N-Type Mg$_3$Sb$_2$–$x$Bi$_x$ Alloys as Promising Thermoelectric Materials

Hongjing Shang, Zhongxin Liang, Congcong Xu, Jun Mao, Hongwei Gu, Fazhu Ding, and Zhifeng Ren

1Department of Physics and Texas Center for Superconductivity at the University of Houston (TcSUH), University of Houston, Houston, TX 77204, USA
2Key Laboratory of Applied Superconductivity and Institute of Electrical Engineering, Chinese Academy of Sciences, Beijing 100190, China
3University of Chinese Academy of Sciences, Beijing 100049, China

Correspondence should be addressed to Fazhu Ding; dingfazhu@mail.iee.ac.cn and Zhifeng Ren; zren2@central.uh.edu

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N-type Mg$_3$Sb$_2$–$x$Bi$_x$ alloys have been extensively studied in recent years due to their significantly enhanced thermoelectric figure of merit ($zT$), thus promoting them as potential candidates for waste heat recovery and cooling applications. In this review, the effects resulting from alloying Mg$_3$Bi$_2$ with Mg$_3$Sb$_2$, including narrowed bandgap, decreased effective mass, and increased carrier mobility, are summarized. Subsequently, defect-controlled electrical properties in n-type Mg$_3$Sb$_2$–$x$Bi$_x$ are revealed. On one hand, manipulation of intrinsic and extrinsic defects can achieve optimal carrier concentration. On the other hand, Mg vacancies dominate carrier-scattering mechanisms (ionized impurity scattering and grain boundary scattering). Both aspects are discussed for Mg$_3$Sb$_2$–$x$Bi$_x$ thermoelectric materials. Finally, we review the present status of, and future outlook for, these materials in power generation and cooling applications.

1. Introduction

Thermoelectric technology, which can achieve reversible conversion between electricity and heat, holds great potential for alleviating the energy and environmental crises [1, 2]. However, large-scale commercialization of thermoelectric technology has yet to be implemented, mainly due to the low energy-conversion efficiency of existing thermoelectric materials. The thermoelectric energy-conversion efficiency is contingent on the materials’ dimensionless figure of merit ($zT$), thus promoting them as potential candidates for waste heat recovery and cooling applications. In this review, the effects resulting from alloying Mg$_3$Bi$_2$ with Mg$_3$Sb$_2$, including narrowed bandgap, decreased effective mass, and increased carrier mobility, are summarized. Subsequently, defect-controlled electrical properties in n-type Mg$_3$Sb$_2$–$x$Bi$_x$ are revealed. On one hand, manipulation of intrinsic and extrinsic defects can achieve optimal carrier concentration. On the other hand, Mg vacancies dominate carrier-scattering mechanisms (ionized impurity scattering and grain boundary scattering). Both aspects are discussed for Mg$_3$Sb$_2$–$x$Bi$_x$ thermoelectric materials. Finally, we review the present status of, and future outlook for, these materials in power generation and cooling applications.

Materials are still not widely applied in viable thermoelectric applications due to the high cost of tellurium (Te) and some unresolved engineering issues (e.g., high contact resistance between the contact materials and the thermoelectric legs when nanostructured materials are considered for making the modules).

Recently, the n-type Mg$_3$Sb$_2$–$x$Bi$_x$ alloys have attracted significant attention because of their promising thermoelectric performance and good mechanical properties, the abundance and low cost of their constituent elements, etc. Mg$_3$Sb$_2$ has a CaAl$_2$Si$_2$-type crystal structure, which consists of an octahedrally coordinated cation Mg$_2^+$ layer and a tetrahedrally coordinated anion structure (Mg$_2$Sb$_2$)$_2^-$ that form a nearly isotropic three-dimensional (3D) chemical bonding network with an interlayer bond that is mostly ionic and partially covalent (Figure 1(a)) [16]. These crystallographic characteristics lead to decent electrical properties, intrinsically low lattice thermal conductivity, and good mechanical properties. Actually, Mg$_3$Sb$_2$–$x$Bi$_x$ alloys have long been regarded as persistent p-type semiconductors, and their n-type counterparts were considered to be impossible to synthesize, which should be attributed to the negatively charged
Mg vacancies that pin the Fermi level around the valence band [17–19]. This was the case until n-type Mg$_3$Sb$_{2-x}$Bi$_x$ with high thermoelectric performance was reported by Tamaki et al. [17] through the addition of excess Mg and doping with Te, although Zhang et al. [20] soon after reported similar results with Te doping only. The extra Mg can effectively suppress the Mg vacancies, thus rendering n-type conduction in Mg$_3$Sb$_{2-x}$Bi$_x$ [17, 21, 22]. Since the discovery of n-type Mg$_3$Sb$_{2-x}$Bi$_x$, notable advancements have been made, and its state-of-the-art average $zT$ has been raised up to ~1.1 in the range of 300-500 K, comparable to that of the Bi$_2$Te$_3$-based materials [23–29].

This review focuses on these n-type Mg$_3$Sb$_{2-x}$Bi$_x$ alloys with promising thermoelectric performance. We first summarize the effects of alloying Mg$_3$Sb$_2$ with Mg$_3$Bi$_2$ on the band structure (e.g., bandgap, effective mass, and carrier mobility). The defect-controlled electronic transport in Mg$_3$Sb$_{2-x}$Bi$_x$ thermoelectric materials will then be discussed, including defect-chemistry-inspired dopant exploration and the defect-induced near-room-temperature shift in the carrier-scattering mechanism. Furthermore, promising applications in power generation and cooling are also discussed. The strategies mentioned here are believed to be equally applicable to many other thermoelectric materials. Some ideas for possible further improvement of thermoelectric performance in n-type Mg$_3$Sb$_{2-x}$Bi$_x$ materials are also presented.

2. Electronic Structure

Alloying of Mg$_3$Sb$_2$ with Mg$_3$Bi$_2$ has a significant impact on the thermoelectric transport properties and band structures of the alloys. Zhang et al. [30] calculated the band alignments of Mg$_3$Sb$_{2-x}$Bi$_x$ alloys and found that Mg$_3$Bi$_2$ alloying results in a moderate increase in the energy separation between the conduction band minima K and CB$_1$, decreasing the

Figure 1: (a) Crystal structure of Mg$_3$Sb$_2$. Reproduced with permission from Ref. [17]. Copyright 2016 John Wiley and Sons. (b) Bandgap energy of Mg$_3$Sb$_{2-x}$Bi$_x$ as a function of composition [23, 32]. (c) Density of state effective mass ($m^*_d$) for n-type Mg$_3$Sb$_{2-x}$Bi$_x$ as a function of composition [23, 28, 34, 35].
contribution of the secondary band minimum K to the electrical transport. Since Mg,Bi is a semimetal [31] and Mg,Sb is a semiconductor, the bandgap of Mg,Sb,Bi will be reduced with increasing Mg,Bi content (Figure 1(b)), leading to an enhanced bipolar contribution for the Bi-rich compositions [23, 32]. Thus, such compositions are not suitable for applications at higher temperatures. Considering the empirical trend of bandgap dependence on the application temperature range, the room temperature thermoelectric materials exhibit similar bandgaps, so the bandgap of Bi₃Te₅–ₓSₓTeₓ provides a hint for choosing MgₓSb₂₋ₓBiₓ compositions with the proper Bi/Sb ratios [32].

In addition, the effective mass will be reduced with increasing Mg,Bi₂ concentration [31]. Theoretically, with increasing Bi content in Mg,Sb₂,Bi, the density of states effective mass (m*ₑ) is reduced from ~1.53 m₀ (Mg,Sb₂) to ~1.23 m₀ (Mg,Sb(Bi)) to ~0.87 m₀ (Mg,Bi₂) based on the simulation from the BoltzTraP software package with spin orbit coupling (SOC) (300 K, carrier concentration: ~4 × 10¹⁹ cm⁻³), leading to a smaller Seebeck coefficient and higher carrier mobility [31]. Such a trend has been verified experimentally although the values seem to be lower than the theoretical calculation, as shown in Figure 1(c). It is clear that Bi alloying significantly reduces the density of states effective mass, indicating that it is an effective strategy to enhance the carrier mobility of Mg,Sb₂,Bi alloys. Therefore, the alloying concentration of Mg,Bi₂ is crucial for balancing the carrier mobility and the Seebeck coefficient, as well as the bipolar effect. Pan et al. [33] showed the band evolution from Mg₃Bi₂ to Mg,Sb₂ through angle-resolved photoemission spectroscopy (ARPES) combined with density functional theory (DFT) calculations, which also indicated the effectiveness of adjusting the Bi/Sb ratio in improving thermoelectric performance.

3. Chemical Doping

Defect chemistry has been widely investigated in thermoelectric Zintl compounds in order to understand their intrinsic defects and to explore effective extrinsic dopants that can optimize their electronic transport properties [36–38]. In Mg₅Sb₂₋ₓBiₓ alloys, native Mg vacancies caused by the low defect formation energy and high vapor pressure of Mg result in p-type conduction and abnormal electronic transport behavior near room temperature. Recent studies have shown that adding excess Mg could suppress the formation of such vacancies, leading to a reduction in hole concentration and further resulting in n-type conduction behavior [22]. However, due to the intrinsic doping limit, the electron concentration achieved is only ~10¹⁹ cm⁻³, which is significantly lower than the optimal carrier concentration (~10²⁰ cm⁻³) needed to maximize the zT. Thus, further optimization of the electron concentration via extrinsic doping at the Mg or Sb/Bi sites is especially necessary in this case.

Gorai et al. [39, 40] used first-principle defect calculations to study n-type doping strategies for MgₓSb₂₋ₓBiₓ alloys, including (i) Sb substitution by mono- (Br, I) or divalent (Se, Te) anions, (ii) Mg substitution by trivalent or higher valence cations (La, Y, Sc, Nb), and (iii) insertion of cation interstitials (Li, Zn, Cu, Be), which are represented by black spheres and denoted by i(1), i(2), and i(3) in Figure 2(a). The chemical trends of various dopants have been revealed in terms of their solubility and maximum achievable electron concentration, and the discussion here mainly focuses on Sb and Mg substitution. For the Sb substitution strategy, the defect formation energy around the conductive band minimum in Te₅Sb is lower than that in Se₅Sb under the Mg-rich condition (Figure 2(b)), indicating that Te may have a higher doping limit and greater efficiency, both of which have been confirmed experimentally [20, 35, 41]. On the other hand, substitution by La, Y, and Sc at the cation site has been also explored. It has been found that the defect formation energy values of La₅Mg(1)ₓY₅Mg(1)ₓ and Sc₅Mg(1)ₓ are each lower than that of Te₅Sb, indicating that Mg substitution is even more effective than Sb substitution by Se or Te. The predicted carrier concentration in (La, Y, Sc)-doped Mg,Sb₂ could exceed ~10²⁰ cm⁻³. The relationship between the dopant concentration and the measured electron concentration of Mg,Sb₂,Bi, for different dopants, i.e., La [42], Y [43], Sc [34], Se [35, 44], and Te [45], is illustrated in Figure 2(c). For each dopant, the carrier concentration gradually saturates at a given value with increasing doping level, which is slightly different from the theoretical predictions (dashed lines). This may be closely related to the limited solubility of dopants in Mg,Sb₂,Bi alloys. Additionally, the optimized carrier concentration for power generation is in the range of ~3 – 5 × 10¹⁹ cm⁻³, and it is slightly lower for cooling, and such carrier concentrations can be achieved by doping with Te, Y, Sc, and La. Actually, most studies reported thus far have focused on how to improve the zT value, ignoring the structural origin: e.g., how the electronic and atomic structures of the alloys, including the chemical bonding and the chemical state, evolve after introducing the dopant; how the band structures vary due to doping; and whether a chemical reaction occurs at high temperature. Such lack of structural understanding limits further improvement in the thermoelectric performance of the Mg₅Sb₂₋ₓBiₓ alloys.

Additionally, it should be noted that dopants may affect the thermal stability of the n-type Mg₅Sb₂₋ₓBiₓ alloys, with studies suggesting that degradation in performance would occur with their long-term operation at high temperatures (≥673 K) and that cation-site doping (Y, La, Yb, etc.) via replacing excess Mg may improve their thermal stability and delay such decline in the thermoelectric properties [42, 46, 47]. This can be explained by the changing defect energetics and the fewer Mg deficiencies. Considering the differences in vapor pressure between Mg and Bi/Sb, the decreasing thermal stability has been attributed to the significant Mg loss (defects) at high temperature [48]. Cation-site doping can effectively eliminate Mg deficiencies and improve the thermal stability. On the other hand, by applying coating (such as boron nitride, etc.) on the surfaces of the MgₓSb₂₋ₓBiₓ alloys, their thermal stability can be also effectively improved since such coating prevents Mg loss. Thus, both cation-site doping and coating technology are beneficial for improving thermal stability and promoting practical applications, especially power generation at elevated temperatures.
4. Manipulating the Carrier-Scattering Mechanism

In addition to tuning the carrier concentration, suppression of Mg vacancies in n-type Mg$_3$Sb$_2$ could also be employed to manipulate the carrier-scattering mechanism, thereby enhancing carrier mobility and improving the $zT$, which is particularly significant near room temperature. By exploring the Hall carrier mobility ($\mu_H$) temperature ($T$) relation, ionized impurity scattering was found to dominate the electron transport around room temperature, resulting in low carrier mobility [45]. In order to reduce Mg vacancies and suppress ionized impurity scattering in Mg$_{3.2}$Sb$_{1.5}$Bi$_{0.49}$Te$_{0.01}$, Mao et al. [25] introduced transition-metal elements (Fe, Co, Hf, Ta) into the material matrix, eventually increasing the room-temperature carrier mobility from ~16 cm$^2$ V$^{-1}$ s$^{-2}$ to ~81 cm$^2$ V$^{-1}$ s$^{-2}$ (Figure 3(a)). Similarly, other transition-metal elements, such as Nb [24] and Mn [5, 32, 44], have also been shown to have a dominant effect in shifting the scattering mechanism from ionized impurity scattering to a mixture of ionized impurity scattering and acoustic phonon scattering around room temperature. Additionally, since defects are highly sensitive to preparation conditions, Mao et al. [50] reported that manipulating the hot-pressing temperature could also tune the carrier-scattering mechanism and thereby substantially enhance the carrier mobility of Mg$_{3.2}$Sb$_{1.5}$Bi$_{0.49}$Te$_{0.01}$.

On the other hand, grain boundary scattering has also attracted increasing attention as a carrier-scattering mechanism other than ionized impurity scattering because samples with large grain size have been shown to demonstrate higher carrier mobility, which is particularly noticeable around...
The Mg$_3$Sb$_{1.5}$Bi$_{0.49}$Te$_{0.01}$ samples prepared at a higher sintering temperature show noticeably enlarged grain size as well as higher electrical conductivity (Figure 3(b)). For example, the room-temperature electrical conductivity is ~4 x 10$^4$ S m$^{-1}$ for the sample with an average grain size of ~7.8 μm, and it is ~1 x 10$^4$ S m$^{-1}$ for the sample with an average grain size of ~1.0 μm [53]. Similarly, the grain size of Mg$_3$Sb$_{2-x}$Bi$_x$ alloys was increased by annealing [54] or hot deforming [27, 34, 55], and improvement in mobility was also observed. It should be noted that the defects would be also reduced, in addition to the increasing grain size, by increasing the sintering temperature or by annealing. Thus, in these cases, the ionized impurity scattering was also reduced, eventually leading to the increased electrical conductivity. Kuo et al. explored the defect compositions near the grain boundary of Mg$_3$Sb$_{1.99}$Te$_{0.01}$ (nominal composition) using 3D atom-probe tomography (APT) (Figure 3(c)), from which the planar defect is clearly noticeable (as marked by the arrow), and it is a maximum 5 at. % Mg deficiency [56]. As discussed above, a Mg deficiency could easily induce a high Mg vacancy ($V_{Mg}^{2-}$) concentration in the vicinity of the boundary and result in the depletion of free n-type carriers since $V_{Mg}^{2-}$ serves as an effective electron-killing defect (Figure 3(d)). Single-crystal n-type Mg$_3$Sb$_2$ was thus grown and used to investigate the underlying charge-scattering mechanism [33, 57, 58]. As indicated in Figure 3(e), acoustic phonon scattering dominates the charge transport in the single-crystal sample that lacks grain boundary electrical resistance, resulting in the sample’s significantly increased weighted mobility near room temperature. This may support the proposition that grain boundary scattering dominates the carrier transport of n-type Mg$_3$Sb$_{2-x}$Bi$_x$ alloys in the near-room-temperature range but does not exclude the ionized impurity scattering existing in the samples that do have lots of defects. Actually, in comparison to polycrystal Mg$_3$Sb$_{2-x}$Bi$_x$, not only grain boundaries but also defects are reduced in the single-crystal sample. Thus, additional
details are needed to clarify the carrier-scattering mechanism, which is also crucial for further improving the thermoelectric performance of n-type Mg$_3$Sb$_2$-Bi$_x$.

5. Power Generation and Cooling Applications

Mg$_3$Sb$_2$-Bi$_x$ alloys have shown promise for applications in power generation and cooling due to their high performance. Generally, the Sb-rich compositions (Mg$_3$Sb$_2$-based alloys) are promising for power generation at medium temperature although they may lack good stability due to Mg loss at high temperature (>673 K). For example, Zhu et al. [59] reported that the conversion efficiency of Mg$_3.2$Sb$_{0.5}$Bi$_{1.5}$Te$_{0.02}$ could be up to ~10.6% at a temperature difference of 400 K in the range from 300 K to 700 K, suggesting good potential for midtemperature heat conversion. The Bi-rich compositions (Mg$_3$Bi$_2$-based materials), on the other hand, show more potential for cooling applications. In this case, concerns regarding thermal stability can be ignored due to the low temperature range. Mao et al. [23] reported that optimized Mg$_3.2$Sb$_{0.5}$Bi$_{1.498}$Te$_{0.02}$ exhibits a room temperature $zT$ of more than 0.7 and that the unipolar Mg$_3$Sb$_{0.5}$Bi$_{1.498}$Te$_{0.02}$ and Mg$_3.5$Sb$_{1.5}$Te$_3$ exhibits a large temperature difference of ~91 K at the hot-side temperature of 350 K, comparable to that of commercial coolers based on the Bi$_4$Te$_3$ alloys. Imasato et al. [26] also fabricated n-type Mg$_3$Sb$_{0.5}$Bi$_{1.4}$ with a $zT$ of 1.0-1.2 at 400-500 K, which surpasses that of the n-type Bi$_4$Te$_3$. Furthermore, Mg$_3$Sb$_{2.5}$Bi$_x$ alloys are inexpensive compared to Bi$_4$Te$_3$-based materials because they minimize the need for expensive elemental Te, largely reducing the material cost. In addition, unlike the nanostructured n-type Bi$_4$Te$_3$-based materials that suffer from high contact resistance between the thermoelectric legs and the electrodes, such contact resistance can be greatly reduced for Mg$_3$Sb$_{2.5}$Bi$_x$ by forming a sandwiched structure of Fe/Mg$_3$Sb$_2$-Bi$_x$/Fe. All of these examples show the great potential that the Mg$_3$Sb$_{2.5}$Bi$_x$ alloys have for becoming good candidates to replace the traditional Bi$_4$Te$_3$, promoting their application in thermoelectric technology. In particular, the high cooling performance of Mg$_3$Bi$_{2.5}$-based alloys inspires researchers to explore these semimetals as potential thermoelectric materials for cooling.

6. Conclusions

In summary, strategies like alloying, as well as defect-controlled carrier-concentration optimization and manipulation of the carrier-scattering mechanism, have been successfully used to improve the thermoelectric performance of Mg$_3$Sb$_2$-Bi$_x$ alloys. Further research efforts are warranted to explore other effective and inexpensive dopants for wider temperature application such as in power generation and solid-state cooling, including the structural variation induced by these dopants, and effective strategies to improve thermal stability. In addition, the carrier-scattering mechanism needs to be clarified (whether ionized impurity scattering or grain boundary scattering can better explain the dramatic increase in mobility around room temperature) in the near future in order to further enhance the $zT$. Even so, Mg$_3$Sb$_{2.5}$Bi$_x$ alloys show great potential for power generation and cooling applications.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions

Hongjing Shang and Zhongxin Liang contributed equally to this work.

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