Hadronic decays of $B$ involving a tensor meson

through a $b \to c$ transition

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Abstract

We re-analyze hadronic decays of $B$ mesons to a pseudoscalar ($P$) and a tensor meson ($T$), or a vector meson ($V$) and a tensor meson, through a $b \to c$ transition. We discuss possible large uncertainties to branching ratios (BR’s) of the relevant modes, mainly arising from uncertainties to the hadronic form factors for the $B \to T$ transition. The BR’s and CP asymmetries for $B \to PT$ and $VT$ decays are then calculated by using the form factors given in the ISGW2 model (the improved version of the original Isgur-Scora-Grinstein-Wise (ISGW) model). We find that the estimated BR’s of many modes are increased by an order of magnitude, compared to the previous results calculated within the ISGW model.

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I. INTRODUCTION

In the next few years plenty of new experimental data on rare decays of $B$ mesons will be available from $B$ factory experiments such as Belle, BaBar, BTeV, LHC-B and so on. Experimentally several tensor mesons have been observed [1], such as the isovector $a_2(1320)$, the isoscalars $f_2(1270), f_2'(1525), f_2(2010), f_2(2300), f_2(2340), \chi_{c2}(1P), \chi_{b2}(1P)$ and $\chi_{c2}(2P)$, and the isospinors $K_2^*(1430)$ and $D_2^*(2460)$. Experimental data on the branching ratios (BR’s) for $B$ decays involving a tensor meson ($T$) in the final state provide only upper bounds [1]: for instance, for a $b \to c$ transition,

$$B(B^+ \to \pi^+ D_2^*(2460)^0) < 1.3 \times 10^{-3},$$
$$B(B^0 \to \pi^+ D_2^*(2460)^-) < 2.2 \times 10^{-3},$$
$$B(B^+ \to \rho^+ D_2^*(2460)^0) < 4.7 \times 10^{-3},$$
$$B(B^0 \to \rho^+ D_2^*(2460)^-) < 4.9 \times 10^{-3}. \quad (1)$$

Recently the process $B \to K_2^* \gamma$ has been observed for the first time by the CLEO Collaboration with a branching ratio of $(1.66^{+0.59}_{-0.53} \pm 0.13) \times 10^{-5}$ [2], and by the Belle Collaboration with $B(B \to K_2^* \gamma) = (1.50^{+0.58+0.11}_{-0.53-0.13}) \times 10^{-5}$ [3].

Two-body hadronic $B$ decays involving a tensor meson $T$ ($J^P = 2^+$) in the final state have long been studied [4–8] using the non-relativistic quark model of Isgur, Scora, Grinstein and Wise (ISGW) [9] with the factorization ansatz. Some of those works [4–6] studied $B$ decays involving a $b \to c$ transition, which include the Cabibbo-Kobayashi-Maskawa (CKM)-favored $B$ decays and the CKM-suppressed $B$ decays. The estimated branching ratios of those decay modes strongly depend on the properties of hadronic form factors. A characteristic feature of the form factors given in the original ISGW model [9] is that values of the form factors decrease exponentially as a function of $(t_m - t)$, where $t \equiv (p_B - p_T)^2$ is the momentum transfer and $t_m \equiv (m_B - m_T)^2$ is the maximum possible momentum transfer in the $B$ meson rest frame for a $B \to T$ transition. The authors in Ref. [4] used the form factors calculated at the maximum momentum transfer $t_m$ for allowed transitions, assuming that in the relevant
transitions the momentum transfer \(t\) is close to the maximum momentum transfer \(t_m\). In contrast, other authors [6–8] used the form factors with their exponentially decreasing behavior as a function of \((t_m - t)\). In particular, in our previous works [7,8], the exponentially decreasing behavior of the form factors was assumed to predict the BR’s of charmless decays \(B \to PT\) and \(B \to VT\) \((P\) and \(V\) denote a pseudoscalar and a vector meson, respectively)\(^1\).

Because the exponentially decreasing behavior of the form factors in the ISGW model is less justified, and the assumption of \(t_m \approx t\) seems to be too naive, it is very important to carefully study all the relevant processes using more reliable and consistent values of the form factors, if available. In fact, the ISGW model has been improved to the ISGW2 model [12], whose feature includes a more accurate parametrization of the form factors which have a more realistic behavior at large \((t_m - t)\) by making the replacement of the exponentially decreasing term to a certain polynomial term. The improved ISGW2 model also incorporates more reliable features, say the constraints of heavy quark symmetry, relativistic corrections, hyperfine distortions of wave functions, and so forth [12].

In this work we re-analyze \(B \to PT\) and \(B \to VT\) decays through a \(b \to c\) transition\(^2\), using the hadronic form factors calculated in the ISGW2 model. We first discuss possible large uncertainties to the BR’s of the relevant modes, mainly arising from uncertainties to

\(^1\)Recently the Belle Collaboration measured the BR of \(B^+ \to K^+\pi^+\pi^−\), where two known candidate states for a \(\pi^+\pi^−\) invariant mass around 1300 MeV are \(f_2(1270)\) and \(f_0(1370)\) [10]. Because our previous result using the ISGW model predicts a rather small BR for \(B^+ \to f_2(1270)K^+\) [7], they concluded that the measurements would provide evidence for a significant nonfactorizable effect, if the peak were due to \(f_2(1270)\). However, our recent result using the improved version of the model (ISGW2) shows that the BR of \(B^+ \to f_2(1270)K^+\) is enhanced by an order of magnitude [11].

\(^2\)In addition, we also study a few \(B \to PT\) and \(VT\) modes involving a \(b \to u\) transition, such as \(B \to D_s^{(*)}a_2\), \(B \to D_s^{(*)}f_2^{(l)}\), and \(B \to D^{(*)}K_2^*\).
the hadronic form factors which are heavily model-dependent. Then, using the form factors obtained in the ISGW2 model, we calculate the BR’s, ratios of $\mathcal{B}(B \to VT)/\mathcal{B}(B \to PT)$ and CP asymmetries for $B \to PT$ and $B \to VT$. We make comments on the difference between our results and the previous results obtained using the relevant form factors calculated in the original ISGW model.

This work is organized as follows. In Sec. II we discuss uncertainties relevant to the hadronic form factors. Our framework is introduced in Sec. III. We present our analysis of $B \to PT$ and $B \to VT$ decays in Sec. IV. Finally, our results are summarized in Sec. V.

II. UNCERTAINTIES RELEVANT TO HADRONIC FORM FACTORS

The decay rate $(\Gamma)$ of $B \to PT$ or $B \to VT$ strongly depends on the relevant hadronic form factors for $B \to T$ transitions. For instance, in $B \to PT$ decays, the decay rate is

$$\Gamma(B \to PT) \propto (F^{B\to T})^2,$$

(2)

where

$$F^{B\to T} = k + (m^2_B - m^2_T)b_+ + m^2_Pb_-.$$

(3)

(See the next section for definitions of the form factors $k$, $b_+$, and $b_-$. ) Table I shows the values of the form factors $F^{B\to T}$ calculated in three cases: (i) at $q^2 = m^2_D$ ($q^\mu \equiv p^\mu_B - p^\mu_T$), (ii) at the maximum momentum transfer $t_m \equiv (m_B - m_T)^2$ in the ISGW model, and (iii) at $q^2 = m^2_D$ in the ISGW2 model. We note that $|F^{B\to T}| \approx 0.2$ at $t_m$, while $|F^{B\to T}| \approx 0.05$ at $q^2 = m^2_D$ in the ISGW model, where $T = a_2$, $f_2$, $f_2'$. The value of $|F^{B\to T}|$ calculated at $t_m$ is about 4 times larger than that calculated at $q^2 = m^2_D$. Thus, the decay rate of a relevant process (e.g., $B \to Da_2$, $Df_2$, $Df_2'$, etc) evaluated by using the former value of the form factor (evaluated at $t_m$) would be roughly 16 times larger than that obtained using the latter value of the form factor (at $q^2 = m^2_D$). For $B \to VT$ decays, the similar argument holds. It is obvious that the uncertainty relevant to the hadronic form factors can seriously
spoil theoretical estimates of the BR’s of $B \to PT$ and $B \to VT$ decays. More reliable values of the form factors are definitely needed.

As previously mentioned, a crucial improvement of the ISGW2 model is that the form factors in this model have a more realistic and reasonable behavior at large $(t_m - t)$. Thus, one no longer needs to naively assume $t \approx t_m$ in $B \to PT$ and $VT$ processes. The value of $|F^{B\to T}|$ obtained at $q^2 = m_D^2$ in the ISGW2 model is in between that obtained at $q^2 = m_D^2$ and that calculated at $t_m$ in the ISGW model (except $|F^{B\to K^*}|$). In fact, from Table I, we see that for $B \to a_2$ and $B \to f_2$ transitions, $|F^{B\to T}|$ obtained at $t_m$ is about 2 times larger than that obtained at $q^2 = m_D^2$ in the ISGW model, which would lead to overestimation of the relevant decay rates. Compared to $|F^{B\to T}|$ obtained at $q^2 = m_D^2$ in the ISGW model, the values obtained in the ISGW2 model are about $2 - 6$ times larger, which would result in roughly $4 - 36$ times larger decay rates.

### III. FRAMEWORK

The relevant $\Delta B = 1$ effective Hamiltonian for hadronic $B$ decays can be written as

$$H_{eff}^q = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{uq}^* (c_1 O_{1u}^q + c_2 O_{2u}^q) + V_{cb} V_{cq}^* (c_1 O_{1c}^q + c_2 O_{2c}^q) 
- \sum_{i=3}^{10} \left( V_{ub} V_{uq_i}^* c_i^u + V_{cb} V_{cq_i}^* c_i^c + V_{tb} V_{tq_i}^* c_i^t \right) O_i^q \right] + H.C.,$$

(4)

where $O_i^q$'s are defined as

$$O_{1f}^q = \bar{q} \gamma_\mu L f \bar{f} \gamma^\mu L b, \quad O_{2f}^q = \bar{q} \gamma_\mu L f \bar{f} \gamma^\mu L b,$$

$$O_{3(5)}^q = \bar{q} \gamma_\mu L b \sum_q q' \gamma^\mu L(R) q', \quad O_{4(6)}^q = \bar{q} \gamma_\mu L b \sum_q q' \gamma^\mu L(R) q',$$

$$O_{5(9)}^q = \frac{3}{2} \bar{q} \gamma_\mu L b \sum_q c_q q' \gamma^\mu R(L) q', \quad O_{8(10)}^q = \frac{3}{2} \bar{q} \gamma_\mu L b \sum_q c_q q' \gamma^\mu R(L) q',$$

(5)

where $L(R) = (1 \mp \gamma_5)$, $f$ can be $u$ or $c$ quark, $q$ can be $d$ or $s$ quark, and $q'$ is summed over $u, d, s,$ and $c$ quarks. $\alpha$ and $\beta$ are the color indices. $T^a$ is the SU(3) generator with the
normalization \( \text{Tr}(T^a T^b) = \delta^{ab}/2 \). \( G^\mu_\alpha \) and \( F^\mu_\nu \) are the gluon and photon field strength, and \( c_i \)’s are the Wilson coefficients (WC’s). We use the improved effective WC’s given in Ref. [13], where the renormalization scheme- and scale-dependence of the WC’s are discussed and resolved. The regularization scale is taken to be \( \mu = m_b \) [14]. The operators \( O_1, O_2 \) are the tree level and QCD corrected operators, \( O_{3-6} \) are the gluon induced strong penguin operators, and finally \( O_{7-10} \) are the electroweak penguin operators due to \( \gamma \) and \( Z \) exchange, and the box diagrams at loop level.

We use the improved ISGW2 quark model to analyze two-body nonleptonic decay processes \( B \to PT \) and \( VT \) in the framework of generalized factorization. We describe the parameterizations of the hadronic matrix elements in \( B \to PT \) and \( VT \) decays: [9,12]

\[
\langle 0 | A^\mu | P \rangle = i f_P p^\mu_P , \tag{6}
\]

\[
\langle 0 | V^\mu | V \rangle = f_V m_V e^\mu , \tag{7}
\]

\[
\langle T | j^\mu | B \rangle = i h (m^2_P) \epsilon^{\mu\nu\rho\sigma} \epsilon_{\nu\rho} p^\alpha_B (p_B + p_T) (p_B - p_T) \delta^{\sigma} + k (m^2_P) \epsilon^{\mu\nu} (p_B) ,
\]

\[
+ \epsilon_{\beta}^* B^\alpha B^\beta [b_+ (m^2_P) (p_B + p_T)^\mu + b_- (m^2_P) (p_B - p_T)^\mu] , \tag{8}
\]

where \( j^\mu = V^\mu - A^\mu \). \( V^\mu \) and \( A^\mu \) denote a vector and an axial-vector current, respectively. \( f_P \) (\( f_V \)) denotes the decay constant of the relevant pseudoscalar (vector) meson. \( h (m^2_P(V)) \), \( k (m^2_P(V)) \), \( b_+ (m^2_P(V)) \), and \( b_- (m^2_P(V)) \) express the form factors for the \( B \to T \) transition, which have been calculated at \( q^2 = m^2_P(V) \) \( (q^2 \equiv p_B^\mu - p_T^\mu) \) in the ISGW2 quark model. \( p_B \) and \( p_T \) denote the momentum of the \( B \) meson and the tensor meson, respectively. Using the above parameterizations, the decay amplitudes for \( B \to PT \) and \( B \to VT \) are [7,8,15]

\[
A(B \to PT) \sim F^{B \to T} (m^2_P) , \quad A(B \to VT) \sim \epsilon^{\alpha\beta} F^{B \to T}_{\alpha\beta} (m^2_V) , \tag{9}
\]

where

\[
F^{B \to T} (m^2_P) = k(m^2_P) + (m^2_B - m^2_P) b_+ (m^2_P) + m^2_P b_- (m^2_P) , \tag{10}
\]

\[
F^{B \to T}_{\alpha\beta} (m^2_V) = \epsilon^\mu (p_B + p_T) \rho [i h (m^2_V) \cdot \epsilon^{\mu\nu\rho} g_{\alpha\nu} (p_V) \beta (p_V) \sigma + k(m^2_V) \cdot \delta^\mu \delta^\rho
\]

\[
+ b_+ (m^2_V) \cdot (p_V) \alpha (p_V) \beta g^{\mu\rho} ] . \tag{11}
\]
For our numerical analysis, we use the following values of the decay constants (in MeV) [16]:

\[ f_\pi = 132, \; f_K = 162, \; f_D = 252, \; f_{D^*} = 280, \; f_{\eta_c} = 393, \; f_\rho = 216, \; f_{K^*} = 222, \; f_{D^*} = 249, \]

\[ f_{D_s^*} = 270, \; f_{J/\psi} = 405. \] The running quark masses (in MeV) at \( m_b \) scale are used as follows [17]: \( m_u = 3.6, \; m_d = 6.6, \) and \( m_s = 100. \)

An important feature of the ISGW2 model is that a more accurate parametrization of the form factors \( h, k, b_+, \) and \( b_- \) is adopted by making the replacement, for \( B \to T \) transition,

\[ \exp[-(\text{constant}) \cdot (t_m - t)] \Rightarrow [1 + (\text{constant}) \cdot (t_m - t)]^{-3}, \tag{12} \]

where \( t \equiv (p_B - p_T)^2 \) is the momentum transfer and \( t_m \equiv (m_B - m_T)^2 \) is the maximum possible momentum transfer in the \( B \) meson rest frame. As a result, the form factors have a more realistic behavior at large \( t_m - t \).

We note that the matrix element \( \langle 0 | j^\mu | T \rangle \) vanishes:

\[ \langle 0 | j^\mu | T \rangle = p_\nu \varepsilon^{\mu\nu}(p_T, \lambda) + p_T^{\nu} \varepsilon_{\nu\nu}(p_T, \lambda) = 0, \tag{13} \]

because the trace of the polarization tensor \( \varepsilon^{\mu\nu} \) of the tensor meson \( T \) vanishes and the auxiliary condition holds, \( p_T^{\mu} \varepsilon_{\mu\nu} = 0 \) [18]. Thus, in the generalized factorization scheme, any decay amplitude for \( B \to PT \) (or \( VT \)) is simply proportional to the decay constant \( f_P \) (or \( f_V \)) and a certain linear combination of the form factors \( F_{B \to T} \) (or \( F_{\alpha \beta \gamma}^{B \to T} \)), i.e., there is no such amplitude proportional to \( f_T \times F_{\alpha \beta}^{B \to P} \) (or \( F_{\alpha \beta \gamma}^{B \to T} \)) (see Appendix).

IV. ANALYSES AND RESULTS

We calculate the BR’s of \( B \to PT \) and \( B \to VT \) decays, whose quark level processes are the \( b \to c \) transition. Among the relevant decay modes, many processes involve a tree diagram only; their decay amplitudes are proportional to the CKM elements \( V_{cb}V_{ud} \) or \( V_{cb}^*V_{us} \) (see Tables II–V). Other processes involve both tree and (strong and electroweak) penguin diagrams; their tree amplitudes are proportional to the CKM elements \( V_{cb}^*V_{cd} \) or \( V_{cb}^*V_{cs} \). But, the penguin diagram contribution is much smaller than the tree contribution.
Expressions for all the amplitudes having both the tree and penguin terms are presented in Appendix, calculated in the generalized factorization scheme. (For expressions of the other amplitudes, see Refs. [4–6].) Among the relevant modes, some processes, such as $B^{+ (0)} \to \pi^+ \bar{D}_2^{*0(-)}$, $B^{+ (0)} \to D_2^+ \bar{D}_2^{*0(-)}$, etc., are the CKM-favored decays whose amplitudes are proportional to the CKM elements $V_{cb}^* V_{ud}$ (or $V_{cb}^* V_{cs}$ at tree level). Other processes, such as $B^{+ (0)} \to K^+ \bar{D}_2^{*0(-)}$, $B^{+ (0)} \to D^+ \bar{D}_2^{*0(-)}$, etc., are the CKM-suppressed decays whose amplitudes are proportional to $V_{ub}^* V_{us}$ (or $V_{ub}^* V_{cd}$ at tree level). As commented in the footnote of Sec. I, we also calculate the BR’s of some CKM-suppressed processes involving the $b \to u$ transition, such as $B^{+ (0)} \to D_s^+ a_2^{0(-)}$, $B^{+ (0)} \to D^0 K_2^{*-0(+)}$, and so on; these processes involve a tree diagram only and their amplitudes are proportional to $V_{ub}^* V_{cs}$.

Tables II and III show the BR’s of $B \to PT$ processes for $\Delta S = 0$ and $|\Delta S| = 1$, respectively ($S$ denotes the strangeness quantum number). Similarly, Tables IV and V show the BR’s of $B \to VT$ for $\Delta S = 0$ and $|\Delta S| = 1$, respectively. In the tables, the results are shown for three different values of the parameter $\xi \equiv 1/N_c$ ($N_c$ denotes the effective number of color): $\xi = 0.1$, 0.3, 0.5. For comparison, the BR’s are also calculated using $a_1 = 1.15$ and $a_2 = 0.26$ whose values are obtained from a fit to $B \to PP$ and $B \to PV$ data [19], where the QCD coefficients are $a_1 \equiv c_1 + \xi c_2$ and $a_2 \equiv c_2 + \xi c_1$ ($c_1$ and $c_2$ are the effective WC’s).

The decay amplitudes of all the modes shown in Tables II–V are (dominantly) proportional to either $a_1$ (color-favored) or $a_2$ (color-suppressed) only. The value of $a_1 \equiv c_1 + \xi c_2$ does not vary much as $\xi$ varies: $a_1 = 1.132$ for $\xi = 0.1$, $a_1 = 1.059$ for $\xi = 0.3$, and $a_1 = 0.986$ for $\xi = 0.5$. In contrast, the value of $a_2 \equiv c_2 + \xi c_1$ varies as follows: $a_2 = -0.248$ for $\xi = 0.1$.

3In the frameworks of the QCD factorization and the perturbative QCD approaches, nonfactorizable effects vary for different four-quark operators: e.g., $\xi$ is different for tree- and penguin-dominated processes. But within our generalized factorization framework, the $\xi$ is assumed universal.
\(a_2 = -0.015\) for \(\xi = 0.3\), and \(a_2 = 0.219\) for \(\xi = 0.5\). We note that the value of \(a_2\) for \(\xi = 0.3\) is about an order of magnitude smaller than that for \(\xi = 0.1\) or \(\xi = 0.5\). It would lead to the estimation that the BR’s of the decay modes, whose amplitudes are proportional to \(a_2\), are very small (i.e., about two orders of magnitude smaller) for \(\xi = 0.3\). However, compared with the values of \(a_1\) and \(a_2\) obtained from \(B \to PP\) and \(B \to PV\) data, the value of \(a_2\) for \(\xi = 0.3\) seems to be too small, while the values of \(a_1\) and \(a_2\) for \(\xi = 0.5\) are quite consistent with those values. (For \(\xi = 0.1\), the value of \(a_1\) fits well to that obtained from the data, but \(a_2\) has the opposite sign to that deduced from the data. However, the sign of \(a_2\) has no (or negligible) effect on our results since each decay amplitude is (dominantly) proportional to only one QCD coefficient (i.e., either \(a_1\) or \(a_2\)).)

As expected, the BR’s of both the CKM-favored and color-favored processes are generally large. In \(B \to PT\) decays, the BR of \(B^{+(0)} \to \pi^+ \bar{D}^{*0(-)}_2\) is about \(3 \times 10^{-4}\), and the BR of \(B^{+(0)} \to D^+_s \bar{D}^{*0(-)}_2\) is about \(4 \times 10^{-4}\). In \(B \to VT\) decays, the BR of \(B^{+(0)} \to \rho^+ \bar{D}^{*0(-)}_2\) is \((7 - 9) \times 10^{-4}\), and the BR of \(B^{+(0)} \to D^{+_s \bar{D}^{*0(-)}_2}\) is about \(1 \times 10^{-3}\). The BR’s of the CKM-favored and color-suppressed modes are \(O(10^{-4}) - O(10^{-5})\), except \(\mathcal{B}(B^0 \to \bar{D}^0 f'_2) \sim O(10^{-7})\) and \(\mathcal{B}(B^0 \to \bar{D}^{*0} f'_2) \sim O(10^{-6})\). The BR’s of the CKM-suppressed modes are relatively smaller, \(O(10^{-5}) - O(10^{-8})\). From Tables II–V, we see that the BR’s of the decay modes such as \(B^{+(0)} \to \bar{D}^{(*)0} a_2^{+(0)}\), \(B^0 \to \bar{D}^{(*)0} f'_2\), \(B^{+(0)} \to \eta_c(J/\psi) a_2^{+(0)}\), \(B^0 \to \eta_c(J/\psi) f'_2\), etc, for \(\xi = 0.3\) are about two orders of magnitudes smaller than those for \(\xi = 0.1\) or \(\xi = 0.5\). This occurs because the decay amplitudes of all those modes are (dominantly) proportional to \(a_2\) (see Appendix), as explained above.

We note that for many processes our predictions are larger than the BR’s given in Ref. [6]. In particular, for the processes whose amplitudes are proportional to \(V_{us}^* V_{cs}\), our results are about an order of magnitude larger than the BR’s given in [6]; for instance, for \(B \to PT\) such as \(B^{+(0)} \to D^+_s a_2^{0(-)}\), \(B^+ \to D^+_s f'_2\), \(B^{+(0)} \to D^0 K^{*+}(0)\), and for \(B \to VT\) such as \(B^{+(0)} \to D^{*_s} a_2^{0(-)}\), \(B^+ \to D^{*_s} f'_2\), \(B^{+(0)} \to D^{*0} K^{*+}(0)\).

In Table VI, we show the ratios of \(\mathcal{B}(B \to VT)/\mathcal{B}(B \to PT)\). The ratios are roughly 3 for the processes which involve a tree diagram only and whose amplitudes are propor-
tional to $a_1$ (via the external $W$ emission); for instance, \( \mathcal{B}(B^{+}(0) \to \rho^{+}\bar{D}^{0}_{2})/\mathcal{B}(B^{+}(0) \to \pi^{+}\bar{D}^{0}_{2}) \approx 3 \). This is naively expected from the fact that massive vector particles have three polarization states. But, for the processes which involve both tree and penguin diagrams, the ratios deviate from 3; e.g., \( \mathcal{B}(B^{+}(0) \to D^{+}\bar{D}^{0}_{2})/\mathcal{B}(B^{+}(0) \to D^{+}\bar{D}^{0}_{2}) \approx 2.3 \). For the processes whose amplitudes are proportional to $a_2$ (via the internal $W$ emission), the ratios are \( \sim 1.6 \), except the processes involving $J/\psi$ or $\eta_c$ in the final state; e.g., \( \mathcal{B}(B^{+}(0) \to D^{+}\bar{D}^{0}_{2})/\mathcal{B}(B^{+}(0) \to D^{+}\bar{D}^{0}_{2}) \approx 1.6 \). We note that for the processes involving $J/\psi$ or $\eta_c$ in the final state, the ratios substantially vary as $\xi$ varies from 0.1 to 0.3 to 0.5. This is because the penguin contribution to the decay amplitudes involving $\eta_c$ differs from that to the amplitudes involving $J/\psi$; the penguin effect to the former amplitudes is proportional to the combination of the QCD coefficients \( (a_3 - a_5 + a_7 - a_9) \), while the penguin effect to the latter amplitudes is proportional to \( (a_3 + a_5 + a_7 + a_9) \). We also compute CP asymmetries $A_{CP}$ for $B \to PT$ and $B \to VT$ decays, defined by

\[
A_{CP} = \frac{\mathcal{B}(B \to f) - \mathcal{B}(\bar{B} \to \bar{f})}{\mathcal{B}(B \to f) + \mathcal{B}(\bar{B} \to \bar{f})},
\]

where $f$ and $\bar{f}$ denote a generic final state and its CP-conjugate state. Since in the relevant modes the tree contribution is very much dominant compared to the penguin contribution, the asymmetries are relatively small\(^4\). We note that for a non-vanishing $A_{CP}$ for a process and its CP-conjugate process, there should exist both the weak phase and the strong phase differences between their tree and penguin amplitudes. Thus, $A_{CP}$’s vanish for the processes involving $V_{cb}^*V_{cs}$ and $V_{ub}^*V_{ts}$, since there is no weak phase in their amplitudes; e.g.,

\(^4\)In addition to the strong phases, there can be other possible sources for the strong phases: for example, in the QCD factorization a large strong phase for the WC, $a_2$, can be induced by hard gluon exchange between final meson states, and in the perturbative QCD approach large absorptive parts can be generated from the weak annihilation diagrams. But, because in the relevant modes the effect from the tree is much larger than that from the penguin, as just mentioned in the text, the resultant asymmetries would remain relatively small.
\[ \mathcal{A}_{CP}(B^{+0} \rightarrow D^{+}_{s} \bar{D}^{0(-)}_{2}) = 0. \] We present our result in Table VII. The CP asymmetries are shown for different values of \( \xi \). For all the relevant modes, the CP asymmetries are expected to be a few percent.

V. CONCLUSIONS

We have studied the decay modes \( B \rightarrow PT \) and \( B \rightarrow VT \) whose quark level processes are the \( b \rightarrow c \) transition. Due to large uncertainties to the relevant hadronic form factors which are model-dependent, the previously estimated BR’s could be spoiled by large uncertainties. Using more reliable and consistent values of the form factors given in the improved version (ISGW2) of ISGW model, we re-calculate the BR’s of all the relevant modes and find that for many modes our results are much larger than those given in the previous work using the ISGW model.

Our results show that the BR’s of some processes are quite large: in \( B \rightarrow VT \), the BR’s of \( B^{+0} \rightarrow D^{+}_{s} \bar{D}^{0(-)}_{2} \) and \( B^{+0} \rightarrow \rho^{+} \bar{D}^{0(-)}_{2} \) are \( \sim 10^{-3} \), and in \( B \rightarrow PT \), the BR’s of \( B^{+0} \rightarrow D^{+}_{s} \bar{D}^{0(-)}_{2} \) and \( B^{+0} \rightarrow \pi^{+} \bar{D}^{0(-)}_{2} \) are \( (3 - 4) \times 10^{-4} \). (These results are roughly consistent with those obtained under the naive assumption of \( t \approx t_{m} \) in the ISGW model.) The estimated BR’s of \( B^{+0} \rightarrow \pi^{+} \bar{D}^{0(-)}_{2} \) and \( B^{+0} \rightarrow \rho^{+} \bar{D}^{0(-)}_{2} \) are about a factor of \( (4 - 5) \) smaller than the present experimental upper bounds shown in Eq. (1), and so far there is no known experimental data on the modes \( B \rightarrow D^{+}_{s(s')} \bar{D}^{0}_{2} \). Observations of these processes in \( B \) experiments such as Belle, BaBar, BTeV and LHC-B will be crucial in testing the ISGW2 model as well as validity of the factorization scheme.

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APPENDIX

In this Appendix, we present expressions for the decay amplitudes of $B \to PT$ and $VT$ modes which have both the tree and penguin contributions (shown in Tables II–V). Below we use $F^{B\to T}$, $F_{\alpha\beta}^{B\to T}$ and $X_{qq'}$, defined by Eqs. (10) and (11), and

$$X_{qq'} = \frac{m_p^2}{(m_b + m_{q'}) (m_q + m_{q'})}.$$  \hspace{1cm} (15)

(1) $B \to PT$ ($\Delta S = 0$) decays.

$$A(B^+ \to D^+ \bar{D}_2^{*0}) = i \frac{G_F}{\sqrt{2}} f_{D^+} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to D_2^{*0}} (m_{D^+}^2) \{V_{cb}^* V_{ca1}$$
$$-V_{tb}^* V_{td}[a_4 + a_{10} - 2(a_6 + a_8) X_{dc}]\},$$  \hspace{1cm} (16)

$$A(B^0 \to D^+ D_2^{*-}) = i \frac{G_F}{\sqrt{2}} f_{D^+} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to D_2^{*-}} (m_{D^+}^2) \{V_{cb}^* V_{ca1}$$
$$-V_{tb}^* V_{td}[a_4 + a_{10} - 2(a_6 + a_8) X_{dc}]\},$$  \hspace{1cm} (17)

$$A(B^+ \to \eta_c a_2^+) = i \frac{G_F}{\sqrt{2}} f_{\eta_c} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to a_2^+} (m_{\eta_c}^2) \{V_{cb}^* V_{ca2} - V_{tb}^* V_{td}(a_3 + a_5 + a_7 + a_9)\},$$  \hspace{1cm} (18)

$$A(B^0 \to \eta_c a_2^-) = i \frac{G_F}{\sqrt{2}} f_{\eta_c} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to a_2^-} (m_{\eta_c}^2) \{V_{cb}^* V_{ca2} - V_{tb}^* V_{td}(a_3 + a_5 + a_7 + a_9)\},$$  \hspace{1cm} (19)

$$A(B^0 \to \eta_c f_2) = i \frac{G_F}{\sqrt{2}} \cos \phi_T f_{\eta_c} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to f_2} (m_{\eta_c}^2) \{V_{cb}^* V_{ca2}$$
$$-V_{tb}^* V_{td}(a_3 + a_5 + a_7 - a_9)\},$$  \hspace{1.4cm} (20)

$$A(B^0 \to \eta_c f_2') = i \frac{G_F}{\sqrt{2}} \sin \phi_T f_{\eta_c} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to f_2'} (m_{\eta_c}^2) \{V_{cb}^* V_{ca2}$$
$$-V_{tb}^* V_{td}(a_3 + a_5 + a_7 - a_9)\}.$$  \hspace{1cm} (21)

(2) $B \to PT$ ($|\Delta S| = 1$) decays.

$$A(B^+ \to D_s^+ \bar{D}_2^{*0}) = i \frac{G_F}{\sqrt{2}} f_{D_s^+} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to D_2^{*0}} (m_{D_s^+}^2) \{V_{cb}^* V_{csa1}$$
$$-V_{td}^* V_{ts}[a_4 + a_{10} - 2(a_6 + a_8) X_{se}]\},$$  \hspace{1cm} (22)

$$A(B^0 \to D_s^+ D_2^{*-}) = i \frac{G_F}{\sqrt{2}} f_{D_s^+} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to D_2^{*-}} (m_{D_s^+}^2) \{V_{cb}^* V_{csa1}$$
$$-V_{td}^* V_{ts}[a_4 + a_{10} - 2(a_6 + a_8) X_{se}]\},$$  \hspace{1cm} (23)

$$A(B^+ \to \eta_c K_s^{*+}) = i \frac{G_F}{\sqrt{2}} f_{\eta_c} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to K_s^{*+}} (m_{\eta_c}^2) \{V_{cb}^* V_{csa2} - V_{ts}^* V_{ts}(a_3 - a_5 + a_7 - a_9)\},$$  \hspace{1cm} (24)

$$A(B^0 \to \eta_c K_s^{*0}) = i \frac{G_F}{\sqrt{2}} f_{\eta_c} \epsilon_{\mu\nu} p_B^\mu p_B^\nu F^{B\to K_s^{*0}} (m_{\eta_c}^2) \{V_{cb}^* V_{csa2} - V_{ts}^* V_{ts}(a_3 - a_5 + a_7 - a_9)\}.$$  \hspace{1cm} (25)
(3) $B \rightarrow VT \ (\Delta S = 0)$ decays.

$$A(B^+ \rightarrow D^{*+} \bar{D}_2^{*0}) = i \frac{G_F}{\sqrt{2}} (m_{D^{*+}} f_{D^{*+}} \epsilon^{* \alpha \beta} F^{B \rightarrow \bar{D}_2^{*0}}_{\alpha \beta} (m_{D^{*+}}^2)) \{V_{cb} V_{cd} a_1 - V_{tb} V_{ts} (a_4 + a_{10})\}, \quad (26)$$

$$A(B^0 \rightarrow D^{*+} \bar{D}_2^{*-}) = i \frac{G_F}{\sqrt{2}} (m_{D^{*+}} f_{D^{*+}} \epsilon^{* \alpha \beta} F^{B \rightarrow \bar{D}_2^{*-}}_{\alpha \beta} (m_{D^{*+}}^2)) \{V_{cb}^* V_{cd} a_1 - V_{tb}^* V_{td} (a_4 + a_{10})\}, \quad (27)$$

$$A(B^+ \rightarrow J/\psi a_2^+) = i \frac{G_F}{\sqrt{2}} (m_{J/\psi} f_{J/\psi} \epsilon^{* \alpha \beta} F^{B \rightarrow a_2^+} (m_{J/\psi}^2)) \{V_{cb}^* V_{cd} a_2 - V_{tb}^* V_{td} (a_3 + a_5 + a_7 + a_9)\}, \quad (28)$$

$$A(B^0 \rightarrow J/\psi a_2) = \frac{G_F}{2} (m_{J/\psi} f_{J/\psi} \epsilon^{* \alpha \beta} F^{B \rightarrow a_2} (m_{J/\psi}^2)) \{V_{cb}^* V_{cd} a_2 - V_{tb}^* V_{td} (a_3 + a_5 + a_7 + a_9)\}, \quad (29)$$

$$A(B^0 \rightarrow J/\psi f_2) = \frac{G_F}{2} \cos \phi_T (m_{J/\psi} f_{J/\psi} \epsilon^{* \alpha \beta} F^{B \rightarrow f_2} (m_{J/\psi}^2)) \{V_{cb}^* V_{cd} a_2 - V_{tb}^* V_{td} (a_3 + a_5 + a_7 + a_9)\}, \quad (30)$$

$$A(B^0 \rightarrow J/\psi f_2') = \frac{G_F}{2} \sin \phi_T (m_{J/\psi} f_{J/\psi} \epsilon^{* \alpha \beta} F^{B \rightarrow f_2'} (m_{J/\psi}^2)) \{V_{cb}^* V_{cd} a_2 - V_{tb}^* V_{td} (a_3 + a_5 + a_7 + a_9)\}. \quad (31)$$

(4) $B \rightarrow VT \ (|\Delta S| = 1)$ decays.

$$A(B^+ \rightarrow D_s^{*+} \bar{D}_2^{*0}) = i \frac{G_F}{\sqrt{2}} (m_{D_s^{*+}} f_{D_s^{*+}} \epsilon^{* \alpha \beta} F^{B \rightarrow \bar{D}_2^{*0}}_{\alpha \beta} (m_{D_s^{*+}}^2)) \{V_{cs} V_{cs} a_1 - V_{ts} V_{ts} (a_4 + a_{10})\}, \quad (32)$$

$$A(B^0 \rightarrow D_s^{*+} \bar{D}_2^{*-}) = i \frac{G_F}{\sqrt{2}} (m_{D_s^{*+}} f_{D_s^{*+}} \epsilon^{* \alpha \beta} F^{B \rightarrow \bar{D}_2^{*-}}_{\alpha \beta} (m_{D_s^{*+}}^2)) \{V_{cs}^* V_{cs} a_1 - V_{ts}^* V_{ts} (a_4 + a_{10})\}, \quad (33)$$

$$A(B^+ \rightarrow J/\psi K_2^{*+}) = \frac{G_F}{\sqrt{2}} (m_{J/\psi} f_{J/\psi} \epsilon^{* \alpha \beta} F^{B \rightarrow K_2^{*+}}_{\alpha \beta} (m_{J/\psi}^2)) \{V_{cb}^* V_{cs} a_2 - V_{tb}^* V_{ts} (a_3 + a_5 + a_7 + a_9)\}, \quad (34)$$

$$A(B^0 \rightarrow J/\psi K_2^{*0}) = \frac{G_F}{\sqrt{2}} (m_{J/\psi} f_{J/\psi} \epsilon^{* \alpha \beta} F^{B \rightarrow K_2^{*0}}_{\alpha \beta} (m_{J/\psi}^2)) \{V_{cb}^* V_{cs} a_2 - V_{tb}^* V_{ts} (a_3 + a_5 + a_7 + a_9)\}. \quad (35)$$
TABLE I. Form factors for $B \rightarrow T$ transitions calculated at $q^2 = m_D^2$ ($q^\mu \equiv p_B^\mu - p_T^\mu$), at the maximum momentum transfer $t_m \equiv (m_B - m_T)^2$ in the ISGW model, and at $q^2 = m_D^2$ in the ISGW2 model, respectively.

| Form factor for $B \rightarrow T$ | ISGW($m_D^2$) | ISGW($t_m$) | ISGW2 |
|----------------------------------|---------------|--------------|--------|
| $F^{B\rightarrow a_2}$           | -0.046        | -0.203       | 0.101  |
| $F^{B\rightarrow f_2}$           | -0.045        | -0.205       | 0.099  |
| $F^{B\rightarrow f'_2}$          | -0.052        | -0.191       | 0.134  |
| $F^{B\rightarrow K_2^*}$         | -0.049        | -0.111       | 0.131  |
| $F^{B\rightarrow D_2^*}$         | -0.060        | 0.378        | 0.367  |
TABLE II. Branching ratios of $B \to PT$ with $\Delta S = 0$ in units of $10^{-6}$, calculated in the ISGW2 model.

| Decay mode | $\xi = 0.1$ | $\xi = 0.3$ | $\xi = 0.5$ | $a_1 = 1.15, a_2 = 0.26$ |
|------------|-------------|-------------|-------------|--------------------------|
| $\propto V_{cb}^* V_{ud}$ |                  |             |             |                           |
| $B^+ \to \pi^+ \bar{D}_2^{*0}$ | 339.63 | 297.22 | 257.64 | 350.83 |
| $B^+ \to \bar{D}^0 a_2^{+}$ | 92.82 | 0.32 | 72.27 | 101.86 |
| $B^0 \to \pi^+ \bar{D}_2^{*-}$ | 318.96 | 279.13 | 241.96 | 329.48 |
| $B^0 \to \bar{D}^0 a_2^{0}$ | 43.55 | 0.15 | 33.91 | 47.79 |
| $B^0 \to \bar{D}^0 f_2$ | 48.56 | 0.17 | 37.81 | 53.29 |
| $B^0 \to \bar{D}^0 f'_2$ | 0.57 | 0.002 | 0.44 | 0.62 |
| $\propto V_{cb}^* V_{cd}$ |                  |             |             |                           |
| $B^+ \to D^+ \bar{D}_2^{*0}$ | 22.23 | 19.45 | 16.86 | 22.68 |
| $B^+ \to \eta_c a_2^{+}$ | 4.17 | 0.004 | 3.73 | 4.89 |
| $B^0 \to D^+ \bar{D}_2^{*-}$ | 20.87 | 18.27 | 15.83 | 21.30 |
| $B^0 \to \eta_c a_2^{0}$ | 1.96 | 0.002 | 1.75 | 2.30 |
| $B^0 \to \eta_c f_2$ | 2.27 | 0.002 | 2.03 | 2.67 |
| $B^0 \to \eta_c f'_2$ | 0.019 | 0.00002 | 0.017 | 0.02 |

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TABLE III. Branching ratios of $B \to PT$ with $|\Delta S| = 1$ in units of $10^{-6}$, calculated in the ISGW2 model.

| Decay mode | $\xi = 0.1$ | $\xi = 0.3$ | $\xi = 0.5$ | $a_1 = 1.15, a_2 = 0.26$ |
|------------|-------------|-------------|-------------|--------------------------|
| $\propto V_{cb}^* V_{cs}$ |                      |             |             |                          |
| $B^+ \to D_s^+ \bar{D}_s^0$ | 493.04       | 431.56      | 373.61      | 493.04                   |
| $B^+ \to \eta_c K_2^{*-}$  | 81.19        | 0.042       | 88.71       | 105.39                   |
| $B^0 \to D_s^+ D_s^-$       | 462.95       | 405.22      | 350.81      | 462.95                   |
| $B^0 \to \eta_c K_2^{*0}$  | 74.46        | 0.038       | 81.35       | 96.64                    |
| $\propto V_{cb}^* V_{us}$ |                      |             |             |                          |
| $B^+ \to K^+ \bar{D}_2^0$  | 24.64        | 21.56       | 18.69       | 25.45                    |
| $B^+ \to D^0 K_2^{*+}$      | 6.69         | 0.023       | 5.21        | 7.34                     |
| $B^0 \to K^+ D_2^-$         | 23.14        | 20.25       | 17.56       | 23.91                    |
| $B^0 \to \bar{D}_2^0 K_2^{*0}$ | 6.19 | 0.021       | 4.82        | 6.80                     |
| $\propto V_{ub}^* V_{cs}$ |                      |             |             |                          |
| $B^+ \to D_s^+ a_2^0$       | 9.14         | 8.00        | 6.93        | 9.44                     |
| $B^+ \to D_s^+ f_2$         | 10.20        | 8.96        | 7.74        | 10.54                    |
| $B^+ \to D_s^+ f_2'$        | 0.12         | 0.10        | 0.09        | 0.12                     |
| $B^+ \to D^0 K_2^{*+}$      | 1.07         | 0.004       | 0.83        | 1.17                     |
| $B^0 \to D_s^+ a_2^-$       | 17.15        | 15.01       | 13.01       | 17.71                    |
| $B^0 \to D^0 K_2^{*0}$      | 0.99         | 0.003       | 0.77        | 1.08                     |
TABLE IV. Branching ratios of $B \rightarrow VT$ with $\Delta S = 0$ in units of $10^{-6}$, calculated in the ISGW2 model.

| Decay mode            | $\xi = 0.1$ | $\xi = 0.3$ | $\xi = 0.5$ | $a_1 = 1.15, a_2 = 0.26$ |
|-----------------------|------------|------------|------------|-------------------------|
| $\propto V_{cd}^*V_{ud}$ |            |            |            |                         |
| $B^+ \rightarrow \rho^+ \bar{D}_s^0$ | 950.15     | 831.51     | 720.77     | 981.48                  |
| $B^+ \rightarrow \bar{D}^*0_a^+$      | 151.77     | 0.53       | 118.16     | 166.54                  |
| $B^0 \rightarrow \rho^+ D^-_2$        | 892.23     | 780.82     | 676.83     | 921.64                  |
| $B^0 \rightarrow \bar{D}^*0_a^0$      | 71.20      | 0.25       | 55.44      | 78.14                   |
| $B^0 \rightarrow \bar{D}^* f_2$       | 76.82      | 0.27       | 59.81      | 84.30                   |
| $B^0 \rightarrow \bar{D}^* f'_2$      | 0.95       | 0.003      | 0.74       | 1.05                    |
| $\propto V_{cd}^*V_{cd}$              |            |            |            |                         |
| $B^+ \rightarrow D^{*+} \bar{D}_s^0$  | 50.05      | 43.78      | 37.94      | 53.25                   |
| $B^+ \rightarrow J/\psi a^+_2$       | 14.21      | 0.059      | 10.78      | 16.41                   |
| $B^0 \rightarrow D^{*+} D_s^{-}$      | 46.98      | 41.10      | 35.61      | 49.99                   |
| $B^0 \rightarrow J/\psi a^0_2$       | 6.67       | 0.028      | 5.60       | 7.70                    |
| $B^0 \rightarrow J/\psi f_2$         | 7.28       | 0.03       | 5.53       | 8.41                    |
| $B^0 \rightarrow J/\psi f'_2$        | 0.074      | 0.0003     | 0.056      | 0.09                    |
TABLE V. Branching ratios of $B \to VT$ with $|\Delta S| = 1$ in units of $10^{-6}$, calculated in the ISGW2 model.

| Decay mode                      | $\xi = 0.1$ | $\xi = 0.3$ | $\xi = 0.5$ | $a_1 = 1.15$, $a_2 = 0.26$ |
|--------------------------------|-------------|-------------|-------------|-----------------------------|
| $\propto V_{cb}^* V_{cs}$       |             |             |             |                             |
| $B^+ \to D^{*+}_s \bar{D}^0_2$ | 1080.17     | 944.61      | 818.12      | 1200.8                     |
| $B^+ \to J/\psi K^{*+}_2$      | 307.66      | 1.66        | 224.02      | 383.62                     |
| $B^0 \to D^{*+}_s D^{*-}_2$     | 1013.89     | 886.64      | 767.92      | 1127.12                    |
| $B^0 \to J/\psi K^{*0}_2$      | 284.10      | 1.53        | 206.87      | 354.25                     |
| $\propto V_{ub}^* V_{cs}$       |             |             |             |                             |
| $B^+ \to K^{*+} \bar{D}^{*0}_2$ | 50.64       | 44.31       | 38.41       | 52.31                      |
| $B^+ \to D^{*0} K^{*+}_2$      | 11.04       | 0.038       | 8.59        | 12.11                      |
| $B^0 \to K^{*+} D^{*-}_2$      | 47.55       | 41.61       | 36.07       | 49.12                      |
| $B^0 \to D^{*0} K^{*0}_2$      | 10.25       | 0.035       | 7.98        | 11.24                      |
| $\propto V_{ub}^* V_{us}$       |             |             |             |                             |
| $B^+ \to D^{*+}_s a^{0}_2$      | 15.00       | 13.13       | 11.38       | 15.49                      |
| $B^+ \to D^{*+}_s f_2$         | 16.17       | 14.15       | 12.26       | 16.70                      |
| $B^+ \to D^{*+}_s f'_2$        | 0.20        | 0.17        | 0.15        | 0.21                       |
| $B^+ \to D^{*0} K^{*+}_2$      | 1.76        | 0.006       | 1.37        | 1.93                       |
| $B^0 \to D^{*+}_s a^{0}_2$      | 28.15       | 24.63       | 21.35       | 29.08                      |
| $B^0 \to D^{*0} K^{*0}_2$      | 1.63        | 0.006       | 1.27        | 1.79                       |
### TABLE VI. Ratios of $\mathcal{B}(B \rightarrow VT)/\mathcal{B}(B \rightarrow PT)$

| Ratio | $\xi = 0.1$ | $\xi = 0.3$ | $\xi = 0.5$ |
|-------|-------------|-------------|-------------|
| $\mathcal{B}(B^+(0) \rightarrow \rho^+ \bar{D}_2^0(0^+)/\mathcal{B}(B^+(0) \rightarrow \pi^+ \bar{D}_2^0(0^+))$ | 2.80 | 2.80 | 2.80 |
| $\mathcal{B}(B^+(0) \rightarrow D^0 a_2^{+(0)})/\mathcal{B}(B^+(0) \rightarrow D^0 a_2^{+(0)}))$ | 1.64 | 1.64 | 1.63 |
| $\mathcal{B}(B^0 \rightarrow \bar{D}^0 f_2)/\mathcal{B}(B^0 \rightarrow \bar{D}^0 f_2)$ | 1.58 | 1.58 | 1.58 |
| $\mathcal{B}(B^0 \rightarrow \bar{D}^0 f_2)/\mathcal{B}(B^0 \rightarrow \bar{D}^0 f_2)$ | 1.68 | 1.68 | 1.68 |
| $\mathcal{B}(B^+(0) \rightarrow D^+(0) \bar{D}_2^0(0^+)/\mathcal{B}(B^+(0) \rightarrow D^+(0) \bar{D}_2^0(0^+))$ | 2.25 | 2.25 | 2.25 |
| $\mathcal{B}(B^+(0) \rightarrow J/\psi a_2^{+(0)})/\mathcal{B}(B^+(0) \rightarrow \eta_c a_2^{+(0)}))$ | 3.41 | 14.34 | 2.89 |
| $\mathcal{B}(B^0 \rightarrow J/\psi f_2)/\mathcal{B}(B^0 \rightarrow \eta_c f_2)$ | 3.21 | 13.50 | 2.72 |
| $\mathcal{B}(B^0 \rightarrow J/\psi f_2)/\mathcal{B}(B^0 \rightarrow \eta_c f_2)$ | 3.83 | 16.11 | 3.24 |
| $\mathcal{B}(B^+(0) \rightarrow J/\psi K_2^{x(0)})/\mathcal{B}(B^+(0) \rightarrow \eta_c K_2^{+x(0)}))$ | 3.79 | 39.82 | 2.53 |
| $\mathcal{B}(B^+(0) \rightarrow K^+ \bar{D}_2^0(0^+))/\mathcal{B}(B^+(0) \rightarrow K^+ \bar{D}_2^0(0^+))$ | 2.06 | 2.06 | 2.06 |
| $\mathcal{B}(B^+(0) \rightarrow \bar{D}^0 K_2^{x(0)})/\mathcal{B}(B^+(0) \rightarrow \bar{D}^0 K_2^{x(0)})$ | 1.65 | 1.65 | 1.65 |
| $\mathcal{B}(B^+(0) \rightarrow D_s^+ a_2^{0(-)})/\mathcal{B}(B^+(0) \rightarrow D_s^+ a_2^{0(-)})$ | 1.64 | 1.64 | 1.64 |
| $\mathcal{B}(B^+ \rightarrow D_s^+ f_2)/\mathcal{B}(B^+ \rightarrow D_s^+ f_2)$ | 1.58 | 1.58 | 1.58 |
| $\mathcal{B}(B^+ \rightarrow D_s^+ f_2)/\mathcal{B}(B^+ \rightarrow D_s^+ f_2)$ | 1.69 | 1.69 | 1.69 |
| $\mathcal{B}(B^+(0) \rightarrow D^0 K_2^{x(0)})/\mathcal{B}(B^+(0) \rightarrow D^0 K_2^{x(0)})$ | 1.65 | 1.65 | 1.65 |

### TABLE VII. CP asymmetries for $B \rightarrow PT$ and $B \rightarrow VT$

| Decay mode | $\xi = 0.1$ | $\xi = 0.3$ | $\xi = 0.5$ |
|------------|-------------|-------------|-------------|
| $B^+(0) \rightarrow D^+ \bar{D}_2^0(0^-)$ | 0.001 | 0.001 | 0.001 |
| $B^+(0) \rightarrow D^{++} \bar{D}_2^0(0^-)$ | -0.004 | -0.004 | -0.004 |
| $B^+(0) \rightarrow J/\psi a_2^{+(0)}$ | -0.0082 | -0.0045 | -0.0087 |
| $B^0 \rightarrow J/\psi f_2(0^-)$ | -0.0082 | -0.0045 | -0.0087 |