Study of Precision Calibration of 3TPS Type Parallel Robot

Ji-Man LUO\textsuperscript{1,a}, Xiao-Dan ZHANG\textsuperscript{2,b,*}, Dong-Yue ZHANG\textsuperscript{3,c}, Ze-Ming WEI\textsuperscript{4,d}

\textsuperscript{1,2,3,4}Shenyang Jianzhu University, Shenyang, Liaoning, China
\textsuperscript{a}syljm2004@163.com, \textsuperscript{b}zxdyana@hotmail.com, \textsuperscript{c}syzdy1201@163.com, \textsuperscript{d}894931520@qq.com

*Corresponding author

Keywords: parallel robot, kinematic calibration, Least Square Method, accuracy.

Abstract. Structural parameters error due to the manufacture and installation of the robot components affect the positioning accuracy and precision of the agency. The approach of kinematic calibration will be used to assure the accuracy of the 3TPS parallel manipulators in the paper. Error model of kinematics calibration base on kinematics equation is set up to identify sources of error, and the point coordinates of tool nose are measured by the SGC- 4.2 grating ruler etc. The nonlinear error equation is solved to identify the error parameter by using the Least Square Method, and the error will be compensated. Calibration results are verified with precision, standard deviation, root mean square error, maximum error. The kinematic calibration experiment is performed on the prototype of a 3-TPS-PT machine tool. Experiment results show that the kinematic calibration method can lead to a better accuracy of the manipulator. The study of this paper is meaningful in kinematic calibration of other parallel manipulators.

Introduction

Parallel mechanism has a deviation between theory and actual structure parameters due to machining error and assembly error etc. influence factors. Kinematics model of parallel mechanism is not accurate which affect the working accuracy. Therefore, it is necessary to improve the machining precision by using calibration techniques. Robot calibration can be divided into three different levels[2]. The first level is joint level that purpose is to confirm the value relationship between the sensor and the actual joint; The second level is to calibrate complete kinematics model of robot; The third level is dynamics calibration of inertia characteristics of different connecting rods, etc. The first two stage is referred to static calibration or kinematic calibration. Kinematic calibration of parallel mechanism generally needs a kinematics model that can accurately represent the actual parameters, repeating measure, identifying parameter and compensating error until ultimately the precision requirement of the actual geometrical parameters [1] is satisfied.

Kinematics Calibration Model

Inverse Kinematics Equation of 3TPS Parallel Mechanism

This 3TPS parallel mechanism is the head device of a hybrid robot for engraving. The robot is shown in figure 1. One end of the telescopic rod is connected to the static platform by hooke joint, the other side is connected to the moving platform by spherical hinge. Constraint chains is
connected with the static platform by hooke joint, on the other side is maintaining fixed by a flange to the moving platform.

Three telescopic rod chain (TPS) is the drive chains. The rotation of servo motor transforms into the translation of the telescopic rod through the whorl to control drive chain to elongate or shorten. Constraint chain (TP) is driven chain, and it controls the movement along the Z axis direction and the rotation around the X, Y axis. The tool nose of machine tool is vertically fixed on the moving platform and always keep on a straight line with constraint chain. The kinematic model of 3TPS parallel mechanism is shown in figure 2.

![Figure 1. 3 TPS/TP hybrid robot.](image1)

![Figure 2. The kinematic model of 3TPS parallel mechanism.](image2)

Static platform and moving platform of the parallel mechanism are equilateral triangle. The circumcircle radius of static platform is \( R_1 \). Origin \( O \) and the centroid of the static coordinate system are coincident. The \( X_0 \) axis goes through the point \( A_1 \) and the \( Z \) axis is vertically goes through the origin \( O \). The circumcircle radius of moving platform is \( R_2 \). Origin \( P \) of moving coordinate system is located in the center of mass of moving platform. The \( X_P \) axis goes through the point \( B_1 \) and the \( Z \) axis is vertical. Assuming that the flatness error of moving platform and the static platform is 0; Flatness error of the plane \( OPB_1A_1 \) is 0.

Where \( \theta_1 \) is the angle between \( OA_2 \) and \( X \) axis, \( \theta_2 \) is the angle between \( OA_3 \) and \( X \) axis, then

\[
A_1 = (−R_1, 0, 0), A_2 = (R_1 \cos \theta_1, R_1 \sin \theta_1, 0), A_3 = (R_1 \cos \theta_2, −R_1 \sin \theta_1, 0), O = (0, 0, 0)
\]

In the moving platform coordinate system, it has

\[
B_1 = (−R_2, 0, 0), B_2 = (R_2 \cos \theta_1, R_2 \sin \theta_1, 0), B_3 = (R_2 \cos \theta_2, −R_2 \sin \theta_1, 0), P = (0, 0, 0)
\]

Use algorithm of \( D-H \) coordinate conversion to map \( P \) of moving platform to the static coordinate system.

\[
R_p^O = T_p^O × R(Y, \beta) R(X, \alpha) = \begin{bmatrix}
c \beta & s \beta \alpha & s \beta c \alpha & -d s \beta c \alpha \\
0 & c \alpha & -s \alpha & d s a \\
-s \beta & c \beta s \alpha & c \beta c \alpha & -d c a c \beta \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  \( (1) \)
Where $d$ is length of constraint chain $OP$. The point coordinates of tool nose is

$$
D = \begin{bmatrix}
X_D \\
Y_D \\
Z_D
\end{bmatrix} = \begin{bmatrix}
-(d + L_{PD})s\beta c\alpha \\
(d + L_{PD})s\alpha \\
-(d + L_{PD})c\alpha \beta
\end{bmatrix}
$$

(2)

Where $L_{PD}$ is fixed distance between tool nose and the center of moving platform.

Inverse kinematics of three drive chains is

$$L_i^2 = (X_{Ai} - X_{Bi})^2 + (Y_{Ai} - Y_{Bi})^2 + (Z_{Ai} - Z_{Bi})^2; \quad i = 1, 2, 3
$$

(3)

From (3), there three separate explicit equations. When the basic size ($d$, $R$ and $r$) and the position and orientation of tool nose ($x$, $y$, $z$, $\alpha$ and $\beta$) is known, length range of drive rod is got.

**Error Model of Kinematics Calibration**

Formula (3) is the model of kinematics calibration of 3TPS parallel mechanism. From (2), $\alpha$, $\beta$, $d$ can be represented by $X = [X_D, Y_D, Z_D]^T$, then formula (3) can be written as

$$F_i(X, R_1, R_2, \theta_1, \theta_2, L_i) = 0
$$

(4)

Where $L_i$ is length of telescopc rod. Then formula (4) can be wrote as:

$$
\frac{\partial F_i}{\partial X} dX + \frac{\partial F_i}{\partial R_1} dR_1 + \frac{\partial F_i}{\partial R_2} dR_2 + \frac{\partial F_i}{\partial \theta_1} d\theta_1 + \frac{\partial F_i}{\partial \theta_2} d\theta_2 + \frac{\partial F_i}{\partial L_i} dL_i = 0
$$

(5)

Where $dX$ is vector of position error between the actual output and nominal output; $dR_1$, $dR_2$ are respectively the error vector between the nominal value and the actual value of the static and moving platform; $d\theta_1$, $d\theta_2$ are error vector of hinge position which is due to manufacture and installation etc; $dL_i$ is error vector of stretch of telescopc rod, here error contains the error caused by manufacture and installation.

From (5) the error model of calibration is

$$dX = -ABdR_1 - ACdR_2 - ADd\theta_1 - AEd\theta_2 - AFdL_i
$$

(6)

where $A = (\frac{\partial F_i}{\partial X})^{-1}, B = \frac{\partial F_i}{\partial R_1}, C = \frac{\partial F_i}{\partial R_2}, D = \frac{\partial F_i}{\partial \theta_1}, E = \frac{\partial F_i}{\partial \theta_2}, F = \frac{\partial F_i}{\partial L_i}$, one has:

$$
\begin{bmatrix}
dx \\
dy \\
dz
\end{bmatrix} = \begin{bmatrix}
dR_1 \\
dR_2 \\
d\theta_1 \\
d\theta_2 \\
dL_i
\end{bmatrix}
$$

(7)
Experiment of Kinematics Calibration

Experimental Instrument

The SGC - 4.2 TM 800 5μ grating ruler ranged 800mm is used to measure displacement of the X axis. It is shown in figure 3. The SGC - 5M 1500 5μ grating ruler ranged 1500mm is used to measure displacement of the Y axis. Its resolution ratio is 5 microns and its response speed is 60 m/min. It is shown in figure 4. Accuracy kTM-250mm displacement sensor is used to measure displacement of the Z axis. Device of displacement is shown in figure 5. Since sensor is greatly affected by external factors, its value fluctuations up and down that can not be used as experimental calibration data. Kalman filter[6] is used to process data of sensor.

Selection of Test Point

Motion space of the 3TPS parallel mechanism is ball cone. This experiment selects 6 plane, each plane has the distance of 5mm between two planes. The center of plane and four points that are on the angle split line of four quadrant in each plane are chosen. Schematic diagram is shown in figure 6. The absolute coordinate of the 30 points is measured and compared with the theoretical value. The error of the X, Y, Z is calculated and is shown in Figure 7.
Analysis of Experimental Results

First 25 points are used to calculate compensation value by using Least Square Method. The last 5 points are used to inspect the result. The calibration errors of each drive chain are $\Delta R_1, \Delta R_2, \Delta \theta_1, \Delta \theta_2, \Delta L_i$. The three drive chains is marked by $i=1,2,3$. There are total 15 errors. Results of identifying 15 errors are shown in table 1, the linear unit: mm, angle unit: deg

| Rod | $\Delta R_1$ | $\Delta R_2$ | $\Delta \theta_1$ | $\Delta \theta_2$ | $\Delta L_i$ |
|-----|--------------|--------------|-------------------|-------------------|--------------|
| i=1 | 0.035        | 0.029        | 0                 | 0                 | 0.069        |
| i=2 | 0.048        | -0.103       | 0.027             | 0.03              | 0.058        |
| i=3 | -0.033       | 0.033        | -0.039            | 0.055             | -0.093       |

The result is substituted into theoretical kinematics model. The compensation of center of the moving platform is calculated by calculating the average value of first 25 points. The compensation value of x direction is 0.077, compensation of y is -0.004 and compensation of z direction is -0.125.

From literature[7], positioning accuracy of robot is the evaluation of motion accuracy and its computational formula is $\Delta = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$. The error of last 5 text points are used to inspect the calibration result. The comparison of position error before and after calibration is shown in table 2.

|          | Before | After | Before | After | Before | After | Before | After |
|----------|--------|-------|--------|-------|--------|-------|--------|-------|
| $\Delta X$ | -0.11  | 0.01  | 0.269  | 0.290 | -0.033 | 0.006 | 0.144  | 0.147 |
| $\Delta Y$ | -0.125 | 0.16  | 0.264  | 0.333 | -0.048 | 0.156 | 0.139  | 0.214 |
| $\Delta Z$ | -0.125 | -0.175| 0.266  | 0.342 | -0.048 | -0.179| 0.141  | 0.233 |
| $\Delta$  | -0.005 | -0.175| 0.425  | 0.459 | 0.072  | -0.179| 0.300  | 0.357 |
| $\Delta$  | -0.005 | 0.165 | 0.208  | 0.265 | 0.072  | 0.161 | 0.083  | 0.195 |

Among them, comparison of position error $\Delta$ before and after calibration is shown in figure 8.

![Figure 8. The comparison of position error before and after calibration.](image)

Calibration results are verified with standard deviation, root mean square error, maximum error. They are the evaluation index of positioning accuracy. The results of calibration are shown in table 3.
Table 3. The results of calibration [mm].

| Index                  | Before | After |
|------------------------|--------|-------|
| Standard variance      | 0.067  | 0.070 |
| root mean square error | 0.345  | 0.240 |
| maximum error          | 0.459  | 0.357 |

According to table 3, one can find standard variance changes 4.28%. Here standard variance only represents the floating of error. Root mean square error reduces 30.48% and maximum error reduces 22.30%. Errors are decreased greatly. This shows this method is able to improve the positioning accuracy of robot.

Conclusion

In this paper, kinematics calibration of 3TPS parallel machine is studied by calculating inverse kinematics, establishing kinematic calibration model and using positive kinematics to derivate jacobian matrix of parameter error. The absolute coordinates of the tool nose is measured. Least Squares Method is used to identify parameter, The error is compensated by combining with the actual structure. The comparison of error before and after calibration is given. Results shows that standard error changes 4.28%, root mean square error reduces 30.48% and maximum error reduces 22.30%. It can be seen that kinematics calibration is an effective method for parallel machine tool to improve static precision machine.

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