Non-Gaussianity from the bispectrum in general multiple field inflation

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Abstract. We study the non-Gaussianity from the bispectrum in multi-field inflation models with a general kinetic term. The models include multi-field K-inflation and multi-field Dirac–Born–Infeld (DBI) inflation as special cases. We find that, in general, the sound speeds for the adiabatic and entropy perturbations are different and they can be smaller than 1. Then the non-Gaussianity can be enhanced. Multi-field DBI inflation is shown to be a special case where both sound speeds are the same due to a special form of the kinetic term (Langlois et al 2008 Preprint 0804.3139 [hep-th]). We derive the exact second-and third-order actions including metric perturbations. In the small sound speed limit and at leading order in the slow-roll expansion, we derive the three-point function for the curvature perturbation which depends on both adiabatic and entropy perturbations. The contribution from the entropy perturbations has a different momentum dependence if the sound speeds for the entropy and adiabatic perturbations are different, which provides a possibility to distinguish multi-field models from single-field models. On the other hand, in the multi-field DBI case, the contribution from the entropy perturbations has the same momentum dependence as the pure adiabatic contributions and it only changes the amplitude of the three-point function (Langlois et al 2008 Preprint 0804.3139 [hep-th]). This could help to ease the constraints on the DBI-inflation models.

Keywords: cosmological perturbation theory, string theory and cosmology, inflation

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1. Introduction

The inflationary scenario succeeds in explaining the origin of temperature fluctuations of the cosmic microwave background (CMB). The increasing precision of CMB measurements enables us to distinguish between many inflationary models. The primordial fluctuations generated during inflation are nearly scale-invariant and Gaussian. Thus the deviation from the exact scale invariance and Gaussianity will give valuable information in discriminating many possible models. In particular, non-Gaussianity of the primordial fluctuations will provide powerful ways to constrain models (see, e.g., [1] for a review). The simplest single-field inflation models predict that the non-Gaussianity of the fluctuations will be very difficult to be detected even in future experiments such as Planck [2]. The detection of large non-Gaussianity would mean that the simplest models of inflation are rejected.

There are a few models where the primordial fluctuations generated during inflation have a large non-Gaussianity. In the single-field case, if the inflaton field has a non-trivial
kinetic term, it is known that the non-Gaussianity can be large. For example, in K-inflation models where the kinetic term of the inflaton field is generic, the sound speed of the perturbations can be much smaller than 1 \cite{3,4}, which leads to large non-Gaussian fluctuations. Additionally, Dirac–Born–Infeld (DBI) inflation, motivated by string theory, can also give large non-Gaussian fluctuations \cite{5–8}. The inflaton is identified with the position of a moving D3 brane whose dynamics is described by the DBI action. Again, due to the non-trivial form of the kinetic term, the sound speed can be smaller than 1 and the non-Gaussianity becomes large \cite{9,10}. The third- and fourth-order actions for a single inflaton field with a generic kinetic term have been calculated by properly taking into account metric perturbations. Three-and four-point functions have also been obtained \cite{10–12}. For the detailed observational consequences of single-field DBI inflation see \cite{13–20}.

Multi-field inflation models where the curvature perturbation is modified on large scales due to the entropy perturbations have also been recently studied \cite{21}. In the case of the standard kinetic term, it is not easy to generate large non-Gaussianity from multi-field dynamics \cite{22–27} (see, however, \cite{28}). In the DBI-inflation case, the position of the brane in each compact direction is described by a scalar field. Then DBI inflation is naturally a multi-field model \cite{29}. The effect of the entropy perturbations in the inflationary models based on string theory constructions in a slightly different context is also considered in \cite{30,31}. Recently, Huang et al calculated the bispectrum of the perturbations in multi-field DBI inflation with the assumption that the kinetic term depends only on $X = -G^{IJ} \partial_\mu \phi^I \partial^\mu \phi^J/2$, where $\phi^I$ are the scalar fields ($I = 1, 2, \ldots$) and $G_{IJ}$ is the metric in the field space, as occurs in K-inflation \cite{32}. They found that, in addition to the usual bispectrum of adiabatic perturbations, there exists a new contribution coming from the entropy perturbations. Then they showed that the entropy field perturbations propagate with the speed of light and the contribution from the entropy perturbations is suppressed. This property can also be confirmed by the analysis of a more general class of multi-field models where the kinetic terms are given by arbitrary functions of $X$ \cite{33,34}. However, Langlois et al pointed out that their assumption cannot be justified for the multi-field DBI inflation \cite{35}. Even though the action depends only on $X$ in the homogeneous background, there exist other kind of terms which contribute only to inhomogeneous perturbations. They find that this dramatically changes the behavior of the entropy perturbations. In fact, it was shown that the entropy perturbations propagate with the same sound speed as the adiabatic perturbations. Reference \cite{35} also calculated the bispectrum by generalizing the work of Huang et al \cite{11}.

In this paper, we study a fairly general class of multi-field inflation models with a general kinetic term which includes K-inflation and DBI inflation. We study the sound speeds of the adiabatic perturbations and entropy perturbations and clarify the difference between K-inflation and DBI inflation. Then we calculate the third-order action by properly taking into account the effect of gravity. We continue to obtain the three-point functions at leading order in slow-roll and in the small sound speed limit. We can recover the results for K-inflation and DBI inflation easily from this general result.

The structure of this paper is as follows. In section 2, we describe our model and derive equations in the background. In section 3, we study the perturbations using the ADM formalism. Additionally, the second-and third-order actions are derived by properly taking into account the metric perturbations. Then we decompose the perturbations into
adiabatic and entropy directions and write down the action in terms of the decomposed fields. In section 4, we study the sound speed in several models including K-inflation and DBI inflation. It is shown that, in general, adiabatic and entropy sound speeds are different and both can be smaller than 1. In section 5, the third-order action at leading order in slow-roll and in the small sound speed limit is obtained in terms of the decomposed fields. Then the three-point functions are derived for a generalized model which includes K-inflation and DBI inflation as particular cases. Section 6 is devoted to the conclusion.

2. The model

We consider a very general class of models described by the following action:

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ M_{Pl}^2 R + 2P(X^{IJ}, \phi^I) \right],$$

(1)

where $\phi^I$ are the scalar fields ($I = 1, 2, \ldots, N$), $M_{Pl}$ is the Planck mass that we will set to unity hereafter, $R$ is the Ricci scalar and

$$X^{IJ} \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J$$

(2)

is the kinetic term, while $g_{\mu\nu}$ is the metric tensor. We label the fields’ Lagrangian by $P$ and we assume that it is a well-behaved function so that derivatives of $P$ with respect to $X^{IJ}$ can be defined. Greek indices run from 0 to 3. Lower case Latin letters ($i, j, \ldots$) denote spatial indices. Upper case Latin letters denote field indices.

The Einstein field equations in this model are

$$G_{\mu\nu} = P g_{\mu\nu} + P_{,X^{IJ}} \partial_\mu \phi^I \partial_\nu \phi^J \equiv T_{\mu\nu},$$

(3)

where $P_{,X^{IJ}}$ denotes the derivative of $P$ with respect to $X^{IJ}$. The generalized Klein–Gordon equation is

$$g^{\mu\nu} \left( P_{,X^{IJ}} \partial_\nu \phi^I \right)_{,\mu} + P_{,J} = 0,$$

(4)

where $;$ denotes covariant derivative with respect to $g_{\mu\nu}$ and $P_{,J}$ denotes the derivative of $P$ with respect to $\phi^J$.

In the background, we are interested in flat, homogeneous and isotropic Friedman–Robertson–Walker universes described by the line element

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j,$$

(5)

where $a(t)$ is the scale factor. The Friedman equation and the continuity equation are

$$3H^2 = E_0,$$

(6)

$$\dot{E}_0 = -3H (E_0 + P_0),$$

(7)

where the Hubble rate is $H = \dot{a}/a$, $E_0$ is the total energy of the fields and it is given by

$$E_0 = 2X_0^{IJ} P_{0,X^{IJ}} - P_0,$$

(8)

where a subscript zero denotes background quantities, $X_0^{IJ} = 1/2 \dot{\phi}_0^I \dot{\phi}_0^J$. The equations of motion for the scalar fields reduce to

$$P_{0,X^{IJ}} \ddot{\phi}_0^I + \left( 3H P_{0,X^{IJ}} + \dot{P}_{0,X^{IJ}} \right) \dot{\phi}_0^I - P_{0,J} = 0.$$  

(9)

\footnote{Strictly speaking, we adopt the symmetrized derivative, $P_{X^{IJ}} \equiv (\partial P/\partial X^{IJ} + \partial P/\partial X^{JI})$.}
3. Perturbations

In this section, we will consider perturbations of the background (5) beyond linear order. For this purpose, we will construct the action at second and third order in the perturbations and it is convenient to use the ADM metric formalism [10,33,34], [36]–[39]. The ADM line element is

\[ ds^2 = -N^2 \, dt^2 + h_{ij} \left( dx^i + N^i \, dt \right) \left( dx^j + N^j \, dt \right), \]

where \( N \) is the lapse function, \( N^i \) is the shift vector and \( h_{ij} \) is the 3D metric. The action (1) becomes

\[ S = \frac{1}{2} \int \, dt \, d^3 x \sqrt{hN \left( (3)R + 2P(X^{IJ}, \phi^I) \right)} + \frac{1}{2} \int \, dt \, d^3 x \sqrt{hN} \left( E_{ij} E^{ij} - E^2 \right). \]

The tensor \( E_{ij} \) is defined as

\[ E_{ij} = \frac{1}{2} \left( h_{ij} - \nabla_i N_j - \nabla_j N_i \right), \]

and it is related to the extrinsic curvature by \( K_{ij} = N^{-1} E_{ij} \). \( \nabla_i \) is the covariant derivative with respect to \( h_{ij} \). \( X^{IJ} \) can be written as

\[ X^{IJ} = \frac{1}{2} h^{ij} \partial_i \phi^I \partial_j \phi^J + \frac{N^{-2}}{2} v^I v^J, \]

where \( v^I \) is defined as

\[ v^I \equiv \dot{\phi}^I - N^j \nabla_j \phi^I. \]

The Hamiltonian and momentum constraints are, respectively,

\[ (3)R + 2P - 2N^{-2} P_{X^{ij} v^I v^J} - N^{-2} \left( E_{ij} E^{ij} - E^2 \right) = 0, \]

\[ \nabla_j \left( N^{-1} E^j_i \right) - \nabla_i \left( N^{-1} E \right) = N^{-1} P_{X^{ij} v^I} \nabla_i \phi^J. \]

We decompose the shift vector \( N^i \) into scalar and intrinsic vector parts as

\[ N_i = \tilde{N}_i + \partial_i \psi, \]

where \( \partial_i \tilde{N}^i = 0 \); here and in the rest of the section indices are raised with \( \delta_{ij} \).

3.1. Perturbations in the uniform curvature gauge

In the uniform curvature gauge, the 3D metric takes the form

\[ h_{ij} = a^2 \delta_{ij}, \]

\[ \phi^I (x, t) = \phi^I_0 (t) + Q^I (x, t), \]

where \( Q^I \) denotes the field perturbations. In the following, we will usually drop the subscript ‘0’ on \( \phi^I_0 \) and simply identify \( \phi^I \) as the homogeneous background fields unless otherwise stated.
We expand $N$ and $N^i$ in powers of the perturbation $Q^I$:

\begin{align}
N &= 1 + \alpha_1 + \alpha_2 + \cdots, \\
\tilde{N}_i &= \tilde{N}_i^{(1)} + \tilde{N}_i^{(2)} + \cdots, \\
\psi &= \psi_1 + \psi_2 + \cdots,
\end{align}

where $\alpha_n$, $\tilde{N}_i^{(n)}$ and $\psi_n$ are of order $(Q^I)^n$. At first order in $Q^I$, a particular solution for equations (15) is

\begin{equation}
\alpha_1 = \frac{P_{X^{IJ}}}{2H} \phi^I Q^J, \quad \tilde{N}_i^{(1)} = 0,
\end{equation}

\begin{equation}
\partial^2 \psi_1 = \frac{a^2}{2H} \left[ -6H^2 \alpha_1 + P_{,K} Q^K - \frac{1}{2} \phi^I \phi^J P_{X^{IJ}} Q^K \\
+ \left( P_{X^{IJ}} + \phi^L \phi^M P_{X^{LM}X^{IJ}} \right) \left( \phi^I \phi^J \alpha_1 - \phi^I \dot{Q}^J \right) \right].
\end{equation}

The second-order action is calculated as

\begin{equation}
S_{(2)} = \int dt \, d^3x \frac{a^3}{2} \left[ X_{1I}^{IJ} X_{1I}^{LM} P_{,X^{IJ}X^{LM}} + P_{X^{IJ}} Q^I \dot{Q}^J \\
- a^2 P_{X^{IJ}} \partial^I \dot{Q}^J \partial^I Q^J - \frac{a^2}{2} \phi^I \dot{J}^L P_{X^{IJ}} P_{X^{LM}Q^I} Q^M \\
+ \frac{\phi^J P_{X^{IJ}} Q^I}{H} \left( P_{X^{LM}X^{IJ}} + P_{,K} Q^K \right) \\
+ 2P_{X^{IJ}} Q^K \left( -\dot{\phi}^I \dot{\phi}^J \alpha_1 + \phi^I \dot{Q}^J \right) + P_{,KL} Q^K Q^L \\
+ P_{X^{IJ}} \left( 3\phi^I \dot{J}^J \alpha_1^2 - 4\alpha_1 \phi^I \dot{Q}^J \right) \right],
\end{equation}

where

\begin{align}
X_{1I}^{IJ} &\equiv -\alpha_1 \dot{\phi}^I \dot{\phi}^J + \dot{\phi}^I (\dot{Q}^J).
\end{align}

After integrating by parts in the action and employing the background field equations, the second-order action can be finally written in the rather simple form

\begin{equation}
S_{(2)} = \frac{1}{2} \int dt \, d^3x \frac{a^3}{3} \left[ (P_{X^{IJ}} + P_{X^{IK}X^{JL}} \dot{\phi}^K \dot{\phi}^L) \dot{Q}^I \dot{Q}^J \\
- \frac{1}{a^2} P_{X^{IJ}} \partial^I \dot{Q}^J \partial^I Q^J - \mathcal{M}_{IJ} Q^I Q^J + \mathcal{N}_{IJ} Q^I Q^J \right],
\end{equation}

with the effective squared mass matrix

\begin{align}
\mathcal{M}_{IJ} &= -P_{IJ} + \frac{X^{LM}}{H} \dot{\phi}^K (P_{X^{IK}P_{X^{JL}}} + P_{X^{IK}P_{X^{JL}}} \\
- \frac{1}{H^2} X^{MN} X^{PQ} P_{X^{MN}X^{PQ}} P_{X^{IK}P_{X^{JL}}} \dot{\phi}^K \dot{\phi}^L \\
- \frac{1}{a^2} \frac{1}{a^2} \frac{1}{a^2} \left[ \frac{a^3}{H} P_{X^{IK}P_{X^{JL}}} \dot{\phi}^K \dot{\phi}^L \right],
\end{align}

\begin{equation}
\mathcal{N}_{IJ} = 2 \left( P_{X^{IK}} - \frac{X^{MN}}{H} P_{X^{IK}X^{MN}} P_{X^{JL}} \dot{\phi}^L \right) \dot{\phi}^K.
\end{equation}
In the same way, the third-order action is given by

$$S_{(3)} = \int dt \, d^3 x a^3 \left[ 3H^2 \alpha_1^2 + \frac{2H}{a^2} \alpha_1 \partial^2 \psi_1 + \frac{1}{2a^2} \left( \partial^2 \psi_1 \partial^2 \psi_1 - \partial_i \partial_j \psi_1 \partial^i \partial^j \psi_1 \right) \right] \alpha_1$$

$$+ \left[ -\frac{1}{2} \alpha_1^2 \dot{Q}^I \dot{Q}^J \alpha_1^2 \dot{Q}^J \right] + a^{-2} \alpha_1 \dot{Q}^I \partial_i \psi_1 Q^J \alpha_1^{-2} \partial_i Q^J \right] P_{X^I J}$$

$$+ \left[ \alpha_1^2 \dot{Q}^I \dot{Q}^J - \frac{3}{2} \alpha_1 \dot{Q}^I \dot{Q}^J + \frac{1}{2} \dot{Q}^I \dot{Q}^J - a^{-2} \partial_i Q^J \frac{\alpha_1}{2} \partial^i Q^J \right] P_{X^I K} Q^K$$

$$+ \frac{1}{2} X_1^J P_{X^I J} Q^L Q^M + \frac{1}{8} P_{X^I J K} Q^L Q^M \right]. \quad (27)$$

### 3.2. Decomposition into adiabatic and entropy perturbations

We can decompose the perturbations into the instantaneous adiabatic and entropy perturbations, where the adiabatic direction corresponds to the direction of the background fields’ evolution while the entropy directions are orthogonal to this [40]. For this purpose, following [33], we introduce an orthogonal basis $e^I_n$ ($n = 1, 2, \ldots, N$) in the field space. The orthonormal condition is defined as

$$P_{X^I J} e^I_n e^J_m = \delta_{nm}, \quad (28)$$

so that the gradient term $P_{X^I J} \partial_i Q^I \partial^i Q^J$ is diagonalized. Here we assumed that $P_{X^I J}$ is invertible and it can be used as a metric in field space. The adiabatic vector is

$$e^I_n = \frac{\dot{Q}^I}{\sqrt{P_{X^I K} \dot{Q}^I \dot{Q}^K}}, \quad (29)$$

which satisfies the normalization given by equation (28). The field perturbations are decomposed on this basis as

$$Q^I = Q_n e^I_n. \quad (30)$$

We defined the matrix $Z_{mn}$ which describes the time variation of the basis as

$$\dot{e}^I_n = e^I_m Z_{mn}, \quad (31)$$

which satisfies $Z_{mn} = -Z_{nm} - \dot{P}_{X^I J} e^I_n e^J_m$ as a consequence of $(P_{X^I J} e^I_n e^J_m) = 0$.

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4 It is worth noting that the normalization given in [33, 35, 45] is $G_{IJ} e^I_n e^J_m = \delta_{nm}$, unless $P_{X^I J} = G_{IJ}$, that is, the case with the canonical kinetic terms, our results written in terms of $Q_n$ are different from the results of [33, 35, 45]. But it can be shown that, when written in terms of $v_n$, which are the canonical variables for the quantum theory, our results are consistent with the results of [33, 35, 45].
In terms of the decomposed fields, the second-order action (24) can be rewritten as
\[
S_{(2)} = \frac{1}{2} \int dt \, d^3x \, a^3 \left[ \mathcal{K}_{mn}(D_t Q_m)(D_t Q_n) - \frac{1}{a^2} \delta_{mn} \partial_i Q_m \partial_i Q_n \right. \\
- \mathcal{M}_{mn} Q_m Q_n + \mathcal{N}_{mn} Q_n (D_t Q_m) \right],
\]
where
\[
D_t Q_m \equiv \dot{Q}_m + Z_{mn} Q_n,
\]
\[
\mathcal{K}_{mn} \equiv \delta_{mn} + (P^{MN} \delta^i_n \delta M^j n) P M^{IK} X^{JL} e_1^I e_1^J e_1^L,
\]
\[
\mathcal{M}_{mn} \equiv \mathcal{M}_{IJ} e_m^I e_n^J,
\]
\[
\mathcal{N}_{mn} \equiv \mathcal{N}_{IJ} e_m^I e_n^J.
\]
From the constructions, \( \mathcal{K}_{mn} \), \( \mathcal{M}_{mn} \) and \( \mathcal{N}_{mn} \) are symmetric with respect to \( m \) and \( n \). The explicit form of the effective squared mass matrix in this representation is
\[
\mathcal{M}_{mn} = -P_{mn} + \frac{1}{H} (P_{XKL} \dot{\phi}^K \dot{\phi}^L)^{3/2} (P_{mXMN} e_1^M e_1^N) \delta_{n1}
\]
\[
- \frac{1}{4H^2} (P_{XKL} \dot{\phi}^K \dot{\phi}^L)^{3/2} (P_{XMN} P_{Q} e_1^M e_1^N e_1^P e_1^Q) \delta_{m1} \delta_{n1}
\]
\[
- \frac{1}{a^3} \left[ \frac{a^3}{H} (P_{XMN} \dot{\phi}^M \dot{\phi}^N) P_{JL} e_1^I e_1^J e_1^L e_1^J e_1^L \right] \delta_{m1} \delta_{n1},
\]
\[
\mathcal{N}_{mn} = - \frac{1}{H} (P_{XN} \dot{\phi}^N \dot{\phi}^N)^2 (P_{XKLXMN} e_1^I e_1^J e_1^K e_1^L e_1^J e_1^L) \delta_{n1}
\]
\[
+ 2 \sqrt{P_{XLM} \dot{\phi}^L \dot{\phi}^M} (P_{XN} e_1^I e_1^M e_1^J),
\]
where \( P_{mn} \equiv P_{IJ} e_m^I e_n^J \) and \( P_{nXIK} \equiv P_{nXIK} e_1^J \).

The equation of motion is obtained as
\[
\frac{1}{a^3} \frac{d}{dt} \left[ a^3 (2 \mathcal{K}_{mn} D_t Q_m + \mathcal{N}_{mn} Q_m) \right] - (2 \mathcal{K}_{mn} Z_{mn} + \mathcal{N}_{mn}) D_t Q_m \\
- (2 \mathcal{M}_{mn} + \mathcal{N}_{mn} Z_{mn}) Q_m + \frac{1}{a^2} \partial^2 Q = 0.
\]

4. Linear perturbations

In this section, we study the linear order perturbations using the second-order action derived in the previous section.

4.1. K-inflation

Let us consider K-inflation models where \( P(X^{IJ}, \phi^I) \) is a function of only the trace \( X = X^{IJ} G_{IJ}(\phi^K) \) of the kinetic terms where \( G_{IJ}(\phi^K) \) is a metric in the field space:
\[
P(X^{IJ}, \phi^I) = \tilde{P}(X, \phi^I).
\]
The derivatives of $P$ can be evaluated as

$$P_{XIJ} = G_{IJ} \tilde{P}_X,$$

$$P_I = \frac{1}{2} G_{JK,I} \dot{\phi}^J \dot{\phi}^K \tilde{P}_X + \tilde{P}_I,$$

$$P_{XIIK} = G_{IJ} G_{KL} \tilde{P}_{XX},$$

$$P_{XJK} = \frac{1}{2} G_{LM,K} \dot{\phi}^L \dot{\phi}^M G_{IJ} \tilde{P}_{XX} + G_{IJ,K} \tilde{P}_X + G_{IJ,K} \tilde{P}_{XX},$$

$$P_{IJ} = \frac{1}{4} G_{KL,I} G_{MN,J} \dot{\phi}^K \dot{\phi}^L \dot{\phi}^M \dot{\phi}^N \tilde{P}_{XX} + \frac{1}{2} G_{KL,I} \dot{\phi}^K \dot{\phi}^L \tilde{P}_X + \frac{1}{2} \dot{\phi}^M \dot{\phi}^N \{G_{MN,J} \tilde{P}_{XI} + G_{MN,I} \tilde{P}_{XJ}\} + \tilde{P}_{IJ},$$

and the sound speed is defined as

$$c_s^2 = \frac{\dot{P}_X}{P_X + 2XP_{XX}}.$$  (46)

In terms of the decomposed field, the second-order action can be written as [33]

$$S_{(2)} = \frac{1}{2} \int dt d^3 x d^3 a^3 \left\{ \left[ \delta_{mn} + \left( \frac{1}{c_s^2} - 1 \right) \delta_{im} \delta_{jn} \right] \left( D_i Q_m \right) \left( D_j Q_n \right) - \frac{1}{a^2} \delta_{mn} \partial_i Q_m \partial_j Q_n - \mathcal{M}_{mn} Q_m Q_n + \mathcal{N}_{mn} Q_m Q_n \right\},  \quad (47)$$

where we do not show the explicit forms of $\mathcal{M}_{mn}$ and $\mathcal{N}_{mn}$.

The sound speed agrees with the adiabatic sound speed defined by $c_s^2 = dP/dE$. The fact that the sound speeds for the entropy perturbations are unity has been recognized in [33]. This is because the non-trivial second derivative of $P$ only affects the adiabatic perturbations.

### 4.2. DBI inflation

An interesting class of models is the DBI inflation which describes the motion of a D3 brane in a higher-dimensional spacetime. The DBI action is given by [41]

$$S = - \int d^4 x \frac{1}{f(\phi^K)} \sqrt{-\det[g_{\mu\nu} + f(\phi^K) \partial_\mu \phi^I \partial_\nu \phi^I]},$$  \quad (48)$$

where $\det$ denotes a determinant, $G^{IJ}$ is a metric in field space and $\phi^I$ corresponds to positions of a brane in higher-dimensional spacetime. Recently it was pointed out by reference [35] that multi-field DBI inflation is not included in multi-field K-inflation discussed in the previous subsection. Indeed, $P(X^{IJ})$ is not a function of $X$, but is given by

$$P(X^{IJ}, \phi^I) = \tilde{P}(\tilde{X}, \phi^I), \quad \tilde{X} = \frac{1 - \mathcal{D}}{2f},$$  \quad (49)$$

where

$$\mathcal{D} = \det(\delta^I_J - 2f X^{I}_J)$$

$$= 1 - 2f G_{IJ} X^{IJ} + 4f^2 X^{[I}_J X^{J]} - 8f^3 X^{[I}_J X^{J] X^{[K}_L X^{K]} + 16f^4 X^{[I}_J X^{J] X^{[K}_L X^{K]} X^{L]}},$$  \quad (50)$$
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where [] denotes the antisymmetric bracket and we used $G^{IJ}$ to raise the indices. In the background, $\dot{X} = X$. However, this does not mean that the full action is a function of $X$ only. The DBI action takes a specific form of $\hat{P}$:

$$\hat{P}(\dot{X}, \phi^I) = -\frac{1}{f} \left( \sqrt{1 - 2f \dot{X}^2} - 1 \right) - V(\phi^I),$$

(51)

where we allow for a potential $V(\phi^I)$. The sound speed is defined as

$$c_s^2 \equiv \frac{\hat{P}_{\dot{X}}}{\hat{P}_{\dot{X}^2} + 2X \hat{P}_{\dot{X}X}}.$$

(52)

The derivatives of $P$ evaluated on the background can be calculated as

$$P_{,X^{IJ}} = \hat{P}_{,X} \left( \frac{d \dot{X}}{dX^{IJ}} \right),$$

(53)

$$P_{,X^{IJ}X^{KL}} = \hat{P}_{,XX} \left( \frac{d \dot{X}}{dX^{IJ}} \right) \left( \frac{d \dot{X}}{dX^{KL}} \right) + \hat{P}_{,\dot{X}} \left( \frac{d^2 \dot{X}}{dX^{IJ}dX^{KL}} \right),$$

(54)

where

$$\frac{d \dot{X}}{dX^{IJ}} = c_s^2 G_{IJ} + 2f X_{IJ},$$

(55)

$$\frac{d^2 \dot{X}}{dX^{IJ}dX^{KL}} = -2f \left( G_{IJ} G_{KL} - \frac{1}{2} G_{IK} G_{JL} - \frac{1}{2} G_{IL} G_{JK} \right) + O(X^{IJ}).$$

(56)

Here we do not explicitly write down the higher-order terms in $X^{IJ}$ in the second derivative as they will not contribute to the final result. In the following, we will omit these terms. We can also show that

$$P_{,I} = \frac{1}{4} G_{JKL} \phi^J \phi^K \hat{P}_{,X} + \hat{P}_{,I},$$

(57)

$$P_{,IJ} = \frac{1}{4} G_{KLM} \phi^K \phi^L \phi^M \phi^N \hat{P}_{,XX} + \frac{1}{2} G_{KL} \phi^K \phi^L \hat{P}_{,X} + \frac{1}{2} \phi^M \phi^N (G_{MN,J} \hat{P}_{,XI} + G_{MN,I} \hat{P}_{,XJ}) + \hat{P}_{,IJ},$$

(58)

$$P_{,X^{IJ}K} = ((1 - 2f X)G_{IJ,K} - 2f_{,K} X G_{IJ} - 2f G_{LM,K} X^{LM} G_{IJ}$$

$$\times 2f_{,K} X_{IJ} + 2f G_{IL,K} X^L_{IJ} + 2f G_{JM,K} X^M_{IJ}) \hat{P}_{,X}$$

$$+ (c_s^2 G_{IJ} + 2f X_{IJ}) \left( \frac{1}{2} G_{LM,K} \phi^L \phi^M \hat{P}_{,XX} + \hat{P}_{,XX} \right).$$

(59)

It is worth noting that, even though $P_{,X^{IJ}K}$ seems to be a bit complicated, we can show that

$$P_{,X^{IJ}K} \phi^J = \frac{1}{2} G_{LM,K} \phi^L \phi^M \phi_{,K} \hat{P}_{,XX} + \phi_{,K} \hat{P}_{,XX} + G_{IJ,K} \phi^J \hat{P}_{,X},$$

(60)

which is just the same form as the K-inflation case. We can also show that

$$P_{,X^{IJ}} \phi^J \phi^I = 2X \hat{P}_{,X}.$$

(61)
The orthonormality conditions for the basis give
\[ e_n^T e_m = \frac{1}{P_x} \delta_{mn} - \frac{1}{P_{\dot{x}}} \frac{1 - a^2}{c_s^2} \delta_{m1} \delta_{n1}. \] (62)

Using these results, the second-order action can be written in terms of the decomposed perturbations as
\[
S^{(2)} = \frac{1}{2} \int dt d^3 x a^3 \left[ \frac{1}{c_s^2} \delta_{mn} (D_t Q_m)(D_t Q_n) - \frac{1}{a^2} \delta_{mn} \partial_i Q_m \partial^i Q_n 
- \mathcal{M}_{mn} Q_m Q_n + \mathcal{N}_{mn} Q_n (D_t Q_m) \right].
\] (63)

Unlike K-inflation models, all field perturbations have the same sound speeds as was pointed out by reference [35]. In order to understand the difference between K-inflation and DBI inflation, we will consider a generalized model where both cases are included.

4.3. Generalized case

Let us consider models described by
\[ P(X^{IJ}, \phi^I) = \tilde{P}(Y, \phi^I), \] (64)
where
\[ Y = G_{IJ}(\phi^K) X^{IJ} + \frac{b(\phi^K)}{2} (X^2 - X^I J X^I J). \] (65)

The functional form of \( Y \) is chosen so that \( Y = X \equiv G_{IJ} X^{IJ} \) in the background as in the DBI-inflation model. This model includes as particular cases the K-inflation model for \( b = 0 \) and the DBI-inflation for \( b = -2f \) and if \( \tilde{P} \) has the DBI form. This might be surprising as the DBI action contains additional terms of order \( f^2 \) and \( f^3 \) in \( \tilde{X} \) (see equations (49) and (50)), but it turns out that these terms do not contribute to the second-order action and the leading order third-order action.

Following a similar procedure to the previous subsection, the second-order action can be written in terms of the decomposed perturbations as
\[
S^{(2)} = \frac{1}{2} \int dt d^3 x a^3 \left[ \delta_{mn} + 2 \tilde{P}_{YY} \delta_{1m} \delta_{1n} + \frac{bX}{1 + bX} (\delta_{11} \delta_{m1} - \delta_{mn}) \right] (D_t Q_m)(D_t Q_n)
- \frac{1}{a^2} \delta_{mn} \partial_i Q_m \partial^i Q_n 
- \mathcal{M}_{mn} Q_m Q_n + \mathcal{N}_{mn} Q_n (D_t Q_m) \right].
\] (66)

Now we are in a position to explain the difference between K-inflation and DBI inflation. As in the K-inflation case, the non-trivial second derivative of \( P \) affects only the adiabatic perturbations. On the other hand, the nonlinear terms of \( X^{IJ} \) in \( Y \) only affects the entropy perturbations. This can be seen from the fact that the sound speeds for adiabatic perturbations \( c_{ad}^2 \) and for entropy perturbations \( c_{en}^2 \) are given by
\[ c_{ad}^2 \equiv \frac{\tilde{P}_Y}{\tilde{P}_Y + 2 \tilde{X} \tilde{P}_{YY}}, \quad c_{en}^2 \equiv 1 + bX, \] (67)
and they are independently determined by $\tilde{P}_{YY}$ and $d^2Y/(dX^{IJ}dX^{KL})$, respectively. Thus in general they are different. Let us derive the condition under which the two sound speeds are the same, i.e. $c^2_{ad} = c^2_{en}$. This condition is given by

$$2X \frac{\tilde{P}_{YY}}{\tilde{P}_Y} = -\frac{bX}{1+bX}.$$  \hspace{1cm} (68)

Then we find that the DBI action is a solution of this equation where $b = -2f$, although we should note that it has not been proved that multi-field DBI inflation is the only case where the sound speeds are the same.

5. The leading order in slow-roll three-point function

In this section, we will calculate the leading order in slow-roll third-order action for the generalized model of the previous subsection and then we shall calculate the leading order three-point function for both adiabatic and entropy directions. Finally we will obtain the three-point function of the comoving curvature perturbation.

5.1. Approximations: slow-roll

In order to control the calculations and to obtain analytical results we need to make use of some approximations. We will use the slow-roll approximation, where we define a set of parameters and assume that these parameters are always small until the end of inflation. We define the slow-roll parameters as

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{X\tilde{P}_Y}{H^2}, \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H},$$  \hspace{1cm} (69)

$$\chi_{ad} \equiv \frac{\dot{c}_{ad}}{c_{ad}H}, \quad \chi_{en} \equiv \frac{\dot{c}_{en}}{c_{en}H}.$$  \hspace{1cm} (70)

It is important to note that these slow-roll parameters are more general than the usual slow-roll parameters and that their smallness does not necessarily imply that the fields are rolling slowly. Assuming that the parameters $\chi_{ad}$ and $\chi_{en}$ are small implies that the rates of change of the adiabatic and entropy sound speeds are small, but the sound speeds themselves can have any value and we assume that they are between zero and one.

It is convenient to define a parameter that describes the nonlinear dependence of the Lagrangian on the kinetic term as

$$\lambda \equiv \frac{2}{3}X^3\tilde{P}_{YYY} + X^2\tilde{P}_{YY}.$$  \hspace{1cm} (71)

We will also assume that the rate of change of this new parameter is small, as given by

$$\dot{l} \equiv \frac{\lambda}{\lambda H}.$$  \hspace{1cm} (72)

We thank Gianmassimo Tasinato for a discussion on this point. While we were writing up this work, there appeared a paper on the arXiv [45]. It is argued that the specialty of the DBI action comes from the fact that it describes the fluctuations of the positions of the brane in the higher-dimensional spacetime and they propagate at the speed of light. From the point of view of observers on the brane, the sound speeds are smaller than 1 due to the Lorentz factor $1/c_s^2$.
At the end of this section, we will show that the size of the leading order three-point function of the fields is fully determined by five parameters evaluated at horizon crossing: $\epsilon$, $\lambda$, $H$ and both sound speeds.

It turns out that the equations of motion for both adiabatic and entropy perturbations at first order form a coupled system of second-order linear differential equations (see the appendix for details). In general, the coupling (denoted by $\xi$ in equation (A.10)) between adiabatic and entropy modes cannot be neglected but in this work we will study the simpler decoupled case, where we assume that $\xi$ is small when the scales of interest cross outside the sound horizons, i.e. we will assume that $\xi \sim \mathcal{O}(\epsilon)$. With these approximations the adiabatic and entropy modes are decoupled and the system of equations of motion can be solved analytically. For simplicity, we will also assume that the mass term present in the entropy equation of motion is small, i.e. $\mu_s^2/H^2 \ll 1$ (refer to the appendix for more details). When calculating the leading order three-point functions, we assume that the quantities related to the time derivatives of the basis vectors given by $Z_{mn}$ are also slow-roll-suppressed. Finally, the calculation of the three-point functions in the next subsection is valid in the limit of small sound speeds. Our results will also include sub-leading terms of $\mathcal{O}(1)$ but these terms will, in general (for small sound speeds), receive corrections coming from terms of the order of $\epsilon/\epsilon_s^2$ that we have neglected.

5.2. Third-order action at leading order

At leading order in the previous approximations, we can neglect terms containing $\alpha_1$, $\psi_1$ and derivatives of $P$ with respect to the fields. Then the third-order action for the general model (1) is calculated as

$$S_{(3)} = \frac{1}{2} \int dx^3 \, dt a^3 \left[ P_{XIKXJL} \dot{\phi}^{(I} \dot{Q}^{K)} \dot{Q}^L + \frac{1}{3} P_{XIKXJLMN} \dot{\phi}^{(I} \dot{Q}^{K)} \dot{\phi}^{(J} \dot{Q}^{L)} \dot{\phi}^{(M} \dot{Q}^{N)} - \frac{1}{a^2} P_{XIKXJL} \dot{\phi}^{(I} \dot{Q}^{K)} \partial_i \dot{Q}^j \partial^j \dot{Q}^L \right].$$

(73)

After decomposition into the new adiabatic/entropy basis the third-order action can be written as

$$S_{(3)} = \int dx^3 \, dt a^3 \left[ \frac{1}{2} \Xi_{nml} \dot{Q}_n \dot{Q}_m \dot{Q}_l - \frac{1}{2a^2} \Upsilon_{nml} \dot{Q}_n \partial_i Q_m \left( \partial^j Q_l \right) \right],$$

(74)

where we define the coefficients $\Xi_{nml}$ and $\Upsilon_{nml}$ as

$$\Xi_{nml} = P_{XIKXJL} \sqrt{P_{XMN} \dot{\phi}^M \phi^N \epsilon_1^{(I} \epsilon_2^{K) \epsilon_3^L}}$$

$$+ \frac{1}{3} P_{XIKXJLMN} \left( P_{XPOQ} \dot{\phi}^O \phi^P \right)^{3/2} \epsilon_1^{(I} \epsilon_2^{K) \epsilon_3^L} e_1^O e_1^P e_1^Q e_1^R;$$

(75)

$$\Upsilon_{nml} = P_{XIKXJL} \sqrt{P_{XMN} \dot{\phi}^M \phi^N \epsilon_1^{(I} \epsilon_2^{K) \epsilon_3^L}}.$$

We shall now give some useful formulae of the previous quantities for the different inflationary models considered in this work.
5.2.1. K-inflation. For the K-inflation model we have

\[ \Xi_{nml} = (2X \dot{P}_X)^{-1/2} \left( \frac{2X \dot{P}_{XX}}{P_X} \delta_1(\delta_{nl}) + \frac{4X^2 \ddot{P}_{XXX}}{P_X} \delta_{n1} \delta_{ml} \right), \]  

(77)

\[ \Upsilon_{nml} = (2X \dot{P}_X)^{-1/2} \frac{2X \dot{P}_{XX}}{P_X} \delta_{n1} \delta_{ml}. \]  

(78)

5.2.2. DBI inflation. For the DBI-inflation scenario they are given by

\[ \Xi_{nml} = (2X \dot{P}_X)^{-1/2} \frac{1 - c^2_s}{c^2_s} \delta_1(\delta_{nl}), \]  

(79)

\[ \Upsilon_{nml} = (2X \dot{P}_X)^{-1/2} \left( \frac{1 - c^2_s}{c^2_s} \delta_{n1} \delta_{ml} - 2 \frac{1 - c^2_s}{c^2_s} (\delta_{n1} \delta_{ml} - \delta_{n(m)l1}) \right), \]  

(80)

where \( c^2_s \) should be understood as the sound speed defined in equation (52).

5.2.3. Generalized case. For the generalized case of section 4.3, \( \Xi_{nml} \) and \( \Upsilon_{nml} \) can be written as

\[ \Xi_{nml} = (2X \dot{P}_Y)^{-1/2} \left[ \frac{(1 - c^2_{ad})}{c^2_{ad} c^2_{en}} \delta_1(\delta_{nl}) + \left( \frac{4X^2 \ddot{P}_{YYY}}{P_Y} - \frac{(1 - c^2_{ad})(1 - c^2_{en})}{c^2_{ad} c^2_{en}} \right) \delta_{n1} \delta_{ml} \right], \]  

(81)

\[ \Upsilon_{nml} = (2X \dot{P}_Y)^{-1/2} \left( \frac{1 - c^2_{ad}}{c^2_{ad}} \delta_{n1} \delta_{ml} - \frac{2(1 - c^2_{en})}{c^2_{en}} (\delta_{n1} \delta_{ml} - \delta_{n(m)l1}) \right), \]  

(82)

and it is obvious that the DBI inflation is a specific case of the general model with \( c^2_{ad} = c^2_{en} = c^2_s \).

5.3. The three-point functions of the fields

In this subsection, we derive the three-point functions of the adiabatic and entropy fields in the generalized case and at leading order in slow-roll and in the small sound speeds limit. We consider the two-field case with the adiabatic field \( \sigma \) and the entropy field \( s \).

The perturbations are promoted to quantum operators as

\[ Q_n(\tau, \mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3k Q_n(\tau, k) e^{i \mathbf{k} \cdot \mathbf{x}}, \]  

(83)

where

\[ Q_n(\tau, k) = u_n(\tau, k) a_n(k) + u_n^*(\tau, -k) a_n^\dagger(-k). \]  

(84)

\( a_n(k) \) and \( a_n^\dagger(-k) \) are the annihilation and creation operators, respectively, that satisfy the usual commutation relations:

\[ [a_n(k_1), a_m^\dagger(k_2)] = (2\pi)^3 \delta^{(3)}(k_1 - k_2) \delta_{nm}, \]

\[ [a_n(k_1), a_m(k_2)] = [a_n^\dagger(k_1), a_m^\dagger(k_2)] = 0. \]  

(85)
At leading order the solution for the mode functions is (see the appendix for details)

$$u_n(\tau, k) = A_n \frac{1}{k^{3/2}} (1 + ikc_n \tau) e^{-ikc_n \tau},$$

(86)

where $c_n$ stands for either the adiabatic or the entropy sound speeds.

The two-point correlation function is

$$\langle 0|Q_n(\tau = 0, k_1)Q_m(\tau = 0, k_2)|0 \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2) P_{Q_n} \frac{2\pi^2}{k_1^3} \delta_{nm},$$

(87)

where the power spectrum is defined as

$$P_{Q_n} = \frac{|A_n|^2}{2\pi^2}, \quad |A_\sigma|^2 = \frac{H^2}{2c_{ad}}, \quad |A_s|^2 = \frac{H^2}{2c_{en}},$$

(88)

and it should be evaluated at the time of horizon crossing, $c_{ns}k_1 = a_*H_*$. The three-point operator in the interaction picture (at first order) is \([37, 42]\)

$$\langle \Omega|Q_l(t, k_1)Q_m(t, k_2)Q_n(t, k_3)|\Omega \rangle = -i \int_{t_0}^t dt'[0| \left[ Q_l(t, k_1)Q_m(t, k_2)Q_n(t, k_3), H_I(\tau) \right]|0 \rangle,$$

(89)

where $t_0$ is some early time during inflation when the field's vacuum fluctuation is deep inside the horizons and $t$ is some time after horizon exit. $|\Omega \rangle$ is the interacting vacuum which is different from the free theory vacuum $|0 \rangle$. If one uses conformal time, it is a good approximation to perform the integration from $-\infty$ to 0 because $\tau \approx -(aH)^{-1}$. $H_I$ denotes the interaction Hamiltonian and it is given by $H_I = -L_3$, where $L_3$ is the Lagrangian obtained from the action (74).

At this order, the only non-zero three-point functions are

$$\langle \Omega|Q_\sigma(0, k_1)Q_\sigma(0, k_2)Q_\sigma(0, k_3)|\Omega \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) \frac{2c_{ad}|A_\sigma|^6}{H} \frac{1}{\Pi_1 k_i^3} \frac{1}{k},$$

$$\times \left[ 6c_{ad}(C_3 + C_4) \frac{k_2^2 k_3^2 k_1^2}{K^2} - C_1 k_1^2 k_2 \cdot k_3 \left( 1 + \frac{k_2 + k_3}{K} + 2 \frac{k_2 k_3}{K^2} \right) \right] + 2 \text{ cyclic terms},$$

(90)

$$\langle \Omega|Q_\sigma(0, k_1)Q_\sigma(0, k_2)Q_\sigma(0, k_3)|\Omega \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) \frac{|A_\sigma|^2|A_s|^4}{H} \frac{1}{\Pi_1 k_i^3} \frac{1}{K},$$

$$\times \left[ C_2 c_{en} k_3^2 k_1 \cdot k_2 \left( 1 + \frac{c_{ad} k_1 + c_{en} k_2}{K} + \frac{2 c_{ad} c_{en} k_3 k_2}{K^2} \right) + (k_2 \leftrightarrow k_3) \right] + 4C_3 c_{ad}^2 c_{en} \frac{k_2 k_3^2 k_1^2}{K^2} - 2(C_1 + C_2) c_{ad}^2 k_1^2 k_2 \cdot k_3$$

$$\times \left( 1 + \frac{k_2 + k_3}{K} + 2 \frac{k_2 k_3}{K^2} \right),$$

(91)

where $K = k_1 + k_2 + k_3$, $\tilde{K} = c_{ad} k_1 + c_{en} (k_2 + k_3)$, cyclic terms means cyclic permutations of the three wavevectors and $(k_2 \leftrightarrow k_3)$ denotes a term like the preceding one but with $k_2$
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and $k_3$ interchanged. The pure adiabatic three-point function is evaluated at the moment $\tau_*$ at which the total wavenumber $K$ exits the horizon, i.e. when $K_{\text{ad}*} = a_* H_*$. Because of the different propagation speeds, the adiabatic and entropy modes become classical at different times: however, at leading order we assume that the background-dependent coefficients of (91) do not vary with time and so they can also be evaluated at the moment $\tau_*$. The pure adiabatic three-point function is evaluated at the moment $\tau_*$ at which the total wavenumber $K$ exits the horizon, i.e. when $K_{\text{ad}*} = a_* H_*$. Because of the different propagation speeds, the adiabatic and entropy modes become classical at different times: however, at leading order we assume that the background-dependent coefficients of (91) do not vary with time and so they can also be evaluated at the moment $\tau_*$. The different constants $C_N$ are given by

\begin{align}
C_1 &= (2H^2\epsilon)^{-1/2} \frac{1 - c_{\text{ad}}^2}{c_{\text{ad}}^2}, & C_2 &= -2(2H^2\epsilon)^{-1/2} \frac{1 - c_{\text{en}}^2}{c_{\text{en}}^2}, \\
C_3 &= (2H^2\epsilon)^{-1/2} \frac{1 - c_{\text{ad}}^2}{c_{\text{ad}}^2c_{\text{en}}^2}, & C_4 &= (2H^2\epsilon)^{-1/2} \left( \frac{2\lambda}{H^2\epsilon} - \frac{1 - c_{\text{ad}}^2}{c_{\text{ad}}^2c_{\text{en}}^2} \right) .
\end{align}
(92)

5.4. The three-point function of the comoving curvature perturbation

In this subsection, we calculate the leading order in slow-roll three-point function of the comoving curvature perturbation in terms of the three-point function of the fields obtained in the previous subsection.

During the inflationary era the comoving curvature perturbation is defined as

\[ R = \frac{H}{E_0 + P_0} \delta q \]
(93)
in the uniform curvature gauge where $\delta q_i = \delta T_{ij}^0$. Using the constraint equation $\delta q = -2H\alpha_1$ (21) and equation (7), we get

\[ R = \frac{H}{\dot{\sigma}} \frac{Q_\sigma}{\sqrt{\tilde{P}_{\gamma}}} . \]
(94)

It is convenient to define the entropy perturbation $S$ as

\[ S = \frac{H}{\dot{\sigma}} \frac{Q_s}{\sqrt{\tilde{P}_{\gamma}}} \sqrt{\frac{c_{\text{en}}}{c_{\text{ad}}}} , \]
(95)

so that $P_{S_*} \simeq P_{R_*}$, where the subscript * means that the quantity should be evaluated at horizon crossing.

In this work we will ignore the possibility that the entropy perturbations during inflation can lead to primordial entropy perturbations that could be observable in the CMB. But we shall consider the effect of entropy perturbations on the final curvature perturbation. We will follow the analysis of Wands et al [43], where it has been shown that, even on large scales, the curvature perturbation can change in time because of the presence of entropy perturbations. The way the entropy perturbations are converted to curvature perturbations is model-dependent but it was shown that this model dependence can be parameterized by a transfer coefficient $T_{RS}$ [43] as

\[ R = R_* + T_{RS}S_* = A_* Q_{\sigma*} + A_s Q_{\sigma s*} , \]
(96)
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\[ A_\sigma = \left( \frac{H}{\dot{\sigma} \sqrt{P_Y}} \right)_s, \quad A_s = T_{RS} \left( \frac{H}{\dot{\sigma} \sqrt{P_Y} \sqrt{\frac{C_{en}}{C_{ad}}}} \right)_s. \]  

(97)

Using the previous expressions we can now relate the three-point function of the curvature perturbation to the three-point functions of the fields obtained in the previous subsection. The three-point function of the curvature perturbation is given by

\[ \langle R(k_1)R(k_2)R(k_3) \rangle = A_\sigma^3 \langle Q_\sigma(k_1)Q_\sigma(k_2)Q_\sigma(k_3) \rangle + A_\sigma A_s^2 \langle (Q_\sigma(k_1)Q_s(k_2)Q_s(k_3)) + 2 \text{perms} \rangle. \]  

(98)

For the DBI-inflation case the previous equation can be simplified and the total momentum dependence of the three-point function of the comoving curvature perturbation is the same as in single-field DBI [35]. For our general model this is no longer the case, i.e. the different terms of the previous equation have different momentum dependence. Once again one can see that DBI inflation is a very particular case and, more importantly, it provides a distinct signature that enables us to distinguish it from other more general models.

6. Conclusion

In this paper, we studied the non-Gaussianity from the bispectrum in general multi-field inflation models with a generic kinetic term. Our model is fairly general including K-inflation and DBI inflation as special cases. We derived the second-and third-order actions for the perturbations including the effect of gravity. The second-order action is written in terms of adiabatic and entropy perturbations. It was shown that the sound speeds for these perturbations are in general different. In K-inflation the entropy perturbations propagate at the speed of light. DBI inflation is a special case where the sound speeds for the entropy and adiabatic perturbations are the same.

Then we derive the three-point function in the small sound speed limit at leading order in slow-roll expansion. In these approximations there exists a three-point function between adiabatic perturbations \( Q_\sigma \) and entropy perturbations \( Q_s \), \( \langle Q_\sigma(k_1)Q_s(k_2)Q_s(k_3) \rangle \), in addition to the pure adiabatic three-point function. This mixed contribution has a different momentum dependence if the sound speeds for the entropy and adiabatic perturbations are different. This provides a possibility to distinguish between multi-field models and single-field models. Unfortunately, in the multi-field DBI case, the sound speeds for the entropy and adiabatic perturbations are the same and the mixed contribution only changes the amplitude of the three-point function. This could help to ease the constraints on DBI inflation as discussed in [35].

In order to calculate the effect of entropy perturbations on the curvature perturbation at recombination, we need to specify a model that describes how the entropy perturbations are converted to the curvature perturbations. In addition, even during inflation, if the trajectory in field space changes non-trivially, the entropy perturbations can be converted to the curvature perturbation. In this paper, we modeled this transition by the function \( T_{RS} \). It would be interesting to study this mixing in specific string theory motivated models.
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Appendix. Equations of motion for the fluctuations

Here, we derive the equations of motion for linear perturbations for the generalized model introduced in section 4.3. In terms of the field space ‘covariant quantities’ [44] which are given by

\begin{align}
D_t \dot{\phi}^I &\equiv \ddot{\phi}^I + \Gamma^I_{JK} \dot{\phi}^J \dot{\phi}^K, \\
D_t Q^I &\equiv \dot{Q}^I + \Gamma^I_{JK} \dot{\phi}^J Q^K, \\
D_t D_J \dot{P} &\equiv \ddot{P}_{IJ} - \Gamma^K_{IJ} \ddot{P}_{,K}, \\
\mathcal{R}^I_{KIJ} &\equiv \Gamma^I_{KJ,L} - \Gamma^I_{KL,J} + \Gamma^I_{LM} \Gamma^M_{JK} - \Gamma^I_{JM} \Gamma^M_{LK},
\end{align}

(\Gamma^I_{JK} denotes the Christoffel symbols associated with the field space metric \(G_{IJ}\)) and the second-order action can be expressed as

\begin{align}
S_{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[ \left( \dot{P}_Y G_{IJ} + \ddot{P}_Y \phi_I \phi_J \right) D_t Q^I D_t Q^J \\
- \frac{1}{a^2} \dot{\phi}_Y \left[ (1 + bX) G_{IJ} - bX_{IJ} \right] \partial_i Q^I \partial^i Q^J - \ddot{M}_{IJ} Q^I Q^J + 2 \ddot{P}_Y \phi_I Q^J D_t Q^J \right],
\end{align}

with the effective squared mass matrix

\begin{align}
\ddot{M}_{IJ} = -D_I D_J \ddot{P}_Y - \ddot{P}_Y \mathcal{R}_{IJKL} \phi^K \phi^L + \frac{X \ddot{P}_Y}{H} (\ddot{P}_{YJ} \phi_I + \ddot{P}_{YI} \phi_J) \\
+ \frac{X \ddot{P}_Y^3}{2 H^2} \left( 1 - \frac{1}{c_{ad}^2} \right) \phi_I \phi_J - \frac{1}{a^3} D_t \left[ \frac{a^3}{2 H} \ddot{P}_Y^2 \left( 1 + \frac{1}{c_{ad}^2} \right) \phi_I \phi_J \right].
\end{align}

It is worth noting that, except for the coefficients of the kinetic term and the gradient term, this action is the same as the K-inflation case and DBI-inflation case which are derived in [33] and [35], respectively.

From now on we will derive the equations of motion for the fluctuations. For simplicity, let us now restrict our attention to the two-field case (\(I = 1, 2\)). Then, the perturbations can be decomposed into \(Q^I = Q^I_\sigma e^I_\sigma + Q^I_\epsilon e^I_\epsilon\), where \(e^I_\sigma = e^I_1\) and \(e^I_\epsilon\) is the unit vector orthogonal to \(e^I_\sigma\). As in standard inflation, it is more convenient to use conformal
The solutions of (A.8) and (A.9) with the Bunch–Davies vacuum initial conditions are
between $Q$ thus given by

$$v_{\sigma} = \frac{a}{c_{ad}} Q_{\sigma}, \quad v_{s} = \frac{a}{c_{en}} Q_{s}. \quad (A.7)$$

From the similar calculations with K-inflation and DBI-inflation cases analyzed by [33]
and [35], respectively, we find the equations of motion for $v_{\sigma}$ and $v_{s}$ as

$$v''_{\sigma} - \xi v'_{\sigma} + \left( c_{ad}^2 k^2 - \frac{z''}{z} \right) v_{\sigma} - \frac{(z\xi)'}{z} v_{s} = 0, \quad (A.8)$$

$$v''_{s} + \xi v'_{s} + \left( c_{en}^2 k^2 - \frac{\alpha''}{\alpha} + a^2 \mu_2^2 \right) v_{s} - \frac{z'}{z} \xi v_{\sigma} = 0, \quad (A.9)$$

where the primes denote the derivative with respect to $\tau$ and

$$\xi \equiv \frac{a}{\dot{\sigma} P_Y c_{ad}} \left[ (1 + c_{ad}^2) \tilde{P}_{,s} - c_{ad} \dot{\sigma}^2 \tilde{P}_{Y,s} \right], \quad (A.10)$$

$$\mu_2^2 \equiv \frac{\tilde{P}_{ss}}{P_Y} \left( 1 + \frac{1}{2c_{ad}^2} \frac{\dot{P}_{ss}^2}{P_Y^2} + 2 \frac{\tilde{P}_{Y,s} \tilde{P}_{ss}}{P_Y^2}, \right) (A.11)$$

$$z \equiv \frac{a \dot{\sigma}}{c_{ad} H} \sqrt{\tilde{P}_Y}, \quad \alpha \equiv a \sqrt{\tilde{P}_Y}, \quad (A.12)$$

with

$$\dot{\sigma} \equiv \sqrt{2X}, \quad \tilde{P}_{,s} \equiv \tilde{P}_{,s} \epsilon_s \sqrt{P_Y c_{en}}, \quad \tilde{P}_{Y,s} \equiv \tilde{P}_{Y,s} \epsilon_s \sqrt{P_Y c_{en}}, \quad (A.13)$$

and $\tilde{R}$ denotes the Riemann scalar curvature of the field space.

If we assume that the effect of the coupling $\xi$ can be neglected when the scales
of interest cross the sound horizons the two degrees of freedom are decoupled and
the system can be easily quantized. If we further assume the slow-roll approximations,
the time evolution of $H$, $c_{ad}$ and $\dot{\sigma}$ are small with respect to that of the scale factor and the
relations $z''/z \simeq 2/\tau^2$ and $\alpha''/\alpha \simeq 2/\tau^2$ hold (see section 5.1 for these approximations).
The solutions of (A.8) and (A.9) with the Bunch–Davies vacuum initial conditions are
thus given by

$$v_{\sigma k} \simeq \frac{1}{\sqrt{2k c_{ad}}} e^{-ik c_{ad} \tau} \left( 1 - \frac{i}{kc_{ad} \tau} \right), \quad (A.14)$$

$$v_{sk} \simeq \frac{1}{\sqrt{2k c_{en}}} e^{-ik c_{en} \tau} \left( 1 - \frac{i}{kc_{en} \tau} \right), \quad (A.15)$$

when $\mu_2^2/H^2$ is negligible for the entropy mode.

Therefore, the power spectra for $Q_\sigma$ and $Q_s$ are obtained as

$$\mathcal{P}_{Q_\sigma} \simeq \frac{H^2}{4\pi^2 c_{ad}}, \quad \mathcal{P}_{Q_s} \simeq \frac{H^2}{4\pi^2 c_{en}}, \quad (A.16)$$

$^6$ Since the definitions of $Q_\sigma$ and $Q_s$ in our paper are different from the definitions in [33, 35, 45] the relations
between $Q_\sigma$ and $v_\sigma$, and $Q_s$ and $v_s$ are also different from the analogous relations in those papers.
which are evaluated at the sound horizon crossing. Here for adiabatic perturbations, the sound horizon is determined by $c_{ad}$ and for entropy perturbations it is determined by $c_{en}$. The ratio of the power spectra for the adiabatic and entropy modes is thus given by $P_{Qs}/P_{Qσ} = c_{ad}/c_{en}$.

Note added. While we were writing up this work, similar results appeared on the arXiv [45].

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