Simplified PMT Model

Karim Zbiri*

*Department of physics, Drexel University, 3141 Chestnut St, Philadelphia, PA 19104, USA.

Abstract

A simplified model is proposed based on the characteristics of the photomultiplier tube (PMT). The Model is compared to the available data, and it succeeds to reproduce the saturation profile of the PMT. The model also adds clarification about how the PMT works.

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1 Introduction

At Drexel University, I was testing the 8-inch, 13 stages, R1408 PMTs which will be used for the inner vetos of the Double Chooz experiment [1]. These PMTs were used in the IMB [2] and SuperKamiokande [3] experiments. This testing allowed a better understanding of their working. In this paper I propose a model inspired from observations during the testing period. The first aim of this model is to explain in simple way the saturation characteristics of the PMT. The Model gives a global description of how the different PMTs characteristics, like the quantum efficiency (QE) or the collectivity efficiency (CE), are linked together to the PMT gain (G). The model is compared to the data obtained from three different PMTs, which have different single photoelectron (spe) distributions.

2 The general case

The basic idea is at a fixed applied high voltage, the PMT works as charge amplifier, which will amplify below a certain charge limit $q_{sat}$ and beyond this
limit the PMT will continue to amplify the signal but at lower gain. As the signal in the input of the PMT is increased, the PMT gain decreases.

A general profile which matches this description of the gain variation is given by:

\[ G = \frac{G_0}{\sqrt{1 + q^\alpha / q_{sat}^\alpha}} \]  

(1)

Where \( \alpha \) is a real coefficient which can be determined from the comparison with the data, \( q \) is the collected charge at the anode and \( G_0 \) is the gain at the limit as \( q \to 0 \), and depends only on the applied high voltage like following [4]:

\[ G_0 = kV^{\beta N} \]  

(2)

Where \( k \) is a constant, which depends on the material of the dynodes and the voltage division between them, \( N \) is the number of the dynodes and \( \beta \) is between 0.7 and 0.8. \( G_0 \) is usually set up by appropriate high voltage at low light level (single photoelectron level). The \( q_{sat} \) is an intrinsic constant to each PMT, and it depends only on the type of the voltage divider. \( q_{sat} \) fixes the amplification performance of the PMT at a given high voltage, and it determines the collectivity efficiency of the PMT. The amplification performance can be adjusted by the use of the appropriate voltage divider.

3 Comparisons of the model with the measurements

3.1 The R1408 PMT case

The 8-inch Hamamatsu R1408 PMT with 13 stages does not have a single photoelectron peak. Figure 5 shows the R1408 single photoelectron (spe) charge distribution. The main method to determine the gain \( G \) for this kind of PMTs is the photo-statistics method [5]. A series of LED/laser runs are taken at varying light levels, and for each run the statistical mean and variance of the spectra are measured as shown in Fig. 1, after that plot the variance versus the mean, and fit the linear region with a polynomial of the first order like in Fig. 2. The gain is extracted from the slope of the fit following the equation [5]:

\[ \frac{d\sigma^2}{dE} = 2eG \]  

(3)
Where $E$ and $\sigma^2$ are the mean and the variance of the ADC charge distribution, given by the following relations:

\begin{align*}
    E &= q + \mu_p \\
    \sigma^2 &= 2qeG + \sigma^2_p,
\end{align*}

with $e = 1.6 \times 10^{-19} C$ is the electron charge, $q$ is the mean charge at the input of the ADC. $\mu_p$ and $\sigma^2_p$ are the mean and the variance of the ADC pedestal distribution. One important feature of the photo-statistics method is that it is ADC pedestal independent.

The R1408 PMT was put under high voltage inside a dark box, illuminated by an LED driven by a fast pulser. The trigger output of the pulser was sent to Lecroy 222 Gate and Delay units, first to set a delay, and then to produce a 200 ns gate which enclosed the PMT pulse that occurred when the LED was flashed, the using setup is shown in Fig. 3. The PMT signal was sent to the input of a Lecroy 2249W ADC, while the gate was coming from the 200 ns gate and delay output. The LED was flashed at 20 Hz at a variety of driving voltages from 1V to 8V, all with very narrow (20 ns) driving pulse widths. In all cases, the PMT pulse was observed on an oscilloscope to be completely contained within the 200ns ADC gate. For each of the ADC distributions, the mean and variance were calculated, as it discussed above, and the ADC variances were plotted as a function of the ADC means which allowed determination of the gain $G_0$. The value of $G_0$ was calculated from the linear region obtained at low LED driving voltages.

To compare the model to the data, the variance need to be written as function of the measured charge $q$. If the equation 5 is combined with the equation 1, we obtain the following relation:

\begin{equation}
    \sigma^2 = 2eG_0 \frac{q}{\sqrt{1 + \frac{q}{q_{sat}}}}.
\end{equation}

Because the equation 6 is the theoretic variance, it does not contain any ADC pedestal variance term.

For large $q$, the variance can be written like following:

\begin{equation}
    \sigma^2_{\infty} = 2eG_0 q_{sat}^{\frac{\alpha}{2}} q^{1-\frac{\alpha}{2}}.
\end{equation}

There are three cases of interest:

(1) $\alpha < 2 : \sigma^2_{\infty} \to \infty$. 

\[
\alpha = 2 : \sigma_\infty^2 \to 2eG_0q_{\text{sat}}.
\]

\[
\alpha > 2 : \sigma_\infty^2 \to 0.
\]

Figure 4 shows the variation of the variance obtained from equation 6 as function of \( q \), for different values of \( \alpha \).

As shown in the Fig. 2 the variance shows a kind of saturation for high collected charge \( q \), which corresponds to the case \( \alpha = 2 \).

Figure 6 shows the comparison of the model with the data obtained from the R1408 PMT at different applied high voltages, this PMT has a linear (or equally distributed) voltage divider. The reference [4] has more details about the voltage dividers and their different types, especially chapter 5. The value of \( G_0 \) depends on the applied high voltage, and it is determined as explained above from the slope of the linear region. The value of \( q_{\text{sat}} \) is adjusted to reproduce the data, and the model reproduces the measurements with \( \alpha = 2 \).

### 3.2 Effect of the high voltage divider on the saturation level

To improve the saturation performance of the PMT, a tapered voltage divider [4] is used instead of the linear one, but the gain is expected to become lower for the same total applied high voltage. Details about the linear and tapered voltage divider, which were used during the measurements, will be given in a separate paper about the Double Chooz inner veto PMTs testing performed at Drexel University [5]. Figure 7 shows the comparison of the model with the data obtained from the measurements with the R1408 PMT at different applied high voltages, this time the PMT has a tapered voltage divider. As expected the tapered voltage divider increases the value of \( q_{\text{sat}} \), which became at least 4 times larger. In addition, The model reproduces the measurements with \( \alpha = 2 \).

I want conclude this section with one remark about the effect of the voltage divider. Since the tapered divider enhances the collectivity efficiency, the gain can be expected to become lower at the same applied high voltage the gain due to the increasing of the collected charge at the anode. The assumption will be true if the gain depends only on the collected charge at the anode. However as it proposed by the equation 1, the gain does not depend on the collected charge \( q \), but it depends on the ratio \( \frac{q}{q_{\text{sat}}} \). The tapered voltage divider will increase \( q \) and \( q_{\text{sat}} \) at the same time, then if they are increased in the same way, their ratio will remain constant and the gain will not be different from the one obtained with the linear voltage divider at the same applied total high voltage.
3.3 The R5912 PMT case

The 8-inch Hamamatsu R5912 PMT has a single photoelectron peak as shown in the Fig. 8. In order to apply the model to the case, a general formulation of the variance has to be found. The equation 5 can be written in a general way like following [5]:

\[ \sigma^2 = \gamma q e G + \sigma_p^2. \]  \hspace{1cm} (8)

\( \gamma \) is a factor depending on the single photoelectron charge distribution of the PMT. Since the interest of this paper is to explain the saturation profile of the PMT, it is enough to express the equation 6 like following:

\[ \sigma^2 = S \frac{q}{\sqrt{1 + \frac{q}{q_{sat}}}^\alpha}. \]  \hspace{1cm} (9)

where \( S = \gamma e G_0 \), \( S \) is the slope of the linear region when the variance is plotted as function of the measured charge \( q \).

Figure 9 shows the comparison of the model with the data obtained from the R5912 PMT, this PMT came already encapsulated with its own voltage divider. The procedure for the gain measurement is exactly the same as for the R1408 PMT like described above. The model reproduces the data with \( \alpha = 2 \) in this case, too.

3.4 The MCP-PMT case

In this section, I compare the model to the data obtained with a new kind of PMT, the MCP-PMT. The ref. [4] contains a general description of this type of PMTs. The data which I used for the comparison with the model is published in the ref. [6]. To compare to the data, the following equation is used:

\[ \frac{G}{G_0} = \sqrt{\frac{-1 + \sqrt{1 + 8 \frac{N^2}{N_{\gamma sat}^2}}}{4 \frac{N^2}{N_{\gamma sat}^2}}}. \]  \hspace{1cm} (10)

The equation 10 gives the gain variation as function of the number of the incident photons \( N_\gamma \). \( N_{\gamma sat} \) is the number of incident photons at which the PMT saturation occurs. \( N_{\gamma sat} \) is directly determined from the data as the value of \( N_\gamma \) which corresponds to the ratio \( \frac{G}{G_0} = \frac{1}{\sqrt{2}} \). The demonstration of
the equation 10 is made in the appendix. Figure 10 shows the results of the comparison. Also in this case, the model reproduces the data.

4 CONCLUSION

To describe some of the known properties of the photomultiplier tubes, a PMT model is proposed. The PMT characteristics like the quantum efficiency, the collectivity efficiency and the gain are linked together in natural way as demonstrated in the appendix. The model is successful to describe the saturation profile measured for three different PMTs with different single photoelectron charge distributions. The comparisons with the measurements show that the value of the coefficient $\alpha$ is 2. The effect of the voltage divider - on the gain variation and the saturation performance - is explained in the model by the variation of the ratio $\frac{q}{q_{sat}}$.

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APPENDIX: Formulation of the PMT characteristics

The collected Charge at the anode and the gain:

The collected charge at the anode can be expressed as function of the number of the photoelectrons, which are collected at the first dynode $\mu$ and the gain $G$ as following:

$$q = \mu Ge,$$

(A-1)

Where $e = 1.6 \times 10^{-19} C$ is the electron charge.

Now, if the equation 1 with $\alpha = 2$ is taken in account, the following equation will be obtained:

$$\frac{e^2 \mu^2}{q_{sat}} G^4 + G^2 - G_0^2 = 0.$$  

(A-2)
The equation A-2 can be considered as the master equation of the PMT, which links the observables of the PMT to each other, the quantum efficiency via $\mu$, the collectivity efficiency represented by $q_{sat}$ and the gain $G$.

The equation A-2 has like solution the following expression:

$$ eG = \sqrt{-1 + \sqrt{1 + \frac{4e^2G_0^2}{q_{sat}^2}\mu^2} - \frac{2\mu^2}{q_{sat}}} \quad (A-3) $$

The equations A-1 and A-3 let to express the collected charge at the anode, like following:

$$ q = \sqrt{-1 + \sqrt{1 + \frac{4e^2G_0^2}{q_{sat}^2}\mu^2} - \frac{2\mu^2}{q_{sat}}} \quad (A-4) $$

In the case of the cathode current saturation, $\mu$ becomes constant which makes the collected charge at the anode and the gain constants, too.

**The quantum efficiency:**

If the equations 1 and A-1 are put together, $\mu$ can be expressed as function of the collected charge at the anode, $q$, as following:

$$ \mu = \sqrt{q^2(1 + \frac{q^2}{q_{sat}^2})} \quad eG_0 \quad (A-5) $$

$\mu$ is linked to the number of incident photons, $N_\gamma$, by the following relation:

$$ \mu = \rho\eta N_\gamma \quad (A-6) $$

Where $\rho$ is the collectivity efficiency between the photocathode and the first dynode, $\eta$ is the quantum efficiency of the PMT.

The equations A-5 and A-6 let to express the product $\rho\eta$ as following:

$$ \rho\eta = \sqrt{q^2(1 + \frac{q^2}{q_{sat}^2})} \quad eG_0N_\gamma \quad (A-7) $$
The number of photoelectrons at the saturation:

For $q = q_{\text{sat}}$, the equation A-5 gives the number of photoelectrons at the saturation, $\mu_{\text{sat}}$, collected at the first dynode:

$$\mu_{\text{sat}} = \sqrt{2} \frac{q_{\text{sat}}}{eG_0},$$  \hspace{1cm} (A-8)

$\mu_{\text{sat}}$ characterizes the saturation level of the PMT at a given high voltage. If the equation 2 is used with the equation A-8, $\mu_{\text{sat}}$ can be expressed as function of the applied high voltage:

$$\mu_{\text{sat}} = \sqrt{2} \sqrt{\frac{q_{\text{sat}}}{eV\beta N}}.$$  \hspace{1cm} (A-9)

Since the value of $q_{\text{sat}}$ depends only on the type of the voltage divider, the number of the photoelectrons at the saturation decreases with increasing applied high voltage for a given PMT with a given voltage divider.

Using the equations A-3, A-6 and A-8, the gain can be written as function of the number of incident photons $N_\gamma$:

$$G = \frac{G_0}{\sqrt{-1 + \sqrt{1 + 8 \frac{N_\gamma^2}{N_{\gamma_{\text{sat}}}^2}}}}\frac{4 \frac{N_\gamma^2}{N_{\gamma_{\text{sat}}}^2}}{\frac{N_\gamma^2}{N_{\gamma_{\text{sat}}}^2}},$$  \hspace{1cm} (A-10)

Where $N_{\gamma_{\text{sat}}} = \mu_{\text{sat}} \rho_\gamma$.

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Fig. 1. ADC distribution of the PMT pulses for LED flashing with 3V from pulser.

Fig. 2. ADC variance($\sigma^2$) versus ADC mean($\bar{q}$) for a variety pulse heights. The slope of the fit of the linear region gives a gain/channel=slope/2=24.9 channel/photoelectron.
Fig. 3. The test setup scheme.

Fig. 4. The ratio $\sigma^2_e G_0$ as function of the mean of the collected charge at the anode $\bar{q}$, as given by the equation 6 with $q_{sat} = 500$. 
Fig. 5. The single photoelectron (spe) charge distribution for the 8-inch, 13 stages, R1408 PMT. The ADC pedestal is not removed. The signal from the PMT output is amplified 10 times.
Fig. 6. Comparison of the model with the data obtained at different high voltages, $q_{sat}$ and $eG_0$ are in ADC channels. The stars represent the data, and the dashed line represents the model with $\alpha = 2$ for all.
Fig. 7. Comparison of the model with the data obtained at different high voltages, $q_{sat}$ and $eG_0$ are in ADC channels. The stars represent the data, and the dashed line represents the model with $\alpha = 2$ for all.
Fig. 8. The single photoelectron (spe) charge distribution for the 8-inch R5912 PMT. The ADC pedestal is not removed. The signal from the PMT output is amplified 10 times.
Fig. 9. Comparison of the model with the data obtained at different high voltages, $q_{sat}$ and $S$ are in ADC channels. The stars represent the data, and the dashed line represents the model with $\alpha = 2$ for all.
Fig. 10. Relative gain $\frac{G}{G_0}$ vs photon rate $N_\gamma$. The dashed line represents the model (i.e. the equation 10 with $N_{\gamma_{\text{sat}}} = 2.5 \times 10^7 \text{ count/cm}^2/\text{s}$), and the stars represent the data from the ref. [6].