Grand unification in the heterotic brane world

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Based on:
M. Blaszczyk, S. Groot Nibbelink, M. Ratz, F. Ruehle, M. Trapletti and P. V.: arXiv:09xx.xxxx
Outline

Motivation

Orbifold MSSMs
- $\mathbb{Z}_6$-II Mini-Landscape
- Full Blow-up

Non-local GUT Breaking
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold
- Example
- vacuum search
- $B - L$ and matter parity

Conclusion
Motivation

- Heterotic string theory: $E_8 \times E_8$ gauge group in 10D
- Aim: connection to observable world; MSSM
- Compactify six spatial dimensions on a compact space (e.g. 6-torus)
- 6D Orbifolds: compact space; flat like torus, except for some singularities (called fixed points)
- In this talk focus on $\mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_6$-II orbifolds
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Orbifold MSSMs
\( \mathcal{O}(100) \) \( \mathbb{Z}_6 \)-II orbifold models with

- \( \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \) times hidden sector
- 3 generations of quarks and leptons + vector-like exotics
- exotics decouple
- (potentially) realistic flavor structure, e.g. heavy top

O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. V., A. Wingerter 2006, 2007

- see talk by S. Ramos-Sanchez
Can these models be obtained from a CY construction?

⇒ No, at least not easily!

\(\mathbb{Z}_6\)-II Mini-Landscape at special (symmetry enhanced) point in moduli space:

- Wilson line breaks GUT to SM (locally) at fixed points
- In full blow-up, SM gauge group (e.g. hypercharge) broken at these fixed points
- (fixed points with only SM charged states ⇒ blow-up mode breaks SM)

S. Groot Nibbelink, J. Held, F. Ruehle, M. Trapletti, P. V 2009

Important: full blow-up of Mini-Landscape models not necessary
Can MSSM orbifold models have a corresponding CY description in principle?

or

Can MSSM orbifold models be blown-up completely?
Non-local GUT Breaking

Non-local GUT Breaking
Non-local GUT Breaking

- One possibility: GUT broken to SM non-locally: freely acting orbifold
- In this talk: $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with freely acting twists
  
  R. Donagi and K. Wendland 2008

- Gauge coupling unification and $M_{GUT}$ vs. $M_{string}$
  
  A. Hebecker and M. Trapletti 2004
\( \mathbb{Z}_2 \times \mathbb{Z}_2 \) Orbifold with Freely Acting Twist

(1-1) \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold by Donagi, Wendland:

- \( T^6 = T^2 \times T^2 \times T^2 \) spanned by orthogonal lattice \( e_i, \ i = 1, \ldots, 6 \)
- \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) generated by

\[
\begin{align*}
\nu_1 & = \left( 0, \frac{1}{2}, -\frac{1}{2} \right) \\
\nu_2 & = \left( -\frac{1}{2}, 0, \frac{1}{2} \right)
\end{align*}
\]

- freely acting twist: \( \tau = (\frac{1}{2}e_2, \frac{1}{2}e_4, \frac{1}{2}e_6) \)

R. Donagi and K. Wendland 2008

See also talks by Faraggi and Rizos
$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with freely acting twist

$T^6$ torus
\( \mathbb{Z}_2 \times \mathbb{Z}_2 \) Orbifold with Freely Acting Twist

twist \( \nu_1 \) acting on \( T^6 \) torus

\[ \nu_1 = \left( 0, \frac{1}{2}, -\frac{1}{2} \right) \]
Motivation
Orbifold MSSMs
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Conclusion

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

twist $\nu_1$ acting on $T^6$ torus

$\nu_1 = \left(0, \frac{1}{2}, -\frac{1}{2}\right)$

$\Rightarrow$ 16 fixed points
\( \mathbb{Z}_2 \times \mathbb{Z}_2 \) Orbifold with Freely Acting Twist

twist \( v_2 \) acting on \( T^6 \) torus

\[
v_2 = \left( -\frac{1}{2}, 0, \frac{1}{2} \right)
\]
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\[ \mathbb{Z}_2 \times \mathbb{Z}_2 \text{ Orbifold with Freely Acting Twist} \]

\[ \text{twist } \nu_2 \text{ acting on } T^6 \text{ torus} \]

\[ \nu_2 = \left( -\frac{1}{2}, 0, \frac{1}{2} \right) \]

\[ \Rightarrow 16 \text{ fixed points} \]
$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

twist $\nu_1 + \nu_2$ acting on $T^6$ torus

$$\nu_1 + \nu_2 = \left( -\frac{1}{2}, \frac{1}{2}, 0 \right)$$
$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

twist $v_1 + v_2$ acting on $T^6$ torus

$v_1 + v_2 = \left( -\frac{1}{2}, \frac{1}{2}, 0 \right)$

$\Rightarrow$ 16 fixed points
$Z_2 \times Z_2$ Orbifold with Freely Acting Twist

freely acting twist $\tau$ acting on $T^6$ torus

$\Rightarrow$ half the number of fixed points: $(16+16+16)/2 = 24$
\[ \mathbb{Z}_2 \times \mathbb{Z}_2 \text{ Orbifold with Freely Acting Twist} \]

Action of freely acting twist in 2d:

\[ T^2 / \mathbb{Z}_2 \]

⇒ half the number of fixed points: \[ 4/2 = 2 \]
$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

**setup:**

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with 6 generations of SU(5)

freely acting $\mathbb{Z}_2$

3 generations of $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$

- where freely acting Wilson line induces GUT breaking
- Potentially: one SM singlet per fixed point $\Rightarrow$ full blow-up
Example

MSSM from $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold with Freely Acting Twist

3 generations of quarks and leptons
plus vectorlike exotics
Input data

- **Shifts**

\[
V_1 = \left( \frac{1}{2}, \frac{1}{2}, 2, 0, 0, 0, 1, -1, 0, 1, 1, 0, 1, 0, 0, -1 \right)
\]
\[
V_2 = \left( \frac{5}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{2}, 0^5, 4 \right)
\]

- **Wilson lines**

\[
A_1 = 0
\]
\[
A_2 = \left( -1, -1, 0, -1, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4} \right)
\]
\[
A_3 = \left( 1, -1, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right)
\]
\[
A_5 = \left( -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 0, 0, 0, -\frac{1}{2}, -\frac{1}{2} \right)
\]
\[
A_6 = A_4 = A_2
\]

- **Freely acting Wilson line**

\[
A_\tau = \frac{1}{2} A_2
\]
(1) without freely acting Wilson line

- 4d gauge group: $\text{SU}(5) \times \text{U}(1)^4 \times [\text{SU}(4)^2 \times \text{U}(1)^2]$
- massless spectrum

| 15 | $(5, 1, 1)$ | 9 | $(\overline{5}, 1, 1)$ |
|----|-------------|---|----------------------|
| 6  | $(\overline{10}, 1, 1)$ | 52 | $(1, 1, 1)$ |
| 6  | $(1, 4, 1)$ | 6 | $(1, \overline{4}, 1)$ |
| 8  | $(1, 1, 4)$ | 8 | $(1, 1, \overline{4})$ |
| 2  | $(1, 1, 6)$ | | |

- (remark: generically, SU(5) vector-like exotics decouple linear in VEVs!)
(2) with freely acting Wilson line

- 4d gauge group:
  \[ \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \times \text{U}(1)^4 \times [\text{SU}(3) \times \text{SU}(2)^2 \times \text{U}(1)^4] \]

- Massless spectrum:

|   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | (3, 2, 1, 1, 1)_{1/6} & q |   |   |   |   |   |   |   |   |   |   |
| 8 | (3, 1, 1, 1, 1)_{1/3} & \bar{d}, \bar{\delta} |   |   |   |   |   |   |   |   |   |   |   |
| 7 | (1, 2, 1, 1, 1)_{-1/2} & \ell, h_d |   |   |   |   |   |   |   |   |   |   |   |
| 3 | (1, 1, 1, 1, 1)_{1} & \bar{e} |   |   |   |   |   |   |   |   |   |   |   |
| 5 | (1, 1, \overline{3}, 1, 1)_{0} & \bar{x} |   |   |   |   |   |   |   |   |   |   |   |
| 6 | (1, 1, 1, 1, 2)_{0} & y |   |   |   |   |   |   |   |   |   |   |   |

- (Remark: generically, vector-like "exotics" still decouple linear in VEVs! Draw back: also the Higgs!)
String couplings

The superpotential

\[ \mathcal{W} \supset m \bar{\delta} \delta + Y_u q \bar{u} h_u + \bar{u} \bar{d} \bar{d} + \ldots \]
String couplings

String states carry some “charges” that must be conserved:

- Space group selection rule (geometric)
- R-charge conservation
- Gauge Invariance

⇒ allowed terms in superpotential $\mathcal{W}$
String couplings

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\[ \Rightarrow \text{allowed terms in superpotential } \mathcal{W} \]
String couplings

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$\Rightarrow$ allowed terms in superpotential $\mathcal{W}$
String states carry some “charges” that must be conserved:

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\[ \Rightarrow \text{allowed terms in superpotential } \mathcal{W} \]
Effective couplings

- couplings like

\[ \mathcal{W} = \overline{\delta}_2 \delta_1 s_1 \]
\[ + q_1 \overline{u}_2 h_u s_4 s_5 s_{26} s_{30} \]
\[ + \overline{u}_1 \overline{d}_1 \overline{d}_2 s_2 s_3 s_{11} s_{30} + \ldots \]

- induce effective operators if SM singlets \( s_i \) develop (large) VEVs

\[ \mathcal{W} = \overline{\delta}_2 \delta_1 \langle s_1 \rangle \]
\[ + q_1 \overline{u}_2 h_u \langle s_4 \rangle \langle s_5 \rangle \langle s_{26} \rangle \langle s_{30} \rangle \]
\[ + \overline{u}_1 \overline{d}_1 \overline{d}_2 \langle s_2 \rangle \langle s_3 \rangle \langle s_{11} \rangle \langle s_{30} \rangle \]

- like Froggatt-Nielsen:

(hierarchical) Yukawa couplings + Proton decay operator

\[ \Rightarrow \text{vacuum selection very difficult} \]
Vacuum selection criterions

- SUSY preserving vacua!
  - F-Terms (global SUSY)
    \[ F_i \sim \frac{\partial W}{\partial \phi_i} = 0 \]
  - D-Terms
    \[ \sum_i q_i |\langle \phi_i \rangle|^2 + FI = 0 \]
- decoupling of exotics
- (hierarchical) Yukawa couplings
- strongly suppressed Proton decay

\[ \Rightarrow \text{Symmetries help!} \]
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\[ \mathbb{Z}_2 \times \mathbb{Z}_2 \text{ Orbifold} \]
Example
vacuum search
\[ B - L \text{ and matter parity} \]

- **U(1)_{B-L}** generator (from U(1)^9)

\[
t_{B-L} = \left(0, 0, 0, 1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \frac{7}{2}, -\frac{5}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, -\frac{1}{2}, -\frac{1}{2}\right)
\]

- **SM charged spectrum** (distinguish between \( \ell \) & \( h_d \) and \( \bar{d} \) & \( \bar{\delta} \))

| 3 | \((3, 2)_{1/6, 1/3}\) | \(q\) | 3 | \((1, 2)_{-1/2, -1}\) | \(\ell\) |
|---|-----------------|---|---|-----------------|---|
| 3 | \((3, 1)_{-2/3, -1/3}\) | \(\bar{u}\) | 3 | \((3, 1)_{1/3, -1/3}\) | \(\bar{d}\) |
| 3 | \((1, 1)_{1, 1}\) | \(\bar{e}\) | 3 | \((3, 1)_{-1/3, -2/3}\) | \(\bar{\delta}\) |

- **U(1)_{B-L}** forbids dangerous dim. 4 operators:

\[
\bar{u} \bar{d} \bar{d}, \quad q \bar{d} \ell, \quad \ell \ell \bar{e} \quad \text{and} \quad \ell h_u \quad (\text{total} \quad B - L = -1)
\]

and allows for wanted couplings (Yukawas), e.g.

\[
q \bar{u} h_u, \quad q \bar{d} h_d, \quad \ell \bar{e} h_d
\]
SM $\times U(1)_{B-L}$ singlets obtain VEVs:
\[
\{S_1, S_2, S_3, S_4, S_5, S_9, S_{10}, S_{15}, S_{16}, S_{18}, S_{19}, S_{20}, S_{22}, S_{23}, S_{25}, S_{26}, S_{28}, S_{30}, S_{31}, S_{33}, y_1, y_2, y_3, y_4, y_5, y_6, z_1, z_2, z_3, z_4, z_5, z_6\}
\]

- symmetry breaking: hidden SU(2)$^2 \times U(1)^6$ broken
- exotics decouple
- $U(1)_{B-L}$ broken to matter parity by VEVs of fields with even $B-L$ charge:
\[
\{S_7, S_8, S_{17}, S_{21}, S_{27}, S_{32}, x_3, x_4, x_5, \bar{x}_2, \bar{x}_4, \bar{x}_5\}
\]

- symmetry breaking: only SM $\times$ matter parity unbroken

$\Rightarrow$ Unwanted couplings remain forbidden!
Unbroken symmetries help for $F = 0$

- Consider set of fields $\{\phi_i\}$ and a symmetry group $G$ (e.g. $G = U(1)$)
- Split the set:
  \[ \{\phi_i\} = \{s_i\} \cup \{r_i\} \]
  where $s_i$ uncharged and $r_i$ charged w.r.t $G$
- Choose VEV: $\langle r_i \rangle = 0$ and $\langle s_i \rangle \neq 0 \Rightarrow G$ unbroken
- Then, the $F$-terms $F(\phi_i) \sim \frac{\partial W}{\partial \phi_i}$ split:
  \[ \langle F(r_i) \rangle = 0 \text{ no term linear in } r_i \]
  \[ \langle F(s_i) \rangle \neq 0 \text{ (model dependent)} \]
  i.e. number of VEVs $\langle s_i \rangle$ equals number of potential non-trivial $F$-terms $\langle F(s_i) \rangle$

$\Rightarrow$ Generically, $F = 0$ has solutions!
$D = 0$

- Gauge invariant monomial involving all $s_i$ ensures $D = 0$ configuration

Buccella, Derendinger, Ferrara, Savoy
Exotics decouple linear in VEVs: $\delta_i M^\delta_{ij} \delta_j$

$$M^\delta = \begin{pmatrix}
  s^3 & \langle s_1 \rangle & s^3 & s^3 & s^3 \\
  \langle s_2 \rangle & s^3 & s^5 & \langle s_{16} \rangle & \langle s_{20} \rangle \\
  s^5 & s^3 & s^5 & \langle s_{26} \rangle & \langle s_{31} \rangle \\
  \langle s_{28} \rangle & s^3 & \langle s_{19} \rangle & \langle s_{10} \rangle & s^3 \\
  \langle s_{33} \rangle & s^3 & \langle s_{23} \rangle & s^3 & \langle s_{10} \rangle
\end{pmatrix}$$

But (generically) so do the Higgs: $(h_d)_i M^h_{ij} (h_u)_j$

$$M^h = \begin{pmatrix}
  s^3 & \langle s_3 \rangle & s^3 & s^3 \\
  \langle s_{15} \rangle & s^5 & \langle s_{19} \rangle & \langle s_{23} \rangle \\
  s^3 & \langle s_{26} \rangle & \langle s_{10} \rangle & s^3 \\
  s^3 & \langle s_{31} \rangle & s^3 & \langle s_{10} \rangle
\end{pmatrix}$$
\[
\begin{bmatrix}
\langle h_u \rangle s^4 & \langle h_u \rangle s^4 & 0 \\
\langle h_u \rangle s^4 & \langle h_u \rangle s^4 & 0 \\
0 & 0 & \langle (h_u)_1 \rangle \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & \langle (h_d)_3 \rangle \\
0 & 0 & \langle (h_d)_4 \rangle \\
\langle (h_d)_3 \rangle & \langle (h_d)_4 \rangle & 0 \\
\end{bmatrix}
\]

- \( q_i \ M^u_{ij} \overline{u}_j \)
- \( q_i \ M^d_{ij} \overline{d}_j \)

- \( q_i \ M^e_{ij} \overline{e}_j \)

- SU(5) relation survives freely acting Wilson line
- \( D_4 \) family symmetry with geometrical origin: third generation: singlet; first/second: doublet

... and there are many more vacua!
Conclusion
Summary

- $\mathbb{Z}_6$-II Mini-Landscape at special (symmetry enhanced) point, but full blow-up not necessary
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ with freely acting twist $\Rightarrow$ non-local GUT breaking
- Example: promising model (potentially also in full blow-up)