Spatiotemporal pattern of periodic rhythms in delayed Van der Pol oscillators for the CPG-based locomotion of snake-like robot

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Abstract This is the further research on the delayed half-center oscillator (DHCO) neural system presented in our previous paper (Song and Xu in Nonlinear Dyn 108:2595–2609, 2022. https://doi.org/10.1007/s11071-022-07222-y). The DHCO is used to construct a CPG (central pattern generator) neural system to control locomotion of a snake-like robot with pitch-yaw connecting configuration. To this end, we firstly give an improved model of the VDP (Van der Pol) oscillator. Employing mutually coupled delay, a pair of VDP oscillator is connected to produce an half-center oscillator (HCO) module with time delay that is called as a DHCO (delayed HCO) model. Based on the analysis of the Hopf bifurcation, periodic rhythm and their spatiotemporal patterns of the DHCO are illustrated in the different regions of parameters. The DHCO presents periodic rhythms with synchronous and anti-synchronous patterns, which is to control joint actuators combined in snake-like robot with pitch-yaw connecting configuration. To realize a backward propulsive wave to promote snake-like robotic locomotion, based on the DHCOs, we construct a chain type of the CPG neural system combined with a new unidirectional delay in which phase difference can be regulated. Numerical simulations are illustrated that the CPG neural system can control snake-like robot to move with serpentine, rectilinear, and side-winding patterns in the forward and backward directions. The results show that the snake-like robot can be controlled in expected locomotion patterns for a region but not a fixed value of the controlling parameters. Further, the corresponding regions of the parameters are obtained by using theoretical dynamical analysis but not a trial-and-error method. The snake-like robot gets smooth and stable gait transition with parameter changing.

Keywords Central pattern generator (CPG) · Half-center oscillator · Time delay · VDP oscillator · Snake-like robot · Gait transition

1 Introduction

In recent decades, with the development of biological neurosciences, some effective control strategies inspired by biological mechanism have been attracting great attentions for engineers and scientists to solve locomotion control of biomimetic robot in
unstructured environment \[1, 2\]. In fact, natural animals including humans have high stability movement. They can walk, crawl, swim, and even worm in any occasion and at any time, which is produced and controlled by a special neural network—central pattern generator (CPG) neural system located in spinal cord \[3, 4\]. Generally, the biological CPG model is a neural network system including some special types of neurons \[5, 6\]. Rhythmic activity and pattern transition are simulated by employing neural models and their connections \[7, 8\]. In the application of the robot engineering, a variety of CPGs are designed to control the locomotion of bio-inspired robots. The CPG controllers can produce continuous and smooth output signals due to self-adjusting ability when the turning parameters are changed. As a result, the bio-inspired robots can smoothly and rapidly switch their states based on controlling parameters or external environment \[9, 10\].

Snake-like robot is an interesting and exciting masterpiece inspired by nature snake. It has a redundant and flexible body. The limbless structure with number of joints gives snake the superiority to move in wide range of environments. Snakes can travel through narrow space, slither on dry/muddy ground, and swim insider water by employing many different types of motion patterns \[11\]. To achieve locomotion patterns of snake-like robots, some CPG-based biomimetic strategies have been proposed \[12–14\]. Matsuoka model is a typical motif oscillator to construct controller of robotic locomotion. It has definite biological meanings, such as sensory feedback, external input, and even control signals from brainstem. For the snake-like robotic locomotion, Lu et al. \[15\] used a bidirectional cyclic inhibitory (BCI) CPG model and achieved three types of rhythmic patterns called as the serpentine, concertina (rectilinear), and side-winding locomotion. Wu and Ma \[16, 17\] presented that the snake-like robot with BCI-CPG controller has adaptive creeping locomotion in a complex environment. However, rhythmic signals of Matsuoka model are obtained through a dual- or tri-neuron interconnection consisting with extensor and flexor neurons. This structure involves so many parameters to cause difficulties in selecting proper parameters \[18\].

On the other hand, to simplify dynamic relationship of system parameters and output signals, phase oscillator, i.e., Kuramoto model is introduced to construct CPG controller in the robotic engineering applications because of the specified parameters of frequency and amplitude. Ijspeert \[19\] achieved gaits transition of swimming and walking locomotion for a snake-like salamander robot. The serpentine and side-winding motions of snake-like robot were generated to fit with an unstructured environment \[20\]. Based on the Kuramoto oscillator, Nor and Ma \[21\] presented a CPG controller for a planar snake-like robot. Bing et al. \[22\] designed a lightweight CPG model and realized the snake-like robot to obtain smooth slithering gait transition. However, the Kuramoto-based CPG controller is just the coupled phase oscillators through sine function, which lacks of well-defined biological meanings. The phase difference between adjacent oscillators must be cooperatively adjusted by multiple parameters. Further, to obtain sidewinding locomotion in complex terrains, Qiao et al. \[23\] suggested a 3D motion control method based on triple-layered CPG by using Kuramoto oscillators. Recently, Manzoor et al. \[24, 25\] constructed a CPG model to generate rhythmic patterns and obtain smooth transition of snake-like robot locomotion. The proposed Kuramoto-based CPG system proposed a triple-layer structure and became more complicated.

In fact, the basic function of motif oscillator in the CPG system is to generate rhythm activity \[26, 27\]. The Hodgkin-Huxley (H–H) types of neural models have many parameters and intractable transcendental functions of channel conductance \[28–30\]. To obtain a biological-like CPG system with fewer parameters, the nonlinear oscillator is a suitable mathematical model to construct the CPG controller of robotic locomotion. The Van der Pol (VDP) oscillator was originally proposed as an electronic circuit model \[31, 32\] and extensively studied as a well-known self-sustained prototype in a great number of applications including heartbeats, circadian rhythms, and biological rhythms \[33, 34\]. Dutra, et al. \[35\] used coupled VDP oscillators to obtain a bipedal locomotor and control hips and knees of human. The VDP-based CPG controllers for quadruped and hexapod robots were proposed \[36, 37\]. For the hexapod robot, Yu et al. \[38\] verified that the VDP-based CPG controller can generate walking gaits and smooth transition. To the best of our knowledge, there is no published information on constructing CPG neural controller by using the delayed oscillators. In this paper, we will give a special network structure of CPG neural system.
based on coupling delays. The mapping function from modulating parameters to output signals is presented by theoretical dynamical analysis, but not a trial-and-error simulation method. The snake-like robot presents some locomotion gaits with smooth switching based on the controlling parameters. This motivates our present research.

In fact, the HCO (half-center oscillator) is a basic and pivotal motif in biological CPG neural system [39, 40]. In our previous paper, we have proposed the DHCO (delayed HCO) neural model and analyzed symmetric patterns of rhythm activity [41, 42]. The DHCO system presents many types of rhythm activities with self- and mutual-symmetric patterns by different bifurcation mechanisms [43]. In this paper, to imitate biological CPG system, we firstly exhibit a DHCO neural module based on the mutually delayed VDP oscillators. Employing the Hopf bifurcation analysis, we present a series of parameter regions, in which the DHCO system exhibits periodic rhythms with synchronous and anti-synchronous spatiotemporal patterns. For the pitch-yaw connecting snake-like robot, we regulate joint actuators by periodic rhythms. At last, we construct a chain type of CPG system by using unidirectional coupling delay, where the delay is to regulate phase difference between the adjacent DHCO modules. The snake-like robot can propose the serpentine, rectilinear, and side-winding movement gaits in the forward and backward directions, respectively.

Generally, based on the above-mentioned CPG controlling strategy to control the snake-like robotic locomotion, there are some following advantages. Firstly, the VDP oscillator used in the paper presents well-defined corresponding parameters to regulate its frequency, amplitude, and phase difference. Secondly, periodic rhythm activities of the DHCO module present synchronous and anti-synchronous spatiotemporal patterns. Time delay varied in different parameter regions can rapidly switch the spatiotemporal patterns without any intermediate states. The snake-like robot presents shorter stage of the locomotion transition process. Thirdly, there exists just one state of rhythm activity in the fixed delay regions. The time delay varied in the fixed region cannot destroy the spatiotemporal patterns. The CPG system has a good robustness to control the snake-like robotic locomotion. Further, to regulate phase difference of the DHCO modules and get a suitable traveling wave, a unidirectional delay is introduced in the chain type of CPG system. So, all activity joints can be regulated by the unidirectional delay, which reduces many redundant controlling parameters. In a word, the proposed CPG neural system based on the delayed VDP oscillator can be used to control snake-like robot with fewer controlling parameters and richer locomotion patterns. A group of parameter values is chosen for an expected motion based on the theoretical dynamical analysis. In addition, there is a very interesting phenomenon in the delayed CPG neural system. The unidirectional delay can multiply switch the spatiotemporal patterns of periodic rhythms from synchrony to anti-synchrony and then return to synchrony. Snake-like robots controlled by the CPG system will multiple switch locomotion in the forward and backward direction when the unidirectional delay increases.

The paper is organized as follows. In Sect. 2, an improved VDP oscillator is proposed to regulate frequency and amplitude of the periodic rhythm. In Sect. 3, the DHCO module is designed to present controlling signals by using a pair of VDP oscillator with mutually coupling delay. The unit actuators in the pitch-yaw connecting snake-like robot can be controlled by the DHCO module system. In Sect. 4, employing the DHCO modules, we construct a chain type of CPG neural system with a unidirectional delay. Numerical simulations show that the CPG system can control the snake-like robotic locomotion with the serpentine, rectilinear, and side-winding patterns in the forward and backward directions, respectively. Finally, Sect. 5 gives some conclusions.

2 An improved VDP oscillator model

Mathematic model of the VDP oscillator is a second-order differential equation described by:

$$\ddot{x} + \varepsilon (x^2 - 1)\dot{x} + \beta x = 0,$$

where \(x\) denotes time-dependent position of the oscillator, and \(\varepsilon\) is a real number of nonlinear damping ratio. The parameter \(\varepsilon > 0\) is introduced to determine oscillation amplitude, and \(\beta > 0\) is to natural frequency of the oscillation.

As is known to all, system (1) has a trivial equilibrium \((0, 0)\). The characteristic equation
corresponding to the trivial equilibrium is 
\( \lambda(\lambda - \varepsilon x) + \beta = 0 \). We obtain the linearized system’s
eigenvalues \( \lambda_{1,2} = (\varepsilon x \pm \sqrt{\varepsilon^2 x^2 - 4\beta})/2 \). It implies
the dynamic behavior of system (1) is stable when \( x < 0 \) since the eigenvalues have Re(\( \lambda_{1,2} \)) < 0, while
unstable when \( x > 0 \). The time history and phase
diagram are shown in Fig. 1. It follows from Fig. 1a, b
that the trajectory of system (1) is closed to the trivial
equilibrium for \( \varepsilon = -0.1 \). However, the trivial
equilibrium is unstable for \( \varepsilon = 0.1 \). The system
exhibits a stable periodic orbit surrounding the unsta-
bilal trivial equilibrium, as shown in Fig. 1c, d. It is a
self-excited oscillation.

The parameters \( x > 0 \) and \( \beta > 0 \) are introduced to
regulate the frequency and amplitude of the periodic
rhythm in system (1). In fact, the snake-like robot
controlled by the CPG neural system can move with
different velocity and patterns by adjusting the
frequency and amplitude. The improved VDP oscilla-
tor can regulate the frequency of rhythm by adjusting
\( x \) and amplitude by \( \beta \) independently, as shown in
Fig. 2 for \( \varepsilon = 0.1 \). It follows that we can freely change
the frequency and amplitude of periodic rhythm
activity in the improved VDP oscillator.

3 Delayed half-center oscillator

In this section, we propose a delayed half-center
oscillator (DHCO) module employing the above-
mentioned VDP oscillator, where two VDP oscillators
connect each other by mutually coupled delays. Different periodic rhythms with synchronous and
anti-synchronous spatiotemporal patterns are exhib-
ted using theory analysis and numerical simulation.
The DHCO model is given by the following differen-
tial equation.

\[
\begin{align*}
\frac{dx}{dt} + \varepsilon(x(t) - x) + \beta x_1(t) &= k(x_2(t - \tau) - x(t)), \\
\frac{dx_1}{dt} + \varepsilon(x_1(t) - x) + \beta x_2(t) &= k(x_1(t - \tau) - x_2(t)),
\end{align*}
\]

where \( k \) is a coupling strength, and \( \tau > 0 \) is time delay.
It follows that the dynamic behavior of system (2) is
determined by time delay \( \tau \) and coupling strength \( k \).
Here, we present periodic rhythms and their regions of
system’s parameters by using the Hopf bifurcation.
The time delay can induce periodic rhythm having
synchrony and anti-synchrony patterns. To this end,
we rewrite system (2) by transformation \( x_1 = u_1, \ x_2 = u_2 \) and \( x_3 = u_3, \ x_4 = u_4 \), which is

![Fig. 1 Time histories and phase diagrams of the VDP oscillator for a, b \( \varepsilon = -0.1 \), and c, d \( \varepsilon = 0.1 \), where \( x = 1 \) and \( \beta = 1 \) in system (1)](image_url)
Fig. 2 Numerical simulations show a amplitude increasing with \( \varepsilon \) for the fixed \( \beta = 1 \) and b frequency increasing with \( \beta \) for \( \varepsilon = 1 \), where the system parameter \( \varepsilon = 0.1 \) in system (1).

\[
\begin{align*}
\dot{u}_1(t) &= u_2(t), \\
\dot{u}_2(t) &= -\beta u_1(t) - \varepsilon (u_1(t) - x) \cdot u_2(t) + k(u_1(t - \tau) - u_2(t)), \\
\dot{u}_3(t) &= u_4(t), \\
\dot{u}_4(t) &= -\beta u_3(t) - \varepsilon (u_3(t) - x) \cdot u_4(t) + k(u_2(t - \tau) - u_4(t)).
\end{align*}
\]

System (3) has a trivial equilibrium \((u_1, u_2, u_3, u_4) = (0, 0, 0, 0)\). The linearized system of (3) at the trivial equilibrium leads to

\[
\begin{align*}
\dot{u}_1(t) &= u_2(t), \\
\dot{u}_2(t) &= -\beta u_1(t) + (\varepsilon - k)u_2(t) + ku_4(t - \tau), \\
\dot{u}_3(t) &= u_4(t), \\
\dot{u}_4(t) &= -\beta u_3(t) + (\varepsilon - k)u_4(t) + ku_2(t - \tau).
\end{align*}
\]

The characteristic equation of system (4) is given by

\[
G(\lambda, \tau) = \lambda^4 + 2(k - \varepsilon x)\lambda^3 + (k^2 + 2\beta - 2k\varepsilon x + \varepsilon^2x^2 - \varepsilon^22\lambda^2)\lambda + 2\beta(k - \varepsilon x)\lambda + \beta^2 = 0.
\]

Let \( \tau = 0 \) in Eq. (5), we have the following four roots, which is

\[
\lambda_{1,2} = \frac{\varepsilon x \pm \sqrt{\varepsilon^2x^2 - 4\beta}}{2},
\]

\[
\lambda_{3,4} = \frac{\varepsilon x - 2k \pm \sqrt{(2k - \varepsilon x)^2 - 4\beta}}{2}.
\]

When \( \varepsilon > 0 \), the trivial equilibrium is always unstable for the case \( \tau = 0 \). For \( \tau > 0 \), we assume characteristic Eq. (5) has a pure imaginary root. Letting \( \lambda = \imath \omega \) (\( \omega > 0 \)) and separating the real and imaginary parts, one has

\[
\begin{align*}
\omega^4 - (2\beta + (k - \varepsilon x)^2)\omega^2 + \beta^2 + k^2\omega^2 \cos 2\tau\omega &= 0, \\
2(k - \varepsilon x)(\beta - \omega^2)\omega \cos 2\tau\omega &= 0.
\end{align*}
\]

(7)

It follows that \( \omega \) is satisfied with

\[
H(\omega) = \omega^6 + 2(k - \varepsilon x)^2 - 2\beta \omega^6 + 2\beta^2(k - \varepsilon x)^2 - 2\beta \omega^4 + (4\beta^2 - 4k^2\beta - 4k\varepsilon(k^2 - 2\beta)^2) + 2(3k^2 - 2\beta)\varepsilon^2x^2 - 4k\varepsilon^3x^3 + \varepsilon^4x^4)\omega^4 = 0.
\]

(8)

Equation (8) at most has four positive roots \( \omega_i \) \((i = 1, \ldots, 4)\). It follows from Eq. (7) we obtain the following critical delays

\[
\tau_j^i = (\phi_i + 2j\pi)/2\omega_i, \quad i = 1, \ldots, 4;
\]

\[
j = 0, 1, 2, \ldots
\]

where \( \phi_i \in (0, 2\pi] \) and is satisfied with

\[
\begin{align*}
\omega_i^4 - (2\beta + (k - \varepsilon x)^2)\omega_i^2 + \beta^2 + k^2\omega_i^2 \cos 2\tau\omega_i &= 0, \\
2(k - \varepsilon x)(\beta - \omega_i^2)\omega_i - k^2\omega_i^2 \sin 2\tau\omega_i &= 0.
\end{align*}
\]

(10)

To determine the transversality condition of the Hopf bifurcation, we differentiate Eq. (5) with \( \tau \), which yields

\[
\frac{d\lambda}{d\tau} = \frac{k^2\lambda^2 e^{-2\lambda\tau}}{(k - \varepsilon x + 2\lambda)(\beta + \lambda(k - \varepsilon x + \lambda)) + k^2\lambda(\lambda\tau - 1)}.
\]

(11)

Employing the Hopf bifurcation theory, we can draw the following conclusions. When Eq. (8) presents no positive root, the dynamic behavior of system (3) near the trivial equilibrium is locally stable. When
Eq. (8) presents one positive root, there exists a critical delay $\tau_c$ satisfied with Eq. (9). The trivial equilibrium is locally stable for $\tau \in [0, \tau_c)$. System (3) exhibits a Hopf bifurcation $\tau = \tau_c$ with transversality condition $\text{Re}(\lambda) = 0$ when $\tau = \tau_c$. A stable periodic orbit will be presented when time delay increases and crosses through $\tau_c$. It implies that the DHCO module exhibits a periodic rhythm. Further, when Eq. (8) has at least two positive roots, there are a series of critical delay $\tau_j^i$, $i = 1, 2; j = 0, 1, 2, \ldots$ satisfied with Eq. (9), which divides the parameter plane into a series of islands of amplitude death (AD). In AD islands, the system has a stable trivial equilibrium, as shown in Fig. 3. Moreover, the DHCO system will exhibit periodic rhythm activities with different spatiotemporal patterns when time delay varies and switches from these AD islands.

In fact, the spatiotemporal patterns of periodic rhythm produced by time delay in system (3) can be analyzed through the equivariant Hopf bifurcation. However, the method is very intractable and complicated because of the normal form and center manifold in the Banach function space. So, for the convenience of reader’s understanding, we just exhibit some numerical simulations to exhibit the spatiotemporal patterns. To this end, we firstly give the critical values of time delay defined by Eq. (9) when $H(\omega) = 0$ has two positive roots. The critical delays will determine a series of parameter regions. Then, for these regions, we illustrate time histories of the DHCO system to show the spatiotemporal patterns of periodic rhythms. It should be noticed that there is just one type of spatiotemporal pattern (synchronous or anti-synchronous) for the chosen delayed regions. The initial functions are fixed as $u_1(t) = 0.1$, $u_2(t) = 0.2$, $u_3(t) = 0.3$, $u_4(t) = 0.4$ for $t \in (-\tau, 0]$. Taking the system parameters as $\alpha = 1$, $\beta = 1$, $\varepsilon = 0.03$ and $k = 0.5$, we obtain two oscillating frequencies such as $\omega_1 = 0.843343$ and $\omega_2 = 1.18576$ employing $H(\omega) = 0$ in Eq. (8). It follows from Eq. (9) we have the critical values of time delay $\tau_j^i$, $i = 1, 2; j = 0, 1, 2, \ldots$, that is

$$
\tau_0^1 = 0.3791, \tau_1^1 = 3.8001, \tau_2^1 = 7.2210,
\tau_3^1 = 10.6420, \ldots,
$$

and

$$
\tau_0^2 = 2.5653, \tau_1^2 = 5.4503, \tau_2^2 = 8.3354,
\tau_3^2 = 11.2204, \ldots.
$$

We rearrange the critical delays as

$$
\tau_0^0 < \tau_0^1 < \tau_1^1 < \tau_2^1 < \tau_1^2 < \tau_2^2 < \tau_3^2 < \ldots.
$$

The periodic rhythms and their spatiotemporal patterns are shown in Fig. 4. It follows the DHCO system presents a stable synchronous rhythm activity when time delay is less than $\tau_0^0$, as shown in Fig. 4a for $\tau = 0.2$. If time delay $\tau$ passes through $\tau_0^1$ and enters into the first AD island, system (3) presents a stable trivial equilibrium by the reverse Hopf bifurcation. Further, the stable trivial equilibrium loses its stability and enters into a periodic rhythm employing the Hopf bifurcation when time delay crosses through the critical delay $\tau_1^1$. At this time, the rhythm activity exhibits an anti-synchronous pattern, as shown in Fig. 4b for $\tau = 3.0$. Moreover, when $\tau$ crosses through the critical delay $\tau_1^1$ and enters into the second AD island, the anti-synchronous rhythm activity will evolve into the stable trivial equilibrium. Then the trivial equilibrium loses its stability and evolves into the synchronous rhythm activity as time delay $\tau$ is being away from the second AD island and passes through the critical delay $\tau_1^2$, as shown in Fig. 4c for $\tau = 6.0$. Following this way, the periodic rhythm activity evolves into anti-synchronous pattern from the synchronous state undergoing the stable trivial equilibrium, as shown in Fig. 4d for $\tau = 10$. In a word, the DHCO model presents the periodic rhythms with synchronous and anti-synchronous patterns when time delay increases to cross through the series of critical delays employing the reverse and forward Hopf bifurcation, respectively. Figure 5 exhibits the

![Fig. 3](image-url)

*Fig. 3* Time history shows the DHCO model of system (3) presents a stable trivial equilibrium, where $\alpha = 1$, $\beta = 1$, $\varepsilon = 0.03$, $k = 0.5$ and $\tau = 2$ located in the AD island.
parameter regions of AD islands and rhythm activities with synchronous and anti-synchronous patterns.

The DHCO model is a unit and motif of the CPG neural system. To control the snake-like robotic locomotion with different patterns, we use the above-mentioned DHCO model to construct a CPG neural system. The controlling relation of the DHCO and the robotic module is shown in Fig. 6a. The snake-like robot is designed as a symmetrical structure with pitch-yaw connecting modules rotating around the pitch and yaw axis, respectively. In fact, the pitch-yaw configuration presents much more kinds of locomotion patterns, such as serpentine, rectilinear and side-winding locomotion compared with the pitch-connecting or yaw-connecting configuration. Further, based on the biological mechanism of excitatory and inhibitory, the output signals of the DHCO are designed as $P_{out} = \frac{x_1 + x_2}{2}$ and $Y_{out} = \frac{x_1 - x_2}{2}$, where $P_{out}$ and $Y_{out}$ are the controlling signals to drive the joint actuators rotating around the pitch and yaw axis. The periodic rhythms of the VDP oscillators, i.e., $x_1$ and $x_2$, present synchronous and anti-synchronous patterns in different parameter regions.

The output signals of the DHCO model with different delayed values are shown in Fig. 6b, c. When time delay is chosen as $\tau \in (0, \tau_0^1) \cup (\tau_1^1, \tau_1^2) \cup \cdots$, the periodic rhythms of the VDP oscillators in the DHCO model present synchronous states, that is $x_1 = x_2$. The output signal of the DHCO just has the periodic rhythm.
Pout_1 = x_1 = x_2. The Yout_1 is just a quiescent state, as shown in Fig. 6b for \( \tau = 0.2 \) belonging \((0, \tau_0^1)\). At this time, the snake-like robotic modules are in silent state around the yaw axis and rotate around the pitch axis, which induces rectilinear locomotion pattern by phase difference of the adjacent DHCO models. Furthermore, when time delay is fixed in \( \tau \in (\tau_2^0, \tau_1^1) \cup (\tau_2^1, \tau_1^2) \cup \cdots \), the VDP oscillator exhibits the anti-synchronous rhythms. The output of the DHCO just has periodic rhythm activity Yout_1. The Pout_1 signal is a resting state, as shown in Fig. 6c for \( \tau = 3 \) belonging \((\tau_2^0, \tau_1^1)\). The snake-like robotic module rotates around the yaw axis and achieves serpentine locomotion pattern. In the next section, we will explain in detail the different locomotion patterns. By using the DHCO module, the CPG neural system is constructed to control the pitch-yaw connecting snake-like robot. It is the periodic rhythm pattern for the combined actuators rotating around the pitch and yaw axis, respectively.

### 4 CPG-based Locomotion of the Snake-like Robot

In this section, based on the above-mentioned DHCO modules, we propose a CPG neural system to control snake-like robot with many types of locomotion patterns including serpentine, rectilinear and side-winding locomotion in the forward and backward directions, respectively. The schematic diagram of the snake-like robot is designed in Fig. 7, where the pitch-yaw connecting modules rotate \( \theta_i \) \((i = 1, \ldots, 2n)\) around pitch and yaw axis. Considering the configuration of the snake-like robot, we choose equal-length distance between pitch and yaw modules to avoid the effect of the movement stability. The DHCO modules are connected in series from the head to the tail by excitatory synapse connections with time delay \( \mu \). The new delay \( \mu \) is introduced to regulate phase difference of the adjacent DHCO modules. The chained-type CPG neural system and their control principle are illustrated in Fig. 7. The control parameters can be used to steer the movement pattern and their speed of the snake-like robot. The CPG neural system has \( n \) DHCOs. Each DHCO consists of two VDP oscillators corresponding to the combined joints with pitch-yaw connecting configuration. The corresponding
The mathematical model is determined by the following delayed differential equation, that is

\[
\begin{align*}
\dot{x}_1 + \varepsilon(x_1^2 - \alpha_1) \cdot \dot{x}_1 + \beta_1 x_1 &= k_1(\dot{x}_2(t - \tau) - \dot{x}_1), \\
\dot{x}_2 + \varepsilon(x_2^2 - \alpha_2) \cdot \dot{x}_2 + \beta_2 x_2 &= k_2(\dot{x}_1(t - \tau) - \dot{x}_2), \\
\dot{x}_3 + \varepsilon(x_3^2 - \alpha_3) \cdot \dot{x}_3 + \beta_3 x_3 &= k_3(\dot{x}_4(t - \tau) - \dot{x}_3) + k_1(\dot{x}_1(t - \mu)), \\
\dot{x}_4 + \varepsilon(x_4^2 - \alpha_4) \cdot \dot{x}_4 + \beta_4 x_4 &= k_4(\dot{x}_3(t - \tau) - \dot{x}_4) + k_2(\dot{x}_2(t - \mu)), \\
&\vdots \\
\dot{x}_{2n-1} + \varepsilon(x_{2n-1}^2 - \alpha_{2n-1}) \cdot \dot{x}_{2n-1} + \beta_{2n-1} x_{2n-1} &= k_{2n-1}(\dot{x}_{2n}(t - \tau) - \dot{x}_{2n-1}) + k_{2n-3}(\dot{x}_{2n-3}(t - \mu)), \\
\dot{x}_{2n} + \varepsilon(x_{2n}^2 - \alpha_{2n}) \cdot \dot{x}_{2n} + \beta_{2n} x_{2n} &= k_{2n}(\dot{x}_{2n-1}(t - \tau) - \dot{x}_{2n}) + k_{2n-2}(\dot{x}_{2n-2}(t - \mu)).
\end{align*}
\]

where \(x_i\) is the output of the VDP oscillator \(i, (i = 1, 2, \ldots, 2n)\), \(\alpha_i > 0\) is to control the amplitude of the periodic rhythm, \(\beta_i > 0\) is to determine the natural frequency, and \(k_i\) is coupling weight. Time delay \(\tau > 0\) is to control spatiotemporal patterns in the DHCO module, and \(\mu > 0\) is to the phase difference of the adjacent DHCO modules.

Numerical simulations are illustrated in Fig. 8 to show their phase difference. The output signal of the VDP oscillator \(x_i\) \((i = 1, \ldots, 5)\) is regulated by time delay \(\mu\), where the DHCO number is \(n = 5\) in the CPG neural system. The parameters of the VDP oscillator are fixed as \(\varepsilon = 0.03\), \(\alpha = 1\), \(\beta = 1\), \(k = 0.1\) and \(\tau = 0.1\). It follows that the VDP \(i\) oscillator \((x_1\) in black) has a preemptive phase when \(\mu = 5.5\). The output signals of the CPG system have a phase lag, which achieves backward locomotion of the snake-like robot. It follows from Fig. 8 that the phase difference of the DHCO module decreases with time delay increasing. When time delay increases to \(\mu = 6.2\), all signals of the DHCO modules almost share a same phase. The phase difference is almost zero. However, the phase difference switches from phase lag to phase lead when time delay changes to \(\mu = 7\). At this time, the first oscillator (VDP 1) has a lagging phase, and the end one, i.e., VDP 9, has a
preemptive phase. The output signals of the CPG system realize forward locomotion. It implies that the new time delay $\mu$ regulates the snake-like robot locomotion in forward and backward direction. Further, the internal parameters of the DHCO module, such as $\alpha_i$ and $\beta_i$, adjust the amplitude and frequency of periodic rhythm. The time delay $\tau$ determines the spatiotemporal pattern of the periodic rhythms. In a word, by adopting suitable parameters of the CPG neural system, we can obtain different locomotion patterns including serpentine, rectilinear and side-winding patterns in the forward and backward directions. The corresponding simulation experiments will be illustrated to demonstrate the snake-like robotic locomotion.

4.1 Serpentine locomotion

Serpentine, also called as lateral undulation locomotion, is a most common and realest pattern of the snake movement, which is described as a sinusoidal waveform from the top view of the body. In this pattern, the joint actuators combined in the horizontal direction will rotate around the yaw axis. The vertical units will be to remain stationary. The pitch-yaw snake-like robot moves like a real snake propelled by the lateral wave thrusting from the tail to the head. To achieve the serpentine locomotion of snake-like robot, the output signals of the CPG neural system should be satisfied with $\text{Pout}_i = 0$. The periodic rhythms $\text{Yout}_i$, loaded the horizontal units form backward or forward wave propulsion. The snake-like robot moves along a given curve path and maintains its longitudinal axis with the same orientation.

Fig. 8  Translation of phase difference between the adjacent DHCO modules with time delay $\mu$ increasing, where $\varepsilon = 0.03$, $\alpha = 1$, $\beta = 1$, $k = 0.1$ and $\tau = 0.1$

Fig. 9 The output signals of the CPG neural system with $\tau = 3$ and $\mu = 7$ generated the forward serpentine locomotion
The CPG output signals are shown in Fig. 9 for the fixed time delays $\tau = 3$ and $\mu = 7$. It follows that the CPG outputs $P_{outi} = 0$. The DHCO systems are chosen as $a = 1$, $b = 1$, $e = 0.03$ and $k = 0.5$. The joint actuators combined in vertical direction are all in silent states. The rhythm activities $Y_{outi}$ configure a backward moving wave with the identical frequency and increasing amplitude. The pitch-yaw connecting snake-like robot presents a forward serpentine locomotion. Fixed time delay as $\tau = 3$ and decreased the delay to $\mu = 5.5$, the CPG outputs are illustrated in Fig. 10, where $P_{outi} = 0$. The periodic rhythms $Y_{outi}$ generate a forward-moving propulsive wave. It follows that the first HCO unit of the CPG neural system has a lagging phase, while the last one presents an early phase. The snake-like robot will adopt the backward serpentine movement. The MATLAB-ADAMS co-simulation is used to describe the detailed patterns of the serpentine locomotion, as shown in Fig. 11, where the swing angles $\theta_i$ of the joints are represented as a linear mapping function of the CPG outputs signals, i.e., $\theta_i = m_i$. $Y_{outi} \ (i = 1, 3, \ldots, 2n-1)$
and $\theta_i = n_i$. $P_{out_i}$ ($i = 2, 4, \ldots, 2n$). It follows that the snake-like robot controlled by the CPG neural system performs the serpentine locomotion.

4.2 Rectilinear locomotion

Rectilinear, also called as straight-line locomotion, is a special type of movement pattern, which is presented by a sinusoidal waveform from the side view of the snake body. From the top view, the whole body of the snake is to maintain a straight-line state when it moves. For the pitch-yaw connecting snake-like robot, the locomotion depends on the vertical actuators rotating around the pitch axis. The rhythm activities $P_{out_i}$ of the CPG neural system form backward or forward wave propulsion. The horizontal actuators remain stationary, that is $Y_{out_i} = 0$. The snake-like robot moves like a worm along with a straight-line path propelled by the touching friction between the snake body and ground.

To obtain the rectilinear locomotion, we choose time delays as $\tau = 0.2$ and $\mu = 7$ for the fixed parameters $\alpha = 1$, $\beta = 1$, $\epsilon = 0.03$ and $k = 0.5$. It follows that the VDP oscillators in the DHCO module present a synchronous pattern for $\tau = 0.2$. The output signals of the CPG neural system propose $Y_{out_i} = 0$, as shown in Figs. 12 and 13. When time delay is fixed as $\mu = 7$, the periodic rhythms $P_{out_i}$ configure a backward moving wave with the identical frequency and increasing amplitude. The head actuators illustrate small amplitude and early phase. At this time, the forward rectilinear locomotion is accompanied by means of the backward-moving wave from head to tail. On the other hand, to generate a backward rectilinear locomotion, the CPG units in the vertical direction should produce a forward-moving propulsive wave. The horizontal units should be silent. Thanks to freely adjust the phase difference of the DHCO modules by time delay $\mu$, we can illustrate the backward control signals, as shown in Fig. 13 for $\mu = 5.5$. It follows that the output signals of CPG illustrate a reversed phase difference. The first unit at the head has a lagging phase, while the last one at the tail presents an early phase. The snake-like robot moves by adopting the backward rectilinear locomotion. To make it easier to understand for the reader, we present the ADAMS simulation as shown in Fig. 14, where the snake-like robot exhibits the rectilinear locomotion under the control of the CPG neural system.

4.3 Side-winding locomotion

Side-winding locomotion is characterized by two sinusoidal waveforms from the top and the side view of the snake body, which is a typical 2D movement. It has a more high efficiency for the snake-like robot. In the pitch-yaw connecting snake-like robot, the joint actuators combined in the horizontal and vertical directions rotate around the pitch and yaw axis, respectively. The snake-like robot can move in the side-winding locomotion by using the combination of the horizontal waves and vertical waves. To simulate the side-winding locomotion, we choose the different parameter values of the VDP oscillators in DHCO system. In fact, $\alpha_i > 0$ is to control the amplitude of the

![Fig. 12](image-url) The output signals of the CPG neural system with $\tau = 0.2$ and $\mu = 7$ generated the forward rectilinear locomotion.
periodic rhythm in the DHCO modules. So in this section, we fix $\alpha_i = 1$ for the odd number of the VDP oscillators and $\alpha_i = 4$ for the even ones. The output signals of the CPG neural system are shown in Fig. 15 for time delays $\tau = 0.2$ and $\mu = 7$. It follows that the periodic rhythm activities $P_{out_i}$ and $Y_{out_i}$ configure two different sinusoidal waveforms, which drives the horizontal and vertical actuators rotating around the pitch and yaw axis, respectively. The ADAMS simulation is shown in Fig. 16. The snake-like robot moves with the side-winding locomotion pattern.

4.4 Turning maneuver

Turning maneuver is a necessary ability to change movement direction for the snake-like robot. To obtain turning maneuver, we should construct the asymmetrical undulation outputs of the CPG neural system. Replacing $x_i$ with $x_i - b_i, (i = 1, \cdots, 2n)$ in Eq. (15), we rewrite the CPG dynamical system having the bias parameters $b_i$, which is added to modulate oscillating center. Whether in forward and backward movement, we can adjust the bias parameter to turn locomotion direction, as shown in Fig. 17. It follows that the output signals regulate its oscillating center from
Fig. 15 The output signals of the CPG neural system generate the side-winding locomotion pattern with the different VDP oscillators $a_i = 1 (i = 1, 3, 5, 7, 9)$ and $a_i = 4 (i = 2, 4, 6, 8, 10)$

Fig. 16 The screenshots of the ADAMS simulation for the side-winding locomotion

Fig. 17 The output signals of the CPG neural system to obtain turning maneuver by changing the bias from $b_i = 0$ before $t = 150$ to $b_i = 1$
$b_i = 0$ to $b_f = 1$ at $t = 150$. The backward locomotion changes its direction to achieve the turning maneuver.

In the end of this section, we present multiple switching of movement direction. In fact, with time delay $\mu$ increasing, the locomotion direction of snake-like robot can multiply switch from forward to backward and then back to forward, as shown in Fig. 18 for $\varepsilon = 0.1$, $\alpha = 1$, $\beta = 1$, $k = 0.1$ and $\tau = 1$. It follows that the CPG system generates a backward movement pattern when $\mu = 6$, where the head joint has small amplitude and early phase, as shown in Fig. 18a. The backward locomotion will switch its direction and present a forward movement pattern when $\mu = 7$. At this time, the head joint exhibits lagging phase, as shown in Fig. 18b. Further, the backward and forward movement will be presented in succession when time delay continuously increases to $\mu = 12.5$ and $\mu = 13.5$, as shown in Fig. 18c, d. The CPG neural system offers an effective and interesting method to realize the locomotion transitions of the snake-like robot.

5 Conclusions

In this paper, based on the DHCO modules, we presented a VDP-based CPG neural system to control a snake-like robot with pitch-yaw connecting configuration, which moves with different locomotion patterns by changing coupled delays. The improved VDP oscillator model was adopted to adjust the amplitude and frequency of periodic rhythms. Employing mutually coupled delay, we firstly constructed a DHCO module consisting with a pair of VDP oscillators. By analyzing the Hopf bifurcation of the DHCO system, we obtained different periodic rhythms with synchronous and anti-synchronous spatiotemporal patterns. The corresponding parameter regions were exhibited by Hopf bifurcation curves. Based on the DHCO modules, we constructed a CPG network system with a chain-type configuration to control a snake-like robot. The different locomotion patterns were obtained by adjusting coupling delays. The pitch-yaw connecting snake-like robot achieved the serpentine, rectilinear and side-winding patterns locomotion in the forward and backward directions. Simulation results show that the presented CPG neural
system can be used to control the snake-like robot with different locomotion patterns. The DHCO modules coupling with time delay is a useful method to achieve smooth transition when the speed or direction of the snake-like robot locomotion is changed.

As a matter of fact, in this paper, we just presented researching results of theoretical analysis and numerical simulations. The MATLAB-ADAMS co-simulation was performed to prove the validity and feasibility of locomotion control by using the proposed CPG neural system. Further work will focus on experiment researches of the snake-like robot. Since the output of the CPG system is dimensionless, it cannot be used directly as a joint control signal. A simple and common method is to give a mapping function to transform the output of the model from the dimensionless outputs to the swinging angle of the module joints. In the experiment, host computer will calculate the CPG model presented by Eq. (15) and obtain the corresponding locomotion gaits to control the snake-like robot. Meanwhile, actuator unit receives the instructions from the host computer and completes the whole movement action according to the CPG neural system. The module runs a PD controller to control the servo so that the joint can reach the desired position.

Conflict of interest  The authors declare that they have no conflict of interest.

Data availability  Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

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