Modeling Adoption of Competing Products and Conventions in Social Media

Isabel Valera  
University Carlos III  
ivalera@tsc.uc3m.es

Manuel Gomez-Rodriguez  
MPI for Intelligent Systems  
manuelgr@tue.mpg.de

Krishna Gummadi  
MPI for Software Systems  
gummadi@mpi-sws.org

ABSTRACT

The emergence and wide-spread use of social networks and microblogging sites has led to a dramatic increase on the availability of users’ activity data. Importantly, this data can be exploited to solve some of the problems that have captured the attention of economists and marketers for decades as, e.g., product adoption, product competition and product life cycle. In this paper, we leverage on users’ activity data from a popular microblogging site to model and predict the competing dynamics of products and social conventions adoptions.

To this aim, we propose a data-driven model, based on continuous-time Hawkes processes, for the adoption and frequency of use of competing products and conventions. We then develop an inference method to efficiently fit the model parameters by solving a convex program. The problem decouples into a collection of smaller subproblems, thus scaling easily to networks with hundred of thousands of nodes. We validate our method over synthetic and real diffusion data gathered from Twitter, and show that the proposed model does not only present a good predictive power but also provides interpretable model parameters, which allow us to gain insights into the fundamental principles that drive product and convention adoptions.

Categories and Subject Descriptors: H.2.8 [Database Management]: Database Applications – Data mining

General Terms: Algorithms; Experimentation.

Keywords: Competing diffusion, social networks, social contagion.

1. INTRODUCTION

There is a long history of work in economics and marketing on studying product adoption, product competition and product life cycle [35]. This work has typically developed mathematical models, such as, the Bass diffusion model [6], the probit model [37], or models based on information cascades [8], that attempt to capture the macroscopic evolution of a product or set of competing products but not its (their) microscopic evolution, i.e., product adoptions and recurrent usage by specific individuals. This has been in part due to the lack of large-scale fine-grained product adoption and product usage data, which would allow to propose and validate microscopic models. However, the emergence and widespread use of social networks and microblogging sites has led to a dramatic increase on the availability of large-scale fine-grained users’ activity data, in which all individual adoptions and subsequent uses are observable. Even more, there are some products, such as url shortening services [3], which are only used in social networks or microblogging sites. This availability opens up a great opportunity for a paradigm shift, where we attempt to capture both the macroscopic and microscopic dynamics.

In this paper, we aim at exploiting the availability of fine-grained users’ activity data to find and analyze the factors that contribute to the success of a particular product above others. To this end, we develop a data-driven model that enables us to predict individual adoptions and frequency of use of a set of competing products. Moreover, we also apply the same methodology to model competing social conventions, where, until very recently [30], large scale coarse- and fine-grained data was very scarce.

Our approach to adoption and frequency of use of competing products and conventions. In this article, we introduce a data-driven model for the adoption and frequency of use of competing products and conventions in social networks and microblogging sites based on a special type of continuous-time point processes, multidimensional Hawkes processes [24]. Hawkes processes have recently received an increasing amount of attention in the machine learning community [10, 27, 42, 43]. Their most characteristic property is that they allow for self-excitement, in other words, they allow an increase in the probability of a (recurrent) event to happen at a given time due to the occurrence of the same type of event in the past. In our scenario, this property is important since it allows us to model cooperation – if a user has used a product or observes that one or more of her neighbors have used the product in the past, this may have a positive contribution to the probability that the user will continue (or start) using the product in the future.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Copyright 20XX ACM X-XXXXXX-X/XX/XX $10.00.
frequency of use. In our work, we generalize Hawkes processes to support both cooperation and competition [40]. Then, we fit the parameters of the model using social activity data, by solving a maximum likelihood estimation problem, which reduces to solving a convex program. Importantly, the problem decouples in several smaller problems, allowing for natural parallelization so that we can fit the parameters of the model for networks with hundreds of thousands of nodes.

We validate our model on both synthetic and real social activity data. First, we show the effectiveness of our maximum likelihood estimation method at recovering the parameters of the model using synthetic data. Second, we validate the predictive power of our model on real data gathered from Twitter [13], which comprises of 1.7 billion public tweets posted by 52 million users during a three year period, from March 2006 to September 2009. In particular, we apply our model to the adoption and frequency of use of one type of competing products, url shortening services [3], and one type of conventions, the way Twitter users indicated back in 2009 that a tweet was being retweeted (or forwarded) [30]. We find that our Hawkes based model always provides not only a better fit to the data, in terms of test log-likelihood, but also a better trade-off between the goodness of fit and the model complexity than a Poisson model, which we use as baseline. Moreover, we give empirical evidence that our model typically outperforms the two considered baselines, the Poisson model and linear regression, at predicting the number of uses of products and conventions per user, measured in terms of Spearman correlation. In addition, our model is able to predict the true ranking (in terms of frequency of use) of products and conventions per user for more than 70% of the users and, importantly, accurately predict the overall cumulative number of uses of each product and convention across users.

Finally, we show that our model allows us to gain insights into the fundamental principles that drive product and convention adoptions by different Twitter users, shedding light on the factors that contribute to the success of particular products or conventions across them. For example, we find that the usage of more popular products and conventions is triggered by self-excitement or cooperation, while less popular products are adopted and used spontaneously. Moreover, using a less popular product or convention has a stronger inhibiting effect on future uses of a more popular product or convention than vice versa.

**Related work.** The works most closely related to ours [7, 17, 22] study competition between firms in a social network. They assume each firm has a budget to seed the initial adoption of its products by some consumers located in a social network. The payoffs to the firms are the eventual number of adoptions of their product through a competitive stochastic diffusion process in the network. They model diffusion as a sequential discrete-time process and use game theory to perform a theoretical analysis of competition in the context of influence maximization. In particular, Bharathi et al. [7] extend the independent cascade model, Dubey et al. [17] propose a local quasilinear model, and Goyal et al. [22] consider a broad class of local influence processes. However, there are several important differences between the above mentioned models and our work.

First, previous work models diffusion as a sequential discrete-time process, which are typically not data-driven and ignore the complex underlying temporal dynamics governing diffusion [19]. In contrast, we model diffusion as an asynchronous continuous time point process with time-varying use rates, designed to naturally fit the diffusion data we record (i.e., the times in which users use products). There is a twofold rationale behind this continuous-time modeling choice: i) Since events occur asynchronously, continuous variables seem more appropriate to represent them, and artificially discretizing the time axis into bins introduces additional tuning parameters, like the bin size, which are not easy to choose optimally; ii) discrete time models can only model inter-event time delays which obey an exponential density, and hence can be too restricted to capture the rich temporal dynamics in the data. Several recent works have shown, by extensive experimental comparisons on both synthetic and real world data, that continuous-time models yield significant improvement in settings such as recovering hidden diffusion network structures from cascade data [16, 19], predicting the timings of future events [20, 27], or estimating and maximizing influence [15, 21]. However, up to our knowledge, this line of research considers ideas, information, behavior, products, or, more generally, contagions to spread simultaneously but independently of each other, ignoring the emergence of competition or cooperation among them [38].

Second, we allow the users to use a product once or several times. In other words, we model how frequently users use a product, and how this frequency depends on what users observe. Hence, we are able to model non-progressive phenomena [28], in which a user starts using a product and then possibly stop at some point. Moreover, in our framework, a user can use different competing products simultaneously, each with different time-varying frequency. In contrast, previous works consider each user to use only one product, the one she adopted first, and then assume the user never stops using the product. We believe these differences are key to accurately fit the observed temporal patterns of use of competing products and conventions over networks and, ultimately, understand, predict and control adoption and frequency of use of competing products and conventions.

Competitive diffusion has been also studied in other contexts: meme diffusion [38], rumor propagation [31] and spread of misinformation [12]. In particular, Myers et al. [38] study url diffusion in Twitter and propose a data-driven discrete-time model which accounts for cooperation and competition between pairs of urls. Kostka et al. [31] study the propagation of two competing rumors as a strategic game, using concepts from game theory and location theory. Finally, Budak et al. [12] address the problem of influence limitation where a bad campaign starts propagating from a certain node in a social network and use the notion of limiting campaigns to counteract the effect of misinformation. However, there are several fundamental differences between this piece of work and the current work: they model diffusion as a sequential discrete time process while we propose an asynchronous continuous time model; they consider each contagion to be adopted (or used) once while we allow for recurrent uses; and, they only allow for pairwise interactions between contagions while we consider n-ary interactions.

Last, Hawkes processes have been also applied to a large variety of problems in which self-excitement plays a fundamental role: earthquake prediction [36], crime prediction [18], computational neuroscience [32], or high frequency trading [5].
2. MODEL AND PROPOSED METHOD

In this section, we describe our model, starting from the data it is designed for, and then propose an inference method to efficiently fit its parameters from social activity data.

2.1 Data

Given a network $G = (V, E)$ and a set of related competing products $P$, we observe the times in which each node $u \in V$ use a product $p \in P$ during an observation window of length $T$. A node does not necessarily use all products and can use a specific product once or several times. Our data is thus a (temporally) ordered 3-tuple $\{(t_1, p_1, u_1), \ldots, (t_N, p_N, u_N)\}$ recording each time $t_i$ when a node $u_i$ use a product $p_i$, where $t_i \in [0, T]$. Lengthening the observation window $T$ increases the number of observed events and results in a more representative sample of the underlying dynamics. However, these advantages must be weighed against the cost of observing for longer periods. To lighten the notation, we will use a more compact representation of our data, $\{(t_i^u, p_i, u_i)\}_{(u,p,i)}$, where $t_i^u$ denotes the time of the $i$th event, in which a node $u$ used a product $p$.

2.2 Adoption and frequency of use as a multidimensional counting process

Consider a node $u$, a product $p \in P$, and a function $N_u^p(t)$ which indicates the number of times node $u$ has used product $p$ by time $t$. Then, we define the filtration $\mathcal{H}_t^u$ as the times, up to time $t$, in which node $u$ and her neighbours $\mathcal{N}^- (u) = \{v \in V : (v, u) \in E\}$ have used any of the products in $P$, i.e., $\mathcal{H}_t^u = \{t_i^u : t_i^u < t, (u,v) \in \mathcal{H}_t^u\} \cup \{t_i^u : t_i^u < t, v \in \mathcal{N}^- (u)\}$. We can think of $\mathcal{H}_t^u$ as the history of products usage that node $u$ has observed up to time $t$. Our goal is to understand the influence of a user’s history on her current and future products usage. Ultimately, we will leverage the user’s history to predict which products a user will use as well as how frequently she will use them.

By definition, since $N_u^p(t)$ is a nondecreasing counting process, it is a submartingale and satisfies $\mathbb{E}(N_u^p(t) \mid \mathcal{H}_t^u) = N_u^p(t')$ for any $t > t'$.

Then we can decompose $N_u^p(t)$ uniquely as $N_u^p(t) = \Lambda_u^p(t) + \mathcal{M}_u^p(t)$, where $\Lambda_u^p(t)$ is a nondecreasing predictable process, called cumulative incidence process and $\mathcal{M}_u^p(t)$ is a mean zero martingale. This is called the Doob-Meyer decomposition of a submartingale [1]. Consider $\Lambda_u^p(t)$ to be absolutely continuous, then there exists a predictable nonnegative intensity function $\lambda_u^p(t)$ such that

$$N_u^p(t) = \int_0^t \lambda_u^p(s) \, ds + \mathcal{M}_u^p(t). \quad (1)$$

Here, we can think of the intensity function as user $u$’s instantaneous frequency of use of product $p$. At any given time $t$, the intensity function may depend on the user’s history or filtration by that time, $\mathcal{H}_t^u$. Our goal now is to infer an intensity function $\lambda_u^p(t)$ for each user $u$ and product $p$ using the digital traces of products usage $\{(t_i^u, p_i, u_i)\}_{(u,p,i)}$. Importantly, although the intensity functions for different (connected) users and products are coupled, given a user’s history up to time $t$, we will be able to compute the user’s intensity function for a particular product at time $t$ independently of the other intensity functions. This will allow us to predict future usage of a product $p$ by a user $u$ using the cumulative probability $F_u^p(t) \mid \mathcal{H}_t^u$, which is given by [1]:

$$F_u^p(t) \mid \mathcal{H}_t^u = 1 - e^{-\int_0^t \lambda_u^p(s) \, ds}, \quad (2)$$

where $t_0$ is the last time user $u$ adopted product $p$ before $t$, if so, or zero otherwise. Moreover, by analyzing the structure of the intensity functions, we will investigate the emergence of competition between products. Next, we continue by proposing a model of product competition based on Hawkes processes [24].

2.3 Nodes’ intensities and multidimensional Hawkes processes

Multidimensional Hawkes processes are point processes in which the intensity function at a current time can increase due to the occurrence of events in the past [24]. Therefore, they allow us to capture the self-exciting nature of the intensity functions associated to product $p$, $\lambda_u^p(t) \mid \mathcal{H}_t^u$, i.e., if a user $u$ or one or more of her neighbors have used a product $p$ in the past, this may have a positive contribution to the event intensity associated to product $p$ and thus increase its frequency of use. However, we would also like to capture the competitive nature of the intensity functions associated to product $p$, $\lambda_u^p(t) \mid \mathcal{H}_t^u$, i.e., if a user $u$ or one or more of her neighbors have used a product $q \neq p$ in the past, this may have a negative contribution to the event intensity associated to product $p$ and thus decrease its frequency of use. In other words, we would like to model not only excitation but also inhibition. Although the original definition of Hawkes processes does not account for this scenario, previous work has studied the properties of such extension [40]. Then, we can write the intensity function for user $u$ using product $p$ at a given time $t$ as

$$\lambda_u^p(t) \mid \mathcal{H}_t^u = \mu_u^p + \sum_{l=1}^{P} a^l_{pp} \sum_{n_i^u \leq t} g(t - t_{i}^u)$$

$$+ \sum_{l=1}^{P} b^l_{pp} \sum_{u' \in \mathcal{N}^- (u)} \sum_{n_i^u \leq t} g(t - t_{i}^u'), \quad (3)$$

where $\mathcal{N}^- (u) = \{v \in V : (v, u) \in E\}$ are user $u$’s neighbors, $\mu_u^p \geq 0$ is the base intensity (i.e., the spontaneous adoption of a product), $a^l_{pp} \in \mathbb{R}^+$ (or $b^l_{pp} \in \mathbb{R}^+$) corresponds to the influence that a previous use of a product $l$ by user $u$ (by a neighbor of user $u$) has on user $u$’s intensity function associated to product $p$, and $g(\cdot)$ is a positive kernel, which models the decay of the influence of an event over time. Moreover, since an event cannot influence the past, $g(t) = 0$ for $t < 0$.

In this article, we will carry out the experimental evaluation using an exponential kernel $g(t) = e^{-\lambda t}$. However, since our inference method does not depend on this particular choice, more complicated positive kernels can easily be chosen.

In Eq. (3), the parameters $a^l_{pp}$ ($b^l_{pp}$) model the (positive) influence of a previous use of product $p$ by user $u$ (by one of her neighbors), i.e., the level of self-excitement, while parameters $a^l_{pp}$ ($b^l_{pp}$) with $l \neq p$ model the (negative) influence of a previous use of product $l$ by user $u$ (by one of her neighbors), i.e., the level of cross-product inhibition. Here, we force the parameters $a^l_{pp} \in \mathbb{R}^+$ and $b^l_{pp} \in \mathbb{R}^+$ to be non-negative, while we allow the cross-product parameters to be positive or negative, $a^l_{pp} \in \mathbb{R}$ and $b^l_{pp} \in \mathbb{R}$. We will fit the parameters in Eq. 3 using maximum likelihood and, therefore, will never happen that $\lambda_u^p(t) \mid \mathcal{H}_t^u$ is negative for the events data we fit the model to, despite allowing negative cross-product parameters. However, for particular (unseen) histories, it may happen that $\lambda_u^p(t) \mid \mathcal{H}_t^u < 0$. In those cases,
Algorithm 1 Ogata’s Algorithm

Initialization: \( n^{|P|} = 0 \) for \( u = 1, \ldots, |V|, p = 1, \ldots, |P| \)

1: \( I^* \leftarrow I^{|V| \times |P|}(t_0) \leftarrow \sum_u |\sum_p \lambda^u_p(t_0) \)

Generate first event:

2: Generate \( q \sim \mathcal{U}_{(0,1)} \) and \( s \leftarrow t_0 - \frac{1}{t} \ln(q) \)
3: if \( s > T \), then go to last step.
4: else Attribution Test:
   i) Sample \( d \sim \mathcal{U}_{(0,1)} \)
   ii) Choose \( u \) and \( p \) such that \( \frac{I^p(t_0)}{I^p_{t_0}} < d \leq \frac{I^p(t_0)}{I^p_{t_0}} \)
   iii) Set \( t_1 \leftarrow t^p_{u,l} \), \( i \leftarrow 1 \) and \( n^{|P|} \leftarrow 1 \)

General subroutine:

5: while \( s < T \) do
6: \( I^* \leftarrow I^{|V| \times |P|}(t_1) + \sum_p |\sum_{u \in \mathcal{N}_u \setminus \{u\}} | \lambda^u_p \)
7: Generate \( q \sim \mathcal{U}_{(0,1)} \)
8: Update \( s \leftarrow s + \frac{1}{t} \ln(q) \)
9: if \( s > T \), then go to last step.
10: else Attribution-Rejection Test:
   i) Sample \( d \sim \mathcal{U}_{(0,1)} \)
   ii) if \( d \leq \frac{I^p(t_0)}{I^p_{t_0}} \), then
      - Choose \( u \) and \( p \) such that \( \frac{I^p_{t_0}}{I^p_{t_0}} < d \leq \frac{I^p_{t_0}}{I^p_{t_0}} \)
      - Set \( t_{i+1} \leftarrow t^p_{u,l} \), \( i \leftarrow i + 1 \)
      and \( n^{|P|} \leftarrow 1 \)
   iii) else
      - Update \( I^* \leftarrow I^{|V| \times |P|}(s) \) and go to step 8.
end while

11: Output: Retrieve the simulated process \( \{(t^u_p)\}_{u=1,\ldots,|V|,p=1,\ldots,|P|} \) on \([t_0, T]\)

as proposed by Ogata [40], we will set \( \lambda^u_p(t|H^u_{t^*}) \) to zero.

Additionally, note that different variations of the model in Eq. 3 can be readily obtained just by setting some of the parameter to zero. In the present study, we consider the two following (more restrictive) variations of the model:

1) A model that does not consider competition among products. To this end, we set the cross-product parameters to zero, i.e., \( a^u_p = 0 \) and \( b^u_p = 0 \) for all \( l \neq p \), resulting in the following intensity function:

\[
\lambda^u_p(t|H^u_{t^*}) = \mu^u_p + a^u_p \sum_{n,l \mid t^l_n < t} g(t - t^l_n) + b^u_p \sum_{u' \in \mathcal{N}_u \setminus \{u\}} \sum_{n,l \mid t^l_n < t} g(t - t^l_{n,l}).
\] (4)

Under this model, we assume that the use of a product is independent of previous uses of other products.

2) A model that does not consider the influence of the network. In this model, we assume that either the users are not influenced by their neighbors or the network connectivity is unobserved. Hence, this model assumes that users’ activity is independent of the activity of her neighbors and thus the intensity function is given by

\[
\lambda^u_p(t|H^u_{t^*}) = \mu^u_p + \sum_{l=1}^{P} a^u_p \sum_{n,l \mid t^l_n < t} g(t - t^l_n).
\] (5)

Throughout the rest of the paper, for conciseness we will also stack all the parameters for user \( u \), i.e., \( \mu^u_p, a^u_p \) and \( b^u_p \) for \( p = 1, \ldots, |P| \) and \( l = 1, \ldots, |P| \), in a vector \( \mu^u \) and in matrices \( A^u \) and \( B^u \), respectively.

2.4 Likelihood of product uses

In order to fit the model parameters by maximum likelihood, we need to compute the likelihood of each use of a product \( p \) by user \( u \). This can be readily done by taking the derivative of the cumulative probability \( F_p^u(t|H^u_{t^*}) \) [1], given by Eq. 2, i.e.,

\[
f_p^u(t) = \frac{dF_p^u(t)}{dt} = \lambda^u_p(t|H^u_{t^*}) e^{-\int_{t_0}^{t} \lambda^u_p(t'|H^u_{t^*}) dt'},
\] (6)

where \( t_0 \) is the last time user \( u \) used product \( p \) before \( t \), if so, or zero otherwise. Taking logarithm in Eq. 6, we obtain

\[
\log f_p^u(t) = \log (\lambda^u_p(t|H^u_{t^*})) - \int_{t_0}^{t} \lambda^u_p(t'|H^u_{t^*}) dt'.
\] (7)

Next, since the probability that a user \( u \) uses a product \( p \) at time \( t \) is independent of future uses, given the user’s history \( H^u_{t^*} \), i.e., the previous events of the user and her neighbors, we can factorize the log-likelihood of the set of uses of user \( u \) as

\[
\mathcal{L}(t_u; \mu^u, A^u, B^u) = \sum_{p=1}^{P} \sum_{l=1}^{T} \log f_p^u(t^u_p) + \sum_{p=1}^{P} \log S^u_p(T),
\] (8)

where the first term corresponds to each of the user’s uses and the second terms accounts for the survival time from the last use of each product \( p \), if any, to the time of the observation window cut-off \( T \).

Finally, we group the integral terms in the first and second terms, yielding

\[
\mathcal{L}(t_u; \mu^u, A^u, B^u) = \sum_{p=1}^{P} \sum_{l=1}^{T} \log \left( \lambda^u_p(t^u_p|H^u_{t^*}) \right)
- \sum_{p=1}^{P} \left( T\mu^p + \sum_{l=1}^{T} a^u_p \sum_{n,l} G(T - t^u_{n,l}) \right)
+ \sum_{u' \in \mathcal{N}_u \setminus \{u\}} \sum_{n,l} b^u_p \sum_{n,l} G(t^u_{n,l}),
\] (9)

where \( G(t) = \int_{t_0}^{t} g(t') dt' \), and define the competing adoption inference problem.

2.5 Competing adoption inference problem

Given an observed set of events \( \{(t^u_p)\}_{u \in V, p \in |P|} \), our goal is to find the optimal parameters \( \mu^u_p, a^u_p \) and \( b^u_p \) by solving for each user \( u \in V \) the maximum likelihood (ML) optimization problem

\[
\min_{\mu^u, A^u, B^u} \quad f(\mu^u, A^u, B^u)
\]

subject to \( \mu^u_p \geq 0, a^u_p \geq 0, b^u_p \geq 0 \).

where \( \mu^u \), \( A^u \) and \( B^u \) are the variables and the objective function is given by

\[
f(\mu^u, A^u, B^u) = \sum_{u} \left( -\mathcal{L}(t_u^{|U^u|}) + \beta(\|\mu^u\|^2 + \|A^u\|^2 + \|B^u\|^2) \right),
\] (11)
where the first term is the negative log-likelihood of the events and the second term is the regularization term, being $\beta$ the parameter that controls the trade-off between these two terms. In a network with $|V|$ users and $|P|$ products, we will need to find the optimal value for $|V|(2|P|^2 + |P|)$ variables. Fortunately, we can find the optimal solution to Eq. 10 efficiently:

**Theorem 1.** The competing diffusion problem defined in Eq. 10 is jointly convex in $\mu^u, A^u, B^u$.

**Proof.** Convexity follows trivially from linearity, composition rules for convexity, and concavity of the logarithm. $\square$

Moreover, it is easy to show that the optimization problem decouples into a collection of $|V| \times |P|$ independent smaller subproblems, one per node $u$ and product $p$, in which we find the optimal values for $p_u^v, \{a_{lp}^u\}_{l=1}^{|V|}$ and $\{b_{lp}^u\}_{l=1}^{|P|}$. We can solve each subproblem in parallel, obtaining local solutions that are globally optimal, and thus our method scales to networks on the order of hundreds of thousands of nodes.

In our experimental evaluation, we solved Eq. 10 with CVX, a software package for specifying and solving convex programs [23]. Developing highly efficient customized solvers for Eq. 10 appears as an interesting venue for future work.

3. EXPERIMENTAL EVALUATION

In this section, we first validate our inference method on synthetic networks that mimic the structure of social and information networks, and then compare the performance of our model to several baselines on over 1.7 billion public tweets posted by 52 million Twitter users [13] during a three year period, from March 2006 to September 2009.

3.1 Experiments on synthetic data

**Experimental setup.** We first generate synthetic networks using a well-known model of directed social networks, the Kronecker graph model [33]. In particular, we generated a core-periphery Kronecker network [34] with parameter matrix $[0.9, 0.5; 0.5, 0.3]$, a hierarchical Kronecker network [14] with parameters $[0.9, 0.1; 0.1, 0.9]$, and a random Kronecker network with parameter matrix $[0.5, 0.5; 0.5, 0.5]$. We set the number of nodes in the three networks to 512 users and the number of edges to 2,040, 4,608 and 7,669, respectively, for the core-periphery, hierarchical and random networks.

Then, for each network, we assume there are two competing products, and set the influence parameters for each product and node in the networks as follows. First, we only allow for a small set of users to have a baseline parameter greater than zero, and draw their baseline parameters from the uniform distribution $U(0, 1)$. In this way, we account for the fact that only a few nodes acquire products spontaneously. Second, we draw every user’s product parameters $\{a_{lp}^u\}$ and $\{b_{lp}^u\}$ from $U(0, 1)$ and every user’s cross-product parameters $\{a_{lp}^u\}_{l \neq p}$ and $\{b_{lp}^u\}_{l \neq p}$ from $U(-1, 1)$.

Finally, for each network, we generate a set of 100,000 events using a procedure similar to Ogata [39] (refer to Algorithm 1). Note that, since we allow negative values for the cross-product parameters, a hazard function could be negative at some $t$ during the simulation and thus become ill defined. In that case, we simply trim it to zero, and assume that a node cannot draw samples as long as it is zero, as proposed previously [40].

Now, given the times when each user used any of the two products, our goal is to find the true model parameters $\mu^u, A^u$, and $B^u$ for each user $u$, by solving the optimization problem defined in Eq. 10.

**Accuracy of our algorithm.** We evaluate our algorithm by comparing the inferred and true parameters in terms of the mean squared error (MSE) across users, $E [(x - \hat{x})^2]$, where $x$ is the true parameter and $\hat{x}$ is the estimated parameter. In particular, for each of the three networks, we compute the MSE across users, products and all model parameters. Figure 1 shows the MSE for each of the networks with respect to the average number of events per user, where we set the regularization parameter $\beta = 10$ and the decay function parameter $w = 1$. We observe that the MSE decreases as the average number of events per user increases, reaching values below 0.18 for the three networks. It is important to note that even though the networks have very different global network structure, the performance of our inference procedure is remarkably stable and does not seem to depend on the structure of the network.

3.2 Experiments on real data

**Dataset description.** We use data gathered from Twitter as reported in previous work [13], which comprises of 1.7 billion public tweets posted by 52 million users during a three year period, from March 2006 to September 2009. Impor-
Experimental setup. Our goal is to estimate each user’s susceptibility to adopt (and repeatedly use) every product or social convention. To this aim, we first build each user’s neighborhood using the interactions via @-messages; we create a directed edge \((i,j)\) as soon as user \(j\) mentions user \(i\) in a tweet, since this provides evidence that user \(j\) is paying attention to user \(i\) [26]. Then, we assume node \(i\) got exposed only to tweets from node \(j\) posted later than this first mention, as argued in previous work [41]. Following this procedure, the neighborhood network for url shortening services has over 67 million directed active edges and the network for retweet conventions has over 47 million active directed edges.

For our experiments, we record the times 3,000 users and their neighbors used url shortening services (retweet conventions) during four consecutive months in 2009. Note that we create two datasets, one for url shortening services and one for retweet conventions, and only consider users with more than 100 uses across the whole three years of data. We then employ the first three months of data as training sets, and the last month as test sets. In both cases, the training and test sets are disjoint and each event belongs either to a training or test set. For url shortening services, the training set spans from March 1 to May 31 and the test set spans from April 15 to May 15. For retweet conventions, the training set spans from January 15 to April 15 and the test set spans from April 15 to May 15. For retweet conventions, the training set spans from March 1 to May 31 and the test set spans from June 1 to June 30.

Performance evaluation. In this section, we compare the performance of the three Hawkes based models: the general model, defined in Eq. 3 (‘Full’), the model that does not consider competition among products, defined in Eq. 4 (‘WCP’), and the model that does not take into account the influence of the network, defined in Eq. 5 (‘WNET’), to two baselines: a memoryless Poisson model (‘Poiss’) [29] and linear regression (‘L.R.’) [9].

For each user, we first fit the Hawkes based models and baselines using the training data as follows. For the three Hawkes based models (‘Full’, ‘WCP’ and ‘WNET’), we solve the optimization problem defined in Eq. 10, where we set the
regularization parameter $\beta$ and the decay function parameter $w$ via cross-validation. In the cross-validation step, we select the parameters $\beta$ and $w$ that minimize the relative absolute error per user $u$ on the last month of the training set data, defined as $\text{RAE}(u) = \frac{1}{n^u} \sum_{t_d=1}^{N^u(t_d)} \frac{|N^u(t_d) - \hat{N}^u(t_d)|}{N^u(t_d)}$, where $N^u(t_d)$ is the total true number of uses (of all the products) by user $u$ at time $t_d$ and $\hat{N}^u(t_d)$ is the average (over 100 independent simulations) predicted number of uses by user $u$ at time $t_d$. Moreover, we force the cross-product parameters $a_{lp}$ and $b_{lp}$ for all $l \neq p$ to be equal or smaller than zero, since, in practice, this provides more stable solutions. For the memoryless Poisson model, we fit one model per user $u$ and product $p$ using maximum likelihood. For linear regression, we fit the number of uses of product $p$ by user $u$ in a 24h bin using as features the number of uses of each product $l$, including $p$, by user $u$ and by user $u$’s neighbors in the previous 24h bin using least square.

Then, we evaluate the goodness of fit and predictive power of our models and the two baselines, learned using the training sets, on the test sets in four different ways:

1. For each user, we compute the Akaike information criterion (AIC) [2] for the three Hawkes based models and the Poisson model. The AIC is given by $\text{AIC} = 2N_p - 2\mathcal{L}$, where $N_p$ is the number of model parameters and $\mathcal{L}$ is the log-likelihood function evaluated in the training set. Note that the AIC is a measure that quantify the trade-off between the goodness of fit and the complexity of the model, and therefore, it is a measure for model selection for which the preferred model is the one with the minimum AIC value.

2. For each product, compute the average test log-likelihood per event for the Poisson and the three Hawkes based models. Note that likelihoods can be larger than 1, and thus log-likelihoods larger than 0. This allows us to assess how well each model fits the data – in other words, its goodness of fit. Note that we compute the log-likelihood over the test data, not the training data, to measure how well each model generalizes to the data.

3. For each user, we rank the products by their number of true usages in the corresponding test set and by their predicted number of uses$^1$, and count the number of users for which both rankings coincide.

4. For each product, we divide the 30 days of the test set in 24h bins, compute the Spearman correlation [25] between the true and the predicted number of events in each bin for every user and product, and average across bins$^1$.

In all models and baselines, when we predict each user’s events during the test set, we assume the neighbor’s true events during the test set to be given, but the user’s true events to be unknown. Moreover, for the three Hawkes based models, we generate events for each user in the test period by using Algorithm 1. For the full and the WCP models, since we assume neighbor’s true events to be known, we need to modify Algorithm 1 by updating the intensity function after every event of user’s neighbors.

Table 3 shows the percentage of users for which each model provides the minimum AIC. In this table, we observe that for more than 90% of the url shortening services users and 80% of the retweet conventions users, the best model is one of the Hawkes based models, being the WCP, followed by the full model, the best performer. The WNET model is rarely the best performer, due to its relatively large number or parameters and lower log-likelihood for the most popular products and conventions. However, it will later emerge as a model that performs well in terms other performance measures. Next, evaluate the performance of the models in the test set by computing the test log-likelihood.

Table 4 shows the average test log-likelihood per event across users. Figures 2 and 3 show the average test log-likelihood per event for the five most popular url shortening services and retweet conventions separately. Perhaps surprisingly, all three Hawkes based models outperform the Poisson model in all shortening services and retweet conventions except one (‘RT’), where, nevertheless, one of the Hawkes models, WNET, still outperforms the Poisson model. In other words, Hawkes models seem to provide a better fit to the data than Poisson. If we now compare the Hawkes based models, we do not find a clear winner. For some url shortening services and retweet conventions, all three provide comparable results (‘twURL’ and ‘snURL’; or, ‘retweeting’), while for other products, one of the three models clearly outperforms the others. For example, the WCP model, which ignores competition, provides the best fit for the url shortening service ‘tinyURL’, which was the default service until the 6th May of 2009, and therefore, was suffering much less competition than other shortening services.

---

$^1$Since the Hawkes based models and the Poisson model are probabilistic, we take the average number of uses over 100 independent simulations.
The full model provides the best fit for two retweet conventions, ‘via’ and ‘retweeting’, while the retweet convention ‘RT’ is better captured by the WNET model. This seems to indicate that some products or conventions may be affected by competition and network influence more (or less) than others, perhaps due to its own inherent properties or the time in their product cycle when its usage was recorded. To some extent, however, average test log-likelihood does not tell us how much predictive power each model has. Next, we evaluate how well our models and the baselines allow us to estimate each user’s rank (in terms of number of uses) for url shortening services and retweet conventions.

Table 5 shows the percentage of users for which the rank estimated by the Hawkes based models and the baselines coincides with the true ranking of url shortening services and retweet conventions. All models recover the true ranking for more than 72% of the users for both url shortening services and retweet conventions. In this case, WNET and Poisson, which are the best performers, achieve indistinguishable accuracy. Rank gives us an estimation of which products and conventions are more or less used by each user. However, are our models also able to predict the overall cumulative number of uses of each product on the test set? Figure 4 answers this questions positively, by showing the cumulative number of estimated and true test events over time for the WNET model.

Figures 5 and 6 shows the average Spearman correlation between the true and the predicted number of events in 24h bins for every user and the five most popular url shortening services and for the five most popular retweet conventions, respectively. At least one of the Hawkes based models outperforms both the Poisson model and linear regression in all shortening services and retweet conventions except two (‘tinyURL’ and ‘Retweeting’), where the Poisson model is the best performer. If we now compare the Hawkes based models, we find that WNET is the best performer for three url shortening services (‘bit.ly’, ‘Isgd’ and ‘snURL’) and three retweet conventions (‘RT’, ‘via’ and ‘Retweet’), while the full model performs best for one url shortening service (‘TwiURL’) and one retweet convention (‘HT’). This is coherent with our findings using the test log-likelihood; some products or conventions seem to be affected by competition and influence more (or less) than others.

Performance under external interventions. Now we study whether sudden exogenous changes, independent of the network dynamics, affect the predictive performance of our models and, if so, to which extent. In particular, we evaluate the predicted product ranking before and after May 6th, 2009, date when Twitter changed the default url shortening service from ‘tinyurl’ to ‘bitly’. Notably, this change was falling by our model after May 6th, 2009. Table 6 shows this percentage of users for which the neighbor influence parameters $a_{pp}$ and $b_{pp}$ are greater than zero. In both url shortening services and retweet conventions, we clearly distinguish two groups. The first group is composed by the more popular services (‘bitly’ and ‘tinyurl’) or conventions (‘RT’, ‘via’ and ‘HT’). In this group, the main force that drives users’ uses are their own self-excitement or the influence of their neighbors. The second group is composed by products (‘isgd’, ‘twurl’ and ‘snurl’) or conventions (‘retweet’ and ‘retweeting’) that are less popular. Here, in contrast, users’ uses are mainly driven by spontaneous adoption.

Further, Tables 8 and 11 show the percentage of users for which the self-excitement parameters $a_{lp}$ (for $l 
eq p$) are smaller than zero, and Tables 9 and 12 show the percentage of users for which the neighbor influence parameters $b_{lp}$ (for $l 
eq p$) are smaller than zero. Here, we can think of the values on each row as a measure of the degree of inhibition that an event of the product associated to the row entails on the usage of the products in each column. We find several interesting patterns, which we summarize next.

First, if we pay attention to the last rows of each table, which correspond to less popular products and conventions, we conclude that using one of these products or conventions has a strong inhibitory effect on using more popular product or conventions. Second, we find that ‘bit.ly’ has a stronger inhibitory effect on ‘tinyurl’ but ‘tinyurl’ does not have almost any effect on ‘bit.ly’. This may indicate that even before the change of default service from ‘tinyurl’ to ‘bit.ly’ on May 6th, ‘bit.ly’ was already a strong competitor to ‘tinyurl’. Finally, if we pay attention to the inhibitory effect of ‘via’ and ‘HT’ on ‘RT’, we find that the former has a much stronger inhibitory effect than the later. A possible explanation is that ‘HT’ is very similar to ‘RT’, and thus users do not switch from ‘HT’ to ‘RT’, but they do switch more from ‘via’ to ‘RT’.

Implementation and scalability. We developed an efficient distributed implementation of our optimization prob-
Table 7: URL shortening services. Percentage of users with $\mu_p > 0$, $a_{pp} > 0$ and $a_{pp} > 0$.

| Product | $\mu_p > 0$ | $a_{pp} > 0$ | $b_{pp} > 0$ |
|---------|-------------|--------------|--------------|
| Bitly   | 22.53%      | 41.88%       | 42.17%       |
| Tiny URL| 42.42%      | 96.65%       | 89.91%       |
| Isgd    | 21.19%      | 17.92%       | 25.59%       |
| TwURL   | 13.65%      | 13.46%       | 11.66%       |
| SnURL   | 3.94%       | 3.89%        | 3.02%        |

Table 8: URL shortening services. Percentage of users with $a_{ip} < 0$.

| $l$ | $p$ | Bitly | Tiny URL | Isgd | TwURL | SnURL |
|-----|-----|-------|----------|------|-------|-------|
| Bitly | 61.18% | 10.22% | 7.29% | 1.47% |
| Tiny URL | 1.59% | 1.01% | 0.42% | 0.21% |
| Isgd    | 29.31% | 85.51% | 9.38% | 1.47% |
| TwURL   | 35.01% | 86.68% | 23.07% | 3.02% |
| SnURL   | 38.78% | 95.63% | 26.01% | 14.41% |

Table 9: URL shortening services. Percentage of users with $b_{ip} < 0$.

| Convention | $\mu_p > 0$ | $a_{pp} > 0$ | $b_{pp} > 0$ |
|------------|-------------|--------------|--------------|
| RT         | 23.3%       | 98.6%        | 97%          |
| via        | 41.9%       | 49.9%        | 68.3%        |
| HT         | 37.3%       | 37%          | 69.1%        |
| retweeting | 11.6%       | 4%           | 8.6%         |
| retweeting | 5.2%        | 2.7%         | 4.8%         |

Table 10: Retweet conventions. Percentage of users with $\mu_p > 0$, $a_{pp} > 0$ and $a_{pp} > 0$.

5. REFERENCES

[1] O. Aalen, O. Borgan, and H. Gjessing. Survival and event history analysis: a process point of view. Springer Verlag, 2008.
[2] H. Akaike. A new look at the statistical model identification. Automatic Control, IEEE Transactions on, 19(6):716–723, Dec. 1974.
[3] D. Antoniades, I. Polakis, G. Kontaxis, E. Athanasopoulos, S. Ioannidis, E. P. Markatos, and T. Karagiannis. we. b: The web of short urls. In WWW, 2011.
[4] E. Bacry, K. Dayri, and J.-F. Muzy. Non-parametric kernel estimation for symmetric hawkes processes. application to high frequency financial data. The European Physical Journal B, 85(5):1–12, 2012.
[5] E. Bacry, S. Delattre, M. Hoffmann, and J.-F. Muzy. Modelling microstructure noise with mutually exciting point processes. Quantitative Finance, 13(1), 2013.
[6] F. M. Bass. A new product growth for model consumer durables. Management Science, 15(1):215, 1969.
[7] S. Bharathi, D. Kempe, and M. Salek. Competitive influence maximization in social networks. Internet and Network Economics, pages 306–311, 2007.
[8] S. Bikhchandani, D. Hirshleifer, and I. Welch. A theory of fads, fashion, custom, and cultural change as informational cascades. The Journal of Economic Perspectives, 12(3):151–170, 1998.
[9] C. Bishop. Pattern Recognition and Machine Learning (Information Science and Statistics). Springer-Verlag New York, Inc., Secaucus, NJ, USA, 2006.
[10] C. Blundell, J. Beck, and K. A. Heller. Modelling reciprocating relationships with hawkes processes. In NIPS, 2012.
[11] P. Brémaud and L. Massoulié. Stability of nonlinear hawkes processes. The Annals of Probability, pages 1563–1588, 1996.
[12] C. Budak, D. Agrawal, and A. El Abbadi. Limiting
Table 11: Retweet conventions. Percentage of users with $a_{ip}$ < 0.

| $t$ | $P$ | RT | via | HT | Retweet | Retweeting |
|-----|-----|-----|-----|-----|---------|------------|
| 0.81% | 0.63% | 0.07% | 0.11% |     |         |            |
| 41.64% | 10.5% | 3.4% | 2.11% |     |         |            |
| 15.72% | 5.84% | 1.55% | 2.11% |     |         |            |
| 54.84% | 40.83% | 38.76% | 1.52% |     |         |            |
| 47.86% | 38.98% | 39.98% | 5.21% |     |         |            |

Table 12: Retweet conventions. Percentage of users with $b_{ip}$ < 0.

| $t$ | $P$ | RT | via | HT | Retweet | Retweeting |
|-----|-----|-----|-----|-----|---------|------------|
| 0.7% | 0.48% | 0.15% | 0.07% |     |         |            |
| 34.28% | 2% | 0.55% | 0.37% |     |         |            |
| 8.95% | 0.89% | 0.37% | 0.41% |     |         |            |
| 58.14% | 39.42% | 28.37% | 1.48% |     |         |            |
| 47.3% | 38.46% | 35.95% | 2.66% |     |         |            |

the Spread of Misinformation in Social Networks. In 4th ACM International Conference on Web Search and Data Mining, 2011.
[13] M. Cha, H. Haddadi, F. Benevenuto, and P. K. Gummadi. Measuring User Influence in Twitter: The Million Follower Fallacy. In 4th AAAI Conference on Weblogs and Social Media, 2010.
[14] A. Clauset, C. Moore, and M. E. J. Newman. Hierarchical structure and the prediction of missing links in networks. Nature, 453(7191):98–101, 2008.
[15] N. Du, L. Song, M. Gomez-Rodriguez, and H. Zha. Scalable influence estimation in continuous-time diffusion networks. In NIPS, 2013.
[16] N. Du, L. Song, A. Smola, and M. Yuan. Learning networks of heterogeneous influence. In NIPS, 2012.
[17] P. Dubey, B. De Meyer, and R. Garg. Competing for customers in a social network. Cowles Foundation Discussion Paper, 2006.
[18] M. Egesdal, C. Fatihauer, K. Louie, J. Neuman, G. Mohler, and E. Lewis. Statistical modeling of gang violence in los angeles. SIAM Undergraduate Research Online, 3, 2010.
[19] M. Gomez-Rodriguez, D. Balduzzi, and B. Schölkopf. Uncovering the Temporal Dynamics of Diffusion Networks. In ICML, 2011.
[20] M. Gomez-Rodriguez, J. Leskovec, and B. Schölkopf. Modeling Information Propagation with Survival Theory. In ICML, 2013.
[21] M. Gomez-Rodriguez and B. Schölkopf. Influence Maximization in Continuous Time Diffusion Networks. In ICML, 2012.
[22] S. Goyal and M. Kearns. Competitive contagion in networks. In STOC, 2012.
[23] M. Grant and S. Boyd. CVX. http://cvxr.com/cvx, 2013.
[24] A. G. Hawkes. Spectra of some self-exciting and mutually exciting point processes. Biometrika, 58(1):83–90, 1971.
[25] R. Hogg, J. McKean, and A. Craig. Introduction to mathematical statistics. Pearson education international. Pearson Education, 2005.
[26] B. A. Huberman, D. M. Romero, and F. Wu. Social networks that matter: Twitter under the microscope. First Monday, 14(1), 2009.
[27] T. Iwata, A. Shah, and Z. Ghahramani. Discovering latent influence in online social activities via shared cascade poisson processes. In KDD, 2013.
[28] D. Kempe, J. M. Kleinberg, and E. Tardos. Maximizing the spread of influence through a social network. In KDD, 2003.
[29] J. Kingman. Poisson Processes. Oxford studies in probability. Clarendon Press, 1992.
[30] F. Kooti, H. Yang, M. Cha, P. K. Gummadi, and W. A. Mason. The emergence of conventions in online social networks. In 6th AAAI Conference on Weblogs and Social Media, 2012.
[31] J. Kostka, Y. A. Oswald, and R. Wattenhofer. Word of mouth: Rumor dissemination in social networks. In SIROCCO, 2008.
[32] M. Krumm, I. Reutsky, and S. Shoham. Correlation-based analysis and generation of multiple spike trains using hawkes models with an exogenous input. Frontiers in computational neuroscience, 2010.
[33] J. Leskovec, D. Chakrabarti, J. Kleinberg, C. Faloutsos, and Z. Ghahramani. Kronecker graphs: An approach to modeling networks. JMLR, 2010.
[34] J. Leskovec, K. J. Lang, A. Dasgupta, and M. W. Mahoney. Statistical properties of community structure in large social and information networks. In WWW, 2008.
[35] V. Mahajan, E. Muller, and F. M. Bass. New product diffusion models in marketing: A review and directions for research. The Journal of Marketing, 1990.
[36] D. Marsan and O. Lengline. Extending earthquakes’ reach through cascading. Science, 319(5866):1076–1079, 2008.
[37] D. McFadden. Econometric models for probabilistic choice among products. Journal of Business, 53(3):S13–S29, 1980.
[38] S. Myers and J. Leskovec. Clash of the Contagions: Cooperation and Competition in Information Diffusion. In ICDM, 2012.
[39] Y. Ogata. On lewis’ simulation method for point processes. IEEE Transactions on Information Theory, 27(1):23–31, 1981.
[40] Y. Ogata and H. Akaike. On linear intensity models for mixed doubly stochastic poisson and self-exciting point processes. In Selected Papers of Hirotugu Akaike, pages 269–274. Springer, 1998.
[41] D. M. Romero, B. Meeder, and J. Kleinberg. Differences in the mechanics of information diffusion across topics: idioms, political hashtags, and complex contagion on twitter. In WWW, 2011.
[42] K. Zhou, L. Song, and H. Zha. Learning social infectivity in sparse low-rank networks using multi-dimensional hawkes processes. In AISTATS, 2013.
[43] K. Zhou, H. Zha, and L. Song. Learning triggering kernels for multi-dimensional hawkes processes. In ICML, 2013.