THE EFFECT OF BARS AND TRANSIENT SPIRALS ON THE VERTICAL HEATING IN DISK GALAXIES

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ABSTRACT

The nature of vertical heating of disk stars in the inner as well as the outer region of disk galaxies is studied. The galactic bar (which is the strongest non-axisymmetric pattern in the disk) is shown to be a potential source of vertical heating of the disk stars in the inner region. Using a nearly self-consistent high-resolution $N$-body simulation of disk galaxies, the growth rate of the bar potential is found to be positively correlated with the vertical heating exponent in the inner region of galaxies. We also characterize the vertical heating in the outer region where the disk dynamics is often dominated by the presence of transient spiral waves and mild bending waves. Our simulation results suggest that the non-axisymmetric structures are capable of producing the anisotropic heating of the disk stars.

Key words: galaxies: evolution – galaxies: halos – galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure

Online-only material: color figures

1. INTRODUCTION

The random velocities of disk stars in the solar neighborhood are known to increase with time since the 1940s and this fact has been verified by recent Hipparcos observations. The analysis of the Hipparcos data (Binney et al. 2000) reveals that the stellar velocity dispersions increase continuously (rather than episodically as claimed by Edvardsson et al. 1993) with time which is the well-known disk heating problem in our Galaxy. The phenomenon of disk heating is not specific for our Galaxy since the heating of stellar disks is also known to persist in external galaxies such as in NGC 2985, NGC 2460, and NGC 2775 (Gerssen et al. 2000; Shapiro et al. 2003). Although mechanisms have been identified for the disk heating, a full understanding is yet to be achieved. It is known that disk heating is anisotropic; that is, disk stars are preferentially heated along the plane in comparison to its perpendicular direction. Thus, the main problem in disk heating is two-fold. The first is the requirement to explain the continual heating of disk stars especially in the case of isolated galaxies for which tidal heating or external perturbations can be ruled out. Second, such a mechanism must be anisotropic in nature. Taken together, disk heating remains a fundamental problem in galactic dynamics.

Historically, Spitzer & Schwarschild (1951, 1953) first showed that the secular increase in the stellar velocity dispersions could arise as a result of the scattering of disk stars with giant molecular clouds (GMCs) based on two-dimensional calculations. Subsequently, there has been considerable development and progress in understanding the disk heating problem in galaxies. For example, Lacey (1984) extended the previous theoretical work to a three-dimensional disk and found that the velocity dispersions increase with time, $t$, as $t^\gamma$, where $\gamma = 0.25$. In addition, it was demonstrated that transient stochastic spiral arms (Barbanis & Woltjer 1967; Carlberg & Sellwood 1985) could also produce secular heating of the disk stars, however, such effects were only effective in the plane of the galaxy. The effect of giant molecular clouds in combination with the effect of stochastic spirals could scatter the stars off the plane (Jenkins & Binney 1990) to account for both the radial and vertical heating. Alternatively, using an extensive orbit analysis of the stars subject to a three-dimensional galactic bar potential, Pfenniger (1984, 1985) has shown that a large fraction of the phase space is chaotic and suggested that all the disk stars trapped in the chaotic phase space would be heated up in all directions. Other possible candidate heating mechanisms include massive dark halo objects, e.g., black holes or dark clusters of mass $\sim 10^6 M_\odot$ (Lacey & Ostriker 1985; Carr & Lacey 1987) or a combination of GMCs and halo black holes (Hanninen & Flynn 2002, 2004). Within the framework of standard cosmological models, a spectrum of subhalos and their substructures could heat up the disk (Sanchez-Salcedo 1999; Ardi et al. 2003). However, Font et al. (2001) found that the subhalos are not efficient perturbers for disk heating because the orbits of the subhalos rarely take them near the disk. Finally, satellite infall onto the galactic disk could produce an abrupt heating of the disk (Quinn et al. 1993) or moderate heating could arise due to sinking satellites on prograde orbits (Velazquez & White 1999). The many possible heating mechanisms of the disk stars has been reviewed by Pfenniger (1993). Note that while most mechanisms produce radial heating of the disk stars they have not been shown to provide a satisfactory explanation of the detailed nature of vertical heating. In fact, studies focusing on the vertical heating are comparatively lacking. To be more precise, a generic and robust internal source remains to be identified which could be responsible for the continual vertical heating of the disk.

Here, we primarily concentrate on the issue of vertical disk heating. Galactic disks often consist of various non-axisymmetric patterns such as bars, spirals, warps, corrugations, and rings. These patterns may be growing or transient and, in general, may be time dependent. It is natural to ask whether such patterns contribute to the disk vertical heating. That is, what is the nature of vertical heating due, for example, to a growing bar? How does a growing bar heat the disk stars in the vertical direction? Heating due to non-axisymmetric patterns is important for isolated galaxies as even for the Milky Way, there are prominent and characteristic signatures of non-axisymmetric patterns (Dehnen 1998) in the phase space in the solar neighborhood.
In this paper, we explore possible mechanisms giving rise to continuous heating of the disk stars in the vertical direction. In particular, we show that stellar bars that are formed nearly spontaneously in disk galaxies are capable of efficiently scattering stars in the vertical direction. Since transient spiral arms are often associated with bars, they also contribute to the disk heating. In order to understand the nature of vertical heating of the disk stars, we have performed a large number (statistically meaningful) of N-body simulations of model disk galaxies constructed over a wide range of parameter space.

The paper is organized as follows. Section 2 describes the galaxy models used in the simulation. A description of the N-body simulation and the physical basis for the choice of our parameter space is presented in Section 3. The heating model used to interpret the numerical simulations is described in Section 4. Section 5 describes the heating due to non-axisymmetric structures. The results on the correlation studies are presented in Section 6. The discussion and conclusions are presented in Sections 7 and 8, respectively.

2. GALAXY MODELS

The construction of a very thin equilibrium disk model involves a non-trivial procedure in N-body simulation. However, using the self-consistent bulge–disk–halo model of Kuijken & Dubinski (1995, hereafter KD95), we are able to construct extremely thin equilibrium model disks (with the ratio of scale height to scale length \( \sim 0.01 \)). Their prescription provides nearly exact solution of the collisionless Boltzmann and Poisson equations which are suitable for studying disk stability related problems, allowing one to construct a wide range of initial models from a large parameter space. All the components in our models are active (i.e., the gravitational potential of each component can respond to an external or internal perturbation) and, hence, provide a realistic representation for the evolution and structure of the galaxies. Below we briefly describe each component of the model separately for the sake of completeness. For more details, the reader is referred to KD95.

A spherical live bulge is constructed from the King model (Binney & Tremaine 1987) and the corresponding distribution function (DF) is given by

\[
q_r = \begin{cases} 
\frac{\rho_0 (2\pi \sigma_0^2)^{3/2} e^{(\Psi_0 - \Psi_c)/\sigma_0^2}}{\Psi_c} & \text{if } E < \Psi_c, \\
0 & \text{otherwise.}
\end{cases}
\]

Here, the bulge is simulated using three parameters, namely the cutoff potential (\( \Psi_c \)), central bulge density (\( \rho_0 \)), and \( \sigma_0 \) governing the central bulge velocity dispersion. The depth of the potential well is measured by \( \Psi_0 \).

An axisymmetric live dark matter halo is simulated using the DF of a lowered Evans model and is given as

\[
q_{dm}(E, L_z^2) = \begin{cases} 
[(A L_z^2 + B) e^{-(E/\sigma_v^2) + C}] \\
\times (e^{-E/\sigma_v^2} - 1) & \text{if } E < 0, \\
0 & \text{otherwise.}
\end{cases}
\]

The velocity and density scales are given by \( \sigma_h \) and \( \rho_1 \), respectively. The halo core radius \( R_c \) and the flattening parameter \( q \) are contained in the parameters \( A, B, \) and \( C \). All the simulated halos are oblate in shape and kept at a constant value \( q = 0.8 \) for simplicity.

The disk DF is constructed using the approximate third integral given by \( E_z = \frac{1}{2} v_z^2 + \Psi(R, z) - \Psi(R, 0) \), the energy of the vertical oscillations. This third integral is approximately conserved for orbits near the disk mid-plane. The radial density of the disk is approximately exponential with a truncation and the vertical density is chosen to depend exponentially on the vertical potential \( \Psi_z(R, z) = \Psi(R, z) - \Psi(R, 0) \). The volume density of the axisymmetric disk is given by

\[
\rho_d(R, z) = \frac{M_d}{8\pi h_z R_d^2} e^{-R/R_d} \text{erfc} \left( \frac{R - R_{out}}{\sqrt{2}(R_{out} - R_{run})} \right)
\]

where \( f_d(z) = \exp(-0.8676\Psi_z(R, z)/\Psi_z(R, h_z)) \) governs the vertical structure of the disk, and erfc is the complementary error function. In the above equation, \( M_d \) is the disk mass, \( R_d \) is the scale length, and \( h_z \) is the scale height.

There are 13 parameters required to construct a particular galaxy model. The galactic disk produced in this manner remains in equilibrium as long as the simulation runs. The dark matter halo generated using the lowered Evans model (Evans 1993) has a constant density core which is probably appropriate for the low surface brightness (LSB) galaxies which we simulate. Each model galaxy is constructed such that an initially almost flat rotation curve is produced. The disk outer radius (\( R_{out} \)) is fixed at about 6.5 \( R_d \) and a truncation width \( \sim 0.3 \) \( R_d \) is adopted within which the disk density smoothly decreases to zero at the outer radius.

3. N-BODY SIMULATION

The primary aim of the present study is to achieve an understanding of the nature and origin of vertical heating of disk stars. In order to gain the necessary insight, we perform a large number of simulations of isolated galaxies. The models are evolved for a sufficiently long period of time so as to examine the nature of vertical heating in different spatial regions of the disk.

Using the KD95 method, we build the initial conditions for all model galaxies. The method is not fully self-consistent because the disk DF is not known exactly. Each model galaxy consists of a bulge, disk, and dark matter halo as described above. Since the initial conditions are constructed based on DFs, they are suitable for studying the long-term evolution of the non-axisymmetric patterns in the disk. Building initial galaxy models with prescribed galaxy properties such as the Toomre \( Q \) parameter is \( \sigma(v) \) or \( Q = 3.36G\Sigma(r) \), ratio of dark-to-disk mass (\( M_h/M_d \)), or ratio of velocity dispersions (\( \sigma_z/\sigma_r \)) is not straightforward because the bulge and the halo models are not derived using their mass profiles but from their DFs. Hence, the relation between the Toomre \( Q \) parameter and \( M_h/M_d \) or between \( \sigma_z/\sigma_r \) and \( M_h/M_d \) is not known from the outset. Here, \( \sigma_z, \sigma_r, \), and \( \Sigma \) denote the velocity dispersion in the radial and vertical direction, epicyclic frequency, and surface density of stars at a given radius, respectively.

The disk, bulge, and halo parameters are chosen such that an almost flat rotation curve is produced in the outer region. We assume a scale length for the disk to be \( R_d = 4.0 \) kpc. The radial and vertical forces are normalized such that it produces the circular velocity \( V_c = 200 \) km s\(^{-1}\) at about 2 disk scale lengths. The unit of time is \( 2.1 \times 10^7 \) yr. The initial disk scale height (\( h_z \)) varies from 40 pc to 400 pc in our sample of model galaxies. The softening length for the disk particles is 10 pc, for the bulge particles 40 pc, and for the halo particles the softening length is 36 pc. A total of \( 2.2 \times 10^5 \) to \( 1.0 \times 10^7 \) particles has been used to simulate the model galaxies. The dark matter halo mass varies from 2 to 40 \( M_d \) which gives, on average, a mass resolution of 40 pc.
The orbital timescale at the disk half mass radius is \( \sim \) 2000. We explore a wide region of the parameter space spanned by \([Q, \sigma_z/\sigma_r, M_h/M_d]\) is considered. The exact relationship between these parameters is dependent on how an active dark matter halo interacts with the other active disk and bulge components. However, we notice that if the central radial velocity dispersion is held constant, there appears to be a positive correlation between the Toomre \( Q \) parameter and \( \sigma_z/\sigma_r \) at the disk half mass radii (see Figure 1) and provide here the parameter space for the simulations below.

3.1. Selection of the Parameter Space

A wide range of parameters in the three-dimensional space spanned by \([Q, \sigma_z/\sigma_r, M_h/M_d]\) is considered. The exact relationship between these parameters is dependent on how an active dark matter halo interacts with the other active disk and bulge components. However, we notice that if the central radial velocity dispersion is held constant, there appears to be a positive correlation between the Toomre \( Q \) parameter and \( \sigma_z/\sigma_r \) at the disk half mass radii (see Figure 1) and provide here the parameter space for the simulations below.

![Parameter space for all the models. Most of the galaxies in the sample are radially hot.](image1)

\(~10^3 M_\odot\). We have observed that with such a mass resolution the shot noise in the simulation reduces considerably and bending instabilities do not grow sufficiently to induce an outer disk warp during the evolution. Each model galaxy in our simulation has been evolved with the Gadget code (Springel et al. 2001) which uses a variant of the leapfrog method for the time integration. The forces between the particles are calculated basically using the BH tree (with some modification) algorithm with a tolerance \( \theta_{tol} = 0.7 \). The integration time step is \(~0.4\) Myr for each model galaxy is evolved for more than 5 Gyr in our simulation. The orbital timescale at the disk half mass radius is \(~0.5\) Myr for model mk97 (see Table 1). For convenience, we describe the parameter space for the simulations below.

### Table 1

| Galaxy | \( Q \) | \( \sigma_z/\sigma_r \) | \( M_h/M_d \) | \( \alpha_{in} \) | \( \alpha_{out} \) |
|--------|--------|-----------------|----------------|--------------|--------------|
| mk51A  | 1.21   | 0.611           | 6.52           | 1.01         | 0.326        |
| mk56A  | 1.45   | 0.594           | 10.15          | 2.59         | 0.557        |
| mk107  | 1.66   | 0.666           | 9.83           | 2.29         | 2.00         |
| mk104  | 1.77   | 1.480           | 15.80          | 1.67         | 0.67         |
| mk97   | 1.84   | 0.447           | 7.88           | 2.31         | 0.420        |
| mk15   | 2.06   | 0.237           | 5.34           | 1.13         | 0.809        |
| mk53   | 2.31   | 0.294           | 11.97          | 0.828        | 0.289        |
| mk52   | 2.33   | 0.618           | 14.47          | 1.81         | 0.707        |
| mk17   | 2.41   | 0.151           | 5.10           | 0.724        | 0.236        |
| dmA    | 2.37   | 0.433           | 12.97          | 0.776        | 0.588        |
| mk112  | 2.47   | 0.344           | 9.44           | 2.105        | 0.215        |
| mk5    | 2.50   | 0.149           | 9.85           | 0.406        | 0.448        |
| mk25   | 2.53   | 0.181           | 7.07           | 1.02         | 0.459        |
| mk63   | 2.64   | 1.160           | 14.12          | 0.865        | 0.902        |
| mk27   | 2.71   | 0.187           | 7.16           | 1.13         | 0.30         |
| mk64   | 3.00   | 1.110           | 16.50          | 0.723        | 1.39         |
| mk33   | 3.01   | 0.260           | 8.75           | 1.82         | 0.41         |
| mk43   | 3.05   | 0.606           | 11.52          | 1.30         | 1.63         |
| mk34   | 3.15   | 0.238           | 6.50           | 1.20         | 0.597        |
| mk58   | 3.16   | 0.850           | 16.75          | 1.27         | 1.02         |
| mk48   | 3.33   | 0.143           | 10.47          | 0.987        | 0.27         |
| mk55   | 3.73   | 0.540           | 27.60          | 1.16         | 1.10         |

Figure 1. Parameter space for all the models. Most of the galaxies in the sample are radially hot.

Figure 2. Rotation curve for model galaxy mk97. The solid line is the total rotation curve, dashed line is due to the dark matter halo, dash-dot-dashed line for the disk and dotted line is for the bulge.

Table 1

Initial Conditions for the Representative Model Galaxies (Arranged in Ascending order of \( Q \)) and the Heating Exponents Computed at the End of Each Simulation

indicate that these galaxies are dynamically hot (with high \( Q \) as most of the galaxies are in our sample) to prevent them from disk instabilities. Even if the disks suffer from instabilities, the resulting non-axisymmetric features could be young. In fact, the low metallicities, high gas-to-star mass ratios, and blue colors of most LSB galaxies indicate that these systems are probably younger than their HSB counterparts (Vorobyov et al. 2009).

4. HEATING MODEL

We use the following simple expression (Equation (4)) to fit all our simulation data, treating this functional form to quantify the effects of heating. It will be applied to the central region and
to the outer region of the galaxy.
\[ \sigma_z(r, t) = \sigma_0(r) + \sigma_1(r) \times (t/\tau_{\text{orb}})^\alpha. \] (4)

In the above equation, \( \sigma_0 \) and \( \sigma_1 \) are independent of time, and \( \tau_{\text{orb}} \) is the orbital timescale measured at the disk half mass radius. \( \alpha \) is the heating exponent and is the logarithmic time derivative of the vertical velocity dispersion \( \alpha = d \ln \sigma_z / d \ln t \). We determine the time evolution of \( \sigma_z \) in the inner region (\(< \sim 4 R_d \)) and in the outer regime (\(\sim 4 R_d \)) of the disk. The definitions for the two regions of the disk are maintained throughout the paper. The heating in each of these regions of the disk, characterized by apparently distinctive dynamical behaviors, is described by the resulting heating exponents given as \( \alpha_{\text{in}} \) and \( \alpha_{\text{out}} \), respectively. Higher values of \( \alpha \) imply that the rate of heating is steeper as the galaxy evolves, whereas lower values indicate a gradual heating process. A thorough knowledge of these values is crucial for understanding the type of heating processes at work in the galaxy and possibly for identifying the mechanism responsible for the heating. The fitting is performed using a robust nonlinear least-squares fitting algorithm due to Levenberg and Marquardt (Markwardt 2009; More 1978). The best-fit model parameters are chosen based on the minimum value of the chi-square \( \chi^2_{\text{min}} \). Using the resulting parameters, a useful estimate for the timescale of heating can be obtained from the above formula and this can also be compared with the relaxation timescale for the model under consideration.

Let us consider that in time \( \tau_h \), the change in the vertical velocity dispersion \( \Delta \sigma_z \sim \sigma_0(r) \) at a particular radius in the disk. Then differentiating Equation (4), it can be shown that
\[ \tau_h = \left[ \frac{\sigma_0(r)}{\sigma_1(r)} \right]^{1/\alpha} \tau_{\text{orb}}. \] (5)

We evaluate this timescale for different models presented here. In comparison, the energy relaxation timescale for the N-body model with a total number of particles \( N \), size \( R_s \), and softening \( \epsilon_s \) can be written following Huang et al. (1993) as
\[ \tau_{\text{relax}} = \frac{N \times R_s}{2 \pi \epsilon_s^2} \exp \left( \frac{R_s}{\epsilon_s} \right) \tau_{\text{orb}}, \] (6)
where we have used the crossing timescale \( \tau_{\text{cross}} = \frac{R_s}{2 \pi \epsilon_s^2} \tau_{\text{orb}} \) assuming that the circular velocity \( V_c \) remains almost flat beyond \( R_{1/2} \); \( R_{1/2} \) denotes the half-mass radius. Considering \( N = 5 \times 10^6 \), \( R_s = 10 \), \( R_{1/2} = 2.5 \), and softening parameter \( \epsilon_s = 0.003 \), we have \( \tau_{\text{relax}} = 5.6 \times 10^4 \tau_{\text{orb}} \).

Previously, disk heating has been modeled as a diffusion process in the velocity space. If the diffusion coefficient \( D_z \) remains constant in time, it can be shown that the vertical velocity dispersion evolves (Lacey 1991) as
\[ \sigma_z = \left( \sigma_{z0}^{1/\gamma} + D_z t/\tau_{\text{orb}} \right)^{\gamma}. \] (7)

Note that the above formula is applicable to the radial dispersion also. The exponent \( \gamma = 1/2 \) denotes heating due to the halo black holes (Lacey & Ostriker 1985). In addition to Equation (4), we also apply the diffusion model to fit our simulation data to some models for comparison and evaluate the corresponding diffusion coefficient \( D_z \). Let us consider that \( \tau_d \) is the timescale over which the change in the vertical velocity dispersion \( \Delta \sigma_z \sim \sigma_{z0} \). Then \( \tau_d \) can be obtained as follows
\[ \tau_d = \left( \sigma_{z0}^2 / D_z \right) \tau_{\text{orb}}. \] (8)

We denote \( \tau_d \) as the diffusion timescale. In general, \( \tau_d \) and \( \tau_{\text{relax}} \) are comparable. Internal evolution of a disk galaxy is driven by various non-axisymmetric perturbations such as bars or spirals which redistribute the energy and angular momentum within the disk and/or between the halo and the disk. The timescale for this process is longer than the dynamical timescale \( \tau_{\text{dyn}} \) and is usually known as the secular evolution timescale \( \tau_{\text{sec}} \). \( \tau_{\text{sec}} \) could be a few to 10 s of orbital timescales \( \tau_{\text{orb}} \). In ordering these timescales, \( \tau_{\text{dyn}} < \tau_{\text{orb}} < \tau_{\text{sec}} < \tau_d < \tau_{\text{relax}} \). We will use these definitions and compare the relevant timescales in the following to determine the relation between \( \tau_h \) and the other timescales.

5. NON-AXISYMMETRIC STRUCTURES AND DISK HEATING

It is well established that stars undergo continuous heating in both the radial and vertical directions in galaxies as they age. Although there has been substantial progress in understanding the radial heating, there is little progress for the vertical heating. Spiral structures have played a significant role in the studies of disk heating in the radial direction. We note, however, that a steadily rotating quasi-stationary spiral pattern heats the disk stars only negligibly (Binney & Tremaine 1987). In this case, sufficient heating of disk stars requires the spiral structures to be stochastic in nature. Transient, swing-amplified spiral density waves (Fuchs 2001) or multiple spiral density waves (Minchev & Quillen 2006) have recently been shown to be effective in the radial heating of disk stars.

The primary mechanisms suggested for the vertical heating of the disk stars in early works range from a combination of stochastic spirals and GMCs (Jenkins & Binney 1990) to resonant heating of the trapped stars in the chaotic phase space in the presence of a three-dimensional bar potential (Pfenniger 1985). More recently, other mechanisms have been suggested including a bending instability of the entire disk or of the bar (Sotnikova & Rodionov 2003) and repeated impact of globular clusters (Vande Putte et al. 2009), although it produces only a small change in the vertical velocity dispersion. Disk galaxies are rich in non-axisymmetric structures, which may form either by means of mergers, tidal interaction, or internal instabilities. Examples include bars, transient spirals, rings (e.g., NGC 1433, NGC 6300, Buta et al. 2001), lopsidedness (Saha et al. 2007), bending waves manifested in the form of a warp (e.g., ESO121-G6, NGC 4565, Saha et al. 2009), or corrugation (e.g., in IC 2233, Matthews & Uson 2008; in NGC 5907, Saha et al. 2009) of the disk mid-plane. These non-axisymmetric instabilities often drive the disk away from equilibrium and are generally time dependent. Each of these features are, in principle, a potential source of heating of the disk stars. Since our understanding of the interactions of these features with the surrounding live dark matter halo is not well understood, it is difficult to disentangle the signatures of each of these non-axisymmetric modes on the net heating of a group of stars. However, some progress has been made especially for our Galaxy in which it has been shown based on the analysis of the Hipparcos data that there exists a gap due to the 2:1 resonance with the bar in the Galactic UV plane (Dehnen 2000).

In the following, we study the nature of the vertical heating of disk stars due to bar growth and transient spirals. We also briefly discuss the effect of halo noise and the corrugation and/or bending waves on the net heating process. We carry out a series of N-body simulations to first systematically identify the sources of disk heating in our model galaxies which are free from
tidal interaction. About 70 simulations have been performed covering a wide range of the abovementioned simulation space and a statistical attempt to characterize and understand the nature of vertical heating in various regions of the disk has been carried out.

5.1. Vertical Heating due to Bar Growth

Although most of the galaxies in our sample are radially hot, they form a bar at the center during its time evolution. Some of the model galaxies form a bar within about 3–4 rotation timescales, while for others it takes more than about 10 rotation timescales to reach nearly the same amplitude. It is found that a bar forms even for galaxies characterized by \( Q \sim 3.7 \) (model mk55) in our simulation, implying that it is difficult for a rotating disk within a live dark matter halo to avoid forming bars at the center. Hence, bar formation appears to be quite common and generic in disk galaxies and a live dark matter halo plays a significant role. As shown by Athanassoula (2002), a live dark matter halo supports, rather than suppresses, the bar instability in the disk. In contrast, bar formation in the disk could be suppressed in the presence of a non-responsive rigid dark matter halo (Hohl 1976). The growth of the bar depends on the initial condition and grows via non-linear processes as the disk stars interact with the live dark matter halo particles. The nature of this interaction is yet to be completely understood although recent N-body simulations by Dubinski et al. (2009), Sellwood & Debattista (2009), and Klypin et al. (2009) have revealed insights into the nature of bar dynamics in disk galaxies. For our choice of parameters and initial conditions, we do not find any unique trends for the bar growth in our simulation. The sample of galaxies in our N-body simulations differs from typical galaxy models studied previously (where \( Q \) is normally held constant throughout the disk or \( Q \ll 2.0 \) in general). Overall, we observe that the growth of bars in our sample falls under two broad distinct categories. In the first category, the bar grows quickly to a peak amplitude within \( \sim 5 \) rotation timescales which is about an order of magnitude less than \( \tau_{\text{sec}} \) and nearly reaches saturation or starts growing again (for convenience, we call these type-I bars). In the second category, the bar continues to grow although slowly as compared to type-I and does not show any tendency to saturate (we call these type-II bars). Type-II bars grow on a secular evolution timescale to reach the same amplitude as in type-I. These two main features of bar growth are illustrated in the appropriate figures below, although there are some intermediate cases present in our simulation. In many cases, the bar undergoes the well-known buckling instability (Combes & Sanders 1981; Pfenniger & Friedli 1991; Raha et al. 1991; Debattista et al. 2006; Martinez-Valpuesta et al. 2006, and references therein) following which the bar takes the form of a peanut and/or X-shaped bulge. The buckling instability driven by the anisotropy in the velocity dispersions is an important phase of the bar evolution as it leads to the formation of a pseudo-bulge (Kormendy & Kennicutt 2004) directly influencing the secular evolution of disk galaxies. However, the condition for the onset of this instability and the exact underlying cause of this phase remains a matter of further investigation. For example, recent work by Martinez-Valpuesta & Shlosman (2004) has shown that the buckling instability weakens the bar and the resulting peanut-shaped phase lasts for several Gyr in their simulation. We also find that the peanut-shaped phase of the bar is long lasting, typically surviving till the end of our simulation (>5 Gyr) producing boxy/peanut/X-shaped structure in the central region. The morphological evolution of the disk in model mk97 is depicted through Figures 3 and 4. These snapshots are taken at time \( T = 0 \) and \( T = 5.4 \) Gyr, which denote the beginning and the end of the simulation, respectively. The initially axisymmetric disk evolves to form a bar which eventually transforms into a peanut morphology. Inspection of the evolution of the bar amplitude reveals a diverse behavior while it is in the peanut phase; in some cases, the amplitude continually grows and in others, it saturates. While the bar strength continues to increase, the evolution of its pattern speed is not always correlated in the sense that the pattern speed does not always decrease. In particular, the bar pattern speed in some cases remains nearly unchanged with time while its amplitude grows (as in mk17, mk33, mk104). A similar anti-correlation between the bar growth and pattern speed evolution has been discussed by Valenzuela & Klypin (2003) and more recently by Villa-Vargas et al. (2009). However, there are also the "normal" cases where the pattern speed decreases with time, while the bar grows to a higher amplitude, by losing angular momentum to the halo via dynamical friction. We notice that the bar growth is slow (typically a factor of 5–10 less) in cases where the pattern speed is observed to remain nearly constant in time. Overall, the emerging diverse behavior of the bar characteristics precludes detailed understanding for inferring gross physical properties (e.g., bar size, velocity dispersions) of the disk based on a few simulation results. As a consequence, the results of a large number of simulations have been used to approach the problem in a statistical sense.

For each of the model galaxies, we compute the \( m = 2 \) Fourier component (\( A_2 \)) of the particle distribution from each N-body snapshot using the following formula:

\[
A_m(r, t) = \frac{1}{N_r} \sum_{j=1}^{N_r} e^{i m \phi_j(r, t)}.
\] (9)

In the above equation, \( \phi_j(r, t) \) is the phase of the \( j \)th particle at position \( r \) and time \( t \), and \( N_r \) represents the number of particles used. Since \( A_m \) is a complex number, the modulus
of the \( m \)th Fourier component is obtained by \(|A_m| = \sqrt{\text{Re}(A_m)^2 + \text{Im}(A_m)^2}\) (Sellwood & Debattista 2009) and the corresponding phase is given by \( \phi_m = \frac{1}{2} \tan^{-1} \text{Im}(A_m)/\text{Re}(A_m), \) \((m \neq 0)\). In general, the radial variation of \( A_2 \) shows a pronounced peak corresponding to a bar in the central region of the disk and often a second peak indicating the presence of a spiral perturbation in the outer parts of the disk. The variation of the peak amplitude of \( A_2 \) in the inner region with time reveals the growth of a bar in the galaxy. The amplitude \( A_2 \) is normalized to the amplitude of the axisymmetric mode \((m = 0\) Fourier component). In all the subsequent figures, we present the smoothed normalized \( A_2 \). For a wide range of model bars, the following linear regression model is fit to the time evolution

\[
\log A_2(r, t) = A_2^0 + \beta \log(t/\tau_{\text{orb}}),
\]

where the slope \( \beta \) is the logarithmic time derivative of the bar amplitude. The linear model in the log–log plot translates to the bar amplitude evolving as \( A_2(r, t) = A_2^0(t/\tau_{\text{orb}})^\beta \). In the following, \( \beta_b \) represents the growth rate of a bar and \( \beta_r \) represents the growth rate of a spiral (in the outer region).

The time evolution of the normalized bar amplitude in model mk97 is illustrated in Figure 5. The bar amplitude nearly saturates through a series of high-frequency oscillations while undergoing the buckling phase. Such oscillations have also been reported in previous simulations by Valenzuela & Klypin (2003) and Villa-Vargas et al. (2009). Our estimate shows that the typical period of such oscillations is \( \sim 1/2 \times \tau_{\text{orb}} \) or less than \( \tau_{\text{orb}} \) in general. This roughly implies that the amplitude oscillates with a frequency close to \( \kappa \), the radial epicyclic frequency. The vertical velocity dispersion \( \sigma_z \) of the stars in the inner region of the disk is plotted as a function of time in Figure 6 for model mk97. From Figure 6, it can be seen that \( \sigma_z \) changes its slope at \( \sim 12 \) rotation timescales, nearly following the overall amplitude evolution of the bar. The best-fit parameters of the heating model (Equation (4)) are \( \alpha_0 = 0.09 \), \( \alpha_1 = 1.7 \times 10^{-4} \), and \( \alpha_{\text{in}} = 2.31 \). Using these parameters in Equation (5), we obtain \( \tau_b = 10.56 \times \tau_{\text{orb}} \ll \tau_{\text{relax}} \). The heating timescale due to the bar growth is found to lie in the range \( \tau_{\text{dyn}} < \tau_b \ll \tau_{\text{relax}} \). We have also applied the diffusion model given by Equation (7) to the inner region of mk97 and find that the heating model given by Equation (4) provides a better fit than the diffusion model. In particular, the diffusion coefficient \( D_z = 5.9 \times 10^{-4} \) and \( \gamma = 109 \) are found to be absurdly large. The resulting timescale \( \tau_d = 1.5 \times 10^9 \tau_{\text{orb}} \).

Interestingly, the temporal variation of \( \sigma_z \) does not reflect any such oscillation while the radial velocity dispersion undergoes oscillation with nearly the same frequency as the bar amplitude beyond about 10 rotation timescales.

The growth of a bar in model mk112 is similar to that in model mk97, although the disk in model mk112 is hotter \((Q = 2.47)\) than mk97. Figure 7 shows the normalized bar amplitude and the time evolution of \( \sigma_z \) in the disk inner region is shown in Figure 8. The best-fit parameters from model mk112 are found to be similar to mk97.

In the case of model mk107, the bar does not reach saturation and continues to grow as shown in Figure 9. The vertical velocity dispersion in the inner region of the disk grows more gradually (Figure 10) following the evolution of the bar. Note
that the overall increase in the vertical velocity dispersion is less than that of models mk97 and mk112. We find that the heating model given by Equation (4) better approximates the simulation data than the diffusion model. The resulting diffusion coefficient $D_z = 2.2 \times 10^{-4}$ is similar to model mk97 and $\tau_d = 4.4 \times 10^3 \tau_{orb}$. For models mk97, mk107, and mk112, we find that, in general, the diffusion model overestimates the dispersion values in the inner region of the disk.

In all our simulations, the vertical velocity dispersion increases following the growth of the bar. Note that all the three galaxy models (e.g., mk97, mk107, and mk112) undergo a phase of buckling instability during their evolution, but the disk stars are continually vertically heated prior to and during/after the buckling instability phase. This fact suggests that the vertical heating of the disk stars in our simulations is not strictly related to the bar buckling phase (e.g., the disk in model mk17 does not go through buckling instability within the simulation time period but still shows continuous heating in the vertical direction). However, we note that the rate of vertical heating during the bar buckling phase is generally higher than in phases without such buckling.

In the presence of a rotating bar ($m = 2$) potential, the planar motion of the disk stars can be coupled with the vertical oscillation (parametric resonance) at the location of the vertical resonances $\nu / (\Omega(r) - \Omega_b) = m/n$, where $\nu$ is the vertical frequency, $\Omega_b$ is the bar pattern speed, and $n$ is the number of vertical oscillations. For the $m = 2$ perturbation, $n = \pm 1$ represent the vertical Lindblad resonances. For the $n = \pm 2$ resonances, the retrograde orbits in the inner region of the galaxy couple efficiently with the vertical motion through the “Binney instability strips” (Binney 1981). Pfenniger (1985) has shown that indeed these 2:1 resonances inside the bar play a significant role in trapping stars, leading to rapid diffusion in the vertical direction. In the presence of a growing bar potential, the disk stars are subjected to a complex instability and the main effect of such an instability is to promote fast diffusion of the low angular momentum, high-$z$ amplitude and energetic stars to the nearby halo. The instability is shown to grow as the bar perturbation grows. Basically, the presence of a growing bar potential (i.e., a time-dependent potential in the galaxy) breaks the time invariance symmetry (because bar growth is an irreversible process) and hence the Jacobi integral, which in turn enhances the ergodicity considerably allowing the stars to visit most of the chaotic phase space (Pfenniger 1984, 1985). This provides a basic understanding of the heating mechanism in the presence of a growing bar. Of course, in $N$-body simulation the nature of bar growth is quite diverse as mentioned in the beginning of this section. The
resonant heating of the disk stars in the presence of a rotating (with changing pattern speed) growing (in amplitude) bar potential is probably more complex, leading to shifting locations of the resonances in the inner part of the galaxy and at the same time promising in the context of vertical heating in the disk. It is natural to examine how the vertical heating of the disk stars is correlated with the growth of bar, the nature of the vertical heating due to a growing bar, and the typical exponent for the vertical heating. In Section 6, we seek the possible correlations between the bar growth rate ($\beta_b$) and the inner heating exponent ($\alpha_{in}$).

5.2. Outer Disk Heating and Transient Spirals

Strong two-armed spirals are difficult to excite and form in radially hot disk galaxies. Using the Fourier decomposition of the disk surface density, however, many transient spirals (often with no steady pattern speed) are detected in our model galaxies. These transient spirals are often diffuse and generally weak in radially hot disks (Figures 5 and 7) as compared to the relatively strong spiral formed in the radially cold disk model mk107 (see Figure 9). These spirals are primarily confined to the outer region of the stellar disk, and we examine whether they play a role in the vertical heating of the disk stars in this region. In Figures 11–13, we present the time evolution of the vertical velocity dispersion in the outer region of the disk for models mk97, mk112, and mk107, respectively. The slope of the heating curve in model mk107 (concave) is quite different from the other two models (convex) presented here. In the later stages of the evolution, $\sigma_z$ in model mk107 increases faster than that in mk97 and mk112 (where $\sigma_z$ nearly saturates). Using the best-fit parameters of our heating model fitted to the outer region of mk97, we find that $\tau_h = 100 \tau_{orb}$ where $\sigma_0 = 0.025$, $\sigma_1 = 8.6 \times 10^{-3}$, and $\alpha = 0.42$. Applying the diffusion model in the outer region of mk97, we obtain a diffusion coefficient $D_z = 6.35 \times 10^{-4}$ and $\gamma = 0.214$ and the corresponding timescale $\tau_d = 1.3 \times 10^4 \tau_{orb}$, which is similar to the relaxation timescale ($\tau_{relax}$) quoted in Section 4.

In the case of model mk107, the best-fit parameters from our heating model yields $\tau_h = 33.7 \tau_{orb}$. The diffusion model in this case gives $D_z = 8.48 \times 10^{-4}$ and $\tau_d = 1.15 \times 10^3 \tau_{orb}$. These timescales indicate that the process of vertical heating in the outer region of the disk is slower in comparison to the inner region. We find that in the presence of a relatively strong spiral in the outer region, disk stars are heated to a greater degree in the vertical direction and the heating rate is found to be faster. The actual heating timescale, $\tau_h$, (in Gyr) in mk107 is lower by a factor of 6 in comparison to models mk97 or mk112. Although, the exact mechanism through which stars are vertically heated in the outer region is not clear, growing spirals definitely play a significant role.

The outer region of the stellar disk is not just simply described by the transient spirals, as corrugation waves or mild warps of the disk mid-plane are often present. It is known that the transient spirals can heat the disk efficiently in the radial direction, but poorly in the vertical direction. However, GMCs could redistribute the random energy of the stars in the vertical as well as in the radial direction through scattering processes. In the absence of GMCs or massive objects (such as halo black holes or dark clusters) in the outer parts of the galaxy, one seeks a process whereby the heating due to spirals or some other feature in the disk is redistributed in the vertical direction. Possible candidates, for example, include corrugation waves, large-scale warps of the stellar disk or, in general, vertical motion of the disk stars coupled with the transient spirals. The growth and development of these bending waves or rather warps is known to a large extent to depend on the noise in the N-body system. We
σz/σr < raises many questions. For example, why do disk galaxies have to occur for the solar neighborhood in the Galaxy. This fact dispersions in the inner and outer region of the model galaxy Fig. 14 and Fig. 15 the evolution of the radial and vertical velocity value? What causes the stars to be heated anisotropically?

have verified that as the number of particles is increased (say by a factor of 10) in the system, the growth of these bending waves is significantly reduced because of the substantial decrement in the Poisson noise arising from particle discreteness. With an average mass resolution of \( \lesssim 10^5 M_\odot \), only weak bending waves or mild warping of the stellar disk are present; comparatively strong warps can arise with a poorer mass resolution \( \gtrsim 10^6 M_\odot \).

Thus, it is unlikely that the bending waves alone can heat the disk stars in the outer region with higher mass resolution. However, the possibility exists that the nonlinear coupling between the weak transient spirals and the weak bending waves (Masset & Tagger 1997) could redistribute energy in the vertical direction. As mentioned earlier, an interesting behavior about the bars in radially hot disks inside a live dark matter halo was noticed. It was found that the type-I and type-II bars trigger transient spirals with distinctive characteristics in the outer disk. In the case of a type-II bar, the transient spiral continues to grow in amplitude, and this process continues as the bar grows (see Fig. 9). This is found to be the case even in one of our hottest disk models (mk55, for which \( Q = 3.73 \); see Table 1). Because of the growing transient spirals, we find that the radial heating dominates over the vertical one throughout the disk. On the other hand, the amplitude of the spirals nearly saturates in the case of models mk97 and mk112 (see Figs. 5 and 7), where the bar is of type-I. Note that in this case, the radial heating nearly saturates throughout the disk (see Figs. 14 and 15).

Overall, there is a parallel development prevailing in the inner and the outer regions of the disk whereby a bar and spiral grow, respectively.

5.3. Anisotropic Heating

An important aspect associated with the heating of disk stars is its anisotropic nature as can be inferred from the fact that the observed ratio of the vertical to radial velocity dispersion i.e., \( \sigma_z/\sigma_r < 1 \) (as in NGC 488, Gerssen et al. 1997), as found to occur for the solar neighborhood in the Galaxy. This fact raises many questions. For example, why do disk galaxies have \( \sigma_z/\sigma_r < 1 \)? Can the heating mechanism preserve an initial \( \sigma_z/\sigma_r \) value? What causes the stars to be heated anisotropically?

To gain some insight into these questions, we illustrate in Figs. 14 and 15 the evolution of the radial and vertical velocity dispersions in the inner and outer region of the model galaxy mk112. It can be seen that the overall nature and the rate of heating is different for the radial and the vertical directions. In the inner region of mk112, the vertical heating is more effective than the radial heating and in the outer regime the radial heating saturates while the vertical heating continues slowly. We find this trend to be the case for most of the galaxy models which are radially hot and in which the bar evolves to saturation. On the other hand, for model mk107 which is relatively cold (\( Q = 1.66 \)), the radial heating dominates over the vertical heating throughout the disk due to the presence of a relatively strong spiral perturbation. It is clear that the non-axisymmetric structures (such as bars or spirals) are essential for disk heating.

5.4. Halo Noise

The role of a live or responsive dark matter halo has been emphasized by Athanassoula (2002) in the context of bar formation in N-body simulations. Unlike the case of a rigid dark matter halo, a live halo facilitates the growth of the bar in an N-body disk, likely through resonant interactions with the disk particles. On the other hand, the finite number of particles employed in the N-body simulation can be an important source of disk heating as well. Because of the finite number of particles, two-body relaxation processes (Velazquez 2005) can be effective in heating the disk stars efficiently. It is highly desirable to eliminate or, at least, minimize the Poisson fluctuation in the halo particle distribution by increasing the number of halo particles employed in the simulation as permitted by the available computational resources. To quantify this effect, we performed four additional simulations on the galaxy model dmA (see Table 1) with different number of halo particles \( N_h \). There is an order of magnitude variation in the number of halo particles between models with the poorest resolution (0.5 million (M) particles) and the highest resolution. The intermediate two simulations use 2 and 3 million halo particles. In Figs. 16 and 17, we show the four vertical heating curves in the inner region and in the outer region, respectively. It is found that the vertical heating curves nearly converge for models with \( N_h > \text{few} \times 10^5 \). The convergence on the number of particles in our N-body simulation is in accordance with the conclusions reached in the recent work of Dubinski et al. (2009) in the context of bar formation and evolution. Comparison of Fig. 16 and Fig. 17 reveals that the effect of halo noise is more important in the central region than in the outer region. In order to quantify

![Figure 14](image-url) Comparison of radial and vertical heating in the inner region in model mk112. \( \sigma_z(0) \) and \( \sigma_r(0) \) are the values of initial dispersions at that radius. Solid line denotes vertical heating and dashed line the radial heating.

![Figure 15](image-url) Comparison of radial and vertical heating in the outer region of model mk112. \( \sigma_z(0) \) and \( \sigma_r(0) \) are the values of initial dispersions at that radius. The solid line denotes vertical and the dashed line the radial heating.
this trend, the timescales \( \tau_b \) in the inner and the outer region were determined. We find that an order of magnitude increase in the halo particle number increases \( \tau_b \) by at least a factor of 10 in the inner region and \( \sim 2 \) in the outer region. In contrast, we find a different behavior when the temporal variation of the radial velocity dispersion in the disk was examined. Because these models are dominated by spiral arms in the outer region and we find that by increasing the number of halo particles (from 0.5 M to 5 M) the spiral arms became even stronger. This resulted in a lower radial dispersion in models with a lower number of halo particles compared to the models with a higher number of halo particles. In the case of model mk112, the spiral arms are rather weak and the radial heating of the disk stars could have resulted from the halo noise. However, we find that the radial dispersion remains nearly constant over more than 10 rotation timescales (see Figures 14 and 15). Based on these facts, it is possible to conclude that the noise due to the halo (with \( N_h > \) few million particles) is neither effective nor the dominant source of vertical heating in the outer region of the disk.

In summary, we find a positive correlation between the growth of the bar at the galactic center and the vertical heating taking place in the central few kpc region. The formation of a bar is almost unavoidable in a stellar disk embedded in a live dark matter halo under the wide range of physical parameter space that has been explored. Bar formation occurs even in a very hot galaxy with its time for onset delayed at higher numerical resolution. In addition, analysis based on the four simulations on model dmA reveals that the strength of the spiral arms also increases with higher mass resolution. It has been noted that the vertical heating exponent is generally larger in the outer regions of a galaxy in the presence of strong spirals. Hence, the transient spirals in the three-dimensional disk which are common in our simulations not only play a significant role in radial heating, but also the vertical heating of disk stars.

6. CORRELATIONS

In the absence of a clear understanding of the physical processes responsible for the vertical heating of disk stars, we approach the problem by seeking possible correlations between the average physical properties (e.g., bar strength versus vertical heating rate) from our fairly large sample of N-body galaxies. To obtain a measure of the strength and slope of possible relations, we carry out a statistical correlation, measuring it between two variables \( x \) and \( y \) by the Pearson’s product–moment correlation coefficient defined as follows:

\[
\text{Corr}(x, y) = \frac{\text{cov}(x, y)}{\delta_x \delta_y} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{(N_x - 1)\delta_x \delta_y}. \tag{11}
\]

In the above equation, \( \text{cov} \) represents the covariance and \( \delta \) denotes the standard deviation of the particular variable in the subscript. The summation is carried over the sample size \( N_x \). According to the Cauchy–Schwarz inequality the correlation cannot exceed 1. If the value of the correlation is positive, it indicates an increasing linear relationship and in the case of negative value, it denotes a decreasing linear relationship.

In Figures 6, 8, and 10, the time evolution of the vertical velocity dispersion in the inner region of the disk is illustrated. In comparison with the evolution of the bar, it appears that the vertical velocity dispersion of the stars roughly follows the time evolution of the bar. Thus, for individual galaxies, there is a trend for the vertical heating, and it is interlinked with the growth of the bar. However, the exact relationship between the two and the reason for the vertical heating of the disk stars closely following the growth of the bar is unknown. In the absence of a definitive answer, we seek to determine whether the trend found in individual galaxies is generic. For example, does it depend on the initial condition, and is this trend robust when examined on a large sample of galaxies with different initial conditions? We note that these are difficult questions to answer because galaxies with varying initial conditions evolve quite differently. As pointed out recently by Sellwood & Debattista (2009), a live dark matter halo interacts with the disk in a more complicated way than previously envisioned.

In Figure 18, the dependence of the vertical heating exponent (\( \alpha_{\text{heating}} \)) in the inner region of the galaxy on the growth rate of the bar (\( \beta_b \)) is illustrated for our sample of model N-body galaxies.
The figure clearly shows that the heating exponent in the inner region of the disk is strongly positively correlated with growth rate of the bar with a value of Corr\( (\alpha_{\text{in}}, \beta_b) = 0.647 \). As pointed out earlier, when examined on a case-by-case basis, greater vertical heating results in galaxies which host stronger bars (e.g., models mk56A, mk52, mk112). Our analysis suggests that as the strength of the bar increases, the disk stars continue to be heated vertically, irrespective of the bar’s pattern speed evolution. The heating exponents tend to be higher in models with comparatively less massive dark matter halos. However, for more massive halos, the vertical heating exponent \( \alpha_{\text{in}} \) tends close to unity. On the other hand, in the absence of a strong bar, the heating exponent is lower, \( \alpha_{\text{in}} \sim 0.5 \).

A weak correlation is found between the growth of spirals and the heating exponent in the outer region as can be seen from Figure 19, corresponding to a value of Corr\( (\alpha_{\text{out}}, \beta_s) = 0.177 \). Its low value indicates that there may be more than one mechanism operating. Although the correlation between the heating exponents and the growth exponents for the spirals is not strong, higher exponents are associated with the presence of stronger spirals (e.g., model mk43, mk55, mk64) and the overall vertical heating is low when the spiral arms are more diffuse and weak (e.g., models mk5, mk51A, mk48). The vertical heating in the outer region is less extensive than the central region. The heating exponents in the outer region in many models are very different from \( \alpha_{\text{out}} = 0.5 \) indicating that the disk stars are not heated purely by massive objects in the halo or by the transient spirals; however, the exact source(s) which are responsible for the vertical heating in the outer region have yet to be identified.

On the other hand, we find that there is a relatively strong correlation (see Figure 20) between the strength of the bars and the strength of the spirals in the disk with a value of Corr\( (\beta_b, \beta_s) = 0.565 \). This correlation indicates that in the presence of a bar there is always at least a weak spiral present in the outer disk. This may also suggest that such weak transient spirals are triggered by the presence of a bar in the disk central region. However, the vertical heating in the inner region and the outer region is not well correlated. As the correlation between the two is very weak, Corr\( (\alpha_{\text{in}}, \alpha_{\text{out}}) = 0.10 \), the source of vertical heating in the inner and outer regions of the disk is likely to differ.

7. DISCUSSION

We have studied the nature of vertical heating of the disk stars in a large sample of radially hot galaxies as a function of halo mass and \( \sigma_z/\sigma_r \) spanning a wide range. For an individual galaxy, we find that the stars in the inner region are heated vertically as long as a bar continues to grow. Such correlation is quite generic and found to be robust when examined on a larger sample of model galaxies constructed from different initial conditions. Bars form in galaxies with diverse initial conditions but do not evolve in a comparable fashion likely reflecting their nonlinear interaction with the surrounding dark matter distribution. However, we find that on a broad category there are two kinds of bars: type-I (growing rapidly and approaching saturation) and type-II (growing slowly with no evidence of saturation) as mentioned in Section 5.1. Heating due to the growth of a bar differs from that due to a diffusion process since it occurs on a timescale of a few rotations in the disk central region. The diffusion timescale in a stellar system is similar to the relaxation timescale because diffusion mainly causes the stars velocity to change via two body encounters. Thus from our analysis, it can be said that the diffusion approximation is likely
to be inappropriate for the vertical heating of disk stars in the central region.

7.1. Growth of Boxy/Peanut Bulge

The boxy/peanut bulges are commonly believed to originate from the galactic bar via the vertical heating (Combes et al. 1990; Pfenniger & Norman 1990; Pfenniger 1993; Athanassoula 2005; Athanassoula & Martinez-Valpuesta 2009, and references therein). The growth of a bar in the $N$-body simulation with a live halo is a complex process because of the nonlinearity in the growing process and its interaction with the dark matter halo particles. The growing bars in our simulation are, at times, associated with a nearly constant pattern speed and, at other times, with a decreasing pattern speed. Such a growing bar is likely to drive the inner disk to a non-equilibrium state and depending on its strength and pattern speed, the inner stellar disk may evolve on a few dynamical timescales. Fundamentally, the growth of the bar facilitates the capturing of the disk stars into its various vertical resonances, e.g., the 1:1; 2:1; 4:1 vertical inner Lindblad resonances (ILRs) etc., which are often present in the bar region of the disk (Combes et al. 1990). As shown by Pfenniger (1985), the 4:1 vertical resonance becomes stronger as the bar grows, capturing disk stars to form the boxy bulge seen in many $N$-body simulations, although such studies do not explicitly include the interaction of the disk stars with a live dark matter halo. Furthermore, the presence of weakly dissipative orbits in a barred galaxy potential is shown to produce appreciable $z$-amplification as resonances are crossed, hence facilitating the growth of boxy bulges (Pfenniger & Norman 1990). For example, a simple Hamiltonian model of the growth of a bar perturbation (Quillen 2002) illustrated the effect of resonant capture of the stars into the 2:1 vertical ILR, lifting them out of the disk plane to explain the growth of the boxy/peanut bulge. One common result amongst these studies is that the vertical resonances (when present) are efficient in capturing disk stars, facilitating their motion perpendicular to the disk plane, essential for the growth of a boxy bulge. We note that, in this context, the disk stars in our simulation are not only vertically heated at the resonance locations but throughout the bar region of the disk and often with similar magnitude. This could imply either the resonant heating is not the only mechanism responsible for the overall vertical heating or broad resonances reflecting strong chaos are present throughout the bar region (Pfenniger 1985). Although a growing bar with/without a decreasing pattern speed could promote resonance sweeping through the bar region, it appears unlikely for the disk to accommodate the latter scenario making it unclear how the non-resonant stars are vertically heated. In any case, it would be useful to have further insight into how the non-resonant stars might be contributing to the growth of the bulge.

7.2. Superthin Galaxies

The disks of many galaxies in our sample are superthin and are dark matter dominated. From the galaxy formation point of view, it is extremely important to understand how these superthin galaxies are formed and how they evolve. Can they preserve their initial superthinness? In the hierarchical structure formation scenario, a galaxy grows via a large number of mergers and as a result, the $N$-body simulations of galaxy formation underestimate the number of thin galaxies. So, it remains a puzzle how superthin galaxies remain superthin. Assuming hydrostatic equilibrium, it can be shown that the disk half-thickness $h_z \propto \sigma_z/(G\rho_{mid})$, where $\rho_{mid}$ is the mid-plane volume density. Because of various unavoidable heating processes $\sigma_z$ would always increase with time and hence the disk thickness ($h_z$). Thus, either superthin galaxies somehow maintained their initial thickness or they evolved from an even thinner state to the present day superthin state. In the absence of environmental influences, the evolution of these galaxies would depend on the non-axisymmetric structures produced in the disk through internal instabilities. Our simulation shows that the non-axisymmetric structures such as bars or spirals are spontaneously formed in the disk and heat the disk stars in their respective regime of dominance. We briefly mention here that normally in radially hot and vertically very cold galaxies (e.g., mk25, mk27, mk48 for which initial $h_z/R_d \sim 0.01-0.02$), the disk thickness remains in the superthin regime with the final half thickness $h_z/R_d \sim 0.06$ after 5–6 Gyr of evolution. Most of these galaxies form a type-II bar but there are exceptions. If the initial thickness is greater than 0.03 or 0.04, then the final thickness evolves to be in the “thin” ($h_z/R_d > 0.1$) regime.

7.3. Common Heating Mechanism?

The vertical heating in the outer region is comparatively more complex because of the presence of spiral arms and, at times, mild warps or corrugations. The presence of these perturbations complicate the identification of the heating mechanism since more than one may operate simultaneously. It is known that a stationary spiral pattern does not lead to heating (except at resonance locations) and transient spirals heat more effectively in the radial direction. However, our analysis indicates that the transient spiral arms also grow (e.g., model mk107) i.e., they are time dependent and, hence, a similar situation may prevail in the outer disk as in the central region. In both regions, the disk stars respond to the time-dependent potentials. Thus, a common mechanism where the growing bars and spirals interact resonantly with the dark matter halo particles and contribute to the vertical heating in their respective regions of dominance may be operating.

8. CONCLUSIONS

Our investigation of the 70 $N$-body models of disk galaxies reveals a positive correlation between the growth of a bar and the vertical heating of the disk stars in the central region. A growing bar seems to contribute significantly to the vertical heating of the disk stars. Overall, the heating exponent $\alpha_m \geq 1$ for the various galaxy models studied. The disk stars in the central region are generally heated in the vertical direction by a factor $\sim 3$ to 4 above their initial values over 5–6 Gyr.

We find that the transient spirals are always present whenever a growing bar is present at the center. Most of these spirals are weak and diffuse, and it is likely due to the fact that the disks in our simulations are relatively radially hot. The numerical results show that the amount of vertical heating in the outer region is lower compared to the inner region with the disk stars in the outer region generally heated vertically by a factor of $\sim 2$ above their initial values over a time period of $\sim 4$ Gyr (if there is a growing spiral present) or more (otherwise).

From the analysis of our simulations, we find that, in general, in radially hot galaxies the vertical heating in the central region dominates over radial heating and in the outer region the relative importance is reversed. In contrast, radial heating dominates over the vertical heating throughout the disk for radially cold galaxies. We conclude that heating due to non-axisymmetric
structures appears to be most promising in the context of disk heating problem in general.

Our simulation results suggest that there is likely a common physical process through which the disk stars are heated vertically, which is active throughout the disk from inner to the outer region. Such a process should be investigated in detail by studying the vertical motion of the disk stars in the presence of a time dependent perturbing potential which could arise due to a growing bar or spiral arms inside a live dark matter halo.

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