Local description of the Aharonov-Bohm effect with a quantum electromagnetic field

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In the Aharonov-Bohm effect, the interference pattern formed by charged particles that propagate in an interferometer where the two possible paths enclose an infinite solenoid depends on the magnetic flux in the solenoid even if the particles only propagate in regions where the electromagnetic fields are zero. The usual interpretations are based on a local interaction of the gauge-dependent potential vector with the particles or on a nonlocal influence of the solenoid on the particles behavior. But here we show how the Aharonov-Bohm effect can be described as the result of virtual photon exchanges between the charged particles and the solenoid, where the particles and the solenoid interact locally with the quantum electromagnetic field. On this way, one of the the principal features of the electromagnetic field concept, which is to avoid action at a distance, is sustained with the Aharonov-Bohm effect. We predict a local and gauge-independent Aharonov-Bohm phase generation for the particles propagation in each path of the interferometer and provide an experimental proposal that could test this prediction.

In classical electrodynamics, the behavior of classical charged particles interacting with a classical electromagnetic field can be fully described in terms of local interactions between the particles and the gauge-independent electric and magnetic fields. However, in a quantum scenario the interference pattern formed by charged particles that propagate through an interferometer that encloses a long solenoid, like the one represented in Fig. 1, depends on the magnetic flux inside the solenoid even if the particles propagate only in regions where the electromagnetic fields are zero [1]. This is the magnetic version of the Aharonov-Bohm (AB) effect [1], that was experimentally observed with many different systems [2–7]. The original description of the AB effect involves a local interaction of the quantum particles with the gauge-dependent potential vector associated to the solenoid magnetic field [1]. Other explanations for the AB effect relies on the superposition between the charged particles electromagnetic field and the solenoid electromagnetic field, that would affect the particles behavior in a nonlocal way [8–13]. A few years ago, Vaidman deduced the AB phase using a quantum mechanical treatment for the charges of the solenoid interacting with the particles field [14], an idea that was further developed by Pearle and Rizzi [15, 16]. But in these works no local mechanism by which the charged particles acquire the AB phase is presented, such that a description of the effect in terms of local interactions between quantum particles and gauge-independent classical fields is not present so far. Since one of the principal ideas behind the introduction of the electromagnetic field concept is to avoid action at a distance, this fact may be disappointing. So there is still a vivid debate on locality issues of the AB effect [12–21].

Here we explain the AB effect as resulting from local interactions of the solenoid and of the quantum particles with the quantized electromagnetic field. We use second-order perturbation theory to see how the energy of the electromagnetic vacuum depends on the particle path in the interferometer, causing the AB phase. The AB effect can then be interpreted as the result of a virtual exchange of photons between the particles and the solenoid generating the AB phase. So, when the more fundamental quantum description of the electromagnetic field is used instead of the classical one, we sustain one of the the principal features of the electromagnetic field concept, which is to avoid action at a distance. These results generalizes a similar treatment from Santos and Gonzalo, that was constructed for a specific model for the solenoid and was valid only for small AB phases [22]. We also show here that the electric version of the AB effect [1] has a similar interpretation.

In a very interesting recent work, Marletto and Vedral used a quantized electromagnetic field to discuss that the AB phase is locally generated, such that it is built up locally and can be detected without the possible paths of the quantum particle closing a loop around the solenoid [21]. However, they do not provide an explicit physical model for the interaction between the charged particle and the solenoid mediated by the quantum field. Here we...
provide such a model and discuss another experimental proposal that could verify the locality of the AB phase, different from the ones proposed in Ref. [21], which we believe is easier to be implemented.

Consider the setup of Fig. 1, an interferometer for quantum charged particles with a solenoid enclosed by the two possible paths. The magnetic field $\mathbf{B}$ is confined inside the solenoid and the vector potential $\mathbf{A}$ is linked to the magnetic field through the relation $\mathbf{B} = \nabla \times \mathbf{A}$. The particles can propagate only outside the solenoid, such that they experience no magnetic field and no Lorentz force, but are subjected to a nonzero vector potential. It can be shown that there is a contribution for the phase difference between the paths given by $q\Phi_0/h$ (the AB phase), where $\Phi_0$ is the magnetic flux in the solenoid and $h$ is Planck’s constant divided by $2\pi$. So the particles interference pattern depends on the enclosing magnetic flux even if they only propagate in regions with zero magnetic field. Some authors have argued that a longitudinal force could act on the particle causing lag between the electrons path that justifies the AB effect [8,10], but experiments ruled out this hypothesis [23]. However, the presence of transverse forces in the AB scheme, as predicted in Refs. [24, 25], recently received an important indirect experimental confirmation by the measurement predicted in Refs. [24, 25], recently received an important indirect experimental confirmation by the measurement

\begin{equation}
V_2 = -\frac{q}{m}p \cdot A(r) + qU(r)
\end{equation}

in the system Hamiltonian, where $q$, $m$, $p$ and $r$ are the particle charge, mass, momentum and position respectively, and $U$ is the scalar potential operator of the quantized electromagnetic field. This operator can be written as

\begin{equation}
U(r) = c \int d^3k \sqrt{\frac{\hbar}{2\pi\omega(2\pi)^3}} \left[ a_0(k)e^{ik\cdot r} + a_0^*(k)e^{-ik\cdot r}\right].
\end{equation}

where $a_0(k)$ is the annihilation operator for a scalar photon mode with wavevector $k$ and $a_0^*(k) = -a_0^\dagger(k)$. The definition of $a_0(k)$ and its use in the above equation is necessary for having a consistent norm for the scalar photon states [27]. Although there can be no real scalar photons, there may be virtual scalar photons. These are not relevant for the present problem, since the solenoid is assumed to be free of charge densities and so do not couple to scalar photons, but virtual scalar photons are used, for instance, to deduce the Coulomb law from quantum electrodynamics [27]. Virtual scalar photons are also relevant in the electric version of the AB effect, to be treated later.

The idea of our treatment is to compute the change on the energy of the vacuum state of the electromagnetic field perturbed by the terms of Eqs. (1) and (3) in the system Hamiltonian, with the use of second-order perturbation theory. We show that the perturbed energy depends on the particle path in Fig. 1 and compute the phase difference for the particle propagation in the two possible paths.

The terms of Eqs. (1) and (3) change the energy of the unperturbed electromagnetic vacuum state $\ket{\text{vac}}$. Since all terms of $V_1$ and $V_2$ have annihilation or creation operators due to the presence of the operators $\mathbf{A}$ from Eq. (2) and $U$ from Eq. (4), the first-order correction of the electromagnetic vacuum energy is zero. The second-order correction can be written as

\begin{equation}
\Delta E = \sum_{n \neq 0} \left| \langle \phi^0_n | (V_1 + V_2) | \text{vac} \rangle \right|^2 \frac{E_0^n - E_n^0}{E_0^n - E_n^0},
\end{equation}

where $\phi^0_n$ is a unitary polarization vector, $a_\sigma(k)$ is the annihilation operator for a mode with wavevector $k$ and polarization index $\sigma$, and $\omega = ck$ is the mode angular frequency. In the traditional quantum optics treatments in the Coulomb gauge [28], the polarization index $\sigma$ assumes two values for each wavevector $k$, corresponding to two polarizations orthogonal to $k$. However, in the complete quantum electrodynamics formalism constructed in the Lorentz gauge [27], there are also photon modes with longitudinal polarization in the direction $k$, such that we have 3 indexes for $\sigma$ for each value of $k$. Although there can be no real photons with longitudinal polarizations, the presence of virtual photons with longitudinal polarization is important in the present treatment.
where \( E_0 \) is the energy of the unperturbed vacuum, \( |\phi_n\rangle \) represent eigenstates of the unperturbed Hamiltonian with energies \( E_n \), and the index \( i \) is associated to the degeneracy of these eigenstates. Terms with the product of matrix elements of \( V_1 \) with \( V_1 \) and of \( V_2 \) with \( V_2 \) result in self-energies of the solenoid and of the particle respectively, not being relevant for the present study.

Considering only the correction in energy due to the interaction between the solenoid and the particle mediated by the field, note that only field states with one photon are relevant, that can be represented as \( |k, \sigma\rangle \), \( k \) being the wavevector and \( \sigma \) the polarization index of the photon (scalar photons do not contribute). So we can write the part of the vacuum energy change which is relevant for the present study as

\[
\Delta E' = 2 \text{Re} \left[ \int d^3k \sum_{\sigma} \langle k, \sigma | V_1 | \text{vac}\rangle \langle \text{vac} | V_2 | k, \sigma \rangle \frac{\hbar \omega}{\hbar \omega} \right], \tag{6}
\]

\( \hbar \omega = \hbar c k \) being the photon energy. Using Eqs. (1) and (2), we obtain

\[
\langle k, \sigma | V_1 | \text{vac}\rangle = - \int d^3r' \sqrt{\frac{\hbar}{2\varepsilon_0 \omega (2\pi)^3}} \mathbf{J}(r') \cdot \mathbf{\varepsilon}_{k\sigma} \ e^{-i \mathbf{k} \cdot \mathbf{r}'}. \tag{7}
\]

Using Eqs. (3), (2), and (4), we obtain

\[
\langle \text{vac} | V_2 | k, \sigma \rangle = - \frac{q}{m} \sqrt{\frac{\hbar}{2\varepsilon_0 \omega (2\pi)^3}} \mathbf{p} \cdot \mathbf{\varepsilon}_{k\sigma} \ e^{i \mathbf{k} \cdot \mathbf{r}}. \tag{8}
\]

So Eq. (6) becomes

\[
\Delta E' = - \frac{q}{mc^2 \varepsilon_0} \text{Re} \left\{ \int d^3r' \left[ \int d^3k \frac{e^{i \mathbf{k} \cdot (\mathbf{r}' - \mathbf{r})}}{(2\pi)^3 k^2} \right] \mathbf{J}(r') \cdot \mathbf{p} \right\}. \tag{9}
\]

The term inside brackets can be written as

\[
\int d^3k \frac{e^{i \mathbf{k} \cdot (\mathbf{r}' - \mathbf{r})}}{(2\pi)^3 k^2} = \frac{1}{4\pi |\mathbf{r} - \mathbf{r}'|}. \tag{10}
\]

So we can write

\[
\Delta E' = - \frac{q}{m} \mathbf{p} \cdot \mathbf{A}(\mathbf{r}), \tag{11}
\]

where

\[
\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \frac{\mathbf{J}(r')}{|\mathbf{r} - \mathbf{r}'|} \tag{12}
\]

is the potential vector generated by the current in the solenoid in the Lorentz gauge. The phase difference between the two paths of Fig. 1 can then be written as

\[
\phi = - \int \frac{(\Delta E_b - \Delta E_a')}{\hbar} dt, \tag{13}
\]

where \( \Delta E_a \) and \( \Delta E_b' \) are the corrections in energy on paths \( a \) and \( b \) respectively, given by Eq. (11). By writing

\[
\dot{\mathbf{p}}/m = d\mathbf{l}/dt, \text{ where } d\mathbf{l} \text{ is an infinitesimal displacement around the path, we can write}
\]

\[
\phi = \frac{q}{\hbar} \int_{\text{path } b} \mathbf{A} \cdot d\mathbf{l} - \frac{q}{\hbar} \int_{\text{path } a} \mathbf{A} \cdot d\mathbf{l} = \frac{q}{\hbar} \int \mathbf{A} \cdot d\mathbf{l} = \frac{q\Phi_0}{\hbar}, \tag{14}
\]

the AB phase.

The treatment we used here to arrive at Eq. (11) is analogous to the deduction of the Coulomb interaction between two charges from similar changes on the vacuum electromagnetic energy due to the coupling of the quantized electromagnetic field with the charges [27]. A difference is that the scalar photons are the relevant ones in this case. The usual interpretation is that the Coulomb attraction or repulsion is the result of the exchange of virtual photons between the charges [27]. A similar interpretation can be used here. The energy variation obtained from Eq. (6) has the contribution from terms with a creation operator in \( V_1 \) and an annihilation operator in \( V_2 \) for the same photon mode, that can be interpreted as the result of a virtual photon emission by the solenoid which is absorbed by the quantum particle. In an analogous way, terms with a annihilation operator in \( V_1 \) and a creation operator in \( V_2 \) for the same photon mode can be interpreted as the result of a virtual photon emission by the quantum particle which is absorbed by the solenoid. So we interpret the AB effect in the quantum electrodynamics formalism as the result of an interaction between the quantum particle and the solenoid mediated by the exchange of virtual photons, where both the solenoid and the particle interact locally with the quantum electromagnetic field. In the AB scheme of Fig. 1, there will be entanglement between the particle path and the energy of the vacuum state of the electromagnetic field, which is the responsible for the appearance of the AB phase.

An interesting point is that the quantity \( \mathbf{A} \) that appears in Eq. (11) is a gauge-independent quantity defined by the expression (12), which coincides with the potential vector generated by the solenoid current density in the Lorentz gauge. This is the case because the quantity \( \Delta E' \) from Eq. (11) cannot be gauge-dependent, since it represents a physical energy variation in the system. So a well defined gauge-independent phase is acquired by each part of the particle wave function while it is propagating through each path. In the following we discuss a possible experiment that could verify if the AB phase is indeed locally generated.

Consider the situation depicted in Fig. 2. A source releases an ion that falls in the \(-\hat{y}\) direction due to gravity. Its wave function spreads in the process, and while it is falling the ion traps A and B are turned on. If the ion is not detected on the detection screen, its wave function will be coherently superposed in traps A and B. If the traps are now turned off, the ion wave function evolves falling again until the ion is detected on the screen. By repeating the experiment many times, an in-
terference pattern is formed. Since the solenoid is not enclosed by the paths, the interference pattern does not depend on its magnetic flux.

Consider now a situation where, while the ion wave function is superposed in the two traps A and B, the magnetic field of the solenoid is turned off. By Faraday’s law, an electric field will then be produced in the position of the traps A and B. But if the solenoid has a cylindrical symmetry and the traps are positioned at the same distance from it, the magnitude of the electric field at each trap will be the same. Considering also that the traps are identical and have a symmetric potential in the $x - y$ plane, this induced electric field (considered to be small) will change the energy of the portion of the ion wave function in each trap in a perturbative way equally, producing no phase difference in the process. If, after the magnetic field of the solenoid is reduced to zero, the traps are turned off, the expected interference pattern on the screen should depend on an intermediate AB phase. This is because there is an accumulated phase difference while the ion goes from the source to traps A and B, which can be calculated from Eqs. (11), (12), and (13), but no extra phase when the ion propagates from the traps to the screen, since the solenoid current is zero in this case. An experiment like this should then show an intermediate AB phase even if the particle paths do not enclose the solenoid, demonstrating that the AB phase is locally generated as predicted by the present treatment.

Now let us discuss the electric version of the AB effect $\Pi$, represented in Fig. 3. The experiment should be made with quantum charged particles sent one by one to the interferometer. The scalar potentials of the conductor tubes in each path are zero when the particle is outside them. When the quantum particle is in a superposition of being inside each of the tubes, the potential of the tubes is varied and comes back to zero before the particle wave function exits each of them. In this case the particles experience no electric field and no Lorentz force, but experience different electric potentials in each path. Again, this results in a phase difference between the paths. If the potential of the tube of path $a$ is $U_a(t)$ and the one of path $b$ is $U_b(t)$, it can be shown that the phase difference is $\int dt[U_a(t) - U_b(t)]q/\hbar$, affecting the interference pattern $\Pi$. This behavior was also experimentally verified $\Pi$, although not with the charged particles propagating only in free-field regions.

The treatment of the electric version of the AB effect with a quantized electromagnetic field is completely analogous to the magnetic AB effect treated previously. But now the term in the Hamiltonian that represents the interaction of the quantum electromagnetic field with the charges on the tubes is

$$V_1 = \int d^3r' \rho(r') U(r'),$$

where $\rho(r')$ is the charge density of the tubes and $U(r')$ is the scalar potential operator given by Eq. (4). The interaction of the charged particle with the electromagnetic field is still given by Eq. (4). Following the same steps as before, it is straightforward to show that the change on the electromagnetic vacuum energy when the particle is in a position $r$, disregarding self-energy terms, is given by

$$\Delta E' = qU(r) \text{ with } U(r) = \int d^3r' \frac{\rho(r')}{4\pi\varepsilon_0|\mathbf{r} - \mathbf{r}'|}.$$ (16)

It is straightforward to show that this result leads to the predicted phase difference between the paths in the electric version of the AB effect.

To summarize, we have shown that the AB effect in the scheme depicted in Fig. 1 (Fig. 3) can be described
as the result of an interaction between the charged particles and the solenoid (conductor tubes) mediated by the quantum electromagnetic field in a local way, with the exchange of virtual photons. The particles locally acquire an AB phase due to the change of the energy of the vacuum electromagnetic field, which depends on the particle path. Our treatment also predicts that, in principle, intermediate AB phases can be measured even if the quantum particles possible paths do not enclose the solenoid, what would demonstrate that the AB phase is locally generated.

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[1] Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959).
[2] R. G. Chambers, Phys. Rev. Lett. 5, 3 (1960).
[3] A. Tonomura et al., Phys. Rev. Lett. 56, 792 (1986).
[4] M. Peshkin and A. Tonomura, Lecture Notes in Physics (Springer, New York, 1989), Vol. 340.
[5] R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, Phys. Rev. Lett. 54, 2696 (1985).
[6] A. Bachtold et al., Nature 397, 673 (1999).
[7] H. Peng et al., Nature Mater. 9, 225 (2010).
[8] B. Liebowitz, Nuovo Cimento 38, 932 (1965).
[9] T. H. Boyer, Phys. Rev. D 8, 1679 (1973).
[10] T. H. Boyer, Found. Phys. 32, 41 (2002).
[11] M. Peshkin, Phys. Rep. 80, 375 (1981).
[12] K. Kang, arXiv:1308.2093
[13] P. L. Saldanha, Braz. J. Phys. 46, 316 (2016).
[14] L. Vaidman, Phys Rev. A 86, 040101 (2012).
[15] P. Pearl and A. Rizzi, Phys. Rev. A 95, 052123 (2017).
[16] P. Pearl and A. Rizzi, Phys. Rev. A 95, 052124 (2017).
[17] Y. Aharonov, E. Cohen, and D. Rohrlich, Phys. Rev. A 92, 026101 (2015).
[18] L. Vaidman, Phys. Rev. A 92, 026102 (2015).
[19] Y. Aharonov, E. Cohen, and D. Rohrlich, Phys. Rev. A, 93, 042110, (2016).
[20] K. Kang, J. Korean Phys. Soc. 71, 565 (2017).
[21] C. Marletto and V. Vedral, arXiv:1906.03440
[22] E. Santos and I. Gonzalo, Europhys. Lett. 45, 418 (1999).
[23] A. Caprez, B. Barwick, and H. Batelaan, Phys. Rev. Lett. 99, 210401 (2007).
[24] A. L. Shelankov, Europhys. Lett. 43, 623 (1998).
[25] M. V. Berry, J. Phys. A. Math. Gen. 32, 5627 (1999).
[26] M. Becker, G. Guzzinati, A. Béché, J. Verbeeck, and H. Batelaan, Nat. Comm. 10, 1700 (2019).
[27] C. Cohen-Tannoudji et al., Photons and Atoms, (John Wiley & Sons, New York, 1989).
[28] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics (Cambridge, New York, 1995).
[29] C. Cohen-Tannoudji, B. Diu, and F. Laloe, Quantum Mechanics, 2nd ed. (Hermann by John Wiley & Sons, Paris, 1977).
[30] J. D. Jackson, Classical Electrodynamics, 3rd ed. (John Wiley & Sons, New York, 1999).
[31] G. Matteucci and G. Pozzi, Phys. Rev. Lett. 54, 2469 (1985).
[32] A. van Oudenaarden, M. H. Devoret, Y. V. Nazarov, and J. E. Mooij, Nature 391, 768(1998).