Sea Contributions and Nucleon Structure

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Abstract

We suggest a general formalism to treat a baryon as a composite system of three quarks and a ‘sea’. In this formalism, the sea is a cluster which can consists of gluons and quark-antiquark pairs. The hadron wave function with a sea component is given. The magnetic moments, related sum rules and axial weak coupling constants are obtained. The data seems to favor a vector sea rather than a scalar sea. The quark spin distributions in the nucleon are also discussed.

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I. INTRODUCTION

Historically, the static SU(6) quark model provided a good description of hadrons: Baryons (mesons) are color-singlet combinations of three quarks (quark antiquark pairs) in the appropriate flavor and spin combination. The space-time part of a hadron wave function can be determined by using a specific model of confinement, e.g. bag model \[1,2\], simple harmonic oscillator model \[3,4\], or other phenomenological models \[5\]. Although the naive SU(6) quark model works successfully in explaining various properties of hadrons, departures from the naive SU(6) results have been observed. The naive valence picture of hadron structure is a simplification or a first order approximation to the real system. Within the framework of QCD, quarks interact through color forces mediated by vector gluons. The QCD interaction Hamiltonian \[H_I(x) = g \bar{\psi}(x) \gamma^\mu (\lambda^a / 2) \psi(x) A^a_\mu(x)\] has several consequences:

First of all, spin-dependent forces (e.g. color-hyperfine interactions \[7\]) between the quarks due to one (or multi-) gluon exchange lift the SU(6) mass degeneracy and explain the basic pattern of baryon and meson spectroscopies. The spin dependent forces also cause different space-time distributions for different quark flavors and provide a good description of baryon magnetic moments and form factors \[8,9\]. Secondly, the existence of quark-gluon interaction implies that quark-antiquark (q\(\bar{q}\)-)pairs can be created by the virtual gluons emitted from valence quarks. These q\(\bar{q}\)-pairs are the so called sea quarks. Usually, the ‘sea’ means a combination of the virtual gluons and sea quark-antiquark pairs. Although deep inelastic muon nucleon scattering shows that the sea components (q\(\bar{q}\)-pairs and gluons) indeed exist and play a very important role (e.g. gluons carry about one half of the nucleon momentum and the sea dominates small\(-x\) behaviour of structure functions), it is commonly believed that in the low energy regime, static properties of hadrons are dominated by their valence components. However, it has been shown \[10,11\] that the sea contributions may change the structure of hadrons and modify their low energy properties. Using the QCD interaction Hamiltonian and the MIT bag model, Donoghue and Golowich (DG) \[10\] (comments see cf \[11\]) calculated the probabilities of different sea quark components in the proton. Several
models \[12-15\] have been suggested to study the gluon component in hadrons. In these models, a mixing of \(q^3\) and \(q^3\)-gluon, in which a color \(8_c\) gluon coupled to a \(8_c\) \(q^3\) state to form a color singlet, has been discussed. However, the “sea” could be a gluon (as discussed in \[12-15\]) or a quark-antiquark pair (as discussed in \[10,11\]), or even more complicated, for instance a multi-gluon state, multi-(\(q\bar{q}\)) pairs or gluon(s) plus \((q\bar{q})\) pair(s). In this paper, we study the sea contributions in a more general formalism and treat the “sea” as a cluster which can consist of two-gluon and a gluon plus a \((q - \bar{q})\) pair or some admixture of both (which may be described by the generic term “flotsam”). Since the baryon should be colorless and a \(q^3\) state can be in color states \(1_c, 8_c,\) and \(10_c,\) the “sea” should also be in corresponding color states to form a color singlet baryon. In addition, the “sea” spin is not required to be one (as in the single-gluon case). Furthermore, if the sea is in a S-wave state relative to the \(q^3\) system, conservation of the angular momentum restricts that sea spin can only be 0, 1 or 2 to give a spin-1/2 baryon. If the sea is in a P-wave state, then its spin could be 0, 1, 2, or 3. In this paper, we only discuss the S-wave case. In section II, a more general wave function of the baryon, which consists of \(q^3\) and a “sea”, is given. In section III, the magnetic moments and related sum rules are derived and compared with the data. In section IV, axial weak coupling constants and first moments of nucleon spin structure functions are calculated. A discussion of the sea contribution, numerical results and several conclusions are given in section V, VI and VII respectively.

**II. HADRON WAVE FUNCTION WITH A SEA COMPONENT.**

The three (valence) quark wave function of the baryon can be written as

\[
\Psi = \Phi(|\phi > \cdot |\chi > \cdot |\psi >) \cdot (|\xi >)
\]

(2.1)

where \(|\phi >, |\chi >, |\psi > and |\xi >\) denote flavor, spin, color and space-time \(q^3\) wave functions. For the lowest-lying hadrons, quarks appear to be in S-wave states and the space-time \(q^3\) wave function \(|\xi >\) is total symmetric under permutation of any two quarks. Hence the
flavor-spin-color part $\Phi$ should be total antisymmetric under $q_i \leftrightarrow q_j$. In the conventional quark model, the color wave function $\psi$ is taken to be total antisymmetric, i.e. a color singlet. But in general this is not necessary if baryon is considered to have a sea component in addition to the $q^3$. Let superscripts $S$ and $A$ denote total permutation symmetry and antisymmetry, and $\lambda$, $\rho$ denote symmetric and antisymmetric under quark permutation $q_1 \leftrightarrow q_2$. Then the $q^3$ wave functions for a flavour octet baryon, are

$$\Phi^{(1/2)}_{1} \equiv \Phi(8, 1/2, 1_c) = F_S \psi^A_1$$

(2.2)

$$\Phi^{(1/2)}_{8} \equiv \Phi(8, 1/2, 8_c) = \frac{1}{\sqrt{2}} (F_{MS} \psi^\rho_8 - F_{MA} \psi^\lambda_8)$$

(2.3)

$$\Phi^{(1/2)}_{10} \equiv \Phi(8, 1/2, 10_c) = F_A \psi^S_{10}$$

(2.4)

$$\Phi^{(3/2)}_{8} \equiv \Phi(8, 3/2, 8_c) = F_A' \chi^{(3/2)}$$

(2.5)

where

$$F_S = \frac{1}{\sqrt{2}} (\phi^\lambda \chi^\lambda + \phi^\rho \chi^\rho)$$

(2.6)

$$F_{MS} = \frac{1}{\sqrt{2}} (\phi^\rho \chi^\rho - \phi^\lambda \chi^\lambda)$$

(2.7)

$$F_{MA} = \frac{1}{\sqrt{2}} (\phi^\rho \chi^\rho + \phi^\lambda \chi^\lambda)$$

(2.8)

$$F_A = \frac{1}{\sqrt{2}} (\phi^\rho \chi^\rho - \phi^\rho \chi^\lambda)$$

(2.9)

and

$$F_A' = \frac{1}{\sqrt{2}} (\phi^\lambda \psi^\rho_8 - \phi^\rho \psi^\lambda_8)$$

(2.10)

where the detail expressions for $\phi^\lambda$, $\phi^\rho$, $\chi^\lambda$ and $\chi^\rho$ can be found in Ref. [16], and $\chi^{(3/2)}$ is the totally symmetric $q^3$ spin wave function with spin $3/2$. 

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We note that $\Phi^{(1/2)}$ in (2.2) is the standard $q^3$ wave function which transforms as 56 of SU(6) and was denoted by $|N_0>$ in Ref. [14]. Our $\Phi^{(1/2)}_8$ and $\Phi^{(3/2)}_8$ correspond to the notation $|^2N_g>$ and $|^4N_g>$ in Ref. [15] respectively, they transform as 70 of SU(6). There is no $\Phi^{(1/2)}_{10}$ term in previous works.

We consider a flavorless sea, which has spin (0, 1, 2 if we assume sea is in a S wave state) and color (1c, 8c and $\bar{10}_c$). Let $H_{0,1,2}$ and $G_{1,8,10}$ denote spin and color sea wave functions, which satisfy

$$<H_i|H_j> = \delta_{ij}, \quad <G_k|G_l> = \delta_{kl} \quad (2.11)$$

The possible combinations of $q^3$ and sea wave functions, which can give a spin 1/2, flavour octet, color singlet state, are:

$$\Phi^{(1/2)}_1 \cdot H_0 \cdot G_1, \quad \Phi^{(1/2)}_8 \cdot H_0 \cdot G_8, \quad \Phi^{(1/2)}_{10} \cdot H_0 \cdot G_{10} \quad (2.12)$$

$$\Phi^{(1/2)}_1 \cdot H_1 \cdot G_1, \quad \Phi^{(1/2)}_8 \cdot H_1 \cdot G_8, \quad \Phi^{(1/2)}_{10} \cdot H_1 \cdot G_{10} \quad (2.13)$$

and

$$\Phi^{(3/2)}_8 \cdot H_1 \cdot G_8, \quad \Phi^{(3/2)}_8 \cdot H_1 \cdot G_8 \quad (2.14)$$

The total flavor-spin-color wave function of a spin up baryon which consists of three valence quarks and a sea component can be written as

$$|\Phi^{(1)}_{1/2}> = \frac{1}{N} \left[ \Phi^{(1/2)}_1 \cdot H_0 \cdot G_1 + a_8 \Phi^{(1/2)}_8 \cdot H_0 \cdot G_8 + a_{10} \Phi^{(1/2)}_{10} \cdot H_0 \cdot G_{10} 
+b_1(\Phi^{(1/2)}_1 \otimes H_1)^\dagger \cdot G_1 + b_8(\Phi^{(1/2)}_8 \otimes H_1)^\dagger \cdot G_8 + b_{10}(\Phi^{(1/2)}_{10} \otimes H_1)^\dagger \cdot G_{10} 
+c_8(\Phi^{(3/2)}_8 \otimes H_1)^\dagger \cdot G_8 + d_8(\Phi^{(3/2)}_8 \otimes H_2)^\dagger \cdot G_8 \right] \quad (2.15)$$

where

$$N^2 = 1 + a_8^2 + a_{10}^2 + b_1^2 + b_8^2 + b_{10}^2 + c_8^2 + d_8^2 \quad (2.16)$$

Although there are seven correction terms in (2.15), they are not equally important. Some arguments are given in section V to show that main modifications come from the vector sea,
in particular $b_8$, $b_1$ and $c_8$ terms, and minor contributions come from the scalar sea, e.g. $a_{10}$ term.

The first three terms in (2.15) come from a spin 1/2 $q^3$ state coupled to a spin 0 (scalar) sea. The next three terms in (2.15) come from spin 1/2 $q^3 \otimes$ spin 1 (vector) sea and in more detail we have

\[(\Phi_{1/2}^{(1/2)} \otimes H_1)^\dagger \equiv \Phi_{b_1}^{(1/2)} \psi_A^{1} \tag{2.17}\]

\[(\Phi_{8}^{(1/2)} \otimes H_1)^\dagger \equiv \Phi_{b_8}^{(1/2)} \tag{2.18}\]

\[(\Phi_{10}^{(1/2)} \otimes H_1)^\dagger \equiv \Phi_{b_{10}}^{(1/2)} \psi_{10}^{S} \tag{2.19}\]

where

\[\Phi_{b_1}^{(1/2)} = \sqrt{\frac{2}{3}} H_{1,1} F_{S}^{(1/2)} - \sqrt{\frac{1}{3}} H_{1,0} F_{S}^{(1/2)} \tag{2.20}\]

\[\Phi_{b_8}^{(1/2)} = \sqrt{\frac{2}{3}} \left[ \Phi_{b_{8S}}^{(1/2)} \psi_{8}^{P} - \Phi_{b_{8A}}^{(1/2)} \psi_{8}^{A} \right] \tag{2.21}\]

\[\Phi_{b_{10}}^{(1/2)} = \sqrt{\frac{2}{3}} H_{1,1} F_{A}^{(1/2)} - \sqrt{\frac{1}{3}} H_{1,0} F_{A}^{(1/2)} \tag{2.22}\]

In (2.21), $\Phi_{b_{8S}}^{(1/2)}$ and $\Phi_{b_{8A}}^{(1/2)}$ are

\[\Phi_{b_{8S}}^{(1/2)} = \sqrt{\frac{2}{3}} H_{1,1} F_{M_{S}}^{(1/2)} - \sqrt{\frac{1}{3}} H_{1,0} F_{M_{S}}^{(1/2)} \tag{2.23}\]

\[\Phi_{b_{8A}}^{(1/2)} = \sqrt{\frac{2}{3}} H_{1,1} F_{M_{A}}^{(1/2)} - \sqrt{\frac{1}{3}} H_{1,0} F_{M_{A}}^{(1/2)} \tag{2.24}\]

The final two ($c_8$, $d_8$) terms in Eq.(2.15) come from spin 3/2 ($q^3$) $\otimes$ spin 1 (sea) and spin 3/2 ($q^3$) $\otimes$ spin 2 (tensor sea) respectively. Their expressions are

\[(\Phi_{8}^{(3/2)} \otimes H_1)^\dagger \equiv \Phi_{c_8}^{(1/2)} \tag{2.25}\]

\[(\Phi_{8}^{(3/2)} \otimes H_2)^\dagger \equiv \Phi_{d_8}^{(1/2)} \tag{2.26}\]
where

\[ \Phi_{c8}^{(1/2\uparrow)} = \left[ \frac{1}{\sqrt{2}} H_{1,-1} \chi_{3/2} - \frac{1}{\sqrt{3}} H_{1,0} \chi_{3/2} + \frac{1}{\sqrt{6}} H_{1,1} \chi_{3/2} \right] \]

(2.27)

\[ \Phi_{d8}^{(1/2\uparrow)} = \left[ \sqrt{\frac{2}{5}} H_{2,2} \chi_{-3/2} - \sqrt{\frac{3}{10}} H_{2,1} \chi_{3/2} + \sqrt{\frac{1}{5}} H_{2,0} \chi_{1/2} - \sqrt{\frac{1}{10}} H_{2,-1} \chi_{3/2} \right] F'_A \]

(2.28)

The wave function used in Ref. [15] (see Eq.(3.9) in [15]) can be obtained from (2.15) by taking \( a_{8,10} = b_{1,10} = d_8 = 0 \) and \( b_8 = c_8 = - \delta \). However, we would not like to restrict ourself to this special case.

### III. MAGNETIC MOMENTS AND RELATED SUM RULES

For any operator \( \hat{O} \) which only depends on quark flavor and spin and does not depend on the color and space-time, we have

\[
\begin{align*}
\langle \Phi_{1/2}^{(\uparrow)} | \hat{O} | \Phi_{1/2}^{(\uparrow)} \rangle &= \frac{1}{N^2} \left[ \langle \Phi_{1/2}^{(\uparrow)} | \hat{O} | \Phi_{1/2}^{(\uparrow)} \rangle \\
&+ \sum_{i=8,10} a_i^2 \langle \Phi_{i}^{(1/2\uparrow)} | \hat{O} | \Phi_{i}^{(1/2\uparrow)} \rangle \\
&+ \sum_{i=1,8,10} b_i^2 \langle \Phi_{bi}^{(1/2\uparrow)} | \hat{O} | \Phi_{bi}^{(1/2\uparrow)} \rangle \\
&+ 2b_8c_8 \langle \Phi_{ds}^{(1/2\uparrow)} | \hat{O} | \Phi_{ds}^{(1/2\uparrow)} \rangle \\
&+ c_8^2 \langle \Phi_{c8}^{(1/2\uparrow)} | \hat{O} | \Phi_{c8}^{(1/2\uparrow)} \rangle \\
&+ d_8^2 \langle \Phi_{ds}^{(1/2\uparrow)} | \hat{O} | \Phi_{ds}^{(1/2\uparrow)} \rangle \right] \quad (3.1)
\end{align*}
\]

the first term is the conventional quark model result. The \( a_8, a_{10} \) terms are the corrections coming from the scalar sea, \( b_{1,8,10}, c_8 \) and \( b_8c_8 \) terms are from the vector sea and the \( d_8 \) term is from the tensor sea.

If operator \( \hat{O} \) has a form like \( \hat{O} = \sum_i \hat{O}_j^i \sigma_z^i \) where \( \hat{O}_j^i \) depends only on the flavor of the ith quark and \( \sigma_z^i \) is the spin projection (z direction) operator of ith quark, from (3.1) we obtain

\[
\begin{align*}
\langle \Phi_{1/2}^{(\uparrow)} | \hat{O} | \Phi_{1/2}^{(\uparrow)} \rangle &= \frac{1}{N^2} \left[ a \sum_i | \langle O_j^i > | \lambda \langle \sigma_z^i > | \lambda \lambda^\uparrow + | \langle O_j^i > | \rho \langle \sigma_z^i > | \rho \rho^\uparrow \\
&+ 2 | \langle O_j^i > | \lambda \rho < \sigma_z^i > | \lambda^\uparrow \rho^\uparrow \right]
\end{align*}
\]
\[ \sum_i (<O_j^i >^{\lambda\lambda} + <O_j^i >^{\rho\rho})(<\sigma_z^i >^{\lambda\lambda\uparrow} + <\sigma_z^i >^{\rho\rho\uparrow}) \]
\[ + c \sum_i [<O_j^i >^{\lambda\lambda} <\sigma_z^i >^{\rho\rho\uparrow} + <O_j^i >^{\rho\rho} <\sigma_z^i >^{\lambda\lambda\uparrow}] \]
\[ -2 <O_j^i >^{\lambda\rho} <\sigma_z^i >^{\lambda\rho\uparrow} \]
\[ + d [\sum_i <O_j^i >^{\lambda\lambda} + \sum_i <O_j^i >^{\rho\rho}] \]
\[ + e [\sum_i (<O_j^i >^{\rho\rho} - <O_j^i >^{\lambda\lambda}) <\sigma_z^i >^{\lambda\lambda\uparrow/2}] \]
\[ + 2 \sum_i <O_j^i >^{\lambda\rho} <\sigma_z^i >^{\rho\rho\uparrow}] \]  
\[(3.2)\]

There are only five combinations of seven parameters appear in (3.2):

\[ a = \frac{1}{2}(1 - \frac{b^2}{3}) \], \[ b = \frac{1}{4}(a_8^2 - \frac{b_8^2}{3}) \], \[ c = \frac{1}{2}(a_{10}^2 - \frac{b_8^2}{3}) \]  
\[(3.3)\]

\[ d = \frac{1}{15}(5c_8^2 - 3d_8^2) \], \[ e = \frac{\sqrt{2}}{3}b_8c_8 \]  
\[(3.4)\]

and \(<O_j^i >^{\lambda\lambda}\equiv< \phi^\lambda |O_j^i |\phi^\lambda >, <\sigma_z^i >^{\lambda\lambda\uparrow}\equiv< \chi^\lambda |\sigma_z^i |\chi^\lambda >\). Similar notations are used for \(<O_j^i >^{\rho\rho}, <\sigma_z^i >^{\rho\rho\uparrow}\) etc. All matrix elements for octet baryons are listed in appendix 1.

For magnetic moments, \(O_j^i = e^i / 2m_i\) (i=u, d, s). The baryon magnetic moments can be expressed in terms of the quark magnetic moments (\(\mu_u, \mu_d, \mu_s\)) and two parameters \(\alpha\) and \(\beta\) as follows

\[ \mu_p = 3(\mu_u\alpha - \mu_d\beta) \], \[ \mu_n = 3(\mu_d\alpha - \mu_u\beta) \]  
\[(3.5)\]

\[ \mu_\Lambda = \frac{1}{2}(\alpha - 4\beta)(\mu_u + \mu_d) + (2\alpha + \beta)\mu_s \]  
\[(3.6)\]

\[ \mu_{\Sigma^+} = 3(\mu_u\alpha - \mu_s\beta) \], \[ \mu_{\Sigma^-} = 3(\mu_d\alpha - \mu_s\beta) \], \[ \mu_{\Sigma^0} = \frac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-}) \]  
\[(3.7)\]

\[ \mu_{\Xi^0} = 3(\mu_s\alpha - \mu_u\beta) \], \[ \mu_{\Xi^-} = 3(\mu_s\alpha - \mu_d\beta) \]  
\[(3.8)\]

Also, the transition moment

\[ \mu_{\Sigma\Lambda} = -\frac{\sqrt{3}}{2}(\alpha + 2\beta)(\mu_u - \mu_d) \]  
\[(3.9)\]
where $\mu_q = e/2m_q$ ($q = u, d, s$) and

$$\alpha = \frac{1}{N_f} \left( \frac{4}{9} \right) \left( 2a + 2b + 3d + \sqrt{2}e \right)$$  \hspace{1cm} (3.10)$$

$$\beta = \frac{1}{N_f} \left( \frac{1}{9} \right) \left( 2a - 4b - 6c - 6d + 4\sqrt{2}e \right)$$  \hspace{1cm} (3.11)$$

One may ask why the seven parameters ($a_i$, $b_i$ etc.) in the wave function contribute only through the combinations given by $\alpha$ and $\beta$. The physical reason is that $\alpha$ and $\beta$ are connected with the number of spin-up ($n(q^\uparrow)$) and spin-down ($n(q^\downarrow)$) quarks in the spin-up proton. If, $\Delta q \equiv n(q^\uparrow) - n(q^\downarrow) + n(\bar{q}^\uparrow) - n(\bar{q}^\downarrow)$, $q = u, d, s$ then $\Delta u = 3\alpha$ and $\Delta d = -3\beta$. This can be directly checked from the wave function given in (2.15). Also, as there are no explicit antiquarks or s-quarks in the wave function, $n(\bar{q}^\uparrow) - n(\bar{q}^\downarrow) = 0$ and $\Delta s = 0$. Further, because of in-built flavour SU(3) symmetry in the wave function $\alpha$ and $\beta$ determine the other magnetic moments. If there is no sea contribution, $2a = 1$ and $b = c = d = e = 0$, then $\alpha = 4/9$ and $\beta = 1/9$, and the simplest quark model result is reproduced [17]. A class of models [18,19] have been recently considered in which the magnetic moments have been expressed in terms of $\mu_q$ and $\Delta q$ ($q = u, d, s$) without giving an explicit wave function. Their expressions reduce to ours on putting $\Delta u = 3\alpha$, $\Delta d = -3\beta$ and $\Delta s = 0$ (see Ref. [18,19]).

At first sight, (3.5)−(3.9) seem to contain five parameters $\mu_q$ ($q = u, d, s$), $\alpha$ and $\beta$. However, as these always appear as products there are only four effective parameters which we take to be $\tilde{U} \equiv 3\alpha \mu_d$, $\tilde{D} \equiv -3\beta \mu_d$, $2p \equiv -\mu_u/\mu_d > 0$ and $r \equiv \mu_s/\mu_d > 0$. The numerical results for this four-parameter fit to the magnetic moments are discussed later. Here we note the following four relations or sum rules between the eight magnetic moments:

$$\mu_p - \mu_n = \mu_{\Sigma^+} - \mu_{\Sigma^-} - (\mu_{\Xi^0} - \mu_{\Xi^-}) \hspace{1cm} (4.15 \pm 0.07)$$  \hspace{1cm} (3.12)$$

$$\mu_{\Lambda} = \mu_{\Sigma^+} + \mu_{\Sigma^-} - 2(\mu_p + \mu_n + \mu_{\Xi^0} + \mu_{\Xi^-}) \hspace{1cm} (3.36 \pm 0.09)$$  \hspace{1cm} (3.13)$$

$$\mu_{\Sigma^+} - \mu_{\Sigma^-} - (\mu_{\Xi^0} - \mu_{\Xi^-}) = \mu_p^2 - \mu_n^2 \hspace{1cm} (4.14)$$  \hspace{1cm} (3.14)$$

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\[(5.58 \pm 0.28) - 2\sqrt{3}\mu_{\Sigma} = 2(\mu_p - \mu_n) - (\mu_{\Sigma^+} - \mu_{\Sigma^-}) (5.83 \pm 0.06) \quad (3.15)\]

The value of the two sides taken from data [17] are shown in parenthesis. The three sum rules in (3.12)–(3.14) are not new and hold in the class of models with \(\Delta s \neq 0\) referred to above. A discussion of why they are poorly satisfied can be found in Ref. [18]. The sum rule in (3.15), a consequence of the 4–parameter model, is surprisingly well satisfied.

The simpler case with three effective parameters \(\mu_0\alpha, \mu_0\beta\) and \(r (\mu_0 \equiv e/2m_u)\) is of interest since it makes the natural assumption \(m_u = m_d\) or \(\mu_u = -2\mu_d\). This implies an additional sum rule (apart from the (3.12)–(3.15))

\[(1.61 \pm 0.08) \mu_{\Sigma}\Lambda = \sqrt{\frac{3}{2}} \mu_n \quad (1.66)\]

which is quite well satisfied [20,21].

The important point to note is that because of the sea contribution \(\alpha\) and \(\beta\) are free parameters and not restricted to the simple quark model value. Finally, we note that the failure of the data to satisfy the relations (3.12)–(3.14) implies that one can only obtain, at best, an approximate fit to the magnetic moment data in all the above cases.

**IV. WEAK DECAY CONSTANTS AND SPIN DISTRIBUTIONS**

For the weak decay constant \((g_A/g_V)\), \(O_f^i = 2I_3^i\) and we obtain

\[(g_A/g_V)_{n\rightarrow p} = \frac{1}{N^2}(\frac{5}{3})(2a + 4b + 6c + 6d + 8\sqrt{2}e)\]

\[= 3(\alpha + \beta) \quad (4.1)\]

\[(g_A/g_V)_{\Xi^0 \rightarrow \Xi^0} = \frac{1}{N^2}(-\frac{1}{3})(2a - 4b - 6c - 6d + 4\sqrt{2}e)\]

\[= -3\beta \quad (4.2)\]

Using (3.5) and (4.1), (4.2) we obtain

\[(\mu_{\Xi^0} - \mu_{\Xi^0})/(\mu_p - \mu_n) = (g_A/g_V)_{\Xi^0 \rightarrow \Xi^0}/(g_A/g_V)_{n\rightarrow p} \quad (4.3)\]
Note that the relation Eq. (4.3) continues to hold in models \([18, 19]\) with \(\Delta s \neq 0\) mentioned above. For the 3-parameter model (i.e. with \(\mu_u = -2\mu_d\)) in addition to (4.3), one obtains

\[
\frac{(\mu_p + 2\mu_n)}{\mu_p - \mu_n} = \frac{(g_A/g_V)_{\Xi^- \Xi^0} / (g_A/g_V)_{n \rightarrow p}}{(g_A/g_V)_{n \rightarrow p}}
\]  

(4.4)

The relations (4.3) and (4.4) cannot be checked directly with data as \((g_A/g_V)_{n \rightarrow p}\) is not measured. However, we can predict (see Table 1) the \((g_A/g_V)_{n \rightarrow p}\) for various semi-leptonic decays since they can be expressed, using flavour SU(3) symmetry, in terms of \(F\) and \(D\) or \(\alpha\) and \(\beta\).

In fact \((g_A/g_V)_{n \rightarrow p} = F + D\) and \((g_A/g_V)_{\Xi^- \Xi^0} = F - D\), from (4.1) and (4.2) we have

\[
F = 3\alpha/2 \quad ; \quad D = 3(\alpha + 2\beta)/2 \quad ; \quad F/D = \alpha/(\alpha + 2\beta)
\]

(4.5)

It is easy to verify that when there is no sea contribution (i.e. \(a_{8,10} = b_{1,8,10} = c = d = 0\)) and \(\mu_u = -2\mu_d\), the standard SU(6) quark model results, e.g. \(\mu_n/\mu_p = -2/3\), \((g_A/g_V)_{n \rightarrow p} = 5/3\) and \(F/D = 2/3\) follow.

For spin distributions in the proton and neutron, we have

\[
I_p^1 = \frac{1}{2} < \sum_i e_i^2 \sigma_i^1 >_p = \frac{1}{3N_f}(\frac{5}{3}a + 2b + \frac{c}{3} + 3d + \frac{2}{3}\sqrt{2}e)
\]

(4.6)

\[
I_n^1 = \frac{1}{2} < \sum_i e_i^2 \sigma_i^1 >_n = \frac{1}{3N_f}(\frac{4}{3}b + \frac{4}{3}c + 2d - \frac{2}{3}\sqrt{2}e)
\]

(4.7)

where \(I_i^p \equiv \int g_i^p(x)dx\) etc. Similarly, one can obtain

\[
I_{\Lambda}^1 = \frac{1}{2} < \sum_i e_i^2 \sigma_i^1 >_{\Lambda} = \frac{1}{3N_f}(\frac{1}{3}a + \frac{4}{3}b + c + 2d - \frac{2}{3}\sqrt{2}e)
\]

(4.8)

Using the parameters \(\alpha\) and \(\beta\), they are

\[
I_p^1 = \frac{1}{6}(4\alpha - \beta) \quad ; \quad I_n^1 = \frac{1}{6}(\alpha - 4\beta) \quad ; \quad I_{\Lambda}^1 = \frac{1}{4}(\alpha - 2\beta)
\]

(4.9)

One can see that the standard SU(6) result gives \(\int g_1^p(x)dx = 5/18\), \(\int g_1^n(x)dx = 0\) and \(\int g_{\Lambda}^1(x)dx = 1/18\). Including the sea contributions, however, \(\int g_1^p(x)dx\), \(\int g_1^n(x)dx\) could be different from their SU(6) value and also \(\int g_{\Lambda}^1(x)dx\) could be nonzero. One can verify, however, that the Bjorken sum rule is still satisfied.
\[ \int [g_1^n(x) - g_1^p(x)] dx = \frac{1}{6}(g_A / g_V)_{n \rightarrow p} \] (4.10)

In addition, we have

\[ \int [g_1^n(x) - g_1^\Lambda(x)] dx = \frac{1}{12}[(g_A / g_V)_{n \rightarrow p} + (g_A / g_V)_{\Lambda \rightarrow p}] \] (4.11)

In our model, \( \int_0^1 g_1^\Lambda(x) dx \) will be less than its SU(6) value if sea contributions are taken into account (see Table 1). It is interesting to note that an experiment to measure the spin structure function of the \( \Lambda \)-particle has been suggested recently \[22\].

V. DISCUSSION OF THE SEA CONTRIBUTION

For simplicity, we consider the case when the magnetic moments are given by three parameters \( \alpha, \beta \) and \( r \) (i.e. put \( \mu_u = -2\mu_d \) in (3.5)–(3.9)). The discussion for the case when \( \mu_u \neq -2\mu_d \) can be carried out on similar lines and suggests that \( -\mu_u / 2\mu_d < 1 \) for both pure scalar and vector sea.

A. Scalar sea

If sea spin is zero, \( a_8 \neq 0 \) and \( a_{10} \neq 0 \), but \( b_1 = b_8 = b_{10} = c_8 = d_8 = 0 \), one obtains

\[
\mu_n / \mu_p = \left(-\frac{2}{3}\right) \frac{1-a_8^2}{1+\frac{1}{3}(a_8^2-a_{10}^2)} \approx \left(-\frac{2}{3}\right) \left(1 - \frac{1}{3}a_8^2 - \frac{2}{3}a_{10}^2 + ...\right)
\] (5.1)

since \( 0 \leq a_8^2, a_{10}^2 \leq 1 \). It is obvious that the contribution from the scalar sea leads to a wrong correction to the ratio of neutron and proton magnetic moments. For \( F/D \) ratio, one obtains

\[ F/D = \frac{2}{3} (1 + \frac{1}{2}a_8^2 + a_{10}^2 + ...) \] (5.2)

which also disagrees with the data. Furthermore, for scalar sea, the first moment of the neutron spin structure function

\[ \int g_1^n(x) dx = \frac{1}{g_A^2} (a_8^2 + 2a_{10}^2) \] (5.3)

is positive which seems to contradict the negative value indicated by the earlier analysis of the EMC result \[23\] and the latest data given by the SMC Collaboration \[24\].
B. Vector sea

We first look at a special vector sea as discussed in Ref. [15]. Assuming \(a_{8,10}=b_{1,10}=d_8=0\) and \(b_8=c_8=-\delta\), it is easy to see, from (3.5) that

\[
\mu_p = \mu_0 \frac{1 + \frac{4}{3} \delta^2}{1 + 2 \delta^2}, \quad \mu_n = \mu_0 \left( -\frac{2}{3} \right) \frac{1 + \frac{4}{3} \delta^2}{1 + 2 \delta^2},
\]

(5.4)

where \(\mu_0 = e/2m_u\). Hence the relation \(\mu_n/\mu_p = -2/3\) is preserved as given in [15]. Similarly, from (4.1) and (4.2), we have

\[
\left( \frac{g_A}{g_V} \right)_{n \rightarrow p} = \left( \frac{2}{3} \right) \frac{1 + \frac{4}{3} \delta^2}{1 + 2 \delta^2},
\]

(5.5)

\[
\left( \frac{g_A}{g_V} \right)_{\Xi^{-} \rightarrow \Xi^{0}} = \left( -\frac{1}{3} \right) \frac{1 + \frac{4}{3} \delta^2}{1 + 2 \delta^2},
\]

(5.6)

one can see that the conventional SU(6) result \(\left( \frac{g_A}{g_V} \right)_{n \rightarrow p}/(g_A/g_V)_{\Xi^{-} \rightarrow \Xi^{0}} = -5\) is also preserved. However, using parameter \(\delta = -0.35\) given in [15], we obtain \(\left( \frac{g_A}{g_V} \right)_{n \rightarrow p} = 1.727\), which is inconsistent with the data [17] \(\left( \frac{g_A}{g_V} \right)_{n \rightarrow p} = 1.257 \pm 0.003\). This disagreement is not unexpected. Because the perturbative calculation of the mixing parameters and its result \(b_8 = c_8 = -\delta\) are questionable. It is obvious that the nonperturbative effects, which are dominant in the low energy region, would change the relative weight of these mixing parameters significantly. Therefore, we prefer to discuss a more general vector sea and to look if there is another appropriate parameter set, in which the nonperturbative and perturbative effects are taking into account, can lead to a better agreement with the low energy baryon properties. We will show below that this parameter set not only gives a right modification to the ratio \(\mu_n/\mu_p\) but also gives a very good result for axial coupling constants.

As we mentioned above, the mixing parameters basically come from the nonperturbative interactions between quarks and gluons. Hence we do not attempt to calculate these parameters, but rather estimate them by the required agreement with the low energy data. Before doing this, we give some arguments as motivations for choosing the parameters. Since the sea basically comes from the emission of virtual gluons, the \(b_8\) term would be dominant and we would expect
\[ b_2^2, b_{10}^2 \text{ (two – gluon sea)} << b_8^2 \text{ (one – gluon sea)} \] (5.7)

The \( c_8 \) term is expected to be small due to another reason

\[ c_8^2 \text{ (quark spin – flip)} << b_8^2 \text{ (quark spin – nonflip)} \] (5.8)

The scalar sea \( a_8 \) and \( a_{10} \) terms are expected to be also small because they can only come from the two-gluon sea. The tensor sea \( (d_8) \) term comes from two-gluon sea and quark spin-flip process, hence it should be highly suppressed. Assuming no scalar and tensor sea contribution and neglecting the \( c_8^2 \) term (since \( c_8^2 << b_8^2 \)), we have

\[
\mu_n/\mu_p = (-2/3) \frac{1 - \frac{1}{3}b_1^2}{1 - \frac{1}{3}b_1^2 - \frac{1}{2}b_8^2} \simeq (-\frac{2}{3})(1 + \frac{1}{9}b_8^2) \] (5.9)

thus the sea contribution gives a correction in the right direction.

VI. NUMERICAL RESULTS

To obtain numerical results, we use the data on magnetic moments and weak decay coupling constants to determine the parameters. In particular, the values of \( \alpha \) and \( \beta \) so obtained should be reproducible by choice of the seven basic parameters \( a_8, a_{10}, b_1, b_8 \) etc. which determine the sea contribution. It is clear from (3.10) and (3.11) that there are many ways of choosing \( a_8 \) etc. to give the same \( \alpha \) and \( \beta \). However, guided by the qualitative discussion of section V, we will assume the sea is mainly vector with a small scalar component. The tensor sea is neglected \( (d_8=0) \). We shall see that the parameters \( (b_8, c_8 \text{ etc.}) \), which determine the contribution of such a sea to the baryon structure, can be chosen to give the \( \alpha \) and \( \beta \) determined from the data.

A. Four-parameter fit

The magnetic moments in (3.5)–(3.9) are given in terms of four effective parameters

\[
\bar{U} \equiv 3\alpha\mu_d, \quad \bar{D} \equiv -3\beta\mu_d, \quad 2p \equiv -\mu_u/\mu_d > 0 \text{ and } r \equiv \mu_s/\mu_d > 0. \]

Using \( \mu_p, \mu_n, \mu_{\Sigma,\Lambda} \) as inputs
one can directly determine $U = -1.348$, $D = 0.306$ and $p = 0.922$ as these do not involve
the parameter $r$. The value of $\mu_\Lambda$ is used as input to fix $r = 0.6255$. Knowledge of the ratio
$\alpha/\beta = 4.406$ immediately predicts (see (4.5))

$$F/D = 0.6878$$

(6.1)

A more realistic model with a small $\Delta s \neq 0$ could easily modify this value. Note that in
the models of Refs. and Refs. with extra parameter ($\Delta s$) they obtain 0.726 and 0.585 for this
ratio. To separate out the parameters $\alpha$ and $\beta$, we use the axial coupling constant data to obtain

$$\alpha = 0.3415, \quad \beta = 0.0775$$

(6.2)

The values obtained for the quark magnetic moments (in nuclear magnetons $\mu_\mathcal{N}$) are

$$\mu_u = 2.428, \quad \mu_d = -1.316, \quad \mu_s = -0.823$$

(6.3)

A choice of sea parameters which reproduce the parameters $\alpha$ and $\beta$ given in (6.2) are

$b_1^2 = 0.0039, \quad b_8^2 = 0.22, \quad c_8^2 = 0.027$ (for vector sea) and $a_{10}^2 = 0.0975$ (for scalar sea) with

$b_8c_8 > 0$.

The values obtained for magnetic moments and other quantities are displayed in column
4 of Table I. It can be seen that the fit to the magnetic moments and the axial coupling
constants is quite reasonable except for $\mu_{\Sigma^+}$. For the quark spin distributions our cal-
culation suggests a small non-zero negative value for $\int_0^1 g_1^n(x)dx$, however, the result for

$\int_0^1 g_1^p(x)dx = 0.2147$ is much larger than the experimental value [23] of $0.126 \pm 0.018$. One
must note, however, that the EMC experiment gives this value for $< Q^2 > = 10.7$ (GeV/c)$^2$
and this can be very different from the very low $Q^2$ result predicted by our $q^3$+sea model.

**B. Three-parameter fit**

The natural assumption $m_u = m_d$ implies the relation $\mu_u = -2\mu_d$. Implementing relation
in (3.5)–(3.9) gives $\mu_p = \mu_0(2\alpha + \beta)$, $\mu_n = -\mu_0(\alpha + 2\beta)$ etc. where $\mu_0 \equiv e/2m_u$. The
magnetic moments are given in terms of three effective parameters $\mu_0\alpha$, $\mu_0\beta$, and $r$. Guided by 4-parameter fit we choose sea parameters similar to that case, namely $b_1^2 = 0.1, b_8^2 = 0.22, c_8^2 = 0.027$ and $a_{10}^2 = 0.02$ with $b_8c_8 > 0$. Basically we have enhanced the vector sea with a larger value of $b_1$ and reduced the scalar sea with a smaller value of $a_{10}$. This choice immediately gives $\alpha = 0.3264$ and $\beta = 0.0927$. Using $\mu_p$ and $\mu_\Lambda$ as inputs then determines $\mu_0 = 3.7465\mu_N$ and $r = 0.6286$. The results of magnetic moments etc. are listed in column 5 of Table I. Since the ratio $\alpha/\beta = 3.521$ one obtains

$$F/D = 0.6380$$

which is fairly close to the experimental value \[25\]. Since $F/D$ increases monotonically with increasing $\alpha/\beta$, for the simple quark model ($\alpha/\beta = 4$) the value of $F/D=2/3$ lies between those in (6.1) and (6.4). The results for quark spin distributions are similar to the 4-parameter case.

For comparison, in column 3 of Table I the results for the simple quark model are given. In this case there is no sea contribution and the baryons are given by standard $q^3$ wave function which fixes $\alpha = 4/9$ and $\beta = 1/9$. The magnetic moments are given in terms of 3-parameters $\mu_u$, $\mu_d$ and $\mu_s$. This fit with $\mu_p$, $\mu_n$ and $\mu_\Lambda$ as inputs gives $\mu_u/(-2\mu_d) = p=0.953$ and $r = \mu_s/\mu_u=0.63$. We have used the same inputs in all three cases for a meaningful comparison. From Table I one can see that the 4-parameter gives a somewhat better overall fit.

**VII. SUMMARY**

In summary, we have suggested a general formalism to treat a baryon as a composite system of $q^3$ plus a flavorless sea. The modifications of the different properties of spin 1/2 baryon, by the sea, are given. Numerical fits to the individual magnetic moments, $\Sigma\Lambda$-transition moment and axial weak coupling constants for the baryon octet have been obtained. These results seem to favour a dominantly vector sea.
It should be noted that our results and conclusions are subject to the following points:
(i) the sea and the 3−quarks are considered to be in a relative $S$−state, possible higher angular momentum states have been neglected; (ii) the sea is assumed to be flavorless and has been specified only by its total quantum numbers; (iii) further, modification of baryon wave function is needed to have non−zero $\Delta s$ in the nucleon; (iv) relativistic corrections have been neglected although the internal motion of the light quarks in the baryon is expected to be relativistic; (v) all calculations have been performed in the baryon rest frame. This may be reasonable for the magnetic moments and the weak decay constants, but may not be appropriate for comparing the spin distribution calculated by us (at low $Q^2$−scale) with the EMC data at much high momentum transfer. All these points need to be considered in future work to fully understand baryon structure.

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Appendix. Matrix Elements for Different Operators

(i) Spin Projection Operator

\[ <\sigma_z^{(1)}>^{\lambda\lambda} = <\sigma_z^{(2)}>^{\lambda\lambda} = 2/3 ; \quad <\sigma_z^{(3)}>^{\lambda\lambda} = -1/3 \]  

\[ <\sigma_z^{(1)}>^{\rho\rho} = <\sigma_z^{(2)}>^{\rho\rho} = 0 ; \quad <\sigma_z^{(3)}>^{\rho\rho} = 1 \]  

\[ <\sigma_z^{(1)}>^{\lambda\rho} = - <\sigma_z^{(2)}>^{\lambda\rho} = 1/\sqrt{3} ; \quad <\sigma_z^{(3)}>^{\lambda\rho} = 0 \]  

It is easy to see that the matrix elements in (1) and (2) satisfy

\[ \sum_{i=1}^{3} <\sigma_z^{(i)}>^{\lambda\lambda} = \sum_{i=1}^{3} <\sigma_z^{(i)}>^{\rho\rho} = 1 \]  

In addition,

\[ <\sigma_z^{(1)}>^{\lambda\lambda/2} = <\sigma_z^{(2)}>^{\lambda\lambda/2} = -\sqrt{2}/3 ; \quad <\sigma_z^{(3)}>^{\lambda\lambda/2} = 2\sqrt{2}/3 \]  

\[ <\sigma_z^{(1)}>^{\rho\rho/2} = - <\sigma_z^{(2)}>^{\rho\rho/2} = \sqrt{2}/3 ; \quad <\sigma_z^{(3)}>^{\rho\rho/2} = 0 \]  

The matrix elements in (3), (5) and (6) satisfy

\[ \sum_{i=1}^{3} <\sigma_z^{(i)}>^{\lambda\rho} = \sum_{i=1}^{3} <\sigma_z^{(i)}>^{\lambda\lambda/2} = \sum_{i=1}^{3} <\sigma_z^{(i)}>^{\rho\rho/2} = 0 \]  

(ii) Isospin Projection Operator

For the proton, we have

\[ <I_3^{(1)}>^{\lambda\lambda} = <I_3^{(2)}>^{\lambda\lambda} = 1/3 ; \quad <I_3^{(3)}>^{\lambda\lambda} = -1/6 \]  

\[ <I_3^{(1)}>^{\rho\rho} = <I_3^{(2)}>^{\rho\rho} = 0 ; \quad <I_3^{(3)}>^{\rho\rho} = 1/2 \]  

\[ <I_3^{(1)}>^{\lambda\rho} = - <I_3^{(2)}>^{\lambda\rho} = 1/2\sqrt{3} ; \quad <I_3^{(3)}>^{\lambda\rho} = 0 \]  

for the neutron, all matrix elements get an opposite sign. For \( \Sigma^+ \)-hyperon, we have
\[ < I_3^{(1)} >_{\Sigma^+}^{\lambda \lambda} = < I_3^{(2)} >_{\Sigma^+}^{\lambda \lambda} = 5/12 ; \quad < I_3^{(3)} >_{\Sigma^+}^{\lambda \lambda} = 1/6 \quad (11) \]

\[ < I_3^{(1)} >_{\Sigma^+}^{\rho \rho} = < I_3^{(2)} >_{\Sigma^+}^{\rho \rho} = 1/4 ; \quad < I_3^{(3)} >_{\Sigma^+}^{\rho \rho} = 1/2 \quad (12) \]

\[ < I_3^{(1)} >_{\Sigma^+}^{\lambda \rho} = - < I_3^{(2)} >_{\Sigma^+}^{\lambda \rho} = 1/4\sqrt{3} ; \quad < I_3^{(3)} >_{\Sigma^+}^{\lambda \rho} = 0 \quad (13) \]

similarly, for \( \Sigma^- \) the matrix elements reverse their signs. For \( \Xi^0 \)-hyperon, we have

\[ < I_3^{(1)} >_{\Xi^0}^{\lambda \lambda} = < I_3^{(2)} >_{\Xi^0}^{\lambda \lambda} = 1/12 ; \quad < I_3^{(3)} >_{\Xi^0}^{\lambda \lambda} = 1/3 \quad (14) \]

\[ < I_3^{(1)} >_{\Xi^0}^{\rho \rho} = < I_3^{(2)} >_{\Xi^0}^{\rho \rho} = 1/4 ; \quad < I_3^{(3)} >_{\Xi^0}^{\rho \rho} = 0 \quad (15) \]

\[ < I_3^{(1)} >_{\Xi^0}^{\lambda \rho} = - < I_3^{(2)} >_{\Xi^0}^{\lambda \rho} = -1/4\sqrt{3} ; \quad < I_3^{(3)} >_{\Xi^0}^{\lambda \rho} = 0 \quad (16) \]

for \( \Xi^- \), all matrix elements reverse their signs. Finally, all isospin matrix elements for \( \Lambda \) and \( \Sigma^0 \) hyperons are zero.

(iii) Charge Operator With Symmetry Breaking Effect

For the proton, we have

\[ < e^{(1)} m_{m_1} >_p^{\lambda \lambda} = < e^{(2)} m_{m_2} >_p^{\lambda \lambda} = 1/2 ; \quad < e^{(3)} m_{m_3} >_p^{\lambda \lambda} = 0 \quad (17) \]

\[ < e^{(1)} m_{m_1} >_p^{\rho \rho} = < e^{(2)} m_{m_2} >_p^{\rho \rho} = 1/6 ; \quad < e^{(3)} m_{m_3} >_p^{\rho \rho} = 2/3 \quad (18) \]

\[ < e^{(1)} m_{m_1} >_p^{\lambda \rho} = - < e^{(2)} m_{m_2} >_p^{\lambda \rho} = 1/2\sqrt{3} ; \quad < e^{(3)} m_{m_3} >_p^{\lambda \rho} = 0 \quad (19) \]

where \( m = m_u = m_d \). We note that the matrix element \( < e^{(3)} m_{m_3} >^{\lambda \rho} \) vanishes for all octet baryons.

For the neutron, the matrix elements in (19) reverse their signs. But in (18) the first two matrix elements do not change the sign, i.e. \( < e^{(i)} m_{m_i} >_n^{\rho \rho} = < e^{(i)} m_{m_i} >_p^{\rho \rho} \) (i=1,2) and the third one becomes \( < e^{(3)} m_{m_3} >_n^{\rho \rho} = -1/3 \). For the neutron matrix elements in (17), we have
\[ < e^{(1)} \frac{m}{m_1} > \lambda^\lambda_n = < e^{(2)} \frac{m}{m_2} > \lambda^\lambda_n = -1/6; \quad < e^{(3)} \frac{m}{m_3} > \lambda^\lambda_n = 1/3 \quad (20) \]

For \( \Sigma^+ \), we obtain

\[ < e^{(1)} \frac{m}{m_1} > \lambda^\lambda_{\Sigma^+} = < e^{(2)} \frac{m}{m_2} > \lambda^\lambda_{\Sigma^+} = (10 - r)/18; \quad < e^{(3)} \frac{m}{m_3} > \lambda^\lambda_{\Sigma^+} = 2(1 - r)/9 \quad (21) \]

\[ < e^{(1)} \frac{m}{m_1} > \rho^\rho_{\Sigma^+} = < e^{(2)} \frac{m}{m_2} > \rho^\rho_{\Sigma^+} = (2 - r)/6; \quad < e^{(3)} \frac{m}{m_3} > \rho^\rho_{\Sigma^+} = 2/3 \quad (22) \]

\[ < e^{(1)} \frac{m}{m_1} > \lambda^\rho_{\Sigma^+} = - < e^{(2)} \frac{m}{m_2} > \lambda^\rho_{\Sigma^+} = (2 + r)/6\sqrt{3} \quad (23) \]

while for \( \Sigma^- \), we have

\[ < e^{(1)} \frac{m}{m_1} > \lambda^\lambda_{\Sigma^-} = < e^{(2)} \frac{m}{m_2} > \lambda^\lambda_{\Sigma^-} = -(5 + r)/18; \quad < e^{(3)} \frac{m}{m_3} > \lambda^\lambda_{\Sigma^-} = -(1 + 2r)/9 \quad (24) \]

\[ < e^{(1)} \frac{m}{m_1} > \rho^\rho_{\Sigma^-} = < e^{(2)} \frac{m}{m_2} > \rho^\rho_{\Sigma^-} = -(1 + r)/6; \quad < e^{(3)} \frac{m}{m_3} > \rho^\rho_{\Sigma^-} = -1/3 \quad (25) \]

\[ < e^{(1)} \frac{m}{m_1} > \lambda^\rho_{\Sigma^-} = - < e^{(2)} \frac{m}{m_2} > \lambda^\rho_{\Sigma^-} = -(1 - r)/6\sqrt{3} \quad (26) \]

For \( \Xi^0 \), we have

\[ < e^{(1)} \frac{m}{m_1} > \lambda^\lambda_{\Xi^0} = < e^{(2)} \frac{m}{m_2} > \lambda^\lambda_{\Xi^0} = (2 - 5r)/18; \quad < e^{(3)} \frac{m}{m_3} > \lambda^\lambda_{\Xi^0} = (4 - r)/9 \quad (27) \]

\[ < e^{(1)} \frac{m}{m_1} > \rho^\rho_{\Xi^0} = < e^{(2)} \frac{m}{m_2} > \rho^\rho_{\Xi^0} = (2 - r)/6; \quad < e^{(3)} \frac{m}{m_3} > \rho^\rho_{\Xi^0} = -r/3 \quad (28) \]

\[ < e^{(1)} \frac{m}{m_1} > \lambda^\rho_{\Xi^0} = - < e^{(2)} \frac{m}{m_2} > \lambda^\rho_{\Xi^0} = -(2 + r)/6\sqrt{3} \quad (29) \]

and for \( \Xi^- \)

\[ < e^{(1)} \frac{m}{m_1} > \lambda^\lambda_{\Xi^-} = < e^{(2)} \frac{m}{m_2} > \lambda^\lambda_{\Xi^-} = -(1 + 5r)/18; \quad < e^{(3)} \frac{m}{m_3} > \lambda^\lambda_{\Xi^-} = -(2 + r)/9 \quad (30) \]

\[ < e^{(1)} \frac{m}{m_1} > \rho^\rho_{\Xi^-} = < e^{(2)} \frac{m}{m_2} > \rho^\rho_{\Xi^-} = -(1 + r)/6; \quad < e^{(3)} \frac{m}{m_3} > \rho^\rho_{\Xi^-} = -r/3 \quad (31) \]

\[ < e^{(1)} \frac{m}{m_1} > \lambda^\rho_{\Xi^-} = - < e^{(2)} \frac{m}{m_2} > \lambda^\rho_{\Xi^-} = (1 - r)/6\sqrt{3} \quad (32) \]
For $\Lambda^0$, we obtain

$$< e^{(1)} \frac{m}{m_1} >_{\Lambda^0} > \sum_{\rho \rho}^\Lambda > (1 - 2r) / 12 ; \quad < e^{(3)} \frac{m}{m_3} >_{\Lambda^0} > \sum_{\rho \rho}^\Lambda = 1 / 6$$

(33)

$$< e^{(1)} \frac{m}{m_1} >_{\Lambda^0} > \sum_{\rho \rho}^\Lambda > (5 - 2r) / 36 ; \quad < e^{(3)} \frac{m}{m_3} >_{\Lambda^0} = (1 - 4r) / 18$$

(34)

$$< e^{(1)} \frac{m}{m_1} >_{\Lambda^0} = - < e^{(2)} \frac{m}{m_2} >_{\Lambda^0} = -(1 + 2r) / 12 \sqrt{3}$$

(35)

and for $\Sigma^0$, one obtains

$$< e^{(1)} \frac{m}{m_1} >_{\Sigma^0} = < e^{(2)} \frac{m}{m_2} >_{\Sigma^0} = (5 - 2r) / 36 ; \quad < e^{(3)} \frac{m}{m_3} >_{\Sigma^0} = (1 - 4r) / 18$$

(36)

$$< e^{(1)} \frac{m}{m_1} >_{\Sigma^0} = - < e^{(2)} \frac{m}{m_2} >_{\Sigma^0} = (1 + 2r) / 12 \sqrt{3}$$

(37)

Finally, for $\Sigma^0 \rightarrow \Lambda^0$ transition elements we have

$$< e^{(1)} \frac{m}{m_1} >_{\Sigma^0 \rightarrow \Lambda^0} = < e^{(2)} \frac{m}{m_2} >_{\Sigma^0 \rightarrow \Lambda^0} = -1 / 4 \sqrt{3} ; \quad < e^{(3)} \frac{m}{m_3} >_{\Sigma^0 \rightarrow \Lambda^0} = 1 / 2 \sqrt{3}$$

(39)

$$< e^{(1)} \frac{m}{m_1} >_{\Sigma^0 \rightarrow \Lambda^0} = < e^{(2)} \frac{m}{m_2} >_{\Sigma^0 \rightarrow \Lambda^0} = 1 / 4 \sqrt{3} ; \quad < e^{(3)} \frac{m}{m_3} >_{\Sigma^0 \rightarrow \Lambda^0} = -1 / 2 \sqrt{3}$$

(40)

$$< e^{(1)} \frac{m}{m_1} >_{\Sigma^0 \rightarrow \Lambda^0} = - < e^{(2)} \frac{m}{m_2} >_{\Sigma^0 \rightarrow \Lambda^0} = -1 / 4$$

(41)

(iv) Charge Square Operator

We only discuss the nucleon case, for the proton we obtain

$$< e^{(1)}^2 >_{p} = < e^{(2)}^2 >_{p} = 7 / 18 ; \quad < e^{(3)}^2 >_{p} = 2 / 9$$

(42)

$$< e^{(1)}^2 >_{p} = < e^{(2)}^2 >_{p} = 5 / 18 ; \quad < e^{(3)}^2 >_{p} = 4 / 9$$

(43)

$$< e^{(1)}^2 >_{p} = - < e^{(2)}^2 >_{p} = 1 / 6 \sqrt{3} ; \quad < e^{(3)}^2 >_{p} = 0$$

(44)

while for the neutron, one obtains

$$< e^{(1)}^2 >_{n} = < e^{(2)}^2 >_{n} = 1 / 6 ; \quad < e^{(3)}^2 >_{n} = 1 / 3$$

(45)

$$< e^{(1)}^2 >_{n} = < e^{(2)}^2 >_{n} = 5 / 18 ; \quad < e^{(3)}^2 >_{n} = 1 / 9$$

(46)

$$< e^{(1)}^2 >_{n} = - < e^{(2)}^2 >_{n} = -1 / 6 \sqrt{3} ; \quad < e^{(3)}^2 >_{n} = 0$$

(47)
TABLES

TABLE I. Comparison of the calculated magnetic moments and axial coupling constants of baryons with data and other models.

| Baryon  | Data\textsuperscript{16} | SQM\textsuperscript{a} | Set I\textsuperscript{b} | Set II\textsuperscript{c} |
|---------|--------------------------|------------------------|---------------------------|---------------------------|
| P       | 2.7928                   | 2.793\*                | 2.7928\*                  | 2.793\*                  |
| n       | −1.9130                  | −1.913\*               | −1.913\*                  | −1.917\*                 |
| Λ       | −0.613±0.004             | −0.613\*               | −0.613\*                  | −0.613\*                 |
| Σ\textsuperscript{0}\Lambda | −1.61±0.08              | −1.63                  | −1.61\*                   | −1.66                    |
| Σ\textsuperscript{+}   | 2.42±0.05                | 2.674                  | 2.678                     | 2.664                    |
| Σ\textsuperscript{0}  |                         | 0.791                  | 0.761                     | 0.830                    |
| Σ\textsuperscript{−}   | −1.160±0.025             | −1.092                 | −1.156                    | −1.004                   |
| Ξ\textsuperscript{0}  | −1.250±0.014             | −1.435                 | −1.408                    | −1.463                   |
| Ξ\textsuperscript{−}   | −0.6507±0.0025           | −0.493                 | −0.537                    | −0.421                   |

\((g_A/g_V)_{n\to p}\) 1.2573±0.0028 1.666 1.2571 1.2573
\((g_A/g_V)_{\Lambda\to p}\) 0.718±0.015 1.000 0.7605 0.7455
\((g_A/g_V)_{\Sigma^-\to n}\) −0.340±0.017 −0.333 −0.2325 −0.2781
\((g_A/g_V)_{\Xi^-\to \Lambda}\) 0.25±0.05 0.333 0.2640 0.2337
\((g_A/g_V)_{\Xi^-\to \Xi^0}\) − 0.333 −0.2325 −0.2781

\(\int g_f^p(x) dx\) 0.126±0.010±0.015 0.278 0.2147 0.202
\(\int g_f^n(x) dx\) −0.08±0.06\textsuperscript{d} 0.0 −0.0052 −0.007
\(\int g_f^\Lambda(x) dx\) 0.0556 0.0466 0.0353

a) Standard quark model result, e.g. see Ref. 16, VIII.59
b) Four-parameter fit
c) Three-parameter fit
d) Ref. [24]
* ) Inputs