Vertical structure of conventionally neutral atmospheric boundary layers

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Conventionally neutral atmospheric boundary layers (ABLs) are frequently encountered in nature, and their flow dynamics affect the transfer of momentum, heat, and humidity in the atmosphere. Therefore, insight into the flow structure in conventionally neutral ABLs is necessary to further improve models for long-term weather and climate forecast, while it provides further insight for atmospheric applications like the wind industry. The structure of conventionally neutral ABLs is complicated due to the coexistence of shear- and buoyancy-generated turbulence, and therefore analytical descriptions have been limited to the mean wind speed. Here we introduce an innovative model based on the Ekman equations and the basis function of the universal potential temperature flux profile that allows one to describe the vertical profiles of the horizontal components of wind and shear stress and hence capture features like the wind veer profile. Our formulation in terms of departure from the geostrophic wind allows us to describe the profiles as a function of one control parameter, although the description of wind speed profile still needs two. We find excellent agreement between analytical predictions, high-fidelity simulations, and field measurement campaigns. These findings advance the fundamental understanding of the ABL structures and atmospheric turbulence.

The atmospheric boundary layer (ABL) is the lower part of the troposphere where most human activity and biological processes occur. The flow dynamics in the ABL are influenced by the Earth’s surface, while they are also affected by the Coriolis force and thermal stratification (1). The typical timescale in which the boundary layer dynamics depend on changes in the surface forcing is 1 h or less (2). In the lower part of the ABL, which is also known as the surface layer, the Coriolis force can be neglected, while the vertical turbulent fluxes of momentum, heat, and humidity are relatively constant with height (3). This suggests that these turbulent fluxes mainly depend on their mean vertical gradients. According to the famous Monin–Obukhov similarity theory (MOST), these profiles can be expressed as functions that depend on a single stability parameter $L_s$, where $L_s$ is height and $L_o$ is the surface Obukhov length (4, 5). For the last seven decades, the MOST has served as a cornerstone for studies of the atmospheric surface layer and therefore is regarded as the starting point of modern micrometeorology (6). For neutral stability conditions, the MOST reduces to the logarithmic law for wall-bounded turbulent shear flows (7, 8). However, above the surface layer the applicability of the MOST breaks down (9–12).

Large-scale weather phenomena determine the internal structure of the ABL. Simultaneously, the large weather phenomena are influenced by the flow structure and heat, momentum, and humidity transfer in the boundary layer (1). A solid analytical model for ABLs is crucial to accurately capture the coupling between meso- and microscales processes, which is a long-standing challenge in numerical weather predictions (13, 14) and was dubbed “terra incognita” (unknown territory) by Wyngaard (15). Greater understanding of this interaction is essential for improving climate models (16–19) and long-term weather forecast models (20–22). Furthermore, advancements in modeling these boundary layer dynamics can inform site assessments for offshore wind farms (14, 23–26).

When the potential temperature flux on the surface is approximately negligible, the ABL is considered to be neutrally stratified (27, 28), which can be further distinguished as truly neutral and conventionally neutral, depending on whether the flow develops against a neutrally stratified free flow or a background stable stratification (29). Although truly neutral ABLs are rare in reality, conventionally neutral ABLs are commonly observed, for example, over sea, above large lakes, and over land during the transition period after sunset or on cloudy days with powerful winds (30). As a concrete example, the conventionally neutral ABL was observed from about 1:40 to 5:30 PM on 7 to 9 September 2010 in the measurement campaign performed at Høvsøre, Denmark (31). The flow dynamics inside
these so-called conventionally neutral ABLs depend on the height of the capping inversion and the potential temperature gradient in the free atmosphere. It follows from dimensional analysis that the dynamics in these conventionally neutral ABLs in equilibrium are governed by just two external dimensionless parameters (22, 29), namely the Rossby number \( \text{Ro} = u_\infty / (\text{f} z_0) \) and the Zilitinkevich number \( \text{Zl} = N / f \), where \( u_\infty \) is the friction velocity, \( z_0 \) is the surface roughness height, \( f \) is the Coriolis parameter, and \( N \) is the free-atmosphere Brunt–Väisälä frequency. Fig. 1 shows a schematic representation of the flow dynamics and parameter definitions for conventionally neutral ABLs.

Conventionally neutral ABLs have been studied extensively (22, 29, 30, 32–40) and the derivation of the mean wind speed profile has received considerable attention (22, 35, 36, 39). In the surface layer, the wind speed profile is captured very well by the logarithmic law of the wall, and effects of free-atmosphere stratification are generally parameterized using an additional quadratic term with height (35, 36). Although such corrections can give improved predictions, they do not allow one to describe the wind speed profile in the entire boundary layer. Based on the similarity-theory derivations, Kelly et al. (39) derived a model that accounts for the effect of the Coriolis parameter and free-atmosphere stratification to obtain an analytical expression that captures the wind speed profiles from large-eddy simulations up to \( z / z_0 = 0.7 \). Using dimensional analysis and a perturbation method approach, Liu et al. (22) derived an analytical expression for the potential temperature flux over the entire boundary layer. Using this universal flux expression, the insights of the MOST (4, 5) with a local scaling hypothesis, and the similarity-theory derivations of Kelly et al. (39), Liu et al. (22) derived an analytic expression for the wind speed profile over the entire boundary layer height. However, fundamental issues remain open, such as predicting the vertical boundary layer structure based on a rigorous mathematical derivation. The increasing height of wind turbines has resulted in considerable attention to develop analytical descriptions to extend the vertical structure of the ABL above the surface to provide more reliable estimates for the expected wind speeds at higher elevations (36, 41, 42). Important progress on the influence of unsteadiness (43, 44), baroclinicity (45–48), and buoyancy (12, 49) has been obtained. For the conventionally neutral ABL considered in the present study, Csanady (50) developed an analytical model that assumes that the eddy viscosity is constant throughout the boundary layer (34). Additionally the model assumes that there is no disturbance above the inversion base. However, both assumptions are not strictly valid over the entire boundary layer, and hence the model does not capture the structure of the entire boundary layer.

In the remainder of this paper, we first present our innovative approach for conventionally neutral ABLs, which is based on the Ekman equations and the basis function of the universal potential temperature flux profile (22). We provide an analytical prediction of the vertical structure of the horizontal velocity and turbulent shear stress components, which are validated against high-fidelity computer simulations and data from field measurement campaigns. The excellent agreement with simulations and observations supports the correctness of the presented analytical profiles, which provides an innovative approach to our understanding of the ABL structures and atmospheric turbulence.

### Analytical Model

For simplicity, we focus on the Northern Hemisphere where the Coriolis parameter \( f > 0 \). The coordinate system is oriented such that the \( x \) axis is parallel to the wind direction at the surface, the \( z \) axis is pointing upward, and \( (x, y, z) \) form a right-handed coordinate system (Fig. 1).

In stationary and horizontally homogeneous conditions, the mean wind components \((U, V)\) in the ABL satisfy the Ekman equations (51, 52),

\[
\frac{d\tau_x}{d\xi} = -fh(V - V_g), \quad \frac{d\tau_y}{d\xi} = fh(U - U_g),
\]

where \( \tau_x \) and \( \tau_y \) are the horizontal components of the stress tensor, \((U_g, V_g)\) are the geostrophic wind components in the free atmosphere, \( h \) is the boundary layer height, and \( \xi = z / h \) is the dimensionless altitude. Note that many different definitions of \( h \) are used in the literature (22, 32, 37, 40). In this work, we define the boundary layer height \( h \) as the lowest height at which the total turbulent shear stress vanishes (Fig. 1). Thus, the boundary conditions of Eq. 1 are

\[
\tau_x(0) = u_\infty^2, \quad \tau_x(1) = \tau_y(0) = \tau_y(1) = 0.
\]

Integrating the Ekman Eq. 1 subject to the shear stress boundary conditions 2 gives

\[
\int_0^1 \frac{U - U_g}{u_\infty} \, d\xi = 0, \quad \int_0^1 \frac{V - V_g}{u_\infty} \, d\xi = \frac{u_\infty}{fh} \equiv b.
\]

These integral constraints must be satisfied when the ABL is in a quasi-stationary state. It directly follows from the first integral of Eq. 3 that there must be a supergeostrophic wind \( U > U_g \) in the upper part of the boundary layer since \( U < U_g \) in the lower part of the boundary layer. The second integral of Eq. 3 is nothing but the inverse dimensionless boundary layer height \( b \). Initially, one may argue that \( b \) can be defined as the Rossby number. However, this definition does not work here since \( b \) can be parameterized as a function of \( Zl \) (35). Taking the definition of the boundary layer height as...
1/(1 - 0.05^2/3) times the height at which the total shear stress reaches 5% of the surface value (22), we find that

\[ b = (1 - 0.05^2/3) \beta, \quad \beta = (C_{RL}^{-2} + C_{NL}^{-2} Z_i)^{1/2}, \]

where \( C_{RL} = 0.5 \) and \( C_{NL} = 1.6 \) are empirical constants (40). The geostrophic wind components \((U_g, V_g)\) can be predicted by the geostrophic drag law (35, 40),

\[ \frac{kU_g}{u_w} = \ln(Ro - A(Zi)), \quad \frac{kV_g}{u_w} = -B(Zi), \]

where \( k = 0.4 \) is the von Kármán constant, and \( A \) and \( B \) are the geostrophic drag law coefficients that depend only on \( Z_i \). The functional forms of \( A \) and \( B \) have been proposed analytically (35) and confirmed in high-fidelity simulations (40). Specifically, the parameterizations of \( A \) and \( B \) are

\[ A = -A_1 m + \ln(A_0 + m) + \ln \beta, \quad B = (B_0 + B_1 m^2) \beta^{-1}, \]

where \( A_1 = 0.65, A_0 = 1.3, B_1 = 7, B_0 = 8 \) are dimensionless constants (40), and \( m \) is the composite stratification parameter,

\[ m = (1 + C_m Z_i^2)^{1/2} \beta^{-1}, \]

where \( C_m = 0.1 \) is an empirical coefficient (35, 40). Fig. 2 shows the dependence of the parameters \((A, B, b)\) on the Zilitinkevich number \( Z_i \), where the solid lines are predictions given by Eqs. 4 and 6 and the solid circles are the simulation data taken from Liu et al. (40). The excellent agreement between the predictions and simulations confirms the parameterizations of \((A, B, b)\).

To determine the functional forms of the streamwise and spanwise velocity components \((U, V)\) over the entire boundary layer height, we assume

\[ \frac{kU}{u_w} = \ln \left( \frac{z}{z_0} \right) + f_u(\xi, Z_i), \quad \frac{kV}{u_w} = f_v(\xi, Z_i), \]

where \( f_u, f_v \) are regular functions such that \( f_u, f_v \to 0 \) as \( \xi \to 0 \). At the ground (or more strictly at \( z = z_0/h \ll 1 \)) the wind vanishes,

\[ f_u(0, Z_i) = f_v(0, Z_i) = 0. \]

At the top of the boundary layer the wind recovers to the geostrophic wind [this can be seen more clearly by combining Eqs. 3, 5, and 8 and recalling that \( \ln(Ro) = \ln(b + \ln(h/z_0)) \)],

\[ f_u(1, Z_i) = \ln b - A \equiv -a(Z_i), \quad f_v(1, Z_i) = -B. \]

Previous studies (51, 52) assumed that the profiles of \((f_u, f_v)\) had a power law dependence on the coordinate \( \xi \). However, this assumption is not valid near the inversion layer, which has been shown to be mathematically a singular perturbation problem (22). Instead, we assume \((f_u, f_v)\) have the functional forms

\[ f_u(\xi, Z_i) = -a(Z_i)\xi + a_\psi(Z_i)\psi(\xi), \]

\[ f_v(\xi, Z_i) = -B(Z_i)\xi + b_\psi(Z_i)\psi(\xi), \]

where

\[ \psi = \xi - \frac{\epsilon^2}{\epsilon^2 + 1}, \quad \psi(0) = \psi(1) = 0. \]

We emphasize that Eqs. 11 and 12 are formulated in the present form such that the boundary conditions 9 and 10 are satisfied. Furthermore, we note that \( \psi \sim u_w/(NL) \), where \( L = -(u_w^2) / (\kappa g q) \) is the local Obukhov length with \( \theta_0 \) the reference potential temperature, \( g \) the gravitational acceleration, and \( q \) the potential temperature flux (22). Thus, the nonlinear term \( \psi \) represents the contribution of the potential temperature flux. The parameter \( \epsilon \) relates the thickness of the inversion layer to the boundary layer depth as

\[ \epsilon \equiv \frac{h - z_i}{2h} \ll 1, \]

where \( z_i \) is the inversion base height that is well below the inversion layer (Fig. 1). In principle, \( \epsilon \) depends on \( Z_i \). However, it turns out that for moderate \( Z_i \) numbers \( \epsilon = 0.12 \) is a good approximation (22). Substituting Eqs. 11 and 12 into Eq. 3 and omitting small terms of \( O(1/\epsilon^2 - 1) \) gives

\[ a_\psi = \frac{2 - a}{1 - 2\epsilon}, \quad b_\psi = \frac{2 \kappa b - B}{1 - 2\epsilon}. \]

Therefore, the geostrophic wind departure is

\[ \frac{kU - U_g}{u_w} = \ln \xi + a(1 - \xi) + a_\psi \psi, \]

\[ \frac{kV - V_g}{u_w} = B(1 - \xi) + b_\psi \psi. \]

The functional forms of \((\tau_x, \tau_y)\) follow by substituting Eqs. 11 and 17 into Eq. 1,

\[ \tau_x = 1 - \frac{k b}{\epsilon^2} \left\{ B \left( \xi - \frac{1}{2} \epsilon^2 \right) + b_\psi \left[ \epsilon(\psi - \xi) + \frac{1}{2} \epsilon^2 \right] \right\}, \]

and

\[ \tau_y = \frac{1}{\epsilon^2} \left\{ \xi(\ln \xi - 1) + a \left( \xi - \frac{1}{2} \epsilon^2 \right) + a_\psi \left[ \epsilon(\psi - \xi) + \frac{1}{2} \epsilon^2 \right] \right\}. \]

We emphasize that the functional forms of Eqs. 16–19 are independent of the Rossby number \( Ro \) and depend only on the Zilitinkevich number \( Z_i \) since the parameters \((A, B, b)\) are only functions of the latter (Fig. 2). The Rossby number \( Ro \) dependence is revealed only when the geostrophic wind \((U_g, V_g)\) has to be related to the friction velocity \( u_w \). Therefore, even though the departure from the geostrophic wind can be described in terms of one control parameter \( Z_i \), the wind components \((U, V)\) themselves are still dependent on the two control parameters \((Ro, Z_i)\).
Comparison to High-Fidelity Simulations

Fig. 3 shows the mean profiles of the geostrophic wind deficit and the turbulent shear stress in conventionally neutral ABLs in comparison with simulations under idealized conditions. The solid symbols are the large eddy simulation data from Liu et al. (22), where Zi = 51 (case A), 89 (cases B to E), and 154 (case F) and Ro = 4.5 \times 10^4 (case B), 3.7 \times 10^5 (case C), 3.2 \times 10^6 (case D), and 2.7 \times 10^7 (cases A, E, and F). The solid lines are the theoretical predictions of Eqs. 16–19 with \( \epsilon = 0.12 \) and the geostrophic drag law coefficients (A, B, b) given by Eqs. 4 and 6. The excellent agreement between the theoretical predictions and simulation results confirms the validity of the proposed profiles for the studied Zi range.

Although \( U, V \) themselves depend on both Ro and Zi, their normalized deficits are only functions of Zi. This fact has already been revealed by Eqs. 16 and 17, and now it is also confirmed by simulation data (yellow symbols in Fig. 3 A and B). Interestingly, all profiles of the normalized longitudinal wind deficit \( (U - U_g)/u_* \) are nearly collapsed (Fig. 3A). The reason is that \( (a, a_0) \approx (0.2, -2.4) \) remains approximately constant for the Zi range found in typical conventionally neutral ABLs at midlatitudes to high latitudes (22). In contrast, Fig. 3B reveals that the normalized transverse wind deficit \( (V - V_g)/u_* \) profile depends significantly on Zi as \( (B, b_0) \) vary significantly with Zi. Similar phenomena are observed in the turbulent shear stress profiles. For example, profiles of the normalized longitudinal turbulent shear stress \( \tau_x/u_*^2 \) almost collapse (Fig. 3C), while the normalized transverse shear stress \( \tau_y/u_*^2 \) profile changes significantly with Zi (Fig. 3D). In particular, the magnitude of the peak of the transverse stress \( \tau_y/u_*^2 \) decreases with increasing Zi (Eq. 19 and Fig. 3D), because b increases with increasing Zi and \( (a, a_0) \) are approximately constant. On the other hand, the approximate collapse of the longitudinal stress \( \tau_x/u_*^2 \) can be explained directly by Eq. 18 since the ratio \( b/B \approx 1.5 \) for the studied Zi range. We note that the behavior of \( \tau_x/u_*^2 \) can also be explained partially by introducing the concept of eddy viscosity \( \nu_t \), such that \( \tau_x/u_*^2 = \left| \nu_t/(u_*h) \right|(U - U_g)/u_* \right) \left| dU \right|/d\xi \). Suppose the dimensionless eddy viscosity \( \nu_t/(u_*h) \) is a constant or does not change significantly with Zi; then \( \tau_x/u_*^2 \) should also mainly depend on \( \xi \) if \( (U - U_g)/u_* \) can be predicted very well by Eq. 16 with \( (a, a_0) \) being regarded as constant. However, this explanation is invalid when \( z > z_i \), where \( dU/d\xi < 0 \). We emphasize that all these theoretically predicted trends are observed in the simulation results, indicating that our proposed analytical profiles provide a powerful tool to describe the flow (including the wind veer, see below) and stress profiles in conventionally neutral ABLs. We emphasize again that the collapse of longitudinal wind deficit and shear stress does not indicate that the flow structures in the boundary layer do not depend on Zi. Instead it indicates that \( (a, a_0) \) and \( b/B \) are approximately constant for the considered Zi range.

Fig. 4 shows the mean profiles of the wind speed \( U_{mag} = \sqrt{U^2 + V^2} \), normalized by the friction velocity \( u_* \), and the wind

![Fig. 3](image-url) Mean profiles of (A and B) geostrophic wind deficit and (C and D) turbulent shear stress in conventionally neutral ABLs. Lines, predictions by Eqs. 16–19; symbols, numerical data of Liu et al. (22), where Zi = 51 (case A), 89 (cases B to E), and 154 (case F). The good agreement between the theoretical predictions and simulation results confirms the validity of the proposed profiles for the Zilitinkevich number range under idealized atmospheric conditions.
angle $\alpha = \tan^{-1} V / U$, normalized with the angle between the surface stress and the geostrophic wind $\alpha_0 = \tan^{-1}|V_0 / U_0|$ in conventionally neutral ABLs. Note that in the chosen coordinate system $\alpha(0) = 0$ and $\alpha(1) = -\alpha_0$, where $\alpha_0 = \alpha_0(\text{Ro}, Zi)$ can be determined analytically from the geostrophic drag law (i.e., Eq. 5). The solid symbols in Fig. 4 are the simulation data taken from Liu et al. (22). The solid lines are the theoretical predictions of Eqs. 5, 16, and 17. Consistent with Fig. 3, we find that the analytical description captures the simulation results well. Fig. 4A shows the Ro dependence of the wind speed (48) and reveals that the wind magnitude is nearly independent of Zi for the studied Zi range. Fig. 4B on the other hand reveals the Zi dependence of the normalized wind angle. Fig. 4B shows that the normalized wind angle $\alpha/\alpha_0$ is nearly independent of Ro. Note that the Ro independence of the quantity $\sin(\alpha + \alpha_0)/\sin \alpha_0$ has also been shown for the truly neutral cases (48).

**Comparison to Field Measurement Data**

To further validate our model, we also compare its predictions with various state-of-the-art field measurement campaigns (2, 53, 54). Compared to the idealized simulations, the flow dynamics in the actual ABL are further complicated by various factors, such as clouds, radiation, humidity, baroclinicity, inhomogeneity, and unsteadiness (3, 21). This, in general, complicated comparison between ABL theories and field measurement data. Nevertheless, we show that our model agrees well with various field measurement campaigns for conventionally neutral ABLs.

Fig. 5 shows the mean profiles of geostrophic wind deficit and turbulent shear stress in conventionally neutral ABLs in comparison with atmospheric observations (31, 55–57) and simulations (40). For the geostrophic wind deficit (Fig. 5A and B), the solid triangles are the Joint Air-Sea Interaction (JASIN) experiment data taken from Nicholls (56) and the solid squares are the Høvsøre experiment observations taken from Pêna et al. (31). The information of Zi and the wind profile in the inversion layer are not known in these measurements. However, the geostrophic drag law coefficients $A = 1.4 \pm 0.8$ and $B = 4.2 \pm 0.6$ for the JASIN experiment (56) are known. Therefore, in the analytical model we select $Zi = 136$, where the predicted values of $A = 2.0$ and $B = 4.1$ (Eq. 6 and Fig. 2). The predicted geostrophic wind deficit agrees with the state-of-the-art atmospheric measurements of the Høvsøre experiment (31) and captures the transition between the surface and inversion layer well. This agreement confirms our proposed model. Note that even for the Høvsøre experiment, the data farther above the inversion layer are still unavailable. Therefore, accurate measurements near and in the inversion layer are urgently needed to further evaluate our modeling approach.

For the turbulent shear stress (Fig. 5C and D), the solid triangles are the Marine Stratocumulus experiment data taken from Brost et al. (55) and the solid squares are the Konvektions- und Turbulenz (KONTUR) experiment observations taken from Grant (57). In these atmospheric measurements, the dimensionless parameter $\tau_z/u_*$ varies between $[0.07, 0.14]$. Therefore, in our proposed model, we select $Zi = 235$ such that the predicted value of $\tau_z/u_* = 0.09$ represents the mean value of the field observations (Eqs. 4, 6, and 14). Although the atmospheric data have some unavoidable scatter, the overall trends align with the analytical predictions. Therefore, the proposed model of the normalized shear stresses has also been validated against field measurement data for conventionally neutral ABLs.

**Conclusions**

In summary, we derived analytic expressions for the horizontal components of the wind and turbulent shear stress in conventionally neutral ABLs. The obtained profiles satisfy the Ekman equations and capture the vertical structure over the entire boundary layer height, including the capping inversion. Although the description of the wind speed profile needs two control parameters, our formulation in terms of departure from the geostrophic wind allows us to describe the profiles as a function of one control parameter instead of two and can also capture features like the wind speed and wind veer profile. The validity of the profiles has been confirmed with high-fidelity simulations and state-of-the-art atmospheric observations. Therefore, our model, which describes the flow structure in conventionally neutral ABLs, provides a powerful tool for better understanding these boundary layers’ flow dynamics.

**Data Availability.** All study data are included in the main text.

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