ON THE ORIGIN OF THE SHORT RANGE NN REPULSION

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Abstract

We calculate S-wave singlet and triplet NN phase shifts stemming from the short-range flavor-spin hyperfine interaction between constituent quarks using the resonating group method approach. A strong short-range repulsion is found in both waves. A fair comparison is performed between the traditional picture, relying on the colour-magnetic interaction, and the present one, relying on the Goldstone boson exchange dynamics. It is shown that the latter one induces essentially stronger repulsion, which is a very welcome feature.

We also study a sensitivity of phase shifts and wave function to extension from the one-channel to three-channel resonating group method approximation.

One of the crucial questions of the low-energy QCD is which physics, inherent in QCD, is responsible for the low-energy properties of light and strange baryons and their interactions. At the very high momenta transfer (the ultraviolet regime of QCD) the nucleon is viewed as a system of the weakly interacting partons, which justifies here the use of the perturbative QCD tools. The low-energy properties, like masses or low-energy interactions, is much harder, if impossible, to understand in terms of the original QCD degrees of freedom.

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and one obviously needs effective theories. One can borrow a wisdom of the many fermion system physics, e.g. in condensed matter, which suggests that in such a situation the concept of the quasiparticles in Bogoliubov or Landau sense becomes very useful. The idea of quasiparticles is that in some circumstances one can approximately absorb complicated interactions between bare fermions, i.e. in our case current quarks, into static properties of quasiparticles, e.g. their masses, and what is left beyond that should be treated as residual interactions between quasiparticles. Such a concept should be helpful to understand the low-energy properties of light baryons where a typical momentum of quarks is below the chiral symmetry breaking scale, $\Lambda_\chi \sim 1$ GeV, which implies that the low-energy characteristics of baryons, such as masses, are formed by the nonperturbative QCD dynamics, which is responsible for the chiral symmetry breaking and confinement, but not by the perturbative QCD interactions which should be active at much higher scale, where the quasiparticles do not exist. At low momenta the scalar part of the nonperturbative gluonic interaction between current quarks, which triggers the chiral symmetry breaking (i.e. pairs the left quarks and right antiquarks and vice versa in the QCD vacuum), can be absorbed into the mass of quasiparticles - constituent quarks. At the same time this nonperturbative interaction, iterated in the $qq$ t-channel in baryons, leads to the poles which can be identified as Goldstone boson exchange between valence quarks in baryons [1]. This is a general feature and does not depend on details or nature of this nonperturbative interaction. If so, the adequate residual interactions between the constituent quarks in baryons at low momenta, $q < \Lambda_\chi$, should be effective confining interaction and the Goldstone boson exchange (GBE).

By now it is established that such a picture is very successful in light and strange baryon spectroscopy [2,3]. Similar conclusions have been obtained recently in the lattice studies of $N - \Delta$ splitting [4], in the large $N_c$ [5] and phenomenological [6] analyses of $L=1$ spectra. If such a physical picture is satisfactory, it should also explain baryon-baryon interaction. It is rather evident that at medium and large distances in the baryon-baryon system, where the Pauli principle at the constituent quark level does not play any role, it is fully compatible
with the wisdom of nuclear physics, where the $NN$ interaction is determined by the Yukawa tail of the pion exchange and the two-pion exchange ($\rho$- and $\sigma$- exchange interactions). The explanation that the short-range repulsion is due to the central spin-independent part of the $\omega$ exchange is not satisfactory, however, as in this case the $\omega N$ coupling constant should be increased by a factor 3 compared to its empirical value. One takes it for granted that the origin of the short-range $NN$ repulsion should be the same as the origin of the nucleon mass and its lowest excitations. If so, the Fermi nature of constituent quarks and specific interactions between them should be of crucial importance to understand the short range $NN$ repulsion.

Traditionally the repulsive core in the $NN$ system within the constituent quark picture was attributed to the colour-magnetic part of the one gluon exchange (OGE) interaction combined with quark interchanges between 3$Q$ clusters (for reviews and earlier references see [7,8]). However, as it follows from the previous discussion it is dubious to use a language of constituent quarks and at the same time of perturbative one gluon exchange. So the important question is whether one can understand the short-range $NN$ repulsion in terms of residual interactions like GBE.

The first simple analysis of possible effects for the S-wave $NN$ system from the short-range part of the pion-exchange interaction between quarks was based on the assumption that the 6$Q$ wave function in the nucleon overlap region has a flavor-spin symmetry $[33]_{FS}$, which is the only possible symmetry in the nonexcited $s^6$ configuration [13]. In that paper, as well as in the subsequent hybrid models [9–12], it was assumed that the pion-exchange produces only some insignificant part of the short-range $NN$ repulsion. However, when the GBE-like hyperfine interaction is made strong enough to produce the $\Delta - N$ splitting and describe the low-lying spectrum, the situation is different. The GBE-like interaction is more attractive within the 6$Q$ configuration with the symmetry $[51]_{FS}$ and thus the spatially excited configuration $s^4p^2[51]_{FS}$ is more favourable and becomes the lowest one [14,15]. The
energy of this configuration is, however, still much higher than the energy of two infinitely separated nucleons and that is why there appears a strong short-range repulsion in the $NN$ system. While this result demonstrates that the GBE-like hyperfine interaction could indeed explain the short-range repulsive core in the $NN$ system, it is still only suggestive (the phase shifts have not been calculated) as it is based on the adiabatic approximation and neglects a smooth transition to the distances with the well-clustered $6Q$ system. In the present work we go beyond the adiabatic approximation and construct our basis in such a way that it includes not only the lowest important $s^4p^2$ and $s^6$ configurations like in [14,15], but also the well clustered states at medium and long distances. We calculate both the $^3S_1$ and $^1S_0$ phase shifts and prove that the GBE-like flavor-spin hyperfine interaction does supply a very strong short-range repulsion in the $NN$ system. We compare this repulsion with the one induced by the colour-magnetic interaction within the traditional picture and find the former one to be much stronger. This is a very welcome feature as the models of the short-range $NN$ repulsion based on the OGE interaction [7–12] fail to describe phase shifts above the lab energy of about 300 MeV because of the lack of the strong enough short-range repulsion in those models.

The convenient basis for solving the Schrödinger equation which comprises both the short-range $6Q$ configurations, incorporates the $NN$ asymptotics as well as a smooth transition from the nucleon overlap region to the medium ranges is suggested by the resonating group method (RGM) approximation. The most simple one-channel ansatz for the six-quark two-nucleon wave function is

$$\psi = \hat{A}\{N(1, 2, 3)N(4, 5, 6)\chi(\vec{r})\}, \quad (1)$$

$$\hat{A} = \frac{1}{\sqrt{10}}(1 - 9\hat{P}_{36}),$$

$$\vec{r} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3} - \frac{\vec{r}_4 + \vec{r}_5 + \vec{r}_6}{3}.$$  

Here $N(1, 2, 3)$ is $s^3$ harmonic oscillator wave function of the nucleon with a standard $SU(6)_{FS}$ spin-isospin part, $\hat{A}$ is an antisymmetrizer at the quark level and the trial func-
tion $\chi(\vec{r})$ is obtained by solving the Schrödinger equation, for review see [7,8]. We remind, however, that at short range the trial function $\chi(\vec{r})$ by no means should be interpreted as a relative motion wave function: in the nucleon overlap region because of the antisymmetrizer the function (1) is intrinsically 6-body function and contains a lot of other “baryon-baryon components”, such as $\Delta\Delta$, $NN^*$, $N^*N^*$,... and hidden colour components [16].

The trial function (1) completely includes the symmetric short-range $s^6$ shell-model configuration provided that the harmonic oscillator parameter for the $s^3$ nucleon and for $s^6$ configuration coincide. This is because of the well-known identity

$$\hat{A}\{N(1, 2, 3)N(4, 5, 6)\phi_{0s}(\vec{r})\}_S = \sqrt{\frac{10}{9}}|s^6[6]_O[33]_{FS}>.$$  

Here and below $\phi_{Ns}(\vec{r})$ denotes the S-wave harmonic oscillator function with $N$ excitation quanta, $[f]_O$ and $[f]_{FS}$ are Young diagrams (patterns) describing the permutational orbital and flavour-spin symmetries in 6Q system, which are necessary to identify the given configuration in the shell-model basis. It is always assumed that the center-of-mass motion is removed from the shell-model wave function.

However, the ansatz (1) contains only a fixed superposition of different shell-model configurations from the $s^4p^2$ shell [14]:

$$\hat{A}\{N(1, 2, 3)N(4, 5, 6)\phi_{2s}(\vec{r})\}_S = \frac{3\sqrt{2}}{9}(|(\sqrt{5}\frac{5}{6}s^52s - \sqrt{1}\frac{1}{6}s^4p^2)[6]_O[33]_{FS} >$$

$$- \frac{4\sqrt{2}}{9}|s^4p^2[42]_O[33]_{FS} >$$

$$- \frac{4\sqrt{2}}{9}|s^4p^2[42]_O[51]_{FS} >.$$  

One can extend the ansatz (1) and include in addition two new channels, “the $\Delta\Delta$” and the “hidden colour channel CC” [20,21]:

$$\psi = \hat{A}\{N(1, 2, 3)N(4, 5, 6)\chi_{NN}(\vec{r})\}$$

$$+ \hat{A}\{\Delta(1, 2, 3)\Delta(4, 5, 6)\chi_{\Delta\Delta}(\vec{r})\}$$

$$+ \hat{A}\{C(1, 2, 3)C(4, 5, 6)\chi_{CC}(\vec{r})\},$$  

5
where $\Delta(1, 2, 3)$ is $s^3$ harmonic oscillator $SU(6)_{FS}$ wave function of the $\Delta$-resonance and the hidden-colour $CC$ channel includes the $C =$ colour-octet $s^3$ cluster. Here we followed a definition of the hidden-color channel $CC$ in ref. [21]. The hidden-color state is constructed so that it contains only the flavor-spin $[33]_{FS}$ symmetry. It must be noted that $C$ is the color octet but it does not have a definite spin and isospin. Note that all three channels in (4) are highly non-orthogonal because of the antisymmetrizer. This can be easily seen from the fact that identities similar to (2) can be written also for the $\Delta\Delta$ and $CC$ channels. This redundancy in the subspace $N = 0$ is only a technical one and can be easily avoided by diagonalizing the norm RGM matrix and removing all “forbidden states”. However, in the subspace with $N = 2$, these three channels become linearly independent since the following identities are also valid for the $\Delta\Delta$ and $CC$ channels:

$$\hat{A}\{\Delta(1, 2, 3)\Delta(4, 5, 6)\phi_{2s}(\vec{r})\}_{SI} =$$

$$-\frac{6\sqrt{10}}{45} |(\sqrt{\frac{5}{6}}s^5 2s - \sqrt{\frac{1}{6}}s^4 p^2)[6]_{O}[33]_{FS} > + \frac{8\sqrt{10}}{45} |s^4 p^2[42]_{O}[33]_{FS} > - \frac{2\sqrt{10}}{9} |s^4 p^2[42]_{O}[51]_{FS} > , \hspace{1cm} (5)$$

$$\hat{A}\{C(1, 2, 3)C(4, 5, 6)\phi_{2s}(\vec{r})\}_{SI} =$$

$$\frac{2\sqrt{10}}{5} |(\sqrt{\frac{5}{6}}s^5 2s - \sqrt{\frac{1}{6}}s^4 p^2)[6]_{O}[33]_{FS} > + \frac{2\sqrt{10}}{15} |s^4 p^2[42]_{O}[33]_{FS} > . \hspace{1cm} (6)$$

Because the trial functions $\chi_{NN}, \chi_{\Delta\Delta}$ and $\chi_{CC}$ are independent full Hilbert space trial functions (i.e. they completely include $\phi_{2s}$), then the compact shell-model configurations $|\sqrt{\frac{5}{6}}s^5 2s - \sqrt{\frac{1}{6}}s^4 p^2[6]_{O}[33]_{FS} >, |s^4 p^2[42]_{O}[33]_{FS} >$ and $|s^4 p^2[42]_{O}[51]_{FS} >$ are relaxed and participate as independent variational configurations when one applies the ansatz (4), in contrast to the ansatz (1). The other possible compact $6Q$ configurations from the $s^4 p^2$ shell, such as $[411]_{FS}, [321]_{FS}$ and $[2211]_{FS}$ are not taken into account, but they play only a very modest role when one applies the interaction (6) [14,15].
First we study the effect of the GBE-like flavor-spin short range interaction. This interaction can be parametrized as

\[ V_\chi = -\sum_{i<j} \frac{a_\chi}{m_i m_j} \vec{\tau}_i \cdot \vec{\tau}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \Lambda^2 e^{-\Lambda r} / r, \]  

where \( \vec{\tau} \) and \( \vec{\sigma} \) are the quark isospin and spin matrices respectively. In this qualitative paper we confine ourselves to the \( \pi \)-exchange between \( u,d \) quarks as the contribution of the \( \eta \) exchange is much smaller.

The minus sign of the interaction (7) is related to the sign of the short-range part of the pseudoscalar meson-exchange interaction (which is opposite to that of the Yukawa tail), crucial for the hyperfine splittings in baryon spectroscopy. It is significant that this short-range part appears at the leading order within the chiral perturbation theory (i.e. in the chiral limit), while the Yukawa part contributes only in the subleading orders and vanishes in the chiral limit \([17]\). The parameter \( \Lambda \), which determines a range of this interaction, is fixed by the scale of spontaneous breaking of chiral symmetry, \( \Lambda \simeq 1 \text{ GeV} \). The Yukawa part of the interaction, on the other hand, is determined by the pion mass and is not important for the interaction of quarks at distances of 0.5 – 0.8 fm, which is a typical distance between quarks in the nucleon and which is important for the short-range \( NN \) interaction. Note that the short-range interaction of the form (7) comes also from the \( \rho \)-exchange \([18]\), which can also be considered as a representation of the correlated two-pion exchange \([19]\). The parameter \( a_\chi \), which determines the total strength of the pseudoscalar and vector-like hyperfine interactions with \( s^3 \) ansatz for both nucleon and \( \Delta \) wave function is fixed to reproduce the \( \Delta - N \) mass splitting. The constituent masses are taken to have their typical values, \( m = \frac{1}{3} m_N \).

When the confining interaction between quarks is assumed to be colour-electric and pair-wise and has a harmonic form, it does not contribute at all to the two nucleon problem as soon as the ansätze (1) or (4) are used and the two-nucleon threshold, calculated with the same Hamiltonian, is subtracted. Hence within the given toy model we have only one free parameter, the nucleon matter root-mean-square size \( b \), which coincides with the harmonic
oscillator parameter of the $s^3$ wave function. We fix it to be $b = 0.5$ fm. The parameters used in the calculation are summarized in Table I.

The second model is a traditional one, based on the colour-magnetic component of OGE

$$V_{cm} = -\sum_{i<j} \frac{a_{cm}}{m_i m_j} \lambda_i^C \cdot \lambda_j^C \bar{\sigma}_i \cdot \bar{\sigma}_j \Lambda 2 e^{-\Lambda r}, \quad (8)$$

where $\lambda^C$ are color Gell-Mann matrices with an implied summation over $C = 1, \ldots, 8$. We want to make a fair comparison between two models and thus use exactly the same $b$ and $\Lambda$. The effective OGE coupling constant $a_{cm}$ is determined from the $\Delta - N$ mass splitting, which is also given in the Table. Thus we can study a difference between repulsion implied by the flavor-spin and color-spin structures of the hyperfine interactions.

In Figures 1 and 2, we show the S-wave triplet and singlet phase shifts as a function of the center of mass momentum for both models, which are negative and thus indicate repulsion in both cases. However, it is immediately seen from comparison that the flavour-spin hyperfine interaction (7) supplies essentially stronger repulsion than the colour-magnetic interaction (8). One of the reasons is that while the colour-magnetic interaction contributes to the short-range repulsion exclusively via the quark-exchange terms which vanish when one approaches the nucleon size to zero, the repulsion in the NN system stemming from the flavor-spin interaction is supported by both direct and quark-exchange terms and does not vanish in this limit.

Next we address the issue whether an extension from the one-channel ansatz (1) to the three-channel ansatz (4) is important for phase shifts and six-quark wave function. We employ the model (7).

In Fig. 3 we compare phase shifts calculated with the one-channel and the three-channel ansätze. While there is some difference, it is not significant. Note that inclusion of the $\Delta\Delta$ channel will produce an important effect as soon as the long- and intermediate-range
attraction between quarks (π, 2π or σ exchanges) is included.

There is, however, a difference in the short-range 6Q wave functions. Unfortunately it is not possible to show the six-body wave function in both cases, but we can compare a projection of the wave function onto the given baryon-baryon component. There is no unique definition of such a projection, because it is not an observable and does not make a direct physical sense (for a discussion on this issue see ref. [16]). Only a full 6-body wave function can be used to calculate any observable, which includes both the direct and the quark-interchange terms. We shall use two different definitions, one of them via the first power of the norm kernel (this correspond to that one used in [9,16])

\[
\bar{\chi}(\vec{r}'') = \int d\vec{r}' N_{\beta\alpha}(\vec{r}'\vec{r}'')\chi_\beta(\vec{r}'),
\]

(9)

\[
N_{\beta\alpha}(\vec{r}',\vec{r}'') = <B_\beta(1,2,3)B_\beta(4,5,6)\delta(\vec{r} - \vec{r}')|1 - 9\hat{P}_{36}|B_\alpha(1,2,3)B_\alpha(4,5,6)\delta(\vec{r} - \vec{r}'')>,
\]

(10)

where \(B_\alpha = N, \Delta, C\). The other definition uses a square root of the norm kernel

\[
\bar{\chi}'(\vec{r}'') = \int d\vec{r}' N_{\beta\alpha}^{1/2}(\vec{r}'\vec{r}'')\chi_\beta(\vec{r}').
\]

(11)

Sometimes the latter projection is interpreted as a probability density for a given channel, which is, however, not correct, since only the full 6-body wave function has a direct and clear probability interpretation.

Both types of projections would give an identical result if one used a multichannel ansatz for wave function with all possible baryon states. Then the closure relation

\[
\sum_\alpha |B_\alpha(1,2,3)B_\alpha(4,5,6)><B_\alpha(1,2,3)B_\alpha(4,5,6)| = I
\]

(12)

would be satisfied and one would obtain \(\hat{N} = (\hat{N}^{1/2})^2\) (which is satisfied on the subspace \(N = 0\) but not satisfied on the subspace \(N = 2\) with \(B_\alpha = N, \Delta, C\)).

In Figs. 4 we show projections onto \(NN\) using both one-channel and three-channel ansätze and both definitions of projections. It is indeed well seen that different definitions
give different behaviour of projections at short range. While there is a node with the definition (11), such a node is absent with the definition (9), which illustrates a very limited physical sense of projections. Still, when we compare the projections obtained with different ansätze (4) or (1) within the same definition (11), one observes a significant difference, which is of no surprise since the ansatz (4) is much richer at short distances in the NN system.

In conclusion we summarize. The short-range flavour-spin hyperfine interaction between the constituent quarks implies a strong short-range repulsion in the NN system. This repulsion is essentially stronger than that one supplied by the colour-magnetic interaction within the traditional model. This is a welcome feature as the traditional models, based on the colour-magnetic interaction, do not provide a strong enough short-range repulsion and fail to describe the phase shifts above the lab energy of 300 MeV. Another significant difference is that the interaction (7) implies a repulsion of the same strength in both singlet and triplet partial waves, while the colour-magnetic interaction supplies a repulsion of different strength, which makes it difficult to describe both partial waves at the same time. While both models imply a repulsion in the $NN$ system, their implications are dramatically different in the ”H-particle” channel. The colour-magnetic interaction, reinforced by the the Yukawa parts of the meson exchanges, leads to a deeply bound H-particle [22], while the interaction (7) tends to make the $6q$ system with ”H-particle” quantum numbers unbound or loosely bound [23]. The existing experimental data exclude the deeply bound H-particle [24].

Thus the chiral constituent quark model has a good potential to explain not only baryon spectroscopy, but also the baryon-baryon interaction. The next stage is to add a long-range Yukawa potential tail from one-pion and two-pion (sigma + rho) exchanges (and possibly from omega-exchange) and provide a realistic description of NN system including all the necessary spin-spin, tensor and spin-orbit components. This task is rather involved and all groups with the corresponding experience are invited.
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TABLES

TABLE I. Parameters of models GBE and OGE interaction. $a_c$ is the strength of the harmonic confinement.

| Model | $m$ [MeV] | $b$ [fm] | $\Lambda$ [GeV] | $a_{cm}$ | $a_\chi$ | $a_c$ [MeV/fm$^2$] |
|-------|-----------|----------|-----------------|---------|---------|----------------|
| GBE   | 313       | 0.5      | 1.0             | 0.0     | 0.068   | 47.7           |
| OGE   | 313       | 0.5      | 1.0             | 0.051   | 0.0     | 93.7           |
FIGURES

FIG. 1. Phase shifts for the NN $^3S_1$ channel. Phase shifts given by the single channel calculation are shown for the models GBE and OGE as a function of the wave number $k$. 

PHASE SHIFT for $^3S_1$

$\delta$ (rad)

Solid: GBE
Dashed: OGE

$k$ [fm$^{-1}$]
FIG. 2. Phase shifts for the $NN^{1}S_{0}$ channel

See Fig. [ ] for explanation

PHASE SHIFT for $^{1}S_{0}$

\[ \delta \text{ (rad)} \]

Solid: GBE
Dashed: OGE

$k$ [fm$^{-1}$]

\[ 0 \quad 1 \quad 2 \quad 3 \]

\[ 0 \quad -1 \]

\[ 0 \quad 1 \quad 2 \quad 3 \]
FIG. 3. Phase shifts for the NN $^3S_1$ channel. Phase shifts given by the single and three channel calculations are shown for the models GBE and OGE interaction as a function of the wave number $k$. Two curves correspond to single channel and three channel (weaker repulsion) calculations.
FIG. 4. Projection of wave function on the NN $^3S_1$ channel. The results of the single and three channel calculations using the model GBE are shown. $N$ and $N^{1/2}$ correspond to the projections using the norm kernel and a square root of the norm kernel, respectively.
Table 1: Parameters of models OPEP and OGEP.

| Model | $m_q$ [MeV] | $b$ [fm] | $\Lambda$ [GeV] | $a_{cm}$ | $a_Y$ | $a_c$ [MeV/fm$^2$] | $M_N$ |
|-------|------------|----------|-----------------|----------|-------|-------------------|------|
| OPEP  | 313        | 0.5      | 1.0             | 0.0      | 0.068 | 47.7              | 1604 |
| OGEP  | 313        | 0.5      | 1.0             | 0.051    | 0.0   | 93.7              | 2101 |