Complete Quantum Communication with Security

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Abstract

The long-standing problem of quantum information processing is to remove the classical channel from quantum communication. Introducing a new information processing technique, it is discussed that both insecure and secure quantum communications are possible without the requirement of classical channel.

Around 1970, Wiesner first realized [1] quantum state can be used to ensure security of private data. Security reason apart, the idea of using quantum state for information processing was new, but because of delayed publication it then silently laid the foundation of modern quantum information. However, quantum information received widespread attention after the discovery of quantum key distribution (QKD) [2, 3], quantum computation [3], teleportation
Except the local quantum computation, all are basically quantum communication protocols. To execute these protocols a supplementary classical channel is required. In other words, in quantum communication, classical channel is needed to transmit message. One may inquire why quantum mechanics cannot provide a quantum channel to transmit message.

In their famous paper on teleportation [4], Bennett et al have apparently given some answer to this query. They argued that unknown quantum state cannot be teleported without classical channel. If it is possible it will prove the existence of superluminal signal. The intended meaning and reasoning can be easily understood but the problem is, they have implicitly assumed that classical channel is the only legitimate tool for sending logical 0 and 1 to transmit message. And now the assumption has almost become irrefutable theorem. Here we shall see that logical 0 and 1 can also be coded and decoded in quantum fashion. It means, a qubit can represent logical 0 and 1. It implies that quantum channel can alone transmit message, and in quantum computation, controlled gates can be controlled quantum mechanically by receiving human instructions.

Before constructing our quantum channel let us classify the classical channels. This might help explore the reason behind their assumption.

1. Deterministic classical channel: Here every bit can be deterministically recovered. Example: existing classical channel in absence of noise.
2. Probabilistic classical channel: Here every bit can be statistically recovered. Example: two induced noise levels greater than the environmental noise can represent the two bit values (this channel [5] can provide computational security).

Let us construct channels using quantum states.
1. Channel A: Two orthogonal quantum states represent the two bit values.
2. Channel B: Two nonorthogonal quantum states represent the two bit values.
3. Channel C: Two ensembles of quantum states represent the two bit values (in ensemble interpretation, however, individual state has no place as perceived by Einstein[6]).

The channel A is a deterministic channel and classical message can be sent through it. The channel B is a probabilistic quantum channel and message cannot be sent through it [7]. The channel C is another kind of probabilistic channel but message can be sent through it. The channel A is quantum counterpart of deterministic classical channel and the channel C is quantum counterpart of probabilistic classical channel. These two channels cannot provide quantum encryption where no-cloning principle [8] provides security. For Channel A, no-cloning principle breaks down and for channel C it is not applicable. As a whole, these two channels do not carry any signature of quantum information processing, and therefore, they may not be called as truly quantum channel (like quantum states do not necessarily perform quantum computation). From the above discussion it seems that truly quantum channel does not exist to transmit plain or secure message.

We shall see that such channel do exist. To construct such quantum channel we need two sufficiently distinguishable sequences of quantum states to represent the two bit values. The most simple sequences are the sequences of two nonorthogonal states, say $0^\circ$ and $45^\circ$ single photon polarized states. Suppose in the sequence, representing bit 0, $0^\circ$ and $45^\circ$ photons are at the even and odd positions respectively, and in the sequence representing bit 1, the $0^\circ$ and $45^\circ$ photons are at odd and even positions respectively. Let us also assume that receiver share the information of these two sequences
and only a single bit - a sequence - is sent. The question is, how receiver will definitely recover the bit value by measurements. Suppose receiver sets the analyzer at $0^\circ$ for every even event. If he/she gets "yes" results at every even position then the bit is definitely 0. If he does not get so, the bit is definitely 1. This is a new type of probabilistic quantum channel but not secure quantum channel, and message can be sent through it. Message can be seen as a sequence of two operating sequences of quantum states. For quantum security, legitimate users have to generate a arbitrarily long sequence of randomly operating two sequences of random states by secret sharing of information of the two operating sequences, although the randomness of states of the two operating sequences is not a stringent criteria.

As for example, let us take the following two operating sequences of two different pairs of random quantum states.

\[ S_0 = \{ |A\rangle_1, |A\rangle_2, |B\rangle_3, |B\rangle_4, |A\rangle_5, |B\rangle_6, |A\rangle_7, ...... |A\rangle_n \}; \]

\[ S_1 = \{ |C\rangle_1, |D\rangle_2, |C\rangle_3, |D\rangle_4, |C\rangle_5, |D\rangle_6, |C\rangle_7, |D\rangle_8, ...... |D\rangle_n \}, \]

where $S_0$ and $S_1$ stand for bit 0 and 1 respectively and $n$ is moderately large number. Information of these two sequences $S_0$ and $S_1$ are shared between sender Alice, and receiver Bob. Key, the sequence of random sequences of random quantum states, is, $K_N = \{ S_0, S_1, S_0, S_1, S_0, S_1, S_0, S_1, ...... \}$, where $N$ is the number of bits in the key. Obviously, $N$ is greater than $2n$ since $2n$ bits (standard meaning) are shared.

First, we shall present a QKD protocol using superposition states for the preparation of the two operating sequences. The reason is, if we can construct the channel by superposition states and if it gives quantum security then we do not have any ambiguity about the quantumness of the channel. This particular QKD protocol can be modified to accomplish more sophisticated cryptographic tasks such as key splitting [9,10] and quantum bit commitment [11].
Firstly, we describe the preparation procedure of the two sequences. Suppose, in a secret place, Alice and Bob have $2n$ horizontally polarized ($|↔⟩$) incoherent photons. To prepare a sequence they use $n$ photons. To prepare $S_0$, they split the wave function of each photon with a symmetric (50:50) beam splitter. After splitting they do one of the two things in one of the path, called $s$: toss a coin, and if "tail" they do nothing ($|↔⟩_s \rightarrow |↔⟩_s$) and if "head", unitarily rotates the polarization by $90^\circ$ ($|↔⟩_s \rightarrow |↕⟩_s$). In the other path, called $r$, they do nothing ($|↔⟩_r \rightarrow |↔⟩_r$). They repeat this procedure for $n$ photons. The states are:

$$|A⟩_i = \frac{1}{\sqrt{2}}(|↔⟩_r + |↔⟩_s)$$
$$|B⟩_i = \frac{1}{\sqrt{2}}(|↔⟩_r + |↕⟩_s)$$

To prepare $S_1$, similarly after splitting a wave function they do one of the two things in the path $s$: toss a coin; if "heads", unitarily rotates by $45^\circ$ ($|↔⟩_s \rightarrow |ʅ⟩_s$) and if "tail", unitarily rotates by $135^\circ$ ($|↔⟩_s \rightarrow |ยาย⟩_s$). Similarly in the other path $r$, they do nothing. They repeat this procedure for $n$ photons. The states are:

$$|C⟩_i = \frac{1}{\sqrt{2}}(|↔⟩_r + |ʅ⟩_s)$$
$$|D⟩_i = \frac{1}{\sqrt{2}}(|↔⟩_r + |ยาย⟩_s)$$

By the above operation, they essentially prepare the two operating sequences of superposition states. These superposition states are mutually nonorthogonal states and they can be represented by the following base states:

$$|↔⟩_r, |↕⟩_r, |↔⟩_s, |↕⟩_s$$
In this basis, the density matrix of the two sequences is,

\[ \rho = \frac{1}{4} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

Now they are separated. Alice will transmit the two operating sequences at random. For clarity, let us think that Alice transmits a single bit, either \( S_0 \) or \( S_1 \). Bob’s task is to recover the bit. As density matrices are same, one may think, how Bob will recover it. Bob can independently recover the bit value in different ways since he knows the preparation code of the both types of possible bits, although their density matrices are same (note that, \( \rho_0 = \rho_1 \) for earlier example). Whatever be the identification processes, Bob’s objective is to recover the bit value from the shared information. Basically there are two types of measurement tricks:

1. Sequence of measurements is predetermined according to the preparation codes.
2. Sequence of measurements is not predetermined according to the preparation codes.

In this protocol, we shall use conclusive which path (WP) information to recover the bit value by using the second method. But mere WP information is not enough to identify the states and bit values. Bob needs which-path of which-state (WPWS) information to identify the individual states and bit values. But WPWS information is also not enough, he needs which-path of which-state of which density matrix (WPWSWD) information to identify the states and the bit values (here which density matrix means how it was prepared). Next we shall see how to get WPWS information.

Suppose there are two sets of dual analyzers (DA) on the end of the two resulting paths. Suppose the orientations of DA are: i)
$DA_0 = \{0^\circ : 0^\circ\}$ ii) $DA_1 = \{0^\circ : 45^\circ\}$. The measurements produce three types ($\alpha, \beta, \gamma$) of results: $\alpha = (\sqrt{r} : x_s), \beta = (x_r : \sqrt{s}), \gamma = (x_r : x_s)$ where " $\sqrt{\ }$ " and " $x$ " stand for "yes" and "no" results respectively. The probabilities of these three kinds of results corresponding to the four different superposition states are given in table 1 and 2 considering the statistical weight of the states and orientations of DA. The results $\alpha$ and $\beta$ provide which-path (WP) information and the result $\gamma$ gives no-path (NP) information. The result $\alpha$ does not give any WPWS information for any of the above two settings of DA. The NP information, corresponding to the result $\gamma$, is always inconclusive for any of the two settings of the DA. The only result $\beta$ provides conclusive WPWS information for proper choice of above two settings of DA. The WPWS information conclusively determines the state $|A\rangle_i$ for $DA_0$ and the state $|C\rangle_i$ for $DA_1$. But we need WPWSWD information to recover the bit value.

As Bob does not know the bit value in advance, he uses two sets of DA at random according to the second trick. The measurements yield two sets of random results. Firstly, Bob discards inconclusive results ($\alpha$ and $\gamma$) from both the sets. For the time being, let us assume that the states $|A\rangle_i$ are at even positions and the states $|C\rangle_i$ are at odd positions in the sequence $S_0$ and $S_1$ respectively. If the bit is 0, the reduced sequences of results will like this (see the 2nd col. of the tables),

$P_{n/8} = \{..., \beta_4, ..., \beta_{10}, ..., \beta_{18}, ..., \beta_{32}, ....\}$ for $DA_0$.

If the bit is 1 then it will look like:

$Q_{n/8} = \{..., \beta_3, ..., \beta_{11}, ..., \beta_{21}, ..., \beta_{27}, ....\}$ for $DA_1$

where $n/8$ is the reduced length of the sequences (75% results are discarded). It means Bob will only check whether $\beta$ is missing in odd or even positions of his two reduced sequences of results to recover the bit value. But the state $|A\rangle_i$ and $|C\rangle_i$ will not have regular distributions; they are randomly distributed. Yet the technique of
recovery of bit value is almost same. Bob first discards the states corresponding to the discarded events from both of the shared sequences. Now with these two reduced sequences of results \( P^{n/8} \) and \( Q^{n/8} \) and two reduced sequences of states \( S_0^{n/8} \) and \( S_1^{n/8} \), he performs four correlation tests to identify the bit. Out of these tests, only one of the reduced sequences of results will be totally correlated (assuming noise is not present) with one of the reduced sequences of states. Recovery of the bit value means recovery of WPWSWD information.

**Security**: First, we want security directly from no-cloning principle. It means we need bit by bit by (sequence by sequence) security as this is a repetitive code. In bit by bit security, Alice will not send the next sequence, until she is confirmed that Bob’s received sequence was uncorrupted. Therefore, after transmission of each bit, the feedback communication is necessary. This feedback communication should be authentic communication. Like existing QKD schemes, if we want to use classical authentication [12] we need some additional secret data. It implies that a single created data can be made secure by using more secure data! This is totally unrealistic. We need new kind of authentication technique.

**Quantum authentication (QA)**: For authentication, following feedback technique can be taken. If the received sequence is uncorrupted, Bob can send back the same sequence or the other to Alice. If Alice also found it uncorrupted she will send the next sequence to Bob. Otherwise Alice will stop transmission and reject the two operating sequences. There is no room for peaceful co-existence with eavesdropper. In contrast, in existing QKD, one can compromise with eavesdropper.

**Silent features**:
1. Security and authentication are simultaneously achieved by the
protocol itself. Nowhere it has been revealed.
2. For QKD, classical authentication and classical channel cannot be used (at least classical channel can be used for QA in one of our entanglement-based scheme [13])
3. If the users do not share information of the two operating sequences, still key/message can be recovered at the cost of security. As if receiver impersonates as eavesdropper to clone the key.

Key splitting: Key splitting is one of the important task of cryptography. The purpose is to distribute a key securely to two (or many) receivers to make them mutually dependent on each other. The above two-party protocol can be extended to perform this task.

Suppose two receivers, Bob and Sonu, in the two resulting paths, where Bob is on the path $r$ and Sonu on the path $s$ and both of them share information of the same two operating sequences with Alice. Notice that, only $s$ is the bit-carrying path. So Sonu independently can identify the bit values. That is, she can have WPWSWD information using single analyzer with proper orientation [see discussion in ref. 11]. But Bob cannot. Bob always gets the same truncated state $|↔⟩_r$ which never carries any bit values. To give equal opportunity to Bob, Alice can make the path $r$ as bit-carrying path.

Then the new superposition states are:

\[
\begin{align*}
|A⟩_i &= \frac{1}{\sqrt{2}}(|↔⟩_s + |↔⟩_r) \\
|B⟩_i &= \frac{1}{\sqrt{2}}(|↔⟩_s + |↕⟩_r) \\
|C⟩_i &= \frac{1}{\sqrt{2}}(|↔⟩_s + |rtle⟩_r) \\
|D⟩_i &= \frac{1}{\sqrt{2}}(|↔⟩_s + |-opacity⟩_r)
\end{align*}
\]

Note that Alice does not change their shared secrets, she will use the same two shared sequences to transmit the bit values to either
Bob or Sonu. Note that, the positions of the states in the two operating sequences are not changed only the preparation of the states are changed. Due to this action, both of them are in similar position. Now if Alice randomly selects (randomness is meant for key-splitting) paths to transmit bit values both of them will get 50% bits. So they have to co-operate to access the full key.

Splitting the state vector into many paths and making every path as bit-carrying path at random (randomness is meant for key splitting) the protocol can be extended to distribute the key among many users. As for example, input state (0\text{\tiny s} photon) can be split up into three parts \textbf{r}, \textbf{s} and \textbf{t} (1:1:1) in four following ways, simply using triple-slit and single polarization rotator.

\[
|A\rangle_i = \frac{1}{\sqrt{3}}(|↔\rangle_r + |↔\rangle_s + |↔\rangle_t)
\]
\[
|B\rangle_i = \frac{1}{\sqrt{3}}(|↔\rangle_r + |↔\rangle_s + |↕\rangle_t)
\]
\[
|C\rangle_i = \frac{1}{\sqrt{3}}(| ↔ \rangle_r + | ↔ \rangle_s + | ↑ \rangle_t)
\]
\[
|D\rangle_i = \frac{1}{\sqrt{3}}(| ↔ \rangle_r + | ↔ \rangle_s + | ↔ \rangle_t)
\]

These states can be used to prepare two operating sequences. The density matrix of the sequence of states \(|A\rangle_i\) and \(|B\rangle_i\) (1:1) and the sequence of states \(|C\rangle_i\) and \(|D\rangle_i\) (1:1) in the representation \(R\) corresponding to the following sequence of base states - \(|↔\rangle_r, |↕\rangle_r, |↔\rangle_s, |↕\rangle_s, |↔\rangle_t, |↕\rangle_t\) - is,

\[
\rho = \frac{1}{6} \begin{pmatrix}
2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

Three receivers are at the end of three paths. Here bit-carrying path is \textbf{t}, so only receiver on path \textbf{t} will get the bit. If Alice randomly changes the bit-carrying path giving equal importance to
each path, then each of the three receivers will get 33.33% bits of the key. Of course, who will get the bit it is his/her duty to pursue QA. Here op-operation is required only to access the full key. This is accomplished by the same apparatus. Therefore, the protocol will be more powerful than other key splitting protocols which use separate apparatus, since any denial of receiving bit values can be legally challenged by sender and other receiver(s). Clearly this advantage arises from superposition states. In each bit level, co-operation can be guaranteed in our entanglement-based scheme [14].

In all the above protocols, states are mutually non orthogonal and density matrices of the two sequences are same. Mutual nonorthogonality and equivalence of density matrices are the most powerful combinations for this alternative QKD on disentangled state. Suppose the two sequences, having same density matrices, \( \rho_0 = \rho_1 \) are prepared by two different pair of orthogonal states (say, one sequence is made by 0° and 90° and the other is made by 45° and 135° polarized states). This is another interesting quantum channel however, but so far security is concerned this will be a weak quantum channel. Intercepting a single sequence, Eve will not get the bit value but can evade detection if she fortunately chooses the correct orthogonal basis. On the other hand, in a protocol where two sequences of two same or different pairs of non orthogonal states, having unequal density matrices, \( \rho_0 \neq \rho_1 \) are used, Eve can get the bit value from the single sequence but cannot evade detection (for the above protocols \( \rho_0 \neq \rho_1 \) if coherent states are used). If two criteria are imposed i.e. density matrices are same and states are non orthogonal for both sequences, then Eve will not get the bit value still she will introduce error. As if bank robber is caught as soon as he/she proceeds towards the bank. The system is extremely sensitive. In contrast, in the existing QKD protocols eavesdropping is detected only after leakage of some of the bit values.
On the basis of indistinguishability of differently prepared density matrix, once Park claimed [15] to have refuted the legitimacy of individual quantum state description. Here we saw that same density matrix can be quantum mechanically distinguished (in his book, D’espagnat [16] presented a classical separation method). But we would not like to make any counter claim, rather we leave the issue of quantum state description to be freshly reviewed in the light of this information processing.

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Table 1. Joint probabilities when DA at \((0^\circ : 0^\circ)\)

| States | \(P_{(\sqrt{x} : x)}\) | \(P_{(\times r : \sqrt{x})}\) | \(P_{(\times r : x)}\) |
|--------|-----------------|-----------------|-----------------|
| \(|A\rangle_i = 1/\sqrt{2}(|\leftrightarrow\rangle_r + |\leftrightarrow\rangle_s)|\ | 1/4 | 1/4* | 0 |
| \(|B\rangle_i = 1/\sqrt{2}(|\leftrightarrow\rangle_r + |\downarrow\rangle_s)|\ | 1/4 | 0 | 1/4 |
| \(|C\rangle_i = 1/\sqrt{2}(|\leftrightarrow\rangle_r + |\uparrow\rangle_s)|\ | 1/4 | 1/8 | 1/8 |
| \(|D\rangle_i = 1/\sqrt{2}(|\leftrightarrow\rangle_r + |\uparrow\rangle_s)|\ | 1/4 | 1/8 | 1/8 |

* Only this result (\(\beta\)) provides conclusive WPWS information.

Table 2. Joint probabilities when DA at \((0^\circ : 45^\circ)\)

| States | \(P_{(\sqrt{x} : x)}\) | \(P_{(\times r : \sqrt{x})}\) | \(P_{(\times r : x)}\) |
|--------|-----------------|-----------------|-----------------|
| \(|A\rangle_i = 1/\sqrt{2}(|\leftrightarrow\rangle_r + |\leftrightarrow\rangle_s)|\ | 1/4 | 1/8 | 1/8 |
| \(|B\rangle_i = 1/\sqrt{2}(|\leftrightarrow\rangle_r + |\downarrow\rangle_s)|\ | 1/4 | 1/8 | 1/8 |
| \(|C\rangle_i = 1/\sqrt{2}(|\leftrightarrow\rangle_r + |\uparrow\rangle_s)|\ | 1/4 | 1/4* | 0 |
| \(|D\rangle_i = 1/\sqrt{2}(|\leftrightarrow\rangle_r + |\uparrow\rangle_s)|\ | 1/4 | 0 | 1/4 |

* Only this result (\(\beta\)) provides conclusive WPWS information.