Some effects of the quantum field theory in the early Universe

A. A. Grib∗ and Yu. V. Pavlov†

A. Friedmann Laboratory for Theoretical Physics,
30/32 Griboedov can, St.Petersburg, 191023, Russia

Abstract

Effects of the vacuum polarization leading to change of the effective gravitational constant and particle creation in the early Universe are discussed. Gauss-Bonnet type coupling to the curvature is considered. Renormalization methods are generalized for such coupling and the N-dimensional space-time. Calculations of creation of entropy and visible matter of the Universe by the gravity of dark matter are made.

1 Introduction

Quantum field theory in curved space-time is the generalization of well developed theory in Minkowski space-time, so it has some features of the standard theory together with new ones due to the curvature. It was actively developed in the 70-ties of the last century (see books [1, 2]) however some new results were obtained just recently and in this paper we’ll concentrate on these new results as well as on some physical interpretation of the old ones.

One of the main results of the theory for the early Friedmann Universe was the calculation of finite stress-energy tensor for particle creation. Let us begin from some remarks on what is meant by particle creation in curved space-time? In spite of the absence of the standard definition of the particle as the representation of the Poincaré group in curved space-time where Poincaré group is not a group of motions, one still has physically clear idea of the particle. The particle is understood as the classical point like object moving along the geodesic of the curved space-time. This classical particle however is the quasiclassical approximation of some quantum object, so the main mathematical problem is to find the answer to the question: what is the Fock quantization of the field giving just this answer for particles in this approximation? The answer to this question was given by us in the 70-ties for conformal massive scalar particles and spinor particles in Friedmann space-time by use of the metrical Hamiltonian diagonalization method. Creation of particles in the early Universe from vacuum by the gravitational field means

∗E-mail: andrei_grib@mail.ru
†E-mail: yuri.pavlov@mail.ru
that for some small time close to singularity the stress-energy tensor has the geometrical form, expressed through some combinations of the Riemann tensor and its derivatives while for the time larger than the Compton one it has the form of the dust of pointlike particles with mass and spin defined by the corresponding Poincaré group representation.

So our use of quantum field theory in curved space-time with its methods of regularization (dimensional regularization and Zeldovich-Starobinsky regularization) was totally justified by the obtained results. However still unclear was the situation with the minimally coupled scalar field where our method led to infinite results for the density of created particles, it is only recently that Yu.V.Pavlov could find the transformed Hamiltonian, diagonalization of which leads to finite results for this case.

The effect of particle creation in the early Universe can have more general meaning concerning the origination of space-time itself. It is reasonable to consider macroscopic Universe as classical approximation of the quantum Universe. But what is quantum Universe? One can speculate that not only the metric but even the very existence of space-time points described by some coordinates is due to existence of operators of coordinates for some massive particles. In the “empty” quantum Universe there are no points! This is in some sense return to the Leibniz idea that space and time are relations between particles and don’t exist without them. It is important to note that space with space-like intervals between particles and time are necessary in order to measure or have information for Boolean – minded classical observer about observables described by noncommuting operators. Noncommuting observables become commuting ones if one has some “copying” mechanism for the quantum system, when the same (identical) system is observed in different points of space separated by the space-like interval or at different moments of time. Time is needed for measuring noncommuting observables. This “copying” process makes possible contemplation of the quantum Universe with its complementary characteristics by the classical observer. “Copying” is the same as particle creation. That is why particle creation can be considered as origination of the classical Universe with its space and time from the quantum one. There is no necessity for quantization of gravity from this point of view, because space, time and metric are just some forms used by the classical observer dealing with the quantum Universe. Entropy of the Universe arising due to such “measurement” process is the other side of this process of particle creation and origination of space-time.

We use the system of units where \( \hbar = c = 1 \). The signs of the curvature tensor and Ricci tensor are chosen such that

\[
R^i_{jkl} = \partial_l \Gamma^i_{jk} - \partial_k \Gamma^i_{jl} + \Gamma^i_{mn} \Gamma^m_{jk} - \Gamma^i_{nk} \Gamma^m_{jl},
\]

\[
R_{ik} = R^l_{ilk}, \quad R = R^l_l, \quad \text{where } \Gamma^i_{jk} \text{ are Christoffel symbols.}
\]
2 Some remarks on the physical interpretation of terms in vacuum polarization dependent on mass

The calculations of the vacuum expectation value of the stress-energy tensor of the quantized conformal massive scalar, spinor and vector fields \[1,2\] led to the expressions different for strong and weak external gravitational field. By strong field one understands the field with the curvature much larger than the one defined by the mass of the particle. For strong gravitational field it is basically polarization of vacuum terms which become zero if the gravitation is zero, for weak gravitation it is defined by particles created previously, so that if gravitation becomes zero particles still exist. Vacuum polarization for strong gravitation consists of three terms: the first is due to the conformal anomaly and it does not depend on mass of the particle at all, the second is due to the Casimir effect and is present for the closed Friedmann space, the third is vacuum polarization dependent on mass of the particle which is for scalar conformal particles \[1,2\]

\[
\langle T_{ik}\rangle_m = \frac{m^2}{144\pi^2} G_{ik} + \frac{m^4}{64\pi^2} g_{ik} \ln\left(\frac{\mathcal{R}}{m^4}\right).
\]

(1)

Here \(G_{ik} = R_{ik} - R g_{ik}/2\) is the Einstein tensor, \(g_{ik}\) is the metrical tensor, \(\mathcal{R}\) is some geometrical term of the dimension of the fourth degree of mass. It is interesting that the first term is the illustration of the Sakharov’s idea \[5\] of gravitation as the vacuum polarization. If one takes the Planckian mass one has just the standard expression for the term in the Einstein equation. One can also think that gravitation is the manifestation of the vacuum polarization of all existing fields with different masses. However for weak gravitation for the time of evolution of the Friedmann Universe larger than the Compton one defined by the mass there is no Sakharov term, it is compensated. It is just manifestation of the fact, known also in quantum electrodynamics that vacuum polarization is different in strong and weak external fields being asymptotics of some general expression. So the physical meaning of this term in strong field is finite change of the gravitational constant, so that the new effective gravitational constant is different from that in the weak field, being

\[
\frac{1}{8\pi G_{\text{eff}}} = \frac{1}{8\pi G} + \frac{m^2}{144\pi^2}.
\]

(2)

So the larger is \(m\), the smaller is \(G_{\text{eff}}\). If \(m\) is macroscopic \[6\], then \(G_{\text{eff}} \to 0\) which means some kind of “asymptotic freedom” for gravity if one deals with macroscopic masses (like the mass of the Universe) quantum mechanically.

The second term on the right side \[1\] describes what is now called “quintessence” – cosmological constant, dependent on time. This “quintessence” is different for different stages of the evolution of the Friedmann Universe. We calculated it previously for the radiation dominated Universe, so that together with the
Sakharov term one has

\[ \langle T^0_0 \rangle_m = -\frac{m^2}{48\pi^2} \left( \alpha \right)^2 - \frac{m^4}{16\pi^2} \ln(ma) + C + \frac{1}{4a^3} \int_0^{\eta_1} d\eta_1 \frac{d\alpha^2}{d\eta_1} \int_0^{\eta_2} d\eta_2 \frac{d\alpha^2}{d\eta_2} \ln |\eta_1 - \eta_2| \] .

(3)

Here \( a \) is the scale factor of the Friedmann space-time, \( \eta \) is the conformal time, \( C = 0.577 \ldots \) is the Euler constant.

3 Creating of scalar particles in curved space-time

One of unsolved problems of quantum theory in curved space is related to the definition of an elementary particle and vacuum in a curved space-time. The reason is that in the case of a curved space there is no group of symmetries similar to the Poincaré group in the Minkowski space. If we believe that a particle is associated with a quantum of energy, then according to quantum mechanics, the observation of particles at an instant of time implies finding an eigenstate of the Hamiltonian. The Hamiltonian diagonalization method [2] takes this into account automatically.

The use of the Hamiltonian constructed from the metrical energy-momentum tensor, first proposed by A.A.Grib and S.G.Mamayev [7] was successful in the homogeneous isotropic space-time for the conformal scalar fields. But such Hamiltonian leads to the difficulties related to an infinite density of created quasiparticles in the nonconformal case [8]. The energy of such quasiparticles differs from the oscillator frequency of the wave equation [9, 10]. The nonconformal case is important for investigation by many reasons. Vector mesons and gravitons can be such particles. The additional nonconformal terms can be dominant in the vacuum expectation values of the energy-momentum tensor [11]. In Ref. [3] the modified Hamiltonian was found so that the density of the particles corresponding to its diagonal form and created in the nonstationary homogeneous isotropic space-time is finite. In this section one gives the generalization of the method, proposed in Ref. [3], for the cases of conformally static spaces and coupling with curvature of the general form (see Ref. [12]).

We consider a complex scalar field \( \varphi(x) \) of the mass \( m \) with Lagrangian

\[ L(x) = \sqrt{|g|} \left[ g^{ik} \partial_i \varphi^* \partial_k \varphi - (m^2 + V_g) \varphi^* \varphi \right] , \]

(4)

and corresponding equation of motion

\[ (\nabla^i \nabla_i + V'_g + m^2)\varphi(x) = 0 , \]

(5)

where \( \nabla_i \) are the covariant derivatives in \( N \)-dimensional space-time, \( g = \det(g_{ik}) \), \( V_g \) is the function of the invariant combinations \( g_{ik} \) and curvature tensor \( R^{i}_{jkl} \).
The equation (5) is conformally invariant if $m = 0$ and $V_g = \xi_e R$, where $\xi_e = (N - 2) / [4(N - 1)]$ (conformal coupling). The case $V_g = 0$ is the minimal coupling. In this section we investigate the case of conformally static metric of the form

$$ds^2 = dt^2 - a^2(t) \gamma_{\alpha\beta}(x) dx^\alpha dx^\beta = a^2(\eta) \left( d\eta^2 - \gamma_{\alpha\beta}(x) dx^\alpha dx^\beta \right),$$

where $a', \beta = 1, \ldots, N - 1$. It is realized, in particular, for the homogeneous isotropic space-time. The equation (5) for the functions $\tilde{\varphi} = a^{(N-2)/2} \varphi$ in coordinates $(\eta, x)$ takes the form

$$\tilde{\varphi}'' - \Delta_{N-1} \tilde{\varphi} + \left[ \left( m^2 + V_g \right) a^2 - \frac{N - 2}{4} \left( 2c' + (N - 2)c_0^2 \right) \right] \tilde{\varphi} = 0,$$

where the prime denotes the derivative with respect to "conformal" time $\eta$, $c \equiv a'/a$, $\Delta_{N-1} \equiv \gamma^{-1/2} \partial_\alpha (\sqrt{\gamma} \gamma^{\alpha\beta} \partial_\beta)$, $\gamma = \det(\gamma_{\alpha\beta})$. Let us assume that Laplace-Beltrami operator $\Delta_{N-1}$ has the complete orthonormal set of the eigenfunctions $\Phi_J(x)$

$$\Delta_{N-1} \Phi_J(x) = -\lambda^2(J) \Phi_J(x),$$

where $J$ is set of $N - 1$ indices (quantum numbers). The full set of the eq. (7) solutions can be found in the form $\tilde{\varphi}(x) = g_J(\eta) \Phi_J(x)$, where

$$g''_J(\eta) + \Omega^2(\eta) g_J(\eta) = 0,$$

$\Omega$ is the oscillator frequency:

$$\Omega^2(\eta) = \left( m^2 + V_g \right) a^2 - \frac{N - 2}{4} \left( 2c' + (N - 2)c_0^2 \right) + \lambda^2(J).$$

The expression $g_J(\eta) g_J^{*'}(\eta) - g_J'(\eta) g_J^*(\eta)$ is the first integral of eq. (8). Further one normalizes the solutions of eq. (8) by condition

$$g_J g_J^{*'} - g_J' g_J^* = -2i.$$

For quantization, we expand the field $\tilde{\varphi}(x)$

$$\tilde{\varphi}(x) = \int d\mu(J) \left[ \tilde{\varphi}^{(+)}_J a^{(+)}_J + \tilde{\varphi}^{(-)}_J a^{(-)}_J \right],$$

where $d\mu(J)$ is the measure on the space of $\Delta_{N-1}$ eigenvalues,

$$\tilde{\varphi}^{(+)}_J(x) = \frac{1}{\sqrt{2}} g_J(\eta) \Phi_J^*(x), \quad \tilde{\varphi}^{(-)}_J(x) = \left( \tilde{\varphi}^{(+)}_J(x) \right)^*,$$

and impose the usual commutation relations on the operators $a^{(\pm)}_J, a^{(\pm)}_J$:

$$\left[ a^{(-)}_J, a^{(+) \dagger}_J \right] = \left[ a^{(-)}_J, a^{(+) \dagger}_J \right] = \delta_{JJ'}, \quad \left[ a^{(\pm)}_J, a^{(\pm)}_J \right] = \left[ a^{(\pm)}_J, a^{(\pm)}_J \right] = 0.$$

We construct Hamiltonian as canonical one for variables $\tilde{\varphi}(x)$ and $\tilde{\varphi}^*(x)$. If we add $N$-divergence ($\partial J^i / \partial x^i$) to Lagrangian density (4), where in the coordinate
system \((\eta, x)\) the \(N\)-vector \((J^i) = (\sqrt{N} c \tilde{\varphi}^* \tilde{\varphi} (N - 2)/2, 0, \ldots, 0)\), the motion equation is invariant under this addition. By using the Lagrangian density \(L^A(x) = L(x) + (\partial J^i/\partial x^i)\), we obtain by integration on hypersurface \(\Sigma\) : \(\eta = \text{const}\) of Hamiltonian density \(h(x) = \varphi' (\partial L^A)/(\partial \dot{\varphi}') + \tilde{\varphi}'' (\partial L^A)/(\partial \dot{\varphi}'') - L^A(x)\), the modified Hamiltonian

\[
H(\eta) = \int_{\Sigma} h(x) d^{N-1}x = \int_{\Sigma} d^{N-1}x \sqrt{\gamma} \left\{ \tilde{\varphi}'' \tilde{\varphi}' + \gamma^{\alpha\beta} \partial_\alpha \tilde{\varphi}^* \partial_\beta \tilde{\varphi} + \right.
\]
\[
+ \left. \left[ (m^2 + V_\alpha) a^2 - \frac{N - 2}{4} (2c' + (N - 2)c^2) \right] \tilde{\varphi}^* \tilde{\varphi} \right\}.
\]

Hamiltonian is expressed in terms of the operators \(a_j^{(\pm)}, \tilde{a}_j^{(\pm)}\) by

\[
H(\eta) = \int d\mu(J) \left\{ E_J(\eta) \left( \tilde{a}_j^{(\pm)} a_j^{(-)} + \tilde{a}_j^{(-)} a_j^{(\pm)} \right) + F_J(\eta) \tilde{a}_j^{(\pm)} a_j^{(\pm)} + F_J^*(\eta) \tilde{a}_j^{(-)} a_j^{(-)} \right\},
\]

where

\[
E_J(\eta) = \frac{1}{2} \left[ |g_j|^2 + \Omega^2 |g_j|^2 \right], \quad F_J(\eta) = \frac{\partial J}{2} \left[ g_j^2 + \Omega^2 g_j^2 \right],
\]

here we used the normalization condition and the special choice of the eigenfunctions so that for an arbitrary \(J\) such \(\tilde{J}\) exists, that \(\Phi_\eta(x) = \partial J \Phi_J(x)\). In accordance with condition of the completeness such set can be selected by the redefinition of eigenfunctions \(\Phi_J(x)\). The Hamiltonian diagonalization for initial instant \(\eta_0\) and normalization condition give (under \(\Omega^2(\eta_0) > 0\)):

\[
g_j'(\eta_0) = i \Omega(\eta_0) g_j(\eta_0), \quad |g_j(\eta_0)| = \Omega^{-1/2}(\eta_0).
\]

The Hamiltonian diagonalization for an arbitrary instant \(\eta\) is realized in terms of the operators \(\tilde{b}_j^{(\pm)}\), \(b_j^{(\pm)}\) related to the operators \(a_j^{(\pm)}, \tilde{a}_j^{(\pm)}\) via the time-dependent Bogoliubov transformations

\[
a_j^{(\pm)} = \alpha_j(\eta) b_j^{(\pm)}(\eta) - \beta_j(\eta) \partial J b_j^{(\pm)}(\eta), \quad \tilde{a}_j^{(\pm)} = \alpha_j^*(\eta) \tilde{b}_j^{(\pm)}(\eta) - \beta_j(\eta) \partial J \tilde{b}_j^{(\pm)}(\eta),
\]

where the functions \(\alpha_j, \beta_j\) satisfy the initial conditions \(|\alpha_j(\eta_0)| = 1\), \(\beta_j(\eta_0) = 0\) and the identity \(|\alpha_j(\eta)|^2 - |\beta_j(\eta)|^2 = 1\). Substituting expansion in and requiring that the coefficients of the nondiagonal terms \(\tilde{b}_j^{(\pm)} b_j^{(\pm)}\) vanish, we obtain

\[
-2 \alpha_j \beta_j \partial J E_J + \alpha_j^2 F_J + \beta_j^2 \partial J^2 F_J^* = 0, \quad |\beta_j|^2 = \frac{1}{4\Omega} \left( |g_j|^2 + \Omega^2 |g_j|^2 \right) - \frac{1}{2},
\]

\[
H(\eta) = \int d\mu(J) \Omega(\eta) \left( \tilde{b}_j^{(\pm)} b_j^{(-)} + \tilde{b}_j^{(-)} b_j^{(\pm)} \right).
\]

Therefore, the energies of quasiparticles corresponding to the diagonal form of the Hamiltonian are equal to the oscillator frequency \(\Omega(\eta)\).

The vacuum state for the instant \(\eta\) is defined by \(b_j^{(-)}(\eta)|0_\eta\rangle = \tilde{b}_j^{(-)}(\eta)|0_\eta\rangle = 0\). The state \(|0\rangle = |0_{\eta_0}\rangle\) contains \(|\beta_j(\eta)|^2\) quasiparticle pairs corresponding to the
operators $b_{J}^{(\pm)}(\eta)$ in every mode $[2,13]$. The density of created particles is proportional to $\int d\mu(J) |\beta_{J}|^{2}$. The function $S(\eta) = |\beta_{J}(\eta)|^{2}$ obeys the integral equation

$$S(\eta) = \frac{1}{2} \int_{\eta_0}^{\eta} d\eta_1 \int_{\eta_0}^{\eta_1} d\eta_2 \ w(\eta_1) \ w(\eta_2) \ (1 + 2S(\eta_2)) \ \cos[2 \Theta(\eta_2, \eta_1)],$$

(22)

where

$$w(\eta) = \Omega'(\eta)/\Omega(\eta), \quad \Theta(\eta_2, \eta_1) = \int_{\eta_2}^{\eta_1} \Omega(\eta) \ d\eta.$$  

Using the first iteration and taking into account that $\Theta(\eta_2, \eta_1) \rightarrow \lambda(\eta_1 - \eta_2)$ in $\lambda \rightarrow \infty$ one can represent (22) as

$$S(\eta) \approx \frac{1}{4} \left| \int_{\eta_0}^{\eta} w(\eta_1) \exp(2i\lambda \eta_1) \ d\eta_1 \right|^{2}.$$  

(23)

Consequently, one can see, that $S \sim \lambda^{-6}$ in $\lambda(J) \rightarrow \infty$.

Therefore, in 4-dimensional space-time with metric (6) the density of created particles is finite. So it is shown differently from $[8,13]$, that even for minimal coupling the result is finite! From eq. (22) for $S(\eta)$ one can see that under $V_g = \xi R$ the densities of scalar particles created in Friedmann radiative-dominant Universe don’t depend on $\xi$. So the results will be for $t \gg m^{-1}$ the same as for conformal coupled particles obtained earlier $[1,2]$ (see eq. (67) in this paper).

### 4 Scalar field with Gauss-Bonnet type coupling to the curvature

Usually for scalar field in curved space-time one writes the eq. (5) with $V_g = \xi R$. The condition $\xi = \xi_c$ consistent with conformal invariance of the equations for massless field is considered sometimes as preferable (see for example $[14,15]$). For scalar fields in inflation models $[16]$ minimal coupling $\xi = 0$ usually is used. In general case one can take arbitrary values of $\xi$ $[11,17]$. Renormalization of interacting fields in curved space-time is inconsistent with conformal invariance not only of the effective action but also of usual action $[13]$. Models with arbitrary $\xi$ without conformal invariance on the classical level can be generalized by adding of the quadratic in curvature terms

$$V_g = \xi R + \zeta R^2 + \kappa R_{ij} R^{ij} + \chi R_{ijkl} R^{ijkl} + \ldots$$

(24)

Nonzero coupling constants in these terms in $[24]$ having the dimension (mass)$^{-2}$ lead usually to derivatives of the third and fourth order in the metrical energy-momentum tensor (EMT) and in Einstein equations. As it is known $[18]$ such terms with higher derivatives even for small coefficients lead to the radical change of the theory. If one has the condition that metrical EMT of the scalar field does not contain derivatives of metric higher than the second order, one can take for $V_g$

$$V_g = \xi R + \zeta R^{2}_{GB}, \quad \text{where} \quad R^{2}_{GB} \overset{\text{def}}{=} R_{impq} R^{impq} - 4 R_{im} R^{im} + R^{2}.$$  

(25)
On even-dimensional spaces due to the Gauss-Bonnet theorem \[19\] Euler characteristic of compact orientable manifold \(M\) with Riemann metric is equal to

\[
\chi(M) = \int_M E(x) \sqrt{g} \, d^N x ,
\]

where \(E(x) = -(4\pi)^{-1} R(x)\) for \(N = 2\) and \(E(x) = R_{GB}^2/(128\pi^2)\) for \(N = 4\). So the case with \(V_q\) defined by \[20\] can be called Gauss-Bonnet type coupling.

The condition of absence of higher derivatives of metric was considered earlier \[20\] as basic for multidimensional generalizations of the gravity theory. It was shown that the demands of symmetry in indices, covariant conservation and the condition that the tensor generalizing Einstein tensor \(G_{ik}\) is constructed from \(g_{ik}\) and only its first and second derivatives give the Einstein equations with cosmological constant in dimension four. But in dimension larger than four the corresponding equations can be obtained from the action which is a sum of contributions associated with continuation to this dimension of Euler characteristics of all lower even dimensions \[20\]. The demand of absence of ghosts in low energy approximations of string theories gives the Einstein-Gauss-Bonnet gravity with \(R_{GB}^2\) density in action for higher dimensions \[21\]. The Gauss-Bonnet coupling of scalar field with gravity was also considered in dilaton theories (see for example \[22\]).

Taking variational derivatives of the action for scalar field with coupling \[25\] one obtains the following expression for metrical EMT:

\[
T_{ik} = \partial_i \varphi^* \partial_k \varphi + \partial_k \varphi^* \partial_i \varphi - g_{ik} \partial^l \varphi^* \partial_l \varphi + g_{ik} m^2 \varphi^* \varphi - 2\zeta \left( G_{ik} + \nabla_i \nabla_k - g_{ik} \nabla^l \nabla_l \right) (\varphi^* \varphi) - 2\zeta (E_{ik} + P_{ik}) (\varphi^* \varphi) ,
\]

where

\[
E_{ik} = \frac{\delta \int R_{GB}^2 \sqrt{|g|} \, d^N x}{\sqrt{|g|} \delta g_{ik}} = 2R_{ilmp} R_{k}^{lmp} - \frac{g_{ik}}{2} R_{lmpq} R^{lmpq} - 4R_{lm} R_{l}^{lm} - 4R_{ik} R_{i}^{k} + 2g_{ik} R_{lm} R_{l}^{lm} + 2RR_{ik} - \frac{g_{ik}}{2} R^2 = \]

\[
= 2C_{ilmp} C_{k}^{lmp} - \frac{g_{ik}}{2} C_{lmpq} C_{k}^{lmpq} - (N - 4)(3)H_{ik} ,
\]

\[
P_{ik} = 2 \left[ R \nabla_i \nabla_k + 2R_{ik} \nabla_l \nabla^l + 2g_{ik} R_{lm} \nabla^l \nabla^m - g_{ik} R \nabla_l \nabla_l - 4R_{l(i} \nabla_k \nabla^l \right] = -4C_{iklm} \nabla^l \nabla^m +
\]

\[
+ 4 \frac{N-3}{N-2} \left( R_{ik} \nabla_l \nabla^l + g_{ik} R_{lm} \nabla^l \nabla^m - 2R_{l(i} \nabla_k \nabla^l + \frac{NR}{2(N-1)} \left( \nabla_i \nabla_k - g_{ik} \nabla_l \nabla^l \right) \right) ,
\]

\[
(3)H_{ik} = \frac{4C_{iklm} R_{lm}^i}{N-2} + \frac{2(N-3)}{(N-2)^2} \left[ 2R_{ik} R_{l}^{l} - \frac{NR R_{ik}}{N-1} - g_{ik} \left( R_{lm} R_{l}^{lm} - \frac{(N+2) R^2}{4(N-1)} \right) \right] ,
\]
\[ C_{iklm} = R_{iklm} + \frac{2}{N-2} \left( R_{m[i} g_{k]l} - R_{l[i} g_{k]m} \right) + \frac{2}{(N-1)(N-2)} R g_{[i} g_{k]m}, \]  

(parentheses (square brackets) in indices mean symmetrization (antisymmetrization). Expressions for EMT using conformal Weyl tensor \( C_{iklm} \) are convenient for calculation as in conformally flat \( (C_{iklm} = 0) \), for example homogeneous isotropic spaces, as in Ricci flat \( (R_{ik} = 0) \) spaces. For \( N = 2,3 \) one has \( R^2_{GB} \equiv 0 \) and there are no new effects due to \( \zeta \neq 0 \). In 4-dimensional space-time \( E_{ik} = 0 \) (see Ref. [23]), but \( P_{ik}(\varphi^* \varphi) \neq 0 \) for general metric and \( \varphi(x) \neq const. \)

Vacuum averages of EMT for quantized fields are divergent. For the analysis of the geometrical structure of divergences of vacuum averages of EMT one can use dimensionally regularized effective action. For complex scalar field with eq. \([31]\) in loop approximation the effective action can be written \([24, 25]\) as

\[ S_{eff} = \int L_{eff}(x) \sqrt{|g|} d^N x, \]  

where

\[ L_{eff}(x) = (4\pi)^{-N/2} \left( \frac{M}{m} \right)^{2\epsilon} \sum_{j=0}^{\infty} a_j(x) m^{N_0-2j} \Gamma\left( j - \frac{N}{2} \right), \]  

\[ a_0(x) = 1, \quad a_1(x) = \frac{1}{6} R - V_g = \left( \frac{4 - N}{12(N-1)} + \Delta \xi \right) R - \zeta R^2_{GB}, \]  

\[ a_2(x) = \frac{R_{lmpq} R^{lmpq}}{180} - \frac{R_{lm} R^{lm}}{180} + \frac{R^2}{72} - \frac{\nabla^l \nabla_l R}{30} - \frac{R V_g}{6} + \frac{V_g^2}{2} + \frac{\nabla^l \nabla_l V_g}{6}, \]  

\( N \) is space-time dimension which is taken as some variable, analytically continued in complex plane, \( \epsilon \) is some complex parameter, \( M \) is some constant of the dimension of mass \([20]\), necessary for correct dimension of \( L_{eff}, \) \((\text{length})^{-N_0}\) for \( N = N_0 - 2\epsilon \), \( \Gamma(z) \) is gamma-function, \( \Delta \xi \equiv \xi_c - \xi \). Using \([31]\) for Gauss-Bonnet coupling \([25]\) one can write \( a_2 \) as

\[ a_2(x) = \left( \frac{(N-4)(N-6)}{480(N-1)^2} - \frac{\Delta \xi(N-4)}{12(N-1)} + \frac{(\Delta \xi)^2}{2} \right) R^2 + \frac{(N-2) C_{lmpq} C^{lmpq}}{240(N-3)} + \]

\[ + \frac{(N-6) R^2_{GB}}{720(N-3)} + \left( \frac{(N-4)}{12(N-1)} - \Delta \xi \right) \zeta R R^2_{GB} + \frac{\zeta^2 R^4_{GB}}{2} - \frac{1}{6} \nabla^l \nabla_l \left( \frac{1}{5} R - V_g \right). \]  

First \([N_0/2]+1\) terms in \([33]\) are excluded to get the renormalized \( L_{eff} \) \([b]\) denotes the integer part of the number \( b \). By variation in \( g_{ik} \) of terms \( j = 0,1,2 \) in effective action one obtains terms subtracted from vacuum EMT

\[ T_{ik;\epsilon}[0] = - \frac{m^{N_0}}{2^{N_0} \pi^{N_0/2}} \left( \frac{4\pi M^2}{m^2} \right)^{\epsilon} \Gamma\left( \epsilon - \frac{N_0}{2} \right) g_{ik}, \]  

(37)
\[ T_{ik,\varepsilon}[1] = \frac{m^{N_0-2}}{2^{N_0-1} \pi^{N_0/2}} \left( \frac{4\pi M^2}{m^2} \right)^{\varepsilon} \Gamma\left(1 - \frac{N}{2}\right) \left[ \left( \frac{1}{6} - \xi \right) G_{ik} - \zeta E_{ik} \right] = \]
\[ = \frac{m^{N_0-2}}{2^{N_0-1} \pi^{N_0/2}} \left( \frac{4\pi M^2}{m^2} \right)^{\varepsilon} \times \]
\[ \times \left[ \Delta \xi \Gamma\left(1 - \frac{N}{2}\right) G_{ik} - \frac{\Gamma\left(3 - \frac{N}{2}\right)}{(N-2)} \left( \frac{G_{ik}}{3(N-1)} + 4\zeta E_{ik} \right) \right] \]

\[ T_{ik,\varepsilon}[2] = \frac{m^{N_0-4}}{(4\pi)^{N_0/2}} \left( \frac{4\pi M^2}{m^2} \right)^{\varepsilon} \left\{ \frac{\Gamma\left(2 - \frac{N}{2}\right)}{360(N-3)} \left[ (N-6) E_{ik} + 3(N-2) W_{ik} \right] + \right. \]
\[ + \left. \left[ \frac{\Gamma\left(2 - \frac{N}{2}\right)}{60(N-1)^2} + \Delta \xi \frac{\Gamma\left(3 - \frac{N}{2}\right)}{3(N-1)} + (\Delta \xi)^2 \Gamma\left(2 - \frac{N}{2}\right) \right] \right\}^{(1)H_{ik}} - \]
\[ - \zeta \frac{\Gamma\left(3 - \frac{N}{2}\right)}{3(N-1)} \left[ \left( R_{ik} + \nabla_i \nabla_k - g_{ik} \nabla^l \nabla^l \right) R_{GB}^2 + \left( E_{ik} + P_{ik} \right) R \right] + \]
\[ + 2 \zeta \Gamma\left(2 - \frac{N}{2}\right) \left[ \zeta \left( \frac{g_{ik}}{4} R_{GB}^2 + E_{ik} + P_{ik} \right) R_{GB}^2 - \right. \]
\[ \left. - \Delta \xi \left( R_{ik} + \nabla_i \nabla_k - g_{ik} \nabla^l \nabla^l \right) R_{GB}^2 + \left( E_{ik} + P_{ik} \right) R \right] \right\}, \tag{39} \]

where

\[ ^{(1)H_{ik}} = \frac{\delta \int R^2 \sqrt{|g|} d^N x}{\sqrt{|g|} \delta g^{ik}} = 2 \left( \nabla_i \nabla_k R - g_{ik} \nabla^l \nabla_l R \right) + 2R \left( R_{ik} - \frac{1}{4} R g_{ik} \right), \tag{40} \]

\[ W_{ik} = \frac{\delta \int C_{lmpq} C_{lmpq} \sqrt{|g|} d^N x}{\sqrt{|g|} \delta g^{ik}} = E_{ik} + 4(N-3) \left( 2R_{lm} R_{lkm} - \frac{g_{ik}}{2} R_{lm} R_{lm} - \right. \]
\[ \left. - \frac{NRR_{ik}}{2(N-1)} + \frac{NR^2 g_{ik}}{8(N-1)} + \frac{N-2}{2(N-1)} \nabla_i \nabla_k R + \frac{g_{ik}}{2(N-1)} \nabla^l \nabla_l R - \nabla^l \nabla_l R_{ik} \right). \tag{41} \]

In conformally flat case \( C_{iklm} = 0, \ E_{ik}/(4-N) = ^{(3)H_{ik}}, \ W_{ik} = 0 \) and so

\[ T_{ik,\varepsilon}[1] = \frac{m^{N_0-2}}{2^{N_0-1} \pi^{N_0/2}} \left( \frac{4\pi M^2}{m^2} \right)^{\varepsilon} \left[ \Delta \xi \Gamma\left(1 - \frac{N}{2}\right) G_{ik} - \frac{\Gamma\left(3 - \frac{N}{2}\right)}{(N-2)} \left( \zeta^4 (3)H_{ik} - \frac{G_{ik}}{3(N-1)} \right) \right], \tag{42} \]
Some effects of the quantum field theory in the early Universe

\[ T_{ik,e}[2] = \frac{m^{N_0-4}}{(4\pi)^{N_0/2}} \left( \frac{4\pi M^2}{m^2} \right)^{\xi} \left\{ \begin{array}{l} (3)H_{ik} \left[ -\Gamma \left( 4 - \frac{N}{2} \right) \right] + \\
+ \zeta 4\Gamma \left( 3 - \frac{N}{2} \right) \left( \left( \frac{N - 4}{12(N - 1)} - \Delta \xi \right) R + \zeta R_{GB}^2 \right) + \\
+ (1)H_{ik} \left[ \frac{\Gamma \left( 4 - \frac{N}{2} \right)}{60(N - 1)^2} + \Delta \xi \frac{\Gamma \left( 3 - \frac{N}{2} \right)}{3(N - 1)} + (\Delta \xi)^2 \Gamma \left( 2 - \frac{N}{2} \right) \right] - (43) \\
\end{array} \right. \]

Supposing that vacuum averages of EMT, i.e. \( \langle T_{ik} \rangle \), are sources of the gravity field \( 2 \mathbf{[13]} \)
\[ G_{ik} + \Lambda g_{ik} = -8\pi G \left( T_{ik}^b + \langle T_{ik} \rangle \right), \]

where \( \Lambda, G \) are cosmological and gravitational constants, \( T_{ik}^b \) is EMT of the background, and taking into account eqs. \( 37 \)–\( 39 \) one comes to the following conclusion. First three subtractions from the vacuum EMT in \( N \)-dimensional space-time correspond to renormalization of the cosmological and gravitational constants and parameters in quadratic, cubic and fourth degrees in curvature terms in the bare gravitational Lagrangian

\[ L_{gr,e} = \sqrt{\lvert g \rvert} \left[ \frac{R - 2\Lambda_e}{16\pi G_e} + \alpha_e R_{GB}^2 + \beta_e R^2 + \gamma_e C_{imnpq}C^{imnpq} + \delta_e R R_{GB}^2 + \theta_e R_{GB}^4 \right]. \]

Subtraction \( T_{ik,e}[0] \) due to \( 37 \), leads to infinite renormalization of the cosmological constant \( \Lambda_e \). From \( 38 \) it follows that subtraction \( T_{ik,e}[1] \) correspond to renormalization of the gravitational constant \( G_e \) (finite for \( N \to 4, \xi = \xi_e \)) and infinite renormalization of \( \alpha_e \) (for \( \xi \neq 0 \)). Subtraction \( T_{ik,e}[2] \) due to \( 36 \) and \( 39 \) corresponds to renormalization of the parameters \( \alpha_e, \beta_e, \gamma_e, \delta_e, \theta_e \). For \( \xi = \xi_e \) and \( N \to 4 \), the parameters \( \beta_e, \delta_e \) have finite renormalization.

Note that introduction of the term \( \alpha_e R_{GB}^2 \) in \( 45 \) is consistent with dimensional regularization (\( E_{ik} \equiv 0 \) only for integer \( N = 2, 3, 4 \)).

For \( N \to 4 \) the products \( E_{ik} \Gamma(1-(N/2)) \) and \( E_{ik} \Gamma(2-(N/2)) \) have finite limits for arbitrary metric of space-time because the dependence of \( E_{ik} \) on \( N \) is fractional-rational in analytic continuation on dimension, \( E_{ik} = 0 \) for \( N = 4 \) and gamma-function has pole of the first order in points \( 0, -1 \). So the corresponding terms in \( 38 \), \( 39 \) are finite and there is no necessity for subtracting them to get finite terms in \( 44 \) leading to back reaction of the quantized field in metric. However the effective action is divergent without such subtractions and the anomalous trace of vacuum EMT is different from the standard even for \( \zeta = 0 \). Due to the appearance
of such terms for different regularization procedures [2, 13] it is convenient to have them in counterterms in vacuum EMT.

Values of renormalized parameters are fixed by experiment and it can be [1, 2] that renormalized parameters for non-Einstein terms in the gravity Lagrangian are zero.

Now consider calculating of the renormalized vacuum EMT for \( N \)-dimensional quasi-Euclidean space-time with metric

\[
ds^2 = dt^2 - a^2(t) d\mathbf{x}^2 = a^2(\eta) \left( d\eta^2 - d\mathbf{x}^2 \right).
\]  

(46)

In this case one can choose

\[
\Phi_J(x) = (2\pi)^{-(N-1)/2} e^{-i\lambda_\alpha x^\alpha},
\]

(47)

\( J = \{\lambda_1, \ldots, \lambda_{N-1}\}, \quad -\infty < \lambda_\alpha < +\infty, \quad \lambda = \sqrt{\sum_{\alpha=1}^{N-1} \lambda_\alpha^2}, \quad g_J(\eta) = g_\lambda(\eta). \)

It is convenient to express vacuum averages of the EMT operator for Fock vacuum \(|0\rangle\) annihilated by operator \( a^{(-)}_J, \ a^{(\ast)}_J \) (14) through linear combinations of functions \( g_\lambda, g_\ast_\lambda \):

\[
S = \frac{|g_\ast_\lambda|^2 + \Omega^2 |g_\lambda|^2}{4 \Omega} - \frac{1}{2}, \quad U = \frac{\Omega^2 |g_\lambda|^2 - |g_\ast_\lambda|^2}{2 \Omega}, \quad V = -\frac{d(g_\ast_\lambda g_\lambda)}{2 d\eta},
\]

(48)

which due to (3) satisfy differential equations

\[
S' = \frac{\Omega'}{2 \Omega} U, \quad U' = \frac{\Omega'}{\Omega} (1 + 2S) - 2 \Omega V, \quad V' = 2 \Omega U.
\]

(49)

Taking initial conditions \( S(\eta_0) = U(\eta_0) = V(\eta_0) = 0 \) following from (13) the eqs. (49) can be written as integral equations of the Volterra type (22) and

\[
U(\eta) + iV(\eta) = \int_{\eta_0}^{\eta} \omega(\eta_1) (1 + 2S(\eta_1)) \exp[2i \Theta(\eta_1, \eta)] d\eta_1.
\]

(50)

Substituting expansion (12) in eq. (27) and using (14), (28)–(30), (48), one obtains the following (diverging) expression for vacuum EMT averages

\[
\langle 0 | T_{ik} | 0 \rangle = B_N \frac{a^N}{a^{N-2}} \int_0^\infty \tau_{ik} \lambda^{N-2} d\lambda,
\]

(51)

where \( B_N = \left[ 2^{N-3} \pi^{(N-1)/2} \Gamma((N-1)/2) \right]^{-1} \),

\[
\tau_{00} = \Omega \left( S + \frac{1}{2} \right) + (N-1) \left( \Delta \xi - \tilde{\zeta}^2 \right) \left[ cV + \left( c' + (N-2)c^2 \right) \frac{1}{\Omega} \left( S + \frac{1}{2} U + \frac{1}{2} \right) \right],
\]

(52)
\[
\tau_{\alpha\beta} = \delta_{\alpha\beta} \left\{ \frac{1}{(N-1)\Omega} \left[ \lambda^2 \left( S + \frac{1}{2} \right) - (\Omega^2 - \lambda^2) \frac{U}{2} \right] + \right. \\
+ \left[ \Delta \xi(N-1) - \tilde{\zeta} \left( (N+1)c^2 - 2c' \right) \right] cV - 2 \left( \Delta \xi - \tilde{\zeta} c^2 \right) \Omega U - \left. \frac{1}{\Omega} \left( S + \frac{1}{2} U + \frac{1}{2} \right) \left[ \Delta \xi(N-1)c' - \tilde{\zeta}(N-1)c^2(3c' - 2c^2) \right] \right\},
\]

\[\tilde{\zeta} \equiv \zeta a^{-2} 2(N-2)(N-3).\]

The vacuum expectation (51) has \([N/2] + 1\) different types of divergences: \(\sim \lambda^N, \lambda^{N-2}, \ldots, \ln \lambda\) if \(N\) is even, and \(\sim \lambda^N, \lambda^{N-2}, \ldots, \lambda\) if \(N\) is odd. For renormalization we are using \(n\)-wave procedure proposed by Zeldovich and Starobinsky [27].

\[
\langle 0 \mid T_{ik} \mid 0 \rangle_{\text{ren}} = \frac{B_N}{a_{N-2}} \int_0^\infty \lambda^{N-2} \left[ \tau_{ik} - \sum_{l=0}^{[N/2]} \tau_{ik}[l] \right] d\lambda, \quad (54)
\]

\[
\tau_{ik}[l] = \frac{1}{l!} \lim_{n \to \infty} \frac{\partial^l}{\partial (n-2)!} \left( \frac{1}{n^l} \tau_{ik}(n\lambda, nm) \right), \quad (55)
\]

To obtain the explicit expression \(\tau_{ik}[l]\) write the series for \(S, U, V\) in reverse degrees of \(n\) for \(\lambda \to n\lambda, m \to nm, n \to \infty\): \(S = \sum_{k=1}^\infty n^{-k}S_k, \ldots\) Taking subsequent iterations in integral eqs. (22), (50) and using the stationary phase method one obtains for first different from zero terms

\[
V_1 = W, \quad U_2 = DW, \quad S_2 = \frac{1}{4} W^2, \quad S_3 = \frac{1}{2} W^3 - D^2W - \frac{\omega}{2} D \left( \frac{q}{\omega^3} \right), \quad (56)
\]

\[
U_4 = \frac{3}{2} W^2 DW - D^3W - D \left( \frac{\omega}{2} D \left( \frac{q}{\omega^3} \right) \right) + \frac{q}{2\omega^2} DW, \quad (57)
\]

\[
S_4 = \frac{3}{16} W^4 + \frac{1}{4} (DW)^2 - \frac{1}{2} WD^2W - \frac{1}{4} \omega WD \left( \frac{q}{\omega^3} \right), \quad (58)
\]

where

\[
q = \left( \Delta \xi R - \zeta R_{GB}^2 \right) a^2, \quad \omega = (m^2a^2 + \lambda^2)^{1/2}, \quad W = \frac{\omega'}{2\omega^2}, \quad D = \frac{1}{2\omega} \frac{d}{d\eta}. \quad (59)
\]

Note that in (56)–(58) terms nonlocal on time (dependent on \(\eta\) and \(\eta_0\)) are excluded. These terms are absent if \(V_1(\eta_0) = V_3(\eta_0) = U_2(\eta_0) = U_4(\eta_0) = 0\) which is supposed further. In particular, nonlocal terms are absent if first \(2[N/2]\) derivatives of the scale factor \(a(\eta)\) of metric are zero in the initial moment of time.

Using (52), (53), (56)–(58) for \(\tau_{ik}[l]\) one obtains

\[
\tau_{00}[0] = \frac{\omega}{2}, \quad \tau_{0\beta}[0] = \delta_{0\beta} \frac{\lambda^2}{2(N-1)\omega}, \quad (60)
\]

\[
\tau_{00}[1] = \omega S_2 + (N-1) \left( \Delta \xi - \tilde{\zeta} c^2 \right) cV_1 + \frac{N-1}{8\omega} \left[ \Delta \xi 2(N-2)c^2 + \tilde{\zeta}(4-3N)c^4 \right], \quad (61)
\]
\[ \tau_{\alpha\beta}[1] = \delta_{\alpha\beta} \left\{ \frac{1}{(N-1)\omega} \left[ \lambda^2 S_2 - \frac{m^2 a^2}{2} \left( U_2 + \frac{q}{2\omega^2} \right) \right] + 
abla \right\} \]
\[ + \left[ \Delta \xi (N-1) - \tilde{\xi} \left( (N+1)c^2 - 2c' \right) \right] cV_1 - 2 \left( \Delta \xi - \tilde{\xi} c^2 \right) \omega U_2 + \right) \]
\[ + \frac{1}{4\omega} \left[ \Delta \xi (N-2) \left( c^2 - 2c' \right) + \tilde{\xi} c^2 (3N-4) \left( 2c' - \frac{3}{2} c^2 \right) \right] \right\}, \]
\[ \tau_{00}[2] = \omega \left( S_2 + \frac{q}{4\omega^2} U_2 + \frac{q^2}{16\omega^4} \right) + (N-1) \left( \Delta \xi - \tilde{\xi} c^2 \right) cV_3 + \]
\[ + \frac{N-1}{4\omega} \left[ \Delta \xi 2(N-2) c^2 - \tilde{\xi} (3N-4) c^4 \right] \left( S_2 + \frac{1}{2} U_2 + \frac{q}{4\omega^2} \right), \]
\[ \tau_{0\beta}[2] = \delta_{0\beta} \left\{ \frac{1}{N-1} \left[ \frac{\lambda^2}{\omega} \left( S_2 + \frac{qU_2}{4\omega^2} + \frac{q^2}{16\omega^4} \right) - \frac{m^2 a^2}{2\omega} \left( U_4 + \frac{q^2}{4\omega^4} + \frac{qS_2}{\omega^2} \right) \right] + \right\}
\[ + \left[ \Delta \xi (N-1) - \tilde{\xi} \left( (N+1)c^2 - 2c' \right) \right] cV_3 - \left( \Delta \xi - \tilde{\xi} c^2 \right) \left( 2\omega U_4 - \frac{qU_2}{\omega} \right) + \right) \]
\[ + \left[ \Delta \xi (N-2)(c^2 - 2c') + \tilde{\xi} (3N-4) c^2 \left( 2c' - \frac{3}{2} c^2 \right) \right] \frac{1}{2\omega} \left( S_2 + \frac{1}{2} U_2 + \frac{q}{4\omega^2} \right) \right\}. \]

These expressions give all information on subtractions for \( N = 4, 5 \). New counterterms appear for \( N \geq 6 \). For conformal scalar field they are given in Ref. [28]. The renormalized due to (54) vacuum EMT is covariantly conserved. This is seen from equations \( \nabla^i (\tau_{ik}/a^{N-2}) = 0 \) and \( \nabla^i (\tau_{ik}[l]/a^{N-2}) = 0 \) following from (49), (52), (53), (60)-(64).

To see the geometrical structure of the \( n \)-wave procedure counterterms let us make as in [29] the dimensional regularization. For calculation of integrals in dimensionally regularized counterterms

\[ T_{ik,\varepsilon}[l] = \frac{B_N}{a^{N-2}} (M)^{2\varepsilon} \int_0^{\infty} \lambda^{N-2} \tau_{ik,\varepsilon}[l] d\lambda, \]

with \( \tau_{ik,\varepsilon}[l] \) defined by (50)-(64) with putting \( N \rightarrow N_0 - 2\varepsilon \), one uses the equality

\[ \int_0^{\infty} x^k (1 + x^2)^{-p} dx = \frac{\Gamma \left( \frac{k+1}{2} \right) \Gamma \left( p - \frac{k+1}{2} \right)}{2 \Gamma (p)}. \]

In cases when the integral in the left hand side of (65) does not exist in usual sense it is taken as the analytic continuation of the right side of (66) for corresponding values of \( k \) and \( p \). As the result of calculations one obtains expressions (37), (42), (43) for the 0-th, 1-st and 2-nd counterterms \( n \)-wave procedure. So the geometrical structure of first three subtractions in the \( n \)-wave procedure and in the effective action method occurs to be the same.

In conclusion note that Gauss-Bonnet coupling terms \( R_{GB}^2 \) can play important role in the early Universe. Effects of the nonzero value of \( \zeta \) for scalar field can be observed in black hole radiation, in parameters of the so called bosonic stars [30]. Explicit value of \( \zeta \) can be checked by experiment.
5 Superheavy particles in the early Universe

It is known [2, 31, 32] that the number of particles with mass of the order of the Grand Unification scale created by gravitation in the early Universe described by the radiation dominated Friedmann metric is of the Dirac-Eddington order, i.e. of the observable order for the visible mass. On the other side it is clear that if superheavy particles after their creation continued to be stable for large enough time they will lead to the collapse of the Universe governed by the radiation dominated metric in the short on the cosmological scale time for closed Friedmann space or lead to the unrealistic scale factor for the open space. So the idea was proposed that these superheavy particles must decay on quarks and leptons with CP-noninvariance leading to the observable baryon charge of the Universe before the time when the energy density of the created superheavy particles will become equal to that creating the background metric. If superheavy particles have nonzero baryon charge then their decay in analogy with decay of neutral K-mesons will go as decay of some short living and long living components. Supposing that the lifetime of long living components is of the cosmological order but their number was diminished in comparison with the number of the short living components due to their interaction with the baryon charge created previously similar to the well known regeneration mechanism for K-mesons one can speculate about their existence today as cold dark matter. Rare events of their decays can be identified as experimental observations of high energetic cosmic rays [33] with the energy higher than the Greizen-Zatsepin-Kuzmin limit [34]. Here we shall discuss different possibilities of the role of superheavy particles with the mass of the Grand Unification scale in the early Universe.

1) It is natural to think that some inflation era took place before the Friedmann stage. Some inflaton field probably manifesting itself as the quintessence in the modern epoch after the quasi de Sitter stage led to the dust like or to the radiation dominated Friedmann Universe. Usually it is supposed that the inflaton field does not interact with ordinary particles and can be some manifestation of the non Einsteinian gravity for example due to high order corrections. So even if it decayed on some light “inflaton” particles the primordial inflaton field can form hot dark matter but not the visible matter and entropy present in background radiation. Our idea is that inflaton field was the source of Friedmann metric with some small inhomogeneities, but visible matter and the entropy of the Universe were created not by the inflaton field itself but by the gravitation of this inflaton field. That gravitation created pairs of superheavy particles. Short living components decayed in time of the Grand Unification scale and led to the nonzero baryon charge observed today as visible matter. If long living components had the lifetime of the order of the “early recombination era” then the energy density of created long living particles soon became equal to that of the background inflaton field (hot dark matter). Then the decay of all long living components led to the observable entropy of the Universe. Here it is supposed that the energy density of the inflaton field led to the observed cosmological scale factor, so it is evident that the created
entropy due to our mechanism will be of the observable order.

2) The other possibility is to put the hypothesis discussed by us earlier [35] that not all long living components decayed and formed the entropy but some part of them survived up to modern time as cold dark matter and superheavy particles are observed in cosmic rays events. Then it is natural to suppose that the lifetime of the long living component is of the cosmological order but the large part of them regenerated into short living components due to interaction with the baryon charge in time shorter or equal to that of the “early recombination era” and entropy appeared due to this decay.

Now let us give some numerical estimates. Total number of massive particles created in Friedmann radiation dominated Universe (scale factor \( a(t) = a_0 t^{1/2} \)) inside the horizon is as it is known [1, 2]:

\[
N = n(s)(t) a^3(t) = b(s) M^{3/2} a_0^3 ,
\]

(67)

where \( b(0) \approx 5.3 \cdot 10^{-4} \) for scalar and \( b(1/2) \approx 3.9 \cdot 10^{-3} \) for spinor particles (\( N \sim 10^{80} \) for \( M \sim 10^{14} \) GeV, see Ref. [2]). Radiation dominance in the end of inflation era dark matter is important for our calculations. If it is dust like the results will be different (see further). For the time \( t \gg M^{-1} \) there is an era of going from the radiation dominated model to the dust model of superheavy particles

\[
t_X \approx \left( \frac{3}{64\pi b(s)} \right)^2 \left( \frac{M_{Pl}}{M} \right)^4 \frac{1}{M} .
\]

(68)

If \( M \sim 10^{14} \) GeV, \( t_X \sim 10^{-15} \) s for scalar and \( t_X \sim 10^{-17} \) s for spinor particles.

Let us call \( t_X \) — “early recombination era”. For dust like ending of inflation era one has \( N \sim M \) (see Ref. [35]) and therefore the ratio of the \( X \)-particles energy density \( \varepsilon_X \) to the critical density \( \varepsilon_{crit} \) does not depend on time (\( \varepsilon_X < \varepsilon_{crit} \) for \( M < M_{Pl} \)).

Let us define \( d \) — the permitted part of long living \( X \)-particles — from the condition: on the moment of recombination \( t_{rec} \) in the observable Universe one has

\[
d \varepsilon_X(t_{rec}) = \varepsilon_{crit}(t_{rec}) .
\]

It leads to

\[
d = \frac{3}{64\pi b(s)} \left( \frac{M_{Pl}}{M} \right)^2 \frac{1}{\sqrt{M} t_{rec}} .
\]

(69)

For \( M = 10^{13} - 10^{14} \) GeV one has \( d \approx 10^{-12} - 10^{-14} \) for scalar and \( d \approx 10^{-13} - 10^{-15} \) for spinor particles. So the life time of main part or all \( X \)-particles must be smaller or equal than \( t_X \).

Now let us construct the model which can give: a) short living \( X \)-particles decay in time \( \tau_q \approx t_X \) (more wishful is \( \tau_q \approx t_C \approx 10^{-38} - 10^{-35} \) s, i.e. Compton time for \( X \)-particles) b) long living particles decay with \( \tau_l \approx t_X \). Baryon charge nonconservation with \( CP \)-nonconservation in full analogy with the \( K^0 \)-meson theory with nonconserved hypercharge and \( CP \)-nonconservation leads to the effective Hamiltonian of the decaying \( X, \bar{X} \) - particles with nonhermitean matrix.
For the matrix of the effective Hamiltonian $H = \{H_{ij}\}$, $i, j = 1, 2$ let $H_{11} = H_{22}$ due to $CPT$-invariance. Denote $\varepsilon = (\sqrt{H_{12}} - \sqrt{H_{21}}) / (\sqrt{H_{12}} + \sqrt{H_{21}})$. The eigenvalues $\lambda_{1,2}$ and eigenvectors $|\Psi_{1,2}\rangle$ of matrix $H$ are

$$\lambda_{1,2} = H_{11} \pm \frac{H_{12} + H_{21}}{2} \frac{1 - \varepsilon^2}{1 + \varepsilon^2},$$  \hspace{1cm} (70)$$

$$|\Psi_{1,2}\rangle = \frac{1}{\sqrt{2(1 + |\varepsilon|^2)}} [(1 + \varepsilon) |1\rangle \pm (1 - \varepsilon) |2\rangle].$$ \hspace{1cm} (71)$$

In particular

$$H = \begin{pmatrix} E - \frac{i}{4} (\tau_q^{-1} + \tau_l^{-1}) & \frac{1+i\varepsilon}{1-i\varepsilon} \left[ A - \frac{i}{4} (\tau_q^{-1} - \tau_l^{-1}) \right] \\ \frac{1-i\varepsilon}{1+i\varepsilon} \left[ A - \frac{i}{4} (\tau_q^{-1} - \tau_l^{-1}) \right] & E - \frac{i}{4} (\tau_q^{-1} + \tau_l^{-1}) \end{pmatrix}.$$ \hspace{1cm} (72)$$

Then the state $|\Psi_1\rangle$ describes short living particles $X_q$ with the life time $\tau_q$ and mass $E + A$. The state $|\Psi_2\rangle$ is the state of long living particles $X_l$ with life time $\tau_l$ and mass $E - A$. Here $A$ is the arbitrary parameter $-E < A < E$ and it can be zero, $E = M$.

So for the scenario 1) it is sufficient to take $\tau_l \approx t_X$.

In scenario 2) the small $d \sim 10^{-15} - 10^{-12}$ part of long living $X$-particles with $\tau_l > t_U \approx 4.3 \cdot 10^{17}$ s ($t_U$ is the age of the Universe) is forming the dark matter. The decay of these superheavy particles in modern epoch can give observed ultra high energy cosmic rays. Using the estimate for the velocity of change of the concentration of long living superheavy particles $|\dot{n}_x| \sim 10^{-42}$ cm$^{-3}$s$^{-1}$, and taking the life time $\tau_l$ of long living particles as $2 \cdot 10^{22}$ s, we obtain concentration $n_X \approx 2 \cdot 10^{-20}$ cm$^{-3}$ at the modern epoch, corresponding to the critical density for $M = 10^{14}$ GeV.

Let us use the model with effective Hamiltonian (72) where $\tau_l > t_U$ and take into account transformations of the long living component into the short living one due to the presence of baryon substance created by decays of the short living particles in analogy with the regeneration mechanism for $K^0$-mesons.

Let us investigate the model with the interaction which in the basis $|1\rangle, |2\rangle$ is described by the matrix

$$H^d = \begin{pmatrix} 0 & 0 \\ 0 & -i\gamma \end{pmatrix}.$$ \hspace{1cm} (73)$$

The eigenvalues of the Hamiltonian $H + H^d$ are

$$\lambda_{1,2}^d = E - \frac{i}{4} (\tau_q^{-1} + \tau_l^{-1}) - i\frac{\gamma}{2} \pm \sqrt{(A - \frac{i}{4} (\tau_q^{-1} - \tau_l^{-1}))^2 - \gamma^2 \frac{1}{4}}.$$ \hspace{1cm} (74)$$

In case when $\gamma \ll \tau_q^{-1}$ for the long living component one obtains

$$\lambda_2^d \approx E - A - \frac{\gamma}{2},$$ \hspace{1cm} (75)$$
\[ \| \Psi_2(t) \|^2 = \| \Psi_2(t_0) \|^2 \exp \left[ \frac{t_0 - t}{\tau_l} - \int_{t_0}^{t} \gamma(t) \, dt \right]. \] (76)

The parameter \( \gamma \), describing the interaction with the substance of the baryon medium, is evidently dependent on its state and concentration of particles in it. For approximate evaluations take this parameter as proportional to the concentration of particles: \( \gamma = \alpha n^{(0)}(t) \). Putting \( \tau_l = 2 \cdot 10^{22} \) s, \( t \leq t_U \), \( a(t) = a_0 \sqrt{t} \) by (67) one obtains

\[ \| \Psi_2(t) \|^2 = \| \Psi_2(t_0) \|^2 \exp \left[ \alpha 2 b^{(s)} M_3^{3/2} \left( \frac{1}{\sqrt{t}} - \frac{1}{\sqrt{t_0}} \right) \right]. \] (77)

So the decay of the long living component due to this mechanism takes place close to the time \( t_0 \). One can think that this interaction of \( X_l \) with baryon charge is effective for times, when the baryon charge becomes strictly conserved, i.e. we take to the time \( t \).

The parameter \( \alpha \) value is given by

\[ T(t) = \left( \frac{30 b^{(s)}}{\pi^2 N_l} \right)^{1/4} \frac{M^{5/8} \tau_q^{1/8}}{k_B \sqrt{t}}, \] (78)

where \( k_B \) is Boltzmann constant, \( N_l \) is defined by the number of boson \( N_B \) and fermion \( N_F \) degrees of freedom of all kinds of light particles: \( N_l = N_B + \frac{7}{8} N_F \) (see Ref. 38). At time \( t_X \) this temperature is equal to

\[ T(t_X) = \frac{64 \sqrt{\pi}}{3} \left( \frac{30 b^{(s)}}{N_l} \right)^{1/4} \left( \frac{b^{(s)}}{M_3} \right)^{5/4} \left( M_3 \right)^{1/8} \frac{M^3}{k_B M_{P_l}^2}. \] (79)

If \( \tau_q = 1/M \) and \( N_l \sim 10^2 - 10^4 \), then for spinor \( X \)-particles \( T(t_X) \approx 300 - 100 \) GeV, i.e. the electroweak scale for created particles (which is however different from that for the background).

So let us suppose \( t_0 \approx t_X \). If \( d \) is the part of long living particles surviving up to the time \( t \) \( (t_U \geq t \gg t_C) \) then from (79) and (77) one obtains the evaluation for the parameter \( \alpha \)

\[ \alpha = \frac{-3 \ln d}{128 \pi (b^{(s)})^2 M_{P_l}^2}. \] (80)

For \( M = 10^{14} \) GeV and \( d = 10^{-15} \) one obtains \( \alpha \approx 10^{-30} \) cm/s. If \( \tau_q \sim 10^{-38} - 10^{-35} \) s then the condition \( \gamma(t) \ll \tau_q^{-1} \) used in Eq. (75) is valid for \( t > t_X \). For this value \( \alpha \) we have \( \gamma(t_U) \approx 10^{-36} \) s\(^{-1} \) \( \ll \tau_l^{-1} \). So one can neglect this mechanism for the decay of the long living component of \( X \)-particles for the modern epoch while for early universe at \( t_0 \approx t_X \) it was important. The observable entropy in this scenario is created due to decay of \( X_l \) on quarks and antiquarks at the time \( t_X \) when the Grand Unification symmetry is totally broken. Baryon charge is created at \( t_q \) which can be equal to Compton time for \( X \)-particles \( t_C \sim 10^{-38} - 10^{-35} \) s.

Our scheme is the same for the scalar particles and the fermions. The superheavy fermions are used, for example, in some models of neutrino mass generation.
Some effects of the quantum field theory in the early Universe

The see-saw mechanism in Grand Unification theories [39, 40]. New experiments on high energetic particles in cosmic rays surely will give us more information on their structure and origin.

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