Gluonium states and the Pomeron trajectory

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Abstract

Pomeron is modeled as a system of two interacting by Cornell potential massive gluons. In bound state region, relativistic wave equation for the potential is analyzed. Two exact asymptotic solutions of the equation are used to derive an interpolating mass formula for gluonium states and the Pomeron trajectory in the whole region. The trajectory obtained is linear at large timelike \( t \) and flattens off at \(-1\) in the scattering region at large \(-t\). Parameters of the trajectory are found from the fit of recent HERA data for \( \alpha_P(t) \).

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1 Introduction

As known many features of hadron interactions at high energies are remarkably well reproduced by Regge calculus and, in particular, the high-energy behavior of hadron scattering is very well predicted by the Pomeron [1]. The well known “soft” Pomeron is responsible for most of the total cross section in hadron-hadron collisions, for small \(-t\) elastic scattering and for diffractive dissociation. Hadron amplitude at high energy is proportional to \( s^{\alpha_P(t)} \), where \( \alpha_P(t) \) is the Pomeron trajectory.

Last years the phenomenology of processes dominated by the perturbative Regge kinematics attracts increasing interest [2]. The most prominent example is the so-called BFKL-Pomeron (“hard” Pomeron) [3] [4] which has been found very useful in describing the rise of nucleon structure function \( F_2(x,Q^2) \) at small \( x \) [5], gives excellent description of data for \( J/\Psi \) photoproduction and the charm structure function \( F_c^2 \) [6]. It is expected also to show up in jet-inclusive final states with rapidity gaps in deep-inelastic, hadron-hadron interactions and other processes [7].

Most important parameters of the Pomeron in high-energy hadron physics are the intercept \( \alpha_P(0) \) and the slope \( \alpha'_P(0) \) of the Pomeron trajectory. Usually, the parameters are determined from experiment [8] [9]. However, the ideas proposed in Ref. [10] were rather successful. They have succeeded, at \( t = 0 \), to describe the Pomeron in terms of modified tree-level two-gluon exchange, thus providing a connection with QCD. This approach has shown, that the Pomeron trajectory could be obtained at non zero \( t \), when resumed to higher orders, whereas perturbative QCD (pQCD) seems to fail in that respect.

Most recent small \(-t\) ZEUS data for exclusive \( \rho \) and \( \phi \) photoproduction [11] [12] lead to a slope \( \alpha'_P(0) \) for the trajectory of the “soft” Pomeron that differs significantly from the classical value \( \alpha'_P(0) = 0.25 \text{GeV}^{-2} \). The results of these experiments have been discussed in Ref. [13]. It was pointed out that the slope \( \alpha'_P(0) \) of the “soft” Pomeron should be determined from the data at small \(-t\), \(-t \simeq 0.4 \text{GeV}^{-2} \) at HERA energy [14], where “soft”-Pomeron exchange dominates the differential cross section; but recent ZEUS measurements [11] extend to rather large \(-t\). The ZEUS data have been explained in Ref. [6] by adding in a flavor-blind “hard”-Pomeron contribution, whose magnitude is calculated from the data.

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for exclusive $J/\Psi$ photoproduction. These data can be explained by nonlinear behavior of the Pomeron trajectory we obtain in this work.

In this work, we obtain the Pomeron trajectory in the whole region. We use the method suggested earlier to calculate the Reggeon trajectories \cite{15} \cite{16}. The Pomeron is considered as the $t$-channel process. Then we take a picture where Pomeron is dual to glueball (gluonium) states, i.e. it is a bound state in $s$-channel. We work in the framework of the potential approach, which is a natural framework for the studying Regge trajectories and their properties.

In our model, two massive gluons interact by means of the QCD motivated funnel type potential \cite{17}. In the scattering region ($t \leq 0$), this system corresponds to a ladder-type diagrams, i.e. tree-level two-gluon exchange. In bound state region ($t > 0$) we have a bound state problem for two massive gluons, i.e., we consider gluonium and its excited states.

Gluonium masses are calculated with the use of the interpolating mass formula we obtain from two exact asymptotic solutions of relativistic wave equation \cite{16}. Inverting this formula, we derive an analytic expression for the Pomeron trajectory, $\alpha_P(t)$. In the scattering region, at $-t \gg \Lambda_{QCD}$, the trajectory flattens off at $-1$, i.e., it has asymptote $\alpha_P(t \rightarrow -\infty) = -1$.

In bound state region, at large timelike $t$, the Pomeron trajectory is linear in accordance with the string model.

2 The “soft” and “hard” Pomerons

There exist different approaches to investigating an object such as the Pomeron; the issue of “soft” and “hard” Pomeron has been discussed extensively in the literature \cite{3} \cite{4} \cite{18}. The classic “soft” Pomeron is constructed from multi-peripheral hadronic exchanges and has intercept $\alpha_P(0) = 1$. Because this is not compatible with the rising hadronic cross sections at high energies, the “soft” Pomeron was replaced by a “soft” supercritical Pomeron with an intercept $\alpha_P(0) > 1$, i.e. $\alpha_P(0) - 1 = \Delta > 0$ \cite{19} \cite{20}.

In the framework of QCD (in the scattering region), the Pomeron is understood as the exchange of two (or more) gluons \cite{21}. The perturbative QCD approach to the Pomeron has been discussed in Refs. \cite{3} \cite{4}. The Balitski-Fadin-Kuraev-Lipatov (BFKL) Pomeron \cite{3} is built out of multiperipheral high transverse momentum gluon exchanges and predicts a different Pomeron, which in complex $j$ plane has a series of poles at $1 < j < 1 + \Delta$ with $\Delta \simeq 0.45$. This approach gives good results where the leading logarithmic approximation (LLA) holds.

The leading and subleading logarithmic summation in QCD \cite{4} predicts asymptote $\alpha_P(t) = 1$ for the “hard” BFKL-Pomeron at large spacelike $t$ \cite{3} \cite{4},

$$\alpha_P(t) \simeq 1 + O(\tilde{g}^2(t)), \quad t \rightarrow -\infty,$$  

(1)

where $\tilde{g}^2(t)$ stands for the running coupling of QCD.

This prediction for the Pomeron trajectory contradicts to present ISR and Tevatron data, and most recent ZEUS data \cite{11} \cite{12} on $\alpha_P(t)$. The shrinkage of the forward elastic differential cross section peak in $pp/\bar{p}p$ elastic scattering from ISR to Tevatron energies means that the Pomeron trajectory is approximately linear with a slope $\propto 0.25\,\text{GeV}^{-2}$ out to $t \simeq -1.0\,\text{GeV}^2$. The ISR diffractive dissociation data indicate that this approximate linearity continues at least out to $t \simeq -2.0\,\text{GeV}^2$ by which time the value of the trajectory is $\propto 0.6$, i.e. considerably below 1. Recent ZEUS data \cite{11} demonstrate similar behavior.

In the last years difficulties have emerged with the BFKL equation (see \cite{22} and references therein). The LLA BFKL predictions overestimate the $\gamma^*\gamma^*$ cross section by a large factor \cite{22}. In the BFKL formalism, there is a problem at LL order in setting the two mass scales on which the cross section depends: the mass $\mu^2$ at which the strong coupling $\alpha_s$ is evaluated and the mass $Q^2$ which provides the scale for high-energy logarithms; the results are very sensitive to these parameters \cite{23}. An additional uncertainty is due to the correct treatment of the production of massive charm quarks. An attempt to overcome the scale problem reduces both the size of the BFKL cross section and its energy dependence \cite{24}.

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A comparative investigation of various Pomeron models was carried out in the impact parameter space through their predicted values of $\sigma_{\text{tot}}$, slope $B$, and $\sigma_{\text{el}}/\sigma_{\text{tot}}$ in high energy $pp$ and $p\bar{p}$ scattering [25]. The main result of this investigation is that the data is compatible with a smooth transition from a “soft” to a “hard” Pomeron contribution which can account for the rise of $\sigma_{\text{tot}}$ with $s$. Note, however, that the singularity of the “hard” Pomeron is not a pole but a cut in the $\alpha_P(t)$ plane, starting at $\Delta = 12\alpha_s\ln 2/\pi$ [3, 26]. Then, if “soft” and BFKL Pomeron have a common origin, the discontinuity across the cut in the $\alpha_P(t)$ plane must have a strong $t$ dependence [26] that points out on nonlinearity of the Pomeron trajectory.

Both the “soft” and the “hard” QCD Pomeron predict a powerlike rise of the total cross section, $\sigma_{\text{tot}} \propto s^{\Delta}$. However, the physics of the “soft” Pomeron is much less clear; any realistic pQCD attempts lead to the introduction of an “infrared” cutoff in gluon propagator that generates the main part of Pomeron exchange. The derivation of nonperturbative (NP) gluon propagator from QCD [19, 27, 28] contains many assumptions to be proved. In many of these studies, the low-momentum singularity of the gauge field propagator is softened. A soft infrared behavior can be also obtained in a Yang-Mills theory with Higgs mechanism, where the gluon acquires a mass, or for some solutions of the Dyson-Schwinger equation [27], or simply in the presence of a cutoff.

There are currently no any theoretical estimations of the Pomeron trajectory. The behavior of the Pomeron trajectory $\alpha_P(t)$ in the whole region is unknown. The linear dependence,

$$\alpha_P(t) = \alpha_P(0) + \alpha_P'(0)t,$$

is usually assumed, which is a good approximation in small $-t$ region. It is a goal of this work to reproduce the Pomeron trajectory in the whole region and calculate the parameters $\alpha_P(0)$ and $\alpha_P'(0)$.

There are good bases to believe that the “soft” Pomeron can be considered as an exchange of two NP gluons, whose properties are dictated by the expected structure of the QCD vacuum [19]. In recent time several successful models based on the theory of the “soft” supercritical Pomeron have been developed in which the Pomeron is modeled as an exchange of two NP gluons dynamically generated mass whose propagator is finite at $q^2 = 0$ [27, 29]. These models and their modification [30] provide a reasonable description of data in and above CERN Intersecting Storage Rings (ISR) energy range.

### 3 Some general properties of Regge trajectories

Let us remind some general results obtained for the Regge trajectories in recent time. This will allow us to understand many features of such a Reggeon known as the Pomeron.

It is well-known experimental fact that hadrons populate linear Regge trajectories in $s$-channel (or $t > 0$). That means that the square of the mass of a state with orbital angular momentum $l$ is proportional to $l$: $M^2(l) = \beta l + \text{const}$, with the same slope, $\beta \simeq 1.2\text{GeV}^2$, for all trajectories. There exists a conviction, that the Regge trajectories $\alpha(l)$ are linear in the whole region, that is, not only in the bound state region ($t > 0$) but in the scattering region ($t < 0$), too.

Main characteristics of “soft” hadronic processes in the scattering region can be understood in terms of the exchange of particles, which lie on linear Regge trajectories. This approximate linearity encouraged the dual model approach to strong interactions which in tree approximation assumes exactly linear trajectories. However, the conception of linear Regge trajectories is not consistent with experimental data and expectations of pQCD at large spacelike momentum transfer $-t \gg \Lambda_{\text{QCD}}$. In the experiment [31] far more complicated behavior of the $\rho$ meson trajectory, $\alpha_{\rho}(t)$, was discovered; the $\rho$ trajectory flattens off at about $-0.6$.

There are different approaches to investigate the Regge trajectories [2, 3, 4]. Significant efforts were undertaken in order to obtain information on the behavior of Regge trajectories...
at large spacelike $t$ \cite{32,33,34}; the asymptotic behavior at $-t \to \infty$ has been discussed by many authors \cite{33,36,37,38}. As shown in \cite{33}, exclusive processes whose cross-sections are determined by Regge pole trajectory exchange, $\alpha(t)$, at small momentum transfers, $t$, are controlled by these same exchanges at very large spacelike $t$, too. Hereby trajectory must to be nonlinear and one of the most crucial distinction between small $-t$ behavior of $\alpha(t)$ and large $-t$ behavior of $\alpha(t)$ involves the asymptotic form of Regge trajectories at $-t \to \infty$.

Important information on large $-t$ behavior of trajectories can be obtained from the comparing the predictions for the scattering amplitude, $T(s,t)$, of the ”quark counting rule” \cite{39} and the Regge pole approach at $s \to \infty$, $-t$ fixed. From the demand of a smooth interpolation between these two predictions the condition was obtained \cite{36},

$$\alpha(t) = \text{const}, \quad t \to -\infty. \quad (3)$$

Such asymptotic behavior of the Regge trajectories was proposed by various authors \cite{33,36,37,39}, and seems does not contradict to experimental data \cite{31}.

There have been undertaken considerable efforts to extend the constituent interchange model (CIM) (see Ref. \cite{40}) from the fixed-angle region into the fixed $t$-region. These efforts have resulted in the prediction for the large $-t$ behavior of $\rho$ trajectory

$$\alpha_{\rho}(t) = -1, \quad t \to -\infty. \quad (4)$$

The same asymptotic behavior \cite{1} for all leading $S = 1$ quarkonium Regge trajectories was obtained in our Ref. \cite{15} on the basis of analysis of the relativistic quasipotential equation with the QCD motivated potential, and in Ref. \cite{16} on the basis of solution of the Klein-Gordon equation. The Regge trajectories have been determined as the function $l(E^2)$, where $E^2$ is the squared quarkonium mass obtained from solution of the eigenvalue problem for two bound quarks and $l$ is the relative orbital angular momentum of the quarks. Main result of these investigations is that, that the Regge trajectories are nonlinear and have the asymptote \cite{1}.

The Pomeron trajectory can be obtained similar way as the Reggeon ones. However, instead of two interacting quarks we consider two interacting massive gluons. We deal with the gluon system primarily in bound state region and solve relativistic wave equation for the funnel-type QCD motivated potential. Then, using the same technic as for Reggeons, we obtain the Pomeron trajectory $\alpha_{P}(t)$ in the whole region $-\infty < t < \infty$.

4 The model

Let us outline the main features of our model. Two interacting particles can be considered both in the scattering region ($t \leq 0$) and in bound state region ($t > 0$). The wave equation describes these two interacting particles in the whole region of $t$, $-\infty < t < \infty$, with the corresponding boundary conditions for the wave function. Most important aspect in this approach is the form of the potential.

The potential approach is a more convenient and, at the same time, simpler way to reconstruct the Regge trajectories. Moreover, the Regge theory itself was proposed by Regge at the dealing with solutions of the Schrödinger equation for the nonrelativistic potential scattering. In other words, the potential approach and studying solutions of the wave equation is a natural framework to investigate Regge trajectories and its properties, in spite of the phenomenological nonrelativistic nature of the potential.

Because of the intrinsically nonperturbative nature of bound-state problem in non-Abelian gauge theories, it is up to now, not possible to derive the forces acting between the quarks and gluons from first principles. It has been well tested that hard processes are governed by short range part of the strong interaction. It is generally agreed that, in pQCD, as in QED the essential interaction at small distances is instantaneous Coulomb exchange; in QCD, it is $qq$, $qg$, or $gg$ Coulomb scattering \cite{26}. The dynamics is the Coulomb interaction,

$$V_S(r) = -\frac{\alpha}{r}, \quad r \to 0, \quad (5)$$

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where $\tilde{\alpha}$ is the effective strong coupling constant ($\tilde{\alpha} = \frac{4}{3}\alpha_s$ for mesons). In momentum space, the Coulomb potential (5) is $	ilde{V}(t) = 4\pi\tilde{\alpha}/t$, that is the Fourier transform of $\tilde{V}(t)$ leads to the potential (5). At large $-t = -q^2$, the potential (5) corresponds to the scattering amplitude in the Born approximation with the gluon propagator $D(q^2) \propto 1/q^2$ [26].

For large distances, in order to be able to describe confinement, the potential has to rise to infinity. From lattice-gauge-theory computations [34] follows that this rise is an approximately linear, that is,

$$V_L(r) \simeq \kappa r, \quad r \to \infty,$$

(6)

where $\kappa \simeq 0.15\text{GeV}^2$ being the string tension.

The potential is more poorly understood in an intermediate region. In this region, many well known potentials give reasonable results for hadron masses (see [41] and Refs. therein), but these results do not depend very strong on the form of the potential. The most reasonable possibility to construct an interquark potential, which satisfies both of the above constraints, is to simply add these two contributions. This leads to the so-called funnel-shaped (or Cornell) potential [42]:

$$V(r) = -\frac{\tilde{\alpha}}{r} + \kappa r + V_0.$$  

(7)

A closer inspections reveals that all phenomenologically acceptable "QCD-inspired" potentials are only variations around the funnel potential. Its parameters are directly related to basic physical quantities: the universal Regge slope $\alpha' \simeq 0.9 \text{(GeV/c)}^{-2}$ for trajectories and one-gluon-exchange coupling strength $\alpha_s$ at small distances. As for constant $V_0$, usually, it is added to the confining contribution.

It is usually supposed a distinctly different dynamical origin of Pomeron and Reggeons. This suggests a different dependence $\alpha_P(t)$ and the ordinary Reggeons. The physics of ordinary Reggeon (such as the $\rho$) is exchange of a ladder, for which the sides are generally regarded to be constituent quarks. The rungs of the ladder represent the binding potential between quarks (nonperturbative gluons), i.e. Reggeons can be obtained as orbital (and radial) excitations of bound system of interacting quarks [26]. Similarly, due to the non-abelian nature of gluonic field, gluons interact each other. This (and strong believe that gluon has nonzero mass) makes it possible to consider the bound state problem for two (or more) gluons interacting by means of QCD motivated potential, i.e. gluonium excited states (glueballs).

Glueballs were suggested theoretically in Refs. [43] and then have been extensively studied in the framework of different approaches (see [44]), and potential-like models [45], too, where a notation of the constituent gluon mass $\mu_0$ was introduced. There is no overall agreement between the results of different approaches in the theory (and also in experiment); moreover, many theoretical approaches contain basic model assumptions which are difficult to prove starting from the QCD Lagrangian.

Gluonium masses have been calculated in [17] within a new method called the Vacuum Correlator Model (VCM) [46]. In this model, all nonperturbative and perturbative dynamics of quarks and gluons in the Pomeron is universally described by lowest cumulants, i.e. gauge invariant correlators of the type $\langle F_{\mu\nu}(x_1) \ldots F_{\lambda\sigma}(x_\nu) \rangle$. Linear confinement naturally arises due to the presence of specific structures in the correlators, which are nonzero for Abelian fields when monopole condensate is formed, and for nonabelian fields due to their specifics nonabelian structure.

The gluon correlators (cumulants) are characterized by a specific decay time (correlation time), which is a universal feature of the QCD vacuum [46]. For quarks and gluons the situation is the same. All the methods and formulae derived for light quarks apply also for gluons some self-evident replacements [17]. The Regge trajectories for mesons and baryons have been found with good phenomenological properties, i.e. the slope is the same for mesons and baryons and is equal to $1/8\sigma$, where $\sigma$ is the string tension [17]. In this way one connects the properties of the QCD Pomeron with the Regge phenomenology.
The physical picture of gluonium can be formulated as follows. In the vacuum, two or three gluons are excited. Each of gluons propagates through the Pomeron feeling both perturbative and nonperturbative interactions with this medium and another gluon. This gives rise to the Coulomb-like and string-type interactions, which are handled as in the case of light quarks [47].

In this model all dependence on Pomeron gluonic fields \( A_\mu \) is contained in the adjoined Wilson loop \( W_{\text{adj}}(C) \), where the closed contour \( C \) runs over trajectories \( z_\mu(\sigma) \) and \( \bar{z}_\mu(\sigma) \) of both gluons. For the nonperturbative part and Coulomb interactions the ratio of correlators at least at small distance in adjoined and fundamental representation is the number \( N_2 = C_2^{\text{adj}}/C_2^{\text{fund}} \), where \( C_2^{\text{adj}} = N_c = 3, C_2^{\text{fund}} = (N_c^2 - 1)/2N_c \). In the adjoined and fundamental representation, the final form of interaction of two gluons is given by [17]

\[
V^{\text{adj}}(r) = -\frac{\alpha_a}{r} + \sigma_a r - C_0, \tag{8}
\]

where \( \alpha_a = \alpha_a^{\text{adj}} = \frac{2}{9} \alpha_s^{\text{fund}} \equiv 3 \alpha_s^{\text{fund}} \), \( \sigma_a = \sigma_a^{\text{adj}} = \frac{2}{9} \sigma^{\text{fund}} \); \( \alpha_s^{\text{fund}} \) is the strong coupling, \( \sigma^{\text{fund}} \equiv \sigma \simeq 0.15 \text{GeV}^2 \) is the string tension, and \( C_0 \) is the arbitrary parameter. We see that the slope of the gluonium trajectory at large \( l \) is \( \sigma^{\text{adj}} \) of that of light meson trajectories. Important feature is the occurrence of the constituent gluon mass \( \mu_0 \) in the dynamical equation, which is state dependent. For lowest state \( (l = 0, n_r = 0) \) this model gives \( \mu_0 = \frac{4}{9} m_q \), where \( m_q \) is the quark mass.

In hadron physics, the nature of the potential is very important. Concept of the scalar-like potential is especially important in hadron physics. In relativistic potential models of quarkonia based on a Dirac-type equation with a local potential is a sharp distinction between a linear potential \( V(r) \) which is vectorlike and one which is scalarlike.

There are normalizable solutions for a scalarlike \( V(r) \) but not for a vectorlike \( V(r) \) [18, 19]. No any problems arise and no any difficulties encountered with the numerical solution if the confining potential is purely scalarlike. It was shown in many works that the effective interaction has to be scalar in order to confine particles inside the hadrons (see, for example, Refs. [49]).

The same is true for the short-range Coulomb potential. In Ref. [50], we have considered nonrelativistic semiclassical wave equation for central potentials, and in Ref. [51] - solution of relativistic one for vectorlike and scalarlike Coulomb potentials. We have obtained two results for the Coulomb potential: the known exact result for spinless particles which coincides with one obtained from solution of the Klein-Gordon equation and another result obtained from solution of the semiclassical equation for the scalarlike Coulomb potential. We have shown that, unlike the known relativistic wave equations for the Coulomb potential, the semiclassical equation with the scalarlike potential has the regular solution at the spatial origin [51]. Thus, in this work, for the \( gg \) interaction, we use the scalarlike Cornell potential [8].

We use the potential [8] to reproduce the gluonium masses. For \( gg \) system, to be able reproduce the Pomeron trajectory, we need to obtain an analytic expression for the squared gluonium mass \( E^2 \). For this, we solve the relativistic semiclassical wave equation [50, 51]: for two interacting gluons of equal masses \( \mu_1 = \mu_2 = \mu_0 \), the semiclassical wave equation with the scalarlike potential [8] is

\[
(-i)^2 \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right) \bar{\psi}(\vec{r}) = \left[ \frac{E^2}{4} - [\mu_0 + V^{\text{adj}}(r)]^2 \right] \bar{\psi}(\vec{r}). \tag{9}
\]

It is hard to find the analytic solution of equation (9) for the potential [8]. But we can find exact analytic solutions of this equation for two asymptotic limits of the potential [8], the Coulomb and the linear potentials [51].

\[
E^2_n = 4 \mu_0^2 \left[ 1 - \frac{\alpha_a^2}{n_r + \frac{9}{2} + \sqrt{(l + \frac{9}{2})^2 + \alpha_a^2}} \right], \tag{10}
\]
\[ E^2_n = 8\sigma_a \left( 2n_r + l - \alpha_a + \frac{3}{2} \right). \]  
(11)

At small distances, where the Coulomb type contribution dominates, the effective strong coupling, \( \tilde{\alpha} \), is a small value and Eq. (10) can be written in the simpler form

\[ E^2_n \simeq \mu_0^2 \left[ 1 - \frac{\alpha_a^2}{(n_r + l + 1)^2} \right]. \]  
(12)

Note, that equation (12) for the squared invariant mass of the system of two particles has the correct relativistic form, i.e. \( E^2_n = 4(p_n^2 + \mu_0^2) \), where \( p_n^2 = -\alpha_a^2\mu_0^2/(n_r + l + 1)^2 \) or \( p_n = \pm \alpha_a\mu_0/(n_r + l + 1) \).

To find gluonium energy eigenvalues we use the same approach as in Refs. [15, 16], i.e., we derive an interpolating mass formula for \( E^2 \), which satisfies both of the above constraints. A justification is that we do not know the exact form of the QCD potential in the intermediate region and exact expression for \( E^2 \).

Two exact analytic formulae (11) and (12) correspond to the linear and Coulomb terms of the potential (8), respectively. The formulae describe interaction of two gluons at large and small distances. Two formulae (11) and (12) represent two asymptotes of the exact analytical formula for \( E^2 \) which is unknown. To derive an interpolating mass formula, the two-point Padé approximant [52] can be used,

\[ [K/N]_f(z) = \frac{\sum_{i=0}^{K} a_i z^i}{\sum_{j=0}^{N} b_j z^j}, \]  
(13)

with \( K = 3 \) and \( N = 2 \). This results in the interpolating mass formula (16),

\[ E^2_n = 8\sigma_a \left( 2n_r + l + \frac{3}{2} - \alpha_a \right) - \frac{4\sigma_a^2 \mu_0^2}{(n_r + l + 1)^2} + 4\mu_0^2. \]  
(14)

Expression (14) is an Ansatz [as the potential (8)] containing the appropriate asymptotic limits, i.e. the two exact asymptotic formulae (11) and (12). It allows us to reproduce the pomeron trajectory in the whole region.

5 The Pomeron trajectory

Regge trajectories being the objects connecting bound state and the scattering regions. Expression (11) has been derived in the bound state region. It can be used to derive the Pomeron trajectory in the whole region.

The invariant gluonium mass \( E^2 \) in the bound state region turns into the invariant variable \( t \) (transfer momentum) in the scattering region, i.e. \( -E^2 = t \). Let us transform Eq. (14) into the cubic equation for the angular momentum \( J \),

\[ J^3 + c_1(t)J^2 + c_2(t)J + c_3(t) = 0, \]  
(15)

where \( c_1(t) = 2\tilde{n} + \lambda(t), \ c_2(t) = \tilde{n}^2 + 2\tilde{n}\lambda(t), \ c_3(t) = \tilde{n}^2\lambda(t) - \alpha_a^2\mu_0^2/2\sigma_a, \ \tilde{n} = n_r + 1, \ \lambda(t) = 2\tilde{n} - 1/2 - \alpha_a + (4\mu_0^2 - \mu_0^2)/8\sigma_a, \) and \( t = E^2 \). Equation (15) has three (complex in general case) roots, \( J_1(t), \ J_2(t), \) and \( J_3(t) \). The real part of the first root, \( \text{Re}J_1(t) \), gives the Pomeron trajectory \( \alpha(t) \),

\[ \alpha_P(t) = \begin{cases} \sqrt[3]{-q(t)} + \sqrt[3]{Q(t)} + \sqrt[3]{-q(t) - \sqrt[3]{Q(t)}} - \sqrt[3]{c_1(t)}, & Q(t) \geq 0; \\ 2\sqrt{-p(t)} \cos \left( \frac{1}{3} \beta(t) \right) - \frac{1}{3} c_1(t), & Q(t) < 0, \end{cases} \]  
(16)

where

\[ Q(t) = p^2(t) + q^2(t), \quad p(t) = -\frac{1}{9}c_1^2(t) + \frac{1}{3}c_2(t), \quad q(t) = \frac{1}{9}c_1^2(t) + \frac{1}{3}c_2(t). \]
Expression (16) supports existing experimental data and reproduces the “soft” Pomeron trajectory in the whole region of \( t \) (see below); the corresponding parameters \( \alpha_a, \sigma_a \) and \( \mu_0 \) are listed in Table 1. We calculate the Pomeron trajectory for three different sets of parameters (methods): I) the typical for light mesons parameter values \( \alpha_a = 0.816 \), string tension \( \sigma = 0.15 \text{GeV}^2 \), and quark mass \( m_q = 0.330 \) GeV and calculate (according to Ref. [17]) the gluonium parameters, \( \alpha_a = 3 \alpha_s = 2.448 \), \( \sigma_a = \frac{4}{3} \sigma = 0.338 \text{GeV}^2 \), and gluon mass \( \mu_0 = 1.5 m_q = 0.495 \) GeV (see also Ref. [27] for the gluon mass); II) the parameters \( \alpha_a, \sigma_a \), and \( \mu_0 \) are found from free fit of ZEUS data for the Pomeron trajectory [11]; III) include into the fit a \( 2^{++} \) glueball candidate at \( M = 1.710 \) GeV [53] by supposing that the glueball trajectory is the “soft” Pomeron trajectory.

| Method | \( J \) | \( E_{n}^{Gl} \) | Glueball parameters | \( \alpha_P(t) \) and \( \alpha'_P(t) \) at \( t = 0 \) |
|--------|-------|--------------|---------------------|------------------------------|
| I      | 2     | 1.740        | \( \alpha_a = 2.448 \) | \( \alpha_P(0) = 1.085 \) |
|        | 3     | 2.452        | \( \sigma_a = 0.338 \text{GeV}^2 \) | \( \alpha'_P(0) = 0.250 \text{GeV}^{-2} \) |
|        | 4     | 2.974        | \( \mu_0 = 0.495 \) GeV | \( \alpha'_P(0) = 0.151 \text{GeV}^{-2} \) |
|        | 5     | 3.408        | \( \alpha_a = 2.276 \pm 0.041 \) | \( \alpha_P(0) = 1.084 \) |
|        | 6     | 3.789        | \( \sigma_a = 0.294 \pm 0.003 \text{GeV}^2 \) | \( \alpha'_P(0) = 0.265 \text{GeV}^{-2} \) |
| II     | 2     | 1.984        | \( \alpha_a = 2.442 \pm 0.044 \) | \( \alpha'_P(0) = 0.265 \text{GeV}^{-2} \) |
|        | 3     | 2.689        | \( \sigma_a = 0.323 \pm 0.071 \text{GeV}^2 \) | \( \alpha_P(0) = 1.113 \) |
|        | 4     | 3.164        | \( \mu_0 = 0.478 \pm 0.084 \) GeV | \( \alpha'_P(0) = 0.265 \text{GeV}^{-2} \) |
|        | 5     | 3.549        | \( \alpha_a = 2.276 \pm 0.041 \) | \( \alpha_P(0) = 1.084 \) |
|        | 6     | 3.884        | \( \sigma_a = 0.294 \pm 0.003 \text{GeV}^2 \) | \( \alpha'_P(0) = 0.265 \text{GeV}^{-2} \) |
| III    | 2     | 1.695        | \( \alpha_a = 2.442 \pm 0.044 \) | \( \alpha_P(0) = 1.113 \) |
|        | 3     | 2.393        | \( \sigma_a = 0.323 \pm 0.071 \text{GeV}^2 \) | \( \alpha'_P(0) = 0.265 \text{GeV}^{-2} \) |
|        | 4     | 3.904        | \( \mu_0 = 0.478 \pm 0.084 \) GeV | \( \alpha_P(0) = 1.113 \) |
|        | 5     | 3.330        | \( \alpha_a = 2.276 \pm 0.041 \) | \( \alpha_P(0) = 1.084 \) |
|        | 6     | 3.703        | \( \sigma_a = 0.294 \pm 0.003 \text{GeV}^2 \) | \( \alpha'_P(0) = 0.265 \text{GeV}^{-2} \) |

From this table, we see that the methods I and III reproduce the trajectory with the properties of the classic “soft” Pomeron. The intercept and slope estimated by these three methods are: \( \alpha_P(0) = 1.09 \pm 0.02 \) and slope \( \alpha'_P(0) = 0.22 \pm 0.03 \text{GeV}^{-2} \). Parameters, \( \alpha_a, \sigma_a \), and gluon mass \( \mu_0 \) close to those predicted by different authors [27, 46]. The corresponding mass of the \( 2^{++} \) glueball candidate is around 1.81 GeV. Masses of gluonium leading states, \( E_{n}^{Gl} \), have been calculated with the help of the interpolating mass formula [14].

In Fig. [1] we show the Pomeron with parameters of the Method III. The trajectory is linear at \( t \to \infty \) with the slope \( \alpha'_P = 1/8 \sigma_a \text{GeV}^{-2} \). In the scattering region, the trajectory flattens off at \( -1 \) for \( t \to -\infty \). The first derivative, \( \alpha'_P(t) \), is positive in the whole region, \( -\infty < t < \infty \). We see that the experimental data and simple calculations in the framework of the potential approach support the conception of the “soft” supercritical Pomeron as observed at presently available energies.

We can also obtain asymptote for the BFKL-Pomeron trajectory predicted by pQCD [4]. If we take into account gluons’ spins with the total spin of two interacting gluons \( S = 2 \), then the formula [53]

\[
\alpha(E^2) = l(E^2) + S \tag{17}
\]
gives for the leading “hard” pomeron trajectory at large spacelike \( t \),

\[
\alpha_P(t) = 1 + \frac{\bar{\alpha}(|t|)}{\sqrt{1 - \frac{t}{4m_s^2}}} \quad t \to -\infty. \tag{18}
\]
Pomeron with such properties has been also used to describe the ZEUS data on the charm structure function \( F_2^c \) \[^{[55]}\]. It was shown that the two-Pomeron picture (“soft” plus “hard” Pomeron) gives a very good fit to the total cross section for elastic \( J/\Psi \) photoproduction and the charm structure function \( F_2^c \) over the whole range of \( Q^2 = -t \) \[^{[32]}\]. The results of these experiments and the found higher order corrections \[^{[56]}\] make it quite unclear what a “hard” Pomeron is.

There is another explanation of the small \( x \) charm production data at HERA. In many Regge models (see, for instance, Refs. \[^{[20, 30]}\]), one-Pomeron exchange gives only dominant contribution into the cross section. With energy growth, multiple “soft” Pomeron (MSP) exchanges and sea quark contributions become important; these contributions are important just at small \( x \). Combined with the eikonal model the MSP exchanges give the correct energy dependence of total and total inelastic cross sections \[^{[20]}\] and allow to describe hard distributions of secondary hadrons \[^{[30]}\]. From this point of view, the required “hard” Pomeron discussed in Ref. \[^{[6]}\] effectively accounts for the MSP exchange contributions.

6 Conclusion

After a long period the dominant point of view that Pomeron is a t-channel process, not a particle, researchers took more seriously a picture where Pomeron is dual to glueball (gluonium) states, or even it is a bound state in s-channel. From one side, these considerations were encouraged by new data for hard diffraction and discovering of Pomeron parton structure. From the other, it was caused by a significant theoretical progress with duality concept.

Our previous results obtained for Reggeon trajectories in the framework of the quark potential model \[^{[15, 16]}\] are in agreement with the existing experimental data and [for appropriate definition of the Regge trajectory (see Eq. \[^{[17]}\]) with the pQCD predictions on asymptotic behavior of the trajectories. In this work, an endeavor to investigate the properties of the Pomeron trajectory in the framework of the same potential approach has been undertook. This approach assumes a unify consideration of both the scattering problem and bound state problem on the basis of solution of the wave equation for the QCD motivated potential.

It is known, that using the-fixed-number of particles with a potential description can not be used for strict relativistic description. Strict description of Pomeron presuppose multiparticle description of the system. For perturbative regime with the Pomeron scattering, the dominant contribution comes from BFKL Pomeron. In this work, we have constructed Pomeron as a system of two relativistic massive gluons interacting by Cornell potential and obtained the interpolating mass formula for the squared mass.

The trajectory obtained is linear at large timelike \( t \) and flattens off at \( -1 \) in the scattering region at large \( -t \). To reproduce the Pomeron trajectory in the intermediate region, we have used the interpolating mass formula \[^{[11]}\] for the squared energy eigenvalues, \( E_n^2 = E^2(l, n_r) \), of the two-gluon system. This can be justified because we do not know the exact form of the potential in this region. The analytic dependence \( E^2(l, n_r) \) has allowed us to reproduce the Pomeron trajectory and calculate its intercept and slope from the fit of recent HERA data on \( \alpha_P(t) \). These parameters are in agreement with ones obtained earlier by Landshoff and Nachtmann for the “soft” Pomeron.

Perturbative QCD predicts \[^{[37]}\] different asymptotic behavior of the Regge trajectories which contradicts to experimental data. The resolution of this contradiction for the \( \rho \) trajectory was proposed in Ref. \[^{[37]}\], namely that the hard QCD part of the trajectory is weakly coupled and that its contribution will be hidden until much high energy. However, given the much higher energies at which the Regge trajectories are known, this argument does not appear to be of much help and the contradiction remains. A more realistic explanation can be connected with nonperturbative nature of hadronic interactions.

Dealing with Regge trajectories, one needs to mention the Regge cuts. From the Regge viewpoint cuts are expected to become important at large \( -t \). As shown in Ref. \[^{[33]}\], only in
a rather limited intermediate angular region the Regge cuts may be important, $1 < -t < 3$ GeV$^2$ at low energies, $s < 60$ GeV$^2$. At ISR energies the two Pomeron cut appears to control the whole measured high $-t$ region, $1.4 < -t < 14$ GeV$^2$. However, at larger $-t$ the curvature of the trajectories enable the poles again to become dominant. The factorization property of the Pomeron coupling has also been tested in fixed target experiments and found to hold. If the Pomeron was a cut it would not necessarily lead to factorizable coupling. The experimental data indicate that it is more likely that the Pomeron is a pole rather than cut. Therefore, the existing data and analysis performed in this work confirm the existence of the Pomeron whose trajectory is nonlinear and coincide with the classic “soft” Pomeron at small spacelike $t$.

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Figure 1: The Pomeron trajectory. Solid curve is the trajectory \[(16)\] with the parameters found from the fit of combined ZEUS $\rho$ (triangles) and $\phi$ (circles) data \[11\], and $2^{++}$ glueball candidate $f_0(1710)$ (cross) \[53\]. Other lines show the classic “soft”, BFKL, and Donnachie-Landshoff “hard” Pomerons.