricted to have a wavevector $2k_F$ from the phase-space consideration, can occur through phonons not just with the bare frequency $2ck_F$, but with all possible frequencies, in particular, with arbitrarily low frequencies. Strictly speaking, the simple exponentially activated low temperature behavior of the resistivity suggested above, which is based on the existence of a single phonon energy scale of $2\hbar c k_F$, is lost. The questions that need to be addressed now are what quantitative effect the phonon renormalization has and how significant it is. Firstly, the phonon renormalization increases the resistivity compared with the bare phonon situation at low temperatures because the scattering from the low energy phonons located at the tail of the broadened phonon spectrum, which will be referred to as virtual phonons hereafter, is not exponentially suppressed at any non-zero temperature because low energy renormalized phonons are always available. Secondly, the enhancement of the resistivity becomes very important at sufficiently low temperatures for one-dimensional electron systems. The electron-phonon interaction creates arbitrarily low energy virtual phonons, but these virtual phonons carry a very tiny fraction of the total phonon spectral weight. Most of the spectral weight is still retained by the bare phonon, so that the broadening is completely unimportant at high temperatures, and at any temperature in higher dimensional systems. Because phase-space consideration in 1D severely restricts the wavevectors of the scattering phonons to $2k_F$, the contribution to the
resistivity from the bare phonons becomes effectively frozen out at \( k_B T \ll \hbar \omega_{2k_F} = 2\hbar ck_F \), leaving the scattering from the virtual phonons as the only remaining contribution to the resistivity of quantum wires at low temperatures. Many-body renormalization of the phonon spectral function thus becomes the dominating factor in determining the low temperature resistivity. For higher dimensional electron systems, the restriction on the wavevector of the phonons in the scattering events is not present, so the scattering by phonons of arbitrarily small wavevectors is allowed. The contribution from the bare phonons is not frozen out at any finite temperatures in higher dimensions. Due to the large spectral weight of the bare phonons, their contribution to the resistivity therefore always dominates in higher dimensions with virtual phonon scattering being negligibly (orders of magnitude) small. Thus, in higher dimensional electron systems the many-body phonon renormalization is an entirely negligible effect, and the familiar Bloch-Grüneisen behavior \(^{3}\) remains valid for phonon scattering.

The above discussion can be represented by Fig. 1, where the excitation spectrum of a 1DEG at zero temperature is shown along with the bare phonon dispersion. The most distinct feature for a 1DEG is that the low energy part of the phase-space, the region below the dotted-line \( BCD \) in Fig. 1, is excluded from the particle-hole excitation spectrum. The acoustic phonon dispersion line and the particle-hole excitation region overlap only in a narrow strip around \( q \sim 2k_F \), where the electron-phonon scattering can take place. This is not the case in higher dimensions. The particle-hole excitation spectrum for a two- or three-dimensional electron gas, for example, contains all the low energy regions between the dotted lines \( AB \) and \( DE \) in Fig. 1. The phonon dispersion line with \( q \leq 2k_F \) is completely within the particle-hole excitation region, so electron-phonon scattering can occur for arbitrary wavevectors \( q \leq 2k_F \). Due to the large spectral weight carried by the bare phonon, the many-body broadening of the phonon spectrum is always negligibly small in higher dimensions.

In the following, we first briefly describe our calculation and then present the results. The resistivity of a 1DEG with acoustic phonon scattering is calculated by using the memory
function formalism [4–8]. One significant advantage of the memory function formalism is that vertex corrections are automatically incorporated in the theory, and one does not need to work with complicated electron-phonon vertex equation [9]. In the memory function formalism [4–6], the resistivity is given by 

$$\rho = -\frac{1}{\hbar n^2 e^2} \lim_{\omega \to 0} \frac{\text{Im} R^{ret}(\omega)}{\omega},$$

where $n$ is the average density of the 1DEG and $e$ is the electron charge. $R^{ret}(\omega)$ is the retarded force-force correlation function

$$R(t) = -\frac{i}{L} < [F(t), F] >,$$

where $L$ is the length of the 1DEG. For phonon scattering the force operator $F$ is

$$F = \sum_q i q \varrho(q) \frac{M_q}{L^{1/2}} (a_{-q} + a_q^\dagger),$$

where $\varrho(q)$ is the density operator of the 1DEG, $a_q(a_q^\dagger)$ is the phonon annihilation (creation) operator, $M_q$ is the electron-acoustic-phonon deformation potential coupling matrix element [10]: $M_q^2 = \hbar D_s^2 q^2 / (2 \varrho_s ab \omega_q)$, where $D_s$ is the deformation potential, $\varrho_s$ is the density of the substrate material, $a$ and $b$ are the widths of the 1DEG wire, which is taken to be of rectangular cross-section within the infinite well confinement approximation [10]. Combining Eq.(1)-(3), and after a standard procedure of contour integration and analytic continuation, we obtain the following expression for the acoustic phonon scattering resistivity

$$\rho = \frac{\hbar \beta}{2 \pi^2 n^2 e^2} \int_0^\infty dqq^2|M_q|^2 \int_0^\infty d\omega \frac{\text{Im} \chi^{ret}(q, \omega) \text{Im} D^{ret}(q, \omega)}{\sinh^2(\hbar \beta \omega/2)},$$

where $\beta = 1/(k_B T)$, $\chi^{ret}(q, \omega)$ is the retarded density-density response function of the 1DEG, and $D^{ret}(q, \omega)$ is retarded phonon propagator. In the random-phase approximation (RPA), $\chi(q, \omega) = \chi_o(q, \omega) /[1 - v_q \chi_o(q, \omega)]$, with the finite temperature polarizability $\chi_o(q, \omega)$ of a free 1DEG obtained from its zero-temperature counterpart by an integration over chemical potential [11]. For a 1DEG with finite wire widths, the Coulomb interaction potential is [10] $v_q = (e^2/\epsilon_s) \int d\eta |I(\eta)|^2 H(q, \eta) / (q^2 + \eta^2)^{1/2}$, where $\epsilon_s$ is the dielectric constant of the
substrate material. The form factor $I(\eta)$ and $H(q, \eta)$ used in our calculation are taken from the infinite well confinement model [10]. The phonon propagator is

$$D(q, \omega) = \frac{2\omega_q}{\omega^2 - \omega_q^2 - 2\omega_q|M_q|^2\chi(q, \omega)}. \quad (5)$$

The last term in the denominator is the phonon self-energy arising from the electron-phonon interaction. This self-energy gives rise to the broadening of the phonon spectrum, whose effect is the main concern of the present work. When the phonon renormalization is ignored, one has $\text{Im}D^{\text{ret}}_0(q, \omega) = \pi[\delta(\omega+\omega_q)-\delta(\omega-\omega_q)]$ with $\omega_q$ being the bare acoustic phonon frequency. Inserting this into Eq. (4), one obtains the Drude-Boltzmann resistivity due to bare phonons

$$\rho_o = \frac{\hbar\beta}{2\pi n^2 e^2} \int_0^\infty dq q^2 |M_q|^2 \frac{(-1)\text{Im}\chi^{\text{ret}}(q, \omega)}{\sinh^2(\hbar\beta\omega_q/2)}. \quad (6)$$

The characteristic of the bare phonon resistivity is an approximate exponential temperature dependence $\rho_o \propto \exp(-\hbar\omega_{2k_F}\beta)$, which comes from the hyperbolic function $\sinh^2(\hbar\beta\omega_q/2)$ in the above expression. The effect of many-body broadening of the phonon spectrum can be inferred by comparing the results of Eq. (4) and (6).

The numerical results of our calculation are presented in Fig. 2 to 4, where we take the parameters which are appropriate for semiconductor quantum wires in GaAs materials: deformation potential $D_s = 7.0$ eV, density $\rho_s = 5.307$ g cm$^{-3}$, speed of sound $c = 4.73 \times 10^5$ cm s$^{-1}$, effective mass $m/m_o = 0.069$, dielectric constant $\epsilon_s = 12.9$. In Fig. 2, the results for calculated resistivities, expressed in terms of mobilities $\mu = (n_e\rho)^{-1}$, are shown as functions of temperature $T$. The bare phonon result of Eq. (4), the dotted-line in Fig. 2, shows an approximate exponential temperature dependence as we mentioned above. In addition to the bare phonons, the broadened phonon spectrum also contains the virtual phonons. The bare phonons carry large spectral weight and have high energy, while the virtual phonons carry small spectral weight but have arbitrarily low energy. At high temperatures ($k_B T \geq \hbar\omega_{2k_F}$), the bare phonon contribution dominates because of the large spectral weight. The total resistivity is essentially the same as that of the bare phonons. As the temperature is lowered,
the resistivity due to the bare phonons drops exponentially and the contribution from the virtual phonons begins to dominate. The temperature where the deviation from the bare phonon result starts to become appreciable is $k_B T / \hbar \omega_{2k_F} < 0.1$, which is $T < 1K$ for a 1DEG of density $n = 10^6 \text{cm}^{-1}$. The temperature dependence of the total resistivity, including the phonon renormalization effect, in the low temperature regime is shown in the insert of Fig. 2. One can see that an approximate power law behavior $\rho \sim T^\nu$ is found, with $\nu \sim 1.15$ indicated by the slope of the curve in the insert. It should be noted that it is only an approximate power law dependence. The exact temperature dependence is actually very complicated because of the logarithmic divergence of the density-density response function of a 1DEG at zero temperature. This low temperature behavior of resistivity can also be inferred analytically by performing the $T \to 0$ expansion on Eq. (4).

The relative importance of the contributions to the resistivity from the bare phonons and from the virtual phonons is compared in Fig. 3 at high ($k_B T \sim \hbar \omega_{2k_F}$) and low temperatures ($k_B T \ll \hbar \omega_{2k_F}$), where $I(\omega)$ is defined by rewriting Eq. (4) as

$$\rho = \int_0^\infty I(\omega) d\omega.$$  

It is clearly seen that the contribution to the resistivity is dominated by the bare phonons with $\omega \sim \omega_{2k_F}$ at high temperatures and by the virtual phonons with $\omega \ll \omega_{2k_F}$ at low temperatures. Finally in Fig. 4 we show the calculated resistivities from the bare phonons and from the renormalized phonons as functions of the quantum wire carrier density in the low temperature regime. The bare phonon result shows an exponential dependence on the density because of $\rho_o \sim e^{-2\hbar c k_F}$, while the renormalized phonon result shows a very weak dependence on the density on the scale of this figure. Again, as in the temperature dependence of the resistivity, the renormalized phonon theory gives the usual “metallic” behavior of weak density dependence, whereas the usual Drude-Boltzmann transport theory, which includes only scattering by bare phonons, leads to unphysically strong exponential density dependence.

We conclude by summarizing our rather dramatic finding with respect to the low temperature acoustic phonon scattering limited resistivity of 1D semiconductor quantum wire structures: The usual Drude-Boltzmann transport theory, which considers scattering by
bare phonons only, results in extremely “non-metallic” exponentially strong temperature and density dependence of low temperature resistivity due to severe 1D phase space restrictions, whereas a more refined theory, which incorporates many-body renormalization of the phonon spectral function, restores the usual Bloch-Grüneisen power law temperature dependence and weak density dependence in the low temperature resistivity. We believe that we have discovered a very peculiar and rather subtle purely one dimensional many-body phenomenon which should be experimentally observable.

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FIGURES

FIG. 1. Excitation spectrum of a one-dimensional electron gas at zero temperature. The particle-hole excitations are allowed in the region surrounded by the dotted line $ABCDE$. The acoustic phonon dispersion is shown as the solid line.

FIG. 2. Resistivities of a quantum wire from bare phonon scattering (dotted-line) and from the renormalized phonon scattering (solid line) as functions of temperature. The inset shows the power-law behavior of the resistivity from the renormalized phonon scattering in the low temperature regime. The density of the electron gas is $n = 10^6 \text{cm}^{-1}$. The widths of the electron gas wire is $a = b = 200 \text{Å}$.

FIG. 3. Intensity of phonon scattering as functions of phonon frequency $\omega$ at both high temperature (solid-line) and low temperature (dotted-line), where $I(\omega)$ is defined in the text. The dotted-line represents the low temperature result multiplied by a factor of $10^7$. The density of the electron gas is $n = 10^6 \text{cm}^{-1}$. The widths of the electron gas wire is $a = b = 200 \text{Å}$.

FIG. 4. Resistivities of a quantum wire from bare phonon scattering (dotted-line) and from the renormalized phonon scattering (solid line) as functions of electron density at low temperature. The widths of the electron gas wire is $a = b = 200 \text{Å}$. The temperature is $k_B T = 0.02\hbar\omega_{2k_F}$. 
Fig.1 Zheng & Das Sarma

\[ \frac{\hbar \omega}{\epsilon_F} \]

\[ \frac{q}{k_F} \]

\[ \omega_q = cq \]

PHE
Fig. 3  Zheng & Das Sarma

\[ I(\omega) \]

\[ k_B T = 0.05 \hbar \omega_{2k_F} \]

\[ k_B T = 0.9 \hbar \omega_{2k_F} \]

\[ \times 10^7 \]

\[ \omega / \omega_{2k_F} \]
