Mathematical modelling of heat transfer in a gas radiant of “dark” type

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Abstract. The technique and results of mathematical modelling of a complex of internal and external processes of conjugate radiant-convective heat transfer are showed in conditions of laminar long-flame combustion inside the radiating tubes under different modes of operation of the "dark" gas radiators. The conditions of the numerical experiment performed on the example of series serial products made by Sibschwank company for estimating the radiant and full efficiency factors in variable regimes are described. The results of the investigation are summarized in the form of multifactor computational dependencies obtained by the method of orthogonal planning of the first order with the change of variables by power functions. The indices of the longitudinal uniformity of the temperature distribution over the surface of the radiating tubes are quantitatively estimated when the main significant factors are varied. The conditions for the effective operation of the devices studied are substantiated, taking into account a set of basic requirements for ensuring longitudinal uniformity, energy efficiency, and not exceeding the permissible temperature of the metal wall in the zones of existence of local maxima.

1. Introduction

"Dark" gas infrared radiants (GIR) play a huge role in the market of heating equipment today, as they are able to provide significant energy savings and operating costs for heating industrial buildings of great height and volume by implementing the principle of radiant heating. However, despite obvious advantages, there are a number of reasons that make it difficult to calculate, design and implement gas infrared heating systems on existing and newly created industrial facilities. The main of these reasons is the information policy of the main producers of GIR, aimed only at advertising its products with a minimal amount of information of a technical nature. This policy is aimed at protecting copyright on its own "know-how" in competition. Therefore, on Internet sites and technical datasheets, "dark" radiators, as a rule, do not provide data on a number of the most important quantitative indicators necessary for performing qualified engineering calculations for gas radiant heating systems. At the same time, all design and design work is performed by dealers of manufacturing firms using simplified methods, the essence of which is not disclosed.

In particular, the technical documentation of the manufacturers of GIR does not contain information on the coefficients of excess air; temperatures of outgoing flue gases; volumes and compositions of combustion products; the actual degree of blackness and the area of the radiating surfaces; internal and external diameters of radiating tubes; uneven distribution of temperatures and radiant fluxes along the length of the radiators; reflex reflectance factors; spatial radiation indicatrices and other important indicators. Quantitative dependencies necessary for the estimation of radiant and
general (radiant-convective) efficiency under the GIR regimes other than the nominal is also not presented.

All this does not allow to adequately estimate the actual irradiations of surfaces located even directly under the radiators when designing the systems of gas radiant heating, because of the presence of a very significant (but quantitatively indefinite) longitudinal nonuniformity of the radiant fluxes along the length of the radiating tubes.

Due to the lack of information on the efficiency of GIRs on reduced thermal loads, it is not possible to calculate with sufficient accuracy the annual fuel costs necessary to provide the required heat output of radiators equipped with modulating gas burners [1] whose heat output is automatically regulated by the condition of maintaining the specified temperature and radiation mode of heated rooms with any changes in outdoor temperature.

The main aim of this article is to obtain the basic design dependencies characterizing the operation of "dark" gas radiators under variable loads on the basis of mathematical modeling of the conjugate processes of internal and external radiant-convective heat transfer under conditions of long-flame combustion of gas in radiating tubes.

To achieve the aim, the following tasks were formulated. Conjugate heat transfer processes in radiation tubes of gas radiators of a "dark" type are complicated by interaction with the diffusion part of a burning gas flare. In this case, the local intensity of heat exchange is determined by the joint action of a number of factors affecting each other and changing in the direction of the longitudinal axis of the radiator, which contributes to the uneven distribution of temperatures and the outgoing radiative heat flux along the length of the radiating surface. Under such conditions, it is not possible to directly derive analytical dependencies reflecting the change in operating parameters and energy efficiency indexes of GIR "dark" type with varying thermal power.

However, the calculated dependencies in this case can be obtained by numerical method on the basis of analysis of the results of a multifactorial computing experiment. To perform a computational experiment, it is necessary to develop a mathematical model capable in a complex to describe the interrelated processes of internal and external radiant-convective heat transfer during diffusion-kinetic combustion of gas inside radiating tubes, taking into account the interfactor interactions in variable regimes. The creation of a mathematical model of gas infrared emitters of a "dark" type, as well as an analysis of the results of a computational experiment performed, are the main objectives of this study.

To ensure comparability of results, modeling should be performed using an example of a particular design. In this case, a series of GIR TM products was selected as the object of research, the main technical characteristics of which are listed on the sites of the Russian-German enterprise of JSC Sibschwank [2] and its official representative LLC NPF RASKO [3].

The data sets obtained during the computational experiment should be represented by graphs of the dependencies of local and integral indicators on a number of significant factors reduced to a dimensionless or specific type.

The final results of the study should be generalized in the form of multifactor approximating formulas that are convenient for engineering calculation.

2. Theory
The process of long-flame gas combustion in the GIR TM series emitters is organized as follows: the primary combustion of a part of the fuel takes place in the combustion chamber of a special gas burner patented by Schwank, which is located before the beginning of the radiating tube. A schematic diagram of such a burner is available on the website [3]. The analysis of the presented scheme allows drawing a conclusion that a mixture of natural gas with a small amount of primary air is supplied to the combustion chamber, sufficient by the condition of observing the concentration limits of combustion.

Secondary air in an amount sufficient to burn the residual portion of natural gas not burned in the chamber is fed directly to the radiating tube by a peripheral annular flow adjacent to the cylindrical wall. At the beginning of the radiating tube, mixing of the secondary air with the unburned part of the
natural gas and high-temperature flue gases from the combustion chamber begins. As a result, a long diffusion-kinetic flame forms within the radiating tube, the development of which terminates at some distance, $L_f$, from the inlet cross-section, by the complete completion of the burning process of the secondary gas-air mixture. Further, high-temperature products of complete combustion of the gas move along the radiating tube to the outlet, giving off their heat to the walls, and after cooling to the temperature of the outgoing gases are released into the atmosphere.

Therefore, the entire length of the radiating tube can be conditionally divided into two parts: the initial (the zone of the flame development, $x \leq L_f$) and the main working zone (the zone of cooling of the combustion products, $L_f < x \leq L$). Here, the symbol $x$, m, denotes the linear coordinate counted from the input section, and the symbol $L$, m, is the total length of the radiating tube.

In this case, within each of the allocated zones, the distribution of the local temperatures of the radiating surface along the longitudinal axis of the tube has a different character, determined by the peculiarities of the heat exchange conditions and the varying relationships between the radiant and convective components of the internal and external heat fluxes.

The surface of the radiating tube, heated by flue gases, simultaneously undergoes a cooling effect both due to external heat transfer, and by cooling the inner wall with a secondary air stream fed along the periphery.

As a result of cooling by the peripheral air flow, the temperature of the radiating surface in the initial section of the pipe ($t_{0rad}$) is less than the maximum temperature ($t_{rad\ max}$), which is observed at the end of the zone of development of the flame before the beginning of the main working zone of the radiating tube.

The increase in temperature in the initial zone of development of the flame from $t_{0rad}$ to $t_{rad\ max}$ is explained by the gradual increase in the total amount of heat released as the fuel is burned in the diffusion part of the flare. In parallel with this, the effect of the internal cooling effect is reduced, due to a gradual decrease in the consumption of the "cold" peripheral air participating in the diffusion combustion reaction, which also contributes to the growth of the wall temperature. However, as the temperature of the radiating surface increases, the external cooling effect due to the radiant-convective heat transfer of the pipe increases and contributes to intensive cooling of the flow of flue gases moving through the pipe. Therefore, the maximum temperature of the surface of the radiating tube is observed at a point in the initial zone in which the increment of the total cooling effect becomes quantitatively equal to the increment in heat release from the combusted part of the fuel gas.

![Figure 1. The computational scheme of the gas radiant of “dark” type.](image)
After reaching a maximum, the temperature of the radiating surface begins to gradually decrease due to the predominance of the external cooling effect over the heat release from the flame.

The scheme shown in Fig. 1, was taken as a basis for the development of the mathematical model of the "dark" radiator used to perform the computational experiment in this paper. According to the adopted scheme, the radiating tube with a known relative diameter \( d/L = \text{const} \) is divided into \( n \) design sections of the same length \( l_i = L \times n^{-1}, m \), arranged in series along the gases. The numbering of the calculated sections starts from the input section of the radiating tube, which adjoins the exit from the combustion chamber and has a relative coordinate \( x/L = 0 \).

In constructing the mathematical model of the "dark" radiator, the following assumptions were made in this paper.

1. Combustion of the gas-air mixture begins in the volume of the combustion chamber and is completely completed in the radiating tube.
2. Heat transfer through the outer walls of the combustion chamber is not taken into account, and all the heat obtained from burning a part of the fuel in the combustion chamber is completely supplied to the entrance of the radiating tube with heated flue gases. The temperature of the gas mixture in the input section of the radiating tube is established by the heat balance, which takes into account the heat released as a result of the primary combustion of a part of the gas in the chamber, and also the volumes and specific enthalpies of all other components of the mixture, including secondary air.
3. The dynamics of fuel burn-up along the length of the radiating tube obeys the dependence of the development of the constrained diffusion-kinetic flare [4], which, applied to the conditions of this problem, is reduced to the form:

\[
\frac{B_x}{B} = 1 - \exp \left[ -7.41 \left( \frac{x}{L} + \frac{L_{cc}^{\text{spec}}}{L} \right)^{2.73} \right]
\]  

(1)

where \( B_x/B \) is the fraction of fuel burned from the inlet section to any point with relative coordinate \( x/L \), counted from the beginning of the radiating tube; \( L_{cc}^{\text{spec}}/L \) - the conventional length of the combustion chamber, which is a dimensionless parameter, depending on the fraction of fuel burned in the chamber and determined as a result of solving equation (1) under the condition \( x/L = 0 \):

\[
\frac{L_{cc}^{\text{spec}}}{L} = \left[ -\frac{1}{7.41} Lg \left( 1 - \frac{B_{cc}}{B} \right) \right]^{1/2.73}
\]  

(2)

where \( B_{cc}/B \) is the fraction of fuel burned in the combustion chamber before entering the radiating tube.

4. Radiation of triatomic components of flue gases inside the pipe is taken into account in each calculation section, provided that the concentration of CO\(_2\) and H\(_2\)O increases along the length of the zone of development of the flare, taking into account the calculated change in the average temperature of the combustion products in each subsequent section.

5. Within each \( i \)-th section, the average temperature of flue gases \( t_{\text{av},i}^{\text{fg}} \), °C, is considered constant and quantitatively determined by the arithmetic mean value between the temperatures of the combustion products at the inlet and outlet from this section:

\[
t_{\text{av},i}^{\text{fg}} = 0.5 \cdot \left( t_{\text{in},i}^{\text{fg}} + t_{\text{out},i}^{\text{fg}} \right)
\]  

(3)

where, \( t_{\text{in},i}^{\text{fg}} \) and \( t_{\text{out},i}^{\text{fg}} \), the temperatures of incoming and outgoing flue gases in the \( i \)-th section, ° C, respectively.

6. The temperatures of the flue gases leaving each \( i \)-th section are determined by the thermal balances of the simultaneous processes of internal heat supply and external heat removal through the
pipe wall into the environment. At the same time, all the sections located in the zone of flare development take into account the heat input not only from the heated flue gases from the previous section but also from the heat released directly in the given section due to the burning out of a certain fraction of the fuel $\Delta B_i$, which is calculated as

$$\Delta B_i = \left( B_{x,i} - B_{x,(i-1)} \right) / B$$

7. The input temperatures for each subsequent section are assumed to be equal to the temperatures of the flue gases leaving the previous section.

8. The wall thickness is considered to be infinitesimally small in comparison with the diameter of the radiating tube, and the thermal conductivity of the metal of the radiating tube is considered to be infinitely large.

9. The conditional temperature of heat perception, which determines the temperature of the ambient air and all radiation-receiving surfaces, is assumed to be constant: $t_0^{spec} = 15^\circ C$.

10. A useful amount of radiant heat extracted from each design section of the surface of the radiating tube, $Q_{use rad,i}$, kW, is only that portion of the radiant flux that, with allowance for the reflection coefficient of the reflector, is directed to the working area of the room

$$Q_{use rad,i} = k_{ref} \cdot Q_{rad,i}$$

where $Q_{rad,i}$ is the calculated amount of radiant heat, kW, which could be diverted from the i-th section of the pipe at the same temperature, area and blackness of the radiating surface in the absence of a reflector; $k_{ref}$ is the reflection coefficient of the reflector, depending on the material, the geometric shape of the reflecting surface, and the efficiency of the thermal protection of the outer part of the reflector.

11. The reflection coefficient of the reflector does not depend on temperature conditions and is constant at all sections of the radiating tube: $k_{ref} = const$.

12. The part of the radiant heat flux perceived by the reflector (i.e., not reflected) after the appearance of the stationary heat transfer regime increases the overall flow of convective heat due to heat transfer from the external surface of the reflector. Therefore, the total convective heat flow, $Q_{sum conv,i}$, kW, diverted to the upper zone of the room within the length of each i-th section of the radiating pipe, should be determined taking into account the additional heat transfer of the reflector:

$$Q_{sum conv,i} = Q_{conv,i} + (1 - k_{ref}) \cdot Q_{rad,i}$$

where $Q_{ref,i}$ - convective heat flow from the i-th section of the radiating tube, calculated with no reflector, kW.

Taking into account the above assumptions, a finite-difference mathematical model of the "dark" radiator was developed and programmed on the basis of the calculation scheme presented in Fig. 1. The theoretical basis of the mathematical model was the well-known criterial equations of convective heat transfer, the classical heat transfer equations, the basic laws of radiation of gases and solids [5], the standard methods for calculating the combustion of gaseous fuels [6], and the heat and material balance equations. In the developed mathematical model, built-in cycles of iterative calculations were provided, which ensured a high degree of convergence of thermal balances at each design section of the radiating tube.

3. Numerical approach
Computational experiment was carried out on the example of products of JSC "Sibschwank" brand GIR TM of four standard sizes, the data of which are given in Table 1.
Table 1. Information about GIR TM [2].

| Parameters                                      | Typical size of GIR |
|------------------------------------------------|---------------------|
| Rated thermal power, $Q^\text{n}_{\text{gas}}$, kW | 20L                 |
|                                                  | 30L                 |
|                                                  | 40L                 |
|                                                  | 50L                 |
| Rated gas, $B^\text{n}$, m$^3$/h *               | 1.907               |
| Length of radiating tube, L, m                   | 6.10                |
| Diameter of radiating tube, d, m                 | 0.1                 |
| Specific rated thermal power **, $Q^\text{n}_{\text{gas}}/F$, kW/m$^2$ | 9.92              |

Notes for Table 1: * here and further there are normal cubic meters; ** specific heat capacities obtained by calculation at nominal values, $Q^\text{n}_{\text{gas}}$, kW, at the length of radiating pipes $L$, m, and diameter $d$, m.

In addition, the following data were used, independent of the size of the emitters: the number of computed sections $n=20$; the degree of blackness of the radiating surface is $\varepsilon = 0.95$; reflection coefficient of the reflector $k_{\text{ref}}=0.84$; calculated air excess ratio $\alpha=1.05$ (total taking into account primary and secondary air); density of natural gas $\rho_{g}=0.798$ kg/m$^3$; lower working heat of combustion $Q_{\text{lhv}}=35876.37$ kJ/m$^3$; theoretically the required amount of air $L_T=9.517$ m$^3$/m$^3$. The composition of natural gas is shown in Table 2.

Table 2. Natural gas composition (% volume).

| CH$\text{}_4$ | C$_2$H$_6$ | C$_3$H$_8$ | C$_4$H$_{10}$ | C$_4$H$_{12}$ | N$_2$ | CO$_2$ | H$_2$O | Σ       |
|--------------|------------|------------|---------------|--------------|------|-------|-------|--------|
| 87.5         | 3.86       | 1.39       | 0.41          | 0.23         | 5.54 | 0.1   | 1.0   | 100%   |

When performing multivariate calculations, all the numerical values presented above were considered constants. Variable factors were three parameters: $Q_{\text{gas}}/F$ - specific heat output of the radiator, kW/m$^2$, determined by the ratio of the amount of heat calculated from the complete combustion of the variable gas flow to the area of the cylindrical surface of the radiating tube; $B_{cc}/B$ is the fraction of the primary fuel burned in the combustion chamber, which affects the length of the diffusion part of the flare and thereby changes the longitudinal distribution of the local temperatures and densities of the radiant flux along the length of the radiating tube; $L/d$ is the factor of the radiator shape, determined by the ratio of the length and diameter of the radiating tube.

The purpose of the computational experiment was to obtain dependencies that reflect the thermal work of the "dark" radiators when the main influencing factors change. The scope of the study was limited to the following intervals of variation of the parameters:

$$4.58 \leq Q_{\text{gas}}/F \leq 13.75 \text{ kW/m}^2; \quad 0.1 \leq B_{cc}/B \leq 0.7; \quad 61.0 \leq L/d \leq 136.2.$$  (7)

4. Results and discussion

The results are presented in the form of a series of sampling plots that clearly illustrate the nature of the dependences obtained. In Fig. 2 and 3 show graphs of the distribution of local temperatures and densities of radiant heat fluxes along the longitudinal axis of the radiating tube, corresponding to the conditions of operation of the GIR TM-50L at the nominal value of the specific thermal power.

Analogous dependencies are obtained for radiators of all sizes shown in Table 1, and the graphs of Fig. 2 and 3 are shown for exemplary purposes only. As a result of the mutual comparison of all the graphs, the identical character of the dependences obtained for different $Q_{\text{gas}}/F$ and $L/d$ is established. The differences between them are only in the numerical values of the local temperatures and the
densities of the radiant fluxes at similar points on the surfaces. The shape and mutual arrangement of the curves reflecting the distribution of these parameters along the length of the radiating tubes does not change significantly.

![Figure 2. Distribution of local temperatures of the radiating surface along the tube axis by the example of GIR TM-50L at $Q_{gas}/F=11.46$ kW/m²; $L/d=136.2$.](image2)

An analysis performed on the example of the graphs in Fig. 2 and 3 shows that a decrease in the proportion of fuel burned in the chamber always leads to a decrease in the surface temperature in the initial section of the radiating tube and substantially reduces the temperature maxima, the position of which is shifted towards the center of the radiating tube due to the increase in the length of the flame. Consequently, a decrease in the proportion of primary fuel burned in the chamber contributes to the creation of more favorable conditions for the operation of radiators, reducing the probability of formation of burn-throughs in the thin walls of radiating tubes, which, as is known from practice [4], can occur in places of local excess of the permissible temperature of the wall metal, as well as in the turning areas of U-shaped radiators.

The obtained data made it possible to quantify the longitudinal uniformity of the temperature distribution with the aid of the dimensionless exponent $\chi(t)$, determined by the ratio of the average
integral temperature of the radiating surface \( t_{av}, ^\circ\text{C} \), to the corresponding temperature maximum \( t_{rad}^{\text{max}}, ^\circ\text{C} \),

\[
\chi(t) = \frac{t_{av}}{t_{rad}^{\text{max}}}.
\]  

(8)

As an example, Fig. 4 shows the graphs of the variation of the longitudinal uniformity index of the temperature of the radiating surface, \( \chi(t) \), as a function of \( B_{cc}/B \) for varied values of specific heat power \( Q_{\text{gas}}/F = \text{var} \) and a constant value of the form factor \( L/d = \text{const} \).

Analysis of the graphs in Fig. 4 shows that at any constant value of specific thermal power, the maximum uniformity of the longitudinal temperature distribution can be achieved with a 20 per cent share of the primary fuel burned in the chamber. The increase in the share of the burned primary fuel to 30% with constant heat output (\( Q_{\text{gas}}/F = \text{const} \)) leads to a slight decrease in the uniformity index \( \chi(t) \) with respect to its maximum value. The sharpest decrease in uniformity occurs only after an increase in \( B_{cc}/B > 40\% \). This makes it possible to limit the operating range to \( 20\% \leq B_{cc}/B \leq 40\% \) according to the conditions for uniformity in the distribution of the temperature of the radiating surface (and hence radiant fluxes) along the length of the tubes of the "dark" radiators.

It is also seen that the index \( \chi(t) \) deteriorates substantially as the specific heat output decreases for any constant values of \( B_{cc}/B \). Consequently, when the heat load is reduced to half of the design value corresponding to the thermal power rating of the radiators of each type (which actually can occur during the heating period when controlling the heat transfer), the unevenness of the temperatures and radiant fluxes along the length of the radiating tubes must inevitably increase.

Further analysis of the results of the computational experiment made it possible to quantify the values of the radiant (\( \eta_{\text{rad}} \)) and full (\( \eta_{\text{sum}} \)) efficiency. The values of these indicators were calculated with all the varying operating modes of each of the four standard sizes of the "dark" radiators shown in Table 1, according to the expressions:

\[
\eta_{\text{rad}} = \frac{Q_{\text{rad}}}{Q_{\text{gas}}},
\]

(9)

\[
\eta_{\text{sum}} = \frac{Q_{\text{rad}} + Q_{\text{conv}}}{Q_{\text{gas}}}.
\]

(10)

The graphs of the corresponding dependences for the GIR TM-50L emitter are presented as an example in Fig. 5 and 6.
A joint analysis of the graphs in Fig. 4, 5 and 6 shows that a decrease in the share of combusted primary fuel (in order to achieve maximum uniformity of temperature distribution) can negatively affect the energy efficiency of radiators, contributing to a reduction in both radiant and overall efficiency. This fact should be taken into account when justifying the recommended value of $B_{cc}/B$, which along with longitudinal uniformity and acceptable energy efficiency should also ensure that the maximum temperature of the metal wall of the radiating tubes is not exceeding 600°C.

The local maxima of the wall temperature of the radiating tube are shown in the graph of Fig. 2, from which it can be seen that at $B_{cc}/B = 0.4$ the local temperature maximum in the zone of the flare development is $t_{\text{rad max}} = 460 \, ^\circ\text{C}$. With an increase in $B_{cc}/B > 0.4$, this maximum begins to increase, reaching almost 600 °C at $B_{cc}/B = 0.7$. This can adversely affect the longevity of the products, leading to premature pipe burn-ups in local overheating zones. Therefore, the value $B_{cc}/B = 0.4$ can be recommended, both at the same time satisfying the set of all necessary conditions determined by the requirements for ensuring uniformity, energy efficiency and not exceeding the permissible wall temperature of the radiating tubes.
On the graphs of Fig. 5 and 6 show the results of the computational experiment, and the solid lines show the curves constructed from the multifactor approximating relationships (11) and (12), which are valid for the entire range of investigated radiators:

\[
\eta_{\text{rad}} = \left[0.1968 + 0.06294 \left( \frac{B_{\text{cc}}}{B} \right)^{1.103} - 0.0995 \left( \frac{L}{d} \right)^{0.337} \left( \frac{Q_{\text{gas}}}{F} \right)^{-1}\right] + 0.395 + 0.0945 \left( \frac{B_{\text{cc}}}{B} \right)^{1.103} + 0.0373 \left( \frac{L}{d} \right)^{0.337} \left( \frac{Q_{\text{gas}}}{F} \right)^{-0.056}
\]

\[
\eta_{\text{sum}} = \left[0.9674 + 0.1169 \left( \frac{B_{\text{cc}}}{B} \right)^{0.289} - 0.1548 \left( \frac{Q_{\text{gas}}}{F} \right)^{0.535} + 0.0255 \left( \frac{L}{d} \right)^{0.174}\right] + 0.0317 \left( \frac{B_{\text{cc}}}{B} \right)^{0.289} \left( \frac{Q_{\text{gas}}}{F} \right)^{0.535} - 0.0418 \left( \frac{B_{\text{cc}}}{B} \right)^{0.289} \left( \frac{L}{d} \right)^{0.174}
\]

Approximating dependences (11) and (12) were obtained by orthogonal first-order scheduling with preliminary replacement of the natural variables \(Q_{\text{gas}}/F\), \(B_{\text{cc}}/B\) and \(L/d\) by the corresponding power functions: \((Q_{\text{gas}}/F)^m\), \((B_{\text{cc}}/B)^n\) and \((L/d)^z\). The numerical values of the exponents \((m, n, z)\) in these functions were previously determined on the basis of least-squares processing of an array of data from 147 elements uniformly distributed throughout the area of the computational experiment [7].

A histogram of the error distribution of the approximating function (11) within the accepted domain of the computational experiment is shown in Fig. 7.
The statistical analysis by the Pearson criterion [7], performed on the basis of the 9-bit histogram in Fig. 7, showed that the actual error spread of the obtained dependence (11) within the entire region of the computational experiment is in good agreement with the law of the normal Gaussian distribution ($\chi^2=4.09 < \chi^2_{\text{tab}}=12.592$, with a confidence level of $P=95\%$ and the number of degrees of freedom $m=6$).

The mathematical expectation of the absolute error of approximation is 0.0042%, the standard deviation is $\pm 0.134\%$, and the confidence interval does not exceed $\pm 0.26\%$ for $P=95\%$.

The approximate dependence (12), obtained for estimating the total efficiency, has even higher accuracy indices. Consequently, both dependences (11) and (12) have an accuracy sufficient to perform engineering calculations.

The function (11) and the graphs of Fig. 5 show that the dependence of the radiant efficiency on the specific thermal power for any constant value $B_{cc}/B=\text{const}$ is of an extreme nature. As a result of the analysis of this function $\frac{\partial \eta_{\text{rad}}}{\partial (Q_{\text{gas}}/F)}=0$, the following two equations are obtained for the extremum, performed by the classical method by the condition, determining respectively the optimal specific thermal power $(Q_{\text{gas}}/F)_{\text{opt}}$ for given $L/d$ and $B_{cc}/B$ or the optimal shape factor $(L/d)_{\text{opt}}$ for given $Q_{\text{gas}}/F$ and $B_{cc}/B$, which can ensure the achievement of the maximum of the radiant efficiency within the entire range of parameters studied:

$$\left(\frac{Q_{\text{gas}}}{F}\right)_{\text{opt}} = \left[\frac{1.777 \cdot (L/d)^{0.337} - 1.124 \cdot (B_{cc}/B)^{1.103} - 3.514}{0.0373 \cdot (L/d)^{0.337} + 0.0945 \cdot (B_{cc}/B)^{1.103} + 0.3951}\right]^{1/0.944}; \quad (13)$$

$$\left(\frac{L}{d}\right)_{\text{opt}} = \left[\frac{3.514 + 0.0945 \cdot (B_{cc}/B)^{1.103} \cdot (Q_{\text{gas}}/F)^{0.944} + 0.3951 \cdot (Q_{\text{gas}}/F)^{0.944} + 1.124 \cdot (B_{cc}/B)^{1.103}}{1.777 - 0.0373 \cdot (Q_{\text{gas}}/F)^{0.944}}\right]^{1/0.337}. \quad (14)$$

Using the equations obtained, an optimum line is constructed, shown in Fig. 8 and determining the optimum length of the radiating tubes for the GIR TM series burners, which are additionally marked with the points of optimum thermal power calculated at the passport lengths of the pipes specified in the technical documentation [2].

Figure 8. Dependences of the optimum and actual length of radiating tubes on the thermal power of the radiators of the GIR TM series at $B_{cc}/B = 0.4$ and $d=0.1 \text{ m}$.
Fig. 8 shows that the actual lengths of the radiating tubes of all the four types of radiators of the GIR TM series, with the installed thermal power of 19; 29; 39 and 49 kW, have values less than optimal. This means that combinations of geometric characteristics and nominal operating parameters, established for the radiators of this series by the manufacturer's technical documentation, do not correspond to the optimum conditions ensuring the achievement of the maximum of the radiant efficiency.

Visual analysis allows us to conclude that, in order to achieve optimal conditions, all the points constructed in Fig. 8 according to the technical documentation, should be combined with the optimum line, changing for this either the thermal power of the radiators (in arrow 1, at ) or the length of the radiating tubes (arrow 2, with ).

The predicted values of the radiant and full efficiencies of the products of this series, calculated with the two indicated methods of achieving optimal conditions using the dependences (11), (12), (13) and (14) obtained with the previously recommended fraction of combustible primary fuel of 0.4, are presented respectively in Table 3 and 4.

**Table 3.** The maximum attainable values of efficiency at the optimum () and nominal () heat output of the radiators, calculated taking into account the actual dimensions () of the radiating tubes specified in the manufacturer's technical documentation.

| Mark and type of radiator | According to technical documentation [2] | Calculated values |
|----------------------------|------------------------------------------|-------------------|
|                            |                                | Efficiency at | Efficiency at |
|                            |                                | \( Q_{\text{gas}}^{\text{nom}} \) and \( L_{\text{act}} \) | \( Q_{\text{gas}}^{\text{opt}} \) and \( L_{\text{opt}} \) |
|                            | \( L_{\text{act}}, \text{m} \) | \( d, \text{m} \) | \( Q_{\text{gas}}^{\text{nom}}, \text{kW} \) | \( \eta_{\text{rad}*} \) | \( \eta_{\text{sum}}** \) | \( Q_{\text{gas}}^{\text{opt}}, \text{kW} \) | \( \eta_{\text{rad}} \) | \( \eta_{\text{sum}} \) | \( \eta_{\text{rad}} \) | \( \eta_{\text{sum}} \) |
| GIR TM-20L                  | 6.1                          | 0.1          | 19             | 11.57                       | 49.37%                      | 89.30%                      | 49.09%                      | 84.72%                      |
| GIR TM-30L                  | 9.15                         | 0.1          | 29             | 22.52                       | 50.49%                      | 88.55%                      | 50.41%                      | 86.27%                      |
| GIR TM-40L                  | 12.2                         | 0.1          | 39             | 35.24                       | 51.49%                      | 88.38%                      | 51.47%                      | 87.50%                      |
| GIR TM-50L                  | 13.62                        | 0.1          | 49             | 41.64                       | 51.92%                      | 88.40%                      | 51.88%                      | 86.97%                      |

Note for Table 3: * The radiant factor of the GIR TM series, which according to [3] was determined by the laboratory that conducted the certification tests; ** The value of the total efficiency obtained by summing up the proportion of radiant and convective heat based on the data [3] on the distribution of the energy fluxes of Schwank's "dark" heaters of the GIR TM series.

**Table 4.** Maximum achievable efficiency at the optimum () and actual () length of the radiating tubes, calculated at the thermal power of the radiators () specified in the manufacturer's technical documentation.

| Mark and type of radiator | According to technical documentation [2] | Calculated values |
|----------------------------|------------------------------------------|-------------------|
|                            |                                | Efficiency at | Efficiency at |
|                            |                                | \( Q_{\text{gas}}^{\text{nom}} \) and \( L_{\text{opt}} \) | \( Q_{\text{gas}}^{\text{opt}} \) and \( L_{\text{act}} \) |
|                            | \( L_{\text{act}}, \text{m} \) | \( d, \text{m} \) | \( Q_{\text{gas}}^{\text{nom}}, \text{kW} \) | \( \eta_{\text{rad}*} \) | \( \eta_{\text{sum}}** \) | \( L_{\text{opt}}, \text{m} \) | \( \eta_{\text{rad}} \) | \( \eta_{\text{sum}} \) | \( \eta_{\text{rad}} \) | \( \eta_{\text{sum}} \) |
| GIR TM-20L                  | 6.1                          | 0.1          | 19             | 8.23                        | 50.16%                      | 88.68%                      | 49.09%                      | 84.72%                      |
| GIR TM-30L                  | 9.15                         | 0.1          | 29             | 10.75                       | 51.03%                      | 88.42%                      | 50.41%                      | 86.27%                      |
| GIR TM-40L                  | 12.2                         | 0.1          | 39             | 13.04                       | 51.74%                      | 88.39%                      | 51.47%                      | 87.50%                      |
| GIR TM-50L                  | 13.62                        | 0.1          | 49             | 15.18                       | 52.36%                      | 88.47%                      | 51.88%                      | 86.97%                      |

Analysis of the data contained in these tables allows us to draw the following conclusions.

1) Comparison of the radiant and full efficiencies taken according to [3], with the corresponding values calculated according to the developed procedure for all four types of emitters in nominal modes...
(for $L_{\text{act}}$ and $Q_{\text{gas, nom}}$), shows good agreement. This can serve as an indirect confirmation of the adequacy of the developed mathematical model and the reliability of the results and conclusions obtained.

2) The numerical values of the radiant and full efficiency calculated in the optimal modes for the first and second options for achieving the optimum are higher than the corresponding values calculated using the same procedure for the nominal parameters specified in the technical documentation. This confirms the reliability of the optimization results and allows us to recommend the developed methodology for use in practical engineering calculations.

5. Conclusion
A finite-difference mathematical model has been developed for the complex description of long-flame combustion of gas together with the processes of conjugate radiant-convective heat exchange occurring on the inner and outer surfaces of radiating tubes of "dark" gas radiators.

Based on the results of multivariate calculations performed using the developed mathematical model, the curves of the distribution of temperatures and densities of radiant heat fluxes along the length of radiating tubes were obtained using the example of linear "dark" radiators of Sibschwank a series of GIR TM-L. The conditions for achieving maximum uniformity distribution of temperatures and densities of radiant heat fluxes were determined.

On the basis of mathematical processing of the results of the computational experiment, adequate approximate dependences of the radiant and total efficiency were obtained from three main factors: the specific thermal power, the shape factor (determined by the ratio of the length to the diameter of the radiating tubes), and the fraction of the primary fuel burned in the combustion chamber.

Comparison of the numerical values of the radiant and full efficiency, calculated from the dependences obtained, with the data of the manufacturer's technical documentation indicates a good quantitative coincidence of the results, which confirms the adequacy of the developed mathematical model and the reliability of the conclusions obtained.

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