Modeling microscopic behavior of geotextile-wrapped soil by discrete element method

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ABSTRACT

Geosynthetic is gaining great attention in civil and geotechnical engineering. Wrapping soils in geotextile bags (soilbag) as earth reinforcement gives astonishing rise in bearing capacity. To understand the fundamental mechanism and facilitate the development of soilbag’s constitutive relation, the authors develop and validate a computational tool for studying the micro-mechanical behavior of soilbag using the Discrete Element Method (DEM). Plate loading on a woven geotextile bag is considered. Spatial difference in the wrapped soil’s stress states and fabric anisotropies due to geometrical and mechanical reasons are investigated. Linear and symmetrical stress paths are found at different probed locations. Upon the rupture of the woven structure, the geometrical fabric anisotropy is high and persistent. Analyzing the evolutions of the fabric anisotropies due to normal and tangential force and their relation with the geometrical fabric anisotropy help unveil the first and secondary mechanism of such reinforcement method, i.e. confinement and interlocking.

Keywords: earth reinforcement, geotextile, DEM, anisotropy

1 INTRODUCTION

Geosynthetic endowed with tensile strength, water drainage/separation, interlocking, and anchorage effect is widely implemented in the development of infrastructures where proper cost-effective earth or subgrade reinforcement is anticipated (Bhandari & Han, 2010). To facilitate its application in civil engineering practice one needs to understand the complex interface mechanism between soil particles and geosynthetic, which is both discontinuous and heterogeneous in nature. Woven geotextile, in particular, is of peculiar mechanical behaviors due to ease relative motion between yarns, impregnated resin and abrasion on the textile fabric. Discrete approach like the Discrete Element Method (DEM) is suitable to mimic the macroscopic behavior of geotextile-reinforced soil, while providing insight at the micro scale.

In most cases, geotextiles are employed as separation, filtration and reinforcement layers in road and slope construction. Wrapping soil in geotextile bags (soilbag) is not a common approach and yet the engineering values in this ancient technique should not be underestimated. For economic, technical, sustainable and environmental reasons, soilbag has reclaimed its role in earth reinforcement. Through decades of laboratory tests and engineering experience (Matsuoka & Liu, 2006), soilbag earth reinforcement have been found able to:

- Improve the bearing capacity of a soft foundation by five to ten times,
- Provide high compressive strength by wrapping inside the soils of various types,
- Dissipate energy under traffic and seismic vibration,
- Prevent frost heaving if granular soils are backfilled.

Matsuoka (Matsuoka & Liu, 2006) proposed an analytical solution of soilbag’s compressive strength in plain strain condition, introducing an apparent cohesion $c$ which takes into account of the uniformly distributed tension in textile. Matsuoka and Yamamoto (Yamamoto, Arakawa, & Yano, 2005) later extended it, considering the evolution of tension and $\sigma_{10}/\sigma_{30}$ for the prediction of compressive deformation. These analytical solutions are supported by laboratory tests evidence (Arakawa, Yamamoto, & Yano, 2005) whereas numerical models using the Finite Element Method (FEM) are still not able to fully capture soilbag’s the complex behavior in laboratory nor present its reinforcement mechanism.

The previous FEM attempts considered geotextile using truss elements and homogenous inside soil. However, the experimentation evidence, e.g. bag’s top and bottom faces worn out near the center, suggested a certain degree of non-uniformity in soil. Moreover, the authors believe the fabric structure of wrapped soil must be essentially anisotropic. It may vary spatially
and temporally due to the evolution of soilbag confinement and interface interlocking. Getting this knowledge could facilitate the development of constitutive models for geosynthetic reinforced soils. For this purpose, the authors seek a rigorous numerical investigation on the microscopic behavior of geotextile wrapped soil under unconfined compression. Problems that shall be addressed in present work: 

- Explore stress path and its spatial variation in geotextile wrapped soil under compression;
- Unveil the soilbag reinforcement mechanism taking advantage of the fabric anisotropies of soil structure.

The authors have conducted an extensive range of compression and shear tests on soilbag assemblies, both monotonic and cyclic. In these tests, the effect of stress history was observed. In the 50-cycle cyclic shear tests with and without initial shear, stable shear stress-strain hysteretic loops were obtained with some degree of hardening (Cheng, Yamamoto, Jin, & Okano, 2013). These phenomena could be explained by the evolution of the wrapped soil’s fabric anisotropy due to geometrical and mechanical reasons. With the DEM, one can easily acquire this fabric information if the macroscopic response is correctly reproduced.

The DEM is drawing attentions in various fields. For example, in the analysis of the shape forming, discrete models consists of rigid bars at nodes with diagonal and rotational springs are found effective in capturing some macroscopic behavior (Ben Boubaker, Haussy, & Ganghoffer, 2007). Similar approach for studying rockfall protective system was initiated in (Thoeni, Lambert, Giacomini, & Sloan, 2013) where the double-twisted hexagonal wires were modeled. Their approach generally suits any woven system. In present work, the authors use remote stretch springs and imaginary spheres to construct the orthogonal structure of a 3D soilbag. A packing with the scaled particle size distribution (PSD) of Toyoura sand is prepared to model the inside soil. Macroscopic response of the textile and the soil are calibrated against a series of laboratory tests. The numerical and experimental results are compared to validate the proposed model. This numerical study is conducted using the open-source frame work YADE (Šmilauer et al., 2014).

2 WOVEN GEOTEXTILE CHARACTERISTIC

In present work the mechanical property of a 40 cm × 40 cm × 10 cm woven geotextile soilbag with 0.25 mm fabric thickness is investigated. The tensile behavior according to the width wide tensile test (D35 Committee, 2011) is shown in Fig. 1(b). A regular shape shown in Fig. 2(a) is assumed to be preserved after the initial compaction so that the cross section along one axis satisfies the boundary in Fig. 2(b).

3 DISCRETE MODELING OF SOILBAG

Numerical modeling of a geotextile soilbag using the FEM requires the soil-textile interface to be dealt explicitly. In addition, granular soils wrapped inside the bag may undergo large deformation with intense strain localization (Tantono, 2010). Above all the soilbag system is anisotropic and heterogeneous in nature. The aforementioned problems can be solved radically by considering soil consists of polydisperse particles taking into account the local behaviors of soil such as sliding and rolling/twisting.

The discrete element method (DEM) describes such behaviors of discontinuous bodies in a rigorous manner. At local scale inter-particle forces are computed from detected overlaps between neighboring particles with force-displacement laws (e.g. Hertzian contact laws). After computing all resultant forces applied on each particle, the dynamic behavior of the system is solved numerically through Newton’s second law in a time progress scheme. In YADE, this is done using an explicit finite difference algorithm considering constant velocities and accelerations in each time step. Detailed derivation of the algorithms can be found in YADE’s documentation (Šmilauer et al., 2014).
Transfer Law (MTL) was adopted for describing the local behavior between spherical particles. The tensile behavior of the geotextile is defined using remote springs implemented by Thoeni in Yade (Thoeni, Lambert, Giacomini, & Sloan, 2013). This section deals with the calibration of the models' parameters.

### 3.1.1 Sand
Toyoura sand was chosen as inside material as in previous experimental investigations (Cheng et al., 2013). It is uniformly graded (D60/D10 = 1.3) with the specific gravity of 2.65 and 0.2 mm diameter in average. The maximum and minimum void ratios are 0.95 and 0.58 respectively. To lower computational cost, a scaled packing (scaled particle size distribution with \( r_{\text{mean}} = 3 \text{ mm}, G_s = 2.65 \text{g/cm}^3 \)) is considered. In the past laboratory tests, the bags were filled with 24 kg Toyoura sand and compacted thoroughly until the height of one bag reached 8 cm. Note that if the 3D boundary illustrated in Fig. 2(a) perfectly holds, the volume of one soilbag will be 0.015 m³, i.e. an average density about 1.6 g/cm³. Fig. 3 shows the drained triaxial compression results for saturated Toyoura sand under different levels of initial confining pressure (Sun, Huang, Sheng, & Yamamoto, 2007). The cylindrical specimen was 5 cm in diameter and 10 cm in height with an initial void ratio of 0.68 (\( \rho = 1.608 \text{ g/cm}^3 \)).

| Young’s modulus (10 GPa) | Poisson ratio | Rolling/twisting stiffness (kg/m²) | Density (kg/m³) |
|--------------------------|--------------|----------------------------------|----------------|
| Soil                     | 4            | 0.33                             | 2650           |
| textile                  | 3.63         | 0.33                             | 444            |
| Wall                     | 400          | 0.33                             | 7850           |

Table 1. The local parameters for the contact laws and interfaces

Fig. 4. Schematic view of shearing box test setup

The fabric-soil interface behavior was calibrated by shearing a sand box on a piece of geotextile sheet. Fig. 4 illustrates the schematic view of shearing box test setup. The 20 cm × 20 cm PE sheet is affixed to the concrete base. Double-layered Teflon films with silicone grease lubrication are sandwiched between the geotextile sheet and the side wall’s bottom to exclude friction. A DEM simulation with 21° interface frictional angle was found closest to the laboratory results.

### 3.2 Soil model generation
With the 3D boundary of soilbag properly assumed in Fig 2a, it is easy to fill the volume with a proper packing by the radius expansion method. The question is how to prepare a DEM specimen that preserves an equivalent fabric structure as in the test. The following sections will try to address this problem.

Fig. 3. DEM and test results of the curves of stress ratio and volumetric strain versus axial strain

The DEM specimen (later used as the representative volume (RV) for model generation in section 3.2.1) was of cubic shape (5 cm × 5 cm × 10 cm). The dense packing (\( e = 0.68 \)) was prepared by applying 0.1 MPa isotropic pressure first. After the packing stabilized the inter-particle frictional angle was reduced gradually until 0.68 was reached for void ratio. The curves of stress ratio and volumetric strain versus axial strain under different confining pressure are shown together with the experimental measurements in Fig. 3, using the parameters given in Table 1. The contact frictional angle among soil particles is 29 degree. For the soil-textile contacts, the angle is 21 degree (determined in section 3.1.2). The wall is assumed frictionless to exclude its other confinement on the soilbag.

### 3.1.2 Geotextile
Discrete modeling of woven structure considers fabric as an assembly of their constituent yarns, giving insights into the effect of the yarns properties and organization on the macro-scale behavior. The main objective of current modeling is to investigate dry woven fabric’s interactions with soil. Therefore, the woven fabric is modeled in such a scale that the fabric’s structural mechanics can be described. This is done by linking the physical nodes where the warp and the weft meet as shown in Fig 1(a). In addition the important influence of apertures and undulations on the fabric-soil interface is taken into account by the built-in roughness introduced from the linked imaginary spheres and local contact friction. Because the fabric is continuously subjected to large tensile force, it can be assumed that no relative sliding occurs between warp and weft. Stretching springs are sufficient for current modeling purpose. In addition, the yarn property is assumed uniform so that one tensile stress-strain relationship (Fig. 1(b)) can be used for all stretching springs.

Fig. 3. DEM and test results of the curves of stress ratio and volumetric strain versus axial strain
3.2.1 Generate a packing with particular void ratio
Coordinate number, void ratio and fabric tensor are generally used to specify the particular structure of a specimen. All three can be measured in laboratory but void ratio is most conventional and handy which is of great significance in classical soil models. Therefore, the authors seek a model generation method that creates a packing with a preferred void ratio.

For this purpose, the periodic boundary condition (PBC) was used to generate a RV consists of 1,000 spherical particles with the scaled Toyoura sand PSD. Unlike the flip technique (Dang & Meguid, 2010) the packing that consists of 100 RV duplicates generated in the PBC (10 along x-axis and 10 along z-axis) has intrinsic fabric consistency. Note that, the same RV was used elsewhere in section 3.1.1 for calibrating purpose. The authors believe that this packing can well represent the mechanical behavior of the inside soil.

3.2.2 Conform to the boundary
The packing was filtered to satisfy the soilbag boundary. To make the remaining packing sufficiently conform to the boundary, the soil spheres were allowed to grow with a small multiplier \((E_w = 1.0001)\). While searching for the equilibrium, the system was given a high damping coefficient (0.9) so that the induced numerical viscosity can prevent the spheres deviating too much from its original fabric structure. The expansion process stopped when the soilbag packing’s volume (computed by the Delaunay triangulation) ceased to grow and the mean overlap between soil spheres was lower than \(2.0 \times 10^{-8}\) (same as in the original RV). At last, a packing of 0.67 void ratio was generated. It should be noted that in the PBC the void ratio is based on the PBC cell. Analogously, the authors base the void ratio of soilbag on the Delaunay cells.

![Fig. 5. Axial stress-strain relation in test and simulation](image)

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![Fig. 6. Void ratio \(\varepsilon\) and deviatoric stress \(q\) versus \(\varepsilon_q\)](image)

Fig. 6. Void ratio \(\varepsilon\) and deviatoric stress \(q\) versus \(\varepsilon_q\)

4 RESULTS AND DISCUSSIONS
4.1 Validation of the soilbag model
The stress-strain curve obtained from the plate loading test was used to validate the proposed DEM model. In the test, a triple-layered soilbag stack was fully compacted beforehand (8.5 cm height for each layer) and compressed vertically with the strain rate increasing from zero to 0.1%/s. In current numerical study, to maintain low computational cost, only one bag was considered. The strain rate was kept constant at 10%. Gravitational field is excluded in the simulation. The unbalanced force was periodically checked to ensure the quasi-static state according to (Lin & Ng, 1995). Local parameters are given in Table 1.

The evolution of the axial stress \(\sigma_{ax}\) on the plate and the \(\sigma_z\) of the average stress tensor over the soil domain are both shown in Fig. 5. The curves of \(\sigma_{ax}\) from the experiment and the simulation generally agree, except that the bearing capacity in the simulation is not as high as in the test. This is due to the assumption of the frictionless wall and the uniform yarn property which doesn’t account for the contribution of the sewing seams. In a special case where the spheres representing textile nodes on the seams were fixed in the \(x-y\) plane, the bearing capacity almost doubled but the textile’s rupture was brought forward and occurred along the seams. However both simulations and experiments showed geotextile’s failure on the top and the bottom surfaces of the bag. When the seams were fixed, the stress-strain behavior was rather brittle because of the sudden loss of confinement upon failure, while in other cases \(\sigma_z\) drops down mildly. The relationship between void ratio \(\varepsilon\) and deviatoric strain \(\varepsilon_q\) is given in Fig. 6. It suggests phase transformation occurred upon 2% \(\varepsilon_q\) which was confirmed elsewhere in Fig. 7, indicating soil reaching the critical state \((\frac{q}{p} = 1.6\) close to the \(M_c = 1.76\) of the Toyoura sand at 90% relative density).

4.2 Spatial difference in stress paths
The particle-based stress tensor can be computed using equation (1) (Guo & Zhao, 2013). Weighting the tensors over the soilbag volume gives the average stress tensor whose \(c\) component \(\sigma_c\) is shown in Fig. 5.

\[
\sigma = \frac{1}{V} \sum_{N_c} d^c \otimes f^c
\]

where \(\sigma\) is the bulk stress tensor, \(d^c\) and \(f^c\) are the branch vector and the contact force vector. \(V\) is the volume of Voronoi cell for particle-based stress tensors. Averaging over the stress tensors in a preferred domain, one can easily acquire the regional values for the mean stress \(p\) and deviatoric stress \(q\). As the wrapped soil is derived from the original RV, it is reasonable to base the regional \(p, q\) on these RV duplicates.

Stress tensors were averaged in five volumes #1 ~ #5 aligning along y axis. Each occupies four RVs as
shown in Fig. 7. #0 represents the overall volume. The resulting stress paths are depicted in Fig. 8 together with the overall tension path. Fig. 8 shows interesting linear stress paths along the critical state line prior to the soilbag failure in all probed locations. Moreover, these stress paths in the wrapped soil were symmetrical with higher values near the center (#2 ~ #4), as shown on the distribution of \( p \) in Fig. 10(a). Though not shown in here similar symmetry was also found in tension distribution. Note that this non-uniformity in stress state doesn’t exist initially. This characteristic and \( \sigma_c \) grew together with sustained confinement before the failure and dropped thereafter with progressing rupture in geotextile. The development of large interconnecting ruptures like those shown on the bottom of the bag in Fig. 10(b) led to the loss of vertical confining pressure. At last, as Fig. 8 shows, all stress paths collapsed to the overall path. It is interesting to see that after the failure the wrapped soil underwent triaxial unloading \( (\alpha p \approx 3) \). This may be attributed to the sustained tension on the lateral faces (see Fig 10(b)). With interlocking between soil and the textile fabric, the average tension didn’t drop abruptly as the tension path in Fig. 8 illustrates.

![Fig. 7. Five selected volumes #1 ~ #5 along y axis](image)

![Fig. 8. Overall and regional stress paths in volumes #0 ~ #5 and overall tension path](image)

![Fig. 9. Evolution of anisotropy characteristics \( a_c, a_n, \) and \( a_t \)](image)

### 4.3 Evolution of fabric structure

Following the pioneering work by (Oda, 1972), the authors seek the fabric anisotropy characteristics of the wrapped soil. Getting this knowledge may unveil the soilbag reinforcement mechanism. With accessible information on particles’ spatial configuration and internal force chain, equation (2) ~ (4) give the corresponding fabric tensors namely \( \varphi^*, \chi^n, \) and \( \chi^t \).

\[
\varphi^* = \frac{1}{N_c} \sum_{N_c} n^* \otimes n^*
\]

\[
\chi^n = \frac{1}{N_c} \sum_{N_c} f^s n^* \otimes n^*
\]

\[
\chi^t = \frac{1}{N_c} \sum_{N_c} f^t n^* \otimes n^*
\]

where \( n^*, t^c \) and \( c^t \) are the unit vector normal and tangential to the contact plane, \( N_c \) the total number of the contacts between soil and textile, \( f^s_c \) and \( f^t_c \) are the length of inter-particle contact force along normal and tangential direction. \( a^t = 15\text{deg} \alpha^t/2 \) is the fabric anisotropy tensor. Analogically with the averaged contact normal force \( f^0 = \text{tr}\chi^0 \), one can easily define the other two anisotropy tensors \( a^n = 15\text{deg} \chi^n/(2 f^0) \), \( a^t = 15\text{deg} \chi^t/(3 f^0) \). The second invariants of these tensors can be used to represent the magnitude of anisotropy as in equation (5)

\[
a_c = \text{sign}(S_c) \sqrt{\frac{3}{2} a^n : a^t}
\]

where \( * \) stand for \( c, n, t \) accordingly, \( S_c \) calculates the inclination between a specific anisotropy tensor with the stress deviator. Fig. 9 shows the evolution of the fabric anisotropy characteristics \( a_c, a_n, \) and \( a_t \) with \( e_q \).

It can be seen that \( a_c \) reached the peak right after the loading started, indicating the onset of confinement on the wrapped soil. With the yarns continuously strained and softened, the confinement became less dominate in the vertical direction starting from 5% \( e_q \). On the other hand \( a_c \), closely related to soil’s strength, was kept constant until some yarns ruptured starting at about 10% \( e_q \) (Fig. 10(a) and the average tension stopped increasing (Fig. 8). It seems that the confinement, as \( a_t \), macroscopically and \( p \) macroscopically, is the primary mechanism which gives rise to fabric anisotropy \( a_c \). Tension serves to sustain the level of \( a_c \). Once tension breaks, \( a_c \) drops subsequently. In the post peak stage, \( a_c \) fluctuated with an average of 2.5. This fluctuation may be contributed by the interfacial tangential force whose anisotropy \( a_t \) was always small suggesting few mobilized contacts on the top and bottom interface. It can be understood that interlocking is not a primary mechanism in the pre peak stage, but it helps relieve the drop of the post peak confinement as indicated by the steady drop of \( a_c \), and some post peak high values of \( a_c \). However these high instantaneous \( a_c \) cannot make up for the lost of strong
contacts near interconnecting ruptures shown in Fig. 10.

![Fig. 10. Force chain (cylinder radii proportional to $f_{rc}$ and color legend denotes $f_{rc}$), p distribution in the wrapped soil and average T distribution on the geotextile bag at (a) peak stress state $\varepsilon_a = 12.9\%$, (b) final state $\varepsilon_a = 20\%$](image)

**5 CONCLUSIONS**

In present work, a woven geotextile soilbag is modeled using the DEM. The parameters for contact laws are carefully calibrated with test data from triaxial, width-wise tensile and shearing box tests (section 3.1). Upon a proper geometrical assumption, the soilbag model is rigorously prepared by using RV generated in PBC (section 3.2.1). The filtered packing is expanded in radii to conform to the soilbag boundary with a high damping ratio to keep fabric consistency (section 3.2.2). The proposed model has been validated by a simple plate loading test. Strong seams are not recommended if a ductile stress-strain behavior is anticipated. A quality controlled bag should have strong top and bottom faces. Section 4.2 and 4.3 analyses the spatial difference in stress states and fabric anisotropies due to the variation in geometrical arrangement and contact force network, the following conclusions can be drawn.

- Stress is concentrated in the middle and the paths are almost linear prior to the rupture of geotextile.
- Confinement is the soilbag reinforcement method’s the primary mechanism, giving rise to geometrical fabric anisotropy which is sustained by tension.
- Interlocking effect takes a secondary role in the post peak stage, prolonging the confinement effect.

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