Azimuthal anisotropy in Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

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The azimuthal anisotropic flow of identified and unidentified charged particles has been systematically studied in Cu+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV for harmonics \( n = 1-4 \) in the pseudorapidity range \( |\eta| < 1 \). The directed flow in Cu+Au collisions is compared with the rapidity-odd and, for the first time, the rapidity-even components of charged particle directed flow in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV. The slope of the directed flow pseudorapidity dependence in Cu+Au collisions is found to be similar to that in Au+Au collisions, with the intercept shifted toward positive pseudorapidity values, i.e., the Cu-going direction. The mean transverse momentum projected onto the spectator plane, \( \langle p_x \rangle \), in Cu+Au collision also exhibits approximately linear dependence on pseudorapidity with the intercept at about \( \eta \approx -0.4 \) (shifted from zero in the Au-going direction),

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closer to the rapidity of the Cu+Au system center-of-mass. The observed dependencies find natural explanation in a picture of the directed flow originating partly due the “tilted source” and partly due to the asymmetry in the initial density distribution. A charge-dependence of $(p_t)$ was also observed in Cu+Au collisions, consistent with an effect of the initial electric field created by charge difference of the spectator protons in two colliding nuclei. The rapidity-even component of directed flow in Au+Au collisions is close to that in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, indicating a similar magnitude of dipole-like fluctuations in the initial-state density distribution. Higher harmonic flow in Cu+Au collisions exhibits similar trends to those observed in Au+Au and Pb+Pb collisions and is qualitatively reproduced by a viscous hydrodynamic model and a multi-phase transport model. For all harmonics with $n \geq 2$ we observe an approximate scaling of $v_n$ with the number of constituent quarks; this scaling works as well in Cu+Au collisions as it does in Au+Au collisions.

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I. INTRODUCTION

The study of the azimuthal anisotropic flow in relativistic heavy-ion collisions has been making valuable contributions to the exploration of the properties of the hot and dense matter – quark-gluon plasma (QGP) – created in such collisions. Anisotropic flow is usually characterized by the coefficients, $v_n$, in the Fourier expansion of the particle azimuthal distribution measured relative to the so-called flow symmetry planes: $dN/d\phi \propto 1 + 2 \sum_n v_n \cos[n(\phi - \Psi_n)]$, where $\phi$ is the azimuthal angle of a produced particle, and $\Psi_n$ is the azimuthal angle of the $n^{th}$-harmonic flow plane. The first harmonic (directed flow) and second harmonic (elliptic flow) coefficients have been measured most often and compared to the theoretical models [1–3]. According to recent theoretical calculations, the higher harmonic flow coefficients appear to provide additional and sometimes even stronger constraints on the QGP models and on the initial conditions in heavy-ion collisions [4, 5].

Elliptic flow, $v_2$, has been extensively studied both at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) energies. For low transverse momentum ($p_T < 2$ GeV/$c$), $v_2(p_T)$ is well described by the viscous hydrodynamic models. A comparison of the elliptic flow measurement to hydrodynamic model calculations led to the finding that the QGP created in nuclear collisions at RHIC and LHC energies has extremely small ratio of shear viscosity to entropy density, $\eta/s$, and behaves as an almost ideal liquid [2–4]. The centrality dependence of elliptic flow, and in particular flow fluctuations, provided detailed information on the initial conditions and their fluctuations.

While the experimental results on the elliptic flow are mostly understood, there exists no single model that satisfactorily explains the directed flow dependencies on centrality, collision energy, system size, rapidity, transverse momentum, and even more, on the particle type [1]. This clearly indicates that an important piece in our picture of ultrarelativistic collisions is still missing. This could affect many conclusions made solely on the elliptic flow measurements, as the initial conditions that would be required for a satisfactory description of the directed flow, could lead to stronger (or weaker) elliptic flow. Possible effects of that have been mostly ignored so far in part due to complication of 3+1 hydrodynamical calculations compared to 2+1 calculations assuming Bjorken scaling. The directed flow originates in the initial-state spatial and momentum (initial collective velocity fields) asymmetries in the transverse plane. The directed flow might be intimately related to the vorticity in the system, and via that to the global polarization of the system and to chirality flow – two of the most intriguing directions in current heavy ion research [6, 7].

RHIC has been very successful in providing data on symmetric collisions of approximately spherical nuclei such as Cu+Cu and Au+Au, and non-spherical nuclei such as U+U, as well as asymmetric Cu+Au collisions. Since the anisotropic flow originates from the anisotropy of the initial density distribution in the overlap region of the colliding nuclei, these collisions provide important complementary information on both the geometry and fluctuations in the initial density distributions. In particular, Cu+Au collisions are characterized by a large asymmetry in the average initial density distribution in the transverse plane, leading to significant $v_1$ and $v_2$ flow coefficients even at midrapidity. Measurements of $v_1$ and $v_3$ in Cu+Au collisions can be compared to the corresponding measurements in symmetric collisions, where they can originate only in density fluctuations, thus providing additional information on the role of the initial density gradients. Asymmetric collisions, with their strong electric fields in the initial stages due to the charge difference of spectator protons in the colliding nuclei, offer a unique opportunity to study the electric conductivity of the created matter and provide access to the time development of quark and antiquark production [8–11].

In symmetric collisions, such as Au+Au, the directed flow measured relative to the reaction plane (a plane defined by the impact parameter vector and the beam direction) is an odd function of (pseudo)rapidity. Note that while in symmetric collisions there exist an ambiguity/freedom in which of the nuclei is called a projectile and which a target, there is not any ambiguity in the results. The impact parameter is always defined as a vector in the transverse plane from the center of the target nucleus to the center of the projectile nucleus. The projectile velocity defines the positive $z$-direction, and, correspondingly, positive (pseudo)rapidity. The directed
flow measured relative to the reaction plane has a characteristic “≈-”-shape, crossing zero three times, with negative slope at midrapidity (for a review, see [1]), where the sign of the directed flow is conventionally defined to be positive for projectile spectators at forward rapidity. The origin of such a dependence is not totally clear. In hydrodynamic models, it is often produced through “tilted” source initial conditions [12–14], as shown in Fig. 1(a), with parameters of the tilt obtained from a fit to the data [14, 15]. In a pure “tilted source” scenario [12, 13], \( v_1(p_T) \) is a monotonic function of \( p_T \) and the pseudorapidity dependence of \( \langle p_T \rangle (\eta) \equiv \langle p_T \cos(\phi - \Psi_1) \rangle \), where \( \langle \rangle \) means an average over particles in an event and then an average over all events, can be directly related to that of \( v_1(\eta) \) (see Appendix). In asymmetric collisions, as well as in symmetric collisions away from midrapidity, the initial transverse density distribution has dipole-like asymmetry. This leads to an additional contribution to anisotropic flow, interpreted either as shadowing [16], or due to the difference in pressure gradients in different directions within the transverse plane [17]. The first harmonic term, often called dipole flow after a dipole-like density asymmetry, contributes to directed flow. The sign of the dipole flow contribution appears to be similar to that of “tilted source”. However there exist a significant difference between the two contributions – the contribution to \( \langle p_T \rangle \) from dipole flow is zero [18]. This fact can be used to disentangle the relative contributions to directed flow from the “tilted source” and initial density asymmetries. The condition \( \langle p_T \rangle \equiv \langle p_T \cos(\phi - \Psi_1) \rangle = 0 \) also leads to a characteristic \( v_1^{\text{dipole}}(p_T) \) shape which crosses zero at \( p_T \sim \langle p_T \rangle \) [18]. Higher \( p_T \) particles tend to be emitted in this direction, while lower \( p_T \) particles are emitted in the opposite direction to balance the momentum in the system. The sign of the average contribution to \( v_1 \) is determined by the low \( p_T \) particles.

The fluctuations in the initial density distribution, in particular those leading to a dipole asymmetry in the transverse plane, lead to non-zero directed flow, i.e. dipole flow, even at midrapidity [18]. The direction (azimuthal angle) of the initial dipole asymmetry, \( \Psi_1^{\text{dipole}} \), determines the direction of flow. The dipole flow angle \( \Psi_1^{\text{dipole}} \) can be approximated by \( \Psi_{1,3} = \arctan((r^3 \sin \phi)/(r^3 \cos \phi)) + \pi \) [18] where \( r \) and \( \phi \) are the polar coordinates of participants and a weighted average is taken over the overlap region of two nuclei, with the weight being the energy or entropy density. The angle \( \Psi_{1,3} \) points in the direction of the largest density gradient. Very schematically, the modification to \( v_1(\eta) \) for a particular fluctuation leading to positive dipole flow is shown in Fig. 1(b).

The difference in the number of participating nucleons (quarks) in the projectile and target nuclei also leads to the change in rapidity of the “fireball” center-of-mass relative to that of nucleon-nucleon system. In symmetric collisions such a difference would be a consequence of fluctuations in the number of participating nucleons event-by-event [19], while in asymmetric collisions the position of the center-of-mass of participating nucleons will be shifted on average, depending on centrality. In this case, one would expect that the overall shape of \( v_1(\eta) \) to be mostly unchanged, but the entire \( v_1(\eta) \) curve to be shifted in the direction of rapidity where more participants move, as schematically indicated in Fig. 1(c).

Finally, we note that the dipole flow is found to be less sensitive to the shear viscosity over entropy \( \eta/s \) [20] than \( v_2 \) and \( v_3 \), therefore it provides a better constraint on the geometry and fluctuations of the system in the initial state.

In Pb+Pb and Au+Au collisions the initial dipole-like asymmetry in the density distribution at midrapidity is caused purely by the fluctuations, while Cu+Au collisions have an intrinsic density asymmetry due to the asymmetric size of colliding nuclei. In addition to the directed flow of the “tilted source” (Fig. 1(a)), one might expect the dipole flow to be produced by the asymmetric density gradient (Fig. 1(b)) and the center-of-mass shift in asymmetric collisions (Fig. 1(c)). Therefore it is of great interest to study the different components of directed flow in Cu+Au collisions to improve our understanding of the role of gradients in the initial density distributions and the hydrodynamic response to such an initial state.

Experimentally, the directed flow is often studied with the first harmonic event plane determined by the spectator neutrons [21–23]. Recent study [10] shows that in ultra-relativistic nuclear collisions the spectators on average deflect outward from the center of the collision, e.g. projectile spectators deflect in the direction of the impact parameter. By combining the measurements relative to the projectile, \( \Psi_{SP}^P \), and target, \( \Psi_{SP}^T \), spectator planes, the ALICE Collaboration reported the rapidity-odd and even components of directed flow in Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV [24]:

\[
\begin{align*}
\langle \Psi \rangle &= v_1^{\text{odd}} + v_1^{\text{even}}, \\
v_1^{\text{odd}} &= (v_1\{\Psi_{SP}^P\} - v_1\{\Psi_{SP}^T\})/2, \\
v_1^{\text{even}} &= (v_1\{\Psi_{SP}^P\} + v_1\{\Psi_{SP}^T\})/2.,
\end{align*}
\]

where the “even” component might originate in the fluctuation of the initial density. Note that the “projectile” nucleus defines the forward direction and \( \langle \cos(\Psi_{SP}^P - \Psi_{SP}^T) \rangle < 0 \). Since the target spectator plane \( \Psi_{SP}^P \) points in the opposite direction to \( \Psi_{SP}^T \), in the ALICE paper [24], directed flow relative to the target spectator plane was defined as \( v_1\{\Psi_{SP}^P\} = -\langle \cos(\phi - \Psi_{SP}^T) \rangle \), resulting in Eqs. (2) and (3) having the opposite sign convention from Ref. [24].

A finite \( v_1^{\text{even}} \) was observed in Pb+Pb collisions with little if any rapidity dependence [24]. It is believed that the origin of this component is in finite correlations between the direction of spectator plane and the direction of the initial dipole asymmetry at midrapidity. Such a correlation is expected to be weak, \( \langle \cos(\Psi_{SP}^P - \Psi_{1,3}) \rangle \ll 1 \), which would explain the small magnitude of \( v_1^{\text{even}} \) of the order of a few per mil. The \( v_1^{\text{dipole}} \) can be measured.
FIG. 1. (Color online) Cartoon illustrating different contributions to the directed flow and their effect on the (pseudo)rapidity dependence of mean $v_1$. Panel (a) shows the effect of the “tilted source”, while panels (b) and (c) include additional effects of asymmetric density distribution and asymmetry in number of participating nucleons. In panels (b) and (c), the dashed lines represent the effect of the “tilted source” only and the solid lines represent the two effects combined.

via two-particle correlation ($v_1^{\text{dipole}}$ relative to participant plane) \cite{25,26} taking into account the momentum conservation effect which requires model-dependent treatment. The $v_1^{\text{dipole}}$ measured using two-particle correlation \cite{25} shows $\sim 40$ times larger magnitude than $v_1^{\text{even}}$ measured with spectator planes. This difference can be explained by the weak correlation of $\langle \cos(\Psi_{\text{sp}}^P - \Psi_{1,3}) \rangle$ as discussed in Ref. \cite{24}.

Following a similar approach to that of ALICE Collaboration, we study directed flow in midrapidity region relative to the target (Au) and projectile (Cu) spectator planes (see Fig. 2). We identify two components of the directed flow: the one determined by the directed flow relative to the (true) reaction plane, $\Psi_{\text{RP}}$, and the component due to the initial density fluctuations. The first component is similar to the “odd” component in symmetric collisions, but in Cu+Au collisions it also includes a contribution due to non-zero average dipole-like asymmetry in the initial density distribution. The second component, due to the initial density fluctuations, is similar to the “even” component in the ALICE analysis. In addition to the results obtained from correlations to the spectator planes, we also present the results from 3-particle correlations \cite{3,21,27}, $v_1 \{3\}$, which are interpreted as projection of the directed flow onto the second harmonic event plane, $\Psi_2$, that is defined by participants. See the schematic view of a collision with different event planes identified in Fig. 2. Model calculations \cite{18} suggest that the dipole flow might be correlated more strongly with $\Psi_2$ (second harmonic participant event plane) than with the spectator plane (which is very close to the reaction plane), and thus one can expect that the dipole flow contribution to $v_1 \{3\}$ might be slightly larger than that with the spectator plane.

Elliptic and higher harmonic flow measurements in asymmetric collisions are also extremely interesting. While in symmetric collisions, the odd harmonics originate from the initial density fluctuations \cite{28}, in asymmetric collisions the intrinsic geometrical asymmetry in the initial state may lead to significant odd components of the flow. Thus the measurements of higher harmonic flow as well as the directed flow in Cu+Au collisions provide an opportunity to study the interplay of the two effects and provide additional constraints on hydrodynamic models.

A quark number scaling was observed for the elliptic flow \cite{3,29,30}, suggesting collective behavior at a partonic level. Recently PHENIX reported that the quark number scaling also works for higher harmonic flow in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV \cite{31} by considering the order of the harmonics in the scaling rule, al-
though the interpretation is still under discussion. It is very interesting to study if such a scaling is also held in asymmetric collisions having a potentially different origin for the odd component of the higher harmonic flow.

In this paper we present the measurements of the higher harmonic (up to $n = 4$) anisotropic flow of unidentified and identified charged particles in Cu+Au collisions. Results from Cu+Au collisions are compared with those from Au+Au collisions, as well as with hydrodynamic and transport models. We discuss the quark number scaling for $v_2$, $v_3$, and $v_4$ of charged pions, charged kaons, and (anti)protons. Compared to the previous measurements, a better accuracy of $v_3$ results and new data on $v_4$ provide a more detailed view on the scaling properties of anisotropic flow in asymmetric collisions and the physics behind it.

This paper is organized as follows: Section II provides a brief explanation of the experimental setup. The details of data reduction and analysis method are described in Sec. III. Results for the directed flow are presented in Sec. IV and results for higher harmonic flow are presented in Sec. V. For charged particles, we compare our results to theoretical models. For the higher harmonic flow of identified particles, we also discuss the number of constituent quark (NCQ) scaling. Section VI summarizes the results and findings.

II. EXPERIMENTAL SETUP

The STAR detector system is composed of central detectors performing tracking and particle identification, and trigger detectors located at the forward and backward directions. The Zero Degree Calorimeters (ZDC) [32] and the Vertex Position Detector (VPD) [33] are used to determine the minimum-bias trigger. The ZDCs are located at forward and backward angles of $|\eta| > 6.3$ and measure the energy deposit of spectator neutrons. The VPD consists of two identical detectors surrounding the beam pipe and covering the pseudorapidity range of $4.24 < |\eta| < 5.1$. The VPD provides the start time of the collision and the position of the collision vertex along the beam direction.

The Time Projection Chamber (TPC) [34] is used for the tracking of charged particles. It covers the full azimuth and has an active pseudorapidity range of $|\eta| < 1$. The TPC is also used for particle identification via specific ionization energy loss, $dE/dx$. Particle identification also utilizes the Time-Of-Flight detector (TOF) [35]. The TOF consists of multigap resistive plate chambers and covers the full azimuth and has a pseudorapidity range of $|\eta| < 0.9$. The timing resolution of the TOF system with the start time from the VPD is $\sim 100$ ps.

III. DATA ANALYSIS

The analysis is based on the minimum-bias data for Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV collected in 2012 and Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV collected in 2010. The collision vertex was required to be within $\pm 30$ cm from the center of the TPC in the beam direction. Additionally, the difference between the two $z$-vertex positions determined by TPC and VPD was required to be less than $\pm 3$ cm to reduce the beam-induced background (pileup). The vertex position in the transverse plane was required to be within $2$ cm from the beam center. These criteria select forty-four million minimum-bias triggered events for Cu+Au collisions and ninety-five million minimum-bias triggered events for Au+Au collisions. Centrality was defined based on the measured charged particle multiplicity within $|\eta| < 0.5$ and a Monte Carlo Glauber simulation in the same way as in previous studies [36]. The effect of the trigger efficiency was taken into account in the results by appropriate weights for both Cu+Au and Au+Au collisions.

In the following subsections, the details of analysis are described. Analysis procedures are basically the same as in previous STAR publications [27, 37]. The only difference in the analysis between asymmetric and symmetric collisions is the way to evaluate the resolution of the event plane because one cannot assume equal subevents in forward and backward rapidities (two subevent method) in asymmetric collisions, as explained in Sec. III B and III C.

A. Track selection and particle identification

Good quality charged tracks were selected based on the TPC hit information as follows. The number of hit points used in track reconstruction was required to be greater than 14, with the maximum possible number of hit points of 45. The ratio of the number of hits to the maximum possible for that track was required to be larger than 0.52. These requirements ensure better momentum resolution and allow to avoid track splitting and merging effects. The track distance of closest approach to the primary vertex (DCA), was required to be less than 3 cm to reduce contributions from secondary decay particles. The tracks within $0.15 < p_T < 5$ GeV/$c$ and $|\eta| < 1$ were analyzed in this study.

Particle identification was performed using the TPC and TOF information as mentioned above. For the TPC, the particles were identified based on the $dE/dx$ distribution normalized by the expected energy loss given by the Bichsel function [38], expressed as $n\sigma^{\text{TPC}} = \log([dE/dx]^{\text{meas}}/[dE/dx]^{\exp})/\delta_{dE/dx}$, where $\delta_{dE/dx}$ is the $dE/dx$ resolution. The distribution of $n\sigma^{\text{TPC}}$ is nearly Gaussian for a given momentum and is calibrated to be centered at zero with a width of unity for each particle species [39, 40]. $\pi^+ (\pi^-)$, $K^+ (K^-)$, and $p(\bar{p})$ samples were obtained by requiring $|n\sigma^{\text{TPC}}| < 2$ for particles of interest and $|n\sigma^{\text{TPC}}| > 2$ for other particle species.
To increase the purity of the kaon and proton samples, we applied the more stringent pion rejection requirement $|n\sigma_{\text{TPC}}| > 3$. When the track has hit information from the TOF, the squared mass ($m^2$) can be calculated from the momentum, the time of flight, and the path length of the particle. The $\pi^+(\pi^0)$, $K^+(K^-)$, and $p(\bar{p})$ were selected from a 2$\sigma$ window relative to their peaks in the $m^2$ distribution. Additionally, the selected particles were required to be away from the $m^2$ peak for other particles. When the TOF information was used in the particle identification, the TPC selection criterion was relaxed to $|n\sigma_{\text{TPC}}| < 3$ for the particle of interest. The purity of selected samples drops down to ~90% at higher $p_T$. However we found that the variation of particle selection cuts does not affect the results beyond the uncertainties as described in Sec. III D.

**B. Event plane determination**

The event plane angles were reconstructed based on the following equations [3]:

$$n\Psi_n^{\text{obs}} = \tan^{-1} \left( \frac{Q_{n,y}}{Q_{n,x}} \right),$$

$$Q_{n,x} = \sum_i w_i \cos(n\phi_i),$$

$$Q_{n,y} = \sum_i w_i \sin(n\phi_i),$$

where $\phi_i$ is the azimuthal angle of the charged track and $w_i$ is the $p_T$ weight (used only for the event plane determined in the TPC). The $\Psi_n^{\text{obs}}$ is an estimated $n$th-order event plane and $Q_{n,x,y}$ is referred to as the flow vector. Corrections for the detector acceptance were applied following Ref. [41]. The tracks measured in the TPC acceptance were divided into three subevents ($-1 < \eta < -0.4$, $|\eta| < 0.2$, and $0.4 < \eta < 1$). The track selection criteria mentioned above were applied but only tracks with $p_T < 2$ GeV/c were used for the event plane reconstruction.

The Beam-Beam Counters (BBC) [42] and the Endcap-Electromagnetic Calorimeter (EEMC) [43] were also used for the event plane determination in addition to the TPC. The BBCs are located at forward and backward angles ($3.3 < |\eta| < 5$) and consist of scintillator tiles. When using the BBCs for the event plane determination, the azimuthal angle of the center of each tile was used for $\phi_i$ in Eqs. (5) and (6), and the ADC value in that tile was used as the weight, $w_i$. The EEMC covers the pseudorapidity range of 1.086 $< \eta < 2$ and consists of 720 towers ($60 \times 12$ in $\phi - \eta$ plane). When using the EEMC for the event plane determination, the azimuthal angle of each tower center was used as $\phi_i$, and the transverse energy, $E_T$, was used as $w_i$. If $E_T$ exceeded 2 GeV, a constant value of 2 was used as the weight.

For the first-order event plane, the ZDCs with Shower Maximum Detectors (SMD) [21] were used. Each SMD is composed of two planes with scintillator strips aligned with the $x$ or $y$ directions and sandwiched between the ZDC modules. Therefore, the SMD measures the centroid of the hadronic shower caused by the interaction between spectator neutrons and the ZDC. The $x$ and $y$ positions of the shower centroid was calculated for each ZDC-SMD on the event-by-event basis as follows:

$$\langle X \rangle = \frac{\sum_i X_i \cdot w_{X_i}}{\sum_i w_{X_i}},$$

$$\langle Y \rangle = \frac{\sum_i Y_i \cdot w_{Y_i}}{\sum_i w_{Y_i}},$$

where $X_i(Y_i)$ denotes the position of a vertical (horizontal) scintillator strip in the SMD and $w_{X_i}(w_{Y_i})$ denotes the ADC signal measured in each strip. Then the first-order event plane was determined as $\Psi_1 = \tan^{-1}(\langle Y \rangle / \langle X \rangle)$. The angle determined by the target spectators points into the opposite direction ($+\pi$) to that of the projectile spectator plane, then the combined event plane of ZDC-SMD east and west can be obtained by summing Eqs. (7) and (8) from each ZDC-SMDs flipping the sign for one of them.

![Event plane resolution as a function of centrality in Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV](image.png)

The event plane resolution defined as $\text{Res}(\Psi_n) = \langle \cos(\Psi_n - \Psi_n^{\text{obs}}) \rangle$ was estimated by the three-subevent method [44]. Here $\Psi_n^{\text{obs}}$ denotes the azimuthal angle of a measured (“observed”) event plane. For the first-order event plane, either BBC in the west (BBCW) or east (BBCE) sides was used as a third subevent along with the two ZDCs. For higher harmonic event plane, three subevents from TPC were used. In the case of using the EEMC, one of the TPC subevents was replaced with EEMC subevent. In Au+Au collisions, both the
two-subevent and the three-subevent methods were used. The results are reported using the reaction plane resolution from the two-subevent method, with the difference in results between the two methods included in the systematic uncertainty. Figure 3 shows the estimated event plane resolution, $\text{Res}(\Psi_n) = \langle \cos(\Psi_n - \Psi_{\text{obs}}^n) \rangle$ ($2 \leq n \leq 4$), for TPC and EEMC, and $\text{Res}(\Psi_1)$ for ZDC-SMD in Cu+Au collisions. Note that the forward direction or the west side (ZDCW and BBCW) is the going direction. The resolution of $\Psi_1$ with ZDC-SMD in Au+Au collisions can be found in Ref. [45]. Results for wide centrality bins in this study were obtained by taking averages of results measured with 10% step centrality bins.

C. Flow measurements

Azimuthal anisotropy was measured with the event plane method using the following equation:

$$v_n = \frac{\langle \cos[n(\phi - \Psi_{\text{obs}}^n)] \rangle}{\text{Res}(\Psi_n)}, \quad (9)$$

where $\langle \rangle$ means an average over particles in an event, followed by the averaging over all events. We study $v_n$ as a function of $p_T$ for different centralities, as well as the (pseudo)rapidity dependence of $v_1$. For the event plane determined by TPC, the $v_n$ of charged particles were measured using an $\eta$-gap of 0.4 from the subevent used for the event plane determination, i.e. particles of interest were taken from $-1 < \eta < 0$ ($0 < \eta < 1$) when using the event plane determined in the subevent from the forward (backward) rapidity. The results from these two subevents are found to be consistent and the average of the two measurements is used as the final result.

Directed flow can be also measured by the three-point correlator with the use of the second harmonic event plane [27]:

$$v_1\{3\} = \frac{\langle \cos(\phi + \Psi_{\text{obs}}^1 - 2\Psi_{\text{obs}}^2) \rangle}{\text{Res}(\Psi_1) \times \text{Res}(\Psi_2)}, \quad (10)$$

where $\Psi_{\text{obs}}^1$ and $\Psi_{\text{obs}}^2$ were taken from different subevents and $\phi$ is the azimuthal angle of particles of interest in the rapidity region different from those subevents to avoid self-correlation. In our analysis, $\Psi_{\text{obs}}^1$ was taken from the east BBC and $\Psi_{\text{obs}}^2$ from either the TPC or EEMC subevents. The results for $v_1\{3\}$ obtained with TPC subevents from the backward and forward rapidities are statistically consistent in the overlapping region, and were further combined to cover the same $\eta$ range for particles of interest as used in the event plane method. The difference between results obtained from TPC or EEMC subevents was taken into account as a systematic uncertainty. Note that Eq. (10) was calculated without any spectator information, and thus provides information on the directed flow projected onto the second harmonic participant plane.

For the higher harmonic flow measurements, the scalar product method [46–48] was tested for comparison with the event plane method. The scalar product method is equivalent to the two-particle correlation method with corresponding $\eta$ gap between two particles and particle of interest. Three subevents were used to calculate the flow coefficients based on the following equation:

$$v_n = \frac{\langle u \cdot Q^A_n/N^A \rangle}{\sqrt{\langle Q^B_n/N^B \rangle \langle Q^C_n/N^C \rangle \langle Q^A_n/N^A \rangle}} \cdot (11)$$

where $Q_n$ is the flow vector defined in Eqs. (5) and (6) and the superscripts $A$, $B$, and $C$ denote different subevents with a finite rapidity gap from the other subevent. The subevents were taken from TPC and/or EEMC. We denote by $u$ a unit vector in the direction of the particle transverse momentum; $N$ denotes the sum of weights used for reconstructing the flow vectors in each subevent.

The tracking efficiency was accounted for in $p_T$-integrated observables, although the effect of that is much smaller than other systematic uncertainties discussed below.

D. Systematic uncertainties

The systematic uncertainties were estimated by varying the track quality cuts described in III A and by varying collision $z$-vertex cut. The effect of the track quality cuts becomes largest at low $p_T$ in central collisions and was found to be <4% for $v_2$, <6% for $v_3$, and <8% for $v_4$. The effect of the $z$-vertex cut is <1%. For identified particles, the effect of particle identification purity was also considered. The effect for charged pions is <1% in $v_2$ and $v_3$ and <3% in $v_4$. The effects for charged kaons and (anti)protons are <3% in $v_2$, <5% in $v_3$, and <10% in $v_4$. The combined estimated uncertainty was found to be $p_T$-uncorrelated; namely all data points do not move in the same direction over $p_T$, and was assigned as a point-by-point systematic uncertainty.

Along with the TPC event plane, the event plane determined by the EEMC was used for the $v_n$ ($n \geq 2$) measurements and the difference in $v_n$ obtained with the two methods was included in the systematic uncertainty. The latter was found to be $p_T$-correlated: it was <2% (<10%) for $v_2$ and $v_3$ ($v_4$) in central collisions, and increased up to ~5% (16%) for $v_2$ ($v_3$ and $v_4$) in peripheral collisions. For $v_1$, the details of the systematic uncertainty estimation can be found in our previous study [11]. As mentioned before, $v_1\{3\}$ was measured without the spectator information, but one can also use the ZDCs for $\Psi_1$ in Eq. (10) for a cross check. We found that $v_1\{3\}$ measured using the ZDCs was consistent with $v_1\{3\}$ measured using the BBC within the uncertainties.
The notion of “odd” and “even” $v_1$ components can be justified only for symmetric collisions. Therefore, the following definitions are used for Cu+Au collisions:

$$v_1^{\text{conv}} = (v_1 \langle \Psi_{SP}^p \rangle - v_1 \langle \Psi_{SP}^s \rangle)/2$$  \hspace{1cm} (12)

$$v_1^{\text{fluc}} = (v_1 \langle \Psi_{SP}^p \rangle + v_1 \langle \Psi_{SP}^s \rangle)/2$$  \hspace{1cm} (13)

where “projectile” (Cu) spectators go into the forward direction. The term $v_1^{\text{conv}}$ and $v_1^{\text{fluc}}$ denotes “conventional” and “fluctuation” components of directed flow, respectively. Note that the right-hand side of Eq. (12) and Eq. (13) represents the same definitions as Eq. (2) and Eq. (3).

The mean transverse momentum projected onto the spectator plane defined as

$$\langle p_x \rangle = \frac{\langle p_T \cos(\phi - \Psi_{\text{obs}}^T) \rangle}{\text{Res}(\Psi_1)}$$  \hspace{1cm} (14)

is also shown in the bottom panels (b,d) of Fig. 4. There seems to be small difference between results with two spectator planes in Cu+Au but not in Au+Au. The terms “conv (odd)” and “fluc (even)” are also used for $\langle p_x \rangle$ in the following discussion, with analogous definitions to Eqs. (12) and (13).

The top panels of Fig. 5 present the pseudorapidity dependence of $v_1^{\text{odd}(\text{conv})}$ and $v_1^{\text{even}(\text{fluc})}$, defined according to Eqs. (2), (3), (12), and (13). The $\langle p_x \rangle$ normalized by the mean $p_T$ is also shown in the bottom panels. The lines represent linear fits to guide the eye. The conventional component of directed flow, $v_1^{\text{conv}}$, in Cu+Au has a similar slope to $v_1^{\text{odd}}$ in Au+Au, with the intercept shifted to the forward direction. The mean transverse momentum component $\langle p_x^{\text{conv}} \rangle$ in Cu+Au might deviate from linear dependence (observed in Au+Au) with the slope slightly increasing at backward rapidities. This trend in $\langle p_x^{\text{conv}} \rangle$ might reflect the momentum balance between particles produced in the forward and backward hemispheres – in
Cu+Au collisions more charged particles are produced in the Au-going direction, and therefore the particles at forward rapidity need to have a larger $p_x$ on average to compensate the asymmetric multiplicity distribution over $\eta$. Results from Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV measured by the ALICE experiment [24] are also shown in Fig. 5. The slope of $v_1^{\text{odd}}$ in Pb+Pb collisions is about 3 times smaller than that in Au+Au collisions. This trend, i.e. the energy dependence of the $v_1$ slope, is consistent with that observed in the RHIC Beam Energy Scan [50]. Calculations from an event-by-event hydrodynamic model with two different values of $\eta/s$ ($\eta/s = 0.08$ and 0.16) for Cu+Au collisions [49] are also compared to the data. Despite the model’s successful description of elliptic flow and triangular flow (see Section V), it cannot reproduce either the magnitude of the directed flow nor its pseudorapidity dependence.

The even component of directed flow, $v_1^{\text{even}}$, in Au+Au does (Fig. 5(c)) not depend on pseudorapidity (within error bars) and is very similar in magnitude to $v_1^{\text{even}}$ in Pb+Pb collision at LHC energies. The $\langle p_x^{\text{even}} \rangle$ in both Au+Au and Pb+Pb collisions is consistent with zero, which indicates zero net transverse momentum in the systems. This agrees with the expectation that the even component of $v_1$ originates from event-by-event fluctuations of the initial density. The magnitude of $v_1^{\text{fluc}}$ in Cu+Au is larger than that of $v_1^{\text{even}}$ in Au+Au. This would be due either to larger initial density fluctuations in Cu+Au collisions or to stronger correlations between the spectator and dipole fluctuation planes.

The results presented in Figs. 4 and 5, and in particular a positive intercept of $v_1(\eta)$ and negative intercept of $\langle p_x \rangle$, are consistent with a picture of directed flow in Cu+Au collisions as a superposition of that from a “tilted source” (shifted in rapidity to the system center-of-mass rapidity) and dipole flow due to non-zero average density gradients. Compared to the $v_1(\eta)$ dependence in symmetric collisions, the first mechanism shifts the function toward negative rapidities, and the second moves the entire function up (note that the Cu nucleus is defined as the projectile) as shown in Fig. 1(a,b). This picture receives further support from the study of the centrality dependence of the corresponding slopes and intercepts presented in Fig. 6. Very similar slopes of $v_1$ and $\langle p_x \rangle/\langle p_T \rangle$ would be a natural consequence of a “tilted source”. The intercepts of $\langle p_x \rangle$ follow very closely the shift in rapidity center-of-mass of the system shown with the solid line in Fig. 6(b), which was calculated by a Monte-Carlo Glauber model based on the ratio of Au and Cu participant nucleons:

$$y_{CM} \approx \frac{1}{2} \ln \left( \frac{N_{Au}^\text{Au}}{N_{Cu}^\text{Cu}} \right),$$

where $N_{part}^{Au(Cu)}$ is the number of participants from Au or Cu nuclei. The centrality dependence of $v_1$ intercept (more exactly, in this picture the difference in $v_1$ and
The slopes of $v_1^{\text{odd(conv)}}$ and $<p_{T}^{\text{conv}}>/ <p_T>$ in Fig. 5 agree within 10% both in Au+Au and Cu+Au collisions. In Pb+Pb collisions at the LHC energy the $v_1$ slope is almost a factor of two larger in magnitude than that of $<p_{T}^{\text{conv}}>/ <p_T>$. This clearly indicates that both mechanisms, “tilted source” (for which one would expect the slope of $<p_{x}^{\text{conv}}>/ <p_T>$ to be about 50% larger than that of $v_1^{\text{odd(conv)}}$, see Appendix), and initial density asymmetries (for which $<p_{x}^{\text{conv}}> = 0$), play a significant role in the formation of the directed flow even in symmetric collisions. The relative contribution of the “tilted source” mechanism to the $v_1$ slope, $r$, can be expressed as (see Appendix):

$$r = \frac{\langle dv_1 \rangle^{\text{tilt}}}{\langle dv_1 \rangle} \approx \frac{2}{3} \frac{<p_{T}>}{<p_{x}>},$$

where $\langle \cdot \rangle^{\text{tilt}}$ denotes a contribution from the “tilted source”. The relative contribution $r$ is about 2/3 at the top RHIC collision energies decreasing to about 1/3 at LHC energies. From the centrality dependence of slopes shown in Fig. 6 one can conclude that the relative contribution of the “tilted source” mechanism is largest in peripheral collisions where the $<p_{T}^{\text{conv}}>/ <p_T>$ slope is approximately 1.5 times larger than that of $v_1^{\text{odd(conv)}}$. This dependence might be due to the stronger decorrelation between spectator and dipole flow planes in peripheral collisions. Figure 7 shows the even (fluctuation) components of $v_1$ and $<p_x>$ as a function of centrality. The $v_1^{\text{even}}$ for Au+Au has a weak centrality dependence and is consistent with $v_1^{\text{even}}$ for Pb+Pb except in most peripheral collisions. Furthermore, $p_{x}^{\text{even}}$ in both Au+Au and Pb+Pb are consistent with zero. This may indicate that the dipole-like fluctuation in the initial state has little dependence on the system size and collision energy. $v_1^{\text{fluc}}$ and $<p_x>^{\text{fluc}}$ for Cu+Au has a larger magnitude than in symmetric collisions over the entire centrality range; it is smallest in the 30%-40% centrality bin.

The reference angle of dipole flow can be represented by $\Psi_{1,3}$, but $v_1^{\text{even}}$ or $<p_x>^{\text{fluc}}$ are the projections of dipole flow onto the spectator planes. Therefore, the measured even (or fluctuation) components of $v_1$ should be decreased by a factor $\langle \cos(\Psi_{1,3} - \Psi_{\text{SP}}) \rangle$. Such a “resolution” effect may
The signals of both particles with the negative rapidity dependence in sign between viscous hydrodynamic model calculation is also compared in its magnitude for the entire region.

Anisotropic flow of charged pions, kaons, and (anti)protons was measured based on the particle identification with the TPC and TOF, as explained in Sec. III A. Figure 10 presents directed flow of $\pi^+ + \pi^-$, $K^+ + K^-$, and $p + \bar{p}$ measured with respect to the target (Au) spectator plane ($v_1 = -v_1\{\Psi_{SP}\}$ in the 10%-40% centrality bin. For $p_T < 2$ GeV/c, there is a clear particle type dependence, likely reflecting the effect of particle mass in interplay of the radial and directed flow [51, 52]. In $p_T > 2$ GeV/c region, there is no clear particle type dependence due to the large uncertainties. Measurement of identified particle $v_1$ with the projectile (Cu) spectator plane is difficult due to small statistics of identified particles and poor event plane resolution; therefore we do not decompose the $v_1$ into the conventional and fluctuation components. The presented $v_1$ of $\pi^+ + \pi^-$, $K^+ + K^-$, and $p + \bar{p}$ includes both components. The observed mass dependence in the $v_1$ of identified particles is consistent with results from the PHENIX Collaboration [53].

B. Directed flow of identified hadrons

Anisotropic flow of charged pions, kaons, and (anti)protons was measured based on the particle identification with the TPC and TOF, as explained in Sec. III A. Figure 10 presents directed flow of $\pi^+ + \pi^-$, $K^+ + K^-$, and $p + \bar{p}$ measured with respect to the target (Au) spectator plane ($v_1 = -v_1\{\Psi_{SP}\}$ in the 10%-40% centrality bin. For $p_T < 2$ GeV/c, there is a clear particle type dependence, likely reflecting the effect of particle mass in interplay of the radial and directed flow [51, 52]. In $p_T > 2$ GeV/c region, there is no clear particle type dependence due to the large uncertainties. Measurement of identified particle $v_1$ with the projectile (Cu) spectator plane is difficult due to small statistics of identified particles and poor event plane resolution; therefore we do not decompose the $v_1$ into the conventional and fluctuation components. The presented $v_1$ of $\pi^+ + \pi^-$, $K^+ + K^-$, and $p + \bar{p}$ includes both components. The observed mass dependence in the $v_1$ of identified particles is consistent with results from the PHENIX Collaboration [53].

C. Charge dependence of directed flow

In our previous study [11], a finite difference in directed flow between positively and negatively charged particles was observed in asymmetric Cu+Au collisions. These results can be understood as an effect of the electric field due to the asymmetry in the electric charge of the Au and Cu nuclei. Similarly, one would expect a difference in $\langle p_x \rangle$ between positive and negative particles. Figure 11 shows the centrality dependence of charge-dependent $\langle p_x \rangle$ and the difference $\Delta\langle p_x \rangle$ between positive and negative particles in Au+Au and Cu+Au collisions. The difference is consistent with zero for Au+Au collisions, but a finite difference is observed in Cu+Au collisions ($\Delta\langle p_x \rangle \sim 0.3$ MeV/c). The direction of the electric field is expected to be strongly correlated to the direction of the Cu (projectile) spectator deflection, which should
lead to a positive \( \langle p_x \rangle \) by the convention used in this analysis. The results are consistent with these expectations.

The magnitude of the momentum shift can be roughly estimated based on the equation of motion, i.e. \( \Delta p_x = e|\vec{E}|/m_{\pi}^2 \times m_{\pi}^2 \times \Delta t \) where \( \vec{E} \) denotes the electric field, \( m_{\pi} \) is a pion mass, and \( \Delta t \) is the lifetime of the electric field. If one takes \( e|\vec{E}|/m_{\pi}^2 \sim 0.9 \) and \( \Delta t \sim 0.1 \text{ fm}/c \) [9], assuming that the time dependence of the electric field approximates a step function, the resulting \( \Delta p_x \) is \( \sim 9 \text{ MeV}/c \) which is \( \sim 30 \) times larger than the observed \( \Delta p_x \). The charge-dependence of \( \Delta (p_x) \) is determined by the number of charges, i.e. the number of quarks and antiquarks, at the time when the initial electric field is strong after the collisions. Therefore a difference in \( \Delta (p_x) \) between the data and our estimate might indicate a smaller number of quarks and antiquarks at early times \( (t < 0.1 \text{ fm}/c) \) compared to the number of quarks in the final state, as discussed in Ref. [11]. The lifetime of the electric field depends on the model and could be longer if the medium has a larger conductivity. Also note that the observed \( \Delta (p_x) \) might be smeared by the fluctuations between the direction of the electric field and the spectator plane, and by hydrodynamic evolution and hadron rescattering at later stages of the collisions.

For a more detailed view of the quark-antiquark production dynamics, as well as to understand the role of baryon stopping in the development of directed flow at midrapidity we also extended our measurements to identified particles. In the so-called “two-wave” scenario of quark production [54], the number of \( s \) quarks approximately remains the same during the system evolution while the number of \( u \) and \( d \) quarks sharply increases at the hadronization time. In this case, one might expect a relatively larger effect of the initial electric field for \( s \) quarks than that for \( u \) and \( d \) quarks. Therefore the measurement of charge-dependent \( v_1 \) for pions and kaons might serve as a test of such a quark production scenario. The difference in number of protons and neutrons in the colliding nuclei in combination with the baryon stopping might also contribute to the charge dependence of directed flow. In this case one can expect a significantly larger effect measuring the flow of baryons itself. For that we measure the charge dependence of directed flow of protons and antiprotons.

Top panels in Fig. 12 show \( p_T \) dependence of \( v_1 \) separately for \( \pi^+ \) and \( \pi^- \), and \( K^+ \) and \( K^- \), and \( p \) and \( \bar{p} \) for 10%-40% centrality in Cu+Au collisions. Bottom panels show difference in \( v_1 \), \( \Delta v_1 \), between positively and negatively charged particles for each species. Similarly as observed for charged hadrons [11] and in agreement with results presented in Fig. 11, \( v_1 \) of \( \pi^+ \) is larger than that of \( \pi^- \) in the \( p_T < 2 \text{ GeV}/c \) region, which is consistent with the expectation from the initial electric field effect. For charged kaons and (anti)protons, no significant difference are observed within the current experimental precision.
FIG. 9. (Color online) Directed flow of charged particles as a function of $p_T$ (a) and $\eta$ (b) in the 10%-40% centrality bin measured with the ZDC-SMD event planes and three-point correlator in Cu+Au collisions. The $p_T$ dependence was measured in $|\eta|<1$ and the $\eta$ dependence was integrated over $0.15<p_T<5$ GeV/c. Open boxes represent systematic uncertainties.

V. ELLIPTIC AND HIGHER HARMONIC FLOW

A. Unidentified charged particles

Higher harmonic anisotropic flow coefficients, $v_n$, of charged particles were measured with TPC $\eta$ subevents as a function of $p_T$ up to $n=4$. Results for six centrality bins (0%-5%, 10%-20%, 20%-30%, 30%-40%, 40%-50%, and 50%-60%) are shown in Fig. 13. Results for $v_2$ and $v_3$ from the PHENIX experiment [53], shown for comparison, agree well with our results within uncertainties. The small difference in $v_2$ for $p_T > 2$ GeV/c can be explained by a different contribution from non-flow correlations – PHENIX measured $v_2$ with a larger $\eta$ gap ($\Delta\eta > 2.65$) between the particles of interest and those used for the event plane determination, while our TPC $\eta$ subevents have $\Delta\eta > 0.4$. To confirm that explanation, we also calculated $v_2$ with respect to the BBC event plane, which ensures $\Delta\eta > 2.3$. Those results, while having larger statistical uncertainties, are consistent with the PHENIX

FIG. 10. (Color online) Directed flow of $\pi^+ \pi^-$, $K^+ K^-$, and $p + \bar{p}$ as a function of $p_T$ for $|\eta| < 1$ in the 10%-40% centrality bin. The $p_T$-uncorrelated systematic uncertainties are shown with lines around $v_1 = 0$ for each particle species. $p_T$-correlated systematic uncertainty is shown only for pions with a shaded band.

FIG. 11. (Color online) Positively and negatively charged particles $\langle p_x \rangle$ and the difference $\Delta \langle p_x \rangle$ as a function of centrality in Au+Au and Cu+Au collisions. Open and shaded boxes show systematic uncertainties.
The comparison of the data to model calculations provided valuable constraints on the shear viscosity over entropy density $\eta/s$ [4, 5]. Further constraints can be obtained from a similar comparison for asymmetric collisions. Figure 15 compares $v_2$ and $v_3$ in Cu+Au collisions to the viscous hydrodynamic calculations [49]. The model employs the Glauber (participant nucleons) initial density distribution and applies the event-by-event viscous hydrodynamic model with $\eta/s = 0.08$ or 0.16. Both $v_2$ and $v_3$ are reasonably well described by the model at $p_T < 2$ GeV/c. The calculation with $\eta/s = 0.08$ seems to work better in the 0%-5% centrality bin, while the 20%-30% centrality results might need a larger $\eta/s$. In the same figure we also compare $v_2$ and $v_3$ measured with the scalar product method to the corresponding measurements obtained with the event plane method. Both methods use TPC $\eta$ subevents. The results are in a very good agreement with each other.

Figure 16 compares our results to a multi-phase transport (AMPT) model [59] ($v1.26t5$ for the default version and $v2.26t5$ for the string melting version). The initial conditions in this model are determined by the Heavy Ion Jet Interaction Generator (HIJING) [60] which is based on the Glauber model and creates minijet partons and excited strings. In the AMPT default version, the strings are converted into hadrons via string fragmentation, while in the string melting version the strings are first converted to partons (constituent quarks) and the created partons are converted to hadrons via a coalescence process after the subsequent parton scatterings.

The event plane and centrality in the model calculations were determined in the same way as in the real data analysis. Flow measurements were also performed in the same way. Figure 16 shows $v_n$ for the 0%-5%, 10%-20%, and 30%-40% centrality bins compared to the AMPT model in the default and string melting versions. The parton cross section in the string melting version was set to $\sigma_{\text{parton}} = 1.5 \text{ mb}$ [61, 62]. The AMPT calculations with the default version and the string melting version with $\sigma_{\text{parton}} = 1.5 \text{ mb}$ qualitatively describe the data of $v_2$, $v_3$, and $v_4$ for $p_T < 3$ GeV/c. The data is between the default and string melting with $\sigma_{\text{parton}} = 1.5 \text{ mb}$ results, similar to the observation in Ref. [37, 62].

### B. Flow of identified hadrons and NCQ scaling

Anisotropic flow of charged pions, kaons, and (anti)protons was also measured for higher harmonics ($n = 2-4$). Figure 17 presents $v_2$ and $v_3$ of $\pi^+ + \pi^-$, $K^+ + K^-$, and $p + \bar{p}$ for different centralities. A particle mass dependence is clearly seen at low transverse momenta ($p_T < 1.6$ GeV/c) similar to that seen in $v_1$ in Fig. 10. In the $p_T$ range $1.6 < p_T < 3.2$ GeV/c, the splitting between baryons and mesons is observed in $v_2$ and $v_3$. Results for a wide centrality bin (0%-40%) are shown in Fig. 18, along with results for $v_4$ that show similar trends to $v_2$ and $v_3$.

The baryon-meson splitting in the flow coefficients was already observed in symmetric collisions and indicates the collective flow at a partonic level, which can be tested...
by the number of constituent quark (NCQ) scaling. The idea of the NCQ scaling is based on the quark coalescence picture of hadron production in intermediate \( p_T \) [63, 64]. In this process, hadrons at a given \( p_T \) are formed by \( n_q \) quarks with transverse momentum \( p_T/n_q \), where \( n_q = 2 \) (3) for mesons (baryons). Figure 19(a-c) shows \( v_n/n_q \) for \( \pi^+ + \pi^- \), \( K^+ + K^- \), and \( p + \bar{p} \) as a function of \( p_T/n_q \). The scaled \( v_2 \), \( v_3 \), and \( v_4 \) as a function of \( p_T/n_q \) seem to follow a global trend for all particles species, although there are slight differences for each \( v_n \). For example, the pion \( v_2 \) seems to deviate slightly from the other particles at low \( p_T \) region. This difference might be due to the effect of resonance decays or related to the nature of pions as Goldstone bosons [65, 66]. Unlike the \( v_2 \), kaons seem to deviate from the other particles in \( v_3 \) and \( v_4 \).

An empirical NCQ scaling with the transverse kinetic energy, defined as \( m_{T} - m_{0} \), is known to work well for \( v_2 \) [29, 30]. \( m_{T} \) is defined as \( m_{T} = \sqrt{p_{T}^{2} + m_{0}^{2}} \) and \( m_{0} \) denotes the particle mass. The idea of the NCQ scaling with the transverse kinetic energy comes from an attempt to account for the mass dependence of \( p_{T} \) shift during the system radial expansion. Figure 20(a-c) shows the NCQ scaling with the transverse kinetic energy for \( v_2 \) in 0%-40% centrality bin. The scaling works well for \( v_2 \) as reported in past studies for symmetric collisions [27, 67], but it does not work for higher harmonics. A modified NCQ scaling for higher harmonics, \( v_n/n_q/2 \), was proposed in Ref. [68]. It works better for \( v_3 \) and \( v_4 \), as seen in Fig. 20(d,e), as it did in Au+Au collisions [31]. Hadronic rescattering might be responsible for the modified scaling, but the underlying physics is still under discussion [69, 70].

VI. SUMMARY

We have presented results of azimuthal anisotropic flow measurements, from the first- up to the fourth-order harmonics, for unidentified and identified charged particles in Cu+Au collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). as well as the directed flow of charged particles in Au+Au collisions.
The similar slopes of $v_1$ in Cu+Au and Au+Au collisions, comparing to results in Au+Au from the PHENIX experiment [5]. Open and shaded bands represent systematic uncertainties.

For inclusive charged particles, the directed flow, $v_1$, was measured as functions of $\eta$ and $p_T$ over a wide centrality range. The slope of the conventional $v_1(\eta)$ in Cu+Au is found to be similar to that in Au+Au, but is shifted toward the forward rapidity (the Cu-going direction), while the $\langle p_T \rangle$ in Cu+Au has a slightly steeper slope and is shifted towards backward rapidity (the Au-going direction). The similar slopes of $v_1$ likely indicate a similar initial tilt of the created medium. Such a tilt seems to depend weakly on the system size but does depend on the collision energy. The slight difference in slope of $\langle p_T \rangle$ could be explained by the momentum balance of particles between the forward and backward rapidities and the asymmetry in multiplicity distribution over $\eta$ in Cu+Au collisions. The shift of the intercept in $\langle p_T \rangle$ is close to the expectation based on the shift in the center-of-mass rapidity estimated by the number of participants in Au and Cu nuclei in a Monte-Carlo Glauber model (Eq. (15)). Comparing slopes of $v_1(\eta)$ with those of $\langle p_T \rangle$, we conclude that in mid-central collisions the relative contribution to conventional directed flow from the initial tilt is about 2/3 with the rest coming from rapidity dependence of the initial density asymmetry. The fluctuation component of $v_1$ in Au+Au agrees with that in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and shows a weak centrality dependence. This indicates that the initial dipole-like fluctuations do not depend on the system size, the system shape (overlap region of the nuclei), or the collisions energy.

The mean transverse momentum projected onto the spectator plane, $\langle p_T \rangle$, shows charge dependence in Cu+Au collisions but not in Au+Au collisions, similarly as observed in charge-dependent directed flow reported in our previous publication [11]. The observed difference can be explained by the initial electric field due to the charge difference in Cu and Au spectator protons. The charge-dependent $v_1(p_T)$ was also measured for pions, kaons, and (anti)protons. The pion results are very similar to our previous results of inclusive charged particles. The charge difference of $v_1$ for kaons and protons is no larger than that of pions and consistent with zero within larger experimental uncertainties. These results may indicate that the number of charges, i.e., quarks and antiquarks, at the early time when the electric field
FIG. 16. (Color online) Higher harmonic flow coefficients $v_n$ of charged particles as a function of $p_T$ comparing to the AMPT model [59], where solid lines are for default AMPT setup and dashed lines are for the string melting version with $\sigma_{\text{parton}} = 1.5$ mb.

FIG. 17. (Color online) The second and third harmonic flow coefficients of $\pi^+ + \pi^-$, $K^+ + K^-$, and $p + \bar{p}$ as a function of $p_T$ for four centrality bins. Solid lines represent $p_T$-uncorrelated systematic uncertainties for each species. Shaded bands represent $p_T$-correlated systematic uncertainties for pions.

is strong ($t < 0.1$ fm/$c$) is smaller than the number of charges in the final state.

Higher harmonic flow coefficients, $v_2$, $v_3$, and $v_4$, were also presented as functions of $p_T$ in various centrality bins, showing a similar centrality dependence to those in Au+Au collisions. The $v_2$ in Cu+Au is smaller than that in Au+Au for the same number of participants because of different initial eccentricities. Meanwhile, $v_3$ scales with the number of participants between both systems, supporting the idea that $v_3$ originates from density fluctuations in the initial state. For $p_T < 2$ GeV/$c$, $v_2$ and $v_3$ were found to be reasonably well reproduced by
the event-by-event viscous hydrodynamic model with the shear viscosity to entropy density $\eta/s = 0.08 - 0.16$ with the Glauber initial condition. The AMPT model calculations also qualitatively reproduced the data of $v_2$, $v_3$, and $v_4$.

For identified particles, a particle mass dependence was observed at low $p_T$ for all flow coefficients ($v_1$-$v_4$), and a baryon-meson splitting was observed at intermediate $p_T$ for $v_2$, $v_3$, and $v_4$, as expected from the collective behavior at the partonic level. The number of constituent quark scaling with $p_T$, originating in a naive quark coalescence model, works within $\sim 10\%$ for all $v_n$. The empirical number of constituent quark scaling with the kinetic energy works well for elliptic flow but not for higher harmonics, where the modified scaling works better. This is similar to what has been observed in Au+Au collisions. The exact reason for that is still unknown; our new data should help in future theoretical efforts in answering this question.
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Appendix: Directed flow from a “tilted source”

In this Appendix we derive the relation between the rapidity slopes of $v_1$ and $\langle p_x \rangle$ in the “tilted source” scenario. The approach used here is very similar to the one developed in [52]. Let us denote the invariant particle distribution as:

$$d^3n = J_0(p_T, y). \quad (A.1)$$

A small “tilt” in $xz$ plane by an angle $\gamma$ leads to a change in the $x$ component of the momentum $\Delta p_x = \gamma p_x = \gamma p_T \tan(\theta) = \gamma p_T \sinh \eta$, where $\eta$ is the pseudorapidity. Then the particle distribution in a tilted coordinate system would read

$$J \approx J_0 + \frac{\partial J_0}{\partial p_T} \frac{\partial p_T}{\partial \eta} \Delta p_x
= J_0 \left( 1 + \frac{\partial \ln J_0}{\partial p_T} \cos \phi p_T \gamma \sinh \eta \right). \quad (A.2)$$

**FIG. 20.** (Color online) NCQ scalings of $v_2$, $v_3$, and $v_4$ of $\pi^+ + \pi^-$, $K^+ + K^-$, and $p + \bar{p}$ as a function of $(m_T - m_0)/n_q$ in the 0%-40% centrality bin. Solid lines represent $p_T$-uncorrelated systematic uncertainties for each species. Shaded bands represent $p_T$-correlated systematic uncertainties for pions.
From here one gets

\[ v_1(p_T) = \frac{1}{2} \gamma p_T \sinh \eta \frac{\partial \ln J_0}{\partial p_T}. \]  

(A.3)

Heavier particle spectra usually have less steep dependence on \( p_T \), which would lead to the mass dependence of \( v_1(p_T) \) – particles with large mass would have smaller \( v_1 \) at a given \( p_T \). Integrating over \( p_T \), and using \( p_T \) weight for \( \langle p_z \rangle \) calculation leads to the following ratio of slopes:

\[ \frac{1}{p_T} \frac{d \langle p_z \rangle}{d \eta} = \frac{1}{p_T} \left( \frac{\partial \ln J_0}{\partial p_T} \right). \]  

(A.4)

For both the exponential form of \( J_0(p_T) \) (approximately describing the spectra of light particles) and the Gaussian form (better suited for description of protons), this ratio equals 1.5.

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