Filtering out the cosmological constant in the Palatini formalism of modified gravity

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Abstract  According to theoretical physics the cosmological constant (CC) is expected to be much larger in magnitude than other energy densities in the universe, which is in stark contrast to the observed Big Bang evolution. We address this old CC problem not by introducing an extremely fine-tuned counterterm, but in the context of modified gravity in the Palatini formalism. In our model the large CC term is filtered out, and it does not prevent a standard cosmological evolution. We discuss the filter effect in the epochs of radiation and matter domination as well as in the asymptotic de Sitter future. The final expansion rate can be much lower than inferred from the large CC without using a fine-tuned counterterm. Finally, we show that the CC filter works also in the Kottler (Schwarzschild-de Sitter) metric describing a black hole environment with a CC compatible to the future de Sitter cosmos.

Keywords  Cosmological constant · Modified gravity · Palatini formalism · Dark energy · Vacuum energy

1 Introduction

The starting point of this work is a CC or equivalently a vacuum energy density \( \Lambda \) of enormous magnitude. This expectation is suggested by contributions to the CC coming from phase transitions in the early universe, zero-point energy in quantum field theory or even from quantum gravity. In general, all these parts are of different magnitude and probably unrelated to each other. Hence, the sum \( \Lambda \) of all terms is dominated by the largest contribution. Since other energy sources dilute with the expansion of the
universe, the CC will eventually take control over the cosmos. Depending on its sign the CC would induce in the very early universe either a Big Crunch or an eternal de Sitter phase with a very high Hubble rate $H \propto \Lambda$. Obviously, the standard Big Bang evolution does not happen in this case.

The simplest way to avoid this problem is the introduction of a CC counterterm $\Lambda_{ct}$, which makes the sum $|\Lambda + \Lambda_{ct}|$ smaller than the currently observed critical energy density $\rho_{c0} \sim 10^{-47} \text{GeV}^4$. For concreteness let us assume that $\Lambda \sim -M_{\text{ew}}^4$ were related to the electroweak phase transition at the energy scale $M_{\text{ew}} \sim 10^2 \text{GeV}$. Then the counterterm must be extraordinarily close to $(-\Lambda)$ requiring an enormous amount of fine-tuning,

$$\left| 1 + \frac{\Lambda_{ct}}{\Lambda} \right| < \frac{\rho_{c0}}{\Lambda} \sim 10^{-55}.$$  (1)

Apart from the fine-tuning of the classical counterterm, the situation is even more involved when quantum corrections are included, cf. Ref. [1] for an elaborated discussion in the context of the electroweak sector of the standard model of particles. Moreover, the problem worsens when $\Lambda$ is dominated by higher energy scales, possibly originating from grand unified theories where $\Lambda \sim (10^{16} \text{GeV})^4$ or quantum gravity with $\Lambda \sim (10^{19} \text{GeV})^4$ for instance. Summing up, the fine-tuning of the CC is considered to be one of the most severe problems in theoretical physics [2,3]. In addition, the current accelerated expansion of the universe [4–6] can be explained very well by a tiny CC of the same magnitude as the energy density of matter, giving rise to the so-called coincidence problem. For the latter problem many explanations have been proposed [7–11], which induce late-time accelerated expansion. However, most of these models tacitly assume that the large $\Lambda$ has been fine-tuned away and thus they do not address the big CC problem.

Without fine-tuning we have to accept the existence of the presumed huge CC, and we have to find a way to neutralise its effects in order to obtain a reasonable cosmological evolution. Along this line, several proposals have been made, e.g. relaxation models for a large CC in the context of matter with an inhomogeneous equation of state (EOS) [12], or in the LXCDM framework [13] with a variable cosmological term [14,15], see also [16–21]. Removing or filtering out vacuum energy has been investigated e.g. in Refs. [22–25], and it is a feature in unimodular gravity [26–30]. Recently, a CC relaxation model has been discussed in the context of modified gravity with an action functional $f(R, G)$ involving the Ricci scalar $R$ and the Gauß-Bonnet invariant $G$ in the metric formalism [1,31], where the action is varied with respect to the metric $g_{ab}$ only.

In this work, we also consider a modified gravity model with an action functional $f$ in terms of the Ricci scalar $R$ and the squared Ricci tensor $Q = R_{ab}R^{ab}$. However, here we apply the Palatini formalism, where the metric $g_{ab}$ and the connection $\Gamma^a_{bc}$ are treated independently by the variation principle. In contrast, the metric formalism requires from the beginning that

$$\Gamma^a_{bc}[g] = \frac{1}{2} g^{ad} (g_{dc,b} + g_{bd,c} - g_{bc,d}).$$  (2)