Geometric interpretation of modeling calculation of cylinders axial vibration effect on transfer of viscous incompressible fluid

A V Panichkin\textsuperscript{1}, L G Varepo\textsuperscript{2}

\textsuperscript{1}Sobolev Institute of Mathematics SB RAS, 13, Pevtsova st., Omsk, 644043, Russia
\textsuperscript{2}Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia

E-mail: panich@ofim.oscsbras.ru

Abstract. The transfer of viscous incompressible fluid between contacting cylinders is influenced by a large number of factors that can be investigated by numerical simulation based on the use of the most complete mathematical models. One of the factors affecting the transfer of viscous fluid between the contacting cylinders is the axial vibration of one of the cylinders. At the same time, experimental studies cannot be performed to the full because of the insufficiency and complexity of monitoring fast processes. Along with a number of other factors, a mathematical model was developed and applied for the numerical study of the effects of axial vibrations (oscillations) on the indicators characterizing the transfer of a viscous incompressible fluid on the test example in a two-dimension setting. The practical implementation of the computer geometric modeling algorithm is shown. A computer visualization of the calculation data on viscous incompressible fluid transfer between the rotating cylindrical surfaces on the example of the printing system is presented.

Keywords: computer geometric modeling algorithm, numerical modeling, cylinder axial vibration, deformation

1. Introduction
The processes of a viscous incompressible fluid interaction with the structure during its transfer make a direct impact on both the dynamic characteristics and the behavior of the fluid itself [1, 2]. These processes are reflected in technology, science, printing systems, medicine and other fields and modeled by various methods. It is noted that there are still many problems in the development, primarily, in terms of the accuracy and stability of the process.

Various mathematical models of the system node are known. These models allow one to determine the change in the quality indicators of the transfer process of a viscous incompressible fluid from the surface of the cylinder to the surface of the cylinder perceiving it. With regard to the printing system, vibrations of an offset printing press are a serious problem [3], which causes many difficulties and the solution of which is still relevant.

2. Problem statement
Among the many sources of vibration in printing presses, the most important sources appear to be the cylinders of the printing system. The block of the printing system consists mainly of three cylinders, and two of them are in direct contact and generate unwanted vibrations.

Measurements of cylinder vibrations on a real printing press were presented in [4]. The work [5] is devoted to calculations of cylinder displacements caused by CGS. The authors [6] focus on studying the
problem of contact with large deformation using the finite elements method. The authors note that the layer compressibility prevails in the process of feeding on the cylinder with a rubber cloth. In [6] a model of offset printing press is presented. The model is described by the system of two parametric differential equations. Computer simulation of the behavior of the printed block was performed.

In the works [7-12] the results of numerical simulation of the viscous incompressible fluid transfer on the ink-receiving surface of the material printed cylinder in the printed contact zone are presented. The implementation of the approach is conducted using the developed algorithm for numerical solution of Navier-Stokes equations of incompressible fluid using finite-difference approximations of differential operators on a compact template taking into account the deformation of the rubber cloth and paper of the offset and printed cylinders of the printing system (Figure 1). The visualization of the geometric modeling was performed.

Despite the positive solutions obtained in the analyzed works, the problem of computer interpretation of changes in the layer of viscous incompressible fluid taking into account the axial vibration of the cylinders does not have a complete solution and is the purpose of this study.

3. Research results
Geometric interpretation of the problem solutions makes it possible to visualize their structure, to identify specific features. The solution area and the Navier-Stokes equations in polar coordinates with the center of the cylinder in p. O1 with variables is considered. The details of the area are shown in figure 1.

![Figure 1. Geometric diagram of viscous incompressible fluid transfer between contacting cylinders taking into account the axial vibration of the cylinders on the example of the printing system](image-url)
After the transition to the convected system of coordinates with the transformation \( \theta = \phi - \omega \), where \( \omega \) is the angular velocity of rotation of the cylinders 1 and 2 in different directions, counterclockwise and clockwise, respectively. In the new variables \((r, \theta)\) with velocity components \((U_r, U_\theta)\) in the presence of angular acceleration of this system \( \varepsilon \), the equations have the following form:

\[
\begin{align*}
\frac{\partial U_r}{\partial t} + \frac{U_r}{r} \frac{\partial U_r}{\partial r} + \frac{U_\theta R}{r} \frac{\partial U_r}{\partial \theta} - \frac{(U_\theta + \omega r)^2}{r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left( \frac{\partial^2 U_r}{\partial r^2} + \frac{2}{r} \frac{\partial U_r}{\partial r} - \frac{2R}{r^2} \frac{\partial U_\theta}{\partial \theta} \right), \\
\frac{\partial U_\theta}{\partial t} + \frac{U_r}{r} \frac{\partial U_\theta}{\partial r} + \frac{U_\theta R}{r} \frac{\partial U_\theta}{\partial \theta} + \frac{U_r U_\theta}{r} + 2U_r \omega + \varepsilon r &= -\frac{1}{\rho} \frac{\partial p}{\partial \theta} + \nu \left( \frac{\partial^2 U_\theta}{\partial r^2} - \frac{2R}{r^2} \frac{\partial U_r}{\partial \theta} \right), \\
\frac{\partial U_r}{\partial r} + \frac{U_r}{r} + \frac{r}{\rho} \frac{\partial U_\theta}{\partial \theta} &= 0,
\end{align*}
\]

where \( \nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \) is Laplace operator, \( \nu \) is the kinematic viscosity, \( \rho \) is the fluid density, \( P \) is the pressure, \( \omega \) is the angular velocity of rotation.

To represent the simulation results of the system in the future, the transition to other coordinates \((x, y) = (R \theta, R - r)\) will be used.

For the area of fluid \( \Omega \) in the convected system at the initial period of time \( t = 0 \), the coordinates \((r, \theta)\) are taken within the following limits: \( r \in [R, R + \delta_2]; \theta \in [-\delta_L/(2R), \delta_L/(2R)] \). In the entire area \( U_r(0, r, \theta) = 0; U_\theta(0, r, \theta) = 0 \), except the coordinate of the initial contact point with cylinder 2: \((R + \delta, \delta_L/(2R))\). For this coordinate from the conditions of adhesion and non-flow, the velocity components are determined by the formulas:

\[
\begin{align*}
V_\theta &= V_c \cos(\varphi_{m1}) + V_y \sin(\varphi_{m1}), \\
V_r &= V_x \sin(\varphi_{m1}) - V_y \cos(\varphi_{m1}),
\end{align*}
\]

for which

\[
\begin{align*}
x_m &= x_c + (2R + \delta) \sin(\varphi_1) - r_2 \sin(\varphi_2), \\
y_m &= y_c - (2R + \delta) \cos(\varphi_1) + r_2 \cos(\varphi_2), \\
V_x &= -\omega(2R + \delta) \cos(\varphi_1) - 2\omega r_2 \cos(\varphi_2), \\
V_y &= -\omega(2R + \delta) \sin(\varphi_1) + 2\omega r_2 \sin(\varphi_2), \\
\varphi_1 &= \varphi_0 - \omega t, \\
\varphi_2 &= \varphi_{m0} - 2\omega t,
\end{align*}
\]

where \((x_m, y_m), (x_c, y_c)\) are the coordinates of this point and the center of cylinder 1 in the Cartesian coordinate system \(OXY\) associated with cylinder 1; \( \varphi_0, \varphi_{m1} \) is the initial value \( \theta \) for the center of cylinder 2 and the current value of the point \( \theta \); \( \varphi_1, \varphi_2 \) are the angles of the two centers of the cylinders between the point of cylinder 2 and the vertical guides, depending on the time and angular velocity of rotation; \( \varphi_{m0} \) is the initial value \( \varphi_2 \).

The coordinates of the circle points of radius \( R \) for cylinder 2, the center of which \( O_2 \) is at a distance \((2R + \delta)\) from \( O_1 \), will have coordinates \((r, \psi_2)\) where \( \psi_2 \) is the angle from the center \( O_1 \) between the point of the circle of cylinder 2 and the segment connecting the centers of the cylinders \( O_1 \) and \( O_2 \). These coordinates have the form:

\[
r = R \sqrt{\sin^2 \psi_2 + (2 + \delta/R - \cos \psi_2)^2} = R \sqrt{5 + 4\delta/R + \delta^2/R^2 - 2(2 + \delta/R) \cos \psi_2};
\]
\begin{align*}
\psi_1 &= \arcsin \left( \frac{\sin \psi_2}{\sqrt{5 + 4\delta/R + \delta^2/R^2 - 2(2 + \delta/R)\cos \psi_2}} \right), \tag{4}
\end{align*}

where $\psi_2$ is the angle of the corresponding arc in the second cylinder.

In the convected system of coordinates, the movement of the center of the second cylinder center around the first one will be at an angular velocity $\omega$, and its rotation around its center will be at an angular velocity $2\omega$ in the opposite direction.

For changing positions with time for points $(x_m, y_m)$ in the corners $\varphi_1, \varphi_2$, their coordinates in polar system of coordinates $(r, \theta)$ are determined as follows. First $r$ after defining $\psi_2 = \varphi_2 - \varphi_1$, is determined from (3), then $\psi_1$ is determined from (4), and finally $\theta = \varphi_{m1} = \varphi_1 - \psi_1$. In such a case the radii of cylinders 1 and 2 will be $r_1 = R \Omega t_1 = R$.

Let us consider the vibration effect of the cylinders axes on the example of axis vibration of cylinder 2. To do this, we consider the change in the distance between the cylinder centers in time from $2R + \delta$ to $2R + \delta_2$, where $\delta_2$ is represented as an oscillation according to the following law with the amplitude $a$ and frequency $\nu$:

$$
\delta_2 = \delta + a \cdot \sin(2\pi \nu t). \tag{5}
$$

Changing the distance value between the centers of the cylinders, it is essential to consider the magnitude of $r_2$ instead of the magnitude $r$ (Figure 1). This leads to the change in the values for $\psi_1$, $\varphi_{m1}$, which are recalculated according to (3), (4). In such a case, variation of other values, such as $\psi_2$, $\varphi_1$, will be presented in the same form according to the specified formulas.

### 4. Results and discussion

The problem posed in the work is solved and implemented in practice by the example of viscous incompressible fluid (ink) transfer between the cylinders of the printing system. The developed algorithm for the computer interpretation of the calculation simulation results showing the influence of the cylinder axial vibration on the transfer of viscous incompressible fluid is presented in figure 2.

To assess the detection problem solution quality in the tracking mode, a computer experiment was performed. Computer simulation of calculation results in the form of graphical representations is shown in table 1.

**Table 1. Computer visualization of the calculation of the cylinders axial vibration influence on viscous incompressible fluid transfer**

| Calculation with vibrations: $r_2 = r_2 + a \cdot \sin(2\pi \nu t)$ | Calculation of vibrations: $r_n = r_n + a \cdot \sin(2\pi \nu t)$ |
|---------------------------------------------------------------|---------------------------------------------------------------|
| $r_2 + a \cdot \sin(2\pi \nu t)$, amplitude $a = 0.3 \cdot 10^{-6}$m, angular velocity $k = 0.05 \cdot 10^{-3}$s, frequency $\nu = 3.18 \cdot 10^{-3}$c$^{-1}$, period $T = 2\pi k = 1/\nu = 3.14 \cdot 10^{-3}$s | $r_n + a \cdot \sin(2\pi \nu t)$, amplitude $a = 0.0 \cdot 10^{-6}$m, angular velocity $k = 0.0 \cdot 10^{-3}$c$^{-1}$, frequency $\nu = 0.0 \cdot 10^{-3}$c$^{-1}$, period $T = 1/(2\pi k) = 1/\nu = \infty \cdot 10^{-3}$c |
| $t = 0.00010$ s | $t = 0.00010$ s |
Calculation with vibrations: 
\[ r'_2 = r_2 + a \cdot \sin(k \cdot t) = r_2 + a \cdot \sin(2\pi \frac{t}{\nu}), \]
amplitude \( a = 0.3 \cdot 10^{-6} \) m, angular velocity \( k = 0.045 \cdot 10^{-3} \) s, frequency \( \nu = 3.50 \cdot 10^{3} \) s\(^{-1}\), period \( T = 2\pi \cdot k = 1/\nu = 2.85 \cdot 10^{-4} \) s,

\[ a = 0.3 \cdot 10^{-6} \] m,
angular velocity \( k = 0.033 \cdot 10^{-3} \) s,
frequency \( \nu = 4.77 \cdot 10^{3} \) s\(^{-1}\),
period \( T = 2\pi \cdot k = 1/\nu = 2.1 \cdot 10^{-4} \) s,

\( t = 0.00050 \) s \hspace{1cm} \( t = 0.00044 \) s

**Figure 2.** Algorithm of the computer simulation for the calculation of viscous incompressible fluid transfer with axial vibration.
Computer implementation of this algorithm demonstrates the changes that occur in the viscous incompressible fluid layer at different stages of the process and in different time periods in a fraction of a second under the influence of the considered variables. The images (table 1) obtained by computer geometric modeling, respectively, simulate:

- the presence of deformations in the layer and their growth with increasing time;
- the form of rupture of viscous incompressible fluid from the degree of impact on the axial vibration process;
- the phenomenon of "ink misting" when it breaks at the exit of the contact zone.

Computer visualization of the calculation results based on the proposed mathematical model illustrates the deformation in the fluid film. The growth of this deformation is caused by an increase in vibration of the printing system cylinder. In this example, there is a transverse rupture of the ink coat, and it is not longitudinal as is typical of the process with minimal or complete absence of vibration. This has a negative impact on the quality of the final product.

5. Conclusion

The graphic images of the changes in the viscous incompressible fluid layer are represented. These changes depend on the cylinders axial vibration value as a function of random variables. The images are obtained by the numerical methods and computer geometric modeling.

The results suggest that the developed and software-implemented model of the viscous incompressible fluid transfer process in the cylinder contact zone adequately simulates the operation of the printing system.

Computer visualization of the calculation results based on the proposed model extends the theoretical understanding of the contact mechanics of viscous incompressible fluid with the cylinders surface of the examined unit.

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