Experiments on disordered alloys\textsuperscript{1–3} suggest that spin glasses can be brought into low-energy states faster by annealing quantum fluctuations than by conventional thermal annealing. Owing to the importance of spin glasses as a paradigmatic computational testbed, reproducing this phenomenon in a programmable system has remained a central challenge in quantum optimization\textsuperscript{4–13}. Here we achieve this goal by realizing quantum-critical spin-glass dynamics on thousands of qubits with a superconducting quantum annealer. We first demonstrate quantitative agreement between quantum annealing and time evolution of the Schrödinger equation in small spin glasses. We then measure dynamics in three-dimensional spin glasses on thousands of qubits, for which classical simulation of many-body quantum dynamics is intractable. We extract critical exponents that clearly distinguish quantum annealing from the slower stochastic dynamics of analogous Monte Carlo algorithms, providing both theoretical and experimental support for large-scale quantum simulation and a scaling advantage in energy optimization.

The study of spin glasses initiated an enormously productive exchange between physics and computer science\textsuperscript{14,15}. One key by-product of this exchange was the invention of simulated annealing (SA)\textsuperscript{16}, a method of optimization that simulates a gradually cooling system as it settles into a low-energy state. Recent decades have seen annealing brought to bear against countless multivariate optimization applications, seeking low-energy states that translate to low-cost solutions\textsuperscript{17}.

Passing through a thermal phase transition, as in SA, is not the only way to evolve a spin system from an 'easy' disordered phase into a 'hard' glassy phase. One can also pass through a quantum phase transition (QPT), in which the ground state undergoes a macroscopic shift in response to changing quantum fluctuations. Both experiments\textsuperscript{1} and simulations\textsuperscript{4,5} suggested that quantum annealing (QA) can guide a spin glass towards equilibrium faster than thermal annealing. Thus, QA originated as a means of both studying quantum-critical phenomena and optimizing quadratic objective functions\textsuperscript{18,19}. Simulating the Schrödinger dynamics of QA with a classical computer is an unpromising optimization method, as memory requirements grow exponentially with system size. But, as Feynman famously asked\textsuperscript{20}, "Can you do it with a new kind of computer—a quantum computer?".

This question motivated not only the development of programmable QA processors\textsuperscript{5,21–25}, but a more general effort to probe the capabilities of near-term quantum devices via quantum simulation, including in trapped ions\textsuperscript{26–27}, ultracold atoms\textsuperscript{28} and Rydberg arrays\textsuperscript{29} (the latter was recently explored as an annealing optimizer\textsuperscript{29}). Although D-Wave QA processors have already been used to simulate quantum spin glasses in a decohering thermal bath\textsuperscript{31}, it was only recently shown that they can simulate QPTs with negligible interaction with the thermal environment\textsuperscript{22}. QA processors can realize the coherent dynamics of quantum-critical phenomena despite the finite temperature of the apparatus itself. Here we use a QA processor to study the critical dynamics of a spin-glass QPT. The exceedingly slow dynamics of the spin-glass state make this phase transition vitally important in the study of quantum optimization. We compare these dynamics against SA and simulated quantum annealing (SQA), an algorithm based on path-integral Monte Carlo\textsuperscript{33} that reproduces the thermal equilibrium statistics of QA\textsuperscript{34}.

Quantum annealing and Schrödinger dynamics

We use a D-Wave Advantage QA processor (Fig. 1a) whose pairwise-coupled superconducting flux qubits can be programmed to realize a transverse-field Ising model described by the Hamiltonian

\[ \mathcal{H}(s) = \mathcal{H}_0 + \mathcal{A}(s) \mathcal{H}_I \]  

where

\[ \mathcal{H}_0 = -\sum_i \sigma_i^z \]  

\[ \mathcal{H}_I = \sum_{ij} J_{ij} \sigma_i^x \sigma_j^x \]
Here $\sigma^x, \sigma^z$ are Pauli operators on qubit $i, s$ is a unitless normalized time, the transverse field $h(s)$ imparts quantum fluctuations through the driver Hamiltonian $\gamma_0$ and $X(s)$ is the energy scale of the classical Ising Hamiltonian $\gamma_c$. Over annealing time $t$, $s = t/t_a$ increases from 0 to 1, annealing the system from a quantum paramagnet dominated by $\gamma_0$, to a classical Ising model dominated by $\gamma_c$, following an annealing schedule as given in Fig. 1d. The coupling coefficients $J_{ij}$ can be programmed into a variety of two-dimensional (2D) and three-dimensional (3D) geometries, among others $^{12,31,35–38}$ (the QA processor also provides programmable biases, which we set to zero in this study).

Although our main focus is on large spin glasses, we first seek evidence of coherent quantum dynamics in an ensemble of small spin glasses. Taking the 16-spin graph in Fig. 1b, we generate spin-glass realizations with each coupling set to $J_{ij} = +1$ or $-1$ uniformly at random. We select 100 spectrally unique realizations in which $\gamma_0$ has two ground states and many first excited states. At this scale we can numerically evolve the time-dependent Schrödinger equation

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \gamma_c (t/t_a) |\psi(t)\rangle,$$

(4)

for the wavefunction $|\psi(t)\rangle$, where $\gamma_c (t/t_a)$ is given by equation (1). Let $|\phi_s(1)\rangle$ denote instantaneous eigenstates of $H(s)$ with eigenvalues $E_s(s)$. Sufficiently slow evolution results in adiabatic quantum optimization $^{39}$ into the twofold-degenerate final ground states $|\phi_s(1)\rangle$ and $|\phi_1(1)\rangle$. The relevant timescale is proportional to $\delta_{\text{min}}$, where

$$\delta_{\text{min}} = \min_s |E_s(s) - E_0(s)|$$

(5)

is the minimum parity-preserving eigengap. Faster anneals have a higher probability of excitation.

In Fig. 2a, we show spectral gaps $E_s(s) - E_0(s)$ for three 16-qubit examples: one with a small gap, one with a moderate gap and one with a large gap. For fixed $t_a$ we define

$$P_{\text{final}}(s) = \sum_{n=0,1} \langle |\phi_n(1)\rangle |\psi(s)\rangle|^2,$$

(6)

$$P_{\text{final}}(s) = \sum_{n=0,1} \langle |\phi_n(1)\rangle |\psi(s)\rangle|^2,$$

(7)

which measure instantaneous probabilities of being in the ground or first excited state of $\gamma_c(s)$ and $\gamma_c(1)$, respectively. Because the classical ground states are twofold degenerate, $P_{\text{final}}(1)$ gives the success probability of QA under Schrödinger dynamics. Figure 2b tracks $P_{\text{final}}(s)$ and $P_{\text{final}}(s)$ through anneals with $t_a = 14$ ns. The wavefunction begins as concentrated on the easily prepared ground state at $s = 0$, and this probability decreases via Landau–Zener excitation in the vicinity of a small gap.

We run QA experiments on each of these 100 instances using 192 disjoint sets of qubits in parallel. The Schrödinger excitation probability $1 - P_{\text{GS}} = 1 - P_{\text{final}}(1)$ is compared against experimental QA excitation probability in Fig. 2c; the probabilities are in close agreement with no fitting parameters used, showing an approximately exponential form. Figure 2d compares $P_{\text{GS}}$ for $t_a = 14$ ns across the entire 100-instance ensemble. In Extended Data Fig. 1 we compare probability distributions among ground and first excited states for QA, SA, SQA and Schrödinger dynamics, and find that experimental QA data are better explained by Schrödinger dynamics than by SA and SQA. The quantitative agreement between QA experiment and Schrödinger evolution up to $t_a = 30$ ns provides strong evidence for coherent quantum dynamics at small scale. We now consider critical dynamics in large 3D spin glasses.

**Critical spin-glass dynamics**

When a system is brought slowly (annealed) through a continuous phase transition, its dynamics slow down due to diverging correlations, and its macroscopic properties follow universal behaviour described by critical exponents. Here we use extensions of the Kibble–Zurek (KZ) mechanism $^{40–48}$, which describes the generation of excitation as an annealed system falls out of equilibrium. We use a dynamic finite-size scaling ansatz $^{49}$ (Supplementary Information section I) to relate time and the growth of correlations as functions of two critical exponents: $\nu$, which describes the divergence of correlation length at a phase transition, and $z$, which describes divergence of the characteristic timescale of domain fluctuations. We investigate three annealing dynamics—QA, SQA and SA—and their corresponding phase transitions.

Spin-glass order is quantified via the overlap between two replicas (independently annealed $N$-spin states $S$ and $S'$) of a given realization (set of couplings $J_{ij}$):

$$q = \frac{1}{N} \sum_{i=1}^{N} S_i S'_i, S_i, S'_i \in \{-1, 1\}.$$  

(8)

The mean-squared Edwards–Anderson order parameter is given by $\langle q^2 \rangle$, with $\langle \cdot \rangle$ denoting an average over both independent replicas and realizations.
Owing to restrictions on the available coupling geometry, we program spin glasses in the 3D layout shown in Fig. 1c, which differs from the simple cubic lattice: it has two qubits at every (x, y, z) coordinate, coupled as a dimer with strong ferromagnetic coupling $J_{G}(0 < J_{c} ≤ 1)$—this coupling uses one coupler in the x and y directions, and in the z direction the coupling is equally divided between two couplers. We use open x and y boundaries and periodic z boundaries.

In this model, contracting the strongly correlated dimers into individual spins yields the $+ J_{F}$ Ising spin glass on a simple cubic lattice, so we expect experimental results to reflect criticality in the same universality class. Figure 1e shows the phase diagram of the model in the $T$-$P$ plane, where a spin-glass (SG) phase is separated from a disordered paramagnetic (PM) phase. At $T = 0$ there is a quantum-critical point $T_c = 0$. At $T = 0$ there is a finite-temperature classical transition at $T_{c} > 0$. Tuning $J_{F}$ while keeping $J_{G} = 2$ varies the details of the phase diagram, but not the qualitative picture.

We measure $\langle q^2 \rangle$ for a range of $\tau_{a}$ with linear system size $L$ ranging from 5 to 12, in QA (Fig. 3a), SA (Fig. 3b) and SQA (Extended Data Fig. 2), for $J_{c} = 1$ ($J_{c} = 1/2$ data are shown in Extended Data Fig. 3). For Monte Carlo (MC) methods, $\tau_{a}$ is measured in MC sweeps (MCS). Although in all cases we anneal through the critical point rather than stopping at the critical point (Methods), the system experiences a critical slowing down at the QPT. Owing to slower dynamics in the glass phase, measurements in the final state approximately reflect the relevant critical dynamics. The Binder cumulant

$$U = \frac{1}{2} \left( 3 - \frac{\langle q^4 \rangle}{\langle q^2 \rangle^2} \right)$$

provides a statistical signature of phase transitions and, like $\langle q^2 \rangle$, also grows with $\tau_{a}$ and $1/L$, as seen in Fig. 3c, d. For $U$, any post-critical effective scaling dimensions in $\langle q^2 \rangle^2$ and $\langle q^2 \rangle$ cancel, but the functional dependence on $\tau_{a}$ remains. Thus, under the dynamic finite-size scaling ansatz, $U(L, \tau_{a})$ is expected to collapse onto a common curve for all system sizes when $\tau_{a}$ is rescaled by $L^{-\xi_{F}}$ (Supplementary Information section I(B)), reflecting the fact that the annealing time required for the system to remain adiabatic up to a correlation length of $L$ scales as $\tau_{a} \propto L^{d_{F}}$, $\mu = \xi_{F}^{-1/2}$.

Thus we estimate the Kramers–Wannier (K–W) exponent $\mu$ via best-fit collapse of $U$ horizontally along the time axis. Figure 3c, d shows collapse of $U$ from which we extract $\mu$ as a fitting parameter. Our analysis assumes, and is consistent with, a $z < \infty$ scenario, although there are some suggestions that this may not be the case $^{47}$.

In Fig. 3e we show these estimates for QA, SA and SQA for a range of coupling energies $J_{c}$. In all cases, $U(L, \tau)$ approximately follows power-law scaling when far from equilibrium $^{46}$. The extracted K–W exponents are clearly distinct, with smaller values indicating faster dynamics. We find $\mu_{\text{QA}}$ between 2.9 and 3.1 depending on $J_{c}$. Adding our MC estimate $1/\sqrt{\text{QA}} = 1.55$ (Supplementary Information section III) to $z_{\text{QA}} = 1.3$ (ref. 51) gives $\mu_{\text{QA}} = 2.85$, in close agreement with experimental results.

For QA we find $\mu_{\text{QA}} = 5.3$, much larger than QA but smaller than estimates from simple cubic lattices; for SA we found a value of 6.0 (dashed line) compared to the previously reported $^{46}$ value of 6.3. These deviations are largely explained by finite-size effects, and dimers delaying the onset of asymptotic behaviour (see Extended Data Figs. 2–4 and Supplementary Information section IV(A)). No previous estimate of $\mu_{\text{SA}}$ has been reported; the dashed line in Fig. 3e indicates the extracted value $\mu_{\text{SA}} = 4.39$ for simple cubic lattices (Extended Data Fig. 4). As QA and SQA share an equilibrium exponent, $\nu$, this corresponds to $z_{\text{SA}} = 2.8$, which reflects the MC dynamics.

To better understand the role of frustration in the glassy dynamics, we increase the ‘doping’ probability $p$ of a random interdimer coupling being antiferromagnetic ($-J_{d}$). The SG phase has been shown to persist until $p_{c} = 0.778$ (vertical line) for $T = 0$ and $\Gamma = 0$, beyond which the system is a disordered antiferromagnet $^{46–48}$—we might expect that $\Gamma = 0$ slightly reduces $p_{c}$ (ref. 55). Figure 3f shows the $p$ dependence of $\mu$ extracted by collapsing for even values of $L$; all three dynamics are insensitive to changes in $p$ until it approaches $p_{c}$. For $p$ close to 1, QA, SA and SQA all have $\mu = 2$, consistent with coarsening dynamics in the antiferromagnetic phase $^{47}$. In this scenario, post-critical dynamics eliminate small domains, replacing the K–W exponent $\mu$ with a universal exponent 2. The latter corresponds to correlation length scaling as $\xi \propto \tau^{1/2}$, expected for diffusive dynamics present in the antiferromagnetic phase. Owing to the rough potential landscape in the SG phase, the post-critical dynamics have negligible effect on $\mu$, although they affect energy decay as we discuss next.
Energy decay

The smaller values of critical exponents $z$ and $\mu$, obtained via data collapse, indicate faster critical dynamics in QA compared to SA and SQA. We now ask whether this leads to a speed-up in approximating the ground-state energy in classical Ising models. We first answer this question theoretically by considering a hypothetical annealing protocol that is measured at the critical point. Although this is not the real schedule of QA, it makes the energy decay dependent only on the dynamics approaching the critical point, and therefore enables connecting the relevant critical exponents. We define dimensionless residual Ising energy density at the critical point as

$$\rho_E^c = \langle (\gamma_1 - E) / (N\gamma_c) \rangle.$$  \hspace{1cm} (11)

Here $N$ is the number of spins, and $E_c$ is the equilibrium expectation of $\gamma_1$ at $T = T_c^f$ and $f = 0$ for SA, and at $T = 0$ and $f = f_c$ for QA and SQA. Notice that $\rho_E^c \rightarrow 0$ as $t_c \rightarrow \infty$. It is shown in Supplementary Information Section I that $\rho_E^c$ follows a power-law relation:

$$\rho_E^c \propto t_c^{\kappa_c}, \hspace{0.5cm} \kappa_c = (d_s - 1/v)/\mu,$$  \hspace{1cm} (12)

where $d_s = d$ for SA and $d_s = d + z_{sq}$ for QA and SQA. As expected, $\kappa_c$ is inversely related to the KZ exponent $\mu$, therefore faster critical dynamics (smaller $\mu$) leads to faster decay of energy. Moreover, $d_s$ is larger for quantum than classical decay, making a larger contribution to the numerator.

However, $\rho_E^c$ is not very relevant to optimization because $E_c$ is far from the ground-state energy $E_0$ of $\gamma_1$. We therefore consider the corresponding final quantities $\rho_f^c$ and $\kappa_c$ obtained by annealing to the low-temperature classical point $T = T_c^f, f = 0$:

$$\rho_f^c = \langle (\gamma_1 - E) / (N\gamma_c) \rangle,$$  \hspace{1cm} (13)

and fitting $\rho_f^c \propto t_c^{\kappa_c}$. Again, the average is over realizations and samples, with annealing according to the full QA schedule. For very long (adiabatic) anneals, we expect $\rho_f^c \rightarrow 0$, thus optimal solutions are asymptotically reached.

Figure 4a shows $\rho_f^c$ as a function of annealing time for QA, SQA and SA, for 3D spin glasses on $N = 5,374$ spins ($15 \times 15 \times 12$ dimers, with some vacancies). The ground-state energy $E_c$ is estimated by exchange MC (Supplementary Information Section II(C)). Each dynamics follows a power-law scaling within a window of $t_c$ (Supplementary Information Section V) but deviates outside the window, most notably for QA due to the onset of thermal effects; this deviation from coherent behaviour is expected for longer anneals \(^{32}\). For SA, the decay of $\rho_f^c$ settles onto a consistent exponent only after several hundred MCS.

We estimate $\kappa_c$ for the three dynamics with varying $f_c$ from power-law fits. Figure 4b shows, as a function of $f_c$, the fit values of $\kappa_c$ (symbols) as well as the critical values $\kappa_c$ (horizontal lines) obtained using independent MC estimates of $z$ and $1/v$, corresponding to the lines in Fig. 3e. Deviations $\kappa_c - \kappa_c$ are expected beyond the critical point (Supplementary Information Section C(4)); we find a modest correction $\kappa_c - \kappa_c = 0.1$ for both SA and SQA (Supplementary Information Section IV(B)), which one might also see in QA if it could be measured at the critical point.

QA shows the fastest energy decay $\kappa_c$, followed by SQA and SA. For sufficiently large $f_c$, $\rho_f^c$ decays roughly quadratically faster in QA than in SA, with SQA in between the two. This experimentally observed scaling advantage is consistent with the theoretical speed-up in critical ordering dynamics (smaller $\mu$ in Fig. 3e) and faster critical energy decay (larger $\kappa_c$ in Fig. 4b). In Fig. 4c we show the annealing time (in MCS) required by SA and SQA to match the energy achieved by a given $t_c$ in QA; within the coherent QA regime, the approximation speed-up of QA over SA and SQA increases as a function of annealing time.

Outlook

We have experimentally demonstrated quantum-critical dynamics in programmable spin glasses on thousands of qubits, observing the expected scaling in system size and annealing time. Simulation accuracy was confirmed via comparison to numerical simulation of
Schrödinger dynamics at the 16-qubit scale. For large 3D spin glasses, the simulated many-body quantum dynamics are far beyond the reach of current exhaustive or tensor-based techniques; the former is limited to tens of qubits and the latter can be applied to moderately sized 2D models. We therefore appeal to critical exponents via finite-size scaling analysis, finding good agreement with independent MC estimates. Thus we have presented both microscopic and macroscopic evidence for a coherently annealed programmable quantum spin glass. These exponents indicate, in theory and experiment, that quantum annealing has a dynamical advantage over simulated annealing and simulated quantum annealing in penetrating the spin-glass phase. The predicted speed-up was experimentally demonstrated through a scaling advantage in approach to the ground-state energy. These results point to the utility of programmable quantum annealers both as quantum simulators and optimization tools.

For sufficiently large spin systems, the extent of ideal quantum-critical scaling is limited in time by qubit decoherence, disorder and noise, and the results shown here indicate that improvements in these areas would pay great dividends. Extending the region of critical scaling would not only facilitate the further study of these dynamics, but also extend their utility in real-world applications, helping QA reach lower energy solutions. These efforts must be balanced with improvements in qubit connectivity, which enable more flexible problem embeddings, and high coupling energy, which can protect against control error and thermal excitation outside the coherent limit.

Spin glasses represent a paradigmatic hard optimization problem, and provide a robust theoretical framework for understanding and demonstrating quantum-critical dynamics. They were instrumental in motivating the field of quantum annealing itself, via magnetic experiments, and here we have answered in the affirmative the foundational question raised over 20 years ago: Is it possible to engineer a programmable quantum system, in which quantum annealing imparts a dynamical advantage over simulated annealing? Extending this characterization of quantum dynamics to industry-relevant optimization problems, which generally do not enable analysis via universal critical exponents or finite-size scaling, would mark an important next step in practical quantum computing.

Online content
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Methods

Spin-glass instances

The 3D lattices have open x and y dimensions and periodic z dimension. Instances up to size $9 \times 9 \times 9 \times 2 (L = 9)$ were fully yielded, with no site vacancies. Larger instances had over 99.5% site yield. The inputs were heuristically embedded into the qubit connectivity graph of the QA processor, with a structure shown in Extended Data Fig. 5. Input construction is discussed in detail in the Supplementary Information.

Quantum annealing system and methods

All QA data, except specifically indicated temperature variation experiments discussed in the Supplementary Information, were taken using one D-Wave Advantage system operating at 12 mK (QPU1). The variable-temperature experiments were performed using a second D-Wave Advantage system of the same design, between 12 mK and 21 mK (QPU2).

Calibration refinement methods were used to balance qubits at degeneracy and to synchronize annealing lines. We followed the same method as described in the supplementary material for ref. 32, but with no tuning of coupling strengths. We describe these methods in detail in Supplementary Information section VII.

Each call to the QA system resulted in 200 anneals. QA data on 3D spin glasses were generated from 900 calls, cycling through 300 disorder realizations. For MC methods, between 100 and 300 realizations were used. Thus the data points for $(q^2)$, $U$ and $\rho_s$ represent the average over 100–300 spin-glass realizations, with error bars capturing variation between instances. QA Binder cumulants were computed by comparing overlap between annealing samples generated in different QA calls, but the same seed and embedding, thus suppressing the effect of correlated biases. The experiments were sufficiently extensive that the Binder cumulants and overlaps computed from samples within the same QA programming gave similar results.

Measuring annealing time. The anneal of the Hamiltonian from $s = 0$ to $s = 1$ over annealing time $t_s$ was achieved through a rapid change in qubit control current. Of the factors limiting the minimum value of $t_s$, the two most important are: the ability to reliably quench $s$ at a known rate and with tolerable non-linearity and distortion from filtering; and the ability to synchronize the qubits to within a reasonable deviation in terms of $s$.

Annealing times slower than 20 ns were reliably realized. Annealing times faster than 20 ns deviate significantly from their requested values due to low pass filter bandwidth and resolution of digital control electronics, and must be measured independently. We did so in two ways. The recent demonstration of the KZ mechanism in a one-dimensional (1D) Ising chain showed very good agreement with theory, in particular a scaling of residual energy $\rho_s^f = t_s^{1/2}$. Therefore extrapolating 1D data from $t_s = 20$ ns to faster anneals provides a reliable measurement of effective annealing time. The second measurement we performed was a direct measurement in which one qubit is quenched and measured by a witness qubit, enabling us to estimate the effective annealing time. Data for these two measurements are compared in Extended Data Fig. 6. We took the average of the two measurements as our value of $t_s$. Error bars were obtained by adding, in quadrature, the difference between the two measurements and the deviation in annealing slope before and after software filtering (less than 0.05 relative error).

Quantum annealing schedule. Simulating the time-dependent Schrödinger equation requires an accurate annealing schedule $(f(s), \langle \sigma(s) \rangle)$. To achieve this, we diagonalized a time-dependent many-body flux-qubit Hamiltonian for a small representative system of coupled qubits, using a flux-qubit model whose parameters are given in Extended Data Table 1. Then, for each $s$ in our range of interest, we computed $(f(s)$ and $\langle \sigma(s) \rangle$ as best-fit parameters that give an Ising model whose eigengaps closely match those of the diagonalized superconducting quantum interference device Hamiltonian.

Diagonalizing the many-body flux-qubit Hamiltonian to high accuracy is computationally challenging even at small scales because the flux qubits have multiple energy levels, unlike the model two-level Ising spins. Thus, as a representative system, we took eight qubits, and divided them up into four dimers such that the two qubits in each dimer have similar interactions with the other qubits. We then treated each dimer as a six-level object, and diagonalized the system of four dimers.

Reflecting the frustration in spin glasses, our eight-spin system, shown in Extended Data Fig. 7, has frustration in its twofold-degenerate classical ground state. Performing a two-parameter fit with $f(s)$ and $\langle \sigma(s) \rangle$ becomes numerically unstable for $s > 0.40$ due to the small first gap. For $s > 0.40$ we fit $f(s)$ to the expected exponential decay form and extract only $\langle \sigma(s) \rangle$ as a fitting parameter. Extended Data Fig. 7 shows the nominal and extracted schedules.

Classical Monte Carlo dynamics

Simulated annealing. SA uses a geometric annealing schedule with $\beta = 0.001J_z$ to $\beta = 10J_z$. Each MCS processes $N$ spins, sampled randomly with replacement. Spin updates are accepted or rejected in a Metropolis–Hastings algorithm.

Simulated quantum annealing. SQA uses an annealing schedule based on the QA schedule; to accelerate our experiments we begin when $f / f_z = 6$ – a fast-equilibrating regime in which we assume the system is quasi-static with respect to the changing Hamiltonian—and perform ten MCS before proceeding through the QA schedule until $f / f_z = 1/25$, far into the ordered phase for the models studied. Inverse temperature $\beta$ is 64 except where specified, is relative to $f(s) = \langle \sigma(s) \rangle$ at the crossing point of $f$ and $f_z$. Swendsen–Wang cluster updates were used in imaginary time.

Time-dependent Schrödinger evolution

For the data in Fig. 2 we used an iterative method that follows the annealing schedule from $s = 0.1$, where the QA wavefunction $\psi$ is concentrated on the instantaneous ground state, and $s = 0.7$, where dynamics are negligible. We tracked populations for only the lowest 50 of $2^s$ eigenstates; this exceeds the number of ground and first excited states for the classical models studied. Step sizes in $s$ were determined adaptively on the basis of the minimum eigengap, and ranged between 0.00008 and 0.01.

Statistical methods and error analysis

Error bars on the order parameter $(q^2)$, Binder cumulant $U$ and residual energy $\rho^f_s$ were generated by treating each random seed as an independent experiment and performing a resampling bootstrap on the set of statistics. This bootstrapping method gives a population for each statistic estimate. We used this population in two ways. First, we took the middle 95% of the populations as our confidence interval for the statistics. Second, from the population we computed a variance on the logarithm, which we used to determine error weights for our data collapses.

Data collapse

To collapse measurements of the Binder cumulant $U$ for varying system sizes onto a common target curve, we needed to find a best-fit value for $\mu$. This fit minimizes a weighted sum of squared distances (in the logarithm) from the target curve. Weights in the sum are inversely proportional to the variance of the logarithm of the estimator. The form of the target curve must capture a crossover between the power-law form of the KZ regime and the equilibrium $(t_s \rightarrow \infty)$ limit. To achieve this, for each putative value of $\mu$ we found a best-fit target curve (nested within the $\mu$ optimizing method) whose power-law slope varies as a logistic function. Our target curve has the form
\[ f(x) = a_0 + a_1 \log(1 + \exp(a_2(x - a_3))), \quad (14) \]

which fits \( \log(U) \) as a function of \( \log(t_a) \).

To generate error estimates for \( \mu \) we performed a jack-knife on the measurements in \( L \) and \( t_a \) and added the resulting standard errors in quadrature. To approximate the 95\% confidence interval in the data points we use error bars that span \( \pm 2\sigma \).

Collapse in \( \langle q^2 \rangle \) was achieved with the same approach. Selection of ranges of \( t_a \) over which we collapsed data is described in the Supplementary Information.

Data availability
Data supporting the findings are available in the Zenodo online repository https://doi.org/10.5281/zenodo.7640779.

Code availability
An open-source version of the SQA code used in this work is available at https://github.com/dwavesystems/dwave-pimc.

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Author contributions
A.D.K., J.R., T.L., R.H., A.Z., A.W.S. and M.H.A. conceived and designed the experiments and analysed the data. A.D.K., J.R., T.L. and A.W.S. performed the experiments and simulations. T.L., R.H., F.A., A.I.B., K.B., S.E., C.E., E.H., S.H., E.L., A.I.R.M., G.M., R.M., T.O., G.P.-L., M.R., C.R., Y.S., N.T., M.V., J.D.W., J.Y. and M.H.A. contributed to the design, fabrication, deployment and calibration of the QA system. A.D.K., J.R., R.H., A.W.S. and M.H.A. wrote the manuscript.

Competing interests
A.W.S. declares no competing interests. All other authors have received stock options in D-Wave as current or former employees.

Additional information
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Extended Data Fig. 1 | State probabilities for 16-spin glasses. a, We show observed probabilities for ground states and first excited states in Schrödinger evolution ($t_\text{e} = 14$ ns), SA ($t_\text{e} = 200$ MCS), and SQA ($t_\text{e} = 100$ MCS) compared with experimental measurements from QA ($t_\text{e} = 14$ ns). The three columns contain data for the three exemplary instances, with colors corresponding to those in Fig. 2. Annealing times for SA and SQA were chosen to have good agreement with Schrödinger evolution in average ground state probability. Each dynamics was run 19,200 times; dashed lines indicate the statistical floor, i.e., the probability if a state is seen exactly once. Unobserved states are represented by points below the statistical floor. b, Kullback-Leibler (KL) divergence in the probability distribution among first excited states, with QA used as a reference distribution, measures deviation between two dynamics for a given realization. Empirical CDF (proportion of 100 realizations below a given KL divergence) is shown. Data indicate that coherent quantum (Schrödinger) dynamics agrees more closely with experimental data better than does SA or SQA.
Extended Data Fig. 2 | Data collapse for 3D spin glasses with $J_G = 1$. Best-fit exponent $\mu$ collapses $U$ by rescaling data horizontally based on $L$, and $r$ collapses $\langle q^2 \rangle$ by scaling vertically based on $L$, given a horizontal rescaling by $L^\mu$. 
Extended Data Fig. 3 | Data collapse for 3D spin glasses with $J_G = \frac{1}{2}$. Best-fit exponent $\mu$ collapses $U$ by rescaling data horizontally based on $L$, and $r$ collapses $\langle q^2 \rangle$ by scaling vertically based on $L$, given a horizontal rescaling by $L^\mu$. 
Extended Data Fig. 4 | Data collapse for 3D spin glasses on simple cubic lattices. Best-fit exponent $\mu$ collapses $U$ by rescaling data horizontally based on $L$, and $r$ collapses $\langle q^2 \rangle$ by scaling vertically based on $L$, given a horizontal rescaling by $L^\mu$. 
Extended Data Fig. 5 | 3D lattice structure in qubit connectivity graph. \(L \times L \times \text{max}(L, 12) \times 2\) lattices are found by heuristic search given a basic structure in which horizontal and vertical (long) couplings form two dimensions, and the interior of unit cells forms the third dimension. One dimer (thick gray line) and its six neighboring dimers are shown as thick gray lines. As in Fig. 1, purple and yellow lines represent glass couplings of \(\pm J_G\) and \(\pm J_G/2\) respectively.
Extended Data Fig. 6 | Measurement of effective QA annealing time.

Two independent measurement methods are used to estimate $t_a$ for fast anneals ($<20$ ns). First is a direct measurement using a witness qubit. Second is an extrapolative measurement that assumes a quantum KZ scaling in a 1D chain and assumes a kink density $n \propto t_a^{-1/2}$ for $t_a < 20$ ns. For the fastest anneals, $t_a$ deviates significantly from the values requested from the control electronics. However, the two independent measurement approaches give consistent results.
Extended Data Fig. 7 | Extracting Ising model from flux-qubit model.

a, Eight-qubit gadget used to extract an effective annealing schedule in the transverse-field Ising model based on a many-body flux-qubit Hamiltonian. Dimers indicated by dashed ellipses are treated as six-level objects and combined to diagonalize a many-body Hamiltonian. b, General-purpose (nominal) annealing schedule based on single-qubit measurements, and extracted many-body effective schedule, used for Schrödinger evolution.
Extended Data Table 1 | Physical qubit properties for the QA processor

| Parameter                           | Symbol | Value   |
|-------------------------------------|--------|---------|
| Mutual inductance for $J = 1$       | $M_{AFM}$ | 1.65 pH |
| Initial external applied flux       | $\Phi_{CCJJ}^i$ | -0.621 $\Phi_0$ |
| Final external applied flux         | $\Phi_{CCJJ}^f$ | -0.717 $\Phi_0$ |
| Qubit inductance                    | $L_q$  | 371 pH  |
| Qubit capacitance                   | $C_q$  | 118 fF  |
| CJJ loop capacitance                | $C_l$  | 5 fF    |
| CJJ junction capacitance            | $C_{CJJ}$ | 25 fF  |
| Qubit critical current              | $I_c$  | 2.10 $\mu$A |

These parameters are input to a flux-qubit model for generating the best-fit many-body annealing schedule.