Determination of the refractive index of water and glass using smartphone cameras by estimating the apparent depth of an object

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1 Abstract

A smartphone camera can be used for measuring the width and distance of an object by taking its photographs. The focal length of the camera lens can be determined very accurately by finding the width of the image of an object on the camera sensor to micron level accuracy. The level of accuracy achieved with the help of camera sensors, allows us to determine the refractive index of water up to four significant digits by finding the apparent depth of an object immersed in it. We have also measured the refractive index of glass by the same method, using three glass slides of different thicknesses, the smallest being 1.2 mm.

2 Introduction

A smartphone is a powerful instrument in the hands of a large section of people around the world. Students of physics in particular, can use it to perform many experiments, as is evident from different publications in educational journals from all over the world. We have performed an experiment on optics using the cameras of smartphones. Camera sensors in smartphones use sophisticated technology to produce images in a two dimensional array, called pixels, with length dimensions accurate to microns. We have taken the advantage of this accuracy to measure the
refractive index of water and glass with great accuracy, by taking photographs of an object and its apparent positions when kept beneath water or glass.

In some recent works [1,2] the method of determining the focal length of the lens of a smartphone camera, has been discussed. In the present paper we have first determined the focal lengths of lenses in two smartphone cameras used in this experiment. By photographing an object from a suitable position, we can find the image size on the camera sensor in terms of pixel, using software called Paint and Preview, available freely from the internet for Windows and Apple Mac operating systems, respectively. If we know the transverse dimensions of the object, we can easily find its transverse magnification. From the magnification and the focal length of the lens, we can accurately estimate the distance of the object [3]. This allows us to find the apparent depth of an object placed beneath water or glass, very accurately. From this we determine the refractive index of water or glass by the standard formula [4-9]

$$\mu = \frac{\text{Real Depth}}{\text{Apparent Depth}}.$$  

We have measured the refractive indices of water and glass with Apple smartphones of model IPhone 12-Mini and IPhone 12 Pro Max respectively.

3 Determination of the focal length of the lens

We can easily show that the object distance $u$ from a convex lens of focal length $f_c$ can be related to its transverse magnification $m$ as [3]

$$u = f_c \left( \frac{1}{m} - 1 \right).$$  

In our sign convention, the focal length is positive and the object distance for the real image is negative and the magnification is negative. If we know the object size and the focal length of the lens, we can find the distance of the object using the algebraic equation (2).

For an accurate determination of the focal length of the lens, we photograph the object from two positions of the camera shifted by a distance $D$ along the line
of sight of the lens. If \( u_1 \) and \( u_2 \) are the distances of the object from two positions of the camera with transverse magnifications \( m_1 \) and \( m_2 \) respectively, we get

\[
D = f_c \left( \frac{1}{m_2} - \frac{1}{m_1} \right) \tag{3}
\]

where \( D = u_2 - u_1 \). This displacement should be along the line of sight of the camera lens. In terms of the object and image sizes we can write \( m_1 = \frac{I_1}{O} \) and \( m_2 = \frac{I_2}{O} \) where \( O \) is the size of the object and \( I_1 \) and \( I_2 \) are the sizes of the images formed on the camera sensor. We can write equation (3) as

\[
f_c = \frac{|D|}{\left( \frac{1}{m_2} - \frac{1}{m_1} \right)}. \tag{4}
\]

In equation (3) the displacement \( D \) can be both towards or away from the object photographed. In figure 1, we show the lens configuration. The lens in the displaced position has been shown with dash marks. In this figure \( D \) is positive and \( m_2 \) is less than \( m_1 \). So, \( f_c \) becomes positive as it should. Similarly, when the camera is brought closer to the object, \( D \) is negative and the denominator in equation (3) is also negative. So, we have written equation (4) with absolute signs. We determine \( f_c \) using equation (4) by determining \( I_1, I_2 \) of an object of known width \( O \) and known displacement \( D \) of the lens. Once we determine the focal length, we can determine the distance of an object using equation (2).

4 Experiment

We first determine the focal lengths of the two camera lenses. In table 1 we present the data for one lens. We take a plastic ruler and photograph it from two distances with displacement \( D \) along the line of sight of the lens. \( u_1 \) and \( u_2 \) are the two distances of the camera from the ruler. When the image of the ruler is opened by the software Paint, one can select a certain part of the image using the cursors of the software and can find the pixel difference between its two ends. We select 7cm equivalent length on the image plane and the pixel equivalent has been presented as the pixel diff. in the table. From the internet one can find the pixel length
for the particular model of the camera. For our lens, 1 pixel was equal to 1.4 µm [10]. From this we determine the magnifications of the images which are obtained by taking the ratio of the image size to the object size which was 7cm. From this and the displacement of the camera, we determine the focal length very accurately using equation (3). There is an uncertainty in the actual position of the lens inside the smartphone. We displace the smartphone by a known distance in order to eliminate this uncertainty. Here the displacement of the lens is the same as the displacement of the phone. In exactly the same way, we determine the focal length of the other camera lens for which 1 pixel is equal to 1.7 µm [10].

To determine the refractive index of water we take an empty bucket and keep a small ruler at the bottom. From a distance about 10cm above the bucket we photograph the ruler with a smartphone held by a stand. By analysing the photograph with the Paint software, we can easily find the image size in terms of pixels. Specifically we find the image width for two marks on the ruler separated by 4cm and find the magnification. Using equation (2) we get the distance of the ruler from the camera. Now putting some water in the bucket we can, in the same way, find the distance of the virtual object. Finally, we float the ruler on the surface of water in the bucket and hence find the distance of the surface of water from the camera. All these three photographs are taken from the same position of the camera. From these three measurements we can determine the real depth as well as the apparent depth of the object. We then determine the refractive index of water using equation (1).

In a similar way, we determine the refractive index of glass. We take three slides of the same glass material, of nominal thicknesses of 1,4,6 mm. We take a strip of a graph sheet secured on a wooden stool. We take a photograph of the graph sheet from a fixed distance with a smartphone held by a stand. Then we put the glass slide, of a definite thickness, on the graph sheet. We take the photograph of the graph paper through the glass slide. Finally we remove the graph paper from the bottom of the glass slide and secure it on top of the glass. Then we take the third photograph of the graph sheet. Here also the camera position is kept
fixed for the three photographs. From these photographs we determine the real
depth which is nothing but the thickness of the glass slide. For each glass slide
we determine the apparent depth inside the glass. From these we determine the
refractive index of glass.

5 Experimental Results

The first row of table 1 shows that our camera is at a distance of $u_1 = 30.0 \text{cm}$
from the ruler. The camera was then shifted by 100.0 cm to take the second pho-
tograph from a distance $u_2 = 130.0 \text{cm}$. Magnifications in these two positions
are determined and using equation (4) we determine the focal length of the lens. The
process is repeated for 9 more times varying $u_1$ and $D$. The average focal length
turns out to be $f_c = 0.423 \pm 0.001 \text{cm}$. Similarly we get the focal length of the
second camera lens $f_c = 0.531 \pm 0.001 \text{cm}$. We have not shown the data for this
camera in tabular form.

In tables 2 and 3 we discuss the measurement of the refractive index of water.
In table 2, $u_1$ denotes the distance of the ruler, placed at the bottom of the empty
bucket, from the camera. The bucket is partially filled with water. The distance of
the apparent position of the ruler, as seen by the camera, is $u_3$. After taking two
photographs we take up the ruler and float it carefully on the water. Once again
it is photographed. Its distance from the camera is denoted by $u_2$. Each row of
table 2 corresponds to the same row of table 3. Image sizes are determined from
pixel readings and the transverse magnifications are obtained. Using equation (2)
we calculate all distances $u_1$, $u_2$, and $u_3$. From these we determine the real depth
and the apparent depth of the ruler and hence the refractive index. We get the
average value of the refractive index of water as $\mu = 1.327 \pm 0.004$. The refractive
index of pure water with respect to air at 20°C has been given by [11] as 1.3312 at
656 nm. Our sample of water is not pure. The refractive index of water goes down
with the rise in temperature [12]. In our case the room temperature was 30°C.
Assuming the presence of a small amount of dissolved material in our sample, we
can claim that our result upto 4 significant figures matches that given in [11] within experimental uncertainty.

We have determined the refractive index of glass using a second smartphone camera. In tables 4 and 5 we show our data with the three glass slides. Here also each row of table 4 corresponds to the corresponding row of table 5. For each slide we determine \( u_1, u_2, u_3 \) from the same position of the camera in the same way as described for water. For each slide we have taken photographs from 4 positions. As shown in table 4 the average measured thicknesses of the slides turn out to be 1.2 mm, 4.0 mm and 6.0 mm. In table 5 we show the average refractive index of glass slides which we determine to be 1.50 ± 0.01.

6 Conclusions

Taking advantage of the accuracy to micron level in the image sizes on the sensor of smartphone cameras, we have been able to determine the refractive index of water to 4 significant digits. We have measured the apparent depth of an object immersed in water by taking photograph of the object, looking vertically downwards. Using the same technique we have determined the refractive index of glass taking glass slides with thickness as small as 1.2 mm. The accuracy of the refractive index also indicates that our measurement of the thickness of the glass slides is also accurate. So the photographic technique introduced here is an alternative method for determination of the thickness of a thin sheet. The technique is easy and fast. We can determine the refractive index of any transparent liquid or a transparent solid, in the shape of slides, very accurately, using this photographic method.
Table 1: Pixel data for the image sizes on the camera sensor from two distances:

Object size = 7cm

Smartphone : Apple IPhone 12-Mini ; 1 pixel=1.4\(\mu\)m

| obs no. | cm diff. 1 | pixel | \(I_1\) cm | cm diff. 2 | \(I_2\) cm | \(f_c\) cm |
|--------|------------|-------|-------------|------------|-------------|------------|
| 1      | 30.0       | 712   | 0.0997      | 130.0      | 163         | 0.0228     | 0.422      |
| 2      | 40.0       | 537   | 0.0752      | 140.0      | 152         | 0.0213     | 0.424      |
| 3      | 30.0       | 712   | 0.0997      | 140.0      | 152         | 0.0213     | 0.426      |
| 4      | 40.0       | 537   | 0.0752      | 150.0      | 142         | 0.0199     | 0.425      |
| 5      | 50.0       | 427   | 0.0598      | 160.0      | 132         | 0.0185     | 0.421      |
| 6      | 40.0       | 537   | 0.0752      | 160.0      | 132         | 0.0185     | 0.421      |
| 7      | 50.0       | 427   | 0.0598      | 170.0      | 124         | 0.0174     | 0.421      |
| 8      | 30.0       | 712   | 0.0997      | 160.0      | 132         | 0.0185     | 0.422      |
| 9      | 40.0       | 537   | 0.0752      | 180.0      | 118         | 0.0165     | 0.423      |
| 10     | 30.0       | 712   | 0.0997      | 180.0      | 118         | 0.0165     | 0.424      |

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Table 2: Data for refractive index of water

\( u_1 = \) Distance to the bottom of the bucket from the camera

\( u_2 = \) Distance to the top of the water surface from the camera

Object size: 4 cm

Focal length of the lens \( f_c = 0.423 \) cm

| obs no. | diff. 1 m1 \( \text{cm} \) | \( u_1 \) pixel | diff. 2 m2 \( \text{cm} \) | \( u_2 \) pixel | Depth of water \( (u_1-u_2)\text{cm} \) |
|---------|----------------|----------------|----------------|----------------|----------------|
| 1       | 303 0.01060   40.33 | 333 0.01166   | 36.70           | 3.63           |
| 2       | 240 0.00840   50.78 | 272 0.00952   | 44.86           | 5.92           |
| 3       | 303 0.01060   40.33 | 373 0.01306   | 32.81           | 7.52           |
| 4       | 316 0.01106   38.67 | 401 0.01404   | 30.55           | 8.12           |
| 5       | 329 0.01152   37.14 | 449 0.01572   | 27.33           | 9.81           |
| 6       | 329 0.01152   37.14 | 601 0.02104   | 20.53           | 16.61          |
| 7       | 329 0.01152   37.14 | 772 0.02702   | 16.08           | 21.06          |
| 8       | 329 0.01152   37.14 | 1065 0.03728  | 11.77           | 25.37          |

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Table 3: Data for refractive index of water

u3= Distance of the virtual object from the camera

Object size :4cm

Focal length of the lens \( f_c = 0.423 \text{cm} \)

| obs no. | pixel diff. | cm | u3 (u3-u2)cm | \( \mu = \frac{u1-u2}{u3-u2} \) | Av \( \mu \) |
|---------|-------------|----|--------------|-----------------|---------|
| 1       | 310         | 0.01085 | 39.41        | 2.71            | 1.339   |
| 2       | 247         | 0.00864 | 49.38        | 4.52            | 1.310   |
| 3       | 318         | 0.01113 | 38.43        | 5.62            | 1.338   |
| 4       | 333         | 0.01166 | 36.70        | 6.15            | 1.320   | 1.327 ± 0.004 |
| 5       | 352         | 0.01232 | 34.76        | 7.43            | 1.320   |
| 6       | 371         | 0.01298 | 33.01        | 12.48           | 1.331   |
| 7       | 383         | 0.01340 | 31.99        | 15.91           | 1.324   |
| 8       | 397         | 0.01390 | 30.85        | 19.08           | 1.330   |
Table 4: Data for refractive index of glass

Object size=6cm

Smartphone: Apple IPhone 12 pro max

Pixel size=1.7\mu m; Focal length of the camera lens $f_c = 0.531 \pm 0.001\text{cm}$

$u_1=$Distance of the graph sheet from the camera

$u_2=$Distance to the top of glass slide

| obs no. | Nominal thickness of slides (mm) | pixel m1 cm | u1 cm | pixel m2 cm | u2 cm | Thickness of slides (u1-u2) cm |
|---------|---------------------------------|-------------|-------|-------------|-------|-----------------------------|
| 1       | 1749                            | 0.04956     | 11.24 | 1770        | 0.05015 | 11.12                      | 0.12 |
| 2       | 1606                            | 0.04550     | 12.20 | 1623        | 0.04598 | 12.08                      | 0.12 |
| 3       | 1504                            | 0.04261     | 12.99 | 1519        | 0.04304 | 12.87                      | 0.12 |
| 4       | 1321                            | 0.03743     | 14.72 | 1333        | 0.03777 | 14.59                      | 0.13 |
| 5       | 1659                            | 0.04700     | 11.83 | 1719        | 0.04870 | 11.43                      | 0.40 |
| 6       | 1435                            | 0.04066     | 13.59 | 1479        | 0.04190 | 13.20                      | 0.39 |
| 7       | 1282                            | 0.03632     | 15.15 | 1317        | 0.03732 | 14.76                      | 0.39 |
| 8       | 1207                            | 0.03420     | 16.06 | 1238        | 0.03508 | 15.67                      | 0.39 |
| 9       | 1659                            | 0.04700     | 11.83 | 1751        | 0.04961 | 11.23                      | 0.60 |
| 10      | 1435                            | 0.04066     | 13.59 | 1503        | 0.04258 | 13.00                      | 0.59 |
| 11      | 1282                            | 0.03632     | 15.15 | 1336        | 0.03785 | 14.56                      | 0.59 |
| 12      | 1207                            | 0.03420     | 16.06 | 1255        | 0.03556 | 15.46                      | 0.60 |
Table 5: Data for refractive index of glass

u3= Distance of the virtual object from the camera

Object size :6cm

| obs no. | pixel | m3 | u3 | Apparent depth cm | (u3-u2) cm | Av µ |
|---------|-------|----|----|--------------------|------------|------|
| 1       | 1756  | 0.04975 | 11.20 | 0.08 | 11.20 | 0.08 | 1.50 |
| 2       | 1612  | 0.04567 | 12.16 | 0.08 | 12.16 | 0.08 | 1.50 |
| 3       | 1509  | 0.04276 | 12.95 | 0.08 | 12.95 | 0.08 | 1.50 |
| 4       | 1325  | 0.03754 | 14.68 | 0.09 | 14.68 | 0.09 | 1.44 |
| 5       | 1679  | 0.04757 | 11.69 | 0.26 | 11.69 | 0.26 | 1.54 |
| 6       | 1449  | 0.04106 | 13.46 | 0.26 | 13.46 | 0.26 | 1.50 |
| 7       | 1293  | 0.03664 | 15.02 | 0.26 | 15.02 | 0.26 | 1.50 |
| 8       | 1217  | 0.03448 | 15.93 | 0.26 | 15.93 | 0.26 | 1.50 |
| 9       | 1689  | 0.04786 | 11.62 | 0.39 | 11.62 | 0.39 | 1.54 |
| 10      | 1457  | 0.04128 | 13.39 | 0.39 | 13.39 | 0.39 | 1.51 |
| 11      | 1299  | 0.03680 | 14.96 | 0.40 | 14.96 | 0.40 | 1.48 |
| 12      | 1223  | 0.03465 | 15.86 | 0.40 | 15.86 | 0.40 | 1.50 |
Fig. 1 Image formation at the two positions of the lens