An Optimum EOQ model for buyer-vendor with price breaks and fixed holding cost

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Abstract—In this paper, an EOQ model is developed for deteriorating product where constant demand depends on the buyer-vendor inventory level with price breaks. In traditional inventory problems, it is considered that the holding cost is fixed and setup cost is not subject to control. An inventory problem is formulated for deteriorating items, further, the optimal solutions are derived. Numerical examples presented to demonstrate the developed model. An algorithm is presented to determine the optimal inventory level while minimizing the present value of the total cost.

Key words—Inventory Level, Deteriorating products, Price Breaks, Coordination, Fixed life time product, Quantity discount.

1. Introduction

In general products with fixed lifetime like pharmaceutical items, perishable items, and beverage, etc., are produced by large and small scale producers. Sometimes the same product may be produced by small and large scale producers. In this situation, the small producer (First producer) may decide to purchase the same product from the large scale producer (Second producer) instead of manufacturing by him. At that time, second producer frequently offers a discounted price to Manufacturer1 if a large-enough quantity of an item is purchased.

Fujiwara et al. [2] developed a model using optimal policy for perishable products of the two-stage inventory system. Goyal and Gupta [1] analyzed minimum inventory cost both buyer vendor. Kaj - Mikael Bjork [4] investigated many finite production quantity inventory problems. Kit Nam Francis Leung [3] developed an integrated production inventory problem in many scenario multi-firm supply chain. Mahdi Tajbakhsh et al. [6] analyzed an inventory model using a random discount. Ravithammal et al. [9] considered production inventory to obtain minimum average cost with a quantity discount, shortages for an item. Muniappan et al. [7] developed a back-ordering product inventory model for with quantity discount. Ravithammal et al. [10] analyzed the perishable inventory model with production. Muniappan et al. [5] obtained economic order quantity to minimize inventory costs with deteriorating items. Muniappan et al. [8] developed an incentive inventory model for exponential function of cost of deteriorating products. Saoussen Krichen et al. [13] discussed single and multiple suppliers, cooperative retailer’s inventory models, a quantity discount. Umamaheshwareni et al. [15] obtained the design and analysis of an optimal inventory model for perishable goods with a fixed lifetime. Sundararaj et al. [11] discussed the article analyzes on providing a general deterministic inventory model in which the rate of demand is determined by price and time over the...
ordering cycle time. Sharma Vikas et al. [12] analyzed perishable items to obtain optimum replacement policy.

Yongrui et al. [14] discussed buyer-vendor inventory problems for various stages to analyze discount incentives for the product. In this model developed two-stage manufacturer produces the same product. This paper is discussed Mathematical Formulation, assumption and notation, and analyzed different manufacturers using the same product with two stages, numerical analysis procedure.

2. Assumptions and Notations

2.1 Assumptions
(1) Production rate is greater than or equal to the demand of a product.
(2) Both producers meet shortages without coordination, shortages are not allowed, with coordination.

2.2 Notations
D1, D2 Annual demand of First producer and Second producer
P1, P2 Both Producers rate of production per annum
L Life time of product
k1, k2 Both Producers setup costs per annum
h1, h2 Both Producers holding costs per unit per annum
p1, p2 Both Producers price per unit per annum
Q Q = Q1 + Q2, order size of First producer
Q1 The inventory level after fulfilling the shortage demand
Q2 Inventory size which is immediately taken to satisfy unfilled demand
Q0 Order quantity of Second producer
m In the absence of coordination order multiple of Second producer
n Under coordination order multiple of Second producer
K Under coordination order multiple of First producer.
KQ0 New order quantity of First producer
d(K) Discount order of First producer per unit per item, every order
TCm1 Without coordination, Total cost of the First producer
TCm2 Without coordination, Total cost of the Second producer
TCm1(m) With coordination, Total cost of the First producer
TCm2(n) With coordination, Total cost of Second producer

3. Systematic Solution Procedure

3.1 Development of Inventory Model without coordination
In this phase both producers produce the same product, shortages are allowed.

\[ T_{Cn} = \frac{D_1k_1}{2} + \frac{1}{2y_2} \left( \frac{h_1}{Q_1-D_1} \right) \left[ b_2 (Q_1 - D_2) + Q_2 \right]. \]

The optimality conditions are \( \frac{dT_{Cn}}{Q_1} = 0 \) and \( \frac{dT_{Cn}}{Q_2} = 0 \).

implies \( Q = \sqrt{\frac{2D_1k_1(h_2+P_1)}{b_2 y_2}} \) and \( Q_2 = Q \left( 1 - \frac{D_2}{P_2} \right) \frac{b_2}{h_2+y_2}. \)
When \( Q = \sqrt{\frac{2D_k b_2 s_2}{h_1 s_1}} \frac{P_1}{P_1 - D_1} \) gives \( TC_{M_1} = \sqrt{\frac{2D_k b_2 s_2}{h_1 s_1}} \frac{(1 - D_1)}{P_1} \).

When \( Q_0 = \sqrt{\frac{2D_k b_2 s_2}{h_1 s_1}} \frac{P_2}{P_2 - D_2} \), order size = \( mQ_{x_0} = \sqrt{\frac{2D_k b_2 s_2}{h_1 s_1}} \frac{P_2}{P_2 - D_2} \).

Average inventory cost for Second producer

\[
\frac{(m-1)Q_0 + (m-2)Q_0 + \ldots + Q_0 + 0Q}{m} = \frac{(m-1)Q_0}{2}
\]

\( TC_{M_2}(m) = \frac{D_k b_2 s_2}{mQ_{x_0}} + \frac{(m-1)Q_0}{2} \frac{1}{P_1 - D_1} = \frac{k_1}{m} \sqrt{\frac{2D_k b_2 s_2}{h_1 s_1}} \frac{P_2 - D_2}{P_2} \) + \( (m-1) \frac{b_1}{2h_1} \sqrt{\frac{2D_k b_2 s_2}{h_1 s_1}} \frac{P_2 - D_2}{P_2} \)

without coordination, the Second producer model developed as follows

\[
\min TC_{M_2}(m) \quad \text{s.t} \quad \begin{cases} \frac{m t_0}{m} \leq L, \\ m \geq 1, \end{cases} \tag{1}
\]

here \( mt_0 \leq L \) shows that the product is not overdue before they are sold up by the first producer.

**Lemma 1:**

Let us consider \( m^* \) be the optimum of (1), if \( L^2 \geq \frac{2k_1}{D b_2} \left( \frac{P_2}{P_2 - D_2} \right) \) then

\[
m^* = \min \left\{ \left[ \frac{b_1 b_2 + \frac{1}{4} - \frac{1}{2}}{b_1 b_2} \right] \left[ \frac{L}{\sqrt{\frac{2D_k b_2 s_2}{h_1 s_1}} \frac{P_2 - D_2}{P_2}} \right] \right\} \tag{2}
\]

\([x] \) is the least integer greater than or equal to \( x \), \( L^2 \geq \frac{2k_1}{D b_2} \left( \frac{P_2}{P_2 - D_2} \right) \) is to ensure that \( m^* \geq 1 \)

**Proof**

\[
\frac{d^2TC_{M_2}(m)}{dm^2} = k_1 \frac{2D_k b_2 s_2}{m^\frac{3}{2}} \frac{P_2 - D_2}{P_2} > 0, \quad TC_{M_2}(m) \text{ is strictly convex in } m.
\]

Considered \( m_1^* \) be the optimum of \( \min TC_{M_2}(m) \), then

\[
m_1^* = \max \left\{ \min \left\{ \frac{m}{m} / TC_{M_2}(m) \leq TC_{M_2}(m+1), 1 \right\} \right\} = \max \left\{ \min \left\{ \frac{m}{m + 1} \geq \frac{2D_k b_2 s_2}{Q_0 (1 - \frac{D_2}{P_2}) h_1} \right\}, 1 \right\}
\]

\[
= \left[ \frac{b_1 b_2 + \frac{1}{4} - \frac{1}{2}}{b_1 b_2} \right] \geq 1
\]

Substituting the value of \( t_0 \) into the constraints in (1), then

\[
m = \frac{2k_1}{D b_2} \left( \frac{P_2}{P_2 - D_2} \right) \leq 1
\]

if \( m_2^* = \frac{L}{\sqrt{\frac{2D_k b_2 s_2}{h_1 s_1}} \frac{P_2 - D_2}{P_2}} \geq 1 \), is true since \( L^2 \geq \frac{2k_1}{D b_2} \left( \frac{P_2}{P_2 - D_2} \right) \).

\( m^* = m_1^* \) where \( m_1^* \leq m_2^* \), otherwise \( m^* = m_2^* \). Hence \( m^* = \min(m_1^*, m_2^*) \), if \( L^2 \geq \frac{2k_1}{D b_2} \left( \frac{P_2}{P_2 - D_2} \right) \).
Scenario 1: Without coordination, the First producer optimum total cost is $T_{C_M_1}$, order size is
\[ \sqrt{\frac{2D_1k_1k_2(3L_1+L_2)}{h_1(1-D_1)}} \]
and the Second producer optimum total cost is $T_{C_M_2}(m^*)$, order size is
\[ m^* \sqrt{\frac{2D_2k_2}{h_2} \left( \frac{P_2}{P_1-D_1} \right)} \]

3.2 Development of model with coordination

In coordination scheme, First producer stop their production and purchase the product of Second producer and no shortages for First producer. In this strategy, Second producer given quantity discount with the discount factor $d(K)$, if First producer change his lot size by $KQ_0$. Now the Second producer lot size is $nKQ_0$ where $n$ is a positive integer and $KQ_0$ is the First producer new order quantity.

Now, the First producer order quantity $Q_1 = \sqrt{\frac{2D_1k_1k_2}{h_1(1-D_1)}}$ and optimum total cost $T_{C_M_1} = \sqrt{2D_1k_1h_1(1-D_1)}$.

Therefore, Second producer total cost $T_{C_M_2(n)} = \frac{D_2k_2}{nKQ_0} + \frac{KQ_0(1-\frac{D_2}{P_1})}{2} + p_2D_2d(K)$ (3)

In coordination discount strategy, the problem can be developed as follows

\[ \min T_{C_M_2(n)} \]

subject to
\[ \begin{cases} \frac{D_2k_2}{nKQ_0} + \frac{KQ_0(1-\frac{D_2}{P_1})}{2} - \sqrt{2D_2k_2h_2(1-D_2)} & \leq p_2D_2d(K) \\ n \geq 1, \end{cases} \] (4)

Now $nKt_c \leq L$ shows that the product is not overdue before they are sold up by the First producer. The second constraint shows that the First producer cost under coordination cannot exceed that without coordination.

Scenario 2

$T_{C_M_1}(m^*) \leq T_{C_M_2}(m^*)$ is true, if $m^*$ is optimum of (1) and $n^*$ be the optimum of (4).

Proof

If the second constraint must be an equation, then $p_2D_2d(K)$ takes smallest value and $T_{C_M_2(n)}$ is optimized.

\[ i.e., \frac{D_2k_2}{nKQ_0} + \frac{KQ_0(1-\frac{D_2}{P_1})}{2} - \sqrt{2D_2k_2h_2(1-D_2)} = p_2D_2d(K) \]
If \( K = 1 \), then \( d(1) = \frac{\sqrt{2D_2 k_2 (1 - \frac{D_2}{P_2})} - \frac{\sqrt{2D_2 k_2 (1 - \frac{D_2}{P_2})}}{P_2 D_2}}{P_2 D_2} = 0. \)

So if \( K = 1 \), then (4) is equivalent to (1). Therefore, \( TC_{M_2}(n^*) \leq TC_{M_2}(n^*) \) is true.

**Result 2:** Scenario (2), ensures that Second producer will get more benefit to compare with First producer if the first producer order size is \( Q_0, K > 0 \) because optimum total cost under coordination is less than without coordination.

Put equation (5) into equation (3), we have

\[
TC_{M_2}(n) = \frac{D_2 k_2}{nQ_0} + \frac{(n-1)(1 - \frac{D_2}{P_2}) k_2 Q_0}{2} \left( P_2 D_2 \right) + P_2 D_2 \left( \frac{D_2 k_2}{Q_0} + \frac{k_2 (1 - \frac{D_2}{P_2}) h_2}{2} \right) \left( \frac{1}{P_2 D_2} - \frac{1}{P_2 D_2} \right)
\]

Let \( K^* \) be the optimum of \( TC_{M_2}(n) \), we have

\[
K^*(n) = \frac{1}{Q_0} \sqrt{\frac{2D_2 (k_2 + h_2)}{\left(1 - \frac{D_2}{P_2}\right)(n-1) h_2 - h_2}}
\]

From first constraint of (4), we have

\[
\left( \frac{k_2}{n} + k_2 \right) n^2 \leq \frac{1}{4 k_2} \left( \frac{D_2}{P_2} \right)^2 \left( (n-1) h_2 + h_2 \right)
\]

\[
Ta \quad g(n) = -k_2 n^2 + \left( \frac{D_2}{2} \left( \frac{P_2 - D_2}{P_2} \right) h_1 - k_1 \right) n + \frac{D_2^2}{2} \left( \frac{P_2 - D_2}{P_2} \right) (h_2 - h_1)
\]

Substituting (7) and \( t_0 = \sqrt{\frac{2D_2 (k_2 + h_2)}{D_2 (P_2 - D_2)}} \) into (3), we have

\[
TC_{M_2}(n) = \sqrt{2D_2 \left[ k_1 \left( 1 - \frac{D_2}{P_2} \right) h_1 + \frac{k_2 (1 - \frac{D_2}{P_2}) h_2}{n} + nk_2 \left( 1 - \frac{D_2}{P_2} \right) h_1 + \left( 1 - \frac{D_2}{P_2} \right) k_2 h_2 - h_1 \right]}
\]

\[
- \sqrt{2Dh_2k_2 \left( 1 - \frac{D_2}{P_2} \right)}
\]
Therefore, (4) becomes

\[
\min TC_{M2}(n)
\]

subject to \( \frac{g(n)}{n} \geq 0, \quad n \geq 1 \) \hspace{1cm} (10)

for \( x \geq 0, \sqrt{x} \) is a strictly increasing so the above equation is equivalent to

\[
\min TC_{\overline{M2}}(n)
\]

\[
= D_2 \left[ k_1 \left(1 - \frac{D_2}{P_2}\right) h_3 + \frac{k_1 \left(1 - \frac{D_2}{P_2}\right) [b_2 - h_3]}{n} + nk_2 \left(1 - \frac{D_2}{P_2}\right) h_2 + \left(1 - \frac{D_2}{P_2}\right) k_1 [b_2 - h_3] \right]
\]

subject to \( \frac{g(n)}{n} \geq 0, \quad n \geq 1 \) \hspace{1cm} (11)

Here, \( TC_{\overline{M2}}(n) \) is convex when \( h_2 \geq h_3 \), since \( TC_{\overline{M2}}'(n) = \frac{2D_2k_1 \left[1 - \frac{D_2}{P_2}\right] [b_2 - h_3]}{n^2} > 0 \), otherwise it is concave. \( g(n) \) is strictly concave because \( g''(n) = -2k_2 < 0 \).

**Conclusion**

Let \( n^*_1 \) be the optimum of \( TC_{\overline{M2}}(n) \) for \( n \geq 1 \), then

\[
n_1^* = \left\{ \begin{array}{ll}
\left[ \frac{k_2 [b_2 - h_3]}{k_2 h_3} + \frac{1}{4} \right], & k_2 [b_2 - h_3] \geq 2 \\
\text{otherwise} & 
\end{array} \right. \hspace{1cm} (12)
\]

**Proof**

\( TC_{\overline{M2}}(n^*_1) \leq \min \{ TC_{\overline{M2}}(n^*_1 - 1), TC_{\overline{M2}}(n^*_1 + 1) \} \) because \( n^*_1 \) is the minimum of \( TC_{\overline{M2}}(n), n \geq 1 \)

Now

\[
TC_{M2}(n^*_1) - TC_{\overline{M2}}(n^*_1 - 1) = \frac{-D_2k_1 \left[1 - \frac{D_2}{P_2}\right] [b_2 - h_3]}{n(n^*_1 - 1)} + D_2k_2 \left(1 - \frac{D_2}{P_2}\right) h_2 \leq 0
\]

\[
\left( n_1^* - \frac{1}{2} \right)^2 \leq \frac{k_2 [b_2 - h_3]}{k_2 h_3} + \frac{1}{4}
\]

Similarly, by \( TC_{M2}(n^*_1) - TC_{\overline{M2}}(n^*_1 + 1) \leq 0 \), we have

\[
\left( n_1^* + \frac{1}{2} \right)^2 \geq \frac{k_2 [b_2 - h_3]}{k_2 h_3} + \frac{1}{4}
\]

Hence, if \( \frac{k_2 [b_2 - h_3]}{k_2 h_3} + \frac{1}{4} < 0, TC_{M2}(n^*_1) \leq TC_{\overline{M2}}(n^*_1 + 1) \) for any given \( n \), then \( n^*_1 = 1 \).

If \( \frac{k_2 [b_2 - h_3]}{k_2 h_3} + \frac{1}{4} \geq 0 \) by (13) & (14),
\[ n^*_1 = \sqrt{\frac{k_1 (b_1 - h_1) + \frac{1}{4}}{k_2 h_1}} + \frac{1}{2} \leq n^*_1 \leq \sqrt{\frac{k_1 (b_2 - h_2) + \frac{1}{4}}{k_2 h_1}} + \frac{1}{2} \]

So \( n^*_1 = \sqrt{\frac{k_1 (b_2 - h_2) + \frac{1}{4}}{k_2 h_1} - \frac{1}{2}} \) Also note that,

if \( 0 < \frac{k_1 (b_2 - h_2)}{k_2 h_1} < 2 \) then \( n^*_1 = 1 \), so (12) holds.

4. Numerical Example
We now study the effects of changes in the value of system parameters \( h_1, h_2, k_1, k_2, s_2 \) on the First producer and Second producer minimum total relevant cost per unit time \( TC_{M1}, TC_{M2}, TC_{M1}(m^*), \) and \( TC_{M2}(n^*) \) of the Example 1. The sensitivity analysis is performed by taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in Table 1.

**Table 1**

| Decision Variables | Cost/Unit/ Dollar | \( k^*_1 \) | \( d(k^*_1) \) | \( TC_{M1}(m^*) \) | \( TC_{M2}(n^*) \) | \( TC_{M1} \) | \( TC_{M2} \) |
|--------------------|------------------|------------|----------------|-------------------|-------------------|-------------|-------------|
| \( k_1 \) 500 | 4.3541 | 0.0031 | 3582.0 | 3256.7 | 1350 | 1469.3 |
| 550 | 4.5178 | 0.0033 | 3738.6 | 3456.0 | 1350 | 1469.3 |
| 600 | 4.6742 | 0.0036 | 3886.8 | 3646.4 | 1350 | 1469.3 |
| \( k_2 \) 100 | 4.9686 | 0.0122 | 3236.1 | 3124.7 | 965 | 995.25 |
| 150 | 4.0000 | 0.0015 | 3236.1 | 2825.7 | 1250 | 1369.3 |
| 200 | 3.5981 | 0.0012 | 3236.1 | 2656.9 | 1431 | 1577.1 |
| \( s_2 \) 35 | 3.0000 | 0.0015 | 2236.1 | 1825.7 | 1281 | 1369.3 |
| 45 | 3.0000 | 0.0015 | 2236.1 | 1825.7 | 1299 | 1369.3 |
| 55 | 3.0000 | 0.0015 | 2236.1 | 1825.7 | 1311 | 1369.3 |
| \( h_1 \) 12 | 3.0000 | 0.0015 | 2449.5 | 1825.7 | 1250 | 1369.3 |
| 14 | 3.0000 | 0.0015 | 2645.8 | 1825.7 | 1250 | 1369.3 |
| 16 | 3.0000 | 0.0015 | 2828.4 | 1825.7 | 1250 | 1369.3 |
| \( h_2 \) 6 | 3.0000 | 0.0013 | 2236.1 | 2000.0 | 1347 | 1500.0 |
| 7 | 3.0000 | 0.0018 | 2236.1 | 2160.2 | 1432 | 1620.2 |
| 8 | 3.0000 | 0.0019 | 2236.1 | 2309.4 | 1508 | 1732.1 |

Computational results shows that

a) The optimum total cost of Second producer under coordination is lower than that without coordination. i.e., Second producer is relatively highly benefited than First producer in spite of giving more quantity discount.

b) A raise in holding cost for Second producer, the optimal total cost of First producer and Second producer remain same or rise.

c) A decrease value of holding cost for First producer tends to decrease in total cost for First producer and Second producer.

d) The set up cost for First producer and Second producer reduce, automatically the total cost of First producer and Second producer gets reduced.

5. Results and Discussions
This study develops Mathematical analysis on single product multiple manufacturing supply chain model with various cost for perishable product. This model assumes coordination and non coordination system. In absence of coordination, First producer and Second producer produce the similar product and shortages allowed for First producer. First producer profited more without coordination. Under coordination system First producer stop their production and purchase items from Second producer with quantity discount and good relationship. This model concludes coordination
system is more profited to Producer 2 even though he offers more quantity discount to First producer, in this case both producers are profited. This paper discussed to optimize the total inventory cost, with various decision variables. Numerical analysis and Sensitivity analysis on the parameter changes is also discussed. The future model can be extended by considering various factors like multiple products, random discount offering, credit periods, varying holding cost, exponential demand etc.,

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