N=1 and N=2 Supersymmetric Non-Abelian Born-Infeld Actions from Superspace

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Abstract

New non-abelian supersymmetric generalizations of the four-dimensional Born-Infeld action are constructed in N=1 and N=2 superspace, to all orders in $\alpha'$. The proposed actions are dictated by simple (manifestly supersymmetric and gauge-covariant) non-linear constraints.

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1 Introduction

The abelian Born-Infeld (BI) action is known to be the low-energy part of any (gauge-fixed, world-volume) D-brane effective action. The BI action is merely dependent upon the abelian vector field strength, being independent upon spacetime derivatives of the field strength by definition. The supersymmetreric BI actions with linearly realized N=1 or N=2 supersymmetry in four dimensions describe the low-energy (world-volume) dynamics of a single D3-brane propagating in four or six dimensions, respectively. The D3-brane action is supposed to have the Goldstone-Maxwell interpretation associated with partial (1/2) spontaneous supersymmetry breaking, with the Goldstone fields in a (Maxwell) vector supermultiplet with respect to unbroken (linearly realized) supersymmetry. Hence, a supersymmetric BI action should be the low-energy part of the corresponding Goldstone-Maxwell action. The unbroken N=1 and N=2 supersymmetry can be made manifest in superspace. The N=1 manifestly supersymmetric BI action was first formulated in ref. [2], while its Goldstone-Maxwell interpretation was later established in ref. [3]. The N=2 manifestly supersymmetric BI action was first formulated in ref. [4], whereas its relation to (yet to be fully determined) N=2 Goldstone-Maxwell action is briefly discussed in sect. 2, see also ref. [5] for more.

As was pointed out by Witten [6], there is a non-abelian gauge symmetry enhancement when \( N \) parallel D3-branes coincide. A supersymmetric abelian BI action is then supposed to be replaced by a Non-abelian Born-Infeld (NBI) action where the world-volume fields are valued in the Lie algebra of \( U(N) \). Both abelian and non-abelian BI actions are the effective actions, defined modulo local field redefinitions. Nevertheless, the bosonic abelian BI action (with tension \( T_3 \))

\[
S_{BI} = -T_3 \int d^4x \sqrt{-\det (\eta_{\mu\nu} + 2\pi \alpha' F_{\mu\nu})}
\]

is unambiguous, being only dependent of the abelian field strength \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) but not of spacetime derivatives of it (\( \partial F \)). In contrast, a bosonic NBI action is not well-defined, while there are two principal sources for ambiguities [1]. The first type of non-abelian ambiguities is related to the obvious fact that the terms dependent of the gauge-covariant derivatives of the non-abelian field strength cannot be unambiguously separated from the \( F \)-dependent commutators since \([D_\mu, D_\nu]F_{\lambda\rho} = [F_{\mu\nu}, F_{\lambda\rho}]\). Any concrete proposal for an NBI action has to specify an order of the \( F \)-matrices and, hence, it may effectively include some of the \( DF \)-dependent terms, even if they do

\[^3\text{See ref. [1] and references therein for a review.}\]
not explicitly appear in the action. Though the full (abelian or non-abelian) D3-brane effective action certainly includes the derivative-dependent terms, it does not make much sense to keep some of them while ignoring other possible terms. Perhaps, the best one can do with a bosonic NBI action is to define it for almost covariantly constant gauge fields with almost commuting field strengths, which does not seem to be very illuminating. The second (related) type of ambiguities is connected to the trace operation over the gauge group. For example, when using the abelian identity \( (2\pi\alpha' = b) \)

\[
- \det (\eta_{\mu\nu} + bF_{\mu\nu}) = 1 + \frac{b^2}{2} F^2 - \frac{b^4}{16} (F^2)^2 , \quad F^\mu{}^\nu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho} , \quad (2)
\]

one gets two natural candidates for the bosonic NBI action,

\[
S_{(a)} = - T_3 \int d^4x \sqrt{1 + \frac{b^2}{2} \text{tr}(F^2) - \frac{b^4}{16} \text{tr}(F^2)^2} \quad (3a)
\]

and

\[
S_{(b)} = - T_3 \int d^4x \text{Str} \sqrt{- \det(\eta_{\mu\nu} + bF_{\mu\nu})} , \quad (3b)
\]

where \( F_{\mu\nu} = F_{\mu\nu}^a t_a \), \( \{t_a\} \) are the hermitian generators of the gauge group, \([t_a, t_b] = if_{ab}^c t_c\), \( \text{tr}(t_a t_b) = \delta_{ab} \), and \( \text{Str} \) is the symmetrized trace,

\[
\text{Str} (t_{a_1} \cdots t_{a_k}) = \frac{1}{k!} \sum_{\text{permutations}} \text{tr} \left( t_{\pi(a_1)} \cdots t_{\pi(a_k)} \right) . \quad (4)
\]

The \( F \)-matrices effectively commute under the symmetrized trace, so that the formal definition (2) of the determinant still applies in eq. (3b). It is not difficult to verify that the equations of motion in the NBI theory (3b) on self-dual (Euclidean) configurations \((F^2 = \tilde{F}^\mu{}^\nu)\) coincide with the ordinary Yang-Mills equations, so that they have the same BPS solutions \[8\], though the existence of a BPS bound is not obvious in the non-abelian case. Away from self-dual configurations the action (3a) is much simpler than (3b), while it is also known to admit solitonic (glueball) solutions \[9\].

The gauge-invariant actions (3a) and (3b) are obviously different, so that further resolution requirements are needed. Some extra conditions are provided by string theory, because the BI action is well-known to represent the effective action of slowly varying gauge fields in open string theory. The most basic requirement of string theory is the overall single trace of the non-abelian gauge field strength products \[1\]. The overall symmetrized trace advocated by Tseytlin \[1\] is a stronger condition based on the observation that it reproduces the \( F^4 \)-terms in the non-abelian effective action of open superstrings in ten dimensions \[7\]. In this Letter I show that adding

\[\text{...}\]
supersymmetry unexpectedly gives rise to some more constraints on supersymmetric NBI actions in four dimensions, which are not apparent in the bosonic case.

At first sight, it seems to be straightforward to supersymmetrize any NBI action, so that supersymmetry would not add anything new towards its intrinsic definition. However, in fact, supersymmetry does tell us something more about the BI actions. For example, linearly realized supersymmetry apparently prefers the parametrization of the abelian BI actions in terms of the (anti)self-dual combinations, $F^\pm = \frac{1}{2}(F \pm \tilde{F})$, rather than in terms of the naively expected tensors $F$ and $\tilde{F}$. More importantly, it is the spontaneously broken (non-linearly realized) supersymmetry on top of the unbroken (linearly realized) supersymmetry that is responsible for the complicated non-linear structure of the D3-brane action to be considered as the Goldstone-Maxwell action. Hence, the non-linear structure of the supersymmetric abelian BI actions [1, 2, 4] should also be dictated by similar features. Though a Goldstone interpretation of the supersymmetric NBI actions is far from being obvious, if any, the well-established Goldstone form of the N=1 supersymmetric abelian BI action in superspace gives us the natural starting point for a construction of its generalizations, either non-abelian or with extended (linearly realized) supersymmetry.

The paper is organized as follows. In sect. 2 the N=1 and N=2 supersymmetric abelian BI actions are reviewed by emphasizing their relation to the Goldstone-Maxwell actions. In sect. 3 the non-abelian generalizations of the BI actions are proposed in N=1 and N=2 superspace. Sect. 4 is my conclusion.

2 N=1 and N=2 abelian BI actions

The abelian bosonic BI Lagrangian $L_{BI}(F)$ can be thought of as the unique non-linear generalization of the Maxwell Lagrangian, $-\frac{1}{4}F^2$, under the conditions of preservation of causality, positivity of energy, and electric-magnetic duality. In particular, the duality invariance of an abelian Lagrangian $L(F)$ amounts to the constraint [10]

$$G_{\mu\nu}\tilde{G}^{\mu\nu} + F_{\mu\nu}\tilde{F}^{\mu\nu} = 0 , \quad \text{where} \quad \tilde{G}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\lambda\rho}G_{\lambda\rho} = 2\frac{\partial L}{\partial F_{\mu\nu}} . \quad (5)$$

Supersymmetry is known to be consistent with all these physical properties, so that the supersymmetric abelian BI actions enjoy similar features.
The N=1 supersymmetric abelian BI action reads

\[ S_{1BI} = \frac{1}{2} \left( \int d^4 x d^2 \theta W^2 + \text{h.c.} \right) + \int d^4 x d^4 \theta Y(K, \bar{K}) W^2 \bar{W}^2, \]

where the structure function \( Y \) is given by

\[ Y(K, \bar{K}) = \frac{1}{1 - \frac{1}{2}(K + \bar{K}) + \sqrt{1 - (K + \bar{K}) + \frac{1}{4}(K - \bar{K})^2}}, \]

and

\[ K = \frac{1}{2} D^2 W^2, \quad D^2 = D^\alpha D_\alpha, \quad W^2 = W^\alpha W_\alpha, \quad \bar{W}^2 = \bar{W}_\alpha \bar{W}^\alpha, \]

in terms of the abelian N=1 chiral spinor superfield strength \( W_\alpha, \alpha = 1, 2 \), satisfying the off-shell superspace constraints (N=1 Bianchi identities)

\[ \bar{D}_\alpha W_\alpha = 0, \quad D^\alpha W_\alpha = \bar{D}_\alpha \bar{W}^\alpha. \]

The action (6) can be rewritten in the ‘non-linear sigma-model’ form

\[ S_{1BI} = \int d^4 x d^2 \theta \Phi + \text{h.c.}, \]

where the N=1 chiral Lagrangian \( \Phi \) is the perturbative solution to the non-linear superfield constraint

\[ \Phi = \frac{1}{2} \Phi D^2 \Phi + \frac{1}{2} W^2. \]

It is worth mentioning that the constraint (11) is Gaussian in \( \Phi \), while its perturbative solution is unambiguously constructed in superspace by iterations.

In fact, the simple constraint (11) is most useful in proving the invariance of the action \( S_{1BI} \) under the second (non-linearly realized or spontaneously broken) supersymmetry with the rigid anticommuting spinor parameter \( \eta^\alpha \)

\[ \delta_2 \Phi = \eta^\alpha W_\alpha, \quad \delta_2 W_\alpha = \eta_\alpha \left( 1 - \frac{1}{2} D^2 \Phi \right) + i \eta^\alpha \bar{D}_\alpha \Phi, \]

where the second equation follows from the first one after the use of eqs. (9) and (11). The constraint (11) generating the full action (6) is also quite useful in proving the electric-magnetic self-duality of \( S_{1BI} \). The duality invariance amounts to another non-local constraint

\[ \int d^4 x d^2 \theta (W^2 + M^2) = \int d^4 x d^2 \bar{\theta} (\bar{W}^2 + \bar{M}^2), \quad i \bar{M}_\alpha = \frac{\delta S_{1BI}}{\delta W^\alpha}, \]

which is the straightforward N=1 generalization of eq. (5). Thus, \( S_{1BI} = S_{GM: N=2/N=1} \).

\(^4\)The deformation parameter \( b \) is set to be one. The dependence upon \( b \) can be easily restored for dimensional reasons. I also ignore \( T_3 \) for simplicity.
Similarly, the N=2 supersymmetric Abelian BI action in N=2 superspace reads

\[ S_{2BI} = \frac{1}{2} \int d^4x d^4\theta \mathcal{W}^2 + \frac{1}{4} \int d^4x d^8\theta \mathcal{Y}(\mathcal{K}, \bar{\mathcal{K}})\mathcal{W}^2\bar{\mathcal{W}}^2 , \]  

(14)

with the same structure function

\[ \mathcal{Y}(\mathcal{K}, \bar{\mathcal{K}}) = \frac{1}{1 - \frac{1}{4}(\mathcal{K} + \bar{\mathcal{K}}) + \sqrt{1 - (\mathcal{K} + \bar{\mathcal{K}}) + \frac{1}{4}(\mathcal{K} - \bar{\mathcal{K}})^2}} , \]  

(15)

but

\[ \mathcal{K} = D^4\mathcal{W}^2 , \quad D^4 = \prod_{i,\alpha} D^i_\alpha = \frac{1}{12} D_{ij} D^{ij} , \quad D_{ij} = D^i_\alpha D^j_\beta = D^{ji} , \quad i, j = 1, 2 , \]  

(16)

in terms of the N=2 restricted chiral gauge superfield strength \( \mathcal{W} \) satisfying the off-shell constraints (N=2 Bianchi identities)

\[ \bar{D}^i_\alpha \mathcal{W} = 0 , \quad D^4\mathcal{W} = \Box \mathcal{W} . \]  

(17)

The action (14) can be rewritten (modulo \( \partial\mathcal{W} \)-dependent terms) in the ‘non-linear sigma-model’ form

\[ S_{2BI} = \frac{1}{4} \int d^4x d^4\theta \mathcal{X} + h.c . , \]  

(18)

whose N=2 chiral Lagrangian \( \mathcal{X} \) satisfies the non-linear N=2 superfield constraint

\[ \mathcal{X} = \frac{1}{4} \mathcal{X} \bar{D}^4 \mathcal{X} + \mathcal{W}^2 . \]  

(19)

Similarly to the N=1 abelian BI action, the non-linear constraint (19) gives us the convenient way of handling the complicated N=2 BI abelian action (14). For example, as was demonstrated in ref. [11], electric-magnetic self-duality of an N=2 action \( S(\mathcal{W}, \bar{\mathcal{W}}) \) amounts to the following N=2 supersymmetric extension of the N=1 non-local constraint (13):

\[ \int d^4x d^4\theta (\mathcal{W}^2 + \mathcal{M}^2) = \int d^4x d^4\theta (\bar{\mathcal{W}}^2 + \bar{\mathcal{M}}^2) , \quad \frac{i}{4} \mathcal{M} = \frac{\delta S}{\delta \mathcal{W}} , \]  

(20)

while it appears to be satisfied in the case of \( S_{2BI} \) defined by eqs. (18) and (19).

Unlike its N=1 BI counterpart, the N=2 BI action (14) or (18) does not give the full N=2 Goldstone-Maxwell action, but rather represents its low-energy part, i.e. \( S_{GM: N=4/N=2} = S_{2BI} + \mathcal{O}(\partial\mathcal{W}, \partial\bar{\mathcal{W}}) \). The infinitesimal parameters of the spontaneously broken (non-linearly realised) rigid symmetries can be naturally unified into a single (spacetime-independent) superfield \( \Lambda = \lambda + \theta^a_\alpha \lambda^i_\alpha + \theta_{ij} \lambda^{ij} \), where \( \lambda \) is the complex parameter of the Peccei-Quinn-type symmetry associated with two spontaneously broken translations (from the viewpoint of a D3-brane propagating in
six dimensions), $\lambda^i_\alpha$ are the spinor parameters of two spontaneously broken supersymmetries, whereas $\lambda^{ij}$ are the parameters of spontaneously broken R-symmetry $SU(2)$. The natural Ansatz for the transformation laws of the non-linearly realised symmetries is given by an N=2 analogue of eq.(12) as follows [5]:

$$\delta_2 \mathcal{X} = 2\Lambda \mathcal{W} , \quad \delta_2 \mathcal{W} = \Lambda \left( 1 - \frac{1}{4} \bar{D}^4 \mathcal{X} \right) + \ldots ,$$

(21)

where $\mathcal{X}$ is the perturbative solution to the non-linear constraint (19) by iterations, to all orders in $\mathcal{W}$ and $\bar{\mathcal{W}}$, while the dots stand for some $\partial \mathcal{W}$-dependent terms needed for consistency with the second Bianchi identity (17). A variation of the N=2 BI action (18) under the non-linear transformations (21) does not vanish, but it appears to be only dependent upon higher spacetime derivatives of $\mathcal{W}$ and $\bar{\mathcal{W}}$. This may not be surprising since the BI action and its supersymmetric extensions were defined modulo such terms. However, it also means that the N=2 BI action has to be modified, order by order in $\partial \mathcal{W}$ and $\partial \bar{\mathcal{W}}$, in order to get the full N=2 Goldstone-Maxwell action. A derivation of the derivative-dependent terms is beyond the scope of this Letter, and we do not need them for our main purpose formulated in the title.

A manifestly N=4 supersymmetric abelian BI action is not known (see, however, ref. [1] and references therein for some partial results).

### 3 N=1 and N=2 supersymmetric NBI actions

Having understood the fact that the simple non-linear constraints (11) and (19) fully determine the structure of the highly complicated abelian BI actions (6) and (14), respectively, it is natural to define the N=1 and N=2 supersymmetric NBI actions by non-abelian generalizations of eq. (11) and (19).

The non-abelian (Yang-Mills) N=1 chiral superfield strength is given by the well-known formula (see, e.g., ref. [13] for a review or an introduction) [6]

$$W_\alpha = \frac{1}{8} D^2 \left( e^{-2V} D_\alpha e^{2V} \right) , \quad D_\alpha W_\alpha = 0 ,$$

(22)

where the real scalar gauge superfield potential $V$ transforms under gauge transformations with the chiral parameter $\Lambda(x, \theta, \bar{\theta})$ in the standard way [13]:

$$e^{2V} \rightarrow e^{-2i\Lambda} e^{2V} e^{2i\Lambda} , \quad \bar{D}_\alpha \Lambda = 0 ,$$

(23)

---

5 All superfields are now Lie algebra-valued.
so that $W_\alpha$ and $\bar{W}_\alpha$ transform covariantly, \textit{viz.}

$$W_\alpha \rightarrow e^{-2i\Lambda} W_\alpha e^{2i\Lambda}, \quad \bar{W}_\alpha \rightarrow e^{-2i\bar{\Lambda}} \bar{W}_\alpha e^{2i\bar{\Lambda}}.$$ \hfill (24)

The non-abelian gauge-covariant generalization of eq. (11) is given by

$$\Phi = \frac{1}{2} \Phi \bar{D}^2 \left( e^{-2V} \Phi e^{2V} \right) + \frac{1}{2} W^2,$$ \hfill (25)

where $\Phi$ is the N=1 chiral superfield Lagrangian that transforms like $W_\alpha$ under the gauge transformations. The invariant action reads

$$S_{1\text{NBI}} = \int d^4x d^2\theta \, \text{tr} \Phi + \text{h.c.},$$ \hfill (26a)

or

$$S_{1\text{NBI}}^{(s)} = \int d^4x d^2\theta \, \text{Str} \Phi + \text{h.c.}$$ \hfill (26b)

The NBI actions (26a) and (26b) are supersymmetric and gauge-invariant, while they both have the single overall trace. The symmetrized trace in eq. (26b) is supposed to be applied to the gauge-covariant operators only, by definition.

It is instructive to take a look at the structure of the quartic ($F^4$) terms in the actions (26), which arise from the standard ‘adjoint chiral matter’ term,

$$\int d^4x d^4\theta \, (S) \text{tr} \phi e^{-2V} \bar{\phi} e^{2V}, \quad \phi = W^2.$$ \hfill (27)

It is straightforward to verify that taking the trace as in eq. (26a) results in the non-abelian generalization of the Euler-Heisenberg Lagrangian, in the bosonic sector of the component expansion of the action (26a),

$$\frac{1}{4} \left[ \text{tr}(F^2)^2 + \text{tr}(F \bar{F})^2 \right].$$ \hfill (28)

In contrast, taking the symmetrized trace, as in eq. (26b), exactly yields the $F^4$-terms appearing in the expansion of the bosonic NBI Lagrangian (3b) \cite{14}. Hence, if one insists on the choice (3b) of the bosonic NBI action, its supersymmetric extension in compact form is provided by eq. (26b) — cf. ref. \cite{14}. Supersymmetry alone does not provide a resolution between the two different actions (26a) and (26b), so that more physical input is apparently needed. One natural option is to restrict the gauge group $G$ to its (abelian) Cartan subgroup and then impose the condition of the generalized (non-abelian) self-duality for the resulting supersymmetric field theory of rank $G$ (abelian) gauge fields along the lines of ref. \cite{12}. As was argued in ref. \cite{12}, this may ultimately support the symmetrized trace. Still, in the absence of more physical reasons, the ordinary trace in eq. (26a) is much simpler, being dependent of
only two matrix building blocks, $W^2$ and $\hat{W}^2$ (or $F^2$ and $F\tilde{F}$), and their covariant derivatives (see below). The action (3a) does not seem to have a nice supersymmetric generalization.

It is possible to rewrite the action (26) into the manifestly gauge-invariant and N=1 supersymmetric form, by using the N=1 supersymmetric gauge-covariant derivatives in superspace, which satisfy the standard N=1 super-Yang-Mills constraints: 

\[ \{\nabla_\alpha, \nabla_\beta\} = \{\tilde{\nabla}_\alpha, \tilde{\nabla}_\beta\} = 0, \quad \{\nabla_\alpha, \tilde{\nabla}_\beta\} = -2i\nabla_{\alpha\beta}, \]
\[ [\nabla_\alpha, \nabla_{\beta\gamma}] = 2i\varepsilon_{\alpha\beta\gamma} \hat{W}_\gamma, \quad [\tilde{\nabla}_\alpha, \nabla_{\beta\gamma}] = 2i\varepsilon_{\alpha\beta\gamma} \hat{\tilde{W}}_\gamma, \]

(29)

where $\hat{W}_\alpha$ is the N=1 covariantly-chiral gauge superfield strength.

\[ \tilde{\nabla}_\alpha \hat{W}_\alpha = 0, \quad \tilde{\nabla}_\alpha \hat{W}^\alpha = \nabla^\alpha \hat{W}_\alpha. \]

(30)

Equation (26) then takes the form

\[ S_{1NBI} = \int d^4x d^2\theta \text{(S)} \text{tr} \hat{\Phi} + \text{h.c.} \]

(31)

where the N=1 covariantly-chiral Lagrangian $\hat{\Phi}$ is the perturbative (iterative) solution to the manifestly gauge-covariant and supersymmetric nonlinear superfield constraint

\[ \hat{\Phi} = \frac{1}{2} \hat{\Phi} \nabla^2 \hat{\Phi} + \frac{1}{2} \hat{W}^2. \]

(32)

It is not difficult to generalize eqs. (31) and (32) further to the case of N=2 supersymmetry, by doing a similar construction in N=2 superspace. The standard N=2 superspace constraints, defining the off-shell N=2 supersymmetric Yang-Mills theory, are given by

\[ \{\nabla_{\alpha i}, \nabla_{\beta j}\} = -2\varepsilon_{\alpha i j} \hat{W}_{\beta j}, \quad \{\nabla_{\alpha i}, \nabla_{\beta j}\} = -2\varepsilon_{\alpha j \beta i} \hat{\tilde{W}}_{\alpha i}, \]
\[ [\nabla_{\alpha i}, \tilde{\nabla}_{\beta j}] = i\varepsilon_{\alpha i j} \nabla_{\alpha i} \hat{W}_{\beta j}, \quad [\nabla_{\alpha i}, \nabla_{\beta j}] = i\varepsilon_{\alpha i j} \nabla_{\beta j} \hat{W}_{\alpha i}, \]
\[ \{\nabla^i_{\alpha i}, \nabla^j_{\beta j}\} = -2i\delta^i_j \nabla^j_{\alpha i}. \]

(33)

where the non-abelian N=2 gauge superfield strength $\hat{W}$ obeys the off-shell constraints (N=2 Bianchi identities)

\[ \tilde{\nabla}_{\alpha i} \hat{W}_\alpha = 0 \quad \text{and} \quad \nabla_{ij} \hat{W} = \nabla_{ij} \hat{\tilde{W}}. \]

(34)

I use the following book-keeping notation:

\[ \nabla_{\alpha i} = \nabla_{(i\alpha} \nabla_{j)} = \nabla_{ji}, \quad \nabla_{ij} = \nabla_{(i\alpha} \nabla_{j)} = \nabla_{ji}, \]
\[ \nabla_{\alpha\beta} = \nabla_{i(\alpha} \nabla^i_{\beta)} = \nabla_{\beta\alpha}, \quad \tilde{\nabla}_{\alpha i} = \tilde{\nabla}_{(i\alpha} \tilde{\nabla}_{j)} = \tilde{\nabla}_{ji}, \]
\[ \tilde{\nabla}_{\alpha i} = \tilde{\nabla}_{i(\alpha} \tilde{\nabla}_{j)} = \tilde{\nabla}_{ji}. \]

(35)
where all symmetrizations have unit weight. The non-abelian generalization of the $D^4$ operator, which converts the (covariantly) anti-chiral N=2 superfields into (covariantly) chiral N=2 superfields, is most easily (and unambiguously) identified in the $SL(4, \mathbb{C})$ notation of ref. [18], by combining fundamental $SL(2, \mathbb{C})$ and $SU(2)$ indices into a single (fundamental) $SL(4, \mathbb{C})$ index $a = (\bar{\alpha}, i) = 1, 2, 3, 4$. The $\bar{\nabla}$-algebra of eq. (33) in the $SL(4, \mathbb{C})$ notation takes the familiar Dirac-type-form

$$\{\bar{\nabla}_a, \bar{\nabla}_b\} = 2C_{ab}\hat{W}, \quad \bar{\nabla}_a\hat{W} = 0,$$

with the constant metric $C$, $C^2 = 1$ and $C^T = C$. The desired gauge-covariant operator is just given by the ‘$\gamma_5$-type’ top product

$$\bar{\nabla}^4 = \frac{1}{4!}e^{abcd}\bar{\nabla}_a\bar{\nabla}_b\bar{\nabla}_c\bar{\nabla}_d .$$

In the notation (35) it reads

$$\bar{\nabla}^4 = \frac{1}{24} \left( \bar{\nabla}_{ij}\bar{\nabla}^{ij} - \bar{\nabla}_a\bar{\nabla}^a \right) - \frac{2}{3}\hat{W}^2 .$$

The N=2 supersymmetric non-abelian Born-Infeld action is given by

$$S_{2NBI} = \frac{1}{4} \int d^4x d^4\theta \text{tr} \hat{X} + \text{h.c.} ,$$

whose N=2 covariantly chiral Lagrangian $\hat{X}$ is the perturbative (iterative) solution to the N=2 superfield constraint

$$\hat{X} = \frac{1}{4} \hat{X} \bar{\nabla}^4 \hat{X} + \hat{W}^2 .$$

### 4 Conclusion

The proposed N=1 and N=2 supersymmetric NBI actions in components contain only even powers of $F$, while they reduce to the known super-Born-Infeld actions in the abelian case. Both actions enjoy ‘auxiliary freedom’ by keeping the auxiliary fields $\vec{D}$ (in the Wess-Zumino gauge) away from propagation, with $\vec{D} = 0$ being a solution to their equations of motion.

Unlike the supersymmetric abelian BI actions (sect. 2), their supersymmetric non-abelian counterparts (sect. 3) are dependent of the gauge superfields not only via their gauge superfield strengths but also directly (via the gauge-covariant derivatives). This does not allow us to extend the notion of abelian electric-magnetic duality to the supersymmetric NBI actions. Similarly, it is unclear to us whether our supersymmetric non-abelian BI actions admit any Goldstone-Yang-Mills interpretation.
It would be also interesting to investigate the structure of BPS solutions to the new supersymmetric NBI actions and find a precise relation between these actions and the non-abelian Dirac-Born-Infeld actions describing clusters of D3-branes with ‘deformed’ (non-linear) supersymmetry. A connection to noncommutative geometry seems to exist along the lines of ref. [7] too.

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