We describe a procedure for accurately determining luminosity distances to Type Ia supernovae (SNe Ia) without knowledge of redshift. This procedure, which may be used as an extension of any of the various distance determination methods currently in use, is based on marginalizing over redshift, removing the requirement of knowing $z$ a priori. We demonstrate that the Hubble diagram scatter of distances measured with this technique is approximately equal to that of distances derived from conventional redshift-specific methods for a set of 60 nearby SNe Ia. This indicates that accurate distances for cosmological SNe Ia may be determined without the requirement of spectroscopic redshifts, which are typically the limiting factor for the number of SNe that modern surveys can collect. Removing this limitation would greatly increase the number of SNe for which current and future SN surveys will be able to accurately measure distance. The method also may be able to be used for high-$z$ SNe Ia to determine cosmological density parameters without redshift information.

Subject headings: distance scale — methods: data analysis — supernovae: general

1. INTRODUCTION

The use of Type Ia supernovae (SNe Ia) as a tool for cosmological studies became feasible with the realization that there is a relationship between luminosity and light-curve shape (Phillips 1993). With this knowledge and the advent of wide-field CCD arrays that allow for the efficient discovery of large numbers of SNe Ia, cosmological investigations into the fundamental composition of the universe using SNe Ia as probes have revealed the acceleration of the expansion of the universe (Riess et al. 1998; Perlmutter et al. 1999).

In order to derive distance from observed photometry, numerous factors that alter the light curve must be taken into account. Any distance measurement technique must calculate the probability of a fit for an SN light curve in the parameter space of $(t_0, A_v, R_{de}, z, d)$, where $t_0 = $ time of maximum, $A_v = $ extinction, $R_{de} = $ decline rate (using any of the several current parameterizations of SNe Ia light-curve shapes; see below), $z = $ redshift, and $d = $ distance. We then either marginalize over these factors or take cuts through parameter space at specific values in order to obtain a measurement for $d$, the primary quantity of interest.

The initial discovery of the relationship between light-curve shape and brightness led to parameterization by the rate of decline in $B$-band brightness over the 15 days after maximum light ($\Delta m_{15}$). A second method, the multicolor light-curve shape (MLCS) method (see Riess et al. 1996, 1998) is based on $\Delta$, the difference in peak brightness between an observed SN and a fiducial light-curve template. The “stretch” method (Perlmutter et al. 1997) parameterizes the light curve by a factor $s$ that broadens or narrows a template light curve in order to modify the light-curve shape. The Bayesian Adapted Template Match (BATM) method introduced by Tonry et al. (2003) measures distances through comparison with a large set of well-observed nearby SNe rather than a parameterized template. These methods all seem to be fundamentally equivalent for the purposes of measuring accurate distances and resolving uncertainties between the effects mentioned above. Another recently introduced distance measurement technique is CMAGIC (Wang et al. 2003), which uses the relationship between light-curve shape and brightness in a more indirect way than the above methods. It utilizes an observed linear relationship in color-magnitude space for extended periods of time after maximum light to estimate distance. As noted by Wang et al. (2003), CMAGIC remains under development, and a more thorough demonstration of its utility as a distance estimator is necessary.

Redshift is an especially problematic parameter. It broadens the light curve by $(1+z)$, which affects what we can deduce about luminosity and also modifies how the spectral energy distribution (SED) maps into observed bandpasses, thereby changing the color of the SN Ia. It thus exhibits considerable covariance with other fit parameters and needs to be known accurately. Redshift is normally measured spectroscopically, which confers the added benefit of confirming that an object is indeed an SN Ia. However, for large surveys (see Barris et al. 2004), far more SNe are discovered than can possibly be observed spectroscopically. The situation will get worse with larger surveys such as ESSENCE (Smith et al. 2002), the Canada-France-Hawai`i Legacy Survey, and Pan-STARRS (Kaiser et al. 2002).

There have been several attempts to determine redshift and type for SNe through photometric means. Barris et al. (2002) concluded that over a wide range of redshift, SNe Ia lie in a narrow region of color-magnitude space, indicating that it is likely that tests discriminating between SNe Ia and II may be reliable. Riess et al. (2004) constructed cuts based on SEDs of SNe to optimize the selection of high-$z$ candidates for spectroscopy, distinguishing between the various SN types at high redshifts based on the UV deficit of SNe Ia relative to core-collapse SNe. In combination with the photometric colors of the host galaxy, they were also able to make a rough estimate of the redshift of the SNe. Barris et al. (2004) used a similar, although simplified, comparison to argue for the identification of several SNe as Type Ia, despite lacking spectral confirmation.

Currently it is not apparent that photometric observations are sufficient to accurately determine the redshift and/or distances of SNe Ia. Comparisons with SEDs of SNe Ia may allow constraints on $(1+z)$ on the order of $\approx 0.1$, but this is a poor constraint on $z$ for $z < 0.5$. Without knowledge of redshift, the standard methods for determining distance cannot be mean-
implemented with a uniform step in \( \Delta z \), weighting by \( \Delta z \). A \( z \)-step in redshift, weighting each by \\
scale by volume, which can be accomplished using a uniform \\
\( z \)-step they may be meaningfully combined. A suitable prior for \\
After marginalizing out parameters \( \theta, \phi, \rho \), one is left with \\
of \( \theta, \phi, \rho \), \( z \) or marginalized over, in order to obtain a measurement of \\
distance is possible by considering magnitude. The redshift-

2. MARGINALIZATION OVER REDSHIFT

As described above, all distance measurement techniques for 
SNe Ia calculate the probability of a fit for an SN as a function of 
\( (t_p, A_v, R_p, z, d) \). The parameters are either fitted as a group 
or marginalized over, in order to obtain a measurement of \( d \). After 
marginalizing out parameters \( (t_p, A_v, R_p) \), one is left with 
probability as a function of \( (z, d) \). For some questions knowl-
ege of \( (z, d) \) probability contours is very interesting. The most 
immediately obvious is for probing cosmological density pa-
rameters, which led to the accelerating universe result. For all 
conventional distance determination methods, \( z \) is required to 
be an input parameter for the reasons described above, pro-
viding a final estimate of \( d \) by evaluating at the known redshift.

One could alternatively treat \( z \) as a free parameter to be mar-
ginalized out. For many questions there is no added benefit to 
knowing \( z \) in addition to distance. For example, inquiries into rates 
as a function of distance, galaxy type, location within the host, or 
host galaxy properties can be answered based solely on \( d \) rather 
than \( z \). Even with a poor constraint on \( z \), in principle we may still 
be able to measure accurate distances since the effect of redshift 
on distance comes from dependence on \( (1 + z) \).

Implementation of the redshift marginalization involves se-
veral minor modifications to any fitting process. The most im-
portant is that care must be taken to place calculated fit prob-
abilities for each redshift on an equal footing, so that at each 
step they may be meaningfully combined. A suitable prior for 
\( z \) must also be applied. The most obvious to consider is to 
scale by volume, which can be accomplished using a uniform 
redshift step, weighting each by \( z \). The same prior could be 
implemented with a uniform step in \( \log z \), weighting by \( z^2 \). A 
different prior to consider is one constant in \( z \), rather than 
volume, which could be obtained by a linear step in redshift 
without scaling by \( z \). One then iterates over the range of \( z \) 
contributing nonnegligible values of probability, combining the 
results to produce a final estimate of \( d \).

It is important to note that this redshift marginalization 
method is not simply a restatement of the photometric redshift 
procedure typically used for galaxies. Photometric redshifts use 
the observed colors of galaxies in order to produce a likelihood 
range for the redshift based on template SEDs. Since galaxies 
are not standard candles, only a very weak constraint on 
distance is possible by considering magnitude. The redshift-
independent implementation of an SN Ia distance estimator uses 
observed light curves in order to produce a constraint on both 
distance and redshift by considering the magnitude and light-
curve shape as well as the color. This additional information 
greatly strengthens the power of the method in comparison to 
galaxy photometric redshifts.

It is also important to stress that this redshift marginalization 
procedure may be implemented with any method for measuring 
distances to SNe Ia (stretch, \( \Delta m_{15} \), MLCS, BATM, or any other 
equivalent method). Rather than evaluating \( d \) for the measured 
spectroscopic redshift, derived probabilities should be mea-
ured for a wide range of \( z \), with \( z \) then marginalized over as 
is done with other fit parameters.

3. ACCURATE REDSHIFT-INDEPENDENT DISTANCES

The BATM method for calculating luminosity distances for 
SNe Ia was briefly described by Tonry et al. (2003) and Barris 
et al. (2004), with a more complete treatment to appear in Barris 
(2004). It is based on an idealized set of representative SN Ia 
light curves that are photometrically and spectroscopically well 
sampled in time and for which accurate distances are known. 
With such a spectrophotometric template set, predicted light 
curves could be produced to compare to observations. How-
ever, data of this quality are extremely uncommon at present, 
so we use a set of light curves that have excellent temporal 
coverage over a range of wavelengths and span a wide range 
of luminosity and a large set of observed spectra. For a given 
redshift, the SEDs are shifted and warped so that they match 
the observed photometry of each template light curve. BATM 
treats the “template” and “unknown” in a fundamentally dif-
f erent manner from the previous methods. The SEDs and light-
curves are shifted to the redshift of the SN to be measured, so 
that redshift effects are \textit{introduced} to the template set rather 
than \textit{removed} from the observational data. The idea is to com-
pare the observed SN to what we would expect the template to 
look like at a given redshift, as opposed to comparing the 
template to what the SN would look like were it at the redshift of 
the template.

Since in BATM it is the template data that are transformed, 
rather than the observed data, the operation naturally can be 
performed over a range of redshift, facilitating the implementa-
tion of the redshift marginalization. It should be stressed again 
that any method for measuring distances may similarly be ex-
tended by merely iterating the procedure over a range of red-
shifts, treating \( z \) as an additional free parameter and marginal-
izing it out.

We have performed the redshift-marginalization procedure 
in combination with the BATM method for a set of 60 SNe Ia 
taken from Hamuy et al. (1996), Riess et al. (1999), and Jha 
(2002). This is a well-observed sample for which we know 
relative distances via spectroscopic redshifts. Results are shown 
on a Hubble diagram in Figure 1. It is important to note that the 
redshifts used to construct these Hubble diagrams were 
measured spectrophotometrically, \textit{not} photometrically. The rms 
scatter about the best-fit line is 0.21 mag for the \( z \)-free BATM 
distances, compared to 0.19 mag for distances taken from the 
recent compilation of Tonry et al. (2003), who calculated dis-
tances by combining the results from as many methods as 
possible. We observe no significant difference in performance 
between the redshift priors mentioned in the previous section, 
indicating that neither the redshift prior nor details of its im-
plementation are limiting factors for the method for this sample.

We have therefore used distances calculated with a linear step 
of \( z = 0.01 \) in redshift and weighted by \( z \), beginning at \( z = \)
to the data but is to guide the eye. or the redshift marginalization procedure. The dotted diagonal line is due to properties of the SNe rather than some aspect of the BATM method $z$ the inset, with photometry. Plots of residuals relative to the Hubble diagrams are shown in distances in order to produce for the set of 60 SNe.2 Uncertainties in Figure 1 reflect this re-scaling. The distance uncertainties so that they more accurately reflect the scatter has yet to be performed. At present it seems sensible to rescale has from each of the BATM template light curves and determine how to optimally calculate and combine distance methods will both alleviate this discrepancy and reduce the scatter about the Hubble diagram. Such a procedure would for BATM similar to that used to produce templates for other value of 40.0 for 60 objects. We expect that a "training" process for this set of objects, but we have left them as reported.

The residuals with respect to a Hubble line for the two sets of distances are compared in the inset of Figure 1. There is a clear correlation between the two methods, indicating that the residuals are due to intrinsic properties of the SNe rather than some feature of the redshift marginalization or the BATM analysis. This is further evidence that the redshift marginalization procedure is recovering the same distance information as the established $z$-specific techniques. It is natural to worry about additional possible systematic biases that might be introduced by this method. In Figure 2 we compare the Hubble diagram residuals as a function of several SN properties. There are no evident biases in the redshift-independent distances as a function of extinction, redshift, or light-curve shape. It is interesting to note that for SNe Ia at the high ends of the sample distribution of $A_v$, $z$, and MLCS $\Delta$, the redshift-independent distances appear to be slightly less biased than those using redshift-specific methods.

It should in principle also be possible to examine the $(z, d)$ probabilities without marginalizing over $z$. Figure 3 shows con-

0.01 and typically truncated at $z = 0.20$, where the probability has become negligible.

We have scaled the uncertainties for the $z$-free BATM distances in order to produce $\chi^2/N_{\text{dof}} \sim 1$ for the set of 60 SNe. Our initial uncertainties were overestimated in comparison to their scatter about the Hubble line, as shown by a total $\chi^2$ value of 40.0 for 60 objects. We expect that a “training” process for BATM similar to that used to produce templates for other methods will both alleviate this discrepancy and reduce the scatter about the Hubble diagram. Such a procedure would determine how to optimally calculate and combine distance estimates from each of the BATM template light curves and has yet to be performed. At present it seems sensible to rescale the uncertainties so that they more accurately reflect the scatter and yield $\chi^2/N_{\text{dof}} \sim 1$. Uncertainties in Figure 1 reflect this rescaling. The distance uncertainties from Tonry et al. (2003) appear to be underestimated for this set of objects, but we have left them as reported.

4. CONCLUSIONS

We have described a procedure for measuring accurate luminosity distances for SNe Ia independent of knowledge of redshift by marginalizing over $z$. This procedure may be used in combination with any distance measurement technique to remove the limitations on SN surveys imposed by the need for spectroscopic redshifts. When applied as an extension to the
BATM method, the redshift marginalization formalism produces a Hubble diagram scatter of $\sim0.2$ mag, comparable to that using conventional implementations of distance methods, in which knowledge of redshift is required. Further refinement of the BATM technique through a training process, or use with other methods, is likely to reduce the scatter to values equal to that of redshift-specific methods.

Distances measured with this method will allow investigation of various properties of SNe as a function of distance, which is all that is necessary to answer many fundamental questions. The rates of SNe Ia as a function of redshift are still not well constrained (see Pain et al. 1996, 2002; Tonry et al. 2003) and likely will remain so because of the difficulty in collecting large numbers of spectroscopic redshift. However, rates as a function of distance are just as fundamental, and the lack of redshift knowledge will not prevent investigations into the dependence of rates on numerous host-galaxy properties, for instance. For investigations in which cosmological parameters may be considered as known, the conversion from distance to redshift is determined, so accurate distances are all that is necessary.

The method also may be used to produce probability contours as a function of $(d, z)$ for high-$z$ SNe. This will allow cosmological parameters to be measured based purely on photometric observations. Such a development will be an important step for cosmological studies based on SN surveys. In order to accomplish this, it will be necessary to obtain a sufficient number of observations to properly determine the multi-wavelength light-curve shape to isolate the effects of $(d, z)$ from other fit parameters (particularly $A_v$ and $R_p$) and sufficiently constrain the solution space. The achievable accuracy and the observational strategy required to reach it are influenced by both the intrinsic properties of SNe Ia (which are not equally “standard candles” at all wavelengths; see Jha 2002) and the quality of temporal and wavelength coverage of nearby SNe Ia used for distance calibration.

The method described here is more powerful than the photometric redshift procedure used to estimate redshifts for galaxies based on their observed colors, because it also takes into account the magnitude and light-curve shape to produce constraints on both redshift and distance. It promises to remove the limitations to SN studies caused by the requirement for spectroscopic redshifts for the determination of distances for SNe Ia. The ongoing ESSENCE (Smith et al. 2002) and Canada-France-Hawaii Legacy Survey and future projects such as Pan-STARRS (Kaiser et al. 2002) and the proposed SuperNova Acceleration Probe (Nugent 2000) will also be able to greatly increase their yields using this method, enhancing their ability to investigate fundamental cosmological questions.

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