Dark matter accretion wakes of high-redshift black holes

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Abstract

Anisotropic emission of gravitational waves during the merger or formation of black holes can lead to the ejection of these black holes from their host galaxies. A recoiled black hole which moves on an almost radial bound orbit outside the virial radius of its central galaxy, in the cold dark matter background, reaches its apapsis in a finite time. The low value of dark matter velocity dispersion at high redshifts and the black hole velocity near the apapsis passage yield a high-density wake around these black holes. Gamma-ray emission can result from the enhancement of dark matter annihilation in these wakes. The diffuse high-energy gamma-ray background from the ensemble of such black holes in the Hubble volume is also evaluated.

Key words: Black holes, high-energy gamma-rays, dark matter

1. Introduction

Accretion onto a black hole (BH) from a non-dissipative medium, such as a dark matter (DM) dominated Universe, is rather inefficient, mainly due to the absence of a cooling mechanism. However, dark matter distribution can be highly modified in the presence of a black hole. Enhancement of dark matter density and spike formation in the process of adiabatic accretion of dark matter onto black holes has been studied extensively (Gondolo & Silk 1999). Here, we study a different mechanism and study the response of dark matter to the BHs that are ejected from their host galaxies and move in a cold dark matter background on bound orbits (Mohayaee, Colin & Silk 2008). Indeed, in this article we review in more detail the results already presented in a short letter previously (Mohayaee, Colin & Silk 2008).

Assuming a zero velocity dispersion for DM and a relative motion between the BH and DM, and assuming the flow in the frame of the BH is steady and uniform at infinity, the density diverges on the downstream symmetry axis. This comes about because rings of dark matter, concentric with the symmetry axis, shrink down to points on the axis and this loss of dimension manifests itself in a singularity, i.e. a caustic. This caustic is merely a focal line and forms due to axial symmetry and it is not a stable catastrophe as classified by Arnol’d (Arnol’d 1986). Unlike a shock which forms in a dissipative medium, the particles cannot be trapped in a caustic and hence a caustic has very little mass in spite of its high density. However, due to the finite primordial dark matter velocity dispersion, a high density wake, rather than a line singularity, forms. In this case some of the dark matter particles with velocities smaller than escape velocities move on bound orbits around the BH instead of arriving from infinity on hyperbolic trajectories.

If dark matter consisted of weakly-interacting particles, such as those which arise in supersymmetry or extra-dimension extensions of the standard electroweak model, they would annihilate in pair and produce a host of particles among which are very high-energy gamma-rays. The flux of these gamma-rays depends on local dark matter density.
Fig. 1. The trajectory of a test particle in a cold (zero velocity dispersion) medium past a perturber, e.g. a black hole.

Hence, the enhanced density of the wake leads to an ever-more enhanced flux of high-energy gamma-rays. It is this flux which we aim to determine in this work.

How can black holes attain a velocity relative to the DM environment? And can the velocity dispersion of the background dark matter and the BH velocity be small enough for the wakes' over-density to be significantly high?

Black holes (BHs) can gain recoil velocity during their formation if their pregenitor stars collapse asymmetrically (Bekenstein 1973) or they can have gravitational recoil during their mergers with other BHs (Fitchett 1983). The kick velocity can be as large as a few hundreds of km/s during formation (Bekenstein 1973) and has recently been shown to reach a few thousands of km/s for maximally rotating BHs, with a particular spin configuration, during the merger phase (Campanelli et al 2007).

In spite of the recoil velocity, the density enhancement in the wake can be extremely low, because the velocity dispersion of dark matter is usually very high. As dark matter collapses into a halo which then evolves by accretion and merger towards a final virialised state, its effective velocity dispersion increases. Our galactic halo is assumed to have a velocity dispersion of a few hundreds of km/s. This can be compared with the present primordial velocity dispersion of dark matter which is only a few cm/s for neutralinos (that of axions is 7 orders of magnitude smaller). However, as we go back in time, dark matter becomes less and less clumpy and although the primordial velocity dispersion of dark matter increases linearly with redshift (its present value is about 0.03 cm/s for neutralinos), its average velocity dispersion in the clumps falls as on average they become less massive and smaller. Due to this low velocity dispersion, many of the BHs with a moderate kick velocity can escape from their host haloes at high redshifts (Favata et al 2004, Merritt et al 2004, Portegies Zwart 2000). The recoil velocity does not depend on the masses of the merging BHs but only on the ratio of their masses and on the spin configurations. Hence, at high redshifts where dark matter haloes and escape velocities from them are small, only a small recoil velocity is sufficient to set a BH on its orbit outside the virial radius of its host halo. In addition, at high-redshifts the velocity dispersion of dark matter is mostly small and hence the wakes could be significantly overdense if the BH also has a small velocity. The latter inevitably happens near the apapses of the radial orbits.

The physics of the early Universe is unveiling fast (Barkana & Loeb 2001). To explain the quasar population at high redshifts, a large number of early massive BHs must have existed (see e.g. Madau & Rees 2001). These BHs are believed to have formed in dark matter haloes and grown by merger with other BHs and hence could attain recoil velocities. These BHs can travel beyond the virial radii of their host haloes and move through colder and colder environments. At the apapses passages these BHs come to rest and there a substantial density enhancement can occur. Dark matter annihilation in these high density regions could open a new window into the early Universe for high-energy gamma-ray explorers.

In Section 2, we review the previous results on the calculation of the wake density and obtain the density profile in the limit as the velocity of the BH goes to zero. In Section 3, we evaluate the time duration a BH spends around the apapsis of its orbit. In Section 4, we evaluate the absolute luminosity of a BH in high-energy gamma-ray, as a function of its mass and redshift. We also evaluate the boost factor due to the BH wake relative to the background and also relative to the luminosity of the parent halo assuming a NFW profile. In Section 5, we evaluate the diffuse background, using Press-Schechter formalism and compare our results with the minimum flux from the host haloes. In Section 6, we conclude.

2. Density of the wake

We consider the black hole to be a point mass moving in a cold and almost homogeneous dark matter fluid. Due to this motion a wake forms behind the moving object which has an enhanced density w.r.t. the background. Indeed, the enhanced density of the wake has been suggested as the underlying
reason for dynamical friction (Chandrasekhar 1949, Kalnaj 1971, Hénon 1973). In this section we evaluate the wake density for the three following cases: (I) a medium with zero velocity dispersion, (II) when black hole velocity is comparable or smaller than the velocity dispersion of its environment and (III) when the BH velocity is far larger than the velocity dispersion of the dark matter medium.

2.1. zero velocity dispersion: $\sigma_{\text{DM}} = 0$

In the case of zero velocity dispersion, the density of the wake can be evaluated by solving a two-body problem (Binney & Tremaine 1987). The orbit in polar coordinates (as shown in Fig. 1) is given by

$$\frac{1}{r} = \frac{G M_{\text{BH}}}{R^2 V_{\text{BH}}^2} (1 - \cos \theta) + \frac{1}{R} \sin \theta,$$

where $R$ is the impact parameter as shown in Fig. 1. In this case, the density profile can be obtained analytically, using mass-conservation and can be written in the following convenient form

$$\left( \frac{\rho}{\bar{\rho}} \right)_{|\sigma=0} = \frac{1}{\sqrt{1 - \left( 1 + \frac{r V_{\text{BH}}^2 (1 + \cos(\theta))}{2 G M_{\text{BH}}} \right)^{-2} }},$$

where $\bar{\rho}$ is the background density, $M_{\text{BH}}$ is the mass of the BH moving with velocity $V_{\text{BH}}$ and the distance $r$ and angle $\theta$ are as shown in Figures 1 and 3. Evidently a singularity forms on the axis behind the BH for $\theta = \pi$ where the density diverges (see Fig. 2). The density can also be written in terms of the cartesian coordinates as,

$$\left( \frac{\rho}{\bar{\rho}} \right)_{|\sigma=0} = \frac{1}{2} \left( \xi + \frac{1}{\xi} \right),$$

where

$$\xi = \sqrt{1 + 4 \frac{G M_{\text{BH}}}{V_{\text{BH}}^2} \frac{x^2 + \sqrt{x^2 + y^2}}{y^2}}.$$  

and the profile is shown in Fig. 2 where the coordinates are shown in Figs. 1 and 3. The situation considered here is unrealistic and dark matter particles do indeed have a finite primordial velocity dispersion. This case is studied in the next subsection.

2.2. Density of the wake: $\sigma_{\text{DM}} \neq 0$

In this case, we assume that the DM particles have non-zero temperature and their velocities obey Maxwellian distribution. The integral form of the density profile can be found by using Jeans’ theorem. The calculation is too detailed to be reviewed here (Danby & Camm 1957, Griest 1988). The final expression for the density enhancement due to a moving point mass in a thermal environment is

$$\frac{\rho}{\bar{\rho}} = \int_{u=0}^{\infty} \frac{u \sqrt{u^2 + q^2}}{(2\pi)^{3/2}} \int_{\lambda=0}^{\pi} \sin \lambda d\lambda \int_{\nu=0}^{2\pi} d\nu e^{-F/2},$$

where

$$F = \frac{1}{2} \left( \frac{u}{m} \right)^2 - \frac{\mathcal{L}^2}{2m^2} - \frac{\mathcal{L}^2}{2m^2} + \frac{\mathcal{L}^2}{2m^2},$$

and

$$\mathcal{L} = \frac{G M_{\text{BH}}}{r}.$$
very low in the limit \( V \approx \omega \), and the density enhancement is given by

\[
\rho = \frac{\rho}{\rho_0} = \frac{1}{1 - \cos \psi} + \frac{1}{1 - \cos \psi} + \frac{1}{1 - \cos \psi} + \frac{1}{1 - \cos \psi},
\]

and subsequently

\[
W^2(1 - \cos \psi) = \left( V - V_0 \right) \left( 1 + \frac{W}{V_0} \right).
\]

Under the condition that \( \rho \gg \rho_0 \), one can demonstrate, using expression (7) that away from the negative symmetry axis (downstream) the density approaches that for zero velocity dispersion (2). After a suitable re-arrangement of expression (7) one obtains (see Sweatman & Heggie 2004 and also Sikivie & Wick 2002)

\[
\rho(M_{BH}, z, r, \theta) = \frac{2\rho(z)V_{BH}^2}{\pi \sigma_{DM}^2} \left( \int_0^\pi \int_0^{\pi/2} n \sin \alpha \right) - 1,
\]

where the function \( f(M_{BH}, z, n, \alpha) \) is given by

\[
f = 1 + \frac{rV_{BH}^2}{2Gm_{BH}} \left( 1 + \cos \theta - 2\sqrt{1 + \cos \theta n \cos \alpha + \eta^2} \right).
\]

We integrate expression (10) numerically and show the density contours in Fig. 3. However, one can show analytically that for \( V_0 \gg \sigma_{DM} \), the density (10) along the symmetry axis (\( \theta = \pi \)) attains the following maximum value:

\[
\rho_{\alpha = \pi} \approx \frac{\bar{\rho}(z)}{\sigma_{DM}(z)} \sqrt{\frac{\pi Gm_{BH}}{r}}, \quad \text{for} \quad \theta = \pi
\]

as long as \( \sigma_{DM}^2 \ll GM_{BH}/r \). That the wake density is independent of the velocity of the BH, downstream along the symmetry axis, might be surprising. However, we recall that the density is infinite there for a zero velocity dispersion and a cut-off to this divergence is put by finite dark matter velocity dispersion.

The density enhancement along the symmetry axis can be obtained by direct integration of (10). The density increases with squared-root of the mass of the black hole, grows quadratically with \( z \) and can be shown to also fall slowly with distance (as \( 1/\sqrt{r} \)). These results are confirmed by the simple expression (12).

In the case of zero velocity dispersion, the density enhancement given by (2) has a 1/\( V_0 \) dependence and hence diverges as \( V_0 \to 0 \). For a non-vanishing velocity dispersion, the density enhancement given by the integral (10) is independent of velocity along the symmetry axis as shown by expression (12) (see

\[
W^2(1 - \cos \psi) = \left( V - V_0 \right) \left( 1 + \frac{W}{V_0} \right).
\]
Fig. 3. Off the symmetry axis the density falls almost linearly with increasing velocity, and flattens at large velocities as predicted by the exponential term in expression (10).

2.4. Density of the wake: \( V_{\text{BH}} \to 0 \)

The zone of influence of BH decreases with increasing its velocity and the velocity dispersion of its environment. In the limit as \( V_{\text{BH}} \to 0 \), the density profile of the wake [4] reduces to

\[
\frac{\rho}{\bar{\rho}} = \sqrt{\frac{4}{\pi}} \frac{1}{r} + e^{r^2/r} \text{Erfc} \left( \sqrt{\frac{r}{r_0}} \right),
\]

(13)

where Erfc is the complementary error function and \( r_0 \) is the radius of influence of the BH: \( r_0 = GM_{\text{BH}}/\sigma_{\text{DM}}^2 \). We emphasis that [13] is the limit \( V_{\text{BH}} \to 0 \) of [4], and is not a unique density profile for stationary BHs. Here, we use [13] only as an approximation to [4] for slowly-moving BHs. Fig. 4 shows the dependence of the density enhancement on the BH velocity. When the BH is moving fast with respect to the background, a significant density enhancement only arises in a small zone around the symmetry axis (downstream) of the BH. The density enhancement also decreases with increasing velocity dispersion of dark matter environment. The highest density enhancement and largest radius of influence are achieved for BHs moving slowly (\( V_{\text{BH}} \leq \sigma_{\text{DM}} \)) in a cold background.

Expressions [4], or [13], have been obtained by assuming that the BH velocity remains constant, which is questionable in our situation where the BH is both slowed down by the pull of its parent halo and also by the dynamical friction of the wake itself. Since the BH is most luminous near the apapsis where its velocity is very small, we expect our assumption to be indeed valid near the apapsis. This is justified by Fig. 4 once the BH velocity is less that the velocity dispersion of the background, the constant velocity approximation is valid.

3. Time spent around the apapsis

The BH remains bound to its central halo if it is ejected with a velocity less than the escape velocity (measured from the virial radius). Because it is ejected from the centre (and also when with a large velocity), the BH is on almost radial orbit. The BH initial velocity is set as follows. We assume that the halo mass is about \( 2 \times 10^4 \) times the mass of the BH (Madau & Rees 2001), keeping in mind that the validity of the Magorrian relation (Magorrian et al 1998) at high redshifts is yet to be confirmed. Thus, for a BH of mass \( M_{\text{BH}} \), the virial radius of the halo, from which it was ejected, can be determined using \( M_{\text{halo}} = 4\pi/3\Delta_{\text{vir}}(z)\bar{\rho}(z)R_{\text{vir}}^3(z) \) and noting that \( \Delta_{\text{vir}}(z) = (18\pi^2 + 82x - 39x^2)/\Omega(z) \) and \( x = \Omega(z)/\Omega(z) \) and \( \Omega(z) = \Omega_0(1+z)^3/\Omega_m(1+z)^3 + \Omega_A + \Omega_k(1+z)^2 \) (see Bullock et al 2001 for details). Having evaluated the virial radius, we can then evaluate the escape velocity from the virial radius of the halo, using \( V_{\text{escape}} = \sqrt{2GM_{\text{halo}}/R_{\text{vir}}} \).

The ejected BH is slowed down by the gravitational pull of its parent halo and also by the dynamical friction of dark matter background as

\[
\frac{dV_{\text{BH}}}{dt} = - \left[ \frac{(2E + V_{\text{BH}}^2)}{4GM_{\text{halo}}} + \frac{4\pi G^2 M_{\text{BH}} \bar{\rho} \ln(\Lambda)}{V_{\text{BH}}} \right]
\]

(14)

where \( E = -V_i^2/2 + GM_{\text{halo}}/R_{\text{vir}} \) is the absolute value of the energy with which a bound BH leaves the virial radius with velocity \( V_i \). Since dynamical friction plays a sub-dominant role in braking the BH, the values of \( \ln(\Lambda) \) and the background density \( \bar{\rho} \) marginally affect the value of (14) for a BH in its initial outward journey.

By comparing the dynamical friction force to the force of the parent halo in (4), we can find the range of values of the velocity for which the former dominates, give by the inequality

\[
V_{\text{BH}}(V_{\text{BH}}^2 + 2E) \leq 400GM_{\text{BH}}\sqrt{2\pi\bar{\rho}}
\]

(15)

For a BH with ejection velocity (from virial radius) of about half the escape velocity, the inequality becomes

\[
V_{\text{BH}} \leq 4 \times 10^{-4} \left( \frac{M_{\text{BH}}}{M_\odot} \right)^{1/3} \sqrt{1 + z}
\]

(16)

in km/s, for which the dynamical friction dominates over the pull of the parent halo.

Time spent at the apapsis is defined to be the time during which the velocity of BH reduces from the background DM velocity dispersion to zero (at the apapsis), i.e. \( 0 < V_{\text{BH}} < \sigma_{\text{DM}}(z) \) where \( \sigma(z) \) is the velocity dispersion of dark matter in the field outside the halo. If the dynamical friction dominates over the gravitational pull of the halo, in bringing the BH to rest at apapsis. This yields

\[
\Delta t_{\text{DF}} = \frac{V_{\text{BH}}^3}{12\pi G^2 M_{\text{BH}} \bar{\rho} \ln(\Lambda)}
\]

(17)
where Coulomb logarithm is set to unity here and is not expected to be significantly greater than this value.

If the pull of the halo is the dominant force in bringing the BH to rest then the time during which the BH can be considered stationary [hence its velocity $0 < V_{BH} < \sigma_{DM}(z)$] is

$$\Delta t_{halo} = \frac{G M_{\text{halo}}}{E} \left( \frac{\sigma_{DM}}{2E + \sigma_{DM}^2} + \frac{\text{Arctan} \left( \frac{\sigma_{DM}}{\sqrt{2E}} \right)}{\sqrt{2E}} \right).$$  \hspace{1cm} (18)

However, the density enhancement is most significant when the velocity of the BH and the velocity dispersion of DM background are very low and hence when the dynamical friction force dominates over the gravitational pull of the host halo. Subsequently, $\Delta t_{DF}$ is shorter than $\Delta t_{halo}$.

The important issue we have not discussed here is that the wake does not form instantaneously and the time scale for the formation of the wake has to be compared to the time the BH spends at the apapsis. This would give us a meaningful estimate of the radius of the wake. However, we postpone this issue to the forthcoming work and here leave the radius of the wake, $R_{\text{cutoff}}$, as a free parameter. We estimate the suitable range of this parameter which yields a sufficient luminosity. The criteria here are

that BH luminosity would dominate over that of its central halo and the boost over the background luminosity evaluated within the same radius, $R_{\text{cutoff}}$, be far greater than unity.

4. Gamma-ray flashes from BHs around apapses

Cold dark matter if composed of neutralinos or Kaluza-Klein particles would annihilate in pairs and produce a host of secondary products, including energetic photons (e.g. see Bertone, Hooper, Silk 2005 for a recent review). The absolute luminosity, in units of $\gamma^{-1}$ of a BH of mass $M$ at redshift $z$ is:

$$L(M, z) = \frac{N_{\gamma} (\sigma v)}{2m_{\chi}} \left[4 \pi \int_{r_s}^{R_{\text{cutoff}}} r^2 dr \left( \rho(M, z, r) \right)^2 \right]$$  \hspace{1cm} (19)

where $r_s$ is the Schwarchild radius, $r_*$ is the radius of influence of the BH, $m_{\chi}$ is the neutralino mass ($\sim 100 \text{ GeV}/c^2$), $\langle \sigma v \rangle$ is the interaction cross-section [which we fix at $2 \times 10^{-26} \text{ cm}^3/\text{s}$] and $N_{\gamma}$ is the number of photons produced per annihilation. Note that the integral [(19)] is independent of angle $\theta$ for stationary BHs.

We had previously found that the wake density of a slowly-moving BH ($V_{BH} \leq \sigma_{DM}$) is well-approximated by the wake density of a stationary
Fig. 5. The minimum radius $R_{\text{cutoff}}$ at apapsis required for the BH luminosity to dominate over that of its parent halo is shown by the lower solid lines for different masses. For reference, the minimum virial radius, $R_{\text{vir}}$, of the halo (corresponding to ejection of a BH of mass $100M_\odot$) is also plotted.

BH. By inserting (13) [keeping only the first term] in (19) we obtain the analytic expression (in units of $\gamma s^{-1}$)

$$L_{\text{BH}} = 1.3 \times 10^{-15} (1+z)^6 \left( \frac{M_{\text{BH}}}{M_\odot} \right) R_{\text{cutoff}}^2 \sigma_{\text{DM}}^2$$

(20)

for the absolute luminosity of a BH where $R_{\text{cutoff}}$ and $\sigma_{\text{DM}}$ are given in the same length units. For dark matter with primordial velocity dispersion of $\sigma = 0.03(1+z)$ cm/s, expression (20) reduces to

$$L_{\text{BH}} = 1.4 \times 10^{25} (1+z)^6 \left( \frac{M_{\text{BH}}}{M_\odot} \right) R_{\text{cutoff}}^2$$

(21)

for the absolute luminosity of a BH, where $R_{\text{cutoff}}$ is in unit of cm. We recall that in (20) and (21), $R_{\text{cutoff}}$ is the radius within which the luminosity of the BH is evaluated.

Next, the BH luminosity is compared to the background luminosity, $L_{\text{BG}}$, within the same radius in the absence of BH, which is given by

$$L_{\text{BG}} = 5.36 \times 10^{-42} (1+z)^6 R_{\text{cutoff}}^3$$

(22)

where $R_{\text{cutoff}}$ is in unit of cm.

The BH luminosity shall also be compared to the absolute luminosity of its central dark matter halo of mass $2 \times 10^4 M_\odot$, assuming it has a NFW density profile (Navarro, Frenk & White 1997)

$$\rho = \frac{(1+z)^3}{\Omega_m} \left( \frac{c r / R_{\text{vir}}}{(1 + (c r / R_{\text{vir}}))^2} \right) \left( \frac{\delta_c}{\Omega_\Lambda + \Omega_k + \Omega_m(1+z)^3} \right)$$

(23)

where $\Omega(z) = \Omega_m(1+z)^3/\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^2$, $\delta_c = (\Delta_{\text{vir}}/3)c^2/(\ln(1+c) - c/(1+c))$, and $c$ is the concentration parameter, for which we use the fit $(10/(1+z))(M_{\text{BH}}/M)^{-0.13}$ which agrees well with Bullock et al 2001, Hennawi et al 2007, and is slightly lower than Comerford & Natarajan 2007. The halo mass within the virial radius is $M_{\text{halo}} = M_{\text{vir}}$.

The absolute luminosity $L$ (19), in unit of $\gamma s^{-1}$, of a NFW halo of mass $M_{\text{halo}}$ can then be evaluated using (23) in (19) [after setting $M = M_{\text{halo}}$, $r_s = R_{\text{vir}}$, $r_s = 0$] and fitted by the following functions

$$L_{\text{NFW}}(z > 1) = \frac{M_{\text{halo}}}{M_\odot} 0.7 (1+z)^{0.075}$$

$$L_{\text{NFW}}(z \leq 1) = \frac{M_{\text{halo}}}{M_\odot} 0.7 (1+z)^{0.7}$$

(24)

A lower limit can be put on the parameter $R_{\text{cutoff}}$ in (21), by requiring that $L_{\text{BH}}/L_{\text{NFW}} > 1$. This is shown in Fig. 5. The figure shows that at high redshifts, very small cutoff radii are sufficient for the black holes to be more luminous than their host haloes.

However, in comparing the BH luminosity to that of its parent halo, we have assumed a primordial velocity dispersion for DM. This assumption becomes less realistic at lower redshifts. The velocity dispersion of dark matter outside haloes has not yet been studied in numerical simulations. Observations report on different values, depending on the environment. For example in the local group, the velocity dispersion has been reported to be as low as 40 km/s (Karachentsev et al. 2003). Furthermore, it is not clear how the velocity dispersion in the field evolves with redshift. A high luminosity BH studied here, requires a low velocity dispersion environment. Here we put bound on this velocity dispersion by requiring the BH luminosity to dominate over that of the background and also over that of its central halo, i.e.,

$$L_{\text{BH}} \gg L_{\text{BG}} \quad \text{and} \quad L_{\text{BH}} \geq L_{\text{NFW}},$$

(25)

using expressions (20) for the black hole luminosity $L_{\text{BH}}$, (22) for the background luminosity $L_{\text{BG}}$, and (24) for the luminosity of the parent halo $L_{\text{NFW}}$. The relationship (25) puts an upper-bound on the velocity dispersion. The velocity dispersion of a dark matter halo as a function of the redshift and the
mass of its orbiting BH (recalling that $M_{\text{halo}} = 2 \times 10^4 M_{\odot}$) is given by

$$\sigma_{\text{halo}} = \left( \frac{M_{\text{BH}}}{M_{\odot}} \right)^{1/3} \sqrt{\frac{1}{1 + z}} \left[ 12 + \frac{17}{4\Omega(z)} - 12 \Omega(z) \right]^{1/6}$$

(26)

where again $\Omega(z) = 0.3(1/(1 + z) + 0.3(1 - 1/(1 + z)) + 0.7(1/(1 + z)^3 - 1/(1 + z)))^{-1}$. The result is plotted in Fig. 6. In this figure, the lower three curves are upper-bounds [see expression (25)] to the velocity dispersion corresponding to BHs of masses $10^7M_{\odot}$, $10^8M_{\odot}$ and $10^9M_{\odot}$ as marked on the plot. The upper three dashed-dotted (blue) curves are plotted for reference and represent the velocity dispersion of the dark matter haloes from which the BHs were ejected, given by (26).

Evidently the maximum velocity dispersion outside the halo would be smaller than that inside the halo. Here we have given only an approximate outline and we emphasise that a better method to clarify the problem of DM velocity dispersion would be to study the velocity dispersion outside DM haloes at different redshifts in N-body simulations. Although less decisive, analytic studies can also be made through for example Press-Schechter formalism.

Further important constraint on $R_{\text{cutoff}}$ can be put by studying the formation time of the wake. Indeed, on one hand a finite time is required for a wake to form within a certain radius and on the other hand the time spent at the apapsis is not infinite. By comparing these two time scales, namely time for the formation of the wake and the time the BH spends at the apapsis (during which it can be considered stationary) we can better determine the radius of the wake. However, we do not expect this radius to be much smaller than the minimum values of $R_{\text{cutoff}}$ given in Fig. 5. More detailed works on this problem is postponed to the forthcoming article and here the wake radius is left as a free parameter.

5. Diffuse gamma-ray background

Next, we study in a cosmological scenario, ejected BHs near their apapses passages, especially those at high redshifts, where the merger rate is higher, the escape velocities are lower and the recoiled velocities are as large as now. The recoil velocity depends on the mass ratio of the BHs and not on the masses of the individuals, which indicates that ejected BHs are expected to be more abundant at high redshifts. Hence the ensemble of recoiled BHs might yield an observable diffused background flux. The total flux is given by the integral

$$\Phi = \int \int_M \frac{L(M,z)}{4\pi r^2(z)} N(M,z) dM dV(z)$$

(27)

where $M$ can be either the BH mass $(M_{\text{BH}})$ or the halo mass $(M_{\text{halo}})$, $r(z) = R_H \left( 1 - \frac{1}{\sqrt{1+z}} \right)$ with Hubble radius $R_H = 4000\text{Mpc}$, and $N(M,z)$ is the number density of the BHs or haloes in the calculation for NFW haloes and the luminosity of a single BH $L(M,z)$ (or the parent NFW haloes at $z$) is given by (21) or (24) and the volume element is $dV = r(z)^2 dr(z) d\psi d\phi$.

The physical number density of the BHs is assumed to follow the Press-Schechter formalism (Press & Schechter 1974, Bower 1991),

$$N(M,z) = \frac{n_0}{\sqrt{2\pi}} \left( \frac{n + 3}{3} \right) \left( \frac{M_{\text{halo}}}{M_*(0)} \right)^{\frac{n+3}{2}} \left( 1 + z \right)^4 \frac{1}{M^2_{\text{halo}}} \frac{n+3}{2} \exp \left[ -\frac{1}{2} \left( \frac{M_{\text{halo}}}{M_*(0)} \right)^{\frac{n+3}{2}} \right] (1 + z)^2$$

(28)
where $n$ is the power spectrum index $-2 < n < -1$. For BHs, the above expression (25) has to be multiplied by the relative time a BH spends at apapsis, i.e. $\Delta t_{DF}/t_0$ where $t_0$ is the age of the Universe.

The time spent at apapsis, (17), is itself a function of BH mass. Thus only the fraction $\Delta t_{DF}/t_0$ of the ejected BHs can be considered to be actually on their apapses passage in a Hubble time. We can evaluate the total flux by performing the integrals in (24) [multiplied by $\Delta t_{DF}/t_0$ for BHs]. In using (17), we assume that the BH velocity $V_{BH}$ is of the same order as the dark matter velocity dispersion $\sigma_{DM}$ whose current value is $0.03 \text{ cm/s}$ and increases linearly with $(1 + z)$. We expect this assumption about the velocity dispersion to be more valid at very high redshifts.

For spectral index $n = -1$ and $M_\star \sim 10^{12} M_\odot$ and for the ensemble of BHs at their apapses passages and their central haloes (assuming NFW profiles) we obtain $\Phi_{NFW} \sim 10^{-6} \gamma \text{ cm}^{-2}\text{sr}^{-1}$. The flux from the BHs is lower than this value, since we have evaluated our parameter $R_{\text{cutoff}}$ by requiring $L_{NFW} = L_{BH}$. The BHs only spend a fraction of time at the apapse which yields approximately $\Phi_{BH} \sim 10^{-14} \gamma \text{ cm}^{-2}\text{sr}^{-1}$. The flux would be attenuated due to interaction of photons which however would affect approximately equally $\Phi_{BH}$ and $\Phi_{NFW}$.

We have assumed that only BHs produced in $3\sigma$ peaks of the density perturbation can undergo effective mergers. We have assumed that all ejected BHs orbit their central haloes outside the virial radius; however, were this not the case, we do not expect any significant overall decrease in the flux which is already underestimated by our moderate choices of parameters and also by assuming that there is only one apapsis passage for a BH. We have also ignored the effect of multiple density-enhancement for a BH which is on its inward journey through an already high-density wake.

6. Conclusion

Black holes can be ejected from their host haloes due to anisotropic emission of gravitational waves in the merger of their progenitors. If ejected with velocities below the escape velocity they move on bound orbits around their host haloes. Since they are ejected from the centres of the haloes they move on radial orbits and their velocities come to zero at the apapses passages. Around their apapses, these BHs have low velocities and move in a cold background and very high density wakes can form around them. If dark matter was to consist of self-annihilating particles, these BHs would be powerful sources of high-energy $\gamma$-rays, both individually as resolved sources and collectively as diffuse background. The results here indicate that the globular clusters in the outskirts of our halo or field galaxies in our local Universe devoid of central BHs can have orbiting BHs which during their apapsis passages would produce flashes of high-energy $\gamma$-rays, although this effect is expected to be most significant for massive objects at high redshifts. The validity of dynamical friction formulae has been very rarely studied for radial orbits (Gualandris & Merritt 2007). The fact that there is no mass loss makes BHs a rare case for dynamical friction theory. Throughout this work we have assumed a homogeneous background and a constant-velocity approximation, both of these assumptions are questionable for the problem considered here. The validity of these assumptions remains to be checked in high-resolution numerical simulations.

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1 This corrects the typo in Mohayee, Colin & Silk 2008 where this $\Phi_{BH}$ was mistakingly written as $\Phi_{NFW}$. 

2 This was mistaken as $\Phi_{NFW}$.
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