Berry Phase Coupling and the Cuprate Neutron Scattering Resonance

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We examine the influence of coupling between particle-hole and particle-particle spin fluctuations on the inelastic neutron scattering resonance (INSR) in cuprate superconductors in both weak and strong interaction limits. For weak-interactions in the particle-hole channel, we find that the interchannel coupling can eliminate the resonance. For strong interactions which drive the system close to a \( Q = (\pi, \pi) \) magnetic instability, the resonance frequency always approaches zero but its value is influenced by the interchannel coupling. We comment on constraints imposed on cuprate physics by the INSR phenomenology, and a comparison between the cuprates and the newly-discovered iron pnictide superconductors is discussed.

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I. INTRODUCTION

The low-temperature properties of cuprate superconductors seem to be well described by mean-field-theory with effective interactions which lead to \( d \)-wave superconductivity, except possibly in the extreme underdoped limit. At the same time the apparently ubiquitous presence of a low-frequency spin-resonance near \( Q = (\pi, \pi) \) in inelastic neutron scattering experiments and the appearance of spin order in some systems\textsuperscript{3} when an external magnetic field is applied, suggest that superconducting cuprates are close to an antiferromagnetic instability. In previous work\textsuperscript{3,4} we have argued that coupling\textsuperscript{5,6} between particle-hole and particle-particle spin fluctuations likely contributes to the suppression of the superfluid density\textsuperscript{7} observed in underdoped cuprates. In this article we specifically address, within time-dependent mean-field theory, the influence of particle-particle to particle-hole coupling on the inelastic neutron scattering resonance (INSR) position. When the interchannel coupling is ignored, an arbitrarily weak particle-hole channel interaction will induce a resonance. We find that the interchannel coupling can eliminate weak-interaction resonances. For stronger interactions the system can be driven close to an antiferromagnetic instability. In this limit we find that the way in which the resonance frequency approaches zero is influenced by the interchannel coupling. There is no limit in which RPA-type theories without the interchannel coupling yield reliable resonance frequency predictions.

II. RELATIONSHIP BETWEEN THE DIAGONAL AND OFF-DIAGONAL RPA SUSCEPTIBILITIES

In time-dependent mean-field theory the most general linear response of a system consists of correlated particle-hole and particle-particle excitations. For square-lattice models with on-site and near-neighbor electron-electron interactions we have shown previously\textsuperscript{2,6} that at \( Q = (\pi, \pi) \) correlations occur between between four different weighted particle-hole transition sums. The most general response function can therefore be evaluated by inverting four by four matrices. For a model (motivated by the cuprate superconductivity literature) with only on-site \( (U) \) and near-neighbor \( (V) \) spin-independent interactions and nearest-neighbor Heisenberg \( (J) \) spin interactions we find that:

\[
\hat{\chi}^{-1}(\vec{Q}, \omega) = \hat{\chi}_{qp}^{-1}(\vec{Q}, \omega) - \hat{V}
\]

(1)

where \( \hat{V} = \text{diag}(-U - 2J, J/2 - 2V, V + J/4, V + J/4) \) is the interaction kernel,

\[
\hat{\chi}_{ab,qp}(\vec{Q}, \omega) = \frac{1}{N} \sum_{\vec{k}} \left( \frac{f_a(\vec{k}) f_b(\vec{k})}{\omega - E(\vec{k}) - E(\vec{k}')} - \frac{(-1)^{a+b} f_a(\vec{k}) f_b(\vec{k})}{\omega + E(\vec{k}) + E(\vec{k}')} \right)
\]

(2)

is the bare mean-field-quasiparticle response function, and \( \vec{k}' = \vec{k} + \vec{Q} \) reduced to the first Brillouin-zone. The indices \( a \) and \( b \) refers to the four coupled two-particle excitation channels and \( f_a \) is the corresponding coherence factor defined in Ref. [7]. As shown in previous work\textsuperscript{1,2} the INSR mode is dominated by two channels, the spin flip \( (a = 1) \) and the \( d \)-wave pair phase \( (a = 4) \) modes, when parameters are in a range broadly consistent with experiment. The same conclusion can be reached on the basis of earlier approximate RPA theories\textsuperscript{1,2} which omitted the second channel. The qualitative discussion of INSR physics below is based on a model in which only these two modes are retained, although the full 4-channel model is required for quantitative accuracy.

In the truncated two-channel model the INSR solves

\[
K^s(\omega) K^\phi(\omega) = C^2(\omega) \omega^2
\]

(3)

where \( K^s(\omega) = K^s_{11}(\omega) - V_a, \ K^\phi(\omega) = K^\phi_{qp}(\omega) + V_a, \ V_a = U + 2J, \ V_a = \hat{V} + J/4, \) and the frequency-dependent stiffness and Berry-phase coupling parameters are defined by:

\[
K^s_{qp}(\omega) = \frac{1/\chi_{11,qp}(\omega)}{1 - R(\omega)}
\]
\[ K^\phi_{qp}(\omega) = \frac{1/\chi_{44,qp}(\omega)}{1 - R(\omega)} \]

\[ C(\omega) = \frac{-1}{\omega \chi_{11,qp}(\omega)} \left( \frac{\chi_{14,qp}(\omega)}{\chi_{44,qp}(\omega)(1 - R(\omega))} \right) \]

\[ R(\omega) = \frac{\chi_{34,qp}(\omega)}{\chi_{11,qp}(\omega)\chi_{44,qp}(\omega)}. \tag{4} \]

\( K^\phi(\omega = 0) \) and \( K^\phi(\omega = 0) \) are\(^a\) proportional to the energetic cost of static spin-density and spin-polarized electron-pair density fluctuations, as estimated by mean-field-theory while \( C(\omega = 0) \) is\(^b\) the Berry-curvature associated with adiabatic evolution of electron-electron and electron-hole pair mean-fields. Stability of the \( d \)-wave superconducting state requires that both \( K^\phi(\omega = 0) \) and \( K^\phi(\omega = 0) \) be positive. In the two-channel approximation the interchannel coupling is captured by the function \( \chi_{14,qp}(\omega) \). If \( \chi_{14,qp}(\omega) \) was negligible, Eq. (3) would be satisfied at both the spin-exciton resonance frequency \( \omega_{ex} \) defined by

\[ 1/\chi_{11,qp}(\omega_{ex}) + V_\phi = 0, \] \( \text{and at the } \pi\text{-resonance frequency } \omega_\pi \text{ defined by} \]

\[ 1/\chi_{44,qp}(\omega_\pi) - V_\phi = 0. \]  

When coupling between particle-hole and particle-particle channels is neglected both \( R(\omega) \) and the Berry phase coupling \( C(\omega) \) are set to zero. It is therefore important to study how \( R(\omega) \) and \( C(\omega) \) depend on the model parameters. Since both \( R \) and \( C \) are dimensionless, the unbiased justification of whether or not the interchannel coupling is negligible should be based on these two quantities. It is evident from the definition of \( R(\omega) \) and from the way in which it appears in the above equations, that the interchannel coupling certainly can not be neglected when it approaches 1. To judge the importance of the Berry-phase coupling \( C(\omega) \), we should compare \( |C| \times \omega_{ex} \) with typical values for \( \sqrt{K_x \times K_\phi} \), which indicates that values of \( |C| \) comparable to 1 also correspond to strong interchannel coupling.

The mean-field-theory of INSR is defined by the hopping parameters of the square-lattice tight-binding model and by the interaction parameters \( U, V, \) and \( J \). We emphasize that all these parameters should be viewed to a large degree as effective parameters which depend on microscopic many-body physics in ways which are not fully understood. Their values are therefore best determined phenomenologically. Luckily the quantities \( R(\omega) \) and \( C(\omega) \) depend only on the normal state band-structure model, and on the maximum of the mean-field \( d \)-wave energy gap \( \Delta \). At the current mature state of cuprate experimental research, the information we require is therefore available with adequate accuracy.

Fig. 1(a) plots \( R(\omega) \) for \( \omega \leq \Omega_0 \), where \( \Omega_0 \) is the minimum two-particle excitation energy, with two very different band structures. Fig. 1(b) plots \( C(\omega) \). Although the association of \( C \) with Berry phases is not strictly appropriate away from the adiabatic limit, it is appropriate to retain the terminology because \( C(\omega) \) has weak frequency dependence up to quite high frequencies. Model I in Fig. 1 refers to the band structure model obtained by Norman et al.\(^{11} \) by fitting to angle-resolved-photoemission data\(^2\) and used by us in our previous work.\(^7\) Model II refers to a model used by Hao and Chubukov\(^{12} \) in a related recent work. The main difference between these two models is that the nearest-neighbor hopping \( t \) is almost 2.5 times bigger in Model II than that in Model I. Since the maximum energy gaps in both models are fitted to be around \( 35 - 50 \) meV in the underdoped regime, the ratio of the energy gap to the kinetic energy is much smaller in Model II than that in Model I. This difference is responsible for the generally weaker interchannel coupling effects seen in both the \( R(\omega) \) and \( C(\omega) \) in Model II as shown in Fig. 1. We note however that \( R(\omega) \rightarrow 1 \) as \( \omega \) approaches \( \Omega_0 \) in both models, and indeed as we discuss below, for any band-structure model. Although Model I seems to be much more sophisticated and better fitted with
experiments\textsuperscript{11}, we study both models to ease comparison with Ref. \textsuperscript{10} and to provide some quantitative indication of the sensitivity of our conclusions to very-different band-structure model choices.

III. WEAK INTERACTION LIMIT

Weak interactions can compete with band energies for \( \omega \) smaller than but close to the minimum two-particle excitation energy \( \Omega_0 \) because the energy denominator in Eq. \textsuperscript{3} becomes small when \( \mathbf{k} \) approaches \( \mathbf{p} \), the point at which \( E(\mathbf{k}) + E(\mathbf{k'}) \) is minimized. By expanding the energy denominator around its minimum we find that all elements of the quasiparticle response function diverge logarithmically as \( \omega \to \Omega_0 \) from below:

\[
\chi_{11,qp}(\omega = \Omega_0) \sim f_1^2(\mathbf{p}) \nu_0 \ln(1 - \Omega_0/\omega) \\
\chi_{44,qp}(\omega = \Omega_0) \sim f_2^2(\mathbf{p}) \nu_0 \ln(1 - \Omega_0/\omega) \\
\chi_{14,qp}(\omega = \Omega_0) \sim f_1(\mathbf{p}) f_4(\mathbf{p}) \nu_0 \ln(1 - \omega/\Omega_0). \tag{7}
\]

Here \( \nu_0 \) is the joint density of states per site just above the minimum excitation energy. Note that all off-diagonal elements of the quasiparticle response matrix approach the geometric mean of the corresponding diagonal elements, so that the response matrix becomes singular and \( R(\omega = \Omega_0) \to 1 \). As we explain below, the familiar BCS-like weak-interaction result for the spin-exciton resonance frequency is qualitatively altered as a consequence.

When the interchannel coupling is neglected, the spin exciton energy \( \omega_{ex} \) is obtained\textsuperscript{12} by inserting Eq. \textsuperscript{7} in Eq. \textsuperscript{6} to obtain

\[
1 - \frac{\omega_{ex}}{\Omega_0} = \exp \left( -\frac{1}{\nu_0 V_s f_1^2(\mathbf{p})} \right) \tag{8}
\]

An undamped resonance appears just below the two-particle continuum edge for an infinitesimally small positive \( V_s \). To account for the interchannel coupling for \( \omega \) near \( \Omega_0 \) we rewrite Eq. \textsuperscript{8} as

\[
( -\chi_{44,qp}(\omega) - D(\omega)V_s)(-\chi_{11,qp}(\omega) + D(\omega)V_0) = \chi_{14,qp}^2 \tag{9}
\]

where

\[
D(\omega) = (\chi_{11,qp}(\omega) \chi_{44,qp}(\omega) - \chi_{14,qp}^2(\omega)). \tag{10}
\]

Note that, although each matrix element diverges logarithmically the geometric product structure of the matrix implies that \( D(\omega) \) diverges like \( \ln \), not like \( \ln^2 \). After some algebras, we obtain

\[
1 - \frac{\omega_{INSR}}{\Omega_0} = \exp \left( -\frac{1}{\nu_0 (V_s f_1^2(\mathbf{p}) - V_0 f_1^2(\mathbf{p}))} \right). \tag{11}
\]

If \( V_0 > 0 \), \( V_s \) must be greater than a critical value in order to have a resonance and \( \omega_{INSR} > \omega_{ex} \) in agreement with Ref. \textsuperscript{10}. If \( V_0 \leq 0 \), \( \omega_{INSR} \leq \omega_{ex} \) and an infinitesimal positive \( V_s \) is again sufficient to guarantee a resonance below the two-particle continuum.

IV. STRONG INTERACTION LIMIT

Strong interactions drive the system close to the antiferromagnetic instability, \textit{i.e.} into the regime which appears to be relevant for optimally doped and moderately underdoped cuprates. Given the instability criterion, \( K^s(\omega = 0) = K^s_{qp}(\omega = 0) - V_s \) with \( V_s = U + 2J \) we see that either on-site Coulomb interactions \( U \), or Heisenberg near-neighbor spin-dependent effective interactions \( J \), or a combination of the two, could be responsible. Since, as we mention again below, experiments imply that \( K^s(\omega = 0) \) is around an order of magnitude smaller than \( K^s_{qp}(\omega = 0) \), the value of \( V_s \) is tightly constrained on a relative basis. It has been argued\textsuperscript{10} that \( \omega_{ex} \) should match \( \omega_{ex} \) accurately in a relative sense in this limit because the frequency which appears in the Berry phase coupling term is small. As we will prove in detail later, this argument is not necessarily correct because the leading frequency dependent term in the low-frequency expansion of Eq.\textsuperscript{3} varies as \( \omega^2 \), and all \( \omega^2 \) terms, including the Berry phase coupling term, should be retained.

Let’s consider the case in which the system is close to the antiferromagnetic instability so that the INSR frequency is small. We expand the quasiparticle-response dependent quantities at low frequencies as follows: \( \chi_{11,qp}(\omega) \approx -(R_0^s + R_2^s \omega^2) \), \( \chi_{44,qp}(\omega) \approx -(R_0^s + R_2^s \omega^2) \), \( R(\omega) \approx R_{14} \omega^2 \), and \( C(\omega) \approx C \). Detailed expressions for the positive-definite coefficients \( R^{s,\phi} \), \( R^{\phi,\phi} \) and \( R_{14} \) can be found in Ref. \textsuperscript{7}. \( R_{14} \) and \( C \) are related by \( C = R_{14}/D \) where \( D \) is the low frequency limit of \( \chi_{14,qp}(\omega)/\omega \). The spin-exciton energy can be obtained by setting \( V_s \chi_{11,qp}(\omega) = -1 \). We find that

\[
\omega_{ex} = \sqrt{\frac{1}{V_s R_2^s} - \frac{\delta}{V_s R_2^s}} \equiv \gamma \tag{12}
\]

where \( \delta \) is defined by the second equality. The above expression is valid close to the antiferromagnetic instability, \textit{i.e.} for \( \delta \to 0^+ \). When the interchannel coupling is included the corresponding resonance energy \( \omega_{INSR} \) is obtained by solving Eq. \textsuperscript{3} retaining terms up to order \( \omega^2 \). After some algebras we find:

\[
\frac{\omega_{INSR}}{\omega_{ex}} = \frac{1}{\sqrt{1 + \gamma_1 + \gamma_2}} \equiv \gamma \tag{13}
\]

where

\[
\gamma_1 = \frac{D^2}{R_0^s R_2^s} \left( \frac{1}{1 - \delta} \right) \left( 1 + \frac{V_0^2 R_0^s}{(1 + V_0^2 R_0^s)} - 1 \right) \tag{14}
\]

and

\[
\gamma_2 = \frac{-\delta (R_2^s - R_0^s R_{14} V_0^2)}{(1 + V_0^2 R_0^s) R_2^s V_0}. \tag{15}
\]

Since \( \gamma_2 \propto \delta \to 0 \) near the spin-density instability, the discrepancy between \( \omega_{INSR} \) and \( \omega_{ex} \) is mainly associated with the parameter \( \gamma_1 \). We comment further on this point in the next section. Clearly the value of \( \gamma_1 \) depends strongly on model parameters.
V. DISCUSSION

As emphasized in our previous work\textsuperscript{2}, because we have three interaction parameters ($U$, $V$, $J$) but only two experimentally-determined properties, the anti-nodal gap and the INSR frequency, experiment does not uniquely determine the effective interaction model. The two combination of interaction parameter combinations that are tightly constrained by experiment are the spin-interaction $V_e = U + 2J$ (INSR experiment) and the pair-potential $V_{pair} = 3J/2 - 2V$ (ARPES and other measures of the $d$-wave antinodal gap). We have argued previously that the $V = 0, J \neq 0$ ($J$-scenario) possibility (pairing driven by effective Heisenberg interactions) is more likely than that $V \neq 0, J = 0$ ($V$-scenario) possibility (pairing driven by attractive scalar near-neighbor interactions). Our argument appealed to the Occam’s razor assumption that one type of effective interaction is likely to play a dominant role in the low-energy physics. We then observed that the $J$-scenario can account for both $V_e$ and $V_{pair}$ scales whereas the $V$-scenario requires in addition a finely-tuned on-site effective interaction. ($U$ would have to be at least 350meV in model I and 800meV in model II if the $V$-scenario was adopted to fit the experimental data\textsuperscript{10,18.}) It is in fact clear from the outset that the near-neighbor scalar interaction $V$ cannot succeed in accounting for both scales because it contributes to $V_{pair}$ but not to $V_e = U + 2J$. On the other hand, the $J$-scenario is not trivially guaranteed to explain both experimental energy scales. This fact does appear to us to be strongly suggestive.

Eqs. (13,14,15) demonstrate that interchannel coupling can alter the resonance frequency even in the limit $K_s \to 0$ and that the resonance frequency always depends on both $V_e$ and $V_s$. In the $J$-scenario we favor $V_e$ is small and positive, and $\gamma$ tends to be negative in models which are close to the antiferromagnetic instability. For this reason $\gamma$ is slightly larger than 1 in both models we have examined. For Model I, we find $\gamma \approx 1.1$ while for Model II we find $\gamma \approx 1.04$. The approximate coincidence between $\omega_{ex}$ and $\omega_{INSR}$ for some model parameters does not imply that particle-hole and particle-particle modes are weakly coupled in the elementary excitations. In fact, it is evident in Fig. (1) that $C(\omega)$ is weakly frequency dependent and remains comparable to 1 in both models, which already guarantees the strong interchannel coupling as discussed above. The close match between $\omega_{ex}$ and $\omega_{INSR}$ only reflects that the choice of interaction parameters leads to not only $K^s(\omega) \to 0$ (i.e. $\delta \to 0$) but also the large ratio of $K^s(\omega)/K_s(\omega)$ at low frequency. For realistic experimental situations in which the resonance frequency is always a substantial fraction of the antinodal gap, this ratio is always larger than approximately 3, which does favor $\omega_{INSR} \approx \omega_{ex}$. This analysis proves that the ratio of $\omega_{INSR}/\omega_{ex}$ can not serve as a reliable criterion to judge the importance of the interchannel coupling. We have referred to this resonance as a magnetic plasmon to suggest this aspect of its character, namely that it represents the coupled quantum fluctuations of canonically conjugate degrees of freedom as long as $|C|$ is comparable to 1. Our conclusions on this point are in disagreement with those of Ref. \textsuperscript{10}.

In Ref. \textsuperscript{19} we have argued that the quantum zero-point energy of cuprates makes a negative contribution to the superfluid density of cuprates, and that it therefore contributes to the decrease of the critical temperature in the underdoped regime. The basic idea is that superconductivity is weakened by phase gradients, allowing the antiferromagnetic instability to creep closer and reducing the quantum zero-point energy associated with spin-fluctuations. Similar physics occurs in bilayer quantum Hall superfluids\textsuperscript{12,13}. Microscopic caculations suggest that in cuprates this effect is large enough to account for the decreasing superfluid density, in contrast to the situation in typical superconductors in which correlation effects are largely independent of the configuration of the superfluid condensate. It will be interesting to examine how this idea plays out when applied to the recently-discovered iron pnictide superconductors. Current experiments and theories seem to favor extended-$s$-wave gap symmetry. An INSR has been observed\textsuperscript{14,15,16} in the pnictides with a phenomenology which is surprisingly similar to that of the cuprates. Because the the ratio $2\Delta/k_BT_c$ is found to be remain close to the BCS prediction\textsuperscript{17}, it seems that the supression of superfluid density by correlations is less important in this material. To account for this difference within our theory, we will need to understand why phase gradients in the superconducting condensate have a weaker effect on spin-fluctuations and on the correlation energy. The multiband character and the peculiar Fermi surface pockets of pnictides would seem likely to produce this result. The large effect in cuprates rests strongly on the importance of fluctuations near the wavevector which connects antinodal momenta for spin-response functions, a coincidence that is not likely to be repeated in iron pnictide superconductors. A recent RG analysis\textsuperscript{18} which showed that the particle-hole and particle-particle channels are decoupled at energy scales below the Fermi energy $E_F$ supports this view. A more detail calculations for the iron pnictides using an appropriate multiband model which aims to resolve this issue is currently in progress.

Based on these analyses, we conclude that Berry phase coupling between the particle-hole and particle-particle channels can never be ignored in any RPA-type theory of the inelastic neutron scattering resonance mode for cuprates. As a result, the inelastic neutron scattering resonance mode observed in cuprates should be better interpreted as a magnetic plasmon than a spin exciton. The importance of coupling in the pnictides has not yet been adequately analyzed. Nevertheless, the spin resonance mode observed in the inelastic neutron scattering measurements could be a purer spin exciton, and the magnetic and superconducting instabilities less coupled in the pnictide case.

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1. J. Rossat-Mignion et al., Physica. C 185, 86 (1991); H. Mook et al., Phys. Rev. Lett. 70, 3490 (1993); H.F. Fong et al., Nature 398, 588 (1999); H. He et al., Science 295, 1045 (2002); G. Yu et al, to appear.

2. B. Lake et al., Nature 415, 299 (2002); J. Chang et al. arXiv:cond-mat/0712.2182; Ying Zhang, Eugene Demler, and Subir Sachdev, Phys. Rev. B 66, 094501 (2002).

3. Wei-Cheng Lee, Jairo Sinova, A.A. Burkov, Yogesh Joglekar, and A.H. MacDonald, Phys. Rev. B. 77, 214518 (2008).

4. E. Demler, H. Kohn, and S. C. Zhang, Phys. Rev. B 58, 5719 (1998) and work cited therein.

5. Y.J. Uemura et al., Phys. Rev. Lett. 62, 2317 (1989); V.J. Emery and S.A. Kivelson, Nature 374, 434 (1995).

6. M. Eschrig, Adv. Phys. 55, 47 (2006) and references therein.

7. Wei-Cheng Lee and A.H. MacDonald, Phys. Rev. B 78, 174506 (2008).

8. O. Tchernyshyov, M. R. Norman, and A. V. Chubukov, Phys. Rev. B 63, 144507 (2001).

9. A. Damascelli, Z. Hussain, and Z.X. Shen, Rev.Mod. Phys. 75, 473 (2003).

10. Zhihao Hao and A.V. Chubukov, arXiv:0812.2697 (2008).

11. M.R. Norman, M. Randeria, H. Ding, and J.C. Campuzano, Phys. Rev. B 52, 615 (1995).

12. Yogesh N. Joglekar and Allan H. MacDonald, Phys. Rev. B 64, 155315 (2001).

13. J.P. Eisenstein and A.H. MacDonald, Nature 432, 691 (2004).

14. A.D. Christianson, E.A. Goremychkin, R. Osborn, S. Rosenkranz, M.D. Lumsden, C.D. Malliakas, I.S. Todorov, H. Claus, D.Y. Chung, M.G. Kanatzidis, R.I. Bewley, T. Guidi, Nature 456, 930 (2008).

15. M.D. Lumsden, A.D. Christianson, D. Parshall, M.B. Stone, S.E. Nagler, G.J. MacDougall, H.A. Mook, K. Lokshin, T. Egami, D.L. Abernathy, E.A. Goremychkin, R. Osborn, M.A. McGuire, A.S. Sefat, R. Jin, B.C. Sales, and D. Mandrus, Phys. Rev. Lett. 102, 107005 (2009).

16. Songxue Chi, Astrid Schneidewind, Jun Zhao, Leland W. Harriger, Linjun Li, Yongkang Luo, Guanghan Cao, Zhu’an Xu, Micheal Loewenhaupt, Jiangping Hu, and Pengcheng Dai, Phys. Rev. Lett. 102, 107006 (2009).

17. T.Y. Chen, Z. Tesanovic, R.H. Liu, X.H. Chen, C.L. Chien, Nature 453, 1224 (2008).

18. A.V. Chubukov, arXiv:0902.4188 (2009).