A Dynamic Scheduling Policy for a Network with Heterogeneous Time-Sensitive Traffic

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Abstract

In 5G and beyond systems, the notion of latency gets a great momentum in wireless connectivity as a metric for serving real-time communications requirements. However, in many applications, research has pointed out that latency could be inefficient to handle applications with data freshness requirements. Recently, the notion of Age of Information (AoI) that can capture the freshness of the data has attracted a lot of attention. In this work, we consider mixed traffic with time-sensitive users; a deadline-constrained user, and an AoI-oriented user. To develop an efficient scheduling policy, we cast a novel optimization problem formulation for minimizing the average AoI while satisfying the timely throughput constraints. The formulated problem is cast as a Constrained Markov Decision Process (CMDP).

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relax the constrained problem to an unconstrained Markov Decision Process (MDP) problem by utilizing Lyapunov optimization theory and it can be proved that it is solved per frame by applying backward dynamic programming algorithms with optimality guarantees. Simulation results show that the timely throughput constraints are satisfied while minimizing the average AoI. Also, simulation results show the convergence of the algorithm for different values of the weighted factor and the trade-off between the AoI and the timely throughput.

I. INTRODUCTION

With the advent of 5G communication networks, the metric of latency plays a vital role in wireless connectivity for addressing the requirements of real-time communications, such as autonomous vehicles, wireless industrial automation, environmental, and health monitoring, to name a few [1], [2]. In real-time communications, information is required to arrive at the destination within a certain period (deadline-constrained) due to stringent requirements in terms of latency, while in other cases, it is required to keep the information at the destination as fresh as possible. The notion of packets with deadlines is connected with the timely-throughput, that is the average number of successful packet deliveries before their deadline expiration [3]. Information “freshness” is captured by a new metric called Age of Information (AoI) [4], [5]. It was first introduced in [6], and it is defined as the time elapsed since the generation of the status update that was most recently received by a destination. Furthermore, time-sensitive applications with different requirements co-exist in the same network and share the same resources. Therefore, it is important to allocate the resources efficiently in order to satisfy the requirements of the heterogeneous traffic.

In order to enhance our understanding of such systems, we consider a system with an AoI-oriented user and a user with timely throughput requirements. Our goal is to minimize the average AoI while satisfying the timely throughput requirements. We consider both time-correlated channel model and independent and indentically distributed (i.i.d) channel model. We utilize tools from Lyapunov optimization theory in order to transform the initial Constrained Markov
Decision Process (CMDP) problem into a Markov Decision Process (MDP) problem, and we provide a dynamic programming algorithm that solves the problem optimally.

Systems with deadlines have been considered almost two decades ago [7]. An extensive survey that provides an overview of the mathematical tools that are used in the area of resource control for delay-sensitive networks can be found in [8]. Recently, there has been a renewed interest in studying the performance of systems with deadline-constrained traffic [9]–[13], especially due to the ongoing automation of traditional manufacturing and industrial practices under the fourth industrial revolution. Packets with deadlines are connected with the notion of timely throughput. Timely throughput is first introduced in [3], and it is defined as the average number of the successfully delivered packets before their deadlines expiration. In [3], the authors propose an algorithm that can satisfy any feasible timely throughput constraint. An extensive study is provided in [14]. The authors analyze the fundamental limits for networks with timely throughput constraints.

Real-time scheduling optimization for deadline-constrained traffic has been extensively studied in the literature. In [15], the authors provide a dynamic algorithm for minimizing the packet drop rate while satisfying the average power constraints. In [16], the authors consider a joint scheduling and power allocation problem for a network with real-time traffic, i.e., packets with deadlines, and non-real time traffic. Furthermore, a mixed type of traffic is considered in [17]. The authors consider a joint scheduling and power allocation problem for a network with deadline-constrained users and users with minimum throughput requirements. Furthermore, it has been often shown that it is natural to formulate this kind of problem as an MDP [18]–[20]. For example, in [19], the authors formulate the problem of minimizing the packet drop rate while providing queueing stability as an MDP problem. Several approximations and also a methodology are provided that can be applied to general problems of this kind. In [20], the authors propose an optimal dynamic programming algorithm for users with energy and deadline constraints. In addition, there are works that consider packets with deadlines in multihop network or in fog-based systems [21],
Recently, the optimization and control of average or peak AoI has been attracted a lot of attention for a plethora of scenarios \([23]–[32]\). There are two cases for generation of the status updates: i) status updates arrive randomly at the users \([23]\), \([24]\), ii) *generate-at-will* mechanism that allows the user to sample fresh data at will \([25]–[27]\). In \([23]\), \([24]\), the authors consider the problem of AoI minimization in single-hop networks with stochastic arrivals and they provide several scheduling policies. In \([25]\), the problem of AoI minimization with throughput constraints in a multi-user network is considered. In \([28]\), the problem of joint sampling and scheduling is considered for multisource systems. Furthermore, there is a line of works that considers transmission and sampling costs \([26]\), \([29]\). In these works, if the sampled information failed to be transmitted due to channel errors, it may be retransmitted in next slots to reduce the sampling cost. In \([30]\), the AoI minimization problem with energy constraints is considered. It is shown that the optimal policy is a mixture of two stationary deterministic policies. In \([27]\), \([31]\), \([32]\), the optimization of AoI in Internet of Things (IoT) and energy harvesting systems has been studied. In these works, the problems are formulated as MDPs and they are solved by using tools from dynamic programming and reinforcement learning.

Although there are many works that consider the AoI optimization or analysis, there are few works that consider AoI optimization in a system with heterogeneous traffic \([33]–[36]\). In \([33]\), the authors consider a wireless network with a AoI-oriented user and a user with random packet arrivals. The problem of AoI minimization under stability constraints is considered. In \([34]\), the authors provide the stochastic analysis to show the interplay between delay violation probability and average AoI. In \([35]\), expressions that show the interplay between the packet drop rate and the average AoI are provided in a shared multi-access channel with two users.

The work that is closer to our work is \([36]\). The authors in \([36]\) consider a wireless network including AoI-oriented users and deadline-constrained users. The goal is to minimize the average AoI while satisfying the timely throughput constraints. The authors in \([36]\) also consider that the
time is divided into frames and the frames into slots. However, they additionally assume that the AoI-oriented user can be scheduled in any time slot within the frame and the value of the AoI remains 1, if the transmission succeeds, during the whole frame. Furthermore, it is assumed that the channel remains fixed during a frame. On the contrary, in our work, we consider that AoI is 1 only when the AoI-oriented user transmits a packet successfully. Furthermore, the channel of a user can change from slot to slot unlike from frame to frame. These assumptions make the problem considered in our paper fundamentally different and more realistic.

In this work, we consider two users that send their information over an error-prone channel to a common receiver. The first user is AoI-oriented and the second user has timely throughput requirements. We consider two channel model cases: i) i.i.d. channels over time slots, ii) time-correlated channels. Our goal is to minimize the average AoI while satisfying the timely throughput requirements. The problem is formulated as a CMDP problem which is known to be a difficult problem to solve and standard approaches, such as the method of Lagrange multipliers, cannot be directly applied. To solve this problem, we first apply tools from Lyapunov optimization theory to transform the CMDP into an MDP. It is shown that the infinite CMDP can be reduced to an unconstrained weighted stochastic shortest path problem, i.e., a finite-horizon MDP, that is easier to be solved. Note that our approach can be applied in more general stochastic optimization problems of that type. The relaxed problem is then solved by invoking backward dynamic programming. In addition, simulation results show that considering a large enough threshold for the AoI will not affect the performance of the system because we observe the value of AoI never reaches this threshold. Therefore, the threshold makes the analysis easier without affecting the performance of the system. Furthermore, simulation results show that the optimal policy schedules the AoI user multiple times within a frame with some probability. That means that the optimal decision is not to schedule the AoI only at specific slots of the frame with high probability, e.g., at the beginning or the end of the frame. Instead, it is more beneficial to spread the scheduling time across all the slots within the frame.
II. SYSTEM MODEL

We consider two users transmitting their information in the form of packets to a single receiver over a wireless fading channel, as shown in Fig. 1. Let \( i \in \{1, 2\} \) denote the \( i \)th user of the system. Time is assumed to be slotted, and let \( t \in \mathbb{Z}_{\geq 0} \) denote the \( t \)th slot, where \( t \in \mathbb{Z}_{\geq 0} \) is the set of nonnegative integer numbers. We consider a centralized scheduler that decides every slot to schedule up to one user. Let \( u_i(t) \) denote the decision of the scheduler, where

\[
u_i(t) = \begin{cases} 1, & \text{if user } i \text{ is scheduled at time slot } t, \\ 0, & \text{otherwise,} \end{cases}
\]  

(1)

and \( \mathbf{u}(t) = [u_1(t) \ u_2(t)]^T \). Note that \( \sum_i u_i(t) \leq 1, \ \forall t \). Due to the wireless nature of the channels, we assume that a packet is successfully transmitted from user \( i \) to the receiver with some probability. Let \( d_i(t) \) denote the successful packet reception of user \( i \), given that \( u_i(t) = 1 \), where

\[
d_i(t) = \begin{cases} 1, & \text{successful packet reception for user } i, \\ 0, & \text{otherwise,} \end{cases}
\]  

(2)

and \( \mathbf{d}(t) = [d_1(t) \ d_2(t)]^T \).
User 1 is an AoI-oriented user who either samples and transmits fresh information to the receiver or remains silent depending on the scheduling policy. Let $A(t) \in \mathbb{Z}_{>0}$ associated with AoI of user 1 at the receiver. We assume that the value of the AoI is bounded by $A_{\text{max}}$. This assumption is considered for the following two reasons:

1) In practical applications, values of AoI that are larger than a threshold will not provide us additional information about the staleness of the packet, [26], [27], [31].

2) Assuming unbounded AoI will complicate significantly the solution of the optimization problem without giving us additional insights for the performance of the system.

The evolution of the AoI at the receiver is described as

$$A(t + 1) = \begin{cases} 1, \text{ successful packet transmission of user 1, i.e., } d_1(t) = 1, \\ \min\{A_{\text{max}}, A(t) + 1\}, \text{ otherwise.} \end{cases}$$

(3)

The time average AoI is defined as

$$\bar{A} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t} \mathbb{E}\{A(\tau)\},$$

(4)

where the expectation is with respect to the scheduling policy and the channel randomness. Note that we use a generate-at-will policy. Furthermore, since we do not consider sampling cost, user 1 does not retransmit a packet if the transmission fails. Instead, user 1 samples new information whenever it is scheduled in one of the following slots. We consider that the sampling and transmission process needs one time slot to be performed.

User 2 is deadline-constrained, and it includes packets that must be transmitted within a specific time frame, i.e., before a deadline. More specifically, we consider that $K$ packets arrive to the queue of user 2 every $T$ slots, where $K \leq T$. We consider that a packet needs one slot to be transmitted. The time between every packets arrival is a time frame of which the length is $T$ time slots. Let $m \in \mathbb{Z}_{\geq 0}$ denote the $m^{th}$ frame, and $t_m = mT$ be the first slot of frame $m$. An example with the first three time frames, and $T = 6$ slots, is shown in Fig. 2. The packets that arrive at the beginning of each frame $m$ must be transmitted before the end of the frame,
K packets arrive
1st frame 2nd frame 3rd frame ...

Fig. 2: A snapshot of for the first three frames with $T = 6$ slots.

i.e., $T$ slots after their arrival. At the end of the last slot of each frame, the remaining packets in the queue are dropped from the system. We also denote by $f_m(t)$ the time interval between the beginning of the current frame and slot $t$, i.e., $f_m(t) = t - \lfloor \frac{t}{T} \rfloor T$. Let $Q(t)$ denote the number of packets that are in the queue of user 2 in time slot $t$. The evolution of the queue is described as

$$Q(t + 1) = \max\{Q(t) - d_2(t), 0\} \mathbb{1}_{\{f_m(t) \neq 0\}} + K \mathbb{1}_{\{f_m(t) = 0\}} , \forall t. \tag{5}$$

A. Frame-based Timely Throughput Requirements

The timely throughput measures the average number of successful deliveries, i.e., the packets delivered before the deadline [3], [14]. Timely-throughput was proposed in [3] as an analytical metric for evaluating both throughput and Quality of Service for deadline-constrained traffic. In this work, we are interested in keeping frame-based timely throughput above a threshold, for user 2. The frame-based timely throughput is defined as

$$\lim_{M \to \infty} \frac{1}{M} \sum_{m=0}^{M} \left( \sum_{\tau=mT}^{(m+1)T-1} \mathbb{E} \{d_2(\tau)\} \right). \tag{6}$$

Our motivation for defining per frame average timely throughput is that it is not only important to serve as many packets is possible, but also to keep high QoS for every frame. In other words, to ensure a high average number of successful packets delivery before their deadline expiration, i.e., before the end of the frame. Consider a real-time video transmission where videos consist
Fig. 3: Two-state Markov chain for channel modeling that captures correlation in time.

of frames. We care to have high-quality video transmission, and also a smooth transmission of
the video from frame to frame which can be captured by ensuring that the value of the expression
in (6) is above a threshold.

B. Channel Model

In this work, we consider two cases for the channel model: 1) i.i.d channel over slots, 2) correlated channels over slots.

1) i.i.d channels: In the i.i.d case, the channels of user 1 and user 2 are considered as Bernoulli processes with success probabilities $p_1$ and $p_2$, respectively. Therefore, if we schedule user $i$, the probability of the successful transmission is $p_i$. The success or failure of a transmission does not depend on the channel state of the previous slot.

2) Correlated channels: The channel of each user $i$ is assumed to be a time-correlated fading channel and each one evolves as a two-state Gilbert-Elliot model. The evolution of the channel states can be modeled as a Markov chain as shown in Fig. 3. Let $h_i(t)$ denote the channel state of user $i$ at time slot $t$, which is modeled as a Markov chain with two states as shown in Fig. 3 and $\mathbf{h}(t) = [h_1(t) \ h_2(t)]^T$. “Bad” state represents deep fading of the channel and any transmission will fail. “Good” state represents mild fading of the channel and any transmission will succeed. The channel transition probabilities are given by $\Pr\{h_i(t+1) = 1|h_i(t) = 1\} = p_{11,i}$, $\Pr\{h_i(t+1) = 1|h_i(t) = 0\} = p_{01,i}$. We consider delayed channel sensing for both users.
More specifically, the channel state for each user \( i \) is known at the receiver only at the end of each slot. Therefore, at the beginning of each slot, the scheduler knows the channel state based on the previous Channel State Information (CSI) and with some probability.

### III. Problem Formulation

In this work, our target is to find a policy \( \pi \) that solves the following optimization problem

\[
\min_{\pi} \quad \bar{A}^\pi \tag{7a}
\]

subject to

\[
\lim_{M \to \infty} \frac{1}{M} \sum_{m=0}^{M} \left( \sum_{\tau=mT}^{(m+1)T-1} \mathbb{E}\{d_2^\pi(\tau)\} \right) \geq q, \tag{7b}
\]

where \( q \) is the minimum per frame average timely throughput requirements of user 2, \( 0 \leq q \leq K \), and \( \pi \) is the scheduling policy.

**Remark 1.** The optimization problem in (7) belongs to the general class of CMDPs, which are usually difficult to solve [37]. In [38], [39], authors utilize the Lagrange multiplier approach to treat power minimization for single-queue systems with an average delay constraint. Our approach that is described below, while similar to the Lagrange multiplier approach, can handle the stochastic nature of the shortest path problem over frames, which is otherwise hard to achieve. This specific characteristic of our problem makes the Lagrange multiplier approach inappropriate. Additionally, the use of virtual queues provide information about the stability of the overall system.

In order to proceed to the solution of the optimization problem, we consider two important slackness assumptions described below. We consider a class of stationary randomized policies denoted by \( \Omega \). A policy \( \omega \), that belongs in \( \Omega \), takes probabilistic decisions independently of the state of the queue of user 2 and the channel conditions.

**Assumption 1:** There is an \( \omega \)-only policy, denoted by \( \omega_1 \), that satisfies the following, over any
renewal frame:

\[
\mathbb{E} \left[ \sum_{\tau=mT}^{(m+1)T-1} A^{\omega_1}(\tau) \right] - A_{\text{opt}} = A_{\text{opt}}, \tag{8}
\]

\[
q - \mathbb{E} \left[ \sum_{\tau=mT}^{(m+1)T-1} d_{2}^{\omega_1}(\tau) \right] \leq 0, \tag{9}
\]

where \( A^{\omega_1}, \) and \( d_{2}^{\omega_1}, \) are the values of \( A(t) \) and \( d_{2}(t) \) obtained by applying policy \( \omega_1. \) Note that Assumption 1 is mild and holds whenever problem (7) is feasible. We now make a stronger assumption that can guarantee that the constraint in (7b) is met with \( \epsilon \)-slackness. In the following assumption, we focus only on the satisfaction of the constraint.

**Assumption 2:** There is a value \( \epsilon > 0 \) and an \( \omega_2 \) policy that satisfies the following over any renewal frame:

\[
q - \mathbb{E} \left[ \sum_{\tau=mT}^{(m+1)T-1} d_{2}^{\omega_2}(\tau) \right] \leq -\epsilon, \tag{10}
\]

where \( d_{2}^{\omega_2}(t) \) is the value of \( d_{2}(t) \) obtained by applying policy \( \omega_2. \) In the next section, we describe our proposed dynamic control algorithm.

**IV. Dynamic Control Algorithm**

Before describing our proposed algorithm that solves (7), let us recall a definition and a basic theory that comes from the theory of stochastic processes [40]. Consider a system with \( N \) queues. The number of unfinished jobs of queue \( n \) is denoted by \( q_n(t), \) and \( q(t) = [q_1(t) \ldots q_N(t)]^T. \) The Lyapunov function and the Lyapunov drift are denoted by \( L(q(t)) \) and \( \Delta(L(q(t))) \triangleq \mathbb{E} \{L(q(t+1)) - L(q(t))|q(t)\}, \) respectively, and they are defined below.

**Definition 1 (Lyapunov function).** A function \( L : \mathbb{R}^N \rightarrow \mathbb{R} \) is a Lyapunov function if it has the following properties

- \( L(x) \geq 0, \forall x \in \mathbb{R}^N \)
- It is non-decreasing in any of its arguments
• \( L(x) \to \infty \), as \( ||x|| \to +\infty \)

**Theorem 1** (Lyapunov Drift). *If there exists positive values \( B \) and \( \epsilon \) such that for all time slots we have \( \Delta(L(q(t))) \leq B - \epsilon \sum_{n=1}^{N} q_n(t) \), then the system \( q(t) \) is strongly stable.*

The intuition behind Theorem 1 is that if we have a queueing system and we provide an algorithm for which the Lyapunov drift becomes negative for large queue sizes, then the Lyapunov function decreases and subsequently the queue sizes. As a consequence, the queues remain bounded and the overall system is stable.

We define a virtual queue \( Z(t) \) associated with the constraint in (7b), where \( Z(0) = 0 \). We update the value of the virtual queue as shown below

\[
Z(t + 1) = \max [Z(t) - d_2(t), 0] + \frac{q}{T}.
\] (11)

Process \( Z(t) \) can be seen as a queue with “service rate” \( \bar{d}_2 \) and “arrival rate” \( \frac{q}{T} \). We will show that the average constraint in (7b) is transformed into a queue stability problem.

**Definition 1.** A discrete time process \( Q(t) \) is rate stable if \( \lim_{t \to \infty} \frac{Q(t)}{t} = 0 \) with probability 1.

**Lemma 1.** *If \( Z(t) \) is rate stable, then the constraint in (7b) is satisfied.*

**Proof:** Using the basic sample property [41, Lemma 2.1, Chapter 2], we have

\[
\frac{Z(t_m)}{t_m} - \frac{Z(0)}{t_m} \geq \frac{1}{M} \sum_{m=0}^{M} \left( \sum_{\tau=mT}^{(m+1)T-1} d_2(\tau) - \frac{q}{T} \right).
\] (12)

Therefore, if \( Z(t_m) \) is rate stable, so that \( \frac{Z(t_m)}{t_m} \to 0 \), with probability 1, then the constraint in (7b) is satisfied.

**A. Lyapunov Drift**

We define the following quadratic Lyapunov function as

\[
L(t) \triangleq \frac{1}{2} Z^2(t).
\] (13)
We define the frame-based Lyapunov drift as
\[
\Delta(t_m) \triangleq \mathbb{E} [L(t_m + T) - L(t_m)|Z(t_m)],
\]
where \( t_m = mT \) is the start of the \( m \)th frame.

**Lemma 2.** Under any policy \( u(\tau) \) for all slots during a renewal frame \( \tau \in \{t_m, \ldots, t_m + T - 1\} \), we have
\[
\Delta(t_m) \leq B + \mathbb{E} [G(t_m)|Z(t_m)],
\]
where \( G(t_m) \) is defined as
\[
G(t_m) \triangleq Z(t_m) \sum_{\tau=t_m}^{t_m+T-1} (q - d_2(\tau)),
\]
and \( B \) is finite constant defined as
\[
B \triangleq \frac{Tq^2 + T(T - 1)}{2}.
\]

*Proof:* See Appendix A.

B. Frame-Based Drift-Plus Penalty Algorithm

In order to provide a solution to the optimization problem in (7), we implement a policy over the course of the frame to minimize the following expression
\[
\min_{u(t)} \mathbb{E} \left[ G(t_m) + V \sum_{\tau=t_m}^{t_m+T-1} A(\tau)|Z(t_m) \right],
\]
where the expectation is with respect to the policy and the randomness of the channel. The problem in (17) is a stochastic shortest path problem which usually is solved approximately [19]. In the next subsection, we analyze the performance of the algorithm under the assumption that we have a policy that can approximate (17).
C. Approximation Theorem

Assumption 3: For constants, \( C \geq 0, \delta \geq 0 \), define a \((C, \delta)\)-approximation of (17) to be a policy for choosing \( u(t) \) over a frame \( \tau \in \{t_m, \ldots, t_m + T - 1\} \) such that
\[
\mathbb{E} \left[ G(t_m) + V \sum_{\tau=t_m}^{t_m+T-1} A(\tau)|Z(t) \right] \leq \\
\mathbb{E} \left[ G^*(t_m) + V \sum_{\tau=t_m}^{t_m+T-1} A^*(\tau)|Z(t) \right] + C + \delta Z(t_m) + V\delta,
\]
where \( A^* \) and \( G^* \) are the optimal values.

Theorem 2. Suppose that Assumptions 1, 2, hold for a given \( \epsilon > 0 \), and suppose we use a \((C, \delta)\)-approximation every frame so that Assumption 3 holds. If \( \epsilon > \frac{\delta}{T} \), then constraint (7b) is satisfied and
\[
\lim_{M \to \infty} \frac{1}{M} \sum_{m=0}^{M} \mathbb{E}[Z(t_m)] \leq \frac{B + C + V(A_{\text{max}} + \delta - 1)}{\epsilon T - \delta}, \tag{19}
\]
and
\[
\lim_{t \to \infty} \sup_{t} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[A(\tau)] \leq \frac{B}{V T} + \gamma(A_{\text{max}} - 1) + (1 - \gamma)A^* + \frac{C}{V T} + \frac{\delta}{T}, \tag{20}
\]
where \( \gamma = \frac{\delta}{\epsilon T} \).

Proof: See Appendix C.

V. Solution of the MDP

The problem in (17) is an MDP problem. Let \( \mathcal{A} = \{1, 2, \ldots, A_{\text{max}}\} \) denote the set of possible values of AoI of user 1. Furthermore, let \( \mathcal{Q} = \{0, 1, 2, \ldots, L\} \) be the set of possible values of the queue of user 2. Then, \( \mathcal{A}(t) \in \mathcal{A} \), and \( \mathcal{Q}(t) \in \mathcal{Q} \). A transmission policy \( u(t) \) specifies the decision rules every time slot \( t \). Note that the described MDP problem is a finite-horizon problem. We solve the optimization problem at every frame. At the beginning of each frame, we know the channel conditions of the previous slot for each users, the state of the queue (it is always \( L \) packets at the beginning of the frame), and the value of the AoI of user 1.
The next state depends on both the scheduler’s decision and the channel states. Note that we schedule user 2 only if it has remaining packets in its queue, and recall that at the end of frame $m$, we drop all the remaining packets, if there are any.

A. Transition Probabilities

The transition probabilities of the system states depend on the channel model. Below we describe the transition probabilities for each channel model, and the corresponding system states.

1) i.i.d channels: In this case, the system state in time slot $t$ is described by $s(t) = (A(t), Q(t))$. The transition probabilities for the i.i.d channel case are described below.

$$P_{s_t ightarrow s_{t+1}} = \begin{cases} 
  p_1, & \text{if } \mathbf{u}(t) = [1 \ 0]^T \text{ and } s(t + 1) = (1, Q(t)), \\
  1 - p_1, & \text{if } \mathbf{u}(t) = [1 \ 0]^T \text{ and } s(t + 1) = (\min \{A(t) + 1, A_{\max}\}, Q(t)), \\
  p_2, & \text{if } \mathbf{u}(t) = [0 \ 1]^T \text{ and } s(t + 1) = (\min \{A(t) + 1, A_{\max}\}, Q(t) - 1), \\
  1 - p_2, & \text{if } \mathbf{u}(t) = [0 \ 1]^T \text{ and } s(t + 1) = (\min \{A(t) + 1, A_{\max}\}, Q(t)). 
\end{cases}$$ (21)

2) Time-correlated channels: In this case the system state in time slot $t$ is described by $s(t) = (A(t), Q(t), h(t))$. The transition probabilities for the case of Gilbert-Elliot channel model are described below.
\[
P_{s_t \rightarrow s_{t+1}} = \begin{cases} 
  p_{11,1}, & \text{if } u(t) = [1 0]^T, \ h_1(t) = 1, \ \text{and } s(t+1) = (1, Q(t), (1, x)), \\
  1 - p_{11,1}, & \text{if } u(t) = [1 0]^T, \ h_1(t) = 1, \ \text{and } s(t+1) = (\min \{A(t) + 1, A_{\max}\}, Q(t), (0, x)), \\
  p_{01,1}, & \text{if } u(t) = [1 0]^T, \ h_1(t) = 0, \ \text{and } s(t+1) = (1, Q(t), (1, x)), \\
  1 - p_{01,1}, & \text{if } u(t) = [1 0]^T, \ h_1(t) = 0, \ \text{and } s(t+1) = (\min \{A(t) + 1, A_{\max}\}, Q(t), (0, x)), \\
  p_{11,2}, & \text{if } u(t) = [0 1]^T, \ h_2(t) = 1, \ \text{and } s(t+1) = (\min \{A(t) + 1, A_{\max}\}, Q(t) - 1, (x, 1)), \\
  1 - p_{11,2}, & \text{if } u(t) = [0 1]^T, \ h_2(t) = 1, \ \text{and } s(t+1) = (\min \{A(t) + 1, A_{\max}\}, Q(t) - 1, (x, 0)), \\
  p_{01,2}, & \text{if } u(t) = [0 1]^T, \ h_2(t) = 0, \ \text{and } s(t+1) = (\min \{A(t) + 1, A_{\max}\}, Q(t) - 1, (x, 1)), \\
  1 - p_{01,2}, & \text{if } u(t) = [0 1]^T, \ h_2(t) = 0, \ \text{and } s(t+1) = (\min \{A(t) + 1, A_{\max}\}, Q(t), (0, x)),
\end{cases}
\]

where \( x \) is used to show that the value of the corresponding element does not affect the state transition.

\section*{B. Backward dynamic programming algorithm}

Initially, we drop the frame indices and take \( t \in \{0, 1, \ldots, T - 1\} \). As a first step, we consider that the transmission error probabilities are fixed, i.e., the channels are i.i.d over the slots. In our system model, we take an action in time slot \( t \), and we observe the cost in time slot \( t + 1 \). This happens because of the randomness of the channels for both i.i.d and correlated channels. If we transmit a packet, it will successfully be transmitted with some probability. We know whether the transmission is successful or not at the end of the slot due to ACK/NACK. Below we define the costs for the different channel models.

\subsection*{1) i.i.d channels:} We denote the cost received in slot \( t + 1 \) given the state \( s(t) \), the decision \( u(t) \), and the information received \( W_{t+1} \) (successful transmission or failure), as \( \hat{C}_{t+1}(s(t +}
The Bellman’s equation is described below:

\[ C_t(s(t), u(t)) = \mathbb{E} \left\{ \hat{C}_{t+1}(s(t+1), W_{t+1}) | s(t), u(t) \right\} \]

\[ = \begin{cases} 
Zq + V(p_{1,1} + (1 - p_{1,1}) \min\{A(t) + 1, A_{\max}\}), & \text{if } u_1(t) = 1, \\
Z(q - p_{2}) + V(\min\{A(t) + 1, A_{\max}\}), & \text{if } u_2(t) = 1.
\end{cases} \]  \quad (23)

The Bellman’s equation is described below:

\[ V_t(s(t)) = \min_{u(t)} \mathbb{E} \left\{ \hat{C}_{t+1}(s(t + 1), W_{t+1} | s(t), u(t)) \right\} \]

\[ = \min_{u(t)} \left( C_t(s(t), u(t)) + \gamma \sum_{s' \in S} \Pr(s(t + 1) = s' | s(t), u(t)) V_{t+1}(s') \right), \]  \quad (24)

where \(0 < \gamma < 1\).

2) Time-correlated channels: In this case, the instantaneous cost at time slot \(t\) is described as

\[ C_t(s(t), u(t)) = \mathbb{E} \left\{ \hat{C}_{t+1}(s(t + 1), W_{t+1} | s(t), u(t)) \right\} \]

\[ = \begin{cases} 
Zq + V(p_{11,1} + (1 - p_{11,1}) \min\{A(t) + 1, A_{\max}\}), & \text{if } u_1(t) = 1 \text{ and } h_1(t) = 1, \\
Zq + V(p_{01,1} + (1 - p_{01,1}) \min\{A(t) + 1, A_{\max}\}), & \text{if } u_1(t) = 1 \text{ and } h_1(t) = 0, \\
Z(q - p_{11,2}) + V(\min\{A(t) + 1, A_{\max}\}), & \text{if } u_2(t) = 1 \text{ and } h_2(t) = 1, \\
Z(q - p_{01,2}) + V(\min\{A(t) + 1, A_{\max}\}), & \text{if } u_2(t) = 1 \text{ and } h_2(t) = 0,
\end{cases} \]  \quad (25)

where \(\hat{C}_{t+1}(s(t), u(t), W_{t+1} | s(t), u(t), h(t))\) is the cost received in time slot \(t\). The Bellman’s equation is described below:

\[ V_t(s(t)) = \min_{u(t)} \mathbb{E} \left\{ \hat{C}_{t+1}(s(t + 1), W_{t+1} | s(t), u(t)) \right\} \]

\[ = \min_{u(t)} \left( C_t(s(t), u(t)) + \gamma \sum_{s' \in S} \Pr(s(t + 1) = s' | s(t), u(t)) V_{t+1}(s') \right), \]  \quad (26)

where \(0 < \gamma < 1\).
We can solve the recursions in (26) and (24) by using backward dynamic programming. We denote by $S$ the set with all possible states. The algorithm is shown below.

**Algorithm 2: Backward Dynamic Programming**

**Step 0. Initialization:**

Initialize the terminal contribution $V_T(s_T)$. Usually we set the value of 0 to $V_T(s_T)$. Set $t = T - 1$.

**Step 1. Calculate:**

$$V_t(s(t)) = \min_{u(t)} \left( C_t(s(t), u(t)) + \gamma \sum_{s' \in S} \Pr(s(t + 1) = s'|s(t), u(t))V_{t+1}(s') \right),$$

$\forall s(t) \in S$.

**Step 2.** If $t > 0$, decrement $t$ and return to step 1. Else stop.

We can implement the dynamic described above for both channel cases. The idea of the algorithm is quite simple. The algorithm runs over the duration of each frame starting at the last slot, i.e., the $T^{th}$ of each frame. We initialize the value of being at each state at the last slot, and then, we calculate the value of each state at every time slot by going backward in time as shown in Step 1.

A diagram that summarizes the steps taken for solving the initial problem is shown in Fig. 4.
In this section, we provide results to study the performance of our proposed algorithm in terms of the average value of the AoI and the convergence regarding the timely throughput requirements. We investigate how different values of the weight factor $V$ can affect both the

VI. SIMULATION RESULTS

In this section, we provide results to study the performance of our proposed algorithm in terms of the average value of the AoI and the convergence regarding the timely throughput requirements. We investigate how different values of the weight factor $V$ can affect both the
value of AoI and the convergence of the algorithm. For the following results, we consider the Gilbert-Elliot channel, with $p_{11,1} = p_{11,2} = 0.9$, and $p_{01,1} = p_{01,2} = 0.6$. The length of the frame, $T$, is 20 time slots, and the number of arrived packets at the beginning of every frame, $K$, is equal to 15 packets. We consider that the maximum value of the AoI, $A_{\text{max}} = 20$. The timely throughput requirements are $q = 12$ packets/frame or $q/T = 0.6$ packet/slot. We run each experiment for $0.5 \times 10^6$ time slots, and we use MATLAB environment to perform our simulations. In Fig. 5, we provide the average value of AoI for different values of $V$ as well as the convergence of the timely throughput constraints. In Fig. 5a, we compare the average value of AoI of five sample paths with that of one sample-path. We observe that the values are quite close to each other. Therefore, the algorithm offers high performance regarding robustness. Furthermore, it is shown that the AoI reaches its minimum value even for small values of $V$. We see that for values of $V$ larger than 5 the change of the value of the average AoI is negligible. On the other hand, we observe that the convergence of the algorithm regarding the timely throughput constraints changes dramatically as $V$ increases, as shown in Fig. 5b and Fig. 5c. The performance of the
algorithm regarding the convergence is also shown in Figs. 6a and 6b. We see that for large values of $V$, for example, $V = 150$, the algorithm needs long time to stabilize the virtual queue because the value of the virtual queue becomes larger than the term of AoI after many slots. Therefore, the algorithm starts scheduling user 2 when the virtual queue takes a large value. More specifically, the length of the virtual queue remains around a value after $2.5 \times 10^4$ time slots. Therefore, it is more beneficial for the system to choose a value of $V$ that is small, for example, 5.

In Fig. 7, we provide results for the distribution of the AoI obtained by simulations for different values of $V$. In Fig. 7a, we observe that the AoI reaches the maximum value because its weight is zero. Therefore, the algorithm schedules user 2 as long as its queue is non empty. After emptying the queue of user 2, the scheduler schedules user 1 if there are remaining slots. For values larger or equal than 5, in Figs. 7b, 7c, 7d, we observe that the AoI never reaches its maximum value. Instead, the values of AoI fluctuate mainly in the range of $1 - 5$.

In Fig. 8, we provide results that show the scheduling time percentage per slot within a frame for each user. In Fig. 8a, the value of $V$ is 0. For the first 15 slots the scheduler schedules only user 2. The number of packets is 15, therefore, the scheduler needs at least 15 slots to empty the queue of user 2 because a packet may be failed to be transmitted and the transmitter has to retransmit it. After emptying the queue of user 2, if there are remaining slots, the scheduler allocates slots to user 1. In Figs. 8b, 8c, 8d, we observe that for different values of $V$ the scheduling time for every user changes. However, as we observe in Fig. 5a, the values of average AoI are quite close to each other. We observe that for $V = 5$, the scheduling time for user 1 is spread within the frame. That means that the percentage of scheduling time does not change significantly from slot to slot. On the other hand, for larger values of $V$, we observe that the percentage of the scheduling time for user 1 changes from slot to slot, especially after the 10th slot and for $V = 100$ because the AoI is multiplied by a large weight and if the value of AoI starts increasing as time passes by the corresponding term becomes quite large. Therefore, the
scheduler schedules the user 1 in order to minimize the objective function.

VII. CONCLUSION & FUTURE WORK

In this work, we considered a wireless network consisting of time-critical users with different requirements under uncertain environments. We studied how an AoI-oriented user and a deadline-
Fig. 8: Scheduling time percentage per slot within a frame.

constrained user can share the same resources to satisfy their requirements. To this end, we formulated a stochastic optimization problem for minimizing the average AoI while satisfying the timely-throughput constraints which is a CMDP problem. In order to solve the problem, we utilized tools from Lyapunov optimization and MDP. With this approach, we reduced the CMDP to an unconstrained weighted stochastic shortest path problem. We implemented backward
dynamic programming to solve the unconstrained problem. Simulation results showed that the timely-throughput constraint is satisfied while minimizing the average AoI. Furthermore, we provided the trade-off between the minimum value of AoI and the convergence of the average constraint.

As a future work, we consider that an interesting direction is the case with variable length of the frames and random packet arrivals of the deadline-constrained user. Also, a realistic scenario is the multi-user scenario that will make our problem even more interesting but more difficult to be solved. In this case, backward dynamic programming has prohibitive complexity and approximation algorithms could give a good solution.

APPENDIX A

DRIFT BOUND: PROOF OF LEMMA 2

Proof: By utilizing the following formula [41],

\[
\max [Q - b, 0] + A \leq Q^2 + A^2 + b^2 + 2Q(A - b),
\]

we get

\[
Z^2(t + 1) \leq Z^2(t) + q^2 + d_2^2(t) + 2Z(t)(q - d_2(t)),
\]

by adding and substituting the term \(2Z(t_m)(q - d_2(t))\), we get

\[
Z^2(t + 1) \leq Z^2(t) + q^2 + d_2^2(t) + 2Z(t)(q - d_2(t)) + 2Z(t_m)(q - d_2(t)) - 2Z(t_m)(q - d_2(t))
\]

\[
= Z^2(t)q^2 + d_2^2(t) + 2Z(t_m)(q - d_2(t)) + 2[Z(t) - Z(t_m)](q - d_2(t))
\]

\[
\leq Z^2(t) + q^2 + 1 + 2Z(t_m)(q - d_2(t)) + 2(t - t_m),
\]

diving by 2, using telescoping sums, and using that \(\sum_{\tau=t_m}^{t_m+T-1} (\tau - t_m) = \frac{T(T+1)}{2}\), we get,

\[
\frac{Z^2(t_m + T) - Z^2(t_m)}{2} \leq \frac{Tq^2 + T(T - 1)}{2} + Z(t_m) \sum_{\tau=t_m}^{t_m+T-1} (q - d_2(\tau)),
\]
taking conditional expectations, we get the result,

\[
\Delta(t_m) \leq B + \mathbb{E} [G(t_m)|Z(t_m)].
\] (33)

**APPENDIX B**

**VIRTUAL QUEUE STABILITY**

From (50), we get

\[
\frac{1}{R} \sum_{m=0}^{R} \mathbb{E} [Z(t_m)] \leq D,
\] (34)

where \(D\) is a bounded constant, if (34) holds, then

\[
\lim_{t \to \infty} \sup \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E} [Z(\tau)].
\] (35)

**Proof:** Since every frame is at least one slot, we have

\[
\sum_{\tau=0}^{R-1} Z(\tau) \leq \sum_{r=0}^{R-1} \sum_{\tau=t_m}^{T-1} Z(\tau)
\] (36)

\[
\leq \sum_{r=0}^{R-1} T [Z(t_m) + T].
\] (37)

By taking the expectations and using (34), we get

\[
\sum_{\tau=0}^{R-1} \mathbb{E} [Z(\tau)] \leq RT^2 + DT.
\] (38)

By taking the limit \(R \to \infty\), and dividing by \(R\), we get,

\[
\lim_{R \to \infty} \sup \frac{1}{R} \sum_{\tau=0}^{R-1} \mathbb{E} [Z(\tau)] \leq T^2 + DT.
\] (39)

That proves the result.
APPENDIX C

PROOF OF THEOREM 2

Proof: Part I - Queue bound. From (33), we have

\[
\Delta(t_m) + E \left[ V \sum_{\tau=t_m}^{t_m+T-1} A(\tau) | Z(t) \right] \leq B + E \left[ G^*(t_m) + V \sum_{\tau=t_m}^{t_m+T-1} A^*(\tau) \right] + C + \delta Z(t_m) + V\delta. \tag{40}
\]

Note that \( |A^*(\tau) - A(\tau)| \leq A_{max} - 1 \), therefore, we have

\[
\Delta(t_m) \leq B + E \left[ G^*(t_m) \right] + E \left[ V \sum_{\tau=t_m}^{t_m+T-1} A^*(\tau) - A(\tau) \right] + C + \delta Z(t_m) + V\delta \tag{41}
\]

\[
\Delta(t_m) \leq B + E \left[ G^*(t_m) \right] + V(A_{max} - 1) + C + \delta Z(t_m) + V\delta. \tag{42}
\]

Now consider that the policy \( u_2(t) \) from Assumption 2,

\[
E \left[ G^*(t_m) | Z(t) \right] \leq -\epsilon TZ(t_m), \tag{43}
\]

Substituting (43) into (42), we get

\[
\Delta(t_m) \leq B - \epsilon TZ(t_m) + V(A_{max} + \delta - 1) + C + \delta Z(t_m) + V\delta \tag{44}
\]

\[
= B + C + V(A_{max} + \delta - 1) + (\delta - \epsilon T)Z(t_m) \tag{45}
\]

\[
E[L(t_m + 1)] - E[L(t_m)] \leq B + C + V(A_{max} + \delta - 1) + (\delta - \epsilon T)Z(t_m). \tag{46}
\]

Summing over \( m \in \{0, \ldots, R-1\} \) and using the fact that \( E[L(t_0)] = 0 \), and using the telescoping sums, we get

\[
\frac{E[L(t_m)]}{R} \leq B + C + V(A_{max} + \delta - 1) + \frac{\delta - \epsilon T}{R} \sum_{\tau=t_m}^{t_m+T-1} Z(\tau) \tag{47}
\]

\[
\frac{\epsilon T - \delta}{R} \sum_{\tau=t_m}^{t_m+T-1} Z(t_m) \leq \frac{-E[L(t_m)]}{R} + B + C + V(A_{max} + \delta - 1), \tag{48}
\]

26
By ignoring the negative part, we get
\[ \frac{\epsilon T - \delta}{R} \sum_{\tau = t_m}^{t_m + T - 1} Z(t_m) \leq B + C + V(A_{\text{max}} + \delta - 1), \] (49)
\[ \frac{1}{R} \sum_{r=0}^{R} E[Z(t_m)] \leq \frac{B + C + V(A_{\text{max}} + \delta - 1)}{\epsilon T - \delta}. \] (50)

From (50), we can prove that the virtual queue is strongly stable, and therefore, the constraints in (7b) are satisfied (See Appendix A).

Part II - Approximation theorem. Define the probability \( \gamma = \frac{\delta}{\epsilon T} \). This is a valid probability by assumption (\( \epsilon T > \delta \)). We consider a policy \( \omega \) performed over the frame \( \tau \in \{t_m, \ldots, t_m + T - 1\} \). The policy \( \omega \) is a randomized mixture from Assumptions 1 and 2. At the beginning of each frame, flip a coin with probabilities \( \gamma \) and \( 1 - \gamma \), and apply one of the policies as following

- With probability \( \gamma \), apply policy \( \omega_2 \) from Assumption 2,
- with probability \( 1 - \gamma \), apply policy \( \omega_1 \) from Assumption 1.

From (8), we have
\[ E \left[ \sum_{\tau = t_m}^{t_m + T - 1} A^*(\tau) \right] \leq T(\gamma A_{\text{max}} + (1 - \gamma)A_{\text{opt}}), \] (51)
we also have from (10),
\[ E \left[ \sum_{\tau = t_m}^{t_m + T - 1} q^* - d_2^*(\tau) \right] \leq -\gamma \epsilon T = -\delta, \] (52)
plugging (51), (52), into (18), we get
\[ \Delta(t_m) + E \left[ V \sum_{\tau = t_m}^{t_m + T - 1} A(\tau)Z(t_m) \right] \leq B + T(\gamma A_{\text{max}} - 1) + (1 - \gamma)A_{\text{opt}} + C + V\delta \]
\[ \Delta(t_m) + E \left[ V \sum_{\tau = t_m}^{t_m + T - 1} A(\tau)Z(t_m) \right] \leq B + T(\gamma A_{\text{max}} - 1) + (1 - \gamma)A_{\text{opt}} + C + V\delta \]
\[ \frac{1}{MT} \sum_{\tau=0}^{T-1} A(\tau) \leq \frac{B}{VT} + (\gamma A_{\text{max}} - 1) + (1 - \gamma)A_{\text{opt}} + \frac{C}{VT} + \frac{\delta}{T}, \] (55)
by taking $M \to \infty$, we get the result.

REFERENCES

[1] M. A. Abd-Elmagid, N. Pappas, and H. S. Dhillon, “On the role of age of information in the internet of things,” *IEEE Communications Magazine*, vol. 57, no. 12, pp. 72–77, 2019.

[2] T. Shreedhar, S. K. Kaul, and R. D. Yates, “An age control transport protocol for delivering fresh updates in the Internet-of-Things,” in *Proc. IEEE WoWMoM*, pp. 1–7, 2019.

[3] I. Hou, V. Borkar, and P. R. Kumar, “A theory of QoS for wireless,” in *Proc. IEEE INFOCOM 2009*, pp. 486–494, 2009.

[4] A. Kosta, N. Pappas, and V. Angelakis, “Age of information: A new concept, metric, and tool,” *Foundations and Trends in Networking*, vol. 12, no. 3, pp. 162–259, 2017.

[5] Y. Sun, I. Kadota, R. Talak, and E. Modiano, “Age of information: A new metric for information freshness,” *Synthesis Lectures on Communication Networks*, vol. 12, no. 2, pp. 1–224, 2019.

[6] S. Kaul, R. Yates, and M. Gruteser, “Real-time status: How often should one update?,” in *Proc. IEEE INFOCOM*, 2012.

[7] S. Shakkottai and R. Srikant, “Scheduling real-time traffic with deadlines over a wireless channel,” *Wireless Networks*, vol. 8, no. 1, pp. 13–26, 2002.

[8] Y. Cui, V. K. Lau, R. Wang, H. Huang, and S. Zhang, “A survey on delay-aware resource control for wireless systems—Large deviation theory, stochastic Lyapunov drift, and distributed stochastic learning,” *IEEE Transactions on Information Theory*, vol. 58, no. 3, pp. 1677–1701, 2012.

[9] L. You, Q. Liao, N. Pappas, and D. Yuan, “Resource optimization with flexible numerology and frame structure for heterogeneous services,” *IEEE Communications Letters*, vol. 22, no. 12, pp. 2579–2582, 2018.

[10] E. Fountoulakis, N. Pappas, Q. Liao, V. Suryaprakash, and D. Yuan, “An examination of the benefits of scalable TTI for heterogeneous traffic management in 5G networks,” in *Proc. WiOpt*, pp. 1–6, 2017.

[11] S. ElAzzouni, E. Ekici, and N. Shroff, “Is deadline oblivious scheduling efficient for controlling real-time traffic in cellular downlink systems?,” in *Proc. IEEE INFOCOM*, pp. 49–58, 2020.

[12] C. Tsanikidis and J. Ghaderi, “On the power of randomization for scheduling real-time traffic in wireless networks,” *IEEE/ACM Transactions on Networking*, 2021.

[13] A. Destounis, G. S. Paschos, J. Arnaud, and M. Kountouris, “Scheduling URLLC users with reliable latency guarantees,” *Proc. WiOpt*, pp. 1–8, 2018.

[14] S. Lashgari and A. S. Avestimehr, “Timely throughput of heterogeneous wireless networks: Fundamental limits and algorithms,” *IEEE Transactions on Information Theory*, vol. 59, no. 12, pp. 8414–8433, 2013.

[15] E. Fountoulakis, N. Pappas, Q. Liao, A. Ephremides, and V. Angelakis, “Dynamic power control for packets with deadlines,” *Proc. IEEE GLOBECOM*, 2018.

[16] A. E. Ewaisha and C. Tepedelenlioglu, “Optimal power control and scheduling for real-time and non-real-time data,” *IEEE Trans. Vehic. Tech.*, vol. 67, no. 3, pp. 2727–2740, 2018.
[17] E. Fountoulakis, N. Pappas, and A. Ephremides, “Dynamic power control for time-critical networking with heterogeneous traffic,” *ITU Journal on Future and Evolving Technologies*, March 2021.

[18] N. Master and N. Bambos, “Power control for packet streaming with head-of-line deadlines,” *Performance Evaluation*, vol. 106, pp. 1–18, 2016.

[19] M. J. Neely and S. Supittayapornpong, “Dynamic markov decision policies for delay constrained wireless scheduling,” *IEEE Transactions on Automatic Control*, vol. 58, no. 8, pp. 1948–1961, 2013.

[20] A. Fu, E. Modiano, and J. N. Tsitsiklis, “Optimal transmission scheduling over a fading channel with energy and deadline constraints,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 630–641, 2006.

[21] S. Saraswat, H. P. Gupta, T. Dutta, and S. K. Das, “Energy efficient data forwarding scheme in fog-based ubiquitous system with deadline constraints,” *IEEE Transactions on Network and Service Management*, vol. 17, no. 1, pp. 213–226, 2020.

[22] Z. Mao, C. E. Koksal, and N. B. Shroff, “Optimal online scheduling with arbitrary hard deadlines in multihop communication networks,” *IEEE/ACM Transactions on Networking*, vol. 24, no. 1, pp. 177–189, 2014.

[23] R. Talak and E. Modiano, “Age-delay tradeoffs in queueing systems,” *IEEE Transactions on Information Theory*, 2020.

[24] I. Kadota and E. Modiano, “Minimizing the age of information in wireless networks with stochastic arrivals,” *IEEE Transactions on Mobile Computing*, vol. 20, no. 3, pp. 1173–1185, 2021.

[25] I. Kadota, A. Sinha, and E. Modiano, “Scheduling algorithms for optimizing age of information in wireless networks with throughput constraints,” *IEEE/ACM Transactions on Networking*, vol. 27, no. 4, pp. 1359–1372, 2019.

[26] B. Zhou and W. Saad, “Joint status sampling and updating for minimizing age of information in the internet of things,” *IEEE Trans. Commun.*, vol. 67, no. 11, pp. 7468–7482, 2019.

[27] M. A. Abd-Elmagid, H. S. Dhillon, and N. Pappas, “AoI-optimal joint sampling and updating for wireless powered communication systems,” *IEEE Trans Vehic. Tech.*, vol. 69, no. 11, pp. 14110–14115, 2020.

[28] A. M. Bedewy, Y. Sun, S. Kompella, and N. B. Shroff, “Optimal sampling and scheduling for timely status updates in multi-source networks,” *IEEE Transactions on Information Theory*, pp. 1–1, 2021.

[29] E. Fountoulakis, M. Codreanu, A. Ephremides, and N. Pappas, “Joint sampling and transmission policies for minimizing cost under aoi constraints,” *arXiv preprint arXiv:2103.15450*, 2021.

[30] G. Yao, A. M. Bedewy, and N. B. Shroff, “Age minimization transmission scheduling over time-correlated fading channel under an average energy constraint,” *arXiv preprint arXiv:2012.02958*, 2020.

[31] E. T. Ceran, D. Gündüz, and A. György, “Reinforcement learning to minimize age of information with an energy harvesting sensor with HARQ and sensing cost,” *in Proc. IEEE INFOCOM*, pp. 656–661, Apr. 2019.

[32] G. Stamatakis, N. Pappas, and A. Traganitis, “Optimal policies for status update generation in an IoT device with heterogeneous traffic,” *IEEE Internet Things J.*, 2020.

[33] Z. Chen, N. Pappas, E. Björnson, and E. G. Larsson, “Optimizing information freshness in a multiple access channel with heterogeneous devices,” *IEEE Open Journal of the Communications Society*, vol. 2, pp. 456–470, 2021.
[34] N. Pappas and M. Kountouris, “Delay violation probability and age of information interplay in the two-user multiple access channel,” Proc. IEEE SPAWC, 2019.

[35] E. Fountoulakis, T. Charalambous, N. Nomikos, A. Ephremides, and N. Pappas, “Information freshness and packet drop rate interplay in a two-user multi-access channel,” in Proc. ITW, pp. 1–5, 2021.

[36] J. Sun, L. Wang, Z. Jiang, S. Zhou, and Z. Niu, “Age-optimal scheduling for heterogeneous traffic with timely throughput constraints,” IEEE Journal on Selected Areas in Communications, vol. 39, no. 5, pp. 1485–1498, 2021.

[37] E. Altman, Constrained Markov decision processes, vol. 7. CRC Press, 1999.

[38] N. Salodkar, A. Bhorkar, A. Karandikar, and V. S. Borkar, “An on-line learning algorithm for energy efficient delay constrained scheduling over a fading channel,” IEEE Journal on Selected Areas in Communications, vol. 26, no. 4, pp. 732–742, 2008.

[39] D. V. Djonin and V. Krishnamurthy, “Q-learning algorithms for constrained markov decision processes with randomized monotone policies: Application to mimo transmission control,” IEEE Transactions on Signal Processing, vol. 55, no. 5, pp. 2170–2181, 2007.

[40] S. P. Meyn and R. L. Tweedie, Markov chains and stochastic stability. Springer Science & Business Media, 2012.

[41] M. J. Neely, Stochastic Network Optimization with Application to Communication and Queueing Systems. Morgan & Claypool, 2010.