Theoretical Turbine Model with Hydraulic Losses

B Svingen$^{1,2}$, A F Reines$^1$, T K Nielsen$^1$, P T Storli$^1$

$^1$ Department of Energy and Process Engineering, NTNU, Norway  
$^2$ Hymatek Controls AS, Oslo, Norway

bjoernar.svingen@hymatek.no, anefr@stud.ntnu.no, torbjorn.nielsen@ntnu.no, pal-tore.storli@ntnu.no

Abstract. When doing transient and dynamic analysis of powerplants, a mathematical representation of the turbine is needed. These models can be fully based on empirical data, or they can be based on a first principles approach. A first principles approach is practical in many circumstances because any known and unknown turbine can be modelled, and the model can easily be simplified and linearized without losing generality or physical correctness. The model can be used for general transient studies, also where the turbines are not yet specified, or when doing control system analysis and analysis of the grid.

Using the Euler turbine equations together with the definition of the turbine opening degree, one obtains a model that is geometrically and physically correct. The development of this model started several years ago, and this paper presents the current development. The model shows good agreement of the efficiency as a function of speed, as part of the Euler turbine equation. However, the efficiency as a function of flow is over predicted because hydraulic loss phenomenon such as turbulence and dissipation are not inherently included. There are several ways to include the hydraulic losses. This can be done in a simplified manner, or more elaborate, which also could involve empirical relations for exact fit with a measured turbine. This paper discusses these possibilities, along with some examples.

1. Introduction

When doing analysis of hydropower plants, a mathematical model of the turbine is needed. The accuracy and detail of the model needed, varies according to the accuracy and detail required for the results. Typical analysis are analysis of the grid (including hydropower plants); stability analysis of the turbine and turbine governor, and transient analysis of the turbine and waterways. The required detail and accuracy of the turbine model will typically increase accordingly. In later years, the addition of so called digital twins and similar concepts, with their own sets of requirement regarding accuracy and detail, have emerged [3] [4].

In 1990, Nielsen introduced a new model [6]. This model has been studied and developed ever since, for instance, but not limited to [5] [8] [9] [10] [11] [12]. This is also the main model used in LVTrans [1] [2]. This model is based on a first principles approach using the Euler Turbine equation and the equation for the opening degree. Being a first principles approach, it lends itself naturally to studying and modelling of the details of the physical losses in a turbine.

The Nielsen model includes "Euler losses". These losses are caused by the turbine spinning too fast or too slow in relation to nominal speed. The model also includes effects due to geometry. However, in its most basic form, the model does not include losses caused by irreversible phenomenon, such as turbulent losses, losses in draft tube etc. This makes the basic model able to accurately predict turbine
behaviour along constant opening curves and along the "speed axis", but not the irreversible losses that typically occur along the "flow axis" [8] [9] [11] [12]. Whether these losses are of importance depends on the nature of the analysis. The implication of these losses, as well as modelling alternatives are presented and discusses in this paper. The discussion is mainly about Francis turbines, but is also relevant for Pelton turbines. It does not include Kaplan turbines as of the time being.

2. Governing equations

The derivation of the model, which is not trivial, can be found in [6] [8] [9] [12], and in particular [11]. The main starting point is the Euler turbine equation and the related inlet/outlet diagrams, Figure 1

\[ \eta_h = \frac{1}{gH_e} (c_{u1}u_1 - c_{u2}u_2) = \frac{\omega_e}{gH_e} (c_{u1}r_1 - c_{u2}r_2) \]  

(1)

\[ H_e = (H_0 + \frac{c_0^2}{2g} + Z_0) - (H_3 + \frac{c_3^2}{2g} + Z_3) \]  

(2)

Combining the above 3 equations yields one equation for the flow and head (4) and one equation for the torque (5).
\[ g(H_0 - H_3) = gH_R \left( \frac{Q}{Q_R} \right)^2 + s_D (\omega^2 - \omega^2_R) \]  
\[ T = \rho Q (c_{u1} r_1 - c_{u2} r_2) = \rho Q (t_s - r^2_2 \omega) \]
\[ t_s = r_1 c_1 \cos(\alpha_1) + r_2 A_z \cot(\beta_2) c_1 \sin(\alpha_1) \]

Here \( t_s \) is the starting torque when \( \omega = 0 \) [7]. \( A_z \) is the inlet area divided by the outlet area perpendicular to the shaft.

Equation (4) and (5) are made non-dimensional to form equation (6) and (7) respectively. A term, \( \eta_i \), called the incipient efficiency, is included [5] [8] [12]. This incipient efficiency can be regarded as a general term describing all the irreversible losses in the turbine model. This term is the focus of this paper. It's worth noting that equation (6), (7) and (8) are capable of fully describing a turbine hill chart as long as a suitable \( \eta_i \) is constructed and included.

\[ t = q \eta_i (m_s - \psi \sigma) \]
\[ h_e = \left( \frac{q}{k} \right)^2 + \sigma (\sigma^2 - 1) \]
\[ \bar{\eta}_h = \frac{\eta_h}{\eta_R} = \frac{t \sigma}{q h_e} = \frac{\eta_i (m_s - \psi \sigma) \sigma}{h_e} = \frac{\eta_i (m_s - \psi \sigma) \sigma}{\left( \frac{q}{k} \right)^2 + \sigma (\sigma^2 - 1)} \]

All variables are “per unit” (pu) and subscript “R” denotes "rated" value at best efficiency point. \( t, q \) and \( h_e \) are torque, flow and head correspondingly. Thus \( \bar{\eta}_h \in [0, 1], \sigma = s_D \frac{\omega^2_R}{gH_R}, \psi = \frac{u^2_R}{gH_R}, \kappa = \frac{\sin(\alpha_1)}{\sin(\alpha_1 R)}, m_s = \xi \frac{q}{k} \left( \cos(\alpha_1) + \tan(\alpha_1 R) \sin(\alpha_1) \right) \) and \( \xi = \frac{1 + \psi}{\cos(\alpha_1 R) + \tan(\alpha_1 R) \sin(\alpha_1 R)} \).

3. Model behaviour with \( \eta_i = 1 \)

When \( \eta_i \) is set to the numerical constant value 1, then no effect of this terms is included. This is useful because it will show what is missing in the basic model.

Figure 3 shows a sketch of a hill chart. With \( \eta_i = 1 \), the model is able to give a good prediction of all the variables along the line that represents a constant opening degree, \( \kappa = 1 \). The reason for this is that along
this line, the losses are mostly represented by the turbine running too fast or too slow, and this already included in the basic model from the Euler turbine equation. For a Pelton turbine, this also extends where the flow is different from 1, or $\kappa \neq 1$.

What is missing for Francis turbines, are mainly two different irreversible losses: Losses due to wrong inflow angles, and losses due to spin in the draft tube. Other losses such as friction and turbulence can implicitly be included but are considered minor in comparison.

For a Pelton, the hill chart is already rather good for $\eta_i = 1$, which is not unreasonable knowing that a Pelton does not suffer from wrong inflow angles or swirl in a draft tube. There still are errors compared with measured diagrams, particularly for $q > 1$. This can be explained by too much flow for each bucket to empty before it enters a new jet, and/or the jet becomes too wide for each bucket.

Also worth discussing is when the basic turbine equations in general can be used, even with $\eta_i = 1$. For both Francis and Pelton, the equations can be used "as is", at least when going along the line where $\kappa = 1$. Thus for most grid analysis and stability analysis, the model will be a substantial improvement from using a simple valve equation, as is often used [14], [15]. Specifically the model will include the variation of flow with respect to speed, which is important for stability analysis. The model/equations can be used without any knowledge of the actual turbine, other than nominal flow and head [8], [9], [12].

4. Previous model improvements

Already in Nielsen [6] several first principle improvements of the model were made. Specifically, the losses due to wrong inflow angles and draft tube losses were addressed. Both these losses were modelled as head losses using the same basic equation:

$$h_{loss} = R(Q - Q_c)^2 \quad (9)$$

where

$$Q_c = F(\kappa, \omega) \quad (10)$$

The function, $F$, itself is an analytical function describing the flow $q$ at BEP. Equation (9) is subtracted from equation (7) to include this in the model. However, several such losses make verifications very difficult, as they are impossible to differentiate from each other. It all becomes very convoluted for practical calculations when model tests or site measurements are needed to find good values for all the $R$s, and at the same time they are all related to a head loss.

5. Different forms of $\eta_i$

In Nielsen [5] the first use of the incipient efficiency was found. This made it possible to clearly divide the "pure" model equations from all the irreversible losses. In addition, the incipient efficiency modified the turbine efficiency instead of the head losses [8], [9]. This resulted in the basic equations (6), (7) and (8), and where the incipient efficiency, $\eta_i$, is an arbitrary loss function that can take any shape and form. Moreover, it makes the model equations more consistent, in the sense that for any turbine, there should exist an $\eta_i$ that will make the model equations fit with measurements [9].

As shown in [8], [9], [11], [12], even the simplest and fully generic 1D function of $\eta_i$ will make huge improvements.

From a more fundamental point of view, this kind of model is interesting. The basic physics is correct, as far as one can be certain, but there are certain loss terms that must be modelled separately because they are not intrinsically a part of the model. This is similar to turbulence modelling in CFD or the Moody diagram in pipe flow. The losses cannot be modelled in the domain of the model, 1D in this case, but they can be approximated by simpler functions that are part of the same domain.
5.1. The simplest $\eta_i$

The simplest form of $\eta_i$ is found in [5] [8] [11]. It is expressed as:

$$\eta_i = q(2 - q)$$

This is a function of the flow, $q$ where: $\eta_i = 0$ for $q = 0$; $\eta_i = 1$ for $q = 1$; $\eta_i = 0$ for $q = 2$. The shape is based on the observation from measurements that the efficiency must go towards zero when $q$ approaches zero [6]. The efficiency must be equal to nominal efficiency when $q = 1$ (nominal flow), and it should decrease when $q > 1$. This is shown in the figures below comparing real measurements with the mathematical model.

**Figure 4** Measured vs calculated efficiency for a high head Francis turbine

**Figure 5** Measured vs calculated efficiency for a medium head Francis turbine

**Figure 6** Measured vs calculated efficiency for a low head Francis turbine

This very simple generic $\eta_i$ improves the basic model considerably [8]. However, it is a bit "too generic" for the entire range from low head Francis, Figure 6, to high head Francis, Figure 4, and further to Pelton[12]. It must be noted that equation (11) is a rather good approximation for low head Francis, Figure 5. The curve for $\eta_i = 1$ in the figures above represents the basic model with no irreversible loss terms.
5.2. Best fit incipient efficiency $\eta_i$ for low head to high head Francis

Assuming that $\eta_i$ can be approximated by a function of $q$, then finding the best $\eta_i$ that fits to several different turbines with different specific speeds, it should be possible to create generic curves that fit with those speeds and interpolate between them.

Further, there are some pure practical considerations that has to be considered. When installing new turbine governors on existing plants, the complete hill chart usually cannot be found. Even if it is found, it is an elaborate process to transfer paper charts to a well-behaved interpolating function for analysis. A more efficient method would be to simply use readily available nominal values, and to build a mathematical model automatically from that.

As shown in [12] this is indeed possible. Measured hill charts from 3 model turbines: high head Francis; medium head Francis; low head Francis, were used to find the best curve for each.

With this method the hill chart will be accurate for all $\kappa$ and $q$ when $\omega = 1$ and for all $\omega$ when $\kappa = 1$. The model can be used for any analysis where $\omega \approx 1$, for instance grid analysis and stability analysis. Since it is also accurate for all $\omega$ at $\kappa = 1$, it can be used for simple transient analysis.

The model is slightly inaccurate where both $\omega$ and $\kappa$ are largely different from 1 at the same time. Regarding transient analysis, this would typically be along the curve of the runaway speed, which is
important when calculating load rejection. This can be seen comparing Figure 10 and Figure 11. However, it is worth noting that for \( \kappa = 1 \) or for \( \omega = 1 \), assuming constant head, \( h = 1 \) also for the measured chart, the runaway curve is quite correct in these points.

![Figure 10 Measured hill chart for a medium head Francis](image1)

![Figure 11 Medium head Francis hill chart using the turbine model and \( \eta \) from Figure 8 and representing the model tests in Figure 10](image2)

### 6. Discussion

Using the simplified design procedure as shown in [9] and [12] together with the incipient efficiency as a function of \( q \) and speed number, any unknown (Francis) turbine can easily be modelled just using nominal values as flow, speed and head. This is useful for grid analysis and stability analysis of turbine governors, as no additional information is needed [8] [12]. The model is easy to linearize, even analytically [9] [11].

Data from only 3 model tests were used: low head, medium head and high head Francis. The assumption is therefore that these three specific Francis turbines give representative curves for all other Francis turbines. This may seem as a rather big assumption. However, when studying how turbine characteristics changes with speed number, and noting the fact that the basic equations already include most of these characteristics, this cannot be considered an unreasonable assumption. Still, including many more model tests would of course be one of the next steps further with this model.

More work must be done with the \( \eta_i \) function to make it fully usable for general transient analysis including load rejection, since the runaway curve is not represented well [12].

LVTrans has used the original model [6] with the R functions and \( Q_c \), Equation (9), with very good results for more than 15 years [1] [2] [4], although the problem of easily and methodically finding suitable R values for arbitrary turbines do remain. The idea to use an incipient efficiency function of the form:

\[
\eta_i = F(q, q_c) = F(q, \kappa, \omega)
\]

is therefore tempting and should be investigated. It is believed that simple but effective forms may be found that are adequate for the purpose of anticipating the runaway curve when this is needed.

Linearizing this model is straight forward [9] [11]. The block diagram will change slightly in comparison to a traditional diagram. The reason for this can be seen directly from equation (7) and (6) where \( \omega \) is an independent parameter. This creates the block diagram as shown in Figure 12.
7. Conclusion
The turbine equations with the incipient efficiency seems to be very promising. Most of the shortcomings with the original simple form, equation (11), has been solved by using more elaborate forms as shown in Figure 7, Figure 8 and Figure 9. These new forms do not add any complexity in practical numerical calculations.

This new form of incipient efficiency is accurate around best efficiency point and along the curves of nominal speed and nominal turbine opening in the turbine hill chart. Thus, the turbine equations can be used for grid calculations and stability analysis, as well as simple transient calculations for Francis turbines of literally the full range of speed numbers.

As found in [12] and seen in when comparing Figure 10 and Figure 11, the new form of incipient efficiency does not give a good representation of the runaway curve for Francis turbines. Further work is needed in this area.

References
[1] Svingen B, 2005, Application of LabVIEW for dynamic simulation of Hydraulic piping systems, SIMS 2005 46th Conf. on Simulation and Modeling, 325-333
[2] Svingen B, Brekke T, Skåre P E, Nielsen T, 2006, Minimizing oscillations and transients for large and rapid changes in power output from hydro power plants by new control algorithms, 23rd IAHR Symposium
[3] Svingen B, Francke H, 2015, Large and rapid set-point adjustment of hydro power plants using embedded transient hydraulic simulations of the plant as a model predictive method, 12th Int. Conf Pressure Surges, BHR Group
[4] Svingen B, 2016, A predictive controller based on transient simulations for controlling a power plant, 28th IAHR Symposium on Hydraulic Machinery and Systems
[5] Nielsen T K, 2015, Simulation model for Francis and Reversible Pump Turbines, International Journal of Fluid Machinery and Systems Vol. 8 No. 3
[6] Nielsen T K, 1990, Transient characteristics of high head Francis turbines, Dr. Thesis, ISBN 82-7119-242-6
[7] Wylie E B, Streeter, V L, 1993, Fluid Transients in Systems, Prentice Hall, ISBN 0-13-322173-3
[8] Svingen B, Nielsen T K, 2018, First principles approach linear model for hydraulic turbines suitable for use in available simulation platforms, 13th International Conference on Pressure Surges, Bordeaux, France, 14th - 16th November 2018
[9] Svingen B, Nielsen TK, Transient Hydraulic Calculations with a Linear Turbine Model derived from a Nonlinear Synthetic Model, 29 IAHR Symposium on Hydraulic Machinery and Systems, Kyoto, Japan, November 2018
[10] Storli P, Nielsen T K, Simulation and discussion of models for hydraulic Francis turbine simulations, IFAC PapersOnLine, 51(2):109-114, 2018
[11] Reines A F, 2019, Modelling of hydro power plants, Project Thesis, Institutt for Termisk Energi og Prosessteknikk. NTNU, Norway
[12] Reines A F, 2020, *Analysis and improvement of mathematical turbine model*, Master Thesis, Institut for Termisk Energi og Prosessteknikk, NTNU, Norway

[13] Woodward J L, 1968, Hydraulic-turbine transfer function for use in governing studies. Proc. IEE 115, No. 3, 424–426

[14] Wang C, Goa H, 2006, *Effect of detailed hydro turbine models on power system analysis*, PCSE ’06, IEEE.

[15] Monuz-Hernandez G A, Mansoor S P, Jones D I, 2013, *Modelling and Controlling Hydropower Plants*, Springer