Multi-resolution progressive computational ghost imaging

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Abstract
Most studies on ghost imaging focus on high-quality and high-resolution imaging with a few measurements. However, as far as we know, continuous multi-resolution imaging is rarely mentioned. In this work, we both theoretically and experimentally demonstrate a method that uses the Hadamard derived pattern to realize continuous multi-resolution imaging simply and quickly, whereby both the reconstruction time and measurements required for multi-resolution images can be significantly reduced. This approach improves the flexibility of ghost imaging, and can be extended to multi-resolution image-dependent practical applications, such as target tracking and recognition.

Keywords: ghost imaging, multi-resolution imaging, Hadamard derived pattern

(Some figures may appear in colour only in the online journal)

1. Introduction

Ghost imaging (GI) is realized by correlating the light field reflected (or transmitted) from the object with the reference light field [1]. Because all the photons transmitted (or reflected) from the object illuminate the same bucket detector and the measurement method is globally random, this technique has the superiority of higher sensitivity in detection and higher efficiency in information extraction than traditional optical imaging [2]. Also, GI has aroused increasing interest in applications like remote sensing [3–5], super-resolution [6], and optical encryption [7]. In order to simplify the schematic of GI, computational ghost imaging (CGI) that requires only one optical path was theoretically proposed by Shapiro [8] and later was demonstrated by Bromberg [9]. In 2009, Katz et al introduced compressive sensing into image reconstruction of CGI, where a better reconstruction result can be obtained by even using the measurement far below the Nyquist limit [10]. In order to enhance the imaging characteristics of compressive GI, most works were focused on the imaging scenes and sparse representation of the targets [11–13].

Recently, the optimization coding of speckle pattern for GI was considered [14–18]. These schemes are important ways to reduce the number of measurements, reconstruction time and computing resources. Averbuch et al [14] proposed a high-performance imaging scheme which directly uses the patterns that form the sparse basis to replace classical random speckle patterns. Then, a more optimized method named compressive
2. Theoretical analysis

The schematic diagram of CGI is presented in figure 1, showing a projector as the light modulator projects the modulated light field onto an object with a reflection coefficient \(O(x, y)\). The total reflected signals are collected by a bucket detector. The \(m\)th light field and bucket signal are expressed as \(f^m(x, y)\), \(B^m\), respectively.

For conventional GI, the reflected coefficient can be obtained by computing the correlation between \(f^m(x, y)\) and \(B^m\):

\[
O_{\text{CGI}}(x, y) = \frac{1}{M} \sum_{m=1}^{M} \left[ (B^m(y) - \langle B^m \rangle) \cdot (f^m(x, y) - \langle f^m \rangle) \right],
\]

where \(\langle B^m \rangle = \frac{1}{M} \sum_{m=1}^{M} B^m\) and \(\langle f^m(x, y) \rangle = \frac{1}{M} \sum_{m=1}^{M} f^m(x, y)\).

We can transform the \(f^m(x, y)\) (dimensions \(p \times q\)) of \(M\) measurements into a matrix form:

\[
\Phi = \begin{bmatrix}
I^{(1)}(1, 1) & I^{(1)}(1, 2) & \cdots & I^{(1)}(p, q) \\
I^{(2)}(1, 1) & I^{(2)}(1, 2) & \cdots & I^{(2)}(p, q) \\
\vdots & \vdots & \ddots & \vdots \\
I^{(M)}(1, 1) & I^{(M)}(1, 2) & \cdots & I^{(M)}(p, q)
\end{bmatrix}.
\]

Here, each row of the matrix \(\Phi\) is converted from a row vector of length \(p \times q\), which is obtained by reshaping the \(m\)th light field \(f^m(x, y)\).

Thus, equation (1) can be rewritten into a matrix form:

\[
O_{\text{CGI}}(x, y) = \frac{1}{M} \langle \Phi - I(\Phi) \rangle^T (B - I(B))
\]

\[
= \frac{1}{M} \langle \Phi - I(\Phi) \rangle^T \Phi \hat{O}
\]

\[
= \frac{1}{M} \Phi^T \hat{O} \Phi,
\]

where \(B\) is an \(M \times 1\) vector and composed by \(M\) times of measurement, that is \(B = [B^{(1)}, B^{(2)}, \ldots, B^{(M)}]^T\). Similarly, \(\hat{O}\) is an \(M \times 1\) vector made up of the reflection coefficient \(O(x, y)\) of object, \(\hat{O} = [O(1, 1), O(1, 2), \ldots, O(p, q)]^T\). In addition, \(\Phi = \Phi - I(\Phi)\), \(\Phi \hat{O} = B\), and \(B = \langle \Phi \rangle \hat{O}\). \(\Phi\) represents an \(M \times 1\) column vector of all elements with a value of 1. \(\Phi\) is a \(1 \times N\) row vector, which denotes the average of each column of \(\Phi\). Theoretically, a high quality reconstructed image will be obtained by equation (3), if \(\Phi^T \Phi\) is a diagonal matrix. Additionally, in a CGI system, the full-width at half maximum of \(\Phi^T \Phi\) diagonal nonzero elements determines the spatial...
transverse resolution of the preset light field [21], which is proportional to the resolution on the object plane. Hence, to realize the multi-resolution progressive computational ghost imaging, a multi-resolution \( \Psi \Psi' \Psi \) with different measurements number is extremely essential.

### 2.2. Reordering methods for Hadamard derived pattern

To enable high-speed multi-resolution progressive computational ghost imaging, we optimized the sequence of the Hadamard derivative pattern to achieve real-time high-resolution imaging. Then, we show how to select and use the Hadamard derived pattern to actualize the multi-resolution progressive imaging.

The Hadamard basis is a square matrix composed of +1 and −1, and can be generated rapidly by the Kronecker product, that is

\[
H^k = \begin{bmatrix}
+1 & +1 \\
+1 & -1
\end{bmatrix},
\]

\[
H^k = H^2 \otimes H^{k-1} = \begin{bmatrix}
+H^{k-1} & +H^{k-1} \\
+H^{k-1} & -H^{k-1}
\end{bmatrix},
\]

where \( 2 < k \) (integer), and \( \otimes \) denotes the Kronecker product. Hence, a Hadamard matrix of size \( M \times M = N \),

\[
H^2(m, n) = \begin{bmatrix}
H(1, 1) & H(1, 2) & \cdots & H(1, N) \\
H(2, 1) & H(2, 2) & \cdots & H(2, N) \\
\vdots & \vdots & \ddots & \vdots \\
H(M, 1) & H(M, 2) & \cdots & H(M, N)
\end{bmatrix}.
\]

To construct a modulation matrix of the light field for GI, we will select an arbitrary row of \( H^2(m, n) \) to obtain a two-dimensional Hadamard derived pattern \( H^{(m)}_2(x, y) \).

To get a set of Hadamard derived pattern \( H^{(m)}_2(x, y) \), we first generate a high-order Hadamard matrix \( H^2(m, n) \) that can satisfy the imaging resolution (taking \( k = 2 \) for example, as shown in the left panel of figure 2(a)). Then, we extract each row of \( H^2(m, n) \) to obtain the corresponding single row vectors (as shown in the middle panel of figure 2(a)). At last, we acquire the two-dimensional matrix \( H^{(m)}_2(x, y) \) of \( x \) rows and \( y \) columns. Starting from the top (all the way) down to the bottom of the columns in the right extreme in figure 2(a), we give the derived pattern \( H^{(m)}_2(2, 2) \) with \( m = 1, 2, 3, 4 \), respectively.

Since the Hadamard matrix is a direct product of \( H^2 \), the higher order Hadamard matrix naturally contains the distribution information of a lower order Hadamard matrix. For example, with \( k = 4 \), the 16 pixel \( 16 \times 16 \) pixel Hadamard pattern \( H^2(16, 16) \) is shown in the left subfigure of figure 2(b). The corresponding Hadamard derived pattern in the right subfigure of figure 2(b) contains the 4-order Hadamard derived matrix \( H^{(4)}_2(4, 4) \), which is labelled by brown frames. For the purpose of reordering, we enlarge the latter matrix to the same size as the former one. In a similar derived approach, every pattern \( H^{(m)}_2(x, y) \) is a proper subset of the \( H^{(m)}_2(x, y) \) for \( k' > k \), and the relationship between the sets is shown in figure 2(c).

With the lower and higher derived pattern in our hand, we are now ready to reorder them. For a high order derived pattern \( H^{(m)}_2(x, y) \), \( k = 2 \times \kappa \), we first extract the lowest order derived pattern, i.e. \( H^{(m)}_2(2, 2) \) and prepose it. Then, we extract Hadamard derived patterns of order 16
[H^{(m)}_{2^{2}\times 2}(4, 4)] from the remaining Hadamard derived patterns [H^{(m)}_{2^{2}\times 2}(x, y) = H^{(m)}_{2^{2}\times 2}(2, 2)] and place it behind the previously extracted derived patterns [H^{(m)}_{2^{2}\times 2}(2, 2)], and deal with the rest of the lower order patterns [H^{(m)}_{2^{2}\times 2}(8, 8), H^{(m)}_{2^{2}\times 2}(16, 16), H^{(m)}_{2^{2}\times 2}(32, 32), …, H^{2\times (2^{k}-1)}_{2^{2}}] according to priority in the same way.

Concretely, in the language of the sets theory, H_{2^{2}\times 2} is a proper subset of H_{2^{2}\times 2} which is notated as H_{2^{2}\times 2} \subset H_{2^{2}\times 2} (to make it simple, we omit the superscript m in what follows), and the complementary set as \overline{H}_{2^{2}\times 2} = H_{2^{2}\times 2} - H_{2^{2}\times 2}, which is shown by the green area in figure 2(c). As for the higher order, every two adjacent even orders will produce a corresponding complementary set \overline{H}_{2^{2}\times 2} = H_{2^{2}\times 2} - H_{2^{2}\times 2}, whose elements will be reordered in our scheme (as shown in figure 2(d)). In a word, by enlarging the complementary set constantly we completed the Hadamard derived pattern reordering, H_{2^{2}\times 2}^{(m)}(x, y)_{new}, and the two dimensional form can be expressed as H_{2^{2}\times 2}^{(m), new} which is equivalent to \Phi in equation (2). At this point, \Psi = H_{2^{2}\times 2}^{(m), new} - I \{H_{2^{2\times 2}}^{(m), new}\}, the different full-widths at half-maximum of \Psi with different measurements number M = 2^{2\times \kappa} (\kappa = 1, 2, \cdots) can be efficiently calculated, as shown in figure 3. If the data in the 8380th row of the matrix figure 3(d) is reshaped as a 128 pixel \times 128 pixel image, as displayed in figure 3(i), the full-width at half-maximum of the peak image determines the spatial transverse resolution of GI [21]. So, as can be seen from figures 3(a)–(g), we can get seven different full-width at half-maximum images when M = 2^{2\times 7}. Hence, we can achieve the multi-resolution progressive imaging quickly.

To demonstrate the advantages of this method, we select an aircraft image with the image size of 128 pixel \times 128 pixel as the target object for the numerical simulation experimental. The comparison results with traditional computational ghost imaging (TCGI) using a natural order Hadamard derived pattern are shown in figure 4. By comparing the results TCGI and MPCGI in figure 4, it is noteworthy that the MPCGI can obtain 3, 4, 5, 6 and 7 multi-resolution images respectively when the measurement times is 64 (M = 2^{2\times 3}), 256 (M = 2^{2\times 4}), 1024 (M = 2^{2\times 5}), 4096 (M = 2^{2\times 6}), and 16384 (M = 2^{2\times 7}). Obviously, the MPCGI can obtain \kappa images with different resolutions when the measurement times are M = 2^{2\times \kappa}, \kappa = 1, 2, 3, \cdots. With this, continuous multi-resolution imaging is realized. The results also show that this scheme can be used for imaging at low measurement times. For example, the MPCGI method can perform low-resolution imaging on the target object of 128 pixel \times 128 pixel when M = 2^{2\times 4}, but TCGI did not.

3. Experimental results

To verify the feasibility of MPCGI, we conducted a series of experiments. In the experiments, the object is an aircraft model (see figure 5(h)) with the size of 20 cm \times 17 cm and positioned about 0.72 m, 2.42 m away from the projector (XGIMI Z4 Air miniature projector) and bucket detector (Thorlabs, PDA100A-EC, 320-1100nm, 2.4 MHz BW, 100 mm²), respectively.

We adopted a set of reordered Hadamard derived patterns H_{2^{2}\times 2}^{(m)}(128, 128)_{new}, m = 1, 2, 3, \cdots, 16384, i.e. we have done a set of seven resolutions MPCGI experiments and the results are shown in figure 5. With the increase of the number of the measurements, the aircraft image information of the reconstructed image gradually becomes clear, i.e. the image resolution steadily increased with measurement times. When the imaging resolution of MPCGI is 4 \times 4 (figure 5(b) M = 2^{2\times 2}), the position of the target object can be clearly identified. Hence, the target location can be locked by a low-resolution image with a small number of measurements. Even for M = 2^{2\times 5} (32 \times 32 resolution), we can clearly distinguish the clear outline of the target object (aircraft), which is enough for the military reconnaissance. As the number of measurements is further increased, the details of the reconstructed image gradually emerge. For example, the engines on both sides of the aircraft have been reconstructed by

![Figure 3](image-url)  
Comparison of the full-widths at half-maximum of \Psi^T\Psi with different measurements number. (a) M = 2^{2\times 1} (2 \times 2 resolution); (b) M = 2^{2\times 2} (4 \times 4 resolution); (c) M = 2^{2\times 3} (8 \times 8 resolution); (d) M = 2^{2\times 4} (16 \times 16 resolution); (e) M = 2^{2\times 5} (32 \times 32 resolution); (f) M = 2^{2\times 6} (64 \times 64 resolution); (g) M = 2^{2\times 7} (128 \times 128 resolution); (h) reordered Hadamard pattern; (i) the data in the 8380th row of the matrix (d) is reshaped as a 128 pixel \times 128 pixel image.
measuring $2^{3/6}$ times, as shown in figure 5(f). To achieve the same efficiency, 5460 times are needed in the TCGI scheme. Hence, 1364 measurement times can be reduced by our MPCGI scheme. Furthermore, after a 4-fold increase in resolution, we find out that the reconstructed image in figure 5(g) is much the same as figure 5(f), which shows that, in a few cases, high imaging resolution is indispensable to CGI, but there is a waste of resources in ultra-high imaging resolution. The results of MPCGI experiments verify the feasibility of the multi-resolution progressive imaging and low resolution location.

To evaluate the performance of our scheme under background light noise, we introduce the detection signal-to-noise ratio (DSNR) which is defined as

$$DSNR = 10 \log_{10} \frac{\langle B \rangle}{\sqrt{\langle (E - \langle E \rangle)^2 \rangle}}.$$  

where $\langle B \rangle$ is the mean signal power and $\langle E \rangle$ is the mean background light noise power [22]. For the 16384 measurements (figure 5), the DSNR is close to positive infinity (without noise), which exceeds the criterion in applicable practical application scenarios. Therefore, we add a laser and modulate it by a rotating ground glass (as shown in figure 6), and we weaken the intensity of the laser by attenuator to get a different DSNR so as to discuss the multi-resolution progressive imaging performance of the proposed scheme.

| Measurement times | Imaging scheme | Reconstructed multi-resolution images ($m \times n$ resolution) |
|-------------------|----------------|---------------------------------------------------------------|
| 16384 ($M = 2^{2^{3/6}}$) | TCGI | ![Reconstructed images](image1) |
| 4096 ($M = 2^{2^{3/6}}$) | TCGI | ![Reconstructed images](image2) |
| 1024 ($M = 2^{2^{3/6}}$) | TCGI | ![Reconstructed images](image3) |
| 256 ($M = 2^{2^{3/6}}$) | TCGI | ![Reconstructed images](image4) |
| 64 ($M = 2^{2^{3/6}}$) | TCGI | ![Reconstructed images](image5) |

Figure 4. Numerical simulation experimental results of TCGI and MPCGI schemes with different measurement times.
In figure 7, we show MPCGI results for different DSNR and resolution (equal to the measurement times). In the low DSNR case (DSNR < 32.71 dB), the low-resolution images can be effectively reconstructed (figures 7(a) and (b)). Unlike low resolution imaging, the high-resolution imaging can only get blurred reconstructed images (figures 7(c) and (d)), and, if the DSNR is too low (DSNR < 8.62 dB), the experimental system will be invalid. By contrast, when the DSNR is high enough (DSNR > 32.71 dB), the method can effectively reconstruct the multi-resolution images (figures 7(a)–(d)), and it has a fine image formation ability. Also, when the DSNR is 53.86 dB, the image quality with $M = 16384$ (figure 7(d), $128 \times 128$ resolution) and $M = 4096$ (figure 7(c) $64 \times 64$ resolution) are almost equal in vision. The results show that high-resolution images are difficult to obtain at low DSNR (i.e. the greater the resolution, the worse the noise resistance). When the DSNR is sufficient and there is no big demand for high-resolution, a large number of measurement times are not required to obtain the higher resolution images. Simultaneously, we find that a higher resolution is combined with a high DSNR for an ideal image reconstruction, and low resolution imaging robust performance is superior to high resolution. Moreover, a high resolution imaging is still the optimal choice in the case of high DSNR.

In the GI experiment, some evaluation indicators (such as signal-to-noise ratio, visibility, contrast to noise ratio, etc.) require the target image information and reconstructed image after a large number of measurements, and it makes the obtained reference evaluation values lag behind. In contrast, DSNR can be directly calculated based on the echo signal power and background light noise power received by the detector and the image information of the target object is not required. Because of that, we can refer to the DSNR to analyze whether the experimental echo signal power meets the imaging requirements in advance.

4. Conclusion

In this paper, we have proposed and demonstrated a new method named multi-resolution progressive computational ghost imaging which uses the Hadamard derived pattern to realize continuous multi-resolution imaging simply and quickly. Both numerical simulations and experimental realizations have been used to demonstrate its exceptional features. First of all, continuous multi-resolution images can be retrieved without additional detection; second, the number of measurements is much less than for the TCGI scheme; third, the complexity of Hadamard derived pattern reordering and multi-resolution imaging implementation is greatly reduced, compared with the ‘Russian dolls’ ordering of the Hadamard [19] method; fourth, studies on the effect of DSNR for different resolution can provide some reference value for practical application. Furthermore, if the order of Hadamard can be further optimized and the ultra high speed spatial light modulator (such as digital micromirror devices or LED-array [23]) is used, the MPCGI method can approximate the real-time imaging requirements of video frame rate. In some practical
application scenarios, like ground-to-air or air-to-air imaging, especially air surveillance applications, it is necessary to quickly feedback the results of target detection and recognition based on the feature information of multi-resolution images. In this case, the MPCGI method can play a key role.

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Figure 7. Results of different DSNR. (a) $M = 2^{2 \times 4}$ (16 × 16 resolution); (b) $M = 2^{2 \times 5}$ (32 × 32 resolution); (c) $M = 2^{2 \times 6}$ (64 × 64 resolution); (d) $M = 2^{2 \times 7}$ (128 × 128 resolution).
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