Application of Optimization Method in the Design of Shock Vector Control Nozzle

Li LI, Xichang LV, Linlin ZHANG, Dongming LI
School of Mechanical Engineering, Dalian Jiaotong University, Dalian, Liaoning 116028, China
wejoy_lily@hotmail.com

Abstract. The optimization of shock vector control nozzle are analyzed with design of experiment method, and effects of four parameters of secondary jet in the two-dimensional converging-diverging nozzle are simulated. The approximate parameters model is obtained by Latin hypercube design method, and the optimum solution is achieved by the multi-island genetic algorithm. The numerical simulation results show that the width of the secondary jet is the biggest influence on the thrust pitching angle, the distance of the secondary jet to the nozzle exit is the second, the length of the secondary jet is the third, and the angle of the secondary jet is the least.

1. Introduction
The Fluidic thrust vectoring technology can improve aircraft maneuverability and agility, enhance the stealth and combat effectiveness. The technology uses a secondary injection in the nozzle diverging section to produce the oblique shock wave, thus forcing the mainstream to deflect [1-3].

The detail optimization method is used in the optimization steps of thrust vector nozzle: set parameter regions of secondary jet; design four parameters of secondary jet; get the effect of parameter ratios on the thrust vector angle; extract and calculate the bigger ratio parameters; obtain the optimization solution of parameters with the aim of enlarging thrust vector angle.

2. Nozzle Model and Mesh Partition
The throat diameter is 0.02 m, the entrance radius is 0.06 m, and the length of the diverging wall is 0.08 m. The secondary jet related parameters include the distance of secondary jet to the nozzle exit $e$, the width of the secondary jet $w$, the height of the secondary jet $h$, and the angle of the secondary jet $\theta$. The nozzle flow field and the parameters of secondary jet are shown in Fig.1 [4-5].
Figure 1 Mesh of Flow field and parameters of secondary jet

The flow field is divided into 4 areas: the nozzle converging region, the nozzle diverging region, the secondary jet region and the nozzle exit region. The grids in the shock region and near the outlet are encrypted, and the total number of the grid elements is around 250 thousands. The range of parameters are \(2 \text{ mm} \leq e \leq 15 \text{ mm}, 0.8 \text{ mm} \leq w \leq 1.2 \text{ mm}, 4 \text{ mm} \leq h \leq 12 \text{ mm},\) and \(-10^\circ \leq e \leq 30^\circ\). The angle of the secondary jet normal to the nozzle diverging wall is set to zero, the counterclockwise is positive, and the clockwise is negative.

3. Optimization Method and Process

3.1. Design of Experiments (DOE)
DOE is mainly used to identify the key test factors, determine the best combination of parameters, analyze the relationship and trends between input parameters and output parameters, and build empirical formula and provide data for the approximate modeling [6-7].

Latin hypercube design algorithm, which can effectively reduce the number of simulation, collects the average sample points in the design space. In this algorithm, each design variable is divided into several levels, and each level is extracted from each variable.

The every designed parameter is evenly divided into \(n\) parts, so the number of factors of \(n+1\). All the factors were combined at random, pick one point in different horizontal and vertical coordinates, so there is a \(n \times n\) matrix is designed. In the optimization model, the maximum thrust pitching angle \(\delta_p\) of the main flow is as the optimization goal, so the factors is five and the \(5 \times 5\) matrix is generated based on the four design parameters. In order to solve the following response surface, more than 25 groups of DOE tests are needed. There are 80 groups of sample points are chosen to calculate.

3.2. Response Surface Methodology (RSM)
The approximate model method establishes the relationship between input variables and output variables by mathematical model. In this study, the RSM is chosen to model and establish the polynomial based on the surface curve [8-9].

\[
F(x) = a_0 + \sum_{i=1}^{N} b_i x_i + \sum_{i=1}^{N} c_{ij} x_i^2 + \sum_{i=1}^{N} d_i x_i^3 + \sum_{i=1}^{N} e_i x_i^4
\]

Here \(N\) is the number of the variable, \(x\) is the input points, and \(a, b, c, d, e\) are represent polynomial coefficients.

Suppose the \(N\) is the polynomial number, the at least \(N+1\) samples are needed, and at least \(2N\) samples are needed for the two order linear fitting. The polynomial coefficient with little relation can be ignored based on the Pareto.
In the figure 2, the red color means the variables have negative effect on the thrust vector angle; the blue color means variables have positive effect on the thrust pitching angle. The variables of $e$, $w$ have positive tendency, and the variables of $h$, $\theta$ have negative tendency, and the effects of $w$, $\theta$, $e$, $h$ on the thrust pitching angle reduce gradually.

The four order polynomial function is used to establish the response surface, which is used to fit the polynomial coefficients:

$$F(x) = a_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + c_1 x_1^2 + c_2 x_2^2 + c_3 x_3^2 + c_4 x_4^2 + c_{12} x_1 x_2 + c_{13} x_1 x_3 + c_{14} x_1 x_4 + c_{23} x_2 x_3 + c_{24} x_2 x_4 + c_{34} x_3 x_4 + d_1 x_1^3 + d_2 x_2^3 + d_3 x_3^3 + d_4 x_4^3 + e_1 x_1^4 + e_2 x_2^4 + e_3 x_3^4 + e_4 x_4^4$$

(2)

The $a$, $b$, $c$, $d$, $e$ are represent polynomial coefficients, $x_1$ represents $h$, $x_2$ represents $e$, $x_4$ represents $w$, $x_2$ represents the $\theta$. The determination coefficient of the approximate model of the nozzle is calculated and the results are shown in table 1:

| Polynomial term | Coefficient | Polynomial term | Coefficient | Polynomial term | Coefficient |
|-----------------|-------------|----------------|-------------|----------------|-------------|
| Constant        | -250.3992709 | $w^2$          | -1722.193643 | $w^0$          | -0.012222054 |
| $h$             | 3.521240731  | $0^2$          | 0.012393159  | $e^3$          | 0.039049236  |
| $e$             | 11.09285443  | $h^e$          | -0.007368612 | $w^3$          | 0.157976607  |
| $w$             | 1050.955067  | $h^w$          | 0.388349003  | $0^3$          | 1255.530698  |
| $0$             | -0.06887694  | $h^0$          | -0.00518408  | $e^4$          | -339.8934054  |
| $h^2$           | 0.572609064  | $e^w$          | -0.489849314 | $w^4$          | 0.000996909   |
| $e^2$           | -1.997492396 | $e^0$          | -0.006133179 | $0^4$          | -0.004500545  |

After the coefficients of response surface polynomial are obtained, the accuracy of polynomial can be used to check by R-Squared ($R^2$). $R^2$ regression is used to check the effect of decision coefficient function, and determine the relationship between the polynomial and real data.
The more the value of $R^2$ is close to 1, the closer the relationship of variables is to the real. As to $R^2$ is 0.96, as shown in Fig. 3, so the response surface polynomial equation can be used for reference.

### 3.3. Multi-lands Genetic Algorithm (MGA)

The multi-island genetic algorithm is improved from genetic algorithm. The algorithm divides a large group into several sub groups, known as the ‘island’. After several generations of evolution on the island, the selection, crossover and mutation are done between the islands, so the algorithm can increase the diversity of individuals, the overall and computing efficiency [10]. According to the sub individual generated by the algorithm, the formula for the crossover operation is:

$$x_{i}^{(2,i)} = \frac{1 + \beta_{0,i}}{2} x_{i}^{(1,i)} + \frac{1 - \beta_{0,i}}{2} x_{i}^{(2,i)} \quad \text{(3)}$$

The formula for the crossover and mutation are:

$$x_{i}^{(2,i)} = x_{i}^{(1,i)} + \delta_{i} \left( x_{i}^{UB} - x_{i}^{LB} \right) \quad \text{(4)}$$

$$\delta = \min(x_{i} - x_{i}^{LB}, x_{i}^{UB} - x_{i})/(x_{i}^{UB} - x_{i}^{LB}), u \in [0,1]$$

The sub groups are obtained by crossover and mutation of the parent groups, and the optimal objective function is established on the response surface to get the optimal solution.

The number of the subgroups and the islands decides the number of the basis optimization, and the genetic algebra, crossover rate, mutation rate, and migration interval time determine the number of solutions in response surface. The optimal solution is shown in Fig. 4:
Based on the established response surface, the coupled optimal solution and point map of maximum thrust pitching angle calculation are obtained by 16000 times. The series of black spots mean calculation, and the green point means the optimal solution. The optimization results of RSM response surface are $h = 8.268 \text{mm}$, $w = 1.1394 \text{mm}$, $e = 3.045 \text{mm}$, and $\theta = -6.8069^\circ$. The thrust pitching angle is $3.66^\circ$ before optimization, and the angle is $7.93^\circ$ after optimization which increases $4.28^\circ$, which is shown in Table 2.

### Table 2 Compared before and after Optimization

| Parameter | h/mm | w/mm | e/mm | $\theta$(º) | $\delta p$(º) |
|-----------|------|------|------|-------------|--------------|
| Initial   | 10   | 1    | 10   | 0           | 3.66         |
| Optimized | 8.268| 1.1394| 3.045| -6.8069     | 7.93         |

### 4. Conclusion

The detail optimization method is used in the analysis of the shock vector nozzle, and the effect of the secondary injection width, the length of the secondary injection, the angle of the secondary injection, the distance of the secondary injection to the nozzle exit on the nozzle are investigated. The response surface formula is obtained by the response surface model, and optimization solution is gotten by the multi Island genetic algorithm.

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