Second-order terminal sliding mode control based on perturbation estimation for nanopositioning stage

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Abstract: This study proposes a robust second-order terminal sliding mode control with perturbation estimation (2OTSMCPE) strategy with application to trajectory tracking control of the flexure-based nanopositioning system. The proposed controller advantages not only lie on its finite-time convergence but also can provide a high tracking precision with a chattering alleviation which is attend by employing a second-order sliding surface with the switching function. The model of the piezo-driven nanopositioning system is presented first. Second, the sliding variable is designed such as proportional–integral–derivative form to enhance the dynamic response of the control system. Then, a non-singular terminal sliding function (NTSM) is used to achieve the finite-time convergence of the linear sliding variable. Next, a perturbation estimation technique is integrated with the control structure for online estimation of the system uncertainties, thus the prior knowledge of the bounds of system uncertainties are not needed in the proposed control design. Afterwards, the theoretical analysis of the 2OTSMCPE with stability proof is investigated herein. Finally, the system performance with the proposed controller is experimentally verified. The results reveal that the 2OTSMCPE has stronger robustness and also has smoother control signals in comparison with both conventional sliding mode control and the NTSM controller.

1 Introduction

Micro/nanopositioning systems have been extensively employed in diverse applications where the high precision is essential. Commonly, these systems are driven by piezoelectric actuators (PEAs). Thus, PEAs are considered an essential part in such system due to their characteristics of managing low distance and fast response without mechanical issues, for instance, micromanipulators [1], atomic force microscopes [2] and many others. However, the main challenges with PEAs are the hysteresis effect and drift phenomena [3]. This non-linear behaviour is quite obvious when the PEAs are set in open-loop applications, and it is presented that such drawbacks greatly influence the stability and performance of these actuators in high precision positioning systems [4]. Thus, a proper control technique has to be used in order to account for these limitations and achieve the required performance. In literature, a huge number of control strategies have been proposed to cope with PEAs non-linear behaviour [4]. Owing to their characteristics of robustness and the simplicity of implementation, sliding mode control (SMC) strategy has been frequently employed for this task [5].

SMC is a very well-known robust application control method that has been extensively used for the control of uncertain systems. The SMC surface is designed to enforce the system state trajectories to reach the surface and to maintain on it persistently. Once the sliding mode is satisfied, the system state becomes insensitive to the parameters variation and external disturbance [6]. Owing to the above attractive features, SMC has been widely used in the tracking control of nanopositioning systems; see, for example [7, 8]. Despite the extraordinary SMC features, it has some drawbacks which may deter its practical usage. As a switching control, the chattering phenomenon would be the main drawback. Chattering appears in the control signal as high-frequency oscillations which may excite the unmodelled dynamics. Such undesirable oscillations degrade the system performance and significantly reduce the life span of the moving mechanical parts (i.e. actuators) [9]. In order to address the chattering phenomena in terms of its effect and treatment methods, considerable efforts have been done in the literature [10]. The boundary layer technique is initially proposed to handle the chattering effect and it was successfully employed for trajectory tracking applications; for instance, piezo-driven micro/nanopositioning systems [11], this technique is simply used the saturation function instead of the signum function to achieve a smooth control signal at the cost of robustness [12]. Another control strategy like the adaptive technique also was adopted to deal with the chattering effect [13, 14]; the essence of this control strategy is to make the control gain changing continuously based on the system uncertainties or perturbation magnitudes. However, in both these control methods, either the system robustness or the system performance will be sacrificed. Thus, to preserve the useful SMC feature and mitigate the chattering effect, the high order sliding mode control (HOSMC) was devised in [15], and it was successfully employed to address the trajectory tracking issues in high precision positioning applications [16].

The concept of HOSMC is to preserve the main advantages of conventional SMC (CSMC) and to move the switching action into the high order of the sliding variable (i.e. $\sigma$). For instance, the ‘sign’ function acts on the $r$th order SMC (i.e. $\sigma^r$) that would happen when the $\sigma^r = \sigma^r - 1 = \cdots = \sigma = \sigma = 0$ is satisfied. Hence, the control action will be smoothed, and the chattering effect will be eliminated inherently [15]. While the CSMCs are designed by using a linear sliding mode where the only asymptotic convergence of the tracking errors to zero can be achieved rather than finite-time convergence [17]. In order to overcome the aforementioned shortcoming, a new non-linear generation of SMC called a terminal sliding mode control (TSMC) was initially proposed by Zhong et al. [18]. Developed versions of TSMC are proposed by the authors of [19, 20], where a significant enhancement in terms of convergence rate and singularity (i.e. a faster convergence and singularity free) has been added to ordinary TSMC. Therefore, with such huge improvements, many researchers have been encouraged to apply these control methods for piezo-driven nanopositioning to attain a rapid response speed as well as high robustness against system uncertainties [21, 22].

However, TSMC is also suffering from the chattering issue [20]. In [23, 24], the TSMC and the HOTSMC techniques have been merged to realise the second-order TSMC (2OTSMC). Thus, with the 2OTSMC both chattering-free control signal and finite-
The author proposed a new design that builds on a 2OTSMC based on a certain issue. Employing the LMI condition can guarantee the state errors to be in the predictable bounds and the parameters of the controller known. Hence, the real-time implementation will be simpler and more accurate.

Unlike the previous work, the resultant 2OTSMCPE is completely robust control effort. These are the main merits of the proposed control structure named as the second-order terminal system to verify the effectiveness and ability of 2OTSMCPE over other controllers in terms of track following problem. The most distinguished contribution in this work is the proof of stability. Theoretically, the stability of the whole control system has been proved in the sense of Lyapunov and some remarks have been written for further exploitation. In addition, we have provided a guideline of the parameters selection of the proposed controller to attain optimal performance. The rest of the paper is organized as follows. Section 2 describes the model of PEA with parametric uncertainties. Section 3 elaborates the design of the proposed controller. Section 4 presents and discusses the implementation results of the 2OTSMCPE controller. Finally, Section 5 provides the conclusion of the paper.

2 Dynamic modelling of nanopositioning system

The internal design of nanopositioning systems is commonly built on a flexure-based driven by PEAs. The dynamic model of the nanopositioning stage will be elaborated in this section. Fig. 1 depicts the experimental setup of the PEA system. This platform consists of a lead zirconate titanate (PZT) micro-actuator and moving stage driven by the PZT with a ±12.5 μm moving range. Besides, there is a built-in high resolution (i.e. 14 nm) capacitive which used to feed the control system with a precise position signal (i.e. y).

Generally, the dynamic model of the PEA with non-linear hysteresis can be expressed as follows [30]:

\[ y = \frac{k_w}{s^2 + 2\zeta w_n s + w_n^2}u + h \]  

where \( y \) and \( h \) are the position output, control input and the non-linear hysteresis of the PEA, respectively. The model parameters \( \zeta \), \( \alpha_h \) and \( k \) represent the damping ratio, resonant frequency and plant gain, respectively. The non-linear hysteresis effect of the PEA is commonly considered as a bounded input disturbance [31]. Thus, the linear PEA model is considered in this work because it simplifies the control system design and implementation. In addition, the transfer function of (1) can be written in a linear differential equation form with bounded disturbance as follows:

\[ m\ddot{y} + a_2\dot{y} + a_1y = u + \Psi \]  

where \( m = 1/k_w \), \( a_1 = 2\zeta w_n \), \( a_2 = 1/k \) and the variable \( \Psi(t) \) represents lumped perturbation (i.e. hysteresis effect, external disturbance and modelling error) which is supposed to be bounded such that \( |\Psi(t)| \leq \Psi \). It should be mentioned that the values of \( \Psi(t) \) and \( \Psi \) are unknown. Thus, we will employ a proper estimation method in this work to estimate these parameters as will be illustrated later. Accordingly, the dynamic model (2) becomes

\[ m\ddot{y} + a_1\dot{y} + a_2y = u(t) + \dot{\Psi} + \Psi \]  

where \( \dot{\Psi}(t) \) is the perturbation estimation (PE) error which will be presented later and \( \dot{\Psi} \) is the online estimated value of perturbation. The identification of model parameters in (3) can be experimentally implemented by using frequency responses data. To this end, the system has been excited by a sinusoidal chirp signal and has a varying range of frequency (i.e. 10 Hz to 1 kHz). Then, the frequency responses of the output \( y \) are collected with a dynamic signal analyser (HP 35670A) which is used here as a specialist instrument. Finally, the identification toolbox from MATLAB Simulink is employed to identify the model parameters as follows: \( m = 3.2982 \times 10^{-3}, a_1 = 5.9683 \times 10^{-4} \) and \( a_2 = 0.33 \). Fig. 2 shows the identified linear model versus the measured model to demonstrate the accuracy of the identified model.

3 2OTSMCPE control design

The 2OTSMCPE control law is a combination of two controllers, which consists of a 2OTSMC controller and a PE. This section describes the design process of the two controllers.
Obviously, the first and second derivatives of \( y \) yields that the total time \( t \) follows:

\[
\sigma = s + y \dot{s} = 0
\]

where \( \sigma \) is the total time \( t \) from any initial time \( t_i \) to the time when \( s = 0 \) given by

\[
t_i = t_i + \frac{p}{r(p - q)} \int_{t_i}^{t}(y)^{(p-q)}
\]

This implies that the convergence can be ensured at finite-time \( t_i \) once \( \sigma = 0 \) and hence the sliding variable \( s \) and \( \dot{s} \) can reach 0 within \( t_i \). Proceeding to control design, we need to find the derivative of TSM function \( \sigma \) in (8) which can be written as follows:

\[
\sigma = \frac{q}{p} \dot{s} \dot{s}^{-1} \left[ \frac{q}{p} \dot{s}^{-1} + \dot{s} \right]
\]

Next, an equivalent controller \( u_{eq} \) can be derived by solving \( \sigma = 0 \). By considering nominal values of the model parameters in (2) with \( \Psi = 0 \), and replacing \( s \) by \( u_{eq} \). Then, substituting \( s \) from (7) into (11) and solving \( \sigma = 0 \) for \( u_{eq} \) yields

\[
u_{eq} = (\alpha y + \alpha y y) + \eta y
\]

where \( y \) is the second derivative of the desired reference. Furthermore, a robust controller \( u \) is designed to deal with the uncertainties, whose form is given by

\[
u_r = - \eta \int \text{sign}(\sigma) \, d\tau - \Psi
\]

where \( \Psi \) is the online perturbation estimator output as will be discussed in the next subsection, sign(·) is the signum function and \( \eta > 0 \) is the control gain given in (21). Thus, the final 2OTSMCPE controller is with the form of

\[
u = u_{eq} + u_r
\]

3.2 Perturbation estimation technique

In practical circumstances, the system uncertainties are impossible to be known accurately. Therefore, the control strategies are usually designed based on the upper bound of system perturbation. Due to overestimated bounds on system perturbations, such an assumption can cause to conservative feedback gains [32]. Some estimation methods have been proposed to overcome these limitations in control design. For instance, we can see the time-delayed estimators [33] and disturbance observer [34]. In the aforementioned methods, the low-pass filter (LPF) has to be used. However, the system performance and/or robustness will be compromised with the use of the LPF. Thus, the LPF should be designed properly. In [29], a new perturbation estimator (PE) was proposed where its design only relied on system states. Owing to its simplicity and flexibility in the design and implementation, the latter estimator has been used in many practical applications [8, 11]. Motivated by these advantages, we exploit the PE in this work to estimate the system perturbation. Hence, the proposed 2OTSMCPE controller is designed and implemented without predefined information of the system perturbation. The PE design presents in this section. Suppose that the non-linear uncertain system can be written in the following form:

\[
y^{(n)} = A(y_i, \ldots , y_m) + \Delta A(y_i, \ldots , y_m) + [B(y_i, \ldots , y_m) + \Delta B(y_i, \ldots , y_m)] + u(t) + d(t)
\]

where \( y_i = [y_i, \ldots , y^{(m)}] \in \mathbb{R}^n \) and \( u(t) \) is the control input and \( d(t) \) is the external disturbance. Then, system perturbation may be written by the following formula:

\[
\Psi = \Delta A + \Delta Bu(t) + d(t)
\]

It is proved that the online estimation of \( \Psi \) can be done by using the following equation [35]:

\[
\hat{\Psi}(t) = y^{(i)} - A - Bu(t - \tau)
\]

where \( \hat{\Psi}(t) \) and \( \Psi(t) \) are the estimated actual values of system perturbation, \( \tau \) is the sampling time, \( y^{(i)} \) indicates system state vectors, and \( u(t - \tau) \) is one sample delayed input. In a nanopositioning system, the sampling frequency is usually more than 10 kHz which is fast enough to guarantee that \( u(t - \tau) \approx u(t) \) is valid. It should be noted that the full state (i.e., \( y, \dot{y}, \gamma \)) is needed with PE implementation. Thus, a model-free robust exact differentiator [36] will be used for that purpose based on the only
measurable position signal (i.e. $y$). Hence, $\Psi$ in (2) can be easily estimated by applying (17) as
\[
\dot{\Psi} = m \dot{y} + a_i \ddot{y} + a_2 y - u(t - \tau)
\] (18)
where $\dot{\Psi}$ is the online estimated perturbation of nanopositioning platform. Thus, the difference between the actual perturbation $\Psi$ and estimated one $\dot{\Psi}$ can be introduced as
\[
\dot{\Psi} = \Psi - \dot{\Psi}.
\] (19)
Based on [14], it has been shown that the change rate of perturbation $\dot{\Psi}$ is bounded. Thus, in the view of bounded $\dot{\Psi}$ and (19), the following assumption can be deduced.

**Assumption 1**: The PE error rate change $\dot{\Psi}$ is bounded which can be expressed in the following formula:
\[
\|\dot{\Psi}\| < \kappa
\] (20)
where $\kappa$ is the bound of the rate of change of the PE error.
Consequently, the control gain in (13) must satisfy the following condition:
\[
\kappa < \eta
\] (21)
in order to ensure system stability.

3.3 Stability analysis

**Lemma 1**: Consider the PEA nanopositioning system (2) under the 2OITSMCPE control law defined by (14), where the PE output $\Psi$ is given by (18). Then, the PID-type sliding variable $s$ (5) approaches to zero in finite-time $t_f$. Consequently, the tracking error $e(t)$ in (4) will asymptotically converge to zero.

**Proof**: Let a Lyapunov candidate function be defined as
\[
V = \frac{1}{2} \sigma^2.
\] (22)
Taking the first derivative of (22) yields
\[
\dot{V} = \sigma \dot{\sigma}.
\] (23)
Substituting $\sigma$ (11) into (23) yields
\[
\dot{V} = \sigma_0 \left\{ \frac{q}{\gamma p} \sigma^{-\gamma p} \sigma + \sigma \right\}.
\] (24)
where $\sigma_0 = \gamma (p / q)^{s / r - 1}$.
By substituting $s$ from (7) into (24), we get
\[
\dot{V} = \sigma_0 \left\{ \frac{q}{\gamma p} \sigma^{-\gamma p} \sigma + \frac{d}{dt} \xi + \xi \dot{e} + \xi \ddot{e} \right\}.
\] (25)
From the definition of tracking error in (4), we can deduce that
\[
\frac{d}{dt} \xi = \frac{d}{dt} \left\{ - \frac{1}{m} \left[ a_i y + a_2 y - \Psi - \dot{\Psi} - u - \dot{y}_d \right] \right\}.
\] (26)
Thus, by combining the last two equations (25) and (26), we get
\[
\dot{V} = \sigma_0 \left\{ \frac{q}{\gamma p} \sigma^{-\gamma p} \sigma + \xi \dot{e} + \xi \ddot{e} \right\}
\]
\[
+ \frac{d}{dt} \left\{ - \frac{1}{m} \left[ a_i y + a_2 y + \Psi(t) + \dot{\Psi}(t) + u(t) - \dot{y}_d \right] \right\}.
\] (27)
Then, by substituting the control law of (14) into (27) with some basic algebraic operations, we can write (27) in the following form:
\[
\dot{V} = \sigma_0 \left\{ \frac{q}{\gamma p} \sigma^{-\gamma p} \sigma + \xi \dot{e} + \xi \ddot{e} \right\}
\]
\[
- \frac{1}{m} \left[ a_i y + a_2 y + \Psi(t) + \dot{\Psi}(t) + u(t) - \dot{y}_d \right]
\] (28)
Using Assumption 1 for (28) yields
\[
\dot{V} \leq \frac{\eta_1}{m} \left( \sigma \right)^2
\] (29)
\[
\leq - \frac{\eta_1}{m} \sigma^2 \left( \eta - \kappa \right).
\] (30)
Thus, the time derivative of the $V$ is negative definite (i.e. $\dot{V} \leq 0$) if the condition (21) is held, the control gain is designed such that $\eta \geq \kappa$. Hence, the proof of stability is completed. □

**Remark 1**: It is notable that $\eta$ has two possible cases:
- When $s \neq 0$, $\eta \geq 0$ always holds [19], as $\gamma > 0$, $p$ and $q$ are selected to be positive odd integers.
- When $s = 0$ and $s \neq 0$, it is proved that this point is not an attractor. Hence, the system trajectory will leave this point in finite-time [19].

**Remark 2**: From the equivalent control law in (12), it can be seen that the high derivative of the reference signal (i.e. $y_d$) is needed. Since the nanopositioning systems are usually used to track a predefined smooth signal. Thus, the assumption of $y_d$ is still valid.

3.4 Controllers for comparison

In order to conduct a comparative study, two additional controllers are designed and implemented on the real nanopositioning platform which are listed below:

- The first CSMC controller is simply designed based on the equivalent control principle. That is, solving $s = 0$ to get the equivalent controller $u_{eq}$, where $s$ is the PID-type sliding variable defined in (5) and we suppose that there are no disturbances or system uncertainties, then $u_{eq}$ can be written as
\[
u_{eq} = (a_i y + a_2 y) + m \ddot{y}_d - m \dot{y}_d \left( \dot{y} + \dot{y}_d \right).
\] (31)

By including the robust controller part, the final form of the CSMC can be expressed as
\[
u_{eq} = u_{eq} + \eta \nu_{eq} \text{sign}(s)
\] (32)
where $\eta > 0$ is the control gain, which has to be greater than the upper bound of the system uncertainties to ensure the system stability.

- The second controller is the non-singular terminal sliding function (NTSM) [19], which is designed as
\[
u_{ntsm} = u_{eq} + \eta \text{sign}(s)
\] (33)
where $\eta > 0$ is the control gain, $s$ represents the NTSM variable is given by $s = e + (1 / 2 p)^2$, with $1 < a < 2$, $\gamma > 0$. $u_{eq}$ is the equivalent control given by
\[
u_{eq} = a_i y + a_2 y + m \left( y_d - \frac{\gamma}{\alpha} \dot{y}_d \right)
\] (34)

**Remark 3**: It can be observed from (32), (33) and (14) that the three controllers have a switching part which is $\text{sign}(s)$. However, it is well-known that the sign function induces severe chattering.
issue. Thus, in the CSMC and NTSM, the boundary layer technique is used to alleviate this issue, where the sign will be replaced by a sat which is defined below, and hence, the robust term of these two methods is defined as \( \Psi \), where sat(s) is given by

\[
\text{sat}(s) = \begin{cases} 
\text{sign}(s), & \text{for } |s| > \Delta \\
\frac{s}{\Delta}, & \text{for } |s| \leq \Delta 
\end{cases}
\]

where \( \Delta > 0 \) is the boundary layer thickness, which is used to guarantee boundedness of \( s \). Meanwhile, in the 2OTS as the sign is not changed, as the first derivative of the control signal \( u \) in the 2OTS is used as a virtual control signal. Thus, the real control signal will be substantially smoothed by the integration part, and hence its chattering effect will be alleviated without the need for replacing the sign function. This is the main merit of the proposed 2OTS in comparison with others.

Remark 4: The control gains in CSMC and NTSM are designed based on the upper bound of system perturbation \( \Psi \), where the gains have to be greater than these bounds to guarantee the system stability. However, in 2OTS the gain is designed such that \( \eta \geq \kappa \). Indeed, the upper bound of the rate change of the PE error is much less than the actual perturbation \( \Psi \) and its upper bound [11]. Consequently, the conservative control gain issue will be avoided in 2OTS. Moreover, the prior knowledge of the system uncertainties is not required in the control design.

3.5 Parameter selection

The controller performance is determined by the selection of the parameters. Practically, the tracking performance is also affected by the measurement noise and the limited control efforts.

3.5.1 PID-CSMC parameters: The PID-CSMC surface has two parameters, \( \zeta_1 \) and \( \zeta_2 \). As aforementioned, these two parameters are strictly positive such that the characteristic polynomial of \( s + \zeta_1s + \zeta_2s = 0 \) satisfies the Hurwitz condition, that is, all roots have to be allocated in the left hand side of the complex plane. In addition, fast dynamic response with weak robustness can be obtained when system roots are located close to zero. However, the opposite action can be obtained (i.e. slower response with stronger robustness) when roots are located far away from the origin. Thus, the final chosen values are 1000, 5000 for \( \zeta_1 \) and \( \zeta_2 \), respectively. On the other hand, \( \eta = 1.25 \) is the robust gain, which is selected to satisfy the Lyapunov criteria as it is explained in Section 3.3.

3.5.2 NTSM parameters: The NTSM parameters are extensively discussed in the previous work [37]. Thus, by following the guidelines therein the NTSM parameters are selected as \( \gamma = 1.4 \), \( p/q = 1.2 \) and \( \varepsilon = 1e^{-3} \).

4 Experimental results and analysis

In this section, several experimental studies, as listed in Table 1, are conducted to verify and compare the performance of the designed controller. It should be noted that in addition to the typical sinusoidal, two more difference signals (i.e. varying sinusoidal and triangular signal) are employed here for track following test. It is proved that applying the two latter signals can cause a larger width of the output hysteresis [38] which means a more uncertainties is deduced and hence, a high chattering is expected to address by the proposed controller. Such tests would further show the validity and the ability of the proposed controller in terms of robustness.

4.1 Track following

To testify the tracking performance, tests 1, 2 and 3 are conducted first. For the sake of brevity, only test 2 will be presented here. Fig. 4 shows the tracking profile of the sinusoidal signal with varying amplitudes (8–0.5  \( \mu \)m). It can be noted from Fig. 4b that the CSMC produces larger tracking errors, that is, a maximum and root-mean-square (RMS) errors of 0.15 and 0.05  \( \mu \)m, respectively. Meanwhile, the NTSM performance is much better than the CSMC and is very close to the proposed 2OTS. However, the control signals of NTSM and CSMC controllers in Fig. 4c exhibit larger chattering, which is obviously observed through the frequency spectra of the control signals as shown in Fig. 5. It is clear that the 2OTS control signal is smoother and has less chattering than the other controllers in the high frequency range. Despite using the boundary layer technique in CSMC and NTSM,
**4.2 Robustness verification**

Aiming to evaluate the controller robustness, an additional experimental study is conducted. First, the test 4 is done to investigate the system robustness against the system uncertainties. Next, we also examined the system performance under an external disturbance in test 5. Finally, a combination between the system uncertainties and external disturbance is also elaborated in test 6.

**4.2.1 System uncertainties:** For robustness verification in test 4, a 1 kg steel is mounted on the nanopositioning stage. Fig. 1b depicts the experimental setup with mounted mass for robustness verification. The first three experiments are repeated herein, but with system uncertainties (i.e. test 4 in Table 1). Thus, only the triangular tracking profile is presented in Fig. 6. From these results, we observed that the CSMC and NTSM exhibit large oscillations in the tracking profile. The oscillations are more obvious in the error profile as depicted in Fig. 6b. On the contrary, the proposed 2OTSMCPE behaviour has much fewer oscillations than the others. There are two reasons for that. First, the 2OTSMCPE is a chattering free controller as it has been presented in the previous section. The second reason is that the control gain in 2OTSMCPE is designed based on the rate of PE error rather than the upper bound of the actual perturbation. Hence, a small converging region would be achieved; this yields a less tracking error with less chattering than the other controllers. This is the main merit of using the PE [11].

**4.2.2 Disturbance rejection:** To test the control system robustness for the external disturbance, a real random external disturbance is applied to PEA nano-stage. Two experiments are conducted for this purpose. First, the random disturbance is applied to the nominal system (i.e. no mounted weight). Next, the first experiment is repeated in the presence of system uncertainties (i.e. 1 kg weight is mounted on the nanopositioning stage). Only the latter test is presented here. It should be noted that our system setup is a part of a dual stage actuator (DSA) combined by a linear motor (LM) and the PEA. Hence, the LM is the main source of the external disturbance in the application of DSA, such issue in DSA is called a coupling force. Thus, in these experiments, the random disturbance is simply generated by turning on the LM. Fig. 7 depicts the sinusoidal tracking profile of all controllers with decoupling force and 1 kg added mass. From Fig. 7, CSMC and NSTM both have significant chattering which mainly comes from the switching part and the system uncertainties. Hence, the disturbance rejection would also be oscillated and take a long time to settle down. Meanwhile, the proposed 2OTSMCPE can reject the external disturbance in a short time period with fewer oscillations in comparison with the other controllers. In order to further clarify these results and show the smoothness of the control signals, the frequency spectra of control signals are presented in Fig. 8. From this figure, there is no doubt that the 2OTSMCPE has the smoothest control signal, and hence it has the superiority in terms of system robustness and the disturbance rejection. Thus, the proposed 2OTSMCPE can be considered as an alternative way to solve the coupling issue in DSA applications.

**4.2.3 Summary and discussion:** In this section, the results of the experiments that are tabulated in Table 1 will be summarised and discussed. To quantify and analyse the system performance of the 2OTSMCPE with other controllers, we define the maximum (Max($e$)) and the root-mean-square (RMS($e$)) of the tracking error as follows:

\[ \text{Rms} (e) = \sqrt{\frac{1}{M} \sum_{k=1}^{M} e_k^2} \]  \hfill (36)

\[ \text{Max} (e) = \max |e| \]  \hfill (37)

where $M$ is the number of datasets. After that, these values are presented in Figs. 9 and 10, respectively. In addition, the improvement ratio of the system tracking performance is also

![Fig. 6 Tracking response of all controllers for 10 Hz triangular reference signal with varying amplitude (8–0.5) μm without system uncertainties](image)

(a) Tracking profiles, (b) Tracking errors, (c) Control signals

![Fig. 7 Tracking response of all controllers for 10 Hz sinusoidal reference signal with 2 μm P-P amplitude with random decoupling disturbance and 1 kg added mass](image)

(a) Tracking profiles, (b) Tracking errors, (c) Control signals

![Fig. 8 Frequency spectra of the control signals of test 8](image)
included in these two figures for further clarification. Referring to these results, it can be easily noted that the proposed 2OTS MCM PE shows superiority in terms of the RMS(e) and Max(e) in comparison with CSMC and NTSMC, particularly, in those tests under the system uncertainties and external disturbance. For instance, if we investigate these eight experiments quantitatively, we can find that the 2OTS MCM PE has the smallest value in terms of RMS(e) and Max(e), respectively, and also improved by 38.26 and 21.62% compared with CSMC in terms of RMS(e) and Max(e), respectively.

5 Conclusion

In this paper, the design of 2OTS MCM PE structure for nanopositioning stage system is presented. It has an important advantage which alleviates the chattering effect. Besides, the proposed scheme requires no prior knowledge of system uncertainties. In addition, with the proposed 2OTS MCM PE, the finite-time convergence of the PID-type sliding function is guaranteed and hence the tracking error converges asymptotically. Moreover, the 2OTS MCM PE scheme has been theoretically analysed and the stability proof proved based on Lyapunov criteria. Finally, the proposed control approach has been experimentally verified on a real nanopositioning system. The results demonstrated the superiority of the 2OTS MCM PE structure over the NTS M and CSMC in terms of the chattering alleviation and system robustness.

6 References

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