Chapter 17

$U(2)$ and Minimal Flavour Violation in Supersymmetry

J. Jones Pérez

Abstract

In SUSY, the MFV framework is usually called upon in order to ameliorate the New Physics contribution to FCNC. However, this framework, based on a $U(3)^3$ flavour symmetry, is insufficient to solve current tensions in $\Delta F = 2$ processes related to CP Violation. In this work, we analyze the consequences of reducing the symmetry down to a $U(2)^3$ acting on the two lighter generations. We shall outline the $U(2)^3$ framework, and show how it can resolve the current tension between $K^0 \to \bar{K}^0$ and $B^0 \to \bar{B}^0$ mixing, predicting at the same time a larger phase in $B^0_s \to \bar{B}^0_s$ mixing.

17.1. Introduction

One of the most popular frameworks for low-scale SUSY studies is the so-called CMSSM. Here, sfermions have universal masses at the unification scale, and the RGE-generated flavour structures are of Minimal Flavour Violation (MFV) type [1]. However, there currently exist many reasons to consider other frameworks for the MSSM. To begin with, the lack of new physics signals at the LHC [2,3] indicate that first generation squark masses should be large, putting at risk the solution of the Higgs mass hierarchy problem, which requires a low mass scale. Moreover, there exists a tension between CP violation observables in the $K$, $B$ and $B_s$ sectors, which MFV structures cannot solve [4].

Nevertheless, the lack of significant new physics signals in flavour changing neutral currents (FCNCs) requires the approximate degeneracy of the first two generation squark masses, as well as a MFV-like flavour structure. Furthermore, in addition to LHC data, electric dipole moment experiments also suggest that these squark masses should be very high [5]. Thus, all of these facts motivate us to consider a new framework for the MSSM, which should provide some sort of large-mass universality for the first two generations as well as a MFV-like structure, with the third generation being able to have lower masses and structures somewhat different from MFV.

To this end, in this work we describe a $U(2)^3$ flavour symmetry framework acting on the first two quark
superfield generations, and briefly show its phenomenological consequences on the $K$, $B$ and $B_s$ sectors. The full details of the framework can be found in [6].

17.2. Framework

We consider the quark superfields to transform under a $U(2)_Q \times U(2)_u \times U(2)_d$ group, following:

$$Q \equiv (Q_1, Q_2) \sim (2, 1, 1), \quad u_R^i \equiv (u_1^i, u_2^i)^T \sim (1, 2, 1), \quad d_R^i \equiv (d_1^i, d_2^i)^T \sim (1, 1, 2), \quad (17.1)$$

with singlet third generation superfields $q_3$, $t^c_R$, and $b^c_R$. Considering also a $U(1)_b$ symmetry, under which only $d_R^i$ and $b_R^i$ transform, the only term in the Superpotential in the limit of unbroken symmetry is:

$$W = y_t q_3 t^c_R H_u,$$

where $y_t$ is the $O(1)$ top Yukawa coupling.

The first step in the construction of the Yukawa matrices lies on the introduction of a spurion $V$ transforming as a $(2, 1, 1)$, which breaks the symmetry in the $U(2)_Q$ direction. In addition, the $U(1)_b$ symmetry can be broken by the introduction of a $y_b$ spurion. With this, we can write:

$$Y_u = y_t \left( \begin{array}{cc} 0 & x_t \lambda \hat{\epsilon} V \\ 0 & \hat{\epsilon} \end{array} \right), \quad Y_d = y_b \left( \begin{array}{cc} 0 & x_b \lambda \hat{\epsilon} V \\ 0 & \hat{\epsilon} \end{array} \right).$$

(17.3)

In $Y_u$ ($Y_d$), everything above the horizontal line is subject to the $U(2)_Q$ symmetry, while everything to the left of the vertical line is subject to the $U(2)_u$ ($U(2)_d$) symmetry. The parameters $x_t, x_b$ are complex, of $O(1)$. The size of $y_b$ depends on the value of $\tan \beta$, and in the following we shall assume $\tan \beta \approx 10$.

In order to build the masses and mixing of the first two generations we introduce two additional spurions, $\Delta Y_u$ and $\Delta Y_d$, transforming as $(2, 2, 1)$ and $(2, 1, 2)$, respectively. Combining the various symmetry breaking terms, the Yukawa matrices end up with the following pattern:

$$Y_u = y_t \left( \begin{array}{cc} \Delta Y_u & x_t \lambda \hat{\epsilon} V \\ 0 & \hat{\epsilon} \end{array} \right), \quad Y_d = y_b \left( \begin{array}{cc} \Delta Y_d & x_b \lambda \hat{\epsilon} V \\ 0 & \hat{\epsilon} \end{array} \right),$$

(17.4)

where we have absorbed $O(1)$ couplings by redefining $\Delta Y_u$ and $\Delta Y_d$.

In order to understand how this structure generates the mass hierarchy and mixings, we shall now go to an explicit parametrization. The leading spurion can always be decomposed as:

$$V = \epsilon U_V \hat{s}_2, \quad \hat{s}_2 = \left( \begin{array}{c} 0 \\ 1 \end{array} \right),$$

(17.5)

where $U_V$ is a $2 \times 2$ unitary matrix and $\epsilon$ is a real suppression parameter that we require to be of $O(\lambda_{\text{CKM}}^2)$. Next, the $\Delta Y_u$ and $\Delta Y_d$ spurions can be written in the following way:

$$\Delta Y_u = U_{Q_u}^\dagger \Delta Y_u^d U_d^d, \quad \Delta Y_d = U_{Q_d}^\dagger \Delta Y_d^d U_d^d, \quad (17.6)$$

where $\Delta Y_u^d = \text{diag}(\lambda_{u1}, \lambda_{u2})$, and $\Delta Y_d^d = \text{diag}(\lambda_{d1}, \lambda_{d2})$, and $U_d$ are $2 \times 2$ unitary matrices. We require the $\lambda_i$ to be real, and their size is such that the largest entry is $\lambda_{d2} \approx m_s/m_b = O(\epsilon)$. It is now possible to absorb $U_d^d$ and $U_d^d$ through a redefinition of $u_R^i$ and $d_R^i$, respectively. In addition, we can absorb $U_V$ through a redefinition of $Q_L$, $U_{Q_u}^\dagger$, and $U_{Q_d}^\dagger$. In such base, the Yukawa matrices assume the explicit form:

$$Y_u = \left( \begin{array}{cc} U_{Q_u}^\dagger & \Delta Y_u^d \epsilon x_t \hat{s}_2 \\ 0 & \hat{\epsilon} \end{array} \right) y_t, \quad Y_d = \left( \begin{array}{cc} U_{Q_d}^\dagger & \Delta Y_d^d \epsilon x_b \hat{s}_2 \\ 0 & \hat{\epsilon} \end{array} \right) y_b,$$

(17.7)
We shall now address the relevant CP phases. We can parametrize each unitary matrix as:

\[
U_Q = \begin{pmatrix}
  e^{i\omega_f} & 0 & c_f \\
  0 & e^{i\omega_f} & s_f e^{i\alpha_f} \\
  -s_f e^{-i\alpha_f} & c_f & 0
\end{pmatrix},
\]

where \(c_f\) and \(s_f\) are, respectively, the cosine and sine of a mixing angle \(\theta_f\). The \(\omega_f\) phases can be removed through the rephasing of the components of \(u_R^f\) and \(d_R^f\), leaving only the phases \(\alpha_u\) and \(\alpha_d\).

In addition, we also have the phases \(\text{arg}(y_t), \text{arg}(y_u), \text{arg}(x_t)\) and \(\text{arg}(x_b)\). With a suitable rephasing of the superfields we can remove all of these phases but one. However, in order to maintain a symmetric notation for both Yukawas, we keep the latter two, denoting \(x_f e^{i\delta_f}\), with \(x_f\) real and positive. Thus, we are left with four phases: \(\alpha_u, \alpha_d, \phi_t\) and \(\phi_b\), where only three combinations of them shall be physical.

The Yukawas can now be diagonalized using:

\[
Y_u = U_uL Y_u U_uR, \quad Y_d = U_dL Y_d U_dR,
\]

where, to a very good approximation, the left-handed up-type diagonalization matrices are:

\[
U_uL = \begin{pmatrix}
  c_u & s_u e^{i\alpha_u} & -s_u e^{i(\alpha_u + \phi_u)} \\
  -s_u e^{-i\alpha_u} & c_u & s_u e^{i\phi_u} \\
  0 & -s_u e^{i\phi_u} & c_u
\end{pmatrix}.
\]

Here we have \(s_u/c_u = \epsilon x_t\). An analogous matrix diagonalizes the down-type sector (with \(s_u, c_u \rightarrow s_d, c_d, x_t e^{i\phi_t} \rightarrow x_b e^{i\phi_b}\)). For both sectors, the right-handed diagonalization matrices are approximately diagonal.

The four parameters \(s, c, \phi\) are attributed to the smallness of \(s, c\), and \(\phi\) is attributed to the smallness of \(\phi\). The \(\lambda\) phases do not appear on the CKM, but shall be important when we discuss the soft SUSY masses. This means that only the \(\alpha\) phases are relevant for the CKM phase.

The four parameters \(s, c, \phi\) can be determined completely (up to discrete ambiguities) in terms of the four independent measurements of CKM elements. In particular, using tree-level inputs we get:

\[
s = |V_{cb}| = 0.0411 \pm 0.0005, \quad \frac{s_u}{c_u} = \frac{|V_{ub}|}{|V_{cb}|} = 0.095 \pm 0.008, \quad s_d = -0.22 \pm 0.01\),
\]

As a consequence of the \(U(2)Q\) symmetry, \(|V_{td}/V_{cs}|\) is naturally of \(\mathcal{O}(\lambda_{\text{CKM}})\) and the smallness of \(|V_{ub}/V_{td}|\) is attributed to the smallness of \(s_u/s_d\).
17.3. Consequences

Following the pattern outlined above for the Yukawa matrices, we can build the respective soft SUSY masses. If the symmetry is unbroken, all mass matrices shall have the following structure:

\[
m_f^2 = \begin{pmatrix}
m_f^2  & 0 & 0 \\
0  & m_f^2  & 0 \\
0  & 0  & m_f^2
\end{pmatrix}
\]  

(17.16)

where the \(m_f^2\) are real parameters. This structure is modified by the inclusion of spurions, giving:

\[
m_Q^2 = m_Q^2 \begin{pmatrix}
1 + c_Q e^{i\Phi} Y^T & c_Q e^{i\Phi} Y^T + c_Q d e^{i\Phi} Y^T & c_Q d e^{i\Phi} Y^T \\
1 + c_Q d e^{i\Phi} Y^T & 1 + c_Q e^{i\Phi} Y^T & c_Q e^{i\Phi} Y^T + c_Q d e^{i\Phi} Y^T \\
1 + c_Q d e^{i\Phi} Y^T & c_Q e^{i\Phi} Y^T & 1
\end{pmatrix} \frac{1}{m_Q^2},
\]  

(17.17)

\[
m_d^2 = m_d^2 \begin{pmatrix}
1 + c_d e^{i\Phi} Y^T & c_d e^{i\Phi} Y^T + c_d d e^{i\Phi} Y^T & c_d d e^{i\Phi} Y^T \\
1 + c_d d e^{i\Phi} Y^T & 1 + c_d e^{i\Phi} Y^T & c_d e^{i\Phi} Y^T + c_d d e^{i\Phi} Y^T \\
1 + c_d d e^{i\Phi} Y^T & c_d e^{i\Phi} Y^T & 1
\end{pmatrix} \frac{1}{m_d^2},
\]  

(17.18)

\[
m_u^2 = m_u^2 \begin{pmatrix}
1 + c_u e^{i\Phi} Y^T & c_u e^{i\Phi} Y^T + c_u d e^{i\Phi} Y^T & c_u d e^{i\Phi} Y^T \\
1 + c_u d e^{i\Phi} Y^T & 1 + c_u e^{i\Phi} Y^T & c_u e^{i\Phi} Y^T + c_u d e^{i\Phi} Y^T \\
1 + c_u d e^{i\Phi} Y^T & c_u e^{i\Phi} Y^T & 1
\end{pmatrix} \frac{1}{m_u^2},
\]  

(17.19)

where the \(c_i\) and the \(x_i\) are real \(O(1)\) parameters. This structure again gives almost diagonal right-handed squark mixings. On the basis where \(Y_d\) is diagonal, the left-handed mixing matrix is given by \(W_L^d m_Q^2 W_L^d = \text{diag}(m_Q^2, m_Q^2, m_Q^2)\), resulting in:

\[
W_L^d \approx \begin{pmatrix}
c_d & -s_d e^{-i(c+\beta)} & -s_d e^{-i(c+\beta)} \\
-s_d e^{i(c+\beta)} & c_d & -s_d e^{i(c+\beta)} \\
0 & 1 & 1
\end{pmatrix} = \begin{pmatrix}
c_d & \kappa^* & -s_d e^{i(c+\beta)} \\
-\kappa & c_d & -s_d e^{i(c+\beta)} \\
0 & 1 & 1
\end{pmatrix},
\]  

(17.20)

where \(\kappa = c_d V_d e^{i\beta} \text{ and } s_d e^{i\beta} = e^{-i\beta_s} e^{-i\phi} + s_Q e^{i\phi}\). Notice that the phase \(\gamma\) does not involve the phases \(\alpha_f\), related to the CKM phase, meaning it can be treated as a new physics phase.

From this matrix, we can derive simple expressions for the SUSY contribution to FCNC observables in \(\Delta F = 2\) processes. We find:

\[
\epsilon_K = \epsilon_K^{\text{SM}(l)} \times (1 + x^2 F_0) + \epsilon_K^{\text{SM}(2)} \\
S_{\psi K_s} = \sin(2\beta + \text{arg}(1 + x F_0 e^{-2i\gamma})) \\
\Delta M_d / \Delta M_s = \Delta M_d^{\text{SM}} / \Delta M_s^{\text{SM}}
\]  

(17.21)

\[
(17.22)
\]

where \(x = s_L^2 c_d^2 / |V_{ts}|^2\). The function \(F_0\) is:

\[
F_0 = \frac{2}{3} \left( \frac{g_s}{g} \right) ^4 \frac{m_W^2}{m_Q^2} \frac{1}{S_0(x_t)} \left[ f_0(x_g) + \mathcal{O} \left( \frac{m^2_{Q_i}}{m^2_{Q_h}} \right) \right], \quad x_g = \frac{m^2_{Q_i}}{m^2_{Q_h}},
\]  

(17.24)

\[
f_0(x) = \frac{11 + 8x + 19x^2 + 26x \log(x) + 4x^2 \log(x)}{3(1 - x)^3}, \quad f_0(1) = 1.
\]  

(17.25)

where \(S_0(x_t = m^2_{Q_i} / m^2_{Q_h}) \approx 2.4\) is the SM one-loop electroweak coefficient function.

In [8], we showed that the modifications to these observables allowed us to solve the tensions between \(\epsilon_K\), \(S_{\psi K_s}\) and \(\Delta M_d / \Delta M_s\). This would require a non-zero value for \(\gamma\), and would restrict the values of \(F_0\) and \(x\) to
those shown on the left panel of Figure 17.1. This implies that, to solve the flavour tension, we require values of squark and gluino masses under $1 \sim 1.5$ TeV.

Moreover, a non-zero value for $\gamma$ means that our framework also provides a contribution to $S_{\psi \phi}$. The requirement of having the correct $S_{\psi K_s}$ restricts the values of $S_{\psi \phi}$ to those shown on the right panel of Figure 17.1. Notice that having a large value of this observable is favoured by the dimuon anomaly observed by Tevatron [7], and shall be accurately probed by the LHCb experiment in the near future.

17.4. Conclusions

We have described a $U(2)^3$ flavour symmetry framework acting on the quark superfields, and shown that it can successfully accommodate the observed masses and mixings. The framework maintains the degeneracy between the first two generation squark masses, allowing a light third generation squark mass. This is consistent with the LHC and FCNC bounds, and in principle allows the solution of the Higgs mass hierarchy problem.

In addition, the framework presents deviations from MFV, such that the tensions in the flavour sector can be solved. This requires squarks and gluinos to have masses of a value at most $1 \sim 1.5$ TeV, and predicts a large $S_{\psi \phi}$. Thus, the framework proves testable, and a good alternative for MFV.

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