Generation of equal spin-triplet Cooper pairings by magnetic barriers in superconducting junctions

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We investigate the proximity effect in an $s$-wave superconductor/ferromagnetic metal with Rashba spin-orbit coupling/diffusive normal metal junction and an $s$-wave superconductor/noncollinear magnetic metal/diffusive normal metal junction. We show the generation of equal spin-triplet pairings in the diffusive normal metal due to spin-flip scattering in the intermediate magnetic regions. The emergence of the spin-triplet odd-frequency Cooper pairings can generate zero energy peak in quasiparticle density of states in the diffusive normal metal.

I. INTRODUCTION

Recently, the emergence of spin-triplet pairings in superconductor (SC)/ferromagnet junctions has received much attention\textsuperscript{[1–4]}. In ferromagnet/SC junctions, nonunitary spin-triplet pairings can be generated due to spin-flip scattering by inhomogeneous magnetization\textsuperscript{[5, 6]}. The generation of the spin-triplet pairing has been confirmed by observing Josephson current through strong ferromagnets\textsuperscript{[7–9]}. Equal spin-triplet pairings are an important ingredient for superconducting spintronics. For example, in a current-biased ferromagnetic Josephson junctions, one can realize spin-polarized supercurrent due to the generation of equal spin-triplet pairings\textsuperscript{[10]}. Since equal spin-triplet pairings have a spin polarization, they can be also used to exert spin transfer torques and induce magnetization dynamics\textsuperscript{[11–15]}. Even in uniform ferromagnets, spin-triplet pairings can be generated by spin-orbit coupling due to spin rotation or precession of spin of Cooper pairs\textsuperscript{[16–19]}. The interplay between spin-orbit coupling and superconductivity leads to various phenomena such as a zero energy peak in the density of states\textsuperscript{[20, 21]}, \(\phi_0\) junctions\textsuperscript{[22–24]}, magnetoelectric effects\textsuperscript{[25–26]}, and enhanced spin pumping\textsuperscript{[27]}

Spin-triplet pairings are also generated with the use of ferromagnetic insulators. Inserting a ferromagnetic insulator at the interface between a normal metal and an SC, spin-triplet pairings are generated by spin-flip scattering at the interface\textsuperscript{[28–30]}. Recent experiments on superconducting tunnel junctions with a magnetic insulator GdN or EuS have indicated the emergence of odd-frequency spin-triplet pairings\textsuperscript{[31, 32]} which manifests itself as a zero energy peak in the local density of states\textsuperscript{[33–37]}. Also, it has been predicted that the coupling between a magnon in a ferromagnetic insulator and Cooper pairs can lead to magnon spin current\textsuperscript{[38]} and a formation of magnon-Cooparons\textsuperscript{[39]}

In this paper, we extend the previous works on the generation of spin-triplet Cooper pairings by magnetic interface with uniform magnetization in superconducting junctions\textsuperscript{[28, 29]} by including more complicated magnetic (spin) structures\textsuperscript{[30]}. We show that the equal spin-triplet Cooper pair amplitude is generated in the presence of magnetic barriers with nontrivial magnetizations in two kinds of junctions: an $s$-wave SC/ferromagnet with Rashba spin-orbit coupling (RSOC)/diffusive normal metal (DN) junction and an $s$-wave SC/noncollinear magnet/DN junction. We clarify the generation of equal spin triplet pairings in the DN due to spin-flip scattering in the magnetic regions. The emergence of this odd-frequency spin-triplet pairings is manifested as a zero energy peak in the local density of states.

The organization of this paper is as follows. In Section II, we explain our model and the method of theoretical calculations. We show the numerically calculated results in Section III. We summarize our results in Section IV.

II. MODEL AND METHOD

We consider two systems: the spin-singlet $s$-wave SC/Ferromagnetic metal with Rashba spin-orbit coupling (FR)/DN junction (Sec. II A) and the spin-singlet $s$-wave SC/noncollinear ferromagnetic metal (NCF)/DN junction (Sec. II B).
The Hamiltonian for the two-dimensional SC/FR/DN junction on a two-dimensional square lattice [Fig. 1] is
\[ H_l = H_l + H_{SC} + H_{FR} + H_{DN,l}, \]
\[ H_l = -t \sum_{\langle i,j \rangle, \sigma} (c_{i,j}^\sigma c_{j,i}^\sigma + H.c.), \]
\[ H_{SC} = -\mu_{SC} \sum_{j_x \leq 0, j_y} n_{j_x,j_y} + \Delta \sum_{j_x \leq 0, j_y} (c_{j_x,j_y}^\dagger c_{j_x,j_y}^\dagger + H.c.), \]
\[ H_{FR} = \sum_{1 \leq j_x \leq L_{FR}, j_y, \alpha, \beta} \left[ h(\sigma_x)_{\alpha,\beta} - \mu_{FR}(\sigma_0)_{\alpha,\beta} \right] c_{j_x,j_y}^\dagger c_{j_x,j_y}^\dagger \]
\[ + i\lambda \sum_{1 \leq j_x \leq L_{FR}, j_y, \alpha, \beta} \left[ c_{j_x,j_y}^\dagger (\sigma_y)_{\alpha,\beta} c_{j_x,j_y}^\dagger + H.c. \right] \]
\[ + \lambda \sum_{1 \leq j_x \leq L_{FR}, j_y, \alpha, \beta} \left[ c_{j_x,j_y}^\dagger (\sigma_z)_{\alpha,\beta} c_{j_x,j_y}^\dagger + H.c. \right], \]
\[ H_{DN,l} = \sum_{L_{FR} \leq j_x \leq L_{FR} + L_{DN}, j_y, \sigma} (V_{j_x,l} - \mu_{DN}) n_{j_x,j_y}, \]
with \( n_{j_x,j_y} = c_{j_x,j_y}^\dagger c_{j_x,j_y} \). Here, \( c_{j_x,j_y}^\dagger \) (\( c_{j_x,j_y} \)) is an annihilation (creation) operator on the \( j \)-th site with spin \( \sigma \), \( t \) is a hopping integral, \( \mu_{SC} \) is a chemical potential in the s-wave SC region, \( \Delta \) is an s-wave pair potential, \( h \) is an exchange field, \( \mu_{FR} \) is a chemical potential in the FR region, \( \lambda \) is a Rashba spin-orbit coupling, \( V_{j_x,l} \) is an impurity potential in the DN region, \( \mu_{DN} \) is a chemical potential in the DN region, and \( \sigma_0, \sigma_x, \sigma_y, \sigma_z \) is a Pauli matrix in the spin space.

We use a lattice constant as a unit of length, \( (i,j) \) in Eq. 2 denotes a sum of nearest neighbor pairs, and \( e_{x(y)} \) denotes the unit vector in the \( x(y) \)-direction. As a random potential \( V_{j_x,l} \), we use uniformly distributed random number ranging from \(-t\) to \(t\) for each \( j \) and \( l \). The index \( l \) denotes the \( l \)-th impurity sample. To calculate physical quantities, we averaged over the impurity samples from \( l = 1 \) to \( l = N_{sample} \). We impose the periodic boundary condition in the \( y \)-direction with \( y \)-sites and the open-boundary condition at \( j_x = L_{FR} + L_{DN} \) in the \( x \)-direction.

### B. SC/NCF/DN junction

The Hamiltonian for the two-dimensional SC/NCF/DN junction [Fig. 2] is
\[ H_l = H_l + H_{SC} + H_{NCF} + H_{DN,l}, \]
with
\[ H_{NCF} = \sum_{1 \leq j_x \leq L_{NCF}, j_y, \alpha, \beta} \left[ \hat{h}_{j_x} - \mu_{NCF}(\sigma_0)_{\alpha,\beta} c_{j_x,j_y}^\dagger c_{j_x,j_y} \right], \]
\[ \hat{h}_{j_x} = [\tilde{\sigma}_x \sin(j_x - 1) \theta + \tilde{\sigma}_z \cos(j_x - 1) \theta] \]

The schematic picture of the direction of the magnetic field is shown in Fig. 2. We also impose the periodic boundary condition in the \( y \)-direction with \( y \)-sites and the open-boundary condition at \( j_x = L_{NCF} + L_{DN} \) in the \( x \)-direction.

In the following, we set \( \mu_{SC} = \mu_{FR} = \mu_{NCF} = \mu_{DN} = -t, \Delta/t = 0.1, L_{FR} = L_{NCF} = 5 \) and \( L_{DN} = 50 \) throughout this paper.

### C. LDOS and Cooper pair amplitude

In order to clarify the emergence of spin-triplet pairings and their manifestation, we calculate the local density of states (LDOS) and the anomalous Green function. We mainly focus on the physical quantities at the center of the DN: \( j_x = L_{FR} + L_{DN}/2 \) for the SC/FR/DN junction and \( j_x = L_{NCF} + L_{DN}/2 \) for the SC/NCF/DN junction.

The Green functions \( \hat{G}_l(\tilde{z}) \) of the systems are defined as
\[ \hat{G}_l(\tilde{z}) = (\tilde{z} - H_l)^{-1}, \]
with \( \tilde{z} = E + i\eta \) (small positive infinitesimal \( \eta \)) for the retarded Green function and \( \tilde{z} = i\omega_n \) for the Green function with Matsubara frequency \( \omega_n = (2n+1)\pi/\beta \) with inverse temperature \( \beta \) and \( n \in \mathbb{Z} \) representation. Here, the index \( l \) stands for the \( l \)-th impurity sample. The Green functions are calculated by using the recursive Green function
method \cite{42}. The LDOS is obtained from the normal part of the Green function:

$$\rho_j(E) = -\frac{1}{N_{\text{sample}}\pi} \sum_{l=1}^{N_{\text{sample}}} \text{ImTr}[PG_l,j_j(E + i\eta)],$$

(10)

with $\eta/t = 10^{-3}$,

$$\hat{G}_l(z) = \begin{pmatrix} G_l(\tilde{z}) & F_l(\tilde{z}) \\ \bar{F}_l(\tilde{z}) & \bar{G}_l(\tilde{z}) \end{pmatrix},$$

(11)

and the $j$-th lattice site. Here, $P$ is a projection on the particle space: $P = (\bar{\tau}_0 + \tau_z)/2$ with a Pauli matrix $\tau_{0,x,y,z}$ in the particle-hole space. We discuss the averaged value of the LDOS in the $y$-direction

$$\bar{\rho}_j(y) = \frac{1}{L_y} \sum_{j_y=1}^{L_y} \rho_j(E).$$

(12)

In the result section, we show the LDOS normalized by zero energy LDOS for a normal metal (N)/FR/DN or an N/NCF/DN junction. Here, the Hamiltonian for the normal metal is $H_l$ with $\Delta = 0$, and the other parameters are the same as the parameters for SC/FR/DN and SC/NCF/DN junctions, respectively. We denote the LDOS for the N/FR/DN or the N/NCF/DN junction as $\bar{\rho}_j,N(E)$.

The Cooper pair amplitude is given by the anomalous Green functions $[\text{Eqs. (15) and (16)}]$, and even-frequency spin-singlet Cooper pair amplitude $[\text{F}_l,s(\tilde{z})]$ given by Eq. (16).

Also, we calculate the following quantity \cite{46, 47}:

$$Q_{l,\alpha,j}(\tilde{z}) = i [\bar{f}_{l,j,j}(\tilde{z}) \times \bar{f}_{l,j,j}(\tilde{z})]_\alpha,$$

(17)

with $\bar{f}_{l,j,j}(\tilde{z}) = (f_{l,x,j,j}(\tilde{z}), f_{l,y,j,j}(\tilde{z}), f_{l,z,j,j}(\tilde{z}))$. $Q_{l,\alpha,j}(\tilde{z})$ is by definition a real quantity and expresses the equal (polarized) spin-triplet component of the Cooper pair amplitude: e.g.,

$$Q_{l,z,j}(\tilde{z}) = -\frac{1}{2} |F_{l,\uparrow\uparrow,j,j}(\tilde{z})|^2 - |F_{l,\downarrow\downarrow,j,j}(\tilde{z})|^2 .$$

(18)

We discuss the averaged value of $Q_{l,j}(\tilde{z}) = (Q_{l,x,j}(\tilde{z}), Q_{l,y,j}(\tilde{z}), Q_{l,z,j}(\tilde{z}))$:

$$\bar{Q}_{l,j}(\tilde{z}) = \frac{1}{L_y N_{\text{sample}}} \sum_{l=1}^{N_{\text{sample}}} \sum_{j_y=1}^{L_y} Q_{l,j}(\tilde{z}).$$

(19)

From the definition of $Q_{l,\alpha,j}(\tilde{z})$ [Eq. (17)], $Q_{l,\alpha,j}(\tilde{z})$ is a product of two odd-frequency spin-triplet Cooper pair amplitudes. Then, $Q_{l,\alpha,j}(\tilde{z})$ is an even function of frequency $\tilde{z}$.

### III. RESULTS

In this section, we discuss the LDOS [Eq. (11)], the anomalous Green functions [Eqs. (15) and (16)], and $\bar{Q}_{l,j}(\omega_n)$ [Eq. (17)]. The results for SC/FR/DN junction are shown in Sec. III A and those for SC/NCF/DN junction are shown in Sec. III B. In both junctions, we obtained the equal spin-triplet Cooper pair amplitude in the DN due to spin-flip scattering of Cooper pairs in the FR or the NCF regions.
A. SC/FR/DN junction

In this subsection, we consider the system shown in Fig. 1. In Fig. 3(a) we show several quantities at the center of the DN: $j_x = L_{\text{FR}} + L_{\text{DN}}/2$. In Fig. 3(a), the LDOS normalized by its normal state value is shown as functions of $h/t$ and $\lambda/t$. The normalized LDOS becomes the largest at approximately $(\lambda/t, h/t) = (0, 0.3)$. In Fig. 3(b), the absolute value of the spin-triplet component $|\bar{F}_{j_x,t}(i\omega_n)| = |\bar{F}_{j_x,s}(i\omega_n)|$ is shown. Qualitatively, $|\bar{F}_{j_x,t}(i\omega_n)|$ at a small frequency is very similar to the LDOS: the spin-triplet Cooper pair amplitude is generated when there is a zero energy peak in the LDOS. In Fig. 3(c), the absolute value of the spin-triplet component $|\bar{F}_{j_x,s}(i\omega_n)|$ with $\omega_n/t = 10^{-3}$ is shown. $|\bar{F}_{j_x,s}(i\omega_n)|$ has a small value where the zero energy LDOS has a large value.

In Fig. 4(a), the energy dependence of the normalized LDOS is shown for $(\lambda/t, h/t) = (0.4, 0.5)$, where the normalized LDOS is larger than unity at zero energy. We can see that there is a zero energy peak and corresponding to this zero energy state, the spin-triplet component of the anomalous Green function is largely enhanced toward zero frequency [Fig. 4(b)]. The absolute value of the spin-singlet component of the anomalous Green function also increases for small frequency but it approaches a finite value [Fig. 4(c)]. We also calculate the normalized LDOS and the anomalous Green function at $(\lambda/t, h/t) = (0.8, 0.5)$ where we observe a gap-like structure in the normalized LDOS (Appendix A).
value of the spin operator in Appendix C1 to which the equal spin-triplet Cooper pairs can contribute. We find that \( \mathbf{Q}_{j_x}(i\omega_n) \) and the expectation value of spin operator are almost independent since the quasiparticles also contribute to the spin polarization.

In Fig. 5, we show \( \mathbf{Q}_{j_x}(i\omega_n) \), which reflects the equal spin-triplet Cooper pair amplitude. In Fig. 5(a), we show \( |\mathbf{Q}_{j_x}(i\omega_n)| = \sqrt{\sum_{x,y,z} \mathbf{Q}^2_{j_x,z}}(i\omega_n) \). It is zero at \( h = 0 \) or \( \lambda = 0 \) axes. This means that both \( h \) and \( \lambda \) must be non-zero to generate the equal spin-triplet Cooper pairing [16][17]. We show each component of \( \mathbf{Q}_{j_x}(i\omega_n) \) as functions of \( h \) and \( \lambda \) in Figs. 5(b)–(d) and their spatial dependences for \( (\lambda/t,h/t) = (0.4,0.5) \) in Figs. 5(e)–(g). In Figs. 5(b)–(d), all the components have non-zero values in some regions but the \( y \)-component is very small. From Figs. 5(c) and (g), we can see that the equal spin-triplet Cooper pair amplitudes penetrate the DN. Also, the \( y \)-component [Figs. 5(f)] has a non-zero value but it approaches zero for a large number of impurity sample average. The \( N_{\text{sample}} \) and \( L_y \) dependences of the normalized LDOS, \( |\mathbf{F}_{j_x,t}(i\omega_n)| \), \( |\mathbf{F}_{j_x,s}(i\omega_n)| \), and \( |\mathbf{Q}_{j_x,t}(i\omega_n)| \) are discussed in Appendix B. We discuss an expectation

Here, we show the results for the SC/NCF/DN junction. The normalized LDOS shown in Fig. 6(a) exceeds unity in some regions due to the presence of the odd frequency spin-triplet pairings [Fig. 6(b)]. The normalized LDOS for the SC/FR/DN junction becomes the largest when \( \lambda = 0 \), but for the SC/NCF/DN junction, the normalized LDOS becomes the largest with non-zero \( \theta \). The spin-singlet Cooper pair amplitude becomes also small when the spin-triplet Cooper pair amplitude has a large value [Fig. 6(c)].
The normalized LDOS and the anomalous Green function at ωn → 0. We also show the normalized LDOS and the anomalous Green function at (θ/2π, h/t) = (0.15, 0.5) in Appendix A.

In Fig. 8(a), we show the energy dependence of the normalized LDOS at (θ/2π, h/t) = (0.075, 0.5) and see that it has a zero energy peak. In Figs. 8(b) and (c), frequency dependences of the spin-triplet and the spin-singlet Cooper pair amplitudes are shown, respectively. The spin-triplet Cooper pair amplitude is also largely enhanced toward zero frequency and the spin-singlet one has a non-zero value for ωn → 0. We also show the normalized LDOS and the anomalous Green function at (θ/2π, h/t) = (0.15, 0.5) in Appendix A.

The equal spin-triplet Cooper pair amplitudes are generated in the SC/NCF/DN junction (Fig. 8). In Fig. 8(a), the absolute value of Q_{j_\theta}(iω_\theta) with ω_\theta/t = 10^{-3} is shown and it is zero for θ = 0 or h = 0. The h = 0 case is trivial: there is no field and the Hamiltonian has spin rotational symmetry. Here, the equal spin-triplet Cooper pair amplitude has only x and z components [Figs. 8(b)–(d)] and it might correspond to the fact that the noncollinear spin structure in the NCF lies in the x–z plane 50, 51. We also show the j_x dependence of Q_{j_\theta}(iω_\theta) in Figs. 8(e)–(g). Similar to the SC/FR/DN junction, the equal spin-triplet Cooper pair amplitudes penetrate the DN. The y-component of Q_{j_\theta}(iω_\theta) is almost zero [Figs. 8(f)]. The dependences of these results on N_{sample} and L_y are discussed in Appendix B 2. We also discuss the expectation value of spin operator in Appendix C 2 and we do not find a direct relationship between the expectation value of spin operator and Q_{j_\theta}(iω_\theta).
IV. SUMMARY

In this paper, we show that the equal spin-triplet Cooper pair amplitude is generated from the spin-singlet $s$-wave superconductor in two kinds of junctions: the $s$-wave SC/ferromagnetic metal with Rashba spin-orbit coupling/diffusive normal metal junction and the $s$-wave SC/noncollinear ferromagnetic metal/diffusive normal metal junction. We have clarified the generation of the spin polarized triplet pairings in the diffusive normal metal due to spin-flip scattering in the magnetic regions. The emergence of the triplet pairings is manifested as a zero energy peak in the density of states.

In this paper, we have chosen the spin-singlet $s$-wave pairing as a symmetry of Cooper pair in the SC. If we choose spin-singlet $d$-wave pairing, zero energy Andreev bound states \[52–54\] and the resulting odd-frequency pairing \[37\] protected by the spectral bulk-edge correspondence are generated at the interface \[55\]. It is an interesting issue to study the proximity effect in $d$-wave superconductor junctions \[40\] in the presence of ferromagnetic metal with Rashba spin-orbit coupling or non-collinear ferromagnetic metal.

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Appendix A: Energy or frequency dependence of LDOS and anomalous Green function

In Figs. 9 (SC/FR/DN junction) and 10 (SC/NCF/DN junction), we show the energy dependence of the normalized LDOS, and $\omega_n$ dependences of $\bar{F}_{j,\uparrow \downarrow}(i\omega_n)$ and $\bar{F}_{j,\downarrow \uparrow}(i\omega_n)$ at the parameter where the normalized LDOS at zero energy is smaller than unity. In both graphs, we can see that there is a gap-like structure at zero energy for the normalized LDOS [Figs. 9(a) and 10(a)]. The spin-triplet and the spin-singlet components of the anomalous Green function are shown in Figs. 9(b), (c) and 10(b), (c). The absolute value of the spin-singlet component is larger than that of each spin-triplet component, and the spin-triplet components are linear functions at $\omega_n = 0$. 
FIG. 10. (a) The normalized LDOS is plotted as a function of $E$ (b) $\bar{F}_{x,t}(i\omega_n)$ is shown as a function of $\omega_n$. (c) $\text{Re}\bar{F}_{x,s}(i\omega_n)$ is shown as a function of $\omega_n$. $\text{Im}\bar{F}_{x,t,\uparrow\uparrow}(i\omega_n) = \text{Im}\bar{F}_{x,t,\downarrow\downarrow}(i\omega_n) = \text{Re}\bar{F}_{x,t,\uparrow\downarrow}(i\omega_n) + \text{Re}\bar{F}_{x,t,\downarrow\uparrow}(i\omega_n) = 0$ within numerical accuracy. $(\theta/2\pi, h/t) = (0.15, 0.5)$, $j_x = L_{\text{NC}} + L_{\text{DN}}/2$, $L_y = 100$ and $N_{\text{sample}} = 10^2$ samples averaged.

Appendix B: System size and sample number dependence

Here we show the $N_{\text{sample}}$ and $L_y$ dependences of the normalized LDOS, $|\bar{F}_{x,t}(i\omega_n)|$, $|\bar{F}_{x,s}(i\omega_n)|$, and $|\bar{Q}_{j_x}(i\omega_n)|$ in order to demonstrate the robustness of our results.

1. SC/FR/DN junction

We show the $L_y$ dependences of the normalized LDOS at zero energy, $|\bar{F}_{x,t}(i\omega_n)|$, and $|\bar{F}_{x,s}(i\omega_n)|$ at a small frequency in Fig. 11. Their system size dependences are not significant. In Figs. 12 and 13, we show $N_{\text{sample}}$ and $L_y$ dependences, respectively, at fixed $\lambda$ and $h$. The $N_{\text{sample}}$ and $L_y$ dependences are also not significant in these plots.

FIG. 11. The normalized zero energy LDOS, $|\bar{F}_{x,t}(i\omega_n)|$, and $|\bar{F}_{x,s}(i\omega_n)|$ are plotted as a function of $\lambda$ with $h/t = 0.5$ and $\omega_n/t = 10^{-3}$ for several values of $L_y$. $j_x = L_{\text{FR}} + L_{\text{DN}}/2$ and $N_{\text{sample}} = 10^2$ sample averaged.

FIG. 12. The normalized zero energy LDOS, $|\bar{F}_{x,t}(i\omega_n)|$, and $|\bar{F}_{x,s}(i\omega_n)|$ with $\omega_n/t = 10^{-3}$ are plotted as a function of $1/\sqrt{N_{\text{sample}}}$ for (a) $(\lambda/t, h/t) = (0.35, 0.5)$ and (b) $(0.7, 0.5)$. $j_x = L_{\text{FR}} + L_{\text{DN}}/2$ with $L_y = 100$ and 200.
In Fig. 14, we show the $|\tilde{Q}_{j_s}(i\omega_n)|$ for $(L_y, N_{sample}) = (100, 10^2), (100, 10^3), (200, 10^2)$, and $(200, 10^3)$ at $\omega_n/t = 10^{-3}$. Here we show the standard error as an error bar. The averaged values $|\tilde{Q}_{j_s}(i\omega_n)|$ have a large fluctuation for $0.1 \lesssim \lambda/t \lesssim 0.5$. It might correspond to the fact that the normalized LDOS at zero energy has large values close to these parameters. For $N_{sample} = 10^3$, $|\tilde{Q}_{j_s}(i\omega_n)|$ for $L_y = 100$ and $L_y = 200$ have almost the same value. In Fig. 15, we show the $N_{sample}$ dependence of $|\tilde{Q}_{j_s}(i\omega_n)|$ with $h/t = 0.5$. $|\tilde{Q}_{j_s}(i\omega_n)|$ for $L_y = 100$ and $200$ have almost the same value but for large $N_{sample}$. $|\tilde{Q}_{j_s}(i\omega_n)|$ with $L_y = 100$ has slightly smaller value at $\lambda/t = 0.35$ [Fig. 15(a)]. In Fig. 16, we show the $L_y$ dependence of $|\tilde{Q}_{j_s}(i\omega_n)|$ at $h/t = 0.5$ for $N_{sample} = 10^2$ and $10^3$. There are a bit large statistical errors for $N_{sample} = 10^2$ but the size of the error bar is sufficiently small for $L_y = 100$ and $N_{sample} = 10^3$.
2. SC/NCF/DN junction

We show the $N_{\text{sample}}$ and $L_y$ dependence for the SC/NCF/DN junction. We show the $L_y$ dependence of the normalized LDOS, $|F_{jx,t}(i\omega_n)|$, and $|F_{jx,s}(i\omega_n)|$ at a small frequency in Fig. 17. In this case, similar to the SC/FN/DN junction, their system size dependences are weak. In Figs. 18 and 19, we show $N_{\text{sample}}$ and $L_y$ dependences, respectively, at fixed $\theta$ and $h$. The $N_{\text{sample}}$ and $L_y$ dependences are also not so strong in these plots.

FIG. 16. $|Q_{jx}(i\omega_n)|$ is plotted as a function of $1/\sqrt{L_y}$ for (a) $(\lambda/t,h/t) = (0.35,0.5)$, and (b) $(0.7,0.5)$ with $\omega_n/t = 10^{-3}$. $j_x = L_{\text{FN}} + L_{\text{DN}}/2$.

FIG. 17. The normalized LDOS, $|F_{jx,t}(i\omega_n)|$, and $|F_{jx,s}(i\omega_n)|$ are plotted as a function of $\lambda$ with $h/t = 0.5$ and $\omega_n/t = 10^{-3}$ for several values of $L_y$. $j_x = L_{\text{NCF}} + L_{\text{DN}}/2$ and $N_{\text{sample}} = 10^2$ sample averaged.

FIG. 18. The normalized zero energy LDOS, $|F_{jx,t}(i\omega_n)|$, and $|F_{jx,s}(i\omega_n)|$ with $\omega_n/t = 10^{-3}$ are plotted as a function of $1/\sqrt{N_{\text{sample}}}$ for (a) $(\theta/2\pi,h/t) = (0.08,0.5)$ and (b) $(0.12,0.5)$. $j_x = L_{\text{NCF}} + L_{\text{DN}}/2$.
In Fig. 20 we show the $|\mathbf{Q}_{jx}(i\omega_n)|$ for $(L_y, N_{\text{sample}}) = (100, 10^2), (100, 10^3), (200, 10^2)$, and $(200, 10^3)$ at $\omega_n/t = 10^{-3}$. Here we also show the standard error as a error bar. We also observe large error at 0.06 $\lesssim \theta/2\pi \lesssim 0.09$. $|\mathbf{Q}_{jx}(i\omega_n)|$ for $N_{\text{sample}} = 10^2$ and $10^3$ have almost the same value but $L_y = 100$ and $200$ are different for $0.06 \lesssim \theta/2\pi \lesssim 0.09$. In Fig. 21 we show the $N_{\text{sample}}$ dependence of $|\mathbf{Q}_{jx}(i\omega_n)|$ with $h/t = 0.5$. $|\mathbf{Q}_{jx}(i\omega_n)|$ for $L_y = 100$ is smaller than that for 200 even for large $N_{\text{sample}}$ at least for $\theta/2\pi = 0.08$ [Fig. 21(a)]. In Fig. 22 we show the $L_y$ dependence of $|\mathbf{Q}_{jx}(i\omega_n)|$ at $(\theta/2\pi, h/t) = (0.08, 0.5)$ and (0.12, 0.5) for $N_{\text{sample}} = 10^2$ and $10^3$. The $L_y$ dependence is not small for $\theta/2\pi = 0.08$ [Fig. 22(a)] but we expect that the qualitative behaviors of the results do not change.
FIG. 22. $|Q_{jx}(i\omega_n)|$ is plotted as a function of $1/\sqrt{L_y}$ for (a) $\theta/2\pi = 0.08$, and (b) 0.12 with $h/t = 0.5$ and $\omega_n/t = 10^{-3}$.

In the following, we choose $\beta t = 10^3$ which reduces the computational cost. Here we use an IR basis to calculate the Matsubara frequency sum \([56]\) which reduces the computational cost. In the following, we choose $\beta t = 10^3$. We confirmed that the results are almost the same as the results with $\beta t = 10^4$ and then, $\beta t = 10^3$ is sufficiently small temperature.

1. SC/FR/DN junction

In Fig. 23 we show the expectation value of spin operator [(a)–(d)] and its spatial dependence [(e)–(g)] for the SC/FR/DN junction. $|\bar{s}_{jx}|$ has a small value in the region where the normalized LDOS at zero energy exceeds unity and has a large value [Fig. 3(a), see also Fig. 24]. This might be understood as follows: the LDOS at zero energy might have a maximum when both the spin up and the down components of the LDOS have large values and as a consequence, the expectation value of spin becomes small (minimum) in the DN.

$|\bar{s}_{jx}|$ becomes the non-zero at $\lambda = 0$ in some region where the equal spin triplet Cooper pair amplitude is zero. In general, both the quasiparticles and the Cooper pairs contribute to $\bar{s}_{jx}$. Thus, the non-zero value of $\bar{s}_{jx}$ at $\lambda = 0$ comes from spin polarization of the quasiparticles. Therefore, the non-zero value of $\bar{s}_{jx}$ is not direct evidence of the presence of the equal spin-triplet Cooper pair amplitude. $\bar{s}_{jx}$ has only $x$ and $z$-components similar to $Q_{jx}(i\omega_n)$ shown in Fig. 5(b)–(d). In Fig. 23(e)–(g), we show the spatial dependence of $\bar{s}_{jx}$ for the SC/FR/DN and the normal metal (N)/FR/DN junction (For N, we set the pair potential $\Delta = 0$ and the other parameters are the same as in the corresponding SC junctions). We can see that the presence of the SC is crucial for the non zero value of $\bar{s}_{jx,x}$ and $\bar{s}_{jx,z}$ in the DN.

Appendix C: Expectation value of spin operator

In this appendix, we discuss an expectation of spin operator:

$$\bar{s}_{jx,\alpha} = \frac{1}{L_y N_{\text{sample}}} \sum_{j_y=1}^{L_y} \sum_{n=1}^{N_{\text{sample}}} \text{Tr}[\hat{\sigma}_\alpha G_{l,j_y}(i\omega_n)],$$

(C1)

with $\alpha = x, y, z$ and the inverse of the temperature $\beta$. Here we use an IR basis to calculate the Matsubara frequency sum \([56]\) which reduces the computational cost. In the following, we choose $\beta t = 10^3$. We confirmed that the results are almost the same as the results with $\beta t = 10^4$ and then, $\beta t = 10^3$ is sufficiently small temperature.
2. SC/NCF/DN junction

We can see similar behaviors for the SC/NCF/DN junction [Figs. 25 and 26]. In this case, $|\bar{s}_{jx}|$ also becomes small in the region where the normalized LDOS at zero energy exceeds unity [Fig. 25(a) and Fig. 26]. Also, $\bar{s}_{jx}$ penetrates the DN due to the superconducting proximity effect [Figs. 25(e)–(g)].
FIG. 26. The normalized zero energy LDOS (left y-axis) and $|s_{jx}|$ (right y-axis) are plotted as a function of (a) $h/t$ for $\theta/2\pi = 0.05$, and (b) $\theta/2\pi$ for $h/t = 0.5$ at $j_x = L_{SCF} + L_{DN}/2$. $t\beta = 10^3$, $L_y = 100$ and $N_{\text{sample}} = 10^2$ sample averaged.

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