Electrohydrodynamic thrust with no combustion emissions and noises in a centimeter-scale point-to-grid configuration

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Abstract. Alternative propulsion systems are needed to overcome the combustion emissions and noises caused by the fossil-fuel combustion-based gas turbines and propeller-driven propulsion systems. The electrohydrodynamic (EHD) thrust produced by the corona induced ionic wind is an attractive choice because its generation needs no mechanical moving part and emits no combustion emissions and noises. In this investigation, the electrohydrodynamic thrust provided by the positive point-to-grid corona discharge at the centimeter scale is theoretically and experimentally analyzed. The previous one-dimensional theory is reviewed to understand the fundamental characteristics of the EHD thrust. The thrust performance of a 4.1 mg, centimeter-scale, laser-micromachined EHD thruster is experimentally quantified. The measured thrust reaches up to 0.68 mN, corresponding to a thrust density of 8.7 N/m\textsuperscript{2}, a thrust-to-weight ratio of 17 and a thrust-to-power ratio of 2.1 N/kW.

1. Introduction
Aircraft is propelled by mechanical moving parts, such as fossil-fuel combustion-based gas turbines and propeller-driven propulsion systems, which results in harmful combustion emissions and noises in the environment. With the wide usage of small-scale aircraft, this issue brings increasing concerns about environmental protection. Electrohydrodynamic (EHD) propulsion is an alternative to in-atmosphere electric propulsion that produces propulsive thrust by inducing ionic wind with no need for mechanical moving parts, nearly silently without combustion emissions \cite{1,2}. The most commonly applied method for the generation of ionic wind is corona discharge \cite{3}. The corona discharge is a self-sustaining atmosphere discharge that can operate stably at small scales \cite{4}. When applying a high potential difference across two asymmetrical electrodes, in which an electrode with a small radius of curvature (emitter) is stressed by a high voltage and the other electrode with a relatively large radius of curvature (collector) is grounded, the air molecules around the emitter are ionized due to the large potential gradient, and the produced ions are propelled by the likely charged emitter at the drift velocity towards the collector. Then the momentum is transferred from the drifting ions to the neutral air molecules via collisions between the ions and air molecules in the electrode gap, resulting in an
acceleration of airflow in the direction from the emitter to the collector. The result is the ionic wind that produces a propulsive thrust in the opposite direction to the ionic wind. This process requires no mechanical moving parts and emits no combustion emissions and noises.

Aiming at exploring the possibility for in-atmosphere propulsion, the EHD thrust has been studied theoretically and experimentally. Early study found that poor efficiency makes the EHD thrust impossible for in-atmosphere propulsion [5]. Researchers from NASA conclude that if a thrust-to-power ration of 20 N/kW and a thrust density of 20N/m² are simultaneously achieved, the EHD thrust would be practically useful, and unfortunately, they reported the failure to reach this goal [6]. However, the unique properties of the EHD thrust are still attractive, especially for environmental protection at present. Studies have been focusing on improving the performance of the EHD thrust. Different electrode configurations are designed and investigated, including wire-to-cylinder electrodes [7], wire-to-grid electrodes [8], needle-to-grid electrodes [9] and even more complex configurations such as a wire-to-cylinder-to-plate configuration [10] and a dual-stage thruster [11]. Masuyama & Barrett quantified EHD thruster performance and obtained a thrust-to-power ratio as high as approximately 100 N/kW [11], comparable to that of current gas turbines, where typically performance is 2 N/kW. With these efforts, the potential for EHD thrust at the unmanned aerial vehicle (UAV) scale and the micro aerial vehicle (MAV) was demonstrated recently. Using a launch system, researchers from MIT have flown an EHD-powered fixed-wing UAV with an onboard battery for the first time [2]. A 10mg, 1.8cm by 1.8cm “ionocraft” powered by four microfabricated EHD thrusters with a thrust-to-weight ratio of 10 by was proved to be capable of taking off [12].

The EHD propulsion is promising for small aircraft at the centimeter scale or even smaller. This is due to the inherent mechanical and principle simplicity that make the EHD thruster can effortlessly be scaled to centimeter scales and keep functionality for a high thrust-to-weight ratio [8,13]. The point-to-grid configuration seems to be an effective option at the centimeter scale due to the following reasons: 1) grid s have a smaller drag coefficient and a lighter weight compared with plates and cylinders; 2) grid-like collectors performer better in mechanical power generation; 3) the point-to-grid configuration is suitable for the microfabrication technologies such as laser micromachining. However, the detailed investigation on the EHD thrust for a point-to-grid configuration at the centimeter scale is scarce. The purpose of this paper is to fulfill it.

2. Theoretical analysis

2.1. Fundamental characteristics of the EHD Thrust

A simple one-dimensional theory has been widely used for predicting the EHD thrust generated by various types of corona discharges, which states that the EHD thrust is equal to the sum of the Coulomb forces acted on the ions existing in the electrode gap [10,12,14,15]. To estimate the EHD thrust, the following assumptions should be made: (1) the electric field in the electrode gap is uniform and one-dimensional; (2) the space charge effects do not significantly disturb the current and the electric field. Refer to Figure 1 for the terms related to geometrical and physical parameters.

The emitter is at a voltage $V$ and the collector is grounded. The corona discharge is featured by two regions in the electrode gap: the ionization region and the drifting region. The ionization process occurs in the ionization region that is located near the tip of the emitter. The drifting region occupies the majority of the electrode gap, where the ion-molecule collisions occur. The steady current conservation law is expressed as:

$$ \nabla \cdot \mathbf{j} = \nabla \cdot \left[ \rho_i \left( \mu \mathbf{E} + \mathbf{u}_0 \right) - D \nabla \rho_i \right] = 0 $$

(1)

where $\mathbf{j}$ is the current density, $\mathbf{u}_0$ is the neutral fluid velocity, $\mathbf{E}$ is the electric field, $D$ is ion diffusion coefficient, $\rho_i$ is the space charge density, $\mu$ is the ion mobility and approximately equals $2 \times 10^{-4}$ m²/(Vs) for positive ions. The first term in the right of Eq.(1) is the drift current, the second term is the convective current and the third term is the diffusion current. A common method to simplify Eq.(1) is neglecting the convection and diffusion terms, which is valid in centimeter-scale
flight where the flight velocity is much less than the drift velocity. Making this assumption and simplifying to one-dimension reduces Eq.(1) to:

\[ \frac{d(\rho, \mu E)}{dx} = 0 \]  

(2)

where \( x \) is a coordinate pointing the collector from the emitter. Assuming that ion mobility \( \mu \) is constant, Eq.(2) implies that the body force \( \rho_c E \) on the ions is constant along the electrode gap. Defining a characteristic area \( A \) to describing the area perpendicular to the thrust direction occupied by ions, i.e. the surface area of the collector, we have:

\[ I = \int j \, dA = \rho_c \mu E A \]  

(3)

The one-dimensional theory states that the EHD thrust equals the sum of the Coulomb forces acted on the ions existing in the electrode gap, which yields:

\[ T_{\text{EHD}} = \int \rho_c E dU = \frac{j}{\mu} A d = \frac{I d}{\mu} \]  

(4)

where \( U \) is the volume defined by extruding the area \( A \) by the electrode gap distance \( d \) in the thrust direction. Eq.(4) implies that the EHD thrust is proportional to the corona current and electrode gap distance and inversely proportional to the ion mobility. The thrust density (thrust per unit area) \( \psi \) is then given by:

\[ \psi = \frac{T_{\text{EHD}}}{A} = \frac{I d}{\mu A} \]  

(5)

Recall that \( A \) is the surface area of the collector and also represents the frontal area of the thruster. The effect of \( A \) on the EHD thrust is complicated. On one hand, from an electrical point of view, a larger surface area of the collector is preferred for enlarging the produced corona current and thrust. On the other hand, from an aerodynamic point of view, a larger area will increase the drag force on the collector, causing a thrust degradation. Experimental research by Moreau et al [7] and theoretical analysis by Gilmore & Barrett [14] both demonstrated that the area of the collector does not play a key role in the current-to-thrust conversion compared with the electrode gap distance.

\[ I = CV(V - V_c) \]  

(6)

**Figure 1.** Schematic illustration of the one-dimensional EHD thrust model.

**Figure 2.** (a) Schematic illustration and (b) photograph of the experimental setup.
where $C$ is a geometrical parameter and $V_i$ is the corona discharge inception voltage. Substituting Eq.(6) into Eq.(4) gives:

$$T_{EHD} = CV(V - V_i) \frac{d}{\mu}$$  \hspace{1cm} (7)

This indicates that the EHD thrust increases quadratically with increasing applied voltage. Since the EHD thrust is proportional to $d$ and $d$ significantly affects $C$ and $V_i$, $d$ is an important geometrical parameter that needs to understand its overall effect on the EHD thrust.

2.2. Efficiency of the EHD thrust

Like conventional in-atmosphere propulsion using either gas turbines or propellers, the overall efficiency of the EHD propulsion can be separated into two parts, the mechanical efficiency $\eta_m$ and the propulsive efficiency $\eta_p$. The mechanical efficiency, which is analogous to the thermal efficiency of conventional in-atmosphere propulsion, is the efficiency of converting electrical energy into kinetic energy of the propulsive stream via generation of the ionic wind:

$$\eta_m = \frac{\frac{1}{2}\dot{m}(u_e^2 - u_0^2)}{P} = \frac{1}{2} \frac{T_{EHD}(u_e + u_0)}{P}$$  \hspace{1cm} (8)

where $P = V I$ is the input electrical power, $u_0$ and $u_e$ represent the airflow velocities at the inlet and outlet respectively. Propulsive efficiency is the efficiency of converting the kinetic energy of the propulsive stream into aircraft power:

$$\eta_p = \frac{T_{EHD}u_0}{\frac{1}{2}\dot{m}(u_e^2 - u_0^2)}$$  \hspace{1cm} (9)

Then the overall efficiency is given by:

$$\eta = \eta_m \eta_p = \frac{T_{EHD}u_0}{P} = \frac{u_0 d}{\mu V} = \frac{u_0}{\mu E}$$  \hspace{1cm} (10)

which is defined as the thrust power delivered to the aircraft divided by the consumed electrical power. In the static tests, the overall efficiency equals zero because of $u_0 = 0$. As is quantified by Masuyama and Barrett [11], an alternative performance metric for thrust efficiency is the thrust-to-power ratio $\phi$ (the ratio of produced thrust and consumed electrical power). In the case of a uniform electric field as assumed:

$$\phi = \frac{T_{EHD}}{P} \frac{d}{\mu V} = \frac{d}{\mu V} = \frac{1}{\mu E}$$  \hspace{1cm} (11)

Eq.(11) implies a trade-off between the thrust and thrust efficiency. The thrust-to-power ratio can be increased by reducing the applied voltage at a fixed electrode gap distance, but the thrust will decrease. Moreover, too low an electric field strength results in no corona inception, then no thrust will be generated. This also indicates a fundamental limit of the achievable thrust-to-power ratio, which is achieved at the corona discharge inception voltage.

3. Experimental setup

To quantify the performance of the EHD thrust, experiments are performed by using a centimeter-scale positive point-to-grid corona discharge to generate EHD thrust as shown in Figure 2, applying a high positive voltage across the electrodes and measuring the current and thrust simultaneously. An equal-armed lever is designed to convert the EHD thrust to a compression force that acts on a force transducer (Aurora Scientific Inc., 403A, resolution 0.1 μN). Then thrust is computed by time-averaging the results of the force transducer. The corona current is measured by an electrometer (Keithley, Model 6514) with an accuracy of 1 pA and a range of 20 mA. The electrical power is supplied by a positive high DC voltage source (Kikusui, TOS5101) of a voltage up to 10 kV and a current up to 5 mA. In this paper, the upper limit of the applied voltage is 10 kV or at the onset of spark breakdown, whichever is lower. The emitter is positively stressed and the collector is grounded.
in all experiments. To avoid the current leakage due to corona discharge occurs around the connecting wires, copper wires with a thin layer of insulation are used to connect the high voltage to the emitter. Current and thrust data are recorded simultaneously at a frequency of 10 kHz. Each data point plotted is the average value of over 1000 data points. Experiments conducted in the controllable environment, where the temperature is approximately 24℃ and relative humidity ranges from 40~50%. The variation of the relative humidity in this range does not affect the corona discharge characteristics [7]. The grid-like collector is 10 mm in diameter and the holes distributed on it are 1 mm in diameter. The electrode gap distance can be varied by moving the emitter. The electrodes are laser-micromachined into desired shapes using a computer-controlled laser (UV laser, wavelength 355nm). The materials chosen for the emitter and the collector are copper and carbon fiber, respectively. This simple, fast and cheap fabrication process reduces the mass of the thruster to 4.1 mg (the total mass of the emitter and the collector). All measurements are conducted under the static condition ($u_0=0$).

4. Results and discussion

Figure 3(a) shows the measured EHD thrust versus the applied voltage for various electrode gap distances. The measured results agree well with Eq.(7), showing the EHD thrust increases quadratically with applied voltage. No thrust is obtained at lower voltages below the corona inception voltage, but beyond that, the thrust increases quadratically, as predicted by Eq.(7). Moreover, Figure 3(a) indicates that a smaller thrust is produced for a given applied voltage at larger gap distances. The maximum measured thrust in this investigation is 0.68 mN obtained by applying 10 kV across a 10-millimeter-long electrode gap distance, achieving a thrust density of 8.7 N/m², a thrust-to-weight ratio of 17 and a thrust-to-power ratio of 2.1 N/kW.

Figure 3(b) shows the measured thrust versus corona current for various electrode gap distances. The measured results are fitted correctly by Eq.(4), demonstrating the EHD thrust increases linearly as the corona current increases at a given electrode gap distance. The slope of fit lines increases from $6.92 \times 10^{-3}$ mN/μA to $58.6 \times 10^{-3}$ mN/μA as the electrode gap distance increases from 4 mm to 20 mm.

Although able to generate large thrusts at low voltages, small electrode gap distances are limited in their maximum thrust by the low onset voltage of the electrical breakdown, beyond which no thrust is generated due to the transition from corona discharge to spark. For example, the electrical breakdown occurs at 5~6 kV for $d = 4$ mm case and 9~10 kV of applied voltage for $d = 8$ mm case. Then we consider an average electric field strength of approximately $1.1~1.3 \times 10^6$ kV/m as a limit, beyond which it will cause an electrical breakdown.

Figure 3(b) shows the measured thrust versus corona current for various electrode gap distances. The measured results are fitted correctly by Eq.(4), demonstrating the EHD thrust increases linearly as the corona current increases at a given electrode gap distance. The slope of fit lines increases from $6.92 \times 10^{-3}$ mN/μA to $58.6 \times 10^{-3}$ mN/μA as the electrode gap distance increases from 4 mm to 20 mm.
Figure 4 presents the efficiency performance of the EHD thrust, where the thrust-to-power ratio as a function of applied voltage and measured thrust are plotted. By Eq.(11), the thrust-to-power ratio is predicted to be directly proportional to the electrode gap distance and inversely proportional to the applied voltage, which is observed in Figure 4(a). Figure 4(b) also confirms the prediction of Eq(11) that a trade-off exists between thrust and thrust-to-power ratio. Take \( d=10 \) mm case as an example, the thrust-to-power ratio is 5.4 N/kW at 0.04 mN of thrust and reduces to 2.1 N/kW at 0.68 mN of thrust. The achieved thrust-to-power ratio ranges from 1.2 N/kW to 7.6 N/kW among various electrode gap distances.

The thrust formula Eq.(4) predicts that the EHD thrust for \( d=10 \) mm at 10 kV is 1.6 mN, which is larger than the test results (0.68 mN). It has been proved that the thrust formula derived from the 1-D theory overestimates the magnitude of the EHD thrust compared with the test results for small electrode gaps [15]. By considering the nonuniformity of the electric field and the drag force on the collector, the thrust formula can be improved.

Figure 4. (a) Thrust versus power and (b) thrust-to-power ratio versus voltage for varying electrode gap distance.

For the EHD thrust in the positive point-to-grid configuration, an electrode gap distance of 10 mm is expected as an optimal choice with the limit of applied voltages up to 10 kV. Specifically, applying 10 kV across a 10-millimeter-long electrode gap distance in the point-to-grid configuration, the measured thrust is 0.68 mN, according to a thrust density of 8.7 N/m\(^2\), a thrust-to-weight ratio of 17 and a thrust-to-power ratio of 2.1 N/kW. This performance is attractive as a propulsion system for small aircraft [16]. Improvements are needed to increase efficiency. The fundamentally effective way is to reduce the applied electric field strength that maintains a corona discharge, but too low an electric field strength results in no corona inception. Since there is a limit to the achievable efficiency using corona discharge, further study may focus on exploring alternative ionization methods such as a dielectric barrier discharge that of a thrust efficiency reaches up to 20 mN/W. Another remaining challenge is the high operating voltage of the EHD propulsion. This would require an onboard high-voltage power converter that steps up the battery voltage to thousands of volts. No such a device has been provided, future progress in the integrated circuits may address this issue.

5. Conclusions
This paper analyzed and quantified the performance of the EHD thrust in a centimeter-scale point-to-grid configuration. The one-dimensional theory gave the fundamental characteristics of the EHD thrust with particular considerations of the interactions between various parameters. The performance of a 10.2-milligram, centimeter-scale, laser-micromachined EHD thruster was tested at various electrode gap distances and applied voltages. The experimental results confirmed the predicted trends
of the one-dimensional theory. Specifically, (1) for a given electrode gap distance, beyond corona inception voltage, the thrust increases quadratically and linearly with applied voltage and corona current, respectively; (2) the thrust-to-power ratio is directly proportional to electrode gap distance and inversely proportional to applied voltage; (3) a trade-off exists between thrust and efficiency as the thrust-to-power decreases with increasing thrust. In particular, the measured thrust reaches to 0.68 mN at a 10 mm electrode gap distance and a 10 kV applied voltage, achieving a thrust density of 8.7 N/m², a thrust-to-weight ratio of 17 and a thrust-to-power ratio of 2.1 N/kW. The achievement described in this paper represents a major potential for EHD-propelled flight at the centimeter scale with no combustion emissions and noises. For practical applications, further studies may explore alternative ionization methods to achieve acceptable efficiency and develop onboard high-voltage power converters.

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