A NOTE ON REGULAR TYPE 0 SOLUTIONS 
AND CONFINING GAUGE THEORIES

F. Bigazzi\textsuperscript{a}, L. Girardello\textsuperscript{a}, and A. Zaffaroni\textsuperscript{a,b}

\textsuperscript{a}Universit\`a di Milano-Bicocca, Dipartimento di Fisica
\textsuperscript{b} INFN - Sezione di Milano, Italy

Abstract

We discuss some features of the regular supergravity solution for fractional branes on a deformed conifold, recently found by Klebanov-Strassler, mostly adapting it to a type 0 non-supersymmetric context. The non-supersymmetric gauge theory is $SU(M) \times SU(M)$ with two bi-fundamental Weyl fermions. The tachyon is now stabilized by the RR antisymmetric tensor flux. We briefly discuss the most general non-supersymmetric theory on electric, magnetic and fractional type 0 D3-branes on a conifold. This includes the pure $SU(N)$ theory.
1 Introduction

Supergravity solutions dual to non-conformal gauge theories have been extensively discussed in the past years. They have been obtained as type 0 solutions [1, 2, 3, 4], as deformations of conformal backgrounds [5, 6, 7, 8, 9] or using fractional branes [10, 11, 12, 13, 14]. These backgrounds are dual to strongly coupled gauge theories. The IR dynamics of such theories can be different from the familiar one of their weakly coupled cousins, but they are nevertheless expected to exhibit familiar phenomena as confinement, chiral symmetry breaking etc. Unfortunately, the IR region of the supergravity background, which should describe these phenomena, is generically plagued with naked singularities.

There are known examples of stringy resolution mechanisms. The enhancon mechanism [15] might serve for curing a generic class of repulsion singularities, while the expansion of dielectric branes into five-branes [16] is adapt for describing some deformations of conformal theories. An interesting solution that does not involve extra brane-like sources and it is completely regular has been proposed in [12]. It represents the dual of an $N = 1$ gauge theory, realized by wrapping D5-branes on a 2-cycle of a conifold. Regularity is achieved by deforming the conifold without introducing physical D3-brane sources.

It is the purpose of this note to study the Klebanov-Strassler (KS) solution [12] in a non-supersymmetric context. The natural candidates are type 0 solutions. The simplest examples are orbifolds of the KS solution. Since type 0B is an orbifold of type IIB, every type IIB brane solution leads to a type 0B solution for orbifolded branes in the regular representation. The $AdS_5 \times S^5$ dyonic solution of [2] is an example of this correspondence [17].

The tachyon instability of type 0 needs to be cured. Fortunately, for this class of regular solutions the type 0 tachyon is stabilized by the RR antisymmetric tensor flux. As in [1, 2], the stabilization mechanism is effective only for large or intermediate curvature. In this regime, we expect to recover the string dual of weakly coupled gauge theories. We can speculate that at large 't Hooft coupling a large N phase transition has occurred [4], making the theory unstable. The supergravity solution nevertheless is a good starting point for the formulation of the string model problem and for a qualitative analysis of the dynamics.

We will also discuss more general solutions with running tachyon and dilaton, corresponding to the non-regular representation. These are the analogous of the purely electric type 0 solutions discussed in [1]. Most of the gauge theories discussed in this paper have no massless scalar fields, thus avoiding some complications in [1]. We start an analysis of the type 0 dual of the pure $SU(M)$ gauge theory. We mostly focus on the IR behavior. Under the assumption that the tachyon relaxes to zero and its instability is cured by the RR flux, we find that the KS solution can be easily adapted to the IR description of the
pure glue theory. We work at large $N$. We may expect corrections to the gauge theory behavior already at the first order in $1/N$ [18].

2 The KS solution

Let us start with a brief review of the type IIB solution. There is a class of exact type IIB solutions [19, 20], that do not necessarily request supersymmetry. Consider the general class of black D3-brane solutions

$$
\begin{align*}
    ds^2 &= Z^{-1/2}dx_\mu dx^\mu + Z^{1/2}ds_K^2 \\
    F_5 &= da_4 + *da_4, \quad a_4 = \frac{1}{4Z}dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3
\end{align*}
$$ (1)

where $K$ is a Calabi-Yau manifold with a closed self-dual 3-form $\omega_3$, $*K\omega_3 = i\omega_3$. If the 3-form $G = H_{(3)} + ig_5F_{(3)}$ is proportional to $\omega_3$, its equation of motion is automatically satisfied and we obtain a class of exact solutions by imposing

$$
-\Box_K Z = \rho_{D3}(x) + G_{mnp}G^{(K)mn}p
$$ (2)

where $\rho(x)$ is a general density of D3-branes [19]. Notice that the Laplacian and the metric in $G^2$ are those of $K$.

Regular and fractional branes on a conifold nicely fit into eq. (1), as noted in [19, 20]. Let us briefly discuss some details of the solution [23, 14, 1, 12]. N D3-branes on a conifold give rise to a conformal theory with $SU(N) \times SU(N)$ gauge group, whose supergravity dual is $AdS_5 \times T^{1,1}$ [22]. Since a D5-brane wrapped on a 2-cycle of $T^{1,1}$ (fractional brane) carries D3-charge, we have a system with two types of D3-branes. The gauge theory corresponding to N physical and M fractional D3-branes is $SU(N) \times SU(N + M)$ [22, 23, 10]. In the $AdS_5 \times T^{1,1}$ background, we turn on M units of RR-flux on $T^{1,1}$. The solution was originally found by enforcing supersymmetric-inspired first order equations of motion [1]. Remarkably, this solution satisfies all the requests for the class of exact solutions [1] [19]. Supersymmetry or imposition of the black D3-brane ansatz [1] are extremely useful for solving the equations of motion of type IIB supergravity, but they do not guarantee by themselves regularity of the resulting solution. The $SU(N) \times SU(N + M)$ theory flows to the IR region by forgetting shells of regular branes via an enhancon mechanism [13, 10, 12]. In the far IR, the dynamics is that of $SU(M)$ SYM without regular branes and the background is made regular by the hypothesis that the conifold is replaced by a deformed conifold [12]. The explicit solution can be found in [12] and the asymptotic IR behavior is reviewed in Section 4. We just quote the behavior of the conifold metric and the $Z$ function in the IR (small $\tau$).

$$
\begin{align*}
    ds^2_K &= \frac{d\tau^2}{2} + \frac{\tau^2}{4}d\Omega_2^2 + d\Omega_3^2
\end{align*}
$$
\[ Z \sim (g_s M)^2 + O(\tau^2) \]  \hspace{1cm} (3)

In the deformed conifold at small \( \tau \) the 2-cycle is collapsed but it remains a finite three-sphere of radius squared \( O(g_s M) \), supporting the RR flux. The complex field \( G_{(3)} = H_{(3)} + i g_s F_{(3)} \) is self-dual \( *_K G_{(3)} = iG_{(3)} \), and the R-R and NS-NS three-forms satisfy

\[ g_s^2 F_{(3)}^2 = H_{(3)}^2 . \]  \hspace{1cm} (4)

\( g_s M \) plays the role of the gauge theory t’Hooft parameter and it has to be large for the supergravity approximation to be trusted.

Notice also that another regular supersymmetric solution for \( N = 1 \) SYM appears in [13]. The crucial ingredients, a vanishing 2-cycle and a finite-size 3-cycle supporting the RR flux, are the same.

The class of exact solutions (1) does not require supersymmetry. We want to study the solutions (1) in the context of type 0B.

### 3 Type 0 branes on a conifold

Let us start with a general discussion of the theory of 3-branes on a conifold with general charges.

It is useful to think about type 0B as an orbifold of type IIB by the spacetime fermion number. The untwisted sector of the orbifold contains all the bosonic fields of type IIB, which we denote as \( g^U_{\mu\nu}, B^U_{\mu\nu}, \phi^U, F^U_{(n)} \), while the twisted sector contains a NS-NS tachyon \( T \) and a new set of R-R fields \( F^T_{(n)} \). The tachyon and R-R part of type 0B Lagrangian is better formulated in terms of the combinations \( F_{(n)}^\pm = (F^U_{(n)} \pm F^T_{(n)}) \)

\[
L = \int d^{10}x \sqrt{g} \left( -2e^{-2\phi} T^2 + \sum_{n=1,3,\pm} \frac{1}{2} (1 \pm T + \frac{T^2}{2}) |F^\pm_{(n)}|^2 + (1 + T + \frac{T^2}{2}) |F_{(5)}|^2 \right) \]  \hspace{1cm} (5)

The Lagrangian is written in string frame and is expanded to quadratic order in \( T \). \( F_{(5)} \) has now both self-dual or antiself-dual components and appears in the Lagrangian as the electric variable. Bianchi identities now read

\[ dF_{(5)} = H_{(3)} \wedge F^+_{(3)}, \quad d* F_{(5)} = H_{(3)} \wedge F^-_{(3)} \]  \hspace{1cm} (6)

Since all the R-R fields are doubled, also D-brane are. On a conifold, we now have a total of 4 different 3-brane charges, electric and magnetic D3 charges [1] and wrapped
for the type 0 solution along the lines of ref. [12]. The metric ansatz describes both a kinetic term. It has been explicitly checked [24] that complex bosons and dashed lines Weyl fermions. Arrows distinguish between \((N_i, \bar{N}_j)\) and \((N_j, \bar{N}_i)\) representations.

D5\(^+\) and D5\(^-\) charges

The generic theory containing all the four types of 3-brane charges can be determined as in [1, 25, 26]. It is \(\prod_{i=1}^{4} SU(N_i)\) with the matter content indicated in the quiver diagram of Figure 1. There is in addition a complicated non-renormalizable potential for the matter field obtained from the projection of the superpotential in [24].

Incidentally, we notice that, as in type II [27], the brane configuration has a type 0A T-dual description consisting of D4\(^\pm\) and NS branes. The NS world-volume is, say, along \((0, 1, 2, 3, 4, 5)\) and \((0, 1, 2, 3, 8, 9)\) and the NS branes are at the position \(0, x_6^{(0)}\) in the compact direction \(x_6\). There are \(N_1, N_2\) D4\(^\pm\) branes stretching in \(x_6\) between the NS branes along \((0, x_6^{(0)})\) and \(N_3, N_4\) D4\(^\pm\) branes along \((x_6^{(0)}, 2\pi R_6)\). Standard rules for brane configurations, adapted to type 0 [1] reproduce the gauge theory and matter content in Figure 1.

We are also interested in studying the deformed conifold. Let us introduce an ansatz for the type 0 solution along the lines of ref. [12]. The metric ansatz describes both a conifold and a deformed conifold. The antisymmetric tensor ansatz is the most general one compatible with the \(Z_2\) conifold symmetry and the expected RR flux\(^4\). The 10

---

1It is possible that a generic function \(g(T) = 1 + \alpha T^2 + \ldots\) multiplies the NS-NS antisymmetric field kinetic term. It has been explicitly checked [24] that \(\alpha = 0\). Higher terms in \(T\) do not affect our conclusion and, therefore, we set \(g(T) = 0\) in the following.

2An electric or magnetic D3-brane in type 0 can be considered as a fractional brane of the \((-1)^F\) orbifold. To avoid confusion, we reserve the name fractional for the wrapped D5-branes.

3Some example of similar type 0 brane configurations can be found in [28].

4The RR flux is supported by the conifold 3-cycle in the UV and by the \(S^2\) that replaces the singularity in the deformed conifold in the IR.
dimensional solution is

\begin{align}
    ds^2 &= e^{-5q}(dt^2 + e^{2A(r)}dx_\mu dx^\mu) + ds_5^2, \\
    ds_5^2 &= e^{3q-8f}g_5 + e^{3q+2f+y}(g_1^2 + g_2^2) + e^{3q+2f-y}(g_3^2 + g_4^2), \\
    B_{(2)} &= g(r)g_1 \wedge g_2 + k(r)g_3 \wedge g_4, \\
    F_{(3)}^\pm &= M_{(3)}g_5 \wedge g_3 \wedge g_4 + d[F(r)(g_1 \wedge g_3 + g_2 \wedge g_4)], \\
    F_{(5)} &= K_+ (r)g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5 + *(K_-(r)g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5)
\end{align}

where \( K_\pm = N_\pm + (k-g)F_\pm + 2M_\pm g \). Explicit expressions for the \( g_i \) forms can be found in [12]. Notice that, while the Bianchi identities for the 5-form are automatically satisfied, no self-duality condition is imposed on the 3-forms. The requirement of self-duality in the class of solutions (1) amounts to a precise relation between \( Z \) and the unknown functions in the 3-form ansatz. In [12], this relation is imposed by requesting first order equations. From the point of view of [19], it is not so strictly related to supersymmetry but it is a part of the exact solution (1). The field \( y \) allows for deformations of the conifold. \( N_\pm \) are the number of electric and magnetic physical D3 branes and \( M_\pm \) are the number of fractional branes. With these notations the gauge group is \( SU(N_+) \times SU(N_-) \times SU(N_+ + M_+) \times SU(N_- + M_-) \).

Using the results in [1, 11, 29], it is easy to write a five-dimensional effective action for the relevant fields,

\[
L = \int d^5x e^{4A} \left( 3(\partial A)^2 - \frac{1}{2} G_{ab} \partial \phi^a \partial \phi^b - V(\phi) \right)
\]

\[
G_{ab} \partial \phi^a \partial \phi^b = 15(\partial q)^2 + 10(\partial f)^2 + \frac{\partial (T)^2}{8} + \frac{\partial (y)^2}{2} + \frac{\partial (\phi)^2}{4} + e^{-\phi - 6q - 4f}(e^{-2y}2 + e^{2y}2) + \sqrt{3} \sum_{\pm} f_{\pm}(T)e^{\phi - 6q - 4f}(\partial F_{\pm})^2
\]

\[
V(\phi) = e^{-8q} \left( e^{-12f} - 6e^{-2f} \cosh y + \frac{9}{4}e^{8f} \sinh y^2 \right) - \frac{T^2}{8} e^{\phi/2 - 5q} + \sum_{\pm} \left[ \frac{9\sqrt{3}}{8} e^{4f - 14q + \phi} f_{\pm}(T)(e^{-2y}F_{\pm}^2 + e^{2y}(2M_+ - F_+))^2 + \frac{27}{4} e^{-20q} f_{\pm}(T)K_\pm^2 \right] + \frac{9\sqrt{3}}{8} e^{4f - 14q} e^{-\phi}(g - k)^2
\]

where \( f_{\pm} = 1 \pm T + T^2/2 + \ldots \). Einstein equations also impose the constraint \( 3(\partial A)^2 - \frac{1}{2} G_{ab} \partial \phi^a \partial \phi^b + V(\phi) = 0 \). The undeformed conifold is obtained for \( y = 0, k = g, F_{(\pm)} = 0 \).

We analyze the potential in various steps. First, set \( M_\pm = 0, N_+ = N_+ = N \) and \( T = 0 \) and identify all \pm fields: \( \lambda_- = \lambda_+ \). The potential is now indistinguishable from the potential for \( N \) type IIB D3-branes at a conifold. By rescaling fields, we can set
Consider first the type IIB potential. For $q = f = y = g = k = F = 0$ we have an $N = 1$ critical point corresponding to $AdS_5 \times T^{1,1}$. From eq. (8), we can extract the mass of the fields and the dimensions of the corresponding operators: $\Delta_q = 8, \Delta_f = 6, \Delta_y = 3, \Delta_{k-g,F} = 3, 7$. $k + g$ does not appear in the potential. It is the massless field $f_{s2} B_{(2)}$ associated with a marginal direction in the CFT [22, 30, 10]. The corresponding operator is $\frac{1}{g^2}(F_{(1)}^2 - F_{(2)}^2)$. Using the results in [31], we can tentatively identify these fields with operators in the multiplets:

$$
q, f \rightarrow \text{Tr}(W^2 \bar{W}^2)
$$

$$(k - g, F)_{\Delta=7} \rightarrow \text{Tr}(A \bar{A} + B B)W^2
$$

$$(k - g, F)_{\Delta=3} \rightarrow \text{Tr}(W_{(1)}^2 + W_{(2)}^2)
$$

$$
y \rightarrow \text{Tr}(W_{(1)}^2 - W_{(2)}^2)
$$

The terms proportional to $M_\pm$ come from the introduction of wrapped D5-branes. If we still set $\lambda_- = \lambda_+$ and $T = 0$ we find the potential for the $N = 1$ configuration of regular and fractional type IIB branes [11]. Finally, the type 0 reduction is accomplished by turning on the tachyon field $T$ and by doubling all the fields associated with RR forms.

### 4 A conformal solution

We can consider configurations of D3 and wrapped D5-branes in type 0B as projections of similar configurations in type IIB. The simplest configuration is obtained by using the regular representation in the orbifold projection [22]. Every type IIB brane is doubled. The $AdS_5 \times S^5$ dyonic solution of [4] is an example of this procedure [17]. The type 0B solution is obtained by identifying the untwisted fields with their parents in type IIB and setting to zero all the twisted fields.

We expect a non-supersymmetric conformal solution as a projection of the $N = 1 AdS_5 \times T^{1,1}$ background. This solution is also discussed in [23, 26]. It does not contain fractional branes. Every IIB D3-brane is now doubled into an electric and magnetic type 0 D3-brane. The resulting gauge theory is $SU(N)^4$ with the matter content indicated in figure 1 [23, 26] and a non-renormalizable potential. We expect the theory to flow in the IR to an interacting fixed point as in [22].

It is easy to verify that for $M_\pm = 0$ and $N_- = N_+ = N, q = f = T = \phi = F_\pm = g = k = 0$ is an $N = 0$ critical point of the potential (8) corresponding to a 10 dimensional solution $AdS_5 \times T^{1,1}$.

There are a total of four independent coupling constants in the gauge theory. We can identify the real part of three of these couplings with the dilaton, the value of $B_{(2)}$ on the 2-cycle and the tachyon. The constant value of the dilaton and $B_{(2)}$ do not appear
explicitly in the 10 dimensional equations of motion. There are therefore two exactly
marginal direction in the CFT as in \[22\], at least in the large N limit. The existence of
such marginal directions in a non-supersymmetric theory is easily explained. In the large
N limit, the correlation functions for untwisted fields are identical to those in the parent
type IIB theory \[33\]. However, we do not expect that the coupling constant associated
with the tachyon, which is a twisted field, is a marginal parameter. The dimension of the
 corresponding operator is computed below. It would be interesting to identify the fourth
coupling within the supergravity modes.

The spectrum of operator dimensions can be easily extracted from eq. (8). The fields
considered before eq. (9) are still in the spectrum with the same mass as in type IIB.
The type 0 operators are obtained by replacing fermions/scalars with bi-fundamental
fermions/scalars and considering particular combinations of the traces in the four gauge
groups. We have two more fields from the type 0 twisted sector, the tachyon \(T\) and a
combination \(t\) of \(F^{(\pm)}\), \(k\), \(g\). The corresponding operators have mass-squared

\[
T \rightarrow m^2 = 16 - 2\sqrt{x}
\]
\[
t \rightarrow m^2 = 9, \quad \Delta = 2 + \sqrt{13}
\]

where \(x\) is the t’Hooft coupling of the overall coupling constant \(\xi\). As in \[4\], the tachyon
is unstable at large \(x\). It is tempting to speculate as in \[4\] that the theory is perfectly
unitary at small \(x\) (as the extrapolation of formula (11) indicates) and undergoes a phase
transition at larger \(x\), as typical of many gauge theories in the large N limit.

5 A non-conformal “self-dual” solution

The next example is obtained by introducing D5\(^\pm\) branes but still enforcing an orbifold
projection with the regular representation. The \(N = 1\) solution for a \(SU(N+M) \times SU(N)\)
gauge theory \[11, 12\] has been briefly reviewed in the Introduction. It descends to a non-
supersymmetric type 0 solution by putting \(T = 0\) and \(\lambda_- = \lambda_+\) for all the relevant fields
and parameters. The gauge theory is \(SU(N)^2 \times SU(N + M)^2\). We have \(N\) self-dual
D3-branes, \(M\) wrapped D5\(^+\) and \(M\) wrapped D5\(^-\). The supersymmetric solution flows
in the IR to the pure \(N = 1\) SYM with a cascade mechanism \[12\]. The corresponding
IR solution contains no D3-branes and is defined on a deformed conifold \[12\]. We expect
that the cascade mechanism has a non-supersymmetric analogous. The gauge theory
dynamics is obviously much more difficult to analyze due to the lack of supersymmetry.
However, we may expect that, in the large N limit, many supersymmetric results based
on duality should continue to hold \[33, 34\]. Notice that there are no reasons for imposing

\[
^5\text{Notice that we rescaled a factor of } N, \text{ so that } x = e^\phi.
\]
first order equations in a non supersymmetric context. The cascade solution is just one particular solution of type 0 out of infinitely many. The fact that it is completely regular indicates that it actually corresponds to a possible quantum field theory RG flow.

The IR gauge theory is an $SU(M) \times SU(M)$ gauge theory with 2 bi-fundamental Weyl fermions. The IR dynamics of this theory is described by the type 0 restriction of the deformed conifold solution. For completeness, we quote from [12] the IR behavior of the solution (small $\tau$).

$$ds^2_K = \frac{d\tau^2}{2} + \frac{\tau^2}{4}d\Omega^2_2 + d\Omega^2_3$$

$$Z \sim (g_s M)^2 + O(\tau^2)$$

$$g \sim \tau^3, \quad k \sim \tau, \quad F \sim \tau^2$$ (11)

The complete solution can be found in [12]. It is also derived in [29] using the 5-dimensional formalism of eq. (8). The radial coordinate $\tau$ is related to the coordinate $r$ of the five-dimensional effective theory by $d\tau = -e^{4f-4q}dr$. Notice that in the five-dimensional approach, the metric fields $q, f, y, A$ are singular, behaving as $\sim \log(r)$. The five-dimensional metric has therefore a naked singularity $ds^2 \sim dr^2 + r^{4/5}dx_\mu dx^\mu$. This is just an artifact of the dimensional reduction, the ten-dimensional solution being completely regular for all $r$.

The solution exhibits confinement and spontaneous chiral symmetry breaking [12]. Incidentally, we notice that the deformation of the conifold is requested for studying vacua with non-zero fermionic condensate. We discuss the type IIB solution, all considerations being applicable to type 0 too. We use standard techniques [5, 6, 35], with a proviso. Since the gauge theory is not a deformation of a CFT, the supergravity solution is not asymptotic to a purely AdS vacuum. We can however consider a regime, for example $M << N$, where modifications of the AdS solution are small. The space has still a boundary and we can reasonably use the machinery of AdS/CFT for identifying operators. Adapting the results in [29], we can write a superpotential [6] for the type IIB supersymmetric theory

$$W = -3e^{4f-4q} \cosh y - 2e^{-6f-4q} - 3\sqrt{3} e^{-10q} (N + F(k-g) + 2Mg)$$ (12)

The tadpole term $2Mg$, which is the same for a conifold or a deformed conifold, is due to the fractional branes. This term is responsible for the change of gauge group $SU(N) \to SU(N + M)$. The running of fields due to the remaining terms in the superpotential can be interpreted as a deformation or a change of vacuum in the gauge theory [36, 30]. Substitution of the conifold with the deformed conifold introduces the fields $y, F, (k-g)$. According to eq. (9), these fields correspond to the gaugino condensates of the two gauge groups. We can decide whether the running of these fields corresponds to a deformation
or a choice of a different vacuum by looking at the asymptotic behavior \[36, 30, 35\]. The expansion of \( W \) near the origin determines the asymptotic behavior using \( 2G^{ab} \dot{\lambda}_b = \partial W / \partial \lambda_a \). For canonically normalized fields \( \lambda_C \), \( W = -3 + \alpha \lambda_C^2 + ... \) implies \( \lambda_C \sim e^{-\alpha r} \). \( \alpha = \Delta \) corresponds to a vacuum of the theory, \( \alpha = 4 - \Delta \) to a deformation. In our case, for canonically normalized fields,

\[
W \sim -3 + 4q_C^2 - 6f_C^2 - 3y_C^2 - 6F_C(k - g)_C + ... \tag{13}
\]

We can easily see that the fields associated with the gaugino condensates, \( y_C \), and a combination of \( F_C, (k - g)_C \) behave as \( e^{-\Delta r} = e^{-3r} \). The corresponding operators have non-zero vacuum expectation value. Therefore, one motivation for introducing the deformation of the conifold is to study vacua with non-zero gaugino condensate, as it is the case for \( N = 1 \) SYM. Similar results apply to the fermionic condensates of type 0 solutions.

There is still a fundamental question. The tachyon vacuum expectation value is zero. Is the background stable? In type 0 models, the stabilization of the tachyon is due to the D3-brane charge, which is absent here. Nevertheless, we have an antisymmetric tensor background that may serve the purpose. Let us qualitatively investigate the effective mass for the tachyon field. In our background, since \( F^T = 0 \), we have \( F^+_{(3)} = F^-_{(3)} = H_{(3)}/g_s \) and the effective potential for the tachyon follows from eq. \( (F) \)

\[
L = -2 e^{-2\phi} T^2 + (1 + \frac{T^2}{2}) |F_{(3)}^+|^2 \tag{14}
\]

From this equation we see that, as anticipated, \( T \) has no tadpoles and can be consistently set to zero. Moreover, its effective mass is \( \uparrow \)

\[
m_T^2 \sim -\frac{2}{g_s^2} + \frac{M^2}{2(g_s M)^3} \tag{15}
\]

where we used the fact that the \( S^3 \) has radius-square \( (g_s M) \). We see that the stabilization condition is \( -2 + \frac{1}{2g_s M} \geq 0 \).

If \( g_s M \) is sufficiently small, the background is stable. Notice that this is the limit where supergravity is no longer valid. The same happens in the known type 0 solutions \([1, 2]\). The tachyon instability may correspond to some pathological behavior for the strongly coupled gauge theory. The description of the weakly coupled \( SU(M)^2 \) theory with bi-fundamental fermions requires small \( g_s M \) \( \uparrow \). The supergravity approximation is then invalidated and a string sigma-model analysis is requested. Extrapolating from eq. \([15]\), we may conclude that the associated string background is stable. We can image

\[
\uparrow \text{Notice that the NS-NS two-form does not contribute to the tachyon mass}[24].
\]

\[
\uparrow \text{The focus into the IR by decoupling the UV region and the infinite cascade needs small } g_s M \ [2].
\]
a situation where the stabilization mechanism, made more precise, just requires an intermediate (of order 100) value for the t’Hooft parameter and corrections can be still estimated to be small. Alternatively, we should request the knowledge of the string sigma-model. The absence of $F(5)$ background may simplify the problem.

6 The pure glue theory

We now turn to the system with only one type of fractional brane, say D5+. This is the most interesting theory, since it is pure glue $SU(M)$.

A solution for D5+ branes corresponds to the non-regular representation in the orbifold reduction of the type IIB solution. An analogous configuration in type 0, consisting of purely electric D3 branes with gauge group $SU(N)$ and 6 massless bosons, was considered in [4]. By avoiding the existence of massless bosons, we also avoid all the related complications and fine tuning problems [1, 3]. The IR behavior of the equations of motion for electric D3-branes was considered in [4] and, as expected, is generically singular. Such asymptotic behavior contains very few information about what kind of solution is really selected by physics and how the singularity is resolved by dynamics.

Here we propose a possible regular background describing the IR dynamics of the pure glue theory. Once again, we look for regular solutions on a deformed conifold. Since $F_{(3)}^{-} = 0$, we see from eq. (5) that the tachyon has now a tadpole and has to run. It may induce a running of the dilaton.

We propose that the tachyon relaxes to zero in the IR. Most likely, this is a fine-tuning. We also make the assumption that the dilaton goes to a constant value. All the other fields retain the same value as in the type II deformed conifold solution [12], modulo a trivial rescaling of the R-R fields. We can now easily prove that all these assumptions are compatible with the equations of motion. Consider the dilaton and tachyon equations of motion in Einstein frame,

\[
\Box \phi \sim \frac{1}{2} e^\phi f_+(T) |F_{(3)}|^2 - e^{-\phi} |H_{(3)}|^2 + \frac{T^2}{2} e^{\phi/2} \\
\Box T \sim \frac{1}{2} e^\phi f'_+(T) |F_{(3)}|^2 + f'_+(T) |F_{(5)}|^2 + T e^{\phi/2}
\]

For $T \to 0$ the last term in the dilaton equation is negligible. Since $f_+(0) = 1$, the equation reduces to the type IIB dilaton equation after a rescaling of the R-R fields. The dilaton may remain constant. After neglecting subleading terms, there is a constant term $e^\phi f_+(T) |F_{(3)}|^2 \sim 1/(g_s^2 M)$ in the right hand side of the equation of $T$. For the solution \(\Box T \sim \frac{1}{\tau^2} \partial_\tau (\tau^2 \partial_\tau T)\). The tachyon equation is then satisfied for $T \sim \tau^2$. We can similarly check that Einstein equations are still satisfied.
Alternatively, we can use the five-dimensional effective theory (8). For $T = 0$, a simple rescaling $(N, M, F) \rightarrow \sqrt{2}(N, M, F)$ maps the type 0 potential to the type IIB one. The change of variables $r \rightarrow \tau$ is useful. It is then possible to systematically expand the potential and the solution in power of $\tau^2$.

We expect that the gauge theory coupling constant blows up in the far IR. We can qualitatively identify the behavior of the coupling constant by looking at the world-volume of a D5$^+$ probe wrapped on the 2-cycle. The effective coupling constant is

$$\frac{1}{g(\tau)^2} = e^{\phi(T)} \sqrt{g(2)} \sqrt{1 + |B|^2_{(2)}}$$

(17)

where $k(T)$ testifies the coupling of the worldvolume theory to the tachyon field [1, 38] and the metric and B-field are evaluated on the 2-cycle. Exactly as in [12], the effective coupling constant (17) will diverge due to the metric and B-field on the 2-cycle, $1/g^2 \sim \tau^2$.

We can systematically expand the solution near the IR in power series of $\tau^2$. We expect that, being regular, such solution describes a consistent RG flow. We may conjecture that the previous asymptotic solution correctly describes the IR behavior of the pure $SU(M)$ theory. However, the full solution describing the theory at all scales is certainly difficult to find. Since the solution involves a fine-tuning $T \rightarrow 0$, it is not clear if it can really describe the pure $SU(M)$ theory decoupled from its UV completion. We could speculate that $T = 0$ is an IR attractor for all solutions. We envisage different possibilities for the UV theory. The IR solution can be connected via a cascade mechanism to the $SU(N + M) \times SU(N)$ theory with bi-fundamental scalar fields, obtained with $N$ D3$^+$ and $M$ D5$^+$ branes. Alternatively, we could find a pure $SU(M)$ theory exhibiting asymptotic freedom in the UV, as in [11]. A third option is a UV completion along the lines of [13]. In all cases we face the problem of extrapolating our Lagrangian (5) to a region where $T$ is not small. The exact form of $f_+(T)$ and of the potential $V(T)$ for $T$ could become crucial for correctly capturing the UV behavior. Moreover the dilaton will start to run. It would be quite interesting to see if the class of exact solutions (1) has a type 0 extension to the case of non-constant dilaton and arbitrary $f_+(T), V(T)$.

---

8 Notice that in the D3-branes type 0 solutions [1] the dilaton necessarily runs. Here, we are saved by the 2-cycle. Notice also some analogies. An interesting solution in [1] flows to an IR fixed point at strong coupling. Such a particular flow exists also for the more general theories $SU(Q) \times SU(N)$ of $N$ electric and $Q$ magnetic D3-branes [39]. In this (fine-tuned) solution, the tachyon goes to zero while the dilaton blows up in a very precise combination, with most of the fields retaining the same value as in type II. In the conifold case, since the dilaton affects differently the terms $F^2_{(5)}$ and $F^2_{(3)}$, we have no solution like that.
7 Conclusions

We have seen that the deformed conifold solution can be easily adapted to describe non-supersymmetric solutions. The use of a Calabi-Yau manifold may suggest supersymmetry, but the mechanism in [12] seems more general. The necessary ingredients are RR fluxes supported by finite-size cycles. From eq. (2), we see that, in the absence of D3 sources and with mild assumption on K, the warp factor $Z$ is regular.

These ingredients may define a larger class of solutions. We may expect to study interesting non-supersymmetric theories by using generic manifolds. Type 0 is extremely useful for constructing non-supersymmetric gauge theories with branes. However, in type 0, we are more restrained by the absence of a sensible Lagrangian for generic T. It would be interesting to extend the class of solutions (1) to type 0. We noticed that, under the assumption that $T$ relaxes to zero, the solutions (1) still provide an IR asymptotic description of the physics.

Moreover, in type 0 solutions, the stabilization of the tachyon requires small t’Hooft coupling. Ultimately, this could be not so dangerous, at least for our most ambitious goals. The discovery of completely regular supergravity theories can be useful for studying the dynamics of strongly coupled gauge theories. We can typically describe with supergravity only gauge theories with finite cut-off and coupled to extra-modes. Eventually, we will be interested in duals of weakly coupled theories. As usual, we were prepared to the unpleasant situation that, in the seek of a description of strong dynamics for weakly coupled gauge theories, we need to cross a region where the effective t’Hooft parameter is small. This invalidates the supergravity approximation and requires a string sigma-model analysis.

Acknowledgments

We would like to thank A. Armoni, S. Ferrara, A. Pasquinucci and A. Tseytlin for useful discussions and suggestions. L.G., F. B. and A. Z. are partially supported by INFN and MURST, and by the European Commission TMR program HPRN-CT-2000-00131, wherein they are associated to the University of Padova.

References

[1] I.R. Klebanov and A. A. Tseytlin, Nucl. Phys. B546 155 (1999), hep-th/9811035;
    Nucl. Phys. B547 143 (1999), hep-th/9812089.

[2] I.R. Klebanov and A. A. Tseytlin, JHEP 9903 015 (1999), hep-th/9901101.
[3] J. A. Minahan, JHEP 9901:020 (1999), hep-th/9811156; JHEP 9904:007 (1999), hep-th/9902074.

[4] I. R. Klebanov, Phys. Lett. B466 166 (1999), hep-th/9906220.

[5] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, JHEP 9812:022 (1998), hep-th/9810126; JHEP 9905:026 (1999), hep-th/9903026; Nucl. Phys. B569:451 (2000), hep-th/9909047.

[6] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, hep-th/9904017; JHEP 2000:038 (2000), hep-th/9906194.

[7] S. S. Gubser, hep-th/0002160.

[8] A. Brandhuber and K. Sfetsos, Phys.Lett.B488 373 (2000), hep-th/0004148; N. Evans and M. Petrini, hep-th/0006048; A. Khavaev and N. P. Warner, hep-th/0009159.

[9] K. Pilch and N. P. Warner, hep-th/0004063; hep-th/0006066.

[10] I. R. Klebanov and N. Nekrasov, Nucl. Phys. B574 263, (2000), hep-th/9911096.

[11] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B578 123 (2000), hep-th/0002159.

[12] I. R. Klebanov and M. J. Strassler JHEP 0008:052 (2000), hep-th/0007191.

[13] J. M. Maldacena and C. Nuñez, hep-th/0008001.

[14] K. Oh and R. Tatar, JHEP 0005 (2000) 030, hep-th/0003183.

[15] C. V. Johnson, A. W. Peet and J. Polchinski, Phys. Rev.D61 086001 (2000), hep-th/9911161.

[16] J. Polchinski and M. J. Strassler, hep-th/0003136.

[17] N. Nekrasov and S. L. Shatashvili, Phys. Rept. 320 127 (1999), hep-th/9902110.

[18] C. Angelantonj and A. Armoni, Nucl. Phys. B578 (2000) 239, hep-th/9912257; Phys. Lett. B482 (2000) 329, hep-th/0003050.

[19] M. Graña and J. Polchinski, hep-th/0009211.

[20] See M. Cvetic, H. Lu and C.N. Pope, hep-th/0011023 for generalizations.

[21] S. S. Gubser, hep-th/0010010.
[22] I. R. Klebanov and E. Witten, Nucl. Phys. B536 199, (1998), hep-th/9807080.
[23] S. S. Gubser and I. R. Klebanov, Phys. Rev. D58 125025 (1998), hep-th/980807.
[24] A. A. Tseytlin, proceedings of QFTHEP 99, Published in In Moscow 1999, High energy physics and quantum field theory 427.
[25] M. Alishahiha, A. Brandhuber and Y. Oz, JHEP 9905:024 (1999) hep-th/9903186.
[26] R. Blumenhagen, A. Font and D. Lust, Nucl. Phys. B560, 66 (1999), hep-th/9906101.
[27] A. M. Uranga, JHEP 9901:022 (1999), hep-th/9811104; K. Dasgupta and S. Mukhi, Nucl. Phys. B551 (1999) 204, hep-th/9811133; JHEP 9907 (1999) 008, hep-th/9904131.
[28] A. Armoni and B. Kol, JHEP 9907:011 (1999), hep-th/9906081.
[29] L.A. Pando Zayas, A.A. Tseytlin, hep-th/0010088.
[30] I. R. Klebanov and E. Witten, Nucl. Phys. B556 89 (1999), hep-th/9905104.
[31] A. Ceresole, G. Dall’Agata, Riccardo D’Auria and Sergio Ferrara, Phys. Rev. D61 066001 (2000), hep-th/9905226.
[32] M. R. Douglas and G. Moore, hep-th/9603167.
[33] S. Kachru and E. Silverstein, Phys. Rev. Lett 80, 4855 (1998), hep-th/9802183; A. Lawrence, N. Nekrasov and C. Vafa, Nucl. Phys. B533 199 (1998), hep-th/9803013; M. Bershadsky, Z. Kakushadze and C. Vafa, Nucl. Phys. B523 59 (1998), hep-th/9803076.
[34] M. Schmaltz, Phys. Rev. D59 (1999) 105018, hep-th/9805218.
[35] See M. Petrini and A. Zaffaroni, hep-th/0002172 for a review.
[36] V. Balasubramanian, P. Kraus and A. Lawrence, Phys. Rev. D59:046003 (1999), hep-th/9805174.
[37] R. Grena, S. Lelli, M. Maggiore and A. Rissole, JHEP 0007:005 (2000), hep-th/0005213.
[38] M. R. Garousi, Nucl. Phys. B550 225 (1999), hep-th/9901083.
[39] Work in progress.