Quantum field aspect of Unruh problem

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Abstract

It is shown using both conventional and algebraic approach to quantum field theory that it is impossible to perform quantization on Unruh modes in Minkowski spacetime. Such quantization implies setting boundary condition for the quantum field operator which changes topological properties and symmetry group of spacetime and leads to field theory in two disconnected left and right Rindler spacetimes. It means that "Unruh effect" does not exist.

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1. The Unruh problem \cite{1} closely associated with the Hawking effect \cite{2} is known for more than twenty years. It is asserted that from the point of view of a uniformly accelerated observer in Minkowski spacetime (MS) the usual vacuum state $|0_M\rangle$ looks like a mixed state described by the thermal density matrix with Davies – Unruh \cite{1, 3} effective temperature $T = a/(2\pi)$ where $a$ stands for the proper acceleration of the observer. More precisely (see e.g. Refs.\cite{4-9} and citation therein) one has the relation

$$\langle 0_M|\mathcal{R}|0_M\rangle = \text{Sp} (\rho_R \mathcal{R}), \quad \rho_R = Z^{-1} \exp(-2\pi H_R). \quad (1)$$

Here $\mathcal{R}$ is an arbitrary observable which depends on ”values of the field” $\phi(x)$ only for $x$ from the right Rindler wedge $R$ (see Fig.1) which contains the world line of the observer, $\rho_R$ is the density matrix and $H_R$ is the secondly quantized Hamiltonian with respect to timelike variable $\eta$ in $R$.

An evristic explanation of Unruh effect is the following. Spacetime in the Rindler reference frame (with respect to which the accelerated observer is at rest) possesses event horizons. Therefore the Rindler observer looses a part of information accessible for inertial observer in MS. Hence he perceives the Minkowski vacuum state as a mixed state.

Mathematically correct consideration of Unruh problem is possible in the frame of algebraic approach to quantum field theory (see reviews \cite{10, 11} on algebraic approach and Ref.\cite{12} for it’s application to Unruh problem) which allows one to consider pure and mixed states on the unified grounds. In this approach a notion of Kubo – Martin – Schwinger (KMS) state \cite{13} is used as a definition of thermal equilibrium state.

In the current paper we will show that eq.(1) as well as it’s analog in algebraic approach cannot serve as a proof of Unruh effect. The reason is the existance of boundary condition \cite{14} for the field operator in Rindler spacetime (RS). We will also point out a generalization of this boundary condition in algebraic treatment. But first we will discuss an equivalent interpretation of the Unruh effect in terms of Fulling – Unruh ”particles”. The latter arises when the so – called Unruh modes \cite{1, 15} are used for quantization of the field. We will show that the Unruh modes can be used as a basis for quantization only in double RS rather than in MS.

2. Since the Rindler observer world line coincides with one of the orbits

\footnote{We use units $\hbar = 1, c = 1.$}
of Lorentz rotation we consider quantization of neutral scalar field in the basis of boost generator eigenfunctions

\[ \Psi_{\kappa}(t, z) = 2^{-3/2} \pi^{-1} \int_{-\infty}^{\infty} d\theta \exp \{-im(t \cosh \theta - z \sinh \theta) - i\kappa \theta\}. \tag{2} \]

These modes are positive frequency with respect to global Minkowski time solutions of the Klein – Gordon (KG) equation. They are orthonormal relative to the KG inner product

\[ \langle f, g \rangle_M = i \int_{-\infty}^{\infty} dz f^*(x) \frac{\partial}{\partial t} g(x), \quad x = (t, z), \]

form a complete set and hence may be used for the field quantization \(^3\)

\[ \phi(x) = \int d\kappa \{ \Psi_{\kappa}(x)b_{\kappa} + \Psi^*_{\kappa}(x)b^\dagger_{\kappa} \} \]

\[ [b_{\kappa}, b^\dagger_{\kappa'}] = \delta(\kappa - \kappa'), \quad b_{\kappa}|0_M\rangle = 0, \quad -\infty < \kappa < \infty. \tag{3} \]

For \( b_{\kappa} \) we have

\[ b_{\kappa} = \langle \Psi_{\kappa}, \phi \rangle_M. \tag{4} \]

Using light cone coordinates \( x_\pm = t \pm z \) the modes (2) can be represented in the form corresponding to splitting of MS into domains invariant under Lorentz rotation in \((z, t)\) plain, see Fig.1,

\[ \Psi_{\kappa}(x) = \theta(x_+)\theta(-x_-)\Psi^R_{\kappa}(x) + \theta(x_+)\theta(x_-)\Psi^F_{\kappa}(x) + + \theta(-x_+)\theta(x_-)\Psi^L_{\kappa}(x) + \theta(-x_+)\theta(-x_-)\Psi^P_{\kappa}(x). \tag{5} \]

The first term in the r.h.s. of Eq.(5) relates to the right sector of MS, see Fig.1. Nevertheless due to the presence of Heaviside \( \theta \) – functions it obeys the KG equation with sources localized on the horizons. Therefore we will consider the open domain \( R \) not containing boundary \( h^+ \cup h_0 \cup h^+_0 \) – the Rindler wedge. This manifold is covered by Rindler coordinates \( \eta, \rho \)

\[ z = \rho \cosh \eta, \quad t = \rho \sinh \eta, \quad -\infty < \eta < \infty, \quad \rho > 0, \tag{6} \]

\(^2\)We restrict ourselves to the case of two dimentional spacetime. This assumption is choosen only to simplify notation and does not affect the results.

\(^3\)Quantization of scalar field performed in Ref.[16] by analitical continuation of Green functions is equivalent to the one defined by Eqs.(2, 3).

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Fig. 1. Splitting of Minkowski spacetime into submanifolds stable under Lorentz rotations in \((z, t)\) plane: \(R\) and \(L\) – right and left wedges; \(F\) and \(P\) – future and past wedges; \(h_\pm = h_\pm^+ \cup h_\pm^-\) – event horizons; \(h_0\) (two dimensional plane \(z = t = 0\)) – trivial orbit. Variables \(\eta\) and \(\rho\) are Rindler coordinates in the right Rindler wedge. The undeshed area is the double Rindler space where Unruh and Fulling modes coincide and quantization leads to the concept of non-interacting right and left Fulling particles.
and KG equation takes form
\[ \left\{ \frac{\partial^2}{\partial \eta^2} + K_R(\rho) \right\} \phi_R(\xi) = 0, \quad K_R(\rho) = -\rho \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + m^2 \rho^2, \quad \xi = (\eta, \rho). \] (7)

Positive frequency solutions of Eq. (7) with respect to timelike variable \( \eta \) (Fulling modes) are [17]
\[ \Phi_\mu(\xi) = (2\mu)^{-1/2} \varphi_\mu(\rho) e^{-i\mu \eta}, \quad \varphi_\mu(\rho) = \frac{\sqrt{2\mu \sinh \pi \mu}}{\pi} K_{i\mu}(m \rho), \quad \mu > 0. \] (8)

They are orthonormal relative to KG inner product for RS
\[ \langle f, g \rangle_R = i \int_0^\infty d\rho \rho f^*(\xi) \frac{\partial}{\partial \eta} g(\xi). \] (9)

Fulling modes \( \Phi_\mu \) constitute a complete set of positive frequency solutions for KG equation and therefore may be used for quantization of the field \( \phi_R(x) \) in RS:
\[ \phi_R(\xi) = \int_0^\infty d\mu \{ \Phi_\mu(\xi) c_\mu + h.c. \}, \quad [c_\mu, c_\mu^\dagger] = \delta(\mu - \mu'), \quad c_\mu |0_R\rangle = 0, \quad \mu > 0, \] (10)

where the state \( |0_R\rangle \) is called Fulling vacuum. Annihilation operators of Fulling particles \( c_\mu \) may be expressed in terms of the field \( \phi_R(\xi) \) by
\[ c_\mu = \langle \Phi_\mu, \phi_R \rangle_R = \frac{i}{\sqrt{2\mu}} \int_0^\infty d\rho \rho \varphi_\mu(\rho) \left[ \frac{\partial}{\partial \eta} \phi_R(\xi) - i\mu \phi_R(\xi) \right] \bigg|_{\eta=0} \] (11)

Crucial point for the concept of Fulling particles is the requirement for the field \( \phi_R(\xi) \) to obey the boundary condition [14] at \( \rho = 0 \)
\[ \lim_{\rho \to 0} \phi_R(\rho, \eta) = 0, \] (12)

besides a trivial null condition at \( \rho = \infty \). Because of unboundedness of the operator \( \phi_R(\eta, \rho) \) the relation [12] should be understood as a condition for matrix elements between physically realizable states [4]. It is worth noting that

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4 By physically realizable states we mean the states corresponding to finite mean values of operators \( H_R = \int_0^\infty d\mu \mu c_\mu^\dagger c_\mu \) and \( H_R^{-1} \). The second requirement arises due to absence of the mass gap for Fulling particles, see, e.g., sec. 4 of Ref. [14].
the substitution $\rho = m^{-1}e^\mu$ mapping the point $\rho = 0$ into $\rho = \infty$ reduces the condition (12) to the usual requirement for vanishing of the field at spatial infinity.

3. To perform in MS quantization similar to Fulling one in RS Unruh suggested [1, 15] to use modes which are superpositions of boost modes with positive and negative frequencies. In notation of Ref.[14] they read

$$R_\mu = \frac{\left[e^{\pi \mu/2} \Psi_{\mu} - e^{-\pi \mu/2} \Psi_{-\mu}\right]}{\sqrt{2 \sinh \pi \mu}}, \quad L_\mu = \frac{\left[e^{\pi \mu/2} \Psi_{-\mu} - e^{-\pi \mu/2} \Psi_{\mu}\right]}{\sqrt{2 \sinh \pi \mu}}, \quad \mu > 0.$$  

(13)

Although Unruh modes $R_\mu, (L_\mu)$ are not positive frequency with respect to Minkowski time $t$ they are positive (negative) frequency in $R$ and $L$ wedges of MS with respect to timelike variables $\frac{1}{2} \ln(\pm x_+/\mp x_-)$ respectively. Unruh modes have remarkable properties

$$R_{\mu}(x) = 0 \text{ in } L, \quad R_{\mu}(x) = \Phi_{\mu}(x) \text{ in } R, \quad \langle R_{\mu}, R_{\mu'} \rangle_{M} = \delta(\mu - \mu'),$$

$$L_{\mu}(x) \equiv 0 \text{ in } R, \quad L_{\mu}(x) \equiv \Phi_{\mu}(-x) \text{ in } L, \quad \langle L_{\mu}, L_{\mu'} \rangle_{M} = -\delta(\mu - \mu').$$

(14)

Note that $R$ – modes are not analitical continuation of Fulling modes (8) because different rules are used for continuation of $\Psi_{\mu}(x)$ and $\Psi_{-\mu}(x)$ when passing around branch points $x_{\pm} = 0$. Inverting Eqs.(13) and substituting the result into Eq.(3) one obtains for $x \neq 0$ (when it is possible to split the integral in Eq.(3) into two integrals over $\kappa > 0$ and $\kappa < 0$)

$$\phi(x) = \int_{0}^{\infty} d\mu \left\{R_{\mu}(x)r_{\mu} + L_{\mu}(x)l_{\mu}^\dagger + h.c.\right\},$$

(15)

where operators

$$r_{\mu} = \left[\frac{e^{\pi \mu/2}b_{\mu} + e^{-\pi \mu/2}b_{-\mu}^\dagger}{\sqrt{2 \sinh \pi \mu}}\right], \quad l_{\mu} = \left[\frac{e^{\pi \mu/2}b_{-\mu} + e^{-\pi \mu/2}b_{\mu}^\dagger}{\sqrt{2 \sinh \pi \mu}}\right], \quad \mu > 0,$$

$$[r_{\mu}, r_{\mu'}^\dagger] = [l_{\mu}, l_{\mu'}^\dagger] = \delta(\mu - \mu').$$

(16)

Unfortunately Eqs.(13),(16) does not define valid quantization in the whole MS because splitting of the integral over $\mu$ into separate terms describing creation and anihilation parts of the field cannot be performed in $F$ and $P$ wedges. Indeed, using explicit expression for boost modes [14] one
finds
\[ R^F_\mu r_\mu = -L^F_\mu l_\mu^\dagger = -\frac{i}{2\sqrt{2\pi\mu}} J_0(m\sqrt{x_+x_-}) \phi(0,0), \quad \mu \to 0, \] (17)
where according to Eqs.(2), (3)
\[ \phi(0,0) = \frac{1}{\sqrt{2}} (b_0 + b_0^\dagger) \neq 0. \] (18)
Hence the integral over \( \mu \) of these expressions diverges logarithmically on the low limit while in the sum of these terms in Eq.(15) singularities cancel. Of course the Eqs.(17), (18) should be understood as relation between matrix elements, see discussion after Eq.(12).

Using Eqs.(3), (16) we obtain the relation
\[ \langle 0_M | r_\mu^\dagger r_{\mu'} | 0_M \rangle = \frac{1}{\exp(2\pi\mu) - 1} \delta(\mu - \mu'). \] (19)
The Eq.(19) is usually interpreted as a special case of Eq.(1) for \( R = r_\mu^\dagger r_{\mu'} \) under assumption that \( r_\mu = c_\mu \). Moreover Eq.(19) is sometimes used for derivation of the general result (1) and thus is equivalent to it. Nevertheless such interpretation is not valid since according to Eqs.(15),(17) the operators \( r_\mu, l_\mu, r_\mu^\dagger, l_\mu^\dagger \) can not be considered as anihilation and creation operators of Fulling - Unruh particles in MS where the vaccum state \( |0_M \rangle \) is defined. This is not a surprise since it is impossible to find any time – like variable in MS relative to which the Unruh modes (13) correspond to frequency of definite sign. Quantization of the field in the basis of Unruh modes could be performed only if the boundary condition
\[ \lim_{z \to 0} \phi(0, z) = 0 \] (20)
existed. The condition (20) is equivalent to the boundary condition for the field in RS. Hence quantization of the field in the basis of Unruh modes can be performed only in double RS (a disjoint union of wedges \( R \) and \( L \), see Fig.1) rather than in MS and the r.h.s. of the Eqs.(1), (13) can not be considered as thermal equilibrium expectation values.

4. Let us turn back now to the discussion of Eq.(1) which encounter mathematical difficulties in the conventional formalism of quantum field theory. The representation of cannonical commutation relations in terms of
Unruh operators (16) is unitary inequivalent to the one in terms of operators $b_\kappa$, see Eq.(3). It is a direct consequence of divergency of $Z$ in Eq.(1) [17].

There are two ways to avoid this difficulty. The first one is to place the field in the box which may in this problem be constructed by two uniformly accelerated mirrors moving in right and left Rindler wedges [18]. However such regularization again leads to consideration of double RS as a physical space-time of the observer. The second opportunity is to use algebraic approach and a notion of KMS state as a definition of thermal equilibrium state.

To reformulate the Eq.(1) in terms of algebraic approach let us introduce the required definitions. Let $D$ be a linear symplectic space of solutions of KG equation for $C^\infty$ Cauchy data with compact support on some surface $\Sigma$ (a classical phase space of field theory). An algebra $\mathcal{U}$ of observables of the field is a $C^*$ algebra with generators $W(\Phi)$, $\Phi \in D$ satisfying usual Weyl relations

$$W(\Phi_1)W(\Phi_2) = \exp(-i\sigma(\Phi_1, \Phi_2)/2)W(\Phi_1 + \Phi_2), \quad W(\Phi)^* = W(-\Phi)$$ (21)

with $\sigma$ being a symplectic product on $D$. The states in algebraic approach are linear functionals on $\mathcal{U}$. Vacuum state and the corresponding representation of $\mathcal{U}$ by operators acting in Hilbert space can be constructed by using a one particle structure which maps $D$ onto the Hilbert space of physically realizable positive frequency solutions [1]. Let $D_R, D_L$ be subspaces of $D$ consisting of those solutions which vanish in closed wedges $\bar{L}$, $\bar{R}$ respectively and $\tilde{D} = D_L \oplus D_R \subset D$. Note that $\tilde{D}$ is a subspace of those solutions which vanish in a neighbourhood of $h_0$, see Fig.1. Finite linear combinations of elements from $\mathcal{U}$ of the form $W(\Phi)$ with $\Phi \in D_R$ constitute an open (rather than $C^*$) subalgebra $\mathcal{U}_R$ of $\mathcal{U}$ which is called the right wedge algebra. The left wedge algebra $\mathcal{U}_L$ and the double wedge algebra $\tilde{\mathcal{U}}$ are defined similarly by changing $D_R$ to $D_L$ and $\tilde{D}$ respectively.

According to the Bisognano–Wichmann theorem [19] the Minkowski vacuum state $\omega_M$ when restricted to the right wedge algebra $\mathcal{U}_R$ satisfies KMS condition with temperature $(2\pi)^{-1}$ with respect to boost time $\eta$. But in order to give a physical interpretation to this theorem one must relate it to the procedure of measurement by pointing out the quanta which are thermally distributed. Such interpretation is given by the notion of double KMS state.

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5 In this construction the space $D$ plays a role of a complete set of quantum numbers in the usual formalism.
Let $\omega_M$ be usual Minkowski vacuum state and $\tilde{\omega}_F^{(2\pi)}$ be a $(2\pi)^{-1}$ temperature double KMS state over "double linear system" $U = U_L \otimes U_R$ with respect to the boost evolution on $U_R$ and "antiboost" evolution on $U_L$ (see Ref. [12], section 1 for exact definitions and eqs. (2.9)–(2.11) for explicit construction). The main result (proposition 2.1 of Ref. [12]) is that

$$\omega_M = \tilde{\omega}_F^{(2\pi)} \text{ on } \tilde{U}.$$  \hspace{0.2cm} (22)

This equation is an analog of eq. (1) in algebraic approach.

Let us explicitly evaluate eq. (22) and check that it cannot be extended to the whole algebra of observables of the free field $U$. In the case of theory in MS an expectation value of Weyl generator in Minkowski vacuum state is defined by [12]

$$\omega_M(W(\Phi)) = \exp \left( -\frac{1}{2} ||K_M\Phi||^2 \right),$$  \hspace{0.2cm} (23)

where $K_M$ is a ground one particle structure for this case which is a map extracting a positive frequency part of the solution, $K_M : \Phi \mapsto \Phi^{(+)}$ and

$$||K_M\Phi||^2 = \langle \Phi^{(+)}, \Phi^{(+)} \rangle_M = \int_{-\infty}^{\infty} d\kappa \ |\langle \Psi_\kappa, \Phi \rangle_M|^2,$$  \hspace{0.2cm} (24)

where we have used a complete set of boost modes $\Psi_\kappa$ (2) to extract positive frequency part. By inverting relations (13) one can rewrite eq. (24) in terms of Unruh modes. The result is

$$||\tilde{K}_M\Phi||^2 = \int_0^{\infty} d\mu \ \coth \pi \mu \ {||R_\mu, \Phi||_M^2 + ||L_\mu, \Phi||_M^2} + \ldots$$ \hspace{0.2cm} (25)

(here and below dots denote the correlation term).

Now let us evaluate expectation value of Weyl generator in a double KMS state with temperature $\beta^{-1}$ with respect to Fulling ground one particle structure. This expectation value may be written as [12]

$$\tilde{\omega}_F^{(\beta)}(W(\Phi)) = \exp \left( -\frac{1}{2} ||\tilde{K}_F^{(\beta)}\Phi||^2 \right),$$  \hspace{0.2cm} (26)

where $\tilde{K}_F^{(\beta)}$ is a double KMS one particle structure. According to definition of $\tilde{K}_F^{(\beta)}$ as a map into a direct sum of two copies of Hilbert space of positive frequency solutions in RS one has

$$||\tilde{K}_F^{(\beta)}\Phi||^2 = ||K_F\Phi_R||_\beta^2 + ||K_F\Phi_L||_\beta^2 + \ldots.$$  \hspace{0.2cm} (27)

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where $\Phi_{R,L}$ is the restriction of $\Phi$ to the right (left) Rindler wedge, $\mathcal{F}\Phi_L(\xi) = \Phi_L(-\xi)$. Quantity $||K_F\Phi_R||^2_\beta$ is defined by:

$$||K_F\Phi_R||^2_\beta = \int_{\Sigma \cap R} d\sigma \int_{\Sigma \cap R} d\sigma' W^{(\beta)}_F(\xi, \xi') \frac{1}{\rho} \frac{\partial}{\partial \eta} \Phi_R(\xi) \frac{\partial}{\partial \eta'} \Phi_R(\xi'),$$

where $W^{(\beta)}_F(\xi, \xi')$ is the thermal Wightman function for theory in RS and Cauchy surface $\Sigma$ is chosen to be $\eta = 0$. For explicit calculation let us use a complete set of Fulling modes [8]. One can express thermal Wightman function in terms of Fulling modes by:

$$W^{(\beta)}(\xi, \xi') = \int_0^\infty d\mu \frac{\exp(\beta \mu) - 1}{\exp(\beta \mu) + 1} \{\Phi_\mu(\xi)\Phi_\mu^*(\xi') \exp(\beta \mu) + \Phi_\mu^*(\xi)\Phi_\mu(\xi')\}. \text{ (29)}$$

Using Eqs. (28) – (29) one obtains

$$||\tilde{K}^{(\beta)}_F\Phi||^2 = \int_0^\infty d\mu \coth\left(\frac{\beta \mu}{2}\right) \{|\langle \Phi_\mu, \Phi_R \rangle_R|^2 + |\langle \Phi_\mu, \mathcal{F}\Phi_L \rangle_R|^2\} + \ldots \text{ (30)}$$

Taking into account the relation [14]

$$\langle R_\mu, \Phi \rangle_{\mathcal{M}} = \langle \Phi_\mu, \Phi_R \rangle_R + \frac{i}{2\pi} \sqrt{\sinh \pi \mu} \lim_{z \to 0} \Phi(0, z) \times \Gamma(\mu) \left(\frac{mz}{2}\right)^{-\mu} - \Gamma(-\mu) \left(\frac{mz}{2}\right)^{\mu} \right\} \text{ (31)}$$

one concludes after comparing Eqs. (30), (25) that equation

$$\omega_{\mathcal{M}}(W(\Phi)) = \tilde{\omega}_{F}^{(2\pi)}(W(\Phi)) \text{ (32)}$$

(and hence by linearity Eq. (22)) holds if and only if $\Phi(0, 0) = 0$ or in other words only for $\Phi \in \tilde{D}$.

We see that Eq. (22) holds only on the dense subalgebra $\tilde{\mathcal{U}} \subset \mathcal{U}$, which corresponds to the space of those solutions for the field equation which satisfy

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6 We refer to sect. 1.4 of Ref. [12] for detailed explanations on construction of the map $\tilde{K}^{(\beta)}_F$.

7 The second term is responsible for canceling of divergent part of $\langle \Phi_\mu, \Phi_R \rangle_R$ if $\Phi(0, 0) \neq 0$. Note that we improved in Eq. (31) an obvious misprint made in Eq. (20) of the Ref. [14].
boundary condition at the plain \( h_0 \). Therefore the l.h.s. of Eq.\( (22) \) admits continuation to the whole \( \mathcal{U} \) while the r.h.s. does not.

Let us consider two opportunities to interpret Eq.\( (22) \). The first one is to treat \( \mathcal{U} \) as the true algebra of observables for the accelerated observer. In this case Eq.\( (22) \) does not hold for all observables and therefore Minkowski vacuum does not coincide with the thermal state \( \tilde{\omega}^{(2\pi)}_F \).

The second opportunity is to insist that \( \tilde{\mathcal{U}} \) should be the true algebra of observables for accelerated observer (although this is not \( C^* \) algebra as it is usually required). In this case Poincaré invariance is lacking and hence the true Minkowski vacuum state \( \omega_{\mathcal{M}} \) does not exist. Then Eq.\( (22) \) is satisfied for all physical observables and hence the restriction \( \omega_{\mathcal{M}}|_{\tilde{\mathcal{U}}} \) of the state \( \omega_{\mathcal{M}} \) to \( \tilde{\mathcal{U}} \) coincides with the state \( \tilde{\omega}^{(2\pi)}_F \) and admits interpretation in terms of Fulling – Unruh quanta. But the restriction \( \omega_{\mathcal{M}}|_{\tilde{\mathcal{U}}} \) of Minkowski vacuum state to the subalgebra \( \tilde{\mathcal{U}} \) is not Minkowski vacuum state any more because of the other domain of definition.

5. Presence of boundary condition \( (20) \) shows that Unruh quantization can not be performed in MS. From the physical point of view the meaning of Eq.\( (20) \) is that the plain \( h_0 \) does not affect any physically realizable measurements and therefore should be considered as being removed from the spacetime. But such removal crucially changes topological properties of the spacetime. Therefore the known general relation \( (22) \) between Minkowski vacuum state and thermal Fulling – Unruh state is by no way physically related to the behaviour of accelerated detector in empty MS.

A separate aspect of Unruh problem is whether a concrete accelerated detector with known structure would behave as if having been immersed in thermal bath with Davies – Unruh temperature. A rather complete treatment of this problem was done \( [22] \) for the case when elementary particles are used as detectors and constant electric field is employed as accelerating force. It occurred that only for some set of values of parameters of the model such detectors demonstrate Unruh type behaviour. An example of utilization of a composite system (a heavy atom or ion) as an accelerated detector

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8The point of view that Unruh effect arises if the true spacetime is \( \mathcal{M}\setminus h_0 \) has been previously discussed in Refs.\( [20],[21] \).

9Lacking of Poincaré invariance for the field theory with boost time evolution is a consequence of the fact that boost transformations do not constitute a normal subgroup in Poincaré group.
was considered nonrelativistically in Ref. 23. It was shown that due to the
tunneling ionization process in accelerating electric field such detector will
be destroyed long before it comes to thermal equilibrium state with Davies
– Unruh temperature.

Because a systematic relativistic theory of bound states has not been
created yet a question of behaviour of accelerated detector in general case is
still open.

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