Inductive Energy Harvesting for Rotating Sensor Platforms

T T Toh, S W Wright, M E Kiziroglou, P D Mitcheson and E M Yeatman
Department of Electrical and Electronic Engineering, Imperial College London, SW7 2AZ, U.K.

E-mail: tzern.toh02@imperial.ac.uk

Abstract. An inductive energy harvesting concept for structures rotating in proximity to a stationary body is proposed. The performance of such devices is studied analytically and numerically, and an experimental proof of concept is presented, demonstrating energy output density of 1 mW/cm\(^2\) from a typical geometry and rotation scale. The proposed approach may be suitable for powering retrofitted wireless sensors on engine bodies and also on rotating parts where complex stator-rotor wiring solutions would otherwise be required.

1. Introduction
A range of applications for energy autonomous sensors involves rotating structures. Such applications include airborne and terrestrial vehicles as well as industrial facilities. Harvesting energy from rotary motion has been limited to locations in the vicinity of the rotation axis, mainly because the high centrifugal force that appears off-axis locks a suspended proof mass into a fixed position [1]. For the same reason, other approaches such as the combination of gravity and the Coriolis acceleration require complicated mechanical structures that are impractical to implement within the typical sizes of sensor packages. Here, an inductive device concept is introduced, for applications where relative motion between proximal structures is available. This is usually the case in rotating engines such as motors, generators, turbines etc. A schematic description of the operation principle is presented in Figure 1. The device consists of one or more permanent magnets attached to one of the structures, while one or more coils are attached to the other. During motion, as a coil passes by magnet, a voltage is induced to it. This voltage can be exploited by appropriate electronics to harvest energy from the relative motion.

2. Analysis
In order to investigate the performance of particular implementations for inductive harvesting, a formulation of the field of a permanent magnet (PM) is introduced, including the geometry of a travelling coil. A PM block of rectangular, \(y_m - z_m\) cross section, with length \(x_m\) and magnetization \(M\) along the \(x\) – axis is assumed. A rectangular, \(y_c - z_c\) cross section coil of \(N\) turns is aligned in the \(z\)-direction with the PM and can travel along the \(y\) direction at a distance \(x_g\) from the closest PM face. A schematic of this setup is given in Figure 1 (right top).

An analytical calculation of the magnetic flux density \(x\)-axis component, \(B_x\) along the central \(x\)-axis of the PM is possible by calculation of the magnetic scalar potential \(V_m\) that is imposed at a position \(x\) by each discrete PM volume element \(dV\) [2]. A vector algebraic analysis, with reference to Figure 1 (top right), gives the result:

\[
V_m = \int_V \frac{1}{4\pi} \frac{M}{|r|^3} dV = \frac{M}{4\pi} \int_{x_g}^{x_m} \int_{y_m/2}^{y_m/2} \int_{z_m/2}^{z_m/2} \frac{x}{\sqrt{x^2+y^2+z^2}} dx dy dz
\]
Figure 1. Left: Schematic of the device concept. Right: Vector diagrams for magnetic field calculation along the PM axis (top) and by treating the PM as a single magnetic dipole (bottom).

Then $B_x$ can be found by calculating the integral for $y$ and $z$, and differentiating $V_x$ with respect to $x$:

$$B_x = \mu \nabla V_m \cdot \hat{x} = \frac{\mu M}{\pi} \left( \arctan \frac{y m z m}{2 g (x g + y m + z m)} - \arctan \frac{y m z m}{2 (x g + z m) (x g + z m + y m + z m)} \right)$$

(2)

where $\mu$ is the magnetic permeability of air. A first estimate of induction could be obtained by assuming a uniform $B_x$ field abruptly appearing in front of the PM. This approach provides a reliable prediction of the induction dependence on PM – coil distance $x_g$ and an overestimation of the induction magnitude. However, it doesn’t provide information about the actual profile of the induced voltage. Another approach would be to treat the PM as a single magnetic dipole. Such an approach has low quantitative accuracy at the vicinity of a PM but provides a more detailed description of the fringing flux. In this case, the $x$-axis magnetic flux density at a distance $r$, and angle $\theta$ from the centre (Figure 1 right bottom) can be written as [3]:

$$B = \frac{\mu m_r}{4 \pi r^3} (2 \cos \theta \hat{r}_0 + \sin \theta \hat{\theta}_0)$$

(3)

where $m_r=M \cdot V$ is the total magnetic moment of the PM, $V$ being its volume. If $d$ is the distance between the coil and the centre of the PM, then its $x$-axis component in Cartesian coordinates and the corresponding induced voltage will be:

$$B_x = \frac{\mu m_r}{4 \pi r^3} (2 \cos^2 \theta - \sin^2 \theta) \hat{x}_0 = \frac{\mu m_r}{4 \pi} \frac{2 d^2 - x^2 - y^2}{(d^2 + x^2 + y^2)^2}$$

(4)

$$V = -N \cdot \frac{\partial \Phi}{\partial t} = -N \cdot U \cdot \frac{\partial \Phi}{\partial y} = -N \cdot U \cdot 2 (F \left( \frac{z c}{2}, y + y_c \right) - F \left( \frac{z c}{2}, y \right))$$

(5)

Here, $U=y/t$ is the speed of the coil with coil-PM alignment occurring at $t = 0$. The last term of (5), including the factor of 2, represents the flux gradient along $y$. The function $F$ is the general integral of $B_x$ from equation (4) with respect to $z$:

$$F(z, y) = \frac{\mu m_r}{4 \pi} \frac{(d^2 - y^2)^2 z + 2 (d^2 - y^2)(d^2 + y^2) z}{(d^2 + y^2)^2 (d^2 + z^2 + y^2)^3} (d^2 + z^2 + y^2)^2$$

(6)
From equation (5) one can calculate the induced voltage and power output for different setups. Results for a particular harvester with a $10 \times 10$ cm, 100 turns coil passing by a $10 \times 10 \times 40$ mm neodymium PM at 10 mm distance with speed corresponding to 1600 rpm at 0.5 m from the rotation axis are shown in Figure 2. For the calculation of power, a resistive load matching a coil resistance of 8.56 $\Omega$ was considered, which corresponds to a 0.1 mm diameter Cu wire. As expected, a smooth alternating voltage profile is observed with amplitude exceeding 1 V for the particular device and conditions. The total energy from a single coil-magnet interaction is over 12 $\mu$J. If one such interaction per full rotation is taken into account, an average harvesting power of 0.32 mW is obtained. Although the total PM magnetic moment was taken into account, the field is underestimated by this method, because the assumption of a single magnetic dipole leads to an effective increase of the PM-coil distance.

3. Simulation

As a complementary method to the above analysis, the magnetic field of PMs in geometries relevant to the device design was calculated using finite element analysis in 3D. The simulation software used was COMSOL. A $10 \times 10 \times 40$ mm neodymium PM with $M = 1$ MA/m and $\mu_r = 1$ was assumed. The flux through a $y-z$ frame moving along the $y$-axis at a distance of 10 mm from the magnet’s edge was studied, with and without a soft magnetic core. The frame size was $10 \times 10$ mm with negligible turn dimensions. The core size was $20 \times 10 \times 10$ mm with its closer-to-magnet small face coinciding to the frame. A constant relative permeability of 4000 was assumed. This geometry is illustrated in Figure 3 along with x-axis flux density distribution results for two different core positions.

For a given $y$-axis frame velocity, the total flux through the frame was calculated as a function of time. Results are plotted in Figure 4 for the case of a 1600 rpm, 1 m diameter rotation (velocity 83.8 m/s). An over two-fold increase of the peak flux is observed with the introduction of the magnetic core. From these simulations, the open-circuit induced voltage can be calculated as the negative derivative of total flux. The corresponding results are also presented in Figure 4. In the same figure, the corresponding analytical calculation, as obtained by equation (5) is plotted for comparison. These results provide a quantitative demonstration of the performance enhancement that is achieved by employment of a soft magnetic core. In addition, they demonstrate the correct prediction of voltage profile by the proposed analytical model.

The underestimation of the magnetic field density in equation (4) is studied further by comparing the simulated $B_x$ profile on the central PM x-axis with the analytical ones. The results are presented in Figure 5. Excellent matching between the precise expression (2) and the simulated values is observed, supporting the validity of the numerical model. As expected, the single magnetic dipole assumption of equation (4) leads to significant underestimation of $B_x$ in the PM vicinity. This is because all the magnetic moment is assumed to be concentrated to the PM centre, effectively increasing the distance between field-source and coil. For distances larger than the PM length, all three methods converge. In the same diagram, the case of using a core at a distance of 10 mm from the PM edge is also plotted, illustrating the field concentration that is accomplished. At the core edge, the $B_x$ maximum is increased to 135 mT, from 77 mT without core, a 75% improvement. The corresponding total flux improvement, as illustrated in Figure 4, is from 6.3 $\mu$Wb to 15.2 $\mu$Wb, or 141%.
Figure 3. Geometry and x-axis flux density distribution of a magnet with \( M = 1 \text{ MA/m} \) along the x-axis and a core with \( \mu_r = 4000 \), for 2 different core positions. \( Y_{\text{CORE}} \) is the magnet-core distance (centre-to-centre) along y.

Figure 4: Left: Simulated flux through a 10 × 10 mm frame travelling at 83.8 m/s (1600 rpm, diameter: 1 m) with respect to a PM at a distance of 10 mm (Figure 3). Right: Corresponding induction on a 100 turn coil.

Figure 5: Simulated flux density as a function of distance from magnet along the x-axis at \( y = z = 0 \), with and without a soft magnetic core in comparison with analytical calculations. All parameters are nominal (no fitting).

4. Experimental Results
A prototype implementation was built and tested as illustrated in Figure 6. The coil – magnet gap is adjustable from 0.2 to 10 cm, and radial distance can be up to 15 cm. Two magnetic cored coils wound with 1000 turns of 0.1 mm diameter Cu wire and 79 \( \Omega \) resistance are attached to the baseplate. Two 1 cm diameter, 1 cm length N42 Nd magnets are mounted on the rotor, which is spun by a motor controllable up to 1200 rpm. The voltage induced in the coil is observed on an oscilloscope and the available power into a matched resistive load calculated. The magnets are rotating and the coils are fixed in this implementation, but the converse arrangement is also possible and would allow energy harvesting on a rotating structure.
Figure 7 shows (left) a typical voltage trace observed from a 200 turn coil during rotation as a magnet passes by the coil assembly. A voltage profile shape matching that in Figure 2 is seen. On the right, measured induced open circuit voltage and the corresponding power as a function of rotation speed are shown, from a 2 PM, single coil arrangement. The linear fit seen for the voltage is in agreement with the theory of equation (5). The power levels of 4 mW at 600 rpm and 16 mW at 1200 rpm demonstrate the possibilities of harvesting significant power on a rotating system at speeds typical of much industrial and transport machinery. Such power levels are sufficient for operation of wireless sensors on moving structures where the provision of power by cabling would not be possible. Power capability can be raised by adding more magnets to the system, with the same coils and electronics.

5. Conclusion
An inductive concept for energy harvesting from proximal relative motion was introduced. The operation of such devices was studied analytically and simulated numerically, revealing an expected output exceeding the requirements of typical low-power sensor node implementations. Experimental results demonstrated several mW of continuous power supply, from a device implementation of around 2 cm$^3$ of magnet volume and 1000 coil turns at a 150 mm distance from the rotation axis. Further experimental results and a more detailed analysis will be presented in a following paper.

6. References
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