Parton Densities from Collider Data

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Abstract. Collider data can play an important role in determining the parton distribution functions of the nucleon. I outline a formalism which makes it possible to use next-to-leading order calculations in such an extraction, while minimizing the amount of numerical computation required.

INTRODUCTION

The search for new physics underlying the standard model requires a precise understanding of known physics. At hadron colliders, this translates into the requirement for a precise understanding of perturbative QCD, along with knowledge of the non-perturbative inputs: $\alpha_s$ and the parton distribution functions of the nucleon. In recent years, our detailed knowledge of perturbative QCD — in the form of matrix elements [1], techniques for combining virtual and real corrections in next-to-leading order calculations [2,3,5,4] — has grown tremendously. The same is true of our knowledge of $\alpha_s$ [6], and of the parton densities at small $x$ [7].

On the other hand, issues which might have been thought settled — the parton distributions, and in particular the gluon distribution, at moderate $x$ — have become unsettled. Investigations by the CTEQ group [8], in light of the supposed excess of large-$E_T$ jets claimed by CDF [9], have shown that deeply inelastic scattering does not constrain the gluon distribution well enough to allow a claim of new physics to be made on the basis of the inclusive jet distribution. (The claim of an excess is not, however, supported by the DØ data [10].) More recently, CTEQ has pointed out [11] that similar statements apply to the high-$x$ valence quark distributions and resulting implications of ZEUS high-$Q^2$ data [12].

To date, the parton densities have been taken solely as inputs in calculations of collider processes. To constrain the gluon distribution better, however, it would

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be desirable to include collider data in the global fits. It is the triply-differential distribution \[13\] in dijet events, \( d^3\sigma/dE_T d\eta_1 d\eta_2 \), that will presumably give us the most useful information on the gluon distribution. To obtain sensible results using jet data, however, the use of theory at next-to-leading order is a bare minimum. From a theoretical point of view, this results from the presence of multiple scales: a jet cross section contains not only a hard-scattering scale, but also a scale characterizing the ‘size’ of the jet. The perturbative expansion is thus not merely an expansion in \( \alpha_s \), but rather in \( \alpha_s \ln^2 y_{IR} \) and \( \alpha_s \ln y_{IR} \), where \( y_{IR} \) is a ratio of scales, as well. In order to ascertain that these potentially dangerous logarithms are under control, and that the perturbative calculation is reliable, we must calculate (at least) at next-to-leading order. We must therefore perform fits to collider data using NLO theoretical calculations.

**FITTING TO DATA**

The parton densities depend on both the momentum fraction \( x \) at which they are evaluated, and on a factorization scale \( Q^2 \). Their evolution with changes in \( Q^2 \) is governed by the Altarelli-Parisi equation [14],

\[
Q^2 \frac{\partial f(x, Q^2)}{\partial Q^2} = \int_0^1 dy \int dz \, \delta(x - yz) \, P(y, Q^2) f(z, Q^2),
\]

whose kernel \( P \) can be computed perturbatively. We may thus take the non-perturbative input to the parton densities to be their values at some fixed scale \( Q^2_0 \), and it is these values that we must fit to data. This is usually done by picking a parametrization of the form [17],

\[
f_i(x, Q^2_0) = A_i x^{-\lambda_i} (1 - x)^{\beta_i} (1 + \epsilon_i \sqrt{x} + \delta_i x).
\]

For the traditional observables in deeply-inelastic scattering data (e.g. \( F_2 \)), one has relatively simple analytic NLO formulæ in terms of the parameters \( \{ A_i, \lambda_i, \beta_i, \cdots \} \), and one can iteratively find a best fit by adjusting parameters, re-evaluating the theoretical prediction, and comparing with measurements.

For jet distributions, such an iterative procedure would be extremely slow using existing NLO jet programs, since each iteration involves re-running the jet program, a matter of several hours if not a day even with present-day workstations. A better way of organizing the computation would thus be desirable.

We may expect that a better organization should be possible, because most of the calculation — computation of the perturbative matrix elements, applying cuts, clustering, and the jet algorithm — doesn’t involve the parton densities in an essential way. I now give an outline of such a re-organization. It bears certain similarities to a formalism proposed by Gradenz, Hampel, Vogt, and Berger [15] for jet production in deeply-inelastic scattering, but is fully general and free of certain limitations present in that approach.
I shall explain the formalism in the context of a toy problem, leading-order glueball-glueball scattering in quarkless QCD. The same formalism, with an appropriate sprinkling of indices and division into the different contributions that arise at NLO, carries over to NLO fitting of both DIS and hadron-hadron collider differential cross sections. The $n$-jet cross section in glueball-glueball scattering, subject to experimental cuts, is given by

$$\sigma_n = \int_0^1 \int_0^1 dx_1 dx_2 \int d\text{LIPS}(x_1 k_G + x_2 k'_G \rightarrow \{k_i\}_{i=1}^n)$$

$$\times f_{g \rightarrow G}(x_1, \mu_F^2(\{k_i\}, x_{1,2}))) f_{g \rightarrow G}(x_2, \mu_F^2(\{k_i\}, x_{1,2})))$$

$$\times \alpha_s^n(\mu_R^2(\{k_i\}, x_{1,2}))) \hat{\sigma}(gg \rightarrow \{k_i\}) J_{n \leftarrow n}(\{k_i\})$$

where LIPS stands for the Lorentz-invariant phase-space measure, $f_{g \rightarrow G}$ is the gluon distribution inside the glueball, and $\hat{\sigma}$ stands for the usual leading-order partonic differential cross section with the running coupling $\alpha_s$ set to 1. Note that the $k_i$ are implicitly dependent on $x_1$ and $x_2$ as well. The renormalization and factorization scales $\mu_R$ and $\mu_F$ — typically something like a jet $E_T$ — also depend on $x_{1,2}$ and the final-state momenta. The jet algorithm is represented by $J_{n \leftarrow n}$, which evaluates to 1 if the original $n$-parton configuration yields $n$ jets satisfying the experimental cuts, and 0 otherwise.

It is easiest to write down the Mellin transform of the solutions to the evolution equation (1), using a universal evolution operator [16],

$$f^z_{g \rightarrow G}(Q^2) = E^z(\alpha_s(Q^2), \alpha_0) f^z_{g \rightarrow G}(Q_0^2),$$

where $z$ is the conjugate variable to $x$. An inverse Mellin transform gives us back the parton distribution itself,

$$f_{g \rightarrow G}(x, Q^2) = \frac{1}{2\pi i} \int_C dz \ x^{-z} E^z(\alpha_s(Q^2), \alpha_0) f^z_{g \rightarrow G}(Q_0^2),$$

where the contour $C$ lies to the right of all singularities in $f$.

All the parameters (except $\alpha_0$) that we wish to fit are contained in $f^z_{g \rightarrow G}(Q_0^2)$. This function is independent of all integration variables except $z$, and thus can be pulled out of the numerical integrations in eqn. (3). The remaining $z_{1,2}$ contour integrals are to be performed during the fitting procedure, but this reduces to just a double sum of the gluon distribution function multiplied by precomputed numerical coefficients, and is vastly less time-consuming than the evaluation of the cross-section (3). That is, each step of a fitting procedure requires computing,

$$-\frac{1}{4\pi^2} \int_{c-i\infty}^{c+i\infty} dz_1 \int_{c-i\infty}^{c+i\infty} dz_2 \ f^z_{g \rightarrow G}(Q_0^2) f^z_{g \rightarrow G}(Q_0^2) \Sigma^{z_1, z_2},$$

where $\Sigma^{z_1, z_2}$ are precomputed coefficients given by
\[\Sigma^{z_1,z_2} = \int_0^1 \int_0^1 dx_1 dx_2 \int d\text{LIPS}(x_1 k_G + x_2 k'_G \to \{k_i\}_{i=1}^n) x_1^{-z_1} x_2^{-z_2} \]
\[
\times E^{z_1}(\alpha_s(\mu_F^2(\{k_i\}, x_{1,2}), \alpha_0) E^{z_2}(\alpha_s(\mu_F^2(\{k_i\}, x_{1,2}), \alpha_0) \]
\[
\times \alpha_s^n(\mu_F^2(\{k_i\}, x_{1,2})) \sigma(gg \to \{k_i\}) J_{n \to n}(\{k_i\}). \]

We see that \(\Sigma\) is a ‘cross section’ computed with \(x^{-z} E^z\) replacing the usual parton distribution functions.

To fit the remaining parameter, \(\alpha_0\), the running coupling at \(Q_0^2\), we can proceed as follows. Generate the coefficients \(\Sigma^{z_1,z_2}\) for a set of \(\alpha_0\) around a “canonical” value (e.g. \(\alpha_s(M_Z^2) = 0.118\)), and then fit using interpolation. While this approach would be vastly too time-consuming for a large number of parameters, it is acceptable for a lone parameter.

As mentioned above, a similar decomposition works for differential cross sections in full QCD.

The contour integrals in eqn. (6) must be performed numerically. We must first choose the contours, and then a set of points along each contour; the integral is then approximated, as usual, by a sum of the integrand evaluated at these points. The contour in eqn. (5) can be deformed freely into the left half-plane for \(z \to \infty\), since all the singularities of the integrand are along the real axis; we can choose an efficient quadratic contour using the same ideas as described in ref. [18] for evolving parton distributions.

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