Measurement of the higher-order anisotropic flow coefficients for identified hadrons in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

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Introduction. The quark-gluon plasma (QGP) is a novel phase of nuclear matter at high temperature and energy density, whose existence is predicted by quantum chromodynamics \[^{[1]}\]. A wide variety of experimental observations at the Relativistic Heavy Ion Collider (RHIC) \[^{[2],[3]}\] provide strong evidence for the formation of a QGP in ultra-relativistic heavy ion collisions, particularly (1) the magnitude of the observed suppression of high-\(p_T\) (\(p_T \gtrsim 4 \text{ GeV/c}\)) particles, relative to the scaled yield from \(p+p\) collisions; and (2) the large azimuthal anisotropy or anisotropic flow of the low-\(p_T\) (\(p_T \lesssim 3–4 \text{ GeV/c}\)) bulk of hadrons in the final state. The flow of low-\(p_T\) particles has been attributed to anisotropic expansion of the QGP \[^{[4],[5]}\], and consequently the measured strength of anisotropic flow should be sensitive to the transport properties of the QGP and the mechanism for its space-time evolution.

The magnitude of anisotropic flow can be quantified by the Fourier coefficients \(v_n\{\Psi_m\} = \langle \cos(n(\phi - \Psi_m)) \rangle\) of the azimuthal distribution of produced particles \[^{[5],[12]}\], where \(n\) and \(m\) are the order of the harmonics, \(\phi\) is the azimuthal angle of the particles, and \(\Psi_m\) is the azimuthal angle of the \(m^{th}\) order event plane. In early studies with symmetric systems, \(v_n\{\Psi_m\}\) was presumed to be zero for odd \(n\) owing to the assumption that initial-state energy densities were smooth and symmetric across the transverse plane. The recent observations of sizable \(v_n\{\Psi_m\}\) values for odd \(n\) \[^{[13],[17]}\] confirms the important role of fluctuations in the initial-state collision geometry \[^{[18]}\].

Model-dependent analyses of higher-order harmonics for inclusive hadrons measured in Au+Au and Pb+Pb collisions at RHIC and the Large Hadron Collider have indicated that such measurements can provide simultaneous constraints for initial-state fluctuation models and the ratio of shear viscosity to entropy density of the QGP \[^{[8],[13],[19],[20]}\]. The new data on higher-order \(v_n\{\Psi_m\}\) for identified particles presented here provides additional information about the initial conditions and hydrodynamic properties. Here, we show that our \(v_n\{\Psi_m\}\) measurements for different particle species provide (1) further tests for the constituent quark number scaling and quark coalescence models \[^{[21],[23]}\] by extending our previously observed scaling for \(v_2\{\Psi_2\}\) \[^{[24],[25]}\] to higher harmonics \[^{[26]}\]; and (2) freeze-out parameters for hydrodynamic expansion with anisotropic blast-wave (BW) model fits \[^{[27],[30]}\].

Data taking and particle identification. The results presented here for Au+Au collisions at \(\sqrt{s_{NN}} = 200 \text{ GeV}\) are obtained with the PHENIX experi-
iminent from an analysis of $4.14 \times 10^9$ minimum-bias events taken during the 2007 running period. Collision centrality is determined with the beam-beam counters [31]. Charged hadrons are reconstructed in a pseudorapidity ($\eta$) range of $|\eta| < 0.35$ using the drift-chamber and pad-chamber subsystems [32], which achieve the momentum resolution $\delta p / p \approx 1.3\% \oplus 1.2\% \times p$ (GeV/c) [33]. The ring imaging Čerenkov counter is employed to veto conversion electrons. Time-of-flight detectors in both the east (TOFE, $\Delta \phi = \pi/4$ rad) and west (TOFW, $\Delta \phi = 0.342$ rad) arms are used for $\pi^\pm$, $K^\pm$, and $p + \bar{p}$ identification after the conversion electron veto [34]. The timing resolution of TOFE (TOFW) is 133 (84 ± 1) ps. For $p_T < 3$ GeV/c both TOFE and TOFW detectors were used. For $p_T > 3$ GeV/c particle identification utilizes the TOFW in conjunction with the Aerogel Čerenkov Counter (ACC). The two detectors have a common azimuthal acceptance of $\Delta \phi = 0.171$ rad. With these detectors, a $p + \bar{p}$ purity of greater than 97% was achieved for $p_T < 4$ GeV/c, and purity for $\pi^\pm$ and $K^\pm$ greater than 98% for $p_T < 3$ GeV/c and 90% for $3 < p_T < 4$ GeV/c were also achieved, as detailed in [33]. The purity and efficiency of particle identification (PID) are independent of the relative azimuthal angle between particles and the event plane $\phi - \Psi_m$.

Experimental technique. Measurements of the flow coefficients $v_2(\Psi_2)$, $v_3(\Psi_3)$, $v_4(\Psi_4)$, and $v_4(\Psi_2)$ as a function of centrality and $p_T$ for $\pi^\pm$, $K^\pm$, and $p + \bar{p}$ (i.e. with charge signs combined) are obtained with both the event plane (EP) and the long-range two-particle correlation (2PC) methods. In the EP method, a measured event plane direction $\Psi_m^{obs}$ is determined for every event and for each order $m$, using the south and north reaction-plane detectors (RXN), covering $\Delta \phi = 2\pi$ and $1 < |\eta| < 2.8$ [34]. Each is made of plastic scintillator paddles with lead converter in front and with optical fibers guided to photo multiplier tubes. Each RXN detector is segmented into 12 sections in $\varphi$ and two rings in $\eta$. The $\Psi_m^{obs}$ are determined via a sum over the azimuthal angle $\phi_i$ of each RXN element in both the arms with its charge $w_i$ deposited by particles for that event, as $\text{tan}(m \Psi_{m}^{obs}) = \sum_i w_i \sin(m \phi_i) / \sum_i w_i \cos(m \phi_i)$. The flow magnitudes $v_n(\Psi_m) = \langle \cos n(\phi - \Psi_m^{obs}) \rangle / \text{Res}(n, \Psi_m)$ are then measured with respect to each harmonic event plane, where $\phi$ is the azimuthal angle of the hadron and $\text{Res}(n, \Psi_m) = \langle \cos n(\Psi_m - \Psi_m^{obs}) \rangle$ is the event plane resolution, which is estimated for each centrality by the standard sub-event method as described in [10, 33, 34]. The best resolution of each harmonic is measured to be $\text{Res}(2, \Psi_2) \sim 0.75$ and $\text{Res}(4, \Psi_2) \sim 0.5$ (Res(3, $\Psi_3$) $\sim 0.3$ and Res(4, $\Psi_4$) $\sim 0.15$) in 20%-30% (0%-10%) central collisions.

The 2PC method pairs the hadrons (HAD) with deposited charges in the RXN segments. The distribution of the relative azimuthal angles of particle hits in separate $\eta$ ranges $A$ and $B$, $\Delta \phi \equiv \phi^A - \phi^B$, reflects the product of the $v_n$'s via $dN / d\Delta \phi \propto 1 + \sum_{n=1}^{2} \varepsilon_n v_n^A v_n^B \cos(n \Delta \phi)$ [10, 37, 38]. We analyze the $\Delta \phi$ correlations using the mixed-event technique for two pair combinations; $(A, B)$=(HAD,RXN) and $(A, B)$=(RXN-N,RXN-S). These correlations then fix the event-averaged products $\langle v_n^{HAD,RXN} \rangle$ and $\langle v_n^{RXN,RXN} \rangle$, and allow us to obtain $v_n^{HAD} = \langle v_n^{HAD,RXN} \rangle / \sqrt{\langle v_n^{RXN,RXN} \rangle}$. Note that flow harmonics extracted with the 2PC method are not measured with respect to event planes. Thus, from this point forward we refer to flow harmonics in the 2PC methods as $v_n(2PC)$. We use $v_n$ in cases when the discussion is generically about either method. In both of the analysis methods used, the results for wider centrality ranges are obtained by averaging across several smaller ranges, weighted by the multiplicity of the selected particle [39].

The systematic uncertainties in the $v_n$ measurements were estimated for: (1) $\eta$ acceptance variation of the RXNs, in the EP and 2PC methods; this is correlated among $v_n(p_T)$ for each hadron species with the same fractional $v_n$ amount in the entire $p_T$ range, except for $v_4(\Psi_4)$ where it tends to decrease as $p_T$ increases; (2) detector acceptance effects of TOFE and TOFW, including occupancy; these are correlated among $v_n(p_T)$ for each hadron species with the same $v_n$ constant in the entire $p_T$ range; (3) hadron track/hit matching cut; and (4) particle identification purity. The systematic uncertainties (1) and (2) are $p_T$-correlated, while (3) and (4) are $p_T$-uncorrelated. These uncertainties are similar between the EP and 2PC methods. Table I summarizes typical systematic uncertainties on the different $v_n(\Psi_m)$ measures in the EP method for $\pi^\pm$ at $p_T = 2$ GeV/c.

| Type Source | $v_2(\Psi_2)$ | $v_3(\Psi_3)$ | $v_4(\Psi_4)$ | $v_4(\Psi_2)$ |
|-------------|---------------|---------------|---------------|---------------|
| (1) RXN $\eta[\%]$ | 4.3(3.0) | 4.7(12.5) | 16(31) | 34(7.0) |
| (2) Acceptance$^{[10-3]}$ | 5.0(1.0) | 0.5(2.0) | 0.7(2.5) | 0.1(0.2) |
| (3) Matching$^{[\%]}$ | 1.4(0.3) | 0.7(1.0) | 2.6(2.8) | 7.7(1.7) |
| (4) PID$^{[\%]}$ | 0.3(0.1) | 0.3(0.3) | 0.8(1.0) | 2.7(0.4) |

Results for 0%-50% centrality bin. Figures (1a)-(c) show a comparison of $v_2(p_T)$, $v_3(p_T)$, and $v_4(p_T)$ for $\pi^\pm$, $K^\pm$, and $p + \bar{p}$ for the EP (solid points) and 2PC (open points) methods in a 0%-50% centrality sample; they indicate very good agreement between the two methods. Shown in Fig. (1d) is $v_4(\Psi_2)$, i.e., the fourth harmonic coefficient with respect to the second-order harmonic event plane. It can be seen that $v_4(\Psi_2)$ is smaller than $v_4(\Psi_4)$ but still sizable, indicating significant correlations between $\Psi_2$ and $\Psi_4$ [40], which can be ascertained through the trigonometric identity $v_4(\Psi_2) / v_4(\Psi_4) = \langle \cos 4(\Psi_2 - \Psi_4) \rangle$ [41]. There are two trends common to all $n$ in Fig. (1) in the low-$p_T$ region the anisotropy appears largest for the lightest
The evolution of baryon-meson splitting at intermediate- to low-mass ordering from the common velocity field (central Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV. Different \( p_T \) bins were used for the EP and 2PC methods. The green bands indicate the \( p_T \)-correlated systematic uncertainties of the \( \pi^\pm \) results from the EP method. The shaded boxes around the data points are \( p_T \)-uncorrelated systematic uncertainties, which are smaller than the symbols in many cases.

FIG. 1. (Color online) Fourier coefficients for charge-combined \( \pi^\pm \), \( K^\pm \), and \( p+p \overline{p} \) at midrapidity for 0%–50% central Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV. Different \( p_T \) bins were used for the EP and 2PC methods. The green bands indicate the \( p_T \)-correlated systematic uncertainties of the \( \pi^\pm \) results from the EP method. The shaded boxes around the data points are \( p_T \)-uncorrelated systematic uncertainties, which are smaller than the symbols in many cases.

Results for finer centrality bins. The \( \nu_n(\Psi_m) \) of \( \pi^\pm \), \( K^\pm \), and \( p+p \overline{p} \) measured with the event plane method are shown in Fig. 2 for the centrality selections 0%–10% and 30%–50%. The same mass dependence of \( \nu_n(\Psi_m) \) is seen in the low-\( p_T \) region for all harmonics and centralities. The evolution of baryon-meson splitting at intermediate- \( p_T \) is also observed for all centralities in \( v_2(\Psi_2) \) and \( v_3(\Psi_3) \) but could not be confirmed for \( v_4(\Psi_4) \) in the most central and more peripheral events, or for \( v_3(\Psi_2) \) in the most central events owing to the lower statistical significance of the measurements in those bins.

Quark-number scaling. The baryon-meson splitting in the intermediate-\( p_T \) region can be taken as an indication that the number of constituent valence quarks \( N_q \) is an important determinant of final-state hadron flow in this range. Indeed, the \( v_2(\Psi_2) \) data for identified hadrons had previously been seen to scale such that \( v_2(\Psi_2)/N_q \) was the same for different particle species when evaluated at the same transverse kinetic energy per constituent quark number in the range \( K E_T/N_q \lesssim 1 \) GeV (\( K E_T \equiv m_T - m_0 \) and \( m_T \equiv \sqrt{p_T^2 + m_0^2} \), where \( m_0 \) is the hadron mass) i.e. “quark-number scaling” [24, 33].

Any explanation of the underlying physics needs to match this scaling over this \( K E_T \) range, and neither hydrodynamics [11, 24, 43, 44] nor naive quark coalescence alone [45] predicts this scaling for the higher moments. It is notable that for \( v_2(\Psi_2) \), there are deviations from valence-quark scaling at higher \( p_T \) with mesons and baryons having comparable anisotropies [33]. Reconciling the different physics as a function of \( p_T \) remains an outstanding challenge.

Blow-wave fit. The BW model [27–30] is a description of a fluid freeze-out state characterized by its temperature \( T_f \) and its \( \phi \)-averaged maximal radial flow rapidity \( \rho_0 \). Here we extend the BW description to incorporate azimuthal anisotropies in both radial rapidities \( \rho_n(\Psi_m) \) and spatial density \( s_n(\Psi_m) \) for \( n = 2, 3, 4 \), using the empirically defined quantities

\[
\rho(n, m, \phi, r) = \rho_0 (1 + 2 \rho_n(\Psi_m) \cos(n\phi)) \times r/R_f^{\max}
\]

and

\[
s(n, m, \phi) = 1 + 2s_n(\Psi_m) \cos(n\phi).
\]

The spectra and anisotropies of all hadrons freezing out of the fluid can then be predicted via [28–29]

\[
\frac{dN}{d\phi d\rho T d\rho T} \propto \int_{R_f^{\max}}^{R_f} dr d\phi m_T I_0(\alpha_1 K_1(\beta_1)) \\
\frac{dN}{d\phi d\rho T d\rho T} \propto \int_{R_f^{\max}}^{R_f} dr d\phi I_0(\alpha_1 K_1(\beta_1)) S(n, m, \phi),
\]

where \( I_n \) and \( K_1 \) are modified Bessel functions of the first and second kind, \( \alpha_1 = (p_T/T_f) \sinh \rho(n, m, \phi, r) \), and \( \beta_1 = (m_T/T_f) \cosh \rho(n, m, \phi, r) \). Using single particle spectra from [10] together with the present \( \nu_n(\Psi_m) \) data, BW parameters \( T_f, \rho_0, \rho_n(\Psi_m) \), and \( s_n(\Psi_m) \) are extracted via simultaneous fitting of the \( \pi^\pm, K^\pm \), and \( p+p \overline{p} \) data with a minimization of global \( \chi^2 \), separately for each centrality selection and each \( \nu_n(\Psi_m) \). The fit ranges used for the \( \pi^\pm, K^\pm \), and \( p+p \overline{p} \) central collisions, together with the global \( \chi^2/ndf \) of the fits determined using the quadrature sum of the statistical and systematic uncertainties of the data. The
global $\chi^2/\text{ndf}$ in 10%–20% and 20%–30% central collisions is similar to that in 0%–10% and 30%–50% central collisions.

The results for the BW parameters are shown in Fig. 3. The freeze-out temperatures $T_f$ and radially averaged flow rapidities $\langle \rho \rangle = \int [\rho_0 \times r/R_{\text{max}}] r dr/\int r dr$ are in good agreement for the fits at different $n$, as would be required for a model of freeze-out. $T_f$ and $\langle \rho \rangle$ are primarily determined by the single particle spectra $|N_q\rangle$, while $\rho_n(\Psi_m)$ and $s_n(\Psi_m)$ are determined by $v_n(\Psi_m)$ measurements including $p_T$ and particle mass dependences.

The radial rapidity and spatial density anisotropies $\rho_n(\Psi_m)$ and $s_n(\Psi_m)$ extracted from the fits are shown against the average initial-state spatial participant-plane (PP) anisotropy $\varepsilon_n(\Psi_m^\text{PP}) = \langle \{r^2 \cos n(\phi^\text{part} - \Psi_m^\text{PP})\}/r^2 \rangle$, where $r$ and $\phi^\text{part}$ are the polar coordinate positions of

FIG. 2. (Color online) Fourier coefficients for charge-combined $\pi^\pm$, $K^\pm$, and $p + \bar{p}$ at midrapidity in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Coefficients are determined using the event plane method. The curves illustrate the fits from the BW model. Systematic uncertainties are shown as in Fig. 1.

FIG. 3. (Color online) Quark-number ($N_q$) scaling for 0%–50% central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, where $N_q$ is the constituent valence quark number of each hadron. Systematic uncertainties are shown as in Fig. 2.

FIG. 4. (Color online) BW model fit parameters extracted for each $v_n(\Psi_m^+) +$ spectra across different centrality classes. The gray bands in (a)–(b) and shaded boxes in (c)–(d) indicate systematic uncertainties on the fitting $p_T$ range and those propagated from the measurements. The width of the shaded boxes in $\varepsilon_n(\Psi_m^+)$ direction in (c)–(d) indicates systematic uncertainties from Glauber models. Systematic uncertainties in (a) and (b) are similar among different fittings.
collision participant nucleons defined by Glauber models [18, 48], and $\Psi_{m}^{PP}$ is the angle determined as $\tan\left(\frac{m\Psi_{m}^{PP}}{n}\right) = \frac{\langle r^2 \sin m \theta \rangle}{\langle r^2 \cos m \theta \rangle}$. Here, the brackets $\langle \rangle$ and $\{ \}$ denote averages over events and participants, respectively. The amplitude of $\varepsilon_{n}\{\Psi_{m}^{PP}\}$ is smallest for the most-central collisions and increases with centrality percentile.

Eccentricity of the medium at freeze out. The $\rho_{n}\{\Psi_{m}\}$ and $s_{n}\{\Psi_{m}\}$ are generally smaller than the $\varepsilon_{n}\{\Psi_{m}^{PP}\}$. The $\rho_{n}\{\Psi_{m}\}$ has a positive finite value and generally follows a common increasing curve as a function of $\varepsilon_{n}\{\Psi_{m}^{PP}\}$ for $n = 2, 3, 4$. The $s_{2}\{\Psi_{2}\}$, $s_{3}\{\Psi_{3}\}$, and $s_{4}\{\Psi_{4}\}$ also show a common increasing trend in $\varepsilon_{n}\{\Psi_{m}^{PP}\} \gtrsim 0.1$. We can interpret relative oscillations of event-plane dependent Hanbury-Brown-Twiss (HBT) radii with respect to averaged radii as the eccentricity of the medium at freeze-out if the direction of the radii is selected perpendicular to beam and pair momentum ($R_{side}$), where these radii are less influenced by the emission duration and position-momentum correlations.

Spatial information. Finite final eccentricities for $n = 2$ and $n = 3$ are observed by both the BW fit to $v_{n}\{\Psi_{m}\}$ and the event plane dependent HBT radii measurements using positive and negative pion pairs [19]. The $s_{n}\{\Psi_{m}\}$ therefore could reflect physical effects at the freeze-out of the medium. The finite $s_{n}\{\Psi_{m}\}$ could be interpreted as a residual effect of initial state anisotropy $\varepsilon_{n}\{\Psi_{m}^{PP}\}$, especially the contribution of initial-state fluctuations for $n = 3, 4$, after its dilution by the medium expansion. For $\varepsilon_{n}\{\Psi_{m}^{PP}\} \lesssim 0.1$, $s_{3}\{\Psi_{3}\}$, $s_{4}\{\Psi_{4}\}$, and $s_{4}\{\Psi_{4}\}$ are consistent with zero within systematic uncertainties. Comparisons of these small $s_{n}\{\Psi_{m}\}$ to the finite $\rho_{n}\{\Psi_{m}\}$ and $v_{n}\{\Psi_{m}\}$ in this $\varepsilon_{n}\{\Psi_{m}^{PP}\}$ range indicate that the anisotropic expansion velocity $\rho_{n}\{\Psi_{m}\}$ is a dominant source of the observed $v_{n}\{\Psi_{m}\}$ for higher harmonics. We expect this spatial information could provide new insights into freeze-out conditions in hydrodynamic calculations.

Summary and conclusions. In summary, the anisotropy strengths $v_{2}\{\Psi_{2}\}$, $v_{3}\{\Psi_{3}\}$, $v_{4}\{\Psi_{4}\}$, and $v_{4}\{\Psi_{4}\}$ for $p^{+}$, $K^{\pm}$, and $p + \bar{p}$ produced at midrapidity in Au+Au collisions at RHIC have been presented. The higher-order harmonics $v_{n}\{\Psi_{m}\}$ show particle mass splitting at low-$p_{T}$ and baryon-meson difference at intermediate-$p_{T}$, very similar to what has been seen already for $v_{2}\{\Psi_{2}\}$. The anisotropies obey a modified quark number scaling, where $v_{n}\{\Psi_{m}\}/(N_{q})^{n/2}$ falls on a common trend against $KE_{T}/N_{q}$ for each $n$. The data can be fit with a generalized BW model with empirically defined anisotropies in radial rapidity and spatial density at higher harmonic orders, which could provide a geometrical view of the hydrodynamical expansion at the end of freeze out. Future analyses combining the results in this letter with similar results from HBT and jet-like correlations with respect to higher-order event planes will further constrain the conditions and properties of the matter created at RHIC.

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