Abstract. We study the merging of massive stars inside a common envelope for binary systems consisting of a red supergiant with a mass of $15-20 M_\odot$ and a main-sequence companion of $1-5 M_\odot$. We are particularly interested in the stage when the secondary, having overfilled its Roche lobe inside the common envelope, starts to transfer mass to the core of the primary at a very high mass-transfer rate and the subsequent nucleosynthesis in the core-impact region. Using a parametrized model for the structure of the envelope at this stage, we perform 2-dimensional hydrodynamical calculations with the Munich Prometheus code to calculate the dynamics of the stream emanating from the secondary and its impact on the core of the primary. We find that, for the lower end of the estimated mass-transfer rate, low-entropy, hydrogen-rich material can penetrate deep into the primary core where nucleosynthesis through the hot CNO cycle can take place and that the associated neutron exposure may be sufficiently high for significant s-processing. For mass-transfer rates at the high end of our estimated range and higher densities in the stream, the stream impact can lead to the dredge-up of helium, but the neutron production is too low for significant s-processing.

1. Introduction

The merging of two stars inside a common envelope is a common, but very poorly understood phase of binary evolution. It is the consequence of dynamical mass transfer and typically occurs when the mass-losing star is a (super-)giant expanding more rapidly as a result of mass loss than its Roche lobe (for a general discussion see, e.g., Podsiadlowski 2001). The secondary then orbits around the core of the primary within a common envelope which is composed of material from the giant’s envelope. Due to drag forces, the orbit of the binary slowly decays, and the envelope is spun up as a result of the transfer of angular momentum from the orbital motion to the envelope. The spiral-in stage ends either when the envelope is ejected or, as in the case considered here, when the spiraling-in secondary starts to fill its own Roche lobe and begins to transfer mass to the core of the primary. Eventually this leads to the complete merger of
the two stars. We are particularly interested in the question whether the merger of two massive stars (with masses $M_1 \simeq 15-20M_\odot$ and $M_2 \simeq 1-5M_\odot$) is accompanied by unusual nucleosynthesis, leading to a merger product, a rapidly rotating supergiant, with anomalous chemical abundances (as is, for example, observed in the progenitor of SN 1987A; Podsiadlowski 1992).

In this contribution, we give a brief outline of how we model the merger of two massive stars. In Sections 2 and 3 we describe how we determine the structure of the common envelope during the spiral-in phase and how we calculate the stream-core interaction, respectively. In Section 4 we present some of the results of our study to date.

### 2. Common-Envelope Evolution: the Spiral-in Phase

In order to follow the spiral-in of a star inside a common-envelope phase after the initial rapid phase (see Podsiadlowski 2001), we calculate the structure of the primary by simulating the effects of a spiraling-in secondary (i.e., the additional energy sources due to the frictional luminosity caused by the differential rotation between the spiraling-in co-rotating binary core and the envelope, and the accretion luminosity caused by the accretion of envelope material onto the secondary, limited to the Eddington luminosity). We also include the effect of the additional gravity of the secondary on the structure of the envelope.

To follow the evolution of the secondary and determine its structure, we perform a separate stellar calculation, imposing as outer boundary conditions those appropriate for the conditions at the position of the secondary inside the
common envelope and taking into account the contribution to gravity from the primary.

We obtain the angular-velocity profile in the envelope by solving the equations of motion for the angular velocity, produced only by angular momentum transport. These equations are averaged over azimuthal and polar angles. The viscosity is assumed to be produced by turbulent convection in convective regions (Meyer & Meyer-Hofmeister 1979) and to be the ordinary viscosity (i.e., molecular plus radiative) in radiative regions. The equations are solved with the explicit conservation of the total angular momentum $J$ of the binary system, simultaneously with the evolution of the common envelope (the stellar structure is updated at each time step where we use the frictional energy as determined from the current angular-velocity profile). It is further assumed that the primary star initially has a small angular velocity compared to the orbital angular velocity.

3. Stream-Core Interaction

3.1. Conditions at $L_1$ and the Ballistic Stage

When the secondary starts to fill its own Roche lobe inside the common envelope, we can estimate the mass-transfer rate, $\dot{M}_{\text{sec}}$, assumed to be adiabatic, according to (Ritter 1996)

$$\frac{\dot{M}_{\text{sec}}}{M_{\text{sec}}} = \frac{1}{\zeta_{\text{ad}} - \zeta_R} \frac{\partial \ln A}{\partial t}$$

where $M_{\text{sec}}$ is the mass of the secondary, $A$ the binary separation, $\zeta_{\text{ad}}$ the adiabatic mass-radius exponent and $\zeta_R$ the mass-radius exponent of the Roche radius. This estimate leads to characteristic mass-loss rates of $10^{-100} M_\odot \, \text{yr}^{-1}$ for the binary parameters considered.

For the stream cross-section we use

$$S_{\text{CS}} = 1.5 \cdot 10^{20} s_{15}^{1/2} \Omega_{-3}^{-4/3} \dot{M}_{\text{sec}}^{-1/3} \text{ cm}^2,$$

where $s_{15} = \frac{P}{\rho} \cdot 10^{-15}$, $\Omega_{-3} = \Omega \cdot 10^3$ (both in cgs units), and $\dot{M}_{\text{sec}}$ is the mass-loss rate in solar masses per year.

The entropy of the stream material is taken to be the same as the entropy in the secondary. Following the results of 3-dimensional numerical simulations for the stream at the inner Lagrange Point (Bisikalo et al. 1998), we take the density profile of the stream as being composed of a dense core with exponentially decreasing density outside the core. The velocity at $L_1$ is taken to be the local sound speed of the stream material at $L_1$ (this speed varies through the stream cross-section).

Initially, we take the stream trajectory to be ballistic up to the point where the density of the ambient matter (the matter in the common envelope) can no longer be ignored. The stream parameters at this point are determined from the assumption that the Bernoulli integral is conserved. The angle, $\Theta$, at which the stream leaves $L_1$ is calculated using the formulae from Lubow & Shu (1975). This approximation works well for the co-rotating frame and helps to avoid numerical problems at $L_1$ at the start of the hydrodynamical calculations. Some
uncertainty is introduced by the possibility that the inner region surrounding the primary core is not in complete co-rotation with the orbit and will have a backward motion with respect to the co-rotating frame. This relative rotation causes a force which reduces the interception angle between the falling stream and the normal direction to the core as compared to what a ballistic trajectory would predict.

3.2. The Hydrodynamical Stage

For the hydrodynamical simulations of the stream-core interactions, we use the hydrodynamical PROMETHEUS code (Fryxell, Müller & Arnett 1989) with the following modifications: an optional nuclear reactions network (a full network including several hundred heavy elements and a smaller network for the reactions of the hot CNO-cycle) and the inclusion of the external gravitational field. We perform 2-dimensional calculations (using a cylindrical polar coordinate system). The common envelope of the star is treated as a stratified ambient medium with a power-law dependence for the temperature and the pressure distribution:

\[ T(r) = T(r_0) \left( \frac{r}{r_0} \right)^{\alpha_T} \]
\[ P(r) = P(r_0) \left( \frac{r}{r_0} \right)^{\alpha_P}, \]

where \( \alpha_T = 0.9 - 1.5 \) and \( \alpha_P = 4.2 - 5.5 \) well describe stellar models at this evolutionary stage in the region we are interested in.

The boundary conditions which we use to describe the stream inflow are taken to be the Mach number of the gas inflow, \( M_{str} \), the Mach number with respect to the ambient matter, \( M_{amb} \) (the ratio of the stream velocity to the local ambient sonic speed), and the ratio between the central flow density and the ambient density at the top of the box, \( \eta_0 \). Radial and azimuthal velocities of the stream are defined by the interception angle \( \Theta \). For the chemical composition of the gas inflow we use the abundances obtained directly from the secondary star. Boundary conditions for the rest of the box in the radial direction are outflow boundaries (except in the gas inflow zone) with the condition of hydrostatic equilibrium imposed, and in the azimuthal direction are outflow conditions for non-symmetrical cases. Alternatively one of the boundaries is taken to be a reflection boundary (for symmetrical cases with \( \Theta = 0 \)).

4. Results

We have modeled the common-envelope phase for systems composed of 20 + 5, 20 + 1, 15 + 5 \( M_\odot \) components, respectively, taking into account the accretion luminosity of the secondary, the frictional luminosity and the effect of the gravity from the secondary. Parametrized results of these calculations were then used as initial conditions and as boundary conditions for the hydrodynamical calculations. Together with calculations of the ballistic trajectories and the Bernoulli integrals we estimated the parameter space of Mach numbers typically found in our problem. We then performed hydrodynamical calculations for a number of parameter combinations \( (M_{str}, M_{amb}) \). In addition, for some Mach numbers, we performed calculations which included nuclear burning. Purely hydrodynamical calculations showed that the stream is able to penetrate deep into the dense region of the primary for the assumed mass-loss rates and stream material entropies. On the other hand, when nuclear burning is included, this leads to the fast expansion of these streams for high \( \dot{M} \) gas inflow. In this case, at least
in 2-dimensional calculations, hydrogen-rich material does not penetrate deep enough to start a hot CNO cycle. Instead, there is effective mixing in an intermediate zone, which can lead to the dredge-up of He. In the case of gas inflow with lower $\dot{M}$ and lower entropy material (e.g., as would be found for a $1M_\odot$ secondary), a colder stream can penetrate deeper before the temperature in the hydrogen-rich stream increases. Therefore in this case the hot-CNO cycle with corresponding neutron production and subsequent s-process may be possible.

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