Day-Ahead Scheduling for Economic Dispatch of Combined Heat and Power With Uncertain Demand Response

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ABSTRACT This paper presents an energy management method for the interconnected operation of power, heat, Combined Heat and Power (CHP) units to settle the Day-Ahead market in the presence of a demand response program (DRP). A major challenge in this regard is the price uncertainty for DRP participants. First, the definitive model of the problem is introduced from the perspective of the Regional Market Manager (RMM) in order to minimize the total supply cost in the presence of TOU program, which is a type of DRP. Furthermore, a market-oriented tensile model is presented in the form of a combination of over-lapping generations (OLG) and price elasticity (PE) formulations to determine the amount of electricity demand in the TOU program. Then, a price uncertainty model of the proposed problem is introduced according to the IGDT risk aversion and risk-taking strategies considering information gap decision theory (IGDT). The above problem is solved through the use of the co-evolutionary particle swarm optimization (C-PSO) algorithm and the proposed model is implemented on a standard seven-unit system for a period of 24 hours.

INDEX TERMS Combined heat and power, time of use, diamond’s OLG model, price uncertainty, information gap decision theory, co-evolutionary particle swarm optimization.

NOMENCLATURE

Index of time, power-only, CHP and heat-only units, respectively

Sets of peak load time, flat load time, off-peak load time and load time, respectively

Set of power demand
Sets of power-only, CHP and heat-only units, respectively

Cost coefficients for ith CHP unit
Cost coefficients for kth heat-only unit
Initial prices and Initial prices at time t ($/MWh)
Critical cost for opportunity function
Minimum and maximum initial power demand (MW)
Percentage increase in cost for RMM
Initial power and heat demand at time t (MWh and MWh)
Power Price Elasticity Matrix in the peak, flat and off-peak hours

 cost coefficients for jth CHP unit
 cost coefficients for kth heat-only unit
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\[ p_{C \min}^{j,t}, p_{C \max}^{j,t} \]

Minimum and maximum power and heat outputs for CHP units, respectively

\[ p_{d \text{-} peak}^0 (t), p_{d \text{-} flat}^0 (t), p_{d \text{-} Off - peak}^0 (t) \]

correspond to power consumption before implementing DRP at time \( t \) during the peak, flat and off-peak hours (MWh)

\[ B_0, B_{00}, \delta_{\text{min}}, \delta_{\text{max}} \]

loss coefficient of the \( i \)th unit, loss coefficient parameter

\[ \rho_{\min}^0, \rho_{\max}^0, \rho_{\min}^{j,t}, \rho_{\max}^{j,t} \]

Minimum and maximum value of Price fluctuation limit

\[ \tilde{P}_0 (t) \]

Forecasted uncertainty variable at time \( t \)

\[ C_r \]

Critical cost for robustness function

\[ C_b \]

Minimum expected cost of RMM

\[ \gamma \]

Percentage decrease in cos for RMM

\[ p_{\text{peak}}, p_{\text{flat}} \]

price after implementing DRP during the peak, flat and off-peak hours ($/MWh$)

\[ \theta \]

Relative risk aversion

\[ \rho \]

uncertainty radius

\[ P_{d,t}, H_{d,t} \]

Power demand after implementing DRP at time \( t \) (MWh and MWhth)

\[ \rho_i \]

prices at time \( t \) ($/MWh$)

\[ P_{i,t}^r \]

real power output for power-only of the \( i \)th unit at time \( t \) (MWh)

\[ P_{C_i}^r, H_{C_i}^r \]

real power and heat output for CHP of the \( j \)th unit at time \( t \) (MWh and MWhth)

\[ P_{L,t} \]

power transmission loss at time \( t \) (MWh)

\[ H_{k,t}^r \]

real heat output for heat-only of the \( k \)th unit at time \( t \) (MWhth)

\[ P_{d, \text{peak}}, P_{d, \text{flat}}, P_{d, \text{Off - peak}} \]

power consumption after implementing DRP during the peak, flat and off-peak hours (MWh)

\[ \tilde{P}_d, \tilde{P}_{d, \text{flat}}, \tilde{P}_{d, \text{Off - peak}} \]

Power load reduction after implementing DRP during the peak, flat and off-peak hours (MWh)

\[ P_{d, \text{peak}} (t), P_{d, \text{flat}} (t), P_{d, \text{Off - peak}} (t) \]

power consumption after implementing DRP at time \( t \) during the peak, flat and off-peak hours (MWh)

\[ \text{OF} \]

Total cost function of RMM in $
\[ \hat{\beta} (C_O) \]

Opportunity function

\[ \hat{\alpha} (C_r) \]

Robustness function

I. INTRODUCTION

A. MOTIVATIONS

The shortage of energy sources of fossil fuels, low efficiency of the power plants as well as the high amount of wasted energy in the form of heat resulted in [1] applying CHP units that make use of the residual heat for increasing the efficiency of the energy conversion [2]. In this regard, Combined Heat and Power Economic Dispatch (CHPED) is considered an important optimization problem for determining the optimal output of power and the heat of each unit in order to minimize the production costs considering the system constraints. In other words, it can be said that the RMM uses CHPED to settle the leading day market [3]. On the other hand, increasing energy on the demand side increases the daily load curve, overloads transmission lines, causes unpredictable accidents, and leads to system instability and a sharp rise in electricity prices. For this purpose, energy consumption pattern optimization and management policies are performed by demand-side management (DSM) [4]. It should be noted that DRPs as one of the most useful solutions for DSM systems can play a complementary mechanism in controlling the fluctuations caused by sharp increases in electricity prices [5]. In this study, the day-ahead market settlement framework is presented with several market participants, including heat-only, power-only and CHP units in the presence of the TOU program. Here, RMM, as the leader of the regional market, aims to minimize the total supply costs. Therefore, it is necessary to create an optimization model based on the supply cost function of the heat-only, power-only and CHP units and TOU in the day-ahead market to minimize the total supply cost. Here, the TOU cost function is the amount of demand reduction when the subscribers reduce their electricity demand during peak hours or transfer to other
periods due to the implementation of the TOU program. Moreover, since the sharp fluctuations of electricity prices in the day-ahead market is one of the serious challenges in the TOU program and also because definite methods are not able to provide acceptable and accurate analysis to solve this problem, the IGDT uncertainty method has been used in this study to solve this challenge.

**B. LITERATURE REVIEW**

In recent years, significant studies have been conducted on the impact of DRPs on the CHPED issue called DR-CHPED, which has often been used for determining the price and demand levels based on definitive and random models. However, only a few articles have investigated the impact of DRPs price uncertainty on the CHPED issue. For example, references 6 to 12 used the definitive DRP model in the CHPED problem. In [6], an operating cost model of interconnected MGs (Microgrids) was presented in the presence of DRP and CHP using the distributed (decentralized) energy management method. In addition, a distributed iterative algorithm was also proposed based on subgradient with a dynamic search direction. In [7], the demand response aggregator (DRA) interaction with ISO was checked through CHP systems, wind turbine (WT), energy storage system (ESS) and heat buffer tank (HBT) systems to maximize participants’ profits in the concurrent electricity and heat markets. In [8], an MG including CHP, ESS and DRP was presented using DRP as the virtual generation units. The first and second objective functions of the optimization problem include minimizing the total operating cost of the MG and minimizing the emission of DGs, respectively. In [9], a chaotic fast convergence evolutionary programming (CFCEP) was investigated to solve the CHPED problem with DSM through the use of renewable energy sources and pumped storage hydraulic units. In [10], short-term hourly planning of the industrial and commercial customers was presented in the presence of CHP, HBT and DRP units. Furthermore, DRP has been used to minimize the providing cost of the power and heat services for the customers. In [11], planning of the CHP systems under TOU electricity tariffs was presented by minimizing the subscribers’ electricity costs and the cost of operating CHP under CHP restrictions. Problem modelling and optimization was performed using Mixed-Integer Nonlinear Programming (MINLP) and PSO, respectively.

In [12], a framework was presented for the optimal performance of a CHP energy system according to the TOU program, in which a robust optimization has been used to determine CHP risk. References 13 to 15 used the random DRP model in the CHPED problem. In [13], a bi-level programming approach was presented for the MG-based CHP. An upper-level model and a lower-level model were proposed to maximize the profits of the MG operator and the profits of CHP owners, respectively. Furthermore, the demand and price of DRP were evaluated based on the random model of Scenario Reduction and the wind speed was considered based on the random programming technique of the autoregressive moving average (ARMA). In [14], a model was presented for the optimal control and the long-term evaluation of CHP and HBT in the presence of market price uncertainty. Price uncertainty was solved using the Least Squares Monte Carlo regression (LSMCR) random control model. In [15], a scenario-based (SB) stochastic programming framework was used to model the load and wind uncertainty in the combined problem of CHP, WT and DRP. Furthermore, the PSO algorithm was also used to achieve the optimal solution to the problem. In [16], thermal generators (i.e., spinning reserves), demand-side resources and also battery storage are considered as reserves. Besides, optimal dynamic and sequential reserve activation plans are developed by using the optimal coordination of fast reserves (i.e., demand-side resources and battery storage) and slow reserves (i.e., spinning reserves). The optimization problem is solved using the MATLAB optimization toolbox (Fmincon function). The effectiveness and suitability of the proposed approach is tested on IEEE 30, 118 and 300 bus test systems. Reference [17] optimizes the risk-constrained scheduling of a wind-integrated smart multi-carrier energy hub and evaluates its operation in combination with a compressed air energy storage system, an electrical demand response program and a thermal demand response program. The wind turbine generation and electrical/thermal demands are modelled as a scenario-based stochastic problem using the Monte Carlo simulation method. Moreover, the conditional value-at-risk (CVaR) algorithm is merged with the proposed model to propagite the risk of the high costs relevant to the worst scenarios.

Ref. [18] used the DRP uncertainty model in the CHPED problem. In [18], due to the limitations of minimizing the number of start-ups and shutdowns, ramp rate limits and minimum up / down-time limits of generation, DRP price uncertainty has been addressed through the use of robust optimization in order to minimize the CHP costs.

In addition, in the combined issue of DRP and CHPED, a comprehensive and accurate model is not considered for DRPs. For example, references [6]–[8] have modelled customer behaviour characteristics based on the linear demand model and the PE Matrix, and references [9]–[18] have not defined a precise model for DRPs.

In [19], an optimal integrated/dynamic post-contingency reserve activation plan has been developed by utilizing fast and slow reserves from generating units and demands. In this approach, an attempt is made to remove overloads, following a contingency, using coordinated action of fast and slow reserves, in order to restore secure operation at the minimum overall cost. In this paper, the reserves are supplied by the conventional thermal generators (spinning reserves), hydro power units and load demands (demand-side reserves). In [20], congestion management is performed using the generator rescheduling by taking into account the minimization of generation scheduling cost while satisfying all the line flow limits and customers’ demand response. [19],
have not used models that characterize the behaviour of demand-side subscribers and only considered a limited number of changes for the responsive load.

The reason for using the PE Matrix in some DRP and CHPED combination issues is that this model is simple and highly efficient. The concept of PE Matrix is defined as the ratio of the relative change in demand to the relative change in the electricity prices. One problem with using the PE Matrix is that the price elasticity depends on the operating points on the load curve, which means that the PE Matrix must be recalculated as the result of any change in the load curve. As an alternative, Diamond’s Over Lapping Generations (OLG) model has been used to avoid reducing the simulation accuracy of DRPs, which is more flexible in DRPs than the PE Matrix model [21].

Reference [22] develops a multi-objective day-ahead market clearing mechanism considering with demand response offers and realistic voltage dependent load models. Some objectives are considered in Social Welfare Maximization such as demand response offers/ load reduction cost, load reduction minimization, and load served error minimization. However, the dependence of the voltage is the main drawback of this load model, and it does not consider the effects of price signals on subscribers and the sensitivity among the consumption patterns involved in the PE Matrix.

In [23], the DRP model was represented by using Diamond’s OLG formula. This model is market-oriented and indicated an effective function for modelling consumer behaviour. In [24], the OLG Diamond’s model was used in the TOU program to test the flexibility of this model at different levels of load with respect to the price signals. The disadvantage of Diamond’s OLG model is that customers only change their demand from period to period in response to the price signals, and the loads that are self-sensitive, i.e. reduce their consumption in a targeted time period are not used. In this model, the relative risk limit means the additional compensation that a person expects to achieve during the next period at the cost of a unit reduction regarding his consumption in the current period. In the same regard, the time preference rate indicates that the benefits that the person will gain in the next period are more valuable than what is earned in the current period. In order to overcome the limitations of the mentioned models, a combination of PE Matrix and Diamond’s OLG models has been used in this study for evaluating customer behaviour.

In [25], objectives such as maximizing social welfare including demand-side reserves, energy reserves, demand and minimizing load-service error by considering actual voltage-dependent load modelling are taken into account. They are cleared simultaneously through a participatory optimization process. In such circumstances, a multi-objective market clearing mechanism with DR recommendations is essential. To solve the optimization problem, the Pareto evolution algorithm of multi-objective power (SPEA 2+) has been used. Reference [26] presents a short-term DR trading model using a bilevel optimisation framework. At the upper level, the retailer’s problem is to maximise expected payoff, i.e. revenues earned by selling energy to end-users minus the expected cost of purchasing from the wholesale energy pool and the DR aggregators. The evolution of mean reverting volatility in pool electricity prices is captured as a stochastic jump-diffusion process. The CVaR-based risk metric approach is used to trade off the expected payoff in the presence of electricity demand and pool price uncertainties. The effect of bidding strategies, user willingness, and the DR capacity limits on optimal DR share is investigated. Reference [27] proposes a bi-level market model for wind-integrated electricity market, where the DR requirement is paired with the wind profile to deal with wind variability. At the upper level, an electricity market operator aims to minimise the day-ahead operation cost considering plausible wind generation scenarios. At the lower level, the DR exchange operator aims to maximise social welfare by trading aggregated DR among several aggregators. The solution at this level determines the optimal DR amount and price setting for each aggregator. The DR from the flexible loads is modelled from the end-users perspective considering their willingness parameter. The market model is formulated as a bi-level optimisation problem using Lagrangian relaxation with Karush–Kuhn–Tucker optimality conditions. Considering the above research and other studies, there are four main gaps on the impact of DRPs on the CHPED which can be listed as follows:

- Most studies [6]–[12] have used a definite model to solve the DRP problem, but it was noted that various parameters such as load demand and electricity price in DRP are considered as uncertainty parameters [13]–[18].
- Some studies such as [13]–[19] have used the random DRP model in the CHPED problem. However, this model requires a large number of scenarios and knowledge about the probability density function (PDF) for achieving the right answer. Hence, the design of the proposed problem is complicated and the method of solving it will be difficult [23].
- Research [13]–[23] has provided various frameworks for examining the characteristics of uncertainty, including scenario-based, Monte Carlo, fuzzy optimization, Scenario Reduction and robust optimization. These methods depend on the historical data of the uncertainty variables and if these data are incorrect or unavailable, decisions cannot be reliable. In addition, other drawbacks of these methods are that if the number of scenarios increases the computational load of the problem increases as well or there is a two-level optimization problem in the robust optimization that is usually difficult to solve.
- Research [6]–[8] has used the modelling of these programs by PE Matrix for investigating the DRPs. However, it was pointed out that the main drawback of PE Matrix is the dependence of the price upon the operating point on the load curve.
Table 1 explains the different classification of recent studies about the impact of DRPs on CHP.

Table 1. Classification of recent studies about the impact of DRPs on CHP.

| Ref. No. | Objective function | DRP Model | DRP | Uncertainty parameters DRP | Uncertainty model |
|----------|--------------------|-----------|-----|--------------------------|------------------|
| [6]      | MFC & MDRC        | PE Matrix | TBP | NO                        | NO               |
| [7]      | Maximize Genco & DR profit | PE Matrix | IBP | NO                        | NO               |
| [8]      | MFC & MDRC        | PE Matrix | IBP | NO                        | NO               |
| [9]      | MFC & MDRC        | NO        | TOU | NO                        | NO               |
| [10]     | Maximize Genco & DR profit | NO        | IBP | NO                        | NO               |
| [11]     | MFC & MDRC        | NO        | TOU | NO                        | NO               |
| [12]     | MFC, MPE & MDRC  | NO        | IBP | NO                        | NO               |
| [13]     | MFC & MDRC        | NO        | TOU | NO                        | NO               |
| [14]     | Maximize Genco & DR profit | NO        | IBP | Yes                       | ARIMA Stochastic |
| [15]     | Maximize Genco & DR profit | NO        | IBP | Yes                       | Scenario Based Stochastic |
| [16]     | Minimize cost MFC | Constant  | TBP | NO                        | NO               |
| [17]     | Minimize cost    | Constant  | TBP | Yes                       | scenario-reduction and CVaR |
| [18]     | MFC & MDRC        | NO        | IBP | NO                        | LSMCR Stochastic  |
| [19]     | Maximize social welfare | DR offers/DR reserves | IBP | YES                       | NO               |
| [20]     | MFC & MDRC        | NO        | IBP | Yes                       | SB Stochastic    |
| [21]     | MFC & MDRC        | NO        | IBP | YES                       | Robust uncertainty |
| [22]     | Proposed method   | MFC & MDRC | PE & Diamond’s OLG | TOU | NO | Yes | IGDT uncertainty |

*MFC: Minimum Fuel Costs  *MPE: Minimum Pollutant Emissions  *TBP: Time-based Programs  *IBP: Incentive-based Programs

Table 1 explains the different classification of recent studies about the impact of DRPs on CHP.

To address the first, second and third issues, this paper presents the electricity price uncertainty using IGDT theory in order to answer TOU program on the CHPED issue. Firstly, in order to minimize the fuel costs and to determine the optimal price and demand in the TOU, the definitive DR-CHPED model has been investigated for determining the optimal output of power and heat of each unit by considering some technical constraints such as power and heat balance constraint, power and heat generation limits, price fluctuation limit, demand ceiling. To address the fourth problem, a market-oriented tensile model is presented in the form of a combination of PE & Diamond’s OLG formulations for determining the amount of demand in the TOU. Next, according to the concept of IGDT theory, a model of electricity price uncertainty is introduced in the TOU program for the proposed problem. RMM uses DR-CHPED based on risk aversion and risk-taking strategies for minimizing the total supply cost in the leading day market. As a result, RMM can choose the best strategy considering the desired level of risk that is mutually beneficial for the manufacturers and subscribers.

It was noted that the IGDT method is used to determine the uncertainty of electricity prices in the proposed problem since it can be implemented with minimal information on uncertainty parameters. In addition, IGDT provides a definite framework for different budgets that reduce the burden and time of calculations. Furthermore, the IGDT method is well able to investigate severe uncertainties in mathematical programming problems. For example, in [28], the reliability-based unit commitment was solved and then the obtained results were used to clear the reserve market in the presence of DRP based on the IGDT theory while considering the uncertainty of the load demand parameter. In [29], IGDT theory was used to plan DRP electricity price uncertainty for maximum profits of the retailers. In [30], the optimal performance of a microgrid including photovoltaic, fuel cell, the battery in the presence of DRP was evaluated taking into account the uncertainty of the electric charge. In this regard, the IGDT theory has been used to model the electric charge uncertainty in order to minimize the total cost of the microgrid. Moreover, the examination of the references mentioned in the previous paragraphs reveals that the PSO algorithm has a good performance in achieving the optimal solution of the problems [11], [15]. The performance of this algorithm is based on searching for particles (such as a flock of birds) for finding the optimal answer (for example, their food). This algorithm, while being simple, has a relatively acceptable efficiency in optimization [31]. Among the modifications of this algorithm, it can be referred to the improvement of the method of assigning a random number to each particle in every iteration of the algorithm, the particle velocity adjustment and the mirror effect (reflection) [32], [33]. In the current study, a combination of the co-evolutionary theorem with the PSO algorithm or the C-PSO algorithm is applied. Co-evolution means the ability of the algorithm to solve the optimization problem during every hour of a time interval.

C. CONTRIBUTIONS

This paper focuses on the interconnected operation of power, heat, CHP units to settle the Day-Ahead market in the presence of TOU under different price uncertainty conditions.
Furthermore, a market-oriented tensile model is presented in the form of a combination of over-lapping generations (OLG) and price elasticity (PE) formulations for determining the amount of electricity demand in the TOU program.

It is clear that the uncertainty related to price results in the volatility of the operation cost. Therefore, the modelling of risk of operation cost variation can be useful for the RMM. In addition, the IGDT method is applied to model the risk of operation cost variation of the DR-CHPED problem.

The above problem is solved through the use of the co-evolutionary particle swarm optimization (C-PSO) algorithm and the proposed model is implemented on a standard seven-unit system for a period of 24 hours. Consequently, the main contribution of the present paper can be summarized as follows:

A Modelling of the TOU program for the day-ahead electricity market through using the combined model of Diamond’s OLG and PE, this model gives a more intuitive sense of the decision-making process for the consumer responses.

B Creating an objective function for minimizing the total supply cost in order to clear the leading day market with several market participants including heat-only, power-only and CHP units and DRP with technical limitations such as demand ceiling, relative risk limit, etc.

C A robust strategy for the TOU price uncertainty in the CHPED problem using IGDT robustness function.

D Opportunistic strategy for the TOU price uncertainty in the CHPED problem using IGDT opportunity function.

II. DEFINITIVE MODEL FORMULATION OF THE DESIRED PROBLEM

In the proposed DR-CHPED problem, the CHPED section includes three types of units including power-only, CHP and heat-only and the DR section includes the demand power load. Taking constraints into consideration, the objective function of DR-CHPED is minimizing the total supply cost from the RMM point of view.

A. OBJECTIVE FUNCTION

DR-CHPED is one of the important optimization problems from the perspective of the RMM in order to minimize the total supply cost in the presence of TOU program, which is used in heat and power systems operation to obtain the optimal scheduling of the generation units over the entire dispatch period. The equation of the DR-CHPED objective function is as the following (1), shown at the bottom of the page.

In (1), the first, the second, the third and the fourth parts are the cost power-only unit, the cost co-generation (power and heat) unit, the cost heat-only unit and the cost TOU, respectively. The power is generated by power-only units and cogeneration units while the heat is generated by cogeneration units and heat-only units and the cost of implementing TOU is by the fourth part.

In (1), the CHP and TOU problem formulas are given in [33] and [34], respectively. In addition, to calculate the optimal price of power in different periods, the \( \delta \) parameter is applied in order to change the price in different periods. In other words, one of the main goals of (1) is the optimal determination of these parameters [35].

\[
\rho_t = \begin{bmatrix}
\rho_{\text{peak}} \\
\rho_{\text{flat}} \\
\rho_{\text{off-peak}}
\end{bmatrix} = \begin{bmatrix}
\rho_t^{0} + \delta \\
\rho_t^{0} \\
\rho_t^{0} + \delta
\end{bmatrix}
\]

By increasing \( \delta \), \( \rho_{\text{peak}} \) increases and \( \rho_{\text{off-peak}} \) decreases.

This procedure will continue as long as the objective function has the lowest possible value in (1), and thus the optimal...
value of $\delta$ and power price in TOU are determined. In fact, it allows customers to change their consumption patterns so that they can reduce their consumption during the peak period or transfer it to flat or off-peak periods.

**B. CONSTRAINTS**

The following equality and inequality constraints should be met by the proposed DR-CHPED model.

### 2.2.1 Power and heat balance constraint: The sum of the generated power and heat should meet the power and heat demands, respectively [36].

$$\begin{align*}
P_{t,i}^d + P_{t,i}^h &= P_{d,t} + P_{L,t} \\
H_{f,t}^i + H_{f,t}^h &= H_{d,t}
\end{align*}$$

(3)

The mathematical expression of power transmission loss between the units is given by formula (4):

$$P_{L,t} = \sum_{i \in \psi_{NP}} \sum_{j \in \psi_{NP}} P_{i,t}^d B_{ij} P_{j,t}^d$$

$$+ \sum_{i \in \psi_{NP}} \sum_{j \in \psi_{NC}} P_{i,t}^d B_{ij} P_{j,t}^d$$

$$+ \sum_{i \in \psi_{NC}} \sum_{j \in \psi_{NC}} P_{i,t}^d B_{ij} P_{j,t}^d$$

$$+ \sum_{i \in \psi_{NP}} B_{0i} P_{i,t}^d$$

$$+ \sum_{i \in \psi_{NC}} B_{0i} P_{i,t}^d + B_{00}$$

(4)

### 2.2.2 Power and heat generation limits: The generated electric power and heat should be within the acceptable limits for each unit [36].

$$\begin{align*}
P_{i,t}^d_{\text{min}} &\leq P_{i,t}^d \\ P_{i,t}^d_{\text{max}} &\leq P_{i,t}^d \\
P_{j,t}^d_{\text{min}} &\leq P_{j,t}^d \\ P_{j,t}^d_{\text{max}} &\leq P_{j,t}^d \\
H_{f,t}^i_{\text{min}} &\leq H_{f,t}^i \\
H_{f,t}^i_{\text{max}} &\leq H_{f,t}^i \\
H_{k,t}^h_{\text{min}} &\leq H_{k,t}^h \\
H_{k,t}^h_{\text{max}} &\leq H_{k,t}^h
\end{align*}$$

(5)

### 2.2.3 Demand ceiling: Consumer demand is assumed to have the following range:

$$P_{d,t}^0_{\text{min}} \leq P_{d,t} \leq P_{d,t}^0_{\text{max}} \quad \forall t \in \psi_T$$

(6)

It should be noted that this equation is used to limit the demand in the TOU program in order to prevent the power price fluctuations. It could be expressed in the other words as the limitation of the maximum demand during peak hours to prevent further demand increase, as well as the limitation of minimum demand during non-peak hours to prevent further demand decline.

### 2.2.4 Price fluctuation limit: The limit of power price changes in different $\delta$ periods is defined based on the following range:

$$\delta_{\text{min}} \leq \delta \leq \delta_{\text{max}}$$

(7)

$\delta_{\text{min}}$ and $\delta_{\text{max}}$ are considered to be 0.25 and 25 in terms of $\$/MWh, respectively.

It should be noted that this equation is used to limit the demand in the TOU program in order to prevent price fluctuations.

#### 2.2.5 Relative risk limit: This restriction is defined based on the coefficient that has the following range:

$$\theta_{\text{min}} \leq \theta \leq \theta_{\text{max}}$$

(8)

Reference [18] is usually considered to be 0.2 and 0.9, respectively.

### C. PROPOSED MODEL FOR TOU

In the current study, it is assumed that the TOU program is used to smooth the load curve. Moreover, Diamond’s OLG model is used to model the load shifts of the subscribers and the logarithmic model and PE Matrix are used to model subscriber’s load removal. The reason for choosing the logarithmic model is that the previous study [37] showed that this model gives more conservative answers than the other models so that the values of this model are in the middle of the values of other models.

Taking into consideration the price of power in different periods, consumers try to shift their demand from peak hours to non-peak hours in order to reduce their power consumption. According to references [21], [38], $P_{d,\text{flat}}$ and $P_{d,\text{off}-\text{peak}}$ of the TOU program are expressed with three specific time periods in Diamond’s OLG model as the following:

$$P_{d,\text{flat}} = \left( \frac{\rho_{\text{peak}}}{\rho_{\text{flat}} (1 + \rho_1)} \right)^\frac{1}{\theta} P_{d,\text{peak}}$$

$$P_{d,\text{off}-\text{peak}} = \left( \frac{\rho_{\text{peak}}}{\rho_{\text{off}-\text{peak}} (1 + \rho_2)} \right)^\frac{1}{\theta} P_{d,\text{peak}}$$

(9)

Here, $\theta$ parameter determines the willingness of consumers to participate in the TOU program. The higher the value of $\theta$, the less likely the consumers will be to participate in the TOU program. In other words, as $\theta$ increases, the consumers are less inclined to shift their load from one period to another.

According to the DRP model [39], the difference in the energy consumption before and after the implementation of the TOU program is equal to the load removal in the sum of the three time periods, where the load removal for the power will be described as the followings:

$$P_{d,\text{peak}} + P_{d,\text{flat}} + P_{d,\text{off-peak}}$$

$$= P_{d,\text{peak}}^0 + P_{d,\text{flat}}^0 + P_{d,\text{off-peak}}^0$$

$$- \left[ \bar{P}_{d,\text{peak}} + \bar{P}_{d,\text{flat}} + \bar{P}_{d,\text{off-peak}} \right]$$

(10)
By substituting $P_{d,\text{flat}}$ and $P_{d,\text{off-peak}}$ in relation (9), there will be:

$$P_{d,\text{peak}} = \frac{\left( P_{d,\text{peak}}^0 + \frac{P_{d,\text{flat}}^0}{P_{d,\text{peak}}^0} + \frac{P_{d,\text{off-peak}}^0}{P_{d,\text{peak}}^0} \right)}{\left( 1 + \frac{\rho_{\text{peak}}}{\rho_{\text{flat}} (1 + \rho_1)} \right)^\frac{1}{\gamma}}$$

(11)

In the TOU program, the load removal uses a logarithmic model and the PE Matrix [37] for the three periods of peak, flat and off-peak, where the power load for these three periods will be as the followings:

$$\overrightarrow{P}_{d,\text{peak}} = P_{d,\text{peak}}^0 \times E_{P,\text{peak}} \times \ln \left( \frac{\rho_{\text{peak}}}{\rho_0} \right) \forall t \in \varphi_{\text{peak}}$$

$$\overrightarrow{P}_{d,\text{flat}} = P_{d,\text{flat}}^0 \times E_{P,\text{flat}} \times \ln \left( \frac{\rho_{\text{flat}}}{\rho_0} \right) \forall t \in \varphi_{\text{flat}}$$

$$\overrightarrow{P}_{d,\text{off-peak}} = P_{d,\text{off-peak}}^0 \times E_{P,\text{off-peak}} \times \ln \left( \frac{\rho_{\text{off-peak}}}{\rho_0} \right) \forall t \in \varphi_{\text{off-peak}}$$

(12)

The following equations are used to obtain the power demand in each hour of a period:

$$P_{d,\text{peak}}(t) = P_{d,\text{peak}}^0 \times \frac{P_{\text{peak}}(t)}{P_{d,\text{peak}}^0} \forall t \in \varphi_{\text{peak}}$$

$$P_{d,\text{flat}}(t) = P_{d,\text{flat}}^0 \times \frac{P_{\text{flat}}(t)}{P_{d,\text{flat}}^0} \forall t \in \varphi_{\text{flat}}$$

$$P_{d,\text{off-peak}}(t) = P_{d,\text{off-peak}}^0 \times \frac{P_{\text{off-peak}}(t)}{P_{d,\text{off-peak}}^0} \forall t \in \varphi_{\text{off-peak}}$$

(13)

D. MODELING OF CHPS

CHPs includes CHP units, boiler units, and the heat buffer tank. There are two types of feasible operation regions (FOR) for the CHP unit, as shown separately in Figs 12-13 in appendix. From Fig.12, it can be observed that the FOR is enclosed by the ABCDEF boundary curve, and it is constrained by three operational factors: maximum fuel consumption, minimum fuel consumption, and maximum heat extraction. The minimum and maximum fuel consumption is set at the amount that meets 40-50% and 115% of the rated power under normal conditions, respectively.

Equations (13-17) model the FOR of type 1 CHP unit, as shown in Fig.12 in appendix [7]:

$$P_{i,t}^c - P_{i,A}^c \left( \frac{P_{i,A}^c - P_{i,B}^c}{H_{i,A}^c - H_{i,B}^c} \right) \left( H_{i,t}^c - H_{i,A}^c \right) \leq 0$$

$$\forall i \in N_c, \ t \in T$$

(14)

$$P_{i,t}^c - P_{i,C}^c \left( \frac{P_{i,C}^c - P_{i,B}^c}{H_{i,C}^c - H_{i,B}^c} \right) \left( H_{i,t}^c - H_{i,C}^c \right) \geq - (1 - V_{i,t}) \times M \ \forall i \in N_c, \ t \in T$$

(15)

$$P_{i,t}^c - P_{i,A}^c \left( \frac{P_{i,A}^c - P_{i,D}^c}{H_{i,A}^c - H_{i,D}^c} \right) \left( H_{i,t}^c - H_{i,A}^c \right) \geq 0 \ \forall i \in N_c, \ t \in T$$

(16)

$$0 \leq P_{i,t}^c \leq P_{i,A}^c \times V_{i,t} \ \forall i \in N_c, \ t \in T$$

(17)

$$0 \leq H_{i,t}^c \leq H_{i,A}^c \times V_{i,t}$$

(18)

The FOR of type 2 CHP unit (Fig.13 in appendix) is surrounded by the ABCDEF polygon, which is a non-convex feasible domain. To make it simpler, only the FOR (Fig.14 in appendix) is considered. In fact, the FOR of the non-convex Fig.14 in the appendix can be processed by the method indicated in [7]. The main idea of this method is to solve the problem by splitting the non-convex feasible domain into two convex sub-regions by introducing integer auxiliary variables.

III. THE PROCESS OF IMPLEMENTING THE PROPOSED METHOD

A. ANALYSIS OF THE UNCERTAINTY BASED ON THE IGD T TECHNIQUE

A definite model of the proposed problem, which is comprised of (1-13), is obtained by determining the optimal output of power and the heat of each unit in order to minimize the DR-CHPED costs considering the problem constraints. Given that the electricity price is a parameter of uncertainty in the TOU program, the IGD T method is used to address it in this issue. This method uses the opportunity and robustness functions for determining the risk aversion and risk-taking of the proposed problem. This means that after determining the value of the objective function of the definite model of the proposed problem, the minimum and maximum ranges of the value that the objective function is allowed to change is determined by the opportunity and robustness functions, respectively [38]. IGD T consists of three parts: system model, operation requirements and uncertainty modelling.

1) SYSTEM MODEL

The system model requires the input/output structure of the system. In other words, the OF system model evaluates the system response in terms of the variable and parameter of the uncertainty.

2) UNCERTAINTY MODELING

Here, the IGD T method is used by the finite envelope model [40], [41] for determining the price uncertainty at time $t$, which its equation is as the following:

$$U (\alpha, \tilde{p}_0 (t)) = \left\{ \tilde{p}_0 (t) : \frac{|\tilde{p}_0 (t) - \tilde{p}_0 (t)|}{\tilde{p}_0 (t)} \leq \alpha \right\} \ \alpha \geq 0$$

In IGD T, $\alpha$ must be determined in such a way that the value of the objective function does not exceed higher than a certain level from the base value.
3) OPERATING REQUIREMENTS

In this section, the operating requirements of the studied system are presented in the form of two objective functions. These operating requirements may lead to higher or lower OF. Operating requirements are evaluated based on the robustness and the opportunity functions and these two functions must be set for the current problem. In addition, for the OF, these functions are introduced as the followings:

$$\hat{a} (OF_r) = \max_{\alpha} \{ \alpha : \max_{i \in N_c, t \in T} (OF) \leq OF_r \} \quad \forall i \in N_c, t \in T$$

$$\hat{\beta} (OF_0) = \min_{\alpha} \{ \alpha : \min_{OF} (OF) \leq OF_0 \} \quad (20)$$

Based on the risk-seeking strategy and the risk-averse strategy, two different operations of the objective function can be defined in an IGDT model. A risk-averse decision-maker wants to plan to tolerate the adverse deviations of the uncertainty parameter. In the IGDT method, immunity against such adverse deviations is modelled using the robustness function. In this regard, $\hat{a} (OF_r)$ refers to the degree of resistance to the rising of electricity prices. This means that the risk-taking decision-maker wants to plan in a way to be resistant to the undesirable deviations of the uncertainty parameter of the price, and also the robustness objective function will be less than a pre-defined OF. Conversely, a risk-seeking decision-maker wants to take advantage of the desired deviations of the uncertainty from the expected value. The information gap method uses the opportunity function to model these potential benefits from the viewpoint of a risk-seeking decision-maker. Therefore, $\hat{\beta} (OF_0)$ is the amount of opportunity against the reduction of the electricity prices. In other words, it means that the risk-aversion decision-maker is willing to take advantage of the expected value of the favourable deviations of the price uncertainty parameter. It should be noted that the opportunity objective function is smaller than a pre-defined OF. In addition, in the above equation, $OF_r$ is greater than $OF_0$. 

a: ROBUSTNESS FUNCTION

In order to be safe against the maximum degree of the fluctuations of the price evaluation, the robustness function models the harmful aspect of the price uncertainty parameter considering the risk-taking strategy. The $\hat{a} (OF_r)$ function is as follows [42]:

$$\hat{\alpha} (OF_r) = \max \left\{ \alpha : \max_{t \in U(\alpha, \tilde{p}_0 (t))} OF \leq OF_r = (1 + \mu) OF_b \right\} \quad (21)$$

The value of the robustness function is obtained by maximizing $\alpha$ as the following:

$$\hat{\alpha} (OF_r) = \max \alpha$$

Subject to: $\hat{\alpha} (OF_r) = \max \{ Eq(1) \}$

$$\leq (1 + \mu) OF_b \quad (22)$$

Equations (2) – (18) \quad (23)

Since in the robustness function, the maximum increase of the uncertainty parameters of $\tilde{p}_0 (t) = (1 - \alpha) \tilde{p}_0 (t)$ is obtained, the robustness function is formulated as the following:

$$\hat{\alpha} (OF_r) = \max \alpha$$

Subject to: $\hat{\alpha} (OF_r) = \max \{ Eq(1) \}$

$$\leq (1 + \alpha) \tilde{p}_0 (t) \quad (24)$$

Equations (2) – (18) \quad (25)

b: OPPORTUNITY FUNCTION

Any reduction in the uncertainty parameters will be useful for ISO so that these positive effects of the uncertainty are modelled using the opportunity function. The $\hat{\beta} (OF_0)$ function is as follows [42]:

$$\hat{\beta} (OF_0) = \min \left\{ \alpha : \min_{l \in U(\alpha, \tilde{p}_0 (t))} OF \leq OF_0 = (1 - \gamma) OF_b \right\} \quad (26)$$

The value of the opportunity function is obtained by minimizing $\alpha$ as the following:

$$\hat{\beta} (OF_0) = \min \alpha$$

Subject to: $\hat{\beta} (OF_0) = \min \{ Eq(1) \}$

$$\leq OF_0 \quad (27)$$

$$\leq (1 - \alpha) \tilde{p}_0 (t) \quad (28)$$

Equations (2) – (18) \quad (29)

Since in the opportunity function, the least reduction of the uncertainty parameters $\tilde{p}_0 (t) = (1 + \alpha) \tilde{p}_0 (t)$ is obtained, the opportunity function is formulated as the following:

$$\hat{\beta} (OF_0) = \min \alpha$$

Subject to: $\hat{\beta} (OF_0) = \min \{ Eq(1) \}$

$$\leq (1 - \gamma) OF_b \quad (30)$$

$$\tilde{p}_0 (t) = (1 + \alpha) \tilde{p}_0 (t) \quad (31)$$

Equations (2) – (18) \quad (32)

B. IMPLEMENTATION STEPS

According to the flowchart shown in Fig. 1, the implementation steps of the proposed method are as the followings:

Step 1: Initial data for the objective function, constraints of the load curves, power prices, C-PSO algorithm parameters, $T = 1, \alpha = 0, \delta = 0, \mu = 0.2, \gamma = 0.19$ and so on is entered.

Step 2: If $\alpha = 0$, go to step 6 and minimize the proposed cost function of DR-CHPED (1) with respect to constraints (2)–(18), where the RMM cost without considering IGDT is determined as OFb.
Step 3: If the RMM is looking for a risk-taking or risk-aversion strategy regarding the uncertainty parameter of the electricity price, the applied IGDT-based robustness optimization and the applied IGDT-based opportunity optimization should be chosen, respectively.

Step 4: If the repetition is not higher than its upper limit, go to step 5; Otherwise, determine the best value for DR-CHPED cost for the robustness optimization according to (27) and for the opportunity optimization according to (36) and then end the process.
### STEP 5: Day-Ahead Scheduling for Economic Dispatch of CHP With Uncertain Demand Response

**Step 5:** Put (28) instead of $\rho_0(t)$ in the robustness optimization of the electricity price, and put (37) instead of $\rho_0(t)$ in the opportunity optimization. Then go to step 6.

**Step 6:** $\delta$ and $\theta$ are determined by

$$
\delta = \delta_{\text{min}} + (\delta_{\text{max}} - \delta_{\text{min}}) \times \text{rand} 
$$

and

$$
\theta = \theta_{\text{min}} + (\theta_{\text{max}} - \theta_{\text{min}}) \times \text{rand} 
$$

respectively, taking into consideration (7) and (8) constraints, respectively.

**Step 7:** The amount of power demand is determined in the TOU program for a period of 24 hours using (9) - (13) taking into consideration the constraint (6).

**Step 8:** Solve the DR-CHPED every hour using the C-PSO algorithm and calculate the losses as well as the objective function using the relation (1), while taking into consideration constraints according to (2)–(5) and (14) to (18).

**Step 9:** Save the best (planning units, power demand at T hour) for each $\delta$ and $\theta$. Then, consider the next $a + 0.025$ return to step 4. Otherwise, if $T$ is higher than 24, go to step 10.

**Step 10:** Save $\delta$, the best value for the DR-CHPED cost, and the price if the DR-CHPED cost is lower than its previous value.

### IV. Results and Discussion

#### A. Hypotheses

Studies in this section are conducted based on the proposed formulas with the following hypotheses: This test system consists of 7 units, including four power-only units (unit1-4), two cogeneration units (unit5-6) and a heat-only unit (unit7) [43]. The valve-point effects and the transmission loss between networks are considered in this system. The daily power and heat demand of the system initial and time period are indicated in Table 5 and 6, respectively and the parameters units and co-generation feasible operation region are shown in Table 7 and Figs. (13 and 14), respectively. Moreover, the characteristics of parameters customers are indicated in Table 8 and network loss coefficients are detailed in the Appendix.

- The initial value of $\alpha$ is considered as zero and increases to $\alpha + 0.025$ in each iteration.
- The electricity price is 14 $/\text{MWh}$ before the implementation of the TOU.
- For the C-PSO algorithm, the number of iterations is limited to 200, the population size is 100, and 9 scenarios are considered as shown in Table 2.

#### B. Simulation Results

In this section, the results of evaluating the implementation of the DR-CHPED problem are investigated in a standard seven-unit system for a period of 24 hours with the aim of minimizing the total supply cost from the RMM point of view using the C-PSO algorithm. Simulations have been implemented by writing codes in MATLAB 2015. The applied PC specifications are 2.2 GHz Intel Core i5 and 6 GB RAM.

In order to describe the performance of the proposed method, five case studies have been conducted according to Table 2. Case 1 is about the presence or absence of the TOU in the CHPED problem according to scenarios 2 and 3, case 2 is related to the doubling of the parameter $E$ in the DR-CHPED program according to scenario 4 and case 3 is the implementation of the DR-CHPED program without power transmission loss limit by scenario 5. Furthermore, case 4 is the implementation of the DR-CHPED program according to the TOU of references [18] and [37] in accordance with scenarios 6 and 7 and case 5 is the price uncertainty in the DR-CHPED program according to IGDT model under Scenarios 8 and 9.

1) **CASE STUDY 1: EVALUATING THE PRESENCE AND ABSENCE OF TOU IN THE CHPED PROBLEM**

This case study includes scenarios 2 and 3, in which the presence and absence of the TOU in CHPED program according to scenarios 2 and 3, case 2 is related to the doubling of the parameter $E$ in the DR-CHPED program according to scenario 4 and case 3 is the implementation of the DR-CHPED program without power transmission loss limit by scenario 5. Furthermore, case 4 is the implementation of the DR-CHPED program according to the TOU of references [18] and [37] in accordance with scenarios 6 and 7 and case 5 is the price uncertainty in the DR-CHPED program according to IGDT model under Scenarios 8 and 9.
TABLE 3. Effects of implementing DR-CHPED in different scenarios.

| Scenario   | Cost of Manufacturing Units ($) | TOU Cost ($) | Total Cost ($) |
|------------|---------------------------------|--------------|----------------|
| Scenario 1 | 528656.7                        | 546467       | 536348.9       |
| Scenario 2 | 282395.3                        | 406560       | 412003.7       |
| Scenario 3 | 33188258                        | 522835.5     | 519793.6       |
| Scenario 4 | 1091.2                          | 1068.9       | 1016.2         |
| Scenario 5 | 1008.1                          | 1013.2       | 1001.6         |
| Scenario 6 | 1020.1                          | 983.0195     | 1083.8         |

Scenario 2 is equivalent to $148,434.4, which ammount is only the cost of the manufacturing units. In Scenario 3, the implementation of the TOU in the CHPED issue results in an additional cost (TOU cost), a reduction in the cost of the manufacturing companies and a reduction in the total cost (equivalent to $147,872.3) in comparison with Scenario 2. In this scenario (3), the total cost includes the cost of the manufacturing units and the cost of the TOU program. The reason for the total cost reduction in Scenario 3 is that consumers participate in the TOU and reduce their consumption during peak hours or transfer it to the flat or off-peak periods. In other words, the implementation of TOU reduces the load of power consumers during peak hours and reduces the use of high-cost production units during these hours, thus leading to a reduction in the total cost. Figures 2 and 12 indicate that the implementation of TOU in the CHPED problem improves the power load curve and reduces the power load peak by 129,3643 MW. Table 3 indicates that the optimal determination of $\delta$ determines the optimal price at different hours and thus causes the subscribers to reduce their power load during the peak hours or transfer it to the flat or off-peak periods. In this regard, the implementation of the CHPED causes the amount of power and heat generated by the units to be selected optimally, leading to a reduction in the amount of losses and total cost. On the other hand, determining the optimal $\theta$ increases the consumer’s desire to participate in the TOU program, that is, with the optimal reduction of $\theta$, the subscribers’ desire to transfer their load from peak hours to other times increases. In other words, the result of this case study is that the implementation of TOU causes the customers to transfer their consumption during the hours with high electricity price to the hours when the electricity is cheaper. By doing this, the peak of the power load curve improves and reduces, and on the other hand, the implementation of the CHPED causes the generation of electricity and heat to be more cost-effective, while reducing the losses and network congestion. The order of the optimal selection of power and heat of the manufacturing units in this case study can be seen in Figures 3-5.
TABLE 4. The best dispatch found by DR-CHPED considering IGDT (opportunity) for scenario 8, after the implementation of TOU.

| Hour | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|------|---|---|---|---|---|---|---|---|---|----|----|----|
| Power demand | 1340 | 1286.4 | 1232.8 | 1179.2 | 1232.8 | 1286.4 | 1243.5 | 1266.5 | 1315.2 | 1363.9 | 1510 | 1599 |
| \(P^1\) (MW) | 410.19 | 312.84 | 469.04 | 281.64 | 538.6 | 374.92 | 254.04 | 367.4 | 379.35 | 458.8 | 464.29 | 533.62 |
| \(P^2\) (MW) | 303.82 | 250.5 | 200.15 | 255.86 | 200.16 | 254.71 | 335.96 | 338.76 | 287.91 | 337.9 | 377.89 | 387.79 |
| \(P^3\) (MW) | 180.81 | 262.51 | 199.46 | 157.71 | 95.069 | 189.82 | 173.98 | 136.12 | 172.66 | 104.16 | 171.51 | 207.42 |
| \(P^4\) (MW) | 149.75 | 158.13 | 64.322 | 160.63 | 101.44 | 167.64 | 162.58 | 82.998 | 175.37 | 128.14 | 169.76 | 130.18 |
| \(P^5\) (MW) | 217.71 | 220.11 | 218.05 | 219.83 | 217.94 | 217.83 | 220.06 | 247 | 220.16 | 247 | 246.73 | 246.99 |
| \(P^6\) (MW) | 113.16 | 113.01 | 113.19 | 113.2 | 113.03 | 113.2 | 125.8 | 125.8 | 113.18 | 125.8 | 125.75 | 125.8 |
| \(P_e\) (MW) | 35.42 | 30.69 | 31.38 | 25.651 | 33.382 | 31.693 | 28.882 | 31.599 | 33.44 | 38.045 | 45.89 | 52.775 |
| VPB Constraint | 0.0001 | 0.0023 | 0.0055 | 0.0029 | 0.0399 | 0.0252 | 0.0109 | 0.0161 | 0.0351 | 0.1459 | 0.0440 | 0.0001 |
| Heat demand | 760 | 750 | 777 | 790 | 810 | 850 | 950 | 1000 | 1030 | 1070 | 1110 | 1140 |
| \(H^1\) (MW-h) | 148.24 | 130.85 | 148.61 | 150.54 | 148.49 | 148.39 | 150.79 | 180 | 150.9 | 180 | 179.71 | 179.9 |
| \(H^2\) (MW-h) | 115.62 | 115.41 | 115.67 | 115.69 | 115.42 | 115.69 | 115.65 | 115.35 | 135.52 | 135.59 |
| \(H^3\) (MW-h) | 496.14 | 483.75 | 505.72 | 523.77 | 546.13 | 585.92 | 663.64 | 684.7 | 766.45 | 744.77 | 824.43 |
| VHBM Constraint | 0.0096 | 0.0052 | 0.0007 | 0.0002 | 0.0350 | 0.0118 | 0.0214 | 0.0023 | 0.0004 | 0.0184 | 0.0021 | 0.0038 |

FIGURE 4. Power generation result of Scenario3.

Figures 2 & 12 and Table 3 show that the implementation of TOU in the CHPED problem results in the reduction in total cost by 1478106.6MW due to the doubling of PE, reduction in the losses of the transmission lines by 1008.1Mw, improvement of the power load curve and reduction in the peak power load by 131.6562MW compared to scenario 3. In other words, by adjusting PE in the proposed TOU model, customers reduce their peak load consumption or shift it to other hours in order to avoid the higher costs.

3) CASE STUDY 3: ELIMINATING THE LINE LOSSES

This case study includes scenario 5, where the TOU program in the CHPED problem evaluates \(\rho_1\) and \(\rho_2\) for the day heading market and the C-PSO algorithm based on the values of PE and rate of time preference, in order to determine the optimal power and heat output of each unit while considering all technical constraints except transmission line losses. Compared to Scenario 3, the capacity of the manufacturing units and the total cost (equivalent to 14,605,521$) decrease in this scenario due to ignoring the reduction in the transmission line losses. In other words, if the line losses are ignored, the manufacturing units do not need to generate power for overcoming the line losses. Fig. 6 indicates the capacity of the manufacturing units, which the shown contradiction is due to the difference in the power consumed by the consumer between various time periods.

4) CASE STUDY 4: EVALUATION EFFECTS OTHER ALREADY-USED TOU MODELS IN DR-CHPED

This case study includes scenarios 6 and 7, where the TOU program according to references [18] and [37] in the CHPED.

power load during peak hours or transfer it to the flat or off-peak periods.

![Figure 5. Daily heat demand & heat generation result of Scenario2.](image-url)
problem evaluates E and (ρ₁, ρ₂) for the leading day market and the C-PSO algorithm based on the values of PE and rate of time preference to determine the optimal output power and heat of each unit considering all technical constraints. According to Table 3 and Figures 7 and 8, it can be observed that in scenarios 6 and 7 the costs of TOU, the costs of manufacturing companies and total costs have increased and also have less peak reduction and improvement of the power curve in comparison with scenario 3. This issue indicates a better performance of the proposed TOU model than the TOU models of [18] and [37] in terms of improving the power curve and reducing the production costs. In other words, from RMM’s point of view, a model is more successful than TOU only when it can transfer less power load to a period of time with high energy consumption, and thus less high-cost and high-consuming manufacturing units will be used at that time period.

5) CASE STUDY 5: EVALUATING THE PRESENCE OF IGDT IN THE DR-CHPED ISSUE
This case study includes scenarios 8 and 9, where the uncertainty (electricity price) using IGDT in the TOU program and the CHPED problem evaluate E and (ρ₁, ρ₂) for the day-ahead market and C-PSO algorithm based on the values of PE and rate of time preference to determine the optimal output power and heat of each unit considering all technical limitations. According to Table 3 and Figures 10-12, Scenario 8 uses a risk-taking strategy for determining the maximum resistance to lower electricity prices using the robustness function. It is observed that compared to Scenario 3, the price of electricity decreases and subscribers reduce their power load to a lesser degree in the peak hours or transfer it to the flat or off-peak periods, thus weakening the power load curve, increasing the peak power load curve and leading to the increase of transmission line losses and total costs. In Scenario 9, a risk-aversion strategy is used for determining the minimum resistance to the rising of the electricity prices using the opportunity function. It can be observed that compared to Scenario 3, the electricity price has increased and subscribers reduce their power load to a higher degree during peak hours or transfer it to flat or off-peak periods, thus improving the power load curve, reducing
the power load curve peak and leading to the reduction of the transmission line losses and total cost.

![Figure 11. Power generation result of Scenario 8.](image)

**FIGURE 11.** Power generation result of Scenario 8.

![Figure 12. Peak reduction in different scenarios.](image)

**FIGURE 12.** Peak reduction in different scenarios.

Fig. 9 indicates the ratio of alpha to robustness cost function. As observed, the RMM increases the robust cost function by choosing a lower power price (higher alpha) than Scenario 3 (neutral power price) and the RMM has to pay more in order to have a stronger strategy, and vice versa. In other words, if the RMM chooses a higher cost, its decision will be stronger. Fig. 9 also includes the ratio of alpha to the opportunity cost function, in which the RMM decreased the opportunity cost function by selecting a higher power price (higher alpha) than Scenario 3 (neutral power price), and the RMM pays less to have a more risk-averse strategy. In other words, examining price uncertainty in the DR-CHPED issue helps the RMM to understand the limitations of the optimal load changes in exchange for price changes, thus providing an acceptable basis for planning and deciding to operate new power plants to avoid possible blackouts during peak hours. Furthermore, ignoring price uncertainty in this issue makes manufacturing units more vulnerable to price changes, which can greatly affect the economic benefits and safe operation of the system. It is worth noting that in Scenario 8, the optimal output power of power-only units, cogeneration units and the heat-only unit after TOU implementation, power & heat parity errors (to show the accuracy of DR-CHPED in removing power and heat balance constraints) and power transmission losses are given (as shown in Table 4).

**V. CONCLUSION**

In the current study, at first, the DR-CHPED problem was presented using C-PSO algorithm to determine the optimal output power and heat of each unit according to some real constraints of system performance such as valve point effect and transmission line losses in a 7-unit test system. Furthermore, in order to determine the power and heat demand in the TOU based on the combination of Diamond’s OLG and PE models, a flexible model was proposed that was more efficient in modelling the customers’ behaviour. It has been shown that if the optimal pricing in DR-CHPED is carried out intelligently, a number of benefits will happen including energy savings, reduced production costs and improved load curves. Then, due to the existence of electricity price uncertainty in the DR-CHPED problem, the positive aspects of the electricity price uncertainty using the IGDT opportunity function modelling and the negative aspects of electricity price uncertainty using the IGDT robustness function modelling were presented. It can be observed that in all scenarios where the DR-CHPED issue has been implemented, the subscribers have tried to shift their loads from the peak to the off-peak periods, which resulted in reducing the total cost of RMM.

Moreover, considering the risk-taking strategy in the DR-CHPED, it was found that the customers were less inclined to reduce the peak load, total cost of RMM and to improve the power load curve, while considering the risk-aversion strategy in the DR-CHPED, power customers were more inclined to reduce the peak load, total cost of RMM and the power load curve improvement. It should be noted that paying attention to the price uncertainty in this issue can help RMM to withstand the risks of load changes, which greatly affects the economic benefits and the safe operation of the system. On the other hand, paying attention to TOU in the CHPED issue improves the characteristics of the load curve, which makes it unnecessary to build a new power plant for supporting the peak load times. The findings of this study can be useful for RMM to prevent unrealistic decisions and financial losses.

Regarding the limitations and the possibility of the extension of the proposed model, the following points can be mentioned. The proposed model applies a step by step approach and hence all of the formulations have not been solved together in one optimization problem. Moreover, the proposed model does not guarantee the best possible solution and therefore accuracy of the model in relation to problem-solving from the perspectives of RMM and the consumers using two-level optimization needs further evaluation.

Suggestions for further work in this area include: 1) Expanding the formula by adding more sentences to the objective function and adding more constraints 2) Use of new meta-heuristic algorithms 3) Improving the handling of the price uncertainty in the IGDT methods 4) Considering the possibility of the consumers’ interactions with the RMM.

**APPENDIX**

Appendixes, if needed, appear before the acknowledgment.
$B = \begin{bmatrix} 49 & 14 & 15 & 15 & 20 & 25 \\ 14 & 45 & 16 & 20 & 18 & 19 \\ 15 & 16 & 39 & 10 & 12 & 15 \\ 15 & 20 & 10 & 40 & 14 & 11 \\ 20 & 18 & 12 & 14 & 35 & 17 \\ 25 & 19 & 15 & 11 & 17 & 39 \end{bmatrix} \times 10^{-6}$,

$B_{00} = 0.056$

$B_0 = \begin{bmatrix} -0.3908 & -0.1297 & 0.7047 & 0.0591 & 0.2161 & -0.6635 \end{bmatrix} 5 \times 10^{-3}$

**TABLE 5.** Initial power & heat demand of the customers.

| Power Demand (MW) | 1250 | 1200 | 1250 | 1300 | 1450 | 1400 |
|-------------------|------|------|------|------|------|------|
| Heat Demand (MWh) | 760  | 750  | 770  | 790  | 810  | 850  |

| Hour             | 7    | 8    | 9    | 10   | 11   | 12   |
|------------------|------|------|------|------|------|------|
| Power Demand (MW) | 1450 | 1500 | 1550 | 1600 | 1650 | 1700 |
| Heat Demand (MWh)| 950  | 1000 | 1030 | 1070 | 1110 | 1140 |

| Hour             | 13   | 14   | 15   | 16   | 17   | 18   |
|------------------|------|------|------|------|------|------|
| Power Demand (MW) | 1750 | 1700 | 1650 | 1600 | 1550 | 1500 |
| Heat Demand (MWh)| 1180 | 1230 | 1180 | 1140 | 1090 | 1060 |

| Hour             | 19   | 20   | 21   | 22   | 23   | 24   |
|------------------|------|------|------|------|------|------|
| Power Demand (MW) | 1450 | 1400 | 1350 | 1250 | 1200 |     |
| Heat Demand (MWh)| 1030 | 1010 | 1000 | 950  | 900  | 830  |

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