Probabilistic Settlement Analysis of Granular Soft Soil Foundation in Southern China Considering Spatial Variability

Lin-Chong Huang, Shuai Huang and Yu Liang

Abstract

In this chapter, the method of combining the theory of random field and numerical analysis was used to systematically analyze the settlement probability of the soft soil foundation in the south of China, considering the effect of spatial variability of soil parameters. Based on the midpoint discretization and Cholesky decomposition, the cross-correlated non-Gaussian random field of cohesion and internal friction angle was constructed, which had considered the cross-correlation, and a single parameter random field of modulus was also constructed. The Monte-Carlo stochastic finite element program for two-dimensional foundation probabilistic settlement was developed in APDL language. The influence of spatial variability of soil parameters on probability foundation settlement was studied. The results indicate that the foundation settlement increases with the increase of coefficient variation and correlation distance. Modulus is the most important parameter for foundation settlement. The settlement of foundation is more sensitive to the correlation distance in vertical direction. Based on exponential square autocorrelation function, the continuity of random fields is obviously better, and the foundation settlement is larger. On the contrary, the fluctuation of random fields is larger, and the foundation settlement is smaller with single exponential autocorrelation function.

Keywords: foundation settlement, soil spatial variability, random field, autocorrelation function, midpoint discretization

1. Introduction

The soft soil is widely distributed in the coastal areas of southern China, which exhibits high compressibility and low shear strength [1]. With the acceleration of infrastructure construction
in the region, many structures are built on soft soil foundation. Therefore, it is of great significance to study the settlement prediction of soft soil foundation. At present, the prediction methods of foundation settlement are mainly classical formula [2, 3] and numerical analysis [4, 5]. However, these two traditional methods have neglected the spatial variability of soil parameters as a result of mineralogical composition, stress history, and deposition process [6]. At present, many scholars have considered the spatial variability of soil parameters when studying on geotechnical engineering. Yan et al. [7] used the field data of Tianjin Port to establish the random field model of the foundation soil, analyzed and obtained the general law of determining the reduction function with the completely unrelated distance method. Li et al. [8] proposed a noninvasive stochastic finite element method for the reliability analysis of underground caverns; the accuracy and efficiency of calculation were improved. Jiang et al. [9] used random field model to characterize the spatial variability of soil hydraulic conductivity, effective cohesion, and internal friction angle. The effects of rainfall intensity, variability of soil parameters, and cross-correlation between parameters on slope reliability were studied. Kenarsari and Chenari [10] simulated soil mass as an anisotropic random field, combined with FLAC2D finite difference model to study the influence of soil spatial variability on settlement of shallow ground. Lo and Leung [11] used Latin hypercube sampling with dependence to simulate the random field, which was coupled with polynomial chaos expansion to approximate the probability density function of model response, and applied it to the reliability analysis of strip foundation and slope. Johari [12] presented a reliability-based analysis of strip-footing settlement by stochastic finite-element method and combined with random finite-element method to improve computational efficiency.

The above researches are to introduce random field theory into geotechnical engineering, considering the spatial variability of soil. There is spatial autocorrelation of soil between any two points in space, which is usually characterized by correlation distance. And the correlation is inversely proportional to the distance between two points. Autocorrelation functions are generally used to solve the correlation distance. Common autocorrelation functions include single exponential (SNX), exponential square (SQX), cosine exponential (CSX), second-order Markov (SMK), and binary noise (BIN) [13, 14]. Unfortunately, many random field researches in geotechnical engineering were assumed to the autocorrelation function of random field simulation. In order to simplify the calculation, the single exponential autocorrelation function was used to characterize the spatial correlation of the soil parameters. There are few studies considering the influence of the selection of autocorrelation function on foundation settlement. In this chapter, the cross-correlated non-Gaussian random field of South China soft soil was simulated by the Cholesky decomposition technique with midpoint discretization, and then a Monte-Carlo stochastic finite element program for probability settlement of two-dimensional foundation was developed, to study the quantitative evaluation of different autocorrelation functions. This chapter mainly studied the influence of the type of autocorrelation function on foundation settlement when considering the variation of parameter variability, correlation distance, and cross-correlation of parameters.
2. Theoretical basis of random field

The spatial variability of soil parameters reflects the unity of correlation and randomness. This characteristic of soil can be well described with the theory of the random field.

2.1. Numerical characteristics

A random field $S(u)$ can be defined as a curve in vectoral space, which is a collection of random variables indexed by a continuous parameter. For the random field, the most important three numerical characteristics are mean ($\mu$), variance ($\sigma^2$), and correlation distance ($\delta$) [15].

The variability of parameters and spatial correlation of soil are all the basic properties of geomaterials. Parameter variability is generally described with coefficient of variation, and the correlation can be described by the correlation distance which is expressed as Eq. (1). Its physical meaning is to measure the size of closely related element in the soil. Within the correlation distance, the soil property of two points is completely correlated, and the geotechnical properties of two points are independent outside the related distance. For the homogeneous random field, the mean and variance are constant, and correlation distance depends only on the distance between two points in the space [13].

\[
\delta = \lim_{u \to \infty} u \Gamma^2(u)
\]  

(1)

where $\Gamma^2()$ is the variance reduction function, which represents the ratio of the mean variance in the range $u$ space to the point variance of the random field.

2.2. Autocorrelation function

Based on a large number of measured data, the autocorrelation of soil random fields can be directly derived with the sample autocorrelation function, which is expressed as Eq. (2) [16].

\[
\rho_{S(\Delta u)} = \rho[S(u), S(u + (\Delta u))] = \frac{\text{COV}[S(u), S(u + \Delta u)]}{\sqrt{\text{var}[S(u)]}\sqrt{\text{var}[S(u + \Delta u)]}}
\]  

(2)

The limited number of field measured data is usually difficult to directly characterize the spatial correlation of soil parameters. Therefore, the theoretical autocorrelation function is used to fit the sample autocorrelation function. Common autocorrelation functions include single exponential (SNX), exponential square (SQX), cosine exponential (CSX), second-order Markov (SMK), and binary noise (BIN). Such five kinds of two-dimensional autocorrelation function expressions and function images are shown in Table 1. The difference of these autocorrelation functions is small when the distance between any two points in the space is large. SQX and SMK are isotropic, and their surfaces are smooth. The edges and corners of SNX, CSX, and BIN are clear, and the continuity is poor.
| Types | Functional expressions $\rho(\tau_x, \tau_y)$ | Function graph ($\delta_x = \delta_y = 1$) |
|-------|------------------------------------------|------------------------------------------|
| SNX   | $\rho(\tau_x, \tau_y) = \exp\left[-2\left(\frac{\tau_x^2}{\delta_x^2} + \frac{\tau_y^2}{\delta_y^2}\right)\right]$ | ![SNX Function Graph](image) |
| SQX   | $\rho(\tau_x, \tau_y) = \exp\left[-\pi\left(\frac{\tau_x^2}{\delta_x^2} + \frac{\tau_y^2}{\delta_y^2}\right)\right]$ | ![SQX Function Graph](image) |
| CSX   | $\rho(\tau_x, \tau_y) = \exp\left[-\left(\frac{\tau_x^2}{\delta_x^2} + \frac{\tau_y^2}{\delta_y^2}\right)\right] \cos\left(\frac{\tau_x}{\delta_x}\right) \cos\left(\frac{\tau_y}{\delta_y}\right)$ | ![CSX Function Graph](image) |
| SMK   | $\rho(\tau_x, \tau_y) = \exp\left[-4\left(\frac{\tau_x^2}{\delta_x^2} + \frac{\tau_y^2}{\delta_y^2}\right)\right] \left(1 + \frac{4\tau_x}{\delta_x}\right) \left(1 + \frac{4\tau_y}{\delta_y}\right)$ | ![SMK Function Graph](image) |
| BIN   | $\rho(\tau_x, \tau_y) = \begin{cases} \left(1 - \frac{\tau_x}{\delta_x}\right) \left(1 - \frac{\tau_y}{\delta_y}\right) & \tau_x \leq \delta_x \quad \text{and} \quad \tau_y \leq \delta_y \\ 0 & \text{else} \end{cases}$ | ![BIN Function Graph](image) |

$\tau_x, \tau_y$ respectively, represent the relative distance between horizontal and vertical directions of any two points. $\delta_x, \delta_y$ respectively, represent the correlation distance between the horizontal and vertical directions.

**Table 1.** Common analytical models for autocorrelation functions.
3. Random field simulation of soft soil in South China

In practical engineering, the soil generally obeys non-Gaussian distribution, and there is some cross-correlation in the soil parameters. For example, there is a significant negative correlation between soil cohesion and internal friction angle. In this chapter, the cross-correlated non-Gaussian random fields of soft ground in South China were simulated, based on Cholesky decomposition technique with midpoint discretization [17–20].

3.1. Simulation process

The variability of Poisson’s ratio and density of soft soil is relatively small. Therefore, the spatial variability of modulus, cohesion, and internal friction angle is only considered in this chapter. The random field considering the cross-correlation between cohesive and internal friction angle is introduced below. Cross-correlated non-Gaussian distribution of random field simulation needs to generate the cross-correlated standard Gaussian random field. The cross-correlated logarithmic random field can be expressed as Eq. (3) [19].

\[
S_i(x, y) = \exp \left( \mu_{lni} + \sigma_{lni} \cdot S_i^D(x, y) \right) \quad (i = c, \varphi)
\]  

(3)

where \((x, y)\) represents the position coordinate of the random field space point; \(\mu_{lni}\), \(\sigma_{lni}\) represent the mean and variance of the normal variable \(ln_i\), respectively, which is solved by Eq. (4); \(S_i^D(x, y)\) represents the relevant standard Gaussian random field.

\[
\sigma_{lni} = \sqrt{\ln \left( 1 + \left( \frac{\sigma_i}{\mu_i} \right)^2 \right)}
\]

\[
\mu_{lni} = \ln \mu_i - \frac{1}{2} \sigma_{lni}^2
\]

(4)

The cross-correlated non-Gaussian random field simulation focuses on the generation of Gaussian distribution of the relevant standard Gaussian distribution field, \(S_i^D(x, y)\). The process is as follows:

(1) The autocorrelation between any two points of the soil is considered, which is characterized by the autocorrelation coefficient matrix \(K\) of the soil. \(K\) is solved by the theoretical autocorrelation function. The Cholesky decomposition of the autocorrelation coefficient matrix \(K\) is performed, \(K = L_1 L_1^T\), and the lower triangular matrix \(L_1\) is obtained.

\[
K_i = \begin{bmatrix}
1 & \rho_{12}^i & \cdots & \rho_{1n_e}^i \\
\rho_{12}^i & 1 & \cdots & \rho_{2n_e}^i \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{1n_e}^i & \rho_{2n_e}^i & \cdots & 1
\end{bmatrix} \quad (i = c, \varphi)
\]

(5)

where \(n_e\) represents the number of random field elements.
(2) Considering the cross-correlation between cohesion and internal friction angle, the cross-correlation coefficient matrix $R$ is used to represent it. Cholesky decomposition of the cross-correlation matrix, $R = L_2 L_2^T$, leads to the lower triangular matrix $L_2$. Due to the transformation in the random field simulation, theoretically, the correction of $R$ and $K$ needs to be modified according to the Nataf model. However, the difference of the correlation coefficient matrix between Gaussian and lognormal random fields is very small [18]. Take the correction coefficient of 1.

$$R = \begin{bmatrix} 1 & \rho_{c,\varphi} \\ \rho_{c,\varphi} & 1 \end{bmatrix}$$ (6)

(3) A set of related standard normal random sample matrices $\alpha$ was derived using Latin hypercube sampling, $\alpha_i = \{\alpha_{1i}^1, \alpha_{2i}^2, \ldots, \alpha_{ni}^n\}, \ (i = c, \varphi)$. According to Eq. (7), the cross-correlated standard Gaussian random field $S_i^D(x, y)$ is obtained.

$$S_i^D(x, y) = L_1 \cdot \alpha \cdot L_2^T$$ (7)

The cross-correlated non-Gaussian random field simulation is completed with the cohesion and friction angle, by taking Eq. (7) into the Eq. (3). The simulation of modulus random field is consistent with the above process, which will not be repeated here. Because it is a single parameter random field, the cross-correlation coefficient need not be considered in the calculation process, and the simulation process is simpler.

3.2. Typical realizations of random fields

Based on MATLAB software, the random field procedure was written according to the process above. A typical South China homogeneous soft soil foundation was adopted to simulate. The size and soil parameter of this foundation were introduced in the Section 4.1. The coefficient of variation of modulus, cohesion, and internal friction angle are 0.3. The cross-correlation coefficient of cohesion and internal friction angle is $-0.5$. The size of random field elements is 0.5 m, the correlation distance in horizontal, and vertical directions are 40 m and 3 m, respectively. Figure 1 shows typical realizations of random field of $c$ and $\varphi$ for five autocorrelation functions.

Figure 1(a), (b), (c), (e) and (f) shows the typical realizations of random fields of cohesion with five autocorrelation functions, respectively. In these figures, the red regions denote a larger strength parameter value, while the blue regions indicate a smaller strength parameter value. The continuity of random fields based on SQX and SMK is obviously better than the other three kinds of autocorrelation functions. And the fluctuation of the SNX is the largest. This conclusion is consistent with the continuity of the theoretical autocorrelation function in Table 1. For Figure 1(c) and (d), the distribution of random fields of $c$ and $\varphi$ is approximately the opposite, where the value of cohesive is large, and the value of internal friction angle is small. The overall trend is negative correlated. The difference between the random fields
4. Example of foundation settlement analysis

In this chapter, a typical southern soft soil ground in China was selected. First, a deterministic model was established (mean value of soil parameters), and then, the probabilistic analysis of ground settlement with the random field finite element model of the soil parameters was carried on. The influence of spatial variability of soil parameters and selection of autocorrelation function on foundation settlement was studied.
4.1. Deterministic analysis

Deterministic calculation does not consider the spatial variability of the parameters, which assigns the same soil parameters to each element. Based on ANSYS software, a two-dimensional foundation plane strain model was established. The horizontal width of this model is 20 m, and the vertical depth is 10 m. There is a rigid strip foundation above the foundation soil with a foundation width of 2 m. Foundation geometry and finite element mesh division are shown in Figure 2. To facilitate the randomness analysis, the mesh size is consistent with the size of the random field in Section 3.2 (0.5 m), which consisted of 800 elements and 861 nodes. Drucker-Prager criterion is adopted to represent the stress-strain behavior of the soil. The contact surface and target surface are simulated by CONTA172 and TARGE169, respectively [23]. Both lateral boundaries are rollers, and the base is full fixity. There is a concentrated load $P = 100$ kN on the foundation. Calculated parameters are as follows: cohesion 20 kPa, internal friction angle $12^\circ$, unit weight 18 kN/m$^3$, modulus of deformation 4 MPa, and Poisson’s ratio 0.25.

Figure 3 shows the vertical displacement cloud for deterministic calculation. From Figure 3, the maximum settlement is 41.18 mm, which occurs just below the rigid strip foundation. In order to verify the accuracy of the model calculation results, the traditional hierarchical design method was adopted, and the theoretical result is 39.1 mm, which is close to the simulated one, with the error of 5.3%. It shows that the numerical simulation result is reliable.

4.2. Randomness analysis

The spatial variability of modulus, cohesion, and internal friction angle was mainly considered in this chapter [12]. About 30 calculation conditions were designed as shown in Table 2. In each condition, the random fields of $E$, $c$, and $\phi$ were simulated by five kinds of autocorrelation functions. Based on APDL language, the Monte-Carlo stochastic finite element calculation program for two-dimensional foundation was constructed. Specifically, $E$, $c$, and $\phi$ were defined as input variables, and the values at each random field were brought into the finite

![Figure 2. Finite element model of foundation settlement.](image-url)
element calculation. Then, the results of the finite element calculation were obtained. The maximum vertical displacement (Umax) is the output variable, and the statistics of Umax are required.

Take the RF-E3 condition as an example, where the type of autocorrelation function is SNX. Figure 6 shows the result of randomness analysis for foundation settlement within the confidence limit of 95%. In Figure 4(a), the mean of maximum settlement of the foundation tends to be stable when the times of simulation reach to 1000. The rest of the calculation conditions also costs the same simulation times. The mean of random analysis in RF-E3 condition is 45.096 mm, which is slightly larger than the result of deterministic analysis. Figure 4(b) shows the cumulative distribution curve of the maximum settlement of the foundation. The probability of the maximum settlement of the foundation between the 30 and 60 mm interval is 95%. The foundation settlement can be predicted by probability. If the value of settlement is used as an index of foundation reliability, the failure probability of foundation can be read from the figure.

4.2.1. Analysis of parameter variability

The variability of soil parameters is represented by coefficient of variation (COV) in statistics. The influence of spatial variability on foundation settlement is analyzed by 15 kinds of calculation conditions of RF-E1~RF-φ5. At the same time, the influence of autocorrelation function on foundation settlement is studied.

The effects of coefficient of variation on ground settlement with E, c, and φ are given in Figure 3, respectively. It can be seen from the figure that with the increase of coefficient of
variation of soil parameters, the mean of the maximum settlement also increases, and all the mean of randomness analysis are larger than the result of deterministic analysis, which indicates that the parameter variability of soil has an important influence on foundation

| Variable | Mean   | Coefficient of variation | δ/m |
|----------|--------|--------------------------|-----|
| **E**    | 4 MPa  | 0.1 0.3 0.3 40 3        | −0.5| RF-E1  |
|          |        | 0.2 0.3 40 3            |     | RF-E2  |
| c        | 20 kPa | 0.3 0.3 40 3            | −0.5| RF-c1  |
| φ        | 12°    | 0.3 0.3 40 3            | −0.5| RF-φ1  |
| δₓ       | —      | 0.3 0.3 20 3            | −0.5| RF-x1  |
| δᵧ       | —      | 0.3 0.3 40 1            | −0.5| RF-γ1  |
| ρₓ,ᵧ     | —      | 0.3 0.3 40 3            | −0.7| RF-r1  |

Table 2. Calculation conditions.
settlement. In other words, traditional deterministic analysis underestimates foundation settlement. It is necessary to consider the variation of soil parameters in engineering practice. Contrast the rangeability of the mean of maximum settlement in Figure 5(a)–(c), the curve of modulus changes larger than cohesion and internal friction angle obviously, which means that the parameter sensitivity, \( E > \varphi > c \). The influence trend of different autocorrelation function on foundation settlement is basically the same. The mean of maximum settlement obtained by SQX was largest and the SNX was smallest. With the increase of coefficient of variation, the difference of the calculated results with the five autocorrelation functions becomes greater. In Figure 5(a), the difference of settlement calculated by different autocorrelation functions is only 0.1 mm when \( \text{COV}_E = 0.1 \). The difference value increases to 2 mm when \( \text{COV}_E = 0.5 \), which accounts for 20% of the settlement variation value (10 mm) caused by parameter variability. This indicates that the influence of autocorrelation function should be considered when the coefficient of variation becomes larger. As the coefficient of parameter variation increases, the discreteness of random fields increases. These facts indicating the increase in the probability of the appearance of element with low value will cause the increase of foundation settlement. Besides, the smoothness and continuity of the random field by SNX is poor; thus, the elements with low value are discrete. The stability of foundation calculated by SNX is improved, and the foundation settlement calculated by it comes to the smallest.

4.2.2. Analysis of spatial correlation

Correlation distance is one of the important parameters to characterize the spatial variability of soil parameters [24]. The influence of horizontal correlation distance \( (\delta_x) \) and vertical correlation distance \( (\delta_y) \) on foundation settlement is studied. About 10 calculation conditions of RF-x1~RF-y5 are set. The random field model is degraded into random variable model when the correlation distance of all directions approaches infinity. Thus, the parameters are completely correlated to the model area.
Figure 6 shows the effect of correlation distance on the mean of maximum settlement. The black line in the figure represents the result of the random variable model. The mean of maximum settlement increases with the increase of the correlation distance, which gradually reaches to convergence. The influence of vertical correlation distance on settlement is more significant than that of horizontal correlation distance. It is necessary to simulate the spatial variability of soil parameters with the anisotropic random field. The results of random fields are less than that of the random variable model (46.91 mm). It indicates that ignoring the spatial variability of the soil will lead to the overestimation of the settlement of the foundation. The mean of maximum settlement obtained by SQX was largest and the one obtained by SNX was smallest. As the correlation distance increases, the continuity of the random field will be significantly improved. The elements with low value are also distributed continuously, which is equivalent to the formation of weak intercalated layer in the foundation. The stability of foundation is reduced and the foundation settlement increases. Compared with other

Figure 5. Curve of the mean of maximum settlement with coefficient of variation. (a) Modulus, (b) cohesion, and (c) internal friction angle.
autocorrelation functions, the continuity of the random field by SNX autocorrelation function is poor. Thus, the foundation settlement by SNX comes to the smallest.

In order to incorporate the dependence between the strength parameters, the cross-correlation coefficient ($r_{c,\varphi}$) is needed. The study shows that there is a significant negative correlation between $c$ and $\varphi$ [25]. Figure 7 shows the effect of cross-correlation between cohesion and friction angle on foundation settlement. With the increase of cross-correlation coefficient, the mean of

![Figure 6](image-url)

**Figure 6.** Curve of the mean of maximum settlement with correlation distance. (a) Horizontal correlation distance and (b) vertical correlation distance.

![Figure 7](image-url)

**Figure 7.** Effect of cross-correlation between cohesion and friction angle on settlement.
maximum settlement increases. This indicates that neglecting the negative correlation between cohesion and internal friction angle will overestimate the settlement of foundation. Considering the negative correlation between cohesion and friction angle, the increase of cohesion corresponds to the decrease of friction angle, which leads to the decrease of the total shear strength variance of soil. The stronger the negative correlation is, the smaller the variance of total shear strength parameters is, which means the small scale of fluctuation of random fields. Thus, the foundation settlement is decreased. The maximum settlement value can be obtained by SQX; the value of SNX is smaller than the other autocorrelation functions obviously.

In summary, the selection of autocorrelation function has obvious influence on the analysis of foundation settlement. The influence trend is basically consistent with the change of statistical parameters of random fields. The settlement value selected by SQX is the largest, and the settlement value selected by SNX is the smallest. In other words, the results of foundation settlement are safer for the designers based on SQX.

5. Conclusions

This chapter combined Cholesky decomposition midpoint method with Monte-Carlo method. The calculation method of two-dimensional ground settlement was obtained based on random field theory. Considering the influence of the autocorrelation function selection in the random field simulation, several conclusions are drawn from this study:

1. Based on the Cholesky decomposition technique with midpoint discretization, the cross-correlated non-Gaussian random fields considering cross-correlation and the independent non-Gaussian random fields are convenient to simulate. The random fields are easier to be introduced into the stochastic finite element model. By changing the type of autocorrelation function in simulation, the influence of the selection of autocorrelation function on foundation settlement is studied. Combined with the typical realization of the random field in Section 3.2, the mechanism of influence on foundation settlement caused by statistics of soil parameters and the type of autocorrelation function can be further explored.

2. The variability of soil parameters has a significant influence on the calculation results of foundation settlement, and the results of randomness analysis are larger than the results of deterministic analysis. The mean value of maximum settlement increases with the variation coefficient of the parameters, and the modulus E of soil affects the calculated value of foundation settlement most. Therefore, the variability of soil parameters should be considered in the calculation of foundation settlement.

3. Spatial correlation of soil has a significant impact on the calculation of foundation settlement. The larger the correlation distance is, the larger the maximum settlement of foundation is. The settlement of foundation is more sensitive to the correlation distance in vertical direction. The mean of maximum settlement increases with the increase of the cross-correlation coefficient between cohesion and internal friction angle.
4. The selection of different autocorrelation functions has a significant effect on foundation settlement; the values of settlement based on SQX and SMK are larger, and that based on SNX and BIN is smaller. The result of SNX is significantly smaller than that of the other types. With the increase of coefficient of variation, the influence of the selection of autocorrelation function on the settlement value also increases.

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Author details

Lin-Chong Huang, Shuai Huang and Yu Liang*

*Address all correspondence to: liangyu25@mail.sysu.edu.cn

Sun Yat-sen University, Guangzhou, China

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