Marked drag reduction in non-affine viscoelastic turbulence in homogeneous isotropic and pipe flows

K. Horiuti, K. Matsumoto & M. Adati
Department of Mechano-Aerospace Engineering, Tokyo Institute of Technology, 2-12-1 O-okayama, Meguro-ku, Tokyo 152-8552, Japan
E-mail: khoriuti@mes.titech.ac.jp

Abstract. Effect of non-affinity of the molecular motions to the macroscopic deformation in the polymer-diluted flow on turbulent drag reduction (DR) is studied using the DNS data for homogeneous isotropic turbulence and pipe flow. The polymer stress is obtained by solving the non-affine Johnson-Segalman constitutive equation. In both flows, DR is maximal when non-affinity is either minimum or maximum, but the largest reduction is achieved when non-affinity is maximum. As an extreme case, in pipe flow, the mean velocity profile exceeds the Virk’s maximum DR limit and almost complete relaminarization of turbulent state is achieved. The normal-stress difference (NSD) is obtained on the basis of new eigenvectors which span the isosurfaces of vortex tube and sheet. It is shown that the first NSD is predominantly positive, while the second NSD is negative along the sheets and tubes. Thus, an extra tension is exerted on the sheet and tube. With an increase of effective viscosity by an addition of elongation viscosity, resistance of the sheet and tube to their stretching is enhanced. The principal mechanism for DR is that the transformation of the sheet into the tube is restrained and the energy cascade is diminished leading to the reduction of drag.

1. Introduction

Phenomenon of drag reduction (DR) in the polymer-diluted fluids is well known [1]. One of the theoretical concepts is due to Lumley [2]. He proposed that randomly coiled polymer molecules stretch in regions of strong deformation, increasing the elongation viscosity $\eta$ of the solution. This in turn results in damping of small turbulent eddies and DR. De Gennes [3] considered that Lumley scheme may hold for (rigid) rods, and in the coil-stretch transition, stretched chains behave elastically and it leads to modifications of the turbulent cascade. It is generally assumed that the Newtonian fluid which surrounds the bead-spring configuration of the polymers moves affinely with an equivalent continuum [4]. However, in the fluid diluted with stretched polymer or rod, molecular motions may not precisely correspond to the macroscopic deformation. In fact, in the fluids diluted with other additives such as the double-stranded DNA [5] and the cationic surfactant with high concentration [6], DR was remarkably enhanced. In the latter, DR even exceeded the Virk’s maximum DR limit in pipe flow, which is considered to be attributable to an appearance of the shear-induced-structures (SIS). When rigid structures such as SIS are formed, the polymer molecules may not follow the local deformation affinely.
DNS has been devoted to analyze DR, in which the polymer stress $\tau_{ij}$ was added to the molecular stress due to the solvent in the Navier-Stokes equation as [7]

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - \frac{\partial \tau_{ij}}{\partial x_j},$$ (1)

where $\nu$ denotes the kinematic viscosity, $\beta$ the ratio of solvent viscosity contribution to total viscosity of solution. The most commonly used constitutive equation to evaluate the polymer stress is the upper convective Maxwell (Oldroyd-B) constitutive equation [8] and its variants [4].

The Oldroyd-B equation was derived assuming that the Newtonian fluid which surrounds the bead-spring configuration of the polymers moves affinely with an equivalent continuum, i.e., the deformation of the polymer strand is identical to the macroscopically-imposed deformation [4, 9]. With a speculation that molecular motions do not precisely correspond to the macroscopic deformation, Gordon & Schowalter [10] proposed a continuum theory in which the strand motion does not deform affinely with the macroscopic deformation [9]. This theory yields the Johnson-Segalman (JS) model [11] as

$$\frac{D\tau_{ij}}{Dt} = (\tau_{ik} \frac{\partial u^*_{j}}{\partial x_k} + \frac{\partial u^*_{k}}{\partial x_k} \tau_{kj}) - \frac{1}{\lambda} \tau_{ij} - \frac{\nu(1-\beta)}{\lambda} S_{ij},$$ (2)

where $\lambda$ denotes the relaxation time and $\alpha$ is the slip parameter, i.e., the proportion of contribution of the non-affine part to the constitutive equation ($0 \leq \alpha \leq 1$), $D/Dt$ the material derivative, $S_{ij}$ the strain-rate tensor, $(\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2$.

$(\partial u^*_{j}/\partial x_k)$ in (2) is the effective convective-velocity gradient [12],

$$\frac{\partial u^*_{j}}{\partial x_k} = \frac{\partial u_j}{\partial x_k} - 2\alpha S_{jk}.$$ (3)

From (3), we get

$$\frac{\partial u^*_{j}}{\partial x_k} - \frac{\partial u^*_{k}}{\partial x_j} = \frac{\partial u_j}{\partial x_k} - \frac{\partial u_k}{\partial x_j},$$ (4)

$$\frac{\partial u^*_{j}}{\partial x_k} + \frac{\partial u^*_{k}}{\partial x_j} = (1 - 2\alpha)(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j}).$$ (5)

The Oldroyd derivative in (2) is reduced to the contravariant derivative with no slippage when $\alpha = 0$ (the Oldroyd-B equation), the corotational derivative with total slippage when $\alpha = 0.5$, and the covarient derivative when $\alpha = 1$ (the Oldroyd-A equation). The rotation of the polymer strands incurred by this effective-velocity gradient exhibits no change from that provided by the solvent, while the straining of the motion of the polymer strands is modified. When $\alpha = 0.5$, the polymer strands are not strained and only rigid rotation is applied to the strands. When $\alpha = 1$, the direction of the straining field is opposite to that provided by the solvent.

The aim of the present study is to elucidate the effect of introduction of this non-affinity in the constitutive equation on occurrence of DR. Assessment is carried out in the forced homogeneous isotropic turbulence (HIT) and the pipe flow.

2. Results of DNS in homogeneous isotropic turbulence

In the homogeneous isotropic turbulence, the periodic boundary conditions are used in the three $(x, y, z)$ directions and the size of the computational domain is $2\pi$ in each direction. 128 grid points are used in each direction, $\nu$ is set equal to 0.004, $\beta = 0.8$, $\lambda = 0.45$. A statistically stationary state is maintained by an imposition of a solenoidal Gaussian random forcing [13].
Isotropic

\[ \langle \varepsilon \rangle \]

\[ \langle u_i f_i \rangle \]

\[ \langle Pe \rangle \]

\| MINMOD \| 2nd order

| Newtonian | 0.419 | 0.470 | - | - | - |
| \( \alpha = 0.0 \) | 0.109 | 0.465 | 0.306 | 35.3 | 22.1 |
| \( \alpha = 0.1 \) | - | - | - | 17.5 | 3.2 |
| \( \alpha = 0.5 \) | 0.386 | 0.446 | 0.011 | 0.2 | 0.1 |
| \( \alpha = 0.9 \) | - | - | - | 22.8 | 7.5 |
| \( \alpha = 1.0 \) | 0.045 | 0.457 | 0.360 | 86.0 | 86.1 |

Table 1. Computed cases and parameters

\(| A_{ij} | + \) (gray color) and \( Q \) (red) from HIT. (a) Newtonian; (b) \( \alpha = 0.0 \); (c) \( \alpha = 1.0 \).

Figure 1. Three dimensional rendering of the isosurfaces of \( A_{ij} \) and \( Q \) from HIT. The tube is identified using the second-order invariance of the velocity gradient tensor \( Q = - (S_{ik} S_{kj} + \Omega_{ik} \Omega_{kj}) \) (shown using red color in the figure). The sheet is identified using the eigenvalue of \( A_{ij} \) (shown using white color). \( \Omega_{ij} \) is the vorticity tensor. One of the eigenvalues of \( A_{ij} \) is chosen as \( A_{ij} \) when the corresponding eigenvector is maximally aligned with the vorticity vector, the largest remaining eigenvalue as \( A_{ij} \), and the smallest one as \( A_{ij} \). The corresponding eigenvectors are denoted as \( a_s, a_+, a_- \), respectively. In the case of \( \alpha = 0.0 \), smaller number of the vortex tubes are created than in the Newtonian case. This result is consistent with [7], and DR is related to the reduction of generation of vortical...
structures. In $\alpha = 1.0$, this reduction is more remarkable, and only few fat tubes are identified and the vortex sheets dominate. The result in the case of $\alpha = 0.5$ is similar to that from the Newtonian case (figure not shown).

The normal-stress difference (NSD) is considered to be an important parameter to identify the orientation and alignment of the polymer strands with due consideration that NSD is zero in the Newtonian fluid. Besides, the elongation viscosity can be obtained when the NSDs are available. To derive the first NSD and second NSD, it is needed to determine the streamline direction of fluid velocity and the direction in which the fluid velocity varies [4]. Figures 2 (a), (b), (c) show the distributions of $a_s$, $a_+\, a_-$ along the vortex sheet, respectively. It is seen that $a_+$ is mostly perpendicular to the vortex sheet and the sheet is spanned by $a_s$ and $a_-$ [15]. $a_s$ is oriented in the stretching direction and aligned in the streamline direction, while $a_+$ is aligned along the direction of velocity variation in HIT. Therefore, the $\tau_{ij}$ terms on the basis of

![Figure 2](image1.png)

**Figure 2.** Three dimensional rendering of the isosurfaces of $[A_{ij}]_+$ (gray color) and the eigenvectors of $[A_{ij}]_+$ along the vortex sheets in HIT. (a) $a_s$; (b) $a_-$; (c) $a_+$.

![Figure 3](image2.png)

**Figure 3.** Isocontours of NSDs plotted on the isosurfaces of vortex sheets from $\alpha = 1.0$ case in HIT. (a) first NSD ($\tau_{ss} - \tau_{++}$); (b) second NSD ($\tau_{++} - \tau_{--}$).
the eigenvectors of \([A_{ij}], E^T(\tau_{ij})E\), provide an accurate estimate of NSD, where \(E\) denotes the orthogonal matrix whose rows are \(a_+, a_-, a_s\). Along the sheet in HIT, the first NSD is given by \((\tau_{ss} - \tau_{++})\) and the second NSD by \((\tau_{++} - \tau_{--})\).

Figure 3 shows the contours of the first NSD and second NSD plotted upon the isosurfaces of the vortex sheets obtained from \(\alpha = 1.0\) case of HIT. First NSD is predominantly positive on the vortex sheets, while second NSD is predominantly negative. This anisotropy implies that preferential stretching and alignment of the polymer molecules along the streamlines occurs [4] and elastic effect is caused on the vortex sheets. Extra tension is exerted on the sheets and the sheets tend to snap back to the original form. Thus, annihilation of creation of the tubes by rolling-up of the vortex sheet occur as is evidenced in figure 1 (c).

The elongation viscosity \(\eta\) can be evaluated as \((\tau_{ss} - \tau_{++})/\sigma_s\) (\(\sigma_s\) is the stretching rate). Figure 4 (a) shows the contours of elongation viscosity plotted upon the isosurfaces of the vortex sheets obtained from \(\alpha = 1.0\) case. Its distribution is similar to that of the first NSD and it is seen that intense \(\eta\) is generated along the sheets when \(\alpha = 1.0\). With an increase of effective viscosity by an addition of elongation viscosity, resistance of the sheet to their stretching is enhanced and cascade of the energy into the small scales is annihilated. As a result, the schemes due to Lumley [2] and de Gennes [3] are basically equivalent in the case of \(\alpha = 1.0\) and both schemes are considered to be valid.

In the cylindrical coordinate system \((z - r - \theta)\) around the vortex tube, the pressure gradient is governed by the equation as [4]

\[
r \frac{d(p - \tau_{zz})}{dr} = r \frac{d(\tau_{rr} - \tau_{zz})}{dr} + (\tau_{rr} - \tau_{\theta\theta}).
\]

Similarly as in the sheet, along the vortex tube, \(a_s\) is aligned along the axial \((z-)\)direction, \(a_+\) is in the radial \((r-)\)direction and perpendicular to the tube axis, and \(a_-\) is in the azimuthal \((\theta-)\)direction of the tube. Therefore, \((\tau_{rr} - \tau_{\theta\theta}) \approx (\tau_{++} - \tau_{--})\). Figure 5 (a) shows the isocontours of the NSD \((\tau_{++} - \tau_{--})\) plotted on the isosurfaces of vortex tubes in the case of \(\alpha = 0.0\). \((\tau_{++} - \tau_{--})\) is predominantly negative along the vortex tube, i.e. \(dp/dx < 0\) according to (6). Thus, the pressure bulges out in tube core region, which leads to the reduction of lowering of pressure in the tube core and subsequent reduction of stretching of the tube. the vortex generation is inhibited by this NSD when \(\alpha = 0.0\). The contours of elongation viscosity \(\eta = (\tau_{ss} - \tau_{++}))/\sigma_s\) plotted upon the isosurfaces of the vortex tubes obtained from the case of
Figure 5. Isocontours of NSD plotted on the isosurfaces of vortex tubes from $\alpha = 0.0$ case. (a) NSD ($\tau_{++} - \tau_{--}$) in HIT; (b) NSD ($\tau_{--} - \tau_{++}$) in pipe flow.

Figure 6. Mean velocity profiles obtained in pipe flow. (a) the profiles in the cases of $\alpha = 0.0, 0.0, 0.5, 0.9$; (b) the profiles in the cases of $\alpha = 0.0, 0.5, 1.0$.

$\alpha = 0.0$ is shown in figure 4 (b). It is seen that intense $\eta$ is generated along the tubes when $\alpha = 0.0$, and resistance of the tube to their stretching is enhanced.

3. Results of DNS in Pipe flow

In pipe flow, $256 \times 64 \times 64$ grid points are used in the streamwise ($x$), radial ($r$) and circumferential ($\theta$) directions. The Reynolds number based on the wall-friction velocity in Newtonian case $Re_{\tau 0}(\equiv \rho u_{\tau}/\eta_0)$ is set equal to 180 and the Weissenburg number $We_{\tau 0}(\equiv \lambda \rho u_{\tau}^2/\eta_0)$ is 25.0 ($\eta_0$ is the zero-shear rate viscosity), and $\beta = 0.9$. The Oldroyd derivatives in the JS model is approximated using the 2nd-order upwind scheme and the Min-Mod method [16]. Artificial stress diffusive terms [7] are added to (2) to retain the stability when the 2nd-order upwind
scheme is used. The second-order central finite difference method is used for other spatial
derivatives. The Peterlin damping function \[ f(\tau) = \frac{L^2}{L^2 - 3} + \frac{W e_{\tau_0}}{Re_{\tau_0}} \frac{1}{E^2} |(1 - 2\alpha)\tau_{kk}| \] (7)
is multiplied to the second term in the right-hand side of (2) in the JS model, where \( L \) is the
maximum extensibility and set equal to 30.0. The Crank-Nicolson implicit method is used for
the viscous terms in (1), and other terms are advanced in time using a third-order Runge-Kutta
scheme. The length of the computational domain in the \( x \)-direction is 10
\( R \) in the Newtonian
case and 20\( R \) in the viscoelastic case (\( R \) is the radius of the pipe).

Computed cases in pipe flow are included in Table 1. \%\( DR \) denotes the drag reduction rate
based on the bulk flow velocity [17]. Similarly as in the homogeneous isotropic turbulence,
the degree of DR does not monotonously depend on the value of \( \alpha \). When \( \alpha = 0.0 \) and \( \alpha = 1.0 \), \%\( DR \)
is maximal, while when \( \alpha = 0.5 \), \%\( DR \) is minimum and close to the Newtonian case. The results
from the cases of \( \alpha = 0.1 \) and \( \alpha = 0.9 \) are intermediate between the cases of \( \alpha = 0.0, 0.5, 1.0 \).
Large DR occurs in both \( \alpha = 0.0 \) and \( \alpha = 1.0 \) cases, but DR in \( \alpha = 1.0 \) is more remarkable.
We note that \%\( DR \) obtained using the Min-Mod method is larger than that obtained using
the second-order upwind method, which indicates that numerical dissipation introduced by the
second-order upwind method and artificial stress diffusive terms is very large. In the following,
we present the results obtained using the Min-Mod method.

Figure 6 shows the mean velocity profiles obtained in pipe flow. These profiles are consistent
with the degree of DR shown in Table 1. \%\( DR \) denotes the drag reduction rate
based on the bulk flow velocity [17]. Similarly as in the homogeneous isotropic turbulence,
the degree of DR does not monotonously depend on the value of \( \alpha \). When \( \alpha = 0.0 \) and \( \alpha = 1.0 \), \%\( DR \)
is maximal, while when \( \alpha = 0.5 \), \%\( DR \) is minimum and close to the Newtonian case. The results
from the cases of \( \alpha = 0.1 \) and \( \alpha = 0.9 \) are intermediate between the cases of \( \alpha = 0.0, 0.5, 1.0 \).
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the second-order upwind method, which indicates that numerical dissipation introduced by the
second-order upwind method and artificial stress diffusive terms is very large. In the following,
we present the results obtained using the Min-Mod method.

Figure 6 shows the mean velocity profiles obtained in pipe flow. These profiles are consistent
with the degree of DR shown in Table 1. It should be noted that DR in the case of \( \alpha = 1.0 \) is
remarkable and it even exceeds the Virk’s maximum DR limit which is plotted using the dotted
line in the figure. The profile is close to the laminar profile shown in the figure using the one-
point dashed line, and almost complete relaminarization of turbulent state is achieved. In our
study, this almost complete relaminarization was not achieved using the Oldroyd-B equation.

Figure 7 shows the isosurfaces of the vortex tubes and sheets obtained in the cases of
\( \alpha = 0.0, 0.5, 1.0 \) in pipe flow. The result in the Newtonian case is similar to that from the
case of \( \alpha = 0.5 \) (figure not shown). As in HIT, when \( \alpha = 0.0 \), smaller number of the vortex
tubes are created in comparison to the case of \( \alpha = 0.5 \), while in the case of \( \alpha = 1.0 \), this reduction
is more remarkable, and only few tubes which are rather compressed in the axial direction are
identified and the vortex sheets dominate.

Figure 7. Three dimensional rendering of the isosurfaces of \([A_{ij}]_+\) (gray color) and \( Q \) (red) in
pipe flow. (a) \( \alpha = 0.0 \); (b) \( \alpha = 0.5 \); (c) \( \alpha = 1.0 \).
In pipe flow, \( \mathbf{a}_s \) is mostly perpendicular to the vortex sheet and aligned along the direction of velocity variation, while the sheet is spanned by \( \mathbf{a}_s \) and \( \mathbf{a}_n \) as in HIT (figure not shown). It is \( \mathbf{a}_n \) which is oriented and aligned along the streamline direction on the vortex sheets unlike in HIT, while \( \mathbf{a}_s \) is oriented in the spanwise direction of the sheets. The presence of the streaky structures is responsible for causing this configuration in the pipe flow. Because the sheets mostly collapse with the streaks, the intense spanwise vorticity distributes along the sheets, and the stretching (s–) direction is in the spanwise direction. Therefore, along the sheet in pipe flow, the first NSD is given by \( (\tau_{--} - \tau_{++}) \) and the second NSD by \( (\tau_{++} - \tau_{ss}) \). The isocontours of the first NSD \( (\tau_{--} - \tau_{++}) \) and second NSD \( (\tau_{++} - \tau_{ss}) \) plotted on the isosurfaces of vortex sheets in the case of \( \alpha = 1.0 \) are shown in figures 8 (a) and 8 (b), respectively. Similarly as in HIT, the first NSD on the vortex sheet is predominantly positive, while the second NSD is predominantly negative. The distribution of the elongation viscosity is similar to that of \( (\tau_{--} - \tau_{++}) \) shown in figure 8 (a) (figure not shown). DR occurs due to the same mechanism as in HIT in the case of \( \alpha = 1.0 \), i.e., stretching and alignment of the polymer molecules along the streamlines, appearance of extra tension along the sheets and annihilation of creation of the tubes.

Included in figure 5 (b) is the isocontours of the NSD \( (\tau_{--} - \tau_{++}) \) on the isosurfaces of vortex tubes in the case of \( \alpha = 0.0 \) in pipe flow. \( (\tau_{++} - \tau_{--}) \) is predominantly negative along the vortex tube, and \( dp/dx < 0 \) (note on the reversal of the sign in \( (\tau_{++} - \tau_{--}) \)). Thus, similarly as in HIT, lowering of pressure is reduced in the tube core, and vortex generation is inhibited by this NSD when \( \alpha = 0.0 \).

4. Summary

Effect of non-affinity on turbulent drag reduction in the polymer-diluted flow is studied using the Johnson-Segalman constitutive equation in the forced homogeneous isotropic turbulence and pipe flow. Common mechanism of drag reduction in both flows is presented.

A new method to derive the normal-stress difference is developed based on a new eigenvectors basis. The normal-stress difference and the elongation viscosity are derived using this method. When non-affinity is maximum or minimum, large elastic energy is stored in polymer molecules. Strong anisotropy among the normal-stress differences indicates a selective alignment of the polymer strands in the streamline direction and subsequent appearance of intense elastic effect. With an increase of effective viscosity by an addition of elongation viscosity, resistance of the
sheet and tube to their stretching is enhanced. The sheet and tube tend to snap back to the original flat form, and cascade of the energy into the small scales is restricted. Drag reduction is more remarkable in the case in which non-affinity is maximum, and almost complete relaminarization is achieved in pipe flow. This result is similar to the drag reduction in the fluid diluted with high concentration surfactant [6]. When non-affinity is maximum, Lumley and de Gennes schemes are nearly equivalent and the results obtained in this study provides support for both schemes.

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6. References

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