A NEW COMPTONIZATION MODEL FOR WEAKLY MAGNETIZED, ACCRETING NEUTRON STARS IN LOW-MASS X-RAY BINARIES

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ABSTRACT

We have developed a new model for the X-ray spectral fitting package XSPEC that takes into account the effects of both thermal and dynamical (i.e., bulk) Comptonization. The model consists of two components: one is the direct blackbody-like emission due to seed photons that are not subjected to effective Compton scattering, while the other is a convolution of the Green’s function of the energy operator with a blackbody-like seed photon spectrum. When combined thermal and bulk effects are considered, the analytical form of the Green’s function may be obtained as a solution of the diffusion equation describing Comptonization. Using data from the BeppoSAX, INTEGRAL, and RXTE satellites, we test our model on the spectra of a sample of six bright neutron star low-mass X-ray binaries with low magnetic fields, covering three different spectral states. Particular attention is given to the transient power-law–like hard X-ray (≥30 keV) tails, which we interpret in the framework of the bulk motion Comptonization process. We show that the values of the best-fit $\delta$-parameter, which represents the importance of bulk with respect to thermal Comptonization, can be physically meaningful and can at least qualitatively describe the physical conditions of the environment in the innermost part of the system. Moreover, we show that in fitting the thermal Comptonization spectra to the X-ray spectra of these systems, the best-fit parameters of our model are in excellent agreement with those from compTT, a broadly used and well-established XSPEC model.

Subject headings: accretion, accretion disks — stars: individual (Scorpius X-1, GX 17+2, Cygnus X-2, GX 340+0, GX 3+1, GX 354–0) — stars: neutron — X-rays: binaries

1. INTRODUCTION

The study of the transient hard X-ray tails in neutron star (NS) low-mass X-ray binaries (LMXBs) has recently received a strong theoretical push. The initial approach was mainly phenomenological, with the hard tails simply fitted a power-law (PL) model (see review by Di Salvo & Stella 2002). Subsequently, using 0.4–120 keV BeppoSAX data, Farinelli et al. (2005, hereafter F05) attempted a more physical approach, describing the hard tail of GX 17+2 by means of Comptonization of soft ($\sim$0.6 keV) seed photons off a hybrid (thermal plus nonthermal) electron population. More recently, Paizis et al. (2006, hereafter P06), using long-term time-averaged spectra from the IBIS instrument on board the International Gamma-Ray Astrophysics Laboratory (INTEGRAL) ($\gtrsim$20 keV), confirmed the presence of hard X-ray emission in the Z sources Sco X-1, GX 5–1, GX 17+2, GX 340+0, and Cyg X-2 and also discovered it in the atoll source GX 13+1, which, in fact, is characterized by a stable Z-like X-ray continuum (thermal component with low electron temperature $kT_e$ and high optical depth $\tau$).

Motivated by observations of the intermediate and soft states of black hole candidates, P06 proposed that bulk motion Comptonization could be responsible for the formation of the well-known hard X-ray tails in NSs observed in these sources as well. In that analysis, the bulk motion Comptonization (BMC) model in XSPEC, which is described in Titarchuk et al. (1996, 1997, hereafter TMK96 and TMK97, respectively), was used. Moreover, merging these results with those of Falanga et al. (2006, hereafter F06) on the atoll source 4U 1728–34 (GX 354–0), P06 identified four main spectral states for NS LMXBs (see Fig. 4 of P06): the hard/PL state (H1750–440 and H1608–55), the low/hard state (GX 354–0), the intermediate state (e.g., GX 5–1), and the very soft state (e.g., GX 3+1). In this picture, the relative contribution between thermal and bulk Comptonization (TC and BC, respectively), which is in turn related to the accretion rate $M$ and the radiation pressure from the NS, is the main parameter explaining both the different spectral states among sources and the spectral evolution of a single source. However, the lack of data below 20 keV prevented more stringent conclusions from being drawn about the accretion geometry of the systems and their physical parameters.

Reanalyzing broadband BeppoSAX data, Farinelli et al. (2007, hereafter F07) applied the BMC model to GX 17+2, finding that the source spectrum can be described by blackbody-like emission plus an unsaturated TC component plus a power law. The TC component, fitted with the compTT model (see Titarchuk 1994, hereafter T94), was suggested to come from the region between the Keplerian accretion disk and the NS surface (the so-called transition layer [TL]), while the PL-like emission is attributed to bulk Comptonization of the NS seed photons by the infalling material, which is in turn the innermost part of the TL itself. This accretion scenario is supported by results obtained from numerical solutions of the equation for the radial velocity profile in accretion disks (L. Titarchuk & R. Farinelli 2008, in preparation [hereafter TF08], where the quasi–free-fall behavior of $v_R$ inside the TL from some radius $R_R$ to the NS surface is unambiguously shown).

It is thus evident that BC is gaining strong theoretical and observational support in the study of LMXB spectral evolution. This is also true in the case of accretion-powered X-ray pulsars, for which a new theoretical model based on TC and BC, occurring...
in the accreting shocked gas, was recently presented (Becker & Wolff 2007, hereafter BW07). In the BW07 Comptonization model, the effects of the strong magnetic field ($B \sim 10^{12}$ G) are very important, as the accreting gas column is channeled at the NS magnetic poles, and the presence of this field also has important consequences for the photons propagating through the plasma.

From the observational point of view, the energy index $\alpha$ and the cutoff energy $E_c$ are only parameters that can be determined by fitting the observed spectral shapes to the Comptonization models; additional assumptions have to be adopted in order to relate them to the physical parameters of a particular model.

However, in this paper we concentrate on the study of sources in which the magnetic field is low, so that it has second-order effects with respect to radiation pressure in driving the accretion process. For this class of weakly magnetized compact objects, in the current XSPEC package there is available the BMC model, which was used, as mentioned above, in P06. This is in fact a general Comptonization model, that is, widely applicable, given that the output Comptonized spectrum is given by the convolution of an analytical approximation to the Green’s function (the response of the system to injection of a monochromatic line at energy $E_0$) with a blackbody (BB) seed photon spectrum, regardless of the Comptonization process—whether due only to thermal or to combined thermal and bulk effects.

The BMC model parameters are the BB seed photons’ color temperature ($kT_{bb}$) and normalization, the energy index $\alpha$ of the Comptonized spectrum, and the “illumination factor” $\log A$. The information about the efficiency of the Comptonization is carried by the index $\alpha$: the lower its value, the greater the energy exchange from hotter electrons to softer seed photons. As already mentioned, $\alpha$ does not specify which kind of process produces the Comptonization but is simply related to an observable quantity in the photon spectrum. The Green’s function in the model is represented by a broken power law, an approximation that holds when the average energy of the seed photons is much lower than the average energy $E_{\text{av}}$ of the plasma (Sunyaev & Titarchuk 1980, hereafter ST80; TMK97). The BMC model however lacks the electron-recoil term in the Green’s function, which is responsible for the rollover in the spectrum (which occurs, depending on the plasma temperature and on the first- and second-order bulk velocity terms, at about $E_{\text{av}}$).

Thus, the Comptonized part of the emergent spectrum in the BMC model is always a power law, with no cutoff. However, the lack of an energy cutoff in the model becomes critical in fitting the TC spectra of LMXBs in their high-luminosity state, for which a rollover is well documented around 10 keV.

Motivated by these limitations in the currently available version of the BMC model for XSPEC and by the recent observational results on LMXBs (see above), we have developed a new model for XSPEC, which can be considered an extension and completion of BMC. In § 2, we summarize the main results of the bulk motion Comptonization theory and present the mathematical background of our model. In § 3, we show the results of X-ray spectral fitting using our model on a sample of six persistently bright NS LMXBs. In § 4, we discuss the main results and the limits of applicability of the model itself, along with a brief theoretical comparison with the model of BW07, and in § 5 we draw our conclusions.

2. DESCRIPTION OF THE MODEL

2.1. Solution of the Comptonization Equation for Bulk Flow of Nonrelativistic Electrons

The Comptonization equation (a particular type of Fokker-Planck equation; see, e.g., T94) describes the evolution of a photon field due to Compton scattering of photons off nonrelativistic thermal electrons. It is obtained as an approximation of the full kinetic Boltzmann equation for the case in which the photon field is almost isotropic, the average number of scatterings is high, and the average energy exchange per scattering is small, namely, $\Delta E/E \ll 1$. When the divergence of the bulk velocity field $\nabla \cdot \mathbf{v}_b$ of the electrons is zero (i.e., when the electrons are subject to pure Brownian motion), only TC contributes to the total emergent spectrum (ST80). On the other hand, if the plasma flow is subject to inward bulk motion ($\nabla \cdot \mathbf{v}_b < 0$), this will provide a further channel for Comptonization. The Comptonization equation including a bulk motion term was first derived by Blandford & Payne (1981). A slightly modified version, which includes the second-order bulk term $v_{b,2}$, is presented in TMK97. The equation for the photon occupation number $n(x, \tau)$ in the case of a free-fall radial velocity profile ($v_r \propto r^{-1/2}$) reads

$$\tau \frac{\partial^2 n}{\partial \tau^2} - \left( \tau + \frac{3}{2} \right) \frac{\partial n}{\partial \tau} = x \frac{\partial n}{\partial x} - \frac{1}{2} \frac{\partial}{\partial x} \left[ \chi \left( f_{\text{sc}}^{-1} n \right) \frac{\partial f_{\text{sc}}}{\partial x} \right] - \frac{kT_{\text{c}}}{n c},$$

where $x \equiv E/kT_c$ is a dimensionless energy, $\Theta \equiv kT_c/m_e c^2$ is the dimensionless electron temperature, $m$ is the accretion rate in units of the Eddington accretion rate, $\kappa$ is the inverse of the scattering free path of electrons, and $\tau$ is the optical depth. The term $j$ in equation (1) is a function of $x$ and $\tau$ and represents the seed-photon source function, and $\delta_1 \equiv \delta f_s(\tau)$, where $f_s(\tau) = 1 + \frac{1}{\kappa} (v_0/c)^2 \Theta$. The quantity of interest in the above equation is the bulk parameter $\delta$, which is derived in TMK97 as

$$\delta \equiv \frac{\langle E_{\text{bulk}} \rangle}{\langle E_{\text{bb}} \rangle} = \sqrt{1 - \frac{l}{\Theta m}},$$

where $l \equiv L/L_{\text{Edd}}$ is the fractional Eddington luminosity impinging on the flow.

It is worth noting that in equation (1), the term $f_{\text{sc}}^{-1} n$ depends on $\tau$ through the bulk term $v_{b,2}^2$. Given that $f_{\text{sc}}^{-1} n$ depends on $\tau$, it is possible to find an analytical solution to the equation (using the separation-of-variables method) by neglecting only the $v_{b,2}^2/c^2$ term, namely, setting $f_0 = 1$. In this case the Green’s function $G(x, x_0) = N_G(x_0) \tilde{G}(x, x_0)$ (see also eqs. [B4]–[B8] of TMK97), with $\tilde{G}(x, x_0)$ given by

$$\tilde{G}(x, x_0) = \frac{1}{x_0 \Gamma(2\alpha + 4 + \delta)} \times \begin{cases} \frac{1}{\Gamma(\alpha, 4 + 2\alpha + \delta, x)} \times (x/x_0)^{\alpha + 3} e^{-x} J(x_0, \alpha, \omega), & \text{if } x \leq x_0, \\ \Gamma(\alpha, 4 + 2\alpha + \delta, x_0) \times (x/x_0)^{-\alpha} e^{-x} J(x, \alpha, \omega), & \text{if } x \geq x_0. \end{cases}$$

Here $\alpha$ is the energy index, $\omega = \alpha + \delta + 3$, and $x$ and $x_0$ represent the dimensionless scattered and injected photon energies, respectively, namely, $x \equiv E/kT_c$ and $x_0 \equiv E_0/kT_c$. This is in fact the complete form of the Green’s function for the Comptonization equation, while equation (22) of TMK97 applies to the asymptotic case of low-energy ($x_0 \ll 1$) monochromatic line injection. The dependence of the spectral energy index $\alpha$ on the model physical parameters $\Theta, \tau$, and $\delta$ is provided in equations (7) and (23) of TMK96 and TMK97, respectively. We note that the dependence of $\alpha$ on $\Theta$ and $\tau$ in equation (3) is implicit through the presence of the first eigenvalue of the space diffusion operator.
In equation (3), $F_1$ is a confluent hypergeometric function (Abramowitz & Stegun 1970) and $J(x, \alpha, \omega)$ is an integral function expressed through the formula

$$J(x, \alpha, \omega) = \int_0^\infty e^{-i(x+t)}t^{\alpha-1}dt.$$  \tag{4}

Given that Comptonization conserves photon number, this photon conservation law can be rewritten as the integral equation for the Green’s function:

$$\int_0^\infty \frac{G(x,x_0)}{x} dx = \frac{1}{x_0}. \tag{5}$$

Thus, the normalization factor $N_G(x_0)$ in equation (3), which depends on the injected line energy $x_0$, is

$$N_G(x_0) = \left[ x_0 \int_0^\infty \frac{\tilde{G}(x,x_0)}{x} dx \right]^{-1}. \tag{6}$$

When the energy of the injected line is much smaller than the plasma energy, that is, when $(x_0, x) \ll \omega$, the Green’s function is a broken power law and $N_G(x_0) \approx \alpha(\alpha + \delta + 3)$ (see TMK97).

The emergent Comptonized spectrum is given by the convolution of the Green’s function with $S(x_0)$ if the input spectrum, instead of being a simple monochromatic line, has a continuum shape $S(x_0)$. Let us consider seed photons distributed according to

$$S(x_0) = \frac{C_N \gamma^{\gamma}}{e^{T_{x_0}/T_x} - 1}, \tag{7}$$

where $T_x$ and $T_e$ are the electron and seed-photon temperatures, respectively, and $C_N$ is a normalization factor. This is evidently a blackbody spectrum if $\gamma = 3$. The Comptonization spectrum is thus given by

$$f(x) = S(x_0) \ast G(x, x_0) \equiv \frac{J(x, \alpha, \omega)e^{-x/x_0}}{\Gamma(2\alpha + 4 + \delta)} \int_0^\infty N_G(x_0)J(x, \alpha, \omega) \frac{1}{x_0^{\alpha+1}} \frac{x_0^2}{e^{T_{x_0}/T_x} - 1} dx_0 \times \int_0^\infty N_G(x_0)J(x, \alpha, \omega) \frac{1}{x_0^{\alpha+1}} \frac{x_0^2}{e^{T_{x_0}/T_x} - 1} dx_0 + \left[ \frac{e^{-x/x_0^{\alpha+\delta+3}}}{\Gamma(2\alpha + 4 + \delta)} \int_0^\infty N_G(x_0)J(x, \alpha, \omega) \frac{1}{x_0^{\alpha+1}} \frac{x_0^2}{e^{T_{x_0}/T_x} - 1} dx_0 \right]^{x_0} \times \int_0^\infty N_G(x_0)J(x, \alpha, \omega) \frac{1}{x_0^{\alpha+1}} \frac{x_0^2}{e^{T_{x_0}/T_x} - 1} dx_0. \tag{8}$$

Given that the two integrals in equation (8) do not have an analytical form, we have to calculate them numerically. This step is performed using the GNU Scientific Library package.\(^7\) Note also that in this equation the functions $N_G(x_0)$ and $J(x_0, \alpha, \omega)$ are themselves integrals (see eqs. [4] and [6]) and thus should be calculated before being put into the integral. This embedded integration is however very expensive in terms of CPU time, and it makes the whole convolution process very slow in running XSPEC.

To avoid this problem, the integral function in equation (4) is analytically solved at each integration step using the steepest-descent method, as described in Appendix C of TMK97. This method provides very high accuracy (within a few percent) for high $\delta$-values ($\gtrsim 10$), but when $\delta$ is close to zero the discrepancy may reach 10% (depending on the corresponding $\alpha$- and $x$-values).

To better approximate the integral’s value when $\delta$ is low, we produced a three-dimensional correction matrix of $\delta$, $\alpha$, and $x$-values (with $\delta$ in the interval 0–10), which provides a correction factor between the numerical and analytical results for the integral in equation (4). The correction factor is computed using linear interpolation among the $\delta$, $\alpha$, and $x$-bounds of the grid. The same approach is adopted to compute the normalization constant $N_G(x_0)$; for $\delta \gtrsim 10$ we simply use the value $\alpha(\alpha + \delta + 3)$, whereas for lower $\delta$-values we use an interpolation grid. A subsequent numerical check reveals that the photon number is conserved with high accuracy (within a few percent) over a very large span of $\alpha$- and $\delta$-values.

### 2.2. The Free Parameters of CompTB

Following the same approach as the BMC model (TMK96, TMK97), we structure our model as the sum of two components: one represents the emission of the soft seed photons that are not affected by noticeable upscattering in the plasma cloud, while the other is the result of efficient Comptonization. The emergent spectrum, similarly to BMC, can thus be described as

$$F(E) = \frac{C_N}{A + 1} (BB + A \times BB \times G), \tag{9}$$

where $BB \equiv S(x_0)$ (see eq. [7]) and the convolution $BB \times G$ is given by equation (8). The free parameters of our model are the seed-photon temperature $k_{Te}$, the index $\gamma$ (if $\gamma = 3$, the seed photons have a pure BB spectrum), the Green’s function energy index $\alpha$ (see eq. [3]), the bulk parameter $\delta$, the temperature of the electrons $k_{Te}$, the illumination factor $A$, and the normalization constant $C_N$. The latter is chosen in such a way that when $A \to 0$ (and $\gamma = 3$) the model simply reduces to the standard BB model for XSPEC. We note also that $C_N$ must be multiplied by the factor $0.5 \times 10^{23} [k_{Te}/(1 \text{ keV})]^3$ in order to express the flux in physical units (ergs cm$^{-2}$ s$^{-1}$ keV$^{-1}$). In equation (9), the factor $1/(1 + A)$ is the fraction of the seed-photon radiation directly seen by the observer, whereas $A/(1 + A)$ is the fraction upscattered by the Compton cloud.

As can be clearly seen, we have expanded the parameter set with respect to BMC. In principle, this allows one to extract more detailed physical information from the observed spectra. In Figure 1, we demonstrate the effect of bulk motion, presenting the Green’s function $G(x, x_0)$ for a couple of $\alpha$-values (1 and 2.5) and two different $\delta$-values (0 and 30) for each $\alpha$. The cutoff energy is higher for larger $\delta$-values (compare the red and blue curves with the black and green curves, for which $\delta = 0$). Note that the photon number $N_G(x_0)$ is the same for all $G(x, x_0)$. We want also to emphasize, however, that our model is not specific to bulk motion: it is evident indeed from equations (3) and (9) that by setting $A \to 1$ and $\delta = 0$, a pure TC spectrum is reproduced, as in the widely used models compST (ST80) and compTT (T94).

It is also important to point out the difference between the standard models and our Comptonization model. In our case the code provides, as the best-fit parameters, the electron temperature $k_{Te}$ and the energy spectral index $\alpha$ instead of $k_{Te}$ and optical depth $\tau$ as in compST and compTT. Once $k_{Te}$ and $\alpha$ are provided, it is possible to infer the Thomson optical depth $\tau$ using the relations reported in Titarchuk & Lyubarskij (1995, hereafter TL95) for both spherical and slab geometries of the Comptonizing

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\(^7\) The GNU Scientific Library is freely available at http://www.gsl.org/software/gsl.
plasma. For example, in the nonrelativistic limit the following equation holds:

$$\alpha = -\frac{3}{2} + \sqrt{\frac{9}{4} + \frac{\pi^2 m_e c^2}{C_e k T_e (\tau + \frac{3}{2})^2}}, \quad (10)$$

where $\tau$ is the optical radius of the sphere or optical half-thickness of the slab for spherical or plane geometry, respectively, with $C_r = 3$ for a sphere and $C_r = 12$ for a slab. When bulk motion is present ($\delta > 0$), the relation between $(\alpha, \delta)$ and $k T_e$ is not straightforward and must be calculated numerically (see TMK97).

3. APPLICATION TO A SAMPLE OF LMXBs

In order to test our model, we use a data set from a sample of six LMXBs belonging to the Z and atoll classes (Hasinger & van der Klis 1989): sources in the former class are Sco X-1, GX 17+2, Cyg X-2, and GX 340+0, while GX 354−0.6 keV (4U 1728−34) and GX 3+1 belong to the latter. The choice of the first four was mainly motivated by the fact that they show transient, PL-like hard (≥30 keV) X-ray emission over their stable continuum (Sco X-1: D’Amico et al. [2001]; P06; GX 17+2: Di Salvo et al. [2000b]; F05, F07; Cyg X-2: Di Salvo et al. [2002, hereafter DS02]; GX 340+0: Lavagetto et al. [2004, hereafter LO4]). Other sources, such as GX 5−1 (Asai et al. 1994; Paizis et al. 2005), GX 349+2 (Di Salvo et al. 2001), and, most recently, GX 13+1 (P06) have shown the same high-energy transient behavior. The stable continuum spectra of these sources are usually fitted with a two-component model consisting of a photoelectrically absorbed BB plus the TC model compTT; when the hard X-ray tail appears, an additional power law is included to fit the data. The presence of a Gaussian emission line around 6.7 keV is also observed in these sources.

The persistent continuum spectrum of the bursting atoll source GX 34+1 is very similar to that of classical Z sources, with a Comptonized spectrum characterized by $k T_e \sim 3$ keV and $\tau \sim 10$ (Oosterbroek et al. 2001; P06) plus a contribution from a ~1 keV BB-like component. Neither spectral transitions nor hard X-ray tails have been observed to date from this source.

Finally, the bursting atoll source GX 354−0 is characterized by the presence of different spectral states that correlate with its position on the hardness-intensity diagram (HID). Long-term monitoring performed with INTEGRAL by P06 has shown that in the lower “island” state of the HID, the source spectrum is quite hard and can be described by a TC spectrum with $k T_e \sim 30$ keV and optical depth $\tau \sim 1.5$ (assuming slab geometry), with no evidence of soft BB-like emission. As the source moves across the HID, reaching the “banana” state, the TC spectrum becomes softer, with electron temperature dropping to ~3 keV ($\tau$ correspondingly increasing to ~5) and the simultaneous appearance of a soft (~0.6 keV) BB-like component, which was attributed by F06 to the accretion disk.

Selecting a sample of sources such as these allows us to test our model over a wide range of spectral states that are experienced by NS LMXBs, as also noted in P06. Following the terminology of P06, we can apply our model to intermediate-state spectra (Sco X-1, GX 17+2, Cyg X-2, and GX 340+0), in which the combined effect of the thermal and bulk Comptonization is suggested to be the origin of the hard X-ray emission; to very soft state spectra (GX 3+1), in which thermal Comptonization by a relatively cold plasma ($k T_e \sim 3$ keV) dominates; and to low-hard state spectra (GX 354−0), in which the dominant thermal Comptonization component is characterized by a relatively hot electron temperature ($k T_e \sim 30$ keV).

We refer the reader to P06 for a more complete discussion of the interpretation of the evolution among these spectral states. It is worth noting that fitting of the fourth spectral state identified in P06 (hard/PL) with compTB (similarly to compTT) does not provide physically meaningful parameters, because of the presence of the unattenuated power law, which results in very high plasma temperatures (hundreds of keV), out of the regime of validity of the model. The spectral shape can in fact be fitted, but the resulting parameters are not physically meaningful (see discussion in §4.3).

As a complement to the interpretation in P06, in the analysis of GX 17+2, F07 proposed an accretion scenario in which the BB-like component in the X-ray spectrum arises from the NS surface while the TC spectrum originates in the outer part of the TL region. The PL-like hard X-ray emission is then instead the result of bulk Comptonization of the BB-like (NS) seed photons by the infalling material in the innermost part of the TL (see Fig. 2).

In our analysis of the intermediate-state sources, we proceed as follows: First, we fit the spectra with the commonly used multi-component model consisting of a blackbody plus compTT plus a power law plus, when required, Gaussian emission lines (see above). Given that the radial extent of the TL is small compared with its vertical scale height (e.g., Titarchuk & Osherovich 1999; TF08), so that it can be roughly approximated as a geometrically thick equatorial belt around the NS, we assume a slab geometry for compTT for all the sources but GX 354−0, whose spectral state seems to be more related to a quasi-spherical accretion. For this multicomponent model, we do not report the unabsorbed bolometric (0.1–200 keV) source fluxes, because the presence of the PL component gives rise to a strong bias at low energies, where in fact the flux diverges, and this effect becomes worse as the PL steepness increases. It is worth taking note of this problem when one fits spectra with PL components.

Subsequently, we replace compTT with compTB, fixing the illumination factor at $A > 1$ (i.e., $\log A = 8$, corresponding to no direct seed-photon emission; see eq. [9]) and $\delta = 0$ (no bulk contribution); hereafter, we refer to this model as thermal compTB. A second compTB is used instead of the BB and PL components, this time leaving $\log A$ and $\delta$ free to vary during the fit (see Tables 1 and 2). The main idea is that the first model should just reproduce the TC spectrum from the outer TL and presumably most of a Comptonized disk component, while the second takes into

![Diagram](https://via.placeholder.com/150)
account the TC and BC of the NS-surface seed photons, which produce the observed PL-like hard X-ray tail. For the low/hard state spectrum (GX 354–0) and the high/soft state spectrum (GX 3+1), which are characterized by pure TC, the compTT model is replaced with a single thermal compTB (Table 2), with no need for the second, bulk-related, component. For all of the sources, the optical depth \( \tau \) of the TC region is later estimated using the compTB best-fit values of \( \alpha \) and \( kT'_e \), and equations (17) and (24) of TL95 and then compared with the value reported from compTT (see Table 1).

### 3.1. Sco X-1

The spectrum of Sco X-1 was obtained from observations by the INTEGRAL satellite (Winkler et al. 2003), using data from the Joint European X-Ray Monitor (JEM-X; Lund et al. 2003) and the high-energy imager IBIS/ISGRI (Ubertini et al. 2003; Lebrun et al. 2003).

### Table 1

| Parameter | Sco X-1 | GX 17+2 | Cyg X-2 | GX 340+0 | GX 3+1 | GX 354–0 |
|-----------|---------|---------|---------|----------|--------|---------|
| \( N_H^d \) | [0.18]  | 2.45±0.22 | 2.02±0.05 | 7.98±0.12 | 1.73±0.08 | 3.06±0.05 |
| Blackbody: | | | | | | |
| \( kT_0^b \) (keV) | | 1.57±0.09 | 1.18±0.03 | 1.40±0.04 | | |
| \( R_{BB} \) (km) | | 46.0±0.5 | 8.0±0.5 | 6.6±0.5 | | |
| CompTT: | | | | | | |
| \( kT_0^{\text{comp}} \) (keV) | | 1.08±0.07 | 0.55±0.02 | 0.17±0.03 | 0.43±0.09 | 0.69±0.08 | 1.18±0.04 |
| \( kT_e \) (keV) | | 2.75±0.03 | 3.39±0.07 | 2.70±0.04 | 2.80±0.04 | | 31.3±3.1 |
| \( \Gamma \) | | 4.0±0.2 | 5.9±0.2 | 6.5±0.2 | 9.3±1.0 | 5.2±0.27 | 1.5±0.6 |
| Power law: | | | | | | |
| \( N_{PL} \) | 524±152 | 175.1±19 | 0.33±0.0 | 8.4±1.4 | | |
| Gaussian: | | | | | | |
| \( E_0 \) (keV) | | 6.70±0.05 | 6.79±0.07 | | | |
| \( \sigma_0 \) (keV) | | 0.24±0.11 | 0.21±0.15 | 0.20±0.14 | 1.0±0.31 | 0.3±0.0 |
| \( \Gamma_0 \) | | 5.1±1.0 | 2.6±0.1 | 3.3±0.8 | 13.4±3.6 | 1.9±0.3 |
| EW (eV) | | 43±14 | 27±13 | 48±16 | 35±10 | 126±24 |
| \( \chi^2/\text{dof} \) | 212/167 | 155/144 | 196/152 | 174/137 | 50/42 | 151/131 |

**Note:** For GX 17+2, Cyg X-2, and GX 340+0, additional BB and PL components are required, while for Sco X-1 only a power law is required to describe the high-energy part of the spectrum. For GX 3+1 and GX 354–0, the continuum model is simple compTT. Emission lines, where observed, are modeled with a simple Gaussian model. Parameters in square brackets are kept frozen in the fit. Photoelectric absorption is computed using the wabs model in XSPEC. Errors are computed at 90% confidence for a single parameter.

- \( ^a \) Equivalent hydrogen column in units of \( 10^{22} \) cm\(^{-2} \).
- \( ^b \) Temperature of the blackbody spectrum.
- \( ^c \) Computed assuming isotropic emission and a given distance for each source (see text).
- \( ^d \) Temperature of the Wien seed photon spectrum of compTT.
- \( ^e \) Power-law normalization in units of photons cm\(^{-2} \) s\(^{-1} \) keV\(^{-1} \) at 1 keV.
- \( ^f \) Total photons in the line, in units of \( 10^{-3} \) cm\(^{-2} \) s\(^{-1} \).


### TABLE 2

**Best-Fit Parameters of the compTB Model**

| Parameter          | Sco X-1 | GX 17+2 | Cyg X-2 | GX 340+0 | GX 3+1 | GX 354–0 |
|--------------------|---------|---------|---------|----------|--------|----------|
| $N_A^a$            | [0.15]  | 2.10$^{+0.08}_{-0.06}$ | 0.26$^{+0.03}_{-0.04}$ | 6.35$^{+0.07}_{-0.09}$ | [1.7]  | [3.0]    |
| CompTB (thermal, log $A = 8$, $\delta = 0$): |         |         |         |          |        |          |
| $kT_e^b$ (keV)     | [0.4]   | 0.55$^{+0.03}_{-0.04}$ | 0.16$^{+0.04}_{-0.04}$ | 0.35$^{+0.07}_{-0.04}$ | 0.61$^{+0.10}_{-0.14}$ | 1.20$^{+0.05}_{-0.05}$ |
| $kT_b^b$ (keV)     | 2.66$^{+0.06}_{-0.04}$ | 3.38$^{+0.07}_{-0.05}$ | 2.74$^{+0.05}_{-0.05}$ | 2.95$^{+0.07}_{-0.05}$ | 2.61$^{+0.11}_{-0.11}$ | 26$^{+8}_{-8}$ |
| $\alpha$           | 0.96$^{+0.03}_{-0.04}$ | 0.93$^{+0.03}_{-0.04}$ | 0.81$^{+0.03}_{-0.02}$ | 0.86$^{+0.04}_{-0.03}$ | 1.26$^{+0.11}_{-0.09}$ | 1.69$^{+0.08}_{-0.08}$ |
| CAF$^e$            | 2.1     | 2.1     | 3.4     | 2.5      | 1.5    | 1.7      |
| CompTB (thermal plus bulk): |         |         |         |          |        |          |
| $kT_e^b$ (keV)     | 1.21$^{+0.14}_{-0.06}$ | 1.45$^{+0.30}_{-0.10}$ | 0.99$^{+0.10}_{-0.13}$ | 1.38$^{+0.18}_{-0.15}$ | [6.4]  |         |
| log $A$            | $-0.47^{+0.02}_{-0.08}$ | 0.28 ($> -1.00$) | 0.26 ($> -0.14$) | $-0.44^{+0.05}_{-0.04}$ | [2.5]  | [2.4]    |
| $\alpha$           | 3.63$^{+0.07}_{-0.04}$ | 2.37 ($<2.73$) | [2.1]    | 1.76$^{+0.42}_{-0.35}$ |        |          |
| $\delta$           | $>63$   | 89 ($>15$) | 60 ($>27$) | 52 ($>23$) |        |          |
| $kT_b^b$ (keV)     | [=$kT_{eb}^b$] | [=$kT_{eb}^b$] | [=$kT_{eb}^b$] | [=$kT_{eb}^b$] |        |          |
| Gaussian:          |         |         |         |          |        |          |
| $E_0$ (keV)        | ...      | 6.69$^{+0.05}_{-0.05}$ | 6.78$^{+0.09}_{-0.08}$ | 6.73$^{+0.07}_{-0.10}$ | [6.4]  |         |
| $\sigma_0$ (keV)   | ...      | 0.24$^{+0.10}_{-0.09}$ | 0.28$^{+0.18}_{-0.13}$ | 0.34$^{+0.12}_{-0.12}$ | 0.95$^{+0.51}_{-0.51}$ |         |
| $E_{0}^4$          | ...      | 5.5$^{+13}_{-1.0}$ | 2.5$^{+1.5}_{-0.7}$ | 4.3$^{+4.0}_{-3.9}$ | 11$^{+5}_{-5}$ |         |
| EW (eV)            | 47$^{+14}_{-13}$ | 39$^{+18}_{-18}$ | 62$^{+17}_{-17}$ | 286$^{+92}_{-135}$ |        |          |
| $L_{\text{compTB}, 10^{38}}$ erg s$^{-1}$ | 3.65   | 1.53    | 0.96    | 1.42    | 0.53    | 0.06    |
| $\chi^2$/dof$^c$   | 214/165 | 158/143 | 189/152 | 182/136  | 48/42   | 155/131 |

**Note.**—When fitting the source spectra with two compTB components, their electron temperatures $kT_e^b$ (thermal plus bulk component) and $kT_b^b$ (thermal component) are kept equal to each other during the fit. Emission lines, where observed, are fitted with a simple Gaussian model. Parameters in square brackets are kept frozen in the fit.

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2003). The ISGRI spectrum was obtained by summing all the INTARG observations of the source over the period 2003–2005 (see P06), while for JEM–X, because of the very high brightness of the source (which may give rise to problems in the standard observing mode of the instrument), we considered a single pointing ($t_{\text{exp}} \sim 1800$ s). We thus caution the reader that the high-energy source spectrum ($\gtrsim 20$ keV) shows, in fact, the average behavior of the source.

The inclusion of the JEM–X data at lower energies is very important in order to obtain a wide energy coverage, which is of key importance in constraining the model parameters. Using these two instruments we can indeed analyze the source spectrum in the energy range $4–150$ keV. The lack of data below 4 keV, however, prevents XSPEC from determining the photoelectric interstellar absorption $N_A$ along the source direction, so we fixed its value to $0.19 \times 10^{22}$ cm$^{-2}$, as found by the Solid State Spectrometer on board the Einstein Observatory (Christian & Swank 1997). A simple compTT plus PL model formally would not be completely acceptable from a statistical point of view ($\chi^2 = 212$ for 167 degrees of freedom [dof]), even though no systematic deviations are observed in the residuals between the data and the model. Moreover, this result is obtained with the addition of a systematic error of 1% to both the ISGRI and JEM–X spectra, which is likely below the current uncertainty on the instrument calibrations. If the systematics are increased to 2%, we obtain $\chi^2$/dof$^c = 189/167$. The best-fit parameters of the compTT plus PL model are reported in Table 1.

We subsequently used a two-compTB model: in this case, we deal with two seed-photon temperatures, which cannot be determined simultaneously, because the JEM–X low-energy threshold ($\sim 3$ keV) is well above the peak of the softer seed-photon temperature usually observed in LMXBs ($\sim 0.5$ keV; see Barret 2001). We thus fix $kT_e$ related to the thermal compTB to 0.4 keV, instead allowing the seed-photon temperature $kT_b$ to vary, suggested to be related to the NS surface. The results of the fit with the two-compTB model are reported in Table 2: from the best-fit values of $\alpha$ and $kT_b$ for the thermal compTB component and equations (17) and (24) of TL95, we estimate $\tau = 5.6$, which is not far from the value directly found by compTT ($\sim 5$; see Table 1).

As for the second compTB component, aimed to describe the hard X-ray tail, we find that the bulk parameter $\delta$ would be pushed up to unreasonably high values by XSPEC, with no apparent minimum in $\chi^2$-space. It is in fact just possible to put a lower limit on $\delta$, as reported in Table 2, whereas good constraints are instead obtained for the seed-photon temperature $kT_e$ and the energy index $\alpha$. In Figure 3, we show the deconvolved unabsorbed source spectrum and best-fit two-compTB model. In estimating the 0.1–200 keV source luminosity, we assumed a distance of 2.8 kpc (Bradshaw et al. 1999).

3.2. GX 17+2

For GX 17+2, we use BeppoSAX (Boella et al. 1997a) observations performed on 1997 April 3, when the source was in the left part of the horizontal branch (HB; see Fig. 2 of F05) and a hard X-ray tail was present in the source spectrum. The data set is the same as previously used in the analysis of F05 and F07.

Following Di Salvo et al. (2000b) and F05, we fitted the 0.4–120 keV source spectrum with a photoelectrically absorbed BB plus TC (compTT) component, plus a power law to fit the hard X-ray tail, plus a Gaussian emission line around 6.7 keV. The best-fit parameters of the model are reported in Table 1. Assuming a distance to the source of 7.5 kpc (Penninx et al. 1988) and using the best-fit BB temperature and flux, we estimate a BB radius of $\sim 5$ km. Keeping in mind all the limitations in making
such an estimate (i.e., color–to–effective temperature correction, possible anisotropy in the emission), the order of magnitude of the BB radius supports the idea that this component very likely originates close to the NS surface. The PL photon index $\Gamma$ is the same as already reported in Di Salvo et al. (2000b) and F05.

In the next step, we have good constraints on the best-fit parameters of the pure thermal compTB component using the two-compTB model: they are reported in Table 2. Again, from the thermal $\alpha$- and $kT_e$-values, we estimate $\tau = 5.2$, close to the $\tau \sim 6$ found by compTT. The situation is a bit more critical in the thermal plus bulk-related compTB component. In particular, for the illumination factor log $A$ and the bulk parameter $\delta$, only lower limits (at 90% confidence) can be only placed by XSPEC, while, on the contrary, we obtain only an upper limit on the $\alpha$-value. These statistical uncertainties are mainly related to the small amount of high-energy ($\gtrsim 30$ keV) points, as can be seen in Figure 3, where we show the deconvolved source spectrum from the two-compTB model.

3.3. Cyg X-2

For Cyg X-2, we use a BeppoSAX data set that has already been analyzed by DS02. These data are related to a source observation performed on 1996 July 23. During this observation, the source traced the full HB across its color-intensity diagram. The HB was divided into two parts (upper and lower HB, respectively; see Fig. 2 of DS02), for which separate spectral analyses were carried out; in both cases a hard X-ray tail was detected.

In this paper we report results from the upper HB. DS02 fitted the $0.1-100$ keV continuum with a multicolor disk blackbody (DISKBB in XSPEC; Mitsuda et al. 1984) plus compTT, plus a power law, plus two Gaussian emission lines at $1$ and $6.7$ keV, respectively. We note however that DS02 used for the Low-Energy Concentrator Spectrometer (LECS; Parmar et al. 1997) the standard response matrix available online. The latter may give rise to spurious features when the instrumental count rate exceeds $50$ counts s$^{-1}$. For this observation, the LECS count rate is $60$ counts s$^{-1}$, thus requiring the production of an observation-related response function, which was generated using the LEMAT package (ver. 5.0.1).

With the newly produced matrix, we establish that there is in fact no more strong evidence for the $1$ keV emission line. On the other hand, an excess around $2.6$ keV is still found in the residuals from both the LECS and the Medium Energy Concentrator Spectrometer (MECS; Boella et al. 1997b). We fitted this excess with a Gaussian emission line, even though it is not clear whether its origin may be attributed to the source or arises from some instrumental effect. The results of the fit with a BB plus compTT plus PL model are reported in Table 1. The PL index $\Gamma$ comes out

![Figure 3](image-url)
steep than that reported by DS02. This is mainly due to the different response matrix used at low energies (<4 keV), namely, in the region where the interstellar absorption \(N_{\text{H}}\) and PL normalization must be simultaneously determined by XSPEC. We also estimated the BB radius, finding \(\sim 8\) km for a source distance of 8 kpc.

Replacing the above model with the two-compTB one, similarly to Sco X-1 and GX 17+2, we find very good constraints on the best-fit parameters related to the thermal compTB (see Table 2). As for the previous sources, the optical depths inferred from the best-fit \(\alpha\) and \(kT_e\) values and those estimated by compTT are very close (\(\tau \sim 6.3\) and \(\tau \sim 6.9\), respectively).

As concerns the thermal plus bulk parameters, unlike Sco X-1 and GX 17+2, where, even though poorly constrained, they can be left free in the fit, there is no way to perform a serious analysis with all the parameters left free. For Cyg X-2, we thus performed spectral fitting in several steps with \(\alpha\) fixed at each step in the range from 0.5 to 3.5, finding that the minimum in the \(\chi^2\) parameter space is obtained for \(\alpha = 2.5\). Fixing \(\alpha\) just allows us to put lower limits (90% confidence) on \(\delta\) and \(\log A\). In Figure 3, we report the deconvolved source spectrum from the two-compTB models.

### 3.4. GX 340+0

The spectrum of GX 340+0 is from a pointed BeppoSAX observation performed 2001 August 9/10. A more detailed description of the data reduction can be found in L04. The results with the BB plus compTT plus PL model, with an additional Gaussian emission line, are reported in Table 1. It is worth pointing out that the compTT model with slab geometry gives a slightly worse result than that with spherical geometry (for which we obtain \(\chi^2/\text{dof} = 160/137\)). We also emphasize the different values of the direct BB and compTT seed-photon temperatures (\(kT_{bb}\) and \(kT_W\), respectively) reported in Table 1 as compared with the results of L04. As discussed in F05, the BB plus compTT model usually adopted to describe the persistent continuum spectra of LMXBs may give rise to a dichotomy, given that two solutions (statistically indistinguishable) are possible, one with \(kT_{bb} > kT_W\) and the other with \(kT_{bb} < kT_W\). In the former case, the BB-like emission is suggested to mainly come from the NS surface, while in the latter it is presumably related to the accretion disk. These different interpretations are also suggested by the inferred BB radii, which are on the order of the NS radius in the former case and significantly greater (\(\sim 78\) km in L04) in the latter. On the basis of the accretion scenario proposed in F07, we prefer the first solution (see Table 1).

The results with the two-compTB model are instead reported in Table 2. In this case, the general considerations regarding the other three sources can be extended to GX 340+0: the constraints on the best-fit parameters of the thermal compTB component are very good, while they are worse for the thermal plus bulk-related one, except for the seed-photon temperature. Noticeably, the inferred \(\tau\) from the best-fit (thermal) \(\alpha\) and \(kT_e\) is lower (\(\tau \sim 6\)) than that directly estimated with compTT (\(\tau \sim 9\); see § 4). For the estimate of the bolometric (0.1–200 keV) source luminosity, we assumed a distance of 10 kpc (L04).

### 3.5. GX 3+1

For GX 3+1, we use data from the RXTE (Bradt et al. 1993) public archive related to a source observation performed on 2004 August 21. A joint spectrum from the second unit of the Proportional Counter Array (PCA; Jahoda et al. 2006) and the High-Energy X-Ray Timing Experiment (HEXTE; Rothschild et al. 1998) on board the spacecraft was obtained, following the standard procedure for data reduction. At high energies the source was detected by HEXTE only up to 30 keV. Similarly to the Sco X-1 INTEGRAL spectra, the low-energy threshold of 3 keV does not allow us to constrain the photoelectric interstellar absorption \(N_{\text{H}}\) in the source direction, so we fixed its value to \(1.7 \times 10^{22}\) cm\(^{-2}\) (Christian & Swank 1997).

We find that a simple photoelectrically absorbed compTT model is good enough to describe the source continuum spectrum. However, a clear excess in the 6–7 keV region reveals the presence of an iron emission line, which is fitted with a simple Gaussian. The poor instrumental resolution in this energy region, and the uncertainty in the calibration of the instrument because of the diffuse Galactic ridge emission, prevent us from simultaneously constraining all the line parameters. We thus fix the line energy centroid at 6.4 keV during the fit. Because of these uncertainties, both the line broadening \(\sigma\) and equivalent width values must be considered with some caution. The best-fit parameters of the compTT plus Gaussian model are reported in Table 1.

As in the case of Sco X-1 (§ 3.1), the absence of a direct BB-like component in the spectrum is very likely due to a combination of the low-energy PCA threshold (3 keV) and the data quality. Subsequently, replacing compTT with thermal compTB, we find the same statistical result, with the best-fit values reported in Table 2. The inferred optical depth of the TC region, obtained from the best-fit values of \(kT_e\) and \(\alpha\), is \(\tau \sim 4.9\), very close to that found by compTT (\(\tau \sim 5.3\); see Table 1). The bolometric 0.1–200 keV source luminosity is estimated assuming a distance of 5 kpc. We note that this is certainly an underestimate, as it includes the region below 3 keV not covered by the PCA, where a BB-like component has already been observed from the source (Oosterbroek et al. 2001).

### 3.6. GX 354–0

The spectral analysis for GX 354–0 was carried out using data obtained by F06 with JEM-X and ISGRI on board the INTEGRAL satellite. Among the nine spectra extracted by F06 as a function of the source’s position in the HID, we consider the hardest one (see Fig. 3 and Table 2 of F06). Unlike F06, however, we assume a spherical geometry for the plasma in compTT, given that it seems to be more representative for the hard-state accretion scenario. A simple photoelectrically absorbed compTT model is good enough to fit the source spectrum, and no soft X-ray emission is seen in the spectrum.

Replacing compTT with thermal compTB, we obtain the same results as from compTT, both in terms of statistics and as concerns the agreement between the seed-photon and electron temperatures (see Tables 1 and 2). Using equations (17) and (24) of TL95 for the spherical case, from the best-fit \(kT_e\) and \(\alpha\) we derive \(\tau \sim 1.5\), which perfectly matches the best-fit value directly obtained from compTT. The source bolometric 0.1–200 keV luminosity was computed assuming a source distance of 5 kpc (Di Salvo et al. 2000a; Galloway et al. 2003). Figure 3 shows the deconvolved source spectrum and superposed best-fit model.

### 4. DISCUSSION

We have presented a new model for the X-ray spectral fitting package XSPEC, named compTB. The principal aim of this model is to provide a more physical description of the transient PL-like hard X-ray (≥30 keV) tails observed in LMXBs, in the framework of the theory of bulk motion Comptonization. We recall however that compTB is actually not specific to bulk motion. Indeed, with the bulk parameter \(\delta\) set to zero, it reduces to a generalized thermal Comptonization model, in the same fashion as the widely used models compST and compTT.
4.1. Thermal Comptonization Component

In the six sources that we analyzed, the spectra are dominated by a TC component that accounts for most of the total energy budget (see Table 2 and Fig. 3). In the case of GX 3+1 and GX 354–0, this is in fact the only component observed in the X-ray spectrum.

In the GX 3+1 case, this is a result of the low-energy (~3 keV) threshold effect in the PCA, given that BB-like emission has been previously observed with BeppoSAX (Oosterbroek et al. 2001). On the other hand, in the case of GX 354–0 it is possible that this soft emission was intrinsically absent or very weak in the analyzed spectrum, given that it was detected when the source was in the intermediate state (see F06; see also Di Salvo et al. 2000a). F06 attributed the appearance of the soft BB-like emission in the soft state to the accretion disk. Instead, in our accretion scenario (as already reported in F05), the BB-like emission is related to the NS surface. Regardless of the emission region from which it comes, the fact that this component is not observed in the low/hard state spectrum of GX 354–0 is consistent with an accretion geometry in which the TC corona is significantly extended and almost spherical. This spherical configuration should completely cover the central object, intercepting all the seed photons coming from the NS surface.

For all the studied sources, we compute the Compton amplification factor (CAF), defined as the ratio of the Comptonized to seed-photon energy flux. Notably, the CAF takes the highest value in Cyg X-2 (3.5), despite the fact that its TC component provides the lowest fractional contribution to the total spectrum among the six studied sources (see Table 2). This is ultimately related to the fact that the seed-photon energy is softer ($kT_\gamma \sim 0.2$) in Cyg X-2 than in the remaining five sources (see Chakrabarti & Titarchuk 1995 for the CAF dependence on the seed-photon energy).

It is important to emphasize that the energy index $\alpha$ directly describes the efficiency of the Comptonization process (see, e.g., Bradshaw et al. 2007), regardless of the plasma geometry (i.e., slab or sphere). For a given geometry (taking into account hydrodynamic considerations), one can infer the Thomson optical depth $\tau$ using the best-fit $\alpha$- and $kT_\gamma$-values provided by compTT and equations (17) and (24) from TL95.

We find that the derived values are in very good agreement with those obtained from compTT, except for GX 340+0, where the $\tau$ from compTT (~9.2) is higher than that estimated using compTB (~5.8). This difference likely comes from the slightly different description of the Comptonization in the two models.

4.2. Combined Thermal and Bulk Comptonization Component

The bulk motion Comptonization process has been suggested as a possible origin of hard X-ray tails in LMXB systems hosting either a NS or a black hole (BH), on both observational (Shrader & Titarchuk 1998; Borozdin et al. 1999; P06; F07) and theoretical (TMK96, TMK97; Laurent & Titarchuk 1999, 2001; TF08) grounds. A model that can be used to describe these emergent spectra is already present in the XSPEC package (BMC; see TMK97), but as discussed in § 1, it presents some limitations, which are the lack of the electron-recoil term (cutoff) in the Green’s function and the parameterization of the high-energy Comptonized component in terms of just the spectral index $\alpha$ (no $\delta$ bulk parameter). In fact, the index $\alpha$ only carries information about the efficiency of Comptonization, no matter whether it is purely thermal or due to the combined effects of thermal and bulk Comptonization (see TMK97; Bradshaw et al. 2007).

We have tried to overcome these limitations by including the full Green’s function expression (eq. [3]) in our model and using a numerical convolution of the Green’s function with the seed photon spectrum (eq. [8]). One of the advantages of this numerical approach is that the condition $kT_e \ll E_p$ (seed-photon energy much less than plasma energy and Green’s function approximated as a broken power law; see ST80; TMK97) is not required. This model modification is, in fact, important in the spectra of high-luminosity NS LMXBs, where the seed-photon and plasma energy (electron temperature) differ by only a factor of a few (see Tables 1 and 2).

We applied our combined thermal plus bulk Comptonization model ($\delta > 0$) to the four LMXBs in our sample that show a PL-like hard X-ray tail, namely, Sco X-1, GX 17+2, Cyg X-2, and GX 340+0. In particular, we used a two-compTB model for these sources, in which the first compTB takes into account the purely TC part of the spectrum (see § 4.1) while the second describes both the BB-like emission observed at low energies and the PL-like emission above 30 keV. We assumed the same temperature $kT_e$ for the thermal and thermal-plus-bulk Comptonization region. The correctness of this assumption depends on how the two regions are physically and geometrically coupled and whether an actual temperature gradient exists in the region where Comptonization occurs.

In the accretion geometry proposed in F07, bulk Comptonization occurs in the innermost part of the transition-layer region, while TC is dominant in the outer TL and presumably in some extended region located above the accretion disk. The latter statement is suggested by the lack of direct soft (~0.5 keV) BB-like X-ray emission in the spectra, usually attributed to the accretion disk. It is likely that most of the disk emission is upscattered in the TL or somewhat embedded in the total spectrum, without a possibility of splitting it out as a single spectral component. We find some indication of a contribution from disk soft seed photons to the TC component using the best-fit values for the BB temperature of these photons. Note that this temperature is significantly lower than that of the BB-like seed-photon component presumably coming from the NS surface (see Tables 1 and 2 for compTT and compTB, respectively). In particular, for Cyg X-2 we obtain a value $\lesssim 0.2$ keV, in contrast to the higher values found by DS02 and Done & Gierliński (2003). Higher disk temperatures were also found by Gilfanov et al. (2003) and Revnivtsev & Gilfanov (2006).

However, we should point out that all these authors (except DS02) used RXTE PCA data, for which the low-energy band is higher than 3 keV, whereas our best-fit seed-photon temperatures are based on BeppoSAX spectra in the broad energy band from 0.4 to 120 keV. The PCA, with its low-energy threshold above 3 keV, cannot in principle determine any trace of the soft component whose temperature is lower than 1 keV, given that its maximum emission ($\gtrsim 3$ keV) would still fall outside the PCA energy band. On the other hand, the disk photon temperatures of 0.8–1.4 keV found by DS02 were inferred using a response matrix for the low-energy instrument (LECS) that is not correct for such bright sources and thus (as also testified by the strong decrease of the 1 keV emission line in our data) must be considered very cautiously.

The uncertainties in the bulk-related parameters of compTB (see Table 2) do not allow us to check the actual differences among them as far as the innermost bulk-dominated region is concerned, given the presence of only a few points at high energies ($\gtrsim 30$ keV) in the spectra of the four intermediate-state sources that were studied (see the top four panels of Fig. 3). However, we should note that in all cases $\delta$, through which the relative importance of BC to TC is parameterized, is of the order of tens, which values are expected in the case of efficient BC, for which the hard
PL-like emission should be seen. The fact that we cannot constrain the upper value (at 90% confidence) of δ is not surprising, because a measurement of the high-energy cutoff (not observed in the analyzed spectra) is required.

Indeed, the most important effect of the bulk term in the Green’s function is to push forward the rollover energy with respect to the pure TC case: higher δ-values lead to higher energies at which the spectrum exponentially falls. This can be seen clearly in Figure 1. On the other hand, it is possible to provide only a lower limit on δ if the high-energy behavior of the available data follows an unbroken power law. This is exactly what happens in the INTEGRAL IBIS spectrum of Sco X-1 (see Fig. 3).

In the case of GX 17+2, Cyg X-2, and GX 340+0, the thermal plus bulk compTB component predicts a rollover of the spectrum in the energy range 100–200 keV (see Fig. 3). However, it is not possible with the present data to really confirm this prediction; even the fact that XSPEC finds a minimum of δ in the χ² parameter space would not be indicative of this.

In the thermal plus bulk compTB component, the Green’s function energy index α is, for all sources, higher than that obtained for pure TC, with the highest difference observed in Sco X-1. In Figure 1, we plot the Green’s function for a monochromatic input line at energy x₀ = 0.3 (where x₀ ≡ E₀/kT₀) for two different values of α (1 and 2.5, respectively), considering pure thermal (δ = 0) and bulk-dominated (δ = 30) cases. The chosen value of x₀ is, in fact, representative of the NS local-environment conditions in this study, as kT_e ∼ 1 keV and kT_e ∼ 3 keV (see Tables 1 and 2). Looking at Figure 1, one can note that high δ-values push the high-energy cutoff forward, as mentioned above. It is evident that the higher α-values related to the BC region, with respect to those related to TC region (see Table 1), are indicative of the fact that in the innermost part, for all four sources, the TC efficiency is reduced but the BC effect dominates over the TC one, given that the power-law hard tail is detected.

4.3. Limitations and Applicability of the Model

When using compTB to fit the X-ray spectra of NS LMXBs, there are some issues that must be kept in mind. Let us first consider the case of pure TC spectra (δ = 0 in the Green’s function; see TMK97 and eq. [3] here). The Green’s function in the model is derived as a solution of the diffusion equation describing Comptonization, which is in turn obtained as an approximation to the Boltzmann kinetic equation in the diffusive regime. This means that the average photon energy exchange per scattering is small (∆E/E ≪ 1), and photons suffer many scatterings when traveling across the plasma before escaping.

This condition holds when the plasma temperature is about 20–30 keV or less but its optical depth is high, such as is observed in the persistent spectra of Z sources and high-luminosity atoll sources (see Barret 2001 for a review). In general, most of the NS LMXBs obey this condition, as the strong radiation flux coming from the central object acts as a thermostat in controlling, through Compton cooling, the plasma temperature (Barret et al. 2000).

In the case of black hole candidates (BHCs) the situation is quite different, and typical hard states for this class of sources are characterized by plasma temperatures of 60 keV (see, e.g., Remillard & McClintock 2006) and optical depths of a few. In this case, the diffusion approximation should be used with some care (the photon energy exchange for scattering is not so small, but the number of scatterings before photons escape is relatively small), and a different treatment of the Compton scattering should be addressed: models such as compPS (Poutanen & Svensson 1996), which use iterative numerical scattering procedures, are thus more suitable for fitting BHC hard-state spectra. This is one of the reasons why compTB (and also compST or compTT) should preferably be applied to NS LMXB spectra with plasma temperatures below 20–30 keV. In other cases, BHs or NSs in their hard/PL state (see § 3), the model can only fit the shape of the spectrum: while the result may be satisfactory in terms of χ²-value, the output parameters should be taken with caution.

Let us now concentrate on the case in which BC dominates over TC (δ ≪ 1). An analytical solution of the Comptonization equation (eq. [1]), for the Green’s function, does exist, neglecting only the term (1/(v_b/c²)) in equation (1), where Θ ≡ kT₀/mₑc². Whether this assumption is correct enough depends on the environment near the central object. In the case of systems hosting a NS, the presence of a rigid surface plays a key role in determining the global inward motion of the accreting material. At high accretion rates, the NS surface on one side is the source of strong radiation pressure due to kinetic energy release from the matter, but on the other hand its mirror-like behavior (reflecting inner boundary condition) is crucial in determining, at any radius, the pressure gradient due to local gravitational energy release from the infalling material.

In many cases, these competing effects may efficiently stop (or strongly decrease) the infall of matter, thus suppressing the bulk Comptonization effect (and associated hard X-ray tail). The numerical simulations in TF08 show that under these conditions the infall velocity v_b ≤ 0.2c, and typical values close to the NS surface are ∼0.1c. In this case, for an electron temperature kT_e = 3 keV, one has (v_b/c²)/Θ = 0.6.

In the case of BHCs, the different inner boundary condition at the event horizon (see, e.g., Titarchuk & Fiorito 2004) strongly modifies the radiation pressure’s behavior with respect to the NS case and may lead the inward bulk velocity v_b to reach values of ∼(0.3–0.4)c in the proximity of the last marginally stable orbit, even at accretion rates above the Eddington limit. Given that the typical temperatures for these states are similar to those of the NS intermediate and very soft states (e.g., Titarchuk & Fiorito 2004), namely, ∼3–5 keV, we find that (v_b/c²)/Θ may reach values of a few. A full kinetic treatment should be used to determine the Green’s function in this case (see Titarchuk & Zanni 1998; Laurent & Titarchuk 1999), because v_b ≳ 0.3c and the photon energy change is not small in the bulk flow into the BH.

The numerical solution of the Comptonization equation including the second-order bulk term shows that this term produces two net effects in the emergent spectrum: it pushes forward the high-energy cutoff and makes the Comptonized spectrum’s normalization higher (see Fig. 1 of TMK97). In terms of our model parameterization, given that the high-energy cutoff is not observed in the data, we would expect that this higher Compton normalization, which is actually due to (v_b/c²), would be somewhat compensated by a higher value for the illumination factor log A. The model can thus always fit the shape of the spectrum, whereas its best-fit parameters should be taken with some caution, as in the case of purely thermal, high-kT_e, TC spectra (see above).

A second issue to be pointed out is how one should define δ. The analytical form reported in equation (2) is derived for a plasma subject to a pure free-fall, radiation-corrected velocity profile, in which only gravity and radiation pressure from the central object are the concurrent processes. In particular, if the compact object is a BH, only gravity is present.

The condition that only gravity and radiation pressure are at work, in fact, obtains in the case of an optically thin medium,
where the radiation flux (from the NS) follows an $R^{-2}$ law. However, it is known that high-luminosity LMXB systems that exhibit hard X-ray tails (the six known Z sources and, recently, also GX 13+1; see P06) are characterized by high $M$-values (mainly inferred from their luminosity) and, consequently, optically thick environments. In this case, additional (and even dominant) effects due to radiation pressure from local gravitational and kinetic energy releases, viscous energy transport, and gas and magnetic pressure cannot be neglected. The numerical simulations in TF08 show that under these conditions, the radial velocity behavior of the infalling material deviates from a pure free-fall law across the whole TL. Nevertheless, in the innermost part of the TL itself, gravity and radiation pressure actually dominate and the velocity profile is almost that of a free-fall radiation-corrected one, and the definition of $\delta$ in equation (2) can be fairly safely used.

To strengthen this statement, we tried to make an independent estimate of $\delta$ as would be derived in the optically thin case (see Appendix). It is worth noting that in equation (A3), the only parameter that cannot be directly obtained from the observations is the dimensionless accretion rate $\dot{m}$. We find $\delta$-values of $155/\dot{m}$, $145/\dot{m}$, $178/\dot{m}$, and $169/\dot{m}$ for Sco X-1, GX 17+2, Cyg X-2, and GX 340+0, respectively. Keeping in mind the issues discussed above, the order-of-magnitude similarity of these $\delta$-values and those reported in Table 2 is interesting. Higher values of $\dot{m} > 1$ would further reduce this difference.

4.4. Similarities and Differences between CompTB and BW07’s Model

As pointed out in § 1, BW07 developed a TC and BC model for accreting X-ray pulsars. The radiative transfer form reported in equation (34) of BW07 is slightly different from that in equation (14) of TMK97, which we adopt here (eq. [1]). The difference mainly arises because of different assumptions about the velocity profiles in the two works.

A free-fall velocity profile $v(r) \propto r^{-1/2}$ is used in TMK97, which leads to $\tau(r) \propto r^{1/2}$ and $\ell(\tau) \propto \tau$ from the continuity equation (see TMK97; see also TMK96). This result is obtained keeping in mind that the surface area through which matter flows scales as $r^2$.

Instead, in the model of BW07 plasma motion channeled by the strong magnetic field is modeled as a flow passing through a cylindrical column whose area does not change with the distance $r$ from the NS polar cap (see their eq. [19]). Using this particular form of the continuity equation, BW07 show that $\tau(r) \propto r^{1/2}$. In order to analytically treat (by means of separation of variables) the problem, BW07 assume that $\tau(r) \propto \tau$ and thus $\ell(\tau) \propto r^{1/2}$. This direct, instead of inverse, proportionality between $\tau(r)$, $\ell(\tau)$, and $r$ would obviously modify the form of the spatial term in the Comptonization equation when writing it as a function of the optical depth $\tau$ instead of the distance $r$.

Moreover, we note that in BW07 the optical depth $\tau$ is defined such that $d\tau = N_e \sigma_T dr$ (where $\sigma_T$ is the Thomson cross section for photons propagating parallel to the magnetic field lines and $r$ is a radial length coordinate), while TMK97 used an effective optical depth, defined such that $\tau_{\text{eff}}(r_{\text{trap}}) = 1$, where $r_{\text{trap}}$ is the photon trapping radius. Looking at equations (1) and (5) of TMK97, it is evident that with such a parameterization $\tau_{\text{eff}}(r) \propto r^{-1/2}$ but $\ell(\tau_{\text{eff}}) \propto \tau_{\text{eff}}^{1/2}$.

It is worth noting that BW07 also neglect the second-order velocity bulk term $v_b/C_2$, which is essential in order to separate the Comptonization equation in space and energy. Another important difference between the two models is that BW07 study the BC effect in the presence of a strong magnetic field ($B \sim 10^{12}$ G), which modifies the electron cross section in the photon direction parallel and perpendicular to the magnetic field lines. However, this is not our case, as the class of sources we consider are characterized by $B$-fields on the order of $B \leq 10^8$ G (see Titarchuk et al. 2001 for details of the $B$-field determination in LMXBs). For such low magnetic field strengths, the $B$-field’s modification of the electron cross section can be neglected.

However, it is remarkable that for both models the Green’s function is a broken power law in the energy range where the recoil effect can be neglected (compare Fig. 3 here with Fig. 5 of BW07). Also common to these two models is that the high-energy power law is followed by an exponential turnover when the recoil effect is taken into account. This implies that the shape of the emergent spectrum is similar and can also be fitted by the XSPEC models compTT and BMC. The main difference is how the PL index of the Green’s function and the high-energy cutoff are related to the physical parameters of the particular model. We can consider our models to be somewhat complementary, as, starting from the same physical process but in very different environments, they are supposed to describe the emergent spectra for two classes of NS binary systems.

5. CONCLUSIONS

We have developed a new model for the X-ray spectral fitting package XSPEC (compTB), which can be considered a general Comptonization model and, more specifically, an extension of the bulk Comptonization model (BMC) already present in XSPEC. Similarly to BMC, our model consists of two components: one represents the BB-like seed photons that escape the plasma cloud without appreciable energy exchange, and the other gives the effectively Comptonized spectrum, obtained as the convolution of the system Green’s function with the seed input photon spectrum.

Using spectra from the BeppoSAX, INTEGRAL, and RXTE satellites, we have tested the model on a sample of six NS LMXBs that exhibit different spectral states: the intermediate state (Sco X-1, GX 17+2, Cyg X-2, and GX 340+0), in which a PL-like hard X-ray component appears over the stable continuum dominated by a thermal Comptonization, low-$kT_e$, high-$\tau$ spectrum; the very soft state (GX 3+1), similar to the previous one but without the PL-like component; and the low/hard state (GX 354–0), in which only thermal Comptonization from a hot plasma ($kT_e \sim 30$ keV, $\tau \sim 1.5$) is observed in the spectrum.

We find excellent agreement, insofar as fitting the TC components is concerned, between the best-fit parameters from compTB and those from compTT (the most widely used Comptonization model), which confirms the goodness of the code. On the other hand, one of the most important goals of compTB is to attempt a more physical approach, in the framework of the bulk motion Comptonization process, to explain the transient PL-like hard X-ray (\geq 30 keV) emission observed in NS LMXBs. This is what we have done in analyzing the spectra of the four intermediate-state sources. It is worth noting that bulk motion Comptonization is also supposed to be at work, with even higher efficiency, in BH systems in their soft states, where an unbroken power law extending up to MeV energies is usually observed.

We have shown that the values of the $\delta$-parameter, which represents the importance of bulk with respect to thermal Comptonization, obtained from the fits can be physically meaningful and can quantitatively describe the physical conditions of the environment in the innermost part of the NS systems. For three of the intermediate-state sources (GX 17+2, Cyg X-2, and GX 340+0),
the best-fit model predicts a high-energy cutoff in the spectrum in the 100–200 keV energy range, which is however not possible to confirm on the basis of the available data.

The next generation of high-energy missions, with their improved sensitivity and extended energy coverage, will definitively address this issue. On the other hand, the theoretical results coming from a full magnetohydrodynamic treatment of the TL region in NS and BH systems are very promising. We are planning to perform Monte Carlo simulations in which the derived radial velocity profiles for NS systems will be of key importance in predicting the position of the high-energy cutoff due to the BC effect, to then be compared with future observations.

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APPENDIX

ESTIMATION OF THE $\delta$-PARAMETER FROM OBSERVABLE QUANTITIES

The $\delta$-parameter for an optically thin, accreting plasma is given by

$$\delta \equiv \sqrt{1 - \dot{m}/\dot{m}_\text{Edd}},$$

(A1)

where $l \equiv F(R)/F_\text{Edd}(R)$ is the ratio of the actual to the Eddington flux at any distance $R$, $\dot{m}$ is the accretion rate in Eddington units, and $\Theta \equiv kT/e\mu m^2 c^2$ is the dimensionless electron temperature. In the case of an isothermal plasma, given that both $F(R)$ and $F_\text{Edd}(R)$ vary as $R^{-2}$, the parameter $\delta$ is independent of $R$. Let us define $F_\text{obs}$ as the flux measured by an observer at a distance $D$. This is related to the flux at a given distance $R$ as

$$F(R) = (D^2/R^2)F_\text{obs},$$

(A2)

Keeping in mind the definition of the Eddington flux, $F_\text{Edd} = GMc\mu/\mu kR^2$ (where $k = 0.4$ cm$^2$ g$^{-1}$ and $\mu$ is the effective electron molecular weight, which we take equal to 1.14, as for solar abundances), and using equation (A2), we may rewrite equation (A1) as

$$\delta = \left(1 - \frac{F_\text{obs} D^2 k/\mu GM}{\dot{m} c^2} \right)^{1/2} \frac{1}{\dot{m} \Theta}.$$  

(A3)

If we relate $F_\text{obs}$ to the BB-like flux coming from the NS, then we can estimate $\delta$ using the best-fit compTB parameters, by setting to zero the illumination factor $A$ or, correspondingly, $\log A \ll 1$. This would indeed provide just the flux of the direct BB-like component of the spectrum (see eq. [9]). On the other hand, the dimensionless parameter $\Theta$ can be estimated using the best-fit temperature $kT_e$. Thus, assuming a NS mass $M = 1.4 M_\odot$ and if the distance $D$ from the source is known, it is possible to estimate $\delta$ as a function of the mass accretion rate $\dot{m}$, which is the only quantity not directly measurable from observations.

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