Determining Bias with Cumulant Correlators

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ABSTRACT

The first non-trivial cumulant correlator of the galaxy density field $Q_{21}$ is examined from the point of view of biasing. It is shown that to leading order it depends on two biasing parameters $b_1$ and $b_2$, and on $q_{21}$, the underlying cumulant correlator of the mass. As the skewness $Q_3$ has analogous properties, the slope of the correlation function $-\gamma$, $Q_3$, and $Q_{21}$ uniquely determine the bias parameter on a particular scale to be $b = \gamma / \langle Q_{21} - Q_3 \rangle$, when working in the context of gravitational instability with Gaussian initial conditions. Thus on large scales, easily accessible with the future Sloan Digital Sky Survey and the 2 Degree Field Survey, it will be possible to extract $b$, and $b_2$ from simple counts in cells measurements. Moreover, the higher order cumulants, $Q_n$, successively determine the higher order biasing parameters. From these it is possible to predict higher order cumulant correlators as well. Comparing the predictions with the measurements will provide internal consistency checks on the validity of the assumptions in the theory, most notably perturbation theory of the growth of fluctuations by gravity and Gaussian initial conditions. Since the method is insensitive to $\Omega$, it can be successfully combined with results from velocity fields, which determine $\Omega^{1/6} / b$, to measure the total density parameter in the universe.

Key words: large scale structure of the universe — methods: numerical

1 INTRODUCTION

Galaxies are likely to be biased tracers of the underlying density field, since surveys with different selection properties are correlated differently, i.e. they are biased with respect to each other. The amplitude of the two point correlation function is ambiguous and cannot be compared directly with measurements of the cosmic microwave background fluctuations, or dark matter simulations. Although the galaxy density field can be a complicated function of the underlying mass field, on large scales the fluctuations are small and only the leading order contributions are important. Then the two-point function of galaxies becomes $b^2$ times the underlying two-point correlation function, where $b = b_1$ is the first coefficient in the Taylor expansion of the general (local) bias function, the galaxy density field as function of the underlying mass field.

Higher order statistics can be used to alleviate this ambiguity by constraining the unknown value of $b$. The cumulants of the galaxy field are related to the cumulants of the underlying mass field and the biasing parameters (Fry and Gaztañaga 1993, hereafter FG93). For instance the skewness of the galaxy field was shown to be $Q_3 = q_3 / b_2 / b^2$, where $b_2$ is the second order coefficient in the expansion of the bias function, and $q_3$, and $Q_3$ are the skewness of the mass and galaxy field, respectively. The slope of the two-point function determines the theoretical $q_3$ using perturbation theory (Peebles 1980; Fry 1994; Bernardeau 1992; Juszkiewicz, Bouchet, & Colombi 1993; Bernardeau 1994). Unfortunately $Q_3$ alone cannot yield both bias parameters, therefore additional information is needed. Note that adding $Q_4$ does not help, since to leading order it depends on an additional bias parameter, $b_1$, and at higher orders the situation is exactly analogous. Several attempts have been made in the literature to supply the missing piece of information and to constrain the bias parameter by using the bi-spectrum (Fry 1994; Matarrese, Verde, & Heavens 1997; Scoccimarro et al. 1998), configuration dependence of the three-point function (Frieman & Gaztañaga 1998), and the skewness in elongated cells (Scoccimarro, Szapudi, & Frieman 1998). This Letter proposes a simple, robust alternative using the first non-trivial cumulant correlator (Szapudi & Szalay 1997) to provide the needed additional constraint. This technique yields $b$ via counts in cells measurements, thus it is possibly one of the simplest and most unique methods ever put forward. Note that there are alternatives besides higher order statistics for the determination of the bias, most importantly the use of velocity fields (Willick et al. 1997), which, however,
determine the combination of $\Omega^{0.6}/b$. The simple method proposed here is insensitive to the cosmological parameter $\Omega$. Thus in combination with results from velocity flows the total density in the universe can be determined.

Section §2 introduces the theory of biasing for cumulant correlators, §3 details how it can be applied to measure the bias parameters §4 discusses practicality, possible extensions, and limitations of the results.

2 BIASING OF CUMULANT CORRELATORS

Let us consider the moments and joint moments of the (smoothed) fluctuation field $\delta$, and denote the connected moments of the field with $\langle \delta \rangle_c$. The cumulants, $q_N$, can be defined by the moments as $\xi = \langle \delta^2 \rangle$, and $\langle \delta^N \rangle_c = q_N N^{-2} \xi^{N-1} (N \geq 3)$. Similarly the cumulant correlators are defined through the joint moments $\xi_l = \langle \delta_1 \delta_2 \rangle$ and $\langle \delta_1 \delta_2 \delta_3 \rangle_c = q_{LM} N^{-1} M^{-1} \xi^{M+L-2} \xi_l (N + M \geq 3)$. The former is a well established tool for characterization of the non-Gaussianity of the galaxy distribution (Peebles 1980), while the latter were recently introduced by (Szapudi & Szalay 1992; Meiksin, Szapudi, & Szalay 1992; Szalay & Boschán 1992).

If the galaxy density field is a general function of the underlying density field $\delta$, similar quantities can be defined for the galaxies. Note that this Letter uses continuum limit throughout, even though galaxies constitute a discrete process. This discreteness can be simply taken into account by the use of factorial moments (Szapudi & Szalay 1993) under the assumption of infinitesimal Poisson sampling (Peebles 1980), and will not be mentioned further. On large scales, where the mass fluctuations are small, it makes sense to Taylor expand the galaxy density field in terms of the mass field (bias function) $f(\delta) = \sum_{k \geq 0} b_k \delta^k/k!$, where $b_k$ are called the $k$th order bias parameters. FG93 have shown that to leading order it is possible to calculate the cumulants of the galaxy field in terms of cumulants of the underlying density field, and the bias parameters. Next their results are recapitulated briefly. For $N \leq 3$ the connected moments of the galaxy field are

$$\xi_3 = b^2 \xi_1$$
$$Q_3 = q_3/b + b_2/b^2,$$

where $b_1 = b$, the usual bias parameter for simplicity. The bias can be systematically calculated from the generating function $\log(e^{f(\delta)x})$, and explicit results up to sixth order can be found in FG93. The parameter $b_0$ is irrelevant for the higher order cumulants, it can be fixed by requiring $\langle f(0) \rangle = 0$. The hierarchy is approximately inherited by the biased field to lowest non-vanishing order, even for multi-point estimators. Note that several other general calculations were done in the past for the correlations of biased fields, among others (Kaiser 1984; Bardeen et al. 1985; Grinstein, & Wise 1985; Matarrese, Lucchin, & Bonometto 1985; Szalay 1988; Szapudi 1992; Matsubara 1993).

Analogously to FG93, biasing of the cumulant correlators can be calculated from the generating function $\log(e^{f(\xi_1)+f(\xi_2)})$ (Szapudi & Szalay 1993). It is a trivial although tedious matter to derive explicit formulae from it. Here only those results are shown which are needed for the applications in the next section,

$$\xi_{l,g} = b^2 \xi_l$$
$$Q_{21} = q_{21}/b + b_2/b^2,$$

where the notation is analogous to the previous equation. Note that the dependence on the bias parameters is extremely simple and exactly analogous to the corresponding results for $Q_3$. This facilitates the solution of the equations in the next section. The higher order results will be shown explicitly elsewhere. Note that the generating function can be trivially generalized to higher than two-point estimators as well.

3 MEASURING THE BIAS PARAMETER

For models of structure formation where Gaussian initial conditions were amplified via gravity, the above formulae can be combined with perturbation theory results in the weakly non-linear regime (Peebles 1980; Fry 1984; Bernardeau 1994; Coles & Frenk 1991; Gaztañaga 1993; Szapudi & Szalay 1993).

Thus given $\gamma$, $Q_3$, and $Q_{21}$, the equations can be solved for the bias parameters $b_1$, $b_2$. The result is

$$b = \frac{\gamma}{6(Q_{21} - Q_3)}$$
$$b_2 = \frac{\gamma Q_{21}(14\gamma - 6) + 21Q_3(6\gamma - 7\gamma)}{252(Q_{21} - Q_3)^2}$$

These equations constitute the main result of this Letter. They become degenerate when $q_{21} = q_3$, i.e. $\gamma = 0$. For typical CDM spectra and for $b \neq 0$ the solution is well behaved on large scales, where the assumptions of this calculation hold. The above solution provides a simple and unique way of determining the bias parameter $b$. Note that $Q_{21}$ and the $Q_N$’s up to a certain order fully resolve the degeneracy, as the $Q_N$’s depend on $N - 1$ bias parameters, and $\gamma$, thus the $b_{N-1}$’s can be successively calculated. From these further cumulant correlators can be predicted and compared with measurements, providing a set of consistency checks for the theory.

4 DISCUSSION

The previous section showed, that if $-\gamma$, the slope of the correlation function, $Q_3$, the third order cumulant, and $Q_{21}$ the first non-trivial cumulant correlator are measured, the bias parameter $b$ can be calculated. Next the practicality of such a measurement in the near future will be reviewed.

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According to (Colombi, Szapudi, & Szalay 1998), the third and fourth moments of the galaxy distribution will be measured with less than a few %, and 10% error, respectively, in the Sloan Digital Sky Survey (SDSS) on scales between $1 - 50 h^{-1}$Mpc. Errors of the same order can be expected for the future 2 Degree Field Survey (2dF), perhaps in a slightly narrower dynamic range of scales because of its smaller size. Similar accuracy can be achieved for $Q_2$ and $Q_3$, and even better accuracy for the correlation function on these scales. The previous considerations require that $\delta < 1$ both because of the Taylor expansion of the bias function, and reliance on perturbation theory. This still leaves quite a large dynamic range, roughly $10 - 50 h^{-1}$Mpc, in which the bias parameters $b_1$ and $b_3$, and their scale dependence can be determined accurately. Similarly, at higher orders using $Q_N$, $b_{N-1}$ can be calculated successively. Thus on large scales the bias will be accurately measured facilitating the comparison with the fluctuations in the cosmic microwave background and $N$-body simulations. Note that the assumption of large scales is important for both the applicability of the perturbation theory and Taylor expansion. On small scales, where the hierarchical assumption applies, i.e. the three-point function $\zeta = Q_3(\xi_1\xi_2 + \xi_3\xi_3 + \xi_1\xi_1)$ for $N = 3$ (Peebles 1980), $Q_3 = \xi_3/2$, as found in the APM catalog (Szapudi & Szalay 1997a). This leads to an asymptotically small bias formally $b = \gamma/3\zeta_2^2$, because the first term in the Taylor expansion does not describe the bias accurately when the variance is large.

Besides systematic errors, which are expected to be quite small for the SDSS and 2dF, redshift distortions could pose a problem. For the two-point function the theory of redshift distortions is well known (Kaiser 1987), and can be corrected for. For $Q_2$, the effects of redshift distortions were calculated by (Hivon et al. 1995), and they found it to be negligible on large scales; simulations (Szapudi et al. 1998) confirm this on scales when the variance is small, which is anyway the limit of applicability of the theory described above. It is safe to assume that the same holds for $Q_{21}$ as well, since it is a third order cumulant with slightly different smoothing than $Q_2$. Thus redshift distortions do not constitute a real problem for applying the theory of the previous section. The light cone effect is even less of a problem than redshift distortions: higher order moments of the distribution can be corrected for it, and in any case it is not expected to be important for the next generation of wide field surveys where the median redshift is $z \simeq 0.2$ (Matsubara, Suto, & Szapudi 1998).

Practicality of measuring these key quantities was demonstrated by several authors: correlation functions, and cumulants were estimated in numerous galaxy catalogs and simulations (Peebles 1980, Gaztañaga 1992, Szapudi, Szalay & Boschán 1993, Meiksin, Szapudi, & Szalay 1992, Bouchet et al. 1993, Gaztañaga 1994, Szapudi et al. 1995, Colombi et al. 1995, Szapudi, Meiksin, & Nichol 1996), cumulant correlators were determined in the APM catalog (Szapudi & Szalay 1997a), and recently in simulations (Munn & Melott 1996). In fact, the slope $-\gamma$ in the previous equations is precisely the slope of the second moment of fluctuations in the same cells in which the cumulants and cumulant correlators are measured. Therefore in practice, only counts in cells and bincounts (or joint counts) in pairs of cells have to be estimated for obtaining all required quantities. Thus, there is no doubt that this simple method will give a unique determination of the bias on large scales from the SDSS, and from the 2dF.

There are several further developments possible for the simple theory of the previous sections. First of all, it is entirely trivial to calculate the effects of bias on higher order cumulant correlators, given the generating function of $\gamma$. Similarly, the generating function generalizes easily for multipoint statistics. Projected catalogs can provide another way of estimating the needed three quantities, although deprojection in the weakly non-linear regime is far from trivial matter (Bernardeau 1993, Gaztañaga & Bernardeau 1997). Smaller scales could be penetrated as well by using either extended perturbation theory (Colombi et al. 1997), or second order perturbation theory (Scoccimarro, Frieman 1998, Scoccimarro, & Frieman 1999) in combination with expanding the bias correction further than leading order. Note that smoothing in this theory precends biasing as in FG93. While these two operations do not commute, according to David Weinberg (1998, private communication) the effect of this appears to be negligible in simple models on large scales, although it could lead to problems for some exotic biasing schemes. Based on the group properties of the bias function (FG94) it is possible to apply Equations 1-2, and their above mentioned possible generalizations, for biasing between clusters and galaxies, or between different types of galaxies (Kaiser 1984, Bernardeau & Schaeffer 1992, Szapudi & Szalay 1993, Bernardeau 1998) to predict the enhancement of the two-point correlation function on large scales from counts in cells. The mentioned group property enables more then one step as well, i.e. between mass to galaxies to clusters, etc. Note that all the previous considerations were based on local bias models. It is possible to construct more general models which are stochastic and/or non-local in nature. These possible improvements and generalizations, as well as a full scale demonstration of the technique on mock galaxy catalogs (Cole et al. 1998) with different biases will be presented elsewhere.

In summary, a novel method has been presented to determine the bias parameter $b$, as well as the higher order bias parameters. It is simple to use and will become a practical possibility in the near future when the new generation of wide field surveys, the SDSS and 2dF, come on line. The technique involves the estimation of (bi)counts in cells, and from them the slope of the correlation function $-\gamma$, and two third order moments: $Q_2$, and $Q_{21}$. The bias parameter can be obtained explicitly as $b = \gamma/6(Q_{21} - Q_2)$ in the context of gravitational instability with Gaussian initial conditions. It was demonstrated that these quantities can be measured in the near future, and $N > 3$ connected moments will provide additional constraints as internal consistency check. Thus a unique determination of the bias will be a realistic possibility with higher order statistics. Since velocity measurements yield $\Omega h^2/b$, this method can be combined with bulk flow measurements to yield $\Omega h^2$ as well. The general theory presented here can be used to describe bias between different species of galaxies and clusters as well.

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