Off-shell renormalization
in the presence of dimension 6 derivative operators.

II. UV coefficients

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Abstract

We present the full off-shell renormalization of dimension 6 operators in the Abelian Higgs-Kibble model supplemented with a maximally power counting violating higher-dimensional gauge-invariant derivative interaction $\phi^\dagger \phi (D^\mu \phi)^\dagger D_\mu \phi$.

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I. INTRODUCTION

In this paper we continue the study of the off-shell renormalization of the Abelian Higgs-Kibble model supplemented by the maximally power counting violating dimension 6 operator $\phi^\dagger \phi (D^\mu \phi)^\dagger D_\mu \phi$. In particular, we will show here how to evaluate the one-loop divergent coefficients associated to all dimension 6 operators which are radiatively generated.

The general aspects of the formalism needed to achieve this result have been explained in details in [1], to which we refer the reader for a thorough exposition of the technical tools required within the Algebraic Renormalization approach to the problem [2–15] we use. Nevertheless, let us briefly summarize in the following the procedure developed in [1] from an operational point of view.

To systematically compute the (one-loop) UV coefficients in spontaneously broken effective field theories possessing (dimension 6) derivative operators, it is convenient to first renormalize an associated auxiliary model, the so-called $X$-theory, which is obtained by describing the scalar physical degree of freedom in terms of the gauge-invariant field coordinate

$$v X_2 \sim \phi^\dagger \phi - \frac{v^2}{2},$$  \hspace{1cm} (1.1)

$v$ being the vacuum expectation value of the Higgs scalar $\phi$.

Then, in the $X$-theory all higher dimensional operators in the classical action are required to vanish at $X_2 = 0$. Thus, the operator $\frac{\Lambda}{v \lambda} \phi^\dagger \phi (D^\mu \phi)D_\mu \phi$ (with the energy scale $\Lambda$ much higher than the electroweak scale $v$) will be expressed as $\frac{\Lambda}{\lambda} X_2 (D^\mu \phi)D_\mu \phi$; then by going on-shell with the field $X_2$ and an additional Lagrange multiplier $X_1$ enforcing algebraically the constraint in Eq. (1.1), we get back the original operator. Two external sources are then required in order to formulate in a mathematically consistent way the $X$-theory [1]: one is coupled to the constraint $v X_2 - \phi^\dagger \phi - \frac{v^2}{2}$ and is denoted by $\tilde{c}^*$; the second, called $T_1$, is required to close the algebra of operators, implementing the $X_2$-equation of motion at the quantum level.

The important point is that, unlike in the ordinary formalism, in the $X$-theory all 1-PI amplitudes, with the exception of those involving insertions of the $T_1$ source, exhibit a manifest weak power-counting [16]: only a finite number of divergent amplitudes exist at each loop order (although increasing with the loop number, as expected in a general effective field theory setting). As for $T_1$-dependent amplitudes, they can be recovered by resumming
the $T_1$-insertions on the Green’s functions at $T_1 = 0$, which, sometimes, can be even done in a closed form.

Once the renormalization of the $X$-theory is achieved, one goes on shell with the $X_1$ and $X_2$ fields, which amounts to a suitable mapping of the sources $\bar{c}^*$ and $T_1$ onto operators depending on $\phi$ and its covariant derivatives. Then, one can immediately read off the UV coefficients of the higher dimensional gauge-invariant operators in the target theory, as now everything is expressed in terms of the original $\phi$ field. We hasten to emphasize that since we are working off-shell the effects of generalized field redefinitions, that are present already at one-loop order, and are not even polynomial for the model at hand [1], need to be correctly accounted for. This is automatically done through the cohomologically trivial invariants of the $X$-theory. In fact, as we will show, the associated coefficients are gauge-dependent (as we will explicitly check by evaluating all the coefficients both in Feynman and Landau gauge), being instrumental in ensuring crucial cancellations leading to the gauge-independence of the coefficients associated to gauge-invariant operators. Notice in fact that since the ensuing analysis is based cohomological results valid for anomaly-free gauge theories, the computational approach presented here can be readily extended to the electroweak gauge group $\text{SU}(2) \times \text{U}(1)$ and, more generally, to any non-anomalous non-Abelian gauge group.

The paper is organized as follows. Our notations and conventions are described in Sect. II, whereas in Sect. III we proceed to evaluate the coefficients of the cohomologically trivial invariants relevant for dimension 6 operators. Sect. IV, V and VI are then devoted to the evaluation of the coefficients of the three classes of gauge invariant operators appearing in the theory: those only depending on the external sources, those mixing external sources and fields and those that only depend on the fields. We finally apply the mapping to the target theory in Sect. VII therefore computing the coefficients of all the UV divergent operators up to dimension 6 in the original (target) theory. Our conclusions and outlook are presented in Sect. VIII. The paper ends with two appendices: Appendix A contains the list of all the independent invariants needed for renormalizing the theory, while the relevant $X$-theory divergent one-loop amplitudes up to dimension 6 are given in Appendix B.
II. NOTATIONS AND SETUP

We refer the reader to [1] for the classical action and the conventions for the fields and external sources. Let us only remark here that the classical action of the $X$-theory depends on a parameter $m^2$ associated with the quartic potential of the field $\phi$ that compensates with the contribution from the quadratic mass term for $X_2$ once one goes on-shell $X_2 \sim \frac{1}{v} \left( \phi \phi - \frac{v^2}{2} \right)$. Hence, Green’s functions in the target theory have to be $m^2$-independent, a fact that provides a very strong check of the computations, due to the ubiquitous presence of $m^2$ both in the invariants and the Feynman amplitudes.

In what follows subscripts denote functional differentiation w.r.t. fields and external sources. Moreover, if not otherwise stated, amplitudes will $\Gamma^{(1)}_{\chi\chi}$, meaning

$$\Gamma^{(1)}_{\chi\chi} \equiv \left. \frac{\delta^2 \Gamma^{(1)}}{\delta \chi(-p) \delta \chi(p)} \right|_{p=0}. \tag{2.1}$$

A bar denotes the UV divergent part of the corresponding amplitude in the Laurent expansion around $\epsilon = 4 - D$, with $D$ the space-time dimension. Dimensional regularization is always implied, with amplitudes evaluated by means of the packages FeynArts and FormCalc [17, 18]. As already remarked, all amplitudes will be evaluated in the Feynman ($\xi = 1$, with $\xi$ the gauge fixing parameter) and Landau ($\xi = 0$) gauge; this will allow to explicitly check the gauge cancellations in gauge invariant operators.

The UV divergent contributions to one-loop amplitudes form a local functional (in the sense of formal power series) aptly denoted by $\bar{\Gamma}^{(1)}$. In particular, $\bar{\Gamma}^{(1)}$ belongs to the kernel of $S_0$, i.e.

$$S_0(\bar{\Gamma}^{(1)}) = 0, \tag{2.2}$$

where $S_0$ is the linearized ST operator

$$S_0(\bar{\Gamma}^{(1)}) = \int d^4x \left[ \frac{\partial \mu \omega}{\delta A_\mu} \frac{\delta \bar{\Gamma}^{(1)}}{\delta A_\mu} + e \omega (\sigma + v) \frac{\delta \bar{\Gamma}^{(1)}}{\delta \chi} - e \omega \chi \frac{\delta \bar{\Gamma}^{(1)}}{\delta \sigma} + b \frac{\delta \bar{\Gamma}^{(1)}}{\delta \bar{\omega}} \right]
+ \frac{\delta \Gamma^{(0)}}{\delta \sigma} \frac{\delta \bar{\Gamma}^{(1)}}{\delta \sigma^*} + \frac{\delta \Gamma^{(0)}}{\delta \chi} \frac{\delta \bar{\Gamma}^{(1)}}{\delta \chi^*}]
= s \bar{\Gamma}^{(1)} + \int d^4x \left[ \frac{\delta \Gamma^{(0)}}{\delta \sigma} \frac{\delta \bar{\Gamma}^{(1)}}{\delta \sigma^*} + \frac{\delta \Gamma^{(0)}}{\delta \chi} \frac{\delta \bar{\Gamma}^{(1)}}{\delta \chi^*} \right], \tag{2.3}$$

which acts as the BRST differential $s$ on the fields of the theory while mapping the antifields into the classical equations of motion of their corresponding fields. Then, the nilpotency of
\( S_0 \) ensures that \( \Gamma^{(1)} \) is the sum of a gauge-invariant functional \( \mathcal{F}^{(1)} \) and a cohomologically trivial contribution \( S_0(\mathcal{Y}^{(1)}) \):

\[
\Gamma^{(1)} = \mathcal{F}^{(1)}_{gi} + S_0(\mathcal{Y}^{(1)}). \tag{2.4}
\]

As a result, we only need to determine the invariants contributing to \( \mathcal{F}^{(1)}_{gi} \) and \( \mathcal{Y}^{(1)} \) that will induce in the target theory operators of dimension less or equal to 6.

To that end we first need to consider how the mapping affects the external sources \( \bar{c}^* \), \( T_1 \).

From Eq. (6.2) of [1], the latter are replaced effectively (i.e., once the equations of motion for the fields \( X_{1,2} \) are imposed) by

\[
\bar{c}^* \rightarrow -\frac{(M^2 - m^2)}{v^2} (\phi^\dagger \phi - \frac{v^2}{2}) + \frac{g}{v\Lambda} (D^\mu \phi)^\dagger D_\mu \phi; \quad T_1 \rightarrow \frac{g}{v\Lambda} (\phi^\dagger \phi - \frac{v^2}{2}). \tag{2.5}
\]

Since the right-hand side (r.h.s.) of the above equation contains operators of dimension at least 2, in order to obtain target operators of up to dimension 6 it is clear that we need to consider amplitudes with up to 3 external sources \( \bar{c}^* \) and \( T_1 \). Equivalently, we can assign dimension 2 to both \( \bar{c}^* \) and \( T_1 \) and use it in order to identify the mixed fields-external sources invariants that will contribute to target operators of up to dimension 6. For instance

\[
\int d^4x \bar{c}^* \left( \phi^\dagger \phi - \frac{v^2}{2} \right) \] would project onto

\[
\int d^4x \bar{c}^* \left( \phi^\dagger \phi - \frac{v^2}{2} \right) \rightarrow -\frac{(M^2 - m^2)}{v^2} \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2 + \frac{g}{v\Lambda} \int d^4x (D^\mu \phi)^\dagger D_\mu \phi \left( \phi^\dagger \phi - \frac{v^2}{2} \right), \tag{2.6}
\]

whereas \( \int d^4x \bar{c}^* \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2 \) would give rise to

\[
\int d^4x \bar{c}^* \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2 \rightarrow -\frac{(M^2 - m^2)}{v^2} \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^3, \tag{2.7}
\]

where we have neglected the covariant kinetic term in the first term of Eq. (2.5) since it would generate a dimension 8 operator.

Finally, the coefficients of the three possible types of invariants contributing to the \( X \)-theory functional \( \mathcal{F}^{(1)}_{gi} \) will be indicated with \( \lambda_i \) (combinations of the field strength, its derivatives and \( \phi \) and its covariant derivatives of up to dimension 6), \( \theta_i \) (combinations of external sources and fields) and \( \vartheta_i \) (combinations of external sources only). The complete list of invariants is reported in Appendix A.
III. COHOMOLOGICALLY TRIVIAL INVARIANTS

Before addressing the evaluation of the coefficients of the gauge invariants, it is necessary to fix the coefficients \( \rho_1 \) of the cohomologically trivial invariants contributing to \( S_0(\Gamma^{(1)}) \). Taking into account the bounds on the dimensions, this requires to consider two invariants at \( T_1 = 0 \), namely

\[ \rho_0 S_0 \int d^4x [\sigma^*(\sigma + v) + \chi^*\chi]; \quad \rho_1 S_0 \int d^4x (\sigma^*\sigma + \chi^*\chi). \quad (3.1) \]

A. Generalized field redefinitions

To begin with let us observe that Eq. (2.3) implies

\[ \rho_1 S_0 \int d^4x (\sigma^*\sigma + \chi^*\chi) \supset -ev\rho_1 \int d^4x \chi^*\omega. \quad (3.2) \]

Therefore, the coefficient \( \rho_1 \) associated to this invariant is controlled by the single amplitude \( \Gamma^{(1)}_{\chi^*\omega} \). Indeed, Eq. (3.2) demands that

\[ ev\rho_1 = -\Gamma^{(1)}_{\chi^*\omega}, \quad (3.3) \]

or, using the result (B2a),

\[ \rho_1 = \frac{M_\Lambda^2}{8\pi^2v^2} \frac{1}{\epsilon}(1 - \delta_{\xi_0}), \quad (3.4) \]

with \( \delta_{\xi_0} = \delta_{\eta_0} = 1 \) in the Landau gauge and \( \delta_{\xi_0} = \delta_{\eta_0} = 0 \) in the Feynman gauge. Notice that this result implies that there are no pure field redefinitions in Landau gauge, \( i.e. \), the v.e.v. renormalizes in the same way as the fields, as we will soon show.

Finally, repeated insertions of the source \( T_1 \) resum to

\[ \rho_1 S_0 \int d^4x \frac{1}{1 + T_1} (\sigma^*\sigma + \chi^*\chi). \quad (3.5) \]

A comment is in order here. In the standard formalism one should consider the effect of the generalized field redefinitions in the target theory, which, as explained in Ref.[1], is the one induced by Eq. (3.5). This implies that the fields \( \sigma \) and \( \chi \) undergo the transformation

\[ \sigma \rightarrow \sigma + \frac{\rho_1}{1 + \frac{\alpha}{\lambda}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)} \sigma; \quad \chi \rightarrow \chi + \frac{\rho_1}{1 + \frac{\alpha}{\lambda}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)} \chi. \quad (3.6) \]

This would be a rather involved task, which is however simplified in the approach developed here, since all the combinatorics is automatically taken into account via the renormalization of the \( X \)-theory, through the cohomologically trivial invariant Eq. (3.5).
B. Tadpoles

The tadpoles $\overline{\Gamma}^{(1)}_{\sigma}, \overline{\Gamma}^{(1)}_{\bar{c}^*}$ allow to fix the coefficients of three invariants:

$$\rho_0 S_0 \int d^4x \left[ \sigma^* (\sigma + v) + \chi^* \chi \right] + \lambda_1 \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right) + \vartheta_1 \int d^4x \overline{c}^* .$$

Indeed, Eq. (3.7) gives rise to the equations

$$-m^2 v \rho_0 + v \lambda_1 = \overline{\Gamma}^{(1)}_{\sigma} ; \quad (3.8a)$$

$$\rho_0 v^2 + \vartheta_1 = \overline{\Gamma}^{(1)}_{\bar{c}^*} . \quad (3.8b)$$

Direct inspection of the one-loop results (B1a) and (B1c) shows that, in the Feynman gauge, it is consistent to set $\rho_0 |_{\xi=1} = 0$, thus yielding the results

$$\lambda_1 |_{\xi=1} = \frac{1}{v} \overline{\Gamma}^{(1)}_{\sigma} |_{\xi=1} = \frac{1}{16\pi^2 v^2 \epsilon} \frac{1}{\epsilon} \left[ m^2 (M^2 + M_A^2) + 2(M^4 + 3M_A^4) \right] , \quad (3.9a)$$

$$\vartheta_1 |_{\xi=1} = \overline{\Gamma}^{(1)}_{\bar{c}^*} |_{\xi=1} = -\frac{M^2 + M_A^2}{16\pi^2} \frac{1}{\epsilon} . \quad (3.9b)$$

On the other hand, since $\lambda_1$ must be gauge invariant, Eq. (3.8a) implies

$$\rho_0 = \frac{1}{m^2 v} \left( v \lambda_1 - \overline{\Gamma}^{(1)}_{\sigma} \right) = \frac{M_A^2}{16v^2 \pi^2} \frac{1}{\epsilon} \delta \xi_0 , \quad (3.10)$$

whereas Eq. (3.8b) furnishes a consistency condition that can be easily checked. Notice in particular that Eq. (3.8b) shows that $\vartheta_1$ is gauge independent (as it should) since the gauge dependence in $\overline{\Gamma}^{(1)}_{\bar{c}^*}$ is cancelled by the one in $\rho_0$. Finally, using Eq. (3.10) and the gauge independence of $\vartheta_1$, Eq. (3.8b) can be recast in the form

$$-\frac{m^2}{v} \left( \overline{\Gamma}^{(1)}_{\bar{c}^*} |_{\xi=0} - \overline{\Gamma}^{(1)}_{\bar{c}^*} |_{\xi=1} \right) = \overline{\Gamma}^{(1)}_{\sigma} |_{\xi=0} - \overline{\Gamma}^{(1)}_{\sigma} |_{\xi=1} . \quad (3.11)$$

Next, we need to consider the insertion of one and two sources $T_1$ on tadpole amplitudes. Starting from a single insertion, the relevant projection equation becomes

$$\rho_0 T_1 S_0 \int d^4x T_1 [\sigma^* (\sigma + v) + \chi^* \chi] + \theta_2 \int d^4x T_1 \left( \phi^\dagger \phi - \frac{v^2}{2} \right) + \vartheta_1 \int d^4x \overline{c}^* T_1$$

$$\delta \int d^4x \left[ (-m^2 v \rho_0 + v \theta_2) T_1 \sigma + (v^2 \rho_0 + \vartheta_1) \overline{c}^* T_1 \right] . \quad (3.12)$$

As before, one obtains two equations

$$-m^2 v \rho_0 + v \theta_2 = \overline{\Gamma}^{(1)}_{T_1 \sigma} , \quad (3.13a)$$

$$v^2 \rho_0 + \vartheta_1 = \overline{\Gamma}^{(1)}_{\overline{c}^* T_1} , \quad (3.13b)$$
which is most easily solved in the Feynman gauge in which $\rho_{0T_1}|_{\xi=1} = 0$, and therefore, using the results (B2h) and (B2f),

$$\theta_2|_{\xi=1} = \frac{1}{v} \Gamma_{T_1\sigma}^{(1)}|_{\xi=1} = -\frac{1}{8\pi^2v^2} \left[ m^2(M^2 + M_A^2) + 2(M^4 - 3M_A^4) \right] \frac{1}{\epsilon}, \quad (3.14a)$$

$$\vartheta_7|_{\xi=1} = \Gamma_{e^*T_1}^{(1)}|_{\xi=1} = \frac{(M^2 + M_A^2)}{8\pi^2} \frac{1}{\epsilon}. \quad (3.14b)$$

Then, using the fact that $\theta_2$ is gauge invariant, Eq. (3.13a) can be used to fix the coefficient $\rho_{0T_1}$, obtaining

$$\rho_{0T_1} = \frac{1}{m^2v} \left( v\theta_2 - \Gamma_{T_1\sigma}^{(1)} \right) = -\frac{M_A^2}{8\pi^2v^2}\frac{1}{\epsilon} \delta_{\xi=0}, \quad (3.15)$$

which, once inserted in Eq. (3.13b) shows that $\vartheta_7$ is gauge invariant, thus allowing to recast the condition (3.13b) in the form

$$-\frac{m^2}{v} \left( \Gamma_{e^*T_1}^{(1)}|_{\xi=0} - \Gamma_{e^*T_1}^{(1)}|_{\xi=1} \right) = \Gamma_{T_1\sigma}^{(1)}|_{\xi=0} - \Gamma_{T_1\sigma}^{(1)}|_{\xi=1}, \quad (3.16)$$

in complete analogy with Eq. (3.11).

Finally, for the case of two $T_1$-insertions, the relevant projection equation reads

$$\rho_{0T_1} \int d^4x T_1^2 S_0 \int d^4x \left[ \sigma^*(\sigma + v) + \chi^*\chi \right] + \theta_{12} \int d^4x T_1^2 \left( \phi^*\phi - \frac{v^2}{2} \right) + \frac{\vartheta_{11}}{2} \int d^4x e^*T_1^2 \supset \int d^4x \left[ (-m^2v\rho_{0T_1} + v\theta_{12})\sigma T_1^2 + (v^2\rho_{0T_1} + \frac{\vartheta_{11}}{2})e^*T_1^2 \right], \quad (3.17)$$

giving rise to the conditions

$$2(-m^2v\rho_{0T_1} + v\theta_{12}) = \Gamma_{\sigma T_1T_1}^{(1)}, \quad (3.18a)$$

$$2v^2\rho_{0T_1} + \vartheta_{11} = \Gamma_{e^*T_1T_1}^{(1)}. \quad (3.18b)$$

In the Feynman gauge $\rho_{0T_1}|_{\xi=1} = 0$, so that, using Eqs. (B3l) and (B3b)

$$\theta_{12}|_{\xi=1} = \frac{1}{2v} \Gamma_{\sigma T_1T_1}^{(1)}|_{\xi=1} = \frac{1}{16\pi^2v^2} \left[ m^2(3M^2 + 2M_A^2) + 6(M^4 + M_A^4) \right] \frac{1}{\epsilon}, \quad (3.19a)$$

$$\vartheta_{11}|_{\xi=1} = \Gamma_{e^*T_1T_1}^{(1)}|_{\xi=1} = -\frac{3M^2 + 2M_A^2}{8\pi^2} \frac{1}{\epsilon}. \quad (3.19b)$$

Using then the gauge independence of $\theta_{12}$ we obtain, from Eq. (3.18a)

$$\rho_{0T_1}^2 = \frac{1}{2m^2v} \left( 2v\theta_{12} - \Gamma_{\sigma T_1T_1}^{(1)} \right) = \frac{M_A^2}{8\pi^2v^2} \frac{1}{\epsilon} \delta_{\xi=0}, \quad (3.20)$$
which, once inserted in Eq. (3.18b) shows that \( \vartheta_{11} \) is also gauge invariant, so that the condition (3.18b) reads

\[
-\frac{m^2}{v} \left( \Gamma^{(1)}_{e^*T_1T_1} \bigg|_{\xi=0} - \Gamma^{(1)}_{e^*T_1T_1} \bigg|_{\xi=1} \right) = \Gamma^{(1)}_{\sigma T_1 T_1} \bigg|_{\xi=0} - \Gamma^{(1)}_{\sigma T_1 T_1} \bigg|_{\xi=1}. \tag{3.21}
\]

We remark that resummation of the \( T_1 \)-insertions is not at work for the tadpoles in the Landau gauge since the loop with a massless Goldstone field in \( \Gamma^{(1)}_{e^*} \) and \( \Gamma^{(1)}_{\sigma} \) happens to be zero in dimensional regularization.

In the Landau gauge there is no pure field redefinition since \( \rho_1|_{\xi=0} = 0 \). On the other hand the invariant

\[
\rho_0 S_0 \int d^4 x \left[ \sigma^* (\sigma + v) + \chi^* \chi \right], \tag{3.22}
\]

shows that in Landau gauge also the v.e.v. \( v \) renormalizes in the same way as the field \( \phi \). This is a well-known fact in spontaneously broken gauge theories [19].

IV. THE PURE EXTERNAL SOURCES SECTOR

We now move to the pure external sources sector. These invariants, which are reported in Eq. (A1), cannot depend on the gauge, as we will explicitly show.

A. Linear terms

\( \vartheta_1 \) has been already fixed in Eq.(3.8b). \( \vartheta_2 \) can be fixed by looking at the \( T_1 \)-tadpole (B1b):

\[
\vartheta_2 = \Gamma^{(1)}_{T_1} = -\frac{(M^4 - 3M_A^4)}{16\pi^2} \frac{1}{\epsilon}. \tag{4.1}
\]

Notice that there are no contributions from cohomologically trivial invariants since there are no linear couplings for \( T_1 \) at tree-level. Consequently \( \Gamma^{(1)}_{T_1} \) is the same both in Landau and in Feynman gauge.

B. Bilinears

\( \vartheta_3 \) is fixed by the 2-point \( \bar{e}^* \)-amplitude Eq. (B2e):

\[
\vartheta_3 = \Gamma^{(1)}_{\bar{e}^*e^*} = \frac{1}{8\pi^2} \frac{1}{\epsilon}. \tag{4.2}
\]
Notice that $\Gamma_{e^* e^*}$ does not develop momentum-dependent divergences and that it does not depend on the gauge.

This is clearly not the case for $\Gamma_{T_1 T_1}$ as Eq. (B2g) shows; we can then read off the coefficients of the different bilinear invariants, obtaining

$$\vartheta_4 = \frac{3}{16\pi^2}(M^4 + M_A^4) \frac{1}{\epsilon},$$

$$\vartheta_5 = \frac{3}{32\pi^2}(M^2 + M_A^2) \frac{1}{\epsilon},$$

$$\vartheta_6 = \frac{1}{32\pi^2} \frac{1}{\epsilon}.$$  \hspace{1cm} (4.3)

Finally, $\vartheta_7$ has been fixed in Eq. (3.14b), while the $p^2$-coefficient of the amplitude $\Gamma_{e^* T_1}$, see (B2f), is gauge independent and implies

$$\vartheta_8 = \frac{1}{16\pi^2} \frac{1}{\epsilon}. \hspace{1cm} (4.4)$$

### C. Trilinears

While $\vartheta_{11}$ has been fixed in Eq. (3.19b), it turns out that the remaining trilinears do not receive contributions from cohomologically trivial invariants. In particular we find

$$\vartheta_9 = 0 \hspace{1cm} (4.5)$$

since $\Gamma_{e^* e^* e^*}$ is UV finite, and, using the results (B3a) and (B3c)

$$\vartheta_{10} = \frac{1}{4\pi^2} \frac{1}{\epsilon}, \hspace{1cm} \vartheta_{12} = \frac{3M^4}{4\pi^2} \frac{1}{\epsilon}. \hspace{1cm} (4.6)$$

### V. THE MIXED EXTERNAL SOURCES-FIELD SECTOR

#### A. The $\theta_1$ and $\theta_2$ coefficients

The coefficients $\theta_1$ and $\theta_2$ can be fixed by evaluating the three-point functions $\Gamma_{e^* \chi \chi}$ and $\Gamma_{T_1 \chi \chi}$ at zero momentum. Since

$$\rho_0 S_0 \int d^4 x \left[ \sigma^*(\sigma + v) + \chi^* \chi \right] + \rho_1 S_0 \int d^4 x \left[ \sigma^* \sigma + \chi^* \chi \right] + \theta_1 \int d^4 x \left[ 2\phi^* \phi - \frac{v^2}{2} \right]$$

$$\ni \int d^4 x \left( \rho_0 + \rho_1 + \frac{\theta_1}{2} \right) \bar{c} \chi^2, \hspace{1cm} (5.1)$$
one arrives at the relation

\[ 2\rho_0 + 2\rho_1 + \theta_1 = \Gamma^{(1)}_{e\chi\chi}. \]  

(5.2)

Then, using Eqs. (3.4), (3.10) and (B3h), we immediately obtain the result

\[ \theta_1 = \Gamma^{(1)}_{e\chi\chi} - 2(\rho_0 + \rho_1) = -\frac{m^2 + M^2 + M_A^2}{8\pi^2 v^2} \frac{1}{\epsilon}, \]

(5.3)

which, due to the compensation of the gauge parameter dependence between the amplitude and the coefficients \(\rho_0\) and \(\rho_1\) turns out to be gauge independent, as it should. In a similar fashion, considering the combination

\[ \rho_0 T_1 S_0 \int d^4 x T_1 [\sigma^*(\sigma + v) + \chi^*\chi] + \theta_2 \int d^4 x T_1 \left( \phi^\dagger \phi - \frac{v^2}{2} \right) \supset \int d^4 x \left( -\rho_0 T_1 m^2 + \frac{\theta_2}{2} \right) T_1 \chi^2, \]

(5.4)

we get

\[ -\rho_0 T_1 m^2 + \theta_2 = \Gamma^{(1)}_{T_1 \chi\chi}, \]  

(5.5)

or, using the result (B3i)

\[ \theta_2 = -\frac{m^2(M^2 + M_A^2) + 2(M^4 - 3M_A^4)}{8\pi^2 v^2} \frac{1}{\epsilon}, \]  

(5.6)

and again one obtains the gauge independence of this parameter as a result of the cancellation of the gauge-dependence between the 1-PI amplitude and the coefficient \(\rho_0 T_1\).

The validity of these results can be checked against the relations provided by 1-PI amplitudes involving one source and one external \(\sigma\)-field. For example considering the \(e^*\sigma\) case, we find

\[ \rho_0 S_0 \int d^4 x \left[ \sigma^* (\sigma + v) + \chi^* \chi \right] + \rho_1 S_0 \left( \int d^4 x \left( \sigma^* \sigma + \chi^* \chi \right) \right) + \theta_1 \int d^4 x e^* \left( \phi^\dagger \phi - \frac{v^2}{2} \right) \supset \int d^4 x v \left( 2\rho_0 + \rho_1 + \theta_1 \right) e^*\sigma, \]  

(5.7)

yielding the relation

\[ v(2\rho_0 + \rho_1 + \theta_1) = \Gamma^{(1)}_{e^*\sigma}, \]  

(5.8)

which can be checked directly using Eqs. (3.4), (3.10) and (5.3). Notice that \(\Gamma^{(1)}_{e^*\sigma}\) is the same in Feynman and Landau gauge, see Eq. (B2i); therefore, since \(\theta_1\) is gauge independent, so must be the combination \(2\rho_0 + \rho_1\), as can be easily verified.
Considering the $T_1\sigma$ amplitudes, we find instead
\[\rho_{0T_1} S_0 \left( \int d^4x T_1 (\sigma (\sigma + v) + \chi^* \chi) \right) + \theta_2 \int d^4x T_1 \left( \phi^+ \phi - \frac{v^2}{2} \right) \]
\[\supset \int d^4x \left(-vm^2\rho_{0T_1} + v\theta_2 \right) T_1 \sigma. \tag{5.9}\]
Thus we get
\[-vm^2\rho_{0T_1} + v\theta_2 = \Gamma^{(1)}_{T_1\sigma}, \tag{5.10}\]
which can be immediately verified using the one-loop result (B2h).

**B. The $\theta_3$ and $\theta_5$ coefficients**

In order to fix $\theta_3$ and $\theta_5$, we need the amplitude $\Gamma^{(1)}_{\bar{c}\chi\chi}$, which can be decomposed in form factors according to
\[\Gamma^{(1)}_{\bar{c}\chi\chi}(p_1, p_2) = \gamma^0_{\bar{c}\chi\chi} + \gamma^1_{\bar{c}\chi\chi}(p_1^2 + p_2^2) + \gamma^2_{\bar{c}\chi\chi}(p_1 \cdot p_2). \tag{5.11}\]
We find
\[\theta_3 \int d^4x \bar{c}^* (D^\mu \phi)^\dagger D_\mu \phi \supset \theta_3 \int d^4x \frac{\bar{c}^*}{2} \partial^\mu \chi \partial_\mu \chi, \tag{5.12a}\]
\[\theta_5 \int d^4x \bar{c}^* \left[ (D^2 \phi)^\dagger \phi + \text{h.c.} \right] \supset \theta_5 \int d^4x \bar{c}^* \chi \Box \chi, \tag{5.12b}\]
which, using the result Eq. (B3h), implies the following identifications
\[\theta_3 = -\gamma^2_{\bar{c}\chi\chi} = -\frac{1}{16\pi^2} \frac{g}{v\Lambda} \left(2 + \frac{g\varepsilon}{\Lambda} \right), \quad \theta_5 = -\gamma^1_{\bar{c}\chi\chi} = -\frac{1}{16\pi^2} \frac{g}{v\epsilon}. \tag{5.13}\]
Notice that both coefficients are the same in Landau and Feynman gauge, as expected.

In this case a consistency check is provided by the three-point function $\Gamma^{(1)}_{\bar{c}\chi\chi A_\mu A_\nu}$, since one has
\[\theta_3 \int d^4x \bar{c}^* (D^\mu \phi)^\dagger D_\mu \phi + \theta_5 \int d^4x \bar{c}^* \left[ (D^2 \phi)^\dagger \phi + \text{h.c.} \right] \supset \int d^4x \frac{M^2_\Lambda}{2} \left( \theta_3 - 2\theta_5 \right) \bar{c}^* A^2, \tag{5.14}\]
so that
\[M^2_\Lambda (\theta_3 - 2\theta_5) g_{\mu\nu} = \left. \Gamma^{(1)}_{\bar{c}\chi\chi A_\mu A_\nu}(p_1, p_2) \right|_{p_1=p_2=0}, \tag{5.15}\]
as can be easily verified with the help of Eq. (B3e).
C. The $\theta_4$ and $\theta_6$ coefficients

In order to fix $\theta_4$ and $\theta_6$ we need the amplitude $\Pi_{T_1}^{(1)}$, which we decompose as before according to

$$\Pi_{T_1}^{(1)}(p_1, p_2) = \gamma_{T_1}^0 + \gamma_{T_1}^1(p_1 + p_2^2) + \gamma_{T_1}^2(p_1 \cdot p_2) + \mathcal{O}(p_4),$$

and the dots denote terms of order $p^4$, which are not needed.

There are two projections to be considered, namely $T_1 \partial^\mu \chi \partial^\nu \chi$ and $T_1 \chi \square \chi$, to which the cohomologically trivial invariants can also contribute. To begin with, observe that there is no contribution coming from the $\rho_1$ coefficient due to a cancellation between the pre-factor $1/1 + T_1$ and the dependence of the classical vertex functional $\Gamma^{(0)}$ on $T_1$; indeed one has

$$\rho_1 S_0 \int d^4 x \frac{1}{1 + T_1} (\sigma^* \sigma + \chi^* \chi) = \rho_1 S_0 \int d^4 x (1 - T_1 + \cdots) (\sigma^* \sigma + \chi^* \chi) \supset \rho_1 \int d^4 x \left(T_1 \partial^\mu \chi \partial^\nu \chi - T_1 \partial^\mu \chi \partial^\nu \chi\right) = 0. \tag{5.17}$$

On the other hand we have

$$\rho_0 S_0 \int d^4 x (\sigma^* (\sigma + v) + \chi^* \chi) + \rho_0 T_1 S_0 \int d^4 x T_1 [\sigma^* (\sigma + v) + \chi^* \chi]$$

$$\supset \int d^4 x (\rho_0 + \rho_0 T_1) T_1 \partial^\mu \chi \partial^\nu \chi, \tag{5.18}$$

with no contribution generated to $T_1 \chi \square \chi$. Therefore we obtain

$$2(\rho_0 + \rho_0 T_1) + \theta_4 = -\frac{\gamma_{T_1}^2}{\epsilon}; \quad \theta_6 = -\frac{\gamma_{T_1}^1}{\epsilon}, \tag{5.19}$$

from which, using Eq. (B3i), we finally get the values

$$\theta_4 = -\frac{1}{32 \pi^2 \nu^2} \left[4 m^2 + (M^2 - M_A^2) \left(4 + 3 \frac{g^2 \nu^2}{\Lambda^2}\right) \right] \frac{1}{\epsilon},$$

$$\theta_6 = -\frac{1}{16 \pi^2 \nu^2} \left[m^2 - 3 M_A^2 + M^2 \left(1 + 2 \frac{g \nu}{\Lambda}\right) \right] \frac{1}{\epsilon}. \tag{5.20a}$$

Similarly to what we have done in the previous case, we can check the results above using the three-point function $\Pi_{T_1 A_\mu A_\nu}^{(1)}$. Indeed we have

$$\theta_4 \int d^4 x T_1 (D^\mu \phi)^\dagger D^\mu \phi + \theta_6 \int d^4 x T_1 [(D^2 \phi)^\dagger \phi + \text{h.c.}] + \rho_0 S_0 \int d^4 x [\sigma^* (\sigma + v) + \chi^* \chi]$$

$$+ \rho_0 T_1 S_0 \int d^4 x T_1 [\sigma^* (\sigma + v) + \chi^* \chi] \supset \int d^4 x \frac{M_A^2}{2} [\theta_4 - 2 \theta_6 + 2(\rho_0 + \rho_0 T_1)] T_1 A^2, \tag{5.21}$$

implying the consistency condition

$$M_A^2 \left[\theta_4 - 2 \theta_6 + 2(\rho_0 + \rho_0 T_1)\right] g_{\mu \nu} = \Pi_{T_1 A_\mu A_\nu}^{(1)}(p_1, p_2) \bigg|_{p_1 = p_2 = 0}. \tag{5.22}$$

the validity of which can be easily verified with the help of Eq. (B3f).
D. The $\theta_7$ and $\theta_8$ coefficients

In this sector the relevant projections are

$$\rho_0 S_0 \int d^4 x [\sigma^+ (\sigma + v) + \chi^+ \chi] + \rho_1 S_0 \int d^4 x (\sigma^+ \sigma + \chi^+ \chi) + \theta_1 \int d^4 x \bar{c}^{+} \left( \phi^{+} \phi - \frac{v^2}{2} \right)$$

$$+ \theta_7 \int d^4 x \left( \phi^{+} \phi - \frac{v^2}{2} \right) \geq \int d^4 x \left( \rho_0 + \rho_1 + \frac{\theta_1}{2} + v^2 \theta_7 \right) \bar{c} \sigma^{2}.$$

(5.23a)

$$\rho_0 T_1 \int d^4 x T_1 [\sigma^+ (\sigma + v) + \chi^+ \chi] - \rho_1 S_0 \int d^4 x T_1 (\sigma^+ \sigma + \chi^+ \chi) + \theta_2 \int d^4 x T_1 \left( \phi^{+} \phi - \frac{v^2}{2} \right)$$

$$+ \theta_8 \int d^4 x \left( \phi^{+} \phi - \frac{v^2}{2} \right) \geq \int d^4 x \left( m^2 \rho_1 - \frac{5}{2} \rho_0 T_1 m^2 + \frac{\theta_2}{2} + v^2 \theta_8 \right) T_1 \sigma^{2}.$$

(5.23b)

yielding the relations

$$2(\rho_0 + \rho_1) + \theta_1 + 2v^2 \theta_7 = \Gamma_{c^{+} \sigma^+}^{(1)}; \quad 2m^2 \rho_1 - 5\rho_0 T_1 m^2 + \theta_2 + 2v^2 \theta_8 = \Gamma_{T_1 \sigma^{+}}^{(1)}.$$

(5.24)

and, finally, the values

$$\theta_7 = 0; \quad \theta_8 = -\frac{1}{8\pi^2 v^4} \left[ m^4 + 2m^2 (M^2 + M_A^2) + 2(M^4 - 3M_A^4) \right] \frac{1}{\epsilon}.$$

(5.25)

see Eqs. (B3j) and (B3m)

E. The $\theta_9$ and $\theta_{10}$ coefficients

The fact that the function $\Gamma_{c^{+} A_{\mu} A_{\nu}}^{(1)}$ turns out to be momentum independent, see Eq. (B3e), implies immediately that

$$\theta_9 = 0.$$

(5.26)

Next, in order to extract the coefficient $\theta_{10}$ one needs first to change the variables to the contractible pairs basis, as explained in [1]. To this end, one replaces the derivatives of the gauge field with a linear combination of the complete symmetrization over the Lorentz indices and a contribution depending on the field strength:

$$\partial_{\nu_1 \ldots \nu_\ell} A_\mu = \partial_{(\nu_1 \ldots \nu_\ell} A_\mu) + \frac{\ell}{\ell + 1} \partial_{(\nu_1 \ldots \nu_{\ell - 1}} F_{\nu_\ell)\mu},$$

(5.27)

where (…) denote complete symmetrization. In the present case it is therefore sufficient to consider the monomial $T_1 \partial^\mu A^\nu \partial_\mu A_\nu$ since, due to Eq. (5.27)

$$T_1 \partial^\mu A^\nu \partial_\mu A_\nu = T_1 \partial^\mu (A^{(\nu}) A^{\nu)} + \frac{T_1}{4} F^{\mu \nu} F_{\mu \nu}.$$

(5.28)
Then, after we decompose the amplitude $\Gamma^{(1)}_{T_1 A_\mu A_\nu}$ according to

$$\Gamma^{(1)}_{T_1 A_\mu A_\nu}(p_1, p_2) = [\gamma^0_{T_1 AA} - 2\gamma^1_{T_1 AA}(p_1 \cdot p_2) + \gamma^2_{T_1 AA}(p_1^2 + p_2^2)]g^{\mu\nu}$$

$$+ \gamma^3_{T_1 AA}p_1^\mu p_2^\nu + \gamma^4_{T_1 AA}p_1^\nu p_2^\mu,$$

Eq. (B3f) gives

$$\theta_{10} = \frac{\gamma^1_{TAA}}{4} = \frac{M_A^2}{128\pi^2} \frac{g^2}{v A^2} \frac{1}{\epsilon}.$$  \hspace{1cm} (5.30)

**F. The $\theta_{11}$, $\theta_{12}$ and $\theta_{13}$ coefficients**

The coefficient $\theta_{12}$ has been fixed already, see Eq. (3.19a); on the other hand, $\theta_{11}$ is determined by the projection of

$$\theta_{11} \int d^4x \bar{c}^* T_1 \left( \phi^\dagger \phi - \frac{v^2}{2} \right) - \rho_1 S_0 \int d^4x T_1 (\sigma^* \sigma + \chi^* \chi) + \rho_0 T_1 S_0 \int d^4x T_1 [\sigma^*(\sigma + v) + \chi^* \chi]$$

$$\supset \int d^4x \left( v \theta_{11} - v \rho_1 + 2 v \rho_0 T_1 \right) \bar{c}^* T_1 \sigma.$$ \hspace{1cm} (5.31)

yielding

$$\theta_{11} = \frac{1}{v} \left( \bar{\Gamma}^{(1)}_{\bar{c}^* T_1 \sigma} + v \rho_1 - 2 v \rho_0 T_1 \right) = \frac{1}{4\pi^2 v^2} (m^2 + M^2 + M_A^2) \frac{1}{\epsilon},$$ \hspace{1cm} (5.32)

where the one-loop result (B3d) has been used. Finally,

$$\theta_{13} \int d^4x (\bar{c}^*)^2 \left( \phi^\dagger \phi - \frac{v^2}{2} \right) \supset \int d^4x \theta_{13} v \sigma (\bar{c}^*)^2,$$ \hspace{1cm} (5.33)

which implies

$$\theta_{13} = \frac{1}{2v} \bar{\Gamma}^{(1)}_{\bar{c}^* \sigma} = 0,$$ \hspace{1cm} (5.34)

as this amplitude turns out to be UV finite.

**VI. THE GAUGE-IN Variant FIELD SECTOR**

The last sector we need to consider is finally the one of gauge invariants containing only the fields.
A. The $\lambda_2$ and $\lambda_3$ coefficients

While the coefficient $\lambda_1$ has been already fixed, see Eq. (3.7), $\lambda_2$ and $\lambda_3$ can be fixed by considering the two- and three-point $\sigma$ amplitudes. The relevant projection equation are

\[
\rho_0 S_0 \int d^4x \left[ \sigma^*(\sigma + v) + \chi^* \chi \right] + \rho_1 S_0 \int d^4x \left( \sigma^*\sigma + \chi^* \chi \right) + \lambda_1 \int d^4x \left( \phi^+ \phi - \frac{v^2}{2} \right)
\]
\[
+ \lambda_2 \int d^4x \left( \phi^+ \phi - \frac{v^2}{2} \right)^2 \supset \int d^4x \left( v^2 \lambda_2 + \frac{1}{2} \lambda_1 - m^2 \rho_1 - \frac{5}{2} m^2 \rho_0 \right) \sigma^2,
\]  
(6.1a)

\[
\rho_0 S_0 \int d^4x \left[ \sigma^*(\sigma + v) + \chi^* \chi \right] + \rho_1 S_0 \int d^4x \left( \sigma^*\sigma + \chi^* \chi \right) + \lambda_2 \int d^4x \left( \phi^+ \phi - \frac{v^2}{2} \right)^2
\]
\[
+ \lambda_3 \int d^4x \left( \phi^+ \phi - \frac{v^2}{2} \right)^3 \supset \int d^4x \left( 2v^3 \lambda_3 + 2v \lambda_2 - \frac{3m^2}{v} \rho_1 - \frac{4m^2}{v} \rho_0 \right) \sigma^3,
\]  
(6.1b)

yielding

\[
2v^2 \lambda_2 + \lambda_1 - 2m^2 \rho_1 - 5m^2 \rho_0 = \Gamma_{\sigma\sigma}^{(1)},
\]  
(6.2a)

\[
6v^3 \lambda_3 + 6v \lambda_2 - \frac{9m^2}{v} \rho_1 - \frac{12m^2}{v} \rho_0 = \Gamma_{\sigma\sigma\sigma}^{(1)}.
\]  
(6.2b)

Eqs. (B2c) and (B3n) implies then the following results

\[
\lambda_2 = \frac{1}{16\pi^2 v^4} \left[ m^4 + 2m^2(M^2 + M_A^2) + 2(M^4 + 3M_A^4) \right] \frac{1}{\epsilon},
\]
\[
\lambda_3 = 0.
\]  
(6.3a)

The values of these coefficients can be checked by looking at the $\Gamma_{\sigma\chi\chi}^{(1)}$ and $\Gamma_{\sigma\sigma\chi\chi}^{(1)}$ amplitudes, for which the projection equation

\[
\rho_0 S_0 \int d^4x \left[ \sigma^*(\sigma + v) + \chi^* \chi \right] + \rho_1 S_0 \int d^4x \left( \sigma^*\sigma + \chi^* \chi \right) + \lambda_2 \int d^4x \left( \phi^+ \phi - \frac{v^2}{2} \right)^2
\]
\[
\supset \int d^4x \left( v^2 \lambda_2 + \frac{1}{2} \lambda_1 - m^2 \rho_1 - \frac{5}{2} m^2 \rho_0 \right) \sigma^2 \sigma^2 + \int d^4x \left( v^2 \lambda_2 + \frac{1}{2} \lambda_1 - m^2 \rho_1 - \frac{5}{2} m^2 \rho_0 \right) \sigma^2 \chi^2,
\]  
(6.4)

gives rise to the consistency conditions

\[
2v \lambda_2 - \frac{3m^2}{v} \rho_1 - \frac{4m^2}{v} \rho_0 = \Gamma_{\sigma\chi\chi}^{(1)},
\]
\[
2 \lambda_2 - \frac{4m^2}{v^2} \rho_1 - \frac{4m^2}{v^2} \rho_0 = \Gamma_{\sigma\sigma\chi\chi}^{(1)}.
\]  
(6.5)

which, using Eqs. (B3o) and (B4a), can be easily proven to be fulfilled.
B. The $\lambda_4$ and $\lambda_5$ coefficients

These coefficients are fixed by the 2-point Goldstone amplitude, which is controlled by the invariants

$$\rho_0 S_0 \int d^4x [\sigma^* (\sigma + v) + \chi^* \chi] + \rho_1 S_0 \int d^4x (\sigma^* \sigma + \chi^* \chi) + \lambda_1 \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right)$$

$$+ \lambda_4 \int d^4x (D^\mu \phi)^\dagger D_\mu \phi + \lambda_5 \int d^4x \phi^\dagger [D^2 + D^\mu D_\mu + D^\mu D_\nu D_\nu] \phi$$

$$\supset \int d^4x \left[ \frac{1}{2} (\lambda_1 - m^2 \rho_0) \chi^2 + (\rho_0 + \rho_1 + \frac{\lambda_4}{2}) \partial^\mu \chi \partial_\mu \chi + \frac{3}{2} \lambda_5 \chi^2 \right], \quad (6.6)$$

which gives rise to the following projections

$$\lambda_1 - m^2 \rho_0 = \Gamma_{\chi \chi (1)} \bigg|_{p^2 = 0}; \quad 2(\rho_0 + \rho_1) + \lambda_4 = \frac{\partial \Gamma_{\chi \chi (1)}}{\partial p^2} \bigg|_{p^2 = 0}; \quad 3\lambda_5 = \frac{\partial \Gamma_{\chi \chi (1)}}{\partial (p^2)^2} \bigg|_{p^2 = 0}. \quad (6.7)$$

From the one-loop expression reported in (B2b), we then obtain the gauge-independent coefficients

$$\lambda_4 = -\frac{1}{32\pi^2 v^2} \left[ \frac{g v}{\Lambda} \left( 4 - \frac{g v}{\Lambda} \right) M^2 + M_A^2 \left( 16 + 12 \frac{g v}{\Lambda} + 3 \frac{g^2 v^2}{\Lambda^2} \right) \right] \frac{1}{\epsilon}, \quad (6.8a)$$

$$\lambda_5 = \frac{g^2}{96\pi^2 \Lambda^2} \frac{1}{\epsilon}. \quad (6.8b)$$

C. The $\lambda_6$ and $\lambda_7$ coefficients

The relevant Green’s function for fixing these coefficients is the four-point Goldstone amplitude since

$$\rho_0 S_0 \int d^4x [\sigma^* (\sigma + v) + \chi^* \chi] + \rho_1 S_0 \int d^4x (\sigma^* \sigma + \chi^* \chi) + \lambda_2 \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2$$

$$+ \lambda_6 \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right) \left( \phi^\dagger D^2 \phi + (D^2 \phi)^\dagger \phi \right) + \lambda_7 \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right) (D^\mu \phi)^\dagger D_\mu \phi$$

$$\supset \int d^4x \left\{ \left[ \frac{\lambda_2}{4} - (\rho_0 + \rho_1) \frac{m^2}{2 v^2} \right] \chi^4 + \frac{\lambda_6}{2} \chi^3 \Box \chi + \frac{\lambda_7}{4} \chi^2 \partial^\mu \chi \partial_\mu \chi \right\}, \quad (6.9)$$

yielding

$$6\lambda_2 - \frac{12 m^2}{v^2} (\rho_0 + \rho_1) - 3\lambda_6 \sum_{i=1}^4 p_i^2 - \lambda_7 \sum_{i<j} p_i p_j = \Gamma_{\chi \chi \chi \chi (1)} (p_i). \quad (6.10)$$

Notice that we keep the momentum dependence of the four point $\chi$ amplitude on the right-hand side. A remark is in order here. Before attempting to extract the coefficients of the
momenta polynomial on the left-hand side of Eq. (6.10), we need to take into account the fact that FeynArts and FormCalc internally implement momentum conservation, so the amplitude is only known on the hyperplane $\sum_i p_i = 0$. Hence we eliminate $p_4$ in favor of the remaining momenta, $p_4 = -\sum_{i=1}^3 p_i$, so that Eq. (6.10) becomes

$$6\lambda_2 - \frac{12m^2}{v^2} (\rho_0 + \rho_1) - (6\lambda_6 - \lambda_7) \left( \sum_{i=1}^3 p_i^2 + \sum_{i<j} p_ip_j \right) = \Gamma^{(1)}_{\chi\chi\chi\chi}(p_1, p_2, p_3).$$

(6.11)

Then the condition

$$6\lambda_2 - \frac{12m^2}{v^2} (\rho_0 + \rho_1) = \left. \Gamma^{(1)}_{\chi\chi\chi\chi} \right|_{p_i=0},$$

(6.12)

is easily verified, see Eq. (B4b). On the other hand, we notice that the restriction of $\Gamma^{(1)}_{\chi\chi\chi\chi}$ on the momentum conservation hyperplane only fixes the combination $6\lambda_6 - \lambda_7$, and an additional amplitude needs to be considered to fix the two coefficients separately.

To this end, let us consider the two point $\sigma$-amplitude, with the following projection on the derivative-dependent sector

$$\rho_0 S_0 \int d^4x [\sigma^\dagger (\sigma + \v) + \chi^\dagger \chi] + \rho_1 S_0 \int d^4x (\sigma^\dagger \sigma + \chi^\dagger \chi) + \lambda_4 \int d^4x (D^\mu \phi)\dagger D_\mu \phi$$

$$+ \lambda_6 \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right) \left( \phi^\dagger D^2 \phi + (D^2 \phi)^\dagger \phi \right) + \Gamma_\sigma \int d^4x \left[ \left( \frac{\lambda_4}{2} + \rho_0 + \rho_1 \right) \partial^\mu \sigma \partial_\mu \sigma + v^2 \lambda_6 \sigma \Box \sigma \right],$$

(6.13)

leading to the condition

$$2v^2 \lambda_6 - \lambda_4 - 2(\rho_0 + \rho_1) = - \left. \frac{\partial \Gamma^{(1)}_{\sigma\sigma}}{\partial p^2} \right|_{p^2=0}.$$

(6.14)

This gives then the result, see Eq. (B2c)

$$\lambda_6 = \frac{1}{64\pi^2 v^3} \frac{g}{\Lambda} \left[ 4m^2 + (M^2 - 3M_A^2) \left( 4 + \frac{g^2 v}{\Lambda} \right) \right] \frac{1}{\epsilon},$$

(6.15)

which in combination with Eqs. (6.11) and (B4b) yields finally

$$\lambda_7 = \frac{1}{32\pi^2 v^3} \frac{g}{\Lambda} \left[ 2m^2 \left( 2 + \frac{g^2 v}{\Lambda} \right) - M^2 \left( 4 - 5\frac{g^2 v}{\Lambda} \right) - 3M_A^2 \left( 12 + 5\frac{g^2 v}{\Lambda} \right) \right] \frac{1}{\epsilon}.$$

(6.16)

We can check this result against the projections on the monomials $\sigma \chi \Box \chi, \sigma \partial^\mu \chi \partial_\mu \chi$, namely

$$\lambda_6 \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right) \left( \phi^\dagger D^2 \phi + (D^2 \phi)^\dagger \phi \right) + \lambda_7 \int d^4x \left( \phi^\dagger \phi - \frac{v^2}{2} \right) (D^\mu \phi)^\dagger D_\mu \phi$$

$$+ \int d^4x \left( v\lambda_6 \sigma \chi \Box \chi + \frac{v\lambda_7}{2} \sigma \partial^\mu \chi \partial_\mu \chi \right).$$

(6.17)
After eliminating the $\sigma$-momentum in favour of the remaining two by using momentum conservation, the resulting amplitude can be expanded as

$$
\Gamma_{\sigma\chi\chi}^{(1)}(p_1, p_2) = \gamma_{\sigma\chi\chi} + \gamma_{\sigma\chi\chi}^1(p_1^2 + p_2^2) + \gamma_{\sigma\chi\chi}^2 p_1 \cdot p_2 + \mathcal{O}(p^4)
$$

Eq. (6.17) then implies the consistency conditions

$$
v\lambda_6 + \gamma_{\sigma\chi\chi}^1 = 0; \quad -v\lambda_7 + \gamma_{\sigma\chi\chi}^2 = 0,
$$

which can be easily verified using the result Eq. (B3o).

D. The $\lambda_8$ and $\lambda_9$ coefficients

These coefficients are controlled by the $AA$ amplitude which also provides a non-trivial example of the contractible pairs technique. Indeed, the two-point function of the Goldstone field fixes the coefficient $\lambda_5$ via the projection on the monomial $\int d^4 x \chi \square^2 \chi$; on the other hand, the $\lambda_5$ invariant admits also a non-trivial expansion in power of the gauge field, precisely accounting for the non-transverse form factors of $\Gamma_{A\mu A\nu}^{(1)}$.

To see this in detail, observe that the relevant invariants are

$$
\rho_0 S_0 \int d^4 x \left[ \sigma^* (\sigma + v) + \chi^* \chi \right] + \rho_1 S_0 \int d^4 x \left( \sigma^* \sigma + \chi^* \chi \right)
$$

$$
+ \lambda_5 \int d^4 x \phi^* [(D)^2 + D^\mu D^\nu D_\mu D_\nu + D^\mu D^2 D_\mu] \phi + \lambda_8 \int d^4 x F^2_{\mu\nu} + \lambda_9 \int d^4 x \partial_\mu F_{\mu\nu} \partial_\rho F^\rho\nu
$$

$$
\supset \int d^4 x \left[ \left( \rho_0 + \frac{\lambda_4}{2} \right) e^2 v^2 A^2 - \frac{\lambda_5}{2} e^2 v^2 (2 A^\mu \partial_\mu A + A^\mu \square A_\mu) + \frac{\lambda_8}{2} (\partial^\mu A^\nu - \partial^\nu A^\mu)^2
\right.

$$

$$
+ \lambda_9 (\Box A^\mu - \partial^\mu (\partial A)^2) \right]
$$

There are no contribution of order $p^4$ in $\Gamma_{A\mu A\nu}^{(1)}$, see Eq. (B2d), so

$$
\lambda_9 = 0.
$$

The remaining terms give the projection equation

$$
\left[ e^2 v^2 (2 \rho_0 + \lambda_4) + (2 \lambda_8 + e^2 v^2 \lambda_5) p^2 \right] g^{\mu\nu} + 2 \left( e^2 v^2 \lambda_5 - \lambda_8 \right) p^\mu p^\nu = \Gamma_{A\mu A\nu}^{(1)}(p).
$$

Notice that in the right-hand side of the above equation we keep the momentum dependence of the two point gauge amplitude. From Eqs. (3.10), (6.8a), (6.8b) and (B2d), we see that the above equation is verified with

$$
\lambda_8 = - \frac{M_A^2}{96\pi^2 v^2} \left( 2 + 2 \frac{g v}{\Lambda} + \frac{g^2 v^2}{\Lambda^2} \right) \frac{1}{\epsilon},
$$

which implies that $\lambda_8$ is gauge-independent, as it should.
E. The $\lambda_{10}$ coefficient

This coefficient can be obtained in much the same way as $\theta_{10}$, i.e., by the contractible pair method. Parameterize the amplitude $\Gamma^{(1)}_{\sigma A, A, \nu}$ according to

$$\Gamma^{(1)}_{\sigma A, A, \nu} (p_1, p_2) = [\gamma_{\sigma AA}^0 - 2\gamma_{\sigma AA} p_1 \cdot p_2 + \gamma_{\sigma AA}^2(p_1^2 + p_2^2)] g^\mu\nu + \gamma_{\sigma AA}^3 p_1^\mu p_2^\nu + \gamma_{\sigma AA}^4 p_1^\nu p_2^\mu, \quad (6.24)$$

and extract $\lambda_{10}$ through the form factor $\gamma_{\sigma AA}^1$:

$$\lambda_{10} = \frac{\int d^4x \, F^2_{\mu\nu} (\phi^\dagger \phi - \frac{v^2}{2})}{16 \pi^2 v^2} \supset \lambda_{10} \int d^4x \, 2\sigma \partial^\mu A^\nu \partial_\mu A_\nu. \quad (6.25)$$

We obtain, see Eq. (B3g),

$$\lambda_{10} = \frac{\gamma_{\sigma AA}^1}{4v} = \frac{M_A^2}{128 \pi^2} \frac{g^2}{v^2 \Lambda^2} \left( -4 + \frac{g v}{\Lambda} \right) \frac{1}{\epsilon}. \quad (6.26)$$

Notice in particular that the combination

$$\lambda_{10} + \frac{g}{v \Lambda} \theta_{10} = - \frac{M_A^2}{32 \pi^2} \frac{g^2}{\Lambda^2 v^2} \frac{1}{\epsilon}, \quad (6.27)$$
correctly reproduces the coefficient $c^{(1)}_O$ of [1].

VII. MAPPING

We are now in a position to evaluate the renormalization coefficients of the operators of dimension less or equal to 6 in the target theory. For that purpose one simply needs to map the invariants depending on the external sources by applying the substitution rules (2.5) and collecting the contributions to the operator one is interested in.

Notice that all the coefficients obtained must be gauge-invariant (as a consequence of the gauge-invariance of the $\theta_i$, $\vartheta_i$ and $\lambda_i$ coefficients); in addition they must not depend on $m^2$. The latter is a highly non-trivial check of the computations, due to the ubiquitous presence of $m^2$ in the projections as well as in the amplitudes.

In what follows, we list here the results for all possible operators.

- $\phi^\dagger \phi - \frac{v^2}{2}$

$$\frac{1}{v^2} \left[ (M^2 - m^2) \vartheta_1 + \frac{g v}{\Lambda} \vartheta_2 + v^2 \lambda_1 \right] = \frac{1}{16 \pi^2 v^2} \left\{ M^4 \left( 3 - \frac{g v}{\Lambda} \right) + M_A^2 \left[ M^2 + 3M_A^2 \left( 2 + \frac{g v}{\Lambda} \right) \right] \right\} \frac{1}{\epsilon}. \quad (7.1)$$
\[ (\phi^\dagger \phi - \frac{v^2}{2})^2 \]

\[
\frac{(m^2 - M^2)^2}{2v^4} \vartheta_3 + \frac{g^2}{2\Lambda^2 v^2} \vartheta_4 + \frac{g}{\Lambda v} (m^2 - M^2) \vartheta_7 + \frac{m^2 - M^2}{v^2} \vartheta_1 + \frac{g}{\Lambda v} \vartheta_2 + \lambda_2 = \\
\frac{1}{32\pi^2 v^4} \left\{ 4M_A^2 M^2 \left( 1 - \frac{gv}{\Lambda} \right) + 3M_A^4 \left( 4 + 8 \frac{gv}{\Lambda} + \frac{g^2 v^2}{\Lambda^2} \right) + M^4 \left( 10 - 12 \frac{gv}{\Lambda} + 3 \frac{g^2 v^2}{\Lambda^2} \right) \right\} \frac{1}{\epsilon}.
\] (7.2)

\[ (\phi^\dagger \phi - \frac{v^2}{2})^3 \]

\[
\frac{(m^2 - M^2)^3}{6v^6} \vartheta_9 + \frac{g(m^2 - M^2)^2}{2\Lambda^2 v^5} \vartheta_{10} + \frac{g^2 (m^2 - M^2)}{2\Lambda^2 v^4} \vartheta_{11} + \frac{g^3}{6\Lambda^3 v^3} \vartheta_{12} \\
+ \frac{m^2 - M^2}{v^2} \vartheta_7 + \frac{g}{\Lambda v} \vartheta_8 + \frac{g(m^2 - M^2)}{\Lambda v^3} \vartheta_{11} + \frac{g^2}{\Lambda^2 v^2} \vartheta_{12} + \frac{(m^2 - M^2)^2}{v^4} \vartheta_{13} + \lambda_3 \\
= -\frac{1}{16\pi^2 v^5} \frac{g}{\Lambda} \left[ 2M^2 M_A^2 \left( 2 - \frac{gv}{\Lambda} \right) - 6M_A^4 \left( 2 + \frac{gv}{\Lambda} \right) + M^4 \left( 10 - 9 \frac{gv}{\Lambda} + 2 \frac{g^2 v^2}{\Lambda^2} \right) \right] \frac{1}{\epsilon}.
\] (7.3)

\[ (\phi^\dagger \phi - \frac{v^2}{2})(D^\mu \phi)^\dagger D_\mu \phi \]

\[
\frac{g(m^2 - M^2)}{\Lambda v^3} \vartheta_3 + \frac{g^2}{\Lambda^2 v^2} (\vartheta_5 + \vartheta_7) + \frac{2g}{\Lambda v^3} (m^2 - M^2) \vartheta_8 + \frac{g}{\Lambda v} (\vartheta_1 + \vartheta_4) + \frac{m^2 - M^2}{v^2} \vartheta_3 + \lambda_7 \\
= -\frac{1}{32\pi^2 v^3} \frac{g}{\Lambda} \left[ M_A^2 \left( 36 + 8 \frac{gv}{\Lambda} - 3 \frac{g^2 v^2}{\Lambda^2} \right) + M^2 \left( 16 - 14 \frac{gv}{\Lambda} + 3 \frac{g^2 v^2}{\Lambda^2} \right) \right] \frac{1}{\epsilon}.
\] (7.4)

\[ (\phi^\dagger \phi - \frac{v^2}{2})(\phi^\dagger D^2 \phi + \text{h.c.}) \]

\[
\frac{g^2}{2\Lambda^2 v^2} \vartheta_5 + \frac{g}{\Lambda v^3} (m^2 - M^2) \vartheta_8 + \frac{m^2 - M^2}{v^2} \vartheta_5 + \frac{g}{\Lambda v} \vartheta_6 + \lambda_6 = -\frac{1}{16\pi^2 v^2} \frac{g^2 M^2}{\Lambda^2} \frac{1}{\epsilon}.
\] (7.5)

\[ (D^\mu \phi)^\dagger D_\mu \phi \]

\[
\frac{g}{\Lambda v} \vartheta_1 + \lambda_4 = -\frac{1}{32\pi^2 v^2} \left[ M_A^2 \frac{gv}{\Lambda} \left( 6 - \frac{gv}{\Lambda} \right) + M_A^2 \left( 16 + 14 \frac{gv}{\Lambda} + 3 \frac{g^2 v^2}{\Lambda^2} \right) \right] \frac{1}{\epsilon}.
\] (7.6)

\[ \phi^\dagger [(D^2)^2 + D^\mu D^2 D_\mu + D^\mu D^\nu D_\mu D_\nu] \phi \]

\[
\lambda_5 = \frac{g^2}{96\pi^2 \Lambda^2} \frac{1}{\epsilon}.
\] (7.7)

\[ F^{\mu \nu} F_{\mu \nu} \]

\[
\lambda_8 = -\frac{M_A^2}{96\pi^2 v^2} \left( 2 + 2 \frac{gv}{\Lambda} + \frac{g^2 v^2}{\Lambda^2} \right) \frac{1}{\epsilon}.
\] (7.8)
\[ \partial^\mu F_{\mu \nu} \partial^\rho F_{\rho \nu} \]
\[ \lambda_9 = 0. \quad (7.9) \]
\[ \left( \phi^\dagger \phi - \frac{v^2}{2} \right) F_{\mu \nu}^2 \]
\[ - \frac{M^2 - m^2}{v^2} \theta_9 + \frac{g}{v \Lambda} \theta_{10} + \lambda_{10} = - \frac{1}{32 \pi^2} \frac{g^2 M_A^2}{\Lambda^2} \frac{1}{\epsilon}. \quad (7.10) \]

**VIII. CONCLUSIONS**

We have presented the explicit evaluation of all the UV coefficients of dimension less or equal to 6 operators in an Abelian spontaneously broken gauge theory supplemented with a maximally power counting violating derivative interaction of dimension 6. This has been possible by following the methodology put forward in a companion paper [1], in which one constructs an auxiliary theory based on the $X$-formalism in which a power-counting can be established (thus limiting the number of divergent diagrams one has to consider at each loop order) together with a mapping onto the original theory.

In particular, a separation of the gauge-dependent contributions, associated to the cohomologically trivial invariants, from the genuine physical renormalizations of gauge invariant operators has been achieved, and we have explicitly checked in two different gauges (Feynman and Landau) our results in order to explicitly verify the gauge independence of the coefficients of gauge invariant operators. In this respect it should be clear the pivotal role played by the field redefinitions for the correct identification of the gauge dependent coefficients of the cohomologically trivial invariants and, consequently, of the coefficients of the gauge invariant operators. Purely gauge fixed on-shell calculations will completely miss their contributions, running the risk of obtaining gauge dependent results even in the case of ostensibly gauge invariant quantities.

The techniques presented here and in [1] are suitable to be generalized both to higher orders in the loop expansion, as well as to the non-Abelian case, and, in particular, to the Standard Model effective field theory in which dimension 6 operators are added to the usual SU(2)×U(1) action. This latter generalization would be especially interesting, as it would allow to better understand the remarkable cancellations and regularities discovered when evaluating the one-loop anomalous dimensions for this model, and which have been linked
to holomorphicity [20], and/or remnants of embedding supersymmetry [21]. Work in this
direction is currently underway and we hope to report soon on this and related issues.

Appendix A: List of invariants

1. Pure external sources invariants

The invariants in this sector are

\[ \vartheta_1 \int d^4 x \, \bar{c}^* ; \quad \vartheta_2 \int d^4 x \, T_1 , \]
\[ \vartheta_3 \int d^4 x \, \frac{1}{2} (\bar{c}^*)^2 ; \quad \vartheta_4 \int d^4 x \, \frac{1}{2} T_1^2 , \]
\[ \vartheta_5 \int d^4 x \, \frac{1}{2} T_1 \Box T_1 ; \quad \vartheta_6 \int d^4 x \, \frac{1}{2} T_1 \Box^2 T_1 , \]
\[ \vartheta_7 \int d^4 x \, \bar{c}^* T_1 ; \quad \vartheta_8 \int d^4 x \, \bar{c}^* \Box T_1 , \]
\[ \vartheta_9 \int d^4 x \, \frac{1}{3!} (\bar{c}^*)^3 ; \quad \vartheta_{10} \int d^4 x \, \frac{1}{2} (\bar{c}^*)^2 T_1 , \]
\[ \vartheta_{11} \int d^4 x \, \frac{1}{2} (\bar{c}^*) T_1^2 ; \quad \vartheta_{12} \int d^4 x \, \frac{1}{3!} (\bar{c}^*)^3 . \]  

(A1)

Notice that \( \vartheta_6 \) has been inserted for completeness but does not contribute to dim. 6
operators in the target theory.
2. Mixed field-external sources invariants

The invariants in this sector are

\[
\begin{align*}
\theta_1 & = \int d^4x \bar{c}^* \left( \phi^\dagger \phi - \frac{v^2}{2} \right); \\
\theta_2 & = \int d^4x T_1 \left( \phi^\dagger \phi - \frac{v^2}{2} \right), \\
\theta_3 & = \int d^4x \bar{c}^* (D^\mu \phi)^\dagger D_\mu \phi; \\
\theta_4 & = \int d^4x T_1 (D^\mu \phi)^\dagger D_\mu \phi, \\
\theta_5 & = \int d^4x \bar{c}^* \left[ (D^2 \phi)^\dagger \phi + \text{h.c.} \right]; \\
\theta_6 & = \int d^4x T_1 \left[ (D^2 \phi)^\dagger \phi + \text{h.c.} \right], \\
\theta_7 & = \int d^4x \bar{c}^* \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2; \\
\theta_8 & = \int d^4x T_1 \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2, \\
\theta_9 & = \int d^4x \bar{c}^* F^2_{\mu\nu}; \\
\theta_{10} & = \int d^4x T_1 F^2_{\mu\nu}, \\
\theta_{11} & = \int d^4x \bar{c}^* T_1 \left( \phi^\dagger \phi - \frac{v^2}{2} \right); \\
\theta_{12} & = \int d^4x T_1^2 \left( \phi^\dagger \phi - \frac{v^2}{2} \right), \\
\theta_{13} & = \int d^4x (\bar{c}^*)^2 \left( \phi^\dagger \phi - \frac{v^2}{2} \right). 
\end{align*}
\]

(A2)

Notice that the use of the contractible pair basis allows us to re-express the (otherwise present) invariants

\[
\begin{align*}
\theta_{14} & = \int d^4x \bar{c}^* \Box \left( \phi^\dagger \phi - \frac{v^2}{2} \right); \\
\theta_{15} & = \int d^4x T_1 \Box \left( \phi^\dagger \phi - \frac{v^2}{2} \right), \\
\end{align*}
\]

(A3)

in terms of the above, since one has

\[
\Box \left( \phi^\dagger \phi - \frac{v^2}{2} \right) = (D^2 \phi)^\dagger \phi + \phi^\dagger (D^2 \phi) + 2(D^\mu \phi)^\dagger D_\mu \phi,
\]

(A4)

and therefore

\[
\begin{align*}
\theta_{14} & = 2 \theta_3 + \theta_5; \\
\theta_{15} & = 2 \theta_4 + \theta_6.
\end{align*}
\]

(A5)
3. Gauge invariants depending only on the fields

The invariants in this sector are

\[ \lambda_1 \int d^4 x \left( \phi^\dagger \phi - \frac{v^2}{2} \right); \]
\[ \lambda_2 \int d^4 x \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2; \]
\[ \lambda_3 \int d^4 x \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^3; \]
\[ \lambda_4 \int d^4 x \left( D^\mu \phi \right)^\dagger D_\mu \phi, \]
\[ \lambda_5 \int d^4 x \phi^\dagger [(D^2)^2 + D^\mu D^\nu D_\mu D_\nu + D^\mu D^2 D_\mu] \phi; \]
\[ \lambda_6 \int d^4 x \left( \phi^\dagger \phi - \frac{v^2}{2} \right) \left( \phi^\dagger D^2 \phi + (D^2 \phi)^\dagger \phi \right), \]
\[ \lambda_7 \int d^4 x \left( \phi^\dagger \phi - \frac{v^2}{2} \right) (D^\mu \phi)^\dagger D_\mu \phi; \]
\[ \lambda_8 \int d^4 x F^2_{\mu\nu}, \]
\[ \lambda_9 \int d^4 x \partial^\mu F_{\mu\nu} \partial^\rho F_{\rho\nu}; \]
\[ \lambda_{10} \int d^4 x F^2_{\mu\nu} \left( \phi^\dagger \phi - \frac{v^2}{2} \right). \]

(A6)

Appendix B: UV divergent ancestor amplitudes

1. Tadpoles

\[ \Gamma_{\epsilon^*}^{(1)} = - \frac{M^2 + (1 - \delta_{\xi_0})M_A^2}{16\pi^2} \frac{1}{\epsilon}, \]
\[ \Gamma_{T_1}^{(1)} = - \frac{(M^4 - 3M_A^4)}{16\pi^2} \frac{1}{\epsilon}, \]
\[ \Gamma_{\sigma}^{(1)} = \frac{1}{16\pi^2 \nu} \left[ m^2 M^2 + (1 - \delta_{\xi_0}) m^2 M_A^2 + 2(M^4 + 3M_A^4) \right] \frac{1}{\epsilon}. \]
\[ \Gamma^{(1)}_{\chi^* \omega} = \frac{eM_A^2}{8\pi^2 v} \left( \delta_{\xi_0} - 1 \right), \quad (B2a) \]

\[ \Gamma^{(1)}_{\chi \chi} = \frac{1}{32\pi^2 v^2} \left\{ 2m^2(M^2 + M_A^2) + 4(M^4 + 3M_A^4) - \frac{1}{16\pi^2 v^2} M_A^2 (m^2 + 2p^2) \frac{\delta_{\xi_0}}{\epsilon} \right. \]
\[- \left. \left[ \frac{g v}{\Lambda} M^2 \left( 4 - \frac{g v}{\Lambda} \right) + M_A^2 \left( 8 + 12 \frac{g v}{\Lambda} + 3 \frac{g^2 v^2}{\Lambda^2} \right) \right] p^2 + \frac{g^2 v^2}{\Lambda^2} p^4 \right\} \frac{1}{\epsilon}, \quad (B2b) \]

\[ \Gamma^{(1)}_{\sigma \sigma} = \frac{1}{16\pi^2 v^2} \left\{ 2m^2 + m^2(5M^2 + M_A^2) + 6(M^4 + 3M_A^4) \right. \]
\[- \left. \frac{1}{3} \left( 2 + \frac{g v}{\Lambda} \right)^2 p^2 \right\} \frac{g^\mu \nu}{\epsilon} + \frac{M_A^2}{24\pi^2 v^2} \left( 1 + \frac{g v}{\Lambda} + \frac{g^2 v^2}{\Lambda^2} \right) \frac{p^\mu p^\nu}{\epsilon}, \quad (B2c) \]

\[ \Gamma^{(1)}_{e^* e^*} = \frac{1}{8\pi^2 v} \; \frac{1}{\epsilon}; \quad (B2d) \]

\[ \Gamma^{(1)}_{e T_1} = \frac{1}{16\pi^2 v} \left\{ 2M^2 + 2M_A^2 (1 - \delta_{\xi_0}) - p^2 \right\} \frac{1}{\epsilon}, \quad (B2e) \]

\[ \Gamma^{(1)}_{T_1 T_1}(p^2) = \frac{1}{32\pi^2 v} \left\{ 6(M^4 + M_A^4) - 3(M^2 + M_A^2) p^2 + p^4 \right\} \frac{1}{\epsilon}, \quad (B2f) \]

\[ \Gamma^{(1)}_{T_1 \sigma}(p^2) = -\frac{1}{32\pi^2 v} \left\{ 4m^2(M^2 + M_A^2) + 8(M^4 - 3M_A^4) \right. \]
\[- \left. 2 \left( m^2 + M^2 + M_A^2 + 2M^2 \frac{g v}{\Lambda} \right) p^2 + \frac{g v}{\Lambda} p^4 \right\} \frac{1}{\epsilon} + \frac{\delta_{\xi_0}}{8\pi^2 v} M_A^2 (m^2 - p^2) \frac{1}{\epsilon}, \quad (B2g) \]

\[ \Gamma^{(1)}_{e^* \sigma}(p^2) = \frac{1}{16\pi^2 v} \left\{ -2(m^2 + M^2) + \frac{g v}{\Lambda} p^2 \right\} \frac{1}{\epsilon}. \quad (B2i) \]
3. Three-point functions

\[ \Gamma^{(1)}_{e^eT_1} = -\frac{1}{8\pi^2} \frac{1}{\epsilon}, \]

\[ \Gamma^{(1)}_{e^eT_1T_1} \bigg|_{p_1=p_2=0} = -\frac{3M^2 + 2M_A^2}{8\pi^2} \frac{1}{\epsilon} + \frac{M_A^2 M_B}{8\pi^2} \delta_{\xi_0}, \]

\[ \Gamma^{(1)}_{T_1T_1T_1} = -\frac{3M^4}{4\pi^2} \frac{1}{\epsilon}, \]

\[ \Gamma^{(1)}_{e^eT_1\sigma} = \frac{m^2 + M^2 + \frac{M_A^2}{2}}{4\pi^2\epsilon} - \frac{M_A^2 M_B}{8\pi^2\epsilon} \delta_{\xi_0}, \]

\[ \Gamma^{(1)}_{e^{\mu}A_\mu A_\nu(p_1,p_2)} = -\frac{M_A^2 g^2}{16\pi^2 A^2} \frac{1}{\epsilon} \frac{1}{\epsilon}, \]

\[ \Gamma^{(1)}_{T_1A_\mu A_\nu(p_1,p_2)} = \frac{M_A^2}{32\pi^2 v^2} \left\{ \frac{g v}{\Lambda} \left( 8 - 3 \frac{g v}{\Lambda} \right) M^2 - \left( 8 + 4\delta_{\xi_0} - 3 \frac{g^2 v^2}{\Lambda^2} \right) M_A^2 \right. \]

\[ + \frac{2 g v}{3\Lambda} \left( 1 + 2 \frac{g v}{\Lambda} \right) (p_1^2 + p_2^2) + 2 \frac{g^2 v^2}{\Lambda^2} p_1 \cdot p_2 \left[ \frac{1}{\epsilon} \right] g_{\mu\nu}, \]

\[ - \frac{1}{96\pi^2 v^2} \frac{g}{\Lambda} M_A^2 \left( 2 + \frac{g v}{\Lambda} \right) (p_1 p_1 p_1 p_2 + p_2 p_2 p_1 p_2) \left[ \frac{1}{\epsilon} \right] + \frac{M_A^2 g^2}{16\pi^2 v^2} (p_1 p_2) \left[ \frac{1}{\epsilon} \right]. \]

\[ \Gamma^{(1)}_{e^{\mu}A_\mu A_\nu(p_1,p_2)} = \left\{ \frac{m^2 + M^2 + \frac{M_A^2}{2}}{8\pi^2 v^2} + \frac{M_A^2}{8\pi^2 v^2} \delta_{\xi_0} + \frac{1}{16\pi^2 A^2} \left[ \frac{1}{\epsilon} \right] \right\}, \]

\[ \Gamma^{(1)}_{e^{\mu}A_\mu A_\nu(p_1,p_2)} = \left\{ \frac{m^2 + M^2 - \frac{M_A^2}{2}}{8\pi^2 v^2} + \frac{M_A^2}{8\pi^2 v^2} \delta_{\xi_0} + \frac{1}{16\pi^2 A^2} \left[ \frac{1}{\epsilon} \right] \right\} + \mathcal{O}(p^4), \]

\[ \Gamma^{(1)}_{e^{\sigma}\sigma(p_1=p_2=0)} = -\frac{1}{8\pi^2 v^2} \left( m^2 + M^2 - \frac{M_A^2}{2} \right) \frac{1}{\epsilon} - \frac{M_A^2}{8\pi^2 v^2} \frac{1}{\epsilon}. \]
\[ \Gamma_{T_1 T_1}^{(1)} \bigg|_{p_1 = p_2 = 0} = -\frac{1}{8\pi^2 v^2} (2m^4 + 5m^2 M^2 + 6M^4 + 3m^2 M_A^2 - 18M_A^4) \frac{1}{\epsilon} + \frac{3m^2 M_A^2 \delta_{\xi_0}}{8\pi^2 v^2} \frac{1}{\epsilon}, \]  

(B3k)

\[ \Gamma_{\sigma T_1 T_1}^{(1)} \bigg|_{p_1 = p_2 = 0} = \frac{1}{8\pi^2 v^2} \left[ m^2 (3M^2 + 2(1 - \delta_{\xi_0}) M_A^2) + 6(M^4 + M_A^4) \right] \frac{1}{\epsilon}, \]  

(B3l)

\[ \Gamma_{\epsilon \epsilon \sigma}^{(1)} = 0, \]  

(B3m)

\[ \Gamma_{\sigma \sigma \sigma}^{(1)} \bigg|_{p_1 = p_2 = 0} = \frac{3}{8\pi^2 v^3} \left( m^4 + 2m^2 M^2 + 2M^4 - m^2 M_A^2 (1 - \delta_{\xi_0}) + 6M_A^4 \right) \frac{1}{\epsilon}, \]  

(B3n)

\[ \Gamma_{\sigma \chi \chi}^{(1)}(p_1, p_2) = \frac{1}{8\pi^2 v^3} \left( m^4 + 2m^2 M^2 + 2M^4 - m^2 M_A^2 (1 - \delta_{\xi_0}) + 6M_A^4 \right) \frac{1}{\epsilon} - \frac{1}{32\pi^2 v^2} \frac{g}{\Lambda} \left[ 4m^2 + (M^2 - 3M_A^2) \left( 4 + \frac{gv}{\Lambda} \right) \right] (p_1^2 + p_2^2) - \frac{1}{16\pi^2 v^2} \frac{g}{\Lambda} \left[ 3 \frac{gv}{\Lambda} M^2 + m^2 \left( 4 + \frac{gv}{\Lambda} \right) - 3M_A^2 \left( 8 + \frac{3gv}{\Lambda} \right) \right] p_1 \cdot p_2 \frac{1}{\epsilon}. \]  

(B3o)

4. Four-point functions

\[ \Gamma_{\sigma \sigma \chi \chi}^{(1)} \bigg|_{p_i = 0} = \frac{1}{8\pi^2 v^4} \left( m^4 + 2m^2 M^2 + 2M^4 - 2m^2 M_A^2 (1 - \delta_{\xi_0}) + 6M_A^4 \right) \frac{1}{\epsilon}, \]  

(B4a)

\[ \Gamma_{\chi \chi \chi \chi}^{(1)}(p_1, p_2, p_3) = \frac{3}{4\pi^2 v^4} \left( m^2 - M_A^2 \right) M_A^2 \frac{\delta_{\xi_0}}{\epsilon} - \frac{1}{16\pi^2 v^3} \frac{g}{\Lambda} \left[ 3M_A^2 \frac{gv}{\Lambda} + \left( 8 - \frac{gv}{\Lambda} \right) M^2 \right] + \left( 4 - \frac{gv}{\Lambda} \right) \frac{m^2}{2} \left( \sum_{i=1}^{3} p_i^2 + \sum_{i<j} p_i p_j \right) + \mathcal{O}(p_i^4) \]  

(B4b)

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