Flavor-Nonconservation and $CP$-Violation with Singlet Quarks $^1$

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**abstract**

Some aspects are considered on the flavor-nonconservation and $CP$-violation arising from the quark mixings with singlet quarks. In certain models incorporating the singlet quarks, the contributions of the quark couplings to the neutral Higgs fields may become more significant than those of the neutral gauge interactions. Then, they would provide distinct signatures for new physics beyond the standard model in various flavor-nonconserving and $CP$-violating processes such as the neutron electric dipole moment, $D^0-\bar{D}^0$, $K^0-\bar{K}^0$, $B^0-\bar{B}^0$, $b \to s\gamma$, and so on.

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1 Introduction

Some extensions of the standard model may be expected in various points of view. Among such possibilities, electroweak models incorporating $SU(2)_W \times U(1)_Y$ singlet quarks with electric charges $2/3$ and $-1/3$ have been investigated extensively in the literature [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. In the presence of singlet quarks, various interesting issues are presented phenomenologically. In particular, some novel features arise from the mixings between the ordinary quarks ($q$) and the singlet quarks ($Q$): The CKM unitarity in the ordinary quark sector is violated, and the flavor changing neutral currents (FCNC’s) are present at the tree-level in both the gauge and Higgs interactions of the quarks. The $q$-$Q$ mixings even involve new $CP$-violating sources. In this talk, we make further considerations on the flavor-nonconservation and $CP$-violation arising from the quark mixings with singlet quarks. We present some relevant formulations for the quark mixings and the FCNC’s, which are useful for making more precise estimates on the $q$-$Q$ mixing effects in the flavor-nonconserving and $CP$-violating processes such as the neutron electric dipole moment (NEDM), $D^0$-$\bar{D}^0$, $K^0$-$\bar{K}^0$, $B^0$-$\bar{B}^0$, $b \to s\gamma$, and so on. Then, these formulations are applied, for instance, for estimating the neutral Higgs contributions to the NEDM and the $D^0$-$\bar{D}^0$ mixing in the presence of significant mixing between the top quark and singlet $U$ quarks [11]. These estimates may be relevant for investigating the electroweak baryogenesis with $CP$-violating $t$-$U$ mixing. Some variants of the electroweak model incorporating singlet quarks will also be discussed later, in particular, by considering possible parametrizations of the quark mass matrices, relative significance between the gauge and Higgs FCNC’s and the role of the singlet Higgs field.

2 Quark masses and mixings with singlet quarks

We here describe explicitly a specific version of electroweak model incorporating singlet quarks, where the quark Yukawa couplings are given by

$$
\mathcal{L}_{\text{Yukawa}} = - u_0^c \lambda_u q_0 H - u_0^c (f_U S + f'_U S^\dagger) U_0 - U_0^c (\lambda_U S + \lambda'_U S^\dagger) U_0 - d_0^c \lambda_d V_0^\dagger q_0 \bar{H} - d_0^c (f_D S + f'_D S^\dagger) D_0 - D_0^c (\lambda_D S + \lambda'_D S^\dagger) D_0 + \text{h.c.} \quad (1)
$$

with the two-component Weyl fields (the generation indices and the factors representing the Lorentz covariance are omitted for simplicity). Here $q_0 = (u_0, V_0 d_0)$ represents the quark doublets with a unitary matrix $V_0$, and $H = (H^0, H^+)$ is the electroweak Higgs doublet with $\bar{H} = (H^-, -H^0)$. Suitable redefinitions among the $q_0^c$ and $Q_0^c$ fields with the same quantum numbers has been made to eliminate the $U_0^c q_0 H$ and $D_0^c q_0 \bar{H}$ couplings without loss of generality. Then, the Yukawa coupling matrices $\lambda_u$ and $\lambda_d$ have been made diagonal by using unitary transformations among the ordinary quark fields. In this basis, by turning off the $q$-$Q$ mixings with $f_Q, f'_Q \to 0$, $u_0$ and $d_0$ are reduced to the mass eigenstates, and $V_0$ is
identified with the CKM matrix. The actual CKM matrix is slightly modified due to the \( q-Q \) mixings, as shown explicitly later. A complex singlet Higgs field \( S \) is introduced in the present model given by eq. (1) to provide the singlet quark mass terms and the \( q-Q \) mixing terms. Some variants of the model will be considered later by noting whether the singlet Higgs field is introduced or not.

The Higgs fields develop vev’s,

\[
\langle H^0 \rangle = \frac{v}{\sqrt{2}}, \quad \langle S \rangle = e^{i\phi_S} \frac{v_S}{\sqrt{2}},
\]

where \( \langle S \rangle \) may acquire a nonvanishing phase \( \phi_S \) due to either spontaneous or explicit CP violation originating in the Higgs sector. The quark mass matrices are produced with these vev’s as

\[
\mathcal{M}_U = \begin{pmatrix} M_u & \Delta_{u-U} \\ 0 & M_U \end{pmatrix}, \quad \mathcal{M}_D = \begin{pmatrix} M_d & \Delta_{d-D} \\ 0 & M_D \end{pmatrix}.
\]

These quark mass matrices are diagonalized by unitary transformations \( V_Q_L \) and \( V_Q_R \) (\( Q = U, D \)) as

\[
V_{U_L}^\dagger \mathcal{M}_U V_{U_L} = \text{diag.}(m_u, m_c, m_t, m_{U_1}, \ldots),
\]

\[
V_{D_L}^\dagger \mathcal{M}_D V_{D_L} = \text{diag.}(m_d, m_s, m_b, m_{D_1}, \ldots)
\]

with quark mass eigenvalues given by

\[
m_{q_i} = \frac{\lambda_{q_i} v}{\sqrt{2}} \left[ 1 + \mathcal{O}(\epsilon_{q-Q}^2) \right].
\]

Here the parameters \( \epsilon_{q-Q} \sim |f_Q| + |f'_Q| \) represent the mean magnitudes of the \( q-Q \) mixings. The generalized CKM matrix is given by

\[
V = V_{U_L}^\dagger \begin{pmatrix} V_0 & 0 \\ 0 & 0 \end{pmatrix} V_{D_L} = \begin{pmatrix} V & * \\ * & * \end{pmatrix}.
\]

Here the CKM matrix \( V \) is no longer unitary due to the \( q-Q \) mixings. It is found by determinig \( V_{Q_L,R} \) perturbatively that the CKM unitarity violation arises as

\[
(V^\dagger V - 1)_{ij}, \quad (VV^\dagger - 1)_{ij} \sim (m_u m_{u_j}/m_U^2)\epsilon_{u-U}^2 + (m_d m_{d_j}/m_D^2)\epsilon_{d-D}^2,
\]

being related to the FCNC’s coupled to the \( Z \) boson [1, 2, 3, 4, 5, 6, 7, 8, 9]. This relation ensures that the CKM unitarity violation is sufficiently below the experimental bounds [14] for reasonable ranges of the model parameters.

3 FCNC’s

The FCNC’s, which may include \( CP \) violating sources, arise in both the gauge interactions and the neutral Higgs couplings. We here consider these FCNC’s, respectively.
**FCNC’s in the gauge interactions**

It is straightforward to write down the quark gauge interactions coupled to the $Z$ bosons in terms of the quark mass eigenstates:

$$\mathcal{L}_{NC}(Z) = g_Z Z_{\mu} \left[ \sum_{Q=U,D} Q^\dagger \sigma^\mu \mathcal{V}_{Z}(Q) Q + \sum_{Q^c=U^c,D^c} Q^{c\dagger} \sigma^\mu \mathcal{V}_{Z}(Q^c) Q^c \right],$$  \hspace{1cm} (8)

where $g_Z = g/\cos\theta_W$, $g = e/\sin\theta_W$, and the coupling matrices are given by

$$\mathcal{V}_Z(Q) = \mathcal{V}_{Q_L}^\dagger \begin{pmatrix} I_Z(q_0)1 & 0 \\ 0 & I_Z(Q_0)1 \end{pmatrix} \mathcal{V}_{Q_L} = \begin{pmatrix} V_Z(q) & * \\ * & * \end{pmatrix},$$  \hspace{1cm} (9)

$$\mathcal{V}_Z(Q^c) = I_Z(Q^c_0) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \hspace{1cm} (10)$$

with $I_Z(F) = I_3(F) - \sin^2\theta_W Q_{EM}(F) \ [F = Q_0, Q^c_0]$. The FCNC’s do not appear for the right-handed quarks in the gauge interactions, since they have the same SU(2)$_W \times$ U(1)$_Y$ quantum numbers with $I_Z(Q^c_0) = I_Z(q^c_0) = I_Z(Q^c_0)$. The coupling matrices $V_Z(q)$ for the left-handed quarks are actually determined in respective models depending on how the singlet quark mass terms and the $q-Q$ mixing terms are provided, and how the Yukawa couplings and the quark mass matrices are parametrized in certain bases. In the present model given by eq. (1), the $U^c_0 q_0 H$ and $D^c_0 q^c_0 \bar{H}$ couplings have been rotated out by suitable redefinition among the right-handed quarks. Then, the quark mass matrices have the specific form (3), respecting the relation $m_{qi} \sim \lambda_{qi} v$ as given in eq. (5). By making perturbative calculations with these quark mass matrices the $q-Q$ mixing effects on the neutral gauge couplings of the ordinary quarks are found as

$$V_Z(q)_{ij} - V_Z(q_0)_{ij} \sim (m_{qi} m_{q_j} / m_Q^2) \epsilon_{q-Q},$$  \hspace{1cm} (11)

where the $V_Z(q_0)$ represents the usual neutral currents in the absence of $q-Q$ mixings. This modification on the neutral currents is related to the unitarity violation given in eq. (6).

**FCNC’s in the neutral Higgs couplings**

The quark couplings to the neutral Higgs fields are extracted from (4) as

$$\mathcal{L}_{NC}(\text{Higgs}) = - \sum_{Q:a=0,1,2} Q^c \Lambda_a^Q Q \phi_a + \text{h.c.},$$  \hspace{1cm} (12)

where $\phi_0, \phi_1, \phi_2$ are the mass eigenstates of the neutral Higgs fields. Then, the coupling matrices in eq.(12) are given by

$$\Lambda_a^Q = \frac{1}{\sqrt{2}} \mathcal{V}_{Q_R}^\dagger (O_{a0} \Lambda_q + O_{a1} \Lambda^+_Q + i O_{a2} \Lambda_Q) \mathcal{V}_{Q_L} \hspace{1cm} (13)$$
\[
\rho_u = \begin{pmatrix}
\lambda_u & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 
\end{pmatrix},
\rho_d = \begin{pmatrix}
-\lambda_d V_0^\dagger & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 
\end{pmatrix},
\rho^{\pm}_{Q} = \begin{pmatrix}
0 & f_Q \pm f'_Q \\
f_Q \pm f'_{Q} & 0 \\
0 & 0 
\end{pmatrix}.
\]

(14)

Here an orthogonal matrix \( O \) is introduced to parametrize the mass eigenstates of the neutral Higgs fields. It is seen by making perturbative calculations that the ordinary quark couplings to the neutral Higgs fields, in particular, have specific generation dependence as

\[
(\rho^Q_{a})_{ij} \sim \left( \frac{m_{Q_j}}{m_{Q_i}} \right) \varepsilon^2_{Q_{a}}.
\]

(15)

**FCNC’s (Z) versus FCNC’s (Higgs)**

As seen in eqs. (11) and (15), the FCNC’s coupled to the neutral Higgs fields are of the first order of the relevant ordinary quark masses, while the \( q-Q \) mixing effects on the \( Z \) boson couplings appear at the second order. This implies that the FCNC’s (Higgs) are more significant than FCNC’s (Z) in this sort of models with the quark mass matrices of the form given in eq. (3), where the quark mass hierarchy is respected naturally under the relation (3). Then, the neutral Higgs contributions in various flavor-conserving and \( CP \)-violating processes are expected to serve as signals for the new physics beyond the standard model.

**4 Higgs contributions to NEDM and \( D^0-\bar{D}^0 \) with \( u-U \) mixings**

The neutral Higgs contribution to the NEDM was considered earlier [5], claiming that the singlet Higgs mass scale \( |\langle S \rangle| = v_S/\sqrt{2} \) should be in the TeV region or larger. On the other hand, in view point of electroweak baryogenesis [12, 13], the mass scale of the singlet Higgs field to provide the \( CP \)-violating \( q-Q \) mixings is desired to be comparable to the electroweak scale. In order to clarify this apparently controversial situation, detailed analyses have been made recently [11], showing that the NEDM becomes comparable to the experimental bound for the singlet Higgs mass scale \( v_S \sim \text{several} \times 100 \text{GeV} \) even in the presence of significant \( t-U \) mixing. We here describe these analyses briefly, where the \( D^0-\bar{D}^0 \) mixing is also considered.

The total one-loop contribution of the neutral Higgs fields to the \( u \) quark EDM is calculated by a usual formula

\[
d_u(\phi) = -\frac{2e}{3(4\pi)^2} \sum_a \sum_{U_K=U_i,U} \left\{ \text{Im} \left[ (\rho^a_{1})_{1K}(\rho^a_{2})_{K1} \right] \frac{m_{U_K}}{m^2_{\phi^a}} I \left( \frac{m^2_{U_K}}{m^2_{\phi^a}} \right) \right\},
\]

(16)

where \( I(X) \) is a certain function of \( X \). The effective Hamiltonian for the \( D^0-\bar{D}^0 \) mixing, on the other hand, is obtained from the quark couplings to the neutral Higgs fields (13) as

\[
\mathcal{H}^{\Delta=2}_\phi = \sum_a \frac{1}{m^2_{\phi^a}} \left[ \bar{c} \left( \Gamma^S_{a} \right)_{21} + \left( \Gamma^P_{a} \right)_{21} \gamma_5 \right] u^2.
\]

(17)
where
\[
(\Gamma_{a}^{S})_{21} = \frac{1}{2} \left[ (\Lambda_{a}^{U})_{12}^* + (\Lambda_{a}^{U})_{21} \right], \quad (\Gamma_{a}^{P})_{21} = \frac{1}{2} \left[ (\Lambda_{a}^{U})_{12}^* - (\Lambda_{a}^{U})_{21} \right].
\]  

(18)

Systematic analyses have been done in \[11\] for the neutral Higgs contributions (16) and (17) to the NEDM and the \(D^0-\bar{D}^0\) mixing due to the \(u-U\) mixings. There, the quark mass matrices are diagonalized numerically to determine precisely the quark mixing matrices and the quark couplings to the \(Z\) boson and neutral Higgs fields. The relevant coupling parameters are taken in certain reasonable ranges as

\[
\epsilon_{u-U} \sim 0.1, \quad \text{complex phases in } L_{\text{Yukawa}} \text{ and } |S| \sim 1,
\]

\[
m_U \sim \text{several} \times 100\text{GeV},
\]

\[
m_{\phi_0} \sim 100\text{GeV}, \quad m_{\phi_1}, m_{\phi_2} \sim v_S \gtrsim 100\text{GeV}.
\]

The results are given as

\[
|d_u(\phi)| \sim (10^{-25} - 10^{-27})\text{ecm}, \quad \Delta m_D(\phi) \sim (10^{-13} - 10^{-15})\text{GeV},
\]

which are comparable to the present experimental bounds \[14\].

As for the case of \(d-D\) mixings, the neutral Higgs contributions to the \(K^0-\bar{K}^0\) mixing and the NEDM should be investigated as well. It is actually found that the \(CP\) violation parameter \(\epsilon\) for the \(K^0-\bar{K}^0\) mixing, in particular, can be as large as \(10^{-3}\) for \(\epsilon_{d-D} \sim 0.1\) and \(v_S \sim v\).

5 Some variants of the model with singlet quarks

We finally consider some possible variants of the model incorporating the singlet quarks.

alternative form of the quark mass matrices

We have rotated out the \(u_0^c q_0 H\) and \(d_0^c q_0 \bar{H}\) couplings in eq. (11) providing the specific form of the quark mass matrices (3). This respects naturally the mass hierarchy of the ordinary quarks with the relation (4). It is instead possible to eliminate the \(u_0^c U_0 S\) and \(d_0^c D_0 S\) couplings with \(f_U, f_D \rightarrow 0\), while the \(f_U'\) and \(f_D'\) couplings may still have generic forms with the diagonal \(\lambda_u\) and \(\lambda_d\) couplings. Then, alternative form of the quark mass matrices are obtained. Even in this case, if the \(f_U'\) and \(f_D'\) couplings are small enough (not necessarily for the top quark), then the masses of the ordinary quarks are not changed significantly, maintaining the natural relation \(m_{q_i} \sim \lambda_{q_i} v\). It is interesting in this case, as confirmed by numerical calculations, that the FCNC’s (\(Z\)) can be larger than the FCNC’s (Higgs). Then, for instance, a significant contribution of the FCNC’s (\(Z\)) to the \(D^0-\bar{D}^0\) mixing may be obtained, as investigated in [7].
real singlet Higgs field

The complex Higgs field $S$ may be replaced by a real field with $f_Q' = \lambda_Q = 0$. Even in this case, similar contributions are expected from the FCNC’s (Higgs) for the flavor-nonconserving and $CP$-violating processes. It should here be mentioned that with only one real singlet Higgs field the $t$-$U$ mixing is ineffective for the electroweak baryogenesis. This is because the complex phases in the $t$-$U$ couplings to the real singlet Higgs field are eliminated away by rephasing the $U_0$ and $U_0^c$ fields. The alternative form of the quark mass matrices,

$$
\mathcal{M}_Q = \begin{pmatrix}
M_q & 0 \\
\Delta'_{t-Q} & M_Q
\end{pmatrix},
$$

are also possible, as mentioned above, by redefining the right-handed quark fields.

no singlet Higgs field

The singlet Higgs field $S$ may be absent with explicit mass terms $\Delta_{u-U}$ and $M_U$ in eq. (3). Even in this case, the significant FCNC’s with $CP$-violating phases may still be present in the quark couplings to the $Z$ boson and the standard neutral Higgs field. The one-loop neutral Higgs contribution to the NEDM is, however, vanishing, just as the one-loop $Z$ boson contribution. This is because the standard neutral Higgs field and the Nambu-Goldstone mode couple to the quarks in the same way.

6 Summary

Some aspects have been considered on the flavor-nonconservation and $CP$-violation arising from the quark mixings with singlet quarks. In certain models incorporating the singlet quarks, the contributions of the quark couplings to the neutral Higgs fields may become more significant than those of the neutral gauge interactions. Then, they would provide distinct signatures for new physics beyond the standard model in various flavor-nonconserving and $CP$-violating processes such as the NEDM, $D^0-\bar{D}^0$, $K^0-\bar{K}^0$, $B^0-\bar{B}^0$, $b \rightarrow s\gamma$, and so on. It is, for instance, found that the neutral Higgs contributions to the NEDM and the neutral $D$ meson mass difference can be comparable to the present experimental bounds for the case where the singlet Higgs mass scale is of order of the electroweak scale and a significant $CP$-violating $t$-$U$ mixing is present. This situation may be desired for the electroweak baryogenesis with $t$-$U$ mixing.

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