Effectiveness of Variable Rotor Blades Technique & Variable Stator Vanes Technique in Adjusting Axial-flow Compressor

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Abstract. Multi-stage Axial-flow compressor is the main driver of continuous transonic wind tunnel and it usually has ultra-wide operating range. High performance wind tunnel generally requires its multi-stage axial-flow compressor has high economic performance and stability in its ultra-wide operating range. The combination of variable compressor rotational speed and variable geometry technique, variable rotor blades technique (VRBT) and variable stator vanes technique (VSVT), has been widely adopted in extending the stable working range with high efficiency of multi-stage axial-flow wind tunnel compressor. Adjusting Effectiveness was defined in this paper to quantify the effectiveness of VRBT and VSVT in extending stable and high aerodynamic efficiency operating range of wind tunnel axial-flow compressor under various operating conditions. The results show that the adjusting effectiveness of VRBT and VSVT is a function of reaction degree, flow coefficient, loading coefficient and the amount of loading coefficient change.

1. Introduction
Multi-stage axial-flow compressor is extensively adopted in continuous transonic wind tunnel applications to provide air flow through the tunnel circuit ducting for establishing test conditions, and it has a direct and vital influence on the aerodynamic and economic performance of wind tunnel. Testing Mach number range of continuous transonic wind tunnels is wide and is generally range from 0.2 to 1.3 or even higher, which leads to a ultra-wide loading line for axial-flow compressor (Figure 1). The general aerodynamic design requirements for wind tunnel compressor can be stated that multi-stage axial-flow compressor with high adiabatic efficiency and sufficient surge margin in its ultra-wide operating range. Thus, how to extend working range with high adiabatic efficiency and satisfactory surge margin of wind tunnel compressor with such a wide operating range is the main challenge that compressor designer has to deal with.

In general, variable geometry method, variable rotor blades technique (VRBT) or variable stator vanes technique (VSVT), is used together with variable speed technique to extend the stable high efficiency working range of wind tunnel compressor.

A three-stage axial-flow compressor with variable stator vanes was used in the Arnold Engineering Development Complex (AEDC) 16-foot transonic wind tunnel[1] and the National Wind Tunnel Complex (NWTC) project[2]. R.A.E. considered the VRBT was not realistic in wind tunnel compressor applications in 1950s[3]. In the early 1960s, however, German adopted the manual variable
rotor stagger technique in the multi-stage axial-flow compressor for the AVA supersonic wind tunnel[4].

Although variable geometry technique has long been used in various wind tunnel compressor applications, few public literature can be found in the area of comparative analysis of these two kinds of variable geometry method. In this paper, a variable -Adjusting Effectiveness- was defined and a mathematical model was developed for the purpose of quantifying the effectiveness of these two kinds of variable geometry method in adjusting wind tunnel compressor. This article is organized as follows. The first section describes the derivation of the mathematical model. A subsequent section presents the comparative analysis results and analysis. This article ends with some conclusions.

![Figure 1. Typical Working Line of Wind Tunnel Compressor](image)

**2. Mathematical Model**

According to the basic theory of compressor aerodynamics, flow coefficient ($\phi$) and loading coefficient ($\psi$) were defined as follows:

$$
\psi = \frac{V_1 \sin (\alpha_1) - V_2 \sin (\alpha_2)}{U}, \quad \phi = \frac{V}{U}
$$

Flow coefficient determines the incidence as well as the mass/volume flow rate into blade row. Loading coefficient is a direct reflection of total pressure ratio. Thus, a typical constant rotational speed performance line of axial-flow compressor, as it was show in Figure 2, can be presented using flow coefficient and loading coefficient. Working condition A, B and C in Figure 3 represent the optimum incidence condition, negative incidence condition and positive condition respectively. For a fixed compressor rotational speed, regulating the stagger angle of IGV/stator vanes or rotor blades can change the working condition of compressor and achieve the purpose of extending the axial-flow compressor operating range.

![Figure 2. Typical Axial-flow Compressor Performance Line](image)

Adjusting Effectiveness ($\eta_{adjusting}$) was defined to quantify the effectiveness of the VRBT and VSVT in adjusting wind-tunnel compressor. As can be seen from equation 2, Adjusting Effectiveness
is the ratio of the amount of loading coefficient change ($\Delta \psi$) to the amount of stagger angle change ($\Delta \gamma$).

$$\eta_{\text{adjusting}} = \frac{\Delta \psi}{\Delta \gamma}$$  \hspace{1cm} (2)

Two dimensional cascade model and velocity triangle, which were presented in Figure 3, were used in the process of developing the mathematical model.

In axial-flow compressor aerodynamic design, the inlet/outlet axial velocity ratio is given as an aerodynamic design input and it generally equal or close to 1.0. Thus, in order to simplify the derivation of the analysis model, the axial velocity at the inlet of the cascade was assumed to equal to that at the outlet. Re-plot the inlet and outlet velocity triangle as Figure 4. For a given rotational speed ($U$), a given loading coefficient and a given flow coefficient, $\Delta P_{1AB}$ and $\Delta P_{2AB}$ denote the inlet and outlet velocity triangle respectively.

For a given change in loading coefficient is ($\Delta \psi$). If variable IGV/stator vanes method was used, the inlet velocity triangle will change from $\Delta P_{1AB}$ to $\Delta P'_{1AB}$, the variation of the IGV/stator stagger angle is $\angle P_{1}BP'_{1}$. If variable rotor blade technique was adopted, the outlet velocity triangle will change from $\Delta P_{2AB}$ to $\Delta P'_{2AB}$, the variation of the rotor stagger angle is $\angle P_{2}AP'_{2}$. The edge length of each velocity triangle is as follows:

$$P_{PB} = \sqrt{1 + (\tan \alpha)^2 \phi}, \quad P_{PB} = \sqrt{\phi^2 + (\Delta \psi + \phi \tan \alpha)^2}, \quad P_{P_{1}} = \Delta \psi$$ \hspace{1cm} (3)

$$P_{P_{2}}A = \sqrt{1 + \phi \tan \alpha - \psi}^2 + \phi^2, \quad P_{P_{2}}A = \sqrt{(1 + \phi \tan \alpha - \psi - \Delta \psi)^2 + \phi^2}, \quad P_{P_{2}} = \Delta \psi$$ \hspace{1cm} (4)

The absolute flow angle ($\alpha$) was eliminated so that the Adjusting Effectiveness can be presented as a function of more commonly used aerodynamic design inputs of axial-flow compressors. According to the definition of the reaction degree ($R$)[5], equation 5 and 6 can be derived.

$$\cos \beta = \phi \left( \phi^2 + (R + \psi / 2)^2 \right)^{-1/2}, \quad \tan \alpha = \frac{\phi \tan \beta - 1}{\phi}, \quad -90^\circ < \alpha, \beta < 90^\circ$$ \hspace{1cm} (5)
\alpha = \arctan \left( \frac{R + 0.5\psi - 1}{\phi} \right) \quad (6)

Substitute equation 6 into equation 3 and 4, rewrite equation 3 and 4 as follows:
\begin{align*}
P_B &= \sqrt{\phi^2 + (1 - R - 0.5\psi)^2}, \quad P'_B = \sqrt{\phi^2 + (-1 + R + 0.5\psi + \Delta\psi)^2}, \quad P'_B = \Delta\psi \\
P_A &= \sqrt{(R - 0.5\psi)^2 + \phi^2}, \quad P'_A = \sqrt{(R - 0.5\psi - \Delta\psi)^2 + \phi^2}, \quad P'_A = \Delta\psi \quad (7)
\end{align*}
\begin{align*}
P_{A} &= \sqrt{(R - 0.5\psi)^2 + \phi^2}, \quad P'_{A} = \sqrt{(R - 0.5\psi - \Delta\psi)^2 + \phi^2}, \quad P'_{A} = \Delta\psi \quad (8)
\end{align*}

The variation of rotor stagger angle and IGV/stator stagger angle can be calculated using equation 9 and 10 respectively.
\begin{align*}
\Delta\gamma_{\text{rotor}} &= \arccos \left( \frac{(P_B)^2 + (P'_B)^2 - (P'_A)^2}{2(P_B)(P'_B)} \right) \quad (9)
\end{align*}
\begin{align*}
\Delta\gamma_{\text{stator}} &= \arccos \left( \frac{(P_A)^2 + (P'_A)^2 - (P'_B)^2}{2(P_A)(P'_A)} \right) \quad (10)
\end{align*}

It is clear, from the definition of adjusting effectiveness as well as from equation 7 to 10, that adjusting effectiveness is a function of reaction degree, flow coefficient, loading coefficient and the amount of loading coefficient change and can be expressed using the following mathematical function:
\[
\eta_{\text{adjusting}} = f(\psi, \phi, \Delta\psi, R) \quad (11)
\]

3. Results and Analysis
In the following analysis, the range of reaction degree, loading coefficient and flow coefficient were chosen base on the Lewis Chart and are as follows:
\[
R \in [0.5 \ 0.9], \phi \in [0.2 \ 0.7], \psi \in [0.15 \ 0.45] \quad (12)
\]

The adjustable range was chosen so that the loading coefficient of compressor after adjusting still lies within the range from 0.15 to 0.45

3.1. Adjusting Effectiveness of VRBT
Figure 5 to Figure 7 represent the change of the Adjusting Effectiveness of VRBT with the variation of reaction degree, flow coefficient, loading coefficient and the amount of loading coefficient change.

As can be seen from Figure 5, for compressor with a reaction degree of 0.5, the Adjusting Effectiveness of VRBT decreases with the increase of the amount of loading coefficient change in conditions with flow coefficient and loading coefficient lie in the range defined in equation 12.

For working condition whose flow coefficient range from 0.4 to 0.7, it is clear from Figure 5 that the Adjusting Effectiveness of VRBT decrease with the decrease of flow coefficient. Figure 5 also demonstrates that, for a given amount of loading coefficient change and a given flow coefficient, the Adjusting Effectiveness of VRBT increase with the decrease of loading coefficient.

From Figure 5 to Figure 7, it is easy to infer that the trend of the variation of Adjusting Effectiveness with the variation of the amount of loading coefficient change is similar for compressor with reaction degree in the range of 0.5 to 0.9.

Figure 5 to Figure 7 also show that the Adjusting Effectiveness of VRBT increase with the increase of reaction degree, and the amplitude of the Adjusting Effectiveness change increase with the decrease of flow coefficient for given amount of loading coefficient change and a given loading coefficient.

As can be seen from Figure 7, for compressor with a reaction degree of 0.9, the Adjusting Effectiveness of VRBT increases with the decrease of flow coefficient.

3.2. Adjusting Effectiveness of VSVT
Figure 8 to Figure 10 show the change of the Adjusting Effectiveness of VSVT with the variation of reaction degree, flow coefficient, loading coefficient and the amount of loading coefficient change.
Figure 8 clearly demonstrates that, for compressor with reaction degree of 0.5, the Adjusting Effectiveness of VSVT increases with the increase of the amount of loading coefficient change in conditions with flow coefficient and loading coefficient lie in the range defined in equation 12. As the last chart in Figure 10 shows, for high-reaction highly loaded compressor, the Adjusting effectiveness of VSVT decreases with the increase of the amount of loading coefficient change.

Figure 5. Adjusting Effectiveness via VRBT (R=0.5)
The Adjusting Effectiveness of VSVT, as can be seen from Figure 8 to Figure 9, decreases with the increase of loading coefficient for the same amount of loading coefficient change and the same flow coefficient condition. Figure 10, however, presents a contrary trend. Figure 8 to Figure 10 demonstrates that for low-reaction low-loaded compressor, VSVT has larger Adjusting Effectiveness in small flow coefficient conditions. For high-reaction highly-loaded compressor, however, VSVT has greater Adjusting Effectiveness under large flow coefficient conditions. Figure 8 to Figure 10 also shows that the Adjusting Effectiveness of VSVT decreases with the increase of reaction degree.

3.3. Adjusting Effectiveness Difference Analysis
Figure 11 to Figure 13 show the change of the Adjusting Effectiveness difference between VRBT and VSVT with the variation of reaction degree, flow coefficient, loading coefficient and the amount of loading coefficient change.
It is clear from Figure 11 to Figure 13 that the difference between the adjusting effectiveness of VRBT and VSVT change linearly with the variation of the amount of loading coefficient change.

As can be seen from Figure 11, for compressor with reaction degree of 0.5, VSVT is more effective than VRBT in any working conditions except when the amount of loading coefficient change is smaller than 0.0. When the amount of loading coefficient change is larger than 0.0, Figure 11 demonstrates that the smaller the flow coefficient the greater superiority the VSVT has.

For any working condition with flow coefficient in the range of 0.2 to 0.7 and loading coefficient in the range of 0.15 to 0.45, it is easy to infer from Figure 12 and Figure 13 that, within the adjustable range, VRBT is more effective than VSVT in adjusting compressor with reaction degree of 0.7 to 0.9. Figure 12 and Figure 13 also demonstrate that the smaller the flow coefficient, the larger superiority margin the VRBT has.

![Figure 8. Adjusting Effectiveness via VSVT (R=0.5)](image-url)
Figure 9. Adjusting Effectiveness via VSVT (R=0.7)

Figure 10. Adjusting Effectiveness via VSVT (R=0.9)
Figure 11. Adjusting Effectiveness Difference (R=0.5)

Figure 12. Adjusting Effectiveness Difference (R=0.7)
Figure 13. Adjusting Effectiveness Difference (R=0.9)

4. Conclusion
Adjusting Effectiveness was defined to quantify the effectiveness of the VRBT and the VSVT in adjusting wind tunnel compressor. It is found that the adjusting effectiveness if a function of reaction degree, flow coefficient, loading coefficient and the amount of loading coefficient change;

For low-reaction low-loaded wind tunnel compressor, VRBT has higher Adjusting Effectiveness in conditions with large flow coefficient; while for high-reaction highly-loaded wind tunnel compressor, VRBT has higher Adjusting Effectiveness in conditions with small flow coefficient;

For low-reaction low-loaded wind tunnel compressor, VSVT has higher Adjusting Effectiveness in conditions with small flow coefficient; while for high-reaction highly-loaded wind tunnel compressor, VSVT has higher Adjusting Effectiveness in conditions with large flow coefficient;

The adjusting effectiveness of VRBT increase with the increase of reaction degree, while the Adjusting Effectiveness of VSVT decrease with the increase of reaction degree; For high-reaction compressor adjusting, VRBT is more effective than VSVT from the aerodynamic point of view.

References
[1] Estabrooks B B, Robinson R A, and Jones M H 1961 Aerodynamic Performance of A Third Stage Plenum Evacuation Compressor of The ARNOLD CENTER Propulsion Wind Tunnel. AEDC-TN-61-90.
[2] Jim Young, John Dison, Mike Chaney, et al. 1996 Compressor & Drive System Segment Specification. NWT-SS-B-300-0000-01.
[3] Cheshire L J, Evans J Y G, Goodsell W A, et al. 1958 The design and construction of the compressor for the 8ft by 8ft high-speed wind tunnel at R.A.E. Bedford PROC. INST. MECH. E.172 549-583
[4] Hans Hofmann and Heidenheim. 1967 Constructive details of a 8-stage axial compressor for the transonic wind tunnel of aerodynamic testing facility Gottingen, Voith Research and construction. Issue 17, Item 5 (In German)
[5] N.A. Cumpsty 2004 Compressor Aerodynamics(KRIEGER PUBLISHING COMPANY, Malabar Florida)