An MHD Study of Large-Amplitude Oscillations in Solar Filaments

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Abstract
Quiescent filaments are usually affected by internal and/or external perturbations triggering oscillations of different kinds. In particular, external large-scale coronal waves can perturb remote quiescent filaments leading to large-amplitude oscillations. Observational reports have indicated that the activation time of oscillations coincides with the passage of a large-scale coronal wavefront through the filament, although the disturbing wave is not always easily detected. Aiming to contribute to understanding how—and to what extent—coronal waves are able to excite filament oscillations, here we modelled with 2.5D magnetohydrodynamic simulations a filament floating in a gravitationally stratified corona disturbed by a coronal shock wave. This simplified scenario results in a two-coupled-oscillation pattern of the filament, which is damped in a few cycles, enabling a detailed analysis. A parametric study was carried out varying parameters of the scenario such as height, size, and mass of the filament. An oscillatory analysis reveals a general tendency for periods of oscillations, amplitudes, and damping times to increase with height, whereas filaments of larger radius exhibit shorter periods and smaller amplitudes. The calculation of forces exerted on the filament shows that the main restoring force is the magnetic tension.

Keywords Prominences, quiescent · Magnetohydrodynamics · Magnetic fields, corona

1. Introduction
Prominence seismology is a powerful diagnostic tool that combines the observation of filament oscillations with theoretical models to infer coronal-plasma parameters and analyse the dynamical response associated with the restoring forces involved in the motion (see, e.g., Tripathi, Isobe, and Jain, 2009; Arregui, Oliver, and Ballester, 2018). In particular, large-amplitude oscillations of filaments can be classified into longitudinal oscillations, with the motion parallel to the filament’s axis, and transverse oscillations, moving perpendicular to the axis direction. Transverse oscillations can also be divided into horizontal and vertical oscillations, and they have been related to the winking-filament phenomenon observed in Hα.
Ranges of periods, velocity amplitudes, and damping times of 11–29 minutes, 6–41 km s\(^{-1}\), and 25–180 minutes (Shen et al., 2014b), respectively, have been reported for transverse oscillations. Whereas longitudinal oscillations are associated with larger periods, velocity amplitudes, and damping times in the ranges of 44–160 minutes, 30–100 km s\(^{-1}\), and 115–600 minutes, respectively. In this work we will use the term filament as a synonym for prominence.

The determination of driving mechanisms for large-amplitude oscillations is still a subject of research. Nevertheless, several studies have suggested that the triggering disturbances are fast magnetohydrodynamic (MHD) shocks, such as chromospheric Moreton waves (Eto et al., 2002; Francile et al., 2013), and coronal EUV waves produced by distant flares and/or coronal mass ejections (Okamoto et al., 2004; Isobe and Tripathi, 2006; Asai et al., 2012; Shen and Liu, 2012; Shen et al., 2014a,b). Subflares or jets are also able to produce large-amplitude oscillations (Jing et al., 2003; Luna and Moreno-Insertis, 2021). Francile et al. (2013) pointed out that the activation time of two distant winking filaments coincided with the passage of a large-scale wavefront through them, which was not detected at coronal heights. Also, Shen et al. (2014a), analysing four winking filaments excited by a weak coronal EUV wave coming from an X-flare region, suggested that the oscillation parameters resulted from the normal-mode excitation of each filament triggered by a single disturber. Previously, Ramsey and Smith (1966) proposed that the oscillation parameters depend on the intrinsic properties of the filament rather than the external disturbance features. In fact, if the external driver is a pulse, i.e. not a forced-oscillation mechanism, the system should respond to the perturbation with its natural frequency. Moreover, considering the fluctuation-dissipation theorem (Kubo, 1966), small-amplitude oscillations will be spontaneously excited at the natural frequencies of the system.

Several observational studies on the properties of filament oscillations have contributed to the understanding of the physical mechanisms involved in the motion. For instance, Shen et al. (2014b) suggested that a filament oscillates like a linear vertical solid body with one end tied to the solar surface. The analysis performed by Liu et al. (2013) and Pant et al. (2015) on the measurements of transverse oscillations also supported the idea that filaments oscillate as a whole. However, Hershaw et al. (2011), by analysing this type of arched filament oscillations, suggested that the filament presented a global kink mode, although there were some discrepancies indicating that the filament oscillates as a collection of separated but interacting threads rather than as a rigid body. Concerning the forces, Shen et al. (2014b) proposed that the restoring forces of the transverse oscillations are most likely due to the coupling of gravity and magnetic tension of the supporting magnetic field. Furthermore, by analysing transverse oscillations, Gilbert et al. (2008) sustained the hypothesis that the main restoring force is the magnetic tension.

From another perspective, several numerical studies have been performed to contribute to the understanding of prominence seismology. Among several models available to emulate filament dynamics, the main ones are two simplified cases associated with the magnetic structures of Kippenhahn and Schlüter (1957) and Kuperus and Raadu (1974). In the first model, the filament is sustained by the magnetic-field lines (magnetic dip), whereas in the second one it is totally contained in the interior of a closed helical magnetic structure that isolates the filament mass. For example, performing 3D MHD simulations, Adrover-González and Terradas (2020) analysed filament oscillations generated by instantaneous velocity perturbations in a relaxed system based on the Kippenhahn–Schlüter model, and identified the magnetic force as the restoring process for transverse oscillations. Also, they showed that the periods increase with the filament density and width, but decrease with the magnetic-field strength. On the other hand, in line with Kuperus and Raadu, Zhou et al.
(2018) carried out 3D ideal MHD simulations of a filament in an initially relaxed state, where the oscillations were triggered by perturbing the velocity field independently in each direction. These authors found that the periods of horizontal and vertical transverse oscillations are different, and they attributed this result to the shape of the filament, which is wider in the vertical direction. In addition, they determined for transverse oscillations that the main restoring force is the magnetic tension. Liakh, Luna, and Khomenko (2020) used 2.5D simulations to model oscillations excited independently by horizontal and vertical internal perturbations, and also by an external perturbation. In the former case, considering a single filament, the authors found a weak dependence of the transverse oscillation periods on the density contrast and on the shear angle, suggesting that the periods are almost constant with height. For the latter case, they found that the displacement and deformation of the magnetic-field lines produced by a coronal shock wave generates the filament oscillation. In turn, Luna and Moreno-Insertis (2021) by 2.5D MHD modelling demonstrated that coronal jets are capable of producing large-amplitude oscillations. Alternatively, in an analytical study representing a filament by a current-currying wire, which is similar to the present model, Kolotkov, Nisticò, and Nakariakov (2016) developed a linear model of the transverse oscillations of a filament, determining periods and stability conditions. That article was followed by a weakly nonlinear and a fully nonlinear study considered by Kolotkov et al. (2018). Beyond the differences between the models or the proposed disturbances, there is a consensus that the magnetic force plays an important role as a restoring force characterising the oscillations. However, the analysis of oscillation properties is a topic still open to discussion.

With regard to the damping of oscillations, several theoretical mechanisms were proposed. An important aspect is the interaction between a moving filament and the ambient coronal plasma. This motion gives rise to a process where a transfer of energy from large scales to small ones takes place until it finally dissipates due to viscous effects at smaller scales. Viscous processes were also described by Hyder (1966), and the damping contribution by the emission of magnetoacoustic waves, known as wave leakage, was described by Kleczek and Kuperus (1969) and van den Oord and Kuperus (1992). For example, Schutgens and Tóth (1999), considering filament oscillations in terms of a solid body, suggested that vertical oscillations would lead to fast-wave emission carrying momentum away from the filament and so damping the motion, whereas horizontal oscillations would lead to the emission of slow waves, but in a less effective process. Arregui, Oliver, and Ballester (2018) noticed that emitted slow waves propagating along magnetic-field lines are unable to take energy out of an environment with closed magnetic lines. Under these conditions, wave leakage is only possible by fast waves. Another important damping mechanism is the resonant-wave process (see, e.g., Goossens, Erdélyi, and Ruderman, 2011), where the energy of the global oscillation is transferred to an inhomogeneous transition layer between the filament and the solar corona. For instance, Hershaw et al. (2011) observationally studying large-amplitude transverse oscillations of a filament, where periods and damping times were determined for different heights, found that the damping times showed a linear dependence on the periods, where the resonant absorption was suggested as the main damping mechanism. In particular, through 3D numerical simulations Terradas et al. (2016) showed evidence that the energy transfer from global oscillations to continuum Alfvén modes at the filament’s edge is the main damping mechanism.

With the aim of contributing to the understanding of the quiescent-filament oscillations, in this article we perform 2.5D ideal MHD simulations. Our motivation is to study large-amplitude, transverse oscillations driven by an external coronal shock wave produced by a distant event, e.g. a jet or a flare. For this purpose, we consider different scenarios that
vary the height, size, and mass of the filament. The interaction between the filament and the coronal shock wave is analysed by means of the main motion characteristics, i.e. period of oscillation, amplitudes, damping time, and restoring forces.

The article is organised as follows: In Section 2 we present the governing equations, the solar-atmospheric scenario, and the magnetic model used to represent the filament, detailing how the initial equilibrium conditions were found in a stratified medium with variable gravity. The main features of the numerical code used to run different configurations and the perturbing method are developed. In Section 3 we present the results and discussion. We start with the analysis of the filament dynamics in terms of the forces involved and continue with a parametric study evaluating the different configurations. Finally, the main conclusions are laid out in Section 4.

2. The Model

In numerical experiments, we study the transverse oscillatory properties of quiescent filaments triggered by a large-scale coronal wave coming from a remote site. The scenario is depicted in Figure 1. The quiescent filament is assumed to be located in a quiet coronal region and long enough that it can be adequately represented by a 2.5D modelling in a perpendicular plane of symmetry. Based on the model of Forbes (1990), the scenario includes: a filament floating in the corona at a certain height, a variable gravity, and a stratified coronal background in hydrostatic equilibrium, with the chromosphere and a thin transition region at the base. The large-scale coronal-wave perturbation is emulated by a blast. This scenario allows us to carry out an oscillatory analysis, measuring periods of oscillations, amplitudes, and damping times.

An ideal MHD scenario is considered with a compressible plasma in the presence of a gravitational field. In conservative form, the ideal MHD equations read (CGS units):

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,
\]
\[
\frac{\partial \rho v}{\partial t} + \nabla \cdot \left( \rho v \otimes v - \frac{1}{4\pi} B \otimes B \right) = -\nabla p + \frac{1}{c^2} \mathbf{j} \times B + \rho \mathbf{g},
\]

\[
\frac{\partial B}{\partial t} + \nabla \cdot (v \otimes B - B \otimes v) = 0,
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot \left[ (E + p + \frac{B^2}{8\pi})v - \frac{1}{4\pi} B (v \cdot B) \right] = \rho \mathbf{g} \cdot v,
\]

(1)

where \( \rho \) is the mass density, \( v \) is the plasma velocity, \( B \) is the magnetic field, \( j \) is the current density, and \( g \) is the acceleration of gravity. Also, \( E \) is the total energy, \( e \) is the internal energy, and \( p \) is the thermal pressure, whose expressions are

\[
E = \frac{1}{2} \rho v^2 + e + \frac{B^2}{8\pi},
\]

\[
e = \frac{p}{(\gamma - 1)},
\]

\[
p = \frac{R_g}{\mu} \rho T,
\]

(2)

with \( T \) the plasma temperature, \( \gamma = 5/3 \), and \( R_g \) the gas constant. The ideal plasma is assumed to be fully ionised and with a solar abundance of 70.7% H + 27.4% He + 1.9% heavier elements (Prialnik, 2000), which imply a mean atomic mass \( \bar{\mu} = 0.613 \). Ampère’s law is \( j = \frac{c}{4\pi} \nabla \times B \), with \( c \) the speed of light. Note that when the MHD Equations 1 are written in conservative form, the tensor products \( v \otimes v, v \otimes B, B \otimes v \), and the magnetic part of the Maxwell stress tensor \( B \otimes B \) appear in the second and third expressions.

2.1. Stratified Atmosphere

To describe our model setup, two reference systems with different origins were used. First, a Cartesian coordinate system \((x, y, z)\) was used to describe the setup as a whole, with its origin fixed at the base of the chromosphere, the \( y \)-axis pointing radially away from the solar surface and the \( x \)- and \( z \)-axes parallel to the surface (neglecting the surface curvature). Second, to describe the filament structure locally, it is convenient to use a cylindrical coordinate system \((r, \phi, z')\) with its origin fixed at the filament’s centre, for the radial, azimuthal, and axial directions, respectively; see Figure 1. Note that as the \( z' \)- and \( z \)-axes are parallel, the \( z' \) coordinate can be omitted.

The simulated solar atmosphere is in hydrostatic equilibrium with gravitational stratification in the vertical direction including a temperature profile for the chromosphere, transition region, and corona. The chromosphere has a height \( h_{cr} = 2.5 \) Mm with a constant temperature \( T_{cr} = 1.2 \times 10^4 \) K, the transition region extends to a height \( h_{tr} = 0.5 \) Mm with a linearly varying temperature, and the corona is based at \( y = h_{cr} + h_{tr} \) extending upwards with a constant temperature \( T_0 = 1 \) MK. Therefore, the adopted atmospheric temperature is

\[
T(y) = \begin{cases} 
T_{cr} & 0 \leq y \leq h_{cr}, \\
\frac{(T_0 - T_{cr})}{h_{tr}} (y - h_{cr}) + T_{cr} & h_{cr} < y < h_{cr} + h_{tr}, \\
T_0 & y \geq h_{cr} + h_{tr}.
\end{cases}
\]

(3)
The initial pressure due to the hydrostatic equilibrium condition, \( \frac{dp}{dy} = -\rho g \), is

\[
p(y) = \begin{cases} 
  p \exp \left( \frac{\alpha}{R\circ} \left[ \frac{1}{y + R\circ} - \frac{1}{h_{cr} + R\circ} \right] \right) & 0 \leq y \leq h_{cr}, \\
  p_0 \exp \left( -\alpha \int_{h_{cr} + h_u}^{y} \frac{dy'}{T'(y') (y' + R\circ)^2} \right) & h_{cr} < y < h_{cr} + h_u, \\
  p_0 \exp \left( \frac{\alpha}{R\circ} \left[ \frac{1}{y + R\circ} - \frac{1}{h_{cr} + h_u + R\circ} \right] \right) & y \geq h_{cr} + h_u,
\end{cases}
\]  

(4)

with the reference pressure \( p_0 = \frac{R\mu}{\mu} \rho_0 T_0 \) fixed at the coronal base and

\( p_u = p_0 \exp \left( -\alpha \int_{h_{cr} + h_u}^{h_{cr}} \frac{dy'}{T'(y') (y' + R\circ)^2} \right) \)

(5)

being the extrapolated pressure at the base of the transition region. The gravity acceleration, \( g(y) = -\frac{GM\odot}{(y + R\circ)^2} \hat{j} \), \( \alpha = \frac{\mu GM\odot}{Rg} \), is a constant, where \( G \) is the gravitational constant, \( M\odot \) is the solar mass, and \( R\circ \) is the solar radius. The density \( \rho \) and internal energy \( e \) in the atmosphere are determined by the Equations 2. Finally, the integral in Equations 4 and 5 has a closed analytic solution if \( T(y) \) is a linear function, which is

\[
\int_{y_0}^{y} \frac{dy'}{T'(y') (y' + R\circ)^2} = \frac{1}{R\circ a - b} \left( \frac{1}{y + R\circ} - \frac{1}{y_0 + R\circ} \right) + \frac{a}{(R\circ a - b)^2} \ln \left( \frac{T(y) y_{0} + R\circ}{T(y_0) y + R\circ} \right).
\]

(6)

Constants \( a \) and \( b \) are obtained by writing the linear temperature variation of the transition region in Equation 3 as \( T(y) = ay + b \), having \( a \approx \frac{T_0 - T_{cr}}{h_u} \) and \( b \approx T_{cr} - ah_{cr} \).

2.2. The Filament

In line with Forbes (1990), but adding the gravity force, the flux rope is modelled as an electric current-carrying wire of radius \( R \) floating in the corona. The magnetic configuration consists of three components: the current-carrying wire located at height \( y = h \); a mirror current at depth \( y = -h \); and a line dipole of relative strength \( M_{dip} \) at depth \( y = -d \), representing the photospheric field. The initial magnetic field, considering each contribution respectively, is:

\[
B_x(x, y) = -\frac{(y - h)}{r_-} B_\phi(r_-) + \frac{(y + h)}{r_+} B_\phi(r_+) + \frac{(x^2 - (y + d)^2)}{r_d^4} M_{dip} d R_3 B_\phi(R_3),
\]

\[
B_y(x, y) = \frac{x}{r_-} B_\phi(r_-) - \frac{x}{r_+} B_\phi(r_+) + \frac{2x(y + d)}{r_d^4} M_{dip} d R_3 B_\phi(R_3),
\]

\[
B_z(x, y) = B_z(r),
\]

(7)

with \( r_- \), \( r_+ \), and \( r_d \) being the distances from the filament, the mirror current, and the dipole, respectively, to the observed point, where

\[
r_\pm = \sqrt{x^2 + (y \pm h)^2},
\]

(8)
\[ r_d = \sqrt{x^2 + (y + d)^2}. \] (9)

\( B_\phi, B_z, R_3, \) and \( M_{\text{dip}} \) are defined next.

We assume a transition layer of thickness \( \Delta \) to relate the internal structure of a filament of radius \( R \) with the outer coronal region. Thus, three filament zones are distinguished: an inner zone \([\text{Z1}]\) for radius \( 0 \leq r \leq R_2 \); a transition zone \([\text{Z2}]\) for radius \( R_2 < r < R_3 \); and the external zone \([\text{Z3}]\) for \( r \geq R_3 \). High-resolution observations have revealed the existence of inhomogeneous fine structures of the filaments (Lin et al., 2005), making it reasonable to include in the model a non-uniform layer between the filament inner zone and the coronal external zone (e.g. as done by Soler et al., 2009, and references therein).

The assumed electric-current density inside the filament is \( j(r) = j_\phi \hat{\phi} + j_z \hat{k} \), with components:

\[
j_z(r) = \begin{cases} 
  j_0 & \text{in } \text{Z1}, \\
  \frac{j_0}{2} \left( \cos \left[ \frac{\pi}{\Delta} (r - R_2) \right] + 1 \right) & \text{in } \text{Z2}, \\
  0 & \text{in } \text{Z3}, 
\end{cases}
\] (10)

\[
j_\phi(r) = \begin{cases} 
  \frac{2 \pi j_0}{c} r & \text{in } \text{Z1}, \\
  \frac{2 \pi j_0}{c} \frac{1}{r} \left( \frac{r_2^2 + R_3^2 - (\frac{\Delta}{2})^2}{\frac{r_2^2}{2} + \frac{R_3^2}{2} - (\frac{\Delta}{2})^2} \cos \left[ \frac{\pi}{\Delta} (r - R_2) \right] \right) & \text{in } \text{Z2}, \\
  \frac{2 \pi j_0}{c} \frac{1}{r} \left( \frac{R_3^2}{2} + \frac{R_2^2}{2} - 2 (\frac{\Delta}{2})^2 \right) & \text{in } \text{Z3}, 
\end{cases}
\] (11)

and the axial component is

\[
B_z(r) = \begin{cases} 
  \frac{\sqrt{8 \pi}}{c} j_1 \sqrt{R_2^2 - r^2} & \text{in } \text{Z1}, \\
  0 & \text{in } \text{Z2}, \\
  0 & \text{in } \text{Z3}. 
\end{cases}
\] (12)

The initial thermal pressure inside the filament, intended to be in equilibrium as close as possible with its neighbourhood and satisfying \( \nabla p - \frac{1}{c} j \times B \approx 0 \), results in

\[
p(r) = p_c(r) - \frac{1}{c} \int_r^\infty j_\phi(r') B_z(r') \, dr' + \frac{1}{c} \int_r^\infty j_z(r') B_\phi(r') \, dr',
\] (13)

where \( p_c \) is the background coronal pressure given by the last expression of Equation 4. Considering the different shells in the radial direction throughout the filament, the pressure is

\[
p(r) = \begin{cases} 
  p_c(r) + \frac{\pi}{c^2} \left( j_0^2 - j_1^2 \right) (R_2^2 - r^2) + \frac{1}{c} \int_{R_1}^{R_2} j_z(r') B_\phi(r') \, dr' & \text{in } \text{Z1}, \\
  p_c(r) + \frac{1}{c} \int_{R_2}^{R_3} j_z(r') B_\phi(r') \, dr' & \text{in } \text{Z2}, \\
  p_c(r) & \text{in } \text{Z3}. 
\end{cases}
\] (14)
This pressure balance was obtained taking into account the azimuthal magnetic field $B_\phi$ generated by the filament, written in Equation 11, but neglecting the external contributions to the azimuthal component by the mirror wire and the dipole. The prescribed filament temperature is

$$T(r) = \begin{cases} T_{\text{fil}} & \text{in Z1,} \\ \frac{(T_0 - T_{\text{fil}})}{\Delta_1} (r - R_2) + T_{\text{fil}} & \text{in Z2,} \\ T_0 & \text{in Z3,} \end{cases}$$

(15)

with the constant inner temperature of the filament $T_{\text{fil}} = 0.3$ MK. Finally, the density $\rho(r)$ of the filament is obtained by introducing the expressions for the pressure and temperature, Equations 14 and 15, into Equation 2.

Figure 2 shows an enlargement of the initial conditions for a particular filament case floating in the stratified atmosphere. The density pattern is shown in Panel a, and the thermal pressure with the total magnetic-field lines superimposed is shown in Panel b.

### 2.3. Equilibrium Condition

In order to establish equilibrium conditions, the forces acting on the filament are analysed. The forces are: the upward force exerted by the mirror current $F_{\text{mir}}$, and the downward forces of the dipole $F_{\text{dip}}$ and the weight $F_g$. Thus, the initial total force $[F]$, per unit length $[L]$, in the vertical direction is

$$\frac{F}{L} = F_{\text{mir}} - F_{\text{dip}} - F_g$$

$$\approx \frac{1}{c^2} \frac{I^2}{h} - 2 \frac{Im}{c^2 (h + d)^2} - (m_{\text{fil}} - m_{\text{buo}}) g,$$

(16)

where approximations were made on the magnetic-force expressions and are valid for a small radius $[R]$. $I$ is the electric current flowing inside the filament and its mirror, $m$ is
the dipole strength, \( m_{\text{fil}} \) is the mass of the filament (per unit length), and \( m_{\text{buo}} \) is the background mass enclosed in a filament-like volume. The weight term includes the buoyancy force \( m_{\text{buo}} g \), which is smaller than the weight: \( \approx 1\% \). The axial electric current going through the transverse filament area \( A_{\text{fil}} \) is

\[
I = \int_{A_{\text{fil}}} j \cdot dA' = \frac{\pi j_0}{2} \left( R_3^2 + R_2^2 - \left( \frac{2\Delta}{\pi} \right)^2 \right).
\]

To satisfy the balance \( (F = 0) \) the following relation must hold:

\[
h \frac{d}{d} = \frac{(M_{\text{dip}} - (1 - M_g)) \pm \sqrt{M_{\text{dip}}^2 - 2M_{\text{dip}}(1 - M_g)}}{1 - M_g},
\]

which gives the equilibria in terms of the height and depth as a function of two parameters: \( M_{\text{dip}} \) and \( M_g \). The dimensionless parameter \( M_{\text{dip}} = \frac{m_{\text{Id}}}{L} \) gives the relative strength between the dipole force \( [F_{\text{dip}}] \) and the mirror force \( [F_{\text{mir}}] \). Similarly, \( M_g = \frac{c^2 h}{j} F_g \) is the relative strength between the weight and the mirror forces. Solutions with minus (plus) signs represent stable (unstable) equilibria (Forbes, 1990). An example of these equilibria is shown in Figure 3, where the ratio \( \frac{h}{d} \) between the filament height and the dipole depth is plotted as a function of \( M_{\text{dip}} \). The black-solid portion of the line represents stable locations while the black-dashed portion represents unstable ones. To get an idea of the weight contribution, the red curve represents the equilibria neglecting the weight force. Note that the weight force allows us to consider smaller values of the relative dipole strength \( M_{\text{dip}} \), i.e. considering gravity the minimum stable location occurs for \( M_{\text{dip}} = 1.24 \), while neglecting gravity the minimum stable location occurs for \( M_{\text{dip}} = 2 \). In Figure 3, the equilibria neglecting the weight force represented in red colour are equivalent to the stable equilibrium case 11 of Forbes (1990).

When searching for equilibrium configurations, the mass of the filament plays an important role, but its value \( m_{\text{fil}} = \int_{V_{\text{fil}}} \rho(r') \, dV' \) does not have a closed expression. Therefore, we obtain the following approximated expression

\[
m_{\text{fil}} / L \approx \frac{1}{T_{\text{fil}}} \left[ \pi \rho_0 T_0 R^2 + \epsilon R^4 \left( j_0^2 - \frac{j_1^2}{2} \right) \right],
\]

with \( \epsilon = \frac{\pi \tilde{\mu}}{2R_k c^2} \) being a constant, which is helpful in understanding the mass dependence on the setup parameters. The filament mass increases for larger values of the radius \( R \) and the axial current density \( j_1 \), while it decreases with \( j_0 \). In addition, the mass is inversely proportional to its temperature \( T_{\text{fil}} \), resulting in heavier filaments at colder temperatures (and vice versa). To a lesser degree the mass also depends on the coronal background density \( \rho_0 \) and temperature \( T_0 \).

### 2.4. Perturbation

After a relaxation period, the filament was perturbed using a blast mechanism (Balsara, 2004) placed far from it, the domain’s border. This configuration emulates a standing quiescent filament perturbed by an external large-scale coronal wave coming from a remote energetic flaring site. More details on the simulations will be given below in Section 2.5.1. The wavefront, with a small inclination angle from the horizontal axis towards the solar surface, hits the filament and simultaneously excites horizontal and vertical transverse oscillations. The downward inclination of the wave is in line with the observations (Liu et al.,
Figure 3  The locations of the equilibria for the ratio $h/d$ as a function of the relative strength of the dipole $M_{\text{dip}}$. The black-solid portion of the line represents stable equilibria, while the black-dashed portion unstable ones. The red (solid and dashed) lines display the same example, but neglecting the gravity. The equilibria indicated by the black lines and the stable point chosen at $M_{\text{dip}} = 2.25$ correspond specifically to Case 3 defined in Section 2.5.2.

2013). An inclination angle of $\approx 7^\circ$ was used in all cases. The instantaneously applied blast was imposed at $t = 30.08$ minutes, placed at coordinates $(x, y) = (95, h + 6)$ Mm, expressed as a function of filament height $h$, with a diameter of 20 Mm, and the blast released an energy of $7 \times 10^{19}$ erg. The average propagation speed of the perturbing wavefront was 486 km s$^{-1}$, which in terms of Alfvén Mach number $[M_A]$, the wavefront speeds (for the different cases considered, see Section 2.5.2 and Figure 4), were $M_A = 1.8$ for filaments with small radius and $M_A = 0.85$ for large-radius filaments. They were both measured at coordinates $(x, y) = (49, 19)$ Mm, where the ambient magnetic-field magnitude behind the wave was 3.0 G in Case 3 and 5.8 G in Case 4.

Zhou et al. (2018) did not use simulations with shock waves as external perturbations of the filament, by assuming that a perturbation of the velocity field would be representative of anything that results in a bulk motion of the filament. Following this idea, as an alternative to the blast mechanism, we have also tried to perturb the velocity field inside and around the filament, which is much less demanding numerically. This mechanism is supposed to mimic the impulsive velocity phase exerted by the blast on the filament. However, we found that the resulting oscillations were not equivalent to those obtained from the blast, the main difference being that the damping is stronger when the velocity mechanism was used. Hence, to model large-scale coronal-wave perturbations, we discarded the velocity mechanism.

### 2.5. Numerical Simulations

#### 2.5.1. Code

To carry out 2.5D ideal-MHD numerical simulations, the set of Equations 1 was solved using the FLASH code (Dubey et al., 2014, release 4.5). This code uses Godunov-type schemes in a co-located regular grid with finite volumes to solve the coupled compressible MHD equations with adaptive mesh refinement (AMR) and high-performance computing capabilities.

For our simulations, we chose the unsplit, staggered-mesh solver with a second-order monotone upstream centred-scheme for conservation laws (MUSCL)-type reconstruction. This solver, which is based on the scheme by Lee and Deane (2009), uses a directionally unsplit technique for the evaluation of the numerical fluxes and implements the constrained-transport method and the corner-transport-upwind method for the treatment of the magnetic field, which leads to a better numerical behaviour of the divergence free condition ($\nabla \cdot B = 0$). The Riemann problems at interfaces of computational cells were calculated using Roe’s solver and the MC slope-limiter was used for reconstructions of cell-centred variables.
Distance–magnetic-field plot along the thick-dashed line of Figure 2a. The values correspond to the magnetic-field magnitude in the plane \((x, y)\) for the filaments of Case 3 (black-dashed line) with small radius \(R = 2.5\) Mm and Case 4 (red-solid line) with large radius \(R = 3.5\) Mm.

The 2D physical domain was delimited by \([-120, 120]\) Mm \(\times\) \([0, 120]\) Mm and discretised by a Cartesian grid with eight levels of refinement with \(20 \times 10\) cells per block, giving a maximum spatial resolution of \(\delta x = \delta y = 0.094\) Mm. The boundary conditions were set as follows: for the left and right borders zero-gradient (outflow) conditions were chosen for the thermodynamic variables and velocity, allowing the waves to leave the domain without reflection. For the magnetic field at the lateral and upper ends, we extrapolated the initial condition to ghost cells in order to avoid spurious magnetic forces generated by the violation of the force-free condition of the border background magnetic field. Obviously, this implementation is valid provided that the magnetic perturbations do not reach these borders, although relatively small numerically induced forces can be tolerated since the region of interest is at the filament location. The lower and upper borders require more detailed treatment due to the presence of gravity and the tying of the magnetic-field lines to the chromosphere. For the vertical extrapolation of the thermodynamic variables, we implemented the constant-temperature hydrostatic boundary conditions proposed by Krause (2019), while the zero-gradient condition was imposed for the velocity at the upper end. On the other hand, the line-tied boundary conditions proposed by Robertson and Priest (1987) were applied to the magnetic field and velocity at the bottom.

Taking into account that our study requires an accurate model of the filament equilibrium at the unperturbed condition, we needed to improve the ability of the numerical code to preserve the hydrostatic equilibrium. As explained by Krause (2019), the traditional MUSCL-type scheme is unable to satisfy that condition since the vertical exponential decay of the pressure cannot be accurately approximated by a polynomial reconstruction. In addition, the numerical pressure gradient is not cancelled by the gravitational source term when a simple cell-centred evaluation is used. Consequently, spurious momentum fluxes are numerically induced producing non-physical velocities. To avoid this undesirable behaviour, we implemented the local hydrostatic reconstruction scheme and improved the equilibrium preservation during the simulation.

2.5.2. Simulations

The simulations consisted of two stages in total. The first was the relaxation period, which started with the initial condition \(t_0 = 0\) (see Figure 2) and lasted for 30 minutes to allow an equilibrium adjustment between the filament and its background environment. This time lapse is necessary because initially the filament is not in exact equilibrium. After this time, the system reaches a pseudo-equilibrium position that is stationary enough to be considered
still. At the end of this first period, the non-equilibrium velocity of the filament is smaller than 1 km s$^{-1}$, which means an order of magnitude smaller than the velocity exerted by the blast. The second stage corresponds to the oscillating scenario, which starts when the blast is discharged, at $t_{\text{pert}} = 30.08$ minutes, and lasts until the simulation ends. During this stage, the shock wave generated by the blast passed through the filament and the oscillating motion is triggered.

Aiming to make a seismological study of the system as a function of height, size, and mass of the filament, seven simulations were performed: six of them for three different heights ($h = [14, 19, 25]$ Mm) and two radii ($R = [2.5, 3.5]$ Mm), and the remaining one in which the mass was varied by means of the current density $j_1$. These three parameters were assumed to be the independent ones, while the remaining parameters, i.e. $j_0$, $\rho_0$, $\Delta$, $M_{\text{dip}}$, $T_0$, $T_{\text{cr}}$, $T_{\text{fil}}$, $h_{\text{cr}}$, and $h_{\text{tr}}$, have fixed values for all cases. The details for each case and the fixed parameters are listed in Table 1 (Set-up parameters). For all cases: $j_0 = 600$ statAmpere cm$^{-2}$ (statA cm$^{-2}$); $\rho_0 = 3.2 \times 10^{-15}$ g cm$^{-3}$; $\Delta = 1.25$ Mm; and $M_{\text{dip}} = 2.25$. Note that the azimuthal current density $j_1 = 600$ statA cm$^{-2}$ is the same in all cases, except in Case 5 that is equal to $j_1 = 797.5$ statA cm$^{-2}$. It should be taken into account that the black equilibrium lines displayed in Figure 3 are different for all cases.

For all cases, the dipole depth $[d]$ and strength $[m]$ are dependent parameters determined by the balance Equation 18. Thus, to set up equilibrium configurations, higher filaments correspond to deeper magnetic dipoles, but interestingly with larger strengths $[m]$. On the other hand, increasing the filament radius $[R]$ implies larger values of the magnetic field in the filament surroundings (as displayed in Figure 4) and larger repelling mirror magnetic forces $[F_{\text{mir}}]$. In addition, a larger radius produces a shallower dipole with greater strength $[m]$, and also increases the filament mass $[m_{\text{fil}}]$, thus balancing the repelling mirror force. Finally, given the total pressure balance between the filament’s interior and exterior by Equation 14, a rise in the azimuthal current density $[j_1]$, as in Case 5, results in a decrease in the filament mass, generating a shallower dipole with smaller strength $[m]$.

3. Results and Discussion

3.1. Dynamics

As a result of the interaction with the shock wave, the filament exhibits a damped oscillatory motion coupled in the horizontal and vertical directions. Figure 5 displays the evolution of the wavefront–filament interaction (see the accompanying animation). Panel a shows the shock wave triggered by the blast, travelling at a coronal level and disturbing the chromosphere. In the figure, the black lines represent the total magnetic field and the colour map the thermal pressure. On the right side, the presence of waves compressing the plasma and deforming the magnetic field can be seen. At $t = 33$ minutes, both the main coronal wavefront and its subsequent chromospheric reflection impact the filament and destabilise its equilibrium state (see Panel b). The main wavefront hits the filament with a small downward inclination angle. Then the shock wave continues propagating beyond the filament position, dragging it towards the bottom-left direction pointed by the arrow in Panel b. Later on, the filament starts to oscillate during a few cycles (Panels c–d). By $t = 42.5$ minutes the external perturbation has left the domain.

To analyse the oscillatory motion of the filament, the displacement and velocity of the centre of mass were measured as a function of time for each case. The horizontal and vertical components are represented by $\Delta x$, $\Delta y$, $v_x$, and $v_y$, respectively. Afterwards, the displacements were fitted using an exponentially decayed harmonic function
Figure 5  Temporal evolution of the interaction between the quiescent filament and a large-scale coronal wave coming from a remote site. The images correspond to the enlarged snapshots of Case 3. (a) In the colour map, the pressure is plotted at time $t = 32$ minutes and before the filament–shock interaction begins. The black lines in (a) display the field lines of the total magnetic field. (b) It is shown just a moment after the interaction begins at $t = 35$ minutes. The arrow points to the inclination angle of the shock and the dashed line represents the horizontal direction. (c) When the filament oscillation reaches its maximum left horizontal displacement at $t = 41$ minutes. Here, the vertical dashed line indicates the slice used to evaluate the pressure stack Figure 9b. (d) Same as (c) but for the maximum right horizontal displacement at $t = 59$ minutes. (See the animation in the Electronic Supplementary Material accompanying this figure.)
Figure 6  The filament oscillations for Case 3. (a–b) The horizontal and vertical displacements are shown as a function of time, just before and after the interaction was triggered: the black-squared lines represent the simulation data, while the red lines represent the fitted curves. Note that the displacements are relative to the initial position. (c–d) Corresponding horizontal and vertical oscillation speeds. 

\[ f_i(t) = A_i \sin \left( \frac{2\pi (t-t_{fit})}{P_i} + \phi \right) \exp \left( -\frac{t-t_{fit}}{\tau_i} \right) + C_i(t - t_{fit}) + f_{0i}, \]

where \( f_i \) stands for the displacements \( \Delta x \) and \( \Delta y \), \( A_i \) is the amplitude of the oscillation, \( P_i \) is the period, \( \tau_i \) is the decay constant, \( C_i \) is the slope, \( f_{0i} \) is the initial position, and \( t_{fit} = 35 \text{ minutes} \) is the initial fitted time. The index \( i \) indicates the fitted variable. The results obtained for all cases are exhibited in Table 1 (Fitted Parameters).

As an example, Figure 6 shows in black-squared lines the displacements and speeds measured for Case 3 together with the fitted curves in red lines. The speed fits are obtained from the temporal derivative of the fitted displacement functions \( \frac{df_i(t)}{dt} \); however, the corresponding speed amplitudes displayed in Table 1 are calculated as half the difference between the minimum and maximum values. The fits show that there is a main frequency that describes the oscillation with a certain degree of accuracy. Note that the displacements and speeds are nearly zero before the interaction begins and then show a damped oscillation with the vertical motion exhibiting a small upward slope. This upward slope is mainly due to a slight decrease in filament density with time due to numerical diffusion. Comparing the left Panels a–c with the right ones b–d, it is clearly seen that the horizontal period is larger than the vertical one. In the same way, the horizontal damping time is larger than the vertical one (see Table 1). This behaviour is qualitatively similar for all cases.

Figure 7 displays the trajectories for Cases 3 and 4 during the oscillating stage. As the displacements refer to the initial position, both trajectories begin almost at the origin, indicated by their corresponding times \( t = 30 \text{ minutes} \). The numerical points are arranged se-
quentially in time, indicating the temporal evolution. Comparing the trajectories with those determined analytically by Kolotkov et al. (2018, see their Figures 3 and 4) using a model similar to ours, we can see that the trajectories in Figure 7 resemble Lissajous-like curves of an hourglass shape, suggesting that the coupled transverse oscillations are close to resonant regimes. Note that the patterns shown in Figure 7 are distorted with respect to the symmetric Lissajous-like curves because the filament is finite; it oscillates embedded in an inhomogeneous plasma; and due to the action of dissipative effects (numerical diffusion and damping, see Section 3.1.1). Not all simulated cases exhibit a Lissajous-like pattern, especially Case 5, which is strongly damped.

3.1.1. Numerical Diffusion and Damping

It is important to verify that the filament’s attenuated oscillations (e.g. as seen in Figure 6 for Case 3) are not due to artificial viscosity produced by truncation errors of the numerical scheme used, the magnitude of which depends on the order of accuracy of the method and is generally reduced by increasing the grid resolution. However, a direct evaluation of the numerical diffusion is not a simple task because the modified set of Equations 1 (i.e. the governing equations affected by the truncation-error terms of the numerical method used) is too difficult to obtain for high-order MUSCL type schemes. Numerical estimation is also difficult since the diffusion effects are different along the domain due to gradient changes and variations in spatial resolutions when using AMR meshes.

For the analysis, we chose Case 3 as the reference run and performed simulations at different spatial resolutions to evaluate changes in the numerical results. If the damping time is a consequence of numerical diffusion, it will be shorter for coarser grids, because the numerical-diffusion coefficient increases when the grid resolution is reduced. This means that the filament motion should be damped in fewer cycles for coarser grids than for finer ones. Figure 8 displays the numerical results for the displacements using four different progressively refined grids ($\delta x = \delta y = [0.37, 0.19, 0.094, 0.059]$ Mm), corresponding to relatively coarser grids of 300%, 100%, and 0%, with respect to the standard resolution of Case 3 (see Section 2.5.1), and a relatively finer grid of −40%. The AMR meshes used for the runs 300%, 100%, and Case 3 consisted of six, seven, and eight levels of refinement, respectively, with $20 \times 10$ cells per block; while the −40% run consisted of eight levels with $32 \times 16$ cells per block. Taking into account that the upward slopes exhibited by the vertical displacements $\Delta y$, which are smaller at higher grid resolutions, are due to the numerical imbalances between the filament and its background and do not affect the oscillatory part of the motion, we conclude that the numerical results of Case 3 are not significantly influenced by the artificial viscosity. In fact, we can see that there is no direct correlation between the damping level and the grid resolution, since, for example, the motion along the $x$-axis for the reference run damps a bit faster than for the coarser ones. Therefore, for all cases studied with the reference grid resolution, we can assume that numerical diffusion has a minimal impact on the damping process and can reasonably be ignored.

With regards to the filament’s attenuated oscillations, since we do not consider non-adiabatic processes (as radiative losses or heat conduction) and according to the geometry of the problem, we discuss two possible damping mechanisms: resonant absorption and wave leakage.

First, we consider the resonant absorption of infinitely long thread oscillations (see, e.g., Arregui, Oliver, and Ballester, 2018, and references therein). This mechanism relies on the energy transfer from transverse kink modes to small-scale Alfvén waves because of the plasma inhomogeneity located at the transition layer between the filament and the surrounding corona. Assuming a small transition layer ($\Delta / R \ll 1$), analytical expressions for the
Figure 7 Filament trajectories during the oscillating stage corresponding to Case 3 (Panel a) and Case 4 (Panel b). The displacements are referred to the initial positions \((x, y) = (0, h)\). The sequential numeric annotations indicate the times (in minutes) corresponding to those positions, helping to understand the temporal evolution.

Figure 8 Test to determine whether the damped motion exhibited by the filament is due to numerical diffusion or has a physical origin. Four runs were made varying the spatial resolution, corresponding to relatively coarser grids of 300%, 100%, and 0% with respect to the standard resolution of Case 3, and a relatively finer grid of −40%. We concluded that the actual damping process has a significant physical meaning. Panels a and b show the horizontal and vertical displacements for the different resolutions, respectively.

damping time scale for the kink modes were obtained (see, e.g., Goossens, Hollweg, and Sakurai, 1992; Goossens, Andries, and Aschwanden, 2002; Ruderman and Roberts, 2002), claiming a linear dependence between periods and damping times. In order to explore a possible relation, we fitted a linear function \(\tau = aP + b\) using the values in Table 1. Figure 9a shows the damping times versus periods for horizontal (circles) and vertical (diamond) oscillations. Case 5 is pointed out with a cross and an asterisk. The error bars are in grey, for the periods these errors are comparable with the symbol size. The small circles and diamonds correspond to cases with filament radius \(R = 2.5\) Mm and the large symbols to \(R = 3.5\) Mm. The black-dashed line displays the linear fit with \(a = (1.6 \pm 0.5)\) and \(b = (20.8 \pm 13.7)\) minutes. For the analysis, we excluded the damping times of Cases 6 and 7 because they are out of range. This crescent-shaped linear tendency looks similar to the behaviour addressed by Hershaw et al. (2011), suggesting that resonant absorption may act as a damping mechanism. As a complementary study, taking into account the work of Terradas et al. (2016), in which they found an energy enhancement in a thin region near the filament’s edge attributed
Table 1  Setup and fitted parameters for all numerical cases. Varying parameters: the initial filament height $h$, filament radius $R$, azimuthal current density $j_1$, and maximum simulation time $t_{\text{max}}$. $\bar{\rho}_{\text{fil}}$ is the averaged filament density. Fixed parameters for all cases: the axial current density $j_0$, density at the coronal base $\rho_0$, filament transition layer $\Delta$, dipole intensity $M_{\text{dip}}$, coronal temperature $T_0$, chromospheric temperature $T_{\text{cr}}$, filament temperature $T_{\text{fil}}$, chromospheric width $h_{\text{cr}}$, and transition region width $h_{\text{tr}}$. Fitted parameters in the horizontal $x$-axis: the period $P_x$, displacement amplitude $A_x$, the damping time $\tau_x$, and the speed amplitude $A_{v_x}$. The same is displayed for the vertical $y$-axis.

|                  | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 | Case 7 |
|------------------|--------|--------|--------|--------|--------|--------|--------|
| **Setup parameters** |        |        |        |        |        |        |        |
| $h$ [Mm]         | 14     | 14     | 19     | 19     | 25     | 25     |        |
| $R$ [Mm]         | 2.5    | 3.5    | 2.5    | 3.5    | 2.5    | 2.5    | 3.5    |
| $j_1$ [statA cm$^{-2}$] | 600    | 600    | 600    | 600    | 797.5  | 600    | 600    |
| $t_{\text{max}}$ [minutes] | 120    | 120    | 120    | 120    | 120    | 150    | 150    |
| $\bar{\rho}_{\text{fil}}$ [g cm$^{-3} \times 10^{-13}$] | 4.0    | 7.2    | 4.0    | 7.2    | 2.5    | 4.0    | 7.1    |
| $j_0$ [statA cm$^{-2}$] |        |        |        |        | 600    |        |        |
| $\rho_0$ [g cm$^{-3}$] |        |        |        |        | 3.2 $\times 10^{-15}$ |        |        |
| $\Delta$ [Mm] |        |        |        |        | 1.25   |        |        |
| $M_{\text{dip}}$ | 2.25   |        |        |        |        |        |        |
| $T_0$ [MK]      | 1      |        |        |        |        |        |        |
| $T_{\text{cr}}$ [K] | 1.2 $\times 10^4$ |        |        |        |        |        |        |
| $T_{\text{fil}}$ [MK] | 0.3    |        |        |        |        |        |        |
| $h_{\text{cr}}$ [Mm] | 2.5    |        |        |        |        |        |        |
| $h_{\text{tr}}$ [Mm] | 0.5    |        |        |        |        |        |        |
|                  |        |        |        |        |        |        |        |
| **Fitted parameters** |        |        |        |        |        |        |        |
| $P_x$ [minutes] | 21.0   | 22.6   | 44.9   | 35.3   | 26.9   | 66.9   | 51.7   |
| $A_x$ [Mm]      | 6.0    | 1.5    | 11.2   | 3.1    | 15.2   | 22.2   | 6.7    |
| $\tau_x$ [minutes] | 22.1   | 56.2   | 63.6   | 92.8   | 16.1   | 197.2  | 354.4  |
| $A_{v_x}$ [km s$^{-1}$] | 19.8   | 7.2    | 27.1   | 9.9    | 36.0   | 29.9   | 13.5   |
| $P_y$ [minutes] | 13.7   | 12.5   | 18.8   | 16.1   | 15.6   | 27.5   | 23.0   |
| $A_y$ [Mm]      | 2.7    | 1.4    | 3.3    | 2.2    | 5.1    | 2.6    | 2.9    |
| $\tau_y$ [minutes] | 32.7   | 47.5   | 45.7   | 27.1   | 17.4   | 66.8   | 53.7   |
| $A_{v_y}$ [km s$^{-1}$] | 15.6   | 8.9    | 14.5   | 9.1    | 18.7   | 6.7    | 9.7    |

to resonant absorption, we explored the evolution of the kinetic energy in a shell surrounding the filament’s core ($R_2 < r < 5$ Mm). The results did not show a definite kinetic-energy enhancement associated with the surrounding shell that would support the linear tendency found in Figure 9a. Thus, more studies are required to investigate the extent to which the resonant absorption may be responsible for the oscillation damping.

Second, to study whether the wavefronts observed emanating from the filament (such as in Figures 5c–d) are evidence of a wave leakage, we constructed a pressure stack plot for Case 3 for each measured time along a vertical slice located at $(x, y) = (0, 35–65)$ Mm (highlighted by a dashed line in Figure 5c). The stack-plot results are shown in Figure 9b, where the wave signatures propagating with speeds of approximately $52$ km s$^{-1}$ at an early time, and later with $\approx 25$ km s$^{-1}$ and $\approx 8$ km s$^{-1}$, can be seen. Also, the period of these waves, which can be...
Figure 9 (a) Damping times versus periods for the cases exhibited in Table 1 except for Cases 6 and 7. The values for the horizontal (vertical) direction are displayed with circle (diamond) symbols. Case 5 is displayed with cross symbols. The width of symbols are according to the radius of the filament. The grey lines correspond to the errors of the variables. The black-dashed line shows a linear fit. (b) Pressure stack plot of Case 3 along the vertical dashed line in Figure 5d. The black-solid lines outline the speed of the slow waves propagating away from the filament. The colour map represents ln(p).

deduced from the separation in time between the fronts, is \( \approx 20 \) minutes, hence associated with the periods of the filament oscillation. On the slice domain, the spatial average sound speed \( \bar{c}_s = 150 \text{ km s}^{-1} \), the average Alfvén speed \( \bar{v}_A = 185 \text{ km s}^{-1} \) and the average plasma \( \bar{\beta} = 0.94 \). From Figures 5c–d and Figure 9b we identify compressive, slow magneto-acoustic waves emanating from the filament with an oblique propagation angle with respect to the magnetic field, which seems to be the reason why the wavefront speeds are markedly smaller than the sound speed. Thus, it seems that the filament acts as a forcing driver during oscillation, transferring energy into the surrounding plasma. On the other hand, the filament has lost \( \approx 3\% \) of its mass to the surrounding environment during the oscillation stage. Thus, we conclude that the main process responsible for the damping of oscillations seems to be the wave leakage.

3.1.2. Restoring Forces

Figures 10a–b display the calculation of the thermal-pressure force \( -\nabla p \) (blue lines) and the Lorentz force \( \frac{1}{c} j \times B \), decomposed into the magnetic tension \( \frac{B \cdot \nabla B}{4\pi} \) (cyan lines) and the magnetic-pressure force \( -\nabla (\frac{B^2}{8\pi}) \) (red lines), along the horizontal and vertical axes, respectively. The green lines represent the net forces. Also, as a reference, in black lines the filament displacements were added with an appropriate scale, to facilitate a visual comparison. The Figure 10 corresponds to Case 3 and displays the averaged forces over the grid cells belonging to the filament structure \( (r < R_3) \) evolving in time. With respect to the horizontal direction, Panel a shows that, before the interaction begins (at \( t < 33 \) minutes), all forces are almost null, as expected. Then, the forces behave noisily during a very short time, which is due to the strong interaction with the shock waves. Later, the magnetic tension becomes the main restoring force (more smoothly drawn by the net force in green colour). Meanwhile, at early times, the thermal-pressure force seems to follow the displacement, and
the magnetic-pressure force does not play a significant role. Regarding the forces acting in the vertical direction, Panel b shows that the magnetic tension is always positive, and points upward, opposing the almost constant and negative weight force (orange line). The correspondence between the magnetic tension and the vertical displacement is noticeable. Note that the main restoring force in the upward direction is the magnetic tension, while in the downward direction it is the weight. In turn, the thermal- and magnetic-pressure forces play secondary roles, the thermal-pressure force becomes negative for later times, while the magnetic-pressure forces pushes upward until it finally disappears, by $t \approx 80$ minutes.

### 3.2. Parametric Study

Figure 11 summarises the fitted parameters of all cases for the horizontal and vertical displacements (also listed in Table 1). The left (right) panel corresponds to the $x$- ($y$-)axis. In each panel, the circles indicate the periods as a function of height, and the colour map describes the amplitudes of the oscillation. The small circles represent the triad of Cases 1 – 3 – 6 with small filament radius $R = 2.5$ Mm, connected by dashed lines; while the large circles indicate the triad of Cases 2 – 4 – 7 with large radius $R = 3.5$ Mm, linked by dotted lines. We also show the damping times, depicted by squared symbols following the same scheme. Case 5, with a different filament mass, is highlighted with different symbols, a star for the period and a diamond for the damping time.

In the following we itemise the main trends obtained for the seven cases. Beginning with the $x$-direction represented by Figure 11a:

i) Comparing cases of the same height but different radii: large radii are generally associated with smaller periods, smaller amplitudes (darker blue), and larger damping times than small radius cases. This tendency is seen comparing the pairs of Cases 1 – 2, 3 – 4, and 6 – 7, with the exception of the case pair 1 – 2 with similar periods.

ii) Comparing cases of the same radius but different heights: larger heights are associated with larger periods, larger amplitudes, and larger damping times. This occurs for the triads of Cases 1 – 3 – 6 and 2 – 4 – 7.
iii) Comparing cases of the same radius and height but different filament masses: Case 5 shows a smaller period, a larger amplitude, and a considerably smaller damping time than the heavier Case 3.

With respect to the $y$-direction (see Figure 11b), the fitted parameters show similar trends as just described for the $x$-direction, but with a less regular behaviour:

i) Comparing cases of the same height but different radii: the pairs of Cases 1–2, 3–4, and 6–7 show that the increase of the radius implies smaller periods, smaller amplitudes (except the pair 6–7), and smaller damping times (except the pair 1–2).

ii) Comparing cases of the same radius but different heights: the triads of Cases 1–3–6 and 2–4–7 show that the increase in height commonly leads to larger periods, larger amplitudes (except the pair 3–6), and larger damping times (except the pair 2–4).

iii) Comparing cases of the same radius and height but different filament masses: Case 5 exhibits a smaller period, a larger amplitude, and a somewhat smaller damping time than the heavier Case 3.

Also, comparing the results between the horizontal and vertical oscillations, we note that the horizontal amplitudes, periods, and damping times are larger than the vertical ones, in agreement with Schutgens and Tóth (1999).

To summarise, there is a general tendency of periods, displacement amplitudes, and damping times to increase with height. This tendency is clearly observed for horizontal oscillations, although it is not always valid for vertical ones. The same behaviour is followed by the velocity amplitudes. The general tendency to increase with height is in agreement with the fact that the magnitude of the net force acting on the filament, along with its restoring frequency, decreases with height (the net-force frequency can be seen in Figure 10).
Increasing the radius generally implies shorter periods and smaller amplitudes. However, while this clearly implies longer damping times for horizontal oscillations, we found shorter damping times for vertical oscillations. Note that increasing the radius produces an increase in the net force frequency acting on the filament, which explains the shorter periods found. Finally, exploring variations of the filament mass, while keeping the remaining independent parameters of the setup fixed, the lower mass Case 5 exhibits shorter periods and larger amplitudes in both axes, although more noticeable is that Case 5 shows a stronger damped motion.

As described above, the horizontal and vertical oscillation properties of the filament are, in general, different. Thus, projection effects along the line-of-sight may play an important role in the results of observations using the Doppler technique to measure the oscillation properties. For instance, in the framework drawn in Figure 1, a particular observer whose line-of-sight is that of a prominence seen at the limb ($\phi = 0$) will measure the horizontal component of the oscillation; whereas an observer with a top-down view of a filament at the centre of the solar disc ($\phi = 90$) will measure the vertical component. The Doppler velocity measured by intermediate observers corresponds to the filament velocity vector projected along the line-of-sight of the observer. Therefore, as the winking phenomenon related to the Doppler shift is more likely to be detected in filaments with large velocity amplitudes, in the framework of our results this corresponds to Cases $[1, 3, 5, 6]$ and with lines-of-sight near the limb, that is when the horizontal-velocity component, which is frequently larger than the vertical one, contributes more to the Doppler effect.

4. Conclusions

In order to contribute to the understanding of the oscillatory phenomenon of quiescent filaments, we have performed a parametric study of large-amplitude, transverse oscillations carrying out 2.5D ideal MHD simulations. The model of Forbes (1990) was adapted to numerically represent a filament floating in a gravitationally stratified atmosphere. After the initialisation of the setup and a relaxation process, the quiescent filament was perturbed using a blast device that excites coupled horizontal and vertical oscillations for a few cycles.

Although our model differs from that of other authors, either in the magnetic configuration (Kippenhan and Schlüter or Kuperus and Raadu) or in the perturbation mechanism, in analyzing the forces we have also found that the main restoring force is the magnetic tension. As suggested by Shen et al. (2014b), the main restoring force along the vertical direction is the coupling between gravity and magnetic tension, whereas along the horizontal direction the main force is magnetic tension. In agreement with Zhou et al. (2018), we found that the magnetic pressure plays a secondary role and the gas-pressure contribution is negligible.

The oscillation properties present different behaviours along the horizontal and vertical axes. The periods, amplitudes, and most of the damping times are larger in the horizontal direction. The filament exhibits a vertical-oscillation frequency about twice that of the horizontal (listed in Table 1), corresponding to coupled transverse oscillations that in some cases seem close to a resonant regime (see Figure 7), as indicated by the resemblance of the trajectory pattern to the Lissajous-like curves obtained by Kolotkov et al. (2018). This behaviour may indicate that the vertical motion is a reaction to the horizontal oscillation, as pointed out by Liakh, Luna, and Khomenko (2020).

Also, our results (Cases 1 and 5) are in good agreement with the results of Tripathi, Isobe, and Jain (2009) and Shen et al. (2014b) who found a range of periods of 11 – 29 minutes, velocity amplitudes of $6 – 41$ km s$^{-1}$, and damping times of 25 – 180 minutes. However,
Cases 6 and 7 present some agreement with the events studied by Pant et al. (2015), who reported values of 61–67 minutes, 12–22 km s$^{-1}$, and 92–117 minutes, respectively.

In order to explore the role of the different parameters on the oscillatory motion, a parametric study was carried out performing seven simulations varying the height, size, and mass of the filament. The fitted parameters revealed, as a first general rule, that periods, amplitudes, and damping times increase with height (Figure 11). This tendency is clearly observed in the horizontal direction, but has some exceptions in the vertical direction. Given that both the net force exerted on the filament and its restoring frequency decrease with height, this results in larger periods and amplitudes measured at higher filaments.

This tendency for periods to increase with height partially agrees with the analytical model of Kolotkov, Nisticò, and Nakariakov (2016, see their Figure 3), which considers uncoupled linear oscillations. In both, the vertical periods increase with height, whereas for the horizontal periods these authors found a tendency to diminish with height.

Our first general tendency is in agreement with observational cases, for example: Francile et al. (2013) analysed a filament with a low height of $\approx 7$ Mm (Gilbert et al., 2008) oscillating with a short period of $\approx 4$ minutes; Shen et al. (2014b) analysing a higher filament (40 Mm) measured a larger period of $\approx 13$ minutes and an amplitude of 10 Mm; moreover, Liu et al. (2013) reported oscillations of an even higher filament (63 Mm) with an even larger period of $\approx 30$ minutes and an amplitude of 22 Mm. Considering the damping time, this trend is also seen in observational studies: Gosain and Foullon (2012) for a filament at a height of 36 Mm reported a transverse oscillation period of 28 minutes and a damping time of 44 minutes. Following the increasing tendency, Liu et al. (2013) measured global oscillations in a filament of height 63 Mm determining a period of 30 minutes and a damping time of 47 minutes. This trend is also followed by the filament studied in the EUV by Hershaw et al. (2011), who for a height of 80 Mm reported a period of 100 minutes and a damping time of 240 minutes.

As a second general rule, the results in both directions showed that larger filaments are associated with shorter periods and smaller amplitudes. As before, this tendency partially agrees with the linear model of Kolotkov, Nisticò, and Nakariakov (2016); taking into account that in our model an increase in filament radius implies an increase in filament current (Equation 17), these authors concluded that vertical periods diminish with larger filament currents but, conversely, horizontal periods increase with current. On the other hand, it is more difficult to compare this tendency with observations due to the lack of joint measurements of radius, period, and amplitude. It seems that as larger filaments are subjected to stronger net forces and higher restoring frequencies, it would imply that larger filaments oscillate with shorter periods and smaller amplitudes.

Concerning the damping mechanism obtained from the simulations, we examined the main processes discussed by several authors and found that wave leakage seems to be the main damping mechanism.

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Declarations

Disclosure of Potential Conflicts of Interest The authors declare that they have no conflicts of interest.

References

Adrover-González, A., Terradas, J.: 2020, 3D numerical simulations of oscillations in solar prominences. Astron. Astrophys. 633, A113. DOI. ADS.

Arregui, I., Oliver, R., Ballester, J.L.: 2018, Prominence oscillations. Liv. Rev. Solar Phys. 15, 3. DOI. ADS.

Asai, A., Ishii, T.T., Isobe, H., Kitai, R., Ichimoto, K., UeNo, S., Nagata, S., Morita, S., Nishida, K., Shiota, D., Oi, A., Akioka, M., Shibata, K.: 2012, First simultaneous observation of an Hα Moreton wave, EUV wave, and filament/prominence oscillations. Astrophys. J. Lett. 745, L18. DOI. ADS.

Balsara, D.S.: 2004, Second-order-accurate schemes for magnetohydrodynamics with divergence-free reconstruction. Astrophys. J. Suppl. 151, 149. DOI. ADS.

Dubey, A., Antypas, K., Calder, A.C., Daley, C., Fryxell, B., Gallagher, J.B., Lamb, D.Q., Lee, D., Olson, K., Reid, L.B., Rich, P., Ricker, P.M., Riley, K.M., Rosner, R., Siegel, A., Taylor, N.T., Weide, K., Timmes, F.X., Vladimirova, N., ZuHone, J.: 2014, Evolution of FLASH, a multi-physics scientific simulation code for high-performance computing. Int. J. High Perform. Comput. Appl. 28, 225. DOI.

Eto, S., Isobe, H., Narukage, N., Asai, A., Morimoto, T., Thompson, B., Yashiro, S., Wang, T., Kitai, R., Kurokawa, H., Shibata, K.: 2002, Relation between a Moreton wave and an EIT wave observed on 1997 November 4. Publ. Astron. Soc. Japan 54, 481. DOI. ADS.

Forbes, T.: 1990, Numerical simulation of a catastrophe model for coronal mass ejection. J. Geophys. Res. 95, 11919.

Francile, C., Costa, A., Luoni, M.L., Elaskar, S.: 2013, Hα Moreton waves observed on December 06, 2006. A 2D case study. Astron. Astrophys. 552, A3. DOI. ADS.

Gilbert, H.R., Daou, A.G., Young, D., Tripathi, D., Alexander, D.: 2008, The filament-Moreton wave interaction of 2006 December 6. Astrophys. J. 685, 629. DOI. ADS.

Goossens, M., Andries, J., Aschwanden, M.J.: 2002, Coronal loop oscillations. An interpretation in terms of resonant absorption of quasi-mode kink oscillations. Astron. Astrophys. 394, L39. DOI. ADS.

Goossens, M., Erdélyi, R., Ruderman, M.S.: 2011, Resonant MHD waves in the solar atmosphere. Space Sci. Rev. 158, 289. DOI. ADS.

Goossens, M., Hollweg, J.V., Sakurai, T.: 1992, Resonant behaviour of magnetohydrodynamic waves on magnetic flux tubes - Part three. Solar Phys. 138, 233. DOI. ADS.

Gosain, S., Foullon, C.: 2012, Dual trigger of transverse oscillations in a prominence by EUV fast and slow coronal waves: SDO/AIA and STEREO/EUVI observations. Astrophys. J. 761, 103. DOI. ADS.

Harrison, C., Krishnan, H.: 2012, Python’s role in VisIt. In: Ahmadia, A., Millman, J., van der Walt, S. (eds.) 11th Python in Science Conf. (scipy 2012). DOI.

Hershaw, J., Foullon, C., Nakariakov, V.M., Verwichte, E.: 2011, Damped large amplitude transverse oscillations in an EUV solar prominence, triggered by large-scale transient coronal waves. Astron. Astrophys. 531, A53. DOI. ADS.

Hyder, C.L.: 1966, Winking filaments and prominence and coronal magnetic fields. Zeit. Astrophys. 63, 78. ADS.

Isobe, H., Tripathi, D.: 2006, Large amplitude oscillation of a polar crown filament in the pre-eruption phase. Astron. Astrophys. 449, L17. DOI. ADS.

Jing, J., Lee, J., Spiroock, T.J., Xu, Y., Wang, H., Choe, G.S.: 2003, Periodic motion along a solar filament initiated by a subflare. Astrophys. J. Lett. 584, L103. DOI. ADS.

Kippenhahn, R., Schlüter, A.: 1957, Eine Theorie der solaren Filamente. Zeit. Astrophys. 43, 36. ADS.

Kleczek, J., Kuperus, M.: 1969, Oscillatory phenomena in quiescent prominences. Solar Phys. 6, 72. DOI. ADS.
Kolotkov, D.Y., Nisticò, G., Nakariakov, V.M.: 2016, Transverse oscillations and stability of prominences in a magnetic field dip. *Astron. Astrophys.* 590, A120. DOI. ADS.

Kolotkov, D.Y., Nisticò, G., Rowlands, G., Nakariakov, V.M.: 2018, Finite amplitude transverse oscillations of a magnetic rope. *J. Atmos. Solar-Terr. Phys.* 172, 40. DOI. ADS.

Krause, G.: 2019, Hydrostatic equilibrium preservation in MHD numerical simulation with stratified atmospheres. Explicit Godunov-type schemes with MUSCL reconstruction. *Astron. Astrophys.* 631, A68. DOI. ADS.

Kubo, R.: 1966, The fluctuation-dissipation theorem. *Rep. Prog. Phys.* 29, 255. DOI. ADS.

Kuperus, M., Raadu, M.A.: 1974, The support of prominences formed in neutral sheets. *Astron. Astrophys.* 31, 189. ADS.

Lee, D., Deane, A.E.: 2009, An unsplit staggered mesh scheme for multidimensional magnetohydrodynamics. *J. Comp. Phys.* 228, 952. DOI. ADS.

Liakh, V., Luna, M., Khomenko, E.: 2020, Numerical simulations of large-amplitude oscillations in flux rope solar prominences. *Astron. Astrophys.* 637, A75. DOI. ADS.

Lin, Y., Engvold, O., Rouppe van der Voort, L., Wiik, J.E., Berger, T.E.: 2005, Thin threads of solar filaments. *Solar Phys.* 226, 239. DOI. ADS.

Liu, R., Liu, C., Xu, Y., Liu, W., Kliem, B., Wang, H.: 2013, Observation of a Moreton wave and wave-filament interactions associated with the renowned X9 flare on 1990 May 24. *Astrophys. J.* 773, 166. DOI. ADS.

Luna, M., Moreno-Insertis, F.: 2021, Large-amplitude prominence oscillations following impact by a coronal jet. *Astrophys. J.* 912, 75. DOI. ADS.

Okamoto, T.J., Nakai, H., Keiyama, A., Narukage, N., UeNo, S., Kitai, R., Kurokawa, H., Shibata, K.: 2004, Filament oscillations and Moreton waves associated with EIT waves. *Astrophys. J.* 608, 1124. DOI. ADS.

Pant, V., Srivastava, A.K., Banerjee, D., Goossens, M., Chen, P.-F., Joshi, N.C., Zhou, Y.-H.: 2015, MHD seismology of a loop-like filament tube by observed kink waves. *Res. Astron. Astrophys.* 15, 1713. DOI. ADS.

Prialnik, D.: 2000, *An Introduction to the Theory of Stellar Structure and Evolution*, Cambridge University Press, Cambridge. ADS.

Ramsey, H.E., Smith, S.F.: 1966, Flare-initiated filament oscillations. *Astron. J.* 71, 197. DOI. ADS.

Robertson, J.A., Priest, E.R.: 1987, Line-tied magnetic reconnection. *Solar Phys.* 114, 311. DOI. ADS.

Ruderman, M.S., Roberts, B.: 2002, The damping of coronal loop oscillations. *Astrophys. J.* 577, 475. DOI. ADS.

Schutgens, N.A.J., Tóth, G.: 1999, Numerical simulation of prominence oscillations. *Astron. Astrophys.* 345, 1038. ADS.

Shen, Y., Liu, Y.: 2012, Evidence for the wave nature of an extreme ultraviolet wave observed by the atmospheric imaging assembly on board the solar dynamics observatory. *Astrophys. J.* 754, 7. DOI. ADS.

Shen, Y., Ichimoto, K., Ishii, T.T., Tian, Z., Zhao, R., Shibata, K.: 2014a, A chain of winking (oscillating) filaments triggered by an invisible extreme-ultraviolet wave. *Astrophys. J.* 786, 151. DOI. ADS.

Shen, Y., Liu, Y.D., Chen, P.F., Ichimoto, K.: 2014b, Simultaneous transverse oscillations of a prominence and a filament and longitudinal oscillation of another filament induced by a single shock wave. *Astrophys. J.* 795, 130. DOI. ADS.

Soler, R., Oliver, R., Ballester, J.L., Goossens, M.: 2009, Damping of filament thread oscillations: effect of the slow continuum. *Astrophys. J. Lett.* 695, L166. DOI. ADS.

Terradas, J., Soler, R., Luna, M., Oliver, R., Ballester, J.L., Wright, A.N.: 2016, Solar prominences embedded in flux ropes: morphological features and dynamics from 3D MHD simulations. *Astrophys. J.* 820, 125. DOI. ADS.

Tripathi, D., Isobe, H., Jain, R.: 2009, Large amplitude oscillations in prominences. *Space Sci. Rev.* 149, 283. DOI. ADS.

van den Oord, G.H.J., Kuperus, M.: 1992, The effect of retardation on the stability of current filaments. *Solar Phys.* 142, 113. DOI. ADS.

Zhou, Y.H., Xia, C., Keppens, R., Fang, C., Chen, P.F.: 2018, Three-dimensional MHD simulations of solar prominence oscillations in a magnetic flux rope. *Astrophys. J.* 856, 179. DOI. ADS.

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Large-Amplitude Oscillations in Solar Filaments

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