Orbital evolution under action of fast interstellar gas flow

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\textbf{Abstract}

Orbital evolution of an interplanetary dust particle under action of an interstellar gas flow is investigated. Secular time derivatives of the particle orbital elements, for arbitrary orbit orientation, are presented. An important result concerns secular evolution of semi-major axis. Secular semi-major axis of the particle on a bound orbit decreases under the action of fast interstellar gas flow. Possible types of evolution of other Keplerian orbital elements are discussed. The paper compares influences of the Poynting-Robertson effect, the radial solar wind and the interstellar gas flow on dynamics of the dust particle in outer planetary region of the Solar System and beyond it, up to 100 AU.

Evolution of putative dust ring in the zone of the Edgeworth-Kuiper belt is studied. Also non-radial solar wind and gravitational effect of major planets may play an important role. Low inclination orbits of micron-sized dust particles in the belt are not stable due to fast increase of eccentricity caused by the interstellar gas flow and subsequent planetary perturbations – the increase of eccentricity leads to planet crossing orbits of the particles.

Gravitational and non-gravitational effects are treated in a way which fully respects physics. As a consequence, some of the published results turned out to be incorrect. Moreover, the paper treats the problem in a more general way than it has been presented up to now.

The influence of the fast interstellar neutral gas flow might not be ignored in modeling of evolution of dust particles beyond planets.

\textbf{Keywords.} ISM: general, Celestial mechanics, Interplanetary medium

1. Introduction

Motion of stars relative to their local interstellar medium is frequent/usual process in galaxies. Neutral atoms penetrate into the Solar System due to the relative motion of the Sun with respect to the interstellar medium. This flow of neutral atoms through a heliosphere has been investigated in many papers, e.g. Fahr (1996), Lee et al. (2009), Möbius et al. (2009). Motion of dust in interplanetary space can be affected by the neutral gas penetrating into the heliosphere. Influence of this effect on dynamics of dust particles is usually ignored in literature. The Poynting-Robertson effect, the radial solar wind and the gravitational perturbation of planet(s) are usually taken into account (Šidlichovský & Nesvorný 1994; Liou & Zook 1997; Liou & Zook 1999; Kuchner & Holman 2003).

Scherer (2000) has calculated secular time derivatives of angular momentum and Laplace-Runge-Lenz vector of a dust particle under the action of interstellar gas flow. But Scherer’s calculations contain several incorrectnesses. He has come to the conclusion that semi-major axis of the dust particle increases exponentially (Scherer 2000, p. 334). This paper presents that semi-major axis of the dust particle decreases under the action of interstellar gas flow, in the framework of the perturbation theory.

Motion of dust particles in the zone of the Edgeworth-Kuiper belt under the action of the interstellar flow of gas has been investigated by Kláčka et al. (2009a). The authors have calculated secular time derivatives of orbital elements only for the case when interstellar gas velocity vector lies in the orbital plane of the dust particle and direction of the velocity vector is parallel with y-axis. This paper overcomes these restrictions. Moreover, it presents some main properties of dust dynamics under the action of the interstellar gas.

2. Secular evolution

Acceleration of a spherical dust particle caused by the flow of neutral gas can be given in the form (Scherer 2000)

\[ \frac{dv}{dt} = - c_D \gamma_H |v - v_H| (v - v_H) , \]

where \( v_H \) is velocity of the neutral hydrogen atom, \( v \) is velocity of the dust grain, \( c_D \) is the drag coefficient, \( \gamma_H \) is the collision parameter. For the collision parameter we can write

\[ \gamma_H = n_H \frac{m_H}{m} A , \]

where \( m_H \) is mass of the neutral hydrogen atom, \( n_H \) is the concentration of interstellar neutral hydrogen atoms, \( A = \pi R^2 \) is the geometrical cross section of the spherical dust grain of radius \( R \) and mass \( m \). The concentration of interstellar hydrogen \( n_H \) is not constant in the entire heliosphere. For heliocentric distances \( r \) less than 4 AU \( n_H \) decreases precipitously from its value in the outer heliosphere toward the Sun, due to ionization (Lee et al. 2009). But in the outer heliosphere, \( r \in (30 \text{ AU}, 80 \text{ AU}) \), we can assume that the concentration of the neutral hydrogen atoms is constant \( n_H = 0.05 \text{ cm}^{-3} \) (Fahr 1996). The same assumption can be used also behind the solar wind termination.
shock. The shock was crossed by Voyager 1 at a heliocentric distance 94 AU and by Voyager 2 at 84 AU (Richardson et al. 2008).

We will assume that the speed of interstellar gas is much greater than the speed of the dust grain in the stationary frame associated with the Sun \((v < \ll v_H)\). This approximation leads to approximately constant value of \(c_D \approx 2.6\) (Baines et al. 1965; Banaszkiewicz et al. 1994; Klačka et al. 2009a).

We want to find influence of the flow of interstellar gas on secular evolution of particle’s orbit. We will assume that the dust particle is under the action of solar gravitation and the flow of neutral gas. Hence we have equation of motion

\[
\frac{dv}{dt} = -\frac{\mu}{r^3} r - c_D \gamma_H \left| v - v_H \right| (v - v_H),
\]

where \(\mu = GM_\odot\), \(G\) is the gravitational constant, \(M_\odot\) is mass of the Sun, \(r\) is position vector of the dust particle with respect to the Sun and \(r = |r|\). The acceleration caused by the flow of interstellar gas will be considered as a perturbation acceleration to the central acceleration caused by the solar gravity. In order to compute secular time derivatives of Keplerian orbital elements (\(a\) - semi-major axis, \(e\) - eccentricity, \(\omega\) - argument of perihelion, \(i\) - longitude of ascending node, \(i\) - inclination) we want to use Gauss perturbation equations of celestial mechanics. To do this, we need to know radial, transversal and normal components of acceleration given by Eq. (1). Orthogonal radial, transversal and normal unit vectors of the dust particle on the Keplerian orbit are seen from Fig. 1 and e.g. Pástor 2009

\[
e_R = (\cos \Omega \cos (f + \omega) - \sin \Omega \sin (f + \omega) \cos i),
\]

\[
\sin \Omega \sin (f + \omega) + \cos \Omega \sin (f + \omega) \cos i),
\]

\[
\sin (f + \omega) \sin i),
\]

\[
e_T = (-\cos \Omega \sin (f + \omega) - \sin \Omega \cos (f + \omega) \cos i),
\]

\[
-\sin \Omega \sin (f + \omega) + \cos \Omega \cos (f + \omega) \cos i),
\]

\[
\cos (f + \omega) \sin i),
\]

where \(f\) is true anomaly. Thus, we need to calculate the values of \(a_R = dv/dt \cdot e_R, a_T = dv/dt \cdot e_T\) and \(a_N = dv/dt \cdot e_N\). Velocity of the particle in an elliptical orbit can be calculated from

\[
v = \frac{dr}{dt} = \frac{d}{dt} (re_R)
\]

\[
= r \frac{e \sin f}{1 + e \cos f} \frac{df}{dt} e_R + r e_T \frac{df}{dt} e_T,
\]

where

\[
r = \frac{p}{1 + e \cos f}
\]

and \(p = a(1 - e^2)\). In this calculation also the second Kepler’s law \(df/dt = \sqrt{\mu/mr^2}\) must be used. Now, one can easily verify that

\[
(v - v_H) \cdot e_R = v_H \sigma e \sin f - v_H \cdot e_R = v_H \sigma e \sin f - A,
\]

\[
(v - v_H) \cdot e_T = v_H \sigma (1 + e \cos f) - v_H \cdot e_T = v_H \sigma (1 + e \cos f) - B,
\]

\[
(v - v_H) \cdot e_N = -v_H \cdot e_N = -C,
\]

where

\[
\sigma = \frac{\sqrt{\mu/m}}{v_H},
\]

and \(v_H = |v_H|\). Hence

\[
|v - v_H|^2 = \frac{\mu}{p} (1 + 2e \cos f + e^2)
\]

\[
- 2 \sqrt{\frac{\mu}{p}} [B + e(A \sin f + B \cos f)] + v_H^2,
\]

where the identity \(\sqrt{A^2 + B^2 + C^2} = v_H\) was used.

If we denote components of the velocity vector of hydrogen gas in the stationary Cartesian frame associated with the Sun as \(v_H = (v_{HX}, v_{HY}, v_{HZ})\), then we obtain

\[
A \sin f + B \cos f = (-\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i) v_{HX}
\]

\[
+ (-\sin \Omega \sin \omega \cos i) v_{HY}
\]

\[
+ \cos \Omega \cos \omega \sin i v_{HZ} = I.
\]

Now we consider only such orbits for which \(\sigma \ll 1\),

or, more precisely, the value \(\sigma^2\) is negligible in comparison with \(\sigma\). This is reasonable for orbits with not very large eccentricities, since \(v \ll v_H\). Using this approximation, Eqs. (13)-(14) yield

\[
|v - v_H| = v_H [1 - \frac{\sigma}{v_H} (B + eI)].
\]

For radial, transversal and normal components of acceleration we obtain from Eq. (1), Eqs. (9)-(11) and Eq. (16)

\[
a_R = -c_D \gamma_H v_H^3 \left[ \frac{A}{v_H} \frac{\sigma e I}{v_H - 1} \right].
\]
The perturbation equations have the form

\[ \sigma \left( e \sin f + \frac{AB}{v_H^2} \right), \]

\[ a_T = - c_D \gamma_H v_H^2 \left( \frac{\sigma a}{v_H} \left( \frac{\sigma a}{v_H} - 1 \right) \right) + \sigma \left( 1 + e \cos f + \frac{B^2}{v_H^2} \right), \] \tag{17}

\[ a_N = - c_D \gamma_H v_H C \left( \frac{\sigma a}{v_H} - 1 + \frac{B}{v_H} \right). \] \tag{18}

Now we can use Gauss perturbation equations of celestial mechanics to compute time derivatives of orbital elements. The perturbation equations have the form

\[ \frac{da}{dt} = \frac{2}{1 - e^2} \sqrt{\frac{\mu}{a}} \left[ a_R \sin f + a_T \left( \cos f + 1 + e \cos f \right) \right], \]

\[ \frac{de}{dt} = \sqrt{\frac{\mu}{2a}} \left[ a_R \sin f + a_T \left( 2 + e \cos f \sin f \right) \right], \]

\[ \frac{d\omega}{dt} = -\frac{r^2}{\mu} \frac{a_N}{\sin i} \sin \left( f + \omega \right), \]

\[ \frac{d\Omega}{dt} = \frac{r^2}{\mu} \frac{a_N}{\sin i} \cos \left( f + \omega \right). \] \tag{20}

Time average of any quantity \( g \) during one orbital period \( T \) can be computed using

\[ (g) = \frac{1}{2} \int_0^T g \ dt = \frac{\sqrt{\mu}}{2\pi a^{3/2}} \int_0^{2\pi} \frac{df}{dt} \left( \frac{df}{dt} \right)^{-1} \ dt \]

\[ = \frac{\sqrt{\mu}}{2\pi a^{3/2}} \int_0^{2\pi} g \left( \frac{df}{dt} \right)^{-1} \ dt \]

\[ = \frac{1}{2\pi a^{3/2}} \sqrt{1 - e^2} \int_0^{2\pi} g \ r^2 \ df, \] \tag{21}

where the second \( \left( \frac{\sqrt{\mu}}{r^2} = \frac{1}{df/dt} \right) \) and the third \( \left( 4\pi^2 a^3 = \mu T^2 \right) \) Kepler’s laws were used. From Eqs. (17)-(21) we finally obtain for the secular time derivatives of the Keplerian orbital elements

\[ \frac{da}{dt} = - 2 a c_D \gamma_H v_H^2 \left[ \frac{\sqrt{\mu}}{2a} \sigma \left( 1 + \frac{1}{\mu} \right) \right], \]

\[ \times \left( I^2 - (I^2 - S^2) \frac{1 - \sqrt{1 - e^2}}{e^2} \right), \] \tag{22}

\[ \frac{de}{dt} = c_D \gamma_H v_H \left( \frac{3I}{2} + \frac{\sigma(I^2 - S^2)(1 - e^2)}{v_H^2} \right), \]

\[ \times \left( 1 - \frac{e^2}{2} - \sqrt{1 - e^2} \right), \]

\[ \frac{d\omega}{dt} = c_D \gamma_H v_H \left( \frac{3S}{e} \right), \]

\[ + \frac{\sigma S}{v_H^2 e} \left[ e^4 - 6e^2 + 4(1 - e^2)^{3/2} \right]. \] \tag{23}

Fig. 2. A schematic representation of the values \( S \), \( I \) and \( C \) for a given orbit.

\[ S \cos \omega - I \sin \omega = \cos \Omega v_H X + \sin \Omega v_H Y = e_{PA} \cdot v_H, \]

\[ S \sin \omega + I \cos \omega = - \sin \Omega \cos i v_H X \]

\[ + \cos \Omega \sin i v_H Y, \]

\[ C = \sin \Omega \sin i v_H X - \cos \Omega \cos i v_H Y, \] \tag{24}

where the quantities

\[ S = (\cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i) v_H X \]

\[ + (\sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i) v_H Y, \]

\[ I = (- \cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i) v_H X \]

\[ + (- \sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i) v_H Y, \]

\[ C = \sin \Omega \sin i v_H X - \cos \Omega \cos i v_H Y + \cos \omega, \] \tag{27}

are values of \( A = v_H \cdot e_R, B = v_H \cdot e_T \) and \( C = v_H \cdot e_N \) at perihelion of particle’s orbit \( (f = 0) \), respectively. The value of \( C \) is a constant on a given oscular orbit. The values of \( S, I \) and \( C \) are depicted in Fig. 2. One can use also the relations

\[ e_{PA} \cdot e_{PE} = 0, \quad e_N \times e_{PA} = e_{PE}. \] \tag{28}

Vector \( e_{PA} \) is directed from the Sun to the ascending node. \( e_{PA} \cdot v_H = S \cos \omega - I \sin \omega \) is magnitude of \( v_H \) component parallel with the line of nodes. The orbital plane is defined by its normal unit vector \( e_N. e_N \times e_{PA} = e_{PE}. \)
Fig. 3. Secular time derivatives of $S$ (see Eqs. 27) for a dust particle on prograde orbit in planar case. Origins of these Cartesian coordinate systems are at the Sun and vertical axes are aligned with the direction of the hydrogen gas velocity vector. $e_{RP}$ and $e_{TP}$ are unit radial and transversal vectors in perihelion of the particle orbit (see text).

\[ e_{PE} \cdot \mathbf{v}_H = S \sin \omega + I \cos \omega \] is magnitude of $\mathbf{v}_H$ component perpendicular to the line of nodes and lying in the orbital plane.

3. Theoretical discussion

Eqs. (22)-(26) enable to deduce some properties of secular evolution of the dust particle under the action of the flow of interstellar gas. $C = 0$ for a special case when the velocity of hydrogen gas $\mathbf{v}_H$ lies in the orbital plane of the particle. In this case we get that the inclination and the longitude of ascending node are constant. Secular time derivatives for a special case of this kind ($i = \Omega = 0, v_{HX} = v_{HZ} = 0, v_{HY} = v_H$ in Eq. 27) have been derived in Kláčka et al. (2009a). Secular time derivatives of $a, e$ and $\omega$ are given in Kláčka et al. (2009a) without generalization represented by Eqs. (27) and Fig. 2.

Putting $\sigma = 0$ in Eqs. (22)-(26) one obtains a solution equivalent to the solution of Eq. (1) with the RHS independent of the particle’s velocity (constant force). Eq. (22) yields constant semi-major axis for the unrealistic case $\sigma = 0$. We will show, using Eq. (22), that secular semi-major axis decreases under the action of interstellar gas flow for $\sigma > 0$. If secular increase of the semi-major axis would occur, then the value of curl braces in Eq. (22) would be negative. The value of $1 - \sqrt{1 - e^2}$ is always positive or zero. Thus, the curly braces could be negative only for $I^2 - S^2 > 0$. Since $I$ and $S$ are radial and transversal components of the constant vector $\mathbf{v}_H$ in the perihelion (Fig. 2) of particle’s orbit, we obtain maximal value of $I^2 - S^2$ for orbit orientation characterized by $S = 0$. Using these assumptions, the minimal value (MV) in the curl braces is

\[
MV = 1 + \frac{1}{v_H^2} \left( I^2 - I^2 \right) \left( \frac{1}{e^2} - \frac{1}{e^2} \right) = 1 + \frac{I^2}{v_H^2} \sqrt{1 - e^2} \frac{1}{e^2} - \frac{1}{e^2} > 0 .
\]

The positiveness of $MV$ means that the secular semi-major axis is a decreasing function of time. Eq. (22) was derived under the assumption that Solar gravity represents dominant acceleration in comparison to the interstellar gas disturbing acceleration. Since gravitational acceleration decreases with square of a heliocentric distance, the assumption about the disturbing interstellar gas acceleration may not be valid far from the Sun. Behind the heliocentric distance at which solar gravitational acceleration equals the interstellar gas acceleration, the particle’s secular semi-major axis can also be an increasing function of time. The two accelerations are equal at heliocentric distance $\approx 1 \times 10^4$ AU for dust particle with $R = 1 \mu m$ and $\varrho = 2 g/cm^3$, and, at heliocentric distance $\approx 7 \times 10^4$ AU for dust particle with $R = 1 \mu m$ and $\varrho = 1 g/cm^3$. If Eq. (22) can be used, then secular time derivative of the semi-major axis is proportional to the value of semi-major axis (value of $\sqrt{p/\mu} \sigma$ is independent of semi-major axis). Our result differs from the Scherer’s statement (semi-major of the particle can increase exponentially) at least due to an error in his expression for secular time derivative of magnitude of angular momentum.

Defining the function $w(e) \equiv 1 - e^2 / 2 - \sqrt{1 - e^2}, e \in [0,1)$, present in Eq. (23), we obtain

\[
\frac{dw}{de} = \frac{d}{de} \left( 1 - \frac{e^2}{2} - \sqrt{1 - e^2} \right) = -e + \frac{e}{\sqrt{1 - e^2}} \geq 0 .
\]

Thus, $w(e)$ is an increasing function of eccentricity. We obtain that $w(e) \geq 0$ for all $e \in [0,1)$, since $w(e)$ is an increasing function of eccentricity and $w(0) = 0$. Hence, the sign of the second term in the square braces in Eq. (23) will depend on the sign of $I^2 - S^2$.

For small values of the eccentricity we get that the secular time derivative of eccentricity is, approximately, proportional only to the first term multiplied by $I$ in the square braces in Eq. (23).

Eq. (24) yields that the argument of perihelion is constant for the planar case $C \equiv 0$ and for the orbit orientation characterized by $S = 0$. The longitude of the ascending node and the inclination are constant in this case. Hence differentiation of the second equation of Eqs. (27) with respect to time gives

\[
\frac{dI}{dt} = -S \frac{dw}{dt} .
\]

Thus, $I$ is also constant in this case. Since $\sigma$ is a small number, the first term in square braces in Eq. (23) is the dominant term on evolution eccentricity.
Now, let us consider the planar case \((C \equiv 0)\) with \(S \neq 0\). The differentiation of the first of Eqs. (27) with respect to time gives
\[
\frac{dS}{dt} = I \frac{d\omega}{dt}.
\]
This quantity can be averaged using Eq. (21). We get
\[
\left\langle \frac{dS}{dt} \right\rangle = I \left\langle \frac{d\omega}{dt} \right\rangle.
\]
If \(\sigma\) is a small number and \(I\) is not close to zero, then we can use the following approximations of Eqs. (23)-(24)
\[
\left\langle \frac{dc}{dt} \right\rangle \approx 3\frac{c_d \gamma_H v_H \sqrt{\mu S}}{2} I,
\]
\[
\left\langle \frac{d\omega}{dt} \right\rangle \approx -3\frac{c_d \gamma_H v_H \sqrt{\mu S}}{2} e.
\]
Inserting Eqs. (34)-(35) into Eq. (33) one can obtain
\[
\left\langle \frac{de}{dt} \right\rangle \approx -e\left\langle \frac{dS}{dt} \right\rangle.
\]
This equation leads to the differential equation
\[
\frac{de}{dt} \approx -dS \frac{S}{e},
\]
with the solution
\[
e \approx D\frac{e}{|S|}.
\]
\(D\) is a constant which can be determined from initial conditions. Thus, eccentricity is close to its minimal value if the major axis of the orbit is aligned with the direction of the hydrogen gas velocity vector.

If we consider the system Eqs. (23)-(26) with \(\sigma = 0\), then we obtain \((dS/dt) = -3\frac{c_d \gamma_H v_H \sqrt{\mu S}}{2} I S / (2e)\) from the first of Eqs. (27). If we use Eq. (23) with \(\sigma = 0\), then we get equation \((de/dt) = -e (dS/dt) / S\). This equation leads to the solution \(e = D / |S|\) also for this non-planar case. But approximation \(\sigma \approx 0\) is not allowed in Eqs. (24)-(26) since the first terms in square brackets multiplied by \(C\) are multiplied by \(e, \sin \omega\) and \(\cos \omega\) which can be close to zero.

The case \(\sigma = 0\) (the RHS of Eq. 1 is independent of the particle’s velocity – constant force) is also significant for motion of a rocket in a central gravitational field with a constant-reaction acceleration vector. We obtain the following important result: if the constant acceleration can be considered as a perturbation acceleration to the central gravitation and \(S \neq 0\), then equation \(e = D / |S|\) holds during orbital motion of the rocket. This result fills up the results of Betelsky (1964) and Kunitsyn (1966).

Parameters \(S, I\) and \(C\) determine position of the orbit with respect to the hydrogen gas velocity vector. Therefore their time derivatives are useful for description of evolution of the orbit position in space. Putting Eqs. (24)-(26) into averaged time derivatives of the quantities \(S, I\) and \(C\) defined by Eqs. (27), one obtains
\[
\frac{dS}{dt} = \frac{c_d \gamma_H v_H S \sqrt{\mu}}{2} \left\{ -\frac{3I}{e} - \frac{\sigma}{v_H} \left[ C^2 - \frac{I^2}{e^4} \left( e^4 - 6e^2 + 4 - 4(1 - e^2)^{3/2} \right) \right] \right\},
\]
\[
\frac{dI}{dt} = \frac{c_d \gamma_H v_H S \sqrt{\mu}}{2} \left\{ -\frac{3eC^2}{2} - \frac{3S^2}{e} - \frac{\sigma I}{v_H} \left[ C^2 + \frac{S^2}{e^4} \left( e^4 - 6e^2 + 4 - 4(1 - e^2)^{3/2} \right) \right] \right\},
\]
\[
\frac{dC}{dt} = \frac{c_d \gamma_H v_H C \sqrt{\mu}}{2} \left\{ -\frac{3eI^2}{e^2} - \frac{\sigma I^2}{v_H} (k(e)) \right\}.
\]
Function \(k(e)\) is a decreasing function of eccentricity for \(e \in (0, 1)\). This can be shown in a similar way as we used in Eq. (30) for the function \(w(e)\). The procedure must be used more than once for the function \(k(e)\). The function \(k(e)\) obtains values from \(\lim_{e \to 0} k(e) = -0.5\) to \(\lim_{e \to 1} k(e) = -1\), for \(e \in (0, 1)\). For the eccentricity \(e = 1\) we obtain \(\lim_{e \to 1} k(e) = \infty\). Hence we use maximal value of eccentricity \(e_m\) for which Eq. (15) holds. The first term in parenthesis in Eq. (42) is minimal for \(e = e_m\) for a given value of \(I\). Value of the parenthesis in Eq. (42) will be maximally affected by the second term for \(e = e_m\). The second term in parenthesis yields for \(e = e_m\)
\[
\frac{\sigma I^2}{v_H} k(e_m) \geq -\frac{\sigma I^2}{v_H} \geq -\sigma |I| \geq -|I| \geq -3 \frac{|I|}{e_m},
\]
since \(\sigma \ll 1\) and \(|I| \leq v_H\). Thus, the sign of \((dS/dt)\) depends only on the sign of the first term in the parenthesis in Eq. (42). Fig. 3 depicts unit vectors \(e_{RP}\) and \(e_{TP}\) in the orbital plane of a particle with prograde orbit. The hydrogen gas velocity vector \(v_H\) for the planar case lies in the orbital plane. Direction of \(v_H\) is also shown. The unit vector \(e_{RP}\) has direction and orientation from the Sun to perihelion of the particle orbit. In Fig. 3a \(e_{RP}\) lies in the first quadrant of the Cartesian coordinate system with origin in the Sun and vertical axis aligned with the direction of the hydrogen gas velocity vector. Both \(S\) and \(I\) are greater than 0, for these positions of the unit vectors \(e_{RP}\) and \(e_{TP}\). Hence \((dS/dt)\) is negative. If \(e_{RP}\) lies in the second quadrant (Fig. 3b), then \((dS/dt)\) is positive. If \(e_{RP}\) lies in the third quadrant (Fig. 3c), then \((dS/dt)\) is negative. Finally, if \(e_{RP}\) lies in the fourth quadrant (Fig.}
3d), then \(\langle dS/dt \rangle\) is positive, as it is in the second quadrant. Therefore, the vector \(e_{RP}\) rotates counterclockwise in the first and the second quadrants, and, clockwise in the third and the fourth quadrants. If the vector \(e_{RP}\) is parallel with vertical axis in Fig. 3 \((I = 0 \text{ and } C = 0)\), then Eq. (40) yields \(\langle dI/dt \rangle > 0\). Thus, positions of the vector \(e_{RP}\) parallel with the vertical axis in Fig. 3 are not stable. However, if \(e_{RP}\) is parallel with the horizontal axis \((S = 0 \text{ and } C = 0)\), then \(\langle dS/dt \rangle = 0\) and \(\langle dI/dt \rangle = 0\). Thus, \(e_{RP}\) parallel with the horizontal axis yields stable positions of the vector \(e_{RP}\). The stable position of the vector \(e_{RP}\) parallel with the horizontal axis and directed to the left in Fig. 3 is of theoretical importance, only. In reality, no particles should be observed with perihelia in this direction. However, all unit vectors \(e_{RP}\) of particles in prograde orbits will approach the right direction in Fig. 3. For retrograde orbits we have to use transformation \(S \rightarrow -S, I \rightarrow -I\). We obtain \(\langle dS/dt \rangle \rightarrow -\langle dS/dt \rangle\). Hence, all unit vectors \(e_{RP}\) of particles in retrograde orbits will approach the left direction in Fig. 3. This result was obtained, in a different way, also by Scherer (2000). Scherer states that the approaches of unit vectors \(e_{RP}\) to one direction holds also for the non-planar case. However, the statement of Scherer is incorrect, in general.

If \(I = 0 \text{ and } C \neq 0\), then Eq. (39) implies that \(\langle dS/dt \rangle\) is

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**Fig. 4.** Two evolutions of orbital elements of a dust particle with \(R = 2 \mu m\) and mass density \(\varrho = 1 \text{ g/cm}^3\) under the action of interstellar gas flow. Evolution depicted by a solid line is calculated from the equation of motion. Evolution depicted by a dashed line corresponds to Eqs. (22)-(26).
proportional to $SC^2$ and not to $SI$. This leads to a behavior which differs from the behavior discussed above using Fig. 3.

Scherer (2000, p. 332) furthermore states that orbital plane under the effect of the interstellar gas flow will be rotated into a plane coplanar to the flow vector $v_H$, independent of initial position of the orbital plane. We showed, using numerical integrations, that the statement of Scherer is not correct. Retrograde orbits were not discussed by Scherer. We found an interesting orbit behavior. It depends on the orbit orientation with respect to the hydrogen gas velocity vector $v_H$. If $v_H$ lies in a plane $i = 0$ and $v_H$ is perpendicular to the vector $e_{RP}$ (in this case $S = 0$ and $\langle dS/dt \rangle = 0$), then the interstellar gas flow can change prograde orbit into a retrograde one (even more times for one particle) and inclination does not approach the value $i = 0$.

4. Numerical results

4.1. Comparison of the numerical solution of equation of motion and the solution of Eqs. (22)-(26)

We numerically solved Eq. (3) and the system of differential equations represented by Eqs. (22)-(26). Solutions are compared in Fig. 4. We assumed that the direction of the interstellar gas velocity vector is identical to the direction of the velocity of the interstellar dust particles entering the Solar System. The interstellar dust particles enter the Solar System with a speed of about $v_\infty = 26$ km/s (Landgraf et al. 1999) and they are arriving from direction of $\lambda_{cen} = 259^\circ$ (heliocentric ecliptic longitude) and $\beta_{cen} = 8^\circ$ (heliocentric ecliptic latitude) (Landgraf 2000). Thus, components of the velocity in the ecliptic coordinates with x-axis aligned toward the equinox are $v_H = -26$ km/s ($\cos(259^\circ) \cos(8^\circ)$, $\sin(259^\circ) \cos(8^\circ)$, $\sin(8^\circ)$). As an initial conditions for a dust particle with $R = 2 \mu$m and mass density $\varrho = 1$ g/cm$^3$ we used $a_{in} = 60$ AU, $e_{in} = 0.5$, $\omega_{in} = 90^\circ$, $\Omega_{in} = 90^\circ$, $i_{in} = 20^\circ$ for Eqs. (22)-(26). The initial true anomaly of the dust particle was $f_{in} = 180^\circ$ for Eq. (3). Fig. 4. shows that the obtained evolutions are in a good agreement. Evolutions begin separate as the eccentricity approaches 1. This is caused by the fact that approximation $\sigma \ll 1$, see Eq. (15), does not hold for large eccentricities. Detailed numerical solution of the equation of motion (Eq. 3) yields that the secular semi-major axis is a decreasing function of time also when eccentricity approaches 1.

We can summarize, on the basis of the previous paragraph. If there is no other force, besides solar gravity and the flux of interstellar gas, then the semi-major axis of an interplanetary dust particle decreases and the particle can hit the Sun. However, the particle can hit the Sun also by another possibility: particle’s eccentricity increases to 1. These mathematical possibilities probably do not occur in reality, since other forces can act on the dust particle and the interstellar gas is ionized below the heliocentric distance of about 4 AU.

Let us return, once again, to the planar case ($C \equiv 0$) in which $S = 0$ and the dominant term in the square braces in Eq. (23), the term $(3/2) I$, is negative. Numerical integration of Eq. (3) shows that if the eccentricity decreases to 0, then the argument of perihelion $\omega$ "shifts" its value to the value $\omega + \pi$. This means that the negative value of $I$ changes to positive and the eccentricity begins to increase with the same slope.

The approximative solution represented by Eq. (38) is in a good agreement with the detailed numerical solution of Eqs. (22)-(26) for the planar case with $S \neq 0$. This holds for the whole time interval, also for $I$ close to zero. Eq. (38) holds, approximately, also for the non-planar evolution depicted in Fig. 4. In this case $i$ is close to zero, $v_H \approx v_{HY}$ and $\Omega \approx 90^\circ$ at the eccentricity minimum. Eq. (38) gives $e \approx D / |v_H \cos \omega|$. The evolutionary minimum of eccentricity occurs for the case when $\omega$ is close to 180°. This is in accordance with Eq. (38).
4.2. Comparing of influences of interstellar gas flow, Poynting-Robertson effect and radial solar wind on dynamics of dust particles

We have considered only the effect of interstellar gas flow, up to now. In reality, some other non-gravitational effects play non-negligible role. Thus, we want to compare the effect of the interstellar gas flow with the other effects influencing dynamics of dust grains in the Solar System. For this purpose we included the Poynting-Robertson effect (P-R effect) and the radial solar wind into the equation of motion. The P-R effect is electromagnetic radiation pressure force acting on a spherical particle (Klačka 2004; arXiv:astro-ph/0807.2915; arXiv:astro-ph/0904.0368). Equation of motion for the P-R effect, the effect of the radial solar wind and the effect of gas flow has the form (e.g. Klačka et al. 2009b)

\[
\frac{dv}{dt} = - \frac{\mu}{r^2} (1 - \beta) \, e_R - \beta \frac{\mu}{r^2} \left(1 + \frac{\eta}{Q_{pr}} \right) \left(\frac{v \cdot e_R}{c} e_R + \frac{v}{c} \right) - c_D \gamma_H |v - v_H| (v - v_H),
\]

(44)

where decrease of particle’s mass (corpuscular sputtering) and higher orders in \(v/c\) are neglected. \(c\) is the speed of light in vacuum. Parameter \(\beta\) is defined as the ratio of the radial component of the electromagnetic radiation pressure force and the gravitational force between the Sun and the particle in rest with respect to the Sun:

\[
\beta = \frac{3L_\odot Q_{pr}'}{16\pi c G M_\odot R_\odot^2} \simeq 5.763 \times 10^{-4} \frac{Q_{pr}'}{R \ [\text{m}]} [\text{g} / \text{kg/m}^3].
\]

(45)

\(L_\odot\) is the solar luminosity, \(L_\odot = 3.842 \times 10^{36} \text{ W}\) (Bahcall 2002), \(Q_{pr}'\) is the dimensionless efficiency factor for radiation pressure integrated over the solar spectrum and calculated for the radial direction (\(Q_{pr}' = 1\) for a perfectly absorbing sphere) and \(\eta\) is mass density of the particle, \(\gamma\) is the ratio of solar wind energy to electromagnetic solar energy, both radiated per unit of time

\[
\gamma = \frac{4\pi^2 u^2}{L_\odot} \sum_{i=1}^{N} n_i m_i c^2,
\]

(46)

where \(u\) is the speed of the solar wind, \(u = 450 \text{ km/s}\), \(m_i\) and \(n_i\), \(i = 1\) to \(N\), are masses and concentrations of the solar wind particles at a distance \(r\) from the Sun, \(\gamma = 0.38\) for the Sun (Klačka et al. 2009b). Four numerical integrations of Eq. (44) are shown in Fig. 5. We used dust particle with \(R = 2 \mu m\), mass density \(\rho = 1 \text{ g/cm}^3\) and \(Q_{pr}' = 0.75\). We took into account only planar case when the interstellar gas velocity lies in the orbital plane of the dust particle (\(C = 0\)), for the sake of simplicity. Initial position is \(r_{in} = (0, -90 \text{ AU}, 0)\) and initial velocity vector is \(v_{in} = (2 \text{ km/s}, 0, 0)\). Orbital evolution is given by evolution of orbital elements calculated for the case when a central acceleration is defined by the first Keplerian term in Eq. (44), namely \(-\mu (1 - \beta) e_R / r^2\). The evolution depicted by the dash-dotted line is for the P-R effect alone (\(\gamma_H = 0, \eta = 0\) in Eq. 44). The evolution depicted by the dotted line is for the P-R effect with the flow of interstellar gas (\(\eta = 0\) in Eq. 44). The evolution depicted by the solid line is for the P-R effect and the radial solar wind (\(\gamma_H = 0\) in Eq. 44). Finally, the evolution depicted by the solid line holds for the case when all three effects act together.

The evolution of semi-major axis depicted in Fig. 5 shows that the flow of interstellar gas is more important than the radial solar wind, as for the effects on the dynamics of the dust particle.

The secular eccentricity is always a decreasing function of time for the P-R effect and the radial solar wind (e.g., Wyatt & Whipple 1950, Klačka et al. 2009b). The growth in eccentricity depicted in Fig. 5 is due to the interstellar gas. Fast decrease of the semi-major axis in Fig. 5 may also be, at least partially, caused by the fact that higher eccentricities decrease the value of \(\langle dv/dt\rangle_{PR}\) and the P-R effect becomes stronger. The secular evolution of eccentricity can be also an increasing function of time if the flow of interstellar gas is taken into account. We have

\[
\frac{d\dot{e}}{dt} = - \frac{5}{2} \beta \left(1 + \frac{\eta}{Q_{pr}'} \right) \frac{\mu}{c} \frac{e_\beta}{a_\beta^2(1 - e_\beta^2)^{1/2}} \left[3\beta \frac{\sigma(I_B - S_B^2)(1 - e_\beta^2)}{v_H e_\beta^2} \times \left(1 - \frac{e_\beta^2}{2} - \sqrt{1 - e_\beta^2}\right)\right],
\]

if also Eq. (23) is used. The subscript \(\beta\) denotes that the central acceleration \(-\mu (1 - \beta) e_R / r^2\) is used for calculation of the oscillating orbital elements. We remind that the transformation \(\mu \rightarrow \mu (1 - \beta)\) has to be done on the RHS sides of Eqs. (20)-(26). If we use definition of the oscillating orbital elements, then the physical central acceleration is given by the gravitational acceleration of the Sun, \(-\mu e_R / r^2\). In this case, the secular evolution of eccentricity is given by Eq. (103) in Klačka (2004), assuming that \(e_\beta\) is calculated from Eq. (47).

If optical properties of the dust particle are constant, then the secular time derivative of the argument of perihelion equals to zero for the P-R effect and the radial solar wind (Klačka et al. 2007, 2009b). If the flow of interstellar gas is included into the equation of motion, then, even in the planar case, the secular time derivative of the argument of perihelion may not be equal to zero, in general (see Fig. 5).

The evolutions of eccentricity and argument of perihelion shown in Fig. 5 are significantly affected by the flow of interstellar gas.

4.3. Dust ring in the Edgeworth-Kuiper belt zone

Real situation in the Edgeworth-Kuiper belt zone may be much more complicated than the situation discussed in Secs. 4.1 and 4.2. In particular, gravitation of planets may have an important influence on dynamics of dust in the zone. For this reason we included gravitation of four major planets into the final equation of motion. Observations from Helios 2 during its first solar mission in 1976 (Bruno et. al. 2003) show that the angle between the radial direction and the direction of the solar wind velocity does not significantly depend on heliocentric distance. If the value of
this angle is approximately constant, then the non-radial solar wind can also have an important influence on dynamics of dust in outer parts of the Solar System. We took into account the non-radial solar wind with constant value of the angle. Influence of precession of the rotational axis of the Sun on the non-radial solar wind was also considered. Hence, equation of motion of the dust particle has the form

\[
\frac{dv}{dt} = -\frac{\mu}{r^2} e_R + \beta \frac{\mu}{r^2} \left[ \left(1 - \frac{v \cdot e_R}{c}\right) e_R - \frac{v}{c} \right] - \frac{\beta \eta}{Q_{pr}} \frac{\mu}{c} u |v - u| (v - u) - c_D \frac{\gamma_H}{|v - v_H|} (v - v_H) - \sum_{i=1}^{4} \frac{GM_i}{|r - r_i|^3} (r - r_i),
\]

(48)

where \( u \) is solar wind velocity vector, \( M_i \) are masses of the planets and \( r_i \) are position vectors of the planets with respect to the Sun. The non-radial solar wind velocity vector was calculated from equation

\[
u = u \left( e_R \cos \varepsilon + \frac{k \times e_R}{|k \times e_R|} \sin \varepsilon \right),
\]

(49)

where \( \varepsilon \) is the angle between the radial direction and the direction of the solar wind velocity and \( k \) is unit vector with direction/orientation corresponding to the direction/orientation of solar rotation angular velocity vector. Vector \( k \) for a given time can be calculated from

\[
k = (\sin \Omega_s \sin i_s, -\cos \Omega_s \sin i_s, \cos i_s) ,
\]

\[
i_s = 7^\circ 15', \quad \Omega_s = 73^\circ 40' + 50.25'' (t[\text{years}] - 1850) .
\]

(50)
Fig. 7. Evolution of orbital elements of dust particles in the Edgeworth-Kuiper belt zone during 96 numerical solutions depicted in Fig. 6. Initial values of orbital elements are: $a_i = 60$ AU, $e_i = 0.1$, $\omega_i \in \{0^\circ, 45^\circ, 90^\circ, ..., 270^\circ, 315^\circ\}$, $\Omega_i \in \{0^\circ, 90^\circ, 180^\circ, 270^\circ\}$, $i_i \in \{5^\circ, 10^\circ, 15^\circ\}$ and $f_i = 0^\circ$. Evolution during the first 750 000 years is influenced mainly by interstellar gas, and, later on, mainly by gravitation of planets (see text).

While Eqs. (49)-(50) are consistent with Klačka (1994) and Abalakin (1986), the value of $\varepsilon$, $\varepsilon = 2.9^\circ$, used in our numerical calculations, is in accordance with Bruno et al. (2003) and Klačka et al. (2007). The observed neutral hydrogen gas velocity vector in the ecliptic coordinates with $x$-axis aligned toward the equinox is $v_H = -26 \text{ km/s} (\cos(259^\circ) \cos(8^\circ), \sin(259^\circ) \cos(8^\circ), \sin(8^\circ))$. As for the initial conditions of dust particles we did not use random positions and velocities. We assumed that putative dust ring in the Edgeworth-Kuiper belt has approximate circular shape and contains lot of particles with approximately equal optical properties. We assumed that the ring contains such large amount of particles that one can choose, approximately, a given value of semi-major axis and a given radius of the particles. Accelerations caused by the P-R effect, the solar wind and the interstellar neutral hydrogen gas are inversely proportional to particle’s radius and mass density. Therefore, a large particle is less influenced by the non-gravitational effects than a small particle of the same mass density. The evolution of the large particle under action of the non-gravitational effects is slower than the evolution of the small dust grain. We used uniformly distributed initial
values of the argument of perihelion and longitude of the ascending node. Furthermore we assumed that particles in the ring orbit prograde in low inclination orbits. We used particles with $R = 2 \mu m$, $q = 1 \text{ g/cm}^3$ and $Q_{pr} = 0.75$. Exact initial values of orbital elements were $\omega_{in} = 60 \text{ AU}$, $\varepsilon_{in} = 0.1$, $\Omega_{in} \in \{0^\circ, 45^\circ, 90^\circ, ..., 270^\circ, 315^\circ\}$, $i_{in} \in \{5^\circ, 10^\circ, 15^\circ\}$ and $f_{in} = 0^\circ$. Hence, we obtained $8 \times 4 \times 3 = 96$ individual orbits. Results of numerical solutions of Eq. (48) are depicted in Figs. 6 and 7. Fig. 6 depicts evolution of the dust ring viewed from perspective. Orbits of the planets are also shown. Time span between various pictures in Fig. 6 is 250 000 years. As we can see, the ring becomes more and more eccentric because of a fast increase of eccentricity caused by the interstellar gas flow (see also eccentricity evolution in Fig. 7). Perihelia of orbits are shifted in accordance with the behavior discussed in Fig. 3. This is caused by the facts that influence of interstellar gas is dominant and the solved problem is almost coplanar. The term multiplied by $C^2$ in Eq. (39) does not have large influence on the first term in the curly braces in Eq. (39), in almost coplanar case. Therefore, the lines connecting the Sun with the perihelia of particles orbits are approaching the direction perpendicular to the interstellar gas velocity vector. Time evolution of the orbital elements of the dust particles in the ring, considered in Fig. 6, is depicted in Fig. 7. Due to the approach of the perihelia to one direction, the evolutions of the argument of perihelion $\omega$, beginning with a given initial value $\omega_{in}$, are divided into four branches. Each of them corresponds to one initial value of the ascending node. If the time is less than 750 000 years, then: i) the concentration of the particles in the ring is smallest in the direction (from the Sun) into which perihelia of the orbits are approaching, and, ii) the concentration of the particles is greatest in exactly opposite direction. If the time is greater than 750 000 years, then the orbits of particles in the dust ring are getting close to the orbits of the planets due to the increase of particles eccentricities. The situation after 750 000 years can be seen in Fig. 7. The dust particles with $R = 2 \mu m$, $q = 1 \text{ g/cm}^3$ and $Q_{pr} = 0.75$ are characterized by the value $\beta \approx 0.216$ (see Eq. 45). For this value of $\beta$, one obtains $a \approx 57.7 \text{ AU}$ for the location of the exterior mean motion 3/1 resonance with Neptune; it follows from $a = a_0 (1 - \beta)^{1/3} (3/1)^{2/3}$, where $a_0$ is semi-major axis of Neptune. We can see, from the evolution of semi-major axis in Fig. 7, that the secular semi-major axis is a decreasing function of time during the first 750 000 years. Thus the semi-major axis can evolve from an initial value of 60 AU to the location close to the mean-motion 3/1 resonance. Particles are influenced both by the vicinity of Neptune orbit and the exterior mean motion 3/1 resonance with Neptune. Evolution during the first 750 000 years is influenced mainly by the neutral interstellar hydrogen gas and, later on, mainly by the gravitation of planets.

Inclusion of the P-R effect, the non-radial solar wind and the interstellar gas into the equation of motion of the dust particle without planets can stabilize the particle’s orbit. The stabilization is characterized by stable values of orbital elements. This stabilization is discussed in Kláčka et al. (2009a) for sin $\varepsilon = 0.05$. The process of stabilization requires about $1 \times 10^8$ years for the dust particle with $\beta = 0.01$. This time is not very sensitive to the efficiency factor for radiation pressure $Q_{pr}$. However, for lower values of $Q_{pr}$ the stabilization occur with larger probability, because stabilization effect of the non-radial solar wind is stronger (see Eq. 48). If also planets are considered in the equation of motion, then the stabilizing value of dust particle eccentricity is usually sufficiently high to get the particle close to one of the planets during a long time span. As a consequence, gravitation of the planet can change orbit of the particle and cancel the stabilization process.

5. Conclusion

We investigated orbital evolution of a dust grain under the action of interstellar gas flow. We presented secular time derivatives of the grain’s orbital elements for arbitrary orientation of the orbit with respect to the velocity vector of the interstellar gas, which is a generalization of several results presented in Kláčka et al. (2009a). The secular time derivatives were derived using assumptions that the acceleration caused by the interstellar gas flow is small in comparison with gravitation of a central object (the Sun), eccentricity of the orbit is not close to 1 and the speed of the dust particle is small in comparison with the speed of the interstellar gas. These assumptions lead to secular decrease of semi-major axis of the particle. The secular time derivative of the semi-major axis is negative and proportional to $a$. This result is not in accordance with Scherer (2000) who has stated that the semi-major of the particle increases exponentially. Scherer’s statement is incorrect and our analytical result is confirmed by our detailed numerical integration of equation of motion (see also Fig. 4).

If the hydrogen gas velocity vector $v_H$ lies in the particle’s orbital plane and the major axis of the orbit is not perpendicular to $v_H$, then the product of (secular) eccentricity and magnitude of the radial component of $v_H$ measured in perihelion is, approximately, constant during orbital evolution.

We considered simultaneous action of the P-R effect, the radial solar wind and the interstellar gas flow. Numerical integrations showed that the action of the flow of interstellar gas can be more important than the action of the electromagnetic and corpuscular radiation of the Sun, as for the motion of dust particles orbiting the Sun in outer parts of the Solar System (see Fig. 5). Physical decrease of semi-major axis can be more than 2-times greater than the value produced by the P-R effect and radial solar wind. The evolution of eccentricity can also be an increasing function of time when we consider the P-R effect and the radial solar wind together with the flow of the neutral interstellar gas. This is also relevant difference from the action of the P-R effect and the radial solar wind when secular decrease of eccentricity occurs. Simultaneous action of all three effects yields that the secular time derivative of the argument of perihelion may not be equal to zero, in general.

Gravitation of four major planets was also directly added into the equation of motion, see Eq. (48). This access correctly describes capture of dust grains into mean motion resonances with the planets. Our physical approach differs from the Scherer’s approach (Scherer 2000), who has used some kind of secular access to gravitational influence of the planets.

Assumption on an existence of dust ring in the zone of the Edgeworth-Kuiper belt is in contradiction with rapid increase of eccentricity of the ring due to an acceleration caused by the interstellar gas flow. Speed of the eccentricity
increase (time derivative of eccentricity) is roughly inversely proportional to the particle’s size and mass density. As the eccentricity of the particles increases, the particles approach the planets. The particles in the ring are under the gravitational influence of the planets. The particles evolve also in semi-major axis and they can be temporarily captured into a mean motion resonance. The particles can remain in chaotic orbits between orbits of the planets, or, the particles are ejected to high eccentric orbits due to close encounters with one of the planets. Only particles with greater size and mass density should survive in the dust ring for a long time.

A relevant result of the paper is that equation of motion in the form of Eq. (44) (or, Eq. 48) and Eqs. (45)-(46) have to be used in modeling of orbital evolution of dust grains in the Solar System. The influence of the fast interstellar neutral gas flow might not be ignored in general investigations on evolution of dust particles in the zone of the Edgeworth-Kuiper belt.

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