Multi-scale flow patterns during immiscible displacement of oil by water in a layer-inhomogeneous porous media

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Abstract. This paper presents the results of numerical study of the relationship between micro- and macroscale flows during immiscible displacement in a two-layer porous medium. A feature of the proposed approach is the allowance for large-scale capillarity induced flow due to curvature of the displacement front in macro-inhomogeneous porous medium. The physical mechanisms determining the development of viscous instability in a layer-inhomogeneous porous medium are considered, the methods for suppressing viscous fingers formation based on the stabilization of the displacement front due the action of capillary forces are proposed.

1. Introduction

The preferable method for enhanced oil recovery is the injection of water and reagent solutions through the wells, which contribute to a decrease in the number of production wells and an increase in their productivity. Despite the high efficiency of the waterflooding for enhancement of oil recovery, there are a number of problems that reduce its efficiency, especially for layered heterogeneous formations with high viscosity oil. This is due to the uneven movement of the water-oil contact and formation of the oil fields in water-washed zones [1]. It is well known that layered oil and gas reservoirs represent the most common structure of sedimentary rocks and the layering is observed at various scales from a few millimeters to tens of meters [2].

In the case of immiscible displacement of oil by water in the layered heterogeneous porous medium, the displacement front becomes unstable and water, which is a wetting phase, penetrates firstly into the layer with high permeability. This has been observed experimentally [3, 4] and in the numerical simulations [5]. At low displacement rates, water imbibition into the layer with lower permeability induced by capillarity can stabilize the displacement and large finger does not occur in highly permeable layer. At the same time, the dimensionless criteria for finger suppression available in the literature characterize the conditions for stabilization of the displacement in packed sand; their transfer to other types of porous media is not justified.

The aim of the current study is numerical investigation of the relationship between micro- and macro scale flows during immiscible displacement in a two-layer porous medium based on the Musket-Leverett equations of unsteady two-phase filtration. A feature of the proposed approach is the allowance for large-scale capillarity induced flow due to curvature of the displacement front in macro-inhomogeneous porous medium.

2. Basic equations and numerical method

The development of macroscale fingers during the displacement of a viscous hydrocarbon liquid by
water in inhomogeneous porous media was studied numerically within the framework of the Muskat-Leverett mathematical model. Immiscible displacement of oil by water is considered in two-layer porous medium using two-dimensional formulation without taking into account the gravity. The flow occurs in a rectangular region with low-permeability interlayer $\Omega = \{0 \leq x \leq L/H, -h \leq y \leq 0\}$ and high-permeability interlayer $\Omega = \{0 \leq x \leq L/H, 0 \leq y \leq 1 - h\}$ which is shown in figure 1, where $h$ is the dimensionless thickness of the low-permeability interlayer. At lateral boundaries $y = -h$ and $y = 1 - h$, no-flow conditions are set, at the boundaries of the interlayers, the conditions for the equality of fluid flow rates and capillary equilibrium are applied, the pressure of the displaced fluid continuous throughout the entire flow region and $P_2 = 0$ at $x = L/H$.

The system of equations for the two-phase flow in porous media, written for the normalized saturation of displacing fluid (water) $S$ and effective pressure of the displaced fluid $P$ for a highly permeable interlayer was formulated in [6] as follows

$$
\frac{\partial S}{\partial t} = \text{div}(\epsilon_H a(S) \nabla S \cdot \vec{v} F(S))
$$
(1)

$$
\text{div}(\vec{v}) = 0, \quad \vec{v} = -M(S) \nabla P
$$
(2)

$$
P = P_2 + \epsilon_H \int_S F_1(S) \frac{d\ell}{dS} dS
$$
(3)

where $S = (S_1 - S_{1,0})/(S_{1,1} - S_{1,0})$, $F_1(S) = k_1(S)/(k_1(S) + \mu_0 k_2(S))$, $\epsilon_H = s \sqrt{k_h m_h} / \mu_0 \mu_2 H$, $P_2 = \overline{P}_2 k_h k_1(s_{1,0}) / \nu_0 \nu_2 H$, $t = \tilde{t} v_0 / m_h H (S_{1,1} - S_{1,0})$, $x = \tilde{x} / H$, $y = \tilde{y} / H$, $v = \vec{v} / v_0$.

For a low-permeability interlayer, the system of equations for two-phase flow in porous media has the form

$$
\frac{\partial S}{\partial t} = \text{div} \left( \epsilon_H \sqrt{k m_a} a(S) \nabla S \cdot \vec{v} F(S) \right)
$$
(4)

$$
\text{div}(\vec{v}) = 0, \quad \vec{v} = -k M(S) \nabla P
$$
(5)

$$
P = P_2 + \frac{\epsilon_H}{\sqrt{k m_a}} \int_S F_1(S) \frac{d\ell}{dS} dS
$$
(6)

Here $\vec{v}$ is the total filtration rate of liquids, $M(S)$ is the mobility of the mixture at saturation $S$, referred to the initial mobility $S = 0$: $F(S) = (S_1 - S_{1,0})/(S_{1,1} - S_{1,0})$. $F_1(S)$ is the Leverett function, $k_1(S)$ is the relative phase permeabilities, $J(S)$ is the Leverett function, $\mu_0 = \mu_1 / \mu_2$ is the ratio of viscosities of liquids; $k_1 = k_1 / k_h$, $m = m_t / m_h$ is the ratio of permeabilities and porosities of low-permeability and high-permeability interlayers, $\tilde{t}$, $\tilde{x}$, $\tilde{y}$, $\tilde{P}$, $\tilde{v}$ are the dimensional time, coordinates, pressure and filtration rate of the mixture, $H$ is the total width of the interlayers, $v_0$ is the filtration rate of the mixture at $x = 0$, $m_h$ and $k_h$ are the porosity and the permeability of a highly permeable interlayer, $S_{1,0}$ and $S_{1,1}$ are the initial and final values of water saturation, indices 1 and 2 refer to displacing and displaced fluids, $\sigma$ is the interfacial tension. The relative phase permeabilities and Leverett function which determine the capillary forces action in microscale (pore level) were selected according to data [7] as follows

$$
k_1(S) = 0.305 S^{0.23} + 0.31 S^{1.9}
$$
(7)

$$
k_2(S) = (1 + 20 S)^{0.125} (0.12(1-S)^{1.6} + 0.305(1-S)^{3.6})
$$
(8)
\[ J(S) = -0.226(S + 0.01)^{0.25} - (1 + (S + 0.019)^{3.0}) \] (9)

The displacement occurs in the direction of the \( x \) axis, at a given total flow rate of the injected fluid at \( x = 0 \) or given pressure drop \( \Delta P \). At the initial moment of time, the flow region is completely filled with the displaced liquid and \( S = 0 \). Displacement is considered with small values of the dimensionless parameter \( \varepsilon_L = \varepsilon_H H/L \ll 1 \), so that a narrow stabilized zone with large longitudinal gradients of water saturation can be distinguished in the solution at \( H/L \ll 1 \).

During simulations the finite-difference scheme was used where the saturation field was determined using an explicit conservative difference scheme of the second order of accuracy with automatic step selection. The equation for pressure was approximated by a conservative difference scheme of the second order of accuracy on a five-point stencil. The resulting system of algebraic equations was solved by the sequential upper relaxation method.

3. Numerical results
Displacement with small values of the dimensionless parameter \( \varepsilon_L = \varepsilon_H H/L \ll 1 \) is considered, so that a narrow stabilized zone with large longitudinal water saturation gradients can be distinguished in the solution at \( L/H \gg 1 \). Here \( \varepsilon_H = \frac{\sigma \sqrt{k_1 m_1}}{v_0 \mu k_1^*} \), \( v_0 \) is the filtration rate at the inlet, \( k_1 \) and \( m_1 \) are the permeability and the porosity of the highly permeable interlayer, \( \sigma \) is the surface tension. The simulation results have shown that during displacement in a two-layer porous medium, the displacement front moves at a higher speed in the highly permeable than in the low permeable interlayers. Variation of frontal saturations for a curved displacement front as compared to the solution of the Buckley - Leverett problem (\( S_c = 0.3 \)) is shown in figure 2. In this figure the structures of the saturation jump for the head (solid lines) and tail (dashed lines) of the finger are shown for \( t = 0.27 \) at \( h = 0.5 \), \( \mu_0 = 0.1 \), \( k_* = 0.5 \), \( H/L = 0.1 \) and \( \varepsilon_H \sqrt{k_*} = 0 \), 0.14 and 0.68. As it is seen, large scale capillary cross-flows reduce the frontal saturation in highly permeable interlayer and increase it in low permeable interlayer.

The results of calculating the water saturation field (isolines) for the dimensionless time \( t = 0.38 \) are shown in figure 3 at \( h = 0.5 \), \( \mu_0 = 0.1 \), \( k_* = 0.1 \), \( \varepsilon_H \sqrt{k_*} = 0.37 \), \( H/L = 0.1 \). Here the curves 1, 2, 3, 4 are correspond to the isolines of constant saturation of displacing fluid \( S = 0.1; 0.2, 0.3, 0.4 \) respectively. As it is seen, the displacement front is stabilized by large-scale capillary flows and the length of the finger does not increase with time at \( t > 0.2 \). Isolines of the stream function show that the

**Figure 2.** Distribution of dimensionless water saturation during displacement in high permeability (continuous lines) and low permeability (dotted lines) interlayer, \( t = 0.27 \) and \( \varepsilon_H \sqrt{k_*} = 0, 0.14, 0.68 \).
Figure 3. Water saturation field for dimensionless time $t=0.38$ at $h = 0.5$, $\mu_0 = 0.1$, $k_\ast = 0.1$, $\varepsilon H \sqrt{k_\ast} = 0.37$, $H/L = 0.1$.

transverse component of the mixture filtration rate is small in the entire displacement region, especially near the displacement front, and the displacement front is aligned in the interlayers due to capillary cross-flows along the front. The length of the fingers obtained in numerical calculations allows us to estimate the conditions of stabilized displacement with a given length of the finger. For the water saturation field shown in figure 3, the values of front saturation in highly permeable and low permeable interlayers are close to 0.1 and 0.35. This variation in the front saturation causes variation of mobility and the limiting length of the finger.

4. Conclusions
The numerical calculations show that during immiscible displacement of oil by water in two-layer porous medium, the displacement front moves with higher velocity in highly permeable interlayer than in low permeable interlayer. The deformation of water saturation field on macroscale promotes the increase in capillary cross-flows along the displacement front caused by microscale capillary forces action that stabilizes the length of the finger. It was found that the limiting length of the finger is determined by the parameters characterizing the intensity of the capillary forces in microscale for the low permeable interlayer (Leverett function and phase permeabilities) and the ratio of the interlayer thicknesses, their permeabilities and liquid viscosities. At low capillary cross-flows, the fingers reach a great length which is unfavorable for enhanced oil recovery.

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