Neutrino propagation in a random magnetic field

Sarira Sahu\textsuperscript{1}

Theory Group, Physical Research Laboratory,
Navrangpura, Ahmedabad-380 009
India

Abstract

The active-sterile neutrino conversion probability is calculated for neutrino propagating in a medium in the presence of random magnetic field fluctuations. Necessary condition for the probability to be positive definite is obtained. Using this necessary condition we put constraint on the neutrino magnetic moment from active-sterile electron neutrino conversion in the early universe hot plasma and in supernova.
1 Introduction

Neutrino propagation in the presence of a strong magnetic field is of great interest from the point of view of astrophysics as well as early universe hot plasma\cite{1}. The presence of primordial magnetic fields ($B \simeq 10^{-6} \text{G}$) over galactic scales, when extrapolated back can give very large fields and these large fields might have affected the particle interactions in the early universe\cite{2}. It is believed that the primordial plasma was chaotic in nature and the magnetic flux lines moving along with this plasma might get mixed up creating randomness in the fields itself. It is also believed that the magnetic field inside a newly born neutron star is huge and random in nature\cite{3, 4, 5, 6}. This strong random magnetic fields can influence the neutrino propagation as well as their conversion in the early universe hot plasma and in a supernova medium\cite{2, 4, 7, 9}. Neutrinos having magnetic moment or transition magnetic moment can flip their chirality and left handed neutrinos becomes right handed sterile neutrinos in the presence of an external magnetic field. These right handed neutrinos being sterile to weak interaction stream out from the sun or supernova core. The mechanism of helicity flipping in the presence of a magnetic field is one of the explanation for the supernova cooling mechanism or the solar neutrino deficit\cite{10}.

Recently Pastor et. al.\cite{8} have derived bounds on the transition magnetic moment of the Majorana neutrinos in the presence of random magnetic fields from the supernova energy loss as well as from nucleosynthesis. In this paper we discuss about the active-sterile electron neutrino conversion ($\nu_{eL} \to \nu_{eR}$) in the presence of a random magnetic field in early universe hot plasma and in supernova medium. From the positive definiteness of the average conversion probability we put constraint on the neutrino magnetic moment.

The paper is organized as follows: In Section 2, we have derived the average probability equation for the active sterile/active ($\nu_a \to \nu_x$) neutrino conversion in the presence of a random fluctuation over the constant background magnetic field. For this derivation we have used the simple delta correlation for the uncorrelated random magnetic field domains. We show that, the fluctuation in the transverse and longitudinal components of the magnetic field are mixed up. We obtain the solution for the probability equation and found the necessary condition for the conversion probability to be positive definite. We have considered the neutrino propagation in the early universe plasma and in the supernova medium. Assuming the magnetic fields in the early universe hot plasma and inside the core of a newly born neutron star to be strong and purely random in nature we put constraints on the magnetic moment for active sterile neutrino conversion, which are discussed in Section 3 and 4 respectively. In discussion Section 5, we briefly summarise our results and discuss
about the further improvement of the bound on the neutrino magnetic moment.

2 Neutrino propagation

The wave equation for the propagation of two neutrinos, one active and one sterile (or active) in a plasma in the presence of a magnetic field is given by

\[
\frac{i}{\hbar} \frac{d}{dt} \begin{pmatrix} \nu_a \\ \nu_x \end{pmatrix} = \begin{pmatrix} H_{aa} & H_{ax} \\ H_{xa} & H_{xx} \end{pmatrix} \begin{pmatrix} \nu_a \\ \nu_x \end{pmatrix}
\]  

(1)

where \( x = s, b \) (sterile, active) and \( b = e, \mu, \tau \). In the evolution eqn. (1) the diagonal components are \( H_{aa} = V_{\text{vec}} - \Delta + V_{\text{axial}} \) and \( H_{xx} = 0 \) and the off-diagonal entries \( H_{ax} = H_{xa} = \mu_B \perp (t) \). The \( \Delta \) is given as \( \Delta = \cos(2\theta) \frac{(m_2^2 - m_1^2)}{2E} \) and vanishes for degenerate Majorana neutrino and \( E \) is the neutrino energy. For Dirac neutrino \( \Delta = 0 \) and \( \mu \) corresponds to its diagonal magnetic moment. On the other hand for Majorana neutrino \( \mu \) is the transition magnetic moment. \( V_{\text{vec}} \) is the difference of neutrino vector interaction potentials, \( V_{\text{vec}} = V_{\nu a} - V_{\nu x} \). Here we consider neutrinos without mixing, which implies \( \cos(2\theta) = 1 \). The axial vector potential \( V_{\text{axial}} = \mu_{\text{eff}} kB/\hbar \) is generated by the mean axial vector current of charged leptons in an external magnetic field [4, 11, 12].

Let us define the functions \( R = Re(< \nu_a^* \nu_x >) \) and \( I = Im(< \nu_a^* \nu_x >) \). Then using these in eqn. (1) we obtain the following set of equations:

\[
\frac{dP(t)}{dt} = -2H_{ax}(t)I(t),
\]

(2)

\[
\frac{dI(t)}{dt} = H_{aa}(t)R(t) + H_{ax}(t)(2P(t) - 1),
\]

(3)

and

\[
\frac{dR(t)}{dt} = -H_{aa}(t)I(t).
\]

(4)

The function \( P(t) \) is the neutrino conversion probability \( P_{\nu_a \rightarrow \nu_x} (t) \). The magnetic field can be written as \( B(t) = B_0 + \tilde{B}(t) \), where \( B_0 \) is the constant background field and \( \tilde{B}(t) \) is the random fluctuation over it. Putting this in \( H_{aa}(t) \) and \( H_{ax}(t) \) we get

\[
H_{aa}(t) = H_{aa}(0) + \tilde{H}_{aa}(t) = (V_{\text{vec}} - \Delta + \mu_{\text{eff}} B_{\parallel 0}) + \mu_{\text{eff}} \tilde{B}_{\parallel}(t)
\]

(5)

and

\[
H_{ax}(t) = H_{ax}(0) + \tilde{H}_{ax}(t) = \mu B_{\perp 0} + \mu \tilde{B}_{\perp}(t).
\]

(6)

For the neutrino conversion length greater than the domain size \( (l_{\text{conv}} >> L_0) \) one can average the the equations (2), (3) and (4) over the random magnetic field.
distribution. Let us define the average of the functions \( < P(t) > = \mathcal{P}(t), < R(t) > = \mathcal{R}(t) \) and \( < I(t) > = \mathcal{I}(t) \). Using these in eqns. (3) and (4) we obtain

\[
\frac{d\mathcal{I}(t)}{dt} = < H_{ax}(t)(2P(t) - 1) > + < H_{aa}(t)R(t) >
\]

\[
= H_{ax}(0) < (2P(t) - 1) > + < \tilde{H}_{ax}(2P(t) - 1) >
\]

\[
+ H_{aa}(0) < R(t) > + < \tilde{H}_{aa}(t)R(t) > ,
\]

(7)

and

\[
\frac{d\mathcal{R}(t)}{dt} = -H_{aa}(0) < I(t) > - < \tilde{H}_{aa}(t)I(t) >
\]

(8)

respectively. The magnetic field in different domains is randomly oriented with respect to the neutrino propagation direction. So the neutrino conversion probability depends on the root mean square (rms) value of the random magnetic field. With the use of the delta correlation for uncorrelated magnetic field domain of size \( L_0 \), the average of the random magnetic field is \([13, 14, 15]\),

\[
< B_\parallel(t) >= < B_\perp(t) >= < B_\parallel(t)B_\perp(t) >= 0,
\]

(9)

\[
< B_\parallel(t)B_{j\parallel}(t_1) >= < B_\parallel^2 > \delta_{ij}L_0\delta(t - t_1),
\]

(10)

and

\[
< B_{i\perp}(t)B_{j\perp}(t_1) >= < B_\perp^2 > \delta_{ij}L_0\delta(t - t_1).
\]

(11)

The rms value of the averaged magnetic field is given as \( B_{\text{rms}} = \sqrt{< B^2 >} \). Neglecting the higher order correlation of \( H_{aa}(t) \) and \( H_{ax}(t) \) with \( I(t) \), \( R(t) \) and \( P(t) \) and using the above equations in eqn. (2) to (4) we obtain

\[
\frac{d\mathcal{I}(t)}{dt} = H_{aa}(0)\mathcal{R}(t) + H_{ax}(0)(2\mathcal{P}(t) - 1) - 2(\Gamma_\perp + \Gamma_\parallel)\mathcal{I}(t),
\]

(12)

\[
\frac{d\mathcal{R}(t)}{dt} = -H_{aa}(0)\mathcal{I}(t) - 2\Gamma_\parallel\mathcal{R}(t),
\]

(13)

and

\[
\frac{d\mathcal{P}(t)}{dt} = -2H_{ax}(0)\mathcal{I}(t) - \Gamma_\perp (2\mathcal{P}(t) - 1).
\]

(14)

For convenience we define

\[
\mathcal{I}(t) = e^{-2(\Gamma_\parallel + \Gamma_\perp)t}\mathcal{I}_1(t),
\]

(15)

and

\[
\mathcal{R}(t) = e^{-2\Gamma_\parallel t}\mathcal{R}_1(t),
\]

(16)

where

\[
\Gamma_\perp = \frac{4}{3}\mu^2 < B^2 > L_0,
\]

(17)
and
\[ \Gamma_\parallel = \frac{1}{6} \mu^2_{\text{eff}} < B^2 > L_0. \] (18)

\( \Gamma_\perp \) and \( \Gamma_\parallel \) are the transverse and longitudinal magnetic field damping parameters. Putting these in eqn.(14) and differentiating two times with respect to \( t \) we obtain
\[
\frac{d^3 \mathcal{P}(t)}{dt^3} + 4 \left( \Gamma_\perp + \Gamma_\parallel \right) \frac{d^2 \mathcal{P}(t)}{dt^2} + 4 \left( 3 \Gamma_\perp \Gamma_\parallel + \Gamma_\parallel^2 + \Gamma_\perp^2 + H^2_{ax}(0) + \frac{H^2_{aa}(0)}{4} \right) \frac{d \mathcal{P}(t)}{dt} \\
+ 8 \left( \Gamma_\perp^2 \Gamma_\parallel + \Gamma_\perp \Gamma_\parallel^2 + H^2_{ax}(0) \Gamma_\parallel + \frac{H^2_{aa}(0) \Gamma_\perp}{4} \right) \mathcal{P}(t) \\
- 4 \left( \Gamma_\perp^2 \Gamma_\parallel + \Gamma_\perp \Gamma_\parallel^2 + H^2_{ax}(0) \Gamma_\parallel + \frac{H^2_{aa}(0) \Gamma_\perp}{4} \right) = 0, \tag{19}
\]

with the boundary conditions \( \mathcal{P}(t)/t=0 = 0 \), \( \frac{d \mathcal{P}(t)}{dt}/t=0 = \Gamma_\perp \) and \( \frac{d^2 \mathcal{P}(t)}{dt^2}/t=0 = 2H^2_{ax}(0) - 2\Gamma^2_\perp \). Switching off the damping terms in eqn.(19), will give MSW type solution with magnetic field[7, 10, 16]. For \( H_{aa}(0) = H_{ax}(0) = \Gamma_\parallel = 0 \), the eqn.(19) reduces to
\[
\frac{d^2 \mathcal{P}(t)}{dt^2} + 4\Gamma_\perp \frac{d \mathcal{P}(t)}{dt} + 4\Gamma^2_\perp \mathcal{P}(t) - 2\Gamma^2_\perp = 0, \tag{20}
\]

and it has the same solution as shown by Pastor et. al.[8]. In this the effect of strong random magnetic field on the neutrino transition magnetic moment is studied both in the early universe hot plasma and in supernova. The eqn.(19) can be written in a simplified form as
\[
\frac{d^3 \mathcal{P}(t)}{dt^3} + A_0 \frac{d^2 \mathcal{P}(t)}{dt^2} + B_0 \frac{d \mathcal{P}(t)}{dt} + 2C_0 \mathcal{P}(t) - C_0 = 0, \tag{21}
\]

where the quantities \( A_0, B_0 \) and \( C_0 \) are
\[
A_0 = 4 \left( \Gamma_\perp + \Gamma_\parallel \right), \tag{22}
\]
\[
B_0 = 4 \left( 3 \Gamma_\perp \Gamma_\parallel + \Gamma_\perp^2 + \Gamma_\parallel^2 + H^2_{ax}(0) + \frac{H^2_{aa}(0) \Gamma_\perp}{4} \right), \tag{23}
\]

and
\[
C_0 = 4 \left( \Gamma_\perp^2 \Gamma_\parallel + \Gamma_\perp \Gamma_\parallel^2 + H^2_{ax}(0) \Gamma_\parallel + \frac{H^2_{aa}(0) \Gamma_\perp}{4} \right) \tag{24}
\]

respectively. Eqns.(23) and (24) shows that the damping terms are mixed up among themselves and also with the terms \( H_{aa}(0) \) and \( H_{ax}(0) \). The solution of eqn.(21) is
\[
\mathcal{P}(t) = y(t) + \frac{1}{2}, \tag{25}
\]

where \( y(t) \) is given as
\[
y(t) = e^{-(\Gamma_\perp + A_0)t} \left[ A_1 \left\{ e^{\frac{\Gamma_\parallel}{B_0}t} - \cos(Z_4 t) - \frac{3 Z_1}{2 Z_4} \sin(Z_4 t) \right\} \\
- \frac{\cos(Z_4 t)}{2} + \left( \frac{\Gamma_\perp - \frac{A_0}{2} - \frac{Z_4}{4} \right) \sin(Z_4 t) \right]. \tag{26}
\]
The coefficient $A_1$ is given as

$$A_1 = -\frac{(A_0^2 - 12A_0\Gamma_+ + 36\Gamma_+^2 - 36H_{ax}^2(0) + 3A_0Z_1 - 18\Gamma_+Z_1 + \frac{9}{4}Z_1^2 + 9Z_4^2)}{(22Z_1^2 + 18Z_4^2)},$$

(27)

and $Z_1$ and $Z_4$ are

$$Z_1 = \left( -\frac{q}{2} + \sqrt{\left( \frac{p}{3} \right)^3 + \left( \frac{q}{2} \right)^2} \right)^{\frac{1}{3}} - \frac{\left( \frac{q}{2} \right)}{-\frac{q}{2} + \sqrt{\left( \frac{p}{3} \right)^3 + \left( \frac{q}{2} \right)^2}}^{\frac{1}{3}},$$

(28)

and

$$Z_4 = \frac{\sqrt{3}}{2} \left[ \frac{\left( \frac{p}{3} \right)}{-\frac{q}{2} + \sqrt{\left( \frac{p}{3} \right)^3 + \left( \frac{q}{2} \right)^2}}^{\frac{1}{3}} + \left( -\frac{q}{2} + \sqrt{\left( \frac{p}{3} \right)^3 + \left( \frac{q}{2} \right)^2} \right)^{\frac{1}{3}} \right],$$

(29)

respectively. The quantities $p$ and $q$ are defined as

$$p = (B_0 - \frac{A_0^2}{3}),$$

(30)

and

$$q = (2C_0 - \frac{A_0B_0}{3} + \frac{2}{27}A_0^3).$$

(31)

The solution eqn.(25) for the average neutrino conversion probability $P(t)$ is very complicated. So here we will consider condition for the existence of the solution rather than going into the details of it. Since the probability is positive definite ($0 \leq P(t) \leq 1$), the following condition,

$$-\frac{q}{2} + \sqrt{\left( \frac{p}{3} \right)^3 + \left( \frac{q}{2} \right)^2} > 0,$$

(32)

has to satisfy, otherwise $Z_1$ and $Z_4$ will be complex so also $P(t)$. Then putting values of $p$ and $q$ in eqn.(32) we obtain the condition

$$4H_{ax}^2(0) + H_{aa}^2(0) > \frac{4}{3}(\Gamma_+^2 + \Gamma_\parallel^2 - \Gamma_+\Gamma_\parallel).$$

(33)

Thus for the average neutrino conversion probability to be positive definite, the above condition has to satisfy, irrespective of the form of the neutrino potential and the magnetic field. Let us consider the neutrino propagation in the early universe hot plasma and the core of the newly formed neutron star, where the above condition can be satisfied.
# 3 Early Universe hot plasma

In the early universe when the temperature $T \gg 1$ MeV, all the particles are in thermal equilibrium\(^{[17]}\). In the early universe the magnetic flux lines are moving along with the hot plasma. Because of the chaotic motion of the plasma, the flux lines will be mixed up and twisted thus creating the randomness in the magnetic field. So the constant background field $B_0$ must be very small compared to the random fluctuation term. Thus we can safely assume $B_0 \simeq 0$ and then $H_{ax}(0) \simeq 0$ and $\mu_{eff}B_{\parallel 0} \simeq 0$. For small particle-anti particle asymmetry in the early universe hot plasma the axial vector potential contribution can be very small as in a relativistic plasma the charged lepton/anti-lepton masses and their chemical potentials are small compared to the kinetic energy term, thus the factor $\mu_{eff}$ will also be small\(^{[4, 11]}\). So we can neglect the longitudinal damping term for the magnetic field. Then eqn.\((33)\) will be

$$H_{aa}(0) > \frac{2}{\sqrt{3}} \Gamma_\perp.$$  \(34\)

The vector potential for a neutrino in the early universe hot plasma is\(^{[18]}\)

$$V_{vec} = \sqrt{2}G_F n_\gamma(T) \left[ L - A \frac{T^2}{M^2_W} \right]. \quad (35)$$

Here $n_\gamma(T) \approx 0.244T^3$ is the photon number density and $A \approx 55$. For temperature $T \gg m_e$ the second non-local term is greater than the first term for very small particle-anti particle asymmetry and the second term is

$$V_{vec} \approx -3.45 \times 10^{-20} \left( \frac{T}{MeV} \right)^5 MeV, \quad (36)$$

for electron neutrino. For $\nu_{eL} \rightarrow \nu_{eR}$ process we assume that the right handed neutrino produced is sterile and decouple from the system.

Irrespective of its origin, the primordial magnetic field had a very large value in the early universe. We assume that the primordial plasma consists of magnetic domain structure with a size $L_0$ and the magnetic field is uniform and constant within each domain and the field in different domains are randomly aligned\(^{[4, 13]}\). For homogeneous magnetic fields, the flux conservation indicates that $B \propto T^2$. But detailed structure of random magnetic field profile in the early universe hot plasma will depend on the complicated nature of the magneto-hydrodynamic equations\(^{[19]}\). For the root mean square field $B_{rms} = \sqrt{\langle B^2 \rangle}$, averaged over a volume $L^3 \gg L_0^3$ we assume a power law behavior\(^{[20, 21]}\),

$$B_{rms} = B_n \left( \frac{T}{T_0} \right)^2 \left( \frac{L_0}{L} \right)^n,$$  \(37\)
where $T_0$ is the temperature at some reference epoch and $B_n$ is the corresponding field strength within a domain of size $L_0$. The maximal scale we have chosen is the horizon length $L = l_H = M_{pl}/T^2$, where $M_{pl} = 1.22 \times 10^{19}$ GeV is the Planck mass. We take here $B_n T_0^2 L_0^n$ to be constant. If the primordial magnetic fields are to be the seed fields for the galactic dynamo, then it should survive till the recombination epoch and this leads to a minimal domain size

$$L_0 \geq L_0^{\text{min}} \simeq 10^3 \text{ cm} \times \left( \frac{\text{MeV}}{T} \right).$$

(38)

Here we take $B_n = 10^{24}$ Gauss and $T_0 = T_{EW} = 10^5$ MeV the electroweak phase transition temperature\textsuperscript{[13]}. Putting all these in eqn.(34) we obtain

$$\mu \leq \frac{1.15 \times 10^{-15+12n}}{(4.15)^n} \left( \frac{T}{\text{MeV}} \right)^{(1-n)} \left( \frac{L_0}{\text{cm}} \right)^{-\left(\frac{1}{2}+n\right)}. $$

(39)

The index parameter $n = 0$ corresponds to the uniform magnetic field, which is not physically very likely in the early universe hot plasma. For the random magnetic field along the neutrino trajectory the index parameter would be $n = 1/2$ and $n = 3/2$ corresponds to 3-dimensional elementary cells\textsuperscript{[4, 8, 13]}. As the magnetic field is random in nature we take $n = 1/2$. For QCD phase transition temperature $T = T_{QCD} \simeq 200$ MeV we obtain $\mu \leq 8 \times 10^{-12} \mu_B$.

4 Supernova Core

Now let us consider the neutrino propagation in the supernova medium. As shown in eqn.(3) and eqn.(4) we have $H_{aa}(0) = (V_{vec} - \Delta + \mu_{eff} B_{\parallel 0})$ and $H_{ax}(0) = \mu B_{\perp 0}$. Thomson and Dunkan\textsuperscript{[5]} have shown that, magnetic fields as strong as $10^{14}$ to $10^{16}$ Gauss might be generated inside the core of the supernova due to a small scale dynamo mechanism. If these fields are generated after core collapse, then it could be viewed as random superposition of many small dipole of size $L_0 \sim 1$ Km. So we neglect here the $B_0$ term in the diagonal and non-diagonal parts. Thus $H_{aa}(0) = (V_{vec} - \Delta)$ and $H_{ax}(0) = 0$. For $\nu_eL \rightarrow \nu_eR$ the vector potential experience by neutrino in the supernova medium is

$$V_{vec} = 4 \times 10^{-6} \rho_{14} f(Y_e) \text{ MeV},$$

(40)

where $f(Y_e) = (3Y_e - 1)$ is the electron neutrino abundance factor and $\rho_{14}$ is the density in units of $10^{14}$ $g/cm^3$. The right-handed electron neutrino being sterile stream away from the supernova core. The $\Delta = 5 \times 10^{-15}/E_{100} (\Delta m^2/eV^2)$ MeV, where the neutrino energy $E_{100}$ is in units of 100 MeV. We can see that $\Delta << V_{vec}$ even for large $\Delta m^2$. Inside the neutron star core, there are less number of electrons.
so the damping term $\Gamma_\parallel$ can be very small compared to the vector potential or the transverse damping term $\Gamma_\perp$. Assuming $\Gamma_\parallel$ to be very small inside the core we have $H_{aa}(0) \simeq V_{vec}$. Then eqn. (33) is reduced to the same condition as in the early universe hot plasma $H_{aa}(0) > 2\Gamma_\perp/\sqrt{3}$. Inside the core the damping term is

$$\Gamma_\perp = 2.2 \times 10^{-18} \mu_0^2 \left( \frac{B_{rms}^2}{G^2} \right) \left( \frac{L_0}{cm} \right) MeV,$$

(41)

where $\mu_0$ is in units of $\mu_B$ the Bohr magneton. Putting the values of $H_{aa}(0)$ and $\Gamma_\perp$ in eqn. (34) we obtain for the magnetic moment

$$\mu \leq 1.25 \times 10^6 \left( \rho_{14} |f(Y_e)| \right)^{1/2} \left( \frac{B_{rms}}{G} \right) \left( \frac{L_0}{cm} \right).$$

(42)

For $B_{rms} \sim 10^{16}$ Gauss, $L_0 \sim 1$ Km, $\rho_{14} \sim 8$ and the electron abundance factor $Y_e \sim 0.3$ inside the neutron star, the magnetic moment comes out to be $\mu \leq 3.5 \times 10^{-13} \mu_B$. For smaller value of the magnetic field, the magnetic moment will be larger.

5 Discussion

Assuming the magnetic field has a random fluctuation over the mean value, we have derived the average probability equation for neutrino conversion/spin precession in the magnetized plasma. As a consequence of the random fluctuation in the magnetic field the transverse and longitudinal magnetic field damping are getting mixed up. We have assume a delta correlation for the random magnetic field domains and obtain the solution for the probability equation. The definiteness of the probability is invalid if the condition in eqn. (33) is not satisfied. We assume the magnetic field to be purely random in nature, in the early universe hot plasma as well as in supernova medium (inside the newly born neutron star core) and consider the process $\nu_{eL} \rightarrow \nu_{eR}$. At the QCD phase transition temperature $T \simeq 200$ MeV we obtain the neutrino magnetic moment $\mu \leq 8 \times 10^{-12} \mu_B$. Inside the supernova core we consider the same process $\nu_{eL} \rightarrow \nu_{eR}$ and for $B_{rms} \simeq 10^{16}$ Gauss, obtain $\mu \leq 3.5 \times 10^{-13} \mu_B$. In this estimate we have neglected the contribution from the axial vector potential as well as the contribution due to the constant magnetic field in the plasma. We have also neglected the contribution from the longitudinal damping term. So inclusion of all these terms might improve the constraints on the magnetic moment. Apart from that the magnetic field profiles for the supernova and early universe are very much speculative, so the upper limit might change for other magnetic field profiles.

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