Superconducting and chiral-glass to insulator transition in phase-glass models in a magnetic field

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Abstract. We investigate the quantum phase transitions of a phase-glass model in a magnetic field with frustration parameter $f = 1/2$, describing the effects of random negative Josephson-junction couplings in two-dimensional superconductors. The critical behavior is obtained by a scaling analysis of path-integral Monte Carlo simulations at zero temperature, including corrections to finite-size scaling. A single superconducting and chiral-glass to insulator transition occurs above a disorder threshold with divergent nonlinear magnetic susceptibility, unaffected by the additional magnetic-field frustration. The relevance of this transition for nanohole superconducting thin films doped with magnetic impurities is discussed.

1. Introduction
Negative Josephson junctions distributed randomly lead to striking effects in the transport and magnetic properties of superconducting systems. In high-$T_c$ superconductors, these effects have been studied by XY spin-glass or chiral-glass models, where they represent the $\pi$ junctions arising from grain misorientations [1, 2]. A quantum version of these models, the phase-glass model, which includes the charging energy of local superconducting regions, has been employed [3, 4] to explain a Bose metal phase separating the superconductor and insulator phases in thin films, appearing experimentally as an anomalous metallic phase [5, 6]. Although recent numerical simulations [7, 8] of this model in two dimensions found no evidence of such metallic phase, the model finds applications for nanohole superconducting thin films doped with magnetic impurities in an applied magnetic field [9].

Here, we investigate in further details the quantum phase transitions of the phase-glass model in a magnetic field with frustration parameter $f = 1/2$. From a scaling analysis of the Monte Carlo (MC) data including finite-size scaling corrections, we find a single superconducting and chiral-glass to insulator transition above a disorder threshold, unaffected by the additional magnetic-field frustration. It is also argued that the nonlinear magnetic susceptibility diverges at transition for decreasing temperatures. Recent measurements on nanohole superconducting thin films are interpreted in terms of this chiral-glass regime.

2. Phase-glass model and MC simulation
We consider a two-dimensional phase-glass model given by the Hamiltonian [3, 7]

$$\mathcal{H} = \frac{E_C}{2} \sum_i n_i^2 - \sum_{<ij>} E_{ij} \cos(\theta_i - \theta_j - A_{ij}),$$

(1)
where $\theta_j$ is the phase operator at site $j$ of a square lattice with lattice spacing $a$ and $n_j = -i \partial / \partial \theta_j$ is the corresponding canonically conjugate operator. $E_C = 4e^2/C$ is charging energy, where $e$ is the electron charge and $C$ is an effective capacitance to the ground. $E_{ij}$ is the Josephson-junction coupling of nearest-neighbor grains, with an asymmetric probability distribution $P(E_{ij}) = x \delta(E_{ij} + E_J) + (1 - x) \delta(E_{ij} - E_J)$, where $x$ is the fraction of negative junctions, a measure of the degree of disorder. The gauge variables $A_{ij}$ are constrained by $\sum_{ij} A_{ij} = 2\pi f$, where the summation is a direct sum around a lattice plaquette. The frustration parameter $f = Ba^2/\Phi_o$ corresponds to the average number of flux quantum, $\Phi_o = hc/2e$, per plaquette, due to the external magnetic field $B$. The presence of negative junctions can introduce quenched vortices [1] even when $f = 0$, due to frustration effects. This can be described by a chiral order parameter $[2], \kappa_p = \frac{1}{\kappa_0} \sum_{<ij>} E_{ij} \sin(\theta_i - \theta_j - A_{ij})$, where $p$ labels a plaquette and $\kappa_0$ is a normalization factor.

The MC simulations were implemented in the equivalent (2+1) dimensional classical model [10, 7], with reduced Hamiltonian

$$H = -\frac{1}{g} \sum_{\tau, i} \cos(\theta_{\tau,i} - \theta_{\tau+1,i}) + \sum_{<ij>, \tau} \epsilon_{ij} \cos(\theta_{\tau,i} - \theta_{\tau,j} - A_{ij}),$$

where $g = (E_C/E_J)^{1/2}, \epsilon_{ij} = E_{ij}/E_J$ and $\tau$ labels the sites in the time direction. MC simulations employed the parallel tempering method with periodic boundary conditions [7] in systems of linear size $L$.

### 3. Results and discussion

For increasing disorder, a transition from a superconducting phase to a chiral-glass phase occurs at a critical disorder value $x_C$. This threshold value can be obtained from the behavior of the domain-wall energy [7] $E_W$ in the classical model of Eq. (2). Figure 1a shows the disorder averaged $[E_W]_d$ for $f = 1/2$. At small disorder, it increases with $L$, corresponding to long-range phase coherence while at large disorder it decreases with $L$ in the disordered glass phase. From the change of behavior, we estimate a disorder threshold $x_C \sim 0.9$.

We now consider the quantum phase transitions in the chiral-glass regime ($x > x_C$) at $f = 1/2$, tuned by the coupling parameter $g$. The critical properties can be obtained from the scaling behavior of the size-dependent correlation length $\xi$ in the imaginary time direction, defined as

$$\xi(L, g) = \frac{1}{2 \sin(k_0/2)} |S(0)/S(k_0) - 1|^{1/2}.$$

$S(k)$ and $k_0$ are the Fourier transform of the correlation function and the smallest nonzero wave vector, respectively [11]. The phase correlation length $\xi_G$ is obtained from the correlation function of the overlap order parameter [12, 7] $q_{\tau,j} = \exp(i(\theta_{\tau,j}^1 - \theta_{\tau,j}^2))$, where 1 and 2 are two different replicas with the same coupling parameters. Likewise, the chiral correlation length $\xi_C$ is obtained as $q_{\tau,p}^c = \kappa_{\tau,p}^1 e^{2\pi i p}$. For a continuous phase transition, $\xi(L, g)$ is expected to satisfy the scaling form $\xi/L = A(L^{1/\nu} \delta g)$, where $\delta g = g - g_c$ and $\nu$ is the critical exponent. The scaled correlation length $\xi(L, g)/L$ for different $L$, plotted as a function of $g$, should then cross at the critical coupling $g_c$.

Figures 1b and 1c show the correlation lengths $\xi_G$ and $\xi_C$ at $x = 0.2$, with the dynamic exponent $z = 1.2$, the same value used [7, 8] when $f = 0$. For large system sizes, the curves for $\xi_G/L$ and $\xi_C/L$ as a function of $g$ cross at a common point $g_c$. However, there are significant corrections to finite-size scaling since the curves for smaller system sizes cross at different points. From data for large system sizes ($L = 16$ to 30), estimates of $g_c = 1.50(2)$ and $\nu = 1.4(2)$ for phase-coherence and $g_c = 1.42(2)$ and $\nu = 1.1(2)$ for chiral-glass transitions have already been obtained [8] in absence of scaling corrections. There is a small difference in the estimates of $g_c$.
Figure 1. (a) Domain-wall energy $[E_W]_d$ at $f = 1/2$ for increasing disorder $x$ and system sizes $L$. (b) Scaled phase correlation length $\xi_{G,\tau}/L$ near the chiral-glass to insulator transition at $x = 0.2$. (c) Same as (b) but for the chiral correlation length $\xi_{cG,\tau}/L$.

Figure 2. Finite-size behavior of critical couplings $\tilde{g}_c(L)$ and exponent $\nu$. Dotted lines are fittings according to Eq. (4).

for the two transitions. As the chiral variables are obtained from phase correlations of nearest-neighbors sites, chiral-glass disorder leads to phase incoherence. It is then expected that the critical couplings for chiral-glass order and phase coherence should be equal or the chiral-glass transition occurs at a larger coupling [13]. This discrepancy of the estimates of $g_c$ is probably the effect of corrections to scaling and the two transitions occur as a single transition.

To check this possible scenario, we performed an approximate extrapolation to the infinite system of finite-size estimates of the critical coupling $\tilde{g}_c(L)$ including the smaller system sizes of Figs 1b and 1c. $\tilde{g}_c(L)$ was obtained from the crossing points of $\xi(L, g)/L$ for successive system sizes, averaged over six independent calculations, and required to follow the finite-size scaling [11]

$$g_c(L) = g_c + a/L^{\omega+1/\nu},$$

where $a$ is constant and $\omega$ is a correction-to-scaling exponent. As shown in Fig. 3a, an extrapolation is possible for both phase and chiral variables with a common critical coupling $g_c = 1.46$ and critical exponents $\omega + 1/\nu = 1.07$. The corresponding estimates of the critical exponent $\nu$ are shown in Fig 3b. Although the estimates of $\nu$ fluctuate with $L$, there is no particular trend and for the largest systems sizes they are consistent with previous estimates [8, 7] for $f = 1/2$ and $f = 0$, considering the errorbars, indicating that the superconducting chiral-glass to insulator transition is unaffected by the additional magnetic-field frustration. Qualitatively, this effect results from the disorder of the quenched vortices, which dominate in the chiral-glass regime for $x > x_C$. On the other hand, for $x < x_C$, magnetic-field frustration reduces the critical coupling of the superconductor-insulator transition. In particular, at $x = 0$, it changes its universality class [14, 15].

The superconducting chiral-glass transition should also affect the magnetic properties, like the linear and nonlinear magnetic susceptibility $[2] \chi$ and $\chi_{nl} = \delta \chi / \delta H^2$. Since the chirality $\kappa$ is an Ising-like variable, the singular contributions to $\chi$ and $\chi_{nl}$ due to the random circulating currents in the phase-glass model, should have the same scaling behavior of the quantum Ising-spin glass. Using the results for the quantum Ising-spin glass; [12], we then expect that $\chi \sim 1/L^\beta z^{\nu - 1}$ and $\chi_{nl} \sim 1/L^{(2\beta - 2\nu)/\nu - 3}$, at the transition. From the estimates of $\beta$, $\nu$ and $z$ for the phase-
glass model [7, 8], and the scaling of $L\tau \propto 1/T$ with the temperature $T$, we find that these quantities have a power-law dependence with the temperature at the transition as $\chi \sim T^{-0.19}$ and $\chi_{nl} \sim T^{-2.28}$. Thus, as for the thermal transition in absence of quantum fluctuations [2], the zero-temperature chiral-glass transition shows a divergence of the nonlinear magnetic susceptibility with decreasing temperatures.

The phase-glass model can explain some magnetic-field effects in superconducting thin films perforated with nanoholes, containing magnetic impurities [9]. Since magnetic impurities can introduce negative junctions ($\pi$ junctions) [16], the transition to the insulating phase can be described by the phase-glass model. The activation energy for the resistance observed experimentally in the insulating phase corresponds to the energy gap $E_G$, which vanishes near the transition as $E_{G,f} \propto (g - g_{c,f})^{2\nu}$. For small disorder, the magnetic-field frustration reduces significantly the critical coupling for the superconductor-insulator transition, $g_{c,1/2} << g_{c,0}$, with the corresponding reverse behavior for the energy gap, $E_{G,1/2} >> E_{G,0}$. However, above the chiral-glass threshold $x_{C}$, a single superconducting and chiral-glass to insulator transition occurs, unaffected by the additional magnetic-field frustration, leading to $g_{c,1/2} \sim g_{c,0}, E_{G,1/2} \sim E_{G,0}$, with essentially the same critical behavior. The absence of magnetic-field frustration effects at large doping in the experimental system is therefore a signature of such chiral-glass regime [8].

4. Conclusions
We have studied the quantum phase transitions of a phase-glass model with frustration parameter $f = 1/2$. From a finite-size scaling analysis of MC data at zero temperature including corrections to scaling, we find evidence of a single superconducting and chiral-glass to insulator transition above a disorder threshold. Therefore, an intervening Bose metal phase between the superconductor and insulator phases is absent, in sharp contrast with predictions from mean-field theory. This single transition also affects the magnetic properties, leading to a nonlinear magnetic susceptibility diverging with decreasing temperatures, which could be tested experimentally. The absence of magnetic-field frustration effects at large magnetic-impurity doping in superconducting nanohole thin films [9] is consistent with such chiral-glass regime.

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