The Nonlocal Correlation of Spin in High Energy Physics

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Abstract

Non-locality is a key feature of quantum theory and is reflected in the violation of Bell inequalities for entangled systems. The experimental tests beyond the electromagnetism and massless quanta are of great importance for understanding the non-locality in different quantum interactions. In this work, we develop a generalized Clauser-Horne inequality pertaining especially to the high energy physics processes, which is quantum mechanical intervene free. We find, in the process of pseudoscalar quarkonium exclusive decay to entangled $\Lambda\bar{\Lambda}$ pair, the inequality could be violated and is verifiable in high energy experiments, like BES III or BELLE II.

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1 Introduction

Quantum non-locality, which distinguishes classical physics from the quantum one, lies at the heart of quantum mechanics. In the studies of quantum theory, the non-locality is normally testified by the violation of Bell inequalities [1–3]. Many beautiful experiments have been carried out to this end, most of which rely on the entanglement of photons [4]. Being a fundamental nature of quantum physics, the non-locality should be examined in various of extreme situations, rather than limiting to low energy electromagnetic interaction with massless quanta. Investigation of the non-locality in high energy physics attracts more and more attention in recent years [5, 6], though the corresponding experiments tend to be tough and delicate schemes are needed to avoid various loopholes.

Testing the nonlocal correlation using spin or polarization in high energy physics has been sorted into four classes based on the interactions in the decay processes [7]. Since the work of Ref.[8], the study on correlation in the baryon decays has been a hot topic in particle theory [9, 10] and high energy experiment [11]. However, up to date, controversial issues still remain in testing the non-locality via high energy process. First, dichotomic observables are not explicitly available in correlation functions [12]. Second, when embedding the correlation functions into Bell’s inequalities, some of the coefficients may experience re-normalization in the framework of quantum mechanics, which is somehow self-attesting and improper for the testimony of quantum nonlocality [13]. Third, conclusive Bell’s test requires an active control of measurement settings, the so-called free will, but in high energy phenomena, for instance, spontaneous decays, that is to say the “measurements” performed are usually passive [14].

In this work, we explore the application of Clauser-Horne (CH) [3] inequality to high energy experiment, by which the above problems one and two can be avoided since here
the probabilities rather than correlations are employed. Another superiority by means of probability density lies in its easiness in experimental measurement. As for the problem three, we will remark on it in the conclusion. In high energy physics experiment, there are a huge number of quarkonium states being accumulated, at e.g. BES III, BELLE II or even LHCb detectors. We find the exclusive process of $\eta_c$ to $\Lambda\bar{\Lambda}$ is an ideal process for our aim, where the $\Lambda$-pair is predominantly entangled in spin degrees of freedom with s-wave orbital angular momentum. Noticing the spin measurement in the $\Lambda$ decay amplitude is not simply a probability distribution ranging from 0 to 1, we generalize the original CH inequality to a novel form, which is suitable for any kind of decay amplitude.

2 Generalized CH inequality

According to the measurement postulate of quantum mechanics, a general quantum measurement is described by a collection $\{M_m\}$ of measurement operators, and the probability for getting the outcome $m$ is given by $p_m = \langle \psi | M_m^{\dagger} M_m | \psi \rangle$ [15]. The probabilities are positive semi-definite and normalized, i.e., $0 \leq p_m \leq 1$ and $\sum_m E_m = 1$ where $E_m \equiv M_m^{\dagger} M_m$. Considering the spin 1/2 system, a measurement $M_m$ may be performed by an apparatus set along the direction $\vec{n}$, in which each particle triggering the apparatus gives the result of $m$. As we are only interested in the possibility of each measurement outcome, discussions on the technical details and the configurations of the apparatus are not our main concern. A realistic description of the possibility of measurement outcome may be formulated as [3]

$$P(\vec{n}) = \int_{\Gamma} p_m(\lambda, \vec{n}) \rho(\lambda) \, d\lambda \, , \, a \leq p_m(\lambda, \vec{n}) \leq b \, .$$

Here $\lambda$ is the hidden variable determining the possibility $p_m(\lambda, \vec{n})$, by which the particle triggers the apparatus, and $\Gamma$ denotes the space of the hidden variable with a normalized
distribution $\rho(\lambda)$; $a$ and $b$ are the lower and upper bounds of the possibility, $0 \leq a \leq b \leq 1$, and the subscript of $P_m(\vec{n})$ is omitted for convenience. Within a local and realistic theory, we have the following

**Lemma 1** In a bipartite system of particles 1 and 2, if we perform the measurement $M_m$ with $p_m(\vec{n}_1) \in [a_1, b_1]$ and $p_m(\vec{n}_2) \in [a_2, b_2]$ on each side, the local realism leads to the following inequality

$$P(\vec{n}_1, \vec{n}_2) - P(\vec{n}_1, \vec{n}_2') + P(\vec{n}_1', \vec{n}_2) + P(\vec{n}_1', \vec{n}_2') - (a_2 + b_2)P(\vec{n}_1') - (a_1 + b_1)P(\vec{n}_2) + a_1b_2 + b_1a_2 \leq 0.$$  

(2)

where $P(\vec{n}_1, \vec{n}_2) = \int_{\Gamma} p_m(\lambda, \vec{n}_1)p_m(\lambda, \vec{n}_2)\rho(\lambda)\,d\lambda$ is the joint distribution, and $0 \leq a_i \leq b_i \leq 1$.

Lemma 1 can be regarded as a generalized CH inequality for the measurements. The proof of Lemma 1 is straightforward by taking advantage of the following inequality

$$x_1y_2 - x_1y_1 + x_2y_1 + x_2y_2 - (a_2 + b_2)x_2 - (a_1 + b_1)y_1 + a_1b_2 + b_1a_2 \leq 0$$  

(3)

for parameters $x_{1,2} \in [a_1, b_1]$ and $y_{1,2} \in [a_2, b_2]$.

### 3 Violation of new CH inequality in entangled $\Lambda\bar{\Lambda}$

First we show the local realism predictions for the joint distribution of momenta of $p$ and $\bar{p}$ in the decays of $\Lambda\bar{\Lambda}$ system. The differential decay width of $\Lambda \to p\pi^-$ with $p$ moving in direction $\vec{n}_p$ writes

$$\frac{d\sigma_{\Lambda \to p\pi^-}}{d\vec{n}_p} = \frac{1}{4\pi}(1 + \alpha_s\bar{s}_{\Lambda} \cdot \vec{n}_p).$$  

(4)
Here $\vec{s}_\Lambda$ denotes the spin operator of the hyperon $\Lambda$ and $\vec{n}_p$ is a unit vector; $\alpha_-$ is the decay parameter for $\Lambda$ \[16\]. Without loss of generality, we may choose the polarization direction of $\Lambda$ to be the $z$-axis, then the probability of finding the proton leaving in the direction $\vec{n}_p$ with polar angle $\theta$ is

$$p(\vec{n}_p) = 2\pi \frac{d\sigma_{\Lambda \to p\pi^-}}{d\Omega_p} = \frac{1}{2} (1 + \alpha_- \cos \theta) ,$$

(5)

which ranges from $\frac{1-\alpha_-}{2}$ to $\frac{1+\alpha_-}{2}$. Analogously, the distribution for the polarized $\bar{\Lambda}$ can also be measured with decay parameter $\alpha_+$. In view of the (approximate) CP parity conservation arguments, the decay parameters satisfy $\alpha = \alpha_- = -\alpha_+ \simeq 0.750$ \[16\], hence, $0 \leq \frac{1-\alpha_-}{2} \leq \frac{1+\alpha_-}{2} \leq 1$. From Lemma \[1\] we have:

**Corollary 1** In bipartite system consisting of $\Lambda$ and $\bar{\Lambda}$, the local realism predicts that the joint distribution of the momenta of $p$ and $\bar{p}$ satisfies

$$P(\vec{n}_1, \vec{n}_2) = P(\vec{n}_1, \vec{n}_2') + P(\vec{n}_1', \vec{n}_2) + P(\vec{n}_1', \vec{n}_2')$$

$$- P(\vec{n}_1') - P(\vec{n}_2) + \frac{1 - \alpha^2}{2} \leq 0 .$$

(6)

Here $\vec{n}_1, \vec{n}_1'$ are directions of momenta of $p$ and $\vec{n}_2, \vec{n}_2'$ are that of $\bar{p}$; $\alpha$ is the decay parameter of $\Lambda(\bar{\Lambda})$ decay.

In quantum theory, the distribution of equation (5) may be explained by the following measurement process. The Hilbert spaces of the spin of $\Lambda$ and the momentum of proton are coupled by a unitary interaction $U$

$$U : |\psi\rangle \otimes |\vec{n}_p\rangle \mapsto M_+(\vec{n}_p)|\psi\rangle \otimes |\vec{n}_p\rangle + M_-(\vec{n}_p)|\psi\rangle \otimes | - \vec{n}_p\rangle ,$$

(7)

where $\vec{n}_p$ is the unit vector of the momentum and

$$M_\pm(\vec{n}_p) \equiv \frac{1}{\sqrt{2(|S|^2 + |P|^2)}} [S + P\vec{s}_\Lambda \cdot (\pm \vec{n}_p)] .$$

(8)
Here $S$ and $P$ are the decay amplitudes of $\Lambda$ for the $S$ and $P$ waves and $\alpha_- = (S^*P + SP^*)/(|S|^2 + |P|^2)$. For $\Lambda$ spinning along the z-axis, i.e. $|\psi\rangle = |z\rangle$, the probability for the proton going along $\vec{n}_p$ is

$$p(\vec{n}_p) = \langle z|E_+|z\rangle = \frac{1}{2}(1 + \alpha_- \cos \theta),$$  \hspace{1cm} (9)$$

where $E_+ = M_+^1 M_+ = \frac{1}{2}(1 + \alpha_- \vec{s}_\Lambda \cdot \vec{n}_p)$ and the argument $\vec{n}_p$ in $M_+$ is suppressed for simplicity. The probability for $p$ coming from the reverse direction of $\vec{n}_p$ is

$$p(-\vec{n}_p) = \langle z|E_-|z\rangle = \frac{1}{2}(1 - \alpha_- \cos \theta).$$  \hspace{1cm} (10)$$

Here $E_- = M_-^1 M_-$ and $E_+ + E_- = 1$. For the $\Lambda \bar{\Lambda}$ system described by a bipartite state $\rho_{12}$, the joint distribution for proton $p$ coming along $\vec{n}_1$ and anti-proton $\bar{p}$ coming along $\vec{n}_2$ is

$$P(\vec{n}_1, \vec{n}_2) = \text{Tr} \left[ \rho_{12} \left( E_+^{(1)} \otimes E_+^{(2)} \right) \right],$$  \hspace{1cm} (11)$$

where $E_+^{(i)}$ are the measurement operators for particles $\Lambda$ and $\bar{\Lambda}$, and the one-side distribution

$$P(\vec{n}_1) = \text{Tr} \left[ \rho_{12} \left( E_+^{(1)} \otimes 1^{(2)} \right) \right] = P(\vec{n}_1, \vec{n}_2) + P(\vec{n}_1, -\vec{n}_2).$$  \hspace{1cm} (12)$$

Here $1^{(2)} = E_-^{(2)} + E_-^{(2)}$ is employed.

In the process of $\eta_c \to \Lambda \bar{\Lambda}$, we have $\rho_{12} = |\psi_{12}\rangle \langle \psi_{12}|$ with

$$|\psi_{12}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle).$$  \hspace{1cm} (13)$$

Substitue $M_+$ into equation (11) and consider the CP conservation decay parameter $\alpha$, we have

$$P(\vec{n}_1, \vec{n}_2) = \frac{1}{4}(1 + \alpha^2 \vec{n}_1 \cdot \vec{n}_2), \quad P(\vec{n}_1) = \frac{1}{2}.$$  \hspace{1cm} (14)$$
Figure 1. The violation of the generalized CH inequality in the hyperon decay. The generalized CH inequality has an upper bound of 0 which is violated by the quantum mechanics (shaded and doubly shaded region) in a wide range of the parameter \( \theta \). For the massive quanta in \( \eta_c \rightarrow \Lambda \bar{\Lambda} \), the upper bound turns to \( (1 - \beta_\Lambda) \frac{\alpha^2}{2} \), which is violated only in the doubly shaded region.

With the quantum prediction of equation (14), the inequality (6) becomes

\[
\frac{\alpha^2}{4} (\cos \theta_{12} - \cos \theta_{1'2'2} + \cos \theta_{1'2} + \cos \theta_{1'2'}) - \frac{\alpha^2}{2} \leq 0 ,
\]  

where \( \theta_{ij} \) are the angles between \( \vec{n}_i \) and \( \vec{n}_j \). For \( \theta_{12} = \theta_{1'2} = \theta_{1'2'} = \theta \) and \( \theta_{1'2'} = 3\theta \), we arrive at the following inequality

\[
\alpha^2 \left[ \frac{3\cos \theta - \cos(3\theta)}{4} - \frac{1}{2} \right] \leq 0 .
\]  

(16)

The violation of the inequality (16) is plotted in Figure 1. The maximal violation happens at \( \theta = \pi/4 \) where

\[
\alpha^2 \left( \frac{\sqrt{2}}{2} - \frac{1}{2} \right) \leq 0 .
\]  

(17)

Hence the quantum predictions violate the local realism and the violation scales with the square of the decay parameter \( \alpha \), see the shaded and doubly shaded region for \( \theta \) in Figure 1.
4 Testing the violation in experiments

4.1 Measurement of the joint distribution

From equation (9), we may rewrite equation (4) as
\[ \frac{d\sigma_{\Lambda \rightarrow p\pi^-}}{d\vec{n}_p} = \frac{1}{2\pi} \langle \psi | M_+^\dagger(n_p^-)M_+(\vec{n}_p)|\psi \rangle, \tag{18} \]
where $|\psi\rangle$ is the polarization state of $\Lambda$. In the process of $\eta_c \rightarrow \Lambda\bar{\Lambda} \rightarrow (p\pi^-)(\bar{p}\pi^+)$, the joint distribution may be expressed as
\[ \frac{d\sigma_{\eta_c \rightarrow \Lambda\bar{\Lambda} \rightarrow (p\pi^-)(\bar{p}\pi^+)}{d\vec{n}_p d\vec{n}_{\bar{p}}} = \frac{1}{4\pi^2} \text{Tr} \left[ \rho_{12} (M_+^{(1)}\dagger M_+^{(1)}) \otimes (M_+^{(2)}\dagger M_+^{(2)}) \right] = \frac{1}{4\pi^2} P(\vec{n}_p, \vec{n}_{\bar{p}}). \tag{19} \]

Here $\rho_{12}$ is the polarization state of the bipartite system of $\Lambda\bar{\Lambda}$; $\vec{n}_p$ and $\vec{n}_{\bar{p}}$ are the unit directions of the momenta of $p$ and $\bar{p}$ in the rest frame of the $\Lambda$ and $\bar{\Lambda}$ respectively. According to equation (19), the joint distribution of $P(\vec{n}_p, \vec{n}_{\bar{p}})$ can be measured from the differential cross section. When taking the joint distributions into equation (6), a violation is expected from the quantum mechanics.

Note the operators $M_\pm$ are regarded as general measurement with dichotomic outcomes in the polarized $\Lambda$ decay. Nonetheless, how the measurements are actually implemented during the weak decay $\Lambda \rightarrow p\pi^-$ is not relevant in testing the local realism via the generalized CH inequality. Another important requirement in testing the local realism versus the predictions of quantum mechanics is that one cannot refer to the results deduced from quantum theory itself. That is, in the bipartite $\Lambda\bar{\Lambda}$ system, re-normalization of the spin correlation function by the asymmetry parameter $\alpha$ is not allowed [13]. It is clear that no such re-normalization problem exists in equation (19). Finally, we note that the active control problem still remains, and readers may refer to Ref.[17] for a recent discussion.
4.2 Space-like separation

In order to test the non-local correlation, any possible classical communications should be excluded, i.e., the decays need to be space-like separated. In the process of $\eta_c \to \Lambda \bar{\Lambda}$, because $\Lambda$ and $\bar{\Lambda}$ are flying apart at a speed $v < c$, not all the subsequent decay events $\Lambda \to p\pi^-$ and $\bar{\Lambda} \to \bar{p}\pi^+$ are space-like separated. Suppose they decay at position $x_1$ and $x_2$ respectively, the two events are space-like separated if there is a time interval during which information cannot be communicated at the speed of light $c$

$$c \left| \frac{x_1}{v} - \frac{x_2}{v} \right| \leq x_1 + x_2 .$$

(20)

Here $v$ is the speed of $\Lambda(\bar{\Lambda})$ in the rest frame of $\eta_c$. Equation (20) can be simplified to

$$\frac{1}{k} \leq \frac{x_1}{x_2} \leq k ,$$

(21)

where $k = \frac{1 + \beta_\Lambda}{1 - \beta_\Lambda}$, $\beta_\Lambda = v/c$. The fraction of the space-like separated events to the total events of hyperon pairs goes as

$$F = \int_0^\infty e^{-x_2} dx_2 \int_{\frac{1}{k}x_2}^{kx_2} e^{-x_1} dx_1 = \beta_\Lambda .$$

(22)

For time-like events (fraction of $1 - \beta_\Lambda$) the left hand side of inequality (6) may reach the maximal value of

$$\frac{1 + \alpha^2}{4} - \frac{1 - \alpha^2}{4} + \frac{1 + \alpha^2}{4} - \frac{1}{2} - \frac{1}{2} + \frac{1 - \alpha^2}{2} = \frac{\alpha^2}{2} .$$

(23)

Therefore the realism prediction of equation (6) now turns to

$$P(\vec{n}_1, \vec{n}_2) - P(\vec{n}_1, \vec{n}_2') + P(\vec{n}_1', \vec{n}_2) + P(\vec{n}_1', \vec{n}_2')$$

$$- P(\vec{n}_1') - P(\vec{n}_2) + \frac{1 - \alpha^2}{2} \leq \beta_\Lambda \cdot 0 + (1 - \beta_\Lambda) \frac{\alpha^2}{2} .$$

(24)
From equation (17), we find that the contradiction between the quantum prediction and the local realism is still observable in case

\[ \beta_\Lambda \cdot 0 + (1 - \beta_\Lambda) \frac{\alpha^2}{2} < \alpha^2 \frac{\sqrt{2} - 1}{2}, \] (25)

which gives \( \beta_\Lambda > 2 - \sqrt{2} \approx 0.586 \), while the ratio of space-like \( \Lambda \bar{\Lambda} \) from \( \eta_c \) is \( \beta_\Lambda = 0.664 \) [16]. The violation of equation (24) is also presented in Figure 1 where the upper bound changes from 0 to \( (1 - \beta_\Lambda) \frac{\alpha^2}{2} \).

5 Discussions

In this work, we present a generalized CH inequality for the measurements whose outcome probabilities do not span the whole range of \([0, 1]\). Since the differential decay amplitudes in high energy physics usually have the similar behavior as that of \( \Lambda \to p\pi^- \) concerned in this work, the generalized CH inequality provides a proper formalism to compare the local realism and quantum predictions, and is applicable to a wide range of interactions. The massive entangled quanta experience both weak and strong interactions here, hence the quantum nonlocality in \( \eta_c \to \Lambda \bar{\Lambda} \) induced process per se is typically important.

By embedding the polarization distributions of entangled \( \Lambda \bar{\Lambda} \) pairs into the generalized CH inequality and taking typical correlation angles, it is found that the quantum theory calculation leads to an evident violation of the generalized CH inequality. This can be readily examined in experiment, like at BESIII, where millions of \( \eta_c \) have been collected. It is worth emphasizing that the novel CH inequality has the merit being not subject to the result deduced from quantum theory calculation like Bell or CHSH inequalities which are about correlation functions.
Finally, we note that although the high energy physics experiments are generally lack of free-will in testing quantum correlation, which impairs the steerability to refute the local realism, the quantum Bell non-locality can still be examined.

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References

[1] J. S. Bell, On the Einstein Podolsky Rosen paradox, Physics 1, 195-200 (1964).

[2] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Proposed experiment to test local hidden-variable theories, Phys. Rev. Lett. 23, 880-884 (1969).

[3] J. F. Clauser and M. A. Horne, Experimental consequences of objective local theories, Phys. Rev. D 10, 526-535 (1974).

[4] A. Aspect, Bell’s Theorem: The naive view of an experimentalist, Quantum [Un]speakables-From Bell to Quantum information, edited by R. A. Bertlmann and A. Zeilinger (Springer, 2002).

[5] R. A. Bertlmann, Entanglement, Bell inequalities and decoherence in particle physics, Lecture Notes in Physics 689, 1-45 (Springer, Berlin, Heidelberg, 2006).

[6] Yi-Bing Ding, Jun-Li Li, and Cong-Feng Qiao, Bell inequalities in high energy physics, High Ener. Phys. Nucl. Phys. 31, 1086 (2007).
[7] Jun-Li Li and Cong-Feng Qiao, Testing local realism in $P \rightarrow VV$ decays, Science China: Physics, Mechanics & Astronomy 53, 870-875 (2010).

[8] N. A. Törnqvist, Suggestion for Einstein-Podolsky-Rosen experiments using reactions like $e^+e^- \rightarrow \Lambda\Lambda \rightarrow \pi^-p\pi^+\bar{p}$, Found. Phys. 11, 171-177 (1981).

[9] S. P. Baranov, Bell’s inequality in charmonium decays $\eta_c \rightarrow \Lambda\Lambda$, $\chi_c \rightarrow \Lambda\Lambda$ and $J/\psi \rightarrow \Lambda\Lambda$, J. Phys. G: Nucl. Part. Phys. 35, 075002 (2008).

[10] S. Chen, Y. Nakaguchi, and S. Komamiya, Testing Bell’s inequality using charmonium decays, Prog. Theor. Exp. Phys. 063A01, 1-13 (2013).

[11] M. Ablikim, et al., Polarization and entanglement in baryonantibaryon pair production in electronpositron annihilation, Nat. Phys. 15, 631-634 (2019).

[12] A. Afrial and F. Selleri, The Einstein, Podolsky and Rosen Paradox in Atomic, Nuclear, and Particle Physics (Springer Science+Business Media, New York, 1999).

[13] B. C. Hiesmayr, Limits of quantum information in weak interaction processes of hyperons, Sci. Rep. 5, 11591 (2015).

[14] R. A. Bertlmann, A. Bramon, G. Garbarino, and B. C. Hiesmayr, Violation of a Bell inequality in particle physics experimentally verified? Phys. Lett. A 332, 355-360 (2004).

[15] M. A. Nielsen and I. L. Chuang, Quantum computation and quantum information, (Cambridge Universtiy Press 2010).

[16] M. Tanabashi, et al. Particle Data Group, Phys. Rev. D 98, 030001 (2018).
[17] Y. Shi and Ji-Chong Yang, Entangled baryons: Violation of inequalities based on local realism assuming dependence of decays on hidden variables, arXiv:1912.04111