Corrigendum: Modelling the bending behaviour of novel fibre-reinforced sandwich structures with polyurethane foam core

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Abstract. In contrast to conventional manufacturing methods for sandwich structures, in this paper a foamable polyurethane system is used to impregnate textile layers and to create the sandwich foam core in one single process step. Given this simultaneous process, a high adaptability of the resulting mechanical properties can be achieved by varying the textile reinforcement of the sandwich top layers, the foam core density and the sandwich thickness, respectively. Within this paper, a design of experiment approach is implemented by using statistical analysis of variance (ANOVA) to investigate the influence of selected geometrical and material parameters on the resulting bending properties of these novel sandwich structures. A partial factorial design helps to identify the sensitivity of individual parameters and their interaction, as well as the correlation between sandwich morphology and bending performance. Based on the gained results, statistical models will help to identify parameters with statistical significance. Therefore, optimized sandwich structures according to stress-related requirements and geometrical constraints can be designed.

1. Introduction
Sandwich structures are considered being excellent weight-optimized structural components for the use in lightweight applications [1]. In this context, fibre-reinforced plastics with their excellent adjustable mechanical properties are established materials for the use as sandwich face sheets. In many cases, low-density polymer foams, such as polyurethane, serve as core material by stabilizing the sandwich structure and contributing with an improved bending and buckling resistance as well as high energy absorption abilities [2, 3]. Conventional manufacturing technologies for sandwich structures usually include hand lay-up methods (e.g. VARI or VARTM) by draping of textiles on a prefabricated polymer foam core [4, 5, 6] or the reaction injection moulding of the foam material into a predefined cavity made of fibre-reinforced face sheets [7, 8]. Other processing technologies such as the long-fibre injection method involve the spraying of a foamable polyurethane resin with cut glass fibres on a paper honeycomb core [9]. In contrast to these processing technologies, a foamable polyurethane resin was used in the current work to impregnate textile structures and create the sandwich foam core simultaneously.

Although sandwich structures are already well established in numerous applications, many research studies focus on design optimization. In this context, the limitations of local failure modes, e.g. face-core debonding [10] or foam core collapse [11] and the improvement of mechanical properties [12, 13] are among the main motivation. Comprehensive studies were performed to predict the failure behaviour of sandwich panels by developing analytical and numerical models taking different face-sheet–core material combinations into account [14, 15, 16]. However, most of these works did not focus on a sound statistical investigation of the gained experimental data using ANOVA.
In the present work the bending performance of these novel sandwich structures is analysed using a statistical design of experiments approach. Based on the experimental results, geometrical parameters with statistical significance are selected and statistical models are developed to predict the mechanical properties bending stiffness and maximum bending moment. The main motivation for using these models is anticipated for the load adapted design of sandwich components with geometrical constraints, such as limited wall thickness or limitations in total component mass.

2. Materials and manufacturing of sandwich structures

The process of sandwich manufacture is divided into several steps, which include the preparation of textile preforms (1), the mixing and spraying of the polyurethane foam inside an open mould (2), the insert of a second textile layer (3) as well as chemical cross-linking and curing (4, see figure 1) [17]. Initially, glass-fibre plain weave textiles (Type 92105), provided by P-D Interglas Technologies GmbH, with a grammage of 163 g/m² were inserted into the heated open mould (~ 80 °C).

![Figure 1. Manufacturing process of sandwich structures using a polyurethane spray coat method](image)

The processing of the reactive polymer was performed using a polyurethane spray coat method (KraussMaffei Technologies GmbH). As matrix material serves the polyurethane system Elastoflex E3851/102 (BASF Polyurethanes GmbH) [18]. Mould height and foam matrix density were adapted according to the statistical design plan between 8 to 12 mm and 650 to 900 kg/m³, respectively. After a reaction time of 300 seconds, the sandwich plates were demoulded and stored at room temperature for post-curing.

3. Statistical experimental design, plan of experiments, 4-Point-bending-tests

Experimental design using statistical approaches is a common method to improve and optimize scientific and engineering issues. In the current work, a 3²-factorial design was implemented, involving three identified factors, each with at least two levels of interest. For the factor amount of layers, all three levels were included. Beyond that, one additional experimental run is included, being the central point of the design. Preliminary processing studies helped to estimate the magnitude of the response change of each factor and to identify predominant processing restrictions [17]. In the current work, sandwich structures with three different thicknesses (8.0 mm, 10.0 mm and 12.0 mm), three foam core densities (650 kg/m³, 775 kg/m³ and 900 kg/m³) and sandwich top layers with each up to 3 layers of fibre-reinforcement were manufactured. Both factors foam core density and sandwich thickness are considered being numerical and continuous. In contrast, the number of textile layers have numerical and nominal characteristics.

After final curing of the polyurethane foam, specimens for mechanical testing with dimensions according to the German standard DIN 53 293 were produced using water jet cutting. The 4-point-bending test was performed using a ZWICK universal testing machine. The applied force was measured with a 10 kN load cell. At the given load speed of 5 mm/min, the deformation of the sandwich was measured with an ARAMIS 5M optical system (GOM GmbH) and local deflections at
the maximum point (fm) and opposite of the compression die (fs) were determined. Analysis and evaluation of the experimental data including calculation of bending stiffness and bending moment were performed using ORIGINPro 2016G (OriginLab Corporation). For statistical analysis, evaluation and modelling the software JMP Pro 12.1.0 (SAS Institute GmbH) was used.

4. Results and discussion

Bending stiffness was calculated using data in the linear elastic region of the force-deflection curve, bending moment at the point of maximum force measured. Three replicates were tested for each configuration. Table 1 presents the average value of all replicates including statistical scatter and their specific density-related value. However, for statistical modelling the individual results were used instead of the average values.

Table 1. Average results of mechanical investigations.

| Run | Amount of layers [1] | Foam density [kg/m³] | Sandwich thickness [mm] | Bending stiffness [kNmm²] | Specific bending stiffness | Bending moment [Nm] | Specific bending moment |
|-----|----------------------|----------------------|-------------------------|---------------------------|--------------------------|---------------------|------------------------|
| 1   | 1                    | 650                  | 8                       | 3,614 ± 1.5 %             | 5,405 ± 1.5 %            | 18,039 ± 8.5 %      | 26.97                  |
| 2   | 1                    | 650                  | 12                      | 13,064 ± 4.1 %            | 20,156 ± 4.1 %           | 33,330 ± 12.2 %     | 51.42                  |
| 3   | 1                    | 775                  | 10                      | 9,263 ± 5.7 %             | 15,072 ± 5.7 %           | 23,686 ± 5.2 %      | 26.64                  |
| 4   | 1                    | 900                  | 8                       | 15,072 ± 3.5 %            | 26,753 ± 3.5 %           | 37,953 ± 6.4 %      | 43.94                  |
| 5   | 1                    | 900                  | 12                      | 7,226 ± 1.4 %             | 17,451 ± 1.4 %           | 25,908 ± 4.8 %      | 36.45                  |
| 6   | 2                    | 650                  | 8                       | 17,916 ± 4.4 %            | 26,753 ± 4.4 %           | 32,343 ± 6.2 %      | 63.22                  |
| 7   | 2                    | 650                  | 12                      | 19,116 ± 4.4 %            | 26,753 ± 4.4 %           | 42,337 ± 6.2 %      | 63.22                  |
| 8   | 2                    | 900                  | 8                       | 8,991 ± 5.4 %             | 10,167 ± 5.4 %           | 25,908 ± 3.3 %      | 33.79                  |
| 9   | 2                    | 900                  | 12                      | 20,136 ± 9.1 %            | 22,006 ± 9.1 %           | 51,232 ± 5.3 %      | 55.99                  |
| 10  | 3                    | 650                  | 8                       | 8,669 ± 8.6 %             | 11,776 ± 8.6 %           | 29,314 ± 3.2 %      | 38.92                  |
| 11  | 3                    | 650                  | 12                      | 24,562 ± 9.0 %            | 34,801 ± 9.0 %           | 50,901 ± 4.8 %      | 72.12                  |
| 12  | 3                    | 900                  | 8                       | 10,708 ± 9.3 %            | 10,500 ± 9.3 %           | 36,992 ± 5.4 %      | 36.27                  |
| 13  | 3                    | 900                  | 12                      | 26,637 ± 4.4 %            | 26,753 ± 4.4 %           | 51,579 ± 3.5 %      | 55.72                  |

Initially, the influence of sandwich thickness (T), foam core density (D) and number of layers (L) was studied through statistical design for the bending stiffness of the sandwich structures. The analysis of variance was performed for a level of confidence of 95 %. In table 2, the parameters T, D and L were evaluated considering their F-distribution and p-value. Critical F-distribution was determined being $F_c \approx 2.2$ for $df_1 = 11$ (parameter degrees of freedom) and $df_2 = 27$ (error degrees of freedom). Factors with “Test F” < $F_c$ and p > 0.005 were rejected because the effects have a level of significance below 95 %. Thus, the factors D*L, D*T and D*L*T were rejected and a new analysis of variance was performed (table 3).

Table 2. Analysis of variance for bending stiffness.

| Source | SDQ                  | Mean square | df | Test F  | p-values |
|--------|----------------------|-------------|----|---------|----------|
| D      | 4,787E13             | 4,787E13    | 1  | 24.035  | 0.0001   |
| L      | 4,256E14             | 2,128E14    | 2  | 106.848 | 0.0001   |
| T      | 1,240E15             | 1,240E15    | 1  | 622.730 | 0.0001   |
| D*L    | 1,979E12             | 0,989E12    | 2  | 0.4969  | 0.6139   |
| D*T    | 3,788E11             | 3,788E11    | 1  | 0.1902  | 0.6662   |
| L*T    | 8,347E13             | 4,173E13    | 2  | 20.9558 | 0.0001   |
| D*L*T  | 2,590E12             | 1,295E12    | 2  | 0.6502  | 0.5299   |
| Error  | 5,377E13             | 1,991E12    | 27 | -       | -        |
| Total  | 1,856E15             | -           | 38 | -       | -        |

Table 3. Analysis of variance for stiffness with significance effects.
The corresponding model describing the bending stiffness for the studied range in relation to the geometrical factors with statistical significance is given by:

\[
Eb(D, L, T) = 13,658,126 + 1,153,124 \cdot \left(\frac{(D - 775)}{125}\right) + \begin{cases} 
-3,946,071 \text{ (for } L = 1) \\
-90,423 \text{ (for } L = 2) \\
4,036,494 \text{ (for } L = 3)
\end{cases} + 
\begin{cases} 
+ 5,869,471 \cdot \left(\frac{(T - 10)}{2}\right) + \left(\frac{(T - 10)}{2}\right) \cdot \begin{cases} 
-1,625,222 \text{ (for } L = 1) \\
-410,885 \text{ (for } L = 2) \\
2,036,108 \text{ (for } L = 3)
\end{cases}
\end{cases} 
\quad (Eq. 1)
\]

Comparing the experimental and predicted results, the statistical model can be described with \( r^2 = 0.9714 \) and a normalized RMSE = 0.104. The statistical model indicates that the bending stiffness is depending on all studied parameters, but each having a different statistical significance. In particular, the predominant factor is the sandwich thickness followed by the number of textile layers. Since the increase of all parameters contributes to a rising bending stiffness, concurrent effects of the factors \( T \) and \( L \) with an almost equal statistical significance to the foam density were detected. The corresponding response surface for the bending stiffness (left) and their relation to the structure density (right) are given in figure 2.

**Figure 2.** Response curve surfaces for the variation of bending stiffness as a function of foam density, sandwich thickness and amount of textile layers (left) and the bending stiffness related to the sandwich structure density (specific bending stiffness, right).

Since the parameter “\( L \)” (number of textile layers) is assumed to be a nominal factor, each diagram shows three planes, each resembling sandwich structures with the particular amount of textile layers. It is evident from the gained results, that the increase of foam density causes only a slight increase in bending stiffness. In contrast, the sandwich thickness has a major impact on the bending stiffness. Also, each additional layer of textile reinforcement contributes to an improved stiffness significantly. However, the gain in stiffness depends on the interaction of several factors. For instance, the increase in stiffness from a single layer to three layers has a larger impact at a higher sandwich thickness of 12 mm compared to 8 mm. By taking the structural density into account, the specific bending stiffness in
relation to the studied factors is analysed (see figure 2, right). Here, the reduction of the foam core density from 900 kg/m³ to 650 kg/m³ overcompensates the contribution of the foam core density to the bending stiffness. Thus, optimized configurations for lightweight sandwich structures can be found at lower foam core density.

The maximum bending moment is the second studied mechanical property with high relevance for sandwich structures. An analysis of variance of the factors D, L, T and their interaction was conducted in analogy to the bending stiffness (table 4). Again, factors with “Test F” < Fc and p > 0.005 were rejected because of insufficient levels of significance below 95 %.

### Table 4. Analysis of variance for bending moment.

| Source | SDQ      | Mean square | df | Test F  | p-values |
|--------|----------|-------------|----|---------|----------|
| D      | 288,257,011 | 288,257,011 | 1  | 49.6178 | 0.0001   |
| L      | 1,370,538,453 | 685,269,226 | 2  | 117.9557| 0.0001   |
| T      | 2,552,768,888 | 2,552,768,888 | 1  | 439.4091| 0.0001   |
| D*L    | 19,478,985   | 9,739,492   | 2  | 1.6765  | 0.2059   |
| D*T    | 7,741,007    | 7,741,007   | 1  | 1.3325  | 0.2585   |
| L*T    | 19,421,701   | 9,710,850   | 2  | 1.6715  | 0.2068   |
| D*L*T  | 34,336,189   | 17,168,094  | 2  | 2.9552  | 0.0691   |
| Error  | 156,857,840  | 5,809,549   | 27 | -       | -        |
| Total  | 4,449,400,077 | -           | 38 | -       | -        |

Also for the bending moment, all studied main parameters D, L and T are assumed to be statistically significant. However, the remaining interacting factors appearing in table 4 (D*L, D*T, L*T and D*L*T) are rejected and a new analysis of variance was performed (table 5).

### Table 5. Analysis of variance for bending moment with significance effects.

| Source | SDQ      | Mean square | df | Test F  | p-values |
|--------|----------|-------------|----|---------|----------|
| D      | 288,257,011 | 288,257,011 | 1  | 41.2080 | 0.0001   |
| L      | 1,370,538,453 | 685,269,226 | 2  | 97.9632 | 0.0001   |
| T      | 2,552,768,888 | 2,552,768,888 | 1  | 364.9332| 0.0001   |
| Lack of fit | 81,969,339 | 10,246,167 | 8  | 1.7092  | 0.1436   |
| Error  | 237,835,725  | 9,147,527   | 26 | -       | -        |
| Total  | 4,449,400,077 | -           | 38 | -       | -        |

Based on the experimental data, a statistical model considering the significant factors D, L and T with the following equation was determined:

\[
M_b(D, L, T) = 36,177.4 + 2,829.7 \left( \frac{(D-775)}{125} \right) + \left( -7,796.9 \text{ (for } L = 1) \right) + \left( 1,777.7 \text{ (for } L = 2) \right) + \left( 8,420.8 \times \left( \frac{(T-10)}{2} \right) \right) \quad (\text{Eq. 2})
\]

The retrieved model is described by \( r^2 = 0.9465 \) and a normalized RMSE = 0.074. The response surface of the bending moment as a function of the considered geometrical parameters is plotted in figure 3 (left). Again, the results indicate that the sandwich thickness has the main impact on the bending performance, followed by the number of textile layers and lastly the foam density. The results also show a significant increase in maximum bending moment as the number of textile layers increases from a single to double layer at a low sandwich thickness. The gain in bending moment by further increase on up to three textile layers is considerably lower, especially at a sandwich thickness of 12 mm.
Figure 3. Response curve surfaces for the variation of bending moment as a function of foam density, sandwich thickness and amount of textile layers (left) and the bending moment related to the sandwich structure density (specific bending moment, right)

The response surface of the density-related bending moment is plotted in figure 3 (right). Here, the results indicate a significant increase of the specific bending moment by reducing the foam core density, especially at a higher sandwich thickness. In analogy to the results of the specific bending stiffness, the reduction of the foam core density reduces the component density significantly and therefore overcompensates the contribution of the foam core to the bending stiffness.

5. Conclusions
In this paper, the influence of thickness, amount of reinforcing textile layers and foam core density on the bending performance of novel sandwich structures was examined using a design of experiments approach. Evaluation of the experimental data revealed a statistical significance of the parameters T, L and D as well as of the interacting parameter L*T on the bending stiffness and the bending moment, respectively. With the gained statistical models, reliable predictions of the bending performance can be obtained. The plotted response surfaces provided for bending stiffness, bending moment and their density-related values could help to optimize the design of these novel sandwich structures.

References

[1] Schaedler T A and Carter W B 2016 Architected cellular materials Annual Review of Materials Research 46:1 187-210
[2] Gibson L J and Ashby M F 1997 Cellular Solids. Structure and properties Cambridge University Press
[3] Dogan A and Arikan V 2017 Low-velocity impact response of E-glass reinforced thermoset and thermoplastic based sandwich composites Composites Part B 127 63-69
[4] Mohamed M, Anandan S, Huo Z, Birman V, Volz J and Chandrashekara K 2015 Manufacturing and characterization of polyurethane based sandwich composite structures Composite Structures 123 169-79
[5] Nunes J P and Silva J F 2016 Sandwich composites in aerospace engineering in Advanced composite materials for aerospace engineering (Cambridge: Woodhead Publishing) p 129-174
[6] Calabrese L, Bella G D and Fiore V 2016 Manufacture of marine composite sandwich structures in Marine applications of advanced fibre-reinforced composites ed J Graham-Jones and J Summerscales (Cambridge: Woodhead Publishing) p 57-78
[7] Rohleder M and Jakob F 2016 Foam injection molding in Specialized injection molding techniques ed H P Hein (Oxford: William Andrew Publishing) p 53-106
[8] Karlsson K F and Åström B T 1997 Manufacturing and applications of structural sandwich components Composites Part A: Applied Science and Manufacturing 28:2 97-111
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Modelling the Bending Behaviour of Novel Fibre-Reinforced Sandwich Structures with Polyurethane Foam Core

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Abstract. In contrast to conventional manufacturing methods for sandwich structures, in this paper a foamable polyurethane system is used to impregnate textile layers and to create the sandwich foam core in one single process step. Given this simultaneous process, a high adaptability of the resulting mechanical properties can be achieved by varying the textile reinforcement of the sandwich top layers, the foam core density and the sandwich thickness, respectively. Within this paper, a design of experiment approach is implemented by using statistical analysis of variance (ANOVA) to investigate the influence of selected geometrical and material parameters on the resulting bending properties of these novel sandwich structures. A partial factorial design helps to identify the sensitivity of individual parameters and their interaction, as well as the correlation between sandwich morphology and bending performance. Based on the gained results, statistical models will help to identify parameters with statistical significance. Therefore, optimized sandwich structures according to stress-related requirements and geometrical constraints can be designed.

1. Introduction

Sandwich structures are considered being excellent weight-optimized structural components for the use in lightweight applications [1]. In this context, fibre-reinforced plastics with their excellent adjustable mechanical properties are established materials for the use as sandwich face sheets. In many cases, low-density polymer foams, such as polyurethane, serve as core material by stabilizing the sandwich structure and contributing with an improved bending and buckling resistance as well as high energy absorption abilities [2, 3]. Conventional manufacturing technologies for sandwich structures usually include hand lay-up methods (e.g. VARI or VARTM) by draping of textiles on a prefabricated polymer foam core [4, 5, 6] or the reaction injection moulding of the foam material into a predefined cavity made of fibre-reinforced face sheets [7, 8]. Other processing technologies such as the long-fibre injection method involve the spraying of a foamable polyurethane resin with cut glass fibres on a paper honeycomb core [9]. In contrast to these processing technologies, a foamable polyurethane resin was used in the current work to impregnate textile structures and create the sandwich foam core simultaneously.

Although sandwich structures are already well established in numerous applications, many research studies focus on design optimization. In this context, the limitations of local failure modes, e.g. face-core debonding [10] or foam core collapse [11] and the improvement of mechanical properties [12, 13] are among the main motivation. Comprehensive studies were performed to predict the failure behaviour of sandwich panels by developing analytical and numerical models taking different face-
sheet–core material combinations into account [14, 15, 16]. However, most of these works did not focus on a sound statistical investigation of the gained experimental data using ANOVA.

In the present work the bending performance of these novel sandwich structures is analysed using a statistical design of experiments approach. Based on the experimental results, geometrical parameters with statistical significance are selected and statistical models are developed to predict the mechanical properties bending stiffness and maximum bending moment. The main motivation for using these models is anticipated for the load adapted design of sandwich components with geometrical constraints, such as limited wall thickness or limitations in total component mass.

2. Materials and manufacturing of sandwich structures
The process of sandwich manufacture is divided into several steps, which include the preparation of textile preforms (1), the mixing and spraying of the polyurethane foam inside an open mould (2), the insert of a second textile layer (3) as well as chemical cross-linking and curing (4, see Figure 1) [17]. Initially, glass-fibre plain weave textiles (Type 92105), provided by P-D Interglas Technologies GmbH, with a grammage of 163 g/m² were inserted into the heated open mould (~ 80 °C).

![Figure 1. Manufacturing process of sandwich structures using a polyurethane spray coat method](image)

The processing of the reactive polymer was performed using a polyurethane spray coat method (KraussMaffei Technologies GmbH). As matrix material serves the polyurethane system Elastoflex E3851/102 (BASF Polyurethanes GmbH) [18]. Mould height and foam matrix density were adapted according to the statistical design plan between 8 to 12 mm and 650 to 900 kg/m³, respectively. After a reaction time of 300 seconds, the sandwich plates were demoulded and stored at room temperature for post-curing.

3. Statistical experimental design, plan of experiments, 4-Point-bending-tests
Experimental design using statistical approaches is a common method to improve and optimize scientific and engineering issues. In the current work, a 3²-factorial design was implemented, involving three identified factors, each with at least two levels of interest. For the factor amount of layers, all three levels were included. Beyond that, one additional experimental run is included, being the central point of the design. Preliminary processing studies helped to estimate the magnitude of the response change of each factor and to identify predominant processing restrictions [17]. In the current work, sandwich structures with three different thicknesses (8.0 mm, 10.0 mm and 12.0 mm), three foam core densities (650 kg/m³, 775 kg/m³ and 900 kg/m³) and sandwich top layers with each up to 3 layers of fibre-reinforcement were manufactured. Both factors foam core density and sandwich thickness are considered being numerical and continuous. In contrast, the number of textile layers have numerical and nominal characteristics.

After final curing of the polyurethane foam, specimens for mechanical testing with dimensions according to the German standard DIN 53 293 were produced using water jet cutting. The 4-point-
bending test was performed using a ZWICK universal testing machine. The applied force was measured with a 10 kN load cell. At the given load speed of 5 mm/min, the deformation of the sandwich was measured with an ARAMIS 5M optical system (GOM GmbH) and local deflections at the maximum point (fm) and opposite of the compression die (fs) were determined. Analysis and evaluation of the experimental data including calculation of bending stiffness and bending moment were performed using ORIGINPro 2016G (OriginLab Corporation). For statistical analysis, evaluation and modelling the software JMP Pro 12.1.0 (SAS Institute GmbH) was used.

4. Results and discussion

Bending stiffness was calculated using data in the linear elastic region of the force-deflection curve, bending moment at the point of maximum force measured. Three replicates were tested for each configuration.

Table 1 presents the average value of all replicates including statistical scatter and their specific density-related value. However, for statistical modelling the individual results were used instead of the average values.

| Run | Amount of layers [1] | Foam density [kg/m³] | Sandwich thickness [mm] | Bending stiffness [kNm²] | Specific bending stiffness | Bending moment [Nm] | Specific bending moment |
|-----|----------------------|----------------------|-------------------------|--------------------------|---------------------------|---------------------|-------------------------|
| 1   | 1                    | 650                  | 8                       | 3,614 ± 1.5 %            | 5,405                     | 18,039 ± 8.5 %      | 26.97                   |
| 2   | 1                    | 650                  | 12                      | 13,064 ± 4.1 %           | 20,156                    | 33,330 ± 1.2 %      | 51.42                   |
| 3   | 1                    | 775                  | 10                      | 9,263 ± 5.7 %            | 11,779                    | 28,895 ± 3.9 %      | 36.74                   |
| 4   | 1                    | 900                  | 8                       | 7,545 ± 6.7 %            | 8,484                     | 23,686 ± 5.2 %      | 26.64                   |
| 5   | 1                    | 900                  | 12                      | 15,072 ± 3.5 %           | 17,451                    | 37,953 ± 6.4 %      | 43.94                   |
| 6   | 2                    | 650                  | 8                       | 7,226 ± 1.4 %            | 10,167                    | 25,908 ± 4.8 %      | 36.45                   |
| 7   | 2                    | 650                  | 12                      | 17,916 ± 4.4 %           | 26,753                    | 42,337 ± 6.2 %      | 63.22                   |
| 8   | 2                    | 900                  | 8                       | 8,991 ± 5.4 %            | 9,392                     | 32,343 ± 3.3 %      | 33.79                   |
| 9   | 2                    | 900                  | 12                      | 20,136 ± 9.1 %           | 22,006                    | 51,232 ± 5.3 %      | 55.99                   |
| 10  | 3                    | 650                  | 8                       | 8,869 ± 8.6 %            | 11,776                    | 29,314 ± 3.2 %      | 38.92                   |
| 11  | 3                    | 650                  | 12                      | 24,562 ± 9.0 %           | 34,801                    | 50,901 ± 4.8 %      | 72.12                   |
| 12  | 3                    | 900                  | 8                       | 10,708 ± 9.3 %           | 10,500                    | 36,992 ± 5.4 %      | 36.27                   |
| 13  | 3                    | 900                  | 12                      | 26,637 ± 4.4 %           | 28,775                    | 51,579 ± 3.5 %      | 55.72                   |

Table 2. Analysis of variance for bending stiffness.

| Source | SDQ      | Mean square | df | Test F | p-values |
|--------|----------|-------------|----|--------|----------|
| D      | 4,787E13 | 4,787E13    | 1  | 24.035 | 0.0001   |
| L      | 4,256E14 | 2,128E14    | 2  | 106.848| 0.0001   |
| T      | 1,240E15 | 1,240E15    | 1  | 622.730| 0.0001   |
| D*L    | 1,979E12 | 0,989E12    | 2  | 0.4969 | 0.6139   |
| D*T    | 3,788E11 | 3,788E11    | 1  | 0.1902 | 0.6662   |
| L*T    | 8,347E13 | 4,173E13    | 2  | 20,9558| 0.0001   |
| D*L*T  | 2,590E12 | 1,295E12    | 2  | 0.6502 | 0.5299   |
| Error  | 5,377E13 | 1,991E12    | 27 | -      | -        |
| Total  | 1,856E15 | -           | 38 | -      | -        |
Initially, the influence of sandwich thickness (T), foam core density (D) and number of layers (L) was studied through statistical design for the bending stiffness of the sandwich structures. The analysis of variance was performed for a level of confidence of 95%. In Table 2, the parameters T, D and L were evaluated considering their F-distribution and p-value. Critical F-distribution was determined being $F_c \approx 2.2$ for $df_1 = 11$ (parameter degrees of freedom) and $df_2 = 27$ (error degrees of freedom). Factors with “Test F” $< F_c$ and $p > 0.005$ were rejected because the effects have a level of significance below 95%. Thus, the factors D*L, D*T and D*L*T were rejected and a new analysis of variance was performed (Error! Not a valid bookmark reference.).

Table 3. Analysis of variance for stiffness with significance effects.

| Source   | SDQ    | Mean square | df | Test F   | p-values |
|----------|--------|-------------|----|----------|----------|
| D        | 4,787E13| 4,787E13    | 1  | 26.0864  | 0.0001   |
| L        | 4,256E14| 2,128E14    | 2  | 115.9650 | 0.0001   |
| T        | 1,240E15| 1,240E15    | 1  | 675.8645 | 0.0001   |
| L*T      | 8,347E13| 4,174E13    | 2  | 22.7439  | 0.0001   |
| Lack of fit | 5,702E12| 0,950E13    | 6  | 0.4660   | 0.8270   |
| Error    | 5,872E13| 1,835E12    | 32 | -        | -        |
| Total    | 1,856E15|             | 38 |          | -        |

The corresponding model describing the bending stiffness for the studied range in relation to the geometrical factors with statistical significance is given by:

$$E(D, L, T) = 13,658,126 + 1,153,124 \cdot \frac{(D-775)}{125} - 3,946,071 \left(\text{for } L = 1\right) - 90,423 \left(\text{for } L = 2\right) + 5,869,471 \cdot \frac{(T-10)}{2} +$$

$$\begin{cases} 
-1,625,222 \left(\text{for } L = 1\right) \\
-410,885 \left(\text{for } L = 2\right) \\
2,036,108 \left(\text{for } L = 3\right)
\end{cases}$$

(Eq.1)

Comparing the experimental and predicted results, the statistical model can be described with $r^2 = 0.9714$ and a normalized RMSE = 0.104. The statistical model indicates that the bending stiffness is depending on all studied parameters, but each having a different statistical significance. In particular, the predominant factor is the sandwich thickness followed by the number of textile layers. Since the increase of all parameters contributes to a rising bending stiffness, concurrent effects of the factors T and L with an almost equal statistical significance to the foam density were detected. The corresponding response surface for the bending stiffness (left) and their relation to the structure density (right) are given in Figure 2.
Figure 2. Response curve surfaces for the variation of bending stiffness as a function of foam density, sandwich thickness and amount of textile layers (left) and the bending stiffness related to the sandwich structure density (specific bending stiffness, right)

Since the parameter “L” (number of textile layers) is assumed to be a nominal factor, each diagram shows three planes, each resembling sandwich structures with the particular amount of textile layers. It is evident from the gained results, that the increase of foam density causes only a slight increase in bending stiffness. In contrast, the sandwich thickness has a major impact on the bending stiffness. Also, each additional layer of textile reinforcement contributes to a improved stiffness significantly. However, the gain in stiffness depends on the interaction of several factors. For instance, the increase in stiffness from a single layer to three layers has a larger impact at a higher sandwich thickness of 12 mm compared to 8 mm. By taking the structural density into account, the specific bending stiffness in relation to the studied factors is analysed (see Figure 2, right). Here, the reduction of the foam core density from 900 kg/m³ to 650 kg/m³ overcompensates the contribution of the foam core density to the bending stiffness. Thus, optimized configurations for lightweight sandwich structures can be found at lower foam core density.

The maximum bending moment is the second studied mechanical property with high relevance for sandwich structures. An analysis of variance of the factors D, L, T and their interaction was conducted in analogy to the bending stiffness (Table 4). Again, factors with “Test F” < Fc and p > 0.005 were rejected because of insufficient levels of significance below 95 %.

Table 4. Analysis of variance for bending moment.

| Source | SDQ       | Mean square | df  | Test F   | p-values |
|--------|-----------|-------------|-----|----------|----------|
| D      | 288,257,011 | 288,257,011 | 1   | 49.6178  | 0.0001   |
| L      | 1,370,538,453 | 685,269,226 | 2   | 117.9557 | 0.0001   |
| T      | 2,552,768,888 | 2,552,768,888 | 1   | 439.4091 | 0.0001   |
| D*L    | 19,478,985  | 9,739,492   | 2   | 1.6765   | 0.2059   |
| D*T    | 7,741,007   | 7,741,007   | 1   | 1.3325   | 0.2585   |
| L*T    | 19,421,701  | 9,710,850   | 2   | 1.6715   | 0.2068   |
| D*L*T  | 34,336,189  | 17,168,094  | 2   | 2.9552   | 0.0691   |
| Error  | 156,857,840 | 5,809,549   | 27  | -        | -        |
| Total  | 4,449,400,077 | 5,809,549  | 38  | -        | -        |

Also for the bending moment, all studied main parameters D, L and T are assumed to be statistically significant. However, the remaining interacting factors appearing in table 4 (D*L, D*T, L*T and D*L*T) are rejected and a new analysis of variance was performed (Table 5).

Table 5. Analysis of variance for bending moment with significance effects.

| Source   | SDQ       | Mean square | df  | Test F   | p-values |
|----------|-----------|-------------|-----|----------|----------|
| D        | 288,257,011 | 288,257,011 | 1   | 41.2080  | 0.0001   |
| L        | 1,370,538,453 | 685,269,226 | 2   | 97.9632  | 0.0001   |
| T        | 2,552,768,888 | 2,552,768,888 | 1   | 364.9332 | 0.0001   |
| Lack of fit | 81,969,339 | 10,246,167  | 8   | 1.7092   | 0.1436   |
| Error    | 237,835,725  | 9,147,527   | 26  | -        | -        |
| Total    | 4,449,400,077 | -           | 38  | -        | -        |

Based on the experimental data, a statistical model considering the significant factors D, L and T with the following equation was determined:
\[ E(\text{D, L, T}) = 36.177.4 + 2.829.7 \times \left( \frac{\text{D-775}}{125} \right) + \begin{cases} -7.796.9 & \text{(for } \text{L = 1}) \\ 1.777.7 & \text{(for } \text{L = 2}) \\ 6.019.2 & \text{(for } \text{L = 3}) \end{cases} + 8.420.8 \times \frac{(\text{T-10})}{2} \quad \text{(Eq. 2)} \]

The retrieved model is described by \( r^2 = 0.9465 \) and a normalized RMSE = 0.074. The response surface of the bending moment as a function of the considered geometrical parameters is plotted in Figure 3 (left). Again, the results indicate that the sandwich thickness has the main impact on the bending performance, followed by the number of textile layers and lastly the foam density. The results also show a significant increase in maximum bending moment as the number of textile layers increases from a single to double layer at a low sandwich thickness. The gain in bending moment by further increase on up to three textile layers is considerably lower, especially at a sandwich thickness of 12 mm.

![Figure 3](image-url)

**Figure 3.** Response curve surfaces for the variation of bending moment as a function of foam density, sandwich thickness and amount of textile layers (left) and the bending moment related to the sandwich structure density (specific bending moment, right).

The response surface of the density-related bending moment is plotted in Figure 3 (right). Here, the results indicate a significant increase of the specific bending moment by reducing the foam core density, especially at a higher sandwich thickness. In analogy to the results of the specific bending stiffness, the reduction of the foam core density reduces the component density significantly and therefore overcompensates the contribution of the foam core to the bending stiffness.

### 5. Conclusions

In this paper, the influence of thickness, amount of reinforcing textile layers and foam core density on the bending performance of novel sandwich structures was examined using a design of experiments approach. Evaluation of the experimental data revealed a statistical significance of the parameters T, L and D as well as of the interacting parameter L*T on the bending stiffness and the bending moment, respectively. With the gained statistical models, reliable predictions of the bending performance can be obtained. The plotted response surfaces provided for bending stiffness, bending moment and their density-related values could help to optimize the design of these novel sandwich structures.

### 6. References

[1] Schaedler T A and Carter W B 2016 *Architected cellular materials* Annual Review of Materials Research 46:1 187-210

[2] Gibson L J and Ashby M F 1997 *Cellular Solids. Structure and properties* Cambridge University Press

[3] Dogan A and Arikan V 2017 *Low-velocity impact response of E-glass reinforced thermoset and*
thermoplastic based sandwich composites Composites Part B 127 63-69

[4] Mohamed M, Anandan S, Huo Z, Birman V, Volz J and Chandrashekhar K 2015 Manufacturing and characterization of polyurthane based sandwich composite structures Composite Structures 123 169-7

[5] Nunes J P and Silva J F 2016 Sandwich composites in aerospace engineering in Advanced composite materials for aerospace engineering (Cambridge: Woodhead Publishing) p 129-174

[6] Calabrese L, Bella G D and Fiore V 2016 Manufacture of marine composite sandwich structures in Marine applications of advanced fibre-reinforced composites ed J Graham-Jones and J Summerscales (Cambridge: Woodhead Publishing) p 57-78

[7] Rohleder M and Jakob F 2016 Foam injection molding in Specialized injection molding techniques ed P Heim (Oxford: William Andrew Publishing) p 53-106

[8] Karlsson K F and Åström B T 1997 Manufacturing and applications of structural sandwich composites Composites Part A: Applied Science and Manufacturing 28:2 97-111

[9] Hufenbach W, Gude M and Geller S 2012 Function integrated fibre-reinforced polyurethane composite with cellular matrix for intelligent lightweight structures Conference on Cellular Materials (Dresden)

[10] Sun Z, Li D, Zhang W, Shi S and Guo X 2017 Topological optimization of biomimetic sandwich structures with hybrid core and CFRP face sheets Composites Science and Technology 142 79-90

[11] Sun Z, Shi S, Guo X, Hu X and Chen H 2016 On compressive properties of composite sandwich structures with grid reinforced honeycomb core Composites Part B: Engineering 94 245-52

[12] Zhang J, Supernak P, Mueller-Alander S and Wang C 2013 Improving the bending strength and energy absorption of corrugated sandwich composite structure Materials & Design 52 767-73

[13] Padmanabhan K 2014 Strength-based design optimization studies on rigid polyurethane foam core-glc and carbon-glass fabric face sheet/epoxy matrix sandwich composites Mechanics of Advanced Materials and Structures 21:3 191-6

[14] Vitale J P, Francucci G, Xiong J and Stocci A 2017 Failure mode maps of natural and synthetic fiber reinforced composite sandwich panels Composites Part A: Applied Science and Manufacturing 94 21' 25

[15] Martins R, Reis L and Matr-Mendes R 2016 Finite element prediction of stress-strain fields on sandwich composites Procedia Structural Integrity 1 66-73

[16] Steeves C A and Fleck N A 2004 Collapse mechanisms of sandwich beams with composite faces and foam core loaded in three-point bending. Part II: Experimental investigation and numerical modelling International Journal of Mechanical Sciences 46:4 585-08

[17] Gude M, Geller S and Weissenborn O 2014 Integral manufacture of fiber-reinforced sandwich structure with cellular core using a polyurethane spray-coat method Conference on Cellular Materials (Dresden)

[18] Elastoflex E 3851/102 Version 15.02.2007 Material Data Sheet BASF Polyurethanes GmbH

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