Optimization Design of an Aircraft Wing Structure based on Response Surface Method

A A G Hanif¹, H S Li¹, M A Raza², M Kamran³ and M Abdullah³

¹College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China.
²Centres of Excellence in Science & Applied Technologies (CESAT), No. 2801, Islamabad, Pakistan.

E-mail address: ahmad@outlook.com

Abstract. Wing is the most critical part of an aircraft whose structural weight holds prime importance. It is always desirable to reduce the weight of an aircraft. In this work, a detailed analysis of wing is carried out and results pertaining to stress distribution are recorded. Commercial software ANSYS is used to perform simulations related to stress, strain and displacement distributions. Moreover, the optimization design of an aircraft wing was conducted using design of experiments (DOE) and response surface method (RSM) technique for the reduction of computational cost. The aircraft wing is efficiently optimized by changing thickness under the same loading and flight conditions. Results show that this optimization technique is useful to reduce stresses as well as overall weight of the structure with minor variation in thickness of components.

1. Introduction

An aircraft wing is mainly used to produce required lift while drag force resists the forward motion. Therefore, wing is the most critical part of an aircraft and its failure is unaffordable. The wing structures of aircraft are generally made up of alloys and metals, thus ensuring a cantilever design where no exterior support is necessary. Mostly the skin of wings also serves as a component of wing frame thus allowing it to bear flight loads and generated stresses in flight. The wing consists of ribs, spars and stringers. The ribs are located chord wise while the spars are positioned span wise. Spars are key structural members of the aircraft wing also known “beams”.

Usually, the wing consists of front and rear spars alongside the span. The two spars used in the structure of wing are identified as “two-spar” construction. Spar is very crucial and vital structural component of the wing which starts inside the fuselage up to the wing tip. The spar is mainly responsible to cater for the bending strength in the wing. Ribs are positioned crossways in between the spars, which allow maintaining the airfoil shape of the wing. First of all the flight load is experienced by the wing skin which is shifted to the ribs and then the ribs transfer some load to the spars. In this way the loads are distributed among each structural component.

It is observed that the analysis can be improved or refined by including ribs and spars in geometric model as performed by Kavya and Reddy [1].

The analysis of aircraft wing is a versatile, challenging and time demanding process. Optimization of wing structure is one approach to cope up with this intricacy which can be successfully attained by numerical method technique such as finite element methods. Numerical analysis is conducted in a...
commercially available tool named as ANSYS, as used by Sureka and Meher [2] in their study regarding A300 aircraft wing consisting of ribs and spars. Standard airfoil NACA 64215 was modelled for analysis. Senthilkumar et.al [3] also conducted structural analysis in ANSYS. FE model of the problem used shell elements. Kumar et.al [4] studied stress distribution, using shell elements for surface modelling. They also performed optimization technique on FE model.

To maximize or minimize a required objective function in the presence of existing constraints is termed as an optimization procedure [5]. Mazhar and Khan [6] optimized stiffness and strength of UAV wing using FEA ANSYS software. They used Artificial Neural Networks approach to decide the best possible arrangement through optimum stiffness and minimized cost and weight.

As far as optimization is concerned, it has always been a quest for designers to improve any desired parameter but not at the cost of other aspects. In 1951, Box and Wilson introduced RSM for the first time. It is characterized as a scheme to build global estimates to the system performance based on outcome considered at diversified points in the design space [7]. RSM reduces the computational costs acquired in solving such scenarios. Structural optimization using ANSYS and RSM was performed by Ajay Kumar Menon [8]. The stress reduction was successfully achieved in his work.

In this study we used ribs and spars for geometric modelling to emulate near practical situation. Shell elements were employed in FE model. Boundary and loading conditions are applied to perform static structural analysis. RSM is subsequently used to optimize the maximum stress value over the wing.

2. The Aircraft Wing Model

In this paper, we used, twin-spar wing geometry consisting of ribs and skin at top and bottom surface of the wing. In order to select the material structural steel, stainless steel and aluminium alloy were considered as shown in Table 1.

| Material         | Density (kg/m³) | Young’s Modulus (MPa) | Poisson’s Ratio |
|------------------|-----------------|-----------------------|-----------------|
| Structural steel | 7.85 x 10³      | 2.00 x 10⁵            | 0.30            |
| Stainless steel  | 7.75 x 10³      | 1.93 x 10⁵            | 0.31            |
| Aluminium alloy  | 2.77 x 10³      | 7.10 x 10⁴            | 0.33            |

The size of aircraft wing corresponds to that of Boeing 767, a twin-aisle commercial aircraft. A supercritical airfoil Boeing BACXXX, was selected for this analysis, as used by other researchers too. [9]-[11] The dimensions of this aerofoil were recorded from the UIUC database (University of Illinois at Urbana-Champaign). The normalized dimensions are shown in Figure 1. [12] The waterline to span for the top and bottom surface of wing is given in Figure 2. In addition, the wing model design calculations are in agreement with the method, as recommended by Torenbeek. [13]
The “.igs” file of 3D model created in CAD software is then imported in Ansys 15.0 software. The geometry was “cleaned up” and load condition was applied. 1G-load was considered as a load (self-weight of the wing). FE model is then imposed by a boundary condition as side constraint, thus cantilevering the wing. The meshing was performed using shell elements. In total, there are 3902 shell elements and 10676 nodes. The weight of current wing model is 6266 kg. The FE model is prepared to perform static structural analysis to calculate stresses and deformation produced as shown in Figure 3 below.

Static structural analysis was performed to calculate the stress, strain and the displacement in the wing with all the three suggested materials. The results of all the three materials are tabulated in Table 2. Hence, for the purpose of this study, we opted Aluminium alloy, as the material of the wing structure.

| Material           | Max. Stress (Pa) | Total Deformation (mm) | Total Weight(kg) |
|--------------------|------------------|------------------------|------------------|
| Structural steel   | 84.91 x 10^6     | 67                     | 6266.04          |
| Stainless steel    | 82.57 x 10^6     | 65                     | 5612.26          |
| Aluminium alloy    | 30.20 x 10^6     | 65                     | 1411.30          |

### 3. Optimization Design Of Aircraft Wing

The finite element process is a mathematical method for the analysis of structures. [14] After obtaining the results from structural analysis, DOE and RSM are used for the reduction of computational costs. In the field of design optimization, widespread research is being conducted using simulation tool of FEA (finite element analysis). Nevertheless, structural optimization usually engages costly function evaluations. For example a simulation of a traveller automobile for crash acquires around 27 hours with an probable computational cost of about US Dollars 5,200/- [15]. As a result, alternate technique of function evaluations; for instance, DOE and RSM are generally engaged in design engineering to minimize the computational costs.

**Figure 3.** Finite element model of the wing for structural analysis.

**Figure 4.** CCD representation for 2 design variables

### 3.1. Design of experiments (DOE)

The method to select a group of samples in a given design space, with the aim to maximize the amount of information gained from an inadequate number of samples is referred as Design of experiments (DOE) [16]. The objective of DOE study is to approximate and envisage the trend in the response data. In this method, the design variables are considered to be uniformly distributed along the upper and lower limits. The measured response quantity in DOE is expressed as:

\[ y_m(x) = y_r(x) + \varepsilon \]  \hspace{1cm} (1)

where \( y_m(x) \) is the measured response, \( y_r(x) \) is the true response and \( \varepsilon \) represents error term.
Central composite design (CCD) is a DOE technique which has been used in this study. In CCD the numbers of samples are given as $2^n + 2n + 1$. $n$ represents number of design variables, whereas $2^n$ samples shows corner points in the design space and $2n$ samples shows the points outside the design space. For a two dimensional CCD problem, the design boundaries are scaled from 0 to +1, 8 out of the 9 samples are lying on or outer surface of the border of the design space, while only the center point, lies inside the design space as presented in Figure 4, above.

### 3.2. Response Surface Method (RSM)

RSM is a compilation of procedures used in mathematics and statistics which are helpful for various processes such as improving, developing and optimizing. In the industrial world the use of RSM is very common and efficient. In general, the relationship of a method linking a response $u$, that involves the input parameters $\xi_1, \xi_2, \ldots \xi_n$ is written as follows:

$$u = f(\xi_1, \xi_2, \ldots, \xi_n) + \varepsilon$$

where $f$ is the response function and the term $\varepsilon$ represents error (other sources of variability):

$$E(u) = \eta = E[f(\xi_1, \xi_2, \ldots, \xi_n)] + E[\varepsilon] = f(\xi_1, \xi_2, \ldots, \xi_n)$$

It is handy to change these variables to the coded variables $v_1, v_2, \ldots v_n$. The coded variables are usually dimensionless with standard normal distribution and it can be denoted as:

$$\eta = f(v_1, v_2, \ldots, v_n)$$

Since $f$ is unknown, therefore, an approximation is needed. We often use first or second order polynomial when the small zone of independent variable space is available. The first-order model expressed in the form of coded variables is expressed as:

$$\eta = \beta_0 + \beta_1 v_1 + \beta_2 v_2$$

If there is an interaction between the parameters, the main effects model is used which can be written as:

$$\eta = \beta_0 + \beta_1 v_1 + \beta_2 v_2 + \beta_{12} v_1 v_2$$

Sometimes in order to define the response function, we need the second-order model. The following equation describes interaction between the variables:

$$\eta = \beta_0 + \beta_1 v_1 + \beta_2 v_2 + \beta_{12} v_1 v_2 + \beta_{11} v_1^2 + \beta_{22} v_2^2$$

The second-order model is flexible and has the ability to implement the estimation of the $\beta$ values conveniently [17].

### 4. Results And Discussion

In this study, the design objective is to curtail the weight of aircraft wing, restricted to stress and displacement constraints. The weight has a pivotal role in the structure of any aircraft. It is a well known fact that that the incremental factor is 4.525 for any weight bearing part; meaning thereby, 1 lbs (pound) decrement in the weight of a structure represents 4.525 lbs reduction in the gross take-off weight of an aircraft [18]. In the presented work, stress limits are 30 MPa in tension and compression, while; allowable $z$-displacement component within 0.065 m. The thickness of upper and lower skin, thickness of ribs and thickness of spars are considered as design variables. DOE was performed to fix and ascertain the thicknesses of spars and ribs. The optimum thickness of spars and ribs is found 27.3 mm and 8.5 mm respectively. The equivalent stress, equivalent strain and total deformation in the wing are determined using the static structural analysis as shown in Figures. 5, 6 and 7, correspondingly.
The trade-off chart of response surface optimization is presented in Figure 8; where the feasible area is indicated in green colour. In this technique, the behaviour of stress over the wing in response to thickness variation of wing skin is observed. It reveals that the value of stress can be reduced from 30.2 MPa to 29.7 MPa by optimizing the thickness of wing. Results were optimized so that the maximum stress reduction is achieved with minimum increment in thickness. Figure. 9 depicts the response chart showing the behaviour of maximum stresses against thickness of wing skin. The optimum value of design variable i.e. wing skin is found to be 3.1 mm.

In our work, it is revealed that optimized values of design variables that is; thickness of spar, thickness of rib, upper skin thickness and lower skin thickness for least weight should be 27.3 mm, 8.5 mm, 3.1 mm and 3.1 mm respectively, subjected to the stresses within the allowable limit. It is pragmatic here that with the presented optimization method, the procedure converged to the minimum value of weight = 1031.1 kg having the above quoted optimum values of design variables in altogether eleven iterations. The function convergence plot is presented in Figure. 10.

5. Conclusion
In this study, an aircraft wing was optimized considering the stress and displacement constraints. The thickness of the wing ribs, spars and skins were selected as design variable. Commercial software ANSYS was utilized to calculate the structural response of the wing, i.e. stress, strain and displacement distributions. The objective of optimization design is set to reduce the maximum stress as well as overall weight of the wing. For the purpose of reduction in the computational cost, response surface method technique has been employed to build up surrogate for optimization problem. After the process of optimization, the equivalent maximum stress is successfully reduced. The structural weight is also reduced by 27% using DOE and RSM technique, which is a remarkable weight saving that requires very little effort. The weight reduction could be very beneficial in terms of carrying more payloads. Furthermore, the amount of fuel to power up the aircraft will also be reduced, thus plummeting the running costs of the aircraft.
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