Mixed convection heat transfer of nanofluid over microscale vertical duct preceded with a double-step expansion using Lattice Boltzmann Method

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Abstract
A numerical investigation of laminar mixed convection flow through a water–alumina nanofluid in a microscale vertical duct preceded with a double-step expansion has been executed. The governing equations are solved by using Lattice Boltzmann equation (LBE) with multiple-relaxation-time (MRT) collision model. The thermal conductivity and effective viscosity of nanofluid have been calculated by Brinkman and Maxwell models, respectively. To examine the effects of nanoparticles concentrations on the heat transfer and the flow behavior, the study has been carried out for the Reynolds number Re=10 to 40, Richardson number Ri=0 to 1.0 and the solid volume fraction 0 to 20%. The results obtained from Lattice Boltzmann modeling clearly show that the inclusion of nanoparticles into the base fluid produces a significant enhancement of the convective heat transfer, especially in the channel entry region. This enhancement increases as function of growing Reynolds number. In addition, the increase in Richardson number leads to decrease the solid concentration effect. Results also show that adding solid particles decreases significantly the fanning friction factor in mixed convection case.

Key words: Mixed convection, Nanofluid, LBE, MRT, Microscale, Vertical duct, Double-step expansion

1. Introduction

It is well known that nanofluids present major technological and economic challenges especially in heat transfer area. Recently, they have attracted great deal of interest and several studies are concerned with nanofluid flow (Wang and Mujumdar, 2007). Thus, it has been shown that adding nanometer-sized conductive particles, usually called nanoparticles, to a conventional fluid as water, ethylene glycol or engine oil leads to increase heat transfer coefficients. In order to evaluate the potential benefits of using nanofluids and to study their thermal and dynamical behavior, some experimental and numerical studies appears in recent years. Buongiorno, et al. (2009) conducted an international nanofluid property benchmark exercise (INPBE) by over 30 organizations worldwide. They used a variety of experimental approaches, including the transient hot wire method, steady-state methods, and optical methods to measure the thermal conductivity of identical samples of colloidaly stable dispersions of nanoparticles. Their results showed that the thermal conductivity of the nanofluids increases with particle concentration and aspect ratio as expected from classical theory and small systematic differences in the absolute values of the nanofluid thermal conductivity among the various experimental approaches. They also found that the effective medium theory developed for dispersed particles by Maxwell Garnett (1904) is in good agreement with the experimental data. In the same context, Kedzierski et al. (2010) reported viscosity data on a series of colloidal dispersions collected as part of INPBE. Their data are examined for seven different fluids that include
dispersions of metal-oxide nanoparticles in water, and in synthetic oil. They observed that enhancement in thermal conductivity was slightly larger for the spherical particle fluids and significantly lower for the rod-shaped particle fluids than predicted by effective medium theory. They also concluded that adding nanoparticles to the base fluid is negative in terms of heat transfer performance. Yu et al. (2012) conducted an experimental study to investigate the thermo physical properties and convective heat transfer of Al2O3-polyalpahophefin (PAO) nanofluids containing both spherical and rod-like nanoparticles. Their results showed that many parameters have significant impact on the effective properties of the nanofluids such as the particle volume fraction, the aspect ratio, the dispersion state and the aggregation of nanoparticles. Wen et al. (2004) reported an experimental work on the laminar convective heat transfer of nanofluids made of water and γ-Al2O3 nanoparticles. They concluded that the use of nanoparticles enhances the heat transfer and that enhancement increases significantly with Reynolds number.

Heat transfer in nanofluids has also attracted the attention of several numerical studies. Kherbeet al. (2014) conducted numerical study to investigate the laminar mixed convection flow of nanofluids over a 3D horizontal microscale forward-facing step. Their results revealed that the SiO2 nanofluid had the highest Nusselt number, which increased with decreasing nanoparticle material density, increasing volume fraction and decreasing nanoparticles diameter. Talebi et al. (2010) used finite volume method to study mixed convection flows in a square lid-driven cavity utilizing copper-water nanofluid. Their study has been carried out for different solid volume fraction Rayleigh and Reynolds number. They found that at fixed Reynolds number, for a higher Rayleigh number, the solid concentration affects the flow pattern and thermal behavior particularly. Their results showed also that the effect of solid concentration decreases by the increase of Reynolds number. The problem of laminar forced convection flow of nanofluids has been investigated using the finite volume method by Maiga et al. (2010). Their results showed that the inclusion of nanoparticles into the base fluids produced a considerable augmentation of the heat transfer coefficient. This enhancement increases considerably with an augmentation of the flow Reynolds number. They also provided correlations for computing the Nusselt number for the nanofluids considered in terms of the Reynolds and the Prandtl numbers. Rehena et al. (2012) analyzed heat transfer and fluid flow of natural convection in a vertical closed chamber filled with Al2O3-water nanofluid. Their results highlighted the range where the heat transfer uncertainties can be affected by the volume fraction of the nanoparticles. They also developed a correlation for the average Nusselt number as a function of the cavity aspect ratio. Abu-Nada (2008) used the same technique (FVM) to investigate heat transfer over a backward facing step. Numerical results registered an enhancement in Nusselt number at the top and bottom walls except in the primary and secondary recirculation zones. His study showed also an increase in the average Nusselt number with the volume fraction of nanoparticles for the whole range of Reynolds number. Parvin et al. (2012) studied numerically the thermal conductivity variation on natural convection flow of water–alumina nanofluid in an annulus. They discussed the presence of nanoparticles, the Prandtl number and the Grashof number effects on the flow and heat transfer characteristics for the Chon model (Chon et al., 2005) and the Maxwell Garnett (MG) model (Maxwell Garnett, 1904) two nanofluid models. They found that the heat transfer improvement accentuated with nanoparticles volume fraction and Prandtl number at moderate and large Grashof number. Nemati et al. (2010) investigated numerical simulation of mixed convection flows lid-driven cavity by using Lattice Boltzmann for nanofluid. Their results indicated that the effects of solid volume fraction grow stronger sequentially for Al2O3, CuO and Cu, and also they found that the increases of Reynolds number leads to decrease the solid concentration effect. Etminan-Farooji et al. (2012) studied numerically forced convection for a laminar and steady free stream flow of nanofluids past an isolated square cylinder. Their results showed that the heat transfer improvement is more evident in flows with higher Peclet numbers and higher particle volume concentration. Sarkar et al. (2013) studied mixed convective flow of nanofluids past a square cylinder in vertically upward flow by a stabilized SUPG based finite element technique. They used dynamic mode decomposition (DMD) technique (Schmid, 2010) to analyze the dynamics of coherent structures of the flow. Their analysis showed that Energy content in the mean flow of the base fluid at Richardson number of -0.5 is maximum compared to that of nanofluid. Sarkar and Ganguly (2013) used finite volume method to study the buoyancy driven mixed convective flow and heat transfer characteristics of water-based nanofluid past a square cylinder in vertically upward flow. Copper (Cu) and alumina (Al2O3) particles was used with volume fractions 0<φ<20%. They studied the effect of aiding and opposing buoyancy flow by considering the Richardson number (Ri) range -0.5 < Ri < 0.5. They found that at a fixed φ the time averaged local Nusselt number is higher for Cu-water nanofluids, as compared to Al2O3-water nanofluids. They also found that the average Nusselt number
increases with the concentration. Cu-water based nanofluids show higher magnitudes of $\overline{Nu}$ compared to Al$_2$O$_3$-water nanofluids.

Although heat transfer of nanofluids including thermal conduction and convective heat transfer has been well studied and reviewed in the literature, little attention has been paid in the past decade to the rheological behavior of nanofluids. Chen et al. (2009) conducted experimental study on ethylene glycol (EG)-based nanofluids containing 0.5-8.0 % spherical TiO$_2$ nanoparticles and the theoretical analyses on the high shear viscosity, shear thinning behavior and temperature dependence. Their results show that the EG-based nanofluids are Newtonian under the conditions of their work with the shear viscosity as a strong function of temperature and particle concentration. However, the relative viscosity of the nanofluids is independent of temperature. Sarit et al. (2003) used pool boiling in Al$_2$O$_3$-water to characterize the rheological behavior of the nanofluids. Their results indicate that the nanoparticles have pronounced and significant influence on the boiling process deteriorating the boiling characteristics of the fluid. Their results also confirm a Newtonian behavior of the fluid between 1% and 4% particle volume concentration. Other reported studies found non-Newtonian behavior of nanofluids. Tseng and Lin (2003) studied the suspension structure and the rheological behavior of Anatase titanium dioxide (TiO$_2$) nanoparticles dispersed in pure water with a range of volumetric concentrations. Their results revealed that the nanoparticle suspensions generally exhibited pseudo plastic flow behavior, indicating an existence of particle aggregations in the liquid medium. It must be mentioned that there is some inconsistency of results reported by the researchers in these topics. The reader can refer to Chen et al. (2007) and Yu et al. (2008) to find out more details.

Thermal performance and rheological behavior of nanofluids needs further research works to provide evidence of their effectiveness and feasibility. However, no general correlations have been reported in the literature due to lack of common understanding on heat transport mechanisms of nanofluid. On the other hand, separated flows encountered in backward facing step affect considerably the performance of heat transfer in microchannels. This aspect has been thoroughly investigated for conventional fluids. Nevertheless, little importance has been accorded to this flow configuration using nanofluids. The present study deals with laminar mixed convective nanofluid flow over microscale vertical duct preceded with a double-step expansion. The main objective of this study is to illustrate the effect of nanofluid concentration, Reynolds number and Richardson number on the flow and heat transfer characteristics. The velocity and temperature fields are solved by Lattice Boltzmann Method with multiple relaxation time (MRT-LBE). This method has been considered by many studies as useful method to simulate heat transfer and fluid flow. Therefore, the secondary objective of this work is to investigate the model effectiveness for mixed convection nanofluid flow and to predict physical phenomena, such as the heat transfer, pressure drop, and flow patterns, in microchannels flow configuration.

Nomenclature

- $c$ lattice streaming speed.
- $c_i$ discrete velocity vector.
- $C_p$ isobaric specific heat of the fluid.
- $g$ gravitational acceleration.
- $Gr$ Grashof number $Gr = g \beta \Delta T h^3 / \nu^2$.
- $h$ step height.
- $f$, $f_i$ local and overall average fanning friction factor.
- $f_i$ particle distribution function.
- $f_i^{eq}$ equilibrium distribution function for $f_i$.
- $g$ gravitational acceleration.
- $g_i$ energy distribution function.
- $g_i^{eq}$ equilibrium distribution function for $g_i$.
- $Ma$ Mach number in the simulation.
- $c_s$ speed of sound.
- $e$ internal energy.
- $F$ external force term.
- $N_x$, $N_y$ grid size in x and y-direction, respectively.
- $Nu$, $\overline{Nu}$ local and average Nusselt number on wall surface.
- $k_i$ thermal conductivity ($W.m^{-1}.K^{-1}$).
- $Pr$ Prandtl number $Pr = C_p \mu / k$.
- $q_x$ heat flux in x direction.
- $r$ position vector of lattice node.
- $Re$ Reynolds number $Re = u_0 h / \nu$.
- $Ri$ Richardson number $Ri = Gr / Re^2$.
- $T$ temperature ($K$).
- $X$ dimensionless transverse coordinate $X = x/h$. 

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2. Problem description and macroscopic equations

The problem under investigation is, as sketched in Fig. 1 and detailed as the following. An alumina-water nanofluid is injected into the channel at a cooler temperature \( T_0 \) and with a fully developed velocity profile. The length of inlet channel is equal to \( 5 \times h \) and the larger channel width is \( 3 \times h \). The upstream portion of the two stepped walls have the length of \( h \), the channel expansion ratio is taken equal to 30 and the overall length of the computational domain is \( L=35 \times h \).

The centerline velocity \( u_0 \) at the inlet and the fluid viscosity is taken so that the Mach number in the simulation \( Ma \) is within the limit of incompressible flow. Thus, the magnitude velocity for both the buoyant and forced flows are taken small compared with the speed of sound so that the \( Ma<<1 \). The walls downstream of the step are maintained at a constant temperature \( T_1 \), while the other walls are treated adiabatically. Although in this study we will focus on dimensionless numbers rather than on the actual values of physical dimensions, the step height and the total channel length will be considered in (μm) based on the previous experimental and numerical studies on microchannels published by Kherbeet et al. (2014), Hong et al. (1993) and Bianco et al. (2009). We assume that the channel height is sufficiently small so that the flow can be considered as laminar and hydrodynamically fully developed at the entrance region of the channel. An example of application of such geometric configuration is the cooling of electronic equipment like microprocessors and Micro-Electro-Mechanical Systems. The reader can refer to Saidur et al. (2011) and references therein for further applications.

Water and solid particles are assumed to be in a thermal equilibrium state and flow with the same velocity. Therefore, the mixture (AL\(_2\)O\(_3\)-water) behaves similar to a single phase liquid. The physical properties are taken to be constant except for the density. The effective density is given by (Corcione, 2010):

\[
\rho_n = (1 - \varphi) \rho_f + \varphi \rho_s
\]  

(1)

Where \( \varphi \) is solid volume fraction, \( \rho_f \) and \( \rho_s \) are respectively the water and nanoparticles density, the specific heat of nanofluid is calculated as (Corcione, 2010):

\[
\left( \rho C_p \right)_n = (1 - \varphi) \left( \rho C_p \right)_f + \varphi \left( \rho C_p \right)_s
\]  

(2)

To determine the dynamic viscosity of the nanofluid, we choose the Brinkman relation (Brinkman, 1952).

\[
\mu_n = \frac{\mu_f}{(1 - \varphi)^{2.5}}
\]  

(3)
Assuming that spherical particles are well-dispersed in the water, the effective thermal conductivity is approximated by the Maxwell-Garnetts model (Maxwell Garnett, 1904), also known as the Wasp model (Eastman et al. 2004), as follow:

$$k_s = k_f + 2k_f - 2\varphi(k_f - k_s)$$

The thermal expansion coefficient of nanofluid is calculated as follow:

$$\beta_n = (1 - \varphi)\beta_f + \varphi\beta_s$$

Using the previous expressions, the governing equations in dimensional form used in the present study are as follows:

Continuity equation:

$$\nabla \cdot \mathbf{u} = 0.$$  

Momentum equation:

$$\mathbf{u} \nabla \cdot \mathbf{u} = -\frac{1}{\rho_n} \nabla p + \nu_n \nabla^2 \mathbf{u} + \beta_n (T - T_n) \mathbf{g}.$$  

Energy equation:

$$\mathbf{u} \nabla T = \alpha_n \nabla^2 T.$$  

Where $\nu_n$ and $\alpha_n$ are the effective kinematic viscosity and thermal diffusivity respectively.

3. Numerical technique

As it was mentioned before, the numerical technique used in this study is the Lattice Boltzmann Method which is a relatively modern approach in Computational Fluid Dynamics. In this section, a brief explanation seems to be needed to clarify the main reason for employing this method in solving such problems of mixed convection in nanofluids.

The lattice Boltzmann method, commonly called LBM, is a new efficient alternative for numerical simulation of single and multi-phase flow problems (Mohamad, 2007 and Arcidiacono et al., 2007). Compared with other existing numerical techniques, the advantages of LBM include easy handling of complex geometries and boundary conditions (Chang et al., 2009) and efficient implementation for parallel computation (Yasuda et al., 2011). Thus, several studies have been carried out in recent years to validate the approach for many heat transfer (Mishra and Roy, 2007) and reactive flow cases (Hou et al., 1997). However, more studies are still needed to improve the stability of the mass, momentum and energy-conserving LB models. The single-relaxation time (SRT) approach based on Bhatnagar-Gross-Krook (BGK) approximation is the most popular lattice Boltzmann model. This approach derived from the Enskog equation (Mohamad, 2007 and references therein). In this model all variables (density, momentum, energy, Diagonal and off-diagonal stress) require the same time to return to their equilibrium state. However, this model is restricted to Pr=1. To avoid this restriction, some authors for example, (Hou et al., 1997) and (Ginzburg et al., 2008) used thermal lattice Boltzmann with two relaxation times that contained also an error in the energy equation due to the fact that dissipation and transport of energy by viscous forces was multiplied by thermal conductivity rather than by the viscosity. To remove these defects, Mai et al. (2010) proposed a thermal BGK lattice Boltzmann model for flows with viscous heat dissipation. The viscous dissipation rate is obtained by computing the second-order moments of non-equilibrium distribution function. Although their numerical results showed that the second order accurate LBM scheme is not degraded by the thermal BGK model, the SRT-LBM with most of its schemes still remains suffering from numerical instability at high Reynolds number and (or) Rayleigh number (McNamara et al., 1995). To solve this problem, the multiple relaxation times (MRT) model presents the superior scheme in terms of accuracy, stability, and computational efficiency (Luo et al., 2011). In such model each moment (that is to say, each physical parameter in momentum and energy equations) has a characteristic time. Dubois and Lallemand (2009) extended the Taylor expansion method and established equivalent partial differential equations of the LB scheme proposed previously by D’Humières (1992) at an arbitrary order of accuracy. They derive the associated dynamical equations for classical thermal and linear fluid models in one to three space dimensions. They obtain an explicit
formula, which enable them to tune some parameters of the LB scheme to reach, up to fourth-order accuracy, shear waves. Stokes flow problems are used to compare their results with analytical references. Mezrhab et al. (2010) developed two-dimensional double Multiple Relaxation Time-Thermal Lattice Boltzmann Equation (2-MRT-TLBE) method to predict convective flows in a square differentially heated cavity. Their results showed that their method is suitable and numerically stable at high Rayleigh numbers. Wang et al. (2013) employed the lattice Boltzmann equation with MRT collision model to simulate incompressible thermo-hydrodynamics with Pr = 0.71 and Pr=7.0. They demonstrated that the MRT-TLBE scheme is second-order accurate for incompressible thermo-hydrodynamic flows with the Boussinesq approximation. Their results also showed that the Mach number has little effect on the accuracy of the MRT-TLBE simulations.

In spite of its remarkable success, MRT-LBM has several aspects that need to be tested on nanofluid flow. Thus, a few investigations of heat transfer in combined natural and forced convection problems using this model are found in the literature and the majority of the published works focus on either natural convection (Re=0) or forced convection (Gr=0). In this study, we aim to investigate the MRT-LBM capability to simulate the buoyancy assisted flow utilizing Al₂O₃-water nanofluid and to predict the effects of the presence of solid particles on the flow and thermal behavior. In the following section, we describe the solution procedure of recovering Navier Stockes and energy equations using the lattice Boltzmann equation (LBE) with multiple-relaxation-time (MRT) collision model.

### 3.1. MRT LBE model for dynamic and thermal fields

In the LBM, the movement of particles is restricted to a limited number of directions. We consider the case of two-dimensional model with nine discrete velocities, commonly called D2Q9 model. In this study, we use the D2Q9 model on a square lattice $\delta_x \times \delta_y$, of which the discrete velocity set $c_i$ is as follows:

$$c_i = \begin{cases} 
(0,0), & i = 0, \\
(\pm 1,0)c, (0,\pm 1)c & i = 1-4, \\
(\pm 1,\pm 1)c, & i = 5-8.
\end{cases}$$  \hspace{1cm} (7)

The time step $\delta_t$ is chosen so that the reference speed $c = \delta_x / \delta_t = \delta_y / \delta_t$ (in lattice units) remains constant throughout the study. The particle distributions defined for the finite set of the discrete particle velocity vectors $c_i$ at a site $r$ at time $t$ is denoted $\{f_i (r,t), i = 0-8\}$. The discrete evolution describing mass and momentum conservations is obtained by the streaming from the nearest neighbors and the collision. These two steps are described by the governing LBE in a concise vector form (Dubois and Lallemand, 2009).

$$f(r + \vec{c}_i \delta_t, t + \delta_t) = f(r,t) - M^{-1} . S . \{m - m^{eq}\}(r,t) + \vec{F}(r,t)$$  \hspace{1cm} (8)

The used notations are defined as follows:

$$f(r,t) = (f_0(r,t), f_1(r,t), ..., f_8(r,t))^T,$$
$$m(r,t) = (m_0(r,t), m_1(r,t), ..., m_8(r,t))^T,$$
$$m^{eq}(r + \vec{c}_i \delta_t, t) = (m_0^{eq}(r + \vec{c}_i \delta_t, t), m_1^{eq}(r + \vec{c}_i \delta_t, t), ..., m_8^{eq}(r + \vec{c}_i \delta_t, t))^T,$$
$$\vec{F} = (F_0, F_1, ..., F_8)^T.$$

In this evolution equations, $\dagger$ is the transpose operator, $f(r,t)$ is the 9-dimensional vectors for the distribution functions, $m(r,t)$ and $m^{eq}(r,t)$ are the velocity moments and their equilibrium functions, respectively. $M \in \mathbb{R}^{9\times 9}$ is the transformation matrix such that $f = M^{-1} m$ and $S = diag(s) \in \mathbb{R}^9$ is the diagonal relaxation matrix. The nine macroscopic moments are arranged in the following order:

$$m = (\rho, e, \varepsilon, j_x, j_y, q_x, j_z, q_y, p_{xx}, p_{yy})^T$$  \hspace{1cm} (10)
Where $\rho$ is the density, $e$ is the second-order moments corresponding to energy, $\varepsilon$ is the fourth-order moment of energy square, $(j_x, j_y) = \rho (u, v)$ is the flow momentum taken in LBM unit, $(q_x, q_y)$ is related to the energy flux. $p_{xx}$ and $p_{yy}$ are the $x$ and $y$ components of diagonal and off-diagonal of the stress tensor, respectively.

Since we are only interested in incompressible fluids, the fluctuation of density $\rho$ about $\rho_0$ and the flow momentum $j$ are obtained through moment summations in the velocity space as follows:

$$\rho - \rho_0 = \sum_i f_i (r, t)$$

$$j = \rho_0 u = \sum_i c_i f_i (r, t)$$

This approximation is used for incompressible flows i.e. especially when $\varepsilon / \rho_0 \ll 1$.

Using this order of moments $M$ maps the distribution $\{f_i\}$ to the velocity moments $\{m_i\}$ and can be easily constructed:

$$M = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\
4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\
0 & -2 & 0 & 2 & 0 & 1 & -1 & 1 & 1 \\
0 & 0 & 1 & 0 & -1 & 1 & 1 & 1 & 1 \\
0 & 0 & -2 & 0 & 2 & 1 & -1 & 1 & 1 \\
0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 1
\end{bmatrix}$$

Since the row vectors of $M$ are orthogonal, $M$ is a diagonal matrix and its inverse can be computed easily via the Gram–Schmidt orthogonalization procedure according to the formula:

$$M^T (M M^T)^{-1} = M^{-1}$$

The equilibrium moments are chosen to be polynomial functions of the conserved moments whose expressions result from calculating the moments of the Maxwell-Boltzmann distribution function. To ensure symmetry, isotropy and Galilean invariance, they can be written concisely as follows:

$$e^{(eq)} = \frac{3}{\rho_0} (j_x^2 + j_y^2) - 2\delta \rho,
\quad e^{(eq)} = -\frac{3}{\rho_0} (j_x^2 + j_y^2) + \delta \rho.$$

$$q_x^{(eq)} = -j_x,
\quad q_y^{(eq)} = -j_y.$$

$$p_{xx}^{(eq)} = \frac{1}{\rho_0} (j_x^2 - j_y^2),
\quad p_{yy}^{(eq)} = \frac{1}{\rho_0} j_x j_y.$$ 

The mean density $\rho_0$ is usually set to be unity (in lattice unit) in all simulations. The equilibria of the conserved moments are themselves, thus $m_0^{(eq)} = \delta \rho$ and $(m_0^{(eq)}, m_x^{(eq)}) = (j_x, j_y)$. The diagonal relaxation matrix is built from the relaxation rates $s_i$ and with the ordering of $\{m_i\}$ described in (10), it is given by:

$$S = \text{diag} (0, s_x, 0, 0, s_y, 0, s_x, s_y, s_x, s_y)$$

The choice of the relaxation parameters $s_i$ are determined by a linear stability analysis (Hou et al., 1997) and can be written as the following:

$$s_x = \frac{2}{1 + 6 \zeta},
\quad s_y = \frac{2}{1 + 6 \upsilon},
\quad s_x = \frac{2 - s_x}{8 - s_x}.$$
\( \nu \) and \( \zeta \) are the shear and the bulk viscosities. If we set \( s_1 = s_2 = s_3 = s_4 = s_5 = s_6 = s_7 = s_8 = 1/\tau \), the MRT-LBE model reduces to the SRT-LBE, also known as the BGK Model of which the equilibrium distribution functions are:

\[
f^{(m)}_i = \omega_i \left[ \delta \rho + \delta c_i \frac{\langle c_i u \rangle}{c_i} + \frac{1}{2} \left( \frac{\langle c_i u \rangle^2}{c_i^2} - \frac{\langle c_i u \rangle}{c_i} \right) \right]
\]

(18)

The weighting factors \( \omega_i \) are given by:

\[
\omega_0 = 4/9 , \omega_{1,2,3,4} = 1/9 \quad \text{and} \quad \omega_{5,6,7,8} = 1/36
\]

(19)

\( c_s \) is the lattice speed of sound, it remains constant throughout this study with \( c_s^2 = 1/3 \). The forcing term \( F \) in the LBE is implemented by using a splitting scheme Wang and al. (2013) and will be described later.

To solve advection–diffusion equation for the temperature, we use thermal MRT-D2Q5 model. We consider only five discrete velocities \( c_i, i = \{0 - 4\} \) and the corresponding distribution functions \( g_i \). The thermal evolution equation can be written as the following:

\[
g (r + \delta t + \delta \xi) = g (r,t) - N^{-1} Q [n (r,t) - n^{eq} (r,t)]
\]

(20)

The transformation matrix \( N \) is given by:

\[
N = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 \\
-4 & 1 & 1 & 1 & 1 \\
0 & 1 & -1 & 1 & -1
\end{pmatrix}
\]

(21)

The diagonal relaxation matrix \( Q \) in Eq. (20) is given by:

\[
Q = \text{diag} (0, \sigma_a, \sigma_a, \sigma_v, \sigma_v)
\]

(22)

where \( \sigma_a \) and \( \sigma_v \) are a relaxation rates satisfying the constraint \( 0 < \{\sigma_a, \sigma_v\} < 2 \) to get a numerically stable scheme. Temperature in LBM unit is computing by conserving only the first moment:

\[
n_0 = T (r,t) = \sum_{i=0}^{4} g_i (r,t)
\]

(23)

The equilibrium moments can be expressed as follows:

\[
\begin{align*}
n_0^{(m)} &= T, \quad n_1^{(m)} = uT, \quad n_2^{(m)} = vT, \quad n_3^{(m)} = aT, \quad n_4^{(m)} = 0
\end{align*}
\]

(24)

Where \( u = (u_s, u_v) \) is computed from Eq. (12) and \( a \) is a constant to be determined. Based on the Taylor expansion method, the TLBE scheme describes the following partial differential equation (d'Humières, 1992).

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = c_s \frac{\partial^2 T}{\partial x^2} + \frac{1}{\sigma_a} - 0.5 \frac{\partial^2 T}{\partial y^2} + 0 (\delta^2)
\]

(25)

Since we want to find the advection–diffusion equation, we take the following relationship between \( \sigma_a \), \( a \) and the thermal diffusivity \( \alpha \):

\[
\alpha = c_s \frac{\delta (4 + a)}{10} \left( \frac{1}{\sigma_a} - 0.5 \right)
\]

(26)

In order to avoid the so-called "checkerboard" type instability and to attain the isotropy for the fourth-order (error) term resulting from Eq. (25), we take the following relationship between the two relaxation rates \( \sigma_a \) and \( \sigma_v \).

\[
\frac{1}{\sigma_a} = \frac{1}{\sigma_v} = 0.5 + \sqrt{3}/3
\]

(27)
With the above choice and in the particular case \( \delta = c = 1 \) (in lattice unit) the relationship between \( a \) and \( \alpha \) is given by:

\[
a = -4 + 60\alpha / \sqrt{3}
\]  

(28)

In this way, the LBE scheme describes the following advection-diffusion equation:

\[
\frac{\partial T}{\partial t} + \mathbf{u} \nabla T = \alpha \Delta T + 0(\varepsilon^2)
\]  

(29)

Since we work with small temperature differences, it is reasonable to neglect the viscous dissipation and pressure work. Therefore, the density is approximated by the linear Boussinesq model.

\[
\frac{\rho - \rho_0}{\rho_0} = \beta (T - T_0), \quad \beta = \frac{1}{\rho_0} \frac{\partial \mu}{\partial T},
\]  

(30)

\( T_0 \) is the mean temperature and \( \beta \) is the thermal expansion coefficient. The buoyancy force term is then given by:

\[
F = \rho_0 \beta (T - T_0) \mathbf{g} = -\rho_0 \beta (T - T_0) \mathbf{g} y.
\]  

(31)

\( \mathbf{g} \) is the gravity with \( g = 9.81 \text{ m s}^{-2} \) and \( y \) is the unit vector in the \( y \) axis in Fig.1. The body force \( F \) is projected onto velocity space as:

\[
F_i = -3\omega_i \frac{\mathbf{c} \cdot F}{c^2} = 3\omega_i \rho_0 g \beta (T - T_0) \frac{\mathbf{c} \cdot y}{c^2}
\]  

(32)

### 3.2. Boundary conditions

As detailed in section 2, at the inlet, a fully developed parabolic profile is enforced. The boundary condition proposed by Zou and He (1997) is applied. It is based on the idea of bounce-back on non-equilibrium part. At the inlet, \( f_0, f_1, f_3, f_4, f_7 \) and \( f_8 \) are known from the streaming step. The bounce back rule for the non-equilibrium part of the particle distribution normal to the inlet gives:

\[
\begin{align*}
    f_2 &= f_4 + \frac{2}{3} \rho_0 v_0 \\
    f_5 &= f_7 + \frac{1}{2} (f_7 - f_1) + \frac{1}{6} \rho_0 v_0 \\
    f_6 &= f_8 + \frac{1}{2} (f_8 - f_1) + \frac{1}{6} \rho_0 v_0
\end{align*}
\]  

(33)

At all walls, bounce-back boundary conditions were applied. In this case, the incoming distribution functions turned back to the site where they are from. Thus, incoming boundary populations in the direction \( \mathbf{\hat{i}} \) are equal to out-going populations in the opposite direction \( \mathbf{-\hat{i}} \), i.e., on lattice boundary sites, one reverses the velocity of a particle which wants to enter a solid area.

\[
f_i(\mathbf{r}, t) = f_i(\mathbf{r}, t), \quad i = 1, \ldots, 8
\]  

(34)

The streamwise gradient of the velocity in the outlet the artificial open boundary condition is set to zero by proportionally copying the fields from the before last column. The densities to be specified on the outlet boundary are computed from the following relationship:

\[
f^n_{i} \left( \mathbf{r}_{o_i} \right) = f^n_{i} \left( \mathbf{r}_{o_{i-1}} \right)
\]  

(35)

For thermal boundary conditions, two types of boundary conditions are used. One is the isothermal boundaries and the other is the adiabatic boundaries. For isothermal boundaries (Dirichlet type boundary condition) the normal equilibrium condition (Zou and He, 1997) was used to determine the unknown densities. For example, at the left wall boundary, the density to be specified is \( g_1 \). So, to determine these densities we use the following condition:

\[
g_1 + g_3 = g_1^{eq} + g_3^{eq}
\]  

(36)
Since the equilibrium distributions at the boundary nodes are:

\[ g_n^a = \left[ N^{-1} \right]_{i,j} (T_i, uT_j, vT_i, aT, 0)^T \]  

(37)

The above equation becomes:

\[ g_i = -g_i + \left( \frac{2}{5} + \frac{1}{10} a \right) T_i \]

(38)

Note that this implementation guarantees constant temperature and null heat conduction along the boundary i.e.

\[
\left\{ \begin{array}{l}
T^{bc} = T_i \\
\sum_{i=0}^{4} \Delta T^{bc} = 0
\end{array} \right.
\]

(39)

Where the superscripts be denote before collisions.

For adiabatic walls, the heat flux is given by \( \frac{\partial T}{\partial y} = 0 \), the following condition is adopted:

\[ g_z (r,t) = g_s (r,t) \]

(40)

4. Results and discussions

The computational domain of size \((35h) \times (3h)\) was discretized with a uniform Cartesian grid \(N_x \times N_y\), where \(N_x\) and \(N_y\) are the numbers of lattice nodes in \(x\) and \(y\)-direction, respectively. Initially the flow is isothermal and quiescent, i.e., \(\theta = 0\) and \(u = 0\). Results are assumed to be converged when the following conditions are satisfied.

\[
\sum_{i,j} \left| u ((i,j), t + 1000) - u ((i,j), t) \right| < 10^{-4}, \quad (41.a)
\]

\[
\max \left| (i,j), t + 1000 \right| \theta ((i,j), t) - \theta ((i,j), t) \right| \leq 10^{-6}. \quad (41.b)
\]

Where \(\theta\) is obtained with the scaling \(\theta = (T - T_0)/\Delta T\) and the summation is over the entire system. The quantities to be computed in this study include Nusselt number and fanning friction coefficient. Since the MRT-LBM is second order accurate (Wang et al., 2013), we use the Taylor expansion of \(\theta\) around the wall location up to second order. Therefore the local heat flux in the horizontal \(x\) direction requires values of \(\theta\) at three successive points that are \(\theta(0,j), \theta(1,j)\) and \(\theta(2,j)\) i.e.,

\[
q_x = -\frac{\partial \theta}{\partial x} \bigg|_{x=0} = \frac{3\theta(0,j) - 4\theta(1,j) + \theta(2,j)}{2\delta x} + O(x^2)
\]

(42)

With \(\delta x = 1\) (in lattice unit), Eq. (42) will be used to compute the local and the average Nusselt numbers on wall, i.e., the local and mean heat flux through the left side wall and along the channel height:

\[
Nu = \frac{k_s}{\lambda} \frac{\partial V}{\partial X} \bigg|_{x=0} \approx \frac{k_s}{\lambda} \frac{3\theta(0,j) - 4\theta(1,j) + \theta(2,j)}{2} \quad (43.a)
\]

\[
\overline{Nu} = \frac{k_s}{\lambda} \frac{1}{(M+1)} \sum_{j=N_y}^{N} \frac{3\theta(0,j) - 4\theta(1,j) + \theta(2,j)}{2} \quad (43.b)
\]

Where \(M\) is the \(y\)-location (lattice number in \(y\) direction) of the two horizontal adiabatic walls. Then the same procedure, as for the Nusselt number, has been taken for determining the local and average fanning friction factors.

\[
f = -\frac{\partial V}{\partial X} \bigg|_{x=0} \approx -\frac{V(2,j) + 4V(1,j) - 3V(0,j)}{2}. \quad (44.a)
\]
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\[
\overline{f} = \frac{1}{(M+1)} \sum_{j=N_y-M}^{N_y} \frac{-V(2, j) + 4V(1, j) - 3V(0, j)}{2}
\] (44.b)

The heat transfer enhancement in the channel \( E(\%) \) and the reduction in fanning friction factor are obtained from the following expression:

\[
E(\%) = \frac{\overline{Nu}(\varphi) - \overline{Nu}(\varphi = 0)}{\overline{Nu}(\varphi = 0)} \times 100; \quad R(\%) = \frac{\overline{f}(\varphi = 0) - \overline{f}(\varphi)}{\overline{f}(\varphi = 0)} \times 100
\] (45)

4.1. Code validation and grid size effect

In order to validate the MRT-LBM code, the results for mixed convection flow in a vertical duct preceded with a double-step expansion filled with air (Pr=0.71) have been compared with numerical results reported by Tsui and Shu (1997) using finite-volume method. Dimensionless velocity profiles at the particular transverse position \( Y = 5 \) for fixed Reynolds number \( Re=114 \) and for the Grashof number ranging from \( Gr=10^2 \) (\( Ri=0.007 \)) to \( Gr=10^4 \) (\( Ri=0.76 \)) has been compared with those obtained by Tsui and Shu (1997) in Fig. 2. It is shown that our numerical results compares well with the numerical data.

Fig.1. Sketch of the problem geometry.         Fig.2 Velocity profiles at \( Y = 5 \) and \( Re = 114 \) (Pr=0.71).

The streamlines for \( Gr = 10^2, 2 \times 10^3 \) and \( 10^4 \) and with \( Re = 114 \) are illustrated in Fig. 3. As was observed by El Alimi et al. (2005) the asymmetric flow is detected at the lowest Grashof number \( Gr = 10^2 \). This asymmetry is caused by the instability of the shear layers separating the main stream and the two recirculation zones behind the two steps. For \( Gr=2 \times 10^3 \), the flow becomes symmetric and the central region becomes nearly stagnant. However, for the highest Grashof number (\( Gr=10^4 \)), the fluid zone near the hot walls is accelerated by the increased buoyancy forces which causes a deviation in the flow symmetry, the flow reversal takes place near the centerline region.
The steady state isotherms are shown in Fig. 4. As depicted, isotherms follow the flow structure represented by the streamlines. It must be noted that similar results are reported in literature (Durst et al., 1974) and Tsui and Shu, 1997) as well. Excellent agreement is obtained between our results and previous works.

As pointed out in section 3, the stability of thermal LB models with medium or high Prandtl number, even with low Rayleigh and (or) Reynolds number is a challenging task. Using the same grid size, thermal LB scheme may converge with a Prandtl number close to the unity and diverge for other values (even if Gr=0). To test the validity of the numerical scheme for Prandtl number close to 7 (water flow), with the same grid size used for air flows, we increase gradually the Reynolds number until no converged solution can be found. Computations were performed for different successively sized grids, 32×352, 62×702, 92×1052 and 122×1402 at Re=40 and Ri=0.1. Water is used as the working fluid (Pr=6.57). We tabulate the average Nusselt number and fanning friction factor for different grid sizes in Table 1.

The relative difference between the results obtained by 1052 × 92 and 1402 × 122 grid is about 0.18% for $\overline{N_u}$ and about 0.74 % for $\overline{f}$ . One can deduce that there is no significant difference among the results for the last two used meshes. Considering both computational time and the numerical accuracy, the 1052 × 92 grid is sufficiently fine to give an accurate solution. In all the following simulations, a uniform rectangular mesh (1052 × 92) will be used.
4.2. Nanofluid flow

In this section, numerical simulations have been performed for a water suspension of alumina nano-sized particles flow. Table 2 presents thermo physical properties of water and alumina at the reference temperature 22°C. The nanofluid properties were determined from those of water and nanoparticles as detailed in previous section. The volume fraction, Reynolds number and Richardson number ranges from 0% to 20%, 10 to 40 and 0 to 1.0 respectively.

Table 2: Thermophysical properties of the base fluid (water) and nanoparticles (Oztop and Abu-Nada, 2008) and (Abu-Nada, 2008).

| Physical properties | Fluid phase (Water) | Solid phase Al₂O₃ |
|---------------------|---------------------|-------------------|
| \( \rho = \frac{[kg.m^{-3}]}{} \) | 997.1 | 3970.0 |
| \( k = \frac{[W.m^{-1}.K^{-1}]}{} \) | 0.613 | 25.0 |
| \( C_p = \frac{[J.kg^{-1}.K^{-1}]}{} \) | 4179.0 | 765.0 |
| \( \beta = \frac{[10^{-5}K^{-1}]}{} \) | 21.0 | 0.85 |

The main purpose of this section is to analyze the effects of combining such parameters on the flow structure and heat transfer in the microchannel. The results presented below are divided into two parts: Forced convection case (\( Ri=0 \)) and mixed convection case (\( Ri\neq0 \)).

4.2.1. Case 1: Forced convection flow

In this limiting case, we keep the Richardson number fixed at zero and we vary the Reynolds number. Streamlines and isotherms at \( Re=20 \) for pure fluid (water without dispersed nanoparticles) and nanofluid with \( \varphi = 20\% \) are shown in Fig.5. The two recirculation zones after the two-step expansion for pure fluid and nanofluid are of the same size. Adding nanoparticles into the base fluid has not a significant effect on the streamlines distribution along the channel while the difference between the two temperatures contours is clear especially in the region away from the steps. Adding the solid particles leads a higher temperature gradient near the wall and enhances heat transfer in the entire channel. In fact, in the case of forced convection i.e. in the absence of buoyancy forces, the fluid velocity plays an important role on the heat transferred through the walls and, consequently, improves the heat transfer performance. As shown previously, the thermal properties of the fluid are significantly modified when adding nanoparticles. However, the nanofluid behaves more like a single phase fluid than a solid–liquid mixture. This assumption is used in order to compute the physical properties in Eq. (1-5). Shown in Fig. 6 are the relative values of physical properties of the liquid-solid suspension with varying volume fraction. As it can be observed, effects of added Al₂O₃ nanoparticles are noticeable on the density and the thermal conductivity. Thus, the physical properties of the nanofluid show a significant increment with the increase of volume fractions, except the viscosity for some values. This increment is with different slopes. The thermal conductivity increases with the increase of the volume fraction and the enhancement reaches \( k_n/k = 1.70 \) with \( \varphi = 0.2 \) while the relative density reaches \( \rho_n/\rho = 1.59 \) with the same volume fraction. From Fig. 6, we can even derive a strong indication: when \( \varphi < 10\% \), the viscosity of the nanofluid showed a decrease. This result is explained by looking at the Brinkman relation in Eq. (3). It is important to note that the Brinkman relation, only considers the liquid-particle interactions, and assumes that the spherical particles are with only short range interactions. In fact, the viscosity of nanofluid strongly depends on its rheological behavior which may, according to its composition, either be non-Newtonian with a shear-thinning characters (Ding et al, 2006) or Newtonian (Sahoo et al., 2009). In fact, not only the volume fraction that affect the physical properties of the nanofluid, but also, the shape and size of nanoparticles, the material, and the temperature had a great impact on the rheological behavior and the thermophysical properties of the nanofluid.
Fig. 5 Streamlines and isotherms for two solid volume fractions at Re=20.

Fig. 6 The relative physical properties of the nanofluid for different volume fractions.

Fig. 7 presents the evolution of the local bulk Nusselt number for $\varphi = 0$ and $\varphi = 20\%$ with Reynolds number varying from 10 to 40. Nusselt number increases in the initial stage near the entrance of the channel (just after the channel expansion) and decreases until achieving the outlet. The location of the peak of Nusselt number corresponds to the reattachment point of the two smaller recirculation zones after the steps. After that location, the flow emerging from the inlet behaves like a jet. The decrease of the Nusselt number is more slowly when adding nanoparticles. The figure illustrates also an enhancement in Nusselt number by adding the nanoparticles. The enhancement rate is not of the same order across the entire duct. For Re=10 and Re=20 we note that the Nusselt Number values for pure water are under values of nanofluid with $\varphi = 20\%$, a slight decrease in the
Nusselt number of nanofluid from $Y=12.5$ and $Y=25$ for $Re=10$ and $Re=20$ respectively. By looking at the computed Nusselt number in Eq. (43), nanoparticles concentration affects the thermal conductivity ratio $k_{p}/k$ and the temperature gradient near the wall. One can deduce that, for low Reynolds number, the increase in the thermal conductivity could not cover the decrease of the temperature gradient. This phenomenon is observed away from the channel expansion. As known, the areas under the Nu-curves and the Y-axis represent the total heat transfers into the duct. One can affirm that adding nanoparticles into the pure fluid encourage heat transfer more effectively at the entrance of the channel and so in the wake region. For $Re=30$ and $Re=40$, the increase in Nusselt number is along the entire channel and there is not an intersection between $Nu(\varphi = 0)$ and $Nu(\varphi = 20\%)$. Just after the channel expansion, the enhancement is of the same order for all chosen Reynolds number, while, away from the two steps, the heat transfer enhancement decreases for the two Reynolds number.

The enhancement in the average Nusselt number and the reduction in the average fanning friction factor for various Reynolds number and for $\varphi = 0$ and $\varphi = 20\%$ are shown in Table 3. The table reveals an enhancement in the average Nusselt number up to 36% and a slight decrease in the fanning friction factor when volume fraction is about 20%. The increase of the average Nusselt number is predictable with the Maxwell Garnett model (Maxwell-Garnett, 1904) which increases linearly the conductivity. While the slow decline of the fanning friction factor and so that the slow decrease in the velocity gradient near the wall is approved by taking into account the used Brinkman relation in Eq. (3), the viscosity of the nanofluid decreased with the volume fraction. When volume fraction of nanoparticles in the water is 20%, the average Nusselt number enhancement is about 9.66% for $Re=10$, 26.07% for $Re=20$, 33.28% for $Re=30$ and is about 36.54% for $Re=40$. The most basic observation here is the slow growing of the friction factor with all tested volume fractions and Reynolds numbers. The friction factor increased because of the increasing in the fluid viscosity which reduces the moving of the fluid. The viscosity plays an important role on both the resulting flow dynamic and heat transfer characteristics. The increase in the Reynolds number enhances significantly the nanoparticles effect in the heat transfer along the channel. When increasing Reynolds number, the fluid velocity become higher, the thermal boundary layer thickness is reduced and this sets a higher temperature gradient and leading to enhance heat transfer into the fluid in the duct. This improvement is offset by the decrease in the averaged friction factor. Thus, for $Re=10$ the decrease in $f$ is about 0.09% against 0.19% for $Re=40$ and using the same nanoparticles volume fraction $\varphi = 20\%$.

Table 3: Effects of nanoparticles volume fraction on averaged Nusselt number and fanning friction factor in forced convection case.

| $Re$ | $\overline{Nu}(\varphi)$ | $\overline{f}(\varphi)$ |
|------|----------------|----------------|
| Pure water | $\varphi = 20\%$ | Pure water | $\varphi = 20\%$ |
| 10   | 0.628          | 0.688 (9.66%) | 18.448     | 18.465 (0.09%) |
| 20   | 0.905          | 1.141 (26.07%) | 18.270     | 18.297 (0.14%) |
| 30   | 1.062          | 1.415 (33.28%) | 18.139     | 18.170 (0.16%) |
| 40   | 1.175          | 1.604 (36.54%) | 18.031     | 18.067 (0.19%) |

The variation of $\overline{Nu}$ with volume fraction $\varphi = 0–20\%$ for $Re=10–40$ is presented in Fig. 8. The figure shows that adding nanoparticles into the water enhance the averaged Nusselt number and the slope of $\overline{Nu}$ still positive for all chosen Reynolds number. While, the use of nanoparticles improves slightly heat transfer in the microchannel for $Re=10$. For low Reynolds number, increasing volume concentration has not a significant effect in heat transferred into the fluid by the hot walls. For $Re=10$ adding nanoparticles increases the averaged Nusselt number without doing exceed unity. The function $\overline{Nu}(\varphi)$ has a low slope while the average Nusselt number increases as the Reynolds number increases. These results were also observed by Kalteh and al. (2012) in forced convection of nanofluid inside a wide microchannel. This significant improvement in the Nusselt number is due to the fact that heat transferred to the nanofluid depends mainly on the thermophysical properties of the
nanoparticles. For $Re=40$, the enhancement is amplified relatively with an important positive slope compared to other Reynolds number.

4.2.2. Case 2: Mixed convection flow

In this case, the motion of the fluid has both mechanical and thermal origin. The flow is assisted by the buoyancy forces induced by the variations in density with temperature. We increase gradually the Richardson number until $Ri=1.0$ and the Reynolds number from 10 to 40. The steady state streamlines for water without nanoparticles and nanofluid with $\phi=20\%$ solid volume fractions at $Re=40$ and for two Richardson number $Ri=0.1$ and $Ri=1.0$ are shown in Fig.9. As can be observed, the streamlines differs from pure fluid to nanofluid for two selected Richardson number especially away from the expansion. The main typical phenomenon observed for $Ri=0.1$ is the suppression of the reversed flow in the centerline of the duct. Thus, the presence of nanoparticles affects considerably the flow pattern in the microchannel. In the case of pure water, the flows near the walls are accelerated by the buoyancy forces. Adding nanoparticles to the water with $\phi=20\%$ assists to stabilize the flow near the two plates and render the flow symmetric. The flow just after the expansion is not affected and the two recirculation regions near the wall are of the same size. When increasing Richardson number from 0.1 to 1.0 the recirculation detected at the centerline of the channel becomes larger, however, the role of adding nanoparticles is to reduce the intensity of the reversed flow.

The shown temperature contours in Fig.10 revealed that, as for forced convection case, adding nanoparticles increased heat penetration and the heat from the wall has more time to diffuse, and the fluid gets hotter, which leads to an increase in the maximum temperature in the channel. For $Ri=1.0$, adding nanoparticles makes the shapes of the temperature patterns more convex and reduces the buoyancy forces effects on temperature pattern. The convection effects are minimized due to the increase of the nanofluid viscosity.

Fig.11 shows the axial variation of the calculated Nusselt number for the base fluid and nanofluid with $\phi=20\%$ for $Re=10-40$ with $Ri=0.1$. The values reached by the Nusselt number are higher than those obtained in forced convection case and adding nanoparticles enhances the bulk Nusselt number for all Reynolds number. This enhancement is accentuated in the wake region. The decrease in the Nusselt number just after the expansion is more slow when adding nanoparticles and one can note some axial locations where $Nu(\phi=0.2) < Nu(\phi=0)$ for $Re=10$ and $Re=20$. 

Fig.7 Nusselt number distribution for different Reynolds number in forced convection case.

Fig.8 Average Nusselt number in forced convection case.

Fig.9 Streamlines for water without nanoparticles and nanofluid with $\phi=20\%$ for $Re=40$ and for two Richardson number $Ri=0.1$ and $Ri=1.0$.
Fig. 9 Streamlines for base fluid and nanofluid with \( \phi = 20\% \) solid volume fractions at \( Re=40 \) and for two Richardson number \( Ri=0.1 \) (Left) and \( Ri=1.0 \) (Right).

Fig. 10 Isotherms for base fluid and nanofluid with \( \phi = 20\% \) solid volume fractions at \( Re=40 \) and for two Richardson number \( Ri=0.1 \) (Left) and \( Ri=1.0 \) (Right).

The heat transfer enhancement and the fanning friction factor reduction with Reynolds number ranging from 10 to 40 and for \( \phi = 0 \) and \( \phi = 20\% \) are shown in Table 4. For \( Re=10 \), Nusselt number is about 0.66 for pure
water and is about 0.733 for $\varphi = 20\%$ i.e. an improvement about 11%. The increase is in order of 26% using the same nanoparticles volume fraction for $Re=40$. The heat transfer enhancement increases with Reynolds number. With increasing the Reynolds number, the fanning friction coefficient value decreases. As Reynolds number value increases with fixed Richardson number, the Grashof number increases, the velocity gradient near the wall increases and therefore, the fanning factor value increases. Unlike forced convection case, a significant decrease in the fanning friction factor is observed by adding nanoparticles. It changes from 25.68 for pure water to 22.29 for nanoparticles volume fraction 20% for $Re=10$ with a decrease of 13.18% and from 46.29 for pure water to 33.0% for $\varphi = 20\%$ for $Re=40$ with a decrease of 28.71%. It can be concluded that the effect of adding nanoparticles on the velocity gradient near the wall increase with the flow Reynolds number.

Table 4: Enhancement in the average Nusselt number and reduction in and fanning friction factor for various Reynolds number and solid volume fraction $Ri=0.1$.

| $Re$ | $\bar{Nu}(\varphi)$ | $\bar{f}(\varphi)$ |
|------|---------------------|--------------------|
| Pure water | $\varphi = 20\%$ | Pure water | $\varphi = 20\%$ |
| 10 | 0.660 | 0.733 (+11.02%) | 25.685 | 22.299 (-13.18%) |
| 20 | 0.954 | 1.182 (+23.93%) | 32.177 | 25.424 (-20.98%) |
| 30 | 1.164 | 1.476 (+26.72%) | 39.269 | 29.133 (-25.81%) |
| 40 | 1.349 | 1.704 (+26.28%) | 46.294 | 33.001 (-28.71%) |

Table 5 summarizes the influence of $Ri$ on the heat transfer enhancement using pure water and nanofluid for $\varphi = 0$ and $\varphi = 20\%$ for the same Reynolds number $Re=40$. A better enhancement about 36.54% is seen at forced convection case. Beyond this enhancement one can note a decline in heat transfer enhancement until 15.08% for $Ri=1.0$, while there is a small increase about 1% at $Ri =1.0$ compared to $Ri=0.5$. In general, the particles volume fraction effects in the averaged Nusselt number enhancement decreases with Richardson number. The decrease in the average fanning friction factor is about 0.19% for forced convection case and 38.6% at $Ri=0.5$, while there is a decrease about 1% for $Ri=1.0$. The friction factor characteristic is strongly related to the shear thinning effect of the fluid, thus, as noted previously, adding nanoparticles assisted the stability of the flow and so to decrease the velocity gradient near the wall. Ding et al. (2005) also observed the shear thinning behaviors of the Carbon nanotube (CNT) nanofluids. The shear thinning have also been observed by Xu et al. (2005) using the carbon nanofiber suspensions.

Table 5: Enhancement in the average Nusselt number and reduction in and fanning friction factor for various Richardson number and solid volume fraction $Re=40$.

| $Ri$ | $\bar{Nu}(\varphi)$ | $\bar{f}(\varphi)$ |
|------|---------------------|--------------------|
| Pure water | $\varphi = 20\%$ | Pure water | $\varphi = 20\%$ |
| 0.0 | 1.175 | 1.604 (+36.54%) | 18.031 | 18.067 (+0.19%) |
| 0.1 | 1.349 | 1.704 (+26.28%) | 46.294 | 33.001 (-28.71%) |
| 0.5 | 1.761 | 2.010 (+14.12%) | 148.377 | 91.099 (-38.6%) |
| 1.0 | 1.946 | 2.239 (+15.08%) | 241.367 | 151.795 (-37.11%) |

The variation of $\bar{Nu}$ with volume fraction $\varphi = 0 – 20\%$ for $Re=10-40$ is depicted in Fig.12. The figure showed that the function $\bar{Nu}(\varphi)$ has the same shape as in forced convection case. However the values reached by heat transfer coefficients are important compared with forced convection case. Moreover, Fig.12 shows that increasing the nanofluid concentration increases the energy transport rate inside the fluid, while the effects of the nanoparticles concentration decrease slightly at low Reynolds number ($Re=10$ and $Re=20$).
5. Conclusion

In this paper, Lattice Boltzmann Method with Multi-relaxation-time (MRT-D2Q9) collision model was used to study the thermal and dynamic effects of adding Al₂O₃ nanoparticles into the water in a microscale vertical duct preceded with double-step expansion. Brinkman and Maxwell Garnett models are used to predict the nanofluid viscosity and thermal conductivity, respectively. First results of this study show that the MRT-LBM reproduces all the known features of the buoyancy assisted nanofluid flow and heat transfer phenomena in microchannels. The obtained numerical results indicate the feasibility and the reliability of the corresponding code in solving both flow and temperature fields. Based on the analysis, it is found that heat transfer coefficient of aluminum oxide nanofluid is higher than that of the base fluid. This property increases with the increase of volume fraction and Reynolds number. These results largely confirm those reported by Duangthongsuk and Wongwises (2010). In forced convection case, heat penetration increases when adding nanoparticles into the base fluid especially in the entrance region of the microchannel. At Re=40, we reached an enhancement in the averaged Nusselt number up to 36% with particle volume fraction \( \phi = 20\% \). However, the dynamic behavior of the fluid is still without significant change by increasing the particle volume fraction. In mixed convection case, the flow reversal in the centerline of the microchannel is eliminated by the presence of nanoparticles. Adding nanoparticles stabilizes the flow near the two plates and renders the flow symmetric. Although we reached important values of heat transfer coefficient in mixed convection case, the growth rate of heat transfer decreases with Richardson number. In both forced and mixed convection flow, the effect of adding nanoparticles increases further by increasing the Reynolds number. The results illustrate that the increase of heat transferred into the fluid is mitigated by the decrease of the averaged friction factor.

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