Supersymmetry and F-theory realization
of the deformed conifold with three-form flux

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Abstract

It is shown that the deformed conifold solution with three-form flux, found by Klebanov and Strassler, is supersymmetric, and that it admits a simple F-theory description in terms of a direct product of the deformed conifold and a torus. Some general remarks on Ramond-Ramond backgrounds and warped compactifications are included.

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1 Introduction

It is believed on general grounds [1] that a string theory capable of describing the low-energy limit of QCD should be defined on a warped background in some dimension larger than four:

\[ ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - e^{-2A(y)} \tilde{g}_{IJ} dy^I dy^J, \tag{1} \]

where we use mostly minus signature for the 3 + 1-dimensional Minkowski metric \( \eta_{\mu\nu} dx^\mu dx^\nu \) and for the higher dimensional metric \( ds^2 \), and all plus signature for the extra-dimensional metric \( \tilde{g}_{IJ} dy^I dy^J \). The choice of warp factor \( e^{-2A(y)} \) on the extra dimensions will prove convenient later, although it could have been absorbed into \( \tilde{g}_{IJ} \). In seeking for supersymmetric solutions of this form in critical string theory (so that \( \tilde{g}_{IJ} dy^I dy^J \) represents a six-dimensional metric), Ramond-Ramond fields seem a necessary ingredient.

There does not seem to be a clean general argument to the effect that type II string backgrounds dual to gauge theories must involve Ramond-Ramond fields. There is one, however, if we assume minimal supersymmetry in 3 + 1 dimensions. Supersymmetric solutions with only the bosonic fields in the NS-NS sector turned on (namely, the metric, the dilaton, and the two-form potential) have been shown to have an unwarped string metric [2]. This amounts to the statement that the worldsheet CFT factorizes into a 3 + 1-dimensional part and an additional piece from the six extra dimensions. Zig-zag symmetry cannot be implemented unless there is a warp factor which either diverges or vanishes at some point [1]. Thus we must consider Ramond-Ramond backgrounds if a string dual to super-Yang-Mills theory is desired.

An interesting solution exhibiting some of the features of confinement (namely the area law for Wilson loops, screening for appropriately defined magnetic flux, and a mass gap) was recently exhibited in [3]. A somewhat similar solution appeared in [4] as a lift of a solution to seven-dimensional supergravity [5]. Supersymmetry was not demonstrated in [3]. The more abstract and general treatment via superpotentials that appeared in [6] makes it clear that a supersymmetric solution exists. The aim of this note is to demonstrate that the solution of [3] is indeed supersymmetric, that the simple first order equations appearing in [3] are precisely the conditions for supersymmetry, and that the geometry is an explicit example of an F-theory compactification on the product of a non-compact Calabi-Yau three-fold and a torus. These results were independently derived in [7].
2 Supersymmetry in type IIB supergravity

The tools for studying bosonic backgrounds of ten-dimensional type IIB supergravity are the bosonic equations of motion and the fermionic supersymmetry variations. They are, in a unitary gauge for the $SL(2, \mathbb{R})/U(1)$ coset,

\[
\hat{D}^M P_M = \frac{\kappa^2}{24} G_{MNP} G^{MNP}
\]

\[
\hat{D}^P G_{MNP} = P^P G^*_{MNP} - \frac{2i}{3} \kappa F_{MNPQR} G^{PQR}
\]

\[
\hat{R}_{MP} = P_M P_P^* + P_P^* P_M + \frac{\kappa^2}{6} F_{Q_1 \cdots Q_4 M} F_{Q_1 \cdots Q_4 P}
\]

\[
+ \frac{\kappa^2}{8} \left( G_{M^R Q^R P^S Q^S} G^*_{P^S Q^S} + G^*_{M^R Q^R P^S Q^S} G_{P^S Q^S} - \frac{1}{6} \hat{g}_{MP} G^{QRS} G^*_{QRS} \right)
\]

\[
F_{(5)} = \hat{*} F_{(5)}
\]

\[
\delta \lambda = \frac{i}{\kappa} P_M \hat{\gamma}^M e^* - \frac{i}{24} G_{MNP} \hat{\gamma}^M N P e
\]

\[
\delta \psi_M = \frac{1}{\kappa} \hat{D}_M e + \frac{i}{480} F_{P_1 \cdots P_5} \hat{\gamma}^M P_1 \cdots P_5 e + \frac{1}{96} (\hat{\gamma}_M N P Q G_{NPQ} - 9 \hat{\gamma}^N P G_{MPN}) e^*
\]

where

\[
F_{(5)} = dA_{(4)} - \frac{\kappa}{8} \text{Im} A_{(2)} \wedge F^*_{(3)} \quad F_{(3)} = dA_{(2)}
\]

\[
G_{(3)} = \frac{F_{(3)} - B F^*_{(3)}}{\sqrt{1 - |B|^2}} \quad P_M = \frac{\partial_M B}{1 - |B|^2}
\]

\[
\hat{\gamma}_{11} \lambda = \lambda \quad \hat{\gamma}_{11} \psi_M = -\psi_M \quad \hat{\gamma}_{11} e = -e.
\]

Except for some typographic alterations, the conventions used in this note are those of [8]; in particular, the metric signature is mostly minus. All explanation of notation is relegated to the Appendix.

In [9], solutions to the supersymmetry transformations laws were considered where the three-form was set to zero but the scalars could vary. Here we wish to do the opposite and consider constant scalars. This means $G_{MNP} G^{MNP} = 0$, which is certainly satisfied if $\hat{*} G_{(3)} = i G_{(3)}$. This latter “self-dual” ansatz is the one we wish to focus on. In purely ten-dimensional terms, $\hat{*} G_{(3)} = i e^A \text{vol}_4 \wedge G_{(3)}$. Without loss of generality we can choose $B = 0$ and then apply a global $SL(2, \mathbb{R})$ transformation to restore arbitrary $B$.

A compact rewriting of the gravitino variation in (3) is as follows:

\[
\delta \lambda = \frac{i}{\kappa} P_{(1)} e^* - \frac{i}{4} G_{(3)} e
\]

\[
\delta \psi_M = \frac{1}{\kappa} \hat{D}_M e + \frac{i}{4} F_{(5)} \hat{\gamma}_M e - \frac{1}{16} (2 G_{(3)} \hat{\gamma}_M + \hat{\gamma}_M G_{(3)}) e^*.
\]
The first equation is easy to solve because \( P_1(1) = 0 \) and \( G_3(1-\gamma_7) = G_3 \) (this last equation is a re-writing of \( \tilde{\delta}G_3 = iG_3 \)). The result is that any spinor with \( \tilde{\gamma}_7 \epsilon = -i \epsilon \) will satisfy the dilatino variation equation. The largest possible holonomy group for the internal manifold is \( SO(6) \approx SU(4) \), and from \( \tilde{\gamma}_7 \epsilon = -i \epsilon \) we learn that \( \epsilon \) falls in the 4 of \( SU(4) \) rather than the \( \bar{4} \). Note that complex conjugation of a spinor reverses the eigenvalue of \( \tilde{\gamma}_7 \), as does multiplication by any \( \hat{\gamma}_I \).

The next equation to look at is the gravitino variation in the extra dimensions, which simplifies to

\[
\delta \psi_I = \frac{1}{\kappa} \hat{D}_I \epsilon + \frac{i}{4} \tilde{F}(5) \hat{\gamma}_I \epsilon - \frac{1}{16} \hat{\gamma}_I G_3(3)^* \epsilon^* .
\]  

(6)

The form of \( F(5) \) is restricted by 3 + 1-dimensional Poincaré invariance and self-duality: it is

\[
F(5) = h(5) + \tilde{\delta}h(5)
\]

(7)

where \( h(5) = -\tilde{\delta}h(1) \) is a five-form on the extra dimensions and \( \tilde{\delta}h(5) = e^{iA} \text{vol}_4 \wedge h(1) \). To satisfy the Bianchi identity for \( F(5) \) we must have

\[
dh(5) = -\frac{\kappa}{8} \text{Im} F(3) \wedge F^*(3)
\]

(8)

\[
d\tilde{\delta}h(5) = 0 .
\]

Multiplying (6) by \( 1 \pm i \tilde{\gamma}_7 \) one sees that the first two terms must cancel against one another, and the last term must vanish on its own. Thus we can separate the conditions on \( G_3 \),

\[
\tilde{\delta}G_3 = iG_3, \quad G_3(3)* \epsilon = G_3(3)^* \epsilon = 0 ,
\]  

(9)

from the rest of the conditions for supersymmetry,

\[
\delta \psi_M = \left[ \frac{1}{\kappa} \left( \partial_M + \frac{1}{4} \hat{\omega}_{MNP} \hat{\gamma}^{NP} \right) - e^{4A} \frac{i \gamma_7}{2} h_J \hat{\gamma}^J \hat{\gamma}_M \right] \epsilon = 0 ,
\]  

(10)

which, in terms of a rescaled spinor \( \tilde{\epsilon} = e^{-A/2} \epsilon \) whose eigenvalue under \( \gamma_5 \) is \(-i\), read

\[
\delta \psi_\mu = e^{A/2} \hat{\gamma}_\mu \left[ \frac{1}{2\kappa} \partial_I A - \frac{e^{4A}}{2} h_J \right] \tilde{\epsilon} = 0
\]

\[
\delta \psi_I = e^{A/2} \left[ \frac{1}{\kappa} \hat{D}_I + \left( \frac{1}{2\kappa} \partial_I A - \frac{e^{4A}}{2} h_I \right) - \hat{\gamma}_J \left( \frac{1}{2\kappa} \partial_J A - \frac{e^{4A}}{2} h_J \right) \right] \tilde{\epsilon} = 0 .
\]  

(11)

Equivalent to (11) are the conditions

\[
h(1) = -\frac{1}{4\kappa} de^{-4A} , \quad \tilde{D}_I \zeta = 0 .
\]  

(12)

Thus we learn that \( \tilde{g}_{IJ} \) is a metric of (at most) \( SU(3) \) holonomy, which is to say a Calabi-Yau metric. If the holonomy is exactly \( SU(3) \), then the only possible choice for
\( \tilde{\epsilon} \) is the \( SU(3) \) singlet with eigenvalue \(-i\) under \( \tilde{\gamma}_7 \), multiplied by an arbitrary spinor in the \( \mathbf{2} \) of \( SO(3,1) \). Thus four supercharges are preserved: \( \mathcal{N} = 1 \) in four dimensions, as implicitly claimed in \([3]\). Indeed, (81) and (82) of \([3]\) are precisely the supersymmetry conditions \( \tilde{s}G_{(3)} = iG_{(3)} \) and \( h_{(1)} = -\frac{1}{4\kappa} de^{-4A} \). If the holonomy is \( SU(2) \), then eight supercharges are preserved.

It remains only to check the Bianchi identities, (8). Using (12) one obtains \( \hat{s}h_{(5)} = \frac{1}{4\kappa} \text{vol}_4 \wedge de^{4A} \), which is obviously closed. Thus the first equation in (8) gives us our only constraint: in two equivalent forms,

\[
\begin{align*}
    d\hat{s}h_{(1)} &= \frac{\kappa}{8} \text{Im} F_{(3)} \wedge F_{(3)}^* \\
    \hat{\mathbf{D}} e^{-4A} &= \frac{\kappa^2}{2} \hat{s} \text{Im} F_{(3)} \wedge F_{(3)}^* = -\frac{\kappa^2}{12} \tilde{g}^{I_1 J_1} \tilde{g}^{I_2 J_2} \tilde{g}^{I_3 J_3} F_{I_1 I_2 I_3} F_{J_1 J_2 J_3}.
\end{align*}
\]

To obtain the second form we have used (12) and \( \hat{\mathbf{D}} = \hat{D}_I \hat{D}^I = -\hat{s} \hat{s} d \) acting on scalars. Keeping track of all the signs is somewhat difficult, but there is a consistency check: the second form of (13) is exactly the trace of the Einstein equations. Unsurprisingly, it is impossible to satisfy (13) on a compact manifold unless \( F_{(3)} = 0 \): the right hand side is negative and the left hand side is a total derivative.

We have stated in complete detail the conditions for supersymmetry in equations (9), (12), and (13). Because the the six extra dimensions form a complex manifold, the conditions (9) can be simplified. The equation \( \tilde{s}G_{(3)} = iG_{(3)} \) is satisfied precisely if \( G_{(3)} \) is a sum of a \((2,1)\) form and a \((0,3)\) form (both closed of course). The \( SU(3) \) singlet spinor can conveniently be defined as the Fock space ground state annihilated by \( \tilde{\gamma}_p \), where the holomorphic vector index \( p \) runs from 1 to 3. If \( G_{(3)} \) contains a \((0,3)\) component, then \( \tilde{G}_{(3)} \) fails to annihilate the Fock vacuum, since it contains a term proportional to the product \( \tilde{\gamma}_1 \tilde{\gamma}_2 \tilde{\gamma}_3 \) of all three creation operators. So \( G_{(3)} \) must be a \((2,1)\) form.

Generally speaking, it is not trivial to find a closed \((2,1)\) form for which (13) can be solved to obtain \( e^{-4A} \) which is everywhere non-singular and vanishes at infinity. This is exactly what the authors of \([3]\) did for the case of the deformed conifold. It would be quite interesting to inquire in what generality non-singular solutions to (13) exist. Whenever one is found, it is always possible to add an arbitrary function to \( e^{-4A} \) which is harmonic except for delta function sources and vanishes at infinity. This corresponds to adding D3-branes at arbitrary locations, and doesn’t break any additional supersymmetry.
3 Relation to F-theory

A number of authors [10, 11, 12, 13] have considered warped F-theory compactifications related to M-theory on Calabi-Yau four-folds with G-flux [14]. It is straightforward to see how the solutions described in the previous section fit into this rubric. The general prescription is that F-theory on an elliptically fibered CY$_4$ is equivalent to type IIB string theory on the base of the fibration, where the IIB coupling is identified with the modular parameter of the torus. In our case, this coupling is constant, so we must be considering F-theory on a product of $T^2$ and the Calabi-Yau three-fold with metric $\tilde{g}_{I,J}dy^I dy^J$. We have restricted attention to the case where the coupling is $B = 0$, or equivalently $\tau = i$: this corresponds to a square $T^2$. Other complex structures can be obtained through a global $SL(2, \mathbb{R})$ rotation.

In the construction of M-theory compactifications on eight-manifolds with G-flux [14], it was shown that $G^{(4)}$ had to be a $(2,2)$-form. In translating the related F-theory compactification into type IIB language, the following formula is standard (see for instance [10, 11, 13]):

$$G^{(4)} = \frac{\pi}{i \text{Im } \tau} (H \wedge d\tilde{z} - \tilde{H} \wedge dz), \quad (14)$$

where $H = H^R - \tau H^\text{NS}$ and $z$ is the holomorphic coordinate on the $T^2$. Since $\tau = i$ for us, we may identify $H = G^{(3)} = F^{(3)}$. From (14) we learn that for $G^{(4)}$ to be a $(2,2)$ form, $G^{(3)}$ must be a $(2,1)$ form—as concluded earlier from a direct analysis in the type IIB language. Furthermore, the equation for the warp factor, (13), descends from an analogous formula ((2.57) of [14]), which for the warped M-theory geometry

$$ds^2 = e^{-\phi(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{\phi(y)/2} g_{I,J} dy^I dy^J \quad (15)$$

reads

$$\Box e^{3\phi/2} = * \left[ 4\pi^2 X_8(R) - \frac{1}{2} G \wedge G - 4\pi^2 \sum_{i=1}^{n} \delta^8(y - y_i) \right] \quad (16)$$

where $\Box$ and $*$ are defined with reference to the Calabi-Yau metric $g_{I,J}$ on the eightfold, as is

$$X_8 = \frac{1}{(2\pi)^4} \left[ -\frac{1}{768} (\text{tr } R^2)^2 + \frac{1}{192} \text{tr } R^4 \right]. \quad (17)$$

In translating (16) through F-theory to type IIB, the last term changes from source terms for M2-branes to source terms for D3-branes. The first term goes away when $\tau$ is constant because $X_8$ vanishes for the product of a Calabi-Yau three-fold and $T^2$. Thus F-theory considerations do not lift the topological obstruction to solving (13) on a compact manifold, unless we lift the assumption that the complex coupling is constant. In a more general F-theory compactification on an elliptically fibered CY$_4$

*Type IIB vacua with three-form flux have also been studied in [15].
with nonzero Euler number (and, necessarily, nontrivial fibration of the $T^2$), the global constraint that arises from the generalization of (13) is

$$\frac{\chi}{24} = n_{D3} + \int \text{Im} \, G_{(3)} \wedge G^*_{(3)}$$

in appropriate units. (The Euler number $\chi$ vanishes when the eight-manifold is a Calabi-Yau times $T^2$, and then there is a problem because both terms on the right hand side are positive). Such compactifications have been studied [13, 12] as possible realizations of the Randall-Sundrum scenario [16, 17].

Minimally supersymmetric compactifications of F-theory lie at the center of a fascinating locus of ideas. As warped geometries, they offer the hope of obtaining a hierarchy of scales from geometry.† It was observed in [11] that the conditions for supersymmetry are difficult to satisfy and admit few moduli compared to usual Calabi-Yau compactifications. The discovery [3] that a deformation of the conifold was necessary in order to have G-flux resolve the naked singularity found in [18] can be viewed as an example of this moduli fixing, since the size of the $S^3$ at the tip of the deformed conifold is determined by the G-flux. Warped compactifications of F-theory have even been speculated to offer a solution [19] to the cosmological constant problem along the lines of [20]. The eventual hope is to find an isolated compactification, with strong warping to account for the hierarchy between the gravitational and electroweak scales, and broken supersymmetry without a large cosmological constant.

The properties of F-theory compactifications, and in particular the claims of [3], suggest that there is a big chunk missing from our understanding of type II string theory. It was suggested in [3] that the deformed conifold with three-form represented a duality cascade of $SU(M) \times SU(N)$ gauge theories. The logic originated with a study of related geometries in AdS/CFT [21, 22, 23]: first there was the idea [24] that D5-branes wrapped on a two-cycle of $T^{11}$ represented domain walls between a $SU(N) \times SU(N)$ gauge theory and a $SU(N) \times SU(N + 1)$ gauge theory; then there were supergravity treatments of the geometry that would arise from a small number of such “fractional branes” (in the sense of [25]) added to many D3-branes [26]; then there came the extension [18] to the case of only fractional branes, which finally led [3] to a wholly non-singular geometry without any D-branes at all, just three-form flux. The origins of the construction lead us to believe that the final geometry has a large hidden gauge symmetry, which originated in the Chan-Paton factors of the open strings attached to the fractional branes that are smoothed away in the end. And it is natural to believe that this hidden gauge symmetry persists in a compactification of F-theory which locally looks like the deformed conifold solution of [3]. Intuitively, the open strings are there, just confined—or, perhaps more appropriately, “dualized” into

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†The generality of this idea as an extension of [16] has been emphasized to me by H. Verlinde.
a smooth closed string background. AdS/CFT thus seems to lead us toward a vastly more general open-closed string duality, applicable (one would hope) to compact as well as non-compact geometries. Studies of tachyon condensation [27, 28] seem to point in the same direction.

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**Appendix**

We follow the notation of [8] except for trivial changes in typography. The metric signature is mostly minus, and the Clifford algebra is \( \{ \hat{\gamma}^M, \hat{\gamma}^N \} = 2\eta^{MN} \). The notation \( \hat{\gamma} \) indicates a ten-dimensional quantity, while \( \tilde{\gamma} \) indicates a six-dimensional quantity and \( \gamma \) indicates a four-dimensional quantity. We will use \( M, N \) to indicate ten-dimensional curved space indices, \( \mu, \nu \) four four-dimensional indices, and \( I, J \) for six-dimensional indices. If the six-dimensional manifold is always assumed to have complex structure, then \( p, q \) denote three-valued holomorphic indices while \( \bar{p}, \bar{q} \) denote anti-holomorphic indices. Flat tangent space indices are indicated by \( \underline{M} \). Symmetrization and antisymmetrization are carried out with “weight one:” for example, \( [ab] = \frac{1}{2}(ab - ba) \). For \( k \)-forms we use the notation \( F_{(k)} = \frac{1}{k!} F_{M_1...M_k} dx^{M_1} \wedge ... \wedge dx^{M_k} \) where the summation over the \( M_i \) is unrestricted.

A warped product geometry is a direct product of two spaces, endowed with a metric which respects the product structure except for a conformal factor on one of the factors which depends on the coordinates of the other. In our case,

\[
d\hat{s}^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu - e^{2B(y)} \tilde{g}_{IJ} dy^I dy^J,
\]

where the minus sign allows us to have a six-dimensional metric with positive signature, and for the sake of generality we have not required \( B = -A \). (This \( B \) is not to be confused with the complex coupling in the main text.) Hodge duals are defined as

\[
(\hat{\ast} \omega)_{\underline{P}_1...\underline{P}_k} = \frac{1}{(10 - k)!} \hat{\epsilon} \underline{P}_1...\underline{P}_k \omega_{\underline{P}_{k+1}...\underline{P}_{10}}
\]

and similarly for the four- and six-dimensional Hodge duals \( \ast \) and \( \tilde{\ast} \). We take the
convention $\epsilon^{01\cdots 9} = 1$ in order to agree with [8]; also $\epsilon^{0123456789} = 1$. Finally,
\begin{align*}
\text{vol}_4 &= \sqrt{|\det g_{\mu\nu}|} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \\
\text{vol}_6 &= \sqrt{|\det \tilde{g}_{IJ}|} dy^4 \wedge \cdots \wedge dy^9 \\
\text{vol}_{10} &= e^{4A+6B} \text{vol}_4 \wedge \text{vol}_6 .
\end{align*}
(21)

The obvious choice of 10-bein for (19) is $\hat{e}_\mu^\nu = e^A e_\mu^\nu$ and $\hat{e}^I_J = e^B \tilde{e}^I_J$. The non-vanishing components of the Christoffel connection and the spin connection are
\begin{align*}
\hat{\Gamma}_{\nu}^{\mu\lambda} &= \Gamma_{\nu}^{\mu\lambda} \\
\tilde{\Gamma}_{IJK}^J &= \Gamma_{IJK}^J + 2 \delta^I_J \tilde{g}_{KL} B + 2 \delta^J_K \tilde{g}_{IL} B - \tilde{g}_{IK} \tilde{g}_{JL} \partial K A \\
\hat{\omega}_{\mu}^{\nu\lambda} &= \omega_{\mu}^{\nu\lambda} \\
\hat{\omega}_{\mu}^{\nu\lambda} &= -\hat{\omega}_{\mu\nu\lambda} = e^{A-B} \epsilon_{\mu\nu} \tilde{e}^L_K \partial L A \\
\hat{\omega}_{ILK} &= -\hat{\omega}_{LIK} - \tilde{e}_L^I \tilde{e}^L_K \partial L B + \tilde{e}_L^I \tilde{e}^L_K \partial L B.
\end{align*}
(22)

The signs in last line looks peculiar, but they are only the result of the fact that ten-dimensional flat indices (which appear on the left-hand side) are lowered with $\hat{\eta}^{MN}$, while six-dimensional flat indices (which appear on the right-hand side) are lowered with $\delta_{IJ}$, and $\hat{\eta}^{IJ} = -\delta_{IJ}$. This is the penalty we pay for adopting mostly minus signature. It would not have been a problem if we had quoted results for $\hat{\omega}_{IJK}$: these components of the ten-dimensional spin connection are precisely the same as the spin connection for the six-dimensional metric $e^{2B} \tilde{g}_{IJ}$ with 6-bein $e^B \tilde{e}^I_J$.

The ten-dimensional gamma matrices can be expressed as
\begin{align*}
\hat{\gamma}^\mu &= \gamma^\mu \otimes 1 \\
\hat{\gamma}^5 &= \gamma_5 \otimes \gamma^L \\
\gamma_5 &= \gamma_0 \gamma_1 \gamma_2 \gamma_3 \\
\gamma_7 &= \gamma_4 \gamma_5 \gamma_6 \gamma_7 \\
\gamma_11 &= \gamma^0 \gamma^1 \cdots \gamma^9 \\
\gamma_7 &= \gamma_5 \otimes \gamma_7 .
\end{align*}
(23)

By convention, the matrices $\hat{\gamma}^L$ are the same whether one is thinking of $I$ as a true six-dimensional index or as a ten-dimensional index restricted to the internal manifold. This means that $\hat{\gamma}^L$ is ambiguous in sign due to the choice of mostly minus signature the ten-dimensional flat metric and positive signature for the six-dimensional flat metric. Let us choose $\hat{\gamma}^L = \delta^L_K \tilde{\gamma}^K$. Finally, if $\omega_{(p)}$ is a $p$-form, then we define
\begin{equation}
\psi_{(p)} = \frac{1}{p!} \omega_{M_1 \cdots M_p} \hat{\gamma}^{M_1 \cdots M_p} .
\end{equation}
(24)

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