A simple holographic model for spontaneous breaking of translational symmetry

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Abstract

It has been shown that holographic massive gravities can effectively realize the states with spontaneous breaking of translational symmetry in a homogenous manner. In this work, we consider a toy model of such category by adding a special gauge-axion coupling to the bulk action. Firstly, we identify the existence of spontaneous breaking of translations by the UV analysis. In the absence of explicit breaking, the black hole solution is simply the same as the Reissner-Nodström(RN) black holes, regardless of the non-trivial profile of the bulk axions. The associated Goldstone modes exist only when the charge density is non-zero. Then, we investigate the optical conductivity both analytically as well as numerically. Our numerical result perfectly agrees with that for a clean system, while the incoherent part gets modified due to the symmetry breaking. The transverse Goldstone modes are dispersionless, which reflects the fact that the solution is dual to a liquid state. Finally, the effect of momentum relaxation to the transverse modes is also considered. In this case, the would-be massless modes are pinned at certain frequency, which is another key difference from unbroken phases.

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I. INTRODUCTION

In most of real-world materials, the translational symmetry in spatial dimensions are inevitably broken both spontaneously and explicitly (In this paper, we call them SSB and ESB for short.) due to the existence of periodic lattice, striped orders, impurities, defects, etc. For crystalline states, the Goldstone bosons associated with the SSB of translations are usually called phonons, all of which have linear dispersion relations and propagate freely at certain speed in the clean materials. While, for liquid states with SSB of translations, there is only one longitudinal phonon since the shear stresses cannot be supported.

In strongly interacting electronic systems, Goldstone modes and electrons can be mightily coupled which gives rise to novel collective behaviors and exotic transport properties[1–3]. To have a deeper understanding on such patterns, building a framework beyond the conventional perturbative methods has already become an important mission in condensed matter physics.

Holographic duality provides a tractable approach to the physics of strong correlated systems by mapping the many-body problems to classical gravity problems. Recently, some holographic models for solid states that spontaneously break the translations have been constructed[5–12]. A common feature in these models is that the translations are spontaneously broken in a homogenous manner(For inhomogeneous models, one can refer to
This significantly simplifies the holographic calculations and makes it possible to dissect the key properties of the system, say, the transport, in analytic ways. In this paper, we mainly consider a new simple holographic models which can realize the liquid states with SSB of the translations, by introducing a special gauge-axion coupling. In the absence of relaxation, it is found that the background metric as well as the gauge field are exactly the same as the Reissner-Nodström(RN) black holes, while the profile of the bulk axions plays the role of the scalar condensate that breaks the translations. On top of this, we investigate the imprints of the transverse Goldstone modes on the electric conductivity.

This plan of this work is as follows: In section II, we construct the simplest holographic model with a gauge-axion coupling and explain how the SSB of translations can happen in this model via analyzing the UV expansion of the bulk fields. In section III, we compute the electric conductivity in the purely SSB pattern both analytically and numerically. In section IV, we consider the pinned modes in the presence of relaxation. In section V, we conclude.

II. GOLDSTONE MODES BY GAUGE-AXION COUPLING

To break the translations in an isotropic and homogeneous manner, one can introduce massless axion fields with a bulk profile $\phi^I = k \delta^I_i x^i$. Essentially, these scalar fields give the gravitons an effective mass, hence breaking the diffeomorphism invariance of the bulk theory[21]. In this paper, we will focus on the following simple holographic model with a special gauge-axion coupling:

$$ S = \int d^4x \sqrt{-g} \left( R - 2 \Lambda - \lambda XX - \frac{1}{4} F^2 - \frac{J}{4} Tr[XXF^2] \right) $$

where the U(1) gauge field $F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu$,

$$ Tr[XXF^2] \equiv XX_{\mu}^{\nu} F_{\rho \sigma}^{\nu} F_{\rho \sigma}^{\mu}, \quad \text{with} \quad XX_{\mu}^{\nu} = \frac{1}{2} \sum_{I=x,y} \partial^\mu \phi^I \partial^\nu \phi^I, $$

and $X \equiv Tr[X]$. We require that $\lambda \geq 0$ for and a necessary condition $0 \leq J \leq 2/3$ for unitarity and causality[22]. For convenience, we set the cosmological constant $\Lambda = -3$ which means a normalized AdS radius. When taking $J = 0$, it reduces to the simplest linear axion model.
From the action, the covariant form of the equations of motion are given by

$$\nabla_\mu \left[ F^{\mu\nu} - \frac{J}{2} ((\mathcal{X}F)^{\mu\nu} - (\mathcal{X}F)^{\nu\mu}) \right] = 0, \quad (3)$$

$$\nabla_\mu \left[ \lambda \nabla^\mu \phi^I + \frac{J}{4} (F^2)^{\mu}_\nu \nabla^\nu \phi^I \right] = 0, \quad (4)$$

and

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{\lambda}{2} \nabla_\mu \phi^I \nabla_\nu \phi^I - \frac{1}{2} \left( 6 - \frac{\lambda}{2} \nabla_\sigma \phi^I \nabla^\sigma \phi^I \right) g_{\mu\nu}$$

$$= \frac{1}{2} \left( F_\mu^{\sigma} F_\nu^{\rho} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) + \frac{J}{4} \left( \frac{1}{2} \nabla_\mu \phi^I \nabla_\nu \phi^I (F^2)^{\sigma}_\nu \right)$$

$$+ F_{(\mu|\sigma}(F\mathcal{X})^{\sigma|\nu)} + F_{(\mu|\sigma}(\mathcal{X}F)^{\sigma|\nu}) - \frac{1}{2} g_{\mu\nu} Tr[\mathcal{X}F^2]. \quad (5)$$

As is known that this model has the following isotropic charged black hole solutions:

$$ds^2 = -D(r) \, dt^2 + B(r) \, dr^2 + C(r) \, dx^i dx_i, \quad A_\mu = A_t(r) \, dt, \quad \phi^I = (0, 0, kx, ky), \quad (6)$$

where $i = 2, 3$ denotes the two spatial directions. Choosing such radial coordinate $r$ that the AdS boundary is located at $r = 0$. Then, in the asymptotic region the background solution behaves like

$$D(r) = \frac{1}{r^2} \left( 1 - d_{(3)} r^3 + \ldots \right),$$

$$B(r) = \frac{1}{r^2} \left( 1 + d_{(3)} r^3 + \ldots \right),$$

$$C(r) = \frac{1}{r^2},$$

$$A_t(r) = \mu - \rho r + \ldots, \quad (7)$$

where $d_{(3)}$ is associated with the energy density, $\mu$ and $\rho$ are the chemical potential and the charge density of the boundary theory.

To explain why SSB of translations can be realized in the holographic model (2), we firstly need to explain what role the profile of the scalars $\phi^i = k x^i$ plays in the following different two cases. Without loss of generality, let us assume that $\phi^I$ depend on the full coordinates $x^\mu$ first. If we set $\lambda \neq 0$ and $J = 0$ in (4), the asymptotic behavior of $\phi^I$ near the UV boundary is

$$\phi^I = \phi^I_{(0)}(t, x^i) + \phi^I_{(3)}(t, x^i) r^3 + \ldots \quad (8)$$
Then, the $r-$independent term $\phi^i_{(0)}(t, x^i)$ dominates the second one and should play the role of an external source in the standard quantization. Then, the profile $\phi^i = kx^i = \phi^i_{(0)}$ introduces an ESB of translations. This case has been widely investigated in previous holographic studies on momentum relaxation[23–25].

Conversely, if we instead set $\lambda = 0$ and $\mathcal{J} \neq 0$, the scalars behave as

$$\phi^I = \frac{\phi^I_{(-1)}}{r} + \phi^I_{(0)}(t, x^i) + \ldots . \quad (9)$$

In this case, the $r-$independent term is subleading and corresponds to the expectation value of a dual operator $\mathcal{O}^I$. Then, $\phi^i = kx^i$ should be interpreted as the expectation value $\langle \mathcal{O}^i \rangle = kx^i$ with a zero source, i.e. $\phi^i_{(-1)} = 0$. Since such a scalar condensate $\langle \mathcal{O}^i \rangle$ is not uniform in $x^i$, the translational symmetry is broken spontaneously. The Nambu-Goldstone theorem tells there should exist gapless excitations in the low energy description which are called Goldstone modes. With $\lambda = 0$, the background solution is given by

$$D(r) = \frac{f(r)}{r^2} = \frac{1}{r^4 B(r)}, \quad (10)$$

$$f(r) = 1 - \frac{r^3}{r_h^3} - \frac{\mu^2 r^3}{4 r_h} \left( 1 - \frac{r}{r_h} \right),$$

$$C(r) = \frac{1}{r^2}, \quad A_t(r) = \mu - \rho r,$$

where $r_h$ denotes the location of the horizon. Note that, unlike the cases of explicitly breaking, the background metric does not depend on $k$. Requiring the gauge field to be regular on the horizon gives $\mu = \rho r_h$. Finally, the Hawking temperature is

$$T = \frac{3}{4\pi r_h} - \frac{\mu^2 r_h}{16\pi}, \quad (11)$$

which is also the same as the RN black holes. If we separate the scalars as $\phi^I = kx^i + \chi^i$, the Lagrangian of the fluctuating fields is given by

$$\mathcal{L}_\chi = -\frac{\mathcal{J} r^4 \rho^2}{8}(\partial^\nu \chi^i \partial_\nu \chi^i + \partial^\nu \chi^i \partial_\nu \chi^i). \quad (12)$$

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1. Here, we have assumed that $A'_t(0) \sim \text{const} \neq 0$, i.e., the system is at a state with a finite density of net charge.
2. For this case, the non-canonical kinetic term of the scalars implies the theory is strongly coupled near the trivial solution of the scalars. Around the non-trivial solution $\phi^i = kx^i$, the fluctuating scalars do have a canonical quadratic kinetic term.
3. The similar UV expansion was firstly found in an earlier study of the holographic massive gravity theories with a nonlinear kinetic term of the axion, $V(X) = X^n$ with $n > 5/2$ in [7].
We find that the transverse fields \( \chi^i \) are massless and \( \partial_j \chi^i = 0 \) are satisfied. Then, the dynamics of the transverse Goldstone modes is encoded in the eom of \( \chi^i \). The prefactor of \( \mathcal{L}_\chi \) implies that the Goldstone modes only exist at finite density. This looks kind of like the gapless sliding modes of charge density waves\(^{26}\), in contrast to the acoustic phonons which do not carry \( U(1) \) charges. With a zero net charge, the \( \chi^i \) will be decoupled from the other spin-1 fluctuations in the bulk and will not affect the charge transport, as expected. In the next section, we will investigate the electric conductivity and reveal that the condition \( \partial_j \chi^i = 0 \) implies the transverse modes are dispersionless.

### III. ELECTRIC CONDUCTIVITY

We now turn to study small fluctuations around the background solution. We denote \( g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \), \( A_\mu = \bar{A}_\mu + \delta A_\mu \) and \( \phi^i = \bar{\phi}^i + \chi^i \), where the quantities with bars are evaluated on the background, and introduce the time-dependent perturbations as follows

\[
\delta A_\mu(t, r, x^i) = \int_{-\infty}^{+\infty} \frac{d\omega d^2 p_\parallel}{(2\pi)^3} e^{-i\omega t + ip_\parallel x^i} a_\mu(r),
\]

\[
\delta g_{\mu\nu}(t, r, x^i) = \int_{-\infty}^{+\infty} \frac{d\omega d^2 p_\parallel}{(2\pi)^3} e^{-i\omega t + ip_\parallel x^i} r^{-2} h_{\mu\nu}(r),
\]

\[
\chi^i(t, r, x^i) = \int_{-\infty}^{+\infty} \frac{d\omega d^2 p_\parallel}{(2\pi)^3} e^{-i\omega t + ip_\parallel x^i} \psi^i(r). \tag{13}
\]

To derive the conductivity, we focus on the homogeneous vector modes, namely setting all the momenta \( p_i = 0 \). The system is isotropic and does not violate parity, the parity-even and parity-odd sectors decouple. This means we only need to consider the \( x \)-component of the vector modes, namely \( a_x \), \( h_{tx} \), \( h_{rx} \) and \( \psi^x \). The linearized Maxwell, scalar equation and Einstein equations read

\[
fa''_x - \frac{J}{4} k^2 r^2 f a'_x = \frac{1}{4} \mathcal{J} k^2 r^2 a'_x - \frac{1}{2} \mathcal{J} k^2 r^2 a'_x + \frac{J}{4} k^2 r^2 \omega^2 a_x + \frac{\omega^2 a_x}{4 f} + \frac{i k \omega a_x}{r} - \frac{2 k f h_{rx}}{r} - \frac{2 f \psi^x}{r} - \frac{\omega^2 \psi^x}{f} = 0, \tag{14}
\]

\[
f\psi'''^x - \frac{2 i k \omega a_x}{r} - \frac{1}{4} \mathcal{J} k^2 r^2 h_{tx}' - \frac{1}{4} \mathcal{J} k^2 r^2 h_{tx}' - \frac{2 k f h_{rx}}{r} - \frac{2 f \psi^x}{r} - \frac{\omega^2 \psi^x}{f} = 0, \tag{15}
\]

\[- \frac{i \mathcal{J} k^2 r^2 \omega a_x}{4 f} + \frac{2 i r^2 \omega a_x}{f} + \frac{\omega^2 h_{rx}}{f} = 0, \tag{16}
\]

\[ fh''_{tx} + \frac{J}{4} k^2 r^2 a'_x - pr^2 a'_x + \frac{i \omega f h_{rx}}{r} = 0, \tag{17}
\]
In this case, the mass of the spin-1 gravitons should be identified as $M(r)^2 = J k^2 \rho^2 r^4$, which flows non-trivially along the radial direction\(^{22}\). In particular,

$$M(0) = 0, \quad \text{and} \quad M(r_h) = \left( J \frac{\pi k^2 \mu^2}{s} \right)^{1/2} \sim \text{finite}. \quad (18)$$

From the point of view of the bulk theory, this is exactly a condition for realizing the *gapless* Goldstone bosons\(^{6}\). The optical conductivity can be achieved numerically by taking the infalling boundary conditions at the horizon. Since the electric current is a vector operator, the conductivity is sensitive to the transverse Goldstone modes but cannot mix with the longitudinal component which is a scalar mode. Then, we can read some information about the transverse modes directly from the conductivity.\(^4\)

Without explicit breaking, the optical conductivity for a *relativistic* system can be written as\(^{1}\)

$$\sigma(\omega) \xrightarrow{\omega \to 0} \sigma_0 + \frac{\chi_{JP}^2 i}{\chi_{PP} \omega} \quad (19)$$

where $\chi_{JP} = \rho$, $\chi_{PP} = \epsilon + P$ and the finite part $\sigma_0$ is the incoherent conductivity that is theory-dependent and irrelevant to the momentum. Unlike the DC conductivity, the incoherent one can always be computed via the membrane paradigm, hence is UV insensitive from the RG perspective. In our model, it can be obtained similarly as in \(^{8, 9}\):

$$\sigma_0 = \left( \frac{sT}{sT + \mu \rho} \right)^2 \left( 1 - J \frac{\pi k^2}{s} \right), \quad (20)$$

where the thermodynamic relation $sT + \mu \rho = \epsilon + P$ has been applied. Eq.(19) means that the real part of conductivity has a delta infinity at zero frequency in the purely SSB pattern due to the absence of momentum relaxation.

In Fig.1, we show that the numeric result of our holographic model with $\lambda = 0$ agrees with (19) and (20) very well. This again indicates that the background profile of the scalars $\bar{\phi}^i = k x^i$ should be interpreted as breaking the translational symmetry spontaneously rather than explicitly. And the Goldstone modes contribute the correction term in (20) to the incoherent conductivity that is controlled by the parameters $J$ and $k$.

The Goldstone modes associated with the SSB of translations in crystals are usually called phonons. Nevertheless, the transverse gapless modes in this model are not phonon like. For

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\(^4\) We would like to thank M. Baggioli for pointing out this.
transverse and longitudinal phonons, they have the linear dispersion relations \( \omega \sim v_{T,L} p \) with finite sound speeds \( v_{T,L} \). Setting \( p_y = p \neq 0 \) and repeat the numerical calculations (To do so, we adopt the gauge invariant formulation of the bulk fields like in [27], and solve the coupled equations of the shear modes numerically.), we find that the infinite peak does not move away from \( \omega = 0 \) as the momentum \( p \) is tuned. In Fig. 2, we show the imaginary part of the conductivity with the varying momentum. Therefore, the Goldstone modes are not dispersive, which means there is not freely propagating transverse phonons. This can be easily understood in another way: For phonons, the sound speeds are related to the shear modulus \( G \) and the bulk modulus \( K \) by [28]

\[
v_T^2 = \frac{G}{\chi_{PP}}, \quad v_L^2 = \frac{G + K}{\chi_{PP}}. \tag{21}
\]

One can check that in our model the \( J \) coupling does not contribute a finite mass term to the spin-2 gravitons. Then, this gives the result that the shear viscosity obeys the KSS bound \( \eta = \frac{s}{4\pi} \) and the shear modulus \( G = 0 \) which is the case for a fluid instead of a solid. According to the first relation in (21), we have \( v_T = 0 \), the transverse modes are not phonon like. This does not conflict with the well-known fact that there is no transverse phonons in a fluid due to the lack of shear stress. Therefore, we believe that this holographic model provides a low energy description for the gapless Goldstone modes coupled with a conformal fluid. In the next section, we will further study the optical conductivity in the presence of the explicit source that breaks the translations. The numeric result of the holographic model captures another key feature of the SSB of translations which is called pinning effect.

IV. PINNING EFFECT

Now, we consider how the peak of the goldstone bosons moves in the presence of ESB. According to the UV analysis in section II, such a pinning effect can be realized by setting a non-zero value of \( \lambda \). In this case, it is expected to be that the would-be massless modes get gapped, reminiscent of the QCD story that pions associated the SSB of chirality is gapped because of the small current mass of quarks. In consequence, the infinite delta at zero frequency is removed and there will be a sharp but finite peak at a certain frequency (called pinning frequency) in the optical conductivity.
FIG. 1: Left plot: Im(\(\sigma\)) as the function of \(\omega/\mu\) for different temperature. The inset plot shows the Drude weight, \(\chi_{JP}^2/\chi_{PP}\), as the function of \(T/\mu\). Blue line is the analytical result obtained from Eq.(19) and the red dots are the numerical result. Right plot: The incoherent conductivity \(\sigma_0\) as the function of \(T/\mu\). Blue line is the analytical result obtained from Eq.(20) and the red dots are the numerical result. Here, we have set \(J = 1/3\) and \(k/\mu = 1\).

FIG. 2: Im(\(\sigma\)) as the function of \(\omega/\mu\) for different momenta. Here, we have set \(J = 1/3\), \(k/\mu = 1\) and \(T/\mu = 1\). For all the cases, there is always an infinite pole standing still at zero frequency.

When \(\lambda \neq 0\), the blackfactor \(f(r)\) becomes

\[
f(r) = 1 - \frac{r^3}{r_h^3} - \left( \frac{\lambda k^2 r^2}{2} + \frac{\mu^2 r^3}{4 r_h^3} \right) \left( 1 - \frac{r}{r_h} \right).
\]

(22)

And the linearized equations of motion:
From the equations, the effective mass of the spin-1 gravitons should be identified as by the membrane paradigm [22].

The gapped modes can be read from the optical conductivity in the two plots of Fig.3. As is expected, the pinning frequency becomes higher as the increase of \( \lambda \), i.e., the rate of momentum relaxation. Moreover, it is obvious to see from the left plot of Fig.4 that the pinning effect introduces a mechanism of transition from a metallic state (when \( J = 0 \)) to an insulating state, which is quite similar as what happens in a doped Mott insulator [29].

Even though, the commensurability effect is in general absent in the holographic systems with homogeneity [30].

The quantitative relation between \( \lambda \) and the relaxation rate can be in principle checked by a full analysis on the quasi-normal modes of the black hole like in [6, 31, 32], which is however not a target in this work. Now, we consider how the propagation of the transverse modes would change when we vary the value of \( \lambda \). We turn on \( p \neq 0 \) and obtain the optical conductivity with finite momentum in the right plot of Fig.4. The numerical result shows that the peak of gapped modes becomes milder as increasing the momentum. However, these modes are still dispersionless, in contrast to the solid holographic massive gravity model [6, 27].

\[
fa''_x - \frac{J}{4} k^2 r^2 fa''_x - \frac{1}{4} \mathcal{J} k^2 r^2 a'_f + a'_x f' - \frac{1}{2} \frac{J k^2 r^2 a_x}{4 f} - \frac{\mathcal{J} k^2 r^2 \omega^2 a_x}{4 f} + \frac{\omega^2 a_x}{f} = \frac{i}{4} \frac{J k^2 r^2 \omega h_{rx} - i \rho \omega h_{rx} + \frac{1}{4} \mathcal{J} k^2 r^2 h'_{tx} - \rho h_{tx} + \frac{1}{4} \mathcal{J} k^2 r^2 h_{tx} + \frac{1}{4} i \mathcal{J} k \rho r \omega \psi^x = 0,}{\mathcal{J} r^2 f^4}
\]

\[
fpsi''_x - 2 i k \omega a_x - k h_{rx} f' + f' \psi^x - k f h_{rx} + \frac{2 k f h_{rx}}{r} - \frac{i k \omega h_{tx}}{r} - \frac{2 f \psi^x}{r} + \frac{\omega^2 \psi^x}{f} = \frac{4 \lambda k f' h_{rx}}{\mathcal{J} r^2 f^4} + \frac{4 \lambda f' \psi^x}{\mathcal{J} r^2 f^4} - \frac{4 \lambda f \psi^x}{\mathcal{J} r^2 f^4} - \frac{8 \lambda f \psi^x}{\mathcal{J} r^2 f^4} + \frac{4 \lambda \omega \psi^x}{\mathcal{J} r^2 f^4} + \frac{4 \lambda \omega \psi^x}{\mathcal{J} r^2 f^4} = 0,
\]

\[
fh''_{tx} + \frac{J}{4} k^2 r^4 f a'_x - \rho r^2 f a'_x + i \omega f h_{tx} + \frac{2 i \omega f h_{rx}}{r} - \frac{2 f h_{tx}}{r} = \frac{J}{4} k^2 r^4 h_{tx} + \frac{i J}{4} k \rho r^4 \omega \psi^x - \lambda k^2 h_{tx} - i \lambda k \omega \psi^x = 0,
\]

From the equations, the effective mass of the spin-1 gravitons should be identified as \( M(r)^2 \equiv k^2 \left( \lambda + \frac{\mu^2}{4} \right) \). With \( \lambda \neq 0 \), the DC conductivity can be directly computed by the membrane paradigm [22]

\[
\sigma_{DC} = \left( 1 - \frac{\mathcal{J} \pi k^2}{s} \right) + \left( 1 - \frac{\mathcal{J} \pi k^2}{s} \right)^2 \frac{\mu^2}{M(r)^2},
\]

The gapped modes can be read from the optical conductivity in the two plots of Fig.3. As is expected, the pinning frequency becomes higher as the increase of \( \lambda \), i.e., the rate of momentum relaxation. Moreover, it is obvious to see from the left plot of Fig.4 that the pinning effect introduces a mechanism of transition from a metallic state (when \( J = 0 \)) to an insulating state, which is quite similar as what happens in a doped Mott insulator [29].

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\begin{align*}
\lambda &= 5 \times 10^{-4} \\
\lambda &= 1 \times 10^{-3} \\
\lambda &= 5 \times 10^{-3} \\
\lambda &= 1 \times 10^{-2} \\
\lambda &= 5 \times 10^{-2} \\
\lambda &= 1 \times 10^{-1}
\end{align*}

\begin{align*}
\omega/\mu &\approx 0.272532 \lambda^{0.125} \text{ in the region } \lambda \in [5 \times 10^{-4}, 3 \times 10^{-2}]. \text{ Here, we have fixed } J = 1/3, k/\mu = 1 \text{ and } T/\mu = 0.005.
\end{align*}

(FIG. 3: Left plot: Re(\sigma) as the function of \omega/\mu for different values of \lambda. Right plot: The pinning frequency as the function of \lambda. The solid lines are the fitting results which show that \omega_0/\mu \approx 0.272532 \lambda^{0.125} \text{ in the region } \lambda \in [5 \times 10^{-4}, 3 \times 10^{-2}]. \text{ Here, we have fixed } J = 1/3, k/\mu = 1 \text{ and } T/\mu = 0.005.)

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\end{align*}

(FIG. 4: Left plot: Re(\sigma) as the function of \omega/\mu for \lambda = 0.1 and various values of T/\mu. It shows that \frac{d\sigma_{dc}}{dT} > 0. Right plot: The optical conductivity for \lambda = 0.01, T/\mu = 0.005 and various values of p/\mu. Note that the pinned modes are dispersionless which is the same as the purely SSB pattern. In both plots, we have fixed J = 1/3, k/\mu = 1.)

V. CONCLUSION AND OUTLOOK

In this paper, we introduce a simple holographic model that can realize both the spontaneous and explicit breaking of the translational symmetry in the dual field theory. In this model, the SSB is induced by a gauge-axion coupling \( JTr[\lambda F^2] \), while the ESB can be realized by turning on the linear axion term \( \lambda X \).
When we turn off the external scalar source by setting $\lambda = 0$, the condensate of the dual operators that breaks the translations should be identified as the bulk profile of the axions, via the UV analysis. In this case, the background metric and gauge field is the same as the RN black holes. And the dynamics of the transverse Goldstone modes is encoded in the eoms of the spatial components of axions. Our numeric result of electric conductivity matches with that of a fluid with SSB of translations. We then turn on the explicit source to see its pinning effect on the Goldstone modes. It is found that the pinning frequency becomes higher as we increase the value of $\lambda$. And the gapped modes are still dispersionless.

In this short paper, the analysis on the bulk mode is lacking. In fluids, there exists longitudinal phonons whose speed is related to both of the shear and bulk modulus. Then, the second relation in (21) can be checked by studying the coupled spin-0 fluctuations, which is more complicated.\(^5\) We will leave this for future work\(^{[34]}\).

Our model can be generalized, including the higher derivative terms like $\sum_{n=2}^{\infty} Tr[X^n F^2]$. One can, however, check that such terms do not change the UV expansion (9), hence will not change the story a lot, albeit further modifications on the incoherent conductivity. One can also consider another class of gauge-axion couplings, for instance, $K Tr[X] F^2[22]$. This term will change the background solution and gives the system a non-zero shear modulus when $\rho \neq 0$. Then the dual system may be interpreted as some kind of “electronic crystals”, whose impacts on the transport or elastic properties are also worth studying.

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\(^5\) Since the $J$ coupling does not affect the background, we cannot directly express the bulk modulus $K$ in terms of the background quantities as in \([11, 33]\).
this work.

[1] S. A. Hartnoll, A. Lucas, and S. Sachdev, “Holographic quantum matter”, MIT press.
[2] D. Bandurin et al., “Negative local resistance caused by viscous electron backflow in graphene”, Science 351 (2016), no. 6277 1055-1058, [arXiv:1509.04165].
[3] L. Delacrétaz, B. Goutéraux, S. Hartnoll, A. Karlsson, SciPost Phys. 3, 025 (2017), arXiv:1612.04381 [cond-mat.str-el]; L. Delacrétaz, B. Goutéraux, S. Hartnoll, A. Karlsson, Phys. Rev. B 96, 195128 (2017), arXiv:1702.05104 [cond-mat.str-el]
[4] A. Esposito, S. Garcia-Saenz, A. Nicolis, R. Penco, “Conformal solids and holography”, JHEP 1712 (2017) 113, arXiv:1708.09391[hep-th].
[5] T. Andrade, M. Baggioli, A. Krikun, N. Poovuttikul, “Pinning of longitudinal phonons in holographic spontaneous helices”, JHEP 1802 (2018) 085, arXiv:1708.08306 [hep-th].
[6] L. Alberte, M. Ammon, M. Baggioli, A. Jiménez, O. Pujolàs, “Black hole elasticity and gapped transverse phonons in holography”, JHEP 1801(2018)129, arXiv:1708.08477 [hep-th].
[7] L. Alberte, M. Ammon, M. Baggioli, A. Jiménez, O. Pujolàs, “Holographic Phonons”, Phys. Rev. Lett. 120, 171602 (2018), arXiv:1711.03100 [hep-th].
[8] A. Amoretti, D. Areán, B. Goutéraux, D. Musso, “Effective holographic theory of charge density waves”, Phys. Rev. D 97, 086017 (2018), arXiv:1711.06610 [hep-th].
[9] A. Amoretti, D. Areán, B. Goutéraux, D. Musso, “DC resistivity of quantum critical, charge density wave states from gauge-gravity duality”, Phys. Rev. Lett. 120, 171603 (2018), arXiv:1712.07994 [hep-th].
[10] S. Grozdanov, N. Poovuttikul, “Generalised global symmetries in states with dynamical defects: the case of the transverse sound in field theory and holography”, Phys. Rev. D 97, 106005 (2018), arXiv:1712.07994 [hep-th].
[11] M. Baggioli, A. Buchel, “Holographic Viscoelastic Hydrodynamics”, arXiv:1805.06756 [hep-th].
[12] G. Filios, P. A. González, X.-M. Kuang, E. Papantonopoulos, Y. Vásquez, “Spontaneous Momentum Dissipation and Coexistence of Phases in Holographic Horndeski Theory”, arXiv:1808.07766 [hep-th].
[13] M. Rozali, D. Smyth, E. Sorkin, J. Stang, “Holographic Stripes”, Phys. Rev. Lett. 110, 201603
[14] A. Donos, JHEP 1305, 059 (2013) [arXiv:1303.7211 [hep-th]]; A. Donos, J. Gauntlett, Phys. Rev. D 87, 126008 (2013), arXiv:1303.4398 [hep-th].

[15] Y. Ling, C. Niu, J.-P. Wu, Z. Xian, H. Zhang, “Metal-insulator Transition by Holographic Charge Density Waves”, Phys. Rev. Lett. 113, 091602 (2014), arXiv:1404.0777 [hep-th].

[16] S. Cremonini, L. Li, J. Ren, Phys.Rev. D95 (2017) no.4, 041901, arXiv:1612.04385 [hep-th]; S. Cremonini, L. Li, J. Ren, JHEP 1708 (2017) 081, arXiv:1705.05390 [hep-th].

[17] R.-G. Cai, L. Li, Y.-Q. Wang, J. Zaanen, “Intertwined Order and Holography: The Case of Parity Breaking Pair Density Waves”, Phys. Rev. Lett. 119 (2017) no.18, 181601, arXiv:1706.01470 [hep-th].

[18] N. Jokela, M. Jarvinen, M. Lippert, “Holographic pinning”, Phys. Rev. D 96, 106017 (2017), arXiv:1708.07837[hep-th].

[19] A. Donos, J. Gauntlett, T. Griffin, V. Ziogas, “Incoherent transport for phases that spontaneously break translations”, JHEP 1804(2018)053, arXiv:1801.09084 [hep-th].

[20] B. Goutéraux, N. Jokela, A. Pönni, “Incoherent conductivity of holographic charge density waves”, JHEP 1807 (2018) 004, arXiv:1803.03089 [hep-th].

[21] D. Vegh, Holography without translational symmetry, arXiv:1301.0537 [hep-th].

[22] B. Goutéraux, E. Kiritsis, W.-J. Li, “Effective holographic theories of momentum relaxation and violation of conductivity bound”, JHEP 1604 (2016) 122, arXiv:1602.01067[hep-th].

[23] T. Andrade and B. Withers, “A simple holographic model of momentum relaxation”, JHEP 05 (2014) 101, arXiv:1311.5157 [hep-th].

[24] B. Goutéraux, “Charge transport in holography with momentum dissipation”, JHEP 04 (2014) 181, arXiv:1401.5436 [hep-th].

[25] K.-Y. Kim, K. K. Kim, M. Park, “A Simple Holographic Superconductor with Momentum Relaxation”, JHEP 04 (2015) 152, arXiv:1501.00446 [hep-th].

[26] G. Grüner, “The dynamics of charge density waves”, Rev. Mod. Phys. 60. 1129.

[27] M. Baggioli, O. Pujolàs, “Electron-Phonon Interactions, Metal-Insulator Transitions, and Holographic Massive Gravitest”, Phys. Rev. Lett. 114, 251602 (2015), arXiv:1411.1003 [hep-th].

[28] L. P. Kadanoff and P. C. Martin, “Hydrodynamic equations and correlation functions”, Annals of Physics 24 (1963) 419-469.

[29] T. Andrade, A. Krikun, K. Schalm, J. Zaanen, and P. C. Martin, “Doping the holographic
Mott insulator”, Nature Physics 14, 1049?1055 (2018), arXiv:1710.05791 [hep-th].

[30] T. Andrade, A. Krikun, “Commensurability effects in holographic homogeneous lattices”, JHEP 05 (2016) 039, arXiv:1512.02465 [hep-th].

[31] M. Baggioli, K. Trachenko, “Solidity of liquids: How Holography knows it”, arXiv:1807.10530 [hep-th].

[32] M. Baggioli, K. Trachenko, “Maxwell interpolation and close similarities between liquids and holographic models ”, arXiv:1808.05391 [hep-th].

[33] M. Baggioli, V. Castillo, O. Pujolàs, S. Petel, to appear.

[34] M. Baggioli, W.-J. Li, J.-P. Wu, in preparation.