Effective Theory for QCD in the LOFF Phase

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1 Introduction

Recently, the old idea of color superconductivity [1] has been given a new life [2], after the discovery that QCD at high density and low temperature can undergo a phase transition to a color super-conducting state characterized, for three light quarks, by Color Flavor Locking: CFL (for recent reviews see [3]). In this talk I wish to examine a different phase of QCD also present at low temperature and high baryonic chemical potential [4]; for two flavors this phase is characterized by isospin breaking and the emergence of a crystalline pattern of the diquark condensate. In a recent paper [5] it has been shown that in this phase there exists a quasi particle (phonon) associated to spontaneous breaking of rotational and translational invariance. Its properties can be studied by an effective lagrangian approach [6], [7], [8]; they will be reviewed here together with a short discussion on possible astrophysical implications.

2 Crystalline colour superconductive phase

In any possible application of the color superconductivity, for instance in the inner core of neutron stars, flavor symmetry is likely to be broken not only explicitly by quark mass terms, but also by weak interactions. For example in compact stars, considering only two flavors, isospin is broken by $\delta \mu = \mu_u - \mu_d \neq 0$, due to the process:

$$d \rightarrow u e \nu .$$

(1)

When the two quarks in the Cooper pair have different chemical potentials, the vacuum is characterized, for certain values of $\delta \mu$, by a non vanishing expectation value of a quark bilinear breaking translational and rotational invariance. The appearance of this condensate is a consequence of the fact that in a given range of $\delta \mu$ [4], the formation of a Cooper pair with a total momentum

$$\vec{p}_1 + \vec{p}_2 = 2\vec{q} \neq \vec{0}$$

(2)
is energetically favored. A similar phenomenon was already observed many years ago in the context of the BCS theory for superconducting materials in presence of magnetic impurities by Larkin, Ovchinnikov, Fulde and Ferrel and the corresponding phase is named LOFF state [9].

The exact form of the order parameter (diquark condensate) breaking space-time symmetries in the crystalline phase is not yet known. In [4] the following ansatz is made:

$$\Delta(\vec{x}) = \Delta e^{i2\vec{q} \cdot \vec{x}} = \Delta e^{i2\vec{n} \cdot \vec{x}}.$$  \hspace{1cm} (3)

The value of $|\vec{q}|$ is fixed by the dynamics, while its direction $\vec{n}$ is spontaneously chosen. In the sequel I will assume the ansatz (3) as well. The order parameter (3) induces a lattice structure given by parallel planes perpendicular to $\vec{n}$:

$$\vec{n} \cdot \vec{x} = \frac{\pi k}{q} \quad (k = 0, \pm 1, \pm 2, \ldots).$$  \hspace{1cm} (4)

We can give the following physical picture of the lattice structure of the LOFF phase: Due to the interaction with the medium, the Majorana masses of the red and green up and down quarks have a periodic modulation in space, reaching on subsequent planes maxima and minima. For the Cooper pair to be formed one needs a color attractive channel, which is the antisymmetric channel; therefore it must be in antisymmetric flavor state if it is in antisymmetric spin state ($S = 0$). The condensate has therefore the form

$$- <0|\epsilon_{ij}\epsilon_{\alpha\beta}\bar{\psi}_i^{\alpha}(\vec{x})C\psi_j^{\beta}(\vec{x})|0> = 2\Gamma_A e^{i2\vec{q} \cdot \vec{x}}.$$  \hspace{1cm} (5)

The analysis of [4] shows that, besides the condensate (5) (scalar condensate), another different condensate is possible, i.e. one characterized by total spin 1 (vector condensate) and by a symmetric flavor state:

$$i <0|\sigma_{ij}\epsilon_{\alpha\beta}\bar{\psi}_i^{\alpha}(\vec{r})C\sigma^{03}\psi_j^{\beta}(\vec{x})|0> = 2\Gamma_B e^{i2\vec{q} \cdot \vec{x}}.$$  \hspace{1cm} (6)

Note that in the BCS state the quarks forming the Cooper pair have necessarily $S = 0$.

It goes without saying that the hypotheses of [4] are rather restrictive, as these authors assume only two flavors and make the ansatz of a plane wave behavior, Eq. (3). In any event their results are as follows. Assuming a pointlike interaction as the origin of the Fermi surface instability, the LOFF state is energetically favored in a small range of values of $\delta\mu$ around $\delta\mu \sim 0.7\Delta$. The actual value of the window range compatible with the presence of the LOFF state depends on the calculation by which the crystalline color state is computed. While the small interval is based on a local interaction, assuming gluon exchange, as in [10], the window opens up considerably.

The order parameters (5) and (6) spontaneously break rotational and translational symmetries. Associated with this breaking there will be Nambu Goldstone Bosons (NGB) as in a crystal; these quasi-particles are known as phonons. Let us discuss them in some detail.
3 Phonons in the LOFF phase

For a generic lattice structure it is known that there are three phonons associated to the breaking of space symmetries. However one can show [5] that in order to describe the spontaneous breaking of space symmetries induced by the condensates (5) and (6) one NGB is sufficient. The argument (see Fig. 1) is as follows: Rotations and translations are not independent transformations, because the result of a translation plus a rotation, at least locally, can be made equivalent to a pure translation. The NGB

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form the vector $\vec{\nabla} \Phi$; therefore we get
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\vec{R} = \frac{\vec{\nabla} \Phi}{|\vec{\nabla} \Phi|},
\]
which satisfies $|\vec{R}| = 1$ and $< \vec{R} >_0 = \vec{n}$. In terms of the phonon field $\phi$ the field $\vec{R}$ is
given at the first order in $\phi$ by the expression
\[
\vec{R} = \vec{n} + \frac{1}{2fq} \left[ \vec{\nabla} \phi - \vec{n} (\vec{n} \cdot \vec{\nabla} \phi) \right].
\]
The interaction term between the NGB and the quarks is contained in
\[
\mathcal{L}_{\text{int}} = -\frac{1}{2} e^{i\Phi} \sum_{\vec{v}} \left[ \Delta^{(s)} \epsilon_{ij} + \Delta^{(v)} (\vec{v} \cdot \vec{R}) \sigma_{ij} \right] \epsilon^{\alpha\beta\gamma} \psi_{i,\alpha,\vec{v}} C \psi_{j,\beta,-\vec{v}}, + \text{h.c.} \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber \nonumber 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core of neutron star; in this case the crystalline color superconductivity could be partly responsible for the glitches of the pulsar. A detailed analysis of this scenario is however premature as one should first complete the study of the LOFF phase in two directions, first by including the strange quark and, second, by sorting out the exact form of the color lattice.

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Discussion

J. Madsen (University of Aarhus): How would you visualize the LOFF phase? I.e.: What are the ”ions” in the crystal and how are they organized?

Nardulli: In the LOFF phase the condensate varies periodically in space with minima and maxima. Maxima are places where the condensate is bigger and therefore the fermions have the largest Majorana mass. The lattice of ”ions” could be visualized as parallel planes where the Cooper pairs are more tightly bounded.

D. K. Hong (Pusan National University): When you derived the effective theory of the LOFF phase, you used the same Fermi velocity for up and down quarks. But it would be much easier to analyze the effective theory using different Fermi velocities that satisfy the constraint $\mu_u \vec{v}_u + \mu_d \vec{v}_d = 2 \vec{q}$.

Nardulli: The use of one velocity has the advantage of a Fermi velocity superselection rule, which reduces the complexity of the loop integrals. It arises because in the $\mu \to \infty$ limit the rapid oscillations of the exponential factor in the lagrangian cancel all the contributions except those satisfying $\vec{v}_u + \vec{v}_d = 0$. 