NAO robots as context to teach numerical methods

Sergio Rolando Cruz-Ramírez · Moisés García-Martínez · José Manuel Olais-Govea

Received: 2 February 2021 / Accepted: 22 September 2022 / Published online: 11 October 2022
© The Author(s), under exclusive licence to Springer-Verlag France SAS, part of Springer Nature 2022

Abstract
This article examines the contextual role that NAO humanoid robots play in the learning environment of a numerical methods course through a design-based research approach. Under the design of intentional and reflective planning that emanates from the socio-critical paradigm in education, the feasibility of using this type of robot in the classroom as a motivation element to engage students in understanding numerical analysis issues is addressed. In particular, the active learning of engineering students is achieved by studying such robots’ forward and inverse kinematics. The didactical sequence to teach the generalized Newton–Raphson method for solving non-linear equation systems is validated as part of a Hypothetical Learning Trajectory proposed. Besides, this research provides a brief theoretical discussion on reflection-in-action of the teacher of Higher Education level.

Keywords
NAO humanoid robots · Design-based research approach · Socio-critical paradigm · Active learning · Engineering teacher reflection · Educational innovation

1 Introduction
Robotics avoid exposing humans to dirty, complicated, and dangerous jobs [1]. These advanced machines replaced people who carried out repetitive, tedious, and even expendable tasks. Today there is a wide variety of industrial robots. From robots that carry out low-precision tasks (e.g., painting large surfaces) to those that assemble parts at high speeds, with great precision, and various programmed movements. Such is the case of serial, cartesian, and collaborative robots distinguished by sharing workspace with humans [2].

On the other hand, the area of service robotics refers to those robots that operate partially or autonomously at the service of human beings’ welfare, excluding manufacturing operations, as defined by the International Federation of Robotics (IFR) [3]. Service robots can be found in a complete sort of sector, such as the so-called precision agriculture, where crop variability observations are through drones. In medicine, telepresence robots (mobile robots composed of a microphone, speakers, screen, and controlled remotely) are increasingly being used in healthcare facilities around the world to offer a quick check-up to patients safely. In building and construction automation, robotic systems equipped with human–machine interfaces, different sensors, and artificial intelligence have been applied to assist humans with hard work [4, 5]. Technological advances in robotics, of course, do not exclude the educational field. Here, students interact with robots that are commonly commercial platforms whose purpose is to facilitate them in understanding subjects such as mathematics and developing their logical thinking [6, 7]. Mostly, in the last decade, applied robotics has played a significant role in all educative levels [8]. It is a critical element in Science, Technology, Engineering, and Mathematics (STEM), where it has adopted different educational roles: being a learning object, being a learning environment, or being a didactic tool [9]. These advances in science and technology have generated new fields of study and research. Immediate examples of this are biotechnology, nanotechnology, big data, robotics itself, among others. These new disciplines are naturally linked to the educational field since they are fields of knowledge with a high social impact [10]. Education is the bridge between this new knowledge, the qualification in these specific domains, and the productive sector. These new problems, which we can localize typically at higher education, have an inherent social value and give
rise to so-called socio-scientific issues [11]. From educational research [12, 13], these kinds of problems are a fertile area of work that must be exploited within universities as social studies concerning teaching and learning. However, being problems loaded with a high degree of specialization results in a complex task for traditional educational research to address them as entirely social problems [14]. Herein lies the importance of higher education teachers joining, as experts in techno-scientific disciplinary areas, into the action research [15, 16]. To be possible, it is strictly necessary that these teachers develop reflection strategies on their educational practice in higher education [17]. An authentic educational approach does not occur before teachers reflect on it. It appears as a consistent action on teaching’s educational fact, and it is seen as an objective and intentional social praxis [18]. From this social praxis approach, meanings, perceptions, and the actions of all the agents involved in the process: educational institutions, teachers, the object of knowledge, students, and society, converge. One of the most used tools through which teachers approach the reflective processes of practice is educational intervention planning [19]. In this process, all educational agents’ perspectives intersect and find their instrumentation in the teachers’ lesson plans. Planning is the heart of the educational process conducted in the classroom; that is, it constitutes the hidden curriculum’s desirable functioning [20].

In the present work, we show a part of a Hypothetical Learning Trajectory (HLT) [21, 22] set up in robotics to teach specific content on numerical analysis and reflect on the teaching–learning process that is triggered by this planning process. Through design-based research [23] and content analysis [24], we implement one of the didactic sequences of a said learning trajectory with fourth-semester engineering students. Using this, we carry out the solution of non-linear system equations in numerical methods courses, under the assumption that the robotics context facilitates the learning of the numerical methods. This assumption happens due to the tangible mirror embodied in robots simplifying the abstract objects typically studied in this course. In the first phase, this research is derived from the socio-critical paradigm [25, 26] by using teacher reflection as a mechanism that incorporates theoretical approaches to educational practice. From the teacher’s role, the transformation of a specific social reality happens, namely, learning of numerical methods mediated by the inclusion of humanoid robots [27]. In a second phase, this work analyzes the active learning [28] of students in numerical methods, particularly the analysis of the response to the contextual stimulus of robotics. Under this research design, we partially but concretely answer the question: does the programming of NAO humanoid robots favor the active learning of numerical methods? The particular case of the direct and inverse kinematics of these robots is presented. Scientific instruction based on socio-scientific problems proves to have positive results both in learning the content by the students and in their argumentation and reasoning skills [12].

In general, the instructional base used in this work is useful for students to frame their learning in science and technology within a social context characterized by the dynamics of the reciprocal nature of interpretation and action. In the second phase of this research, we provide evidence of the active learning scheme that students carry out. In particular, the cognitive actions of modeling, programming, and simulating are analyzed so that they are later able to analyze and formulate solutions. In other words, we interpret from the symbolic interactionism that the students are permanently exposed, a pragmatic understanding of the abstract objects with which they work. They carry out active learning within the context of robotics induced by thoughtful and intentional teaching.

The present work is structured as follows. Section 2 describes the theoretical-methodological approach that is followed to analyze students’ active learning from the teacher’s reflective practice within a socio-critical pedagogical approach. It also describes the typical process followed in design-based research. Section 3 shows, in length, the specific part of the HLT developed to teach non-linear equation systems, using humanoid robots as a context to solve the kinematics of the arm of a robot of this nature. In this section, at the same time as the teacher’s lesson plans are described, the partial results that are the product of his critical reflection and typical of the cyclical iteration of design-based educational research are given. At the end of this section, we describe how students use numerical methods in subsequent courses. These methods are no longer an object of study but a tool to tackle problems of greater complexity. For example, problems are attached to real situations in which even training partners from the productive sector can intervene. In Sect. 4, this intervention’s results are presented and discussed, based on the content analysis of the information units collected during the research. We use a textual reconstruction of this information based on codes that come from the critical pedagogical action. The intentional questions asked to the students as part of the design itself serve as the analysis guides and demand the reflection of the student’s learning. As a bonus of this methodology’s iterative process, in a final application of the sequence presented, we discuss the notion of a natural critical learning environment, a goal founded on the socio-constructivist approach to education. Finally, in Sect. 5, the conclusions and perspectives of this research work are addressed to validate a didactic model as a part of the HLT proposed. Additionally, we include Annex A, which intends to explain the theoretical features from which the qualitative methodology followed in Sect. 2 emerges.
2 Approach to the educational problem

Teaching at a higher level suffers from proper educational structures and methodologies due to the predominantly lecture style [29, 30]. This teaching approach is justified by the broad domain of the contents taught and the super specialization that university professors usually have due to the common immersion in disciplinary research. However, in-depth knowledge of the object of study is not enough since it becomes sterile within the teaching–learning process (a social nature process) without a set of didactic strategies [31, 32] associated with said content. These strategies only have the student’s function of having a more friendly interaction with the object of knowledge. Still, they do not fulfill a process of social interaction with the teacher. For this to happen, the teacher must have at least a glimmer of understanding about how his/her students learn [33]. This construct will not occur until teachers reflect on their pedagogical practice. Pedagogy and knowledge are a teaching dimension that enables it to have communication channels with students on the specific topic of the object of knowledge [34, 35]. This last awakens one of the essential elements for the teacher to understand how students learn. This will never be possible without a permanent teacher’s reflection mediated by the practice on the educational process (not only on what he/she is teaching but on the social system in which the teacher is immersed) [17]. Finally, the teacher’s reflection, as such, would have no meaning unless there is a social frame of reference that indicates that what it teaches has a purpose [14]. Curricular knowledge is a reference system that connects the isolated classroom with society itself [36]. The curriculum’s instrumentation is educational planning, and its premises are educational institutions, although it does not necessarily emanate from them. From this social perspective of educational phenomena [37], these elements provide quality and meaning to the teaching intervention and ensure the quality of the teaching–learning process.

It is the combination of these kinds of knowledge (content, pedagogy, curriculum) which gives rise to a glimpse of the teaching construct. However, the motor of pedagogical intervention, which lies within teaching, lies in reflecting on educational practice carried out by the teacher [33]. Ultimately, this metacognitive process involves reflection on disciplinary fields, reflection on didactic and pedagogical knowledge of the discipline, and curricular reflection on the knowledge taught [34]. Here, symbolic interactionism [36] is translated as a pragmatic understanding of educational events [37], whatever the scale of the phenomenon is analyzed. A natural product of this process is the teacher’s formulation of what it means to teach. Reflection on this construct allows modifying (and eventually improving) the teaching intervention.

The reflection arises from the educational practice to improve the teaching intervention, giving value to the teaching experiences. It is a process of routine action aimed at personal growth in awareness of one’s educational actions. To achieve a true Reflective Practice (RP), this position must be inscribed within an analytical relationship with the action. For this reason, the registration of teaching experiences (for example, a binnacle) is an essential tool that allows the construction of one’s own teaching styles, promoting the close relationship between theory and practice [38]. Thus, teaching aims to achieve its students’ learning through the development of strategies, methods, and a series of self-regulated behaviors to achieve this objective. The relation practice-theory enables the teacher to correct and improve the teaching–learning process. Generally speaking, RP is based on an intentional, systematic analysis. Due to this feature, it can be implemented and, therefore, learned. A natural reflection can then become a RP if it seeks to promote a learning outcome that helps the knowledge created through theoretical foundations. Assuming a reflective attitude contributes to accepting a greater variety of options to help achieve, as already mentioned, learning in students. Teachers who practice and exercise RP continuously assume cycles of observation, planning, action, and reflection in the classroom. These are usually nested with future sessions, thereby acting as an upward spiral, in which there is no interruption, thereby achieving continuous improvement in the teaching–learning process. From this frame of reference, we understand RP as a special form of reasoning that teachers must carry out during (1) the knowledge of the action, (2) in the action, and (3) about the action [39]. In this way, educational research from a qualitative paradigm is in charge of solving pedagogical problems through critical thinking that arises from the socio-critical method [40]. This method assumes a crucial stance in front of educational phenomena (whatever the scale), considering the action from reflection, self-learning, reasoning, and the analysis of reality, necessary aspects in a competency curriculum. To amplify and deepen these ideas linked to the paradigm of qualitative research, you can consult Annex A of this manuscript, in which concepts typical of social science research applied to educational phenomena are discussed.

The development of this work occurs in Tecnologico de Monterrey (from now on Tec de Monterrey), a university whose educational model is based on competency-based curricular approaches [41, 42]. The elements mentioned above, for this work, only impact the teacher and constitute the first methodological phase of this research: the formulation of lesson plans that, together, make up a part of the HLT [43], outlined in Fig. 1 (octagon four from left to right). Despite having a didactic model for implementing a course of numerical methods in robotics, this research only addresses the
section on the solution of systems of nonlinear equations. Validation of the HLT is beyond the scope of this paper.

As already mentioned, the implementation of the RP in education is the planning of the teaching intervention [44]. This, in turn, is the product of a close relationship between the ontological and epistemological positions of the teacher. These postures are a combination of what is believed to be researchable and what is known about what is taught. All this leads to adopting a methodological approach to understand the impact of said ontology and teacher epistemology.

The elements taken into account in this work, in each of the aforementioned dimensions, correspond to numerical analysis as a didactic object added to robotics as a didactic means and learning context (see Fig. 2). Assessing these dimensions in teaching practice methodologically leads to formative research based on design. The core product of this approach is an instrument we call thoughtful and intentional planning (explained in Sect. 3) that, in the long run, allows the teacher to partially answer the questions: How to teach numerical methods to engineering students? How do these students learn numerical analysis?

The intervention carried out involved 75 engineering students throughout three Numerical Methods courses (January–May 2020, January–May 2019, August–December 2018) at Tec de Monterrey, San Luis Potosí campus.

Currently, Tec de Monterrey has a competency-based curricular approach, where the adoption of this word is defined as “the conscious integration of knowledge, skills, attitudes, and values that allow facing both structured and uncertain situations successfully. Competencies integrate both knowledge and procedures of a specific discipline, as well as the attitudes and values that allow the training of participatory professionals” [45, 46]. This competency model has a pedagogical approach that actively involves the student in a relevant problem situation related to the environment through activities designed to promote disciplinary and transversal competences.

The population studied here was composed of students enrolled in various engineering programs such as Mechatronics, Technology, Industrial and Systems Engineering, Mechanics, and Innovation and Development. This variability in the study population ensures the complexity and integrity of the analyzed social system. Under the aforementioned intentional design, active learning was proposed as an intervention model for students to model the robotic arm’s movement. In deriving the motion equations, the general Newton–Raphson (NR) numerical method was implemented, which iteratively solves systems of nonlinear equations. Once the inputs required for this numerical method are working properly, students iterate the algorithm three times to refer to programming the numerical method with the appropriate computational tools.

Figure 3 shows the active learning carried out during the didactic sequence, detailing the cognitive processes that the students carry out within the development of the activities, the learning objectives, the teaching intentions, the deliverables, and, in general, the fundamental dynamics of the teaching–learning process that we have designed. Note that the didactic sequence aims to analyze the actions of modeling, programming, and simulating. The first of these arises from a triggering question whose purpose is to visualize and sketch the programming logic on paper. The action of programming consists of carrying out the program itself that will solve the kinematic equations that govern the robot arm’s movement. The simulated action is the first verification step that the student must know if the first two actions were carried out correctly. In these phases, there is a permanent visual apprehension that detracts from the complexity of the student’s abstraction processes but strengthens their
Fig. 2  Theoretical and methodological elements of this research. From right to left, we can see the interpretation made here of the socio-critical approach that the teacher adopts to carry out his intervention and how this collectivized approach has a natural relationship with the active learning approach carry out within the described intervention. The elements of the competency-based curricular approach of Tec de Monterrey are included, and the phases of the design research are carried out in this research. Content analysis is used as an analysis technique of the collected information units.

Fig. 3  The didactic design has the objective of teaching systems of nonlinear equations in a numerical methods course, part number 4 of the HLT of the designed class.

metacognitive processes. Once the student achieves a correct simulation of the robot arm’s movement, he/she will be able to analyze and formulate conjectures of the problem he/she has solved. The analysis phase is critical in two ways: knowing if the previous cognitive stages were carried out successfully and reflecting on those things that simplify the problem-solving process due to the robots’ context. Finally, the student formulates decisions based on correct solutions based on what he/she wants the robot to do or what the activity by design asks him to do. At this point, the students have a marked disengagement from the teacher’s accompaniment due to technological mediation and the correct passage through the first three cognitive processes, where there is a marked accompaniment (almost dependency) on the teacher.

The next section details the didactic sequence that the students carried out while showing the planning process that the teacher designed, according to what they want to obtain in class.

3 Design of the educational experience: the link between reflective practice and active learning.

The socio-critical method described as the first methodological phase of this research consists of (1) self-reflection, (2) dialogue with academic peers as a way of sharing the ideas that come from the self-reflection process, (3) the consensus...
of the focus group (regardless of its size) and (4) the critical conclusions and the formulation of general resolutions, keeping practice as a criterion of truth. This set of steps enables reflection and research to be activated from the observation of reality in the application of pedagogical praxis [38]. Pedagogical praxis is understood as the set of didactic procedures that awakens and activates knowledge conditioned by social becoming laws and is inextricably linked to practical activity. This method raises awareness of the surrounding reality in the human brain’s cognitive activity [47, 48]. The focus group in this research is made up of the authors of this proposal. It is summarized in the didactic model for teaching numerical methods in engineering through the context of robotics, as illustrated in Fig. 3.

3.1 NAO robots

The NAO robot is a humanoid developed by Aldebaran Robotics and is designed to simulate the shape and movements. NAO has 58 cm high and is fully programmable. It has different sensors such as vision, gyroscopes, sonars (among others), 25 degrees of freedom based on DC servo motors, and advanced functions such as voice recognition [49]. One of the two robots currently on campus is shown in Fig. 4, along with its programming environment. NAO robots are handy in-service robotics. An example of this is its use as a therapeutic exercise instructor in children with autism [50, 51].

In what follows, we refer to the concrete experience of solving non-linear systems of equations through the kinematic analysis of the movement of the NAO robot’s arm.

3.2 General statement of the problem

In this learning experience, we decided to analyze a robotic arm with two degrees of freedom $\theta_1$ and $\theta_2$, in a planar configuration, with ends $(x, y)$ that relates the lengths $L_1$ and $L_2$ of the robot, as seen in Fig. 5a and whose equations contain the forward kinematics of the robotic arm, namely,

$$
\begin{align*}
x &= L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) \\
y &= L_1 \sin \theta_1 + L_2 \sin(\theta_1 + \theta_2)
\end{align*}
$$

(1)

Inverse kinematics, as is usual in all serial robots, has no trivial and unique solution, as can be seen by trying to transpose $\theta_1$ and $\theta_2$ from Eq. (1) as a function of $L_1$, $L_2$, $x$ and $y$. In Fig. 5b, the results of programming the kinematic equations in Matlab are shown together with the real-time simulation of the movements of the NAO robot in its programming environment (Fig. 5c).

3.3 Numerical solution to the inverse kinematics problem

A generalized Newton–Raphson method is an essential tool for iteratively solving systems of nonlinear equations, which are present when trying to solve the inverse kinematics of a robotic arm. In the case at hand, the system of nonlinear equations that results from considering the target position $(x = 0.5, \ y = 1.87)$ is

$$
\begin{align*}
L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) - 0.5 &= 0 \quad f_1 \\
L_1 \sin \theta_1 + L_2 \sin(\theta_1 + \theta_2) - 1.87 &= 0 \quad f_2
\end{align*}
$$

(2)

with an initial approximation of $\theta_1$ and $\theta_2$, resulting in the following iterating equations

$$
\begin{align*}
\left( \frac{\partial f_1}{\partial \theta_1} \right) h_1 + \left( \frac{\partial f_1}{\partial \theta_2} \right) h_2 &= -f_1 \\
\left( \frac{\partial f_2}{\partial \theta_1} \right) h_1 + \left( \frac{\partial f_2}{\partial \theta_2} \right) h_2 &= -f_2
\end{align*}
$$

(3)

that we must solve iteratively, until $h_1$ and $h_2$ are less than a given value that is a function of the required precision.

3.4 The learning sequence

The activity’s general objective is to understand the link with relevant technology in solving systems of nonlinear equations applying the generalized NR method. The robotics’ real application is achieved using NAO humanoid robots, the Matlab program as the programming environment, and the robot program itself to visualize the results obtained.
final activity design was developed in 3 sessions of the numerical methods course, each lasting 80 min (in three different years). Each one is detailed below, including the activities before, during, and after each session, for both the teacher and the student.

3.4.1 Session 1

Before the session, the teacher must prepare information related to NAO robots, such as infographics, manuals, and videos, so that students become familiar with the context they intend to carry out the activity that the intentional class plan defines. For their part, students should review the materials from the previous class in which the generalized NR method for solving systems of nonlinear equations was made known.

Demonstration of the capabilities and characteristics of the NAO humanoid robot On the first day that students meet the NAO humanoid robot, an almost immediate hookup occurs. They seem to be surprised when the robot executes certain routines pre-programmed. For example, doing movements of the Chinese martial art Tai-Chi in full coordination with certain music, following a red ball using your vision system, and telling stories while interacting with the audience.

To have the first contact with the students and the object of study, the robot is programmed to tell the students the following message:

Hello students of numerical methods, today you will try to solve the kinematic analysis of one of my arms.

This simple interaction with the robot, which from an educational perspective is nothing more than a technological element that is incorporated into the learning environment, motivates students even more, who make the first connection between school knowledge and real-world applications. In Fig. 6, the NAO robot can be seen executing one of its routines. More than one student can be seen taking photos or videos, demonstrating the interest that this type of robotic platform arouses in them.

Specifically, during January-May 2020, this demonstration using the real robot could not be carried out due to the sanitary confinement of the Covid-19 pandemic. Still, it was worked with videos and questions-based on the reflection of learning. A general taste for the activity can be appreciated due to the relationship between scholarly topics and their real applications.

Coordinate systems associated with the robot and kinematic configuration of one of the arms Like any motion system, the establishment of coordinate systems is essential to reference both angular and linear displacements. In the NAO robot’s case, these systems are already predefined, but it is important to show them to the students using the reference manuals.

As the first activity, in groups of three, the students work on the direct and inverse kinematic analysis of one of the NAO robot’s arms. This activity aims meaning to the idea they formulated the relationship between scholarly knowledge and real applications. In the first part of this activity, the teacher acts as a guide and uses trigger questions and keywords that help students deduce the NAO robot arm’s direct kinematics, considering two degrees of freedom. A test done in the three study groups was to provoke the students with extra points in the final grade of the course if they solved, analytically and in time no longer than 5 min, the inverse kinematics from the equations of the direct kinematics. Although the problem looks quite approachable (and has an analytical solution, but in our teaching experience it takes around an hour for an average engineering student to solve it), it is not trivial at all due to the non-linearity of the robot’s set of equations of motion—still, students failing in all cases. Contrary to the fact that this causes demotivation during the session, the effect is inverse. It causes curiosity about finding that solution and with what tools it is possible to do it. This is of great help to understand the need to solve the inverse kinematics using the iterative method of generalized NR, that is, to recognize the importance of the object of study. As long as the sequence (hooking
Fig. 6 NAO robot executing one of its movement routines where students are quickly hooked. The faces of the students were blurred to protect their identity.

= > motivation = > curiosity = > challenge) does not exist, it is difficult for students to have meaningful learning.

In the second part of this activity, students think about the necessary elements that feed the numerical method (initial conditions, partial derivatives, among others) and then manually perform at least three iterations. This positively conditions the next session since there is a reference to the results obtained when programming the method. Also, the use of programming is valued, which saves considerable time and effort when you want a large number of iterations, greater precision in the results and, even to be able to have the results in graphical form. This moment in which numerical methods are valued as an ally tool in problem-solving motivates students to incorporate these theoretical-practical elements into their “abstract toolbox,” essential in science and engineering. Finally, in addition to the teacher’s closing conclusions, the students are asked to have the Matlab programming environment ready so that, in the next session, they can carry out activity 2, which consists of programming the method. The concatenation of the activities, both carried out in class and outside of it, is essential in the design of teaching intervention since it strongly depends on this to sustain the interest and, therefore, the reasoning about the problems being solved. The appropriate level of challenge and socialization with group mates allow the sessions’ enjoyment and strongly promote symbolic interactionism within the class, under the appropriate symbolization registers [52, 53]. All activity must have at least one symbolization record since these are resources that help in the construction of knowledge, fixing the new objects of knowledge to the cognitive structure of the learning subject, after varying periods of time in which the subject is in interaction and use of said objects of knowledge.

3.4.2 Session 2

This session’s objective is to program the numerical method that solves nonlinear systems of equations considering their real application in robotics. Before the session, the teacher must have the Matlab programming environment ready, prepare the manuals and equations for the NAO robot, and the specific activities that the student will develop. It is always essential to have clear and achievable objectives to maintain the balance within their learning process. An adequate communication flow occurs between the teacher, the student, and the object of knowledge itself. Students are asked to revisit activity 1 that they completed in the previous session and have the programming environment ready. Notice that there are communication and specific tasks outside of class time, necessary not to interrupt the reasoning built around the learning object. To get this point, the teacher does a quick review of what was seen in the previous class. This part is essential to remember the equations analyzed in the last session regarding the NAO robot arm and as an instant feedback mechanism to the student. Feedback is part of the evaluation process. It helps the teacher make informed decisions in their class, have good judgment about their students, and take appropriate actions so that the flow of learning occurs. Within this process, students’ solutions to the previous activity are also analyzed, in which they manually carried out three iterations of the generalized NR method. With this, the students will have a reference for the results they obtain from their programming. Here we are in a new phase of their active learning; we have reached metacognition aspects since students can self-regulate their learning process by knowing if their results are correct. If not, they will know that they must modify their codes. Additionally, the teacher explains the new commands in Matlab that will be necessary to complete the activity. It is of major importance to note that (i) the teacher begins to lose presence in the session concerning the
students who, due to the metacognition processes, no longer depend strictly on the teacher’s accompaniment and (ii) that we have added to the active learning phases (engagement = > motivation = > curiosity = > challenge), feedback actions = > self-regulation.

**Activity 2: programming the numerical method**  This activity, unlike the previous two, is carried out individually. In the first part, the teacher asks the students to create a code that gives a solution to the generalized NR method from scratch. The above was the suggestion of some students in past activities not to have “so much support from the teacher.” This demarcates the teacher’s decentralization within the learning process and opens up another metacognitive process not seen until then: autonomy. Again, there was impressive educational progress. However, during the activity, some students admit not having reached a level of understanding about the programming logic to be able to conclude their code. According to certain taxonomies of reasoning, the students reflect on their learning process, one of the most complex cognitive actions [54, 55]. Thinking from the contents and objectives of the activity, this phenomenon may not be impressive, the elaboration of the code demands to consider multiple conditions of stoppage and evaluation of different functions, and this gives it a certain complexity that indicates that some students will not be able to complete the code. However, thought from learning, we have added two new links to the chain of active learning actions: (engagement = > motivation = > curiosity = > challenge = > feedback = > self-regulation) = > reflection = > self-evaluation.

In the second part of the activity, the teacher provides a template for the method’s programming. This template only includes the “skeleton” of the program, along with some commands. An empty box method is used to graduate the complexity of the logic in the method programming. As teachers, we should not allow the activity’s challenge to exceed the student if we are interested in maintaining the Optimal Flow Learning [56]. After this support, some of the students begin to obtain results similar to what they obtained manually.

This action generates the confidence to increase the difficulty of programming. For example, ask for a greater number of iterations and changes in the method’s initial conditions.

### 3.4.3 Session 3

This session’s objective is to include a graphic display of results to the previous program that solves the numerical method and gives solutions to nonlinear systems of equations, considering their real application in robotics. Before the session, the teacher must have the Matlab programming environment ready, prepare the support program for the graphic viewer, and the activities to be carried out by the students. Again, students should review the previous sessions before the activity and have the programming environment ready as well.

Before starting the activity, the teacher asks three students (randomly) to share their results from the previous activity. They programmed the code that provides a solution to the generalized NR method. One more finding that we found in this intervention: Although most students decided to use the teacher’s template, certain differences were observed that open up a natural space for socialization and discuss the best solution to the problem. A better solution to this problem may mean fewer lines of code, functionality, speed of convergence, among others. Naturally, socialization and decision-making were present in the active learning sequence that the students are carrying out. Returning to the list of actions or phases that students carry out, we can add to the chain (engagement = > motivation = > curiosity = > challenge = > feedback = > self-regulation = > reflection = > self-evaluation) = > decision-making = > socialization.

We must interpret this last action in a supremely collective sense and not limited to work teams. This is aimed at creating natural critical learning environments [57], often considered the goal of the teaching–learning process.

After the indicated socialization process, the teacher shows the elemental use of the support program regarding the graphic display. The teacher developed this graphic viewer for the students to observe the positions of a robotic arm with two degrees of freedom, considering as input parameters only the angles of each of its joints.

As a practice activity, the students are asked to add said graphic viewer to the program developed in the previous class (the numerical method that gives solutions to systems of nonlinear equations). In the first part of the activity, the teacher allows the students to carry out the requested activity individually. After a few minutes, a survey is carried out on the progress of the activity and, those students who have not yet been able to implement what is required are supported. In this phase of the intervention, we noticed that the students did not necessarily ask the teacher for support but that, in many cases, they did not care to ask other classmates for support. This phenomenon describes the teaching figure’s decentralization as the possessor of knowledge, typically observed in traditional basic instruction. It reinforces the observation of previous sessions on autonomy and, even more, the formation of collaborative learning environments. Once again, to the learning flow chain that students build, a final element is added. Namely, we already had (engagement = > motivation = > curiosity = > challenge = > feedback = > self-regulation = > reflection = > self-evaluation = > decision-making = > socialization) and we added, not an individualized action but a collective one, = > collaborative critical learning environments.
Fig. 7 Evidence provided by one of the students. a Manual solution applying the NR method. b Table of results when programming the numerical method in Matlab. c Graphical evolution of the results in the iterations of the method.

Once the graphic viewer is correctly linked to the students’ program, it can be observed that there is a better compression of the results. Due to the visual apprehension, a better understanding is happening between each iteration of the method. In the end, the students are asked to generate initial conditions such that they obtain different solutions. Since the programmed method only provides one solution at a time and, since those are systems of nonlinear equations, there may be more than one solution to the system. The above enriches the topic since each student decides on different values and shares their results with the group. In Fig. 7, evidence of a student chosen randomly is shown in each of the sequence’s parts described. Finally, reflection is made on how what was learned can be applied to analyze more complex mechanisms, such as an industrial robot with 6 degrees of freedom where more variables appear in its motion equations, ending the didactic sequence.

3.5 Application of abstract tools in real situations: examples of courses after the numerical methods course

As stated before, the course on numerical methods in science and engineering becomes a frequently used reasoning, an abstract tool within the engineering student’s “toolbox.” As a bonus in this study, we analyze this tool’s use by observing the study groups, which are now in the last third of their engineering career. Here, we can describe how these students interact in real learning environments, real-world problems, and even training partners from the productive sector [52].

3.5.1 Manufacturing systems automation

In this subject of the 8th semester, core topics such as industrial robotics, numerical control machines (CNC), and programming of programmable logic controllers (PLCs), among others, are addressed. These types of courses involve
students in real-life projects, where the development of competencies plays a leading role. For example, students must integrate an industrial-type production cell (see Fig. 8) using industrial robots of different configurations, PLC programming, conveyor belts, and CNC machine programming. Likewise, they simulate an automated production line, from the raw material’s arrival through its manufacturing processes to its final output as a finished product and ready for packaging. This real-world situation is successfully achieved once students analyze the kinematic model (equations of motion) of the robotic arms, as were done with the NAO robot arms in the Numerical Methods course. Furthermore, the robots must be programmed and controlled to optimally complete the pick-and-place tasks, interacting with each other and with the other machines in the production cell through the central PLC. In this way, the situations analyzed in the classroom become palpable.

### 3.5.2 Industrial robotics

From the 9th semester, this course has a direct connection with the analysis carried out on the NAO robot arms in the Numerical Methods course. In addition to analyzing the characteristics of different industrial robots and their programming environments, this course addresses the analysis of the generation of trajectories to move the end of the robot with some movement desired (linear, sinusoidal, among others), as can be seen in Fig. 9. The analysis is necessary to complete the kinematic solution (direct and inverse) for the robot under study. Sometimes it is necessary to implement the generalized NR algorithm, as in Numerical Methods, since the more degrees of freedom a robotic arm has, the more complex its kinematic analysis, becomes and numerical approximations are a good alternative solution. In addition to practicing with real industrial robots, where the students control these machines and program them, which excellently complements the students’ training, is visiting companies in the industrial zone that have robots in their production lines. During the visits to companies, something fundamental is the explanations of the robotics expert engineers, where the students listen to problems and advice like those mentioned in class.

### 3.5.3 Industrial networks project

In more advanced courses (7th semester of their training as engineers), students must learn automation concepts, industrial devices (PLC), their differences, the correct connection of inputs and outputs, and the necessary software development for its optimal performance process. At this point, students must develop programming skills, logical-mathematical thinking, and Boolean logic. These competencies come from numerical methods courses, so it is only necessary to focus knowledge on new areas and, in this way, take advantage of the skills they have developed.

The development of the course project in question can be divided into three parts: hardware configuration, programming, and the execution of the program through simulators. In the first part, the students know the devices in-depth, including their technical specifications, connection modes, and their configuration. Figure 10a shows some of the hardware used in the laboratory.

In the second part, the students use the TIA (Totally Integrated Automation) Portal development environment of the Siemens company to develop the code responsible for the entire automation process. It is essential to mention that there are different languages. However, the most common programming language is used, known as ladder programming. This language is a visual language in which the processes are represented using electrical components, making it very easy and intuitive to define the logic you want to achieve. Besides, to give the subject more formalism, students are asked to represent the diagrams represented in equations. Figure 10b shows part of a ladder programming code.

Finally, students must validate that the proposed programming works correctly; for this, there are simulators developed...
**Fig. 9** Kinematic analysis of an industrial robot with 6 degrees of freedom. Left: Assignment of coordinate systems. Right: Simulation in Matlab Robotics Toolbox environment.

**Fig. 10** Industrial networks project course. a) Siemens S7-1500 series PLC. b) Ladder programming blocks. c) Virtual simulator of a classification line.
by Tec de Monterrey that focus on these types of problems. We must point out that there are specialized laboratories with the necessary hardware to run 3D simulations in real-time (see Fig. 10c). However, due to the contingency caused by Covid-19, a 2D platform was developed which can be run on practically any computer since it does not require a specialized CPU with high hardware features. In this way, students can have these simulators on their personal computers, which we call Lab @t Home.

There are four different simulators. One of which presents more possibilities to achieve the development of skills and competencies is the simulator of a classification line. This simulator contains virtually all the sensors and motors required for its operation. The advantage of using this type of tool is that students can validate their proposals without putting their own safety or the production line components at risk. They can detect programming errors in your model quickly and accurately. This allows that, on the one hand, they can self-manage and self-regulate with the help of the teacher under specifications about the programmed iterations. On the other hand, it allows students to continue developing their specific skills and competencies within an environment close to reality. They can see direct applications of the knowledge learned. With this, they gain experience in daily problems that occur with a certain frequency in companies’ automation processes.

3.6 Preliminary conclusions of the didactical design into the HLT

The above described only corresponds to the fourth topic indicated in Fig. 1. The didactic model proposed to teach numerical methods from a robotics context can be validated for the rest of the hypothetical learning trajectory (HLT) points. This can serve as a didactic prototype or platform that can be used or reproduced in other universities’ numerical methods courses. We can anticipate, concerning the context of robotics and its relationship with the rest of the topics listed in Fig. 1, that didactic sequences can be created where:

1. Speeds and positions of a robot or any other object are calculated based on accelerometer readings. Here, numerical integration can be applied using methods such as the Riemann sum or the trapezoidal rule.
2. The mechanical design of robots is optimized, where numerical methods such as Bisection and Secant can be used to find the solution of non-linear equations and polynomials that arise in the analysis.
3. To analyze the position, orientation, and dynamics of robots with $N$ degrees of freedom requires matrix algebra.
4. The trajectories that a robotic arm or mobile robot will follow are generated, even with obstacles to avoid.

Lagrange polynomial interpolation is a good numerical method to achieve the goal.

5. Sensors onboard a mobile robot, such as vision cameras, sonars, and laser range finders, are calibrated. The calibration process can be accomplished by applying a least-squares regression method to the sensors’ raw data.

4 Results

4.1 Theoretical saturation of the didactic sequence

The described sequence responds to intentional and reflective planning designed to teach the solution of systems of nonlinear equations, under a socio-critical method framed within the collectivized teaching-learning methods, to which the active learning approach belongs, frequently used in the training of engineers. As shown in Sect. 3, this method causes students to go through the personal, group, intergroup, and debate-conclusion stages that generate in students a training process in which they assume a critical attitude towards the proposed activities, considering their performance based on reflection, self-learning, reasoning, and real analysis. These elements are derived directly from any competency-based curriculum, whose main mission is to prepare students in training for their professional-working life to be successful. This first result is focused on teaching-in-action. It comes from a temporally longitudinal itinerant reflection (whose analysis memos are the class plans of 5 previous years and the registry logs of those years of teaching the course of numerical methods to students of engineering) that converges to the planning presented in Fig. 1 and which is linked to student learning in Fig. 2. Reflection from this qualitative research paradigm leads the teacher to understand that this topic’s development should offer the students a significant number of objects with which they could interact and build their learning. In the concrete case presented, we applied this to the generalized NR method for the solution of systems of nonlinear equations. The objects of the robotics learning environment reduce the complexity inherent in the cognitive processes of abstraction. This technological context fulfills various functions that are aligned. The student reveals the development of a sequence of cognitive actions, each more complex (from paper models to computational simulation) and collectivized (from individual actions to collective environments critical learning). Specifically, this methodological phase strongly focused on teaching, produced a learning flow that we have synthesized in the cognitive processes chain (engagement = motivation = curiosity = challenge = feedback = self-regulation = reflection = self-evaluation = taking decision-making = socialization = collaborative critical learning environments). Regarding
the disciplinary knowledge, firstly, the numerical programming of the equations was used, the solution of which could be judged as correct or incorrect according to the simulation response obtained. Finally, once the programming and simulation gave accurate results, the kinematic equations were programmed in the NAO robot programming environment. This didactic tool (tangible in this case) flattened the abstraction process inherent in the problem posed since it had a simple way of being verified by observing the robot’s movements that, by construction, had been modeled, programmed, and simulated. The actions on the objects of knowledge that the students carried out (modeling, programming, simulation) made the subject of study tangible for promoting its better understanding and facilitating analysis and formulation of results. This process confirms that visualization is a means to move to abstraction and that collective analysis is a means of reaching argumentation. All the results described above are based on the systematization and registration of the teacher’s symbolic interactionism with his students until reaching a theoretical saturation that allows him to find an instructional basis as a teaching method on a specific object of study.

Regarding the second phase of this methodology, namely, the didactic model’s assessment through the content analysis collected in each iteration of the design methodology used. In this case, three iterations (3 years) were made after the two years it took to formulate the didactic model.

Content analysis is often called interpretive procedures of products that come from singular communication processes previously registered. Based on measurement techniques, sometimes quantitative (statistics based on the count of units) and other qualitative (logic based on the combination of dimensions). This methodological technique aims to elaborate and process relevant data on the conditions in which it is used or on the situations that may arise for their subsequent use. In this research, we used a count of units (averaged over the three courses indicated in Sect. 2 from a previous analysis guide that consisted of two questions: (1) Do you consider that the use of robots in class will motivate you to develop the class for numeric methods? And (2) Do you consider it useful to learn some aspects of robotics in your training as an engineer (regardless of the program you are enrolled in)? The positive cases were 96.8% and 83.9%, which served to establish the communication channel within this methodological phase. It is assumed that the context is desirable for the students, and the design is applied, with an analysis guide that consisted of 3 questions, coded in two types. The first code addressed the NAO robots’ context and had the guiding questions associated with it: P1-NAO: What important decisions did you have to make when solving the problem? And P2-NAO: What new abstract tools do you have now?

Regarding this category, we found a strong identification with the codes of the learning flow (chain of words) of the sequence described in Sect. 3, which always contained questions with the same semiotic content as those of the analysis guides. The codes in italics for this part of the guide are shown below, and the results are synthesized based on a textual reconstruction of the study group analyzed.

P1-NAO What important decisions did you have to make when solving the problem?

The use of NAO Robots helped me to know how to start the problem […] since it was easier to think about the problem statement. […] The NAO programming and simulation environment was useful because I understood the use of the “while” command […] necessary many times in the programming of the method to solve nonlinear systems of equations. […] The movement of the robot and the program given to graph it was useful to incorporate the graph that shows the movement of the robot to our solution. […] Realizing that the initial values led to different movements of the robot and I think that improved my logic to obtain the value of the angles, […] all this helped me to properly handle other commands within the code. […] The mathematical approach to the problem was not as difficult as finding values that satisfy the desired arm position. […] in this way I was able to transfer the NAO system to the Matlab program to understand the structure of the program […], to know where everything was going, for our program to work, […] and to ask it the correct results […], which would not have been possible without taking into account the logic of the robot’s operation. […] The method seen in class helped me to arrange my equations well in order to arrive at a solution and to know what was the maximum number that my values had to reach for the conditions and iterations to be fulfilled. […] However, I had to make the decision to use another method to solve it or to use the one recommended by the teacher to solve the system of equations. […] I understood how to return the angles so that the program that showed the arm could read it […] and thus find the best way to program the exercise.

P2-NAO: What new abstract tools do you have now?

I have a better handle on Matlab to solve application problems. In addition, the use of matrices as a “shortcut” to facilitate the planning and development of the generalized NR method. […] I improved my skills in the graphical environment of Matlab and I understood a little about how the robot works. […] The graph allows me to observe the movement of the NAO robot, making it easier to detect my errors. […] With the application of numerical methods to solve systems of non-linear equations by the NR method, you can find solutions that give guidelines to true applications, […] now I
know that with Matlab you can see how the program is going to work before applying it definitely. […] Before, I didn’t know that the NAO robot works that way […] now I can reason similar problems with the NR method for other systems of non-linear equations, […] and I even have a subroutine in Matlab to do it. […] Not only did my programming improve but also how the coordinates work in the position analysis of a robotic arm […], this aroused interest in technology and less fear of making mistakes in the programming […], since these activities were helpful in using my imagination, creativity, and understanding of the method more. […] I consider that I acquired more analytical and critical thinking, […] in terms of knowledge I was able to use Matlab to solve problems with matrices and […] that led me to use new commands in the program. […] I was able to solve a problem that at the beginning was apparently impossible by means of numerical methods […] that can be applied to many real situations […], as we apply it to the movement of the NAO robot arm with equations and methods such as that of generalized NR […]. Now I can solve more complex problems and with graphical Matlab environments that I did not know within a completely new topic for me.

The second category of analysis in this phase was related to satisfaction and enjoyment of the activities. Let’s not forget that the pedagogical principles for an Optimal Flow Learning (OFL) to occur are a challenge, curiosity, concentration, and enjoyment. So, the last question in our discussion guide (again on written grammar-based memos) was.

**P1-OFL: Did using NAO humanoid robots as a context for teaching numerical methods help make your learning experience successful?**

I quite liked it. […] It is an excellent idea to complement the subject studied with application problems because learning can be faster and, on occasions, friendlier. […] I enjoyed them a lot, I quite like to see the class topics reflected in a real problem, and additionally, from the program carried out, one can be made that generalizes or solves other problems. […] It was a better way to learn, […] it was enriching to appreciate that the knowledge acquired is applicable. […] Great activity! It will be interesting to include NAO doing the movements in the next face-to-face classes. […] The subject is interesting to me and I feel that having activities of this type makes the knowledge more enriching. […] I would have liked to know how to transfer code from Matlab to NAO. I have always been curious to know how you incorporate a program into a robot, system, etc. […] It was an interesting challenge because it involved robotics concepts. I also liked the level of the challenge, because I had difficulties, but it could be solved. […] Very interesting to know how the Robot works through mathematics and programming. […] My perspective on robotics changed, in the sense that I didn’t see myself attempting a challenge of this type, […] I really liked this challenge, […] and this type of activities to see applications that we are learning, and […] to know what it will serve us for in the future when we are engineers. […] It was very cool to be able to achieve the iterations by hand and also in Matlab. […] I really enjoyed the activities, learning an interesting topic with a real representation of the application makes it very enjoyable and makes it easier to understand and remember the topic. […] I really like having problems that we could apply or that do have an application in real life.

In textual reconstruction, very commonly used in this type of qualitative methodologies, we can find the codes associated with the chain of active learning actions, a product of the first phase, and that is indicated in Fig. 3. In this way, we have validated intentional, and reflective planning based on the socio-critical paradigm of collectivized learning focused on the pedagogy of teaching intervention that, in this specific case, is in the context of NAO robots. This analysis obviates the conclusion that interaction with various technological elements facilitates the learning of abstract objects. In particular, the robotics context greatly favored the understanding of numerical analysis topics within a class of numerical methods with engineers due to the inclusion of NAO robots in the teaching intervention design. In the next section, we discuss the results of applying the described learning sequence to a new group of 32 engineering students, again in the numerical methods course.

### 4.2 Revisiting the didactic sequence: an active teaching intervention methodology that promotes natural environments of critical thinking

Once the theoretical saturation occurred in the chain of action verbs that the students executed during the sequence described in Fig. 3, the sequence of activities was tested in a new group of 32 engineering students who attended the class of numerical methods. The application occurred during October 2020 and was followed in an identical manner to what is described in Sect. 3. The intention of this new application is to show that the proposed design is robust enough to reproduce the condensed learning pattern in the action verb chain (engagement = > motivation = > curiosity = > challenge = > feedback = > self-regulation = > reflection = > self-evaluation = > taking decision-making = > socialization = > collaborative critical learning environments).

In this application, we take as the unit of analysis a written report that each student submitted and that contains (1)
the numerical solution of Eq. (1), (2) the programming code for the simulation of the NAO robot arm kinematics, and (iii) a conclusion of the work done in the sequence. This integrated report of the phases described in the class sessions in Sect. 3.4 allowed us to refine the verb sequence’s previous analysis, leading to its categorization. Namely, the Learning Flow (LF) category made up of verbs (engagement = > motivation = > curiosity = > challenge), the Metacognition (M) category made up of verbs (feedback = > self-regulation = > reflection = > self-evaluation = > taking decision-making) and the Natural Critical Learning Environment (NCLE) category made up of verbs (socialization = > collaborative critical learning environments). In this way, we can synthesize the original string notation in (LF = > M = > NCLE). In this way, it is an attempt to note that the didactic sequence presented is based on the intrinsic motivation that students develop in the first phase of the sequence. The notion of utilitarianism that the students develop when solving this problem is latent throughout the sequence. Motivation is born and prevails due to the practical nature of the results and the possible relationships they find there with the real world’s problems. Given that the sequence presented underlies active learning and competency-based education, the value of students’ behavior is determined, a priori, by the praxis of knowledge. However, in the second phase of the activity, in which the verbs in category M clearly awaken, this practical value dominates the “hands-on” activity approach is gradually combined with a “mind-on” activity approach. Students begin to find with themselves validity and significance criteria for the object of study they analyze. It is not the robot and its kinematics that they study, but rather that which is responsible for its functionality, namely, the numerical solution of Eq. (1). At this stage, students begin to extrapolate the activity’s content to new and possible applications of the numerical method that they explore, to the point that they begin to generalize about the use of that knowledge. Possibly, category M is where the greatest impact of this proposal is reflected since it is clear that the teacher’s feedback changes to personal feedback and, in turn, to collective feedback. At this point, both the teacher and the students are academic peers on the new questions that the students ask. This is a deeply constructivist act: the teacher launches reasoning in the form of a problem, and the student responds with a new problem based on the solution of the original problem. Cognitively, answering a question is just as complex as asking new questions. In fact, this last action encompasses the idea of knowledge generation. The student who goes through category M, at the same time, goes from praxis to knowledge. Only when that collective learning state is achieved is it feasible to say that the NCLE category is present. This category and its action verbs lead to a learning phase characterized by socialization, collaboration, and critical learning. In this context, demarcated by knowledge construction, socializing implies sharing common and accepted meanings within the learning environment. Only this condition makes collaborative work possible, in the most meaningful sense that it has: a communication channel between academic peers that solve and pose new problems in the real world. This way, we will have identified the last transition in this sequence: self-induced socio-constructivist learning by the study group. This is the meaning of the adjectives natural and critical on the NCLE category label.

5 Concluding remarks and future perspectives

Formative research in education constitutes an opportunity for teachers to carry out design research based on the reflection of their educational intervention. The university teacher must consider various dimensions when executing his/her practice, such as disciplinary knowledge, pedagogical knowledge of content, and curricular knowledge of content. Permanent reflection on the relationship between these dimensions will give rise to constructing the meaning of teaching. In the case presented, we can affirm that the engineering students’ learning analyzed responded favorably to the use of new technologies in class. The incorporation of NAO robots to numerical methods shows the intentional and reflective condition of the designed activities, leading the teaching to a well-founded practice. On the one hand, the intentional quality lies in the intervention plan (conditioned by the teacher’s continuous reflection) that results from the analysis of years of experience duly documented. On the other hand, students experience this quality by interpreting that they have the potential for practical, real, and current applications. This experience leads us to think about the possibility that students prefer to have an active role in the teaching–learning process and that technology can be a pedagogical intervention tool in training engineers of any specialty. New technological tools, such as the NAO humanoid robots, strengthened student motivation and learning. The robot was used to solve systems of nonlinear equations linked to the context of direct and inverse kinematics of the movement of the robot arm, solved by the generalized NR numerical method. With this didactic sequence, we validate one of the six parts (see Fig. 1) of the hypothetical learning trajectory (HLT) that, due to theoretical saturation, reveals that, in this design, active learning consisted of the chain of learning actions (engagement = > motivation = > curiosity = > challenge = > feedback = > self-regulation = > reflection = > self-assessment = > decision-making = > socialization = > collaborative critical learning environments).

This analysis of the teaching–learning process interposed by the didactic sequence and the robotics context allows us to understand and verify that, even though teacher planning
starts from an instructional basis of active learning ("hands-on"), the evolution of a teaching–learning process can indeed be characterized in other phases ("mind-on") in which other pedagogical principles predominate, not necessarily dictated by the teaching focus. The dynamism between teaching, knowledge, and learning often takes on other nuances. In the present case, we observe a socio-constructivist phase of knowledge characteristic of natural environments for critical learning. The methodological approach that is described here, separated into the reflection of the teaching practice that is poured into a design investigation to support a HTL, and into the active learning of students who are under an educational approach based on competencies, leads to the formation of such learning environments.

The context of robotics in this work is only a witness to the abstraction of the students: they create criteria within their learning process to decide if the task carried out is on the right track or not. However, the focus of this report falls on the analysis of the teaching–learning process. On the one hand, the teacher iteratively refines his lesson plans until they are intentional and reflective. From the beginning, the sequence encourages the development of critical and complex thinking of the student during the construction that he does on an object of knowledge: the numerical solution of non-linear equations systems. In this sense, the transitions (praxis = > constructivism = > socio-constructivism) and (individual learning = > collaborative learning = > collectivized learning), which emanate from this design, reveal a pedagogical innovation focused on the teacher and not on the context. Various studies on the teaching of numerical methods with a project-based approach assume success in educational practice (or in learning) due to concrete achievements within the context (e.g., the design of a robotic work cell [58]). Some allude to educational success in the management and application of transversal concepts (e.g., the combination of mathematics and robotics [59]). Furthermore, success metrics can fall into mere general observations made to students about using methodologies to transfer knowledge to real contexts (e.g., kinematic analysis, simulations, and commercial platforms for building robots [60]). Although these works have important contributions in engineers’ training, they relegate the scope of the concept of educational innovation to use (often exacerbated) to the inclusion of new objects and technologies in the learning environment. Based on this, this work constitutes a pedagogical innovation that registers the evolution of learning through the dynamism between the teacher’s reflection and the student’s reflection on an object of knowledge on which a problem of practical application is mounted. Describing a metric for that dynamic is well beyond this report.

One perspective of this work is to design the didactic sequence of each point of which the HLT is composed. The validation of the learning flow found here could not correspond to the rest of the trajectory. On the other hand, another learning flows, which produces an optimal learning experience in engineering students over a wide range of semesters, could be found. The validation of the didactic sequences by qualitative methods occurs in extensive time windows since, unlike quantitative methods, we categorize aspects of learning centered on students’ cognitive processes. This methodology is quite akin to competency-based curricular models. It promotes formative teaching (centered on the teacher and in the construction of knowledge) and formative evaluation (centered on the student and developing their metacognitive processes). Beyond the didactic sequence and the learning of numerical methods, this type of research addresses curricular innovation. In the case of engineers’ training, the designs must approximate it to solving real-world problems and for the student to interact with training partners incorporated into their learning process. All this brings the school reality (curricular) closer to the professional reality (work).

Acknowledgements The authors acknowledge the technical support of Writing Lab, Institute for the Future of Education and the GIEE—Optimization and Data Science, Tecnologico de Monterrey, Mexico, in the production of this work.

Author contributions All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by SRC-R, MG-M, and JMO-G. Methodology and Formal analysis and investigation was performed by JMO-G. The first draft of the manuscript was written by JMO-G and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript. Writing—review and editing, and Supervision was performed by JMO-G.

Appendix A: On the methodological approach of this work

It is correct to place this research within the qualitative paradigm of social research. Notably, in the social praxis approach [61] that we adopt, the meanings, perceptions, and actions of various agents involved in the social phenomenon converge. Some of the main elements that are linked in educational research are teachers and students (located in a social microscale), the School (located in a social mesoscale), and society (situated in a social macroscale); addition to the objects of knowledge (social epistemology). It is well known that the complexity of the analysis of these phenomena is due, in part, to the interaction and correlation that the different spatial and temporal scales that harbor the study problem have [62]. The processes of reflection in this research are its backbone and are responsible for assigning the appropriate space–time coordinates to the object of study. These processes are responsible for integrating the elements mentioned.
to give rise to a framework for local interpretive theory [63]. Some of these theoretical-methodological elements (which in the social sciences have this double face and use) in our work are the reflection of educational practice, research based on design, content analysis, and hypothetical learning trajectories, among others. All this leads us to be able to expose a concrete reality as praxis and as theory–practice dualities, integrating knowledge, values, and actions. This adoption of a global and dialectical vision of educational reality leads this qualitative research along the route of the sociocritical paradigm, in a sense proposed by Thomas S. Popkewitz in his social epistemology [64].

Under this perspective, we place our work in the category of socio-scientific problems within a much broader socio-constructivist dimension. In social constructivism [65], scientific research is understood as a process of inquiry influenced by the researcher and the context of the study. This philosophical perspective holds that individuals socially construct reality, and that this social construction leads to multiple meanings. Hence the importance of emphasizing the reflection processes of the teacher and the metacognitive processes of the students [66]. The correlation of these processes and the combination of the participants’ experiences originate in the sense of interpretation, which motivates the spiral design [67] described in Sect. 2 of our work. The acceptance of multiple realities leads social constructivists to insist that a set of initial questions asked in a study is likely to change or be modified as these multiple realities are discovered or reconstructed during the research process. For example, what does it mean to teach numerical methods? How do teach numerical methods…? Do NAO humanoid robots promote motivation…? The only accurate way to achieve this understanding is for the researcher to engage with the reality of the participants and interact with them in meaningful ways. This provides an opportunity for mutual influence and allows the researcher to see the world through the eyes of the participants, as pointed Lincoln and Guba in [68] when mentioned that

The researcher and the 'object' of the research interact to influence each other; the knower and the known are inseparable, p. 37.

This approach requires researchers to use data collection methods (in our case, grammatically coded student productions) that bring them closer to the participants using appropriate interpretation techniques (in our case, content analysis).

With this description of the nature of our work, we hope that readers can appreciate the scientific structure that emerges from the qualitative paradigm and the resignification that must be given to the adjective "scientific" within this approach to knowledge generation. Going deeper into this last aspect, we must say that qualitative research approaches collect data through observations, interviews, and document analysis and summarize the findings mainly through narrative or verbal means. In grounded theory research [69], content analysis emerges as instrumentation of symbolic interactionism and Glaser and Strauss’s operative principle of theoretical saturation [70]. The researcher uses the data collected through qualitative techniques to develop a theory based on the data. In essence, the researcher constructs an approach from the "background" or narrative data produced in the study. As theory emerges, the researcher returns to collect more data to confirm or challenge the initial findings (see Sect. 3.4). In some way, a grounded theory researcher [71] tries to use the results generated in a particular context and develop a theory that can be generalized to other contexts. Like research in different fields, in this (educational) research, we use inductive reasoning to build an abstraction or describe a picture of the phenomenon under study.

References

1. Sanneman, L., Fourie, C., Shah, J.A.: The state of industrial robotics: emerging technologies, challenges, and key research directions. Found. Trends Robot. 8(3), 225–306 (2021)
2. Universal Robots. Recovery from: https://www.universal-robots.com/ (2022)
3. International Federation of Robotics. Recovery from: https://ifr.org/service-robots (2022)
4. Cruz-Ramírez, S.R., Mac, E., Araai, T., Takubo, T., Ohara, K.: Vision-based hierarchical recognition for dismantling robot applied to interior renewal of buildings. Comput.-Aided Civ. Infrastruct. Eng. 26(5), 336–355 (2011)
5. López-Belmonte, J., Segura-Robles, A., Moreno-Guerrero, A.J., Parra-González, M.E.: Robotics in education: a scientific mapping of the literature in Web of Science. Electronics 10(3), 291 (2021)
6. Zhong, B., Xia, L.: A systematic review on exploring the potential of educational robotics in mathematics education. Int. J. Sci. Math. Educ. 18(1), 79–101 (2020)
7. Hadad, S., Shamir-Inbal, T., Blau, I., Leykin, E.: Professional development of code and robotics teachers through small private online course (SPOC): teacher centrality and pedagogical strategies for developing computational thinking of students. J. Educ. Comput. Res. 59(4), 763–791 (2021)
8. Jawaid, I., Javed, M.Y., Jaffery, M.H., Akram, A., Saﬁder, U., Hassan, S.: Robotic system education for young children by collaborative-project-based learning. Comput. Appl. Eng. Educ. 28(1), 178–192 (2020)
9. Alimisis, D.: Teacher training in educational robotics: the ROBOESL project paradigm. Technol. Knowl. Learn. 24(2), 279–290 (2019)
10. Almeida, F., Simoes, J.: The role of serious games, gamification and Industry 4.0 tools in the Education 4.0 paradigm. Contemp. Educ. Technol. 10(2), 120–136 (2019)
11. Fleming, R.: Adolescent reasoning in socio-scientific issues: I. Social cognition. J. Res. Sci. Teach. 23, 677–687 (1986)
12. Hancock, T.S., Friedrichsen, P.J., Kinslow, A.T., Sadler, T.D.: Selecting socio-scientific issues for teaching. Sci. Educ. 28(4), 639–667 (2019)
13. Chen, L., Xiao, S.: Perceptions, challenges and coping strategies of science teachers in teaching socioscientific issues: a systematic review. Educ. Res. Rev. 32, 100377 (2021)
59. Caldeira, A., Faria, A., Barbosa, R., Brás, H., Figueiredo, I., Gavina, A.: Engaging students in engineering courses with mathematics and robotics. In: 2020 IEEE Global Engineering Education Conference EDUCON, pp. 1392–1399 (2020)

60. González-García, S., Rodríguez-Arce, J., Loreto-Gómez, G., Montaño-Serrano, V.M.: Teaching forward kinematics in a robotics course using simulations: transfer to a real-world context using LEGO Mindstorms™. Int. J. Interact. Des. Manuf. 14(3), 773–787 (2020)

61. Lather, P.: Research as praxis. Harv. Educ. Rev. 56(3), 257–278 (1986)

62. Neumann, M.: Time scales of socio-cultural dynamics. Cybern. Hum. Knowing 21(1–2), 66–79 (2014)

63. Ford, C.L., Yore, L.D.: Toward convergence of critical thinking, metacognition, and reflection: illustrations from natural and social sciences, teacher education, and classroom practice. In: Zohar, A., Dori, Y.I. (eds.) Metacognition in Science Education, pp. 251–271. Springer, Dordrecht (2012)

64. Popkewitz, T.: Social epistemology, the reason of “reason” and the curriculum studies. Educ. Policy Anal. Arch. 22, 22–22 (2014)

65. Adams, P.: Exploring social constructivism: theories and practicalities. Education 34(3), 243–257 (2006)

66. Carlson, H.L.: From practice to theory: a social constructivist approach to teacher education. Teach. Teach. 5(2), 203–218 (1999)

67. Kemmis, S., Wilkinson, M.: Participatory action research and the study of practice. In: Action Research in Practice, pp. 47–62. Routledge (2002)

68. Lincoln, Y.S., Guba, E.G.: Naturalistic inquiry. Newbury Park, CA: Sage (1985)

69. McCallin, A.M.: Designing a grounded theory study: some practicalities. Nurs. Crit. Care 8(5), 203–208 (2003)

70. Glaser, B.G., Strauss, A.L.: The Discovery of Grounded Theory. IL Aldine, Chicago (1967)

71. Suddaby, R.: From the editors: what grounded theory is not. Acad. Manag. J. 49(4), 633–642 (2006)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.