STRONG LENSING PROBABILITY FOR TESTING TeVeS THEORY

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ABSTRACT

We calculate the strong lensing probability as a function of the image separation $\Delta \theta$ in TeVeS (tensor-vector-scalar) cosmology, which is a relativistic version of MOND (MOdified Newtonian Dynamics). The lens, often an elliptical galaxy, is modeled by the Hernquist profile. We assume a flat cosmology with $\Omega_m = 1 - \Omega_b = 0.04$ and the simplest interpolating function $\mu(x) = \min(1, x)$. For comparison, we recalculated the probabilities for lenses by singular isothermal sphere (SIS) galaxy halos in LCDM with a Schechter fit velocity function. The amplification bias is calculated based on the magnification of the second bright image rather than on the total of the two brighter images. Our calculations show that the Hernquist model predicts insufficient but acceptable probabilities in flat TeVeS cosmology compared with the results of the well-defined combined sample of the Cosmic Lens All-Sky Survey (CLASS) and the Jodrell Bank/Very Large Array Astrometric Survey (JVAS); at the same time, it predicts higher probabilities than the SIS model in LCDM at small image separations.

Subject headings: cosmology: theory — dark matter — galaxies: luminosity function, mass function — gravitational lensing — methods: numerical

Since Bekenstein (2004) proposed the relativistic MOND theory (dubbed TeVeS), it has become possible to investigate the MOND phenomena in the cosmological sense. In particular, after determining the geometry and background evolution of the universe, and calculating the deflection of light due to a weak gravitational field, one can test TeVeS, and thus MOND, with gravitational lensing (Chiu et al. 2002; Zhao et al. 2006; Angus et al. 2006). Before TeVeS, strong gravitational lensing in the MOND regime could only be manipulated by extrapolating nonrelativistic dynamics (Qin et al. 1995; Mortlock & Turner 2001), in which the deflection angle is only half the value in TeVeS (Zhao & Qin 2006).

 Needless to say, comparing the predicted results of gravitational lensing with observations is of key importance in testing TeVeS. Zhao et al. (2006) first examined the consistency of the strong lensing predictions in the TeVeS regime for galaxy lenses in the CfA-Arizona Space Telescope Lens Survey (CAS- TLES). In this Letter, we investigate the statistics of strong lensing in the TeVeS regime, and we compare the predicted lensing probabilities to the well-defined sample of CLASS/JVAS. We adopt the mass function of the stellar component of galaxies (Panter et al. 2004). As a first approximation, we do not consider galaxy cluster lenses; the lenses in the well-defined sample in CLASS/JVAS are believed to be produced by galaxies rather than galaxy clusters, although a cluster lens, SDSS J1004, was discovered in Sloan Digital Sky Survey (SDSS) (Inada et al. 2003; Oguri et al. 2004). We consider the simplest MOND interpolating function $\mu(x)$ and use the Hernquist profile (Hernquist 1990) to model the galaxy lenses. It is now established that, in standard cosmology (LCDM), when galaxies are modeled by an SIS and when galaxy clusters are modeled by a Navarro-Frenk-White (NFW) profile, the predicted strong lensing probabilities can match the results of CLASS/JVAS quite well (e.g., Chae 2003; Chen 2003a, 2003b, 2004a, 2004b; Li & Ostriker 2002; Mitchell et al. 2005; Oguri et al. 2002, 2003; Oguri & Keeton 2004; Peng et al. 2006; Sarbu et al. 2001; Wang 2004; Zhang 2004; Zhang et al. 2005). For comparison, we recalculated the lensing probabilities predicted by the SIS-modeled galaxy lenses in LCDM cosmology with the velocity function. Note that, in LCDM, the baryon infall effect (e.g., Kochanek & White 2001; Keeton 2001) has been well described by the SIS model for galaxies (Rusin & Kochanek 2005; Koopmans et al. 2006), at least statistically; furthermore, the effects of substructures (Oguri 2006) are also considered since we use the velocity function to account for the number density of lensing galaxies. Throughout this Letter, we assume the source QSOs have a redshift of $z_s = 1.27$.

TeVeS cosmology and deflection angle.—As in Bekenstein (2004) and Zhao et al. (2006), we adopt the Friedmann-Robertson-Walker (FRW) metric in TeVeS; i.e., $ds^2 = -ct^2 dt^2 + a(t)^2 (dx^2 + f^2(x)(d\theta^2 + \sin^2 \theta d\varphi^2))$, all in physical coordinates, where $c$ is the speed of light and $f(x) = \chi$ for a flat universe. The proper distance from the observer to an object at redshift $z$ is $D^r(z) = c \int_0^z [(1 + z)H(z)]^{-1} dz$, where the Hubble parameter at redshift $z$ is $H(z) = H_0(1 + z)^{3/2} + H_0 z^{1/2}$, where $H_0$ and $\Omega_b$ are the constant density parameters for baryon and dark energy, respectively, and we set the contribution from the scalar field to be zero by approximation (Bekenstein 2004). The angular diameter distance from an object at redshift $z_1$ to an object at redshift $z_2$ is $D(z, z_2) = [c/(1 + z)] \int_{z_1}^{z_2} H(z) dz$ for a flat universe. We assume cosmologies with a baryon density $\Omega_b = 0.04$ and a Hubble parameter $h = 0.73$.

In TeVeS, the lensing equation has the same form as in general relativity (GR), and for a spherically symmetric density profile

$$\beta = \theta - D_{\text{LS}} \alpha, \quad \alpha(b) = \int_0^b \frac{4b d\Phi(r)}{c^2 r} dr$$

(Zhao et al. 2006), where $\beta, \theta = b/D_a$, and $\alpha(\theta)$ are the source position angle, image position angle, and deflection angle, respectively; $b$ is the impact parameter; $D_a$, $D_s$, and $D_{\text{LS}}$ are the angular diameter distances from the observer to the lens, to the source, and from the lens to the source, respectively; and $g(r) = d\Phi(r)/dr$ is the actual gravitational acceleration [here $\Phi(r)$ is the spherical gravitational potential of the lensing galaxy, and $l$ is the light path]. It is well known that the stellar component...
of an elliptical galaxy can be well modeled by a Hernquist profile, 
\( \rho(r) = \frac{M_{}\text{r}}{2\pi r (r + r_{o})^3} \), with the mass interior to \( r \) as 
\( M(r) = \frac{1}{2}\pi r^2 M_{\text{r}}(r + r_{o})^3 \), where \( M_{\text{r}} = \int_0^r 4\pi r^2 \rho(r) dr \) is the total mass 
and \( r_{o} \) is the scale length. The corresponding Newtonian acceleration 
\( g_{N}(r) = GM(r)/r^2 = GM(r + r_{o})^2 \). According to 
MOND (Milgrom 1983; Sanders & McGaugh 2002; Sanders 
2006), \( g(r) = g_{N}(r/a_{\text{s}}) = g_{s}(r) \). We choose the simplest interpo-
lating function \( f(x) \) with \( f(x) = x \) for \( x < 1 \) and \( f(x) = 1 \) for \( x > 1 \). Thus, the deflection angle is 
\[
\alpha(b) = \int_{0}^{\sqrt{b^2 - b}} \frac{4GM}{c^2} \frac{b dl}{r(r + r_{o})^2} + \int_{\sqrt{b^2 - b}}^{\infty} \frac{4\sqrt{r(v)}}{c^2} \frac{b dl}{r(r + r_{o})}, \tag{2a}
\]
for \( b < r_{o} \), and 
\[
\alpha(b) = \int_{0}^{\infty} \frac{4\sqrt{r(v)}}{c^2} \frac{b dl}{r(r + r_{o})}, \tag{2b}
\]
for \( b > r_{o} \), where \( r_{o} \) and \( v_{o} \) are defined by \( GM(r_{o} + r_{s}) = v_{o}^2(r_{o} + r_{s}) = a_{o} = 1.2 \times 10^{-4} \text{ cm s}^{-2} \), so that \( r_{o} \) is a transition 
radius from the Newtonian to the MONDian regime and \( v_{o} \) is the 
flat part of the circular velocity (i.e., the circular velocity in the 
MONDian regime). The above deflection angle has an analytical 
but cumbersome expression (Zhao et al. 2006), so we calculate it 
umERICally.

We also need a relationship between the scale length \( r_{o} \) and 
the mass \( M_{\text{r}} \), which could be determined by observational data. 
First, the scale length is related to the effective (or half-light) 
radius \( R_{e} \) of a luminous galaxy by \( r_{o} = R_{e}/1.8 \) (Hernquist 
1990). It has been recognized that there exists a correlation 
between \( R_{e} \) and the mean surface brightness \( (I_{e}) \) interior to 
\( R_{e} \) (Djorgovski & Davis 1987): \( R_{e} \propto (I_{e})^{0.83 \pm 0.06} \). Since the 
luminosity interior to \( R_{e} \) (half-light) is \( L_{e} = L/2 = \pi(I_{e})R_{e}^2 \), 
we immediately finds \( R_{o} \propto L^{1/6} \). Second, we need to know the mass-to-light ratio \( T = M/L \) for elliptical galaxies. The 
observed data give \( p = 0.35 \) (van der Marel 1991); according to 
MOND, however, we should find \( p \approx 0 \) (Sanders 2006). In 
any case, we have \( L \propto M^{1/(1+p)} \). Therefore, the scale length 
should be related to the stellar mass of a galaxy by \( r_{o} \propto 
M^{1/(1+p)} \). In our actual calculations, we need to know \( r_{o}/R_{e} \). 
Since \( (r_{o} + r_{s}) \propto M^{1/2} \), we have \( (r_{o} + r_{s})/R_{e} = AM^{-p^2} \), where 
\( p^2 = -0.5 + 1.26/(1 + p) \), and the coefficient \( A \) should be further 
determined by observational data. Without a well-defined 
sample at our disposal, we use the galaxy lenses that have an 
observed effective radius \( R_{e} \), and thus \( r_{o} \) in CASTLES 
(Muñoz et al. 1998) and that are listed in Table 2 of Zhao et al. (2006).

The fitted formulae are 
\[
\frac{r_{o}}{r_{s}} + 1 = \begin{cases} 
2f_{1}(M) = 16.2M^{0.43} & \text{, for } p = 0.35, \\
2f_{2}(M) = 16.2M^{0.76} & \text{, for } p = 0, 
\end{cases}
\tag{3}
\]
where \( M_{*} = 7.64 \times 10^{10} \text{ h}^{-2} \text{ M}_{\odot} \) is the characteristic mass of 
galaxies (Panter et al. 2004).

Figure 1 shows us the cases when a lens is located at redshift 
\( z = 0.05 \) but with different values of \( r_{o}/R_{e} \) and mass \( M_{\text{r}} \). Here 
we allow \( \beta \) and \( \theta \) to take negative values due to symmetry. 
Generally, three images are produced when \( \beta < \beta_{c} \), where \( \beta_{c} \) is the 
critical source position determined by \( \theta \beta / d\theta = 0 \) and \( \theta < 0 \). 
For a plausible range of \( p = 0.35 \), Figure 1 shows us that 
a smaller scale length results in a larger value of \( \beta_{c} \), as expected.

**Lensing probability.**—Usually, the lensing cross section 
defined in the lens plane with image separations larger than \( \Delta \theta \) is 
\( \sigma(\Delta \theta) = \pi D_{l}D_{v} \Theta(\Delta \theta) \), where \( \Theta(x) \) is the 
Heavyside step function. This is true only when \( \Delta \theta(M) \) is approxi-
mately constant within \( \beta_{c} \), and the effect of the flux density 
ratio \( q_{s} \) between the outer two brighter and fainter images 
can be ignored. From Figure 1 we see that this is not true, in 
particular for low-mass galaxies. As usual, we consider the outer 
two images (the central image is very faint). For a given \( \beta \), the left one (with \( \theta < 0 \) is closer to the center and is 
fainter; the right one (with \( \theta > 0 \) is farther away from center and is 
brighter. On the other hand, when \( \beta \) increases, the fainter image 
approaches to the center and becomes fainter, and the opposite 
is true for the brighter image. Therefore, a larger \( \beta \) corresponds 
to a larger flux density ratio, as is well known. We thus intro-
duce a source position quantity \( \beta_{c} \), determined by 
\[
\left( \frac{\theta(\beta)}{\beta} \right) \left( \frac{d\theta(\beta)}{d\beta} \right)_{\beta_{c}} = q_{s}, \tag{4}
\]
where \( \theta_{s} = \theta(0) < 0 \), the absolute value of which is the Einstein 
radius, and \( \beta_{c} \) is determined by \( \theta \beta / d\theta = 0 \) for \( \theta < 0 \). Equation 
(4) means that when \( \beta_{c} < \beta < \beta_{c} \), the flux density ratio would 
be larger than \( q_{s} \), which is the upper limit of a well-defined sample. 
For example, in the CLASS/IVAS sample, \( q_{s} \leq 10 \). The flux den-
sity ratio effect is strongest for intermediate-redshift and low-mass lensing galaxies; e.g., for $z \sim 0.5$ and $f_1(M)$, $\beta_z/\beta_{z,0}\sim 0.35$ at $M = 0.1M_\odot$, and $\beta_z/\beta_{z,0}\sim 0.15$ at $M = 0.01M_\odot$. This effect can be ignored when the redshift of lensing galaxies is $z \sim 0$ or $z \sim z_\odot$. On the other hand, we adopt the suggestion that the amplification bias should be calculated based on the magnification of the second bright image of the three images rather than on the total of the two brighter images (Lopes & Miller 2004). For the source QSOs having a power-law flux distribution with slope $\gamma$ (Patnaik et al. 1992; King et al. 1999) and the predicted results of CLASS/JVAS (Myers et al. 2003; Browne et al. 2003; Patnaik et al. 1992; King et al. 1999) and the predicted probability for lensing by SIS halos in LCDM with different amplification biases ($B = 3.976$ for dashed line and $1.09$ for dash-dotted line) are also shown.

We thus write the lensing cross section with an image separation larger than $\Delta \theta$ and a flux density ratio less than $q_\star$, and combined with the amplification bias $B(\beta)$ as

$$
\sigma(\Delta \theta, q_\star) = 2\pi D_l^2 \int_0^{\Delta \theta_{\star}} \beta \mu^{-1}(\beta) d\beta, \quad \text{for } \Delta \theta \leq \Delta \theta_{\star},
$$

$$
\int_0^{\Delta \theta_{\star}} \beta \mu^{-1}(\beta) d\beta, \quad \text{for } \Delta \theta_{\star} < \Delta \theta \leq \Delta \theta_{\psi},
$$

$$
0, \quad \text{for } \Delta \theta > \Delta \theta_{\psi},
$$

(Schneider et al. 1992; Chen 2004a), where $\beta_{\Delta \theta}$ is the source position at which a lens produces the image separation $\Delta \theta$, $\Delta \theta_{\psi} = \Delta \theta(0)$ is the separation of the two images that are just on the Einstein ring, and $\Delta \theta_{\star} = \Delta \theta(\beta_{\star})$ is the upper limit of the separation above which the flux ratio of the two images will be greater than $q_\star$.

Now we can calculate the lensing probability with an image separation larger than $\Delta \theta$ and a flux density ratio less than $q_\star$, in TeVeS cosmology, for the source QSOs at mean redshift $z_s = 1.27$ lensed by foreground elliptical stellar galaxies by

$$
P(\Delta \theta, q_\star) = \int_0^{q_\star} \frac{dD^*(z)}{dz} dz \int_0^{\mu_{\max}} n(M, z)(1 + z)^3 \sigma(\Delta \theta, q_\star) dM
$$

(e.g., Wu 1996), where $M_{\max}$ is the upper limit of the mass for the lensing galaxies and $n(M, z)$ is the comoving number density of galaxies for which we use the well-fitted mass function of the stellar component of galaxies in SDSS given by Panten et al. (2004): $n(M) dM = n_s(M/M_\star)^{q} \exp \left( -M/M_\star \right) dM/M_\star$, where $n_s = (7.8 \pm 0.1) \times 10^{-3} h^3 Mpc^{-3}$, $\alpha = -1.159 \pm 0.008$, and $M_\star = (7.64 \pm 0.09) \times 10^{10} h^2 M_\odot$. The exact value of $M_{\max}$ is unimportant, but we adopt $M_{\max} = 10M_\odot$, so that we do not consider the contribution from galaxy clusters; unlike the galaxies, these are dominated by gas of mass $>10^{12} M_\odot$ with a $\beta$-profile.

The numerical results of equation (6) are shown in Figure 2. The solid line represents the probabilities when $r_{\Delta \theta}/r_{\Delta \psi} = 6.3(M/M_\odot)^{-0.70} - 1$ for $MIL = \text{constant supported by MOND}$, and the dotted line represents $r_{\Delta \theta}/r_{\Delta \psi} = 9.5(M/M_\odot)^{-0.43} - 1$ for $MIL = L_{0.3}$ from observations. For comparison, the survey results of CLASS/JVAS (Myers et al. 2003; Browne et al. 2003; Patnaik et al. 1992; King et al. 1999) and the predicted probability for galaxy lensing by SIS profiles in LCDM are also shown. The observational probability $P_{\text{obs}}(\Delta \theta)$ (Chen 2003b, 2004a, 2005) is plotted as a thick solid line in Figure 2. We recalculate the lensing probability with an image separation larger than $\Delta \theta$ and a flux density ratio less than $q_\star$, in flat LCDM cosmology ($\Omega_m = 0.3$ and $\Omega_{\lambda} = 0.7$), for the source QSOs at mean redshift $z_s = 1.27$ lensed by foreground SIS-modeled galaxy halos (Chae et al. 2002; Ma 2003; Mitchell et al. 2005): $P_{\text{sys}}(\Delta \theta, q_\star) = \int_0^{q_\star} \int_0^{\Delta \theta_{\max}} n(v, z)\sigma_{\text{sys}}(v, z)B dv' dz$, where $n(v, z)dv = n_s(1 + z)\left( v/v_\star \right)^{\gamma} \exp \left( -v/v_\star \right) \left( v/v_\star \right)^{\text{exp} \left( -v/v_\star \right) \left( v/v_\star \right)}$, is the number density of galaxy halos at redshift $z$ with a velocity dispersion between $v$ and $v + dv$ (Mitchell et al. 2005), $\sigma_{\text{sys}}(v, z) = 16\pi^3(v/c)^2(D_{1}\Delta D_{2}/D_0)^2$ is the lensing cross section, $r_{\Delta \psi} = 4.4 \times 10^{-3}(c/v_\star)(D_\Delta D_2/D_0)^2$ is the minimum velocity for lenses to produce an image separation $\geq \Delta \theta$, and $B$ is the amplification bias. We adopt $(n_s, v_\star, \alpha, \gamma) = (0.0064 \ h^1 Mpc^{-3}, 198 \ km \ s^{-1}, -1.0, 4.0)$ for early-type galaxies from Chae et al. (2002). According to equations (4) and (5), for the SIS model and the CLASS sample, $B = 2 \left| \beta \mu(\beta)^{-1}d\beta \right|$, with $\beta = 9/11$ and $\gamma = 2.1$ (see also Mitchell et al. 2005). Therefore, $B = 1.091$ based on the magnification of the fainter image (dash-dotted line in Fig. 2), and $B = 3.976$ based on the total magnification of two images (dashed line).

**Discussion.**—It has been held that, in the MOND regime, the effect of lensing is inefficient, in particular, that strong lensing never occurs (e.g., Scarpa 2006). Our calculations shown in Figure 2 indicate, however, that this is not true. Although the Hernquist model predicts insufficient lensing probabilities in a flat TeVeS cosmology compared with the result of CLASS/JVAS, the result is acceptable considering, at least, that the lensing galaxy can be modeled by steeper slopes and more efficient MOND $\mu$-functions.
Our results argue that TeVeS (and thus MOND) generates lenses with higher efficiency than CDM if the latter is modeled by an SIS profile and if, in both cases, the amplification bias is calculated based on the magnification of the second bright image (for SIS, the fainter image). Usually, $B$ is calculated based on the total magnification of the two images considered. Because we introduced a cutoff $\beta_p$ due to the flux ratio $q$, the total magnification is $2 \sim q + 1$ times larger than that of the second bright image (depending on $\beta$ when $\beta \leq \beta_p$), which results in a corresponding value of $B$ that is $\sim 4$ times larger (for both SIS in LCDM and Hernquist in TeVeS). As shown in Figure 2, the lensing probabilities for SIS halos in LCDM with $B = 3.976$ (total) match the results of CLASS/JVAS quite well (dashed line). Similarly, if we apply the total magnification to $B$ for a Mondian Hernquist model, the final lensing probabilities would be overpredicted compared with the results of CLASS/JVAS.

Note that an SIS profile is more concentrated in mass than a Hernquist profile, so if both profiles are applied in the same regime (LCDM or TeVeS), the SIS profile would be more effective at lensing than Hernquist. Therefore, the fact that the probabilities for the SIS model in LCDM ($B = 1.09$, dash-dotted line) are lower than the Hernquist model in TeVeS shown in Figure 2 implies that MOND demonstrates a higher lensing efficiency than CDM. This phenomena is in fact not difficult to understand. It is well known that MOND, as an alternative to dark matter gravitation or Mondian luminous matter gravitation. We know that a CDM halo is assumed to have nonzero density and its acceleration is proportional to the square of the source position angle $\beta$ (when $\beta < \beta_p$), as is well known in the SIS model.