Phase Transitions in Dense Baryonic Matter and Cooling of Rotating Neutron Stars

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New astrophysical instruments such as skA (square kilometer Array) and IXO (formerly Constellation X) promise the discovery of tens of thousands of new isolated rotating neutron stars (pulsars), neutron stars in low-mass X-ray binaries (LMXBs), anomalous X-ray pulsars (AXPs), and soft gamma repeaters (SGRs). Many of these neutron stars will experience dramatic density changes over their active lifetimes, driven by either stellar spin-up or spin-down, which may trigger phase transitions in their dense baryonic cores. More than that, accretion of matter onto neutron stars in LMXBs is believed to cause pycno-nuclear fusion reactions in the inner crusts of neutron stars. The associated reaction rates may be drastically altered if strange quark matter would be absolutely stable. This paper outlines the investigative steps that need to be performed in order to explore the thermal response of neutron stars to rotationally-driven phase transitions in their cores as well as to nuclear burning scenarios in their crusts. Such research complements the exploration of the phase diagram of dense baryonic matter through particle collider experiments, as performed at RHIC in the USA and as planned at the future Facility for Antiproton and Ion Research (FAIR) in Darmstadt, Germany.

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1. Introduction

On the Earth, particle collider experiments enable physicists to cast a brief glance at the properties of ultra-dense and hot baryonic matter (Fig. 1). On the other hand, it is estimated that galaxies like our Milky Way contain between $10^8$ and $10^{10}$ collapsed stars known as neutron stars, which harbor ultra-dense matter permanently in their cores. This key feature together with the unprecedented progress in observational astrophysics, which

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Fig. 1. Schematic phase diagram of strongly interacting matter [17]. The dashed circle indicates the portion of the phase diagram that can be explored by studying either hot and newly formed proto neutron stars (arrow pointing downwards), or cold rotating neutron stars (vertical double-headed arrow) such as millisecond pulsars and neutron stars in low-mass X-ray binaries (LMXBs).

is expected to be excelled by future observatories such as the square kilometer Array (skA) and Constellation-X, make neutron stars superb astrophysical laboratories for a wide range of physical studies. These studies concern nuclear fusion processes on the stellar surface, pycnonuclear reactions in electron degenerate matter at sub-nuclear densities, and the possible formation of boson condensates and other novel states of baryonic matter—like color superconducting quark matter—at super-nuclear densities. (For overviews, see, for instance [1, 2, 3, 4, 5, 6, 7, 8, 9, 10].) More than that, there is the very intriguing theoretical suggestion that strange quark matter could be more stable than atomic nuclei, known as the strange quark matter hypothesis, in which case neutron stars should be largely composed of pure strange quark matter [11, 12, 13]. If quark matter exists in neutron stars it ought to be a color superconductor [5, 14, 15, 16]. Other testable implications of the strange quark matter hypothesis concern the possible existence of a new class of white dwarfs, known as strange white dwarfs [18, 19], and the drastic alteration of heavy-ion reaction rates in the deep crustal layers of neutron stars [20], if strange quark matter nuggets should be present in these layers.

2. Phase transitions in the cores of neutron stars

Rotating neutron stars are called pulsars. Three distinct classes of pulsars are currently known. These are (1) rotation powered pulsars, where the
Fig. 2. Composition of a rotating neutron star in equatorial direction (left panel) and polar direction (right panel) [21]. The star’s mass at zero rotation is $1.40 M_\odot$.

loss of rotational energy powers the emitted electromagnetic radiation, (2) accretion-powered (X-ray) pulsars, where the gravitational potential energy of the matter accreted from a low-mass companion is the energy source, and (3) magnetars, where the decay of a ultra-strong magnetic field powers the radiation. Depending on star mass and rotational frequency, the matter in the core regions of neutron stars may be compressed to densities that are up to an order of magnitude greater than the density of ordinary atomic nuclei. This extreme compression provides a high-pressure environment in which numerous subatomic particle processes are likely to take place [1, 2, 3, 9]. The most spectacular ones stretch from the generation of hyperons and baryon resonances ($\Sigma, \Lambda, \Xi, \Delta$), to quark ($u, d, s$) deconfinement, to the formation of boson condensates ($\pi^-, K^-, H$) [1, 2, 4, 6, 8, 9]. Rapid rotation can change the structure and composition of neutron stars substantially, depending on the equation of state of ultra-dense baryonic matter. The most rapidly rotating, currently known neutron star is pulsar PSR J1748-2446ad, which rotates at a period of 1.39 ms (which corresponds to a rotational frequency of 719 Hz) [22]. It is followed by PSRs B1937+21 [23] and B1957+20 [24] whose rotational periods are 1.58 ms (633 Hz) and 1.61 ms (621 Hz), respectively. Finally, we mention the recent discovery of X-ray burst oscillations from the neutron star X-ray transient XTE J1739–285 [25, 26], which could suggest that XTE J1739–285 contains an ultrafast neutron star rotating 1122 Hz. Rotating neutron stars appear as much better probes for the structure of dense baryonic matter than non-rotating neutron stars, primarily because the particle compositions in rotating neutron stars are not frozen in, as it is the case for non-rotating neutron stars, but are varying with time. The associated density changes can be as large 60% [2] in neutron stars in binary stellar systems (e.g., LMXBs), which are being spun up to high rotational frequencies, or isolated rotating neutron
stars (e.g. isolated millisecond pulsars) which are spinning down to low frequencies because of the emission of gravitational radiation, electromagnetic dipole radiation, and a wind of electron-positron pairs. As an example, we show the rotationally-driven restructuring effects in the cores of standard neutron stars in Figs. 2 and 3. Qualitatively similar restructuring effects were obtained for neutron stars containing quark matter [2, 6].

3. Thermal evolution of rotating neutron stars

Figure 4 illustrates schematically the differences of the cooling behavior of rotating and non-rotating neutron stars. Computing the thermal evolution of rotating neutron stars, however, is considerably more complicated than computing the cooling behavior of non-rotating neutron stars, for several reasons [2]. First, stellar rotation requires solving Einstein’s field equations for rotationally deformed fluid distributions, which renders the problem 2-dimensional. Second, the general relativistic frame dragging (Lense-Thirring) leads to the appearance of additional non-linear field equation. Third, the determination of the general relativistic Kepler frequency, which sets an absolute limit on stable rapid rotation, leads to an additional self-consistency condition [2]. Fourth, the thermal transport equations need to be solved for general relativistic, non-spherical fluids that may experience anisotropic heat transport. Because of all these complications, a fully self-consistent general relativistic treatment of the cooling of rotating neutron stars has not been achieved yet. However, first steps toward this goal were made in [28]. The basic cooling features of a neutron star are easily grasped by considering the energy conservation relation of the star in the Newtonian limit [29]. This equation is given by

\[ \frac{dE_{th}}{dt} = C_V dT/dt = -L_\nu - L_\gamma + H, \]
where $E_{\text{th}}$ is the thermal energy content of a neutron star, $T$ its internal temperature, and $C_V$ its total specific heat. The energy sinks are the total neutrino luminosity, $L_{\nu}$, and the surface photon luminosity, $L_{\gamma}$. The source term $H$ includes all possible heating mechanisms [29], which, for instance, convert magnetic or rotational energy into heat. The dominant contributions to $C_V$ come from the core whose constituents are leptons, baryons, boson condensates and possibly deconfined superconducting quarks. When baryons and quarks become paired, their contribution to $C_V$ is strongly suppressed at temperatures smaller than the critical temperatures associated with these pairing phases. The crustal contribution is in principle dominated by the free neutrons in the inner stellar crust but, since these are extensively paired, practically only the nuclear lattice and electrons contribute. Extensive baryon, and quark, pairing can thus significantly reduce $C_V$. In order to derive the general relativistic version of Eq. \((1)\) for rotating stars, one needs to solve Einstein’s field equations using the metric of a rotationally deformed fluid [2],

$$
\begin{align*}
\text{\textit{ds}}^2 &= -e^{2\nu} dt^2 + e^{2\phi} (d\varphi - N^\varphi dt)^2 + e^{2\omega} (dr^2 + r^2 d\theta^2),
\end{align*}
$$

\text{\textit{ds}}^2 = -e^{2\nu} dt^2 + e^{2\phi} (d\varphi - N^\varphi dt)^2 + e^{2\omega} (dr^2 + r^2 d\theta^2),
\tag{2}
$$

where $e^{2\phi} \equiv e^{2(\alpha+\beta)} r^2 \sin^2 \theta$ and $e^{2\omega} \equiv e^{2(\alpha-\beta)}$. The quantities $\nu$, $\phi$ and $\omega$ denote metric functions, and $N^\varphi$ accounts for frame dragging caused by
the rotating fluid. All these functions are to be computed self-consistently from Einstein’s field equation, \( G^{\bar{\alpha}\bar{\beta}} = 8\pi T^{\bar{\alpha}\bar{\beta}} \), where \( T^{\bar{\alpha}\bar{\beta}} \) denotes the fluid’s energy momentum tensor. In the case of neutron star cooling, the equations which describe the conservation of energy and momentum in a co-moving reference frame follow from \( T^{\bar{\alpha}\bar{\beta}} \).

\[
T^{\bar{\alpha}\bar{\beta}} = \begin{pmatrix}
J & H_r & H_\theta & H_\varphi \\
H_r & J/3 & 0 & 0 \\
H_\theta & 0 & J/3 & 0 \\
H_\varphi & 0 & 0 & J/3
\end{pmatrix}, \quad q^{\bar{\alpha}} = \begin{pmatrix}
-\epsilon \\
-H_r/\Lambda \\
-H_\theta/\Lambda \\
-H_\varphi/\Lambda
\end{pmatrix}.
\]

The quantity \( q^{\bar{\alpha}} \) describes the interaction of photons with matter, \( J = aT^4 \) is the energy density of the photon field, \( H_i \) is the \( i \)th component of the heat flux, \( \epsilon \) the emitted energy, and \( \Lambda \) the mean free path of photons. From Eq. (3) one obtains

\[
q^{\bar{\alpha}} = e^{\beta}_{\bar{\alpha}} T^{\bar{\alpha}\beta} + \gamma^{\bar{\alpha}}_{\bar{\beta}} T^{\bar{\gamma}\bar{\beta}} + \gamma^{\bar{\beta}}_{\gamma\bar{\beta}} T^{\bar{\alpha}\gamma},
\]

where the \( \gamma \)'s denote the Ricci rotation coefficients given by \( \gamma^{\bar{\alpha}}_{\bar{\beta}\gamma} = e^{\beta}_{\bar{\alpha}} e^{\alpha}_{\gamma}. \) Combining Eqs. (3) and (4) leads to the general relativistic transport equations

\[
\partial_r \tilde{H}_r + \frac{1}{r} \partial_\theta \tilde{H}_\theta = -r e^{\phi+2\omega} \left( \frac{1}{\Gamma} e^{2\nu} \epsilon + \Gamma C_V \partial_t \tilde{T} \right) - r \Gamma U e^{\nu+2\phi+\omega} \left( \partial_r \Omega + \frac{1}{r} \partial_\theta \Omega \right),
\]

\[
\partial_r \tilde{T} = -\frac{1}{r\kappa} e^{-\nu-\phi} \tilde{H}_r - \Gamma^2 U e^{-\nu+\phi} \tilde{T} \partial_r \Omega,
\]

\[
\frac{1}{r} \partial_\theta \tilde{T} = -\frac{1}{r\kappa} e^{-\nu-\phi} \tilde{H}_\theta - \Gamma^2 U e^{-\nu+\phi} \tilde{T} \frac{1}{r} \partial_\theta \Omega,
\]

\[
\Gamma U \partial_t \tilde{T} = -\frac{1}{r\kappa} e^{-\omega-\phi} \tilde{H}_\varphi,
\]

where \( \tilde{H}_i \equiv r e^{2\nu+\phi+\omega} H_i/\Gamma \), \( \tilde{T} \equiv e^{\nu} T/\Gamma \), and the Lorentz factor \( \Gamma \equiv (1 - U^2)^{-1/2} \). In the case of uniform rotation, \( \Omega = \text{const} \), Eqs. (5) to (8) reduce to

\[
\partial_r \tilde{H}_r + \frac{1}{r} \partial_\theta \tilde{H}_\theta = -r e^{\phi+2(\alpha-\beta)} \left( \frac{1}{\Gamma} e^{2\nu} \epsilon + \Gamma C_V \partial_t \tilde{T} \right),
\]

\[
\partial_r \tilde{T} = -\frac{1}{r\kappa} e^{-\nu-\phi} \tilde{H}_r,
\]

\[
\frac{1}{r} \partial_\theta \tilde{T} = -\frac{1}{r\kappa} e^{-\nu-\phi} \tilde{H}_\theta.
\]
The standard cooling equations of spherically symmetric, non-rotating neutron stars are obtained from Eqs. (9) to (11) for $\Omega = 0$ and $\partial_\theta \tilde{T} = 0$ [2]. This project aims at solving Eqs. (5) to (7) for the temperature distribution $T(r, \theta; t)$ of non-spherical, (possibly differentially) rotating neutron stars. The boundary condition are given by defining $\tilde{H}_r$ at $r = 0, R$, and $\tilde{H}_\theta$ at $\theta = 0, \pi/2$ and at $r = R$, with $R$ denoting the stellar radius. The choice for the star's initial temperature, $T(r, \theta; t = 0)$, is typically chosen as $\tilde{T} \equiv 10^{11}$ K. Equations (6) and (7) can be solved for $\tilde{H}_r$ and $\tilde{H}_\theta$, differentiated with respect to $\partial_r$ and $\partial_\theta$, respectively, and substituted into Eq. (5), which leads to the following parabolic differential equation,

$$
\partial_t \tilde{T} = -\frac{1}{\Gamma e^{2\nu}} \frac{e^{\nu+\gamma-\xi}}{C_V} \left( \partial_r \Omega + \frac{1}{r} \partial_\theta \Omega \right)
+ \frac{1}{r^2 \sin \theta} \frac{1}{e^{2\nu-\gamma-2\xi}} \frac{1}{C_V} \left( \partial_r \left( r^2 \kappa \sin \theta e^{\gamma} \left( \partial_r \tilde{T} + \Gamma^2 U e^{-2\nu+\gamma} \tilde{T} \partial_\theta \Omega \right) \right) \right)
+ \frac{1}{r^2} \partial_\theta \left( r^2 \kappa \sin \theta e^{\gamma} \left( \partial_\theta \tilde{T} + \Gamma^2 U e^{-2\nu+\gamma} \tilde{T} \partial_r \Omega \right) \right),
$$

(12)

with the definitions $r \sin \theta e^{-\nu+\gamma} = e^\phi$ and $e^{-\nu+\xi} = e^{\alpha-\beta}$. It is this differential equation that needs be solved numerically in combination with a general relativistic stellar rotation code, as illustrated schematically in Fig. 5. A 2-dimensional, general relativistic stellar rotation code is to be used to determine the metric functions, frame dragging frequency, pressure and density gradients, and particle compositions of a deformed neutron star as a function of rotational frequency. Depending on the cooling channels that are active at a given rotational frequency, the numerical outcome obtained from the rotation code then serves as an input for the general relativistic 2-dimensional thermal evolution code, which is used to determine the luminosity, and thus the surface temperature, of a deformed rotating neutron star. As outlined in Sect. 2, because of stellar spin-down or spin-up, the density in such stars may change dramatically so that new cooling channels open up (or close) with time (Fig. 4). The new cooling channels imply neutrino emissivities, heat capacities, and thermal conductivities that are different from the original ones, thus altering the stars thermal response. It is this response in the thermal behavior of rotating neutron stars that carries information about the properties of the matter in the dense baryonic stellar cores and the deep crustal layers. Several key research activities that can be addressed with these codes are briefly summarized next:

- Study the impact of latent heat, released by rotation-driven quark-hadron phase transitions in the cores of isolated rotating neutron stars, on the thermal evolution of such objects [28].
• Simulate the impact of latent heat release on the thermal evolution of accreting neutron stars in LMXBs. Because of the spin-up of such neutron stars, these objects becomes progressively more decompressed with time, which may lead to drastic changes in their core compositions, causing the destruction of certain states (K- condensate, quark matter) of dense baryonic matter that were initially present at slow stellar rotation rates. This modifies the star’s thermal evolution and may lead to signals indicative of the existence of phase transitions in the star.

• Study the impact of energy released by pycnonuclear burning of matter in the inner crusts of neutron stars. In particular, the dependence of the stellar surface temperature on the possible presence of strange quark matter nuggets in the inner crusts of neutron stars can be explored. As shown in Fig. 6, the nuclear reactions rates are strongly modified if strange quark matter nuggets would be present in the crusts of neutron stars [20].

• The thermal evolution of neutron stars undergoing episodes of intense accretion, alternated by long periods of quiescence, i.e. soft X-ray transients (SXRTs) such as SAX J1808.4–3658, can be explored. This neutron star is the first known accretion powered millisecond pulsars rotating rapidly (at 2.5 ms, 400 Hz). At such small spin periods certain phases of dense matter may have been spun out of the core,
altering the neutrino loss. The quiescent emission of SAX J1808.4–
3658, therefore, opens the possibility of using this object as a tool
for probing the core compositions of neutron stars [29, 30]. Another,
and still poorly explored, aspect of SXRTs is their short time-scale
thermal response to the accretion phases. This is an aspect of the
problem from which extremely important information about both the
structure of the neutron star crust and the thermal state of its core
can be obtained [31, 32], and about which intriguing observational
results were found [33].

• Another interesting problem concerns the thermal evolution of neu-
tron stars immediately after they went through the proto-neutron star
phase. Of particular interest is the crystallization behavior of such
matter and the occurrence of thermoelectric instabilities at the core–
crust boundary.

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