Damping and coherence in a high-density magnon gas

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We report on the fast relaxation behavior of a high-density magnon gas created by a parametric amplification process. The magnon gas is probed using the technique of spin-wave packet recovery by parallel parametric pumping. Experimental results show a damping behavior which is in disagreement with both the standard model of exponential decay and with earlier observations of non-linear damping. In particular, the inherent magnon damping is found to depend upon the presence of the parametric pumping field. A phenomenological model which accounts for the dephasing of the earlier injected magnons is in good agreement with the experimental data.

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I. Introduction

Several decades after the first phenomenological descriptions, the nature of spin-wave dissipation is still a subject of intense scientific research. An understanding of spin-wave damping mechanisms is crucial both for the furtherance of the field of fundamental spin dynamics and practical applications in novel magnetic memory, microwave, and spin-wave logic devices. Recent developments in the study of energy transfer in magnon systems, including the discovery of Bose-Einstein condensation of magnons, have attracted particular interest to the problem of nonlinear damping and decoherence of densely populated magnon states, created by the effect of parametric pumping.

Up to now there has been no access to the information on relaxation of highly populated short-wavelength magnon states as these spin waves are too short to be directly detected by conventional microwave and optical methods. Only measurements of thresholds of parametric instability provide us with information about the decay characteristics of these magnons at low population level. However, even in this case the magnons are influenced by an external microwave pump field. To overcome this obstacle and to access the freely evolving highly populated magnon groups, we used the recently developed technique of spin-wave signal recovery.

In this work we demonstrate that free evolution of a dense group of parametrically injected magnons can be described neither by its monotonic exponential relaxation nor by possible nonlinear decay. The phase decoherence of the frequency smeared magnon group should be taken into account in order to understand the experimental data. We chose a single crystal yttrium-iron-garnet (YIG) ferrimagnetic film as a test object in our experiments. Due to its extremely low natural magnetic damping, this material is widely used in works on magnon gases and condensates. Furthermore, currently one observes a clear revival of interest to YIG films due to the recent discovery of the spin Hall and spin Seebeck effects in Platinum/YIG bi-layers. Moreover, submicron-thick polycrystalline YIG films are now promising as candidates for novel microwave devices, and conventional single-crystal micron-thick films have been recently used to demonstrate microwave encoding.

II Experiment

A 100 ns-long traveling spin-wave packet (τinput = 100 ns) is excited at a frequency of 7 GHz by a microstrip input antenna in a 1.5 mm-wide and 5 µm-thick YIG stripe. An external magnetic field \( \vec{H}_0 \) of 143.64 kA/m (1805 Oe) is applied in the plane of the stripe, perpendicular to the spin-wave propagation (Fig. 1a), i.e. in the Damon-Eshbach (DE) geometry. The excited spin-wave packet traverses the magnetic sample, is detected by the output antenna placed 12 mm apart from the input one, amplified, and observed with an oscilloscope. After 450 ns since the passage by the wave packet of the central area of the YIG stripe (τ0 = 450 ns), a pumping microwave field \( \vec{H}_p \) at 14 GHz produced by a dielectric resonator is applied parallel to the bias magnetic field. As a result of the pump action, an additional “recovered” pulse is observed at the output antenna at a frequency of 7 GHz (see output waveform in Fig. 1a and dashed line in Fig. 2). Since the DE packet has already left...
the area of parametric interaction, this is not the result of the
direct amplification of the signal pulse\textsuperscript{19,20}. Rather, the
parametric pumping acts on the standing spin-wave (SSW) modes
(see Fig. 1b) which are excited across the film thickness by
the traveling wave packet through a two-magnon scattering
mechanism. These modes are then amplified by parametric
pumping and are scattered back to form a new traveling DE
wave, which is finally detected at an output antenna in a form
of the “recovered” pulse (Fig. 1).

Unlike in previous studies\textsuperscript{12,21,22}, in the experiment reported
here, two consecutive parametric pump pulses are used rather
than one. The response of the magnonic system to the second
pulse is utilized as a probing tool for extracting information
about the free relaxation behavior of parametrically excited
magnons during the pump free pause. Therefore, we cut off
the first recovered pulse by stopping the parametric pumping.
After a certain delay time $\tau_{\text{delay}}$, a second pump pulse is
applied and a second recovered pulse is observed (see continu-
ous line in Fig. 2). We register the peak amplitude $A_{\text{max}}$
and the arrival time $t_{\text{max}}$ of the second recovered pulse.

As one sees from Fig. 2, the behavior of the second re-
covered pulse strongly depends on the duration of the first
pumping pulse $\tau_{\text{pump1}}$ for small $\tau_{\text{pump1}} \leq 450$ ns as well
as on the delay time $\tau_{\text{delay}}$ between two consecutive pump-
ing pulses. It is remarkable that the latter dependence is non-
monotonic: with an increase in $\tau_{\text{delay}}$ the second recovered pulse first grows, reaches a maximum, and then decreases in
intensity (Fig. 2a). In particular, one observes a gain in the
amplitude of the second recovered pulse compared to the exper-
iment in which the pumping is uninterrupted ($\tau_{\text{delay}} \to 0$
in Fig. 2).

III Theoretical model

One easily finds that neither the initial increase in the am-
plitude nor the subsequent non-exponential decrease (note the
logarithmic scale) in the peak amplitude can be explained in
the framework of the standard approach based on association
of a specific relaxation time with each decay process. For
this reason we make an attempt to employ a more detailed
model. The model we use is an extension of the theory in
Ref.\textsuperscript{22}. The latter succeeded in describing the single-pulse
pumping. Treating a two-pulse pumping with a significant
length of the pause between the two pulses requires consider-
ing dephasing within the parametrically excited wave packet
during the pause. Thus, in contrast to Ref.\textsuperscript{22}, one cannot use
ensemble-averaged equations for magnon densities and has
to consider each magnon group as consisting of a number of
pairs of waves with different eigenfrequencies.

We start the description of the developed theory by noting
that the recovered pulse has a pulse-like shape. This sug-
gests that competition of two magnon groups for energy pro-
duced by the parametric pumping process\textsuperscript{22} takes place in
the magnon system, since a pulse-like shape of the observed sig-
nal is a typical result of this competition. This process can
be explained in the following way. The parametric amplifica-
tion is frequency selective: only magnon groups with eigen-
frequencies $\omega_k$ which lie inside the narrow frequency band
$\omega_p/2 - \nu < \omega(k) < \omega_p/2 + \nu$ whose width is equal to the
parametric amplification gain $\nu = h_p V$ are amplified ($V$ is
the parametric coupling coefficient). They are driven at half the
frequency $\omega_p/2$ of the applied microwave field. Importantly,
there is no restriction on the magnitude and the direction of
the magnon wave vector: the process allows creation of magnons
with arbitrary wave vectors, provided their frequencies are in-
side this frequency band. Thus, given the large number of
frequency-degenerate magnon dispersion branches, magnon
groups over a large range of wave vectors are amplified. This
includes externally excited oscillations, hereafter called the
signal group, as well as thermally activated magnons with
wave vectors up to $10^5$ cm$^{-1}$. The latter magnons, which
are considerably decoupled from structural defects of the YIG
film because of their short wavelength, experience the lowest
two-magnon decay and consequently show the highest para-
As the phase shift between the external pumping field \( \vec{h}_p \) and a microwave magnetic field induced by the parametrically pumped magnons (“internal pumping”) increases with increase in the total magnon density, the resulting effective pumping field inside the sample consecutively diminishes. At some point in time the total magnon density reaches a critical threshold level \( A_{cr} \) at which the effective pumping becomes so small that it is able to further support just one magnon group, the one which has the lowest decay rate and the highest efficiency of parametric interaction: the dominant group. This leads to the suppression of the signal group, whilst the dominant group reaches saturation at \( A_{cr} \). From this time on the system stays in a quasi-equilibrium state wherein the magnon density of the dominant group is saturated. The structure of obtained equations allows us to renormalize the spin-wave amplitudes \( C_\omega \) such that \( S = 1 \) and only the ratio \( |S/T| \) matters \( (T < 0) \). Initial conditions for the dominant group are random distribution of amplitudes for its frequency components. Initial conditions for the signal group follow from the expression for the frequency spectrum of a rectangular pulse:

\[
C_\omega(t_1) = C_0 \exp(i\omega t_1) F(\delta\omega) \frac{\sin(\omega t_{input}/2)}{\omega t_{input}/2},
\]

is the effective pumping for the signal and the dominant groups respectively. In Eqs. (3-6) and below the upper indices “s” and “d” denote the signal and the dominant group respectively. Double indices of type \( \alpha\beta \) denote the action of the group \( \beta \) on the group \( \alpha \). If both indices are the same the respective coefficient or magnitude describes self-action.

In our calculations we make a number of important simplifications which allow us to considerably decrease the computation time. First we use a usual assumption that the non-linear frequency shift is not important for the dynamics of the dominant group \( (\Delta\omega_{\text{dd}} = 0) \). Second, we do not need to take into account the contribution to the total nonlinear frequency shift by the signal group \( (\Delta\omega_{\text{ss}} = \Delta\omega_{\text{ds}} = 0) \), since its amplitude is considerably smaller than the amplitude of the dominant group at the time, when the effective pumping saturates. Thus, the only term we have to account for is

\[
\Delta\omega_{\text{sd}} = T_{\text{sd}} \sum_{\omega', \omega'', \omega'''} C_{\omega'}^d C_{\omega''}^s C_{\omega'''}^s \delta(\omega' - \omega'' + \omega''' - \omega), \quad (7)
\]

where \( \delta \) denotes the Kronecker Delta, and \( T_{\text{sd}} \) is the respective nonlinear coefficient. In the following we will use a short hand notation \( T_{\text{sd}} = T \).

Similar considerations apply to the contributions to the total pumping which consists of the external pumping \( (\nu) \) and the internal pumping given by the remainder of the terms in Eqs. (5, 6). We obtain: \( P_{\text{ss}} = P_{\text{ds}} = 0 \),

\[
P_{\text{sd}} = S_{\text{sd}} \sum_{\omega', \omega'', \omega'''} C_{\omega'}^d C_{\omega''}^s C_{\omega'''}^s \delta(\omega' - \omega'' + \omega''' - \omega), \quad (8)
\]

\[
P_{\text{dd}} = S_{\text{dd}} \sum_{\omega', \omega'', \omega'''} C_{\omega'}^d C_{\omega''}^d C_{\omega'''}^d \delta(\omega' - \omega'' + \omega''' - \omega), \quad (9)
\]

where \( S_{\alpha\beta} \) are the respective nonlinear S-coefficients.

Test numerical calculations show that calculation results do not qualitatively change with the variation in the difference \( S_{\text{dd}} - S_{\text{sd}} \) in reasonable limits. Therefore, to minimize the number of degrees of freedom we set \( S_{\text{dd}} = S_{\text{sd}} \equiv S \) and, as previously, we assume that \( S > \nu^2 \).

The last simplification we make is removing the frequency-mixing terms from Eqs. (7-9). This simplification does not lead to a qualitative change in the results of our numerical calculations, but enormously decreases the computation time. It reduces Eq. (4) to a simple formula for the effective pumping:

\[
P_{\omega}^s = P_{\omega}^d = P_{\omega} = \nu + S \sum_{\omega} \left( C_{\omega}^d \right)^2, \quad (10)
\]

For the pump-free period the same equations (1-10) are valid, one just assumes \( \nu = 0 \), so that only the internal pumping is on during the pause.

The structure of obtained equations allows us to renormalize the spin-wave amplitudes \( C_\omega \) such that \( S = 1 \) and only the ratio \( |S/T| \) matters \( (T < 0) \). Initial conditions for the dominant group are random distribution of amplitudes for its frequency components. Initial conditions for the signal group follow from the expression for the frequency spectrum of a rectangular pulse:
The light red areas indicate the time intervals when the pump is on. The gray area in panel b) indicates the input spin-wave pulse. The system of equations (1,2) was solved numerically. We used 200 discrete values of eigenfrequency detuning $\omega$ from the half-pump frequency in the range from $-2\nu$ to $2\nu$. This formed a system of 200 coupled nonlinear equations which was resolved by using 4th-order Runge-Kutta method.

We do numerical calculations for $\tau_{\text{delay}} = 400$ ns and for a number of ratios $|S/T|$, a number of values of rates of parametric gain, and a number of decay rates for both magnon groups. Each time all these parameters, including $C_0$, are chosen such that the restored pulse peaks at the same time as in the experiment when the parametric pumping is not interrupted ($\tau_{\text{delay}} = 0$). A natural constraint we impose on the validity of simulation results is a single-peaked shape for the restored pulse for all experimental lengths of the pump-free pause $\tau_{\text{delay}}$.

**IV Discussion**

A number of simulation runs allows us to locate the area in the parameter space, where the simulated behavior is close to the experimental one displayed in Fig. 3. Figure 4a shows the best fit we obtain. One sees a fair agreement with the experimental values of peak amplitudes of the restored pulse and of the times for the peak arrival $t_{\text{max}}$ as a function of the pause length $\tau_{\text{delay}}$. This calculation shows that the terms involving $S$ and $T$ coefficients do not contribute to the dynamics during the pump-free pause, as the parametric amplification is quite far from saturation during this time interval. Note that analytical solutions exist for $S = T = 0$ and they are in full agreement with our simulations for the first two stages of the considered process ($t < 0$ in Fig. 4b-4d).

Figures 4b-4d show the behavior of the macroscopic amplitude of the signal group $A_s = 10 \log(|\sum C_0^2|)$ and of the strength of the internal pumping $A_d = 10 \log(|\sum C_0^{d2}|)$. One sees that during the pump pause both magnitudes do not vary linearly on the logarithmic scale. The signal-group behavior is close to parabolic on this scale. Note the increase in the amplitude for the signal group during the first 200 ns after switching off the first pump in Fig. 4d. A careful analysis of this stage shows that the increase is due to phase reversal in a parametric-echo-like process. Recall that the coherence of the originally deterministic signal is lost due to dephasing during the time interval $\tau_0$ between the passage of the traveling wave pulse and the application of the first pump pulse. In the phase reversal process the coherence is restored. The full phase restoration occurs for $\tau_{\text{delay}} = \tau_0$.

The behavior of $A_d$ is more complicated: for the first 150 ns after the first pumping has been switched off, the internal pump decreases parabolically on the logarithmic scale, then its behavior switches to more or less linear on the same scale. This non-monotonic behavior of the internal pump also originates from dephasing. During the first pump pulse the frequency width of the dominant group reduces significantly due to the frequency selective amplification of the initially thermal signal. The phases of the waves which belong to this group are locked to the external pumping and the behavior of the internal...
ternal pumping follows the linear behavior of the number of magnons, which is in full agreement with L'vov's S-theory. However, once the first pumping has been switched off, the phase coherence within the group is lost and the magnitude of $A_{q}$ starts to decrease. As a result, at large times ($t > 300$ ns) the magnitude of $A_{q}$ essentially follows the exponential decrease in the number of magnons, but deviates significantly from this law because of the dephasing for $t < 300$ ns.

Both echo-like behavior of the signal group and dephasing in the dominant group contribute to the non-monotonic behavior of the peak amplitude of the restored signal. From Fig. 4 one clearly sees that for $\tau_{\text{delay}} = 300$ ns (Fig. 4c) the amplitude of the signal at the end of the pause ($t = 0$ in this graph) is quite close to the amplitude of this group at $t = 0$ for the uninterrupted pump. However, the internal pump by the dominant group at $t = 0$ for $\tau_{\text{delay}} = 300$ ns is smaller than for $\tau_{\text{delay}} = 0$ by 25 dB. This allows the recovered signal developing a larger peak amplitude for $\tau_{\text{delay}} = 300$ ns than for $\tau_{\text{delay}} = 0$. Obviously, a larger $\tau_{\text{delay}}$ requires a larger time for the restored signal to peak, which results in a linear dependence of $t_{\text{max}}$ on $\tau_{\text{delay}}$ in Fig. 3a.

In agreement with experiment, the model shows a variation of $A_{\text{max}}$ and $t_{\text{max}}$ with $\tau_{\text{pump}}$. However, in the model the effect is not so pronounced as in the experiment. We also checked if nonlinear damping in the form of four-magnon scattering processes involving all spin waves existing at the half-pump frequency can contribute to the non-monotonic response. Our calculations for $\tau_{\text{pump}} = 400$ ns have shown that the nonlinear damping is negligible during the pump pause for reasonable values of the nonlinear damping coefficient. The amplitudes of both signal and dominant groups are just too small during the pause to develop the non-linear damping. Accounting for the nonlinear damping during the second pump pulse does not lead to qualitative changes in the behavior either, unless one assumes that the coefficient of nonlinear damping is larger than $T$ by several orders of magnitude, which is unreasonable.

As a final note of this section we want to comment on applicability of the two-pulse method for other materials. YIG is a unique material: the linear relaxation time for YIG is 100 ns. This allows dephasing of the wave packet in our experiment well before amplitudes of its constituents drop to the thermal level (see Fig. 3, where the typical time scale is hundreds of nanoseconds, which is given by the rate of dephasing.) For comparison, the best metallic magnetic material - Permalloy - has magnetic relaxation time $< 5$ ns. This material is operational in the same frequency range as YIG, thus the dephasing rate would be the same, be this experiment conducted with Permalloy. Obviously, for the reason of the much stronger decay rate we would not be able to register any signal recovered by the second pump pulse, as on the time scale of hundred nanoseconds no coherent signal would survive in a Permalloy film to the time of its arrival. However, the new materials, like Heusler alloys and other half-metals, theoretically may have magnetic losses of the same order as YIG. If quality of these materials is improved in the nearest future, it will be possible to use the two-pulse approach for these materials as well.

V Conclusion

In this work we have investigated the relaxation of a free evolving gas of previously parametrically pumped magnons. The experimental results show a clear deviation from the standard exponential spin-wave decay model. In particular, the inherent magnon damping is found to depend upon the presence of the parametric pumping field. The results are in agreement with the model which accounts for variation of phase coherence for parametrically injected magnon groups during the pump-free pause.

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1. H. Suhl, IEEE Trans. Mag. 34, 1834 (1998).
2. T. Gilbert, IEEE Trans. Mag. 40, 3443 (2004).
3. A. Azevedo, A. B. Oliveira, F. M. de Aguiar, and S. M. Rezende, Phys. Rev. B 62, 5331 (2000).
4. S. Rezende, A. Azevedo, M. Lucena, and F. de Aguiar, Phys. Rev. B 63, 214418 (2001).
5. A. Y. Dobin and R. Victora, Phys. Rev. Lett. 90, 167203 (2003).
6. K. Gilmore and M. D. Stiles, Phys. Rev. B 81, 174414 (2010).
7. S. O. Demokritov, V. E. Demidov, O. Dzyapko, G. A. Melkov, A. A. Serga, B. Hillebrands, and A. N. Slavin, Nature 443, 430 (2006).
8. A. Khitun, M. Bao, and K. L. Wang, J. Phys. D: Appl. Phys. 43, 264004 (2007).
9. O. Dzyapko, V. E. Demidov, S. O. Demokritov, G. A. Melkov, and A. N. Slavin, New Jour. Phys. 9, 64 (2007).
10. H. Suhl, J. Phys. Chem. Solids 1, 209 (1957).
11. R. M. White and M. Sparks, Phys. Rev. 130, 632 (1963).
12. A. A. Serga, A. V. Chumak, A. André, G. A. Melkov, A. N. Slavin, S. O. Demokritov, and B. Hillebrands, Phys. Rev. Lett. 99, 227202 (2007).
13. V. Cherepanov, I. Kolokolov, and V. L'vov, Phys. Rep. 229, 81 (1993).
14. Y. Kajiwara, K. Harri, S. Takahashi, J. Ohe, K. Uchida, M. Mizuguchi, H. Umezawa, H. Kawai, K. Ando, K. Takanashi, S. Maekawa, and E. Saitoh, Nature, 464, 262 (2010).
15. K. Uchida, J. Xiao, H. Adachi, J. Ohe, S. Takahashi, J. Ieda, T. Ota, Y. Kajiwara, H. Umezawa, H. Kawai, G. E. W. Bauer, S. Maekawa, and E. Saitoh, Nature Mater. 9, 894 (2010).
16. S. A. Manoulov, R. Fors, S. I. Khatsev, and A. M. Grishin, J. Appl. Phys. 105, 033917 (2009).
17. O. V. Kolokolsev, C. L. Ordóñez-Romero, N. Qureshi, O. Cortes-Pérez, G. López-Maldonado, and M. Avendaño-Alejo, Electron. Lett. 46, 1387 (2010).
18. R. W. Damon and J. R. Eshbach, J. Appl. Phys. 31, 104 (1960).
19. B. A. Kalinikos, N. G. Kovshikov, M. P. Kostylev, and H. Benner, JETP Letters 64, 171 (1996).
20. B. A. Kalinikos and M. P. Kostylev, IEEE Trans. Mag. 33, 3445 (1997).
21. S. Schäfer, A. V. Chumak, A. A. Serga, G. A. Melkov, and B. Hillebrands, Appl. Phys. Lett. 92, 162514 (2008).
22. A. V. Chumak, A. A. Serga, B. Hillebrands, G. A. Melkov, V. Tiberkevich, and A. N. Slavin, Phys. Rev. B 79, 014405 (2009).
23. V. S. L’vov, Wave Turbulence Under Parametric Excitation (Springer, Berlin, 1994).
24. V. E. Zakharov, V. S. L’vov, and S. S. Starobinets, Sov. Phys. Uspekhi 17, 896 (1975).
25. G. A. Melkov, Y. V. Kobljanskyj, A. A. Serga, V. S. Tiberkevich, and A. N. Slavin, Phys. Rev. Lett. 86, 4918 (2001).
26. G. F. Herrmann, R. M. Hill, and D. E. Kaplan, Phys. Rev. B 2, 2587 (1970).
27. M. M. Scott, C. E. Patton, M. P. Kostylev, and B. A. Kalinikos, J. Appl. Phys. 95, 6294 (2004).
28. T. Kubota, S. Tsunegi, M. Oogane, S. Mizukami, T. Miyazaki, H. Naganuma, and Y. Ando, Appl. Phys. Lett. 94, 122504 (2009).
29. C. Liu, C. K. A. Mewes, M. Chshiev, T. Mewes, and W. H. Butler, Appl. Phys. Lett. 95, 022509 (2009).
30. S. Trudel, O. Gaier, J. Hamrle, and B. Hillebrands, J. Phys. D: Appl. Phys. 43, 193001 (2010).