Vortex States of Chiral p-wave Superconductors

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Abstract. In chiral p-wave superconductors a flux lattice of doubly quantized vortices is shown to be energetically stable for fields \( H_c^1 < H < H_c^2 \), while at low fields a lattice of singly quantized vortices is stable. Here we report self-consistent calculations by Eilenberger theory for spatial structures of the pair potential and current density for vortex states of single- and double-winding vortices in chiral p-wave superconductors with a cylindrical Fermi surface.

1. Introduction

Chiral p-wave superconductivity is an exotic state in which both space- and time-inversion symmetries are spontaneously broken by the pair condensate. This type of pairing is realized in superfluid \(^3\)He-A, and is a candidate for the ground state of the superconducting phase of Sr\(_2\)RuO\(_4\) [1]. For tetragonal symmetry chiral, p-wave superconductivity belongs to a two-dimensional representation \((E_{1u})\) of the point group with basis functions, \((p_x,p_y)\). A chiral ground state, e.g. \(p_- = p_x - ip_y\) with internal orbital angular momentum \(L_z^\text{int}/\hbar = -1\), breaks time-reversal symmetry, and is degenerate with the time-reversed orbital state, \(p_+ = p_x - ip_y\) and \(L_z^\text{int}/\hbar = +1\). This leads to novel inhomogeneous states in chiral, p-wave superconductors. In particular, for a ground state with chirality \(p_-\) (the “main” chiral component), a spatially varying state such as in vortex will have the time-reversed chiral component, \(p_+\) (the “induced” chiral component) induced in the vicinity of the vortex core [2]. Furthermore, the winding number \((m)\) of the induced chiral component is tied to that of the main chiral component \((n)\) by the orbital winding numbers of the two chiral components, \(n - 1 = m + 1\) [3]. For the doubly quantized vortex in \(p_-\) \((n = 2)\) the induced chiral component has zero-winding number \((m = 0)\). Thus, the amplitude of the induced component is large - of order the bulk order parameter even at the vortex center, which leads to a “coreless” double quantum vortex of almost uniform condensate density and very low vortex core energy density compared to that for singly quantized vortices [4]. Observation of a vortex state of doubly quantized vortices would provide strong evidence for a chiral superconducting ground state.

Here we report results for the structure and stability of flux lattice phases formed from doubly quantized vortices in a chiral p-wave superconductor as a function of temperature and magnetic field. In Sec. 2 we outline the formulation of the Eilenberger’s equations used for this purpose, and in Sec. 3 we present results illustrating the differences in the pair potential and vortex currents between vortex lattices formed from singly- and doubly quantized vortices. In Sec. 4 we present results for the field-dependence of the free energy of these vortex lattice states. We conclude with a summary and discussion of these results and the current status of known vortex states in Sr\(_2\)RuO\(_4\).
2. Eilenberger’s equations

Self-consistent calculations of the spatial structure of the pair potential, $\Delta(p, r)$ are carried out in the clean limit using Eilenberger’s transport equations [2],

$$
\{\omega_n + v \cdot (\nabla + iA)\} f = \Delta g, \quad \{\omega_n - v \cdot (\nabla - iA)\} f^\dagger = \Delta^* g, \quad (1)
$$

where $f(\omega_n, p, r)$ and $f^\dagger(\omega_n, p, r)$ are the anomalous pair propagators and $g(\omega_n, p, r)$ is the quasiparticle propagator, determined here by Eilenberger’s normalization condition, $g = (1 - ff^\dagger)^{1/2}$. The pair propagators determine the mean field order parameter (‘pair potential’), $\Delta(p, r)$, while the quasiparticle propagator determines the single-particle spectrum and related observables, e.g. the supercurrent density $j(r) = -\kappa^{-2} 2T \sum_{\omega_n>0} \langle v \text{Im} g \rangle_p$ associated with the vortex state, where $\langle \cdots \rangle_p$ indicates an average over the Fermi surface, and $\kappa = \lambda/\xi$ is the Ginzburg-Landau ratio penetration depth to coherence length.

For vortex states in a chiral $p$-wave superconductor the pair potential takes the general form,

$$
\Delta(p, r) = \Delta_+(r)\phi_+(p) + \Delta_-(r)\phi_-(p),
$$

where we use the $E_{1u}$ basis functions for a cylindrical Fermi surface given by $\phi_{\pm}(p) = (p_x \pm ip_y)/p_F = e^{\pm i\theta}$, and $v = p/p_F = (\cos \theta, \sin \theta, 0)$ defines the direction of the trajectory in Eq. (1) with the relative momentum, $p$, of a Cooper pair on the Fermi surface, and $r$ is the center of mass coordinate of the pair. Hereafter, we scale length, temperature, magnetic field, and energies in units of $\xi_0$, $T_c$, $B_0$, and $\pi k_B T_c$, respectively, where $\xi_0 = h v_F / 2\pi k_B T_c$, $B_0 = \phi_0/2\pi \xi_0^2$ and $\phi_0 = h c/2 e$ is the superconducting flux quantum [5].

In the calculations reported here we assume $\kappa \gg 1$. Then, for a uniform magnetic field $H = (0, 0, H)$, the vector potential is given by $A(r) = \frac{1}{2}H \times r$ in the symmetric gauge. For a $c$-axis field with $H > 0$ we choose the ground state with chirality $\pm$. The amplitude $\Delta_-(r)\phi_-(p)$ is then the main component in describing the vortex state, while the time-reversed chiral state with amplitude $\Delta_+ r\phi_+(p)$ is induced by the spatial and orbital phase variations of main component $\Delta_-(r)\phi_-(p)$. The pair potential is self-consistently calculated from the weak-coupling gap equations,

$$
\Delta_{\pm}(r) = \lambda_0 2T \sum_{\omega_n>0} \langle \phi_{\pm}^*(p) f \rangle_p,
$$

where $\langle \cdots \rangle_p$ indicates an average over the Fermi surface, and $\lambda_0 = N_0 g_0$ is the dimensionless pairing interaction in the low-energy band, $|\omega_n| \leq \omega_c$, defined by the cutoff energy, $\omega_c$. In the weak-coupling limit, $T_c \ll \omega_c \ll E_F$, $\omega_c$ and $\lambda_0$ can be eliminated in favor of $T_c$. We calculate using the cutoff, $\omega_c = 20k_B T_c$, and divide out the pairing interaction with $1/\lambda_0 = \ln T + 2T \sum_{\omega_n>0} \omega_1^{-1}$.

We assume a triangular vortex lattice for $\Delta(r)$ with unit vectors $u_1 = (a_x, 0, 0)$, $u_2 = (a_x/2, a_y, 0)$ and $a_y/a_x = \sqrt{3}/2$. For vortex states with a single flux quantum per unit cell (“single-winding”), $a_x a_y H = \phi_0$. We start the calculation by initializing $\Delta_{\pm}(r) = 0$ and $\Delta_{\pm}(r) = \Phi(r)$, where $\Phi(r)$ is the Abrikosov solution for a triangular vortex lattice in a conventional s-wave superconductor. We solve Eq. (2) for quasiclassical propagators, then re-calculate the pair potentials, $\Delta_{\pm}(r)$, from Eq. (2) and iterate this procedure until we obtain a self-consistent solution for $f$, $f^\dagger$, $g$ and $\Delta_{\pm}$. A similar procedure is followed for vortex states with two flux quanta per unit cell (“double-winding”), $a_x a_y H = 2\phi_0$. For this case we initialize the lattice with $\Delta_{\pm}(r) = 0$ and $\Delta_{\pm}(r) = \Phi(r)^2$.

3. Vortex states of single- and double-winding vortices

In Fig. 1, we show the spatial structure of the pair potentials for a vortex lattice of single quantum vortices at $H = 0.05B_0$ and $T = 0.5T_c$. Note that $H_{c2} \sim B_0$. The main component $\Delta_{\pm}(r)$ includes a vortex with phase winding $+1$ at the center of unit cell where $|\Delta_{\pm}(r)|$ is
Figure 1. Spatial structure of the single-winding vortex state is presented in a square domain that includes the hexagonal unit cell for $T = 0.5T_c$ and $H = 0.05B_0$. (a) Amplitude of the main component ($p_-$) for the pair potential, $|\Delta_-(r)|$. (b) Amplitude of the induced component ($p_+$) of the pair potential, $|\Delta_+(r)|$. (c) Magnitude of the supercurrent $|j(r)|$ around the vortex.

Figure 2. Spatial structure of the double-winding vortex state for the same $T$ and $B$ as that for Fig. 1. The size of unit cell is twice larger than that of single-winding vortex state.

suppressed to zero as shown in Fig. 1(a). Figure 1(b) shows the amplitude of the induced component, $\Delta_+(r)$, for the time-reversed chiral phase. This amplitude has phase winding $-1$ about the vortex center, and also $+1$ winding about the corners of the hexagonal unit cell, so that total phase winding for a unit cell is $+1$ [2]. In Fig. 1(c) we show the magnitude of the supercurrent in the unit cell. The hexagonal structure is apparent in both Figs. 1(b) and 1(c).

The spatial structure of the double-winding vortex states is presented in Fig. 2 for $H = 0.05B_0$ and $T = 0.5T_c$. In this case the main chiral component $\Delta_-(r)$ has phase winding $+2$, which leads to $|\Delta_-(r)| \propto r^2$ near the the vortex center as shown in Fig. 2(a). Since the induced chiral-component ($p_+$) has phase winding, $m = n - 2$, $\Delta_+(r)$ develops with finite amplitude and zero phase-winding at vortex center, filling the core and generating a “coreless” structure with nearly uniform condensate density as shown in Fig. 2(b). The supercurrent shown in Fig. 2(c) consists of the regular vortex flow far from the vortex center, but the flow reverses direction at a radius $r_{DW} \approx 5\xi_0$ from the vortex center. The node in the current density at $r_{DW}$ corresponds to a circular domain wall separating the two time-reversed chiral ground states. The vortex lattice structures for the double-winding vortex states reported here are consistent to those obtained by previous calculations for isolated vortices [4].

4. Free energy of flux lattices of singly doubly quantized vortices

Here we report calculations of the free energy as a function of magnetic field $H$ in order to determine the equilibrium vortex phase for chiral ground states. The standard approach is to use Eilenberger’s free energy functional (c.f. Ref. [5]). The free energy relative to the normal state is then

$$\Omega_E/\Omega_0 = \frac{1}{\lambda_{0}} \left\langle |\Delta_+(r)|^2 + |\Delta_-(r)|^2 \right\rangle_r - T \sum_{\omega_n \leq \omega_{c}} \left\langle \langle I(r, p, \omega_n) \rangle_p \right\rangle_r$$

(3)
Figure 3. (a) Magnetic field $H$ dependence of free energy $\Omega_E/\Omega_0$ (circles) and $\Omega_{\text{LW}}/\Omega_0$ (lines) for single- and double-winding vortex states. (b) Results of (a) are replotted as free energy difference from single-winding vortex state. Within accuracies of these figures, $\Omega_E \sim \Omega_{\text{LW}}$.

with $I = \Delta f^\dagger + \Delta^* f + (g - \text{sgn}(\omega_n))\{f^{-1}(\omega_n + v \cdot (\nabla + iA))f + f^\dagger^{-1}(\omega_n + v \cdot (\nabla - iA))f^\dagger\}$ in dimensionless units, $\Omega_0 \equiv (\pi k_B T_c)^2 N_0 \xi_0^3$ and $\langle \cdots \rangle_r$ indicates spatial average. The stationary point of Eilenberger’s functional reduces to Eq. (1). The resulting stationary value for $\Omega_E$ reduces to an evaluation of $I(r, p, \omega_n) = (1 + \text{sgn}(\omega_n)g)^{-1}(\Delta f^\dagger + \Delta^* f)$ with the self-consistently calculated propagators and pair potentials. We also calculated the stationary free energy using the quasiclassical reduction of the Luttinger-Ward functional, $\Omega_{\text{LW}}$, as described in Ref. [6]. In the clean limit the results we obtain based on the Eilenberger functional and the quasiclassical Luttinger-Ward functional agree within our numerical accuracy.

As shown in Fig. 3 the double-winding vortex state is energetically favored for $H > H_{F_L} = 0.005 B_0$. This relative stability arises from the reduction in core energy for the double-winding vortex compared that for a lattice of single-winding vortices. The reduced core energy outweighs the higher kinetic energy density of doubly quantized vortices compared to singly quantized vortices in the region far from the vortex center. However, at sufficiently low fields the single-winding vortex state becomes stable because the kinetic energy far from the cores dominates the lattice energy. As a result there is an intermediate field $H_{F_L}$ corresponding to a first-order transition between single- and double-winding vortex states.

5. Conclusion

The double-winding vortex state is a characteristic phase of chiral $p$-wave superconductors, predicted to be energetically stable over a wide range of fields for cylindrically symmetric $E_{1u}$ pairing functions. For Sr$_2$RuO$_4$, SANS experiments [7] show diffraction patterns consistent with a square flux lattice of singly quantized vortices for $H || \hat{c}$. In order to understand the stability and structure of the vortex phase(s) of Sr$_2$RuO$_4$ within the chiral $p$-wave model, or to rule out this model for Sr$_2$RuO$_4$, we must consider anisotropic, and multi-band, models for the Fermi surface and basis functions for the $E_{1u}$ symmetry class. These results will be reported elsewhere.

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