The flow regimes of the annular swirling turbulent jet

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Abstract. We perform Large-eddy simulations (LES) of an annular swirling turbulent jet. The swirl parameter is considered within the range \( S = 0.3-0.6 \). The Reynolds number based on the bulk velocity and outer pipe diameter is 8900 while the outer to inner diameter ratio is 2. We obtain different flow regimes without and with the vortex breakdown occurring for \( S = 0.4 \). Coherent vortical structures for each regime are indentified using the azimuthal Fourier decomposition and the Proper orthogonal decomposition (POD). The turbulent kinetic energy accumulates in the first POD modes with the azimuthal number \( m = 1 \).

1. Introduction
Annular jets are characterized by a recirculation zone, which is formed behind the bluff body [1, 2]. The reversal flow increases the level of turbulent fluctuations and, thus, the overall mixing rate. Due to this fact, annular jets are widely used in industry, for example, in combustion technology. Industrial burners have annular nozzles that allow one to stabilize the flame and reduce the level of pollutant emissions into the atmosphere [3, 4].

In round jets without bluff body the reversal flow region can be organized by adding the integral swirl. In this case, the formation of the recirculation zone leads to the vortex breakdown phenomenon. The flow regime in swirling round jets depends on the ratio of the azimuthal to axial velocity. When this ratio is low the flow due to the centrifugal force begins to diverge from the axis resulting in the pressure decreases. As the azimuthal velocity exceeds a critical value, the reversal flow region appears [5]. The structure of swirling flows strongly depends on the problem parameters: the geometry, the Reynolds number, the swirl parameter etc. The experiments with a swirling flow passing through an expanding pipe reveal up to seven different regimes of the vortex breakdown [5–8]. In swirling annular jets, the flow structure is complicated by the central recirculation zone behind the bluff body. The reversal flow caused by the vortex breakdown and the central recirculation area merge when the swirl increases [9].

Another feature of swirling flows is the precessing vortex core [10]. For laminar low-swirling flows streamlines are spirals, and the isosurfaces of velocity are axisymmetric elongated coherent structures, that rotate about the axis. During the transition to the turbulent regime or the increase of swirl, sinusoidal instabilities appear. This leads to the transformation of the coherent structures into single or double helix structures, precessing around the axis [11–13]. These structures indicate the presence of the vortex core in the flow. The temporal and spatial characteristics of the precessing vortex core have a critical effect on combustion regimes, both in the industrial burners and furnaces [14–17].

The present paper deals with the annular swirling jet of an incompressible fluid with the Reynolds number \( Re = 8900 \) within the range of the swirl parameter \( S = 0.3-0.6 \). A gradual increase of the swirl...
number allows us to investigate the evolution of the flow structure and detect the characteristic flow regimes. The azimuthal Fourier decomposition is used to identify coherent vortical structures. Their spatial and temporal characteristics are investigated using the Proper orthogonal decomposition (POD).

2. Computational details

We perform Large-eddy simulations (LES) of the annular turbulent swirling jet of an incompressible fluid. We use LES model that employs the filtering of two highest harmonics [18]. The jet flows out into a cylindrical domain $12D \times 17D$ in size (see Fig. 1) with a co-flow $U_c = 0.04U_b$, where $D$ and $U_b$ are the pipe outer diameter and bulk velocity, respectively. The pipe length is $2D$ and the diameter ratio is $d/D = 0.5$, while the wall thickness is $0.03D$. The Reynolds number based on the bulk velocity and the outer diameter is 8900. A turbulent velocity profile for the supplying pipe is generated using an auxiliary periodic annular $2.5D$ channel simulations. The instantaneous realizations of the fully developed velocity profile from some $r-\theta$ plane are copied to the inflow of the main domain on every time step. On the other boundaries the Neumann boundary conditions were imposed.

![Figure 1.](https://example.com/figure1.png)

Figure 1. The main computational domain showing the axial instantaneous velocity field for the case $S = 0.3$. The blue isosurface of the time-averaged axial velocity ($\bar{u} = 0$) indicates the reversal flow region. The green coherent structure is identified using the isosurface of the instantaneous pressure for $p - p_\infty = -0.35$.

The Navier–Stokes equations are solved numerically in a weak form using Nek5000 [19]. The code is based on the spectral–element method (SEM) [20]. For the space discretization Galerkin approximation is performed using $N$th-order Lagrange polynomial interpolants based on the Gauss–Lobatto–Legendre points for both the velocity and pressure field ($P_N$–$P_N$ formulation). The results reported below correspond to the polynomial order $N = 7$. The semi-implicit third-order accuracy scheme is used for the time discretization. The accuracy of Nek5000 was previously validated for a number of configurations including the channel flow and non-swirling annular jet [21, 22].

The swirl is set by adding the tangential velocity at the inflow plane of the pipe with a parabolic velocity profile $w/U_b = \alpha(1 - 2r/D)(r/D - 1)$, where $\alpha$ is the magnitude of the azimuthal velocity described below. The swirl number definition is as follows:
\[ S = \frac{\int \overline{u} \overline{w} r^2 dr}{R_2 \int \overline{u}^2 r dr}, \]

where \( R_1 \) and \( R_2 \) are the inner and outer radius and \( \overline{u}, \overline{w} \) are the axial and azimuthal velocity components, while the overline denotes the time-averaging. We imposed the swirl parameters \( S = 0.3, 0.4, 0.5, 0.6, \) corresponding to \( \alpha = 4.55, 6.07, 7.59, 9.1, \) respectively. The total number of the spectral elements was 5120 (20×8×32 for directions) for the auxiliary domain and about 125 thousand (90×40×32) for the main domain. Polynomial order \( N = 7 \) provides 512 computational nodes inside each of the spectral element, so the total number of computational nodes was about \( 6.4 \times 10^6 \) for the main simulation and about \( 2.6 \times 10^6 \) for the auxiliary domain. The near wall resolution inside the pipe corresponds to \( \Delta r^+ = 0.76, r(\Delta \theta)^+ = 6.4, \Delta x^+ = 64, \) for radial, azimuthal and axial direction, satisfying the near-wall resolution criteria of the well-resolved LES [23]. The superscript “+” denotes the non-dimensional units in terms of the friction velocity and viscosity.

3. Results

Figure 2. The flow regimes in the annular turbulent swirling jet.

Typical flow patterns for the considered swirl numbers are shown in Fig. 2. We visualize a part of the main computational domain with the instantaneous axial velocity field. The isosurfaces of the average axial velocity represent the reversal flow region (the red one corresponds to the \( \overline{u} = 0 \)) in the center. The helical vortical structures are identified with the use of isosurface of the instantaneous pressure field indicating the precessing vortex core (PVC) for the highest swirl number. When the swirl intensity is low the flow near the axis decelerates and the instantaneous axial velocity may be negative but the average flow is positive. The vortex breakdown phenomenon occurs only for the swirl number \( S = 0.4, \) where the second recirculation zone is formed. The further increase of \( S \) up to 0.6 leads to the merging of the reversal flow regions, resulting in one large recirculation zone.
Figure 3. The energy distribution among the Fourier/POD modes. The energy values shown in the right top part for each the plots are the turbulent kinetic energy corresponding to the first POD modes with $m = 1$.

To obtain some quantitative information about helical structures we apply the Proper orthogonal decomposition (POD) to the velocity fields in the near region [24, 25]. We represent the velocity as a sum over a spatial basis $\psi_q^m(r, x)$ with time coefficients $a_q^m(r, x)$ and eigenvalues $\lambda_q^m(r, x)$:

$$u(r, x, m) = \sum a_q^m(r, x) \lambda_q^m(r, x) \psi_q^m(r, x)$$

Since the geometry has the homogeneous azimuthal direction the Fourier modes are exact solutions of the eigenvalues problem. So we combine the snapshot POD with the azimuthal Fourier decomposition that additionally allows reducing the dimension of the problem and the calculation time $\tilde{u}(r, x, m) = FT[u(r, x, \phi)]$, where FT stands for the Fourier transform.

Figure 4. The real part of the spatial functions for the first POD mode with $m = 1$.

The analysis of the eigenvalues (Fig. 3.) reveals the accumulations of the energy in the first POD ($q = 1$) modes with $m = 1$, with an increase of swirl. The spatial distribution of these modes with streamlines is presented in Fig. 4. The characteristic size of the first POD mode as well as their distance
from the axis increases. For $S = 0.6$ the additional helical structure is observed in the outer shear layer. The time coefficients determine the evolution of the POD modes. For the analysis we represent them in the geometrical form: $a^m_q(t) = \rho^m_q \times e^{i\phi t}$. Figure 5 shows that the phase linearly depends on time, while the rotation frequency decreases more than twice for $S = 0.6$. However, the distance between the POD modes and the axis shown in Fig. 4 increases more rapidly than the rotation frequency decreases resulting in the growth of the linear velocity of the coherent structures.

![Figure 5](image.png)

**Figure 5.** The time coefficient of the first POD mode with $m = 1$ (the blue solid lines) and linear approximation functions (the red dashed lines).

**Conclusion**

We studied the annular swirling turbulent jet at the Reynolds number of 8900. The analysis shows that for low swirl numbers there is the recirculation zone behind the bluff body while the second reversal flow region is strongly unsteady. With the increase of swirl these two regions are present, while a double helix coherent structure is formed. For $S = 0.6$ this double helix structure transforms into a single helical structure typical for vortex breakdown with merging of the reversal flow regions. The POD analysis reveals that the turbulent kinetic energy accumulates in the first POD modes with $m = 1$, and the linear velocity of this POD mode increases.

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