Estimating the Sensitivity of the LHC to Electroweak Symmetry Breaking:
Longitudinal-Goldstone Boson Equivalence as a Criterion

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Abstract

Based upon our recent study, we reveal the profound physical content of the longitudinal-Goldstone boson equivalence theorem as being able to discriminate physical processes which are sensitive/insensitive to probing the electroweak symmetry breaking (EWSB) sector. We then develop a precise electroweak power counting rule (a generalization from Weinberg’s counting method) to separately count the power dependences on the energy \(E\) and all relevant mass scales. With these, we analyze the complete set of the bosonic operators in the electroweak chiral Lagrangian and systematically estimate and classify the sensitivities for testing all these effective operators at the CERN LHC via the weak-boson fusions and quark-anti-quark annihilations. These two kinds of processes are shown to be complementary in probing the EWSB mechanism.

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1. Introduction

Despite the impressive success of the Standard Model (SM) over the years, its scalar part, the electroweak symmetry breaking (EWSB) sector, remains as the greatest mystery. Due to Veltman’s screening theorem \[1\], the current low energy data, allowing the SM Higgs boson mass to range from 65.2 GeV to about $O(1) \text{TeV}$ \[2\], tell us little about the EWSB mechanism. Therefore, it is important to probe all possible EWSB mechanisms, either weakly or strongly interacting as long as the light Higgs particle remains undetected. Even if a light resonance is detected at future colliders, it is still crucial to further test whether it is associated with a strong dynamics, because it is unknown \textit{a priori} if such a resonance simply serves as the SM Higgs boson or comes from a more complicated mechanism \[3\]. If the EWSB is driven by a strong dynamics with no new resonance well below the TeV scale, the probe will be more difficult at the future high energy colliders. It is this latter case that we shall currently investigate for the CERN Large Hadron Collider (LHC).

While the transverse components $V_T$ of $W^{\pm}$, $Z^0$ are irrelevant to the EWSB mechanism, the longitudinal weak-bosons ($V_L^a = W_L^{\pm}$, $Z_L^0$), as the products of the spontaneously symmetry-breaking mechanism, are expected to be sensitive to probing the EWSB sector. However, even for the strongly coupled case, studying the $V_L$-scatterings does not guarantee probing the EWSB sector in a sensitive and unambiguous way because the spin-0 Goldstone bosons (GB’s) are invariant under the proper Lorentz transformations, while, on the contrary, both $V_L$ and $V_T$ are Lorentz non-invariant (LNI). After a Lorentz transformation, the $V_L$ component can mix with or even turn into a pure $V_T$. Thus a conceptual ambiguity arises: How can the LNI $V_L$-amplitudes be used to probe the EWSB sector of which the physical mechanism should evidently be independent of the choices of the Lorentz frames? This motivated our recent precise formulation of the electroweak longitudinal-Goldstone boson Equivalence Theorem (ET) in Ref. \[4\]. In the high energy region ($E \gg M_W$), the ET provides a general and quantitative relation between any $V_L$-amplitude and its corresponding GB-amplitude to all loop orders \[4\]- \[8\];

\[^{\text{a}}\] A simple pedagogical discussion on the ET was given at the tree level for amplitudes with one external $V_L$-line in a new textbook by Peskin and Schroeder \[9\]. For an earlier textbook introduction of the ET, see Ref. \[10\].
mer is physically measurable while the latter carries information about the EWSB sector. Hence, the ET allows us to probe the EWSB sector by relating it to the $V_L$-scattering experiments. As will be shown later, the difference between the $V_L$- and GB-amplitudes is intrinsically related to the ambiguous LNI part of the $V_L$-scattering which has the same origin as the $V_T$-amplitude and is thus insensitive to probing the EWSB sector. When the LNI contributions can be safely ignored and the Lorentz invariant (LI) scalar GB-amplitude dominates the experimentally measured $V_L$-amplitudes, the physical $V_L$-scatterings can then sensitively and unambiguously probe the EWSB mechanism. Since the ratio of the LI GB-amplitude to the LNI contributions is process-dependent, it can thus provide a useful theoretical determination on the sensitivities of various scattering processes to probing the EWSB sector.

At the scale below new heavy resonances, the EWSB sector can be parametrized by means of the electroweak chiral Lagrangian (EWCL) in which the $SU(2)_L \otimes U(1)_Y$ gauge symmetry is nonlinearly realized. Without experimental observation of any new light resonance in the EWSB sector, this effective field theory approach provides the most economic description of the possible new physics effects and is thus complementary to those specific model buildings. In the present analysis, taking this conservative and general EWCL approach, we shall concentrate on studying the effective bosonic operators among which the leading order operators are universal and the next-to-leading-order (NLO) operators describe the model-dependent effects in the EWCL. We show in this paper that, for a given process, the ratio of the scalar GB-amplitude to the LNI part of the $V_L$-amplitude, which determines the validity of the ET, varies for different effective operators. The larger this ratio is, the more sensitive this process will be to an operator. Therefore, this ratio can be used to discriminate sensitivities to all the NLO effective operators as well as to the scattering processes for probing the EWSB sector. By formulating the ET as a physical criterion, we shall systematically classify the sensitivities to all these effective operators at the LHC\textsuperscript{b}. We show that the ET is not just a technical tool for

\textsuperscript{b} The actual sensitivity of the LHC to probing these operators will also depend on the detection efficiency for suppressing the backgrounds to observe the specific decay mode of the final state for each given process (as discussed in Ref. [11]). This is beyond the scope of our present theoretical global analysis. In this paper we take the same spirit as Ref. [12] and leave the detection issue to a future detailed and precise numerical study with this work as a useful guideline.
explicitly computing $V_L$-amplitudes via GB-amplitudes; as a criterion, it has an even more profound physical content for being able to theoretically discriminate sensitivities to different effective operators via different processes for probing the EWSB mechanism [1].

In performing such a global analysis, in contrast to just studying a few operators, we need to estimate the contributions of all the NLO operators to various high energy scattering processes. For this purpose, we construct a precise electroweak power counting rule for the EWCL formalism through a natural generalization of Weinberg’s counting method for non-linear sigma model [13]. This simple power counting rule is proven to be extremely convenient and useful for our global analysis.

This paper is organized as follows. We first formulate the ET as a criterion for probing the EWSB mechanism in Sec. 2, and derive a precise electroweak power counting rule for the EWCL formalism in Sec. 3. Then, based upon these we classify the sensitivities of all effective operators at the level of the $S$-matrix elements in Sec. 4. Finally we further analyze, in Sec. 5, the probe of the EWSB sector at the LHC via weak-boson fusions and quark-anti-quark annihilations. Conclusions are given in Sec. 6. Also, a detailed analysis on the the validity of the ET in some special kinematic regions and its implication in probing the EWSB sector is presented in Appendix A. Appendix B is devoted to derive a set of power counting rules for the linearly realized effective Lagrangian formalism which includes the light Higgs SM at the lowest order.

2. Formulating the ET as a Criterion for Probing the EWSB

The precise conditions for the longitudinal-Goldstone boson equivalence, i.e. for the validity of the ET, have been derived in our recent study [4]. In this section, after a further exploration on the physical implications of these conditions we formulate the ET as a criterion for probing the EWSB mechanism at the level of both the $S$-matrix elements and the total cross sections.

We start from the following general identity for the renormalized $S$-matrix elements (cf. the 2nd paper in Ref. [7] for a rigorous derivation):

$$T[V_L^{a_1}, \cdots, V_L^{a_n}; \Phi_\alpha] = C \cdot T[-i\pi^{a_1}, \cdots, -i\pi^{a_n}; \Phi_\alpha] + B \ , \quad (2.1)$$
\begin{align}
C & \equiv C_{\text{mod}}^{a_1} \cdots C_{\text{mod}}^{a_n} = 1 + O(\text{loop}) , \\
B & \equiv \sum_{l=1}^{n} (C_{\text{mod}}^{a_{l+1}} \cdots C_{\text{mod}}^{a_n} T[v^{a_1}, \ldots, v^{a_l}, -i\pi^{a_{l+1}}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + \text{permutations}) , \\
v^a & \equiv v^\mu V^a_\mu , \quad v^\mu \equiv \epsilon^\mu_L - k^\mu/M_V = O(M_V/E) , \quad (M_V = M_W, M_Z) , 
\end{align}

where \( \pi^a \)'s are GB fields, \( \Phi_\alpha \) denotes other possible physical in/out states. The finite constant modification factor \( C_{\text{mod}}^a \) has been systematically studied in Ref. \[7,4\]. From the above identity, we see that the LNI \( V_L \)-amplitude can be decomposed into two parts: the 1st part is \( C \cdot T[-i\pi; \Phi_\alpha] \) which is LI; the 2nd part is the \( v_\mu \)-suppressed \( B \)-term which is LNI because it contains the external spin-1 \( V_\mu \)-field(s). (Without losing generality \[4\], here we have assumed that \( \Phi_\alpha \) contains possible physical scalars, photons and light fermions.) Such a decomposition shows the essential difference between the \( V_L \)- and the \( V_T \)-amplitudes: the former contains a LI GB-amplitude that can yield a large \( V_L \)-amplitude in the case of strongly coupled EWSB sector, but the latter does not. We note that only the LI part (the GB-amplitude) of the \( V_L \)-amplitude is sensitive to probing the EWSB sector, while its LNI part, containing a significant Lorentz-frame-dependent \( B \)-term (which is related to the \( V_L \)-\( V_T \) mixing effects under proper Lorentz transformations), is insensitive to the EWSB mechanism. Thus, for a sensitive and unambiguous probe of the EWSB, we must find conditions under which the LI GB-amplitude dominates the \( V_L \)-amplitude and the LNI \( B \)-term is negligible. It is obvious that one can technically improve the prediction for the \( V_L \)-amplitude from the right-hand side (RHS) of (2.1) by including the complicated \( B \)-term ( or part of \( B \) ) \[14\] or even directly calculate its left-hand side (LHS) of (2.1) despite the complexity. However, this is not an improvement of the longitudinal-Goldstone boson equivalence and thus the sensitivity of probing the EWSB mechanism via \( V_L \)-scattering experiments. The physical content of the ET is essentially independent of how to numerically compute the \( V_L \)-amplitude.

In Ref. \[4\], we estimated the \( B \)-term from a detailed analysis on the LNI \( V_L \)-amplitude.

\footnote{The loop factor \( C_{\text{mod}}^a - 1 = O \left( s^2/\sqrt{\pi} \right) \) for the EWCL formalism if the wavefunction renormalization constant \( Z_\pi \) is subtracted at a scale of \( O(M_W) \); and \( C_{\text{mod}}^a = 1 \) to all orders in some convenient renormalization schemes for all \( R_\xi \)-gauges \[5\].}
and concluded

\[ B \approx O \left( \frac{M_W^2}{E_j^2} \right) T[-i\pi^{a_1}, \cdots, -i\pi^{a_n}; \Phi_\alpha] + O \left( \frac{M_W}{E_j} \right) T[V_{cr_1}^{a_1}, -i\pi^{ar_2}, \cdots, -i\pi^{ar_n}; \Phi_\alpha] . \]

(2.2)

We see that the condition \( E_j \sim k_j \gg M_W, \ (j = 1, 2, \cdots, n) \) for each external longitudinal weak-boson is necessary for making the GB-amplitude much larger than the \( B \)-term (and its Lorentz variation). We thus deduced the general and precise formulation of the ET as \([4]\):d

\[ T[V_L^{a_1}, \cdots, V_L^{a_n}; \Phi_\alpha] = C \cdot T[-i\pi^{a_1}, \cdots, -i\pi^{a_n}; \Phi_\alpha] + O(M_W/E_j)-\text{suppressed} \ , \]

(2.3)

\[ E_j \sim k_j \gg M_W, \quad (j = 1, 2, \cdots, n) \ , \quad (2.3a) \]

\[ C \cdot T[-i\pi^{a_1}, \cdots, -i\pi^{a_n}; \Phi_\alpha] \gg B \ . \quad (2.3b) \]

Here, (2.3a,b) are the precise conditions for the validity of the equivalence in (2.3). We emphasize that, in principle, the complete set of diagrams (including those with internal gauge boson lines) has to be considered when calculating \( T[-i\pi^{a_1}, \cdots, -i\pi^{a_n}; \Phi_\alpha] \), as already implied in (2.3). If not, this equivalence might not manifest for scattering processes involving \( t \)- or \( u \)-channel diagram in either forward or backward direction. A detailed discussion on this point is given in Appendix A. Furthermore, the ET (2.3) and its high energy condition (2.3a) indicate the absence of infrared (IR) power divergences (like \( (\frac{E_j}{M_W})^r, \ r > 0 \) in the \( M_W \to 0 \) limit for fixed energy \( E_j \sim k_j \) \[8\]). This can be understood by noting that the limit \( M_W \to 0 \) implies \( g \to 0 \) after fixing the physical vacuum expectation value (VEV) at \( f_\pi = 246 \text{ GeV} \). Taking \( g \to 0 \) limit leads to the well-defined un-gauged linear or non-linear sigma-model which suggests turning off the gauge coupling to be a smooth procedure. The smoothness of the \( g \to 0 \) limit indicates the absence of IR power divergences for \( M_W \to 0 \). In our present formalism, we shall fix the gauge boson mass \( M_W(M_Z) \) at its experimental value and take the validity of the

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\[ \text{d Note that the } B-\text{term on the RHS of (2.3), as specified, is only } O(M_W/E_j)-\text{suppressed relative to the leading contributions in the GB-amplitude and is therefore not necessarily of the } O(M_W/E_j) \text{ in magnitude. (2.2) explicitly shows that the magnitude of the } B-\text{term depends on the size of the amplitudes } T[-i\pi^{a_1}, \cdots] \text{ and } T[V_{cr_1}^{a_1}, -i\pi^{ar_2}, \cdots] \text{ so that it can be either larger or smaller than } O(M_W/E_j) \text{ [cf. Eq. (2.5)].} \]
ET as a physical criterion for sensitively probing the EWSB sector in the specified high energy regime of a given collider.

The amplitude $T$, to a finite order, can be written as $T = \sum_{\ell=0}^{N} T_\ell$ in the perturbative calculation. Let $T_0 > T_1, \ldots , T_N \geq T_{\text{min}}$, where $T_{\text{min}} = \{T_0, \ldots , T_N\}_{\text{min}}$, then condition (2.3b) and eq. (2.2) imply

$$T_{\text{min}}[-i\pi^{a_1}, \ldots , -i\pi^{a_n}; \Phi_\alpha] \gg \mathcal{O}(\frac{M_R}{E_j}) T_0[-i\pi^{a_1}, \ldots , -i\pi^{a_n}; \Phi_\alpha] + \mathcal{O}(\frac{M_W}{E_j}) T_0[V_{\alpha 1}^{\alpha 1}, -i\pi^{a_2}, \ldots , -i\pi^{a_n}; \Phi_\alpha].$$

(2.4)

Note that the above formulation of the ET discriminates processes which are insensitive to probing the EWSB sector when either (2.3a) or (2.3b) fails. Furthermore, as a necessary criterion, condition (2.4) determines whether or not the corresponding $V_L$-scattering process in (2.3) is sensitive to probing the EWSB sector to the desired precision in perturbative calculations.

From (2.2) or the RHS of (2.4) and the precise electroweak power counting rule (cf. Sec. 3.2), we can directly estimate the largest model-independent $B$-term to be $B_{\text{max}} = O(g^2 f_\pi^{4-n})$ in the EWCL formalism, which comes from the $n$-particle pure $V_L$-amplitude. (This conclusion also holds for the heavy Higgs SM.) It is crucial to note that $B_{\text{max}}$ is of the same order of magnitude as the leading $V_T$-amplitude:

$$B_{\text{max}} \approx T_0[V_T^{a_1}, \ldots , V_T^{a_n}] = O(g^2 f_\pi^{4-n}) .$$

(2.5)

Since both the largest $B$-term and the leading $V_T$-amplitude are of $O(g^2)$, they are therefore irrelevant to the EWSB mechanism as pointed out in the above discussion. Thus, (2.4) provides a useful criterion for discriminating physical processes which are sensitive, marginally sensitive, or insensitive to the EWSB sector.

In conclusion, our formulation of the ET provides a necessary criterion for probing the EWSB sector as follows. If the ET is valid for a scattering process to the order of $T_{\text{min}}$ [i.e. $T_{\text{min}} \gg B_0$, cf. (2.4), where $B_0$ is the tree-level leading contribution to $B$.], this process is classified to be sensitive to probing $T_{\text{min}}$. Otherwise, we classify this process to be either marginally sensitive (for $T_{\text{min}} > B_0$ but $T_{\text{min}} \gg B_0$) or insensitive (for $T_{\text{min}} \leq B_0$) to testing $T_{\text{min}}$. This classification is given at the level of the $S$-matrix elements. Up to the next-to-leading order, $T_{\text{min}} = T_1$. For this case, by simply squaring both side of the condition (2.4) and integrating over the phase space, we can
easily derive the corresponding condition (criterion) at the level of the constituent cross sections for $\hat{\sigma}_1 \simeq \int_{\text{phase}} f_{T_0 T_1}$ and $\hat{\sigma}_B \simeq \int_{\text{phase}} f_{T_0 B_0}$, where $f_{\text{phase}}$ denotes the phase space integration. The constituent cross sections ($\hat{\sigma}$) are functions of the invariant mass ($\sqrt{s}$) of the final state weak bosons. Defining the differential parton luminosity (for either the incoming light fermion or the weak boson) as $\frac{d L_{\text{partons}}}{d \hat{s}}$, the total cross section is thus given by

$$\sigma = \int d \hat{s} \frac{d L_{\text{partons}}}{d \hat{s}} \hat{\sigma}(\hat{s}). \quad (2.6)$$

Using (2.6) we can further derive the corresponding conditions for total event rates $R_1$ (calculated from $\hat{\sigma}_1$) and $R_B$ (calculated from $\hat{\sigma}_B$), and then define the corresponding criterion for testing the sensitivities of various operators and processes at the level of event rates. They are: (i). Sensitive, if $R_1 \gg R_B$; (ii). Marginally sensitive, if $R_1 > R_B$, but $R_1 \gg R_B$; (iii). Insensitive, if $R_1 \leq R_B$. A specific application to the LHC physics is given in Sec. 5. We note that, at the event rate level, the above criterion is necessary but not sufficient since the leading $B$-term, of the same order as the LNI $V_L - V_T$ mixing effects [cf. (2.2)] and as an intrinsic background to any strong $V_L - V_L$ scattering process, denotes a universal part of the full backgrounds [15]. The sufficiency will of course require detailed numerical analyses on the detection efficiency for suppressing the full backgrounds to observe the specific decay mode of the final state (as discussed in Ref. [11]). This is beyond our present first step theoretical global study (see also footnote-b).

Before concluding this section, we note that in the following power counting analysis (cf. Secs. 3-5), both the GB-amplitude and the $B$-term are explicitly estimated (cf. Tables 1-4 in Sec.4). The issue of numerically including/ignoring $B$ in an explicit calculation is essentially irrelevant here. If $T_1 \leq B$, this means that the sensitivity is poor so that the probe of $T_1$ is experimentally harder and requires a higher experimental precision of at least the order of $B$ to test $T_1$.

3. Electroweak Chiral Lagrangian and A Generalized Precise Power Counting Rule

In this section, we first define the EWCL which we investigate in this paper and analyze the current constraints on the next-to-leading order EWCL parameters (whose values are model-dependent and reflect the underlying dynamics). Then, we generalize Weinberg’s counting method [13] and develop a precise counting rule for the EWCL in the energy region $M_W, m_t \ll E < \Lambda$, where the effective cutoff $\Lambda$ is the upper limit of $E$
at which the EWCL formalism ceases to be applicable. [The generalization of Weinberg’s
counting method to the linearly realized effective Lagrangian (including the light Higgs
SM) is given in Appendix B.] As will be shown in Sec. 4 and 5, such a precise electroweak
power rule is particularly convenient and useful for correctly estimating all high energy
scattering amplitudes in performing a global analysis.

3.1. Electroweak Chiral Lagrangian and the Current Constraints

The electroweak chiral Lagrangian (EWCL) gives the most economical
description of the EWSB sector below the scale of new heavy resonance and can be constructed as
follows [16,17]:

\[
\mathcal{L}_{\text{eff}} = \sum_n \ell_n \frac{f_\pi}{\Lambda_{a_n}} O_n(W_{\mu\nu}, B_{\mu\nu}, D_\mu U, f, \bar{f}) = \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_F
\]

where

\[
D_\mu U = \partial_\mu U + igW_\mu U - ig'B_\mu,
\]

\[
U = \exp[i\tau^a \pi^a / f_\pi], \quad W_\mu \equiv W^a_\mu \frac{\tau^a}{2}, \quad B_\mu \equiv B_\mu \frac{\tau^3}{2}.
\]

\(f(\bar{f})\) is the SM fermion with mass \(m_f \leq O(m_t) \simeq O(M_W)\). \(\mathcal{L}_G, \mathcal{L}_S\) and \(\mathcal{L}_F\) denote
gauge boson kinetic terms, scalar boson interaction terms (containing GB self-interactions
and gauge-boson-GB interactions), and fermion interaction terms, respectively. Here we
concentrate on probing new physics from all possible bosonic operators so that we shall
not include the next-to-leading order fermionic operators in \(\mathcal{L}_F\). For clearness, we have
factorized out the dimensionful parameters \(f_\pi\) and \(\Lambda\) in the coefficients so that the
dimensionless factor \(\ell_n \sim O(1)\). This makes our definitions of the \(\ell_n\)'s different from the
\(\alpha_i\)'s in Ref. [16] by a factor of \((f_\pi/\Lambda)^2\). We note that \(f_\pi\) and \(\Lambda\) are the two essential
scales in any effective Lagrangian that describes the spontaneously broken symmetry.
The former determines the symmetry breaking scale while the latter determines the scale
at which new resonance(s) besides the light fields (such as the SM weak bosons, would-
be Goldstone bosons and fermions) may appear. In the non-decoupling scenario, the
effective cutoff scale \(\Lambda\) cannot be arbitrarily large: \(\Lambda = \min(M_{SB}, \Lambda_0) \leq \Lambda_0\)
where \(\Lambda_0 \equiv 4\pi f_\pi \simeq 3.1\) TeV and \(M_{SB}\) is the mass of the lightest new resonance in the EWSB
sector. In (3.1.1), \(r_n = 4 + a_n - D_{O_n}\), where \(D_{O_n} = \text{dim}(O_n)\). The power factor
\(\Lambda^{a_n}\) associated with each operator \(O_n\) can be counted by the naive dimensional analysis
For the bosonic part of EWCL, we have [16]:

\[ \mathcal{L}_G = - \frac{1}{2} \text{Tr}(W_{\mu \nu} W^{\mu \nu}) - \frac{1}{4} B_{\mu \nu} B^{\mu \nu}, \]

\[ \mathcal{L}_S = \mathcal{L}^{(2)} + \mathcal{L}^{(2)'} + \sum_{n=1}^{14} \mathcal{L}_n, \]

\[ \mathcal{L}^{(2)} = \ell_0 \left( \frac{f_\pi}{\Lambda} \right)^2 \left( \frac{f_\pi}{4} \text{Tr}(V_{\mu}) \right)^2, \]

\[ \mathcal{L}^{(2)'} = \ell_1 \left( \frac{f_\pi}{\Lambda} \right)^2 \frac{g}{2} \text{Tr}(W_{\mu \nu}), \]

\[ \mathcal{L}_1 = \ell_2 \left( \frac{f_\pi}{\Lambda} \right)^2 \text{Tr}(\mathcal{V}_{[\mu, \nu]}), \]

\[ \mathcal{L}_2 = \ell_3 \left( \frac{f_\pi}{\Lambda} \right)^2 i g \text{Tr}(W_{\mu \nu}), \]

\[ \mathcal{L}_3 = \ell_4 \left( \frac{f_\pi}{\Lambda} \right)^2 \left( \text{Tr}(\mathcal{V}_{[\mu, \nu]}) \right)^2, \]

\[ \mathcal{L}_4 = \ell_5 \left( \frac{f_\pi}{\Lambda} \right)^2 \left( \text{Tr}(\mathcal{V}_{\mu \nu}) \right)^2, \]

\[ \mathcal{L}_6 = \ell_6 \left( \frac{f_\pi}{\Lambda} \right)^2 \left( \text{Tr}(\mathcal{V}_{\mu \nu}) \right)^2 \text{Tr}(\mathcal{V}_{[\mu, \nu]}), \]

\[ \mathcal{L}_7 = \ell_7 \left( \frac{f_\pi}{\Lambda} \right)^2 \left( \text{Tr}(\mathcal{V}_{\mu \nu}) \right)^2 \text{Tr}(\mathcal{V}_{[\mu, \nu]}), \]

\[ \mathcal{L}_8 = \ell_8 \left( \frac{f_\pi}{\Lambda} \right)^2 g \left( \frac{f_\pi}{4} \text{Tr}(\mathcal{W}_{\mu \nu}) \right)^2, \]

\[ \mathcal{L}_9 = \ell_9 \left( \frac{f_\pi}{\Lambda} \right)^2 g \frac{g'}{2} \text{Tr}(\mathcal{W}_{\mu \nu}) \text{Tr}(\mathcal{W}_{[\mu, \nu]}), \]

\[ \mathcal{L}_{10} = \ell_{10} \left( \frac{f_\pi}{\Lambda} \right)^2 g \left( \frac{f_\pi}{4} \text{Tr}(\mathcal{V}_{\mu}) \right)^2 \text{Tr}(\mathcal{V}_{[\mu, \nu]}), \]

\[ \mathcal{L}_{11} = \ell_{11} \left( \frac{f_\pi}{\Lambda} \right)^2 g \text{Tr}(\mathcal{V}_{\mu \nu}) \text{Tr}(\mathcal{V}_{\mu \nu}), \]

\[ \mathcal{L}_{12} = \ell_{12} \left( \frac{f_\pi}{\Lambda} \right)^2 g \text{Tr}(\mathcal{V}_{\mu \nu}) \text{Tr}(\mathcal{W}_{\mu \nu}), \]

\[ \mathcal{L}_{13} = \ell_{13} \left( \frac{f_\pi}{\Lambda} \right)^2 g \text{Tr}(\mathcal{W}_{\mu \nu}) \text{Tr}(\mathcal{W}_{\mu \nu}), \]

\[ \mathcal{L}_{14} = \ell_{14} \left( \frac{f_\pi}{\Lambda} \right)^2 g \text{Tr}(\mathcal{W}_{\mu \nu}) \text{Tr}(\mathcal{W}_{\mu \nu}). \]

\[ (3.1.2) \]

\[ e \]

In this paper, the NDA is only used to count the \( \Lambda \)-powers ( \( \Lambda^{a_n} \) ) associated with the operators \( O_n \)'s in the chiral Lagrangian (3.1.1). This is irrelevant to the derivation of the power counting rule for \( D_E \) in the following (3.2.4).
where \( W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu] \), \( \mathcal{V}_\mu \equiv (D_\mu U)U^\dagger \), and \( \mathcal{T} \equiv U\tau_3 U^\dagger \). There is certain arbitrariness in choosing the complete set of operators which can be related to another set after applying the equation of motion, but this will not affect the physical results [18]. Eq. (3.1.2) contains fifteen bosonic NLO effective operators among which there are twelve \( CP \)-conserving operators \( (\mathcal{L}^{(2)'}_6, \mathcal{L}_{1~1~11}) \) and three \( CP \)-violating operators \( (\mathcal{L}_{12~14}) \). Furthermore, the operators \( \mathcal{L}_{6,7,10} \) violate custodial \( SU(2)_C \) symmetry (even after \( g' \) being turned off) contrary to \( \mathcal{L}_{4,5} \) which contain \( SU(2)_C \)-invariant pure GB interactions. The coefficients \( (\ell_n)'s \) of all the above operators are model-dependent and carry information about possible new physics beyond the SM. The dimension-2 custodial \( SU(2)_C \)-violating operator \( \mathcal{L}^{(2)'}_6 \) has a coefficient at most of \( O(f_2^2/\Lambda^2) \) since it is proportional to \( \delta\rho = O(m_t^2/(16\pi^2f_2^2)) \approx O(f_2^2/\Lambda^2) \) for the top Yukawa coupling being of \( O(1) \).

In the non-decoupling scenario [19,20], the coefficients for all NLO dimension-4 operators are suppressed by a factor \( (f_2/\Lambda)^2 \approx 1/(16\pi^2) \) relative to that of the universal dimension-2 operator \( \mathcal{L}^{(2)} \), because of the derivative expansion in terms of \( (D_\mu/\Lambda)^2 \). If we ignore the small \( CP \)-violating effects from the Cabibbo-Kobayashi-Maskawa mixings in the lowest order fermionic Lagrangian \( \mathcal{L}_F \), all the one-loop level new divergences generated from \( \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)} \) are thus \( CP \)-invariant. Therefore, the \( CP \)-violating operators \( \mathcal{L}_{12~14} \) are actually decoupled at this level, and their coefficients can have values significantly larger or smaller than that from the naive dimensional analysis [19]. Since the true mechanism for \( CP \)-violation remains un-revealed, we shall consider in this paper the coefficients \( \ell_{12~14} \) to be around of \( O(1) \).

Before proceeding further, let us discuss the current constraints on these EWCL parameters which will be useful for our study in Sec. 5. First, the coefficients \( \ell_{0,1,8} \) are related to the low energy \( S, T, U \) parameters [21] through the oblique corrections:

\[
\ell_0 = \left( \frac{\Lambda}{\Lambda_0} \right)^2 8\pi^2 \alpha T = \left( \frac{\Lambda}{\Lambda_0} \right)^2 8\pi^2 \delta\rho = \left( \frac{\Lambda}{\Lambda_0} \right)^2 \frac{32\pi^3 \alpha}{c_w^2 s_w^2 M_Z^2} [\Pi_{11}^{\text{new}}(0) - \Pi_{33}^{\text{new}}(0)], \\
\ell_1 = -\left( \frac{\Lambda}{\Lambda_0} \right)^2 \pi S = \left( \frac{\Lambda}{\Lambda_0} \right)^2 8\pi^2 \Pi_{33}^{\text{new}}(0), \\
\ell_8 = -\left( \frac{\Lambda}{\Lambda_0} \right)^2 \pi U = \left( \frac{\Lambda}{\Lambda_0} \right)^2 16\pi^2 [\Pi_{33}^{\text{new}}(0) - \Pi_{11}^{\text{new}}(0)],
\]

(3.1.3)

where \( \Lambda_0 \equiv 4\pi f_\pi \simeq 3.1 \text{ TeV} \), and \( s_w = \sin\theta_W \), \( c_w = \cos\theta_W \), \( \alpha = e^2/4\pi \) are measured at \( Z \)-pole in the \( \overline{\text{MS}} \) scheme. The factor \( \left( \frac{\Lambda}{\Lambda_0} \right)^2 \) in (3.1.3) reduces to one for the case
$\Lambda = \Lambda_0$. The updated global fit to the low energy data gives [22]:

\begin{align*}
S &= -0.36 \pm 0.19, \\
T &= -0.03 \pm 0.26, \\
U &= -0.31 \pm 0.54,
\end{align*}

for $s_w^2 = 0.2311 \pm 0.0003$ and $m_t = 181 \pm 12$ GeV. For the present analysis, we have specified the reference value of the SM Higgs mass as $m_H = 1$ TeV. Results for other values of $m_H$ can be found in Ref. [22]. Since the experimental errors in (3.1.4) are still quite large, the parameters $T$ and $U$ ($\ell_0$ and $\ell_8$) can be either positive or negative within 1σ error and the parameter $S$ ($\ell_1$) is only about $-1.89\sigma$ below the SM value.\(^\dagger\)

From (3.1.3) and (3.1.4), for $\Lambda \simeq 3.1$ TeV, we get the following 1σ level constraints on $\ell_{0,1,8}$ at the scale $\mu = M_Z$: \(^9\)

\begin{align*}
-0.18 &\leq \ell_0 \leq 0.14, \\
0.53 &\leq \ell_1 \leq 1.73, \\
-0.72 &\leq \ell_8 \leq 2.67.
\end{align*}

We can further deduce the bounds at the TeV scale (e.g., $\mu = 1$ TeV) by incorporating the running effects from the renormalization log-terms. By the one-loop calculation [24], we find, for instance, the coefficients $\ell_{0,1,8}$ are renormalized as:

\begin{align*}
\ell_0(\mu) &= \ell_0^b - \left(\frac{\Lambda}{\Lambda_0}\right)^2 \frac{3}{4} (g')^2 \left(\frac{1}{\hat{\epsilon}} + c_0\right), \\
\ell_1(\mu) &= \ell_1^b - \left(\frac{\Lambda}{\Lambda_0}\right)^2 \frac{1}{6} \left(\frac{1}{\hat{\epsilon}} + c_1\right), \\
\ell_8(\mu) &= \ell_8^b - \left(\frac{\Lambda}{\Lambda_0}\right)^2 c_8,
\end{align*}

where $\frac{1}{\hat{\epsilon}} \equiv \frac{1}{4 - n} - \ln \mu$ and the superscript “\(^b\)” denotes the bare quantity. In (3.1.6), the $c_i$’s are finite constants which depend on the subtraction scheme and are irrelevant.

\(^\dagger\) This is at the same deviation level as the present $R_b$ anomaly which is $+1.75\sigma$ above the SM value [23].

\(^9\) The $2\sigma$ bounds at $\mu = M_Z$ are: $-0.34 \leq \ell_0 \leq 0.30$, $-0.063 \leq \ell_1 \leq 2.33$, $-2.42 \leq \ell_8 \leq 4.37$, which allow $\ell_{0,1,8}$ to be either positive or negative at $O(1)$ or larger.
to the running of $\ell_n(\mu)$’s. From (3.1.5) and (3.1.6) we can deduce the 1$\sigma$ bounds at other scales, e.g., at $\mu = 1$ TeV for $\Lambda = \Lambda_0 = 3.1$ TeV,

\begin{align}
0.045 & \leq \ell_0(1 \text{ TeV}) \leq 0.37, \\
0.93 & \leq \ell_1(1 \text{ TeV}) \leq 2.13, \\
-0.72 & \leq \ell_8(1 \text{ TeV}) \leq 2.67,
\end{align}

where the ranges for $\ell_{0,1}$ are slightly moved toward positive direction due to the running effects. (3.1.7) shows that $\ell_{0,1,8}$ are allowed to be around of $O(1)$ except that the parameter space for $\ell_0$ is about a factor of $5 \sim 10$ smaller than the others. All those NLO coefficients $\ell_n$’s in (3.1.2) varies for different underlying theories and thus must be independently tested since the real underlying theory is unknown and these operators are inequivalent by the equation of motion. For example, the $SU(2)_C$-violating operator $\mathcal{L}^{(2)\prime}$ (containing two $\mathcal{T} = U T^3 U^\dagger$’s) is constrained to be significantly below $O(1)$ within 1$\sigma$ bound [cf. (3.1.7)], while the updated data still allow the coefficients of other $SU(2)_C$-violating operators such as $\mathcal{L}_1$ (containing one $\mathcal{T}$ operator) and $\mathcal{L}_8$ (containing two $\mathcal{T}$ operators) to be around of $O(1)$ or even larger [cf. (3.1.5) and (3.1.7)].

Besides the bounds from the oblique corrections, the tests on triple gauge boson couplings (TGCs) at LEP and Tevatron \cite{27} impose further constraints on more operators at the tree level. In the conventional notation \cite{27,28}, the TGCs are parameterized as

\begin{equation}
\frac{\mathcal{L}_{WVW}}{g_{WVW}} = ig^V \left[ W^\mu_\mu W^{-\mu_\nu} V^{\nu} - W^{\mu_\nu} W^{-\mu_\mu} V^{\nu} + i \kappa_V W^\mu_\mu W^{-\mu_\nu} V^{\nu} + i \lambda_V W^\mu_\mu W^{-\mu_\nu} V^{\nu} \right] \\
- g^V_4 W^\mu_\mu W^{-\mu_\nu} \left[ \partial^{\mu} V^{\nu} + \partial^{\nu} V^{\mu} \right] + g^V_5 \epsilon^{\mu_\rho_\nu_\lambda} W^{\mu_\rho_\nu_\lambda} V^{\nu} + i \kappa_V W^\mu_\mu W^{-\mu_\nu} V^{\nu} + i \lambda_V W^\mu_\mu W^{-\mu_\nu} \tilde{V}^{\rho_\nu},
\end{equation}

where $V = Z$, or $\gamma$, $W^\pm_\mu = \partial_\mu W^{\pm}_\nu - \partial_\nu W^{\pm}_\mu$, $\tilde{V}^{\mu_\nu} = \frac{i}{2} \epsilon^{\mu_\nu_\rho_\lambda} V^{\rho_\lambda}$.

We summarize the tree level contributions from all 15 NLO order operators listed in
(3.1.2) to the TGCs defined in (3.1.8) as follows:

\[ g_1^Z - 1 \equiv \Delta g_1^Z = \frac{f_\pi^2}{\Lambda^2} \left[ \frac{1}{c^2 - s^2} \ell_0 + \frac{e^2}{c^2(c^2 - s^2)} \ell_1 + \frac{e^2}{s^2 c^2} \ell_3 \right] , \quad g_1^\gamma - 1 = 0 , \]

\[ g_4^Z = -\frac{f_\pi^2}{\Lambda^2 s_w^2 c_w^2} \ell_{12} , \quad g_4^\gamma = 0 , \]

\[ g_5^Z = \frac{f_\pi^2}{\Lambda^2 s_w^2 c_w^2} \ell_{11} , \quad g_5^\gamma = 0 , \]

\[ \kappa_Z - 1 \equiv \Delta \kappa_Z = \frac{f_\pi^2}{\Lambda^2} \left[ \frac{1}{c^2 - s^2} \ell_0 + \frac{2e^2}{c^2} \ell_1 - \frac{e^2}{c^2} \ell_2 + \frac{e^2}{s^2} (\ell_3 - \ell_8 + \ell_9) \right] , \]

\[ \kappa_\gamma - 1 \equiv \Delta \kappa_\gamma = \frac{f_\pi^2 e^2}{\Lambda^2} \left[ -\ell_1 + \ell_2 + \ell_3 - \ell_8 + \ell_9 \right] , \]

\[ \tilde{\kappa}_Z = \frac{f_\pi^2}{\Lambda^2} \left[ \frac{e^2}{c_w} \ell_{13} - \frac{e^2}{s_w^2} \ell_{14} \right] , \quad \tilde{\kappa}_\gamma = -\frac{f_\pi^2 e^2}{\Lambda^2 s_w^2} [\ell_{13} + \ell_{14}] , \]

which coincide with Ref. [25] after taking into account the difference in defining the coefficients. (3.1.9) shows that, at the first non-trivial order [i.e., \( O \left( \frac{f_\pi^2}{\Lambda^2} \right) \)], only operators \( L^{(2)'} \) and \( L_{1,2,3,8,9,11-14} \) can contribute to anomalous triple gauge couplings while \( L^{(4)}_{4,5,6,7,10} \) do not. Among \( L^{(2)'} \) and \( L_{1,2,3,8,9,11-14} \), \( L^{(2)'} \) and \( L_{1,8} \) can be constrained by the oblique corrections [cf. (3.1.5) and (3.1.7)] so that we are left with seven operators \( L^{(4)}_{2,3,9,11,12,13,14} \). There are just seven independent relations in (3.1.9) by which the coefficients \( \ell_{2,3,9,11,12,13,14} \) can be independently determined in principle if the seven TGC parameters \( g_{1,4,5}^Z \) and \( \kappa_{Z,\gamma}, \tilde{\kappa}_{Z,\gamma} \) can all be measured. From (3.1.9), we derive

\[ \ell_2 = \frac{1}{e^2} \frac{s_w^2 c_w^2}{c_w^2 - s_w^2} \ell_0 + \frac{c_w^2}{c_w^2 - s_w^2} \ell_1 + \frac{\Lambda^2 s_w^2 c_w^2}{f_\pi^2 e^2} (\Delta \kappa_\gamma - \Delta \kappa_Z) , \]

\[ \ell_3 = -\frac{1}{e^2} \frac{s_w^2 c_w^2}{c_w^2 - s_w^2} \ell_0 - \frac{c_w^2}{c_w^2 - s_w^2} \ell_1 + \frac{\Lambda^2 s_w^2 c_w^2}{f_\pi^2 e^2} \Delta g_1^Z , \]

\[ \ell_9 = \ell_8 + \frac{\Lambda^2 s_w^2 c_w^2}{f_\pi^2 e^2} \left( \Delta \kappa_Z + \frac{s_w^2}{c_w^2} \Delta \kappa_\gamma - \Delta g_1^Z \right) , \]

\[ \ell_{11} = \frac{\Lambda^2 s_w^2 c_w^2}{f_\pi^2 e^2} g_5^Z , \]

\[ \ell_{12} = -\frac{\Lambda^2 s_w^2 c_w^2}{f_\pi^2 e^2} g_4^Z , \]

\[ \ell_{13} = \frac{\Lambda^2 s_w^2 c_w^2}{f_\pi^2 e^2} (\tilde{\kappa}_Z - \tilde{\kappa}_\gamma) , \]

\[ \ell_{14} = -\frac{\Lambda^2 s_w^2 c_w^2}{f_\pi^2 e^2} \left( \tilde{\kappa}_Z + \frac{s_w^2}{c_w^2} \tilde{\kappa}_\gamma \right) . \]
Inputting the experimentally measured $S$, $T$, $U$ and $g_{1,4,5}^{Z,\gamma}$, $\kappa_{Z,\gamma}$ and $\tilde{\kappa}_{Z,\gamma}$, we can derive constraints on all $\ell_n$’s from (3.1.3) and (3.1.10). For example, a recent global fit at LEP [27] gives the following $1\sigma$ (i.e. 68.27% confidence level) bounds, for allowing only one TGC to be nonzero each time:

$$-0.064 \leq \Delta g_1^Z \leq -0.002 , \quad -0.070 \leq \lambda_\gamma \leq -0.002 , \quad 0.004 \leq \lambda_Z \leq 0.094 ,$$

$$-0.046 \leq \Delta \kappa_Z \leq 0.042 , \quad 0 \leq \Delta \kappa_\gamma \leq 0.112 .$$

(3.1.11)

From the above result we can estimate the constraints on $\ell_{2,3,9}$ as

$$-12.1 \left( \frac{\Lambda}{\Lambda_0} \right)^2 \leq \ell_2 \leq 32.3 \left( \frac{\Lambda}{\Lambda_0} \right)^2 ,$$

$$-18.5 \left( \frac{\Lambda}{\Lambda_0} \right)^2 \leq \ell_3 \leq 0.61 \left( \frac{\Lambda}{\Lambda_0} \right)^2 ,$$

$$-13.3 \left( \frac{\Lambda}{\Lambda_0} \right)^2 \leq \ell_9 \leq 18.5 \left( \frac{\Lambda}{\Lambda_0} \right)^2 ,$$

(3.1.12)

where $\Lambda \leq \Lambda_0 \equiv 4\pi f_\pi \simeq 3.1$ TeV.

At the FermiLab Tevatron, the TGCs can be directly measured at the tree level instead of at the loop level. For instance, the CDF group gives, at the 95% confidence level (C.L.) [27],

$$-1.1 < \Delta \kappa_V < 1.3 , \quad (\text{for } \lambda_V = \Delta g_Y^V = 0 ) ,$$

$$-0.8 < \lambda_V < 0.8 , \quad (\text{for } \Delta \kappa_V = \Delta g_Y^V = 0 ) ,$$

$$-1.2 < \Delta g_1^Z < 1.2 , \quad (\text{for } \lambda_V = \Delta \kappa_V = 0 ) ,$$

(3.1.13)

where $\Delta \kappa_\gamma = \Delta \kappa_Z$ and $\lambda_\gamma = \lambda_Z$ are assumed. Thus we can estimate the 95% C.L. constraints on $\ell_{3,9}$ as

$$-346 \left( \frac{\Lambda}{\Lambda_0} \right)^2 < \ell_3 < 346 \left( \frac{\Lambda}{\Lambda_0} \right)^2 , \quad -412 \left( \frac{\Lambda}{\Lambda_0} \right)^2 < \ell_9 < 488 \left( \frac{\Lambda}{\Lambda_0} \right)^2 ,$$

(3.1.14)

which gives, for $\Lambda = 2$ TeV

$$-145 < \ell_3 < 145 , \quad -173 < \ell_9 < 204 , \quad (\Lambda = 2$ TeV ) .

(3.1.14a)

As shown above, the indirect $1\sigma$ bounds from LEP/SLC allow $\ell_{2,3,9}$ to be around of $O(10)$, and the direct 95% C.L. bounds from Tevatron on $\ell_{3,9}$ are also too weak to be useful in discriminating different dynamical models whose effects to these coefficients $\ell_n$’s are theoretically expected to be of $O(1)$ [19].
Since the operators $\mathcal{L}_{4,5,6,7,10}^{(4)}$ contain only quartic vertices, they cannot be constrained at tree level by any low energy data. The current experiments can only constrain these operators at one-loop level [i.e., of $O(1/\Lambda^4)$]. By calculating the one-loop logarithmic contributions (with all the constant terms ignored) to the low energy data from these operators, one can roughly estimate the indirect experimental bounds on their coefficients $[29,30]$. Since the ignored constant terms are of the same order of magnitude as the logarithmic contributions, we should keep in mind that some uncertainties (like a factor of 2 or so) may naturally exist in these estimated bounds. It was found in Ref. $[29]$ that, at the 90% C.L., the LEP data constraints (allowing only one non-zero coefficient at a time) are

$$-11 < \ell_4 < 11 \ , \quad -28 < \ell_5 < 26 \ ,$$

for the cut-off scale $\Lambda = 2$ TeV. In another recent study $[30]$, for $\Lambda = 2$ TeV and $m_t = 170$ GeV, the following LEP constraints are derived at the 90% C.L.:

$$-3.97 \leq \ell_4 \leq 19.83 \ , \quad -9.91 \leq \ell_5 \leq 50.23 \ ,$$

$$-0.66 \leq \ell_6 \leq 3.50 \ , \quad -5.09 \leq \ell_7 \leq 25.78 \ , \quad -0.67 \leq \ell_{10} \leq 3.44 \ .$$

The above results show that the low energy bounds on the $SU(2)_C$-violating operators $\mathcal{L}_{6,10}$ are close to their theoretical expectation for $\ell_n \sim O(1)$, which are stronger than that for $\mathcal{L}_7$ and the $SU(2)_C$-conserving operators $\mathcal{L}_{4,5}$ (when turning off the $U(1)_Y$ gauge coupling). Thus, $\mathcal{L}_{6,10}$ are more sensitive to the low energy data. But these numerical values should not be taken too seriously (except as a useful guideline) since all non-logarithmic contributions are ignored in the above estimates and the correlations among different operators are not considered for simplicity. We also note that the sensitivities of the $SU(2)_C$-violating operators to the low energy data do not have a naive power-like dependence on the number of $\mathcal{T}$-operators. (3.1.16) shows that the operators $\mathcal{L}_{6,10}$ (containing two and four $\mathcal{T}$-operators, respectively) have quite similar sensitivities to the low energy data and their bounds are much stronger than that for $\mathcal{L}_7$ (containing two $\mathcal{T}$’s). When further looking at the LEP 68.27% C.L. bounds (3.1.12) for the triple gauge boson couplings (TGCs) from the $SU(2)_C$-violating operators $\mathcal{L}_{2,9}$, we find that they are similar and are both much weaker than the 90% C.L. bounds for quartic couplings from $\mathcal{L}_{6,10}$ in (3.1.16) despite $\mathcal{L}_{2,9,6,10}$ containing one, two, two and four $\mathcal{T}$-operators, respectively. Intuitively, it would be natural to expect that the $SU(2)_C$-violating operators may get stronger bounds than the $SU(2)_C$-conserving ones [as implied in (3.1.16) for
the quartic couplings of \( \mathcal{L}_{6,10} \) when we consider the \( SU(2)_C \) as a good approximate symmetry. But the real situation is more involved. From the LEP bounds (3.1.12) and the Tevatron bounds (3.1.14,14a), we see that, for TGCs, the \( SU(2)_C \)-violating operators \( \mathcal{L}_{2,9} \) have weaker bounds than that of the \( SU(2)_C \)-conserving operator \( \mathcal{L}_3 \). Ref. [29] also estimated the 90% C.L. LEP bounds for \( \mathcal{L}_{2,3} \) as \(-47 < \ell_2 < 39\) and \(-8 < \ell_3 < 11\) which impose stronger constraint on \( \ell_3 \). [The relations \( L_{9R} = -2\ell_2, \, L_{9L} = -2\ell_3, \, L_5 = \ell_4, \, L_4 = \ell_5 \) have been used to translate the eq. (39) of Ref. [29] into our notations.]

In summary, the results in (3.1.12), (3.1.14,14a), (3.1.15) and (3.1.16) indicate that the current bounds on \( \ell_{2,3,9} \) and \( \ell_{4,5,7} \) are still too weak. Concerning the \( SU(2)_C \)-violating operators, the bound on \( \ell_0 \) (\( T \)) is most stringent while that on \( \ell_{1,8,6,10} \) are all around \( O(1) \) (or larger) as shown in (3.1.7) and (3.1.16). However, the updated constraints on other \( SU(2)_C \)-violating operators \( \mathcal{L}_{2,9,7} \) can be of \( O(10) \) or larger, implying that the current low energy tests do not well probe the \( SU(2)_C \)-violation effects from these operators. Although LEPII and the upgraded Tevatron are expected to improve the current bounds somewhat, to further improve the precision on \( \ell_n \)'s and to fully probe the EWSB sector require finding the most sensitive high energy scattering processes to independently test all those coefficients in (3.1.2) at the LHC and the future linear colliders (LC).

3.2. A Generalized Precise Electroweak Power Counting Rule

We want to separately count the power dependences of the amplitudes on the energy \( E \), the cutoff scale \( \Lambda \) of the EWCL and the Fermi scale (vacuum expectation value) \( f_\pi = 246 \text{ GeV} \) (\( \sim M_W, m_t \)).\(^h\) This is crucial for correctly estimating the order of magnitude of an amplitude at any given order of perturbative calculation. For instance, an amplitude of order \( \frac{E^2}{\Lambda^2} \) differs by two orders of magnitude from an amplitude of order \( \frac{E^2}{\Lambda^2} \) in spite that they have the same \( E \)-dependence. Also, the amplitudes \( \frac{E^2}{\Lambda^2} \) and \( \frac{E^2}{\Lambda^2} \) have the same sum for the \( E \) and \( \Lambda \) powers, but are clearly of different orders in magnitude. E.g., in the typical case \( E = 1 \text{ TeV} \) and \( \Lambda \approx 4\pi f_\pi \approx 3.1 \text{ TeV} \), they differ by a large factor \( \sim 10 \). Since the weak-boson mass \( M_W = gf_\pi/2 \) and the fermion mass \( m_f = y_f f_\pi/\sqrt{2} \), we can count them through powers of the coupling constants \( g \) and \( y_f \) and the vacuum

\(^h\)This is essentially different from the previous counting result in the literature for the heavy Higgs SM \cite{31} where only the sum of the powers of \( E \) and \( m_H \) has been counted.
expectation value $f_\pi$. The $SU(2)$ weak gauge coupling $g$ and the top quark Yukawa coupling $y_t$ are around of $O(1)$ and thus will not significantly affect the order of magnitude estimates. The electromagnetic $U(1)_{em}$ coupling $e = g \sin \theta_W$ is smaller than $g$ by about a factor of 2. The Yukawa couplings of all light SM fermions other than the top quark are negligibly small. In our following precise counting rule, the dependences on coupling constants $g$, $g'$ (or $e$) and $y_t$ are included, while all the light fermion Yukawa couplings $[y_f (\neq y_t) \ll 1]$ are ignored.

Weinberg’s power counting method was derived for only counting the energy dependence in the un-gauged nonlinear $\sigma$-model as a description of low energy QCD interaction [13]. But some of its essential features are very general: (i). The total dimension $D_T$ of an $S$-matrix element $T$ is determined by the number of external lines and the space-time dimension; (ii). Assume that all mass poles in the internal propagators of $T$ are much smaller than the typical energy scale $E$ of $T$, then the total dimension $D_m$ of the $E$-independent coupling constants included in $T$ can be directly counted according to the type of vertices contained. Hence, the total $E$-power $D_E$ for $T$ is given by $D_E = D_T - D_m$.

Here, we shall make a natural generalization of Weinberg’s power counting method for the EWCL in which, except the light SM gauge bosons, fermions and would-be GB’s, all possible heavy fields have been integrated out. It is clear that in this case the above conditions (i) and (ii) are satisfied. The total dimension of an $L$-loop $S$-matrix element $T$ is

$$D_T = 4 - e \ ,$$  \hspace{2cm} (3.2.1)

where $e = e_B + e_F$, and $e_B$ ($e_F$) is the number of external bosonic (fermionic) lines. Here the dimensions of the external spinor wave functions are already included in $D_T$. For external fermionic lines, we only count the SM fermions with masses $m_f \leq m_t \sim O(M_W) \ll E$. So the spinor wave function of each external fermion will contribute an energy factor $E^{1/2}$ for $E \gg m_f$, where the spinor wave functions are normalized as $\bar{u}(p, s)u(p, s') = 2m_f \delta_{ss'}$, etc.

Let us label the different types of vertices by an index $n$. If the vertex of type $n$ contains $b_n$ bosonic lines, $f_n$ fermionic lines and $d_n$ derivatives, then the dimension of the $E$-independent effective coupling constant in $T$ is

$$D_m = \sum_n V_n \left(4 - d_n - b_n - \frac{3}{2}f_n\right) \ .$$  \hspace{2cm} (3.2.2)
where $V_n$ is the number of vertices of type $n$. Let $i_B$ and $i_F$ be the numbers of internal bosonic and fermionic lines, respectively. ($i_B$ also includes possible internal ghost lines.) Define $i = i_B + i_F$, we have, in addition, the following general relations

\[ \sum_n b_n V_n = 2i_B + e_B, \quad \sum_n f_n V_n = 2i_F + e_F, \quad L = 1 + i - \sum_n V_n. \] (3.2.3)

These can further simplify the terms in (3.2.2).

Note that external vector-boson lines may cause extra contributions to the power of $E$ in $D_E$ due to the $E$-dependence of their polarization vectors since each longitudinal polarization vector $\epsilon_L^\mu$ is of $O(E/M_W)$ for $E \gg M_W$. Thus, if we simply count all external $V_L$-lines directly, the relation between $D_E, D_T$ and $D_m$ will become $D_E = D_T - D_m + e_L - e_v$, where $e_L$ and $e_v$ denote the numbers of external $V_L$ and $v^a$ lines, respectively. [As shown in (2.1c), each external $v^a$-line is a gauge-line $V^a_\mu$ suppressed by the factor $v^\mu = O(M_W/E)$.] However, when this relation is applied to the $V_L$-amplitudes with $D_m$ given in (3.2.2), it does not lead to the correct results. To see this, let us take the $V_LV_L \rightarrow V_LV_L$ scattering amplitude as an example, in which $e_L = 4$ and $e_v = e_F = 0$. To lowest order of the EWCL, the leading powers of $E$ in the amplitudes $T[V_L^{a_1}, \cdots, V_L^{a_4}], T[\pi^{a_1}, \cdots, \pi^{a_4}]$ and the $B$-term [cf. (2.1)] are $E^4, E^2$ and $E^0$, respectively. This is not consistent with the prediction of the ET (2.3). The reason for this inconsistency is that this naive power counting for the $V_L$-amplitude only gives the leading $E$-power for individual Feynman diagrams. It does not reflect the fact that gauge invariance causes the cancellations of the $E^4$-terms between different diagrams, and leads to the final $E^2$-dependence of the whole $V_L$-amplitude. Thus directly counting the external $V_L$-lines in the $V_L$-amplitudes for $D_E$ does not give the correct answer. This problem can be elegantly solved by implementing the ET identity (2.1). We see that the power counting of the GB-amplitude plus the $B$-term does give the correct $E$-dependence because, unlike in the $V_L$-amplitude, there is generally no large $E$-power cancellations in the GB-amplitudes and the $B$-term. Therefore based upon the ET identity (2.1), the correct counting of the powers of $E$ for the $V_L$-amplitude can be given by counting the corresponding GB-amplitude plus the $B$-term. Thus, in the following generalized power counting rule, we do not directly count the the external $V_L$-lines in a given diagram. Instead, they will be counted through counting the RHS of the ET identity (2.1). We shall therefore drop the $e_L$ term in the above relation between $D_E, D_T$ and $D_m$, and make the convention that the number of external vector-boson lines $e_V$ counts only the number of external $V_T$-lines and photon lines. Then from (3.2.1), (3.2.2) and (3.2.3), the feasible formula for the leading
energy power in $T$ is

$$D_E = D_T - D_m - e_v = 2L + 2 + \sum_n \mathcal{V}_n \left( d_n + \frac{1}{2} f_n - 2 \right) - e_v \ .$$

(3.2.4)

This is just the Weinberg’s counting rule [13] in its generalized form with the gauge boson, ghost and fermion fields and possible $\nu_{\mu}$-factors included. (3.2.4) is clearly valid for any gauge theory satisfying the above conditions (i) and (ii).

To correctly estimate the magnitude of each given amplitude $T$, besides counting the power of $E$, it is crucial to also separately count the power dependences on the two typical mass scales of the EWCL: the vacuum expectation value $f_\pi$ and the effective cutoff scale $\Lambda$. If the powers of $f_\pi$ and $\Lambda$ are not separately counted, $\Lambda/f_\pi \simeq 4\pi > 12$ will be mistakenly counted as 1. This can make the estimated results off by orders of magnitudes.

Consider the $S$-matrix element $T$ at the $L$-loop order. Since we are dealing with a spontaneously broken gauge theory which has a nonvanishing vacuum expectation value $f_\pi$, $T$ can always be written as $f^{D_T}_\pi$ times some dimensionless function of $E$, $\Lambda$, and $f_\pi$, etc. The $E$-power dependence has been given by our generalized Weinberg formula (3.2.4). We now count the power of $\Lambda$. The $\Lambda$-dependence in $T$ can only come from two sources:

(i). From tree vertices: $T$ contains $\mathcal{V} = \sum_n \mathcal{V}_n$ vertices, each of which contributes a factor $1/\Lambda^{a_n}$ so that the total factor from $\mathcal{V}$-vertices is $1/\left( \Lambda^{\sum_n a_n} \right)$;

(ii). From loop-level: Since each loop brings a factor $(1/4\pi)^2 = (f_\pi/\Lambda_0)^2$, the total $\Lambda$-dependence from loop contribution is $1/\Lambda_0^{2L}$, where $\Lambda_0 \equiv 4\pi f_\pi \geq \Lambda$.

Hence the total $\Lambda$-dependence given by the above two sources is $1/\left( \Lambda^{\sum_n a_n + 2L} \right)$, which reduces to $1/\left( \Lambda^{\sum_n a_n + 2L} \right)$ in the case $\Lambda \simeq \Lambda_0 = 4\pi f_\pi$. For generality, we shall explicitly keep the loop factor $(1/4\pi)^{2L} = (f_\pi/\Lambda_0)^{2L}$ in eq. (3.2.5) because $\Lambda = \min(M_{SB}, \Lambda_0)$ can be somehow lower than $\Lambda_0 = 4\pi f_\pi \approx 3.1$ TeV for strongly coupled EWSB sector, as indicated by model buildings. From the above discussion, we conclude the following precise counting rule for $T$:

$$T = c_T f^{D_T}_\pi \left( \frac{f_\pi}{\Lambda} \right)^{N_\mathcal{O}} \left( \frac{E}{f_\pi} \right)^{D_{E0}} \left( \frac{E}{\Lambda_0} \right)^{D_{EL}} \left( \frac{M_W}{E} \right)^{e_v} H(\ln E/\mu),$$

$$N_\mathcal{O} = \sum_n a_n \ , \ \ D_{E0} = 2 + \sum_n \mathcal{V}_n \left( d_n + \frac{1}{2} f_n - 2 \right) \ , \ \ D_{EL} = 2L \ , \ \ \Lambda_0 = 4\pi f_\pi \ ,$$

(3.2.5)
where the dimensionless coefficient $c_T$ contains possible powers of gauge couplings ($g$, $g'$) and Yukawa couplings ($y_f$) from the vertices in $T$. $H$ is a function of $\ln(E/\mu)$ which arises from loop integrations in the standard dimensional regularization [16,26] and is insensitive to $E$. Here, $\mu$ denotes the relevant renormalization scale for loop corrections. In Ref. [26], it has been specially emphasized that the dimensional regularization supplemented by the minimal subtraction scheme is most convenient for loop calculations in the effective Lagrangian formalism.

It is useful to give the explicit and compact form of $D_{E0}$ in (3.2.5) for the lowest order EWCL $L_G + L^{(2)} + L_F$. Expanding the interaction terms in $L_G + L^{(2)} + L_F$, we find

$$\sum_n \nu_n = \nu = \nu_F + \nu_{d(V)} + \nu_\pi + \nu_4VVVV + \nu_{VV-\pi} + \nu_{c\bar{c}-\pi},$$

$$\sum_n d_n \nu_n = \nu_{d(V)} = 2\nu_\pi,$$

with

$$\nu_F = \nu_{3FV} + \nu_{FF-\pi},$$

$$\nu_{d(V)} = \nu_{3\pi V} + \sum_{n=1}^{\infty} \nu_{2n+1} + \nu_{VVVV} + \nu_{c\bar{c}\bar{c}},$$

$$\nu_\pi = \sum_{n=2}^{\infty} \nu_{2n}.$$

In the above equations, $\nu_\pi$ denotes the number of vertices with pure GB self-interactions; $\nu_{FF-\pi}$ and $\nu_{c\bar{c}-\pi}$ denote the numbers of fermion-GB vertices and ghost-GB vertices, respectively; $\nu_{VV-\pi}$ denotes the $V-V-\pi^n$ ($n \geq 1$) vertices; and $\nu_{3FV}$ denotes the three-point vertex $F-\bar{F}V$, etc. (Note that $\nu_{c\bar{c}-\pi}$ vanishes in the Landau gauge because of the decoupling of GB fields from ghost fields [16].) Hence, for $L_G + L^{(2)} + L_F$, the $D_{E0}$ factor in (3.2.5) is

$$D_{E0} = 2 - (\nu_{d(V)} + \nu_F + 2\nu_4VVVV + 2\nu_{VV-\pi} + 2\nu_{c\bar{c}-\pi}).$$

This clearly shows that the leading energy-power dependence at $L$-loop level ($L \geq 0$) is always given by those diagrams with pure GB self-interactions, i.e., $(D_E)_{max} = (D_{E0})_{max} + D_{EL} = 2 + 2L$, because of the negative contribution from $-(\nu_{d(V)} + \nu_F + 2\nu_4VVVV + 2\nu_{c\bar{c}-\pi})$ in (3.2.8) which includes all types of vertices except the pure GB

---

$i$ It is straightforward to include the higher order operators in the EWCL for counting $D_{E0}$ although the possible vertices in this case are more complicated.
self-interactions. This conclusion can be directly generalized to all higher order chiral Lagrangian operators such as $\mathcal{L}_n$’s in (3.1.2), and is easy to understand since only pure GB self-interaction-vertices contain the highest powers of the momenta in each order of the momentum expansion. The same conclusion holds for pure $V_L$-scattering amplitudes since they can be decomposed into the corresponding GB-amplitudes plus the $M_W/E$-suppressed $B$-term [cf. (2.1)]. We finally conclude that in the EWCL $(D_E)_{\text{max}} = 2L + 2$ which is independent of the number of external lines of a Feynman diagram. To lowest order of EWCL and at the tree level (i.e. $L = 0$), $(D_E)_{\text{max}} = 2$, which is in accordance with the well-known low energy theorem [32]. For example, by (3.2.5) and (3.2.8), the model-independent tree level contributions to $\pi^{a_1} + \pi^{a_2} \rightarrow \pi^{a_3} + \ldots + \pi^{a_n}$ and $V_T^{a_1} + \pi^{a_2} \rightarrow \pi^{a_3} + \ldots + \pi^{a_n}$ ($n \geq 4$) are estimated as

$$T_0[\pi^{a_1}, \ldots, \pi^{a_n}] = O\left(\frac{E^2}{f_\pi^2} f_\pi^{n-4}\right), \quad B_0^{(0)} = g^2 f_\pi^{n-4};$$

$$T_0[V^{a_1}_T, \pi^{a_2}, \ldots, \pi^{a_n}] = O\left(\frac{g E}{f_\pi} f_\pi^{n-4}\right), \quad B_0^{(1)} = O\left(\frac{g^2 M_W}{E} f_\pi^{n-4}\right),$$

(3.2.9)

where $B_0^{(0)}$ and $B_0^{(1)}$ are the leading order $B$-terms contained in the corresponding $V_L$-amplitudes for the above two processes. (3.2.9) also coincides with the lowest order explicit calculations in Appendix A.

4. Classification of sensitivities at the level of $S$-matrix elements

Armed with the above counting rule (3.2.5), we can conveniently estimate contributions from all effective operators in the EWCL to any high energy scattering process. In the literature (cf. Ref. [11]), what usually done was to study only a small subset of all effective operators for simplicity. But, to discriminate different underlying theories for a complete test of the EWSB mechanism, it is necessary to measure all these operators via various high energy processes. As the first step global study, our electroweak power counting analysis makes it possible to quickly grasp the overall physical picture which provides a useful guideline for selecting relevant operators and scattering processes to perform further detailed numerical studies. In this and the next sections, we shall systematically classify all possible NLO effective operators for both the $S$-matrix elements and the LHC event rates.

We concentrate on the high energy weak-boson fusion and quark-anti-quark annihilation processes. As shown in Refs. [11,33], for the non-resonance case, the most
important fusion process for probing the EWSB sector is the same-charged channel: \( W^\pm W^\pm \rightarrow W^\pm W^\pm \), which gets dominant contributions from the 4-GB vertices in the EWCL. In Tables 1a and 1b we estimate the contributions from the lowest order (model-independent) operators in \( \mathcal{L}_{\text{MI}} \equiv \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)} \) up to one-loop and from all the NLO (model-dependent) bosonic operators in (3.1.2) at the tree-level for \( W^\pm W^\pm \rightarrow W^\pm W^\pm \). The contributions of different operators to a given amplitude are different due to their different structures. For instance, the commonly discussed operators \( \mathcal{L}_{4,5} \) contribute the model-dependent leading term of \( O \left( \frac{g^2 E^2}{f_{\pi}^2} \right) \) to the \( T[4W_L] \) amplitude, and the sub-leading term of \( O \left( \frac{g E^2}{f_{\pi}^2} \right) \) to the \( T[3W_L, W_T] \) amplitude, while \( \mathcal{L}_{3,9} \) give their largest contributions to \( T[3W_L, W_T] \) rather than \( T[4W_L] \) at high energies. The model-independent and model-dependent contributions to various \( B \)-terms are summarized in Tables 2a and 2b, in which \( B^{(i)}_t (i = 0, \cdots, 3; \ell = 0, 1, \cdots) \) denotes the \( B \)-term from \( V_L \)-amplitudes containing \( i \) external \( V_T \)-lines with \( B_0^{(i)} \) obtained from the leading order and \( B^{(i)}_t \) from the NLO calculations. We see that the largest \( B \)-term is \( B^{(0)}_t \) from the \( 4W_L \) amplitudes, as given in (2.5). The term \( B^{(0)}_0 \) [of \( O(g^2) \)], is a model-independent constant containing only the SM gauge coupling constants. All the other \( B \)-terms are further suppressed by a factor of \( M_W/E \) or \( (E/\Lambda)^2 \), or their product.

For all the \( q\bar{q}^{(i)} \rightarrow V^aV^b \) processes (with \( q \) or \( q^{(i)} \) being light quarks except the top), which get dominant contributions from \( s \)-channel diagrams (containing the \( V_T \)-GB-GB vertices), the model-independent and the model-dependent contributions are estimated in Tables 3 and 4, respectively. Note that the tree-level \( q\bar{q} \rightarrow ZZ \) annihilation process has no model-dependent NLO contribution, therefore to probe new physics in the EWSB sector, we have to study \( q\bar{q}^{(i)} \rightarrow W^+W^-, \, W^\pm Z \) annihilations. As shown in Tables 4a and 4b, the operators \( \mathcal{L}_{2,3,9} \) give the leading contributions, of \( O \left( g^2 E^2 \right) \), to \( q\bar{q} \rightarrow W^+W^- \) via \( T_1[q\bar{q}; W_L^+W_L^-] \) channel\(^1\) and the operators \( \mathcal{L}_{3,11,12} \) give the same leading contributions to \( q\bar{q} \rightarrow W^\pm Z \) via \( T_1[q\bar{q}'; W_L^\pm Z_L] \) channel. But \( \mathcal{L}^{(2)} \) does not contribute any positive \( E \)-power term to any of the \( V^aV^b \) final states via tree-level quark-anti-quark annihilations. Tables 3 and 4 also show that the largest \( B \)-term is \( B = B^{(1)}_0 = O \left( g^2 \frac{M_W}{E} \right) \) which is model-independent and comes from \( T_0[q\bar{q}^{(i)}; V_T, v] \), a part of the \( T_0[q\bar{q}^{(i)}; V_T, V_L] \) amplitude (cf. Table 3a). All model-dependent \( B \)-terms, as listed in Tables 4a and 4b, are

\[^1\text{We note that the contributions from } \mathcal{L}_2 \text{ (and also } \mathcal{L}_{1,13} \text{) are always associated with a suppressing factor } \sin^2 \theta_W \sim \frac{1}{4}.\]
either constant terms of \( O \left( (g^4, e^2 g^2) \frac{f^2}{\Lambda^2} \right) \) or further suppressed by negative \( E \)-power(s) and are thus negligibly small.

From Tables 1-2, we further classify in Table 5 the sensitivities to all the bosonic operators for probing the EWSB sector either directly (from pure GB interactions) or indirectly (from interactions suppressed by the SM gauge coupling constants). The same classification for all \( q \bar{q}^{(t)} \rightarrow V^a V^b \) annihilation processes is separately given in Table 6. The classifications in Tables 5 and 6 are based upon the following hierarchy in the power counting:

\[
\frac{E^2}{f^2} \gg \frac{E^2}{f^2 \Lambda^2}, \; \frac{g}{F} \gg \frac{E}{f^2 \Lambda^2}, \; g^2 \gg \frac{g^2 E^2}{\Lambda^2}, \; g^3 \frac{f^2}{E} \gg \frac{g^3 E f^2}{\Lambda^2}, \; g^4 \frac{f^2}{E^2} \gg g^4 \frac{f^2}{\Lambda^2}, \; \text{ (4.1)}
\]

In the typical TeV region, for \( E \in (750 \text{ GeV}, 1.5 \text{ TeV}) \), this gives:

\[
(9.3, 37) \gg (0.55, 8.8), (2.0, 4.0) \gg (0.12, 0.93), (0.42, 0.42) \gg \\
(0.025, 0.099), (0.089, 0.045) \gg (5.3, 10.5) \times 10^{-3}, (19.0, 4.7) \times 10^{-3} \gg (1.1, 1.1) \times 10^{-3},
\]

where \( E \) is taken to be the invariant mass of the \( V V \) pair. The numerical values in (4.2) convincingly show the existence of the power counting hierarchy in (4.1). This governs the order of magnitude of the results from detailed numerical calculations. This hierarchy makes it possible to conveniently and globally classify the sensitivities of various scattering processes to the complete set of the effective operators in the EWCL. The construction of this power counting hierarchy is based upon the property of the chiral perturbation expansion and can be understood as follows. The leading term \( \frac{E^2}{f^2} \) in (4.1) comes from the model-independent lowest order \( 4V_L (\neq 4Z_L) \) scatterings. Starting from this leading term, (4.1) is built up by increasing either the number of derivatives (i.e. the power of \( E/\Lambda \)) or the number of external transverse gauge bosons (i.e. the power of gauge coupling constants). The NLO contributions from the derivative expansion are always suppressed by \( E^2/\Lambda^2 \) relative to the model-independent leading term. Also, for each given process, when an external \( V_L \)-line is replaced by a corresponding \( V_T \)-line, a factor \( \frac{E}{f^2} \) in the amplitude would be replaced by a gauge coupling \( g \) (or \( g' \)).

\( k \) The counting on the amplitudes \( T_0[4W_T] \) and \( T_0[q q^{(t)}; V_T V_T] \) are exceptions of this rule since they have a contribution from the tree-level pure Yang-Mills gauge term \( \mathcal{L}_G \). These two similar exceptions can be found at the second line of Tables 1a and 3a, respectively.
the power counting hierarchy takes the form of (4.1).

Tables 5 and 6 are organized in accordance with the power counting hierarchy given in (4.1) for all $VV$-fusion and $q\bar{q}^{(f)}$-annihilation amplitudes. It shows the relevant effective new physics operators and the corresponding physical processes for probing the EWSB sector when calculating the scattering amplitudes to the required precision. For instance, according to the classification of Table 5, the model-independent operator $\mathcal{L}_{\text{MI}}$ can be probed via studying the leading tree-level scattering amplitude $T_0[4V_L] \neq T_0[4Z_L]$ which is of $O\left(\frac{E^2}{\Lambda^2}\right)$. A sensitive probe of $\mathcal{L}_{\text{MI}}$ via this amplitude requires $T_0 \gg B_0$, i.e., $O\left(\frac{E^2}{\Lambda^2}\right) \gg O(g^2)$ or $(2M_W/E)^2 \ll 1$ which can be well satisfied in the high energy region $E \geq 500$ GeV. We note that the test of the leading order operator $\mathcal{L}_{\text{MI}}$ will first distinguish the strongly interacting EWSB sector from the weakly interacting one. To test the model-dependent operators $\mathcal{L}_{4,5,6,7,10}$ demands a higher precision than the leading tree level contribution by a factor of $\frac{E^2}{\Lambda^2}$. As an example, in order to sensitively test the $\mathcal{L}_{4,5}$ operators with coefficients of $O(1)$ via the $4V_L$-processes, the criterion (2.4) requires $O\left(\frac{E^2}{\Lambda^2}\right) \gg O(g^2)$, or, $(0.7\text{TeV}/E)^4 \ll 1$. This indicates that sensitively probing $\mathcal{L}_{4,5}$ via the $4V_L^\pm$-scatterings requires $E \geq 1$ TeV. Thus, we find that, in the TeV region, the $4V_L$ scatterings can sensitively probe $\mathcal{L}_{4,5}$; while, similarly, $\mathcal{L}_{6,7}$ can be probed via $2W_L + 2Z_L$ or $4Z_L$ scattering and $\mathcal{L}_{10}$ can only be tested via $4Z_L$ scattering. As shown in Table 3, to probe the operators $\mathcal{L}_{2,3,9,11,12}$, one has to detect the $3V_L + V_T$ scatterings, which are further suppressed by a factor $\frac{M_W}{E}$ relative to the leading model-dependent contributions from the $\mathcal{L}_{4,5}$ and $\mathcal{L}_{6,7,10}$ via $4V_L$ processes. Since the model-independent leading order $2V_T + 2V_L$ and $4V_T$ amplitudes (from $\mathcal{L}_{\text{MI}}$) and the largest constant $B$-term ($B_0^{(0)}$) are all around of the same order, i.e. $O\left(\frac{g E^2}{\Lambda^2}\right)$ [cf. (4.2)],\(^{1}\) it requires a significantly higher precision to sensitively probe the operators $\mathcal{L}_{2,3,9,11,12}$ which can only contribute the $g$-suppressed indirect EWSB information and therefore are more difficult to be tested. Here the ratio $B_0/T_1 \sim g^2/\left[\frac{g E^2}{\Lambda^2}\right]$ gives $(1.15\text{TeV}/E)^3 \approx 0.45 \ll 1$ for $E = 1.5$ TeV, which shows the probe of these operators is at most marginally sensitive when their coefficients $\ell_n = O(1)$. Finally, the operators $\mathcal{L}_{1,8,13,14}$ can be probed via the amplitude $T_1[2V_L, 2V_T] \neq T_1[2Z_L, 2Z_T]$ which is of

\(^{1}\) They can in principle be separated if the polarization of the external $V$-lines are identified. For the final state $V$’s, one can study the angular distribution of the leptons from $V$-decay. For the incoming $V$’s, one can use forward-jet tagging and central-jet vetoing to select longitudinal $V$’s [34].
\( O \left( g^2 \frac{E^2}{\Lambda^2}, g^3 \frac{f_{\pi}}{E} \right) \) and numerically much smaller [cf. (4.2)] in comparison with the leading \( B \)-term in (2.5). Therefore, \( L_{1,8,13,14} \) should be effectively probed via scattering processes other than the \( VV \)-fusions.

We then look at Table 6 for \( q\bar{q}(t) \)-annihilations. For the lowest order Lagrangian \( \mathcal{L}_{\text{MI}} = \mathcal{L}_G + \mathcal{L}^{(2)} + \mathcal{L}_F \), the model-independent operators \( \mathcal{L}^{(2)} \) and \( \mathcal{L}_G \) can be probed via tree-level amplitudes [of \( O(g^2) \)] with \( V_LV_L \) and \( V_TV_T \) final states, respectively. Thus, the contribution of \( \mathcal{L}^{(2)} \) to \( V_LV_L \) final state is not enhanced by any \( E \)-power in the high energy region, in contrast to the case of \( VV \)-fusions (cf. Table 1a). So, the leading order \( T_0[q\bar{q}(t); V_LV_L] \) amplitude, similar to the \( T_0[q\bar{q}(t); V_TV_T] \) amplitude, is not sensitive to the strongly coupled EWSB sector. We then discuss the contributions of model-dependent NLO operators to the \( q\bar{q}(t) \)-annihilations. We first note that the operators \( L_{4,5,6,7,10} \) cannot contribute to \( q\bar{q}(t) \)-annihilations at \( 1/\Lambda^2 \)-order and thus should be best probed via \( VV \)-fusions (cf. Table 5). Among all other NLO operators, the probe of \( L_{2,3,9} \) are most sensitive via \( q\bar{q} \rightarrow W_L^+W_L^- \) amplitude and the probe of \( L_{3,11,12} \) are best via \( q\bar{q} \rightarrow W_L^+Z_L \) amplitude. For operators \( L_{1,8,13,14} \), the largest amplitudes are \( T_1[q\bar{q}(t); W_L^+W_T^+/W_T^+W_L^-] \) and \( T_1[q\bar{q}; W_L^+Z_T/W_T^+Z_L] \), which are at most of \( O \left( g^3 \frac{Ef_{\pi}}{\Lambda^2} \right) \) and are suppressed by a factor \( \frac{f_{\pi}}{E} \) relative to the model-dependent leading amplitudes of \( O \left( g^2 \frac{E^2}{\Lambda^2} \right) \).

In summary, applying the power counting technique allows us to conveniently estimate contributions of various operators to any scattering amplitude. For a given scattering process, this result tells us which operators can be sensitively probed. Similarly, the same result can also tell us which process would be most sensitive for probing new physics via a given effective operator. In the next section, we shall examine the important \( W^\pm W^\pm \rightarrow W^\pm W^\pm \) fusion and \( q\bar{q} \rightarrow W^\pm Z \) annihilation processes at the LHC to illustrate how to use the electroweak power counting method to estimate the event rates and how to use the ET as a theoretical criterion to classify the sensitivities of these typical scattering processes to the NLO bosonic operators in the EWCL.

5. Probing EWSB Mechanism at the LHC via Weak-Boson Fusions and

\(^m \) \( \mathcal{L}^{(2)} \) just gives the low energy theorem results and thus denotes the model-independent part of the EWSB sector, while \( \mathcal{L}_G \) is the standard tree-level Yang-Mills gauge term which is irrelevant to the EWSB mechanism.
5.1. Preliminaries

In this section, we estimate the production rates for both weak boson fusions and quark-anti-quark annihilations at the LHC (a pp collider with $\sqrt{s} = 14$ TeV and an integrated luminosity of 100 fb$^{-1}$). The gauge-boson fusion process $W^+W^+ \rightarrow W^+W^+$ and the quark-anti-quark annihilation process $q\bar{q}' \rightarrow W^+Z$ will be separately studied.

To calculate the event rates for gauge-boson fusions, we multiply the luminosity of the incoming weak-boson pair $VV$ (by the effective-W approximation (EWA) [35]) and the constituent cross section of the weak-boson scattering (from the amplitude estimated by the power counting analysis in the last section). Note that the validity of the EWA requires the $VV$ invariant mass $M_{VV} \gg 2M_W$ [35], which coincides with the condition in (2.3a) for the validity of the ET. Thus, the EWA and the ET have similar precisions in computing the event rate from $V_LV_L$ fusion process in hadron collisions. As $M_{VV}$ increases, they become more accurate. It is known that the EWA is less accurate for sub-processes involving initial transverse gauge boson(s) [36,15]. Nevertheless, the EWA has been widely used in the literature for computing event rates from gauge-boson (either transversely or longitudinally polarized) fusion processes because it is easy to implement and can be used to reasonably estimate event rates before any exact calculation is available. As to be shown shortly, our power counting results agree well to the existing detailed calculations within about a factor of 2. Hence, it is appropriate to apply the power counting analysis together with the EWA for estimating the event rates from weak-boson fusions at the LHC. The coincidence for the case of the $q\bar{q}'$-annihilation is even better, where the EWA is not needed.

For the purpose of systematically analyzing the sensitivity to each bosonic operator in (3.1.2), in what follows, we separately compare the rates contributed by each individual operator with that by the $B$-term. The actual experiments contain the contributions from all possible operators and are thus more complicated. For simplicity and clearness, we follow the well-known naturalness assumption (i.e., contributions from different operators

\[\text{Here, we reasonably take the typical energy scale } E \text{ of the } VV \text{ scattering to be } M_{VV} \text{ for estimating the event rates.}\]
do not accidently cancel each other) and estimate the contributions from each operator separately.

Let us denote the production rate for the scattering process \( W_\alpha^+ W_\beta^+ \to W_\gamma^+ W_\delta^+ \) as \( R_{\alpha\beta\gamma\delta(\ell)} \), where \( \alpha, \beta, \gamma, \delta = L, T \) label the polarizations of the \( W \)-bosons and \( \ell = 0, 1 \) indicates contributions from leading order and next-to-leading order, respectively. Up to the one-loop level, we define

\[
R_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta(0)} + R_{\alpha\beta\gamma\delta(1)} ,
\]

\[
R_{\alpha\beta\gamma\delta(\pm)} = R_{\alpha\beta\gamma\delta(0)} \pm |R_{\alpha\beta\gamma\delta(1)}| .
\]

Also, \( R_B \) denotes the rate contributed by the largest \( B \)-term. For convenience, we use the subscript “s” to stand for summing up the polarizations of the corresponding gauge boson. For the \( q\bar{q}^{(l)} \to V_\alpha V_\beta \) processes, the rates are denoted by \( R_{\alpha\beta(\ell)} \), and correspondingly we define \( R_{\alpha\beta} = R_{\alpha\beta(0)} + R_{\alpha\beta(1)} \) and \( R_{\alpha\beta(\pm)} = R_{\alpha\beta(0)} \pm |R_{\alpha\beta(1)}| \). We also note that when applying the power counting analysis, we have ignored the angular dependence in the scattering amplitudes (cf. Tables 1~2) because it does not affect the order of magnitude estimates for the total cross sections (or the event rates).

To check the reliability of our power counting method, we have compared our numerical results for the \( W^+ W^+ \to W_L^+ W_L^+ \) fusion and the \( q\bar{q} \to W_L^+ W_L^- \) annihilation with those in Fig. 8 and Fig. 5 of Ref. [12] in which the above constituent amplitudes were explicitly calculated and the polarizations of the initial weak-bosons were summed over for the fusion process. As shown in Figs. 1a and 1b, both results coincide well within about a factor of 2 or better. These are two typical examples showing that the correct physical picture can be consistently and quickly grasped by our power counting analysis.

We conclude this subsection by briefly commenting on a recent paper [37]. The sole purpose of Ref. [37] was to avoid using the ET, but still within the EWA, to increase the calculation precision and to extend the results to lower energy regions. This approach is, however, inconsistent because the validity of the EWA also requires the same high energy condition \( E \gg M_W \) as that of the ET. To study the operators \( \mathcal{L}_{4,5} \) (dominated by pure \( V_L \)-modes), the approach in Ref. [37] cannot really get higher precision than previous studies [12,11] (using the ET combined with the EWA) except making unnecessary complications and confusions in their calculation. To study other operators like \( \mathcal{L}_{1,2,3,8,9,11} \), the \( V_T \)-modes must be included, for which the EWA is much worse [10,15]. Hence, to get the consistent and precise results for these operators, one must go beyond the EWA for
full calculations. This was not contained in Ref. [37]. On the contrary, our present global power counting analysis provides the first complete and consistent estimate for all NLO operators in the EWCL (3.1.2).

5.2. Analyzing the Model-Independent Contributions to the Events Rates

In Fig. 2a we give the power counting estimates for the production rates of the $W_L^+W_L^+$ pairs at the LHC from the initial state $W$-bosons with different polarizations. In this figure, setting the renormalized coefficients $\ell_{0-14}$ to be zero, we include only the model-independent contributions up to one-loop.\(^6\) As clearly shown in Fig. 2a, the rate from $4W_L$ scattering dominates over the rate from $W_T+3W_L$ scattering. The latter is lower by about an order of magnitude for large $M_{WW}$ in spite of the fact that the $W_TW_L$ luminosity is larger than the $W_LW_L$ luminosity in the initial state. Also separately shown in the same figure is the event rate $|R_B|$ contributed by the largest $B$-term [cf. (2.1) and (2.5)] which is even significantly lower than that from the $W_T+3W_L$ scattering by a factor of $2 \sim 7$ for $M_{WW} > 500$ GeV. However, the rate from $W_TW_T$ initial state is lower than that from the $B$-term in the 4$W_L$ amplitude as $E \geq 600$ GeV. This implies that if the contribution from $W_TW_T$ initial state is to be included in calculating the total production rate of the $W_LW_L$ pair, the contribution from the $B$-term in the 4$W_L$ amplitude should also be included because they are of the same order in magnitude. If, however, only the pure Goldstone boson amplitude $T[\pi^+\pi^+ \rightarrow \pi^+\pi^+]$ is used to calculate the 4$W_L$-amplitude (with the $B$-term ignored) the contribution from $T[W_T^+W_T^+ \rightarrow W_L^+W_L^+]$ should also be consistently ignored for computing the total rate of $W_L^+W_L^+$ pair production via the weak-boson fusion mechanism.

As shown in Ref. [11], it is possible to statistically, though not on the event-by-event basis, choose event with longitudinally polarized $W$-bosons in the initial state by applying the techniques of forward-jet tagging and central-jet vetoing. In this work we do not intend to study the details of the event kinematics, and we shall sum over all the initial state polarizations for the rest of discussions. Let us first compare the rates for different polarizations in the final state. Fig. 2b shows that the rate of $W_LW_L$ final state dominates, while the rate of $B$-term and the rates of $W_LW_T$ and $W_TW_T$ final states are of the same

\(^6\)It is understood that the divergent pieces from one-loop calculations have been absorbed by the coefficients of the corresponding NLO effective operators [16,19].
order, and all of them are about an $O(10)$ to $O(10^2)$ lower than the rate of $W_L W_L$ final state in the energy region $E = M_{WW} > 500$ GeV. Therefore, if one wants to increase the precision in calculating the total event rates by including the small contribution from the $B$-term in pure $4W^+_L$ scattering, then the contributions from $W^+_S W^+_S \to W^+_T W^-_T$ and $W^+_S W^+_S \to W^+_L W^-_L$ scatterings should also be consistently included. Otherwise, they must be neglected all together. From Figs. 2a and 2b, we conclude that the scattering process $W^+_L W^+_L \to W^+_L W^+_L$ dominates the $W^+W^+$-pair productions when the model-dependent coefficients $\ell_{0-14}$ in (3.1.2) are set to be zero.

Similarly, we apply the power counting analysis to estimate the model-independent contribution to the rates of polarized $W^+Z^0$ pair produced from $q\bar{q}'$ fusion up to one-loop order. The results are plotted in Fig. 3. It shows that the $W^+_T Z^+_T$ and $W^+_L Z^+_L$ rates (i.e., $R_{TT}$ and $R_{LL}$) are of the same order. They are much larger than the rate $R_{LT}$ (from the $W^+_L Z^+_T$ and $W^+_T Z^+_L$ final states) and the rate $|R_B|$ (from the largest $B$-term contained in the $q\bar{q}' \to W^+_L Z^+_L, W^+_T Z^+_T$ amplitudes). The rate $R_{LT}$ is only slightly above the $|R_B|$ because the leading GB-amplitude $T_0[q\bar{q}'; \pi^+Z^-/\pi^0W^+]$ is of the same order as the term $B_0^{(1)}$ (cf. Table 3a). Here we see that for the lowest order total signal rate, both $R_{TT}$ and $R_{LL}$ have to be included since they are of the same order and larger than the NLO model-dependent contributions. If one wants to further include $R_{LT}$, then $R_{LT}$ should also be included.

5.3. Estimating Sensitivities for Probing the Model-Dependent Operators

In this section, we classify the sensitivities to all the NLO bosonic operators at the LHC. Without knowing the values of the model-dependent coefficients ($\ell_n$'s), we shall take them to vary from $O(1)$ to $O(10)$ except that $\ell_0$, $\ell_1$ and $\ell_8$ are bounded to be of $O(1)$ by the low energy data [cf. (3.1.7)].

We first consider the scattering process $W^+W^+ \to W^+W^+$. Our theoretical criterion for discriminating different sensitivity levels (sensitive, marginally sensitive, or insensitive) to probe a particular operator via the production of $W^+W^+$ pairs is to compare its contribution to the event rate ($|R_{\alpha\beta\delta(1)}|$) with that from the largest model-independent contribution of the LNI $B$-term ($|R_B|$), according to the Sec. 2. In Figs. 4-7, we show the results for varying $|\ell_n|$ from $O(1)$ to $O(10)$ (except $\ell_0$, $\ell_1$ and $\ell_8$). Here, the polarizations of the initial and the final states have been summed over. In Figs. 4a and 4b, we consider the coefficients ($\ell_n$'s) to be naturally of $O(1)$ according to the
naive dimensional analysis \[19\]. Fig. 4a shows that the event rates/(100 fb\(^{-1}\)GeV) from operators \(\mathcal{L}_{4,5}\) are larger than that from the \(B\)-term when \(E = M_{WW} > 600\) GeV, while the rates from operators \(\mathcal{L}_{3,9,11,12}\) can exceed \(|R_B|\) only if \(E = M_{WW} > 860\) GeV. As \(M_{WW}\) increases, the rates contributed by \(\mathcal{L}_{4,5}\) remain flat, while the rates by \(\mathcal{L}_{3,9,11,12}\) and the \(B\)-term decrease. The ratio of the event rates from \(\mathcal{L}_{4,5}\) to \(|R_B|\) is 5.0 at \(E = M_{WW} = 1\) TeV, and rapidly increases to 19.6 at \(E = M_{WW} = 1.5\) TeV. In contrast, the ratio between the rates from \(\mathcal{L}_{3,9,11,12}\) and the \(B\)-term only varies from 1.4 to 3.0 for \(E = M_{WW} = 1 \sim 1.5\) TeV. Fig. 4b shows that for the coefficients of \(O(1)\), the event rates contributed by operators \(\mathcal{L}_{3,9,11,12}\) and the \(B\)-term are all below \(|R_B|\) for a wide region of energy up to about 2 TeV, so that they cannot be sensitively probed in this case. Especially, the contributions from \(\mathcal{L}_{1,13}\) are about two orders of magnitude lower than that from the \(B\)-term. This suggests that \(\mathcal{L}_{1,13}\) must be tested via other processes \[38\]. In Figs. 5a and 5b, different event rates are compared for the coefficients (except \(\ell_0, \ell_1, \ell_8\)) to be of \(O(10)\). Fig. 5a shows that the rates from \(\mathcal{L}_{3,9,11,12}\) could significantly dominate over \(|R_B|\) by an order of magnitude for \(E = M_{WW} \sim 1\) TeV if their coefficients are increased by a factor of 10 relative to the natural size of \(O(1)\). Fig. 5b shows that the rates from \(\mathcal{L}_{13}\) is still lower than \(|R_B|\) by about an order of magnitude, while the rate from \(\mathcal{L}_2\) agrees with \(|R_B|\) within a factor of 2. The contribution from \(\mathcal{L}_{14}\) exceeds \(|R_B|\) by about a factor 2 \(\sim 3\) at \(E = M_{WW} = 1\) TeV and a factor of 3 \(\sim 5\) at \(E = M_{WW} = 1.5\) TeV when its coefficients is of \(O(10)\).

As discussed above, the cutoff scale \(\Lambda\) can be lower than \(\Lambda_0 = 4\pi f_\pi \simeq 3.1\) TeV if there is any new heavy resonance below \(\Lambda_0\). In that case, the signal rates \(|R_1|\) will be higher than that reported in Figs. 4 and 5 by about a factor of \(\left(\frac{\Lambda_0}{\Lambda}\right)^2\). For comparison, we repeat the above calculations for \(\Lambda = 2\) TeV in Figs. 6 and 7. As indicated in Fig. 6a and 7b, the sensitivities to probing \(\ell_{3,9,11,12} \sim O(1)\) and \(\ell_{14,2} \sim O(10)\) via \(W^+W^+ \rightarrow W^+W^+\) process increase for a lower \(\Lambda\) value.

From the above analyses, we conclude that studying the \(W^+W^+ \rightarrow W^+W^+\) process can sensitively probe the operators \(\mathcal{L}_{4,5}\), but is only marginally sensitive for probing \(\mathcal{L}_{3,9,11,12}\) and insensitive for \(\mathcal{L}^{(2)'}\) and \(\mathcal{L}_{1,2,8,13,14}\), if their coefficients are naturally of \(O(1)\). In the case where these coefficients are of \(O(10)\), the probe of \(\mathcal{L}_{14}\) (for lower \(\Lambda\)) and \(\mathcal{L}_{3,9,11,12}\) could become sensitive and that of \(\mathcal{L}_2\) (for lower \(\Lambda\)) could become marginally sensitive, while \(\mathcal{L}_{13}\) still cannot be sensitively or marginally sensitively measured.

Moreover, we note that the operators \(\mathcal{L}_{6,7,10}\), which violate the custodial \(SU(2)_C\)
symmetry, do not contribute to the $W^+W^+$ pair productions up to $O(1/\Lambda^2)$ . They can however contribute to the other scattering channels such as $WZ \to WZ$, $WW \to ZZ$, $ZZ \to WW$ and $ZZ \to ZZ$, cf. Table 3. (Here, $\mathcal{L}_{10}$ only contributes to $ZZ \to ZZ$ channel.) By our order of magnitude estimates, we conclude that they will give the similar kind of contributions to the $WZ$ or $ZZ$ channel as $\mathcal{L}_{4,5}$ give to the $W^+W^+$ channel. This is because all these operators contain four covariant derivatives [cf. (3.1.2)] and thus become dominant in the high energy $VV$-fusion processes.

Let us consider the $W^-W^- \to W^-W^-$ production process. At the LHC, in the TeV region, the luminosity of $W^-W^-$ is typically smaller than that of $W^+W^+$ by a factor of $3 \sim 5$. This is because in the TeV region, where the fraction of momentum ($x$) of proton carried by the quark (which emitting the initial state $W$-boson) is large ($x = E_\sqrt{S} \sim 0.1$), the parton luminosity is dominated by the valence quark contributions. Since in the large-$x$ region, the probability of finding a down-type valence quark in the proton is smaller than finding an up-type valence quark, the luminosity of $W^-W^-$ is smaller than that of $W^+W^+$. However, as long as there are enough $W^-W^-$ pairs detected, which requires a large integrated luminosity of the machine and a high detection efficiency of the detector, conclusion similar to probing the effective operators for the $W^+W^+$ channel can also be drawn for this channel. For $M_{WW} > 1.5$ TeV, the $W^-W^-$ production rate becomes about an order of magnitude smaller than the $W^+W^+$ rate for any given operator. Thus, this process will not be sensitive to probing the NLO operators when $M_{WW} > 1.5$ TeV.

Next, we examine the $W^\pm Z^0$ production rates in the $qq' \to W^\pm Z^0$ channel. As shown in Figs. 8, for $\Lambda = 3.1$ TeV, when the coefficients are of $O(1)$, the probe of $\mathcal{L}_{3,11,12}$ is sensitive when $E > 750$ GeV, while that of $\mathcal{L}_{8,9,14}$ is marginally sensitive if $E > 950$ GeV. The probe of $\mathcal{L}^{(2)}_{1}^{(2)}$ becomes marginally sensitive if $E > 1.4$ TeV, and that of $\mathcal{L}_{1,2,13}$ is insensitive for $E < 1.9$ TeV. When the coefficients other than $\ell_0$, $\ell_1$, $\ell_8$ are of $O(10)$, $\mathcal{L}_{3,11,12,9,14}$ could all be sensitively probed when $E > 500$ GeV, and the probe of $\mathcal{L}_{2,13}$ could be sensitive when $E > 1.2$ TeV. For $\Lambda = 2$ TeV, the sensitivities are increased overall by a factor $\left(\frac{\Lambda_0}{\Lambda}\right)^2 \sim 2.4$, as shown in Fig. 9. The event rate for $qq' \to W^-Z^0$ is slightly lower than that of $qq' \to W^+Z^0$ by only about a factor of 1.5 due to the lower luminosity for producing $W^-$ bosons in pp collisions. Hence, the above conclusion also holds for the $qq' \to W^-Z^0$ process.

We note that the $qq' \to W^\pm Z^0$ annihilation provides complementary information on probing the EWSB sector, in comparison with the $W^\pm W^\pm \to W^\pm W^\pm$ fusion. In the
former, $L_{3,11,12}$ can be unambiguously probed, while in the latter, $L_{4,5}$ can be sensitively probed. Furthermore, as shown in Table 5 and 6, the operators $L_{6,7}$ can be probed from either $T_1[2W_L, 2Z_L]$ or $T_1[4Z_L]$, and $L_{10}$ can only be probed from $T_1[4Z_L]$, while $L_{2,9}$ can be tested from $T_1[qar{q}; W_L^+ W_L^-]$. It is therefore necessary and useful to measure all the gauge boson fusion and quark-anti-quark annihilation processes for completely exploring the EWSB sector.

6. Conclusions

In this work, based upon our recent study on the intrinsic connection between the longitudinal weak-boson scatterings and probing the EWSB sector, we first formulate the physical content of the ET as a criterion for discriminating processes which are sensitive/insensitive to probing the EWSB mechanism [cf. Eqs. (2.3)∼(2.5)]. Then, we develop a precise power counting rule (3.2.5) for the EWCL, from a natural generalization of Weinberg’s counting method for the ungauged non-linear sigma model. For completeness and for other possible applications, in Appendix B, we also generalize our power counting rule for a linearly realized effective Lagrangian [39] which is often studied in the literature. The renormalizable SM with a light Higgs boson is included in the linear effective Lagrangian formalism at the lowest order.

Armed with this powerful counting rule and using the ET as the theoretical criterion for probing the EWSB sector, we further systematically classify the sensitivities of various scattering processes to the complete set of bosonic operators at the level of $S$-matrix elements (cf. Tables 1-6). The power counting hierarchy in (4.1) governs the order of magnitude of all relevant scattering amplitudes.

Finally, based on the above power counting analysis combined with the EWA, we study the phenomenology for probing the EWSB sector at the LHC via the $W^\pm W^\mp \rightarrow W^\pm W^\mp$ fusion and the $q\bar{q}' \rightarrow W^\pm Z^0$ annihilation processes. In this simple power counting analysis, our numerical results for the production rates agree, within about a factor of 2 (cf. Fig. 1.), with the explicit calculations performed in the literature in which only a small subset of the NLO operators were studied. This indicates that our power counting analysis conveniently and reasonably grasps the overall physical picture. With this powerful tool,
we perform the first complete survey on the sensitivities\textsuperscript{p} of all fifteen next-to-leading order $CP$-conserving and $CP$-violating effective operators at the LHC via $W^\pm W^\pm$-fusions and $q\bar{q}' \rightarrow W^\pm Z^0$ annihilations. The results are shown in Figs. 4-7 and Fig. 8-9, respectively. We find that, for $W^+W^+$-channel, when the coefficients $\ell_n$’s are naturally of $O(1)$, $L_{4,5}$ are most sensitive, $L_{3,9,11;12}$ are marginally sensitive, and $L^{(2)}$ and $L_{1,2,8,13,14}$ are insensitive. For the case where the coefficients other than $\ell_0$, $\ell_1$, $\ell_8$ are of $O(10)$, the probe of $L_{14}$ (for lower $\Lambda$) and $L_{3,9,11;12}$ could become sensitive and that of $L_2$ (for lower $\Lambda$) could become marginally sensitive. However, $L_{13}$ cannot be sensitively probed via this process so that it must be measured via other processes.\textsuperscript{q} A similar conclusion holds for the $W^-W^-$ channel except that the event rate is lower by about a factor of 3 ~ 5 in the TeV region because the quark luminosity for producing a $W^-W^-$ pair is smaller than that for a $W^+W^+$ pair in pp collisions. Up to the next-to-leading order, the $SU(2)_C$-violating operators $L_{6,7,10}$ do not contribute to the $W^\pm W^\pm$ channel. They, however, can be probed via the $WZ \rightarrow WZ$, $WW \rightarrow ZZ$, $ZZ \rightarrow WW$, and $ZZ \rightarrow ZZ$ processes\textsuperscript{38}.

For the $q\bar{q}' \rightarrow W^\pm Z^0$ process, the conclusion is quite different. The operators $L_{4,5,6,7,10}$ do not contribute at the tree level. Using this process, $L_{3,11,12}$ can be sensitively probed in the high energy range ($E > 750$ GeV), and the probe of $L_{8,9,14}$ can be marginally sensitive for $E > 950$ GeV if their coefficients are of $O(1)$ and that of $L_{9,14}$ can be sensitive if their coefficients are of $O(10)$. The results are plotted in Figs. 8-9. We conclude that the $VV$-fusion and the $q\bar{q}'$-annihilation processes are complementary to each other for probing the complete set of the NLO effective operators in the electroweak chiral Lagrangian (3.1.2). Extensions of our analysis to the future linear colliders are given in Ref. [38].

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\textsuperscript{p} Cf. footnote-b.

\textsuperscript{q} We note that $L_{13}$ (and $L_{14}$) can be sensitively probed via $e^-\gamma \rightarrow \nu_eW_L^+Z^0$ or $e^-W_L^+W_L^+$ processes at the future TeV linear collider [38].
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Appendix A. Validity of the ET in some special kinematic regions

Here we examine the validity of the ET in some special kinematic regions and its physical implication in probing the EWSB, which often cause confusion in the literature. It is known that there are kinematic regions in which the Mandelstam variables $t$ or $u$ is small or even vanishing despite the fact that $\sqrt{s} \gg M_W$ for high energy scatterings. Therefore, the amplitude that contains a $t$- or $u$-channel diagram with massless photon field can generate a kinematic singularity when the scattering angle $\theta$ approaches to 0° or 180°. In the following, we study in such special kinematic regions whether the $B$-term [cf. (2.1)] can be safely ignored to validate the ET and its physical consequence to probing the EWSB sector.

For illustration, let us consider the tree level $W_L^+ W_L^- \to W_L^+ W_L^-$ scattering in the chiral Lagrangian formalism. Generalization to loop orders is obvious since the kinematic problem analyzed here only concerns the one-particle-reducible (1PR) internal $W$, $Z$ or photon line in the $t$-channel (or $u$-channel) diagram. Both the tree level $W_L^+ W_L^- \to W_L^+ W_L^-$ and $\pi^+ \pi^- \to \pi^+ \pi^-$ amplitudes in the chiral Lagrangian formalism contain contact diagrams, $s$-channel $Z$-exchange and photon-exchange diagrams, and $t$-channel $Z$-exchange and photon-exchange diagrams. In the C.M. frame, the precise tree-level amplitudes $T[W_L]$ and $T[GB]$ are:

$$T[W_L] = i g^2 \left[ -(1 + \kappa)^2 \sin^2 \theta + 2 \kappa (1 + \kappa) (3 \cos \theta - 1) - c_w^2 \frac{4 \kappa (2 \kappa + 3)^2 \cos \theta}{4 \kappa + 3 - s_w^2 c_w^2} \right]$$

$$+ c_w^2 \frac{8 \kappa (1 + \kappa) (1 - \cos \theta) (1 + 3 \cos \theta) + 2 [(3 + \cos \theta) \kappa + 2] [(1 - \cos \theta) \kappa - \cos \theta]^2}{2 \kappa (1 - \cos \theta) + c_w^2}$$

$$+ i e^2 \left[ - \frac{\kappa (2 \kappa + 3)^2 \cos \theta}{\kappa + 1} + 4 (1 + \kappa) (1 + 3 \cos \theta) + \frac{[(3 + \cos \theta) \kappa + 2] [(1 - \cos \theta) \kappa - \cos \theta]^2}{\kappa (1 - \cos \theta)} \right], \quad (A1a)$$

$$T[GB] = i g^2 \left[ -(1 + \kappa)^2 \sin^2 \theta + 2 \kappa (1 + \kappa) (3 \cos \theta - 1) - c_w^2 \frac{4 \kappa (2 \kappa + 3)^2 \cos \theta}{4 \kappa + 3 - s_w^2 c_w^2} \right]$$

$$+ c_w^2 \frac{8 \kappa (1 + \kappa) (1 - \cos \theta) (1 + 3 \cos \theta) + 2 [(3 + \cos \theta) \kappa + 2] [(1 - \cos \theta) \kappa - \cos \theta]^2}{2 \kappa (1 - \cos \theta) + c_w^2}$$

$$+ i e^2 \left[ - \frac{\kappa (2 \kappa + 3)^2 \cos \theta}{\kappa + 1} + 4 (1 + \kappa) (1 + 3 \cos \theta) + \frac{[(3 + \cos \theta) \kappa + 2] [(1 - \cos \theta) \kappa - \cos \theta]^2}{\kappa (1 - \cos \theta)} \right], \quad (A1b)$$
\[T[\text{GB}] = ig^2 \left( \frac{(1 + \cos \theta)\kappa}{2} + \frac{1}{3} + \frac{c_w^2 - s_w^2}{2c_w^2} \right) \left( -\frac{2\kappa \cos \theta}{4\kappa + 3 - s_w^2 c_w^2} + \frac{(3 + \cos \theta)\kappa + 2}{2(1 - \cos \theta)\kappa + c_w^2} \right) \]

\[+ie^2 \left[ -\frac{4\kappa \cos \theta}{4\kappa + 1} + \frac{(3 + \cos \theta)\kappa + 2}{(1 - \cos \theta)\kappa} \right], \]

(A1b)

where \( \kappa \equiv p^2/M_W^2 \) with \( p \) equal to the C.M. momentum; \( s_w \equiv \sin \theta_W \), \( c_w \equiv \cos \theta_W \) with \( \theta_W \) equal to the weak mixing angle; and \( \theta \) is the scattering angle. In (A1a) and (A1b) the terms without a momentum factor in the denominator come from contact diagrams, terms with denominator independent of scattering angle come from \( s \)-channel diagrams and terms with denominator containing a factor \( 1 - \cos \theta \) are contributed by \( t \)-channel diagrams. Let us consider two special kinematic regions defined below.

(i). In the limit of \( \theta \to 0^\circ \):

As \( \theta \to 0^\circ \), the \( t \)-channel photon propagator has a kinematic pole, but both \( W_L \) and \( \text{GB} \) amplitudes have the same pole structure, i.e.

\[(T[W_L] - T[GB])_{\text{pole term}} = -ie^2 \left[ \frac{[(3 + \cos \theta)\kappa + 2] \cos^2 \theta}{(1 - \cos \theta)\kappa} - \frac{(3 + \cos \theta)\kappa + 2}{(1 - \cos \theta)\kappa} \right] \]

\[= -ie^2 (1 + \cos \theta)(3 + \cos \theta + 2\kappa^{-1}) = O(g^2), \]

which is finite.\(^7\) Hence, the \( B \)-term, which is defined as the difference \( T[W_L] - T[GB] \), is finite at \( \theta = 0^\circ \), and is of \( O(g^2) \). This means that when \( \theta \) is close to the \( t \)-channel photon pole, the \( B \)-term is negligibly small relative to the \( \text{GB} \)-amplitude so that (2.3b) is satisfied and the ET works. More explicitly, in the limit of \( \theta = 0^\circ \) (i.e. \( t = 0 \)), and from (A1a,b), the \( W_L \) and \( \text{GB} \) amplitudes are

\[T[W_L] = i \left[ 4(3 - 8c_w^2 + 8c_w^4)\frac{p^2}{f_\pi^2} + 2e^2 \left( 2 + \frac{M_W^2}{p^2} \right) \frac{1}{1 - c_0} \right] + O(g^2), \]

\[T[\text{GB}] = i \left[ 4(3 - 8c_w^2 + 8c_w^4)\frac{p^2}{f_\pi^2} + 2e^2 \left( 2 + \frac{M_W^2}{p^2} \right) \frac{1}{1 - c_0} \right] + O(g^2), \]

\[T[W_L] = T[\text{GB}] + O(g^2), \]

\(^7\) This conclusion can be directly generalized to other \( t \) or \( u \) channel processes.
where \( c_0 \equiv \lim_{\theta \to 0} \cos \theta \). Notice that in this case one cannot make the \( M_W^2/t \) expansion\(^a\) because \( t \) vanishes identically. Since both \( W_L \) and GB amplitudes have exactly the same kinematic singularity and the \( B \)-term is much smaller than \( T[GB] \), the ET still holds in this special kinematic region. We also emphasize that in the kinematic regions where \( t \) or \( u \) is not much larger than \( M_W^2 \), the \( t \)-channel or \( u \)-channel internal gauge boson lines must be included according to the precise formulation of the ET [cf. (2.3) and (2.3a,b)].\(^t\)

(ii). In the limit of \( \theta \to 180^\circ \):

In the kinematic region with \( s, t \gg M_W^2 \), (A1a) and (A1b) yield

\[
T[W_L] = i \left[ 2(1 + \cos \theta) \frac{p^2}{f^2} + O(g^2) \right],
\]

\[
T[GB] = i \left[ 2(1 + \cos \theta) \frac{p^2}{f^2} + O(g^2) \right],
\]

\[
T[W_L] = T[GB] + O(g^2),
\]

where the \( O(g^2) \) term is the largest term we ignored which denotes the order of the \( B \)-term [cf. (2.5)]; all other terms we ignored in (A4) are of \( O(M_W^2/p^2) \) or \( O(e^2) \) which are smaller than \( O(g^2) \) and thus will not affect the order of magnitude estimate of the \( B \)-term. For \( s, t \gg M_W^2 \), the \( W_L \) and GB amplitudes are dominated by the \( p^2 \)-term in (A4), which is actually proportional to \( u \) for this process. When the scattering angle \( \theta \) is close to \( 180^\circ \), \( u \) becomes small and thus this leading \( p^2 \) term is largely suppressed so that both the \( W_L \) and GB amplitudes can be as small as the \( B \)-term, i.e. of \( O(g^2) \). In this case our condition (2.3a) is satisfied while (2.3b) is not, which means that the EWSB sector cannot be sensitively probed for this kinematic region. Since the total cross section of this process is not dominated by this special kinematic region and is mainly determined by the un-suppressed leading large \( p^2 \)-term, so the kinematic dependence of the amplitude will not affect the order of magnitude of the total cross section. Hence, our application of the power counting analysis in Sec. 5 for computing the total event rates remains valid even though we have ignored the angular dependence in estimating the magnitude of the

\(^a\)This expansion is unnecessary for the validity of the ET, cf. (2.3) and (2.3a,b).

\(^t\) This does not imply, in any sense, a violation of the ET since the ET, cf. (2.3) and (2.3a,b), does not require either \( t \gg M_W \) or \( u \gg M_W \).
scattering amplitudes. Neglecting the angular dependence in the amplitude may cause a small difference in the event rate as compared to that from a precise calculation. For the processes such as $W^+_L W^-_L \rightarrow W^+_L W^-_L$ and $W^+_L W^-_L \rightarrow Z_L Z_L$, the leading $p^2$-term is proportional to $s/f_\pi^2$ with no angular dependence, so that the angular integration causes no difference between our power counting analysis and the exact calculation for the leading $p^2$-term contribution. In the above example for $W^+_L W^-_L \rightarrow W^+_L W^-_L$ channel [cf. (A4)], the leading amplitude is proportional to $-u/f_\pi^2$. When applying the power counting method, we ignore the $\theta$-dependence and estimate it as $s/f_\pi^2$. In computing the total rate, we integrate out the scattering angle. This generates a difference from the precise one:

$$\frac{\int_{-1}^{1} u^2 \cos \theta \, du \cos \theta}{\int_{-1}^{1} s^2 \cos \theta \, du \cos \theta} = \frac{1}{3},$$

which, as expected, is only a factor of 3 and does not affect our order of magnitude estimates.

Finally, we make a precise numerical analysis on the equivalence between the $W_L$ and the GB amplitudes to show how well the ET works in different kinematic regions and its implication to probing the EWSB sector. We use the full expressions (A1a,b) for $W_L$ and GB amplitudes as required by the ET, cf. (2.3) and (2.3a,b). In Fig. 10a, we plot the ratio $|B/g^2|$ for scattering angle $\theta = 2^\circ, 10^\circ, 45^\circ, 90^\circ, 120^\circ, 135^\circ, 150^\circ, 180^\circ$. It shows that the LNI $B$-term is always of $O(g^2)$ in the whole kinematic region, and thus is irrelevant to the EWSB sector, in accordance with our general physical analysis in Sec. 2. Hence, to have a sensitive probe of the EWSB mechanism, condition (2.3b) or (2.4) must be satisfied. Fig. 10b shows that for $0^\circ \leq \theta \leq 100^\circ$, the ratio $|B/T[W_L]| \leq 10\%$ when $M_{WW} \geq 500$ GeV. For $\theta \geq 120^\circ$, this ratio becomes large and reaches $O(1)$ when $\theta$ is close to $180^\circ$. This is because the kinematic factor $(1 + \cos \theta)$, associated with the leading $p^2$ term [cf. (A4)], becomes small. This, however, will not alter the conclusion that for $4W_L$-scattering the total cross section from $T[GB]$ is much larger than that from the $B$-term as $M_{WW} \geq 500$ GeV.\footnote{Note that in Fig. 10b, for $\theta \leq 10^\circ$, i.e. close to the}

$^u$ The small difference (a factor of 1.4) in Fig. 1 mainly comes from neglecting the tree level sub-leading terms in our order of magnitude estimate for the amplitudes.

$^v$ In practice, this is even more true after applying the necessary kinematic cuts to require the final state $W$-bosons to be in the central rapidity region of the detector for detecting the signal event [11], so that
$t$-channel photon pole, the ratio $|B/T| [W_L]|$ is below 1% and thus the ET holds very well. In Fig. 10c, we plot both the $W_L$ and GB amplitudes for $\theta = 10^\circ, 45^\circ, 100^\circ, 150^\circ$. The solid lines denote the complete $W_L$ amplitude and the dotted lines denote the GB amplitude. We find that when $\theta \leq 100^\circ$, the GB amplitude is almost indistinguishable from the $W_L$ amplitude. For $\theta = 150^\circ$, the $W_L$ amplitude is of the same order as the $B$-term, i.e. of $O(g^2)$, when $M_{WW} < 1$ TeV. In this case the $W_L$ or GB amplitude is too small and the strongly coupled EWSB sector cannot be sensitively probed. As the energy $E$ increases, we see that the $W_L$ and GB amplitudes rapidly dominate over the $B$-term and agree better and better even for large scattering angles. Finally, using the effective-$W$ method [35], we compare the LHC production rates in Figs. 10d and 10e for the invariant mass ($M_{WW}$) and the polar angle ($\cos \theta$) distributions, respectively. To avoid the $t$-channel photon singularity (at $\theta = 0^\circ$) in the phase space integration, we add an angular cut $-0.1 \leq \cos \theta \leq 0.8$ (i.e., $36.9^\circ \leq \theta \leq 180^\circ$). Fig. 10d shows that the total cross sections computed from the $W_L$ and the GB amplitudes [cf. eq. (A1)] indeed agree with each other very well. From Fig. 10e, we see that the difference clearly appears only for the region of large scattering angle (i.e., $\cos \theta < -0.6$ or $\theta > 127^\circ$) where both the leading $W_L$ and GB amplitudes are suppressed by the kinematic factor $1 + \cos \theta$ and thus the event rates are too low to be sensitive to the EWSB sector. Hence, the difference from the large $\theta$ region has only negligible effects on the total cross sections, as clearly shown in Fig. 10d. This also agrees with our conclusion from Figs. 10a-c. We have also made the comparison with symmetric angular cuts (such as $|\cos \theta| \leq 0.8$) and found similar good agreement to that in Fig. 10d. This is clear since in the large $\theta$ region the event rates become much lower and are close to their difference, i.e., of $O(|R_B|)$.

The above conclusions hold for the tree level contributions from the lowest order operators in $\mathcal{L}_G + \mathcal{L}^{(2)} + \mathcal{L}_F$, cf. (3.1.2). However, independent of the kinematic region considered, not all the contributions from the NLO effective operators can dominate the $B$-term and satisfy the condition (2.3b) [or (2.4)]. This is why the condition (2.3b) [or (2.4)] can serve as the criterion for classifying the sensitivities of these NLO operators in probing the EWSB sector for a given scattering process.

We therefore conclude that for the process considered here the $B$-term, as defined in

the $\theta$ angle cannot be close to either $180^\circ$ or $0^\circ$. 

(2.1), can be at most of $O(g^2)$ for all kinematic regions (cf. Fig. 10a), and is insensitive to the EWSB mechanism, in accordance with our general analysis in Sec. 2. When $t$ or $u$ is not large, the $t$- or $u$-channel internal lines must be included. We find that in certain kinematic region even $t$ (or $u$) is close to zero, the ET still works well [cf. Eq. (A3) and Fig. 10b]. This is because the validity of the ET does not require either $t \gg M_W^2$ or $u \gg M_W^2$ [cf. (2.3) and (2.3a,b)]. For some scattering processes, there may be special kinematic regions in which the GB and the $W_L$ amplitudes are largely suppressed so that the EWSB sector cannot be sensitively probed in these special kinematic regions (cf. Figs. 10b,c and 10e). But, as shown in this work, measuring the total event rates from these processes can still be used to sensitively probe the EWSB sector (cf. Fig. 10d and Figs. 2-9).

Appendix B. Electroweak Power Counting Rule for Linearly Realized Effective Lagrangians

For completeness and for other possible applications, we also generalize Weinberg’s power counting method to another popular effective Lagrangian formalism [39] for the weakly coupled EWSB sector, which is usually called as the decoupling scenario. In this formalism, the lowest order Lagrangian is just the linear SM with a relatively light Higgs boson and all higher order new physics effective operators must have dimensions larger than 4 and are suppressed by the effective cutoff scale $\Lambda$. Even if a relatively light scalar is found in future colliders, it remains important to know whether such a scalar particle trivially serves as the SM Higgs boson or originates from a more complicated dynamics. For instance, the possible new physics effects parametrized in (B1) should be probed in details for discriminating the SM Higgs boson from the non-SM Higgs boson at the LHC and the future linear colliders.

Following Ref. [39], we can generally write the $SU(2)_L \otimes U(1)_Y$ linear effective Lagrangian as follows

$$L_{\text{linear eff}} = L_{\text{SM}} + \sum_n \frac{\ell_n}{\Lambda^{d_n - 4}} O_n$$

(B1)

where $d_n \geq 5$ is the dimension of the effective operator $O_n$. In (B1), the lowest order

\[\text{This large suppression can also arise from the polarization effects of the in/out states.}\]
Lagrangian $\mathcal{L}_{\text{SM}}$ is just the SM Lagrangian with a relatively light Higgs boson. The interesting high energy region considered here is

$$M_W, m_H, m_t \ll E < \Lambda$$  \hspace{1cm} (B2)

in which $m_H = \sqrt{2\lambda f_\pi}$ denotes the Higgs boson mass.

Since the field content of $\mathcal{L}_{\text{eff}}^{\text{linear}}$ in (B1) is the same as that of the SM and the masses of all the known fields are much lower than the typical high energy scale $E$ under consideration [cf. eq. (B2)], it is clear that all the essential features of Weinberg’s counting method hold for this linear case. Following the same reasoning as done in Sec. 3.2 [cf. eqs. (3.2.1)-(3.2.4)], we find that that, for a given $S$-matrix element $T$, the counting formula for the linear case is very similar to Eq. (3.2.5):

$$T = c_T f_\pi^D \left( \frac{f_\pi}{\Lambda} \right)^{N_O} \left( \frac{E}{f_\pi} \right)^{D_{E0}} \left( \frac{E}{\Lambda_0} \right)^{D_{EL}} \left( \frac{M_W}{E} \right)^{\epsilon_v} H(\ln E/\mu) ,$$

where the only difference is that $N_O$ is now determined by the canonical dimensional counting in (B1) instead of the naive dimensional analysis (NDA) [19] for the non-decoupling scenario discussed in Sec. 3.

For $\mathcal{L}_{\text{SM}}$ (i.e., for $\ell_n = 0$ in (B1)), the counting for $D_E$ defined in (B3) or (3.2.4) can be further simplified since we know that the total $E$-power dependence of the SM contributions will not increase as the loop number $L$ increases because of the perturbative unitarity of the light Higgs SM. We shall show that, due to the renormalizable feature of $\mathcal{L}_{\text{SM}}$, the $D_{EL}$ term, $2L$, in (B3) will be cancelled by a counter term from the vertex-contribution in the $D_{E0}$ term.

In $\mathcal{L}_{\text{SM}}$, there are only 3-point and 4-point vertices. Due to the renormalizability of the SM, all the 4-point vertices do not contain partial derivatives, while each 3-point vertex may contain at most one partial derivative. So, we have $d_n = 0$ or 1. Thus,

$$\sum_n \mathcal{V}_n d_n = \mathcal{V}_d , \hspace{0.5cm} \mathcal{V}_d \equiv \mathcal{V}^{VVVV}_3 + \mathcal{V}^{ssVV}_3 + \mathcal{V}^{ccVV}_3 ,$$

(B4)

where $\mathcal{V}_d$ is the number of all vertices containing one partial derivative and $\mathcal{V}^{\chi_1 \cdots \chi_n}_n$ is the total number of $n$-point vertices of type $\chi_1 \cdots \chi_n$ ($\chi$ denotes any possible
field in the theory). The symbol $s$ denotes scalar fields (Higgs or GB), $c$ ($\bar{c}$) denotes (anti-)ghost field and $F$ ($\bar{F}$) denotes (anti-)fermion field. Furthermore, in $\mathcal{L}_{\text{SM}}$,

$$V = \sum_n V_n = V_3 + V_4,$$

$$V_3 \equiv \mathcal{V}_d + \mathcal{V}_F + \bar{\mathcal{V}}_3, \quad V_F \equiv \mathcal{V}_s^{FF} + \mathcal{V}_3^{FF}, \quad \bar{\mathcal{V}}_3 \equiv \mathcal{V}_3^{VV} + \mathcal{V}_3^{s\bar{c}} + \mathcal{V}_3^{ss} \quad (B5)$$

$$V_4 \equiv \mathcal{V}_4^{ssss} + \mathcal{V}_4^{sVV} + \mathcal{V}_4^{VVVV},$$

Substituting (B4), (B5) and the SM relation $V_F = 2i_F + e_F$ into (B3), we obtain

$$D_{E}^{\text{SM}} = D_{E0}^{\text{SM}} + D_{EL}^{\text{SM}} = 2L + 2 - 2\mathcal{V} + \mathcal{V}_d + \mathcal{V}_F - e_v \quad (B6)$$

which, with the aid of another SM relation

$$3V_3 + 4V_4 = e + 2i, \quad \text{or,}$$

$$V_d + V_F = 4V - \bar{V}_3 - 2i - e = 2 - 2L + 2V - \bar{V}_3 - e \quad (B7)$$

can be further simplified as

$$D_{E}^{\text{SM}} = 4 - e - e_v - \bar{V}_3 \quad (B8)$$

where $\bar{V}_3 \equiv \mathcal{V}_3^{VV} + \mathcal{V}_3^{s\bar{c}} + \mathcal{V}_3^{ss}$. Note that the loop-dependence term $2L$ is indeed canceled by the counter term from the vertex-contribution [cf. (B6) and (B7)] as expected. This is the unique feature of the renormalizable SM and this feature is absent in the EWCL with the derivative expansion which has been fully studied in Sec. 3. In summary, for $\mathcal{L}_{\text{SM}}$, a Feynman diagram with its external lines fixed can have the leading energy dependence if it does not contains the trilinear vertices $s-V-V$, $s-c-\bar{c}$ and $s-s-s$, and the $v_\mu$-factor. Equivalently, (B6) shows in another way that at a given $L$-loop level the leading energy behavior of a diagram corresponds to the minimal $(V_d + V_F + 2\bar{V}_3 + 2V_4)$ and the vanishing $e_v$.

The NLO linear operators $\mathcal{O}_n$ in (B1) have been fully compiled in Ref. [39]. Here
are a few typical dimension-6 effective operators:

\[ O_W = -i4\text{Tr}(W_\mu \nu W_\nu \tau \phi_\mu) \]
\[ O_{\partial \phi} = \frac{1}{2} \partial_\mu (\phi^\dagger \partial^\mu \phi) \]
\[ O^{(1,1)}_{qq} = \frac{1}{2} (\bar{q} \gamma_\mu q)(\bar{q} \gamma_\mu q) \]
\[ O^{(1,3)}_{qq} = \frac{1}{2} (\bar{q} \gamma_\mu \gamma^a q)(\bar{q} \gamma_\mu \gamma^a q) \]
\[ O_{\phi W} = (\phi^\dagger \phi)\text{Tr}(W_\mu \nu W_\mu \nu) \]
\[ O_{\phi B} = (\phi^\dagger W_\mu \nu \phi) B^\mu \nu \]
\[ O^{(1)}_{\phi} = \frac{1}{2} (\phi^\dagger \phi)(D_\mu \phi^\dagger D^\mu \phi) \]
\[ O^{(3)}_{\phi} = (\phi^\dagger D^\mu \phi)[(D_\mu \phi)^\dagger \phi] \]

where \( \phi \) denotes the Higgs doublet which contains the linearly realized Higgs field \((H)\) and three would-be Goldstone bosons \((\pi^\pm, \pi^0)\).

It is straightforward to apply our power counting rule \((B3)\) for estimating various scattering amplitudes contributed by \((B1)\). Some typical examples are in order. First, we count the model-independent contributions from \(L_{SM}\) to some \(2 \to 2\) scattering processes:

\[ T[V_1 V_2 \to V_3 V_4] = 0 \]
\[ T[\pi_1 \pi_2 \to \pi_3 \pi_4] = 0 \]
\[ T[V_1 V_2 \to \pi_3 \pi_4] = O\left(\frac{g^4}{16\pi^2}\right) \]

Second, we count the NLO model-dependent contributions from \((B9)\) to some tree-level high energy processes at the \(O(1/\Lambda^2)\):

\[ T_1[V_1 V_2 V_3 V_4](O_W) = O\left(\ell W \frac{gE^2}{\Lambda^2}\right) \]
\[ T_1[\pi_1 \pi_2 \to \pi_3 \pi_4](O_W) = 0 \]
\[ T_1[\pi_1 \pi_2 \to HH](O_{\partial \phi}) = O\left(\ell_{\partial \phi} \frac{E^2}{\Lambda^2}\right) \]
\[ T_1[q\bar{q} \to q\bar{q}](O^{(1,1)}_{qq}) = O\left(\ell^{(1,1)}_{qq} \frac{E^2}{\Lambda^2}\right) \]

The above examples illustrate, in the linear effective Lagrangian formalism, how to conveniently apply our power counting rule \((B3)\) to determine the high energy behavior of any given amplitude and estimate its order of magnitude.
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Table Captions

Table 1. Estimates of amplitudes for $W^\pm W^\pm \to W^\pm W^\pm$ scattering.

Table 1a. Model-independent contributions from $\mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)}$.

Table 1b. Model-dependent contributions from the next-to-leading order operators.

Table 2. Order estimates of $B$-terms for $W^\pm W^\pm \to W^\pm W^\pm$ scattering.

Table 2a. Model-independent contributions.

Table 2b. Relevant operators for model-dependent contributions.\(^{(a)}\)

Table 3. Estimates of amplitudes for $q\bar{q}(t) \to V^a V^b$: Model-independent contributions.

Table 3a. For $q\bar{q}(t) \to W^+ W^-$, $W^\pm Z$.

Table 3b. For $q\bar{q} \to ZZ$.

Table 4. Estimates of amplitudes for $q\bar{q}(t) \to V^a V^b$: Model-dependent contributions.\(^{(a)}\)

Table 4a. For $q\bar{q} \to W^+ W^-$.

Table 4b. For $q\bar{q}' \to W^\pm Z$.

Table 5. Global classification of sensitivities to probing direct and indirect EWSB information from effective operators at the level of $S$-matrix elements (I).\(^{(a)}\)

Notes:

\(^{(a)}\) The contributions from $L_{1,2,13}$ are always associated with a factor of $\sin^2 \theta_W$, unless specified otherwise. Also, for contributions to the $B$-term in a given $V_L$-amplitude, we list them separately with the $B$-term specified.

\(^{(b)}\) MI = model-independent, MD = model-dependent.

\(^{(c)}\) There is no contribution when all the external lines are electrically neutral.

\(^{(d)}\) $B_0^{(1)} \simeq T_0[2\pi,v,V_T]$ (\(\neq T_0[2\pi^0,v^0,Z_T]\)), $B_0^{(3)} \simeq T_0[v,3V_T]$ (\(\neq T_0[v^0,3Z_T]\)).

\(^{(e)}\) $T_1[2V_L,2V_T] = T_1[2Z_L,2W_T]$, $T_1[2W_L,2Z_T]$, or $T_1[Z_L,W_L,Z_T,W_T]$.

\(^{(f)}\) $\mathcal{L}_2$ only contributes to $T_1[2\pi^\pm,\pi^0,v^0]$ and $T_1[2\pi^0,\pi^\pm,v^\pm]$ at this order; $\mathcal{L}_{6,7}$ do not contribute to $T_1[3\pi^\pm,v^\pm]$.

\(^{(g)}\) $\mathcal{L}_{10}$ contributes only to $T_1[\cdots]$ with all the external lines being electrically neutral.

\(^{(h)}\) $B_0^{(2)}$ is dominated by $T_0[2V_T,2v]$ since $T_0[\pi,2V_T,v]$ contains a suppressing factor $\sin^2 \theta_W$ as can be deduced from $T_0[\pi,3V_T]$ (cf. Table 1a) times the factor $v^\mu = O \left( \frac{M_0}{E} \right)$.

\(^{(i)}\) Here, $T_1[2W_L,2W_T]$ contains a coupling $\epsilon^4 = g^4 \sin^4 \theta_W$.\(^{46}\)
(j) $\mathcal{L}_2$ only contributes to $T_1[3\pi^\pm, v^\pm]$.

(k) $\mathcal{L}_{1,13}$ do not contribute to $T_1[2\pi^\pm, 2v^\pm]$.

Table 6. Global classification of sensitivities to probing direct and indirect EWSB information from effective operators at the level of $S$-matrix elements (II).

Figure Captions

Fig. 1. Comparison of the power counting predictions with the corresponding ones in Fig. 8 of Ref. [12] up to one-loop for a pp collider with $\sqrt{s} = 40$ TeV. The solid lines are given by our power counting analysis; the dashed lines are from Ref. [12]. [The meanings of the rates $R_{\alpha\beta\gamma\delta}$'s and $R_{\alpha\beta}$'s are defined in eq. (5.1a,b) and below.]

(1a). $W^+W^+ \rightarrow W^+_LW^+_L$.

(1b). $q\bar{q}' \rightarrow W^+_LW^-_L$.

Fig. 2.

(2a). Comparison of the $W^+_LW^+_L$ production rates up to one-loop (for $\ell_{0-14} = 0$) with $W^+_TW^+_T$, $W^+_ LW^+_L$ and $W^+_TW^+_T$ initial states, at the 14 TeV LHC.

(2b). Comparison of the production rates for different final-state polarizations up to one-loop (for $\ell_{0-14} = 0$) after summing over the polarizations of the initial states, at the 14 TeV LHC.

Fig. 3. Comparison of the production rates for different final-state polarizations up to one-loop (for $\ell_{0-14} = 0$) via $q\bar{q}' \rightarrow W^+Z^0$ at the 14 TeV LHC.

Fig. 4. Sensitivities of the operators $\mathcal{L}^{(2)\nu}$ and $\mathcal{L}_{1,1-14}$ at the 14 TeV LHC with $\Lambda = 3.1$ TeV. The coefficients $\ell_n$'s are taken to be of $O(1)$,

(4a). For operators $\mathcal{L}_{3,4,5,9,11,12}$.

(4b). For operators $\mathcal{L}^{(2)\nu}$ and $\mathcal{L}_{1,2,8,13,14}$.

Fig. 5. Same as Fig. 4, but the coefficients $\ell_n$'s are taken to be of $O(10)$ except $\ell_{0,1,8}$ which are already constrained by low energy data to be of $O(1)$.

(5a). For operators $\mathcal{L}_{3,4,5,9,11,12}$.

(5b). For operators $\mathcal{L}_{2,13,14}$.
Fig. 6 Same as Fig. 4, but with $\Lambda = 2.0$ TeV.

Fig. 7. Same as Fig. 5, but with $\Lambda = 2.0$ TeV.

Fig. 8. Sensitivities of the operators $\mathcal{L}^{(2)'}$ and $\mathcal{L}_{1-14}$ in $q\bar{q}' \rightarrow W^+Z^0$ at the 14 TeV LHC with $\Lambda = 3.1$ TeV.

(8a). The coefficients $\ell_n$'s are taken to be of $O(1)$.

(8b). The coefficients $\ell_n$'s are taken to be of $O(10)$ except $\ell_{0,1,8}$ which are already constrained by low energy data to be of $O(1)$.

Fig. 9. Same as Fig. 8, but with $\Lambda = 2.0$ TeV.

Fig. 10. Examination on the kinematic dependence and the validity of the ET for the $W_L^+W_L^- \rightarrow W_L^+W_L^-$ scattering process.

(10a). The ratio $|B/g^2|$ for $\theta = 2^\circ, 10^\circ, 45^\circ, 90^\circ, 100^\circ, 120^\circ, 135^\circ, 150^\circ, 180^\circ$.

(10b). Same as (10a), but for the ratio $|B/T[W_L]|$.

(10c). Comparison of the $W_L$-amplitude (solid lines) and the corresponding GB-amplitude (dotted lines) for $\theta = 10^\circ, 45^\circ, 100^\circ, 150^\circ$. Here, $B[150^\circ]$ denotes the $B$-term at $\theta = 150^\circ$.

(10d). Comparison of the LHC production rates contributed by the exact $W_L$-amplitude (solid line) and the GB-amplitude (dotted line) from eq. (A1).

(10e). Same as Fig. 10d, but for the angular distributions.
**TABLES**

**Table 1.** Estimates of amplitudes for $W^\pm W^\pm \rightarrow W^\pm W^\pm$ scattering.

**Table 1a.** Model-independent contributions from $L_G + L_F + L^{(2)}$.

| $L_G + L_F + L^{(2)}$ | $T_\ell[4\pi]$ | $T_\ell[3\pi, W_T]$ | $T_\ell[2\pi, 2W_T]$ | $T_\ell[\pi, 3W_T]$ | $T_\ell[4W_T]$ |
|------------------------|----------------|---------------------|---------------------|---------------------|----------------|
| Tree-Level             | $\frac{E^2}{f_\pi}$ | $g\frac{E}{f_\pi}$ | $g^2$               | $e^2 g \frac{f_\pi}{E}$ | $g^2$           |
| ($\ell = 0$)           |                |                     |                     |                     |                 |
| One-Loop               | $\frac{E^2}{f_\pi^2 \Lambda_0^2}$ | $g\frac{E}{f_\pi} \frac{E^2}{\Lambda_0^2}$ | $g^2 \frac{E^2}{\Lambda_0^2}$ | $g^3 \frac{f_\pi E}{\Lambda_0^2}$ | $g^4 \frac{f_\pi^2}{\Lambda_0^2}$ |
| ($\ell = 1$)           |                |                     |                     |                     |                 |
Table 1b. Model-dependent contributions from the next-to-leading order operators.

| Operators | $T_1[4\pi]$ | $T_1[3\pi, W_T]$ | $T_1[2\pi, 2W_T]$ | $T_1[\pi, 3W_T]$ | $T_1[4W_T]$ |
|-----------|--------------|------------------|------------------|-----------------|--------------|
| $L^{(2)\nu}$ | $\ell_0 \frac{E^2}{\Lambda^2}$ | $\ell_0 \frac{g^2 E}{\Lambda^2}$ | $\ell_0 \frac{g^2 f^2}{\Lambda^2}$ | $\ell_0 \frac{g^3 f^2}{E\Lambda^2}$ | / |
| $L_{1,13}$ | / | $\ell_{1,13} \frac{e^2 g f E}{\Lambda^2}$ | $\ell_{1,13} \frac{e^4 f^2}{\Lambda^2}$ | $\ell_{1,13} \frac{e^2 g^2 f E}{\Lambda^2}$ | $\ell_{1,13} \frac{e^2 g^2 f^2}{\Lambda^2}$ |
| $L_2$ | $\ell_2 \frac{e^2 E^2}{\Lambda^2}$ | $\ell_2 \frac{e^2 g f E}{\Lambda^2}$ | $\ell_2 \frac{e^2 E^2}{\Lambda^2}$ | $\ell_2 \frac{e^2 g f E}{\Lambda^2}$ | $\ell_2 \frac{e^2 g^2 f^2}{\Lambda^2}$ |
| $L_3$ | $\ell_3 \frac{g^2 E^2}{\Lambda^2}$ | $\ell_3 \frac{g E^2}{f E \Lambda^2}$ | $\ell_3 \frac{g^2 E^2}{\Lambda^2}$ | $\ell_3 \frac{g^3 f E}{\Lambda^2}$ | $\ell_3 \frac{g^4 f^2}{\Lambda^2}$ |
| $L_{4,5}$ | $\ell_{4,5} \frac{E^2 E^2}{f E \Lambda^2}$ | $\ell_{4,5} \frac{g E^2}{f E \Lambda^2}$ | $\ell_{4,5} \frac{g^2 E^2}{\Lambda^2}$ | $\ell_{4,5} \frac{g^3 f E}{\Lambda^2}$ | $\ell_{4,5} \frac{g^4 f^2}{\Lambda^2}$ |
| $L_{6,7,10}$ | / | / | / | / | / |
| $L_{8,14}$ | / | $\ell_{8,14} \frac{g^3 f E}{\Lambda^2}$ | $\ell_{8,14} \frac{g^2 E^2}{\Lambda^2}$ | $\ell_{8,14} \frac{g^3 f E}{\Lambda^2}$ | $\ell_{8,14} \frac{g^4 f^2}{\Lambda^2}$ |
| $L_9$ | $\ell_9 \frac{g^2 E^2}{\Lambda^2}$ | $\ell_9 \frac{g E^2}{f E \Lambda^2}$ | $\ell_9 \frac{g^2 E^2}{\Lambda^2}$ | $\ell_9 \frac{g^3 f E}{\Lambda^2}$ | $\ell_9 \frac{g^4 f^2}{\Lambda^2}$ |
| $L_{11,12}$ | / | $\ell_{11,12} \frac{g E^2}{f E \Lambda^2}$ | $\ell_{11,12} \frac{g^2 E^2}{\Lambda^2}$ | $\ell_{11,12} \frac{g^3 f E}{\Lambda^2}$ | $\ell_{11,12} \frac{g^4 f^2}{\Lambda^2}$ |
Table 2. Order estimates of $B$-terms for $W^\pm W^\pm \rightarrow W^\pm W^\pm$ scattering.

| $\mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)}$ | $B^{(0)}_\ell$ | $B^{(1)}_\ell$ | $B^{(2)}_\ell$ | $B^{(3)}_\ell$ |
|-----------------|----------------|----------------|----------------|----------------|
| Tree-Level ($\ell = 0$) | $g^2$ | $g^2 \frac{M_W}{E}$ | $g^2 \frac{M_W^2}{E^2}$ | $g^2 \frac{M_W}{E}$ |
| One-Loop ($\ell = 1$) | $g^2 \frac{E^2}{\Lambda_0}$ | $g^3 \frac{E f_\pi}{\Lambda_0}$ | $g^4 \frac{f_\pi^2}{\Lambda_0}$ | $g^4 \frac{f_\pi^2}{\Lambda_0} \frac{M_W}{E}$ |

Table 2a. Model-independent contributions.

Table 2b. Relevant operators for model-dependent contributions.\(^{(a)}\)

| $O\left(g^2 \frac{E^2}{\Lambda_0}\right)$ (from $B^{(0)}_1$) | $O\left(g^3 \frac{E f_\pi}{\Lambda_0}\right)$ (from $B^{(1)}_1$) | $O\left(g^2 \frac{f_\pi^2}{\Lambda_0}\right)$ (from $B^{(0)}_1$) | $O\left(g^4 \frac{f_\pi^2}{\Lambda_0}\right)$ (from $B^{(2)}_1$ or $B^{(0)}_1$) |
|-----------------|----------------|----------------|----------------|
| $\mathcal{L}_{3,4,5,9,11,12}$ | $\mathcal{L}_{2,3,4,5,8,9,11,12,14}$ | $\mathcal{L}^{(2)}_1$ | $\mathcal{L}_{1,7,8,9,11,14}$ ($B^{(2)}_1$) $\mathcal{L}_{1,2,8,13,14}$ ($B^{(0)}_1$) $\mathcal{L}_{2,7,8,9,11,12,14}$ ($B^{(0)}_1$)\(^{(b)}\) |

\(^{(a)}\) We list the relevant operators for each order of $B$-terms.

\(^{(b)}\) Here $B^{(0)}_1$ is contributed by $T_1[2\pi^\pm, 2v^\pm]$. 

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Table 3. Estimates of amplitudes for $q\bar{q}^{(i)} \rightarrow V^aV^b$: Model-independent contributions.

Table 3a. For $q\bar{q}^{(i)} \rightarrow W^+W^-, W^\pm Z$.

| $\mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)}$ | $T_\ell[q\bar{q}^{(i)} \rightarrow \pi\pi]$ | $T_\ell[q\bar{q}^{(i)} \rightarrow \pi V_T]$ | $T_\ell[q\bar{q}^{(i)} \rightarrow V_TV_T]$ | $B_\ell^{(0)}$ | $B_\ell^{(1)}$ |
|--------------------------------|-------------------------------|-----------------------------------|-----------------------------------|----------------|----------------|
| Tree-Level ($\ell = 0$) | $g^2$ | $e^2g\frac{f_\pi}{E}$ | $g^2$ | $g^2\frac{M_W^2}{E^2}$ | $g^2\frac{M_W}{E}$ |
| One-Loop ($\ell = 1$) | $g^2\frac{E^2}{X_0}$ | $g^3\frac{f_\pi E}{X_0}$ | $g^4\frac{f_\pi^2}{X_0}$ | $g^3\frac{f_\pi M_W}{X_0}$ | $g^4\frac{f_\pi^2 M_W}{X_0}$ |

Table 3b. For $q\bar{q} \rightarrow ZZ$.

| $\mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)}$ | $T_\ell[q\bar{q} \rightarrow \pi\pi]$ | $T_\ell[q\bar{q} \rightarrow \pi Z_T]$ | $T_\ell[q\bar{q} \rightarrow Z_T Z_T]$ | $B_\ell^{(0)}$ | $B_\ell^{(1)}$ |
|--------------------------------|-------------------------------|-----------------------------------|-----------------------------------|----------------|----------------|
| Tree-Level ($\ell = 0$) | / | / | $g^2$ | $g^2\frac{M_W^2}{E^2}$ | $g^2\frac{M_W}{E}$ |
| One-Loop ($\ell = 1$) | $g^2\frac{E^2}{X_0}$ | $g^3\frac{f_\pi E}{X_0}$ | $g^4\frac{f_\pi^2}{X_0}$ | $g^3\frac{f_\pi M_W}{X_0}$ | $g^4\frac{f_\pi^2 M_W}{X_0}$ |
Table 4. Estimates of amplitudes for $q\bar{q}(t) \rightarrow V^a V^b$: Model-dependent contributions.$^{(a)}$

Table 4a. For $q\bar{q} \rightarrow W^+W^-$.

| Operators | $T_1[q\bar{q} \rightarrow \pi\pi]$ | $T_1[q\bar{q} \rightarrow \pi V_T]$ | $T_1[q\bar{q} \rightarrow V_T V_T]$ | $B_1^{(0)}$ | $B_1^{(1)}$ |
|-----------|---------------------------------|---------------------------------|---------------------------------|--------------|--------------|
| $\mathcal{L}^{(2')}$ | $\ell_0 g^2 f_2^{E_2}$ | $\ell_0 g^3 f_3^{E_2}$ | / | $g^2 f_2^{E_2} M_0^{(0)}$ | / |
| $\mathcal{L}_{1,13}$ | / | $\ell_{1,13} e^2 g f_4^{E_2}$ | $\ell_{1,13} e^2 g^3 f_3^{E_2}$ | $e^2 g^2 f_2^{E_2}$ | $e^2 g^2 f_2^{E_2} M_{W^\pm}$ |
| $\mathcal{L}_2$ | $\ell_2 e^2 f_4^{E_2}$ | $\ell_2 e^2 g f_4^{E_2}$ | $\ell_2 e^2 g^2 f_3^{E_2}$ | $e^2 g^2 f_2^{E_2}$ | $e^2 g^2 f_2^{E_2} M_{W^\pm}$ |
| $\mathcal{L}_3$ | $\ell_3 g^2 f_4^{E_2}$ | $\ell_3 g^3 f_3^{E_2}$ | $\ell_3 g^4 f_2^{E_2}$ | $g^4 f_2^{E_2}$ | $g^4 f_2^{E_2} M_{W^\pm}$ |
| $\mathcal{L}_{8,14}$ | / | $\ell_{8,14} g^3 f_3^{E_2}$ | $\ell_{8,14} g^4 f_2^{E_2}$ | $g^4 f_2^{E_2}$ | $g^4 f_2^{E_2} M_{W^\pm}$ |
| $\mathcal{L}_9$ | $\ell_9 g^2 f_4^{E_2}$ | $\ell_9 g^3 f_3^{E_2}$ | $\ell_9 g^4 f_2^{E_2}$ | $g^4 f_2^{E_2}$ | $g^4 f_2^{E_2} M_{W^\pm}$ |
| $\mathcal{L}_{11,12}$ | / | $\ell_{11,12} g^3 f_3^{E_2}$ | $\ell_{11,12} g^4 f_2^{E_2}$ | $g^4 f_2^{E_2}$ | $g^4 f_2^{E_2} M_{W^\pm}$ |

$^{(a)}$Here we only consider the light quarks ($q \neq t$) whose Yukawa coupling $y_q \approx 0$. At tree level, $q\bar{q} \rightarrow ZZ$ contains no model-dependent contribution and the operators $\mathcal{L}_{4,5,6,7,10}$ do not contribute to $q\bar{q}(t) \rightarrow W^+W^-, W^\pm Z$.
Table 4b. For $q\bar{q}' \rightarrow W^\pm Z$.

| Operators | $T_1[q\bar{q}' \rightarrow \pi^\pm \pi^0]$ | $T_1[q\bar{q}' \rightarrow \pi V_T]$ | $T_1[q\bar{q}' \rightarrow W_T^\pm Z_T]$ | $B_1^{(0)}$ | $B_1^{(1)}$ |
|-----------|---------------------------------|---------------------------------|---------------------------------|-----------------|-----------------|
| $\mathcal{L}^{(2)'}$ | $\ell_0 \, g^2 \frac{f_2^2}{N^2}$ | $\ell_0 \, g^3 \frac{f_3^2}{N^2}$ | / | $g^2 \frac{f_2^2 M_W^2}{N^2}$ | / |
| $\mathcal{L}_{1,13}$ | / | $\ell_{1,13} \, e^2 \frac{f_2 E}{N^2}$ | $\ell_{1,13} \, e^2 g^2 \frac{f_2^2}{N^2}$ | $e^2 g^2 \frac{f_2^2}{N^2}$ | $e^2 g^2 \frac{f_2^2}{N^2} \frac{M_W}{E}$ |
| $\mathcal{L}_2$ | / | $\ell_2 \, e^2 \frac{f_2 E}{N^2}$ | $\ell_2 \, e^2 g^2 \frac{f_2^2}{N^2}$ | $e^2 g^2 \frac{f_2^2}{N^2}$ | $e^2 g^2 \frac{f_2^2}{N^2} \frac{M_W}{E}$ |
| $\mathcal{L}_3$ | $\ell_3 \, g^2 \frac{E}{N^2}$ | $\ell_3 \, g^3 \frac{f_3 E}{N^2}$ | $\ell_3 \, g^4 \frac{f_3^2}{N^2}$ | $g^4 \frac{f_2^2}{N^2}$ | $g^4 \frac{f_2^2}{N^2} \frac{M_W}{E}$ |
| $\mathcal{L}_{8,14}$ | / | $\ell_{8,14} \, g^3 \frac{f_3 E}{N^2}$ | $\ell_{8,14} \, g^4 \frac{f_3^2}{N^2}$ | $g^4 \frac{f_2^2}{N^2}$ | $g^4 \frac{f_2^2}{N^2} \frac{M_W}{E}$ |
| $\mathcal{L}_9$ | / | $\ell_9 \, g^3 \frac{f_3 E}{N^2}$ | $\ell_9 \, g^4 \frac{f_3^2}{N^2}$ | $g^4 \frac{f_2^2}{N^2}$ | $g^4 \frac{f_2^2}{N^2} \frac{M_W}{E}$ |
| $\mathcal{L}_{11,12}$ | $\ell_{11,12} \, g^2 \frac{E^2}{N^2}$ | $\ell_{11,12} \, g^3 \frac{f_3 E}{N^2}$ | $\ell_{11,12} \, g^4 \frac{f_3^2}{N^2}$ | $g^4 \frac{f_2^2}{N^2}$ | $g^4 \frac{f_2^2}{N^2} \frac{M_W}{E}$ |
### Table 5. Global classification of sensitivities to probing direct and indirect EWSB information from effective operators at the level of S-matrix elements (I). *(a)*

| Required Precision | Relevant Operators | Relevant Amplitudes | MI or MD *(b)* |
|--------------------|--------------------|---------------------|----------------|
| $O \left( \frac{E^2}{\Lambda^2} \right)$ | $\mathcal{L}_{MI}$ ($\equiv \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)}$) | $T_0[4V_L](\neq T_0[4Z_L])$ | MI |
| | $\mathcal{L}_{4,5}$ | $T_1[4V_L]$ | MD |
| | $\mathcal{L}_{6,7}$ | $T_1[2Z_L, 2W_L], T_1[4Z_L]$ | MD |
| | $\mathcal{L}_{10}$ | $T_1[4Z_L]$ | MD |
| | $\mathcal{L}_{MI}$ | $T_0[3V_L, V_T]$ (≠ $T_0[3Z_L, Z_T]$) | MI |
| | $\mathcal{L}_{MI}$ | $T_1[4V_L]$ | MI |
| $O \left( \frac{E^2}{\Lambda^2}, g \frac{E^2}{\Lambda^2} \right)$ | $\mathcal{L}_{3,4,5,9,11,12}$ | $T_1[3W_L, W_T]$ | MD |
| | $\mathcal{L}_{2,3,4,5,6,7,9,11,12}$ | $T_1[2Z_L, 2W_L, Z_T], T_1[2Z_L, W_L, W_T]$ | MD |
| | $\mathcal{L}_{3,4,5,6,7,10}$ | $T_1[3Z_L, Z_T]$ | MD |
| | $\mathcal{L}_{MI}$ | $T_0[2V_L, 2V_T], T_0[4V_T]$ *(c)* | MI |
| | $\mathcal{L}_{MI}$ | $T_1[3V_L, V_T]$ | MI |
| | $\mathcal{L}_{MI}$ | $B_0^{(0)} \simeq T_0[3\pi, v]$ (≠ $T_0[3\pi^0, v^0]$) | MI |
| $O \left( \frac{E^2}{\Lambda^2} \right)$ | $\mathcal{L}^{(2)\nu}$ | $T_1[4W_L], T_1[2W_L, 2Z_L]$ | MD |
| $O \left( g \frac{E^2}{\Lambda^2}, g^2 \frac{E^2}{\Lambda^2} \right)$ | $\mathcal{L}_{MI}$ | $T_0[V_L, 3V_T], T_1[2V_L, 2V_T], B_0^{(1,3)}$ *(c,d)* | MI |
| | $\mathcal{L}_{2,3,9}$ | $T_1[4W_L]$ | MD |
| | $\mathcal{L}_{3,11,12}$ | $T_1[2Z_L, 2W_L]$ | MD |
| | $\mathcal{L}_{2,3,4,5,8,9,11,12,14}$ | $T_1[2W_L, 2W_T]$ | MD |
| | $\mathcal{L}_{1,9,11,14}$ | $T_1[2V_L, 2V_T]$ *(e)* | MD |
| | $\mathcal{L}_{3,4,5,6,7,10}$ | $T_1[2Z_L, 2Z_T]$ | MD |
| | $\mathcal{L}_{MI,2,3,4,5,6,7,9,11,12}$ | $B_0^{(0)} \simeq T_1[3\pi, v]$ *(f,g)* | MI + MD |
| $O \left( g^3 \frac{E^2}{\Lambda^2}, g^4 \frac{E^2}{\Lambda^2} \right)$ | $\mathcal{L}_{MI,1,2,3,8,9,11,14}$ | $T_1[V_L, 3V_T]$ (≠ $T_1[Z_L, 3Z_T]$) | MI + MD |
| | $\mathcal{L}_{4,5}$ | $T_1[4W_L]$ | MD |
| | $\mathcal{L}_{6,7,10}$ | $T_1[V_L, 3V_T]$ (≠ $T_1[W_L, 3W_T]$) *(g)* | MD |
| | $\mathcal{L}_{2,5,8,9,11,12,14}$ | $B_1^{(1)} \simeq T_1[2\pi, V_T, v]$ | MD |
| | $\mathcal{L}_{MI}$ | $B_2^{(2)} \simeq T_0[2V_T, 2v]$ *(c,h)* | MI |
| $O \left( (g^2, g^4) \frac{E^2}{\Lambda^2} \right)$ | $\mathcal{L}^{(2)\nu}$ | $T_1[2V_L, 2V_T], B_1^{(0)} \simeq T_1[3\pi, v]$ *(c)* | MD |
| | $\mathcal{L}_1$ | $T_1[2W_L, 2W_T]$ *(i)* | MD |
| | $\mathcal{L}_{MI,1,5,8,9,11,14}$ | $T_1[4W_T]$ | MI + MD |
| | $\mathcal{L}_{MI,1,9,11,14}$ | $T_1[4W_T]$ (≠ $T_1[4W_T], T_1[4Z_T]$) | MI + MD |
| | $\mathcal{L}_{MI,1,4,5,6,7,10}$ | $T_1[4Z_T]$ | MI + MD |
| | $\mathcal{L}_{1,2,8,13,14}$ | $B_0^{(0)} \simeq T_1[3\pi, v]$ *(c,i)* | MD |
| | $\mathcal{L}_{MI,1,9,11,14}$ | $B_0^{(0)} \simeq T_1[2\pi, 2v]$ *(c,k)* | MI + MD |
| | $\mathcal{L}_{MI,1,4,5,6,7,10}$ | $B_1^{(0)} \simeq T_1[\pi^\pm, 2W_T, v^\pm]$ *(g)* | MI + MD |
| | $\mathcal{L}_{MI,1,5,8,9,11,14}$ | $B_1^{(2)} \simeq T_1[\pi^\pm, 2W_T, v^\pm], T_1[\pi^0, 2Z_T, 0]^0$ | MI + MD |
| | $\mathcal{L}_{MI,1,9,11,14}$ | $B_1^{(2)} \simeq T_1[\pi^0, 2Z_T, 0]^0$ | MI + MD |
Table 6. Global classification of sensitivities to probing direct and indirect EWSB information from effective operators at the level of $S$-matrix elements (II). \(^{(a)}\)

| Required Precision | Relevant Operators | Relevant Amplitudes | MI or MD \(^{(b)}\) |
|--------------------|--------------------|---------------------|------------------|
| \(O(g^2)\)        | \(\mathcal{L}_\text{MI} \equiv \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)}\) | \(T_0[q\bar{q}; V_L V_L], \ T_0[q\bar{q}; V_T V_T]\) | MI |
| \(O\left(\frac{g^2}{\Lambda^2}, \ g^3 f^2 \right)\) | \(\mathcal{L}_{2,3,9}, \ \mathcal{L}_{3,11,12}, \ \mathcal{L}_\text{MI}, \ \mathcal{L}^{(2)}_\text{MI}, \ \mathcal{L}_\text{MI}\) | \(T_1[q\bar{q}; W_L W_L], \ T_1[q\bar{q}; W_L Z_L], \ T_0[q\bar{q}; V_L V_T], \ T_1[q\bar{q}; V_L V_L], \ B_0^{(1)} \simeq T_0[q\bar{q}; V_T, v]\) | MD |
| \(O\left(\frac{g^3 f}{\Lambda^2}, \ g^4 f^2 \right)\) | \(\mathcal{L}_{1,2,3,8,9,11\sim14}, \ \mathcal{L}_\text{MI}\) | \(T_1[q\bar{q}; V_L V_T], \ T_1[q\bar{q}; V_L V_T], \ B_0^{(0)} \simeq T_0[q\bar{q}; 2v]\) \(^{(c)}\) | MI |
| \(O\left(\frac{g^2, g^4}{\Lambda^2}\right)\) | \(\mathcal{L}^{(2)}_\text{MI}, \ \mathcal{L}_{1,2,3,8,9,11\sim14}, \ \mathcal{L}_\text{MI}\) | \(T_1[q\bar{q}; V_L V_L], \ T_1[q\bar{q}; V_T V_T], \ B_1^{(0)} \simeq T_1[q\bar{q}; \pi, v]\) | MD |

\(^{(a)}\) The contributions from \(\mathcal{L}_{1,2,13}\) are always associated with a factor of \(\sin^2 \theta_W\), unless specified otherwise. \(\mathcal{L}_{4,5,6,7,10}\) do not contribute to the processes considered in this table. Also, for contributions to the \(B\)-term in a given \(V_L\)-amplitude, we list them separately with the \(B\)-term specified.

\(^{(b)}\) MI = model-independent, MD = model-dependent.

\(^{(c)}\) Here, \(B_0^{(0)}\) is dominated by \(T_0[q\bar{q}; 2v]\) since \(T_0[q\bar{q}; \pi, v]\) contains a suppressing factor \(\sin^2 \theta_W\) as can be deduced from \(T_0[q\bar{q}; \pi V_T]\) (cf. Table 3a) times the factor \(v^\mu = O\left(\frac{M_W}{\Lambda}\right)\).
Fig. 1.
$W^+W^- \rightarrow W^+_lW^-_l$

$W^+W^- \rightarrow W^+_S W^-_S$

(set $l_{0-14}=0$)
Fig. 3.
$W^*W^* \rightarrow W^*W^*$

$(\Lambda = 3.1 \text{TeV})$

$(l_i = O(1))$

$|\nu_{4,5}|$

$|\nu_{3,9,11,12}|$

$|\nu_{1,13}|$

$|\nu_{8,14}|$

$|\nu_{10}|$

$|\nu_{2}|$

$|\nu_6|$

$|\nu_{1,13}|$

$|\nu_{5,14}|$

$|\nu_{10}|$

$|\nu_{2}|$

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$|\nu_{1,13}|$

$|\nu_{5,14}|$

$|\nu_{10}|$

$|\nu_{2}|$
Fig. 5.
Fig. 6.

$W^+W^+ \rightarrow W^+W^+$

($\Lambda = 2.0\,\text{TeV}$)

($l_i = O(1)$)
\begin{align*}
W^+ W^+ &\rightarrow W^+ W^+ \\
(\Lambda=2.0\text{TeV}) &\quad (|t|=O(10))
\end{align*}

\begin{align*}
|\mathbf{R}_1| &
\begin{cases}
|\nu_{3,9,11,12}| & \\
|\nu_{4,5}| & \\
|t_1| & \\
|t_2| & \\
|t_3| & \\
|t_4| &
\end{cases}
\end{align*}

\begin{align*}
|\mathbf{R}_1| &
\begin{cases}
|\nu_{3,9,11,12}| & \\
|\nu_{4,5}| & \\
|t_1| & \\
|t_2| & \\
|t_3| & \\
|t_4| &
\end{cases}
\end{align*}

\text{Fig. 7.}
Fig. 8.
Fig. 9.

$q\bar{q} \rightarrow W^*Z^0$
$\Lambda=2.0\text{TeV}$
$\Lambda=O(1)$

$q\bar{q} \rightarrow W^*Z^0$
$\Lambda=2.0\text{TeV}$
$\Lambda=O(10)$

$|\mu_{3,11,12}|$
$|\mu_{1,2,13}|$
$|\mu_{8,9,14}|$
$|\mu_R|$

$\mu_{1,2,13}$

$\mu_{8,9,14}$

$\mu_R$

$\mu_{3,11,12}$

$\mu_{1,2,13}$

$\mu_{8,9,14}$

$\mu_R$

Fig. 9.
Fig. 10a.
Fig. 10b.
Fig. 10c.
Figs. 10d and 10e.