On the Design of Secure Full-Duplex Multiuser Systems under User Grouping Method

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Abstract—Consider a full-duplex (FD) multiuser system where an FD base station (BS) is designed to simultaneously serve both downlink users and uplink users in the presence of half-duplex eavesdroppers (Eves). Our problem is to maximize the minimum secrecy rate (SR) among all legitimate users by proposing a novel user grouping method, where information signals at the FD-BS are accompanied with artificial noise to degrade the Eves’ channel. The SR problem has a highly nonconcave and nonsmooth objective, subject to nonconvex constraints due to coupling between the optimization variables. Nevertheless, we develop a path-following low-complexity algorithm, which invokes only a simple convex program of moderate dimensions at each iteration. We show that our path-following algorithm guarantees convergence at least to a local optima. The numerical results demonstrate the merit of our proposed approach compared to existing well-known ones, i.e., conventional FD and non-orthogonal multiple access.

I. INTRODUCTION

By enabling simultaneous transmission and reception on the same channel, full duplex (FD) radio, which offers considerable potential of doubling the spectral efficiency compared to its half-duplex (HD) counterpart, has arisen as a promising technology for 5G wireless networks [1], [2]. The major challenge in designing an FD radio is to suppress self-interference (SI) caused by signal leakage from the downlink (DL) transmission to the uplink (UL) reception on the same device to a potentially suitable level, such as a few dB above background noise. Fortunately, recent advances in hardware design have allowed the FD radio to be implemented at a reasonable cost while canceling out most of the SI [3].

Wireless networks have a very wide range of applications, and an unprecedented amount of personal information is transmitted over wireless channels. Consequently, wireless network security is a crucial issue due to the unalterable open nature of the wireless medium. Physical-layer (PHY) security can potentially provide information security at the PHY layer by taking advantage of the characteristics of the wireless medium. An effective means to deliver PHY security is to adopt artificial noise (AN) to degrade the decoding capability of the eavesdropper (Eve) [4], [5] so that the confidential messages are useless for Eve (security). Notably, with FD radio, we can exploit AN even more effectively [6]. By exploiting FD radio, FD-BS secure communications are of great interest thanks to providing communication secrecy of both UL and DL transmissions. In [7], joint information beamforming and AN beamforming at the FD-BS was investigated to guarantee the security of a single-antenna UL user and DL user. However, this work assumed that there is no SI and co-channel interference (CCI), which is highly idealistic.

Therefore, an extension in [8] by considering both SI and CCI was proposed to mitigate SI and guarantee the physical layer security. The work [9] analyzed the non-trivial trade-off between DL and UL transmit power in FD systems to secure multiple DL users and UL users. Even with recent advances in hardware design for SI cancellation techniques, the harmful effect of SI cannot be neglected if it is not properly controlled, which is proportional to the DL transmission power. The CCI may become strong and uncontrolled whenever an UL user is located near DL users. These shortcomings limit the performance of FD systems [7]–[9].

In this paper, we propose a new transmission design to further resolve the practical restrictions given above. Specifically, the near DL users and far UL users are served in a fraction of the communication time block, and then the FD-BS uses the remaining time to serve the near UL users and far DL users. It is worth noting that the effects of SI and CCI are clearly reduced. There are multiple-antenna eavesdroppers (Eves) that overhear the information signals form both DL and UL transmissions. We are concerned with the problem of jointly designing linear precoders/beamformers at the FD-BS, the UL transmit power allocation, and fractional time (FT) to maximize the minimum secrecy rate (SR) among all users subject to power constraints. In general, such a design problem involves optimization of highly nonconcave and nonsmooth utility functions subject to nonconvex constraints, for which the optimal solution is difficult to compute. The main contributions of the paper are summarized as follows.

1) We propose a new model for FD security to optimize simultaneous DL and UL information privacy by exploring user grouping-based fractional time model.

2) We develop a path-following computational procedure to maximize the SR. The core idea behind our approach is to develop a new inner approximation of the nonconvex problem. The convex program solved at each iteration is of moderate size, and thus is very computationally efficient.

3) Numerical results show that the proposed FD scheme provides a substantial improvement of the SR performance over the conventional FD and non-orthogonal multiple access. It also confirms the robustness of the proposed approach against the significant effect of SI.

Notation: $X^H$, $X^T$ and $\text{Tr}(X)$ are the Hermitian transpose, normal transpose and trace of a matrix $X$, respectively. $\| \cdot \|_r$, $\| \cdot \|$ and $| \cdot |$ denote the Frobenius matrix norm, Euclidean norm of a vector, and absolute value of a complex scalar,
respectively. $\Re\{\cdot\}$ represents the real part of the argument.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Signal Processing Model

Consider a multicarrier communication system illustrated in Fig. 1 where the FD-enabled BS is equipped with $N_t$ transmit antennas and $N_r$ receive antennas to simultaneously serve $2K$ DL users and $2L$ UL users over the same radio frequency band. Each legitimate user is equipped with a single antenna to ensure low hardware complexity. The communications of both DL and UL are observed by $M$ non-colluding Eves, where the $m$-th Eve has $N_{e,m}$ antennas. The users are randomly placed into two zones, such that there are $K$ DL users and $L$ UL users located in a zone nearer to the FD-BS referred to as zone-1, and $K$ DL users and $L$ UL users are located in a zone farther from the FD-BS called zone-2.

In this paper, we split each communication time block, denoted by $T$, into two sub-time blocks orthogonally. As previously mentioned, in order to mitigate the harmful effects of SI and CCI, $K$ near DL users and $L$ far UL users are grouped into group-1, and $K$ far DL users and $L$ near UL users are grouped into group-2. During the first duration time $\tau T$ ($0 < \tau < 1$), the users in group-1 are served, while the users in group-2 are served in the remaining duration time $(1 - \tau)T$. Although each group operates in the FD mode, the inter-group interference, i.e., interference between group 1 and 2, can be eliminated thanks to the fractional time allocation. The communication time block $T$ is normalized to unity ($T = 1$). Upon denoting $K \triangleq \{1, 2, \cdots, K\}$ and $L \triangleq \{1, 2, \cdots, L\}$, the sets of DL users and UL users are denoted as $D \triangleq T \times K$ and $U \triangleq T \times L$ for $T \triangleq \{1, 2\}$, respectively. Thus, the $k$-th DL user and the $\ell$-th UL user in the $i$-th group are referred to as DL user $(i, k)$ and UL user $(i, \ell)$, respectively.

1) Received Signal Model at the FD-BS and DL Users: We consider that the FD-BS deploys a transmit beamformer $w_{i,k} \in \mathbb{C}^{N_r \times 1}$ to transfer the information bearing signal $x_{i,k} \in \mathbb{C}^{N_t \times 1}$ to DL user $(i, k)$. The FD-BS also injects an AN to interfere with the reception of Eves: $x_1 = \sum_{k=1}^{K} w_{1,k} x_{1,k} + v_1$ for DL users in zone-1, and $x_2 = \sum_{k=1}^{K} w_{2,k} x_{2,k} + v_2$ for DL users in zone-2, where $v_1 \sim \mathcal{CN}(0, \mathbf{I})$ and $v_2 \sim \mathcal{CN}(0, \mathbf{I})$ are the AN vector whose elements are zero-mean complex Gaussian random variables, i.e., $v_i \sim \mathcal{CN}(0, \mathbf{V}_i)$ where $\mathbf{V}_i \in \mathbb{C}^{N_t \times N_t}$.

The received signal at DL user $(i, k)$ is expressed as

$$ y_{i,k} = h_{i,k}^H w_{i,k} x_{i,k} + \sum_{j=1, j \neq k}^{K} h_{i,j}^H w_{i,j} x_{i,j} + h_{i,k}^H v_i + \sum_{\ell=1}^{L} f_{i,\ell} \rho_{i,\ell} \hat{x}_{i,\ell} + n_{i,k} $$

(1)

where $h_{i,k} \in \mathbb{C}^{N_r \times 1}$ is the transmit channel vector from the FD-BS to DL user $(i, k)$. The term $\sum_{\ell=1}^{L} f_{i,\ell} \rho_{i,\ell} \hat{x}_{i,\ell}$ represents the CCI from UL users to DL user $(i, k)$, where $f_{i,\ell} \in \mathbb{C}$, $\rho_{i,\ell}$ and $\hat{x}_{i,\ell}$ with $\mathbb{E}\{|\hat{x}_{i,\ell}|^2\} = 1$ are the complex channel coefficient from UL user $(i, \ell)$ to DL user $(i, k)$, transmit power and message of UL user $(i, \ell)$, respectively, and $n_{i,k} \sim \mathcal{CN}(0, \sigma^2 I)$ denotes the additive white Gaussian noise (AWGN) at DL user $(i, k)$. By defining $\tau_1 = \tau$ and $\tau_2 = 1 - \tau$, the information rate decoded by DL user $(i, k)$ in nats/sec/Hz is given by

$$ C^0_{i,k}(X_i, \tau_1) = \tau_1 \ln \left(1 + \frac{|h_{i,k}^H w_{i,k}|^2}{\varphi_{i,k}(X_i)} \right) $$

(2)

where $X_i = \{ [w_{i,k}]_{k \in K} : V_i, \{\rho_{i,\ell} : \ell \in L\} \}$, and $\varphi_{i,k}(X_i) = \sum_{i=1}^{K} \| h_{i,j}^H w_{i,j} \|^2 + \| h_{i,k}^H v_i \|^2 + \sum_{\ell=1}^{L} \| f_{i,\ell} \| \rho_{i,\ell} \hat{x}_{i,\ell} \|^2 + \sigma^2$.

2) Received Signal Model at Eves: The information signal leaked out to the $m$-th Eve in whole communication time block is given by

$$ y_m = \sum_{i=1}^{K} \tau_i h_{m,i,k}^H \left( \sum_{k=1}^{K} w_{i,k} x_{i,k} + v_i \right) + \sum_{i=1}^{K} \tau_i \sum_{\ell=1}^{L} \rho_{i,\ell} w_{m,i,\ell} \hat{x}_{i,\ell} + n_{m,m} $$

(5)

where $h_{m,i,k} \in \mathbb{C}^{N_r \times N_t}$ and $w_{m,i,\ell} \in \mathbb{C}^{N_r \times 1}$ are the wiretap channel matrix and vector from the FD-BS and UL user $(i, \ell)$ to the $m$-th Eve, respectively, and $n_{m,m} \sim \mathcal{CN}(0, \sigma^2 I_{N_m})$ denotes the AWGN at the $m$-th Eve. The information rates at the $m$-th Eve, corresponding to the signal targeted for DL user $(i, k)$ and UL user $(i, \ell)$, are given by

$$ C^\text{ED}_{m,i,k}(X, \tau) = \ln(1 + \| H_{m,i,k}^H w_{i,k} \|^2/\chi_{m,i,k}(X, \tau)), $$

(6a)

and

$$ C^\text{ED}_{m,i,\ell}(X, \tau) = \ln(1 + \rho_{i,\ell}^2 |g_{m,i,\ell}|^2/\chi_{m,i,\ell}(X, \tau)), $$

(6b)
respectively, where $X \triangleq [X_1, X_2]$, $\tau \triangleq [\tau_1, \tau_2]$, and

$$
\psi_{m,i,k}(X, \tau) \triangleq \frac{1}{\tau_i} \sum_{i' = 1}^{2} \sum_{j' = 1}^{K} \left( \sum_{k' = 1}^{K} \|H_{m,k',i'}^H w_{i',k'}\|^2 + \|H_{m,i'}^H V_{i'}\|^2_{F} \right)
$$

and $\chi_{m,i,\ell}(X, \tau) \triangleq \frac{1}{\tau_i} \sum_{i' = 1}^{2} \sum_{j' = 1}^{K} \sum_{k' = 1}^{K} \left( \|H_{m,k',i'}^H w_{i',k'}\|^2 + \|H_{m,i'}^H V_{i'}\|^2_{F} \right)

+ \sum_{(i',\ell') \neq (i,\ell)} \frac{\rho_{\ell',\ell}^2 \|g_{m,i'}^H \|^2}{\tau_i} + N_e m, \sigma^2 / \tau_i.

\[ \text{B. Optimization Problem Formulation} \]

We aim to jointly optimize the transmit information vectors and AN matrices $X$, and the fractional time $\tau$ to maximize the minimum (max-min) SR. The optimization problem is therefore formulated as

$$
\text{maximize } \min_{X,\tau} \left\{ R_{i,k}(X, \tau), R_{i,\ell}(X, \tau) \right\}
$$

subject to

\[ \text{(7a)} \]

where $\mathcal{M} \triangleq \{1, 2, \ldots, M\}$, and

$$
P_{i,k}(X, \tau) \triangleq C_{i,k}(X, \tau) - \max_{m \in \mathcal{M}} C_{m,i,k}(X, \tau), \quad (8a)
$$

$$
P_{i,\ell}(X, \tau) \triangleq C_{i,\ell}(X, \tau) - \max_{m \in \mathcal{M}} C_{m,i,\ell}(X, \tau). \quad (8b)
$$

The constraint (7b) merely means that the total transmit power at the FD-BS cannot exceed the power budget, $P_{\text{max}}$, while the constraints (7c) and (7d) are individual power budgets at UL user $(i, \ell)$, $P_{i,\ell}^{\text{max}}$.  

\[ \text{III. Proposed Optimal Solution} \]

\[ \text{A. Equivalent Transformations for (7)} \]

We first introduce new variables $\eta$ and $\Gamma \triangleq \{\{\eta_{m,i,k}(i, k)\}_{(i, k) \in D}, \{\eta_{m,i,\ell}(i, \ell)\}_{(i, \ell) \in U}\}$ to equivalently rewrite (7) as 6:

$$
\text{maximize } \eta, \Gamma
$$

s.t. \[ (7a), (7c), (7d), (7e), (7f), \]

$$
C_{i,k}(X, \tau) - \Gamma_{i,k} \geq \eta, \quad (9c)
$$

$$
C_{m,i,k}(X, \tau) \leq \Gamma_{m,i,k}, \quad \forall m \in \mathcal{M}, (i, k) \in D, \quad (9d)
$$

$$
C_{i,\ell}(X, \tau) - \Gamma_{i,\ell} \geq \eta, \quad (i, \ell) \in U, \quad (9e)
$$

$$
C_{m,i,\ell}(X, \tau) \leq \Gamma_{m,i,\ell}, \quad \forall m \in \mathcal{M}, (i, \ell) \in U. \quad (9f)
$$

The problem (9) still remains intractable. To solve this problem, we make the variable change:

$$
\tau_1 = 1 / \alpha_1 \quad \text{and} \quad \tau_2 = 1 / \alpha_2
$$

which implies the following convex constraint

$$
1 / \alpha_1 + 1 / \alpha_2 \leq 1, \quad \forall \alpha_i > 0, \quad i \in I
$$

where $\alpha \triangleq \{\alpha_1, \alpha_2\}$ are new variables. Using (10), the constraints (9c) and (9e) become

$$
\frac{\eta}{\alpha_1} \ln \left( \frac{1 + \|h_{i,k}^H w_{i,k}\|^2}{\varphi_{i,k}(X_i)} \right) \geq \eta + \Gamma_{i,k}^\eta, \quad (12a)
$$

$$
\frac{\eta}{\alpha_2} \ln \left( \frac{1 + \|h_{i,\ell}^H \Phi_{i,\ell}(X_i)\|^2}{\varphi_{i,\ell}(X_i)} \right) \geq \eta + \Gamma_{i,\ell}^\eta. \quad (12b)
$$

Analogously, the constraints (9d) and (9f) become

$$
\frac{\eta}{\alpha_1} \ln \left( \frac{\|h_{i,k}^H w_{i,k}\|^2}{\psi_{m,i,k}(X, \alpha)} \right) \leq \Gamma_{i,k}^\eta, \quad (13a)
$$

$$
\frac{\eta}{\alpha_2} \ln \left( \frac{\|h_{i,\ell}^H \Phi_{i,\ell}(X, \alpha)\|^2}{\psi_{m,i,\ell}(X, \alpha)} \right) \leq \Gamma_{i,\ell}^\eta. \quad (13b)
$$

where $\psi_{m,i,k}(X, \alpha)$ and $\chi_{m,i,\ell}(X, \alpha)$ are re-defined as

$$
\psi_{m,i,k}(X, \alpha) \triangleq \alpha_i \sum_{i' = 1}^{2} \sum_{j' = 1}^{K} \left( \sum_{k' = 1}^{K} \|H_{m,k',i'}^H w_{i',k'}\|^2 \right)
$$

+ $\|H_{m,i'}^H V_{i'}\|^2_{F} \right) + \alpha_i N_e m, \sigma^2 / \tau_i.

\[ \text{IV. Proposed Convex Approximation-Based Iterations} \]

The proposed algorithm is mainly based on an inner approximation method 13, under which the nonconvex parts are completely exposed.

\[ \text{Approximation of the Constraints (12)} \]

We first introduce the following inequality which will be useful to develop a convex approximation:

$$
\ln(1 + \gamma) \geq \frac{1}{\gamma} \ln \left( \frac{1 + \gamma}{\gamma} \right) - 1
$$

The proof is omitted due to the space limitation. Let us define $\gamma_{i,k} \triangleq \|h_{i,k}^H w_{i,k}\|^2 / \varphi_{i,k}(X_i)$. In the spirit of [13], for $w_{i,k} = e^{-j arg[h_{i,k}^H w_{i,k}]} w_{i,k}$ with $j = \sqrt{-1}$, it follows that $|h_{i,k}^H w_{i,k}| = \Re\{h_{i,k}^H w_{i,k}\} \geq 0$ and $|h_{i,k}^H w_{i,k}| = |h_{i,k}^H w_{i,k}|$ for all $(i', k') \neq (i, k)$. Thus, $\gamma_{i,k}^\eta$ can be equivalently replaced...
by
\[ \gamma_{i,k}^D = \left( \Re \left( h_{i,k}^H w_{i,k} \right) \right)^2 / \varphi_{i,k}(X_i). \]  
(16)

By using (15), \( C^D_{\ell}(X_i, \alpha_i) \) in (12a) is lower bounded at a feasible point \((X_i^{(k)}, \alpha_i^{(k)})\) by
\[ \ln \left( 1 + \gamma_{i,k}^D \right) \geq A^{(k)}_{i,k} - \frac{\gamma_{i,k}^D}{\Re \left( h_{i,k}^H w_{i,k} \right)^2} - C^{(k)}_{i,k} \alpha_i \]  
(17)

where
\[ A^{(k)}_{i,k} \triangleq 2 \ln \left( 1 + \gamma_{i,k}^D \right) \alpha_i \left( \gamma_{i,k}^D + 1 \right) > 0, \]
\[ B^{(k)}_{i,k} \triangleq \frac{\left( \gamma_{i,k}^D \right)^2}{\alpha_i \left( \gamma_{i,k}^D + 1 \right)} > 0, \]
\[ C^{(k)}_{i,k} \triangleq \frac{\ln \left( 1 + \gamma_{i,k}^D \right)}{\alpha_i^2} > 0, \]
\[ \gamma_{i,k}^D \triangleq \Re \left( h_{i,k}^H w_{i,k} \right)^2 > 0. \]

We make use of the inequality \(|x|^2 \geq 2|x(1 - |x|)|, \forall x \in \mathbb{C}, \bar{x} \in \mathbb{C}\) due to the convexity of function \(|x|^2\) to further expose the hidden convexity of the right-hand side (RHS) of (17) as
\[ \ln \left( 1 + \gamma_{i,k}^D \right) \geq A^{(k)}_{i,k} - B^{(k)}_{i,k} \Re \left( h_{i,k}^H w_{i,k} \right) \frac{\varphi_{i,k}(X_i)}{\varphi_{i,k}(X_i)} - C^{(k)}_{i,k} \alpha_i \]
\[ := C^{D}_{i,k}(X_i, \alpha_i) \]  
(18)

over the trust region
\[ 2 \Re \left( h_{i,k}^H w_{i,k} \right) - \Re \left( h_{i,k}^H w_{i,k} \right) > 0, \quad (i, k) \in D \]  
(19)

where \( \psi_{i,k}(X_i) \triangleq 2 \Re \left( h_{i,k}^H w_{i,k} \right) - \Re \left( h_{i,k}^H w_{i,k} \right) \). Note that \( C^{D}_{i,k}(X_i, \alpha_i) \) is a lower bounding convex function of \( C^D_{\ell}(X_i, \alpha_i) \), which also satisfies
\[ C^{D}_{i,k}(X_i, \alpha_i) = \frac{1}{\alpha_i} \ln \left( 1 + \frac{\psi_{i,k}(X_i)^2}{\varphi_{i,k}(X_i)^2} \right). \]  
(20)

As a result, (12a) can be iteratively replaced by the following convex constraint:
\[ C^{D}_{i,k}(X_i, \alpha_i) \geq \eta_i + \Gamma^0_{i,k}, \quad (i, k) \in D. \]  
(21)

Following a similar procedure, the nonconvex constraint (12b) can be approximated as follows. For
\( \gamma_{i,k}^D \triangleq \rho_{i,k}^D \Phi_{i,k}(X_i) - g_{i,k} \), the left-hand side (LHS) of (12b) is lower bounded at the feasible point \((X_i^{(k)}, \alpha_i^{(k)})\) by
\[ \ln \left( 1 + \gamma_{i,k}^D \right) \geq A^{(k)}_{i,k} + B^{(k)}_{i,k} \rho_{i,k} - C^{(k)}_{i,k} \alpha_i \]
\[ := C^{D}_{i,k}(X_i, \alpha_i) \]  
(22)

where
\[ A^{(k)}_{i,k} \triangleq 2 \ln \left( 1 + \gamma_{i,k}^D \right) - \gamma_{i,k}^D / \rho_{i,k} \rho_{i,k} / \alpha_i \]
\[ B^{(k)}_{i,k} \triangleq \ln \left( 1 + \gamma_{i,k}^D \right) \rho_{i,k} \left( \rho_{i,k}^2 G_{i,k} \Phi_{i,k}(X_i) \right)^{-1} \]
\[ \phi_{i,k}(X_i) \triangleq \sigma^2 \text{Tr} \left( \Omega_{i,k}^D \right) + \sigma_{SI} \sum_{k=1}^{K} w_{i,k}^H G_{i,k} \Omega_{i,k} G_{i,k}^H w_{i,k} \]
\[ + \sigma_{SI} \text{Tr} \left( V_{i}^H G_{i} \Omega_{i,k} G_{i,k}^H V_{i} \right) + \sum_{j=\ell}^{L} \rho_{i,j}^2 \delta_{i,j} \Omega_{i,k} \delta_{i,j}, \]
\[ \Omega_{i,k}^D \triangleq \Phi_{i,k}(X_i)^{-1} \]
\[ - \left( \rho_{i,k}^2 G_{i,k} + \Phi_{i,k}(X_i) \right)^{-1} \geq 0. \]

For the concave function \( C^{D}_{\ell}(X_i, \alpha_i) \), the constraint (12b) can be iteratively replaced by
\[ C^{D}_{i,k}(X_i, \alpha_i) \geq \eta_i + \Gamma^0_{i,k}, \quad (i, k) \in U. \]  
(23)

Approximation of the Constraints (13): For a given feasible point \( x \), the inequality \( \ln(1+x) \leq (1+x)/x \) holds true for \( x \geq 0, x \geq 0 \), which is a result of the concavity of the function \( \ln(1+x) \). Let us consider (13a) first. At a feasible point \((X_i, \alpha_i)\), its LHS is upper bounded by
\[ \ln \left( 1 + \frac{\left( H_{m,i,k} w_{i,k} \right)^2}{\psi_{m,i,k}(X_i, \alpha_i)} \right) \leq \ln \left( 1 + \gamma_{m,i,k}^D \right) + \left( 1 + \gamma_{m,i,k}^D \right)^{-1} \]
\[ \times \left( \frac{H_{m,i,k}^H w_{i,k}^D}{\psi_{m,i,k}(X_i, \alpha_i)} - \gamma_{m,i,k}^D \right) \]  
(24)

where \( \gamma_{m,i,k}^D \triangleq \left( H_{m,i,k} w_{i,k} \right)^2 / \psi_{m,i,k}(X_i, \alpha_i) \). We also introduce new variables \( \mu_{m,i,k} > 0, \forall m \in \mathcal{M}, (i, k) \in D \), (25a)
\[ \mu_{m,i,k} \leq \psi_{m,i,k}(X_i, \alpha_i), \forall m \in \mathcal{M}, (i, k) \in D \]  
(25b)

For the nonconvex constraint (25b), we can express it in more tractable form as
\[ f(X, \alpha, \mu_{m,i,k}) \leq N_{m,i}^2 \sigma^2 \]  
(26)

where
\[ f(X, \alpha, \mu_{m,i,k}) = \frac{\mu_{m,i,k} - \gamma_{m,i,k}^D}{\alpha_i} - \sum_{l=1}^{L} \left( \sum_{i'=1}^{K} (i' \neq i, k') \frac{H_{m,i',k'}^H}{\psi_{m,i',k'}(X_i, \alpha_i)} \right)^2 + \sum_{l=1}^{L} \psi_{m,i',k'}^D \| g_{m,i',k'}^H \|^2 \]  
(27)

As a consequence, the convex approximation of the constraint (25b) reads as
\[ f^*(X, \alpha, \mu_{m,i,k}) \leq N_{m,i}^2 \sigma^2, \forall m \in \mathcal{M}, (i, k) \in D. \]  
(28)

Similarly, the nonconvex constraint (13b) is iteratively approximated by
\[ C^{D}_{m,i,k}(X_i, \alpha_i, \mu_{m,i,k}) \leq \Gamma^0_{m,i,k}, \forall m \in \mathcal{M}, (i, k) \in U \]  
(29)

with the additional convex constraint:
\[ g^*(X, \alpha, \mu_{m,i,k}) \leq N_{m,i}^2 \sigma^2, \forall m \in \mathcal{M}, (i, k) \in U \]  
(29)

where \( \mu_{m,i,k} > 0, \forall m, i, k \) are new variables, and
\[ C^{D}_{m,i,k}(X_i, \alpha_i, \mu_{m,i,k}) := \ln \left( 1 + \gamma_{m,i,k}^D \right) + \left( 1 + \gamma_{m,i,k}^D \right)^{-1} \]
\[ \times \left( \frac{\left( H_{m,i,k}^H w_{i,k} \right)^2}{\psi_{m,i,k}(X_i, \alpha_i)} - \gamma_{m,i,k}^D \right) \]  
(29)

for \( \gamma_{m,i,k}^D \triangleq \left( H_{m,i,k}^H w_{i,k} \right)^2 / \psi_{m,i,k}(X_i, \alpha_i) \). The function
\[ g^*(X, \alpha, \mu_{m,i,k}) \]  
(29)

is the convex approximation of \( g(X, \alpha, \mu_{m,i,k}) \) in (29) is the convex approximation of nonconvex constraints
\[ C_{m,i,k}(X_i, \alpha_i, \mu_{m,i,k}) \]  
(14a)

and (14d):

The inner convex approximations for nonconvex constraints
Algorithm 1: Proposed path-following algorithm to solve (7)

Initialization: Set $\kappa := 0$ and solve (32) to generate an initial feasible point $(X(0), \alpha(0), \mu(0))$.
1: repeat
2: Solve (31) to obtain the optimal solutions $(X^*, \alpha^*, \mu^*)$.
3: Update $X^{(k+1)} := X^*, \alpha^{(k+1)} := \alpha^*, \mu^{(k+1)} := \mu^*$.
4: Set $\kappa := \kappa + 1$.
5: until Convergence

(14c) and (14d) are given as
\[
\sum_{k=1}^{K} \|w_{1,k}\|^2 + \|V_1\|^2 + \frac{1}{\alpha_2} \left( \sum_{k=1}^{K} \|w_{2,k}\|^2 + \|V_2\|^2 \right) + \frac{\alpha_2}{\alpha_2^2} \left( \sum_{k=1}^{K} \|w_{1,k}\|^2 + \|V_1\|^2 \right) \leq P_{\text{max}} \cdot \rho \cdot 10^\eta.
\]

Using the above equivalent transformations and approximations, the following convex program is solved at the feasible point $(X^{(k)}, \alpha^{(k)}, \mu^{(k)})$:
\[
\text{maximize } \eta \\
\text{subject to } (21), (11), (14c), (19), (21), (23), (25a), (27), (28), (29), (30),
\]
where $\mu \defeq \{\mu_{m,i,k}, \mu_{m,i,k} \}_{m \in M, i \in \mathcal{I}, k \in \mathcal{K}}$ represents the collection of all auxiliary variables. A pseudo-code used to solve (7) is given in Algorithm 1. This algorithm yields a non-decreasing sequence of the objective, i.e., $\eta^{(k+1)} \geq \eta^{(k)}$, which converges to a Karush-Kuhn-Tucker point of (7) [10].

Practical Implementations: Initialized by any feasible $(X(0), \alpha(0))$ to the convex constraints (23), (25a), (19), (21), (23), (25a), (30), the following convex program
\[
\text{maximize } \{\eta - \bar{\eta}_{\text{min}}\} \\
\text{subject to } (7a), (11), (14c), (19), (21), (23), (30)
\]
without imposing Eves’ constraints, is successively solved until reaching: $\{\eta - \bar{\eta}_{\text{min}}\} \geq 0$. Herein, $\bar{\eta}_{\text{min}} > 0$ is a given value to further improve the convergence speed of solving (14). The initial feasible $\mu^{(0)}$ is then found by calculating: $\mu_{m,i,k}^{(0)} = \psi_{m,i,k}(X^{(0)}, \alpha^{(0)})$ and $\mu_{m,i,k}^{(0)} = \chi_{m,i,k}(X^{(0)}, \alpha^{(0)})$. We have numerically observed that it usually takes about 2 iterations to generate an initial feasible point of (13).

IV. NUMERICAL RESULTS

A small cell topology with 4 DL users ($K = 2$), 4 UL users ($L = 2$) and $M = 2$ Eves is used in the numerical examples. The radius of the small cell is set to 100 m with inner circle radius of 50 m. 2 DL users and 2 UL users are randomly located in zone-1 (inner zone) and the remaining 2 DL users and 2 UL users are randomly located in zone-2 (outer zone). There is one Eve with $N_{e,m} = 2$ antennas that is randomly placed in each zone. Unless stated otherwise, the most important parameters based on the settings in [10], [12] are specified in Table I for ease of cross-referencing. The entries of the fading loop channel $G_{S1}$ are generated as independent and identically distributed Rician random variables with Rician factor $K_{S1} = 5$ dB. The CCI channel coefficient from UL user $(i, \ell)$ to DL user $(i, k)$ at a distance $d$ (in km) is assumed to undergo the path loss (PL) model for non-line-of-sight (NLOS) communications as $f_{i,\ell,k} = 10^{-P_{\text{max}}/10} f_{i,\ell,k}^+$. The PL is in dB and $f_{i,\ell,k}^+$ follows $\mathcal{CN}(0, 1)$. All other channels encounter the PL model for line-of-sight (LOS) communications as $h = 10^{-P_{\text{max}}/10} h$, where $h \in \{h_{m,i,k}, g_{m,i}, h_{m,i,\ell}\}$ and $h$ follows $\mathcal{CN}(0, 1)$. For comparison, the following three existing schemes are considered:

- The FD system in [13] without considering fractional times and user grouping, which is referred to as “Conventional FD.”
- Under the same system model with “Conventional FD,” the DL transmission can adopt non-orthogonal multiple access (NOMA) [16] to further improve its own performance. This scheme is called “FD-NOMA.”
- Additionally, an HD system is considered. Here, the HD-BS uses all antennas $N = N_t + N_e$ to serve all DL users in the DL and all UL users in the UL, albeit in two separate communication time blocks. In such a case, there are no SI and CCI, but the time fractions for DL and UL transmissions will decrease into a half of the FD counterpart.

Fig. 2(a) depicts the average max-min SR versus the transmit power at the FD-BS for different resource allocation schemes. The observations from the figure are as follows. First, one can see that the SR of the FD systems is better than that of the HD system at a high transmit power $P_{\text{max}}$ and the SR of HD is nearly unchanged. The reasons for these results are three-fold: 1) The effective SR of the HD per resource block is divided by two; 2) The DL transmission in HD dominates the UL one, as the UL transmission is free of AN; 3) In the FD systems, the FD-BS can better protect both DL and UL transmissions by using AN. Second, the SR of FD-NOMA outperforms the conventional HD, which is a result of canceling out intra-cluster interference. Third,

| Parameter | Value |
|-----------|-------|
| Carrier center frequency/ System bandwidth | 2 GHz/10 MHz |
| Distance between the FD-BS and nearest user | $\geq 10$ m |
| Noise power spectral density, $\sigma^2$ | $-174$ dBm |
| Path loss model for LOS, $P_{\text{Pl,los}}$ | $103.8 + 20.9 \log_{10}(d)$ dB |
| Path loss model for NLOS, $P_{\text{Pl,nlos}}$ | $145.4 + 37.5 \log_{10}(d)$ dB |
| Power budget at the FD-BS, $P_{\text{max}}$ | 26 dBm |
| Power budget at UL users, $P_{\text{max}}^+$ | $23$ dBm |
| FD residual SI, $\sigma_s$ | $-75$ dBm |
| Number of antennas at the FD-BS, $N_t$ | $5$ |

**TABLE I**

**SIMULATION PARAMETERS**
the SR of the proposed FD is fully superior to the others and an improvement of almost 1.51 bps/Hz (≈ 38.2%) over HD is achieved at the practical value of $P_{bs}^{max} = 26$ dBm defined in 3GPP TS 36.814. We recall that the proposed FD-NOMA and at least 78 dB loss in the system performance. In addition, Fig. 2(b) further the SR of the proposed FD to guarantee a better SR per user compared to HD. Interestingly, the SR of the proposed FD always outperforms the HD for whatever value of $\sigma_{SI}$, which confirms its robustness against the significant effect of SI.

V. CONCLUSION

We have addressed the problem of secure FD multiuser wireless communication. To handle the unwanted interference (multiuser interference, SI and CCI), a novel user grouping-based fractional time model has been proposed. We have developed a new path-following optimization algorithm to jointly design the fractional times and power resource allocation to maximize the secrecy rate per user in both DL and UL directions. Numerical results with realistic parameters have revealed that the proposed FD scheme not only provides substantial improvement in terms of secrecy rate over the existing schemes, but also confirms its robustness against the significant effect of SI.

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