Radiation Dominated Universe for Jordan-Brans-Dicke Cosmology

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Jordan-Brans-Dicke cosmology with a standard kinetic term for the scalar field and no mass term has the same radiation dominated solution as standard Einstein cosmology without the cosmological constant. Because of this, the primordial nucleosynthesis (Big - Bang nucleosynthesis) result obtained for standard cosmology remains the same for Jordan-Brans-Dicke cosmology. We show that Jordan-Brans-Dicke cosmology with a mass term for the scalar field as well as explaining dark energy for the present era, can also explain radiation dominated cosmology for the primordial nucleosynthesis era.

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I. INTRODUCTION

Big Bang Nucleosynthesis (BBN) is a theory based on the Standard Model which also has some observational proof that a few minutes after the Big-Bang firstly the stable low mass nuclei such as $^2H$, $^3He$, $^4He$, $^6Li$, $^7Li$ started to form [1-5]. This era is known as the radiation dominated nucleosynthesis period of the Universe. So far the observational data confirm that universe is accelerating but the reasons are not exactly known [6-8]. In cosmology it is agreed that Freidmann-Lemaitre spacetime is used for the metric of universe [9]. General Relativity (GR) is successful in easily explaining the accelerated expansion by introducing a cosmological constant. This is achieved by using the Friedmann equations which Friedmann derived from Einstein’s GR by considering the possibility of an expanding universe [10-11].

As well as GR there have been various studies done by considering the Jordan-Brans-Dicke (JBD) theory [12-15] for the acceleration of universe [16-19]. Friedmann equation for JBD scalar tensor theory of gravitation has been derived [20] and it has been shown that
JBD can make a correction to the matter density component of the Friedmann equation. The main parameter of JBD is $\omega$ which is a dimensionless constant and bigger than $10^4$ \cite{21}. One attractive and simple possibility is to consider a JBD cosmology based on a scalar field with a standard kinetic term and a mass term in the action. Previously a work done investigated the possibility of the primordial inflation in JBD cosmology for a closed universe and for a massive scalar field \cite{22} which joint smoothly with the early radiation dominated era. In this paper we show the radiation dominated solution for JBD.

The Friedmann equation is a relation between the Hubble expansion rate $H = \frac{\dot{a}}{a}$ of the universe and the energy density. For GR with cosmological constant, flat, space-like sections can be expressed as

$$\left(\frac{H}{H_0}\right)^2 = \Omega_\Lambda + \Omega_M \left(\frac{a_0}{a}\right)^3 + \Omega_{rad} \left(\frac{a_0}{a}\right)^4$$ \hspace{1cm} (1)

where $\Omega_\Lambda$ is the density parameter for dark energy, $\Omega_M$ is the density parameter for matter including dark matter and $\Omega_{rad}$ is density parameter for radiation and in total they add up to unity. $a$ is the scale size of the universe where $H_0$ and $a_0$ denotes to today values. The radiation contribution is taken negligible for this work.

The standard JBD action in the canonical form is given by

$$S = \int d^4x \sqrt{g} \left[ -\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 + L_M \right].$$ \hspace{1cm} (2)

The metric signature is (+ - - -). $g$ is minus the determinant of the metric and $d^4x \sqrt{g}$ is the four-dimensional volume form, $\phi$ is the scalar field, $R$ is the Ricci scalar, $g^{\mu\nu}$ is the metric tensor and $L_M$ is the matter Lagrangian density except the scalar field $\phi$. Note that the standard mass term for the scalar field can be combined with the curvature term so that this is sometimes called the cosmological term for JBD theory \cite{23-26}. The field equations derived from the variation of the action (2) with respect to Robertson - Walker metric are

$$\frac{3}{4\omega} \phi^2 \left(\dot{\phi}^2 + \frac{k}{a^2}\right) - \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m^2 \phi^2 + \frac{3}{2\omega} \dot{\phi} \phi = \rho_M$$ \hspace{1cm} (3)

$$-\frac{1}{4\omega} \phi^2 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \frac{1}{\omega} \dot{\phi} \phi - \frac{1}{2\omega} \ddot{\phi} \phi - \frac{1}{2\omega} \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2}\right) \ddot{\phi}^2 + \frac{1}{2} m^2 \phi^2 = p_M$$ \hspace{1cm} (4)

$$\ddot{\phi} + 3\frac{\dot{a}}{a} \dot{\phi} + \left[m^2 - \frac{3}{2\omega} \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)\right] \phi = 0$$ \hspace{1cm} (5)
where $k$ is the curvature parameter which $k = -1, 0, 1$ correspond to open, flat, closed universes respectively, $a(t)$ is the scale factor of the universe, $\dot{a}(t)$ denotes the derivative of that factor and $M$ denotes everything except the scalar field. For $k=0$, $m=0$, $a \sim t^\alpha$, $\phi \sim t^\beta$, $\rho \sim a^{-4}$ and $p = \rho/3$ in (3-5) it gives

$$a \sim t^{1/2}, \phi = constant.$$  

(6)

Since $m$ is different than zero for dark energy, to generalize this solution a more general treatment is needed. Instead of working with the field equations (3-5) which are given in terms of $\phi(t), a(t)$ and their derivatives with respect to cosmological time $t$, the fractional rate of change of $\phi$ and $a$ is taken as

$$F(a) = \frac{\dot{\phi}}{\phi}$$  

(7)

and the Hubble parameter is taken as

$$H(a) = \frac{\dot{a}}{a},$$  

(8)

and rewritten the left hand side of the field equations in terms of $H(a), F(a)$ and their derivatives with respect to $a$

$$H^2 - \frac{2\omega}{3} F^2 + 2HF + \frac{k}{a^2} - \frac{2\omega}{3} m^2 = \left(\frac{4\omega}{3}\right) \frac{\rho}{\phi^2}$$  

(9)

$$H^2 + \left(\frac{2\omega}{3} + \frac{4}{3}\right) F^2 + \frac{4}{3} HF + \frac{2a}{3} \left(H H' + HF'\right) + \frac{k}{3a^2} - \frac{2\omega}{3} m^2 = \left(-\frac{4\omega}{3}\right) \frac{p}{\phi^2}$$  

(10)

$$H^2 - \frac{\omega}{3} F^2 - \omega HF + a \left(\frac{HH'}{2} - \frac{\omega}{3} HF'\right) + \frac{k}{2a^2} - \frac{\omega}{3} m^2 = 0.$$  

(11)

From these three equations it is shown that the continuity equation for the matter-energy excluding the JBD scalar field is as follows

$$\dot{\rho} + 3 \left(\frac{\dot{a}}{a}\right) (p + \rho) = 0.$$  

(12)

The continuity equation is used in place of (10) because (9,11 and 12) is used to derive (10) provided $\rho \neq 0$. After some arrangements equation (9) is received purely in terms of $H(a), F(a), \rho(a)$ and their derivatives with respect to $a$. 


\[ H' (H^2 + HF) + F' \left( H^2 - \frac{2\omega}{3} HF \right) = \frac{H^3}{2} \left( \frac{\rho'}{\rho} \right) + \frac{2\omega}{3a} F^3 + H^2 F \left[ \frac{\rho'}{\rho} - \frac{1}{a} \right] \]

\[ + F^2 H \left[ -\frac{2}{a} - \frac{\omega}{3} \left( \frac{\rho'}{\rho} \right) \right] + \frac{k}{a^2} \left[ H \left( \frac{\rho'}{2\rho} + \frac{1}{a} \right) - \frac{F}{a} \right] - \omega m^2 \left[ H \left( \frac{\rho'}{3\rho} \right) - \frac{2F}{3a} \right] \] (13)

After rewriting (11) in the following form

\[ 3aH H' - 2\omega H aF' = -6H^2 + 2\omega F^2 + 6\omega HF - \frac{3k}{a^2} + 2\omega m^2 \] (14)

(13,14) are solved for \( H', \ F' \)

\[ H' = \frac{\omega a \left( \frac{\rho'}{\rho} \right) - 6}{(2\omega + 3) aH} H^2 - \frac{4\omega^2 + 2\omega + 2a\omega^2 \left( \frac{\rho'}{3\rho} \right)}{(2\omega + 3) aH} F^2 + \frac{8\omega + 2a\omega \left( \frac{\rho'}{\rho} \right)}{(2\omega + 3) aH} HF \]

\[ - \frac{2\omega a \left( \frac{\rho'}{\rho} \right) - 2\omega}{(2\omega + 3) aH} m^2 + \frac{2\omega + \omega a \left( \frac{\rho'}{\rho} \right) - 3}{(2\omega + 3) a^3 H} \] (15)

\[ F' = \frac{3a \left( \frac{\rho'}{2\rho} \right) + 6}{(2\omega + 3) aH} H^2 - \frac{8\omega + a\omega \left( \frac{\rho'}{\rho} \right) + 6}{(2\omega + 3) aH} F^2 - \frac{6\omega - 3a \left( \frac{\rho'}{\rho} \right) - 3}{(2\omega + 3) aH} HF \]

\[ - \frac{\omega a \left( \frac{\rho'}{\rho} \right) + 2\omega}{(2\omega + 3) aH} m^2 + \frac{6 + 3a \left( \frac{\rho'}{2\rho} \right)}{(2\omega + 3) a^3 H} \] (16)

Using (12), it is seen that the energy density \( \rho \) evolves with \( a \) in the same manner as in standard Einstein cosmology when the universe is solely governed by radiation,

\[ \rho = \frac{C}{a^4} \] (17)

where \( C \) is an integration constant.

II. A PERTURBATIVE METHOD TO FIND THE SOLUTION FOR \( m \neq 0 \)

To discuss the solutions of (9-11) firstly the vacuum solution is considered where \( \rho \) and \( p \) are taken to be equal to zero and \( k = 0 \) set for flat, space-like sections.
\[ H^2 - \frac{2\omega}{3} F^2 + 2HF - \frac{2\omega}{3} m^2 = 0 \]  \hspace{1cm} (18)

\[ H^2 + \left( \frac{2\omega}{3} + \frac{4}{3} \right) F^2 + \frac{4}{3} HF - \frac{2\omega}{3} m^2 = 0 \]  \hspace{1cm} (19)

\[ H^2 - \frac{\omega}{3} F^2 - \omega HF - \frac{\omega}{3} m^2 = 0. \]  \hspace{1cm} (20)

The terms with \( m \) are carried to the right hand side of the equations and (20) is multiplied by a factor of 2. A solution such as \( H = H_0 \) and \( F = F_0 \) is considered where \( H \) and \( F \) are constant.

\[ H_0^2 - \frac{2\omega}{3} F_0^2 + 2H_0 F_0 = \frac{2\omega}{3} m^2 \]  \hspace{1cm} (21)

\[ H_0^2 + \left( \frac{2\omega}{3} + \frac{4}{3} \right) F_0^2 + \frac{4}{3} H_0 F_0 = \frac{2\omega}{3} m^2 \]  \hspace{1cm} (22)

\[ H_0^2 - \frac{\omega}{3} F_0^2 - \omega H_0 F_0 = \frac{\omega}{3} m^2. \]  \hspace{1cm} (23)

Since the left hand side equations are homogeneous in \( H_0 \) and \( F_0 \), \( H_0 = cF_0 \) is taken and it is found that

\[ c = 2\omega + 2, \]

\[ H_0 = \sqrt{\frac{(4\omega^2 + 8\omega + 4)\omega m^2}{6\omega^2 + 17\omega + 12}}, \]  \hspace{1cm} (24)

\[ F_0 = \frac{H_0}{2\omega + 2}. \]

To consider how radiation modifies this solution, \( \rho = 3p \) is taken. Two equations are obtained from the main three equations (9-11).

\[ \frac{4H^2}{3} + \left( \frac{4\omega}{9} + \frac{4}{3} \right) F^2 + 2HF + \frac{2a}{3} (HH' + HF') - \frac{8\omega}{9} m^2 = 0 \]  \hspace{1cm} (25)

\[ H^2 - \frac{\omega}{3} F^2 - \omega HF + a \left( \frac{HH'}{2} - \frac{\omega}{3} HF' \right) - \frac{\omega}{3} m^2 = 0 \]  \hspace{1cm} (26)
To investigate a perturbative solution $H = H_0 + H_1 a^n + ...$ and $F = F_0 + F_1 a^n + ...$ are taken where dots denote terms of $a^{2n}$ and higher, and $n$ is determined from the above equations. The equations below are obtained.

\[
a^{2n} \left[ H_1^2 \left( 1 + \frac{n}{2} \right) - F_1^2 \frac{\omega}{3} \right] - \omega H_1 F_1 \left( 1 + \frac{n}{3} \right) \]
\[+ a^n \left[ H_0 H_1 \left( 2 + \frac{n}{2} \right) - \frac{2\omega}{3} F_0 F_1 - \omega H_0 F_1 \left( 1 + \frac{n}{3} \right) - \omega H_1 F_0 \right] + \]
\[\left[ H_0^2 - \frac{\omega}{3} F_0^2 - \omega H_0 F_0 - \frac{\omega}{3} m^2 \right] = 0 \quad (27)\]

\[
a^{2n} \left[ \frac{H_1^2}{4} (2 + n) + \frac{F_1^2}{2} \left( \frac{\omega}{3} + 1 \right) + \frac{H_1 F_1}{4} (3 + n) \right] + \]
\[a^n \left[ F_0 F_1 \left( \frac{\omega}{3} + 1 \right) + H_0 H_1 \left( 1 + \frac{n}{4} \right) + \frac{H_0 F_1}{4} (3 + n) + H_1 F_0 \frac{3}{4} \right] + \]
\[\left[ \frac{H_0^2}{2} + \frac{F_0^2}{2} \left( \frac{\omega}{3} + 1 \right) + H_0 F_0 \frac{3}{4} - \frac{\omega}{3} m^2 \right] = 0 \quad (28)\]

Setting the coefficients of $a^n$ equal to zero we obtain

\[
H_0 H_1 \left( 2 + \frac{n}{2} \right) - \frac{2\omega}{3} F_0 F_1 - \omega H_0 F_1 \left( 1 + \frac{n}{3} \right) - \omega H_1 F_0 = 0 \quad (29)\]

\[
F_0 F_1 \left( \frac{\omega}{3} + 1 \right) + H_0 H_1 \left( 1 + \frac{n}{4} \right) + \frac{H_0 F_1}{4} (3 + n) + H_1 F_0 \frac{3}{4} = 0 \quad (30)\]

$H_0 = cF_0$ is inserted, then

\[
H_1 \left( 2 - \frac{\omega}{c} + \frac{n}{2} \right) - F_1 \left( \frac{2\omega}{3c} + 1 + \frac{\omega}{c} \right) = 0 \quad (31)\]

\[
H_1 \left( 1 + \frac{3}{4c} + \frac{n}{4} \right) + F_1 \left( \frac{\omega}{3c} + \frac{1 + \frac{3}{4} + \frac{n}{4}}{c} \right) = 0 \quad (32)\]
are obtained. For a nontrivial solution the determinant of the coefficients must vanish and two solutions are obtained.

\[ H_1 = -\frac{2F_1}{3} \] (33)

and

\[ H_1 = \frac{2\omega F_1}{3}. \] (34)

For the solution \( H_1 = -\frac{2F_1}{3} \), \( n = -\frac{3\omega + 1}{\omega + 1} \) is obtained from (20) and (21) whereas for \( H_1 = \frac{2\omega F_1}{3} \), \( n = -\frac{4\omega + 5}{\omega + 1} \) is obtained. Since standard radiation dominated cosmology with cosmological constant is \( H = H_0 + H_1 a^{-4} \) only the solution \( F_1 = \frac{3H_0}{2\omega} \), \( n = -\frac{4\omega + 5}{\omega + 1} \approx -4 \) is considered.

Then we move one step further and we search for higher order terms. The equations, \( H = H_0 + H_1 a^n + H_2 a^{2n} + ... \) and \( F = F_0 + F_1 a^n + F_2 a^{2n} + ... \) are used in (25) and (26). Two equations which contains \( a^{2n}, a^{3n}, a^{4n} \) are obtained and only the lowest order \( a^{2n} \) are taken from these equations.

\[ a^{2n} \left[ \frac{4\omega^2}{9} F_1^2 \left( \frac{4}{3} + \frac{2n}{3} \right) + F_1^2 \left( \frac{4}{3} + \frac{4\omega}{9} \right) + (2\omega + 2) H_2 F_0 \left( \frac{4n}{3} + \frac{8}{3} \right) \right. \]

\[ + F_2 F_0 \left( \frac{8}{3} + \frac{8\omega}{9} \right) + (2\omega + 2) F_2 F_0 \left( \frac{4n}{3} + 2 \right) + \frac{2\omega}{3} F_1^2 \left( \frac{2n}{3} + 2 \right) \]

\[ + 2H_2 F_0 = 0 \] (35)

\[ a^{2n} \left[ \frac{4\omega^2}{9} F_1^2 \left( 1 + \frac{n}{2} \right) - \frac{\omega}{3} F_1^2 + (2\omega + 2) H_2 F_0 (2 + n) - \frac{2\omega}{3} F_2 F_0 \right. \]

\[ - (2\omega + 2) F_2 F_0 \left( \omega + \frac{2\omega n}{3} \right) - \frac{2\omega}{3} F_1^2 \left( \omega + \frac{\omega n}{3} \right) - \omega H_2 F_0 \right] = 0 \] (36)

(35) and (36) are separated in parentheses of \( H_0 \) and \( F_0 \).
\[ \left( \frac{4}{3} H_2 + \frac{2n}{3} H_2 + F_2 + \frac{2n}{3} F_2 \right) H_0 + \left( \frac{4\omega}{9} F_2 + \frac{4}{3} F_2 + H_2 \right) F_0 = \left( -\frac{2}{3} H_1^2 - \frac{2\omega}{9} F_1^2 - \frac{2}{3} F_1^2 - H_1 F_1 - \frac{n}{3} H_1^2 - \frac{n}{3} H_1 F_1 \right) \] (37)

\[ \left( 2H_2 + nH_2 - \omega F_2 - \frac{2\omega n}{3} F_2 \right) H_0 + \left( -\frac{2\omega}{3} F_2 - \omega H_2 \right) F_0 = \left( -H_1^2 + \frac{\omega}{3} F_1^2 + \omega H_1 F_1 - \frac{n}{2} H_1^2 + \frac{\omega n}{3} H_1 F_1 \right) \] (38)

The results for $H_2$ and $F_2$ are obtained as below where terms which are lower order in $\omega$ are neglected.

\[ H_2 \approx -\frac{H_2}{H_0} \left[ \frac{1}{2} + \frac{6}{5\omega} \right] \] (39)

and

\[ F_2 \approx -\frac{F_2}{H_0} \frac{\omega}{3} = -\frac{3H_2}{4\omega H_0}. \] (40)

To obtain the Friedmann equation for radiation dominated era the result for $H$ is written as

\[ H = H_0 + H_1 a^{-4} - \frac{H_1^2}{H_0} \left[ \frac{1}{2} + \frac{6}{5\omega} \right] a^{-8} + ... \] (41)

which gives

\[ H^2 = H_0^2 + 2H_0 H_1 a^{-4} - \frac{12H_1^2}{5\omega} a^{-8} + ... \] (42)

For the radiation dominated era we thus obtain a correction to the standard result obtained when Einstein-Hilbert action with cosmological constant is used. This is important since BBN depends critically on how the scale parameter ($a$) changes as a function of time during the radiation dominated era. The conditions for the constant term and the $a^{-8}$ term to be negligible give
which is satisfied since $\omega \gg 10^4$. Note that $H_0$ is given by (24) whereas $H_1$ is a free parameter for our solution. Furthermore our method gives a way to calculate the corrections to the standard model which we have shown to be negligible.

III. CONCLUSION

JBD theory with $m = 0$ admits the same radiation dominated solution as classical Einstein solutions. It is important to extend this solution to the $m \neq 0$ case since this case corresponds to dark energy. By using a power series approach to obtain the Friedmann equation we have shown that there are two corrections to the standard $a^{-4}$ term. As expected, one of them is the constant term which corresponds to dark energy. The other is the $a^{-8}$ term which follows from the power series expansion. We have shown that both terms may be chosen to be small during the radiation dominated era. In fact from equation (43) it can be seen that if the scale size $a$ changes by a factor of $f$ during the radiation dominated era then the Brans-Dicke parameter $\omega$ must be greater than $f^4$. Our results are important because they say that JBD theory can explain dark energy without contradicting the successful BBN predictions of the standard cosmological model.

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