Light scalar mesons in QCD sum rules with inclusion of instantons

J. Zhang\textsuperscript{1}, H.Y. Jin\textsuperscript{1}, Z. F. Zhang\textsuperscript{2}, T.G. Steele\textsuperscript{3} and D. H. Lu\textsuperscript{1}

\textsuperscript{1}Institute of Modern Physics, Zhejiang University, Hangzhou, Zhejiang, China
\textsuperscript{2}Department of Physics, Ningbo University, Ningbo, Zhejiang, China
\textsuperscript{3}Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, Saskatchewan, Canada S7N 5E2

(Dated: March 2009)

Abstract

The light scalar meson nonet above 1 GeV (i.e. the $a_0$, $K_0^*$ and $f_0$) are studied within the framework of QCD sum rules. In conventional QCD sum rules, the calculated masses of this nonet are degenerate, and the mass of $K_0^*$ is always larger than the $a_0$ in contradiction with the observed spectrum. After improving the correlation function by including instanton effects, the masses are well separated from each other. In particular, our results shows glueball content plays an important role in the underlying structure of $f_0(1500)$. The decay constants are also discussed.

PACS numbers: 12.38.Lg, 12.39.Mk, 14.40.Cs, 14.40. Ev
I. INTRODUCTION

The $SU(3)$ classification of strongly-interacting particles originally proposed by Gell-Mann and Zweig has been a very successful paradigm in particle physics. It is observed from the hadronic spectrum that the results following from this naive quark model (NQM) agree better with the heavy meson systems than the light ones. This is understandable because the heavy quarks inside a heavy meson are non-relativistic, and hence we can deal with their kinematics in the framework of nonrelativistic quantum mechanics as a good approximation. However, for light mesons where the light components are relativistic, it is hard to say whether the nonrelativistic approximation is applicable. We can see that there is more complexity in light mesons than the heavy ones from the observed spectrum. The situation is even worse in combination with the proliferation of light scalars and their production in charmless $B$ decays. In order to accommodate these light mesons in theories consistent with QCD, models beyond the naive quark model have been developed, including glueballs, multiquark states and hybrid states. One hopes that these models can supply some reasonable, or at least qualitative, explanation of the observed light mesons.

The underlying structure of mesons with mass near 1 GeV attract much attention. It has widely been suggested that the light scalars below or near 1 GeV [the isoscalars $f_0(600), f_0(980)$, the isodoublet $K^*_0(800)$ (or $\kappa$) and the isovector $a_0(980)$] form a $SU(3)$ flavor nonet, while scalar mesons above 1 GeV [$f_0(1370), a_0(1450), K^*_0(1430)$ and $f_0(1500)/f_0(1710)$] form another nonet. Refs. suggest that the light scalar nonet above 1 GeV can be accommodated in the conventional $\bar qq$ model with some gluonic component, while the light scalars around 1 GeV are dominated by $\bar qq\bar qq$ states with some $0^+ \bar qq$ and glueball states. But this interpretation is still far from deciphering the puzzle presented by the light scalars.

It is obvious that the starting point of all the models mentioned above concentrates on the kinematic aspect of the component inside the scalars, i.e., in order to reproduce the spectrum in theories consistent with QCD, the complexity of the light scalars is attributed to their constituents. Maybe one can refer to this as a kinematics-dependent approach. There is another viewpoint we can adopt. We should recognize that in the hadronic region perturbative

---

1 Following Ref. we refer to this model as the naive quark model.
QCD breaks down, and the nonperturbative aspects of QCD are dominant. It is well-known the nonperturbative aspect of QCD is difficult to analyze. The nonzero quark condensate signals that the QCD vacuum is nontrivial and has a complex structure. In other word, the dynamics in QCD vacuum is very different from the trivial one. It is possible that the complexity of the light scalar mesons can be attributed to the enigmatic QCD vacuum, in which the particle treated as the excitation of the QCD vacuum from the viewpoint of quantum field theory. The nonzero value of QCD vacuum expectation values is one of the main ingredients of QCD sum rules [11, 12, 13] which deals with the low energy nonperturbative aspects of QCD. In conventional QCD rules the physical quantities are expressed by a dominant perturbative part and corrections associated with vacuum expectation values of various operators. This method works well in many cases, but when we apply this method to the pions, it is difficult to obtain reasonable results. This difficulty was solved by introducing instanton contribution into the QCD sum rules [14]. Instantons—the nontrivial solution to the Yang-Mills field equation [15]—play an important role in solving the puzzle. Recent work involving QCD sum rules with instanton effects include the electromagnetic pion form factor [17] and glueballs [18]. Furthermore, instanton effects within QCD the non-strange sum-rules for scalar currents have previously been shown to split the degeneracy between the $a_0$ and $f_0$ [19]. All these works pave a new way to resolving the controversy concerning the nature of the light scalars.

Comparing with the kinematics-dependent approach, we refer to the instanton effects as a dynamics-dependent approach, because here one attempts to solve the problem by further investigating low energy QCD itself. Keeping these motivations in mind, in this paper we investigate the masses of the scalar nonet above 1GeV [i.e, $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$ and $f_0(1500)/f_0(1710)$] from QCD sum rules based on scalar interpolating fields including the corresponding instanton contribution. Because there is no mixing between $a_0$, $K_0^*$ meson and the glueball, these two members are ideally suited to investigate the role of instantons in QCD sum rules, and we will analyze them in the naive quark model. The situation is more complicated for the $f_0$ because it is widely accepted that there is mixing with the isoscalar $0^{++}$ glueball ground state [3] around 1500MeV.

---

2 For details on instantons in QCD, see the excellent review by T. Schäfer and E. V. Shuryak [16].
3 We note that these works did not consider the structure of the entire nonet.
Because of this mixing, a more consistent analysis should consider the mixing of quark and gluonic content in analyzing \( f_0 \) meson. So we will employ a mixed quark-glueball current to discuss \( f_0 \) meson if necessary. Specifically, we assign \( f_0(1500) \) and \( f_0(1710) \) to be a mixed current of quark and gluonic content, while \( f_0(1370) \) is still assumed to be purely of quark content. As will be demonstrated below, the validity of these assignments is upheld by the results of the QCD sum-rule analysis. The instanton contributions to the sum-rules are calculated using the semiclassical approximation with quark zero modes. As a byproduct, the decay constants of the states are obtained naturally.

In Section II we derive the QCD sum rules with scalar interpolating fields in the absence of instantons and note the shortcomings associated with the results of this analysis. In Section III we present the sum rule including instanton contribution based on scalar current or its mixing with gluonic current, and the masses and decay constants of the nonet are calculated. Section IV is devoted to our conclusions.

II. SUM RULES WITHOUT INSTANTONS

In this section we will discuss QCD sum-rules without instantons and the results following it. The starting point is the following correlator defined in terms of scalar interpolating current:

\[
\Pi(q^2) = i \int d^4 x e^{iq \cdot x} \langle 0 | j(x) j^\dagger(0) | 0 \rangle,
\]

where \( j(x) \) is a scalar composite operator defined as:

\[
j(x) = \bar{q}_1(x) q_2(x).
\]

compared with the definition in [14], we have suppressed the renormalization invariant factor \((\ln(\mu/\Lambda))^{-4/b}\), with \( \mu \) is the normalization pint and \( b = (11N_c - 2n_f)/3 \). The correlator can be expressed in terms of operator product expansion, up to order-\( \alpha_s \) perturbative correction and

\[\text{Lattice gauge calculations predict a glueball mass of 1400 to 1800 MeV [20].}\]

4 Lattice gauge calculations predict a glueball mass of 1400 to 1800 MeV [20].
dimension-six, the operator product expansion we get is \[ \Pi_{\text{OPE}}(q^2) = -\frac{3}{8\pi^2}(1 + \frac{11}{3}\frac{\alpha_s}{\pi}) q^2 \ln \frac{-q^2}{\mu^2} + \frac{3}{4\pi^2} m_1 m_2 \ln \frac{-q^2}{\mu^2} - \frac{1}{8\pi \frac{1}{q^2}}(\alpha_s G^2) \]

\[ - \frac{1}{q^2} \left( \frac{m_1}{2} + m_2 \right) \langle \bar{q}_1 q_1 \rangle - \frac{1}{q^2} \left( \frac{m_2}{2} + m_1 \right) \langle \bar{q}_2 q_2 \rangle \]

\[ - \frac{1}{2q^4} m_2 \langle g_s \bar{q}_1 \sigma G q_1 \rangle - \frac{1}{2q^4} m_1 \langle g_s \bar{q}_2 \sigma G q_2 \rangle \]

\[ - \frac{16\pi}{27} \frac{\alpha_s}{q^4} \left[ \langle \bar{q}_1 q_1 \rangle^2 + \langle \bar{q}_2 q_2 \rangle^2 \right] \]

\[ - \frac{48\alpha_s}{9} \frac{1}{q^4} \langle \bar{q}_1 q_1 \rangle \langle \bar{q}_2 q_2 \rangle \]

This is the theoretical side of the QCD sum rule from the quark-gluon dynamics point of view.

On the other hand, the correlator can also be derived phenomenologically:

\[ \Pi(q^2) = \frac{1}{\pi} \int_0^\infty ds \ \frac{\text{Im} \Pi_{\text{ph}}(s)}{s - q^2} + \text{subtraction constants}. \quad (4) \]

The quantity \( \text{Im} \Pi_{\text{ph}}(s) \) obtained by inserting a complete set of quantum states \( \Sigma |n\rangle \langle n| \) into Eq. (1), which reads:

\[ \text{Im} \Pi_{\text{ph}}(q^2) = m_S^2 f_S^2 \pi \delta(q^2 - m^2) + \left[ \frac{3}{8\pi^2} \pi \left( 1 + \frac{11}{3} \frac{\alpha_s}{\pi} \right) q^2 - \frac{3}{4\pi^2} m_1 m_2 \pi \right] \theta(q^2 - s_0), \quad (5) \]

where \( s_0 \) represents the onset of the QCD continuum. The decay constant in Eq. (5) is defined as:

\[ \langle S | \bar{q}_2 q_1 | 0 \rangle = m_S f_S. \]

By equating both the theoretical and phenomenological sides, we obtain the total dispersion integral:

\[ \Pi_{\text{OPE}}(q^2) = \frac{1}{\pi} \int_0^\infty ds \ \frac{\text{Im} \Pi_{\text{ph}}(s)}{s - q^2} + \text{subtraction constants}, \quad (6) \]

After Borel transform and subtracting the perturbative continuum contributions, we obtain the following sum rule:

\[ m_S^2 f_S^2 \exp\left[ -\frac{m_S^2}{M^2} \right] = \int_0^{s_0} ds \left[ \frac{3}{8\pi^2} \pi \left( 1 + \frac{11}{3} \frac{\alpha_s}{\pi} \right) s - \frac{3}{4\pi^2} m_1 m_2 \right] \exp\left[ -\frac{s}{M^2} \right] + \frac{1}{8\pi} \langle \alpha_s G^2 \rangle \]

\[ + \left( \frac{m_1}{2} + m_2 \right) \langle \bar{q}_1 q_1 \rangle + \left( m_1 + \frac{m_2}{2} \right) \langle \bar{q}_2 q_2 \rangle \]

\[ - \frac{1}{2M^2} m_2 \langle g_s \bar{q}_1 \sigma G q_1 \rangle - \frac{1}{2M^2} m_1 \langle g_s \bar{q}_2 \sigma G q_2 \rangle \]

\[ + \frac{16\pi}{27} \frac{\alpha_s}{M^2} \left[ \langle \bar{q}_1 q_1 \rangle^2 + \langle \bar{q}_2 q_2 \rangle^2 \right] - \frac{48\alpha_s}{9} \frac{1}{M^2} \langle \bar{q}_1 q_1 \rangle \langle \bar{q}_2 q_2 \rangle. \quad (7) \]
where scale dependence of decay constant $f_S$ is:

$$f_S(M) = f_S(\mu) \left( \frac{\alpha_s(\mu)}{\alpha_s(M)} \right)^{4/\beta},$$

The parameters in Eq. (7) are as follows [24, 25]:

$$\alpha_s = 0.517, \quad \langle \frac{g^2}{2} \rangle = 0.012 \pm 0.006 \text{GeV}^4,$$

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -(0.24 \pm 0.1)^3 \text{GeV}^3, \quad \langle \bar{s}s \rangle = (0.8 \pm 0.2) \langle \bar{u}u \rangle,$$

$$\frac{m_u + m_d}{2} = 5 \text{MeV}, \quad m_s = 120 \text{MeV},$$

$$\langle g_s \bar{u} \sigma G u \rangle = \langle g_s \bar{d} \sigma G d \rangle = 0.8 \text{GeV}^2 \langle \bar{u}u \rangle, \quad \langle g_s \bar{s} \sigma G s \rangle = 0.8 \langle g_s \bar{u} \sigma G u \rangle.$$
FIG. 1: Mass of $K^*_0$(solid line) and $a_0$ meson (dashed line) from sum rule Eq. (7) based on naive quark model as function of Borel parameter $M^2$ without instanton.

degeneracy broken by a tiny difference resulting from the SU(3) flavor symmetry breaking.\textsuperscript{5}

Even worse, the mass of $K^*_0$ with underlying structure $sd$ is always larger than the $a_0$ with underlying structure $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$. For definiteness, when we select $s_0 = 4.1\text{GeV}^2$ and $M^2$ within the range $[1.3, 1.6]\text{GeV}^2$, the calculated mass of $K^*_0$ and $a_0$ is shown in Figure 1. One can see the mass of the $K^*_0$ is above the $a_0$, which is inverted compared to the experimental results. All the results following from Eq. (7) are unsatisfactory.

To summarize Section II, we conclude that in the conventional QCD sum rule analysis based on the naive quark model, one cannot separate the this nonet with the same threshold and Borel window, the masses following from Eq. (7) are degenerate, and the results for the $K^*_0$ and $a_0$, are in contradiction with experiment. This suggests that important effects have been neglected in Eq. (7).

\textsuperscript{5} In addition, the degeneracy could also be broken when mass corrections proportional to $\alpha_s^2m_{\mu(s)}^2$ are taken into account. These corrections are negligible compared with instanton effects which we will consider in the next section.
III. SUM RULE WITH INCLUSION OF INSTANTON CONTRIBUTIONS

A. Basic formula

It has been known for a long time that the instanton plays an important role in nonperturbative QCD. The starting point on this subject is the solution of classical field equations in four dimension Euclidean gauge-field theories given by Belavin et al. Subsequently t’Hooft derived the instanton with topological quantum number \( n = 1 \) in Euclidean space:

\[
A^a_\mu(x) = \frac{2}{g} \eta_{a\mu\nu} \frac{(x - x_0)\nu}{(x - x_0)^2 + \rho^2},
\]

\[
G^a_{\mu\nu}(x) = -\frac{4}{g} \eta_{a\mu\nu} \frac{\rho^2}{[(x - x_0)^2 + \rho^2]^2},
\]

where \( \rho \) is instanton size, \( \eta_{a\mu\nu} \) is the t’Hooft \( \eta \) symbol, \( x_0 \) is any point in Euclidean space. The density \( n(\rho) \) of instanton with size \( \rho \) in the vacuum can be parameterized as:

\[
n(\rho) = n_c \delta(\rho - \rho_c),
\]

with two parameters \( n_c \) and \( \rho_c \), called the average instanton density and size:

\[
n_c = 8 \times 10^{-4} \text{GeV}^4, \quad \rho_c = \frac{1}{0.6} \text{GeV}^{-1}.
\]

When we include the instanton contribution in the correlator (1), there is a new term:

\[
\Pi_{\bar{q}q, \text{inst}}(q^2) = \left| \int d^4x e^{i q \cdot x} \bar{q}_{10}(x) q_{20}(x) \right|^2 \frac{n_c}{m_1^* m_2^*},
\]

where \( q_{10} \) and \( q_{20} \) is the t’Hooft quark zero mode, respectively, \( m_1^* \) and \( m_2^* \) is the effective mass correspondingly. Similarly applying dispersive relation to Eq. (11) we can rewrite Eq. (11) as:

\[
\Pi_{\bar{q}q, \text{inst}}(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im}\Pi_{\bar{q}q, \text{inst}}(s)}{s - q^2},
\]

After the Borel transformation we get the desired form of the instanton contributions of the current with isospin \( I \):

\[
\Pi_{\bar{q}q, \text{inst}}(M^2) = (-1)^I \frac{n_c \rho^4 M^6}{2 m_1^* m_2^*} \exp\left[ -\frac{M^2 \rho^2}{2} \right] \left[ K_0\left(\frac{M^2 \rho^2}{2}\right) + K_0\left(\frac{M^2 \rho^2}{2}\right) \right],
\]
and the instanton continuum contribution is:

$$\Pi^{\bar{q}q,\text{inst},\text{cont}}(s_0, M^2) = (-1)^I \frac{\pi n_c \rho^2}{m_1^* m_2^*} \int_{s_0}^{\infty} ds \, s \, J_1(\rho \sqrt{s}) Y_1(\rho \sqrt{s}) e^{-s/M^2},$$

(14)

where $K_0$, $K_0$ are the McDonald functions, and $J_1$, $Y_1$ are the Bessel functions.

When the smoke clears, we get the final result:

$$m_{Sf}^2 f_S^2 \exp[-m_{Sf}^2/M^2] = \Pi^{\text{OPE}}(M^2) - \Pi^{\text{OPE, cont}}(s_0, M^2)$$

$$+ \Pi^{\bar{q}q,\text{inst}}(M^2) - \Pi^{\bar{q}q,\text{inst, cont}}(s_0, M^2).$$

(15)

This is the sum rule we obtained including instanton effects in the correlation function. Similarly we can obtain the mass from Eq. (15) with the same manipulation as the previous section.

As an important parameter in sum rule Eq. (15) it is necessary to discuss the value of the effective mass $m^*$’s. In the mean-field approximation [16]:

$$m_u^* = m_d^* = \pi \rho \left(\frac{2}{3}\right)^{\frac{1}{2}} (N/V)^{\frac{1}{2}} = 170\text{MeV},$$

(16)
The value of $N/V$ is $N/V = 1\text{fm}^{-4}$ phenomenologically. And there is a relation between $\langle \bar{u}u \rangle$ and $\langle \bar{s}s \rangle$:

$$\langle \bar{s}s \rangle = (0.8 \pm 0.2) \langle \bar{u}u \rangle,$$

together with

$$\langle \bar{q}q \rangle = -\frac{N/V}{m_q^*},$$

we obtain the effective mass of the strange:

$$m_s^* = 215^{+68}_{-45}\text{MeV}.$$ (19)

In following we take the central value of $m_s^*$, i.e, $m_s^* = 220\text{MeV}.$

**B. Mass, decay constant of $K^*_0$ and $a_0$ meson**

All the parameters needed in numerical calculation have now been fixed. Firstly we take $j = \bar{d}s$ which corresponds to the $K^*_0$ meson as our “sample”, since in this case there is no mixing with glueball, allowing us to examine the effect of instants without the complications presented by mixing.

Using standard QCD sum-rule methodologies, we obtain the threshold $s_0 = 3.5\text{GeV}^2$, which is just below the next excited state $K^*(1950)$, and the Borel window is within the range $[2.0, 2.3]\text{GeV}^2$. The calculated mass of $K^*_0$ is $m_{K^*_0} = 1436 \sim 1462\text{MeV}$. For the $a_0$, assigning $j = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$ with the same threshold and Borel window, the mass of the $a_0$ is $m_{a_0} = 1432 \sim 1472\text{MeV}$. The results for these two states are shown together in Figure 2.

We can see from Figure 2 that there is a crossover at the point $M^2 = 2.16\text{GeV}^2$ corresponding to the mass $m = 1442\text{MeV}$, which is very close to the experimental value of $m_{K^*_0}$ and $a_0$. We refer to this value of $M^2$ as the “key point” representing the Borel scale where the mass hierarchy between the $a_0$ and $K^*_0$ reverses. The more important aspect observed from figure 2 is that the calculated mass for $K^*_0$ and $a_0$ present the right picture: there is a range where the mass of the $a_0$ is larger than the $K^*_0$, a result which cannot be obtained in the sum rule without instanton contributions shown in figure 1. In other words, the sum rule including instanton contributions can reproduce realistic results which in agreement with the light meson spectrum.

Having determined the mass, it is straightforward to obtain the decay constant from Eq. (15). The decay constants of the $K^*_0$ and $a_0$ are shown in figure 3 and figure 4 respectively. It is obvious
FIG. 3: Decay constants of $K^*_0$ with quark structure $s\bar{d}$ as function of Borel parameter $M^2$ includes instanton.

that the results are very stable within the Borel window, and from the Figures we find the decay constants of $K^*_0$ and $a_0$:

$$f_{K^*_0(1430)}(1\text{GeV}) = 510\text{MeV}, \quad f_{a_0(1450)}(1\text{GeV}) = 514\text{MeV}$$

C. Mass, decay constant of $f_0$ meson with underlying structure $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

Similarly if we set

$$j = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}),$$

with isospin $I = 0$, we immediately get the mass of $f_0$:

$$m_{f_0} = 1314 \sim 1391\text{MeV},$$

At the “key point” i.e, $M^2 = 2.16\text{GeV}^2$, the mass is $m_{f_0} = 1376\text{MeV}$ which is close to the experimental value of $f_0(1370)$, while the more important result is that the mass of this state is no longer degenerate with other states. The decay constant following this mass is:

$$f_{f_0(1370)}(1\text{GeV}) = 520\text{MeV}.$$
FIG. 4: Decay constants of $a_0$ with quark structure $(u\bar{u} - d\bar{d})/\sqrt{2}$ as function of Borel parameter $M^2$ includes instanton.

The mass and decay constant of $f_0$ are shown in figure 5 and figure 6, respectively.

Because there are more controversies for the $f_0$ meson than the two members discussed above, it is useful to mention the model beyond pure quark viewpoint. We also notice the glueball can mix with scalar mesons nearby, so it is possible that there is mixing between $f_0(1370)$ glueball, or in other words, $f_0(1370)$ is not in a pure quark state, and could have some glue content; an idea that was first introduced in Ref. [28] and then generalised in Ref. [29]. The work in Ref. [29] suggested the $f_0(1710)$ is dominated by $s\bar{s}$ content, while $f_0(1500)$ and $f_0(1370)$ share roughly equal amounts glueball ($\simeq 40\%$) ($f_0(1500)$ and $f_0(1710)$ will be detailed in the coming subsection). Here we see when including the instanton effects, $f_0(1370)$ can be accommodated naturally in pure quark model. So we conclude that $f_0(1370)$ may be a pure quark state based when instanton effects are considered.
FIG. 5: Mass of $f_0$ with quark structure $(u\bar{u} + d\bar{d})/\sqrt{2}$ as function of Borel parameter $M^2$ includes instanton.

FIG. 6: Decay constants of $f_0$ with quark structure $(u\bar{u} + d\bar{d})/\sqrt{2}$ as function of Borel parameter $M^2$ includes instanton.
D. Mass, decay constant of $f_0$ meson with underlying structure $s\bar{s}$

The $f_0(1500)$ may be the most controversial object in this nonet. As we have seen the important role of the instanton in giving the mass of the three members in previous subsections. Firstly we do some tentative calculations based on pure quark model in hope that these calculations shed some light on its structure. In order to present a thorough investigation on this object, it is reasonable to write the current with isospin $I = 0$ in a general form:

$$j = c_1(\bar{u}u + \bar{d}d) + c_2\bar{s}s,$$

(20)

which includes two adjustable parameters $c_1$ and $c_2$. In case of $c_1 = 0$, Eq. (20) reduces to pure $s\bar{s}$ state which is free of instanton\[14\] so we can deal it with the conventional QCD sum rule as in section II. The calculated mass is shown in figure 7 from which we can read the mass with pure $s\bar{s}$ structure:

$$m_{f_0} = 1413 \sim 1430\text{MeV},$$

We can also write the current in a more complex form as:

$$j = \frac{1}{\sqrt{6}}[(\bar{u}u + \bar{d}d) - 2\bar{s}s],$$

FIG. 7: Mass of $f_0$ with quark structure $s\bar{s}$ as function of Borel parameter $M^2$ without instanton.
such that there will be instanton effects in the corresponding sum rule. The calculated mass with this quark content is shown in figure 8 from which the following mass is read:

\[ m_{f_0} = 1433 \sim 1457 \text{MeV}, \]

In fact, based on the current given in Eq. (20), we can derive a sum rule involving complete instanton contributions induced by the quarks which depends on the two adjustable parameters \( c_1 \) and \( c_2 \). Unfortunately the results indicate the sum rule is not able to produce reasonable masses for the \( f_0(1500) \) and \( f_0(1710) \) by adjusting these two parameters—the masses are always much lower than the experimental ones.

These simple calculations signal that from the viewpoint of pure quark content, one can not produce the \( f_0(1500) \) in QCD sum rules even including complete instanton contributions, so we must look for other solution to this problem. As mentioned in Ref. [3], if we assume a \( q\bar{q} \) structure, one concludes that \( f_0(1500) \) is dominantly \( s\bar{s} \), while this assignment can not produce reasonable mass theoretically as we can see from previous paragraphs, but also leads to contradictions experimentally [3]. There are some works [18] on this subject that take another
extreme: they try to produce $f_0(1500)$ under the assumption of a pure glueball content. But what is the realistic structure of $f_0(1500)$ is still unknown.

There is another viewpoint that the light nonet above 1GeV can be identified as conventional $\bar{q}q$ states with some possible gluonic content, that is, there is mixing of the pure glueball with the nearby two $N = n\bar{n}$ and $S = s\bar{s}$ scalar mesons as first introduced in Ref. [28], where $n\bar{n} = \frac{1}{2}(u\bar{u} + d\bar{d})$. Based on this model, Ref. [29] obtained the results that $f_0(1710)$ is dominated by $s\bar{s}$ content while there is roughly equal amounts of glue content in $f_0(1500)$. We have seen the key role of instanton in solving the puzzle on $K^*_0$ and $a_0$, $f_0(1370)$, and explore this possibility in the assumed mixing of scalar meson and pure glueball in $f_0(1500)$. With this motivation, we modified the current of $f_0(1500)$ as mixing of quark and gluonic current: $^6$

$$j_{mix} = A\bar{s}s + B\alpha_s G^a_{\mu\nu} G^{a\mu\nu}, \quad (21)$$

and in this case the decay constant is defined as:

$$\langle S | j_{mix} | 0 \rangle = m_S^2 f_S.$$

where $A$, $B$ are both real, and one should notice that the parameter $A$ has dimension one of mass which insures the right dimension in the current. The parameters $A$ and $B$ accompany the Wilson coefficients of operators $\bar{s}s$ and $\alpha_s G^a_{\mu\nu} G^{a\mu\nu}$ respectively, and are therefore renormalization scale dependent. Here we fix the renormalization scale of $A$ and $B$ so that they just are numbers in the following consideration. After this modification, there will be new contributions stemming from the glueball $\alpha_s G^a_{\mu\nu} G^{a\mu\nu}$ OPE, the glueball instanton and the mixing instanton contribution which will be presented below.

With the perturbative corrections and including nonperturbative terms up to dimension

---

6 The renormalization-group invariant gluonic current has been used because the subleading perturbative effects will be included in the correlation function.
eight, the OPE of gluonic current is \([22, 31]\):

\[
\Pi_{GB, \text{OPE}}(q^2) = q^4 \ln \frac{-q^2}{\mu^2} \left\{ -2 \left( \frac{\alpha_s}{\pi} \right)^2 \left[ 1 + \frac{659 \alpha_s}{36 \pi} + 247.48 \left( \frac{\alpha_s}{\pi} \right)^2 \right] + 2 \left( \frac{\alpha_s}{\pi} \right)^3 \left( \frac{9}{4} + 65.781 \left( \frac{\alpha_s}{\pi} \right) \right) \ln \frac{-q^2}{\mu^2} - 10.125 \left( \frac{\alpha_s}{\pi} \right)^4 \ln^2 \frac{-q^2}{\mu^2} \right\} \\
+ \left[ 4 \pi \alpha_s \left( 1 + \frac{175 \alpha_s}{36 \pi} \right) - 9 \pi \left( \frac{\alpha_s}{\pi} \right)^2 \ln \frac{-q^2}{\mu^2} \right] \langle \alpha_s G^2 \rangle \\
- 8 \pi^2 \left( \frac{\alpha_s}{\pi} \right)^2 \frac{1}{q^2} \langle \mathcal{O}_6 \rangle + 8 \pi^2 \frac{\alpha_s}{\pi} \frac{1}{q^2} \langle \mathcal{O}_8 \rangle, \tag{22}
\]

where

\[
\langle \mathcal{O}_6 \rangle = \langle g_s f_{abc} G^a_{\mu \nu} G^b_{\nu \rho} G^c_{\rho \mu} \rangle = (0.27 \text{GeV}^2) \langle \alpha_s G^2 \rangle,
\]

and

\[
\langle \mathcal{O}_8 \rangle = 14 \langle (\alpha_s f_{abc} G^a_{\mu \nu} G^b_{\nu \rho})^2 \rangle - \langle (\alpha_s f_{abc} G^a_{\mu \nu} G^b_{\rho \mu})^2 \rangle = \frac{9}{16} \langle \alpha_s G^2 \rangle^2.
\]

are the dimension-6 and dimension-8 gluonic condensates, respectively. Because there is both quark and gluon current, we have to use the unsubtracted dispersive relation for the gluonic correlation function in order to be consistent with of the whole correlation function. Applying the dispersion relation, and after subtracting the continuum contribution and taking the Borel transform, the glueball contribution is obtained \([31, 32]\):

\[
\Pi_{GB, \text{OPE}}(s_0, M^2) = \int_0^{s_0} ds \ s^2 e^{-s M^2} \left\{ 2 \left( \frac{\alpha_s}{\pi} \right)^2 \left[ 1 + \frac{659 \alpha_s}{36 \pi} + 247.48 \left( \frac{\alpha_s}{\pi} \right)^2 \right] \\
- 4 \left( \frac{\alpha_s}{\pi} \right)^3 \left( \frac{9}{4} + 65.781 \left( \frac{\alpha_s}{\pi} \right) \right) \ln \frac{s}{\mu^2} \\
- 10.125 \left( \frac{\alpha_s}{\pi} \right)^4 \left( \pi^2 - 3 \ln^2 \frac{s}{\mu^2} \right) \right\} \\
+ 9 \pi \left( \frac{\alpha_s}{\pi} \right)^2 \langle \alpha_s G^2 \rangle \int_0^{s_0} ds \ e^{-s M^2} \\
+ 8 \pi^2 \left( \frac{\alpha_s}{\pi} \right)^2 \langle \mathcal{O}_6 \rangle - 8 \pi^2 \frac{\alpha_s}{\pi} \frac{1}{M^2} \langle \mathcal{O}_8 \rangle, \tag{23}
\]

The contribution of glueball instanton after subtracting continuum is given by \([30, 31]\):

\[
\Pi_{GB, \text{inst}}(s_0, M^2) = -2 \pi^2 n \rho_c \int_0^{s_0} ds \ e^{-s M^2} s^2 J_2(\rho \sqrt{s}) Y_2(\rho \sqrt{s}), \tag{24}
\]
FIG. 9: Mass of $f_0$ with structure $0.9\text{GeV} \bar{s}s + \alpha_s G_{\mu
u}^a G^{a\mu\nu}$ as function of Borel parameter $M^2$ include glueball instanton and mixing instanton contributions.

where $J_2$ and $Y_2$ are Bessel and Neumann functions, respectively.

We have independently verified the following instanton contribution to the mixed correlator $\bar{s}s\alpha_s G_{\mu
u}^a G^{a\mu\nu}$:

$$\Pi^{\text{mix, inst}}(s_0, M^2) = \frac{2\pi^2 n\rho^3}{m_s^4} \int_0^{s_0} ds \ e^{-\frac{s}{m_s^2}} s^{\frac{3}{2}} \left[ J_1(\rho \sqrt{s}) Y_2(\rho \sqrt{s}) + Y_1(\rho \sqrt{s}) J_2(\rho \sqrt{s}) \right].$$  \hspace{1cm} (25)

Now we have determined all the terms induced by the current given by Eq. (28). It is convenient to write the whole results in a compact form as follows:

$$m_s^4 f_0^2 \exp\left(-\frac{m_s^2}{M^2}\right) = \sum_X \Pi^X(s_0, M^2),$$  \hspace{1cm} (26)

where $X$ denotes

$$X = \left\{ \{\bar{s}s, \text{OPE}\}, \{\text{GB, OPE}\}, \{\text{GB, inst}\}, \{\text{mix, inst}\} \right\}.$$

and we have absorbed the two parameters $A$ and $B$ in the $\Pi^X$'s for convenience. Taking the same algorithm as the previous section one can obtain immediately the mass corresponding to the current given in Eq. (21). Assigning $A = 0.9\text{GeV}$ and $B = 1$ in Eq. (21), corresponding
FIG. 10: Decay constant of $f_0$ with structure $0.9\text{GeV}\bar{s}s + \alpha_s G_{\mu\nu}^a G^{a\mu\nu}$ as function of Borel parameter $M^2$ include glueball instanton and mixing instanton contributions.

to a large glueball content (since the energy scale is $\sim 1\text{GeV}$), the calculated mass with this underlying structure is $m_{f_0} = 1492 \sim 1504\text{MeV}$ which is very close to the experimental result $f_0(1500)$.

After obtaining the mass, we can deduce the decay constant from Eq. (26)

$$f_{f_0(1500)}(1\text{GeV}) = 1.69\text{GeV}.$$  

There is a strong enhancement of the decay constant after involving glueball and related instanton contributions compared with other multiplets. A similarly enhancement of $f_G$ in pure glueball state including instanton effects was found in instanton vacuum model calculation [33]. This strong enhancement also observed in the work of H. Forkel [30], which gave a value $f_G = 1.14\text{GeV}$ when the glueball instanton contribution was included in the pure glueball correlation function using a unsubtracted dispersive relation. The results for our analysis of the mass and decay constant of this mixed state are shown in figure 9 and figure 10.

Finally we turn to the last state $f_0(1710)$. Generally it is assumed this state dominated by
the $s \bar{s}$ content, so we can write the current as:

$$j = A' \bar{s}s + B' \alpha_s G_{\mu\nu}^{a} G^{a\mu\nu},$$  \hspace{1cm} (27)

subjected the following orthogonality condition:

$$\langle 0 | j | f_0(1500) \rangle = 0.$$  

This orthogonal condition is insignificant here since we are not able to get a value agreeable with $f_0(1710)$ whatever the values of $A'$ and $B'$ are chosen. This can be understood intuitively that the threshold $s_0 = (1.9\text{GeV})^2$, which is adopted here only for the states with the masses around 1450GeV, is too low to reproduce such a large mass.

IV. CONCLUSIONS

In this work we have studied the mass and decay constant of the light nonet $a_0$, $K_0^*$, and $f_0$ within the framework of QCD sum rule with and without instanton contributions. Our main results are as follows:

1. In the conventional QCD sum rule, the masses of this nonet are degenerate, the calculated mass of $K_0^*$ is larger than the $a_0$ for the same threshold and same Borel window.

2. When we include instanton contributions in the sum rule, the masses of the nonet can be well separated, and the mass of $K_0^*$ and $a_0$ agrees well with the observed results. The results suggest the underlying structure: $K_0^*(1430)$ is $s \bar{d}$, $a_0(1450)$ is $\frac{1}{\sqrt{2}}(u \bar{u} - d \bar{d})$, and $f_0(1370)$ is $\frac{1}{\sqrt{2}}(u \bar{u} + d \bar{d})$. For the $f_0(1500)$, our results suggest there is considerable glueball content in its underlying structure. The decay constant of $f_0(1500)$ enhanced considerably after this gluonic-content improvement compared with other multiplets.

3. With a mixing current and the threshold and Borel window common to the multiplet, we cannot obtain the mass of $f_0(1710)$. One reason might be that the threshold suitable for $K^*(1430)$ is too low for $f_0(1710)$.

V. ACKNOWLEDGEMENTS

This work is supported by NNSFC under Projects No. 10775117, 10847148. The author H.Y. Jin also thanks KITPC for its hospitality. TGS is grateful for research support from the
[1] M. Gell-Mann, Phys. Lett. 8, 214(1964).
[2] G. Zweig, CERN preprint 8419/Th412, 8182/Th401 (unpublished).
[3] C. Amsler and N. Törnqvist, Phys. Rep. 389, 61(2004).
[4] M. Gell-Mann, Acta Phys. Austriaca, Suppl. 9, 733 (1972); H. Fritzsch and M. Gell-Mann, 16th International Conference on High Energy Physics, Chicago, 1972, Vol. 2, p. 135.
[5] R. L. Jaffe, Phys. Rev. D 15, 267(1977); 15, 281(1977).
   R. L. Jaffe, F. E. Low, Phys. Rev. D 19, 2105 (1979).
[6] T. Barnes, F. E. Close and F. de Viron, Nucl. Phys. B224, 241 (1983).
   H. Y. Jin, J. G. Körner and T. G. Steele, Phys. Rev. D 67, 014025 (2003).
[7] F. E. Close and N. A. Törnqvist, J. Phys. G 28, R249(2002).
[8] S. Spanier and N. A. Törnqvist, (Particle Data Group) “Note on scalar mesons”; W.-M. Yao, J Phys. G. 33, (2006): p. 546.
[9] S. Godfrey and J. Napolitano, Rev. Mod. Phys. 71, 1411(1999).
[10] D. Delepine, J. L. Lucio M., and C. A. Ramírez, [hep-ph/0501022]
[11] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147, 385, 448(1979).
[12] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rep, 127, 1(1985).
[13] V. A. Novikov, M. A. Shifman A. I. Vainshtein et al, Fortschr. Phys. 32, 585(1984).
[14] E. V. Shuryak, Nucl Phys, B214, 237(1983); Nucl Phys, B203, 93, 116, 140(1982).
[15] A. A. Belavin, A. M. Polyakov A. S. Schwartz et al, Phys. Lett. B598, 85(1975).
[16] T. Schäfer and E. V. Shuryak, Rev. Mod. Phys. 70, 323(1998).
[17] H. Forkel and M. Nielsen, Phys. Lett. B345, 55(1995).
[18] L. Kinsslinger and M. B. Johnson, Phys. Lett. B523, 127(2001).
[19] Fang Shi, T.G. Steele, V.Elias, K.B. Sprague, Ying Xue, A.H. Fariborz, Nucl. Phys. A671 416(2000);
   G. Orlandini, T.G. Steele, D. Harnett, Nucl. Phys. A686 261(2001).
[20] C.J. Morningstar, M. Peardon, Phys. Rev. D60 034509(1999);
A. Vaccarino, D. Weingarten, Phys. Rev. D60 114501(1999);
Y. Chen et al, Phys. Rev. D73 014516(2006).
[21] M. Jamin, M. Munz Z. Phys. C60, 569 (1993).
[22] E. Bagan, T. G. Steele, Phys. Lett. B243, 413(1990).
[23] P. Ball and R. Zwicky, Phys. Rev. D 71, 014029(2005).
[24] A. Khodjamirian T. Mannel and M. Melcher, Phys. Rev. D 70, 094002(2004).
[25] H-Y Cheng, C-K Chua and K-C Yang, Phys. Rev. D 73, 014017(2006).
[26] G. t’Hooft, Phys. Rev. Lett. 37, 8(1976), Phys. Rev. D 14, 3432(1976).
[27] D. I. Diakonov and V. Yu. Petrov, Nucl. Phys. B245, 457(1986).
[28] C. Amsler, F. E. Close, Phys. Lett. B 353, 385 (1995);
C. Amsler F. E. Close, Phys. Rev. D 53, 295 (1996).
[29] F. E. Close, A. Kirk, Eur. Phys. J. C 21, 531 (2001).
[30] H. Forkel, Phys. Rev. D 64, 034015(2001).
[31] D. Harnett, T. G. Steele and V. Elias, Nucl. Phys. A686, 393 (2001).
[32] D. Harnett, K. Moats and T. G. Steele, hep-ph/08042195.
[33] T. Schäfer, E. V. Shuryak, Phys. Rev. Lett. 75, 1707 (1995).