K-essence Emergent Spacetime as Generalized Vaidya Geometry

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We establish a formal connection between the K-essence emergent gravity scenario and generalizations of Vaidya spacetime. Choosing the K-essence action to be of the Dirac-Born-Infeld variety, the physical spacetime to be a general static spherically symmetric black hole and restricting the K-essence scalar field to be a function solely of the advanced or the retarded time, we show that the emergent gravity metric resembles closely the generalized Vaidya metrics for null fluid collapse proposed by Husain. Imposing null energy conditions on the emergent energy-momentum tensor derived from the emergent Einstein equation, restrictions are obtained on the functions characterizing the emergent metric for consistent identification with generalized Vaidya spacetimes. Admissible explicit black hole metrics are discussed as examples.

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I. INTRODUCTION

The identification of specific K-essence emergent gravity models with known spacetimes has been one approach [1]-[7] to better understand the physical origin of Dark Energy, if the phenomenon of Late Time Acceleration of the Universe is assumed to truly exist, on the basis of recent cosmological observations. An investigation undertaken to identify emergent K-essence spacetimes with known gravitational configurations, of course, goes beyond the original intent of unravelling Dark Energy, since new motivations from analog gravity make such an investigation a useful enterprise. Based on a specific Dirac-Born-Infeld [8]-[10] model for the K-essence scalar field and specific standard physical black hole spacetimes, one of us (GM) has co-authored papers [11, 12], establishing conformal invariance of the emergent spacetime with Barriola-Vilenkin and Robinson-Trautman spacetimes. To explore general properties of such a ‘map’ between emergent spacetimes corresponding to physical black hole geometries, and unexpected curved geometries apparently unrelated to those black holes, we focus in this paper on Dirac-Born-Infeld type K-essence scalar field models, in the background of general static spherically symmetric black hole spacetimes. Following the standard construction of the emergent composite metric as a function of the physical spacetime metric and the K-essence scalar, we obtain a class of emergent metrics which, under certain restrictions on the scalar field, resemble null fluid collapse models as generalizations [13], [14] of the Vaidya spacetime [15]-[20]. We do not address the question as to whether such a relationship between seemingly unconnected spacetime geometries has any deeper significance in either clarifying any aspect of Dark Energy, or indeed providing any insight into gravitational collapse. As alluded to in the abstract, at this point the mapping between the disparate spacetimes is quite formal. However, detailed computations for the explicit examples of black hole spacetimes may provide some extra insight for the discerning reader.

The paper is organized as follows: In section 2, we follow ref.s [1]-[7] to briefly review the construction of the composite emergent metric for a very general K-essence scalar field sector in an arbitrary physical spacetime background, not necessarily stationary. Towards the end of this section, we specialize to the precise K-essence scalar field action [11, 12] which we actually use in the present work. In the next section, we construct the emergent spacetime metric for a general static spherically symmetric black hole background, with a scalar field restricted to be an arbitrary function of the advanced or retarded Eddington-Finkelstein time, and to be independent of the other variables of the four dimensional spacetime. Such a choice implies that the composite emergent metric will violate Lorentz invariance. So, at this point it is not clear if the construction will at all lead to anything useful. To investigate this question of utility, we construct the Einstein tensor corresponding to our emergent metric, and compute the components of the emergent energy-momentum tensor by direct substitution into an emergent Einstein equation. This emergent tensor must obey energy conditions if the emergent geometry is to have any interpretation as a curved spacetime. This requirement is shown to lead to certain restrictions on the functions characterizing the composite metric, i.e., on the function characterizing the background spacetime,

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as also on the function of the advanced/retarded time characterising the K-essence sector. This is followed in section 4 by results of computations involving several explicit black hole metrics corresponding to the spacetime background. We conclude in section 5.

II. REVIEW OF K-ESSENCE AND EMERGENT GRAVITY

In this section, we present a short review of the construction of the effective metric for the emergent spacetime corresponding to a general background geometry and a very general K-essence scalar field sector. The K-essence scalar field $\phi$ minimally coupled to the background spacetime metric $g_{\mu\nu}$ has action [1]-[7]

$$S_k[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} L(X, \phi)$$

where $X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$. The energy-momentum tensor is

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_k}{\delta g^{\mu\nu}} = L_X \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} L$$

$$L_X = \frac{dL}{dt}, \quad L_X = \frac{\phi L}{\phi}, \quad L_{\phi} = \frac{dL}{d\phi}$$

and $\nabla_\mu$ is the covariant derivative defined with respect to the gravitational metric $g_{\mu\nu}$. The scalar field equation of motion is

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_k}{\delta \phi} = \tilde{G}^{\mu\nu} \nabla_\mu \nabla_\nu \phi + 2X L_X \phi - L_{\phi} = 0$$

where

$$\tilde{G}^{\mu\nu} = L_X g^{\mu\nu} + L_{XX} \nabla^\mu \phi \nabla_\nu \phi$$

and $1 + \frac{2XL_{XX}}{L_X} > 0$.

We make the conformal transformation $G^{\mu\nu} = \frac{L_X}{c_s^2 L_X} \tilde{G}^{\mu\nu}$, with $c_s^2(X, \phi) \equiv (1 + 2X L_{XX} / L_X)^{-1}$. Then the inverse metric of $G^{\mu\nu}$ is

$$G_{\mu\nu} = \frac{L_X}{c_s} [g_{\mu\nu} - c_s^2 L_X \nabla_\mu \phi \nabla_\nu \phi]$$

A further conformal transformation [11, 12] $\tilde{G}^{\mu\nu} \equiv \frac{L_X}{c_s^2} G^{\mu\nu}$ gives

$$\tilde{G}_{\mu\nu} = g_{\mu\nu} - \frac{L_{XX}}{L_X + 2XL_{XX}} \nabla_\mu \phi \nabla_\nu \phi$$

Here one must always have $L_X \neq 0$ for $c_s^2$ to be positive definite and only then equations (1) - (4) will be physically meaningful.

It is clear that, for non-trivial spacetime configurations of $\phi$, the emergent metric $G_{\mu\nu}$ is, in general, not conformally equivalent to $g_{\mu\nu}$. So $\phi$ has properties different from canonical scalar fields, with the local causal structure also different from those defined with $g_{\mu\nu}$. Further, if $L$ is not an explicit function of $\phi$ then the equation of motion (3) reduces to:

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_k}{\delta \phi} = \tilde{G}^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0$$

We shall take the Lagrangian as $L = L(X) = 1 - V\sqrt{1-2X}$ with $V$ is a constant. This is a particular case of the DBI lagrangian [11, 12], [8]-[10]

$$L(X, \phi) = 1 - V(\phi)\sqrt{1-2X}$$

for $V(\phi) = V = \text{constant}$ and kinetic energy of $\phi >> V$ i.e. $(\phi)^2 >> V$. This is typical for the K-essence fields where the kinetic energy dominates over the potential energy. Then $c_s^2(X, \phi) = 1 - 2X$. For scalar fields $\nabla_\mu \phi = 0$. Thus (6) becomes

$$\tilde{G}_{\mu\nu} = g_{\mu\nu} - \partial_\mu \phi \partial_\nu \phi$$

Note the rationale of using two conformal transformations: the first is used to identify the inverse metric $G_{\mu\nu}$, while the second realises the mapping onto the metric given in (9) for the lagrangian $L(X) = 1 - V\sqrt{1-2X}$.

III. EMERGENT SPACETIME FOR GENERAL SPHERICALLY SYMMETRIC BLACK HOLES

The line element corresponding to a general spherically symmetric static (black hole) spacetime is

$$ds^2 = f(r)dr^2 - f^{-1}(r)dv^2 - r^2d\Omega^2$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$.

We define the tortoise coordinate by the relation $dr^* = f^{-1}(r)dr$ and $v = t + \epsilon r^*$. With these definitions, the line element (10) reduces to the Eddington-Finkelstein line element

$$ds^2 = f(r)dv^2 - 2edvdr - r^2d\Omega^2$$

When $\epsilon = +1$, the null coordinate $v$ represents the Eddington advanced time (outgoing), while when $\epsilon = -1$, it represents the Eddington retarded time (incoming).

Now, from (9) the emergent spacetime is described by the line element

$$ds^2 = f(r)dv^2 - 2edvdr - r^2d\Omega^2$$

We make the assumptions that the scalar field $\phi(x) = \phi(v)$, so that the emergent spacetime line element is

$$ds^2 = [f(r) - \phi_v^2]dv^2 - 2edvdr - r^2d\Omega^2$$

where $\phi_v = \frac{\partial \phi}{\partial v}$. Notice that this assumption on $\phi$ actually violates local Lorentz invariance, since in general, spherical symmetry would only require that $\phi(x) = \phi(v, r)$. The additional assumption that $\phi(v, r) = \phi(v)$ i.e., it is independent of $r$ implies that outside of this particular choice of frame, a spherically symmetric $\phi$ is actually a function of both $v, r$.

We now compare the emergent spacetime (13) with the metric [13, 14] of the generalized Vaidya spacetimes corresponding to gravitational collapse of a null fluid (take $\epsilon = +1$)

$$dS^2_V = \left(1 - \frac{2m(v, r)}{r}\right) dv^2 - 2dvdr - r^2d\Omega^2$$
yielding the mass function
\[ m(v, r) = \frac{1}{2} r \left[ 1 + \phi_v^2 - f(r) \right]. \] (15)

One can now compute the emergent Einstein tensor following [13], with the notation that subscripts on \( m \) and \( f \) denote derivatives with respect to those subscripts.

\[ G^0_0 = G^1_1 = -\frac{2m_r}{r^2}, \quad G^1_1 = \frac{1}{r^2} \left[ rf_r + f - 1 - \phi_v^2 \right], \] (16)
\[ G^1_2 = \frac{2m_v}{r^2}, \quad G^2_2 = \frac{m_{rr}}{r} = \frac{2r^2}{r^2} - \frac{m_{rr}}{r} = 0. \] (17)

Recourse to the ‘emergent’ Einstein equation
\[ G^\mu_\nu = \kappa T^\mu_\nu \] (19)
where, \( \kappa \equiv 8\pi G \) leads to the components \( T^\mu_\nu \), which can be parametrized exactly as in ref. [13, 14] in terms of the components \( \gamma, \rho \) and \( P \) given by
\[ T^\mu_\nu = T^{(n)}_\mu_\nu + T^{(m)}_\mu_\nu = \begin{bmatrix} \gamma \gamma/2 + \rho & 0 & 0 \\ \gamma/2 - \gamma/2 \rho & 0 & 0 \\ 0 & 0 & P \end{bmatrix} \] (20)
where \( T^{(n)}_\mu_\nu = \gamma l_\mu l_\nu; \quad T^{(m)}_\mu_\nu = (\rho + P)(l_\mu n_\nu + l_\nu n_\mu) + PC_{\mu\nu} \) with \( l_\mu \) and \( n_\mu \) are two null vectors. Contractions of all indices are performed through the emergent metric \( G^\mu_\nu \).

The expressions for the three independent components are given by,
\[ \gamma = \frac{2\phi_v \phi_{vv}}{\kappa r}, \] (21)
\[ \rho = \frac{1}{\kappa r^2} \left[ 1 + \phi_v^2 - f - rf_r \right], \] (22)
\[ P = \frac{1}{2\kappa r} [2f_r + rf_{rr}]. \] (23)

It is obvious that energy conditions imposed on \( T^{\mu \nu} \) will in turn constrain \( f(r) \) and \( \phi(v) \) and their derivatives. Thus,
\[ \gamma > 0 \Rightarrow \phi_v \phi_{vv} > 0, \] (24)
\[ \rho > 0 \Rightarrow 1 + \phi_v^2 > f + rf_r, \] (25)
\[ P > 0 \Rightarrow 2f_r + rf_{rr} > 0. \] (26)

IV. EXPLICIT EXAMPLES OF BACKGROUND SPACETIME

A. Schwarzschild Black Hole as background

Now, we may choose \( f(r) = 1 - 2M/r \), i.e., the physical spacetime is an exterior Schwarzschild spacetime. In this case, the emergent spacetime has the line element
\[ ds^2 = [1 - 2M/r - \phi_v^2] dv^2 - 2dv dr - r^2 d\Omega^2. \] (27)

From (15), the mass function is
\[ m(v, r) = M + \frac{r^2}{2} \phi_v^2. \] (28)

Therefore, the non-vanishing components of the Einstein tensors are
\[ G^0_0 = G^1_1 = -\frac{2m_r}{r^2}, \quad G^1_1 = \frac{2m_v}{r^2}, \quad G^1_2 = \frac{m_{rr}}{r} = 0. \] (29)

where all subscripts designate derivatives as in the last section. For the case of Schwarzschild background (27) the values of \( \gamma, \rho \) and \( P \) are
\[ \gamma = \frac{2\phi_v \phi_{vv}}{\kappa r^2}, \quad \rho = \frac{\phi_v^2}{\kappa r^2} \quad \text{and} \quad P = 0 \] (30)
which satisfies the weak and strong energy conditions [14, 21]
\[ \gamma > 0, \quad \rho > 0, \quad P > 0 \quad (\gamma \neq 0) \] (31)
provided \( \phi_v \phi_{vv} > 0. \) Therefore, energy-momentum tensor becomes
\[ T^\mu_\nu = \begin{bmatrix} \frac{1}{\kappa r}(\phi_v \phi_{vv} + \phi_v^2) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \] (32)
This type of energy-momentum tensor belongs to type-II class [21] which have a double null vector.

B. Reissner-Nordstrom Black Hole as background

Again, we choose \( f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \), i.e., the physical spacetime is Reissner-Nordstrom (RN) where \( Q \) is the charge of the RN black hole. In this case the line element of the emergent spacetime is
\[ ds^2 = [1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \phi_v^2] dv^2 - 2dv dr - r^2 d\Omega^2 \] (33)
and the mass function is
\[ m(v, r) = M - \frac{Q^2}{2r} + \frac{r^2}{2} \phi_v^2. \] (34)

For this case the non-vanishing components of Einstein tensors are
\[ G^0_0 = G^1_1 = -\frac{Q^2}{r^4} - \frac{\phi_v^2}{r^2}, \quad G^0_1 = \frac{2\phi_v \phi_{vv}}{r}, \quad G^1_2 = \frac{Q^2}{r^4}. \] (35)

Using the relation (34) we get the values of \( \gamma, \rho \) and \( P \) are
\[ \gamma = \frac{2\phi_v \phi_{vv}}{\kappa r^2}, \quad \rho = \frac{1}{\kappa r^2} \left[ \frac{Q^2}{r^4} + \phi_v^2 \right] \quad \text{and} \quad P = \frac{Q^2}{\kappa r^4} \] (36)
which have also satisfied the weak and strong energy condition (31).
C. de-Sitter Reissner-Nordstrom black hole as background

If we consider \( f(r) = (1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2) \) where \( \Lambda \) is the cosmological constant i.e., the physical spacetime is Reissner-Nordstrom-de Sitter (RNdS) \((\Lambda > 0)\) then the emergent spacetime is

\[
dS^2 = [1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2 - \phi_v^2] dv^2 - 2dvdr - r^2 d\Omega^2
\]

and the mass function becomes

\[
m(v, r) = M - \frac{Q^2}{2r} + \frac{\Lambda}{6} r^3 + \frac{r}{2} \phi_v^2
\]

and non-zero components of Einstein tensors are

\[
G^0_0 = G^1_1 = -[\frac{Q^2}{r^4} + \Lambda + \frac{\phi_v^2}{r^2}], \quad G^0_1 = \frac{2\phi_v \phi_{vv}}{r};
\]

\[G^2_2 = G^3_3 = \frac{Q^2}{r^4} - \Lambda.\] (39)

In this case the values of \( \gamma, \rho \) and \( P \) are

\[
\gamma = \frac{2\phi_v \phi_{vv}}{k \Gamma}; \quad \rho = \frac{1}{\kappa r^2} [\frac{Q^2}{r^2} + \Lambda r^2 + \phi_v^2] \text{ and } \quad P = \frac{1}{\kappa} [\frac{Q^2}{r^4} - \Lambda].\] (40)

which have to satisfy the energy conditions (31) provided \( \frac{Q^2}{r^2} > \Lambda. \)

V. CONCLUSION

The link discerned by us between a specific K-essence emergent gravity model and generalized Vaidya models characterizing gravitational collapse of null fluids with a large class of mass functions is interesting from a purely gravitational theory standpoint. If such a K-essence model describes cosmological observations of Dark Energy comprehensively, that would of course demystify Dark Energy to quite an extent. Unfortunately, however, cosmological implications of a paper such as this, are yet to be worked out with any degree of completeness. Indeed, the 2017 Planck data already imposes stringent constraints on non-Gaussianity of the inflationary perturbation spectrum [22], thereby constraining the K-essence class of models which apparently predicts substantial non-Gaussianity in inflation. But this is still assuming broadly that cosmic late time acceleration is a given, as is assumed widely among the community of cosmologists. However, there are a few voices of dissent. In fact, apparently the larger current database available from cosmological observations have led some cosmologists to argue that the evidence for cosmic late time acceleration itself is of much less statistical significance [23] than what might have been perceived in the last part of the last century. Be that as it may, future work in this direction may additionally focus on implications of relationships such as the one discussed above, to various varieties of analog gravity models where gravitational phenomena difficult to observe in real spacetime may still be scrutinized in terrestrial laboratories.

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