PERFORMANCE COMPARISON OF ENERGY DETECTOR AND A MATCHED FILTER BASED SS OVER RAYLEIGH FADING CHANNEL

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Abstract—In the development process of standards for underutilized bands of spectrum, the spectrum sensing (SS) plays a vital role. In the cognitive cycle the vacant bands, unutilized licensed band, identified in SS phase are analyzed for transmission suitability and then assigned to the unlicensed user for transmission. Energy detector (ED) based spectrum sensing is a noncoherent technique where cognitive radio, also known as the secondary user, finds spectrum holes without any prior information of the original or primary user. A matched filter (MF) based spectrum sensing is a coherent technique where cognitive radio has prior information of the original user. This paper gives a MATLAB implementation of the two techniques over Rayleigh fading channel. The performance of two techniques compared with the help of receiver operating characteristics.

Keywords—Multipath propagation, Rayleigh fading coefficient, energy detector, matched filter, probability of detection, probability of false alarm, receiver operating characteristics.

I. INTRODUCTION

In the development of wireless technologies, we find that some electromagnetic spectrum bands, such as mobile communication near 800MHz/900MHz/1900 MHz, are crowded whereas some spectrum bands are still underutilized. In static spectrum allocation if a licensed user (Primary user, PU) is not active at a particular time and specific location, the allocated band remained free and termed as spectrum hole\cite{1}. The cognitive radio networks, work on the principle of dynamic spectrum allocation where the unlicensed user (secondary user, SU) opportunistically use spectrum holes and improve the spectrum utilization. In\cite{2},\cite{3} energy detector and a matched filter based spectrum sensing techniques, for cognitive radio networks, analyzed and compared over an additive white Gaussian noise channel. Further, motivates research for spectrum sensing in current wireless systems.

A multipath propagation mechanism is used to characterize wireless communication system where the receiver gets message signal in multiple copies from multiple paths. These multiple copies follow different paths due to signal scattering, diffraction, and reflection as shown in figure 1. At mobile receiver, multiple copies reach with random delay and attenuation which may result in constructive or destructive interference. This phenomenon gave a variation of signal strength at receiver and termed as fading. The statistical behavior of the fading is described using various fading models such as Rayleigh, Hoyt, Rice and Nakagami-m fading\cite{4}.

In this paper, we are using Rayleigh fading model to characterize the radio environment. In section II we have characterized Rayleigh fading and developed a mathematical model for Rayleigh fading channel. In section III a matched filter based SS analytically modeled and MATLAB simulated receiver operating characteristics plotted for Rayleigh fading. Section IV follow the work of section III for energy detector. Section V compare the performance of two detectors.
II. RAYLEIGH FADING CHANNEL CHARACTERIZATION

Let there is a receiver which receives L copies \((y_0(t), y_1(t), y_2(t), \ldots, y_{L-1}(t))\) of transmitted signal \(x(t)\). These L copies follow \(L_0, L_1, L_2, \ldots, L_{L-1}\) paths with attenuation coefficients \(a_0, a_1, a_2, \ldots, a_{L-1}\) and delays \(\tau_0, \tau_1, \tau_2, \ldots, \tau_{L-1}\) respectively. We can characterize this scenario as an input-output system as shown in figure.2 with input signal \(x(t)\) and output signal \(y(t)\).

The received signal \(y(t)\) can be given as
\[
y(t) = a_0 x(t - \tau_0) + a_1 x(t - \tau_1) + a_2 x(t - \tau_2) + \cdots + a_{L-1} x(t - \tau_{L-1})
\]

Using the Fourier transform properties we can find \(Y(\omega)\) as
\[
Y(\omega) = (a_0 e^{-j\omega\tau_0} + a_1 e^{-j\omega\tau_1} + a_2 e^{-j\omega\tau_2} + \cdots + a_{L-1} e^{-j\omega\tau_{L-1}})X(\omega)
\]
Or
\[
\frac{Y(\omega)}{X(\omega)} = H(\omega) = (a_0 e^{-j\omega\tau_0} + a_1 e^{-j\omega\tau_1} + a_2 e^{-j\omega\tau_2} + \cdots + a_{L-1} e^{-j\omega\tau_{L-1}})
\]

The \(H(\omega)\), which is ratio of Fourier transform of output and Fourier transform of input, is the Fourier transform of impulse response of linear time invariant system under consideration. Using inverse Fourier transform relation we can get impulse response \(h(t)\) of the channel from (1)
\[
h(t) = \sum_{i=0}^{L-1} a_i \delta(t - \tau_i)
\]

Now let the transmitted signal \(x(t)\) is a baseband signal \(x_b(t)\) with a carrier frequency \(f_c\). Then
\[
x(t) = Re\{x_b(t)e^{j2\pi f_c t}\} = x_b(t) \cos(2\pi f_c t)
\]
For above discussed multipath propagation system the received signal \(y(t)\) can be given as \(y(t) = x(t) * h(t)\). Where \(*\) is convolution operation.
As convolution follows the distributive and commutative properties [5]
\[ y(t) = \left( \sum_{i=0}^{L-1} a_i \delta(t - \tau_i) \right) * \left( \sum_{i=0}^{L-1} y_b(t) e^{j2\pi f_c t} \right) \]

Where \( y_b(t) \) is received complex baseband signal, given as \( y_b(t) = \sum_{i=0}^{L-1} a_i x_b(t - \tau_i) e^{-j2\pi f_c \tau_i} \).

If the maximum frequency component \( (f_m) \), present in baseband signal \( x_b(t) \), is less than \( \frac{1}{\tau_i} \), where \( i \) is number of path (0 to L-1). Then the signal \( x_b(t) \) is considered as narrowband signal, and for a narrowband signal \( x_b(t - \tau_i) = x_b(t) \) [6]. Hence
\[ y_b(t) = \sum_{i=0}^{L-1} a_i x_b(t) e^{-j2\pi f_c \tau_i} = \left( \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} \right) \cdot x_b(t) = h \cdot x_b(t) \]

Where \( h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} \) is known as wireless channel fading coefficient.

\[ h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} = \sum_{i=0}^{L-1} a_i (\cos(2\pi f_c \tau_i) - j \sin(2\pi f_c \tau_i)) \]
\[ h = \sum_{i=0}^{L-1} a_i \cos(2\pi f_c \tau_i) - j \sum_{i=0}^{L-1} a_i \sin(2\pi f_c \tau_i) = X + jY \]

The wireless fading channel coefficient, \( h \) is a complex random variable with two independent quadrature components \( X \) and \( Y \). The values of \( X \) and \( Y \) are random, depends on attenuation and delay associated with different paths, and assumed to be Gaussian. We can write the individual probability distribution function (PDF) of random variable \( X \) and \( Y \) as
\[ f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{(x-\mu_X)^2}{2\sigma_X^2}} \quad \text{and} \quad f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_Y^2}} e^{-\frac{(y-\mu_Y)^2}{2\sigma_Y^2}} \]

For \( \mu_X = \mu_Y = 0 \) (zero mean) and \( \sigma_X^2 = \sigma_Y^2 = \sigma^2 \) the PDF of \( X \) and \( Y \) will be
\[ f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \quad \text{and} \quad f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} \]

The joint PDF \( f_{XY}(x,y) \) of \( X \) and \( Y \), is calculated as
\[ f_{XY}(x,y) = f_X(x).f_Y(y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \]

The polar form representation of wireless fading coefficient \( h \) is \( h = X + jY = a e^{j\phi} \) where \( a = \sqrt{X^2 + Y^2} \) and \( \phi = \tan^{-1} \frac{Y}{X} \).

To get the joint PDF of \( A \) and \( \phi \) we can use the relation [7]
\[ f_A\phi(a,\phi) = f_{XY}(x,y).\det(J_{XY}) \]

where \( J_{XY} \) is Jacobian matrix given as
\[
J_{XY} = \begin{bmatrix}
\frac{\partial X}{\partial a} & \frac{\partial X}{\partial \phi} \\
\frac{\partial Y}{\partial a} & \frac{\partial Y}{\partial \phi}
\end{bmatrix} = \begin{bmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{bmatrix}
\]
\[ \det(J_{XY}) = \begin{vmatrix}
\cos \phi & \sin \phi \\
-a \sin \phi & -\cos \phi
\end{vmatrix} = a \]

Hence \( f_A\phi(a,\phi) = f_{XY}(x,y).a = \frac{a}{2\pi\sigma^2} e^{-\frac{a^2}{2\sigma^2}} \)

The marginal distribution \( f_A(a) \), with respect to \( a \) can be given as
\[ f_A(a) = \int_{-\infty}^{\infty} f_A\phi(a,\phi) \, d\phi = \frac{a}{2\pi\sigma^2} e^{-\frac{a^2}{2\sigma^2}} \]

where \( 0 \leq a \leq \infty \) (3)
The marginal distribution in equation (3) followed Rayleigh Distribution and termed as Rayleigh fading. Fig. 3 shows the MATLAB simulated Rayleigh distributed fading for variance=0.5,1,1.5 and a=0 to 5.

![Figure 3 Rayleigh Distributed Fading of Wireless Channel](image)

**Figure 3 Rayleigh Distributed Fading of Wireless Channel**

### 2.1 Mathematical Model for Rayleigh fading based wireless communication system

Let \(x_1, x_2, x_3, \ldots, \ldots, x_M\) be M transmitted symbol and \(h\) be the fading channel coefficient as discussed in previous section. The received symbol corresponding to M transmitted symbol \(x_1, x_2, x_3, \ldots, \ldots, x_M\) are \(y_1, y_2, y_3, \ldots, \ldots, y_M\) respectively. This scenario can be modeled in vector form as

\[
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_M
\end{bmatrix} = h \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_M
\end{bmatrix} + \begin{bmatrix}
n_1 \\
n_2 \\
\vdots \\
n_M
\end{bmatrix}, \text{ where } \begin{bmatrix}
n_1 \\
n_2 \\
\vdots \\
n_M
\end{bmatrix} \text{ is uncorrelated additive white gaussian noise vector.}
\]

Alternatively, in simple notation

\[
y = h\tilde{x} + \tilde{n}
\]

(4)

### III. A MATCHED FILTER BASED SS OVER RAYLEIGH FADING CHANNEL

A binary hypothesis testing problem as discussed in [8] can be modified for Rayleigh fading wireless communication system given in equation (4) as

\(H_0: \tilde{y} = \tilde{n}\) (Noise only)

\(H_1: \tilde{y} = h\tilde{x} + \tilde{n}\) (Signal with noise)

Where \(H_0\) is noise only or null hypothesis and \(H_1\) is alternate or signal with noise hypothesis. A matched filter based spectrum sensing is a coherent technique where information of transmitted signal along with the fading coefficient is required to detect the presence of the primary user. From linear algebra principles [9], the matched filtering can be performed by multiplying vector \(\tilde{y}\) with

\[
h^* \tilde{x} \frac{1}{\|\tilde{x}\|}
\]

The two hypotheses after matched filtering operation can be given as

\(H_0: \tilde{y} = h^* \tilde{x} \frac{1}{\|\tilde{x}\|}, \tilde{n} = \tilde{n}\) \[\because \tilde{y} = \tilde{n} \text{ Under hypothesis } H_0\]

\(H_1: \tilde{y} = h^* \tilde{x} \frac{1}{\|\tilde{x}\|}, \tilde{n} = h\tilde{x} + \tilde{n}\) \[\because \tilde{y} = h\tilde{x} + \tilde{n} \text{ Under hypothesis } H_1\]
Alternatively, 
\[ H_0 : \bar{y} = \bar{n} \]  
\[ H_1 : \bar{y} = |h|.||\bar{x}|| + \bar{n} \]  
After matched filtering, the detector compares the output \( \bar{y} \) with a threshold \( \gamma \) (for optimum performance) to decide one of the hypothesis as 
(i) If \( \bar{y} \geq \gamma \), decide \( H_1 \) or primary user is present. 
(ii) If \( \bar{y} < \gamma \), decide \( H_0 \) or primary user is absent. 
The test statistic \( \bar{y} \), under hypothesis \( H_0 \) is noise only, which is assumed to be a uncorrelated complex gaussian noise with zero mean and variance \( \sigma^2 \), under hypothesis \( H_1 \) is noise with signal \( |h|.||\bar{x}|| \), which is complex gaussian signal with mean \( |h|.||\bar{x}|| \) and variance \( \sigma^2 \). Hence we can choose threshold \( \gamma = \frac{1}{2}|h|.||\bar{x}|| \). The probability of detection, calculated from [8] as 
\[ P_D = P_r \left\{ \bar{y} > \frac{1}{2}|h|.||\bar{x}||; H_1 \right\}; \quad T(y) = \bar{y} \text{ and } \gamma = \frac{1}{2}|h|.||\bar{x}|| \] 
\[ P_D = P_r \left\{ ||\bar{x}|| + \bar{n} \geq \frac{1}{2}|h|.||\bar{x}||; H_1 \right\} = P_r \left\{ \frac{\bar{n}}{\sigma/\sqrt{2}} \geq \frac{|h|.||\bar{x}||}{\sigma/\sqrt{2}} \right\} = Q \left( \frac{|h|.||\bar{x}||}{\sigma/\sqrt{2}} \right) \]  
(7)  
The probability of false alarm, calculated from [8] as 
\[ P_{FA} = P_r \left\{ \bar{y} \geq \frac{1}{2}|h|.||\bar{x}||; H_0 \right\}; \quad T(y) = \bar{y} \text{ and } \gamma = \frac{1}{2}|h|.||\bar{x}|| \] 
\[ P_{FA} = P_r \left\{ \frac{\bar{n}}{\sigma/\sqrt{2}} \geq \frac{|h|.||\bar{x}||}{\sigma/\sqrt{2}} \right\} = Q \left( \frac{|h|.||\bar{x}||}{\sigma/\sqrt{2}} \right) \]  
(8)  
The theoretical values of \( P_D \) and \( P_{FA} \) is calculated from equation (7) and (8) for different values of SNR which follows the MATLAB simulated curves as shown in figure 4. 

![ROC Plot for MF based SS for Rayleigh Fading channel](image.png)

**Figure 4** ROC curve for MF based SS for Rayleigh Fading channel

### IV. ENERGY DETECTOR BASED SS OVER RAYLEIGH FADING CHANNEL

A binary hypothesis testing problem for Rayleigh fading wireless communication system as given in section III is 
\[ H_0 : \bar{y} = \bar{n} \text{(Noise only)} \quad \text{and} \quad H_1 : \bar{y} = h\bar{x} + \bar{n} \quad \text{(Signal with noise)} \]
Where \( \mathcal{H}_0 \) is noise only or null hypothesis and \( \mathcal{H}_1 \) is alternate or signal with noise hypothesis. Energy detector based spectrum sensing is a non-coherent technique where the fading coefficient is unknown (assumed to be circularly symmetric complex noise with zero mean and variance \( \sigma^2 \)) and energy of the received signal is used to choose one of the two hypotheses. Here the received signal is first matched filtered by multiplying vector \( \tilde{y} \) with \( \frac{\hat{x}^H}{\|\hat{x}\|} \). The two hypotheses after this operation can be given as

\[
\mathcal{H}_0 : \tilde{y} = \frac{\hat{x}^H}{\|\hat{x}\|} \tilde{\bar{x}} = \frac{\hat{x}^H}{\|\hat{x}\|} \bar{n} = \bar{n} \quad \text{[\( \tilde{y} = \bar{n} \) Under hypothesis \( \mathcal{H}_0 \)]}
\]

\[
\mathcal{H}_1 : \tilde{y} = \frac{\hat{x}^H}{\|\hat{x}\|} \tilde{\bar{x}} = \frac{\hat{x}^H}{\|\hat{x}\|} (h\tilde{x} + \bar{n}) = h\|\tilde{x}\| + \bar{n} \quad \text{[\( \tilde{y} = h\tilde{x} + \bar{n} \) Under hypothesis \( \mathcal{H}_1 \)]}
\]

Alternatively,

\[
\mathcal{H}_0 : \tilde{y} = \bar{n} \\
\mathcal{H}_1 : \tilde{y} = h\|\tilde{x}\| + \bar{n}
\]

(9)

The detector compares the output \( |\tilde{y}|^2 \) with a threshold \( \gamma \) to decide one of the hypotheses as

(i) If \( |\tilde{y}|^2 \geq \gamma \), decide \( \mathcal{H}_1 \) or primary user is present.
(ii) If \( |\tilde{y}|^2 < \gamma \), decide \( \mathcal{H}_0 \) or primary user is absent.

The \( \tilde{y} \), under hypothesis \( \mathcal{H}_0 \) is noise only, which is assumed to be a uncorrelated circularly symmetric complex gaussian (CSCG) noise with zero mean and variance \( \sigma^2 \), under hypothesis \( \mathcal{H}_1 \) is noise with signal \( h\|\tilde{x}\| \), which is CSCG noise in addition to signal with zero mean and variance \( (\sigma^2 + \|\tilde{x}\|^2) \) and variance \( \sigma^2 \). The probability of detection is calculated as

\[
P_D = P_r\{|\tilde{y}|^2 > \gamma; \mathcal{H}_1 \} = Q_X^2 \left( \frac{\gamma}{(\sigma^2 + \|\tilde{x}\|^2)^{1/2}} \right) \quad \text{[\( T(y) = |\tilde{y}|^2 \)]} (11)
\]

The probability of false alarm, calculated as

\[
P_{FA} = P_r\{|\tilde{y}|^2 > \gamma; \mathcal{H}_0 \} = Q_X^2 \left( \frac{\gamma}{\sigma^2/2} \right) \quad \text{[\( T(y) = \tilde{y} \)]} (12)
\]

The theoretical values of \( P_D \) and \( P_{FA} \) is calculated from equation (11) and (12) for different values of SNR which follows the MATLAB simulated curves as shown in figure 5.

![Figure 5 ROC curve for ED-based SS for Rayleigh Fading channel](image)

**V. PERFORMANCE COMPARISON OF ED AND MF BASED SS OVER RAYLEIGH FADING CHANNEL**

Figure 6 shows theoretical and MATLAB simulated performance curves of energy detector and a matched filter based spectrum sensing over Rayleigh fading channel for SNR= -20 dB and -15dB.
A matched filter, filter which maximizes the SNR, outperformed in a fading environment in comparison to energy detector based spectrum sensing. In matched filter based SS at low probability of false alarm, the probability of detection is higher for both SNR values than the energy detector based SS. On the other hand, the requirement of the known fading coefficient at receiver increases the complexity of MF based SS. Further, we note that at low SNR the energy detector performance for the probability of detection is abysmal and hence deep fading may cause system failure.

VI. CONCLUSION

In this paper, we developed a mathematical model for the multipath environment which leads to Rayleigh fading. Further, analytical expressions for the probability of detection and probability of false alarm, under Rayleigh fading, are derived. We find that the ED performance, with low computational complexity, sensing time, cost and power consumption in the nonfading environment whereas a matched filter based detector perform reliably at low SNR. This work motivates research for the performance analysis of matched filter based spectrum sensing with different estimation techniques of fading coefficients.

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