Heavy Hadron Chiral Perturbation Theory

Peter Cho †
Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

The formalism and applications of chiral perturbation theory for hadrons containing a single heavy quark are discussed. We emphasize the utility of working directly with the velocity dependent “super” fields which appear in the chiral Lagrangian and whose interactions manifestly preserve heavy quark spin symmetry rather than their individual spin components. Chiral logarithm corrections to meson and baryon Isgur-Wise functions are found using these fields.

We also identify a unique dimension-five operator which couples the axial vector Goldstone current to the heavy antitriplet baryon field $T_Q$. We then compute the differential rate for the spin symmetry violating decay $T_b(v) \rightarrow T_c(v')\ell\nu\pi$. The ratio of this decay rate to that for the corresponding pure semileptonic transition can be studied away from the zero recoil point.

† Address after Sept 21, 1992: California Institute of Technology, Pasadena, CA 91125.
1. Introduction

Chiral Perturbation Theory and the Heavy Quark Effective Theory (HQET) have been widely studied in the past in separate contexts. Recently however, a synthesis of these two effective field theories of hadronic physics has been explored [1–5]. A chiral Lagrangian framework for analyzing the interactions of light Goldstone bosons with hadrons containing a heavy quark has been developed. One can use this formalism to investigate $b \to c$ semileptonic transitions with soft pion or kaon emission. Heavy hadron decays to uncharmed final states with low momentum Goldstone bosons have also been examined [6]. Other applications have included studies of chiral log corrections to heavy meson decay constants [7] and Isgur-Wise functions [8], excited meson state transitions [9], and heavy flavor conserving nonleptonic weak decays [10].

The lineage of this new hybrid theory can be traced back to the standard model. Starting from the underlying full theory, running down in energy from the electroweak scale with the renormalization group, and removing heavy degrees of freedom as their particle thresholds are crossed, one generates the following tower of effective field theories:

- Minimal Standard Model with 6 Quarks
  $\downarrow \mu = m_t$
- Minimal Standard Model with 5 Quarks
  $\downarrow \mu = m_W$
- Four Fermion Theory with 5 Quarks
  $\downarrow \mu = m_b$
- HQET with 4 Light Quarks and 1 Heavy Quark
  $\downarrow \mu = m_c$
- HQET with 3 Light Quarks and 2 Heavy Quarks
  $\downarrow \mu = \Lambda_\chi$

The hybrid chiral theory thus appears as the direct descendant of heavy quark theory. As we shall see, the structure of operators in the former are significantly constrained by their progenitors in the latter. Matching between the two theories occurs at the chiral symmetry breaking scale $\Lambda_\chi$.

In this paper, we are interested in extending both the formalism and applications of Heavy Hadron Chiral Perturbation Theory. We first review the construction of its leading
order Lagrangian. The utility of working directly with the velocity dependent “super” fields which appear in the chiral Lagrangian and manifestly respect heavy quark spin symmetry rather than their individual spin components is emphasized. Chiral and flavor symmetry breaking effects are then discussed, and logarithmic contributions to meson and baryon Isgur-Wise functions are calculated. Finally, $O(1/m_Q)$ corrections to the Lagrangian are incorporated, and the differential rate for antitriplet baryon semileptonic decay with soft Goldstone boson emission is determined.

2. Leading Order Lagrangian

In the limit where the up, down and strange current quark masses are set equal to zero, the QCD Lagrangian respects a global $SU(3)_L \times SU(3)_R$ symmetry. Nonperturbative strong interactions break this chiral symmetry down to its diagonal flavor subgroup $SU(3)_{L+R}$. The Goldstone bosons associated with the spontaneous symmetry breaking appear in the pion octet

$$\pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^0 \\ K^- & K^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}. \quad (2.1)$$

One can build a theory for these massless mesons following the classic phenomenological Lagrangian formalism of Callan, Coleman, Wess and Zumino [11]. The pion octet is first exponentiated into the fields $\Sigma = e^{2i\pi/f}$ and $\xi = \sqrt{\Sigma} = e^{i\pi/f}$ which transform linearly and nonlinearly respectively under $SU(3)_L \times SU(3)_R$:

$$\Sigma \to L \Sigma R^\dagger$$

$$\xi \to L \xi U^\dagger = U \xi R^\dagger. \quad (2.2)$$

Here $L$ and $R$ represent global elements of $SU(3)_L$ and $SU(3)_R$, while $U$ acts like a local $SU(3)_{L+R}$ transformation which depends in a complicated way upon $L, R$ and $\pi(x)$. Chiral invariant terms that describe Goldstone boson self interactions can then be constructed from the fields in (2.2) and their derivatives.

Matter fields representing hadrons containing a heavy quark $Q$ may be included into the chiral theory. Goldstone bosons derivatively couple to such matter fields via the vector and axial vector combinations

$$V^\mu = \frac{1}{2}(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger) = \frac{1}{2f^2} [\pi, \partial^\mu \pi] - \frac{1}{24f^4} [\pi, [\pi, [\pi, \partial^\mu \pi]]] + O(\pi^6)$$

$$A^\mu = \frac{i}{2}(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger) = -\frac{1}{f} \partial^\mu \pi + \frac{1}{6f^3} [\pi, [\pi, \partial^\mu \pi]] + O(\pi^5)$$
which transform inhomogeneously and homogeneously under $SU(3)_{L+R}$ respectively:

$$V^\mu \rightarrow U V^\mu U^\dagger + U \partial^\mu U^\dagger$$

$$A^\mu \rightarrow U A^\mu U^\dagger.$$ 

The interactions of heavy mesons and baryons with the pion octet are fixed by their transformation properties under the unbroken flavor subgroup. In the limit that their $Q$ constituents are infinitely massive, the matter fields travel along straight worldlines and their four-velocities are unaffected by Goldstone boson absorption or emission. The heavy hadrons are consequently described by velocity dependent fields.

We will restrict our attention to the lowest lying heavy hadrons that correspond to ground states in the quark model with zero orbital and radial excitation. In the meson sector, we introduce the fields $P_i(v)$ and $P^{*i}_\mu(v)$ which annihilate pseudoscalar and vector mesons with quark content $Qq$. The heavy quark spin symmetry rotates these operators into one another and is automatically taken into account if they are combined into the $4 \times 4$ matrix fields [2,12]

$$H_i(v) = \frac{1 + \gamma^5}{2} \left[ -P_i(v)\gamma^5 + P^{*i}_\mu(v)\gamma^\mu \right]$$

$$\overline{H}^i(v) = \left[ P^{*i}(v)\gamma^5 + P^{*\dagger\mu}_\mu(v)\gamma^\mu \right] \frac{1 + \gamma^5}{2}.$$ 

$H$ then transforms as an antitriplet matter field under $SU(3)_{L+R}$ and as a doublet under $SU(2)_v$:

$$H_i \rightarrow e^{i\vec{S}_v \cdot \vec{S}_i} H_j (U^\dagger)_i^j.$$ 

The matrix field obeys the LHS and RHS constraints

$$\frac{1 + \gamma^5}{2} H_i(v) = H_i(v) \quad (2.4a)$$

$$H_i(v) \frac{1 - \gamma^5}{2} = H_i(v) \quad (2.4b)$$

which project out its two heavy quark and two light antiquark degrees of freedom. Therefore $H_i$ has a total of four degrees of freedom and precisely accommodates one $J^P = 0^-$ and three $J^P = 1^-$ meson states.

Baryons with quark content $Qqq$ enter into the theory in two incarnations depending upon the angular momentum of their light degrees of freedom ("brown muck"). In the first case, the spectators carry one unit of angular momentum and couple with the heavy
spin-$\frac{1}{2}$ quark to form $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ states. Again it is useful to combine the Dirac and Rarita-Schwinger operators $B^{ij}(v)$ and $B^{*ij}_\mu(v)$ associated with these baryon states into the fields \[13\]

$$S^{ij}_\mu(v) = \sqrt{\frac{1}{3}}(\gamma_\mu + v_\mu)\gamma^5 B^{ij}(v) + B^{*ij}_\mu(v)$$

$$\overline{S}^{ij}_\mu(v) = -\sqrt{\frac{1}{3}}B^{ij}(v)\gamma^5(\gamma_\mu + v_\mu) + \overline{B}^{*ij}_\mu(v).$$

$S$ transforms as a sextet under $SU(3)_{L+R}$, doublet under $SU(2)\_v$, and axial vector under parity:

$$S^{ij}_\mu \rightarrow e^{i\vec{S}_v \cdot \vec{U}}U^i_j U^j_k S^{kl}_\mu.$$  

The constraints obeyed by $S^{ij}_\mu$

$$\frac{1 + \frac{\gamma_5}{2} S^{ij}_\mu}{2} = S^{ij}_\mu$$

$$v^{\mu} S^{ij}_\mu = 0$$  

implies that it has six degrees of freedom which account for its two spin-$\frac{1}{2}$ and four spin-$\frac{3}{2}$ states.

The spectators in the remaining heavy baryons are arranged in a spin zero configuration. The resulting $J^P = \frac{1}{2}^+$ baryons are assigned to the field $T_i(v)$ which is an $SU(3)_{L+R}$ antitriplet and $SU(2)\_v$ doublet:

$$T_i \rightarrow e^{i\vec{S}_v \cdot \overline{T}_j (U^i_j)^{\dagger}}.$$  

The $SU(2)\_v$ symmetry simply rotates the spins of these baryons. The condition

$$\frac{1 + \frac{\gamma_5}{2} T_i(v)}{2} = T_i(v)$$  

projects out the $T_i$ field’s two heavy baryon degrees of freedom.

We can now construct the phenomenological Lagrangian which describes the low energy interactions between light Goldstone and heavy hadron fields in the infinite heavy quark mass limit. The leading terms must be hermitian, Lorentz invariant, and symmetric under $SU(3)_{L+R}$, $SU(2)\_v$ and parity. They can be written down by inspection and appear in $d = 4 - \epsilon$ dimensions as

$$L^{(0)}_\pi = \frac{\Lambda - \epsilon f^2}{4} \text{Tr}(\partial^\mu \Sigma \Sigma^\dagger \partial_\mu \Sigma)$$  

$$L^{(0)}_v = \sum_{Q=c,b} \left\{ -i \text{Tr}(\overline{H}^i v \cdot \partial H^i) - i S^{ij}_\mu v \cdot \partial S^{ij}_\mu + \Delta M S^{ij}_\mu S^{kl}_\mu + i T^i v \cdot \partial T_i + g_1 \text{Tr}(H^i(A)^{\dagger j}_i \gamma^5 \overline{T}^{ij}) + ig_2\epsilon_{\mu \nu \sigma \lambda} S^{ij}_k v^\nu (A^{\sigma})^i_j (S^\lambda)^{jk}_i \\
+ g_3 \left[ \epsilon_{ijk} (A^{\mu})^i_j S^{kl}_\mu + \epsilon^{ijk} \overline{T}^{ij}_k (A^{\mu}_j) T_i \right] \right\}.$$  

(2.8b)
Several points about these lowest order contributions should be noted. Firstly, the parameter $f$ in the pion Lagrangian equals the pion decay constant at leading order. Its original mass dimension varied with $d$. However, the $d$ dependence is now absorbed into the renormalization scale $\Lambda$ which appears alongside the pion decay constant in (2.8a). Henceforth, $f \approx 93$ MeV has mass dimension one while the $g_i$ couplings in the heavy hadron Lagrangian are dimensionless. Secondly, in order to remove all heavy mass dependence from the zeroth order Lagrangian, we have expressed the meson contributions to (2.8b) in terms of the dimension-$\frac{3}{2}$ field $H' = \sqrt{M_H} H$. The interactions of this matrix field are significantly restricted by the constraints on its heavy quark and light antiquark degrees of freedom. In particular, the vanishing of the candidate meson interaction term $\text{Tr}(H'\nu A\gamma^5 H')$ follows immediately from eqn. (2.4b). Thirdly, splitting between the sextet and antitriplet baryon multiplets has been absorbed into the parameter $\Delta M = M_S - M_T$. Although this intramultiplet mass difference is phenomenologically comparable in size to intermultiplet breaking, it remains fixed at a nonzero value in the limit of exact flavor symmetry. This splitting is also independent of the baryons’ heavy quark constituent masses. Finally, observe that there is no interaction term between the antitriplet baryons and axial vector Goldstone field in Lagrangian (2.8b). Such an interaction is forbidden by heavy quark spin symmetry in the infinite mass limit [4,5].

It is important to recall that the rest energies of the heavy hadrons have been removed from their velocity dependent fields. Partial derivatives acting on matter fields inside the covariant derivatives

\[ D^\mu H'_i = \partial^\mu H'_i - H'_j (V^\mu)^{ji} \]
\[ D^\mu S^{ij} \nu = \partial^\mu S^{ij} \nu + (V^\mu)^i_k S^{kj} \nu + (V^\mu)^j_k S^{ik} \nu \]
\[ D^\mu T_i = \partial^\mu T_i - T_j (V^\mu)^j_i \]

therefore yield residual momenta $k = p - m_Q v$. Since mesons and baryons containing a heavy quark propagate almost on shell, their residual momenta are small compared to the chiral symmetry breaking scale. The ratio $k/\Lambda_\chi$ consequently serves as a sensible momentum expansion parameter, and the single derivative terms in (2.8b) represent the dominant contributions to the low energy Lagrangian.

In previous studies of chiral perturbation theory for hadrons containing a heavy quark, investigators have generally decomposed the meson and baryon fields into their individual
spin components. However, this is unnecessary and counterproductive for many applications. It is much simpler to work directly with the $H$, $S$ and $T$ “super” fields whose interactions manifestly preserve heavy quark spin symmetry. As the zeroth order Lagrangian is devoid of gamma matrix structure, these fields’ Feynman rules are significantly easier to manipulate than those for their individual spin components. Moreover, the number of diagrams which contribute at any given order to a particular heavy hadron process is minimized. So the use of these “super” fields significantly simplifies and clarifies calculations [14].

Heavy hadron propagators and vertices are listed in fig. 1. The velocity dependent fields’ propagators are fixed by constraints (2.4), (2.6) and (2.7), while their vertices can be read off from (2.8). We have drawn the $H$ propagator in t’Hooft double line notation in order to keep separate track of the matrix field’s heavy quark and light antiquark spinor indices. We have also portrayed heavy hadron propagators as thick, straight lines. These serve as reminders that the heavy quark constituents of the mesons and baryons barrel through graphs unimpeded while their light degrees of freedom emit and absorb pions.

One could continue to develop the leading order formalism. For example, excited heavy meson states [8] or baryons with quark content $QQq$ [15] can be included into the chiral Lagrangian. Alternatively, one may apply the formalism developed so far to study strong interaction transitions among heavy hadrons with soft Goldstone boson emission [2–5]. However we turn at this point to explore subleading symmetry breaking effects in the following two sections.

3. Chiral Symmetry Breaking

The chiral and flavor symmetries of the QCD Lagrangian are explicitly broken by current quark masses. We incorporate the effects of this chiral symmetry breaking into the low energy theory by introducing a “spurion” field $\mathcal{M}$ which transforms as $(3,\bar{3}) + (\bar{3},3)$ under $SU(3)_L \times SU(3)_R$. We write down all contributions to the effective Lagrangian which are linear in $\mathcal{M}$

$$L_{\pi}^{(\mathcal{M})} = \frac{\Lambda^\epsilon f^2}{2} \text{Tr}(\Sigma^\dagger \mu \mathcal{M} + \mu \mathcal{M}^\dagger \Sigma)$$  \hspace{1cm} (3.1a)$$

$$L_{v}^{(\mathcal{M})} = \lambda_1 \text{Tr}H^i_i(\xi \mathcal{M}^\dagger \xi + \xi^\dagger \mathcal{M}^\dagger \xi^\dagger ) \begin{pmatrix} T^i & T^j \end{pmatrix} + \lambda_2 \text{Tr}(\overline{T}^i H^i_i) \text{Tr}(\mathcal{M}^\dagger + \Sigma^\dagger)$$

$$+ \lambda_3 S^i_{ij}(\xi \mathcal{M}^\dagger \xi + \xi^\dagger \mathcal{M}^\dagger \xi^\dagger ) S^j_{ik} + \lambda_4 S^i_{ij} S^j_{ik} \text{Tr}(\mathcal{M}^\dagger + \Sigma^\dagger)$$

$$+ \lambda_5 \text{Tr}T_i(\xi \mathcal{M}^\dagger \xi + \xi^\dagger \mathcal{M}^\dagger \xi^\dagger ) T^i_j + \lambda_6 \text{Tr}(\overline{T} T_i) \text{Tr}(\mathcal{M}^\dagger + \Sigma^\dagger),$$  \hspace{1cm} (3.1b)$$
and then set the “spurion” field equal to the constant mass matrix

\[ M = \begin{pmatrix} m_u & m_d \\ m_d & m_s \end{pmatrix}. \]

Quadratic and higher order chiral symmetry breaking interactions are suppressed relative to those in (3.1) by powers of \( M/\Lambda_\chi \). The terms in \( L_v^{(M)} \) multiplied by \( \lambda_2, \lambda_4, \lambda_6 \) and which contain no pion fields produce common mass shifts for \( H', S \) and \( T \) respectively. The remaining zero-pion terms split the Goldstone and heavy hadron flavor multiplets.

As a consequence of \( SU(3)_L \times SU(3)_R \) breaking, the Isgur-Wise functions for heavy mesons and baryons are corrected by calculable chiral logarithms. These nonanalytic corrections have already been determined in the meson case [8]. However, we can reproduce the result quite simply by working with the matrix field \( H' \) rather than its pseudoscalar and vector meson components. The baryon computation on the other hand is somewhat more complicated, for mixing among the baryon Isgur-Wise functions is induced at one-loop order. But calculation of this mixing is dramatically simplified if we use the combined \( S \) and \( T \) baryon fields rather than their individual spin components. So determining the chiral log corrections to Isgur-Wise functions represents a nice application of the “super” field formalism discussed in the preceding section.

To begin, we match the HQET and Heavy Hadron Chiral Theory hadronic currents responsible for weak \( b \to c \) transitions [4][3]:

\[
\bar{c}v'\gamma_\mu P_- b_v \to C_{cb} \left\{ - \xi(w) \text{Tr}(H'_c(v')\gamma_\mu P_- H'_b(v)) \\
- \left[ g_{\alpha\beta} \eta_1(w) - v_\alpha v'_\beta \eta_2(w) \right] S_c'(v') \gamma_\mu P_- S^\beta(v) \\
+ \eta(w) T_c(v') \gamma_\mu P_- T_b(v) \right\}. \tag{3.2}
\]

Here \( P_- = \frac{1}{2}(1 - \gamma^5) \) denotes a left-handed projection operator. Known perturbative QCD corrections to the heavy quark current are absorbed into the \( C_{cb} \) prefactor. Unknown nonperturbative dependence of the effective hadron currents on light brown muck is lumped into the Isgur-Wise form factors \( \xi, \eta_1, \eta_2 \) and \( \eta \). These are functions of momentum transfer or equivalently \( w = v v' \). At the zero recoil point, \( \xi, \eta_1 \) and \( \eta \) are normalized to unity while \( \eta_2 \) drops out of (3.2) as \( v_\beta S^\beta(v) = 0 \).

The effective meson and baryon currents are renormalized at one-loop order by the wavefunction and vertex corrections shown in fig. [2] and fig. [3]. We focus on the nonanalytic chiral log terms generated by these graphs which cannot arise at tree level. Details on
extracting the logarithms from Goldstone loop integrals are provided in the appendix.

Here we simply quote the final results for the wavefunction renormalization constants

\[
(Z_H)_j^i = \delta_j^i + 3g_1^2 \sum_a (T_a T_a)_j^i \frac{m_{\pi_a}^2}{16\pi^2 f^2} \log \frac{\Lambda^2}{m_{\pi_a}^2}
\]

\[
(Z_S)_j^i = \delta_j^i + \sum_a \left\{ 2g_2^2 (m_{\pi_a}^2 - 2\Delta M^2)(T_a T_a)_j^i + g_3^2 m_{\pi_a}^2 [(T_a T_a)_k^i \delta_j^k - (T_a T_a)_j^i] \right\} \frac{1}{16\pi^2 f^2} \log \frac{\Lambda^2}{m_{\pi_a}^2}
\]

\[
(Z_T)_j^i = \delta_j^i + 3g_3^2 \sum_a [(T_a T_a)_k^i \delta_j^k - (T_a T_a)_j^i] \frac{m_{\pi_a}^2 - 2\Delta M^2}{16\pi^2 f^2} \log \frac{\Lambda^2}{m_{\pi_a}^2}
\]

and the renormalized meson and baryon Isgur-Wise functions

\[
\xi^R(w)_j^i = \left[ \delta_j^i - 2g_1^2 (r - 1) \sum_a (T_a T_a)_j^i \frac{m_{\pi_a}^2}{16\pi^2 f^2} \log \frac{\Lambda^2}{m_{\pi_a}^2} \right] \xi(w)
\]

\[
\eta^R_1(w)_j^i = \eta_1(w)\delta_j^i + \sum_a \left\{ g_2^2 m_{\pi_a}^2 (T_a T_a)_j^i \left[ (1 - rw)\eta_1(w) + r(w^2 - 1)\eta_2(w) \right] + 2g_2^2 \Delta M^2(T_a T_a)_j^i \left[ \frac{r - w}{w + 1} \eta_1(w) - (r + 1)(w - 1)\eta_2(w) \right] 
\]

\[
+ g_3^2 m_{\pi_a}^2 [(T_a T_a)_k^i \delta_j^k - (T_a T_a)_j^i] \left[ \eta_1(w) - r\eta_2(w) \right] \right\} \frac{1}{16\pi^2 f^2} \log \frac{\Lambda^2}{m_{\pi_a}^2}
\]

\[
\eta^R_2(w)_j^i = \eta_2(w)\delta_j^i + \sum_a \left\{ g_2^2 m_{\pi_a}^2 (T_a T_a)_j^i \left[ \frac{2r - w - rw^2}{w^2 - 1} \eta_1(w) + (2 + rw)\eta_2(w) \right] + 2g_2^2 \Delta M^2(T_a T_a)_j^i \left[ \frac{w^2 + rw + 2(w - r - 1)}{w^2 - 1} \eta_1(w) - \frac{2 + 3w + rw}{w + 1} \eta_2(w) \right] 
\]

\[
+ g_3^2 m_{\pi_a}^2 [(T_a T_a)_k^i \delta_j^k - (T_a T_a)_j^i] \left[ \eta_2(w) + \frac{1 - rw}{w^2 - 1} \eta_2(w) \right] \right\} \frac{1}{16\pi^2 f^2} \log \frac{\Lambda^2}{m_{\pi_a}^2}
\]

\[
\eta^R(w)_j^i = \eta(w)\delta_j^i + g_3^2 \sum_a \left\{ m_{\pi_a}^2 [3\eta(w) - (2r + w)\eta_1(w) + (w^2 - 1)\eta_2(w)] + 2\Delta M^2 \left[ -3\eta(w) + (2 + 2r - rw)\eta_1(w) - (w - 1)(2 + r - rw)\eta_2(w) \right] \right\} 
\]

\[
\times \left[ (T_a T_a)_k^i \delta_j^k - (T_a T_a)_j^i \right] \frac{1}{16\pi^2 f^2} \log \frac{\Lambda^2}{m_{\pi_a}^2}
\]

(3.4)

evaluated at the scale \( \Lambda = \Lambda_\chi \).

The renormalized Isgur-Wise form factors are diagonal matrices in \( SU(3)_{L+R} \) flavor space. The function \( r(w) = \log(w + \sqrt{w^2 - 1})/\sqrt{w^2 - 1} \) appearing in their expressions is familiar from HQET current anomalous dimension computations [11]. Noting that \( r(1) = 1 \),
one can readily verify that the nonanalytic corrections preserve the zero recoil point normalizations of $\xi_R$, $\eta_1^R$ and $\eta^R$ \cite{8}. These normalizations are guaranteed by the effective theory’s flavor and spin symmetries.

In chiral log computation results such as \(3.3\) and \(3.4\), the mass of the pion is often neglected in comparison to the kaon and eta masses. We should comment that this approximation is rather poor for two reasons. Firstly, the infrared logarithms in the effective expansion parameters

\[
\begin{align*}
\varepsilon_\pi &= \frac{m_\pi^2}{16\pi^2 f^2} \log \frac{\Lambda^2_\chi}{m_\pi^2} = .053 \\
\varepsilon_\kappa &= \frac{m_\kappa^2}{16\pi^2 f^2} \log \frac{\Lambda^2_\chi}{m_\kappa^2} = .253 \\
\varepsilon_\eta &= \frac{m_\eta^2}{16\pi^2 f^2} \log \frac{\Lambda^2_\chi}{m_\eta^2} = .265 
\end{align*}
\]

partially offset the large differences between the squared Goldstone masses. In estimating these parameters’ numerical sizes, we have assumed isospin invariance and used the input values $f = 93$ MeV, $m_\pi = 135$ MeV, $m_\kappa = 498$ MeV, $m_\eta = 549$ MeV and $\Lambda_\chi = 1000$ MeV. The discrepancy between the pion, kaon and eta nonanalytic terms is further diminished by group theory factors. In particular, the isospin subgroup Casimir coefficient $3/4$ multiplying $\varepsilon_\pi$ in the combination

\[
(T_a T_a)^{ij} \frac{m_{\pi a}^2}{16\pi^2 f^2} \log \frac{\Lambda^2_\chi}{m_{\pi a}^2} = \left( \begin{array}{c} \frac{3}{4} \varepsilon_\pi + \frac{1}{2} \varepsilon_\kappa + \frac{1}{12} \varepsilon_\eta \\ \frac{3}{4} \varepsilon_\pi + \frac{1}{2} \varepsilon_\kappa + \frac{1}{12} \varepsilon_\eta \\ \varepsilon_\kappa + \frac{1}{3} \varepsilon_\eta \end{array} \right) = \left( \begin{array}{c} .040 + .127 + .022 = .189 \\ .127 + .022 = .189 \\ .253 + .088 = .341 \end{array} \right)
\]

is greater than the corresponding group theory coefficients $1/2$ and $1/12$ in front of $\varepsilon_\kappa$ and $\varepsilon_\eta$ combined. So the pion contributions are small but nonnegligible when compared to the kaon and eta terms.

The small logarithmic splittings of the renormalized Isgur-Wise functions in \(3.4\) will be difficult to detect. The most likely possibility for observing the nonanalytic flavor violations would be in the meson sector. The ratio of the strange to the up and down Isgur-Wise functions

\[
\frac{\xi_R^s(w)}{\xi_R^u,d} = 1 - .304 g_1^2 (r - 1) \quad (3.5)
\]
deviates only slightly from unity \[\square\]. In principle, this variation can be extracted from the differential rates for semileptonic $B_s$ and $B$ decay:

$$\frac{d\Gamma(B_s \to D_s \ell \nu)}{d\Gamma(B \to D \ell \nu)} = \frac{M_{D_s}}{M_{B}} \left[ \frac{M_{B_s} + M_{D_s}}{M_B + M_D} \right]^2 \left( \frac{\xi^R(w)_s}{\xi^R(w)_{u,d}} \right)^2$$

$$\frac{d\Gamma(B_s \to D^*_s \ell \nu)}{d\Gamma(B \to D^* \ell \nu)} = \frac{M_{D_s}}{M_{B}} \left[ \frac{M_{B_s} + M_{D_s}}{M_B + M_D} \right]^2 \left( \frac{\xi^R(w)_s}{\xi^R(w)_{u,d}} \right)^2 \left( \frac{\xi^R(w)_s}{\xi^R(w)_{u,d}} \right)^2 \left( \frac{\xi^R(w)_s}{\xi^R(w)_{u,d}} \right)^2$$

But it is probably more useful to regard the ratio in (3.5) as setting a rough tolerance limit for $SU(3)_{L+R}$ breaking in the HQET picture. If future measurements of bottom hadron decay rates and lifetimes reveal flavor discrepancies like those among charmed mesons which are significantly greater than the suggestion of (3.3), then confidence in the HQET approach will be called into question.

4. Heavy Hadron Spin Symmetry Breaking

The HQET is based upon an $SU(2N_h)$ spin-flavor symmetry where $N_h$ denotes the number of heavy quark flavors \[\square,\square\]. Away from the infinite quark mass limit, this symmetry is broken by the $O(1/m_Q)$ operators

\begin{equation}
O_1 = \frac{1}{2m_Q} \bar{h}^{(Q)}_v (iD)^2 h^{(Q)}_v \tag{4.1a}
\end{equation}

\begin{equation}
O_2 = \frac{\mu^{f/2}}{4m_Q} h^{(Q)}_v \sigma_{\mu\nu} G^{\mu\nu}_{\bar{T}_a} T^a h^{(Q)}_v \tag{4.1b}
\end{equation}

which appear in the Lagrangian

\begin{equation}
\mathcal{L}^{HQET} = \sum_{Q=c,b} \left\{ \bar{h}^{(Q)}_v (i\nu D) h^{(Q)}_v + a_1 O_1 + a_2 O_2 \right\} \tag{4.2}
\end{equation}

with coefficients $a_1$ and $a_2$ that equal unity at tree level \[\square,\square\]. These terms in the Heavy Quark Effective Theory match at the chiral symmetry breaking scale onto infinite strings of operators in the Heavy Hadron Chiral Theory that share the same symmetry properties. We will concentrate in particular on the descendants of the gluon magnetic
moment operator $O_2$ which is responsible for breaking the heavy quark spin symmetry at $O(1/m_Q)$. Following the spurion procedure, we generalize $\sigma^{\mu\nu}$ in (4.14) to an antisymmetric tensor field $\Gamma^{\mu\nu}$ that transforms as $\Gamma^{\mu\nu} \to e^{i\vec{e} \cdot \vec{S}_v} \Gamma^{\mu\nu} e^{-i\vec{e} \cdot \vec{S}_v}$. We then match $O_2$ onto hermitian, parity even and $SO(3,1)$, $SU(3)_{L+R}$ and $SU(2)_v$ invariant operators in the chiral theory. Finally, we set $\Gamma^{\mu\nu} = \sigma^{\mu\nu}$. The resulting $O(1/m_Q)$ terms break $SU(2)_v$ in the low energy theory.

Operator $O_2$ matches onto zero derivative terms which lift the degeneracy between the pseudoscalar and vector mesons in $H$ [2] and the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sextet baryons in $S$:

$$L_v^{(O_2)} = \sum_{q=c,b} \left\{ \frac{\alpha_2^{(H)}}{m_Q} \text{Tr} \left( \bar{H} \sigma_{\mu\nu} H^i \sigma^{\mu\nu} \right) + i\frac{\alpha_2^{(S)}}{m_Q} \vec{S}^{\mu\nu} \sigma_{\mu\nu} S^{i\nu} \right\}.$$ 

After decomposing these operators into their individual spin components and calculating their self energy contributions, one can relate their coefficients to the splittings within the $H$ and $S$ spin multiplets:

$$\frac{\alpha_2^{(H)}}{m_Q} = -\frac{M_{P'} - M_P}{8},$$

$$\frac{\alpha_2^{(S)}}{m_Q} = \frac{M_{B'} - M_B}{2}.$$

The gluon magnetic moment term also matches onto a unique dimension-five operator $O_{TTA}$ which mediates the $SU(2)_v$ violating antitriplet baryon transition $T \to T\pi$ at $O(1/m_Q)$. Such an operator must be linear in the Goldstone axial vector field and contain one additional covariant derivative. Since the antisymmetric combination $D^\mu A^\nu - D^\nu A^\mu$ vanishes, the spurion procedure yields only one possibility for the induced operator:

$$O_{TTA} = \frac{i}{m_Q} \epsilon_{\mu\nu\sigma\lambda} T^{ij} \sigma^{\mu\nu} D^\sigma T_i (A^\lambda)^j.$$ 

Its coefficient $g_{TTA}$ is undetermined but should be of order one at the scale $\Lambda_X$.

Having identified $O_{TTA}$, we can investigate the simplest generalization of the antitriplet baryon semileptonic decay

$$T_b(P; v)_i \to T_c(p_1; v')_i + \ell(p_2) + \bar{\nu}_\ell(p_3) + \pi_a(p_4) \quad (4.3)$$

that contains a low-momentum Goldstone boson in the final state:

$$T_b(P; v)_i \to T_c(p_1; v')_j + \ell(p_2) + \bar{\nu}_\ell(p_3) + \pi^a(p_4). \quad (4.4)$$
These semileptonic transitions are of considerable interest, for an accurate value for the KM matrix element $|V_{cb}|$ may be determined from high precision measurements of their endpoint spectra.

Relations among the form factors that parametrize such antitriplet baryon processes persist beyond the infinite quark mass limit \[24\]. The form factor relations will provide valuable checks on the value for $|V_{cb}|$ extracted from future $T_b$ semileptonic measurements.

Decay (4.4) proceeds through the two pole diagrams illustrated in fig. 4. Adding together these graphs, squaring the resulting amplitude, and averaging and summing over fermion spins, we find the total squared amplitude

$$\frac{1}{2} \sum_{\text{spins}} |A|^2 = 64C_v^2|V_{cb}|^2C_{Vb}^2\eta(w)^2\left(\frac{g_{TT}}{f}\right)^2|(T_0)^2|^2$$

$$\times \left\{ \left(\frac{\Lambda}{m_c}\right)^2 \frac{(v'p_4)^2 - p_4^2}{(v'p_4)^2} v_2p_2v_3 + \left(\frac{\Lambda}{m_b}\right)^2 \frac{(v'p_4)^2 - p_4^2}{(v'p_4)^2} v_2p_2v'p_3 \right. - 2\left(\frac{\Lambda}{m_cm_b}\right)^2 v_2p_2p_4p_3p_4 - v_2p_2v_3v_4p_4 - v_3p_4v_2'p_3 + v_2p_2v_3v_4v'p_4 \right\}. \quad (4.5)$$

The individual contributions from the two diagrams as well as their interference term are clearly labelled in this expression by their $(\Lambda/m_Q)^2$ coefficients. The parameter $\Lambda = M_c - m_c = M_b - m_b$ represents the residual mass of the light brown muck inside a $T_Q$ baryon of mass $M_Q$ which is independent of the heavy quark constituent. One can also see in the squared amplitude expression the interplay between the Goldstone boson derivative coupling and heavy baryon pole. As the pion’s four-momentum $p_4$ tends towards zero, the intermediate baryon approaches going on-shell. The small derivative coupling is consequently offset by the pole in the propagator.

The differential rate for decay (4.4)

$$d\Gamma = \frac{1}{2M_b}\left(\frac{1}{2} \sum_{\text{spins}} |A|^2\right)d\Phi_{1234} \quad (4.6)$$

1. The particular process $\Lambda_b^0 \rightarrow \Lambda_c^+ X\ell^-\overline{\nu}_\ell$ is currently under study at LEP \[21\].

2. There are other pole diagram contributions to the antitriplet semileptonic decay (4.4) that involve intermediate sextet baryon exchange. However, flavor symmetry is violated at the weak vertices in such graphs. In addition, these contributions are prohibited by strong parity conservation in the infinite heavy quark mass limit and only proceed at $O(1/m_Q)$ \[22,23\]. Therefore, intermediate sextet pole diagrams are suppressed compared to those shown in fig. 4.
can be partially integrated over the final state phase space measure

\[
d\Phi_{1234} = (2\pi)^4 \delta^{(4)}(P - \sum_{i=1}^4 p_i)(2M_b)(2M_c) \prod_{i=1}^4 \frac{d^3 p_i}{(2\pi)^3 2E_i}.
\]

Imitating the lepton-hadron cross section decomposition familiar from deep inelastic scattering, we first factor out the lepton momenta \( p_2 \) and \( p_3 \) from the squared amplitude:

\[
\frac{1}{2} \sum_{\text{spins}} |A|^2 \equiv p_2^\alpha p_3^\beta W_{\alpha\beta}(v, v', p_4). \tag{4.7}
\]

Then neglecting lepton masses, we rewrite the lepton phase space factors in Lorentz invariant form

\[
p_2^\alpha p_3^\beta d\Phi_{1234} = (2\pi)^{-2} p_2^\alpha p_3^\beta \delta^{(4)}(P - \sum_{i} p_i) \delta(p_2^2) \delta(p_3^2) \theta(p_2^0) \theta(p_3^0) d^4 p_2 d^4 p_3 \prod_{i=1,4} \frac{d^3 p_i}{(2\pi)^3 2E_i}
\]

and integrate over \( p_2 \) and \( p_3 \). The result is a function that depends only upon the momentum \( p_{23} = P - p_1 - p_4 \) of the virtual \( W^* \) which connects the lepton pair to the hadrons participating in the semileptonic process:

\[
\int p_2^\alpha p_3^\beta d\Phi_{1234} = \frac{1}{96\pi} [p_{23}^2 g^{\alpha\beta} + 2p_2^\alpha p_{23}^\beta] \prod_{i=1,4} \frac{d^3 p_i}{(2\pi)^3 2E_i}. \tag{4.8}
\]

The remaining hadron phase space factors can be simplified to

\[
\prod_{i=1,4} \frac{d^3 p_i}{(2\pi)^3 2E_i} = \frac{M_c^2}{32\pi^3} |\vec{v}'||\vec{p}_4| \theta(vv') \theta(vp_4) d(vv')d(vp_4) d(\cos \theta_{14}) \tag{4.9}
\]

where \( \theta_{14} \) denotes the angle between \( \vec{v}' \) and \( \vec{p}_4 \) in the decaying bottom baryon’s rest frame. As written, this phase space expression manifestly vanishes as the three-momentum of either the charmed baryon or Goldstone boson goes to zero. We may eliminate \( \cos \theta_{14} \) in favor of the Lorentz invariant \( vv'p_4 \) via the relation

\[
\cos \theta_{14} = \frac{vv'vp_4 - v'p_4}{|v'||p_4|}. \tag{4.10}
\]

Eqn. (4.6) is then reduced to the concise, frame independent form

\[
d\Gamma = \frac{M_c^3}{1536\pi^5} W_{\alpha\beta}(v, v', p_4) [p_{23}^2 g^{\alpha\beta} + 2p_2^\alpha p_{23}^\beta] \theta(vv') \theta(vp_4) \theta(v'p_4) d(vv')d(vp_4)d(v'p_4). \tag{4.11}
\]
Assembling together the squared amplitude and phase space factors, we at last obtain the differential rate for the semileptonic process (4.4):

\[
\frac{d\Gamma(T_b(v)_i \rightarrow T_c(v')_j \ell \nu)}{dwdxdy} = \frac{1}{24\pi^3} G_F^2 M_c^3 |V_{cb}|^2 C_{cb}^2 \eta(w)^2 \left(\frac{g_{\gamma A}}{f}\right)^2 |(T_a)_{ij}|^2
\]

\times \left\{ \left[ \frac{\Lambda}{m_c} \right]^2 \frac{y^2 - m_{\pi a}^2}{y^2} + \left[ \frac{\Lambda}{m_b} \right]^2 \frac{x^2 - m_{\pi a}^2}{x^2} \right\} \left[ -2M_b M_c + (3M_b^2 + 3M_c^2 + m_{\pi a}^2)w 
+ 2(M_c x - M_b y) - 4M_b M_c w^2 - 4w(M_b x - M_c y) + 2xy \right]

- 2 \left( \frac{\Lambda}{m_c m_b} \right) \frac{1}{xy} \left[ (M_b^2 + M_c^2 + 3m_{\pi a}^2)(m_{\pi a}^2 - x^2 - y^2) + 2x^2 y^2 
- 2M_b M_c (2w^2 + 1)xy - 2M_b wy (2x^2 - m_{\pi a}^2) + 2M_c wx (2y^2 - m_{\pi a}^2) + 4M_b x (x^2 - m_{\pi a}^2) - 4M_c y (y^2 - m_{\pi a}^2) + (M_b^2 + M_c^2 + m_{\pi a}^2)wxy 
+ 2M_b M_c w (2x^2 + 2y^2 - m_{\pi a}^2) - 2xy (M_c x - M_b y) \right] \right\} \theta(w) \theta(x) \theta(y).
\]

(4.12)

The dotproducts \( w = vv' \), \( x = vp_4 \) and \( y = v'p_4 \) assume values only within a certain kinematic region. The limits on \( w \) are simple

\[
1 \leq w \leq \frac{M_b^2 + M_c^2 - m_{\pi a}^2}{2M_b M_c}
\]

(4.13)

and correspond to zero and maximum recoil of the charmed baryon. The ranges of \( x \) and \( y \) on the other hand are implicitly defined by the complicated conditions

\[
m_{\pi a} \sqrt{M_b^2 - 2M_b M_c w + M_c^2} \leq M_b x - M_c y \leq \frac{1}{2} (M_b^2 + M_c^2 - m_{\pi a}^2) - M_b M_c w
\]

\[
xw - \sqrt{w^2 - 1} \sqrt{x^2 - m_{\pi a}^2} \leq y \leq wx + \sqrt{w^2 - 1} \sqrt{x^2 - m_{\pi a}^2}.
\]

It is important to specify the validity domain of (4.12). The two pole graphs in fig. 4 dominate all other contributions to \( T_b \rightarrow T_c \ell \nu \pi \) only if the pion is emitted slowly in the rest frame of its parent baryon. Therefore \( x = vp_4 \) and \( y = v'p_4 \) must both be small compared to \( \Lambda_\chi \). In contrast, \( w = vv' \) can legitimately range over any value in (4.13) and need not be close to unity. The Isgur-Wise function \( \eta(w) \) is of course only known at \( w = 1 \). However, the dependence of (4.12) on \( \eta \) can be removed by normalizing it to the corresponding differential rate for the pure semileptonic transition (4.3):

\[
\frac{d\Gamma(T_b(v)_i \rightarrow T_c(v')_j \ell \nu)}{dw} = \frac{1}{12 \pi^3} G_F^2 M_c^3 |V_{cb}|^2 C_{cb}^2 \eta(w)^2 \sqrt{w^2 - 1} \delta \left[ -2M_b M_c + 3(M_b^2 + M_c^2)w - 4M_b M_c w^2 \right] \theta(w).
\]

(4.14)
Then the only unknown quantities which enter into the ratio of (4.12) to (4.14) are the coupling $g_{TTA}$ and the parameter $\bar{\Lambda}$. The ratio of the two differential decay rates can thus be studied away from the zero recoil point.

5. Conclusions

Other extensions of the formalism and applications of Heavy Hadron Chiral Perturbation Theory beyond those mentioned or considered here can be investigated. For example, the incorporation of electromagnetic interactions into the theory and the study of radiative transitions among heavy hadrons represent areas of significant theoretical and experimental interest. In short, the synthesis of Chiral Perturbation Theory and the Heavy Quark Effective Theory opens up a number of new directions for hadronic physics exploration.

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Appendix. Chiral Logarithms from One-Loop Integrals

Radiative corrections generally induce nonanalytic structure which is absent at tree level. In Chiral Perturbation Theory, single pion-loop graphs yield nonanalytic terms which include chiral logarithms. Such one-loop infrared logarithms always appear in conjunction with ultraviolet logarithmic divergences. So we adopt the mass independent renormalization scheme of dimensional regularization plus modified minimal subtraction to remove all short distance infinities.

Consider the vertex renormalization diagrams in fig. 3. Since we only wish to extract the chiral log corrections to the zero derivative terms in the effective hadronic currents

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3 The heavy quark mass parameter $m_Q$ can be replaced by the antitriplet baryon mass $M_Q$ in the $O(\bar{\Lambda}/m_Q)^2$ differential decay rate (4.12) as the discrepancy is of higher order.
we ignore external residual momenta in each of these graphs. They are then all proportional to momentum integrals that have the general form

\[ I^{\mu \nu} = \Lambda^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{q^\mu q^\nu}{(q^2 - m_\pi^2)(\nu q - \delta m)(\nu' q - \delta m)}. \]  

(A.1)

With a variation on the HQET method for combining denominators \[12\], we rewrite the integrand’s denominator in terms of the dimensionful and dimensionless Feynman parameters \(\alpha\) and \(\beta\):

\[ \frac{1}{(q^2 - m_\pi^2)(\nu q - \delta m)(\nu' q - \delta m)} = \int_0^\infty \alpha d\alpha \int_{-1}^1 d\beta \frac{4}{[q^2 + \alpha[v + v' + \beta(v - v')]q - (m_\pi^2 + \alpha \delta m)]^3}. \]

Shifting the loop momentum to \(q' = q + \alpha[v + v' + \beta(v - v')]/2\) and performing the \(q'\) integration, we obtain

\[ I^{\mu \nu} = \frac{i}{16\pi^2} \left[1 + \frac{\epsilon}{2} \left(\gamma + \log 4\pi + \log \Lambda^2\right)\right] \times \int_0^\infty \frac{d\alpha}{d} \int_{-1}^1 d\beta \left\{ \frac{\alpha \Gamma(\epsilon/2) g^{\mu \nu}}{[m_\pi^2 + 2\alpha \delta m + \alpha^2 [1 + w + (1 - w)\beta^2]/2]^\epsilon/2} \right. \]

\[ - \frac{1}{2} \alpha^3 \Gamma(1 + \epsilon/2) [(v^\mu + v'^\mu)(v'^\nu + v'^\nu) + \beta^2 (v^\mu - v'^\mu)(v'^\nu - v'^\nu)] \}

\[ \left. \frac{1}{[m_\pi^2 + 2\alpha \delta m + \alpha^2 [1 + w + (1 - w)\beta^2]/2]^{1+\epsilon/2}} \right\}. \]

(A.2)

In the special case when \(\delta m = 0\), one can use the ingenious Schwinger trick to evaluate the generalized \(\alpha\) parameter integral

\[ I \equiv \int_0^\infty \frac{\alpha^n d\alpha}{(m_\pi^2 + c\alpha^2)^{p+\epsilon/2}}. \]  

(A.3)

The definition of the Gamma function is first employed to promote the integrand’s denominator into an exponent:

\[ \frac{1}{(m_\pi^2 + c\alpha^2)^{p+\epsilon/2}} = \frac{1}{\Gamma(p + \epsilon/2)} \int_0^\infty \frac{dt}{t^{(p+\epsilon/2)}} e^{-(m_\pi^2 + c\alpha^2)t}. \]

The integral over \(\alpha\) then becomes the simple gaussian

\[ \int_0^\infty \alpha^n e^{-ct} d\alpha = \frac{1}{2} \Gamma \left( \frac{n + 1}{2} \right) (ct)^{-\frac{n+1}{2}}, \]
while the $t$ integral returns a Gamma function. Thus the solution to (A.3) is essentially a Beta function:

$$I = \frac{1}{2} B\left(\frac{n+1}{2}, p + \frac{\epsilon}{2} - \frac{n+1}{2}\right) \left(\frac{m_\pi^2}{c}\right)^{\frac{n+1}{2}} \Gamma(n+1)\left(m_\pi^2\right)^{-(p+\epsilon/2)}.$$

A more detailed analysis is required to perform the $\alpha$ integration in (A.2) when $\delta m \neq 0$. We find that a valid power series expansion in $\epsilon$ can be developed. The remaining $\beta$ parameter integral is then elementary. After cancelling the ultraviolet divergence and setting $\epsilon \to 0$, we can isolate the exact nonanalytic structure of $I^{\mu\nu}$. However, we only display the integral’s chiral log dependence assuming $m_\pi > \delta m$:

$$I^{\mu\nu} = -\frac{i}{16\pi^2} \log \frac{\Lambda^2}{m_\pi^2} \left\{ (m_\pi^2 - 2\delta m^2) \frac{r + 1}{w + 1} g^{\mu\nu} + \frac{m_\pi^2 r - w}{w^2 - 1} + 2\delta m^2 \frac{(w^2 - 1) + 2(w - r) + (rw - 1)}{(w + 1)(w^2 - 1)} (v^\mu v^\nu + v'^\mu v'^\nu) \right\}$$

where

$$r = \frac{\log(w + \sqrt{w^2 - 1})}{\sqrt{w^2 - 1}}.$$

The two-point graphs in fig. 2 all involve the momentum integral

$$J^{\mu\nu} = \Lambda^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{q^\mu q^{\nu}}{(q^2 - m_\pi^2)[v(q + k) - \delta m]}$$

in which we have restored the external residual momentum $k$. This integral can be evaluated using techniques similar to those described above. Its dominant nonanalytic behavior is given by

$$J^{\mu\nu} = -\frac{i}{16\pi^2} \log \frac{\Lambda^2}{m_\pi^2} \left\{ \left[(m_\pi^2 - 2\delta m^2 - \frac{2}{3}\delta m^3) - (m_\pi^2 - 2\delta m^2) v k + O(v k)^2\right] g^{\mu\nu} \right\}$$

$$+ \left[ (-2m_\pi^2 \delta m + \frac{8}{3}\delta m^3) + (2m_\pi^2 - 8\delta m^2) v k + O(v k)^2\right] v^\mu v^\nu \right\}$$

$$- \frac{2}{3} \frac{i m_\pi^2}{16\pi} \left(g^{\mu\nu} - v^\mu v^\nu\right) + \ldots.$$
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Figure Captions

Fig. 1. “Super” field Feynman rules derived from the leading order heavy hadron Lagrangian. Heavy (light) particles are drawn as thick (thin) lines. $A$, $B$ denote heavy quark spinor indices; $\alpha, \beta$ represent light antiquark spinor indices while $\mu, \nu$ are light vector indices; $i, j, k, l$ represent $SU(3)_{L+R}$ indices.

Fig. 2. One loop contributions to heavy meson and baryon wave function renormalization.

Fig. 3. One loop contributions to heavy meson and baryon flavor changing currents. Solid squares denote weak interaction vertices.

Fig. 4. Pole graphs which mediate the antitriplet baryon semileptonic transition $T_b(v)_i \rightarrow T_c(v')_j + \ell + \bar{\nu}_\ell + \pi^a$. Solid circles represent the $O(1/m_Q)$ operator $O_{TTA}$, while solid squares denote weak interaction vertices.