TIDAL LOVE NUMBERS OF NEUTRON STARS

TANJA HINDERER
Center for Radiophysics and Space Research, Cornell University, Ithaca, NY 14853; tph25@cornell.edu

Received 2007 November 15; accepted 2008 January 22

ABSTRACT

For a variety of fully relativistic polytropic neutron star models we calculate the star’s tidal Love number \( k_2 \). Most realistic equations of state for neutron stars can be approximated as a polytrope with an effective index \( n \approx 0.5–1.0 \). The equilibrium stellar model is obtained by numerical integration of the Tolman-Oppenheimer-Volkov equations. We calculate the linear \( l = 2 \) static perturbations to the Schwarzschild spacetime following the method of Thorne and Campolattaro. Combining the perturbed Einstein equations into a single second-order differential equation for the perturbation to the metric coefficient \( g_{tt} \) and matching the exterior solution to the asymptotic expansion of the metric in the star’s local asymptotic rest frame gives the Love number. Our results agree well with the Newtonian results in the weak field limit. The fully relativistic values differ from the Newtonian values by up to \( \sim 24\% \). The Love number is potentially measurable in gravitational wave signals from inspiralling binary neutron stars.

Subject headings: equation of state — gravitation — relativity — stars: neutron

1. INTRODUCTION AND MOTIVATION

A key challenge of current astrophysical research is to obtain information about the equation of state (EOS) of the ultradense nuclear matter making up neutron stars (NSs). The observational constraints on the internal structure of NSs are weak; the observed range of NS masses is \( M \sim 1.1–2.2 \, M_\odot \) (Lattimer & Prakash 2007), and there is no current method to directly measure the radius. Some estimates using data from X-ray spectroscopy exist, but those are highly model-dependent (e.g., Webb & Barret 2007). Different theoretical models for the NS internal structure predict, for a NS of mass \( M \), different degrees of freedom of the bodies start to appreciably influence the signal, and there have been many investigations of how well the EOS can be constrained using the last several orbits and merger, including constraints from the gravitational wave spectrum (Faber et al. 2002) and from the NS tidal disruption signal for NS–black hole binaries (Vallisneri 2002). Several numerical simulations of the hydrodynamics of NS–NS mergers have studied the dependence of the gravitational wave spectrum on the radius and EOS (see, e.g., Baumgarte & Shapiro 2003 and references therein). However, trying to extract EOS information from this late-time regime presents several difficulties: (1) the highly complex behavior requires solving the nonlinear equations of general relativity together with relativistic hydrodynamics; (2) the signal depends on unknown quantities such as the spins and angular momentum distribution inside the stars; and (3) the signals from the hydrodynamic merger are outside of LIGO’s most sensitive band.

During the early regime of the inspiral, the signal is very clean and the influence of tidal effects is only a small correction to the waveform’s phase. However, signal detection is based on matched filtering, i.e., integrating the measured waveform against theoretical templates, where the requirement on the templates is that the phasing remain accurate to \( \sim 1 \) cycle over the inspiral. If the accumulated phase shift due to the tidal corrections becomes of order unity or larger, it could corrupt the detection of NS-NS signals, or alternatively, detecting a phase perturbation could give information about the NS structure. This has motivated several analytical and numerical investigations of tidal effects in NS binaries (Bildsten & Cutler 1992; Kokkotas & Schaefer 1995; Kokhanev 1992; Taniguchi & Shibata 1998; Mora & Will 2004; Shibata 1994; Gualteri et al. 2001; Pons et al. 2002; Berti et al. 2002). The influence of the internal structure on the gravitational wave phase in this early regime of the inspiral is characterized by a single parameter, namely, the ratio \( \tilde{\lambda} \) of the induced quadrupole to the perturbing tidal field. This ratio \( \tilde{\lambda} \) is related to the star’s tidal Love number \( k_2 \) by \( k_2 \sim 3G\tilde{\lambda}\sim -5/2 \), where \( R \) is the star’s radius. Flanagan & Hinderer (2008) have shown that for an inspiral of two nonspinning \( 1.4 \, M_\odot \) NSs at a distance of 50 Mpc, LIGO II detectors will be able to constrain \( \lambda \sim 2.01 \times 10^{37} \) g cm\(^2\) s\(^2\) with 90% confidence. This number is an upper limit on \( \lambda \) in the case that no tidal phase shift is observed. The corresponding constraint on radius would be \( R \lesssim 13.6 \) km (15.3 km) for \( n = 0.5 \) (\( n = 1.0 \)) fully relativistic polytrope, for \( 1.4 \, M_\odot \) NSs (Flanagan & Hinderer 2008). Because NSs are compact objects with strong internal gravity, their Love numbers could be very different from those for Newtonian stars that have been computed previously, e.g., by Brooker & Hinderer (2005).

Knowledge of Love number values could also be useful for comparing different numerical simulations of NS binary inspiral by focusing on models with the same masses and values of \( \lambda \).
In Flanagan & Hinderer (2008), the \( l = 2 \) tidal Love numbers for fully relativistic NS models with polytropic pressure-density relation \( P = K \rho^{1+1/n} \), where \( K \) and \( n \) are constants, were computed for the first time. The present paper will give details of this computation. Using polytropes allows us to explore a wide range of stellar models, since most realistic models can be reasonably approximated as a polytrope with an effective index in the range \( n \sim 0.5-1.0 \) (Lattimer & Prakash 2007). Our prescription for computing \( \lambda \) is valid for an arbitrary pressure-density relation and not restricted to polytropes. In \( \S \) 2, we start by defining \( \lambda \) in the fully relativistic context in terms of coefficients in an asymptotic expansion of the metric in the star’s local asymptotic rest frame and discuss the extent to which it is uniquely defined. In \( \S \) 3, we discuss our method of calculating \( \lambda \), which is based on static linearized perturbations of the equilibrium configuration in the Regge-Wheeler gauge as in Thorne & Campolattaro (1967). Section 4 contains the results of the numerical computations together with a discussion. Unless otherwise specified, we use units in which \( c = G = 1 \).

2. DEFINITION OF THE LOVE NUMBER

Consider a static, spherically symmetric star of mass \( M \) placed in a static external quadrupolar tidal field \( \mathcal{E}_{ij} \). The star will develop in response a quadruple moment \( Q_{ij} \). In the star’s local asymptotic rest frame (asymptotically mass-centered Cartesian coordinates) at large \( r \) the metric coefficient \( g_{ij} \) is given by (Thorne 1998)

\[
\frac{1-g_{ij}}{2} = -\frac{M}{r} \frac{3}{2} O_{ij} \left( n'n' - \frac{1}{3} \delta_{ij} + O \left( \frac{1}{r^3} \right) \right) + \frac{1}{2} \mathcal{E}_{ij} x^i x^j + O(\rho^3),
\]

where \( n' = x'/r' \); this expansion defines \( \mathcal{E}_{ij} \) and \( Q_{ij} \). In the Newtonian limit, \( Q_{ij} \) is related to the density perturbation \( \delta \rho \) by

\[
Q_{ij} = \int d^3 x \delta \rho(x) \left( x_i x_j - \frac{1}{3} \mathcal{E}_{ij} \right),
\]

and \( \mathcal{E}_{ij} \) is given in terms of the external gravitational potential \( \Phi_{\text{ext}} \) as

\[
\mathcal{E}_{ij} = \frac{\partial^2 \Phi_{\text{ext}}}{\partial x^i \partial x^j}.
\]

We are interested in applications to fully relativistic stars, which requires going beyond Newtonian physics. In the strong field case, equations (2) and (3) are no longer valid, but the expansion of the metric from equation (1) still holds in the asymptotically flat region and serves to define the moments \( Q_{ij} \) and \( \mathcal{E}_{ij} \).

We briefly review here the extent to which these moments are uniquely defined, since there are considerable coordinate ambiguities in performing asymptotic expansions of the metric. For an isolated body in a static situation these moments are uniquely defined: \( \mathcal{E}_{ij} \) and \( Q_{ij} \) are the coordinate-independent moments defined by Geroch (1970) and Hansen (1974) for stationary, asymptotically flat spacetimes in terms of certain combinations of the derivatives of the norm and twist of the timelike Killing vector at spatial infinity. In the case of an isolated object in a dynamical situation, there are ambiguities related to gravitational radiation, for example, angular momentum is not uniquely defined (Wald 1984). For the application to the adiabatic part of a NS binary inspiral, we are interested in the case of a non-isolated object in a quasi-static situation. In this case there are still ambiguities (related to the choice of coordinates), but their magnitudes can be estimated (Thorne & Hartle 1985) and are at a high post-Newtonian order and therefore can be neglected. We are also interested in working (1) to linear order in \( \mathcal{E}_{ij} \) and (2) in the limit where the source of \( \mathcal{E}_{ij} \) is very far away. In this limit the ambiguities disappear.

To linear order in \( \mathcal{E}_{ij} \), the induced quadrupole will be of the form

\[
Q_{ij} = -\lambda \mathcal{E}_{ij}.
\]

Here \( \lambda \) is a constant which is related to the \( l = 2 \) tidal Love number (apsidal constant) \( k_2 \) by (Flanagan & Hinderer 2008)

\[
k_2 = \frac{3}{2} G \lambda R^{-5}.
\]

Note the difference in terminology; in Flanagan & Hinderer (2008) \( \lambda \) was called the Love number, whereas in this paper, we reserve that name for the dimensionless quantity \( k_2 \).

The tensor multipole moments \( Q_{ij} \) and \( \mathcal{E}_{ij} \) can be decomposed as

\[
\mathcal{E}_{ij} = \sum_{m=-2}^{2} \mathcal{E}_m Y_{2m}^{ij},
\]

\[
Q_{ij} = \sum_{m=-2}^{2} Q_m Y_{2m}^{ij},
\]

where the symmetric traceless tensors \( Y_{2m}^{ij} \) are defined by (Thorne 1980)

\[
Y_{2m}(\theta, \varphi) = Y_{2m}^{ij} n_i n_j
\]

with \( n = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \). Thus, equation (4) can be written as

\[
Q_m = -\lambda \mathcal{E}_m.
\]

Without loss of generality, we can assume that only one \( \mathcal{E}_m \) is nonvanishing, this is sufficient to compute \( \lambda \).

3. CALCULATION OF THE LOVE NUMBER

3.1. Equilibrium Configuration

The geometry of spacetime of a spherical, static star can be described by the line element (Misner et al. 1973)

\[
ds_0^2 = g_{\alpha\beta} \, dx^\alpha \, dx^\beta = -e^{2\theta} \, dt^2 + e^{2\varphi} \, dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\]

The star’s stress-energy tensor is given by

\[
T_{\alpha\beta} = (\rho + p) u_\alpha u_\beta + p g^{(0)}_{\alpha\beta},
\]

where \( u = e^{-\nu/2} \partial_t \) is the fluid’s four-velocity and \( \rho \) and \( p \) are the density and pressure, respectively. Numerical integration of
the Tolman-Oppenheimer-Volkmann equations (see, e.g., Misner et al. 1973) for NS models with a polytropic pressure-density relation

\[ P = K \rho^{\gamma} \]

where \( K \) is a constant and \( \gamma \) is the polytropic index, gives the equilibrium stellar model with radius \( R \) and total mass \( M = m(R) \).

3.2. Static Linearized Perturbations

Due to an External Tidal Field

We examine the behavior of the equilibrium configuration under linearized perturbations due to an external quadrupolar tidal field following the method of Thorne & Campolattaro (1967). The full metric of the spacetime is given by

\[ g_{\alpha\beta} = g_{\alpha\beta}^{(0)} + h_{\alpha\beta}, \]

where \( h_{\alpha\beta} \) is a linearized metric perturbation. We analyze the angular dependence of the components of \( h_{\alpha\beta} \) into spherical harmonics as in Regge & Wheeler (1957). We restrict our analysis to the \( l = 2 \), static, even-parity perturbations in the Regge-Wheeler gauge (Regge & Wheeler 1957). With these specializations, \( h_{\alpha\beta} \) can be written as (Regge & Wheeler 1957; Thorne & Campolattaro 1967)

\[ h_{\alpha\beta} = \text{diag} \left[ -e^{i\theta} H_0(r), e^{i\theta} H_2(r), r^2 K(r), r^2 \sin^2 \theta K(r) \right] Y_{2m}(\theta, \phi). \]

The general solution to equation (18) in terms of the associated Legendre equation with \( l = m = 2 \):

\[ H'' + \left( \frac{2}{r} - \lambda^2 \right) H' - \left( \frac{6e^2}{r^2} + \lambda^2 \right) H = 0, \]

and changing variables to \( x = (r/M - 1) \) as in Thorne & Campolattaro (1967) transforms equation (17) to a form of the associated Legendre equation with \( l = m = 2 \):

\[ (x^2 - 1) H'' + 2x H' - \left( 6 + \frac{4}{x^2 - 1} \right) H = 0. \]

The continuity of \( H(r) \) and its derivative across \( r = R \). Outside the star, equation (15) reduces to

\[ H'' + \left( \frac{2}{r} - \lambda^2 \right) H' - \left( \frac{6e^2}{r^2} + \lambda^2 \right) H = 0, \]

where \( c_1 \) and \( c_2 \) are coefficients to be determined. Substituting the expressions for \( Q^2(x) \) and \( P^2(x) \) from Abramowitz & Stegun (1964) yields the exterior solution

\[ H = c_1 \left( \frac{r}{M} \right)^2 \left( 1 - \frac{2M}{r} \right) - \frac{M(M - r)(2M^2 + 6Mr - 3r^2)}{r^2(2M - r)^2} \]

\[ + \frac{3}{2} \log \left( \frac{r}{r - 2M} \right) + c_2 \left( \frac{r}{M} \right)^2 \left( 1 - \frac{2M}{r} \right). \]

The asymptotic behavior of the solution from equation (20) at large \( r \) is

\[ H = \frac{8}{5} \left( \frac{M}{r} \right)^3 c_1 + O \left( \frac{r}{M} \right)^4 + 3 \left( \frac{r}{M} \right) c_2 + O \left( \frac{r}{M} \right)^3, \]

where the coefficients \( c_1 \) and \( c_2 \) are determined by matching the asymptotic solution from equation (21) to the expansion from equation (1) and using equation (9).

\[ c_1 = \frac{15}{8} M^3 \lambda \mathcal{E}, \quad c_2 = \frac{1}{3} M^2 \mathcal{E}. \]
We now solve for \( k \) in terms of \( H \) and its derivative at the star’s surface \( r = R \) using equations (22) and (20), and use equation (5) to obtain the expression

\[
k_2 = \frac{8C^5}{5} \left[ 2 + 2C(y-1) - y \right] \left[ 2C(6-3y+3C \times (5y-8)) + 4C^3[13 - 11y + C(3y - 2) + 2C^2(1+y)] \right. \\
+ \left. 3(1-2C^2)[2-y+2C(y-1)] \log(1-2C) \right]^{-1},
\]

where we have defined the star’s compactness parameter \( C \equiv M/R \) and the quantity \( y \equiv RH'(R)/H(R) \), which is obtained by integrating equation (15) outward in the region \( 0 < r < R \).

### 3.3. Newtonian Limit

The first term in the expansion of equation (23) in \( M/R \) reproduces the Newtonian result

\[
k_2^N = \frac{1}{2} \left( \frac{2-y}{y+3} \right).
\]

Next, we will present the range of dimensionless Love numbers \( k_2 \) obtained by numerical integration for various values of \( n \).

| \( n \) | \( M/R \) | \( k_2 \) |
|------|------|------|
| 0.3  | \( 10^{-3} \) | 0.5511 |
| 0.3  | 0.1  | 0.5401 |
| 0.3  | 0.15 | 0.5691 |
| 0.3  | 0.2  | 0.6146 |
| 0.5  | \( 10^{-3} \) | 0.4491 |
| 0.5  | 0.1  | 0.4260 |
| 0.5  | 0.15 | 0.4349 |
| 0.5  | 0.2  | 0.4489 |
| 0.5  | 0.25 | 0.4589 |
| 0.7  | \( 10^{-3} \) | 0.3626 |
| 0.7  | 0.1  | 0.3373 |
| 0.7  | 0.15 | 0.3369 |
| 0.7  | 0.2  | 0.3363 |
| 0.7  | 0.25 | 0.3267 |
| 1.0  | \( 10^{-3} \) | 0.2599 |
| 1.0  | 0.1  | 0.2405 |
| 1.0  | 0.15 | 0.2363 |
| 1.0  | 0.2  | 0.2282 |
| 1.0  | 0.25 | 0.2081 |
| 1.2  | \( 10^{-3} \) | 0.1936 |
| 1.2  | 0.1  | 0.1900 |
| 1.2  | 0.15 | 0.1811 |

where the superscript \( N \) denotes “Newtonian.” In the Newtonian limit, the differential equation (15) inside the star becomes

\[
H'' + \frac{2}{r} H' + \frac{4\pi \rho}{r^2} = 0.
\]

For a polytropic index of \( n = 1 \), equation (25) can be transformed to a Bessel equation with the solution that is regular at \( r = 0 \) given by \( H = A(r/R)^{1/2} J_{5/2}(\pi r/R) \), where \( A \) is a constant. At \( r = R \), we thus have \( y = RH'/H = (\pi^2 - 9)/3 \), and from equation (23) it follows that

\[
\frac{k_2^N(n=1)}{k_2} = \left( -\frac{1}{2} + \frac{15}{2\pi^2} \right) \approx 0.25991,
\]

which agrees with the known result of Brooker & Olle (1955).

### 4. RESULTS AND DISCUSSION

The range of dimensionless Love numbers \( k_2 \) obtained by numerical integration of equation (23) is shown in Figure 1 as a function of \( M/R \) and \( n \) for a variety of different NS models, and representative values are given in Table 1. These values can be

![Image](image-url)
approximated to an accuracy of $\sim6\%$ in the range $0.5 \leq n \leq 1.0$ and $0.1 \leq (M/R) \leq 0.24$ by the fitting formula

$$k_2 \approx \frac{3}{2} \left( -0.41 + \frac{0.56}{n^{0.33}} \right) \left( \frac{M}{R} \right)^{-0.003}.$$  \hspace{1cm} (27)

Both Figure 1 and Table 1 illustrate that the dimensionless Love numbers $k_2$ depend more strongly on the polytropic index $n$ than on the compactness $C = M/R$.\footnote{Note, however, that LIGO measurements will yield the combination $k_2 R^5$ and, therefore, will be more sensitive to the compactness than the polytropic index.} This is expected since the weak field, Newtonian values $k_2^N$ given by equation (24) just depend on $n$ (through the dependence on $y$). The additional dependence on the compactness for the Love numbers $k_2$ in equation (23) is a relativistic correction to this. For $M/R \sim 10^{-5}$ our results for $k_2$ agree well with the Newtonian results of Brooker & Olle (1955). Figure 2 shows the percent difference $(k_2^N - k_2)/k_2$ between the relativistic and Newtonian dimensionless Love numbers. As can be seen from the figure, the relativistic values are lower than the Newtonian ones for higher values of $n$. This can be explained by the fact that theLove number encodes information about the degree of central condensation of the star. Stars with a higher polytropic index $n$ are more centrally condensed and, therefore, have a smaller response to a tidal field, resulting in a smaller Love number.

Some estimates of the masses and radii of NSs, given in Table 2, have been inferred from X-ray observations (Ozel 2006; Webb & Barret 2007) using the information from three measured quantities: the Eddington luminosity, the surface redshift of spectral lines, and the quiescent X-ray flux. The range of the numbers $\lambda$ for these stars is shown in Figure 3. LIGO II detectors will be able to establish a 90\% confidence upper limit of $\lambda < 2.01 \times 10^{-7} \text{g cm}^2 \text{s}^{-2}$ for an inspiral of two nonspinning $1.4 M_\odot$ NSs at a distance of 50 Mpc in the case that no tidal phase shift is observed (Flanagan & Hinderer 2008).

The author thanks Éanna Flanagan for valuable discussions and comments.

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ERRATUM: “TIDAL LOVE NUMBERS OF NEUTRON STARS” (2008, ApJ, 677, 1216)

Tanja Hinderer
Center for Radiophysics and Space Research, Cornell University, Ithaca, NY 14853, USA; tph25@caltech.edu

In the original paper, there are typographical errors in Equations (20) and (23), and some incorrect entries in Table 1. I thank Ryan Lang for pointing these out.

Equation (20) should read as follows:

\[ H = c_1 \left( \frac{r}{M} \right)^2 \left( 1 - \frac{2M}{r} \right) \left[ -\frac{M(M-r)(2M^2 + 6Mr - 3r^2)}{r^2(2M-r)^2} + \frac{3}{2} \log \left( \frac{r}{r-2M} \right) \right] + 3c_2 \left( \frac{r}{M} \right)^2 \left( 1 - \frac{2M}{r} \right). \]  

Equation (23) should be replaced by the following:

\[ k_2 = \frac{8C^5}{5} (1 - 2C)^3 \left[ (2 + 2C(y-1)) \right] \times \left\{ 2C(6 - 3y + 3C(5y - 8)) + 4C^3 \left[ 13 - 11y + C(3y - 2) + 2C^2(1 + y) \right] \right. \\
\left. + 3(1 - 2C)^2 \left[ 2 - y + 2C(y-1) \log (1 - 2C) \right] \right\}^{-1}. \]  

The corrected values for the Love numbers in Table 1 are given in the table below.

| \( n \)  | \( M/R \) | \( k_2 \) |
|---------|---------|---------|
| 0.3     | \( 10^{-3} \) | 0.5511  |
| 0.3     | 0.1     | 0.294   |
| 0.3     | 0.15    | 0.201   |
| 0.3     | 0.2     | 0.119   |
| 0.5     | \( 10^{-3} \) | 0.4491  |
| 0.5     | 0.1     | 0.230   |
| 0.5     | 0.15    | 0.158   |
| 0.5     | 0.2     | 0.095   |
| 0.5     | 0.25    | 0.0569  |
| 0.7     | \( 10^{-3} \) | 0.3626  |
| 0.7     | 0.1     | 0.1779  |
| 0.7     | 0.15    | 0.1171  |
| 0.7     | 0.2     | 0.0721  |
| 0.7     | 0.25    | 0.042   |
| 1.0     | \( 10^{-3} \) | 0.2599  |
| 1.0     | 0.1     | 0.122   |
| 1.0     | 0.15    | 0.0776  |
| 1.0     | 0.2     | 0.0459  |
| 1.0     | 0.2     | 0.0253  |
| 1.2     | \( 10^{-3} \) | 0.2062  |
| 1.2     | 0.1     | 0.0931  |
| 1.2     | 0.15    | 0.0577  |
| 1.2     | 0.2     | 0.0327  |