Double resonances in Borromean heteronuclear triatomic systems

F. Bringas1, M. T. Yamashita2, T. Frederico3, and Lauro Tomio2,4
1Cooperative Institute for Marine and Atmospheric Studies,
University of Miami, 4600 Rickenbacker Causeway Miami, FL 33149, USA.
2Instituto de Física Teórica, São Paulo State University (UNESP), 01140-070, São Paulo, Brazil.
3Dep de Física, Instituto Tecnológico de Aeronáutica, CTA, 12.228-900, S. J. dos Campos, Brazil.
4Instituto de Física, Universidade Federal Fluminense, 24210-346, Niterói, RJ, Brazil.
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We investigate the occurrence of Borromean three-body continuum $s$-wave resonances, in an $\alpha\alpha\beta$ system for large negative two-body scattering lengths. The energy and width are determined by a scaling function with arguments given by energy ratios of the two-body virtual state subsystem energies with the shallowest three-body bound state. The Borromean continuum resonances emerging from Efimov states present a peculiar behavior for trapped ultracold atoms near a Feshbach resonance: two resonances with equal energies at different values of the scattering length. The corresponding three-body recombination peaks should merge as the temperature is raised, with one moving towards lower values of the scattering length as the other moves to larger values.

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The search for Efimov states was favored in recent years with the development of new techniques in cold-atom laboratories, such as laser-cooling mechanisms and the control of two-body interactions by Feshbach resonance techniques (see for example [1] and references therein). At the Innsbruck laboratory, Efimov states are identified in scattering using ultracold gas of Cesium atoms [3] at 10 nanokelvin. And, more recently, at the laboratory of Università di Firenze [4], by using a mixture of $^{41}$K and $^{87}$Rb it was identified the formation of Efimov resonances in the two heteroatomic channels for KKRb and KRbRb. The asymmetry of masses may turn more favorable the observation of Efimov states in comparison to the case of three identical bosons (see e.g. [5, 6]).

Generic systems of the type $\alpha\alpha\beta$ can be classified according to the interaction of their two-body subsystems [3, 8]. In particular, Borromean systems have been studied in [9]. Progress on calculating three-body continuum states and resonance decay [12] are impressive and they give a step further in our precise understanding of these states. Following these novel methods, applied so far in the nuclear physics context (e.g. the decay of $0^+$ state of $^{12}$C in three $\alpha$’s), a picture of resonances arising from Efimov states are highly demanded in view of the tremendous advances on the experimental setups of ultracold atoms with tunable interactions. Some properties of these states under the change in the interaction parameters are well known, e.g., when at least one of the two-body subsystems is bound, an Efimov state emerges from the two-body analytical cut by changing the two-body binding energy [10]. Such state, in the second energy sheet, is given by a pole in the real axis and identified as a virtual state. In the case of a Borromean three-boson system there is no two-body cut, and the Efimov state arrives from a continuum three-body resonance [11]. As systems of ultracold atoms with different species offer larger opportunities for universal physics than the equal boson case, it is timely to study the fate of Borromean states in heteronuclear systems with tunable interactions.

In the present work, we extend the analysis of Ref. [11], to Borromean $\alpha\alpha\beta$ systems (negative scattering lengths) with two kind of particles. It is shown how the Efimov states emerge from three-body $s$-wave continuum resonances. As the absolute values of the scattering lengths (given, respectively, by $|a_{\alpha\alpha}|$ and/or $|a_{\alpha\beta}|$) increase, an existing three-body continuum resonance disappears at the scattering threshold with the formation of an Efimov bound state. This process can be replicated by further increasing one or both two-body scattering lengths, with the formation of more Efimov excited states. In the exact Efimov limit, a tower with infinite excited states is produced when both, $|a_{\alpha\alpha}|$ and $|a_{\alpha\beta}|$, turn to be infinite.

The parametric region, for which exists a three-body resonance, is presented in the form of a scaling function having scattering lengths measured in units of a three-body length identified with $\sqrt{|m_\alpha B_3/\hbar^2|}^{-1}$, where $m_\alpha$ is the mass of the $\alpha$ particle and $B_3$ is the three-body bound-state that we use as our three-body scaling parameter. In the following, our units will be such that $\hbar = 1$ and $m_\alpha = 1$, with the mass ratio defined as $A \equiv m_\beta/m_\alpha$.

In general terms, the quantum description of such large and weakly bound systems is universal and can be defined by few physical scales, despite the range and details of the pairwise interaction. All the detailed information about the short-range force, beyond the low-energy two-body observables, is retained in only one three-body physical information in the limit of zero-range interaction [3, 6]. The two-body scales are defined by the energies of the two-body virtual states, $E_{\alpha\alpha}$ and $E_{\alpha\beta}$, or by the correspondingly scattering lengths $a_{\alpha\alpha} \simeq -1/\sqrt{|E_{\alpha\alpha}|}$ and $a_{\alpha\beta} \simeq -1/\sqrt{|E_{\alpha\beta}|}$. Such two-body scales are given in units of a three-body length scale $1/\sqrt{|B_3|}$, with $B_3$ de-
fined as the binding energy of the shallowest three-body state. In the present case, the scaling function for the continuum resonance energy $E_3$ can be written as

$$E_3 = B_3 \mathcal{E} \left( a_{\alpha\alpha} \sqrt{B_3}, a_{\alpha\beta} \sqrt{B_3}; A \right).$$ (1)

The corresponding critical boundary, given by $\mathcal{E} \left( a_{\alpha\alpha} \sqrt{B_3}, a_{\alpha\beta} \sqrt{B_3}; A \right) = 0$, in the parametric plane $\left( \sqrt{|E_{\alpha\alpha}|/B_3}, \sqrt{|E_{\alpha\beta}|/B_3} \right)$, was numerically verified in Ref. [13], where all possibilities for bound and virtual two-body states were considered. We note that the calculations done throughout this work were performed by considering only two-body virtual states (negative scattering lengths).

Once the critical boundary is crossed in the direction of infinity negative scattering lengths, an existing continuum three-body resonance will move to an Efimov bound state. Such continuum three-body state was indeed observed in the particular case of identical bosons, through the resonant behavior of three-body recombination rate in an ultracold trapped gas of Caesium [3].

The presence of Efimov resonances has been also reported in Ref. [4] in an ultracold mixture of $^{41}\text{K}$ and $^{87}\text{Rb}$ gases, through the three-body collision of KKRb and RbRbK atoms. In these cases, two resonantly interacting pairs for positive or negative scattering lengths are sufficient to allow Efimov states. Such experiment supports the prediction made in Ref. [13], which is resumed in a critical boundary in the parametric plane defined by $\left( \pm \sqrt{|E_{\alpha\beta}|/B_3}, \pm \sqrt{|E_{\alpha\alpha}|/B_3} \right)$, for the existence of at least one Efimov bound state. Three-body heteronuclear mixtures with resonant interspecies were also studied in [14] using zero-range interaction, where the critical conditions for formation of Efimov states at threshold are discussed. The case presented in this work of continuum resonances were left open.

In order to calculate the scaling function (1), we use subtracted Faddeev equations with zero-range interactions for a three-body system composed by two identical bosons, $\alpha$, and a third one, $\beta$. The following equations, presented for continuum resonant states, were already written in a similar form in the context of halo nuclei systems [8, 11]. After partial-wave projection, the $s$–wave coupled integral equations of the Faddeev spectator functions $\chi_{\alpha\alpha}$ and $\chi_{\alpha\beta}$, can be written as

$$
\chi_{\alpha\alpha}(q) = 4\pi \tau_{\alpha\alpha}(q; E_3) \int_0^\infty p^2 dp \int_{-1}^1 dz G_1(p,q,z;E_3) \chi_{\alpha\beta}(p),
$$

$$
\chi_{\alpha\beta}(q) = 2\pi \tau_{\alpha\beta}(q; E_3) \int_0^\infty p^2 dp \int_{-1}^1 dz \times 
$$

$$
\left[ G_1(p,q,z;E_3) \chi_{\alpha\alpha}(p) + G_2(p,q,z;E_3) \chi_{\alpha\beta}(p) \right],
$$

where the two-body amplitudes, $\tau_{\alpha\alpha}$ and $\tau_{\alpha\beta}$, and the subtracted Green functions, $G_1$ and $G_2$, are defined in Ref. [10]. The subtraction energy is of no physical importance and it is set to $\infty$.

The resonance energies are calculated from the coupled eqs. (2) using a contour deformation method in the complex momentum plane [15]. The momentum variable as $p$ appears as a function of the deformation angle $\theta$ and written as $p \equiv |p|e^{-i\theta}$, with $0 \leq \theta < \pi/4$. The second energy sheet is revealed when the momentum integration path is deformed by a contour along the real axis to the complex plane with a fixed rotation angle chosen to place the contour path far from the scattering singularities. That exposes the complex resonance energy pole of the scattering matrix. For large enough $\theta$, the solution of the coupled eqs. (2) in the complex energy plane is found for $\text{tan}(2\theta) > -\text{Im}(E_3)/\text{Re}(E_3)$, where $\text{Re}(E_3)$ represents the resonance energy and $\text{Im}(E_3)$ its half width.

In the limit when the two-body energies are equal to zero, an infinite number of Efimov states emerges from the solution of the coupled eqs. (2). At this point, by varying the two-body energies, a given energy $E_3$ will have zero imaginary part, becoming purely real and negative. The $N^{th}$ Efimov bound state is defined by $E_3^{(N)} \equiv -B_3^{(N)}$ with $N = 0$ indicating the ground state. When the $N^{th}$ Efimov bound state is moved to a resonance its complex energy will be denoted by $E_3^{(N)}$.

The solution of the homogeneous coupled eqs. (2) in the complex rotated contour for complex energies gives the position and width of the continuum three-body resonances, allowing to construct numerically the scaling function (1). In practice, we note that one can approach quite fast the scaling limit (16) or the limit cycle (17) for $N \to \infty$, such that we will present results for $N = 1$. In two frames, we show in Fig. 1 the real and imaginary parts of the complex energy $E_3^{(N)}$, as

FIG. 1: Real (upper frame) and imaginary (lower frame) parts of the three-body complex energy as a function of the two-body virtual state energy with $E_{\alpha\alpha} = E_{\alpha\beta}$. The results, obtained for $N = 1$, are given for mass ratios $A = 1, 9, 200$. 

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functions of the two-body binding energy $E_{\alpha\alpha}$, for the case that $E_{\alpha\alpha} = E_{\alpha\beta}$. All such energy quantities are dimensionless, given in units of the bound-state energy $B_3^{(N-1)}$. So, in Fig. 1(a), we have $Re(E_3^{(N)})/B_3^{(N-1)}$ versus $E_{\alpha\alpha}/B_3^{(N-1)}$; and, in Fig. 1(b), $Im(E_3^{(N)})/B_3^{(N-1)}$ versus $E_{\alpha\alpha}/B_3^{(N-1)}$. In Fig. 1(a), in the positive part of the plot we have the $N$-th resonance energy given by $Re(E_3^{(N)})/B_3^{(N-1)}$. When the ratio $E_{\alpha\alpha}/B_3^{(N-1)}$ is decreased, this resonance turns into a bound state and the negative part of the plot gives the ratio of the energies of two consecutive states ($B_3^{(N)}$) and ($B_3^{(N-1)}$) three-body bound state for each system. This transition from a resonant to a bound state follows the same general behavior as verified in the case of three identical bosons [11]. For $A = 1, 9$ and $200$, the values of $E_{\alpha\alpha}/B_3^{(N-1)}$ at which we have transitions from resonances to bound-states are, respectively, given by $0.876 \times 10^{-3}$, $1.67 \times 10^{-3}$ and $2.15 \times 10^{-3}$. These values correspond to the ones given the position of the boundary curve in the parameter space $\left(\sqrt{|E_{\alpha\alpha}|/B_3}, -\sqrt{|E_{\alpha\beta}|/B_3}\right)$ (for the case $E_{\alpha\alpha} = E_{\alpha\beta}$) that separates the excited bound Efimov state from the continuum resonance.

In Fig. 1 our plots are determined only for $N = 1$, considering that results for higher $N$’s should coincide with the ones obtained for $N = 1$. The energy value of the resonance (real part) grows, as the virtual two-body energies increases (by reducing the absolute values of the scattering lengths), up to a maximum and then decreases, while the imaginary part always increases. This makes the resonance to dive deeper in the second energy sheet, as it is clearly shown in Fig. 2.

The results shown in Fig. 1 suggest that in an ultracold mixture of heteronuclear atoms the increase of temperature moves a Borromean Efimov continuum resonance towards smaller absolute values of the scattering length (larger two-body virtual energies), while the width $|Im(E_3^{(N)})|$ increases. Indeed, an experiment with ultracold Caesium atoms [3] indicated that the recombination peak due to the continuum tratomic resonance moves towards smaller values of $|a|$ as temperature is raised, therefore such resonance is identified with a state that just arises from an Efimov bound one that dives into the three-boson continuum. Such behavior is a clear consequence of the path followed by the real part of the resonance energy that increases as $|a|$ is decreased (see Fig. 1 and ref. [18]).

The maximum value reached by the real part of the three-body energy, the position of the resonance, presumably creates a curious effect that one could observe in the three-body recombination. As the position of the resonance returns to zero (Re($E_3^{(N)}$) → 0), it may be possible to detect, at a given temperature, the occurrence of two peaks in the three-body recombination, corresponding to two values of the scattering length. The wider peak should be at smaller values of the absolute value of the scattering length, as one can easily verify by looking at Fig. 2.

In Fig. 2 we show a scaling function for $E_3^{(N)} = 0$ of $\sqrt{|E_{\alpha\beta}|/B_3^{(N-1)}}$ versus $\sqrt{|E_{\alpha\alpha}|/B_3^{(N-1)}}$, i.e., these curves are the boundaries for the existence of a resonance, as already obtained in Ref. [12]. In the lower section of the figure, we have three plots carrying the points where a resonance begins to appear, $Re(E_3^{(N)}) = 0$, in Fig. 1 and, in the upper section of the figure, we have the corresponding curves carrying the points where the origin is reached with a finite value of the width. The boundary curves indicate that, at ultralow temperatures, a mixture of heteronuclear gas with two species can have two three-body recombination peaks associated with one Efimov state. As $a_{\alpha\beta}$ is modified with $a_{\alpha\alpha}$ kept fixed, a wider resonance appears when the upper curve in Fig. 2
is crossed, that happens for smaller values of $|a_{\alpha\beta}|$; with a thinner one occurring for larger values. This last case corresponds to an excited bound Efimov state diving into the continuum. As $T$ is raised, the two peaks tend to merge, as the maximum of the resonance is approached.

We found two continuum resonances for Borromean systems at different values of the scattering length and same position in energy and different widths, for heteronuclear and homonuclear systems. The absolute maximum for the resonant energy (associated with the system temperature) occurs in both cases. As we observe in Fig. 1, the resonance energy attains a maximum value and then decreases as $|a|$ is moved further towards small values. Therefore, two resonances are located at the same energy for different values of the negative scattering length. It is expected that the corresponding pair of three-body recombination peaks merges as the temperature is raised, one peak moves towards lower values of the scattering length while the other one to larger values. Using the numerical values of the scattering length for homonuclear systems, one gets that the ratio between the scattering lengths where the real part of the resonance energy is identical stays between 1 and 0.25 as one easily gets by inspection of Fig. 1. Therefore, a second three-body recombination peak appears in a region where the four-body recombination peak is present with a scattering length ratio of 0.43 [2, 19], posing an experimental challenge for separating the three- and four-body recombination peaks.

In conclusion, we studied the universal properties of three-body continuum resonances for heteronuclear three-particle systems within a zero-range renormalized three-body model. We discussed how the continuum resonances for heteronuclear Borromean systems may be observed in mixtures of ultracold gases. The fingerprint of the observation of the scaling considered in the present work, for a Borromean system, is that such resonances will be given by their characteristic displacement with temperature $T$: as $T$ raises the recombination peak, that comes from an Efimov state diving into the continuum, should move to lower absolute values of the scattering lengths with an increasing width. Moreover, we found theoretically two recombination peaks at $T = 0$, with different widths and positioned at different values of the scattering lengths.

For a fixed $\alpha\alpha$ scattering length, with the assumption that three-body scale does not move, the wider resonance should appear for smaller absolute values of the $\alpha\beta$ scattering length. As $T$ is raised the two recombination peaks tend to merge. This physical effect may be observed in ultracold trapped heteronuclear systems near a Feshbach resonance, where the properties of the continuum resonances offer a rich structure to be explored by tuning the large and negative scattering lengths. It is an exciting possibility to be probed in actual experiments with mixtures of ultracold atomic gases.

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