Masses and mixing angles and going beyond the
Standard Model

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Abstract

The idea, following Michel and O’Raifeartaigh, of assigning meaning to the (gauge) group and not only the Lie algebra for a Yang Mills theory is reviewed. Hints from the group and the fermion spectrum of the Standard Model are used to suggest the putting forward of our AGUT-model, which gives a very good fit to the orders of magnitudes of the quark and lepton masses and the mixing angles including the CP-breaking phase. But for neutrino oscillations modifications of the model are needed. Baryogenesis is not in conflict with the model.

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1 Introduction

For the purpose of finding out what comes beyond the Standard Model, it is unfortunate that the latter works so exceedingly well that it actually describes satisfactorily almost all we know and can make experiments about: just extending it with even classical Einsteinian gravity is sufficient to provide well working laws of nature for all practical purposes today. So the true hints for going beyond the Standard Model can, except for purely theoretical aesthetic arguments, only come from the structure and parameters of the Standard Model itself, or from the extremely little information we have about the physics beyond the 1 TeV range where, so far, the Standard Model could potentially work perfectly. The extremely little knowledge we have about very short distance physics comes partly from baryon number being presumably not conserved: 1) If baryon number asymmetry should be cosmologically produced at the weak scale and not, for instance, be due to a $B - L$ asymmetry from earlier time, we would need some new physics; even if it was an earlier $B - L$ asymmetry that caused the observed baryon number, $B - L$ would, at some scale at least, have to be produced unless it is truly primordial. 2) The lack of proton decay gives information that, for example, a naive SU(5) GUT is not correct. In addition we really see direct evidence for non-Standard Model physics in the growing experimental support for the existence of neutrino oscillations.

But apart from these tiny bits of information, we mainly have the structure and coupling constants and masses in the Standard Model from which to try to guess the model beyond it! We (Svend Erik Rugh et al.) estimated that the amount of information in these parameters, as measured so far, and in the Standard Model structure was just around a couple of hundred bits. It could all be written on one line. What is now the inspiring information on this line?

In section 2 we stress how part of the information about the quantum numbers of the quarks and the leptons (really their Weyl components) can be packed into saying what group, rather than only what Lie algebra, is to be represented.

In section 3 we look at another hint: the large ratios of the quark and lepton masses in the various generations and the small mixing angles. With good will, these hints could be taken to point in the direction of the AGUT gauge group which is our favourite model. AGUT stands for anti-grand unification theory and is indeed, in a way to be explained, based on assumptions opposite to the ones leading to the usual SU(5) GUT.

In section 4 we put forward the model, especially the AGUT gauge group which we characterize as the largest group not unifying the fermion irreducible representations of the Standard Model. This AGUT gauge group is the non-simple direct product group $SMG^3 \times U(1)_{f}$, where $SMG \equiv SU(3) \times SU(2) \times U(1)$.

The Higgs fields responsible for breaking the AGUT gauge group $SMG^3 \times U(1)_{f}$ down to the diagonal $SMG$ subgroup, identified as the Standard Model gauge group, are considered in section 5.
The structure of the resulting fermion mass matrices are presented in section 6, together with details of a fit to the charged fermion spectrum. In sections 7 and 8, we briefly discuss the problems of baryogenesis and neutrino oscillations respectively. Finally we mention the relation to the Multiple Point Principle (MPP)—see the contribution by Larisa Laperashvili to these proceedings—in section 9 and the conclusion is in section 10.

2 Gauge Group

Since the Standard Model is a Yang Mills theory, the gauge Lie algebra is an important structural element in the specification of the model. This structure presumably carries a significant amount of information, since it is not so obvious why the effective theory explaining present day experiments should just have this gauge algebra: counting all the many cross products of various Lie algebras, it is not immediately clear why it should be the algebra corresponding to $U(1) \times SU(2) \times SU(3)$ that is selected by God.

Referring to the works by Michel and O’Raifeartaigh [1], we have for a long time suggested [2] that rather than the gauge Lie algebra—which of course specifies the couplings of the Yang Mills fields to each other—we should consider the gauge group. A priori the gauge group is only relevant in so far as its Lie algebra determines the couplings of the Yang Mills particles or fields, the coupling constants being proportional to the structure constants of the Lie algebra. If there were a truly ontologically existing lattice it would be a different matter, because in that case there would be place for specifying a group and not only the corresponding Lie algebra. There is, however, also a phenomenologically accessible way of assigning a meaning to the gauge group and not only the algebra: Different gauge groups with the same Lie algebra allow a different set of matter field representations. Certain groups are thus not allowed if one requires that the experimentally found matter shall be represented under the group.

Now the connection between Lie algebra and Lie group is so that there are several groups corresponding to one algebra in general, but always only one algebra to each group. Considering only connected groups, as is reasonable here, there corresponds to each Lie algebra a unique group, the covering group characterized by being simply connected—i.e. that any closed curve on it can be continuously contracted to a point—from which all the other connected groups with the given Lie algebra can be obtained, by dividing out of the covering group the various discrete invariant subgroups of it. Now it is mathematically so that all representations of the Lie algebra are also representations of the covering group, but for the other groups with the given Lie algebra it is only some of the algebra representations that are also representations of the group. You can therefore never exclude that the covering group can be used, whatever the matter field representations may be, while many of the other groups can easily be
excluded whenever some matter field representation is known. If one has found a large number of matter fields for which the elements \( \{(2\pi, -I, \exp(i2\pi/3)I)^n | n \in \mathbb{Z} \} \) are represented by the identity, as is the case in the Standard Model, then it might be almost surprising if any group other than the covering group has all these representations. For the Standard Model it can in fact rather easily be computed that there is, remarkably enough, a group other than the covering one which contains all the representations found in nature so far! In the light of the relatively “many” matter field representations, we could then claim that there is phenomenological evidence that this group is the GROUP selected by nature as the Standard Model Group, which we write in shorthand as SMG. Indeed the group that in this way deserves to be called the Standard Model Group is

\[
SMG = S(U(2) \times U(3)) = (\mathbb{R} \times SU(2) \times SU(3))/\{(2\pi, -I, \exp(i2\pi/3)I)^n | n \in \mathbb{Z} \}.
\]

It may be described as the subset of the cross product of \( U(2) \) and \( U(3) \) for which the product of the determinant for the \( U(2) \) group element, conceived of as a matrix, and that of \( U(3) \) is unity.

What this putting forward of a special group really means is that a regularity in the system of observed matter field representations can be expressed by the specification of the group. In the case of the Standard Model Group \( SMG = S(U(2) \times U(3)) = (\mathbb{R} \times SU(2) \times SU(3))/\{(2\pi, -I, \exp(i2\pi/3)I)^n | n \in \mathbb{Z} \} \), it is actually the regularity required by the well-known rules for electric charge quantization that can be expressed as the requirement of the representations in nature being representations not only of the Lie algebra but really of this group. The electric charge quantization rule is as follows:

For the colourless particles we have the Millikan charge quantization of all charges being integer when measured in units of the elementary charge unit, but for coloured particles the charges deviate from being integer by \(-1/3\) elementary charge for quarks and by \(+1/3\) for antiquarks. This rule can be expressed by introducing the concept of triality \( t \), which characterizes the representation of the centre \( \{\exp(ni2\pi/3)I^3 | n = 0, 1, 2 \} \subset SU(3) \) (suitably modified according to the representation in question) and is defined so that \( t = 0 \) for the trivial representation or for decuplets, octets and so on, while \( t = 1 \) for triplet (\( \mathbb{3} \)) or anti-sextet etc. and \( t = -1 \) for anti-triplet (\( \mathbb{3}^\ast \)) or sextet etc. Then it is written

\[
Q + t/3 = 0 \pmod{1} \quad (1)
\]

where \( Q \) is the electric charge \( Q = y/2 + t_3/2 \) (here \( t_3 \) is the third component of the weak isospin, \( SU(2) \), and \( y \) is the weak hypercharge). We may write this charge quantization rule as

\[
y/2 + d/2 + t/3 = 0 \pmod{1} \quad (2)
\]

where we have introduced the duality \( d \), which is defined to be 0 when the weak isospin is integer and \( d = 1 \) when it is half integer. It is then easily seen that \( d/2 = t_3/2 \) (mod 1) for all weak isospin representations.

Now the point is that this restriction on the representations ensures that the subgroup \( \{(2\pi, -I, \exp(i2\pi/3)I)^n | n \in \mathbb{Z} \} \) is represented trivially and that,
thus, the allowed representations really are representations of the group $SMG = S(U(2) \times U(3)) = (R \times SU(2) \times SU(3))/\{(2\pi, -I, \exp(i2\pi/3))n| n \in \mathbb{Z}\}$.

After having made sense of the group it would be natural to ask if this group could somehow give us a hint about what goes on beyond the Standard Model. Brene and one of us \cite{3, 4} have argued for two attributes implied by the group chosen by Nature:

a) The charge quantization rule in the Standard Model is in some sense linking the invariant sub Lie algebras more strongly than—in a certain way of counting—any other group would do. To be more specific: There are six different combinations of triality and duality—i.e., really of classes of representations of the non-abelian part of the gauge Lie algebra—that can be specified by providing the abelian charge $y/2$. The logarithm of this number of such classes divided by the dimension of the Cartan algebra—four in the case of the Standard Model group—is larger for the SMG than for any other group (except cross products of SMG with itself, for which the mentioned ratio must have the same value). We called this ratio $\chi$.

b) The SMG has rather few automorphisms and can, to a large extent, be considered specified as being one of the most “skew” groups.

If you would take the point a) to help guess what is the gauge group beyond the Standard Model, you could say that the group should be expected to have a large value for the ratio $\chi$. That requirement points in the direction of having a cross product power of the Standard Model group, because such a cross product has just the same $\chi$ value as the Standard Model group itself.

### 3 The large mass ratios of leptons and quarks

What is the origin of the well-known pattern of large ratios between the quark and lepton masses and of the small quark mixing angles? This is the problem of the hierarchy of Yukawa couplings in the Standard Model (SM). We suggest \cite{3} that the natural resolution of this problem is the existence of some approximately conserved chiral charges beyond the SM. These charges, which we assume to be gauged, provide selection rules forbidding the transitions between the various left-handed and right-handed fermion states (except for the top quark).

For example, we suppose that there exists some charge (or charges) $Q$ for which the quantum number difference between left- and right-handed Weyl states is larger for the electron than for muon:

$$|Q_{eL} - Q_{eR}| > |Q_{\mu L} - Q_{\mu R}|$$  \hspace{1cm} (3)

It then follows that the SM Yukawa coupling for the electron $g_e$ is suppressed more than that for the muon $g_{\mu}$, when $Q$ is taken to be approximately conserved. This is what is required if we want to explain the electron-muon mass ratio.
We shall take the point of view that, in the fundamental theory beyond the SM, the Yukawa couplings allowed by gauge invariance are all of order unity and, similarly, all the mass terms allowed by gauge invariance are of order the fundamental mass scale of the theory—say the Planck scale. Then, apart from the matrix element responsible for the top quark mass, the quark-lepton mass matrix elements are only non-zero due to the presence of other Higgs fields having vacuum expectation values (VEVs) smaller (typically by one order of magnitude) than the fundamental scale. These Higgs fields will, of course, be responsible for breaking the fundamental gauge group $G$—whatever it may be—down to the SM group. In order to generate a particular effective SM Yukawa coupling matrix element, it is necessary to break the symmetry group $G$ by a combination of Higgs fields with the appropriate quantum number combination $\Delta \vec{Q}$. When this “$\Delta \vec{Q}$” is different for two matrix elements they will typically deviate by a large factor. If we want to explain the observed spectrum of quarks and leptons in this way, it is clear that we need charges which—possibly in a complicated way—separate the generations and, at least for $t-b$ and $c-s$, also quarks in the same generation. Just using the usual simple $SU(5)$ GUT charges does not help because both $(\mu_R$ and $e_R)$ and $(\mu_L$ and $e_L)$ have the same $SU(5)$ quantum numbers. So we prefer to keep each SM irreducible representation in a separate irreducible representation of $G$ and introduce extra gauge quantum numbers distinguishing the generations, by adding extra Cartesian-product factors to the SM gauge group.

What the structure of the quark and lepton spectrum really calls for is separation between generations and also between at least the $c$-quark and $t$-quark within their generation. Unification is strictly speaking not called for because, as is well-known, the simplest $SU(5)$ unification can only be made to work either by having complicated Higgs fields replacing the simple Weinberg Salam Higgs field taken as a five-plet—the Georgi-Jarlskog model—or by introducing even more sophisticated $SU(5)$ symmetry breaking mechanisms. The experimental mass ratios predicted by simple $SU(5)$ may work for the case of the $\tau$ and $b$-quark adjusted by SUSY, but then the $\mu$ to $s$-quark and the electron to $d$-quark mass ratios do not agree with a simple $SU(5)$, with only the five-plet Higgs field (or two five-plets if supersymmetric) playing the role of the Weinberg Salam Higgs field.

In other words, separation is called for and not unification!

It is in fact possible to extend the Standard Model with just two extra $U(1)$ groups to get an order of magnitude fit of the quark and lepton masses (actually this is the model of the present contribution with the nonabelian groups amputated; see ref. [9]). However, if one insists on quantum numbers closer to being minimal/small relative to what is allowed by the quantization rules (which are embodied in the choice of representation of the group), it is better to have a larger group extending the SMG.
4 The “maximal” AGUT gauge group

To limit the search for the gauge group beyond the Standard Model, let us take the point of view that we do not look for the whole gauge group \( G \), say, but only for that factor group \( G' = G/H \) which transforms the already known quark and lepton Weyl fields in a nontrivial way. That is to say, we ask for the group obtained by dividing out the subgroup \( H \subset G \) which leaves the quark and lepton fields unchanged. This factor group \( G' \) can then be identified with its representation of the Standard Model fermions, i.e. as a subgroup of the \( U(45) \) group of all possible unitary transformations of the 45 Weyl fields for the Standard Model. If one took as \( G \) one of the extensions of \( SU(5) \), such as \( SO(10) \) or the E-groups which are promising unification groups, the factor group \( G/H \) would be \( SU(5) \) only; the extension parts can be said to only transform particles that are not in the Standard Model (and thus could be pure fantasy \textit{a priori}).

We would like to assume that there shall be no gauge or mixed anomalies. So now we can add some further suggestive properties for \( G' \) that could help us in choosing it:

If we ask for the smallest extension of the Standard Model, unifying as many as possible of the irreducible representations under the Standard Model into irreducible representations under \( G' \), we get, as can relatively easily be seen, \( SU(5) \) in the usual way. That represents all the \( SO(10) \) and E-groups, since we think about having divided out the part \( H \) that transforms the known particles trivially.

But, as we argued in the previous section, empirical indications seem to call for the opposite: separation and a big group!

We have actually calculated that, among the subgroups of the \( U(45) \) group of unitary transformations on the Standard Model Weyl fermions without anomalies, the biggest separating group is the AGUT-group which is the gauge group of the model put forward here. The AGUT model is based on extending the SM gauge group \( SMG = S(U(2) \times U(3)) \) not to the grand unified \( SU(5) \), but rather to the non-simple \( SMG^3 \times U(1)_f \) group.

The \( SMG^3 \times U(1)_f \) group should be understood such that, near the Planck scale, there are three sets of SM-like gauge particles. Each set only couples to its own proto-generation [e.g. the proto- \( u, d, e \) and \( \nu_e \) particles], but not to the other two proto-generations [e.g. the proto- \( c, s, \mu, \nu_\mu, t, b, \tau \) and \( \nu_\tau \) particles]. There is also an extra abelian \( U(1)_f \) gauge boson, giving altogether \( 3 \times 8 = 24 \) gluons, \( 3 \times 3 = 9 \) \( W' \)s and \( 3 \times 1 + 1 = 4 \) abelian gauge bosons. The couplings of the \( SMG_i = S(U(2) \times U(3))_i \approx SU(3)_i \times SU(2)_i \times U(1)_i \) group to the \( i \)’th proto-generation are identical to those of the SM group. Consequently we have a charge quantization rule, analogous to eq. (3), for each of the three proto-generation weak hypercharge quantum numbers \( y_i \).

To first approximation—namely in the approximation that the quark mixing angles \( V_{us}, V_{cb}, V_{ub} \) are small—we may ignore the prefix \textit{proto-}. However we really introduce in our model some “proto-fields” characterized by their cou-
plings to the 37 gauge bosons of the $SMG^3 \times U(1)_I$ group. The physically observed $u$-quark, $d$-quark etc. are then superpositions of the proto-quarks (or proto-leptons), with the proto-particle of the same name dominating. Actually there is one deviation from this first approximation rule that proto-particles correspond to the same named physical particle. In the AGUT fit to the quark-lepton mass spectrum discussed below, we find that to first approximation the right-handed components of the top and the charm quarks must be permuted:

$$c_R \text{ PROTO} \approx t_R \text{ PHYSICAL} \quad t_R \text{ PROTO} \approx c_R \text{ PHYSICAL}$$

But for all the other components we have:

$$t_L \text{ PROTO} \approx t_L \text{ PHYSICAL} \quad b_R \text{ PROTO} \approx b_R \text{ PHYSICAL}$$

and so on.

The AGUT group breaks down an order of magnitude or so below the Planck scale to the diagonal subgroup of the $SMG^3$ subgroup (the diagonal subgroup is isomorphic to the usual SM group). For this breaking we shall use a relatively complicated system of Higgs fields with names $W, T, \xi$, and $S$. In order to fit neutrino masses as well, we need an even more complicated system. See the thesis of Mark Gibson [8] and ref. [9].

It should however be said that, although at the very high energies just under the Planck energy each generation has its own gluons, own W’s etc., the breaking makes only one linear combination of a certain colour combination of gluons “survive” down to low energies. So below circa $1/10$ of the Planck scale, it is only these linear combinations that are present and thus the couplings of the gauge particles—at low energy only corresponding to these combinations—are the same for all three generations.

You can also say that the phenomenological gluon is a linear combination with amplitude $1/\sqrt{3}$ for each of the AGUT-gluons of the same colour combination. That then also explains why the coupling constant for the phenomenological gluon couples with a strength that is $\sqrt{3}$ times smaller if, as we effectively assume, the three AGUT $SU(3)$ couplings were equal to each other. In our model the formula connecting the AGUT fine-structure constants to those of the low energy surviving diagonal subgroup $\{(U,U,U) \mid U \in SMG\} \subseteq SMG^3$ is

$$\frac{1}{\alpha_{\text{diag},j}} = \frac{1}{\alpha_{\text{1st gen.},j}} + \frac{1}{\alpha_{\text{2nd gen.},j}} + \frac{1}{\alpha_{\text{3rd gen.},j}}$$

Here the index $j$ is meant to run over the three groups in an SMG, namely $j = U(1), SU(2), SU(3)$, so that e.g. $j = 3$ means that we talk about the gluon couplings (of the generation in question).

The gauge coupling constants do not, of course, unify, because we have not combined the groups $U(1), SU(2)$ and $SU(3)$ together into a simple group, but their values have been successfully calculated using the so-called Multiple Point
Principle [10], which is a further assumption we put into the model (see for this Larisa Laperashvili’s contribution to these proceedings).

At first sight this $SMG^3 \times U(1)_f$ group, with its 37 generators, seems to be just one among many possible SM gauge group extensions.

However, we shall now argue it is not such an arbitrary choice, as it can be uniquely specified by postulating 4 reasonable requirements to be satisfied by the gauge group $G$ beyond the SM. As a zeroth postulate, of course, we require that the gauge group extension must contain the Standard Model group as a subgroup $G \supseteq SMG$. In addition it should obey the following 4 postulates:

The first two are also valid for $SU(5)$ GUT:

1. $G$ should transform the presently known (left-handed, say) Weyl particles into each other, so that $G \subseteq U(45)$. Here $U(45)$ is the group of all unitary transformations of the 45 species of Weyl fields (3 generations with 15 in each) in the SM.

2. No anomalies, neither gauge nor mixed. We assume that only straightforward anomaly cancellation takes place and, as in the SM itself, do not allow for a Green-Schwarz type anomaly cancellation [11].

But the next two are rather just opposite to the properties of the $SU(5)$ GUT, thus justifying the name Anti-GUT:

3. The various irreducible representations of Weyl fields for the SM group remain irreducible under $G$. This is the most arbitrary of our assumptions about $G$. It is motivated by the observation that combining SM irreducible representations into larger unified representations introduces symmetry relations between Yukawa coupling constants, whereas the particle spectrum does not exhibit any exact degeneracies (except possibly for the case $m_b = m_\tau$). In fact AGUT only gets the naive $SU(5)$ mass predictions as order of magnitude relations: $m_b \approx m_\tau$, $m_s \approx m_\mu$, $m_d \approx m_e$.

4. $G$ is the maximal group satisfying the other 3 postulates.

With these four postulates a somewhat cumbersome calculation shows that, modulo permutations of the various irreducible representations in the Standard Model fermion system, we are led to our gauge group $SMG^3 \times U(1)_f$. Furthermore it shows that the SM group is embedded as the diagonal subgroup of $SMG^3$, as in our AGUT model.

Several of the anomalies involving $U(1)_f$ are in our solution cancelled by assigning equal and opposite values of the $U(1)_f$ charge to the analogous particles belonging to second and third proto-generations, while the first proto-generation particles have just zero charge [12]. In fact the $U(1)_f$ group does not couple
to the left-handed particles and the $U(1)_f$ quantum numbers can be chosen as follows for the proto-states:

\begin{align}
Q_f(\tau_R) &= Q_f(b_R) = Q_f(c_R) = 1 \\
Q_f(\mu_R) &= Q_f(s_R) = Q_f(t_R) = -1
\end{align}

Thus the quantum numbers of the quarks and leptons are uniquely determined in the AGUT model. However we do have the freedom of choosing the gauge quantum numbers of the Higgs fields responsible for the breaking of the $SMG^3 \times U(1)_f$ group down to the SM gauge group. These quantum numbers are chosen with a view to fitting the fermion mass and mixing angle data \[13\], as discussed in the next section.

5 Symmetry breaking by Higgs fields

There are obviously many different ways to break down the large group $G$ to the much smaller SMG. However, we can first greatly simplify the situation by assuming that, like the quark and lepton fields, the Higgs fields belong to singlet or fundamental representations of all non-abelian groups. The non-abelian representations are then determined from the $U(1)_i$ weak hypercharge quantum numbers, by imposing the charge quantization rule eq. (2) for each of the $SMG_i$ groups. So now the four abelian charges, which we express in the form of a charge vector

\[ \vec{Q} = \left( \frac{y_1}{2}, \frac{y_2}{2}, \frac{y_3}{2}, Q_f \right) \]

can be used to specify the complete representation of $G$. The constraint that we must eventually recover the SM group, as the diagonal subgroup of the $SMG_i$ groups, is equivalent to the constraint that all the Higgs fields (except for the Weinberg-Salam Higgs field which of course finally breaks the SMG) should have charges $y_i$ satisfying:

\[ y = y_1 + y_2 + y_3 = 0 \]

in order that their SM weak hypercharge $y$ be zero.

We wish to choose the charges of the Weinberg-Salam (WS) Higgs field, so that they match the difference in charges between the left-handed and right-handed physical top quarks. This will ensure that the top quark mass in the SM is not suppressed relative to the WS Higgs field VEV. However, as we remarked in the previous section, it is necessary to associate the physical right-handed top quark field not with the corresponding third proto-generation field $t_R$ but rather with the right-handed field $c_R$ of the second proto-generation. Otherwise we cannot suppress the bottom quark and tau lepton masses. This is because, for the proto-fields, the charge differences between $t_L$ and $t_R$ are the same as
between $b_L$ and $b_R$ and also between $\tau_L$ and $\tau_R$. So now it is simple to calculate the quantum numbers of the WS Higgs field $\phi_{WS}$:

$$\vec{Q}_{\phi_{WS}} = \vec{Q}_{cR} - \vec{Q}_{tL} = \left( 0, \frac{2}{3}, 0, 1 \right) - \left( 0, 0, \frac{1}{6}, 0 \right) = \left( 0, \frac{2}{3}, -\frac{1}{6}, 1 \right)$$

(10)

This means that the WS Higgs field will in fact be coloured under both $SU(3)_2$ and $SU(3)_3$. After breaking the symmetry down to the SM group, we will be left with the usual WS Higgs field of the SM and another scalar which will be an octet of $SU(3)$ and a doublet of $SU(2)$. This should not present any phenomenological problems, provided this scalar doesn’t cause symmetry breaking and doesn’t have a mass below the TeV scale. In particular an octet of $SU(3)$ cannot lead to baryon decay. In our model we take it that what in the Standard Model are seen as many very small Yukawa-couplings to the Standard Model Higgs field really represent chain Feynman diagrams, composed of propagators with Planck scale heavy particles (fermions) interspaced with order of unity Yukawa couplings to Higgs fields with the names $W, T, \xi$, and $S$, which are postulated to break the AGUT to the Standard Model Group. The small effective Yukawa couplings in the Standard Model are then generated as products of small factors, given by the ratios of the vacuum expectation values of $W, T,$ and $\xi$ to the masses occurring in the propagators for the Planck scale fermions in the chain diagrams [5].

The quantum numbers of our invented Higgs fields $W, T, \xi$ and $S$ are chosen—and it is remarkable that we succeeded so well—so as to make the order of magnitudes for the suppressions of the mass matrix elements of the various mass matrices fit to the phenomenological requirements.

After the choice of the quantum numbers for the replacement of the Weinberg Salam Higgs field in our model, eq. (10), the further quantum numbers needed to be picked out of the vacuum in order to give, say, mass to the b-quark is denoted by $\vec{b}$ and analogously for the other particles. For example:

$$\vec{b} = \vec{Q}_{b_L} - \vec{Q}_{b_R} - \vec{Q}_{WS}$$

(11)

$$\vec{c} = \vec{Q}_{c_L} - \vec{Q}_{tR} + \vec{Q}_{WS}$$

(12)

$$\vec{\mu} = \vec{Q}_{\mu_L} - \vec{Q}_{\mu_R} - \vec{Q}_{WS}$$

(13)

Here we denoted the quantum numbers of the quarks and leptons as e.g. $\vec{Q}_{cL}$ for the left handed components of the proto-charmed quark. Note that $\vec{c}$ has been defined using the $t_R$ proto-field, since we have essentially swapped the right-handed charm and top quarks. Also the charges of the WS Higgs field are added rather than subtracted for up-type quarks.

Next we attempted to find some Higgs field quantum numbers which, if postulated to have “small” expectation values compared to the Planck scale masses of the intermediate particles, would give a reasonable fit to the order of magnitudes of the mass matrix elements. We were thereby led to the proposal:
\[ \vec{Q}_W = \frac{1}{3} (2 \vec{b} + \vec{\mu}) = \left( 0, -\frac{1}{2}, \frac{1}{2}, -\frac{4}{3} \right) \]  \hspace{1cm} (14)

\[ \vec{Q}_T = \vec{b} - \vec{Q}_W = \left( 0, -\frac{1}{6}, \frac{1}{6}, -\frac{2}{3} \right) \]  \hspace{1cm} (15)

\[ \vec{Q}_\xi = \vec{Q}_{dL} - \vec{Q}_{sL} = \left( \frac{1}{6}, 0, 0, 0 \right) - \left( 0, \frac{1}{6}, 0, 0 \right) = \left( \frac{1}{6}, -\frac{1}{6}, 0, 0 \right) \]  \hspace{1cm} (16)

From the well-known Fritzsch relation \[ V_{us} \simeq \sqrt{\frac{m_d}{m_s}} \], it is suggested that the two off-diagonal mass matrix elements connecting the \( d \)-quark and the \( s \)-quark be equally big. We achieve this approximately in our model by introducing a special Higgs field \( S \), with quantum numbers equal to the difference between the quantum number differences for these 2 matrix elements in the down quark matrix. Then we postulate that this Higgs field has a VEV of order unity in fundamental units, so that it does not cause any suppression but rather ensures that the two matrix elements get equally suppressed. Henceforth we will consider the VEVs of the new Higgs fields as measured in Planck scale units and so we have:

\[ < S > = 1 \]  \hspace{1cm} (17)

\[ \vec{Q}_S = [\vec{Q}_{sL} - \vec{Q}_{dR}] - [\vec{Q}_{dL} - \vec{Q}_{sR}] \]
\[ = \left[ \left( \frac{1}{6}, 0, 0, 0 \right) - \left( \frac{1}{3}, 0, 0, 0 \right) \right] - \left[ \left( \frac{1}{6}, 0, 0, 0 \right) - \left( 0, -\frac{1}{3}, 0, -1 \right) \right] \]
\[ = \left( \frac{1}{6}, -\frac{1}{6}, 0, 1 \right) \]  \hspace{1cm} (18)

The existence of a non-suppressing field \( S \) means that we cannot control phenomenologically when this \( S \)-field is used. Thus the quantum numbers of the other Higgs fields \( W, T, \xi \) and \( \phi_{WS} \) given above have only been determined modulo those of the field \( S \).

6 Mass matrices, predictions

We define the mass matrices by considering the mass terms in the SM to be given by:

\[ \mathcal{L} = \overline{Q}_L M_u U_R + \overline{Q}_L M_d D_R + \overline{L}_L M_l E_R + \text{h.c.} \]  \hspace{1cm} (19)

Here \( Q_L \) and \( L_L \) denote the sets of three SU(2) doublets of left-handed quarks and leptons respectively, while \( U_R, D_R \) and \( E_R \) denote the sets of right-handed
SU(2) singlet up-type quarks, down-type quarks and charged leptons respectively. The mass matrices $M_f$ can be expressed in terms of the effective SM Yukawa matrices $Y_f$ and the WS Higgs VEV by:

$$M_f = Y_f \frac{<\phi_{WS}>}{\sqrt{2}}$$  \hspace{1cm} (20)

We can now calculate the suppression factors for all elements in the Yukawa matrices, by expressing the charge differences between the left-handed and right-handed fermions in terms of the charges of the Higgs fields. They are given by products of the small numbers denoting the VEVs in the fundamental units of the fields $W$, $T$, $\xi$ and the of order unity VEV of $S$. In the following matrices we simply write $W$ instead of $<W>$ etc. for the VEVs, but we retain the hermitean conjugation symbols to help keep track of the quantum numbers.

With the quantum number choice given above, the resulting matrix elements are—but remember that “random” complex order unity factors are supposed to multiply all the matrix elements—for the uct-quarks:

$$Y_U \simeq \begin{pmatrix}
S^\dagger W^\dagger T^2 (\xi^\dagger)^2 & W^\dagger T^2 \xi & (W^\dagger)^2 T \xi \\
S^\dagger W^\dagger T^2 (\xi^\dagger)^3 & W^\dagger T^2 & (W^\dagger)^2 T \\
t & 1 & W^\dagger T^\dagger
\end{pmatrix}$$  \hspace{1cm} (21)

the dsb-quarks:

$$Y_D \simeq \begin{pmatrix}
SW(T^\dagger)^2 \xi^2 & W(T^\dagger)^2 \xi & T^3 \xi \\
SW(T^\dagger)^2 \xi & W(T^\dagger)^2 & T^3 \\
SW^2(T^\dagger)^4 \xi & W^2(T^\dagger)^4 & WT
\end{pmatrix}$$  \hspace{1cm} (22)

and the charged leptons:

$$Y_E \simeq \begin{pmatrix}
SW(T^\dagger)^2 \xi^2 & W(T^\dagger)^2 (\xi^\dagger)^3 & (S^\dagger)^2 W T^4 \xi^2 \\
SW(T^\dagger)^2 \xi^5 & W(T^\dagger)^2 & (S^\dagger)^2 W T^4 \xi^2 \\
S^3 W(T^\dagger)^5 \xi^3 & (W^\dagger)^2 T^4 & WT
\end{pmatrix}$$  \hspace{1cm} (23)

We can now set $S = 1$ and fit the nine quark and lepton masses and three mixing angles, using 3 parameters: $W$, $T$ and $\xi$. That really means we have effectively omitted the Higgs field $S$ and replaced the maximal AGUT gauge group $SMG^3 \times U(1)_f$ by the reduced AGUT group $SMG_{12} \times SMG_3 \times U(1)$, which survives the spontaneous breakdown due to $S$. In order to find the best possible fit we must use some function which measures how good a fit is. Since we are expecting an order of magnitude fit, this function should depend only on the ratios of the fitted masses to the experimentally determined masses. The obvious choice for such a function is:

$$\chi^2 = \sum \left[ \ln \left( \frac{m}{m_{\exp}} \right) \right]^2$$  \hspace{1cm} (24)
Table 1: Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

|         | Fitted    | Experimental |
|---------|-----------|--------------|
| $m_u$   | 3.6 MeV   | 4 MeV        |
| $m_d$   | 7.0 MeV   | 9 MeV        |
| $m_c$   | 0.87 MeV  | 0.5 MeV      |
| $m_e$   | 1.02 GeV  | 1.4 GeV      |
| $m_s$   | 400 MeV   | 200 MeV      |
| $m_b$   | 88 MeV    | 105 MeV      |
| $M_t$   | 192 GeV   | 180 GeV      |
| $m_{\tau}$ | 8.3 GeV | 6.3 GeV      |
| $m_{\mu}$ | 1.27 GeV | 1.78 GeV     |
| $V_{us}$ | 0.18      | 0.22         |
| $V_{cb}$ | 0.018     | 0.041        |
| $V_{ub}$ | 0.0039    | 0.0035       |

where $m$ are the fitted masses and mixing angles and $m_{\text{exp}}$ are the corresponding experimental values. The Yukawa matrices are calculated at the fundamental scale, which we take to be the Planck scale. We use the first order renormalisation group equations (RGEs) for the SM to calculate the matrices at lower scales.

We cannot simply use the 3 matrices given by eqs. (21)–(23) to calculate the masses and mixing angles, since only the order of magnitude of the elements is defined. Therefore we calculate statistically, by giving each element a random complex phase and then finding the masses and mixing angles. We repeat this several times and calculate the geometrical mean for each mass and mixing angle. In fact we also vary the magnitude of each element randomly, by multiplying by a factor chosen to be the exponential of a number picked from a Gaussian distribution with mean value 0 and standard deviation 1.

We then vary the 3 free parameters to find the best fit given by the $\chi^2$ function. We get the lowest value of $\chi^2$ for the VEVs:

$$\langle W \rangle = 0.179$$

$$\langle T \rangle = 0.071$$

$$\langle \xi \rangle = 0.099$$

The result of the fit is shown in table 1. This fit has a value of:

$$\chi^2 = 1.87$$

This is equivalent to fitting 9 degrees of freedom (9 masses + 3 mixing angles - 3 Higgs VEVs) to within a factor of $\exp(\sqrt{1.87/9}) \approx 1.58$ of the experimental
Table 2: Best fit using alternative light quark masses extracted from lattice QCD. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

|        | Fitted   | Experimental |
|--------|----------|--------------|
| $m_u$  | 1.9 MeV  | 1.3 MeV      |
| $m_d$  | 3.7 MeV  | 4.2 MeV      |
| $m_c$  | 0.45 MeV | 0.5 MeV      |
| $m_c$  | 0.53 GeV | 1.4 GeV      |
| $m_s$  | 327 MeV  | 85 MeV       |
| $m_d$  | 75 MeV   | 105 MeV      |
| $M_t$  | 192 GeV  | 180 GeV      |
| $m_b$  | 6.4 GeV  | 6.3 GeV      |
| $m_{\tau}$ | 0.98 GeV | 1.78 GeV    |
| $V_{us}$ | 0.15     | 0.22         |
| $V_{cb}$ | 0.033    | 0.041        |
| $V_{ub}$ | 0.0054   | 0.0035       |

value. This is better than might have been expected from an order of magnitude fit.

We can also fit to different experimental values of the 3 light quark masses by using recent results from lattice QCD, which seem to be consistently lower than the conventional phenomenological values. The best fit in this case is shown in table 2. The corresponding values of the Higgs VEVs are:

\[
\langle W \rangle = 0.123 \tag{29}
\]

\[
\langle T \rangle = 0.079 \tag{30}
\]

\[
\langle \xi \rangle = 0.077 \tag{31}
\]

and this fit has a larger value of:

\[
\chi^2 = 3.81 \tag{32}
\]

But even this is good for an order of magnitude fit.

7 Baryogenesis

A very important check of our model is whether or not it can be consistent with baryogenesis. In our model we have just the SM interactions up to about one or two orders of magnitude under the Planck scale. So we have no way, at the electroweak scale, to produce the baryon number in the universe. There is
insufficient CP violation in the SM. Furthermore, even if created, the baryon number would immediately be washed out by sphaleron transitions. Our only chance to avoid the baryon number being washed out at the electroweak scale is to have a non-zero B-L (i.e. baryon number minus lepton number) produced from the high, i.e. Planck, scale action of the theory. That could then in turn give rise to the baryon number at the electroweak scale. Now in our model the B-L quantum number is broken by an anomaly involving the $U(1)$ gauge group. This part of the gauge group in turn is broken by the Higgs field $\xi$ which, in Planck units, is fitted to have an expectation value around $1/10$. The anomaly keeps washing out any net $B - L$ that might appear, due to CP-violating forces from the Planck scale physics, until the temperature $T$ of the universe has fallen to $\xi = 1/10$. The $U(1)$ gauge particle then disappears from the thermal soup and thus the conservation of B-L sets in. The amount of $B - L$ produced at that time should then be fixed and would essentially make itself felt, at the electroweak scale, by giving rise to an amount of baryon number of the same order of magnitude.

The question now is whether we should expect in our model to have a sufficient amount of time reversal symmetry breaking at the epoch when the B-L settles down to be conserved, such that the amount of B-L relative to say the entropy (essentially the amount of 3 degree Kelvin background radiation) becomes large enough to agree with the well-known phenomenological value of the order of $10^{-9}$ or $10^{-10}$. At the time of the order of the Planck scale, when the temperature was also of the order of the Planck temperature, even the CP or time reversal violations were of order unity (in Planck units). So at that time there existed particles, say, with order of unity CP-violating decays. However, they had, in our pure dimensional argument approximation, lifetimes of the order of the Planck scale too. Thus the B-L biased decay products would be dumped at time 1 in Planck units, rather than at the time of $B - L$ conservation setting in. In a radiation dominated universe, as we shall assume, the temperature will go like $1/a$ where $a$ is the radius parameter—the size or scale parameter of the universe. Now the time goes as the square of this size parameter $a$. Thus the time in Planck units is given as the temperature to the negative second power

$$t = \frac{0.3}{\sqrt{g_*} \times T^2}$$

where $g_*$ is the number of degrees of freedom—counted as 1 for bosons but as 7/8 per fermion degree of freedom—entering into the radiation density. In our model $g_*$ gets a contribution of $\frac{7}{8} \times 45 \times 2$ from the fermions and 2×37 from the gauge bosons, and in addition there is some contribution from the Higgs particles. So we take $g_*$ to be of order 100, in our crude estimate of the time $t$ corresponding to the temperature $T = \xi = \frac{1}{10}$ in Planck units, when $B - L$ conservation sets in:

$$t \simeq \frac{0.3}{100^{1/2}} \times \left(\frac{1}{10}\right)^2 = 3$$
By that time we expect of the order of \( \exp(-3) \) particles from the Planck era are still present and able to dump their CP-violating decay products. Of course here the uncertainty of an order of magnitude would be in the exponent, meaning a suppression anywhere between say \( \exp(0) \) and \( \exp(-30) \) and could thus easily be in agreement with the wanted value of order \( 5 \times 10^{-10} \). This result is encouraging, but clearly a more careful analysis is required.

8 Neutrino oscillations, a problem?

At first it seems a problem to incorporate the evidence for neutrino oscillations into our model [9]. The \textit{a priori} prediction of the AGUT model would be that the neutrino masses are so small that they could not be seen with present accuracy. This is because a see-saw mass of the order of the Planck scale, combined with further suppression, leads to too small neutrino masses. However, by changing the system of Higgses and allowing the overall neutrino mass scale to be a fitted number, it has been possible to construct a satisfactory scheme involving further Higgs fields. These extra Higgs fields make short-cuts, in the sense of producing transitions that could already occur using other Higgs field combinations. This extension of the model to neutrinos is not too attractive, but tolerable [8].

9 Connection to MPP

Originally the idea of having an \( SMG^3 \) type model was developed in connection with random dynamics [2] ideas of confusion and, for a long time, in connection with the idea of requiring many phases to meet (multiple point principle MPP), in order to get predictions for the fine-structure constants. This type of calculation even predicted that there be three generations, at a time when it was not known experimentally, by fitting the fine-structure constants! (See e.g. ref. [14].)

10 Conclusion

We have looked at some of the hints in the Standard Model that may be useful in going beyond it and have put forward our own model which we showed to be, to some extent, inspired by such features:

We stressed looking for the gauge group rather than just the Lie algebra, which \textit{a priori} is what is relevant for describing a Yang Mills theory.

We have found surprisingly good fits of masses and mixing angles—and in a related model even of the fine-structure constants—in a model in which the gauge group, near the Planck scale, is the maximal one transforming the already known fermions around, not having anomalies and not unifying the irreducible representations of the Standard Model. Although at first it looked a failure,
it has turned out that even the baryon number generation in the big bang is
not excluded from being in agreement with the present AGUT model. It must,
however, then be achieved by first getting a B-L contribution made at a time
when the temperature of the universe was only an order of magnitude under the
Planck temperature.

To incorporate neutrino oscillations, severe but tolerable modifications of
the AGUT model are needed.

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References

[1] Michel L 1962 Group Theoretical Concepts and Methods in Elementary
Particle Physics, Lectures of the Istanbul Summer School of Theoretical
Physics 1962, ed F Gursey (Gordon and Breach) p 135;
O’Raifeartaigh L Group Structure of Gauge Theories (Cambridge University
Press, 1986).

[2] Froggatt C D and Nielsen H B Origin of Symmetries (World Scientific,
1991).

[3] Nielsen H B and Brene N Skewness of the Standard Model possible implica-
tions In: The Gardener of Eden. Special issue in honor of R. Brout’s 60th
birthday, ed N. Nioletopoulus and J. Orloff, 1990 Phys. Mag. 12 Suppl.,
157 - 172.

[4] Brene N and Nielsen H B 1991 Nucl. Phys. B 359, 406 - 422.

[5] Froggatt C D and Nielsen H B 1979 Nucl. Phys. B 147, 277.

[6] Georgi H and Jarlskog C 1979 Phys. Lett. B 86, 297.

[7] Froggatt C D and Nielsen H B 1998 Proceedings of the Trento Lepton-
Baryon 98 meeting, ed Klapdor-Kleingrothaus H V (Institute of Physics
Publishing, Dirac House, Temple Back, Bristol); hep-ph/9810388.

[8] Gibson M, Ph D. thesis, submitted to Glasgow University, May 1999.

[9] Froggatt C D, Gibson M and Nielsen H B, 1999 Phys. Lett. B 446, 256 -
266;
Froggatt C D, Gibson M, Nielsen H B and Smith D J, To be published in
Proceedings of the 29th Int. Conf. on High Energy Physics (Vancouver,
1998); hep-ph/9810391.
[10] Bennett D L, Froggatt C D and Nielsen H B 1995 Proc. of the 27th Int. Conf. on High Energy Physics (Glasgow, 1994) ed P Bussey and I Knowles (IOP Publishing Ltd) p 557; 1995 Perspectives in Particle Physics ’94 ed D Klabučar et al (World Scientific) p 255, hep-ph/9504294.

[11] Green M B and Schwarz J 1984 Phys. Lett. B 149, 117.

[12] Davidson A, Koca M and Wali K C 1979 Phys. Rev. Lett. 43, 92; 1979 Phys. Rev. D 20, 421.

[13] Froggatt C D, Nielsen H B and Smith D J 1996 Phys. Lett. B 385, 150; Froggatt C D, Gibson M, Nielsen H B and Smith D J, 1998 Int. J. Mod. Phys. A 13 5037 - 5074.

[14] Fritzsch H 1977 Phys. Lett. B 70, 436.

[15] Sarkar U 1998 Proceedings of the Trento Lepton-Baryon 98 meeting, ed Klapdor-Kleingrothaus H V (Institute of Physics Publishing, Dirac House, Temple Back, Bristol); hep-ph/9809208.

[16] Bennett D, Ph D. thesis, Niels Bohr Institute, 1996, Multiple Point Criticality, Nonlocality and Fine-tuning in Fundamental Physics: Predictions for Gauge Coupling Constants gives $\alpha^{-1} = 136.8 \pm 9$. 