Exploring bandgap generation mechanism of phonon crystal

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Abstract

In this paper, based on the theoretical research of structural modal analysis, different types of phonon crystal modal structures are designed for the first time, and the characteristics and the generation mechanism of the bandgap were studied through theoretical calculations and experiments. According to the phenomenon in the experimental results, we can find that the vibration transmission characteristics of phonon crystal structure α3 are the best, and it is also superior to that of phonon crystal structure α10 (full period structure). Therefore, the comparison of theoretical analysis with experimental phenomena shows that the bandgap generation mechanism should be modal resonance instead of local resonance in the finite periodic structure. The profound reason lies in there is no separate Z direction local vibration mode of periodic structure in the vibration mode of finite structure, and the bandgap of finite structure is the mode superposition torsional resonance mechanism between scatterer and substrate mode.

1. Introduction

Phonon crystal is a kind of periodic structure for acoustic vibration control, which is composed of two (or one) more materials with different elastic properties. In 1992, M. M. Sigalas and E. N. Economou verified the elastic wave bandgap characteristics of three-dimensional periodic lattice structures by transfer substrate method [1]. In 1993, M S Kushwaha proposed the concept of phonon crystal first and obtained the complete bandgap of the elastic wave through analysis [2]. In 1995, R Martinez-Sala et al verified the existence of the elastic wave bandgap for the first time through the acoustic test of the ‘flowing melody’ sculpture [3]. In 2000, Liu proposes the localized resonance phonon crystal theory, which can control the propagation of large-wavelength acoustic wave by small size scale phonon crystal [4].

The researches of the formation mechanisms of the phonon crystal bandgap by many scholars show that there are two types of them [4–14]: Bragg scattering mechanism and Local resonance mechanism. Compared with the Bragg scattering mechanism, the localized resonance mechanism considered that the scatterer resonance mode and the substrate vibration mode are coupled to each other to generate a bandgap [4, 9–14], which is the key factor in the bandgap formation [4, 9, 15–18]. The Bragg scattering mechanism considered that the periodic arrangement of structures plays a leading role, which focused on the process analysis of elastic wave or acoustic wave propagation in structure [3–8, 19, 20].

The formation of the bandgap mainly depends on the material characteristics of the substrate, the array form and filling rate of the scatterer, and the physical characteristic difference of the scatterer and the substrate, and its corresponding wavelength has a similar scale with the wavelength of the transverse wave. Ideal phonon crystals have specific acoustic properties. For defective phonon crystals, there are specific acoustic characteristics, such as defective wave confinement or defective wave conduction in the presence of elastic waves of a particular frequency [21]; for the composite materials composed of different materials, negative refraction phenomena will be exist [22], and so on.

However, according to the bandgap theory of phonon crystals, some scholars have found through theoretical and experimental comparisons that the acoustic wave in some passband frequency range is greatly
attenuated in the theoretical calculation, which is called ‘deaf band’ phenomenon, except for the elastic wave that cannot propagate in the bandgap frequency range. It can be seen that different types of phonon crystals have their unique bandgap characteristics. The local resonance phonon crystals mainly focuses on constructing a low frequency resonance resonator, which regulate the bandgap by adjusting the cladding stiffness; while Bragg type phonon crystals regulate the bandgap width by adjusting the elastic constants and density of the substrate and scatterer. It can be seen that, whether Bragg phonon crystals or local resonance phonon crystals, the material or structure type that can generate bandgap is usually constructed first, then the arrangement, material properties, filling rate, etc of the periodic elements are changed to obtain the cause of formation and law of bandgap through experimental or theoretical analysis.

In terms of theoretical analysis, there are some structural bandgap analysis methods for phonon crystals, such as plane wave expansion [23, 24], finite difference time domain [25, 26], finite element method (FEM) [27, 28] and so on. The energy band structure diagrams of one-dimensional, two-dimensional or three-dimensional periodic structures can be solved by Bloch theorem, and the directional or complete bandgap frequency range can be identified from it. For the transfer characteristics of the finite structure, the characteristics of vibration transmission loss were studied by comparing the vibration frequency response function or acoustic absorption coefficient and sound permeability coefficient with the energy band structure diagram of periodic structures [29–33]. Subjected to the difference in scale between the phonon crystal structure of infinite period and finite period, the bandgap range of an ideal phonon crystal has a perfect elastic wave durance effect. However, the vibration transfer characteristics of finite structures are difficult to be accurately indicated with their energy band structure diagrams in the actual analysis. In addition, elastic waves can still pass through the finite structure, although they decay to a certain extent, so it is impossible to realize the active design of the structure with the periodic structure bandgap characteristics.

Since the generation of local resonance bandgap depends on the resonance characteristics of the scatterer itself and the correlation of the elastic wave in the substrate, it can be seen that the low-frequency bandgap characteristics depend on the coupling characteristics of the scatterer local resonance and the substrate vibration. The bandgap widths and the start and stop frequencies of the periodic structure are determined by the coupling vibration strength of the scatterer and substrate, and the detailed demonstrations on the bandgap formation mechanisms in finite structures are lacking. In order to explore the relationship between the structure of the substrate and the coupled vibration of the substrate and the scatterer and extend the bandgap mechanism study of the phonon crystals, a periodic phonon crystal plate structure and the different types of phonon crystal modal structures are designed in this paper, and the theoretical research foundation based on the structural modal analysis is proposed. The bandgap characteristics and its formation mechanisms are studied through theoretical analysis and experiments. Theoretical analysis and experimental results show that the local resonance bandgap mechanism is the modal resonance bandgap mechanism. The phonon crystal structure or modal structure is designed based on the modal analysis, combined with the propagation characteristics of elastic waves or acoustic waves, which opens up a new research situation for the low-frequency vibration and noise reduction of phonon crystals and also lays a foundation for the systematic study and engineering application of the formation mechanisms of phonon crystals and opens up the new theoretical research direction of phonon crystals in the future.

2. Theoretical and experimental study on the mechanism of modal resonance bandgap

2.1. Phonon crystal modal structure design
In this paper, the high-order modal structure (α10) of phonon crystal was constructed as shown in figures 1(f) and low-order modal structure (α11~α9) of phonon crystal as shown in figure 2 (according to the modal analysis results of the thin plate structure, as shown in figure 3, scatters were added at the modal position to construct low-order modal structures of different types of phonon crystals), including the stainless steel substrate plate with the thickness h1 = 1 mm, length l = 418 mm, and width b = 228 mm. The radius of the stainless steel column scatterer r = 15 mm, height d = 24 mm. The length of the periodic structural unit a = 38 mm, fill ratio f = πr^2/a^2 = 0.489.

2.2. Mechanism of modal resonance bandgap
Based on multi-degree of freedom dynamics, combined with modal analysis theory [34–37], the mechanism of modal resonance bandgap formation is as follows:
According to the theory of structural dynamics, it can be seen that for multi-degree of freedom vibration systems, the differential equation of motion is satisfied as:
where \( M \) is the mass matrix, \( C \) is the proportional damping matrix, which is satisfied with \( C = \alpha M + \beta K \) (\( \alpha \), \( \beta \) is the coefficient), \( K \) is the stiffness matrix, \( X \) is the displacement array, \( \dot{X} \) is the velocity array, \( \ddot{X} \) is the acceleration array, and \( F \) is the force array of external excitation.

It can be obtained by Laplace transform on the equation (1) as follows:

\[
(M\dot{s}^2 + C\dot{s} + K)X(s) = F(s),
\]

Let \( s = j\omega \), at this point, the equation of motion of the system is:

\[
(K - \omega^2M + j\omega C)X(\omega) = F(\omega).
\]

As the vibration theory shown, for linear time-invariant systems, the response in the system can be expressed as a linear combination of modal responses of each-order mode, for the response of any point \( f \) is expressed as:

\[
\]
Figure 2. Modal shape diagram of the substrate. (a) 27.247 Hz, 1st order bending mode. (b) 51.175 Hz, 1st order torsional mode. (c) 82.847 Hz, 2nd order bending mode. (d) 106.67 Hz, 2nd order torsional mode. (e) 121.87 Hz, bending and torsional combined mode. (f) 142.13 Hz, bending and torsional combined mode. (g) 181.93 Hz, torsional combined mode. (h) 183.3 Hz, bending and torsional combined mode. (i) 209.08 Hz, bending and torsional combined mode.

Figure 3. Phonon crystal modal structure diagram: (a) $\alpha_1$, 1th modal structure diagram, (b) $\alpha_2$, 2th modal structure diagram, (c) $\alpha_3$, 3th modal structure diagram, (d) $\alpha_4$, 4th modal structure diagram, (e) $\alpha_5$, 5th modal structure diagram, (f) $\alpha_6$, 6th modal structure diagram, (g) $\alpha_7$, 7th modal structure diagram, (h) $\alpha_8$, 8th modal structure diagram, (i) $\alpha_9$, 9th modal structure diagram.

\[ x_i(\omega) = \phi_{i1} q_1(\omega) + \phi_{i2} q_2(\omega) + \cdots + \phi_{iN} q_N(\omega) = \sum_{r=1}^{N} \phi_{ir} q_r(\omega), \]
where \( \phi_p \) is the mode factor of the \( l \)th observation point and the \( r \)-order mode, \( q_r(\omega) \) is the \( r \)-order modal coordinates. For the array composed of the mode-shape system of \( N \) observation points, it is satisfied as follows:

\[
\phi_r = \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_N
\end{bmatrix},
\]

where \( \phi_r \) is the \( r \)th order modal vector, which represents the vibration shape of this mode. The matrix composed of various modal vectors is an \( N \times N \)-order modal matrix \( \Phi \), that is, \( \Phi = \begin{bmatrix} \phi_1 & \phi_1 & \cdots & \phi_N \end{bmatrix} \).

In the same way, the array composed of various modal vectors is an \( N \times 1 \)-order modal matrix \( Q \), that is,

\[
Q = \begin{bmatrix} q_1(\omega) & q_2(\omega) & \cdots & q_N(\omega) \end{bmatrix}^T.
\]

So the system response array can be obtained as \( X(\omega) = \Phi Q \).

Thus equation (3) transform into:

\[
(K - \omega^2M + j\omega C)\Phi Q = F(\omega),
\]

where \( \omega = [\omega_1 \, \omega_2 \, \cdots \, \omega_N] \) is the \( N \)-order natural frequency.

According to the modal analysis theory \([36, 37]\), for any order mode, the stiffness matrix, the mass matrix, and the damping matrix satisfy the orthogonality, namely:

\[
\Phi_r^T K \Phi_r = K_r,
\]

\[
\Phi_r^T M \Phi_r = M_r,
\]

\[
\Phi_r^T C \Phi_r = C_r,
\]

where \( K_r, M_r, C_r \) is the \( r \)th order modal stiffness, modal mass, and damping matrix.

Since the natural frequency and mode shape in the linear elastomer structure are determined by the structural mass and stiffness distribution, and the structural response under dynamic loading is not considered in the modal analysis. Equation (5) can be changed to:

\[
(K - \omega^2M)\Phi Q = 0.
\]

Take equations (6) and (7) into equation (9), for any \( r \)-order mode, the undamped structural characteristic equation satisfies:

\[
(K_r - \omega^2M_r)Q = 0.
\]

Take equations (6)–(8) into equation (5), for any \( r \)-order mode, consider the Rayleigh damping structure characteristic equation to satisfy:

\[
(K_r - \omega^2M_r + j\omega C_r)q_r = \Phi_r^T F(\omega).
\]

It can be seen that the dynamic characteristics of the structural system are determined by modal parameters such as modal frequency, modal mass, modal stiffness, and modal damping.

Transform equation (11) into:

\[
q_r = \frac{\Phi_r^T F(\omega)}{K_r - \omega^2M_r + j\omega C_r} = \frac{\sum_{j=1}^{N} \phi_{jr} f_j(\omega)}{K_r - \omega^2M_r + j\omega C_r}.
\]

Due to the response of any point \( l \) on the structure, it satisfies:

\[
x_l(\omega) = \sum_{r=1}^{N} \phi_{lr} q_r(\omega).
\]

Assuming that its excitation force acts on the point \( P \) of the structure, the excitation vector satisfies \( F = \begin{bmatrix} 0 & \cdots & f_p(\omega) & \cdots \end{bmatrix}^T \), and the modal force \( F_r(\omega) \) is:

\[
F_r(\omega) = \phi_{pr} f_p(\omega).
\]

Taking equations (12) and (14) into equation (13), the system response equation is:

\[
x_l(\omega) = \sum_{r=1}^{N} \frac{\phi_{lr} \phi_{pr} f_p(\omega)}{K_r - \omega^2M_r + j\omega C_r}.
\]

Make equation (15) transform into:

\[
x_l(\omega) = \sum_{r=1}^{N} \Gamma_l \phi_{lr} \phi_{pr} = \sum_{r=1}^{N} \Gamma_l \phi_r (j = 1, 2, \cdots, N).
\]
In the formula

\[
\Gamma_j = \sum_{r=1}^{N} \frac{f_p(\omega)}{K_r - \omega^2 M_r + j \omega C_r} = \sum_{r=1}^{N} \frac{f_p(\omega)}{M_r [\omega_r^2 - \omega^2] + j 2 \omega \zeta_r \omega} (j = 1, 2, \ldots, N). \tag{17}
\]

Consider undamped

\[
\Gamma_j = \sum_{r=1}^{N} \frac{f_p(\omega)}{K_r - \omega^2 M_r} = \sum_{r=1}^{N} \frac{f_p(\omega)}{M_r [\omega_r^2 - \omega^2]} (j = 1, 2, \ldots, N), \tag{18}
\]

where \( \phi_r = \phi_{r1} \phi_{r2} \) is the \( r \)-th order mode shape, \( \Gamma_j \) is the \( r \)-th order modal participation factors, \( \omega_r = \sqrt{\frac{K_r}{M_r}} \) is the \( r \)-th order modal frequency ratio, \( K_{er} \) is Equivalent stiffness of \( r \)-th order mode, \( M_{er} \) is Equivalent mass of \( r \)-th order mode, \( \zeta_r = \frac{C_r}{2M_{er}} \) is the \( r \)-th order modal damping ratio.

From formula (16)–(18), it can be seen that the total response of the system is composed of linear superposition of all modes that may exist in the system. For the periodic or limited structure, when the system excitation frequency \( \omega \) approaches a certain \( r \)-order frequency, the modal participation factor \( \Gamma_j \) becomes larger, the system \( x_j(\omega) \) response reaches the maximum, and the structural system exhibits the \( \Phi_r \) characteristic mode shape.

Equations (17) and (18) show that the contribution of modal participation is also dependent on the applied load. Thus, it can be seen that not all modes participate in the response of the system, and the influence of high-order modes may be weak for the low-frequency response so that the mode far away from the studied frequency range has little influence on the overall system response, which can be ignored.

According to the above-mentioned theoretical analysis, the mechanism of modal resonance bandgap formation is proposed. On the basis of the modal superposition principle, the response of the scatterer and substrate in the phononic crystal structure system is the coupled vibration response formed by the modal superposition. The modal participation factor in the structural system is a parameter to describe the mutual coupling between the external excitation and the substrate, as well as the scatterer. The larger the value of \( \omega_r \) frequency mode modal participation factor, the greater the contribution of the mode modal to the dynamic response of the system structure. The mode shape formed by the substrate and the scatterer is dominated by the modal mode with larger contribution, and is converted in different modes according to the change of the modal participation factor. The band gap may only appear in the substrate due to the applied load. Under the influence of the applied load, the bandgap is only likely to occur at the maximum modal frequency of the substrate and the scatterer, and the bandgap characteristics are determined by the vibration characteristics of the substrate and the scattering mode.

### 2.3. Mechanism analysis of bandgap formation in periodic structure

Take the phononic crystal constructed in this paper as the research object. As shown in figure 4(g), it can be seen that the frequency range between 0 and 2605 Hz is the passband, the frequency range between 2605 and 3219 Hz is the forbidden band, and the frequency range between 3250 and 13760 Hz is the directional bandgap.

According to the modal vibration bandgap theory in section 2.2, the mechanism of the periodic structure bandgap can be explained as follows.

1. When the excitation frequency tends from 0 Hz to the first-order natural frequency \( \omega_1 \) (figure 4(a) 855 Hz), the substrate and the scatterer generate an \( x \)-direction torsional coupled vibration response, which is in the main vibration mode at the same time. At this time, the elastic wave propagates in this vibration modal, the vibration energy has no attenuation, and no bandgap is formed.

2. When the excitation frequency goes from \( \omega_1 \) to the second-order natural frequency \( \omega_2 \) (figure 4(b) 888.98 Hz), is main vibration mode frequency. The substrate and the scatterer generate the \( y \)-direction torsional coupling vibration response, which cannot restrain the elastic wave propagation, and the vibration continues to propagate according to the vibration mode of \( \omega_2 \). Meanwhile, it is shown that there is no bandgap in the propagation of elastic wave in the substrate under the torsional vibration response of the scatterer.

3. When the excitation frequency moves from \( \omega_2 \) to the third-order natural frequency \( \omega_3 \) (figure 4(c) 2605.2 Hz), the scattering body is excited as the main vibration mode. The plane vibration of the substrate and the \( z \)-direction vibration of the scatterer generate modal resonance coupling, which suppresses the vibration of the substrate. The elastic wave propagation in the substrate is cut by the \( z \)-direction modal vibration of the scatterer, and the elastic wave propagation is suppressed to form a complete bandgap.
When the excitation frequency is equal to the natural frequency $\omega_4$ (figure 4(d) 13757 Hz), the substrate is the main vibration mode of the coupling of z direction vibration and torsional vibration of the scatterer. At this time, the elastic wave propagation in the substrate is suppressed to some extent, and no bandgap is generated.

When the excitation frequency is equal to the natural frequency $\omega_5$ (figure 4(e) 15719 Hz), the substrate and the scatterer are in the main vibration mode, resulting in z-axis rotating vibration response. The scatterer cannot suppress the vibration mode of the substrate, the elastic waves propagate according to a given path and there is no the bandgap.

When the excitation frequency is equal to the natural frequency $\omega_6$ (figure 4(f) 24573 Hz), the substrate is the main vibration mode, and the coupling torsional vibration response is generated with the scatterer. At this time, the elastic wave propagates through the established path and is free of the bandgap.

According to the modal vibration bandgap theory, for any periodic structure, any order natural frequency $\omega$ is satisfied $\omega_i = \frac{K_i}{M_i}$, where $K_i$ is the equivalent stiffness of $i$-order modal, $M_i$ is the equivalent mass of $i$-order modal. Therefore, the modal suppression force of the scatterer on the substrate is:

$$F_{si} = K_{si}x_i(\omega) = K_{si}\sum_{r=1}^{N} \Gamma_{rt}\phi_{r}.$$  \hspace{1cm} (19)

It can be seen that when the excitation frequency $\omega$ approaches a certain $i$-order natural frequency $\omega_i$, the substrate and the scatterer change with the modal participation factor, resulting in different types of vibration responses. When the scatterer is excited in the main vibration mode, the suppression force of equation (19) is generated, the vibration of the substrate is suppressed, and the elastic wave propagation is blocked. It can be seen

![Figure 4. Periodic structure structural vibration mode. (a) 885 Hz Mode. (b) 88.98 Hz Mode. (c) 2605.2 Hz Mode. (d) 13757 Hz Mode. (e) 15719 Hz Mode. (f) 24573 Hz Mode. (g) Energy band diagram. (h) Comparison between band structure curve and transmission characteristics.](image-url)
that the periodic structure bandgap start-stop frequency is determined by the modal participation factor (including equivalent mass) and the equivalent stiffness.

As shown in figure 4(h). By comparing the energy band structure diagram of the frequency range of 0 Hz∼1000 Hz and the transmission characteristics of the finite periodic structure, it can be seen that the frequency range of 0~1000 Hz is the passband in the energy band structure diagram. But according to the transmission characteristic curve, a large vibration attenuation region appears in the frequency range of 538–1000 Hz, and the maximum attenuation value is 61.93 dB at a frequency of 631 Hz. This phenomenon indicates that the periodic performance band structure obtained by theoretical analysis cannot accurately describe the finite structure transfer characteristics, and there is a big difference between them. The reason for the difference is that the starting and ending frequency of the bandgap of the periodic structure is determined by the modal participation factor (including the equivalent mass) and the equivalent stiffness. Compared with the size of periodic structure, the finite structure is larger. The modal stiffness of the finite structure system is weaker than that of the periodic structure. Under the condition of equal mass ratio, the natural frequency of the finite structure is lower, which results in the forward shift of the vibration attenuation frequency of the whole structure, the bandgap between middle and low frequency is generated.

2.4. The comparison of theoretical analysis with experimental phenomena of the phonon crystal modal structure

Based on the analysis of the cause of formation of section 2.2 periodic structure bandgap and the difference between energy band structure diagram and finite periodic structure transmission characteristics (figure 4(h)), the theoretical and experimental comparison of phonon crystal periodic and modal finite structure transfer characteristics is carried out. The modal resonance bandgap mechanism of finite structure phononic crystals was verified by experiments combined with theoretical analysis (see section 3).

As shown in figures 2(f) and 3, compared with high order modal periodic structure (α10), the modal structures of the 1st to 9th order phonon crystals (α1~α9) are all Centro-symmetric, where α1 is a four periodic structure; α9 can be seen as a periodic local defect structure; other crystal structures can be regarded as local periodic structures or special-shaped defect local periodic structures. The transmission frequency response curves of different phonon crystal modal structures are obtained by solving the vibration equations of different phonon crystal modal structures through the FEM (taking the frequency as the independent variable, the time history of excitation and response is transformed into frequency amplitude by Fourier transform, and the finite structure vibration transmission spectrum is obtained by the amplitude ratio calculation of the response and excitation at the same frequency). In this paper, the bandgap threshold is taken as −10 dB, and the region below the value above is defined as the bandgap frequency range. The analyses of the experimental results are shown in figures 5 and 1(a). The simulation and experimental results of the maximum attenuation for bandgap vibration frequency with different phonon crystal structures were shown in table 1 as follows:

According to the modal structure theory and experimental comparison data shown in figures 1(a), 5, theoretical calculation comparison data figure 1(b) and experimental comparison data figure 1(c), the maximum vibration attenuation frequency range of the phonon crystal structure α9 is the largest, which is superior to the phonon crystal structure α10; the maximum vibration attenuation amplitude of the phonon crystal structure α10 at the frequency of 631 Hz is −62 dB, and the maximum vibration attenuation amplitude of the phonon crystal structure α9 at the frequency of 731 Hz is −43 dB. On the other hand, according to the experimental results, in the structure α1~α9, the maximum vibration attenuation frequency range of the phonon crystal structure α3 is the largest; the maximum vibration attenuation frequency range of the phonon crystal structures α6 and α9 is similar, and the three are all superior to that of the phonon crystal structure α10. The maximum vibration attenuation of the phonon crystal structure α3 at 385 Hz is −60 dB, and the maximum vibration attenuation of the phonon crystal structure α10 at 665 Hz is −56 dB.

The comparative analysis of theoretical calculation data with experimental data in figures 1(a) and 5 show that the theoretical calculation data are basically consistent with the experimental results for the phonon crystal structures α10, α1, α2, and α9, which have a less error. The theoretical analysis results of the phonon crystal structures α3, α4, α5, α6, α7, α8 are the same as the experimental results, but the errors are larger. There is also a significant difference between the theoretical and experimental results: the maximum attenuation of the experimental test data is significantly better than that of the theoretical calculation data.

The reason why the experimental results of the phonon crystal structures α1, α2, and α9 have less errors compared with the simulation results is that the nonlinear characteristics of the substrate are not obvious due to the little influence of the position of the scatterer on the localized stiffness and mass of the substrate, with regard to the phonon crystal structures α3, α4, α5, α6, α7, and α8. The main reason for the larger error between the experimental results with the simulation results of the phonon crystal structures α3, α4, α5, α6, α7, α8 is caused
by the difference between the simulation model and the experimental object. The main error reasons are analyzed as follows.

(1) In the theoretical calculation model, the stainless steel substrate and the stainless steel cylindrical scatterer are continuous rigid connections; however, in the experiment, the stainless steel base and the stainless steel cylindrical scatterer are glued by glue solution (cyanoacrylate glue (502 glue)). Therefore, the connection rigidity of experimental substrate and the scatterer is weakly compared with the theoretical analysis.

(2) In theoretical calculation, the damping of the system comes from the properties of the material. Due to the influence of the mass matrix and stiffness matrix in the equation, the damping characteristics of the system are ignored. In the experiment, the bonding material between the scattering steel column and the base plate

| Phonon crystal structure | The maximum bandgap range of the theory analysis data | The maximum bandgap range of experimental data |
|--------------------------|-----------------------------------------------------|---------------------------------------------|
| α1                       | 632–1000 Hz                                          | 303–1000 Hz                                 |
| α2                       | 671–1000 Hz                                          | 587–1000 Hz                                 |
| α3                       | 700–1000 Hz                                          | 236–1000 Hz                                 |
| α4                       | 747–1000 Hz                                          | 316–1000 Hz                                 |
| α5                       | 837–1000 Hz                                          | 295–1000 Hz                                 |
| α6                       | 720–773 Hz                                           | 254–1000 Hz, (a small attenuation zone in the frequency range of 609–628 Hz,) |
| α7                       | 653–1000 Hz                                          | 280–1000 Hz                                 |
| α8                       | 574–828 Hz                                           | 418–1000 Hz                                 |
| α9                       | 435–1000 Hz                                          | 285–1000 Hz, (a small attenuation zone in the frequency range of 628–642 Hz) |
| α10                      | 538–1000 Hz                                          | 330–1000 Hz, (a small attenuation zone in the frequency range of 620–636 Hz) |
has certain damping characteristics, and the additional stiffness at the joint is lower than the theoretical estimation because of the material properties vary with the modal frequency.

(3) From the formula (15), for any r-order modal, the transfer function of the system can be obtained by the measuring point l and the exciting point p as follows:

\[
H_{lp}(\omega) = \frac{X_p(\omega)}{F_p(\omega)} = \sum_{r=1}^{N} \frac{\varphi_{pr} \varphi_{pr}}{K_r - \omega^2 M_r + j\omega C_r} = \sum_{r=1}^{N} \frac{\varphi_{pr} \varphi_{pr}}{M_r \left[(\omega^2 - \omega_s^2) + j2\zeta_r\omega_s\omega\right]},
\]

where \(H_{lp}(\omega)\) stand for the excitation force acting on point \(p\), in response to l-point, and independent of the magnitude of the excitation force, just depends on the inherent parameters of the system.

Thus, it can be seen that compared with the homogeneous distribution of the scatterer in the periodic structure, the uneven distribution of the scatterer at the modal position changes the modal stiffness \(K\), the modal mass \(M\) and structure Rayleigh damping \(C\) of the local substrate. The nonlinear characteristic of the dynamic impedance \((-M_r\omega^2 + jC_r\omega + K_r\)) is increased, so that the theoretical calculation value of the vibration transmission frequency response function and the experimental value produce a large difference at a certain modal frequency.

3. Mechanism analysis of vibration characteristic

According to the phenomena in the experimental results, the vibration transmission characteristics of phonon crystal structures \(\alpha_3, \alpha_6, \alpha_9\) are better than that of the phonon crystal structure \(\alpha_{10}\), and among them the ones of the phonon crystal structure \(\alpha_3\) are the best. Consequently, the structure \(\alpha_3\) is selected as the typical structure. According to the modal vibration analysis theory, the bandgap generation mechanism can be illustrated as follows.

(1) As shown in figure 2, when the substrate gradually increases from 0 to 1000 Hz due to the external excitation frequency, the external excitation is the same as the certain natural frequency of the substrate, resulting in the first-order bending vibration as shown in figure 2(a), the first-order torsional vibration as shown in figure 2(b), the second-order bending vibration as shown in figure 2(c), the second-order torsional vibration as shown in figure 2(d), and the high-order coupling mode diagram as shown in figures 2(e)–(f). Therefore, it can be seen that the natural frequency of a certain order of the substrate is excited as the main vibration mode, when the elastic wave of a certain frequency propagates in it. The excited mode dominates the propagation of vibrational energy in the substrate, and the elastic wave propagates according to a certain vibration mode of the substrate structure.

(2) Figure 6 is the vibration shape diagram the phonon crystal structure \(\alpha_3\). When the external excitation frequency gradually approaching a certain natural frequency of the structure \(\alpha_3\), the modal participation factor gradually increases as shown in figure 8(a), and a certain order of corresponding modal vibration shape as shown in figure 6 is generated. As shown in figure 6, when the scatterer is distributed on the substrate according to the structure \(\alpha_3\) (figure 3(c)), and the external excitation frequency increases from 0 to 14.807 Hz, it can be seen from figure 6(a) that the modal factor value is magnified and the \(\beta_1\) mode (first-order bending resonance mode) is excited at the resonance mode position. The scatterer is in the same pattern as the vibration mode of the substrate in the yellow area, and due to the comprehensive influence of the position and mass distribution of the scatterer on the stiffness and mass of the substrate, the first-order bending of the substrate is suppressed and the vibration is weakened to some extent. At the modal node, a coupling motion of torsional vibration is generated by the scatterer and the substrate of the green region in figure 6(a), which suppresses the propagation of elastic wave to a certain extent.

Compared crystal structure with the phonon crystal structure \(\alpha_{10}\) of figure 7(a), due to the uniform distribution of the \(\alpha_{10}\) mass and scatterer position, the stiffness and the mass of the substrate change uniformly. When the modal participation factor increases to the maximum frequency (as shown in figure 8(b)), \(\gamma_1\) mode is excited (first-order bending mode resonance), and the vibration mode of the scatterer at the modal position is consistent with the substrate. However, the vibration suppression at the \(\gamma_1\) mode (first-order bending mode) is weaker than that of the \(\beta_1\) mode (first-order bending mode).

In the non-modal position, the torsional coupling vibration is generated in the green region in figure 6(a), which suppresses the partial elastic wave propagation. The influence of the experimental cantilever structure and the mass inertia of the scattering body amplify the vibration amplitude of the end of structure, and the effect of suppressing vibration transmission is limited. It can be seen that when the vibration...
propagates in the substrate, it can be affected by the modal participation factor, and the larger the value of the order frequency modal participation factor, the greater the contribution of the mode to the response of the structure at this order frequency. The propagation mode of the elastic wave in the substrate is characterized excited substrate mode, which is a dominated vibration transfer mode of the substrate. As shown in figure 6(b), the first-order torsional resonance mode $\beta_2$ is excited when the external excitation frequency approaches 37.09 Hz from 14.807 Hz and the modal participation factor reaches the maximum value at this point (as shown in figure 8(a)). At this time, $\beta_1$ mode (first-order bending resonance mode) is transformed into $\beta_2$ mode (first-order torsional resonance mode), and the substrate is carried the vibration energy transfer in the torsional resonance mode. The scatter in the yellow region (the modal position in figure 6(b)) is consistent with the vibration mode of the substrate. In this vibration mode, the scatterer suppresses the main vibration mode of the substrate to a small extent, and the bandgap cannot be generated. At the modal node, the green region and the intersection of the yellow and green regions are connected to the scatterer and the substrate in a torsional coupling resonance mode, and torsional vibration is partially suppressed by the position of the scatterer. As shown in figure 7(b), when the external excitation frequency approaches 35.265 Hz from 12.107 Hz, the modal participation factor reaches the maximum value at that point, and the $\gamma_1$ mode (first-order bending resonance mode) is transformed into the $\gamma_2$ mode (second-order bending resonance mode). The scatterer at the modal position is consistent with the vibration mode of the substrate. The scatterer and the substrate produce a torsional coupling mode at the modal node, and the scatterer has a greater inhibitory effect on the substrate bending resonance. However, due to the influence of the second-order bending resonance of the substrate, the vibration energy transfer of the substrate is larger, and the vibration propagation is only suppressed to a certain extent, and the bandgap cannot be generated. (4) As shown in figure 6(c), when the external excitation frequency approaches 38.819 Hz from 37.09 Hz, the modal participation factor reaches the maximum at this point (figure 8(a)). The substrate structure transmits vibration in $\beta_3$ mode (second-order bending mode) as the main vibration mode. In the modal
position, the scatterer in the yellow region is consistent with the vibration mode of the substrate. The scatterer at the modal node is mainly in the torsional coupling resonance mode, and the vibration propagation is only suppressed to a certain extent. However, the transmission of the vibration energy is large, and hence the bandgap cannot be generated.

In Figure 7(c), when the external excitation frequency approaches 35.82 Hz from 35.265 Hz, the modal participation factor reaches the maximum at this point in Figure 8(b). At this time, the substrate is dominated by $\gamma_3$ mode (first-order torsional mode). In the red region and the intersection region of red and yellow at the modal position, the vibration mode of the scatterer is consistent with that of the substrate.

**Figure 7.** Phonon crystal $\alpha 10$ vibration shape diagram. (a) $\gamma 1$, 12.107 Hz, one order bending mode. (b) $\gamma 2$, 35.265 Hz, second order bending mode. (c) $\gamma 3$, 43.82 Hz, one order torsional mode. (d) $\gamma 4$, 61.428 Hz bending and torsional combined mode. (e) $\gamma 5$, 72.226 Hz bending and torsional combined mode. (f) $\gamma 6$, 85.011 Hz, bending and torsional combined mode. (g) $\gamma 7$, 97.607 Hz bending and torsional combined mode. (h) $\gamma 8$, 116.7 Hz bending and torsional combined mode. (i) $\gamma 9$, 127.88 Hz bending and torsional combined mode.

**Figure 8.** Modal Participation chart Factor. (a) 0–1000 Hz, Phonon crystal $\alpha 3$ structure, Modal participation factor. (b) 0–1000 Hz, Phonon crystal $\alpha 10$ structure, Modal participation factor.
the modal node, the main resonance mode is torsional coupling vibration. The energy transferred by vibration is large, and the vibration propagation is suppressed to a certain extent, and also there is no bandgap.

(5) As shown in figure 6(d), when the external excitation frequency approaches 58.973 Hz from 38.819 Hz, the modal participation factor become larger (figure 8(a)). The structure is in β4 mode (bending and torsional combined mode). The scattering and substrate vibration is in the form of torsional vibration coupling mode at the modal node, which is the same vibration mode in the modal position. However, the vibration energy transfer of the substrate is large, and hence the bandgap cannot be generated.

Relative to figure 6(d), when the external excitation frequency approaches 61.428 Hz from 43.82 Hz, the modal participation factor become larger (figure 8(b)). The structure is in γ4 mode (second order bending and torsional combined mode). The scattering and substrate vibration is in the form of torsional vibration coupling mode at the modal node, which is the same vibration mode in the modal position. The vibration energy transfer of the substrate is large, and the vibration propagation is suppressed to a certain extent. Therefore, there is no bandgap.

Therefore, according to formula (20), the generation mechanism of bandgap in low frequency region is described as follows:

Let $\lambda = \frac{\omega}{c}$, $\lambda$ is the $r$th order modal damping ratio. $x_{lst} = \frac{f_{r}^{r}(\omega)}{K_{r}}$, $x_{lst}$ is the static deformation of the system under the same magnitude of the dynamic force amplitude, there is:

$$\frac{x_{r}(\omega)}{x_{lst}} = \sum_{r=1}^{N} \frac{\phi_{l} \phi_{pr}}{1 - \lambda^2 + j2\zeta_{r} \lambda} = \sum_{r=1}^{N} \eta \phi_{l} \phi_{pr}$$

where $|\eta|$ is the dynamic magnification factor, it satisfies:

$$|\eta| = \frac{1}{\sqrt{(1 - \lambda)^2 + (2\zeta_{r} \lambda)^2}}$$

which $\theta$ is the phase difference between the vibration response and the excitation force, it satisfies:

$$\theta = \tan^{-1} \frac{2\zeta_{r}}{1 - \lambda^2}.$$  

It can be seen that when the external excitation frequency is low ($\lambda < 1$, $|\eta| \approx 1$, $\theta = 0^\circ$), under the action of excitation force, the scatterer distributed at the modal position is consistent with the vibration mode of the substrate, and the amplitude is close to the static deformation under the action of dynamic force with the same force amplitude. The substrate vibration energy is shared by the scatterer, in the mode node, the scatterer and the substrate are mainly torsional coupling vibration mode. It can inhibit the propagation of elastic wave to a certain extent.

The vibration characteristics of the system depend on the modal stiffness of the system. When the structural rigidity is stronger, the bandgap is difficult to generate in the low frequency region.

(6) As shown in figure 8, the vibration mode of the substrate is formed by the superposition of the vibration in the $x, y, z$ direction, and the main vibration mode is determined by the maximum response of the contribution in a certain direction. As shown in figure 8(a), when the external excitation frequency is greater than 200 Hz, the modal of $x$-direction and $z$-direction participation factor becomes smaller, and also the modal of $y$-direction participation factor increases. Therefore, the main coupling vibration mode is torsional vibration for scatterer and substrate. Compare with the main torsional coupling vibration mode of the scattering body and the substrate (figure 9 high order mode diagram). At this modal frequency, the propagation of high frequency elastic wave is suppressed, resulting in vibration loss as shown in figure 1(b).

(7) As shown in figure 8, the comparison of modal participation factors between the phonon crystal $\alpha 3$ structure and the phonon crystal $\alpha 10$ structure shows that when the substrate is at low frequency 0–200 Hz, the value of $z$-direction modal participation factor is greater than the $x$-direction and $y$-direction value. With the change of the modal participation factor, the scatterer and the substrate at the modal position is the $z$-direction coupling resonance mode, and at the modal nodes is the torsional coupling vibration. The substrate is in bending vibration and torsional vibration or combination mode of bending and twisting. At this frequency modal, the vibration propagation is suppressed by the position of the scatterer distribution, but the attenuation is small, there is substantially no bandgap generation. When the external excitation frequency is above 200 Hz, the value of the modal participation factor in $x$ and $y$-direction is greater than that in $z$-direction. Under this vibration condition, the scatterer is in $x$ and $y$-direction torsional coupling.
vibration. The propagation of waves in substrate, which is in a high-order coupled vibration mode. Under the influence of the scatterer distribution and the coupled vibration mode, the propagation of substrate vibration is suppressed, and the corresponding bandgap is generated.

According to the formula (20), it can be seen from the analysis of the generation mechanism of bandgap in high frequency region. When the frequency of external excitation is very high \( (\lambda \gg 1, |\eta| \approx 0, \theta = 180^\circ) \), under the effect of this excitation force, the substrate has a small amplitude. Because of the mass inertia, it is difficult to synchronize with vibration frequency, and the scatterer is characterized by high dynamic impedance, and hence the mass term of the scatterer in the dynamic impedance \( l \gg 1 \) is proportional to \( l \gg 1 \). Therefore, due to the high-frequency mass inertia impedance of the scatterer, the vibration propagation is suppressed, and the bandgap is generated.

In addition, when the external excitation frequency is close to the natural frequency \( (\lambda \approx 1, |\eta| \gg 1, \theta \approx 90^\circ) \). At this time, the amplitude of structural vibration is much larger than that of static deformation. When \( |\eta| \) reaches the maximum, it can be seen from the formula (22), \( \lambda = \sqrt{1 - 2\zeta^2} \). Where \( \omega = \omega_f \sqrt{1 - 2\zeta^2}, |\eta| \) take the maximum value is \( |\eta| = \frac{1}{2\omega_f \sqrt{1 - 2\zeta^2}}, \omega = \omega_f \sqrt{1 - 2\zeta^2} \) is the modal resonance frequency. For the small damping structure (phonon crystal structure, small scale, damping characteristics are not obvious, \( \zeta \ll 1 \)), the modal resonance frequencies of damped structure and undamped structure are approximately the same. It can be seen that, for phonon crystal structure or finite periodic structure, it is possible to study the resonance characteristics and the interaction of the substrate and the scatterer by means of the modal analysis, to obtain the energy band structure and the characteristics of the vibration transmission.

To sum up, according to the mode resonance bandgap mechanism, for the \( r \)-order mode in the finite-structure system, the response of any \( i \)-point is satisfied:

\[
x_i(\omega_i) = \sum_{r=1}^{N} \frac{f_p(\omega_i)}{M_r[(\omega_i^2 - \omega_f^2) + j2\zeta\omega_f \omega_i]} \Phi_{ir} \quad (j = 1, 2, \ldots, p, \ldots, N). \tag{24}
\]

Therefore, in the system of finite structure, that the \( r \)-mode, any \( i \)-point, the modal force of the scatterer on the substrate is as follows:

\[
F_{\omega_i} = K_{i\omega_i}x_i(\omega_i) = K_{i\omega} \sum_{r=1}^{N} \frac{f_p(\omega_i)}{M_r[(\omega_i^2 - \omega_f^2) + j2\zeta\omega_f \omega_i]} \Phi_{ir} \quad (j = 1, 2, \ldots, p, \ldots, N). \tag{25}
\]

From formula (24), it can be seen that the vibration response of the structure is linear superposition of modal shapes, and the vibration mode of substrate and scatterer is determined by the superimposed main vibration mode.
It can be seen from formula (25) that the modal force of the scatterer on the substrate is distributed by the modal mass ratio of the scatterer to the substrate. When the scatterer responds, it is the main vibration mode to suppress the vibration of the substrate. Under the influence of linear superposition of modes, the vibration of the substrate is weakened and the corresponding frequency bandgap is generated. It is shown that the elastic wave propagates along the substrate structure at a steady velocity, which relates to the elastic waves in homogeneous media propagation. The local stiffness and mass of substrate is changed by the distribution of scatterer, therefore, it affects the propagation of elastic waves, and changes the path of propagation. Under this condition, the propagation of elastic wave produces multiple scattering and diffraction, which promotes the attenuation and average distribution of vibration energy of substrate. Thus, the bandgap is generated (As shown in figures 10(a) and (b)).

4. Conclusions

Based on the structural dynamics analysis and modal analysis theory, the vibration characteristics of different types of phonon crystal modal structures are studied in this paper. Through experimental and theoretical analysis, the generation mechanism of modal resonance bandgap is clarified, and the conclusions are as follows.

(1) The bandgap mechanism of phononic crystal structure shows that the bandgap characteristic is determined by the equivalent mass and the equivalent stiffness of the structural system; the vibration response of the structure is a linear superposition of the modal shape; the vibration characteristics of the low frequency range depend on the system modal stiffness, and the vibration characteristics of the high frequency range depend on the system modal mass. By adjusting the mass of the scatterer and the stiffness of the substrate, the active design of the start-stop frequency point and the bandgap width of the bandgap can be realized.

(2) Theoretical and experimental phenomena show that: in the low frequency range, it is necessary to reduce the stiffness of the substrate or install the periodic scatterer at the modal mode position of high modal participation factor, so as to reduce the transfer of substrate vibration energy. According to the modal superposition theory, higher-order mode is formed by the low-order modal superposition. The vibration characteristics of the high frequency range is controlled by installing the scatterer in the low-order modal mode position of high modal participation factor to suppress the vibration transmission. Therefore, the bandgap characteristics of phonon crystals depend to a great extent on the stiffness of the substrate. In the low frequency range, when the stiffness of the substrate is large, and there is no bandgap of the phonon crystal.

(3) Experimental phenomena of transfer characteristics of finite structures show that: elastic waves propagate in the substrate in the form of P–S waves. Due to the distribution of the scattering body on the substrate and the modal resonance coupling and other factors, the wave propagation is disturbed by scattering and diffraction, etc, which promotes the attenuation of vibration energy of the substrate and the average distribution of vibration energy, and the bandgap characteristics of the structural vibration transmission are then generated.

(4) For a certain engineering structure, the modal modes were obtained through modal analysis. Local periodic structures were designed at the modal positions to analyze the structural vibration transmission characteristics. The bandgap characteristics of different modal structures were compared and analyzed to
obtain the active control design of vibration transmission in a specific bandgap within a certain frequency range. Therefore, modal analysis theory can effectively guide the design of phonon crystal.

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Author contributions

CM.W., WQ.X. proposed the idea and designed the experimental proposal; YY.L. and DH.W. supervised the experiments; CM.W., CML.J, KZ.Y., ZH.X., and KP.Z. prepared the experimental set-up and performed the experiments; CM.W. and WQ.X. carried out the theoretical calculations; CM.W. and WQ.X. write the paper; and all authors analyzed the data, discussed the results and commented on the manuscript.

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