Research on the Bi-level Programming Model for Ticket fare pricing of Urban Rail Transit Based on Particle Swarm Optimization Algorithm

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Abstract

As a public service facility, the social and economic benefits of urban rail transit ticket fare are both important, so reasonable ticket fare is a key for the solid development of urban rail transit. The social and economic benefits should be taken into account under the competitive condition led by various modes of transportation in order to get an optimal strategy in ticket fare pricing of urban rail transit on the premise of meeting the service quality standard. Here, the factors considered in the ticket fares fare pricing of urban rail transit in the domestic and foreign cities are summarized, after which the Logit model of the mode split within the public transit system is established. With considering both the respective benefits of the urban rail transit company and the travellers, a bi-level programming model is established together with the solution idea to the model with the particle swarm optimization algorithm. The example demonstrates the feasibility and effectiveness of the bi-level programming model and the related measures and the particle swarm optimization algorithm is fit for the urban rail transit fare pricing. The suggestions proposed from the result of the example are helpful for the decision making of ticket fare pricing of urban rail transit.

Keywords: Ticket fare pricing; Urban rail transit ; Logit model; Bi-level programming model; Particle swarm optimization algorithm

1. Introduction

As the process of urbanization, motorization and the rapid development of the economy as well as the social modernization develops, the growing contradiction between supply and demand outstanding urban public transit, the main trunk road public transit pressure significantly increased, and many sections are saturated or critical...
saturation. Urban public transit has become one of bottleneck restricting the development of the city. Therefore, the development of large capacity of the rail transport, development and utilization of underground space in order to effectively solve the contradiction between supply and demand in the ground path of resources to guide urban land use and urban transport coordination, meet the growing demand for urban transport, effectively promote the rapid development of urban economy.

With the presentation and active implementation of the priority development strategy in the urban public transportation, all the cities in our country commit to the multi-level three-dimensional integrated public transport system with the rain transit as the skeleton and the traditional public transport as the body as well as the other kinds of travel modes as the supporting parts, to satisfy the demand for traveling of residents. In certain circumstances of the public service level and travel demand, the ticket price level would determine the sharing rate of various traveling modes (Matthew, 2002).

As the city expands, the traveling distance and the complication of the level on the public transit network, residents can complete a public traveling with random origin and destination, which needs a various kinds of public transit system with several transfer lines. As the passenger traffic flow grows, the cost for individual passenger per-kilometer in the system of urban rail transit reduces. Therefore, the average cost in the railway system may fall below that in the traditional system under the situation of the facility in high utilization. Theoretically, the ideal rail transit fare structure and fare levels can make up for the costs of operation and annual capital costs, and make certain profit for the railway operating companies. The problem is that the passenger hopes for the minimum of the generalized travel costs, and the plan obtained by the method of single objective function planning can hardly meet the benefits of both the railway operating companies and the passengers in the same time.

In view of the problems above, the paper considers the interests of the both sides of the public transport companies and the passengers respectively, and establishes the bi-level programming model for ticket fare pricing of urban rail transit.

2. The Review of the Current Research

2.1. The Research on the Ticket Fare Pricing of Urban Public Transit

Based on the sufficient consideration of the start time and arrival time of train, Ngostino (1999) proposed a model of behavior choice to simulate the service features of rail in long range or middle range, as well as the relative ticket fares policy. Van (2002) establishes a theory for fare pricing and analyzed the relationship between passenger demand price elasticity and marginal cost in Dutch railway, which demonstrated that The maximization of social welfare can be obtained with Ramsey pricing. Marvin (2002) proposed the optimization model for fares and service in urban rail including the passenger traveling time, the result of which showed that the increase of passengers results in higher average user cost, and The relationship between service frequency and the total capacity does not meet the principle of conversional square root.

2.2. The Research on Fare Pricing Competition Among the Travel Modes

To analyze the widespread phenomenon of scale economy in the modes of public transit especially the rail transit, Tabuchi (1993) took research on the ticket fare pricing in the system of competition between the private transportation and the public transit as well as its influence on the choice making for travel modes. However, his research failed of focusing on the regular driving gap and consideration of the features for physical contact in compartments as well as the attribution differences among the travelers, which is researched by Huang (2000). Jiang and Hai (2005) researched on the sensitivity analysis methods for transit system with the separate
objects of the private transit road network and public transit network. Meanwhile, he applied the sensitivity analysis methods in the research on the optimal fare pricing problem in the united network to analyze the economic features of traffic scale and congestion. After making analysis of the urban road pricing policy based on the integration of the automatic toll system, Hanna Armelius (2005) not only researched on the crowded costs for different types of passengers, but also focused on the welfare effects with variables of travel modes and travel time. The results presented that the expropriation of congestion cost in tolls is still a way to improve the social welfare even under the situation of congestion.

3. The Establishment of the Bi-level Programming Model for Ticket fare pricing of Urban Rail Transit

3.1. The factors for the ticket fare pricing of urban public transit

The urban public transit has a strong public welfare and externality, as well as a certain natural monopolization, the operation of which is under the supervision and control of the government(Yaron,2006). And therefore, the ticket fare pricing of urban public transit would be influenced by the following factors(Alatas,2008):

1. Operation cost. As the main part in the market under the market economic background, the urban public transit should afford Self-financing so as to set the operation cost as the minimum fare;
2. Travel demand and payment willingness of passengers. Passengers usually make their own choices among the travel modes according to their individual travel purpose, the needed generalized travel costs, and their affordability, and therefore it will be considered as the maximum fare in the fare pricing;
3. Competition among the public transit modes. The positive competition among the public transit modes cannot be avoided as the market share is disputed, which determines that the companies should evaluate the price-performance gap among themselves and their competitors, and their strength and weakness;
4. Financial goal. The financial goals of public transit companies would also have some impact on fare pricing. For example, the profit need to surpass the sum of fixed costs and variable costs, and some revenue should be obtained.
5. Government regulation. Generally speaking, government would implement the fare pricing limitation policy on the companies in the aspects of the urban Public and social welfare, but would not make an adjustment unless it would be some necessary condition.

3.2. The choice of public transit modes based on Logit model

In the urban public transit system, it often happens that various kinds of public transit modes cooperate to meet the demand of passenger transit in the same road section of OD. Meanwhile, passengers can make their own choices according to the different ticket price and perception, which would cause the intense competition among various modes of public transit.

The choice of passengers to various modes can be quantified with logit model, that is, the probability that passengers choose the n-th mode for traveling between station i and j can be expressed as (Tabuchi,1993):

\[ X^n_{ij} = \frac{\exp(-U^n_{ij})}{\sum_{n \in N} \exp(-U^n_{ij})} \]  

(1)

In the function (1), N is the collection of all the public transit modes, \(X^n_{ij}\) is the probability that passengers choose the n-th mode for traveling between station i and j, and \(U^n_{ij}\) is the utility of the n-th mode for public transit. Assume that the choice of passengers is made according to the utility of the public transit modes, which includes the travel costs of passengers (including the time cost and the money cost) and the perceived cost of passengers. Therefore, it can be qualified as the utility function as:
In the function (2), $U_{ij}^n$ is the generalized travel cost of the $n$-th mode for public transit between station $i$ and $j$; $\alpha_i (i=1,2,3)$ is the parameters for influencing the preferences of making choice from passenger travel modes; $\lambda$ is the conversion coefficient for time value of passengers, and the time parameter $T_{ij}^n$ is the sum of the vehicle time and exterior time; $p_{ij}^n$ is the money cost which means the fare of the $n$-th mode for public transit between station $i$ and $j$, and $\epsilon_{ij}^n$ is the perceived cost of passengers. To simplified the calculation, the other kinds of cost can be ignored.

Besides, it is defined that the time needed for the passengers between station $i$ and $j$ to walk to the nearest station, and $f_{ij}^n$ is the frequency of the $n$-th mode for public transit between station $i$ and $j$, $v_{ij}^n$ is the average speed of the $n$-th mode for public transit between station $i$ and $j$. And therefore, $T_{ij}^n$ can be demonstrated as:

$$T_{ij}^n = t_{0ij}^n + \frac{1}{f_{ij}^n} + \frac{D}{V_{ij}^n} + \frac{D}{D_0} s_{ij}^n$$

$Y_{ij}^n$ is taken as the exponential function for the utility of the $n$-th mode for public transit, and $Y_{ij}^n$ is taken as the exponential function for the utility excluding the $n$-th mode for public transit. It is assumed that the total amount of passengers between station $i$ and $j$ is $Q_{ij}$, and then, the amount born by the $n$-th mode for public transit is $Q_{ij}^n$, that is:

$$Y_{ij}^n = \exp(-U_{ij}^n), X_{ij}^n = \frac{Y_{ij}^n}{Y_{ij}^n - Y_{ij}^n}, Q_{ij}^n = Q_{ij} \times \frac{Y_{ij}^n}{Y_{ij}^n - Y_{ij}^n}$$

According to the function (3) and (4), the further function is:

$$p_{ij}^n = -\left(\frac{\alpha_1}{\alpha_2} - \lambda T_{ij}^n + \frac{\alpha_2}{\alpha_2} \epsilon_{ij}^n\right) - \frac{1}{\alpha_2} \left[\ln Q_{ij}^n + \ln Y_{ij}^n - \ln(Q_{ij} - Q_{ij}^n)\right]$$

Define $\omega_{ij}^n$ and $\beta_{ij}^n$ is respectively the variable cost and the fixed cost of the $n$-th mode for public transit between station $i$ and $j$. The revenue of the $n$-th mode for public transit is (Costa, 2001):

$$R_n = \sum_{i,j} \left[Q_{ij}^n \times (p_{ij}^n - \omega_{ij}^n) - \beta_{ij}^n\right]$$

3.3. The establishment of the bi-level programming model

It can be assumed that: (1) there are only two kinds of transit modes in the same section of OD with the fixed amount of passengers, among which the urban rail transit is mode 1 and the conventional public transit is mode 2; (2) the route will be determined as the choices are made by passengers; (3) the operation income only includes the fare without considering the other sources on the condition of free competition.
(1) The lower-level programming. Rational passengers usually choose the travel mode with the minimum general travel cost. However, as the passengers who choose the minimum mode increase, the cost will grow. Until the cost reaches a certain level, some passengers will shift from the mode to others. Finally, the distribution of passengers will achieve a balanced state, and the generalized cost of all modes would tend to be a constant no more than that of the unused modes. Therefore, stochastic user equilibrium model can be used for optional selection and the objective function is the minimum of generalized travel cost.

\[
\min U^n_{ij} = \sum_{n \in N} \sum_{i,j} \alpha_1 T^n_{ij} + \alpha_2 p^n_{ij} + \alpha_3 e^n_{ij}
\]

\[
\begin{align*}
Q_{ij} &= Q^1_{ij} + Q^2_{ij} \\
\alpha_1 T^1_{ij} + \alpha_2 p^1_{ij} + \alpha_3 e^1_{ij} &= \alpha_1 T^2_{ij} + \alpha_2 p^2_{ij} + \alpha_3 e^2_{ij} \\
Q^1_{ij} &\geq 0 \\
Q^2_{ij} &\geq 0
\end{align*}
\] (7)

(2) The upper-level programming. In the game of various kinds of public transit, the goal of all the players is the maximum of revenue, and the total revenue function is the sum of that of both players. The final equilibrium solution of the cooperative game is to maximize the overall revenue function. That is:

\[
\max R = \sum_{k=1,2} \sum_{i,j} R^k_{ij}
\]

\[
\begin{align*}
p_{min}^k &\leq P^k_{ij} \leq P_{max}^k , k = 1,2 \\
\sum_{k=1,2} X^k_{ij} &= 1 \\
X^k_{ij} &\geq 0 , k = 1,2
\end{align*}
\] (8)

In the function (8), \(p_{min}^k\) is the lowest price of \(k\)-th travel mode (railway or tradition bus) and \(p_{max}^k\) is the governmental limitation to the ticket fare of \(k\)-th travel mode. \(X_{ij}^n\) is the ratios of the modes between station \(i\) and \(j\).

With the solution to the bi-level programming, the reasonable fare pricing of urban rail transit and conventional public transit can be determined. (Jiang-qian, 2005)

4. The Solution to the Bi-level Programming Model with Particle Swarm Optimization Algorithm

To solve the bi-level programming model, especially the non-linear bi-level programming model as a strong NP problem, is quite hard and complicated. Although there have not been a kind of strict algorithms for ensuring the solution is optimal to the problem, some kinds of fine algorithms are found to obtain the approximately optimal solution to the bi-level programming problem within a certain level, one of which is genetic algorithms. Having some common features of genetic algorithms, the particle swarm optimization algorithm is also an evolutionary algorithm based on population and a random search algorithm with global convergence. With less complicated structure and less number of control parameters, adopting particle swarm optimization algorithm is considered
to be a meaningful attempt to solve the bi-level programming model (Ferrari, 1999). The basic steps of solution to the bi-level programming model with particle swarm optimization algorithm are described as follows:

Step 1 Initialize the parameters of the particle swarm optimization algorithm: create the initial solution to the lower-level programming model and initialize the particle with location \( X_i \) and speed \( V_i \) at random, in which \( i \in [1, m] \) and \( m \) is the population size, that is, the particle number. Set \( p_i \), the location of the \( i \)-th particle as its current location, and \( p_g \) as the location of the best particle in the initial population;

Step 2 Perform the following operation on all the particles in the population:

1) According to the function (9)~(10), update the parameters of location and speed.

\[
V_i^{k+1} = \omega V_i^k + c_1 r_1 (p_i^k - V_i^k) + c_2 r_2 (p_g^k - V_i^k)
\]

\[
X_i^{k+1} = X_i^k + V_i^{k+1}
\]

\[
\omega = \omega_{\text{Max}} - \text{iter} \times \frac{\omega_{\text{Max}} - \omega_{\text{Min}}}{\text{iterMax}}
\]

Among which, \( r_1 \) and \( r_2 \) are the random number between 0 and 1; generally, being called as the learning factors, \( c_1 = c_2 = 2 \); \( \omega \) is the weight coefficient between 0.1 and 0.9. It is defined that \( \omega \) reduces as the iteration grows, and then \( \omega_{\text{Max}} \) is the maximum of weight coefficient, and \( \omega_{\text{Min}} \) is the minimum; \( \text{iter} \) is the current iteration, and \( \text{iterMax} \) is the final iteration of the algorithm.

2) Substitute \( X_i \), the position of \( i \)-th particle and the solution to the upper-level programming model, into the lower-level programming model, and solve the lower-level programming model with the traditional optimization methods to obtain the optimal solution \( y_i^* \);

3) Substitute \( X_i \) and \( y_i^* \) into the objective function, and calculate the \( F(X_i, y_i^*) \), the fitness of \( i \)-th particle when \( i \in [1, m] \).

4) If the fitness of \( i \)-th particle is better than that of \( P_i \), then update \( P_i \) into \( X_i \), the current location of \( i \)-th particle, and update \( yP_i \), the optimal solution of lower-level programming model relative with \( P_i \), into \( y_i^* \). If the fitness of \( i \)-th particle is better than that of \( P_g \), then update \( P_g \) into \( X_i \), the current location of \( i \)-th particle, and update \( yP_g \), the optimal solution of lower-level programming model relative with \( P_g \), into \( y_i^* \).

Step 3 Judge whether the convergence criteria of algorithm is met. If it is met, turn to Step 5, or turn to Step 4.

Step 4 According to the function (12) update \( p_g \) and calculate \( yP_g \), the solution to the relative with \( p_g \), with traditional optimization methods, then turn to Step 2.

\[
p_g^{k+1} = p_g^{k+1} \times (1 + \eta), \eta \in N(0, 1)
\]

Step 5 Output the optimal solution \( P_g \) and \( yP_g \), and calculate the value of objective function in both the relative upper-level programming model and lower-level programming model.
5. The Numeral Example for Ticket Fare Pricing of Urban Rail Transit

To simplify the calculation, it is assumed that the distance between the origin station and the destination station is 20 kilometers, and the overall passenger volume is 1,000. Both the urban rail transit and conventional public transit are in operation in the city, the relationship of which can be demonstrated in Fig.1 (Zhan, 2009).

![Network Diagram](https://example.com/fig1)

Fig.1 The Network Diagram of Rail Transit and Conventional Public Transit

It is also assumed that all the relative factors, including the needed time, perceived cost and various kinds of operation cost are known and the data is displayed in the table 1 (T^n has been calculated with function 3):

| Mode                          | T^n | \( p_{min}^n \) | \( p_{max}^n \) | \( \beta_{ij}^n \) | \( \omega_{ij}^n \) | \( e_{ij}^n \) |
|-------------------------------|-----|----------------|----------------|-------------------|-------------------|---------------|
| Urban rail transit (n=1)      | 8   | 1.5            | 6              | 200               | 0.2               | 5.5           |
| Conventional public transit (n=2) | 35  | 1              | 3              | 50                | 0.1               | 4.1           |

According to the survey and relative research conclusion, the parameters data is shown in Table 2.

| Parameter | \( \lambda \) | \( \alpha_1 \) | \( \alpha_2 \) | \( \alpha_3 \) |
|-----------|---------------|----------------|----------------|----------------|
| Value     | 0.37          | 15.31          | 3.5            | 4.7            |

Substitute data into the model and solve it with the particle swarm optimization algorithm. And the iteration result for the upper programming model is presented in Fig.2.
As is seen in Fig.2, the calculation of objective function value in the upper model is in convergence in the 87 iterations, and in the solution, the fare set of the urban rail transit and conventional public transit is (2.7,1.6), approaching with the realistic fare pricing of rail transportation. The overall revenue of modes, which is the objective function value in the upper-level programming model, is 2866.46, and respectively in the lower-level programming model, the generalized travel cost is 303.75, which also meets the minimum objective.

The example above shows that the reasonable fare of urban rail transit can be obtained with the bi-level programming model, and the particle swarm optimization algorithm is fit for the rail fare pricing.

6. Suggestion and measures for the ticket fare pricing and the operation of the urban rail transit

According to the analysis of the bi-level programming and the calculation for the numeral example, some suggestion can be proposed for the ticket fare pricing and the operation of the urban rail transit.

In the beginning period of the urban rail operation, the primary objective is to attract the passenger flow and the increasing of market share, which make citizens get familiar of rail traveling and make sufficient usage of transportation capacity. Therefore, some suggestion and strategy should be taken to attract the passenger flows.

1) Strengthen propaganda. The advertisement on television and publicity boards can be applied for getting the rail transit well known. In addition, the propaganda can be taken in the compartment or on the IC card to reduce the cost for advertising and achieve the effect for propaganda.

2) Take concessionary fares. According to the experience at home and abroad, with the adjustment of the fare system, a set of policy for concessionary can be used for attracting the passenger flow, such as monthly ticket, reward discounts, segmented pricing system, group tickets, round-trip ticket and etc.

3) Improve the service quality. The aspects of convenience, agility, safety and affordability should be focused on to fully embody the advantages of rail transit; the information handbook can be obtained for free, and the
schedule, the routine as well as the fare can be informed online. Besides, the suggestion and advice can be proposed by the passengers.

With the gradual improvement of rail transit network, and the upgrade of the service level, great changes would take place in the status of operation management and service groups in the maturation stage of the urban rail operation, and so would the objectives and strategies for railway ticket pricing. To increase rail traffic economic benefit by increasing the fare has become the important content of the development in rail transit. The strategy can be taken as:

1) According to the condition of rail and the passenger flow in each segment, the fare would be adjusted; In respect of the price law, according to market supply and demand, choose the reasonable adjustment parameters.

2) Provide the independent pedestrian trail to connect the streets, the bus stations, and the communities near the railway stations, and separate the bus line; set up the pedestrian crossing facilities as the bar line, the central pedestrian refuge and the public transit signal system;

3) The import and export arrangements of the stations should be conducive to the direction of convergence for passengers to shorten the times for crossing the streets.

4) The rail stations with the park and ride facilities must provide the sufficient bicycle parking space as well as the necessary number of support and shelter facilities; the bicycle parking should be closed to the hubs for import and export in order to facilitate the passenger transfer.

5) The connection of hub way and station needs to meet the needs of not only the convenience, but also the evacuation request, so there is a necessity to have a good guiding identification.

7. Conclusion

The ticket fare pricing of urban rail transit is a vital part in the management and operation of the urban rail transit. Unlike the market pricing of the other kinds of product, the ticket fare pricing of the urban rail transit would influence the individual interest of passenger and the revenue for the rail company, and meanwhile, the social benefit would be reflected in the consideration of the government. Therefore, the ticket fare pricing of urban should be a theoretical system for the balance between the benefit of both passengers and company. Based on the existing researches, the paper considers of the interests of the both sides of the public transport companies and the passengers respectively, and establishes the bi-level programming model for ticket fare pricing of urban rail transit with the solution of the particle swarm optimization algorithm. The example demonstrates the feasibility and effectiveness of the bi-level programming model as well as the particle swarm optimization algorithm. With the analysis of the bi-level programming model and the example, some suggestion is proposed for the operation and ticket pricing fare of the rail transit.

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