Charge and spin currents in magnetic multi layers 
in the presence of both electric field and spin dynamics

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Abstract. The number and the spin angular momentum of electrons on each site, and the charge and the spin currents between adjacent sites are theoretically studied. Considering one dimensional electronic system including impurity atoms with some density, the static electric field (represented by an electric potential) and the s-d interaction with spin dynamics are treated perturbatively. We show that the effect of the applied electric field can be understood from the ideas of the drift by the field and the band shift due to the electric potential.

1. Introduction
In most spintronic devices consisting of magnetic multi layers, an interplay between spin and charge currents is believed to play an essential role for its peculiar phenomena. For that reason, the establishment of methods generating and controlling the currents is the central issue in the study of spintronics, and elucidating the mechanisms of the creation and the control of these currents is strongly desired.

It is confirmed experimentally that spin dynamics induce spin currents [1][2]. In particular, the phenomenon in metallic magnetic multi layers is called the spin-pumping. The spin-pumping, the conversion from spin dynamics to spin currents, have been studied based on phenomenological theories [3][4][5]. Takeuchi et al.[6] have recently discussed a spin current in the dynamical spin system from the microscopic viewpoints. They have shown that a spin current is pumped into non-magnetic layers from magnetic layers with spin dynamics such as a precession due to a magnetic field. In this paper, we consider the case that an external static homogeneous electric field is applied to magnetic multi layers with spin dynamics, and calculate microscopically the charge current, the spin current, the number and the spin angular momentum of electrons.

2. Model
We assume one dimensional electronic system described by the tight-binding model. Our model Hamiltonian is $H(t) := H_0 + H_{\text{imp}} + H_{\text{ex}}(t) + H_{\text{bias}}(t)$ where $H_0 := \sum_k E_k c_k^\dagger c_k$ describes itinerant electrons with $E_k := \varepsilon_0 - 2T \cos ka$ ($\varepsilon_0$ is an on-site energy, $T$ is a transfer integral between nearest-neighbor sites and $a$ is a lattice constant), and $H_{\text{imp}} := \frac{N}{2} \sum_k \sum_{q \neq 0} \sum_{j=1}^{n_i} e^{-iq\alpha j} c_k^\dagger c_{k+q} c_k$ describes the elastic electron scattering by impurity atoms with some density $n_i/N$ ($\alpha$ is a strength of the impurity scattering, $n_i$ is the number of impurities, $N$ is the number of sites), which gives rise to a lifetime of an electron in the Born approximation $\tau := \left( \frac{2\pi m_0 a^2}{\hbar N} \right)^{-1} \rho$ where $\rho$ is the density...
of states per a site and a spin at the Fermi energy $\mu$, and $H_{\text{ex}}(t) := -J \sum_{k,q} S_q(t) \cdot c_{k+q}^\dagger \sigma c_k$ describes the exchange interaction with time-dependent local spins $S_q(t)$ ($J$ is the exchange coupling constant, $\sigma$ is a vector of Pauli matrices and a hat ‘’ denotes a $2 \times 2$ matrix in spin-space), and $H_{\text{bias}}(t) := -e \sum_{k,q} \phi_q(t) c_{k+q}^\dagger \sigma c_k$ ($e > 0$ is the elementary charge) describes at once the time-dependent electric potential $\phi_q(t)$ and the electric field $E_q(t) := -i q \phi_q(t)$, $-e \phi_q(t)$ means the energy difference from the $\mu$.

### 3. Method

The number of electrons at the $l$ site is defined as $n_l(t) := \langle c_l^\dagger(t) c_l(t) \rangle_H$, the spin angular momentum at the $l$ site as $\sigma_l(t) := \langle c_l^\dagger(t) \sigma c_l(t) \rangle_H$, the charge current flowing from the $l$ site to the $l+1$ site as $I^c_l(t) := \frac{i}{\hbar} T \left[ \langle c_{l+1}^\dagger(t) c_l(t) \rangle_H - \text{c.c.} \right]$ and the spin current flowing from the $l$ site to the $l+1$ site as $I^s_l(t) := \frac{i}{\hbar} T \left[ \langle c_{l+1}^\dagger(t) \sigma c_l(t) \rangle_H - \text{c.c.} \right]$, where $\langle \cdots \rangle_H$ is the expectation value estimated by $H(t)$ and we calculate it within the Keldysh-Green function framework. We can express these quantities using a lesser Green function as follows

$$n_l(t) = \sum_q \int \frac{d\omega}{2\pi\hbar} e^{i(qa - \omega t)/\hbar} \text{Tr}_\sigma \left[ \Pi^0_q(\omega) \right],$$

$$\sigma_l(t) = \sum_q \int \frac{d\omega}{2\pi\hbar} e^{i(qa - \omega t)/\hbar} \text{Tr}_\sigma \left[ \sigma \Pi^0_q(\omega) \right],$$

$$I^c_l(t) = \sum_q \int \frac{d\omega}{2\pi\hbar} e^{i(qa - \omega t)/\hbar} \text{Tr}_\sigma \left[ \Pi^1_q(\omega) \right],$$

$$I^s_l(t) = \sum_q \int \frac{d\omega}{2\pi\hbar} e^{i(qa - \omega t)/\hbar} \text{Tr}_\sigma \left[ \sigma \Pi^1_q(\omega) \right],$$

where $\text{Tr}_\sigma [ \cdots ]$ denotes taking the trace in spin-space, and

$$\Pi^0_q(\omega) := \frac{1}{N} \int \frac{dE}{2\pi} \sum_k j_{k}^\dagger \tilde{G}^<_k(E_+, E_-),$$

$$j_{k}^\dagger := \left( 1, \frac{e^{i q a/2}}{a} v_k \right).$$

$v_k := \frac{1}{\hbar} \frac{\partial E_k}{\partial k} = 2 T a \sin ka$ is a group velocity of an electron having an energy $E_k$, $k_{\pm} := k \pm q/2$ and $E_{\pm} := E \pm \omega/2$. $\tilde{G}^<_k(E, E') := \int dt dt' e^{i E t / \hbar} e^{-i E' t' / \hbar} \tilde{G}^<_k(t, t')$, and $\tilde{G}^<_k(t, t')$ is the matrix of the lesser Green function with components $\tilde{G}^<_k(E, E') := \frac{i}{\hbar} \langle c_{k}^\dagger(\tau') c_{k \sigma}(\tau) \rangle_H$, ($\sigma = \uparrow, \downarrow$).

### 4. Result

We discuss the four quantities in perturbative regimes that are showed in figure 1, and take certain extreme limits: the slowly varying limit $\Omega T \ll 1$ where $\Omega$ is a frequency of a local spin, the electric potential and the electric field (note that for the electric field we must take the limit after the uniform limit $E_q(\Omega) \rightarrow \delta q_0 E_q(\Omega_i)$), the metallic limit $\frac{N}{\mu L} \ll 1$, and the low temperature limit. However, to focus on structural dependence of magnetic multi layers, we make an effort to reflect wavenumber dependence of spins in the calculation as much as possible. In particular, we calculate equilibrium components using approximation that $E_k \approx \varepsilon_0 - 2 T (1 - \frac{L^2 E_k^2}{a^2})$ and the scope of $k$ is extended from the first Brillouin zone $[\frac{-\pi}{a}, \frac{\pi}{a}]$ to $(-\infty, \infty)$ instead of assuming a spatially smooth spin structure. Unfortunately, only Fourier components of small
wavenumber $Q$ ($v_F \tau Q \ll 1$, $v_F := v_{k_F}$, $k_F$ is the Fermi wavenumber) of spins are considered for dynamical components, and the diagrams including a maximal number of diffusion propagators are considered.

After straightforward calculations, we obtain

$$n_i(t) = 2n_0 + \frac{e}{2T} \sum_{l'} F_{2k_F a(l-l')}^{(1)} \phi_{l'}(t) - 2 \rho J \sum_{l'} F_{2k_F a(l-l')}^{(2)} \phi_{l'}(t) - 2 \rho J \sum_{l',l''} F_{2k_F a(l-l'),2k_F a(l'-l'')}^{(2)} S_{l'}(t) \cdot S_{l''}(t)$$

$$\nu(t) = \frac{J}{2T} \sum_{l'} F_{2k_F a(l-l')}^{(1)} S_{l'}(t) - 4 \rho e P (k_F a)^2 \sum_{l',l''} F_{2k_F a(l-l'),2k_F a(l'-l'')}^{(2)} \phi_{l'}(t) S_{l''}(t)$$

$$\sigma(t) = \frac{J}{2T} \sum_{l'} F_{2k_F a(l-l')}^{(1)} S_{l'}(t) - 4 \rho e P (k_F a)^2 \sum_{l',l''} F_{2k_F a(l-l'),2k_F a(l'-l'')}^{(2)} \phi_{l'}(t) S_{l''}(t)$$

$$I_i^C(t) = - \frac{\sigma}{e a} E_i(t) + \frac{\sigma \tau J}{e^2 a} P \partial_t \{ S_i(t) \cdot \dot{S}_i(t) \}$$

$$I_i^S(t) = - \frac{J^2}{2h T} \sum_{l',l''} \left[ \pi (l - l') \right] F_{2k_F a(l-l')}^{(1)} S_{l'}(t) \times S_{l''}(t) + \frac{\sigma \tau J}{e^2 a} \partial_t \{ S_i(t) \cdot \dot{S}_i(t) \}$$

where we have introduced quantities, the unperturbative number of electrons $n_0 := \int_0^\infty dE \rho(E)$, the spin polarization $P := \frac{\hat{c}}{\rho} J$, the electric conductivity $\sigma := 2e^2 \rho D$, the diffusion constant $D := v_F^2 \tau$, the sine integral function $\text{Si}(x) := \frac{\pi}{2} \int_0^x dt \frac{\sin t}{t}$, $F_{\alpha}(1) := 1 - \text{Si}(|\alpha|)$, $F_{\alpha,\beta}^{(2)} := \frac{1}{2} \int f(\alpha + \beta) + f(\beta) + \frac{\text{sgn}(\alpha)}{2\pi} \int dy \frac{\partial y}{y} f(y) - f(x) := \cos x - \frac{x}{2} (|x| - \text{Si}|x|)$, and a spatial derivative operator is interpreted by the expression $\partial_i A_i := \frac{A_{i+1} - A_i}{a}$ where $A_i$ is an
arbitrary function of \( l \). \((\cdots)\) means diffusion

\[
\langle A_l(t) \rangle := \sum_{l'} \int \frac{dt'}{\tau} D_{l-l'}(t-t') A_{l'}(t'),
\]

\[
\langle A_l(t) B_{l'}(t) \rangle := \sum_{l''} \int \frac{dt''}{\tau} \int \frac{dt'''}{\tau} D_{l-l''}(t-t') A_{l'}(t') D_{l'-l'''}(t'-t'') B_{l'''}(t'''),
\]

where, \( A_l(t) \) and \( B_{l'}(t) \) are arbitrary functions of both \( t \) and \( t' \). \( D_l(t) \) is the diffusion propagator satisfying \( \left[ \frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2} \right] D_l(t) = \delta(t) \). The results do not break the gauge invariance because \(-e\phi(t)\) is the energy difference from the \( \mu \).

5. Discussion and Summary

First, we focus on the equilibrium components

\[
n^{(eq)}_l(t) = 2n_0 + \frac{6}{2T} \sum_{l'} F^{(1)}_{2k_F a(l-l')} \phi_{l'}(t) - 2\rho J P (k_F a)^2 \sum_{l',l''} F^{(2)}_{2k_F a(l-l'),2k_F a(l''-l')} \mathbf{S}_{l'}(t) \cdot \mathbf{S}_{l''}(t),
\]

\[
\sigma^{(eq)}_l(t) = \frac{J}{2T} \sum_{l'} F^{(1)}_{2k_F a(l-l')} \mathbf{S}_{l'}(t) - 4\rho e P (k_F a)^2 \sum_{l',l''} F^{(2)}_{2k_F a(l-l'),2k_F a(l''-l')} \phi_{l'}(t) \mathbf{S}_{l''}(t),
\]

\[
I^C_l(t) = 0,
\]

\[
I^S_l(t) = \frac{J^2}{2hT} \sum_{l',l''} \text{Si} \left[ \pi(l-l') \right] F^{(1)}_{2k_F a(l-l')} \mathbf{S}_{l'}(t) \times \mathbf{S}_{l''}(t).
\]

Terms containing \( F^{(1)}_{2k_F a} \) in \( n^{(eq)}_l(t) \) and \( \sigma^{(eq)}_l(t) \) express the Friedel oscillation and the Ruderman-Kittel-Yosida oscillation which decay inversely proportional to \( al \), oscillating with the half Fermi wavelength period \( \pi \). The equilibrium spin current \( I^S_l(t) \) describes the exchange interaction between local spins through the Ruderman-Kittel-Kasuya-Yosida interaction. The factor \( \text{Si} \left[ \pi(l-l') \right] \) reflects a situation that the spin current is induced only in areas between non-parallel spins. The second term of \( \sigma^{(eq)}_l(t) \) represents the effect of shift of the local density of states at a site due to the electric potential. Also we can paraphrase that the presence of electric field is essential for this effect because the term exists only in the case that the electric potential possess spatial dependence. The last term of \( n^{(eq)}_l(t) \) represents the interference effect between a local spin and a spin of spin-polarized itinerant electrons. The numerical calculation for \( F^{(2)}_{2a,2l} \) reveals that the number of electrons in areas between local spins is controllable by the relative angle between the spins. These terms including \( F^{(2)}_{2a,2l} \) will be discussed in detail in a separate paper. The equilibrium charge current vanishes identically.

Next, we see non-equilibrium components

\[
n^{(ne)}_l(t) = -2\rho J \tau P \left( \mathbf{S}_l(t) \cdot \dot{\mathbf{S}}_l(t) \right),
\]

\[
\sigma^{(ne)}_l(t) = -2\rho J \tau \left[ \dot{\mathbf{S}}_l(t) + \frac{4\rho J^2 \tau^2}{\hbar} \langle \mathbf{S}_l(t) \times \dot{\mathbf{S}}_l(t) \rangle - 2e\rho \tau P \langle \dot{\phi}_l(t) \dot{\mathbf{S}}_l(t) \rangle, \right.
\]

\[
I^C_{l}(t) = -\frac{\sigma}{ea} E_l(t) + \frac{\sigma \tau J}{e^2 a} \dot{\mathbf{S}}_l(t),
\]

\[
I^S_{l}(t) = \frac{\sigma J \tau}{e^2 a} \dot{\phi}_l(t) \langle \mathbf{S}_l(t) \times \dot{\mathbf{S}}_l(t) \rangle - \frac{\sigma E_l(t)}{ea} P \langle \mathbf{S}_l(t) \rangle + \frac{\sigma \tau}{ea} P \dot{\phi}_l(t) \mathbf{S}_l(t). \]
The first term of $I_l^C(\text{ne})(t)$ is the drift electric current according to the Ohm’s law. This part appears also in the third term of $I_l^S(\text{ne})(t)$, thus the terms describe the spin-polarized electric current induced by the electric field. The last term of $I_l^S(\text{ne})(t)$ represents correction for the first term from the band shift due to the electric potential. Much the same is true on the last term of $\sigma_l^{\text{ne}}(t)$. $n_l^{\text{ne}}(t)$ is the dynamical version of the interference effect, and the second term of $I_l^C(\text{ne})(t)$ is the diffusion current of $n_l^{\text{ne}}(t)$ because these quantities satisfy the Fick’s first law of diffusion $\frac{\partial}{\partial x} \langle S(t) \cdot \dot{S}(t) \rangle = -\frac{D}{a} \frac{\partial}{\partial x} n_l^{\text{ne}}(t)$. Other parts of contributions from the exchange interaction are compatible with Takeuchi et al.[6][7] and Ohe et al.[9].

In summary, we have derived the microscopic expressions for the charge current, the spin current, the number and the spin angular momentum of electrons in the magnetic multi layers in the presence of both electric field and spin dynamics. In particular, the analytic expressions have been derived for the equilibrium components, and the effects that the static homogeneous electric field gives to magnetic layers have been discussed.

References
[1] Mizukami S, Ando Y and Miyazaki T 2002 Phys. Rev. B 66 104413
[2] Saitoh E, Ueda M, Miyajima H and Tatará G 2006 Appl. Phys. Lett. 88 182509
[3] Tserkovnyak Y, Brataas A and Bauer G E W 2002 Phys. Rev. Lett. 88 117601
[4] Tserkovnyak Y, Brataas A and Bauer G E W 2003 Phys. Rev. B 67 140404
[5] Taniguchi T and Imamura H 2008 Mod. Phys. Lett. B 22 2909
[6] Takeuchi A, Hosono K and Tatará G 2010 Phys. Rev. B 81 144405
[7] Takeuchi A and Tatará G 2008 J. Phys. Soc. Jpn. 77 074701
[8] Rikitake Y and Imamura H 2005 Phys. Rev. B 72 033308
[9] Ohe J, Takeuchi A and Tatará G 2007 Phys. Rev. Lett. 99 266603