Application of non-stationary gas dynamic functions for mathematical modeling of gas dynamic processes

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Abstract. At numerical calculations of non-stationary flow in extended channels with use of basic provisions of disintegration of discontinuities method noticeable acceleration of calculations gives application of specially developed non-stationary gas dynamic functions from Mach number. At the same time the greatest advantages of these functions’ application is shown during recording of boundary conditions. Here they are applied in a combination with the known functions of a steady flow as at disintegration of discontinuities, the idea of quasi-steady flow between the waves which passed and reflected from a boundary condition is used. By means of the program complex developed on the basis of non-stationary and stationary functions a number of practical tasks, in particular, of the organization of dynamic pressurization of the piston engine was solved.

1. Introduction

Mathematical model methods enable to assign and solve a wide range of practical gas dynamic problems [1-5]. Among the most efficient numerical calculation methods dedicated to extended channel non-stationary processes is a breakdown of an arbitrary discontinuity (Godunov scheme) [4-8]. This method is successfully used for inertia supercharging wave adjustment of individual inlet pipes of piston engines; description of pneumatic system processes, gas pipes and other gas dynamic problems [7-11].

Non-stationary flow in extended channels is considered in one dimension (length-time coordinates, $x - t$), based on the basic gas dynamic equation system [4-8]:

$$\left\{ \begin{align*}
\int (\rho u) dt - \rho dx &= 0, \\
\int (\rho u^2 + p) dt - \rho u dx &= -\frac{\lambda}{2} \int \int \rho |u| dx dt, \\
\int \rho u \left( e + \frac{u^2}{2} \right) dt - \rho \left( e + \frac{u^2}{2} \right) dx &= -\frac{4}{D} \int \int (T - T_w) dx dt, \\
p &= \rho RT.
\end{align*} \right. $$

Where: $\rho$, $u$, $p$, $T$, are density, velocity, pressure, temperature, specific internal energy ($e = c_v T$) respectively; $D$ and $T_w$ are hydraulic diameter and pipe wall temperature; $\lambda$ and $\alpha$ are coefficients of friction and heat transfer to channel walls [12]. In case of numerical solution, a computational region is divided into intervals $\Delta x$, a calculation time step $\Delta t$ is assumed using a certain maximum possible Courant grid number $Co = a \Delta t / \Delta x$ (where $a$ is a speed of sound),
which can ensure the calculation procedure stability. Mass calculations in most cases involve simplified dependencies (linearization) \[5,6\] to define velocities \(u_{i+1/2}\) and pressures \(p_{i+1/2}\) of a flow crossing a boundary \(i + 1/2\) between cells \(i\) and \(i + 1\) of a numerical solution (fig.1):

\[
p_{i+1/2} - p_i + a_i p_i (u_{i+1/2} - u_i), \quad p_{i+1} - p_{i+1/2} - a_{i+1} p_{i+1} (u_{i+1} - u_{i+1/2}).
\]

\[2\]

Figure 1. Breakdown of an arbitrary discontinuity on a boundary between numerical solution cells.

However, numerical analysis of high amplitude waves processes, which, for example, are involved in inlet pipes of inertia discharging-adjusted piston engines, cannot ensure a necessary accuracy of results even with significantly reduced calculation step \(\Delta t\). The best calculation results may be obtained through Rankine-Hugoniot relations as to forward wave front, i.e. the front shall be considered as a shock front to correspond to basic idea of a breakdown of an arbitrary discontinuity \[4-7\]. Meanwhile, an expansion wave dividing zones \(i\) and \(i + 1/2\), is presented as simple Riemann wave, the parameter variations of which are described by Poisson adiabatic law without a simplifying linearization. By equating velocity expressions \(u_{i+1/2} = u_w\) on both sides of the contact area, we shall obtain the pressure equation \(p_{i+1/2}\):

\[
u_i - \frac{2a_i}{k-1} \left[ \frac{(p_{i+1/2})^{k-1}}{p_i} - 1 \right] = u_{i+1} - \frac{p_{i+1} - p_{i+1/2}}{\sqrt{[(k-1)p_{i+1} + (k+1)p_{i+1/2}] \rho_{i+1/2}}},
\]

and find \(u_{i+1/2}\) using either left or right part. However, the implementation of such calculation pattern leads to some difficulties in using shock-capturing method, since it requires making pre analysis of wave shape happening during a breakdown of an arbitrary discontinuity and is nonuniform in terms of direct and return wave calculation. Besides, the solution of equation (3) requires iterations, thus significantly increasing calculation time.

2. Formulation of problem
The experience has shown that as far as simulation of even high intensity gas dynamic impulses is concerned, a compression wave can also involve non-linearized simple wave relations. Thus, a uniform algorithm can be obtained without additional wave shape analysis and time expenditures for making iterations at each calculation step. Here we can use Mach non-stationary gas dynamic functions \(M = u/a\) \[13,14\].

Thus, Riemann invariant \(r^+\) may be found as follows:

\[
r^+ = \frac{2}{k-1} a + u = \frac{2}{k-1} a'' u,'
\]

where \(a''\) is a speed of sound stopped by \((u = 0)\) a simple wave, traveling backwards (upwind).
This value is obviously definitely related to Riemann invariant value like corresponding temperature \( T'' \), density \( \rho'' \) and pressure \( p'' \). Then the equation of upwind wave front sound speed, temperature, density and pressure variation may be written as follows:

\[
a_{i+1/2} = a_i \frac{\rho''(M_{i+1/2})}{\tau''(M_{i+1/2})} = a_i \frac{\alpha''(M_{i+1/2})}{\alpha''(M_i)}, \quad T_{i+1/2} = T_i \frac{\rho''(M_{i+1/2})}{\tau''(M_{i+1/2})} = T_i \frac{\alpha''(M_{i+1/2})}{\alpha''(M_i)},
\]

\[
\rho_{i+1/2} = \rho_i \frac{\rho''(M_{i+1/2})}{\rho''(M_i)}, \quad \rho_{i+1/2} = \rho_i \frac{\alpha''(M_{i+1/2})}{\alpha''(M_i)}, \quad p_{i+1/2} = p_i \frac{\rho''(M_{i+1/2})}{\rho''(M_i)} = p_i \frac{\alpha''(M_{i+1/2})}{\alpha''(M_i)},
\]

where we shall have corresponding non-stationary gas dynamic functions

\[
\alpha''(M) = 1/(1 + \frac{k-1}{2} M), \quad \tau''(M) = [\alpha''(M)]^2, \quad \epsilon''(M) = [\alpha''(M)]^{\frac{2k}{k-1}}, \quad \pi''(M) = [\alpha''(M)]^{\frac{2k}{k-1}},
\]

The opposite family waves shall involve the equation \( r^- = \frac{2}{k-1} \alpha - u = \frac{2}{k-1} \alpha' \), where is \( \alpha' \) speed of sound stopped by a simple wave, traveling forwards (downwind). Then the equation of downwind wave front sound speed, temperature, density and pressure variation may be written as follows:

\[
a_w = a_{i+1} \frac{\alpha''(M_w)}{\alpha''(M_{i+1})} = a_{i+1} \frac{\alpha''(M_w)}{\alpha''(M_{i+1})} T_w = T_{i+1} \frac{\tau''(M_w)}{\tau''(M_{i+1})} = T_{i+1} \frac{\tau''(M_w)}{\tau''(M_{i+1})},
\]

\[
\rho_w = \rho_i \frac{\rho''(M_w)}{\rho''(M_{i+1})} = \rho_i \frac{\alpha''(M_w)}{\alpha''(M_{i+1})}, \quad p_w = p_i \frac{\rho''(M_w)}{\rho''(M_{i+1})} = p_i \frac{\alpha''(M_w)}{\alpha''(M_{i+1})},
\]

where non-stationary gas dynamic functions are:

\[
\alpha'(M) = 1/(1 - \frac{k-1}{2} M), \quad \tau'(M) = [\alpha'(M)]^2, \quad \epsilon'(M) = [\alpha'(M)]^{\frac{2k}{k-1}}, \quad \pi'(M) = [\alpha'(M)]^2.
\]

A contact area breakdown of an arbitrary discontinuity involves the following relations:

\[
u_{i+1/2} = M_{i+1/2} a_{i+1/2} = u_w = M_w a_w \quad \text{and} \quad p_{i+1/2} = p_w.\]

Pressure variations between wave shape zones can be written as follows:

\[
\frac{p_i}{p_{i+1/2}} = \frac{p_i}{p_{i+1/2}} = \frac{\pi''(M_i)}{\pi''(M_{i+1/2})} = \frac{\alpha''(M_i)}{\alpha''(M_{i+1/2})} = \frac{\alpha''(M_w)}{\alpha''(M_{i+1})}.
\]

Equality of velocities upon contact area is represented as:

\[
M_{i+1/2} a_i \frac{\alpha''(M_{i+1/2})}{\alpha''(M_i)} = M_w a_{i+1} \frac{\alpha''(M_w)}{\alpha''(M_{i+1})}.
\]

Let us designate complexes including initial parameters \( p_i, p_{i+1}, a_i, a_{i+1}, M_i \) and \( M_{i+1} \) as follows: \( F = \alpha'(M_{i+1}) / \alpha'(M_i) \); \( X = F(p_i/p_{i+1})^{\frac{k-1}{2k}} \); \( Y = F a_i/a_{i+1} \). Discovering values \( \alpha'(M) \) and \( \alpha''(M) \) of \( M \) and transformations lead to the following simple relations:

\[
M_{i+1/2} = \frac{2}{k-1} \frac{X-1}{Y+1}, \quad M_w = \frac{Y}{X} M_{i+1/2}.
\]

To continue calculation, we need only \( M_{i+1/2} \). After that using corresponding non-stationary gas dynamic function formula we can find velocity \( u_{i+1/2} \) and pressure \( p_{i+1/2} \). By comparing the test calculation results of shock breakdown of discontinuity in terms of pressure variations
\[ \frac{p_i}{p_{i+1}} = 2 \] and using non-stationary gas dynamic functions we shall obtain insignificant errors \( \delta_p = \delta_u = 0.0005 \) as against using the equation (3). In case of calculation of breakdown of an arbitrary discontinuity with similar pressure variation using linearized equations (2) we shall have errors two degrees higher: \( \delta_p = \delta_u = 0.05 \). Besides, the grid number may be reduced 1.5 - 2 times even as against a linearized pattern. Thus, the calculation time becomes shorter.

The greatest benefits of non-stationary gas dynamic functions are seen in cases of practical computations on one-dimension non-stationary flow calculation regions boundaries, with some local resistances being always present, which are considered as lumped boundary conditions. The most efficient calculation methods for stationary flows over such elements are based on gas dynamic functions [15] of Mach \( M = u/a \) or specific speed \( \lambda = v/a_{kp} \).

A construction of calculation models for interaction of a non-stationary flow and such boundary conditions should involve the idea of breakdown of an arbitrary discontinuity. It is convenient to apply non-stationary gas dynamic functions combined with known stationary flow gas dynamic functions, since at each step a breakdown of an arbitrary discontinuity involves the idea of quasi-stationary flow between the waves, which passed and were reflected by boundary conditions [16, 17].

3. Boundary condition like a piston engine valve

Let us review a calculation of interaction between a non-stationary flow and boundary condition like a piston engine valve in an inlet pipe adjusted for inertia discharge. The calculation diagram of a corresponding discontinuity breakdown is shown in fig. 2, where the last calculation cell has indexed \( m \).

![Figure 2. Discontinuity breakdown calculation diagram in case of flowing into a cylinder through a valve.](image)

The through-the-valve flow in a cylinder head is supposed as adiabatic, i.e. stationary slow down temperature remains constant \( \left( T^*_{m+1/2} = T^*_v \right) \). Besides, fluid and gas dynamics state that full pressure loss \( \left( p^*_m p^*_m + 1/2 \right) \) in short converging duct like a valve inlet pipe is negligible. Then using Mach stationary and non-stationary functions we can obtain the following equation as to pressure variation between calculation areas (fig. 2):

\[
\frac{p_m}{p_c} = \frac{p_m}{p_{m+1/2}} \times \frac{p_{m+1/2}}{p_c} = \frac{\pi'' (M_m)}{\pi'' (M_{m+1/2}) \pi (M_v)}. \tag{7}
\]

For the calculation, the equation (7) is supplemented with the following flow relation [15]:

\[
m \frac{F_m p^*_m p^*_m + 1/2 q (M_{m+1/2})}{\sqrt{T^*_m}} = m \frac{\mu F_v p^*_v q (M_v)}{\sqrt{T^*_v}}. \tag{8}
\]
Here $\mu$ is a flow coefficient reflecting unsteady flow through the valve hole $F_v$. It is defined using experimental valve blowdown. Parameters $p_m, M_m, F_v$ at calculation step are known. Using iteration methods to solve the equation system (7) and (8) with two unknowns $M_{m+1/2}$ and $F_v$, we shall define number $M_{m+1/2}$ at this calculation step, required for making further calculation using discontinuity breakdown method with regard to in-pipe wave process and in-cylinder flow. The description of in-cylinder procedures involved methods described in [8]. Meanwhile, it is necessary to specify mass $G\Delta t$ of a charge coming through the valve and the value of total enthalpy $H^*\Delta t = h^*G\Delta t$.

4. Numerical example

The mentioned methods have been used to simulate a wave process in terms of inertia supercharging of diesel engine 1Ch8,5/11 at $n = 1500\text{rpm}$. Fig.3 shows corresponding wave pictures on entering into a cylinder head obtained in experiment [11] and calculation, as well as dependency of coefficient of admission $\nu$ on inlet pipe length L (fig.4). The calculation results are well consistent with the experimental data and prove that having a pipe 1.65 and 2.15m-long shall ensure inertia supercharging with $\nu$ increased over 1.05. As against the unadjusted system, the coefficient of admission and engine power are increased by 24 %. The results of the calculations mentioned in fig. 3 and 4 prove the validity of the proposed computational simulation methods as to high intense wave processes and can be recommended for solving practical problems of non-stationary gas dynamics, such as design of piston engine intake systems with wave adjustment to ensure inertia supercharging with power much increased.

![Figure 3](image1.png)

**Figure 3.** Inertia discharge intake pressure, $L = 1.65m$.

![Figure 4](image2.png)

**Figure 4.** Admission coefficient $\eta_v$ vs. length $L$ of adjusted inlet pipe.
5. Conclusion
In paper the new modification of a numerical method of disintegration of discontinuities based on specially developed gasdynamic functions of nonstationary flow from Mach number is offered. Application of functions allows to carry out the shock-capturing calculation of wave processes of considerable intensity without use of iterations at the calculation step during setting of increased values mesh Courant number. It increases the speed of calculations and increases the accuracy of results. The greatest advantages of application of these functions are shown at formulation of boundary conditions on local resistance where they during recording of ratios of discontinuities disintegration are combined with the known gasdynamic functions of a stationary flow from Mach number. It simplifies calculations up to obtaining, in certain cases, analytical decisions on boundaries.

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