Probabilistic prediction of electric power extreme load

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Abstract. For load forecasting, the difficulty lies in how to accurately grasp some key points in the process of load fluctuation, such as the highest load and the lowest load. It is advisable to call the forecast of daily maximum load and daily minimum load as “extreme load forecast”. On one hand, we can use the whole day load curve forecasting technology to directly get the forecasting results of extreme load; on the other hand, we need to study the special method of directly forecasting extreme load. At the same time, the conventional extreme load forecasting results are generally deterministic. Only a precise value is given, which cannot estimate the probability of the load value and determine the possible fluctuation range of the forecasting results, ignoring the probability characteristics of the forecasting results themselves. In fact, because of the advance of the prediction problem, the realization of the uncertainty prediction is more in line with the objective needs. According to the probability characteristics of the prediction results, it is helpful for the decision-makers to better grasp the objective laws of the research objects in the aspects of risk analysis and reliability evaluation, so as to achieve more reliable and scientific analysis and evaluation. Therefore, it is of great significance to introduce the idea of uncertainty analysis to realize the probability prediction of extreme load.

1. Overview
Different from general probabilistic load forecasting, the probability density of extreme load is abnormal - the probability density of the maximum value of several random variables with normal distribution is non Gaussian distribution[1-2]. Therefore, it is necessary to establish a mathematical method to form the maximum probability density from any probability density of multiple random variables.
In this paper, the probability prediction method of extreme load is established by using sequence operation theory. In order to express conveniently, this paper takes daily maximum load as an example to analyze, and the forecasting method of daily minimum load is similar to it.

2. Analysis of daily maximum load forecasting
Generally speaking, the daily maximum load series not only has a long-term trend, but also has significant seasonal characteristics. Taking a 220kV transformer substation in City A as an example, the daily maximum load sequence of the whole year is shown in the figure 1 below. Among them, the daily maximum load reaches the local maximum value on the 350th day (December) and the 220th day (August) respectively, and reaches the local minimum value on the 120th day (April) and the 280th day (October), thus forming four obvious increase and decrease stages. At the same time, the daily maximum load is significantly higher than the rest day in the working day, which has significant weekly characteristics.

3. The general idea of bus daily maximum load forecasting
The existing probabilistic load forecasting methods can be divided into two categories [3]: probabilistic forecasting methods based on prediction error and probabilistic forecasting methods based on amplitude statistics. The prediction accuracy of probabilistic prediction method based on prediction error depends on the accuracy of deterministic prediction method, but there is no mature prediction method for the maximum load amplitude for now. The prediction method based on the statistical value of the amplitude requires that the sequence is stable, that is to say, the statistical characteristics of the sequence to be predicted are the same as that of the historical sequence, which cannot be satisfied in the extreme load prediction of the bus.

Considering the prediction of daily maximum load of bus, it could be regard as a single random variable [4], at the same time, it can also be deconstructed into the sum of multiple random variables, such as the sum of predicted value and prediction error, the sum of load increment of the previous day and two days, etc. In view of the above situation, this paper proposes a statistical forecasting method based on historical load increment. This method deconstructs the bus load of the day to be predicted into the sum of the load in the previous day and the load increment of the two adjacent days. As the bus load of the previous day is known, the daily load to be predicted has the same distribution characteristics as the load increment. At the same time, the classification statistical results of load increment can automatically cover the periodicity and trend of load, and can form a stable sequence through appropriate transformation. Therefore, the classification statistical results can be used to predict the daily maximum load of bus.

In the prediction of bus daily maximum load, the amplitude of each sub peak directly determines the amplitude of daily maximum load. Therefore, based on the analysis and prediction of each sub peak, this paper proposes the following prediction process:
1) Firstly, the sub peak amplitude of daily bus load should be found out in the historical load curve, and carried out the statistical analysis;

2) According to the statistical information, the probability density of each sub peak amplitude is predicted respectively, and the probability density of the daily maximum load amplitude is obtained by using the sequence operation;

3) According to the predicted value of probability density of each sub peak amplitude, the probability distribution of daily maximum load appearing in each sub peak is obtained by using sequence operation.

4. Probability prediction of daily maximum load amplitude of bus

In this section, based on the classified statistics of load increment, the probability density of daily maximum load is obtained by using the sequence operation through the prediction of peak amplitude of each sub peak [5]. In this section, the prediction method based on load increment is used in the prediction process of each sub peak amplitude: the probability distribution of each sub peak amplitude is obtained by statistical analysis of the load increment according to the types of seasons and weeks.

4.1. Classified statistics of load increment

The fluctuation, trend and periodicity of daily maximum load are the main difficulties in amplitude prediction. Due to the inertia of load, the daily maximum load between two adjacent days is not easy to change suddenly. Through the analysis of load increment between two adjacent days, the influence of trend is eliminated. Through the classification and statistics of load increment, the periodicity of daily maximum load can be reflected, so as to unify the probability expression of maximum load amplitude. Assuming that the first \( n \) days are historical days, the historical load vector is:

\[
P = [P_1 \ P_2 \ \cdots \ P_n]
\]

(1)

Define load increment sequence as:

\[
\Lambda = [\Delta_1 \ \Delta_2 \ \cdots \ \Delta_{N-1}]
\]

(2)

\[
\Delta_n = l_{n+1} - l_n, \ n = 1, 2, \ldots, N - 1
\]

(3)

Take \( n = 365 \) and divide it into 4 components according to the different seasons \( \Lambda \). For each component, reorganize it into a two-dimensional array according to the different week types, and eliminate the holiday factors. The statistical results are shown in the figure 2 below.

It can be seen from the figure 2 that the mean value of load increment on Saturday is significantly negative compared with Friday, and the mean value of load increment on Monday is positive compared with Sunday, which is a direct reflection of the higher load level on weekdays and the lower load level on weekends. At the same time, the dispersion of summer load increment is obviously greater than other seasons.

By statistical analysis of the results of different seasons and different week types, we can get \( 7 \times 4 \) increment expectation matrix \( E \) and increment variance matrix \( S \) as follows.

Incremental expectation matrix:

\[
E = \begin{bmatrix}
    u_{11} & u_{12} & u_{13} & u_{14} \\
    u_{21} & u_{22} & u_{23} & u_{24} \\
    \vdots & \vdots & \vdots & \vdots \\
    u_{71} & u_{72} & u_{73} & u_{74}
\end{bmatrix}
\]

(4)

Incremental variance matrix:

\[
S = \begin{bmatrix}
    \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\
    \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\
    \vdots & \vdots & \vdots & \vdots \\
    \sigma_{71} & \sigma_{72} & \sigma_{73} & \sigma_{74}
\end{bmatrix}
\]

(5)

Among them, \( \mu_{ij} \) and \( \sigma_{ij} \) are the statistical mean and variance of the load increment of the two adjacent days from day \( i \) to \( i + 1 \) in season \( j \) respectively.
4.2. Probability prediction of peak amplitude

According to the increment expectation matrix and the increment variance matrix, the probability density prediction results of the amplitude of each sub peak can be obtained. In the case of two peaks, set the historical sequence of the first and second sub peaks be:

\[ P' = [P'_1 \ P'_2 \ \ldots \ P'_N] \]
\[ P'' = [P''_1 \ P''_2 \ \ldots \ P''_N] \] (6)

The corresponding increment sequence are:

\[ A' = [\Delta'_1 \ \Delta'_2 \ \ldots \ \Delta'_{N-1}] \]
\[ A'' = [\Delta''_1 \ \Delta''_2 \ \ldots \ \Delta''_{N-1}] \] (7)

Split and reorganize \( A' \) and \( A'' \) by season and week respectively, the corresponding increment expectation matrix and increment variance matrix are \( E', S', E'', S'' \) respectively, set the increment \( \Delta'_i, \Delta''_i \) corresponding to the day to be predicted to meet the normal distribution, the expectation and variance are \( \mu_{ij}', \sigma_{ij}' \), \( \mu_{ij}'' \), \( \sigma_{ij}'' \) respectively. Then the probability density of the first and second sub peak amplitude of the day to be predicted can be expressed as:

\[
p'(x) = \frac{1}{\sqrt{2\pi}\sigma_{ij}'} e^{-\frac{(x - \mu_{ij}')^2}{\sigma_{ij}'^2}}
\]
\[
p''(x) = \frac{1}{\sqrt{2\pi}\sigma_{ij}''} e^{-\frac{(x - \mu_{ij}'')^2}{\sigma_{ij}''^2}}
\] (9)

4.3. Probability prediction of daily maximum load amplitude of bus

The probability prediction of daily maximum load of bus is considered below. Because the daily maximum load of bus is the maximum value of each sub peak, the probability density function of the maximum load amplitude can be directly calculated from the probability density of each sub peak with the help of sequence operation theory.

Firstly, discretizing the possible value space of daily maximum load,
then \( [\underline{l}, \overline{l}] \) can be considered as all possible value spaces, dividing it into \( k \) segments, the \( k \) segment represents the value in the sub interval \([a_k, b_k]\), and then

\[
a_k = \frac{l + \frac{\overline{l} - l}{K}(k - 1)}{2} \\
b_k = \frac{l + \frac{\overline{l} - l}{K} - Lk}{2}
\]

The probability sequence of the first and second sub peak appearing in each sub interval are:

\[
p'(k) = [p_1', p_2', \ldots, p_k'] \\
p''(k) = [p_1'', p_2'', \ldots, p_k'']
\]

\[
p'_k = \int_{a_k}^{b_k} p'(x)dx, p''_k = \int_{a_k}^{b_k} p''(x)dx
\] (13)

5. Example analysis

5.1. Sectional statistical results of bus load increment
Analyze the load data of the bus in 2011 of plant B in city A, forming the load increment matrix, regrouping the load increment in different seasons and weeks, and count the mean value and variance respectively. The results are shown in the following two tables, in which the data dimension is MW. It can be seen from table 1 that the average load increment from Friday to Saturday is negative, indicating a large decrease. The load increment from Sunday to Monday is significantly positive, the mean value of the increment of the second sub peak from Friday to Sunday is significantly lower than that of the first sub peak corresponding to the week type increment, and the variance value of the summer peak load is far greater than that of other seasons, indicating that the fluctuation degree of summer load is higher than that of other seasons.

**Table 1 Mean value of load increment in different weeks and seasons**

| Season Week | Morning peak - Spring | Morning peak - Summer | Morning peak - Autumn | Morning peak - Winter | Evening peak - Spring | Evening peak - Summer | Evening peak - Autumn | Evening peak - Winter |
|-------------|-----------------------|-----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Mon.        | -1.274                | 2.9525                | -0.0112              | 1.1159               | -0.1521              | 3.8148               | 1.218                | 1.2442               |
| Tue.        | 2.1705                | -2.0345               | -1.3352              | 4.2981               | 1.3338               | -0.3902              | -0.7628              | 1.5324               |
| Wed.        | 0.9441                | 0.4754                | 3.2828               | -5.5029              | 1.2255               | -4.396               | 1.9278               | 0.8472               |
| Tur.        | -0.0611               | 0.3412                | 0.7204               | 0.5543               | -1.6111              | 0.7315               | 0.9439               | -0.3499              |
| Fri.        | -5.2171               | -2.1889               | -4.4289              | -1.8785              | -3.1306              | -3.4094              | -4.786               | -3.2833              |
| Sat.        | 0.7932                | -2.9786               | 1.1003               | -4.8607              | -1.0192              | -1.5416              | 1.2543               | -1.8097              |
| Sun.        | 2.4871                | 4.0321                | 2.4186               | 3.7728               | 2.916               | 5.859                | 2.1295               | 2.0936               |

**Table 2 Variance of load increment in different weeks and seasons**

| Season Week | Morning peak - Spring | Morning peak - Summer | Morning peak - Autumn | Morning peak - Winter | Evening peak - Spring | Evening peak - Summer | Evening peak - Autumn | Evening peak - Winter |
|-------------|-----------------------|-----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Mon.        | 12.003                | 30.918                | 20.558               | 26.085               | 9.085                | 68.913               | 21.799               | 35.186               |
| Tue.        | 10.782                | 28.344                | 11.465               | 16.389               | 13.331               | 39.446               | 18.528               | 20.286               |
| Wed.        | 10.938                | 22.442                | 10.187               | 14.747               | 33.310               | 55.652               | 26.56                | 16.534               |
6. Conclusion
The parameters of the probability density function of each sub peak are determined by the above increment mean and increment variance, and the corresponding probability density prediction results are formed. The probability density of the maximum daily load amplitude of the bus is formed by combining the two operations. The figure below shows the extreme value prediction process of a bus in city A.

![Figure 3. Probability density of bus maximum load](image)

For the convenience of comparison, the actual maximum load value of the day is drawn with a vertical line. It can be seen from the figure 3 that the actual maximum load of the day appears near the peak value of the probability density of the maximum load of the day, which shows that such probability prediction has practical value.

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