Optimization model and algorithm on electrical resolver position of high speed railway

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Abstract. The position of resolver of electrified railway affects inter-station run-time and energy consumption of train movement. Considering the constraints on train operation, signal distance as well as the number and distance requirements of electrical resolver, this paper presents a mathematical model to optimize the position of electrical resolvers for the whole line with the objective to minimize the weighted sum of train inter-station run-time and energy consumption. Genetic algorithm is employed to solve the proposed model. Case studies on Wuhan-Guangzhou high speed rail line from Chibi North to Yueyang East show that the proposed model saves 3.14% of train inter-station run-time and 1.28% of train energy consumption compared with the real-world electrical resolver configuration.

1. Introduction
Electrical resolver is an important component to ensure electrical insulation. Since 25 kV single-phase AC power supply mode has been adopted in the electrified railway in China, to guarantee three-phase balance of power system, it is necessary to set electrical resolvers to avoid short circuit between adjacent traction substations. Due to no power supplied in the electrical resolver, trains have to coast when moving into it. Therefore, if the position of electrical resolver is set improperly like on a steep ascent, train speed will decrease significantly, causing safety problems, deviation of run-time and change of energy consumption. In order to improve transportation efficiency and reduce the cost of energy consumption, it is necessary to optimize the location of electrical resolver.

For electrical resolver position optimization, most of the existing researches analyzed the design principles from the safety perspective. Wu and Zhang [1,2] summarized the constraints such as the minimum speed when train exits electrical resolver, the distance between electrical resolver and signal post, but neglect the impact on inter-station run-time and energy consumption. When electrical resolver is set on different slopes, the train coasts in it and lead to different degrees of speed loss, so that the final running time is different too. For this situation, Han and Wen [3,4] optimize the design of single electrical resolver by simulation, and analyze the effect of electrical resolver on train run-time. However, studies mentioned above neglected the impact of electrical resolver position on energy consumption of train movement. Although Su [5] took into account the effect of electrical resolver when calculating train energy consumption, it did not optimize the electrical resolver layout scheme. Therefore, further studies on reducing energy consumption and run-time of trains by optimizing electrical resolver layout scheme in the whole line need to be conducted.
Based on train operation process, considering the constraints of the number and distance requirements of electrical resolver, this paper establishes an electrical resolver position optimization model for the whole line to reduce train run-time and energy consumption. Genetic algorithm is adopted to solve this model. Finally, Chibi North to Yueyang East section on Wuhan-Guangzhou high speed railway is selected in case study to analyze the result.

2. Problem description
Generally, high speed railway will be equipped with one electrical resolver every 20-30km in China. The length of electrical resolver which is called dead zone is approximately the same, so it is assumed to be a fixed value in this paper. For better understand, the position of electrical resolver can be expressed by its central mileage. Taking a section as an example, as shown in Fig. 1, the section length is known in advance, where $S_1$ and $S_2$ represent the starting station and the terminal station of the section, and $x_i (i = 1, 2, 3, ..., n)$ is electrical resolver central mileage.

![Figure 1. Layout scheme of electrical resolver](image)

The scheme of electrical resolver is composed of $x_i$. For the whole line, there are many different combinations of electrical resolver layout. Therefore, electrical resolver layout scheme can be presented by positions of each electrical resolvers, which is $x_i (i = 1, 2, 3, ..., n)$.

3. Model formulation
According to train operation behavior and relevant regulations, following assumptions are proposed in this paper: (1) The train is regarded as a single particle. (2) The train adopts time-saving operation mode [6]. (3) The train will coast for 10s before entering electrical resolver. It is assumed train coasting distance in advance is a fixed value $x_c$, which is roughly calculated according to the speed. In addition, according to the "Technical Regulations for Railway Technology (High-speed Railway Section)”, the power-on sign is set at 400m after the electrical resolver exit position, so it can be assumed that train coasts for 400m after leaving the electrical resolver. (4) According to the foregoing description, it is assumed that the length of the dead zone is a fixed value $x_b$.

3.1. Decision variables
Decision variable in this paper is $U = [x_1, x_2, ..., x_n]$, which represents the center mileage of electrical resolvers in a section. $n$ denotes the number of electrical resolvers, which is known in advance.

3.2. Objective function
This model aims to minimize the run-time and energy consumption of high-speed train movement, which can be formulated by

$$\min C = \alpha \cdot f_e(x_1, x_2, ..., x_n) + \beta \cdot f_t(x_1, x_2, ..., x_n)$$  \hspace{1cm} (1)

where $\alpha$ and $\beta$ are coefficients for converting energy consumption and run-time into money respectively. The calculation formula of $\beta$ can refer to literature [7]; $\alpha$ is the unit price of industrial electricity in China; $f_e$ and $f_t$ denote train energy consumption and run-time respectively, which can be calculated by equation (2) and (3)
where $S$ is the length of the section; $j$ is the distance step; $f_j$ is the traction force of the train under the $j$-th step; $\Delta s$ is the unit step; $p$ is the train auxiliary power; $v_j$ is the initial speed under the $j$-th step; $a_j$ is the acceleration under the $j$-th step, which can be calculated by

$$a_j = \frac{g \cdot c}{1000 \cdot (1 + \gamma)}$$  \hspace{1cm} (4)

$$c = \frac{C(x_j) \cdot 10^3}{(P + C_p \cdot c_a \cdot m_{ave}) \cdot g}$$  \hspace{1cm} (5)

where $c$ is the unit resultant force; $\gamma$ is the rotating mass coefficient; $C(x_j)$ represents the resultant force, which can be calculated by traction, resistance and braking force. Since train will coast in the dead zone, the position of electrical resolver has an impact on the calculation of train resultant force; $P$ denotes train mass; $C_p$ is train capacity; $c_a$ is the load factor of train; $m_{ave}$ is the average mass of a single passenger.

### 3.3. Constraints

Due to the power limitation of traction substation, the distance between two adjacent electrical resolvers should be limited within a range, as shown in equation (6).

$$x_{min} \leq |x_{j+1} - x_j| \leq x_{max}$$  \hspace{1cm} (6)

In order to prevent the electrical resolver from being set in a lower speed area, the distance between the electrical resolver and the station should be limited within a range, as shown in Equations (7) and (8).

$$l_{min} \leq |x_i - x_{station1}| \leq l_{max}$$  \hspace{1cm} (7)

$$l_{min} \leq |x_n - x_{station2}| \leq l_{max}$$  \hspace{1cm} (8)

To ensure that train can restart safely after stopping in front of the signal, it is necessary to set the distance constraint between the signal post and the start point of electrical resolver, as shown in Equations (9), where $x_{istart}$ is the start point of electrical resolver.

$$x_{istart} - x_{signal} > x_a$$  \hspace{1cm} (9)

In the process of train operation, when train stops at the station, its speed equals to 0. Moreover, train speed must be less than the speed limit during operation. The constraints on train speed is described by equations (10) and (11).
4. Algorithm

The proposed model is a nonlinear model with a large searching space. Genetic algorithm has advantages of low mathematical requirements, high robustness, which is suitable for solving complex nonlinear optimization problems. Therefore, it is adopted to solve this problem.

The general procedure of genetic algorithm is as follows:

(1) Chromosome coding and initial solution generating: This paper adopts binary coding, and $x_i$ ($i = 1, 2, 3, \ldots, n$) is encoded by genes on a chromosome. Each chromosome represents a scheme of electrical resolver, the initial solution generation method is shown in Figure 2.

(2) Fitness function calculation: After calculating the objective function, fitness can be obtained by equation (12), where $t$ is the number of iterations, and $k$ is chromosome number.

$$f_k^t = C_{\text{max}} - C_k^t$$  \hspace{1cm} (12)

(3) Genetic strategy: The roulette method is used for selection operation. And single-point crossover method and random mutation method are used to obtain new daughter chromosomes.

5. Case study

In this paper, Chibi North to Yueyang East section on Wuhan-Guangzhou high speed railway is taken as an example to verify the feasibility of proposed model. The length of the section is 81.9km and CRH380A high-speed train composed of 8 cars runs on this line. In addition, other parameters are shown in Table 1. The solver for this case was written in MATLAB2017, and the computer running the program is configured as CPU i7-7700, RAM 8G, Windows 10 (64-bit).

Table 1. Model parameter value.

| $P$ (t) | $C_p$ | $v_m$ (km/h) | $c_a$ (kg) | $m_{\text{ave}}$ (m) | $x_b$ (m) | $x_c$ (m) | $x_{\text{min}}$ (km) | $x_{\text{max}}$ (km) | $l_{\text{min}}$ (km) | $l_{\text{min}}$ (km) |
|--------|------|----------------|---------|----------------------|--------|--------|---------------------|---------------------|-------------------|-------------------|
| 448    | 494  | 300            | 0.7     | 60                   | 300    | 833    | 800                 | 20                  | 30                | 7                 |

Electrical resolver layout scheme obtained by the proposed model, real-world scheme and uniformly distribution scheme are shown in Fig. 2 and Table 2.
Figure 3. Comparison of train trajectories with different layouts of natural zones

Table 2. Optimized results of natural zones in Chibi North-Yueyang East section

| Scheme           | $x_i$ (m) | $f_t$ (s) | $r_1$ (%) | $f_e$ (kw*h) | $r_2$ (%) | $C$ (yuan) | $r_3$ (%) |
|------------------|----------|-----------|-----------|--------------|-----------|------------|-----------|
| Real-world       | [1354, 26407, 56118, 76765] | 1183.27   | -         | 622.87       | -         | 4512.70    | -         |
| Uniform distribution | [16390, 32780, 49170, 65560] | 1153.33   | 2.53      | 624.67       | -0.29     | 4414.33    | 2.18      |
| Optimization     | [9188, 29946, 50193, 72069] | 1146.11   | 3.14      | 614.90       | 1.28      | 4381.42    | 2.91      |

($r_1$, $r_2$, $r_3$ are optimization rates of run-time, energy consumption and total cost respectively.)

As shown in Fig.3, the first and the fourth electrical resolvers are located near the station in the real-world scheme. Moreover, the train pass these two electrical resolvers during its acceleration and braking phase respectively, so the traction and braking time will be extended when the train coasting in electrical resolvers. Besides, the third electrical resolver is located on a down-slope with a gradient of 16.8 ‰, train speed drops obviously.

In turn, electrical resolvers in the uniform distribution scheme and the optimization scheme are located where train is cruising, so the train runs at a high speed. In addition, in the uniform distribution scheme, the distance between two electric resolvers is too close, and some of them are located on the steep slopes. On the contrary, electrical resolvers in the optimization scheme are set on relatively slight slopes and also meet the interval constraints. Even if the train coasts to pass through electrical resolver area, the speed loss and impact on run-time is small.

In Table 2, train run-time is optimized in the uniform distribution scheme but train energy consumption is increased, as some of electrical resolvers are set on the up-slope. Compared with the real-world scheme, the optimization scheme saves 3.14% of run-time and 1.28% of energy consumption, which performs best among the three schemes in total cost of train run-time and energy consumption. If the total train services of the line is 150 pairs/day, 14.38 million yuan will be saved in one year.
6. Conclusion

This paper proposes a model to optimize the layout scheme of electrical resolver for high-speed railway with the objective of saving train run-time and energy consumption. The electrical resolver design and train operation constraints are taken into account. A genetic algorithm is adopted to solve the model. Case studies on Chibi North to Yueyang East section verify the feasibility of the proposed model and algorithm. The results show that the optimization scheme can effectively reduce train run-time and energy consumption. Electrical resolver should not set on the steep slopes and not be too close to the station in order to ensure the safety and efficiency of train operation. In addition, the interaction between passenger demand and operating costs should also be considered during the design process to get the appropriate scheme.

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