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Citation for published version:
Jiang, XL, Yang, T, Zou, Q & Gu, HB 2018, 'Flow Separation and Vortex Dynamics in Waves Propagating over A Submerged Quartercircular Breakwater', China Ocean Engineering, vol. 32, no. 5, pp. 514-523.
https://doi.org/10.1007/s13344-018-0054-5

Digital Object Identifier (DOI):
10.1007/s13344-018-0054-5

Link:
Link to publication record in Heriot-Watt Research Portal

Document Version:
Peer reviewed version

Published In:
China Ocean Engineering

Publisher Rights Statement:
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Flow separation and vortex dynamics in waves propagating over a submerged quartercircular breakwater

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Abstract

The interactions of cnoidal waves with a submerged quartercircular breakwater are investigated by a Reynolds-Averaged Navier-Stokes (RANS) flow solver with a Volume of Fluid (VOF) surface capturing scheme (RANS-VOF) model. The vertical variation of the instantaneous velocity indicates that flow separation occurs at the boundary layer near the breakwater. The temporal evolution of the velocity and vorticity fields demonstrates vortex generation and shedding around the submerged quartercircular breakwater due to the flow separation. An empirical relationship between the vortex intensity and a few hydrodynamic parameters is proposed based on parametric analysis. In addition, the instantaneous and time-averaged vorticity fields reveal a pair of vortices of opposite signs at the breakwater which are expected to have significant effect on sediment entrainment, suspension, and transportation, therefore, scour on the leeside of the breakwater.

Key words: submerged quartercircular breakwater; cnoidal wave; flow separation; vortex dynamics; scour.
1. Introduction

Offshore submerged breakwaters have recently become popular coastal defence to protect maritime infrastructure and retain sediments in the sheltered harbor through premature wave breaking (Zuo et al., 2015; Wang et al., 2016; Zheng et al., 2016; Ju et al., 2017). Better understanding of the flow field around submerged offshore breakwaters, especially small-scale hydrodynamic phenomena, is critical to improve the design of breakwaters.

Generation and shedding of vortices occur during the interaction of waves with submerged structures due to flow separations at the structure. The vortices may interact with sea bed at the submerged structures and therefore cause erosion and scour at the foundation and undermine the structure.

Optic and acoustic measurement techniques have been applied to investigate the vortex dynamics induced by wave-structure interaction in the laboratory recently. Ting and Kim (1994) used Laser Doppler Velocimetry (LDV) to measure regular waves travelling over a submerged rectangular obstacle. Lin et al. (2006) applied Particle Image Velocimetry (PIV) and Laser Induced Fluorescence (LIF) to study a solitary wave interacting with a bottom-mounted rectangular dike. Poupardin et al. (2012) used PIV to investigate the evolution of vortices generated by waves before and after a submerged horizontal plate.

Recent development of novel computational fluid dynamics (CFD) technique for free surface flow has been applied to study hydrodynamics around submerged structures. Examples include the Reynolds Averaged Navier-Stokes (RANS) solver by Chang et al. (2001), Hsu et al. (2004), Jiang et al. (2010), Zarruk et al. (2015), the boundary element method (BEM) by Ning et al. (2014), Wang et al. (2018), and the meshless Lagrangian vortex method by Lin and Huang (2009), as well as Chang et al. (2015).

Various submerged breakwaters have been employed to protect the coast, such as vertical, rubble mound, and circular-shaped breakwaters. Previous studies paid more attention to the flow field around the former two structures, such as vortex generation over submerged trapezoidal and rectangular dikes by Huang & Dong (1999), and vertical breakwater by Hajivalie et al. (2015), and vortex shedding from a submerged rectangular obstacle subject to a solitary wave by Lin & Huang (2010) and Zhang et al. (2010), and vortex evolution in Bragg scattering by Hsu et al. (2014). In the past decades, circular-shaped breakwaters, including semi- and quarter- circular breakwaters, have attracted considerable attentions for their aesthetically pleasing view and economic feasibility, especially in deep water. A comprehensive review of semicircular breakwaters can be found in Dhinakaran & Sundar (2012). Xie et al. (2006) developed the concept of quartercircular breakwaters in China based on semicircular breakwaters. Quartercircular breakwaters are more...
economical than semicircular breakwaters because they consume much less concrete and rubble mound. However, there is a lack of studies on the hydrodynamic processes of wave propagation over a quartercircular breakwater. Previous studies related to the quartercircular breakwater have been focused on wave reflection and transmission (Jiang et al., 2008; Shi et al. 2011 and Hafeeda et al. 2014), wave dynamic pressures (Liu et al., 2006; Qie et al., 2013), wave run-up and run-down (Binumol et al., 2015). The generation and shedding of vortices at a submerged coastal structure have significant impact on the hydrodynamics. Circular-shaped breakwaters reflect less and transmit more wave energy than vertical and rubble mound breakwaters, therefore, may produce greater local scouring (Young & Testik, 2009) and hydrodynamic loading behind the structure (Jiang et al., 2017). In contrary to the recommendation of existing design criteria for the semi- and quarter- circular breakwaters, Jiang et al’s (2017) experimental and numerical studies suggest that wave trough instead of wave crest plays a dominant role in the stability of circular-front breakwaters against seaward sliding. Under wave trough, although a stronger trailing vortex is formed on the leeside of the quartercircular breakwater than the semicircular breakwater, it is away from the rear wall and thus leads to a small impact on the dynamic pressures exerted on the structure.

The present study focuses on the characteristics of flow fields around a submerged quartercircular breakwater with special attentions to the vortex dynamics in the leeside of the breakwater. In the following sections, the setup of numerical flume is first described in section 2. Next, the numerical results are presented in section 3, including the wave generation and validation, the temporal evolution of velocity and vortex fields within a wave period, the effects of hydrodynamic parameters on vortex intensity, and the time-averaged flow field in the vicinity of the submerged quartercircular breakwater. Conclusions are summarized in section 4.

2. Numerical model setup

In this study, a numerical wave flume is developed to investigate the dynamics of vortex around a submerged quartercircular breakwater subject to cnoidal waves. Fig. 1 illustrates the configuration of the numerical flume following the PIV experiment by Chang et al. (2005) for cnoidal waves propagating over a submerged rectangular obstacle. The flume is 36 m long, 0.6 m wide and 0.9 m deep. x coordinate is positive in the wave propagation direction with x=0 at the back wall of the structure and z coordinate is positive upward with z=0 at the still water level. An impermeable quartercircular breakwater with a size of 0.12 m in radius (D), 0.175 m in width (B) is placed in the middle of the flume. A porous beach with a 1/10 seaward slope is placed 6 m upwave towards the wavemaker from the outflow boundary following Chang et al’s (2005) experiment.
The motion of incompressible fluid in the numerical flume is described by the Reynolds-Averaged-Navier-Stokes equations (RANS):

$$\frac{\partial \overline{u}_i}{\partial x_j} = 0$$  \hspace{1cm} (1)

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + g_i + \frac{1}{\rho} \frac{\partial}{\partial x_j} \left( \mu \frac{\partial \overline{u}_i}{\partial x_j} - \rho \overline{u}_i \overline{u}_j \right)$$  \hspace{1cm} (2)

where \(i, j \ (=1, 2)\) refer to the horizontal (\(x\)) and vertical (\(z\)) direction, respectively, \(t\) is time, \(\overline{u}_i\) the mean velocity in the \(i^{th}\) direction, \(\rho\) the fluid density, \(p\) the fluid pressure, \(g_i\) the gravitational acceleration in the \(i^{th}\) direction, \(\mu\) the dynamic viscosity, and \(-\rho \overline{u}_i \overline{u}_j\) the Reynolds stress computed by the nonlinear \(k-\varepsilon\) turbulent model (Lin and Liu, 1998).

In this flume, a porous beach is used to dissipate the incoming wave energy before it reaches the outflow boundary. The fluid domain within the porous media is modeled by the spatially averaged Navier-Stokes equations by including additional frictional forces in RANS (Lin and Karunaratna, 2007),

$$\frac{\partial \overline{u}_i}{\partial x_j} = 0$$  \hspace{1cm} (3)

$$\frac{1}{n} \frac{\partial \overline{u}_i}{\partial t} + \frac{\overline{u}_j}{n} \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + g_i + \frac{1}{\rho} \frac{\partial}{\partial x_j} \left( \mu \frac{\partial \overline{u}_i}{\partial x_j} - \frac{1}{n} \rho \overline{u}_i \overline{u}_j \right) - \overline{f}_i$$  \hspace{1cm} (4)

where the overbar denotes the spatially averaged quantities in porous media, \(n\) the porosity, \(\overline{u}_i \overline{u}_j\) the spatially fluctuated stresses, \(f_i\) the resistance force caused by the presence of porous media in flow. The resistance force is composed of inertia force \(f_{bi}\) and drag force \(f_{Di}\). The spatially averaged inertia force is calculated by \(\overline{f}_{bi} = \frac{(1-n)\gamma_p}{n^2} \frac{\partial \overline{u}_i}{\partial t}\), in which \(\gamma_p\) is the virtual mass coefficient taking the suggested value of 0.34 by Van Gent (1995). The spatially averaged
drag force is estimated by \( f_{di} = \frac{3}{4} C_D \frac{(1-n)}{n^3} d_{50} \), in which \( \vec{u}_c = \sqrt{\nu u_i} \) is the characteristic velocity, \( C_D \) the drag force coefficient, and \( d_{50} \) the median particle size of porous material. The drag force coefficient can be computed using

\[
C_D = c_1 \left( \frac{24.0}{(R_e)_p} \right) + c_2 \left( \frac{3.0}{(R_e)_p} + 0.34 \right) \left( 1.0 + \frac{7.5}{(KC)_p} \right),
\]

where \( c_1 = 7.0 \) and \( c_2 = 2.0 \) are suggested by Lin and Karunarathna (2007) as the modification coefficients for turbulent flow, \((R_e)_p\) and \((KC)_p\) are the local Reynolds and Keulegan-Carpenter numbers in porous media, respectively, \( \vec{u} \) the local spatially averaged fluid velocity, \( u_{\text{max}} \) the maximum water particle velocity, \( T \) the wave period, and \( \nu \) the kinematic viscosity of fluid.

The diameter \( d \) and porosity \( n \) of the porous material for the beach are determined by performing a series of preliminary tests. The test parameters are listed in table 2. Table 1 lists the reflection coefficients obtained from the predicted surface elevation using the three-probe method by Mansard and Funke (1980). It can be seen that over 94% incident wave energy has been eliminated before reaching the outflow boundary, which is consistent with the experiment of Chang et al. (2005). As a result, \( d = 0.1 \)mm and \( n = 0.4 \) are used in this study.

Table 1 Reflection coefficients for the preliminary tests.

| No. | Reflection coefficient, \( K_r = H_r / H_i \) (%) |
|-----|----------------------------------|
| Test1 | 0.54% |
| Test2a | 0.60% |
| Test2b | 5.14% |
| Test2c | 4.00% |
| Test2d | 3.15% |
| Test3 | 1.84% |

Flow field outside the porous media is calculated by solving the RANS equations (1) - (2) whereas that in the porous media is calculated by the spatial-averaged Navier-Stokes equations (3) - (4) and by applying the continuity of velocity and pressure as the boundary condition across the interface of porous media and outside flow. The analytical solution of third-order cnoidal wave theory by Chappelear (1962) is used to specify the velocity components and the free surface displacement at the inlet of the flume on the left-hand side. As suggested by Le Méhauté (1976), cnoidal wave theory is most adequate to describe a shallow water wave. At the outlet of the flume, a Sommerfeld radiation condition is imposed as the boundary condition. At the air-water interface,
zero atmospheric pressure and zero shear stress are applied. The turbulent kinetic energy \( (k) \) and the dissipation rate \( (\varepsilon) \) have a zero normal gradient at the free surface, i.e., \( \partial k / \partial n = 0 \) and \( \partial \varepsilon / \partial n = 0 \), assuming there is no turbulent exchange between air and water. On the fluid-solid interfaces, no-slip velocity and zero normal pressure gradient \( (\partial p / \partial n = 0) \) are imposed.

A non-uniform rectangular grid system is used in this study with cell size varying from locally refined 0.004 m near the obstacle to coarser 0.01 m away from the obstacle in both \( x \)- and \( z \)-directions. Finite difference method is used to discretize equations (1)-(4) on the non-uniform mesh system. The movement of the free surface is captured using VOF scheme by tracking the change of the volume fraction of fluid in each cell (Hirt & Nichols, 1981). The solid boundaries of the internal obstacle are represented by the partial cell technique (Lin, 2008; Peng et al., 2018).

The RANS-VOF model has been applied to investigate wave interactions with a composite porous structure by Liu et al. (1999), wave overtopping rubble mound breakwaters by Losada et al. (2008), and a seaward sloping dike by Peng and Zou (2011), wave transformation over a low-crested structure by Zou et al. (2013) and wave loading on submerged circular breakwaters by Jiang et al. (2015, 2017).

Table 2 lists the key wave and hydrodynamic parameters employed in the simulation, where \( H \) is the wave height, \( d \) the water depth, \( T \) the wave period, \( L \) the wavelength, \( S \) the particle trajectory, \( u_m \) the maximum particle velocity, \( U \) the local characteristic velocity, \( \text{Re} \) the local Reynolds number, \( KC \) the Keulegan-Carpenter number, \( U_r \) the Ursell number, \( D \) the radius (12 cm) of the front wall, and \( \nu \) the kinematic viscosity of fluid. Test 1, 2a, and 3 have the same period but different wave heights, while test 2a, 2b, 2e, and 2d have the same wave height but different periods. During the entire simulation, the water depth was kept at \( d = 0.24 \) m to represent the submerged condition. The duration of each model run is 20 wave periods.

**Table 2 Key wave and hydrodynamic parameters of test cases**

| No. | \( H \) (cm) | \( d \) (cm) | \( T \) (s) | \( L \) (m) | \( S = \frac{H}{\sinh (kh)} \) | \( u_m = \frac{H}{d} \sqrt{g(d + H)} \) | \( U = \frac{u_m d}{d - D} \) | \( \text{Re} = \frac{U d}{\nu} \) | \( KC = \frac{U T}{d} \) | \( U_r = \frac{H E^2}{d^3} \) |
|-----|--------------|--------------|-------------|-------------|-------------------------------|-------------------------------|-----------------|-----------------|-----------------|-----------------|
| Test1 | 1.2 | 24 | 2.0 | 2.93 | 2.3 | 7.86 | 15.72 | 3.77E4 | 1.31 | 7.45 |
| Test2a | 3.6 | 24 | 2.0 | 2.97 | 6.9 | 24.67 | 49.34 | 1.18E5 | 4.11 | 22.97 |
| Test2b | 3.6 | 24 | 1.5 | 2.11 | 4.4 | 24.67 | 49.34 | 1.18E5 | 3.08 | 11.59 |
| Test2c | 3.6 | 24 | 3.0 | 4.63 | 10.8 | 24.67 | 49.34 | 1.18E5 | 6.17 | 55.83 |
| Test2d | 3.6 | 24 | 4.0 | 6.28 | 14.2 | 24.67 | 49.34 | 1.18E5 | 8.22 | 102.70 |
| Test3 | 6.0 | 24 | 2.0 | 3.04 | 11.4 | 42.87 | 85.74 | 2.06E5 | 7.15 | 40.11 |
3. Model results

3.1 Model validation

To validate the numerical wave flume, Chang et al.'s (2005) experiment for cnoidal waves propagating over a submerged rectangular obstacle is reproduced here. Fig. 2 shows good agreement between the predicted and measured surface displacements.

Fig. 2. Comparisons of the numerical and experimental results of surface profiles for cnoidal waves propagating over a submerged rectangular obstacle at (a) \(x=-8.2\) m, (b) \(x=0.15\) m and (c) \(x=1.5\) m. Experimental measurements, ---; numerical predictions, --.

Fig. 3 shows the comparisons of the velocity fields between the present (Right) and previous numerical results using COBRAS (Middle) and the experimental measurements (Left) by Chang et al. (2005) at the leeside of a submerged rectangular obstacle at wave phases A and C indicated in Fig. 4. It is evident from Fig. 3 that the present simulation gives a reasonable description of the observed flow field and vortex dynamics around the submerged obstacle.

Wave phase A
3.2 Velocity field

To elucidate the process of flow separation at the structure, the velocity fields at four representative wave phases over a wave cycle indicated in Fig.4 are extracted from the numerical model results (see Fig. 5 and Fig. 6).

**Fig. 3.** Comparisons of the present (Right) and previous numerical (Middle) and experimental results (Left) of velocity fields at the leeside of a submerged rectangular obstacle for test 2a at wave phases A (Upper) and C (Lower) indicated in Fig.4.

**Fig. 4.** Four representative wave phases A, B, C and D of the free surface profile at the leeside wall (x=0) of the quartercircular breakwater for test 2a.
**Fig. 5** shows the instantaneous velocity field when the wave trough approaches the leeside wall ($x=0$) of the quartercircular breakwater at wave phase A for test 2a. The flow is moving from downwave (same as the wave direction) towards upwave (opposite to the wave direction). As shown in **Fig. 5(a)**, a counterclockwise vortex is created above the obstacle due to the flow separation at the crown of the breakwater. **Fig. 5(b)** and **Fig. 5(c)** illustrate the evolution of the vertical profile of horizontal and vertical velocity components in the vicinity of the leeside wall, respectively. The horizontal velocity at $x=-6.3$ cm in **Fig. 5(b)** changes from negative at $z=-10$ cm to positive at $z=-14$ cm, indicating the flow separation and hence the existence of a vortex. A spatial evolution of the vertical velocity is shown in **Fig. 5(c)**.
Fig. 5. (a) Velocity field and evolution of vertical profile of horizontal (b) and vertical (c) velocity components at \( x = -8.2 \) cm, -6.3 cm and 0.0 cm upwave from the leeside wall of the breakwater when the wave trough approaches the leeside wall \( (x=0) \) at wave phase A for test 2a.

Fig. 6 presents the instantaneous velocity field when the wave crest approaches the leeside wall \( (x=0) \) at wave phase C for test 2a. The flow separation occurs due to the jet flow above the crown of the structure with sharp surface curvature and creates a clockwise vortex just behind the back wall. The center of the vortex locates at about half of the wave particle trajectory (\( S = 6.9 \) cm for test 2a in table 2) from the leeside wall of the structure. The center position of the vortex is determined with the \( \lambda_2 \) factor criteria developed by Jeong and Hussain (1995). As shown in Fig. 6(b), the horizontal velocity above the structure increases with increasing \( x \) and changes from positive above the vortex center to negative below it at \( x = 1.0 \) cm. Fig. 6(c) shows the value of the vertical velocity below the crown of the structure is negative at \( x = 3.6 \) cm. These results indicate the formation of a clockwise vortex.
3.3 Vorticity field

Fig. 7 demonstrates the generation and temporal evolution of vortices over a wave cycle in the vicinity of the leeside wall. At wave phase A, a counterclockwise vortex is formed at the crown due to the flow separation. The vortex is then transported upward and upwave with weakening intensity due to energy dissipation at wave phase B. Meanwhile, a small clockwise vortex is generated at the lee side of the crown due to the jet flow induced by the downwave flow in the same direction as the wave. When the wave crest arrives at the crown of the structure (wave phase C), the clockwise vortex is advected downwave, spread out and strengthened by the increasing downwave flow velocity. At wave phase D, the flow velocity changes direction from downwave to upwave. The
clockwise vortex is shedding from the leeside wall and spreading downwave and advected downward. The velocity and vorticity fields have also been simulated for other tests, but they are not shown here due to their similarity to test 2a.

![Figure 7](image1.png)

**Fig. 7.** Vorticity fields at wave phases A, B, C and D for test 2a. Positive vorticity (counterclockwise) in gray color with solid line; negative vorticity (clockwise) in black color with dashed line; the range of the vorticity is between 40 s⁻¹ and 40 s⁻¹.

### 3.4 Vortex intensity

In this section, the relationships between the vortex intensity and a few nondimensional hydrodynamic and wave parameters (local Reynolds number, Keulegan-Carpenter number and Ursell number) are explored based on the numerical results. We use a nondimensional parameter \( \left( \frac{\Omega U T^2}{d} \right) \) to represent the intensity of the vortex in order to include the effect of wave period explicitly, where \( \Omega \) is the maximum vorticity magnitude, \( U \) the local characteristic velocity, \( T \) the wave period, and \( d \) the water depth. Table 2 lists the values of these parameters. We examined the instantaneous vorticity field within the area of \(-12 \text{ cm} < x < 8 \text{ cm}, -24 \text{ cm} < z < -6 \text{ cm}\) at each time step to obtain the maximum magnitude of counterclockwise (positive) and clockwise (negative)
vorticities for a certain time interval. In comparison, the maximum magnitude of vorticity is used. In order to minimize the effect of the fluctuation of predicted vorticities with time, the maximum vorticity magnitude is derived by averaging over twenty wave periods.

As shown in Fig. 8, the nondimensional vortex intensity increases linearly with the local Reynolds number for both clockwise and counterclockwise vortices. The PIV results of Chang et al. (2005) for a submerged rectangular obstacle are also included in Fig. 8. It can be seen that the quartercircular breakwater has stronger counterclockwise vortex but weaker clockwise vortex than the rectangular obstacle for the same Reynolds numbers owing to the different shape of these structures. This behavior is worth further explorations in the future.

![Fig. 8. Nondimensional vortex intensity versus the local Reynolds number around the quartercircular](image)

Fig. 8. Nondimensional vortex intensity versus the local Reynolds number around the quartercircular (solid circle) and rectangular (solid square) breakwaters with a linear regression for test1, test2a and test3.

![Fig. 9](image)

Fig. 9 shows that the nondimensional vortex intensity increases linearly with the $KC$ number for both clockwise and counterclockwise vortices. A similar relationship between the nondimensional vortex intensity and the $U_c$ number is evident from Fig. 10 as well.

![Fig. 9](image)

Fig. 9. Nondimensional vortex intensity versus the Keulegan-Carpenter number around the quartercircular breakwater with a linear regression for all tests.
Fig. 10. Nondimensional vortex intensity versus the Ursell number around the quartercircular breakwater with a linear regression for all tests.

From the numerical results, we obtain a linear relationship between the nondimensional vortex intensity and the hydrodynamic parameters,

\[
\frac{\Omega UT^2}{d} = 3.56KC + 21.60Ur - 73.15 \quad \text{for clockwise vortex} \quad (5)
\]

\[
\frac{\Omega UT^2}{d} = 230.28KC + 7.60Ur - 351.54 \quad \text{for counterclockwise vortex} \quad (6)
\]

The predictions of the above formula are compared with the numerical results in Fig. 11. The coefficient of determination, \( R^2 \), is 0.9917 for the clockwise vortex and 0.9173 for the counterclockwise vortex.

Fig. 11. Comparison of the predicted vortex intensities by the present empirical formulae (5)-(6) and the numerical model results of the vortex intensity. The diagonal line denotes the former is equal to the latter.
3.5 Time-averaged flow field

The numerical results by RANS-VOF flow model and a partial cell morphological model by Peng et al (2018) indicate that vortex and jet formation play an important role in generating scour in front of a seawall. In order to investigate the accumulated effect of the wave-induced vortices around the quartercircular breakwater on the leeside sediment movement, the time-averaged velocity field was obtained by averaging the instantaneous velocity fields over multiple wave periods (Yeganeh-Bakhtiary, et al., 2010)

$$\overline{V}_i = \frac{1}{N} \sum_{j=1}^{N} \left[ \sum_{t=0}^{T} v_i(t + jT) \right]$$

(7)

where $v_i(t + jT)$ is the instantaneous velocity at wave phase $t$ during $j^{th}$ wave period at $i^{th}$ cell. In this study, the total number of wave periods ($N$) for the calculation of the time-averaged velocity is 20. The time-averaged velocity fields are demonstrated in Fig. 12 together with the corresponding time-averaged vorticity fields. Also, the value of the leeside circulation for each time-averaged velocity field is presented below each subfigure of Fig. 12, which is computed by integrating the vorticity within the range of 0 cm $< x < 8$ cm, -24 cm $< z <-12$ cm.

It is interesting to observe that the time-averaged flow is not symmetric. A pair of vortices of opposite signs exists at the upwave (counterclockwise vortex) and downwave (clockwise vortex) zones of the structure crown. The evolution of the time-averaged vorticity fields implies that the vortex core may move closer towards the bed and causes a greater leeside circulation at larger $KC$ and $U_c$ numbers. If the near-bottom shear stress is strong enough to stir up the sediments from the bottom near the leeside wall, the scoured sediments can be transported upward by the time-averaged clockwise circulation and then onshore and offshore, respectively, by the counterclockwise and clockwise vortex pairs near the crown.

leeside circulation $=-89.3001 \ m^2/s$

leeside circulation $= -402.4211 \ m^2/s$
Fig. 12. The time-averaged velocity and vorticity fields around the leeside wall of the quartercircular breakwater.

4. Conclusion

A Reynolds-Averaged Navier-Stokes (RANS) flow solver with a Volume of Fluid (VOF) surface capturing scheme (RANS-VOF) model is used to investigate the vortex dynamics in the interaction of cnoidal waves with a submerged quartercircular breakwater.

The vertical variation of velocity indicates that the change in flow direction due to negative pressure gradient and viscous effect in the immediate vicinity of the boundary layer causes the flow separation, and hence the vortex generation around the leeside wall of the quartercircular breakwater.

The temporal evolution of the vorticity fields reveals that the generation and shedding of the vortices at the structure depend on the variation of the wave-induced oscillatory flow and the shape of the breakwater. The direction of rotation of the vortices alters between counterclockwise and clockwise when the flow changes direction. The center of the vortex is found to be confined within a wave orbital particle trajectory from the crest of the structure.

The nondimensional vortex intensity is almost linearly proportional to local Reynolds number, Keulegan-Carpenter number and Ursell number. An empirical relationship between the nondimensional vortex intensity and these hydrodynamic parameters is proposed and compares well with the numerical results.

The time-averaged velocity and vorticity fields demonstrate that the vortex pairs before and after the crest of the structure persist over many wave periods, which are expected to play an important role in sediment transport and scour around the quartercircular breakwater.
Acknowledgements

The authors are grateful to Professor Pengzhi Lin from Sichuan University who generously supplied the original RANS-VOF code which was extended for the present work.

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