NUCLEAR COMPOSITION OF GAMMA-RAY BURST FIREBALLS

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ABSTRACT

We study three processes that shape the nuclear composition of gamma-ray burst (GRB) fireballs: (1) neutronization in the central engine, (2) nucleosynthesis in the fireball as it expands and cools, and (3) spallation of nuclei in subsequent internal shocks. The fireballs are found to have a neutron excess and a marginally successful nucleosynthesis. They are composed of free nucleons, α-particles, and deuterium. A robust result is the survival of a significant neutron component, which has important implications. First, as shown in previous works, neutrons can lead to observable multi-GeV neutrino emission. Second, as we show in an accompanying paper, neutrons impact the explosion dynamics at radii up to $10^{17}$ cm and change the mechanism of the GRB afterglow emission.

Subject headings: accretion, accretion disks — dense matter — gamma rays: bursts — gamma rays: theory — nuclear reactions, nucleosynthesis, abundances

1. INTRODUCTION

Cosmological gamma-ray bursts (GRBs) are powerful explosions in distant galaxies. A physical picture of these phenomena has emerged in the past decade (see Mészáros 2002 for a review): the GRBs are generated by compact, dense, and energetic engines, and they are likely related to black hole formations. The typical mass of the central engine is believed to be a few $M_\odot$, its size is $10^6-10^7$ cm, and its temperature is $kT = 1-10$ MeV. The engine ejects a hot outflow ("fireball") made of free nucleons, $e^\pm$ pairs, trapped blackbody radiation, and magnetic fields.

The initial nuclear composition of the fireball and its evolution during expansion turn out to be crucial for the explosion physics. In particular, neutrons were shown to be an important component in the explosion (Derishev, Kocharovsky, & Kocharovsky 1999), and an extreme case of fireballs with a large neutron excess was studied (Fuller, Pruet, & Fuller 2002a). In an accompanying paper (Beloborodov 2003, hereafter Paper II) we show that the presence of neutrons crucially changes the fireball interaction with an external medium and implies a new mechanism of the afterglow production.

The present paper focuses on processes that shape the nuclear composition of GRB fireballs. In § 2 we study the production of neutrons by the central engine and the resulting neutron-to-proton ratio in the ejected material (scale $R \approx 10^7$ cm). In § 3 we calculate nuclear reactions in the expanding fireball and find abundances of survived free nucleons and synthesized helium ($R = 10^7-10^9$ cm). Similar nucleosynthesis calculations, with different codes, have been done recently by Lemoine (2002) and Pruet, Guiles, & Fuller (2002a). In § 4 we study spallation of helium at later stages when internal motions develop in the fireball ($R = 10^9-10^{12}$ cm). The subsequent dynamics of GRB blast waves with the survived neutron component ($R = 10^{15}-10^{27}$ cm) is investigated in Paper II.

2. NEUTRONIZATION

There are various models for the central engines of GRBs. It may be a young neutron star whose rotational energy is emitted in a magnetized wind, a neutron star merger, or a massive star collapse. The latter two scenarios proceed via the formation of a black hole of mass $M \sim M_\odot$ and subsequent disklike accretion of a comparably massive baryonic component of the ejected fireball is then picked up from the accretion disk.

The central engines are sufficiently dense for the electron capture reaction. Below we calculate at what densities and temperatures this process creates a neutron excess (neutron-to-proton ratio above unity) and then show that GRB engines are likely to satisfy these conditions. We illustrate with the accretion-type models of GRBs, where the matter density can be relatively low and neutronization is most questionable.

2.1. The Equilibrium $Y_e$

Consider a dense, $\rho > 10^7$ g cm$^{-3}$, and hot, $kT > m_e c^2$, matter. The rates of photon emission and absorption are huge, and the matter is filled with Planckian radiation. In addition, the rates of $e^\pm$ pair creation and annihilation ($\gamma + \gamma \rightarrow e^- + e^+$) are huge, and the pairs are in perfect thermodynamic equilibrium with the baryonic matter and radiation. The $e^\pm$ number densities are

$$n_{\pm} = \frac{(m_e c)^3}{\pi^2 \hbar^3} \int_0^{\infty} f_{\pm} \left( \sqrt{p^2 + 1} \right) p^2 dp .$$

Here $p$ is particle momentum in units of $m_e c$ and $f_{\pm}$ is the
are not valid. The site nuclei, and the calculations based on the rates given by eqs. (6) and (7) of \cite{4}. In the shadowed region, the baryonic matter is dominated by com- transparent matter. The dashed line shows the degeneracy temperature (eq. (1)), we have the charge neutrality condition,

$$n_\pm - n_\mp = Y_e \frac{\rho}{m_p},$$

where $Y_e$ is the proton-to-nucleon ratio, which would equal the electron-to-nucleon ratio in the absence of $e^\pm$ pairs. Equations (1) and (3) determine $\mu$ and $n_\pm$ for given $T$, $\rho$, and $Y_e$. The electrons become degenerate if $\mu$ exceeds $\theta$, which happens below the characteristic degeneracy temperature,

$$\theta_{\text{deg}} = \frac{\hbar}{m_e c} \left( \frac{\rho}{m_p} \right)^{1/3}, \quad kT_{\text{deg}} = 7.7 \left( \frac{\rho}{10^{11} \text{ g cm}^{-3}} \right)^{1/3} \text{ MeV}. \quad (4)$$

Degeneracy exponentially suppresses the positron density, $n_+/n_\mp \approx \exp(-\mu/\theta)$, because then $e^\pm$ are created only at energies $E/m_e c^2 > \mu > \theta$, in the exponential tail of the thermal distribution.

At temperatures and densities under consideration, the baryonic matter is in nuclear statistical equilibrium, and it is dominated by free nucleons in the unshadowed region of Figure 1. The boundary of this region has been calculated with the Lattimer-Swesty equation-of-state code (Lattimer & Swesty 1991). The free protons and neutrons can capture $e^-$ and $e^+$ via charged current reactions,

$$e^- + p \rightarrow n + \nu, \quad e^+ + n \rightarrow p + \bar{\nu}. \quad (5)$$

These reactions can rapidly convert protons into neutrons and neutrons back into protons and establish an equilibrium $Y_e = n_p/(n_n + n_p)$, where $n_p$ and $n_n$ are number densities of protons and neutrons, respectively. We now calculate the equilibrium $Y_e(T, \rho)$, and in \S 2.2 we show how it applies to GRB central engines.

The exact equilibrium $Y_e$ depends on whether the opposite reactions—reabsorption of the emitted $\nu$ and $\bar{\nu}$—are also significant. We first consider the $\nu$-transparent case, where reabsorption can be neglected, and then address the $\nu$-opaque case.

2.1.1. Neutrino-transparent Matter

The rates of $e^-$ and $e^+$ capture can be derived from the standard electroweak theory \cite[Shapiro & Teukolsky 1983; Brünn 1985]. We assume the nucleons to be non-degenerate, and then the rates are

$$\dot{n}_{e^-} = K_{n e^-} \int_0^\infty f_{e^-} (\omega + Q) (\omega + Q)^2 \left[ 1 - \frac{1}{(\omega + Q)^2} \right]^{1/2} \omega^2 d\omega, \quad (6)$$

$$\dot{n}_{e^+} = K_{n e^+} \int_{\omega + 1}^\infty f_{e^+} (\omega - Q) (\omega - Q)^2 \left[ 1 - \frac{1}{(\omega - Q)^2} \right]^{1/2} \omega^2 d\omega, \quad (7)$$

where $\omega$ is neutrino energy in units of $m_e c^2$, $Q = (m_n - m_p)/m_e = 2.531$, and $K \approx 6.5 \times 10^{-4} \text{ s}^{-1}$. The constant $K$ can be expressed in terms of the mean lifetime of neutrons with respect to $\beta$-decay, $\tau_\beta = 900 \text{ s}$, as $K \approx (1.7 \tau_\beta)^{-1}$ \cite[Shapiro & Teukolsky 1983, pp. 316 and 524]{5}.

An equilibrium $Y_e$ is established when the rates of $e^-$ and $e^+$ captures are equal,

$$\dot{n}_{e^-} = \dot{n}_{e^+}. \quad (8)$$

Equations (1), (3), and (8) determine $Y_e$ for given $\rho$ and $T$. Contours of the function $Y_e(T, \rho)$ on the $T-\rho$ plane are shown in Figure 1.

The transition from a proton excess ($Y_e > 0.5$) to a neutron excess ($Y_e < 0.5$) takes place in the region of mild degeneracy where $\mu < \theta$. We now focus on this region and derive the equilibrium $Y_e$ analytically. At $\mu < \theta$ and $\theta > Q + 1$ equations (6) and (7) simplify,

$$\dot{n}_{e^-} = K_{n e^-} \theta^4 \frac{45}{2} \zeta(5) + \frac{7\pi^4 (2\mu - Q)}{60 \theta}, \quad (9)$$

$$\dot{n}_{e^+} = K_{n e^+} \theta^4 \frac{45}{2} \zeta(5) - \frac{7\pi^4 (2\mu - Q)}{60 \theta}, \quad (10)$$

The neutron decay $n \rightarrow p + e^- + \bar{\nu}$ is a slow process on GRB time-scales and is neglected here.

4. In the $\nu$-transparent regime, neutrinos are sometimes prescribed a zero chemical potential, and the balance $\mu + \mu_e = \mu_n + \mu_p$ is used with $\mu_e = 0$ to determine the equilibrium $Y_e$. In fact, the balance of chemical potentials does not hold because the neutrinos are out of thermodynamic equilibrium. This balance would be valid only in the cold limit $T \rightarrow 0$ \cite[Landau & Lifshitz 1980]{6}.
where $\zeta(5) = 1.037$ is the Riemann $\zeta$-function. We neglected here next-order terms $O(\theta^2), O(\mu^2)$, and $O(1 + \theta^2)$ and used the formula $\int_0^\infty e^x = (1 - 2^{-x}) \times \Gamma(n + 1) \zeta(n + 1) + \Gamma(n + 1) = n!$ for integer $n$. Equating the two rates, we get the equilibrium $Y_e = n_p/(n_n + n_p)$,

$$Y_e = \frac{1}{2} + \frac{7\pi^4}{1350\zeta(5)} \left( \frac{Q}{2 - \mu} \right) \theta = \frac{1}{2} + \frac{4.87}{\theta} \left( \frac{Q}{2 - \mu} \right).$$

(11)

In the nondegenerate limit, $\mu/\theta \to 0$, this gives $Y_e = 0.5 + 3.616/\theta > 0.5$ and implies a proton excess, which is due to the positive difference $Q$ between the neutron and proton masses. A very mild degeneracy $\mu < Q/\theta < \theta$ is sufficient to drive $Y_e$ below 0.5. This happens because the $e^+$ density is reduced by the degeneracy effects and the $e^-$ capture becomes preferential.

It is instructive to write the $e^-$ and $e^+$ densities using the linear expansion of equation (1) in $\mu/\theta$,

$$n_\pm = \frac{1}{2} \left[ \frac{3}{2} \zeta(3) \theta^3 + \frac{\pi^2}{6} \theta^2 \right], \quad \mu < \theta,$$

(12)

where $\lambda = \hbar/m_e c = 3.862 \times 10^{-11}$ cm and $\zeta(3) = 1.202$. Then equation (3) yields

$$\mu = 3Y_e \lambda^3 \rho / m_p \theta^2 \theta.$$  

(13)

The condition $\mu > Q/2$ that defines the neutron-excess region on the $T-\rho$ plane ($Y_e < 0.5$) can now be written as $\theta < \theta_n(\rho)$, where

$$\theta_n = \left( \frac{3\lambda^3 \rho}{Q m_p} \right)^{1/2}, \quad kT_n = 33 \rho^{1/2} \text{ MeV}.$$  

(14)

This simple formula perfectly coincides with the numerical results (Fig. 1). That a very mild electron degeneracy is sufficient to create a neutron excess can also be seen by comparing $T_n$ with $T_{deg}$,

$$\frac{T_n}{T_{deg}} \approx 4.3 \rho^{1/6}.$$  

(15)

A useful explicit formula for the equilibrium $Y_e(T, \rho)$ in $\nu$-transparent matter with mild degeneracy is derived from equations (11) and (13),

$$Y_e(\theta, \rho) = \frac{1}{2} \left[ 1 + \frac{4.87}{\theta} \right] \left( 1 + \frac{1.46 \lambda^3 \rho}{m_p \theta^2} \right)^{-1}. $$

(16)

It agrees with the numerical calculations shown in Figure 1 with a high accuracy, $\delta Y_e/Y_e < 1\%$ at $Y_e > 0.35$. In a more degenerate region, where the equilibrium $Y_e < 0.35$, the formula can still be used, although its error increases to 10% at $Y_e = 0.2$ and 30% at $Y_e = 0.1$.

2.1.2 Neutrino-opaque Matter

If the matter is opaque to the emitted neutrinos, a complete thermodynamic equilibrium is established. A detailed balance now holds, $e^- + p \leftrightarrow n + \nu$ and $e^+ + n \leftrightarrow p + \bar{\nu}$, and the equilibrium $Y_e$ is determined by the condition

$$\mu + \mu_p = \mu_n + \mu_\nu,$$

(17)

where $\mu$, $\mu_p$, $\mu_n$, and $\mu_\nu$ are chemical potentials (in units of $m_e c^2$) of the electrons, protons, neutrons, and neutrinos, respectively, all including the particle rest-mass energy. The antineutrinos have chemical potential $\mu_\nu = -\mu_\nu$, so that $\mu_p + \mu_n = \mu_p + \mu_\nu$ is also satisfied. The neutrons and protons have Maxwellian distributions, which gives $n_p n_p^1 = \exp[(\mu_p - \mu_n - Q)/kT] \exp[-(\mu_p - m_p c^2 + m_\nu)$

$$\mu_n - \mu_p = \theta \log \left( \frac{n_n}{n_p} + Q \right).$$

(18)

The thermalized $\nu$ and $\bar{\nu}$ obey Fermi-Dirac statistics, and they are described in the same way as $e^\pm$ (see eqs. [1] and [2]); the only difference is that the statistical weight of energy states is one for neutrinos and two for electrons. The neutrino chemical potential vanishes if $\nu$ and $\bar{\nu}$ have equal densities, $n_\nu = n_p$. If, however, the matter emits nonequal numbers of $\nu$ and $\bar{\nu}$ (its $Y_e$ is changing), then $n_\nu \neq n_p$ and $\mu_\nu \neq 0$.

Suppose $N_\nu$ neutrinos and $N_\nu$ antineutrinos have been emitted per nucleon. This causes a change of $Y_e$,

$$Y_e - Y_e^0 = N_\nu - N_\nu,$$

(19)

where $Y_e^0$ is an initial value that the matter had before the neutrino emission. If all the emitted neutrinos remain trapped in the matter, then $(n_\nu - n_p)/n_p = Y_e - Y_e^0$, where $n_\nu = n_n + n_p$ is the total nucleon density. If a fraction $\chi$ of the emitted neutrinos diffused out of the matter, then

$$n_\nu - n_p = (1 - \chi) \left( Y_e - Y_e^0 \right)$$

(20)

Let us first consider the case $\chi \to 1$ (efficient neutrino cooling). Then $|n_\nu - n_p| \ll n_\nu - n_\nu$ and the neutrino chemical potential can be neglected compared to that of the electrons. Thus, the chemical equilibrium in $\nu$-cooled, $\nu$-opaque matter reads

$$\mu = \theta \log \left( \frac{n_n}{n_p} + Q \right).$$

(21)

Equation (21) combined with equations (1) and (3) determines an equilibrium $Y_e$ as a function of $T$ and $\rho$. Note however that equation (21) assumes free nucleons, which is invalid in the shadowed region of the $T-\rho$ plane shown in Figure 1 (the boundary of this region is also plotted in Fig. 2). The chemical balance can be extended to this region if $\mu_p$ and $\mu_n$ are corrected for the heavy nuclei formation, which we do using the Lattimer-Sweaty equation-of-state code. Then we find the equilibrium $Y_e$ that satisfies $\mu = \mu_n - \mu_p$. The results are shown in Figure 2.

The neutron excess boundary, $Y_e = 0.5$, lies in the region where all nucleons are free and the electrons are mildly degenerate, $\mu < \theta$. Equation (21) applies here and shows that this boundary is defined by $\mu = Q$. Using the linear expansion in $\mu/\theta$ (eq. [13]), we get a simple equation for $Y_e(\theta, \rho)$,

$$\frac{3Y_e \rho \lambda^3}{Q m_p} = \theta \log \left( 1 - \frac{Y_e}{Y_e^0} \right) + Q.$$  

(22)

In particular, one sees that $Y_e = 0.5$ corresponds to

$$\theta_n = \left( \frac{3\lambda^3 \rho}{Q m_p} \right)^{1/2}, \quad kT_n = 23.1 \rho^{1/2} \text{ MeV}.$$  

(23)
opaque matter with cally found contour Ye. The thick solid curve is the boundary of the free-nucleon region (same as in Fig. 1). Above this curve, the composite nuclei dominate, and therefore the contours Ye = const bend upward. The analytically calculated boundary Ye = 0.5 (eq. [23]) is shown by a dotted line; it coincides with the numerically found contour Ye = 0.5.

It coincides exactly with the contour Ye = 0.5 calculated with the Lattimer-Swesty code (Fig. 2). Note also that \( T_n = 3T_{\text{deg}}\rho^{1/6} \).

Finally, we address the regime where the neutrinos are not only thermalized but also trapped in the matter (x ≈ 0 in eq. [20]). It can happen in GRB accretion flows with high accretion rates, \( M > 1 M_\odot \, s^{-1} \) (e.g., Di Matteo, Perna, & Narayan 2002). Then \( \mu_\nu \) should not be neglected in the chemical balance. Like equation (13) we derive for neutrinos

\[
\mu_\nu = \frac{6\lambda^3(n_\nu - \nu_\nu)}{\theta^2}
\]

and substitute \( n_\nu - \nu_\nu = (Y_e - Y_e^0)(\rho/m_p) \). The chemical balance \( \mu - \mu_\nu = \mu_n - \mu_p \) now reads

\[
\frac{3(2Y_e^0 - Y_e)\rho\lambda^3}{m_p} = \theta \log \left( \frac{1 - Y_e}{Y_e} \right) + Q,
\]

and \( Y_e = 0.5 \) corresponds to

\[
\theta_n = \left[ \frac{3(2Y_e^0 - 0.5)\lambda^3}{Qm_p} \right]^{1/2}.
\]

At \( Y_e^0 = 0.5 \) it coincides with equation (23) as it should: \( Y_e = 0.5 \) requires that \( \nu \) and \( \bar{\nu} \) are emitted in equal numbers and \( \mu_\nu = 0 \).

2.1.3. Effects of Magnetic Fields

Magnetic fields have been neglected in the above calculations, which is a valid approximation as long as the field does not affect the particle distribution functions. Magnetic fields can be generated in the GRB central engines by dynamo, and their energy is likely a fraction \( \epsilon_B \) of the total energy density \( w \), so that

\[
B = \sqrt{8\pi\epsilon_B w}.
\]

Here \( w \) includes the energy of baryons, radiation, and \( e^\pm \); it also includes the neutrino energy if the disk is \( \nu \)-opaque. The field has the strongest effect on light charged particles—\( e^\pm \). It introduces the discrete energy levels (Landau & Lifshitz 1980),

\[
\frac{E_j}{m_e c^2} = \left( 1 + p_z^2 + 2\frac{\hbar\Omega_B}{m_e c^2} \right)^{1/2}, \quad j = 0, 1, \ldots , \tag{28}
\]

where \(-\infty < p_z < \infty \) is the component of the electron momentum parallel to the field and \( \Omega_B = eB/m_e c \). The magnetic field also changes the phase-space factor in equation (1): \( p^2 dp \) is replaced by \( (\hbar\Omega_B/m_e c^2)(dp_z/2) \). Both effects are important if \( (\hbar\Omega_B/m_e c^2) > p_z^2 \). The mean parallel momentum of the relativistic \( e^\pm \) equals \( \sqrt{3kT}/c \), and the condition for the field effects to be important reads

\[
\frac{\hbar\Omega_B}{m_e c^2} > 3\theta^2.
\]

For any plausible \( \epsilon_B < 1 \), the magnetic field is important only where the electrons are degenerate, and the energy density \( w \) is dominated by either baryons, \( w_b = (3/2)kT \times \rho/m_p \), or degenerate electrons, \( w_{\text{deg}} = (9/4)(\pi/3)^{1/3}h c \times (Y_e\rho/m_p)^{1/3} \). Equation (29) can be written as

\[
\theta < \left( \frac{\rho\lambda^3}{m_p} \right)^{1/3} \left\{ \begin{array}{ll}
\left( \frac{4\pi\alpha_f\epsilon_B}{3} \right)^{1/3} \quad \epsilon_B > \left( \frac{\pi^3 Y_e^4}{32\alpha_f} \right)^{1/3}, \\
\left( \frac{\pi}{3} \right)^{7/12} \left( 6\alpha_f\epsilon_B \right)^{1/4} Y_e^{1/3}, \quad \epsilon_B < \left( \frac{\pi^3 Y_e^4}{32\alpha_f} \right)^{1/3},
\end{array} \right. \tag{30}
\]

where \( \alpha_f \) is the fine-structure constant. The upper line corresponds to \( w_b > w_{\text{deg}} \), and the lower line to \( w_{\text{deg}} > w_b \).

2.2. GRB Central Engines

There exists a general constraint on the electron degeneracy in the engine. Its derivation makes use of two facts: (1) The engine is gravitationally bound; otherwise, it would explode, and a high baryon contamination of the fireball would be inevitable. GRB models normally envision a quasi-static engine that liberates a fraction of its gravitational binding energy and passes it to a tiny amount of mass outside the engine, thus creating a highly relativistic outflow. (2) The engine is compact (size \( r < 10^7 \) cm), and it has a relativistic blackbody temperature \( \theta = kT/m_e c^2 > 1 \).

The sound speed in a gravitationally bound object of size \( r \) and mass \( M \) must be smaller than \( (GM/r)^{1/2} \). It gives the constraint

\[
P = \frac{0.1c^2}{3}\left( \frac{r}{3r_s} \right)^{-1}, \tag{31}
\]

where \( r_s = 2GM/c^2 \) is the gravitational radius of the object. Pressure \( P = P_\gamma + P_b + P_\nu + P_\beta \) includes contributions from radiation, \( e^\pm \), neutrinos, and baryons. We have \( P_\gamma + P_b = (11/12)\alpha T^4 \) if the \( e^\pm \) are weakly degenerate and a maximum \( P_\nu = (7/24)\alpha T^4 \) for each neutrino species if it is thermalized, where \( \alpha = (\pi^2k^4/15\hbar^3c^3) = 7.56 \times 10^{-15} \) ergs
cm$^{-3}$ K$^{-4}$ is the radiation constant. Approximately,

$$P \approx aT^4 + \frac{\rho}{m_p}kT.$$  \hspace{1cm} (32)

When comparing the two terms in equation (32), it is easy to see that

$$\frac{aT^4}{(\rho/m_p)kT} = \frac{\pi^2}{15} \frac{\theta^3}{\theta_{\text{deg}}^2}.$$ \hspace{1cm} (33)

Hence, the baryonic pressure gets dominant at $\theta \approx \theta_{\text{deg}}$ and at even lower temperatures, $\theta < (\pi/4)\theta_{\text{deg}}$, the pressure of degenerate electrons $P_{\text{deg}} = \frac{3}{4}(\pi/3)^3/\hbar^2(Y_e\rho/m_p)^{4/3}$ takes over. Our goal is to derive an upper bound on $\theta/\theta_{\text{deg}}$; therefore, we consider $\theta \ll \theta_{\text{deg}}$ with $P \approx aT^4$. Equation (31) then reads

$$\frac{\theta}{\theta_{\text{deg}}} \lesssim 2\rho_\text{crit}^{-1/12} \left(\frac{r}{3r_g}\right)^{-1/4},$$ \hspace{1cm} (34)

which also gives a lower bound on the electron chemical potential (eq. [13]),

$$\frac{\mu}{Q} \lesssim 5Y_e\rho_\text{crit}^{1/2} \left(\frac{r}{3r_g}\right)^{1/2}.$$ \hspace{1cm} (35)

We know from § 2.1 that the condition $Y_e < 0.5$ reads $\mu > Q/2$ for $\nu$-transparent matter and $\mu > Q$ for $\nu$-opaque matter. We now see that any engine of size $r > 10^{10}l_s/3r_g$ g cm$^{-3}$ satisfies this condition and tends to an equilibrium $Y_e < 0.5$. This is evidently the case in models of neutron star mergers (Ruffert et al. 1997), as well as magnetized neutron stars. The collapsar scenario (MacFadyen & Woosley 1999) invokes a relatively low density accretion flow, and here neutronization is questionable. We therefore shall study accretion flows in more detail. We also need to check whether the equilibrium $Y_e$ is achieved on the accretion timescale.

2.2.1. GRB Accretion Flows

All accretion models of GRBs invoke rotation that creates a funnel along which the fireball can escape. The accretion flow is viewed as a rotating disk maintained in hydrostatic balance in the vertical direction, which gradually spirals to the central black hole. A standard model assumes turbulent viscosity in the disk with a stress tensor $W_{\phi\phi} = \alpha c_s^2$, where $c_s = (P/\rho)^{1/2}$ is the isothermal sound speed and $\alpha = 0.01 - 0.1$ (Balbus & Hawley 1998). The same stress tensor can be described in terms of a viscosity coefficient $\nu = \frac{3}{2}\alpha c_s H$, where $H$ is the half-thickness of the disk.

The standard disk theory gives the velocity of accretion at radius $r$

$$u^r = \frac{W_{\phi\phi}}{\Omega_K r^8}.$$ \hspace{1cm} (36)

where $\Omega_K(r) = (GM/r^3)^{1/2}$ is the angular velocity of Keplerian rotation, $M$ is the black hole mass, and $r_{\text{in}}$ is the inner boundary of the disk (the marginally stable orbit); $r_{\text{in}} = 3r_g$ for nonrotating and $r_{\text{in}} = r_g/2$ for extremely rotating black holes.\(^6\) $S(r)$ varies from 0 to 1, and it equals 0.5 at a characteristic radius $r = 4r_{\text{in}}$ where the integrated dissipation rate peaks ($4\pi Hr^2\dot{q}^+ \propto S/r$; see eq. [40]). One expects rotating black holes in GRBs, and $r \approx 3r_g$ is a reasonable characteristic radius.

The vertical hydrostatic balance reads $c_s = H\Omega_K$, which we use to write the accretion time as

$$t_a(r) = \frac{r^2}{\mu} = \frac{S}{\alpha\Omega_K H}r^{-2}$$

$$= 2.9 \times 10^{-3} S \left(\frac{2H}{r}\right)^{-2} \left(\frac{\alpha}{0.1}\right)^{-1} \left(\frac{r}{3r_g}\right)^{3/2} \left(\frac{M}{M_\odot}\right) \text{s}.$$ \hspace{1cm} (37)

We neglect here the disk gravity, which is a valid approximation as long as $M_{\text{tot}} \approx M$, where $M$ is the accretion rate. The typical $t_a$ is much shorter than the burst duration, and accretion is viewed as a quasi-steady process. It can power a relativistic outflow (“fireball”) with luminosity

$$L = \epsilon\dot{M}c^2 \approx 10^{51} \left(\frac{\epsilon}{0.01}\right) \left(\frac{\dot{M}}{10^{32} \text{ g s}^{-1}}\right) \text{ ergs s}^{-1},$$ \hspace{1cm} (38)

where $\epsilon$ is the efficiency of $\dot{M}c^2$ conversion into a fireball, which is below the net efficiency of accretion ($\epsilon \sim 0.1$). Most of the accretion energy is either carried away by neutrinos or advected by the accretion flow (in the case of small neutrino losses).

The disk baryonic density is given by

$$\rho = \frac{M_{\text{tot}}}{4\pi r^2 H}$$

$$\approx 6.6 \times 10^{10} S M_{32} \left(\frac{2H}{r}\right)^{-3} \left(\frac{r}{3r_g}\right)^{-3/2} \times \left(\frac{\alpha}{0.1}\right)^{-1} \left(\frac{M}{M_\odot}\right)^{-2} \text{ g cm}^{-3},$$ \hspace{1cm} (39)

and it is heated viscously with rate

$$\dot{q}^+ = \frac{3\Omega_K^2}{8\pi H} S$$

$$\approx 5.1 \times 10^{33} S M_{32} \left(\frac{2H}{r}\right)^{-1} \left(\frac{r}{3r_g}\right)^{-4} \times \left(\frac{M}{M_\odot}\right)^{-3} \text{ ergs cm}^{-3} \text{ s}^{-1}.$$ \hspace{1cm} (40)

The flow has a huge optical depth for radiation and the radiation is trapped: its diffusion is negligible on the accretion timescale. The only cooling mechanism of the flow is neutrino emission, which becomes efficient at $\dot{\epsilon}_\nu \approx \dot{\epsilon}_\nu$ (e.g., Popham, Woosley, & Fryer 1999; Narayan, Piran, & Kumar 2001; Kohri & Mineshige 2002). An upper bound on the temperature is derived from the assumption that the neutrino cooling is absent. Then the flow does not lose the dissipated energy and instead traps it and advects. Its energy density can be estimated as $w_{\text{max}} \approx \dot{q}^+ t_a \approx \frac{1}{2}(r_g/r)c^2$ (we use eqs. [37] and [40] with $S = 0.5$). The internal energy of such a hot advective flow is dominated by radiation and $e^\pm$, so

$$w_{\text{max}} = \frac{11}{4} aT_{\text{max}}^4 \approx 3r_g \rho c^2.$$ \hspace{1cm} (41)

\(^6\) We give here simple Newtonian estimates and trace the hole spin only through its effect on $r_{\text{in}}$; other relativistic corrections are modest and weakly affect the results.
where factor 11/4 accounts for the contribution of relativistic weakly degenerate \( e^\pm \) (neutrino contribution can further increase this factor if the neutrinos are reabsorbed, and then \( T_{\text{max}} \) will be slightly lower). Equation (41) yields

\[
k_T = 13 \rho_1^{1/4} \left( \frac{r}{3 \rho_b} \right)^{-1/4} \text{MeV.} \quad (42)
\]

The actual temperature can be significantly lower if the neutrino cooling is significant. The main cooling process is \( e^\pm \) capture on nucleons: \( e^+ + p \rightarrow n + \nu \) and \( e^\pm + n \rightarrow p + \bar{\nu} \), which also shapes \( Y_e \) as we discussed in § 2.1. We now evaluate the characteristic accretion rate \( M_{\text{eq}} \) above which the \( e^\pm \) capture is rapid enough to establish an equilibrium \( Y_e \).

The equilibrium \( Y_e \) is achieved when the flow has emitted one neutrino per nucleon. It is easy to see that disks with efficient neutrino cooling must reach the equilibrium. Indeed, the mean energy of the emitted neutrinos, \( E_n \approx 50 T_e \), is below the liberated accretion energy per nucleon, \( E_n = 100-300 \text{ MeV} \), and an efficient cooling implies that more than one neutrino per nucleon is produced. Thus, \( M_{\text{eq}} \) should be looked for in the inefficient (advective) regime with \( T \approx T_{\text{max}} \). Such a flow is only mildly degenerate, and the rates of \( e^\pm \) capture read (in zero order in \( \mu/\theta \))

\[
n_{e^-} \approx 1.5 \times 10^{-2} n_p \theta^5 \text{ cm}^{-3} \text{ s}^{-1}, \quad (43)
\]

\[
n_{e^+} \approx 1.5 \times 10^{-2} n_p \theta^5 \text{ cm}^{-3} \text{ s}^{-1}. \quad (44)
\]

The neutronization timescale is \( t_n = n_p / n_{e^-} \approx 7 \times 10^7 \text{ s} \), and it should be compared with the accretion timescale \( t_a \) (eq. [37]),

\[
t_a \approx 70 \frac{\alpha \Omega K}{S_0 \theta} \left( \frac{H}{r} \right)^2. \quad (45)
\]

We substitute \( T = T_{\text{max}} \), take into account the hydrostatic balance

\[
\frac{H}{r} = \left( \frac{w_{\text{max}}}{3 \rho} \right)^{1/2} \frac{1}{\Omega K} = \frac{1}{2}, \quad (46)
\]

and use equation (39) with \( S = 0.5 \) to get

\[
t_n \approx 1.7 \times 10^{-2} M_{\text{eq}}^{5/4} \left( \frac{r}{3 \rho_b} \right)^{13/8} \left( \frac{\alpha}{0.1} \right)^{9/4} \left( \frac{M}{M_\odot} \right)^{3/2}. \quad (47)
\]

We conclude that disks with

\[
M > M_{\text{eq}} = 3.8 \times 10^{30} \left( \frac{r}{3 \rho_b} \right)^{13/10} \left( \frac{\alpha}{0.1} \right)^{9/5} \left( \frac{M}{M_\odot} \right)^{6/5} \text{ g s}^{-1} \quad (48)
\]

have \( t_n < t_a \) and hence approach the equilibrium \( Y_e \). Radius \( r \) entered this expression in the 13/10 power, which implies that the characteristic \( M_{\text{eq}} \) depends significantly on the black hole spin: the characteristic radius (where viscous dissipation peaks) decreases from \( r \sim 10 \rho_b \) to \( r \sim r_g \) as the hole spin increases from zero to a maximum value.

The equilibrium \( Y_e \) is below 0.5 if the flow temperature is below \( T_n \), which was calculated in § 2.1 (eqs. [14] and [23]). It is instructive to compare \( T_n \) with the maximum accretion temperature \( T_{\text{max}} \) (eq. [42]),

\[
T_n = \kappa \rho_{11} \left( \frac{r}{3 \rho_b} \right)^{1/4}, \quad (49)
\]

where \( \kappa = 2.5 \) for \( \nu \)-transparent and \( \kappa = 1.8 \) for \( \nu \)-opaque flows. The condition for a neutron excess, \( T < T_n \), is most difficult to satisfy in low-\( M \) flows where the neutrino cooling is inefficient and \( T \approx T_{\text{max}} \). Such flows are \( \nu \)-transparent, so \( \kappa = 2.5 \) should be used in equation (49). Substituting equation (39), \( T = T_{\text{max}} \), and \( H/r = 1/2 \), we find that \( T_{\text{max}} < T_n \) if

\[
M > M_n = 7.6 \times 10^{30} \left( \frac{r}{3 \rho_b} \right)^{1/2} \left( \frac{\alpha}{0.1} \right) \left( \frac{M}{M_\odot} \right)^2 \text{ g s}^{-1}. \quad (50)
\]

The “neutronization” accretion rate \( M_{\text{eq}} \) is comparable to \( M_{\text{eq}} \). Plausible \( M \) in GRB accretion flows are \( 10^{32} \text{ g s}^{-1} \) and higher, and they should have a neutron excess.

### 2.3. Deneutronization in the Fireball?

Are outflows from neutron-rich engines also neutron-rich? The fireball picks up baryons from the surface of the central engine, and the surface density is relatively low. \( Y_e \) might change while the matter is escaping into a fireball.

Let us consider fireballs produced by accretion disks. The disk is turbulent (the turbulence is the source of viscosity that is responsible for accretion), and it is mixed in the vertical direction on the sound crossing timescale \( t_{\text{mix}} \approx H/\nu_s \) (the turbulent velocity is somewhat smaller than \( \nu_s \); however, the thickness of surface layers is also smaller than \( H \), and \( t_{\text{mix}} \approx H/\nu_s \) is about right). Given the hydrostatic balance, \( H = \nu_s / \Omega K \), one gets

\[
t_{\text{mix}} \approx \Omega K^{-1}. \quad (51)
\]

The turbulent matter circulates rapidly up to the surface and back to the interior of the disk, and a small portion of it can (also rapidly) escape in each circulation. The escape timescale of the fireball is comparable to \( \Omega K^{-1} \). As an element of matter elevates to the surface its \( Y_e \) would increase if it adjusted instantaneously to a new equilibrium value. However, the time \( \Omega K^{-1} \) the element has before it sinks back into the disk (or escapes) can be too short for the adjustment. Then \( Y_e \) of the escaping matter corresponds to \( \rho \) and \( T \) inside the disk, where it has spent almost all the time before the sudden escape.

To check this picture, let us evaluate the timescale of “deneutronization” of an initially neutron-rich material that has suddenly expanded into a low-density, hot fireball. Neutrons tend to convert back into protons via two charged current reactions: \( e^+ \) capture and \( \nu \) absorption (beta-decay is slow and negligible). The fireball has temperature \( \theta > 1 \) and nondegenerate electrons, \( \mu \ll \theta \). Equation (44) then gives the \( e^+ \) capture timescale

\[
t_e = \frac{n_e}{n_{e^-}} \approx 70 \frac{\theta^5}{\rho^5} \text{ s}, \quad (52)
\]

\[
\frac{1}{t_e} \Omega K = \left( \frac{kT}{8 \text{ MeV}} \right)^{-5} \left( \frac{r}{3 \rho_b} \right)^{-3/2} \left( \frac{M}{M_\odot} \right)^{-1}. \quad (53)
\]

For fireball temperatures up to 8 MeV the \( e^+ \) capture is slow compared to \( \Omega K^{-1} \) and does not affect \( Y_e \) of the escaping material.

The \( \nu \) absorption by neutrons is potentially more important for deneutronization. The cross section of this reaction
is
\[ \sigma_\nu(\omega) = 2.4 \times 10^{-44} [1 - f(\omega + Q)] \]
\[ \times (\omega + Q)^2 \left[ 1 - \frac{1}{(\omega + Q)^2} \right]^{1/2} \text{cm}^2 \]
\[ \approx 2.4 \times 10^{-44} \omega^2 \text{cm}^2, \quad (54) \]
and the corresponding rate of deneutronization is
\[ \dot{n}_n = -cn_n \int_0^\infty n_\nu \sigma_\nu d\omega, \quad (55) \]
where \( n_\nu = dn_\nu/d\omega \) is the spectrum of the neutrino number density. This gives the timescale of deneutronization,
\[ t_\nu = \frac{n_n}{\dot{n}_n} = \frac{1.4 \times 10^{33}}{n_\nu \omega^2} \text{ s}, \quad (56) \]
where the bar signifies an average over a neutrino spectrum. An upper bound on the neutrino density outside the disk is given by
\[ n_\nu < \frac{3M_M \Omega_{\nu} S}{8 \omega n_\nu e^3}, \quad (57) \]
which states that the energy flux of the (electron) neutrino from the two faces of the disk, \( 2F_\nu \approx 2n_\nu \omega n_\nu e^3/\pi \), is smaller than the accretion energy released per unit time per unit area of the disk, \( F = (3/4\pi)GMMS/M^4 \). Thus, we get (with \( S \sim 0.5 \))
\[ t_\nu \Omega_{\nu} > \frac{2}{\omega^2} \frac{M_{\odot}^{1/2}}{M_M^{3/2}} \left( \frac{M}{M_\odot} \right) \left( \frac{r}{3\theta_\nu} \right)^{3/2}. \quad (58) \]
The mean \( \omega \) and \( \omega^2 \) are determined by the state of material that emits the neutrinos. If the disk is \( \nu \)-transparent, the neutrinos have the spectrum (see eq. [6])
\[ n_\nu \propto (\omega + Q)^2 \left[ \exp \left( \frac{\omega + Q - \mu}{\theta} \right) + 1 \right]^{-1} \left[ 1 - \frac{1}{(\omega + Q)^2} \right]^{1/2} \omega^2 \]
\[ \approx \omega^4 \left[ \exp \left( \frac{\omega}{\theta} \right) + 1 \right]^{-1}, \quad (59) \]
where \( \theta \) is temperature inside the disk and \( Q \) and \( \mu \) have been neglected compared to the typical \( \omega = \omega_0 \). This gives \( \omega = (31/6)\zeta(6)/\zeta(5) \approx 5.07\theta \) and \( \omega^2 = (63/2)\zeta(7)/\zeta(5) \approx 30.6\theta^2 \).

The optical depth of the disk for neutrino absorption by neutrons is\(^7\)
\[ \tau_\nu \approx \frac{M_{\odot}^{1/2}}{5\theta} \left( \frac{\omega}{T_{\text{max}}} \right)^{-5} \left( \frac{r}{3\theta_\nu} \right)^{-7/4} \left( \frac{\alpha}{0.1} \right)^{-3/2} \left( \frac{M}{M_\odot} \right)^{-2}. \quad (60) \]

Here we assumed \( T > T_{\text{dep}} \), so that \( \omega_0 \neq 0 \). and \( 2H/r = (T/T_{\text{max}})^4 \). From equations (58) and (60) one can see that \( \nu \)-transparent disks (\( \tau_\nu < 1 \)) have \( t_\nu \Omega_{\nu} > 1 \) for \( \alpha > 0.01 \), and hence their fireballs do not experience any significant absorption of neutrinos.

\(^7\)Absorption makes a major contribution to the disk opacity for neutrinos.

High-\( M \) disks with neutrino luminosity \( L_\nu > 10^{53} \text{ ergs s}^{-1} \) create a sufficiently dense bath of neutrinos that can impact \( Y_e \) of the ejected fireball. High-\( M \) disks are \( \nu \)-opaque, and the emitted neutrinos can be approximated by a blackbody spectrum with \( \omega \approx (7/2)\zeta(4)/\zeta(3) \theta_\nu \approx 3.15\theta_\nu \) and \( \omega^2 \approx 15\zeta(5)/\zeta(3) \theta_\nu^2 \approx 12.9\theta_\nu^2 \), where \( \theta_\nu \) is the temperature of the \( \nu \) photons in the disk. The disk also emits \( \nu \) from a corresponding \( \bar{\nu} \) photosphere. Both \( \nu \) absorption by neutrons and \( \bar{\nu} \) absorption by protons occur at the base of the fireball and set a new equilibrium \( Y_e \) such that the rates of \( \nu \) and \( \bar{\nu} \) absorptions are equal. One can expect the new \( Y_e \) to be below 0.5 (see Qian et al. 1993, where a similar problem is discussed in the context of supernova engines, and Pruet, Fuller, & Cardall 2001). This expectation is based on two facts: (1) the number densities of \( \nu \) and \( \bar{\nu} \) are approximately equal, \( |n_\nu - n_{\bar{\nu}}| \ll n_{\bar{\nu}} \); otherwise, they carry away too large a leptonic number from the disk; and (2) the absorption cross section is proportional to neutrino energy squared (in the limit \( \omega \gg Q \), which is valid for the hot high-\( M \) disks). It gives the equilibrium \( n_n/n_{\bar{\nu}} \approx \theta_\nu^2/\theta_\nu^2 \), where \( \theta_\nu \) and \( \theta_\nu \) are temperatures of the \( \nu \) and \( \bar{\nu} \) photospheres. One expects \( \theta_\nu < \theta_\nu \) because the \( \nu \) photosphere is likely closer to the disk surface (the neutronized disk is more opaque for \( \nu \)) and then \( n_n/n_{\bar{\nu}} > 1 \). This argument assumes, however, that the disk temperature decreases toward its surface. It may not hold if the dissipation rate peaks at the surface and makes it hotter than the interior of the disk.

3. NUCLEOSYNTHESIS

The ejected fireball has a low baryon loading and a high temperature, and its nucleons are initially in the free state. As expansion proceeds, the fireball cools adiabatically, and when its temperature decreases to \( kT_\nu \sim 100 \text{ keV} \), fusion reactions shape the nuclear composition like they do during the primordial nucleosynthesis in the universe (Wagoner, Fowler, & Hoyle 1967). In both cases, we deal with an expanding blackbody fireball with initially free nucleons, and the following three parameters control the outcome of nucleosynthesis: (1) photon-to-baryon ratio \( \phi = n_\gamma/n_b \), (2) expansion timescale \( \tau_\nu \) at the time of nucleosynthesis, and (3) \( n/p \) ratio prior to the onset of nucleosynthesis. In the universe, \( \phi \approx 3 \times 10^6 \), \( \tau_\nu \approx 10^2 \text{ s} \), and \( n_\gamma/n_p \approx 1 \). Below we formulate the nucleosynthesis problem in GRBs and give a qualitative comparison with the big bang. Then we make detailed calculations of nuclear reactions in GRB fireballs.

3.1. Fireball Model

A customary GRB model envisions a central engine that ejects baryonic matter at a rate \( M_\text{b} \) (g s\(^{-1}\)), thermal energy at a rate \( L_{\text{th}} \gg M_\text{b} c^2 \), and magnetic energy at a rate \( L_\text{P} \). The produced fireball expands and accelerates as its internal energy is converted into bulk kinetic energy. The fireball is likely to carry strong magnetic fields, and the Poynting luminosity \( L_\text{P} \) may be higher than the thermalized luminosity. In this case, the magnetic fields prolong the acceleration after all the thermal energy has been converted into bulk expansion. A maximum Lorentz factor of the fireball can be estimated as \( \Gamma_{\text{max}} = (L_{\text{th}} + L_\text{P})/M_\text{b} c^2 \), and it is at least \( 10^9 \) for GRB explosions.

The nucleosynthesis occurs when the fireball temperature drops to about \( kT_\nu \sim 100 \text{ keV} \) at a radius \( R_\gamma \sim 10^7 - 10^9 \) cm. The timescale of expansion to this radius, \( R_\gamma/c \), is
shorter than the duration of the engine activity. Therefore, in the nucleosynthesis calculations, the fireball can be modeled as a quasi-steady outflow.

Let the outflow expand in an axisymmetric funnel with a cross section

\[ S(R) = S_0 \left( \frac{R}{r_0} \right)^\psi. \]  

(61)

For example, \( \psi = 2 \) for a radial funnel (and also for a spherically symmetric explosion), and \( \psi = 1 \) for a parabolic funnel, which may develop in a collapsing progenitor of the GRB (Mészáros & Rees 2001). The outflow is a relativistic ideal fluid with baryon density \( \rho \), pressure \( P \), and energy density \( w = 3P/\rho c^2 \); all these magnitudes are measured in the fluid rest frame. In spherical coordinates \( x^i = (t, R, \theta, \phi) \), the outflow has 4-velocity \( u^\mu = dx^\mu/d\tau = (u^t, u^R, u^\theta, u^\phi) \), where \( \tau \) is proper time. We assume \( u^t = 0 \) and \( R u^R \ll u^t, u^\theta \).

The latter assumption is satisfied at all radii for a radial explosion \( (u^t = 0) \) and at \( R \gg r_0 \) for a collimated explosion \( (\psi < 2) \).

The outflow dynamics is governed by the conservation laws \( \nabla_i (\rho u^i) = 0 \) and \( \nabla_i (T^i_0) = 0 \), where \( T^i_0 = u^i u^j w (\nu + P) + \rho \delta^i_0 \) is the stress-energy tensor. The electromagnetic tensor is not included here, which greatly simplifies the problem (we are interested in the early hot stage when the expansion is likely driven by the thermal pressure even in the presence of strong fields). Then the baryon and energy conservation laws read

\[ \dot{S} \rho u^R = M_b, \quad S(w + P)u^R = L_{th}. \]  

(62)

The high Lorentz factor of the expansion \( \Gamma = u^t/u^R \gg 1 \) implies \( u^R/c \approx \Gamma u^t/c \). Equation (62) then yields

\[ \rho = \frac{M_b}{ST^0 c}, \quad \frac{w}{\rho} = \frac{3}{4} \frac{L_{th}}{M_b r_0^3}. \]  

(63)

We assume that the outflow does not exchange mass or energy with the surroundings, i.e., \( M_b(R) = \text{const} \) and \( L_{th}(R) = \text{const} \). Equation (63) then gives \( w \propto \rho^1/3 \), while the first law of thermodynamics \( d(w/\rho) = -P d(1/\rho) \) gives \( w \propto \rho^{1/3} \). Excluding \( \rho \) from these two relations, one gets \( w^{1/4} \propto 1/\Gamma \) and

\[ \Gamma \sim T_0, \]  

(64)

where \( T_0 \) is a constant. Equation (64) is strictly valid at temperatures \( kT \ll m_c c^2 \approx 511 \text{ keV} \) where the \( e^\pm \) energy density can be neglected. At small \( \Gamma \) (where \( kT \gg m_c c^2 \)), the inclusion of the \( e^\pm \) reduces \( T \) by a modest factor \((11/4)^{1/4}\).

Equation (62) implies \( w = a T^4 \propto (\Gamma^2 S)^{-1} \), and, combining with equation (64), one gets

\[ \Gamma = \frac{T_0}{\Gamma} = \left( \frac{S}{S_0} \right)^{1/2} = \left( \frac{R}{r_0} \right)^{\psi/2}. \]  

(65)

The outflow may be collimated already at its base into a solid angle \( \Omega_0 = S_0/r_0^2 \approx 4\pi \) —its transonic dynamics near the central engine is complicated and unknown. With reasonable accuracy, the simple estimate \( L_{th} \sim S_0 a T_0^4 \gamma c \) gives

\[ kT_0 \approx 1.2 \left( \frac{L_{th}}{10^{31} \text{ ergs s}^{-1}} \right)^{1/4} \left( \frac{r_0}{3 \times 10^6 \text{ cm}} \right)^{-1/2} \left( \frac{\Omega_0}{4\pi} \right)^{-1/4} \]  

(66)

MeV.

The value of \( T_0 \) is most sensitive to the engine size \( r_0 \). It can be as small as \( 3 \times 10^5 \text{ cm} \) (if the outflow is powered by a Kerr black hole via the Blandford-Znajek process) or as large as \( 10^7 \text{ cm} \) (if the outflow is powered by an accretion disk).

A major parameter of the nucleosynthesis problem is the ratio of photon density,

\[ n_\gamma = \frac{30c(3)w}{\pi kT} \approx \frac{w}{2.70kT}, \]  

(67)

to baryon density \( n_b \). This ratio is constant during the expansion, and its value is (see eqs. [63] and [64])

\[ \phi = \frac{n_\gamma}{n_b} = \frac{3 m_p}{4 M_b} \frac{L_{th}}{2.7 k T_0} = 7.7 \times 10^4 \left( \frac{\eta_{\text{th}}}{300} \right) \left( \frac{k T_0}{1 \text{ MeV}} \right)^{-1}, \]  

\[ \eta_{\text{th}} = \frac{L_{th}}{M_b c^2}. \]  

(68)

The typical \( \phi \) is 5 orders of magnitude smaller compared to that of the universe.

The nucleosynthesis must occur during the acceleration stage of the fireball because the acceleration ends at a low temperature \( T = T_0/\Gamma\max < T_{\text{th}} \). The accelerated expansion is described in the comoving time by \( d\tau = dt/\Gamma \approx dR/c \Gamma \), which gives (we use eq. [65])

\[ \tau(T) = \left\{ \begin{array}{ll}
\frac{r_0}{c} \log \left( \frac{T_0}{T} \right), & \psi = 2, \\
\frac{r_0}{c} \left( \frac{T_0}{T} \right)^{(\psi-2)/\psi} & \psi < 2. \end{array} \right. \]  

(69)

The time of nucleosynthesis depends on the shape of the funnel: \( \tau(R_\psi) \approx 3 r_0/c \psi = 2 \) and \( \tau(R_\psi) \approx 2 T_0/T_{\text{th}} \times (r_0/c) \psi = 1 \). The timescale of the density falloff is 3 times shorter than that of the temperature because \( \rho \propto T^3 \). Therefore, a reasonable choice of the characteristic expansion timescale during nucleosynthesis is \( \tau_{\max} \approx r_0/c \psi = 2 \) and \( \tau_{\max} \approx 2 T_0/T_{\text{th}} \psi = 1 \).

Note that \( L_{\text{th}}, M_b, \) and \( T_0 \) enter the nucleosynthesis problem only in combinations that determine \( \phi \) and \( \tau(T) \) and play no other role. For example, the initial temperature is not important as long as \( k T_0 > 200 \text{ keV} \).

3.2. Simple Estimates and Comparison to the Big Bang

First we evaluate the temperature \( T_{\psi} \) at which we expect the nucleosynthesis to begin. Let us remind why \( k T_0 \approx 80 \text{ keV} \) in the big bang. If the nuclear statistical equilibrium (NSE) were maintained, recombination of nucleons into \( \alpha \)-particles would occur at \( k T_{\psi} \approx 100 \text{ keV} \) (e.g., Meyer 1994).

The cause of the nucleosynthesis delay until \( k T_{\psi} \approx 80 \text{ keV} \) is what is sometimes called “deuterium bottleneck.” Before fusing into \( \alpha \)-particles, the nucleons have to form lighter nuclei: deuterium, tritium, or \( ^3 \text{He} \). At \( k T \approx 100 \text{ keV} \), these elements have very low equilibrium abundances, which implies a very long timescale for their fusion, and helium is not formed even though it is favored by the NSE. In particular, the deuterium abundance is suppressed by the very fast

8 It is proportional to entropy per baryon: \( s/k = [2\pi^4/45(3)]n_s/n_b \approx 3.602 n_s/n_b \), where \( k \) is Boltzmann’s constant, and it must be constant in an adiabatic expansion.
photodisintegration $\gamma + d \rightarrow n + p$ that balances the opposite reaction $n + p \rightarrow d + \gamma$ at the equilibrium value

$$Y_d \approx 7 \times 10^{-6} \phi^{-1} Y_n Y_p T_{y_0}^{3/2} \exp\left(\frac{25.8}{T_9}\right)$$  \hspace{1cm} (70)$$

(e.g., Esmailzadeh, Starkman, & Dimopoulos 1991). Here $Y_i = n_i/n_0$ is abundance of the $i$th element and $T_0 = T/10^9 \text{K} = kT/86.17 \text{keV}$. Nucleosynthesis starts when the exponential wins the preexponential factor to give a noticeable $Y_d \sim 10^{-3}$. Substituting $\phi = 3 \times 10^5$, one finds that it happens at $kT_0 \approx 80 \text{keV}$.

In GRB outflows, $\phi$ is 5 orders of magnitude smaller, and theNSE favors the recombination of nucleons into $\alpha$-particles at a temperature as high as 500 keV. For the same reason as in the big bang, the nucleosynthesis is delayed to a lower $T_9$. Using equation (70) with $\phi = 10^5$, we find $kT_0 \approx 140 \text{keV}$, which is almost twice as high as the big bang $T_9$; it depends logarithmically on $\phi$.

At a given $T$, nuclear reaction rates scale as $\dot{Y} \propto \rho \propto \phi^{-1}$ (here $Y$ is an element abundance). The ratio of the expansion timescale $\tau_9$ to a reaction timescale, $\tau_{\text{rec}} = Y/\dot{Y}$, behaves as $\tau_9/\phi$. This combination is $\sim 3-30$ times smaller in GRBs compared to the big bang, which may not seem to be a crucial difference. In fact, the big bang nucleosynthesis was dangerously close to a freezeout of the neutron capture reaction $n + p \rightarrow d + \gamma$, without which nucleosynthesis cannot start. Therefore, the difference by a factor of 10 can be crucial, and it is instructive to compare accurately capture timescale with the expansion timescale in GRBs.

The neutron capture rate varies slowly with temperature (see Fig. 11 in Smith, Kawano, & Malaney 1993), and near 100 keV it is approximately $Y_e \approx 2.5 \times 10^4 \rho Y_n Y_p \text{ s}^{-1}$. The reaction timescale is

$$\tau_e = \frac{\min \{T_0, Y_p, Y_n\}}{Y_e} = \frac{1.2 \times 10^{-9} \phi}{T_0^3 \max \{Y_n, Y_p\}} \text{ s} \hspace{1cm} (71)$$

It should be compared with the timescale of the density fall-off, $\tau_\rho$. For the big bang, $\tau_9/\tau_\rho \approx 30$. For GRBs, we get

$$\tau_\rho \approx 10^{-4} \left(\frac{\phi}{10^5}\right)^{-1} \frac{Y_n}{0.5} \text{ s} \hspace{1cm} (72)$$

$$\frac{\tau_e}{\tau_\rho} \approx \begin{cases} \left(\frac{\phi}{10^5}\right)^{-1} \left(\frac{r_0}{3 \times 10^{-6}}\right) \left(\frac{Y_n}{Y_p}\right)^{1/2}, & \psi = 2, \\ \left(\frac{\phi}{10^5}\right)^{-1} \left(\frac{r_0}{3 \times 10^{-6}}\right) \left(\frac{Y_n}{Y_p}\right) \frac{T_0}{T_e}, & \psi = 1. \end{cases} \hspace{1cm} (73)$$

One can see that, in a radial explosion ($\psi = 2$), the neutron capture rate is marginal for a successful nucleosynthesis. In a collimated explosion, the ratio $\tau_9/\tau_\rho$ is higher by a factor of $T_0/T_e \sim 10$, and nucleosynthesis is efficient.

An important difference between the GRBs and the big bang is the $n/p$ ratio. In the big bang, $n_n/n_p = \frac{4}{3} (Y_e = \frac{1}{2})$, which leads to 25% mass fraction of helium after the $n-p$ recombination, while 75% of mass remains in protons (and a tiny amount of other nuclei). In GRBs, $n_n/n_p > 1$ ($Y_e < 0.5$), and there are leftover neutrons even if all protons are consumed by helium production. The minimum mass fraction of leftover neutrons is

$$X_n = 1 - 2Y_e. \hspace{1cm} (74)$$

### 3.3. Detailed Calculation

#### 3.3.1. The Code

We will keep track of elements with mass numbers less than 5 (like the big bang, the abundances of heavier nuclei are very small). The six elements under consideration are neutrons, protons, $^2\text{H}$, $^3\text{H}$, $^3\text{He}$, and $^4\text{He}$; they are denoted by $n, p, d, t, 3$, and $\alpha$, respectively, and the photons are denoted by $\gamma$. Their abundances are measured by $Y_i = n_i/n_0$ or mass fraction $X_i = A_i Y_i$, where $n_i$ and $A_i$ are the number density and the mass number of the $i$th species, respectively. The photon and matter densities are known functions of temperature,

$$n_\gamma = \frac{2}{\pi^2} \zeta(3) \left(\frac{kT}{\hbar c}\right)^3 = 2.02 \times 10^{28} T_9^3 \text{ cm}^{-3},$$

$$\rho = 3.39 \times 10^4 \phi^{-1} T_9 \text{ g cm}^{-3}. \hspace{1cm} (75)$$

The evolution of nuclear composition is described by the set of equations

$$Y_i = \sum Y_k Y_i \|kklj\| - \sum Y_i Y_j \|ijkl\|, \hspace{1cm} (76)$$

$$Y_n = -Y_d - 2Y_t - Y_3 - 2Y_\alpha - \frac{Y_n}{\tau_\rho}, \hspace{1cm} (77)$$

$$Y_p = -Y_d - Y_t - 2Y_3 - 2Y_\alpha + \frac{Y_n}{\tau_\rho}. \hspace{1cm} (78)$$

Here a dot signifies a derivative with respect to proper time $\tau_i = n_0 \bar{\sigma}_{\|i\rightarrow\|j\|}$ is the rate of reaction $i \rightarrow j + k + l$, and all quantities are measured in the rest frame of the outflow. It is sufficient to calculate the reaction rates for deuterium, tritium, and helium isotopes $^3\text{He}$ and $^4\text{He}$ ($i = d, t, 3, \alpha$ in eq. [76]). Then $Y_n$ and $Y_p$ are found from the neutron and proton conservation laws (eqs. [77] and [78]). The latter include the $Y_n/\tau_\beta$ term: the conversion of neutrons into protons via $\beta$-decay with the mean lifetime of neutrons $\tau_\beta = 900 \text{ s}$. The $\beta$-decay is negligible in GRBs, and we keep it for the code tests on big bang nucleosynthesis.

The sums in equation (76) are taken over all possible reactions with participation of the $i$th nuclei. Not all reactions are important (Smith et al. 1993). For example, reactions that destroy $\alpha$-particles can be neglected as $Y_\alpha$ is far below its equilibrium value. We include the reactions $n + p \rightarrow d + \gamma$, $n + 3 \rightarrow p + t$, $n + 3 \rightarrow \alpha + \gamma$, $p + d \rightarrow 3 + \gamma$, $p + t \rightarrow \alpha + \gamma$, $d + d \rightarrow p + t$, $d + d \rightarrow n + 3$, $d + t \rightarrow n + \alpha$, and $d + 3 \rightarrow p + \alpha$ in the calculations and take their rates from Smith et al. (1993) and Esmailzadeh et al. (1991). We then have

$$\dot{Y}_d = Y_n Y_p \|nduit\| - Y_d Y_3 \|d3\| - Y_d Y_p \|dd3\| - 2Y_\alpha \|ddn3\| \hspace{1cm} (79)$$

$$\dot{Y}_t = Y_n^3 \|d3\| - Y_n Y_3^2 \|d3\| - Y_n Y_p^2 \|d3\| - Y_d Y_3 \|d3\| - Y_n Y_p \|d3\| - Y_d Y_3 \|d3\|, \hspace{1cm} (80)$$

$$\dot{Y}_3 = Y_n Y_3 \|d3\| + Y_d Y_3 \|d3\| + 2Y_\alpha \|dd3\| + Y_n Y_p \|d3\| \hspace{1cm} (81)$$

$$\dot{Y}_\alpha = Y_n Y_3 \|d3\| + Y_d Y_3 \|d3\| + Y_n Y_3 \|d3\| \hspace{1cm} (82)$$

The reaction rates are functions of $T$ and $\rho$, and the set of equations is closed by equation (69) that relates $T$ and $\tau$. We solve equations (77)–(82) numerically with initial
conditions $Y_p^0 = Y_e$, $Y_d^0 = 1 - Y_e$, $Y_\alpha = 0$, and $Y_d = \dot{Y}_d = Y_3^0 = 0$ at an initial temperature $kT_0 \sim 1$ MeV. At high temperatures, the abundances of all elements except $n$ and $p$ are negligibly small. The abundances of deuterium, tritium, and $^3$He are close to quasi-steady equilibrium at $kT > 150$ keV; i.e., the rates of their production $Y^+$ and sink $Y^-$ are much higher than the expansion rate $Y/\tau$, and hence $Y^+$ and $Y^-$ almost balance each other. Where an element abundance $Y$ approaches the equilibrium value, it is calculated by setting the net $\dot{Y} = Y^+ - Y^- = 0$. (Thus, one avoids numerical integration where $Y$ is a small difference of big numbers.)

The code has been tested with the big bang nucleosynthesis. It successfully reproduced the standard evolution of the cosmological nuclear composition.

### 3.3.2. Results

Figure 3 compares the nucleosynthesis in GRBs and the big bang (BB). Parameters of the GRB fireball in this example are chosen as $\phi = 10^5$, $r_0 = 3 \times 10^6$ cm, $\psi = 2$, and $Y_e = 0.5$. As expected, GRB nucleosynthesis occurs at higher temperatures. The freezeout mass fractions of helium and deuterium are $X_\alpha \approx 0.16$ and $X_d \approx 0.03$, and about 81% of mass remains in free nucleons. Interestingly, the deuterium evolution is qualitatively different in GRBs: $X_d$ increases monotonically and freezes out at the 3% level, which is almost 3 orders of magnitude higher than in the big bang (see also Pruet et al. 2002a). This difference is due to a high abundance of neutrons and a negligible $\beta$-decay. As a result, the deuterium production by the neutron capture reaction outweights its burning rate and $X_d$ grows. Furthermore, the freezeout happens quickly in the radial expansion [$T \propto \exp(\tau/r_0)$], where $r_0 = r_0/c$ and $X_d$ could not decrease much below 1% even if the neutron capture reaction were completely switched off at, e.g., $kT < 80$ keV.

The postnucleosynthesis fireball is predominantly composed of free nucleons and helium, and it would be useful to know how their freezeout ratio

$$f = \frac{X_\alpha}{X_n + X_p}$$

depends on the four parameters of the explosion $r_0$, $\phi$, $Y_e$, and $\psi$. Figure 4 shows $f(r_0, \phi)$ for radial explosions with $Y_e = 0.5$. Contours $f = \text{const}$ on the $r_0-\phi$ plane are perfect straight lines $\phi \propto r_0^{0.85}$ (or $r_0 \propto \phi^{1.18}$). This implies that $f$ depends on combination $r_0^{1.18}$. A similar situation takes place for collimated explosions (Fig. 5). The slope of all contours in Figure 5 is 0.90, and $f$ can be viewed as a function of $r_0^{1.11}$. Calculations with $Y_e \neq 0.5$ give different values of

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**Fig. 3.**—Evolution of deuterium and helium abundances with temperature in an expanding fireball. The GRB case is shown by thicker curves for a radial explosion with $Y_e = 0.5$, $\phi = n_e/n_0 = 10^5$, and assuming a central engine of size $r_0 = 3 \times 10^6$ cm ($r_0$ sets the expansion timescale; see eq. [69]). For comparison, the big bang (BB) nucleosynthesis is also shown (with $\phi = 3 \times 10^9$). The dotted curves display the equilibrium (production = sink) abundances of deuterium.

**Fig. 4.**—Contours of freezeout ratio $f = X_\alpha/(X_n + X_p)$ on the $r_0-\phi$ plane for radial explosions with $Y_e = 0.5$. The right-hand axis shows $(L_{\text{th}}/M_{\text{bc}}c^2)(kT_0/\text{MeV})^{-1}$, which is related to $\phi$ by eq. (68). If the fireball is not Poynting flux dominated, $L_p < L_{\text{th}}$, its final Lorentz factor equals $\eta_{\text{th}} = L_{\text{th}}/M_{\text{bc}}c^2$.

**Fig. 5.**—Same as in Fig. 4, but for parabolically collimated explosions.
for both radial and collimated explosions (this explains our choice of constants 0.2 and 7.6 in eq. (84)).

Fireballs with \( Y_e \neq 0.5 \) have an upper bound on the freezeout abundance of helium max \( X_{\alpha} = 2 \text{min} \{ Y_e, 1 - Y_e \} \). For explosions with a neutron excess, \( Y_e < 0.5 \), this gives a maximum \( f \),

\[
\frac{2 Y_e}{1 - 2 Y_e}
\]

(86)

With increasing \( \xi \), the dependence \( f \propto \xi \) switches to \( f = f_{\text{max}} = \text{const} \) (Fig. 6).

The postnucleosynthesis fireball is dominated by \( \alpha \)-particles if \( f > 1 \), which requires \( Y_e > 0.25 \) and \( \xi > 1 \). For example, in a radial explosion with \( Y_e = 0.5 \) it requires \( \phi < 2.5 \times 10^4 (r_0/3 \times 10^6)^{0.3} \), and a similar condition for parabolic explosions reads \( \phi < 6.2 \times 10^4 (r_0/3 \times 10^6)^{0.9} \).

The typical GRB parameters happen to be just marginal for a successful nucleosynthesis: \( f \) varies from \( 10^{-2} \) to \( f_{\text{max}} \) in the expected range of \( \phi \) and \( r_0 \).

4. SPALLATION

Synthesized helium may be destroyed during the subsequent evolution of the explosion. Spallation reactions can occur when an \( \alpha \)-particle collides with another particle with a relative energy exceeding the nuclear binding energy. The fireball temperature at the acceleration stage is too low for such reactions; however, the collisions may become energetic if (1) a substantial relative bulk velocity between the neutron and ion components appears or (2) internal shocks occur and heat the ion fireball to a high temperature, much above the blackbody value.

4.1. Neutron-Ion Collisions during the Acceleration Stage

Near the central engine, the neutron component of the fireball is well coupled to the ion component by elastic collisions with a small relative velocity \( \beta = \vec{v}/c \) (Derishev et al. 1999; Bahcall & Mészáros 2000). The collision cross section can be approximated as \( \sigma_i = \sigma_0/\beta \), where \( \sigma_0 \approx 3 \times 10^{-26} \text{ cm}^2 \) if the ions are protons and \( \sigma_0 \approx 10^{-25} \text{ cm}^2 \) for \( \alpha \)-particles. In the fluid frame, the mean collisional time for neutrons is

\[
\tau_{\text{coll}} = \frac{1}{n_i \sigma_0 \beta c} = \frac{1}{n_i \sigma_0 c}
\]

(87)

where \( n_i \) is the ion density. The ion density is a fraction of the total nucleon density \( n_b \), which behaves as \( n_b = \text{const}/\Gamma^3 \) during the acceleration stage (see eqs. [63] and [65]). The constant of proportionality can be expressed in terms of \( \eta_{\text{nh}} = L_{\text{nh}}/M_{\text{pc}} c^2 \) and \( \phi = n_i/\eta_{\text{nh}} \),

\[
n_b = \frac{453 \zeta^4(3)}{4\pi^{14}} \left( \frac{m_p c}{h} \right)^3 \frac{\eta_{\text{nh}}}{\phi \Gamma^3} = 5.61 \times 10^{38} \left( \frac{\eta_{\text{nh}}}{\phi \Gamma^3} \right) \text{ cm}^{-3}
\]

(88)

In the fixed laboratory frame, the ion fluid is accelerated by radiative or magnetic pressure, and its Lorentz factor is doubled on timescale \( t = R/c \). Neutrons “miss” the ion acceleration by \( \Delta \Gamma/\Gamma \approx \tau_{\text{coll}}/t = \Gamma c \tau_{\text{coll}}/R < 1 \) and have a smaller Lorentz factor \( \Gamma_n = \Gamma - \Delta \Gamma \). The relative velocity of the neutron and ion components is \( \beta = (\Gamma^2 - \Gamma_n^2)/(\Gamma^2 + \Gamma_n^2) \approx (\Gamma - \Gamma_n)/\Gamma \), which is

\[
\beta \approx \frac{t_{\text{coll}}}{t} = \frac{\Gamma}{R n_i \sigma_0} \ll \beta_{\text{nh}} \approx 0.1
\]

(89)

The energy of neutron-ion collisions becomes sufficient for spallation reactions if \( \beta \) exceeds \( \beta_{\text{nh}} \approx 0.1 \). This can happen at late stages of the fireball acceleration, at high \( \Gamma \)
but not exceeding $\Gamma_{\max} = L/M_{\phi}c^2 = \eta_{\phi} + \eta_p$ ($L = L_{\phi} + L_p$ is the total luminosity of the fireball that includes the Poynting flux). In the case of $\Gamma(R) \approx R/r_0$ (radial explosion), $\beta$ reaches $\beta_p$ when $\Gamma$ reaches

$$\Gamma_{sp} = 0.3\Gamma_\max \left( \frac{L_{\phi}}{L} \right) \left( \frac{\phi}{10^5} \right)^{4/3} \left( \frac{\beta_{sp}}{0.1} \right)^{1/3} \left( \frac{r_0}{3 \times 10^6 \text{ cm}} \right)^{1/3} \left( \frac{4t_0}{n_b} \right)^{1/3}.$$  (90)

Spallation takes place if $\Gamma_{sp} < \Gamma_{\max}$, which requires $\phi > \phi_{sp}$,

$$\phi_{sp} \approx 4 \times 10^4 \left( \frac{L_{\phi}}{L} \right)^{3/4}.$$  (91)

(We keep in the last expression only the most uncertain parameter $L_{\phi}/L$, which may be much below unity. Note also that in the case of a nonradial explosion, $r_0$ should be replaced by $R/\Gamma$ in eq. [90]). Using equation (68), it is easy to show that the condition $\phi > \phi_{sp}$ is equivalent to

$$\Gamma_{\max} > 160 \left( \frac{kT_0}{\text{MeV}} \right) \left( \frac{L_{\phi}}{L_{\text{th}}} \right)^{1/4}.$$  (92)

Let $Y_n$ be abundance of neutrons that survived the nucleosynthesis, and suppose that $\phi > \phi_{sp}$. The lifetime of $\alpha$-particles bombarded by neutrons with $\beta \approx \beta_p$ is $t_{\text{life}} \approx (t_{\text{coll}}/Y_n)(\sigma_\alpha/\sigma_{sp})$ (where $\sigma_{sp} \approx \sigma_\alpha$ is the spallation cross section, which is roughly equal to the geometrical size of the nucleus $\pi R_{\text{sp}}^2 = 5.3 \times 10^{-26} A^{2/3}$ cm$^2$ with $A = 4$ for $\alpha$-particles). Hence, a modest $Y_n \sim 0.1$ should be sufficient for a significant spallation.

In case of a very low baryon loading (high $\Gamma_{\max}$) the neutrons decouple from the ions before the end of the acceleration stage and their Lorentz factor $\Gamma_n$ saturates at $\Gamma_{\text{dec}} < \Gamma_{\max}$. The relative velocity $\beta$ approaches unity when the decoupling happens, and the last neutron-ion collisions are energetic enough for pion production, which leads to multi-GeV neutrino emission (see Derishev et al. 1999; Bahcall & Mészáros 2000). The fireball Lorentz factor at this moment, $\Gamma_{\text{dec}}$, is given by equation (90) with $\beta_p$ replaced by unity, i.e., $\Gamma_{\text{dec}} = 10^{1/3} \Gamma_{\text{sp}}$. The decoupling takes place if $\Gamma_{\text{dec}} < \Gamma_{\max}$, which requires $\phi > \phi_{\text{dec}}$

$$\phi_{\text{dec}} \approx 7 \times 10^4 \left( \frac{L_{\phi}}{L_{\text{th}}} \right)^{3/4}.$$  (93)

The condition $\phi > \phi_{\text{dec}}$ is equivalent to $\Gamma_{\max} > 300 (kT_0/\text{MeV})(L/L_{\text{th}})^{1/4}$. The decoupling is always preceded by spallation.

The upper bound on the neutron Lorentz factor due to decoupling is essentially determined by the rate of baryon outflow per unit solid angle $M_\Omega$ (g s$^{-1}$), and it is useful to rewrite $\Gamma_{\text{dec}}$ in terms of $M_\Omega$. We substitute $n_i = n_b = (M_\Omega/\pi R_{\text{sp}}^2 c)$ to equation (87), and then the decoupling condition $\Gamma_{\text{dec}} = R/c$ gives the maximum Lorentz factor of neutrons

$$\Gamma_{\text{dec}} \approx \left( \frac{\sigma_{\alpha} M_\Omega}{4\pi n_b c \gamma} \right)^{1/3} \approx 300 \left( \frac{M_\Omega}{10^{36} \text{ g s}^{-1}} \right)^{1/3} \left( \frac{r_0}{3 \times 10^6 \text{ cm}} \right)^{-1/3}.$$  (94)

### 4.2. Internal Shocks

The $\alpha$-particles can also be destroyed later on, when internal shocks develop in the fireball. Lorentz factor $\Gamma_{\max}$ fluctuates if the central engine is "noisy" during its operation $0 < t < t_b \sim 1$ s. The fluctuations probably occur on timescales $r_0/c \approx 10^{-4}$ s and longer, up to $t_b$. This leads to internal shocks (Rees & Mészáros 1994). Internal dissipation of the velocity fluctuations may give rise to the observed GRB, and this picture is plausible because it easily accounts for the observed short-timescale variability in GRB light curves. The amplitude of the fluctuations is described by the dimensionless rms of the Lorentz factor, $A = \delta\Gamma_{\max}/\Gamma_{\max}$. At modest $A \leq 1$, the internal dissipation proceeds in a simple hierarchical manner (Beloborodov 2000). The internal collisions begin at

$$R_0 = \frac{2\Gamma_\max}{A} \lambda_0 = 5.4 \times 10^{11} \left( \frac{\Gamma_{\max}}{300} \right)^2 \left( \frac{\lambda_0}{3 \times 10^6} \right) \text{ cm},$$  (95)

where $\lambda_0$ (cm) is a minimum length scale of the fluctuations, which is likely comparable to $r_0$. The shortest fluctuations are dissipated first, and fluctuations with $\lambda > \lambda_0$ are dissipated at $R > R_0$ in the hierarchical order.

#### 4.2.1. $n$-$\alpha$ Collisions

Suppose $\Gamma_{\max}$ is small enough, so that the neutrons decouple from the ions after the acceleration stage and have the same $\Gamma_n = \Gamma_{\max}$. As soon as internal shocks develop in the ion component, the neutrons begin to drift with respect to the ions. Their relative velocity $\beta \approx A$ is sufficient for spallation reactions if $A > 0.1$ (and for $A \sim 1$ the pion production and multi-GeV neutrino emission take place; see Mészáros & Rees 2000).

Let $L_\Omega = R^2 n_b m_p c^2 \Gamma_{\max}^2$ be the fireball kinetic luminosity per unit solid angle; then

$$n_b = \frac{L_\Omega}{m_p c^2 R^2 \Gamma_{\max}^2}.$$  (96)

The $\alpha$-particles are destroyed by neutrons with rate $Y_n \approx Y_c Y_{n_0} n_b \sigma_{sp} c$, and their lifetime in the fluid frame is $\tau_{\text{life}} = (Y_n n_b \sigma_{sp} c)^{-1}$.

$$\tau_{\text{life}} = \frac{n_b}{\sigma_{sp} Y_n L_\Omega}.$$  (97)

It should be compared with the timescale of side expansion in the fluid frame, $(R/c\Gamma_{\max})$. If $\tau_{\text{life}} < (R/c\Gamma_{\max})$ at $R = R_0$, then most of the $\alpha$-particles are destroyed by the internal shocks. This condition reads

$$\Gamma_{\max} < \left( \frac{A Y_n \sigma_{sp} L_\Omega}{2 \lambda_0 m_p c^2} \right)^{1/5} \approx 300 A^{1/5} Y_n^{1/5} \left( \frac{L_\Omega}{10^{52}} \right)^{1/5} \left( \frac{\lambda_0}{3 \times 10^6} \right)^{-1/5}.$$  (98)

#### 4.2.2. $\alpha$-$\alpha$ Collisions

The $\alpha$-particles acquire random energy $(A^2/2)4m_p c^2$ in internal shocks, which easily exceeds the spallation threshold. The condition $\tau_{\text{life}} < (R/c\Gamma_{\max})$ for efficient $\alpha$-$\alpha$ spallation is similar to equation (98) (with $Y_n$ replaced by $Y_\alpha$).
There is, however, one more condition. The shocked \( \alpha \)-particles are cooled by Coulomb interactions with \( e^- \) (or \( e^+ \)) on a timescale \( \tau_{\text{Coul}} \), and an efficient spallation requires \( \gamma_{\text{life}} < \tau_{\text{Coul}} \) in addition to \( \tau_{\text{life}} < (R/c\Gamma_{\text{max}})^2 \).

The \( e^\pm \) can be considered as targets at rest for the hot ions because their radiative losses (synchrotron and/or inverse Compton) are rapid compared to the expansion rate. In the fluid frame, hot ions with mass \( m_i \) and charge \( Z_i \) lose their random velocity \( \beta \) on timescale (Ginzburg & Syrovatskii 1964)

\[
\tau_{\text{Coul}} = \beta \left( \frac{d\beta}{d\tau} \right)^{-1} = \frac{2\beta^3 m_i}{3Z^2 \Lambda \sigma_{\text{me}} c n_e},
\]

where \( n_e = n_- + n_+ \) is the total density of \( e^- \) and \( e^+ \) and \( \ln \Lambda \approx 20 \) is Coulomb logarithm. The internal shocks give \( \beta \approx A \), and for \( \alpha \)-particles (\( m_i = 4m_p \), \( Z = 2 \)) we find \( \tau_{\text{life}} = (n_\alpha \sigma_{\text{sp}} m_i)^{-1} < \tau_{\text{Coul}} \) if

\[
\frac{n_e}{n_\alpha} < \frac{2A^3}{3\Lambda \sigma_{\text{sp}}} m_p \approx 10A^3.
\]

A crucial factor in this condition is the \( e^\pm \) density. The postshock matter emits radiation, and \( e^\pm \) pairs can be produced by gamma rays \( (\gamma + \gamma \rightarrow e^- + e^+ \) that have energy \( h\nu > m_e c^2 \) in the fluid frame. Suppose a fraction \( \eta \) of the internal energy density \( (A^2/2)n_h m_e c^2 \) is converted into radiation and a fraction \( f \) of this radiation is above the threshold for pair production. A maximum pair density is evaluated assuming that all photons emitted above the threshold \( m_e c^2 \) are converted into pairs,

\[
n_e \approx f \eta A^2 m_p \frac{m_e}{n_b}.
\]

From equations (100) and (101) we conclude that the Coulomb losses of \( \alpha \)-particles on \( e^\pm \) can prevent the efficient \( \alpha-\alpha \) spallation if \( \eta > \kappa = 3 \times 10^{-3}AX_i \). In this case, only a fraction \( \tau_{\text{Coul}}/\tau_{\text{life}} \approx (\kappa/\eta) \) of the \( \alpha \)-particles are destroyed. The released neutrons can then continue the spallation process via \( n-\alpha \) collisions (§ 4.2.1).

Equation (101) assumes an optical depth for \( \gamma-\gamma \) interactions \( \tau_{\gamma} > 1 \). We now check this assumption. The optical depth can be estimated as \( \tau_{\gamma \gamma} \approx 0.1(\sigma_{\text{me}} m_i c^2) \sigma_{\text{Re}} R/\Gamma_{\text{max}} \), where \( \sigma_{\text{Re}} = f \eta A^2/2 n_h m_p c^2 \). This yields

\[
\tau_{\gamma \gamma} \approx 0.1 f \eta A^2 m_p \frac{m_e}{2 \sigma_{\text{Re}} \Gamma_{\text{max}}} R.
\]

It is easy to see that \( \tau_{\gamma \gamma} > 1 \) at all radii where efficient \( \alpha-\alpha \) spallation can take place [i.e., where \( \gamma_{\text{life}} < (R/c\Gamma_{\text{max}}) \)] if \( f \eta > 5 \times 10^{-4} \).

5. CONCLUSIONS

Our conclusions are as follows:

1. Reactions of \( e^- \) capture on nucleons, \( e^- + p \rightarrow n + \nu \) and \( e^+ + n \rightarrow p + \nu \), operate in GRB central engines and set an equilibrium proton fraction \( Y_p = n_p/(n_n + n_p) \). If the engine is an accretion disk around a black hole mass \( M \), the equilibrium \( Y_p \) is established at accretion rates \( M > M_{\text{eq}} \approx 10^{33}(\alpha/0.1)^{9/5}(M/M_\odot)^{6/5} \) g s\(^{-1} \) (eq. [48]), where \( \alpha = 0.01-0.1 \) is a viscosity parameter of the disk.

2. Of great importance for the explosion dynamics is whether \( Y_e < 0.5 \), i.e., there is an excess of neutrons. A general analysis shows that a neutrino-transparent matter has equilibrium \( Y_e < 0.5 \) if \( \mu > Q/2 \), and a similar condition for a \( \nu \)-opaque matter reads \( \mu > Q \), where \( \mu \) is the electron chemical potential in units of \( m_e c^2 \) and \( Q = (m_n - m_p)/m_e = 2.53 \). This condition is satisfied below a critical “neutrinoization” temperature \( T_{\nu} \) (eqs. [14] and [23]). We find \( T < T_{\nu} \) for plausible central engines of GRBs.

3. Fireballs produced by neutron-rich engines should also be neutron-rich. A major threat for neutrons in the escaping fireball is the neutrino flux from the central engine as an absorption of a neutrino converts the neutron into a proton. This process is slower than the fireball expansion if the neutrino luminosity is below \( 10^{53} \) erg s\(^{-1} \). A neutrino luminosity above \( 10^{53} \) ergs s\(^{-1} \) would require a very powerful and \( \nu \)-opaque central engine, which is possible. In this case, however, absorption of \( \nu \) by the fireball protons takes place, as well as absorption of \( \nu \) by the neutrons. The balance between \( \nu \) and \( \nu \) absorptions establishes a new equilibrium \( Y_e \) in the fireball, which depends on the emitted spectra of \( \nu \) and \( \nu \) and is likely below 0.5 (§ 2.3).

4. As the fireball expands and cools, the ejected free nucleons tend to recombine into \( \alpha \)-particles. This process competes, however, with rapid expansion and can freeze out. For this reason, nucleosynthesis is suppressed in fireballs with a high photon-to-baryon ratio \( \phi = n_\gamma /n_b \) (or, equivalently, high entropy per baryon \( s/k = 3.6\phi \)). We find that, in radiative fireballs, more than half of nucleons can recombine only if \( \phi < 3 \times 10^4 (R_0/3 \times 10^{10})^{1.93} \), where \( R_0/1 \text{MeV} \) (eq. [92]), \( R_0 \) is the size of the central engine. In fireballs with parabolic collimation, the efficient recombination requires \( \phi < 6 \times 10^5 (R_0/3 \times 10^{10})^{0.8} \). The typical GRB parameters \( \phi \sim 10^5 \) and \( R_0 \sim 3 \times 10^{10} \) cm are just marginal for nucleosynthesis.

5. Even in the case of efficient nucleon recombination, there are still leftover neutrons because of the neutron excess \( Y_e < 0.5 \). The minimum mass fraction of leftover neutrons is \( x_n \sim 1 - 2Y_e \).

6. The nucleosynthesis also produces deuterium, which is the next abundant element after the free nucleons and \( \alpha \)-particles. Its typical mass fraction is a few percent. The abundances of tritium, \( ^3\text{He} \), and all elements with mass number greater than 4 are negligible.

7. Synthesized \( \alpha \)-particles can be spalled later on, and then the populations of free neutrons and protons are increased. There are at least two possible mechanisms of spallation: (1) Energetic \( n-\alpha \) collisions with a relative velocity \( \beta > \beta_{\text{sp}} \approx 0.1 \) take place before the end of the fireball acceleration if \( \phi > 4 \times 10^4 (L_{\text{th}}/3 \times 10^{35})^{1/4} \). This mechanism works for fireballs with Lorentz factors \( \Gamma_{\text{max}} > 160(K_{\text{Th}}/1 \text{MeV}) \) (eq. [92]), where \( K_{\text{th}} \) is the initial temperature (eq. [66]). (2) Energetic \( n-\alpha \) and \( \alpha-\alpha \) collisions occur when the fireball is reheated by internal shocks. This spallation mechanism can be efficient at modest \( \Gamma_{\text{max}} > 300(L_{\text{th}}/10^{52})^{1/5} \) (eq. [98]), where \( L_{\text{th}} \) (ergs s\(^{-1} \)) is the fireball kinetic luminosity per unit solid angle.

The presence of a neutron component in GRB fireballs has quite spectacular implications. Beside making the fireball an interesting source of multi-GeV neutrinos (Derishev et al. 1999; Bahcall & Mészáros 2000; Mészáros & Rees 2000), the neutrons survive and play a dramatic role for the explosion development at large radii \( R \sim 10^{16}-10^{17} \) cm. When the fireball begins to decelerate as a result of the
interaction with an external medium, neutrons continue to coast with a high Lorentz factor $\Gamma_n$ and form a leading front. One can easily show that this front has kinetic energy much larger (by a factor of $X_n \Gamma_n \Gamma_{\text{max}}$) than the rest mass of the ambient medium. It leaves behind a relativistic trail loaded with the products of the neutron decay until the neutron front decays completely, which happens at $R \approx 10^{17}$ cm (about $10 \times$ mean decay radius).

An external shock wave—the customary source of GRB afterglows—has to form in the neutron trail rather than in a normal ambient medium. The mechanism of the fireball deceleration in this situation is elaborated in Paper II.

The neutron features of GRB explosions appear inevitably in the standard fireball scenario. They would be absent, however, if the GRBs are produced by magnetized winds with extremely low baryon loading, where Poynting flux carries much more energy than matter (e.g., Usov 1994; Lutikov & Blandford 2002). A detection or nondetection of neutron effects will constrain the level of baryon loading in GRBs.

At the final stages of the preparation of this manuscript, a paper by Pruet, Woosley, & Hoffman (2002b) appeared. They evaluated $\gamma_n$ for numerical accretion models of Popham et al. (1999). The results are consistent with our analysis in § 2 [note a slightly different definition of the viscosity parameter $\alpha_{\text{our}} = (3/2) \alpha_{\text{popham}}$].

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