Evolution of the Yukawa coupling constants and seesaw operators in the universal seesaw model

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The general features of the evolution of the Yukawa coupling constants and seesaw operators in the universal seesaw model with \( \det M_F = 0 \) are investigated. In the model, not only the magnitude of the Yukawa coupling constant \( (Y^u)^{ij}_{33} \) in the up-quark sector but also that of \( (Y^d)^{ij}_{33} \) in the down-quark sector is of the order of one. The requirement that the model should be calculable perturbatively, i.e., \( |Y^f_{ij}|^2/4\pi \lesssim 1 \), puts some constraints on the values of the intermediate mass scales and \( \tan \beta \) (in the SUSY model).

I. INTRODUCTION

Recently, there has been considerable interest in the evolution (energy-scale dependency) of the Yukawa coupling constants of quarks and leptons. If we intend to build a model which gives a unified description of quark and lepton mass matrices, we cannot avoid investigating the evolution of the Yukawa constants. The recent study on the quark masses and mixings has been given, for example, in Ref. [2]. Especially, recently, the evolution of the neutrino seesaw mass matrix has been received considerable attention (for example, see Ref. [2]) in connection with the energy-scale dependence of the large mixing angle.

As one of such unified models, there is a non-standard model, the so-called “universal seesaw model” (USM) [3]. The model describes not only the neutrino mass matrix \( M_\nu \) but also the quark mass matrices \( M_u \) and \( M_d \) and charged lepton mass matrix \( M_\ell \) by the seesaw-type matrices universally: The model has hypothetical fermions \( F_i \) \((F = U, D, N, E; i = 1, 2, 3)\) in addition to the conventional quarks and leptons \( f_i \) \((f = u, d, \nu, e; i = 1, 2, 3)\), and these fermions are assigned to \( F_L = (1, 2), F_R = (1, 1) \) and \( F_R = (1, 1) \) of SU(2)_L \( \times \) SU(2)_R. The 6 \times 6 mass matrix which is sandwiched between the fields \((f_L, F_L)\) and \((f_R, F_R)\) is given by

\[
M^{6 \times 6} = \begin{pmatrix}
0 & m_L \\
m_R & M_F
\end{pmatrix},
\]

where \( m_L \) and \( m_R \) are universal for all fermion sectors \((f = u, d, \nu, e)\) and only \( M_F \) have structures dependent on the flavors \( F \). For \( \Lambda_L < \Lambda_R \ll \Lambda_S \), where \( \Lambda_L = O(m_L), \Lambda_R = O(m_R) \) and \( \Lambda_S = O(M_F) \), the 3 \times 3 mass matrices \( M_F \) for the fermions \( f_i \) are given by the well-known seesaw expression

\[
M_F \simeq -m_L M_F^{-1} m_R^\dagger.
\]

Thus, the model answers the question why the masses of quarks (except for top quark) and charged leptons are so small compared with the electroweak scale \( \Lambda_L \) \((\sim 10^2 \text{ GeV})\).

Recently, in order to understand the observed fact \( m_t \sim \Lambda_L \) \((m_t \text{ is the top quark mass})\), the authors have proposed a universal seesaw mass matrix model with an ansatz \( \det M_F = 0 \) for the up-quark sector \((F = U)\). In the model, one of the fermion masses \( m(U_i) \) is zero \(\text{[say, } m(U_3) = 0]\), so that the seesaw mechanism does not work for the third family, i.e., the fermions \((u_{3L}, U_{3R})\) and \((U_{3L}, u_{3R})\) acquire masses of \( O(m_L) \) and \( O(m_R) \), respectively. We identify \((u_{3L}, U_{3R})\) as the top quark \((t_L, t_R)\). Thus, we can understand the question why only the top quark has a mass of the order of \( \Lambda_L \).

Our interest is as follows: In the conventional model, the Yukawa coupling constants \( y_f \) of the fermions \( f \) are given by \( y_f = m_f/\langle \phi_0^L \rangle \). Only the Yukawa coupling constants \( y_t \) of the top quark \( t \) takes a large value \( y_t = m_t/\langle \phi_0^L \rangle \sim 1 \). The other Yukawa coupling constants \( y_f \) are sufficiently smaller than one. On the contrast to the conventional model, in this USM, the matrices \( m^i_f = Y^f_{ij}\langle \phi_0^L \rangle \) are universal for all fermion sectors \( f = u, d, \nu, e \), i.e., \( Y^u = Y^d = Y^\nu = Y^e \). Therefore, when \((Y^u)^{33} = \text{is of the order of one, the other } (Y^f)^{33} \) will be also of the order of one. We are afraid that in such a model the Yukawa coupling constants have Landau poles at energy scales lower than a unification energy scale \( \mu = \Lambda_X \) (so that the model causes “burst” of Yukawa coupling constants before going to the unification energy scale). One of our interests is to investigate whether such a model can provide or not a set of reasonable parameter values under the conditions that the Yukawa coupling constants (and also the seesaw operators \( m_L M_F^{-1} m_R^\dagger \)) do not have the Landau poles below \( \mu = \Lambda_X \).

We also take interest in the “democratic” USM \([4, 5]\), which is an extended version of USM and has successfully given the quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) \([6]\) matrix parameters in terms of the charged lepton masses. However, the study is only phenomenology at the energy scale \( \mu = m_Z \) \((m_Z \text{ is the neutral weak boson mass})\). Since the model is one of the
promising models of the unified description of the quark and lepton mass matrices, it is important to investigate the evolutions of the mass matrices in the USM.

The democratic USM is as follows:

(i) The mass matrices $m_L$ and $m_R$ have the same structure except for their phase factors

$$ m^f_R = \kappa m^L_R = \kappa m_0 Z^f, \quad \text{where } \kappa \text{ is a constant and } Z^f \text{ are given by} $$

$$ Z^f = \text{diag} \left( z_1 \exp(i\delta^f_1), z_2 \exp(i\delta^f_2), z_3 \exp(i\delta^f_3) \right), \quad (1.3) $$

with $z_1^2 + z_2^2 + z_3^2 = 1$. (The fermion masses $m^f$ are independent of the parameters $\delta^f_i$. Only the values of the CKM matrix parameters $|V_{ij}|$ depend on the parameters $\delta^f_i$).

(ii) In the basis on which the matrices $m^L$ and $m^R$ are diagonal, the mass matrices $M_F$ are given by the form

$$ M_F = m_0 \lambda (1 + 3bfX), $$

$$ 1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (1.5) $$

(iii) The parameter $b_f$ for the charged lepton sector is given by $b_e = 0$, so that in the limit of $\kappa/\lambda \ll 1$, the parameters $z_i$ are given by

$$ \frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}}. \quad (1.6) $$

Then, the up- and down-quark masses are successfully given by the choice of $b_u = -1/3$ and $b_d = -e^{i\beta_u}$ ($\beta_u = 18^\circ$), respectively. The CKM matrix is also successfully obtained by taking

$$ \delta^u_1 - \delta^d_1 = \delta^u_2 - \delta^d_2 = 0, \quad \delta^u_3 - \delta^d_3 \approx \pi. \quad (1.7) $$

However, when we take the evolution of the Yukawa coupling constants into the consideration, we should consider that the assumptions (i) and (ii) are required not at the electroweak energy scale $\mu = \Lambda_L$, but at a unification energy scale $\mu = \Lambda_X$, i.e., the assumptions (i) and (ii) should be replaced with

$$ Y^f_L(A_X) = Y^f_R(A_X) = \xi^f_{LR} Z^f, \quad (1.8) $$

$$ \xi^u_{LR} = \xi^d_{LR}, \quad (1.9) $$

and

$$ Y^f_S(A_X) = \xi^f_S (1 + 3bfX), \quad (1.10) $$

respectively, where mass matrices $m_L, m_R$ and $M_F$ are expressed by

$$ m^f_L = Y^f_L (\phi^0_L), \quad m^f_R = Y^f_R (\phi^0_R), \quad M_F = Y^f_S (\Phi^0), \quad (1.11) $$

respectively, and

$$ \langle \phi^0_L \rangle = \langle \phi^0_R \rangle / \kappa = \langle \Phi^0 \rangle / \lambda \quad (1.12) $$

and $\phi_L, \phi_R$ and $\Phi$ are Higgs scalars whose vacuum expectation values (VEV) break SU(2)_${L,R}$ and an additional U(1) symmetry U(1)$_X$, respectively. (For simplicity, we have assumed that the values of $\langle \phi^0_L \rangle$, $\langle \phi^0_R \rangle$ and $\langle \Phi^0 \rangle$ are real.)

Another interest in the present paper is to check whether or not the phenomenological study in the previous paper [4] is still approximately valid under the evolution of the Yukawa coupling constants. For example, the model with $b_e = 0$ and $b_u = -1/3$ has led to the relation

$$ \frac{m_u}{m_e} \approx \frac{3}{4} \frac{m_u}{m_e}, \quad (1.13) $$

almost independently of the value of the seesaw suppression factor $\kappa/\lambda$. One of the reasons to taking the value of $b_f$ in the up-quark sector as $b_u = -1/3$ exists in the successful relation (1.14). Therefore, we have interest whether the relation (1.14) still holds even when we take the evolution into consideration.

Besides, even apart from such phenomenological interests, it is very important to investigate the general features of the evolution of the Yukawa coupling constants in the universal seesaw model with det$M_F = 0$, because in the present model one of the fermions $F_i$ does not couple from the theory at $\mu < \Lambda_S$, so that the evolution shows peculiar behavior in contrast with the conventional seesaw model.

A similar study has been done in Ref. [8] by one of the authors (Y.K.). However, in Ref. [9], instead of the seesaw operators $K^f_i$ which will be defined later in Eqs. (3.8) corresponding to $m_L M_F^{-1} m_R$, the evolution of the seesaw forms of the Yukawa coupling constants $Y^f_L (Y^f_S)^{-1} (Y^f_R)^i$ were investigated by calculating the Yukawa coupling constants $Y^f_L, Y^f_R$ and $Y^f_S$ individually under the assumption that the heavy particles with the masses of the order of $\Lambda_S \equiv \langle \Phi^0 \rangle$ do not contribute to the evolution of $Y^f_A$ ($A = L, R, S$) below $\mu = \Lambda_S$. In the present paper, we will calculate the evolution of the Yukawa coupling constants $Y^f_A$ above $\mu = \Lambda_S$ and that of the seesaw operators $K^f_i$ below $\mu = \Lambda_S$, except for $(Y^f_L)_{i3}$ as discussed in Sec. [11].

In Sec. [10] we will discuss an additional symmetry which is introduced for the purpose of preventing that the fermions $F$ acquire the masses $M_F$ at the energy scale $\mu = \Lambda_S$. In Sec. [11], we will give the general formulation of the evolution of the seesaw mass matrices with det$M_U = 0$. In Sec. [12], we give the explicit coefficients of the renormalization group equations. In Sec. [13], we
discuss the evolution of an extended version of the USM, the “democratic seesaw model”. The numerical results for a non-SUSY model and for a minimal SUSY model are given in Secs. VI and VII, respectively. It will be emphasized that the energy scale dependencies in the SUSY model are quite different from those in the non-SUSY model. The evolution of the neutrino mass matrix is given in Sec. VIII. It will be showed that, differently from the conventional seesaw model, the present neutrino mass matrix is form-invariant below $\mu = \Lambda_S$. Finally, Sec. IX will be devoted to the conclusions and remarks.

II. $U(1)_X$ SYMMETRY

In the present model, the gauge symmetries are broken as follows:

$$H_{int} = Y_{Lij}^u \tilde{d}_L \phi_L U_{Rj} + Y_{Lij}^d \tilde{d}_L \phi_L D_{Rj} + Y_{Lij}^\nu \tilde{\nu}_L \phi_L N_{Rj} + Y_{Lij}^\nu \tilde{\nu}_L \phi_L E_{Rj}$$

$$+ Y_{Rij}^d \tilde{d}_R \phi_R U_{Lj} + Y_{Rij}^d \tilde{d}_R \phi_R D_{Lj} + Y_{Rij}^\nu \tilde{\nu}_R \phi_R N_{Lj} + Y_{Rij}^\nu \tilde{\nu}_R \phi_R E_{Lj}$$

$$+ Y_{Sij}^\nu \tilde{\nu}_L \Phi U_{Rj} + Y_{Sij}^\nu \tilde{\nu}_L \Phi D_{Rj} + Y_{Sij}^\nu \tilde{\nu}_R \Phi N_{Rj} + Y_{Sij}^\nu \tilde{\nu}_R \Phi E_{Rj} + h.c.,$$

where

$$q_{L/R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L/R}, \quad \ell_{L/R} = \begin{pmatrix} \nu \\ \nu^- \end{pmatrix}_{L/R},$$

$$\phi_{L/R} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_{L/R}, \quad \bar{\phi}_{L/R} = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}_{L/R}. \quad (2.3)$$

From Eq. (2.2), the $U(1)_X$ charge assignment should satisfy the following relations

$$X(U_R) = X(U_L) - X(\Phi), \quad X(D_R) = X(D_L) + X(\Phi), \quad (2.4)$$

$$X(q_L) = \frac{1}{2} [X(U_R) + X(D_R)], \quad X(q_R) = \frac{1}{2} [X(U_L) + X(D_L)], \quad (2.5)$$

$$X(\phi_L) = \frac{1}{2} [X(U_R) - X(D_R)], \quad X(\phi_R) = \frac{1}{2} [X(U_L) - X(D_L)], \quad (2.6)$$

for quark sectors, and equations similar to Eqs. (2.4) - (2.6) for lepton sectors $f = \nu, e$. For simplicity, in the present paper, we choose

$$SU(2)_L \times SU(2)_R \times U(1)_{LR} \times SU(3)_c \times U(1)_X \downarrow \mu = \Lambda_S$$

$$SU(2)_L \times SU(2)_R \times U(1)_{LR} \times SU(3)_c \downarrow \mu = \Lambda_R$$

$$SU(2)_L \times U(1)_Y \times SU(3)_c \downarrow \mu = \Lambda_L$$

$$U(1)_{em} \times SU(3)_c.$$ (2.1)

Here, the symmetry $U(1)_X$, which is spontaneously broken at the energy scale $\mu = \Lambda_S$, has been introduced for the purpose of preventing that the fermions $F$ acquire the masses $M_F$ at $\mu > \Lambda_S$. Hereafter, we call the ranges $\Lambda_L \leq \mu \leq \Lambda_R$, $\Lambda_R < \mu \leq \Lambda_S$, and $\Lambda_S < \mu \leq \Lambda_X$ as the ranges I, II, and III, respectively. In the present paper, the energy scale $\Lambda_X$ does not always mean a gauge unification energy scale. We assume that at the energy scale $\Lambda_X$ the mass matrices (Yukawa coupling constants) take simple forms discussed in the previous section.

The Yukawa coupling constants $Y_L^f$, $Y_R^f$ and $Y_S^f$ are defined as follows:

$$X(q_{L/R}) = X(\ell_{L/R}) = 0, \quad X(\Phi) = +1. \quad (2.7)$$

Then, the quantum numbers of the fermions $f$ and $F$ and Higgs scalars $\phi_L$, $\phi_R$ and $\Phi$ for $SU(2)_L \times SU(2)_R \times U(1)_{LR} \times U(1)_X$ are given in Table 5.

Note that the quantum number of the fermion $N_L$ is identical with that of the fermion $\nu^c_L \equiv (N^c_L)_{SU(2)}$. Therefore, the neutral fermions $N_L$ and $N_R$ can acquire the following Majorana mass terms at $\mu = \Lambda_S$:

$$H_{Majorana} = \left( Y_{Sij}^L \tilde{\nu}_L N_{Lj} + Y_{Sij}^R \tilde{\nu}_R N_{Rj} \right) \Phi + h.c. \quad (2.8)$$

Then, the neutrino mass matrix is given as follows

$$\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & m_L \\ 0 & 0 & m_R & 0 \\ m_L & M_L & M_D & 0 \\ m_R & M_D & M_R & 0 \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_L^c \\ N_R^c \end{pmatrix}, \quad (2.9)$$

where $M_D = Y_{Sij}^c(\Phi)$, $M_L = Y_{Sij}^c(\Phi)$ and $M_R = Y_{Sij}^c(\Phi)$. Since $O(M_D) \sim O(M_L) \sim O(m_R) \gg O(m_L)$, we obtain the mass matrix $M_\nu$ for the active neutrinos

$$M_\nu \simeq -m_L M_R^{-1} m_L^T. \quad (2.10)$$
III. GENERAL FEATURES OF THE EVOLUTIONS

In the present section, we give a general formulation of the evolution of the seesaw matrix with det\( M_F = 0 \). The evolution of the neutrino seesaw mass matrix is well known. However, in such a model with det\( M_F = 0 \) as the present model (the democratic seesaw model), a careful treatment is required.

Without losing the generality, we can express the Yukawa coupling constants \( Y^f_L \) and \( Y^f_R \) \((f = u, d, \nu, e)\) as

\[
Y^f_L(\mu) = \xi^f_L(\mu) Z^f_L(\mu), \quad Y^f_R(\mu) = \xi^f_R(\mu) Z^f_R(\mu), \quad (3.1)
\]

where \( Z^f_A(\mu) \) \((A = L, R)\) are defined by

\[
Z^f_A(\mu) = \text{diag}(z^f_{A1}(\mu), z^f_{A2}(\mu), z^f_{A3}(\mu)), \quad (3.2)
\]

\[
|z^f_{A1}(\mu)|^2 + |z^f_{A2}(\mu)|^2 + |z^f_{A3}(\mu)|^2 = 1, \quad (3.3)
\]
on the basis on which \( Y^f_A(\mu) \) are diagonal. In the present model, the word “universal” means the following initial conditions

\[
\xi^f_L(\Lambda_X) = \xi^f_R(\Lambda_X) \equiv \xi_{LR}, \quad (3.4)
\]

\[
|z^f_{L1}(\Lambda_X)| = |z^f_{R1}(\Lambda_X)| \equiv z_i, \quad (3.5)
\]

for all fermion sectors \( f = u, d, \nu, e \) universally.

In the range III \((\Lambda_S < \mu < \Lambda_X)\), the evolutions of the Yukawa coupling constants \( Y^f_L, Y^f_R \) and \( Y^f_S \) are given by the one-loop renormalization group equations (RGE) as follows:

\[
16\pi^2 \frac{dY^f_A}{dt} = \left( T^f_A - G^f_A + H^f_A \right) Y^f_A, \quad (A = L, R, S), \quad (3.6)
\]

where \( t = \log \mu \), and \( T^f_A, G^f_A \) and \( H^f_A \) \((A = L, R, S)\) denote contributions from fermion loop corrections, vertex corrections due to the gauge bosons, and vertex corrections due to the Higgs boson, respectively. Note that the matrices \( T^f_A \) and \( G^f_A \) are proportional to the unit matrix. As stated in the next section, the coefficients \( H^f_A \) \((A = L, R, S)\) take diagonal forms on the basis on which \( Y^f_A \) are diagonal. Therefore, if we take a basis on which \( Y^f_L \) \((Y^f_R) \) or \( Y^f_S \) are diagonal at \( \mu = \Lambda_X \), then the Yukawa coupling constants \( Y^f_L \) \((Y^f_R) \) or \( Y^f_S \) can keep the forms diagonal in the range III. Sometimes, the basis on which \( Y^f_L \) \((Y^f_R) \) are diagonal is useful, but sometimes, another basis on which \( Y^f_S \) are diagonal is useful, as we discuss later.

In the present model, it is assumed that we can choose a flavor basis on which \( Y^f_S(\Lambda_X) \) are simultaneously diagonal for all \( f = u, d, \nu, e \). Then, on this basis, the Yukawa coupling constants \( Y^f_S(\mu) \) can keep the forms diagonal in the range III, we can find that all \( Y^f_S \) are diagonal at \( \mu = \Lambda_S \). We can denote those as

\[
Y^f_S(\Lambda_S) = \text{diag}(y^f_{1S}, y^f_{2S}, y^f_{3S}). \quad (3.7)
\]

At the energy scale \( \mu = \Lambda_S \), the fermions \( F_i \) \(\text{except for} \ U_3 \) acquire the heavy masses \((M_F)_{ii} = y^f_i(\Phi^0) \). In the conventional seesaw model with det\( M_F \neq 0 \), the energy scale behaviors of the fermion masses in \( \mu < \Lambda_S \) are described by evolutions of the following operators

\[
(K^f)_{ij} = \left[ Y^f_L(Y^f_S)^{-1}(Y^f_R)^T \right]_{ij} = \sum_{k=1}^{\frac{3}{y^f_{kS}}}(Y^f_L)_{ik}(Y^f_R)_{jk}, \quad (3.8)
\]

and

\[
(K^\nu)_{ij} = \left[ Y^\nu_L(Y^\nu_S)^{-1}(Y^\nu_R)^T \right]_{ij} = \sum_{k=1}^{\frac{3}{y^\nu_{kS}}}(Y^\nu_L)_{ik}(Y^\nu_R)_{jk} \quad (3.9)
\]

(Hereafter, for convenience, we will denote the Yukawa coupling constants \( Y^f_S \) in the Majorana mass matrix \( M_R = Y^f_S(\Phi^0) \) \(Y^f_S \). The quark and lepton mass matrices \( M_f \) are given by

\[
M_f = K^f(\phi^0_L)/\langle \Phi^0 \rangle, \quad (f = u, d, e), \quad (3.10)
\]

\[
M_\nu = K^\nu(\phi^0_L)^2/\langle \Phi^0 \rangle. \quad (3.11)
\]

As explicitly shown in Sec. IV, the evolutions of the operators \( K^f \) are described by the one-loop RGE’s with the following forms

\[
16\pi^2 \frac{dK^f}{dt} = \left( T^f_K - G^f_K + H^f_K \right) K^f + H^f_{KL} K^f + H^f_{KR}, \quad (f = u, d, e), \quad (3.12)
\]

\[
16\pi^2 \frac{dK^\nu}{dt} = \left( T^\nu_K - G^\nu_K + H^\nu_K \right) K^\nu + H^\nu_{KL} K^\nu + K^\nu H^\nu_{KL}, \quad (3.13)
\]

where \( T^f_K, G^f_K \) and \( H^f_{KL, KR} \) denote contributions from fermion loop corrections, vertex corrections due to the gauge bosons, and vertex corrections due to the Higgs bosons \( \phi_L, \phi_R \), respectively.

However, in the seesaw mass matrix with det\( M_F = 0 \), since one of the eigenvalues of \( Y^f_S \) \((f = u)\) is zero (say, \( y^f_{3S} = 0 \)), we must calculate the following operator

\[
(K^u)_{ij} = \left[ Y^u_L(Y^u_S)^{-1}(Y^u_R)^T \right]_{ij} = \sum_{k=1}^{\frac{1}{y^u_{kS}}}(Y^u_L)_{ik}(Y^u_R)_{jk}, \quad (3.14)
\]
where $Y^u_S = \text{diag}(y^u_{1S}, y^u_{2S})$. Note that the matrices $Y_U$ and $Y^u_R (Y^u_R)$ in Eq. (3.14) are $2 \times 2$ and $3 \times 2$ matrices, respectively.

Note that in (3.14) we have taken the sum over $k = 1$ and 2 only. In the range II, the evolutions of the Yukawa coupling constants $Y^u_{L3}$ and $Y^u_{R3}$ ($i = 1, 2, 3$) are still described by the equation (3.6). At the energy scale $\mu = \Lambda_R$, we obtain a new mass term

$$H_{\text{mass}} = \sum_i (Y^u_R)^i_{i3} U_{L3} u_{Ri} \langle \phi_R \rangle,$$

(3.15)

By defining a mixing state

$$u'_{R3} = \frac{(Y^u_R)^i_{i3} u_{R1} + (Y^u_R)^i_{i3} u_{R2} + (Y^u_R)^i_{i3} u_{R3}}{\sqrt{||Y^u_R||^2_{i3}||Y^u_R||^2_{i3}||Y^u_R||^2_{i3}}} \quad (3.16)$$

we obtain a mass $m_{u'}$ of the fourth up-quark $t' = (t'_L, t'_R) = (U_{L3}, u'_{R3})$.

$$M^u = \langle \Phi^0 \rangle \begin{pmatrix} -\langle \phi_R \rangle K^u_{11} & \ldots & \langle \phi_R \rangle K^u_{13} \\ \ldots & \ldots & \ldots \\ \langle \phi_R \rangle K^u_{31} & \ldots & \langle \phi_R \rangle K^u_{33} \end{pmatrix}$$

which is sandwiched by the fields $(\mathbf{\Pi}_{L1}, \mathbf{\Pi}_{L2}, \mathbf{\Pi}_{L3}, \mathbf{\Pi}_{L3})$ and $(u_{R1}, u_{R2}, u_{R3}, U_{R3})$, where $\kappa = \langle \phi^0_R \rangle / \langle \phi^0_R \rangle$ and $\lambda = \langle \phi^0_R \rangle / \langle \phi^0_R \rangle$ as defined in Eq. (1.13). Of the Yukawa coupling constants $(Y^u_R)_{ij}$ and $(Y^u_R)_{ij}$, the twelve components $(Y^u_R)_{ik}$ and $(Y^u_R)_{ik}$ ($i = 1, 2, 3; j = 1, 2$) are absorbed into the operator $K^u$ defined by (3.14), while the rest $(Y^u_R)_{i3}$ and $(Y^u_R)_{i3}$ are still described by the equation (3.6).

Finally, we denote the effective Hamiltonian in each range: The effective Hamiltonian $H^{\text{int}}_{\text{III}}$ in the range III ($\Lambda_X \geq \mu > \Lambda_S$) is still given by the form (2.2), and $H^{\text{int}}_{\text{II}}$ in the range II ($\Lambda_S \geq \mu > \Lambda_R$) and $H^{\text{int}}_{\text{I}}$ in the range I ($\Lambda_R \geq \mu > \Lambda_L$) are given by

$$H^{\text{int}}_{\text{III}} = \sum_{i=1}^3 Y^u_{L3} (\mathbf{\Pi}_{Li} \tilde{\phi}_{Li} U_{R3})$$

$$+ \sum_{i=1}^3 Y^u_{R3} (\mathbf{\Pi}_{Ri} \tilde{\phi}_{Li} U_{R3})$$

$$+ \sum_{i,j \neq 3} \frac{1}{\langle \phi^0 \rangle} K^u_{ij} (\mathbf{\Pi}_{Li} \tilde{\phi}_{Li} Q_{Rj})$$

$$+ \sum_{i,j} \frac{1}{\langle \phi^0 \rangle} K^d_{ij} (\mathbf{\Pi}_{Li} \phi_{Li} D_{Rj})$$

$$+ \sum_{i,j} \frac{1}{\langle \phi^0 \rangle} K^c_{ij} (\mathbf{\Pi}_{Li} \phi_{Li} L_{Rj}) + h.c.$$ \quad (3.21)

and

$$H^{\text{int}}_{\text{II}} = \sum_{i=1}^3 Y^u_{L3} (\mathbf{\Pi}_{Li} \tilde{\phi}_{Li} U_{R3})$$

$$+ \sum_{i,j \neq 3} \frac{1}{\langle \phi^0 \rangle} K^u_{ij} (\mathbf{\Pi}_{Li} \phi_{Li} D_{Rj})$$

$$+ \sum_{i,j} \frac{1}{\langle \phi^0 \rangle} K^d_{ij} (\mathbf{\Pi}_{Li} \phi_{Li} L_{Rj}) + h.c.$$ \quad (3.22)

respectively.

**IV. COEFFICIENTS OF THE RGE**

In the present section, we give the coefficients of the renormalization group equations (RGE) (3.6), (3.12) and (3.13).
A. Evolution in the range III

In the non-SUSY model, the terms $T_A^f$, $G_A^f$ and $H_A^f$ \((A = L, R, S)\) are given as follows:

\[
T_A^u = T_A^d = T_A^\nu = T_A^\tau = 3 \text{Tr} \left( Y_A^u Y_A^{u+} + Y_A^d Y_A^{d+} \right) + \text{Tr} \left( Y_A^\nu Y_A^{\nu+} + Y_A^\tau Y_A^{\tau+} \right),
\]

\[
G_A^u = \frac{17}{8} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3 + \frac{3}{4} g_X^2,
\]

\[
G_A^d = \frac{5}{8} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3 + \frac{3}{4} g_X^2,
\]

\[
G_A^\nu = \frac{9}{8} g_1^2 + \frac{9}{4} g_2^2 + \frac{3}{4} g_X^2,
\]

\[
G_A^\tau = \frac{45}{8} g_1^2 + \frac{9}{4} g_2^2 + \frac{3}{4} g_X^2,
\]

\[
H_A^u = -H_A^d = \frac{3}{2} \left( Y_A^u Y_A^{u+} - Y_A^d Y_A^{d+} \right),
\]

\[
H_A^\nu = -H_A^\tau = \frac{3}{2} \left( Y_A^\nu Y_A^{\nu+} - Y_A^\tau Y_A^{\tau+} \right),
\]

where $A = L, R$, and

\[
T_S^u = T_S^d = T_S^\nu = T_S^\tau = 3 \text{Tr} \left( Y_S^u Y_S^{u+} + Y_S^d Y_S^{d+} \right) + \text{Tr} \left( Y_S^\nu Y_S^{\nu+} + Y_S^\tau Y_S^{\tau+} \right),
\]

\[
G_S^u = 4 g_1^2 + 8 g_3 + \frac{3}{2} g_X^2,
\]

\[
G_S^d = g_1^2 + 8 g_3 + \frac{3}{2} g_X^2,
\]

\[
G_S^\nu = 3 g_1^2,
\]

\[
G_S^\tau = 9 g_1^2 + \frac{3}{2} g_X^2,
\]

\[
H_S^f = Y_S^f Y_S^{f+}, \quad (f = u, d, \nu, \tau).
\]

The coefficients $T_A^f$, $G_A^f$ and $H_A^f$ in the minimal SUSY model are given in the Appendix.

As seen from Eq. (4.6), since the matrix $H_A^f$ is diagonal on the diagonal basis of $M_F(L_X)$, the Yukawa coupling constants $Y_S^f(\mu)$ can keep the forms diagonal. Similarly, when we choose the diagonal basis of $M_L(A_X)$ and $M_R(A_X)$, the matrices $Y_L^f(\mu)$ and $Y_R^f(\mu)$ keep their forms diagonal.

For a model with $g_{2L}(\mu) = g_{2R}(\mu)$ and $Y_L^f(\mu) = Y_R^f(\mu)$ at $\mu = A_X$, we can assert that

\[
Y_L^f(\mu) = Y_R^f(\mu),
\]

in the range III ($A_X < \mu \leq A_X$), because on the diagonal basis of $Y_L$ we obtain

\[
16\pi^2 \frac{d}{dt} \ln \left( \frac{Y_L^f}{Y_R^f} \right)_{ii} = (T_L^f - G_L^f + H_L^f - (T_R^f - G_R^f + H_R^f))_{ii}.
\]

The case $g_{2L} = g_{2R}$ is likely in the L-R symmetric model. For convenience, in the numerical evaluation in the present paper, we will take $g_{2L}(\mu) = g_{2R}(\mu)$ in the range III ($A_X < \mu \leq A_X$).

B. Evolution in the ranges I and II

In the ranges I and II, all the fermions $F_i$ except for $U_3$ are decoupled from the equation (3.6): In the present section, we will take the diagonal basis of $M_F$. Therefore, it is convenient that we define a spurion

\[
S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]

Then, the surviving Yukawa coupling constants $Y_A^f_{i3}$ are expressed as $(Y_A^f S)_{i3} = (Y_A^f)_{i3} \delta_{33}$. The evolution of $Y_A^f S$ is still described by the RGE (3.6) by substituting $Y_A^f S$ for $Y_A^f$. Here, the terms $T_A^u$, $G_A^u$ and $H_A^u (A = L, R)$ are expressed as follows $[Y_A^f (f = d, e, \nu)$ are already absorbed into the operators $K^f]$:

\[
T_A^u = 3 \text{Tr} \left( Y_{A}^u S Y_{A}^{u+} \right),
\]

\[
G_A^u = \frac{17}{8} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3 + \frac{3}{4} g_X^2,
\]

\[
H_A^u = \frac{3}{2} Y_A^u S Y_A^{u+},
\]

\[
(A = L, R)
\]

in the range II, and

\[
T_L^u = 3 \text{Tr} \left( Y_{L}^u S Y_{L}^{u+} \right),
\]

\[
G_L^u = \frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3 + \frac{3}{4} g_X^2,
\]

\[
H_L^u = \frac{3}{2} Y_L^u S Y_L^{u+},
\]

in the range I. Here, the coupling constant $g_1 = g_{1LR}$ in the range II is that for the U(1) operator $(1/2) Y_{LR}$ which is defined by the relation

\[
Q = I_{3}^L + I_{3}^R + \frac{1}{2} Y_{LR}.
\]
for the symmetry SU(2)_L × SU(2)_R × U(1)_{LR}, while the coupling constant g_1 \equiv g_{1Y} in the range I is that for the U(1) operator (1/2)Y which is defined by the relation

\[ Q = I_3^Y + \frac{1}{2} Y, \quad (4.17) \]

for the symmetry SU(2)_L × U(1)_Y, and they are connected by

\[ \alpha_{m}^{-1}(A_L) = \alpha_{2L}(A_L) + \frac{5}{3} \alpha_{1LR}(A_L), \quad (4.18) \]

\[ \frac{5}{3} \alpha_{m}^{-1}(A_R) = \alpha_{2R}(A_R) + \frac{2}{3} \alpha_{1LR}(A_R), \quad (4.19) \]

where \( \alpha_i = g_i^2 / 4\pi \).

Similarly, the terms \( T_K^f, G_K^f, H_{KL}^f \) and \( H_{KR}^f \) \( (f = u, d, e) \) are given by

\[ T_K^u = T_K^d = T_K^e = 3 \text{Tr} \left( Y_L^u SY_L^{u+} + Y_R^u SY_R^{u+} \right), \quad (4.20) \]

\[ G_K^u = G_K^d = \frac{5}{2} g_1^2 + \frac{9}{4} g_2^2 L + \frac{9}{4} g_2^2 R + 8 g_3^2, \]

\[ G_K^e = \frac{9}{2} g_1^2 + \frac{9}{4} g_2^2 L + \frac{9}{4} g_2^2 R, \quad (4.21) \]

\[ H_{KA}^u = H_{KA}^d = \frac{3}{2} Y_A^{u} SY_A^{u+}, \quad H_{KA}^e = 0, \quad (A = L, R), \]

in the range II, and

\[ T_K^u = T_K^d = T_K^e = 3 \text{Tr} \left( Y_L^u SY_L^{u+} \right), \quad (4.23) \]

\[ G_K^u = \frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 L + 8 g_3^2, \]

\[ G_K^d = \frac{5}{20} g_1^2 + \frac{9}{4} g_2^2 L + 8 g_3^2, \]

\[ G_K^e = \frac{45}{20} g_1^2 + \frac{9}{4} g_2^2 L, \quad (4.24) \]

\[ H_{KL}^u = H_{KL}^d = \frac{3}{2} Y_L^u SY_L^{u+}, \quad H_{KR}^e = 0, \quad (f = u, d), \]

\[ H_{KL}^e = H_{KR}^e = 0, \quad (4.26) \]

in the range I.

The terms \( T_K^f, G_K^f \) and \( H_{KL}^f \) have rather simple forms in contrast with those in the conventional neutrino seesaw model, because the partners of the fermions \( f_L \) which couple to the Higgs scalar \( \phi_L \) are not \( f_R \), but \( F_R \) which are already decoupled at \( \mu = \Lambda_R \):

\[ T_K^f = 6 \text{Tr} \left( Y_L^f SY_L^{f+} \right), \quad (4.27) \]

\[ G_K^f = 3 g_2^2 L, \quad (4.28) \]

\[ H_{KL}^f = \lambda_{HL}, \quad (4.29) \]

in the ranges I and II, where \( \lambda_{HL} \) is the coupling constant of the Higgs scalar \( \phi_L \) defined by

\[ H_\phi = \frac{1}{2} \lambda_{HL}(\phi_L^0 \phi_L)^2, \quad (4.30) \]

and the mass of the physical Higgs scalar \( H_L^0 \) is given by

\[ m_{HL}^2 = 2 \lambda_{HL}(\phi_L^0)^2. \quad (4.31) \]

The similar coefficients in the minimal SUSY model are given in the Appendix.

**V. CASE OF THE DEMOCRATIC SEESEAW MODEL**

In the democratic seesaw model, on the diagonal basis of \( Y_L^f (A_X) \) and \( Y_R^f (A_X) \), the Yukawa coupling constants of heavy fermions \( Y_S^f (A_X) \) are given by the democratic form (1.11). Since on this basis the Yukawa coupling constants \( Y_S^f \) keep the forms democratic:

\[ Y_S^f(\mu) = \xi_S^f(\mu) \left( 1 + 3 b_f(\mu) X \right), \quad (5.1) \]

we will call this basis the “democratic basis of \( M_F \)” hereafter. On the other hand, if we take a basis on which \( Y_S^f \) are diagonal, i.e., the matrix forms are given by

\[ \tilde{Y}_S^f(\mu) = \xi_S^f(\mu) \left( 1 + 3 b_f(\mu) \tilde{X} \right), \quad (5.2) \]

\[ \tilde{X} = A X A^T = \text{diag}(0, 0, 1), \quad (5.3) \]

\[ A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \quad (5.4) \]

Especially, on this basis, the Yukawa coupling constants \( [\tilde{Y}_S^f]_{ii} \) and \( [\tilde{Y}_S^f]_{ii} \) of the fermions \( E_i \) and \( U_i \) satisfy the relations

\[ [\tilde{Y}_S^f(\mu)]_{11} = [\tilde{Y}_S^f(\mu)]_{22} = [\tilde{Y}_S^f(\mu)]_{33} = \xi_S^f(\mu), \quad (5.5) \]

\[ [\tilde{Y}_S^u(\mu)]_{11} = [\tilde{Y}_S^u(\mu)]_{22} = \xi_S^u(\mu), \quad [\tilde{Y}_S^d(\mu)]_{33} = 0, \quad (5.6) \]

in the range III \( (A_S < \mu \leq A_X) \), i.e.,

\[ b_e(\mu) = 0, \quad b_u(\mu) = -1/3. \quad (5.7) \]

On the other hand, on this basis, the Yukawa coupling constants \( \tilde{Y}_L^f(\mu) \) and \( \tilde{Y}_R^f(\mu) \) are not diagonal. However, we can easily obtain their diagonal forms by \( A^T \tilde{Y}_L^f(\mu) A \) and \( A^T \tilde{Y}_R^f(\mu) A \).

At the energy scale \( \mu = A_S \), the fermions \( F_i \) (except for \( U_3 \)) acquire the heavy masses \( (M_F)_{ii} \). Therefore, for \( \mu < A_S \), the operators \( K^f \) are given as follows:
\[(K^u)_{ij} = \left[ \bar{Y}^u_L (\bar{Y}^u_S)^{-1} \bar{Y}^u_R \right]_{ij} = \frac{1}{\xi^u_S(A_S)} \sum_{k=1,2} \bar{(Y}^u_k)_{ik}(\bar{Y}^u_R)_{jk}, \quad (5.8)\]

\[(K^d)_{ij} = \left[ \bar{Y}^d_L (\bar{Y}^d_S)^{-1} \bar{Y}^d_R \right]_{ij} = \frac{1}{\xi^d_S(A_S)} \left( \sum_{k=1,2} \bar{(Y}^d_k)_{ik}(\bar{Y}^d_R)_{jk} + \frac{1}{1 + 3b_d(A_S)}(\bar{Y}^d_L)_{i3}(\bar{Y}^d_R)_{j3} \right), \quad (5.9)\]

\[(K^e)_{ij} = \left[ \bar{Y}^e_L (\bar{Y}^e_S)^{-1} \bar{Y}^e_R \right]_{ij} = \frac{1}{\xi^e_S(A_S)} \sum_{k=1,2,3} \bar{(Y}^e_k)_{ik}(\bar{Y}^e_R)_{jk}, \quad (5.10)\]

\[(K^\nu)_{ij} = \left( \bar{Y}^\nu_L (\bar{Y}^\nu_S)^{-1} \bar{Y}^\nu_R^T \right)_{ij} = \frac{1}{\xi^\nu_S(A_S)} \left( \sum_{k=1,2} \bar{(Y}^\nu_k)_{ik}(\bar{Y}^\nu_R)_{jk} + \frac{1}{1 + 3b_\nu(A_S)}(\bar{Y}^\nu_L)_{i3}(\bar{Y}^\nu_R)_{j3} \right), \quad (5.11)\]

In Eq. (5.11), we have assumed that the structure of the Majorana mass term \(M_R(A_S) = Y^R_3(A_S)\Phi^0\) for the neutral fermions \(\tilde{\Psi}R\) has a structure similar to the Dirac mass matrices \(M_F\) which is given by Eq. (5.1).

Since the Yukawa coupling constants \(Y^A_f(\mu)\) \((A = L, R)\) in the range III keep their forms diagonal on the democratic basis of \(M_F\), it is convenient to express \(Y^A_f(\mu)\) as follows,

\[Y^A_f(\mu) = \xi^A_f(\mu) Z^A_f(\mu), \quad (5.12)\]

where the diagonal matrix \(Z^A_f(\mu)\) is given by Eq. (3.2). Then, the matrix \(\bar{Y}^A_f(\mu)\) on the diagonal basis of \(M_F\) is given by

\[\bar{Y}^A_f(\mu) = \xi^A_f(\mu) \bar{Z}^A_f(\mu), \quad (5.13)\]

where

\[
\bar{Z}^f = AZ^f A^T = \frac{1}{6} \begin{pmatrix}
3(z_2 + z_1) & -\sqrt{3}(z_2 - z_1) & -\sqrt{6}(z_2 - z_1) \\
-\sqrt{3}(z_2 - z_1) & 4z_3 + z_2 + z_1 & -\sqrt{2}(2z_3 - z_2 - z_1) \\
-\sqrt{6}(z_2 - z_1) & -\sqrt{2}(2z_3 - z_2 - z_1) & 2(z_3 + z_2 + z_1)
\end{pmatrix},
\]

(we have dropped the indices \(A\) and \(f\), and for simplicity, we have taken \(\delta^f_i = 0\)). Although the Yukawa coupling constants \(\bar{Y}^L_f\) and \(\bar{Y}^R_f\) in the range II and \(\bar{Y}^L_f\) in the range I have the physical meaning only for the one column matrix components \((\bar{Y}^L_f)_{i3}\), we still use the expressions (5.12) and (5.13), because the matrix \(K^\nu(\mu)\) \((A_L < \mu \leq A_S)\) which is proportional to \(Y^R_f(\mu)\) is still diagonal on the democratic basis of \(M_F\) as discussed in Sec. \(\overline{\overline{IV}}\) so that we regard that \(Y^A_f(\mu)\) is also “diagonal”. Then, the top quark mass \(m_t(\mu)\) is approximately expressed as

\[m_t(\mu) = \langle \phi^0_L \rangle \sqrt{\sum_i |(\bar{Y}^T_f(\mu))_{i3}|^2} = \langle \phi^0_L \rangle \xi^T_L (\mu) \sqrt{\frac{1}{3} \sum_i |\bar{z}^T_L|^2} = \frac{1}{\sqrt{3}} \xi^T_L (\mu) \langle \phi^0_L \rangle, \quad (5.15)\]

The expression (5.15) is valid in the whole ranges \(A_L < \mu \leq A_X\).

Since

\[
\sum_{i=1}^{3} \sum_{k=1}^{2} (\bar{Z}_{ik})^2 = \frac{2}{3}(z_1^2 + z_2^2 + z_3^2) = \frac{2}{3},
\]

we obtain

\[m_t(\mu) + m_u(\mu) \simeq \frac{2}{3} \xi^u_S(\mu) \xi^T_R(\mu) \langle \phi^0_R \rangle \langle \phi^0_L \rangle, \quad (5.17)\]

from Eq. (5.8). Note that the expression (5.17) is valid only in the range III. In the ranges I and II, the ratio \(\xi^u_S/\xi^u_S\) behaves as an operator \(K^u(\mu)\) which obeys Eq. (3.12). From Eq. (5.17), the ratio \(m_c/m_t\) is given by
\[
\frac{m_e(\mu)}{m_\mu(\mu)} \sim \frac{2}{\sqrt{3}} \frac{\xi_u^e(\mu)}{\xi_u^d(\mu)} \langle \phi_R^0 \rangle .
\] (5.18)

Since \(H_{KL} = H_{KR} = 0\) in the ranges I and II, the form of \(K^e(\mu)\) is invariant in the ranges, i.e.,
\[
Z^e_L(\Lambda_L)Z^e_R(\Lambda_L) = Z^e_L(\Lambda_S)Z^e_R(\Lambda_S) ,
\] (5.19)
especially, since
\[
Z^e_L(\mu) = Z^e_R(\mu) \equiv Z^e(\mu) ,
\] (5.20)
for a model with \(g_{2R}(\Lambda_R) = g_{2L}(\Lambda_R)\), we obtain
\[
Z^e(\Lambda_L) = Z^e(\Lambda_S) .
\] (5.21)

Therefore, in preliminary evaluations prior to fixing the final values of the parameters, we will sometimes use the values of \(z_i(m_Z)\) which are obtained from the observed charge lepton masses \(m^c_i(m_Z)\) by using Eq. (1.7) instead of the values of \(z_i(\Lambda_X)\) which are defined in Eq. (1.9) as the initial condition at \(\mu = \Lambda_X\).

VI. NUMERICAL RESULTS IN THE NON-SUSY MODEL

We define
\[
\Lambda_L = \langle \phi_R^0 \rangle , \quad \Lambda_R = \langle \phi_R^0 \rangle , \quad \Lambda_S = \langle \Phi^0 \rangle .
\] (6.1)

However, for convenience, in the numerical evaluations, instead of physical quantities at \(\mu = \Lambda_L\), we will use those at \(\mu = m_Z\) (\(m_Z\) is the neutral weak boson mass).

First, in order to overlook the behavior of the Yukawa coupling constant \(Y^e_L(\mu)\), we illustrate the behavior of \(\xi_u^e(\mu)\) in the non-SUSY model in Fig. 1. Here, we have used the approximate relation (5.15) and the input values \(m_e(m_Z) = 181\) GeV and \(\langle \phi_R^0 \rangle = 174\) GeV:
\[
\xi_u^e(m_Z) = \sqrt{3} \frac{m_e(m_Z)}{\langle \phi_R^0 \rangle} = 1.80 .
\] (6.2)

In other words, the behavior of \(\xi_u^e(\mu)\) corresponds to that of \(m_e(\mu)\) because of \(\xi_u^e(\mu) = (m_e(\mu)/m_e(m_Z))\xi_u^e(m_Z)\).

In the ranges I and II, since the terms \(T^e_L\) and \(H^e_L\) are expressed only in terms of \(Y^e_L SY^e_L\), the evolution of the factor \(|\xi_L^e|^2\) is described by the equation
\[
16\pi^2 \frac{d}{dt} |\xi_L^e|^2 = 2 \left( \frac{1}{3} |\xi_L^u|^2 - G_L^u \right) |\xi_L^u|^2 + \frac{1}{2} \left[ |\xi_L^u|^2 + |\xi_L^d|^2 \right] .
\] (6.3)

However, in the range III, the terms \(T^e_L\) and \(H^e_L\) contain other factors \(Y^{e_f / y_{f R}}\) in addition to \(Y^e_L Y^{e_R}\), so that the evolution of \(\xi_L^e\) cannot be expressed so simply such as (6.3). For the evaluation of \(\xi_L^e\) in the range III, we have tentatively substituted the values \(z_i(m_Z)\) given by (1.7) for the initial values \(z_i(\Lambda_X)\). For simplicity, as we discussed in (4.7), we have taken as \(g_{2L}(\Lambda_R) = g_{2R}(\Lambda_R)\). In Fig. 1, the ratio \(\Lambda_S/\Lambda_R\) has been taken as \(\Lambda_S/\Lambda_R = 107\), which has determined from the fitting of the observed ratio \(m_u/m_c\) as we discuss later. The behavior of \(\xi_u^e(\mu)\) is insensitive to the ratio \(\Lambda_S/\Lambda_R\). As seen in Fig. 1, in a case with a lower \(\Lambda_S (\Lambda_S < 10^9\) GeV), \(\xi_u^e(\mu)\) has the Landau pole below \(\mu = \Lambda_X\), so that the case is ruled out. On the other hand, a case with a higher \(\Lambda_S (\Lambda_S > 10^{19}\) GeV) causes \(\alpha_1(\mu) \sim \infty\) at \(\mu \rightarrow \Lambda_S\), so that the case is also ruled out.

Taking account of the behavior of \(\xi_L^e(\mu)\) shown in Fig. 1, as a trial, we take
\[
\Lambda_X = 2 \times 10^{16}\) GeV ,
\] (6.4)

which is known as the unification energy scale in the minimal SUSY model. (However, in the present paper, we do not consider the gauge unification.) As a value of \(\Lambda_S\), we tentatively take
\[
\Lambda_S = 3 \times 10^{13}\) GeV ,
\] (6.5)
which leads to the mass-squared difference \(\Delta m^2 = m^2_{2e} - m^2_{2\tau} \sim (10^{-3} - 10^{-2}) eV^2\) as we demonstrate later. For the values (6.4) and (6.5), we obtain \(\xi_L^u(\Lambda_X) = 1.2\).

Next, we determine the values of \(\xi_L^u(\Lambda)\) and \(\Lambda_S/\Lambda_R\). Since we have already obtained the value \(\xi_L^u(\Lambda_X) = 1.2\), it seems that we can fix the value of \(\xi_L^u(\Lambda)\) from the observed value of \(m_u(m_Z)/m_c(m_Z)\) because of the relation (5.18). However, the value of \(\xi_L^u(\Lambda_X) [\text{also } \xi_L^u(\Lambda)]\) is sensitive to the value of \(\xi_L^u(\Lambda_S)\) \(\xi_S^u(\Lambda_S)\) [in other words, a small deviation of \(\xi_L^u(\Lambda_S)\) causes a large deviation of \(\xi_L^u(\Lambda_X)\)]. Therefore, we cannot fix the values \(\xi_L^u(\Lambda_S)\) unless we put a tentative model for \(\xi_L^u\) and \(\xi_L^d\).

The basic assumption in the universal seesaw model is to consider that the mass matrices \(m_L\) and \(m_R\) in Eq. (1.1) are "universal" (common) for all fermion sectors (quarks and leptons). Therefore, we put the following initial condition
\[
\xi_L^u(\Lambda_X) = \xi_L^d(\Lambda_X) = \xi_L^u(\Lambda_X) = \xi_L^d(\Lambda_X) = \xi_L(\Lambda_X) .
\] (6.6)

Then, a model with \(\xi_S^u(\Lambda_X) = \xi_S^d(\Lambda_X) = \xi_S^u(\Lambda_X)\) is obviously ruled out because we cannot give the observed values of quark and charged lepton masses simultaneously. We must consider
\[
\xi_S^u(\Lambda_X) = \xi_S^d(\Lambda_X) = \xi_S^u(\Lambda_X) \neq \xi_S^u(\Lambda_X) .
\] (6.7)

We tentatively put \(\xi_S^u(\Lambda_X) = \xi_L(\Lambda_X)\). The numerical results are as follows:
\[
\xi_L(\Lambda_X) = \xi_S^u(\Lambda_X) = 1.20 , \quad \xi_S^u(\Lambda_X) = 0.80 ,
\] (6.8)
\[
\Lambda_S/\Lambda_R = 107 .
\] (6.9)
\[ z_1 = 0.01617 \], \[ z_2 = 0.2349 \], \[ z_3 = 0.9719 \]. 

(6.10)

In the quark and charged lepton mass expressions (3.19) the factors \( \xi_S^A \) and \( \xi_S^S \) appear only in terms of the combinations \( \xi_S^A \Lambda_S \) and \( \xi_S^S \Lambda_S \), respectively, so that the absolute values of \( \xi_S^A \) and \( \xi_S^S \) depend on the choice of the input value of \( \Lambda_S \). Only the ratio \( \xi_S^A / \xi_S^S \) is substantial for the fitting of the quark and charged lepton mass. (However, as we state in the Sec. [VII], the neutrino mass difference between \( m_{\nu 3} \) and \( m_{\nu 2} \) rapidly varies in the range III. Therefore, in the neutrino mass matrix, the choice of the input value \( \Lambda_S \) is important.) We can obtain

\[ \xi_S^A(\Lambda_X) / \xi_S^S(\Lambda_X) \approx 1.5 \],

(6.11)

for any initial values of \( \xi_S^A(\Lambda_X) \) with \( O(1) \). The values (6.10) are nearly in agreement with the values \( z_1 = 0.01622 \), \( z_2 = 0.2357 \), and \( z_3 = 0.9717 \) which are obtained from Eq. (1.7) at \( \mu = m_Z \). We can see that the effect of the evolution is not so large for \( Z^* \).

The value of the parameter \( b_d(\Lambda_X) \) is determined from the fitting of the observed down-quark mass ratios \( m_d/m_s \) and \( m_s/m_b \) and the CKM matrix parameter \( |V_{us}(m_Z)| = 0.22 \). In Fig. 2, we illustrate the mass ratios \( m_d(\mu)/m_s(\mu) \) and \( m_s(\mu)/m_b(\mu) \) and the CKM parameter \( |V_{us}(\mu)| \) at \( \mu = m_Z \) versus the parameters \( b_d \) and \( \beta_d \), where we have re-defined the complex parameter \( b_d \) by \( b_d e^{i\beta_d} \) with two real parameters. For convenience, in Fig. 2, the quantities are expressed in the unit of the corresponding observed values at \( \mu = m_Z \) (for example, in Fig. 2, the curve \( m_d/m_s \) denotes \( [m_d(\mu)/m_s(\mu)]_{\mu=m_Z}/[m_d/m_s]_{\text{observed}} \)). We obtain

\[ b_d(\Lambda_X) = -1.20 \], \[ \beta_d(\Lambda_X) = 19.2^\circ \],

(6.12)

which give the following predictions at \( \mu = m_Z \):

\[
\begin{align*}
m_u(m_Z) &= 2.60 \times 10^{-3} \text{ GeV}, \\
m_c(m_Z) &= 6.92 \times 10^{-1} \text{ GeV}, \\
m_t(m_Z) &= 182 \text{ GeV}, \\
m_d(m_Z) &= 4.38 \times 10^{-3} \text{ GeV}, \\
m_s(m_Z) &= 9.84 \times 10^{-2} \text{ GeV}, \\
m_b(m_Z) &= 3.02 \text{ GeV}, \\
m_c(m_Z) &= 4.90 \times 10^{-4} \text{ GeV}, \\
m_t(m_Z) &= 1.03 \times 10^{-1} \text{ GeV}, \\
m_\tau(m_Z) &= 1.76 \text{ GeV}.
\end{align*}
\]

The experimental values corresponding to the results (6.13) are as follows \([11]\):

\[
\begin{align*}
m_u(m_Z) &= (2.33^{+0.42}_{-0.40}) \times 10^{-3} \text{ GeV}, \\
m_c(m_Z) &= (6.85^{+0.56}_{-0.61}) \times 10^{-1} \text{ GeV}, \\
m_t(m_Z) &= (181 \pm 13) \text{ GeV}, \\
m_d(m_Z) &= (4.69^{+0.60}_{-0.60}) \times 10^{-3} \text{ GeV}, \\
m_s(m_Z) &= (0.934^{+0.118}_{-0.130}) \times 10^{-1} \text{ GeV}, \\
m_b(m_Z) &= (3.00 \pm 0.11) \text{ GeV}, \\
m_\tau(m_Z) &= (4.8684727 \pm 0.0000014) \times 10^{-4} \text{ GeV}, \\
m_\mu(m_Z) &= (1.0275138 \pm 0.0000033) \times 10^{-1} \text{ GeV}, \\
m_\tau(m_Z) &= (1.7467 \pm 0.0003) \text{ GeV}.
\end{align*}
\]

The results (6.13) are in agreement with the observed values (6.14) within the experimental errors.

The predicted values of \( |V_{ij}| \) depends on the phase parameters \( \delta_f^j \) given by Eq. (1.4). Only when we take those as (1.8) (at \( \mu = \Lambda_X \)), we can obtain reasonable values of \( |V_{ij}| \). For example, for \( \delta_3^d - \delta_3^d = \pi \), we obtain the predictions at \( \mu = m_Z \):

\[
\begin{align*}
|V_{us}| &= 0.220, \\
|V_{cb}| &= 0.0668, \\
|V_{ub}/V_{cb}| &= 0.0558, \\
|V_{td}| &= 0.0177, \\
J &= 3.25 \times 10^{-5}.
\end{align*}
\]

(6.15)

The observed values \([11]\) are

\[
\begin{align*}
|V_{us}| &= 0.2196 \pm 0.0023, \\
|V_{cb}| &= 0.0402 \pm 0.0019, \\
|V_{ub}/V_{cb}| &= 0.090 \pm 0.025.
\end{align*}
\]

(6.16)

Although the results (6.15) are roughly consistent with experiments, the value \( |V_{cb}| = 0.066 \) is somewhat large compared with the observed value \( |V_{cb}| = 0.040 \). This discrepancy can be adjusted by considering a small deviation from \( \pi \) of the relative phase \( \delta_3^d - \delta_3^d \) as demonstrated in Ref. \([11]\).

Related to the phenomenological requirement (1.8), it is interesting to consider that \( Y_L^\nu \) which is the coefficient of the Higgs scalar \( \phi_L \) is related to \( Y_L^d \) which is the coefficient of the scalar \( \phi_d \) as

\[ Y_L^\nu(\Lambda_X) = [Y_L^d(\Lambda_X)]^\dagger. \]

(6.17)

Then, the relations (1.8) mean that \( (Y_L^d)_{11} \) and \( (Y_L^d)_{22} \) are real, while \( (Y_L^d)_{33} \) is almost pure imaginary. We take

\[ (Z^u)^\dagger = Z^d = \text{diag}(z_1, z_2, z_3 e^{i\delta_3}). \]

(6.18)

The parameter \( \delta_3 \) (= \( \delta_3^d - \delta_3^d \)) does not affect the masses, but only the CKM mixings. It is interesting to consider that the parameter \( \delta_3(\Lambda_X) \) takes its value such as the CKM mixings become minimum, i.e., such as the value \( \sum_{i \neq j} |V_{ij}(\Lambda_X)|^2 \) takes the minimum. This requirement gives the initial value \( \delta_3(\Lambda_X) = 84^\circ \) (see Fig. 3).

Then, we obtain the predictions of \( |V_{ij}| \) at \( \mu = m_Z \)

\[
\begin{align*}
|V_{us}| &= 0.220, \\
|V_{cb}| &= 0.0418, \\
|V_{ub}/V_{cb}| &= 0.0726, \\
|V_{td}| &= 0.0109, \\
J &= 2.38 \times 10^{-5}.
\end{align*}
\]

(6.19)

which is in excellent agreement with the experimental values (6.16). In Fig. 3, we illustrate the predicted values \( |V_{ij}(m_Z)| \) versus \( \delta_3(\Lambda_X) \). As seen in Fig. 3, the value of \( \delta_3(\Lambda_X) \) at which \( \sum_{i \neq j} |V_{ij}(\Lambda_X)|^2 \) takes the minimum also gives the minimum of the CKM mixings at \( \mu = m_Z \).
VII. NUMERICAL RESULTS IN THE SUSY MODEL

The behavior of $\xi^u_L(\mu)$ in the SUSY model is somewhat different from that in the non-SUSY model. Since in the SUSY model, the top quark mass $m_t(\mu)$ is given by

$$m_t(\mu) = \frac{1}{\sqrt{3}} \xi^u_L(\mu) \frac{v_t}{\sqrt{2}} \sin \beta ,$$

(7.1)

where $v_t/\sqrt{2} = 174$ GeV and $\tan \beta \equiv \tan \beta_L = v^u_S/v^d_S$, the initial value of $\xi^u_L(m_Z)$ in the SUSY model corresponds to

$$[\xi^u_L(m_Z)]_{\text{SUSY}} = [\xi^u_L(m_Z)]_{\text{non-SUSY}} \frac{1}{\sin \beta}.$$  

(7.2)

However, this does not mean $[\xi^u_L(\Lambda_X)]_{\text{SUSY}} = [\xi^u_L(\Lambda_X)]_{\text{non-SUSY}}/\sin \beta$, because the behavior of $\xi^u_L(\mu)$ in the SUSY model for the case of $\tan \beta = 3.5$. If we take $\tan \beta < 2.5$, the initial value of $\xi^u_L(m_Z)$ becomes $\xi^u_L(m_Z, \tan \beta = 2.5) > \xi^u_L(m_Z, \tan \beta = 2.5)$ from Eq. (7.2), so that the curve of $\xi^u_L(\mu)$ will be illustrated in the upper side of the curve given in Fig. 3. Therefore, for a case with a small value of $\tan \beta$, the Landau pole of $\xi^u_L(\mu)$ appears at a relatively lower energy scale. We consider that the model should be calculable perturbatively, so that a case with such a large value of $\xi^u_L$ should be ruled out. As seen in Fig. 3, since the model gives, in general, $\xi^u_L(\Lambda_X) > \xi^u_L(\mu)$ ($m_Z < \mu < \Lambda_X$), the value $\xi^u_L(\Lambda_X)$ should, at least, be $[\xi^u_L(\Lambda_X)]^2/4\pi < 0$, i.e., $\xi^u_L(\Lambda_X) < \sqrt{4\pi} = 3.54$. However, when we take contributions from the higher order corrections into consideration, even the value $\xi^u_L(\Lambda_X) = 3.0$ is still dangerous. Therefore, we put the constraint $\xi^u_L(\Lambda_X) = 2.0$ for the results of the present one loop calculation. In Fig. 3, we illustrate the predicted value of $m_t(m_Z)$ for the initial values $\xi^u_L(\Lambda_X) = \sqrt{4\pi} = 3.54$ and $\xi^u_L(\Lambda_X) \lesssim 2.0$, where we have used the input values

$$\Lambda_X = 2 \times 10^{16} \text{ GeV}, \quad \Lambda_S = 6 \times 10^{13} \text{ GeV}.$$  

(7.3)

The value of $\Lambda_S$ has been chosen as the neutrino mass-squared difference $\Delta m^2_{32}$ is of the order of $(10^{-3} - 10^{-2})$ eV$^2$.

From Fig. 3, we conclude that the value of $\tan \beta$ must be

$$\tan \beta \gtrsim 3.$$  

(7.4)

Prior to the numerical investigation of the evolutions in the SUSY model, in order to see the difference between the parameter structures in the non-SUSY and SUSY models, let us give a rough sketch for the parameters in the case of the SUSY model by neglecting the evolution effects. The quark mass matrices $M_u$ and $M_d$ are given by

$$\begin{align*}
(M_u)_{ij} &= \sum_{k=1}^{3} \left[ Z \right]_{ik} \left[ Z \right]_{jk} (O^u)_{kk} \frac{\xi^u_{LR} A_L A_R}{\xi^u_S} \frac{\Lambda L A_R}{\Lambda_S} \sin \beta, \\
(M_d)_{ij} &= \sum_{k=1}^{3} \left[ Z \right]_{ik} \left[ Z \right]_{jk} (O^d)_{kk} \frac{\xi^d_{LR} A_L A_R}{\xi^d_S} \frac{\Lambda L A_R}{\Lambda_S} \cos \beta,
\end{align*}$$

(7.5)

where $O^u = \text{diag}(1, 1), O^d = \text{diag}(1, 1, 1/(1 + 3b_d))$, and $\bar{Z}$ is given by Eq. (5.14). Here, for simplicity, we have assumed $\beta_L = \beta_R = \beta \equiv \beta$. For $\tan \beta > 3$, the factors $\sin \beta$ and $\cos \beta$ are approximated as $\sin \beta \sim 1$ and $\cos \beta \sim 1/\tan \beta$, respectively. Obviously, the model with $\xi^u_S = \xi^d_S$ in addition to the constraint

$$\xi^u_S \sim \xi^d_S \sin \beta, \quad \xi^d_S \sim \xi^d_S \cos \beta,$$

(7.6)

is ruled out, because we cannot fit the up- and down-quark masses simultaneously due to the existence of the factor $\cos \beta$. Therefore, we must consider a model with $\xi^u_S \neq \xi^d_S$ differently from the constraint (6.6) in the non-SUSY model. If we consider

$$\xi^u_S \sim \xi^d_S \sin \beta, \quad \xi^d_S \sim \xi^d_S \cos \beta,$$

(7.7)

then the model becomes similar to the case of the non-SUSY model, because

$$\frac{\xi^u_{LR} A_L A_R}{\xi^u_S} \frac{\Lambda L A_R}{\Lambda_S} \sin \beta \sim \frac{\xi^d_{LR} A_L A_R}{\xi^d_S} \frac{\Lambda L A_R}{\Lambda_S} \cos \beta,$$

(7.8)

and we will obtain reasonable fittings for the quark masses and CKM matrix parameters as well as in the non-SUSY model. Note that for a large value of $\tan \beta$, the value of $K^d \equiv \xi^d_S \xi^d_S/\xi^u_S$ becomes large because $K^d \sim K^u \tan \beta$ from the relation (7.9), so that we cannot evaluate the RGE (3.12) perturbatively. We must take the value of $\tan \beta$ near to the lower bound given by Eq. (7.4).

When we take the evolution effects into consideration, the situation is further complicated. The evolutions of $\xi^u_L(\mu)$, $\xi^d_L(\mu)$ and $\xi^u_S(\mu)$ in the SUSY model are quite different from those in the non-SUSY model. We illustrate the behaviors of $m^u_L(\mu)/m^u_L(\Lambda_X)$ which correspond to the behaviors of $[\xi^u_L(\mu)\xi^u_L(\Lambda_X)/\xi^u_L(\Lambda_X)\xi^u_L(\Lambda_X)/\xi^u_S(\Lambda_X)]$ in the non-SUSY model and those in the SUSY model in Figs. 4 and 5, respectively. In Fig. 4, we see that the values $m_u(\mu)$ and $m_d(\mu)$ cause rapid changes in the range III. In the non-SUSY model, the charged lepton mass ratios are almost invariant, i.e., $m_e(\mu)/m_{\mu}(\mu) \simeq$ constant and $m_{\mu}(\mu)/m_e(\mu) \simeq$ constant, while, in the SUSY model, the mass ratio $m_{\mu}(\mu)/m_e(\mu)$ shows a considerable change (although $m_{\mu}(\mu) \simeq m_{\mu}(\mu)$ still holds).

The situation is critical for the input values. If we adhere to the input value $m_t(m_Z) = 181$ GeV, then it
is hard to obtain reasonable values of the other quark mass values $m_c$, $m_s$, $m_b$, $m_s$ and $m_d$ for any parameter values of $A_S/A_R$ and $b_d$. However, if we take a slightly lower value of $m_t(m_Z)$, for example, $m_t(m_Z) = 168$ GeV [cf. $[m_t(m_Z)]_{\text{observed}} = 181 \pm 13$ GeV], we can find the following parameter values

$$\tan \beta = 3.5, \quad \Lambda_S/\Lambda_R = 38,$$  \hspace{0.5cm} (7.10)

$$z_1 = 0.01449, \quad z_2 = 0.2117, \quad z_3 = 0.9772, \quad (7.11)$$

$$\xi^{\nu}_{\nu}(A_X) = \xi^{\nu}_{\nu}(A_X) = \xi^{\nu}_{L}(A_X) = 1.3, \quad \xi^{\nu}_{L}(A_X) = 1.0, \quad (7.12)$$

$$\xi^\nu_S(A_X) = 1.7, \quad \xi^\nu_S(A_X) = 0.50, \quad \xi^\nu_S(A_X) = 1.0, \quad (7.13)$$

$$b_d = -1.2, \quad \beta_d = 19.4^\circ, \quad (7.14)$$

which leads to the following quark and charged lepton masses and CKM matrix parameters:

$$m_u(m_Z) = 2.47 \times 10^{-3} \text{ GeV}, \quad m_c(m_Z) = 6.64 \times 10^{-1} \text{ GeV}, \quad m_t(m_Z) = 167 \text{ GeV},$$

$$m_d(m_Z) = 4.49 \times 10^{-3} \text{ GeV}, \quad m_s(m_Z) = 1.00 \times 10^{-1} \text{ GeV}, \quad m_b(m_Z) = 2.83 \text{ GeV},$$

$$m_e(m_Z) = 4.87 \times 10^{-4} \text{ GeV}, \quad m_\mu(m_Z) = 1.03 \times 10^{-1} \text{ GeV}, \quad m_\tau(m_Z) = 1.75 \text{ GeV}, \quad (7.15)$$

$$|V_{us}| = 0.220, \quad |V_{cb}| = 0.0665,$$

$$|V_{ub}/V_{cb}| = 0.0603,$$

$$|V_{td}| = 0.0179,$$

$$J = 3.38 \times 10^{-5}.$$

The values $|V_{ij}|^2$ are again desirably adjustable by the phase parameter $\delta_3$ defined by (6.18).

VIII. EVOLUTION OF THE NEUTRINO MASS MATRIX

The evolution of the neutrino mass matrix $M_{\nu} = K^{\nu}\langle \Phi^0 \rangle^2/\langle \Phi^0 \rangle$ is described by the RGE (3.13). Since the coefficient $H^K_{KL}$ in the ranges I and II is given by $H^K_{KL} = \lambda_{KL}$, (4.28), for the non-SUSY model, and by $H^K_{KL} = 0$, (A.15) and (A.24), for the SUSY model, the form of the matrix $K^{\nu}$ at $\mu = \Lambda_S$ does not vary from that at $\mu = \Lambda_X$, so that the mass ratios and mixing matrix $U_{\nu}$ are also invariant. Since the coefficients $H^K_{KL}$ and $H^K_{KR}$ in the charged lepton sector are given by $H^K_{KL} = H^K_{KR} = 0$ in the ranges I and II for the non-SUSY and SUSY models, the form of the charged lepton mass matrix $M_e$ is also invariant below $\mu = \Lambda_S$. Therefore, the Maki-Nakagawa-Sakata (MNS) $12$ matrix $U = U_{KL}U_{\nu}$ is invariant in the ranges I and II. Note that in the conventional model, the neutrino seesaw mass matrix can vary the form. The neutrino mass matrix in the present model can vary the form only in the range III ($\Lambda_S < \mu \leq \Lambda_X$). The reason is that in the conventional model the scalar $\phi_{KL}$ couples to $\tau L_{LR}$, while in the present model couples to $\tau L_{LR}$, so that the contribution of $\phi_L$ to $H^K_{KL}$ in the latter case is decoupled below $\mu = \Lambda_S$.

For the numerical study, the case with $b_{\nu} = -1/2$ is most interesting, because the inverse matrix of $Y^{\nu}_{\nu}(A_X) = \xi^{\nu}_{L}(A_X)\{1 + 3b_{\nu}(A_X)X\}$ with $b_{\nu}(A_X) = -1/2$ has the form

$$[Y^{\nu}_{\nu}(A_X)]^{-1} = -\frac{1}{\xi^{\nu}_{L}(A_X)} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{array} \right), \quad (8.1)$$

so that

$$Y^{\nu}_{\nu}(Y^{\nu}_{\nu})^{-1}Y^{\nu}_{LL} = \frac{\xi^{\nu}_{L}(A_X)^2}{\xi^{\nu}_{S}(A_X)} \left( \begin{array}{ccc} 0 & z_1z_2 & z_1z_3 \\ z_2z_1 & 0 & z_2z_3 \\ z_3z_1 & z_3z_2 & 0 \end{array} \right). \quad (8.2)$$

The form (8.2) is well known as the Zee-type $13$ mass matrix, which can lead to a large mixing $14$.

The mass eigenvalues $m_{\nu i}$ and mixing matrix $U$ at $\mu = \Lambda_X$ are given by $14$

$$m_{\nu 1} \simeq -2z_1^2m_0^\nu,$$

$$m_{\nu 2} \simeq -\left[z_2z_3 - \left(1 - \frac{z_3}{2z_2}\right)\frac{z_1^2}{z_1^2}\right]m_0^\nu, \quad (8.3)$$

$$m_{\nu 3} \simeq \left[z_2z_3 + \left(1 + \frac{z_3}{2z_2}\right)\frac{z_1^2}{z_1^2}\right]m_0^\nu,$$

$$m_0^\nu = \frac{(\xi^{\nu}_{L})^2}{\xi^{\nu}_{S}} \frac{\Lambda^2}{\Lambda_S}, \quad (8.4)$$

$$U = \left( \begin{array}{ccc} 1 & -z_2 & \frac{1}{\sqrt{2}}(1 - z_2) \\ -\frac{1}{\sqrt{2}} & \frac{1}{2}z_1 & \frac{1}{2}z_1 \\ -z_2 & -\frac{1}{2}z_1 & \frac{1}{\sqrt{2}} \end{array} \right). \quad (8.5)$$

The model with $b_d = -1/2$ gives highly degenerate mass-squared levels $m_{\nu 1,2} \simeq m_{\nu 3}$ and a large mixing between $\nu_\mu$ and $\nu_\tau$ at $\mu = \Lambda_X$. Therefore, the model has a possibility that it can give a reasonable explanation for the atmospheric neutrino data $16$.

In Figs. $15$ and $16$ we illustrate the behaviors of the mass-squared differences $\Delta m^2_{\nu i} = m^2_{\nu i} - m^2_{\nu 0}$ in the non-SUSY and SUSY models, respectively. As seen in Figs. $15$
and \[\Delta m_{32}^2\] the mass-squared difference \[\Delta m_{32}^2\] rapidly increase according as the energy scale decreases in the range III. The numerical results are given in Table II. We can see that the neutrino mass ratios are invariant in the ranges I and II.

As we stated already, the values \(z_1, z_2, z_3\) (therefore, the mass ratios \(m_\nu/m_\mu\) and \(m_\mu/m_\tau\) are almost invariant in the range III, while the ratio \(\Delta m_{32}^2/\Delta m_{21}^2\) is rapidly vary in the range III. Although the relations (8.3) give \(\Delta m_{32}^2 \simeq 4z_2^2z_3(z_0^2)^2\) and \(\Delta m_{21}^2 \simeq (z_2z_3^2)(\mu_0^2)^2\), the rapid decrease in the ratio \(\Delta m_{32}^2/\Delta m_{21}^2\) does not mean the rapid decrease in the ratio \(z_2^2/2z_3\). The rapid decrease comes from the slight deviation of the parameter \(b_\nu(\mu)\) from the value \(b_\nu(\Lambda_X) = -1/2\). The value of \(b_\nu(\mu)\) is not invariant in the range III, although the form of \(Y_\nu^T(\mu)\), “the unit matrix plus a democratic matrix”, is invariant. When we denote

\[b_\nu(\mu) = -\frac{1}{2}(1 + \varepsilon_\nu(\mu)),\]  

(8.6)

the expression (8.1) is replaced with

\[\left[Y_\nu^T(\mu)\right]^{-1} \simeq \frac{1}{\varepsilon_\nu^2(\mu)}\left(\begin{array}{ccc} -2\varepsilon_\nu & 1 & 1 \\ 1 & -2\varepsilon_\nu & 1 \\ 1 & 1 & -2\varepsilon_\nu \end{array}\right),\]  

(8.7)

so that

\[Y_\nu^T(\nu) = \frac{1}{\varepsilon_\nu^2(\mu)}\left(y_{\nu}^T(\mu)\right)^2 \left(\begin{array}{ccc} -2\varepsilon_\nu & z_1^2z_2 & z_1z_3 \\ z_1z_2 & -2\varepsilon_\nu & z_2z_3 \\ z_1z_3 & z_2z_3 & -2\varepsilon_\nu\end{array}\right).\]  

(8.8)

Therefore, the mass eigenvalues in the range III are given by

\[m_{\nu1} \simeq -2(1 + 3\varepsilon_\nu)z_1^2m_0^\nu,\]  

\[m_{\nu2} \simeq \left[z_2z_3 - \left(1 - \frac{z_3}{2z_2}\right)z_1^2 + \varepsilon_\nu\right]m_0^\nu,\]  

\[m_{\nu3} \simeq \left[z_2z_3 + \left(1 + \frac{z_3}{2z_2}\right)z_1^2 - \varepsilon_\nu\right]m_0^\nu,\]  

(8.9)

instead of (8.3), and the mass squared differences \(\Delta m_{21}^2\) and \(\Delta m_{32}^2\) are given by

\[\Delta m_{21}^2 \simeq (z_2z_3)(m_0^\nu)^2,\]  

\[\Delta m_{32}^2 \simeq 4z_2z_3(z_1^2 - \varepsilon_\nu)(m_0^\nu)^2.\]  

(8.10)

Note that the approximate expression (8.19) tell us that \(\Delta m_{32}^2(\mu)\) takes a zero between \(\mu = \Lambda_X\) and \(\mu = \Lambda_S\) because \(\varepsilon_\nu(\Lambda_X) = 0 < z_1^2(\Lambda_X) \simeq z_1^2(\Lambda_S) < \varepsilon_\nu(\Lambda_S)\), e.g., \(\varepsilon_\nu(\Lambda_X) = 7.3 \times 10^{-2}\) and \(z_1^2(\Lambda_X) \simeq z_1^2(\Lambda_S) \simeq 2.6 \times 10^{-4}\) for the non-SUSY and \(\varepsilon_\nu(\Lambda_S) = 1.1 \times 10^{-2}\), \(z_1^2(\Lambda_X) \simeq 2.1 \times 10^{-4}\) and \(z_1^2(\Lambda_S) \simeq 2.6 \times 10^{-4}\) for the SUSY model. In fact, we can see this at a point which is very close to \(\mu = \Lambda_X\) in Figs. 8 and 10. Thus, the value of \(\Delta m_{32}^2(\mu)\) is highly sensitive to the value of \(\varepsilon_\nu(\mu)\), although \(\Delta m_{21}^2(\mu)\) is not so.

In general, since the mixing angle \(\theta_{23}\) is given by

\[\sin 2\theta_{23} \simeq \frac{2(M_{\nu}^2)_{23}}{m_{\nu3} - m_{\nu2}},\]  

(8.11)

the mixing angle \(\theta_{23}\) in the conventional democratic type neutrino mass matrix model is sensitive to the energy scale \(\Lambda_S\), because \(\Delta m_{32}^2(\mu)\) has a large energy scale dependency. In contrast to the conventional model, the mixing angle \(\theta_{23}\) in the present model does not so drastically vary. The reason is as follows: the neutrino mass matrix \(M_\nu\) in the present “democratic” seesaw model is not democratic, i.e., the form of \(M_\nu\) is given by Eq. (8.2). In fact, the present model gives not \(m_{\nu2} \simeq m_{\nu3}\), but \(m_{\nu2} \simeq -m_{\nu3}\), so that the evolution effect on \(U_{23}\) is not so sensitive as seen in Eq. (8.11).

As seen in Table II, the model can fit the value \(\Delta m_{32}^2\) to the atmospheric neutrino data \[\Delta m_{32}^2(\text{atm}) = 2.4 \times 10^{-3} \text{eV}^2\] by adjusting the value of \(\Lambda_S\), but it cannot give any explanation of the solar neutrino data \(\Delta m_{32}^2(\text{solar}) = 7.1 \times 10^{-5} \text{eV}^2\), because of \(\Delta m_{21}^2 \gg \Delta m_{32}^2 \equiv \Delta m_{21}^2\). We must introduce a further mechanism for the explanation of the solar neutrino data, for example, as discussed in Ref. [9]. However, since the purpose of the present model is not to propose a plausible neutrino mass matrix model in the framework of the USM, but to see the characteristic features of the neutrino mass matrix evolution in contrast to the conventional seesaw model. Therefore, we do not touch the numerical fitting furthermore.

**IX. CONCLUSIONS**

In conclusion, we have investigated the evolutions of the quark and lepton mass matrices \(M_f (f = u, d, \nu, e)\) in the universal seesaw model with \(\det M_F = 0\) in the up-quark sector \(F = U\).

The assumptions which have made in the present paper are classified into the following three categories:

(A) Basic assumptions in the universal seesaw model with \(\det M_U = 0\);

(B) Basic assumptions in the democratic seesaw model \[\Delta m_{32}^2(\mu)\] (we have taken the model as a more concrete one of the universal seesaw model with \(\det M_U = 0\) in order to give an explicit evaluation of the universal seesaw model);

(C) Tentative assumptions for convenience of the numerical evaluation.

The assumptions in the category (A) are as follows:

(A1) \(SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{Y} \times U(1)_{X}\) gauge symmetries with the symmetry breaking pattern (2.1);

(A2) Hypothetical heavy fermions \(F = (U, D, N, E)\) which belong to \((1, 1)\) of \(SU(2)_L \times SU(2)_R\) and acquire masses of the order of \(\Lambda_S\) at the energy scale \(\mu = \Lambda_S\) except for \(U_{3L}\) and \(U_{3L}\).

In the present model, therefore, the quark and charged lepton mass matrices \(M_f (f = u, d, \nu)\) and neutrino mass matrix \(M_\nu\) are given by
\[ M_f = Y_f^I (Y_f^I)^{-1} (Y_f^F)^I (\Lambda_L A_R / \Lambda_S) , \]  
\[ M_\nu = Y_\nu^I (Y_\nu^I)^{-1} (Y_\nu^\nu)^T (\Lambda_L^T A_R / \Lambda_S) , \]

except for the top quark, where \( \Lambda_L = \langle \phi_L^0 \rangle, \Lambda_R = \langle \phi_R^0 \rangle \) and \( \Lambda_S = \langle \phi^0 \rangle \). The evolutions below \( \mu = \Lambda_S \) are described by the RGE (3.12) and (3.13) for the seesaw operators. On the other hand, the top quark mass \( m_t(\mu) \) given by the expression (3.18) is still described by RGE (3.6) for the Yukawa coupling constants below \( \mu = \Lambda_S \). Although the heavy fermions \( F \) do not contribute to the evolutions below \( \mu = \Lambda_S \), the third family “would-be” heavy up-quark \( U_3 \) can contribute to the RGE even below \( \mu = \Lambda_S \). However, as far as the \( H_{KL}^F (F = \nu, e) \) and \( H_{KR}^R \) terms in the lepton sectors are concerned, the would-be heavy quark \( U_3 \) cannot contribute to those, so that the forms of the mass matrices \( M_\nu(\mu) \) and \( M_e(\mu) \) are invariant below \( \mu = \Lambda_S \).

The assumptions in the category (B) are as follows:

(B1) At a unification scale \( \mu = \Lambda_x \), the Yukawa coupling constants \( Y_f^I \) and \( Y_f^F \) have the same form, i.e., 
\[ Y_f^I(\Lambda_x) = Y_f^F(\Lambda_x) \equiv Y_f^{LR}(\Lambda_x) . \]

(B2) At \( \mu = \Lambda_x \), the heavy fermion mass matrices \( M_F \) (therefore, the Yukawa coupling constants \( Y_f^I \)) and also the Majorana masses \( M_L(M_R) \) of the neutral fermions \( N_L(N_R) \) take a simple diagonal form (5.1), the form “the unit matrix pulse a democratic matrix”, on the basis on which the Yukawa coupling constants \( Y_f^{LR}(\Lambda_x) \) are diagonal. Then, the form (5.1) is invariant under the evolution in the range III.

(B3) The values of the parameter \( b_f \) in the matrix \( Y_f^I \) given by Eq. (5.1) are given by \( b_e = 0, b_\nu = -1/2 \) and \( b_\mu = -1/3 \) at \( \mu = \Lambda_x \). (The value \( b_d \) is kept as a free parameter in order to fit the up- and down-quark masses and CKM matrix parameters reasonably.)

In this model, the top quark mass \( m_t(\mu) \) is given by (5.15). The behavior of \( m_t(\mu) \), i.e., \( \xi_f^I(\mu) \), is given in Figs. 1 and 2 for the non-SUSY and SUSY models, respectively. We can obtain the constraint on the values of the intermediate energy scales \( \Lambda_R \) and \( \Lambda_S \) by considering that the model should be calculable perturbatively. In the non-SUSY model, since \( \Lambda_S / \Lambda_R \sim 10^2 \) from the ratio \( m_t/m_c \), we find the constraint

\[ 10^{10} \text{ GeV} < \Lambda_S < 10^{19} \text{ GeV} , \]

for \( \Lambda_X \sim 10^{16} \text{ GeV} \). In the SUSY model, the results highly depend on the input parameter \( \tan \beta \). From the numerical study, we obtain the constraints

\[ 3 < \tan \beta < 4 , \]

\[ 10^{10} \text{ GeV} < \Lambda_S < 10^{19} \text{ GeV} , \]

for \( \Lambda_X \sim 10^{16} \text{ GeV} \). (The above numerical results are slightly dependent on the assumptions stated below, but the dependence is not so large.)

The assumptions in the category (C) are as follows:

(C1) For convenience of the numerical evaluation, \( g_{2L}(\mu) = g_{2R}(\mu) \) has been assumed. Then, we can assert \( Y_f^{I}(\mu) = Y_f^{F}(\mu) \) in the range III (\( \Lambda_S < \mu \leq \Lambda_X \)) as we have shown in Eq. (4.8).

(C2) For evaluation of the non-SUSY model, the initial condition

\[ \xi_{L/R}^e(\Lambda_X) = \xi_{L/R}^e(\Lambda_X) = \xi_{L/R}^e(\Lambda_X) = \xi_{L/R}^e(\Lambda_X) \]

has been assumed together with the initial condition (3.5), i.e.,

\[ Z_{L/R}^u = Z_{L/R}^d = Z_{L/R}^e = Z_{L/R}^\nu . \]

However, since there is no solution of the parameter values for the SUSY model under such a constraint (9.6), the constraint corresponding to (9.6) in the SUSY model has been loosened as

\[ \xi_{L/R}^e(\Lambda_X) = \xi_{L/R}^e(\Lambda_X) \neq \xi_{L/R}^e(\Lambda_X) = \xi_{L/R}^e(\Lambda_X) , \]

although the initial condition (9.7) has still been required in the SUSY model.

(C3) For the non-SUSY model, we have assumed \( \xi_{L}^e(\Lambda_X) = \xi_{R}^e(\Lambda_X) \neq \xi_{L}^e(\Lambda_X) = \xi_{R}^e(\Lambda_X) \), but, for the SUSY model, we have assumed that each value of \( \xi_{S}^e(\Lambda) \) may be different among them, because the previous condition is too strong for the SUSY model and the up–down symmetry is already broken due to the factor \( \tan \beta \neq 1 \) in the SUSY model.

In the conventional model for quark and charged lepton masses (i.e., not seesaw model), the following approximate relations are satisfied in the non-SUSY and SUSY models:

\[ (m_u/m_c)_L \sim (m_d/m_s)_L \sim (m_e/m_\mu)_L \sim (m_\mu/m_\tau)_L \sim 1 , \]

\[ (m_u/m_c)_X \sim (m_d/m_s)_X \sim (m_e/m_\mu)_X \sim (m_\mu/m_\tau)_X \sim 1 , \]

\[ (m_u/m_t)_L \sim (m_d/m_t)_L \sim (m_e/m_t)_L \sim 1 + \varepsilon_u , \]

\[ (m_u/m_t)_X \sim (m_d/m_t)_X \sim (m_e/m_t)_X \sim 1 + \varepsilon_d , \]

\[ \frac{|V_{cb}(\Lambda_L)|}{|V_{cb}(\Lambda_X)|} \sim \frac{|V_{td}(\Lambda_L)|}{|V_{td}(\Lambda_X)|} \sim |V_{td}(\Lambda_L)| \sim 1 + \varepsilon_d , \]

where \( (m_u/m_c)_L \) denotes \( m_u(\Lambda_L)/m_c(\Lambda_L) \), and so on. The relations (9.9)-(9.12) are due to that the Yukawa coupling constant \( y_f \) of the top quark in the conventional model is very large compared with the other Yukawa coupling constants. In the present model, as seen in Figs. 1 and 2, the relations (9.9)-(9.12) are also satisfied in the range I (\( \Lambda_L < \mu \leq \Lambda_R \)) (so that we read the
relations (9.9)-(12.9) as \( X \to R \). The values of \( \varepsilon_u \) and \( \varepsilon_d \) are approximately given by \( \varepsilon_u \sim \varepsilon_d \) for the non-SUSY model, and by \( \varepsilon_u \sim -3 \varepsilon_d \) for the SUSY model. In the range II (\( \Lambda_R < \mu \leq \Lambda_S \)), the relations (9.9)-(12.9) are slightly broken. In the SUSY model, the values show not \( \varepsilon_u \sim 3 \varepsilon_d \), but \( \varepsilon_u \sim \varepsilon_d \) in the range II. However, in the model with \( \Lambda_L/\Lambda_R \gg 1 \), which is required in order to make the neutrino masses tiny, the evolution effects in the range II are not so large, so that we can regard that the relations (9.9)-(12.9) are still satisfied in the range \( \Lambda_L < \mu \leq \Lambda_S \), i.e.,

\[
D_u(\Lambda_L) \simeq \frac{m_u(\Lambda_L)}{m_u(\Lambda_S)}(1 + \varepsilon_u)(1 - \varepsilon_u S)D_u(\Lambda_S),
\]

\[
D_d(\Lambda_L) \simeq \frac{m_d(\Lambda_L)}{m_d(\Lambda_S)}(1 + \varepsilon_d)(1 - \varepsilon_d S)D_d(\Lambda_S),
\]

where \( D_u = \text{diag}(m_u, m_e, m_\mu) \), \( D_d = \text{diag}(m_d, m_s, m_b) \), and \( S \) is defined by Eq. (4.9). In the present model, the value of \( \varepsilon_V \) is not always given by \( \varepsilon_V \sim \varepsilon_d \) because of the presence of the range II.

Also in the ranges I and II, differently from the conventional seesaw model (for example, see Ref. 3), the neutrino mass ratios and mixing angles are not affected by the evolution effects:

\[
\frac{m_{\nu_1}(\Lambda_L)/m_{\nu_1}(\Lambda_S)}{m_{\nu_2}(\Lambda_S)/m_{\nu_2}(\Lambda_S)} \simeq 1,
\]

\[
\frac{V_{ij}(\Lambda_L)}{V_{ij}(\Lambda_S)} \simeq 1.
\]

Note that the relation (9.16) does not mean \( \Delta m^2_{\nu_1}(\Lambda_L)/\Delta m^2_{\nu_1}(\Lambda_S) \simeq 1 \). However, the ratio \( \Delta m^2_{\nu_1}(\Lambda_L)/\Delta m^2_{\nu_2}(\Lambda_S) \) is again invariant in the ranges I and II.

In the range III (\( \Lambda_S < \mu \leq \Lambda_X \)), the relations (9.9)-(12.9) \((9.13)-(9.15)\) and (9.16)-(9.17) are not satisfied at all. For example, the behavior of \( \Delta m^2_{\nu_2}(\mu) \) is highly sensitive to the value \( \varepsilon_\nu(\mu) \) and is given by Eq. (8.10). In other words, the differences of the numerical behaviors of the quark masses, CKM matrix parameters and neutrino mass squared differences from those in the conventional model are substantially formed in the range III.

Note that the mass ratios \( m_c/m_\mu \) and \( m_u/m_c \) are almost constant (although the ratio \( m_u/m_c \) is slightly changed in the SUSY model), so that the phenomenologically well-satisfied relation (1.14) still holds under the evolutions.

For the neutrino mass matrix \( M_\nu \), we have investigate the model with \( b_3(\Lambda_X) = -1/2 \), which leads to a large mixing \( \sin^2 \theta_{23} \simeq 1 \). Although the mass-squared difference \( \Delta m^2_{\nu_2}(\mu) \) is highly sensitive to the energy scale \( \mu \) in the range III (\( \Lambda_S < \mu \leq \Lambda_X \)), the mixing angle \( \theta_{23} \) is not sensitive to the energy scale. In contrast to the conventional seesaw neutrino mass matrix, note that the present neutrino mass matrix \( M_\nu \) is form-invariant below \( \mu = \Lambda_S \), so that the neutrino mass ratios and mixings are invariant below \( \mu = \Lambda_S \).

In the present paper, we have assumed \( SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_X \) symmetries above \( \mu = \Lambda_S \). As seen in Figs. 3 and 4, in general, the rapid increasing of the Yukawa coupling constant \( Y^{\nu}_{\nu}(\mu) \) causes above \( \mu = \Lambda_S \), although we have been able to find a set of the reasonable parameter values without having the Landau pole below \( \mu = \Lambda_X \). The rapid increasing is mainly due to the rapid increasing of the gauge coupling constant \( g_1 \) above \( \mu = \Lambda_S \). If we want to build a unification model with a unified gauge symmetry \( G \), we may consider that the U(1) symmetry is embedded into the unified symmetry \( G \). (For example, see an SO(10) \[ SU(10) \]\times SO(10) \[ R \] model \[ 20 \], where SO(10) \[ L \]\times SO(10) \[ R \] is broken into \( SU(2) \times SU(2)′ \times SU(4) \[ L \]\times [SU(2) \times SU(2)′ \times SU(4)] \[ R \].) Then, the gauge structure above \( \mu = \Lambda_S \) is different from the present model, so that the evolutions will be also different from the present results. (Of course, the evolutions below \( \mu = \Lambda_S \) are still the same as those in the present paper.) It is likely that the gauge structure above \( \mu = \Lambda_S \) is different from the present model. Our next task is to investigate what gauge structure above \( \mu = \Lambda_S \) is promising for a unified description of the quark and lepton masses and mixings.

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**Appendix**

In Secs. 4 and 5, the coefficients of RGE (3.6), (3.12) and (3.13) have been given only for the case of non-SUSY scenario with one SU(2)-doublet Higgs scalar. In the present Appendix, we give the coefficients of RGE in the minimal SUSY scenario.

**[Range III]**

\[
T^u_A = T^u_A = 3\text{Tr}(Y_A^\nu Y_A^\nu \dagger) + \text{Tr}(Y_A^\nu Y_A^\nu \dagger),
\]

\[
T^d_A = T^d_A = 3\text{Tr}(Y_A^d Y_A^d \dagger) + \text{Tr}(Y_A^d Y_A^d \dagger),
\]

\[
G^u_A = \frac{13}{6} g_1^2 + 3 g_2^2 + \frac{16}{3} g_3^2 + g_X^2,
\]

\[
G^d_A = \frac{7}{6} g_1^2 + 3 g_2^2 + \frac{16}{3} g_3^2 + g_X^2,
\]

\[
G^\nu_A = \frac{9}{6} g_1^2 + 3 g_2^2 + g_X^2.
\]
where $A = L, R$.

**[Range I]**

\[ T_u^v = 3 \text{Tr}(Y_L^u S Y_L^{u \dagger}) , \]
\[ G_L^u = \frac{13}{15} g_1^2 + 3 g_2^2 + \frac{16}{3} g_3^2 , \]
\[ H_L^u = 3 Y_u S Y_{u}^{\dagger} , \]
\[ T_K^u = 3 \text{Tr}(Y_L^u S Y_L^{u \dagger}) , \]
\[ H_K^u = 0 , \]
\[ H_{KL}^u = H_{KL}^d = \frac{2}{3} Y_u S Y_{u}^{\dagger} , \]
\[ H_{KR}^u = H_{KR}^d = 0 , \]
\[ H_{KL}^u = H_{KL}^d = 0 . \]

**[Range II]**

\[ T_A^u = 3 \text{Tr}(Y_A^u S Y_A^{u \dagger}) , \]
\[ G_A^u = \frac{13}{6} g_1^2 + 3 g_2^2 + \frac{16}{3} g_3^2 , \]
\[ H_A^u = 3 Y_A^u S Y_A^{\dagger} , \]
\[ T_K^u = 3 \text{Tr}(Y_L^u S Y_L^{u \dagger}) + Y_R^u S Y_R^{u \dagger} , \]
\[ T_K^d = T_K^e = 0 , \]
\[ G_K^u = G_K^d = G_K^e = \frac{9}{2} g_1^2 + \frac{9}{2} g_2^L + g_2^2 R . \]

where \( A = L, R \).

\[ G_A^u = \frac{27}{6} g_1^2 + 3 g_2^2 + g_3^2 , \]
\[ H_A^u = 3 Y_A^u Y_A^{\dagger} + Y_A^d Y_A^{d \dagger} , \]
\[ H_A^d = 3 Y_A^d Y_A^{d \dagger} + Y_A^u Y_A^{u \dagger} , \]
\[ H_A^e = 3 Y_A^e Y_A^{e \dagger} + Y_A^r Y_A^{r \dagger} , \]
\[ T_S^u = T_S^d = 3 \text{Tr}(Y_S^u Y_S^{u \dagger}) + \text{Tr}(Y_S^d Y_S^{d \dagger}) , \]
\[ T_S^d = T_S^e = 3 \text{Tr}(Y_S^d Y_S^{d \dagger}) + \text{Tr}(Y_S^e Y_S^{e \dagger}) , \]
\[ G_S^u = \frac{8}{3} g_1^2 + \frac{16}{3} g_2^2 + 3 g_3^2 , \]
\[ G_S^d = \frac{2}{3} g_1^2 + \frac{16}{3} g_2^2 + \frac{3}{2} g_3^2 , \]
\[ G_S^e = 3 g_3^2 , \]
\[ G_S^r = \frac{18}{3} g_1^2 + 3 g_3^2 , \]
\[ H_S^f = 2 Y_S^f Y_S^{f \dagger} , \]

where $A = L, R$ and $f = u, d, \nu, e$.

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TABLE I. Quantum numbers of the fermions $f$ and $F$ and Higgs scalars $\phi_L$, $\phi_R$ and $\Phi$ for $SU(2)_L \times SU(2)_R \times U(1)_{LR} \times U(1)_X$.

|  | $I_L$ | $I_R$ | $Y_{LR}$ | $X$ |  | $I_L$ | $I_R$ | $Y_{LR}$ | $X$ |
|---|---|---|---|---|---|---|---|---|---|
| $u_L$ | $+\frac{2}{3}$ | 0 | $+\frac{1}{2}$ | 0 | $u_R$ | 0 | $+\frac{1}{2}$ | $+\frac{1}{2}$ | 0 |
| $d_L$ | $-\frac{1}{3}$ | 0 | $+\frac{1}{2}$ | 0 | $d_R$ | 0 | $-\frac{1}{2}$ | $+\frac{1}{2}$ | 0 |
| $\nu_L$ | $+\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $\nu_R$ | 0 | $+\frac{1}{2}$ | $-\frac{1}{2}$ | 0 |
| $e_L$ | $-\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $e_R$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{2}$ | 0 |
| $U_L$ | 0 | 0 | $+\frac{2}{3}$ | $+\frac{1}{2}$ | $U_R$ | 0 | 0 | $+\frac{2}{3}$ | $-\frac{1}{2}$ | 0 |
| $D_L$ | 0 | 0 | $-\frac{1}{3}$ | $-\frac{1}{2}$ | $D_R$ | 0 | 0 | $-\frac{1}{3}$ | $+\frac{1}{2}$ | 0 |
| $N_L$ | 0 | 0 | 0 | $+\frac{1}{2}$ | $N_R$ | 0 | 0 | 0 | $-\frac{1}{2}$ | 0 |
| $E_L$ | 0 | 0 | $-1$ | $-\frac{1}{2}$ | $E_R$ | 0 | 0 | $-1$ | $+\frac{1}{2}$ | 0 |
| $\phi_L$ | $+\frac{1}{2}$ | 0 | $+\frac{1}{2}$ | $-\frac{1}{2}$ | $\phi_R$ | 0 | $+\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | 0 |
| $\phi_R$ | $-\frac{1}{2}$ | 0 | $+\frac{1}{2}$ | $-\frac{1}{2}$ | $\phi_R$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $+\frac{1}{2}$ | 0 |
| $\Psi$ | 0 | 0 | 0 | $+1$ |  |  |  |  |  |

TABLE II. The squared mass difference $\Delta m^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}$. The values of the input parameters are the same as in Figs. 9 and 10. The absolute values of $|\Delta m^2_{ij}|$ should not be taken rigidly, because we can adjust those by the value of $\Lambda_S$.

|  | non-SUSY model |  | SUSY model |  |
|---|---|---|---|---|
|  | at $\mu = \Lambda_L$ | at $\mu = \Lambda_S$ | at $\mu = \Lambda_X$ | at $\mu = \Lambda_L$ | at $\mu = \Lambda_S$ | at $\mu = \Lambda_X$ |
| $|\Delta m^2_{32}|$ [eV$^2$] | $2.39 \times 10^{-3}$ | $9.32 \times 10^{-3}$ | $3.49 \times 10^{-3}$ | $2.72 \times 10^{-3}$ | $2.51 \times 10^{-3}$ | $4.08 \times 10^{-3}$ |
| $|\Delta m^2_{21}|$ [eV$^2$] | $1.83 \times 10^{-3}$ | $7.15 \times 10^{-3}$ | $7.67 \times 10^{-3}$ | $1.35 \times 10^{-2}$ | $1.25 \times 10^{-2}$ | $1.01 \times 10^{-2}$ |
| $|\Delta m^2_{32}/\Delta m^2_{21}|$ | $1.30 \times 10^{-1}$ | $1.30 \times 10^{-1}$ | $4.56 \times 10^{-3}$ | $2.02 \times 10^{-1}$ | $2.02 \times 10^{-1}$ | $4.04 \times 10^{-2}$ |
| $|V_{23}|^2$ | 0.04 | 0.04 | 0.00471 | 0.00492 | 0.00492 | 0.00466 |
| $|V_{12}|^2$ | 0.00484 | 0.00484 | 0.00471 | 0.00492 | 0.00492 | 0.00466 |
FIG. 1. Behaviors of $\xi^u(\mu)$ in a non-SUSY model for the cases (a) $\Lambda_S = 10^6$ GeV, (b) $\Lambda_S = 10^9$ GeV, (c) $\Lambda_S = 10^{12}$ GeV, and (d) $\Lambda_S = 10^{15}$ GeV. The input values are $m_t(m_Z) = 181$ GeV and $\Lambda_S/\Lambda_R = 107$.

FIG. 2. Predictions of $m_d/m_s$, $m_s/m_b$ and $|V_{us}|$, and their dependency on the parameters $b_d$ and $\beta_d$. Here, the mass ratios are denoted in the unit of the corresponding observed values which are quoted from Ref. [10]. The dashed, solid and dotted lines denote $b_d = -1.1$, -1.2 and -1.3, respectively.

FIG. 3. $\sum_{i\neq j} |V_{ij}(\Lambda_X)|^2$ versus $\delta^d_3(\Lambda_X)$.

FIG. 4. Predicted values of the CKM matrix parameters $|V_{ij}(m_Z)|$ versus the parameter $\delta^d_3(\Lambda_X)$. Other input values of the parameters are $\Lambda_S = 3 \times 10^{13}$ GeV, $\Lambda_S/\Lambda_R = 107$, $b_d = -1.2$ and $\beta_d = 19.2^\circ$. 
FIG. 5. Behaviors of $\xi_u(\mu)$ in a SUSY model for the cases (a) $\Lambda_S = 10^6$ GeV, (b) $\Lambda_S = 10^9$ GeV, (c) $\Lambda_S = 10^{12}$ GeV, and (d) $\Lambda_S = 10^{15}$ GeV. The input values are $m_t(m_Z) = 181$ GeV, $\Lambda_S/\Lambda_R = 38$ and $\tan\beta = 3.5$.

FIG. 6. The top-quark mass $m_t(m_Z)$ versus $\tan\beta$ in a SUSY model. The solid and broken lines denote the cases with the initial conditions (a) $\xi_u(\Lambda_X) = 2.0$ and (b) $\xi_u(\Lambda_X) = \sqrt{4\pi} = 3.54$, respectively. The other input values are $\Lambda_S = 6 \times 10^{13}$ GeV and $\Lambda_S/\Lambda_R = 38$. The horizontal solid and broken lines denote the center and lower values of the observed top quark mass at $\mu = m_Z$, respectively.

FIG. 7. Behaviors of $m_i(\mu)/m_i(\Lambda_X)$ ($f = u, d, \nu, e; i = 1, 2, 3$) in the non-SUSY model. The dotted, broken and solid lines denote the first, second and third fermion masses, respectively. The input parameter values are $\Lambda_S = 3 \times 10^{13}$ GeV, $\Lambda_S/\Lambda_R = 107$ and $b_d(\Lambda_X) = -1.2e^{19.2^\circ}$.

FIG. 8. Behaviors of $m_i(\mu)/m_i(\Lambda_X)$ ($f = u, d, \nu, e; i = 1, 2, 3$) in the SUSY model. The dotted, broken and solid lines denote the first, second and third fermion masses, respectively. The input parameter values are $\Lambda_S = 6 \times 10^{13}$ GeV, $\Lambda_S/\Lambda_R = 38$, $\tan\beta = 3.5$ and $b_d(\Lambda_X) = -1.2e^{19.4^\circ}$.
FIG. 9. Behavior of $|\Delta m^2_{ij}(\mu)|$ in the non-SUSY model. The input parameter values are the same as in Fig. 7 with $\xi^\nu_A = \xi^\nu_A$ ($A = L, R, S$).

FIG. 10. Behavior of $|\Delta m^2_{ij}(\mu)|$ in the SUSY model. The input parameter values are the same as in Fig. 8 with $\xi^\nu_A = \xi^\nu_A$ ($A = L, R, S$).