Comparison of two analysis methods on bubble size distribution

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Abstract. The object of this work was to analyze foam bubbles’ diameter distribution. Firstly, Rosin-Rammler distribution function was used in bubble size analysis in some studies. This Rosin-Rammler distribution function should be modified to fit foam bubbles distribution curve. Secondly, coefficient of variation value was introduced in our study. This analysis method could be carried out directly without any modification. Thirdly, these two methods were compared whose characteristics were analyzed. Both analysis methods were effective to analyze foam bubbles figures with relatively obvious difference.

1 Backgrounds

Particle-size distribution of particles could be described by a mathematical function that defines the relative amount such as mass, diameter or density [1]. Particles could be solid powder in air, granular material in biological tissue, foam bubble in liquid, etc. The particle-size distribution is important in understanding particles’ properties.

Liquid foam could be produced by driving air into a foam solution through a generating device. Foam bubbles are formed by trapping pockets of gas in liquid [2]. Then these foams could be applied in different areas. The foam flooding could be used to press out the crude oil from underground. The floating foam could be used to separate ore in mine mountain. The extinguishing foam could be used to put out fire.

To obtain extinguishing foams, there are generally two principles to generate foam as shown in Figure 1 and Figure 2 [3]. To aspirated air foam, the air at normal pressure is aspirated into flowing foam solution by a low expansion foam extinguishing system. Then the aspirated air foams could be used in a close distance. To compressed air foam, the air at higher pressure is pressed into flowing foam solution by a compressed air foam system. Then the compressed air foams could be transferred to the destination long distance away.

![Figure 1. The principle of aspirated air foam generation.](image-url)
These foams are relatively stable system. This system would become unstable as the condition changes including time, environment temperature and air pressure. As time goes by, the liquid between foam membranes will down flow caused by gravity. So this drainage phenomenon is quite common after foam generation. Then the shape, diameter and number of the foam bubbles will change. The particle-size distribution means foam bubbles’ diameter distribution. This distribution will also change during this drainage process [4].

There are many methods to analyze foam bubbles’ diameter distribution. In some studies, the Rosin-Rammler distribution function is modified to fit the foam bubbles’ diameter distribution curve [5]. In our previous study, the coefficient of variation value is also introduced to describe this distribution directly [6]. Therefore, these two methods should be compared in parallel in this paper. The characteristics of these two methods would be also studied.

2 Rosin-Rammler function

2.1 Theory of Rosin-Rammler distribution function

A function is a relation between a set of inputs and a set of permissible outputs. Each input is related to one output. To particle size distribution, some functions could be used to describe distribution curve exactly. Then this distribution function could be used to analyze particle’s property [5].

Most of the distribution functions have been used to study solid particles. Frechet identified Weibull distribution firstly in 1927. Then Rosin & Rammler applied this distribution firstly in 1933 [7]. This distribution could be used to describe solid particle size distribution curve in mineral processing. In some papers, the relation equation between coefficient and index is studied using Rosin-Rammler distribution function below, where D means particle diameter, N means index, A means coefficient, Q(D) means cumulative percentage of D.

\[ Q(D) = 1 - \exp[-A \cdot \ln(D)^N] \]

To describe foam bubble’s diameter distribution more exactly, the Rosin-Rammler distribution function is modified. According to mathematical mechanism, the higher the value of N, the more uniform the foam bubble’s distribution.

2.2 Application of modified Rosin-Rammler distribution function in foam bubbles’ characterization

In some studies, the foam bubble structure images could be obtained using stereo microscope or camera. Then Rosin-Rammler distribution function is modified to fit the foam bubbles’ diameter distribution curve to compare the aspirated air foam and compressed-air foam. Two images were chosen to compare the two analysis methods in parallel as shown in Figure 3 and Figure 4.
Figure 3. The image of aspirated air foam.

Figure 4. The image of compressed air foam.

The modified Rosin-Rammler distributions of aspirated air foam is obtained. The square spots were experimental aspirated air foam bubbles’ number percentage. The solid line was the modified Rosin-Rammler distribution curve. The index \( N_{\text{aspirated}} \) to aspirated air foam was 2.79.

\[
Q(D)=1-\exp\left[-2.65246E-7*\ln(D)^{2.79}\right]
\]

Figure 5. The aspirated air foam bubble distribution

The modified Rosin-Rammler distributions of compressed air foam were obtained as shown in Figure 6. The circle spots were the experimental compressed air foam bubbles’ number. The dotted
line was the modified Rosin-Rammler distribution curve. Then the index $N_{\text{compressed}}$ to compressed air foam was 3.04.

$$Q(D)=1-\exp[-1.28991E-8*\ln(D)^{3.04}]$$

![Figure 6. The compressed air foam bubble distribution](image)

Then, $N_{\text{compressed}}=3.04$ was higher than $N_{\text{aspirated}}=2.79$. According to mathematical mechanism, the higher the value of $N$, the more uniform the foam bubble’s distribution. So compressed air foam bubbles were more uniform in size or diameter than aspirated air foam bubbles. This result was in accordance with exception. For the compressed air foam, the air at higher pressure was pressed into flowing foam solution and the fully mixed at high pressure, leading to more uniform foams in size. So this method, the modified Rosin-Rammler distribution function, could be used to describe foam bubbles’ diameter distribution curve.

## 3 Coefficient of variation

### 3.1 Theory of coefficient of variation

The coefficient of variation is a standardized measure of dispersion of a probability distribution or frequency distribution \[^{[8]}\]. It is equal to the ratio of the standard deviation to the mean. In this coefficient of variation, $\sigma$ is standard deviation, $\mu$ is the mean. $C_V$ is independent of the unit in which the measurement has been taken. This $C_V$ is a dimensionless number.

$$C_V = \frac{\sigma}{\mu}$$

$$\sigma = \left\{ \frac{1}{N} \left[ (D_1-\mu)^2 + (D_2-\mu)^2 + \ldots + (D_N-\mu)^2 \right] \right\}^{0.5}$$

$$\mu = \frac{1}{N} (D_1 + D_2 + \ldots + D_N)$$

To describe foam bubble’s diameter distribution, the coefficient of variation was introduced in our study \[^{[6]}\]. The lower the value of $C_V$, the more uniform the foam bubble’s distribution.

### 3.2 Application of coefficient of variation in foam bubbles’ characterization

For comparison, the foam bubbles were recognized and the data were analyzed aromatically as shown in Figure 6 and Figure 7. Then the particle size percentage and cumulative percentage to aspirated air foam bubble was obtained as shown in Figure 7, Table 1, and Figure 8, Table 2.
**Figure 7.** The recognized foam bubbles to aspirated air foam.

**Figure 8.** The recognized foam bubbles to compressed air foam.

**Table 1.** Diameter range and cumulative percentage to aspirated air foam bubbles.

| Minimum Diameter (µm) | Maximum Diameter (µm) | Cumulative Percentage |
|------------------------|------------------------|-----------------------|
| 0                      | 50                     | 0                     |
| 50                     | 100                    | 0.0063                |
| 100                    | 200                    | 0.2351                |
| 200                    | 250                    | 0.5768                |
| 250                    | 300                    | 0.7513                |
| 300                    | 400                    | 0.8339                |
| 350                    | 450                    | 0.9394                |
| 400                    | 500                    | 0.9592                |
| 450                    | 550                    | 0.9728                |
| 500                    | 600                    | 0.9801                |
| 550                    | 650                    | 0.9864                |
| 600                    | 700                    | 0.9916                |
| 650                    | 750                    | 0.9937                |
| 700                    | 800                    | 0.9958                |
| 800                    | 850                    | 0.9958                |
| 850                    | 900                    | 1                     |
Table 2. Diameter range and cumulative percentage to compressed-air foam bubble.

| Minimum Diameter [μm] | Maximum Diameter [μm] | Cumulative Percentage |
|-----------------------|------------------------|----------------------|
| 0                     | 50                     | 0                    |
| 50                    | 100                    | 0                    |
| 100                   | 150                    | 0.0134               |
| 150                   | 200                    | 0.1119               |
| 200                   | 250                    | 0.2304               |
| 250                   | 300                    | 0.3669               |
| 300                   | 350                    | 0.4966               |
| 350                   | 400                    | 0.6622               |
| 400                   | 450                    | 0.7785               |
| 450                   | 500                    | 0.8508               |
| 500                   | 550                    | 0.9329               |
| 550                   | 600                    | 0.9687               |
| 600                   | 650                    | 0.9866               |
| 650                   | 700                    | 0.9978               |
| 700                   | 750                    | 1                    |
| 750                   | 800                    | 1                    |
| 800                   | 900                    | 1                    |

The values of $C_v$ were 0.497 and 0.396, respectively, for aspirated air foam and compressed air foam. So the compressed-air foam bubbles had a narrower distribution than aspirated air foam bubbles. This result was also in accordance with the result of modified Rosin-Rammler distribution function. So this method, coefficient of variation value, could also be used to describe foam bubbles’ diameter distribution curve.

4 Parallel comparison

For extinguishing foams, two analysis methods were compared. To some images like Figure 3 and Figure 4 with relatively obvious difference, these two methods were both effective.

These two method’s characteristics were also analyzed. To the Rosin-Rammler distribution, this function should be modified to fit the foam’s distribution curve more suitable. The Rosin-Rammler distribution function was mainly used to describe the whole particle distribution curve exactly. So Rosin-Rammler distribution method needed more complex mathematics calculation. For the coefficient of variation, this value could be obtained after relatively simple mathematics calculation. The coefficient of variation was mainly used to describe one value of particle distribution curve. So coefficient of variation needed more foam bubbles as sample target.

In addition, it was still not sure whether these two methods could be used to analyze the figures with less obvious difference. Further study should even focus on whether these two results were opposite to some special images. Therefore further study should focus on various foam bubbles’ images with different distribution situation.

5 Conclusions

In summary, the Rosin-Rammler distribution function and the coefficient of variation value were compared in this paper. Both methods could be used to analyze the foam bubbles’ figures with relatively obvious difference in parallel. This study could be applied to understand the stability of extinguishing foam. The extinguishing effect would be improved by adjusting foam’s stability. In the future, more images with various foam babble distributions should be analyzed using these two methods.

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