Running with the Radius in RS1

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Abstract

We derive a renormalization group formalism for the Randall-Sundrum scenario, where the renormalization scale is set by a floating compactification radius. While inspired by the AdS/CFT conjecture, our results are derived concretely within higher-dimensional effective field theory. Matching theories with different radii leads to running hidden brane couplings. The hidden brane Lagrangian consists of four-dimensional local operators constructed from the induced value of the bulk fields on the brane. We find hidden Lagrangians which are non-trivial fixed points of the RG flow. Calculations in RS1 can be greatly simplified by “running down” the effective theory to a small radius. We demonstrate these simplifications by studying the Goldberger-Wise stabilization mechanism. In this paper, we focus on the classical and tree-level quantum field theory of bulk scalar fields, which demonstrates the essential features of the RG in the simplest context.
I. INTRODUCTION

Physics in warped higher dimensional spacetimes may play an important role in nature, as illustrated by the RS1 model [1, 2]. However, effective field theory calculations are particularly complicated in such contexts. When warp factors vary greatly in the extra dimension(s) it becomes difficult to power-count the size of Feynman diagrams. For examples of such calculations see Refs. [3]. In this paper, we begin a program of honing “holographic renormalization group” ideas [4], applied to the RS scenario in [5], into a tool for simplifying and gaining insight into warped effective field theory. In particular we will apply this tool to understand the robustness of the Goldberger-Wise stabilization mechanism [6] within effective field theory, although the methods are clearly very general. We use the AdS/CFT conjecture to inspire the form of a concrete and explicit renormalization group procedure, but do not rely on any unproven aspects of the conjecture.

The form our RG will take is as a flow of effective field theories in RS1-type geometries of varying radius, $r_c$, theories with smaller $r_c$ having lower warped down effective energy cutoffs. Two effective theories lying on the same flow are matched to describe the same physics at energies below the lower of the two associated cutoffs. In particular this means that matched theories with different radii have the same “IR” or “visible” brane positions, but different “UV” or “hidden” brane positions. The matching is accomplished by tuning the hidden brane localized Lagrangian as a function of $r_c$. This 4D Lagrangian depends on the values of bulk fields induced on the hidden brane. We will show that formally the flow of hidden brane Lagrangians looks like a Wilsonian RG flow of purely 4D effective field theories. A crucial feature of the associated 4D effective Lagrangians is that they are local which greatly simplifies power-counting. We identify a conformal fixed point in the RG as the concrete realization of the AdS/CFT connection within our formalism. Refs. [7] have developed a related RG formalism focusing on renormalization of boundary divergences and properties of the holographic stress tensor. RG flows associated to branes in codimension two were studied in Refs. [8, 9]. While this paper was being completed, Refs. [10, 11] appeared, which also discuss similar RG ideas.

The utility of our RG formalism is this; one can integrate the flow to take low-energy calculations in a highly warped compactification where they are difficult, to simpler calculations in a much smaller compactification (smaller $r_c$) where the effect of warping is much less significant. Of course this requires computing the effective couplings on the hidden brane in the new compactification, but this is facilitated by perturbing about the conformal fixed point and by working systematically in the small couplings of the theory.

In this paper, we deal only with classical and tree-level quantum effective field theory involving a bulk scalar field, deferring a treatment of full effective quantum field theory with varying spins. Already at this level, many of the non-trivial features of the RG procedure come into play. The paper is organized as follows. In Section II, we discuss free field theory and point out the basic RG structure and fixed point. In Section III, we apply our results to rederive in a simple way the Goldberger-Wise stabilization mechanism when the bulk scalar is identified with the one introduced by Goldberger and Wise. (Since we do not make gravity dynamical, our analysis is necessarily restricted to neglect the backreaction of the scalar on the geometry, the same approximation made in the original analysis.) In Section IV, we generalize our RG procedure to include a large class of interactions, in the bulk and on branes. We then use this in Section V to demonstrate the robustness of the Goldberger-Wise mechanism within classical effective field theory in a manner that lends
itself to generalization to the quantum case. In Section VI, we switch from the language of classical field theory to consider tree-level Feynman diagrams, setting the stage for the case of quantum loops, and showing how one deals with general effective interactions. Section VII contains some concluding remarks.

II. FREE THEORY

We consider a bulk 5D scalar field, $\chi$, propagating on a fixed RS1 background with metric,

$$ds^2 = \frac{1}{(kz)^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

(2.1)

The $z$ coordinate parametrizes the direction in a compact orbifolded space. This is a slice of AdS space bounded by the hidden brane at $z = z_h$ and the visible brane at $z = z_v$. We take $1/z_h = k \sim \mathcal{O}(M_{\text{Plank}})$.

In this section we begin by studying a simple action which is exactly quadratic in $\chi$,

$$S = S_{\text{bulk}} + S_{\text{vis}} + S_{\text{hid}},$$

(2.2)

where

$$S_{\text{bulk}} = \int d^4x dz \sqrt{G} \left( \frac{1}{2} (G^{MN} \partial_M \chi \partial_N \chi) - \frac{1}{2} m^2 \chi^2 \right)$$

$$S_{\text{vis}} = - \int d^4x \sqrt{g_v} \left( \frac{1}{2} \chi k \lambda_v \chi \right)$$

$$S_{\text{hid}} = - \int d^4x \sqrt{g_h} \left( \frac{1}{2} \chi k \lambda_h \chi \right).$$

(2.3)

The brane actions are written in terms of the induced brane metrics,

$$g_v^{\mu\nu}(x) \equiv G_{\mu\nu}(x, z = z_v),$$

$$g_h^{\mu\nu}(x) \equiv G_{\mu\nu}(x, z = z_h),$$

(2.4)

and the dimensionless $x$-derivative expansions (the 4D d’Alembertian is $g^{\mu\nu} \partial_\mu \partial_\nu$ for the metric defined in (2.1) and (2.4)).

$$\lambda h_2 = \lambda h_2^{(0)} + k^{-2} \lambda h_2^{(2)} g_v^{\mu\nu} \partial_\mu \partial_\nu + k^{-4} \lambda h_2^{(4)} g_h^{\mu\nu} g_h^{\rho\sigma} \partial_\mu \partial_\nu \partial_\rho \partial_\sigma + \ldots$$

(2.5)

and similarly for $\lambda v_2$. The subscript “2” denotes that the brane terms are quadratic in fields, which we generalize later. We denote the dimensions explicitly in units of the warping constant, $k$. Note that 4D general covariance ensures that in the space of 4-momenta, $q$, the operator $\lambda h_2 \rightarrow \lambda h_2 (q^2 z_h^2)$.

The equation of motion is

$$\left( \partial_M \sqrt{G} G^{MN} \partial_N + \sqrt{G} m^2 + \sqrt{g_v} k \lambda_v \delta(z - z_v) + \sqrt{g_h} k \lambda_h \delta(z - z_h) \right) \chi(x, z) = 0.$$  

(2.6)

In 4-momentum space this is

$$\frac{1}{(kz)^3} \left( -q^2 - \partial_z^2 + \frac{3}{z} \partial_z + \frac{m^2}{(kz)^2} + \frac{\lambda_v}{z_v} \delta(z - z_v) + \frac{\lambda_h}{z_h} \delta(z - z_h) \right) \chi_q(z) = 0.$$  

(2.7)
Strictly in the bulk, $\chi_q(z)$ satisfies the $z_h$ independent differential equation

$$\frac{1}{(kz)^3} \left( -q^2 - \partial_z^2 + \frac{3}{z} \partial_z + \frac{m^2}{(kz)^2} \right) \chi_q(z) = 0, \quad (2.8)$$

with boundary conditions

$$z \partial_z \chi_q(z) \bigg|_{z=z_h} = \frac{\lambda_{h2}}{2} \chi_q(z_h) \quad (2.9)$$
$$z \partial_z \chi_q(z) \bigg|_{z=z_v} = -\frac{\lambda_{v2}}{2} \chi_q(z_v). \quad (2.10)$$

We now define an “effective” hidden brane whose location is a variable, $a \in [z_h, z_v)$. We seek a new set of “effective couplings” on this brane replacing $\lambda_{h2}$ on the original hidden brane,

$$\lambda_2(a) = \lambda_2^{(0)}(a) + k^{-2}\lambda_2^{(2)}(a)g_{\mu\nu}^a \partial_\mu \partial_\nu + k^{-4}\lambda_2^{(4)}(a)g_{\mu\nu}^a g^{\rho\sigma} \partial_\mu \partial_\rho \partial_\sigma + \ldots, \quad (2.11)$$

where $g_{\mu\nu}^a$ is the induced metric on the brane, which imply a new set of boundary conditions at $a$,

$$z \partial_z \chi_q(z) \bigg|_{z=a} = \frac{\lambda_2(q^2a^2, a)}{2} \chi_q(a). \quad (2.12)$$

We require:

1. The matching condition that $\chi(x, z)$ which are solutions to the original equations of motion (including boundary conditions), (2.6), are also solutions to the effective equations of motion when confined to the effective domain $z \in [a, z_v)$.

2. The boundary condition that $\lambda_{2h}(q^2z_h^2) = \lambda_2(q^2a^2, a)$ for $a = z_h$. This is obviously consistent with the previous requirement.

We interpret $\lambda_2$ as a set of running couplings, $\lambda_2^{(m)}(a)$, and associated 4D generally-covariant derivative operators, $(qa)^m$, just as in (2.5). We would like to find the “renormalization group” flow, $a \frac{d}{da} \lambda_j(a)$, of this set. We proceed by taking the logarithmic derivative of the boundary condition (2.12), use the bulk equation of motion (2.8) as $z \to a_+$ to eliminate second $z$-derivatives of $\chi$ at $a$, and (2.12) again to eliminate any first $z$-derivatives of $\chi$ at $a$, thereby obtaining

$$a \frac{d}{da} \lambda_2(q^2a^2, a) = 4\lambda_2 - \frac{\lambda_2^2}{2} + 2\frac{m^2}{k^2} - 2(qa)^2. \quad (2.13)$$

Clearly, the $a$ dependence in the first variable of $\lambda_2(qa, a)$ is entirely fixed by 4D general covariance. We will focus on the non-trivial dependence in the second variable, getting a set of $\beta$ functions,

$$\sum_j (qa)^j \frac{\partial}{\partial a} \lambda_2^{(j)} = -qa \frac{\partial \lambda_2}{\partial (qa)} + 4\lambda_2 - \frac{\lambda_2^2}{2} + 2\frac{m^2}{k^2} - 2(qa)^2. \quad (2.14)$$

Since $q$ varies freely, this represents a set of coupled differential equations for the $\lambda_2^{(j)}(a)$. 
This flow has a set of fixed points. In the vicinity of such fixed points, the RG flow simplifies. When \( a \frac{d}{da} \lambda_2^{(j)} = 0 \) for all \( j \) we solve the differential equation
\[
-qa \frac{\partial \lambda_2}{\partial (qa)} + 4\lambda_2 - \frac{\lambda_2^2}{2} + 2\frac{m^2}{k^2} - 2(qa)^2 = 0
\]
to find the fixed point
\[
\lambda_2^* = (4 - 2\nu) + 2qa \frac{J_{\nu-1}(qa)}{J_\nu(qa)}, \quad \nu = \sqrt{4 + \frac{m^2}{k^2}}.
\]
(2.16)
This can be expanded in powers of \( q^2 \) and therefore corresponds to a local fixed-point effective brane Lagrangian when written in position space. There is another solution, involving \( N_\nu \)'s, which is non-analytic in \( q \) and therefore cannot be interpreted as a local Lagrangian on the effective brane. It is therefore rejected.

For small momentum \( (qa \ll 1) \) \( \lambda_2^* \approx \lambda_2^{(0)} \). As \( q \to 0 \) (2.16) tells us \( \lambda_2^{(0)} = 2(2 + \nu) \). The same result is also obvious from the \( \beta \) function equation (2.13) by first setting \( q \to 0 \). In fact one can solve (2.13) for the fixed point by working order by order in powers of \( qa \). This will generate (2.16) as a series in \( qa \). At leading order (2.13) implies
\[
\frac{1}{2} \left( \lambda_2^{(0)} \right)^2 - 4\lambda_2^{(0)} - 2\frac{m^2}{k^2} = 0.
\]
(2.17)
Taking the positive root to match (2.16) as \( q \to 0 \), we again find
\[
\lambda_2^{(0)} = 2(2 + \nu).
\]
(2.18)
At higher order in \( qa \) we find \( \lambda_2^{(j)} \sim O(1) \). This method of working out the \( \beta \) functions in powers of \( qa \) will be more useful when we add interactions and the coupled RG equations are not solvable in closed form.

Notice that nowhere did the matching procedure specifically require that the bulk metric be AdS space. For example, expressions equivalent to (2.13) and (2.18) can be obtained for a 5D flat spacetime with the extra dimension taken as an orbifolded circle. Since there is no curvature \( k \), we measure the coupling \( \lambda_2 \) with an arbitrary mass scale \( \mu \). We use the metric
\[
d s^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta - \frac{da^2}{(\mu a)^2}
\]
(2.19)
so that the \( \beta \) function has no explicit \( a \) dependence. We find
\[
a \frac{d}{da} \lambda_{2\text{Mink}} = -\frac{1}{2} \lambda_{2\text{Mink}}^2 + 2\frac{m^2}{\mu^2} - 2\frac{q^2}{\mu^2}.
\]
(2.20)
There is no fixed point for \( q^2 > m^2 \). For \( q^2 < m^2 \) the fixed point is
\[
\lambda_{2\text{Mink}}^* = \frac{2m}{\mu} \sqrt{1 - \frac{q^2}{m^2}}.
\]
(2.21)
Thus for massless or sufficiently light bulk fields there is no fixed point about which the RG flow simplifies. In particular there is no guarantee that one can match the original theory to an effective one in which the extra dimension is much smaller, with only small perturbations to the effective hidden brane action. That is a special property of having (nearly) AdS bulk geometry. Carefully taking the flat space limit of RS1, one can check that (2.13) makes a smooth transition to (2.20).
III. GOLDBERGER-WISE RADIUS STABILIZATION

Adding a tadpole term to both branes will allow us to reproduce the Goldberger-Wise stabilization mechanism [6] using the RG analysis. Since in this paper we are keeping a fixed background geometry for simplicity, we will necessarily be making the same approximation as Goldberger and Wise, of neglecting the back-reaction of the scalar profile on the geometry. However, we expect that the generalization to dynamical gravity is straightforward. Exact solutions that exhibit stability have been examined in [12]. A perturbative approach to stability treating the back reaction in a systematic way was discussed in [13].

Consider now the theory

\[ S = S_{\text{bulk}} + S_{\text{vis}} + S_{\text{hid}} \]

where

\[ S_{\text{bulk}} = \int d^4x dz \sqrt{G} \left( \frac{1}{2} (G^{MN} \partial_M \chi \partial_N \chi) - \frac{1}{2} m^2 \chi^2 \right) \]

\[ S_{\text{vis}} = -\int d^4x \sqrt{g_v} \left( \lambda_v \chi + \frac{1}{2} \lambda_v \chi \right) \]

\[ S_{\text{hid}} = -\int d^4x \sqrt{g_h} \left( \lambda_h \chi + \frac{1}{2} \lambda_h \chi \right), \]

where we now use \( k = 1 \) units. Note that \( \lambda_v, \lambda_h \) are constants, not functions of \( q \) like \( \lambda_v, \lambda_h \).

The field \( \chi \) must satisfy the boundary condition

\[ z \partial_z \chi_q(z) \big|_{z=z_h} = \frac{1}{2} (\lambda_h + \lambda_h(qz_h)\chi(z_h)). \]  

(3.2)

As before we attempt to match onto a theory with the new hidden boundary,

\[ z \partial_z \chi_q(z) \big|_{z=a} = \frac{1}{2} (\lambda_1(a) + \lambda_2(qa,a)\chi(a)). \]  

(3.3)

As in the previous section, we take the logarithmic derivative of this boundary condition, \( (3.3) \), and use the bulk equation of motion as well as \( (3.3) \) to eliminate any \( z \)-derivatives of \( \chi \). We arrive at the equation

\[ \left( -a \frac{d}{da} \lambda_1 + 4 \lambda_1 - \frac{1}{2} \lambda_1 \lambda_2 \right) + \left( -a \frac{d}{da} \lambda_2 + 4 \lambda_2 - \frac{1}{2} \lambda_2^2 + 2m^2 - 2q^2 \right) \chi_q(a) = 0. \]  

(3.4)

We will formally treat the hidden boundary value of the field, \( \chi_q(a) \), as allowed to take arbitrary values, leading to RG equations

\[ a \frac{\partial}{\partial a} \lambda_1 = 4 \lambda_1 - \frac{1}{2} \lambda_1 \lambda_2 \]  

(3.5)

\[ a \frac{\partial}{\partial a} \lambda_2 = -qa \frac{\partial}{\partial (qa)} \lambda_2 + 4 \lambda_2 - \frac{1}{2} \lambda_2^2 + 2m^2 - 2q^2, \]  

(3.6)

where the partial derivatives on the left act on the \( a \) argument, not on the \( qa \) argument, which appears on the right. Clearly solutions to these RG equations will solve \( (3.4) \). However it is not actually true that \( \chi_q(a) \) can take on arbitrary values because of the extra boundary conditions imposed by the visible brane. Thus we can view our RG equations as the unique
ones which are independent of the details of the visible brane, very much like the more familiar mass and vev independent RG flows in energy scales.

In the Goldberger-Wise mechanism the scalar field sets up a stabilizing potential for the radion field the vev of which is \( r \sim \ln(z_v/z_h) \). Ignoring the gravitational backreaction as we have done gives the leading approximation in Newton’s constant to the radion potential. We may calculate this potential in the theory defined by (3.1) as is done in Ref. [13]. We may alternatively study the potential of the equivalent effective theory with the hidden brane run down to a location \( z_v \) near the visible brane. The couplings on the hidden brane will be modified from the theory with hidden brane at \( z_h \) as required by the RG flow.

We consider a theory with hidden brane couplings close to the fixed point. We use the linearized flow equations.

\[
\frac{a}{\partial a} \frac{\partial \lambda_1}{\partial a} = (4 - \frac{\lambda_2^*(0)}{2}) \lambda_1 = (2 - \nu) \lambda_1 \tag{3.7}
\]
\[
\frac{a}{\partial a} (\lambda_2(0) - \lambda_2^*(0)) = (4 - \lambda_2^*(0)) (\lambda_2(0) - \lambda_2^*(0)) = -2\nu(\lambda_2(0) - \lambda_2^*(0)) \tag{3.8}
\]

There are flow equations for the coefficients of higher derivative operators, \( \lambda_2^{(j>0)} \). We do not consider these terms as we wish to find the effective radion potential which is evaluated at zero external momentum. Solving the flow equations we have

\[
\lambda_1 \sim z_v \quad (3.9)
\]
\[
\lambda_2^0 \sim \lambda_2^*(0) + (\lambda_2(0) - \lambda_2^*(0)) \left( \frac{z_v}{z_h} \right)^{-2\nu} \tag{3.10}
\]

For such a small effective extra dimension we can neglect the bulk action and treat the field \( \chi \) as constant across the extra dimension. The effective action then becomes

\[
S = S_a + S_{\text{vis}} = \int \frac{d^4x}{z_v^4} \left( \lambda_{v1} + \lambda_{h1} \left( \frac{z_v}{z_h} \right)^{2-\nu} \right) \chi + \left( \lambda_{v2} + \lambda_{h2} \left( \frac{z_v}{z_h} \right)^{-2\nu} \right) \chi \frac{\lambda_2^0}{2} \tag{3.11}
\]

As was found in Refs. [3, 13] a large hierarchy can be generated by a moderately small scalar mass. In this case we will find after radion stabilization that \( z_h/z_v \ll 1 \). We have

\[
2 - \nu \sim -m^2/8, \quad 2\nu \sim 4. \tag{3.12}
\]

Therefore, contributions in the effective potential due to the deviation from \( \lambda_2^0 \) will be suppressed by \( (z_h/z_v)^4 \). We may self-consistently neglect this term.

The tadpole term in the action indicates a vev for \( \chi \). We find

\[
<\chi> = \frac{\lambda_{v1} + \lambda_{h1} (z_v/z_h)^{-m^2/8}}{\lambda_{v2} + \lambda_2^*(0)} \tag{3.13}
\]

Note that this corresponds precisely to the solution for \( \chi \) found in Ref. [13]. There, a solution for \( \chi \) to first order in \( \lambda_{v1} \) and \( \lambda_{h1} \) was found as a function of the extra-dimensional space. Evaluating this solution at the location of the visible brane gives equation (3.13).
Substituting this expression for $\langle \chi \rangle$ into the action we obtain an effective potential for the radion.

$$V_{\text{eff}} = -\frac{\left(\lambda_{v1} + \lambda_{h1} \left(\frac{z_v}{z_h}\right)^{-m^2/8}\right)^2}{2 z_v^4 \left(\lambda_{v2}^{(0)} + \lambda_{h2}^{(0)}\right)}.$$  \hspace{1cm} (3.14)

Minimizing this potential we find that a hierarchy is established.

$$\frac{z_v}{z_h} = \left(-\frac{\lambda_{v1}}{\lambda_{h1}}\right)^{-8/m^2}.$$  \hspace{1cm} (3.15)

A large hierarchy is generated for a modest ratio of $\lambda_{v1}/\lambda_{h1}$.

IV. INTERACTIONS

We now apply the methods of the previous sections to a more general action. Consider interacting theories of the type

$$S_{\text{bulk}}[\chi(x, z)] = \int d^4x \, dz \sqrt{G} \left\{ \frac{1}{2} G^{MN} \partial_M \chi \partial_N \chi - V_{\text{bulk}}(\chi) \right\}$$

$$S_{\text{hid}}[\chi(x, z_h)] = \int d^4x \, \sqrt{g_h} L_h(\chi, \partial_\mu, g_{\mu\nu}^h)$$

$$S_{\text{vis}}[\chi(x, z_v)] = \int d^4x \, \sqrt{g_v} L_v(\chi, \partial_\mu, g_{\mu\nu}^v).$$

\hspace{1cm} (4.1)

The brane localized Lagrangians are taken to be local in $\chi$ and its $x$-derivatives. We will see that the RG flow preserves this general form.

The equations of motion imply the hidden boundary condition

$$z \partial_z \chi(x, z) \bigg|_{z=z_h} = -\frac{1}{2} \frac{1}{\sqrt{g_h}} \frac{\delta S_{\text{hid}}[\psi]}{\delta \psi(x)} \bigg|_{\psi(x)=\chi(x, z_h)}.$$  \hspace{1cm} (4.2)

We will use the dummy variable $\psi$ in the four dimensional variations that remain after integrating over the delta functions in $z$ in the equations of motion emerging from brane terms.

In analogy to (2.12), we look for a new local action $S_a$ which satisfies the boundary condition

$$z \partial_z \chi(x, z) \bigg|_{z=a} = -\frac{1}{2} \frac{1}{\sqrt{g}} \frac{\delta S_a[\psi]}{\delta \psi(x)} \bigg|_{\psi(x)=\chi(x, a)} \quad a \in [z_h, z_v).$$  \hspace{1cm} (4.3)

where

$$S_a[\psi] = \int d^4x \, \sqrt{g} L_a(a, \psi, \partial_\mu, g_{\mu\nu}^a), \quad g_{\mu\nu}^a = G_{\mu\nu}(x, z = a).$$  \hspace{1cm} (4.4)

We require the boundary condition

$$S_a[\chi(x, z_h)] = S_{\text{hid}}[\chi(x, z_h)],$$  \hspace{1cm} (4.5)

and that solutions, $\chi(x, z)$, to the original equations of motion solve the effective equations of motion when restricted to $z \in [a, z_v)$. 
As in previous sections, we calculate the logarithmic $a$-derivative of the effective hidden boundary condition, and eliminate first and second $z$-derivatives at the effective hidden brane using the boundary condition and the equations of motion. Using the obvious result $a\partial_\mu g^{\mu\nu} = 2g^{\mu\nu}$ we arrive at

\[
\frac{1}{2\sqrt{g}} a\partial_\mu \frac{\delta S_a[\psi]}{\delta \psi(x)} \bigg|_{\psi(x) = \chi(x,a)} = \frac{1}{4(\sqrt{g})^2} \int d^4x' \frac{\delta^2 S_a[\psi]}{\delta \psi(x') \delta \psi(x)} \bigg|_{\psi(x) = \chi(x,a)} \times \frac{\delta S_a[\psi]}{\delta \psi(y)} \bigg|_{\psi(y) = \chi(x',a) - \frac{1}{\sqrt{g}} \int d^4x'' \frac{\delta^2 S_a[\psi(x'')]}{\delta g^{\mu\nu}(x'') \delta \psi(x)} \bigg|_{\psi(x) = \chi(x,a)} g^{\mu\nu}(x) - \frac{\partial V_{\text{bulk}}}{\partial \chi(x,a)} - g^{\mu\nu} \partial_\mu \partial_\nu \chi(x,a) \tag{4.6}
\]

where the partial $a$ derivative on the left is taken to act only on the running couplings of the effective theory.

It is important to note that this equation is local in the couplings and the field $\chi(x,a)$ since it is constructed from the variations of the presumed local action, $S_a$. Thus we have shown that local effective actions flow to local effective actions, and the boundary condition for the flow at $a = z_h$ is $S_{\text{hid}}$, which is local. To find the $\beta$ functions for each coupling, we expand (4.3) in powers of $\chi_4(a)$ and the independent momenta, $q$ and require that each coefficient vanish. As in the previous section, this amounts to treating $\chi_4(a)$ as free to take on any value in the set of 5D solutions, which is equivalent to choosing an RG flow which is independent of the details of the visible brane.

Since we have not considered higher-derivative interactions in the bulk, we have still not obtained the greatest generality. There is no obstruction to finding local RG flows in the most general case of higher bulk derivatives, an example of which appears in Section VIB.

As an example of the use of equation (4.6), take the theory with the bulk interaction

\[
S_{\text{bulk}} \supset \int d^4x dz \sqrt{G} (\sigma_3 \chi^3). \tag{4.7}
\]

On the hidden brane we have

\[
S_{\text{hid}} \supset \int d^4x \sqrt{g_h}(\lambda_{h3}^{(0)} \chi^3 + \lambda_{h4}^{(0)} \chi^4). \tag{4.8}
\]

We can calculate the running of $\lambda_3^{(0)}$ and $\lambda_4^{(0)}$. Equation (4.6) gives

\[
\ldots + 3(a \frac{\partial}{\partial a} \lambda_3^{(0)} - 4\lambda_3^{(0)} + \frac{3}{2} \lambda_2^{(0)} \lambda_4^{(0)} + 2\sigma_3^{(0)})\chi^2
+ 4(a \frac{\partial}{\partial a} \lambda_4^{(0)} - 4\lambda_4^{(0)} + 2\lambda_2^{(0)} \lambda_4^{(0)} + \frac{9}{4}(\lambda_3^{(0)})^2)\chi^3 + \ldots = 0 \tag{4.9}
\]

where we take $\lambda_{h1} = 0$. Our RG flow then follows by treating $\chi(z = a)$ as a free variable,

\[
a \partial_a \lambda_3^{(0)} = 4\lambda_3^{(0)} - \frac{3}{2} \lambda_2^{(0)} \lambda_4^{(0)} - 2\sigma_3^{(0)} \tag{4.10}
\]

\[
a \partial_a \lambda_4^{(0)} = 4\lambda_4^{(0)} - 2\lambda_2^{(0)} \lambda_4^{(0)} - \frac{9}{4}(\lambda_3^{(0)})^2. \tag{4.11}
\]

A central result of this present section is the robust retention of a fixed point in the RG flow in the presence of interactions, which is IR-attractive as long as the bulk mass-squared
is positive. To see this recall that we have found in Section II a fixed point for the free theory:

\[ \lambda_2 = \lambda_2^* \] (4.12)

\[ \lambda_n = 0 \] (4.13)

Using (4.6) one can straightforwardly find the linearized \( \beta \) functions for flows near this fixed point when brane interactions are permitted, but bulk interactions are still absent. We find

\[ a \frac{\partial}{\partial a} \lambda = \gamma \cdot (\lambda - \lambda^*) + O((\lambda - \lambda^*)^2). \] (4.14)

The scaling matrix, \( \gamma \), is lower triangular (operators with derivatives only mix with operators with fewer derivatives and \( \lambda_n \) and \( \lambda_m \) do not mix for \( n \neq m \)), so that the eigenvalues are given by the diagonal elements. These eigenvalues are

\[ \gamma_i^{(2j)} = 4 - 2j - \frac{i}{2} (4 + 2\nu) < 0 \] (4.15)

where \( i \) indicates the number of \( \chi \) fields in the operator and \( 2j \) indicates the highest number of derivatives in that operator. The scaling dimensions are negative so this fixed point is attractive.

We now add bulk interactions. From (4.6) one can see that the \( \beta \) function will be corrected by a power series in bulk couplings (actually a linear term at classical order). If these couplings are given the formal small parameter \( \alpha \) they will alter the RG flow by \( O(\alpha) \) terms, which cannot eliminate the fixed point but only shift its position by \( O(\alpha) \). The scaling dimensions will also receive an \( O(\alpha) \) correction. The new fixed point will therefore be attractive for a sufficiently small \( \alpha \).

V. GOLDBERGER-WISE MECHANISM IN THE INTERACTING THEORY

The robustness of the fixed point and the anomalous dimensions when interactions are included translates into the robustness of the Goldberger-Wise mechanism for stabilizing the hierarchy, which we see as follows. As long as our original hidden brane action is sufficiently close to the fixed point action, we are justified in using the linearized RG flow, given by the anomalous dimensions. As long as the strength of bulk couplings is sufficiently small, the \( \alpha \) mentioned above being at most of order the bulk mass-squared (in \( k = 1 \) units), the spectrum of anomalous dimensions is still given by one coupling with anomalous dimension of order \( -m^2 \), while the other couplings have anomalous dimensions which are of order \( -1 \) or more negative.

We will now repeat our Goldberger-Wise analysis as in the previous section, but now using this interacting RG flow. Running the Planck brane down to \( z_v \) as before we drop derivative terms to find a vacuum potential of the form

\[ V_{\text{eff}} = \frac{1}{z_v^4} \left( L_v(\chi) + L_h^*(\chi) + \sum_i (\lambda_{hi} - \lambda_{hi}^*) \left( \frac{z_h}{z_v} \right)^{\gamma_i} \mathcal{O}_i(\chi) \right) \] (5.1)

where the operators \( \mathcal{O}_i \) have scaling dimension \( \gamma_i \). The functions \( L_v \) and \( L_h^* \) have no explicit \( z_v/z_h \) dependence. We will show self-consistently that \( z_h/z_v \ll 1 \). We can then discard all
terms suppressed by order one powers of $z_h/z_v$ as they are negligible. Taking $\gamma_1$ to be of order $-m^2$, only one hidden brane operator is relevant. We have

$$V_{eff} = \frac{1}{z_v^4} \left( L_v(\chi) + L_h(\chi) + (\lambda_{h1} - \lambda_{h1}^*) \left( \frac{z_h}{z_v} \right)^{\gamma_1} \mathcal{O}_1(\chi) \right).$$  \hspace{1cm} (5.2)$$

Notice that this potential has the form,

$$V_{eff} = \frac{1}{z_v^4} P(\chi, (z_v/z_h)^{\gamma_1}),$$  \hspace{1cm} (5.3)$$

where $P$ indicates a polynomial. For quite generic coefficients this indicates that solutions with

$$\langle \chi \rangle = \mathcal{O}(1),$$  \hspace{1cm} (5.4)$$

$$\left( \frac{z_v}{z_h} \right)^{\gamma_1} = \mathcal{O}(1),$$  \hspace{1cm} (5.5)$$

minimize this potential. These relations are explicitly confirmed at leading order in $\alpha$ in Section III were a large hierarchy is generated. We see that a large hierarchy will also be set including the $\mathcal{O}(\alpha)$ (and higher) corrections.

$$\frac{z_v}{z_h} = (\mathcal{O}(1))^{1/\gamma_1}.$$  \hspace{1cm} (5.6)$$

VI. GREEN’S FUNCTIONS

In quantum field theory the quantities of physical interest are the $n$-point functions. Take the theory of equation (4.1) with the hidden brane at $z = z_h$. We may also consider a theory with the hidden brane at a location $a > z_h$. These theories will describe equivalent physics if they give the same $n$-point functions. We require

$$\langle \chi(x_1, z_1) \ldots \chi(x_n, z_n) \rangle_{z_h} = \langle \chi(x_1, z_1) \ldots \chi(x_n, z_n) \rangle_a$$  \hspace{1cm} (6.1)$$

where the final subscript denotes the position of the hidden brane, where $z_i > a$. This method of matching $n$-point functions is done at tree level in this paper and gives an alternate, equivalent way of calculating the flow equations for hidden brane couplings to that of the previous sections. We demonstrate this method through a few examples. Its utility is primarily in that the method may be extended to quantum calculations.

A. The 2 point function

To evaluate $n$-point functions in perturbation theory we must first consider the two-point function.

$$\Delta(x, z; x', z') \equiv \langle \chi(x, z) \chi(x', z') \rangle$$  \hspace{1cm} (6.2)$$

It satisfies

$$\left( \partial_M \sqrt{G} G^{MN} \partial_N + \sqrt{G} m^2 + \sqrt{g_v} \lambda_v \delta(z - z_v) + \sqrt{g_h} \lambda_h \delta(z - z_h) \right) \Delta(x, z; x', z') = \delta^4(x - x') \delta(z - z').$$  \hspace{1cm} (6.3)$$
Explicit solutions for the Green’s function in AdS space have been found in Refs. [14, 15]. Simplifying in 4D momentum space this is

$$\frac{1}{z^3} \left( -q^2 - \partial_z^2 + \frac{3}{z} \partial_z + \frac{m^2}{z^2} + \frac{\lambda_v}{z_v} \delta(z - z_v) + \frac{\lambda_h}{z_h} \delta(z - z_h) \right) \Delta_q(z, z') = \delta(z - z'). \quad (6.4)$$

As usual, this is equivalent to a $z_h$ independent differential equation

$$\frac{1}{z^3} \left( -q^2 - \partial_z^2 + \frac{3}{z} \partial_z + \frac{m^2}{z^2} \right) \Delta_q(z, z') = \delta(z - z'), \quad (6.5)$$

subject to $z_h$-dependent boundary conditions,

$$\partial_z \Delta_q(z, z') \bigg|_{z = z_h} = \frac{\lambda_2(qz_h)}{2z_h} \Delta_q(z_h, z'), \quad (6.6)$$

$$\partial_z \Delta_q(z, z') \bigg|_{z = z_v} = -\frac{\lambda_2}{2z_v} \Delta_q(z_v, z'). \quad (6.7)$$

In the effective theory with the hidden brane at $a$ the 2-point function satisfies the same differential equation and visible boundary condition but with a modified hidden boundary condition.

$$\partial_z \Delta_q(z, z') \bigg|_{z = a} = \frac{\lambda_2(qa, a)}{2a} \Delta_q(a, z'). \quad (6.8)$$

The coupling $\lambda_2(qa, a)$ is chosen so that

$$\Delta_q(z, z')_a = \Delta_q(z, z')_{z_h}, \quad z, z' > a, \quad (6.9)$$

which implies

$$\partial_z \Delta_q(z, z')_a \bigg|_{z = a + \delta a} = \frac{\lambda_2(q(a + \delta a), a + \delta a)}{2(a + \delta a)} \Delta_q(a + \delta a, z')_a. \quad (6.10)$$

Expanding for infinitesimal $\delta a$, using (6.5) to eliminate $\partial^2_z \Delta$ and the effective hidden boundary condition to eliminate $\partial_z \Delta$ at $z = a$, leads to

$$\sum_j (qa)^j a \frac{\partial \lambda_2^{(j)}}{\partial a} = -qa \frac{\partial \lambda_2}{\partial qa} + 4\lambda_2 - \frac{\lambda_2^2}{2} + 2m^2 - 2(qa)^2. \quad (6.11)$$

This is the same flow described in Section II.

What follows are two example flow calculations. The central object will be the free field two point function at the RG fixed point, satisfying the boundary condition

$$\partial_z \Delta_q(z, z') \bigg|_{z = a} = \frac{\lambda_2^*(qa)}{2a} \Delta_q(a, z'). \quad (6.12)$$

The theory will in general not be at this fixed point. We treat contributions to the $n$-point function due to bulk interactions and deviations from $\lambda_2^*$ perturbatively.
B. Bulk Derivatives

In our earlier classical analysis we had restricted ourselves for simplicity to bulk interactions of the form of a potential. As an example of a perturbative calculation using the method of matching Green’s functions consider here the specific bulk derivative coupling

\[ S_{\text{bulk}} \supset \int d^4x \int dz \sqrt{G} \sigma_4^{(2)} (G^{MN} \partial_M \partial_N \chi) (\chi^2), \]  

(6.13)

with coupling constant \( \sigma_4^{(2)} \) \(^1\) Take two theories with the hidden brane at \( a \) and \( a + \delta a \) respectively. We require that the 4-point function be independent of the location of the brane,

\[ \langle \chi(x_1, z_1) \ldots \chi(x_4, z_4) \rangle_a = \langle \chi(x_1, z_1) \ldots \chi(x_4, z_4) \rangle_{a+\delta a}. \]  

(6.14)

For infinitesimal \( \delta a \) this is

\[ a \frac{d}{da} \langle \chi(x_1, z_1) \ldots \chi(x_4, z_4) \rangle_a = 0. \]  

(6.15)

On the hidden brane we need

\[ S_a \supset \int d^4x \sqrt{g_a} (\lambda_4^{(0)} \chi^4 + \ldots). \]  

(6.16)

where in this section \( \ldots \) indicates terms with higher \( x^\mu \) derivatives. In perturbation theory we use the free field propagator at the fixed point \( \lambda_2^* \), as discussed in the previous subsection. Calculating the 4 point function at first order in the couplings gives

\[ \langle \chi(x_1, z_1) \ldots \chi(x_4, z_4) \rangle_a = 4 \int d^4x \int z_0 \sigma_4^{(2)} \sqrt{G} G^{zz} (\partial_z \Delta(x_1, z_1; x, z) \partial_z \Delta(x_2, z_2; x, z) \Delta(x_3, z_3; x, z) \Delta(x_4, z_4; x, z) + \text{perm.}) \]

\[ + 12 \int d^4x \sqrt{g_a} \lambda_4^{(0)} \Delta(x_1, z_1; x, a) \Delta(x_2, z_2; x, a) \Delta(x_3, z_3; x, a) \Delta(x_4, z_4; x, a) + \ldots \]  

(6.17)

where perm. indicates a permutation of the \( z \) derivatives on the propagators. Taking the logarithmic derivative with respect to \( a \) we have

\[ 4 \int d^4x \left\{ - \frac{1}{a^4} \sigma_4^{(2)} a^2 (\partial_a \Delta(x_1, z_1; x, a) \partial_a \Delta(x_2, z_2; x, a) \Delta(x_3, z_3; x, a) \Delta(x_4, z_4; x, a) + \text{perm.}) \right. \]

\[ + 12 \left( \frac{4}{a^4} \lambda_4^{(0)} \Delta(x_1, z_1; x, a) \Delta(x_2, z_2; x, a) \Delta(x_3, z_3; x, a) \Delta(x_4, z_4; x, a) \right. \]

\[ \left. - \frac{1}{a^4} \left( a \frac{d}{da} \lambda_4^{(0)} \right) \Delta(x_1, z_1; x, a) \Delta(x_2, z_2; x, a) \Delta(x_3, z_3; x, a) \Delta(x_4, z_4; x, a) \right. \]

\[ \left. - \frac{a}{a^4} \lambda_4^{(0)} \frac{d}{da} (\Delta(x_1, z_1; x, a) \Delta(x_2, z_2; x, a) \Delta(x_3, z_3; x, a) \Delta(x_4, z_4; x, a))) \right\} = 0. \]  

(6.18)

\(^1\) We will abstain from doing the most general analysis of bulk derivative couplings due to the sheer complexity of notation rather than any intrinsic difficulty. Our example will allow the reader to generalize to any case of their interest. Note that couplings involving more than one \( z \)-derivative acting on the field can be removed by a field redefinition as described in Ref. \[13\].
FIG. 1: Diagrams giving the four point function at tree level. A $\chi^3$ interaction requires a $\chi^4$ “counterterm” at tree level to match theories with different radii.

Using the fact that the two-point functions are at their free-field fixed point values,

$$a \frac{d}{da} \Delta(x, a; x_i, z_i) = \frac{\lambda_2^2}{2} \Delta(x, a; x_i, z_i),$$  

(6.19)

we obtain

$$\frac{12}{a^4} \int d^4x \Delta(x_1, z_1; x, a) \Delta(x_2, z_2; x, a) \Delta(x_3, z_3; x, a) \Delta(x_4, z_4; x, a) \times \left(2\sigma_4^{(2)} a^2 \left(\frac{\lambda_2^{(0)}}{2a}\right)^2 - 4\lambda_4^{(0)} + a \frac{d}{da} \lambda_4^{(0)} + 4a\lambda_4^{(0)} \left(\frac{\lambda_2^{(0)}}{2a}\right) \right) + \ldots = 0$$  

(6.20)

In 4-momentum space we may divide by the external propagators and then set external momentum to zero to obtain

$$a \frac{d\lambda_4^{(0)}}{da} = (4 - 2\lambda_2^{(0)})\lambda_4^{(0)} - 2\sigma_4^{(2)} \left(\frac{\lambda_2^{(0)}}{2}\right)^2,$$  

(6.21)

showing how this bulk derivative interaction modifies the RG flow for a hidden brane coupling.

C. Nonlinear effects of brane $\chi^3$ interactions

As a second example, illustrating nonlinear effects in the RG, consider the brane interactions

$$S_{\text{hid}} \supset \int d^4x \sqrt{g_h}\lambda_h^{(0)} \chi^3.$$  

(6.22)

These will induce 4-point operators through the diagrams of Figure 1. We require hidden brane “counterterms”

$$S_{\text{c.t.}} \supset \int d^4x \sqrt{g_h}(\lambda_4^{(0)} \chi^4 + \ldots).$$  

(6.23)

to maintain invariance of the 4-point function working to second order in $\lambda_h^{(0)}$. Here again ... indicates higher $x_\mu$ derivatives.

We calculate

$$\langle \chi(x_1, z_1) \cdots \chi(x_4, z_4) \rangle_a =$$

$$12 \int d^4x \lambda_4^{(0)}(a) \Delta(x_1, z_1; x, a) \Delta(x_2, z_2; x, a) \Delta(x_3, z_3; x, a) \Delta(x_4, z_4; x, a)$$
FIG. 2: Diagrams contributing to the fourpoint function in addition to those of Figure 1. They give nonlinear contributions to the $\beta$ function of the form $\lambda_2 \lambda_4$ (left fig.) and $\lambda_1 \lambda_5$ (right fig.).

\[ + 9 \int d^4x d^4y \left( \frac{\lambda_3(0)}{a} \right)^2 \left( \Delta(x_1, z_1; x, a) \Delta(x_2, z_2; x, a) \Delta(x, a; y, a) \Delta(x_3, z_3; y, a) \Delta(x_4, z_4; y, a) \right) \\
+ (x_2, z_2 \leftrightarrow x_3, z_3) + (x_2, z_2 \leftrightarrow x_4, z_4) \right) + \ldots \]  

(6.24)

requiring

\[ \frac{a}{da} \langle \chi(x_1, z_1) \ldots \chi(x_4, z_4) \rangle_a = 0. \]  

(6.25)

We use

\[ \frac{a}{da} \Delta(x, a; x_i, z_i) = \frac{\lambda_2^*}{2} \Delta(x, a; x_i, z_i), \]  

(6.26)

\[ \frac{a}{da} \Delta(x, a; y, a) = \lambda_2^* \Delta(x, a; y, a) - a^4 \delta^4(x - y) \]  

(6.27)

where the delta function term in (6.27) is due to the discontinuity in $\Delta$ following from (6.3).

To linear order in $\lambda_3(0)$ it is straightforward to establish

\[ \frac{a}{da} \lambda_3(0) = (4 - \frac{3}{2} \lambda_2^*(0)) \lambda_3(0). \]  

(6.28)

Substituting these expressions into (6.24), transforming to 4-momentum space, dividing by the external propagators and setting external momentum to zero gives

\[ \frac{a}{da} \lambda_4(0) = (4 - 2 \lambda_2^*(0)) \lambda_4(0) - \frac{9}{4} (\lambda_3(0))^2 \]  

(6.29)

This matches the result that is obtained through the methods of Section IV. This demonstrates how $\chi^3$ interactions induce running in $\lambda_4(0)$. Note that other nonlinear contributions in the $\beta$ functions may be calculated through the diagrams of Figure 2.

VII. CONCLUDING REMARKS

The two main directions for generalizing the above RG formalism are including higher spins and including quantum loops. Spins greater than or equal to one raise an interesting subtlety not encountered in this paper. The usual CFT interpretation of a bulk RS1 field is that it corresponds to a CFT operator and to a fundamental 4D field coupled to that CFT. Therefore it is at first sight odd that in our RS1 examples we found exact fixed points without the deviations corresponding to the coupling to a fundamental 4D scalar. However, one can see that the hidden brane fixed point Lagrangian found in this paper contains an
effective-cutoff sized mass for the bulk scalar. The dual of this is a CFT coupled to a cutoff-mass fundamental scalar that is a scalar heavy enough to be completely integrated out of the effective theory leaving the pure CFT dynamics. Now consider higher spins protected by gauge invariance. For concreteness we will consider gravity in the full RS1 model. We may want to keep the zero mode graviton (dual to the fundamental 4D graviton coupled to the CFT). In this case we must accomodate the complication that the effective hidden brane theory must describe “almost” fixed point behavior with small deviations arising from weak coupling to the zero mode graviton.

Quantum loops can be dealt with in a very similar manner to the methods of Section VI. The central technical issue is a proof to all orders of the locality of the running effective hidden brane Lagrangian. (We fully expect this locality.) Once this is demonstrated, the RG approach will undoubtedly provide the simplest and most insightful proof of the stability of the RS-Goldberger-Wise hierarchy to all orders in the effective field theory. The method of proof would be almost identical to the proof of hierarchy robustness given here in Section V for the classical interacting theory, which uses only the most general features of the RG flows. The importance of such a proof is easy to understand when one considers that the Goldberger-Wise effective potential is basically a weak-scale potential for the radius, whereas the RS1 effective theory, has access to Planckian mass scales (especially at the quantum level) which naively threaten to overwhelm weak-scale effects. Understanding how the potential is protected by such effects is a particular case illustrating the more general utility of the RG procedure, which is to identify where large dependences on the warp factor appear within Feynman diagrams. Another important application would be quantum corrections to the masses and decays of low lying Kaluza-Klein resonances.

Work on these fronts is in progress.

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