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Predicting the settlement of geosynthetic-reinforced soil foundations using evolutionary artificial intelligence technique

Muhammad Nouman Amjad Raja, Sanjay Kumar Shukla

Abstract

In order to ensure safe and sustainable design of geosynthetic-reinforced soil foundation (GRSF), settlement prediction is a challenging task for practising civil/geotechnical engineers. In this paper, a new hybrid technique for predicting the settlement of GRSF has been proposed based on the combination of evolutionary algorithm, that is, grey-wolf optimisation (GWO) and artificial neural network (ANN), abbreviated as ANN-GWO model. For this purpose, the reliable pertinent data were generated through numerical simulations conducted on validated large-scale 3-D finite element model. The predictive power of the model was assessed using various well-established statistical indices, and also validated against several independent scientific studies as reported in literature. Furthermore, the sensitivity analysis was conducted to examine the robustness and reliability of the model. The results as obtained have indicated that the developed hybrid ANN-GWO model can estimate the maximum settlement of GRSF under service loads in a reliable and intelligent way, and thus, can be deployed as a predictive tool for the preliminary design of GRSF. Finally, the model was translated into functional relationship which can be executed without the need of any expensive computer-based program.

Keywords:
- Geosynthetics
- Reinforced soil foundation
- Settlement
- Finite element simulations
- Predictive modelling
- Artificial intelligence
- ANN-GWO
- Hybrid model

1. Introduction

In the current construction practice, geosynthetic-reinforced soils are widely used to support shallow foundations. One of the main purposes of constructing the geosynthetic-reinforced soil foundation (GRSF) is to safely transfer the load from the superstructure to the substructure. As a serviceability requirement, the GRSF settlement shall not increase the design level. Therefore, the realistic estimation of the settlement value is imperative for the construction of safe and sustainable GRSFs. In the literature, several theoretical methods are available which can successfully predict the settlement of shallow unreinforced soil foundations on granular soils (Burland and Burbridge, 1985; Gibbs and Holtz, 1957; Meyerhof, 1956; Peck et al., 1974; Schmertmann, 1970; Teng, 1962; Terzaghi and Peck, 1948). However, the settlement estimation of GRSF requires further attention.

In the past, several experimental studies were conducted to investigate the behaviour of reinforced soil foundations. Yamanouchi (1970) successfully carried out an experimental study on the improvement of bearing capacity of soft ground by utilising the polymer net. Binquet and Lee (1975) conducted an experimental study to investigate the load-settlement behaviour of footing resting on soil bed reinforced with aluminium strips. They reported an increase in the bearing capacity of reinforced soil, and for a particular load, decrease in the settlement value, in comparison to unreinforced soil foundation. Thereafter, many experimental studies were performed to evaluate the performance of reinforced soil foundations (e.g., Abu-Farsakh et al., 2013; Chang et al., 2020; Chen et al., 2021; Guido et al., 1986; Khing et al., 1993; Latha and Murthy, 2007; Misir and Laman, 2018; Ouria and Mahmoudi, 2018; Raja and Shukla, 2021a; Shadmand et al., 2018; Tafreshi and Dawson, 2010; Venkateswarlu et al., 2018; Wang et al., 2018; Xu et al., 2019). For the estimation of ultimate bearing capacity of GRSF various analytical formulations are available in the literature (Chen, 2007; Chen and Abu-Farsakh, 2015; Huang and Tatsuoka, 1990; Raja and Shukla, 2020a; Wayne et al., 1998). However, a limited number of studies were also carried out to analytically estimate the settlement of reinforced soil foundations (Huang, 2011; Huang and Hong, 2000). These solutions can only estimate the settlement at peak footing load levels, thus, constituting a handicap particularly from the serviceability point of view. As a result, alternative methods are needed which provide more realistic
prediction of the settlement behaviour of GRSF.

In order to overcome the limitations of analytical modelling methods, artificial intelligence (AI) techniques are gaining popularity among the researchers to solve many complicated engineering problems, including those related to geotechnical engineering (Abuel-Naga and Bouazza, 2014; Ismail and Jeng, 2011; Moayedi et al., 2020; among the researchers to solve many complicated engineering problems, including those related to geotechnical engineering (Abuel-Naga and Bouazza, 2014; Ismail and Jeng, 2011; Moayedi et al., 2020; Moayedi and Hayati, 2018; Nguyen et al., 2020; Raja and Shukla, 2020b; Xiao and Zhao, 2019; Zhang et al., 2020). One of the main advantages of AI methods over traditional approaches is their ability to extract the knowledge from the observed data without any specific assumption. This makes these methods extremely suitable for mapping the non-linear relationships between inputs and outputs variables. Several researchers have successfully predicted the settlement of unreinforced shallow soil foundations by utilising machine learning (ML) models (Rezania and Javadi, 2007; Samui, 2008; Shahin et al., 2002). For reinforced foundations, Soleimanbeigi and Hataf (2006) employed feed forward back propagating neural network, and Raja and Shukla (2021b) utilised multiple adaptive regression splines models to predict the settlement of footings on reinforced sandy ground at peak footing loads. Khosrojerdi et al. (2019) conducted a non-linear regression analysis to predict the load-settlement behaviour of reinforced soil foundations. Among all the computationally intelligent models, ANNs are widely used for predicting the response of a system due to their intrinsic ability of learning the complex relationships between the input and target variables, handling multiple target variables, and efficiently predicting the unseen data (Ghorbani et al., 2020). In spite of the fact that ANN benefits from the gradient decent based approach, yet the local minima trap and slow learning rate are the most commonly known disadvantages of neural networks.

To improve the prediction and generalisation ability of the AI models, researchers and scientists have proposed various innovative techniques. The review of the published literature shows that the use of evolutionary computation methods based on heuristics and artificial intelligence algorithms seems to be the robust technique for improving the efficiency of AI models specially ANNs (e.g., genetic algorithm (GA); artificial bee colony optimisation algorithm (ABCO); particle swarm optimisation (PSO); grey wolf optimisation (GWO); whale optimisation (WO); flower pollination algorithm (FPA) etc.). Amongst these methods, GWO algorithm is a recently developed metaheuristic, and has shown a promising ability in finding the optimum solutions to constrained as well as unconstrained engineering problems (Guha et al., 2016; Mirjalili et al., 2014; Moayedi et al., 2019; Tikhamarine et al., 2020). Therefore, in this study, an attempt is made to develop and implement a new hybrid AI method to simulate the settlement response of geosynthetic-reinforced soil foundations based on an ANN model optimised with GWO (i.e., ANN-GWO hereafter). The database is collected by conducting intensive numerical simulations. For this, large scale field model footing load tests carried out by Adams and Collins (1997) on GRSF were simulated by utilising the PLAXIS 3-D program. This enables the authors to validate their numerical results. Subsequently, a database was generated by varying the soil parameters, footing geometry, and reinforcement characteristics. The performance of the developed ANN-GWO model has been validated and verified using several traditional and modern performance assessment statistical matrices. The sensitivity analysis was also conducted to examine and evaluate the robustness and trustworthiness of the developed model. Furthermore, the predictive veracity of the model is also validated against several independent studies as reported in the scientific literature. The developed model was also converted into trackable formulae for easy hand calculations, and preliminary design purposes. This will prove helpful in saving time and cost (monetary and computational) associated with performing model footing load tests and numerical simulations.

2. Methodology

This section focuses on the development and implementation of a novel hybrid intelligent system for the realistic evaluation and prediction of settlement of geosynthetic-reinforced soil foundations i.e., ANN-GWO. The methodology adopted for the development of the ANN-GWO model for predicting the settlement of GRSFs is explained below.

2.1. Artificial neural network (ANN)

Amongst all the AI based machine learning methods, ANNs are most popular and well-established methods. Originally proposed by McCulloch and Pitts (1943), ANNs aim to imitate the behaviour of human nervous system. The network architecture consists of three parts: single input layer, one or more hidden layers, and an output layer. Each layer consists of logically arranged set of processing element, called neurons (nodes), with different task assignments. In ANN, the data is presented and transmitted to the model through the input layer, thereafter, it is processed in the hidden layer, and sent to the output layer to record the
response of the system. In order to capture the relationship between input dimensions and corresponding target(s), the neurons in each layer interact with each other through weighted connections (Soleimanbeigi and Hataf, 2006; Zhang et al., 2020). The main processing is done in hidden and output layers in which data are multiplied with the weight matrix and added to the bias vector. Afterwards, the transfer function is applied to this weighted sum to produce the output. During training, the weights and biases are adjusted in a way that the error between the measured and the predicted outputs are minimized. In literature, various learning algorithms are available for training the data; however, the backpropagation (BP) algorithm is the most commonly used algorithm for function approximation (Goodfellow et al., 2016). Further details about the historical development of ANNs can be found in many artificial intelligence/machine learning books (e.g., Witten and Frank, 2005).

In this study, ANN trained with BP algorithm is developed to predict the settlement of GRSF, and its architecture is illustrated in Fig. 1.

2.2. Grey wolf optimisation (GWO)

Mirjalili et al. (2014) proposed a grey wolf algorithm based on nature inspired behaviour of pack of grey wolves. The primary idea is to mimic and simulate the predation (hunting) behaviour of grey wolf pack. In grey wolf pack hierarchy, there is leader of the pack, known as alpha (α) wolf. The second and third grey wolves on the hierarchical system are beta (β) and delta (δ) wolves, respectively. The rest of the wolves are categorised as omega (ω). The mathematical model is based on the predation strategy of the pack which consists of encircling, hunting, and attacking the prey. In terms of model simulations, the search process begins with the random generation of grey wolves’ population, that are, candidate solutions for the required problem. The corresponding objective value for each wolf is then calculated. The α, β, and δ wolves track the probable position of the prey and provide the best solution to the problem by continuously updating their positions over the course of iterations. The rest of the population (ω wolves) update their position accordingly, which aids in providing the optimum approximation to the α. The main advantage of GWO over other metaheuristics is its ability to work efficiently with few parameters, less manual interference, and easy implementation. Fig. 2 illustrates the pseudocode for grey wolf optimiser.

2.3. Hybrid ANN-GWO proposed framework

In this section, the architecture of the ANN-GWO for predicting the settlement of reinforced foundations has been proposed and presented. The first step in the model development is the partitioning of data into training and testing subsets. Thereafter, AN is established, that is, initialisation of weights and biases, and selection of hidden layers and nodes through hit and trial procedure. Finally, the GWO algorithm is utilised to reduce the mean square error (MSE) between the observed and predicted values. This is achieved by optimising the connection...
weights and biases of each layer in the ANN model. The final model is selected based on the validation through several statistical indices, model generalisation and robustness, and independent test data prediction. The proposed framework for hybrid ANN-GWO is presented in Fig. 3.

3. Database and model parameters

3.1. Experimental data

Adams and Collins (1997) investigated the behaviour of GRSFs on granular soil by conducting the large-scale field model footing load tests. The tests were conducted in a pit located at Turner-Fairbank Highway Research Centre (T-FHRC), McLean, Virginia, US. The length, width, and depth of pit was 7 m, 5.4 m, and 6 m, respectively. It is noted that those experimental results were also used by Khosrojerdi et al. (2019) to validate their finite difference element framework. Thirty-four large scale load tests were performed on model (precast steel-reinforced concrete) square spread footings (0.30 m, 0.46 m, 0.61 m, and 0.91 m) resting on geosynthetic-reinforced soil. For load tests, entire plan area was covered with three layers of geogrids. The properties of the sand and the geogrid are summarised in Table 1. The load was applied by hydraulic jack and maintained manually with hand pump. The load cell (calibrated within 0.03% of its capacity) along with the strain indicator box was used to measure the load. The corresponding settlement values were measured with four linear variable displacement transducers (LDTVs). The LDTVs were attached to the data-acquisition system to automatically record the settlement values. Moreover, to minimise the interference effect, each foundation was tested separately.

3.2. Numerical modelling and data collection

A finite-element program, PLAXIS 3-D Tunnel, was utilised for the numerical simulations. The program is capable of modelling the behaviour of reinforced soil with built in code for geogrids as reinforcements. For further details regarding the capacities of this soft-

Fig. 3. Proposed scheme for developing hybrid ANN-GWO network for predicting the settlement of GRSF.
of the HSM is not fixed in principal stress space, and can expand with plastic strains (Schanz et al., 1999). The basic parameters of HSM model are unit weight of soil (γ), cohesion (c), friction angle (ϕ), dilatancy angle (ψ), Poisson’s ratio (ν), secant reference modulus at 50% of ultimate deviatoric stress (E<sub>ur</sub>30%), Young’s modulus for unloading and reloading at reference stress (E<sub>ref</sub>30%), and tangent oedometric modulus at reference stress (E<sub>s</sub>ref30%). The power coefficient (m) accounting for material stiffness dependency on stress level, failure ratio (R<sub>f</sub>), and reference stress for stiffness (∑<sub>ref</sub>).

All the values for the backfill soil used in the numerical simulations are summarised in Table 2. Soil and interfaces were modelled as twinned tetrahedral 3D finite elements. For the boundaries, fixed boundary conditions were inherently applied in the PLAXIS 3D program (PLAXIS 3D Tunnels scientific PLAXIS-3D Tunnel scientific manual, 2017). Additionally, a value of cohesion for sand was set to 1.0 kPa, and angle of dilation was estimated using the Bolton’s relationship, that is, ψ = ϕ − 30° (Bolton, 1986). The value of γ for granular soil is usually taken between 0.4 and 0.9 (Schanz and Vermeer, 1998). Benz (2007) reported the m values for various sands (with sub-angular to angular particles) in the range of 0.41–0.57. Similarly, the R<sub>f</sub> should be kept lower than 1. For this study, the value of m and R<sub>f</sub> were taken as 0.45 and 0.85, respectively. For each simulation run, based on soil’s friction angle, the values of E<sub>s</sub>ref and ϕ of soil were kept according to Khosrojerdi et al. (2019). It may be noted that these values were originally suggested by Obrzud and Truty (2010) for granular materials. The footing was modelled as an isotropic plate element with the typical value of unit weight of steel-reinforced concrete as 25 kN/m<sup>3</sup>. Geogrids were modelled as linear elastic material, and already available in the material set in PLAXIS 3D. It is noteworthy that the geogrids can only resist the axial tension and do not provide any resistance to the bending moment. After conducting a sensitivity analysis, a refined coarse sized mesh was selected for model simulations. In order to model the interaction between the soil and geosynthetic, the interface reduction factor (R<sub>int</sub>) is usually applied in the PLAXIS (Brinkgreve et al., 2011). However, based on the research, originally conducted by Jewell et al. (1984) on the soil-geogrid interaction; Holtz and Lee (2002) have demonstrated that if the median grain size (D<sub>50</sub>) is smaller than minimum aperture size of geogrid (D<sub>min</sub>) than full interlocking should be assumed. The value of D<sub>50</sub> is 0.25 mm which is less than d<sub>min</sub> (25 mm); therefore, for this study the value of R<sub>int</sub> was set to unity.

For the validation of the numerical model, the experimental results from Adams and Collins (1997) for the square footing (0.91 m × 0.91 m) were compared for unreinforced and reinforced conditions. The foundation was modelled as rigid foundation. For the reinforced models, the geosynthetic layers were laid down at approximately 0.17B. Baker et al. (1973) proved that if the non-dimensional stress-strain relationships (curves) of homologous materials are similar then the models are constitutively similar. Table 3 presents the numeric values of the settlement at various bearing pressures for both the studies. It can be seen that the results obtained by the numerical simulations were in good agreement with the results obtained by Adams and Collins (1997).
agreement with the experimental results. Therefore, it can be concurred that the developed numerical model is trustworthy in simulating and predicting the settlement response of GRSFs. After the model validation, a database consisting the results of 475 model footing load tests simulations was generated.

3.3. Feature reduction

For data-driven modelling, it is vital to have a good insight of the parameters affecting the response variable. In machine learning, feature reduction is a fundamental step to improve the quality of the performance and generalisation ability of the models (Gao et al., 2019; Mwangi et al., 2014). This has several advantages such as omission of redundant data, less time consumption during the training phase, easy interpretation of the model, and improvement in the prediction performance and generalisation ability of the models (Gao et al., 2019; Blum and Li, 1991). Before model construction, all the data have been normalised between –1 and 1. The dataset of 475 observations were randomly divided into training and testing dataset with a ratio of 75:25. It is noteworthy that the testing dataset was not utilised in model development. Moreover, the statistical properties of all the parameters are tabulated in Table 4.

3.4. Data description and preparation

For this study, 475 data points were collected through 3-D finite element modelling carried out by utilising PLAXIS 3D Tunnels. As a precautionary measure against scour, AASHTO (2012) recommended the minimum embedment depth of 0.6 m (1.96 ft) for shallow foundations. Therefore, all the simulations were performed by considering this recommendation. The similar approach was also utilised by Khosrojerdi et al. (2019). Before model construction, all the data have been normalised between –1 and 1. The dataset of 475 observations were randomly divided into training and testing dataset with a ratio of 75:25. It is noteworthy that the testing dataset was not utilised in model development. Moreover, the statistical properties of all the parameters are tabulated in Table 4.

3.5. Hybrid ANN-GWO model development and implementation

The ANN-GWO model for predicting the settlement of GRSF is developed in a MATLAB environment. It may be noted that, subjected to availability, any other computer program capable of coding the ANN can also be used for the model development. The procedure for simulating the settlement response is already illustrated in Fig. 3. The model includes 9 input variables and $s_p$ is the target variable.

As discussed previously, the first step in constructing the model is the normalisation of the dataset. In this study, the whole dataset is normalised between –1 and 1 by using min-max feature scaling method. The mathematical form is given below:

$$X_n = \frac{X - X_{\text{min}}}{X_{\text{max}} - X_{\text{min}}} - 1$$

where $X_n$, $X_{\text{min}}$, and $X_{\text{max}}$ is the normalised, minimum, and maximum value of the variables, respectively.

For ANN-GWO model development, a few parameters needed to be assigned manually to ANN and GWO. It requires the selection of the model network architecture which includes number of hidden layers, number of hidden nodes, activation function, and selection of optimisation algorithm. Levenberg-Marquardt BP optimisation algorithm is chosen based on its faster convergence rate and smaller mean square errors (MSE) (Ranganathan, 2004). Blum and Li (1991) have shown that a single hidden layer is sufficient for the approximation of any continuous function, given the enough weight connections; therefore, for this study, a single hidden layer with three hidden nodes is considered. Additionally, for easy interpretation and practicability, the developed model is also converted into a trackable mathematical relationship. For hidden layer, tangent-sigmoid activation [-1,1] function is used, and $s_p = f(q, \phi, c, J, z / B, N, B, L / B, d)$

It is noteworthy that these parameter(s) were also considered the most influential affecting the settlement of GRSF (Omar et al., 1993; Soleimanbeigi and Hataf, 2006; Khosrojerdi et al., 2019).

Table 4

| Statistical properties of the database utilised to develop ANN-GWO model. |
|---|---|---|---|---|---|---|---|---|---|
| Mean | 297.4 | 39.7 | 2.4 | 1237.7 | 0.23 | 3.7 | 1.5 | 2.4 | 1.4 | 2.9 |
| Standard Error | 8.9 | 0.2 | 0.1 | 24.2 | 0.004 | 0.1 | 0.0 | 0.1 | 0.0 | 0.2 |
| Median | 300.0 | 40.0 | 1.0 | 1000.0 | 0.20 | 3.0 | 1.0 | 2.0 | 1.2 | 1.9 |
| Mode | 50.0 | 40.0 | 1.0 | 1000.0 | 0.30 | 3.0 | 1.0 | 2.0 | 1.2 | 0.8 |
| Standard Deviation | 194.5 | 4.6 | 2.8 | 528.1 | 0.08 | 1.3 | 0.7 | 1.8 | 0.3 | 3.5 |
| Kurtosis | −1.3 | 0.2 | 1.7 | 1.2 | −0.90 | 0.0 | 0.0 | 5.9 | 1.7 | 20.4 |
| Skewness | 0.2 | 0.2 | 1.7 | 1.3 | 0.001 | 0.7 | 1.2 | 2.3 | 1.6 | 3.7 |
| Range | 550.0 | 20.0 | 10.0 | 2500.0 | 0.30 | 6.0 | 2.0 | 9.0 | 1.6 | 34 |
| Minimum | 50.0 | 30.0 | 0.0 | 500.0 | 0.10 | 1.0 | 1.0 | 1.0 | 0.8 | 0.01 |
| Maximum | 600.0 | 50.0 | 10.0 | 3000.0 | 0.40 | 7.0 | 3.0 | 10.0 | 2.4 | 34 |
| Count | 475 | 475 | 475 | 475 | 475 | 475 | 475 | 475 | 475 | 475 |
given as follows (Ghorbani et al., 2020):

$$
tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{2}{1 + e^{2x}} - 1 \tag{3}
$$

After training and selecting the optimum ANN network, the weights and biases are presented to the grey wolf optimiser in vector form as follows (Mirjalili, 2015):

$$\vec{V} = \{ \vec{W}, \vec{b} \} \tag{4}$$

where $\vec{W}$ is the synaptic weight vector, and $\vec{b}$ is the bias vector. The objective function of the GWO is defined in terms of MSE. The main aim of the algorithm is to reduce the difference between the observed and predicted values by optimising the weight and bias vector. As the problem of training the ANN applies to the whole dataset, therefore, average of MSE is taken as follows (Mirjalili, 2015):

\[
MSE = \frac{1}{n} \sum_{k=1}^{n} \left( s_k^o - s_k^p \right)^2
\]

where $n$ is the number of training data samples, $m$ is the number of outputs, $s_k^o$ is the observed value of the settlement for $k$th training sample, and $s_k^p$ is the predicted value of the settlement for the $k$th training sample. The parameters used for ANN-GWO model are summarised in Table 5. After conducting the optimisation process, the magnitudes of weights and biases of the best model were determined. Fig. 5 illustrates the best ANN-GWO model established for estimating the settlement of GRSF; with 9 input nodes, 3 hidden nodes and 1 output node (9-3-1).

4. Model performance and assessment

It is one of the most important steps is the development of an ANN-GWO model. The scholars have argued that the accuracy and reliability of the AI based models should not be assessed or judged solely based on the statistical criteria (Kingston et al., 2005; Shahin et al., 2009). A good and reliable model is one which provides a good fit to the calibration data and makes forecast that rationally considers the underlying physical behaviour of the investigated system. The third criterion for assessing the strength of the developed data-driven model is its ability to predict the response when presented with entirely independent test datasets. Given the reliability of the input parameters, a good model will predict the independent data realistically, and with reasonable accuracy. Therefore, in this study, the predictive veracity of the developed...
ANN-GWO model has been rigorously assessed based on all these criteria.

4.1. Model performance based on statistical indices

For “goodness of fit”, five statistical indices namely, root mean square error (RMSE), mean absolute error (MAE), mean arctangent absolute percentage error (MAAPE), coefficient of determination ($R^2$), and refined Willmott index (RWI) were used to evaluate the difference between the observed and predicted values. The best performance is marked by the high $R^2$ and RWI (close to unity), and low RMSE, MAE and MAAPE values. The mathematical relationships are given as follows:

\[ \text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (s_o - s_p)^2} \]  \hspace{1cm} (6)

\[ \text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |s_o - s_p| \]  \hspace{1cm} (7)

\[ \text{MAAPE} = \frac{1}{n} \sum_{i=1}^{n} \text{arctan} \left( \frac{s_o - s_p}{s_p} \right) \]  \hspace{1cm} (8)

\[ R^2 = 1 - \frac{\sum_{i=1}^{n} (s_o - s_p)^2}{\sum_{i=1}^{n} (s_o - s_t)^2} \]  \hspace{1cm} (9)

\[ \text{RWI} = 1 - \frac{\sum_{i=1}^{n} |s_o - s_p|}{2 \sum_{i=1}^{n} |s_o - s_t|} \]  \hspace{1cm} (10)

where $s_o$, $s_p$, $s_t$, and $n$ represents the $i$th observed value of settlement, $i$th predicted value of settlement, mean value of observed settlement, and number of data samples, respectively.

In spite of the fact that all these are widely recognised statistical tools to assess the accuracy of any ML model, yet, these indices are associated with certain limitations and biases. Gandomi et al. (2013) and Naser and Alavi (2020) showed that the AI models should be verified using the rigorous multi-criteria process which considers the traditional as well as modern performance fitness and error matrices, and do not give false hype or undermine the model accuracy solely based on a single parameter. Therefore, the relative percentage difference (RPD) indicator and the objective function (OBJ) were also calculated. The mathematical forms of RPD and OBJ are given as follows (Gandomi et al., 2013; Viszarra-Rossel et al., 2006).

\[ \text{RPD} = \frac{SD}{\text{RMSE}} \]  \hspace{1cm} (11)

\[ \text{OBJ} = \left( \frac{No_l - No_t}{No_l + No_t} \right) \times \frac{\text{RMSE} + \text{MAE}}{R_l + 1} + \left( \frac{2No_l}{No_l + No_t} \right) \times \frac{\text{RMSE} + \text{MAE}}{R_l + 1} \]  \hspace{1cm} (12)

where SD is the standard deviation of the measured observations, $No_l$ is the number of learning (training) data, $No_t$ is the number of testing data, $R_l$ is the coefficient of correlation for training data. Similarly, for other parameters the subscript $l$ and $t$ represents learning (training) and testing data points, respectively. The RPD value of greater than 2.5 represents the excellent model predictions and less than 1.4 represents poor or very poor predictions. The intermediary values of 1.4–1.8, 1.8–2.0, and 2.0–2.5 represents fair, good and very good prediction models, respectively (Viszarra-Rossel et al., 2006).

The results of all the statistical indices for training and testing dataset for simple ANN and hybrid ANN-GWO model are given in Table 6. It can be observed that both the models performed reasonably well in determining the settlement of reinforced soil foundation. However, for training dataset, the ANN-GWO have a superior predictive ability with RMSE, MAE, MAAPE, $R^2$, RWI, RPD value of (0.472, 0.833, 17.4, 0.982, 0.933, and 7.53) versus (0.722, 1.27, 25.8, 0.966, 0.890, and 4.93) for ANN model. Similarly, for testing dataset the proposed ANN-GWO yielded better performance (0.612, 0.363, 17.5, 0.962, 0.920, and 5.18).

Table 6: Performance of ANN and ANN-GWO models based on statistical indices.

| Statistical indices | Training data | Testing data | Training data | Testing data |
|--------------------|---------------|--------------|---------------|--------------|
| RMSE               | 0.722         | 0.774        | 0.472         | 0.612        |
| MAE                | 1.27          | 0.49         | 0.833         | 0.363        |
| MAAPE (%)          | 25.8          | 25.4         | 17.4          | 17.5         |
| $R^2$              | 0.966         | 0.943        | 0.982         | 0.962        |
| RWI                | 0.890         | 0.880        | 0.933         | 0.920        |
| RPD                | 4.93          | 4.09         | 7.53          | 5.18         |
| OBJ                | 0.849         | 0.574        |               |              |

Fig. 6. Scatter plots of ANN and ANN-GWO for: (a) training dataset; (b) testing dataset.
5.18) in comparison to ANN model with values of (0.774, 0.49, 25.4, 0.943, 0.880, and 4.09) for the statistical indices mentioned supra. This claim is further substantiated by estimating the OBJ function value which accounts for the combined performance of model in training and testing dataset. The lower value represents more accuracy and vice-versa. Although both the models have yielded good performance, yet ANN-GWO has shown less biasness with OBJ value of 0.574 in comparison to ANN model which has the value of 0.849 for same function.

The correlation between the observed and predicted values for training and testing dataset are shown in Fig. 6 a and b, respectively. The perfect prediction between the observed (abscissa) and simulated values (ordinate) are shown by 1:1 line. As shown in the figures, the ANN-GWO model predicted the output much closer to the observed values for training ($R^2 = 0.983$) and testing ($R^2 = 0.966$) in comparison to ANN’s training ($R^2 = 0.962$) and testing ($R^2 = 0.943$).

4.2. Model robustness and sensitivity

In this section, a sensitivity analysis was carried out to test the robustness of the developed ANN-GWO model. For this, the predicted settlement values of the GRSF were examined against the 9 input variables. Shahin et al. (2009) suggested a procedure for conducting the sensitivity analysis to compute the robustness of the predictive ANN model. In this method, the model was fed with the input values within the range of original data, and the corresponding settlement values were determined. This is also known as one factor at a time (OAT) analysis (Saltelli and Annoni, 2010). Each input variable, one at a time, was increased in the form of incremental steps and rest were kept at their mean value. For this study, twenty incremental steps were chosen for each variable. The robustness of the developed ANN-GWO model is investigated by comparing the general trend predicted by the model with the underlying physical behaviour of settlement prediction of GRSF based on known knowledge pertaining to the field of geotechnical engineering. The model can only be considered suitable if the predictions made by it are realistic, that is, make some sense over the given range of data (Table 3). Fig. 7 represents the results of the sensitivity analysis. Fig. 7a, b, 7c, and 7d, represent the predicted settlement trend with applied load ($q$), soil properties ($c$, $\phi$, and $d$), geosynthetic characteristics ($J$, $z/B$, and $N$), and footing attributes ($B$ and $L/B$), respectively. Fig. 7a shows an increase in the settlement of footing with the applied load...
which is expected. For soil strength properties, the predicted settlement of footing decreases with the increase in the soil strength parameters (c and $\phi$) (Fig. 7b). However, the decreasing trend is significantly more pronounced for the friction angle than the cohesion value. This variation may be explained so that, when the friction angle of the soil increases the soil stiffness also increases (Wu, 2006; Ohrudz and Truty, 2010). Thus, leading to the reduction in the settlement of the footing. Additionally, as expected, it can be observed that the settlement increases with the increase in the compacted depth. The variation of settlement trend with reinforcement characteristics is represented in Fig. 7c. As anticipated, the settlement value increases with the increase in the vertical spacing ratio and decreases with the increase in the tensile modulus of the reinforcement and number of reinforcement layers. Similar observations were made by several researchers in their experimental investigation of GRSFs (e.g., Abu-Farsakh et al., 2013; Chen, 2007; Omar et al., 1993; Yetimoglu et al., 1994). For footing parameters, as one would expect, the settlement increases with the increase in the footing width and footing geometry (aspect ratio) (Fig. 7d). Similar results were also reported by Saha Roy and Deb (2019) when investigating the effect of aspect ratio of footings on the settlement response of geosynthetic-reinforced soil system. This indicates that the developed ANN-GWO is robust and has shown good generalisation ability over the range of training data.

For more detailed insight and understanding of the predictor attributes that influence the predictand (i.e., settlement of GRSF), the sensitivity index (SI) at each incremental step was also calculated. Mathematically, it can be defined as follows Khosrojerdi et al. (2019).

$$SI = \frac{s_{p(t+1)}(v) - s_{p(t)}(v)}{s_{p(t)}(v)} \times \frac{x_{(t)}(v)}{x_{(t+1)}(v) - x_{(t)}(v)}$$ (13)

where for each variable $v$, $s_{p(t+1)}$ is the predicted settlement at step $t+1$, $s_{p(t)}$ is the predicted settlement at step $t$, $x_{(t)}$ is the value of variable at step $t+1$, and $x_{(t)}$ is the value of variable at step $t$. The spider web chart (radar plot) in Fig. 8 represents the average value of the SI (after 20 incremental steps) for each input variable. Regarding the figure, the magnitude of the SI for an input variable represents the intensity of the variable affecting the settlement of GRSFs. The zero-mark line in the web represents the reference value. If a value of SI falls beyond this line it represents the increase in the settlement value with the increase of the corresponding parameter and vice versa. Additionally, the relative importance of each parameter was also calculated according to the Garson’s algorithm (Garson, 1991), and presented in Fig. 9. The Garson’s algorithm for sensitivity analysis is given in appendix A1. From the results of both methods, it can be observed that the applied load and friction angle has the largest effect on the settlement of GRSF with average SI (0.99 and 4.77), and relative importance (23.72% and 41.37%). In comparison to $q$ and $\phi$, the rest of the parameters have moderate to low effect on the settlement of reinforced soil foundations.

### 4.3. Independent validation of ANN-GWO

The third important criterion for evaluating the predictive strength of any data-driven model is its ability to predict the response for entirely independent data. It is noteworthy that the accuracy and reliability of the independent experimental data is an important factor. Considering the limitations associated with small scale experimental model footing load tests regarding to the scale effect, large scale studies were used for model verification. The studies conducted by Adams and Collin (1997), Chen (2007), and Chen and Abu-Farsakh (2011) were considered. The model input parameters are given in Table 7. The comparison between the observed and predicted values for all the studies was given in Fig. 10 (a–c). It can be observed that the developed model has made fair predictions for all the independent studies with absolute average error (AAE0) less than 25% for all the studies. The other statistical parameters such as $R^2$, MAAPE (%), RPD for Adams and Collin (1997), Chen (2007), and Chen and Abu-Farsakh (2011) were (0.999, 11%, and 9.91), (0.989, 12.1%, and 4.9), and (0.962, 23.7%, and 2.7), respectively, confirming the reliability of the model.

From the above results and discussion, it can be concluded that the developed ANN-GWO model forecast the settlement of GRSF in accurate, realistic, and intelligent way. However, the model will remain as
“blackbox” and require a powerful computer program to compute the values internally. Therefore, in the next section the developed model was converted into mathematical/functional relationship.

4.4. ANN-GWO model formulation

For more comprehension and easy calculations, the developed optimal ANN-GWO model was translated into traceable equation for hand or spreadsheet calculations. The equation for ANN-based model is given as (Aamir et al., 2020; Raja et al., 2021):

\[ y = f_{ho} \left( b_o + \sum_{j=1}^{k} w_{jo}f_{ih} \left( b_{hj} + \sum_{i=1}^{m} w_{ij}x_i \right) \right) \]  

(14)

where \( f_{ho} \) is the applied transfer between hidden-output layer, \( b_o \) is the bias at output layer node, \( w_{jo} \) is the synaptic weight between node \( j \) of hidden layer and single output node, \( f_{ih} \) is the applied transfer function between input-hidden layer, \( b_{hj} \) is the bias value for node \( j \) of hidden layer (\( j = 1,h \)), \( w_{ij} \) is the synaptic weight between input \( i \) and node \( j \) of hidden layer, and \( x_i \) is the \( i \)th input node (variable). The weights and biases of the network are summarised in Table 8.

In order to predict the settlement (\( s_p \)) of reinforced soil foundations

![Fig. 10. Comparison of settlement values predicted by the developed ANN-GWO model with the results reported by: (a) Adams and Collin (1997); (b) Chen (2007); and (c) Chen and Abu-Farsakh (2011).](image)

Table 8
Weight and biases of developed ANN-GWO model.

| Weights of input layer - hidden layer, \( w_{ij} \) | Hidden layer bias \( b_{hj} \) |
|-----------------|------------------|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 0.6601 | -1.1885 | 0.0092 | 0.0362 | 0.1919 | 0.4235 | 0.2508 | 0.5716 | 0.4054 | 0.1006 | 2.7945 |
| 0.6931 | 0.8088 | 0.0578 | -0.0265 | -0.1293 | -0.1394 | -0.1195 | 0.0558 | -0.1006 | 0.1310 | 0.1659 |
| 0.4793 | -1.2982 | -0.0737 | -0.0915 | 0.0048 | 0.0011 | 0.1434 | 0.1310 | 0.1659 | -0.2984 | -0.6585 |

| Weights of hidden-output layer, \( w_{jo} \) | Output layer bias \( b_o \) |
|-----------------|------------------|
| 0.0788 | 3.0644 | 3.3078 |
with nine inputs \((q, \phi, c, J, z/B, B, L/B, N, \text{and } d)\), the optimal ANN-GWO model can be expressed as follows:

\[ (s_p)_{\text{ANN-GWO}} = 17(s_p) + 17.01 \]  

(15)

where \(s_p\) is the normalised settlement value and can be estimated as follows:

\[ s_p = \sum_{j=1}^{3} w_{jo} \tanh(\lambda_j) + b_o \]  

(16)

\[ \lambda_j = w_{j1}q + w_{j2}q_2 + w_{j3}c + w_{j4}J + w_{j5}(z/B)_a + w_{j6}B_a + w_{j7}(L/B)_a + w_{j8}N_a + w_{j9}d_a + b_j \]  

(17)

where the subscript \(n\) denotes the normalised values of the corresponding input parameters. The design example is presented in Appendix A2. It may be noted that the AI (ANNs) network predictions are limited to the range of data utilised (see Table 4) to construct the model. Therefore, cautions must be practiced when using this model to predict the settlement of GRSF where the design parameters are not within the range of utilised database. Moreover, only planar geosynthetic-reinforced soil foundations are considered, thus, this expression shall not be used for the settlement estimation of unreinforced soil or for other types of foundations, such as flexible foundation, foundation surrounded by soft soils, or foundations reinforced with other forms of reinforcements, for example, galvanised steel bars, vertical reinforcements, metals, geocells, grid-anchors, geofoams etc.

5. Conclusions

The settlement estimation of geosynthetic-reinforced soil foundation under service load is of paramount importance for practicing geotechnical engineers. In this paper, a novel hybrid AI method, that is, ANN-GWO has been proposed and implemented for predicting the settlement of GRSF. The pertinent database was generated through validated large-scale 3D finite element modelling. The results of this study indicate that the proposed model predict the settlement of GRSF with high accuracy for training (RMSE = 0.472, MAE = 0.833, MAAPE = 17.4%, \(R^2 = 0.982\), RWI = 0.933, RDP = 7.53), and testing (RMSE = 0.612, MAE = 0.363, MAAPE = 17.5%, \(R^2 = 0.962\), RWI = 0.920, RDP = 5.18) dataset. The sensitivity, generalisation capability, and robustness of the developed model are corroborated with the underlying physical behaviour of the settlement prediction of GRSF based on known geotechnical knowledge. Moreover, the predictive veracity of ANN-GWO model has also been substantiated with several independent scientific studies as reported in the literature. The comparison of the forecasted and measured foundation settlement shows that the developed model predicts the settlement values rationally and with reasonable accuracy. More importantly, the developed ANN-GWO model is converted into a mathematical formulation for manual calculations of the maximum settlement values under service loads, and can be useful in the preliminary design stages of GRSF. It may be noted that the prognostic strength of the AI-based models is commonly limited to the range of database and attributes utilised to train the model. The future studies may focus on incorporating more data to increase its predictive capability. Moreover, the development of ensemble learning methods in which the predictive strength of various AI-based models are combined may also be implemented in the future.

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Appendix A1. Garson’ Algorithm for sensitivity analysis

Garson (1991) proposed a sensitivity analysis for calculating the variable importance as follows:

1. Calculate \(P_j\) by multiplying the absolute values of hidden-output weight with the absolute value of input-hidden weight of each input variable \(j\), that is, \(|w_{ji}| \times |w_{ij}|\), e.g. From Table 6 \((P_{11} = 0.661 \times 0.0798 = 0.0520)\)
2. For each hidden neuron, divide \(P_j\) by the sum of all the input variables to obtain \(Q_j\):

\[ Q_j = \frac{P_j}{\sum_{j=1}^{n} P_j} \]  

(18)

e.g., \((Q_{11} = 0.05/(0.052 + 0.0937 + 0.0007 + 0.0029 + 0.0151 + 0.0334 + 0.0198 + 0.045 + 0.0320) = 0.1766)\).
3. For each input neuron, obtain \(S_j\) as the sum of \(Q_j\):

\[ S_j = \sum_{j=1}^{n} Q_j \]  

(19)

e.g., \((S_{11} = 0.1766 + 0.3253 + 0.2006 = 0.7025)\).
4. Calculate the percentage relative importance (RI) of each variable as follows:

\[ RI = 100 \times \left( \frac{S_j}{\sum_{j=1}^{n} S_j} \right) \]  

(20)

e.g., \((RI = 100 \times (0.7025/0.7025 + 1.2411 + 0.0604 + 0.0604 + 0.1141 + 0.1792 + 0.1832 + 0.2339 + 0.2252))) = 23.42\%\)

Complete calculations for variable importance are given below:
Appendix A2. Example of settlement estimation through developed ANN-GWO formulation

Estimate the settlement of a GRSF with the following characteristics:

- Width of footing (B) = 1 m;
- Shape of footing (L/B) = 1;
- Angle of internal friction of soil (ϕ) = 36°;
- Cohesion of soil (c) = 1 kPa;
- Uniform reinforcement spacing ratio (z/B) = 0.25;
- Initial tensile modulus of geogrid reinforcement (J) = 1000 kN/m;
- Number of geogrid layers (N) = 3;
- Compacted depth of soil (d) = 1.5 m;
- Net applied load (q) = 150 kPa.

Solution:

1. Define input parameters:
   - q
   - ϕ
   - J
   - z/B
   - B
   - L/B
   - N
   - d

2. Using Eq. (2), normalise the input parameters:
   
   \[ x = \left\{ q \cdot \phi \cdot J \cdot z/B \cdot B \cdot L/B \cdot N \cdot d \right\} \]

3. Using Eq. (17),
   \[ \lambda_1 = 0.134; \quad \lambda_2 = 2.26; \quad \text{and} \quad \lambda_3 = -2.35. \]

4. Substituting the values into Eq. (16),
   \[ s_p = -0.89789. \]

5. De-normalising using Eq. (15),
   \[ s_p = 1.75 \text{mm}. \]

Similarly, the settlement at 250 and 350 kPa are 2.9 mm and 4.0 mm respectively.

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