Research Article

Takagi–Sugeno State Delayed Feedback and Integral Control for PV Systems: Modeling, Simulation, and Control

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The integration of the large-scale photovoltaic systems has experienced significant growth, which is similarly expected to occur with small-scale photovoltaic systems. Since small-scale systems must be simple in cost-effective components, control strategies must be implemented in low complexity circuits. However, current maximum power point tracking (MPPT) algorithms are generally complex and require electronic components to support variable control gains for different irradiance conditions, preventing simple MPPT implementations suitable for small-scale photovoltaic systems. This paper proposes a new control strategy to tackle the power tracking problem of the power systems. First, a dynamic model of the photovoltaic system is described and converted into a Takagi–Sugeno (T-S) model. Then, an MPPT scheme is proposed in series with a fixed integral and a fuzzy gain state delay feedback controller, which avoids the need for a variable control gain, simplifying the electronic implementation of the control strategy. New delay-dependent stabilization conditions based on the Lyapunov-Krasovskii functional are proposed in terms of a convex optimization problem, where the delayed feedback and integral gains are designed simultaneously. Simulation results using Matlab and Simulink are used to validate the proposed method.

1. Introduction

The increasing demand for electrical energy and climate change has fostered the need for clean energy. Integrating these energies into the power system brings new technological challenges to reevaluate the usual designs, operations, and planning practices. Among clean energy technologies, photovoltaic (PV) systems have experienced significant growth worldwide, due to cost reductions [1] in the last decades. According to [2], the world total PV capacity has increased at an average annual rate of 55% in the period 2009-2013. This way, the development, modeling, and control of PV systems have captured particular attention [3–6].

Although the growth of PV systems is currently associated with large-scale plants, small-scale applications are now gaining interest. Projections of IRENA [1] established small-scale applications like rooftop PV, solar carports, and solar trees as future trends in the industry’s development. In particular, small-scale PV systems usually are close to demands (such as a rooftop PV), increasing its cost-effectiveness as pointed out in [7]. Other projections are in line with this premise of future increase in small-scale PV [8, 9].

One limitation of small-scale PV applications is the low performance of control algorithms. In small-scale applications, the control electronic cost is close to that of the power electronic converter, so current small-scale PV devices usually count with very simple circuits, such as Pulse Width Modulation (PWM) [10]. Nevertheless, this simplicity in the electronic implementation has a negative impact on performance [11]. It is then of interest to find better control algorithms that are simple enough to utilize simple electronic implementation.

In general, MPPT control strategies tend to be complex and MPPT systems are typically implemented in a DC-DC power converter to provide power to different applications [12, 5, 6, 13, 14]. In terms of the MPPT algorithms,
currently, one can find the application of diverse techniques (see, for example, [15–17] and [18, 19]). In [18], the authors have presented a comparative analysis for the perturb and observe (P&O) and the incremental conductance (INC) techniques, which are the most widely used. The MPPT algorithms must be implemented in an electronic converter, and the boost converter has shown to be the simplest [20, 21]. Even in the case of the boost converter, it can be noted that the implementation of the control strategies leads to high-level programming needs, and the use of expensive electronic digital signal processor (DSP) is standard [18, 22]. Advanced MPPT algorithms lead to complex implementation frameworks, preventing their use in small-scale applications.

The advanced MPPT proposals are based on variable control gains, which is one of the reasons why they need complex electronic implementations. Various control strategies, such as neural networks [23], genetic algorithms [24, 25], and fuzzy controllers [26–29], have appeared in the boost converter context. Generally speaking, developing a new strategy to control the PV system is a challenge. In the literature, several methods and technologies have been used and investigated. In [30], the $H_{\infty}$ control approach based on a fuzzy proportional-integral (PI) was used as a state feedback control in the control design and stability conditions of the closed-loop PV system. In [31], the boost converter was controlled using the T-S fuzzy parallel distributed compensation (PDC) to ensure stability and zero tracking error. To achieve MPPT under varying climatic conditions, the authors in [32] propose an $H_{\infty}$ observer-based fuzzy controller. To improve the efficiency of photovoltaic systems, the authors in [29] have proposed to use a fuzzy controller with adaptive gain. Two distinct rule bases were combined in the proposed controller concept. The first rule base is intended for adjusting the boost converter’s duty cycle, while the second rule base is designed for online adjusting of the controller’s gain. It is worth noting that the techniques proposed in [29–32] compute their controller gains offline. Theoretically, the above methods ensure good efficiency, fast PV power convergence to the MPPT, and a smoother steady-state response without oscillation around the MPPT. These provide a stable PV power generation. The traditional PI controller usually is sensitive to parameter variations, resulting in a low stability margin [33]. A common aspect of all these approaches is the variable nature of the different control gains to respond to changing irradiance conditions, which requires a complete processing framework. The reason for variable gains relies on the fact that the underlying nonlinear dynamics is addressed by defining different stability regions, leading to constant control policies within each stability region. The resulting control is then an aggregation of various regions to cover the totality of the control space. Hence, the control algorithm selects the control gain associated with the operating stability region, resulting in variable gain algorithms. These variable gain algorithms are difficult to implement with simple electronics, so they are not generally implemented in small-scale PV systems.

It is well known that frequency control using a fuzzy logic controller (FLC) responds effectively to parameter variations. In [34], FLC is carried out to enhance the system’s performance. It has three stages: fuzzification, processing, and defuzzification. The robust observed-based MPPT control for PV systems with a DC-DC buck converter is developed by [35], using a Lyapunov approach and LMI formulation. In particular, a T-S model-based method has been used for the MPPT of PV systems, resulting in fast convergence to the maximum power and elimination of the oscillations around the maximum. Significant results for maximizing power point tracking of PV systems were reported by [32, 36].

On the other hand, many LMI stabilization conditions based on T-S fuzzy models have been proposed in the literature using the Lyapunov approach to design controllers for nonlinear systems. The authors in [32, 30] present LMI stabilization conditions of the PV model via fuzzy observer-based and a fuzzy PI state feedback control, respectively. In [37], a robust control problem of fuzzy time-delay systems has been investigated throughout the Wirtinger-type integral inequality and convex techniques to estimate the derivative of Lyapunov-Krasovskii functional, ensuring robust asymptotic stability of the closed-loop systems. A robust $H_{\infty}$ static output-feedback controller for discrete T-S fuzzy models with input saturation constraint has been developed by [38]. However, the authors of these works did not consider the delay and the delay-control issues, which can destabilize the closed-loop systems. To the best of our knowledge, a state delayed feedback control has never been applied to PV systems with DC-DC boost converter under disturbance effect. The approximation of nonlinear systems to convex structures makes it possible to formulate stability conditions under LMIAs [39]. Since LMIAs are convex constraints, they are simple to solve using a variety of available solvers, including SeDuMi [40]. As a result, the stability/stabilization criteria of the nonlinear system can be reduced to the feasibility of a set of LMIAs. In general, when the feedback gains have been processed as variable parameters in the LMI feasibility issue, automatic stabilizing control is generated from a set of obtained LMIAs.

This paper proposes a new control approach for a boost converter in an MPPT PV application based on a state delayed feedback approach and integral control. The contribution is a new state delay feedback algorithm, which allows the determination of a unique, constant controller gains for irradiance’s different operating values, including the integral gain. The proposal is based on new sufficient delay-dependent criteria stated as a convex optimization problem to ensure that the closed-loop system is asymptotically stable and minimizes the exogenous impact on the boost converter’s output. The theoretical formulation is verified by numerical results using Matlab and Simulink, where a complete simulation of the system is presented.

The organization of the paper is as follows. In Section 2, the Takagi–Sugeno (T-S) fuzzy model for the PV system is presented. The general control strategy proposed in this paper is presented in Section 3, including the control design approach. In Section 4, a complete simulation result is given, considering the PV panel module: 1Soltech 1STH-FRL-4H-250-M60-BLK. The last section presents the conclusion
and general discussion about the proposed control approach for a boost converter in an MPPT PV application.

Notations. The notations used in this paper are quite standard. \( \mathbb{R}^n \) and \( \mathbb{R}^{n \times n} \) refer to, respectively, the \( n \)-dimensional Euclidean space and \( n \times n \) real matrices. The superscript “\( T \)” means the transpose of a matrix. The notation \( P > 0 \) means the matrix \( P \) is a symmetric and positive definite. The notation \( s(M) \) means \( M + M^T \).

2. T-S Fuzzy Model for the PV System

A first aspect of the analysis is constructing a functional model of the PV system. A T-S model from the complex nonlinear system that describes the PV system global dynamic behavior allows obtaining an exact representation using local submodels. Then, we will present the equations from the PV panel and the DC-DC boost converter’s functioning. The PV panel equations are related to the DC-DC boost converter by a T-S fuzzy model were obtained from [31, 32], which describes the PV system.

2.1. PV System Model. A PV cell is a system that converts light into electricity through the photoelectric effect. A PV cell is generally represented by an equivalent circuit [31, 32] shown in Figure 1.

From Figure 1, the PV current can be defined as follows:

\[
I_{pv} = I_{ph} - I_d - I_{sh},
\]

where \( I_{ph} \) is the photon-current which is dependent on the solar irradiance and can be defined by the following:

\[
I_{ph} = (I_{sc} + K_{sc} \Delta T) \frac{S}{S_0},
\]

where \( I_{sc} \) is the short-circuit current, \( K_{sc} \) is the parameter of PV cell short-circuit current, \( \Delta T = T - T_{ref} \) is the variation temperature, \( T_{ref} \) is the reference temperature of PV cell, \( S \) is the solar irradiation, and \( S_0 \) is the nominal solar irradiation. It is clear that the photon-current increases when the temperature or the solar irradiation increases.

The diode current is defined as follows:

\[
I_d = I_s \left( \exp \left\{ \frac{V_p + R_s I_{pv}}{V_T} \right\} - 1 \right),
\]

where \( V_T \) is the thermal voltage, \( V_{pv} \) is the photovoltaic output voltage, \( R_s \) is the shunt resistance, and \( I_s \) is the saturation current which is represented by [31, 32]

\[
I_s = I_{ns} \left( \frac{T}{T_{ref}} \right)^3 \exp \left\{ \frac{Q E_{by}}{n k T_{ref}} \left( \frac{1}{T_{ref}} - \frac{1}{T} \right) \right\},
\]

where \( Q \) is the electronic charge, \( E_{by} \) is the semiconductor band-gap energy of the PV cell, \( K_B \) is the Boltzmann constant, \( n \) is the ideal PN-junction characteristic, and \( I_{ns} \) is the reverse saturation at \( T_{ref} \) and \( S_{ns} \). The value of reverse satura-

tion current \( I_{ns} \) may be evaluated through the open circuit voltage \( V_{oc} \) and the short circuit current \( I_{sc} \) given by

\[
I_{rs} = \frac{I_{sc}}{\exp \left\{ \frac{V_{oc}}{V_T} \right\} - 1}.
\]

This way, the solar cell can be modeled.

2.2. DC-DC Boost Converter. The DC-DC boost converter (Figure 2) is one of the simplest types of the switch-mode converter [41, 42]. It consists of an inductor, a semiconductor switch, a diode, and a capacitor. The advantage of using the DC-DC converter is that the efficiency is high [43], because all the circuit elements, such as the inductor, capacitor, switch, and diode, present negligible losses. In practice, the efficiency of the DC-DC converter exceeds 90%, which is adequate for an energy converter. Due to the existence of commutations, the boost converter operates in two modes: the inductor stores energy, and the capacitor releases energy when the switch is closed; the inductor releases energy, and the capacitor stores energy when the switch is opened. Additionally, the output voltage can be varied with the duty cycle of the commutator with a Pulse Width Modulation (PWM) strategy. The topology is simple and effective.

Note that the boost converter is normally implemented with a MPPT algorithm, so another converter is necessary to consider a secondary application such as voltage DC source, a battery charger, or a DC/AC inverter [44]. The design of secondary conversion systems is out of the scope of this work.

Regardless of the simplicity of the boost converter, its dynamics are described by nonlinear models [30–32]. A modeling approach based on a T-S fuzzy model is presented below.

2.3. T-S Fuzzy Model. An exact representation of the global dynamic behavior of the PV system can be obtained by a T-S model using local submodels. T-S models are usually defined by if-then fuzzy-rules in the state space. The T-S PV model developed in [31, 32] is described as follows:

\[
x(t) = \sum_{i=1}^{\delta} \theta_i(z(t))(Ax(t) + Bu(t) + Dw(t)),
\]

\[
y(t) = Ex(t),
\]
where \( A \in \mathbb{R}^{n \times n}, B_i \in \mathbb{R}^{n \times m}, D \in \mathbb{R}^{m \times w}, \) and \( E \in \mathbb{R}^{w \times n} \) are constant matrices with

\[
A = \begin{bmatrix}
    \frac{R_i}{L} & -\frac{1}{L} & 1 \\
    \frac{1}{C_2} & -\frac{1}{RC_2} & 0 \\
    -\frac{1}{C_1} & 0 & 0
\end{bmatrix},
B_i = \begin{bmatrix}
    \frac{V_{C_{2\text{max}}}}{L} \\
    \frac{I_{l_{\text{max}}}}{C_2} \\
    0
\end{bmatrix},
E = \begin{bmatrix}
    1 & 0 & 0 \\
    0 & 1 & 0
\end{bmatrix},
\]

and

\[
x(t) = \begin{bmatrix}
    I_L(t) \\
    V_{C_2}(t) \\
    V_{\text{po}}(t)
\end{bmatrix},
\]

\[
\theta_1(z(t)) = U_{1_{\min}} \ast H_{1_{\min}},
\theta_2(z(t)) = U_{1_{\min}} \ast H_{1_{\max}},
\theta_3(z(t)) = U_{1_{\max}} \ast H_{1_{\min}},
\theta_4(z(t)) = U_{1_{\max}} \ast H_{1_{\max}},
\]

with \( U_{1_{\min}}, U_{1_{\max}}, H_{1_{\min}}, \) and \( H_{1_{\max}} \) are the membership functions.

Remark 1. The representation (6) is given by using the sector bounded nonlinearity. This technique is usually applied to obtain linear submodel. Furthermore, the global model (6) is operated in the space \([I_{L_{\min}} I_{L_{\max}}] \times [V_{C_{2\text{min}}} V_{C_{2\text{max}}}].\)

Note that most physical systems involve time delays in their behavior, and in PV systems, delays can occur from the charging and discharging of electronic components. This delay may be small due to the construction of electronic components, but it is still necessary to take them into account. In general, the delay effect is not taken into consideration when using the boost converter directly. Nevertheless, it is known that delays are sources of instability of the system’s performance and a source of uncertainties. Hence, to deal with the delay effect when using the boost converter, we propose to control the boost converter by using state delay feedback and the integral (I) controller. The next section presents the control strategy, including a numerical approach to the control design proposed in this work.

### 3. Control Strategy

The general control strategy proposed in this work can be seen in Figure 3, where the PV panel inputs are the temperature \( T \) and the solar irradiation \( S_r \), while the PV panel outputs are the PV panel current \( I_{\text{pv}} \) and voltage \( V_{\text{pv}} \). The general goal is to design both controllers the integral and the state delay feedback together with the MPPT in order to transfer the maximum power from the PV panel.

In this paper, a reference voltage \( V_{\text{ref}} \) is obtained by using the perturb and observe (P&O) algorithm (see Figure 4) in order to get the maximum power point tracking (MPPT).

The proposed control strategy described in Figure 3 shows that the integral control input is the difference between \( V_{\text{mppt}} \) and \( V_{\text{pv}} \). This implies that the input is the slope, as shown in Figure 4. Moreover, it noted that slope as input does not mean that a disturbance supplied to integral control since the slope parameter is part of the MPPT algorithm, and there is a different way to build it.

The strategy used to control the boost converter is shown in Figure 3. The idea behind using the integral controller (I) and the state delay feedback is that the stability region provided does not display any discontinuity, which...
allows a more flexible selection of the controller’s gains, which can be constant in particular. The integral controller is used to regulate the error between the MPPT output and the PV panel voltage. The state delay feedback is used to command the boost converter and represent the delay’s effect on the boost functioning. The control law is defined as

\[ u(t) = u_1(t) + u_2(t), \]  

where \( u_1(t) \) is the overall state delay feedback described in the fuzzy rule if-then as follows:

\[ u_1(t) = \sum_{i=1}^{4} \theta_i(z(t))K_{di}x(t-h), \]

and \( h \) is the constant delay, \( x(t) \in \mathbb{R}^n \) is the system state, and \( K_{di} \in \mathbb{R}^{m \times n} \) are the delay-feedback controller gains. Moreover, \( u_2(t) \) is the integral controller given by

\[ u_2(t) = K_i \int e(t)dt, \]
with $K_I$ the integral controller gain and $e(t) = V_{\text{mppt}} - V_{\text{pv}}$ the error between the MPPT output and the PV output.

The integral controller produces the output signal given by $u_2(t)$, which is proportional to the integral of the input signal $e(t)$. This $V_{\text{mppt}}$ is compared with the PV voltage ($V_{\text{pv}}$), and the integral controller receives an error signal ($e(t)$). The desired response can be achieved by designing the integral gain ($K_I$). Once the boost converter receives energy from the PV panel, the integral controller starts to function, the value of the duty cycle varies, and the input value sensed by the integral controller changes.

The main advantage of the proposed control is its simplicity, mainly because now it is considered a unique constant integral controller for all levels of irradiation. It is well known that the integral controller is used in many applications since it can be implemented practically with basic electronic components [45–48], which significantly simplifies the implementation and costs.

### 3.1. Control Design

Based on the experimental step response, Ziegler-Nichols and other authors have proposed several rules, for tuning the PI controller [49–51]. Several results associated with the analytical calculation of the gain margin, the phase margin of the delayed systems, and the time-delay ratio have considered [49]. These rules can, of course, be applied to known mathematical models. Such rules suggest a set of values of $K_p$ and $K_I$ that will ensure the stable operation of the system. However, the resulting system may present performance issues, such as excessive overshoots, in which case fine-tuning rules are required until an acceptable result is obtained. The fine-tuning rules of Ziegler-Nichols provide a systematic way of choosing the values of controller gains.

Next, a convex approach is proposed in order to design both the integral and state delay feedback controllers, obtaining the gains $K_I$ and $K_{di}$ for $i = \{1, 2, 3, 4\}$ that will be used to generate the DC-DC boost converter control signal. In order to obtain the latter, replace (10) and (11) into (6):

$$\dot{x}(t) = A(t)x(t) + A_d(t)x(t-h) + D(t)x(t) + Bu(t) + B(t)K_Ie(t),$$

(13)

$$y(t) = Ex(t),$$

(14)
where
\[ A(t) = \sum_{i=1}^{4} \theta_i(t) A_i, \quad B(t) = \sum_{i=1}^{4} \theta_i(t) B_i, \]
\[ A_d(t) = B(t) K_d(t), \quad D(t) = \sum_{i=1}^{4} \theta_i(t) D_i, \quad f(e(t)) = \int e(t) dt. \]

It is also assumed that there exists a positive scalar \( \alpha \) such that \( f^T(e(t))f(e(t)) < \alpha x^T(t)x(t), \forall t > 0. \)

The control technique used helps to display the delay in the system formulation and also to show the effect of the delay on the boost converter. Furthermore, a delay-dependent condition of the closed-loop system can be easily provided. Moreover, to minimize the impact of the exogenous disturbances on the output of the boost converter, the \( H_\infty \) performance index \( J \) is introduced.

\[ J = \int_0^t \left\{ y^T(s)y(s) - \gamma^2 \bar{w}^T(s)\bar{w}(s) \right\} ds, \]

where \( \gamma \) is a positive scalar.

Our aim is to develop a delay-dependent stabilization method, which provides a controller gain \( K(t) \) and integral control parameter \( K_I \) such that the closed-loop system (13) is stable for any positive constant delay \( h \) and the \( H_\infty \) performance index \( J < 0 \) for all \( t > 0 \). The next theorem presents a convex-based approach to solve this problem.

**Theorem 2.** Consider the photovoltaic system described in (6). Based on the control strategy (10) with (11), the closed-loop system (13) is asymptotically stable with \( H_\infty \) performance \( J < 0 \), if
there exist positive matrices $X$, $\tilde{Q}$, and $\tilde{R}$ and matrices $Y$, $i = \{1, 2, 3, 4\}$ and $K_i$ and scalars $\lambda$, $h$, $\rho$, and $\gamma$ such that the following inequalities are satisfied.

$$\tilde{Y} ii < 0, i = \{1, 2, 3, 4\},$$  \hspace{1cm} (17)

with

$$P ii = [A, X B, Y, D, B, K_i],$$

$$\Sigma = \Sigma \Omega = [E, X, 0, 0],$$

$$\Omega = \text{diag} \{X, X, I, I\},$$

$$\tilde{Y} ii = \tilde{Q} - \tilde{R} + \text{sym} \{A, X\},$$

$$\tilde{Y} i2 = \tilde{R} + \tilde{B}, Y, J,$$

$$\tilde{Y} i3 = D, i,$$

$$\tilde{Y} i4 = B, K, i,$$

$$\tilde{Y} 22 = -\tilde{Q} - \tilde{R}.$$  \hspace{1cm} (20)

If the conditions in (17) are satisfied, the controller gains are defined as follows:

$$K_i = Y, X, i, i = \{1, 2, 3, 4\} \text{ and } K, i.$$  \hspace{1cm} (21)

Moreover, $f^T (e(t)) f (e(t)) < 1 / px^T (t) x(t), \forall t > 0.$

Proof. The proof can be found in Appendix.

Theorem 2 proposes sufficient conditions to design the state delay feedback and integral controller in (10), in order to ensure that the T-S PV model is stable and minimizes the impact of exogenous disturbances on the output of the boost converter. Since the proposed stability conditions (17) are Linear Matrix Inequality (LMI) conditions, for a given upper bound delay $h$ and a scalar $\lambda$, the controller gains ($i$) and $K, i$ can easily be determined using effective convex optimization algorithms [39]. On the other hand, note that the traditional state feedback controller (without delay) may lead to more conservative results since they are independent of the delay, which tends to be conservative, especially when the actual delay is small. Although the traditional state feedback controller (without delay) has the advantage of being simple to implement, its performance cannot be better than that of a delayed state-feedback controller, which uses the available information about the size of the delay. The delayed state-feedback controller could be viewed as a compromise between improving performance and implementation simplicity.

Remark 3. Note that the gains of all controllers in the proposed scheme are computed offline. Thus, the need for complex control electronics during the control action is not required. Besides, the simulation test (online control) needed only a couple of gains ($K$ and $K, di$) to get the desired objectives. The considerations set out above show the simplicity and importance of the scheme proposed.

Next, this paper focuses on implementing state delay feedback with an integral controller to transfer the maximum energy from the PV panel to the boost converter’s output. Particular attention has been paid to the design of an integral controller parameter $K, i$ and state delay feedback simultaneously. For this reason, the parameters obtained from Theorem 2 will be used to simulate the PV system. Note that, since the conditions on Theorem 2 are a linear function of $\gamma$, it is possible to solve an optimization problem in order to minimize the $H, co$ performance index $J$.

4. Simulation Result

In order to evaluate the proposed control strategy in Figure 1, an actual PV module is considered, whose specifications are listed in Table 1. Also, we consider the boost converter parameters in Figure 2 as in [30] that are $L = 10 \, mH$, $R, L = 0.01$, $C, 1 = 500 \, \mu F$, $C, 2 = 100 \, \mu F$, and $R = 20 \, \Omega$.

The first step is to obtain the integral gain $K, i$ and state delay feedback gains in order to ensure that the maximum energy from the PV panel to the boost converter’s output. In order to solve this problem numerically using convex optimization algorithms, Theorem 2 has been applied with $h = 0.25$ and $\lambda = 0.005$, and it is also assumed that $K, i < -1.9$, which are values that maintain the convexity of the problem. In this
case, the following gains were simultaneously computed offline: $K_f = -2.0665$ and the T-S fuzzy controller gains.

$$
\begin{align*}
K_d_1 &= 10 - 3[0.1204 - 0.0043 - 0.0003], \\
K_d_2 &= 10 - 3[0.1207 - 0.0035 - 0.0067], \\
K_d_3 &= 10 - 4[-0.2964 - 0.0174 - 0.0035], \\
K_d_4 &= 10 - 3[-0.3905 - 0.0112 - 0.0210]. \\
\end{align*}
$$

(22)

To verify the performance of the boost converter by using the proposed control strategy in the transient and steady-state period, Equation (23) shows the Sigmoidal membership functions used by the state delay feedback. The irradiation changes are proposed in Figure 5. The irradiation profile has transient mode at 0.5 s, 1 s, 1.5 s, and 2 s. Figure 6 shows the PV and boost converter power. Figure 7 shows the evolution of the control law applied. Figure 8 illustrates the Simulink model of the PV model control, and Figure 9 displays the Simulink model of control strategy implemented.

$$
U_{1,\text{min}} = \frac{1}{1 + e^{3/(I_L - 1)}},
$$

(23)

$$
H_{1,\text{min}} = \frac{1}{1 + e^{-2.5(V_{c_2} - 0.3)}},
$$

(24)

From Figure 6, the power generated by the boost converter is close to the MPPT value. It can be seen that the MPPT condition is reached for a variety of irradiance conditions while the dynamic behavior occurs in a smooth manner with low picks and oscillations. Also, output power varies proportional to input irradiance, as normally seen in MPPT applications.

As shown in Figure 10, the dynamic trajectory of system follows a MPPT path. Since the algorithm for MPPT is P&O, an oscillatory behavior is observed, as normally occurs in
these cases. The oscillation is not significant, with a steady-state ripple less than 2%. Furthermore, the proposed MPPT control can detect all the maximum power points, particularly those of small irradiation, while the INC and P&O MPPT control ones have not been detected (see [30, 32] and references therein). This means that the proposed MPPT control can effectively maintain the PV system operating at the MPPT for all the irradiation.

Figure 11 displays the Simulink block used to calculate the efficiency of the PV model based on the control strategy, where "Gain_1" shows the maximum power at 1000 W/m². Figure 12 shows the efficiency of the PV model control calculated. It can be seen that the efficiency decreases while the transient behavior occurs, which shows that the operation is not close to MPPT during the transient behavior. In steady state, the efficiency is close to 95%, which is an acceptable efficiency for these applications.

Note that the controller gains were obtained offline, which implies less computation complexity and less time in computation than using the sliding mode control and fuzzy logic control method. The sliding mode control method used the online calculation and required more accurate time to calculate [5, 6]. Besides, the fuzzy logic control method has a complex implementation [52] and requires the active power at each instant k, which takes more accurate time. Moreover, an additional electronic component needs to be connected between the boost converter and the load that introduces additional cost charges [6, 52], which is not
required for the proposed approach in this paper. Furthermore, the introduction of constant time delay allows to provide a delay-dependent condition of the PV system, which is ignored in other existing works [5, 6, 52].

5. Conclusion

This paper proposes a Takagi–Sugeno state delayed feedback and integral control for a boost converter in a PV application. The proposal obtains constant control gains, regardless of the nonlinear nature of the phenomena that usually leads to variable control gains from stratifications of the stability regions. The control framework is based on delay-dependent stabilization conditions using a Lyapunov-Krasovskii functional, leading to a convex optimization problem, where the delayed feedback and integral gains are obtained simultaneously via LMI. By simulation results, one can see that the oscillations and efficiency of the results are adequate for PV applications, showing the applicability of the method. Note also that the proposed control strategy is simple to implement and achieves acceptable performance. In terms of simplicity, all the gains necessary for the control implementation are constant and do not require online processing; instead, the gains are computed offline and then implemented to be proportional to the input signals. In terms of performance, the proposed method shows accurate MPPT tracking, acceptable steady-state efficiency (about 95%), and low steady-state ripple (less than 2%). These aspects show a proposal that simplifies the electronic implementation of MPPT algorithms for small-scale PV developments.

Appendix

Proof of Theorem 2. Let us consider the Lyapunov-Krasovskii functional described as follows:

\[
V(x(t)) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(s)Qx(s)ds
\]

\[+ h \int_{t-h}^{t} x^T(s)R \dot{x}(s)ds d\lambda.\]  

(A.1)

Calculating the time derivative of (A.1), we obtain

\[
\dot{V}(x(t)) = 2x^T(t)P \dot{x}(t) + x^T(t)Qx(t) - x^T(t-h)Qx(t-h)
\]

\[+ x^T(t)h^2R \dot{x}(t) + h \int_{t-h}^{t} x^T(s)R \dot{x}(s)ds.\]  

(A.2)

Using the Jensen’s inequality, we have

\[
\dot{V}(x(t)) \leq 2x^T(t)P \dot{x}(t) + x^T(t)Qx(t)
\]

\[- x^T(t-h)Qx(t-h) + x^T(t)h^2R \dot{x}(t)
\]

\[+ (x^T(t) - x^T(t-h))R(x(t) - x(t-h)).\]  

(A.3)

Assume that the function \( f((t)) \) is bounded and satisfies the following condition:

\[
f^T(e(t))f(e(t)) < ax^T(t)x(t) \Rightarrow 0 < ax^T(t)x(t) - f^T(e(t))f(e(t)),\]  

(A.4)

where \( a = 1/\rho \). Adding the right side of (A.4) to (A.3), we obtain

\[
\dot{V}(x(t)) \leq 2x^T(t)P \dot{x}(t) + x^T(t)Qx(t) - x^T(t-h)Qx(t-h)
\]

\[+ x^T(t)h^2R \dot{x}(t) + (x^T(t) - x^T(t-h))R(x(t) - x(t-h))
\]

\[+ ax^T(t)x(t) - f^T(e(t))f(e(t)).\]  

(A.5)

Consider now the \( H_\infty \) performance index \( J \) in (16). Under zero initial condition, we have

\[
J < \int_{0}^{\infty} \{ \dot{V}(x(s)) + \gamma^{-1}y^T(s)y(s)\}ds = \int_{0}^{\infty} \xi^T\Psi \xi ds,
\]

(A.6)

where

\[
\Psi = \begin{bmatrix}
\Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} \\
* & \Psi_{22} & 0 & 0 \\
* & * & -\gamma I & 0 \\
* & * & * & -I \\
\end{bmatrix} + h^2\Pi^T\Pi + \gamma^{-1}\Sigma^T\Sigma.
\]

(A.7)

with

\[
\xi^T = \begin{bmatrix} x^T(t)x^T(t-h)t^T(0)f^T(e(t)) \end{bmatrix},
\]

\[
\Pi = [A(t)A_d(t)D(t)B(t)K_d],
\]

\[
\Psi_{11} = Q - R + \alpha I + \text{sym \{PA(t)\}},
\]

\[
\Psi_{12} = R + PA_d(t),
\]

\[
\Psi_{13} = PD(t),
\]

\[
\Psi_{14} = PB(t)K_d,
\]

\[
\Psi_{22} = -Q - R.
\]

To design the controller gains, we define the following matrix \( \Omega = \{X, X, I, \Gamma\} \) with \( X = P^{-1} \). Pre- and post-multiplying \( \Psi \) by \( \Omega \) and its transpose, we obtain

\[
\tilde{\Psi} = \Omega^T\Psi \Omega = \begin{bmatrix}
\tilde{\Psi}_{11} & \tilde{\Psi}_{12} & \tilde{\Psi}_{13} & \tilde{\Psi}_{14} \\
* & \tilde{\Psi}_{22} & 0 & 0 \\
* & * & -\gamma I & 0 \\
* & * & * & -I \\
\end{bmatrix} + \Omega^T \left\{ h^2\Pi^T\lambda (\lambda^2 R^{-1})^{-1}\lambda \Pi + \gamma^{-1}\Sigma^T\Sigma \right\} \Omega.
\]

(A.9)
By using relaxation method, Equation (A.9) can be described as follows:

\[
\dot{\Psi} = \sum_{i=1}^{4} \theta_i^T(z(t))\dot{\Psi}_i + \sum_{i=1}^{4} \sum_{j<i} \theta_i(z(t))\theta_j(z(t))(\dot{\Psi}_{ij} + \dot{\Psi}_{ji}).
\]

(A.10)

By considering the variable changes \(Q = XQX\) and \(R = XRX\), and the controller design \(Y(t) = K(t)\) with Schur Complement Lemma and right-hand side of inequality (A.11), we obtain conditions in (17).

\[
(\lambda R^{-1} - X)R(\lambda R^{-1} - X) > 0 \iff -\lambda^2 R^{-1} < \dot{R} - 2\lambda X.
\]

(A.11)

If conditions in (17) are satisfied, it means that (A.10) is satisfied. This implies that \(J < 0\), and the closed-loop system is asymptotically stable with \(H_{\infty}\) performance. This completes the proof.

**Abbreviations**

DSP: Digital signal processor  
INC algorithm: Incremental conductance algorithm  
FLC: Fuzzy logic controller  
LMIs: Linear matrix inequalities  
MF: Membership function  
MPPT: Maximum power point tracking  
PDC: Parallel distributed compensation  
P&O algorithm: Perturb and observe algorithm  
PI: Proportional-integral  
PV systems: Photovoltaic systems  
PWM: Pulse width modulation  
T-S model: Takagi–Sugeno model.

**Data Availability**

No data were used to support this study.

**Conflicts of Interest**

The authors declare no potential conflict of interests.

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