Mode analysis for a quartz tuning fork coupled to acoustic resonances of fluid in a cylindrical cavity

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Abstract. Quartz tuning forks are precise electromechanical oscillators mass produced in different sizes around one millimeter for the purpose of providing the reference frequency for watches and such. Usually, they are designed to operate at $2^{15} = 32768$ Hz in vacuum at room temperature. When refrigerated to cryogenic conditions, they may show extremely high Q-values. Immersion of such an oscillator to fluid medium changes its response due to inertial forces and dissipation exerted by the medium. This makes it very useful in studies of pure and mixed helium fluids at low temperatures. When the wavelength of sound in the medium, determined by the frequency of oscillation and the speed of sound, corresponds to typical dimensions in the fluid volume, the oscillator may produce standing acoustic waves, observed as strong anomalies in the oscillator response. This can happen in helium fluids for both first and second sound under various conditions. We study the character of such modes by computational methods for typical fork geometries in a cylindrical volume. Reasonable correspondence with measurements in helium mixtures both below and above 1 K is obtained. This is the regime of vigorous second sound resonances, since the speed of this unusual mode compares nicely with the typical dimensions and frequency of the tuning forks. The nontrivial geometry of the fork in the cylinder makes the problem somewhat challenging for computations.

1. Quartz tuning forks in helium

Mechanical oscillators immersed in cryogenic fluids and driven over the resonance by electromagnetic forces are convenient probes sensitive to the density and viscosity (particle mean-free path) of the medium. Temperature, pressure and concentration of helium fluids can be measured over wide ranges by such devices. Usually the medium is assumed incompressible to facilitate easier analysis. The validity of that depends on frequency and size of the oscillator as well as the studied medium [1].

Recently, quartz tuning fork oscillators have become popular due to their easy availability, ease of use and good quality of the data produced. Under certain circumstances it is strikingly obvious that acoustic modes in the medium have to be accounted for proper analysis of the data retrieved. For example, in helium mixtures above and below 1 K, second sound couples strongly to the tuning forks of typical dimensions. The resonance pattern may then become utterly complicated and no simple description or analysis is adequate in such cases. Examples of this can be found in references [2, 3, 4, 5].

In this paper we use computational numerical means to study the acoustic modes excited by a typical tuning fork geometry in a cylindrical cavity filled by the fluid of interest. We
Figure 1. Quartz tuning fork in a cylindrical container. The dimensions are: cylinder diameter 2.6 mm, length 6.8 mm, fork base width 1.5 mm, depth 0.33 mm, height 2.3 mm, fork tines width 0.6 mm, depth 0.33 mm, height 3.8 mm. The cylinder is filled by the fluid of interest. For helium mixtures, speed of second sound varies from about 10 m/s to 40 m/s as the function of temperature giving $\lambda = 0.3$–1.2 mm at 32 kHz.

find resonance patterns very much alike those measured in the corresponding real situations in helium mixtures.

Numerical computations were performed for the tuning fork geometry used in references [1, 2, 3, 4]. The principal shape and the dimensions of the problem are given in figure 1. The medium filling the cylindrical volume is characterized by the density and the speed of sound, which determines the resonance condition for the acoustic waves.

The tines of the tuning fork are assumed to bend as thin rectangular cantilever beams fixed at one end, which is an excellent approximation according to the numerical modeling performed. This view is also supported by the study in reference [6]. The amplitude of oscillation $x$ varies as

$$x/x_o = \frac{1}{3}(z/z_o)^4 - \frac{4}{3}(z/z_o)^3 + 2(z/z_o)^2$$

along the length of the beam with $z$ measuring the distance from the top of the base (tines’ fixing point, $z = 0$) to the tip at $z_o$; the oscillation amplitude at the tip is $x_o$. The periodic motion excites acoustic modes in the medium, whose influence is coupled back to the motion of the tines by momentum transfer upon the surfaces. The necessary coupling constants are obtained by numerical integration of the fluid velocity profile over the surfaces in motion, weighted by the amplitude of oscillation above.

2. Mode analysis for the fluid medium

We find the set of eigenvalues compatible with the geometry of the problem by finite-element method. The solution vector over the discrete mesh gives the values of the fluid velocity or pressure, which can be integrated over the surfaces of the fork to determine the forces acting upon it. The resulting influence on the frequency and width of the mechanical resonance of the oscillator can thus be computed.

The mode analysis as such does not include any dissipation mechanisms for the fluid motion. To achieve a realistic picture, one must account for the viscosity in the bulk fluid and for the acoustic impedance on all surfaces. In our analysis, these are taken into account by assuming a descriptive resonance width for the acoustic modes in the fluid, corresponding to the observed resonances in helium mixtures. This is admittedly quite crude implementation as, for example, we did not incorporate any temperature dependencies at all.

Main body of the computations were made by Comsol Multiphysics software. The dimensions were taken as precise to the real quartz oscillator in its enclosing cylinder as they could be determined. The position of the oscillator was assumed symmetric with respect to the cylinder axis, which may not be entirely true in reality and which is a potential additional variable in real
Some most prominent acoustic modes coupled to a quartz tuning fork oscillating at the center of a cylindrical fluid volume. The antinodes of pressure (or concentration for second sound) show up as blue and red colors. Slices through the cylinder are shown at the top row, while the corresponding solutions at the surface of the cylinder are shown at the bottom row.

cases. The fluid volume was covered by a mesh of 10415 elements, and the numerical solution of the pressure-acoustic system gave a total of 1200 eigenvalues between the wavelengths from 0.42 mm to 1.5 mm. Their relative significance to the present problem depend crucially on the value of the surface integral over the moving tines weighted by the amplitude profile of the mechanical motion (1). Some most potent discrete modes are illustrated in figure 2.

At short wavelengths there are very many modes one right next to the other, but they do not necessarily contribute so much to the total response because they partially cancel out in the integration over the fork’s surface. The resulting apparent absorption pattern integrated over all these modes is shown in figure 3.

The energy carried away from the mechanical oscillator by the acoustic waves is manifested in the measurement by increased apparent width of the mechanical resonance. As the acoustic mode crossing over the fork resonance also stores energy, there is an associated dispersive part of the resonance, which is observed as an apparent change in the resonance position. Therefore an acoustic mode passing by the fork resonance produces a characteristic loop in the resonance frequency-width plane. A general rule for such coupled oscillators implies that the ratio of the change in frequency/width of such loops is 1/2. Since there are many modes next to one another coupling with the fork, the complete pattern is like spaghetti with loops within loops. The computed pattern displayed in figure 4 resembles a lot the actually measured ones but it would be too daring to claim equivalence in full detail.

The constructed model reproduces the observed phenomenon with good correspondence. It is possible to identify with fair certainty some most prominent individual acoustic modes and also distinguish whether the standing waves reside within the structures of the fork itself or within the containing cylinder. However, the detailed pattern depends on variable factors, such
Figure 3. Apparent width of the quartz tuning fork resonator resulting from the fork coupling to acoustic modes in the medium as function of the wavelength of sound. The vertical scale depends on density of the medium and on attenuation of acoustic energy, and corresponds roughly to the measured widths in Hertz under the studied conditions.

Figure 4. Reactive and dissipative contributions of the acoustic modes together produce a complex pattern of loops in the fork’s resonance frequency shift-width plane. Although the units are arbitrary, they correspond to each other on both axes.

as the centricity of the fork oscillator inside the containing cylinder, so that different individual forks even from the same supply probably would produce a unique set of acoustic resonances. Nevertheless, the pattern is reproducible for a given fork, and if untouched, it can be used as an instrument providing repeatable, easily identifiable and very sharp markers as function of temperature or other variables influencing the speed of sound in helium fluids.

Acknowledgments
We thank Academy of Finland, National Graduate School in Material Physics, Finnish Cultural Foundation, Finnish Academy of Science and Letters and Jenny and Antti Wihuri Foundation for financial support.

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