Experimental verification of quantum computation

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Quantum computers are expected to offer substantial speed-ups over their classical counterparts and to solve problems intractable for classical computers. Beyond such practical significance, the concept of quantum computation opens up fundamental questions, among them the issue of whether quantum computations can be certified by entities that are inherently unable to compute the results themselves. Here we present the first experimental verification of quantum computation. We show, in theory and experiment, how a verifier with minimal quantum resources can test a significantly more powerful quantum computer. The new verification protocol introduced here uses the framework of blind quantum computing and is independent of the experimental quantum-computation platform used. In our scheme, the verifier is required only to generate single qubits and transmit them to the quantum computer. We experimentally demonstrate this protocol using four photonic qubits and show how the verifier can test the computer’s ability to perform quantum computation.

The prevalent scientific paradigm of testing physical theories by comparing experimental results with predictions computed on a piece of paper or on a computer assumes that all such predictions are solvable in polynomial time on a classical computer. In present experiments involving quantum particles, such as fundamental tests of quantum mechanics or small-scale quantum computations and simulations\(^1\)–\(^5\), following this paradigm is still possible, as the results can be calculated on a classical computer and verified in experiments involving quantum systems. However, there is an entire class of problems—for example, the simulation of complex quantum systems\(^6\)—that are solvable in polynomial time only on a quantum computer\(^7\).

One of the central conceptual questions in present quantum computing is therefore whether any entity can test the results obtained by a quantum computer, even when that entity is unable to compute these results itself. Alternatively, from a different perspective, can an experimentalist with only classical resources or restricted quantum resources prove that a given device is a quantum computer\(^8\)? Whereas the ultimate answer to such questions is still open, there are several proposals that offer a solution when the verifier is equipped with a range of quantum resources\(^9\)–\(^14\)—quantum memory, two entangled quantum computers, or a large number of qubits—which, however, are outside the reach of present technology.

Here, we demonstrate how to verify a quantum computation on four qubits. Our method is directly applicable to present technology and can be readily extended to more general cases. We show that only minimal quantum resources (specifically, single qubits) are required to certify a quantum-information processor. Our protocol is independent of the physical system on which it is implemented and it can therefore be applied to any quantum-computing platform.

We have implemented the new protocol on a photonic quantum system and demonstrate the necessary components for verifying a quantum device. We also show how our scheme can be used to verify the generation of the archetype of a quantum-computational resource, quantum entanglement, through a violation of Bell’s inequality. In such a verification, the prover remains blind and cannot distinguish the verification procedure from standard quantum-computational tasks such as single- or multi-qubit gates or entire quantum algorithms. To the best of our knowledge, this is the first experiment towards certifying the correctness of a quantum computation.

Interactive proof systems and blind quantum computing

Our protocol combines interactive proof systems and blind quantum computing\(^9\)–\(^11\). Interactive proof systems were originally invented in the field of computer science, to approach questions in classical complexity theory\(^13\)–\(^16\). They have since been extended into the realm of quantum computation\(^1\). A quantum prover interactive proof system addresses the question of whether a prover who has access to quantum-computational resources can convince a classical verifier that he can solve a given problem. Interactive proof systems can therefore be used to address the fundamental questions posed above, provided the traditional scientific paradigm of ‘predicting’ is replaced by ‘verifying’.

In our protocol, the framework of the interactive proof system is given by blind quantum computing (Fig. 1). In this framework, a verifier (or client) with limited quantum computational resources can delegate a quantum computation to a prover (or server) with the full power of quantum computing such that all data and the whole computation remain private\(^10\)–\(^17\). More specifically, the verifier prepares single qubits in the state

$$|\theta_i\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{i\theta} |1\rangle)$$

with $\theta_i \in \{0, \pi/4, \ldots, 7\pi/4\}$ chosen uniformly at random and known only to the verifier. The qubits are then transmitted to the prover who entangles them to create a blind cluster state\(^18\). The actual computation is measurement-based\(^19\)–\(^20\). The verifier calculates for each blind qubit measurement instructions according to

$$\delta_i = \theta_i + \phi_i + \pi r_i$$

where $\delta_i$ is the blind phase of the qubit, $\phi_i$ is the rotation that the verifier wants to perform (including any Pauli corrections) and $r_i \in \{0, 1\}$ is a randomly chosen value to hide

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The verifier wants to find out whether the prover can indeed perform quantum computations. Although the question of whether a classical verifier can test a quantum system is still open, it was shown that a verifier who has access to certain quantum resources can verify quantum computations. Here, in the framework of blind quantum computing, the verifier has to be able to generate single qubits and to transmit them to the prover. After the transmission of the qubits, the verifier and the prover exchange two-way classical communication.

Figure 1 | Concept of a quantum prover interactive proof system based on blind quantum computing. The verifier wants to find out whether the prover can indeed perform quantum computations. Although the question of whether a classical verifier can test a quantum system is still open, it was shown that a verifier who has access to certain quantum resources can verify quantum computations. Here, in the framework of blind quantum computing, the verifier has to be able to generate single qubits and to transmit them to the prover. After the transmission of the qubits, the verifier and the prover exchange two-way classical communication.

the measurement outcome. The prover performs measurements in the basis

\[ |\pm_i\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm e^{i\phi_i}|1\rangle) \]

and delivers the results to the verifier. Without the knowledge of the underlying rotation and the random phase, the prover cannot find out anything about the actual rotation \( \phi_i \)—thus, the computation remains blind. The verifier, in contrast, knows the initial rotation and is able to interpret the results. Blind quantum computing therefore provides a powerful tool to delegate computations and to access the resources of powerful quantum computers without divulging the content of the computation. In the following, we show how this concept can be applied to verify quantum computations.

Verification of a quantum computation

In the framework of blind quantum computing, to test a quantum computation, the correctness of the measurements performed by the server has to be verified. Here we use a new verification procedure based on the creation of trap qubits. Trap qubits are blindly prepared in a well-defined state, which is known only to the verifier, and are isolated from the actual computation. The measurement angle of these trap qubits is chosen such that the measurement result is predetermined by the verifier, and hence any cheating strategy used by the server that alters these measurement outcomes will be detected. By randomly choosing the locations of the trap qubits it is then possible to bound the probability that the server can cheat while remaining undetected. To reduce resource overhead, our protocol forges any encoding of logical qubits into an error detection code, and hence the probability of an error going undetected is polynomially rather than exponentially small, as can be achieved if greater resources are available.

In our setting, we implement the preparation of the trap qubits through a measurement-based computation on the non-trap qubits. The verifier chooses measurement settings on the cluster state such that any of the qubits could become a trap qubit, prepared in a random state \( |\theta_i\rangle \). If the trap qubit is then measured in the basis \( |\theta_i\rangle \), the outcome will always be known to the verifier (Fig. 2). Our measurement-based creation of the trap qubits means that we verify a correlation between a subset of measurements, rather than a single measurement outcome. We therefore have to be careful to ensure that the correctness of these correlations for all trap measurements does imply the correctness of a given computational run. In our demonstration using a four-qubit system, only one error remains undetected (see Supplementary Information). However, this particular error cannot alter the result of the measurement of Bell’s quantity presented in this paper. In general, the use of measurement-based computation to prepare traps can be applied to systems of an arbitrary size. However, the existence of undetectable errors will depend on the particular entanglement graph and the desired computation. This is a distinct disadvantage of the present scheme when compared with scalable unconditionally secure schemes, but the result of making this sacrifice is a reduction in the technological burden.

This verification procedure can be used to verify that the quantum computation was performed correctly. Therefore, we consider multiple runs of the protocol, where the verifier randomly chooses to run an actual computation or a verification test (Fig. 3). This use of multiple runs of the blind-computation protocol lets us make optimal use of the qubits available in our system. Moreover, the server cannot distinguish between an actual computation run or a trap run. Hence, as discussed in detail in the Supplementary Information, this procedure can be used to verify not only the correctness of the measurement outcomes but also of the entire quantum computation. When trap computation and target computation are randomly interspersed, then the probability that the quantum computer produces the correct result for the verification runs but a wrong result for the computation runs is bounded by a value depending on three parameters: the number of computation runs, the number of trap runs, and the total number of qubits in the system (see Supplementary Information). Although error detection using the outcome of various computation runs can lead to an exponentially small probability of failing to detect a logical error, it should be noted that the verification procedure we use here cannot be directly extended to the case of quantum input and output with better than polynomial accuracy.
Entanglement verification

Once the measurement outcomes are verified, we can proceed to use the system to prove the prover’s entangling capabilities and present a Bell test as an example of a small verifiable computation. Quantum correlations are typically confirmed by well-established tests of Bell’s inequality\(^{21}\) (Fig. 4). To do so, combinations of specific measurement settings \(\alpha, \alpha'\) and \(\beta, \beta'\) are performed on the first (a) and on the second qubit (b), respectively, after generating an entangled state \(|\Psi\rangle_{a,b}\). The settings are chosen such that a maximal violation of the Bell inequality of the Clauser–Horne–Shimony–Holt type\(^{22}\) is obtained for an entangled state:

\[
S = |E(\alpha, \beta) - E(\alpha', \beta')| + |E(\alpha', \beta) + E(\alpha, \beta')| \leq 2
\]

The correlation coefficients \(E(\cdot, \cdot)\) are defined by the coincidence counts when measuring qubit a in the basis \(\alpha\) and qubit b in the basis \(\beta\) (for details see Supplementary Information).

To make the Bell test blind, we hide the generation of the entangled state as well as the Bell measurement settings. For this, we base our implementation on a blind zigzag cluster state with four qubits \(|\vartheta\rangle\), which is shown in Fig. 4b. Single-qubit measurements on the blind zigzag cluster state realize a quantum circuit that offers exactly the degrees of freedom that are necessary for our blind Bell test. First, using this type of cluster, the verifier has the possibility to blindly switch between entangled or separable input states by choosing \(\delta_1\) and \(\theta_1\) accordingly. Second, the standard measurement settings for a Bell test are hidden as they are determined by the phases of the blind qubits \(|\vartheta_1\rangle, |\vartheta_2\rangle\) and \(|\vartheta_3\rangle\) and their respective measurement settings of \(\delta_1, \delta_2\) and \(\delta_3\) (see Supplementary Information for details).

As a result, the state generation and the Bell-state measurements are encoded in the phase of blind qubits as well as in the measurement instructions, which remain unknown to the prover at any time. The choice of the cluster-state configuration also remains hidden from the prover. This is a particular advantage of our probabilistic implementation of blind quantum computing, where all qubits are measured coincidentally to verify that a blind cluster state has been created.

Experiment

We use the particular advantages offered by photons to realize a quantum network that can communicate and process quantum information\(^{23}\) within the same physical system\(^{18}\). In our experiments, the blind cluster states are generated from photon pairs entangled in polarization and mode, which originate from spontaneous parametric down-conversion\(^{24}\). Our set-up and the methods used are explained in detail in ref. 10; in the present experiment, we generate blind cluster states for various settings of \(\theta_1\) and use them to implement exemplary runs of trap computations as well as the Bell-test runs—the necessary building blocks of a verified test of Bell’s quantity. We note that the requirement of having single photons can be relaxed to coherent states\(^{25}\) or to the verifier doing only measurements\(^{26}\).

For the demonstration of the measurement verification, we use blind linear cluster states and blind rotated horse-shoe cluster states to prepare traps as shown in Fig. 2a,b. By choosing the blind phases \(\vartheta_1\) and measurement settings...
\[ \delta \] as given in the Supplementary Information, we prepare the trap qubits:

\[ |\text{trap}_i\rangle = |\pm_i\rangle \text{ and } |\pm_i\rangle \]

(1)

\[ |\text{trap}_i\rangle = |\pm_i\rangle \text{ and } |\pm_i\rangle \]

(2)

\[ |\text{trap}_i\rangle = R_z(\pi/4)|\pm_i\rangle \text{ and } R_z(\pi/4)|\pm_i\rangle \]

(3)

\[ |\text{trap}_i\rangle = |\mp_i\rangle \text{ and } |\pm_i\rangle \]

(4)
on qubits 1, 2, 3 and 4, respectively. Here, \([|\pm_i\rangle = (|0_i\rangle \pm |1_i\rangle)/\sqrt{2}\) and \([|\pm_i\rangle = (|0_i\rangle \pm |1_i\rangle)/\sqrt{2}\). The probabilities to find the correct outcome are well above 90%, as shown in Fig. 2c.

For a verification of quantum entanglement, we choose combinations of \(\theta_4\) and \(\delta_4\) to create the entangled state:

\[ |\Psi\rangle_{a,b} = \frac{1}{\sqrt{2}}(|+\rangle_a|0\rangle_b - |i\rangle_a|1\rangle_b) \]

(5)

and to blindly implement the Bell measurements settings we choose:

\[ \alpha = \pi/2, \quad \alpha' = \alpha, \quad \beta = -3\pi/4, \quad \beta' = -\pi/4 \]

(6)

where the bases \(\alpha, \beta\) and \(\beta'\) are defined as \([|0\rangle \pm e^{i\phi}|1\rangle)/\sqrt{2}\] and so on, and \(\alpha'\) is a measurement in the computational basis \([|0\rangle, |1\rangle]\). To obtain the measurement settings given in equation (6), we choose combinations of \(\theta_i\) and \(\delta_i\) as given in detail in the Supplementary Information. From the measured coincidence count rates, we calculate the correlation coefficients to be:

\[ E(\alpha, \beta) = -0.540 \pm 0.084 \]

\[ E(\alpha, \beta') = 0.634 \pm 0.086 \]

\[ E(\alpha', \beta) = -0.646 \pm 0.067 \]

\[ E(\alpha', \beta') = -0.678 \pm 0.079 \]

Those coefficients lead to an S parameter of

\[ S = 2.498 \pm 0.158 \]

which violates the classical bound \((|S| = 2)\) by more than 3 standard deviations.

Although the main role of the Bell test is to serve as an archetypal quantum computation to be verified, the combination of the verification procedure and this violation of Bell’s inequality also provides a measure for the quality of the entangling gates the prover performs. In our experiment, we implement a subset of all possible blind states. The states of qubits 1 and 4 are fixed to \(|+\rangle\), whereas the states of qubits 2 and 3 are fully blind\(^{24}\). The whole verification procedure remains blind, however, if we assume that the prover has no a priori knowledge of our choice of states and measurements.

The violation of Bell’s inequalities is impossible classically, not for the reason of high complexity, but rather on the basis of physical principles. In our implementation, we assume the correctness of quantum mechanics for the verification of the measurement outcomes. Without this assumption, a full demonstration would require the two entangled photons to be sent to two distant laboratories or servers, where only at the very last step of the computation the verifier gives the measurement instructions to the prover. In this way, no classical computer could mimic, even in principle, the output of the computation a priori, while ensuring that the verification procedure would still have a positive outcome.

**Conclusion**

Future large-scale quantum computers and quantum simulators\(^{27–30}\) will require the verification of their experimental results\(^{31}\). Owing to the superior computational capacity of quantum systems the results cannot simply be calculated and checked on a classical device. The development of new methods for the verification of quantum computations is therefore a crucial task. Here, we have developed a new general method for verifying quantum computations that can be readily applied to present small-scale quantum computers.

Finally, how verification mechanisms provide insights into questions of computational complexity and into the foundations of quantum physics is a topic of active research. So far, the limit of high computational complexity is mostly unexplored and it is not impossible that quantum mechanics breaks down at some scale of complexity\(^{32}\).

Verification methods, such as those reported herein, are not only important as a mechanism to certify quantum computers, but also provide an entirely new toolbox for addressing fundamental questions in quantum physics and computer science.

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**Author contributions**

S.B. designed and performed the experiments, acquired the experimental data, carried out theoretical calculations and the data analysis, and wrote the manuscript. J.F.F. and E.K. carried out theoretical calculations, contributed the proofs, and wrote the manuscript. P.W. designed the experiment, edited the manuscript and supervised the project.

**Additional information**

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to S.B. or P.W.

**Competing financial interests**

The authors declare no competing financial interests.