Higher Dimensional Cosmology with Some Dark Energy Models in Emergent, Logamediate and Intermediate Scenarios of the Universe

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We have considered $N$-dimensional Einstein field equations in which four-dimensional space-time is described by a FRW metric and that of extra dimensions by an Euclidean metric. We have chosen the exponential forms of scale factors $a$ and $d$ numbers of $b$ in such a way that there is no singularity for evolution of the higher dimensional Universe. We have supposed that the Universe is filled with K-essence, Tachyonic, Normal Scalar Field and DBI-essence. Here we have found the nature of potential of different scalar field and graphically analyzed the potentials and the fields for three scenario namely Emergent Scenario, Logamediate Scenario and Intermediate Scenario. Also graphically we have depicted the geometrical parameters named statefinder parameters and slow-roll parameters in the higher dimensional cosmology with the above mentioned scenarios.

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I. INTRODUCTION

From recent observations it is strongly believed that the most interesting problems of particle physics cosmology are the origin due to accelerated expansion of the present Universe. The observation from type Ia supernovae [1,2] in associated with Large scale Structure [3] and Cosmic Microwave Background anisotropies(CMB) [4] have shown the evidences to support cosmic acceleration. The theory of Dark energy is the main responsible candidate for this scenario. From recent cosmological observations including supernova data [5] and measurements of cosmic microwave background radiation(CMBR) [4] it is evident that our present Universe is made up of about 4% ordinary matter, about 74% dark energy and about 22% dark matter. Several interesting mechanisms have been suggested to explain this feature of this Universe, such as Loop Quantum Cosmology (LQC) [6], modified gravity [7], Higher dimensional phenomena [8], Brans-Dicke theory [9], brane-world model [10] and many others.

Recently many cosmological models have been constructed by introducing dark energies such as Phantom [11], Tachyon scalar field [12], Hessence [13], Dilaton scalar field [14], K-essence scalar field [15], DBI essence scalar field [16], and many others. After realizing that many interesting of particle interactions need more than four dimensions for their formulation, the study of higher dimensional theory has been revived. The model of higher dimensions was proposed by Kaluza and Klein [17,18] who tried to introducing an extra dimension which is basically an extension of Einstein general relativity in 5D. The activities of extra dimensions also verified from the STM theory [19] proposed recently by Wesson et al [20]. As our space-time is explicitly four dimensional in nature so the ‘hidden’ dimensions must be related to the dark matter and dark energy which are also ‘invisible’ in nature.

Form the cosmological observation the present phase of acceleration of the Universe is not clearly understood. Standard Big Bang cosmology with perfect fluid assumption fails to accommodate the observational fact. Recently, Ellis and Maartens [21] have considered a cosmological model where inflationary cosmologies exist in which the horizon problem is solved before inflation begins, no big-bang singularity exist, no exotic physics is involved and quantum gravity regime can even be avoided. An emergent Universe model if developed in a consistent way is capable of solving the conceptual problems of the big-bang model. Actually the Universe starts out in the infinite past as an almost static Universe and expands slowly, eventually evolving into a hot big-bang era. An interesting example of this scenario is given by Ellis, Murugan and Tsagas [22], for a closed Universe model with a minimally coupled scalar field $\phi$, which has a special form of interaction potential $V(\phi)$. There are several features for the emergent Universe [21,23] viz. (i) the Universe is almost static at the

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finite past, (ii) there is no time like singularity, (iii) the Universe is always large enough so that the classical description of space-time is adequate, (iv) the Universe may contains exotic matter so that the energy condition may be violated, (v) the Universe is accelerating etc.

Here we also consider another two scenarios: (i) “intermediate scenario” and (ii) “logamediate scenario” [24-27] to study of the expanding anisotropic Universe in the presence of different scalar fields. In the first case the scale factors evolves separately as \( a(t) = \exp(At^f) \) and \( b(t) = \exp(Bt^{f2}) \) where \( A > 0, B > 0, 0 < f_1 < 1 \) and \( 0 < f_2 < 1 \). So the expansion of the Universe is slower than standard de Sitter inflation (arises when \( f_1 = f_2 = 1 \)) but faster than power law inflation with power greater than 1. The Harrison - Zeldovich spectrum of fluctuation arises when \( f_1 = f_2 = 1 \) and \( f_1 = f_2 = 2/3 \). In the second case we analyze the inflation with scale factors separately of the form \( a(t) = \exp(A\ln(t)^{\lambda 1}) \) and \( b(t) = \exp(B\ln(t)^{\lambda 2}) \) with \( A > 0, B > 0, \lambda_1 > 1 \) and \( \lambda_2 > 1 \). When \( \lambda_1 = \lambda_2 = 1 \) this model reduces to power law inflation. The logamediate inflationary form is motivated by considering a class of possible cosmological solutions with indefinite expansion which result from imposing weak general conditions on the cosmological model.

In this work, we have considered N-dimensional Einstein field equations in which 4-dimensional space-time is described by a FRW metric and that of the extra d-dimensions by an Euclidean metric. We also consider the Universe is filled with K-essence scalar field, normal scalar field, tachyonic field and DBI essence and investigate the natures of the dark energy candidates for Emergent, Intermediate and Logamediate scenarios of the Universe. Here in extra dimensional phenomenon we have shown the change of the potential \( V(\phi) \) corresponding to the field \( \phi \) for the dark energies mentioned above and also analyze the anisotropic Universe using the “slow roll” parameters in Hamilton-Jacobi formalism and in terms of above mentioned scalar field \( \phi \) and they are given by [24]

\[
\epsilon = \frac{2\dot{H}^2}{H^2\dot{\phi}^2} \quad \text{and} \quad \eta = \frac{2}{H^2} \left[ \frac{\dot{\phi}\ddot{H} - H\dot{\phi}^2}{\dot{\phi}^3} \right]
\]  

(1)

Sahni et al [28] proposed the trajectories in the \( \{r, s\} \) plane corresponding to different cosmological models to depict qualitatively different behavior. The statefinder diagnostic along with future SNAP observations may perhaps be used to discriminate between different dark energy models. The above statefinder diagnostic pair for higher dimensional anisotropic cosmology are constructed from the scale factors \( a(t) \) and \( b(t) \) as follows:

\[
r = 1 + 3 \frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \quad \text{and} \quad s = \frac{r - 1}{3(q - \frac{1}{2})}
\]  

(2)

where \( q \) is the deceleration parameter defined by \( q = -1 - \frac{\dot{H}}{H^2} \) and \( H \) is the Hubble parameter. Since this parameters are dimensionless so they allow us to characterize the properties of dark energy in a model independently. Finally we graphically analyzed geometrical parameters \( r, s \) in the higher dimensional anisotropic Universe in emergent, logamediate and intermediate scenarios of the universe.

II. BASIC EQUATIONS

We consider homogeneous and anisotropic N-dimensional space-time model described by the line element [29,30]

\[
ds^2 = ds^2_{FRW} + \sum_{i=1}^{d} b^2(t) dx_i^2
\]  

(3)

where \( d \) is the number of extra dimensions (\( d = N - 4 \)) and \( ds^2_{FRW} \) represents the line element of the FRW metric in four dimensions is given by

\[
ds^2_{FRW} = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]
\]  

(4)
where \( a(t) \) and \( b(t) \) are the functions of \( t \) alone represent the scale factors of 4-dimensional space time and extra \( d \)-dimensions respectively. Here \( k (= 0, \pm 1) \) is the curvature index of the corresponding 3-space, so that the above Universe is described as flat, closed and open respectively.

The Einstein’s field equations for the above non-vacuum higher dimensional space-time symmetry are

\[
3 \left( \frac{\dot{a}^2 + k}{a^2} \right) = \frac{\dot{D}}{D} - \frac{d^2 \dot{b}^2}{8 \, b^2} + \frac{d \dot{b}^2}{8 \, b^2} + \rho \quad (5)
\]

\[
\frac{2 \dot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = \frac{\dot{a}}{a} \frac{\dot{D}}{D} + \frac{d^2 \dot{b}^2}{8 \, b^2} - \frac{d \dot{b}^2}{8 \, b^2} - p \quad (6)
\]

and

\[
\frac{\ddot{b}}{b} + 3 \frac{\dot{a} \dot{b}}{a \, b} + \frac{\dot{D} \dot{b}}{D \, b} - \frac{\dot{b}^2}{b^2} = -\frac{p}{2} \quad (7)
\]

where \( \rho \) and \( p \) are energy density and isotropic pressure respectively. Here we choose here \( 8\pi G = c = 1 \) and \( D^2 = b^d(t) \), so we have \( \frac{\dot{D}}{D} = \frac{4 \dot{b}}{2 \, b} \) and \( \frac{\dot{b}^2}{b^2} = \frac{4 \dot{b}}{2 \, b} + \frac{d^2 \dot{b}^2}{8 \, b^2} \). Also in this model we define the Hubble parameter as \( H = \frac{1}{a^{1/2}}(3 \frac{\dot{a}}{a} + d \frac{\dot{b}}{b}) \).

### III. EMERGENT SCENARIO

At first we consider Emergent scenario, where the scale factors \( a(t) \) and \( b(t) \) are consider as the power of cosmic time \( t \) are given by [23, 31, 32]

\[
a = a_0 (\beta + e^{\alpha t})^m \quad \text{and} \quad b = b_0 (\mu + e^{\nu t})^n
\]

where \( a_0, \, b_0, \, \alpha, \, \beta, \, \mu, \, \nu, \, m \) and \( n \) are positive constants. So the field equations (5), (6) and (7) become

\[
\frac{3 \mu^2 e^{2\nu t}}{(\beta + e^{\alpha t})^2} + \frac{3k}{a_0^2}(\beta + e^{\alpha t})^{-2m} = \frac{dn e^{2\nu t}}{8(\mu + e^{\nu t})^2}(dn + n + 4\mu e^{-\nu t}) + \rho \quad (9)
\]

\[
\frac{m \alpha^2 e^{2\alpha t}}{(\beta + e^{\alpha t})^2}(3m + 2\beta e^{-\alpha t}) + \frac{k}{a_0^2}(\beta + e^{\alpha t})^{-2m} = \frac{d m \alpha^2 (\alpha + \nu t)}{2(\beta + e^{\alpha t})(\mu + e^{\nu t})} + \frac{d(\alpha - 2)n^2 \nu^2 e^{2\nu t}}{8(\mu + e^{\nu t})^2} - p \quad (10)
\]

and

\[
\frac{n \mu^2 e^{\nu t}}{(\mu + e^{\nu t})^2} + \frac{3 d m \alpha (\alpha + \nu t)}{(\beta + e^{\alpha t})(\mu + e^{\nu t})} + \frac{d n \nu^2 e^{2\nu t}}{2(\mu + e^{\nu t})^2} + \frac{p}{2} = 0 \quad (11)
\]

We now consider K-essence field, Tachyonic field, normal scalar field and DBI essence field. For these four cases we analyze the behavior of the Emergent Universe in extra dimension and finally we analyze the behavior of the statefinder parameters \( r \) and \( s \).

- **K-essence Field:**

  The energy density and pressure due to K-essence field \( \phi \) are given by [15]

  \[
  \rho = V(\phi)(-\chi + \chi^2)
  \]

  and

  \[
  p = V(\phi)(-\chi + 3\chi^2)
  \]
where \( \chi = \frac{\dot{a}}{a} \) and \( V(\phi) \) is the relevant potential for K-essence Scalar field \( \phi \).

Using equations (5)-(7), we can find the expressions for \( V(\phi) \) and \( \phi \) as

\[
V(\phi) = \frac{\frac{d}{1+2d}m^2\phi^2\dot{\phi}^2 + \frac{2d}{\mu+e^\phi} - \frac{24k(\beta+e^\phi)^{-2m}}{a_0^5}}{16 \left( \frac{-12m^2\phi^2\dot{\phi}^2}{(\beta+e^\phi)^2} + \frac{4m^2\phi^2}{(\beta+e^\phi)^2} \right)^2 + \frac{2d}{\mu+e^\phi} + \frac{24k(\beta+e^\phi)^{-2m}}{a_0^5}}
\]

and

\[
\dot{\phi} = \int 2 \left( \frac{-12m^2\phi^2\dot{\phi}^2}{(\beta+e^\phi)^2} - \frac{4m^2\phi^2}{(\beta+e^\phi)^2} + \frac{2d}{\mu+e^\phi} + \frac{24k(\beta+e^\phi)^{-2m}}{a_0^5} \right) dt
\]

From above forms of \( V \) and \( \phi \), we see that \( V \) can not be expressed explicitly in terms of \( \phi \).

- **Tachyonic field:**

The energy density \( \rho \) and pressure \( p \) due to the Tachyonic field \( \phi \) are given by [12]

\[
\rho = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}
\]

and

\[
p = -V(\phi)\sqrt{1 - \dot{\phi}^2}
\]

where \( V(\phi) \) is the relevant potential for the Tachyonic field \( \phi \). Using equations (5)-(7), we can find the expressions for \( V(\phi) \) and \( \phi \) as
\[ V(\phi) = \sqrt{\left( \frac{m_0^2 e^{\gamma t}(2 \beta + 3 \mu e^{\gamma t})}{(\beta + e^{\gamma t})^2} - \frac{d \text{d} \nu_v e^{\nu t}(\alpha + \nu t)}{2(\beta + e^{\gamma t})(\mu + e^{\gamma t})} - \frac{d(d-1) \nu^2 e^{2\nu t}}{8(\mu + e^{\gamma t})^2} + \frac{k(\beta + e^{\gamma t})^{-2m}}{\alpha_0^2} \right)} \]

\[ \times \sqrt{\left( \frac{3m^2 \nu_v e^{2\nu t}}{(\beta + e^{\gamma t})^2} - \frac{d \nu_v^2 e^{3\nu t}(4\mu + (1 + \delta) \nu_v e^{\nu t})}{8(\mu + e^{\gamma t})^2} + \frac{3k(\beta + e^{\gamma t})^{-2m}}{\alpha_0^2} \right)} \]  \hspace{1cm} (18)

and

\[ \phi = \int \left[ 1 + \frac{8m^2 e^{\gamma t}(2 \beta + 3 \mu e^{\gamma t})}{(\beta + e^{\gamma t})^2} + \frac{4d \nu_v e^{2\nu t}(\alpha + \nu t)}{(\beta + e^{\gamma t})(\mu + e^{\gamma t})} + \frac{d(d-1) \nu^2 e^{2\nu t}}{8(\mu + e^{\gamma t})^2} - \frac{8k(\beta + e^{\gamma t})^{-2m}}{\alpha_0^2} \right] dt \]  \hspace{1cm} (19)

From above forms of \( V \) and \( \phi \), we see that \( V \) can not be expressed explicitly in terms of \( \phi \).

- **Normal Scalar field:**

  The energy density \( \rho \) and pressure \( p \) due to the Normal Scalar field \( \phi \) are given by [37]

  \[ \rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) \]  \hspace{1cm} (20)

  and

  \[ p = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]  \hspace{1cm} (21)

  where \( V(\phi) \) is the relevant potential for the Normal Scalar field \( \phi \). Using equations (5)-(7), we can find the expressions for \( V(\phi) \) and \( \phi \) as

  \[ V(\phi) = \frac{3m^2 \nu_v e^{2\nu t}}{(\beta + e^{\gamma t})^2} + \frac{m_0^2 \beta e^{\gamma t}}{(\beta + e^{\gamma t})^2} - \frac{d \text{d} \nu_v e^{\nu t}(\alpha + \nu t)}{4(\beta + e^{\gamma t})(\mu + e^{\gamma t})} - \frac{d^2 \nu^2 e^{2\nu t}}{8(\mu + e^{\gamma t})^2} - \frac{d \nu v^2 e^{\nu t}}{(\mu + e^{\gamma t})^2} + \frac{2k(\beta + e^{\gamma t})^{-2m}}{\alpha_0^2} \]  \hspace{1cm} (22)

  and

  \[ \phi = \frac{1}{2} \int \left( -\frac{8m_0^2 \beta e^{\gamma t}}{(\beta + e^{\gamma t})^2} + \frac{2d \text{d} \nu_v e^{\nu t}(\alpha + \nu t)}{(\beta + e^{\gamma t})(\mu + e^{\gamma t})} - \frac{d^2 \nu^2 e^{2\nu t}}{(\mu + e^{\gamma t})^2} - \frac{2d \nu v^2 e^{\nu t}}{(\mu + e^{\gamma t})^2} - \frac{8k(\beta + e^{\gamma t})^{-2m}}{\alpha_0^2} \right) dt \]  \hspace{1cm} (23)

- **DBI-essence:**

  The energy density \( \rho \) and pressure \( p \) due to the DBI-essence field \( \phi \) are given by [35,36]

  \[ \rho = (\gamma - 1)T(\phi) + V(\phi) \]  \hspace{1cm} (24)

  and

  \[ p = \frac{\gamma - 1}{\gamma} T(\phi) - V(\phi) \]  \hspace{1cm} (25)
Fig. 3 shows the variations of $V$ against $\phi$, for $a_0 = 2$, $\alpha = 1.1$, $\beta = 1.5$, $\mu = 1.2$, $\nu = 1.5$, $m = 1$, $n = 2$, $k = 1$, $d = 5$ and Fig. 4 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $m = 30$, $n = 6$, $\alpha = 1$, $a_0 = 1$, $\nu = 5$, $d = 15$, $k = -1, 1, 0$ respectively in the case of Tachyonic Scalar field for Emergent Scenario.

where $\gamma$ is given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}}}$$

(26)

and $V(\phi)$ is the relevant potential for the DBI-essence field $\phi$.

The energy conservation equation is given by

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0$$

(27)

where $H$ is the Hubble parameter in terms of scale factor as

$$H = \frac{1}{3} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right)$$

(28)

From energy conservation equation we have the wave equation for $\phi$ as

$$\ddot{\phi} - \frac{3T(\phi)}{2T(\phi)} \dot{\phi}^2 + T'(\phi) + \frac{\dot{\phi}}{\gamma} + \frac{1}{\gamma^3}[V'(\phi) - T'(\phi)] = 0$$

(29)

where $'$ is the derivative with respect to $\phi$. Now for simplicity of calculations, we consider two particular cases: $\gamma = \text{constant}$ and $\gamma \neq \text{constant}$.

**Case I**: $\gamma = \text{constant}$.

In this case, for simplicity, we assume $T(\phi) = \sigma \dot{\phi}^2$ ($\sigma > 1$) and $V(\phi) = \delta \dot{\phi}^2$ ($\delta > 0$). So we have $\gamma = \sqrt{\frac{\sigma}{\sigma - 1}}$.

In these choices we have the following solutions for $V(\phi)$, $T(\phi)$ and $\phi$ from equation (29) as

$$\phi = \int \phi_0 a_0^F b_0^G (\beta + e^{\alpha t})^{mF + nG} dt$$

(30)
Fig. 5 shows the variations of $V$ against $\phi$, for $a_0 = 0.5$, $\alpha = 0.5$, $\beta = 0.5$, $\mu = 0.1$, $\nu = 0.1$, $m = 1$, $n = 8$, $k = 1$, $d = 15$ and Fig. 6 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $m = 5$, $n = 4$, $\alpha = 3$, $a_0 = 1$, $\nu = 7$, $d = 15$, $k = -1, 1, 0$ respectively in the case of Normal Scalar field for Emergent Scenario.

from

$$V(\phi) = \delta \phi^2_0 a_0^2 F_0^2 G_0^2 (\beta + e^{\alpha t})^{2mF + 2nG}$$

and

$$T(\phi) = \sigma \phi^2_0 a_0^2 F_0^2 G_0^2 (\beta + e^{\alpha t})^{2mF + 2nG}$$

where $E = \frac{1}{\gamma} [2(\delta - \sigma) + 2(\sigma - 1)^2]$, $\gamma = \sqrt{\frac{1}{2} F - \frac{3}{2}}$, $G = - \frac{d}{2}$ and $\phi_0$ is an integrating constants. From Fig. 7 and Fig. 8, we see that $V(\phi)$ and $T(\phi)$ are both exponentially decreasing with DBI scalar field $\phi$.

**Case II:** $\gamma \neq \text{constant.}$

In this case, we consider $\gamma = \dot{\phi}^{-2}$ and $V(\phi) = T(\phi)$. Using equations (5)-(7) and (24)-(26), we can find the expressions for $V(\phi)$, $T(\phi)$ and $\phi$ as [34]

$$V(\phi) = T(\phi) = \left( \ln \left[ \frac{a_0^3 b_0^3 (\beta + e^{\alpha t})^{(3m + d)n}}{a_0^3 b_0^3 (\beta + e^{\alpha t})^{(3m + d)n}} \right] \right)^{\frac{1}{4}}$$

and

$$\phi = \int \left( 1 - \frac{1}{\ln \left[ \frac{a_0^3 b_0^3 (\beta + e^{\alpha t})^{(3m + d)n}}{a_0^3 b_0^3 (\beta + e^{\alpha t})^{(3m + d)n}} \right]} \right)^{\frac{1}{4}} dt$$

where $C_0$ is an integrating constant.

- **Statefinder parameters:**

The geometrical parameters $\{r, s\}$ for higher dimensional anisotropic cosmology in emergent scenario can be constructed from the scale factors $a(t)$ and $b(t)$ as

$$r = 1 + \frac{(3 + d) \left( 3 \frac{3 \alpha e^{\alpha t} + d \nu e^{\nu t}}{\mu + e^{\mu t}} \right) \left( 3 \frac{\beta e^{\beta t}}{(\beta + e^{\alpha t})^2} + \frac{d \mu e^{\nu t}}{(\mu + e^{\mu t})^2} \right) + (3 + d) \left( \frac{3 \alpha e^{\alpha t} (\beta - e^{\beta t})}{(\beta + e^{\beta t})^3} + \frac{d \mu e^{\nu t} (\mu - e^{\nu t})}{(\mu + e^{\mu t})^3} \right)}{(3 \frac{\beta e^{\beta t}}{\beta + e^{\beta t}} + d \nu e^{\nu t})^3}$$
Fig. 7 shows the variations of $V$ against $\phi$, and Fig. 8 shows the variations of $T$ against $\phi$ for $a_0 = 0.2, b_0 = 0.2, \alpha = 0.5, \beta = 0.6, m = 15, n = 3, k = 1, d = 15, \sigma = 3, \delta = 2, \phi_0 = 2$ Fig. 9 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $a_0 = 0.02, b_0 = 0.03, \alpha = 0.7, \beta = 0.8, m = 2, n = 5, d = 5, \sigma = 2, \delta = 5$ respectively in the 1st case of DBI-essence Scalar field scenario.

The relation between $r$ and $s$ has been shown in Fig.12. From Fig.12, we see that $s$ is negative when $r \geq 1$. The curve shows that the Universe starts from Einstein static era and goes to the $\Lambda CDM$ model ($r = 1, s = 0$).
Fig. 10 shows the variations of $V$ against $\phi$, for $a_0 = 0.02, b_0 = 0.03, C_0 = 2, \alpha = 5, \beta = 6, m = 0.1, n = 0.3, d = 5$ and $b_0 = 0.03, C_0 = 2, \alpha = 5, \beta = 6, m = 0.1, n = 0.3, d = 5$ respectively in the 2nd case of DBI-essence Scalar field for Emergent Scenario.

Fig. 11 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $a_0 = 0.02, b_0 = 0.03, C_0 = 2, \alpha = 5, \beta = 6, m = 0.1, n = 0.3, d = 5$ respectively in the 2nd case of DBI-essence Scalar field for Emergent Scenario.

Fig. 12 shows the variations of $r$ against $s$, for $\alpha = 0.02, \beta = 10, \mu = 10, \nu = 75, m = 10, n = 2, d = 15$ in Emergent Scenario.
behavior of the state finder parameters $r$ and $s$. We analyze the behavior of the Logamediate Universe in extra dimension and finally we analyze the K-essence Scalar field for Logamediate Scenario.

IV. LOGAMEDIATE SCENARIO

Now we consider Logamediate scenario, where the scale factors $a(t)$ and $b(t)$ are consider as the power of cosmic time $t$ are given by [24]

$$a(t) = e^{A(ln t)^{\lambda_1}} \text{ and } b(t) = e^{B(ln t)^{\lambda_2}}$$

where $A$, $B$, $m$ and $n$ are positive constants. So the field equations (5), (6) and (7) become

$$\frac{3A^2\lambda_1^2(ln t)^{2\lambda_1-2}}{t^2} + 3ke^{-2A(ln t)^{\lambda_1}} = \frac{Bd\lambda_2(ln t)^{\lambda_2-2}}{8t^2} \left(4(\lambda_2 - 1) + B\lambda_2(d + 1)(ln t)^{\lambda_2} - 4ln t \right) + \rho$$

$$A\lambda_1(ln t)^{\lambda_1-2}(2(\lambda_1 - 1) + 3A\lambda_1(ln t)^{\lambda_1}) + k e^{-2A(ln t)^{\lambda_1}} = \frac{A\lambda_1(ln t)^{\lambda_1-1}(4 + Bd\lambda_2(ln t)^{\lambda_2-1})}{2t^2} + \frac{B^2\lambda_2^2d(d - 1)(ln t)^{2\lambda_2-2}}{8t^2} - p$$

$$\frac{B\lambda_2(ln t)^{\lambda_2-2}(6A\lambda_1(ln t)^{\lambda_1}) + B\lambda_2(ln t)^{\lambda_2} - 2\ln t + 2\lambda_2 - 2}{t^2} + p = 0$$

We now consider K-essence field, Tachyonic field, normal scalar field and DBI essence field. For these four cases we analyze the behavior of the Logamediate Universe in extra dimension and finally we analyze the behavior of the state finder parameters $r$ and $s$.

- **K-essence Field:**

The energy density and pressure due to K-essence field $\phi$ are given by the equations (12) and (13). Using equations (38)-(40), we can find the expressions for $V(\phi)$ and $\phi$ as

$$V(\phi) = \frac{e^{-2A(ln t)^{\lambda_1}}}{8t^2(ln t)^2} \left[ B^2d\lambda_2^2(2d - 1)(ln t)^{2\lambda_2}e^{2A(ln t)^{\lambda_1}} + 2Bd\lambda_2(ln t)^{\lambda_2}e^{2A(ln t)^{\lambda_1}} (3A\lambda_1(ln t)^{\lambda_1} - \ln t + \lambda_2 - 1) \right]$$
and
\[
\phi = \int \left[ \frac{-2B^2 d^2 \lambda_2^2 (\ln t)^{2\lambda_2} e^{2A(\ln t)^{\lambda_1}} - 4B d\lambda_2 (\ln t)^{\lambda_2} e^{2A(\ln t)^{\lambda_1}} \left( \lambda_1 - 1 - \ln t + \lambda_2 - 1 \right) + 16 \left( 2kt^2 \ln t \right)^2 \right. \\
\left. + 4A \lambda_1 e^{2A(\ln t)^{\lambda_1}} + 2A^2 \lambda_1^2 (\ln t)^{2\lambda_1} e^{2A(\ln t)^{\lambda_1}} \right] \frac{1}{t} dt (42)
\]

- **Tachyonic field:**

The energy density \( \rho \) and pressure \( p \) due to the Tachyonic field \( \phi \) are given by the equations (16) and (17). Using equations (38)-(40), we can find the expressions for \( V(\phi) \) and \( \phi \) as

\[
V(\phi) = \sqrt{\frac{24kt^2 (\ln t)^2 e^{-2A(\ln t)^{\lambda_1}} + 24A^2 \lambda_1^2 (\ln t)^{2\lambda_1} - 4B d\lambda_2 (\ln t)^{\lambda_2} - B^2 d(d + 1) \lambda_2^2 (\ln t)^{2\lambda_2} + 4B d\lambda_2 (\ln t)^{\lambda_2+1}}{64t^4 (\ln t)^4}}
\]

\[
\times \sqrt{8kt^2 (\ln t)^2 e^{-2A(\ln t)^{\lambda_1}} + 16A \lambda_1 (\ln t)^{\lambda_1} + 24A^2 \lambda_1^2 (\ln t)^{2\lambda_1} - 16A \lambda_1 (\ln t)^{\lambda_1+1} - B^2 d(d - 1) \lambda_2^2 (\ln t)^{2\lambda_2}}
\]

\[
- 4AB d\lambda_1 \lambda_2 (\ln t)^{\lambda_1+\lambda_2} \] (43)

and

\[
\phi = \int \left[ 1 - \frac{8kt^2 (\ln t)^2 e^{-2A(\ln t)^{\lambda_1}} + 16A \lambda_1 (\ln t)^{\lambda_1} + 24A^2 \lambda_1^2 (\ln t)^{2\lambda_1} - 16A \lambda_1 (\ln t)^{\lambda_1+1} - B^2 d(d - 1) \lambda_2^2 (\ln t)^{2\lambda_2}}{24kt^2 (\ln t)^2 e^{-2A(\ln t)^{\lambda_1}} + 24A^2 \lambda_1^2 (\ln t)^{2\lambda_1} - 4B d\lambda_2 (\ln t)^{\lambda_2} - B^2 d(d + 1) \lambda_2^2 (\ln t)^{2\lambda_2}} \right]
\]

\[
\frac{-4AB d\lambda_1 \lambda_2 (\ln t)^{\lambda_1+\lambda_2}}{+ 4B d\lambda_2 (\ln t)^{\lambda_2+1}} \] dt (44)

- **Normal Scalar field:**

The energy density \( \rho \) and pressure \( p \) due to the Normal Scalar field \( \phi \) are given by the equations (20) and (21). Using equations (38)-(40), we can find the expressions for \( V(\phi) \) and \( \phi \) as

\[
V(\phi) = \frac{1}{8t^2 (\ln t)^2} \left[ 16kt^2 (\ln t)^2 e^{-2A(\ln t)^{\lambda_1}} + 8A \lambda_1 (\ln t)^{\lambda_1} + 24A^2 \lambda_1^2 (\ln t)^{2\lambda_1} - 8A \lambda_1 (\ln t)^{\lambda_1+1} \\
- 2B d\lambda_2 (\ln t)^{\lambda_2} - B^2 d^2 \lambda_2^2 (\ln t)^{2\lambda_2} + 2B d\lambda_2 (\ln t)^{\lambda_2+1} - 2AB d\lambda_1 \lambda_2 (\ln t)^{\lambda_1+\lambda_2} \right] \] (45)

and

\[
\phi = \int \left[ 1 \right. \\
\left. \frac{1}{4t^2 (\ln t)^2} \left[ 8kt^2 (\ln t)^2 e^{-2A(\ln t)^{\lambda_1}} + 8A \lambda_1 (\ln t)^{\lambda_1} + 8A \lambda_1 (\ln t)^{\lambda_1+1} - 2B d\lambda_2 (\ln t)^{\lambda_2} \right]
\right]
\]

\[
\frac{-B^2 d^2 \lambda_2^2 (\ln t)^{2\lambda_2} + 2B d\lambda_2 (\ln t)^{\lambda_2+1} - 2AB d\lambda_1 \lambda_2 (\ln t)^{\lambda_1+\lambda_2}}{dt} \] (46)
Fig. 15 shows the variations of $V$ against $\phi$, for $A = 1, B = 2, k = 1, \lambda_1 = 5, \lambda_2 = 4, d = 15$ and Fig. 16 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $\lambda_1 = 3, \lambda_2 = 7, d = 5 k = -1, 1, 0$ respectively in the case of Tachyonic Scalar field for Logamediate Scenario.

Fig. 17 shows the variations of $V$ against $\phi$, for $A = 0.01, B = 0.2, k = 1, \lambda_1 = 5, \lambda_2 = 4, d = 15$ and Fig. 18 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $\lambda_1 = 2, \lambda_2 = 3, d = 5 k = -1, 1, 0$ respectively in the case of Normal Scalar field for Logamediate Scenario.
Fig. 19 shows the variations of $V$ against $\phi$ and Fig. 20 shows the variations of $T$ against $\phi$, for $A = 0.3, B = 0.2, \lambda_1 = 4, \lambda_2 = 3, d = 15, \sigma = 3, \delta = 2, \phi_0 = 2$ and Fig. 21 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $A = 0.01, B = 0.02, \lambda_1 = 3, \lambda_2 = 8, \sigma = 8, \delta = 2, \phi_0 = 1, d = 20$ in the 1st case of DBI-essence Scalar field for Logamediate Scenario.

**DBI-essence:**

The energy density $\rho$ and pressure $p$ due to the DBI-essence field $\phi$ are given by the equations (24) and (25).

**Case I:** $\gamma$ = constant.

Using equations (38)-(40), we can find the expressions for $V(\phi)$, $T(\phi)$ and $\phi$ as

$$V(\phi) = \delta \phi_0^2 e^{(2AF(\ln t)^{\lambda_1}+2BG(\ln t)^{\lambda_2})}$$  \hspace{1cm} \text{(47)}$$

$$T(\phi) = \sigma \phi_0^2 e^{(2AF(\ln t)^{\lambda_1}+2BG(\ln t)^{\lambda_2})}$$  \hspace{1cm} \text{(48)}$$

and

$$\phi = \int \phi_0 e^{(AF(\ln t)^{\lambda_1}+BG(\ln t)^{\lambda_2})} \, dt$$  \hspace{1cm} \text{(49)}$$

where $E = \frac{1}{4}[2(\delta - \sigma) + 2(\sigma - 1)\gamma^3]$, $\gamma = \sqrt{\frac{\sigma}{\sigma - 1}}$, $F = -\frac{\delta}{2\epsilon}$, $G = -\frac{\sigma}{2\epsilon}$ and $\phi_0$ is an integrating constants. From above, we see that $V(\phi)$ and $T(\phi)$ are both exponentially decreasing with DBI scalar field $\phi$. 
Fig. 22 shows the variations of $V$ against $\phi$, for $A = 0.3$, $B = 0.2$, $C_0 = 3$, $\lambda_1 = 2$, $\lambda_2 = 3$, $d = 5$ and Fig. 23 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $A = 2$, $B = 3$, $C_0 = 1$, $\lambda_1 = 3$, $\lambda_2 = 4$, $d = 5$ in the 2nd case of DBI-essence Scalar field for Logamediate Scenario.

Case II: $\gamma \neq$ constant.

Using equations (38)-(40), we can find the expressions for $V(\phi)$ and $\phi$ as

$$V(\phi) = \ln \left[ \frac{C_0}{e^{(3A(\ln t)^{\lambda_1} + dB(\ln t)^{\lambda_2})}} \right]$$

and

$$\phi = \int \left( 1 - \frac{1}{\ln \left[ \frac{C_0}{e^{(3A(\ln t)^{\lambda_1} + dB(\ln t)^{\lambda_2})}} \right]} \right) dt$$

Where $C_0$ is an integrating constant.

• Statefinder parameters:

The geometrical parameters $\{r, s\}$ for higher dimensional anisotropic cosmology in Logamediate scenario can be constructed from the scale factors $a(t)$ and $b(t)$ as

$$r = 1 + \frac{(d + 3)(3A\lambda_1(-\ln t + \lambda_1 - 1)(\ln t)^{\lambda_1} + Bd\lambda_2(-\ln t + \lambda_2 - 1)(\ln t)^{\lambda_2})}{(3A\lambda_1(\ln t)^{\lambda_1} + Bd\lambda_2(\ln t)^{\lambda_2})^2}$$

$$+ \frac{(d + 3)^2(3A\lambda_1(\ln t)^{\lambda_1}(2 + \lambda_1(\lambda_1 - 3) + \ln t(2\ln t - 3\lambda_1 + 3)) + Bd\lambda_2(\ln t)^{\lambda_2}(2 + \lambda_1(\lambda_1 - 3) + \ln t(2\ln t - 3\lambda_1 + 3)))}{(3A\lambda_1(\ln t)^{\lambda_1} + Bd\lambda_2(\ln t)^{\lambda_2})^3}$$

$$s = \frac{(d + 3)(3A\lambda_1(\ln t)^{\lambda_1} + Bd\lambda_2(\ln t)^{\lambda_2})(3A\lambda_1(-\ln t + \lambda_1 - 1)(\ln t)^{\lambda_1} + Bd\lambda_2(-\ln t + \lambda_2 - 1)(\ln t)^{\lambda_2})}{[3(3A\lambda_1(\ln t)^{\lambda_1} + Bd\lambda_2(\ln t)^{\lambda_2})^3}$$

$$+ (d + 3)(3A\lambda_1(\ln t)^{\lambda_1}(2 + \lambda_1(\lambda_1 - 3) + \ln t(2\ln t - 3\lambda_1 + 3)) + Bd\lambda_2(\ln t)^{\lambda_2}(2 + \lambda_2(\lambda_2 - 3) + \ln t(2\ln t - 3\lambda_2 + 3)))]$$

$$\left( -\frac{3}{2} + \frac{(d + 3)((-3A\lambda_1(-\ln t + \lambda_1 - 1)(\ln t)^{\lambda_1} - Bd\lambda_2(-\ln t + \lambda_2 - 1)(\ln t)^{\lambda_2}))}{(3A\lambda_1(\ln t)^{\lambda_1} + Bd\lambda_2(\ln t)^{\lambda_2})^2} \right)$$

$$\left( \frac{(d + 3)(3A\lambda_1(\ln t)^{\lambda_1} + Bd\lambda_2(\ln t)^{\lambda_2})(3A\lambda_1(-\ln t + \lambda_1 - 1)(\ln t)^{\lambda_1} + Bd\lambda_2(-\ln t + \lambda_2 - 1)(\ln t)^{\lambda_2})}{[3(3A\lambda_1(\ln t)^{\lambda_1} + Bd\lambda_2(\ln t)^{\lambda_2})^3} \right)$$

(53)
where $A$ and $B$ are cosmic time parameters. We analyze the behavior of the state finder parameters $r$ and $s$ in the following cases:

1. **Intermediate Universe in extra dimension**
2. **K-essence field**
3. **Tachyonic field**
4. **Normal scalar field**
5. **DBI essence field**

We now consider K-essence field, Tachyonic field, normal scalar field and DBI essence field. For these four cases, we analyze the behavior of the Intermediate Universe in extra dimension and finally we analyze the behavior of the state finder parameters $r$ and $s$.

### V. INTERMEDIATE SCENARIO

Finally, we consider Intermediate scenario, where the scale factors $a(t)$ and $b(t)$ are consider as the power of cosmic time $t$ are given by [24]

$$ a(t) = e^{At^{1/3}} \quad \text{and} \quad b(t) = e^{Bt^{2/3}} $$  \hspace{1cm} (54)

where $A$, $B$, $m$ and $n$ are positive constants. So the field equations (3), (4) and (5) become

$$ \frac{1}{8t^2} e^{-2At^{1/3}} (24kt^2 + e^{2At^{1/3}} (24A^2 f_1^2 t^{2f_1} - Bdf_2 t^{f_2}(-4 + f_2(4 + B(d + 1)t^{f_2})))) - \rho = 0 $$  \hspace{1cm} (55)

$$ \frac{1}{8t^2} e^{-2At^{1/3}} (-8kt^2 + e^{2At^{1/3}} (-24A^2 f_1^2 t^{2f_1} + B^2(d - 1) df_2^2 t^{2f_2} - 4Af_1 t^{f_1}(4f_1 - Bdf_2 t^{f_2} - 4))) - p = 0 $$  \hspace{1cm} (56)

$$ Bf_2 t^{f_2 - 2}(6Af_1 t^{f_1} + n(Bdt^{f_2} + 2) - 2) + p = 0 $$  \hspace{1cm} (57)

We now consider K-essence field, Tachyonic field, normal scalar field and DBI essence field. For these four cases, we analyze the behavior of the Intermediate Universe in extra dimension and finally we analyze the behavior of the state finder parameters $r$ and $s$.

#### K-essence Field:

The energy density and pressure due to K-essence field $\phi$ are given by the equations (12) and (13). Using equations (55)-(57), we can find the expressions for $V(\phi)$ and $\phi$ as

$$ V(\phi) = \frac{-e^{-2At^{1/3}} (24kt^2 + e^{2At^{1/3}} (B^2 d^2 (2d - 1) f_2^2 t^{2f_2} + 24Af_1 t^{f_1}(2Af_1 t^{f_1} + f_1 - 1) - 2Bdf_2 t^{f_2} (3Af_1 t^{f_1} + f_2 - 1)))^2}{(8t^2 (16kt^2 + e^{2At^{1/3}} (B^2 d^2 f_2^2 t^{2f_2} - 2Bdf_2 t^{f_2}(Af_1 t^{f_1} + f_2 - 1) + 8Af_1 t^{f_1}(3Af_1 t^{f_1} + f_1 - 1))))} $$  \hspace{1cm} (58)

and

$$ \phi = \int \sqrt{2 (16kt^2 + e^{2At^{1/3}} (B^2 d^2 f_2^2 t^{2f_2} - 2Bdf_2 t^{f_2}(Af_1 t^{f_1} + f_2 - 1) + 8Af_1 t^{f_1}(3Af_1 t^{f_1} + f_1 - 1))) \over (24kt^2 + e^{2At^{1/3}} (B^2 d^2 (2d - 1) f_2^2 t^{2f_2} + 24Af_1 t^{f_1}(2Af_1 t^{f_1} + f_1 - 1) - 2Bdf_2 t^{f_2} (3Af_1 t^{f_1} + f_2 - 1)))} dt $$  \hspace{1cm} (59)

#### Tachyonic field:
Using equations (55)-(57), we can find the expressions for $V$ and $\phi$. Fig. 25 shows the variations of $V$ against $\phi$ for $A = 20, B = 25, k = 1, f_1 = 0.2, f_2 = 0.19, d = 5$ and Fig. 26 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $A = 400, B = 300, d = 75$ and $k = -1, 1, 0$ respectively in the case of K-essence Scalar field for Intermediate Scenario.

The energy density $\rho$ and pressure $p$ due to the Tachyonic field $\phi$ are given by the equations (16) and (17). Using equations (55)-(57), we can find the expressions for $V(\phi)$ and $\phi$ as

$$V(\phi) = \frac{1}{8} \left[ \frac{1}{l^2} \left( 8kt^2 + e^{2Atf_1} \left( -B^2d(d-1)f_2^2t^2f_2 - 4ABdf_1f_2t^{f_1+f_2} + 8Af_1t^{f_1} \right) \right) \right]$$

and

$$\phi = \int \left[ \frac{2 \left( 8kt^2 + e^{2Atf_1} \left( -B^2d(d-1)f_2^2t^2f_2 - 4ABdf_1f_2t^{f_1+f_2} + 8Af_1t^{f_1} \right) \right)}{24kt^2 + e^{2Atf_1} \left( 24A^2f_1^2t^2f_1 - Bdf_2t^{f_2} \right)} \right] dt$$

**Normal Scalar field:**

The energy density $\rho$ and pressure $p$ due to the Normal Scalar field $\phi$ are given by the equations (20) and (21). Using equations (55)-(57), we can find the expressions for $V(\phi)$ and $\phi$ as

$$V(\phi) = \frac{1}{8t^2} e^{-2Atf_1} \left( 16kt^2 + e^{2Atf_1} \left( -B^2d^2f_2^2t^2f_2 - 2Bdf_2t^{f_2}(Af_1t^{f_1} + f_2 - 1) + 8Af_1t^{f_1}(3At^{f_1} + f_1 - 1) \right) \right)$$

and

$$\phi = \int \left[ \frac{1}{2} \left( \frac{1}{l^2} e^{-2Atf_1} \left( 8kt^2 + e^{2Atf_1} \left( 8Af_1t^{f_1}(-Bdf_2t^{f_2} + 4f_1 - 4) + Bdf_2t^{f_2}(-2 + f_2(2 + Bt^{f_2})) \right) \right) \right] dt$$

**DBI-essence:**

The energy density $\rho$ and pressure $p$ due to the DBI-essence field $\phi$ are given by the equations (24) and (25).

**Case I:** $\gamma = \text{constant}$. 
Fig. 27 shows the variations of $V$ against $\phi$, for $A = 0.5, B = 0.02, k = 1, f_1 = 0.5, f_2 = 0.2, d = 5$ and Fig. 28 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $A = 400, B = 200, d = 5$ and $k = -1, 1, 0$ respectively in the case of Tachyonic Scalar field for Intermediate Scenario.

Fig. 29 shows the variations of $V$ against $\phi$, for $A = 1, B = 0.1, k = 1, f_1 = 0.3, f_2 = 0.4, d = 5$ and Fig. 30 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $A = 0.5, B = 0.3, d = 5$ and $k = -1, 1, 0$ respectively in the case of Normal Scalar field for Intermediate Scenario.

Using equations (55)-(57), we can find the expressions for $V(\phi)$, $T(\phi)$ and $\phi$ as

$$V(\phi) = \delta \phi_0^2 e^{(2A F t_1 + 2B G t_2)}$$

and

$$T(\phi) = \sigma \phi_0^2 e^{(2A F t_1 + 2B G t_2)}$$

and

$$\phi = \int \phi_0 e^{(A F t_1 + B G t_2)} dt$$

where $E = \frac{1}{\gamma}[2(\delta - \sigma) + 2(\sigma - 1)\gamma^3]$, $\gamma = \sqrt{\frac{\sigma}{\sigma - 1}}$, $F = -\frac{6}{E}$, $G = -\frac{3}{E}$ and $\phi_0$ is an integrating constants. From above, we see that $V(\phi)$ and $T(\phi)$ are both exponentially decreasing with DBI scalar field $\phi$. 
Fig. 31 shows the variations of $V$ against $\phi$ and Fig. 32 shows the variations of $T$ against $\phi$, for $A = 1, B = 1, f_1 = 0.4, f_2 = 0.1, \sigma = 9, \delta = 10, d = 5, \phi_0 = 2$ and Fig. 33 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $A = 1, B = 2, \sigma = 5, \delta = 2, f_1 = 0.1, f_2 = 0.2, \phi_0 = 2, d = 5$ in the 1st case of DBI-essence Scalar field for Intermediate Scenario.

Case II: $\gamma \neq$ constant.

Again using equations (55)-(57), we can find the expressions for $V(\phi)$ and $\phi$ as

$$V(\phi) = \ln \left[ \frac{C_0}{e^{(3Aft_1 + dBt_2)}} \right] \sqrt{1 - \frac{1}{\ln \left[ \frac{C_0}{e^{(3Aft_1 + dBt_2)}} \right]}}$$  \hspace{1cm} (67)

and

$$\phi = \int \left( 1 - \frac{1}{\ln \left[ \frac{C_0}{e^{(3Aft_1 + dBt_2)}} \right]} \right)^{\frac{1}{2}} dt$$  \hspace{1cm} (68)

where $C_0$ is an integrating constant.
Fig. 34 shows the variations of $V$ against $\phi$, for $A = 1, B = 1, f_1 = 0.9, f_2 = 0.01, C_0 = 1000, d = 5$ and Fig. 35 shows the variation of the slow roll parameters $\epsilon$ against $\eta$ for $A = 1, B = 1, f_1 = 0.02, f_2 = 0.03, C_0 = 3, d = 15$ in the 2nd case of DBI-essence Scalar field for Intermediate Scenario.

Fig. 36 shows the variations of $r$ against $s$, for $A = 0.2, B = 40, f_1 = 0.55, f_2 = 0.02, d = 5$ in Intermediate Scenario.

- **Statefinder parameters:**

  The geometrical parameters $\{r, s\}$ for higher dimensional anisotropic cosmology in Intermediate scenario can be constructed from the scale factors $a(t)$ and $b(t)$ as

  \[
  r = 1 + \frac{3(d + 3)(3A(f_1 - 1)f_1 t^{f_1} + Bdf_2(f_2 - 1)t^{f_2})}{(3Af_1 t^{f_1} + Bdf_2 t^{f_2})^2} + \frac{(3 + d)^2(3Af_1(2 + f_1(f_1 - 3))t^{f_1} + Bdf_2(2 + f_2(f_2 - 3))t^{f_2})}{(3Af_1 t^{f_1} + Bdf_2 t^{f_2})^3} \tag{69}
  \]

  \[
  s = - \frac{[(d + 3)(3Af_1 t^{f_1} + Bdf_2 t^{f_2})(3A(f_1 - 1)f_1 t^{f_1} + Bdf_2(f_2 - 1)t^{f_2}) + (3 + d)(3Af_1(2 + f_2(f_2 - 3))t^{f_1})}{[3(3Af_1 t^{f_1} + Bdf_2 t^{f_2})^3(\frac{3}{2} + \frac{(d + 3)(3A(f_1 - 1)f_1 t^{f_1} + Bdf_2(f_2 - 1)t^{f_2})}{(3Af_1 t^{f_1} + Bdf_2 t^{f_2})^2})]}
  + Bdf_2(2 + f_2(f_2 - 3))t^{f_2})]} \tag{70}
  \]
VI. DISCUSSIONS

In this work, we have considered $N (= 4 + d)$-dimensional Einstein’s field equations in which 4-dimensional space-time is described by a FRW metric and that of the extra $d$-dimensions by an Euclidean metric. We have considered three scenarios, namely, Emergent, Intermediate and Logamediate scenarios where the universe is filled with K-essence, Tachyonic, Normal Scalar Field and DBI-essence types dark energy models. The natures of the potentials as well as dynamics of scalar fields for the dark energy models have been analyzed. The statefinder and slow-roll parameters have been considered and their natures have been investigated for all dark energy models due to three scenarios of the universe.

In the case of Emergent scenario, we have considered a particular forms of scale factors $a$ and $b$ in such a way that there is no singularity for evolution of the anisotropic Universe. We have found $\phi$ and potential $V$ in terms of cosmic time $t$ for K-essence, Tachyonic, Normal Scalar Field and DBI-essence models. Here we have shown that the emergent scenario is possible for open, closed or flat Universe if the Universe contains K-essence, Tachyonic, Normal Scalar Field and DBI-essence field. From figures 1, 3, 5, 7, 8, 10 it has been seen that the potential always increases with K-essence, Tachyonic and decreases with Normal Scalar Field and also with DBI-essence field when $\gamma = constant$ and $\gamma \neq constant$ and also the figures 2, 4, 6, 9, 11 shows the variation of slow-roll parameters $\epsilon$ and $\eta$ in above dark energy model for open,closed and flat universe where they are increase with all dark energy field except Normal scalar field where it increases 1st then decreases. The $\{r, s\}$ diagram (fig.12) shows that the evolution of the emergent Universe starts from asymptotic Einstein’s static era ($r \rightarrow \infty$, $s \rightarrow -\infty$) and goes to $\Lambda$CDM model ($r = 1$, $s = 0$). It is also observed that $r, s$ are independent of the dimension $d$. So, from statefinder parameters, the behavior of different stages of the evolution of the emergent Universe have been generated.

In the case of Logamediate scenario, we have considered a particular forms of scale factors $a$ and $b$ in such a way that there is no singularity for evolution of the anisotropic Universe. We have found $\phi$ and potential $V$ in terms of cosmic time $t$ for K-essence, Tachyonic, Normal Scalar Field and DBI-essence models. Here we have shown that the logamediate scenario is possible for open, closed or flat Universe if the Universe contains K-essence, Tachyonic, Normal Scalar Field and DBI-essence field. From figures 13, 15, 17, 19, 20, 22 it has been seen that the potential are increases with K-essence, Tachyonic and decreases with Normal Scalar Field and also with DBI-essence field when $\gamma = constant$ and $\gamma \neq constant$ and also the figures 14, 16, 18, 21, 23 shows the variation of slow-roll parameters $\epsilon$ and $\eta$ in above dark energy models for open, closed and flat universe where they are increasing with all dark energy field except DBI essence scalar field where it decreases 1st then decreases then again increases. The $\{r, s\}$ diagram (fig.24) shows that the evolution of the Universe starts from asymptotic Einstein static era ($r \rightarrow \infty$, $s \rightarrow -\infty$) and goes to $\Lambda$CDM model ($r = 1$, $s = 0$). It is also observed that $r, s$ are independent of the dimension $d$. So, from statefinder parameters, the behavior of different stages of the evolution of the Logamediate Universe have been generated.

In the case of Intermediate scenario, we have considered a particular forms of scale factors $a$ and $b$ in such a way that there is no singularity for evolution of the anisotropic Universe. We have found $\phi$ and potential $V$ in terms of cosmic time $t$ for K-essence, Tachyonic, Normal Scalar Field and DBI-essence models. Here we have shown that the intermediate scenario is possible for open, closed or flat Universe if the Universe contains K-essence, Tachyonic, Normal Scalar Field and DBI-essence field. From figures 25, 27, 29, 31, 32, 34 it has been seen that the potential are decreases with K-essence, DBI-essence field when $\gamma = constant$ and $\gamma \neq constant$ and 1st increases then decreases with Normal Scalar Field and with Tachyonic and also the figures 26, 28, 30, 33, 35 shows the variation of slow-roll parameters $\epsilon$ and $\eta$ in above dark energy model for open,closed and flat universe where they are increasing with all dark energy field except DBI essence scalar field where it increases 1st then decreases then again increases. The $\{r, s\}$ diagram (fig.36) shows that the evolution of the Universe starts from asymptotic Einstein static era ($r \rightarrow \infty$, $s \rightarrow -\infty$) and goes to $\Lambda$CDM model ($r = 1$, $s = 0$). It is also observed that $r, s$ are independent of the dimension $d$. So, from statefinder parameters, the behavior of different stages of the evolution of the Intermediate Universe have been generated.

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