Dynamics of Tachyonic Dark Matter

James M. Starke

The Catholic University of America
620 Michigan Avenue NE
Washington, D.C. 20064 USA

Ian H. Redmount

Department of Physics
Saint Louis University
3511 Laclede Avenue
St. Louis, Missouri 63103–2010 USA

(Dated: June 15, 2022)

Abstract

Usually considered highly speculative, tachyons can be treated via straightforward Einsteinian
dynamics. Kinetic theory and thermodynamics for a gas of “dark” tachyons are readily con-
structed. Such a gas exhibits density and pressure which, for the dominant constituent of a suitable
Friedmann-Robertson-Walker spacetime, can drive cosmic evolution with features both similar to
and distinct from those of a standard dark-energy/dark-matter model. Hence, tachyons might bear
further consideration as a cosmic dark-matter candidate.

PACS numbers: 03.30.+p, 05.20.Dd, 05.70.Ce, 95.35.+d, 95.36.+x, 98.80.-k

Keywords: tachyons, kinetic theory, equations of state, dark matter, dark energy, cosmology

*Electronic address: cuastrssa@gmail.com
†Electronic address: ian.redmount@slu.edu
I. INTRODUCTION

Tachyons—faster-than-light particles [1], with spacelike four-momenta—are usually considered either unphysical or rather fanciful speculation. (But see, e.g., Clay and Crouch [2].) The extant literature on tachyons is extensive and diverse. A more thorough review than can be accommodated here is in preparation. [3] It is possible, however, to construct straightforward, consistent Einsteinian dynamics for such particles. It is further possible to treat kinetic theory and thermodynamics for a gas of “dark” (noninteracting) tachyons. The density and pressure—i.e., the equation of state—for such a gas have interesting consequences if the gas is taken to be the dominant mass/energy constituent of a Friedmann-Robertson-Walker cosmological model. For suitable choices of parameters, such a model exhibits rapid expansion from an initial singularity, decelerating to a minimum expansion rate, followed by accelerating expansion. This is like the behavior of the dark-energy/dark-matter model which has become the standard model of cosmology, although the late-time behavior of the two models is different enough that they might be distinguished by observation or even experiment. While challenges remain, it may be that tachyons deserve further investigation as a potential dark-matter candidate.

The mechanics of a single tachyon interacting with simple electric and magnetic fields—the simplest interaction consistent with Einsteinian special relativity—is described in Sec. II following. As such dynamics undergirds all that follows, these elementary results are presented in some detail. The kinetic theory and thermodynamics of a gas of “dark” or non-interacting tachyons is worked out in Sec. III. The consequences of the resulting properties for a tachyon-dominated homogeneous and isotropic cosmological spacetime are shown in Sec. IV. Conclusions and further questions are discussed in Sec. V. For conciseness, units with speed of light \( c = 1 \) are used throughout.

II. SINGLE-TACHYON DYNAMICS

The simplest equation of motion, consistent with Einsteinian dynamics, for a single tachyon is the Lorentz force law [4]

\[
\frac{dp^\alpha}{d\tau} = qF_{\alpha\beta} u^\beta, \tag{2.1a}
\]
with Faraday field tensor

$$F^\alpha_\beta = \begin{pmatrix}
0 & E_x & E_y & E_z \\
E_x & 0 & B_z & -B_y \\
E_y & -B_z & 0 & B_x \\
E_z & B_y & -B_x & 0
\end{pmatrix}$$

(2.1b)

Here the tachyon has invariant mass \(m = i\mu\)—imaginary, because the tachyon has space-like four-momentum—real electric charge \(q\), and “proper time” \(d\tau = i\,d\lambda\), with \(\lambda\) a real affine parameter on the spacelike world line of the tachyon. With four-momentum \(p^\alpha = i\mu\,u^\alpha\), this equation can be written

$$\frac{dp^\alpha}{d\lambda} = \frac{q}{\mu} F^\alpha_\beta p^\beta,$$

(2.2)

entirely in terms of real quantities. To illustrate the unambiguously consistent Einsteinian dynamics of tachyons underlying the rest of the results in this paper, several elementary solutions of this equation are presented in detail, following.

A. Tachyon in electric field—one dimension

For a tachyon moving in the \(x\) direction, with a uniform electric field in the \(x\) direction, equation of motion (2.2) reduces to

$$\frac{d\mathcal{E}}{d\lambda} = \frac{q E_x}{\mu} p_x$$

(2.3a)

$$\frac{dp_x}{d\lambda} = \frac{q E_x}{\mu} \mathcal{E}$$

(2.3b)

with \(\mathcal{E} = p^0\) the energy and \(p_x = p^1\) the momentum of the tachyon. The solution to these, choosing \(\lambda = 0\) at \(\mathcal{E} = 0\) and \(p_x = \mu\), is

$$\mathcal{E} = \mu \sinh(\alpha \lambda)$$

(2.4a)

$$p_x = \mu \cosh(\alpha \lambda)$$

(2.4b)

with \(\alpha \equiv q E_x / \mu\). With

$$\mathcal{E} = \pm \frac{\mu}{(v^2 - 1)^{1/2}}$$

(2.5a)

$$p_x = \frac{\mu v}{(v^2 - 1)^{1/2}}$$

(2.5b)
This solution corresponds to
\[ v = \coth(\alpha \lambda) \] (2.6a)
and trajectory
\[ t(\lambda) = \frac{1}{\alpha} \cosh(\alpha \lambda) \] (2.6b)
\[ x(\lambda) = \frac{1}{\alpha} \sinh(\alpha \lambda). \] (2.6c)

For \( \alpha > 0 \), this is the upper branch of an hyperbola, describing a tachyon coming in from \( x \to -\infty \) at near light speed with negative energy (moving backward in time), accelerating to infinite speed and zero energy, then decelerating to near light speed with increasing positive energy, moving forward in time and toward \( x \to +\infty \). For \( \alpha < 0 \) this is the lower branch of the hyperbola, describing a tachyon coming in from \( x \to -\infty \) at near light speed, moving forward in time with positive energy, accelerating to infinite speed and zero energy, then decelerating to near light speed, moving toward \( x \to +\infty \) with negative energy, backward in time.

The Feinberg [1] interpretation of these trajectories imposes a quantum-field-theoretic description of the motion directly on the classical solution: The upper branch of the hyperbola describes the production of a tachyon/antitachyon pair at \( t = 1/\alpha \) and \( x = 0 \), each at zero energy. The tachyon moves rightward, decelerating toward light speed, while the antitachyon moves leftward, also decelerating toward light speed. Each particle maintains zero total energy: energy and momentum are both conserved. The lower branch of the hyperbola describes the corresponding tachyon/antitachyon annihilation process.

**B. Tachyon in electric field—two dimensions**

With two electric-field components and two momentum components, the equation of motion takes the form
\[ \frac{d\xi}{d\lambda} = \frac{q}{\mu} \mathbf{E} \cdot \mathbf{p} \] (2.7a)
\[ \frac{dp_x}{d\lambda} = \frac{q}{\mu} E_x \xi \] (2.7b)
\[ \frac{dp_y}{d\lambda} = \frac{q}{\mu} E_y \xi. \] (2.7c)
These imply
\[ \frac{d^2 \mathcal{E}}{d \lambda^2} = \alpha^2 \mathcal{E}, \]  
with, here,
\[ \alpha \equiv \frac{q}{\mu} (E_x^2 + E_y^2)^{1/2}. \]

These have solution
\[ \mathcal{E} = \mathcal{E}_0 \cosh(\alpha \lambda) + \frac{q (\mathbf{E} \cdot \mathbf{p})_0}{\mu \alpha} \sinh(\alpha \lambda) \]  
\[ p_x = \frac{q^2 (\mathbf{E} \cdot \mathbf{p})_0 E_x}{\mu^2 \alpha^2} \cosh(\alpha \lambda) + \frac{q \mathcal{E}_0 E_x}{\mu \alpha} \sinh(\alpha \lambda) \]  
\[ + p_x^{(0)} \]  
\[ p_y = \frac{q^2 (\mathbf{E} \cdot \mathbf{p})_0 E_y}{\mu^2 \alpha^2} \cosh(\alpha \lambda) + \frac{q \mathcal{E}_0 E_y}{\mu \alpha} \sinh(\alpha \lambda) \]  
\[ + p_y^{(0)} \]  
(2.9a–2.9c)

This result describes three distinct types of motion.

1. **Degenerate case:**

   The values \( \mathcal{E}_0 = 0 \) and \( (\mathbf{E} \cdot \mathbf{p})_0 = 0 \) imply \( \mathcal{E} = 0 \), \( p_x = p_x^{(0)} \), and \( p_y = p_y^{(0)} \). That is, the (infinite-speed) tachyon “evades” the electric field.

2. **One-dimensional case:**

   Values \( \mathcal{E}_0 = 0 \), \( p_y^{(0)} = 0 \), and \( E_y = 0 \) imply that solution (2.9a)-(2.9c) reduces to solution (2.4a)-(2.4b), as expected.

3. **Simplest two-dimensional case:**

   Values \( \mathcal{E}_0 > 0 \), \( p_y^{(0)} = 0 \), \( E_x = 0 \), and \( E_y > 0 \) imply
\[ \mathcal{E} = \mathcal{E}_0 \cosh(\alpha \lambda) \]  
\[ p_x = p_x^{(0)} \]  
\[ p_y = \mathcal{E}_0 \sinh(\alpha \lambda). \]

(2.10a–2.10c)
These correspond to (ordinary) velocities

\[ v_x = v_0 \text{ sech}(\alpha \lambda) \]  
\[ (2.11a) \]

\[ v_y = \text{tanh}(\alpha \lambda) \]  
\[ (2.11b) \]

\[ v = [1 + (v_0^2 - 1) \text{sech}^2(\alpha \lambda)]^{1/2}, \]  
\[ (2.11c) \]

so the tachyon asymptotically approaches the speed of light from above. This corresponds to trajectory

\[ t(\lambda) = \frac{E_0}{\mu \alpha} \text{ sinh}(\alpha \lambda) \]  
\[ (2.12a) \]

\[ x(\lambda) = E_0 v_0 \frac{\mu}{\lambda} \]  
\[ (2.12b) \]

\[ y(\lambda) = \frac{E_0}{\mu \alpha} \text{ cosh}(\alpha \lambda). \]  
\[ (2.12c) \]

Solutions (2.4a)–(2.4b) show that a tachyon pushed in the direction it is moving slows down, approaching the speed of light from above with increasing energy. But solutions (2.12a)–(2.12c) show that a tachyon pushed perpendicular to its direction of motion does not “back up”; it accelerates in the direction it is pushed.

C. Tachyon in magnetic field

For a tachyon in a uniform, purely magnetic field, say in the \( z \) direction, equation of motion (2.1a) reduces to

\[ \frac{d\mathcal{E}}{d\lambda} = 0 \]  
\[ (2.13a) \]

\[ \frac{dp_x}{d\lambda} = \frac{qB_z}{\mu} p_y \]  
\[ (2.13b) \]

\[ \frac{dp_y}{d\lambda} = -\frac{qB_z}{\mu} p_x \]  
\[ (2.13c) \]

\[ \frac{dp_z}{d\lambda} = 0. \]  
\[ (2.13d) \]

For suitable choice of the zero of \( \lambda \), this has general solution

\[ \mathcal{E} = E_0 \]  
\[ (2.14a) \]

\[ p_x = p_{\perp} \cos(\omega \lambda) \]  
\[ (2.14b) \]

\[ p_y = -p_{\perp} \sin(\omega \lambda) \]  
\[ (2.14c) \]

\[ p_z = p_z^{(0)}, \]  
\[ (2.14d) \]
with $p_\perp$ constant and

$$\omega = \frac{qB_z}{\mu}. \quad (2.14e)$$

The tachyon executes uniform helical (or circular, for $p_z^{(0)} = 0$) motion at constant speed about the magnetic field lines. Again the tachyon accelerates in the direction it is pushed, perpendicular to its trajectory, not the opposite direction.

The existence of electrically charged tachyons raises several conundra: Such a tachyon should emit Cerenkov radiation in vacuum. This should be readily detectable [1], and the associated loss of energy should cause the tachyon to accelerate toward infinite speed and zero energy. Furthermore, it would be energetically possible for a free photon to decay into a tachyon/antitachyon pair, if the pair coupled to the electromagnetic field. Unless some other principle suppresses this process, the propagation of photons across cosmic distances should severely constrain the existence of charged tachyons. It is possible, however, to sidestep such questions by considering electrically neutral or “dark” tachyons.

III. KINETIC THEORY AND THERMODYNAMICS OF TACHYON GAS

It is possible to determine thermodynamic properties and an equation of state for a tachyon gas, i.e., for an ensemble of (neutral, dark) tachyons of mass $m = i\mu$ and nonnegative energies $\mathcal{E}$ with probability distribution

$$\mathcal{P}(\mathcal{E}) = \mathcal{N} \exp(-\beta\mathcal{E}), \quad (3.1)$$

where $\mathcal{N}$ is a normalization factor to be determined and $\beta = 1/(k_B T)$ is the inverse temperature of the distribution. This can be done by evaluating the energy-momentum tensor

$$T^{\alpha\beta} = \frac{n}{\mu} \langle p^\alpha p^\beta \rangle, \quad (3.2)$$

with $n$ the number density of the particles in the rest frame of the ensemble. The tensor takes the matrix form

$$\begin{pmatrix}
\langle \mathcal{E}^2 \rangle & 0 & 0 & 0 \\
0 & \frac{1}{3} \langle p^2 \rangle & 0 & 0 \\
0 & 0 & \frac{1}{3} \langle p^2 \rangle & 0 \\
0 & 0 & 0 & \frac{1}{3} \langle p^2 \rangle
\end{pmatrix}, \quad (3.3)$$

angled brackets denoting ensemble averages.
A. Kinetic theory

Thermodynamic averages for this tachyon gas can be expressed as a related sequence of integrals. The normalization constant $N$ for the energy distribution is determined by the condition

$$1 = N \int_{\mu}^{\infty} 4\pi p^2 e^{-\beta\mathcal{E}} \, dp$$

$$= 4\pi \mathcal{N} \int_{0}^{\infty} (\mathcal{E}^2 + \mu^2)^{1/2} \mathcal{E} e^{-\beta\mathcal{E}} \, d\mathcal{E}$$

$$= -4\pi \mathcal{N} \frac{\partial}{\partial \beta} \int_{0}^{\infty} (\mathcal{E}^2 + \mu^2)^{1/2} e^{-\beta\mathcal{E}} \, d\mathcal{E} \ , \quad (3.4a)$$

where the energy/momentum relation $\mathcal{E}^2 - p^2 = -\mu^2$ implies $p \, dp = \mathcal{E} \, d\mathcal{E}$. The average energy is given by

$$\langle \mathcal{E} \rangle = 4\pi \mathcal{N} \int_{0}^{\infty} (\mathcal{E}^2 + \mu^2)^{1/2} \mathcal{E}^2 e^{-\beta\mathcal{E}} \, d\mathcal{E}$$

$$= 4\pi \mathcal{N} \frac{\partial^2}{\partial \beta^2} \int_{0}^{\infty} (\mathcal{E}^2 + \mu^2)^{1/2} e^{-\beta\mathcal{E}} \, d\mathcal{E} \ . \quad (3.4b)$$

The mean squared energy is

$$\langle \mathcal{E}^2 \rangle = 4\pi \mathcal{N} \int_{0}^{\infty} (\mathcal{E}^2 + \mu^2)^{1/2} \mathcal{E}^3 e^{-\beta\mathcal{E}} \, d\mathcal{E}$$

$$= -4\pi \mathcal{N} \frac{\partial^3}{\partial \beta^3} \int_{0}^{\infty} (\mathcal{E}^2 + \mu^2)^{1/2} e^{-\beta\mathcal{E}} \, d\mathcal{E} \ ; \quad (3.4c)$$

the mean squared momentum is found from $\langle p^2 \rangle = \langle \mathcal{E}^2 \rangle + \mu^2$. These can be expressed in forms

$$\mathcal{N} = \frac{\beta}{2\pi^2 \mu^2 \left[ H_2(\mu\beta) - N_2(\mu\beta) - \frac{2\mu\beta}{3\pi} \right]} \ , \quad (3.5a)$$

$$\langle \mathcal{E} \rangle = 3k_B T \frac{H_2(\mu\beta) - N_2(\mu\beta) - \frac{\mu\beta}{3} [H_1(\mu\beta) - N_1(\mu\beta)]}{H_2(\mu\beta) - N_2(\mu\beta) - \frac{2\mu\beta}{3\pi}} \ , \quad (3.5b)$$

$$\langle \mathcal{E}^2 \rangle = 12(k_B T)^2 \frac{H_2(\mu\beta) - N_2(\mu\beta) - \frac{\mu\beta}{4} [H_1(\mu\beta) - N_1(\mu\beta)]}{H_2(\mu\beta) - N_2(\mu\beta) - \frac{2\mu\beta}{3\pi}} - \mu^2 \ , \quad (3.5c)$$

and

$$\langle p^2 \rangle = 12(k_B T)^2 \frac{H_2(\mu\beta) - N_2(\mu\beta) - \frac{\mu\beta}{4} [H_1(\mu\beta) - N_1(\mu\beta)]}{H_2(\mu\beta) - N_2(\mu\beta) - \frac{2\mu\beta}{3\pi}} \ , \quad (3.5d)$$

The average energy is given by

$$\langle \mathcal{E} \rangle = 3k_B T \frac{H_2(\mu\beta) - N_2(\mu\beta) - \frac{\mu\beta}{3} [H_1(\mu\beta) - N_1(\mu\beta)]}{H_2(\mu\beta) - N_2(\mu\beta) - \frac{2\mu\beta}{3\pi}} \ . \quad (3.5b)$$

The mean squared energy is

$$\langle \mathcal{E}^2 \rangle = 4\pi \mathcal{N} \int_{0}^{\infty} (\mathcal{E}^2 + \mu^2)^{1/2} \mathcal{E}^3 e^{-\beta\mathcal{E}} \, d\mathcal{E}$$

$$= -4\pi \mathcal{N} \frac{\partial^3}{\partial \beta^3} \int_{0}^{\infty} (\mathcal{E}^2 + \mu^2)^{1/2} e^{-\beta\mathcal{E}} \, d\mathcal{E} \ ; \quad (3.4c)$$

the mean squared momentum is found from $\langle p^2 \rangle = \langle \mathcal{E}^2 \rangle + \mu^2$. These can be expressed in forms

$$\mathcal{N} = \frac{\beta}{2\pi^2 \mu^2 \left[ H_2(\mu\beta) - N_2(\mu\beta) - \frac{2\mu\beta}{3\pi} \right]} \ , \quad (3.5a)$$

$$\langle \mathcal{E} \rangle = 3k_B T \frac{H_2(\mu\beta) - N_2(\mu\beta) - \frac{\mu\beta}{3} [H_1(\mu\beta) - N_1(\mu\beta)]}{H_2(\mu\beta) - N_2(\mu\beta) - \frac{2\mu\beta}{3\pi}} \ , \quad (3.5b)$$

$$\langle \mathcal{E}^2 \rangle = 12(k_B T)^2 \frac{H_2(\mu\beta) - N_2(\mu\beta) - \frac{\mu\beta}{4} [H_1(\mu\beta) - N_1(\mu\beta)]}{H_2(\mu\beta) - N_2(\mu\beta) - \frac{2\mu\beta}{3\pi}} - \mu^2 \ , \quad (3.5c)$$

and

$$\langle p^2 \rangle = 12(k_B T)^2 \frac{H_2(\mu\beta) - N_2(\mu\beta) - \frac{\mu\beta}{4} [H_1(\mu\beta) - N_1(\mu\beta)]}{H_2(\mu\beta) - N_2(\mu\beta) - \frac{2\mu\beta}{3\pi}} \ , \quad (3.5d)$$

and
featuring Struve functions $H_1$ and $H_2$ and Neumann functions $N_1$ and $N_2$.

These results can be compared with those for a gas of ordinary—slower-than-light or *bradyonic*—particles with real invariant mass $m_b$. In this case the normalization constant is given by

$$N_b = \left( -4\pi \frac{\partial}{\partial \beta} \int_{m_b}^{\infty} (\mathcal{E}^2 - m_b^2)^{1/2} e^{-\beta \mathcal{E}} \, d\mathcal{E} \right)^{-1} = \frac{1}{4\pi m_b^3 K_2(m_b\beta)}, \quad (3.6a)$$

the average energy by

$$\langle \mathcal{E} \rangle_b = 4\pi N_b \frac{\partial^2}{\partial \beta^2} \int_{m_b}^{\infty} (\mathcal{E}^2 - m_b^2)^{1/2} e^{-\beta \mathcal{E}} \, d\mathcal{E}$$
$$= 3k_B T + m_b \frac{K_1(m_b\beta)}{K_2(m_b\beta)}, \quad (3.6b)$$

the mean squared energy by

$$\langle \mathcal{E}^2 \rangle_b = -4\pi N_b \frac{\partial^3}{\partial \beta^3} \int_{m_b}^{\infty} (\mathcal{E}^2 - m_b^2)^{1/2} e^{-\beta \mathcal{E}} \, d\mathcal{E}$$
$$= m_b^2 + 12(k_B T)^2 + \frac{3m_b k_B T K_1(m_b\beta)}{K_2(m_b\beta)}, \quad (3.6c)$$

and the mean squared momentum by

$$\langle p^2 \rangle_b = \langle \mathcal{E}^2 \rangle_b - m_b^2$$
$$= 12(k_B T)^2 + \frac{3m_b k_B T K_1(m_b\beta)}{K_2(m_b\beta)}, \quad (3.6d)$$

here with subscript $b$ distinguishing these bradyonic quantities, and featuring modified Bessel functions $K_1$ and $K_2$.

Results (3.5a)–(3.5d) and (3.6a)–(3.6d) are general, applicable at any temperature $T$. At low temperatures—$k_B T \ll \mu$ and $k_B T \ll m_b$—averages (3.5b)–(3.5d) take the limiting forms

$$\lim_{k_B T \to 0} \langle \mathcal{E} \rangle \sim 2k_B T \left[ 1 + O \left( \frac{(k_B T)^2}{\mu^2} \right) \right], \quad (3.7a)$$

$$\lim_{k_B T \to 0} \langle \mathcal{E}^2 \rangle \sim 6(k_B T)^2 \left[ 1 + O \left( \frac{(k_B T)^2}{\mu^2} \right) \right], \quad (3.7b)$$

and

$$\lim_{k_B T \to 0} \langle p^2 \rangle \sim \mu^2 + 6(k_B T)^2 \left[ 1 + O \left( \frac{(k_B T)^2}{\mu^2} \right) \right] \quad (3.7c)$$
for cold tachyons. Averages (3.6b)–(3.6d) take the familiar forms

$$\lim_{k_B T \to 0} \langle E \rangle_b \sim m_b + \frac{3}{2} k_B T + m_b O \left( \frac{k_B T}{m_b} \right)^2,$$

(3.8a)

$$\lim_{k_B T \to 0} \langle E^2 \rangle_b \sim m_b^2 + 2 m_b \left( \frac{3}{2} k_B T + \frac{15}{8} \frac{(k_B T)^2}{m_b} \right) + \frac{15}{4} (k_B T)^2 + m_b^2 O \left( \frac{k_B T}{m_b} \right)^3,$$

(3.8b)

and

$$\lim_{k_B T \to 0} \langle p^2 \rangle_b \sim 2 m_b \left( \frac{3}{2} k_B T + \frac{15}{8} \frac{(k_B T)^2}{m_b} \right) + \frac{15}{4} (k_B T)^2 + m_b^2 O \left( \frac{k_B T}{m_b} \right)^3 \quad (3.8c)$$

for cold bradyons. At high temperatures—$k_B T \gg \mu$ and $k_B T \gg m_b$—the averages approach limits

$$\lim_{k_B T \to \infty} \langle E \rangle \sim 3 k_B T - \frac{1}{2} \mu \left\{ \frac{\mu}{k_B T} - \frac{\mu^2}{(k_B T)^2} + O \left( \frac{\mu}{k_B T} \ln \left( \frac{\mu}{k_B T} \right) \right) \right\},$$

(3.9a)

$$\lim_{k_B T \to \infty} \langle E^2 \rangle \sim 12 (k_B T)^2 - \frac{5}{2} \mu^2 \left[ 1 + O \left( \frac{\mu}{k_B T} \right) \right],$$

(3.9b)

$$\lim_{k_B T \to \infty} \langle p^2 \rangle \sim 12 (k_B T)^2 - \frac{3}{2} \mu^2 \left[ 1 + O \left( \frac{\mu}{k_B T} \right) \right],$$

(3.9c)

$$\lim_{k_B T \to \infty} \langle E \rangle_b \sim 3 k_B T + m_b \left\{ \frac{m_b}{2 k_B T} + O \left[ \left( \frac{m_b}{k_B T} \right)^3 \ln \left( \frac{m_b}{k_B T} \right) \right] \right\},$$

(3.9d)

$$\lim_{k_B T \to \infty} \langle E^2 \rangle_b \sim 12 (k_B T)^2 \left[ 1 + O \left( \frac{m_b}{k_B T} \right)^2 \right],$$

(3.9e)

and

$$\lim_{k_B T \to \infty} \langle p^2 \rangle_b \sim 12 (k_B T)^2 \left[ 1 + O \left( \frac{m_b}{k_B T} \right)^2 \right].$$

(3.9f)

The behaviors of both hot tachyons and hot bradyons approach that of photons.

**B. Equations of state**

Energy-momentum tensor (3.3) implies tachyon-gas equation of state

$$P = \frac{1}{3} (\rho + \mu n),$$

(3.10)
with \( \rho \) the energy density and \( n \) the particle-number density of the gas. The corresponding bradyon-gas equation of state is

\[
P_b = \frac{1}{3}(\rho_b - m_b n_b) ,
\]

(3.11)
differing only in the sign of the mass term. The low-temperature limit of Eq. (3.10) is

\[
\rho \sim 18P \left( \frac{k_B T}{\mu} \right)^2 ;
\]

(3.12)
the pressure approaches finite limit \( \frac{1}{3} \mu n \), while the energy density approaches zero. In contrast, the low-temperature limit of Eq. (3.11) is

\[
P_b \sim \rho_b \frac{k_B T}{m_b} ,
\]

(3.13)
the familiar Ideal Gas Law. The high-temperature limits of the equations are \( P \sim \frac{1}{3} \rho \) and \( P_b \sim \frac{1}{3} \rho_b \); again, both tachyons and bradyons behave like photons at high temperatures.

**IV. TACHYON-DOMINATED COSMOLOGY**

The equation of state determines the behavior of the gas as a constituent of an homogeneous, isotropic spacetime. The Einstein Field Equations, applied to a Friedmann-Robertson-Walker (FRW) spacetime with scale factor (curvature radius) \( a(t) \) and comoving-observer time \( t \), can be cast in the forms

\[
3 \left( \frac{da}{dt} \right)^2 + k \left( \frac{a}{a_0} \right)^2 = 8\pi G \rho_{\text{tot}}
\]

(4.1a)
and

\[
\frac{d}{dt}(\rho_{\text{tot}} a^2) + p_{\text{tot}} \frac{d}{dt} a^3 = 0 .
\]

(4.1b)
Here \( \rho_{\text{tot}} \) is the total energy density and \( p_{\text{tot}} \) the total pressure of the cosmic fluid, \( k \in \{-1, 0, +1\} \) is the curvature parameter corresponding to an open (hyperbolic), spatially flat, or closed universe, respectfully, and \( G \) is the Newton gravitational constant.

The second of these equations is essentially the First Law of Thermodynamics for a comoving volume of fluid. Bradyonic equation of state (3.11) applied to this equation implies density

\[
\rho_b(a) = \left( \rho_{b0} - M_{b0} \right) \frac{a^4}{a_0^4} + \frac{M_{b0} a_0^3}{a^3} ,
\]

(4.2)
where $\mathcal{M}_b = m_b n_b$ denotes rest-mass density of the gas and subscript 0 identifies values at some chosen fiducial time, e.g., the present time. This is exactly as expected: The rest-mass density undergoes dilution as the spacetime expands, and varies as $1/a^3$; the kinetic-energy density undergoes both dilution and redshift, varying as $1/a^4$. As a source for Friedmann equation (4.1a), this engenders the same behavior as a “traditional” FRW model with matter and radiation constituents. (What is now called the “standard model” of cosmology also includes a “dark energy” term corresponding to a different equation of state, e.g., a vacuum-energy or cosmological-constant term with equation of state $p_\Lambda = -\rho_\Lambda$.)

But tachyonic equation of state (3.10) combined with Eq. (4.1b) implies density
\[ \rho(a) = \frac{(\rho_0 + \mathcal{M}_0) a_0^4}{a^4} - \frac{\mathcal{M}_0 a_0^3}{a^3}, \]
the invariant-mass density terms appear with the opposite sign. This difference is more significant than it may appear. This density will violate the Weak Energy Condition—become negative—for sufficiently large values of the scale factor $a$; this must be regarded as an analytic continuation of form (3.3). The Friedmann equation for a model dominated by this tachyonic density can be cast in the form
\[ \left( \frac{da}{dt} \right)^2 - \frac{8\pi G}{3} \left( \frac{(\rho_0 + \mathcal{M}_0) a_0^4}{a^2} - \frac{\mathcal{M}_0 a_0^3}{a} \right) = -k. \]
This can actually be interpreted in exactly the same way as the energy equation used to determine the speed of a car on a roller coaster, neglecting friction and drag: The terms on the left correspond to kinetic and potential energy, respectively; that on the right is the conserved total energy. The behavior of the “potential energy” term is illustrated in Fig. 1.

It can be parametrized as
\[ U(a) = \frac{2B}{a} - \frac{A^2}{a^2}, \]
with
\[ A \equiv \left( \frac{8\pi G (\rho_0 + \mathcal{M}_0) a_0^4}{3} \right)^{1/2} \]
and
\[ B \equiv \frac{4\pi G \mathcal{M}_0 a_0^3}{3}, \]
both parameters with units of length. The tachyonic potential diverges to $-\infty$ as $a \to 0^+$, crosses from negative to positive values at $a = A^2/(2B)$, reaches a peak at $a = A^2/B$ of height $B^2/(A^2)$, then decreases asymptotically to zero as $a \to +\infty$. Hence, all closed
FIG. 1: Friedmann-equation “potential energy” $U(a)$ for tachyonic (solid curve) and bradyonic (dashed curve) gases. The tachyonic curve corresponds to Eq. (4.5a), with $A = 1.0$ and $B = 0.9$, in the same arbitrary units as scale factor $a$. The bradyonic curve corresponds to the same parameter values, the $B$ term taking a negative sign. Closed, spatially flat, and open FRW models correspond to constant total “energies” -1, 0, and +1, respectively, on this figure.

($k = +1$) and spatially flat ($k = 0$) tachyon-dominated FRW spacetimes must expand from zero scale factor, reach a maximum scale, and recollapse symmetrically. The behavior of open ($k = -1$) models depends on the values of $A$ and $B$, i.e., on the energy and mass densities of the tachyon gas. For $B > A$, the solutions must either expand from $a = 0$ to a maximum and recollapse, or collapse from $a \to +\infty$ to a minimum and reexpand. For $B = A$, there are five possibilities: The spacetime can be static at $a = A = B$; or it can asymptotically approach this value from $a = 0$; or it can collapse to $a = 0$ from this value, departing arbitrarily slowly from the initial value; or it can collapse from $a \to +\infty$ asymptotically to $a = A = B$; or it can expand to $a \to +\infty$ from this value, again departing arbitrarily slowly from the initial value. For $B < A$, the solutions expand from $a = 0$; the expansion slows near $a = A^2/B$, then accelerates. This is precisely the behavior which astronomical
observations demand of the “standard model” of cosmology. These open tachyon-dominated models expand to $a \to +\infty$, the rate of expansion asymptotically approaching unity, i.e., $c$.

All of these behaviors are exhibited in exact solutions of the Friedmann Equation (4.4). In terms of conformal time $\eta$ defined via $d\eta = dt/a(t)$, closed models with $k = +1$ satisfy

$$a(\eta) = A \sin \eta - B(1 - \cos \eta)$$

$$t(\eta) = A(1 - \cos \eta) - B(\eta - \sin \eta)$$

for $0 \leq \eta \leq 2 \tan^{-1}(A/B)$. Spatially flat $k = 0$ models satisfy

$$a(\eta) = A\eta - \frac{1}{2}B\eta^2$$

$$t(\eta) = \frac{1}{2}A\eta^2 - \frac{1}{6}B\eta^3$$

for $0 \leq \eta \leq 2A/B$. Open $k = -1$ models with $B > A$ are described either by

$$a(\eta) = A \sinh \eta - B(\cosh \eta - 1)$$

$$t(\eta) = A(\cosh \eta - 1) - B(\sinh \eta - \eta)$$

for $0 \leq \eta \leq 2 \tanh^{-1}(A/B)$, or

$$a(\eta) = B + (B^2 - A^2)^{1/2} \cosh \eta$$

$$t(\eta) = B\eta + (B^2 - A^2)^{1/2} \sinh \eta$$

for $\eta \in (-\infty, +\infty)$. For $B = A$ the five possibilities are:

$$a(\eta) = B(1 - e^{-\eta})$$

$$t(\eta) = B[\eta - (1 - e^{-\eta})]$$

for $\eta \in [0, +\infty)$, or its time reversal

$$a(\eta) = B(1 - e^{\eta})$$

$$t(\eta) = B[\eta + (1 - e^{\eta})]$$

for $\eta \in (-\infty, 0]$; the static solution

$$a(\eta) = B$$

$$t(\eta) = B\eta$$
or

\begin{align}
   a(\eta) &= B(1 + e^{\eta}) \quad (4.12a) \\
   t(\eta) &= B(\eta + e^{\eta}) \quad (4.12b)
\end{align}

for \( \eta \in (-\infty, +\infty) \), or its time reversal

\begin{align}
   a(\eta) &= B(1 + e^{-\eta}) \quad (4.12c) \\
   t(\eta) &= B(\eta - e^{-\eta}) \quad (4.12d)
\end{align}

for \( \eta \in (-\infty, +\infty) \). For \( B < A \) the solution again takes the form

\begin{align}
   a(\eta) &= A \sinh \eta - B(\cosh \eta - 1) \quad (4.13a) \\
   t(\eta) &= A(\cosh \eta - 1) - B(\sinh \eta - \eta) \quad (4.13b)
\end{align}

for \( \eta \in [0, +\infty) \) in this case. It is this last model which most closely mirrors key features of the standard model.

Solution \((4.13a)-(4.13b)\) can readily be compared with a “standard” ΛCDM (cosmological-constant, i.e., dark-energy/cold-dark-matter) model. For a spatially flat \((k-0)\) model with vacuum energy density \(\rho_V\), radiation density \(\rho_r\), and matter density \(\rho_m\), Eq. \((4.1a)\) can be separated thus:

\[
   dt = \frac{a \, da}{H_0 \left( \Omega_{V0} a^4 + \Omega_{m0} a_0^3 a + \Omega_{r0} a_0^4 \right)^{1/2}},
\]

with Hubble parameter \( H \equiv (1/a)(da/dt) \), density ratios \( \Omega_V \equiv \rho_V/\rho_{\text{tot}} \), etc., and subscript \(0\) again denoting present-time values. This can be integrated:

\[
   t = H_0^{-1} \int_0^{a/a_0} \frac{x \, dx}{(\Omega_{V0} x^4 + \Omega_{m0} x + \Omega_{r0})^{1/2}} \quad (4.14b)
\]

to display the behavior \(a(t)\) of the model. In Fig. 2 an open tachyon-dominated model given by Eqs. \((4.13a)-(4.13b)\), with parameter values \(A = 0.930 \text{ Gpc} \) and \(B = 0.924 \text{ Gpc} \) (hence, \(a/c = 3.03 \text{ Gyr} \) and \(b/c = 3.01 \text{ Gyr} \)), is compared with a spatially flat ΛCDM model with parameters \(H^{-1}_0 = 1.37 \text{ Gyr} \), \(a_0 = 1.40 \text{ Gpc} \), and density ratios \(\Omega_{V0} = 0.7000\), \(\Omega_{m0} = 0.2998\), and \(\Omega_{r0} = 0.0002\). The parameter values are chosen to give the models “typical” features. Each model exhibits the \(a(t) \sim t^{1/2}\) behavior of radiation-dominated models at early times, and each displays the inflection indicating a change from decelerated
FIG. 2: Scale factors $a(t)$ for an open tachyon-dominated model (solid) and a spatially flat dark-energy-dominated $\Lambda$CDM model (dashed). Model parameters are as given in the text.

V. CONCLUSIONS

It is possible to construct straightforward Einsteinian dynamics and thermodynamics for tachyons. Tachyons subject to electromagnetic forces behave exactly as expected, although the existence of charged tachyons may raise several conundras. Kinetic theory for a gas of “dark” tachyons implies an equation of state similar in form to that for a gas of bradyons, save that the invariant-mass density term appears with positive rather than negative sign, corresponding to much greater pressure in a tachyon gas than in a bradyon gas of the same energy density and temperature.

For a constituent of a Friedmann-Roberston-Walker spacetime, the tachyonic equation of state implies a density which varies with scale factor in a manner similar to that of a bradyon
gas, but again with invariant-mass density contributions of the opposite sign. As a dominant source in such a spacetime, this density can give rise to cosmic evolution similar—but not identical—to that in models containing ordinary bradyonic matter and radiation. An open (hyperbolic) tachyon-dominated model can even evolve with decelerating, then accelerating expansion, similar—but not identical—to that of the “standard model” containing bradyonic matter, radiation, and “dark energy.” Whether a tachyon-dominated cosmological model is competitive with the standard model depends on whether detailed features of the tachyonic model can be matched to the extensive array of astronomical observations on cosmological scales, including supernova distances, the cosmic microwave background, and large-scale structure. Such a project is the subject of several ongoing works [6], [7], [8].

As noted, the evolution of a tachyon-dominated model can be similar but not identical to that of the standard model: The latter continues to accelerate exponentially in time, while the former asymptotically approaches expansion at a constant rate. This difference might even affect the dynamics of elementary-particle fields [9], making it possible to test the two models via experiments at the scale of such particles.

Of course, the actual existence of tachyons would raise many other questions. A proper, quantum-field-theoretic description of tachyons would involve a number of subtleties, such as tachyon “spin,” statistics, and interactions with other quantum fields. But such a possibility might not be quite as far fetched as it is usually regarded.

[1] G. Feinberg, Scientific American 222, 68 (1970).
[2] R. W. Clay and P. C. Crouch, Nature (London) 248, 28 (1974).
[3] C. M. Labrador and I. H. Redmount (2022), in preparation.
[4] C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation (W. H. Freeman and Company, San Francisco, 1973), pp. 99–101.
[5] E. A. Milne, Nature (London) 130, 9 (1932).
[6] A. Martin and I. H. Redmount (2019), arXiv:1904.07316, submitted to Astrophysical Journal.
[7] S. L. Kramer and I. H. Redmount (2022), in preparation.
[8] N. Gopal and I. H. Redmount (2022), in preparation.
[9] I. H. Redmount, Physical Review D 60, 104004 (1999).
“Rest mass” is a misnomer for tachyons, as they are never at rest in any inertial reference frame.