Spin-orbit coupling in a Quantum Dot at high magnetic field

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We describe the simultaneous effects of the spin-orbit (SO) perturbation and a magnetic field $B$ on a disk shaped quantum dot (QD). As it is known the combination of electrostatic forces among the $N$ electrons confined in the QD and the Pauli principle can induce a spin polarization when $B$ (applied in the direction orthogonal to the QD) is above a threshold value.

In the presence of an electric field parallel to $B$, coupled to the spin $S$ by a Rashba term, we demonstrate that a symmetry breaking takes place: we can observe it by analyzing the splitting of the levels belonging to an unperturbed multiplet. We also discuss the competitive effects of the magnetic field, the SO perturbation and the electron-electron interaction, in order to define the hierarchy of the states belonging to a multiplet. We demonstrate how this hierarchy depends on the QD’s size. We show the spin texture due to the combined effects of the Rashba effect and the interaction responsible for the polarization.

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I. INTRODUCTION

The physics of mesoscopic devices has attracted a lot of interest in the last two decades. In particular the electronic transport properties of semiconducting Quantum Dots (QDs) were largely investigated. The QDs, in which we are interested, are two-dimensional man-made "droplets" of charge, confined to a small area within a two dimensional electron gas (2DEG), which can contain anything from a single electron to a collection of several thousand ones. Their typical dimensions range from a few nanometers to a few microns and their size, shape and interactions can be precisely controlled through the use of advanced nanofabrication technologies.

The electronic transport through these devices was intensely studied, and it was demonstrated that electron-electron interactions play a central role. In particular, the conductance of a vertical QD was measured, which allowed one to carry out a detailed study of the ground state and the first excited states of a few electrons confined in the QD. The QDs that we have in mind in this paper, are the vertical ones obtained by an electrostatic confinement due to the gate voltages as the ones in the experimental set of ref2,3.

The development of the mesoscopic physics favored the idea of using the electron spin in these devices, both for transmitting and processing information4,5. Datta and Das6 in 1990 described how the electrical field can be used to modulate the current and showed the essential role which the field-dependent spin-orbit (SO) coupling plays in this mechanism. In semiconductors heterostructures, where a 2DEG is confined in a potential well along the $z$ direction, the SO interaction is of the type proposed by Rashba7 and Dresselhaus8: it arises from the asymmetry of the confining potential which occurs in the physical realization of the 2DEG, due to the band offset between AlGaAs and GaAs. Even though the Rashba spin splitting is expected to be very small, nonetheless this perturbation can give rise to a sizeable modification of a semiconductor band structure10,11.

The SO interaction comes from the expansion quadratic in $v/c$ of Dirac equation12 and is due to the Pauli coupling between the spin momentum of an electron and a magnetic field, which appears in the rest frame of the electron, due to its motion in the electric field. It follows that the effects of an electric field ($\mathbf{E}(\mathbf{R})$ where $\mathbf{R}$ is the 3D position vector) on a moving electron have to be analyzed starting from the following hamiltonian:

$$
\hat{H}_{SO} = -\frac{\hbar}{(2M_0c)^2} \mathbf{E}(\mathbf{R}) \left[ \hat{\sigma} \times \left\{ \hat{p} - \frac{e}{c} \hat{A}(\mathbf{R}) \right\} \right].
$$

Here $M_0$ is the free electron mass, $\hat{\sigma}$ are the Pauli matrices, $\hat{A}$ is the vector potential.

The interface electric field ($\mathbf{E}(\mathbf{R}) \approx (0, 0, E_z)$), which accompanies the quantum well asymmetry in semiconductors heterostructures, is directed along the normal to the device plane13 at the interface. Experimentally, in GaAs-AsGaAl interface, values for $\alpha E_z$ of order $10^{-11}$ eV m were observed14, where $\alpha = \frac{\hbar^2}{(2M_0c)^2}$.

Recently the "Rashba interaction" led to an intense research activity also including the effects of disorder and interaction. The theory of transport in the presence of SO interaction including disorder was developed also in the presence of an in-plane magnetic field, yielding a characteristic anisotropic conductivity as a function of the magnetic field15. More recently the so-called spin-Hall effect was proposed16 where the spin current response is due to an applied transverse electric field.

The effects of a strong transverse magnetic field (i.e. applied in the direction orthogonal to the 2DEG) was also analyzed in ballistic Quantum Wires in the presence of Rashba coupling. In a recent article we showed that the magnetic field enhances the spin selection in the current and also gives very singular spin textures in
nanostructures.

In the last years also the case of a QD was analyzed: the effect of spin-orbit coupling on the electronic structure of few-electron interacting QDs is a suppression of Hund’s rule due to the competition of the Rashba effect and the exchange interaction. This behaviour can be measured in vertical QDs while, in lateral semiconductor QDs, weak localization and universal conductance fluctuations were analyzed: in the presence of both SO scattering and a magnetic field the conductance of a chaotic QD is a function of the parallel and perpendicular magnetic field and the SO coupling strength.

In this paper we analyze what happens when a strong transverse magnetic field acts on a vertical QD in the presence of a Rashba coupling. The small strength of the term $\alpha E_z$ allows for a perturbative treatment of the SO coupling. We recall that, when a magnetic field is present, we should take into account also the Zeeman effect. However, because in GaAs systems the band effect renormalizes the electron mass $(m^* = 0.067 M_0)$ the Zeeman splitting is reduced by a factor 4, so that it does not drive any spin polarization in these systems. We will apply our results to two QDs of different radius, in order to emphasize the competitive effects of the magnetic field, the SO perturbation and the electron electron interaction, and hence be able to define the hierarchy of the states belonging to a multiplet.

In sec.II we discuss the single particle problem by showing the relevance of a term in the Rashba hamiltonian neglected in a recent article about this same topic.

In sec.III we discuss the many electron case. In the first subsection we report an overview of the phenomenology without the SO coupling. In the second subsection we investigate how the Rashba coupling split a fully spin polarized multiplet for a 5 electrons QD.

In appendix A we discuss in detail the case of an unperturbed QD with an arbitrary number of electrons, while in appendix B we discuss the simplest case of a 2 electrons QD.

II. SINGLE ELECTRON

Usually a QD is modeled by a two dimensional harmonic confining potential $V(r) = \frac{\hbar^2 \omega_z^2}{2}\left| r \right|^2$ which suggests, with its symmetries, the choice of the symmetric gauge ($A = (-By/2, Bx/2, 0)$).

The unperturbed hamiltonian for a single electron

$$H_T = \frac{1}{2m} \left( \frac{\hat{p}^2}{c^2} + \frac{\hbar^2}{2m} \hat{r}^2 - \frac{\hbar^2 \omega_z^2}{2} \hat{r}^2 - \frac{\hbar}{2} J_z \right),$$

with $\omega_z = \frac{eB}{mc^2}$ commutes with $\hat{L}_z$ and yields the usual Fock-Darwin single particle energy levels and the related eigenfunctions

$$\varepsilon_{n_+, n_-} = n_+ \omega_+ + n_- \omega_- + \hbar \omega_T,$$

where $\omega_+ = \omega_T \pm \frac{\hbar}{2}$ and $\omega_T = \sqrt{\frac{\hbar^2}{4} + \omega_z^2}$.

In a vertical geometry, orbital effects induced by a magnetic field orthogonal to the dot are dominant: the increasing magnetic field gives an orbital polarization, up to a state where all the electrons are in the lowest Pseudo Landau Level (PLL) ($n_+ = 0$). In general the PLLs correspond to a fixed $n_-$ and become the Landau Levels (LLs) in the limit $n_+ \rightarrow 0$. Because the energy levels, when $\omega_d$ vanishes, depend just on $n_-$, the LLs are infinitely degenerate while the PLLs are spaced by $\omega_z$.

The SO hamiltonian in eq.(1) can be expressed in terms of $\hat{a}_\pm, \hat{a}^\dagger_\pm$ operators.

$$H_{SO} = \frac{\hbar}{2m} \left( \frac{\hat{a}_+^\dagger \varepsilon_+ \hat{a}_- - \varepsilon_- \hat{a}^\dagger_- \hat{a}_+}{2} \right) \sigma_+$$

$$+ \frac{\hbar}{2m} \left( \frac{\hat{a}_+^\dagger \varepsilon_+ \hat{a}_- - \varepsilon_- \hat{a}^\dagger_- \hat{a}_+}{2} \right) \sigma_-,$$

where $\sigma_\pm = \frac{1}{2}(1 \pm \gamma)$.

Thus, we can obtain the perturbation to the LLs by setting $\gamma = 1$ in eq.(3) and the QD without magnetic field in the opposite limit, i.e. $\gamma = 0$.

The SO perturbation breaks the symmetries of the QD, so that $m$ and $s_z$ are not any more good quantum numbers for the electron, while eq.(3) still preserves $j_z$.

For a single electron in the lowest PLL the spin-orbit correction to the energy are, to the second order in $\gamma$, 

$$\Delta \varepsilon_{m,0,\pm} \approx \alpha(\omega_z)^2 \left( \frac{j_z + 1/2}{\hbar \omega_-} - \left( \frac{1}{1 - \gamma} \right) \frac{1}{\hbar \omega_z} \right)^2,$$

$$\Delta \varepsilon_{m,0,\uparrow} \approx -\alpha(\omega_z)^2 \frac{(j_z + 1/2)}{\hbar \omega_-}.$$

where

$$\alpha(\omega_z) = \frac{\hbar}{2M_0} \frac{\hbar \omega_T}{2M_0} (1 - \gamma).$$

In the typical QDs that we consider, $\alpha(0) \approx 0.2-0.5meV$.

From eq.(4) and eq.(5) we can conclude how the unperturbed spin-degenerate levels in the lowest PLL are split by the SO perturbation. It is easy to show that the states with spin down $(m, \downarrow)$ have energies lower than those of the corresponding spin up states $(m, \uparrow)$, at both the low and the high limit of the magnetic field, although there might exist an intermediate regime where the spin up states have the lowest energies.

It is also easy to calculate the perturbed single particle eigenfunctions, $\psi(r)$, and then evaluate the local spin density (LSD), as the mean value of the spin vector as a function of the position $r$, i.e. $S(r) = \langle \psi(r) \mid \hat{S} \mid \psi(r) \rangle$. It could also be interesting to analyze the magnetization $\mathbf{m}(r, \varphi) = \frac{\hbar \mathbf{r} \varphi}{mc^2}$, where $\rho$ is the charge density. The
analysis of the magnetization gives a better picture of the spin texture than the one obtained from the LSD, because it describes the local orientation of the spin vector by neglecting the attenuation of the charge density.

The spin texture could be compared with the one which we showed for a Quantum Wire. The spin has to be orthogonal to the velocity (current) at each time as we found in ref [17], but it can reverse its direction going from the center to the edge of the QD. We do not discuss in details this behaviour, but just limit ourselves to point out the relevance of the perturbative term neglected in ref [17], where a gauge noninvariant treatment was implemented.

III. MANY ELECTRON DOT

The many body hamiltonian has in general the form

$$\hat{H}_0 = \sum_\alpha \varepsilon_\alpha \hat{n}_\alpha + \frac{1}{2} \sum_{\alpha,\beta,\gamma,\delta} V_{\alpha,\beta,\gamma,\delta} \hat{c}_\alpha^\dagger \hat{c}_\beta \hat{c}_\gamma \hat{c}_\delta,$$

where \(\alpha \equiv (n_+ n_- s)\) denotes the single particle state. In the single particle energy level \(\varepsilon_\alpha\), \(\hat{c}_\alpha^\dagger\) creates a particle in the state \(\alpha\) and \(\hat{n}_\alpha \equiv \hat{c}_\alpha^\dagger \hat{c}_\alpha\) is the occupation number operator.

Because of the symmetries of eq. (6), due to the properties of electron electron interaction, once the number of electrons in the QD (\(N\)) is fixed, we can characterize the ground state (GS) with its spin \(S\) and angular momentum \(M\) and, starting from eq. (6), calculate in Hartree Fock approximation (HF) the properties of the GS when the magnetic field \(B\) increases.

In the special case of just one filled PLL (\(n_- = 0\)), we can introduce a simplified hamiltonian, which has many analogies with a chiral Luttinger model, with just one branch involved. In this case, we substitute \(V_{\alpha,\beta,\gamma,\delta}\) with constant coupling strengths, \(g^\parallel\) (corresponding to a scattering process involving electrons with the same spin) and \(g^\perp\) (corresponding to a scattering process involving electrons with opposite spins)

$$H_0^S = \hbar v_F \sum_{m,s} m c_{m,s}^\dagger c_{m,s} \quad + \quad \sum_{m,\mu,\sigma} g^\parallel_{\mu,\sigma} \delta_{\mu,\sigma} \left( c_{m+q,\mu,\sigma}^\dagger c_{m,\mu,\sigma} c_{m+q,\mu,\sigma} c_{m,\mu,\sigma} \right) \quad + \quad \sum_{m,\mu,\sigma} g^\perp_{\mu,\sigma} \delta_{\mu,\sigma} \left( c_{m+q,\mu,\sigma}^\dagger c_{m,\mu,\sigma} c_{m+q,\mu,\sigma} c_{m,\mu,\sigma} \right).$$

Here there appear both a field dependent interaction parameter \((1/\ell \equiv \sqrt{\omega_T/\omega_d})\) and a field dependent Fermi velocity \(v_F = \omega_d/\ell\).

The hamiltonian in eq. (7) can be diagonalized analytically and gives many interesting results. The effects of the long range interaction, that sometimes can be quite important, were discussed in a recent paper where we report some details about the derivation of eq. (7).

A. Unperturbed Dot

When there is no SO coupling, the quantum numbers labeling the dot energy levels are the number of electrons \(N\), the total orbital angular momentum along \(z\), i.e. \(M\), the total spin \(S\) and the \(z\)-component of the spin \(S_z\).

The orbital polarization, corresponding to all the electrons in the lowest PLL, was often revealed in the past: if the magnetic field is above a threshold value \(B_s\) (depending on the number of electrons \(N\)), a "singlet" \((S = 0\) or paramagnetic state) is observed where \(N/2\) single electron states in the lowest PLL are doubly filled. As it was clear in the measurements for a 24 electrons dot, by increasing the magnetic field \(B\) above a larger threshold value \(B_p\), both \(M\) and \(S\) increase. The electron system is expected to be polarized, if the reduction in interaction energy (Coulomb exchange) due to creating a finite spin polarization state exceeds the cost in single particle kinetic energy. This spin polarization phase ends when \(B = B^*\) and a fully spin polarized (FSP), \(M = (N-1)/2\) and \(S = N/2\), state is reached. By further increasing \(B\), the electron density reaches a maximum value, so that the QD is in a state usually called the maximum density droplet (MMD)\(^{27,28}\). For larger \(B\) values, the FSP state is disrupted: changes in the density are produced at the edge of the dot and a situation known as dot reconstruction\(^{29}\) occurs by creating a ring of filled states separated from a remaining core of filled states by a ring of empty states.

The theoretically obtained diagram in Fig. (1), usually reported as phase diagram, where the spin properties of the GS of the many electron system are shown as function of the number of electrons \((N)\) and the magnetic field \((B)\), is calculated in appendix A and can also be compared with measurements in\(^{28}\). In this calculation, the long range effects of the interaction are sometimes quite relevant.

B. Introduction of the SO

Here we follow the perturbative approach in the SO coupling, in order to calculate the energy splitting of the FSP multiplet.

In the presence of SO coupling, \(J_z = M + S_z\) becomes the good quantum number and the FSP, which is the GS of the unperturbed dot if \(B\) is above a threshold value, corresponds to a multiplet \(J_z = N(N-1)/2 + S_z\).

In order to explain what happens when the SO coupling acts on the FSP, we have to put the SO hamiltonian in eq. (10) in the second quantization form

$$H_{SO} = \lambda \sum_{m} \left( c_{1, m, \uparrow}^\dagger c_{0, m, \downarrow} - c_{0, m, \uparrow}^\dagger c_{1, m, \downarrow} \right) \frac{1 + \gamma}{1 - \gamma}. $$

Here \(c_{m,-n, \sigma}^\dagger\) are the usual creation operators for the
electrons, $m$ labels $n_+$, so that $m$ is the angular momentum if $n_- = 0$ (i.e. if the state belongs to the lowest PLL).

Now we want to calculate the energy splitting of the states belonging to a multiplet, starting from the wavefunction $\Psi^0_{N,M,S,S_z}$, where $-S \leq S_z \leq S$, up to the second perturbative order. Then, we take in account all the Slater determinants with $N$ electrons $\Psi^{SD}_{N,J_z,l}$, where $l$ labels each different Slater determinant. We calculate the perturbative energy for the states $\Psi^0$

$$\Delta \varepsilon_{N,M,S,S_z} = \sum_{l} \left| \langle \Psi^{SD}_{N,J_z,l} | H_{SO} | \Psi^{0}_{N,M,S,S_z} \rangle \right|^2 \varepsilon_{N,M,S,S_z} - \varepsilon_l,$$

where

$$\varepsilon_l = \langle \Psi^{SD}_{N,J_z,l} | H_0 | \Psi^{SD}_{N,J_z,l} \rangle.$$

Starting from the hamiltonian in eq. (7), the splitting energies for the FSP multiplet can be analytically expressed in terms of $S, S_z$ and $U = (g^\perp - g^\parallel)$

$$\Delta \varepsilon_{N,S_z}(U, \omega_c) \approx -\alpha(\omega_c)^2 \left\{ \frac{(S + S_z)(S + S_z + 1)}{\sqrt{\omega_d}} U \left[ S^2 - (S_z - 1)^2 \right] + \omega_c \right\} + \left( 1 + \gamma \right)^2 \frac{(S - S_z)}{\sqrt{\omega_d}} U \left[ S^2 - (S_z + 1)^2 \right] + \omega_c \right\} + \left( S - S_z \right) \left( S - S_z - 1 \right) / \sqrt{\omega_d} U \left[ S^2 - (S_z + 1)^2 \right] - \omega_c}. \quad (8)$$

The SO interaction lifts the spin degeneracy. From a theoretical point of view, we can analyze how the hierarchy of the states belonging to a multiplet, from ground to top, is due to the competitive effects of the magnetic field, the SO perturbation and the electron electron interaction. So, we can discuss two regimes, i.e. the one of weak SO (WSO) perturbation and the opposite regime, i.e. the weak interaction (WI) one. In fact, the strength of $U$ is responsible for the ordering in energy of the sequence $J_z$, in competition with the magnetic field.

The two states which compete as GS in the $(2S + 1)$-plet are the one with $S_z = -S$ in the WI regime and the one with $S_z = S - 1$ in the WSO limit. The magnetic field favors $J_z = J_{min}$ as the GS if $U$ is not too large. This could also be shown by treating the electron electron interaction as a perturbation. Our results confirm the hierarchy of the multiplet states from the GS $M - S$ to the rather unperturbed $M + S$. These results are shown in Fig.(2).

In the opposite limit, the WSO shown in Fig.(2, right), the presence of the state with $J_z = J_{max} - 1$ as a GS could seem suspicious; however it is a consequence of the strong reduction in the interaction due to the spin polarization. In fact, all the states obtained from it by applying the perturbation are fully polarized states ($S = N/2$) and strongly reduce the interaction energy. This is better
explained in appendix B, where the energy levels of a two electrons dot are discussed.

Both cases discussed above yield a singular spin texture, due to the combined effects of the interaction responsible for the polarization and the Rashba effect.

IV. CONCLUSIONS

In this article we discussed how the presence of a SO coupling affects the charge and the spin polarization in a vertical disk-shaped quantum dot under a strong magnetic field.

When there is no Rashba coupling, the combined effect of the electrostatic forces between the N electrons confined in the QD and the Pauli principle induces a spin polarization, in the presence of a strong magnetic field B. However, also in this case the electron system preserves the quantum number S.

The presence of an electric field parallel to B, coupled to the spin S by a Rashba term, breaks this symmetry. Here we have analyzed the splitting in the levels of the states belonging to a multiplet, and their hierarchy from ground to top.

Now we can discuss the effect of this coupling for two different real QDs filled by 5 electrons, by choosing the ground to top.

When there is no Rashba coupling, the combined effect of the interaction represented by the SO coupling a(0) ≈ p\beta: (left) the state S_z = −5/2, (right) the state (S_z = 3/2). Here r_0 = \sqrt{\hbar/(m\omega)}.

which favors S = 3/2 as GS. The comparison between the two cases is shown in Fig. (2).

Our results can be compared with the ones obtained in a recent paper, where another term of the SO perturbation dominates, obtained by breaking gauge invariance. Hence, we believe that the hierarchy of the states reported in Ref. [22] has to be revised, by including the correct gauge invariance term. We also believe that the inclusion of all the terms in the perturbation modifies the charge density and the LSD, as can be seen in Fig. (3). It could have some hard consequences on the analogy with the "Skyrmion states" discussed in Ref. [22].

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APPENDIX A: SPIN PHASES IN QDS

Here we want to illustrate some simple calculations that allow us to deduce at which value of the magnetic field some phenomena occur. This kind of simplified calculations allows for a good knowledge of the phase diagram shown in Fig. (1). This approach assumes the electron-electron interaction as a perturbation with respect to the kinetic energy and is stopped at first order, so that no correlation effects are appreciable in this way.

We can fix the number of electrons N, write the single Slater Determinants (SDs) Ψ^{SD}_{N,M,S,S_z} and then compare the mean value of the Hamiltonian eq. (7) on these wavefunctions. We can write, in the Lowest PLL limit,

\[ |M,S_z⟩ = \prod_{m,\mu=0}^∞ (\hat{\epsilon}_{0,m,\uparrow}^{\mu})^{n_{0,m,\uparrow}} (\hat{\epsilon}_{0,m,\downarrow}^{\mu})^{n_{0,m,\downarrow}} |0⟩, \quad (A1) \]

where

\[ M = \sum_{m=0}^∞ (n_{0,m,\uparrow}+n_{0,m,\downarrow})m \quad S_z = \frac{1}{2} \sum_{m=0}^∞ (n_{0,m,\uparrow}−n_{0,m,\downarrow}). \]

In the Lowest PLL limit some of these SDs, i.e. those with fixed M, S and S_z = ±S, are eigenstates of the Hamiltonian, because no other states with the same M and S_z can be found. As the eigenstates are not degenerate, there are no correlation effects, hence the HF approach yields exact results. This is the case of the singlet, \( \nu = 2 \) state, the FSP filling \( \nu = 1 \), but also the state of their fundamental excitations.

For singlet, spin-degenerate states belonging to the lowest PLL (Singlet, \( \nu = 2 \)) one has

\[ M = N/2(N/2) S_z = S_z = 0. \]

As B is increased further, it becomes energetically favorable for an electron to flip its spin and move to the edge of the dot (First spin flip).
\( M = N/2(N/2 - 1) + 1 \) \( S = S_z = 1 \).

Then the electrons flip the spins one by one from the edge to the core, until the last spin flip from the state \( M = N(N - 1)/2 - N + 1 \) \( S = S_z = N/2 - 1 \) to the Fully Polarized state (FSP or MDD, \( \nu = 1 \)) \( M = N(N - 1)/2 S_z = S = N/2 \).

When \( B \) is increased further, the angular momentum states shrink in size such that the density of the MDD increases. At some threshold \( B \) value the direct Coulomb interaction has become so large that the MDD breaks apart into a larger, lower density droplet (LDD). The second threshold corresponds to the interaction-induced spin flip at the edge, i.e. the condition for having all the electrons in the lowest quantum numbers of the interacting electrons \( m, \sigma \). So we can assume, starting from a Dirac \( \delta \) model of the interaction,

\[
g_{m,m',\mu,\mu'} = g_{m,m'\mu}\mu' \frac{2m+\mu}{\Upsilon_{m+\mu}} \frac{2m'+\mu'}{\Upsilon_{m'+\mu'}},
\]

where \( \Upsilon_m = \sqrt{\frac{11+m}{2}} \) and \( i = \perp, \parallel \).

A second effect, that the constant interaction model is not able to explain, concerns the transitions from \( \nu = 2 \) to \( \nu = 1 \) observed in the experiments in ref.\( ^2 \) for a 24 electrons Dot. The analysis about the phase corresponding to \( 2 < \nu < 1 \) is one of the central points of the discussion in ref.\( ^2 \), where experimental data were compared with predictions obtained by using the HF approach.

In this model all the spins flip at the same critical field, provided the magnetic field is above the total orbital polarization field obtained from the condition

\[
\omega_-(N - 1) < \omega_+ \Leftrightarrow \omega_c > \omega_d \sqrt{(N-2)^2 / 2(N-2)}. \tag{6}
\]

Now we are ready to plot the phase diagram as shown in Fig.(1). The equations above allow us to predict the magnetic fields corresponding to the transition \( \nu = 2 \rightarrow \nu < 2 \). In agreement with the experimental data of ref.\( ^2 \) we can also deduce the value of the magnetic field for having the transition \( \nu = 2 \rightarrow \nu < 2 \) for fixed \( N \) and \( B \). The experiment in ref.\( ^2 \) also show the transition to FSP (\( \nu > 1 \rightarrow \nu = 1 \)) at a field \( B \approx 6T \). This transition can be predicted in our model (eq.(5)) by putting \( U = (g_{\perp} - g_{\parallel}) \approx 0.1 - 0.15 \hbar \omega_d \).

Although the model gives good results in the description of these two transitions in the limit of not so large \( N \), from the experimental data in ref.\( ^2 \) we can also deduce that at very strong field the FSP can be disrupted by the formation of hole particle excitations at the edge. In order to reproduce this phenomenon, we have to introduce a modification of the interaction parameters, \( g_{m,\mu}^\parallel \) and \( g_{m,\mu}^\perp \), by taking in account their dependence on the quantum numbers of the interacting electrons \( m, \sigma \). So we can assume, starting from a Dirac \( \delta \) model of the interaction,

\[
g_{m,m',\mu,\mu'} = g_{m,m'\mu}\mu' \frac{2m+\mu}{\Upsilon_{m+\mu}} \frac{2m'+\mu'}{\Upsilon_{m'+\mu'}},
\]

where \( \Upsilon_m = \sqrt{\frac{11+m}{2}} \) and \( i = \perp, \parallel \).

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In this case our model with constant interaction can just predict, starting from the values of the two transitions fields \( B_{\nu=2} \approx 1.5T \) and \( B_{\nu=1} \approx 4.5T \), that the QD has a radius larger than the one used in the previous experiment (\( R/r \approx 2 - 3 \)) and a different interaction strength. The modified interaction parameters allow us to calculate also the spin susceptibility, obtaining results in agreement with the ones reported in ref.\( ^2 \).

## II. 2 ELECTRONS DOT

In order to explain why \( J_{max} - 1 \) can be the GS for a strongly interacting electron system, we can discuss what happens for the simplest case of a 2 electrons QD. The unperturbed state is the triplet \( (M = 1 \ S = 1) \)

\[
|a\rangle = c_{0,0,\uparrow}^\dagger c_{0,1,\uparrow}^\dagger |0\rangle
\]
\[ |b⟩ = \frac{1}{\sqrt{2}} \left( c_{0,0,0,0}^r |0⟩ + c_{0,0,0,1}^r |1⟩ \right) \]

\[ |c⟩ = c_{0,0,0}^r |0⟩ \]  

The SO hamiltonian in the second quantization form, if we consider the limit of strong magnetic field (a condition enforced because the FSP states need a magnetic field above a threshold), reads

\[ H_{SO} \approx iα(ω_c) \sum_m (c_{1,m,1}^r c_{0,m,0}^l + c_{1,m,0}^r c_{0,m,1}^l) \frac{(1 + γ)}{(1 - γ)}. \]

It follows that the \( H_{SO}|a⟩ = 0 \), so that the state \( a \) is unperturbed.

The state \( |b⟩ \) has an interesting property, so that the SO hamiltonian acts on it: \( |β⟩ = H_{SO}|b⟩ \) has total spin \( S = 1 \). In fact

\[ H_{SO}|b⟩ = |b⟩ + iα(ω_c) \left( c_{1,0,0,0}^r |0⟩ + c_{1,0,0,1}^r |1⟩ \right) \frac{(1 + γ)}{(1 - γ)} |0⟩ \]

which, in a different formalism, could be written as

\[ H_{SO}|b⟩ = ρ_{M=1}(r_1, r_2)|χ_{1,0}⟩ + ρ_{M=0}(r_1, r_2)|χ_{1,1}⟩. \]

This is a general property of the states with \( J_{max} - 1 \), i.e. under the action of \( H_{SO} \) they preserve the total spin. So, we can apply Hund’s rule, in order to conclude that these states minimize the interaction. This result does not depend either on the formalism nor on the model of interaction.

We just finish by showing that the state \( |c⟩ \) gives

\[ H_{SO}|c⟩ = \rho_{M=1}(r_1, r_2)|χ_{1,1}⟩ - iα(ω_c)\rho_{M=0}(r_1, r_2)\sqrt{\frac{γ}{2}}. \]

So, we can conclude that, in a general case, the interaction splits the perturbed states of a multiplet by favoring the state \( J_{max} - 1 \), and this state can be the lowest energy state if the interaction is quite strong, as we show in Fig.(2.right).

Because in our previous discussion we did not specify which kind of interaction we used, these results are rather general. The central question is: can \( U \) be strong? If we take a model with constant interaction (in configuration space) there is no Coulomb exchange and \( U \) is always 0. In general \( U \) does not vanish, and the factor \( \sqrt{\frac{γ}{2}} \) enforces its action at strong values of the magnetic field.

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