Comment on “A study of phantom scalar field cosmology using Lie and Noether symmetries” [Int. J. Mod. Phys. D 25 (2016) 1650051]

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We show that the recent results of [Int. J. Mod. Phys. D 25 (2016) 1650051] on the application of Lie/Noether symmetries in scalar field cosmology are well-known in the literature while the problem could have been solved easily under a coordinate transformation. That follows from the property, that the admitted group of invariant transformations of dynamical system is independent on the coordinate system.

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In[1] the authors consider the action

\[ S = \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} \lambda(\phi) g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) \right] + S_m \]

where \( S_m \), corresponds to the matter source, and \( \lambda(\phi) \) is an unknown function in a spatially flat Friedmann–Robertson–Walker (FRW) spacetime with signature \((-++,++)\), scale factor \( a(t) \), and a perfect fluid with constant equation of state parameter, \( p = (\gamma - 1) \rho \). From this action for the lapse time \( N(t) = 1 \), and for the comoving observers, \( u^a = \delta^a_t \), it follows that the Lagrangian of the field equations is

\[ L \left( a, \dot{a}, \phi, \dot{\phi} \right) = -3a \dot{a}^2 + \frac{1}{2} \lambda(\phi) a^3 \dot{\phi}^2 - a^3 V(\phi) - \rho_m a^{3(1-\gamma)} \]

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while the corresponding field equations are the “Hamiltonian” function, which is the first Friedmann’s equation

\[ 3a\dot{a}^2 - \frac{1}{2} \lambda(\phi) a^3 \dot{\phi}^2 - a^3 V(\phi) - \rho_m a^{3(1-\gamma)} = 0, \]

(3)

and the Euler-Lagrange equations of Lagrangian \( \mathcal{L} \) for the variables \( \{ a, \phi \} \). In [1] the authors claim that the existence of Lie point symmetries for the field equations or Noether point symmetries for the Lagrangian \( \mathcal{L} \) result in constraints which provide the unknown parameters of the model, that is \( \gamma, V(\phi) \) and the function \( \lambda(\phi) \). The purpose of this short note is to show that this is not true and the correct answer is that the only parameters which can be constrained are \( \gamma \), and \( V(\phi) \), while the last will be \( V(\lambda(\phi)) \), or more precisely, \( V(\int \sqrt{\lambda(\phi)} d\phi) \).

First we note that the Lie algebra of the Lie/Noether symmetries of a dynamical system are independent on the coordinate system. Therefore if in the Lagrangian \( \mathcal{L} \) we define the new field \( \psi \), such as

\[ d\psi = \sqrt{\lambda(\phi)} d\phi \]

we obtain

\[ L(a, \dot{a}, \psi, \dot{\psi}) = -3a\dot{a}^2 + \frac{1}{2} a^3 \dot{\psi}^2 - a^3 V(\psi) - \rho_m a^{3(1-\gamma)} \]

(4)

where \( V(\psi) = V\left( \int \sqrt{\lambda(\phi)} d\phi \right) \). This is the classical Lagrangian of a minimally coupled scalar field cosmological model in a spatially flat FRW spacetime. Furthermore in the case in which \( \psi \to i\psi \), or \( \lambda(\phi) < 0 \), we have the Lagrangian of a phantom field.

Hence the symmetry analysis of [2] or [1] will give the same results, however of a different form of the potentials \( V(\psi), V(\phi) \), but which they will be related under the transformation \( \phi \to \psi \). Furthermore, if the latter transformation is not complex, the solution for the scale factor will be exactly the same.

Concerning the application of Noether symmetries of the gravitational Lagrangian \( \mathcal{L} \), the potential \( V(\phi) \) in [1] the authors of [2] find is the Unified Dark Matter potential (UDM) (for instance see [3–5]). For instance, for \( \lambda(\phi) = \frac{\lambda_0}{\phi^2} \), in [2], they have found the potentials

\[ V(\phi) = V_0 \sinh^2 \left( \frac{3}{8} |\lambda_0| \ln \phi + p_1 \right), \quad \lambda_0 > 0 \]

(5)

\[ V(\phi) = V_0 \sin^2 \left( \frac{3}{8} |\lambda_0| \ln \phi + p_1 \right), \quad \lambda_0 < 0 \]

(6)

which under the coordinate transformation \( \psi = \ln \phi \), which is the same result when \( \lambda(\phi) = \lambda_0 \), i.e. a constant, becomes

\[ V(\psi) = V_0 \sinh^2 \left( \frac{3}{8} \psi + p_1 \right) \]

(7)

\[ V(\psi) = V_0 \sin^2 \left( \frac{3}{8} \psi + p_1 \right). \]

(8)

We would like to remark that the application of the complete Noether’s theorem in scalar field cosmology can be found in [2] and the application of Lie point symmetries in [2] hence the results of [2] are not new in the literature.
Furthermore we wish to draw attention to the paper by Capozziello et al.\(^8\) on Scalar-tensor theories in which an extended discussion on the application of Noether symmetries in cosmology is presented; which includes also the results on Noether symmetries of\(^1\)

Finally we refer the authors of\(^1\) to the original work on symmetries of differential equations of S. Lie\(^9\) and its application to the Action Integral which has been done by E. Noether.\(^10\)

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