Deep meridional circulation below the solar convective envelope

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ABSTRACT

With reasonable assumptions and approximations, we compute the velocity of the meridional flow \( U \) in the convective envelope by modified Chandrasekhar’s (1956) MHD equations. The analytical solution of such a modified equation is found to be

\[
U(x, \mu) = \sum_{n=0}^{\infty} [u_{1n} x^n + u_{2n} x^{-(n+3)}] C_{n}^{3/2}(\mu),
\]

where \( x \) is non-dimensional radius, \( \mu = \cos \vartheta \), \( \vartheta \) is the co-latitude, \( C_{n}^{3/2}(\mu) \) are the Gegenbauer polynomials of order 3/2, and \( u_{1n} \) and \( u_{2n} \) are the unknown constants. By taking a clue from the helioseismic inferences that meridional velocity increases from the surface towards base of the convective envelope, we neglect first part in the series solution and consider the second part \( u_{2n} \) only. Hence the required solution of the meridional velocity in the convective envelope is given as

\[
U(x, \mu) = [(u_{2_1} x^{-4} C_{1}^{3/2}(\mu)) + u_{2_3} x^{-6} C_{3}^{3/2}(\mu)].
\]

In order to solve two unknown constants \( u_{2_1} \) and \( u_{2_3} \) uniquely, we match the observed surface meridional velocity at the two latitudes 5° and 60° with the meridional velocity obtained by the analytical solution.

The results show that meridional velocity flow from the surface appears to penetrate deep below base of the convective envelope and at outer part of the radiative zone. With such a deep flow velocity below the convective envelope and a very high density stratification in the outer part of the radiative zone with likely existence of a strong (\( \sim 10^4 \) G) toroidal magnetic field structure, the velocity of transport of meridional flow is considerably reduced. Hence, it is very unlikely that the return flow will reach the surface (with a period of solar cycle) as required by some of the flux transport dynamo models. On the other hand, deep meridional flow is required for burning of Lithium at outer part of the radiative zone supporting the observed Lithium deficiency at the surface.

1. Introduction

It is believed that meridional velocity flow may be transferring angular momentum and maintaining solar differential rotation in the convective envelope. In the early history of the
stars, especially the sun, meridional flow might have been played a major role in bringing the lithium from the surface towards the outer part of the radiative core where it is easily burnt and supplemented the existing radiative energy flow. This may be one of the reason for the observed lithium depletion over the surface of the sun.

In case of the sun, the analysis from the tracers such as sunspots (Javaraiah 1999 and references there in; Wohl 2002), magnetic field patterns (Snodgrass and Dailey 1996; Meunier 2005 and references there in), inferences from the Doppler measurements (Hathaway et. al. 1996; Nesme-Ribes et. al. 1997) and, the inferences from the local helioseismology techniques (Giles et. al. 1997; Chou and Dai 2001; Gizon, Duvall and Larsen 2001; Beck, Gizon and Duvall 2002; Basu and Antia 2002; Basu and Antia 2003; Haber et. al. 2004; Gizon 2004; Zhao and Kosovichev 2004; Antia and Basu 2007; Gizon and Thompson 2007; Kriger, Roth and Luhe 2007; Shibahashi 2007) show that there exists a meridional velocity flow from the equator towards the pole. On the surface the flow velocity increases from \( \sim 1-2 \) m/sec near the equator to \( \sim 20-50 \) m/sec near the higher latitudes. Some of the inferences (Basu, Antia and Tripathy 1999; Antia and Basu 2007; Gonzalez et. al. 2006) from the local helioseismology show that meridional flow increases from surface towards base of the convective envelope. Unfortunately the helioseismic inferences of meridional flow yield the accurate results only few mega meters just below the surface.

The genesis of meridional circulation in a star was first proposed by Bierman (1958) who discussed extensively in the IAU symposium that pure rotation without meridional circulation is not possible stationary states of motion for the convection zones of stars. Recent studies (Rudiger 1989; Kitchatinov and Rudiger 2005; Tassoul 2000; Rempel 2005) emphasize the turbulent Reynold stresses for maintaining the differential rotation and hence existence of the meridional flow.

In order to reproduce proper solar butterfly diagrams and predict future solar cycles (Dikpati and Gilman 2007; Choudhuri, Chatterjee and Jiang 2007), the flux transport dynamo models require the meridional circulation that needs to penetrate (Nandy Choudhuri 2002) below base of the convective envelope. There are supporting (Rudiger, Kitchatinov and Arlt 2005) and nonsupporting (Gilamn and Miesch 2004) models for the deep penetration of the meridional velocity flow below base of the convective envelope. Very recently Svanda, Zhao and Kosovichev (2007) show that the mean longitudinally averaged meridional flow measurements by helioseismology may not be used directly in solar dynamo models for describing the magnetic flux transport, and that it is necessary to take into account the longitudinal structure of these flows.

Aims of the present study are two fold: (i) with reasonable assumptions and approximations, solve modified Chandrasekhar’s MHD equation for meridional part of the velocity
flow in the convective envelope and, (ii) examine whether meridional velocity flow penetrate deep below base of the convective envelope for the lithium burning and also as required by the flux transport dynamo models. Thus this study also supplements the information of the meridional flow velocity where local helioseismology can not infer reliably so deep below the surface.

2. Reasonable assumptions and approximations

As in our previous work (Hiremath & Gokhale 1995; Hiremath 1994), we assume that, in the convective envelope, the fluid is incompressible and the large-scale magnetic fields and the fluid motions are symmetric about the rotation axis. We also assume that the magnetic eddy diffusivity \( \eta \) and the eddy diffusivity due to viscosity \( \nu \) are constants with values represented by the appropriate averages.

Following Chandrasekhar (1956), the magnetic field \( \mathbf{B} \) and the velocity \( \mathbf{V} \) for the axisymmetric system can be expressed

\[
\mathbf{h} = -\varpi \frac{\partial P}{\partial z} \hat{\mathbf{l}}_z + (\varpi T) \hat{\mathbf{l}}_\varphi + \frac{1}{\varpi} \frac{\partial}{\partial \varpi} (\varpi^2 P) \hat{\mathbf{l}}_z, \tag{1}
\]

\[
\mathbf{V} = -\varpi \frac{\partial U}{\partial z} \hat{\mathbf{l}}_z + (\varpi \Omega) \hat{\mathbf{l}}_\varphi + \frac{1}{\varpi} \frac{\partial}{\partial \varpi} (\varpi^2 U) \hat{\mathbf{l}}_z, \tag{2}
\]

where \( \mathbf{h} = \mathbf{B}/(4\pi \rho)^{1/2}, \rho \) is the density, \( \varpi, \varphi, z \) are the cylindrical polar coordinates, with their axes along the axis of solar rotation; \( \hat{\mathbf{l}}_\varpi, \hat{\mathbf{l}}_\varphi, \) and \( \hat{\mathbf{l}}_z \) are the corresponding unit vectors and; \( P, T, \Omega, \) and \( U \) are the scalar functions that are independent of \( \varphi \).

Further we make the following assumptions and approximations.

Steady parts of the poloidal magnetic field \( P \) and poloidal component of the velocity field \( U \) (meridional velocity) are very weak compared to the steady part of the rotation \( \Omega \). In fact such a steady part of poloidal magnetic field is found to be \( \sim 1 \text{ G} \) from the observation (Stenflo 1993) and \( \sim 0.01 \text{ G} \) from theoretical calculations (Hiremath and Gokhale 1995). Thus, by taking average density of the sun, Alfvén velocity varies between \( \sim 1 - 0.01 \text{ cm sec}^{-1} \) which is very negligible compared to the dominant part of rotational velocity \( \sim 10^5 \text{ cm sec}^{-1} \). This leads us to safely assume that \( P \) is approximately zero and for the sake of making this investigation simple we put \( P = 0 \) in the following Chandrasekhar’s MHD equations. Similarly poloidal part of the velocity (meridional circulation) over the surface is found to be \( \sim 0.001 - 0.01 \) times the rotation velocity. Though we can not neglect the meridional velocity in MHD equations, it can not be equated with the dominant part of the angular velocity \( \Omega \). We also assume that strength of steady part of toroidal field \( T \) is less than (or at most comparable to) that of the steady part of rotation.
3. Modified form of Chandrasekhar’s equations

The afore mentioned assumptions lead to decoupling of poloidal part of velocity equation and, thus we have the following modified Chandrasekhar’s (1956) MHD equations that take into account the eddy viscosity (Nakagawa and Swarztrauber 1969) also

\[ \nu \omega \Delta_5 (\Delta_5 U) + [\omega^2 U, \Delta_5 U] - \omega \frac{\partial \Delta_5 U}{\partial t} = 0, \]

\[ \eta \omega \Delta_5 T + [\omega^2 U, T] - \omega \frac{\partial T}{\partial t} = 0, \]

\[ \omega^3 \nu \Delta_5 \Omega + [\omega^2 U, \omega^2 \Omega] - \omega^3 \frac{\partial \Omega}{\partial t} = 0, \]

\[ \omega \frac{\partial}{\partial z} (T^2 - \Omega^2) = 0, \]

where

\[ [f, g] = \frac{\partial f}{\partial z} \frac{\partial g}{\partial \omega} - \frac{\partial f}{\partial \omega} \frac{\partial g}{\partial z}, \]

and

\[ \Delta_5 = \frac{\partial^2}{\partial z^2} + \frac{3}{\omega} \frac{\partial}{\partial \omega} + \frac{\partial^2}{\partial \omega^2}. \]

In the previous study (Hiremath 2001), we used equations 4-6 to obtain the solution for the toroidal parts of magnetic field and velocity field structures in the convective envelope.

4. Solution and Results

As the equation (3) for the meridional flow velocity \( U \) is decoupled from rest of the equations, there are two unique solutions of this equation: either \( U = 0 \) a trivial solution and, \( \Delta_5 U = 0 \) a non trivial solution.

Nontrivial solution of the equation (3) is \( U(x, \mu) = \sum_{n=0}^{\infty} [u_{1n} x^n + u_{2n} x^{-(n+3)}] C_n^{3/2}(\mu) \), where \( x \) is non-dimensional radius, \( \mu = \cos \theta \), \( \theta \) is the co-latitude, \( C_n^{3/2}(\mu) \) are the Gegenbaur polynomials of order 3/2, \( u_{1n} \) and \( u_{2n} \) are the unknown constants to be determined from the boundary conditions. By taking a clue from the helioseismic inferences (Basu, Antia and Tripathy 1999; Antia and Basu 2007; Gonzalez et. al. 2006) that meridional velocity increases from the surface towards base of the convective envelope, we neglect first part in
the series solution and consider the second part $u_{2n}$ only. For the sake of understanding and simplicity of the problem we consider antisymmetric components $u_2$ and $u_3$ modes only. Hence the required solution of the meridional velocity in the convective envelope is given as $U(x, \mu) = [(u_2 x^{-4} C_1^{3/2}(\mu) + u_3 x^{-6} C_3^{3/2}(\mu)]$. In order to solve two unknown constants $u_2$ and $u_3$ uniquely, we match the observed surface meridional velocity at the two latitudes $5^\circ$ and $60^\circ$ with the meridional velocity obtained by the analytical solution. Finally we get the meridional velocity $U$ from the determined two unknown constants and using above equation.

In Fig 1., we present the iso-meridional velocity flow in the one quadrant of the convective envelope. The results show that: (i) unlike the close isomeridional contours that are required by the flux transport dynamo models, the present solution yields the isomeridional contours that are not closed in the convective envelope and appear to penetrate deep in the outer part of the radiative zone, (ii) on the surface, magnitude of meridional velocity is $\sim 5$ m/sec near the equator and reaches $\sim 25$ m/sec around $45^\circ$ latitude and, (iii) near base of the convective envelope, magnitude of meridional velocity is $\sim 5$ m/sec near the equator and reaches maximum of $\sim 220$ m/sec around $45^\circ$ latitude. The first result that meridional flow penetrates deeply in the outer part of the radiative zone is also consistent with the result obtained by the recent study (Garaud and Brummel 2007).

Let us consider the isomeridional contour that is close to the surface ($\sim 25$ m/sec)
and appears to penetrates deep in the outer part of the radiative zone. Owing to very high density stratification in the outer part of the radiative zone and likely existence (Friedland and Gruzinov 2004i; Rasba et. al. 2007) of strong (∼ 10⁴ G) toroidal magnetic field structure, the velocity of transport of meridional flow is considerably reduced (Rempel 2006). The law of conservation of mass yields the relation \( U_2 = \left( \frac{\rho_1}{\rho_2} \right) U_1 \), where \( U_1 \) and \( \rho_1 \) are meridional flow velocity and density stratification near the surface and, \( U_2 \) and \( \rho_2 \) are meridional flow velocity and density stratification in the outer (\( r/R_\odot=0.5 \)) part of the radiative zone. If we substitute \( U_1 = 25 \) m/sec and the respective density values in this relation, meridional flow velocity in the outer part of the radiative zone is found to be ∼ 1 cm/sec. If we assume that isomeridional velocity flows are circular loops, one can notice from Fig 1 that half of the circular loop is in the convective envelope and rest half lies in the outer part of the radiative core. Thus travel time taken by the longest meridional circular loop that lies in the convective loop is found to be ∼ 3 years and the travel time taken by the longest meridional circular loop that lies in the outer part of the radiative region is found to be 3000 years. That means the meridional flow velocity that starts near the surface and after penetrating deep near base of the convective envelope return back to the surface after 3000 years. This return time scale is nearly 100 times the return time scale as required by the flux transport dynamo models. Hence it is very unlikely that the solar meridional circulation plays any major role in reproducing the proper butterfly diagrams and dictating the next solar cycle.

However, present study supports the idea that the observed surface Lithium depletion can be explained if deep meridional circulation carries the Lithium and penetrates deep in the outer part of the radiative core resulting in burning into helium.

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