TORSIONAL OSCILLATIONS IN THE SUN’S ROTATION CONTRIBUTE TO THE WALDMEIER-EFFECT IN SOLAR CYCLES

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ABSTRACT

Temporal variations in the Sun’s internal velocity field with a periodicity of about 11 years have been observed over the last four decades. The period of these torsional oscillations and their latitudinal propagation roughly coincides with the period and equatorward propagation of sunspots which originate from a magnetohydrodynamic dynamo mechanism operating in the Sun’s interior. While the solar differential rotation plays an important role in this dynamo mechanism by inducting the toroidal component of magnetic field, the impact of torsional oscillations on the dynamo mechanism – and hence the solar cycle – is not well understood. Here, we include the observed torsional oscillations into a flux transport dynamo model of the solar cycle to investigate their effect. We find that the overall amplitude of the solar cycle does not change significantly on inclusion of torsional oscillations. However, all the characteristics of the Waldmeier effect in the sunspot cycle are qualitatively reproduced by varying only the amplitude of torsional oscillations. The Waldmeier effect, first noted in 1935, includes the important characteristic that the amplitude of sunspot cycles is anti-correlated to their rise time; cycles with high initial rise rate tend to be stronger. This has implications for solar cycle predictions. Our results suggest that the Waldmeier effect could be a plausible outcome of cycle to cycle modulation of torsional oscillations and provides a physical basis for sunspot cycle forecasts based on torsional oscillation observations. We also provide a theoretical explanation based on the magnetic induction equation thereby connecting two apparently disparate phenomena.

Keywords: magnetic fields; Sun: activity; Sun: dynamo; Sun: interior

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1. INTRODUCTION

Figure 1. The observed plot of torsional oscillations in the solar rotation (in nHz) on the solar surface from GONG measurements is depicted here. The butterfly diagram of locations of sunspots (denoted by green dots) for cycles 23-24 obtained from the Royal Greenwich Observatory (NASA 1874) is over-plotted. The similar spatio-temporal evolution and periodicity of the sunspot cycle and torsional oscillations indicate a link between the two.

Helioseismic observations over the past four decades have made it possible to study the Sun’s internal velocity fields. The Sun’s azimuthal rotation rate, which is responsible for the generation of sunspot producing toroidal magnetic field, was believed to be constant in time until Howard & Labonte (1980) found torsional waves propagating towards the equator on the solar surface from the rotation rate measurements at Wilcox solar observatory. About a decade later, the addition of Helioseismology to the bag of tools used to study the Sun provided the opportunity to precisely measure the plasma flows inside the Sun. Helioseismic measurements of the azimuthal rotation rate using data from Big Bear Solar Observatory (BBSO; Woodard & Libbrecht (1993)), Michelson Doppler Imager (MDI; Schou et al. (1998)) and Global Oscillations Network Group (GONG; Howe et al. (2000)) confirmed the existence of torsional waves even in the solar interior. These waves, now known as torsional oscillations (see Fig. 1), appear as bands of faster and slower than average rotation propagating towards the equator in the low-latitude region below 60 degrees and towards the poles in the high-latitude region above 60 degrees (Antia & Basu 2001).

The similarity in the spatio-temporal evolution of the sunspot belt and torsional oscillation pattern (Labonte & Howard 1982) has led many researchers to consider direct Lorentz feedback of the magnetic field on plasma flows, (magnetically mediated) geostrophic flows or other indirect energy transfer mechanisms as plausible causes of torsional oscillations (Covas et al. 2000; Spruit 2003; Rempel 2006; Beaudoin et al. 2013; Guerrero et al. 2016). Models based on these theories have been quite successful in producing torsional oscillation patterns similar to the patterns observed.

Past studies have primarily focussed on exploring theories for the origin of torsional oscillations with the aim of reproducing the observed pattern of torsional oscillations based on feedback mechanisms on the solar plasma. But the generation of torsional oscillations is only half of the story. In this paper we explore the other half, that is once the torsional oscillations are taken into account how do they alter the magnetic field induction process. We achieve this by incorporating the helioseismic measurements of torsional oscillations from GONG observations into a flux transport dynamo model, which explores the effect of plasma flows in the solar interior on the magnetic field induction process. The premise of our numerical experimentation is based on two considerations. One that the magnetic feedback on the plasma flows is weak – which is observed and established theoretically (Rempel 2006); this allows for the overall flux transport principle to be effective. Second, variations in the amplitude of torsional oscillation may impact the nature of the sunspot cycle through changes to the Sun’s internal differential rotation profile.

In section 2.1, we give a brief description of the helioseismic data from GONG that has been used to carry out this study. Then we go on to describe the flux transport dynamo model we have used in this study in section 2.2, and how the torsional oscillations have been introduced in this model in section 2.3. We discuss the results obtained in section 3 and provide a theoretical explanation for our findings in section 3.4. The implications of the results are discussed in section 4.

2. METHODS

2.1. Helioseismic data on torsional oscillations

We use the helioseismic data from the Global Oscillations Network Group (GONG) project (Hill et al. 1996) covering the period from 7 May 1995 to 9 September 2012. This data consists of 174 sets each covering a period of 108 days with a shift of 36 days between successive data sets. This period covers the entire solar cycle 23 and the rising phase of cycle 24. Each data set consists of frequencies and splitting coefficients for
p-modes up to a degree $l = 150$ and for each set we performed a 2d Regularised Least Squares (RLS) inversion (Antia et al. 1998) to calculate the rotation rate as a function of radius and latitude. This gives us the temporal variation in the solar rotation rate. To isolate the temporally varying part associated with torsional oscillations observed at the solar surface we subtract the mean rotation rate at each radius and latitude from the value at each time to get the residual:

$$\delta\Omega(r, \theta, t) = \Omega(r, \theta, t) - \langle \Omega(r, \theta, t) \rangle$$

where the angular brackets denote the temporal average over the period of solar cycle 23 which was estimated to be 11.7 years (Antia & Basu 2010) long.

2.2. Flux transport dynamo model of the solar cycle

The solar magnetic cycle originates via a dynamo mechanism which recycles the toroidal and poloidal components of the solar magnetic field relying on solar plasma flows. For this study we utilize a flux transport dynamo model which has been well studied in different contexts (Nandy & Choudhuri 2002; Chatterjee et al. 2004; Yeates et al. 2008; Passos et al. 2014). This model solves for the magnetic induction equation (in the solar convection zone) given by:

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta \nabla \times B).$$

Assuming axisymmetry, one can split the magnetic field into the toroidal and poloidal components as in equation 3, and the velocity as in equation 4

$$B = Be_\phi + \nabla \times (A e_\phi),$$

$$v = v_p + r \sin \theta \Omega e_\phi.$$  

Plugging equations 3 and 4 into the induction equation gives us the equations for the evolution of the toroidal and poloidal field respectively.

$$\frac{\partial A}{\partial t} + \frac{1}{s} (v, \nabla)(sA) = \eta_p (\nabla^2 - \frac{1}{s^2})A + \alpha B,$$  

$$\frac{\partial B}{\partial t} + \frac{1}{r} \left( \frac{\partial}{\partial r} (rv_B) + \frac{\partial}{\partial \theta} (v_\theta B) \right) = \eta_t \left( \nabla^2 - \frac{1}{s^2} \right)B$$

$$+ s (B_p, \nabla) \Omega + \frac{1}{r} \frac{d \eta_p}{dr} \frac{\partial (rB)}{\partial r}.$$  

where $A$ is the magnetic vector potential for the poloidal field ($B_p$), $B$ is the toroidal field, $v$ is the meridional flow velocity, $\Omega$ is the azimuthal rotation velocity, $\eta$ is the turbulent magnetic diffusivity and $s = r \sin \theta$. These two equations govern the two main mechanisms of the solar dynamo:

1. The $\Omega$-effect: The second term on the right hand side of equation 6 suggests that the poloidal field ($B_p$) would be sheared by the gradient of the azimuthal rotation velocity ($\Omega$) and get wrapped around the Sun to produce magnetic field in the toroidal direction. This effect, known as the $\Omega$-effect, was first proposed by Parker (1955).

2. The Poloidal Source: The second term on the right hand side of equation 5 acts as a source term for the poloidal field. In this model it is assumed that the poloidal source arises completely from the Babcock-Leighton mechanism proposed by Babcock (1961) and Leighton (1969). They proposed that magnetically buoyant toroidal flux tubes are tilted due to the Coriolis force (and erupt as tilted bipolar sunspot pairs) such that they produce a net poloidal component of magnetic field. This results in the conversion of the toroidal field back into the poloidal field through near surface flux transport processes.

We use the same diffusivity profile as Muñoz-Jaramillo et al. (2009) which makes our model a high diffusivity model. The meridional flow velocity profile was modelled as in Nandy & Choudhuri (2002), Yeates et al. (2008) and Passos et al. (2014) with a small component of the flow penetrating beneath the base of the solar convection zone. For computational efficiency we limit our simulation domain to the solar northern hemisphere with a boundary condition at the equator which imposes dipolar parity of magnetic field. We note here that the exact nature or values of the parameters above would not impact the results qualitatively in any case as long as they are within the operational regime of the flux transport dynamo. Therefore, for ease of comparison, we have kept the above driving parameters the same as in previous studies. Here we focus only on the impact of inclusion of torsional oscillations (with a variable amplitude) on the solar dynamo.

2.3. Inclusion of Torsional Oscillations in the Flux transport dynamo

Torsional oscillations are temporal variations in the rate of azimuthal rotation ($\Omega$) in the solar convection zone. Since the amplitude of torsional oscillations (20 nHz) is much smaller than the Sun’s average rotation rate (around 400 nHz), we can include these oscillations in the flux transport dynamo (0.55$R_\odot$ to 1.0$R_\odot$ and 0 to 88 degrees latitude) by augmenting the mean azimuthal rotation rate ($\Omega_0$) by a perturbed quantity ($\delta\Omega$) which
represents the torsional oscillations.

\[ \Omega(r, \theta, t) = \Omega_0(r, \theta) + \delta \Omega(r, \theta, t) \]  

(7)

Although the temporal variation in \( \Omega \) is small, the dynamo equations involve the gradient of \( \Omega \) and the temporal variations in both radial and latitudinal gradient of rotation rate can be much larger (Antia et al. 2008). Hence the effect of temporally varying rotation rate on the solar dynamo requires careful analysis.

Since both the torsional oscillations and the solar cycle are periodic phenomena, one needs to handle their phase relationship appropriately, and ensure that the simulations are initiated with the appropriate relative phase. The data for torsional oscillations from GONG start from 29th June 1995 and continue till 17th July 2015. If we look at the polar field measurements by Wilcox Solar Observatory (Hoeksema 1995; Svalgaard et al. 1978), we find that the polar field was maximum in 1995, and was positive in the northern hemisphere. To reproduce the polar field configuration during 1995, we run the dynamo model without torsional oscillations (with just the mean rotation rate) for a few cycles until the solution becomes stable, and then stop it at a time when the polar field in the northern hemisphere becomes maximum and positive. The magnetic field configuration at the end of this simulation is stored and serves as the initial condition for simulations with torsional oscillations included in the model.

The observed period of the solar cycle, as well as the period of torsional oscillations undergoes statistical variations, and is not fixed. Thus, the period of torsional oscillations may not always be exactly equal to the period of the sunspot cycle. The output of the flux transport dynamo model, however, is a uniformly periodic cycle. Thus, we have to prepare a torsional oscillation profile that has a uniform periodicity so that it can retain its phase relationship with the dynamo output. This has been done by repeating a segment of the torsional oscillations data in time. Antia & Basu (2010) have shown that the torsional oscillation pattern had a period of about 11.7 years for the solar cycle 23. With the standard set of parameters the solar dynamo model used here has a cycle period of 11.23 years. Thus, we have compressed 11.7 year long torsional oscillation data starting from 1995 into 11.23 years to match with the cycle output from the dynamo model. This ensures that the phase relationship between the solar cycle and the torsional oscillation pattern is maintained as shown in Fig. 2. Note that this phase-locking is necessary for utilization of the observed torsional oscillations in a theoretical model and does not impact the sanctity of the simulations in any way.

3. RESULTS

With the phase and the period of torsional oscillations matched with the solar cycle, we vary the amplitude of the torsional oscillation to perform a set of computational simulations as a numerical experiment. Nothing is known about the variation of the amplitude of torsional oscillations yet, as we have continuous observations from GONG for only over one and a half solar cycles. Nevertheless, one expects the amplitude of torsional oscillations to be modulated from one cycle to another. It is our aim here to study the impact of this varying torsional oscillation amplitude on the solar cycle. Without making any changes to the spatio-temporal dependence, we simply change the amplitude of the torsional oscillations by multiplying the torsional oscillation input to the dynamo model with a scaling factor (sc).

3.1. Impact of the amplitude of torsional oscillations on the strength of the solar cycle

Charbonneau & Dikpati (2000) have proposed that the magnetic energy density at about 15° latitude at the base of the convection zone \((r = 0.7R_\odot)\) serves as a good proxy for the sunspot number. Since the magnetic energy density is proportional to \(B^2\) (where \(B\) is the toroidal magnetic field), we have considered the peak value of \(B^2\) as a proxy for the strength of the sunspot cycle and explored how this changes with varying amplitude of torsional oscillations. Henceforth, we use the term sunspot proxy to indicate the value of \(B^2\) at 15.3 degrees latitude at the base of the convection zone \((r = 0.7R_\odot)\).
The temporal shape of the solar cycle for different amplitudes of torsional oscillations are shown in Fig. 3. The variation of the peak value of sunspot proxy is shown in Fig. 4 for different scaling factors. The monotonic increase shown by the peak of the sunspot proxy on increasing the amplitude of torsional oscillations indicates that these oscillations enhance the mechanism for the production of toroidal field by increasing the shear in azimuthal rotation.

3.2. Impact of the amplitude of torsional oscillations on the rise and fall times of the solar cycle

Apart from increasing the peak value of the sunspot proxy, increase in the amplitude of torsional oscillations also changes the nature of the sunspot proxy. To learn more about this nature, we would like to define two quantities here:

1. Rise time: Rise time is defined as the time difference between the phases of the cycle when the...
sunspot proxy increases from zero to 85% of its maximum strength.

2. Fall time: Fall time is defined as the time difference between the phases of the cycle when the sunspot proxy decreases from 85% of its maximum strength to zero.

Without torsional oscillations the sunspot proxy is symmetric about its peak, i.e., it has almost equal rise time (4.22 years) and fall time (4.19 years). When torsional oscillations are introduced, the rise time becomes significantly shorter than the fall time. Although the solution for the toroidal field from the model shows some fluctuations after introducing torsional oscillations, the rise time is still always shorter than the fall time for all scaling factors. The rise and fall times for sunspot cycles for a scaling factor of unity are shown in Fig. 5.

To check for robustness, we run the simulations with all scaling factors for seven sunspot cycles. We find that rise time is always shorter than fall time for all scaling factors and in every cycle. The variation of rise time averaged over seven sunspot cycles with the scaling factor is shown in Fig. 6. As we increase the amplitude of torsional oscillations, the rise time becomes shorter and shorter, while the fall time increases (because the total period is fixed by the meridional flow). The ratio of fall time to rise time is thus always greater than unity and it increases with increasing amplitude of torsional oscillations as shown in Fig. 4. Thus, the greater the amplitude of torsional oscillations, greater is the asymmetry in rise and fall times of the sunspot cycle.

3.3. The Waldmeier effect

In 1935, Waldmeier (1935) noted the following intriguing characteristics of the sunspot cycle from observations of sunspot numbers which later came to be known as the Waldmeier effect.

1. The rise time of an average solar cycle is smaller than its fall time.
2. The strength of a cycle is anticorrelated to its rise time. Shorter the rise time, stronger is the cycle.
3. The strength of a cycle is correlated to its rise rate. The higher the rise rate, the stronger is the cycle.

These characteristics are together called the “Waldmeier effect”. We have already seen the first characteristic in section 3.2. In sections 3.1 and 3.2, we have seen that the rise time of a sunspot cycle decreases while its strength increases with increase in the amplitude of torsional oscillations. Thus, if we plot the strength of the sunspot proxy versus its rise time, we can clearly see the second characteristic of the Waldmeier effect (Fig. 7).

3.4. Theoretical Explanation: How do torsional oscillations contribute to the Waldmeier effect?

The rise time of solar cycles depends directly on the magnetic induction time scale whereas their fall time depends mainly on diffusion or flux cancellation time scale. From Fig. 6 it is clear that torsional oscillations impact the rise time much more than the fall time indicating that torsional oscillations enhance the magnetic induction process early in the solar cycle. To analyze their effect on the induction process, we simplify the toroidal field evolution equation (Eq. 6) to focus on the impact of torsional oscillations and recast it in the form

$$\frac{\partial B_\phi}{\partial t} = s(B_p \nabla) \Omega + \chi$$  \hspace{1cm} (8)
where \( \chi \) = contribution from all remaining terms (which do not directly depend on the rotation and are responsible for advection and diffusion of the magnetic field). Expanding the first term related to the impact of rotational shear on the toroidal field induction we get

\[
\frac{\partial B_{\phi}}{\partial t} = r \sin \theta \left( B_r \frac{\partial \Omega}{\partial r} + \frac{B_\theta}{r} \frac{\partial \Omega}{\partial \theta} \right) + \chi
\]  

(9)

\[
\frac{\partial B_r}{\partial t} = B_r \left( r \sin \theta \frac{\partial \Omega}{\partial r} \right) + B_\theta \left( \sin \theta \frac{\partial \Omega}{\partial \theta} \right) + \chi
\]  

(10)

Utilizing Eq. 7,

\[
\frac{\partial B_{\phi}}{\partial t} = B_r \left( r \sin \theta \frac{\partial \delta \Omega}{\partial r} \right) + B_\theta \left( \sin \theta \frac{\partial \delta \Omega}{\partial \theta} \right) + \chi
\]  

(11)

Thus, the difference in growth rate of the toroidal magnetic field (B) between a solar cycle with torsional oscillations and one without torsional oscillations (denoted by subscript “\( o \)”) will be:

\[
\left( \frac{\partial B_{\phi}}{\partial t} \right)_o - \left( \frac{\partial B_{\phi}}{\partial t} \right) \approx B_r \left( r \sin \theta \frac{\partial \delta \Omega}{\partial r} \right) + B_\theta \left( \sin \theta \frac{\partial \delta \Omega}{\partial \theta} \right)
\]  

(12)

Most kinematic flux transport dynamo models generate toroidal magnetic field primarily from \( B_r \) at high latitudes (above 60 degrees) and from \( B_\theta \) (where \( \theta \) is the colatitude) at lower latitudes between 0.7\( R_\odot \) and 0.8\( R_\odot \) inside the Sun (Muñoz-Jaramillo et al. 2009). Since we are considering only the \( \Omega \)-effect here on the sunspot proxy around 15 degrees latitude and at the bottom of the convection zone, \( B_\theta \) dominates over \( B_r \). If one starts with positive polar flux at solar minimum near the north pole, this implies a positive \( B_r \) but a negative \( B_\theta \) in the northern hemisphere. At low latitudes near the base of the convection zone the sign of the co-latitudeal derivative of \( \Omega \) is positive resulting in the production of a negative \( B_\phi \) which is the our proxy for sunspots. The average observed magnitude of \( \frac{\partial \delta \Omega}{\partial \theta} \) from helioseismic measurements is larger during the rising phase of the cycle compared to the declining phase (see Fig. 8) at low latitudes and 0.7\( R_\odot \). Thus, during the rising phase over a significant region of the convection zone, the growth rate of the magnetic field is higher with torsional oscillations than without – which explains both the steeper rise and the increasing strength seen in solar cycles with higher amplitude of torsional oscillations. This direct relationship between the strength of torsional oscillations and the growth rate (and strength) of the toroidal magnetic field – contributes to the Waldmeier effect. This explains why we recover all the characteristics of the Waldmeier effect in our dynamo simulations driven by the observed torsional oscillations without recourse to any other physics.

It should be noted here that the rise-rate of the cycle depends on the product of the strength of the polar field from the previous cycle (\( B_r \), \( B_\theta \)) and the gradient of \( \Omega \). Since we do not see a dramatic change (more than a few percent) in the strength of the solar cycle upon the inclusion of torsional oscillations, most of the modulation in strength of the solar cycle may be arising from modulation of the strength of polar field at the end of the previous cycle. Compelling evidence for such a correlation between the polar field of the previous cycle and the strength of the solar cycle has been presented by various authors (Schatten et al. 1978; Karak & Nandy 2012; Muñoz-Jaramillo et al. 2013). Inclusion of torsional oscillations thus primarily makes the sunspot cycles rise faster, asymmetric around their peak and contributes less towards modulation of their amplitude.

4. CONCLUDING DISCUSSIONS

Previous attempts of simulating the Waldmeier effect with flux transport dynamos have relied on stochastic fluctuations in the dynamo parameters (poloidal source and meridional circulation) (Karak & Choudhuri 2011;
Pipin et al. 2013); see also Charbonneau & Dikpati (2000). While these fluctuations can certainly contribute to the Waldmeier effect, we have shown that the contribution from torsional oscillations needs to be taken into account as well. We have successfully reproduced all the three characteristics of Waldmeier effect qualitatively by incorporating the observed torsional oscillations and modulating their amplitude without recourse to any changes in other dynamo parameters, including cycle period. The analytical explanation based on induction equation convincingly establishes the theoretical basis of how cycle to cycle variations in torsional oscillations can contribute to the observed Waldmeier effect. Using torsional oscillation data for solar cycle 23 we get a fall time to rise time ratio of 1.14 in our simulations which is less than the observed ratio of 1.7 recorded for solar cycle 23. We note that this does not necessarily mean torsional oscillations contribute less to the Waldmeier effect than other processes because making a quantitative comparison requires a model calibrated with observations. One would require the correct turbulent diffusivity profile, meridional flow profile and measurements of $B_r$ and $B_\theta$ in the solar interior to perform a rigorous quantitative analysis of the extent of the contribution of torsional oscillations to the Waldmeier effect. We emphasize that our theoretical explanation is based on the magnetic induction equation and thus independent of any specific dynamo model.

We also conducted some additional numerical simulations to investigate which branch of torsional oscillations has the dominant effect on the shape of the solar cycle. In one set of simulations, we turned off torsional oscillations in the deep interior (below 0.85$R_\odot$) and in another set we turned off the torsional oscillations in the upper half of the convection zone (above 0.85$R_\odot$). The simulations with torsional oscillations only in the upper half of the convection zone did not reproduce the Waldmeier effect and it looked like the torsional oscillations did not alter the solar cycle in any fashion. The set of simulations with torsional oscillations only in the bottom half of the convection zone on the other hand produced results that are identical to the simulations with torsional oscillations introduced in the entirety of the convection zone. This makes sense because the location of magnetic field amplification in these simulations coincides with the deeper branch of torsional oscillations. Another experiment was conducted to independently investigate the effect of the high latitude and low latitude branches of torsional oscillations on the solar cycle. The division between these branches was made at 60° latitude. The low latitude branch seems to be the one which has the dominating effect on the shape of the solar cycle. It produced a fall time to rise time ratio of 1.10 where as the high latitude branch produced a ratio of 1.03 compared to 1.14 - the ratio obtained for torsional oscillations in the entire convection zone. This indicates that magnetic field - plasma flow interactions in the low-latitude and deeper layers of the convection zone contribute more towards the Waldmeier effect.

Our results show that increasing amplitude of torsional oscillations increase the strength of sunspot cycles (Fig. 4) as well as their rise rate and decrease their rise times (Fig. 6). Thus, the amplitude of torsional oscillations may act as the connection between the strength of the cycle and its rise time noted by Waldmeier (1935) (Fig. 7).

These results have important implications for solar cycle predictions. A reasonably successful precursor technique for solar cycle predictions is based on observations of the early growth-rate of the cycle in question (Cameron & Schüssler 2008; Pesnell 2008). There is also an independent, and not yet rigorously proven, understanding emerging within the scientific community that the nature of torsional oscillation patterns of the extended solar cycle (McIntosh et al. 2014) can indicate the strength of upcoming cycles (Howe et al. 2009; Hill et al. 2015). Our theoretical analysis and dynamo simulations causally connect torsional oscillations to the growth rate and amplitude of sunspot cycles, thereby providing a physical basis for solar cycle predictions based on the Waldmeier effect or early observations of torsional oscillations of the extended solar cycle.

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Torsional Oscillations contribute to the Waldmeier effect

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