An alternative approach to the $\sigma$-meson-exchange in nucleon-nucleon interaction

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Through a quantitative comparative study of the properties of deuteron and nucleon-nucleon interaction with chiral quark model and quark delocalization color screening model. We show that the $\sigma$-meson exchange used in the chiral quark model can be replaced by quark delocalization and color screening mechanism.

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I. INTRODUCTION

There has been a long history of studying nucleon-nucleon ($NN$) interaction since the Yukawa theory. In the effective meson-nucleon coupling model, after including pseudo-scalar-, vector- and scalar-meson, especially the $\sigma$-meson, the $NN$ interaction has been described very well\cite{1}. In the traditional picture of $NN$ interaction, the short-range repulsion is provided by vector-meson ($\rho, \omega$) exchange, the intermediate-range attraction is accounted for by $\sigma$-meson exchange, and the long-range tail is attributed to $\pi$ exchange. The pseudo-scalar and vector mesons are well established, whereas the scalar-meson $\sigma$ is still in controversy which had been listed, dropped and relisted in the particle data compilation\cite{2}. With the advancement of Quantum Chromodynamics (QCD), to understand the $NN$ interaction from the fundamental quark-gluon degree of freedom is expected. However there are only very preliminary lattice QCD calculations of the $NN$ interaction due to the complexity of low energy QCD\cite{3}. Various new approaches have been developed since the end of 1970’s. The chiral perturbation theory described the $NN$ interaction very well where only $\pi$ ($1\pi, 2\pi, 3\pi, \ldots$) exchange is employed\cite{4}. The most common used quark model in the study of $NN$ interaction is the chiral quark model\cite{5, 6, 7}. Both in meson-nucleon coupling and quark-meson coupling model (chiral quark model is a typical one) the $\sigma$ meson is indispensable and which is assumed for a long time to be an effective description of the correlated $\pi\pi$ resonance. Recently a very broad $\sigma$-meson as a resonance of $\pi\pi$ was reported\cite{8}. However the results, found by three groups independently, show that the correlated two-pion exchange between two nucleons results in strong short-range repulsion and very moderate long-range attraction which is quite different from the intermediate range attraction resulted from the $\sigma$ meson exchange\cite{9}. This raises a question: Is the $\sigma$-meson used in the one boson exchange model and chiral quark model the correlated $\pi\pi$ resonance or other effective one? Is there an alternative approach to account for the $NN$ intermediate range attraction? In addition, a long standing fact of the similarity of molecular and nuclear force is well-known, but no theoretical explanation so far. Is it accidental or is there a similar mechanism for molecular and nuclear force behind?

in 1990’s, A modified version of quark cluster model: quark delocalization color screening model\cite{10}, is proposed. Two new ingredients were introduced: quark delocalization (to enlarge the model variational space to take into account the mutual distortion or the internal excitations of nucleons in the course of interaction) and color screening (assuming the quark-quark interaction dependent on quark states aimed to take into account the QCD effect which has not been included in the two body confinement and effective one gluon exchange yet). The model has been successfully applied to describe $NN$, hyperon-nucleon scattering\cite{10}. In this model, the intermediate-range attraction is achieved by the quark delocalization, which is like the electron delocalization in molecules. The color screening is needed to make the quark delocalization effective. The model gave a natural explanation of the similarity between molecular force and nuclear force. Since 2000, the model is extended to incorporate the Goldstone-boson-exchange to describe the long-range tail of the $NN$ interaction\cite{11} which is hard to be described with quark-gluon degree of freedom. Then the only difference between the chiral quark model\cite{5} and QDCSM is the intermediate-range attraction mechanism. It is interesting to compare the two approaches to see whether the effect of $\sigma$-meson can be replaced by the new mechanism introduced in QDCSM.

After the Introduction, Section II gives a brief description of the two approaches to $NN$ interaction. The comparison between the two approaches is presented in Section III. A short summary is given in the last section.

II. TWO APPROACHES

A. Chiral quark model

The Salamanca model was chosen as the representative of the chiral quark models, because Salamanca group’s work covers the hadron spectra and nucleon-nucleon interaction, and has been extended to multi-quark states study. The model details can be found in ref.\cite{12}. Here only the Hamiltonian and parameters are given.
The Hamiltonian of Salamanca model in $NN$ sector can be written as \[ H = \sum_{i=1}^{6} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_c + \sum_{i<j} \left[ V^{OGE}(r_{ij}) + V^\pi(r_{ij}) + V^\sigma(r_{ij}) + V^{CON}(r_{ij}) \right], \]

\begin{align*}
V^{OGE}(r_{ij}) &= \frac{1}{4} \alpha_s \xi_i \cdot \xi_j \left[ \frac{1}{r_{ij}} - \frac{\pi}{m^2} \left( 1 + \frac{2}{3} \sigma_i \cdot \sigma_j \right) \delta(r_{ij}) - \frac{3}{4m^2 r_{ij}^2} S_{ij} \right], \\
V^\pi(r_{ij}) &= \frac{1}{3} \alpha_{ch} \frac{\Lambda^2}{\Lambda^2 - m^2} m_\pi \left\{ Y(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3} Y(\Lambda r_{ij}) \right\} \sigma_i \cdot \sigma_j + \left[ H(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3} H(\Lambda r_{ij}) \right] S_{ij} \frac{r_{ij}^2}{r_{ij}}, \\
V^\sigma(r_{ij}) &= -\alpha_{ch} \frac{4m_\sigma^2}{m_\pi^2} \frac{\Lambda^2}{\Lambda^2 - m_\sigma^2} m_\sigma \left\{ Y(m_\sigma r_{ij}) - \frac{\Lambda}{m_\sigma} Y(\Lambda r_{ij}) \right\}, \quad \alpha_{ch} = \frac{g_{ch}^2}{4\pi} \frac{m_\pi^2}{m_u m_d}
\end{align*}

\[ S_{ij} = \frac{\sigma_i \cdot r_{ij} \sigma_j \cdot r_{ij}}{r_{ij}^2} - \frac{1}{3} \sigma_i \cdot \sigma_j. \]

$S_{ij}$ is quark tensor operator. $Y(x)$ and $H(x)$ are standard Yukawa functions. $T_c$ is the kinetic energy of the center of mass. $\alpha_s$ is the chiral coupling constant, which is determined from $\pi$-nucleon coupling constant as usual. All other symbols have their usual meanings. The parameters of Hamiltonian are given in Table I.

\begin{align*}
H_6 &= \sum_{i=1}^{6} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_c + \sum_{i<j=1}^{6} \left[ V^{CON}(r_{ij}) + V^{OGE}(r_{ij}) + V^\pi(r_{ij}) \right], \\
V^{CON}(r_{ij}) &= -\alpha_{ch} \xi_i \cdot \xi_j \frac{r_{ij}^2}{1 - e^{-\mu r_{ij}}/\mu}
\end{align*}

where $V^{OGE}$ and $V^\pi$ are exact the same as Salamanca’s. $\mu$ is the color screening constant to be determined by fitting the deuteron mass in this model. The parameters of the model are given in Table I too.

The quark delocalization in QDCSM is realized by writing the single particle orbital wave function as a linear combination of left and right gaussians, the single particle orbital wave functions in the ordinary quark cluster model.

\[ \psi_\alpha(\vec{s}_i, \epsilon) = \left( \phi_\alpha(\vec{s}_i) + e_\phi \phi_\alpha(-\vec{s}_i) \right) / N(\epsilon), \]

\[ \psi_\beta(-\vec{s}_i, \epsilon) = \left( \phi_\beta(-\vec{s}_i) + e_\phi \phi_\beta(\vec{s}_i) \right) / N(\epsilon), \]

\[ N(\epsilon) = \sqrt{1 + \epsilon^2 + 2\epsilon e^{-S^2_i/4b^2}}, \]

\[ \phi_\alpha(\vec{s}_i) = \left( \frac{1}{\pi b^2} \right)^{3/4} e^{-\frac{1}{4\pi b^2} (\vec{r}_a - \vec{s}_i/2)^2}. \]

\[ \phi_\beta(-\vec{s}_i) = \left( \frac{1}{\pi b^2} \right)^{3/4} e^{-\frac{1}{4\pi b^2} (\vec{r}_a + \vec{s}_i/2)^2}. \]

**B. Quark delocalization, color screening model**

The QDCSM model and its extension were discussed in detail in ref.[10][11]. The Hamiltonian and trial wavefunction of the extended QDCSM are given below.

\[ \phi_{\beta}(-\vec{s}_i) = \left( \frac{1}{\pi b^2} \right)^{3/4} e^{-\frac{1}{4\pi b^2} (\vec{r}_a + \vec{s}_i/2)^2}. \]

| Table I. The parameters of two models. |
|----------------------------------------|
| **Salamanca Model** | **QDCSM** |
|----------------------|-----------|
| Set 1 | Set 2 | Set 3 |
| $m_u, d (MeV)$ | 313 | 313 |
| $\alpha_{ch}$ | 0.027 | 0.027 |
| $m_\pi (MeV)$ | 138 | 138 |
| $\Lambda (fm^{-1})$ | 4.2 | 4.2 |
| $\alpha_s$ | 0.485 | 0.485 | 0.724 | 0.996 |
| $b (fm)$ | 0.518 | 0.518 | 0.56 | 0.60 |
| $m_\sigma (MeV)$ | 675 | - | - | - |
| $\mu$ | - | 0.45 | 0.57 | 1.00 |
III. COMPARISON BETWEEN THE CHIRAL QUARK MODEL AND QDCSM

To compare these two models, the $NN$ scattering and the properties of deuteron are calculated. In order to make the intermediate-range attraction mechanism stand out, one exact the same set (set 1) of parameters: $b, \alpha_s, \alpha_{ch}, m_u, m_\pi, \Lambda$ were used for both models. Thus, the two models have exactly the same contributions from one-gluon-exchange and $\pi$ exchange. The only difference of these two models is how to get the intermediate-range attraction, $\sigma$ exchange for chiral quark model, quark de-localization color screening for QDCSM. To test the sensitivity of the QDCSM to the parameters, other two sets of parameters are used and the gluon contribution in these two cases is a little different from the Salamanca ones.

The resonating-group method (RGM) is used in the calculations. The detail of the method can be found in ref.[13]. For QDCSM, we first fixed baryon size parameter $b$, then the quark-gluon coupling constant $\alpha_s$ is adjusted to fit the N-$\Delta$ mass difference, the color screening parameter $\mu$ is then fixed by deuteron properties. The calculated results for deuteron properties and $NN$ scattering phase shifts are shown in table II and figs.1-4.

| Table II. The properties of deuteron | Salamanca Model | QDCSM set 1 | QDCSM set 2 | QDCSM set 3 |
|--------------------------------------|----------------|--------------|--------------|--------------|
| **B (MeV)**                          | 2.0            | 1.94         | 2.01         | 2.01         |
| $d \sqrt{r^2}$ (fm)                  | 1.96           | 1.93         | 1.92         | 1.94         |
| $P_D(\%)$                            | 4.86           | 5.25         | 5.25         | 5.25         |

**deuteron:** Two models both give a good description of deuteron. For QDCSM, by adjusting the color screening parameter, almost the same results for deuteron can be obtained for the three parameter sets of baryon size $b$. Because of the large separation between the proton and neutron in the deuteron, the properties of deuteron mainly reflect the long-range part of the nuclear force. The same $\pi$-exchange used in the two models assure the properties of deuteron be fitted equally well. Of course, the enough intermediate-range attraction is needed to make the deuteron bound.
**Fig. 4.** The phase shifts for channels $^1D_2$.

**NN scattering phase shifts:**

S-wave: For $^3S_1$, Salamanca model gave an almost perfect description of the experimental data, QDCSM also had a good agreement. For $^1S_0$, QDCSM described the experiment data a little better than Salamanca's single channel approximation did but after including N∆ $^1S_0$ channel coupling Salamanca model also fitted the experimental data well. For QDCSM, the larger the baryon size $b$, the better the agreement. The dominant contribution to the S-wave phase shift comes from the central part of the potentials, the agreement between two models means they have the same behavior, at least for central part of the interaction.

D-wave: For $^3D_1$, Two models fitted the experimental data equally. for $^1D_2$, the Salamanca model gave much better fit to the experimental data than QDCSM did, especially at higher energy. For QDCSM, different $b$ almost gave same results.

Mixing angle $\epsilon_1$: Two models gave a qualitative description of the experimental data. The chiral quark model result is a little closer to the experimental ones, but has the same tendency as QDCSM.

**IV. SUMMARY**

By calculating the deuteron properties and $NN$ scattering phase shifts, we compared the QDCSM with the Salamanca version of the chiral quark model, both models give a good description of the deuteron and $NN$ scattering, although different intermediate-range attraction mechanisms were used. The almost same agreement with experimental data suggest that the $\sigma$-meson exchange can be replaced by quark delocalization and color screening mechanism. If one takes the QDCSM mechanism to describe the $NN$ intermediate and short range interaction, the similarity between molecular and nuclear force obtained a natural explanation.

[1] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. **149**, 1 (1987).
[2] PDG, J. Phys. **G33**, 546 (2006).
[3] N. Ishii, S. Aoki and T. Hatsuda, nucl-th/0611096.
[4] S. Weinberg, Physica **A96**, 327 (1979); J. Gasser and H. Leutwyler, Ann. Phys. **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985); D.R. Entem and R. Machleidt, Phys. Rev. **C68**, 014002 (2003).
[5] A. Valcarce et al., Rep. Prog. Phys. **68**, 965 (2005) and references there in.
[6] Y. Fujiwara, C. Nakamoto and Y. Suzuki, Phys. Rev. **C54**, 2180 (1996).
[7] Z.Y. Zhang, Y.W. Yu, P.N. Shen, et al., Nucl. Phys. **A625**, 59 (1997); L.R. Dai, Z.Y. Zhang, Y.W. Yu, and P. Wang, Nucl. Phys. **A727**, 321 (2003).
[8] BES collaboration, Phys. Lett. **B598**, 149 (2004).
[9] N. Kaiser, S. Grestendorfer and W. Weise, Nucl. Phys. **A637**, 395 (1998); E. Oset, H. Toki, M. Mizobe and T.T. Takahashi, Prog. Theor. Phys. **103**, 351 (2000); M.M. Kaskulov and H. Clement, Phys. Rev. **C70**, 014002 (2004).
[10] F. Wang, G.H. Wu, L.J. Teng and T. Goldman, Phys. Rev. Lett. **69**, 2901 (1992); G.H. Wu, L.J. Teng, J.L. Ping et al., Phys. Rev. **C53**, 1161 (1996); J.L. Ping, F. Wang and T. Goldman, Nucl. Phys. **A657**, 95 (1999); G.H. Wu, J.L. Ping, L.J. Teng et al., Nucl. Phys. **A673**, 279 (2000); H.R. Pang, J.L. Ping, F. Wang and T. Goldman, Phys. Rev. **C65**, 014003(2001).
[11] J.L. Ping, H.R. Pang, F. Wang and T. Goldman, Phys. Rev. **C65**, 044003 (2002); X.F. Lu, J.L. Ping and F. Wang, Chin. Phys. Lett. **20**, 42 (2003).
[12] D.R. Entem, F. Fernández and A. Valcarce, Phys. Rev. **C62**, 034002 (2000); F. Fernández, A. Valcarce, U. Straub and A. Faessler, J. Phys. **G19**, 2013 (1993).
[13] M. Kamimura, Supp. Prog. Theo. Phys. **62**, 236 (1977); M. Oka and K. Yazaki, Prog. Theo. Phys. **66**, 556 (1981).