Vortex proliferation in the Berezinskii-Kosterlitz-Thouless regime on a
two-dimensional lattice of Bose-Einstein condensates

V. Schweikhard, S. Tung, and E. A. Cornell

JILA, National Institute of Standards and Technology and University of Colorado,
and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440

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We observe the proliferation of vortices in the Berezinskii-Kosterlitz-Thouless regime on a
two-dimensional array of Josephson-coupled Bose-Einstein condensates. As long as the Josephson (tunneling) energy \( J \) exceeds the thermal energy \( T \), the array is vortex-free. With decreasing \( J/T \), vortices appear in the system in ever greater numbers. We confirm thermal activation as the vortex formation mechanism and obtain information on the size of bound vortex pairs as \( J/T \) is varied.

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One of the defining characteristics of superfluids is
long-range phase coherence [1], which may be destroyed
by quantum fluctuations, as in the Mott-insulator transition [2, 3], or thermal fluctuations, e.g. in
one-dimensional Bose gases [4, 5] and in a double-well system [6]. In two dimensions (2D), Berezinskii [7], Kosterlitz and Thouless [8] (BKT) developed an elegant description of thermal phase fluctuations based on the unbinding of vortex-antivortex pairs, i.e. pairs of vortices of opposite
circulation. The BKT picture applies to a wide variety
of 2D systems, among them Josephson junction arrays (JJA), i.e. arrays of superfluids in which phase coherence is mediated via a tunnel coupling \( J \) between adjacent sites. Placing an isolated (free) vortex into a JJA is thermodynamically favored if its free energy \( F \) is smaller than the entropy gain \( S \) of vortex-antivortex pairs due to the available vortex sites. In an array of period \( d \) the vortex energy diverges with ar-

temperature is varied. We then transform this system into
a Bose-condensed sample of \(^{87}\text{Rb} \) atoms in a harmonic, ax-

cial symmetric magnetic trap with oscillation frequen-
cies \( \{\omega_x, \omega_y, \omega_z\} = 2\pi \{6.95, 15.0\} \text{ Hz} \). The number of condensed atoms is kept fixed around \( 6 \times 10^5 \) as the temperature is varied. We then transform this system into
a Josephson junction array, as illustrated in Fig. 1. In
a 10 s linear ramp, we raise the intensity of a 2D hexagonal optical lattice [10] of period \( d = 4.7 \mu m \) in the \( x-y \) plane. The resulting potential barriers of height \( V_{OL} \) be-
tween adjacent sites [Fig. 1(b)] rise above the condensate’s chemical potential around \( V_{OL} \approx 250 - 300 \text{ Hz} \), splitting it into an array of condensates which now communicate

\[ \text{FIG. 1: Experimental system. (a) 2D optical lattice intensity profile. A lattice of Josephson-coupled BECs is created in the white-shaded area. The central box marks the basic building block of our system, the double-well potential shown in (b). The barrier height \( V_{OL} \) and the number of condensed atoms per well, \( N_{well} \), control the Josephson coupling \( J \), which acts to lock the relative phase \( \Delta \phi \). A cloud of uncondensed atoms at temperature \( T \) induces thermal fluctuations and phase defects in the array when \( J < T \). (c) Experimental sequence: A BEC (i) is loaded into the optical lattice over 10 s, suppressing \( J \) to values around \( T \). We allow 2 s for thermalization. To probe the system, we ramp off the lattice on a faster timescale \( t_r \) and take images of the recombined condensate. When \( J \) is reduced below \( T \), (ii)-(vii), vortices (dark spots) appear as remnants of the thermal fluctuations in the array.} \]
only through tunneling. This procedure is adiabatic even with respect to the longest-wavelength phonon modes of the array \[18,19\] over the full range of \(V_{OL}\) in our experiments. Each of the \(\approx 190\) occupied sites (15 sites across the BEC diameter \(2 \times R_{TF} \approx 68 \mu m\) \[20\]) now contains a macroscopic BEC, with \(N_{well} \approx 7000\) condensed atoms in each of the central wells at a temperature \(T\) that can be adjusted between \(30 - 70 nK\). By varying \(V_{OL}\) in a range between \(500 Hz\) and \(2 kHz\) we tune \(J\) between \(1.5 \mu K\) and \(5 nK\), whereas the “charging” energy \(E_c\), defined in \[1\], is on the order of a few \(pK\), much smaller than both \(J\) and \(T\). In this regime, thermal fluctuations of the relative phases \(\Delta \phi_{TR} \approx \sqrt{T/J}\) are expected, while quantum fluctuations \(\Delta \phi_Q \approx (E_c/4J)^{1/4}\) are negligible \[1\].

The suppression of the Josephson coupling greatly suppresses the energy cost of phase fluctuations in the x-y plane, between condensates, \(J[1 - \cos(\Delta \phi)]\), compared to the cost of axial (z) phase fluctuations inside the condensates \[21\]. As a result, axial phase fluctuations remain relatively small, and each condensate can be approximated as a single-phase object \[22\].

After allowing \(2 s\) for thermalization, we initiate our probe sequence. We first take a nondestructive thermometry image in the x-z plane, from which the temperature \(T\) and, from the axial condensate size \(R_z\), the number of condensed particles per well, \(N_{well}\), is obtained (see below). To observe the phase fluctuations we then turn down the optical lattice on a time-scale \(t_r\) \[23\], which is fast enough to trap phase winding defects, but slow enough to allow neighboring condensates to merge, provided their phase difference is small. Phase fluctuations are thus converted to vortices in the reconnected condensate, as has been observed in the experiments of Scherer et al. \[24\]. We then expand the condensate by a factor of 6 and take a destructive image in the x-y plane.

Figure \[1(c)\] illustrates our observations: (ii)-(vii) is a sequence of images at successively smaller \(J/T\) (measured in the center of the array \[25\]). Vortices, with their cores visible as dark “spots” in (iii)-(vii), occur in the BEC center around \(J/T = 1\). Vortices at the BEC edge appear earlier, as here the magnetic trap potential adds to the tunnel barrier, suppressing the local \(J/T\) below the quoted value. That the observed “spots” are indeed circulation-carrying vortices and antivortices is inferred from their slow \(\approx 100 ms\) decay after the optical lattice ramp-down, presumably dominated by vortex-antivortex annihilation. From extensive experiments on vortices in our system we know that circulation-free “holes” fill so quickly due to positive mean field pressure, that they do not survive the pre-imaging expansion. Vortices with identical circulation would decay by dissipative motion to the BEC edge, in our trap over \(\gtrsim 10 s\).

To investigate the thermal nature of phase fluctuations, we study vortex activation while varying \(J\) at different temperatures. For a quantitative study, accurate parameter estimates are required. The Josephson-coupling energy \(J\) is obtained from 3D numerical simulations of the Gross-Pitaevskii equation (GPE) for the central double-well system \[6, 26\] [Fig \[1(b)\]], self-consistently including mean-field interactions of both condensed and uncondensed atoms \[27\]. A useful approximation for \(J\) in our experiments is \[25\]: \(J(V_{OL}, N_{well}, T) \approx N_{well} \times 3.15 \times 10^6 nK \exp[N_{well}/3950 - \sqrt{V_{OL}/244Hz}](1 + 0.59 T/100nK)\). The finite-\(T\) correction to \(J\) arises from both the lifting-up of the BEC’s chemical potential and the axial compression by the thermal cloud’s repulsive mean field, but does not take into account the effects of phase fluctuations on \(J\) (in condensed-
matter language, we calculate the bare $J$). $N_{\text{well}}$ is determined by comparison of the experimentally measured $R_s$, to $R_s(V_{\text{OL}}, N_{\text{well}}; T)$ obtained from GPE simulations. Both experimental and simulated $R_s$ are obtained from a fit to the distribution of condensed and uncondensed atoms, to a Thomas-Fermi profile plus mean-field-modified Bose function [27]. In determination of all $J$ values, there is an overall systematic multiplicative uncertainty $\Delta J/J \approx 1.6$, dominated by uncertainties in the optical lattice modulation contrast, the absolute intensity calibration, and magnification in the image used to determine $N_{\text{well}}$. In comparing $J$ for “hot” and “cold” clouds (see Fig. 2) there is a relative systematic error of 15% associated with image fitting and theory uncertainties in the thermal-cloud mean-field correction to $J$.

The qualitative results of our work are consistent whether we use an automated vortex-counting routine or count vortices by hand, but the former shows signs of saturation error at high vortex density, and the latter is vulnerable to subjective bias. As a robust vortex-density surrogate we therefore use the “roughness” $\Delta$ of the condensate image caused by the vortex cores. Precisely, we define $\Delta$ as the normalized variance of the measured column density profile from a fit to a smooth finite-$T$ Bose profile [27], with a small constant offset subtracted to account e.g. for imaging noise. To limit spatial inhomogeneity in $J$, caused by spatially varying condensate density and optical lattice intensity, to $<10\%$, $\Delta$ is extracted only from the central 11% of the condensate area which contains 20 lattice sites [Fig. 2(a)]. Comparison to automated vortex-counts shows that $\Delta$ is roughly linear in the observed number of vortices, irrespective of the sign of their circulation, with a sensitivity of $\approx 0.01$/vortex.

Figure 2 shows results of our quantitative study. In Fig. 2(b), we plot $\Delta$ vs $J$ for two datasets with distinct temperatures. At large $J \geq 200$ nK a background $\Delta \lesssim 0.01$ is observed, that is not associated with vortices, but due to residual density ripples remaining after the optical lattice ramp-down. Vortex proliferation, signaled by a rise of $\Delta$ above $\approx 0.01$, occurs at $J \approx 100$ nK for “hot” BECs and at a distinctly lower $J \approx 50$ nK for “cold” BECs [confirmed by the averaged data shown in Fig. 2(c)], indicating thermal activation as the vortex formation mechanism. Plotting the same data vs $J/T$ in Fig. 2(d) shows collapse onto a universal vortex activation curve, providing strong evidence for thermal activation. A slight residual difference becomes visible in the averaged “cold” vs “hot” data [Fig. 2(e)], perhaps because of systematic differences in our determination of $J$ at different temperatures.

The vortex density $D$ by itself provides no distinction between bound vortex-antivortex pairs and free vortices. In the following we exploit the flexibility of optical potentials to distinguish free or loosely bound vortices from tightly bound vortex-antivortex pairs. A “slow” optical lattice ramp-down allows time for tightly bound pairs to annihilate before they can be imaged. By slowing down the ramp-down duration $\tau$ [inset of Fig. 3(a)], we therefore selectively probe vortices at increasing spatial scales. This represents an attempt to approach the “true” BKT vortex unbinding crossover that is complementary to transport measurements employed so successfully in superconductive and liquid Helium systems.

Figure 3(a) shows vortex activation curves, probed with two different ramp-down times. Two points are worth noticing: First, a slow ramp compared to a fast one shows a reduction of the vortex density $D_\tau$ in arrays with fully randomized phases at low $J/T$. The difference

![Figure 3](image-url)
vortex activation at lower \((J/T)\))50\%, confirming that free or very loosely bound vortices occur only at higher \(T\) (lower \(J\)). Specifically, the data clearly show a range around \(J/T \approx 1.4\) where only tightly bound pairs exist. Figure 3(b) quantitatively shows the shift of \((J/T)\)50\% from 1.4 to 1.0 with slower ramp time. We can make a crude mapping of the experimental ramp-down timescale to theoretically more accessible vortex-antivortex pair sizes as follows: In Fig. 3(c), we see the decrease of the saturated (low-\(J/T\)) vortex density \(D_c\) with increasing ramp timescale \(\tau\). The right axis shows the inferred number of vortices that survived the ramp. We compare this number of surviving vortices to simulations 25 of a 20-site hexagonal array with random phases. In these simulations we find, on average, a total of 10 vortices, 6 of which occur in nearest-neighbor vortex-antivortex pairs [configuration I in Fig. 3(c)], 1.7 (0.4) occur in configuration II (III) respectively, and 1.9 occur in larger pairs or as free vortices. Experimentally \(\approx 11\) vortices are observed for the fastest ramps, in good agreement with the expected total number of vortices. For just somewhat slower ramps of \(\tau \approx 5\) ms, only 3 vortices survive, consistent with only vortices in configuration II & III or larger remaining (indicated in Fig. 3 top axis) 25. For \(\tau \gtrsim 30\) ms ramps less than 2 vortices remain, according to our simulations spaced by more than 2d/\(\sqrt{3}\). Thus we infer that ramps of \(\tau \approx 30\) ms or longer allow time for bound pairs of spacing \(\lesssim 2d/\sqrt{3}\) to decay before we observe them. The downward shift of \((J/T)\)50\% in Fig. 3(b) thus tells us that loosely bound pairs of size larger than 2d/\(\sqrt{3}\), or indeed free vortices, do not appear in quantity until \(J/T \leq 1.0\), whereas more tightly bound vortex pairs appear in large number already for \(J/T \leq 1.4\).

A further interesting observation concerns the width of the vortex activation curve. The relative width, determined from fits to data such as the ones shown in Fig. 3(a), is \(\Delta(J/T)\)27−73/(J/T)50\% \(\approx 0.3\), independent of ramp-down duration. This width is neither as broad as in a double-well system \(6,28\), where the coherence factor rises over a range \(\Delta(J/T)\)27−73/(J/T)50\% \(\approx 1.4\), nor as broad as expected from our simulations 25 of an array of uncoupled phases, each fluctuating independently with \(\Delta \phi_{RMS} = \sqrt{T/J}\), for which we find \(\Delta(J/T)\)27−73/(J/T)50\% \(\approx 0.85\). Presumably collective effects in the highly multiply connected lattice narrow the curve. On the other hand, the width is 3 times larger than the limit due to spatial inhomogeneity in \(J\), suggesting contributions to the width due to finite-size effects or perhaps revealing the intrinsically smooth behavior of vortex activation in the BKT regime.

In conclusion, we have probed vortex proliferation in the BKT regime on a 2D lattice of Josephson-coupled BECs. Allowing variable time for vortex-antivortex pair annihilation before probing the system provides a time-to-length mapping, which reveals information on the size of pairs with varying \(J/T\). We acknowledge illuminating conversations with Leo Radzihovsky and Victor Gurarie. This work was funded by NSF and NIST.

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tions are ∈ (0, π), or if all are ∈ (−π, 0).

[29] The very short annihilation time of configuration I pairs
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d/√3 = 2.8 μm is comparable to the diameter of a vortex
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