NEW IDEAS FOR TRANSVERSE SPIN ASYMMETRIES IN INCLUSIVE REACTIONS

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We will give convincing arguments to view transverse spin asymmetries in inclusive reactions, as very exciting observables, although they were considered as irrelevant for a long time. We look forward to new high energy data in the near future.

1 Motivation

Let us consider the inclusive reaction $pp \rightarrow cX$, with the center-of-mass (c.m.) energy $\sqrt{s}$ and where $c = \pi, \gamma, \Lambda, \text{jet}$, etc, with Feynman variable $x_F$ and transverse momentum $p_T$. We will essentially discuss the relevance of two observables which can be measured in this reaction, when the initial protons are transversely polarized, namely the single-spin asymmetry (SSA), defined as

$$A_N^c(\sqrt{s}, x_F, p_T) = \frac{d\sigma^\uparrow_c - d\sigma^\downarrow_c}{d\sigma^\uparrow_c + d\sigma^\downarrow_c}$$

and the double-spin asymmetry (DSA), defined as

$$A_{NN}^c(\sqrt{s}, x_F, p_T) = \frac{d\sigma^{\uparrow\uparrow}_c - d\sigma^{\uparrow\downarrow}_c}{d\sigma^{\uparrow\uparrow}_c + d\sigma^{\uparrow\downarrow}_c},$$

sometimes also denoted by $A_{TT}$. Clearly, it is legitimate to ask why these spin observables are important at high energy and what is the appropriate kinematic region $x_F, p_T$, where they should be best investigated, both theoretically and experimentally. For the SSA in $pp \rightarrow \pi X$, there are interesting data at $\sqrt{s} = 19.4\text{GeV}$ from FNAL [1] and for $\pi^0$ production at $\sqrt{s} = 200\text{GeV}$ from BNL-RHIC [2]. We will briefly recall the QCD mechanisms which have been proposed to explain these data. We will also consider the case of the $W^\pm$ production and we will show the usefulness of positivity for spin observables.

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2 Leading-twist QCD mechanisms for SSA and $W^\pm$ production

By using the generalized optical theorem, one can write

$$A_N^c d\sigma = \text{Im}[f_+^* f_-],$$

(3)

where $d\sigma = d\sigma_+^c + d\sigma_-^c$ is the corresponding unpolarized inclusive cross section. It is described by means of $f_+$, the forward non-flip $3 \rightarrow 3$ helicity amplitude $ab\bar{c}_\lambda \rightarrow ab\bar{c}_\lambda$, where $\lambda = \pm$ is the same on both sides. Moreover $f_-$ is the forward flip amplitude $ab\bar{c}_\lambda \rightarrow ab\bar{c}_{-\lambda}$. In order to get a non-vanishing $A_N^c$, one needs a non-zero $f_-$ and furthermore it should have a phase difference with $f_+$. This point is important and has to be taken seriously, if we want to have a real understanding of the available experimental data. It is another way to say that a non-zero $A_N^c$ corresponds to a non-trivial situation, which reflects a high coherence effect among many different inelastic channels. However according to naive parton model arguments one expects, $A_N^c = 0$, but several possible mechanisms have been proposed recently to generate a non-zero $A_N^c$. They are based on the introduction of a transverse momentum ($k_T$) dependence of either the distribution functions $q(x, k_T)$, for the Sivers effect [3] or of the fragmentation function $D_q(z, k_T)$, for the Collins effect [4]. These leading-twist QCD mechanisms have been used for a phenomenological study of this SSA [5] and higher-twist effects have been also considered [6, 7]. Once we have identified these two fundamental leading-twist QCD mechanisms to generate SSA, the Sivers and Collins effects, in order to study them in more detail it is important to be able to discriminate between them. One way to achieve this is to consider weak interaction processes, as proposed in Ref. [8]. We recall that with the Collins mechanism, the SSA is obtained from the transversity distribution function $h_1^q$ of a quark of the initial polarized hadron, convoluted with the Collins, $k_T$ - dependent fragmentation function. However in weak interaction processes, such as neutrino DIS on a polarized target or $W$ production in polarized hadron-hadron collisions, since the charged current only couples to quarks of one chirality, $h_1^q$ decouples and thus the observed SSA is not due to the Collins effect. This is not the case for the Sivers effect which will be able to generate a non-zero SSA, as we will see now.

Let us consider the inclusive production of a $W^+$ gauge boson in the reaction $pp \uparrow \rightarrow W^+ X$, where one proton beam is transversely polarized. In the Drell-Yan picture in terms of the dominant quark-antiquark fusion reaction, the unpolarized cross-section reads

$$d\sigma = \int dx_a d^2k_{T_a} dx_b d^2k_{T_b} |u(x_a, k_{T_a})d(x_b, k_{T_b}) + (u \leftrightarrow d)|d\hat{\sigma}^{ab\rightarrow W^+}.$$  (4)
Similarly, the SSA can be expressed such as

\[
\begin{aligned}
\int dx_a d^2k_{Ta} dx_b d^2k_{Tb} [\Delta^N u(x_a, k_{Ta}) d(x_b, k_{Tb})
- \Delta^N \bar{d}(x_a, k_{Ta}) u(x_b, k_{Tb})] d\sigma^{ab \to W^+}. \\
\end{aligned}
\]

(5)

For the Sivers functions [3] we have

\[
\Delta q^N(x, k_T) = q^+(x, k_T) - q^-(x, k_T)
= q^+(x, k_T) - q^+(x, -k_T) = \Delta q^N(x, k_T) S_p \cdot \hat{p} \times k_T.
\]

(6)

Here \(S_p\) denotes the transverse polarization of the proton of three-momentum \(p\) and \(\hat{p}\) is a unit vector in the direction of \(p\). A priori the \(k_T\)-dependence of all these parton distributions is unknown, but as a first approximation one can assume a simple factorized form for the distribution functions and take for example,

\[
q(x, k_T) = q(x) f(k_T),
\]

(7)

where \(f(k_T)\) is flavor independent, and a similar expression for the corresponding Sivers functions. In such a situation, it is clear that the SSA will also factorize and then it reads

\[
A^{W+}_N(\sqrt{s}, y, p_T) = H(p_T) A^+_N(\sqrt{s}, y) S_p \cdot \hat{p} \times p_T,
\]

(8)

where \(p_T\) is the transverse momentum of the \(W^+\) produced at the c.m. energy \(\sqrt{s}\) and \(H(p_T)\) is a function of \(p_T\), the magnitude of \(p_T\). Obviously if the outgoing \(W^+\) has no transverse momentum, the SSA will be zero, as expected.

In the above expression we have now

\[
A^+_N(\sqrt{s}, y) = \frac{\Delta^N u(x_a) \bar{d}(x_b) - \Delta^N \bar{d}(x_a) u(x_b)}{u(x_a) d(x_b) + d(x_a) u(x_b)},
\]

(9)

where \(y\) denotes the \(W^+\) rapidity, which is related to \(x_a\) and \(x_b\). Actually we have \(x_a = \sqrt{\tau e^y}\) and \(x_b = \sqrt{\tau e^{-y}}\), with \(\tau = M_W^2 / s\), and we note that a similar expression for \(A^{W-}_N\), the SSA corresponding to \(W^-\) production, is obtained by permuting \(u\) and \(d\). For the \(y\)-dependent part of the SSA, one gets for \(y = 0\)

\[
A^+ = \frac{1}{2} \left( \frac{\Delta^N u}{u} - \frac{\Delta^N \bar{d}}{\bar{d}} \right) \quad \text{and} \quad A^- = \frac{1}{2} \left( \frac{\Delta^N \bar{d}}{\bar{d}} - \frac{\Delta^N \bar{u}}{\bar{u}} \right)
\]

(10)

evaluated at \(x = M_W / \sqrt{s}\). Moreover a real flavor separation can be obtained away from \(y = 0\), since for \(y = -1\) one has

\[
A^+ \sim -\frac{\Delta^N \bar{d}}{\bar{d}} \quad \text{and} \quad A^- \sim -\frac{\Delta^N \bar{u}}{\bar{u}}
\]

(11)
evaluated at \( x = 0.059 \) and for \( y = +1 \) one has
\[
A^+ \sim \frac{\Delta^N u}{u} \quad \text{and} \quad A^- \sim \frac{\Delta^N d}{d}
\] (12)
evaluated at \( x = 0.435 \), at a c.m. energy \( \sqrt{s} = 500\text{GeV} \). So the region \( y \sim -1 \)
is very sensitive to the antiquark Sivers functions, whereas the region \( y \sim +1 \)
is sensitive to the quark Sivers functions. As well known, the parity violating-asymmetry \( A^{W \pm}_L \) allows the flavor separation of the quark helicity distributions
[9], similarly the measurement of \( A^{W \pm}_N \) is a practical way to separate the \( u \) and \( d \) quarks Sivers functions and their corresponding antiquarks \( \bar{u} \) and \( \bar{d} \). A straightforward interpretation of a non-zero \( A^{W \pm}_L \) is, in fact, a little bit more complicated because of its \( p_T \) - dependence, namely the factor \( H(p_T) \) in Eq. (8), which is unknown. It is possible to avoid this difficulty and to increase statistics by integrating over the \( p_T \) - range of the produced \( W^{\pm} \)'s. This is part of the reason why we cannot make any reliable prediction for these SSA, but the observation of significant effects will be the unambiguous signature for the presence of non-zero Sivers functions. The connection with the spin physics programme at BNL-RHIC and further considerations are also mentioned in Ref. [10].

### 3 What positivity can bring into this game?

The relevance of positivity in spin physics, which puts non-trivial model independent constraints on spin observables, has been largely discussed in Ref. [11]. These positivity conditions are based on the positivity properties of density matrix or Schwarz inequalities for transition matrix elements in processes involving several particles carrying a non-zero spin. This important point was emphasized by means of several examples chosen in different areas of particle physics, in particular, total cross sections in pure spin states, two-body exclusive reactions, polarized deep inelastic scattering, quark transversity distributions, off-shell gluon distributions, transverse momentum dependent distributions, single-particle inclusive reactions, polarized fragmentation functions, off-forward parton distributions, etc.. Very recently [12], new general positivity bounds were obtained, among the spin observables in a single particle inclusive reaction, where the two initial particles carry a spin-1/2. One of the consequences of this result is that, at rapidity \( y = 0 \), one has
\[
1 + A_{NN} \geq 2|A_N|,
\] (13)
for any value of \( \sqrt{s} \) and \( p_T \). In the case of \( pp \) collisions, for some specific reactions like \( pp \to \text{jet}X \) or \( pp \to \gamma X \), an estimate of \( A_{NN} \) for \( y \sim 0 \) was done
and it was found using Ref. [14], that $A_{NN} \sim 0$. As a result one gets a much stronger bound on the corresponding SSA, namely $|A_N| \leq 1/2$. This new non-trivial constraint will be very useful for model builders and to check future data on SSA.

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