The chiral magnetic effect and the chiral spin symmetry in QCD above $T_c$

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Abstract

The chiral magnetic effect (CME) is an exact statement that connects via the axial anomaly the electric current in a system consisting of interacting fermions and gauge field with chirality imbalance that is put into a strong external magnetic field. Experimental search of the magnetically induced current in QCD in heavy ion collisions above a pseudocritical temperature hints, though not yet conclusive, that the induced current is either small or vanishing. This would imply that the chirality imbalance in QCD above $T_c$ that could be generated via topological fluctuations is at most very small. Here we present the most general reason for absence (smallness) of the chirality imbalance in QCD above $T_c$. It was recently found on the lattice that QCD above $T_c$ is approximately chiral spin (CS) symmetric with the symmetry breaking at the level of a few percent. The CS transformations mix the right- and left-handed components of quarks. Then an exact CS symmetry would require absence of any chirality imbalance. Consequently an approximate CS symmetry admits at most a very small chirality imbalance in QCD above $T_c$. Hence the absence or smallness of an magnetically induced current observed in heavy ion collisions could be considered as experimental evidence for emergence of the CS symmetry above $T_c$.

Keywords: QCD; chiral spin symmetry; high temperatures; chiral magnetic effect

1. Introduction

According to the Atiah-Singer theorem local topological fluctuations of the gluonic field in Euclidean space with nonzero topological charge induce creation of chiral quarks such that the number of the right-handed quarks exceeds the number of the left-handed quarks by the topological charge. This process produces a chirality imbalance. If such a system is put into a strong external magnetic field, then this process should give rise to the magnetically induced electric current. This phenomenon is called the chiral magnetic effect (CME)\textsuperscript{1,2} and the exact relation between the external magnetic field and magnetically
induced current based on axial anomaly is

\[ J = \sigma_5 B, \]  

(1)

where the chiral magnetic conductivity \( \sigma_5 \) is expressed in terms of the chiral chemical potential as

\[ \sigma_5 = N_c \sum_f \frac{Q_f^2 e^2}{2\pi^2} \mu_5 \]  

(2)

with \( Q_f e \) being the electric charge of the quark with the flavor \( f \) and

\[ \mu_5 = \frac{\mu_R - \mu_L}{2} \]  

(3)

being the axial chemical potential, i.e. a difference of the chemical potentials for the right- and left-handed quarks. The axial chemical potential parametrises the chirality imbalance induced by local topological fluctuations.

Experimental search of CME at RHIC and LHC suggests that the magnetically induced current is at most very small, though there are many uncertainties and direct extraction of the current from data is impossible.\(^3,4,5\) If the magnetic field that is formed during collision of two nuclei is sufficiently large and at the same time a magnetically induced electric current vanishes or small, as hinted by experimental data, then one infers from (1) that chirality imbalance in QCD is either absent or very small. This obviously requires a convincing explanation.

It was recently suggested\(^6\) and then observed on the lattice\(^7,8,9\) that QCD at RHIC and LHC temperatures is approximately chiral spin (CS) symmetric.\(^10,11\) This symmetry, that includes chiral symmetry \( U(1)_A \) as a subgroup, is not a symmetry of the Dirac Lagrangian but is a symmetry of the Lorentz-invariant fermion charge and consequently is a symmetry of the chromoelectric interactions in QCD in a given reference frame. The chromomagnetic interaction as well as the quark kinetic term break this symmetry. Observation of this symmetry in \( T_c - 3T_c \) interval implies that a dominant physics at this temperatures is due to the chromoelectric interaction between chirally symmetric quarks, that are bound into the color-singlet objects ("strings"), and a contribution of the chromomagnetic interaction as well as of the quark kinetic energy is at least much smaller. The CS symmetry is not a symmetry of the Dirac Lagrangian and consequently implies that there are no free deconfined quarks.

A salient feature of the CS-transformations is that they mix the right- and left-handed quarks. Then exact CS symmetry in QCD above \( T_c \) would require that the chirality imbalance should be absent since this imbalance is proportional the difference of number of the right- and left-handed quarks. Hence an exact CS-symmetry would require vanishing of the magnetically induced electric current in heavy ion collisions. In reality this symmetry is not exact and is broken at a few percent level. Then this approximate CS symmetry admits only a very small magnetically induced current. Stated otherwise a nonobservation
or observation of only small magnetically induced current provided that the external magnetic field is sufficiently large could be considered as an experimental evidence of the CS symmetry observed on the lattice.

This short paper is structured in the following way. In sections 2 and 3 we overview already known results about CS symmetry and its observation on the lattice at high temperatures. This will allow to avoid reading the preceding papers. Then in section 4 we present the key argument of this paper and will conclude in section 5.

2. Chiral spin symmetry

The chiral spin $SU(2)_{CS}$ transformation was defined in ref.\textsuperscript{10} as a transformation that rotates in the space of the right- and left-handed Weyl spinors

$$
\begin{pmatrix}
R \\
L
\end{pmatrix}
\rightarrow
\exp\left(i\frac{\varepsilon^m \sigma^m}{2}\right)
\begin{pmatrix}
R \\
L
\end{pmatrix}.
$$

(4)

In terms of the Dirac spinors $\psi$ this transformation can be written via $\gamma$-matrices\textsuperscript{11}

$$
\psi \rightarrow \psi' = \exp\left(i\frac{\varepsilon^n \Sigma^n}{2}\right) \psi
= \exp\left(i\frac{\varepsilon^n \sigma^n}{2}\right)
\begin{pmatrix}
R \\
L
\end{pmatrix},
$$

(5)

where the generators $\Sigma^n$ of the four-dimensional reducible representation are

$$
\Sigma^n = \{\gamma_0, -i\gamma_5 \gamma_0, \gamma_5\}.
$$

(6)

The $su(2)$ algebra is automatically satisfied for these three generators,

$$
[S^a, S^b] = 2i\epsilon^{abc} S^c.
$$

(7)

The $U(1)_A$ group is a subgroup of $SU(2)_{CS}$.

In Euclidean space with the $O(4)$ symmetry all four directions are equivalent and one can use any Euclidean hermitian $\gamma$-matrix $\gamma_k$, $k = 1, 2, 3, 4$ instead of Minkowskian $\gamma_0$:

$$
\Sigma^n = \{\gamma_k, -i\gamma_5 \gamma_k, \gamma_5\},
$$

(8)

$$
\gamma_i \gamma_j + \gamma_j \gamma_i = 2\delta^{ij}; \quad \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4.
$$

(9)

The $su(2)$ algebra is satisfied with any $k = 1, 2, 3, 4$.

The direct product of the $SU(2)_{CS}$ group with the flavor group $SU(2)_{CS} \times SU(N_F)$ can be extended to a $SU(2N_F)$ group. This group includes the chiral symmetry $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ as a subgroup. The $SU(2N_F)$ transformations are given by

$$
\psi \rightarrow \psi' = \exp\left(i\frac{\varepsilon^m T^m}{2}\right) \psi,
$$

(10)
with \( m = 1, 2, \ldots, (2N_F)^2 - 1 \) and the set of \( (2N_F)^2 - 1 \) generators being

\[
T^m = \{ (\tau^a \otimes 1_D), (1_F \otimes \Sigma^n), (\tau^a \otimes \Sigma^n) \}
\]  

(11)

where \( \tau \) are the flavor generators (with the flavor index \( a \)) and \( n = 1, 2, 3 \) is the \( SU(2)_{CS} \) index.

The fundamental vector of \( SU(2N_F) \) at \( N_F = 2 \) is

\[
\Psi = \begin{pmatrix}
    u_R \\
    u_L \\
    d_R \\
    d_L
\end{pmatrix}.
\]  

(12)

The \( SU(2)_{CS} \) and \( SU(2N_F) \) groups are not symmetries of the Dirac Lagrangian. At the same time they are symmetries of the Lorentz-invariant fermion charge

\[
Q = \int d^3 x \bar{\psi}(x) \gamma_0 \psi(x) = \int d^3 x \bar{\psi}^\dagger(x) \psi(x).
\]  

(13)

This salient feature allows us to use the \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries to distinguish the chromoelectric and chromomagnetic interactions in a given reference frame because the chromoelectric interaction is influenced only by the color charge while the chromomagnetic interaction is dictated by the spatial current. The latter current is not \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetric.

More specifically the (chromo)electric and (chromo)magnetic fields in Minkowski space in a given reference frame are different fields. Interaction of fermions with the gauge field in Minkowski space-time can be split in a given reference frame into temporal and spatial parts:

\[
\bar{\psi} \gamma^\mu D_\mu \psi = \bar{\psi} \gamma^0 D_0 \psi + \bar{\psi} \gamma^i D_i \psi.
\]  

(14)

The covariant derivative \( D_\mu \) includes interaction of the matter field \( \psi \) with the gauge field \( A_\mu \),

\[
D_\mu \psi = (\partial_\mu - ig \frac{t \cdot A_\mu}{2}) \psi.
\]  

(15)

The temporal term contains interaction of the color-octet charge density

\[
\bar{\psi}(x) \gamma^0 \frac{t}{2} \psi(x) = \bar{\psi}(x)^\dagger \frac{t}{2} \psi(x)
\]  

(16)
and $SU(2N_F)$. We conclude that interaction of the chromoelectric and chromomagnetic components of the gauge field with quarks in a given reference frame can be distinguished by symmetry.

Of course, in order to discuss the chromoelectric and chromomagnetic components of the gluonic field one needs to fix a reference frame. The hadron invariant mass is the rest frame energy. Consequently, to address physics of hadron mass one should discuss energy in the hadron rest frame. At high temperatures the Lorentz invariance is broken and a natural frame is the medium rest frame.

3. Emergence of chiral spin and $SU(2N_F)$ symmetries in QCD above $T_c$

Above the chiral restoration pseudocritical temperature $T_c$ one apriori expects in observables chiral $SU(2)_L \times SU(2)_R$ symmetry because the quark condensate vanishes during a very smooth crossover at temperatures between 100 and 200 MeV. A pseudocritical temperature $T_c$ determined from the chiral susceptibility is around 155 MeV in $N_F = 2 + 1$ QCD. This symmetry above the crossover is evidenced by degeneracy of correlators connected by the chiral transformation. While the axial anomaly is a pertinent property of QCD its effect is determined by the topological charge density. There are strong indications from the lattice that the $U(1)_A$ symmetry is also effectively restored above $T_c$, which suggests that the local topological fluctuations in Euclidean space in QCD are at least very strongly suppressed above $T_c$. It is a matter of the present debates whether the $U(1)_A$ restoration happens at the same temperature as of $SU(2)_L \times SU(2)_R$ or at a slightly higher temperature. The $U(1)_A$ restoration is evidenced by degeneracy of correlators connected by the $U(1)_A$ transformation. $SU(2)_L \times SU(2)_R$ and $U(1)_A$ transformation properties of the $J = 1$ operators are given in the left panel of Fig. 1.

In the right panel of the same Fig. we present transformation properties of the same operators with respect to $SU(2)_{CS}$ and $SU(4)$. If one observes on
Figure 2: Temporal correlation functions for $12 \times 48^3$ lattices. The l.h.s. shows correlators calculated with free noninteracting quarks with manifest $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries. The r.h.s. presents full QCD results at a temperature $1.2T_c$, which shows multiplets of all $U(1)_A$, $SU(2)_L \times SU(2)_R$, $SU(2)_{CS}$ and $SU(4)$ groups. The Fig. is from Ref. 9.

the lattice degeneracy of correlators that are connected by $SU(2)_{CS}$ and $SU(4)$ that would be a signal for emergence of these symmetries.

On the r.h.s. of Fig. 2 we show temporal correlators

\[ C_T(t) = \sum_{x,y,z} \langle O_T(x,y,z,t)O_T(0,0) \rangle, \tag{17} \]

at a temperature $T = 1.2T_c$ calculated in $N_F = 2$ QCD with a chirally symmetric Dirac operator. 9 Here $O_T(x,y,z,t)$ is an operator that creates a quark-antiquark pair with fixed quantum numbers. Summation over $x,y,z$ projects out the rest frame.

Correlators of the isovector scalar (S) and isovector pseudoscalar (PS) operators are connected by the $U(1)_A$ transformation and their degeneracy indicates restoration of this symmetry. If there is a tiny splitting of the S and PS correlators then it should be so small so that it cannot be seen in the present lattice data. This strongly suggests that the topological transitions are at least severely suppressed above $T_c$. An approximate degeneracy of the $a_1$, $b_1$, $\rho_{(1,0)+(0,1)}$ and $\rho_{(1/2,1/2)b}$ correlators indicates emergent $SU(2)_{CS}$ and $SU(4)$ symmetries. Their breaking is estimated at the level of less than 5%.

A similar multiplet structure is seen in spatial correlators in the temperature range $T_c - 3T_c$. 7,8

On the l.h.s of Fig. 2 we present correlators calculated with noninteracting quarks on the same lattice. They represent a QGP at a very high temperature where due to asymptotic freedom the quark-gluon interaction can be neglected. Dynamics of free quarks are governed by the Dirac equation and only $U(1)_A$ and $SU(2)_L \times SU(2)_R$ chiral symmetries exist. A qualitative difference between the pattern on the l.h.s. and the pattern on the r.h.s of Fig. 2 is remarkable. The temporal correlators are directly connected to measurable spectral density. $SU(2)_{CS}$ and $SU(4)$ symmetries of the t-correlators imply the same symmetries of spectral densities.
4. Chiral spin transformation properties of the axial chemical potential

Now we arrive at the key point of this note: What are implications of the emerging chiral spin symmetry above $T_c$ on axial chemical potential term in effective action?

The quark chemical potential $\mu \psi(x)\dagger \psi(x)$ and the axial chemical potential $\mu_s \psi(x)\dagger \gamma_5 \psi(x)$ terms can be present in the QCD action,

$$S = \int_0^\beta dt \int d^3x \overline{\psi}[\gamma_\mu D_\mu + \mu \gamma_4 + \mu_s \gamma_5 \gamma_5 + m]\psi,$$

in the $SU(2)_{CS}$ and $SU(2N_F)$ symmetric regime only if they are invariant with respect to the chiral spin transformations, i.e. are the chiral spin singlets. The chemical potential term is indeed invariant, i.e. transforms into itself upon the $SU(2)_{CS}$ transformation (5)

$$\psi(x)\dagger \psi(x) \rightarrow \psi(x)\dagger \psi(x).$$

The axial chemical potential $\psi(x)\dagger \gamma_5 \psi(x)$ term is not invariant under (5) and transforms into a superposition of three terms:

$$\psi(x)\dagger \gamma_5 \psi(x) \rightarrow \alpha \psi(x)\dagger \gamma_4 \psi(x) + \beta \psi(x)\dagger \gamma_4 \gamma_5 \psi(x) + \gamma \psi(x)\dagger \gamma_5 \psi(x),$$

i.e. it transforms under the triplet representation of $SU(2)_{CS}$ with coefficients $\alpha, \beta, \gamma$ being determined by three rotation angles. Then such a term is not allowed in a $SU(2)_{CS}$-symmetric theory. In other words, the $SU(2)_{CS}$ symmetry prohibits existence of a finite axial chemical potential.

The same statement can be understood with a less formal language. The axial chemical potential parametrizes an excess of the right-handed quarks over the left-handed quarks. The $SU(2)_{CS}$ transformation mixes the right- and left-handed quarks, see (4). Consequently a given finite excess of the right- over the left-handed quarks cannot be $SU(2)_{CS}$-symmetric. Only a vanishing axial chemical potential is consistent with chiral spin symmetry.

We conclude that if the emerged chiral spin symmetry were exact, then the chiral magnetic conductivity $\sigma_5$ in (1) must vanish. Exact $SU(2)_{CS}$ symmetry requires vanishing of the magnetically induced electrical current even if the external magnetic field is large.

Of course, in reality the chiral spin symmetry above $T_c$ is not exact: it is broken at a few percent level. Then the magnetically induced electric current can be only very small as it would originate only from the CS symmetry breaking contributions. The topological fluctuations with a nonzero topological charge do indeed break the CS symmetry. However their role in the QCD dynamics above $T_c$ can be only very small because of a very good CS symmetry.

This result explains a vanishing or very small magnetically induced electric current above $T_c$ as it is suggested by the present experimental data. Stated
otherwise, if future experiments confirm a smallness or absence of the magnetically induced current provided that the transient external magnetic field is sufficiently strong, it would be an experimental evidence of the chiral spin symmetry above $T_c$.

5. Conclusions

A formation of multiplets in correlators described by the chiral spin $SU(2)_{CS}$ and $SU(4)$ groups\textsuperscript{10,11} in the range $T_c - 3T_c$ was observed on the lattice.\textsuperscript{7,8,9} These symmetries include the chiral $U(1)_A$ and $SU(2)_L \times SU(2)_R$ as subgroups. These are not symmetries of the free Dirac action and they are not consistent with free deconfined quarks. In the medium rest frame the chromoelectric interaction is invariant under both $SU(2)_{CS}$ and $SU(4)$ transformations, while the chromomagnetic interaction as well as the quark kinetic term break them.

The emergence of these symmetries in the $T_c - 3T_c$ window suggests that the chromomagnetic field disappears or is strongly suppressed, while the confining chromoelectric field is still active. This implies that the physical degrees of freedom are chirally symmetric quarks bound by the chromoelectric interaction into color-singlet objects without chromomagnetic effects. This regime of QCD was named as a "stringy fluid".

These symmetries are broken at a few percent level.

If the $SU(2)_{CS}$ and $SU(4)$ symmetries were exact it would require a vanishing of the chirality imbalance. Consequently the magnetically induced current would exactly vanish even if the external magnetic field be very strong. Then a tiny magnetically induced current can be only related to the CS symmetry breaking dynamics which is however much less important than a confining chromoelectric interaction that binds the chirally symmetric quarks into color singlet objects. This conclusion is drawn from the smallness of the CS symmetry breaking.

Confirmation in experiments of smallness of the magnetically induced current or its absence provided that the external magnetic field is sufficiently strong could be considered as an experimental verification of the chiral spin and $SU(2N_F)$ symmetries in QCD above $T_c$.

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[1] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A \textbf{803} (2008) 227.

[2] K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D \textbf{78} (2008) 074033.

[3] D. E. Kharzeev, J. Liao, S. A. Voloshin and G. Wang, Prog. Part. Nucl. Phys. \textbf{88} (2016) 1.

[4] S. A. Voloshin, Phys. Rev. C \textbf{98} (2018) 054911.
[5] see review plenary talks at Quark Matter 2018 and Quark Matter 2019 conferences.

[6] L. Ya. Glozman, Proceedings of the "Critical point and onset of deconfinement" conference, Wroclaw, Poland, May 30 - June 4, 2016, *Acta Physica Polonica B, Proceedings Supplement*, 10 (2017) 583 [arXiv:1610.00275].

[7] C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, L. Y. Glozman, S. Hashimoto, C. B. Lang and S. Prelovsek, Phys. Rev. D 96 (2017) 094501 Erratum: Phys. Rev. D 99 (2019) 039901.

[8] C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, C. Gattringer, L. Y. Glozman, S. Hashimoto, C. B. Lang and S. Prelovsek, Phys. Rev. D 100 (2019) 014502.

[9] C. Rohrhofer, Y. Aoki, L. Y. Glozman and S. Hashimoto, Phys. Lett. B 802 (2020) 135245.

[10] L. Y. Glozman, Eur. Phys. J. A 51 (2015) 27.

[11] L. Y. Glozman and M. Pak, Phys. Rev. D 92 (2015) 016001.

[12] Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, S. Krieg and K. K. Szabo, JHEP 0906 (2009) 088.

[13] G. Cossu, S. Aoki, H. Fukaya, S. Hashimoto, T. Kaneko, H. Matsufuru and J. I. Noaki, Phys. Rev. D 87 (2013) 114514. Erratum: [Phys. Rev. D 88 (2013) 019901].

[14] A. Tomiya, G. Cossu, S. Aoki, H. Fukaya, S. Hashimoto, T. Kaneko and J. Noaki, Phys. Rev. D 96 (2017) 034509. Addendum: [Phys. Rev. D 96 (2017) 079902].

[15] A. Bazavov et al., Phys. Rev. D 100 (2019) 094510.