Stagnation-Point Flow and Heat Transfer Over an Exponentially Stretching/Shrinking Sheet in Hybrid Nanofluid with Slip Velocity Effect: Stability Analysis

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Abstract. The effect of slip on stagnation point flow and heat transfer over an exponentially stretching/shrinking sheet filled with Copper-Alumina/water nanofluids is investigated numerically in this paper. The governing boundary layer equations are transformed into a set of ordinary differential equations using a similarity transformation and then solved numerically using the bvp4c function in Matlab. The effects of nanoparticle volume fraction, slip parameter and stretching/shrinking parameter on the flow pattern and heat transfer have been studied. It is found that dual solutions exist for hybrid nanofluid in the case of shrinking sheet. Furthermore, slip parameter and Copper nanoparticle acts in widening the range of solution. Hybrid nanofluids have the higher heat transfer rate compared to nanofluid and viscous fluid. A stability analysis showed that the first solution is linearly stable and physically realizable.

1. Introduction

The new evolution in technology demands a new innovation in the area of heat transfer. The latest research on nanofluids introduced the advanced class of fluids with augmented thermal properties named hybrid nanofluids obtained by dispersing the nanocomposite or nanoparticles of different metals into the base fluid. The hybrid nanofluids showed the enhanced thermal properties as compared to the mono nanoparticles based nanofluids and conventional fluids [1-3]. Hybrid nanofluid helps in improving the technology in engineering and industries of heat transfer leading to cost reduction in industrial applications such as nuclear system cooling, electronic cooling, biomedical, cooling and heating in buildings, generator cooling, engine cooling/vehicle thermal management and many more.

A very limited number of analysis have been focusing on the fluid flow and heat transfer with hybrid nanofluid as the working fluid numerically. Olatundun and Makinde [4] investigated the effect of convective boundary condition of a hybrid nanofluid flow over a flat plate using an entropy generation analysis. Chamka et al. [5] have focussed on the investigation of conjugate natural convection of hybrid nanofluid in a semi-circular cavity. Farooq et al. [6] initiated the heat transfer analysis of hybrid nanofluid flow over a nonlinear stretching disk with the effect of viscous dissipation on entropy generation. Rostami et al. [7] are the pioneer researcher that discover nonunique solutions for the study.
of mixed convective stagnation-point flow of hybrid nanofluid numerically. Since then, many researchers have investigated on hybrid nanofluid in their respective fields of study [8-11].

The research topic of exponentially stretching/shrinking sheet in stagnation point flow is often come across in real life problems which engage enormous interest from researchers due to their numerous importance in industries. It seems that the paper by Magyari and Keller [12] are the first researcher that considered the heat transfer and boundary layer flow over an exponentially stretching sheet. While, Bhattacharyya and Vajravelu [13] consider the research over an exponentially shrinking sheet. Nevertheless, the flow and heat transfer in the stagnation region was first considered by Hiemenz [14]. Therefore, Bachok et al. [15] made an attempt to study the problem of exponentially stretching/shrinking sheet in nanofluid in the region of stagnation point. Some excellent investigations in this field were made in the articles [15-19].

The above research shows that there are no numerical studies on hybrid nanofluid flow in exponentially stretching/shrinking sheet. In this research, the effect of slip on the problem of stagnation point flow and heat transfer over an exponentially stretching/shrinking sheet has been investigated numerically with hybrid nanofluid as a new working fluid. To form the required hybrid nanofluid, the composition of Copper nanoparticles is added into 0.1 volume fraction of Alumina/water. The governing systems of boundary value problem are transformed to a set of ordinary differential equations by using similarity variables and numerically solved using the Matlab software. In a way to identify the stability of numerical solutions obtained, the stability analysis is executed.

2. Mathematical Modeling
The steady boundary layer flow of a hybrid nanofluid with slip effect over an exponentially stretching/shrinking sheet is considered in this research. The x-axis is taken along the surface of the sheet, meanwhile the y-axis is normal to it. Follow Bachok et al. [15], the governing boundary layer equations are given by:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \mu_{hnf} \frac{\partial^2 u}{\partial y^2}
\]

(2)

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{hnf} \frac{\partial^2 T}{\partial y^2}
\]

(3)

with boundary equations:

\[
u = U_\infty (x) + N \left( \frac{\partial u}{\partial y} \right), \quad v = 0, \quad T = T_\infty - T_\infty e^{-y/2L} \quad \text{at} \quad y = 0
\]

\[
u \rightarrow U_\infty (x), \quad T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty
\]

(4)

Here, \(U_\infty (x) = ce^{y/L}\) is the stretching/shrinking velocity and \(U_\infty (x) = ae^{y/L}\) is the external flow velocity where \(c\) is the stretching/shrinking velocity rate, \(a\) is a positive constant and \(L\) is a characteristic length of a sheet, respectively. \(N = N_1 e^{-y/2L}\) is the slip factor with \(N_1\) is the initial value of slip factor and \(T_\alpha\) is the constant that estimates the temperature change along the sheet. \(\mu_{hnf}\) is the viscosity, \(\rho_{hnf}\) is the density and \(\alpha_{hnf}\) is thermal diffusivity of hybrid nanofluid which are given by Devi and Devi [20].
\[ \alpha_{hnf} = \frac{k_{hnf}}{(\rho C_p)_{hnf}}, \mu_{hnf} = \frac{\mu_f}{(1 - \varphi_1)(1 - \varphi_2)^{2.5}}, \rho_{hnf} = (1 - \varphi_1)\left[(1 - \varphi_1)\rho_f + \varphi_s\rho_{bf}\right] + \varphi_s\rho_{bf}, \]

\[ (\rho C_p)_{hnf} = (1 - \varphi_1)(1 - \varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{bf} + \varphi_2(\rho C_p)_{bf}, \]

\[ k_{hnf} = k_f + 2k_{bf} - 2\varphi_s(k_{bf} - k_f), \quad k_{bf} = k_f + 2k_{bf} + \varphi_s(k_{bf} - k_f), \]

where \( \varphi \) is the nanoparticle volume fraction, \( (\rho C_p) \) is the effective heat capacity, \( k \) is the thermal conductivity where the subscripts ‘hnf’, ‘f’, ‘bf’ and ‘s’ represent ‘hybrid nanofluid’, ‘fluid’, ‘Al\textsubscript{2}O\textsubscript{3}’ solid fraction’ and ‘Cu solid fraction’, respectively.

To solve Equations (1)-(3) along with (5), the similarity variables are introduced as follows:

\[ \eta = \sqrt{\frac{a}{2\nu_f L}} e^{y/2L}, \quad \psi = (2\nu_f L)^{1/2} f(\eta)e^{y/2L}, \quad \theta(\eta) = \frac{T-T_w}{T_w-T_e} \]

where \( \nu_f \) is the kinematic viscosity of the fluid, \( \eta \) is the similarity variable and \( \psi \) is the stream function defined as \( u = \partial\psi/\partial y \) and \( v = -\partial\psi/\partial x \). A continuity equation (1) is undoubtedly satisfied and upon substitution of (6) in Equations (2)-(4), the governing equations reduced as follows

\[ \frac{\mu_{hnf}}{\mu_f} f'' + ff'' - 2f'^2 + 2 = 0 \]

\[ \frac{1}{Pr} \frac{k_{hnf}}{k_f} \theta'' + f'\theta' - f\theta = 0 \]

with transformed boundary conditions

\[ f(0) = 0, \quad f'(0) = \varepsilon + \sigma f''(0), \quad \theta(0) = 1, \quad f'(\eta) \rightarrow 1, \quad \theta(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty \]

where prime indicates differentiation with respect to \( \eta \), \( \sigma \) is the slip parameter, \( Pr \) is the Prandtl number and \( \varepsilon \) is the stretching/shrinking parameter where \( \varepsilon > 0 \) corresponds to stretching sheet and \( \varepsilon < 0 \) corresponds to shrinking sheet. These parameters are given by:

\[ Pr = \frac{\nu_f}{\alpha_f}, \quad \varepsilon = \frac{c}{a}, \quad \sigma = N_t \left( \frac{a}{2\nu_f L} \right)^{1/2} \]

The practical quantities of interest are the skin friction coefficient, \( C_f \) and Nusselt number, \( Nu_s \) are defined as

\[ C_f = \frac{\tau_w}{\rho_f U_w}, \quad Nu_s = \frac{2Lq_w}{k_f(T_w - T_e)} \]

where shear stress \( \tau_w \) for wall as well as the heat flux \( q_w \) is interpreted as follows:

\[ \tau_w = \mu_{hnf} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{hnf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \]

Dimensionless forms of these quantities are

\[ C_f \text{ Re}_s^{1/2} = \frac{\mu_{hnf}}{\mu_f} f''(0), \quad Nu_s \text{ Re}_s^{1/2} = \frac{k_{hnf}}{k_f} \theta'(0) \]

where \( \text{Re} = 2LU_w/\nu_f \) represents local Reynolds number.
3. Flow Stability

Stability analysis has been executed by several researchers [21-22] to identify which solutions obtained are stable and physically reliable. The momentum and energy equations are rewritten as follows:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_o \frac{dU_o}{dx} + \mu_{nf} \frac{\partial^2 u}{\partial y^2} \tag{14}
\]

where

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} \tag{15}
\]

along with new boundary condition

\[
u = u = U_o(x), \quad v = 0, \quad T = T_w = T_o + T_w e^{i2L} \quad \text{at} \quad y = 0
\]

\[
u \rightarrow U_o(x), \quad T \rightarrow T_o \quad \text{as} \quad y \rightarrow \infty
\]

The new similarities variables are

\[
\eta = y \left( \frac{a}{2\nu L} \right)^{1/2} e^{\frac{x}{\nu L}}, \quad \psi = \left( 2\nu L \right)^{1/2} f(\eta, \tau) e^{\frac{x}{\nu L}}, \quad \theta(\eta, \tau) = \frac{T - T_o}{T_w - T_o}, \quad \tau = \frac{at}{2L e^{L}}
\]

where \( t \) represents time. Hence, equations (14) and (15) can be written as

\[
\frac{\mu_{nf}}{\mu_f} \frac{\partial^3 f}{\partial \eta^3} - 2 \left( \frac{\partial f}{\partial \eta} \right)^2 + \frac{\partial^2 f}{\partial \eta^2} + 2 - 2r \frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \eta \partial \tau} + 2r \frac{\partial f}{\partial \eta} \frac{\partial^3 f}{\partial \eta^3} - \frac{\partial^2 f}{\partial \tau^2} = 0
\]

\[
1 \frac{k_{nf}}{k_f} \left( \frac{\partial^3 \theta}{\partial \eta^3} + f \frac{\partial \theta}{\partial \eta} - \frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial \eta} - 2r \frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial \eta} \frac{\partial \tau}{\partial \eta} - \frac{\partial \theta}{\partial \tau} = 0
\]

and subjected to the boundary conditions

\[
f(0, \tau) + 2 \frac{\partial f(0, \tau)}{\partial \tau} = 0, \quad \frac{\partial f(0, \tau)}{\partial \eta} = \sigma \frac{\partial^2 f(0, \tau)}{\partial \eta^2}, \quad \theta(0, \tau) = 1
\]

\[
\frac{\partial f(\eta, \tau)}{\partial \eta} \rightarrow 1, \quad \theta(\eta, \tau) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty
\]

The stability of the steady flow solution \( f(\eta) = f_o(\eta) \) and \( \theta(\eta) = \theta_o(\eta) \) fulfilling the boundary-value problem (14)-(16) are tested and written as follow

\[
f(\eta, \tau) = f_o(\eta) + e^{i\gamma \tau} F(\eta, \tau), \theta(\eta, \tau) = \theta_o(\eta) + e^{i\gamma \tau} G(\eta, \tau)
\]

where \( \gamma \) is an unknown eigenvalues and \( F(\eta, \tau) \) and \( G(\eta, \tau) \) are relatively small compared to \( f_o(\eta) \) and \( \theta_o(\eta) \), respectively. Substituting (21) into Equations (18) and (19), the following linearized problem are obtained:

\[
\frac{\mu_{nf}}{\mu_f} \frac{\partial^3 F}{\partial \eta^3} - 4 \frac{\partial f_o}{\partial \eta} \frac{\partial F}{\partial \eta} + f_o \frac{\partial^2 F}{\partial \eta^2} + F \frac{\partial^3 f_o}{\partial \eta^3} + \gamma \frac{\partial F}{\partial \eta} - \frac{\partial^2 F}{\partial \tau^2} = 0 \tag{22}
\]

\[
1 \frac{k_{nf}}{k_f} \left( \frac{\partial^3 G}{\partial \eta^3} + f_o \frac{\partial G}{\partial \eta} + F \frac{\partial \theta}{\partial \eta} - \theta_o \frac{\partial F}{\partial \eta} - G \frac{\partial f_o}{\partial \eta} + \gamma G - \frac{\partial G}{\partial \tau} = 0 \right. \tag{23}
\]

along with boundary conditions:

\[
F(0, \tau) = 0, \quad \frac{\partial F(0, \tau)}{\partial \eta} = \sigma \frac{\partial^2 F(0, \tau)}{\partial \eta^2}, \quad G(0, \tau) = 0
\]

\[
\frac{\partial F(\eta, \tau)}{\partial \eta} \rightarrow 0, \quad G(\eta, \tau) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty
\]
As recommended by Weidman et al. [21], the stability of the steady flow are investigated by solving the corresponding linear eigenvalue problem:

\[
\frac{\mu_{nf}}{\mu_f} F''_u + f_s F'' + (4 f_s' - \gamma) F'_u = 0
\]

\[
\frac{1}{\Pr \left( \frac{\rho C_p}{k_f} \right)} G''_s + f_s G'' + F_s \theta' - \theta F_s' - G_s f_s' + \gamma G_s = 0
\]  

with boundary condition

\[ F_u(0) = 0, \quad F_u'(0) = \sigma F''_u(0), \quad G_s(0) = 0 \]

\[ F_u'(\eta) \rightarrow 0, \quad G_s(\eta) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \]  

The set of smallest eigenvalues can be obtained by modify a boundary condition on \( F_u'(\eta) \rightarrow 0 \) as \( \eta \rightarrow \infty \), as proposed by Harris et al. [24]. Hence, the set of equations (25)-(27) with the new boundary condition \( F''_u(0) = 1 \) are solved numerically.

4. Result and Discussions

The system of nonlinear ordinary differential Equations (7) and (8) subject to the boundary conditions (9) was numerically solved for different values of parameters using the bvp4c solver in Matlab software. This solver applies the three stage Lobatto IIIa formula and using a finite difference code (Kierzenka and Shampine [25], Shampine et al. [26]). Additionally, the finite values of \( \eta \rightarrow \infty \), namely \( \eta = \eta_o = 10 \) for the first and second solution branch have been chosen along with the relative tolerance of \( 10^{-7} \). The numerical solutions have been obtained for several values of the physical parameters, such as nanoparticle volume fraction \( \varphi \), slip parameter \( \sigma \) and stretching/shrinking parameter \( \varepsilon \). Table 1 illustrates the thermophysical properties of base fluid and nanoparticles. In this table, \( \varphi_1 \) and \( \varphi_2 \) are the Alumina and Copper nanoparticle volume fractions.

| Physical properties | Water | Al\(_2\)O\(_3\) | Cu |
|---------------------|-------|----------------|----|
| \( \rho \) (kg/m\(^3\)) | 997.0 | 3970 | 8933 |
| \( C_p \) (J/kgK) | 4180 | 765 | 385 |
| \( k \) (W/mK) | 0.6071 | 40 | 400 |

Reduced skin friction \( f''(0) \) and heat transfer \( \theta'(0) \) for different values of Copper nanoparticle \( \varphi_2 \) and slip parameter \( \sigma \) are presented in Figures 1 – 4. It is clearly observed that, for increasing Copper nanoparticle volume fraction, the reduced skin friction increases. This is due to the fact that the probability of nanoparticle agglomeration increases at higher volume fractions thus causing an increase in viscosity of hybrid nanofluid. Increasing value of Copper nanoparticle cause an increasing in heat transfer rate for the case of shrinking sheet while the opposite behaviour shown for the case of stretching sheet. Figures 3 and 4 shows that as slip parameter increases, the values of skin friction are found to increase for shrinking sheet and opposite trend can be observed for stretching sheet. Meanwhile, the values of heat transfer rate are increase with an increasing values of slip parameter. In addition, as the values of Copper nanoparticle and slip parameter increase, the range of solution to exist also increase.
Figures 5 and 6 present the effects of pure water, copper-water nanofluid and alumina-copper-water hybrid nanofluid on reduced skin friction $f''(0)$ and heat transfer $-\theta'(0)$. It is noticed that the similarity solution exists when $\varepsilon \geq \varepsilon_c = -1.6870$ for viscous flow ($\phi_1 = \phi_2 = 0$). However, the range of solution become wider ($\varepsilon \geq \varepsilon_c = -1.7338$) when we considered copper-water nanofluid ($\phi_1 = 0, \phi_2 = 0.1$) and for alumina-copper-water hybrid nanofluid ($\phi_1 = 0.1, \phi_2 = 0.1$), the range of similarity solution have been replaced between the values of pure water and copper–water nanofluid ($\varepsilon \geq \varepsilon_c = -1.7145$), respectively. As can be seen in Figures 1-6, dual solutions exist when $\varepsilon_c < \varepsilon < -1$, i.e., shrinking sheet, where $\varepsilon_c$ is the critical values of $\varepsilon$. While, a unique solution exists when $\varepsilon \geq -1$ and no solution when $\varepsilon < \varepsilon_c$. 
Figure 5. Reduced skin friction with $\varepsilon$ for different values of $\phi_1$ and $\phi_2$

Figure 6. Reduced heat transfer with $\varepsilon$ for different values of $\phi_1$ and $\phi_2$

Figures 7 and 8 display the variations of skin friction coefficient $C_f \Re_x^{1/2}$ and local Nusselt number $Nu_x \Re_x^{1/2}$ with Alumina nanoparticle volume fraction for several values of Copper nanoparticle and slip parameter, respectively for stretching sheet ($\varepsilon = 0.5$). Both figures show that the value of skin friction coefficient and local Nusselt number increases gradually as Alumina nanoparticle increase. However, it is seen that as slip parameter increase, the skin friction coefficient decreases and local Nusselt number increases. The velocity and temperature profiles are shown in Figures 9-12. From these figures, one can see that it satisfies the boundary conditions asymptotically for first and second solutions and thus supporting the graphical results presented in Figures 1-6. It is found that velocity profile significantly decreases with shrinking velocity for the upper branch solution, while increasing for the lower branch solution except for very small $\eta$. Also, it is revealed that the temperature profile increases in the first solution and decreases for the second solution with an increasing value of shrinking velocity. It is worth mentioning that the boundary layer thickness for the second solution are higher than first solution for both profiles.

Figure 7. Variation of skin friction coefficient with $\phi_1$ for different values of $\phi_2$ and $\sigma$

Figure 8. Variation of Nusselt number with $\phi_1$ for different values of $\phi_2$ and $\sigma$
The stability analysis is executed using the bvp4c solver in Matlab by substituting the system of linearized equations (25) and (26) along with boundary conditions (27). By finding a smallest eigenvalue $\gamma$, the stability of the solutions can be determined. The flow is stable if the smallest eigenvalue is positive while if the smallest eigenvalues is negative, the flow is unstable. The smallest eigenvalues for different slip parameter and stretching/shrinking parameter are displayed in Table 2. It is worth noting that as the shrinking rate closer to the critical value, $\varepsilon$, the smallest eigenvalue becomes near to zero. In addition, at the turning point of the first and second solution, the positive smallest eigenvalue changed signs to the negative eigenvalue. It is found that positive smallest eigenvalues are obtained for the first solutions meanwhile the second solutions have negative smallest eigenvalue. Consequently, we can deduce that the first solution is stable and physically reliable while the second solution is unstable.
Table 2. Smallest eigenvalues $\gamma$ at certain values of $\varepsilon$ for different $\sigma$, $\varphi_1$ and $\varphi_2$

| $\sigma$ | $\varphi_1$ | $\varphi_2$ | $\varepsilon$ | First solution | Second solution |
|----------|-------------|-------------|---------------|----------------|----------------|
| 0.1      | 0.1         | 0.1         | -1.5711       | 0.0575         | -0.0574        |
| 0.1      | 0.1         | 0.1         | -1.5819       | 0.0502         | -0.0502        |
| 0.1      | 0.1         | 0.1         | -1.581        | 0.1523         | -0.1519        |
| 0.1      | 0.1         | 0.1         | -1.57         | 0.5263         | -0.5210        |
| 0.2      | 0.1         | 0.1         | -1.5832       | 0.0279         | -0.0279        |
| 0.2      | 0.1         | 0.1         | -1.583        | 0.0732         | -0.0731        |
| 0.2      | 0.1         | 0.1         | -1.57         | 0.5522         | -0.5464        |
| 0.2      | 0.1         | 0.1         | -1.6866       | 0.0428         | -0.0428        |
| 0.2      | 0.1         | 0.1         | -1.686        | 0.1202         | -0.1199        |
| 0.2      | 0.1         | 0.1         | -1.67         | 0.5935         | -0.5870        |
| 0.2      | 0.1         | 0.1         | -1.7145       | 0.0228         | -0.0228        |
| 0.2      | 0.1         | 0.1         | -1.714        | 0.1039         | -0.1037        |
| 0.2      | 0.1         | 0.1         | -1.71         | 0.3053         | -0.3036        |
| 0.2      | 0.1         | 0.1         | -1.7176       | 0.0456         | -0.0456        |
| 0.2      | 0.1         | 0.1         | -1.717        | 0.1199         | -0.1197        |
| 0.2      | 0.1         | 0.1         | -1.71         | 0.3980         | -0.3951        |

5. Conclusions
We have numerically investigated the effect of slip on boundary layer flow and heat transfer over an exponentially stretching/shrinking sheet in hybrid nanofluid along with stability analysis. We can summarize that dual solution obtain for hybrid nanofluid in the case of shrinking sheet. It is found that increasing value of slip parameter widens the range of solution and increase the heat transfer rate and skin friction in hybrid nanofluid. Meanwhile, skin friction coefficient decreases with an increase of slip parameter for the case of stretching sheet. In addition, increasing value of Copper nanoparticle also widen the range of solution to exist. It is seen that hybrid nanofluid is more effective in cooling process compared to nanofluid and viscous fluid. We also observed that the first solution was stable whereas the second solution was unstable solution.

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