Students’ mathematical modeling abilities in interpretation-construction design model

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Abstract. Students’ difficulties in mathematical modeling are in understanding problems, making assumptions in simplifying problem situations into a mathematical problem, recalling concepts in mathematics for solving problems, and checking solution of the problem situation. The study aims at investigating whether interpretation-construction design model (ICON) enhances students’ mathematical modeling abilities. This study is experimental in nature and it was conducted in a public senior high school in Bogor, West Java, Indonesia. The sample of this research consists of the 10th grade students. The teaching materials given were quadratic equations and functions. The instrument used in this research covers mathematical modeling test, and the data were analysed by using Mann-Whitney test and t-test. The results show that students’ mathematical modeling abilities who received learning treatment using ICON is better than those who received conventional learning. The achievement and enhancement of students’ mathematical modeling abilities who received ICON are in medium level. However, the result of the research indicates that the students found difficulties in interpreting solution and mathematical model of situation.

1. Introduction
Mathematical modeling is one of many mathematical abilities that must be possessed by students [1,2] for instance: mathematical argumentation, reasoning, understanding, connection, communication, representation, spatial thinking, and critical thinking.

Mathematical modeling is the process of transforming real world problems into a mathematical model. Through mathematical modeling, students learn using various mathematical representations and apply appropriate mathematical methods or procedures in solving real world problems [3]. Students are expected to find relationships between contextual problems and mathematics knowledge, than translate a problem into a mathematical model through schematization, formulation, and visualization. It can be in the form of mathematical symbols, schemes, graphs, diagrams, or algebraic manipulations.

Mathematical modeling process can be divided into three steps: identifying real world situation, creating mathematical model, and verifying the resulting mathematical model [4]. The situational model is transferred into a mathematical model, within which a mathematical solution is elaborated by applying mathematical techniques or procedures, and the mathematical results need to be interpreted and validated with respect to the real situation [5], mathematical modeling involves messy or open-
ended problems that require students to make genuine choices about how to approach problems mathematically [6].

Previous studies conclude that students are less intelligent in searching for important elements contained in the problems, and verifying the resulted model and solutions [7]. Students faced difficulties not only in reading and understanding the modeling task, but also in recalling mathematics knowledge needed for solving the problem [8]. Students still find difficulties in solving equations and quadratic functions [9,10]. Students understanding of concepts and procedures can affect students ability in solving problems, and students interpret that a quadratic equation is only calculation, without considering the element or variable that are not known as equation properties [10].

This study focuses on the implementation of interpretation-construction design model (ICON), the achievement and enhancement of students’ mathematical modeling ability. ICON consists of seven principles, namely: authentic observation, interpretation construction, contextualization, cognitive apprenticeship, collaboration, multiple interpretation, and multiple manifestation [11,12]. ICON emphasizes the importance of students’ learning activities in constructing interpretations from authentic observations, discussing about interpretations, reflecting, analyzing, and summarizing the interpretations. Teachers as the facilitators, teacher provide learning environment. ICON affect students’ mathematical reasoning abilities [13], and mathematical modeling processes involve mathematical reasoning.

2. Methods
This study is experimental research and it was conducted in a public senior high school in Bogor, West Java, Indonesia. The sample of this research consists of the 10th grade students. This study used pretest-posttest control group design. The experimental group was given interpretation-construction design model (ICON-model) treatment while the control group was given with conventional learning. The instrument used in this research covers mathematical modeling test which refer to Zulkarnaen with having a validity index 0.70 and reliability index 0.87. The data were analysed by using t-test and Mann-Whitney test. The teaching materials given were quadratic equations and functions [14].

3. Result and disussion
The result of the test data of students’ mathematical modeling ability in both groups of samples is presented in table 1.

| Group   | Pretest (Achievement) | Posttest (Achievement) | Gain (Enhancement) |
|---------|-----------------------|------------------------|--------------------|
|         | Min | Max | M  | SD  | Min | Max | M  | SD  | Min | Max | M  | SD  |
| ICON    | 0.10| 4.91| 1.87|      | 10  | 17.12| 3.05|      | 0.24| 0.89| 0.58| 0.15|
| Conventional | 1   | 9.69| 4.69| 1.87|    | 10  | 17.12| 3.05|      | 0.24| 0.89| 0.58| 0.15|

Based on table 1, in beginning the students’ achievement in mathematical modeling abilities between two groups is not different, but their abilities were different after having ICON treatment was implemented. The students’ achievement in mathematical modeling who received ICON treatment are in the medium level (because the average score is above 65% of the maximum score), while the achievement of students’ in mathematical modeling who received conventional learning are in the lower level (because the average score is below 65% of the maximum score). The students’ enhancement in mathematical modeling who received ICON and conventional learning are in the medium level.

For more details, the results of statistical analysis of students' mathematical modeling abilities (MMA) test are presented in table 2.
Table 2. Normality test and comparison test results of students’ MMA (Significance level = 0.05).

| Data group | Normality test | Comparison Test | Conclusion |
|------------|----------------|-----------------|------------|
|            | Sig | Description     | Statistical test | Sig | |
| Pretest ICON | 0.001 | Not normal    | Mann-Whitney | 2094.00 | 0.411 | H₀ accepted |
| Conventional | 0.018 | Not normal    | Mann-Whitney | 342.000 | 0.001 | H₀ rejected |
| Postest ICON | 0.067 | Normal        | Mann-Whitney | 11.698 | 0.001 | H₀ rejected |
| Conventional | 0.023 | Not normal    | t-test | 11.698 | 0.001 | H₀ rejected |
| Gain ICON | 0.198 | Normal        | t-test | 11.698 | 0.001 | H₀ rejected |
| Conventional | 0.187 | Normal        | t-test | 11.698 | 0.001 | H₀ rejected |

Based on table 2, it can be seen that the students’ achievement and enhancement in mathematical modeling who received ICON treatment is better than those who received conventional learning. Therefore, learning mathematics using ICON gives a significant influence on mathematical modeling abilities.

The different between ICON and conventional in term of mathematical modeling abilities as indicated in table 2 caused by several factors, such as characteristics of the learning model. ICON has a constructivism paradigm [11,12] and constructivist perspective contended that students understand mathematics better if they are more actively involved in their own learning. In addition, the other differing factors include teaching strategies, students’ interaction with teachers, students’ interaction with other students, and students’ interaction with the teaching material.

The data of achievement in mathematical modeling both sample groups are analysed from each aspect as presented in table 3.

Table 3. The mean score of achievement in MMA.

| Aspect MMA | Mean Score ICON | Mean Score Conventional (CL) |
|------------|-----------------|-----------------------------|
| Identifying problems | 3.00 | 2.01 | 1.43 |
| Constructing mathematical model | 4.00 | 3.32 | 2.30 |
| Working mathematically | 4.00 | 2.56 | 1.19 |
| Interpreting solutions | 3.00 | 1.85 | 1.21 |
| Interpreting model of situation | 4.00 | 2.09 | 1.22 |
| Validating mathematical model | 4.00 | 2.19 | 1.91 |

Note: MS (Maximum Score)

Based on table 3, it is obvious that the students’ MMA who received ICON are better than the students who received conventional learning almost in every aspect. Among the six aspects measured, the aspect of constructing mathematical model has the highest mean compared to the other aspects of mathematical modeling abilities. The aspect of giving the interpretation of mathematical situation and interpreting solution has the lowest mean. Therefore, the students still have poor ability in providing explanations related to mathematical models.

The different between ICON and conventional in term of achievement and enhancement mathematical modeling abilities was caused by a fundamental difference during the learning process. The students who received conventional learning acquire mathematical knowledge, facts, concepts and procedures from teachers and resource books. In contrast, the students who received ICON, at the beginning of the lesson were confronted with a problem situation. Students' activities in solving problems collaboratively help students to reinforce mathematical ideas and forming new knowledge, and the activity of creating and explaining a mathematical model contained in lesson materials exerts a great affect on the achievement and enhancement of students' mathematical modeling abilities.
Figure 1. An example of contextual problem in students’ worksheet.

Figure 1 shows an example of hands-on activities carried out by the students who work under ICON. Here the students have to answer some posed questions, including: ‘What important information can you find in problem situations?’, ‘Are the data sufficient to determine the length of the house and the width of the house? If there are sufficient, what information be required? Explain!’, ‘Construct the correct mathematical model to explain the length and width of the house!’, 'Solve the problem through the mathematical model that has been constructed’, ‘Is it correct that the length and width of the house consecutively 45 meters and 20 meters? Explain the reason!’.

In mathematics learning under ICON, mathematics concept is not given directly to students. Rather, the students should be involved in constructing mathematics knowledge through solving contextual problem related to the material being studied. By solving the problems, the students will construct of the meaning or the interpretations of the problem situations. Mathematics learning by inductive process accompanied by contextual problems related to lesson material can make the students successful in constructing mathematical model.

The following figure provides some examples of students’ responses in mathematical modeling test.

The computer simulation of highway, gives a function of $y = f(x) = -0.004x^2 - 0.6x$.

Is the quadratic function resulted from the computer simulation relevant with the actual highway? If yes, provide the rationales! If not, make a quadratic function relevant to the actual highway and its rationales as well!

Figure 2. An example question in mathematical modeling test.

The expected students’ answers in figure 2 are correct and they can provide reasoning between quadratic function from the computer simulation and real world problem situation. An example of a student's response presented in figure 3.

Student A-13 tried to testing mathematical models using a routine procedure, but this procedure does not guide the students to the correct solution. Hence, student A-13 responses in verification mathematical model could be produce by him using irrelevant procedure, and it did not guide students to the correct solution. Therefore, most of student still find difficulties in interpret solution and mathematical model of a situation.
Mathematical learning using the ICON requires learning activities related to the real world, through authentic situation observation and contextualized activities [11]. Hands-on activities is structured in such a way as to involve situations and contexts of real world problems that are familiar to students. Mathematics learning using ICON as well as conventional learning (CL) involves the teacher’s involvement in the modeling stage. In both group, the teacher acts as a model on how to solve math problems. Through modeling, students can see how teachers solve problems, and students learn to solve problems in the same way that teachers do or "learning-through-guided-experience" [15].

In ICON and conventional learning, the teacher served as a coaching that 'trains' the students to be skillful or to have the ability in solving problem situation. However, there are differences between coaching by the teacher to the students. In conventional learning, the teacher gave students exercise after the teacher had explained the material of equations and quadratic function, the given problem could be decontextualized. While in ICON, coaching was given as part of learning activities or learning process.

The Hands-on activities it can develop mathematical modeling abilities through mathematization, and makes the students familiar to interpret the appropriateness of mathematical modeling and solution offered (de-mathematization) be accompanied techniques provided by teachers through cognitive apprenticeship that is modeling, scaffolding, coaching, articulation, reflection and collaborative hands-on solutions can improve students' mathematical modeling abilities. The explanations done by students to other students can be performed in small groups as well as explaining in front of the class. Thus, students collaboration helps to develop students' ability to communicate. When they explain their opinions, they should develop clear ideas about concepts that are being used and verbally communicate them to other students [16] which can finally improve students' mathematical modeling abilities.

4. Conclusion
This study attempts to figure out whether learning using interpretation-construction design model (ICON) can enhance the students’ mathematical modeling ability. The results show that the students’ mathematical modeling ability who received ICON is better than those who received conventional learning. However, the students’ are still poor in interpreting solution and model situation. The lesson materials in mathematics learning which was conducted by using ICON hands-on activities to coorporate problem situations or real world problems. These problems should be selected in such way that they can imagined and experienced by students. When the students have difficulties in solving contextual problems presented in hands-on activities, the teacher can asked the other students to explain
the problem solving process. By this effort, the student who have difficulties can gain benefit and do reflection on the process they did.

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