Modeling and vibration control of a smart vibration isolation system based on magneto-sensitive rubber

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Abstract

Magneto-sensitive (MS) rubber is a kind of smart material, the shear modulus of it can be changed rapidly and reversibly by a magnetic field applied. A smart MS rubber-based isolation system and a nonlinear model based on this MS rubber-based vibration isolation system are developed in this paper. The influence of the amplitude, frequency and magnetic dependency for MS rubber, the mechanical inertance of infinite extended foundation, the mass of solid block and the dimension of MS rubber isolators are all considered in this model. The feasibility of two control strategies aimed at reducing the energy transmitted to the foundation and protecting machine against foundation motion, respectively, is investigated based on this smart vibration isolation system. It is found that compared to the traditional passive rubber isolators, an enhanced vibration isolation effect can be achieved by using MS rubber isolators after control strategies applied. Furthermore, the influence of the amplitude dependency and the response time of MS rubber to the isolation effect is studied. The nonlinear model established for MS rubber isolation system, the control strategies developed and the investigation for the amplitude dependency and the response time of MS rubber to the isolation effect in this paper provide fundamentals for the application of MS rubber in the field of vibration reduction.

Keywords: magneto-sensitive rubber, vibration isolation, amplitude dependency, frequency dependency, magnetic dependency, control strategies

(Some figures may appear in colour only in the online journal)

1. Introduction

Due to the advantages of relatively low cost, effectiveness and reliability, rubber-based vibration isolators are widely used in engineering applications to mitigate vibration [1]. However, a major drawback of rubber-based vibration isolators is that their mechanical properties are fixed once installed. Thus, the adaptability of rubber-based vibration isolators to various loading conditions is poor. To address this problem, magneto-sensitive (MS) rubber with changeable mechanical properties, which is an alternative to the traditional rubber, is applied. MS rubber mainly consists of iron particles embedded in a rubber matrix. The modulus of MS rubber can be changed rapidly and reversibly under a magnetic field [2]. Therefore, vibration isolators based on MS rubber have the ability to change their mechanical characteristics and adapt to the change of loading conditions. Consequently, an enhanced vibration isolation effect can be achieved by MS rubber-based vibration isolators [3, 4].

A constitutive model of MS rubber which represents the mechanical behavior of MS rubber accurately is required for the application of MS rubber in vibration reduction area. Initially, the magnetic dipole model was used to represent the
magnetic dependency of MS rubber [5, 6]. Three dimensional models based on continuum mechanics and electromagnetism theory were developed for MS rubber subsequently [7–9]. However, those models are only valid for quasi-static case which are not sufficient to predict the dynamic performance of MS rubber. Consequently, viscoelastic models were developed to describe the frequency dependency of MS rubber [10–13]. In 2005, Blom and Kari [14] performed some dynamic shear modulus measurements on MS rubber with different strain amplitudes in the audible frequency range. Experiment results show that the magnitude and loss factor of the shear modulus of MS rubber are highly dependent on the amplitude of the strain applied. Actually, this dependence of dynamic shear modulus on strain amplitude is a common phenomenon for rubber which is often referred to as the Fletcher–Gent effect [15]. Therefore, constitutive models which only consider the magneto-elastic and viscoelastic properties of MS rubber may not be appropriate for the accurate description of the mechanical properties of MS rubber. Although constitutive models with amplitude, frequency and magnetic field dependency for MS rubber were developed [16, 17] and then a model for MS rubber-based isolation system [18], the effect of amplitude dependency of MS rubber to the vibration isolation effect was not fully discussed. This paper is a continuation of [17] and the main innovation is to study the influence of the mechanical properties (such as amplitude dependency) of MS rubber to the vibration isolation effect.

Besides the research for the constitutive model of MS rubber, there is a large number of studies on the application of MS rubber. Smart tuned vibration absorber based on MS rubber was developed and the frequency shifting properties were investigated [19, 20]. Afterwards, the feasibility of MS rubber-based smart tuned vibration absorber was verified [21–23]. Moreover, studies which explore the possibility of using MS rubber-based sandwich beam to reduce the vibration of vibrating structure were also conducted [24, 25]. The effect of boundary conditions [26] and changing magnetic field [27] to the dynamic performance of MS rubber filled sandwich beam were explored.

Compared to the research for the possible applications of MS rubber in tuned vibration absorbers and smart sandwich beams, the research for the application of MS rubber in vibration isolation is not sufficient. Normally, vibration isolation problem can be divided into two categories. The first type is to protect foundation against the force induced by vibration sources such as rotating machine. The second type is to isolate sensitive equipment from ground motion. For the application of MS rubber in vibration isolation area, research is mainly focused on the second type of vibration isolation problem [28–30] and a fuzzy logical control algorithm is often used. However, it should be noted that the time required for MS rubber to respond has a significant impact on the vibration isolation effect for MS rubber vibration isolation system using fuzzy logic. Consequently, the relation between the response time of MS rubber and the effectiveness of the vibration isolation effect should be studied. At the same time, the first type of vibration isolation problem is also important and needs investigation. Although some researches were conducted to study the effectiveness of using MS rubber-based vibration isolator to mitigate the energy transmitted to the foundation [31, 32], the corresponding control strategy to change the magnetic field for MS rubber is not clearly developed. In addition, the influence of the amplitude dependency for MS rubber on the isolation effect was not fully studied.

In order to facilitate the application of MS rubber in vibration isolation area, a MS rubber isolation system, which consists of a mass block supported by four MS rubber-based isolators with an infinite extended flexible foundation connected, is modeled. A highly nonlinear model for this MS rubber isolation system is developed where the impact of the nonlinearity and magnetic field dependency for MS rubber, the mechanical inerance of the foundation, the dimensions of MS rubber isolators and the mass of solid block on the vibration isolation effect are all considered. After developing the model of MS rubber-based smart vibration isolation system, a numerical simulation is performed and a control strategy is developed for the application of MS rubber in the first type of vibration isolation problem with the goal to protect foundation against external force. The effect of the amplitude dependency for MS rubber to the vibration isolation effect is also investigated. Furthermore, for the application of MS rubber in the second type of vibration isolation problem to isolate sensitive equipment from ground motion, another numerical simulation with fuzzy logical control algorithm is conducted. The feasibility of fuzzy logical control algorithm is investigated and the relation between the response time of MS rubber and the effectiveness of the vibration isolation effect is studied. The work done in this paper provides fundamentals for the application of MS rubber in vibration isolation problem and the control strategies introduced could also be applied to other kinds of stiffness variable vibration reduction components.

2. Model and method

2.1. Configuration and parameters for MS rubber vibration isolation system

The vibration isolation system consists of a mass block mounted upon four MS rubber-based vibration isolators connected to an infinite extended flexible foundation. The schematic diagram is shown in figure 1.

The harmonic force $F_{\text{ext}}$ applies at the center of the upper surface of the solid block. The force generated by each isolator to the solid block is $F_{\text{MS}}$. Only the vertical displacement is considered in this model. The displacement of the solid block and foundation are $u$ and $u_f$, respectively. The distances between MS isolators in $y$-direction and $z$-direction are $2l_y = 0.3 \text{ m}$ and $2l_z = 0.4 \text{ m}$, respectively. The thickness of the infinite extended concrete foundation is $h_l = 0.35 \text{ m}$ and the total mass of the solid block is $m$. The thickness and cross section area of each MS rubber element are $h_{\text{MS}}$ and $A$, respectively. The density, Poisson’s ratio and Young’s
modulus of the infinite flexible foundation are \( \rho_f = 2300 \text{ kg m}^{-3} \), \( \nu_f = 0.2 \) and \( E_f = (1 + 0.02) \times 2.6 \times 10^{10} \text{ N m}^{-2} \), respectively, where \( j = \sqrt{-1} \) is the imaginary unit.

### 2.2. Constitutive model for MS rubber

Each MS isolator consists of two MS rubber elements working in shear deformation mode. The relation between the force \( F_{MS} \) and the shear stress \( \tau \) for each MS rubber isolator is

\[
F_{MS} = 2A\tau. \tag{1}
\]

The relation between the displacement of each MS rubber isolator \((u - u_f)\) and shear strain \( \gamma \) is

\[
\gamma = \frac{u - u_f}{h_{MS}}. \tag{2}
\]

It is found that there is a strong amplitude, frequency and magnetic field dependency of the shear modulus of MS rubber and the amplitude dependency is highly related to the magnetic field applied \([16]\). The relation between the shear stress \( \tau \) and shear strain \( \gamma \) is expressed by a constitutive model of MS rubber. The schematic configuration for the constitutive model of MS rubber is shown in figure 2.

Details of the constitutive model could be found in \([17]\), but for the self-sufficiency of this paper, a brief introduction to the model follows.

The total stress \( \tau(t) \) for MS rubber is divided into \( \tau_e(t), \tau_{ve}(t) \) and \( \gamma(t) \) which represent the elastic, viscoelastic and frictional shear stress, respectively,

\[
\tau(t) = \tau_e(t) + \tau_{ve}(t) + \gamma(t). \tag{3}
\]

The elastic shear stress \( \tau_e(t) \) is linearly related to the shear strain \( \gamma(t) \) by

\[
\tau_e(t) = G_e \cdot \gamma(t), \tag{4}
\]

where \( G_e \) is the elastic shear modulus for MS rubber.

The relation between the viscoelastic shear stress and shear strain is described by a relaxation convolution integral

\[
\tau_{ve}(t) = \frac{b}{\Gamma(1 - a)} \frac{d}{dt} \int_0^t (t - s)^a e^{-s} d\gamma(s) ds, \tag{5}
\]

where \( a \) and \( b \) are the parameters in the relaxation convolution integral and \( \Gamma \) is the Gamma function.

Figure 1. Schematic diagram for MS rubber vibration isolation system.

Figure 2. Schematic configuration for the constitutive model of MS rubber with detail for the stress–strain relation of bounding surface model.

The frictional stress is described by a bounding surface model and the details of the bounding surface model were introduced by Dafalias and Popov \([33]\). For one dimensional case with a vanishing size of elastic range \([34]\), the model can be visualized in the right part of figure 2 where the straight line bounded the curve represents the bounding surface and the constitutive model are as follows.

A bounding surface in stress-space is defined as

\[
f = (\eta - \beta)^2 - S_{yield} = 0, \tag{6}
\]

where \( \tau_f \) is the frictional stress and a bar over \( \tau_f \) indicates stress on the bounding surface, \( \beta \) and \( S_{yield} \) are the center and the radius of the bounding surface, respectively.

The flow rule for strain increment is

\[
\dot{\gamma} = \lambda(\eta - \beta), \tag{7}
\]

where \( \lambda \) is the plastic multiplier, for bounding surface with a zero-elastic range \( \lambda > 0 \).

The hardening functions for \( \beta \) and \( \tau_f \) are

\[
H_p = \frac{d\beta}{d\gamma}, \tag{8}
\]

and

\[
H = \frac{d\tau_f}{d\gamma}. \tag{9}
\]
According to [35], the relation between the plastic modulus $H$ and $H_p$ is

$$ H = H_p \left(1 + \frac{\delta}{\delta_m - \delta}\right), \quad (10) $$

where $\delta$ is the distance between $\tau$ and $\tau_i$ defined as $\delta = |\tau - \tau_i|$ in one dimensional case, parameter $\delta_m$ is the distance to the bounding surface at the previous turning point. Initially, it is assumed that $\delta_m = S_{yield}$ as shown in figure 2. Subsequently, the value of $\delta_m$ is updated when there is a load reversal. Mathematically, the value of $H$ gradually decreases with the decreasing of $\delta$. When there is a change of the loading direction, the plastic modulus surges suddenly and decreases subsequently.

Based on the work by Blom and Kari [16] that the difference in shear modulus of MS rubber under zero and saturation magnetic field is relatively constant at different frequencies and the variation of the loss factor with respect to the magnetic field at different frequencies is relatively small. This leads to the conclusion that the magnetic dependence of the viscoelastic part of MS rubber can be neglected. Moreover, experimental results show that the magnetic field induced shear modulus of MS rubber decreases with the increasing of strain amplitude. It suggests that there is a magnetic dependency for the frictional part of MS rubber. Since loss factor is equal to the loss modulus divided by the storage modulus, in order to ensure the invariance of the loss factor with respect to the magnetic field, the increase of loss modulus by the magnetic dependent frictional part should be balanced by an increase of the storage modulus associated with the magnetic field. Therefore, there is also a magnetic dependency for the elastic part. In summary, the magnetic sensitivity is only introduced for the elastic and frictional parts but not the viscoelastic part. According to Dorfmann et al [36], the magneto-sensitivity is quadratically dependent on magnetic field applied. It is obtained

$$ G_e = G_e^0 \left[1 + \left(\frac{M}{M_e}\right)^2 \delta_1\right], \quad (11) $$

$$ S_{yield} = S_{yield}^0 \left[1 + \left(\frac{M}{M_e}\right)^2 \delta_2\right], \quad (12) $$

and

$$ H_p = H_p^0 \left[1 + \left(\frac{M}{M_e}\right)^2 \delta_3\right], \quad (13) $$

where $G_e^0$, $S_{yield}^0$ and $H_p^0$ are the parameters of MS rubber where no magnetic field is applied. Parameters $M$ and $M_e$ represent the applied and saturation magnetic field respectively, for MS rubber. The parameters $\delta_1$, $\delta_2$ and $\delta_3$ are magnetic influence factors. The details of the numerical implementation are given in [17].

2.3. Governing equations for MS isolation system

For the first type of vibration isolation problem to protect foundation against external loading, the dynamic equilibrium equation for the system is

$$ m\ddot{u} + 4F_{MS}(u - u_t) = F_{ext}, \quad (14) $$

where $\dddot{f}(\cdot)$ denotes the second derivative with respect to time.

The foundation is considerably stiffer compared to MS isolator. Accordingly it is assumed that $u_t \ll u$ and the dynamic equilibrium equation can be simplified to

$$ m\ddot{u} + 4F_{MS}(u) = F_{ext}. \quad (15) $$

In order to calculate $u$ at current time step, the Newmark-beta method based on implicit time integration is applied [37]. Firstly, an initial acceleration is obtained by

$$ \ddot{u}_{trial} = \frac{4\cdot F_{MS}(u_n) - F_{ext}}{m}, \quad (16) $$

where $u_n$ is the displacement at the previous time step and $F_{ext}$ is the external force at current time step.

The prediction of the displacement, velocity and acceleration at current time step are obtained by average acceleration method

$$ u = u_n + \Delta u_n + \frac{\Delta t^2}{4}\ddot{u}_{trial}, \quad (17) $$

$$ \dot{u} = \dot{u}_n + \frac{\Delta t}{2}\ddot{u}_{trial} \quad (18) $$

and

$$ \ddot{u} = 0. \quad (19) $$

Next, the residual force $R$ is obtained as

$$ R = M\ddot{u} + F_{MS}(u) - F_{ext}. \quad (20) $$

A iteration threshold $\kappa = 1 \times 10^{-3}$ is set as the convergence criteria, if the residual force is larger than $\kappa$, a correction is needed, which is

$$ \Delta u = -\frac{R}{K}. \quad (21) $$

where $K$ is the tangent of $R$, which is obtained by

$$ K = \frac{F_{MS}(u) - F_{MS}(u_0)}{u - u_0} + \frac{4M}{\Delta t^2}, \quad (22) $$

where $u_0 = u_n$ at the beginning of iteration.

The displacement, velocity and acceleration are corrected subsequently as

$$ u = u_n + \Delta u, \quad (23) $$

$$ \dot{u} = \dot{u}_n + \frac{2\Delta u}{\Delta t} \quad (24) $$

and

$$ \ddot{u} = \ddot{u}_n + \frac{4\Delta u}{\Delta t^2}. \quad (25) $$

For the next iteration, the value of $u_0$ is updated to $u$.

When the convergence requirement is satisfied, the iteration finishes and $u$ at current time step is obtained and the force $F_{MS}$ can be determined subsequently.

After obtaining the force $F_{MS}$ transmitted to the foundation, the foundation displacement $u_t$ is calculated by

$$ (j2\pi f)^2\ddot{u}(i) = H_{ik}F_{MS(k)}(i), \quad (26) $$
where \( f \) is frequency, \( (\cdot) \) represents the frequency domain quantities of the time domain signal obtained by Fourier transform, \( \tilde{u}_{fr} \) denotes the foundation displacement of point \( i \)th MS rubber isolator in frequency domain. The impact for the force on the \( i \)th point of MS rubber to the acceleration of the foundation on \( i \)th point is reflected by the inertance matrix component \( H_{fr} \).

According to Cremer et al. [38], the inertance at the transfer point with a distance \( r \) to the driving point can be expressed as

\[
H(r) = -\frac{j}{8Bk_r^2} \left[ H_{0}^{(2)}(k_r r) - j2\pi K_0(k_r r) \right],
\]

(27)

where \( B = E_i h_i^3/12(1 - \nu_i^2) \) is the bending stiffness of the foundation. The parameter \( k_r \) is \( (\nu_i h_i \omega^2/B)^{1/4} \) is the bending wave number for the foundation. Lastly, \( H_{0}^{(2)} \) and \( K_0 \) are the zero order Hankel function of the second kind and the zero order modified Bessel function of the second kind, respectively.

For the second type of vibration isolation problem to protect sensitive equipment from ground motion, the excitation force is caused by the foundation motion and the dynamic equilibrium for the whole system is

\[
m\ddot{u}_r + 4 \cdot \mu_{MS}(u_r) = -m\ddot{u}_t,
\]

(28)

where \( u_r = u - u_t \) is the relative displacement of the MS isolators, \( u_t \) is the displacement of the foundation which is known in this case. Therefore, \( u_t \) can be solved by the previously introduced Newmark-beta method based on implicit time integration as well.

### 2.4. Basics for energy flow

Energy flow, which is the time averaged product of force and velocity, describes the changing rate of energy for dynamic system. The fundamentals for this method are based on the law of conservation of energy. Compared with other indicators like force and displacement transmissibility, energy flow assess the vibration isolation effect more properly [39]. In consequence, the time averaged energy flow is used to measure the vibration isolation effect for the first type of vibration isolation problem.

According to signal analysis theory, the time averaged energy flow can be obtained by calculating the inverse Fourier transform of the cross-spectral density between force and velocity with zero-time delay [40].

\[
\langle P \rangle = \int_{-\infty}^{\infty} \mathbf{S}(\tilde{F}, \tilde{v}) e^{j2\pi f \tau} df \bigg|_{\tau=0} = \int_{-\infty}^{\infty} \mathbf{S}(\tilde{F}, \tilde{v}) df,
\]

(29)

where \( \mathbf{S}(\tilde{F}, \tilde{v}) \) is the cross-spectral density between the force (\( \tilde{F} \)) and the velocity (\( \tilde{v} \)) in frequency domain.

By Parseval’s theorem [41], it is known that

\[
\mathbf{S}(\tilde{F}, \tilde{v}) = \lim_{T \to \infty} \frac{1}{T} \mathbf{E}[\tilde{F}^{*}\tilde{v}],
\]

(30)

where \( \mathbf{E}[\cdot] \) is the expected value operator and \( (\cdot)^* \) is the complex conjugate operator. Consequently, the energy flow can be calculated by combing equations (29) and (30). For more details of the calculation of the energy flow, the reader is referred to [31].

In this paper, after obtaining \( F_{MS} \) and \( u_t \) by equations (14)-(27), the energy flow into the foundation is

\[
\langle P \rangle = 4 \int_{-\infty}^{\infty} \mathbf{S}(\tilde{F}_{MS}, \tilde{v}_t) df,
\]

(31)

where \( v_t = \tilde{v}_t \) is the velocity of the point on the foundation which connected to MS isolator.

### 3. Vibration isolation control against harmonic force

#### 3.1. Parameter identification for MS rubber

By using the measurement data conducted by Jonas and Kari [42] for MS rubber under zero magnetic field and at 20 °C, magnetic free parameters \( G_0^0 \), \( S_0^0 \), \( H_0^0 \), \( a \) and \( b \) are obtained. The results are \( G_0^0 = 3.0712 \times 10^6 \text{ N m}^{-2} \), \( S_0^0 = 0.453 \times 10^5 \text{ N m}^{-2} \), \( H_0^0 = 0.102 \times 10^6 \text{ N m}^{-2} \), \( a = 0.2997 \) and \( b = 0.2709 \times 10^6 \text{ N s}^2 \text{ m}^{-2} \). After obtaining the magnetic free parameters of MS rubber, a second round of least square method is conducted and the rest parameters are obtained as \( \delta_1 = 0.2594 \), \( \delta_2 = 0 \), \( \delta_3 = 0.7576 \) and \( M_s = 0.5615 \). The details of the parameter identification method of MS rubber are given in [17].

#### 3.2. Energy flow simulation results

All simulations in this paper are performed with the software MATLAB® (MATLAB Release 2015b, The MathWorks, Inc., Natick, Massachusetts, United States). For the first type of vibration isolation, with the goal to reduce the energy transmitted to the foundation, a numerical study is conducted. Three levels of harmonic forces with magnitude of 3, 10 and 30 N are applied at the center of the upper surface of the solid block and the frequency ranges from 40 to 120 Hz. The mass of the solid block is set to \( m = 50 \text{ kg} \). The thickness and area of each MS rubber element are \( h_{MS} = 0.002 \text{ m} \) and \( A = 0.020 \times 0.020 \text{ m}^2 \), respectively. The resulting magnitude of the relative displacement of the mass (\( u_t \)) and the displacement of the foundation (\( u_f \)) under zero and saturation magnetic field in frequency domain are in figures 3 and 4. It is
found that the displacement of the foundation is considerably smaller compared to the relative displacement of the solid block. Thus, the assumption $u_f = u$ is reasonable.

The energy flow into the foundation at zero and saturation magnetic field cases is different. This is caused by the magnetic dependency of MS rubber. Under saturation magnetic field, MS isolator tends to be stiffer compared to the zero magnetic field case. Furthermore, the resonance frequency decreases with increasing magnitude of the harmonic force. This is caused by the amplitude dependency of MS rubber. The shear modulus of MS rubber decreases with increasing magnitude of the harmonic force and thus, a lower resonance frequency is reached for the MS isolation system.

3.3. Magnetic field control strategy based on coincidence frequency

By comparing the energy flow under zero and saturation magnetic field cases, it is found that at a certain frequency, the energy flow to the foundation at zero and saturation magnetic field cases is the same, which is named coincidence frequency ($f_{\text{coincidence}}$) in this paper. By applying an adaptive frequency dependent magnetic field, a reduced energy flow can be achieved. In specific, saturation magnetic field is applied when the frequency for the harmonic force ($f_h$) is smaller than the coincidence frequency and zero magnetic field is applied when the frequency for the harmonic force is larger than the coincidence frequency. Consequently, a magnetic field control strategy based on coincidence frequency to reduce the energy to the foundation can be obtained. The details of the control strategy are list in table 1.

After applying the control strategy, the energy flow into the foundation is represented in line with symbols in figures 5–7. The maximum energy flow under zero magnetic field is $2.182 \times 10^{-4}$, $2.073 \times 10^{-3}$ and $1.815 \times 10^{-2}$ W for force magnitude 3, 10 and 30 N, respectively. As a consequence, the energy flow into the

![Figure 4](image-url)  
**Figure 4.** The displacement magnitude of foundation (lines with different symbols) and mass (lines without symbol) with saturation magnetic field. Three kinds of harmonic force (3, 10 and 30 N) are considered.

![Figure 5](image-url)  
**Figure 5.** Energy flow to the foundation of zero (dashed line), saturation (solid line) and controlled (line with symbols) magnetic field with force magnitude 3 N.

![Figure 6](image-url)  
**Figure 6.** Energy flow to the foundation at zero (dashed line), saturation (solid line) and controlled (line with symbols) magnetic field with force magnitude 10 N.

![Figure 7](image-url)  
**Figure 7.** Energy flow to the foundation at zero (dashed line), saturation (solid line) and controlled (line with symbols) magnetic field with force magnitude 30 N.

| Criteria for frequency | Magnetic field |
|------------------------|---------------|
| $f_h < f_{\text{coincidence}}$ | Saturation |
| $f_h \geq f_{\text{coincidence}}$ | Zero |

Table 1. Controlled magnetic field based on coincidence frequency.
foundation is reduced by 27.9%, 21.3% and 15.0%, respectively, compared to zero magnetic field case by controlling the magnetic field.

3.4. Effect of the amplitude dependency for MS rubber to the isolation effect

The coincidence frequency is 73.8, 70.4 and 67.4 Hz for the harmonic force magnitude 3, 10 and 30 N, respectively. The coincidence frequency decreases with increasing magnitude of the force. This is caused by the amplitude dependency of MS rubber. It should be noted that different coincidence frequency should be used as indicator to adjust the magnetic field applied to MS vibration isolators at different magnitude of the external harmonic force. Consequently, the magnitude of the harmonic force is an important factor to consider for the implementation of the coincidence frequency based magnetic field control strategy and the directly optimization for the control may not be feasible due to the nonlinear behavior of MS rubber.

4. Vibration isolation control against foundation motion

4.1. Fuzzy logical control strategy

Fuzzy logical control algorithm was initially proposed by Lotfi in 1965 [43]. After that, the research for the application of fuzzy logical control were sparked [44–47]. The notable feature of fuzzy logic control is that the output signals are some specific discrete values like zero and one after the if-then rule of the fuzzy logic controller for continuous valued input signals. Therefore, the time delay of the control algorithm due to the calculation can be reduced to the utmost extent. The main advantage of fuzzy logical control strategy is the ability to address nonlinear phenomenon and uncertainty of the system. For fuzzy logical control strategy used in MS rubber vibration isolator, the relative displacement ($u_r$) and velocity ($\dot{u}_r$) of MS rubber isolator are served as the input signals. The goal is to reduce the relative motion of the solid block with respect to the foundation. The output signal is the magnetic field applied to MS rubber isolators. The details for the if-then rules are described in Table 2. Physically, it means increasing the dynamic stiffness of MS isolators when the solid mass moves away from the equilibrium position and decreasing the dynamic stiffness of MS isolators when the solid mass moves towards the equilibrium position.

Table 2. Controlled magnetic field based on fuzzy logical strategy.

| $u_r \times \dot{u}_r$ | Positive | Negative |
|------------------------|----------|----------|
| Magnetic field         | Zero     | Saturation |

4.2. Feasibility for fuzzy logical control strategy

The mass of the solid block, the thickness and the cross section area of each MS rubber are set to $m = 180$ kg, $h_{MS} = 0.005$ m and $A = 0.010 \times 0.010$ m², respectively. To verify the effectiveness of the fuzzy logical control strategy for MS rubber isolation system, band-limited Gaussian noise with root mean square (rms) value 0.001, 0.003 and 0.010 m s⁻², respectively, is applied as the acceleration.
excitation of the foundation. The frequency of the band-limited Gaussian noise ranges from 5 to 25 Hz. The band-limited Gaussian noise is generated by a fast Fourier transformation method [48]. The sampling frequency and time length of the acceleration signals are 1000 Hz and 3 s, respectively. According to the research by Ginder et al [2] and Zhu et al [49], the response time for MS rubber is at an order of 10 ms. To investigate the vibration isolation effect for MS rubber isolator, for the simplest case, it is assumed that the response time for MS rubber is 10 ms. The response time for the coil to generate the magnetic field and the time required for algorithm implementation are not considered in this paper. At every 10 ms, there is a control instruction for MS rubber to change the magnetic field.

The response for the solid block under zero, saturation and controlled magnetic field by fuzzy logical controller are in figures 8–10. The peak and rms values of the relative displacement of the solid mass under three kinds of band-limited Gaussian noise are compared in tables 3 and 4. It shows that the maximum relative displacement of the block mass under fuzzy logical control is reduced by 24.1%, 29.4% and 33.0%, respectively, for three kinds of band-limited Gaussian noise, compared to the zero magnetic field case. In consequence, an enhanced vibration isolation effect is obtained by MS rubber isolator with fuzzy logical control strategy applied. It is worth noting that the advantage of using fuzzy logic control strategies to reduce vibration can be further demonstrated by MS isolators with larger MS effects, although in this paper the advantage of using fuzzy logic control strategy is not fully reflected compared to the case of saturation magnetic field.

4.3. Limitation of the fuzzy logical control strategy used for MS isolator

The control commands of the fuzzy logic control strategy should be dense enough in order to make the change of the magnetic field sufficient to cope with changes for the external loading. For example, if the relative displacement of the solid block is sin(2πt), there are four control instructions to change the magnetic field for fuzzy logical controller in every period. The details are shown in figure 11. Thus, the frequency for the instructions based on fuzzy logical control strategy ($f_{\text{instruction}}$) should at least four times larger than the maximum frequency component ($f_{\text{max}}$) for the input signal. Mathematically, the relation is

$$f_{\text{instruction}} \geq 4f_{\text{max}}.$$  \hspace{1cm} (32)

Because the response time for MS rubber is at the order of 10 ms, it means that the frequency for the instructions ($f_{\text{instruction}}$) cannot and should not exceed 100 Hz when neglecting the calculation time for the control strategy and the response time for the coil to generate magnetic field. According to equation (32), an improved vibration isolation effect can only be reflected by MS vibration isolator using fuzzy logical control strategy when the highest component of the frequency for the external loading does not exceed 25 Hz. Furthermore, due to the high contribution of the signal in the vicinity of the resonance frequency to the motion of the solid block, the MS rubber-based isolator with fuzzy logical control strategy can exhibit a superior vibration isolation effect compared to the traditional rubber-based isolator only when the resonance frequency for the system is low (less than 25 Hz in this case). Consequently, the limitation of MS vibration isolator to reduce vibration in random excitation case is that it can only be applied to low frequency region. However, for the coincidence frequency-based magnetic field control strategy, there is no limitation of the frequency component for the

| Table 3. Peak value of the relative displacement (m) under different magnetic fields. |
|-------------------------------------------|
| Excitation (m s$^{-2}$) | Rms = 0.001 | Rms = 0.003 | Rms = 0.010 |
|--------------------------|-------------|-------------|-------------|
| Zero magnetic            | 0.0137 × 10$^{-4}$ | 0.0408 × 10$^{-4}$ | 0.1758 × 10$^{-4}$ |
| Saturation magnetic      | 0.0109 × 10$^{-4}$ | 0.0302 × 10$^{-4}$ | 0.1076 × 10$^{-4}$ |
| ON–OFF magnetic          | 0.0104 × 10$^{-4}$ | 0.0288 × 10$^{-4}$ | 0.1177 × 10$^{-4}$ |

| Table 4. Rms value of the relative displacement (m) under different magnetic fields. |
|-------------------------------------------|
| Excitation (m s$^{-2}$) | Rms = 0.001 | Rms = 0.003 | Rms = 0.010 |
|--------------------------|-------------|-------------|-------------|
| Zero magnetic            | 0.0495 × 10$^{-5}$ | 0.1407 × 10$^{-5}$ | 0.4746 × 10$^{-5}$ |
| Saturation magnetic      | 0.0398 × 10$^{-5}$ | 0.1161 × 10$^{-5}$ | 0.3962 × 10$^{-5}$ |
| ON–OFF magnetic          | 0.0379 × 10$^{-5}$ | 0.1075 × 10$^{-5}$ | 0.3875 × 10$^{-5}$ |

Figure 11. Control command for sine signal where dashed line, dashed–dotted line and solid line represent the displacement, velocity and control command, respectively.
signal, the restriction is that it is only valid for periodic loading cases.

5. Conclusion

A highly nonlinear model for MS rubber-based isolation system is developed in this paper. The nonlinearity and magnetic field sensitivity for MS rubber, the mechanical inerterance of the flexible foundation, the size of the MS rubber components and the mass of the solid block are all included in this model. However, the nonlinear model for MS rubber-based isolation system is a one-dimensional model. In order to achieve an improved vibration isolation compared to traditional passive isolators, two control strategies (magnetic field control strategy based on coincidence frequency and fuzzy logical control strategy, respectively) are considered and the feasibility of the two control strategies are investigated based on this MS rubber isolation system. For magnetic field control strategy based on coincidence frequency, the goal is to protect foundation against harmonic force. Simulation results reveal that the energy transmitted to the foundation is reduced by more than 15% compared to passive typed vibration isolators after applying magnetic field control strategy based on coincidence frequency. However, it should be noted that it is only valid for harmonic loading case and the coincidence frequency changes with the magnitude of the harmonic loading.

For fuzzy logical control strategy, the goal is to reduce the relative response for sensitive equipment under foundation motion. The peak value of the relative displacement of the solid block is reduced by at least 24.1% after applying fuzzy logical control strategy. The limitation of fuzzy logical control strategy for MS rubber isolation system is that it is only valid in low frequency range. In particular, the highest frequency component (either the highest frequency component for excitation or the natural frequency for the system) should not exceed 25 Hz based on the current fabrication technology for MS rubber.

The work done in this paper includes the model developed for MS rubber isolation system, the validation for the control strategies used for this MS rubber isolation system and the investigation of the limitations of the two control strategies. The above work involves the main aspects for the application of MS rubber in vibration isolation area and it is helpful to reach an improved vibration isolation effect for vibration isolators. For vibration isolation system with a more complex structure such as cylindrically shaped, three-dimensional constitutive model of MS rubber is needed to simulate the dynamic performance, which is the future work of our research.

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