Superluminal Waves and the Structure of Pulsar Wind Termination Shocks

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Abstract. The termination shock of a pulsar wind is located roughly where the ram pressure matches that of the surrounding medium. Downstream of the shock, MHD models of the diffuse nebular emission suggest the plasma is weakly magnetized. However, the transition from a Poynting-dominated MHD wind to a particle-dominated flow is not well understood. We discuss a solution of this “σ-problem” in which a striped wind converts into a strong, superluminal electromagnetic wave. This mode slows down as it propagates radially, and its ram pressure tends to a constant value at large radius, a property we use to match the solution to the surrounding nebula. The wave thus forms a pre-cursor to the termination shock, which occurs at the point where the wave dissipates. Possible damping and dissipation mechanisms are discussed qualitatively.

1. Introduction - MHD and the σ-problem

The electromagnetic emission from pulsars is only a small fraction of their spin-down power, most of which must be carried away by a relativistic wind, consisting of particles and low-frequency electromagnetic fields. Close to the light cylinder, the latter component is expected to dominate the energetics, in the sense that the ratio σ of the energy flux carried by the fields to that carried by the particles is much larger than unity. Numerical solutions of the force-free magnetosphere problem e.g., Petri (2012) support the idea that the wind is launched in the form of magnetic stripes of alternating polarity frozen into the radial plasma flow (Coroniti 1990).

Locally, the physics of a radial wind depends on the energy flux density it carries, which may be expressed by the dimensionless parameter $a_L = (e^2L/m^2c^5)^{1/2}$. Formally, $L$ is $4\pi$ times the luminosity per unit solid angle in a given radial direction. In the simplest case this equals the spin-down power. Physically, $a = a_L r_L/r$ is the strength parameter that a circularly polarized vacuum wave would need, in order to carry the same energy flux density as the pulsar wind. ($r_L = c/\omega$ is the radius of the light cylinder and $2\pi/\omega$ is the pulsar period. The strength parameter of a vacuum wave is defined as $a = eE/mc\omega$, with $E$ the electric field amplitude.)

But a pulsar is not surrounded by vacuum; the wave carries a finite particle flux. How many particles are available in an outflow is determined by the pair production rate (multiplicity coefficient $\kappa$) in the magnetosphere and can be quantified by the ratio of the luminosity to the mass-loss rate (times $c^3$): $\mu = a_L/4\kappa$ (Lyubarsky & Kirk 2001; Arka & Kirk 2012), which equals the Lorentz factor which each particle would have if the entire luminosity were carried by the particles only.
When it encounters the surroundings, a stellar wind is decelerated and terminates at an (approximately) standing shock, where its ram pressure is balanced by the confining pressure of the medium. At the shock, the energy is deposited in relativistic particles that are responsible for the measured radiation. In the pulsar case, the problem with this scenario is that in an ideal, radial ultrarelativistic MHD wind there is no plausible mechanism of converting the Poynting flux into the particle energy flux. This is often called the $\sigma$-problem.

A solution can be found only by looking beyond the ideal MHD description. One possibility is a scenario in which the MHD wind converts into a strong electromagnetic (EM) wave of superluminal phase speed before reaching the shock (Usov 1975; Melatos & Melrose 1996). The new mode can be thought of as a shock precursor (Kirk 2010), since the point of conversion is causally connected to the external medium. These modes accelerate particles to relativistic energies in a plane transverse to the direction of motion, and so they transfer most of the flow energy from the fields into the plasma. The mode conversion process itself, which is not considered here, can probably only be investigated using two-fluid or PIC simulations. However, just like an MHD shock, it is constrained by jump conditions that follow from the parameters of the MHD wind and the pressure of the external medium.

2. And Beyond MHD

As EM waves can propagate only in an underdense plasma, mode conversion can happen only beyond a certain distance from the star $r > r_c$, where, due to spherical expansion, the particle density drops below a critical value. When this happens, a fraction of the flow energy is available to the transverse degrees of freedom. The critical radius, expressed in terms of the pulsar wind parameters is $r_c \approx (a_L/\mu) r_L = 4 \kappa r_L$ (Arka & Kirk 2012). When a wave is launched far outside this cut-off distance, it resembles a large amplitude vacuum wave, but close to the cut-off the plasma strongly affects the wave properties and a self-consistent solution deviates from a vacuum wave.

The feature which distinguishes strong waves from linear EM waves is that they are able to drive particles to extremely relativistic energy $\gamma \approx a$ in only half a period. To describe the propagation of a strong plane wave in a plasma one has to solve the full nonlinear set of equations of particle motion coupled to Maxwell equations (Akhiezer & Polovin 1956; Max & Perkins 1971). In this self-consistent approach particles are not test particles; their conduction currents contribute to maintaining the wave fields. Since EM waves have a nonvanishing electric field even in the local fluid frame, they are excluded from an MHD description. In pulsar winds, the simplest description that includes them is a cold, two-fluid ($e^\pm$) plasma. A monochromatic solution of these equations can be found that describes a circularly polarized EM wave, propagating in a plasma with superluminal phase velocity, but subluminal group speed $c_\beta^*$. In it, the electron and positron fluids move with equal parallel momenta $p_\parallel$, but have equal amplitude, oppositely directed oscillations in transverse momenta $p_\perp$, which is everywhere perpendicular to the electric field. This generates a conduction current, that, in the frame in which the wave has zero group speed, exactly balances the displacement current. In the general case, the wave group speed does not coincide with the parallel component of the fluid 3-velocity, so that there is a nonvanishing particle flux in the wave frame.
Under pulsar conditions the wave is expected to be radial. At distances $r \gg r_L$, it is, to a first approximation, plane, and the deformation due to spherical geometry can be treated using perturbation analysis, expanding the relevant equations in the small parameter $\epsilon = r_L / r \ll 1$. The first-order equations describe the radial evolution of the phase-averaged quantities associated with the zeroth-order plane wave. These equations are the continuity equation, the energy conservation equation, and an equation for the evolution of the radial momentum flux. In contrast to the MHD wind, the radial momentum flux is not conserved in spherical geometry for the EM modes. However, it can be shown (Mochol 2012) that the third integral of motion for both circularly and linearly polarized modes is the phase-averaged Lorentz factor of the particles, measured in the laboratory frame $\langle \gamma_{\text{lab}} \rangle$.

To find the initial condition, one has to solve jump conditions between the MHD and the EM wave, to ensure that they carry the same particle, energy and radial momentum fluxes (Kirk 2010; Arka & Kirk 2012). In Fig. 1 we show the Lorentz factor of an EM strong wave $\gamma_* = \left(1 - \beta_*^2\right)^{-1/2}$, obtained from the jump conditions (dashed curves), and its radial evolution (solid curves) for different launching points. There are two solutions of the jump conditions that describe two possible EM modes: a free-escape mode (higher branch) and a confined mode (lower branch). Their behaviour is very different: at large distances the free-escape wave accelerates whereas the confined one decelerates. Keeping in mind that the wind solution should be matched to the slowly expanding nebula, we concentrate only on the confined mode. The radial dependence of its ram pressure is shown in Fig. 2, for both linear and circular polarizations.

Since the ram pressure of the confined mode tends to a constant value at large radius, we are able to find an unique solution that matches asymptotically a given pressure $p_{\text{ext}}$ of the external medium. In fact, to constrain a wave at launch uniquely, four quantities have to constrained. These are: the conversion radius $R_0$, initial group speed $\beta_{\text{g0}}$ of a wave and initial particle momenta $p_{\parallel0}, p_{\perp0}$. The jump conditions define three of them, leading to the red curves in Fig. 1 and the external pressure can be used to determine the fourth one — $R_0$. The existence of the third integral of motion $\langle \gamma_{\text{lab}} \rangle$ makes this task easy, and leads to a unique stationary solution for the shock precursor for given MHD wave parameters $\sigma, \mu, a_L$, and external pressure $p_{\text{ext}}$.
3. Damping and Shock Formation

Asymptotic matching of the ram and external pressures leads to a self-consistent solution for the wave into which a given MHD wind converts. However, EM waves are also damped, because the accelerated particles they contain emit photons (Gunn & Ostriker 1971; Asseo et al. 1978) or scatter pre-existing photons from external sources. Damping removes two of the integrals of motion, leaving only particle flux conserved, and the resulting system must be integrated numerically.

However, it has been shown both analytically (Max 1973; Lee & Lerche 1978) and numerically (Romeiras 1978) that strong waves are unstable to small density perturbations in the direction of motion, provided the particles stream through the wave sufficiently slowly. Both the group speed of the wave and the radial component of the particle speed decrease as $1/R^2$ at large $R$. Thus, even if the wave is launched with highly relativistic particle streaming in its rest frame, this streaming speed tends to zero at large $R$. This effect persists when damping is included in the computation, so that parametric instabilities will set in at some stage and destroy the wave. This point is the location of the termination shock.

4. Conclusions

The structure of the pulsar wind termination shock is determined by the physical conditions not only in the magnetosphere, but also in the external medium. Two regimes emerge from the model: the one with high external pressure, in which the EM wave cannot be launched at all and the shock forms rather due to interactions of the external medium with the inner MHD wind; the second one is that with a lower external pressure, in which the EM wave exists as a stationary shock precursor, which, after deceleration, becomes unstable and leads to the formation of a shock front. The damping of the strong precursor wave by photon emission or inverse Compton scattering of external photons can potentially provide an observable test of this scenario.

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