Walking Behavior in Technicolored GUTs

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There exist two ways to obtain walk behavior: assuming a large number of technifermions in the fundamental representation of the technicolor (TC) gauge group, or a small number of technifermions, assuming that these fermions are in higher-dimensional representations of the TC group. We propose a scheme to obtain the walking behavior based on technicolored GUTs (TGUTs), where elementary scalars with the TC degree of freedom may remain in the theory after the GUT symmetry breaking.
I. INTRODUCTION

The nature of the Higgs boson is one of the most important problems in particle physics, and there are many questions that may be answered in the near future by the LHC experiments. Is the Higgs boson, if it exists at all, elementary or composite? What are the symmetries behind the Higgs mechanism? The possibility that the Higgs boson is a composite state instead of an elementary one is more akin to the phenomenon of spontaneous symmetry breaking that originated from the effective Ginzburg Landau Lagrangian, which can be derived from the microscopic BCS theory of superconductivity describing the electron-hole interaction (or the composite state in our case). This dynamical origin of spontaneous symmetry breaking has been discussed with the use of many models, the most popular one being the technicolor (TC) model\textsuperscript{[1]}. 

Unfortunately we do not know the dynamics that forms the scalar bound state, which should play the role of the Higgs boson in the standard-model symmetry breaking, and no phenomenologically satisfactory model along this line has been derived up to now. Most of the models for the spontaneous symmetry breaking of the standard model based on the composite Higgs boson system depend on specific assumptions about the particle content and consequently on the dynamics responsible for the bound-state formation\textsuperscript{[2]}. In theories based in TC the Higgs boson is a composite of the so-called technifermions, and to some extent any model in which the Higgs boson is not an elementary field follows more or less the same ideas as in technicolor models. The beautiful characteristics of TC as well as its problems were clearly listed recently by Lane\textsuperscript{[1, 2]}. Most of the technicolor problems may be related to the dynamics of the theory as described in Ref.\textsuperscript{[1]}. 

Although technicolor is a non-Abelian gauge theory it is not necessarily similar to QCD, and if we cannot even say that QCD is fully understood up to now, it is perfectly reasonable to realize the enormous work that is needed to abstract from the fermionic spectrum the underlying technicolor dynamics. The many attempts to build a realistic model of dynamically generated fermion masses are reviewed in Ref.\textsuperscript{[1, 2]}. Most of the work in this area tries to find the TC dynamics dealing with the particle content of the theory in order to obtain a technifermion self-energy that does not lead to phenomenological problems as in the scheme known as walking technicolor\textsuperscript{[3]}. The idea of this scheme is quite simple. First, we can remember that the expression for the TC self-energy is proportional
to $\Sigma(p^2)_{TC} \propto \langle (\bar{\psi}\psi)_{TC}/p^2 \rangle (p^2/L_{TC}^2)^{\gamma_{TC}}$, where $\langle \bar{\psi}\psi \rangle_{TC}$ is the TC condensate and $\gamma_{TC}$ its anomalous dimension. Secondly, depending on the behavior of the anomalous dimension we obtain different behaviors for $\Sigma(p^2)_{TC}$.

In principle, we could deal with many different models, varying fermion representations and particle content, finding different expressions for $\Sigma(p^2)_{TC}$ and testing them phenomenologically, i.e. obtaining the fermion mass spectra without any conflict with experiment. The anomalous dimension $\gamma_{TC} = 1$ can be obtained in the extreme limit of a walking technicolor dynamics, and a large anomalous dimension may solve many problems in TC models[4].

Usually the walking behavior is obtained assuming a large number of technifermions, $N_{TF} \sim 4N_{TC}$, if technifermions are in the fundamental representation of TC gauge group[3]. Moreover, recently Sannino et al. showed that it is possible to obtain the walking behavior for a small number of technifermions if these are in higher dimensional representations of the TC gauge group[5].

In this work we propose a scheme that combines these two approaches to obtain the walking behavior based on technicolored GUTs, in the sense that we have a reduced number of technifermions in the fundamental representation of TC group, and technicolored Higgs bosons in higher dimensional representations of the TC group. The advantage of this approach in relation the other proposes to generate the walking behavior is that in this case the dynamics associated to the TC would be similar to the QCD until an energy scale of order $O(10^{13} GeV)$.

This paper is composed as follows: in Sect. 2 we discuss the modification of the TC beta function ($\beta_{TC}$) in the presence of scalars that carry technicolor degrees of freedom. After that, in Sect. 3, we propose a toy model for a technicolored GUT (TGUT) to illustrate how we can obtain the walking behavior. In Sect. 4 we draw our conclusions.

\section{The TC Beta Function in the Presence of Scalars}

For asymptotically free gauge theories, with fermions in the fundamental representation, the $\beta$ function at $g^5$ order (two-loop) can by written in the following form[6]

$$\beta(g) = -b_0 \frac{g^3}{16\pi^2} + b_1 \frac{g^5}{(16\pi^2)^2}$$

(1)
where we defined

\[ b_0 = \frac{11}{3} C_2(G) - \frac{4}{3} \sum_{\text{fermions}} T(R) \]

\[ b_1 = -\frac{34}{3} C_2^2(G) + \frac{20}{3} C_2(G) \sum_{\text{fermions}} T(R) + 4 C_2(R) \sum_{\text{fermions}} T(R) + 4 \sum_{\text{scalars}} C_2(G) T(S) + \frac{2}{3} \sum_{\text{scalars}} C_2^2(G) T(S). \]

\( \text{(2)} \)

For \( SU(N) \) and fermions in the fundamental representation these coefficients assume the form \( C_2(G) = N, T(R) = \frac{1}{2}, \) and \( C_2(R) = \frac{N^2 - 1}{2N}. \)

The \( \beta \) function in the presence of scalars at two loop approximation was determinated by Jones\[7\] and the form of the coefficients \( b_0 \) and \( b_1 \) translates to

\[ b_0 = \frac{11}{3} C_2(G) - \frac{4}{3} \sum_{\text{fermions}} T(R) - \frac{1}{3} \sum_{\text{scalars}} T(S) \]

\[ b_1 = -\frac{34}{3} C_2^2(G) + \frac{20}{3} C_2(G) \sum_{\text{fermions}} T(R) + 4 C_2(R) \sum_{\text{fermions}} T(R) + 4 \sum_{\text{scalars}} C_2(S) T(S) + \frac{2}{3} \sum_{\text{scalars}} C_2^2(G) T(S). \]

\( \text{(3)} \)

In the notation used in the equations described above the indices \( R \) and \( S \) respectively denote fermions and scalars in the fundamental representation; in this case the form of the coefficients \( C_2(S) \) and \( T(S) \) is the same for the fermions. However, we can determine the form of the coefficients \( C_2(S) \) and \( T(S) \) for a generic representation \( r \) considering the relations

\[ (C_2(r_1) + C_2(r_2)) d(r_1) d(r_2) = \sum C_2(r_i) d(r_i), \]

\[ T(r) d(G) = d(r) C_2(r) \]

\( \text{(4)} \)

where \( d(r_i) \) is the dimension of the representations \( r_i \) with \( i = 1, 2. \)

We can determine the values of the coefficients \( T(R, S) \) and \( C_2(k) \), where \( k = G, R, S, \) to some TC group. Moreover, it is interesting to consider the case where \( N_{TC} = 3, \) because in the next section we will build a model based on this specific group. Therefore, after we
consider the relations shown in (4), we obtain
\[ C_2(S = 3) = \frac{4}{3}, \quad T(S = 3) = \frac{1}{2} \]
\[ C_2(S = 6) = \frac{10}{3}, \quad T(S = 6) = \frac{5}{2} \]
\[ C_2(S = 8) = 3, \quad T(S = 8) = 3. \]  
(5)

The coefficients \( b_0 \) and \( b_1 \) described by (3) can be written
\[ b_0 = 11 - \frac{2}{3} n_f - \frac{1}{6} n_s(3) - \frac{5}{6} n_s(6) - n_s(8) \]
\[ b_1 = -102 + \frac{38}{3} n_f + \frac{11}{3} n_s(3) + \frac{115}{3} n_s(6) + 42 n_s(8), \]  
(6)

where \( n_f \) is the number of Dirac fermions needed to generate the walking behavior, and \( n_s \) is the number of introduced complex scalars.

In the introductory section we present the expression for the TC self-energy, which for convenience we repeat:
\[ \Sigma(p^2)_{TC} \approx \frac{\langle \bar{\psi} \psi \rangle_{TC}}{p^2} \left( \frac{p^2}{\Lambda^2_{TC}} \right)^{\gamma_{TC}} = \frac{\Lambda^3_{TC}}{p^2} \left( \frac{p^2}{\Lambda^2_{TC}} \right)^{\gamma_{TC}}. \]  
(7)

As we argue, the form for the TC self-energy will depend on the value assumed by the anomalous dimension \( \gamma_{TC} \), and the walking behavior \( \gamma_{TC} \approx 1 \) is obtained in conventional models assuming \( N_{TF} \approx 4 N_{TC} \). For example, to \( SU(N_{TC} = 3) \), the number of technifermions necessary to generate this behavior is \( N_{TF} \approx 12 \). Moreover, in this scenario we have a great proliferation of pseudo-Goldstone bosons (PGBs) and in general large contributions to the parameters of precision S, T and U due to the high number of technifermions introduced.

In the scenario proposed in this work the number of technifermions needed to generate the walking behavior would be much less. Because if we consider (1) and (6), the walking behavior \( \beta(g)_{TC} \approx 0 \), for the case \( SU(N_{TC} = 3) \), is obtained approximately by the following number of fermions\(\textbf{[8]}\):
\[ N_{TF} \approx 12 - \frac{2}{3} n_s(3) - \frac{7}{3} n_s(6) - 3 n_s(8). \]  
(8)

In the next section we will propose a toy model for a technicolored GUT (TGUT) to illustrated our ideas, and in this specific case the gauge group attributed to TC is precisely \( SU(N_{TC} = 3) \).
III. A TOY MODEL FOR A TECHNICOLORED GUT

In this section we propose a toy model for a technicolored GUT (TGUT) based on the group $SO(10)$ in order to illustrate our approach. We shall begin by presenting the usual fermionic content attributed to the $SO(10)$ model,

$$16^T(GUT) = (\psi^T, \phi^T)_L$$

where we denote

$$\psi^T = (u_1^a, u_2^a, \nu^a, d_1^a, d_2^a, l^a)$$

$$\phi^T = (d_1^{a|c}, d_3^{a|c}, l^{a|c}, -u_1^{a|c}, -u_2^{a|c}, -\nu^{a|c})$$

and in this expression $(a, i = 1..3)$ are respectively the family and color indices. To exemplify our purpose of obtaining the walking behavior we shall consider a toy model that just contains a complete generation of leptons $\psi^T_l = (l^a, \nu^a)$ and techniquarks $\psi^T_Q = (U^a, D^a)$. In this case we can consider a model with the fermionic content exhibited in (9) and just make the change $(u, d) \rightarrow (U, D)$.

$$16^T(TGUT) = (\Psi^T, \Phi^T)_L$$

where

$$\Psi^T = (U_1^a, U_3^a, \nu^a, D_1^a, D_3^a, l^a)$$

$$\Phi^T = (D_1^{a|c}, D_3^{a|c}, l^{a|c}, -U_1^{a|c}, -U_3^{a|c}, -\nu^{a|c})$$

In the above expression we can identify $(U, D)$ as the usual techniquarks and the index $i = 1..3$ labels the technicolor. The spontaneous symmetry breaking of the model to $SU(3)_{TC} \times SU(2)_L \times U(1)_Y$ can be promoted assuming a convenient set of scalar fields that develop nonzero vacuum expectation values (VeVs). At the low-energy scale the technicolor theory breaks the electroweak symmetry dynamically by the formation of a condensate $\langle \bar{Q}Q \rangle \sim \Lambda_{TC}^3$. 


The symmetry breaking pattern \( SO(10) \to SU(3)_{TC} \times SU(2)_L \times U(1)_Y \) can be obtained as described in Ref.\[9\]

\[
SO(10)^{54,16} \to SU(3)_{TC} \times SU(2)_L \times U(1)_Y. \tag{13}
\]

For this breaking pattern the VeVs attributed to the Higgs fields 54 and 16 are chosen to be of the same order, \( V_{54} \approx V_{16} \sim \Lambda_{T_{GUT}} \), and we do not have any other intermediate stage. The physical spectrum of the Higgs fields can be extracted from [10] and in Table I we list these fields and these respective masses in terms of the \( SU(3)_{TC} \times SU(2)_L \times U(1)_Y \) decomposition.

In (14) we show the usual expression for the technicolor anomalous dimension, where only the usual technifermions were included. The plot of the behavior of this anomalous dimension, \( \gamma_{TC(A)} \) to \( N_{TC} = 3 \) as a function of \( \Lambda \), is depicted in Fig. I(A):

\[
\gamma_{TC(A)} = \frac{3(N_{TC}^2 - 1)}{4\pi N_{TC}} \alpha_{TC}(\Lambda^2) = \frac{3(N_{TC}^2 - 1)}{4\pi N_{TC}} \left[ \frac{\alpha_{TC}(\Lambda_{TC}^2)}{1 + \left( \frac{b_0}{2\pi} + \frac{b_1}{8\pi^2} \right) \ln \left( \frac{\Lambda^2}{\Lambda_{TC}^2} \right)} \right], \tag{14}
\]

where the coefficients \( b_0 \) and \( b_1 \) are described in (2). However, at high energies the technicolored Higgs bosons responsible for the breaking of the TGUT group contribute to the running of the technicolor coupling and the expression for the technicolor anomalous dimension for \( N_{TC} = 3 \) can be written as

\[
\gamma_{TC(B)} = \frac{3(N_{TC}^2 - 1)}{4\pi N_{TC}} \alpha_{TC}(\Lambda^2) = \frac{3(N_{TC}^2 - 1)}{4\pi N_{TC}} \left[ \frac{\alpha_{TC}(\Lambda_{TC}^2)}{1 + \left( \frac{b_0}{2\pi} + \frac{b_1}{8\pi^2} \right) \ln \left( \frac{\Lambda^2}{\Lambda_{TC}^2} \right)} \right], \tag{15}
\]

in this case, the coefficients \( b_0 \) and \( b_1 \) are described by (6). The plot of the behavior of this anomalous dimension, \( \gamma_{TC(B)} \), as a function of \( \Lambda \) is depicted in Fig. I(B).

From this graph it is possible verify that the number of technifermions necessary to obtain the walking behavior is approximately \( N_{TF} \sim 6 \). Then, for energies of the order \( O(10^{13} GeV) \), the TC self-energy presents a more weaker dependence with momentum

\[
\Sigma(p^2)_{TC} \sim \frac{\Lambda_{TC}^2}{p} \left( \frac{p}{\Lambda_{TC}} \right)^{\frac{4}{3}}. \tag{16}
\]
FIG. 1: Evolution of the anomalous dimension $\gamma_{TC}$, as function of the number of technifermions ($N_{TF}$) and technicoloured heavy scalars ($n_s(r)$, to $r = 3, 6, 8$). To achieve this plot we assumed the following values, $\Lambda_{TC} = 300\text{GeV}$, and $\alpha_{TC}(\Lambda_{TC}^2) \approx \pi/3 \approx 1$.

This behavior is reached just at the scale of energy where the new degrees of freedom associated to the technicolored Higgs bosons appear. The ideas that we presented in this paper are not new. A long time ago the authors of[11, 12] suggested that heavy colored Higgs scalars, which are present in theories such as $SU(5)_{GG}$ should be included in the evolution of the $\beta$ function of QCD at high energies. As the authors of[12] argue, if the masses of these Higgs bosons are of the order $\alpha\Lambda_{GUT}$, where $\alpha \sim O(10^{-2})$, these degrees of freedom would then modify to high energies the classic value obtained for $\sin \theta_W$ in the $SU(5)_{GG}$ because at two loop order the running coupling constants of the Standard Model $\alpha_3, \alpha_2$ and $\alpha_1$ mix[11], where

$$\frac{d\alpha_i^{-1}}{dt} = \frac{\beta_0^i}{4\pi} + \sum_{j=1}^{3} \beta_{ij}^1 \frac{\alpha_j}{(4\pi)^2}$$

and in the expression above $i, j = 1..3$ label the $U(1)$, $SU(2)$ and $SU(3)$ gauge groups.
TABLE I: We present the masses obtained by technicolored Higgs bosons. The constants $\alpha_1$ and $\alpha_2$ are associated to the quadrilinear Higgs coupling involving, respectively, the $54[\chi, \Phi]$ and $16[\eta]$ Higgs fields.

| Higgs Bosons | $SU(3)_{TC} \times SU(2)_L \times U(1)_Y$ | Mass |
|--------------|---------------------------------|------|
| $\chi$      | $(8,1,0)$                        | $\sim \alpha_1 \Lambda_{TGUT}$ |
| $\Phi$      | $(6,1,-2/3)$                     | $\sim \alpha_1 \Lambda_{TGUT}$ |
| $\eta$      | $(3,1,1/3)$                      | $\sim \alpha_2 \Lambda_{TGUT}$ |

In this work we just extended some of these ideas to TC models, showing that it is possible to obtain the walking behavior with a reduced number of technifermions in the fundamental representation of the TC group if we have technicolored scalar bosons resulting from a TGUT.

IV. CONCLUSIONS

In summary, in this paper we presented a possible way to generate the walking behavior in TC models using scalar matter besides ordinary fermionic matter. In the scheme proposed we get walking with a reduced number of technifermions in the fundamental representation of the TC group and the TC dynamics is similar to QCD until an energy scale of order $O(10^{13} \text{GeV})$.

At the energy scale where fermionic mass would be generated, near the TGUT scale, the behavior of the technicolor $\beta$ function would be modified by the presence of the degrees of freedom associated to the heavy technicolored Higgs bosons that result from the TGUT breaking, leading to the walking behavior.

The unification group considered in this work is not realistic. It was used only to illustrate our proposal to obtain walking, though we believe that these ideas can be applied to more realistic models. As an example of a future proposal, which includes the degrees of freedom of color, we want to propose a model along the lines of the Farhi Susskind model[13].
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