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To cite this article: P A Muñoz and J A Araneda 2014 J. Phys.: Conf. Ser. 511 012004

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Nonlinear behavior of double ion beam distributions

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Abstract. We study the stability and nonlinear evolution of differential streaming between core and beam protons based on the analysis of a nonlinear kinetic dispersion relation and using hybrid simulations describing the self-consistent decay of Alfvén-cyclotron waves. The effects of ion trapping by ion acoustic waves and pitch-angle scattering induced by decay of the Alfvén-cyclotron waves are discussed.

1. Introduction
Circularly polarized Alfvén waves are ubiquitous and frequently observed in space plasmas, in particular in the solar wind [1]. In the last four decades, their decay through parametric instabilities has been the subject of many studies by means of analytical and numerical methods, mostly within fluid MHD theory (see references in [2]). On the other hand, the proton-electron plasma in many of these space environments also shows a proton beam, which changes the behavior of the system contributing with another source of free energy for the instabilities and the evolution of both core and beam proton velocity distribution functions (VDF’s). In this sense, the objective of this paper is the analysis of some features related to the presence of the beam not thoroughly understood of these systems within the framework of a hybrid kinetic-fluid dispersion relation, whose results will be compared to simulations which moreover will afford seeing the effects on the evolution of the proton VDF’s.

2. Linearized Dispersion relation
2.1. Non-perturbed equilibrium system
Let us consider a finite amplitude circularly polarized Alfvén-cyclotron wave (the parent or pump wave, denoted by 0) propagating along a background magnetic field $B_s \hat{z}$, in a quasi-neutral plasma composed of two populations of protons: a core (subscript c) and a less dense drifting beam (b), as well as massless isothermal electrons (e). The pump wave has a wave number $k_0$ and frequency $\omega_0$, and satisfies the exact nonlinear solution of the full set of Vlasov-Maxwell equations [3]:

$$k_0^2 = \frac{\mu_0}{B_s^2} \left[ n_{0c}m_p \frac{\omega_{0c}^2}{1 - \frac{\omega_{0c}}{\Omega_p}} + n_{0b} m_b \frac{\omega_{0b}^2}{1 - \frac{\omega_{0b}}{\Omega_b}} \right],$$  (1)

where, if $j = e, c, b$ denotes each specie of the plasma, $\Omega_j = q_j B_s / m_j$, $n_{0j}$ are the densities in equilibrium, and $\omega_j = \omega_0 - k_0 U_j$, with $U_j$ the parallel (to the magnetic field $B_s$) drift speeds.
with respect to the center of mass frame. The protons will be represented by the distribution function (which satisfies the Vlasov equation)

\[ F_j(z, v, t) = n_0 \delta(v - \mathbf{V}_{\perp j}) f_j(z, v_z, t), \]  

where

\[ \mathbf{V}_{\perp j} = -\frac{\omega_0 j/k_0 B_0}{1 - \frac{\omega_0 j}{\Omega_j} B_s} \]  

are the perpendicular bulk velocities and \( f_j(z, v_z, t) \) are spatially uniform drifting Maxwellians.

This model \([4]\) has the advantage it keeps the kinetic effects along the longitudinal direction, while the transverse motion of the protons can be modeled by the simpler Hall-MHD equations.

2.2. Hybrid kinetic dispersion relation

Now, we consider perturbations about the previously described equilibrium in the form \( \delta f \sim \exp(ikz - i\omega t) \) (and analogously for the rest of the quantities), where \( \omega = \omega_r + i\gamma \), with \( \gamma \) the growth rate of the instabilities. Linearizing the Vlasov-MHD-Maxwell set of equations, we obtain a dispersion relation (as appears in \([2]\), with the appropriate changes in the reference system) preserving the same structure as in the pure fluid case, but with a kinetic polytropic coefficient (relating the perturbations of pressure and density) defined as:

\[ \gamma_j = 2 \left( \xi_j^2 - \frac{1}{Z'(\xi_j)} \right), \]  

where \( \xi_j = \omega_j/(kv_{Tj}) \) and \( v_{Tj} = \sqrt{2kB_jT_{||j}/m_p} \) are the thermal speeds (\( m_p \): mass of the proton) and \( Z'(\xi_j) \) is the derivative of the plasma dispersion function. This dispersion relation, with an infinite number of roots (mostly very damped), has the parameters: amplitude \( A = (B_0/B_s)^2 \), \( \beta_j = 2\mu_0 n_j k_B T_{||j}/B_s^2 \) (=proton thermal pressure/background magnetic pressure), normalized beam density \( \eta_b = n_b/n_e \) and relative drift speed \( U := (U_b - U_c)/V_A \), with \( V_A = B_s^2/(\mu_0 n_e m_p) \) the Alfvén speed (it is useful too the notation \( \tilde{\beta}_j = \tilde{v}_{Tj}^2/V_A^2 \), proportional to the temperature). Its ten less damped modes (“daughter” waves) can be identified as:

- \( \pm f \): upper/lower sideband forward propagating
- \( \pm b \): upper/lower sideband backward propagating
- \( \pm beam \): upper/lower beam sideband forward propagating (due to beam)
- \( \pm s(b) \): ion (beam)-acoustic high/low phase speed

3. Numerical solutions and hybrid simulations

3.1. A typical high drift speed, high beam density case

We have performed numerical simulations using a hybrid code (i.e.: electrons considered as massless fluid and protons treated kinetically as particles) 1D in space and 3D in velocity \([4]\), in the protons’ center of mass frame. The chosen parameters are typical for space plasmas like the fast solar wind or the Earth’s bowshock \([1]\).

Thus, in the following figures are shown -by means of the power spectrum of the (longitudinal) density fluctuations- the aforementioned standard modes by comparing with the analytic dispersion relation and their respective growth rates (black lines indicate instabilities) and phase speeds. Colored lines indicate sideband-related waves, and the gray ones are acoustic waves. The parameters used in Figures 1-4 are: \( \beta_c = \tilde{\beta}_c = \tilde{\beta}_b = 0.1 \), \( \eta_b = 0.15 \), \( k_0 V_A/\Omega_p = 0.4 \), \( U = 1.5 \) and \( A = 0.3^2 \). We have chosen a left-hand polarized forward propagating pump wave \((\omega_0/\Omega_p \approx 0.253)\), represented as an open diamond in the diagrams. For the simulation, we
choose the system to have a length \( L_0 = 2\pi m_0/k_0 \), with the mode number of the pump wave \( m_0 = 32 \). The boundary conditions are periodic with 2048 cells and 400 particles/cell.

There are gaps in some branches (whose separation is proportional to \( \eta_b \)) of the dispersion diagram, in particular between the \(-b\) and \(-beam\). Thus, it is only meaningful to assign these names to the asymptotes shown as dashed lines, but in the gap region we shall only denote a determined branch when the usual electron-proton instabilities [4] are recovered.

Four parametric instabilities are visible and in good agreement with simulations, which can be classified in two groups according to its phase speed \( v_\phi = \omega_r/k \):

- Two strong of low phase speed: one for \( k < k_0 \) in the \(-f\) branch (\( v_\phi/V_A \approx 0.39 \)) which extends until \( k = 0 \) (modulational) and other for \( k > k_0 \) in the \(-beam\) branch (\( v_\phi/V_A \approx 0.26 \)), a combined version of the usual beat and decay instabilities.
- Two weak of high phase speed: one for \( k < k_0 \) in the \(-b\) branch (very narrow, \( v_\phi/V_A \approx 1.57 \)) and other for \( k > k_0 \) in the \(-f\) branch (very broad, \( v_\phi/V_A \approx 1.54 \)). These ones are related to the beam-core interaction and its maxima are near to the intersection of the sideband waves with the acoustic \(-sb\) branch. They are also present in the fluid approach [5].

We have also verified that the usual conservation relations between the frequency and number wave of the pump wave and of its respective daughter waves are satisfied. This was obtained by means of the time history of the wave power in the magnetic and density fluctuations calculated from the simulations, by analyzing the respective modes with the greater growth rate as time goes by.

In general, the growth rates of all these instabilities are increasing functions of \( k_0 \) and \( A \). We can even note that there is another instability of very low phase speed in the \(-b\) branch which appears only for a tenous beam and a high \( k_0 \) (For example, \( \eta_b = 0.025 \) and \( k_0 V_A/\Omega_p = 0.7 \)). This is also a increasing function of \( U \).
3.2. Effects of the drift speed

To characterize the instabilities and their relationship with the shape of the proton VDF’s, we made a parameter survey varying the corresponding parameters related to the core and beam as well as to the pump wave. But in this paper, we analyse principally the behavior related to the drift speed $U$. A useful way to visualize this (with the analytic dispersion relation) is by means of plots of the maximum growth rates of each instability (and its respective phase speed) vs. $U$, maintaining the other parameters fixed, in particular $k_0$. But this last condition, by the relation (1), implies a monotonically decreasing $\omega_0$ with respect to $U$. The same previous maximum $U = 2.1$ was chosen to be less than the threshold to trigger the linear right-hand polarized instability [6] for this $A$, because its corresponding growth rate is much higher that the aforementioned parametric instabilities, and therefore it would hide the behavior of these ones.

The symbols used in Figures 5-6 are ⟷ and ◷: high phase speed instabilities; ●: modulational; ◵, ◆ and □: low phase speed instabilities for $k > k_0$.

Figure 5. Maximum growth rates for varying $U$. Same previous parameters (except $\omega_0$) and instabilities colors. Explanation of the symbols in the text.

Figure 6. Phase speeds at the maxima growth rates for varying $U$. See figure 5.

Between $U = 1.2 - 1.7$, the low phase speed instability of the $-beam$ branch has three maxima. In the dispersion diagram $\omega$-$k$, we can associate these three instabilities with the sectors where the $-beam$ branch is located under or upper the $+f$ branch. In the intersections of both branches, they are stable. Two of these (with the lesser $k$) are the usual beat (◆, over $+f$) and decay (□, under $+f$) for an electron-proton plasma [4], and therefore the third one (◇, for the largest $k$, again over $+f$) is related to the beam. Furthermore, this last one only appears after a threshold $U \approx 1.2$.

The growth rates of the modulational and beat are minimally affected by the beam. On the other hand, the decay instability has a maximum growth rate for $U \approx 0.8$, and for higher $U$ it decreases at the same time that the third low phase-speed instability for $k > k_0$ begins to increase (from $U \approx 1.2$). This processes continues until $U \approx 1.5$, at which both have similar growth rates, and, when $U$ takes higher values, this situation reverses, i.e: decay is growing at the expense of the other instability. For low drift speed $U \lesssim 1.1$, there only exists the well-known core-related instabilities, and are well distinguishable the strong decay and the weaker beat, that together with the modulational, are ubiquitous in a wide range of parameters for $\beta_{c,b} < 1$ and a left-hand polarized pump wave [2].

On the other hand, we can justify that the two high phase speed instabilities are related to the beam because both have maximum growth rates that are monotonically increasing functions of $U$ (and also, but it is not shown here, increasing functions of $n_b$). However, these instabilities require a relatively high drift speed to be triggered ($U \gtrsim 1.5$). For the high drift speed case $U = 2.1$, the growth rates of the beam-related instabilities are comparable to the corresponding of the electron-proton instabilities. Furthermore, their phase speeds are even higher that for the $U = 1.5$ case, whereas the other core-related instabilities have somehow lower phase speeds. We
can obtain higher growth rates and higher phase speeds for a lesser $\beta_b$, i.e.: for a colder beam.

We also analyzed the properties of those instabilities for a right-polarized pump wave. As is shown in past studies within fluid theory [7], for the electron-proton plasma instabilities, it is only the decay instability in the $-b$ branch which has important growth rates for $\beta_{c,b} < 1$. In addition, we also observed several minor instabilities related to the beam in various branches with similar behavior to the high phase speed ones previously mentioned, i.e.: they have a minimum threshold around $U \approx 1.5$ and are also increasing functions of the drift speed.

4. Evolution of the VDF’s

The time evolution of the core and beam distributions are shown below in Figures 7-8, for two cases of drift speeds.

![Figure 7. Protons VDF’s. Up: Beam, Down: Core. Left: $t = 0 \Omega_p^{-1}$, Right: $t = 200 \Omega_p^{-1}$. Same previous parameters. High drift speed case $U = 2.1$.](image1)

![Figure 8. Protons VDF’s. Low drift speed case $U = 0.8$. See figure 7](image2)

For $U = 2.1$, we note the beam formation in the core population due to trapping by the ion-acoustic waves [8]. In fact, we have two low phase speed instabilities ($v_\phi/V_A < 0.4$) which affect the core VDF, inducing a elongation in the longitudinal direction due to the resonant interactions with the particles. On the other hand, the high-speed parametric instabilities related to the beam affect the center of the beam VDF (because $v_\phi/V_A \approx 1.5$), but its effects are not easily visible. Instead of that, pitch-angle scattering, induced by the parametric instabilities, produces a reshaping of the protons VDF’s mainly in the transversal direction. This heating of the beam component is clearly visible by the parabolic shell-like trajectories of its particles. On the other hand, the low phase-speed instabilities also involve some particles in the tail of the beam, elongating this distribution in the core direction.

In the low speed case $U = 0.8$, we can note from the Figure 6 and the previous discussion that the beat ($v_\phi/V_A = 0.53$), decay ($v_\phi/V_A = 0.16$) and modulational ($v_\phi/V_A = 0.64$) affects both the core and the beam, elongating the first in the longitudinal direction towards the beam and the second in opposite direction. Moreover, the pitch-angle scattering in the beam population is much less notorious. Thus, both effects produce a more localized (less diffuse) beam VDF in the
velocity space, i.e.: its heating is lower that for the high drift speed case. It is worthwhile to note that the beam VDF’s are closer to the core as the times goes by, while the core goes backwards by momentum conservation in the longitudinal direction [8], but in a lesser proportion due to its larger inertia. On the other hand, the behavior of the core is similar in both cases, although its prolongation is more marked for the low drift-speed case. Therefore, the parametric instabilities plays a crucial role in the morphological evolution of the proton VDF’s, producing in general more diffuse distributions in the transversal direction at higher drift speeds.

5. Summary

In the present paper, we have discussed the drift speed dependence of the parametric instabilities of an electron-core-beam proton plasma, and its relation with the proton VDF’s, complementing similar studies [2] and [9] (for a beam of $\alpha$ particles instead of protons). The hybrid kinetic dispersion relation was solved numerically, identifying the respective modes. We found a threshold in $U$ for two instabilities of high phase speed related to the beam: a narrow one for $k < k_0$ and a broad one for $k > k_0$. On the other hand, we also have the usual parametric instabilities which appears in an electron-plasma, but they have a lesser phase speed. When we analyzed the effects of these instabilities on the evolution of the proton VDF’s, it is possible to note a elongation of the core and beam (in the longitudinal direction) in the places where the excited ion-acoustic waves are resonant with the particles. The main difference when we vary $U$ is for the beam VDF: it is much more diffuse in the velocity space as time goes by for a high drift speed, implicating a greater heating, but mainly due to an induced pitch-angle scattering instead of the effects of ion-acoustic waves resulting from parametric instabilities.

Acknowledgments

One of us (P.A.M.) wants to thank the financial support to attend the ICPP-LAWPP 2010 given by its organizing committee as well by the Dirección de Docencia and Departamento de Física of the Universidad de Concepción.

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