A parametric study on fatigue life of plate girder railway bridge with varying velocity of rolling stock

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Abstract. It is important to measure the fatigue life of a structure as it helps in finding the probable life of a structure before its failure. It also helps in making cost effective decision on existing structure regarding replacement or rehabilitation in time. In this study deterministic approach is used for the estimation of fatigue life of a plate girder railway bridge. Finite element analysis is carried out using software to find the critical region of fatigue failure and the stress developed there. Number of cycles of different stress range during the movement of train is evaluated with the help of S-N curve. The S-N curve is selected based on type of welds and type of stress considered. Rain flow counting method is used for evaluating the number of cycles corresponding to each stress range. The fatigue damage is then evaluated by using Palm Miner damage accumulation rule and corresponding fatigue life is evaluated. The fatigue life corresponding to different velocity of train ranging from 30 kmph to 140 kmph is generated similarly and compared to find the influence of velocity of moving load on fatigue life of structure.

1. Introduction
The majority of structures involve parts that are subjected to fluctuating or cyclic loads. In such loading cases fluctuating or cyclic stresses induced can often result in failure by fatigue. The damage occurred during the fatigue loading is cumulative and is unrecoverable in nature. Same is the case with plate girder railway bridges. It is not economic and is time consuming to build a new structure when the structure starts to crack or fail. Finding the fatigue life helps us get an idea when the structure should be retrofitted to make maximum utilization of the structure and increase the net life span of the structure. Also, considering the need for faster transportation in the current state points out the relevance of the study.

The increased service loads and speed indicates the requirement to find how long the structure would sustain (Pipinato \textit{et al.}, 2012). A study by Zhao \textit{et al.} (1994) found that 80\% to 90\% of steel structure fails by fatigue.

Among various approaches that exist to analyse fatigue damage of welded joints nominal stress concept is least time consuming where as effective notch stress concept is most time consuming (Pettersson and Barsoum, 2012). The structural hotspot stress method and Linear Elastic Fracture Mechanics (LEFM) lies in between them. According to the study conducted by Aygül \textit{et al.}, (2013) in comparing all the methods mentioned before, it was found that in spite of the simplicity the nominal stress approach gave fairly accurate result. But, considering how complex it is to create finite element for numerical analysis, it is difficult to distinguish nominal stress near the welded joints. One
of the available standards by Hobbacher (2009) conveys how to use Hot spot stress for finite element analysis in welded joints. Thus, Hot spot stress approach is used in this study.

S-N curve-based design approach as mentioned in Euro Code is conveniently adopted to find the design life depending on the types of weld, location of joint, loading type, number of lanes etc. The Euro Code defines the effect due to above in based on loading condition at Euro Nations. But it can be adopted with slight variations according to the practical conditions that prevail in certain other country (Goel, 2005). Euro Code also give guidelines on hot spot stress method in addition to nominal stress method.

From a comparative study conducted by Goel and Kumar (2006) on existing provisions of the BS-5400, Part-10 and Indian Railway Standard Steel Bridge Code with respect to fatigue it was concluded that BS-5400 takes in heavier loads compared to IRS Modified Broad Gauge (MBG). This puts the loading condition in Indian aspect at a conservative spot.

Allowable working stress was adopted from the Appendix G of IRS Steel Bridge Code. This is depending on the ratio of minimum stress and maximum stress, also number of stress cycle repetition and the type of connection is a factor. All the details were designed such a way that the stress induced under the influence of design loads are within the allowable limits (Goel and Kumar, 2006). Goel and Kumar (2006) points out the limitations of IRS approach as follows:

- There is no clear basis for adopting counts of 10 million number of cycles to determine allowable stress levels.
- The cumulative phenomena of Fatigue are not clearly reflected in the IRS procedure.
- No S-N curves are used in the current procedure.
- The standard train load can be represented as an equivalent uniformly distributed load. Thus, the actual variation of stresses in members due to the passage of rolling stock is not counted.

Considering the Indian scenario IS 800:2007 fatigue provision is based on the nominal stress approach. There is no indication of finite element modelling or other approaches. The International Institute of Welding (IIW) gives recommendations on fatigue of welded components and structures in their comprehensive code IIW-1823-07 (2008). It gives design S-N curves and clear guidelines on how to determine hotspot stress compared to other design codes.

This study is focused on finding the effect of variation in velocities of the rolling stock on a plate girder railway bridge in Indian scenario using finite element analysis. For this study IRS plate girder bridge of span 12.2 m is adopted.

2. Methodology
The deterministic approach of fatigue analysis based on the stress cycle is adopted in this study. The stress cycle was counted using the principle of Palm-Miner damage accumulation and it was implemented using rain flow counting method with respect to S-N curve. The stress type considered for obtaining the count is hot spot stress. Fig. 1 shows the methodology adopted for this study.
Figure 1. The methodology adopted for fatigue assessment.

With the help of a MATLAB program time histories were developed corresponding to the axle load, distance between loads and velocity of rolling stock. These time histories were then given a load in ANSYS on the imported model of bridge that was modelled using Solid works. Most critical train load was selected to find critical region of the structure, so that that meshing at that region can be refined, the hot spot stress was obtained by standard surface stress extrapolation technique. Then obtained stress history was used as input to the rain flow cycle counting algorithm where number of cycles to failure of different stress range in context to S-N curve defined by IIW-1823-07 (2008) to obtain number of cycles of each stress range. Fatigue damage was then computed with the help of Palm-Miner damage accumulation concept, inverse of which gives the fatigue life.

3. Modelling of plate girder bridge

The bridge used for this study is the IRS welded plate girder bridge as per the Research, Design and Standards Organization (RDSO) drawing no. B16003. The span of the bridge is 40 feet (12.2 m) having a total weight of 10.26 t. The plate girder was modelled with the help of Solid Works and the imported to ANSYS to perform finite element analysis.

The bridge consists of two built up I girders that are placed parallel to each other and is connected using lateral bracings. The web of the girders is stiffened with the help of intermediate and end stiffeners. The intermediate stiffeners and bracings are Indian Standard Angles (ISA) while the web, flange and end stiffener are made up of Indian standard plate sections. The Material that the girder is made is the Structural steel conforming to the Indian standard IS 2062: 2011. It has a density of 7850 kg/m$^3$, yield strength 250 MPa, young’s modulus 200 GPa and poison’s ratio 0.3.

3.1 Meshing and boundary conditions

While modelling plates were modeled using the shell elements and the intermediate stiffeners and bracings were modelled as beam elements. If any one of the dimensions of the geometry that is to be meshed is very small compared to the other two, then 2D elements are preferred for finite element modeling (Gokhale, 2008). By iteration the element size for mesh was found to be around 200 mm which is when the frequency of structure converged. According to Ajmal and Mohammed (2018) the bridge is supported on piers with bearing plate placed between them. The bearing plates are then welded to the bottom flange of the girder and rigidly bolted into abutment concrete. The bearing plate
were modelled using shell elements and the welded connection with the flange were represented by bonded type contact in ANSYS. The support is taken as fixed. The train load acting on the rail is transferred to the bridge through two channel steel sleepers placed back to back. Bearing plates are then sandwiched between the sleepers and top flange of the bridge. The axle loads are applied as pressure on the bearing plate. The standard sleeper density of M+7 was taken which gives 20 sleepers on the bridge span (Saxena and Arora, 2012). Figure. 2. shows the bridge model after initial meshing.

3.2 Fatigue loading
This study mainly considered axle load due to loco and EMU according to values given by IRS MBG loading. The fatigue load model developed by Goel (2007) for Indian railway bridges was used in this study. Goel (2007) classified the Indian railway traffic into four: heavy freight traffic, mixed traffic lines with heavy traffic, suburban traffic and mixed traffic lines with light traffic.

The suburban traffic was chosen for this study as it has the least variety of trains. The most heavily used suburban line in India is at Western Railway, Bombay-Virar, which currently has a traffic of 200 suburban trains in a day and in addition to which 10 other passenger trains also runs in the same route. The two type of passenger trains are composed of 2 locomotives each with 22 and 26 coaches respectively. Table 1 gives an outline of the load model for various types.

| Train       | Train type no. (As per Goel, 2007) | No. of trains per day |
|-------------|------------------------------------|-----------------------|
| 2 Loco_1 + 22 Coach_2 | 2                                  | 5                     |
| 2 Loco_1 + 26 Coach_3 | 3                                  | 5                     |
| EMU         | 4                                  | 200                   |
| Total       |                                     | 210                   |

The fatigue strength of the bridge is mainly governed by the suburban train consisting of Electric Multiple Units (EMU). Table 2 shows axle loads in tons and spacing between them in millimetres for each type of rolling stock units. Since we assume linear damage accumulation, the total fatigue damage caused by these units will be the sum of individual blocks. The stress developed corresponding to each combination is assessed and the total repetition of load is computed over a year while calculating fatigue life.
4. Moving load analysis
According to Imam et al. (2007) principal stress histories can be used to apply miner rule for fatigue damage accumulation of steel railway bridge. IIW recommendations and also the Eurocode 3 advise us to use maximum principal stress as the more appropriate stress component (Heshmati, 2012). Considering that the maximum positive tensile stress is given by the maximum principle stress, it could be used to get more reliable results.

Hot spot stress can be obtained from point of maximum principle stress by refining the mesh size. It was observed that as we refine the mesh size, the point of maximum principle stress tends to change. The refining of maximum point is done until it stays intact. The point is identified by applying the most critical load that could act on the structure, here the load of EMU. It is also considered that instead of loading the full axle load of the whole train, the axle load of the repetitive sets needs to be considered. It is such that the effect of repeated axle load sets imparts same dynamic behavior on the structure.

5. Determination of hot spot stress
The Recommendations for fatigue design of welded joints and components (IIW-1823-07) shows how to determine the hot spot stress by the extrapolation of surface stress, particularly in certain reference points to the weld toe. It was found that the mesh size had great influence on the finite element analysis results. Fig. 3 shows the variation of stresses near weld toe.

As per IIW recommendation there are two types of hotspots according to their location and orientation- Type a in which the weld is located on plate surface and Type b in which the weld is located at plate edge. The procedure is first to identify the type of hotspot. Then establish the reference points required and determine the structural hot spot stress by extrapolation to the weld toe from the stresses of those reference points using the extrapolation equations given by the code. The hotspot in
this problem is identified as Type a. IIW recommendations provide linear extrapolation equations for both cases.

![Figure 3. Vector plot of maximum principal stress at the critical node.](image)

5.1. Surface stress extrapolation
IIW recommendations suggest fine or coarse meshing densities for shell and solid elements. It is a confirmed fact that the finite element model that have finer mesh size generally give more accurate results. Whereas relatively coarse mesh can be used for determining the structural hot spot stress. The coarse mesh enables easier computation. When coarse mesh is used for the FE model, the recommended element size should be strictly followed and no alterations are permitted. These recommendations are as shown in Table 4. The thickness of the web plate was 12 mm and that of stiffeners are 10 mm. The below gives the expression (1) for surface stress extrapolation by IIW-1823-07.

\[
\sigma_{hs} = 1.5\sigma_{0.5t} - 0.5\sigma_{1.5t}
\]  \hspace{1cm} (1)

Where, \(\sigma_{hs}\): Hot spot stress, \(\sigma_{0.5t}\): Stress at a distance of 0.5t from the weld toe, \(\sigma_{1.5t}\): Stress at a distance of 1.5t from the weld toe and t: element size

To satisfy IIW guidelines, the plates were refined near the hotspot area. In order to use the above relation for stress extrapolation, stress values at midpoint of the element is needed. Since ANSYS does not allow to directly extract stress time history from mid node points. It gives only the time history at nodes. Hence it becomes essential to interpolate the stress values at the extraction points 0.5 t and 1.5 t (where t refers to the thickness) from the nodal values. The shape function for the 4 nodded quad shell element shell181 in ANSYS is given as below.

\[
u = \frac{1}{4}(u_i(1-s)(1-t)+u_j(1+s)(1-t)+u_k(1+s)(1+t)+u_l(1-s)(1+t))
\]  \hspace{1cm} (2)

Where \(u\) is the value at any point inside the element, \(u_i\), \(u_j\), \(u_k\) and \(u_l\) are the values at nodes \(i\), \(j\), \(k\) and \(l\) respectively and \(s\), \(t\) are the natural coordinates. The interpolation point lies at the midpoint between the nodes \(i\) and \(l\). As told by Ajmal and Mohammed (2018) the natural coordinates of this point are \(s = -1\) and \(t = 0\) respectively. The stress values have to be interpolated for all time points in the time history. The time histories obtained from ANSYS results for each train load cases with the help of MATLAB coding. This time histories were then used to compute the fatigue damage using a suitable S-N curve. Table 3 show IIW recommendations for meshing and extrapolation.
Table 3. IIW recommendations for meshing and extrapolation (IIW-1823-07).

| Type of model | Relatively coarse mesh | Relatively fine mesh |
|--------------|------------------------|----------------------|
|              | Type a | Type b | Type a | Type b |
| Element size |         |         |         |         |
| Shells       | t x t   | 10 x 10mm | ≤ 0.4 t x t | ≤ 4 x 4 mm |
| Solids       | t x t   | 10 x 10mm | ≤ 0.4 t x t | ≤ 4 x 4 mm |
| Extrapolation points | | | | |
| Shells       | 0.5 t & 1.5 t | 5 & 15 mm | 0.4 t & 1.0 t | 4, 8 & 12 |
| (mid-side points) | (mid-side points) | | | |
| Solids       | 0.5 & 1.5 t | 5 & 15 mm | 0.4 t & 1.0 t | 4, 8 & 12 |
| (surface center) | (surface center) | | | |

6. Calculation of fatigue life

The determination of stress ranges (Δσ) and corresponding number of cycles (n) from the structural hot spot stress time histories was carried out using rain flow cycle counting algorithm. The guidelines for cycle counting are provided by ASTM standard E 1049-85: Standard practices for cycle counting in fatigue analysis (1997). The MATLAB algorithm prepared by Nieslony (2003) is adopted for rain flow counting.

To choose a proper S-N curve is an important part of the fatigue life estimation welded joints. The basic principle of S-N curves has a fatigue limit, below which fatigue failure will not occur and the curve becomes horizontal, named a knee point. It is mostly depicted in terms of the number of cycles on the S-N curve, N=10^7 being the most common assumption. The slope of the curve is taken as 3 below this point. From further study it was found that a constant amplitude fatigue limit does not exist and the S-N curve should continue on the basis of a further decline in stress range at a slope of m=5. Fig. 4 gives the Modified resistance S-N curves of steel defined by IIW-1823-07 for Palm - Miner summation.

![Figure 4. Modified resistance S-N curves of steel for Palm - Miner summation (IIW-1823-07).](image_url)

IIW recommendations defines nine weld types for fatigue life estimation using structural hot spot stress approach. The most suitable fatigue design class for this type of joint as per IIW recommendations is FAT 100. For this FAT class the knee point value is given as 58.5 MPa. The fatigue resistance is limited by the fatigue resistance of the base material for welded joints. The FAT
class of the base material here is FAT 160 and is expected to give higher fatigue resistance than the joint. The equation (3) of an S-N curve is given by,

$$N = C \Delta \sigma^{-m}$$  \hspace{1cm} (3)

Where $\Delta \sigma =$ Stress range and $m$ is the slope of the curve. $C$ is a constant found experimentally which accounts for different factors affecting the fatigue. The S-N curve used for this problem is the two-slope curve proposed by IIW recommendations. The Table 4 gives the design values for the S-N curve for FAT 100.

| FAT class | 100 |
|-----------|-----|
| Stress at $2 \times 10^6$ | 100 MPa |
| Stress at knee point (@ 10^7 cycles) | 58.5 MPa |
| Constant, $C$ | |
| For stress ranges that are above knee point | 2.000x10^{12} |
| For stress ranges that are below knee point | 6.851x10^{15} |
| Slope, $m$ | |
| For stress ranges that are above knee point | 3 |
| For stress ranges that are below knee point | 5 |

The expressions (4-6) for the IIW S-N curve for FAT 100 are given first two below. The $N$ values from the S-N curve and the no of cycles $n$ from the cycle counting results were used in following equations for linear damage accumulation according to Palm-miner rule.

$$N_i = 2.000 \times 10^{12} \Delta \sigma^{-3}, \text{ for } \Delta \sigma \geq 58.5 \text{ MPa}$$  \hspace{1cm} (4)

$$N_i = 6.851 \times 10^{15} \Delta \sigma^{-5}, \text{ for } \Delta \sigma \leq 58.5 \text{ MPa}$$  \hspace{1cm} (5)

$$D = \sum \frac{n_i}{N_i}$$  \hspace{1cm} (6)

The fatigue damage accumulation was done for the structural hotspot stress time histories for all train loads. The damage caused by each train loads were added to get the total damage due to all trains. The fatigue life of the plate girder bridge was then calculated by taking the inverse of the total damage.

7. Fatigue life behaviour for varying velocity

It was found from the analytical analysis using ANSYS that there is mainly a total of 6 critical location for various velocities ranging from 30 kmph to 140 kmph. These locations are considered for the structural hotspot stress extraction as mentioned earlier. When the frequency of loading varies with speed, the location of hotspot stress tends to vary. Even though most of the time most of the same spot repeats as critical for various speed. This makes that spot most vulnerable part in the structure. The critical locations are as shown in the Table 5.
### Table 5. Location of Maximum principal stress.

| Location | Overall View | Isolated View of Critical point |
|----------|--------------|---------------------------------|
| Location 1 | ![Overall View](image1) | ![Isolated View](image2) |
| Location 2 | ![Overall View](image3) | ![Isolated View](image4) |
| Location 3 | ![Overall View](image5) | ![Isolated View](image6) |
| Location 4 | ![Overall View](image7) | ![Isolated View](image8) |
The fatigue life corresponding to each velocity are calculated using a MATLAB program by giving the hotspot stress history as input. The corresponding damage is calculated and its inverse is found to get the fatigue life of structure at that velocity. The results obtained are shown in the Table 6.

Rao et al. (2013) conducted fatigue life analysis of a railway bridge made of plate girders designed as per Indian railway standards using S-N curve approach in frequency domain. The fatigue life estimate of 12.2m span plate girder bridge by Rao et al. (2013) is found to be 45 years. “It is reported that the MBG loading specified by IRS is comparable to RU loading specified in BS 5400: Part 10” (Rao et al., 2013). Hence Rao et al. (2013) had used the RU loading in their study. The fatigue load model used in the present study is based on IRS MBG loading. The geometric properties of the bridge analysed by Rao et al. (2013) are slightly different from the bridge analysed in the present study. Even though the loading conditions and bridge model are not exactly the same for both works, a reasonable comparison may be acceptable.

According to the experience of railway bridge engineers, the time taken for the first fatigue crack to develop could be possibly less than the obtained results. This could be due to many other factors like corrosion that affects the fatigue phenomenon. In Indian railways the toilet wastes splash out of the trains that are highly corrosive in nature. Due to this problem the bridge parts especially, the horizontal bracings corrode and needs replacement after a few years. Other factors like over loading, braking load effects, variations in train traffic, variations in train composition etc. are also not considered in the present study.
Table 6. Fatigue life obtained corresponding to each speed and their critical location.

| Speed (kmph) | Fatigue life           | Critical location |
|--------------|------------------------|-------------------|
| 30           | 62 Years 173 Days      | 1                 |
| 40           | 32 Years 154 Days      | 1                 |
| 50           | 25 Years 28 Days       | 2                 |
| 60           | 45 Years 4 Days        | 1                 |
| 70           | 54 Years 162 Days      | 3                 |
| 80           | 45 Years 4 Days        | 1                 |
| 90           | 45 Years 4 Days        | 1                 |
| 100          | 26 Years 109 Days      | 4                 |
| 110          | 45 Years 4 Days        | 1                 |
| 120          | 45 Years 4 Days        | 1                 |
| 130          | 10 Years 112 Days      | 5                 |
| 140          | 58 Years 277 Days      | 6                 |

8. Conclusion
The fatigue life of a plate girder bridge can be calculated using Palmgren-Miner damage accumulation model through MATLAB coding. The design S-N curve proposed by IIW recommendations was used in the analysis. The results obtained in the present study were found satisfactory.

It is clear from the study that most of the time the bracing tends to have the critical locations, thus periodic repair of the bracing is recommended for healthy service of the plate girder railway bridge. Also considering the unpredictable behaviour of fatigue phenomenon, it is quite hard to find a pattern of effect of velocity on the structure. Though we could accept that for a bridge on which the rolling stock moves at a velocity range between 30 kmph and 140 kmph the theoretical retrofitting should be conducted by 10 years and 112 days. That is taking apart all other factor’s corrosion, unpredicted loads etc. The current study is very complex and sensitive to every parameter considered; thus, a numerical study needs to be validated through actual field observations.

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