Dissecting the $H_0$ and $S_8$ tensions with Planck + BAO + supernova type Ia in multi-parameter cosmologies

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The mismatch between the locally measured expansion rate of the universe and the one inferred from observations of the cosmic microwave background (CMB) assuming the canonical ΛCDM model has become the new cornerstone of modern cosmology, and many new-physics set ups are rising to the challenge. Concomitant with the so-called $H_0$ problem, there is evidence of a growing tension between the CMB-preferred value and the local determination of the weighted amplitude of matter fluctuations $S_8$. It would be appealing and compelling if both the $H_0$ and $S_8$ tensions were resolved at once, but as yet none of the proposed new-physics models have done so to a satisfactory degree. Herein, we adopt a systematic approach to investigate the possible interconnection among the free parameters in several classes of models that typify the main theoretical frameworks tackling the tensions on the universe expansion rate and the clustering of matter. Our calculations are carried out using the publicly available Boltzmann solver CAMB in combination with the sampler Cosmomc. We show that even after combining the leading classes of models sampling modifications of both the early and late-time universe a simultaneous solution to the $H_0$ and $S_8$ tensions remains elusive.

I. INTRODUCTION

The standard Λ-cold dark matter (ΛCDM) cosmological model provides an exceptional description of a wide range of astrophysical and astronomical observations [1]. The parameters governing the ΛCDM cosmology have been constrained with unprecedented accuracy using measurements of galaxy clusters [2–4], weak lensing [5, 6], supernovae type Ia [7], and anisotropies in the cosmic microwave background (CMB) temperature and polarization fields [8, 9]. However, the enhanced precision of the various steps in the local distance-ladder measurements of the Hubble constant, $H_0 \equiv 100~h$ km/s/Mpc, have recently opened a crack in the ΛCDM model. Indeed, a combination of the latest Supernovae $H_0$ for the Equation of State (SH0ES) measurements with constraints of medium-to-high redshift probes have revealed a 4.2σ discrepancy between the ΛCDM inferred $H_0 = 67.27 \pm 0.60$ km/s/Mpc using data from the Planck satellite [8, 9] and the local measured value $H_0 = 73.2 \pm 1.3$ km/s/Mpc [10, 11]. Moreover, these measurements are supported by other early and late time observables, as shown in [12–16] and references therein.

Adding fuel to fire, the ΛCDM inferred value of the amplitude of mass fluctuations $σ_8$ has been consistently higher than the one measured in gravitational lensing [5, 6]. This leads to a tension that is quantified in terms of the $S_8 = σ_8 \sqrt{Ω_m}/0.3$ parameter, where $Ω_m$ is the present day value of the nonrelativistic matter density; see e.g. [17]. Strictly speaking, on the assumption of ΛCDM the Planck Collaboration estimated $S_8 = 0.834 \pm 0.016$, which is in more than 3σ tension with the result reported by KiDS-1000: $S_8 = 0.766^{+0.027}_{-0.029}$ [6]. The tension becomes 3.4σ if we consider a combination of BOSS and KV450: $S_8 = 0.728 \pm 0.026$ [18]. However, some data sets point to higher values of $S_8$, e.g. KiDS-450+GAMA for which $S_8 = 0.800^{+0.029}_{-0.027}$ [19] or HSC SSP finding $S_8 = 0.804^{+0.032}_{-0.029}$ [20].

Models addressing the $H_0$ tension either reduce the size of the sound horizon at recombination modifying the expansion rate in the early-universe, or else shift the matter-dark energy equality to earlier times than it otherwise would in ΛCDM with new physics in the post-recombination universe. Then, to keep the locations of the peaks in the CMB angular power spectrum fixed, $H_0$ increases diminishing the tension. Models addressing the $S_8$ tension are either based on late-universe physics processes that yield a suppression of the linear matter power spectrum, or else decrease the CMB-predicted value of $Ω_m$. It would be appealing and compelling if both the $H_0$ and $σ_8$ tensions were resolved simultaneously, but as yet none of the extant new physics models on this front have done so to a satisfactory degree [21].

In the recent $H_0$ Olympics of [22], spanning both early- and late-time modifications of the universe expansion rate, the “gold medal” for the best scenario has been given to a varying effective electron mass in a curved universe [23]. However, the so-called “interacting dark
energy - dark matter (IDE) models” did not participate in this competition. It has long been suspected that IDE is a compelling framework settling several cosmological issues (see e.g., [24–31]) and recently it gained significant attention for ameliorating the $H_0$ and $S_8$ tensions (see e.g. [32–48]). In this paper we adopt a systematic approach to study the possible interconnection among the free parameters in 24 combinations that typify the various theoretical frameworks tackling the $H_0$ and $S_8$ tensions, including both early- and late-time modifications of the universe expansion rate, as well as IDE and curved-space models. Hence, the results presented herein are complementary to those reported in [22].

The layout of the paper is as follows. We begin in Sec. II by introducing the classes of cosmological models to be explored and discussing the relevant phenomenology. In Sec. III we first describe the observational data sets used in our study together with the method of data analysis and the priors imposed on the cosmological parameters. After that, for each class of models, we use the Boltzmann solver CAMB [49] in combination with CosmoMC [50, 51] to establish which regions of parameter space are empirically viable to resolve the $H_0$ and $S_8$ tensions. Armed with our findings, in Sec. IV we investigate the crosscorrelation between parameters. The paper wraps up with some conclusions in Sec. V.

II. ΛCDM AND BEYOND

Experiments show that the distribution of matter and radiation in the observable universe is almost homogeneous and isotropic. Thus, the evolution of the universe is well-described by the maximally-symmetric Friedmann-Lamaître-Robertson-Walker (FLRW) line element

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \phi \, d\phi^2) \right],$$

where $(t, r, \theta, \phi)$ are comoving coordinates, $a(t)$ is the cosmic scale factor, and $k = -1, 0, 1$ parametrizes the curvature of the homogeneous and isotropic spatial sections [52]. It is often convenient to express the line element as

$$ds^2 = a^2(\eta) \left\{ d\eta^2 - \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \phi \, d\phi^2) \right] \right\},$$

where $\eta = \int_0^t dt / a(t)$ is the “conformal time.” Small deviations from homogeneity and isotropy are generally modeled as perturbations over the background metric given in (2).

The cosmic expansion is driven by the first Friedmann equation for the Hubble parameter $H$,

$$H^2(a) = \frac{8\pi G}{3} \left\{ \sum_i \rho_i(a) \right\} - \frac{k}{a^2},$$

where $G$ is the gravitational constant and the sum runs over the energy densities $\rho_i$ of the various components of the cosmic fluid: dark energy (DE), CDM ($\Lambda$), baryons ($b$), photons ($\gamma$), and three flavors of one helicity state neutrinos (left-handed $\nu_l$ along with their right-handed $\bar{\nu}_R$, that we denote $\nu$ for short). Equation (3) can be rewritten as

$$H^2(z) = \frac{H_0^2}{\Omega_0} \left[ (\Omega_0 + \Omega_b)(1 + z)^3 + \Omega_\gamma (1 + z)^4 + \Omega_L (1 + z)^3 \right. \left. + \Omega_{DE} \exp \left( 3 \int_0^z \frac{1 + w}{1 + z'} dz' \right) + \frac{\rho_\nu(z)}{\rho_{\text{crit,0}}} \right],$$

where $z = a_0/a - 1$ is the redshift, $\rho_{\text{crit,0}} = 3H_0^2/(8\pi G)$ is the present day value of the critical density, and $\Omega_i = \rho_i/\rho_{\text{crit,0}}$ denotes the present-day density parameters. Throughout the article we use the subscript 0 to indicate the quantities evaluated today. Since we always refer to the present day density parameters, we omit the subscript 0 in this case. The energy densities of non-relativistic matter and radiation scale as $a^{-3}$ and $a^{-4}$ respectively, and set the scalings with $(1 + z)$. The scaling of $\Omega_{DE}$ is usually described by an “equation-of-state” parameter $w = p_{DE}/\rho_{DE}$, the ratio of the spatially-homogeneous dark energy pressure to its energy density $\rho_{DE}$. To accommodate the observed cosmic acceleration we should have $w < -1/3$. The most economic explanation for dark energy is the cosmological constant $\Lambda$, for which $w = -1$. An alternative possibility is to consider a cosmic scalar field slowly rolling to the minimum of its potential [53–55], the so-called “quintessence filed” [56]. For this class of models, $-1 < w < -1/3$ and the dark-energy density decreases as $\rho_\nu \propto a^{-3(1+w)}$. Yet another possibility is to consider “phantom dark energy” for which $w < -1$ [57]. A point worth noting at this juncture is that phantom DE models violate the dominant energy condition [58–61] a cherished notion adopted to prohibit wormholes and time machines [62]. However, it is hard to envision how wormholes and time machines could originate from phantom energy. In (4) we have left open the possibility for an arbitrary (albeit constant) value of $w$. For $\rho_\nu$, we could not write a simple scaling with $z$ because the equation-of-state parameter is not constant. The curvature density parameter is defined as $\Omega_k = -k/H_0^2$.

For the benchmark spatially-flat ΛCDM 6-parameter model, we have: $\Omega_b = 0$, $\Omega_{DE} = \Omega_\Lambda$, and $w = -1$. The Hubble constant is inferred from one of these free parameters: the angular size of the sound horizon at recombination $\theta_s$, which is given by the ratio of the comoving sound horizon to the comoving angular diameter distance to last-scattering surface

$$\theta_s = \frac{r_s(z_{LS})}{D_M(z_{LS})},$$

The comoving linear size of the sound horizon and the comoving angular diameter distance are linked to the
expansion history of the universe via
\[ r_s(z) = \int_0^\infty \frac{c_s(z')}{H(z')} \, dz' \] (6)
and
\[ D_M(z) = \int_0^z \frac{1}{H(z')} \, dz' \] (7)
respectively, with \( c_s \) the speed of sound [8]. The set of free parameters that describe the \( \Lambda \)CDM model is:
\[ P_0 = \left\{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s] \right\} , \] (8)
where \( \tau \) is the reionization optical depth, \( n_s \) is the scalar spectral index, \( A_s \) is the amplitude of the scalar primordial power spectrum, and the \( \theta_{MC} \) parameter is an approximation of \( \theta \), (which is adopted in CosmoMC [50, 51] and is based on fitting formulae given in [63]).

A class of spatially-flat extensions to the base \( \Lambda \)CDM model that can reduce the \( H_0 \) tension is based on the addition of relativistic degrees-of-freedom in the early universe. The presence of any additional light species (such as sterile neutrinos [64–67], axions [68–74], massless Goldstone bosons [75], or any other massless fields from the dark sector [76]) can be characterized by the number of “equivalent” light neutrino species
\[ N_{\text{eff}} \equiv \frac{\rho_R - \rho_\gamma}{\rho_\gamma} \] (9)
in units of the density of a single Weyl neutrino \( \rho_\nu \), where \( \rho_R \) is the total energy density in relativistic particles and \( \rho_\gamma \) is the energy density of photons [77]. For three families of massless (Standard Model) neutrinos, \( N_{\text{eff}} = 3.046 \) [78–82]. The set of free parameters describing this class of models is given by
\[ P_1 = \left\{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s] \right\}, \] (10)
\[ N_{\text{eff}} \right\} . \]

Note that by adding dark relativistic degrees-of-freedom into the early universe we are increasing the expansion rate \( H(z) \), which in turn reduces \( r_s(z_{\text{LS}}) \). The accurate measurement of the location of the acoustic peaks by the Planck mission sets the value of the \( \Lambda \)CDM free parameter \( \theta_0 \), and so to maintain the ratio in (5) we must increase \( H_0 \) to decrease \( D_M(z_{\text{LS}}) \). However, adding relativistic degrees-of-freedom into the early universe also affects the damping scale \( \theta_D \) of the CMB power spectrum, with \( \theta_D/\theta_0 \propto \sqrt{H(z_{\text{LS}})} \) [83]. Therefore, while we increase \( N_{\text{eff}} \) for a faster expansion rate \( H(z_{\text{LS}}) \) at \( \theta_0 \), fixed we also increase \( \theta_D \), with the damping contributing at larger scales to reduce the power in the damping tail. The main limiting factor in constraining \( N_{\text{eff}} \) from CMB data is a degeneracy with the primordial helium fraction \( Y_p \equiv n_{\text{He}}/n_0 \). Namely, for fixed \( \Omega_b h^2 \), by increasing \( Y_p \) at the end of big bang nucleosynthesis (BBN) we decrease the number density of free electrons and increase the diffusion length. Altogether this reduces the power in the damping tail [84]. Using \( P_1 \) to accommodate CMB + BAO data and BBN observations [85, 86] the Planck Collaboration reported \( N_{\text{eff}} = 3.12^{+0.25}_{-0.26} \) at the 95% CL [8].

Herein we will take this bound as an external constraint to our numerical analysis for spatially-flat models.

Other attempts to resolve the \( H_0 \) tension involve tweaking \( \Lambda \)CDM somehow to slow down the late-time expansion rate without making radical changes to the early-time expansion rate. Within this class of models the value of \( r_s(z_{\text{LS}}) \) does not differ appreciably from that obtained assuming \( \Lambda \)CDM for the same choice of cosmological parameters. Now, a consistently lower value of \( H(z) \) at low redshifts leads to a larger value of \( D_M(z_{\text{LS}}) \), which in turn would result in a smaller value of \( \theta_0 \). Thus, we must (re-)decrease \( D_M(z_{\text{LS}}) \) to keep \( \theta_0 \) unchanged, and this can be accomplished by increasing \( H_0 \).

A straightforward extension of the standard cosmology within this class of models emerges when considering \( w < -1 \). It is easily seen in (4) that by considering \( w < -1 \) we lower the expansion rate for \( z > 0 \) with respect to the case where the DE is in the form of a cosmological constant. From now on, we denote with \( w_0 \) the equation-of-state parameter of quintessence and phantom models, which are analyzed separately. The sets of free parameters describing these classes of models are given by
\[ P_2 = \left\{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s] \right\}, \] (11)
\[ w_0 \right\} . \]
and
\[ P_3 = \left\{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s] \right\}, \] (12)
\[ w_p \right\} .

It is worth recalling that models with \( w_p < -1 \) violate the null energy density condition and are typically unstable; though with optimistic assumptions the instability time scale can be greater than the age of the universe. Quintessence models for which \( r_s(z_{\text{LS}}) \) is unmodified from \( \Lambda \)CDM exacerbate the \( H_0 \) tension. This is because for \( w \geq -1 \), the DE can only redshift faster than the cosmological constant, yielding a smaller contribution to the energy density today than it would have been in \( \Lambda \)CDM with the same value of \( H(z_{\text{LS}}) \), and so it is easily seen from (4) that \( H_0 \) must also be smaller [87, 88]. A different quintessence scenario appears if the scalar field speeds up \( H(z) \) and reduces the sound horizon during the era leading up to recombination [89–93]. As in the class of models characterized by \( N_{\text{eff}} \), these early dark energy models lead to larger \( H_0 \) values as compared to \( \Lambda \)CDM. However, early dark energy models increase the tension with large scale structure data. It is easy to detect the source that increases \( S_8 \) in this class of models, because
the early dark energy slightly suppresses the growth of perturbations during the period in which it contributes non-negligibly to the cosmic energy density. Therefore, to properly match the CMB data we must increase the CDM component to compensate for the suppression in the efficiency of perturbation growth [94].

A fourth class of models incorporates a coupling between the DE and the dark matter (DM) sectors, altering $\Lambda$CDM late-universe-based predictions [15, 32–35, 37–45, 47, 48, 95–145]. At the background level, the DM-DE coupling modifies the functional form of the continuity equation of the dark fluids

$$\dot{\rho}_c + 3H\rho_c = Q$$  \hspace{1cm} (13)

and

$$\rho_{DE} + 3H(1 + w)\rho_{DE} = -Q.$$  \hspace{1cm} (14)

where the dot denotes derivative with respect to $\eta$, $H \equiv \dot{a}/a$ is the conformal Hubble rate, and $Q$ is the interaction rate or the interaction function which characterizes the transfer of energy or and momentum between the dark sectors, and where $Q < 0$ and $Q > 0$ indicate energy transfer from DE to DM and vice versa. Although the choice of the interaction function is not unique, a classical functional form is given by

$$Q = \xi H \rho_{DE},$$  \hspace{1cm} (15)

where $\xi$ is a dimensionless coupling parameter quantifying the strength of the DM-DE interaction. Since the sign of $\xi$ could be either positive or negative, this defines two sub-classes of models driven by $\xi_+$ (for which $\xi > 0$) and $\xi_-$ (for which $\xi < 0$). The presence of the DM-DE coupling also modifies the evolution at the level of perturbations. Assuming the synchronous gauge, the evolution of the DM and DE density perturbations as well as velocity divergences have been computed in Refs. [97–99]. Following [40], we adopt adiabatic initial conditions in our Boltzmann system for all species. At this stage, it is worthwhile to note that when considering a coupling between the DE and DM sectors, the interacting system could be unstable. Indeed, DE-DM interactions suffer from gravitational instabilities if $w = -1$ [97, 146]. We circumvent the instability problem by taking $w = -0.999$. This approximation is justified because for $w \to -1$, the effect of DE perturbations is basically unnoticeable [34, 41, 100]. Therefore, the interacting system essentially captures the effect of the DM-DE coupling, while at the same time ensuring the absence of gravitational instabilities. In addition, in this case we require $\xi < 0$, in order to avoid the early-time instabilities [97–99, 146–148]. The set of free parameters describing this class of models are given by

$$\mathcal{P}_1 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \xi_-, \}. $$  \hspace{1cm} (16)

Finally, we can open Pandora’s box to construct another class of models that could resolve the $H_0$ tension. By inspection of (4) we can immediately see that the increase of the effective fractional contribution of spatial curvature to the energy budget yields a faster expansion rate. The set of free parameters describing this class of models is given by

$$\mathcal{P}_5 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \Omega_k, \}$$  \hspace{1cm} (17)

Observational data from the Planck mission point to a 3.4$\sigma$ evidence for a closed universe: $-0.095 < \Omega_k < -0.007$ at 99% CL [150–153]. If this were the case, we can immediately infer from (4) that the parameter set $\mathcal{P}_5$ would actually exacerbate the $H_0$ tension.

In what follows we consider the cosmological models that can be characterized by the 18 additional possible combinations of the extra 6 free parameters discussed above ($N_{\text{eff}}, w_q, w_p, \xi_+, \xi_-$, $\Omega_k$). In order to avoid the early-time instabilities [97–99, 146–148] we will have $\xi_-$ when $w_q > -1$, and $\xi_+$ when $w_p < -1$. The set of free parameters describing these classes of models are given by

$$\mathcal{P}_6 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], N_{\text{eff}}, w_q \},$$  \hspace{1cm} (18)

$$\mathcal{P}_7 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], N_{\text{eff}}, w_p \},$$  \hspace{1cm} (19)

$$\mathcal{P}_8 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], N_{\text{eff}}, \xi_+ \},$$  \hspace{1cm} (20)

$$\mathcal{P}_9 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], N_{\text{eff}}, \Omega_k \},$$  \hspace{1cm} (21)

$$\mathcal{P}_{10} \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], w_q, \xi_- \},$$  \hspace{1cm} (22)

$$\mathcal{P}_{11} \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], w_p, \xi_+ \},$$  \hspace{1cm} (23)

$$\mathcal{P}_{12} \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], w_q, \Omega_k \},$$  \hspace{1cm} (24)
\[ \mathcal{P}_{13} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ w_p, \Omega_k \}, \quad (25) \]
\[ \mathcal{P}_{14} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ \xi_-, \Omega_k \}, \quad (26) \]
\[ \mathcal{P}_{15} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ N_{\text{eff}}, w_p, \xi_- \}, \quad (27) \]
\[ \mathcal{P}_{16} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ N_{\text{eff}}, \Omega_k \}, \quad (28) \]
\[ \mathcal{P}_{17} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ N_{\text{eff}}, w_p, \xi_+ \}, \quad (29) \]
\[ \mathcal{P}_{18} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ N_{\text{eff}}, \xi_-, \Omega_k \}, \quad (30) \]
\[ \mathcal{P}_{19} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ N_{\text{eff}}, \xi_+, \Omega_k \}, \quad (31) \]
\[ \mathcal{P}_{20} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ w_p, \Omega_k \}, \quad (32) \]
\[ \mathcal{P}_{21} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ w_p, \xi_-, \Omega_k \}, \quad (33) \]
\[ \mathcal{P}_{22} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ N_{\text{eff}}, w_p, \xi_- \}, \quad (34) \]
and
\[ \mathcal{P}_{23} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \}
\]
\[ N_{\text{eff}}, w_p, \xi_+ \}, \quad (35) \]

Most of the combined models described in (17) to (27) have been previously discussed in the literature, see e.g. [15, 154–166]. In this work we adopt a systematic approach in which we (re)examine all of these models with the goal of establishing possible intercorrelations among the free parameters.

For extensive reviews about possible solutions of the $H_0$ and $S_8$ tensions see [13, 17, 21, 22, 167–170] and references therein.

### III. OBSERVATIONAL DATA, STATISTICAL METHODOLOGY, AND NUMERICAL ANALYSIS

We begin with a brief description of the cosmological data sets used in this work.

- **Planck 2018 CMB data**: The CMB temperature and polarization angular power spectra $plikTT-TEE+lowl+lowE$ from the Planck 2018 legacy release [8, 9].
- **BAO**: Measurements of baryon acoustic oscillations (BAO) from different galaxy surveys: 6dFGS [2], SDSS-MGS [3], and BOSS DR12 [4]. This is the same combination of BAO data considered by the Planck Collaboration in [8].
- **Pantheon**: The 1048 supernovae type Ia data points which are distributed in the redshift interval $0.01 \leq z \leq 2.3$, dubbed the Pantheon sample [7].
- **R20**: A gaussian prior on the Hubble constant in agreement with the measurement obtained by the SH0ES collaboration in [11].

We note in passing that associated to the $H_0$ tension there is the question of the roles played by the sound horizon scale and the local expansion rate as distance anchors [171]. In particular, the so-called “inverse distance ladder calibration” of Pantheon sample [7] based on the CMB-inferred sound-horizon scale as an anchor cannot be made compatible with the direct distance ladder calibration of R20 [11]. This has emerged as a redefinition of the $H_0$ tension gauging the calibration of the intrinsic supernova magnitude $M_B$ adopted in the local distance ladder $H_0$ measurement, while testing the consistency of the corresponding observed fluxes from the Pantheon catalogue with the underlying cosmological model [172, 173]. However, we note that the $M_B$ tension is only referring to the mismatch observed in SH0ES vs Planck data analyses, without considering all the other probes and cannot test modifications of the expansion rate in the early universe. For this reason, herein we center the analysis on the $H_0$ tension.

For the numerical analysis, we adopt a modified version of the well known cosmological package CosmoMC [50, 51], which is publicly available [174]. This package is equipped with a convergence diagnostic based on the Gelman-Rubin criterion [175] and includes the support for the 2018 Planck data release [9]. The flat priors on the free parameters of the classes of models analyzed herein are listed in Table I. The results of our numerical analysis are encapsulated in Tables II to XIII and Figs. 1 to 24.

A point worth noting at this juncture is that the acoustic peaks which are prominently observed in the CMB anisotropy spectra, are also visible as BAO peaks in the galaxy power spectra and carry the footprints of a new standard ruler: the sound horizon at the epoch of baryon decoupling, $r_{\text{drag}}$, when the photon drag on baryons becomes unimportant. The estimated sound horizon at the end of the baryonic-drag epoch is $r_{\text{drag}} = (137 \pm 3^{\text{stat}} \pm 2^{\text{syst}})$ Mpc [176]. This estimate is
based on data from low-redshift probes and a set of polynomial parametrizations which are almost independent of the underlying cosmology. None of the combination of classes of models analyzed herein can accommodate the $r_{\text{drag}}$ estimate of [176] at the 1σ level; see Tables II to XIII.

IV. DISCUSSION OF THE RESULTS

In this section we summarize the results obtained for the different extended cosmological classes of models, highlighting the indication for new physics beyond the $\Lambda$CDM model or the possible solution of the $H_0$ and/or $S_8$ tension. For each case, we show the constraints obtained from Planck alone (CMB), CMB + BAO + Pantheon (labeled by ‘all’), CMB + R20, and CMB +BAO + R20.

A. Planck, BAO and Pantheon

In Table II we show the results obtained for a $\Lambda$CDM model ($P_0$) and two $\omega$CDM classes of models, divided into two different regions based on the DE equation of state parameter: when the DE equation of state lies in the quintessence regime, i.e. $w_q > -1$ ($P_2$) and when the DE equation of state lies in the phantom regime, i.e. $w_p < -1$ ($P_3$). We can see that, with respect to the $\Lambda$CDM model, in the $P_2$ scenario the CMB + BAO + Pantheon dataset combination decreases the tension with the weak lensing data, but there is no indication for $w \neq -1$ and the Hubble constant tension cannot be eliminated. In the $P_3$ scenario, for CMB + BAO + Pantheon dataset combination the tension with the weak lensing data is restored with an indication of a phantom DE at more than 1σ. Additionally, we see a very mild increase in $H_0$ for this dataset compared to the recorded value of $H_0$ in the $P_0$ for the same dataset and hence the $H_0$ tension is very mildly alleviated in this case. This increment in the Hubble constant is mainly driven by the phantom DE.

In order to understand the behaviour of the parameters in these classes of models, we have displayed the one dimensional posterior distributions and the two dimensional joint contours in Figs. 1 (for $P_0$), 3 (for $P_2$), and 4 (for $P_3$).

In Table III we show the cosmological constraints for the classes of models described by $\Lambda$CDM + $N_{\text{eff}}$ model ($P_1$), IDE ($P_4$), and IDE + $N_{\text{eff}}$ ($P_8$). As already shown in the literature, extra relativistic degrees of freedom at recombination are not favored by data, and in the $P_1$ framework the Hubble tension cannot be resolved. On the contrary, $H_0$ is naturally in agreement with SH0ES in the IDE scenario, at the price of a coupling between DM and DE at more than 99% CL for Planck-data alone. The $H_0$ tension is, however, restored above 3σ when BAO and Pantheon data are included, i.e for the dataset CMB + BAO + Pantheon, and the evidence for the coupling is reduced to 1σ. The combination of the two extensions in $P_3$ leaves unaltered the previous results, because $N_{\text{eff}}$ is very well constrained and in agreement with the Standard Model expectation. The tension in the $S_8$ parameter is not addressed by any of these classes of models when BAO and Pantheon are included. In fact, for all these extensions, the $S_8$ parameter takes a higher value when compared to the classes of models characterized by $P_1$ and $P_0$ (see Table II). In a similar fashion, the one dimensional posterior distributions and the two dimensional joint contours for these classes of models are shown in Figs. 2 (for $P_1$), 5 (for $P_4$), and 9 (for $P_8$).

In Table IV we show the cosmological constraints at 68% CL for the classes of models marked out by $k\Lambda$CDM ($P_5$) and $k\omega$CDM, with the latter divided again into two different regions based on the nature of the DE equation of state: quintessence regime, i.e. $w_q > -1$ ($P_{12}$) and phantom regime, i.e. $w_p < -1$ ($P_{13}$). As already shown in the literature, a closed universe is preferred at more than 3σ by Planck-data alone [150–152], but this trade-mark increases the tension with $H_0$. The evidence for a closed universe disappears completely by the inclusion of the BAO and Pantheon data samples. The inclusion of the DE equation of state free to vary in the quintessence region, i.e. $w_q > -1$ ($P_{12}$) yields evidence for a closed universe at more than 3σ along with a $w_q > -1$, at more than 68% CL and an increased Hubble constant discrepancy with respect to the $\Lambda$CDM model, for Planck-data alone. On the contrary, if we consider a phantom DE equation of state (i.e. $w_p < -1$) free to vary in this regime ($P_{13}$), we obtain $\Omega_k < 0$ at more than 99% CL, $w_p$ is in agreement with a cosmological constant, and the Hubble constant is consistent with SH0ES, for the Planck-data alone. However, for all the cases, all these features disappear for the combined dataset CMB + BAO + Pantheon data, which perfectly restores a flat $\Lambda$CDM model. Similarly, aiming to understand the correlations between the model parameters, we again show the one dimensional posterior distributions and the two dimensional joint contours for

| Parameter | Prior |
|-----------|-------|
| $\Omega_b h^2$ | [0.005, 0.1] |
| $\Omega_c h^2$ | [0.001, 0.99] |
| 100$\theta_{\text{MC}}$ | [0.5, 10] |
| $\tau$ | [0.01, 0.8] |
| $n_s$ | [0.8, 1.2] |
| $\ln(10^{10} A_s)$ | [1.61, 3.91] |
| $N_{\text{eff}}$ | [0.05, 10] |
| $w_q$ | [-1, 1] |
| $w_p$ | [-3, -1] |
| $\xi_+$ | [0.1] |
| $\xi_-$ | [-1, 0] |
| $\Omega_k$ | [-0.3, 0.3] |

TABLE I: Flat priors on various cosmological parameters coming from several cosmological scenarios.
TABLE II: 68% CL constraints on various free and derived parameters of the parameter spaces

\[ \mathcal{P}_0 \equiv \{ \Omega_{b}h^2, \Omega_r h^2, \Omega_{MC}, \tau, n_s, \ln(10^{10} A_s) \} \]

\[ \mathcal{P}_1 \equiv \{ \Omega_{b}h^2, \Omega_r h^2, \Omega_{MC}, \tau, n_s, \ln(10^{10} A_s), w_{\phi} \} \]

\[ \mathcal{P}_8 \equiv \{ \Omega_{b}h^2, \Omega_r h^2, \Omega_{MC}, \tau, n_s, \ln(10^{10} A_s), N_{\text{eff}}, \xi \} \]

TABLE III: 68% CL constraints on various free and derived parameters of the parameter spaces

\[ \mathcal{P}_3 \equiv \{ \Omega_{b}h^2, \Omega_r h^2, 100\theta_{\text{MC}}, \tau, n_s, \ln(10^{10} A_s), N_{\text{eff}} \} \]

\[ \mathcal{P}_8 \equiv \{ \Omega_{b}h^2, \Omega_r h^2, 100\theta_{\text{MC}}, \tau, n_s, \ln(10^{10} A_s), N_{\text{eff}}, \xi \} \]

these classes of models in Figs. 6 (for \( \mathcal{P}_3 \)), 13 (for \( \mathcal{P}_{12} \)), and 14 (for \( \mathcal{P}_{13} \)).

In Table V we present the constraint on the cosmological parameters for the classes of models characterized by \( w_{\phi}\text{CDM + N}_{\text{eff}}(\mathcal{P}_6) \), \( w_{\phi}\text{IDE}(\mathcal{P}_{10}) \), and \( w_{\phi}\text{IDE + N}_{\text{eff}}(\mathcal{P}_{13}) \), where the DE equation of state is always in the quintessence regime, i.e. \( w_{\phi} > -1 \). The \( \mathcal{P}_6 \) framework does not show any deviation form the standard ΛCDM scenario. However, in the \( w_{\phi}\text{IDE} \) scenario we find some interesting results. First of all, for the Planck-data alone case, the introduction of a \( w_{\phi} \) free to vary reduces the overlap with the Hubble constant measured by SH0ES, even if this is still in agreement within 1σ. Secondly, even if \( w_{\phi} \) is consistent with a cosmological constant, the evidence for the coupling between DM and DE increases in significance. Lastly, the inclusion of the BAO and Pantheon data does not restore the usual ΛCDM model concordance. In fact, for the combined dataset CMB + BAO + Pantheon we have a coupling still at more than 99% CL, a \( w_{\phi} \neq -1 \) above 95% CL, but \( H_0 \) is still in tension with SH0ES at more than 3 standard deviations. Finally, the extended scenario \( w_{\phi}\text{IDE + N}_{\text{eff}} \) (the last two columns of Table V) does not modify the previous findings about the free parameters, because \( N_{\text{eff}} \) is completely consistent with the standard value of 3.046. However, we have slightly larger error bars for an increased volume of the
TABLE IV: 68% CL constraints on various free and derived parameters of the parameter spaces \( P_5 \equiv \{\Omega_m h^2, \Omega_b h^2, 100\theta_{MC}, \tau, n_s, \ln(10^{10}A_s), \Omega_k\} \), \( P_{12} \equiv \{\Omega_m h^2, \Omega_b h^2, 100\theta_{MC}, \tau, n_s, \ln(10^{10}A_s), w, \Omega_k\} \) and \( P_{13} \equiv \{\Omega_m h^2, \Omega_b h^2, 100\theta_{MC}, \tau, n_s, \ln(10^{10}A_s), w, \Omega_k\} \).

TABLE V: 68% CL constraints on various free and derived parameters of the parameter spaces \( P_6 \equiv \{\Omega_m h^2, \Omega_b h^2, 100\theta_{MC}, \tau, n_s, \ln(10^{10}A_s), N_{\text{eff}}, w_q\} \), \( P_{10} \equiv \{\Omega_m h^2, \Omega_b h^2, 100\theta_{MC}, \tau, n_s, \ln(10^{10}A_s), w_q, \xi\} \) and \( P_{15} \equiv \{\Omega_m h^2, \Omega_b h^2, 100\theta_{MC}, \tau, n_s, \ln(10^{10}A_s), N_{\text{eff}}, w_q, \xi\} \).

parameter space. The one dimensional posterior distributions and the two dimensional joint contours for these classes of models are shown in Figs. 7 (for \( P_6 \)), 11 (for \( P_{10} \)), and 16 (for \( P_{15} \)).

Complementary, in Table VI we present the constraint on the cosmological parameters for the classes of models specified by \( w_q \text{CDM} + N_{\text{eff}} \) (\( P_7 \)), \( w_q \text{IDE} \) (\( P_{11} \)) and \( w_q \text{IDE} + N_{\text{eff}} \) (\( P_{16} \)), where the DE equation of state is always in the phantom regime, i.e. \( w_q < -1 \). In this case, for the scenario \( P_7 \) we find evidence for a phantom \( w_q < -1 \) at more than 99% CL when considering Planck-data alone. In addition, we find that the \( S_8 \) parameter is in agreement with the weak lensing experiments within 1\(\sigma\), and the H0 tension is certainly alleviated within the 95% CL. However, even though the evidence for \( w_q < -1 \) at more than 1\(\sigma\) still persists when the full combination CMB + BAO + Pantheon is considered, the \( S_8 \) and \( S_9 \) tensions strike back. For the \( P_{11} \) scenario, we do not find any indication for a coupling \( \xi \neq 0 \), and our analysis confirms the same salient features present in the \( P_7 \) classes of
models for the Planck data only. On the other hand, when the combined dataset CMB + BAO + Pantheon is considered, \( w_p < -1 \) is still preferred at more than 1\( \sigma \) and the \( S_8 \) tension is alleviated. However, in this case the \( H_0 \) tension is only reduced down to 3.2\( \sigma \). The inclusion of the \( N_{\text{eff}} \) parameter free to vary in the last scenario \( \mathcal{P}_{16} \) does not modify the previous findings, because \( N_{\text{eff}} \) is in complete agreement with the Standard Model value. The one dimensional posterior distributions and the two dimensional joint contours for these classes of models are shown in Figs. 8 (for \( \mathcal{P}_7 \)), 12 (for \( \mathcal{P}_{11} \), and 17 (for \( \mathcal{P}_{16} \)).

In Table VII we show the results for the classes of models heading kLCDM + \( N_{\text{eff}} \) (\( \mathcal{P}_9 \)), kDE (\( \mathcal{P}_{14} \)), and kDE + \( N_{\text{eff}} \) (\( \mathcal{P}_{19} \)). For the Planck-data alone, evidence for a closed universe at more than 95% CL is clearly visible for all three scenarios. However, such an evidence disappears when BAO and Pantheon data samples are included. Both the \( H_0 \) and \( S_8 \) tensions are exacerbated.

### TABLE VII: 68% CL constraints on various free and derived parameters of the parameter spaces

| Model | \( \mathcal{P}_7 \) | \( \mathcal{P}_{11} \) | \( \mathcal{P}_{16} \) |
|-------|----------------------|----------------------|----------------------|
|       | CMB all              | CMB all              | CMB all              |
| \( \Omega_b h^2 \) | \( 0.02227 \pm 0.00022 \) | \( 0.02239 \pm 0.00015 \) | \( 0.02227 \pm 0.00022 \) |
| \( \Omega_c h^2 \) | \( 0.1178 \pm 0.0029 \) | \( 0.1183 \pm 0.0030 \) | \( 0.1178 \pm 0.0029 \) |
| \( 100\theta_{MC} \) | \( 1.04118 \pm 0.00044 \) | \( 1.04113 \pm 0.00044 \) | \( 1.04026 \pm 0.00065 \) |
| \( \tau \) | \( 0.0528 \pm 0.0080 \) | \( 0.0540 \pm 0.0075 \) | \( 0.0539 \pm 0.0078 \) |
| \( n_s \) | \( 0.9597 \pm 0.0083 \) | \( 0.9616 \pm 0.0074 \) | \( 0.9654 \pm 0.0043 \) |
| \( \ln(10^{10}A_s) \) | \( 3.035 \pm 0.019 \) | \( 3.039 \pm 0.018 \) | \( 3.043 \pm 0.016 \) |
| \( \xi \) | \( 0 \) | \( 0 \) | \( 0 \) |
| \( N_{\text{eff}} \) | \( \tau \) | \( 0.014 \) | \( 0.014 \) |
| \( w_p \) | \( -1.65^{+0.19}_{-0.34} \) | \( -1.64^{+0.038}_{-0.016} \) | \( -1.59^{+0.19}_{-0.34} \) |
| \( \Omega_m h^2 \) | \( 0.192^{+0.107}_{-0.051} \) | \( 0.3056 \pm 0.072 \) | \( 0.220^{+0.025}_{-0.066} \) |
| \( \sigma_8 \) | \( 0.891^{+0.091}_{-0.045} \) | \( 0.818 \pm 0.013 \) | \( 0.881 \pm 0.085 \) |
| \( H_0 [\text{km/s/Mpc}] \) | \( > 83.1 \) | \( 86.0 \pm 1.2 \) | \( > 80.9 \) |
| \( S_8 \) | \( 0.771^{+0.024}_{-0.035} \) | \( 0.826 \pm 0.014 \) | \( 0.742 \pm 0.039 \) |
| \( r_{\text{drag}} [\text{Mpc}] \) | \( 148.6 \pm 1.9 \) | \( 148.2 \pm 1.9 \) | \( 147.0 \pm 0.29 \) |
when considering just the Planck data sample, but $N_{\text{eff}}$ is always found to be in agreement with the Standard Model value. In addition, when considering only the CMB-data sample for $P_{14}$ and $P_{19}$, we find evidence for a coupling of DE and DM through an upper limit for $\xi$. However, for the combined dataset CMB + BAO + Pantheon, $\xi \neq 0$ remains true around 1 standard deviation for the $P_{19}$ (kIDE + $N_{\text{eff}}$). Further, we see that for the combined dataset the classes of models typify by $P_{14}$ and $P_{19}$ do not offer any alleviation of the $S_8$ tension, rather the tension in $S_8$ increases significantly. These classes of models, however, lead to a mild alleviation of the $H_0$ tension. For the $P_{14}$ and $P_{19}$ classes of models, the $H_0$ tension is reduced down to 3.1$\sigma$ and 2.8$\sigma$, for the combined CMB +
Table X: 68% CL constraints on various free and derived parameters of the parameter spaces $P_3 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \nu, \ln(10^{10} A_s), w_p, \xi, \Omega_\delta \}$, $P_4 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \nu, \ln(10^{10} A_s), w_p, \xi, \tau, \Omega_\delta \}$, $P_11 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \nu, \ln(10^{10} A_s), w_p, \xi, \tau, \Omega_\delta \}$ and $P_{13} \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \nu, \ln(10^{10} A_s), w_p, \Omega_\delta \}$ for CMB + R20 dataset only.

| Model | $P_3$ | $P_4$ | $P_{11}$ | $P_{13}$ |
|-------|-------|-------|----------|----------|
|       | CMB + R20 | CMB + R20 | CMB + R20 | CMB + R20 |
| $\Omega_b h^2$ | 0.02237 ± 0.00015 | 0.02237 ± 0.00014 | 0.02240 ± 0.00015 | 0.02237 ± 0.00015 |
| $\Omega_c h^2$ | 0.1201 ± 0.0013 | 0.039^{+0.014}_{-0.022} | < 0.0276 | 0.133^{+0.006}_{-0.002} |
| $100\theta_{MC}$ | 1.04090 ± 0.00031 | 1.0463 ± 0.0017 | 1.0479^{+0.020}_{-0.011} | 1.04021^{+0.0091}_{-0.0051} |
| $\tau$ | 0.0540 ± 0.0080 | 0.0533 ± 0.0080 | 0.0539 ± 0.0077 | 0.0536 ± 0.0077 |
| $\nu$ | 0.9650 ± 0.0044 | 0.9653 ± 0.0042 | 0.9658 ± 0.0042 | 0.9647 ± 0.0043 |
| $\ln(10^{10} A_s)$ | 3.044 ± 0.016 | 3.042 ± 0.016 | 3.043 ± 0.016 | 3.043 ± 0.016 |
| $w$ | -1.200 ± 0.048 | -0.999 | -0.939^{+0.020}_{-0.054} | -1.240 ± 0.055 |
| $\xi$ | 0 | -0.60^{+0.11}_{-0.16} | -0.71^{+0.05}_{-0.16} | < 0.162 |
| $\Omega_\delta$ | 0 | 0 | 0 | 0 |

| $\Omega_{\text{tot}}$ | 0.2660 ± 0.0097 | 0.117^{+0.033}_{-0.050} | 0.085^{+0.013}_{-0.040} | 0.291^{+0.015}_{-0.023} |
| $\sigma_8$ | 0.867 ± 0.016 | 2.3^{+0.4}_{-1.1} | 3.2^{+0.0}_{-1.5} | 0.794^{+0.062}_{-0.041} |
| $H_0$[km/s/Mpc] | 73.4 ± 1.3 | 73.4^{+1.1}_{-1.4} | 72.8^{+0.9}_{-1.1} | 73.3 ± 1.3 |
| $S_8$ | 0.816 ± 0.015 | 1.34^{+0.15}_{-0.33} | 1.58 ± 0.28 | 0.780^{+0.032}_{-0.026} |
| $r_{\text{drag}}$ [Mpc] | 147.06 ± 0.29 | 147.07 ± 0.29 | 147.10 ± 0.29 | 147.05 ± 0.29 |

Table XI: 68% CL constraints on various free and derived parameters of the parameter spaces $P_5 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \nu, \ln(10^{10} A_s), w_p, \xi, \Omega_\delta \}$, $P_6 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \nu, \ln(10^{10} A_s), w_p, \xi, \nu, \Omega_\delta \}$ and $P_{18} \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \nu, \ln(10^{10} A_s), N_{\text{eff}}, w_p, \Omega_\delta \}$ when R20 is considered.

| Model | $P_5$ | $P_6$ | $P_{18}$ |
|-------|-------|-------|----------|
|       | CMB + R20 | CMB + BAO + R20 | CMB + BAO + R20 |
| $\Omega_b h^2$ | 0.2224 ± 0.00022 | 0.02236 ± 0.00020 | 0.02260 ± 0.00025 |
| $\Omega_c h^2$ | 0.1179 ± 0.0030 | 0.1207 ± 0.0028 | 0.1180 ± 0.0031 |
| $100\theta_{MC}$ | 1.04118 ± 0.00044 | 1.04085 ± 0.00041 | 1.04119 ± 0.00045 |
| $\tau$ | 0.0532 ± 0.0079 | 0.0539 ± 0.0076 | 0.0528 ± 0.0076 |
| $\nu$ | 0.9591 ± 0.0084 | 0.9643 ± 0.0079 | 0.9620 ± 0.0073 |
| $\ln(10^{10} A_s)$ | 3.036 ± 0.019 | 3.045 ± 0.018 | 3.037 ± 0.018 |
| $w$ | -1.238 ± 0.070 | -1.200 ± 0.048 | -0.90^{+0.15}_{-0.25} |
| $N_{\text{eff}}$ | 2.89 ± 0.19 | 3.06 ± 0.18 | 3.22 ± 0.16 |
| $\xi$ | 0 | -0.65^{+0.18}_{-0.09} | -0.29^{+0.10}_{-0.11} |
| $\Omega_\delta$ | 0 | 0 | 0 |
| $\Omega_{\text{tot}}$ | 0.262^{+0.010}_{-0.002} | 0.2818 ± 0.0090 | 0.264^{+0.010}_{-0.002} |
| $\sigma_8$ | 0.869 ± 0.016 | 0.853 ± 0.016 | 0.954 ± 0.035 |
| $H_0$[km/s/Mpc] | 73.3 ± 1.3 | 71.4 ± 1.1 | 73.2 ± 1.3 |
| $S_8$ | 0.812 ± 0.015 | 0.826 ± 0.013 | 0.895 ± 0.034 |
| $r_{\text{drag}}$ [Mpc] | 148.6 ± 1.9 | 146.9 ± 1.8 | 147.5 ± 2.0 |

BAO + Pantheon dataset. The one dimensional posterior distributions and the two dimensional joint contours for these classes of models are shown in Figs. 10 (for $P_9$), 15 (for $P_{14}$) and 20 (for $P_{19}$).

In Table VIII we show the results of the classes of models featuring kw$_b$CDM + $N_{\text{eff}}$ ($P_{17}$), kw$_b$IDE ($P_{20}$) and the kw$_b$IDE + $N_{\text{eff}}$ ($P_{22}$), where the DE equation of state has been freely varying in the quintessence regime, i.e. $w_q > -1$. For the CMB dataset, we find for all the classes of models evidence for a closed universe at more than 95% CL. However, this evidence disappears when we combine BAO and Pantheon with CMB data. Focusing on the tensions on $H_0$ and $S_8$ parameters, we find that for both CMB and CMB + BAO + Pantheon, the $H_0$ and $S_8$ tensions cannot be resolved. In fact, for CMB alone, both the $H_0$ and $S_8$ tensions significantly
TABLE XII: 68% CL constraints on various free and derived parameters of the parameter spaces $\mathcal{P}_{16} = \{\Omega_0, h^2, \Omega_{0c}, \tau, n_s, \ln[10^{10}A_s], N_{eff}, w, \xi, \Omega_b\}$, $\mathcal{P}_{21} = \{\Omega_0, h^2, \Omega_{0c}, \tau, n_s, \ln[10^{10}A_s], w, \xi, \Omega_b, \Omega\}$ and $\mathcal{P}_{23} = \{\Omega_0, h^2, \Omega_{0c}, \tau, n_s, \ln[10^{10}A_s], N_{eff}, w, \xi, \Omega_b\}$ when R0 is considered.

TABLE XIII: 68% CL constraints on various free and derived parameters of the parameter spaces $\mathcal{P}_{15} = \{\Omega_0, h^2, \Omega_{0c}, \tau, n_s, \ln[10^{10}A_s], N_{eff}, \omega_p, \xi, \Omega_b\}$, $\mathcal{P}_{19} = \{\Omega_0, h^2, \Omega_{0c}, \tau, n_s, \ln[10^{10}A_s], N_{eff}, \xi, \Omega_b\}$ and $\mathcal{P}_{22} = \{\Omega_0, h^2, \Omega_{0c}, \tau, n_s, \ln[10^{10}A_s], N_{eff}, \omega_p, \xi, \Omega_b\}$ when R0 is considered.

increase. However, for CMB + BAO + Pantheon, we see that for $\mathcal{P}_{17}, \mathcal{P}_{20}$ and $\mathcal{P}_{22}$ classes of models, the tension on $H_0$ is reduced down to 3.4$\sigma$, 3.1$\sigma$ and slightly below 3$\sigma$ respectively. Similar to the other cases, the graphical extraction for these scenarios are shown in Figs. 18 (for $\mathcal{P}_{17}$), 21 (for $\mathcal{P}_{20}$), and 23 (for $\mathcal{P}_{22}$).

Finally, in Table IX we compare the classes of models characterized by $\omega_p, cDM + N_{eff} (\mathcal{P}_{18}), \omega_p, IDE (\mathcal{P}_{21})$, and $\omega_p, IDE + N_{eff} (\mathcal{P}_{23})$, where the DE equation of state is always in the phantom regime, i.e. $\omega_p < -1$. Similar to the classes of models summarized in Table VIII, for all the three extended cases, Planck alone suggests an indication for a closed universe and this evidence goes away when BAO and Pantheon are added to CMB, that means for the combined analysis CMB + BAO + Pantheon. However, due to the phantom behaviour of the DE, the Hubble tension is solved within 1$\sigma$, even if $\omega_p$ is consistent with a cosmological constant. Interestingly,
for the CMB + BAO + Pantheon combination and the $P_{21}$ and $P_{23}$ cosmological models, while the agreement with a flat universe is restored, an indication for a $w_p < -1$ appears at more than 68% CL, and at the same time both the $S_8$ and $H_0$ tension are reduced. The one dimensional posterior distributions and the two dimensional joint contours for these models are shown in Figs. 19 (for $P_{16}$), 22 (for $P_{21}$), and 24 (for $P_{23}$).

B. Including the R20 prior

In this section we study the effects of adding a gaussian prior R20 [11] on the classes of models (described in Table II – Table VIII) which alleviate the Hubble tension below 3$\sigma$ for Planck or CMB + BAO + Pantheon datasets. To make this selection we adopt a back-of-the-envelope definition of the $H_0$ tension,

$$T = (x_1 - x_2)/\sqrt{(\text{err } x_1)^2 + (\text{err } x_2)^2},$$

(36)

where $x_1 \pm \text{err } x_1$ is the R20 $H_0$ measurement and $x_2 \pm \text{err } x_2$ is the corresponding $H_0$ prediction from model $P_i$, with $i = 0, \cdots, 23$. To avoid double counting, when using the R20 prior we do not consider the Pantheon catalog.

For the case of study, the data samples are divided into two subgroups: CMB + R20 and CMB + BAO + R20. The results are shown in Tables X to XIII. We can see that in all the selected cases the $H_0$ tension is reduced down to 1$\sigma$, as expected when a gaussian prior is included and datasets are not in strong tension. In particular, the agreement in the CMB + R20 analyses takes place at the price of a phantom DE ($P_3$ and $P_{7,11,16,18}$), a coupling for IDE ($P_4$ and $P_{8,15}$), a quintessence DE interacting with DM ($P_{10}$), and a phantom closed scenario ($P_{13}$ and the extended $P_{21,23}$). Alternatively, the agreement in the CMB + BAO + R20 analyses takes place at the price of a phantom DE ($P_3$), an IDE framework with additional dark radiation ($P_4$ and $P_{19}$), an interacting quintessence DE with DM in presence of additional dark radiation ($P_{15}$ and $P_{22}$), a phantom DE with additional dark radiation ($P_{16}$ and $P_{23}$), and a phantom closed universe ($P_{18}$). However, only the class of models characterized by $P_{16}$ and $P_{23}$ can resolve the $H_0$ tension at 1$\sigma$ level, while relaxing also the $S_8$ and the $r_{\text{drag}}$ tensions for the CMB + BAO + R20 combination.

In closing, we note that the phantom DE can address the $H_0$ problem, but does not solve the $M_8$ tension [172, 173]. On the contrary, the IDE classes of models can solve simultaneously the $H_0$ and $M_8$ tensions [145], but it is not supported by a full dataset combination.

V. CONCLUSIONS

We have investigated the possible interconnection among the free parameters in several classes of cosmological models that typify the main theoretical frameworks tackling the tensions on the universe expansion rate and the clustering of matter. This meta-analysis features interesting results on the global cosmological picture which can be summarized as follows:

- The estimate of $r_{\text{drag}}$ from low-redshift probes [168, 176] poses a challenge for beyond $\Lambda$CDM models trying to address the Hubble tension. Since the baryonic-drag epoch takes place at a somewhat lower redshift than recombination, by comparing the $\Lambda$CDM value of $r_{\text{drag}}$ in Table II with the estimate of Ref. [176] we can conclude that in order to accommodate $r_{\text{drag}} = (137 \pm 3_{\text{stat}} \pm 2_{\text{sys}})$ Mpc we need a modification of the sound horizon at recombination. Actually, for $\Lambda$CDM the two standard rulers are related according to $r_{\text{drag}} \sim 1.0184 r_0$, and the proportionality factor is essentially the same in all analyzed classes of models introducing modifications in the expansion rate pre-recombination. In our study we have considered the latest Planck 2018 data sample which contains both temperature and polarization measurements, together with a new optical depth estimate which strongly correlates with $N_{\text{eff}}$ keeping its value near 3.046, and thereby the sound horizon at the epoch of baryon decoupling near the $\Lambda$CDM prediction, $r_{\text{drag}} = 147.24 \pm 0.24$ Mpc. Altogether our conclusion points towards the need of new-physics at $z_{\text{drag}} \lesssim z \lesssim z_*, or else some unaccounted systematic effects are at play. Note that our conclusion is complementary to the statements presented in [167, 177] because our study includes combination of classes of models modifying both the early and late-time expansion rate of the universe. It should be noticed here that models involving only modifications pre-recombination of early universe physics alone are disfavored [178].

- For considerations of CMB + BAO + Pantheon data, string-inspired models with coupling between the DE and DM sectors characterized by $P_{10}$ tend to fall short of fully resolving the $H_0$ tension [128]. The addition of extra-relativistic degrees of freedom (e.g., by considering 3 right-handed Dirac neutrinos ubiquitous in D-brane constructs [129, 135], or thermal axions [68–74], or sterile neutrinos [64–67], or any other light species) tend to exacerbate the tension; see $P_{15}$ in Table V. However, when considering Planck data and the R20 prior both, the scenarios $P_{10}$ and $P_{15}$ can resolve the $H_0$ tension at the 1$\sigma$ level; see Tables X and XIII. One would expect that the transfer from DM to DE (a.k.a. fading DM) could ameliorate the $S_8$ tension. However, this happens because of the larger error bars instead of a stronger overlap. In particular, for the $P_{15}$ scenario in Table XIII, $N_{\text{eff}}$ saturates the Planck limit. Since $N_{\text{eff}}$ is correlated with $\Omega_m$, the effect of a non-negligible dark radiation shifts the $S_8$ parameter towards higher values, even if with large errors.
FIG. 1: One dimensional posterior distributions and two dimensional joint contours for the parameter space $P_0 \equiv \{\Omega_m h^2, \Omega_l h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10}A_s]\}$ for CMB alone and CMB + BAO + Pantheon (referred to as ‘all’) dataset. The $H_0$ and $r_{\text{drag}}$ parameters are respectively measured in [km/s/Mpc] and [Mpc] in all the triangular plots of this work.

- Frameworks featuring phantom dark energy characterize the classes of models with a potential to accommodate simultaneously the $H_0$ and $S_8$ local measurements (but not the $M_B$ tension [172]). Classes of models with a transfer of energy from DE to DM (e.g. $P_{16}$) keep the value of $N_{\text{eff}}$ consistent with the Standard Model expectation of 3.046, and can resolve the $S_8$ tension while ameliorating the $H_0$ tension, see Table VI and Fig. 17. Notably, the IDE model also accommodates the $M_B$ tension [145]. When including the R20 prior the value of $N_{\text{eff}}$ saturates the Planck limit, but the $H_0$ is resolved at the 1$\sigma$ level and $S_8$ remains consistent with local measurements; see Table XII.

- The latest observations of the Planck satellite have confirmed the presence of an enhanced lensing amplitude in CMB power spectra compared to that predicted in the standard $\Lambda$CDM model. It was noted in [151] that a closed universe can provide a physical explanation for this effect, with the 2018 Planck CMB spectra preferring a positive curvature at more than 99% CL [150–153]. Altogether this motivated our consideration of $\Omega_k$ as a free parameter in the likelihood analysis. Scenarios favoring a closed universe also favor a smaller value of the expansion rate than the $H_0$ measurement by SH0ES, see e.g. Tables VII and VIII. An exception is the class of models featuring extra-relativistic degrees of freedom in the early universe and a phantom DE, which is described by $P_{23}$, and can simultaneously ameliorate the $H_0$ and $S_8$ tensions (i.e. a phantom closed model [179, 180]).

In summary, the $H_0$ and $S_8$ tensions present a daunting challenge. In this paper we have collected some of the best insights to extend the standard $\Lambda$CDM model and studied the interconnections among free parameters of
FIG. 2: One dimensional posterior distributions and two dimensional joint contours for the parameter space $\mathcal{P}_1 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_M, \tau, n_s, \ln[10^{10} A_s], N_{\text{eff}}\}$ for CMB alone and CMB + BAO + Pantheon (referred to as ‘all’) dataset.

these classes of models. So far, all these insights have drawbacks and herein we have shown that the extended multi-parameter cosmologies could only help to narrow down (though not fully eliminate) the tensions. It is crystal-clear that to unlock Pandora’s box a coordinated effort involving theory, interpretation, and data analysis would be needed to exploit the large data sets to be collected by the next-generation experiments [181].

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FIG. 3: One dimensional posterior distributions and two dimensional joint contours for the parameter space $P_2 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], w_q\}$ for CMB alone and CMB + BAO + Pantheon (referred to as ‘all’ dataset).
FIG. 4: One dimensional posterior distributions and two dimensional joint contours for the parameter space $\mathcal{P}_3 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], w_p\}$ for CMB alone, CMB + R20 and CMB + BAO + Pantheon (referred to as ‘all’) dataset.
FIG. 5: One dimensional posterior distributions and two dimensional joint contours for the parameter space $\mathcal{P}_4 \equiv \{\Omega_bh^2, \Omega_ch^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10}A_s], \xi, \ldots\}$ for CMB alone, CMB + R20 and CMB + BAO + Pantheon (referred to as ‘all’) dataset.
FIG. 6: One dimensional posterior distributions and two dimensional joint contours for the parameter space $\mathcal{P}_5 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \Omega_m\}$ for CMB alone, and CMB + BAO + Pantheon (referred to as ‘all’) dataset.
FIG. 7: One dimensional posterior distributions and two dimensional joint contours for the parameter space $\mathcal{P}_6 \equiv \{\Omega_0 h^2, \Omega_L h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], N_{eff}, \bar{w}_q\}$ for CMB alone and CMB + BAO + Pantheon (referred to as ‘all’) dataset.
FIG. 8: One dimensional posterior distributions and two dimensional joint contours for the parameter space $\mathcal{P}_Y \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10}A_s], N_{eff}, w_p\}$ for CMB alone, CMB + R20, CMB + BAO + Pantheon (referred to as ‘all’), and CMB + BAO + R20 datasets.
FIG. 9: One dimensional posterior distributions and two dimensional joint contours for the parameter space $\mathcal{P}_8 \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10}A_s], N_{eff}, \xi \}$ for CMB alone, CMB + R20, CMB + BAO + Pantheon (referred to as ‘all’), and CMB + BAO + R20 datasets.
FIG. 10: One dimensional posterior distributions and two dimensional joint contours for the parameter space $P_\gamma \equiv \{\Omega_B h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10}A_s], N_{eff}, \Omega_k\}$ for CMB alone and CMB + BAO + Pantheon (referred to as 'all') dataset.
FIG. 11: One dimensional posterior distributions and two dimensional joint contours for the parameter space $P_{10} ≡ \{ \Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], w_q, \xi \}$ for CMB alone, CMB + R20 and CMB + BAO + Pantheon (referred to as ‘all’) dataset.
FIG. 12: One dimensional posterior distributions and two dimensional joint contours for the parameter space $P_{11} \equiv \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10} A_s], w_p, \xi_+\}$ for CMB alone, CMB + R20 and CMB + BAO + Pantheon (referred to as 'all') dataset.
FIG. 13: One dimensional posterior distributions and two dimensional joint contours for the parameter space \( P_{12} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln(10^{10} A_s), w_q, \Omega_k \} \) for CMB alone and CMB + BAO + Pantheon (referred to as 'all') dataset.
FIG. 14: One dimensional posterior distributions and two dimensional joint contours for the parameter space $\mathcal{P}_{13} = \{\Omega_b h^2, \Omega_c h^2, \Omega_k, \Omega_{\text{MC}}, \theta, n_s, \ln[10^{10} A_s], w_p, \Omega_{\text{BAO+Pantheon}}\}$ for CMB alone, CMB + R20 and CMB + BAO + Pantheon (referred to as ‘all’) dataset.
FIG. 15: One dimensional posterior distributions and two dimensional joint contours for the parameter space $P_{14} = \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], \xi, \Omega_k \}$ for CMB alone, and CMB + BAO + Pantheon (referred to as ‘all’) dataset.
FIG. 16: One dimensional posterior distributions and two dimensional joint contours for the parameter space $P_{15} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10}A_s], N_{eff}, w_q, \xi \}$ for CMB alone, CMB + R20, CMB + BAO + Pantheon (referred to as ‘all’), and CMB + BAO + R20 datasets.
FIG. 17: One dimensional posterior distributions and two dimensional joint contours for the parameter space \( P_{16} \equiv \{ \Omega_b h^2, \Omega_c h^2, 100 \theta_{MC}, \tau, n_s, \ln[10^{10} A_s], N_{eff}, w_p, \xi \} \) for CMB alone, CMB + R20, CMB + BAO + Pantheon (referred to as ‘all’), and CMB + BAO + R20 datasets.
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