Conditional relaxation of a charge state under continuous weak measurement

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We investigate the conditional evolution of a charge state coupled to a mesoscopic detector under continuous weak measurement. The state suffers relaxation into a particular state with a definite charge when electrons in a particular output lead are monitored in the detector. The process of the conditional relaxation is not restricted by the shot noise of the detector, unlike the case of the back-action dephasing. As a result, the relaxation of conditional evolution is much faster than the current-sensitive part of dephasing. Furthermore, the direction of the relaxation depends on the choice of the output lead. We propose that these properties can be verified in a two-path interferometer containing a quantum dot capacitively coupled to a detector. In this setup, the current-current correlation between the interferometer and the detector reveals characteristic features of conditional relaxation.

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The quantum measurement problem continues to attract interest because a measurement process inevitably causes the “wave function reduction” that cannot be described in terms of the Schrödinger equation \([1]\). Mesoscopic physics has recently progressed into a stage that enables us to treat this issue. In particular, a quantum dot entangled with a mesoscopic conductor undergoes “back-action dephasing” experimentally realized \([2, 3, 4]\). This dephasing has also been a subject of intensive theoretical investigation \([5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]\). The back-action dephasing can be understood in terms of the possibility of acquiring charge-state information. However, it is important to note that the actual measurement has not been performed for the dephasing process. It only refers to the possibility of measurement and is a result of averaging over all possible measurement outcomes. On the other hand, a quantum measurement performed on the detector brings about a sudden reduction of the charge state (or the “wave function collapse”) \([18]\). Continuous measurement on a particular outcome of the detector state results in an evolution of the charge state in a way that depends on the choice of measurement outcome.

The system under study is schematically drawn in Fig. 1. A quantum point contact (QPC) adjacent to the target system (usually a quantum dot) can be used as a charge detector through the charge-sensitivity of the detector current \([2, 4, 19]\). The information of the charge state is transferred to the detector in the form of a quantum entanglement. There are two possible outcomes of measurement in the QPC detector, that is, transmission and reflection, for each of the detector electrons. Transmission through a quantum dot coupled to a QPC detector depends on what detector output current is observed \([20]\), demonstrating the conditional statistics. The nature of electron transport in the detector is stochastic because of random partitioning at the QPC. The stochastic evolution of the charge state under this random selection of the detector state has been studied before \([21, 22]\).

In our study, in contrast, we investigate the evolution of the charge state of the target system with the condition that only one particular lead of the detector is intentionally monitored. Our main observations are: (1) The initial state given as a coherent superposition of two different charge states is relaxed to the one of the fixed charge state. The direction of the relaxation depends on the choice of measurement on the detector. That is, the charge state is relaxed to \(|0\rangle\) (state without an extra charge) conditioned on the selection of the detector electron at \(T\). On the other hand, the charge state is relaxed to \(|1\rangle\) (state with an extra charge) when electrons are continuously selected at lead \(R\). (2) The relaxation rate is the same in both cases and is proportional to the charge sensitivity of the detector transmission. The relaxation rate is much larger than the current-sensitive part of the dephasing rate, which can be regarded as a manifestation of nonlocality in a measurement process.

We propose an experimental setup which can be used to verify this conditional relaxation. In order to monitor the state of the target system, we introduce a quantum dot embedded in a two path interferometer. The electronic Mach-Zehnder interferometer with a quantum Hall edge channel \([23]\) is an ideal system for this purpose, but the conventional type of Aharonov-Bohm interferometer \([24]\) can also be used. For charge detection, a QPC is considered which is capacitively coupled to the quantum dot. We show that, while the current oscillation amplitude in the interferometer is directly related to dephasing via entanglement, the cross correlation of the currents (between a lead of the interferometer and the other in the detector) reveals the characteristic features of the conditional relaxation.

Initially, the charge state of the target system is in general given as a linear superposition, \(a_0|0\rangle + a_1|1\rangle\), of two different charge states, \(|0\rangle\) and \(|1\rangle\), respectively. Electron scattering via QPC detector is affected by the state of
the target system and is accounted for in the scattering matrix (for \( j = 0, 1 \))

\[
S_{pc} = (\delta_{j0} + \delta_{j1}) \begin{pmatrix} r_j & t_j' \\ t_j & r_j' \end{pmatrix},
\]

where its elements depend on the charge state \( |j\rangle \). For a detector bias \( V_{det} \), the average number of electrons injected into the QPC during the time interval \( t \) is \( n = eV_{det}/\hbar \). We are interested in the limit of continuous measurement, that is, \( n \gg 1 \), and neglect the energy dependence of the matrix elements.

The electron creation(annihilation) of energy \( \epsilon \) at lead \( l \) (\( l = S, T, R \)) is represented by the operator \( c^\dagger_l(\epsilon) (c_l(\epsilon)) \).

The initial state is a direct product of the charge state \( |a_00\rangle + |a_11\rangle \) and the detector state \( \prod_{l = 0, \ldots, \epsilon < eV_{det}} c^\dagger_S(\epsilon)|F\rangle \) where \( |F\rangle \) is the Fermi sea of electrons with \( \epsilon < eV_{det} \).

Upon interaction of \( n \) detector electrons with the charge state, the two subsystems get entangled as

\[
|\Psi\rangle = |a_00\rangle \otimes \left[ \prod_\epsilon \chi^0_\epsilon(\epsilon)|F\rangle + \left( \prod_\epsilon \chi^1_\epsilon(\epsilon) \right)|F\rangle \right] + |a_11\rangle \otimes \left[ \prod_\epsilon \chi^0_\epsilon(\epsilon)|F\rangle \right],
\]

where the energy interval \( 0 \leq \epsilon \leq eV_{det} \) is counted. \( \chi^0_\epsilon(\epsilon) \) creates a charge-state-dependent detector electron:

\[
\chi^0_\epsilon(\epsilon) = r_j c^\dagger_R(\epsilon) + t_j c^\dagger_R(\epsilon).
\]

Dephasing of the charge state induced by this type of entanglement is now well understood [12, 13, 14, 15]. First, we briefly review the dephasing properties of the charge state. The charge state is described by a reduced density matrix \( \rho = tr_{det} [\Psi \langle \Psi|] \), where \( tr_{det}[\cdot \cdot \cdot] \) sums over the detector’s degrees of freedom. From this, we can find the time evolution of the density matrix elements,

\[
\ln \rho_{jj'}(t) = \ln \rho_{jj'}(0) + \sum_{0 < \epsilon \leq eV_{det}} \ln [\nu_{jj'}(\epsilon)],
\]

where \( \nu_{jj'}(\epsilon) = r_j^* r_j + t_j^* t_j \) is the quantity that accounts for the effect of charge detection. The initial density matrix is \( \rho_{jj'}(0) = a_j a^\dagger_{j'} \).

Eq. (3) indicates that the diagonal components are unchanged, but the off-diagonal terms decay as a function of time leading to dephasing. In the limit of \( t \gg \hbar/eV_{det} \) with \( |\nu_{01}(\epsilon)| \sim 1 \) (weak continuous measurement), we obtain the asymptotic relation \( |\rho_{01}(t)| = |\rho_{01}(0)| \exp(-\Gamma_{dep} t) \) where the dephasing rate \( \Gamma_{dep} \) is given by \( \Gamma_{dep} = -\hbar^{-1} \ln |\nu_{01}(\epsilon)| \).

Due to the condition of weak measurement (\( |\nu_{01}(\epsilon)| \sim 1 \)), \( \Gamma_{dep} \) can be expanded in terms of the change in the transmission probability \( \Delta T = T_0 - T_1 \) \((T_j \equiv |t_j|^2)\) and the change in the relative scattering phase \( \Delta \phi \equiv \arg(t_0/t_0) - \arg(t_1/t_1) \). We find

\[
\Gamma_{dep} = \frac{eV_{det}}{8\hbar} \left( \frac{\Delta T}{T(1-T)} \right)^2 + \frac{eV_{det}}{2\hbar} T(1-T)(\Delta \phi)^2,
\]

where \( T = (T_0 + T_1)/2 \).

Next, we discuss our main observation of the conditional evolution of the charge state. In the above, we have described dephasing of the charge state by its entanglement with the detector electrons. Actual measurement for the detector is not performed for dephasing of the charge state. In contrast, we can monitor the charge state of the target system under continuous selection of detector electrons at a particular lead. (This corresponds to a continuous projective measurement.) The conditional state is obtained by projecting the total state into a state with a specific outcome of measurement and renormalizing the reduced wave function [18]. It is important to note that, under this circumstance, the charge state is not entangled with the detector state, and remains as a pure state, as long as the initial state of the target system is pure. In the particular setup of Fig. 1, there are two possible outcomes for measurement on the detector, that is, transmission and reflection, for each of the detector electrons. So, there are two different ways of continuous projection for the detector outputs. This measurement is given by the operator

\[
M_y = \left[ \langle \Psi|y\rangle \langle y|\Psi\rangle \right]^{-1/2} \langle y|y\rangle,
\]

where \( |y\rangle = \prod_\epsilon c^\dagger(\epsilon)|F\rangle \). The case \( y = R \) \((y = T)\) corresponds to a continuous projection of the detector state onto lead \( R \) \((T)\). The corresponding state of the composite system evolves as

\[
|\Psi\rangle \rightarrow M_y|\Psi\rangle = |\psi^y(t)\rangle \otimes |y\rangle.
\]

Clearly, the two subsystems are disentangled upon the measurement as a result of the “wave-function collapse”. The conditional state of the target system is

\[
|\psi^y(t)\rangle = A_y(t)|0\rangle + B_y(t)|1\rangle,
\]

where the coefficients \( A_y(t) \) and \( B_y(t) \) satisfy the relations

\[
\frac{B_R(t)}{A_R(t)} = \frac{a_1}{a_0} \prod_\epsilon r_1, \quad \frac{B_T(t)}{A_T(t)} = \frac{a_1}{a_0} \prod_\epsilon t_1.
\]
In the asymptotic limit \( t \gg h/(eV_{\text{det}}R_0) \) for \( y = R \), \( t \gg h/(eV_{\text{det}}T_0) \) for \( y = T \), we find
\[
\frac{B_R(t)}{A_R(t)} = e^{(t^2_{\text{rel}}/2+\xi_R)t^2_0/a_0^2}, \quad \frac{B_T(t)}{A_T(t)} = e^{(-t^2_{\text{rel}}/2+\xi_T)t^2_0/a_0^2},
\]
where the relaxation rates are
\[
\Gamma^R_{\text{rel}} = 2R_0 \int h^{-1} d\epsilon \ln|\epsilon_1/\epsilon_0|, \quad (5f)
\]
\[
\Gamma^T_{\text{rel}} = -2T_0 \int h^{-1} d\epsilon \ln|\epsilon_1/\epsilon_0|. \quad (5g)
\]
Here \( R_j = |r_j|^2 \) is the reflection probability. The measurement also induces the phase shifts \( \xi_R = R_0 eV_{\text{det}} \arg(r_1/\epsilon_0)/h \) and \( \xi_T = T_0 eV_{\text{det}} \arg(t_1/\epsilon_0)/h \), respectively. Imposing conditions for weak measurement, \( (\Delta T/R_0 \ll 1 \text{ for } y = R \text{ and } \Delta T/T_0 \ll 1 \text{ for } y = T) \), we find that \( \Gamma^R_{\text{rel}} = \Gamma^T_{\text{rel}} = \Gamma_{\text{rel}} \), where
\[
\Gamma_{\text{rel}} = \frac{eV_{\text{det}}}{h} \Delta T. \quad (5h)
\]

Implications of these results (Eq. 5) are summarized as follows. First, the charge state evolves into \(|0\rangle \langle 1|\) with the relaxation rate \( \Gamma_{\text{rel}} \) (Eq. (5f)) under continuous projection of detector electrons onto lead \( T(R) \). The direction of the evolution depends on which output lead is selected. It is important to note that the conditional state remains as a pure state as a result of measurement, in contrast to the case of dephasing. We also point out that the conditional relaxation considered here is different from the stochastic evolution under random selection of measurement outcome due to the partition noise of the QPC [21, 22]. In order to observe conditional relaxation under monitoring only one particular output lead, we need to correlate the state of the target system with that of the detector output. (See below for observing this correlation.) Second, \( \Gamma_{\text{rel}} \) is much larger than \( \Delta T \)-dependence \( \Gamma_{\text{dep}} \). Because only one particular output is continuously selected, the conditional relaxation is not restricted by the shot noise of the QPC detector, unlike the dephasing process. Finally, \( \Gamma_{\text{rel}} \) depends only on \( \Delta T \), while \( \Gamma_{\text{dep}} \) depends both on \( \Delta T \) and \( \Delta \phi \). Dephasing is related to the state information transferred to the detector and therefore to the possibility of measurement. On the other hand, by selecting one particular lead in the detector, the phase part \( (\Delta \phi) \) of the state information is erased. In fact, this behavior corresponds to the quantum erasure of the charge state information encoded in the relative scattering phase \( \Delta \phi \) [24].

Next, we propose a possible experiment to observe the effect of the conditional relaxation. For a target system, we consider an electronic two-path interferometer with a quantum dot (QD) embedded in one of the two paths. The QD is capacitively coupled to a QPC detector. (See Fig. 2.) The two-path interferometer can be implemented by constructing a double-slit type Aharonov-Bohm interferometer [24]. Alternatively, it can be built up by two beam splitters (BS-\( \alpha \) and BS-\( \beta \)) with quantum Hall edge state. This is an electronic analogue of the Mach-Zehnder interferometer (MZI) [23]. The electronic transport in the interferometer is characterized by the scattering matrix at BS-\( \alpha \), BS-\( \beta \) and QD,
\[
S_i = \left( \begin{array}{cc}
    r_i & t_i \\
    t_i & r_i
\end{array} \right), \quad (6)
\]
where \( i = \alpha, \beta, \gamma \). The reflection and the transmission probabilities are written as \( R_i = |r_i|^2 \) and \( T_i = |t_i|^2 \), respectively.

Because of the dwell time in the QD (denoted as \( \Gamma^{-1} \)), the dephasing effect due to coupling to the QPC detector appears in the probability \( P_x \) to find an electron at lead \( x \) \((x = A, B)\)
\[
P_x = Tr_{\text{MZI}} \left[ c_x^\dagger c_x \rho(t=\Gamma^{-1}) \right], \quad (7a)
\]
where \( Tr_{\text{MZI}} \left[ \cdots \right] \) sums over the MZI degree of freedom. Eq. (7a) implies that the electron is (on average) collected at lead \( x \) after time \( t = 1/\Gamma \) upon injection. It gives
\[
P_A = R_\alpha R_\beta + T_\alpha T_\beta T_\gamma \beta + 2\nu M \cos(\varphi + \phi_\nu), \quad (7b)
\]
\[
P_B = R_\alpha T_\beta + T_\alpha R_\beta T_\gamma \beta - 2\nu M \cos(\varphi + \phi_\nu), \quad (7c)
\]
where \( \varphi = \arg(t_\alpha t_\gamma t_\beta r_\alpha r_\beta) \), \( M = (R_\alpha R_\beta T_\alpha T_\beta T_\gamma T_\beta T_\gamma)^{1/2} \), and \( \phi_\nu = eV_{\text{det}} \arg(\nu_{\text{det}})/h\Gamma \). The visibility factor \( V \) depends on the dephasing rate of Eq. (4) as
\[
V = e^{-\Gamma_{\text{dep}}/\Gamma} \approx 1 - \Gamma_{\text{dep}}/\Gamma, \quad (7d)
\]
in the limit of \( \Gamma_{\text{dep}}/\Gamma \ll 1 \), which agrees with previous results [3, 12].

The conditional probability \( P_{x|y} \) to find an electron at lead \( x \) \((x = A, B)\) conditioned on a particular detector output \( y \) \((y = T, R)\) is obtained from the relation
\[
P_{x|y} = \langle \Psi | M_y c_x^\dagger c_x M_y | \Psi \rangle = \lambda^2 _y \langle x | x^\dagger | y \rangle, \quad (8)
\]

![Figure 2](image-url)
\[\langle \Psi \vert y \rangle = \| \Psi \|^{-1/2} \text{is the normalization factor of the conditional state. In an experiment,} \ P_{x|y}^\prime \text{is the more relevant quantity for measurement. (See below.)} \]

At time \( t = 1/\Gamma \), it is given as

\[ P_{y|A}^\prime = R_x R_\beta + e^{+i\Gamma_{rf}/\Gamma} T_x T_\beta T_\gamma + 2V_y M \cos \varphi_y, \ (8a) \]

\[ P_{y|B}^\prime = R_x T_\beta + e^{+i\Gamma_{rf}/\Gamma} T_x R_\beta T_\gamma - 2V_y M \cos \varphi_y, \ (8b) \]

where \( \varphi_y = \varphi + \xi_y/\Gamma \ (y = R, T) \). The visibility factor that appears in the interference term is given by

\[ V_y = e^{+i\Gamma_{rf}/2\Gamma} \approx 1 \pm \Gamma_{rel}/2\Gamma. \ (8c) \]

In Eq. \( 8c \), \( + (-) \) sign is for \( y = R \) (\( T \)). In contrast to the case of dephasing in single-particle transport, the visibility of the conditional probability can be enhanced (for \( y = R \)) or reduced (for \( y = T \)) depending on which output lead is chosen in the detector. The enhancement (reduction) of the visibility for \( y = R \) (\( y = T \)) is because selecting detector electrons at lead \( R \) (\( T \)) effectively increases (decreases) the transmission probability through the QD.

In the following, we show that the cross-correlation measurement of current at leads \( x (x = A, B) \) and \( y (y = T, R) \) is directly related to the quantity \( P_{x|y}^\prime \) in Eq. \( 8b \). The bias voltage, \( V \), applied to the MZI is assumed to be much smaller than that of the detector: \( V \ll V_{\text{det}} \). The frequency-dependent current cross correlation \( S_{xy}(\omega) \) is defined by

\[ 2\pi \delta(\omega + \omega')S_{xy}(\omega) \]

\[ = \langle \Psi \vert \Delta I_x(\omega) \Delta I_y(\omega') \Delta I_y(\omega) \Delta I_x(\omega) \vert \Psi \rangle, \ (9) \]

where \( |\Psi\rangle \) is the many-electron transport state of the composite system. \( \Delta I_l \) is the current fluctuation defined by \( \Delta I_l = I_l - \langle I_l \rangle \) where \( I_l \) is the output current operator at lead \( l \).

In evaluating the expectation values in Eq. \( 9 \), we need to calculate quantities such as

\[ \langle \Psi \vert c_l^\dagger c_{l'}(E)c_{l'}^\dagger c_{l''}(E')c_{l''}(E')c_{l''}(E') \vert \Psi \rangle. \]

\( E, E', \epsilon, \epsilon' \) are the energies of electrons injected from the interferometer and the detector, respectively. These energies are in the ranges \( 0 \leq E, E' \leq eV \) and \( 0 \leq \epsilon, \epsilon' \leq eV_{\text{det}} \). In order to calculate such quantities, we made the following assumptions: (i) All of the scattering matrices are independent of the energies. This assumption is valid as long as the bias voltages are not very large to alter the characteristics of the QPCs. (ii) The density matrix of the whole system, \( \bar{\rho} \equiv \langle \Psi \vert \Psi \rangle \) can be written as a direct product

\[ \bar{\rho} \simeq \bar{\rho}_1 \otimes \bar{\rho}_2, \ (10) \]

where \( \bar{\rho}_1 \) is the part of the density matrix that contains energies \( E, E', \epsilon, \epsilon' \), while \( \bar{\rho}_2 \) represents the remaining part. This is a reasonable assumption because the different energy states of electrons are unlikely to interfere with each other. Using these assumptions, we obtain a simple relation of the zero-frequency cross-correlation

\[ S_{xy}(0) = \frac{e^2}{\pi \hbar} V \left[ P_{x|y} P_{y|0} - P_x P_y \right], \ (11) \]

where \( P_{y|0} = R_0, P_{y|0} = T_0, P_R = R_x R_0 + T_0 R_1, \) and \( P_T = 1 - R_R \). Also, it is straightforward to find that the average current \( \langle I_x \rangle \) at lead \( x \) satisfies the Landauer formula: \( \langle I_x \rangle = (e^2/2\pi \hbar)P_x V \). (Similarly, \( \langle I_y \rangle = (e^2/2\pi \hbar)P_y V_{\text{det}} \) for the detector.) Therefore, analyzing the cross correlation \( S_{xy}(0) \) as well as the DC currents reveals the characteristic features of conditional relaxation and dephasing.

In conclusion, we have found that a linearly superposed charge state is conditionally relaxed under continuous measurement by an attached QPC detector. The direction of the relaxation depends on the choice of the detector output lead. It takes place much faster than the current-sensitive part of dephasing. We suggest that this feature can be revealed by constructing an interferometer for the charge state and investigating the current-current correlation between the two subsystems.

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