Risk measurement of global stock markets: a factor copula-based GJR-GARCH approach

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Abstract. Financial crisis in 2008 caused huge loss and one of the accusations is the misprediction of risk measurement. Considering the important role the stock markets play, and the trend of globalization in economy, we propose forecasting Value at Risk of G20’s (except European Union) stock indexes in three periods, pre-crisis, during crisis and post-crisis, via factor copula model. Unlike those models based on multivariate normality, factor copula is based on the assumption that there exists a or several common factors which lead to the change of stock prices. In this paper, different levels of dependence among 19 countries are presented and the results indicate that, during crisis countries with higher values of coefficients tend to have larger loss than others. Also, the large numbers of violations to VaR may be an indicator of the upcoming financial crisis.

1. Introduction

Nowadays, it is becoming a primary concern to measure and monitor the correlation between different stock markets for researchers, investment risk managers, and policy makers because the substantial loss in financial sector in relation to the 2008 global financial crisis has highlighted the importance of the measurement of risk (see Switzer et al. [1]). One of the popular topics to discuss the financial crisis, especially for the financial sector, is to study the correlations of individual stocks or indexes (see Paul & Zalewska, [2]) and the most difficult problem for prediction is model selection, because the best goodness-of-fit model is characteristically for pre-crisis, during crisis and post crisis (see Grundke & Polle [3]). Therefore, it becomes imperative to make risk measurement of global stock markets with suitable models.

In recent years, there have been numerous studies about risks in stock markets and many empirical papers investigated the spillover effects of situation in one market on risk in other stock markets using a wide diversity of methods and models. According to the literature review, most of papers focus on nationwide, like Vijverberg et al. [4] who used the GTL distribution to capture the VaR and ES; comparison between two countries, Hojin Lee et al. [5], propose multifractal VaR model to predict the extreme value. Besides, different tools are adopted to study the effect and causes of the financial crisis. Jun et al. [6] suggested a possible warning signal of the global financial crisis by statistical moments and standardized normalization. Paul & Zalewska [2] claimed that the composition effects in one sector is caused not by this sector itself but other sectors. Andriosopoulos et al. [7], Luchtenberg & Vu...
(2015) [8] studied the contagion effect and found the frontier markets are less vulnerable than mature markets by OLS. Furthermore, papers like Nicholas (2016) [9] proposed a new method called quantile on quantile approach to estimate the correlation between assets; Huang (2009) [10] proposed conditional copula-Garch model to capture the VaR between NASDAQ and TAIEX; Hussain & Li (2018) [11] adopt the copula model to measure the dependence structure between Chinese stock markets and the results indicate the low correlations; Nguyen et al. (2016) [12] found the gold may be a safe heaven asset for some countries during the crisis by different copula families; Reboredo & Ugolini [13] applied Vine copula to obtain the CoVaR for measuring sovereign debt risk for the financial sector; So and Young [14] discovered the tail dependence varies all the time by Vine-copula Garch with five Hong Kong blue chip stock data; Aloui [15] adopted the similar method and find out the dependence structure between oil, stock prices and exchange rates is highly affected by the financial crisis.

The papers from literature review as summarized above reveal the reliability and feasibility of copula-Garch model to study financial crisis, especially correlations between different variables. However, there are also some drawbacks of this model. Firstly, it is difficult to measure the high-dimensional data by Vine-copula. According to our literature review, rare papers use more than 5-dimensional data and none of them uses more than 12-dimensions. Because the number of dependence parameters $O(d^2)$ (where $d$ is the number of variables) is not feasible to current computer calculation when $d$ is large. Secondly, economy performance cannot be interpreted by only few endogenous variables (like interest rate) but also exogenous variables (like climate change). Therefore, it is not reasonable to measure the potential risk of global economy by low-dimensional data and partial financial sectors. If we treat all variables as a whole like latent variable, the prediction will be more reliable. At last, factor copula model can reduce the parameters from $O(d^2)$ to $O(d)$ which make the process practicable(see Krupskii & Genton [16]).

To fix these drawbacks, G20's data are collected to represent the global stock markets. And factor copula model, proposed by Krupskii & Joe [17] is adopted for its simplicity and reliability because it has less numbers of dependence parameters and can be implemented and explained easily.

The main contribution and findings of this paper are threefold. Firstly, it is the first time that factor copula model is applied to measure the global financial risk. Secondly, according to the results, China has the lowest leverage effect and coefficient of common factor meanwhile European countries like Germany, Italy, France, North America countries, like Canada and America, have the largest leverage effect and coefficients of common factor. The estimation results are consistent with the fact that latter countries suffered most during the financial crisis. In addition, VaR predictions are obtained by simulations. The results show the differences among three periods, pre-crisis, during crisis and post-crisis and the number of violations is closed to the theoretical number which indicates the model can be used as a better tool to predict the VaR.

The remainder of the paper is structured as follows. Section 2 presents the data we use. Section 3 presents the definition of factor copula and tail dependence. Section 4 describes an empirical study of daily returns of stock indexes of 19 sovereignties in G20. Section 5 gives a brief conclusion and a discussion for further research.

2. Another section of your paper
In this paper we use the data from 19 countries of G20 (not including EU). The G20 economies account for around 85% of the gross world product, 80% of world trade, two-thirds of the world population, and approximately half of the world land area. Members of G20 like America, Japan, England can represent the advanced economy, China and India can represent emerging market and Indonesia and Mexico can represent developing countries. High dimensional data with representative countries for different levels of economy and large proportion of the gross world products, trade, population and land can surely represent the global stock markets. Separating time into pre-crisis, during crisis and post-crisis is to find the differences among the three periods of time so that we may provide more reliable prediction. Our full sample period spans from May 1, 2003 to May 31, 2018.
Pre-crisis period is from May 1, 2003 to July 31, 2007. Crisis period is from August 1, 2007 to December 31, 2012. Post-crisis is from January 1, 2012 to May 31, 2018. The reason for defining such periods for the whole sample is situational because dot-com bubble and terrorism like 9/11 in early 2000s took place before the occurrence of high volatility in stock markets implying the large effect from exogenous variables. As for the crisis period, followed by the European debt crisis in 2007, the global financial crisis started. We keep some observations for each period for comparison to VaR. We first transform the prices of stock indexes into daily log-return, that is $L_{n_{t,d}} = \log \left( \frac{P_t}{P_{t-1}} \right)$.

We collect daily price of stock indexes of 19 individual countries in G20 from Bloomberg as its representativeness. Names of countries and stock indexes are shown in Table 1.

Table 1. Stock indexes used in the empirical analysis.

| Country   | Name  | Country   | Name  |
|-----------|-------|-----------|-------|
| America   | spx   | Brazil    | ibovespa |
| England   | uxx   | Argentina | merval |
| Japan     | ni225 | Mexico    | s&b/bmv |
| France    | cac40 | Korea     | kospi |
| Germany   | dax30 | Indonesia | jkse |
| Canada    | s&p/tsx | India | bsesensex |
| Italy     | ftmib | Saudi     | tasi |
| Russia    | imoex | South Africa | jti |
| Australia | s&p/asx20 | Turkey | xu100 |
| China     | shcomp |          |        |

We keep 261 observations for each period and use rolling window method to make the prediction. To be specific, the prediction of $R_{t+i}$ is based on the information set $\{R_{t}, R_{t+1}, \ldots, R_{t+i-1}\}$, where $t \in (1,2,\ldots,261)$. And we delete two days’ data due to the dis-convergence situation during post-crisis period. To be specific, in this paper, we initially use the in-sample data to estimate VaR. Take pre-crisis as an example, we use 848 observations to run the estimation and keep 261 for comparison. Then VaR1 is given by the time period from $t=1$ to $t=848$, VaR2 is given by $t=2$ to $t=849$. In the end, we get totally 261 VaRs for pre-crisis and crisis period, and 259 VaRs for post-crisis (due to inconvergence of the model). More details will be provided in the following section.

3. Methodology

At first, we adopt ARMA(1,1)-GJR-GARCH(1,1) models by four different distributions which are normal, skew-norm, skew-t and skew-ged. According to AIC and BIC, skew-t distribution is the most suitable distribution for pre-crisis while skew-norm is suitable for crisis and post-crisis period in this case. Considering the problem of modeling the joint distribution of time series data by copulas, we follow the two-step estimation [18]. We estimate the conditional marginal distribution by ARMA-GJR-GARCH model and get the parameters including leverage effect at the first step. Then we get the dependence parameters(tail dependence and coefficient) of factor copula model by the results from the first step. As for VaR part, we use rolling window approach and generate 10 thousand simulations each time with parameters of factor copula, then we can easily get the VaR by the quantile of the simulation data. In this section, we briefly introduce the ARMA-GARCH model, factor copula, tail dependence and Value at Risk.

3.1. ARMA-GJR-GARCH

Firstly, we model the mean of the marginal distribution by the approach presented by Harvey & Todd [19], the ARMA(r,m)

$$y_t = c + \sum_{k=1}^{r} \varphi_{ik} y_{t-k} + \sum_{k=1}^{m} \rho_{ik} e_{t-k} + e_t$$

(1)

We keep 261 observations for each period and use rolling window method to make the prediction. To be specific, the prediction of $R_{t+i}$ is based on the information set $\{R_{t}, R_{t+1}, \ldots, R_{t+i-1}\}$, where $t \in (1,2,\ldots,261)$. And we delete two days’ data due to the dis-convergence situation during post-crisis period. To be specific, in this paper, we initially use the in-sample data to estimate VaR. Take pre-crisis as an example, we use 848 observations to run the estimation and keep 261 for comparison. Then VaR1 is given by the time period from $t=1$ to $t=848$, VaR2 is given by $t=2$ to $t=849$. In the end, we get totally 261 VaRs for pre-crisis and crisis period, and 259 VaRs for post-crisis (due to inconvergence of the model). More details will be provided in the following section.
where $y_t$ is the conditional mean and $e_t$ is the error term. And the conditional variance model GJR-GARCH(p,q) is defined as

$$\sigma_t^2 = \omega + \sum_{i=1}^{p}(\alpha_i + \gamma_i I_{t-i})e_{t-i}^2 + \sum_{j=1}^{q}\beta_i\sigma_{t-j}^2$$

where

$$I_{t-i} = \begin{cases} 
0 & \text{if } r_{t-i} \geq \mu , \\
1 & \text{if } r_{t-i} < \mu .
\end{cases}$$

$\gamma$ is leverage effect, $e_t = \eta_t \sigma_t$ and $\eta_t$ i.i.d. of standard innovation.

We use the data to fit the different ARMA-GJR-GARCH and get the cdfs of each country. Then we use factor copula to estimate the parameters by the cdfs.

### 3.2. One-factor copula models

Copula can build a unique joint distribution by provided margins which are continuous on their domain. Let $FX$ be the cumulative distribution function of $X$, where $X$ can be either scalar or vector. We assume that $X=(X_1, X_2, \cdots, X_n)$, and $F_{X_j}$ is the marginal distribution of $X_j$ for $j=1, \cdots, n$. According to Sklar [20], $F_X(x_1, x_2, \cdots, x_n) = C_X(F_{X_1}, \cdots, F_{X_n})$. Also, copulas are suitable for modeling data with tail dependence and asymmetry. Moreover, one of the latest models, factor copula model, has several advantages that make the estimators explained easily and nicely. To be specific, economic growth and particularly stock indexes are influenced by one or several common variables which are accumulated by multiple variables and some of which are exogenous variables (such as political environment, interest rate, financial policy, etc.) that cannot be easily measured. If we can measure and estimate the parameters of the common variables, then we can forecast the movement of the stock indexes and eventually we can get a better prediction.

Copula completely describes the dependence among the variables $X$ by combining the univariate margins. After transformation, that is, $FX_i=ui \sim U(0,1)$, we can get the uniform random variable vector $U=(U_1, \cdots, U_n)$. There are several factor copula models (such as Hull and White, Oh and Patton, Vasicek, etc.); here we choose the model proposed by Joe Harry for its simplicity and traceability in the parameter estimation process. Then the joint cdf of $U$ is given by $C(u_1, \cdots, u_n)$. Given p latent variables $V_1, \cdots, V_p$ and assuming that $V_i$ are i.i.d and have uniform distribution, we can get the conditional cdf of $U_i$ given $V_i$, $F_j|V_1, \cdots, V_n$,

$$C(u_1, u_2, \cdots, u_n) = \int_{[0,1]^p} \prod_{j=1}^{q} F_j|v_1, \cdots, v_p(u_j|v_1, \cdots, v_p)dv_1 \cdots dv_p$$

We can explain the dependence of the observed variables via latent variables and (3) is called factor copula model.

When $p=1$, there is only one latent variable. $C_j,V_1$ represents the joint cdf of $(U_j,V_1)$ while $c_j,V_1$ pdf. Since $F_j|V_1= C_j|V_1(u_j|v_1) = \partial C_j|v_1$, then Equation (3) becomes

$$C(u_1, u_2, \cdots, u_n) = \int_{[0,1]} \prod_{j=1}^{q} F_j|v_1(u_j|v_1)dv_1 = \int_{[0,1]} \prod_{j=1}^{q} C_{j|v_1(u_j|v_1)}dv_1$$

then the likelihood can be written as

$$L(u_1, u_2, \cdots, u_n; \theta) = \prod_{i=1}^{n} c(u_i, u_2, \cdots, u_n; \theta)$$

In this case, we adopt Gauss-Legendre quadrature approach to approximate the integral as a summation of weighted integrands at quadrature points.

$$c(u_1, u_2, \cdots, u_n; \theta) \approx \sum_{k=1}^{nq} w_k \sum_{j=1}^{nq} c_j(v_i, x_k; \theta)$$

where $w_k$ stands for the quadrature weights, $n_q$ is the number of quadrature points and $x_k$ is the nodes. When $n_q$ is between 21 and 25, the approximation of the integrals tends to be good. So that from Equation (6) we can easily get the accurate approximation.

### 3.3. Tail dependence

Tail dependence is used in extreme value theory which allows us to obtain the characteristic of tail implied by a given factor copula model. The definition of tail dependence for two variables with marginal distribution $G_i, G_j$ is as follows:
\[ \tau_{ij} \equiv \frac{\Pr[X \leq G_i^{-1}(q), X \leq G_j^{-1}(q)]}{q} \quad \text{(7)} \]

\[ \tau_{ij}^U \equiv \frac{\Pr[X > G_i^{-1}(q), X > G_j^{-1}(q)]}{1-q} \quad \text{(8)} \]

3.4. Value at risk

Value at risk (VaR) is widely used in financial sectors, commercial banks, companies and risk managers to measure and quantify the level of the risk over a certain period of time in order to determine the occurrence ratio of potential losses from firm-wide to larger scope. Mathematically, VaR is defined as

\[ \text{VaR}_\alpha(X) = \inf \{ x \in \mathbb{R} : P(r_t < x) > \alpha \} \quad \text{(9)} \]

where \( \alpha \in (0,1) \) is the quantile, \( r_t \) is the return and VaR means the occurrence of the losses at \( \alpha \) quantile. Every \( R_{t+i} \) can be gotten from information set \( [R_t, R_{t+1}, \cdots, R_{t+i-1}] \) by simulations with factor copula GJR-GARCH models. We run the simulations based on the parameters we get from \( R_t \) to \( R_{t+i-1} \) to get the result for \( R_{t+i} \). We can get the VaR by by the different quantile of simulations.

Additionally, we use univariate coverage test LRUC to check whether the real losses are in line with VaR predictions.

\[ \text{LR} = -2 \text{Log}(1 - P) \times x^T + 2 \times \text{Log} \left( \frac{1-x}{T} \right)^T \times x \quad \text{(10)} \]

Where \( x \) is the number of outliers, \( T \) stands for the total number of predictions.

4. Empirical results for stock indexes

|           | pre-crisis | in-crisis | post-crisis |
|-----------|-----------|-----------|-------------|
| tasi      | -0.046    | shcomp    | 0.022       |
| shcomp    | 0.004     | bsesensex | 0.104       |
| ni225     | 0.050     | imoex     | 0.107       |
| xu100     | 0.058     | merval    | 0.110       |
| spx       | 0.084     | ibovespa  | 0.117       |
| s&p/asx20 | 0.101     | s&p/tsx   | 0.128       |
| ibovespa  | 0.103     | jkse      | 0.128       |
| jtopi     | 0.104     | s&p/bmv   | 0.129       |
| ftsemib   | 0.107     | tasi      | 0.130       |
| dax30     | 0.110     | xu100     | 0.133       |
| cac40     | 0.112     | kospi     | 0.134       |
| kaspi     | 0.118     | spx       | 0.137       |
| s&p/tsx   | 0.122     | jtopi     | 0.147       |
| ukx       | 0.123     | s&p/asx20 | 0.149       |
| s&p/bmv   | 0.125     | ukx       | 0.152       |
| imoex     | 0.141     | ni225     | 0.162       |
| merval    | 0.178     | ftsemib   | 0.167       |
| bsesensex | 0.268     | dax30     | 0.182       |
| jkse      | 0.348     | cac40     | 0.197       |
| spx       | 0.296     |           |             |
Table 2 shows the information of leverage effect in 19 countries which we get from the ARMA-GJR-GARCH model. We can notice that, Chinese stock market got tiny influence in all three periods. Macropic readjustment and government intervention may be the main reason that China did not get much influence. For other countries, leverage effect varied across time. Leverage effect of Canada is high during the three periods while those of America and England increase a lot from crisis to post-crisis. Another interesting thing is that, in general, developed countries tend to have lower leverage effect than other countries.

4.1. Result from 1-factor copula

We use t, Gumbel, Frank and Gaussian copulas to run the test and according to AIC and BIC, t-copula is the most suitable copula for pre-crisis and post-crisis while Gumbel copula is best for in-crisis. The results for AIC and BIC are presented in Table 3.

| Table 3. AIC and BIC for different copula family. |
|-----------------------------------------------|
| pre-crisis | AIC | Gumbel | Frank | Gaussian |
|------------|-----|--------|-------|----------|
| t          | 4363.74 | 4520.161 | 4863.172 | 5138.536 |
| BIC        | 4453.854 | 4610.276 | 4953.287 | 5228.651 |

| crisis     | AIC | Gumbel | Frank | Gaussian |
|------------|-----|--------|-------|----------|
| t          | 11339.012 | 11184.444 | 11834.764 | 12246.414 |
| BIC        | 11434.964 | 11280.396 | 11930.717 | 12342.366 |

| post-crisis| AIC | Gumbel | Frank | Gaussian |
|------------|-----|--------|-------|----------|
| t          | 6057.422 | 6207.554 | 7206.634 | 7229.819 |
| BIC        | 6147.537 | 6297.668 | 7296.749 | 7319.934 |

Bold indicates the best copula with smallest AIC and BIC

Table 4. Estimation result for three periods.

|                | pre-crisis | crisis | post-crisis |
|----------------|------------|--------|-------------|
|                | 0 | s.e | tau | lud | 0 | s.e | ud | TD | 0 | s.e | tau | lud |
| spx            | 0.480*** | 0.030 | 0.319 | 0.301 | 2.071*** | 0.052 | 0.517 | 0.603 | 0.625*** | 0.021 | 0.430 | 0.391 |
| ukx            | 0.840*** | 0.011 | 0.635 | 0.588 | 3.857*** | 0.110 | 0.741 | 0.803 | 0.825*** | 0.011 | 0.618 | 0.569 |
| ni225          | 0.315*** | 0.036 | 0.204 | 0.222 | 1.220*** | 0.027 | 0.181 | 0.235 | 0.247*** | 0.034 | 0.159 | 0.195 |
| cac            | 0.939*** | 0.006 | 0.776 | 0.740 | 5.664*** | 0.202 | 0.823 | 0.870 | 0.927*** | 0.006 | 0.756 | 0.717 |
| dax            | 0.921*** | 0.007 | 0.745 | 0.705 | 4.430*** | 0.135 | 0.774 | 0.831 | 0.905*** | 0.007 | 0.720 | 0.678 |
| sptx           | 0.418*** | 0.032 | 0.275 | 0.269 | 1.718*** | 0.042 | 0.418 | 0.503 | 0.539*** | 0.024 | 0.363 | 0.335 |
| ftsemib        | 0.891*** | 0.009 | 0.699 | 0.655 | 3.460*** | 0.096 | 0.711 | 0.778 | 0.834*** | 0.011 | 0.628 | 0.580 |
| inoex          | 0.246*** | 0.039 | 0.158 | 0.195 | 1.737*** | 0.043 | 0.424 | 0.509 | 0.502*** | 0.026 | 0.335 | 0.314 |
| s&p/asx20      | 0.272*** | 0.037 | 0.175 | 0.205 | 1.280*** | 0.029 | 0.219 | 0.282 | 0.262*** | 0.033 | 0.169 | 0.201 |
| shcomp         | 0.000   | 0.041 | 0.000 | 0.116 | 1.117*** | 0.024 | 0.105 | 0.141 | 0.122*** | 0.036 | 0.078 | 0.152 |
| ibovespa       | 0.327*** | 0.036 | 0.212 | 0.228 | 1.706*** | 0.042 | 0.414 | 0.499 | 0.398*** | 0.030 | 0.261 | 0.260 |
| merval         | 0.202*** | 0.038 | 0.130 | 0.179 | 1.750*** | 0.043 | 0.429 | 0.514 | 0.366*** | 0.031 | 0.238 | 0.245 |
| ibovespa       | 0.427*** | 0.032 | 0.281 | 0.274 | 1.730*** | 0.042 | 0.422 | 0.507 | 0.501*** | 0.026 | 0.334 | 0.313 |
| kospi          | 0.301*** | 0.036 | 0.194 | 0.216 | 1.290*** | 0.030 | 0.225 | 0.289 | 0.269*** | 0.033 | 0.174 | 0.204 |
| jkse           | 0.323*** | 0.039 | 0.149 | 0.189 | 1.286*** | 0.030 | 0.222 | 0.285 | 0.205*** | 0.035 | 0.132 | 0.180 |
| bsesensex       | 0.243   | 0.037 | 0.156 | 0.194 | 1.387*** | 0.033 | 0.279 | 0.352 | 0.404*** | 0.030 | 0.265 | 0.263 |
| tasi           | 0.009*** | 0.044 | 0.006 | 0.119 | 1.123*** | 0.026 | 0.109 | 0.145 | 0.189*** | 0.037 | 0.121 | 0.174 |
| jtopi          | 0.430*** | 0.031 | 0.283 | 0.275 | 1.910*** | 0.047 | 0.476 | 0.563 | 0.553*** | 0.024 | 0.373 | 0.343 |

Lud stands for low and upper tail dependence, ud stands for upper dependence. The symbol *** indicates confidence level of 99%.

After selecting the suitable model, we run the test respectively for the three periods of time. The coefficient, standard error, Kendall’s tau and tail dependence are shown in Table 4. We can notice that the coefficient of Chinese stock index is still the smallest which means the influence it got is smaller than any other G20 countries. Meanwhile, the largest coefficients all belong to Europe (Germany,
England, Italy) except Canada. These four countries are easier to get affected than any other G20s. Another eye-catching point is the coefficient of each country for in-crisis period is significantly larger than pre-crisis and post-crisis which indicates the accuracy of the model since it fits the fact that during financial crisis, most countries are more sensitive about the latent variable (including economy performance, interest rate, political situation, etc).

T-copula has the symmetric tail dependence while Gumbel copula asymmetric. The tail dependence in crisis time is generally relatively larger than in other periods. And the countries with higher coefficients are more likely to have the larger tail dependence. This indicates that countries with higher tau and tail dependence, like America, England, Germany, France, are more vulnerable than others. America experienced huge loss on CDS and CDO while England, Germany and France suffered from the European crisis.

4.2. Value at risk

In this section, we run the program to get the VaR by simulated data set (ten thousand times for each period) based on the previous distribution and copula. Then we compare the computed data with the kept observations.

Table 5 presents the number of violations of each quantile of VaR and LRUC results for three periods respectively. The true data fluctuated frequently but the number of violations are acceptable because the 95% confidence level is \( LRuc=3.84 \), therefore the non-rejection intervals for VaR 1%,2%,5% are [1-6], [2-9], [7-20] of univariate coverage test \( LRuc \).

The number of violations is relatively high during pre-crisis period especially at the 5% level. During crisis period, violation numbers of three quantiles are closer than the other two periods. It fits the fact that when bankruptcies occur, different quantiles of VaR are less sensitive due to the uncertainty of the market. As for the post-crisis period, the numbers of violations are nearly the same as the theoretical ones. Our number of violations is close to the theoretical number which indicates the accuracy of the prediction and can be used to measure the risk in different time periods. It indicates that the predictions of VaR we get from ARMA-GJR-GARCH factor copula models are suitable and accurate, especially for the post-crisis time and this can be the reference for risk manager, policy maker and so on.

| Quantile | Pre-crisis | Crisis | Post-crisis |
|----------|------------|--------|-------------|
| 1%       | 3          | 2      | 0.157       |
| 2%       | 5          | 13     | 0.537       |
| 5%       | 13         | 20     | 0.813       |

\( LRuc=3.84, *P<0.05 \)

Figure 1 shows that, for pre-crisis period the closer to the financial crisis, the more violations appear. We can notice that there are two extreme outliers in pre-crisis time and that may be an indicator for the upcoming financial crisis. On February 27th, 2007, stock markets (including America, China, Canada, Germany etc.) suffered crashes and the loss is more than 3%. Even though our prediction (151th) fails to predict the extreme value, still we capture the downward movement. During crisis period, it shows that our model performs well during financial crisis time. With the frequent fluctuation, outliers are centered at the first half of predictions and less outliers at the second half. A possible explanation is that the economy is recovering so that the returns turned to be more stable. Meanwhile for post-crisis period, it shows the most accurate prediction. Similar with the 151th prediction of pre-crisis period, 180th prediction in post-crisis also successfully captures the plunge of the returns.
5. Conclusions and future research

In this paper, we show the accuracy of the predictions from the ARMA-GJR-GARCH factor copula model. For different period of time, this model can capture the potential movement of the returns. Furthermore, this model simplifies the explanation and calculation procedure by using the latent variable to represent all the endogenous and exogenous variables. As for the empirical results, we find that the leverage effect levels of developed countries are relatively higher than others because countries with open economy are easier to get affected; $\theta$ during crisis is significantly higher than the other two periods which suggests all countries are more susceptible; European countries, America and Canada have higher tau and tail dependence which coincide with the huge loss they got from the crisis because of the correlation between different assets; VaR predictions show the accuracy and sensitivity of the economy performance and our model shows the ability to capture the extreme values.

Figure 1. VaR predictions for three periods. From top to bottom are pre-crisis, crisis, and post-crisis periods.
Since factor copula provides an easier way to dealing with high-dimensional data, we may make a comparison between factor copula with the vine copula in the future research with suitable dimensional data. Besides, different measures of risk, like expected shortfall and contagion effect, can be applied as further research.

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