Measurement time in double quantum dots

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Abstract

In this letter, we have considered an electron in a double quantum dot system interacting with a detector represented by a point contact. We present a dynamical model for the gradual decoherence of the density matrix due to the interaction with the detector. The interaction of the qubit (quantum system) on the quantum point contact (environment) leads to a discrete set of pointer states of the double quantum dot (apparatus). The necessary time for the qubit decoherence process (measurement time) has been calculated through this model. The existence of a minimum time for the quantum measurement has been obtained.

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The problem of understanding whether a measurement process can be analyzed within the quantum mechanical formalism has long been a difficult unresolved issue in the foundations of quantum mechanics \[1\]. On one hand, the quantum theory states that the vector corresponding to a physical system undergoes a continuous evolution governed by Schrödinger equation; on the other hand, the theory prescribes a sudden jump motion to the state of a physical system undergoing a measurement by an external device. Von Neumann’s projection rules \[2\] are indeed to be added to the quantum formalism in order to account for the transition from a pure to a mixed state (the so-called wave function collapse), and this makes quantum mechanics a non-self-contained theory.

The renewed interest in the measurement problem is justified by the development of mesoscopic systems sensitive to the phase of the electronic wave function. Recent proposals suggested using mesoscopic devices, such a Josephson junctions or coupled quantum dots, as quantum bits (qubits), which are the basic elements of quantum computers \[3\]. Among various modern approaches to the measurement problem in mesoscopic structures let us mention the idea of replacing the collapse postulate by the gradual decoherence of the density matrix due to the interaction with the detector \[4\]. Decoherence is the emergence of classical features of a quantum system resulting from its interaction with the environment. Zurek has proposed that the interaction of the quantum system on the environment leads to a preferred and discrete set of quantum states (pointer states), which remain robust. This environment selection of the preferred pointer states was termed einselection \[5\]. In the simplest models that have been used up to now to study decoherence, such pointer states are eigenstates of the pointer observable which commutes with the system-environment interaction Hamiltonian. These interaction Hamiltonians are generally based on spin-atom interaction and such a concept can be generalized using the predictability sieve \[6\].
Solid-state quantum devices have been demonstrated as promising systems for quantum computation [7]. Recently, Gurvitz et. al. [8] have considered a qubit interacting with its environment and continuously monitored by a detector represented by a point contact. In such a case, the decoherence rate $\Gamma_d$ due to interaction with the detector is inversely proportional to the measurement time $\Delta t$. For strong coupling to the detector, i.e., $\Gamma_d \to \infty$ and $\Delta t \to 0$, the measurement is idealized to be instantaneous. Accordingly, the electron in the coupled quantum dot system instantaneously makes the transition $|\psi_i > \rightarrow |\psi_f >$ by measurement (the so-called quantum-jump [9]). Controlled decoherence of electrons has been studied experimentally in recent years [10, 11]. Ferry et. al. [11] have shown that the conductance oscillations exhibited by open quantum dots are governed by a discrete set of stable quantum states which have the properties of the pointer states.

An account of decoherence of a double quantum dot (a qubit), interacting with a measurement device, has become a problem of crucial importance in quantum computing. To build up a quantum computer using quantum dots, it is necessary to know the decoherence time for a qubit when a measurement takes place. [12] This is the aim of the present work. In this letter, we present a dynamical model for the gradual decoherence of the density matrix due to the interaction with the measurement device. In the model, the interaction of the electron (qubit) on the environment (quantum point contact) leads to a discrete set of pointer states, which commute with the system-environment $H_{\text{int}}$ Hamiltonian. The measurement times will be calculated through this model.

Let us now consider electrostatic quantum bit measurements (Fig. 1) in double quantum dots [13]. The qubit is a single electron in a double quantum dot and the detector is a point contact placed near one of the dots [14]. We have also considered a one-dimensional model (quantum wire) for the quantum point contact. If the electron occupies the first
dot, the transmission coefficient of the point contact decreases due to electrostatic repulsion generated by the electron (Fig. 2). Thus, the electron position is monitored by the tunneling current. The qubit can be described by the Hamiltonian $H = H_{DQD} + H_{QPC} + H_{int}$, where

$$H_{DQD} = \varepsilon_1 |A_1 \rangle \langle A_1| + \varepsilon_2 |A_2 \rangle \langle A_2|,$$

(1)

is the double quantum dot Hamiltonian and $|A_{1,2} \rangle$ are the double dot eigenstates (Fig. 2). The term $H_{QPC}$ describe the quantum point contact detector

$$H_{QPC} = \sum_l \varepsilon_l |\psi_l \rangle \langle \psi_l| + \sum_r \varepsilon_r |\psi_r \rangle \langle \psi_r| + \sum_{l,r} t_{lr} (|\psi_l \rangle \langle \psi_r| + |\psi_r \rangle \langle \psi_l|),$$

(2)

where $|\psi_l \rangle$ and $|\psi_r \rangle$ are electron states in the left and right reservoirs, respectively, and $t_{lr}$ is the hopping amplitude between the states $|\psi_l \rangle$ and $|\psi_r \rangle$. The $H_{int}$ term is the interaction with the qubit, i.e.,

$$H_{int} = |A_1 \rangle \langle A_1| \otimes \sum_{l,r} \delta t_{lr} (|\psi_l \rangle \langle \psi_r| + |\psi_r \rangle \langle \psi_l|).$$

(3)

The interaction term generates a change in the hopping amplitude $\delta t_{lr} = t_{lr}' - t_{lr}$ (Fig. 2). We should point out that the interaction term is between each eigenstate in the quantum point contact and the electron in the double quantum dot. Thus the detector current is $I'$ (due to the hopping term $t_{lr}'$) when the electron occupies the first dot state $|A_1 \rangle$, and $I$ (due to $t_{lr}$) when the electron occupies the second dot state $|A_2 \rangle$ (Fig. 2). For simplicity we have considered electrons as spinless fermions and a constant electric field in the double quantum dot system that is generated by the electrons in the quantum point contact. In addition to this, we have neglected the effects of the applied bias voltage and the electron-electron interaction in the quantum point contact.

In order to solve $H_{QPC}$, we have chosen a confining (infinite) potential in the left and right extremes of the point contact (Fig. 2). In such a case, the eigenstates for a particle...
which is moving in a symmetric double quantum well can be written \[15\] as

\[ \varepsilon_n = \varepsilon_n^{qw} \pm \delta \varepsilon_n, \] (4)

where \( \varepsilon_n^{qw} \) is the quantum well eigenvalue and \( \delta \varepsilon_n \) is given by \[15\]

\[ \delta \varepsilon_n \cong 2\varepsilon_n^{qw} \sqrt{2m^*(V - \varepsilon_n^{qw})} \frac{V}{m^*(l_a + l_b/2)} \exp \left[ -\frac{l_b}{\hbar} \sqrt{2m^*(V - \varepsilon_n^{qw})} \right], \] (5)

being \( m^* \) the electron effective mass, \( V \) the barrier potential and \( l_a \) and \( l_b \) the quantum well width and barrier width in the quantum point contact, respectively (Fig. 2). The \( \delta \varepsilon_n \) term splits the energy levels of the quantum well. In obtaining the approximate solution of Eq. (5) we have assumed that the width of the well \( l_a \) is large \[15\], i.e., \( l_a >> l_b \) in our case.

To leave pointer states in the detector untouched, the interaction Hamiltonian should be a function of the pointer observable \( \tilde{A} \) of the apparatus (double quantum dot). We can notice that \( [H_{int}, \tilde{A}] = 0 \) in our model. As a result, pointer states are easy to characterized and are the \(|A_{1,2} >\) eigenstates of the double quantum dot. Now we introduce the environment, represented by a boson bath at zero temperature and interacting with the electron. First, consider the case in which the electron wave function \(|\phi >\) is not coupled to the detector, i.e., \(|\phi > = |A_2 >\). Then, \( H_{int} = 0 \) and the barrier potential takes the lowest value \( V_2 \) (Fig. 2). Now let us consider the interaction with the detector. In such a case, \(|\phi > = |A_1 >\) and the barrier potential takes the highest value \( V_1 \). The eigenstates \(|\psi_{n}^{\pm} >\) and eigenvalues \( \varepsilon_n^{\pm} \) in the electron reservoir can be obtained by solving \( H_{QPC} \) at different barrier heights \( V_1 \) and \( V_2 \) through Eq. (5). In our case, the energy level \( \varepsilon_n^{+} (\varepsilon_n^{-}) \) in the quantum point contact will correspond to \( V_1 \) (\( V_2 \)). We note that the interaction Hamiltonian can be easily diagonalized. In our model, both \(|A_{1,2} >\) and \(|\psi_{n}^{\pm} >\) are stationary states in the double quantum dot and the quantum point contact, respectively. When the electron in the double dot is in the stationary state \(|\phi > = |A_1 >\) (\(|\phi > = |A_2 >\)), each electron in the quantum
point contact occupies the stationary state $|\psi^+_n> (|\psi^-_n>)$. We notice that our Hamiltonian doesn’t depend explicitly on time. In such a case, the initial state of the total hamiltonian

$$|\Phi(0)> = a|A_1> \left( \prod_{n} |\psi^+_n> \right) + b|A_2> \left( \prod_{n} |\psi^-_n> \right), \quad (6)$$

where $a$ and $b$ are two normalization constants, evolves into

$$|\Phi(t)> = ae^{-\frac{i}{\hbar}(\epsilon_1t)}|A_1> \left( \prod_{n} e^{-\frac{i}{\hbar}(\epsilon_1t)}|\psi^+_n> \right) + be^{-\frac{i}{\hbar}(\epsilon_2t)}|A_2> \left( \prod_{n} e^{-\frac{i}{\hbar}(\epsilon_2t)}|\psi^-_n> \right), \quad (7)$$

and the reduced density matrix for the apparatus is

$$\rho_A = |a|^2|A_1><A_1| + ab^*r(t)|A_1><A_2| + a^*b r^*(t)|A_2><A_1| + |b|^2|A_2><A_2|, \quad (8)$$

being

$$r(t) = e^{-\frac{i}{\hbar}(\epsilon_1-\epsilon_2)t} \prod_{n} e^{-\frac{i}{\hbar}(\epsilon_1-\epsilon_2)t} <\psi^-_n|\psi^+_n> \quad (9)$$

a coefficient that determines the relative size of the off-diagonal terms. We note that if each electron in the quantum point contact occupies an specific $a|\psi^+_n> + b|\psi^-_n>$ state in energy space, the off-diagonal terms $r(t)$ will not vanish as time progresses.

We shall now consider that each electron in the quantum point contact will spread out its wave function towards a higher state due to thermal effects. For this purpose, we consider a wave packet which is localized in energy space around a quantum number $\bar{n}$, that is, a coefficient $\epsilon$ assumes only significant values in a small vicinity of $n = \bar{n}$. The approximation is valid for a narrow energy distribution of the wave packet. Only nearest neighbor states will be considered to simplify. In such a case, the initial wave function is

$$|\Phi(0)> = a|A_1> \left( \prod_{n=1}^{N-1} [(1 - \epsilon)|\psi^+_n> + \epsilon|\psi^+_{n+1}> \right) + b|A_2> \left( \prod_{n=1}^{N-1} [(1 - \epsilon)|\psi^-_n> + \epsilon|\psi^-_{n+1}> \right), \quad (10)$$
where $\epsilon$ is a small quantity $|\epsilon| << 1$, that is also normalized, $|1 - \epsilon|^2 + |\epsilon|^2 = 1$, $N$ is the number of electrons in the double point contact and $r(t)$ is

$$r(t) = e^{-\frac{\epsilon}{\hbar}(\epsilon_1 - \epsilon_2)t} \prod_{n=1}^{N-1} \left[ |1 - \epsilon|^2 e^{-\frac{1}{\hbar}(\epsilon_n^+ - \epsilon_n^-)t} + |\epsilon|^2 e^{-\frac{1}{\hbar}(\epsilon_{n+1}^+ - \epsilon_{n+1}^-)t} \right].$$

(11)

To simplify our calculations, we have considered that the wave function forms in the double point contact are slightly modified due to the barrier potential variation. Then, we have approached $<\psi_n^-|\psi_n^+> \sim 1$, $<\psi_{n+1}^-|\psi_{n+1}^+> \sim 1$, $<\psi_n^-|\psi_{n+1}^+> \sim 0$ and $<\psi_{n+1}^-|\psi_n^+> \sim 0$.

For large environments consisting of many $N$ electrons at large times the off-diagonal terms will now vanish. The results have been checked for more than one nearest neighbor state (and for different $\epsilon$ values in the $|\psi_n^+>$ and $|\psi_n^->$ states). In such cases, $r(t)$ vanished faster.

In the quantum point contact, we have considered a GaAs/Ga$_{1-x}$Al$_x$As double quantum well system which consists of two $l_a = 5000\text{Å}$ wide GaAs quantum wells separated by a barrier of thickness $l_b = 100\text{Å}$ (Fig. 2). The barrier height and electron effective mass are taken to be $V_2 = 220\text{meV}$ and $0.067m_0$, respectively [16]. The $V_1$ value has been obtained considering the Coulomb potential generated by a single electron in the $A_1$ quantum well. The distance between the $A_1$ quantum well and the barrier is taken to be $100\text{Å}$. In such a case, $V_2 - V_1 \sim 12\text{meV}$. Fig. 3 and 4 show $z(t) = |r(t)|^2$ versus time at different $N$ and $\epsilon$ values, respectively. It is found that the magnitude of the off-diagonal terms decreases exponentially fast, with the physical size $N$ of the environment effectively coupled to the state of the double quantum dot. As a result, the qubit interaction on the environment leads to two different pointer states $\{|A_1>, |A_2>\}$ of the double quantum dot, which remain robust in the einselection approach. We note that the effectiveness of einselection depends on the initial state of the environment, $\epsilon$. When the environment is in an eigenstate of $H_{int}$, the coherence in the system will be retained ($\alpha = |\epsilon|^2 = 0.001 \sim 0$ in Fig. 4). This special
environment state is, however, unlikely in realistic circumstances.

Finally, we think that it could be possible to monitoring the decoherence process. If the number of electrons in the point contact electrodes is not large enough, the electron wave function in the double quantum dot will not collapse. The $|\epsilon|^2$ parameter could be controlled by decreasing the temperature. In such a case, the quantum measurement will not be obtained. The number of injected electrons in the quantum point contact could be controlled by external contacts. From Fig. 3, we note that it is easy to define a measurement time. The $|r(t)|^2$ term decreases exponentially fast with the number of electrons $N$. If $|r(t)|^2$ takes a value such $|r(t)|^2 < 0.1$ when $t > \tau$, we can define a measurement time $\tau$. In such a case, we have a 0.90 probability of wave function collapse. Fig. 5 shows measurement time versus $N$. The $\tau$ value decreases exponentially with the number of electrons and finally reaches a near constant value, $\tau \sim 175\text{ps}$. It is found the existence of a minimum time for the measurement process at a fixed $\epsilon$ value. If an oscillating current is applied to the electrodes, the measurement process will happen only if the current oscillation period is higher than the measurement time. In this way, it could be also possible to monitoring the wave function decoherence using an external oscillating current.

In summary, in this letter we have considered a qubit (electron) in a double quantum dot interacting with a measurement device. We present a dynamical model for the gradual decoherence of the density matrix due to the interaction with the quantum point contact. The interaction of the qubit on the quantum point contact environment leads to a discrete set of pointer states of the double quantum dot. The necessary time for the measurement has been calculated through this model. It is found the existence of a minimum time for the qubit decoherence process due to the interaction with the detector.
[1] Quantum theory and measurement, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, NJ, 1983.)

[2] J. Von Neumann, Die Mathematische Grundlagen der Quantenmechanik (Springer-Verlag, Berlin, 1932).

[3] A. Shnirman and G. Schön, Phys. Rev. B 57, 15400 (1998).

[4] W. H. Zurek, Rev. Mod. Phys. 75, 715 (2003).

[5] W. H. Zurek, Phys. Rev. D 24, 1516 (1981); Phys. Rev. D 26, 1862 (1982).

[6] M. Schlosshauer, Decoherence and the quantum-to-classical transition (Springer-Verlag, Berlin, 2007.)

[7] L. Tian, Phys. Rev. Lett. 98, 153602 (2007).

[8] S. A. Gurvitz, L. Fedichkin, D. Mozyrsky, and G. P. Berman, Phys. Rev. Lett. 91, 066801 (2003); S. A. Gurvitz, Phys. Rev. B 56, 15215 (1997).

[9] H. S. Goan, G. J. Milburn, H. M. Wiseman and H. B. Sun, Phys. Rev. B 63, 125326 (2001).

[10] P. Sonnentag, and F. Hasselbach, Phys. Rev. Lett. 98, 200402 (2007).

[11] D. K. Ferry, R. Akis, and J. P. Bird, Phys. Rev. Lett. 93, 026803 (2004).

[12] T. Gilad and S. A. Gurvitz, Phys. Rev. Lett. 97, 116808 (2006).

[13] D. Sprinzak, E. Buks, M. Heiblum and H. Shtrikman, Phys. Rev. Lett. 84, 5820 (2000).

[14] S. Pilgram and M. Büttiker, Phys. Rev. Lett. 89, 200401 (2002).

[15] M. Razavy, Quantum theory of tunneling (World Scientific, Singapore, 2003), p. 515.

[16] H. Cruz, Phys. Rev. B 65, 245313 (2002).
I. FIGURES AND TABLES

• **FIG. 1** A schematic illustration of the measurement device. The point contact detector with two separate reservoirs near the double quantum dot. The left and right electrodes are filled up with electrons in the $|\psi_l>$ and $|\psi_r>$ states, respectively. the qubit is an electron in the double quantum dot system, $|\phi> = a|A_1> + b|A_2>$.  

• **FIG. 2** A schematic illustration of the measurement device. The $x$ axis is set longitudinally. A point contact detector monitoring the electron position in the double quantum dot. If the electron wave function is not coupled to the detector, $|\phi> = |A_2>$, the barrier potential takes the $V_2$ value and $\varepsilon_n^-$ are the electron eigenvalues in the reservoir. When $|\phi> = |A_1>$, the barrier potential takes the highest value $V_1$ and $\varepsilon_n^+$ the electrons in the quantum point contact.  

• **FIG. 3** $z(t)$ versus time at different number of electrons $N$ values. The $z$ term is $z(t) = |r(t)|^2$ and $\alpha = |\epsilon|^2 = 0.1$.  

• **FIG. 4** $z(t)$ versus time at different $\alpha = |\epsilon|^2$ values. The $z$ term is $z(t) = |r(t)|^2$ and $N=25$.  

• **FIG. 5** Measurement time versus number of electrons $N$ at $\alpha = |\epsilon|^2 = 0.1$.  

