Two-dimensional Reconstruction of a Time-dependent Mirror Structure from Double-polytropic MHD Simulation

Wai-Leong Teh and Seiji Zenitani

Abstract A new reconstruction method incorporated with pressure anisotropy parameter, \( \alpha(B) \), has recently been developed for magnetohydrostatic equilibria and successfully applied to recovering a two-dimensional (2-D) magnetic field map of mirror structures observed in the Earth’s magnetosheath. Here, \( \alpha(B) = \mu_0 (p_i - p_\perp) / B^2 \) is assumed to be a function of magnetic field strength, \( B \), alone. The fundamental reconstruction theory assumes that the magnetic field and plasma configurations are time-independent and 2-D, which may not be fulfilled in the real applications to satellite observations. When the 2-D structure is time-dependent, the intrinsic field-line invariant \( F_z = (1 - \alpha) B_z \) is violated so that the quantity \( F_z \) is not constant for the same field line. This paper aims to examine the performance of the \( \alpha(B) \) reconstruction of a time-dependent mirror structure, using data from a 2-D, double-polytropic Magnetohydrodynamics (MHD) simulation. With a single-branched fitting function for the field-line invariant, results show that the geometry of time-dependent mirror structure can be reasonably reconstructed, including the distribution maps of gyrotropic pressures \( p_i \) and \( p_\perp \). As expected, the assumption of \( \alpha(B) \) is well satisfied for the mirror structure. Additionally, another two reconstruction methods are also tested, namely, the Grad-Shafranov reconstruction and the \( \alpha(A) \) reconstruction. The former is considered isotropic pressure, while the latter assumes that \( \alpha \) is function of vector potential \( A \) alone. As expected, these two reconstruction methods fail to recover the geometry of the mirror structure. We suggest that use of a single-branched fitting function is more appropriate for reconstruction of a time-dependent, wave-like structure, regardless of which magnetohydrostatic reconstruction method is applied.

1. Introduction

The Grad-Shafranov (GS)-based reconstruction method is a useful tool of satellite data analysis to recover time-independent, two-dimensional (2-D) magnetic field and plasma configurations of a coherent structure in space. The original reconstruction method was based on the well-known GS equation that describes the magnetohydrostatic equilibria with isotropic pressure. Hereafter, it is called the GS reconstruction. The GS reconstruction scheme is to integrate the GS equation as a spatial initial-value problem with data taken from a single spacecraft. The GS reconstruction has been successfully implemented to examine the geometry of magnetopause structures (e.g., Hasegawa et al., 2004, 2006; Hau & Sonnerup, 1999; Hu & Sonnerup, 2000; Sonnerup et al., 2004; Teh et al., 2010; Teh & Hau, 2007), and the magnetic flux ropes and magnetic clouds in the solar wind (e.g., Hu, 2017; Hu & Sonnerup, 2001, 2002).

Recently, the reconstruction theory has been applied to the magnetohydrostatic equilibria with gyrotropic pressure. The degree of the pressure anisotropy can be described by a parameter, \( \alpha = \mu_0 (p_i - p_\perp) / B^2 \), where \( p_\perp \) and \( p_i \) are the plasma pressures perpendicular and parallel to the magnetic field, respectively. Assuming that \( \alpha \) is function of vector potential \( A \) alone, that is, \( \alpha = \alpha(A) \), Teh (2018a, 2018b) derived a GS-like equation in a simple formulation, for which three fitting functions of vector potential \( A \) are required for reconstruction. Later, Teh (2019) realized that the simple GS-like formulation with pressure anisotropy for reconstruction can also be achieved by assuming that the gyrotropic pressure is function of magnetic field strength alone and thus leading to \( \alpha = \alpha(B) \). For convenience, these two different reconstructions are hereafter named as the \( \alpha(A) \) and \( \alpha(B) \) reconstructions. By contrast, the \( \alpha(B) \) reconstruction only requires two fitting functions for \( F_z(A) = (1 - \alpha) B_z \) and \( \alpha(B) \), where \( F_z \) is an intrinsic function of vector potential \( A \) alone and thus leading to a single-branched fitting function. In this study, we apply the two different reconstruction methods, namely, \( \alpha(A) \) and \( \alpha(B) \), to 2-D, double-polytropic MHD simulation data to analyze the time-dependent structure of mirror regions.
field-line invariant and $B_z$ is the magnetic field component along the invariant axis of the structure. In the paper of Teh (2019), the $\alpha(\hat{B})$ reconstruction was applied to recovering 2-D geometry of mirror structures in the Earth's magnetosheath and he found that the assumption of $\alpha(\hat{B})$ is well satisfied for the mirror structure. It is noteworthy that using double-polytropic laws proposed by Hau and Sonnerup (1993), Sonnerup et al. (2006) developed a reconstruction method incorporated with the gyrotropic pressure and field-aligned flow. Recently, Chen and Hau (2018) applied it to the magnetopause structures with field-aligned flows, while Tian et al. (2020) used it to recover the Pc5 compressional waves for magnetohydrostatic condition. This reconstruction method for magnetohydrostatic equilibria requires four fitting functions for four intrinsic field-line invariants and has to deal with a $6 \times 6$ sparse matrix. As compared with the $\alpha(\hat{A})$ and $\alpha(\hat{B})$ reconstructions, it requires more fitting functions and more physical quantities to solve.

The fundamental reconstruction theory assumes that the coherent structure in space is time-independent and 2-D. When the time-independent assumption is violated, the intrinsic field-line invariant, for example, $F_z$, is no longer satisfied. Such a time-dependent structure may happen in the real applications to the satellite observations. Recent works by Liu et al. (2020) demonstrate that the evolution of magnetic cavity structure has a key role in the particle energization and energy dissipation. Understanding the configuration of time-dependent magnetic field structure can therefore provide insights into the process of the energy conversion. This paper aims to examine the performance of the $\alpha(\hat{B})$ reconstruction of a time-dependent mirror structure, using data from a 2-D, ideal, double-polytropic MHD simulation (Teh & Zenitani, 2019). In addition, both the GS and $\alpha(\hat{A})$ reconstruction methods are also tested. We note that the time-aliasing effect on the reconstruction studies by Hasegawa et al. (2014) is not considered in this study. The paper is organized as follows. Section 2 describes the $\alpha(\hat{B})$ reconstruction theory and scheme. Section 3 describes the 2-D, double-polytropic MHD simulation and shows the reconstruction results for the three different reconstruction methods. Finally, summary and discussion are given in Section 4.

### 2. $\alpha(\hat{B})$ Reconstruction Theory and Scheme

With the assumption that the gyrotrropic plasma pressure is solely dependent on the magnetic field strength, a 2-D coherent structure in the magnetohydrostatic equilibria can therefore be described by a GS-like equation as follows,

$$\nabla \cdot (1 - \alpha)\nabla A + B_z dF_z / dA = 0$$

(1)

where $\alpha = \mu_0(p_i - p_\perp) / B^2 = \alpha(\hat{B})$ and $F_z = (1 - \alpha)B_z$. The invariant axis of the 2-D structure is directed along the $z$ axis. The quantity $F_z$ is an intrinsic field-line invariant and thus is function of vector potential $A$ alone, while the pressure anisotropy parameter, $\alpha$, is function of magnetic field strength alone, based on the assumption. Here the magnetic field $\mathbf{B}$ is expressed as $\mathbf{B} = \nabla A \times \hat{z} + \hat{z}B_z$. For the detailed derivation of Equation 1, the reader can find it in the paper of Teh (2019). It is noteworthy that when $\alpha$ goes to zero, that is, the plasma pressure is isotropic, Equation 1 is thus reduced to the GS equation for force-free condition (i.e., the thermal pressure gradient force is neglected), instead of the original GS equation. This is because the gyrotrropic plasma pressure is assumed to be solely dependent on the magnetic field strength. In this study, the absolute value of $\alpha$ is well greater than zero.

To reconstruct a 2-D magnetic field map in the reconstruction plane (i.e., $x-y$ plane), three unknowns are to be solved, namely, $\partial^2 A / \partial y^2$, $\partial B_z / \partial y$, and $\partial A / \partial y$. By rewriting Equation 1 and differentiating $F_z$ and $B^2$ with respect to the variable $y$, three equations for reconstruction are obtained as follows:

$$B_z \partial A / \partial y - B_z \partial A / \partial y + (1 - \alpha) \partial^2 A / \partial y^2 + (1 - \alpha) \partial^2 A / \partial x^2 + B_z dF_z / dA = 0,$$

(2)

$$-B_z \partial A / \partial y + (1 - \alpha) \partial B_z / \partial y = \partial F_z / \partial y = B_z dF_z / dA,$$

(3)

$$\partial B / \partial y = \left(1 / B\right) \left[B_z \partial^2 A / \partial y^2 - B_z \partial B_z / \partial x + B_z \partial B_z / \partial y\right].$$

(4)
where \( B_x = \partial A / \partial y \) and \( B_y = -\partial A / \partial x \) are used. Since \( \alpha \) is function of \( B \) alone, one can get \( \partial \alpha / \partial y = (d\alpha / dB)\partial B / \partial y \). By substituting Equation 4 into Equations 2 and 3, a matrix form for the two unknowns \( \partial^2 A / \partial y^2 \) and \( \partial B_z / \partial y \) is derived, that is, \( \mathbf{M} \mathbf{X}^T = \mathbf{Y} \), where the superscript \( T \) denotes the matrix transpose. The row matrix \( \mathbf{X} = \left[ \partial^2 A / \partial y^2 \quad \partial B_z / \partial y \right] \), and the column matrix

\[
\mathbf{Y} = \begin{bmatrix}
-(1-\alpha)\partial^2 A / \partial y^2 - B_x(d\alpha / dB)\partial B / \partial x - B_y(d\alpha / dB)\partial B_y / \partial A + B(d\alpha / dB)\partial B_x / \partial A - B_y(d\alpha / dB)\partial B_x / \partial A + B(d\alpha / dB)\partial B_y / \partial A
\end{bmatrix}
\]

The 2 x 2 matrix \( \mathbf{M} \) is therefore expressed as

\[
\mathbf{M} = \begin{bmatrix}
(1-\alpha) - B_y^2 / B(d\alpha / dB) & -B_y B_z / B(d\alpha / dB) \\
-B_y B_z / B(d\alpha / dB) & (1-\alpha) - B_z^2 / B(d\alpha / dB)
\end{bmatrix}
\]

The \( dF_z / dA \) and \( d\alpha / dB \) terms in the \( \mathbf{Y} \) and \( \mathbf{M} \) can be determined numerically from the measurements and the \( x \) derivatives in the \( \mathbf{Y} \) can also be calculated. Therefore, the two unknowns in the matrix \( \mathbf{X} \) can be solved by inverting the matrix \( \mathbf{M} \).

When \( \partial^2 A / \partial y^2 \) and \( \partial B_z / \partial y \) are known, the magnetic field map in the \( x-y \) plane can be reconstructed by integrating the vector potential \( A, B_x, B_y, \) and \( B_z \) as follows:

\[
\begin{align*}
A(x, y \pm \Delta y) &= A(x, y) \pm \Delta y \partial A / \partial y + 1 / 2 (\Delta y)^2 \partial^2 A / \partial y^2, \\
B_x(x, y \pm \Delta y) &= B_x(x, y) \pm \Delta y \partial^2 A / \partial y^2, \\
B_y(x, y \pm \Delta y) &= -\partial A(x, y \pm \Delta y) / \partial x, \\
B_z(x, y \pm \Delta y) &= B_z(x, y) \pm \Delta y \partial B_z / \partial y.
\end{align*}
\]

The initial values of vector potential \( A \) at \( y = 0 \) are computed as \( A(x, y = 0) = -\int_{x'=0}^{x'} B_y(x, y = 0) dx' \), where \( dx' = V_0 dt \) and \( V_0 \) is the motion of the structure. Note that the contour of vector potential \( A \) represents the magnetic field line. The distribution map of \( p_{\parallel} \) can be reconstructed by integration of \( p_{\parallel}(x, y \pm \Delta y) = p_{\parallel}(x, y) \pm \Delta y \partial p_{\parallel} / \partial y, \) where \( \partial p_{\parallel} / \partial y = dp_{\parallel} / dB(\partial B / \partial y) = \alpha(B / \mu_0)(\partial B / \partial y). \) Here, \( dp_{\parallel} / dB = (p_{\parallel} - p_{\perp}) / B, \) which is Equation 6 in the paper of Teh (2019). With the \( p_{\parallel} \) distribution, the pressure \( p_{\perp} \) can then be calculated from the \( \alpha \) distribution, which is calculated from the function \( \alpha(B) \).

3. Simulation Data and Reconstruction Results

The mirror structure for reconstruction is produced by the 2-D, ideal, double-polytropic MHD simulation (Teh & Zenitani, 2019). The advantage of using 2-D MHD model for this study is that we can rule out other effects (e.g., three-dimensional and non-ideal MHD) that can violate the fundamental assumptions of the reconstruction method.

The simulation codes are mainly inherited from the 2-D isotropic MHD codes by Zenitani and Miyoshi (2011) and Zenitani (2015). In the double-polytropic laws, two polytropic exponents, \( \gamma_{\parallel} \) and \( \gamma_{\perp} \), are used as parameters to describe various thermodynamic conditions in the gyrostatic plasma (Hau & Sonnerup, 1993). For example, \( \gamma_{\parallel} = 3 \) and \( \gamma_{\perp} = 2 \) for double-adiabatic and \( \gamma_{\parallel} = 1 \) and \( \gamma_{\perp} = 1 \) for double-isothermal. Teh and Zenitani (2019) have demonstrated that using the empirical values of \( \gamma_{\parallel} = 1.14 \) and \( \gamma_{\perp} = 0.94 \) for magnetosheath plasma (Hau et al., 1993), the variations of temperatures \( T_{\parallel} \) and \( T_{\perp} \) in the mirror structures observed by the Magnetospheric Multiscale Mission (MMS; Burch et al., 2016) in the magnetosheath can be reproduced. Note that using the empirical values of \( \gamma_{\parallel} = 1.14 \) and \( \gamma_{\perp} = 0.94 \), the mirror instability
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The plasma unstable regime of the mirror structures observed by the MMS spacecraft in the magnetosheath is consistent for these two conditions. Similar results for the magnetosheath mirror structures are also concluded by Hau et al. (2020). The simulation setup implemented by Teh and Zenitani (2019) is adopted in this study, except that a small and uniform $B_z$ is incorporated into the initial magnetic field profile of a uniform $B_x$, instead of $B_z = 0$. This modification allows us to examine the field-line invariant $F_z$. The initial plasma beta $\beta_0 = 2$ and $\beta_\perp = 4$ and uniform plasma density are used. The simulation box size is $2\pi \times 2\pi$ with grid points of $200 \times 200$. Periodic boundary conditions are employed in the $x$ and $y$ directions. The simulation time is normalized by the inverse of the typical Alfvén speed, $B_0 / \sqrt{\rho_0 \mu_0} = l$, with $B_0 = 1$ and $\rho_0 = 1$.

Figure 1. (a)–(c) Close-up of a mirror structure obtained from the double-polytropic MHD simulation at the time $t = 40$, where solid lines illustrate the magnetic field lines and color codes show the axial magnetic field, perpendicular and parallel pressures. (d)–(f) Simulation data of the mirror structure taken at points along the white dashed line at $y = 3.57$, as shown in the panel (a), for reconstruction. (g)–(i) Results of $\rho \mathbf{V}_x / \mathbf{B}_x$, $\rho (\mathbf{V} \cdot \nabla \mathbf{V}) / \mathbf{B}_x$, and $\mathbf{c} (p_\perp + B_\perp^2 / 2\mu_0) / \mathbf{c}$ shown in color. (j) The time evolution of the average of $(B - B_0)^2 / B_0^2$ for the mirror structures. The simulation data points along the red dashed lines ($y = 3.44$ and $y = 3.72$) are used for comparisons with the reconstruction results.

condition $\gamma_0 \beta_0 < \beta_0^2 / (2 + \gamma_0 \beta_0)$ for double-polytropic MHD is different from the kinetic one (e.g., Hasegawa, 1969). As illustrated in Figure 1b of the paper of Teh and Zenitani (2019), the plasma unstable regime of the mirror structures observed by the MMS spacecraft in the magnetosheath is consistent for these two conditions. Similar results for the magnetosheath mirror structures are also concluded by Hau et al. (2020).
Figures 1a–1c show the close-up of a mirror structure taken at the time $t = 40$, in which solid lines illustrate the magnetic field lines and the axial field $z_B$ and the pressures $p_{\parallel}$ and $p_{\perp}$ are shown in color. Figures 1d–1f show the magnetic field and plasma data of the mirror structure for reconstruction, taken at the data points along the white dashed line at $y = 3.57$ as shown in Figure 1a. The characteristics of the mirror structure are evident in Figures 1e and 1f, that is, (1) the plasma density $\rho$ is anticorrelated with the magnetic field strength, and (2) both the pressures $p_{\parallel}$ and $p_{\perp}$ are enhanced in the magnetic dip but depressed in the magnetic peak (e.g., Balikhin et al., 2009, 2010). Figures 1g–1i show the distribution maps of the time-dependent term $\partial \tau / \partial x V_t \rho$ and the inertial term $\rho (\nabla \cdot V V)$, and the total pressure gradient $\rho (p_{\perp} + B^2 / 2 \mu_0) / \partial x$ in color. It can be found that the time-dependent term is mostly larger than the inertial term, indicating that the time-dependent effect is dominant, the same result for the $y$ component (not shown). In Figure 1j, the time evolution of the average of $p_{\parallel} / B^2$ is demonstrated for the mirror structures. One can find that the mirror structures start to grow around $t = 20$ and the simulation time $t = 40$ is in the transition between the two main growth phases. The mirror structures at $t = 40$ will then be growing in later time.

Figures 2a and 2b show plots of $z_F$ versus the vector potential $A$, with a fitting curve in yellow. The red and blue dots denote the data associated with $B_y > 0$ and $B_y < 0$, respectively. (b) Plot of the pressure anisotropy parameter $\alpha$, versus the magnetic field strength, with a fitting curve in yellow. (c) Plot of the pressures $p_{\parallel}$ and $p_{\perp}$ versus the magnetic field strength.

Figure 2. (a) Plot of the field-line invariant $F_z$ versus the vector potential $A$, with a fitting curve in yellow. The red and blue dots denote the data associated with $B_y > 0$ and $B_y < 0$, respectively. (b) Plot of the pressure anisotropy parameter $\alpha$, versus the magnetic field strength, with a fitting curve in yellow. (c) Plot of the pressures $p_{\parallel}$ and $p_{\perp}$ versus the magnetic field strength.

Figures 1a–1c show the close-up of a mirror structure taken at the time $t = 40$, in which solid lines illustrate the magnetic field lines and the axial field $B_z$ and the pressures $p_{\parallel}$ and $p_{\perp}$ are shown in color. Figures 1d–1f show the magnetic field and plasma data of the mirror structure for reconstruction, taken at the data points along the white dashed line at $y = 3.57$ as shown in Figure 1a. The characteristics of the mirror structure are evident in Figures 1e and 1f, that is, (1) the plasma density $\rho$ is anticorrelated with the magnetic field strength, and (2) both the pressures $p_{\parallel}$ and $p_{\perp}$ are enhanced in the magnetic dip but depressed in the magnetic peak (e.g., Balikhin et al., 2009, 2010). Figures 1g–1i show the distribution maps of the time-dependent term $\rho \partial \tau / \partial x V_t \rho$, the inertial term $\rho (\nabla \cdot V V)$, and the total pressure gradient $\rho (p_{\perp} + B^2 / 2 \mu_0) / \partial x$ in color. It can be found that the time-dependent term is mostly larger than the inertial term, indicating that the time-dependent effect is dominant, the same result for the $y$ component (not shown). In Figure 1j, the time evolution of the average of $(B - B_0)^2 / B_0^2$ is demonstrated for the mirror structures. One can find that the mirror structures start to grow around $t = 20$ and the simulation time $t = 40$ is in the transition between the two main growth phases. The mirror structures at $t = 40$ will then be growing in later time.

Figures 2a and 2b show plots of $F_z$ versus the vector potential $A$ and $\alpha$ versus the magnetic field strength, with a fitting curve in yellow. In Figure 2a, the red and blue dots denote the data associated with $B_y > 0$ and $B_y < 0$, respectively. It is evident that there are two values of $F_z$ for the same vector potential $A$, indicating that the quantity $F_z$ is not constant along the magnetic field line. In the previous GS reconstruction studies of magnetopause structures, Hu and Sonnerup (2003) adopted double-branched function of $p_{\parallel} / B^2$ for the two sides of the magnetopause. The idea of double-branched function is not suitable for the mirror structure, because a clear boundary between two separate plasma regimes does not exist in the mirror structure. Therefore, a single-branched function of $F_z$ is used for reconstruction. A further discussion on this issue will be given in the next section. Unlike $F_z$, the variations of the pressure anisotropy parameter $\alpha$ are well correlated with the magnetic field strength, as shown in Figure 2b, indicating that the assumption of $\alpha(B)$ is well satisfied for the mirror structure. Moreover, Figure 2c indicates that the variations of the pressures $p_{\parallel}$ and $p_{\perp}$ are also well correlated with the magnetic field strength for the mirror structure.

Figure 3 shows the reconstructed magnetic field maps of the time-dependent mirror structure, with $B_z$, $p_{\parallel}$, and $p_{\perp}$ in color, using the magnetic field and pressure data shown in Figures 1d and 1f. The white dashed line denotes the original data line.

Figure 3. Reconstructed magnetic field maps of a time-dependent mirror structure from a 2-D double-polytropic MHD simulation, with $B_z$, $p_{\parallel}$, and $p_{\perp}$ in color, using the magnetic field and pressure data shown in Figures 1d and 1f. The white dashed line denotes the original data line.
white dashed line denotes the original data line. As compared with the simulation results in Figures 1a–1c, the magnetic field configuration of the mirror structure is found to be successfully recovered and the reconstructed distributions of $B_z$, $p_\parallel$, and $p_\perp$ are reasonable and acceptable. Figure 4 shows the quantitative comparisons of the reconstruction results (in red) with the simulation results (in black) for the data points along the two pink dashed lines in Figure 3. The correlation coefficient ($cc$) between them is also calculated and shown for each physical quantity. Overall, high $cc$ values are achieved for the reconstruction results, indicating that the $\alpha(B)$ reconstruction method performs well for the time-dependent mirror structure. We note that when the time-dependent effect becomes large, for example, in the second growth phase at $t = 46$, the deviation of the $F_\parallel$ value at the same field line increases and thus the relationship between $F_\parallel$ and the vector potential $A$ is much less correlated (not shown). These results can therefore degrade the reconstruction performance.

Figure 5 shows the $\alpha(A)$ reconstruction results only for the data associated with $B_\perp > 0$ as shown in Figures 1d and 1f. Three fitting functions of vector potential $A$ for reconstruction are demonstrated by the yellow curves in Figures 5a–5c, where $p_r = (1 - \alpha) p_\perp + \alpha(1 - \alpha)^{-1} (F^2 / 2\mu_0)$ and $F = (1 - \alpha) B$. Note that the initial values of vector potential $A$ are calculated as instead. In the $\alpha(A)$ reconstruction scheme, the vector $F$ is first reconstructed and then the vector $B = F / (1 - \alpha)$ is computed afterward. For more details, the $\alpha(A)$ reconstruction theory and scheme are referred to the paper of Teh (2018a). Note that the three fitting functions also have the double-valued problem. For the $\alpha(A)$ reconstruction, the use of a single-branched
function is not appropriate for the entire data set, where the $\alpha(A)$ has double values. Using a single-fitted value of $\alpha$, the initial magnetic field data have to be modified so as to obtain a consistent result between the measured $B = (1 - \alpha)B$ and the fitted $B = (1 - \alpha_B)B^*$, where $B^*$ is the modified magnetic field. This is the reason that only the leading part of the mirror structure is demonstrated for the $\alpha(A)$ reconstruction.

Figures 5d–5f are shown in the same format as Figure 3. Obviously, the reconstructed magnetic field map and distributions of $zB$, $p\parallel$, and $p\perp$ are different from those for the leading part of the mirror structure in Figure 1. Overall, each reconstructed field component has $cc < 0.6$.

Figure 6 shows the GS reconstruction results using the same data set as the $\alpha(A)$ reconstruction. The physical quantities $p_{TR} = p + B_z^2 / 2 \mu_b$ and $B_z$ are the intrinsic field-line invariants for the magnetohydrostatic equilibria with isotropic pressure $p = (p_\parallel + 2p_\perp) / 3$ and one can find that they also have the double-valued issue. However, there is no problem of using a single-branched function for the GS reconstruction because the $p_{TR}$ and $B_z$ quantities are not required during the integration and only the $dp_{TR} / dA$ is required. As compared with Figure 1a, the GS reconstruction map as well as the $B_z$ distribution are different from those for the leading part of the mirror structure in Figure 1. Overall, each reconstructed field component has $cc < 0.5$. While the GS and $\alpha(A)$ reconstructions fail to recover the geometry of the mirror structure, the GS field map is qualitatively better than the $\alpha(A)$ field map.

4. Summary and Discussion

To examine the performance of the $\alpha(B)$ reconstruction of a time-dependent mirror structure, a 2-D, double-polytropic MHD simulation is implemented. For the time-dependent mirror structure, the intrinsic field-line invariant $F_z$ has double values at the same field line. In our simulation data, the violation of the
field-line invariant is mainly caused by the time-dependent effect rather than the inertial effect. With a single-branched function $F_z$, results show that the geometry of the time-dependent mirror structure can be reasonably reconstructed, including the distribution maps of $B_y$, $p_{\parallel}$, and $p_{\perp}$. As expected, the assumption of $\alpha(B)$ is well satisfied for the mirror structure. Additionally, the GS and $\alpha(A)$ reconstruction methods are also tested. As expected, these two reconstruction methods fail to recover the geometry of the mirror structure.

It is found in Figure 2a that the fitting curve of $F_z$ is not fitted well with the data. How would it affect the $\alpha(B)$ reconstruction results? To answer that, Figure 7 shows the $\alpha(B)$ reconstruction results only for the data associated with $B_y > 0$. As seen in Figure 7a, the fitting curve is now well fitted with the data. One can find that the magnetic field map in Figure 7 is similar to that for the leading part of the mirror structure in Figure 3, including the distribution maps of $B_y$, $p_{\parallel}$, and $p_{\perp}$. For comparison, the two field-line maps are overlaid in Figure 7e, where the yellow dashed lines represent the magnetic field lines for the leading part of the mirror structure in Figure 3. While most small deviations happen near the upper right corner of the map, the two field-line maps are mostly identical. From Equation 1, one can realize that the quantities of $\alpha$ and $B_y$ are required during integration. Note that the quantity $B_y$ is advanced by Equation 10, while the quantity $\alpha$ is advanced by the function $\alpha(B)$. Thus, this examination reveals that the $dF_z / dA$ term, which only appears in the matrix $Y$, plays a minor role in the reconstruction of the mirror structure.

It has been previously mentioned that the use of double-branched function for the field-line invariant is not suitable for the mirror structure. From the data set of reconstruction, the sign of $B_y$ is the indicator for branch selection. However, the sign of $B_y$ is not suitable for branch selection, because the $B_y$ changes sign more than once in the region below the original data line, which is evident in the real map in Figure 1. When using double-branched function for the GS reconstruction, the lower part of the reconstructed field map becomes more different from the real map (not shown). Therefore, it is suggested that use of a single-branched fitting function is more appropriate for reconstruction of a time-dependent, wave-like structure, regardless of which magnetohydrostatic reconstruction method is applied.

**Figure 6.** (a)–(b) Plots of the two intrinsic field-line invariants $p_{tr} = p + B_y^2 / 2 \mu_0$ and $B_z$ versus the vector potential $A$. The red and blue dots denote the data associated with $B_y > 0$ and $B_y < 0$, respectively. (c) The GS reconstruction magnetic field map with $B_y$ in color.
Figure 7. The $a(B)$ reconstruction results using the data associated with $B_y > 0$ only. The panels (c)–(e) are the same format as Figure 3. Two field-line maps are overlaid in panel (e), where the yellow dashed lines represent the magnetic field lines for the leading part of the mirror structure in Figure 3.

Data Availability Statement

Datasets for this reconstruction study are available on the GitHub at https://github.com/wteh5871/Reconstruction-Data.git. The datasets are compiled by Teh and Zenitani (2019).

Acknowledgments

This work was supported by the grant of Universiti Kebangsaan Malaysia (GUP-2018-086).

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