Gauge $U(1)$ Dark Symmetry and Radiative Light Fermion Masses

Corey Kownacki$^1$ and Ernest Ma$^{1,2,3}$

$^1$ Department of Physics and Astronomy, University of California, Riverside, California 92521, USA

$^2$ Graduate Division, University of California, Riverside, California 92521, USA

$^3$ HKUST Jockey Club Institute for Advanced Study, Hong Kong University of Science and Technology, Hong Kong, China

Abstract

A gauge $U(1)$ family symmetry is proposed, spanning the quarks and leptons as well as particles of the dark sector. The breaking of $U(1)$ to $Z_2$ divides the two sectors and generates one-loop radiative masses for the first two families of quarks and leptons, as well as all three neutrinos. We study the phenomenological implications of this new connection between family symmetry and dark matter. In particular, a scalar or pseudoscalar particle associated with this $U(1)$ breaking may be identified with the 750 GeV diphoton resonance recently observed at the Large Hadron Collider (LHC).
In any extension of the standard model (SM) of particle interactions to include dark matter, a symmetry is usually assumed, which distinguishes quarks and leptons from dark matter. For example, the simplest choice is $Z_2$ under which particles of the dark sector are odd and those of the visible sector are even. Suppose $Z_2$ is promoted to a gauge $U(1)$ symmetry, then the usual assumption is that it will not affect ordinary matter. These models all have a dark vector boson which couples only to particles of the dark sector.

Table 1: Particle content of proposed model of gauge $U(1)$ dark symmetry.

| particles | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_D$ | $Z_2$ |
|-----------|-----------|-----------|-----------|-----------|-------|
| $Q = (u,d)$ | 3         | 2         | 1/6       | 0,0,0     | +     |
| $u^c$ | 3$^*$      | 1         | $-2/3$    | 1, $-1,0$ | +     |
| $d^c$ | 3$^*$      | 1         | 1/3       | $-1,1,0$  | +     |
| $L = (\nu, e)$ | 1         | 2         | $-1/2$    | 0,0,0     | +     |
| $e^c$ | 1          | 1         | 1         | $-1,1,0$  | +     |
| $\Phi = (\phi^+, \phi^0)$ | 1         | 2         | 1/2       | 0         | +     |
| $\sigma_1$ | 1         | 1         | 0         | 1         | +     |
| $\sigma_2$ | 1         | 1         | 0         | 2         | +     |
| $N, N^c$ | 1          | 1         | 0         | 1/2, $-1/2$ | $-$ |
| $S, S^c$ | 1          | 1         | 0         | $-3/2, 3/2$ | $-$ |
| $(\eta^0, \eta^-)$ | 1         | 2         | $-1/2$    | 1/2       | $-$ |
| $\chi^0$ | 1          | 1         | 0         | 1/2       | $-$ |
| $\chi^-$ | 1          | 1         | $-1$      | $-1/2$    | $-$ |
| $(\xi^{2/3}, \xi^{-1/3})$ | 3         | 2         | 1/6       | 1/2       | $-$ |
| $\zeta^{2/3}$ | 3         | 1         | $2/3$     | $-1/2$    | $-$ |
| $\zeta^{-1/3}$ | 3         | 1         | $-1/3$    | $-1/2$    | $-$ |

In this paper, it is proposed instead that a gauge $U(1)$ extension of the SM spans both ordinary and dark matter. It is in fact also a horizontal family symmetry. It has a number of interesting consequences, including the radiative mass generation of the first two families of
quarks and leptons, and a natural explanation of the 750 GeV diphoton resonance recently observed \cite{1,2} at the Large Hadron Collider (LHC).

\textit{New Gauge $U(1)_D$ Symmetry:}

The framework that radiative fermion masses and dark matter are related has been considered previously \cite{3}. Here it is further proposed that families are distinguished by the connecting dark symmetry. In Table 1 we show how they transform under $U(1)_D$ as well as the other particles of the dark sector. The $U(1)_D$ symmetry is broken spontaneously by the vacuum expectation value $\langle \sigma_{1,2} \rangle = u_{1,2}$ to an exactly conserved $Z_2$ which divides the two sectors.

![Figure 1: One-loop neutrino mass from trilinear couplings.](image1)

![Figure 2: One-loop neutrino mass from trilinear and quadrilinear couplings.](image2)
The gauge $U(1)_D$ symmetry is almost absent of axial-vector anomalies for each family. The $[SU(3)]^2U(1)_D$ anomaly is zero from the cancellation between $u^c$ and $d^c$. The $[SU(2)]^2U(1)_D$ anomaly is zero because $Q$ and $L$ do not transform under $U(1)_D$. The $[U(1)_Y]^2U(1)_D$ and $U(1)_Y[U(1)_D]^2$ anomalies are cancelled among $u^c$, $d^c$, and $e^c$, i.e.

\[
3 \left( \frac{-2}{3} \right)^2 (1) + 3 \left( \frac{1}{3} \right)^2 (-1) + (1)^2 (-1) = 0,
\]

\[
3 \left( \frac{-2}{3} \right)^2 (1)^2 + 3 \left( \frac{1}{3} \right)^2 (-1)^2 + (1)(-1)^2 = 0.
\]

The $[U(1)_D]^3$ anomaly is not zero for either the first or second family, but is cancelled between the two. This is thus a generalization of the well-known anomaly-free $L_e - L_\mu$ gauge symmetry [4] to the difference of $B - L - 2Y$ between the first two families.

![Figure 3: One-loop electron mass.](image)

![Figure 4: One-loop muon mass.](image)
Radiative Masses for Neutrinos and the First and Second Families:

At tree level, only $t, b, \tau$ acquire masses from $\langle \phi^0 \rangle = v$ as in the SM. The first two families are massless because of the $U(1)_D$ symmetry. Neutrinos acquire one-loop masses through the scotogenic mechanism \[3\] as shown in Figs. 1 and 2. With one copy of $(N, N^c)$, only one neutrino becomes massive. To have three massive scotogenic neutrinos, three copies of $(N, N^c)$ are needed. The one-loop electron and muon masses are shown in Figs. 3 and 4. Note that at least two copies of $(N, N^c)$ are needed for two charged-lepton masses. The mass matrix spanning $(N, N^c, S, S^c)$ is of the form

$$
\mathcal{M}_{N,S} = \begin{pmatrix}
  f_1 u_1 & m_N & f_3 u_1 & f_5 u_2 \\
  m_N & f_2 u_1 & f_6 u_2 & f_4 u_1 \\
  f_3 u_1 & f_6 u_2 & 0 & m_S \\
  f_5 u_2 & f_4 u_1 & m_S & 0
\end{pmatrix}.
$$

(3)

Note that the $f_{1,2,3,4} u_1$ terms break lepton number by two units, whereas the $f_{5,6} u_2$ terms do not. Lepton number $L = 1$ may be assigned to $e, \mu, \tau, N, S$ and $L = -1$ to $e^c, \mu^c, \tau^c, N^c, S^c$.

![Figure 5: One-loop $u$ quark mass.](image)

It is broken down to lepton parity $(-1)^L$ only by neutrino masses. The analogous one-loop $u$ and $d$ quark masses are shown in Figs. 5 and 6. Because the second family has opposite $U(1)_D$ charge assignments relative to the first, the $c$ and $s$ quarks reverse the roles of $u$ and $d$. Two copies of $(S, S^c)$ are needed to obtain the most general quark mass matrices for both the $u$ and $d$ sectors.
To evaluate the one-loop diagrams of Figs. 1 to 6, we note first that each is a sum of simple diagrams with one internal fermion line and one internal scalar line. Each contribution is infinite, but the sum is finite. There are 10 neutral Majorana fermion fields, spanning $3$ copies of $N, N^c$ and $2$ copies of $S, S^c$. We denote their mass eigenstates as $\psi_k$ with mass $M_k$. There are $4$ real scalar fields, spanning $\sqrt{2} \text{Re}(\eta^0), \sqrt{2} \text{Im}(\eta^0), \sqrt{2} \text{Re}(\chi^0), \sqrt{2} \text{Im}(\chi^0)$. We denote their mass eigenstates as $\rho^0_l$ with mass $m_l$. In Figs. 1 and 2, let the $\nu_i \psi_k \bar{\eta}^0$ coupling be $h_{ik}^{\nu}$, then the radiative neutrino mass matrix is given by

$$ (\mathcal{M}_\nu)_{ij} = \sum_k h_{ik}^{\nu} h_{jk}^{\nu} M_k \frac{1}{16\pi^2} \sum_l [(y_l^R)^2 F(x_{lk}) - (y_l^I)^2 F(x_{lk})], \quad (4) $$

where $\sqrt{2} \text{Re}(\eta^0) = \sum_l y_l^R \rho^0_l, \sqrt{2} \text{Im}(\eta^0) = \sum_l y_l^I \rho^0_l$, with $\sum_l (y_l^R)^2 = \sum_l (y_l^I)^2 = 1$, $x_{lk} = m_l^2 / M_k^2$, and the function $F$ is given by

$$ F(x) = \frac{x \ln x}{x - 1}. \quad (5) $$

There are two charged scalar fields, spanning $\eta^\pm, \chi^\pm$. We denote their mass eigenstates as $\rho^\pm_r$ with mass $m_r$. In Fig. 3, let the $e_L \psi_k \eta^+ \eta^0$ and the $e'_L \psi_k \chi^0 \bar{\chi}^-$ couplings be $h_k^{\nu}$ and $h_k^{e_c}$, then

$$ m_e = \sum_k h_k^\nu h_k^{e_c} M_k \frac{1}{16\pi^2} \sum_r y_r^\nu y_r^{e_c} F(x_{rk}), \quad (6) $$

where $\eta^+ = \sum_r y_r^\nu \rho^+ r, \chi^+ = \sum_r y_r^\nu \rho^+ r$, with $\sum_r (y_r^\nu)^2 = \sum_r (y_r^{e_c})^2 = 1$ and $\sum_r y_r^\nu y_r^{e_c} = 0$. A similar expression is obtained for $m_{\mu}$, as well as the light quark masses.
Tree-Level Flavor-Changing Neutral Couplings:
Since different $U(1)_D$ charges are assigned to $(u^c, c^c, t^c)$ as well as $(d^c, s^c, b^c)$, there are unavoidable flavor-changing neutral currents. They can be minimized by the following assumptions. Let the two $3 \times 3$ quark mass matrices linking $(u, c, t)$ to $(u^c, c^c, t^c)$ and $(d, s, b)$ to $(d^c, s^c, b^c)$ be of the form
\begin{align*}
M_u &= U_L^{(u)} \begin{pmatrix}
m_u & 0 & 0 \\
0 & m_c & 0 \\
0 & 0 & m_t
\end{pmatrix}, \\
M_d &= U_L^{(d)} \begin{pmatrix}
m_d & 0 & 0 \\
0 & m_s & 0 \\
0 & 0 & m_b
\end{pmatrix},
\end{align*}
where $U_{CKM} = \left(U_L^{(u)}\right)^\dagger U_L^{(d)}$ is the quark charged-current mixing matrix. However, since $Z_D$ does not couple to left-handed quarks, and its couplings to right-handed quarks have been chosen to be diagonal in their mass eigenstates, flavor-changing neutral currents are absent in this sector. Of course, they will appear in the scalar sector, and further phenomenological constraints on its parameters will apply.

$Z_D$ Gauge Boson:
As $\sigma_{1,2}$ acquire vacuum expectation values $u_{1,2}$ respectively, the $Z_D$ gauge boson obtains a mass given by
\begin{equation}
m_{Z_D}^2 = 2g_D^2 (u_1^2 + 4u_2^2).
\end{equation}
Since $\sigma_{1,2}$ do not transform under the SM, and $\Phi$ does not under $U(1)_D$, there is no mixing between $Z_D$ and $Z$. Using Table 1 and assuming that all new particles are lighter than $Z_D$, the branching fraction of $Z_D$ to $e^-e^+ + \mu^-\mu^+$ is estimated to be 0.07. The $c_{u,d}$ coefficients used in the experimental search [6, 7] of $Z_D$ are then
\begin{equation}
c_u = c_d = g_D^2 (0.07).
\end{equation}
For $g_D = 0.3$, a lower bound of about 3.1 TeV on $m_{Z_D}$ is obtained from LHC data based on the 7 and 8 GeV runs. For our subsequent discussion, let $u_1 = 1$ TeV, $u_2 = 4$ TeV, then $m_{Z_D} = 3.4$ TeV. Note that $Z_D$ does not couple to the third family, so if $\bar{t}t$, $\bar{b}b$, or $\tau^+\tau^-$ final states are observed, this model is ruled out.
Scalar Sector:

There are three scalars with integral charges under $U(1)_D$, i.e. $\Phi$ and $\sigma_{1,2}$. Whereas $\langle \varphi^0 \rangle = v$ breaks the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry as in the SM, $\langle \sigma_{1,2} \rangle = u_{1,2}$ break $U(1)_D$ to $Z_2$, with all those particles with half-integral $U(1)_D$ charges becoming odd under this exactly conserved dark $Z_2$ parity. The relevant scalar potential is given by

$$
V = \mu_0^2 \Phi^\dagger \Phi + m_1^2 \sigma_1^* \sigma_1 + m_2^2 \sigma_2^* \sigma_2 + m_{12} \sigma_1^* \sigma_2^* + m_{12} (\sigma_1^*)^2 \sigma_2^* \\
+ \frac{1}{2} \lambda_0 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_1 (\sigma_1^* \sigma_1)^2 + \frac{1}{2} \lambda_2 (\sigma_2^* \sigma_2)^2 + \lambda_3 (\sigma_1^* \sigma_1) (\sigma_2^* \sigma_2) \\
+ \lambda_4 (\Phi^\dagger \Phi) (\sigma_1^* \sigma_1) + \lambda_5 (\Phi^\dagger \Phi) (\sigma_2^* \sigma_2),
$$

(10)

where $m_{12}$ has been rendered real by absorbing the relative phase between $\sigma_{1,2}$. The conditions for $v$ and $u_{1,2}$ are

$$
0 = \mu_0^2 + \lambda_0 v^2 + \lambda_4 u_1^2 + \lambda_5 u_2^2, \quad \text{(11)}
$$

$$
0 = m_1^2 + \lambda_1 u_1^2 + \lambda_3 u_2^2 + \lambda_4 v^2 + 2 m_{12} u_2, \quad \text{(12)}
$$

$$
0 = m_2^2 + \lambda_2 u_2^2 + \lambda_3 u_1^2 + \lambda_5 v^2 + m_{12} u_1^2 / u_2. \quad \text{(13)}
$$

As in the SM, $\phi^{\pm}$ and $\sqrt{2} Im(\varphi^0)$ become longitudinal components of $W^{\pm}$ and $Z$, and $\sqrt{2} Re(\varphi^0) = h$ is the one physical Higgs boson associated with electroweak symmetry breaking. Let $\sigma_1 = (\sigma_{1R} + i \sigma_{1I}) / \sqrt{2}$ and $\sigma_2 = (\sigma_{2R} + i \sigma_{2I}) / \sqrt{2}$, then the mass-squared matrix spanning $h, \sigma_{1R,2R}$ is

$$
\mathcal{M}_R^2 = \begin{pmatrix}
2 \lambda_0 v^2 & 2 \lambda_4 u_1 & 2 \lambda_5 v u_2 \\
2 \lambda_4 u_1 & 2 \lambda_1 u_1^2 & 2 \lambda_3 u_1 u_2 + 2 m_{12} u_1 \\
2 \lambda_5 v u_2 & 2 \lambda_3 u_1 u_2 + 2 m_{12} u_1 & 2 \lambda_2 u_2^2 - m_{12} u_1^2 / u_2
\end{pmatrix},
$$

(14)

and that spanning $\sigma_{1I,2I}$ is

$$
\mathcal{M}_I^2 = \begin{pmatrix}
-4 m_{12} u_2 & 2 m_{12} u_1 \\
2 m_{12} u_1 & -m_{12} u_1^2 / u_2
\end{pmatrix}.
$$

(15)

The linear combination $(u_1 \sigma_{1I} + 2 u_2 \sigma_{2I}) / \sqrt{u_1^2 + 4 u_2^2}$ has zero mass and becomes the longitudinal component of the massive $Z_D$ gauge boson. The orthogonal component is a pseudoscalar,
call it $A$, with a mass given by $m_A^2 = -m_{12}(u_1^2 + 4u_2^2)/u_2$. In Eq. (14), $\sigma_{1R}$ and $\sigma_{2R}$ mix in general. For simplicity, let $m_{12} = -\lambda_3 u_2$, then for $v^2 << u_{1,2}^2$, we obtain
\begin{align*}
    m_{\sigma_{1R}}^2 &= 2\lambda_1 u_1^2, \\
    m_{\sigma_{2R}}^2 &= 2\lambda_2 u_2^2 + \lambda_3 u_1^2, \\
    m_A^2 &= \lambda_3(u_1^2 + 4u_2^2), \\
    m_h^2 &= 2\left[\lambda_0 - \frac{\lambda_4^2}{\lambda_1} - \frac{2\lambda_5^2 u_2^2}{2\lambda_2 u_2^2 + \lambda_3 u_1^2}\right]v^2.
\end{align*}

(16) \hspace{1cm} (17)

**Relevance to the Diphoton Excess:**

Any one of the three particles $\sigma_{1R}, \sigma_{2R}, A$ may be identified with the 750 GeV diphoton excess. For illustration, let us consider $\sigma_{1R}$. The production cross section through gluon fusion is given by
\begin{equation}
    \hat{\sigma}(gg \to \sigma_{1R}) = \frac{\pi^2}{8m_{\sigma_{1R}}^2}\Gamma(\sigma_{1R} \to gg)\delta(s - m_{\sigma_{1R}}^2).
\end{equation}

(18)

For the LHC at 13 TeV, the diphoton cross section is roughly [8]
\begin{equation}
    \sigma(gg \to \sigma_{1R} \to \gamma\gamma) \simeq (100 \text{ pb}) \times (\lambda_g \text{ TeV})^2 \times B(\sigma_{1R} \to \gamma\gamma),
\end{equation}

(19)

where $\lambda_g$ is the effective coupling of $\sigma_{1R}$ to two gluons, normalized by
\begin{equation}
    \Gamma(\sigma_{1R} \to gg) = \frac{\lambda_g^2}{8\pi} m_{\sigma_{1R}}^3,
\end{equation}

(20)

and the corresponding $\lambda_\gamma$ comes from
\begin{equation}
    \Gamma(\sigma_{1R} \to \gamma\gamma) = \frac{\lambda_\gamma^2}{64\pi} m_{\sigma_{1R}}^3.
\end{equation}

(21)

If $\sigma_{1R}$ decays only to two gluons and two photons, and assuming $\lambda_\gamma^2/8 << \lambda_g^2$, then
\begin{equation}
    \sigma(gg \to \sigma_{1R} \to \gamma\gamma) \simeq (100 \text{ pb}) \times (\lambda_\gamma \text{ TeV})^2/8,
\end{equation}

(22)

which is supposed to be about 6.2 fb from the recent data [11, 12]. This means that $\lambda_\gamma \simeq 2.2 \times 10^{-2} (\text{TeV})^{-1}$, and $\Gamma(\sigma_{1R} \to \gamma\gamma) \simeq 1 \text{ MeV}$.

Now $\sigma_{1R}$ couples to the new scalars $\xi^{2/3}, \xi^{-1/3}, \zeta^{2/3}, \zeta^{-1/3}, \eta^-, \chi^-$ through $\sqrt{2}u_1$ multiplied by the individual quartic scalar couplings. For simplicity, let all these couplings be the same,
say $\lambda$, and all the masses be the same, say $m_0$, then

$$\lambda_\gamma = \frac{\alpha u_1 \lambda_\sigma}{\sqrt{2\pi m_{\sigma_{1R}}^2}} \left[ 6 \left( \frac{2}{3} \right)^2 + 6 \left( -\frac{1}{3} \right)^2 + 2(-1)^2 \right] f \left( \frac{m_0^2}{m_{\sigma_{1R}}^2} \right), \quad (23)$$

where the function $f$ is given by

$$f(x) = 8x \left[ \arctan \left( \frac{1}{\sqrt{4x-1}} \right) \right]^2 - 2. \quad (24)$$

Let $m_0 = 700$ GeV, then $x = 0.87$ and $f = 1.23$. Hence for $u_1 = 1$ TeV and $\lambda_\sigma = 1.1$, the required $\lambda_\gamma \approx 0.022$ (TeV)$^{-1}$ is obtained. For this $\lambda_\sigma$, we find $\lambda_g = 0.128$, hence $\Gamma(\sigma_{1R} \to gg) \approx 0.27$ GeV, which is below the energy resolution of ATLAS and CMS. This narrow width is not favored by the ATLAS data, but cannot be ruled out at this time.

**Dark Matter :**

The lightest neutral particle with odd $Z_2$ is a good dark-matter candidate. In this model, it could be the lightest scalar particle in the sector consisting of $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$ and $\chi^0 = (\chi_R + i\chi_I)/\sqrt{2}$. There are two sectors, the mass-squared matrix spanning $\eta_R, \chi_R$ is given by

$$\mathcal{M}^*_R = \begin{pmatrix} m_{\eta}^2 & A \\ A & m_{\chi}^2 + C \end{pmatrix}, \quad (25)$$

and that spanning $\eta_I, \chi_I$ is

$$\mathcal{M}^*_I = \begin{pmatrix} m_{\eta}^2 & B \\ B & m_{\chi}^2 - C \end{pmatrix}, \quad (26)$$

where $A, B$ come from the $\phi^0 \eta^0 (\chi^0)^*$ and $\phi^0 \eta^0 (\sigma_1)^*$ couplings and $C$ from the $\chi^0 \chi^0 (\sigma_1)^*$ coupling. The phenomenology of the lightest particle in this group is similar to that of the so-called inert Higgs doublet model \[5, 10, 11\]. For details, see for example recent updates \[12, 13, 14\].

**Conclusion :**

A new idea linking family symmetry to dark symmetry is proposed using a gauge $U(1)_D$ symmetry, which breaks to exactly conserved $Z_2$. The first and second families of quarks
and leptons transform under this $U(1)_D$ so that their masses are forbidden at tree level. They interact with the dark sector in such a way that they acquire one-loop finite masses, together with all three neutrinos. The extra $Z_D$ gauge boson may have a mass of order a few TeV, and one particle associated with the breaking of $U(1)_D$ may be identified with the 750 GeV diphoton excess recently observed at the LHC.

Acknowledgement: This work was supported in part by the U. S. Department of Energy Grant No. de-sc0008541.

References

[1] ATLAS Collaboration, ATLAS-CONF-2015-081.

[2] CMS Collaboration, CMS-PAS-EXO-15-004.

[3] E. Ma, Phys. Rev. Lett. 112, 091801 (2014).

[4] X, G. He, G. C. Joshi, H. Lew, and R. R. Volkas, Phys. Rev. D43, 22 (1991).

[5] E. Ma, Phys. Rev. D73, 077301 (2006).

[6] G. Aad et al. (ATLAS Collaboration), Phys. Rev. D90, 052005 (2014).

[7] S. Khachatryan et al. (CMS Collaboration), JHEP 1504, 025 (2015).

[8] J. Ellis, S. A. R. Ellis, J. Quevillon, V. Sanz, and T. You, arXiv:1512.05327 [hep-ph].

[9] Y. Hamada, H. Kawai, K. Kawana, and K. Tsumura, arXiv:1602.04170 [hep-ph].

[10] N. G. Deshpande and E. Ma, Phys. Rev. D18, 2574 (1978).

[11] R. Barbieri, L. J. Hall, and V. S. Rychkov, Phys. Rev. D74, 015007 (2006).
[12] A. Arhib, Y.-L. S. Tsai, Q. Yuan, and T.-C. Yuan, JCAP 1406, 030 (2014).

[13] A. Ilnicka, M. Krawczyk, and T. Robens, Phys. Rev D93, 055026 (2016).

[14] M. A. Diaz, B. Koch, and S. Urrutia-Quiroga, arXiv:1511.04429 [hep-ph].