T-DUALITY AND THE WEAKLY COUPLED HETEROTIC STRING∗ †

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Abstract

T-duality is a symmetry of the heterotic string to all orders in string perturbation theory. This results in an effective four dimensional supergravity theory with desirable features for phenomenology. T-duality, as well as, generically, an anomalous $U(1)$, is broken by quantum anomalies of the effective field theory. The structure of the full anomaly is presented, and the mechanisms for anomaly cancellation are described.

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1 Introduction

When compactified from ten to four space-time dimensions, the weakly coupled heterotic (WCHS) string theory [1] has an invariance under a discrete group of transformations known as “T-duality” or “target space modular invariance” [2]. This leads to several attractive features for phenomenology:

- The Kähler moduli, or T-moduli, are generically stabilized [3] at self-dual points: $t_{sd} \rightarrow t_{sd}$; as a consequence there is no large flavor mixing induced by supersymmetry (SUSY) breaking.

- R-symmetry is protected [4] by T-duality in supergravity (SUGRA), thereby suppressing the mass of the axion. This provides [5] a possible solution to the strong CP problem.

- When combined with $U(1)$ gauge symmetries, T-duality provides [6] a possible mechanism for R-parity or an even stronger discrete symmetry.

I will briefly describe each of these results, which are not new, but serve as motivation for the second part of this talk, namely anomalies and anomaly cancellation in SUGRA.

At the quantum level of the effective supergravity theory, T-duality is broken by quantum anomalies, as is, generically, an Abelian $U(1)_{X}$ gauge symmetry, both of which are exact symmetries of string perturbation theory. It was realized some time ago that these symmetries could be restored by a combination of four dimensional counterparts [7] of the Green-Schwarz (GS) mechanism in 10 dimensions [8] and string threshold corrections [9]. However anomaly cancellation has been demonstrated explicitly only for the coefficient of the Yang-Mills superfield strength bilinear. The entire supergravity chiral anomaly has in fact been determined [10], but the complete superfield form of the anomaly is required to fully implement anomaly cancellation.

Chiral anomalies are ill-defined in the unregulated effective field theory; I use Pauli Villars (PV) regulation [11] to define the theory. Requiring GS anomaly cancellation restricts the form of the anomaly [12], which in turn leads to constraints on soft SUSY-breaking sfermion masses.

2 The benefits of T-duality

2.1 T-moduli stabilization

T-moduli are generically stabilized at self-dual points: $t_{sd} = 1$, $e^{i\pi/6}$. To see this\(^1\) consider a toy model with a single T-modulus superfield $T$ and the dilaton superfield $S$. T-duality and the

\(^1\)See Section 3.1.2 of [3].
shift-symmetry of the axion $\text{Im } S$ require that the Kähler potential $K$ take the form

$$K = k(S + \bar{S}) - 3 \ln(T + \bar{T}).$$

A simple T-duality invariant ansatz for the superpotential $W$, which was often used in the past, is

$$W(S, T) = H(S)\eta^{-6}(T),$$

where $\eta$ is the Dedekind eta function. Minimization of the potential gives two solutions for the vacuum configuration:

$$\langle H_s + k_s H \rangle = \langle F_s \rangle = 0, \quad H_s = \frac{\partial H}{\partial s}, \quad \text{etc.},$$

$$\langle (s + \bar{s})^2 k_{ss} \rangle = \langle -e^{2i\gamma} H^* \left[1 + 24 \text{Re} \left(\text{Re}\zeta(t) + 2 \text{Re} |\zeta(t)|^2\right)\right]\rangle, \quad \zeta = \frac{\partial \ln \eta}{\partial t},$$

where $t = T|, s = S|,$ and $\gamma = \text{arg}(H_s - k_s H)$. For solution (2.3), the self-dual point $t = t_{sd}$ is a local maximum with $\langle F_s \rangle = 0$; this is the only solution in classical limit with $k(S + \bar{S}) = -\ln(S + \bar{S})$. Solution (2.4) instead satisfies $\langle F_t \rangle = 0$, and $t_{sd}$ is a local minimum. For fixed $\langle \text{Re}s \rangle$ we can parametrize the dilaton contribution to the potential as

$$K_{s\bar{s}}^{-1} F^s \bar{F}^\bar{s} = |2 \text{Re} H_s - H|^2 \equiv a|H|^2,$$

with $a = 0$ for solution (2.3). The potential

$$V = \frac{H^2}{16 \text{Re}(\text{Re} t)^3 |\eta(t)|^2} \left[a + 24 \text{Re} \left(2 \text{Re} |\zeta(t)|^2 + \text{Re} \zeta(t)\right)\right]$$

is was studied in the $\text{Re} t$ direction in [3]. It has the oft-cited minimum at $t \approx 1.23$ in the classical limit (2.3) with $a = 0$, but for $a > .05$, the minimum is always at the self-dual point $t = 1, 4 \text{Re} \zeta(2) = -1, \langle F_t \rangle = 0$. This is the result found [13] in more realistic “Kähler stabilization” models for the dilaton.

### 2.2 Is the universal string axion the QCD axion?

If SUSY is broken by a single gaugino condensate $\langle \lambda \lambda \rangle \neq 0$, there is a residual R-symmetry, and the axion remains massless at the SUSY breaking scale in the quantum field theory (QFT) approximation, but in a general SUGRA theory, couplings of the axion to higher powers $\langle (\lambda \lambda)^p \rangle$ of the gaugino condensate may generate a mass that is too large for it to be identified with the Peccei-Quinn axion [14].
T-duality forbids [4] low values of the exponent \( p \). The minimal group of T-duality transformations, namely \( SL(2, \mathbb{Z}) \), requires \( p \geq 4 \), while, for example, the maximal group for a model with just three untwisted Kähler moduli, namely \([SL(2, \mathbb{Z})]^3\), requires \( p \geq 12 \). An analysis [5] of the QCD phase transition shows that the identification of the string axion \( \text{Im} s \) with the QCD axion is possible provided the T-duality group is larger than the minimal one, requiring \( p > 4 \), with the caveat that it also requires a large axion coupling parameter, \( f_a \sim m_{\text{Pl}} \), which may be a viable possibility [15]. Thus a string solution to the strong CP problem implies a mild constraint on the group of T-duality transformations.

### 2.3 R-parity?

If the T-moduli are stabilized at self-dual points, \( \langle t \rangle = t_{sd} = 1 \) or \( e^{i\pi/6} \), there is an unbroken discrete subgroup:

\[
G_R = \mathbb{Z}_4^m \otimes \mathbb{Z}_6^{m'}, \quad m + m' = 3,
\]

under which the gauginos \( \lambda \) and gauge charged chiral superfields transform as

\[
\lambda \rightarrow -\lambda, \quad \Phi^i(\theta) \rightarrow e^{2\pi i \beta_i(q^i_n, q^i_a) \Phi^i(\theta')}.
\]

where \( q^i_n \) is a modular (T-duality) weight. In the presence of an anomalous \( U(1)_X \) the corresponding GS-term generates a D-term, resulting in the breaking of some number \( m \) of \( U(1) \) gauge symmetries and of \( G_R \), but leaving an unbroken discrete subgroup \( G'_R \)

\[
G_R \otimes U(1)^m \rightarrow G'_R \in G_R \otimes U(1)^m,
\]

with the transformation property (2.8) for chiral superfields modified by phase factors that depend on their \( U(1) \) charges \( q^i_n \):

\[
\Phi^i(\theta) \rightarrow e^{2\pi i \beta_i(q^i_n, q^i_a, Y^i) \Phi^i(\theta')}.
\]

Finally, at the electroweak gauge symmetry breaking scale:

\[
SU(2)_L \otimes U(1)_w \rightarrow U(1)_{\text{em}}, \quad \langle H_{u,d} \rangle \neq 0,
\]

the surviving discrete symmetry is the subgroup \( R \) that leaves the Higgs fields invariant:

\[
R \in G'_R \otimes U(1)_w,
\]

with (2.10) now replaced by

\[
\Phi^i(\theta) \rightarrow e^{2\pi i \beta_i(q^i_n, q^i_a, Y^i) \Phi^i(\theta')}, \quad \beta_{H_{u,d}} = n,
\]
where $Y^i$ is weak hypercharge. Requiring nonvanishing quark masses and CKM angles imposes the conditions

$$\beta_Q = -\beta_Q^c \equiv \beta,$$

(2.14)

and imposing nonvanishing lepton masses gives

$$\beta_L = -\beta_E^c \equiv \gamma.$$

(2.15)

There will be no dimension-three operators of the type

$$U^c D^c D'^c, \quad LQD^c, \quad LL'E'^c,$$

provided

$$\beta \neq \frac{n}{3}, \quad \gamma \neq n,$$

(2.16)

and, in contrast to standard R-parity, the dimension-four operator

$$U^c U'^c D'^c E^c$$

(2.17)

will be forbidden provided

$$3\beta + \gamma \neq n.$$

(2.18)

A challenge for string model builders is to find a heterotic string vacuum with the correct modular weights and $U(1)$ charges to satisfy these conditions.

### 3 Anomalies and anomaly cancellation

#### 3.1 Preliminaries

In conventional superspace, the kinetic Lagrangian for SUGRA and matter superfields takes the form

$$\mathcal{L}_{\text{kin}} = -3 \int d^4 \theta \ E_0 \ e^{-\frac{1}{2} K(Z, \bar{Z})},$$

(3.19)

where $E_0$ is the super-determinant of the super-vielbein $E^A_M$. By an appropriate Kähler and super-Weyl transformation, this may be put in the form [16]

$$\mathcal{L}_{\text{kin}} = -3 \int d^4 \theta \ E,$$

(3.20)

giving a canonical Einstein term for the component form of the Lagrangian; this is the Kähler $U(1)$ [$U(1)_K$] superspace formulation of SUGRA. The structure group of $U(1)_K$ geometry contains
the Lorentz, \( U(1)_K \), Yang-Mills (YM) and chiral superfield reparameterization groups. A chiral superfield \( Z \) is covariantly chiral: \( \mathcal{D}_\beta Z = 0 \), where the covariant spinorial derivative \( \mathcal{D}_\beta \) includes the \( U(1)_K \), YM, spin and chiral superfield reparameterization connections. The Kähler potential \( K = \bar{Z}e^V Z + \ldots \) of the standard formulation (3.19) is replaced simply by \( K = |Z|^2 + \ldots \) in the \( U(1)_K \) superspace formulation (3.20). The full Lagrangian takes the form

\[
\mathcal{L} = \frac{1}{2} \sum_i \int d^4 \theta \frac{E_i}{R} r_i + \text{h.c.}, \quad r_{\text{kin}} = -3R,
\]

\[
r_{\text{YM}} = \frac{1}{4} f(Z) W_\alpha^a W_\alpha^a, \quad r_{\text{superpot}} = e^{K/2} W(Z),
\]

(3.21)

where the superfield \( R \) is a component of the super-Riemann tensor, whose lowest component \( R| \) is an auxiliary field of the SUGRA multiplet; its equation of motion reads

\[
R| = \frac{1}{2} e^{K/2} W(z), \quad z = |Z|,
\]

(3.22)

and in the WCHS the gauge kinetic function is just the dilaton superfield:

\[
f(Z) = S.
\]

(3.23)

Local supersymmetry of the Lagrangian (3.21) is assured [16] by the fact that the superfields \( r_i \) have \( U(1)_K \) charge \( w_K(r) = 2 \).

### 3.2 PV regularization

A renormalizable supersymmetric theory is defined by specifying the matter and Yang-Mills chiral superfields \( Z^i \) and \( W_\alpha^a \), respectively, their gauge transformation properties

\[
\delta^a Z^i = i(T^a Z)^i, \quad \delta^a W_\alpha^b = f^{abc} W_{\alpha c},
\]

(3.24)

and the superpotential \( W(Z) \). The (one loop) ultraviolet (UV) divergences of the theory can be regulated [17] by introducing matter chiral PV supermultiplets \( Z^I, Y_I, \varphi^a \) with gauge transformation properties:

\[
\delta^a Z^I = i(T^a Z)^I, \quad \delta_a Y_I = -i(T_a^T Y)_I, \quad \delta^a \varphi^b = f^{abc} \varphi_c,
\]

(3.25)

and superpotential

\[
W_{\text{PV}} = \frac{1}{2} W_{ij} Z^I Z^J + \sqrt{2} g \varphi^a (T_a Z)^i Y_I,
\]

(3.26)
provided the gauge representation of the matter in the SUSY theory satisfies the constraint

\[ C_M^a = \text{Tr} T_a^2 = \text{Tr}(T_a^R)^2 \]  
(3.27)

for some (reducible) real representation \( R \) of the gauge group. The condition (3.27) is indeed satisfied in the MSSM and its extensions, as well as in the hidden sectors of all \( Z_3 \) orbifolds [18], which is the only class of WCHS vacua that has been thoroughly studied. In the case of SUGRA with a dilaton superfield \( S \), additional PV chiral superfields as well as PV Abelian gauge multiplets are needed [11] to cancel all the UV divergences.

The regularized theory would be anomaly free if the PV mass terms respected the classical symmetries of the SUSY theory; this is not possible if the theory is anomalous at the quantum level. The quadratically divergent part of the one-loop corrected Lagrangian contains a term, generated by chiral matter loops,

\[ (L_Q)^\chi \propto \Lambda^2 \text{Tr} L_Q = \Lambda^2 \left[ N \mathcal{D}^\alpha X_\alpha - \mathcal{D}^\alpha \text{Tr} \Gamma_\alpha - 2(\text{Im} s)^{-1} \text{Tr} T^a \right], \]  
(3.28)

where \( \Lambda \) is the UV cut-off, and

\[ X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha K, \quad \Gamma_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\Gamma_{ij}^i \mathcal{D}_\alpha Z^j, \]  
(3.29)

with \( \Gamma_{jk}^i \) the affine connection associated with the Kähler metric. PV fields with T-duality invariant masses give no contribution to the first two terms in (3.28), and those with \( U(1)_X \) invariant masses give no contribution to the last term. One can restore T-duality (but not \( U(1)_X \) invariance) by including a moduli-dependence in the PV mass terms in the superpotential:

\[ \mu \rightarrow \mu(T^i) = \prod_i \eta(T^i)\omega_i \mu_0, \]

which could be interpreted as arising from string threshold corrections; however these are absent [19] in \( Z_3 \) and \( Z_7 \) orbifold compactifications.

### 3.3 The regularized theory

In the PV regulated theory, the contribution (3.28) is replaced by

\[ (L_Q)^\chi \propto \text{Tr} \eta \left[ L_Q |m(z, \bar{z}, V_X)|^2 \right], \]  
(3.30)

where \( \eta \) is the PV signature, \( m \) is the PV mass matrix and \( V_X \) is the \( U(1)_X \) vector superfield. The operator (3.30) is generally not T-duality and \( U(1)_X \) invariant; these noninvariant terms can
be canceled “by hand”, i.e., by imposing conditions on the signatures and overall coefficients of the masses such that the trace in (3.30) vanishes. The cancellation of linear and logarithmic divergences restricts the PV metric:

\[ K_{PV}(z, \bar{z}, V_X) \]

and therefore the PV masses:

\[ m = e^{K/2} K_{PV}^{-1}. \]

Under T-duality and \(U(1)_X\) transformations the regulated one-loop Lagrangian transforms as

\[ \Delta L_{\text{anom}} = - \int d^4 \theta \Omega H(T, \Lambda_X) + \text{h.c.} = \frac{1}{8} \int d^4 \theta \frac{E}{R} \Phi H(T, \Lambda_X) + \text{h.c.}, \]

(3.31)

where \(\Lambda_X\) is the \(U(1)_X\) gauge parameter, and

\[ \Omega = - \text{Tr} \left\{ c_d \left[ M^2 (\bar{D}^2 - 8 \bar{R}) M^{-2} R^m + \text{h.c.} \right] + c_g G_{\alpha\beta} G_{\alpha\beta} + c_r R^m R^m \right\} \]

\[ + c_w \Omega_W + \text{Tr} \left( c_{\bar{w}} \Omega_{YM} - c_X \Omega_X \right), \]

(3.32)

with

\[ (\bar{D}^2 - 8 \bar{R}) \Omega = \Phi, \quad (D^2 - 8 R) \Omega = \bar{\Phi}. \]

(3.33)

The constants \(c_i = c_i(\eta, q_n, q_X)\) depend on the signatures, modular weights \(q_n\) and \(U(1)_X\) charges \(q_X\) of the PV fields, \(M^2\) is a real superfield:

\[ M^2 = |m(z, \bar{z}, V_X)|^2, \]

\[ R^m = - \frac{1}{8} M^{-2} (\bar{D}^2 - 8 \bar{R}) M^2, \quad G_{\alpha\beta} = \frac{1}{2} M [D_\alpha, D_\beta] M^{-1} + G_{\alpha\beta}, \]

(3.34)

and the Chern-Simons (CS) superfields \(\Omega_i\) are defined by

\[ (\bar{D}^2 - 8 \bar{R}) \Omega_W = W^{\alpha\beta\gamma} W_{\alpha\beta\gamma}, \quad (D^2 - 8 R) \Omega_{YM} = \sum_{a \neq X} T_a W^a W^a, \]

\[ (\bar{D}^2 - 8 \bar{R}) \Omega_X^m = X_m X_m^m, \quad X_m^m = \frac{3}{8} (\bar{D}^2 - 8 \bar{R}) D_{\alpha} \ln M^2 + X_{\alpha}. \]

(3.35)

The CS superfield \(\Omega_X^m\) can be explicitly constructed [12] following the procedure [20] used for the construction of \(\Omega_{YM}\). The chiral superfield strength \(W_{\alpha\beta\gamma}\) and the real superfield \(G_{\alpha\beta}\), with \(G_{\alpha\beta}\) a SUGRA auxiliary field, are related to elements of the super-Riemann tensor. The result (3.32) has been obtained by a component calculation [12] and by a superconformal superspace calculation, followed by gauge fixing to \(U(1)_K\) superspace [21].
3.4 The (modified) linear supermultiplet

A linear supermultiplet is defined by the conditions\(^2\)

\[
(D^2 - 8R)\ell = 0, \quad (\bar{D}^2 - 8\bar{R})\ell = 0. \tag{3.36}
\]

It has three components: the dilaton \(\ell = L\), a fermion, the dilatino \(\chi\), and a two-form \(b_{\mu\nu}\) that is dual to the axion \(\text{Im} \, s\); it has no auxiliary field. The modified linearity condition replaces (3.36) by the conditions

\[
(D^2 - 8R)\ell = -\Phi, \quad (\bar{D}^2 - 8\bar{R})\ell = -\bar{\Phi}, \tag{3.37}
\]

where the chiral superfield \(\Phi\) has Kähler and Weyl weights \(w_K(\Phi) = 2, \ w_W(\Phi) = 1\), respectively.

Consider a theory defined by a Kähler potential \(K\) and a Lagrangian \(L\) of the form

\[
K = k(L) + K(Z, \bar{Z}), \quad L = -\frac{3}{2} \int d^4\theta E F(Z, \bar{Z}, L). \tag{3.38}
\]

A canonical Einstein term for this theory requires

\[
F - L \frac{\partial F}{\partial L} = 1 - \frac{1}{3} L \frac{\partial k}{\partial L} = -L^2 \frac{\partial}{\partial L} \left( \frac{1}{L} F \right), \tag{3.39}
\]

which is solved by

\[
F(Z, \bar{Z}, L) = 1 + \frac{1}{3} LV + \frac{1}{3} L \int \frac{dL}{L} \left( \frac{\partial k(L)}{\partial L} \right), \tag{3.40}
\]

where \(V\) is a constant of integration, independent of \(L\). If we take

\[
V = -bV(Z, \bar{Z}) + \delta_X V_X \tag{3.41}
\]

such that under T-duality and \(U(1)_X\) transformations

\[
\delta V = H + \bar{H}, \tag{3.42}
\]

where \(H\) is the holomorphic function introduced in (3.31), there is a shift in the tree level Lagrangian (3.38)

\[
\Delta L = \frac{1}{8} \int d^4\theta \frac{E}{R} (D^2 - 8R)LH + \text{h.c.} = -\frac{1}{8} \int d^4\theta \frac{E}{R} \Phi H + \text{h.c.} = -\delta L_{\text{anom}}, \tag{3.43}
\]

since the first term on the left hand side vanishes under integration by parts [16].

\(^2\)See Section 5 of [16] and references therein.
3.5 Chiral/linear duality

Now consider the Lagrangian

\[ \mathcal{L}_{\text{lin}} = -3 \int d^4 \theta \left[ F(Z, \bar{Z}, L) + \frac{1}{3} (L + \Omega)(S + \bar{S}) \right], \quad (3.44) \]

where \( S = (\bar{\Omega}^2 - 8R)\Sigma \) is chiral, with \( \Sigma \neq \Sigma^\dagger \) unconstrained, \( L = L^\dagger \) is real but otherwise unconstrained, and the chiral and anti-chiral projections of \( \Omega \) are given in (3.33). The equations of motion for \( \Sigma, \Sigma^\dagger \)

\[ \frac{\partial \mathcal{L}}{\partial \Sigma} = \frac{\partial \mathcal{L}}{\partial \Sigma^\dagger} = 0, \quad (3.45) \]

give the constraints (3.37) on \( L \), and (3.44) reduces to

\[ \mathcal{L}_{\text{lin}} \rightarrow -3 \int d^4 \theta E \left[ 1 + \frac{1}{3} \Omega(S + \bar{S}) \right] = -3 \int d^4 \theta E\left( \int d^4 \theta \frac{E}{R} S \Phi + \text{h.c.} \right), \quad (3.52) \]

where the vacuum value \( \langle s(L) \rangle = g^{-2} \) determines the string scale coupling constant \( g \). Alternatively we can use the equation of motion for \( L \):

\[ \frac{\partial \mathcal{L}}{\partial L} = -3E \left\{ \frac{\partial F}{\partial L} + \frac{1}{3}(S + \bar{S}) - \frac{1}{3} \frac{\partial k}{\partial L} \left[ F + \frac{1}{3} L(S + \bar{S}) \right] \right\} = 0, \quad (3.48) \]

to determine

\[ L = L(S + \bar{S} + V). \quad (3.49) \]

Once \( L \) is eliminated, there are only chiral (and \( U(1)_X \) vector) superfields in \( F, L \) and \( K \), and the Einstein normalization condition on (3.44) takes the form (3.20):

\[ F + \frac{1}{3} L(S + \bar{S}) = 1. \quad (3.50) \]

The above duality transformation is valid provided the real superfield \( \Omega \) has Kähler and Weyl weights

\[ w_K(\Omega) = 0, \quad w_W(\Omega) = 2, \quad (3.51) \]

so that \( E\Omega = E_0\Omega_0 \) is Weyl invariant, that is, independent of \( K \) and therefore of \( L \). Combining (3.50) with (3.48), we recover (3.40), and (3.44) now becomes

\[ \mathcal{L}_{\text{lin}} \rightarrow -3 \int d^4 \theta E \left[ 1 + \frac{1}{3} \Omega(S + \bar{S}) \right] = -3 \int d^4 \theta E\left( \int d^4 \theta \frac{E}{R} S \Phi + \text{h.c.} \right) \]
3.6 Strategy for Anomaly Cancellation

A completely regulated supergravity theory was constructed [12] for the case of three untwisted Kähler moduli, which is characteristic of $Z_3$ orbifold compactification. The results can be summarized as follows.

- The Kähler potential (wave function) renormalization and dilaton couplings can be regulated with PV fields with T-duality and $U(1)^X$ invariant masses.
- The remaining divergences can be regulated by PV fields with a simple (T-duality and $U(1)^X$ invariant) Kähler metric.
- The generalized modified linearity condition (3.37) is used to remove (some or all of) the remaining divergences. The operators in (3.32) satisfy the requirements (3.51), as can be shown [21] by identifying Weyl invariants in conformal superspace, and then gauge-fixing to $U(1)_K$ superspace.
- Threshold corrections are incorporated, as appropriate.
- After performing the duality transformation (3.48)–(3.52) to the chiral formulation for the dilaton, the dilaton Kähler potential takes the form

\[ K(S, \bar{S}) = k(S + \bar{S} + V), \]  

(3.53)

which is T-duality and $U(1)^X$ invariant since $L$ in (3.49) is invariant, implying

\[ \Delta S = -H(T, \Lambda_X), \]  

(3.54)

In this formulation the QFT quantum anomaly is canceled (up to threshold corrections) by a shift in the tree level Lagrangian

\[ \mathcal{L}_S = -\int d^4 \theta (S + \bar{S}) \Omega \]  

(3.55)

due to the shift (3.54).

The Lagrangian (3.55) contains new tree level couplings of the dilaton. This is to be expected from superstring theory. The two-form potential $b_{\mu\nu}$ of the linear multiplet defined by (3.36) appears through a three-form field strength, its curl:

\[ b_{\mu\nu\rho} = \partial_{[\mu} b_{\nu\rho]} . \]  

(3.56)
This is modified by (3.37). The supergravity multiplet of 10-d SUGRA contains the three-form

\[ H_{LMN} = \partial\{B_{MN}\} + \omega_{MNL}^{YM} + \omega_{MNL}^{Lor}, \]  

(3.57)

which includes the 10-d Yang-Mills and Lorentz Chern-Simons forms. When the theory is compactified to 4-d SUGRA, the 4-d three-form includes the 4-d Yang-Mills and Lorentz Chern-Simons forms, as well as additional terms that arise from contractions of Lorentz indices in the 6 compact dimensions:

\[ h_{\mu\nu\rho} = \partial_{[\mu}b_{\nu\rho]} + \omega_{\mu\nu\rho}^{YM} + \omega_{\mu\nu\rho}^{Lor} + \text{scalar derivatives} + \ldots \]  

(3.58)

These new couplings can be regulated by PV superfields with invariant mass terms, as is the case for the dilaton coupling to the gauge sector: \( \Phi \rightarrow W^\alpha W_\alpha. \)

Work in progress includes phenomenological applications of the above results, and tightening their connection to the WCHS.

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