Topological phase transition from nodal to nodeless d-wave superconductivity in electron-doped cuprate superconductors

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Abstract - Unlike the hole-doped cuprates, both nodal and nodeless superconductivity (SC) are observed in the electron-doped cuprates. To understand these two types of SC states, we propose a unified theory by considering the two-dimensional $t$-$J$ model in proximity to an antiferromagnetic (AF) long-range ordering state. Within the slave-boson mean-field approximation, the $d$-wave pairing symmetry is still the most energetically favorable even in the presence of the external AF field. In the nodal phase, it is found that the nodes carry vorticity and are protected by the adjoint symmetry of time-reversal and one unit lattice translation. Robust edge modes are obtained, suggesting the nodal $d$-wave SC being a topological weak-pairing phase. As decreasing the doping concentration or increasing the AF field, the nodes with opposite vorticity annihilate and the nodeless strong-pairing phase emerges. The topological phase transition is characterized by a critical point with anisotropic Bogoliubov quasiparticles, and a universal understanding is thus established for all electron-doped cuprates.

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Introduction. – Over thirty-year effort, the consensus has been reached that the rich phase diagram in cuprate superconductors mainly arises from the strong electronic correlations [1,2]. The low-energy theory of the doped Mott insulator is believed to be captured by the single-band $t$-$J$ model [3]. There have been tremendous studies on the $t$-$J$ model [4,5], most of which show $d$-wave pairing superconductivity (SC), which has been confirmed by hole-doped cuprates [6–8]. However, a remarkable asymmetry exists between hole doping ($p$-type) and electron doping ($n$-type) cuprates [9]. One of the most studied $n$-type family $\text{Re}_{2-x}\text{Ce}_x\text{CuO}_4$ (Re is a trivalent rare-earth cation) displays antiferromagnetic (AF) long-range order up to a relatively high dopant concentration 0.14 (refs. [10,11]) before the nodal $d$-wave SC appears. In contrast, the other $n$-type family $\text{A}_{1-x}\text{La}_x\text{CuO}_2$ ($\text{A} = \text{Sr,Ca}$) compound has a nodeless SC gap, and the $d$-wave pairing symmetry is suspected [12–15].

Recently, by virtue of the angular resolved photoemission spectroscopy measurement on the epitaxially stabilized $\text{Sr}_{1-x}\text{La}_x\text{CuO}_3$ thin films by oxide molecular-beam epitaxy, Harter et al. observed that the Fermi surface of the SC samples consists of a large electron pocket around $(\pi,0)$ and a tiny hole pocket surrounding $(\pi/2,\pi/2)$, which perfectly fits into a tight-binding electronic band structure with an AF long-range order [16]. Moreover, in the SC state, a strong coupling between the charge carriers and the AF long-range order can push the nodal quasiparticles below the Fermi level, leading to nodeless $d$-wave SC without a change in the pairing symmetry of the order parameter [16]. Actually, such a feature had been also noticed in the single-crystal $\text{Re}_{2-x}\text{Ce}_x\text{CuO}_4$ samples [17–19].

In the present paper, we will carefully study the Fermi surface evolution as varying the doping concentration and examine the SC pairing symmetry in the two-dimensional $t$-$J$ model in the presence of a staggered magnetic field, which mimics the AF long-range correlations in the $n$-type family cuprates [20,21]. In real materials, the dopant is very likely to be inhomogenous, resulting in the superconducting regions in proximity to some AF regions. With the slave-boson mean-field (MF) approximation in the hole picture, the Fermi surface is composed of a large electron pocket around $(\pi,0)$ and a tiny hole pocket surrounding $(\pi/2,\pi/2)$, which gradually emerges as increasing the
doping concentration. In the superconducting phase, we found that the d-wave pairing symmetry is the most energetically favorable even in the presence of strong AF field. In the nodal phase, the external AF field duplicates the nodes via the AF scattering process, which does not change the nature of the nodal d-wave SC phase. Moreover, the nodes carry vorticity ±2 and are protected by adjoint symmetry \( \mathcal{T} \) of the time-reversal and one unit lattice translation, and the corresponding robust edge states are also obtained. As increasing the AF field or decreasing the doping level, the nodes with opposite vorticity annihilate, and the nodal low-energy excitations are gapped out, leading to a nodeless d-wave SC. A topological phase transition occurs between the nodal phase and the nodeless phase. The critical point is characterized by anisotropic Bogoliubov quasiparticles with quadratic dispersions along the nodal direction and the Dirac line dispersions perpendicular to the nodal direction. We note that distinct from the topological transition of nodal d-wave induced via additional spin-orbital interaction [22], the topological phase transition we address here is intrinsic to the electron-doped cuprate materials.

Model and theory. – Since the doped electrons of the cuprates reside at the Cu 3d_{x^2−y^2} orbital forming the full 3d^{10} configuration, the basic physics is well captured by the t-J model on a two-dimensional square lattice [2]. Experimental measurements suggest that the nearest-neighbor (n.n.), the next-nearest-neighbor (n.n.n.), and the next-next-nearest-neighbor (n.n.n.n.) hopping should be taken into account. By including an external AF field, the model Hamiltonian is defined by

\[
H = t \sum_{\mathbf{r}, \sigma} c_{\mathbf{r}, \sigma}^\dagger c_{\mathbf{r} + \mathbf{e}_\delta, \sigma} + t' \sum_{\mathbf{r}, \sigma} c_{\mathbf{r}, \sigma}^\dagger c_{\mathbf{r} + e_\gamma, \sigma} - \mu_0 \sum_{\mathbf{r}, \sigma} n_{\mathbf{r}, \sigma} + \lambda' \sum_{\mathbf{r}, \sigma} c_{\mathbf{r}, \sigma}^\dagger c_{\mathbf{r} + 2e_\delta, \sigma} + \sum_{\mathbf{r}, \sigma} \sigma \epsilon Q r c_{\mathbf{r}, \sigma}^\dagger c_{\mathbf{r}, \sigma} + \frac{J}{2} \sum_{\mathbf{r}, \delta} \left( \mathbf{S}_\mathbf{r} \cdot \mathbf{S}_{\mathbf{r} + \mathbf{e}_\delta} - \frac{1}{4} n_{\mathbf{r}, \uparrow} n_{\mathbf{r}, \downarrow} \right),
\]

(1)

where \( \delta \) and \( \gamma \) denote the n.n. and n.n.n. vectors, respectively, and \( m_s \) is the external AF field with wave vector \( \mathbf{Q} = (\pi, \pi) \). The model is subject to a local constraint \( \sum_{\mathbf{r}, \sigma} c_{\mathbf{r}, \sigma}^\dagger c_{\mathbf{r}, \sigma} \leq 1 \). The band parameters are adopted by \( t = 215 \text{ meV}, \ t' = -0.16t, \ t'' = 0.2t, \ J = 0.3t \), which are relevant to experiments [16]. Note that this model is expressed in the hole representation so that \( c_{\mathbf{r}, \sigma}^\dagger \) creates a hole in the Cu 3d_{x^2−y^2} orbital and therefore the hopping parameters differ from the hole-doped cases by a sign [23]. Accordingly, in what follows the hole/electron pocket, as seen from the band structure, should correspond to the electron/hole pocket as observed by experiments, and the pocket we address in the following will refer to the experimental pocket to save confusion.

Firstly, we would like to point out that there is a special symmetry in the model Hamiltonian. Since the external AF field breaks the lattice translation and time-reversal symmetries, the square lattice is divided into two sublattices A and B \((r_x + r_y = \text{even} \text{ and } r_x + r_y = \text{odd})\) that are subject to opposite magnetic fields. The sublattice degrees of freedom span a two-dimensional space in terms of Pauli operator \( \tau_\alpha: \tau_3 = 1 \) for the sublattice A and \( \tau_3 = -1 \) for the sublattice B. It is obvious that one unit lattice translation operation is equivalent to applying the operator \( \tau_2 \), which flips the sublattice degree of freedom. It can be proved that the coupling of the electrons with the AF field commutes with an adjoint operator \( \mathcal{T} \equiv \tau_2 \mathcal{T} \), where \( \mathcal{T} \equiv i\sigma_x K \) is the time-reversal operator and \( \tau_2 \) is the one unit lattice translational operator. Therefore, the model Hamiltonian has the adjoint \( \mathcal{T} \) symmetry, which is much similar to the time-reversal symmetry in such an adjoint way due to the zero net magnetization in each unit cell.

To treat the local constraint, we employ the slave-boson decomposition: \( c_{\mathbf{r}, \sigma}^\dagger = b_{\mathbf{r}, \sigma} \) and the constraint is rewritten into \( \delta b_{\mathbf{r}, \sigma} + \sum_{\mathbf{r}, \sigma} f_{\mathbf{r}, \sigma} f_{\mathbf{r} + \mathbf{e}_\delta, \sigma} = 1 \), which can be enforced by introducing a Lagrangian multiplier \( \lambda \). When the holons condense \( \langle b_{\mathbf{r}, \sigma} \rangle = (\delta b_{\mathbf{r}, \sigma})^\dagger = \sqrt{\mathcal{T}} \mathcal{T}, \) the fermionic spinon pairs are left. To decouple the spin superexchange term, MF order parameters are introduced:

\[
\kappa \equiv \frac{J}{4} \left( f_{\mathbf{r}, \uparrow}^\dagger f_{\mathbf{r} + \mathbf{e}_\delta, \uparrow} + f_{\mathbf{r}, \downarrow}^\dagger f_{\mathbf{r} + \mathbf{e}_\delta, \downarrow} \right),
\]

(2)

\[
\Delta_\delta \equiv \frac{J}{4} \left( f_{\mathbf{r}, \uparrow}^\dagger f_{\mathbf{r} + \mathbf{e}_\delta, \uparrow} - f_{\mathbf{r}, \downarrow}^\dagger f_{\mathbf{r} + \mathbf{e}_\delta, \downarrow} \right).
\]

In generally, we assume \( \Delta_\delta = \Delta_\delta f + i\Delta_\delta s, \) and \( \Delta_\delta f = \Delta_\delta s = -\Delta_\delta s = i\Delta_\delta f \), where \( \Delta_\delta s \) and \( \Delta_\delta f \) are amplitudes of n.n. spin-singlet pairing with \( s_{x^2−y^2} \) and \( d_{x^2−y^2} \)-symmetries, respectively. Then, in momentum space the MF Hamiltonian can be written as

\[
H_{\text{MF}} = \sum_{\mathbf{k}, \sigma} \left[ \epsilon_\mathbf{k} f_{\mathbf{k}, \uparrow}^\dagger f_{\mathbf{k}, \uparrow} + \epsilon_\mathbf{k} f_{\mathbf{k}, \downarrow}^\dagger f_{\mathbf{k}, \downarrow} + m_s \sigma (f_{\mathbf{k}, \uparrow}^\dagger f_{\mathbf{k}, \uparrow} + f_{\mathbf{k}, \downarrow}^\dagger f_{\mathbf{k}, \downarrow} + Q, \sigma) \right] + \sum_{\mathbf{k}} \left( \Delta_\delta f_{\mathbf{k}, \uparrow}^\dagger f_{\mathbf{k} + \mathbf{e}_\delta, \uparrow} + \Delta_\delta f_{\mathbf{k}, \downarrow}^\dagger f_{\mathbf{k} + \mathbf{e}_\delta, \downarrow} \right),
\]

(3)

where

\[
\epsilon_\mathbf{k} \equiv 2(tx + \kappa) (\cos k_x + \cos k_y), \mu \equiv \mu_0 - \lambda,
\]

\[
\epsilon'_\mathbf{k} \equiv 4t' \cos k_x \cos k_y + 2tt'' (\cos 2k_x + \cos 2k_y),
\]

\[
\Delta_\delta \equiv 2\Delta_\delta (\cos k_x - \cos k_y) + i2\Delta_\delta (\cos k_x + \cos k_y).
\]

Note that the kinetic energy part is renormalized by the doping concentration but the external AF field \( m_s \) is not, because the AF field in fact couples only to the spinon. This suggests that a small AF field \( m_s \) can have a significant effect on the band structure and SC pairing. One important point is that the MF Hamiltonian still preserves the adjoint symmetry \( \mathcal{T} \).

The MF Hamiltonian can be diagonalized in two steps. First, the normal state band structure determined by the first two terms of eq. (3) can be derived as

\[
\xi_\pm, \mathbf{k} = \epsilon'_\mathbf{k} - \mu \pm \sqrt{\epsilon'_\mathbf{k}^2 + m_s^2},
\]

(4)
in the quasiparticles of
\[
\begin{align*}
\psi_{\pm,k,\sigma}^\dagger &= (\cos \theta_k) f_{\dagger,k,\sigma}^0 + \sigma (\sin \theta_k) f_{\dagger,k,\sigma}^Q, \\
\psi_{\pm,k,\sigma}^\dagger &= (\sin \theta_k) f_{\dagger,k,\sigma}^0 - \sigma (\cos \theta_k) f_{\dagger,k,\sigma}^Q,
\end{align*}
\]
where \( \theta_k \equiv \frac{1}{2} \tan^{-1} \frac{m_x}{t \kappa} \in [0, \pi] \). The normal state spectrum has two-fold Kramer’s degeneracy due to the \( \hat{T} \) anti-unitary symmetry, then the adjoint operation of \( \hat{T} \) on the original fermions is precisely equivalent to the time-reversal \( \hat{T} \) acting on the quasiparticles, i.e.,
\[
\hat{T}^{-1} \left( \begin{array}{c} f_{k,\sigma} \\ G_{k+Q,\sigma} \end{array} \right) \Leftrightarrow \hat{T}^{-1} \left( \begin{array}{c} \psi_{+,k,\sigma} \\ -\psi_{-,k,\sigma} \end{array} \right) \hat{T}.
\]

Physically, this is because the sublattice degrees of freedom are embedded within the quasiparticles.

The next step is to turn on the superconducting pairing between normal quasiparticles. When the pairing matrix is written in the quasiparticle Nambu spinor,
\[
\Psi_{\pm,k} \equiv \left( \begin{array}{c} \psi_{+,k,\uparrow}^\dagger & \psi_{+,k,\downarrow}^\dagger \\ \psi_{-,k,\downarrow} & -\psi_{-,k,\uparrow} \end{array} \right),
\]
the interband pairing is found to be absent, owing to the n.n. pairing and singlet pairing nature. As a result, the two species of quasiparticles are decoupled even in the pairing channel, i.e., \( H_{\text{int}} = \frac{1}{2} \sum_{k,\alpha=\pm} \Psi_{\alpha,k}^\dagger H_{\alpha,k} \Psi_{\alpha,k} \) where
\[
H_{\pm}(k) = \xi_{\pm,k} \rho_0 \sigma_0 \pm (\Delta k \rho_+ + \Delta k \rho_-) \sigma_0,
\]
where \( \rho_+ \), \( \rho_- \), and \( \rho_0 \) denote three \( 2 \times 2 \) Pauli matrices acting in the particle-hole sector. It is straightforward to obtain the Bogoliubov quasi-particle spectrum
\[
E_{\pm,k} = \sqrt{\xi_{\pm,k}^2 + |\Delta k|^2}.
\]

The Bogoliubov quasiparticles are given by
\[
\begin{align*}
\eta_{\pm,k,\sigma} &= (\cos \beta_{\pm,k}) \psi_{\pm,k,\sigma}^\dagger \pm \sigma (\sin \beta_{\pm,k}) e^{i \phi_k} \psi_{\pm,-k,-\sigma}, \\
\eta_{\mp,k,\sigma} &= (\sin \beta_{\pm,k}) \psi_{\pm,k,\sigma}^\dagger \mp \sigma (\cos \beta_{\pm,k}) e^{i \phi_k} \psi_{\pm,-k,-\sigma},
\end{align*}
\]
where \( \beta_{\pm,k} \equiv \frac{1}{2} \tan^{-1} \frac{|\Delta k|}{\xi_{\mp,k}} \in [0, \pi/2] \) and \( \phi_k \equiv -\tan^{-1} \frac{\text{Re}\, m_x}{\text{Im}\, m_x} \).

By filling out the negative-energy states, the ground-state energy density can be written as
\[
\epsilon_\sigma = -\frac{1}{8\pi^2} \int dk_x dk_y \left( (E_{+,k} + E_{-,k}) - \mu \right) + \frac{1}{\pi} \sum_{\sigma=\pm} \left( \kappa^2 + \Delta^2 + \Delta_{d}^2 \right).
\]

Then the saddle point equations can be derived by minimizing the ground-state energy \( \partial \epsilon_\sigma / \partial (\kappa, \Delta, \Delta_d, \mu) = 0 \), from which the MF parameters \( (\kappa, \Delta, \Delta_d, \mu) \) are determined self-consistently. In terms of Bogoliubov quasiparticles, the imaginary-time Green’s function \( G_\sigma(k, \tau) = -(T_\tau f_{k,\sigma}^0(\tau)f_{k,\sigma}^0(0)) \) can be deduced to
\[
G_\sigma(k, i\omega_n) = \frac{\cos^2 \theta_k}{i\omega_n - E_{+,k}} + \frac{\sin^2 \theta_k}{i\omega_n + E_{+,k}}
+ \frac{\cos^2 \theta_k}{i\omega_n - E_{-,k}} - \frac{\sin^2 \theta_k}{i\omega_n + E_{-,k}},
\]
and the corresponding spectral function \( A(k, \omega) = -\frac{1}{\pi} \text{Im} G_\sigma(k, \omega + i0^+) \) is thus obtained:
\[
A(k, \omega) = \frac{1}{4} \sum_{\alpha=\pm} \left( 1 + \alpha \frac{\xi_{\alpha,k}}{\sqrt{\xi_{\alpha,k}^2 + m_x^2}} \right) \left[ \frac{1 + \frac{\xi_{\alpha,k}}{E_{\alpha,k}}}{E_{\alpha,k}} \delta(\omega - E_{\alpha,k}) \\
+ \frac{1 - \frac{\xi_{\alpha,k}}{E_{\alpha,k}}}{E_{\alpha,k}} \delta(\omega + E_{\alpha,k}) \right].
\]
Fermi level around the \((\pi, 0)\) point. The corresponding Fermi surfaces can be shown by the intensity map by integrating out the spectral function near the Fermi level \(\int_{0.06} A(k, \omega) \, d\omega\), displayed in fig. 2(b). Both the hole and electron pockets grow with increasing doping level and tend to be connected, reconstructing a large Fermi surface. Such a feature qualitatively agrees with the experimental observation [16]. Note that with a different value of AF field the feature does not change drastically; for example, a stronger AF field would only require larger dopant concentration for the hole pocket around \(K\) to occur, and the evolution is similar.

In the SC phase, the Bogoliubov quasiparticle spectra \(E_{\pm, k}\) are displayed in fig. 3(a) for given dopant concentrations \(x = 0.06, 0.08, 0.12\) and a reasonable AF field \(m_{s}/t = 0.06\). Due to the presence of the \(d\)-wave pairing, an energy gap opens up in the lower Bogoliubov quasiparticle band \(E_{-\pm, k}\) around the point \((\pi, 0)\), making the Fermi pocket around the anti-nodal point fully gapped. The low-energy excitations are from the upper Bogoliubov quasiparticle band \(E_{\pm, k}\) around the nodal point. Moreover, we also found that the nodal Fermi pocket preserves a pair of nodes residing at the symmetric points \(K_{\pm}\) above a critical doping concentration, below which the nodes are also gapped out. The full phase diagram is calculated and shown in fig. 3(b).

**Topological properties of the nodal \(d\)-wave SC.** – In the nodal \(d\)-wave SC phase, the nodes exist and only exist at the intersections of the Fermi pocket and at the zero line of \(d\)-wave pairing field, i.e., \(\xi_{\pm}(k_{x}, k_{y}) = 0\). There are eight nodes in the first Brillouin zone, as schematically shown in the insert of fig. 3(b), but only two of them are independent, denoted as \(K_{\pm} \equiv (K_{x, K_{y}})\) within the first quadrant Brillouin zone \((0 < K_{x, K_{y}} < \pi/2)\) and \(K_{-} = \pi - K_{+} > \pi/2\). The other six nodes are merely the reflection images of \(K_{\pm}\) and can be neglected for the moment. So when the gapped quasiparticle states are neglected, the low-energy effective Hamiltonian contains two nodal valleys that can be obtained by expanding \(H_{\pm}(k) = (\xi_{+}\pm_{k}p_{x} + \Delta_{k}p_{y})\sigma_{0}\) around the nodal points \(K_{\pm}\). We thus have

\[
H_{\text{eff}}(K_{\pm} + q) = (\pm\nu_{1}q_{x} + v_{1}q_{y})\sigma_{0} \equiv (h_{\pm}(q) \cdot \rho)\sigma_{0},
\]

which is written as two copies of a two-dimensional Dirac Hamiltonian with canonical coordinates \(q_{\pm} \equiv q_{x} \pm q_{y}\) and two characteristic velocities: \(v_{1} = -2\Delta_{d}\sin K_{+}\) and

\[
\nu_{1} = -2(t'x + 2t''x)\sin 2K_{+} - \frac{4(tx + \kappa)^{2}\sin 2K_{+}}{\sqrt{m_{s}^{2} + 16(tx + \kappa)^{2}\cos^{2} K_{+}}},
\]

This two-dimensional Dirac-type Hamiltonian resembles a “magnetic field” \(h_{\pm}(q)\) coupling the Nambu spinor in the momentum space \((h_{\pm}^{x} = v_{1}q_{x}h_{\pm}^{y} = 0, h_{\pm}^{y} = \pm v_{3}q_{y})\). The magnitude of the “magnetic field” \(|h_{\pm}(q)|\) determines the energy spectrum for the low-energy excitations, while its unit direction \(n_{\pm}(q) = h_{\pm}(q)/|h_{\pm}(q)|\) is responsible for pinning the ground state. Moreover, the nodes \(K_{\pm}\) turn out to be the core of vortices, whose vorticity can be calculated by the topological winding number

\[
w_{\pm} = 2\int_{\mathbb{R}^{2}} \frac{d\mathbf{q}}{2\pi} \left[ n_{\pm}^{x}(q)\nabla_{\mathbf{q}} n_{\pm}^{x}(q) - n_{\pm}^{y}(q)\nabla_{\mathbf{q}} n_{\pm}^{y}(q) \right].
\]
Topological phase transition from nodal to nodeless $d$-wave superconductivity etc.

where the winding number is multiplied by 2 due to spin degeneracy. It turns out $w\pm = \pm 2$. The vortices cause the ground-state pairing wave function to experience a nonzero Berry phase for any closed path surrounding each node in the momentum space. And the lower-dimensional model confined to this loop would be fully gapped and topologically nontrivial. Consequently, the vorticity implies a nontrivial topology of the nodal $d$-wave SC, which is supposed to manifest in the edges due to bulk edge correspondence.

For each fixed momentum $k_{(1)}$ along $(1,1)$, the system is effectively a one-dimensional chain along $(1,1)$. The 1D chain that avoids the nodes is always fully gapped and characterized by the topological winding number given by eq. (12). Any two chains whose $k_{(1)}$ embrace the projection of $K_{\pm}$ must have their topological winding number differ by $w_{\pm} = \pm 1$, therefore at least one of them is topologically nontrivial and would show robust edge modes when $(1,1)$ boundary turns open. Then we performed exact diagonalization to the system in cylinder geometry with $(1,1)$ open edges, and compare it with the bulk spectrum given by system of closed boundary. As shown in fig. 4, the spectrum with $(1,1)$ open edges is nothing but the spectrum of the closed system being projected onto the $(1,1)$ edges, except that in the nodal $d$-wave SC phase there appear two additional in-gap edge modes with $k_{(1)}$ residing on the interval between projection of $K_{\pm}$ (the corresponding dispersion shown by the magenta color curve in fig. 4(a)). The in-gap edge modes are topologically protected, which is analogous to the necessary appearance of a Fermi arc on the surface of a three-dimensional Weyl semi-metal. Notice that the $(1,0)$ or $(0,1)$ surface edges do not show edge bound states because the vortices and anti-vortices would collapse upon projection onto those edges. Finally, we mention that it is the AF field that accounts for the energy splitting of the topologically protected edge states away from the exact zero energy of the $d_{x^2-y^2}$ wave SC in the absence of AF studied by Wang and Lee [24].

It is necessary to investigate the robustness of the bulk gap nodes being subject to all possible perturbations. The low-energy effective Hamiltonian (11) is a two-dimensional massless Dirac Hamiltonian though it describes the Bogoliubov excitations rather than the usual $U(1)$ fermionic excitations. To gap out the nodes is equivalent to endowing extra-mass terms on the massless model. Regardless of the symmetry requirements, all the available mass terms are enumerated as the following pairing terms:

$$
\begin{align*}
m_{1}\Psi_{\pm,+}^{\dagger}k\sigma_0i\rho_y\Psi_{\pm,-k}, \\
m_{1}\Psi_{\pm,+}^{\dagger}k\sigma_xi\rho_y\Psi_{\pm,-k}, \\
m_{2}\Psi_{\pm,+}^{\dagger}k\sigma_y\rho_y\Psi_{\pm,-k}, \\
m_{3}\Psi_{\pm,+}^{\dagger}k\sigma_zi\rho_y\Psi_{\pm,-k}.
\end{align*}
$$

The first mass term represents the extra singlet pairing that differs from the existing $d$-wave pairing field by $\pi/2$ phase, transforming the pure $d$-wave pairing symmetry into mixed pairing symmetry, e.g., $(d_{x^2-y^2} + i\sigma_y)$, $(d_{x^2-y^2} + i\sigma_z + i\sigma_y)$, $(d_{x^2-y^2} + i\sigma_z + i\sigma_y)$. However, the mixed pairing phase inevitably breaks the adjoint $T$ symmetry. The other three mass terms stand for triplet pairing channels without the mirror symmetries, which are not energetically favorable in the AF spin superexchange coupling of the $t$-$J$ model. So the only realistic possible mass term that can directly gap out the nodes comes from the first extra-mass term, which is nevertheless forbidden by the adjoint $T$ symmetry. In this sense, the nodal $d$-wave SC phase is protected by $T$.

Moreover, the topology of the nodal $d$-wave SC also manifests in its weak-pairing nature. The low-energy sector has a small Fermi pocket around the nodal points $K_{\pm}$.
and its pairing ground state can be expressed as

$$|\Omega\rangle \propto \exp \left( \sum_k g_k \psi_{k+}^\dagger \psi_{k-}^\dagger \right) |\text{FS}\rangle,$$

which displays a string of poles along the nodal direction inside the pocket,

$$g_k = \frac{1 - n_+^2(k)}{n_-^2(k)} \propto \frac{1}{q_-}.$$  \hspace{1cm} (15)

This result implies a long tail of the pairing function in real space, indicating the weak-pairing nature of the nodal d-wave SC. Therefore, the nodal d-wave SC is a weak-pairing topological superconducting phase due to the adjoint symmetry $\hat{T}$. In contrast, in the nodeless d-wave SC the Fermi pockets around $K_{\pm}$ and its equivalent points are absent, and the pairing function in the momentum space is analytical. These properties suggest that the nodeless d-wave SC is a strong-pairing phase corresponding to BEC limit and thus topologically trivial.

**Topological phase transition.** – In the presence of the symmetry $\hat{T}$, the nodes in the nodal d-wave phase are protected. The only way to kill the nodes is to let the vortex–anti-vortex pairs annihilate each other, which can be done by gradually increasing the AF field or decreasing the dopant concentration. The phase transition from weak-pairing nodal d-wave SC to strong-pairing nodeless d-wave SC is characterized by two nodes with opposite vorticity merging together at the point $(\pi/2, \pi/2)$.

To reveal the detailed features of the phase transition, we have to consider the low-energy excitations near the critical point. Since the effective Hamiltonian is composed of even functions in the Brillouin zone, we can only consider one quadrant Brillouin zone and the other areas of the Brillouin zone are connected by reflections. By expanding $H_+(k) = (\xi_+(k)\rho_+ + \Delta_k\rho_+)\sigma_0$ around $K = (\pi/2, \pi/2)$, the low-energy effective Hamiltonian can be derived as

$$H_v(K + q) = (Aq_+^2 - \mu') \rho_+ \sigma_0 + vq_- \rho_- \sigma_0 \equiv (h_v(q) \cdot \rho)\sigma_0,$$  \hspace{1cm} (16)

where

$$\mu' = \mu - m_s + 4t''x, \quad v = -2\Delta_d, \quad A = \frac{2(t_x + \kappa)^2}{m_s} + (t' + 2t''x);$$

$$h_v^x = vq_-, \quad h_v^y = 0, \quad h_v^z = Aq_+^2 - \mu'.$$

This quasiparticle spectrum has a linear dispersion along the direction $k_x = \pi - k_y$, but quadratic dispersion along the nodal direction $k_x = k_y$, as shown in fig. 5. It is clear that the effective chemical potential $\mu' > 0$ stands for the nodal d-wave phase, while $\mu' < 0$ is for the nodeless d-wave phase, so the critical point is characterized by the effective chemical potential $\mu' = 0$. In the direction $k_x = \pi - k_y$, the phase transition can be viewed as a topological phase transition from negative mass to positive mass and classified by the $Z_2$ quantum number. At the critical point, the low-energy excitations are double-faced, because of the fact that the low-energy excitations of the weak-pairing phase have a massless Dirac spectrum in all directions while the strong-pairing phase shows a nonrelativistic spectrum.

**Discussion and conclusion.** – So far we have discussed the influence of an external AF field on the nodal d-wave SC. It is important to compare the results with those in the absence of the AF field, where the nodal d-wave SC has four nodes connected by the mirror symmetries $M_x$ and $M_y$ and their mirror partners are shown to carry opposite vorticity [24]. The weak AF field induces the $Q \equiv (\pi, \pi)$ vector scattering, and creates a copy for all four nodes. For instance, the node at $K_+ - Q$ induces a node at $K_-$, whose vorticity differs from the original node because the pairing field changes sign under the AF vector $Q$, namely, $\Delta_{K+Q} = -\Delta_K$. The weak AF field does not completely destroy the Fermi pocket that enters into the low-energy sector by crossing the pairing nodal line, and the number of nodes within magnetic Brillouin zone is still four. Therefore, the nodal d-wave SC phase without the AF field is topologically the same phase as that in the presence of a weak AF field, where nodes are protected by $\hat{T}$ symmetry. Only a strong enough AF field would drive the vortex–anti-vortex pairs to annihilate, resulting in the nodeless d-wave SC without breaking $\hat{T}$ symmetry.

On the other hand, it is necessary to address the possibility to have a full-gapped topological SC from the symmetry-protected nodal d-wave SC. Similar to the Haldane’s approach to realizing a nontrivial Chern insulator by gapping out the two-dimensional graphene system [25], it is worth noticing that the gap nodes in the nodal d-wave SC can be directly gapped out by breaking $\hat{T}$ symmetry, which can potentially lead to a nontrivial topological fully gapped SC. There can be two drastically different
ways to introduce mixed singlet pairing channels, depending
upon whether the sign of the mass endowed upon the
two inequivalent nodes is the same or the opposite. In-
truding the masses \( m_\pm \Psi_{\pm, k} \) with the same sign
\((m_+ = m_-)\) would bring the pair of vortex–anti-vortex
into merons that would cancel each other, leading to a
trivial full gapped \((d_{x^2-y^2} + is)\) SC. On the other hand,
introducing masses with the opposite sign \( m_+ = -m_-\)
would drive the vortex–anti-vortex pair into a meron and
an anti-meron which together form a skyrmion, lead-
ing to a topologically full gapped \((d_{x^2-y^2} + is_{x^2+y^2})\) SC.
Actually, the extended \( s\)-wave pairing plays exactly the role
of this nontrivial mass term in the low-energy Dirac-like
Bogoliubov excitations of the nodal \( d\)-wave SC, because
its nodal line is along the direction \( k_x = \pi - k_y \). As a
result, with the first quadrant Brillouin zone contributing
one unit of Chern number, we have found that the weak
pairing \((d_{x^2-y^2} + is_{x^2+y^2})\) SC, as the valley symmetry-
protected topological superconductor, can be realized in
the hole-doped cuprates \([26]\).

In summary, we have developed a unified theory to un-
derstand both nodal and nodeless SC observed in the
electron-doped cuprates by introducing an external AF
field into the two-dimensional \( t-J \) model. Within the slave-
boson mean-field approximation, the \( d\)-wave pairing sym-
metry is the most energetically favorable. In the nodal
\( d\)-wave SC phase, the nodes are protected by the product
of time-reversal and unit lattice translation symmetries.
By increasing the external AF field or decreasing the
doping concentration, the nodes with opposite vorticity anni-
hilate and the nodeless \( d\)-wave SC phase emerges.

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