Probing the neutral edge modes in transport across a point contact via thermal effects in the Read-Rezayi non-abelian quantum Hall states

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Non-abelian quantum Hall states are characterized by the simultaneous appearance of charge and neutral gapless edge modes, with the structure of the latter being intricately related to the existence of bulk quasi-particle excitations obeying non-abelian statistics. In general, it is hard to probe the neutral modes in charge transport measurements and a thermal transport measurement seems to be inevitable. Here we propose a setup which can get around this problem by having two point contacts in series separated by a distance set by the thermal equilibration length of the charge mode. We show that by using the first point contact as a heating device, the excess charge noise measured at the second point contact carries a non-trivial signature of the presence of the neutral mode hence leading to its indirect detection. We also obtain explicit expressions for the thermal conductance and corresponding Lorentz number for transport across a quantum point contact between two edges held at different temperatures and chemical potentials.

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The Read-Rezayi non-abelian quantum Hall states are currently the leading candidates for ground state wavefunctions describing certain Hall plateaus appearing on the first Landau level. The first state of this series coincides with the Pfaffian state suggested by Moore and Read in 1991$^1$ which accounts for the plateau at filling factor $\nu = 2 + 1/2$ $^2$. The rest of the series describes other observed plateaus at filling factors $\nu = 2 + 2/(k+2)$ ($k = 3, 4, \ldots$) $^3$.

The edge states of the Read-Rezayi quantum Hall states are described by two independent 1 + 1 dimensional field theories $^4$: a bosonic field theory carrying U(1) charge and a charge-neutral field theory belonging to a class of conformal field theories (CFT) known as parafermions $^5$. These two theories together are responsible for the low energy transport properties of these quantum Hall states. The structure of the neutral edge is set by the bulk theory, and it directly reflects the non-abelian properties of the quasi-particles $^6$. Hence, it is the properties of this edge that one aims to probe in experimental work when trying to detect signatures of the non-abelian states.

Recently there has been a considerable interest in understanding charge transport across a single quantum point contact (QPC) $^7$ or multiple QPC’s (interferometer geometries) $^8, 9, 10, 11, 12$. These studies mainly focus on tunneling of quasi-particles or electrons across a QPC in the weak-backscattering (WB) limit or in the strong-backscattering (SB) limit, when a voltage is applied across the QPC. Charged quasi-particles or electrons carry the neutral part of the excitation along with them as they tunnel across a QPC. The electric current therefore possesses some of the properties of the neutral field theory. However, past theoretical developments indicate that charge transport measurements in a single QPC cannot confirm the existence of neutral edge modes, and the minimal requirement for probing the non-abelian nature of the state is an interferometer geometry which is yet to be realized experimentally. On the other hand, there has not been much progress in exploring possibilities for the detection of neutral-edge modes keeping limitations of present day experiments in mind.

In this letter, we propose a scenario to probe the neutral edge mode directly through thermal current measurements, and provide the theoretical framework for calculating observables related to such an experiment. In sharp contrast to standard charge transport measurements, a temperature gradient directly couples to the neutral mode, and observables such as the thermal conductance will therefore reflect the presence of the neutral mode. But in view of the fact that controlled thermal conductance measurements are beyond the scope of present day experiments, we propose a setup which does not require any external heating devices, but at the same time indirectly probes the presence of neutral mode via thermal effects. We start with a brief introduction to the physics of the thermal current carried by the edge uninterrupted by a QPC, and find the 2-terminal thermal Hall conductance and Lorentz number (Eq. $^8$). We then perturbatively calculate the charge and energy currents associated with tunneling of particles of scaling dimension $\hbar$ across a QPC to leading order in tunneling amplitude and provide an expression for it as function of the voltage bias, $V$, and temperature bias, $\Delta T$ (see Eq. $^8$). The associated Lorentz number is calculated as well (Eq. $^9$). We then proceed to elaborate on the experimental possibility of using a QPC as a heating device for the charge edge incident on the QPC leaving behind the co-propagating neutral edge at its initial temperature. This is naturally achieved in the presence of a
voltage bias as the applied voltage almost entirely drops between the counter-propagating charge edge modes at the QPC. The out-of-equilibrium charge mode coming out of the QPC region undergoes self-equilibration via the intra-edge Coulomb interaction, hence attaining a new temperature which is different from the temperature of the edge state incident on the QPC due to heating of the charge mode at a rate given by \( P = IV \) (I is the tunneling current and \( V \) is the voltage drop across the QPC). This results in a sustained temperature difference between the co-propagating charge and neutral edge modes coming out of QPC. We show that this temperature difference can be detected via a noise measurement by invoking a second QPC in series with the first one (Eqs. (18), (19)). As the origin of the temperature difference is solely this fact that one of the edges is charge-neutral, its detection can confirm the presence of the neutral mode without resorting to a thermal conductance measurement.

**Thermal Hall conductance for uninterrupted edge.**

The thermal current carried along the edge is given by

\[
I_Q = \frac{\pi^2 k_B^2}{6} \frac{2}{2\pi} T^2, \tag{1}
\]

where \( T \) is the temperature of the edge and \( c = c_n + c_c \) is the total central charge, with \( c_n (c_c) \) being the central charge for the neutral (charge-carrying) edge theory. We have set \( \hbar = 1 \) throughout the letter. For the Laughlin states there are no neutral modes, hence \( c = c_c = 1 \). For the Read-Rezayi series \( c_n = (2k - 2)/(k + 2), \ c_c = 2 + 1 \) \((k = 2, 3, 4, \ldots)\) where the \( 2 \) in \( c_c \) accounts for the two integer Hall edges of the \( \nu = 2 \) state. The corresponding two-terminal thermal Hall conductance is

\[
K_H = \frac{\pi^2 k_B^2}{3} \frac{2}{2\pi} T. \tag{2}
\]

Plugging the value of the electric Hall conductance \( G_H = \nu e^2/2\pi \) into the definition of the Lorentz number we get

\[
L = \frac{K_H}{G_HT} = \frac{c \pi^2 k_B^2}{\nu} = \frac{c}{\nu} \frac{2}{3} L_0, \tag{3}
\]

where \( L_0 = \frac{\pi^2 k_B^2}{4} \) is the Lorentz number for a free electron gas. For the Laughlin series it is given by \( L_{\nu} = L_0/\nu \), where \( \nu = 1/(2p + 1) \) \((p = 0, 1, 2, \ldots)\) \[14\]. For the Read-Rezayi series, \( \nu = 2 + k/(k + 2) \) and \( c = 2 + 3k/(k + 2) \), we get

\[
L_{RR} = \frac{5k + 4}{3k + 4} L_0. \tag{4}
\]

It might be of interest later to consider the Lorentz number for the partially filled upper Landau level, \( \nu = k/(k + 2) \), leaving behind the contribution of the integer Hall edge states \((\nu = 2)\). The Lorentz number is then \( L_{RR} = L_{RR}^n + L_{RR}^c = 3L_0 \) which is independent of \( k \), with the neutral and charge contributions being \( L_{RR}^n = (\frac{2k - 2}{k}) L_0 \) and \( L_{RR}^c = (\frac{k + 2}{k}) L_0 \) respectively. This Lorentz number can be measured in a geometry \[16\] where the \( \nu = 2 \) edges are fully back-scattered while transmitting the fractional edges perfectly at a QPC.

**Perturbative calculation of tunneling thermal currents across a point contact.** We write the Hamiltonian for the edges corresponding to the partially filled upper Landau level as

\[
H = \sum_{i=L,R} (H_i^c + H_i^e) + H_tun. \tag{5}
\]

Here \( H_i^c/R \) describe the charged degrees of freedom in the absence of tunneling,

\[
H_i^e = \frac{v_c}{4\pi} \int dx (\partial \phi_i(x))^2, \tag{6}
\]

where \( v_c \) is the charge edge velocity, and \( H_i^c/R \) describe the neutral degrees of freedom \( (L/R \text{correspond to left/right movers}) \). Although the latter Hamiltonian cannot be written explicitly for all the Read-Rezayi states, it acquires an exact meaning within the formalism of conformal field theory as the zeroth mode in the Laurent expansion of the energy-momentum tensor (see, e.g. \[17\]). Finally, the tunneling term corresponding to QPC which mixes only the edges corresponding to the upper Landau level is given by

\[
H_tun = \lambda \Phi_L^\dagger \Phi_R^c + h.c., \tag{7}
\]

where \( \Phi \) is the most relevant operator for tunneling, as dictated by the experimental scenario. We can decompose the operator into its neutral and charge components, as \( \Phi = \Phi_n \Phi_c \), where \( \Phi_n/c \) has scaling dimension \( h_{n/c} \) respectively. Hence the scaling dimension of \( \Phi \) will be \( h = h_n + h_c \) (Table 1). We next turn to calculate the electric current and thermal current to leading order in \( \lambda \) in perturbation theory, and then calculate corresponding Lorentz numbers. We briefly discuss the electric current, closely following Chamon et al. \[18\], extending the formalism to the case that a temperature difference is maintained between the edges. The electric current is defined as the rate of particle (electron/quasi-particle) transfer between the two edges times the effective charge

\[
I = -e^* \frac{1}{2} \left[ N_L - N_R, H \right]. \tag{8}
\]

Here \( N_{L/R} \) correspond to the number of electrons on the left (right) moving edge and \( e^* \) is the charge of tunneling particle \((i.e. \ e^* \text{ stands for electron charge in the SB limit and quasi-particle charge in the WB limit}) \). To lowest order in the tunneling, the electric current through a single QPC is given by the following expression \[18\]

\[
\langle I_h \rangle = e^*|\lambda|^2 \left[ P_h(\omega_0, T, \Delta T) - P_h(-\omega_0, T, \Delta T) \right]. \tag{9}
\]
Here $\omega_0 = e^* V$ is the Josephson frequency, and $P_h$ is defined as
\[
P_h(\omega, T, \Delta T) = \int_{-\infty}^{\infty} dt e^{i\omega t} G_{T^+}(t) G_{T^-}(t),
\]
with the finite temperature Green’s function being $G_T(t) = [\pi T / \sin(\pi T [\delta + it])]^{1/2}$. Clearly, by expanding $P_h$ in $\Delta T$, there is no contribution linear in $\Delta T$. Hence to order $\Delta T$ the electrical current written in terms of the conformal dimension of the tunneling particle is \[ 13 \]
\[
\langle I_h \rangle = \frac{e^* |\lambda|^2}{(2\pi T)^{1/4} \hbar} \left[ 2h - \frac{i\omega_0}{2\pi T} \right] \sinh \left( \frac{\omega_0}{2T} \right) .
\]
(11)

Here $B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x + y)$ is the beta function. Due to linearity of spectrum of fundamental excitations in the neutral and the charge sector, the electric current vanishes in the zero voltage bias limit even if two edges are maintained at two different temperatures. We now repeat this calculation for the heat current. While only the charge-carrying mode contributes to the electric current, here the neutral mode contributes as well. In the following, we will write the tunneling heat current as a sum of two terms, $I_Q = I_{Q,n} + I_{Q,c}$, where the indices $n$ and $c$ refer to the neutral mode and charge mode contributions respectively. We define $I_{Q,n/c}$ as the rate of energy transfer between the left and right moving edges,
\[
I_{Q,n/c} = -\frac{i}{2}[H^c_{n/c} - H^n_{n/c}, H].
\]
(12)

The expectation values of $I_{Q,n/c}$ can be written as
\[
\langle I_{Q,n} \rangle = \frac{\hbar_n}{\hbar_n + \hbar_c} \langle \hat{I}_Q \rangle + \frac{\hbar_c}{\hbar_n + \hbar_c} \langle \hat{I}_Q \rangle + V \langle \hat{I}_h \rangle.
\]
(13)

Note that $\hat{I}_Q$ goes to zero as $\Delta T \to 0$, and is given by
\[
\hat{I}_Q = -i |\lambda|^2 \frac{1}{2} \left[ Q(\omega_0, T, \Delta T) - Q(-\omega_0, T, -\Delta T) + Q(-\omega_0, T, \Delta T) - Q(\omega_0, T, -\Delta T) \right] ;
\]
with
\[
Q(\omega, T, \Delta T) = \int_{-\infty}^{\infty} dt e^{i\omega t} \left( t \partial_t G_T + \frac{\partial}{\partial t} G_T \right).
\]
(14)

Therefore, in response to a temperature difference, for electron/quasi-particle operators which are composed of fields from both charged and neutral modes, the amount of energy carried by them while tunneling splits among the two modes proportionally to the percentage scaling dimension ($\hbar_n / (\hbar_n + \hbar_c)$) of each constituent field.

To first order in the temperature difference $\Delta T$, we can write $\hat{I}_Q = K \Delta T$, where $K$ is the longitudinal thermal conductance. Using the expressions above, the thermal conductance expressed in terms of the electric current, Eq. (11) is given by
\[
K_h = \frac{1}{2T e^*} \left( 2\hbar \partial_\omega (\langle h_{h+1/2} \rangle - \frac{1}{2} \omega_0 (\langle h_h \rangle) \right).
\]
(15)

| $e^*/e$ | $h_c$ | $h_n$ | $h$ |
|--------|-------|-------|------|
| Electron | $1$ | $k+1/2$ | $k$ | $3/2$ |
| Quasi-particle | $1$ | $2(k+1/2)$ | $k-1$ | $1$ |

TABLE I: The effective charge and scaling dimensions of the electron and quasi-particle in the $v = 2 + k/(k + 2)$ states. Here $h_n$, $h_c$ and $h$ are the neutral, charge and total scaling dimensions respectively.

The corresponding Lorenz number is
\[
L_h = \frac{K_h}{T G_h} = \frac{12h^2}{1 + 4h (e^*)^2} L_0.
\]
(16)

Here $G_h$ is the linear conductance defined as $\langle I_h \rangle / V$ in the $V \to 0$ limit. Plugging the scaling dimension for an electron in the Laughlin states, $h = 1/2\nu$, the Lorentz number coincides with the result given by Kane and Fisher \[14\]. This is one of the central results of this letter. From Eq. (15) the charged and neutral contributions to the Lorentz number are $L_n = \frac{h}{h_c} L_h$, $L_c = \frac{h}{h_c} L_h$. Therefore for tunneling thermal current corresponding to electron tunneling between edge states (which is the case for almost closed QPC) of the upper Landau level, the energy splits among the two modes as $\frac{h_n}{h_c} = \frac{2k-1}{2k+2}$ which is identical to that of the uninterrupted edge, $\frac{c_n}{c_c} = 2\frac{k-1}{k+2}$. However this may be special to the Read-Rezayi series.

Proposed Geometry.- We now consider the experimental proposal depicted in Fig. 1. Consider a situation where a voltage bias is applied at contact-1 with respect to the grounded contact-5 driving a current at contact-2. Here we assume that the edge state emanating out of contacts 1, 2, 5 are at equilibrium and at a common temperature $T$. The current and the associated energy injected into the edge starting at point $B$ (see Fig. 1) and heading towards QPC-2 due to tunneling of electrons at QPC-1 is solely pumped into the charge mode (up to finite size contributions to the neutral mode which go to zero for large system sizes). Now, if the charge mode equilibrates via intra-edge Coulomb interaction, then, under certain conditions on which we elaborate below, it can maintain a temperature different than the co-propagating neutral temperature. There are three relevant time scales which we need to worry about: the time required for the charge bosonic edge to reach equilibrium $\tau_c$, the time required for the neutral edge to reach equilibrium $\tau_n$, and the time required for the two edges to reach mutual equilibrium $\tau_{nc}$. In the case that $\tau_{nc} \gg \tau_c$, as energy is pumped only into the charged mode, the two edges can indeed maintain a different temperatures with $T_c > T$ and $T_n = T$ after the charge edge has undergone self-equilibration. The microscopic mechanisms required to estimate these three timescales are not understood at
this stage. However, on general grounds, we expect that \( \tau_{nc} \) will be much larger than \( \tau_c \), as the charge bosonic edge can interact via the Coulomb interaction, while the neutral edge is more inert. The pumping of charge and energy into the edge emanating out of contact-5 due to tunneling events at QPC-1 will result in increase of both temperature and voltage of this edge after it undergoes equilibration. Assuming both charge and energy currents are conserved at QPC-1, we find

\[
\Delta V = \langle I \rangle / G_H, \quad \Delta T_{nc} = \langle I \rangle V / K_H. \tag{17}
\]

One of the most useful ‘thermometers’ available to experimentalists to measure the electronic temperature on the edge is through shot noise measurements. Here we argue that shot noise measurements can be used to detect the temperature difference \( \Delta T_{nc} \) as well. The purpose of the second QPC in Fig. 1 placed at a distance larger than the charge bosonic edge equilibration length is to generate noise. Following Ref. [19] the noise in the present context either in the weak or the strong backscattering limit can be evaluated to be

\[
S = 2 \varepsilon^* (\omega_0, T, \Delta T_{nc}) \left( \varepsilon^* I_h(\omega_0, T) \right) \coth \left( \frac{\omega_0}{2T} \right), \tag{18}
\]

with the new effective charge being a function of bias, the temperature and the temperature difference \( \Delta T_{nc} \):

\[
\varepsilon^* (\omega_0, T, \Delta T_{nc}) = \varepsilon^* \left( 1 + \frac{\Delta T_{nc}}{T} \frac{h c}{2h \sinh(\omega_0/T)} \right). \tag{19}
\]

This \( \Delta T_{nc} \) dependent \( \varepsilon^* \) is an indirect probe for detection of the neutral mode and in addition the coefficient \( h c / 2h \) which is smaller than 1/2 also indicates the existence of a neutral mode. It should be noted that \( \omega_0 \) represents the voltage bias at QPC-2 which is controlled by the difference between \( \Delta V \) and the voltage applied at contact-2. Hence carrying out a voltage sweep at contact-2 and measuring the corresponding \( \varepsilon^* \) via noise measurement at terminal-3 can provide a direct check of our predictions. For our proposal to work, the most crucial part is to obtain a reasonable \( \Delta T_{nc} \) for typical experimental accessible currents and 2DEG temperatures. Using Eq. (17), for \( T = 40 \times 10^{-3} \) Kelvin, impinging current on QPC-1 to be \( .2 \times 10^9 \) Amps., transmission of QPC to be 1% and assuming terminal-5 be grounded we get \( \Delta T_{nc} = 3 \times 10^{-3} \) Kelvin which is well within resolution of present day experiments.

To conclude, we have perturbatively calculated the thermal conductance and Lorentz number for tunneling of any excitation (quasi-particles/electrons) between the edges of Read-Rezayi quantum Hall states in terms of the scaling dimension of the tunneling operator. We discuss the application of a QPC as a heating device and propose a setup where it can be used to detect the presence of the neutral mode. We also provide an expression for tunneling noise and the corresponding effective charge measured for edge states where the charge and the neutral edges are at two different temperatures.

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[1] G. Moore and N. Read, Nucl. Phys. B 360, 362 (1991).
[2] N. Read and D. Green, Phys. Rev. B 61, 10267 (2000).
[3] N. Read and E. Rezayi, Phys. Rev. B 59, 8084 (1999).
[4] M. Milovanović and N. Read, Phys. Rev. B 53, 13559 (1996).
[5] A. B. Zamolodchikov and V. A. Fateev, Zh. Eksp. Teor. Fiz 89, 380 (1985).
[6] E. Fradkin, C. Nayak, A. Tsvelik, and F. Wilczek, Nucl. Phys. B 516, 704 (1998).
[7] P. Fendley, M. P. A. Fisher, and C. Nayak, Phys. Rev. B 75, 045317 (2007).
[8] A. Stern and B. I. Halperin, Phys. Rev. Lett. 96, 016802 (2006).
[9] P. Bonderson, A. Kitaev, and K. Shtengel, Phys. Rev. Lett. 96, 016803 (2006).
[10] P. Bonderson, K. Shtengel, and J. K. Slingerland, Phys. Rev. Lett. 97, 016401 (2006).
[11] S. B. Chung and M. Stone, Physical Review B 73, 245311 (2006).
[12] R. Ilan, E. Grosfeld, and A. Stern, Phys. Rev. Lett. 100, 086803 (2008).
[13] A. Cappelli, M. Huerta, and G. R. Zemba, Nucl. Phys. B 636, 568 (2002).
[14] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. 76, 3192 (1996).
[15] C. L. Kane and M. P. A. Fisher, Phys. Rev. B 55, 15832 (1997).
[16] S. Das, S. Rao, and D. Sen, arxiv:0808.2249 (2008).
[17] P. D. Francesco, P. Mathieu, and D. Sénéchal, Conformal Field Theory (Springer - Verlag New York Inc., 1997).
[18] C. de C. Chamon et al., Phys. Rev. B 55, 2331 (1997).
[19] C. Bena and C. Nayak, Phys. Rev. B 73 (2006).