First Structure Formation under the Influence of Gas–Dark Matter Streaming Velocity and Density: Impact of the “Baryons Trace Dark Matter” Approximation

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Abstract

The impact of streaming between baryons and dark matter on the first structures has been actively explored by recent studies. We investigate how the key results are affected by two popular approximations. One is to implement the streaming by accounting for only the relative motion while assuming “baryons trace dark matter” spatially at the initialization of simulation. This neglects the smoothing on the gas density taking place before the initialization. In our simulation initialized at $z_i = 200$, it overestimates the gas density power spectrum by up to 40% at $k \approx 10^2 \, h \, \text{Mpc}^{-1}$ at $z = 20$. Halo mass ($M_h$) and baryonic fraction in halos ($f_{bh}$) are also overestimated, but the relation between the two remains unchanged. The other approximation tested is to artificially amplify the density/velocity fluctuations in the cosmic mean density to simulate the first minihalos that form in overdense regions. This gives a head start to the halo growth while the subsequent growth is similar to that in the mean density. The growth in a true overdense region, on the other hand, is accelerated gradually in time. For example, raising $\sigma_8$ by 50% effectively transforms $z \rightarrow \sqrt{1.5} z$ in the halo mass growth history while, at 200 overdensity, the growth is accelerated by a constant in redshift: $z \rightarrow z + 4.8$. As a result, halos have grown more massive in the former than in the latter before $z \approx 27$ and vice versa after. The $f_{bh}-M_h$ relation is unchanged in those cases as well, suggesting that the Population III formation rate for a given $M_h$ is insensitive to the tested approximations.

Unified Astronomy Thesaurus concepts: Cosmology (343); Reionization (1383); Early universe (435); Primordial galaxies (1293)

1. Introduction

Formation of the first stars (Population III stars) is an important milestone in cosmic history, where the primordial density fluctuations from cosmic inflation (Guth 1981; Linde 1982) started collapsing ambient baryons into bound objects from $z \sim 30$, which led to the production of ultraviolet radiation into space for the first time in cosmic history (e.g., Barkana & Loeb 2001; Bromm 2013). According to the standard ΛCDM model of structure formation, structures began collapsing from small scales followed by their assembly into larger structures. Low-mass dark matter halos with $\sim 10^6-10^8 M_\odot$ (i.e., minihalos) are considered as the formation sites of the first collapsed objects. The details of the collapse involve highly nonlinear physics and are an active field of numerical astrophysics (e.g., Yoshida et al. 2003a).

Recently, Tseliakhovich & Hirata (2010) pointed out that the residual velocity fluctuations from the baryonic acoustic oscillation (BAO) resulted in a strong relative motion of typically $\sim 30$ km s$^{-1}$ between baryons and dark matter at cosmic recombination. This motion decayed in time, but it was strong enough to induce the streaming of gas through dark matter potential wells and thus make it more difficult for minihalos to grow their masses and accrete gas at the time of the first star formation. Subsequent numerical studies confirmed that the baryonic fraction in minihalos is highly suppressed by the streaming motion (Greif et al. 2011; O’Leary & McQuinn 2012; Naoz et al. 2013; Richardson et al. 2013; Asaba et al. 2016). Moreover, supersonic motion shock heats the gas, causing cold gas to be even rarer inside halos (Schauer et al. 2019a).

The global impact of the streaming motion on cosmic reionization is being actively explored. The beginning of the reionization is expected to be delayed (Maio et al. 2011; Schauer et al. 2019b) although the impact is considered to be limited at the late stage of reionization ($z \sim 6$), which is driven by more massive ($\gtrsim 10^9 M_\odot$) atomic-cooling halos (Stacy et al. 2011; Fialkov et al. 2014b). In semi-numerical models of star formation and reionization, streaming is considered to raise the minimum halo mass that can form Population III stars (e.g., Greif et al. 2011; Muñoz 2019; Visbal et al. 2020). Also, the effect is expected to vary spatially because the streaming velocity is known to fluctuate at the BAO scale ($\sim 140$ Mpc). It is an interesting possibility that large-scale fluctuations in the Population III star formation rate can leave an imprint on the spin temperature of atomic hydrogen (McQuinn & O’Leary 2012; Visbal et al. 2012; Muñoz 2019), which may be proved by upcoming 21 cm surveys such as the Hydrogen Epoch of Reionization Array and the Square Kilometre Array (Mellema et al. 2013; Fialkov et al. 2014a; DeBoer et al. 2017).

There are also attempts to explain existing tensions between standard cosmology and observation using the streaming motion. Regarding the mystery of high-redshift ($z \sim 6$) supermassive black holes with mass $\sim 10^9 M_\odot$ (Mortlock et al. 2011; Wu et al. 2015; Bañados et al. 2018), several
numerical studies have shown that streaming can induce the formation of direct collapse black holes of $\sim 10^{3-4} M_\odot$ at $z \sim 30$ (Tanaka & Li 2014; Hirano et al. 2017) to give a head start to black hole growth although there are counter-arguments to this scenario (Latif et al. 2014; Visbal et al. 2014). Some studies have attempted to explain the formation mechanisms of missing satellites and globular clusters based on the fact that streaming separates dark matter and baryons (Bovy & Dvorkin 2013; Naoz & Narayan 2014; Popa et al. 2016; Chiou et al. 2019).

Given the increasing number of numerical simulation studies of the streaming motion, it is worth investigating the validity of approximations often made at the initialization of simulations. The first approximation to test is the assumption that baryons trace dark matter (BTD) at the initial conditions. Commonly used initial condition generators mostly assume the initial density/velocity field of baryonic matter is the same as that of the dark matter at initialization. The actual amplitude of baryon density fluctuation is smaller than that of dark matter at $z > 100$, but these two amplitudes are known to converge toward each other due to gravity before the first objects start forming. Thus, many numerical studies applied the streaming effect by simply adding a constant velocity to the baryon velocity field in the initial conditions, while using the same density field for both baryons and dark matter. This BTD assumption, however, is likely to break down when the streaming velocity shifts one component from the other. Also, the streaming effect should be stronger at higher redshift, but this approximation misses the effect taking place between cosmic recombination and the initialization of the simulation. To avoid this issue, one should either account for the effect in the density field at the initialization redshift or simply initialize the simulation at the recombination, as in Hirata (2018).

Another approximation to test is to artificially increase $\sigma_8$, which will amplify the density fluctuation at all scales, to mimic an overdense patch of the universe. This method is frequently used to assimilate the biased formation of the first structures in dense regions of the universe (e.g., Greif et al. 2011; Stacy et al. 2011; Hirano et al. 2017, 2018; Schauer et al. 2019a). In a different context, some early simulation works based on the first-year Wilkinson Microwave Anisotropy Probe (WMAP) results often used $\sigma_8 = 0.9$, which is higher than the currently known value (e.g., Yoshida et al. 2003b). We shall examine how the structure growth compares between such cases and truly overdense cases.

To provide self-consistent initial conditions with the streaming motion in overdensity, Ahn (2016) developed a quasi-linear perturbation theory of small-scale fluctuations under the influence of a large-scale overdensity and the streaming-velocity environment. Ahn & Smith (2018) then developed an initial condition generator, BCCOMICS$^5$ (Baryon–Cold dark matter COsMological Initial Condition generator for Small-scales), which calculates the perturbation equations of Ahn (2016) and generates corresponding three-dimensional initial conditions of dark matter and baryons. BCCOMICS treats a given overdense (underdense) patch as a separate universe with positive (negative) curvature and provides a set of "local cosmology parameters" to account for the local expansion rate being different from the mean cosmic expansion rate. Ahn & Smith (2018) used BCCOMICS to generate a suite of initial conditions for varying streaming-velocity and density environments and then performed $N$-body and hydrodynamic simulations to explore the cosmic variance of high-redshift structure formation.

This study is a continuation of the efforts by Ahn (2016) and Ahn & Smith (2018) to explore the dual impact of the streaming motion and overdensity with correctly generated initial conditions, and an extension of these works to compare the self-consistent approach quantitatively to the two common approximations used in generating initial conditions. Therefore, this work partially revisits the work of O’Leary & McQuinn (2012), which tested the BTD assumption by providing more results on key statistics of Population III star formation.

The paper is organized as follows. In Section 2, we introduce our numerical methods used in this study. In Section 3, we show our results. In Section 4, we summarize our results and give conclusions. For the rest of this paper, we assume $\Lambda$CDM cosmology consistent with the WMAP 9 yr results (Hinshaw et al. 2013): $\Omega_{m,0} = 0.276$, $\Omega_{b,0} = 0.045$, $h = 0.703$, $\sigma_8 = 0.8$ and $n_s = 0.961$.

2. Methodology

2.1. Basics of Streaming Motion

In the absence of the streaming velocity and non-gravitational baryonic physics, the perturbation equation for the overdensity $\delta$ and the peculiar velocity $v$ of matter is given by

$$\begin{align*}
\frac{\partial \delta}{\partial t} & = -\theta \\
\frac{\partial \theta}{\partial t} & = - \frac{3H^2}{2} \Omega_m \delta - 2H\theta,
\end{align*}$$

(1)

where $\theta = a^{-1} \nabla \cdot v$ with the scale factor $a$ and the gradient in the comoving frame $\nabla$, $H$ is the Hubble parameter, and $\Omega_m$ is the cosmic matter fraction of the universe at given cosmic time $t$. Common initial condition generators use the solution from the above equation to set the density and velocity fluctuation amplitudes of both baryons and dark matter.

The perturbation equation in the presence of baryon–dark matter streaming velocity ($V_b = -V_c \equiv V_c - V_b$ with the average peculiar velocities of CDM $V_c$ and baryon $V_b$ inside a patch) was first derived by Tseliakhovich & Hirata (2010); see their Equation (6)). O’Leary & McQuinn (2012) used their initial condition generator, Cosmological Initial Conditions for adaptive mesh refinement and smoothed particle hydrodynamics (SPH) Simulations (CICsASS), to generate initial conditions from the solution of the equation. Then, Ahn (2016) improved on the equation for non-zero overdensity ($\Delta$) as well accommodating the dual impact of $V_b$ and $\Delta$, because the original perturbation equation by Tseliakhovich & Hirata (2010) does not implement the non-zero overdensity environment. Ahn & Smith (2018) developed the initial condition generator BCCOMICS based on Ahn (2016). Their perturbation equation for the Fourier modes of density contrast ($\delta_b$ for baryons and $\delta_c$ for dark matter), velocity divergence ($\theta_b$ for baryons and $\theta_c$ for dark matter) and baryonic temperature fluctuations ($\delta_T \equiv (T - \bar{T})/\bar{T}$, where $T$...
is local baryon temperature) reads
\[
\begin{align*}
\frac{\partial \delta_b}{\partial t} &= -(1 + \Delta_c) \theta_t - \Theta_c \delta_c, \\
\frac{\partial \theta_t}{\partial t} &= -\frac{3H^2}{2} \Omega_m (f_c \delta_c + f_b \delta_b) - 2H \theta_t, \\
\frac{\partial \delta_b}{\partial t} &= -ia^{-4} V_{bc} \cdot k \delta_b - (1 + \Delta_b) \theta_b - \Theta_b \delta_b, \\
\frac{\partial \theta_b}{\partial t} &= -ia^{-4} V_{bc} \cdot k \theta_b - \frac{3H^2}{2} \Omega_m (f_c \delta_c + f_b \delta_b) - 2H \theta_b \\
&\quad \quad + a^{-2} \frac{k \delta_b}{\mu m_H} k^2 ((1 + \Delta_b) \delta_T + (1 + \Delta_T) \delta_b) \\
\frac{\partial \delta_T}{\partial t} &= \frac{2}{3} \left\{ \frac{\partial \delta_b}{\partial t} + \frac{\partial \delta_b}{\partial t} (\delta_T - \delta_b) \right\} \\
&\quad \quad - \frac{\lambda_e}{\tau_c} a^{-4} \frac{P_T}{T} \delta_T, \\
\end{align*}
\]

in the CDM-rest frame ($V_s = 0$). Here, $f_b = \Omega_b/\Omega_m$ and $f_c = (\Omega_m - \Omega_b)/\Omega_m$ are the global baryon and dark matter fraction in matter, respectively, $\delta_c$ is the global ionized fraction, $t_c = 1.17 \times 10^{12}$ yr, and $T_c = 2.725(1 + z)$ K is the mean temperature of the cosmic microwave background at redshift $z$. The bulk quantities of a patch $\Delta_b, \Delta_c; \Theta_b = (a^{-1} \nabla \cdot V_b), \Theta_c = (a^{-1} \nabla \cdot V_c)$, and $\Delta_T$ denote the overdensity of baryons, the overdensity of dark matter, the divergence of $V_b$, the divergence of $V_c$, and the baryon temperature fluctuation, respectively, and their values in Fourier space are identical to the real-space values. Due to the linearity of any perturbative quantities and the smallness of pressure terms at large scales, these bulk quantities satisfy the following linearized equation:

\[
\begin{align*}
\frac{\partial \Delta_c}{\partial t} &= -\Theta_c, \\
\frac{\partial \Theta_c}{\partial t} &= -\frac{3H^2}{2} \Omega_m (f_c \Delta_c + f_b \Delta_b) - 2H \Theta_c, \\
\frac{\partial \Delta_b}{\partial t} &= -\Theta_b, \\
\frac{\partial \Theta_b}{\partial t} &= -\frac{3H^2}{2} \Omega_m (f_c \Delta_c + f_b \Delta_b) - 2H \Theta_b. \\
\end{align*}
\]

$\Delta_b$ and $\Delta_c$ defined at the length scale $4h^{-1}$ Mpc are tightly correlated at $z \lesssim 200$ and almost uncorrelated at $z \sim 1000$ (Ahn 2016; Ahn & Smith 2018). In this work, we shall run several simulations with $\Delta_c = 0.0323$ and $\Delta_b = 0.027$ at $z = 200$, which correspond to $2\sigma$ overdensity for a $4h^{-1}$ Mpc box at that redshift.

The streaming velocity fluctuates spatially at the BAO scale ($\sim 140$ Mpc). Thus, the streaming velocity $V_{bc}$ can be treated as a constant drift within 10 Mpc. $V_{bc}$ follows a Boltzmann distribution with standard deviation $\sigma = 28$ km s$^{-1}$ at $z = 1000$, which decays as $(1 + z)$ with cosmic expansion. We shall take its value at $z = 1000, V_{bc, 1000} = |V_{bc}(z = 1000)|$, as the reference value.

### 2.2. Simulation Setup

#### 2.2.1. Parameter Choice

The list of parameter choices for the simulations in this work is given in Table 1. The fiducial case, d0v0, has the cosmic mean density and zero streaming velocity in a $1h^{-1}$ Mpc box. Several cases are run with a streaming velocity of $56(z/1000)$ km s$^{-1}$, which is twice the root-mean-square of the streaming velocity distribution. We run a streaming case in the cosmic mean density (d0v2) and in the $2\sigma$ overdensity (d0v2; $\Delta = 3.14 \times 10^{-2}$). We also run one simulation with the $2\sigma$ streaming velocity and cosmic mean density in a bigger box of $4h^{-1}$ Mpc to obtain statistics of higher-mass halos (d0v2L) that cannot be captured in a $1h^{-1}$ Mpc box.

We make two cases with the above-mentioned approximations. In d0v2_BTD, we apply the BTD assumption in the initial conditions by assigning the same density field to both baryons and dark matter. In this case, the amplitude of the density/velocity fluctuations is given by Equation (1) and a constant streaming velocity is added to the baryon velocity field. We also run a $4h^{-1}$ Mpc box simulation with the same setup (d0v2L_BTD). In d0v2_IS, we artificially boost the normalization of the initial density power spectrum in d0v2 by raising $\sigma_b$ from 0.8 to 1.2 as done in some previous works to simulate overdense regions.

#### 2.2.2. Initial Conditions

The initial conditions are generated for five cases in $1h^{-1}$ Mpc boxes and two cases in $4h^{-1}$ Mpc boxes at $z = 200$. Simulations with the same box size are initialized with the same random phases to exclude the cosmic variance effect in the comparison.

In Figure 1, we visualize the initial gas/dark matter density fields of d0v0, d0v2_BTD and d0v2 at $z = 200$ generated by BCCOMICS. The similarity in large-scale structures is due to the same random phases used for all three cases. At $z = 200$, the density fluctuation amplitude of gas is smaller than that of dark matter in d0v0 and d0v2. The gas density fluctuation amplitude in d0v2_BTD is highly overestimated due to the BTD approximation. Also, the smoothing effect from the streaming motion is not present in d0v2_BTD.

### Table 1

| Label | $L_{\text{box}}$ [h$^{-1}$ Mpc] | $m_{\text{box}}$ [h$^{-1}$ $M_\odot$] | $m_{\text{gas}}$ [h$^{-1}$ $M_\odot$] | $V_{\text{bc,1000}}$ [km s$^{-1}$] | $\Delta_{c, 200}$ | Baryons Trace Dark Matter | $\sigma_b$ |
|-------|-------------------------------|--------------------------------|--------------------------------|-----------------|---------------|-----------------------------|-----|
| d0v0  | $1$                           | $6.80 \times 10^2$           | $1.32 \times 10^2$           | $0$             | $0$           | No                          | $0.8$ |
| d0v2  | $1$                           | $6.80 \times 10^2$           | $1.32 \times 10^2$           | $56$            | $0$           | No                          | $0.8$ |
| d0v2L | $4$                           | $4.35 \times 10^4$           | $8.45 \times 10^3$           | $56$            | $0$           | No                          | $0.8$ |
| d2v1  | $1$                           | $7.02 \times 10^2$           | $1.35 \times 10^2$           | $56$            | $3.14 \times 10^{-2}$ | No                          | $0.8$ |
| d0v2_BTD | $1$                | $6.80 \times 10^2$           | $1.32 \times 10^2$           | $56$            | $0$           | Yes                         | $0.8$ |
| d0v2L_BTD | $4$          | $4.35 \times 10^4$           | $8.45 \times 10^3$           | $56$            | $0$           | Yes                         | $0.8$ |
| d0v2_IS | $1$                  | $6.80 \times 10^2$           | $1.32 \times 10^2$           | $56$            | $0$           | No                          | $1.2$ |
The power spectra of the initial gas, dark matter, and the total matter density are shown in Figure 2 for d0v0 and d0v2. In both cases, the dark matter density power spectrum is higher than the linear power spectrum while the gas density power spectrum is lower. The total matter power in d0v0 agrees exactly with the linear spectrum. In the case of d0v2_BTD, both gas and dark matter density power spectra are the same as the linear power spectrum. The gas density power spectra of d0v0 and d0v2 agree up to $k = 10^2 h \, \text{Mpc}^{-1}$, above which the spectrum of d0v2 falls off due to suppression from the streaming motion.

We note that we use separate transfer functions for the gas and dark matter components by de-streaming motion. Generating gas and dark matter density fields from the same transfer function is known to cause mild systematic effects that were explored in detail by Yoshida et al. (2003a) and Bird et al. (2020). Our simulation setup is more relevant to the results of Yoshida et al. (2003a), which are from $4 \, h^{-1} \, \text{Mpc}$ boxes at $z \gtrsim 30$. Their work showed that the BTD approximation fails exactly with the linear spectrum. In the case of d0v2_BTD, than the linear power spectrum while the gas density power spectrum is higher.

Figure 1. Initial gas density/velocity field of the simulation at $z = 200$ visualized for d0v0 (left), d0v2_BTD (middle), and d0v2 (right). Density and velocity are described by color contours and arrows, respectively. The density field is color-coded so that over/underdensity is shown in red/blue.

Figure 2. Density power spectrum of gas (solid lines), dark matter (dashed lines), and total matter (dotted lines) density field for the case with (d0v2; cyan) and without the streaming motion (d0v0; black) at $z = 200$. The linear matter–density power spectrum is shown as a gray dashed line for reference.

2.2.3. Hydrodynamics Solver

For the hydrodynamics solver, we adopt the SPH code GADGET-2 (Springel et al. 2001; Springel 2005) to follow the structure formation from $z = 200$ to $\sim 15$. BCCOMICS creates the initial conditions in the ENZO (Bryan et al. 2014) simulation format. We then convert those initial conditions into the GADGET format. We initialize the gas and dark matter particles on two separate grids that are offset by half the average particle distance along all three $(x, y, z)$ coordinates.

2.3. Simulating the Overdense Region

The first collapsed objects likely appeared in overdense patches of the universe. In this work, we simulate one case in an overdense region (d2v2) to study the accelerated growth of structure in true overdensity.

Many numerical studies rely on a multi-resolution adaptive-refinement scheme (e.g., MUSIC; Hahn & Abel 2011) to start from a large box with the cosmic mean density and zoom into a denser subregion where structures develop earlier than in other parts of the volume.

In this work, we present a complementary method, referred to as the “separate universe” approach, which starts from initial conditions of overdense volume using solutions of Equation (2) for a non-zero local overdensity $\Delta$. An advantage of this method is that one can easily create extremely rare density peaks, only a few of which appear in a Gpc$^3$ volume. Creating such an extremely overdense patch would require an excessive number of refinements with the adaptive-resolution scheme.

In an overdense region, the cosmic expansion rate is locally slower than the global rate. We capture this effect by modifying the cosmology parameters in the simulation setup. BCCOMICS provides the local cosmology parameters for volumes with non-zero overdensity. In an overdense volume, cosmology parameters of a closed universe are used to describe the expansion rate. The derivation of the local cosmology parameters is given in Section 3 of Ahn & Smith (2018). A detailed description of this method can also be found from Sirko (2005) for cases without the streaming velocity.

Our method does not capture higher-order gravitational effects like the shear and tidal forces from missing large-scale structures. However, such effects should be negligible at the time of first structure formation. A similar approach was used by Goldberg & Vogele (2004) in their simulations of underdense regions to study cosmic voids.

In volumes with non-zero overdensity, the redshift evolves differently from the global value due to the modified expansion
rate. Thus, one must keep track of the relation between the true and the local redshift in the simulation. We define the local redshift $\tilde{z}$ so that it becomes zero at the end of the simulation: the simulations with non-zero overdensity end at the global redshift of $z = 20$ in this study. In that case, the initial value of the local redshift in d2v2 is $\tilde{z}_i = 7.72$ while the true initial redshift is $z_i = 200$. In Figure 3, we show how $\tilde{z}$ and the box size in the global comoving scale evolve in time. The local cosmological parameters of d2v2 are $\Omega_{\Lambda,0} = 3.58 \times 10^{-4}$, $\Omega_{m,0} = 1.43$, $\Omega_{b,0} = 0.27$ and $h = 31.6$, where tildes are used to denote that the parameters are the local values. Note that the “present” values denoted by the subscript “0” of these cosmological parameters are evaluated at $\tilde{z} = 0$.

It is worth noting that the simulation volume expands by a factor of $\tilde{z}_i + 1 = 8.72$ between $z = 200$ and $\tilde{z}_f = 20$ while the universe globally expands by a factor of $[200 + 1]/[20 + 1] = 9.57$. As a result, the comoving box size of the overdense simulation shrinks by $\sim 9\%$ between $z_i$ and $\tilde{z}_f$ (see also Figure 3) and the mean density of the simulation increases by $[9.57/8.72]^3 = 1.32$.

### 2.4. Halo Identification

We use the publicly available version of Amiga Halo Finder (AHF; Gill et al. 2004; Knollmann & Knebe 2009) to identify halos from the simulation output at $z = 20$. AHF outputs a list of gas, dark matter, and total mass of identified halos. These quantities are used for obtaining the halo mass function and baryonic fraction in halos for a given halo mass, which is considered highly relevant to the Population III star formation rate.

As usual, the virial radius of a halo is chosen to make the mean density within halo $\Delta_{th} = 200$ times the cosmic mean. In the overdense case, the mean density of the simulation box increases when compared to the cosmic mean. We thus compensate for the overdensity by rescaling the density threshold parameter $\Delta_{th}$, which is in units of the mean density of the simulation box as described in Figure 3. For example, the virial radius of a halo in the overdense simulation at $z = 20$ is defined by $\Delta_{th} = 200/1.32 = 152$ times the mean density of the simulation. The details of this mapping process are described in Ahn & Smith (2018).

### 3. Results

#### 3.1. Streaming Effect with BTD

We first examine the BTD approximation by comparing the $z = 20$ snapshots of the no-streaming case (d0v0), the cases with streaming motion applied in the approximate way (d0v2_BTD and d0v2L_BTD), and the cases with the correctly implemented streaming motion (d0v2 and d0v2L), where all cases except d0v0 have a streaming velocity of $56[z/1000] \text{ km s}^{-1}$. The gas particle maps are shown in Figure 4, the gas and dark matter density power spectra are shown for the simulations in $1h^{-1}$Mpc boxes (d0v0, d0v2_BTD, and d0v2) in the left panel of Figure 5, the accumulated halo mass functions are shown in Figure 6, and the baryonic mass fraction as a function of halo mass is shown in Figure 7.

#### 3.1.1. Gas Density Fluctuation Amplitude

The gas density maps in Figure 4 visualize the well-known smoothing effect of the streaming motion: the density field appears much smoother in both d0v2 and d0v2_BTD than in d0v0. Here, it is expected that the smoothing effect is underestimated in d0v2_BTD compared to in d0v2 since the BTD assumption ignores the streaming effect in density taking place between the decoupling of baryons from photons ($z \approx 1000$) and the beginning of the simulation ($z_i = 200$ in this work). The difference between A and B is not evident in the particle maps, but it appears clearly in the gas density power spectra. The gas and dark matter density power spectra diverge above $k \sim 30 h \text{ Mpc}^{-1}$ in d0v2 while they diverge above $k \sim 100 h \text{ Mpc}^{-1}$ in d0v2_BTD. The difference peaks at $k \approx 100 h \text{ Mpc}^{-1}$ and decays toward high-$k$ until it vanishes at $k \approx 500 h \text{ Mpc}^{-1}$. Clearly, the BTD approximation partially misses the streaming effect.

We find that the difference in shape of the gas density power spectrum between d0v2 and d0v2_BTD can be explained from the distance covered by the streaming motion. The distance covered between $z_i$ and $\tilde{z}_f$ is given by

$$d_{cb}(z_i, \tilde{z}_f) = \int_{z_i}^{\tilde{z}_f} a^{-1}V_{cb}(z) \frac{dz}{dz_i} dz_i$$

$$\approx 2.1 \times 10^{-3} |z_i^{0.5} - \tilde{z}_f^{0.5}| \left[ \frac{V_{cb,1000}}{56 \text{ km s}^{-1}} \right] h^{-1} \text{ Mpc},$$

where we approximated the cosmic expansion rate to $\approx z^{-1}$. In d0v2, the streaming velocity is accounted for from the decoupling of gas from the cosmic microwave background at $z_i \approx 1000$, giving $d_{cb}(z_i = 1000, \tilde{z}_f = 20) = 5.8 \times 10^{-2} h \text{ Mpc}^{-1}$ and the gas density power spectrum in d0v2 hence deviates from the dark matter spectrum from roughly half the wavenumber corresponding to this distance $k_{cb}/2 = \pi/d_{cb} = 5.5 \times 10^5 h \text{ Mpc}^{-1}$. In d0v2_BTD, the streaming motion is accounted for from the initialization redshift $z_i = 200$, giving

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7 One may prefer to choose a later epoch to set $z = 0$. We, however, note that high-density peaks may turn around and contract before $z = 0$. For example, the d2v2 patch turns around at $z = 4.6$ and even collapses to a point at $z = 2.5$, which makes it impossible to perform the simulation until $z = 0$. In this study, we therefore set the end of the simulation as the “local present” as in Ahn & Smith (2018).

8 http://popia.ft.uam.es/AHF/Download.html
Figure 4. Gas particle distribution of d0v0 (left), d0v2_BTD (middle), and d0v2 (right) at \( z = 20 \). Density structures are depicted in dark gray. A thin slice of a \( 0.25 \times 0.25 \) \((h^{-1} \text{ Mpc})^2\) region, which has relatively more-grown structures, is chosen for comparison.

Figure 5. Dimensionless power spectrum of gas and dark matter density at \( z = 20 \) plotted as solid and dashed lines, respectively. The left panel compares the results of d0v0 (black), d0v2 (cyan), and d0v2_BTD (blue). Blue and cyan arrows denote \( k_{cb} \) for d0v2_BTD and d0v2, respectively. The right panel compares the results of d0v2 (cyan), d2v2 (red), and d0v2_IS (magenta). To account for the overdensity in d2v2, we multiply the square of the overdensity factor \( \Delta^2(z = 20) = 1.74 \) to the power spectrum. The grey dashed line shows the linear density power spectrum for reference.

Figure 6. Left: accumulated halo mass function at \( z = 20 \) as solid lines for d0v0 (black), d0v2 (cyan), d0v2_BTD (blue), d2v2 (red), and d0v2_IS (magenta). Right: accumulated mass function compared between d2v2 (red) and d0v2_IS (magenta) for \( z = 20 \) (solid), 24.1 (dashed), 29.6 (dotted-dashed), and 33.1 (dotted).
3.1.2. Halo Abundance and Dark Matter Density Fluctuations

The halo mass function in d0v2_BTD is slightly higher than in d0v2 (See Figure 6). For example, the number of halos with \( M_h > 3 \times 10^6 M_\odot \) in d0v2 is reduced to 45.1\% of that in d0v0 and to 52.1\% in d0v2_BTD. This difference is more or less constant throughout the entire range of halo mass in the simulation \((3 \times 10^6 M_\odot)\). A similar difference can be seen from the comparison of dark matter power spectra (solid lines in the left panel of Figure 4) at \( k \gtrsim 100 h \text{Mpc}^{-1} \). Compared to the impact of the streaming velocity, the error caused by the BTD approximation seems to be small in the dark matter sector.

3.1.3. Baryonic Fraction in the Halo

The suppression of the baryonic fraction in halo mass \( f_{b,h} \) is often considered as the most direct impact of the streaming motion on Population III star formation. Hence, \( f_{b,h} \) has been repeatedly modeled with the streaming motion in previous works, most of which were based on the BTD approximation.

Interestingly, the relation between \( f_{b,h} \) and \( M_h \) is not significantly changed by the approximation despite the bias introduced in the other statistics discussed above. Figure 7 shows that \( f_{b,h} \) as a function of \( M_h \) in d0v2 agrees with that in d0v2_BTD within 1\% uncertainty up to \( 10^6 h^{-1} M_\odot \). Similarly, \( f_{b,h} \) in d0v2 agrees with that in d0v2_BTD up to \( 10^7 h^{-1} M_\odot \).

We find that the BTD approximation overestimates both \( M_h \) and \( f_{b,h} \) in a way that the \( f_{b,h}^{-1}M_h \) relation is unchanged. For example, the averages of \( M_h \) and \( f_{b,h} \) for the 100 most massive halos are \( 3.7 \times 10^5 h^{-1} M_\odot \) and 0.19 in d0v2, and \( 4.23 \times 10^5 h^{-1} M_\odot \) and 0.21 in d0v2_BTD. Similarly, the averages of \( M_h \) and \( f_{b,h} \) are 6.95 \( \times 10^5 h^{-1} M_\odot \) and 0.60 in d0v2_BTD, and 7.70 \( \times 10^5 h^{-1} M_\odot \) and 0.61 in d0v2_L_BTD. These changes are described by black arrows in Figure 7, which lie along the direction of the \( f_{b,h}^{-1}M_h \) relation.

3.2. Growth of Structure with Increased \( \sigma_8 \)

Here, we compare the true overdense case (d2v2) to the artificial case where we boosted the initial density/velocity fluctuations in a mean-density volume by raising \( \sigma_8 \) (d0v2_IS). We take d0v2 as the fiducial case so that the three cases (d2v2, d0v2_IS, and d0v2) mentioned have the same streaming velocity \( V_{cb,1000} = 56 \text{ km s}^{-1} \). Such a boost of \( \sigma_8 \) has been commonly used to assimilate an overdense environment inside a mean-density simulation box. Note that increasing \( \sigma_8 \) is solely intended to mimic the overdensity environment regardless of \( V_{cb} \), because \( \sigma_8 \) and \( V_{cb} \) are mutually independent (Ahn et al. 2016; Ahn & Smith 2018). The amount of increase in \( \sigma_8 \) would be identical to that in the case of, e.g., d0v0_IS, if one were to use this scheme to mimic the d2v0 case.

The density power spectrum (right panel of Figure 5) and the halo mass function (left panel of Figure 6) show how much more the structures have evolved in d2v2 and d0v2_IS compared to in d0v2 at \( z = 20 \). Both the density power spectrum and mass function show that the structures have grown more in d2v2 than in d0v2_IS at \( z = 20 \). However, a halo function comparison at higher redshifts in the right panel of Figure 6 shows the opposite: d0v2_IS has more halos than d2v2 does at \( z \gtrsim 30 \).

To compare the time evolution of halo mass function in a convenient manner, we define a mass \( M_{10} \) in a way that the number of halos above that mass is fixed to a certain number.
magenta dashed line is the case where we transformed $M_{10}$ of $d0v2$ by $z \rightarrow \sqrt{1.5} z$ to match that of $d0v2$. Similarly, the black and redshift dotted lines are the results of transforming $d0v2$ with $z \rightarrow \sqrt{1.5} z$ and $z \rightarrow z + 5$ to match $d0v0$ and $d2v2$, respectively.

In Figure 6, this would be the $x$-coordinate of the intersection of mass function and the second gray horizontal grid line from the bottom. Since the halo mass function grows monotonically in time, $M_{10}$ can be used as an indicator of how massive the halos have grown in the simulation. We plot $M_{10}$ for $d0v2$, $d2v2$, and $d0v2$ IS in Figure 8 as a function of redshift.

The redshift evolution of $M_{10}$ shows an interesting difference between $d2v2$ and $d0v2$. $M_{10}$ in $d0v2$ IS (magenta solid) is larger than that of $d2v2$ (red solid) at $z \gtrsim 27$, but smaller at $z \lesssim 27$. This also agrees with the trend in the mass function comparison mentioned above (right panel of Figure 6).

The difference between $d2v2$ and $d0v2$ IS in the time evolution of $M_{10}$ can be understood from how the structure formation is enhanced in those two cases. The structure growth in $d0v2$ IS is given a head start at the beginning of the simulation and then proceeds just as fast as in the mean-density case later on. In contrast, the structure growth in $d2v2$ starts nearly the same as in the mean density case ($d0v2$) and is gradually accelerated over time by a locally slower cosmic expansion rate.

Transforming $z \rightarrow \sqrt{1.5} z$ in $M_{10}(z)$ in $d0v2$ reproduces $M_{10}(z)$ in $d0v2$ IS quite precisely (compare magenta solid and dotted lines in Figure 8). This is explained by the growth rate of structure during the matter-dominated era: $P_{\delta}(k) \propto k^2 z^{-2}$. A factor of 1.5 increment in the initial density power spectrum results in the structure growth accelerated in the way $\sqrt{1.5}$ is multiplied to the redshift. We note that this effect is expected regardless of whether the streaming motion is present or not.

Interestingly, $M_{10}(z)$ in $d0v0$ is similar to transforming $z \rightarrow \sqrt{1.15} z$ in $M_{10}(z)$ in $d0v2$ (compare the black solid and dotted lines). According to the above finding, the impact of the 2σ level streaming velocity on halo mass growth history is similar to lowering $\sigma_8$ by 1.15 at the initialization.

In the true overdense case ($d2v2$), the structure formation is accelerated by roughly a constant in redshift. $M_{10}$ in $d2v2$ is similar to transforming $z \rightarrow z + 4.8$ in $M_{10}(z)$ in $d0v2$ throughout the range we explored ($15 \lesssim z \lesssim 35$; compare the red solid and dotted lines). This constant shift is smaller than the multiplicative shift in $d0v2$ IS down to $z \approx 27$, but larger at lower redshifts.

The difference between $d2v2$ and $d0v2$ IS indicates that self-consistent initial conditions are crucial in studying the effect of local overdensity on structure formation. Note that both the density power spectrum and the halo mass function are evaluated in the global comoving frame, and thus the results from the $d2v2$ simulation reflect the fact that the local patch has been detached and shrunken from the global comoving frame. Even though the boosted $\sigma_8$ of e.g., $d0v2$ IS case can mimic the expedited formation of structures in overdense regions, this scheme cannot reproduce the density bias of halo clustering correctly because the simulation volume still has the same expansion rate as the global value. In terms of the peak-background split scheme (Mo & White 1996), however, a boost of $\sigma_8$ (e.g., $d0v2$ IS) only affects the linear density threshold for halo formation, but a locally collapsing patch (e.g., $d2v2$) affects both the halo-formation density threshold and the clustering scale of halos.

We find that the $f_{b,h}/M_h$ relation in $d2v2$ and in $d0v2$ IS remains the same as in $d0v2$ and $d0v2$ BTD. That is, all the cases with $V_{\text{fbh,1000}} = 56$ km s$^{-1}$ in this study show the same $f_{b,h}/M_h$. Comparing $d0v2$ to $d2v2$ shows that the average baryonic fraction of the 100 most massive halos increases from 0.188 to 0.433 while the average halo mass increases from 0.368 to $1.92 \times 10^6 h^{-1} M_\odot$ (see also the black arrow in the right panel of Figure 7). This suggests that the relation depends only on the streaming velocity.

4. Summary and Discussion

Recently, a number of simulation studies have been performed in the context of assessing the impact of the baryon–dark matter streaming motion on the first collapsed objects. In this study, we have examined two approximations that are often made in those studies. One is the BTD approximation, which ignores the smoothing effect of the streaming motion in gas density before initialization of the simulation. The other is to boost the initial amplitude of density/velocity fluctuations to represent an overdense volume that forms the first collapsed objects.

The BTD approximation overestimates the gas density fluctuation amplitude by up to ~40% in the power spectrum at certain wavenumbers. The distance that the gas is shifted by the streaming motion is underestimated by roughly a factor of three as the streaming motion before the initialization of the simulation ($z_i = 200$ in this study) is unaccounted for. As a result, the minimum wavenumber of the fluctuations suppressed by the streaming motion is overestimated by a similar factor. This results in the gas density power spectrum at $k \sim 100 h$ Mpc$^{-1}$ not being properly suppressed by the streaming motion. We note that this effect is likely to be larger if a simulation is initialized at a lower redshift than in this study.

On the other hand, the impact of the BTD approximation on dark matter structures is limited: the density power spectrum is overestimated by about 5% at $k \gtrsim 100 h$ Mpc$^{-1}$ and the halo abundance is also increased by a similar fraction. The exaggeration in the total matter density fluctuation by the approximation is not severe since baryons do not dominate the gravitational growth of structures.

The baryonic mass fraction $f_{b,h}$ of a halo with mass $M_h$, which is considered to be directly related to Population III star
formation, is not significantly affected by the BTD approximation, justifying a number of previous works based on this approximation (e.g., Hirano et al. 2017; Schauer et al. 2019a). The approximation overestimates both \( M_h \) and \( f_{b,h} \), but the relation between the two quantities remains unchanged. Moreover, the same relation holds for halos in the overdense case \((d^2v^2)\) and the increased \( \sigma_8 \) case \((d0^2v_{2,IS})\) as well, suggesting that \( f_{b,h} \) depends only on \( M_h \) and \( V_{cb} \) and not on the gas density fluctuation amplitude. This can be explained by the fact that the gas density is not a dominant factor in the collapse of a halo: a dark matter clump first begins to collapse and the ambient gas is passively pulled in. \( M_h \) would determine how much gas can potentially be pulled into the halo by gravity and \( V_{cb} \) would determine how much of that gas is blown away.

Increasing \( \sigma_8 \) at the initialization of the simulation is a convenient way of studying a rare density peak that a Population III star is expected to form. While the \( f_{b,h} \)-\( M_h \) relation is not affected by the approximation mentioned above, the growth history of halos is substantially different from that in the true overdense case. Increasing \( \sigma_8 \) gives a head start in the structure growth while the growth in the true overdense volume is gradually accelerated over time. In that case, the structure growth and halo clustering are overestimated at the early time and underestimated at the late time, which would bias the growth history of the halo. For example, a 10\(^6\) \( M_\odot \) minihalo at \( z \approx 30 \) from simulations with an increased \( \sigma_8 \) of 1.2 in Hirano et al. (2017) will likely have a different mass growth history and halo-clustering scale than from in a true overdensity.

Interestingly, the way the halo growth is delayed by the streaming motion is in contrast to the impact of increasing \( \sigma_8 \). Presumably, the characteristic decay of the streaming velocity toward low redshift results in most of the suppression effect finishing much earlier than \( z \sim 30 \), giving final structures similar to the case of starting with a smaller density fluctuation amplitude at the initialization. In our simulation with 2\( \pi \) streaming motion \((V_{cb,1000} = 56 \text{ km s}^{-1})\), the suppression in the halo mass function amounts to what we expect from lowering \( \sigma_8 \) by 13\% or transforming \( z \rightarrow \sqrt{1.15}z \). This can be useful for modeling the impact of streaming on the global Population III star formation rate (e.g., Muñoz 2019).

We note that our simulations do not include the chemical cooling needed to distinguish the cold component from the total minihalo gas. The streaming motion can reduce the cooling fraction in the halo gas by shock heating, further reducing the chance of star formation in minihalos (Schauer et al. 2019a) on top of the reduction in \( f_{b,h} \). Given the small impact of the BTD approximation on halos, it is unlikely that including chemical cooling would introduce a dramatic impact of the approximation on Population III star formation. It is, however, possible that increased \( \sigma_8 \) cases have some impact due to the biased growth history of halos. We aim to explore such issues in future studies to make more direct conclusions about the impact of the streaming motion on Population III star formation.

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