A Case Study of a Hydraulic Servo Drive Flexibly Connected to a Boom Manipulator Excited by the Cyclic Impact Force Generated by a Hydraulic Rock Breaker

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ABSTRACT This study deals with the mathematical modeling, dynamic analysis, hybrid control structure, and experimental tests of a hydraulic servo drive (HSD) flexibly connected by a vibration isolator as a spring damping device (SDD) to a boom manipulator excited by the cyclic impact force generated by a rock breaker. Based on the dynamic model of the HSD-SDD system, the frequency ratios of the rigid and flexible connections to the mass load were determined. The HSD-SDD model was saved as a Hammerstein model with unknown parameters. The dynamic linear part of the HSD-SDD model was adopted as an autoregressive model with an exogenous input (ARX) model. The online proportional integral derivative (PID) controller parameter-tuning algorithm was implemented in several steps. The PID controller tuning process occurs in real time, and the optimal setting of the PID controller depends on the critical ultimate gain and period set at the stability limit. A hybrid control structure consisting of a feedback controller and feedforward controller was proposed. A combination of input shaping and feedforward filters is used, thus the badly damped vibrations are more effectively compensated, resulting in better control accuracy of the HSD-SDD system. The goal of optimizing the hybrid control structure is to determine a feedforward filter coefficient that minimizes the objective function. Finally, the global minimum is calculated from the control error based on the measurement of the input and output signals. The highlight of this study is the development of a new hybrid control structure to compensate for badly damped vibrations.

INDEX TERMS Hydraulic servo drive, crusher manipulator, flexible connection, cyclical load force, hybrid control structure.

I. INTRODUCTION
The dynamic properties and precision of hydraulic servo drive (HSD) control depend on the rigidity of the mounting of its components: actuators (barrel and piston rod), control elements (valves), load mass, and equipment. Although the connections between the hydraulic and mechanical components in hydraulic drives are flexible, they are assumed rigid. It is also necessary to consider clearances in moving joints. The assumptions of rigid connections can be applied to vehicles on rough roads, heavy vehicles, aircraft, construction equipment, road vehicles, road-making machines, heavy-duty machines, agricultural machinery, mining machinery, steel industry machines, lifting equipment, transportation equipment, heavy manipulators and robots. The operating conditions of heavy hydraulic machines with changing load mass and external forces cause the formation of vibrations and deformation of the elastic structural elements and connecting devices of hydraulic systems. Vibrations and flow pulsations were also transmitted to the drive and control components of the hydraulic system. In justified cases, the influence of the non-rigid (flexible) mounting of hydraulic elements on the dynamic properties is considered. It is generally assumed that all hydraulic components are rigidly mounted and that the load mass is rigidly connected to the piston rod of the cylinder. Although this assumption is valid for most industrial applications, there are cases in which a non-rigid connection of a load mass should be considered.
The dynamic properties of the hydraulic drive change dramatically if part of the moving mass is more or less flexibly connected to the piston rod and/or if the actuator stands on a vibrating base. This situation frequently arises in hydraulically actuated machines, such as rolling mills where the hydraulic actuator is fully integrated with the mill stand [1]. Helduser et al. presented the use of hydraulic actuators in four basic versions, considering their non-rigid mounting [2]. The research paper reviewed non-rigid or flexible connections of hydraulic actuators. Cetinkunt presented a hydraulic control system with one degree of freedom, a flexible base and load contact [3]. A single-acting hydraulic actuator connected to a non-rigid base and mass load was considered. In this case, using mobile work equipment, a hydraulic actuator is connected to the machine frame, and the machine frame rides on the flexible wheels. The piston rod end was connected to the mass load through a non-rigid tool mechanism. The interaction between the actuator and mass load was modeled as an inertia spring and damper system. Walters presented various dynamic performances of the linear model of a hydraulic actuator with an elastically mounted mass analyzed when the mass is inside or outside the feedback loop [4]. It has been established that the stiffness of a hydraulic actuator significantly affects the dynamic properties of the hydraulic drive. Feng et al. presented the impacts of various factors on the stiffness properties of the hydraulic actuator, including the modulus of the oil volume modulus, air content of the hydraulic oil, axial deformation of the piston rod, expansion of the barrel volume, expansion of the volume of metal pipes and flexible hoses, and deformation of the seal [5]. Tang and Ren presented spring-damper systems that were equivalent to the dynamic model of hydraulic actuators [6]. This actuator was used to lift the boom from a truck that delivered the concrete. Sochacki and Tomski presented actuator dynamic stability tests to determine the conditions of stability loss [7]. Hydraulic cylinder vibration models are generally adopted as beam models [8], [9], in which a vibration damping analysis is performed on beams that model a barrel and piston rod. Zhang and Zhang presented a method for vibration analysis of a hydraulic pump [10]. The dynamic responses of the hydraulic plunger pump were obtained through numerical simulations. Zhang et al. proposed a novel passive hydraulically interconnected suspension system to achieve improved compromise in the handling of mining vehicles [11]. A lumped-mass vehicle model with a coupled mechanical-hydraulic system was developed by applying the free-body diagram method. Robots for heavy loads and/or wide operating ranges with flexible links have become popular research objects for control engineers owing to their sophisticated properties [12]. The approach to active damping control for flexible multilink robots uses a spring damping element, which is a passive energy-dissipation device [13]. Rydberg demonstrated the weakness of elastic connections in the mounting of a cylinder and piston rod [14]. Hydraulic actuators fail because of physical damage or deterioration of their seals. Physical damage to a hydraulic actuator is usually caused by an external source, such as vibrations excited by a rock breaker. Actuator damage can take the form of a bent piston rod or dented barrel, both of which can prevent a full piston stroke. Physical damage to actuator mounting parts can also occur. Because a hydraulic actuator is a single point of failure (SPOF), its reliability is essential in hydraulic actuation systems [15]. For safety reasons, hydraulic components must be mounted in a manner that is resistant to buckling owing to excitation vibration [16]. Wang presented a comprehensive assessment of the reliability of an aircraft hydraulic power supply and control system [17]. Military standards (MIL-STD) require compliance with good practices for the operation and maintenance of hydraulic systems and components [18]. Hydraulic drives are also used in the vibration test stands. The hydraulic vibrating stand contained a pressure pulsation generator created by the rotary piston of a hydraulic valve with a hydraulic motor and hydraulic cylinder for vibration loads [19]. Hydraulic vibration generators that implement periodic variable motion are used not only in test stands (e.g., vehicle body testbed and, pulse apparatus) but also in technological machines, mining machines, and agricultural and construction equipment. Gao et al. conducted experiments have been carried out to describe hydraulic pipeline systems in which the excitation of fluid pressure in a pipeline is driven by a throttle valve, and the base excitation is produced by a shaker driven by a vibration controller [20].

The contribution of this work in relation to the existing solutions consists of tuning the PID controller in real time and using the hybrid control structure to compensate for badly damped vibration. The optimal real-time PID tuning process depends on the critical ultimate gain and the critical ultimate period set at the stability limit. A hybrid control structure consisting of a feedback controller and feedforward controller was proposed. A combination of an input-shaping filter and a feedforward filter is used; thus, badly damped vibrations are more effectively compensated, resulting in better control accuracy of the HSD-SDD system. The goal of optimizing the hybrid control structure is to determine a feedforward filter coefficient that minimizes the objective function. Finally, the global minimum was calculated from the control error based on the measurement of the input and output signals. The simulation and experimental results of this research provide operational guidelines for designers of hydraulic manipulators working with long-term mechanical vibrations. Design difficulties arise when selecting an HSD-SDD system for the hydraulic manipulators of an industrial crusher.

II. FORMULATING THE RESEARCH PROBLEM

Vibrating mechanisms affect technical devices and machines, leading to failure or even damage to structural elements. Excess vibrations are a common problem during the operation of hydraulic drives. These are not due to hydraulic reasons, but to mechanical reasons resulting from excited vibrations. When designing hydraulic systems, the natural vibrations of the hydraulic components and external mechanical vibrations are generally not considered, although this may
be justified. The mechanical connections and construction structures were assumed rigid and the external mass loads were constant. This implies that the effects of mechanical vibrations on the hydraulic components were not considered. However, designers of hydraulic equipment should anticipate the potential problems of long-term mechanical vibrations and should not be guided by simplified and short-term solutions that can lead to problems with the operation and maintenance of hydraulic components and systems.

In jaw crushers, rocks are crushed between the fixed and moving plates (jaws). Jaw crushers can easily crush rocks of any material or size. Crushers use different jaws with smooth adjustment of the grip angle using a special screw. Blake crushers are commonly used in the rock industry. In the cement industry, a crusher shreds limestone rocks into smaller ones, which are then transported to the mill and calcined into quicklime after milling and separation. During surgery, the jaw crusher, rock conveyor, feed gap, and jaw are often blocked by excessively large rock particles. For crushers without a manipulator, rocks that are too large must be crushed by a worker using a manual hammer under hazardous conditions. This contributes to the downtime of the jaw crusher and reduces productivity. The hydraulic manipulator has a working space that allows the crushing of excessively large stones in the jaw crusher, on the conveyor, in the feed gap, or between the jaws. Hydraulic manipulators contribute to the reduction in downtime caused by material blockage in the jaw crusher. They also significantly improved the safety of crusher operations. Mechanical vibrations from the rock breaker to the crusher manipulator and the components of the hydraulic system were transmitted. It contributes significantly to the troubleshooting and maintenance of hydraulic components and systems. The designer of the crusher manipulator and hydraulic system should anticipate the operational problems related to long-term mechanical vibrations. However, the most common practice is to determine simple design solutions that provide only short-term operational benefits. Rock breakers mounted on hydraulic manipulators break large rocks in the jaw crusher space. Crusher users have reported problems with the operation of hydraulic manipulators. This is related to the mechanical connecting elements and hydraulic components. The design of the hydraulic manipulators used in rock crushers is difficult to modify. It was proposed to constructors of hydraulic manipulators to introduce an elastic connection with damping as a vibro-isolator to reduce the transmission of vibrations from the excitation source (rock breaker) to the HSD. As a result of the cooperation of the authors with the designers of hydraulic manipulators and jaw crusher users in the production of rock aggregates in a cement plant, it was necessary to evaluate the dynamic properties and control system of the HSD that is flexible with the load mass induced by the cyclic force [21].

Fig. 1 shows the model of a Blake jaw crusher with a rock conveyor and mounted hydraulic boom manipulator. A design solution for a hydraulic manipulator for jaw crushers was proposed to crush rocks with an average weight up to 300 kg.

![Figure 1](image_url)

**FIGURE 1.** Hydraulic boom manipulator mounted on a jaw crusher, a) jaw crusher model: 1 – Blake jaw crusher, 2 – rock conveyor, 3 – hydraulic boom manipulator, b) design solution of a hydraulic boom manipulator: 1 – hydraulic rock breaker, 2 – telescopic boom, 3 – clevis bracket, 4 – inner boom actuator, 5 – 4/3 proportional directional control valve, 6 – integrated control electronics (ICE), 7 – center trunnion, 8 – rotation cylinder, 9 – lifting cylinder, SDD – spring damping device.

The hydraulic manipulator has three degrees of freedom (3DoF), and its workspace includes the rock conveyor, feed gap, and jaws. The double telescopic boom consists of a mother boom and internal boom. The mother boom was lifted and rotated by hydraulic cylinders. The inner boom was extended and retracted by the hydraulic cylinder of an HSD, which was the subject of the tests. In the proposed design solution, the hydraulic cylinder was rigidly mounted on the mother boom, and the piston rod was flexibly connected to the inner boom via the SDD vibration isolator. The SDD is a spring device with viscoelastic damping elements that enables effective damping of vibrations under high dynamic loads in hydraulic rock breakers. The SDD parameters were adopted from the following ranges: damping coefficient $d_m = 10 - 25 \text{ kN/s/m}$ and spring stiffness coefficient $k_m = 10 - 100 \text{ kN/m}$. The hydraulic rock breaker was mounted on an internal telescopic boom that was extended by a hydraulic cylinder to a length of 1.2 m. The selected hydraulic rock breaker (SC25 Montabert) had the following parameters: weight of 225 kg, working pressure of 120 bar, flow range of 25-50 L/min, frequency of 1,310 bpm, and tool diameter of 0.055 m. A quantitative estimate of the impact
loads on a hydraulic rock breaker is necessary. The striking forces and vibration frequencies were determined via impact experiments. The use of a hydraulic rock breaker increases the crushing capacity on a Blake jaw crushe by an average of 20%.

Electrohydraulic servo systems are normally integrated units in an assembly, which is specified as an HSD [22]. The HSD consists of a differential hydraulic actuator (cylinder), a proportional directional control valve 4/3 (4-way, 3-position) with electrical position feedback, and integrated control electronics (ICE) called on-board electronics (OBE). The ICE compares the specified command value with the actual position value of the main-valve control spool. Placing a proportional directional control valve with a position controller directly on the hydraulic actuator reduced the volume of oil and improved the dynamic properties of the HSD. When the control valve is to be operated, a positive or negative 0–10 V command signal is used. A positive voltage shifts the spool to position A and a negative voltage shifts the spool to position B. The hydraulic actuator parameters are the diameter of the piston \( D = 0.05 \) m, the diameter of the piston rod \( d = 0.028 \) m, area ratio \( \alpha = 0.69 \), stroke \( h = 1.25 \) m, and working pressure \( p = 25 \) MPa. Technical data of the 4WRSE proportional control valve: nominal volumetric flow rate \( q_{vA} = 35 \) L/min \( (0.583 \times 10^{-3} \) m\(^3\)/s\(^{-1}\) \) at a pressure difference of \( \Delta p = 1 \) MPa, power supply of 24 VDC, command value \( u = \pm 10 \) V, and mineral oil HLP46 (kinematic viscosity \( \nu = 0.46 \times 10^{-4} \) m\(^2\)/s \) at temperature \( T = 313.15 \) K. The hydraulic actuator contained an internal magnetostriuctive linear-position transducer.

### III. DYNAMIC MODEL OF THE HSD–SDD SYSTEM

The complex two degrees of freedom (2DoF) nonlinear dynamic model of the HSD–SDD system was described by differential equations in a semiexplicit manner. Figure 2 shows the computational model of an HSD flexibly connected to the load mass through an SDD.

![Figure 2. A computational model of an HSD flexibly connected to the load mass via an SDD.](image)

The following state coordinates were assumed in the HSD dynamic model:

- displacement \( x \), velocity \( v_x = dx/dt \), and acceleration \( a_x = d^2x/dt^2 \) of the mass load
- displacement \( y \), velocity \( v_y = dy/dt \), and acceleration \( a_y = d^2y/dt^2 \) of the actuator piston
- shift \( z \) and velocity \( v_z = dz/dt \) of the control valve spool, the spool shift \( z \) is related to the command signal \( u \) by the first-order inertial PT1 element

\[
T_z \ddot{z} + z = K_z u \Rightarrow \ddot{z} = -\frac{1}{T_z} z + \frac{K_z}{T_z} u, \quad (1)
\]

where \( T_z \) is the time constant and \( K_z \) is the gain factor of the proportional directional valve [23].

- pressure \( p \) and pressure impulse (pressure–time derivative) \( p_i = dp/dt \) in actuator chambers.

Because a dynamic system with two degrees of freedom was considered, the coordinate differences \( \Delta x = x - y \), \( \Delta y = y - x \) for the dynamic equations were introduced. Finally, the 2DoF dynamic model for an HSD flexibly connected to a mass load is expressed as follows:

\[
\begin{align*}
\ddot{x} &= \frac{1}{m_l} [-d_m \Delta \dot{x} - k_m \Delta x - F_C + F(t)] \\
\ddot{y} &= \frac{1}{m_y} [-d_m \Delta \dot{y} - k_m \Delta y - F_f + F_L] \\
\dot{z} &= -\frac{1}{T_z} z + \frac{K_z}{T_z} u \\
\dot{p}_1 &= \frac{1}{C_1} [-q_{v1} - A_1 \Delta \dot{y} - k_l (p_1 - p_2)] \\
\dot{p}_2 &= \frac{1}{C_2} [-q_{v2} + A_2 \Delta \dot{y} + k_l (p_1 - p_2) - k_{le} p_2],
\end{align*}
\]

where \( m_l \) is the load mass; \( d_m \) is the damping coefficient; \( k_m \) is the spring stiffness coefficient; \( F_C \) is the Coulomb friction force; \( m_y \) is the moving mass of the actuator; \( A_1 \) and \( A_2 \) are the effective areas of the actuator piston; \( k_l \) is the internal leakage flow coefficient between the cylinder chambers; and \( k_{le} \) is the internal leakage flow coefficient in the cylinder piston rod chamber.

\( F(t) \) is the excitation force limited to the harmonic vibration

\[
F(t) = F_0 + F_1 \sin(2 \pi f_e t), \quad (3)
\]

where \( F_0 \) is a constant component of the force, \( F_1 \) is the impact force, and \( f_e \) is the excitation frequency.

\( F_L \) is the load force of the hydraulic actuator

\[
F_L = A_1 p_1 - A_2 p_2 = A_1 (p_1 - \alpha p_2) = A_1 p_L, \quad (4)
\]

where \( \alpha \) is the area ratio, \( \alpha = A_1/A_2 \),

\[
p_L = \frac{F_L}{A_1} = p_1 - \alpha p_2. \quad (5)
\]

\( F_f \) is the friction force as a function of the velocity of the piston

\[
F_f = d_v \dot{y} + F_{st} \exp(-\sigma |\dot{y}|) \text{sign}(\dot{y}(t)) + F_S, \quad (6)
\]

where \( d_v \) is the viscous friction coefficient, \( F_{st} \) is the static friction force, and \( \sigma \) is a variable coefficient.

The total friction force \( F_S \) in the hydraulic actuator seal is the sum of the friction force \( F_f \) due to the squeeze of the seal
and the friction force $F_d$ due to the hydraulic pressure on the seal [24]:

$$F_S = F_i + F_a = f_i L + f_a A,$$  \(\text{(7)}\)

where $f_i$ is the friction force per contact length of the seal; $L$ is the circumference of the piston groove seal ($L = \pi D_o$); $D_o$ is the piston groove outer diameter; $f_a$ is the friction force per projected area of the seal; $A$ is the projected area of the seal in the piston groove ($A = \pi l^2$); and $d_i$ is the piston groove.

$C_1$ and $C_2$ are the hydraulic capacities of the actuator chambers at initial position $y_0$ of the actuator piston

$$C_1 = \frac{V_1}{K} = \frac{V_{10} + A_1 (y_0 + y)}{K},$$  \(\text{(8)}\)

$$C_2 = \frac{V_2}{K} = \frac{V_{20} + A_2 (h - (y_0 + y))}{K},$$  \(\text{(9)}\)

where $K$ is the bulk modulus; $V_{10}$ and $V_{20}$ are the dead volumes of the cylinder chamber; and $h$ is the stroke of the actuator piston.

$q_{v1}$ and $q_{v2}$ are the flow rates through the proportional valve of the proportional directional servo valve $4/3$ ($A + A$, spool overlap $\pm$3%)

$$q_{v1} = K_q (z_o + z) \sqrt{\Delta p_1} \text{sign} (\Delta p_1),$$  \(\text{(10)}\)

$$\Delta p_1 = \begin{cases} p_s - p_1, & y \geq 0, \\ p_1 - p_T, & y < 0, \end{cases}$$  \(\text{(11)}\)

$$q_{v2} = K_q (z_o - z) \sqrt{\Delta p_2} \text{sign} (\Delta p_2),$$  \(\text{(12)}\)

$$\Delta p_2 = \begin{cases} p_1 - p_2, & y < 0, \\ p_2 - p_T, & y \geq 0, \end{cases}$$  \(\text{(13)}\)

where $p_s$ is the supply pressure, $p_T$ is the tank pressure, $z_o$ is the spool of the underlap valve, and $K_q$ is the flow coefficient of the control valve

$$K_q = \frac{q_{vn}}{z_n \sqrt{\Delta p_n}},$$  \(\text{(14)}\)

where $q_{vn}$ is the nominal flow rate at a pressure drop $\Delta p_n = 1$ MPa and $z_n$ is the nominal shift of the servo-valve spool.

The inclusion of the SDD increases the degree of freedom of the dynamic HSD model, which complicates its simulation and analysis. The proposed nonlinear dynamic model was used to evaluate the dynamic properties of an HSD for a rigidly and elastically connected load mass excited by an external cyclic force. The dynamic model, written in the form of explicit differential equations, is used for the numerical calculations performed by the user in a simulation software such as MATLAB–Simulink. Dynamic model (2) was expressed as a nonlinear state-space model (seventh order differential equation).

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{m_L} \left[ -d_m (x_2 - x_4) - k_m (x_1 - x_3) \right] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{1}{m_L} \left[ -d_m (x_4 - x_2) - d_s x_4 - k_m (x_3 - x_1) \right] + A_1 p_6 - a A_1 p_7 + \frac{1}{m_L} F_f (x_4) \\
\dot{x}_5 &= -\frac{1}{T_z} x_5 + \frac{K_v}{T_z} u \\
\dot{x}_6 &= \frac{1}{C_1} \left[ -A_1 (x_4 - x_2) - k_l (x_6 - x_7) \right] + \frac{1}{C_1} q_{v1} (x_5, x_6, x_7) \\
\dot{x}_7 &= \frac{1}{C_2} \left[ A_2 (x_4 - x_2) + k_l (x_6 - x_7) - k_{le} x_7 \right] - \frac{1}{C_2} q_{v2} (x_5, x_6, x_7)
\end{aligned}$$  \(\text{(15)}\)

where: $x_1 = x$, $x_2 = v_x$, $x_3 = y$, $x_4 = v_y$, $x_5 = z$, $x_6 = p_1$, $x_7 = p_2$.

The dynamic model (15) is a complex nonlinear dynamic system that includes the insensitivity and saturation of the control valve, the square pressure drop, and the friction force based on the Striebeck model. Most proportional control valves have undesirably high nonlinearity, asymmetric flow gain, and hysteresis. Nonlinear external disturbances such as the Coulomb friction force and external cyclic force significantly influence the dynamic responses of the HSD-SDD system. The nonlinear behavior of the HSD-SDD system may vary with changes in the system load and operating

**TABLE 1. Main constant parameters of the HSD model.**

| Symbol | Value | Unit | Quantity |
|--------|-------|------|----------|
| $p_s$  | 24    | MPa  | The supply pressure |
| $A_1$  | 1.96 $10^3$ | m$^2$ | Area of the actuator piston in the inlet chamber |
| $A_2$  | 1.3 $10^4$ | m$^2$ | Area of the actuator piston in the outlet chamber |
| $V_{10}$ | 0.1 $10^3$ | m$^3$ | Initial volume in the inlet chamber |
| $V_{20}$ | 0.1 $10^3$ | m$^3$ | Initial volume in the outlet chamber |
| $h$    | 1.2   | m    | Stroke of the cylinder piston |
| $m_o$  | 2500  | kg   | Mass of the cylinder piston |
| $d_i$  | 5     | m    | Diameter of the cylinder piston |
| $C_z$  | 0.56  | m    | Flow coefficient |
| $d_s$  | 6 $10^3$ | m    | Diameter of the valve spool |
| $T_z$  | 0.01  | s    | The time constant of a control valve |
| $\rho$ | 860   | kg   | The density of the oil |
| $K$    | 1600  | MPa  | The bulk modulus of oil |
| $F_f$  | 500   | N    | Coulomb friction |
| $F_s$  | 200   | N    | Static friction force |
| $m_e$  | 250   | Kg   | Load mass |
| $F_L$  | 2500  | N    | External load force |
| $d_a$  | 10000 | N/s/m | Damping coefficient of SDD |
| $k_o$  | 10 $10^4$ | N/m | Spring stiffness of SDD |
conditions. The values of the main constant parameters of the HSD-SDD dynamic model are listed in Table 1.

IV. SELECTION OF THE HSD-SDD SYSTEM

The first step in selecting an HSD-SDD system is to determine the frequency ratios for the case of a rigid and flexible load-mass connection.

A. RIGIDLY CONNECTED LOAD MASS

The excitation frequency \( f_e \) is derived from the rock breaker frequency

\[
 f_e = \frac{f_{rb}}{60},
\]

where \( f_{rb} \) is the frequency of the rock breaker in beats per minute (bpm).

This equation follows

\[
 f_e = \frac{1,310}{60} = 21.8 \text{ Hz}.
\]

The natural frequency \( f_y \) of the actuator excited by the cyclic load is

\[
 f_y = \frac{1}{2\pi} \sqrt{\frac{k_y}{m_y + m_L}}.
\]

The hydraulic oil in the actuator chamber is compressible and thus acts as a stiff spring. The spring stiffness \( k_y(y) \) of the asymmetric actuator resulting from the oil volume elasticity in the series-connected cylinder chambers is

\[
 k_y(y) = \left( \frac{A_1^2}{C_1} + \frac{A_2^2}{C_2} \right) = K \left( \frac{A_1^2}{V_1} + \frac{A_2^2}{V_2} \right)
 = K \left( \frac{A_1^2}{V_{10} + A_1 y} + \frac{A_2^2}{V_{20} + A_2 (h - y)} \right).
\]

The frequency ratio \( f_r \) for a rigid connection is as follows

\[
 f_r = \frac{f_e}{f_y}.
\]

Figure 3 shows the natural frequency \( f_y \) of the asymmetric hydraulic actuator versus the displacement of piston \( y \) over stroke length \( h \), where \( f_e \) is defined as the ratio of the excitation frequency \( f_e \) to the natural frequency \( f_y \) of a hydraulic actuator.

The lowest value of the natural frequency of the hydraulic actuator is \( f_y \approx 30 \text{ Hz} \). The relative frequency \( f_r \) versus piston displacement \( y \) for the asymmetric hydraulic actuator excited by the cyclic load is shown in Fig. 4. A rock breaker is selected to avoid resonant vibrations when the excitation frequency \( f_e \) is close to the natural frequency \( f_y \) of the actuator (\( f_r \approx 1 \)).

The dynamic properties of the HSD depend on the ratio of the actuator frequency \( f_y \) to the proportional servo-valve frequency \( f_z \)

\[
 f_d = \frac{f_y}{f_z},
\]

In this study, a 4WRES proportional valve (Bosch Rexroth) was used to control the direction and size of the volumetric flow. It is a high-response valve with a frequency of \( f_z = 100 \text{ Hz} \) and a phase shift of 90°. For \( f_d = 0.3-0.6 \), the best results are obtained for the state-space controller. This type of controller is used in machines that require highly dynamic operation [25].

B. FLEXIBLE-CONNECTED LOAD MASS

The natural frequency \( f_y \) of the actuator

\[
 f_y = \frac{1}{2\pi} \sqrt{\frac{k_y}{m_y}}.
\]

The natural frequency \( f_m \) of the mechanical components of the SDD

\[
 f_m = \frac{1}{2\pi} \sqrt{\frac{k_m}{m_L}}.
\]

The frequency ratios \( f_{sym} \) and \( f_{rem} \) for a flexible connection are as follows

\[
 f_{sym} = \frac{f_y}{f_m},
\]

\[
 f_{rem} = \frac{f_e}{f_m}.
\]
The damping ratio $\xi_y$ of the actuator

$$\xi_y = \frac{d_y}{2 \sqrt{k_y m_y}}. \quad (25)$$

The damping ratio $\xi_m$ of mechanical components of the SDD

$$\xi_m = \frac{d_m}{2 \sqrt{k_m m_L}}. \quad (26)$$

The relative damping $r_{dny}$ for a flexible connection is as follows

$$r_{dny} = \frac{\xi_m}{\xi_y} = \frac{d_m}{d_y} \sqrt{\frac{k_y}{k_m} \sqrt{\frac{m_L}{m_L}}}. \quad (27)$$

For the assumed setting of the piston stroke, the $y_{set}$ frequency ratios were $f_{fym} = 95.01, f_{rem} = 9.7$, and the relative damping was $r_{dny} = 11.4$.

V. PULSATION OF PRESSURE IN A SHORT HYDRAULIC LINE

The variation in the flow caused by the actuator excitation process leads to flow fluctuations in the hydraulic pipeline. Excess pressure pulsation amplitudes can cause mechanical vibrations and fatigue failures in hydraulic pipelines and connecting elements. The influence of the pressure pulsation in the cylinder on the flow fluctuation was modeled in a short hydraulic line with lumped parameters.

To set the $y_{set}$ of the piston stroke, a mechanical model of an actuator excited by cyclic force was developed.

$$\begin{align*}
\ddot{y} &= \frac{1}{m_y + m_L} \left[ -d_y \dot{y} - (A_1 p_1 - A_2 p_2) + F(t) \right] \\
\dot{p}_1 &= \frac{1}{C_{1set}} A_1 \dot{y} = \left( \frac{K}{V_{10} + A_1 y_{set}} \right) A_1 \dot{y} \\
\dot{p}_2 &= -\frac{1}{C_{2set}} A_2 \dot{y} = -\left( \frac{K}{V_{20} + A_2 (h-y_{set})} \right) A_2 \dot{y}.
\end{align*} \quad (28)$$

For the assumed setting of the piston stroke $y_{set}$, the displacement $y(t)$ and the speed $v_y(t)$ of an actuator rigidly connected to a load mass $m_L = 250 \text{ kg}$ to $r_{y_{set}} = 0.6 \text{ m}$ and the excitation force $F(t)$ at the frequency $f_x = 21.8 \text{ Hz}$ are shown in Fig. 5. For such a piston stroke setting, the pulsation of pressures $p_1(t)$ and $p_2(t)$ in an actuator rigidly connected to a load mass $m_L = 250 \text{ kg}$ for $y_{set} = 0.6 \text{ m}$ and the excitation force $F(t)$ at the frequency $f_x = 21.8 \text{ Hz}$ are shown in Fig. 6. In the analyzed case, the frequency of pressure pulsation in the actuator is $f_{pl} = 1/T = 10 \text{ Hz}$, where $T$ is the time period of pressure pulsation, $T \approx 0.1 \text{ s}$.

Hydraulic short lines can be modeled using lumped circuit elements in the form of resistance, inductance, and capacitance (RLC). A model of a short hydraulic line connecting the actuator and control valve was considered (Fig. 7).

The one-dimensional $R_L L_C h$ model of a short cylindrical hydraulic line with lumped parameters, as shown schematically in Fig. 7, can be expressed using the following differential equation:

$$\frac{1}{C_h} \int q_h \, dt + L_h \frac{dq_h}{dt} + R_h q_h = p_1 - p_h, \quad (29)$$

where $q_h$ is the pipe flow rate; $p_1$ is the pressure at the inlet of the pipe; $p_h$ is the pressure at the outlet of the pipe; $C_h$ is the hydraulic line capacitance, $C_h = V/K$; $L_h$ is the hydraulic line inductance, $L_h = \rho a / \pi^2$; $R_h$ is the hydraulic line resistance, $R_h = (8 \pi l n) / A^2$; $a$ is the velocity of the sound, $a = \sqrt{k / \rho}$; $V$ is the volume of the cylinder chamber.
and pipeline; \( A \) is the pipe cross-section, \( A = \pi r^2 \); \( r \) is the pipe inner diameter; \( l \) is the pipe length; \( K \) is the bulk modulus; \( \rho \) and is the density of the oil.

There is a valve at the end of a short hydraulic line for which the flow rate is \( q = \frac{\sqrt{\rho}}{R} \),
where \( R \) is the flow resistance through the valve at the nominal flow rate \( q_n \) and nominal pressure drop \( \Delta p_n \), and \( R_v = \frac{\Delta p_n}{q_n} \).

Eq. (29) is written as a transition function in the Laplace domain solution

\[
G_p(s) = \frac{q_h(s)}{p_1(s)} = \frac{C_h s}{C_h L_h s^2 + C_h R_h s + 1} = \frac{1}{\omega_h^2 s^2 + 2 \zeta \omega_h s + 1},
\]

where \( \omega_h \) is the natural frequency of the hydraulic line
\[
\omega_h = \sqrt{\frac{1}{v_c L_h}},
\]
\( \zeta \) is the damping coefficient of hydraulic line \( \zeta = \frac{1}{\pi} C_h R_h \omega_h \).

The frequency-domain transfer function was defined as the ratio of the output pressure amplitude to the input pressure amplitude

\[
G_p(j\omega) = \frac{p_h(j\omega)}{p_1(j\omega)} = \frac{R_v C_h j\omega}{1 - \omega^2 + 2 \omega j \omega} = \frac{1}{\alpha [\beta + j(\omega - \omega_h)]},
\]

where \( \alpha \) and \( \beta \) are the constant coefficients of the short hydraulic line, \( \alpha = \frac{1}{R_v C_h \omega_h} \), and \( \beta = 2 \zeta \), respectively.

Ultimately, the amplitude of the transition function (31) was determined

\[
|G_p(j\omega)| = \left| \frac{p_h(j\omega)}{p_1(j\omega)} \right| = \frac{1}{\alpha \sqrt{\beta^2 + (\omega_h - \omega_h)^2}}.
\]

Basic parameters of the short hydraulic pipeline model:
\( V = 0.1 \text{ m}^3 \), \( r = 0.005 \text{ m} \), \( l = 0.2 \text{ m} \). However, for the short pipeline, the frequency \( f_h = \omega_h / 2 \pi = 0.5 \text{ Hz} \) was calculated. For these parameters, Fig. 8 shows the resonance characteristics of the short hydraulic line for different flow rates in the control, and Fig. 9 shows the frequency response of the short hydraulic line.

The presented analysis shows that the pressure pulsations in the actuator are transferred to the flow fluctuations in a short pipeline. The resonance characteristics in Fig. 8 show the maximum vibration amplitudes for a pressure pulsation frequency that is equal to the natural frequency of the short hydraulic line \( f_h = 1 \). Vibration resonance must be avoided because the maximum pressure pulsations that arise can seriously damage the hydraulic system and significantly shorten its life. In the case of near-resonance vibrations, a pulsation damper should be used in the hydraulic lines.

![FIGURE 8. Resonance characteristics of a short hydraulic line for different flow rates in the control: 1 – \( q_n \), 2 – \( q_n/2 \), 3 – \( q_n/4 \).](image)

![FIGURE 9. The frequency response of a short hydraulic line.](image)

**VI. SIMULATION RESULTS**

The industrial use of the HSD was considered, in which the cylinder was rigidly mounted on the main boom and the piston rod was flexibly connected via an SDD to the inner boom of a hydraulic crusher manipulator. The purpose of the simulation tests is to assess the dynamic properties of an HSD that is rigidly and flexibly connected to a load mass excited by a cyclic force. The step response of an HSD rigidly and flexibly connected to a load mass at a constant force or the cyclic force generated by the rock breaker was compared. The HSD-SDD simulation model in MATLAB Simulink using Simscape Fluids™ tools is illustrated in Fig. 10.

**A. THE INITIAL-STATE PARAMETERS**

The initial pressures in the cylinder chamber were established in the steady state of HSD for \( x = 0, v_x = 0, y = 0, v_y = 0, z = 0, \) and \( v_z = 0 \). For the steady state, the flow rate through the proportional directional spool valve can be written as

\[
\begin{align*}
q_1 & = A \widetilde{z} = K_q(\widetilde{z} - \sqrt{\rho}) - P_{10} \\
q_2 & = \alpha A \widetilde{z} = K_{q2}(\widetilde{z} - \sqrt{\rho}/2) \\
q_L & = P_{10} - \alpha P_{20}
\end{align*}
\]

[33]
For $\Delta p_2 = p_s - p_{10}$, $\Delta p_2 = p_{20}$, $p_T = 0$ and using $m = K_{q2}/K_{q1} = 0.5$ and $\alpha = q_2/q_1 = 0.68$ the initial pressures $p_{10}$ and $p_{20}$ in the cylinder were determined as follows

$$p_{10} = \frac{\alpha^3 p_0 + m^2 p_L}{m^2 + \alpha^3}, \quad p_{20} = \frac{\alpha^2 (p_0 - p_L)}{m^2 + \alpha^3}. \tag{34}$$

**B. DYNAMIC RESPONSE**

The dynamic responses of the displacements $y(t)$ and the speeds $v_y(t)$ of an HSD actuator rigidly connected to a load mass $m_L = 250$ kg for $y_{set} = 0.6$ m, a constant force $F_0 = 2500$N, and the excitation force $F(t)$ at frequency $f_e = 21.8$ Hz are compared in Fig. 11.

The dynamic responses of the load pressure $p_L(t)$ in an HSD actuator rigidly connected to a load mass $m_L = 250$ kg for $y_{set} = 0.6$m, a constant force $F_0 = 2500$ N (blue line), and the excitation force $F(t)$ (red line) at frequency $f_e = 21.8$ Hz are compared in Fig. 12.

The opening characteristics of the proportional directional valve for the time constant $T_z = 0.01$ s (100 Hz) and the factor gain $K_z = 1.66 \times 10^{-4}$ m/V are shown in Fig. 13.

Figs. 11, 12, and 13 show that, in hydraulic circuits, the dynamic characteristics of the hydraulic cylinder and control valve are of great importance. Upon sudden switching of the valve and cutoff of the flow to the cylinder chamber, rapid hydraulic pulsation develops, causing significant noise. In the case of fast movements during the vibration of the hydraulic cylinder piston, the position output signals from the magnetostrictive transducer make accurate measurements difficult. This affects the switching disturbance of the control valve. Forced vibrations intensify disturbances in the operation and control of the hydraulic systems.
The dynamic responses of displacement $y(t)$ and speed $v_y(t)$ of an HSD actuator flexibly connected by SDD to a load mass $m_L = 250$ kg for $y_{set} = 0.6$ m, constant force $F_0 = 2500$ N and excitation force $F(t)$ at frequency $f_e = 21.8$ Hz are compared in Fig. 14.

The dynamic responses of the load pressure $p_L(t)$ in an HSD actuator flexibly connected by SDD to a load mass $m_L = 250$ kg for $y_{set} = 0.6$ m, constant force $F_0 = 2500$ N and excitation force $F(t)$ at frequency $f_e = 21.8$ Hz are compared in Fig. 15.

The dynamic responses of the displacements $y(t)$ and the speeds $v_y(t)$ of an HSD actuator rigidly and flexibly connected by SDD to a load mass $m_L = 250$ kg for $y_{set} = 0.6$ m and the excitation force $F(t)$ at frequency $f_e = 21.8$ Hz are compared in Fig. 16.

A comparison of the HSD step response (Figs. 14-17) shows a difference in the dynamic properties for a rigid and flexible connection between a load mass at a constant force and the cyclic force generated by the rock breaker. The simulation results were used to select the SDD and vibration exciter for the test stand.
Knowledge of the influence of mechanical vibrations on the dynamic behavior of hydraulic systems is of key importance in the engineering design of products such as crusher manipulators, as well as in the selection of hydraulic components with the required dynamic properties. When designing, it must be considered that vibration phenomena have an impact on the increase in noise in hydraulic circuits and the reduction in the service life of hydraulic components. When designing, it must be considered that vibration phenomena have an impact on the increase in noise in hydraulic circuits and the reduction in the service life of hydraulic components. In the case of fast movements during the vibration of the hydraulic cylinder piston, the position output signals from the magnetostrictive transducer make accurate measurements difficult. This causes disturbances in the control-valve-switching time. Forced vibrations intensify the disturbances and natural vibrations of the cylinder and the hydraulic valve. Vibrations in a hydraulic system can cause failure or destruction of hydraulic components. Owing to the use of the SDD, the amplitude and frequency of vibrations in the HSD decreased. The sensitivity of the HSD-SDD system to deformations of the manipulator structure and actuator mounting was not analyzed. The simulation of the HSD-SDD includes an analysis of the sensitivity of hydraulic control systems to changes in the internal and external parameters. A sensitivity analysis was performed at the design stage during the simulation tests using the sensitivity function in the Laplace domain. The HSD-SDD is a dynamic system, the output signals of which show periodic vibrations described by a harmonic function. For such a system, the logarithmic sensitivity function is the most appropriate measure of changes in the amplitude characteristics with disturbed parameters.

VII. EXPERIMENTAL TEST STAND

The experimental test aims to select an HSD control system that is flexibly connected to the load mass, ensuring accurate extension of the internal telescopic boom at the cyclic excitation force (impact force). A view of the HSD test stand is shown in Fig. 18.

The mass load of the hydraulic actuator as the technological payload was achieved using weights installed on the linear guide support. A hydraulic loading actuator is used to generate a constant force on the actuator. An electrohydraulic vibration exciter with two rotary valves was used to generate an excitation force. The displacements of the hydraulic actuator (CSM1/MT4/50/28/1200) were measured using a Novostrictive® magnetostrictive linear position sensor TMI 0250 with a CANopen interface. The 4/3 proportional directional valve (4WRSE6V1-35-3X/G24K0/A1) was used for position and speed control of the hydraulic actuator. The technological load on the test stand was measured using a force sensor [25]. The supply pressure ($p_{\text{max}} = 24$ MPa) was set using a proportional pressure-relieving valve (DBETX–1X/250G24–25NZ4M). The test stand also included a computer system for a superior control system equipped with MATLAB/Simulink xPC Target software. The PC had C/A and A/C converter cards from the PCI-type DAS1602/16 Measurement Computing Corporation. A card with an actuator position converter is used to create the measurement system. In the experimental test, the measured signals were taken from the sensors as continuous signals, and then after discretization of the signal, the signal was recorded digitally.

VIII. CONTROL SYSTEM DESIGN

A. PARAMETRIC IDENTIFICATION OF THE HAMMERSTEIN MODEL OF THE HSD-SDD SYSTEM

The HSD-SDD model is considered to be a discrete-time single-input and single-output (SISO) nonlinear system. The identification of unknown parameters in the nonlinear dynamic model of an HSD-SDD is difficult because of the need to measure the internal physical state. When a system’s HSD-SDD output depends nonlinearly on its input, the I/O relationship can be decomposed into two or more series-connected components. In this case, the Hammerstein model, which shows a series connection of static nonlinear parts with a dynamic linear part, was used. A block diagram of the parametric identification of the Hammerstein model (H-model) of the HSD-SDD system is shown in Fig. 19.
The HSD-SDD dynamic model (15) is written in the form of a nonlinear state-space model:

\[
\begin{align*}
\dot{x}(t) &= A x(t) + N [x(t), u(t)] \\
y(t) &= C x(t)
\end{align*}
\]

(35)

where \( \mathcal{A}(x(t)) \) is the state vector, \( \mathcal{B}(u(t)) \) is the input vector, \( \mathcal{C}(y(t)) \) is the output vector, \( \mathcal{A} \in \mathbb{R}^{n+n} \) is a linear matrix, \( \mathcal{B} \in \mathbb{R}^{n+m} \) is a nonlinear matrix, and \( \mathcal{C} \in \mathbb{R}^{m+n} \) is a linear matrix.

The HSD model (35) was transformed into a discrete state space-time model with the separation of a linear transfer function \( G(z^{-1}) \) and a nonlinear function \( f(u(k)) \):

\[
\begin{align*}
x(k) &= f(u(k)) \\
y(k) &= G(z^{-1}) x(k) + \zeta(k)
\end{align*}
\]

(36)

with constraints

\[ u_{\text{min}} \leq u(k) \leq u_{\text{max}}, \quad y_{\text{min}} \leq y(k) \leq y_{\text{max}}, \]

where \( u_{\text{min}} \) and \( u_{\text{max}} \) are vectors of the lower bounds, \( u_{\text{max}} \) and \( y_{\text{max}} \) are vectors of the upper bounds, \( \zeta(k) \) is the additive noise, and \( z^{-1} \) is the discrete-time-domain backward-shift operator.

The HSD-SDD model (36) was saved as the Hammerstein model with unknown parameters consisting of two parts: a static nonlinear part and a dynamic linear part, and \( u(k), x(k), \zeta(k), \) and \( y(k) \) are the identification input, intermediate state, noise sequence, and output of the system [26, 27].

The static nonlinear subsystem is approximated by the set polynomial degree, and is described by the nonlinear function of the \( n \)th order that binds the nonlinear output \( x(k) \) to the input \( u(k) \):

\[
x(k) = \sum_{k=1}^{m} f_k u^k(k) = f_1 u(1) + f_2 u^2(2) + \ldots + f_m u^m(m),
\]

(37)

where \( f_k \) is the nonlinearity parameter and \( m \) is the order of the polynomial.

In the dynamic linear part, the autoregressive model with exogenous inputs (ARX) model was applied, and the relation of input \( x(k) \) to output \( y(k) \) as follows [28]:

\[
y(k) = G(z^{-1}) x(k) + \zeta(k) = z^{-d} \frac{B(z^{-1})}{A(z^{-1})} x(k) + \frac{w(k)}{A(z^{-1})},
\]

(38)

where \( d \) is the discrete delay of the dynamic linear part, \( z^{-1} \) is the backward shift operator in the discrete-time domain \( A(z^{-1}) \), and \( B(z^{-1}) \) are the polynomials of an identified model of the controlled HSD, defined by

\[
\begin{align*}
A(z^{-1}) &= 1 + \sum_{i=1}^{na} a_i z^{-i} \\
&= 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{na} z^{-na} \\
B(z^{-1}) &= \sum_{i=1}^{nb} b_i z^{-i} \\
&= b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{nb} z^{-nb},
\end{align*}
\]

(39)

\( w(k) \) is the noise sequence

\[
w(k) = A(z^{-1}) \zeta(k).
\]

(40)

When designing an ARX model, Equation (38) can be written in the following regression form:

\[
y(k) = -\sum_{i=1}^{na} a_i y(k-i) + \sum_{i=1}^{nb} b_i x(k-i) + w(k).
\]

(41)

The identification of unknown parameters in the HSD-SDD dynamic model is difficult owing to the need to measure the internal physical state. The estimation parameters \( a_i \) and \( b_i \) for the ARX model are shown in Fig. 20.

The measured output of the HSD-SDD plant is given by the general expression:

\[
y_m(k) = y(k) + e(k),
\]

(42)

where \( e(k) \) is the output noise disturbance. Substituting (37) into (41) and then into (42), we obtain the following expression:

\[
y_m(k) = -\sum_{i=1}^{na} a_i y(k-i) \\
+ \sum_{i=1}^{nb} b_i \sum_{k=1}^{m} f_k u^k(k-i) + w(k) + e(k).
\]

(43)

Eq. (43) can be written in the following least squares (LS) format

\[
y_m(k) = \theta^T(k) \phi + w(k) + e(k),
\]

(44)

where \( \theta^T \) is the autoregressive variable

\[
\theta^T(k) = [-y(k-1), \ldots, -y(k-na)],
\]

\[
u(k-1), \ldots, u^m(k-na)],
\]

(45)
where $\varphi$ is the estimated parameter vector

$$\varphi = [a_1, \ldots, a_{na}, b_1 f_1, \ldots, b_{nb} f_m]^T. \quad (46)$$

The estimated value of $\varphi$ was obtained using the least squares method (LSM).

$$\hat{\varphi} = \left( \Phi^T \Phi \right)^{-1} \Phi^T Y, \quad (47)$$

where $\hat{\varphi} = [\hat{a}_1, \ldots, \hat{a}_{na}, \hat{b}_1 f_1, \ldots, \hat{b}_{nb} f_m]^T$, $\Phi = [\theta^T(1), \theta^T(2), \ldots, \theta^T(N)]$, $Y = [y(1), y(2), \ldots, y(N)]^T$.

The parameters of the HSD-SDD model are estimated using the recursive least squares (RLS) method, which is based on minimizing the quality index.

$$\hat{\varphi} = \arg \min_\theta \sum_{k=1}^{n} \left[ y_m(k) - \Phi(k)^T \varphi \right]^2. \quad (48)$$

Examples of the estimated parameters of the nonlinear HSD-SDD model for a third-order polynomial function are shown in Fig. 21.

### B. PID CONTROLLER TUNING

The nonlinear vibration characteristics of the HSD-SDD system make it difficult to properly control it using classic proportional-integral-derivative (PID) controller settings. A method of tuning the PID controller parameters has been proposed using the recursive least squares (RLS) algorithm, which adapts to the vibration variation of the HSD-SDD system. The RLS algorithm with a forget factor was used to assign more weight to recent data, which is suitable for adaptive controller tuning.

Using the backward Euler method for integrals and derivatives in a given sampling period $T$, and modifying the derivative term with a low-pass filter $H_{LPF}$, a $G_{PID}$ transfer function in the $z$-domain of the PID controller is represented by [29]

$$G_{PID}(z) = \frac{u(z)}{e(z)} = K_p + K_i \frac{T_z}{z-1} + K_d \frac{z-1}{N T_z}$$

$$= [K_p \quad K_i \quad K_d] \begin{bmatrix} 1 \\ \frac{T_z}{z-1} \\ \frac{z-1}{N T_z} \end{bmatrix} = \theta^T \varphi, \quad (49)$$

where $K_p$ is the proportional gain; $K_i$ is the integral gain; $K_d$ is the derivative gain; $T$ is the sampling period; $N$ is the filter coefficient of the approximate derivative; $\theta$ and is a PID gain vector to be tuned in the controller, $\theta^T = [K_p \quad K_i \quad K_d]$.

$\phi$ is a vector with the $z$-operator $\varphi = \begin{bmatrix} 1 \\ \frac{T_z}{z-1} \\ \frac{z-1}{N T_z} \end{bmatrix}$.

First, a closed-loop experiment is performed to obtain the input/output data $u(k), y(k)$, $k = 1, \ldots, N$, for an initial controller parameter $\theta_0$, the reference signal $y_{r0}(k)$ is calculated by

$$y_{r0}(k, \theta_0) = G_{PID}(z, \theta_0)^{-1} u(k) + y_0(k). \quad (50)$$

The initial controller parameter $\theta_0$ is then fine-tuned to minimize the performance index

$$J(\theta_0) = \sum_{k=0}^{N} \left( y_0(k) - M(z) y_{r0}(k, \theta_0) \right)^2, \quad (51)$$

where $M(z)$ denotes the fictitious reference control model that expresses an ideal closed-loop system. The aforementioned tuning procedure was performed offline.

In the ideal case

$$y_0(k) - M(z) y_{r0}(k, \theta_0) = 0. \quad (52)$$

After substituting (49) into (50) and then into (52), the tuning error is obtained

$$\tilde{e}(k) = \theta_0^T \varphi (1 - M(z)) y_0(k, \theta_0) - M(z) u_0(k), \quad (53)$$
Then, the tuning method can be applied by minimizing the performance index

\[ \tilde{J}(\theta_0) = \sum_{k=0}^{N} \tilde{e}(k)^2. \]  

(54)

After introducing the parameters \( y(k) \) and \( u(k) \) into (54), the following signals are defined:

\[ \tilde{e}(k) = \theta^T \varphi(1 - M(z)) y(k, \theta) - M(z) u(k) \]

\[ = \theta^T \delta(k) - d(k), \]  

(55)

where \( \delta(k) = \varphi(1 - M(z)) y(k, \theta) \), \( d(k) = M(z) u(k) \).

Tuning of the PID controller leads to the minimization of the performance index

\[ \tilde{J}_k(\theta) = \sum_{i=0}^{k} \lambda^{k-i} \tilde{e}(i)^2. \]  

(56)

Because the RLS algorithm cannot cope with all changes in the HSD-SDD reference model, the forgetting factor \( \lambda \) (0 < \( \lambda \) < 1) is introduced. After introducing the forgetting factor \( \lambda \), performance index (56) is expressed as follows,

\[ \tilde{J}_k(\theta) = \sum_{i=0}^{k} \lambda^{k-i} \tilde{e}(i)^2. \]  

(57)

The RLS algorithm after the introduction of the forgetting factor is as follows

\[ R(k) = \frac{P(k - 1) \delta(k)}{\lambda + \delta(k)^T P(k - 1) \delta(k)}, \]  

(58)

where \( P(k) \) is the covariance matrix

\[ P(k) = (P(k - 1) - r(k) \delta(k)^T P(k - 1)) / \lambda. \]  

(59)

The recursive least squares (RLS) algorithm finds the optimal estimate of \( \hat{\theta}(k) \) the controller gains using \( \hat{\theta}(k - 1) \) at the previous time \( k - 1 \). According to the RLS algorithm, the controller gains \( \hat{\theta}(k) \) are updated each time.

\[ \hat{\theta}(k) = \hat{\theta}(k - 1) + R(k) \left( d(k) - \delta(k)^T \hat{\theta}(k - 1) \right). \]  

(60)

To reduce the variability of the controller parameter by updating the implemented PID controller gains, \( \theta(\alpha) \) was proposed.

\[ \theta(\alpha) = (1 - \alpha) \theta(k - 1) + \alpha \hat{\theta}(k - 1), \]  

(61)

where \( \alpha \) is the positive constant, 0 \( \leq \alpha \leq 1 \).

According to rule (60), \( \hat{\theta}(k) \) is filtered by an \( H_{LPF} \), which can be defined as [30]:

\[ H_{LPF} = \frac{\alpha}{z + \alpha - 1}. \]  

(62)

Figure 22 shows a block diagram for online tuning of the PID controller.

Each online PID controller parameter tuning algorithm is performed in the following steps:

1. The initial parameters are set as follows: gain \( \theta_0 \) of the controller, forgetting factor \( \lambda_0 \) and constant \( \alpha_0 \).

2. Calculating signals:

\[ d(k) = M(z) u(k), \]

\[ \delta(k) = \varphi(1 - M(z)) y(k). \]

3. Obtaining \( \hat{\theta}(k) \) from Eqs.(58), (59) and (60).

4. Obtaining \( \theta(\alpha) \) from Eq.(61).

The proposed online tuning does not provide control stability for the HSD-SDD. The solution to this problem is to ensure the stability of the control system by using the HDS-SDD model.

In the online tuning method, the optimum setting of the PID controller depends on the critical ultimate gain \( K_u \) and critical ultimate period \( T_u \) set at the stability limit (the process of tuning the PID controller occurs in real time). The discrete transfer function of the closed-loop control is considered when the control system depends on the gain factor \( K_p \).

\[ G(z) = \frac{y(z)}{y_r(z)} = \frac{z^{-d} K_p B(z^{-1})}{A(z^{-1}) + z^{-d} K_p B(z^{-1})}, \]  

(63)

where \( y_r \) is the reference set signal, and \( A(z^{-1}) \) and \( B(z^{-1}) \) are polynomials according to Eq.(39).

The denominator of the Equation (63) is used to determine its stable poles,

\[ D(z) = A \left( z^{-1} \right) + z^{-d} K_p B \left( z^{-1} \right) = 0, \]  

(64)

Characteristics equation (64) contains a pair of complex roots \( z_{1,2} = \alpha \pm \beta j \) (where \( \alpha \) is the real part and \( \beta \) is the imaginary part). For a complex space \( \alpha^2 + \beta^2 = r^2 \), where \( r \) is the radius of the circle on the complex plane \( (r = 1) \). The task is to select the gain factor \( K_p = K_u(T_u) \) so that the closed control system is stable, which means that the characteristic equation has roots \( z_i \) and complies with the condition \( |z_i| < 1 \) for \( i = 1, 2, \ldots, m \).

Figure 23 shows the positions of the roots in the unit circle of a discrete transfer function.

If the gain of the controller increases with the gain margin \( K_G \), then the system oscillates with frequency \( \omega_u \). Therefore, \( K_u \) and \( T_u \) can be determined from \( K_G \) and \( \omega_u \) as follows:

\[ K_u = K_G \left( \frac{1}{|G_s(j\omega)|^2} \right). \]  

(65)
where $T_p$ is the sampling time and $\alpha_{Re}$ is the real component depending on the distribution of the roots.

The Ziegler-Nichols frequency response method is used as a PID tuning method containing a proportional controller attached to the HSD-SDD system in a closed-loop configuration. In this classical method of tuning PID controller parameters, the aim is to increase the gain of the controller until a sustained oscillation is recorded, which is called the ultimate gain $K_u$, and to measure the period of the oscillation, which is called the ultimate period $T_u$. This method is effective if the control process can handle oscillations, is a “self-regulating” process, and seeks a steady-state condition. The measured values of the critical ultimate gain $K_u$ and the critical ultimate period $T_u$ of the PID controller for a $\sin$ input frequency of 0.48 Hz are shown in Fig. 24.

The PID controller gains were calculated according to the Ziegler-Nichols frequency response method using the following formula [31]:

$$
\begin{align*}
K_P &= 0.6 K_u \\
K_I &= \frac{1.2 K_u T_p}{T_u} \\
K_D &= \frac{0.075 K_u T_u}{T_p},
\end{align*}
$$

(68)

C. HYBRID CONTROL STRUCTURE

The purpose of the hybrid control structure of the HSD-SDD system, which includes a feedforward controller, feedback controller, and shaping filter, is to compensate for badly damped vibrations after SDD application. Vibration effects can cause deterioration in the response speed, control accuracy, and stability of the HSD. A hybrid feedforward and feedback control method was proposed to improve the control precision and mitigate the effects of nonlinearities and badly damped vibrations of the HSD-SDD system. The hybrid control structure of the HSD-SDD system is shown schematically in Fig. 25.

The hybrid control structure includes a PID feedback controller with a transfer function $G_{PID}(z)$, feedforward controller with an FIR feedforward filter $H_f(z, \theta)$, FIR input shaping filter $H_s(z)$, and HSD-SDD dynamic model $G(z)$ with measurement of the output signal $y$ (displacement of the actuator). This study proposes a combination of input shaping with feedforward control, whereby the oscillations induced by impulse forces are effectively damped, resulting in better control accuracy. The function of the feedforward regulator is to balance the dynamics of the HSD-SDD system using an inverse transition function. The pure inverse of the transfer function $G(z)$ yields an improper transfer function $G_I(z)$:

$$
G_I = G(z)^{-1}.
$$

(69)

In a flexible system such as an HSD-SDD, the transfer function $G(z)$ is usually strictly proper, which makes it impossible to realize the inverse transfer function $G_I(z)$, which may be unstable. Therefore, two finite impulse response (FIR) filters, an input-shaping filter $H_s(z)$ and an inverse filter $H_f(z, \theta)$ of a feedforward controller were used to compensate for the
badly damped vibration in the HSD-SDD system. Note that there are no poles in the FIR filters, therefore there are no instability problems.

After introducing an input-shaping filter \( H_s \), the input signal \( y_s \) to the feedback loop is

\[
y_s = H_s(z) y_r. \tag{70}
\]

An FIR input-shaping filter \( H_s(z) \) was used to reduce the residual vibration of the flexible systems. The FIR input shaping filter in the z-plane can be represented as [32]

\[
H_s(z) = \sum_{n=0}^{M} b_n z^{-n}, \tag{71}
\]

where \( b_n \) is a constant whose value depends on the index \( n \).

An FIR feedforward filter (called a compensator) \( H_f(z, \theta) \) was used to reduce vibrations in the elastic structures. The FIR feedforward filter with variable coefficients has the form [33]:

\[
H_f(z, \theta) = \sum_{n=1}^{N} C_n(z) f_n(\theta), \tag{72}
\]

where \( C_n(z) \) is the function in the z-plane given by the designer and \( f_n(\theta) \) are functions with unknown tunable coefficients \( \theta \in \Re^N \).

The equation resulting from the block diagram in Fig. 25 is considered

\[
y(z) = \frac{G(z) \left( H_f(z, \theta) + G_{PID}(z) H_s(z) \right)}{1 + G_{PID}(z) G(z)} \ y_r(z), \tag{73}
\]

\[
u(z) = \frac{H_f(z, \theta) + G_{PID} H_s(z)}{1 + G_{PID}(z) G(z)} \ y_r(z), \tag{74}
\]

\[
y_r(z) = \frac{H_f(z, \theta) + G_{PID} H_s(z)}{1 + G_{PID}(z) G(z)} \ u(z), \tag{75}
\]

\[
e = H_s(z) y_r(z) - y = \frac{H_s(z) - G(z) H_f(z, \theta)}{1 + G_{PID}(z) G(z)} \ y_r(z)
= \frac{H_s(z)}{K(z)} \ u(z) - \frac{H_f(z, \theta)}{K(z)} \ y(z), \tag{76}
\]

where \( K(z) \) is the factor for the initial filter coefficient \( \theta = \theta_0 \) and \( K(z) = H_f(z, \theta_0) + G_{PID}(z) H_s(z) \).

The coefficients \( \theta \) of the \( H_f \) feedforward filter were determined using the SISO gradient approximation algorithm [33]. The goal of optimizing the hybrid control structure is to determine an FIR feedforward filter coefficient that minimizes the objective function:

\[
\tilde{\theta} = \arg \min_{\theta} V(\theta), \tag{77}
\]

where \( V(\theta) \) is the objective function, which is defined as the square of the 2-norm of the control error \( e(\theta) \), and \( \theta \) is the \( n \)-vector \( \theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \)

\[
V(\theta) = e(\theta)^T e(\theta). \tag{78}
\]

Objective function (78) leads to a convex optimization problem, which means that a global optimal solution is achievable. The gradient vector of the \( V(\theta) \) objective function is given by the partial derivative of each independent variable. Second-order partial derivatives can be represented by a square symmetric matrix called Hessian matrix \( H(\theta) \), which contains \( n \) elements:

\[
\nabla V(\theta) \equiv H(\theta) \equiv \begin{bmatrix}
\frac{\partial^2 V}{\partial \theta_1^2} & \cdots & \frac{\partial^2 V}{\partial \theta_1 \partial \theta_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial^2 V}{\partial \theta_n \partial \theta_1} & \cdots & \frac{\partial^2 V}{\partial \theta_n^2}
\end{bmatrix}. \tag{79}
\]

Then, by minimizing the objective function, a second-order Taylor series expansion of \( V \) about \( \theta_k \) is considered

\[
V(\theta) \approx V(\theta_k) + \nabla V(\theta_k)^T (\theta - \theta_k) + \frac{1}{2}(\theta - \theta_k)^T \nabla^2 V(\theta)(\theta - \theta_k), \tag{81}
\]

where \( k \) is the trial number, five experiments are carried out.

Minimizing the objective function \( V(\theta_k) \) follows from applying a Gauss-Newton update law

\[
\theta_{k+1} = \theta_k - \alpha_k \left( \nabla^2 V(\theta_k) \right)^{-1} \nabla V(\theta_k), \tag{82}
\]

where \( \alpha \) is the step length.

The gradients in formula (82) are defined as follows

\[
\nabla V(\theta_k) \approx 2 \nabla e^T(\theta_k) e(\theta_k), \tag{83}
\]

\[
\nabla^2 V(\theta_k) \approx 2 \nabla^2 e^T(\theta_k) e(\theta_k). \tag{84}
\]

The global minimum is solved for the coefficient

\[
\tilde{\theta} = \theta_{k+1} - \theta_k = -\alpha_k \left( \nabla e^T(\theta_k) e(\theta_k) \right)^{-1} \nabla e^T(\theta_k) e(\theta_k), \tag{85}
\]

Finally, the global minimum is calculated from the control error based on the measurement of the input signal \( u \) and the output signal \( y \).

**IX. EXPERIMENTAL RESULTS**

The effectiveness of the hybrid control system used to compensate for badly damped vibrations was confirmed by an experiment conducted on the test stand. Figure. 26 compares the characteristics of the displacement step response \( y(t) \) of the HSD-SDD system with and without compensation for the badly damped vibration at a frequency of 21 Hz generated by the impact force of the rock breaker.

Furthermore, Fig. 27 compares the frequency characteristics of the HSD-SDD system with and without compensation for the badly damped vibrations.
FIGURE 26. Comparison of the step response characteristics $y(t)$ of the HSD-SDD system without and with compensation for badly damped vibration at frequency 21 Hz.

FIGURE 27. Comparison of the frequency characteristics of the HSD-SDD system without compensation (a) and with compensation (b) for badly damped vibration at frequency 21 Hz.

The characteristics in the time and frequency domains show the effect of the compensation of weakly damped vibrations in the hybrid control system. The characteristics of the frequency domain are more precise than those of the time domain because they exhibit vibration amplitudes with resonant vibrations. The use of the hybrid control structure reduced the amplitude by 50% of the resonant vibration at 21 Hz. The main contribution of this study is a new approach to the hybrid control structure that allows the compensation of vibrations that are badly damped by the cyclic impact force generated by the rock breaker used in jaw crushers. Future work will propose a hybrid control structure for vibration compensation in a wider bandwidth of excitation frequencies.

The influence of the frequency of the hydraulic actuator movement on the quality of HSD control was analyzed. The evaluation was performed according to the following criteria,

- Absolute deviation of the position signal
  \[ \delta_{yi} = |y_r - y_i|, \]  \hspace{1cm} (86)

- Absolute deviation of the speed signal
  \[ \delta_{vi} = |v_r - v_i|, \]  \hspace{1cm} (87)

where $y_r$ is the reference position signal, $y_i$ is the measurement position signal, $v_r$ is the reference speed signal, and $v_i$ is the measured speed signal.

The results of the measurement of the absolute deviations of the position and speed signals for different frequencies of actuator piston movement are shown in Fig. 28.

In the event of a small deviation in position and speed signals, the control system is predictable and stable. As the deviation of the signals increases, the control system becomes less predictable and there may be a problem with stability. At a higher frequency of movement, the unfavorable influence of inertial forces that occur in the hydraulic drive system becomes apparent.

Two indices were adopted for the rating of the HSD-SDD control quality: the integral of square error (ISE) index and the integral of the time-weighted absolute error (ITAE) index. Fig. 29 shows the influence of the piston speed of
The hybrid control structure was designed to operate under real conditions; therefore, hardware was used to compensate for the badly damped vibrations. Reliable results for the operation of such a system are based on experimental research with correct theoretical assumptions. The authors concluded that the actual results were more important for evaluating the performance of the HSD-SDD system. The limitation of the use of the hybrid control structure is the stable behavior of the HSD-SDD system and the effectiveness of the compensation of vibrations that negatively impact the work and operation of the manipulator and hydraulic components of the crusher. The main difficulty in applying the control method to compensate for badly damped vibrations is the variable nature of the impact loads generated by a hydraulic rock breaker.

**X. CONCLUSION**

This study analyzes a difficult and complex construction and scientific problem concerning the control of an HSD flexibly connected by the SDD to the load mass induced by the cyclic impact force generated by a rock breaker mounted on a jaw crusher manipulator boom. An important research contribution is the dynamic model and simulation tests of the HSD-SDD system, which enabled the selection of SDDs to isolate the vibrations generated by the rock breaker. The main scientific contribution was the proposal of a new hybrid control structure of the HSD-SDD system to compensate for badly damped vibrations after using the SDD.

The goal of optimizing the hybrid control structure of the HSD-SDD system is to determine an FIR feedforward filter coefficient that minimizes the objective function. The objective function leads to a convex optimization problem, which means that a global optimal solution is achievable. The global minimum of the objective function was calculated from the measurement of the input signal \( u \) and output signal \( y \). The gradient vector of the objective function is given by the partial derivative of each independent variable. Second-order partial derivatives can be represented by a square symmetric matrix called a Hessian matrix.

The purpose of the hybrid control structure was to compensate for the badly damped vibrations of the rock breaker excited by the cyclic impact force after the application of a spring damping device (SDD). The HSD-SDD model was saved as a Hammerstein model with unknown parameters, which consisted of two parts: a static nonlinear part and a dynamic linear part. The static nonlinear subsystem is approximated by the set polynomial degree and described by a nonlinear function of the nth order that binds the nonlinear output \( x(k) \) to the input \( u(k) \). The dynamic linear part is an autoregressive model with exogenous inputs (ARX), that represents the relationship between input \( x(k) \) and output \( y(k) \). The \( G_{PID} \) transfer function in the z-domain is introduced by applying the Euler backward method to the PID feedback controller. The results of the experiments show that the hybrid control method proposed in this study effectively compensates for the bad-damped vibrations excited by the cyclic impact force, and thus significantly improves the control precision of the HSD-SDD system.

The most important detailed conclusions from the study:

1) Simulation tests carried out in the Simscape Fluids simulation test (MATLAB/Simulink) compared the influence of rigid and flexible HSD connections on its dynamic properties. Simulation tests showed that the omitted flexible connection of an HSD may become a source of unexpected mechanical vibrations. These vibrations can be a source of parasitic vibrations, which are particularly harmful to the mechanical structure and hydraulic components of the manipulators. These observations were confirmed in the user reports of crusher manipulators.

2) The impact of pressure pulsation in a cylinder on the flow fluctuations in a short hydraulic pipeline with lumped parameters was demonstrated. When extreme pressure build-up occurs in the resonance region, the hydraulic pipeline can be damaged.

3) Simulation and experimental studies of HSD-SDD systems are of great importance in the design, control, and operation of heavy hydraulic manipulators with large moving masses and mechanical structures susceptible...
to elastic deformation. These manipulators are used in construction, roads, and mining machines.

The study will be used to conduct research projects on HSD-SDD systems for crusher manufacturers, jaw crusher users, and aggregate producers in the cement industry. The advantages of this research are the operational guidelines for designers of hydraulic manipulators that operate with long-term mechanical vibrations. Design difficulties result from the adaptation of SDD for hydraulic manipulators in stationary crushers.

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