Leptogenesis with supersymmetric Higgs triplets in the TeV region

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The leptogenesis with supersymmetric Higgs triplets is studied in the light of experimental verification in the TeV region. The lepton number asymmetry appears just after the inflation via multiscalar coherent evolution of Higgs triplets and antilepton on a flat manifold. If the Higgs triplet mass terms dominate over the negative thermal-log term for the Hubble parameter $H$ comparable to the Higgs triplet mass $M_\Delta$, the asymmetry is fixed readily to some significant value by the redshift and rotation of these scalar fields, providing the sufficient lepton-to-entropy ratio $n_L/s \sim 10^{-10}$. This can be the case even with $M_\Delta \sim 1$ TeV for the reheating temperature $T_R \sim 10^9$ GeV and the mass parameter $M/\lambda \sim 10^{2-3}$ GeV of the nonrenormalizable superpotential terms relevant for leptogenesis.

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I. INTRODUCTION

The baryogenesis is a very important subject in particle physics and cosmology. In most of the baryogenesis scenarios including those of leptogenesis, however, the participating particles are supposed to be extremely heavy, and hence it seems impossible to verify them experimentally. The electroweak baryogenesis might be promising in this respect, though it is realized in a rather restricted situation within the minimal supersymmetric standard model.

In this paper we study the leptogenesis with supersymmetric Higgs triplets in the light of experimental verification in the TeV region. It is indeed interesting that the neutrino masses may be generated naturally by the exchange of Higgs triplet. In particular, phenomenological implications of the Higgs triplets with mass $M_\Delta \sim 100$ GeV – 1 TeV, such as lepton flavor violating processes, have been investigated recently, which are intimately related to the neutrino masses and mixings $\beta\beta$. It is hence attractive to study the possibility of leptogenesis with Higgs triplets in the TeV region. In this respect we note that in the supersymmetric Higgs triplet model the leptogenesis can be realized after the inflation via multiscalar coherent evolution on a flat manifold of a pair of Higgs triplets $\Delta$, $\bar{\Delta}$ and the antilepton $\tilde{e}^c$ in the manner of Affleck and Dine. The Higgs triplet mass was originally supposed to be very large as $M_\Delta \sim 10^9 - 10^{14}$ GeV, so that it provides a strong driving force for the rotation of scalar fields to fix the lepton number asymmetry to some significant value. We here reexamine this scenario to show that the successful leptogenesis can be obtained even with $M_\Delta \sim 1$ TeV, where the asymmetry fixing becomes a more important issue due to the effects of thermal terms.

This paper is organized as follows. In Sec. II we present the supersymmetric Higgs triplet model, and describe the neutrino mass generation. In Sec. III we recapitulate the essential aspects of the generation of lepton number asymmetry after the inflation via coherent evolution of $\Delta$, $\bar{\Delta}$, $\tilde{e}^c$ on the flat manifold. In Sec. IV we examine the completion of leptogenesis to show that the sufficient lepton-to-entropy ratio $n_L/s \sim 10^{-10}$ can be obtained even for the case of phenomenologically interesting Higgs triplets with $M_\Delta \sim 1$ TeV. The thermal terms provide significant effects on the evolution of $\Delta$, $\bar{\Delta}$, $\tilde{e}^c$ and the lepton number asymmetry for the Hubble parameter $H < M_\Delta$. Section V is devoted to the summary.

II. MODEL

We investigate an extension of the minimal supersymmetric standard model by introducing a pair of Higgs triplet superfields,

$$\Delta = \begin{pmatrix} \Delta^++/\sqrt{2} \\ \Delta^0 \\ -\Delta^+/-\sqrt{2} \end{pmatrix}$$

(1)

$$\bar{\Delta} = \begin{pmatrix} \bar{\Delta}^-/\sqrt{2} \\ \bar{\Delta}^0 \\ -\bar{\Delta}^-/-\sqrt{2} \end{pmatrix}.$$ 

(2)

The lepton doublets $L_i = (\nu_i, l_i)$, anti-lepton singlets $\bar{l}_i$ ($i = 1, 2, 3$) and the Higgs doublets $H_u$, $H_d$ are given as usual. The generic lepton-number conserving superpotential for the leptons and Higgs fields is given by

$$W_0 = h_{ij} L_i H_u \bar{l}_j + \mu H_u H_d + \frac{1}{\sqrt{2}} f_{ij} L_i \Delta L_j + M_\Delta \bar{\Delta} \Delta.$$ 

(3)

The lepton numbers are assigned to the Higgs triplets as

$$Q_L(\Delta) = -2, \quad Q_L(\bar{\Delta}) = 2.$$ 

(4)

The lepton-number violating terms may also be included in the superpotential as

$$W_{\text{LV}} = \xi_1 \bar{\Delta} H_u H_u + \xi_2 \Delta H_d H_d.$$ 

(5)
These terms are $R$-parity conserving by assigning the Higgs triplets to be $R$-parity even. We do not consider $R$-parity violating terms for definiteness.

The Higgs triplets develop nonzero vacuum expectation values (VEV’s) due to the effects of $W_{1\nu}$ as

$$
(A^0) = -c_1 \frac{\xi_1 (H_u)^2}{M_\Delta},
$$

$$
(\bar{A}) = -c_2 \frac{\xi_2 (H_d)^2}{M_\Delta}.
$$

The factors $c_1, c_2 \sim 1$ with $\xi_1 \approx \xi_2$ are determined precisely by minimizing the scalar potential including the soft supersymmetry breaking terms with the mass scale $m_0 \sim 10^{3}\text{GeV}$ ($c_1 = c_2 = 1$ in the supersymmetric limit of $m_0 \to 0$ with the vanishing $F$ terms). The neutrino mass matrix is then generated by the VEV of the Higgs triplet

$$
M_\nu = f \sqrt{2}(A^0).
$$

This neutrino mass matrix should reproduce the masses $m_i$ and mixing angles $\theta_{ui}$ inferred from the data of neutrino experiments [13, 14, 15, 16]. The constraint on the magnitude of $f$ coupling is then placed from $m_i \lesssim 10^{-1}\text{eV}$ roughly as

$$
|f| \lesssim 10^{-1} \frac{\xi}{10^{-10}} \left(\frac{M_\Delta}{10^{6}\text{GeV}}\right).
$$

This constraint, however, does not seem so stringent, allowing even $|f| \sim 1$ and $M_\Delta \sim 1\text{TeV}$ with small enough $(A^0)$. Then, the interesting phenomenology of lepton flavor violation are provided intimately related to the neutrino masses and mixings [2, 3, 4, 5, 6].

The generation of the very small VEV’s of the Higgs triplets in Eq. (6) for the neutrino masses has been described in the literature in terms of the trilinear couplings of Higgs doublets and triplets, as given in Eq. (6), to break explicitly the lepton-number conservation [1]. Specifically, for the Higgs triplets in the TeV region with $M_\Delta \sim 1\text{TeV}$, the small VEV’s of the Higgs triplets are stably generated by the tiny couplings $\xi_1 \sim \xi_2 \sim \xi \sim 10^{-10}$, as given in Eq. (5). We recapitulate below the essential points of this feature, while it is not directly related to the present scenario of leptogenesis, which is described in the following sections.

In the absence of the trilinear couplings $\xi_1$ and $\xi_2$, the stable lepton-number conserving minimum with the vanishing VEV’s $\Delta = \bar{\Delta} = 0$ are generated as usual in a reasonable range of model parameters, since the Higgs triplet mass terms $M_\Delta^2(|\Delta|^2 + |\bar{\Delta}|^2)$ with $M_\Delta \sim 1\text{TeV}$ may dominate over the soft supersymmetry breaking terms with $m_0$. This feature is still valid even if the radiative corrections are included in the effective scalar potential. Then, by adding the small lepton-number violating terms of $W_{1\nu}$, the effective linear terms, $\xi_2 M_\Delta \Delta^* (H_u)^2$, $\xi_1 m_0 \bar{\Delta} \bar{\Delta} (H_d)^2$, etc., are provided for the Higgs triplets. The lepton-number violating part of the radiative corrections are also small, since they should be generated with the original $\xi_1$ and $\xi_2$ couplings. Accordingly the potential minimum is shifted very slightly by these terms to provide the small VEV’s of the Higgs triplets in Eq. (6) breaking the lepton-number conservation. It should be mentioned here that the VEV’s of the Higgs triplets are induced by the explicit lepton-number violation of the $\xi_1$ and $\xi_2$ couplings, rather than the spontaneous violation. This is analogous to the case of explicit $R$-parity violation with the $LH_u$ term, where the small VEV’s of sleptons are induced. The so-called triplet Majoron does not appear in the present case, and all the scalar fields in the Higgs triplets acquire masses $\sim M_\Delta$. It is also seen that the slepton fields $\tilde{l}_i, \tilde{\nu}_i$ do not develop VEV’s since the $R$-parity is not violated by the VEV’s of the Higgs triplets.

In this way, the small VEV’s of the Higgs triplets are attributed to the small lepton-number violating trilinear couplings, while keeping the Higgs triplet mass terms $M_\Delta \sim 1\text{TeV}$ dominant in the scalar potential. This is clearly in contrast with the Coleman-Weinberg type potential. In the Coleman-Weinberg case, while a very small VEV of a scalar field is obtained by the lepton-number violation with the $LH_u$ term, where the small VEV’s of sleptons are induced. The so-called triplet Majoron does not appear in the present case, and all the scalar fields in the Higgs triplets acquire masses $\sim M_\Delta$. It is also seen that the slepton fields $\tilde{l}_i, \tilde{\nu}_i$ do not develop VEV’s since the $R$-parity is not violated by the VEV’s of the Higgs triplets.

### III. GENERATION OF ASYMMETRY

We begin with recapitulating the essential aspects of the generation of lepton number asymmetry with supersymmetric Higgs triplets via multiscalar coherent evolution on a flat manifold after the inflation [3, 8]. In the following, we consider for definiteness the case that one generation of $\tilde{e}^c \equiv \tilde{e}_L$, together with $\Delta$ and $\bar{\Delta}$, participates in the leptogenesis. The essential results are even valid for the case with more than one $\tilde{e}^c$. The nonrenormalizable terms relevant for leptogenesis are given by

$$
W_{LG} = \frac{\lambda_\nu}{2M} \bar{\Delta} \bar{\Delta} e^c e^c + e^{i\phi} \frac{\lambda_\Delta}{2M} \bar{\Delta} \Delta \bar{\Delta},
$$

where $M$ represents some unification scale such as the Planck scale. These terms represent the flat directions, $\Delta \bar{\Delta} e^c e^c (Q_L = 2)$ and $\bar{\Delta} \Delta (Q_L = 0)$, respectively. Then,
if these directions are comparably flat with
\[ 0.3 \lesssim \lambda_f / \lambda_\Delta \lesssim 3, \]
the coherent evolution of the scalar fields, say AD-flatons \( \Phi \) [9, 10, 19], may take place on the complex two-dimensional flat manifold spanned by these directions, starting with large initial values after the inflation. This manifold is specified by the \( D \)-flat condition,
\[ |\Delta|^2 - |\bar{\Delta}|^2 + |\bar{\epsilon}|^2 = 0, \]
and the other fields are negligibly small.

The scalar potential for the AD-flatons is given by
\[
V = (C_1 m_0^2 - c_1 H^2)|\Delta|^2 + (C_2 m_0^2 - c_2 H^2)|\bar{\Delta}|^2 + (C_3 m_0^2 - c_3 H^2)|\bar{\epsilon}|^2
+ \left[ M_\Delta \Delta + \frac{\lambda_f}{M} \Delta \bar{\epsilon} \bar{\epsilon} + e^{i\delta} \frac{\lambda_\Delta}{M} \bar{\Delta} \Delta \bar{\Delta} \right]^2
+ \left[ M_{\bar{\Delta}} \bar{\Delta} + e^{i\delta} \frac{\lambda_\Delta}{M} \bar{\Delta} \Delta \bar{\Delta} \Delta \right] + \frac{\lambda_f}{M} \bar{\Delta} \Delta \bar{\Delta} \Delta
+ \frac{1}{2M} (a_\Delta H + A_\Delta m_0) M \bar{\Delta} \Delta + h.c.
+ \frac{1}{2M} (a_{\bar{\Delta}} H + A_{\bar{\Delta}} m_0) \lambda_f \bar{\Delta} \Delta \bar{\epsilon} \bar{\epsilon} + h.c.
+ \frac{1}{2M} (a_{\bar{\Delta}} H + A_{\bar{\Delta}} m_0) \lambda_\Delta \bar{\Delta} \Delta \bar{\Delta} \Delta + h.c.
+ \theta_0^2 (|\Delta|^2 - |\bar{\Delta}|^2 + |\bar{\epsilon}|^2)^2, \]
where the last term with the \( U(1)_Y \) gauge coupling \( g_1 \) is included to realize dynamically the \( D \)-flat condition [11]. (Henceforth \( \Delta \equiv \Delta^+ \) and \( \bar{\Delta} \equiv \Delta^- \) for simplicity.) The energy density of the universe provides the soft supersymmetry breaking terms with the Hubble parameter \( H [11] \). The AD-flatons \( \phi_a = \Delta, \bar{\Delta}, \bar{\epsilon} \) evolve in time governed by this potential \( V \). Their number asymmetries are given with the time derivatives by
\[ n_a = i \left( \phi_a^* \dot{\phi}_a - \dot{\phi}_a^* \phi_a \right) = \dot{\theta}_a |\phi_a|^2, \]
where
\[ \dot{\theta}_a(t) \equiv e^{i \theta_a(t)} |\phi_a(t)|. \]
Then, the lepton number asymmetry is evaluated by
\[ n_L = 2n_\Delta - 2n_{\bar{\Delta}} - n_{\bar{\epsilon}}, \]
where
\[ |\phi_a| \sim (M/\lambda)^{1/2} H_{\text{inf}}^{1/2}, \]
where \( \lambda \) represents the mean value of \( \lambda_f \) and \( \lambda_\Delta \). The phases \( \theta_a \) of AD-flatons are fixed with constant \( H_{\text{inf}} \), and the number asymmetries \( n_a \) are vanishing. After the end of inflation, the AD-flatons evolve with the decreasing Hubble parameter \( H = (2/3)t^{-1} \) in the matter-dominated universe as
\[ |\phi_a| \sim \bar{\delta} \equiv (M/\lambda)^{1/2} H^{1/2}. \]
Then, their phases \( \theta_a(t) \) begin to vary slowly in time as \[ |d\theta_a/d\ln t| \sim |\dot{\theta}_a|/H \lesssim 1, \]
so that the balance among the phase-dependent terms in the scalar potential, \( \lambda_f \) - \( \lambda_\Delta \) cross term, \( a_f \) term and \( a_\Delta \) term, is changing with the decreasing Hubble parameter \( H \). This causes the gradual fluctuation in \( \ln t \) of the fractions of number asymmetries, \( \epsilon_a(t) \equiv n_a(t)/[(M/\lambda)H^2] \), which is not expected in the usual Affleck-Dine mechanism along the one-dimensional flat direction. Accordingly, even in this very early epoch the lepton number asymmetry really appears by this phase fluctuation of the AD-flatons on the flat manifold. Numerically, we have
\[ |\epsilon_L(t)| \lesssim 0.1 - 1, \]
represents the fraction of lepton number asymmetry.

IV. COMPLETION OF LEPTOGENESIS

For the completion of leptogenesis, the lepton number asymmetry should be fixed to some significant value by the rapid redshift and oscillation of the lepton-number violating terms. In the original scenario [8], it is found that the asymmetry is fixed readily in the early epoch due to the effect of very large Higgs triplet mass terms with \( M_\Delta \gtrsim 10^3 \) GeV, which dominate fairly over the thermal terms [11]. On the other hand, for the phenomenologically interesting case of \( M_\Delta \sim 1 \) TeV, the problem of asymmetry fixing becomes a more important issue due to the effects of thermal terms. We henceforth examine the case of \( M_\Delta \sim m_\nu \sim 1 \) TeV for the completion of leptogenesis.

The condensates of \( \Delta \) and \( \bar{\epsilon} \) (more precisely \( \Delta^+ \) and \( \bar{\epsilon}_1 \), respectively) provide the effective superpotential mass terms for \( L_i \) and \( H_d \) from the superpotential \( W_0 \) in Eq. [4] as
\[ f_{i1} \nu_i \Delta + h_{i1} l_i H_d^0 \bar{\epsilon}_i \bar{\epsilon}_i - h_{i1} H_d^0 \nu_i \bar{\epsilon}_i, \]
where \( l_i, \nu_i, H_d^0 \) and \( H_d^- \) are chiral superfields. The lepton doublet basis is taken with the diagonal \( f_{i1} = f_{1i} \delta_{ij} \) \( (0 < f_1 < f_2 < f_3) \). The \( H \) couplings may be estimated as \( |h_{i1}| \sim m_\nu/(H_d^0) \) for the almost bi-maximal mixing of neutrinos. We hence consider for definiteness the case
\[ f_3 > f_2 \gtrsim 0.05 > f_1 \sim |h_{i1}| \sim 0.01, \]
which will be interesting phenomenologically for the lepton flavor violation with the \( f \) couplings [2, 3, 4, 5].
In this case with the large $|\phi_a|$ in Eq. (17), the lepton doublets $L_2$ and $L_3$ acquire the masses mainly with the $f_2$ and $f_3$ couplings, respectively, while $L_1$ and $H_d$ form a $2 \times 2$ mass matrix with the $f_1$ and $h_{11}$ couplings in a good approximation. Then, for a long period after the inflation the lepton doublets $L_i$, the Higgs doublet $H_d$ and the gauge bosons $W^\pm$ of $SU(2)_W/U(1)_Y$ and $B$ of $U(1)_Y$ are heavy enough to decouple from the dilute thermal plasma of the inflaton decay products. The plasma temperature before the reheating epoch of $H = H_R$ is given by

$$T_p \sim (T_\text{TH}^2 H M_P)^{1/4} \left( H_R < H < H_{\text{int}} \right),$$

where $M_P = 2.4 \times 10^{18} \text{GeV}$ is the reduced Planck mass. The reheating temperature is constrained as $T_R \lesssim 10^{10} \text{GeV}$ to avoid the overproduction of gravitinos with mass $m_\chi \sim m_0 \sim 1 \text{TeV}$.

In this situation, the thermal-log terms appear through the modification of the gauge coupling $g_2$ of unbroken $U(1)_Y$, due to the decoupling of $W^{\pm}$, $L_i$ and $H_d$:

$$V_{\text{thlog}} = a_2 \alpha_2^2 T_p^4 \ln \left[ (|\Delta|^2 + |\bar{\Delta}^2|)/T_p^2 \right] + a_2 \alpha_2^2 T_p^4 \ln \left[ (|\Delta|^2 + \bar{\Delta}^2)/T_p^2 \right] + a_2 \alpha_2^2 T_p^4 \ln \left[ (\bar{e}^2 + e^2)/T_p^2 \right],$$

where

$$a_2 = -6(27/64), \quad a_L = 27/64, \quad a_H = 27/64,$$

and $g_2^2/4\pi \approx 1/30$. The leading contribution to the gauge coupling dependent part of the free energy is calculated as $F = (27/64)g_2^2 T_p^4$ in the supersymmetric $U(1)_Y$ gauge theory, by using the formula in the literature [20] for the chiral superfields of quarks, Higgs doublet ($H_u$) and Higgs triplets. It should be noted here that the thermal-log terms $V_{\text{thlog}}$ act in total as negative one ($a_2 + a_L + a_H < 0$), providing a significant effect on the evolution of AD-flatons for $H < M_\Delta$.

On the other hand, with the decreasing $|\phi_a|$ the lepton doublet $L_1$ and Higgs doublet $H_d$ enter the thermal plasma in a later epoch satisfying the condition $f_1|\Delta|, f_{11}|\bar{\Delta}| < T_p$. Then, $\Delta$ and $\bar{\Delta}$ acquire the thermal mass terms through the couplings with $L_1$ and $H_d$ as

$$V_{\text{thm}} = \frac{1}{4} f_1^2 T_p^2 |\Delta|^2 + \frac{1}{2} f_{11}^2 T_p^2 |\bar{\Delta}|^2.$$  

(25)

The heavier lepton doublets $L_2$ and $L_3$ may also enter the thermal plasma with smaller $|\Delta|$, providing the similar mass terms. These thermal mass terms may also make some effects on the evolution of AD-flatons for $H < M_\Delta$.

### A. Case of $M_\Delta > H_{\text{th}}$

The AD-flatons are scaled as $\phi_a \propto H^{1/2}$ for some period after the inflation, as seen in Eq. (17). Then, the Higgs triplet mass terms and the thermal-log terms, scaled as $H$, eventually become important for the dynamics of AD-flatons. As for the thermal mass terms $V_{\text{thm}}$ with the $f$ and $h$ couplings in Eq. (21), they appear really after the thermal-log terms $V_{\text{thlog}}$ become dominant, as will be seen later. We here consider specifically the case that the Higgs triplet mass terms first dominate over the Hubble induced mass terms for $H \sim M_\Delta \sim m_0$ under the condition

$$M_\Delta > H_{\text{th}} \sim \sqrt{a_\Delta T_R [M_P/(M/\lambda)]^{1/2}}$$

with $a = |a_\gamma + a_L + a_H| = 27/16$. Here, the Hubble parameter $H_{\text{th}}$ is given by the condition $H^2|\phi_a|^2 \sim |V_{\text{thlog}}|$ with Eqs. (17) and (22), for which the thermal-log terms would become comparable to the Hubble induced mass terms if the mass terms with $M_\Delta \sim m_0$ were subdominant. It is estimated as

$$H_{\text{th}} \sim \frac{T_R}{10^9 \text{GeV}} \left( \frac{M/\lambda}{10^{13} \text{GeV}} \right)^{-1/2}.$$  

(27)

Hence, the condition (20) for the dominance of the Higgs triplet mass terms can be satisfied even for $M_\Delta \sim 1 \text{TeV}$ with relatively low reheating temperature, which is favorable for avoiding the gravitino problem.

In this case of $M_\Delta > H_{\text{th}}$, the AD-flatons begin to rotate for $H \sim M_\Delta$ with frequency $\sim M_\Delta \sim m_0$ driven by the mass terms. The AD-flatons are hence redshifted rapidly by rotation for $H \lesssim M_\Delta$ as

$$|\phi_a| \sim |(M/\lambda)/M_\Delta|^{1/2} H.$$  

(28)

Then, after a while the thermal-log terms $\propto T_p^4 \propto H$ catch up the mass terms $\propto |\phi_a|^2 \propto H^2$ for the Hubble parameter

$$H \sim H'_{\text{th}} \sim (H_{\text{th}}/M_\Delta) H_{\text{th}} = (H_{\text{th}}/M_\Delta)^2 M_\Delta.$$  

(29)

Accordingly, some minima are formed by the effect of the negative thermal-log term as

$$\phi_a(1) : |\Delta| \sim |\bar{\Delta}| \sim |\bar{e}^2| \sim \bar{\phi}_{\text{th}},$$

$$\phi_a(2) : |\Delta| \sim |\bar{\Delta}| \sim |\bar{e}^2| = 0,$$

$$\phi_a(3) : |\Delta| = 0, |\bar{\Delta}| \sim |\bar{e}^2| \sim \bar{\phi}_{\text{th}}.$$  

(30-32)

with

$$\bar{\phi}_{\text{th}} = \sqrt{a_\Delta T_R^2 / M_\Delta}$$

(33)

from the condition $M_\Delta^2 |\phi_a|^2 \sim |V_{\text{thlog}}|$. In particular, before the reheating

$$\bar{\phi}_{\text{th}} \sim (H_{\text{th}}/M_\Delta) \bar{\phi} \propto H^{1/2} \left( H_R < H \lesssim H'_{\text{th}} \right)$$

(34)

with Eq. (22) for $T_p$ and Eq. (20) for $H_{\text{th}}$. The main terms to determine these minima are given by

$$V_1 = V_{\text{thlog}} + g_2^2 (|\Delta|^2 + |\bar{\Delta}|^2 + |\bar{e}^2|^2)^2 + (M_\Delta^2 + C_1 m_0^2) |\Delta|^2 + (M_\Delta^2 + C_2 m_0^2) |\bar{\Delta}|^2 + [B_\Delta m_0 M_\Delta \bar{\Delta} \Delta + \text{h.c.}] + C_3 m_0^2 |\bar{e}^2|^2.$$  

(35)
It is really checked that the thermal mass terms \( V_{\text{thm}} \) do not appear in this epoch, since \( f_1|\Delta| \sim h_{11} |\tilde{e}^c| > T_p \) for \( H \sim H_{\text{th}}^\ast \) with \( f_1 \sim h_{11} \sim 0.01 \) and the reasonable range of \( T_{R} \sim 10^{5} - 10^{7} \text{GeV} \) and \( M/\lambda \lesssim 10^{22} \text{GeV} \). Here, it should be noticed that \( V_{\text{th}} \) is degenerate along the circles with radii \( \phi_{\text{th}} \), including the minima \( \phi_{a}(K) \) \((K = 1, 2, 3)\), in the complex planes of \( \phi_{a} \) under the constraint \( \theta_{\Delta} + \theta_{\bar{\Delta}} + \text{arg}(B_{\Delta}) = \pi \) for \( K = 1, 2 \) to minimize the \( B_{\Delta} \) term as \(-|B_{\Delta}|m_{0}|M_{\Delta}||\Delta| \). This degeneracy is slightly lifted by the higher order terms in the whole potential, determining the phases of \( \phi_{a}(K) \) to form actually the minima.

Since the AD-flatons get significant angular momenta by the effect of the mass terms with \( M_{\Delta} \sim m_{0} \) under the condition \( 20 \), they continue to rotate in the epoch of \( H < M_{\Delta} \). Specifically, \( \tilde{e}^c \) may rotate almost freely and redshifted as \( |\tilde{e}^c| \propto H \) toward the origin, separated from \( \Delta \) and \( \bar{\Delta} \). On the other hand, \( \Delta \) and \( \bar{\Delta} \) rotate around the minimum \( \phi_{a}(2) \) linked by the \( B_{\Delta} \) term and the \( D^{2} \) term with \( |\Delta| \sim |\bar{\Delta}| \ll |\tilde{e}^c| \) after the thermal-log terms dominate over the Higgs triplet mass terms. That is, for \( H \ll H_{\text{th}}^\ast \):

\[
|\Delta| \sim |\bar{\Delta}| \sim \phi_{\text{th}}, \quad |\tilde{e}^c| \sim [(M/\lambda)/M_{\Delta}]^{1/2}H. \tag{36}
\]

It seems rather unlikely that the AD-flatons are trapped by the minimum \( \phi_{a}(1) \) or \( \phi_{a}(3) \), once \( |\tilde{e}^c| \propto H \) is reduced sufficiently until \( H \sim H_{\text{th}}^\ast \).

According to this redshift and rotation of AD-flatons, the lepton number asymmetry is fixed \( 7 \) as

\[
\epsilon_{L}(t) \approx \epsilon_{L} \sim 0.1 \quad (t \gg H_{\text{th}}^{-1}). \tag{37}
\]

The fixing of lepton number asymmetry can really be approved by considering the rate equation,

\[
\frac{d\epsilon_{L}}{dt} = -\frac{2}{H^{2}(M/\lambda)} \sum_{a} Q_{L}(a) \text{Im} \left[ \phi_{a} \frac{\partial V}{\partial \phi_{a}} \right] \approx -\frac{2(\lambda_{f}/\lambda)}{H^{2}(M/\lambda)^{2}} \text{Im} \left[ 2M_{\Delta} \Delta \bar{\Delta} \tilde{e}^c \tilde{e}^c \right] -\frac{2(\lambda_{f}/\lambda)}{H^{2}(M/\lambda)^{2}} \text{Im} \left[ A_{f} m_{0} \Delta \bar{\Delta} \tilde{e}^c \tilde{e}^c \right]. \tag{38}
\]

(The thermal terms \( V_{\text{thlog}} + V_{\text{thm}} \) conserve the particle numbers.) The lepton-number violating sources in the right side of Eq. \( 38 \) are given roughly as \((H_{\text{th}}/M_{\Delta})^{2}H \propto t^{-1} \) with Eqs. \( 34 \) and \( 35 \), and oscillate around zero with frequency \( \sim m_{0} \) particularly due to the rapid rotation of \( \tilde{e}^c \). Hence, the lepton number asymmetry is fixed to some significant value for \( t \gg H_{\text{th}}^{-1} \) upon integration of Eq. \( 38 \) in time.

These arguments for the case of \( M_{\Delta} > H_{\text{th}} \) on the evolution of AD-flatons and the lepton number asymmetry \((t \sim H_{\text{inf}}^{-1} \rightarrow t \gg H_{\text{th}}^{-1}) \) can be confirmed by numerical calculations. A typical example is presented in Figs. 1 and 2 with \( M_{\Delta} = m_{0} = 10^{6} \text{GeV} \), \( T_{R} = 5 \times 10^{6} \text{GeV} \), \( M/\lambda = 2 \times 10^{23} \text{GeV} \), \( H_{\text{inf}} = 10^{3} \text{GeV} \) \((H_{\text{th}} \sim 10^{6} \text{GeV}) \) and certain values of the other parameters in the reasonable range. It is clearly seen in Fig. 1 that the trajectories of \( \Delta \) and \( \bar{\Delta} \) in unit of \( \bar{\phi} \propto H^{1/2} \) with Eq. \( 34 \) are converging to the circle with radius \( \sim H_{\text{th}}/M_{\Delta} \sim 0.1 \) satisfying \( |\Delta| \sim |\bar{\Delta}| \sim \phi_{\text{th}} \). The trajectory of \( \tilde{e}^c \) is, on the other hand, shrinking as
The relevant Hubble parameter is estimated from thermalization of the gauge bosons sphaleron process is effective to convert the lepton number asymmetry to complete the leptogenesis while the dominated universe, which is similar to Eq. (38). This is approved by considering the log term. While this situation may continue even after entropy production. This is approved by considering the thermal-log terms.

It is shown for Eq. (36). Then, after a while the lepton doublet \( \tilde{L} \) also enters the thermal plasma, providing the thermal energy density \( n_L/s \) as seen in Eq. (39).

The Hubble parameter in the radiation-dominated epoch is given by

\[
H = \left( \frac{\pi}{\sqrt{90}} \right) g_* T_p^2 / M_p \quad (H \ll H_R)
\]

with \( g_* \approx 200 \). It should be noted here that for a long period of \( H < H_R^1 \) the lepton number asymmetry is still stored in the condensates of AD-flatons rotating around the potential trap \( \phi(2) \) formed by the negative thermal-log term. While this situation may continue even after the reheating, the lepton-to-entropy ratio \( n_L/s \) as given in Eq. (39) remains constant (without significant extra entropy production). This is approved by considering the rate equation for \( n_L/s \) with \( s \approx H^{3/2} \) in the radiation-dominated universe, which is similar to Eq. (39).

The AD-flatons should be liberated anyway from the potential trap to complete the leptogenesis while the sphaleron process is effective to convert the lepton number to the baryon number. This liberation takes place when the negative thermal-log term disappears by the thermalization of the gauge bosons \( W^\pm \). After the thermal-log terms \( V_{thlog} \) become dominant for \( H \approx H_R^1 \), the Higgs doublet \( H_R^1 \) is first thermalized with \( h_{11} \sim f_1 \sim 0.01 \) since \( |\phi|^2 \propto H \) is reduced faster than \( |\Delta| \propto H^{1/2} \), as seen in Eq. (39). Then, after a while the lepton doublet \( L_1 \) also enters the thermal plasma, providing the thermal mass terms \( V_{thm} \). This may occur before or after the reheating depending on whether \( M_{\Delta} > f_1 \sqrt{a_{\alpha_2}} T_R \) or \( M_{\Delta} < f_1 \sqrt{a_{\alpha_2}} T_R \) with \( f_1 \sqrt{a_{\alpha_2}} \sim 10^{-3} \), as seen below. The relevant Hubble parameter is estimated from the condition \( f_1 |\Delta| \sim T_p \) by considering \( |\Delta| \sim |\Delta| \sim \phi_{th} \) with Eq. (39) for \( \phi_{th} \) and Eqs. (22) and (40) for \( T_p \) as

\[
H_{thm} \sim \left\{ \begin{array}{l}
H_R(M_\Delta/f_1 \sqrt{a_{\alpha_2}} T_R)^{1/4} \quad (H_{thm} > H_R) \\
H_R(M_\Delta/f_1 \sqrt{a_{\alpha_2}} T_R)^{1/2} \quad (H_{thm} < H_R)
\end{array} \right.
\]

Here, the Hubble parameter \( H_R \) at the reheating is estimated with Eq. (40) as

\[
H_R \sim 10^{-6} GeV \left( \frac{T_R}{10^{6} GeV} \right)^2.
\]

By considering the condition \( f_1 |\Delta| \sim f_1 \phi_{th} \sim T_p \) with Eq. (39), we have a relation \( f_1 T_p / \phi_{th} \sim M_{\Delta} / \sqrt{a_{\alpha_2}} \) for \( H \sim H_{thm} \). That is, the thermal mass \( (1/2) f_1 T_p \) is fairly larger than \( M_{\Delta} \) by a factor \( \sim 1/(2\sqrt{a_{\alpha_2}}) \approx 10 \). Hence, the potential minimum \( \phi_{th} / (2) \) is shifted by the balance between the thermal mass terms and the thermal-log terms as

\[
|\Delta| \sim \bar{\Delta} \sim (2 \sqrt{a_{\alpha_2}} / f_1) T_p \sim (2 \sqrt{a_{\alpha_2}}) \bar{\phi}_{th} \quad (43)
\]

with the reduction of \( |\Delta| \sim |\Delta| \) from \( \phi_{th} \) by a factor \( \sqrt{a_{\alpha_2}} \approx 0.1 \), where the condition \( f_1 \phi_{th} \sim T_p \) for \( H \sim H_{thm} \) is considered.

It may be expected here that the \( f \) couplings satisfy the condition

\[
f_i < f_{i+1} < f_i / (2 \sqrt{a_{\alpha_2}}) \approx 10 f_i \quad (i = 1, 2),
\]

which is consistent with the hierarchical neutrino mass spectrum, e.g., \( f_1 = 0.01, f_2 = 0.05, f_3 = 0.3 \). Then, since \( f_2 |\Delta| \approx 2 \sqrt{a_{\alpha_2}} (f_2 / f_1) T_p < T_p \) from Eqs. (43), the second lepton doublet \( L_2 \) also enters the thermal plasma, and by the effect of thermal mass term \( (1/4) f_2^2 T_p^2 |\Delta|^2 \) the Higgs triplets are reduced further as \( (2 \sqrt{a_{\alpha_2}} / f_2) T_p \sim (2 \sqrt{a_{\alpha_2}} / f_2) T_p \). In this way, when the lightest \( L_1 \) is thermalized for \( H \sim H_{thm} \), the heavier \( L_2 \) and \( L_3 \) sequentially come into the thermal plasma, providing larger thermal mass terms for \( \Delta \). As a result, if the condition

\[
f_i > g_2 (2 \sqrt{a_{\alpha_2}}) \approx 0.07 \quad (i = 2 \text{ or } 3)
\]

is further satisfied, the \( SU(2)_W / U(1)_P \) gauge bosons \( W^\pm \) with mass \( M_{W} = (g_2 / \sqrt{2}) (|\Delta|^2 + |\Delta|^2)/2 \approx g_2 (2 \sqrt{a_{\alpha_2}} / f_2) T_p < T_p \) (i = 2 or 3) may even be thermalized soon after the lepton doublet \( L_1 \) enters the thermal plasma for \( H \sim H_{thm} \).

It is, on the other hand, considered that the above conditions on \( f_1 \) and \( g_2 \) may not be satisfied. Then, since the thermal mass terms decrease with \( T_p \), the mass terms with \( M_{\Delta} \sim m_0 \) dominate again after a while for \( H < H_{thm} \) so that the minimum returns to \( \phi_{th} / (2) \) from Eq. (45). Even in this case, the negative thermal-log term disappears in a later epoch when the gauge bosons are thermalized satisfying the condition \( M_{W} \approx g_2 |\Delta| < T_p \) (|\Delta| \sim |\Delta|). The relevant Hubble parameter is estimated
with $|\Delta| \sim \phi_{\text{th}} \propto T_p^2$ in Eq. (33), $T_p$ in Eq. (10) and 
\[ g_2^2 a_2^2 \approx 10^{-3} \text{ as } \]
\[ H_{\text{thg}} \sim 10^{-9}\text{GeV} \left( \frac{M_\Delta}{10^{10}\text{GeV}} \right)^2, \]  
(46)
which is really smaller than $H_{\text{R}}$ in Eq. (12).

Once the negative thermal-log term disappears, as seen so far, the minimum is moved to the origin $\phi_a = 0$. Then, the condensates of AD-flatons with energy densities $\sim a_2 T_p^4$ ($\ll T_p^4$) for $\Delta$ and $\Delta$ and a smaller amount for $\tilde{e}^c$ are evaporated through the lepton-number conserving gauge interactions without significant entropy production. Accordingly, the lepton number asymmetry stored in the AD-flatons is released to the thermal plasma through this evaporation process. This occurs for $H \sim H_{\text{thm}}$ or $H_{\text{thg}}$, which is fairly before the sphaleron process is freeze-out for the Hubble parameter $H_{\text{sh}} \sim 10^{-13}\text{GeV}$ with $T_p \sim 10^2\text{GeV}$ of the electroweak phase transition. Then, the lepton number asymmetry is finally converted to the baryon number asymmetry through the sphaleron process as $n_B = -\langle 8/23 \rangle n_L$ [23, 24]. Therefore, the sufficient baryon number asymmetry can be provided for the nucleosynthesis with $\eta = (6.1 \pm 0.2) \times 10^{-10}$ from the lepton number asymmetry as given in Eq. [30].

B. Case of $M_\Delta < H_{\text{th}}$

We in turn examine the case of $M_\Delta < H_{\text{th}}$ with larger $T_R \sim 10^7\text{GeV}$ and smaller $M/\lambda \sim 10^{20}\text{GeV}$ in Eq. (27), where the evolution of AD-flatons appears substantially different from that for the case of $M_\Delta > H_{\text{th}}$. The negative thermal-log term first dominates over the Hubble induced mass terms for $H \sim H_{\text{th}}$, and it soon competes with the other terms in $V_I$, forming the potential minima in Eqs. (30), (31), (32). The AD-flatons are already tracking the instantaneous minimum $|\phi_a| \sim \phi$ with fluctuating phases after the inflation [7]. In this course, $\tilde{e}^c$ is linked with $\Delta$ and $\Delta$ by the quartic terms $\Delta^2 \Delta \tilde{e}^c \tilde{e}^c$ and $\Delta \Delta \tilde{e}^c \tilde{e}^c$ with couplings $\sim M_\Delta/(\lambda/\chi)$ in $V$. Then, for $H \lesssim H_{\text{th}}$ the AD-flatons $\Delta, \tilde{e}^c$ together move gradually toward the minimum $\phi_a(1)$ due to the effect of the negative thermal-log term.

If the Hubble parameter decreases further to $H \sim (M_\Delta/H_{\text{th}})^3 H_{\text{th}}$, the mass term $m_\tilde{e}^c \tilde{e}^c \tilde{e}^c \propto H$ dominates over the quartic terms $\Delta^2 \Delta \tilde{e}^c \tilde{e}^c$ and $\Delta \Delta \tilde{e}^c \tilde{e}^c \propto H^2$ to link $\tilde{e}^c$ with $\Delta$ and $\Delta$. Then, $\tilde{e}^c$ begins to oscillate toward the origin with redshift faster as $\bar{H}$ rather than $\phi_{\text{th}} \propto H^{1/2}$, so that the AD-flatons turn to move from the minimum $\phi_a(1)$ to the minimum $\phi_a(2)$ with $|\tilde{e}^c| = 0$. Since the negative thermal-log term dominates much earlier than the mass terms with $M_\Delta \sim m_\tilde{e}$, the AD-flatons do not get significant angular momenta until $H \sim (M_\Delta/H_{\text{th}})^3 H_{\text{th}}$. Hence, for $H \lesssim (M_\Delta/H_{\text{th}})^3 H_{\text{th}}$ $\Delta$ and $\Delta$ move gradually toward the minimum $\phi_a(2)$ without rotating around the circle including $\phi_a(2)$. On the other hand, $\tilde{e}^c$ with small angular momentum shows a complicated motion around the origin, changing frequently the sign of the time derivative of its phase $\dot{\theta}_{\tilde{e}^c}$, as seen by numerical calculations. This would imply that $\tilde{e}^c$ is not liberated fully from $\Delta$ and $\Delta$ in the presence of the $D^2$ term and the phase-dependent quartic terms $\Delta^4 \Delta^2 \tilde{e}^c \tilde{e}^c$ and $\Delta \Delta \tilde{e}^c \tilde{e}^c$.

Particularly due to this complicated behavior of $\tilde{e}^c$ linked to $\Delta$ and $\Delta$ for the case of $M_\Delta < H_{\text{th}}$, the lepton number asymmetry $\epsilon_L(t)$ oscillates violently for $H < H_{\text{th}}$. Its mean magnitude, on the other hand, tends to be saturated to some large value $\epsilon_L \sim 10^{-2}$ for $H < (M_\Delta/H_{\text{th}})^3 H_{\text{th}}$ after the epoch of transition from $\phi_a(1)$ to $\phi_a(2)$, as seen by numerical calculations. After the reheating, the plasma temperature $T_p$ decreases faster as $H^{1/2}$ rather than $H^{1/4}$, so that $\Delta$ and $\Delta$ trapped by the minimum $\phi_a(2)$ also decrease faster as $T_p \propto H$. Then, by considering the rate equation it would be expected that the lepton number asymmetry is fixed, providing a significant amount of $n_L / s \sim 10^{-10}$ in Eq. (39) with $\epsilon_L \sim 10$ for $T_R \sim 10^7\text{GeV}$ and $M/\lambda \sim 10^{20}\text{GeV}$. In the case of $M_\Delta < H_{\text{th}}$, however, it seems difficult to make a reliable estimate of the asymmetry due to the substantial effect of the negative thermal-log term.

V. SUMMARY

In summary, we have investigated the leptogenesis with the supersymmetric Higgs triplets in the light of experimental verification in the TeV region. The lepton number asymmetry really appears after the inflation via multiscalar coherent evolution of $\Delta, \Delta$ and $\tilde{e}^c$ on the flat manifold. If the Higgs triplet mass terms dominate over the negative thermal-log term for $H \sim M_\Delta$, the asymmetry is fixed readily to some significant value by the redshift and rotation of the AD-flatons, providing the sufficient lepton-to-entropy ratio $n_L / s \sim 10^{-10}$. This can be the case even with $M_\Delta \sim 1\text{TeV}$ for $T_R \sim 10^6\text{GeV}$ and $M/\lambda \sim 10^{22}\text{GeV}$. On the other hand, if the negative thermal-log term dominates first, the evolution of the AD-flatons appears different substantially. Even in this case, a significant amount of $n_L / s \sim 10^{-10}$ might be obtained for $T_R \sim 10^7\text{GeV}$ and $M/\lambda \sim 10^{20}\text{GeV}$, though it seems difficult to make a reliable estimate of the asymmetry.

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