An NSGA-III algorithm for solving multi-objective economic/environmental dispatch problem

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Abstract: The main ambition of utility is to provide continuous reliable supply to customers, satisfying power balance, transmission loss while generators are allowed to be operated within rated limits. Meanwhile, achieving this from fossil fuel fired power plant emission value and fuel cost should be as less as possible. An allowable deviation in fuel cost and feasible tolerance in fuel cost has been additively called as multi objective combined economic emission dispatch (MOCEED) problem. MOCEED problem is applied to newly proposed non dominated sorting genetic algorithm-III (NSGA-III). NSGA-III method is really powerful to handle problems with non-linear characteristics as well as having many objectives. The proposed NSGA-III is firstly applied to unconstraint/constraints multi-objective test functions then applied to solve MOCEED problem with 6-generation unit, IEEE 118 bus 14 generating unit system with a smooth quadratic fuel/emission objective functions and 10-unit with non-smooth/valve point loading effect test system. Statistical results of MOCEED problem obtained by NSGA-III is compared with other well-known techniques proposed in recent literature, validates the effectiveness of proposed approach.

Subjects: Artificial Intelligence; Evolutionary Computing; Power Engineering; Engineering Economics

Keywords: emission constrained economic dispatch; NSGA-III; valve point loading effect; meta-heuristic; multi-objective

1. Introduction
The main ambition of fossil fuel fired power plant management is to optimal scheduling of active power to committed units so as to achieve least possible generation fuel cost as well as emission value. Primarily economic load dispatch problem (ELDP) (Trefny & Lee, 1981) is main objective of electricity generation utilities but with large consciousness towards environment protection and...
passage of the clean air act amendments have forced generation utilities to reduce emission (Le et al., 1995; Talaq, El-Hawary, & El-Hawary, 1994) to a certain level.

Evolutionary techniques are capable to overcome the difficulties associated with classical methods such as multiple run. The target of multi-objective optimization technique is not only to steer the search towards the pareto optimal front but also to preserve population diversity in the set of non-dominated solutions.

Newly proposed NSGA-III (Deb & Jain, 2014; Jain & Deb, 2014) is powerful technique to eliminate the drawbacks of NSGA-II such as lack of uniform diversity and absence of lateral diversity preserving operator among the current best non dominated solutions.

In this paper term used emission constrained economic dispatch (ECED) problem is similar to term combined economic emission dispatch (CEED) problem. Various conventional linear optimization methods (Wood, Wollenberg, & Sheble, 2013) were used to solve the ELD problem, (a) lambda-iteration method, (b) gradient-method, (c) linear-programming-method and (c) Newton’s method. Linear programming techniques are fast and reliable but these methods are failed to obtain the optimal solution for solving highly complex non-linear objective functions.

The multi-objective power system dispatch problem can be transformed into single objective by Secularization Methods (Priori Approach) using these techniques:

• Price penalty factor technique and
• Weighted sum method (WSM)
• Goal Attainment method
• Lexicographic method etc.

The ECED problem consists of either single objective or multi-objective is solved using various algorithms such as: After Scalarization technique applied ECED problem can be classified into two forms with and without considering valve point loading effect of generators further classified into the equation used either quadratic and cubic equation to evaluate fuel cost and emission value. ECED problem is solved without considering valve point effect and with price penalty factors based approach is solved with various computational techniques.

ECED problem solved using “Max-Max” price penalty factor approach by various artificial intelligence (AI) techniques (Jacob Raglend, Veeravalli, Sailaja, Sudheera, & Kothari, 2010) consisting of genetic algorithm (GA), Evolutionary Programming (EP), Particle swarm optimizer (PSO) and Differential evolution (DE) is applied on IEEE-30 Bus system. “Max-Max” price penalty factor is also used to solve CEED/ECED problem with Gravitational Search Algorithm (Güvenç, Sönmez, Duman, & Yürükeren, 2012), Parallelized PSO (PPSO) (Hamedi, 2013), Evolutionary programming (EP), GA and Micro GA (MGA) (Venkatesh, Gnanadass, & Padhy, 2003), Assessment of available transfer capability for practical power system with CEED problem for IEEE-30bus system with 6 generating units and Indian utility system 62-Bus (IUS-62) with nineteen generators (Gnanadass, Padhy, & Manivannan, 2004). Analytical solution for CEED problem with IUS-62 with six generators(Palanichamy & Babu, 2008), comparative study (Krishnamurthy & Tzoneva, 2012b) with “Min-Max” price penalty factor using PSO and Lagrange’s algorithm (LA), with LA (Krishnamurthy & Tzoneva, 2011a) and PSO (Krishnamurthy & Tzoneva, 2012a) taking “Min-Max” and “Max-Max” price penalty factors approach ECED problem is solved. Lagrange’s algorithm is used to solve ECED problem with four penalty factors (Krishnamurthy & Tzoneva, 2012d) with the quadratic equation is considered for evaluating fuel cost and emission value, six penalty factors with cubic equation (Krishnamurthy & Tzoneva, 2012c) used for calculation of ECED problem. Scenario based dynamic economic emission dispatch problem is solved by Fuzzy adaptive improved PSO (FAIPSO) (Aghaei, Niknam, Azizipanah-Abarghoee, & Arroyo, 2013).
The ECED problem with valve point effect is solved by using “Min-Max” and “Max-Max” price penalty factors approach with LA (Krishnamurthy & Tzoneva, 2011b), Maclaurin series based Lagrangian method (Simon & Hemamalini, 2009), Opposition based-GSA (OGSA) (Shaw, Mukherjee, & Ghoshal, 2012).

Various types of economic dispatch problem are solved with weighted sum method (WSM) using PSO (Jeyakumar, Jayabarathi, & Raghunathan, 2006). The ECED problem with WSM technique is solved using Artificial Bee Colony with Dynamic Population size (ABCDP) (Aydin, Özyn, Yaşar, & Liao, 2014) and opposition-based harmony search algorithm (OHS) (Chatterjee, Ghoshal, & Mukherjee, 2012). Hybridization of PSO and GSA computational techniques with weighted sum method considering valve point effect (Jiang, Ji, & Shen, 2014) for ECED problem solution. Neural network, Fuzzy system and Lagrange’s algorithm (LA) (Krishnamurthy & Tzoneva, 2011b) for single and multi-area dispatch problem is investigated, Emission Standards (Guttikunda & Jawahar, 2014), Location of Greenhouse gases (GHG) emission from thermal power plant in India (Sethi, 2015), Dispatch problem on different power system using Stochastic algorithm (Dhillon, Parti, & Kothari, 1993; Kothari & Dhillon, 2011), Security constrained economic scheduling of generation considering generator constraints (Chang, 1995; Gaing & Chang, 2006), Integration of solar and coal fired plant (Parvareh et al., 2014).

Finally, the economic environmental emission dispatch problem is multi-objective (such as Fuel Cost, Emission Value, Transmission Loss, ECED fuel cost, Different gases exhalation etc.) considering at a single time to find an actual operating point of generators to fulfil all objectives efficiently. Multi-objective thermal power dispatch (Dhillon, Parti, & Kothari, 1994), considering more than one objective for ECED problem is solved using various computational techniques such as: multi-objective DE (MODE) (Basu, 2011), MOGSA (Mondal, Bhattacharya, & ree Dey, 2013), Modified Non-dominated sorting genetic algorithm-II (MNSGA-II) (Dhanalakshmi, Kannan, Mahadevan, & Baskar, 2011), NSGA-II with valve point effect (Basu, 2008), BB-MOPSO (Zhang, Gong, & Ding, 2012), Hybrid multi-objective optimization algorithm based on PSO and DE (MO-DE/PSO) (Gong, Zhang, & Qi, 2010), Multi-objective particle swarm optimization algorithm proposed by Coello et al. (CMOPSO) (Coello, Pulido, & Lechuga, 2004), Multi objective particle swarm with the sigma method (SMOPSO) (Mostaghim & Teich, n.d.), time variant multi-objective particle swarm optimization (TV-MOPSO) (Tripathi, Bandyopadhayay, & Pal, 2007) and multi-objective harmony search algorithm proposed by Sivasubramani and Swarup (2011) etc.

Combined Economic Emission Dispatch (CEED) problem is also known as Emission Constrained Economic Dispatch (ECED) problem, Combined Economic and Environmental Dispatch, Environmental/Economic dispatch (EED), Multiobjective CEED (MOCEED) and Constraint Environment Dispatch (CED) problem.

Recently proposed nature based and evolutionary algorithms with various applications like energy management of RES in a Microgrid using Cuckoo Search (CS) Algorithm (Bhoye et al., 2016), constrained engineering design problem by Moth-Flame optimizer (MFO) (Jangir et al., 2016), Voltage stability improvement by BAT optimization algorithm (Trivedi et al., 2016), Seyedali Mirjalili et al. algorithms such as Grey wolf optimizer (GWO) (Mirjalili, Mirjalili, & Lewis, 2014), Whale optimizer (WOA) (Mirjalili & Lewis, 2016), Moth-flame optimizer (MFO) (Mirjalili, 2015), swarm based Dragonfly Algorithm (DA) (Mirjalili, 2016), Gai-Ge Wang et al. elephant herding optimization (EOH) (Wang & Suash Deb, in press) and evolution based NSGA-III. Among these algorithms NSGA-III is chosen because it has comparatively better capability of handling many objectives (up to 50 objectives) and it also has a uniform diversity to obtained Pareto optimal front in a set of non-dominated solutions. Overview of this paper is shown in Figure 1.
2. Mathematical formation of emission constrained economic dispatch (ECED) problem

2.1. Without valve-point loading effect

Mathematic equation for ECED problem is designed from (Aghaei et al., 2013; Aydin et al., 2014; Chatterjee et al., 2012; Gnanadass et al., 2004; Güvenç et al., 2012; Hamedi, 2013; Jacob Raglend et al., 2010; Jiang et al., 2014; Krishnamurthy & Tzoneva, 2011a, 2012a, 2012b, 2012d; Palanichamy & Babu, 2008; Simon & Hemamalini, 2009; Venkatesh et al., 2003). Fuel cost equation is given as follows:

\[ \text{Min}(F_c) = \sum_{i=1}^{NG} a_i P_i^2 + b_i P_i + c_i (\$/h) \]  

(1)

where \( a_i \) = Cost coefficient of ith generator in (\$/MW^2 h), \( b_i \) = Cost coefficient of ith generator in (\$/MWh), \( c_i \) = Cost coefficient of ith generator in (\$/h), \( F_c \) = Generation cost [21] of the ith generator (\$/h), \( NG \) = Number of generators.

Emission standards (Krishnamurthy & Tzoneva, 2011b) and its effect is known as Global warming due to dangerous gases exhalation from power plants, the range of effected area due to GHG (Guttikunda & Jawahar, 2014). Total emission is calculated from quadratic equation given below:

\[ E_T = \sum_{i=1}^{n} (\alpha_i P_i^2 + \beta_i P_i + \gamma_i) \text{ (kg/h)} \]  

(2)
where, $ET = \text{Total emission value}$, $\alpha_i = \text{Emission coefficient of } i\text{th generator in (kg/MW}^2\text{ h)}$, $\beta_i = \text{Emission coefficient of } i\text{th generator in (kg/MWh)}$, $\gamma_i = \text{Emission coefficient of } i\text{th generator in (kg/h)}$.

### 2.2. With valve-point loading effect

\[
\text{Min}(F_f) = \sum_{i=1}^{NG} a_i P_i^2 + b_i P_i + c_i + \left| d_i \sin \left( e_i (P_i^{\min} - P_i) \right) \right|
\]

where, $F_f = \text{Total fuel cost }$/h.

\[
\text{Min}(E_f) = \sum_{i=1}^{NG} a_i P_i^2 + \beta_i P_i + \gamma_i + \eta_i \exp(\delta_i P_i)
\]

where, $E_f = \text{Total emission value kg/h}$.

### 3. Non-dominated sorting algorithm-III

NSGA-III (Deb & Jain, 2014; Jain & Deb, 2014) algorithm start randomly initialized population size $N$, pre-defined more distributed $M$-dim, with reference points $H$ on plane have hyper with a normal vector that cover entire entire regions $RM$ region. Each reference point in hyper-plane is put such of manner its intersect every objective function arises at one that called Das and Dennis’s technique where $H = \left( \frac{M+p-1}{p} \right)$ each Range have $(p + 1)$ points and $N$ is multiple of 4 greater than $H$ both have desired conditions.

At a generation $t$, complete population $Pt$ convert in non-dominated solutions in the same way of NSGA-II algorithm Sorting mechanism, after that $Pt$ produces new offspring population $Q_t$ with the help of mutations and recombination operators in which everyone population member associated with each reference point & any selection operator will allow a competition to be set among different reference points. A combined population $R_t = Pt \cup Q_t$ is then formed. So we have get first non-dominated solution $Pt+1$ until every solution cannot be included from whole fronts. Suppose we have denoted some front that cannot be associated to select $F_i$ after that $P_{t+1}$ and $F_i$ perform niching and normalized mechanism after that each member associated with a specific reference point based on shortest perpendicular distance ($d()$) of each population member with a reference line created by joining the origin with a supplied reference point. At finally niching mechanism choose the $F_i$ member that is linked with minimum reference points in $P_{t+1}$.

The whole process is then expected to find one population member corresponding to each supplied reference point close to the Pareto-optimal front, based on crossover, mutation and recombination operators that are used to develop uniform solutions. The use of a well-spread reference points ensures a well-distributed set of trade-off points at the end.

A better advantage of NSGA-III (Deb & Jain, 2014; Jain & Deb, 2014) is that have not required additional parameter compare to NSGA-II. NSGA-III Algorithm step by step representation shown in Figure 2. Main difference of selection mechanism of both NSGA-II & NSGA-III algorithms given below:

1. NSGA-III algorithm cannot have required another selection operator for $P_p$ to create new operator $q_*$. On the other hand, NSGA-II’s selection operator uses non-dominated rank and a crowding distance value to choose a winner between two feasible individuals from $p_*$. It is worth noting however that, NSGA-III performs selection if and only if at least one of the two individuals being compared is infeasible. In that case NSGA-III prefers feasible over infeasible, and less violating over more violating individuals.

2. To maintain better Coverage of pareto solutions NSGA-III uses reference point mechanism and other side NSGA-II uses crowding distance operator to maintain uniform coverage.
NSGA-III (Deb & Jain, 2014; Jain & Deb, 2014) uses a pre-allocated reference set mechanism to choose better diverse solutions in the size of population in free space, whereas NSGA-II algorithm does not require any pre-allocated methods on the objective space. So, more time taken to generate first solution in spaces, NSGA-III have easily generated first solution so NSGA-III better than NSGA-II algorithm for solving many objective problems. Figure 2 shows step by step procedure of NSGA-III Algorithm has been explained in pseudo code named step 2, step 3 and step 4 respectively.

4. Application and results

The meta-heuristic techniques are implemented to resolve the MOCEED problem for standard test system and for a number of cases with dissimilar objective functions. The software program is written in MATLAB 2014b and applied on a 2.60 GHz i5 PC having 4 GB RAM.
4.1. Unconstraint/constraint test functions
NSGA-III technique has been primarily applied to solve multiobjective unconstraint/constraint test functions such as FON 2, KUR, ZDT 1, ZDT 2, ZDT 3, ZDT 4, ZDT 6, BINH, VNT 3, DTLZ 1, DTLZ 2 and DTLZ 3. Appendix A consisting of objective function of all above multiobjective unconstraint/constraint problems: (a) FON2 (b) KUR (c) ZDT1 (d) ZDT2 (e) ZDT3 (f) ZDT4 (g) ZDT6 (h) BINH (i) VNT3 (j) DTLZ1 (k) DTLZ2 and (l) DTLZ3.
test functions. Statistical value and Pareto front obtained for inverse gradient distance (IGD) and hyper volume (HV) matrices are compared with other techniques (Cheng, Jin, Olhofer, & Sendhoff, 2016) solves same problem like SPEA2, and NSGA-II in Tables 1, 2 and Figure 3 respectively.

4.2. Test systems

NSGA-III technique is applied to three different test systems. In order to represent the effectiveness of proposed algorithm statistical results are compared with NSGA-II (Dhillon et al., 1994), Strength pareto evolutionary algorithm 2 (SPEA2) (Dhillon et al., 1994), pareto differential evolution (PDE) (Dhillon et al., 1994). NSGA-III results are also provided with single objective such as minimum fuel cost and minimum emission value. For each test system internal parameter like population/search agent, maximum/termination count and maximum archive size are 100, 200 and 100 respectively. In the way to check performance of NSGA-III algorithm, it has been tested on: the IEEE 30-bus, the 39-bus New England system network and the IEEE 118-bus test systems. All test systems are used to check the performance of NSGA-III algorithm compare to other algorithms.

4.2.1. Test system 1

In IEEE 30-bus test system contain 30-Buses, 41-Branches, 4-Transformers, 9-Shunt Var Compensators and 6-Generators. Further details, the Cost and emission coefficients, generator data and the minimum and maximum limits for the control variables are given in Table B1. In test system consists of six operational generating unit with simply a quadratic fuel and emission objective function for a power demand of 1,200 MW (Basu, 2011). Input data for operational generating unit loading limits and loss parameters are given in Table B1 of Appendix B. Single line diagram of 6-unit system is shown in Figure 4.
Figure 4. Single line diagram of 6-unit system.

| Problem | NSGA-III Mean  | NSGA-II (Cheng et al., 2016) Mean  | SPEA-2 (Cheng et al., 2016) Mean  |
|---------|----------------|-----------------------------------|----------------------------------|
|         | SD             | SD                                | SD                               |
| FON 2   | 6.1532E−5      | 5.1506E−3                         | 4.1800E−3                        |
| KUR     | 5.9741E−04     | 4.2322E−2                         | 3.4163E−2                        |
| ZDT 1   | 1.3508E−05     | 4.8182E−3                         | 4.1795E−3                        |
| ZDT 2   | 1.1212E−07     | 4.8259E−3                         | 4.1675E−3                        |
| ZDT 3   | 6.4797E−04     | 5.6881E−3                         | 5.5675E−3                        |
| ZDT 4   | 4.4880E−05     | 6.5921E−3                         | 6.5020E−3                        |
| ZDT 6   | 3.8151E−06     | 7.6800E−3                         | 8.3703E−3                        |
| Binh    | 7.0908E−1      | 3.5E−2                            | 5.8733E−1                        |
| VNT 3   | 1.2343E−04     | 4.9851E−2                         | 3.2437E−2                        |
| DTLZ 1  | 0.00030305     | 3.3798E−2                         | 2.2106E−2                        |
| DTLZ 2  | 8.6501E−08     | 6.8952E−2                         | 5.4307E−2                        |
| DTLZ 3  | 0.0018203      | 2.6043E+0                         | 1.6749E+0                        |
It is represented in Table 3 that with the objective of least cost objective minimum fuel cost is 6.41E+04 $ and emission value is 1,346 lb. But fuel cost increases to 6.599E+04 $ and emission value reduced to a numeric value 1,241 lb with the objective of emission minimization. Compromise point or true operating point obtained by NSGA-III for MOCEED problem is as fuel cost is 6.4830E+04 $ that is higher than minimum fuel cost 6.41E+04 $ and lower than 6.599E+04 $ obtained during least cost and emission value objectives respectively. So as with emission value for true operating point is 1,285 lb that is lower than 1,346 lb and higher than 1,241 lb obtained during least cost and emission value objectives respectively. Statistical value obtained for compromise point is compared with other techniques solves same MOCEED problem like SPEA2, NSGA-II and PDE in Table 3. Figure 5 shows 100 non-dominated solutions as true pareto front for 6-opertaional generating for PD = 1,200 MW.

4.2.2. Test system 2

In 39-bus New England test system, contain 39-Buses, 46-Branches, and 10-Generators. Further details, the Cost and emission coefficients, generator data and the minimum and maximum limits for the control variables are given in Table B2. This test system consists of ten operational generating unit with a non-smooth/non-convex fuel and emission objective function for a power demand of

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**Table 3. Statistical performance comparison of NSGA-III for 6-unit system**

| Parameters | NSGA-III | MODE (Basu, 2011) | PDE (Basu, 2011) | NSGA-II (Basu, 2011) | SPEA (Basu, 2011) |
|------------|----------|--------------------|-------------------|-----------------------|-------------------|
| Economic dispatch | Emission dispatch | EED | EED | EED | EED | EED |
| P1 (MW) | 84.6285 | 125 | 107.9932 | 108.6284 | 107.3965 | 113.1259 | 104.1573 |
| P2 (MW) | 93.4213 | 150 | 118.3631 | 115.9456 | 122.1418 | 113.4488 | 122.9807 |
| P3 (MW) | 210 | 201.4824 | 206.7969 | 206.7356 | 217.4191 | 214.9553 |
| P4 (MW) | 225 | 198.8723 | 206.65 | 206.0000 | 203.7047 | 207.9492 | 203.1387 |
| P5 (MW) | 315 | 288.5129 | 306.6592 | 301.8884 | 308.1045 | 304.6641 | 316.0302 |
| P6 (MW) | 325 | 286.2913 | 303.8712 | 304.4127 | 303.3797 | 291.5969 | 289.9396 |
| Cost ($) | 64.099 | 65.992 | 64.830 | 64.843 | 64.920 | 64.962 | 64.884 |
| Emission (lb) | 1,345.9 | 1,240.7 | 1,285 | 1,286.0 | 1,281.0 | 1,281.0 | 1,285 |

It is represented in Table 3 that with the objective of least cost objective minimum fuel cost is 6.41E+04 $ and emission value is 1,346 lb. But fuel cost increases to 6.599E+04 $ and emission value reduced to a numeric value 1,241 lb with the objective of emission minimization. Compromise point or true operating point obtained by NSGA-III for MOCEED problem is as fuel cost is 6.4830E+04 $ that is higher than minimum fuel cost 6.41E+04 $ and lower than 6.599E+04 $ obtained during least cost and emission value objectives respectively. So as with emission value for true operating point is 1,285 lb that is lower than 1,346 lb and higher than 1,241 lb obtained during least cost and emission value objectives respectively. Statistical value obtained for compromise point is compared with other techniques solves same MOCEED problem like SPEA2, NSGA-II and PDE in Table 3. Figure 5 shows 100 non-dominated solutions as true pareto front for 6-opertaional generating for PD = 1,200 MW.
2,000 MW (Basu, 2011). Input data for operational generating unit loading limits and loss parameters are given in Table B2 of Appendix B. Single line diagram of 10-unit system is shown in Figure 6.

It is represented in Table 4 that with the objective of least cost objective minimum fuel cost is 1.115E+05 $ and emission value is 4,562 lb. But fuel cost increases to 1.164E+05 $ and emission value reduced to a numeric value 3,932 lb with the objective of emission minimization. Compromise point or true operating point obtained by NSGA-III for MOCEED problem is as fuel cost is 1.1340E+05 $ that is higher than minimum fuel cost 1.115E+05 $ and lower than 1.164E+05 $ obtained during least cost and emission value objectives respectively. So as with emission value for true operating point is
4,105 lb that is lower than 4,562 lb and higher than 3,932 lb obtained during least cost and emission value objectives respectively. Statistical value obtained for compromise point is compared with other techniques solves same MOCEED problem like SPEA2, NSGA-II and PDE in Table 4. Figure 7 shows 100 non-dominated solutions as true Pareto front for 6-operational generating for PD = 2,000 MW.

4.2.3. Test system 3
In IEEE 118-bus test system contain 118-Buses, 186-Branches, 9-Transformers, 14-Shunt Var Compensators and 14-Generators. Further details, the Cost and emission coefficients, generator data and the minimum and maximum limits for the control variables are given in Table B3. The IEEE 118-bus 14-operational generating unit test system with a smooth quadratic fuel and emission objective function neglecting transmission losses for a power demand of 950 MW. Input data for operational generating unit loading limits and loss parameters are given in Table B3 of Appendix B. Single line diagram of 14-unit system is shown in Figure 8.

| Parameters | NSGA-III | MODE (Basu, 2011) | PDE (Basu, 2011) | NSGA-II (Basu, 2011) | SPEA 2 (Basu, 2011) |
|------------|----------|--------------------|------------------|-----------------------|---------------------|
| P₁ (MW)    | 55       | 55                 | 54.9324          | 54.9487               | 54.9853             | 51.9515             | 52.9761             |
| P₂ (MW)    | 80       | 79.9782            | 80               | 74.5821               | 79.3803             | 67.2584             | 72.8130             |
| P₃ (MW)    | 106.0514 | 82.1289            | 82.8893          | 79.4294               | 83.9842             | 73.6879             | 78.1128             |
| P₄ (MW)    | 99.2176  | 82.3506            | 83.7835          | 80.6875               | 86.5942             | 91.3554             | 83.6088             |
| P₅ (MW)    | 81.5808  | 160                | 149.0664         | 136.8551              | 144.4386            | 134.0522            | 137.2432            |
| P₆ (MW)    | 85.1964  | 240                | 153.8082         | 172.6393              | 165.7756            | 174.9504            | 172.9188            |
| P₇ (MW)    | 299.9843 | 296.1872           | 299.6631         | 283.8233              | 283.2122            | 289.4350            | 287.2023            |
| P₈ (MW)    | 340      | 206.2329           | 317.0490         | 316.3407              | 312.7709            | 314.0556            | 326.4023            |
| P₉ (MW)    | 340      | 397.4092           | 435.7486         | 448.5923              | 440.1135            | 455.6978            | 448.8814            |
| P₁₀ (MW)   | 340      | 392.2266           | 427.0254         | 436.4287              | 432.6783            | 431.8054            | 423.9025            |
| Cost (×10⁵$) | 1.1150   | 1.1643             | 1.1341           | 1.1348                | 1.1351              | 1.1354              | 1.1352              |
| Emission (lb) | 4,562   | 3,932.5            | 4,118.6          | 4,124.90              | 4,111.40            | 4,130.20            | 4,109.10            |
It is represented in Table 5 that with the objective of least cost objective minimum fuel cost is 4,265 $/h and emission value is 446.5 ton/h. But fuel cost increases to 4,485 $/h and emission value reduced to a numeric value 24.09 ton/h with the objective of emission minimization. Compromise point or true operating point obtained by NSGA-III for MOCEED problem is as fuel cost is 4,335.9 $/h that is higher than minimum fuel cost 4,265 $/h and lower than 4,485 $/h obtained during least cost and emission value objectives respectively. So as with emission value for true operating point is 124.6384 ton/h that is lower than 446.5 ton/h and higher than value 24.09 ton/h obtained during least cost and emission value objectives respectively. Statistical value obtained for compromise point is compared with other techniques solves same MOCEED problem like NSGA-II in Table 5. Figure 9 shows 100 non-dominated solutions as true pareto front for 14-opertaional generating for PD = 950 MW.

In order to check the robustness of the NSGA-III algorithm for solving Multi-Objective Economic/Environmental dispatch problem, different standard benchmark functions and the IEEE 30-bus, the 39-bus New England system network and the IEEE 118-bus test systems used for experimental study. Tables 1 and 2 represents the statistical results on benchmark functions achieved by the NSGA-III, NSGA-II and SPEA-2 algorithms for unconstraint and constraint problem in terms of IGD and HV metrics. From these table clear that NSGA-III algorithm have better value compare to NSGA-II and SPEA-2 in terms of SD and mean value. Tables 3–5 represents the statistical results on the IEEE 30-bus, the 39-bus New England system network and the IEEE 118-bus test systems achieved by the NSGA-III, NSGA-II, MODE, PDE and SPEA-2 algorithms for Economic/Environmental dispatch problem in terms of compromise solution point. From these tables clear that NSGA-III algorithm have better value compare to other algorithm reported on literature survey.

4.3. Performance evaluation study of NSGA-III algorithm

In this Section describe why NSGA-III algorithm Comparison with other published techniques to demonstrate the accuracy and the validity of the NSGA-III technique should be presented in details.
In NSGA-III algorithm does not required any additional adjustable parameters compare to NSGA-II. So experimental study on benchmark and real world problem NSGA-III algorithm have required less computational complexity and less time consuming compare to other algorithms reported in literatures.

NSGA-III algorithm are used to many objective means easily implemented more than 10 objectives that future make more power full NSGA-III algorithm compare to other algorithm.
(c) Results of NSGA-III algorithm in term of IGD and HV metrics in term of SD and mean value on standard unconstrained/constraint benchmark functions shown in Tables 1 and 2 represents NSGA-III algorithm represent better solution quality compare to NSGA-II and SPEA-2 algorithms.

(d) Results of NSGA-III algorithm in terms of best value on economic constraint emission dispatch (ECED) problem with the IEEE 30-bus, the 39-bus New England system network and the IEEE 118-bus test systems represents the better effectiveness of NSGA-III algorithm compare to other algorithm shown in Tables 3–5.

5. Conclusion
In this paper, a new NSGA-III algorithm, an improved version of most popular multi-objective algorithm NSGA-II, is successfully applied to standard benchmark functions and economic constraint emission dispatch (ECED) problem on three test systems, such as IEEE 30-Bus, New England 39-Bus, and IEEE 118-Bus, with different fuel cost curve characteristics such as simple quadratic fuel and non-smooth/non-convex fuel with emission value and various constraints. NSGA-III algorithm removes drawbacks of NSGA-II algorithm such as disability to maintaining the diversity among population members that is avoided by supplying and adaptively updating a number of well-spread reference points in NSGA-III. The obtained results using NSGA-III algorithm have been compared to well recognize NSGA-II, SPEA-2 and other multiobjective techniques reported in literatures. The comparative study among that algorithms represent the solution quality in terms of SD and mean value, superiority in terms of IGD and HV metrics and effectiveness in terms of best values of NSGA-III algorithm over other algorithms on standard unconstraint/constraint test functions and economic constraint emission dispatch (ECED) problem.

In future direction of this research work is to improve the coverage and convergence characteristics of NSGA-III algorithm considering integrated with different oppositional strategy, cross over and mutation schemes.

Funding
The authors received no direct funding for this research.

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Citation information
Cite this article as: An NSGA-III algorithm for solving multi-objective economic/environmental dispatch problem, Rajnikant H. Bhesdadiya, Indrajit N. Trivedi, Pradeep Jangir, Narottam Jangir & Arvind Kumar, Cogent Engineering (2016), 3: 1269383.

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Appendix A

(1) FON 2

\[ F = (f_1(x), f_2(x)), \] where

\[ f_1(x) = 1 - \exp \left( -\sum_{i=1}^{n} \left( x_i - \frac{1}{\sqrt{n}} \right)^2 \right), \]

\[ f_2(x) = 1 - \exp \left( -\sum_{i=1}^{n} \left( x_i + \frac{1}{\sqrt{n}} \right)^2 \right), \]

Constraints

\[ -4 \leq x_i \leq 4 \]

(2) KUR

\[ F = (f_1(x), f_2(x)), \] where

\[ f_1(x) = \sum_{i=1}^{n} (-10 e^{-0.2 / \sqrt{x_i^2 + 1}}), \]

\[ f_2(x) = \sum_{i=1}^{n} (-10 e^{-0.2 / \sqrt{x_i^2 + 1}}), \]
\( f_2(x) = \sum_{i=1}^{n} \left( |x_i|^{0.8} + 5 \sin(x_i)^3 \right) \)

### (3) ZDT1

\[ F = (f_1(x), f_2(x)), \text{ where} \]
\[ f_1(x) = x_1, \]
\[ f_2(x, g) = g(x) \left( 1 - \sqrt{\frac{f_1}{g(x)}} \right), \]

and,

\[ g(x) = 1 + \frac{9}{n-1} \cdot \sum_{i=2}^{n} x_i, \]

#### Constraints

\[ 0 \leq x_i \leq 1, \]

\[ n = 30, \ i = 1, 2, \ldots, 30 \]

### (4) ZDT 2

\[ F = (f_1(x), f_2(x)), \text{ where} \]
\[ f_1(x) = x_1, \]
\[ f_2(x, g) = g(x) \left( 1 - \left( \frac{f_1}{g(x)} \right)^{\frac{1}{2}} \right), \]

and,

\[ g(x) = 1 + \frac{9}{n-1} \cdot \sum_{i=2}^{n} x_i, \]

#### Constraints

\[ 0 \leq x_i \leq 1, \]

\[ n = 30, \ i = 1, 2, \ldots, 30 \]

### (5) ZDT 3

\[ F = (f_1(x), f_2(x)), \text{ where} \]
\[ f_1(x) = x_1, \]
\[
f_2(x, g) = g(x) \cdot \left(1 - \sqrt{\frac{f_1}{g(x)}} - \frac{f_1}{g(x)} \cdot \sin(10 \pi f_1)\right),
\]

and,
\[
g(x) = 1 + \frac{9}{n-1} \cdot \sum_{i=2}^{n} x_i,
\]

Constraints

\[0 \leq x_i \leq 1,\]
\[n = 30, \ i = 1, 2, \ldots, 30\]

(6) ZDT 4

\[F = (f_1(x), \ f_2(x)), \text{ where}\]
\[f_1(x) = x_1,\]
\[f_2(x, g) = g(x) \cdot \left(1 - \sqrt{\frac{f_1}{g(x)}}\right),\]

and,
\[
g(x) = 1 + 10(n - 1) + \sum_{i=2}^{n} (x_i^2 - 10 \cos(4 \pi x_i)),
\]

Constraints

\[0 \leq x_i \leq 1,\]
\[-5 \leq x_i \leq 5,\]
\[n = 10, \ i = 1, 2, \ldots, 10\]

(7) ZDT 6

\[F = (f_1(x), \ f_2(x)), \text{ where}\]
\[f_1(x) = 1 - \exp(4x_1) \cdot \sin^6(6\pi x_1),\]
\[f_2(x, g) = g(x) \cdot \left(1 - \left(\frac{f_1}{g(x)}\right)^2\right),\]

and,
\[
g(x) = 1 + \frac{9}{n-1} \left[\frac{\sum_{i=2}^{n} x_i}{9}\right]^{0.25},
\]

Constraints

\[0 \leq x_i \leq 1,\]
\[n = 10, \ i = 1, 2, \ldots, 10\]
(8) BINH

\[ F = (f_1(x, y), f_2(x, y)), \]

where

\[ f_1(x, y) = x^2 + y^2, \]
\[ f_2(x, y) = (x - 5)^2 + (y - 5)^2 \]

Constraints

\[-5 \leq x, y \leq 10\]

(9) VNT 3

\[ F = (f_1(x, y), f_2(x, y), f_3(x, y)), \]

where

\[ f_1(x, y) = 0.5(x^2 + y^2) + \sin(x^2 + y^2), \]
\[ f_2(x, y) = \frac{(3x - 2y + 4)^2}{8} + \frac{(x - y + 1)^2}{27} + 15, \]
\[ f_3(x, y) = \frac{1}{(x^2 + y^2 + 1)} + 1.1e^{-x^2 - y^2} \]

Constraints

\[-3 \leq x, y \leq 3\]

(10) DTLZ 1

\[ F = (f_1(x), f_2(x), f_3(x)), \]

where

\[ f_1(x) = \frac{1}{2}x_1x_2(1 + g(x)), \]
\[ f_2(x) = \frac{1}{2}x_2(1 - x_2)(1 + g(x)) \]
\[ f_3(x) = \frac{1}{2}(1 - x_1)(1 + g(x)) \]

and,

\[ g(x) = 100 \left[ 10 + \sum_{i=3}^{n}(x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5)) \right], \]

Constraints

\[ 0 \leq x_i \leq 1, \]
\[ n = 12, i = 1, 2, ..., 12 \]

(11) DTLZ 2

\[ F = (f_1(x), f_2(x), f_3(x)), \]

where

\[ f_1(x) = \cos(\frac{\pi}{2}x_1) \cos(\frac{\pi}{2}x_2)(1 + g(x)), \]
\[ f_1(x) = \cos\left(\frac{\pi}{2} x_1\right) \sin\left(\frac{\pi}{2} x_2\right) (1 + g(x)), \]
\[ f_2(x) = \sin\left(\frac{\pi}{2} x_1\right) (1 + g(x)) \]
and,
\[ g(x) = \sum_{i=3}^{n} (x_i - 0.5)^2, \]
Constraints
\[ 0 \leq x_i \leq 1, \]
\[ n = 12, \ i = 1, 2, \ldots, 12 \]
(12) DTLZ 3
\[ F = (f_1(x), f_2(x), f_3(x)), \text{ where} \]
\[ f_1(x) = \cos \left(\frac{\pi}{2} x_1\right) \cos \left(\frac{\pi}{2} x_2\right) (1 + g(x)), \]
\[ f_2(x) = \cos \left(\frac{\pi}{2} x_1\right) \sin \left(\frac{\pi}{2} x_2\right) (1 + g(x)), \]
\[ f_3(x) = \sin \left(\frac{\pi}{2} x_1\right) (1 + g(x)) \]
and,
\[ g(x) = 100 \left[10 + \sum_{i=3}^{n} (x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5))\right], \]
Constraints
\[ 0 \leq x_i \leq 1, \]
\[ n = 12, \ i = 1, 2, \ldots, 12 \]

Appendix B
Table B1. Input data for operational generating unit like loading limits and loss parameters of 6-unit system

| Unit | \( p_{\text{min}} \) (MW) | \( p_{\text{max}} \) (MW) | \( a_i \) ($/h) | \( b_i \) ($/MWh) | \( c_i \) ($/MW^2h) | \( \rho_i \) (lb/h) | \( \gamma_i \) (lb/ MW^2h) |
|------|----------------|----------------|---------------|----------------|----------------|----------------|----------------|
| 1    | 10             | 125            | 756.7988      | 38.539         | 0.15247        | 13.8593       | 0.32767        | 0.00419        |
| 2    | 10             | 150            | 451.3251      | 46.1591        | 0.10587        | 13.8593       | 0.32767        | 0.00419        |
| 3    | 35             | 210            | 1,243.531     | 38.3055        | 0.03546        | 40.2669       | −0.54551       | 0.00683        |
| 4    | 35             | 225            | 1,049.998     | 40.3965        | 0.02803        | 40.2669       | −0.54551       | 0.00683        |
| 5    | 125            | 315            | 1,356.659     | 38.2704        | 0.01799        | 42.8955       | −0.51116       | 0.00461        |
| 6    | 130            | 325            | 1,658.57      | 36.3278        | 0.02111        | 42.8955       | −0.51116       | 0.00461        |
Loss parameters:

\[ A_3 = 0 \]

\[ A_1 = [0, 0, 0, 0, 0, 0, 0, 0] \]

\[ A_1 = [0, 0, 0, 0, 0, 0, 0, 0] \]

Table B2. Input data for operational generating unit like loading limits and loss parameters of 10-unit system

| Unit | \( p_{\text{min}} \) (MW) | \( p_{\text{max}} \) (MW) | \( a_i \) ($/h) \) | \( b_i \) ($/MWh) \) | \( c_i \) ($/MW^2h) \) | \( d_i \) ($/h) \) | \( e_i \) (rad/ MW) | \( \beta_i \) (lb/ MWh) | \( \gamma_i \) (lb/ MW^2h) | \( \delta_i \) (lb/h) | \( \omega_i \) (1/ MW) |
|------|---------------|----------------|----------|-----------|----------------|---------|-----------|-----------------|----------------|---------------|-----------------|
| 1    | 10            | 55            | 1,000.40 | 40.5407   | 0.12951       | 33      | 0.0174    | 360.0012        | -3.9864        | 0.04702       | 0.25475         | 0.01234       |
| 2    | 20            | 80            | 950.606  | 39.5804   | 0.10908       | 25      | 0.0178    | 350.0056        | -3.9524        | 0.04652       | 0.25475         | 0.01234       |
| 3    | 47            | 120           | 900.705  | 36.5104   | 0.12511       | 32      | 0.0162    | 330.0056        | -3.9023        | 0.04652       | 0.25163         | 0.01215       |
| 4    | 20            | 130           | 800.705  | 39.5104   | 0.12111       | 30      | 0.0168    | 330.0056        | -3.9023        | 0.04652       | 0.25163         | 0.01215       |
| 5    | 50            | 160           | 756.799  | 38.3055   | 0.03546       | 20      | 0.0152    | 13.8593         | 0.3277         | 0.0068        | 0.2497          | 0.0129        |
| 6    | 70            | 240           | 451.325  | 46.1592   | 0.10587       | 20      | 0.0163    | 13.8593         | 0.3277         | 0.0042        | 0.2497          | 0.012         |
| 7    | 60            | 300           | 1,243.531| 38.3055   | 0.03546       | 20      | 0.0152    | 40.2669         | -0.5455        | 0.0068        | 0.2499          | 0.01203       |
| 8    | 70            | 340           | 1,049.998| 40.3965   | 0.02803       | 30      | 0.0128    | 40.2669         | -0.5455        | 0.0068        | 0.2499          | 0.01203       |
| 9    | 135           | 470           | 1,658.569| 36.3278   | 0.02111       | 60      | 0.0136    | 42.8955         | -0.5112        | 0.0046        | 0.2547          | 0.01234       |
| 10   | 150           | 470           | 1,356.659| 38.2704   | 0.01799       | 40      | 0.0141    | 42.8955         | -0.5112        | 0.0046        | 0.2547          | 0.01234       |

Loss parameters:

\[ A_3 = 0 \]

\[ A_1 = [0, 0, 0, 0, 0, 0, 0, 0] \]

\[ A = \begin{bmatrix}
0.000049 & 0.000014 & 0.000015 & 0.000015 & 0.000016 & 0.000017 & 0.000017 & 0.000018 & 0.000019 & 0.000020 \\
0.000014 & 0.000045 & 0.000016 & 0.000016 & 0.000017 & 0.000015 & 0.000015 & 0.000016 & 0.000018 & 0.000018 \\
0.000015 & 0.000016 & 0.000010 & 0.000010 & 0.000012 & 0.000012 & 0.000014 & 0.000014 & 0.000016 & 0.000016 \\
0.000015 & 0.000016 & 0.000010 & 0.000040 & 0.000014 & 0.000010 & 0.000011 & 0.000012 & 0.000014 & 0.000015 \\
0.000016 & 0.000017 & 0.000012 & 0.000014 & 0.000035 & 0.000011 & 0.000013 & 0.000013 & 0.000015 & 0.000016 \\
0.000017 & 0.000015 & 0.000012 & 0.000010 & 0.000011 & 0.000036 & 0.000012 & 0.000012 & 0.000014 & 0.000015 \\
0.000017 & 0.000015 & 0.000014 & 0.000011 & 0.000013 & 0.000012 & 0.000038 & 0.000016 & 0.000016 & 0.000018 \\
0.000018 & 0.000016 & 0.000014 & 0.000012 & 0.000013 & 0.000012 & 0.000016 & 0.000040 & 0.000015 & 0.000016 \\
0.000019 & 0.000018 & 0.000016 & 0.000014 & 0.000015 & 0.000014 & 0.000016 & 0.000015 & 0.000004 & 0.000019 \\
0.000020 & 0.000018 & 0.000016 & 0.000015 & 0.000016 & 0.000015 & 0.000018 & 0.000016 & 0.000019 & 0.000044
\end{bmatrix} \]
Table B3. Input data for operational generating unit like loading limits of IEEE 118 bus 14-unit system.

| Unit | $P_{\text{min}}^i$ (MW) | $P_{\text{max}}^i$ (MW) | $a_i$ ($$/\text{MW}^2$$) | $b_i$ ($$/\text{MW}$$) | $c_i$ ($) | $\alpha_i$ (ton/MW$^2$) | $\beta_i$ (ton/MW) | $\gamma_i$ (ton) |
|------|-----------------|-----------------|----------------|----------------|-----|----------------|----------------|--------------|
| 1    | 50              | 300             | 0.005          | 1.89           | 150 | 0.016          | −1.5          | 23.333       |
| 2    | 50              | 300             | 0.0055         | 2              | 115 | 0.031          | −1.82         | 21.022       |
| 3    | 50              | 300             | 0.006          | 3.5            | 40  | 0.013          | −1.249        | 22.05        |
| 4    | 50              | 300             | 0.005          | 3.15           | 122 | 0.012          | −1.355        | 22.983       |
| 5    | 50              | 300             | 0.005          | 3.05           | 125 | 0.02           | −1.9          | 21.313       |
| 6    | 50              | 300             | 0.007          | 2.75           | 70  | 0.007          | 0.805         | 21.9         |
| 7    | 50              | 300             | 0.007          | 3.45           | 70  | 0.015          | −1.401        | 23.001       |
| 8    | 50              | 300             | 0.007          | 3.45           | 70  | 0.018          | −1.8          | 24.003       |
| 9    | 50              | 300             | 0.005          | 2.45           | 130 | 0.019          | −2            | 25.121       |
| 10   | 50              | 300             | 0.005          | 2.45           | 130 | 0.012          | −1.36         | 22.99        |
| 11   | 50              | 300             | 0.0055         | 2.35           | 135 | 0.033          | −2.1          | 27.01        |
| 12   | 50              | 300             | 0.0045         | 1.3            | 200 | 0.018          | −1.8          | 25.101       |
| 13   | 50              | 300             | 0.007          | 3.45           | 70  | 0.018          | −1.81         | 24.313       |
| 14   | 50              | 300             | 0.006          | 3.89           | 45  | 0.03           | −1.921        | 27.119       |