The dynamical Casimir effect and energetic sources for gamma ray bursts

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On the analogy of the Casimir effect, we present an effect of quantum-field fluctuations, attributed to gravitational field coupling to the zero-point energy of virtual particles in the vacuum. In the process of black hole’s formation, such an effect could cause tremendous energy release, possibly describing a scenario of energetic sources for observed gamma ray bursts.

PACS numbers: 04.62.+v, 04.70.Bw, 04.70.Dy

The mystery of energetic sources generating gamma ray bursts, a prompt emission of extremely huge energy, has stimulated many studies in connection with electromagnetic properties of black holes. Various astrophysical scenarios are discussed in literatures. We present an alternative idea and scenario on the basis of quantum-field fluctuations, attributed to quantum fluctuations of gravitationally collapsing matter. In arbitrary coordinate systems, the equivalence principle is modified by the gravitational field around a Schwarzschild geometry, resulting in gravitationally collapsing matter, and as a result, an attractive force between two plates is observed as the Casimir effect. Virtual particles in the vacuum are not in mass-shell, and as a result, an attractive force between two plates is observed as the Casimir effect. Virtual particles in the vacuum are not in mass-shell, and as a result, an attractive force between two plates is observed as the Casimir effect.

In quantum field theories, the vacuum is composed of virtual particles whose energy (the vacuum energy or zero-point energy) \( \delta E \) does not vanish. The Casimir effect shows that the vacuum energy \( E \) is modified by boundary conditions and the vacuum gains the Casimir energy,

\[
\delta E_c = -\frac{\pi^2}{720a^3} < 0,
\]

where \( a \) is the distance between two plates. Thus the vacuum becomes energetically unstable and has to quantize mechanically. This leads to releasing the Casimir energy, and as a result, an attractive force between two plates is observed as the Casimir effect.

In this short letter, we consider how the vacuum energy \( E \) is modified by the gravitational field around a gravitationally collapsing matter \( M \), described by the Schwarzschild geometry,

\[
ds^2 = -g(r)dt^2 + g^{-1}(r)dr^2 + r^2d\Omega,
\]

where \( g(r) = (1 - \frac{2M}{r}) \) and \( \Omega \) is the spherical solid angle. Virtual particles in the vacuum are not in mass-shell, and as a result, an attractive force between two plates is observed as the Casimir effect.

In local inertial coordinate systems \( (\bar{r}, \bar{t}) \), where gravitational field is absent, the variations of their energy (\( \delta E_c \)) are described by the Heisenberg uncertainty relationship:

\[
\Delta t \Delta \delta E_c \simeq 1.
\]

In arbitrary coordinate systems, the equivalence principle tells us that the Heisenberg uncertainty relationship is unaffected by the presence of a gravitational field:

\[
\Delta \delta E_c \simeq 1.
\]

These relationships and the gravitational time dilation between local inertial and arbitrary coordinate systems \( \Delta \delta E = g^+(r)\Delta t \) lead us to obtain:

\[
\Delta E = g^+(r)\Delta E_0, \quad \delta E = g^+(r)\delta E_0,
\]

where \( g^+(r) \) (see Eq.2) is the gravitational red-shift factor. This indicates that vacuum energy-level \( E \) and its width \( \Delta E \) of virtual particles are gravitationally red-shifted from corresponding vacuum energy-level \( E_0 \) and its width \( \Delta E_0 \) of virtual particles in the absence of gravitational field. This difference between \( E \) and \( E_0 \) is originated from gravitational field interacting with vacuum energy \( (E_0) \) of virtual particles in the vacuum. The vacuum gains the energy \( E - E_0 < 0 \) from gravitational field. This result implies that the vacuum energy \( E \) varies from one spatial point \( (r_1) \) to another \( (r_2) \),

\[
\delta E = E_1 - E_2 = \Delta E \Delta t \simeq 1.
\]

This could be in principle examined by measuring the Casimir effect at different altitudes above the Earth.

At two different altitudes \( r_2 \) and \( r_1 \) above the Earth, Eq.5 implies the Casimir energy \( \delta E_0 \) should be modified by the gravitational field of the Earth in the following way:

\[
|\Delta E_0| = \left( \frac{g(r_2)}{g(r_1)} \right)^\frac{1}{2} |\Delta E_0|, \quad g(r) = 1 - \frac{2M_\oplus}{r},
\]

where \( M_\oplus \) is the mass of the Earth. This indicates that \( |\Delta E_0| > |\Delta E_0| \) for \( r_2 > r_1 \). Given \( r_1 = r_\oplus, \quad M_\oplus \approx 7.1 \times 10^{-12} \) and \( \Delta r = r_2 - r_1 = 10^8 \text{cm} \), we obtain

\[
|\Delta E_0| = |1 + O(10^{-12})| \Delta E_0.
\]

Test of this very small energy-gain, modifying the Casimir energy and force, seems to be very difficult for current experiments. In the Newtonian regime \( \min(r_2, r_1) \gg 2M \), and \( \delta r = r_2 - r_1 \ll \min(r_2, r_1) \), the variation of the vacuum energy Eq.5 is proportional to \( M\delta r/r^2 \ll 1 \), in practice, too small to be tested.
However, such vacuum energy variations could be enormous, in a gravitational collapse approaching to the formation of black hole’s horizon. To analyze this, at the first we simply model the gravitational collapse as a massive star of mass $M$ and radius $R$ undergoing a spherical collapse. In the gravitationally collapsing process, the surface of the star moves inwards $\delta R > 0$ from

\[
\text{the step 1: the radius star } = R + \delta R, 
\] (9)

to

\[
\text{the step 2: the radius star } = R, 
\] (10)
in the time interval $\delta t$. At the step (9), $M$ is the mass distributed in the sphere $r < R + \delta R$ of the volume $\frac{4\pi}{3} (R + \delta R)^3$; $M'$ is the mass distributed in the sphere $r < R$ of the volume $\frac{4\pi}{3} R^3$. $\delta M = M - M'$ is the mass distributed in the spherical shell $R < r < R + \delta R$ of the volume $4\pi R^2 \delta R$. At the step (10), the mass $\delta M$ falls into the sphere $r \leq R$, and total mass $M$ distributed within the sphere $r \leq R$. Assuming that the matter density is of uniform distribution in this gravitational collapse process, we can compute $\delta M$

\[
\delta M = M (1 - \frac{R^3}{(R + \delta R)^3}). 
\] (11)

Such a gravitational collapse process $\delta R/\delta t$ can be described by the equation $[7]$

\[
\delta t = - \frac{2Mh(R)}{g(R)\sqrt{h^2(R) - g(R)}} \delta R, 
\] (12)
\[
h(R) = 1 - \frac{2M}{4R}. 
\]

At the second, we study how vacuum energy varies in this gravitational collapse process. In the absence of gravitational field (inertial frame), we introduce the surface vacuum-energy on the surface $r = R$ of the area $4\pi R^2$:

\[
\mathcal{E}_s \simeq 4\pi R^2 \Lambda_p^3, 
\] (13)

where $\Lambda_p = \sqrt{\frac{\hbar c}{4\pi G}} = 1$ is the Planck scale. In the presence of gravitational field, the surface vacuum energy (13) is modified according to Eq.(16)

\[
\mathcal{E}_s' = g^\frac{3}{2}(R)\mathcal{E}_s. 
\] (14)

At the step (9) of the gravitational collapse, the surface vacuum-energy on the surface $r = R$ is,

\[
\mathcal{E}_s' = (1 - \frac{2M'}{R})^\frac{3}{2}\mathcal{E}_s. 
\] (15)

While, at the step (10) of the gravitational collapse, the surface vacuum-energy on the surface $r = R$ is,

\[
\mathcal{E}_s = (1 - \frac{2M}{R})^\frac{3}{2}\mathcal{E}_s. 
\] (16)

We find that $M'$ in Eq.(15) is altered to $M$ in Eq.(16), since the mass $\delta M$ in the spherical shell $R < r < R + \delta R$ falls into the sphere $r < R$. The vacuum-energy variation in this gravitational collapse process from the step (9) to the step (10) is

\[
\delta \mathcal{E} = \mathcal{E}_s - \mathcal{E}_s' < 0, 
\] (17)

which indicates the vacuum gains energy from gravitational field.

By using Eqs.(12) and (17), we compute the rate of energy gain $\delta \mathcal{E}/\delta t$ in the spherical shell $4\pi R^2 \delta R$ that the surface of the collapsing star sweeps in the time interval $\delta t$. Given the initial condition that at the moment $t_0 = 0$ of starting the collapsing process, the radial size of the collapsing star $R_0 = 100(2M)$ and star’s mass $M = 10M_\odot$, we compute the rate of vacuum-energy variation (gain) $\delta \mathcal{E}/\delta t$, plotted in Fig.4 as a function of $R$ in the unit of $2M$. The result shows that the rate $\delta \mathcal{E}/\delta t$ rapidly increases to $10^{57}$erg/sec, as the surface $R(t)$ of the collapsing star moves, almost in the speed of light, inwards to the horizon. Whereas, in the vicinity of the horizon, the collapsing process becomes slow and the rate decreases.

Due to this vacuum-energy gain $\delta \mathcal{E}$ (17), vacuum become energetically unstable, have to spontaneously undergo a quantum transition to lower energy states via quantum-field fluctuations. This is exactly analogous to the phenomenon of the Casimir effect. As a consequence, the vacuum-energy $\delta \mathcal{E}$ (17) gained from gravitational field must be released and deposited in the region from $r = 2M$ extending to $r = R_0$.

Which process of quantum transition releases this vacuum-energy $\delta \mathcal{E}$ (17)? One of possibilities is spontaneous photon emission, analogous to the spontaneous photon emission taking place in the atomic physics. Such a spontaneous photon emission can be induced by the four-photon interacting vertex in the theory of Quantum Electromagnetic Dynamics (QED). The rate of the quantum transition must be proportional to $\sim \alpha^4 \omega$, where $\omega$ is the characteristic energy of the process in the time interval $\delta t$. For high-energy $\omega \sim m_e$, where $m_e$ is the electron mass, the rate is very fast [8]. As shown in Fig.4, the spatial density of vacuum energy release can be very large, as the collapsing process approaching to the formation of black hole’s horizon $R = 2M$. The energy of photons spontaneously emitted can be larger than the energy threshold $2m_e c$, so that electron and positron pairs are produced. These pairs, on the other hand, annihilate into two photons. As a consequence, a dense and energetic plasma of photons, electron and positron pairs, called “dyadosphere” or “fireball” in literatures [3], could be formed. Using the rate of collapsing $\delta R/\delta t$ (12) and the rate $\delta \mathcal{E}/\delta t$ of vacuum-energy release process (Fig.4), we
can obtain that total amount energy:

\[ E_{\text{total}} = \int_{2M}^{R_c} dE \lesssim \frac{3}{2\pi} M \simeq 8.6 \cdot 10^{54}\text{erg}, \quad (18) \]

is released in a very short time, about a second for the collapsing process from \( R = R_c \) to \( R = 2M \). These qualitatively agree to the characteristic of energetic sources for gamma ray bursts.

The total energy release Eq. (18) is less than the maximum variation \( \Delta E_g = \frac{1}{2} M \) of gravitation energy \((\text{potential} - \frac{\mu^2}{r})\) in the collapse process from \( r \sim \infty \) to \( r = 2M \). The model and approach that we adopt only to illustrate the idea and scenario, is over simplified for quantitative results and will be elaborated in future work. Note added: after finishing this paper, the author was interested in reading the paper by I.Yu Sokolov (Phys. Lett.A, v.223, p.163, 1996), where conducting electron gas is used as boundary conditions for computing the vacuum energy to discuss possible huge output of cosmic energy accounting for Quasars.

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[8] The details will be presented elsewhere. From Prof. F. Cappasso’s talk in Rome June 2002, I heard that Bell Labs is interested in the phenomenon of photon emissions from the vacuum, called as the dynamical Casimir effect.

FIG. 1: The rate of energy release \( \delta E/\delta t \) (erg/sec) as a function of the radius \( R \) in unit of \( 2M \).