Prediction of Interfacial Heat Transfer Coefficients by Using a Modified Lump Capacitance Method for Aluminum Casting in a Green Sand Mold

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This paper analyzes the heat flux and heat transfer coefficient at the mold/metal interface in the green sand mold casting of a cylindrical aluminum component. In the present casting experiment, thermocouples are used to measure the temperature at various points in the molten metal and the sand mold. Using the acquired temperature data, the heat flux and heat transfer coefficient at the sand-mold/metal interface are computed using a modified lump capacitance method, in which the latent heat term is calculated by considering the time-dependent variation of the solid/liquid interface location. For comparison purposes, the heat flux and heat transfer coefficient are also computed using Beck’s inverse scheme. The results obtained from the two computational methods are found to be similar. The casting process is simulated by entering the values of the interfacial heat transfer coefficients computed by the two schemes into a commercial finite element analysis program (FIDAP). The cooling curve computed using the interfacial heat transfer coefficient determined by the modified lump capacitance method is found to be in good agreement with the experimental results. The predicted solidification time differs from that observed experimentally by just 2.8%. Furthermore, it is found that the Biot number is very small for the present aluminum casting. Therefore, the present results verify the feasibility of using the modified lump capacitance method to compute the interfacial heat flux and heat transfer coefficient in the green sand mold casting of small-sized components.

KEY WORDS: mold/metal interface; air gap; modified lump capacitance method and inverse method.

1. Introduction

In green sand mold casting, molten metal is poured into a mold cavity at room temperature. When the molten metal contacts the mold wall, its temperature drops to the solidification point very rapidly. Therefore, gas holes are easily formed because air rolls into the casting mold with the melt during the filling process and the water originally held in the green sand mold is vaporized by the heat from the melt.1–3) To minimize the occurrence of such casting defects, commercial software such as PROCAST has been developed to support theoretical investigations into various aspects of the casting process. However, the accuracy of the theoretical results is largely dependent on the availability of a suitable mathematical model of the casting process and an appropriate specification of the thermal parameter values.

In the casting process, the contact between the mold and the metal is not perfect, and hence an interfacial heat transfer coefficient is generally employed to evaluate the quantity of heat flux transferred from the metal to the mold. However, an accurate value of the interfacial heat transfer coefficient is not easily obtained. Consequently, the objective of this study is to develop a simple and reliable method for identifying the interfacial heat transfer coefficient in the green sand mold casting process.

Ho and Pehlke4,6) calculated the interfacial heat transfer coefficient by first estimating the interfacial heat flux using Beck’s inverse method6,7) based on temperature data measured at points near the cast/chill interface. Subsequently, two linear transducers were employed to measure the respective displacements of the chill and the cast component during the solidification process such that the average gap size at the interface could be obtained. Based on the computed gap size, the interfacial heat flux and the interfacial heat transfer coefficient were calculated by taking into account the effects of conduction and radiation heat transfer and making an assumption of quasi-steady state conditions. Gafur et al.8) applied Beck’s inverse method and a one-dimensional heat transfer model to calculate the interfacial heat transfer coefficient during the solidification of commercially pure aluminum. Santos et al.9) used Beck’s inverse method to compute the interfacial heat transfer coefficient in chill mold castings. Additionally, the authors applied a heat resistance concept to calculate the interfacial heat transfer coefficient at the interface between the cast and the environment. In their study of the heat transfer coefficient in the casting of A366 aluminum alloy, Hwang et al.10) established four temperature measurement points. The data obtained from two of these points were regarded as boundary conditions, while the data from the other two points were used for calibration purposes. The temperature data were utilized to calculate the interfacial heat transfer coefficient.
flux by applying Beck’s inverse method. Using two highly sensitive displacement gauges, the variation of the gap size between the metal and the mold was measured during the casting process. The heat flux across the gap and the interfacial heat transfer coefficient were then calculated by taking both radiation and conduction effects into account.

Browne and O’Mahoney\textsuperscript{11} developed a novel inverse method for calculating the temperature at the metal/mold interface and computing the interfacial heat transfer coefficient by combining a derivative scheme with an integral scheme. Kobryn and Semiatin\textsuperscript{12} classified castings as either “shrink-off” or “shrink-on” depending on their geometric profiles, i.e. solid cylinders were classified as shrink-off, while hollow cylinders were classified as shrink-on. Based on a guessed or modified interfacial heat transfer coefficient, the authors used PROCAST finite element software to calculate the temperature curve in Ti-6Al-4V castings. The heat transfer coefficient was calculated iteratively by comparing the computed curve with a calibration curve based on measured temperature data until a convergent solution was obtained. Prasanna Kumar and Kamath\textsuperscript{13} used Beck’s inverse method to estimate the interfacial heat flux in bar and plate aluminum alloy castings by dividing the metal/mold interface into one, two and three boundary segments, respectively.

Pehlke et al.\textsuperscript{14,16} employed two different methods to compute the interfacial heat transfer coefficient depending on whether or not an air gap existed at the sand-mold/metal interface. For the case of a perfect contact between the mold and the metal, it was shown that the interfacial heat coefficient was related directly to the size and roughness of the mold/metal contact area. However, when a gap existed between the mold and the metal, the authors calculated the interfacial heat transfer coefficient on the basis of the measured gap size. The computed heat transfer coefficients were then compared to the values obtained from Beck’s method. Narayan Prabhu and Griffiths\textsuperscript{16} proposed a model for computing the interfacial heat transfer coefficient during the solidification of cast iron in a sand mold based on the roughness characteristics of the casting and mold surfaces. Nishida\textsuperscript{17} used the explicit finite difference method to compute the temperature distribution based on a guessed or modified heat transfer coefficient. The value of the computed heat transfer coefficient was then refined iteratively by comparing the computed temperature with the measured temperature until a convergent solution was obtained. The author also utilized an extrapolation method to calculate the temperature of the metal and mold, respectively, at the metal/mold interface based on temperature measurements taken near the interface. The heat flux and heat transfer coefficient were then computed based on the extrapolated interfacial temperature.

In analyzing the heat transfer in a casting process, computing the heat flux transferred from the casting to the mold requires a knowledge of the interfacial heat transfer coefficient. However, the literature contains no generic formulae or simple methods for obtaining the value of this coefficient for different types of mold/metal interface. Generally, the interfacial heat transfer coefficient is estimated using some form of inverse or extrapolation method. However, in inverse methods, the calculations are complicated by a number of factors, including the released latent heat, undercooling during solidification, and movement of the moisture front in the sand mold. Consequently, this study proposes a modified lump capacitance method for calculating the interfacial heat transfer coefficient during the green sand mold casting of an aluminum cylinder. The results are compared with those predicted by Beck’s inverse method. To confirm the validity of the proposed method, the interfacial heat transfer coefficients obtained from the proposed lump capacitance method and Beck’s method, respectively, are entered into a commercial finite element program (FIDAP) to simulate the casting process. The computed cooling curves and solidification times are then compared with the experimental results.

\section{Experimental Methods}

The green sand mold casting experiment performed in this study used the gating system shown in Fig. 1. The casting was a 99.86\% pure aluminum cylinder measuring 14.7 cm in length by 4.6 cm in diameter. As shown, the entrance of the molten metal into the mold cavity was located at the center of the cylinder. Figure 2 indicates the positions of four temperature measurement points, i.e. $T_1$, $T_2$, $T_3$, and $T_4$, located at distances of 2 mm, 12 mm, 30 mm, and 70 mm, respectively, from the mold/metal interface. $T_{\text{sand}}$ and $T_{\text{metal}}$ denote the temperatures of the green sand mold and the metal at the interface, respectively. It is difficult to measure $T_{\text{metal}}$ precisely as the molten metal solidifies. However, since the location of $T_1$ is very close to the interface, $T_1$ is chosen to represent $T_{\text{metal}}$. To calculate the interfacial heat transfer coefficient, this study uses a modified lump capacitance method and Beck’s inverse method, respectively, to compute the heat flux, $q_{\text{metal}}$, transferred
from the metal to the sand mold at the mold/metal interface.

2.1. Modified Lump Capacitance Method

Since aluminum has a high conductivity, in analyzing the heat transfer during the solidification of the present small casting, it can be assumed that the temperature is distributed uniformly throughout the entire casting and varies only with time. Consequently, the mold and casting can be regarded as a lump system, and the heat transfer which takes place during the solidification process can be analyzed by establishing a total heat balance for the system. In a previous study, the current authors proposed a lump capacitance model for the green sand mold casting process. Due to the geometric symmetry of the casting, it is only necessary to consider one half of the casting in the heat transfer analysis. According to the heat balance prescribed in the original lump capacitance method, the heat flux, \( q_{metal} \), can be expressed as:

\[
q_{metal}(t) = \frac{\rho V}{A} \left[ -C_p \frac{dT}{dt} + L_f \frac{df_s}{dt} \right] \tag{1}
\]

where \( \rho \), \( C_p \), \( L_f \), and \( f_s \) are the density, specific heat, latent heat, and solid fraction, respectively. Additionally, \( V \) and \( A \) are the volume and surface area of the half casting, respectively. \( T_s \), located at the center of the half casting, is chosen to represent the temperature of the lump system. Conceptually, the latent heat at \( T_3 \) is calculated using the temperature data acquired at points \( T_1 \), \( T_2 \), \( T_3 \) and \( T_4 \), as shown in Fig. 3, in which \( S \) is the distance of the solid/liquid interface from the mold/metal interface. Accordingly, the fraction of solid, \( f_s \), can be defined as:

\[
f_s = \frac{S}{L} \tag{2}
\]

where \( L \) is the length of the half casting. The latent heat term in Eq. (1) is then calculated using \( f_s \).

2.2. Beck’s Inverse Method

In this study, the heat transfer coefficient at the mold/metal interface was also calculated using Beck’s inverse method, based on the temperature data measured near the interface during the casting experiment. Referring to Fig. 2, \( T_i \) denotes a calibration point, while \( T_2 \) and the guessed or modified \( q_{metal} \) represent the boundary conditions of the direct problem. In this study, \( q_{metal} \) is calculated using the measured or computed temperature at two different time-steps. For a given value of \( q_{metal} \), the accumulated error is defined as:

\[
F(q_{j+1}) = \sum_{i=1}^{2} \left( Y_{\eta_i} - Y_{\eta_i} \right)^2 \tag{3}
\]

where \( T \) and \( Y \) are respectively the computed and measured temperatures at the calibration point. Additionally, \( j \) and \( i \) are time indices, where \( \eta = 2j \) and \( q_{j+1} \) is the estimated or modified value of \( q_{metal} \). To minimize the accumulated error, Eq. (3) is differentiated with respect to \( q_{j+1} \) and set to zero, i.e.

\[
\frac{\partial F(q_{j+1})}{\partial q_{j+1}} = 0 \tag{4}
\]

Substituting Eq. (3) into Eq. (4) gives:

\[
\sum_{i=1}^{2} \left( T_{\eta_i} - Y_{\eta_i} \cdot \frac{\partial T_{\eta_i}}{\partial q_{j+1}} = 0 \tag{5}
\]

For the \( \lambda \)-th iteration, the Taylor series approximation of \( T_{\eta_i} \), has the form:

\[
T_{\eta_i}^\lambda = T_{\eta_i}^{\lambda-1} + \left( \frac{\partial T_{\eta_i}^{\lambda-1}}{\partial q_{j+1}} \right) \left( q_{j+1}^{\lambda-1} - q_{j+1}^{\lambda-1} \right) \tag{6}
\]

In Beck’s method, \( \frac{\partial T_{\eta_i}^{\lambda}}{\partial q_{j+1}} \) represents the sensitivity coefficient, \( \phi_{q_{j+1}} \), and is calculated by:

\[
\phi_{q_{j+1}} = \frac{\partial T_{\eta_i}^{\lambda}}{\partial q_{j+1}} = T_{\eta_i}^{\lambda-1}(1 + \varepsilon) \cdot q_{j+1}^{\lambda-1} - T_{\eta_i}^{\lambda-1}/\varepsilon q_{j+1}^{\lambda-1} \tag{7}
\]

where \( \varepsilon \) is an extremely small quantity and is set to 0.001 in the current case. \( T_{\eta_i}^{\lambda}(q_{j+1}^{\lambda-1}) \) is the computed temperature for \( q_{j+1}^{\lambda-1} \) and similarly \( T_{\eta_i}^{\lambda-1}(1 + \varepsilon) \cdot q_{j+1}^{\lambda-1} \) is the computed temperature for \( q_{j+1}^{\lambda-1} \). Substituting Eq. (7) into Eq. (6) gives:

\[
T_{\eta_i}^\lambda = T_{\eta_i}^{\lambda-1} + \phi_{q_{j+1}} \cdot \delta q_{j+1}^{\lambda-1} \tag{8}
\]

where \( \delta q_{j+1}^{\lambda-1} = q_{j+1}^{\lambda-1} - q_{j+1}^{\lambda-1} \). Combining Eq. (8) and Eq. (6) gives:

\[
\delta q_{j+1}^{\lambda} = \sum_{i=1}^{2} \left( \phi_{q_{j+1}}(Y_{\eta_i} - T_{\eta_i}^{\lambda-1}) \right) \tag{9}
\]

The iteration is convergent as \( \delta q_{j+1}^{\lambda} \cdot q_{j+1}^{\lambda} = 10^{-5} \) and \( q_{j+1} \) is the heat flux at the time step. Repeating the previous steps yields \( q_{metal} \) for the next time step.
2.3. Calculation of Interfacial Heat Flux and Heat Transfer Coefficient

Due to the non-perfect contact between the sand mold and the metal during the casting process, \( q_{\text{metal}} \) is calculated from the interfacial heat transfer coefficient, \( h \). In the present study, two different methods are employed to determine the value of the heat transfer coefficient, namely:

1. It is assumed that the interfacial heat transfer takes place under nearly quasi-steady state conditions such that the heat flux provided by the molten metal is equivalent to the heat flux received by the sand mold. The interfacial heat transfer coefficient, \( h_1 \), can therefore be expressed as:

\[
h_1 = \frac{q_{\text{metal}}}{(T_{\text{metal}} - T_{\text{mold}})} \text{..................................(10)}
\]

where \( q_{\text{metal}} \) indicates the heat flux from the metal at the mold/metal interface and \( T_{\text{metal}} \) and \( T_{\text{mold}} \) are the temperatures of the metal and the sand mold at the interface, respectively.

2. Due to the low thermal conductivity of the sand mold, the solidification process is analogous to a high-temperature body cooling in air. Therefore, Newton’s cooling law can be applied to derive the interfacial heat transfer coefficient, \( h_2 \), as:

\[
h_2 = \frac{q_{\text{metal}}}{(T_{\text{metal}} - T_{\infty})} \text{..........................(11)}
\]

where \( T_{\infty} \) is the ambient temperature and is specified as 28°C in the current case.

3. Numerical Method

In this work, to confirm the validity of the proposed modified lump capacitance method, the finite element computer package FIDAP is used to simulate the casting process by utilizing the \( h_1 \) data obtained from the Beck’s and the proposed methods. The computed cooling curves and solidification times are then compared with the experimental results. Since the casting process is a transient problem of heat transfer and the effective specific heat method is used to handle the release of latent heat during phase change, the governing equation can be expressed as

\[
\rho C_p^{\text{eff}} \frac{\partial T}{\partial t} = k \nabla^2 T \text{..................................(12)}
\]

where \( \rho \) and \( k \) and \( C_p^{\text{eff}} \) are the density and thermal conductivity, respectively. \( C_p^{\text{eff}} \) is the effective specific heat and its relationship with temperature can be written as

\[
C_p^{\text{eff}} = \begin{cases} 
C_{pS} & T < T_i - \Delta T \\
\frac{1}{2} \left( L_i + C_{pS} + C_{pL} \right) & T_i - \Delta T \leq T \leq T_i + \Delta T \\
C_{pL} & T > T_i + \Delta T 
\end{cases} \text{..................................(13)}
\]

where \( C_{pS}, C_{pL} \) and \( L_i \) are the liquid and solid specific heat and latent heat. In the effective specific method, a small temperature interval \( \Delta T \) taken near the solidification temperature \( T_i \) is used to deal with the effect of latent heat and the range from \( T_i - \Delta T \) to \( T_i + \Delta T \) is sometimes called the artificial mushy zone.\(^{[19]}\) The initial condition of the pouring temperature and the boundary condition is given by

\[
-k \frac{\partial T}{\partial n} = h_{\text{eff}}(T - T_{\infty}) \text{..................................(14)}
\]

where \( n \) is the normal direction of the boundary.

In FIDAP\(^{[20]}\) the Galerkin method is utilized to derive the finite element equations and the skyline storage mode and the LU decomposition scheme are applied to solve the resulting algebraic equations. The mesh for the cylindrical casting is shown in Fig. 4, which is built up by using the twenty-node hexahedral elements. In solving the casting problem, the backward Euler time integration method is used and the Newton–Raphson scheme is utilized for the nonlinear iterations.

4. Results and Discussions

The modified lump capacitance method proposed in this work is applied to a small-sized casting. Firstly, in a small casting, it is not convenient to use many thermocouples to measure their temperatures for obtaining the interfacial heat transfer coefficients. Besides, the more thermocouples are utilized, the more interferences in the heat-transfer and solidification process are caused. This will influence the resulting heat transfer coefficients. Secondly, the thermal conductivity of the casting is high, whose corresponding Biot number (shown below) is very small. It is reasonable to use a lump capacitance method to solve a heat transfer problem.

Figure 5 presents the cooling curves at the four temperature measurement positions indicated in Fig. 2 during the aluminum casting process. It is observed that the temperature at each measurement point drops rapidly to the solidification temperature and then remains at this temperature until the latent heat is released. Clearly, the length of the horizontal line segments represents the time for which the temperature at the measurement point remains at the solidification temperature. The length of the line segments increases as the distance of the measurement point from the mold/metal interface increases. Therefore, the length of the line segment is directly related to the heat transfer situation in the casting process.

As shown in Fig. 5(b), the interior temperature of the casting remains essentially uniform in both the liquid and the solid states. Accordingly, it is reasonable to apply the lump capacitance method to compute the energy variation caused by the specific heat in the casting process. The main difference between the four cooling curves shown in Fig. 5(a) is their solidification time, which is related to the release of latent heat. Consequently, the modified lump capacitance method proposed in Sec. 1 is used in the present analysis to take account of the latent heat effect in a more realistic manner.
Figure 6 illustrates the variation of the interfacial heat transfer coefficients $h_1$ and $h_2$ over time, as calculated by the modified lump method. In the casting experiment, liquid metal at a temperature of 720°C was poured into a room-temperature sand mold. After just 12.4 s, the temperature of the molten metal fell to 660.3°C, which is close to the melting point. As shown in Fig. 6, the interfacial heat transfer coefficient increases rapidly as the latent heat is released during the solidification process and then falls equally rapidly once the process is completed.

The interfacial heat transfer coefficient was also calculated using Beck’s inverse method. The corresponding results for $h_1$ and $h_2$ are shown in Fig. 7. Note that in the inverse computations, the initial temperature of the liquid aluminum was specified as 679.1°C. In Fig. 7, it can be seen that the heat transfer coefficient increases rapidly as soon as the metal is poured into the mold since the metal and the mold form a perfect contact initially. However, $h_1$ and $h_2$ then reduce to their minimum values after approximately 14.7 s. At this point, the metal at the mold/metal interface forms a solidified shell, while the metal further from the interface remains in a liquid state. As in Fig. 6 (computed using the modified lump capacitance method), the results calculated by Beck’s inverse method indicate that the interfacial heat transfer coefficient increases to its maximum value as the latent heat is released.

Figure 8 illustrates the time variation of the heat flux at the mold/metal interface as calculated by the modified lump method and Beck’s inverse method, respectively. In the modified lump method, the entire surface of the cylinder casting is considered. However, in Beck’s method, only the side surface of the casting vertical to the axial direction is taken into account. Therefore, in Fig. 8, it can be seen that the peak value of the heat flux computed by the modified lump method is higher than that calculated by the inverse scheme.

Comparing Figs. 6 and 7, it is apparent that the time-dependent variations of $h_1$ and $h_2$, respectively, are similar in both figures. Furthermore, the variations of $h_1$ and $h_2$ in Figs. 6 and 7 resemble those of the interfacial heat flux shown in Fig. 8. Moreover, $h_1$ is consistently higher than $h_2$ because the latter is calculated on the basis of the ambient temperature (see Eq. (11)), while $h_1$ is based on the mold temperature at the mold/metal interface (see Eq. (10)).

To confirm the validity of the lump capacitance method, an effective Biot number is defined with the form:

$$Bi = \frac{(h_2)_{\text{average}} L_s}{k} \quad (15)$$

where $(h_2)_{\text{average}}$ is the average value of $h_2$ over the duration of the solidification process; $L_s$ is the length scale, defined as the ratio of the volume of the casting to its surface area; and $k$ is the thermal conductivity of the casting. The Biot number for the current aluminum casting is found to be 1757 © 2007 ISIJ
T3 derived from the experimental and computed temperature of point T4 using the FIDAP to simulate the casting process. The experimental solidification time at point T3 is 126.6 s, in Beck’s inverse scheme, the value computed for calculated using Beck’s method. This result is reasonable since closer agreement with the experimental curve than that calculated using the modified lump capacitance method in this study, the interfacial heat transfer coefficient, h2, was input to the commercial finite element program FIDAP to simulate the casting process.

Figure 9 shows the cooling curves at measurement point T3 derived from the experimental and computed temperature data, respectively. It is observed that the cooling curve based on the results of the modified lump capacitance method is in closer agreement with the experimental curve than that calculated using Beck’s method. This result is reasonable since in Beck’s inverse scheme, the value computed for h2 is valid only for the casting surface vertical to the axial direction. However, the temperature measurement data required to calculate h2 for the surface vertical to the radial direction in Beck’s method are not easily obtained. Figure 9 shows that the experimental solidification time at point T3 is 126.6 s, while that calculated by the modified lump capacitance method is 122 s. The relative error between these two results is just 2.8%. Except point T3, the computed cooling curve of point T4 using the h2 data from the modified lump method is also compared with the experimental one, as shown in Fig. 10. Point T4 is very close to the center of the casting. In the figure, the computed cooling curve is consistent with the experimental one and so is the solidification time. From these comparisons, it can be concluded that the modified lump capacitance method provides a feasible approach for computing the interfacial heat transfer coefficients in small-sized castings.

5. Summary
This study has calculated the heat flux and heat transfer coefficient at the mold/metal interface in the green sand mold casting of aluminum using a modified lump capacitance method and Beck’s inverse method, respectively. Based on the current experimental and simulation results, the following conclusions can be drawn:

(1) The use of the modified lump capacitance method to calculate the heat flux and the heat transfer coefficient at the mold/metal interface overcomes the problems inherent in Beck’s inverse method of measuring the temperature at the mold/metal interface and obtaining a convergent solution.

(2) The modified lump capacitance method proposed in this study overcomes the limitation of the original lump capacitance method by treating the latent heat effect during the solidification process in a more realistic manner.

(3) The time-dependent variations of the interfacial heat flux and the heat transfer coefficient calculated by the modified lump capacitance method are in good agreement with the results predicted by Beck’s inverse scheme. However, the values of the heat flux and heat transfer coefficient calculated by the modified lump capacitance method are higher than those calculated by Beck’s method.

(4) Both the interfacial heat flux and the heat transfer coefficient reach their peak value as the casting releases most of its latent heat.

(5) The Biot number is found to be very small for the current aluminum casting. By using the h2 heat transfer coefficient data from the modified lump capacitance method, the computed cooling curve and solidification time are found to be in good agreement with the experimental results. Therefore, the feasibility of using the proposed modified lump capacitance method to compute the interfacial heat transfer coefficient in small-sized castings is confirmed.

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