Selective interactions in trapped ions: state reconstruction and quantum logic

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We propose the implementation of selective interactions of atom-motion subspaces in trapped ions. These interactions yield resonant exchange of population inside a selected subspace, leaving the others in a highly dispersive regime. Selectivity allows us to generate motional Fock (and other nonclassical) states with high purity out of a wide class of initial states, and becomes an unconventional cooling mechanism when the ground state is chosen. Individual population of number states can be distinctively measured, as well as the motional Wigner function. Furthermore, a protocol for implementing quantum logic through a suitable control of selective subspaces is presented.

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The interaction of a two-level system with an infinite-dimensional system is one of the most simple and fundamental quantum models, describing the interplay between discrete and continuous variable systems. The case of a two-level atom interacting with a single mode of the electromagnetic field, typically described by the Jaynes-Cummings (JC) model \[ \text{JC} \] in cavity quantum electrodynamics (CQED) \[ \text{CQED} \], can be reproduced with more complexity in the context of an ion trapped in a harmonic potential \[ \text{H} \]. In the latter, two internal levels are coupled to a vibrational mode through an adequately tuned laser system. CQED and trapped ion systems have been studied along different research lines like the generation and measurement of nonclassical states and/or entangled states, state reconstruction and the implementation of quantum processing and computing devices. Typically, the interactions used for those purposes, in resonant or dispersive regimes, involve dynamically all states of the associated atom-field or atom-motion Hilbert space.

In this letter, we propose the implementation of selective interactions in trapped ion systems. Selectivity is associated with the possibility of producing resonant interactions exclusively inside preselected atom-motion Hilbert subspaces, while all others remain in a highly dispersive regime. We describe how to tailor the interaction of a two-level system (like a two-level atom) with an infinite dimensional system (like a harmonic oscillator) into a resonant interaction of a reduced two-level systems. We show that, beyond its fundamental interest, selectivity is a powerful tool for achieving diverse quantum effects, from nonclassical state generation, cooling to the ground state, state reconstruction, and quantum logic.

Selectivity, in the way it is presented here, can be related and eventually applied to quantum effects like blockade, individual (selective) addressing, and turnstile mechanisms, among others. Early attempts to implement similar (selective) devices were developed in recent years in CQED and, quite specifically, in the collective behavior of two trapped ions outside the Lamb-Dicke regime

\[ H_1^{\text{JC}} = \hbar g (\sigma^+ a + \sigma a^+) \] (1)

The interaction of a two-level system with a harmonic oscillator, like atom-field (atom-motion) interactions in CQED (trapped ions), can be described by the JC model, whose Hamiltonian under resonant conditions and in the interaction picture reads

\[ \{ \{g, n\}, \{e, n-1\} \} \] (2)

with \( n = 0, 1, \ldots \), where \( |n\rangle \) is a number state of the harmonic oscillator.

It is possible to produce effectively anti-Jaynes-Cummings (AJC) interactions in CQED and trapped ions, whose Hamiltonian reads

\[ H_1^{\text{JC}} = \hbar g (\sigma^+ a + \sigma a) \] (3)

and produces Rabi oscillations in all AJC subspaces

\[ \{ \{g, n\}, \{e, n+1\} \} \] (4)

with \( n = 0, 1, \ldots \). We call selective interaction to a resonant interaction producing exchange of population (a selective Rabi oscillation) inside a chosen JC or AJC subspace,

\[ \{ \{g, N_0\}, \{e, N_0 \pm 1\} \} \] (5)

with fixed \( N_0 \), while all other subspaces remain strictly off-resonance.

Beyond the fundamental interest in this particular tailoring of the Hilbert space, selectivity provides us with a flexible tool that will prove to be useful in a wide range of applications. Here, we will discuss its implementation in trapped ion systems, although the different examples
tive (resonant) two-photon process involves the excitation of internal states with coupling strength $g$, respectively. The Raman scheme is realized in such a way that the effective (resonant) two-photon process involves the excitation of the first blue vibrational sideband through the $|g\rangle \leftrightarrow |c\rangle$ transition. In the rotating wave approximation, the Hamiltonian describing this system is

$$H = \hbar \omega a^\dagger a + \hbar \omega_c |c\rangle\langle c| + \hbar \omega_c |c\rangle\langle c|$$

$$+ \hbar g_1 \left[ e^{-i(k_1 z - \omega_1 t)} + e^{i(k_1 z + \omega_1 t)} \right] |g\rangle\langle c|$$

$$+ \hbar g_2 e^{-i(k_2 z - \omega_2 t)} |c\rangle\langle c| + H.c.,$$

(6)

where the coupling strengths $g_1$ and $g_2$ are taken as positive and $\omega_i = k_i \omega_i (i = 1, 2)$ are the frequencies of the exciting fields. In the Lamb-Dicke (LD) regime, in the interaction picture, and eliminating adiabatically level $|c\rangle$ under the dispersive condition $\Delta \gg \Omega_{\text{eff}} \equiv 2\eta_2 g_1 g_2 / \Delta$, we can write the effective AJC-like Hamiltonian

$$H_{\text{eff}} = -4\hbar \eta_1 g_1^2 \Delta \{ |g\rangle\langle g| - \hbar \eta_1 g_1^2 \Delta (2a^\dagger a + 1) |g\rangle\langle g|$$

$$+ 2i \hbar \eta_2 \frac{g_1 g_2}{\Delta} |c\rangle\langle c| a^\dagger - |g\rangle\langle c| a\},$$

(7)

where $\eta_i = k_i \hbar / 2m \nu$ are the LD parameters corresponding to each laser field excitation.

Associated with the transition in the AJC subspace $\{ |g, N_0\rangle, |e, N_0 + 1\rangle \}$, the effective Hamiltonian in Eq. (7), $H_{\text{eff}}$, shows a detuning frequency

$$\Delta_{\text{eff}}^{N_0} = -4\eta_1^2 g_1^2 \Delta (2N_0 + 1) + (4\eta_2^2 - \frac{g_2^2}{\Delta}).$$

(8)

This detuning can be compensated for a fixed phonon number $N_0$ by DC Stark shift or by shifting the laser frequencies. Selectivity appears when we tune to resonance a preselected subspace transition $\{ |g, N_0\rangle \leftrightarrow |e, N_0 + 1\rangle \}$, while all other AJC subspaces $\{ |g, n\rangle, |e, n + 1\rangle \}$, with $n \neq N_0$, remain dispersive. Once the correction is done specifically for $\{ |g, N_0\rangle, |e, N_0 + 1\rangle \}$, the remaining detunings associated with other subspaces $(n \neq N_0)$ are

$$\Delta_{\text{eff}}^n \equiv \Delta_{\text{eff}}^0 - \Delta_{\text{eff}}^{N_0} = -8\eta_1^2 g_1^2 \Delta (n - N_0).$$

(9)

If after this reshifting process, the dispersive condition

$$|\Delta_{\text{eff}}^n| \gg |\Omega_{\text{eff}}| \equiv 2\eta_2 \frac{g_1 g_2}{\Delta}$$

(10)

holds for all $n \neq N_0$, we arrive to the selectivity condition

$$S \equiv \frac{4\eta_1^2 g_1 g_2}{\Delta} \gg 1$$

(11)

for the selectivity parameter $S$.

Considering the experimental parameters of the ion experiments at NIST (Boulder) as an example without optimization, and imposing strictly the selectivity condition, $S \gg 1$, it is possible to achieve

$$\Omega_{\text{eff}} \equiv 2\eta_2 \frac{g_1 g_2}{\Delta} \lesssim 10^5 \text{Hz.}$$

(12)

This effective coupling strength produces population inversion of any selected subspace $\{ |g, N_0\rangle \leftrightarrow |e, N_0 + 1\rangle \}$ in $\tau_{\text{inv}} < 0.1 \text{ ms}$. Then, the required times for implementing selectivity in trapped ions are shorter compared to the motional decoherence time, typically $\tau_{\text{dec}} \lesssim 10 \text{ ms}$. For sure, it will be also interesting to design a selective scheme in other ion setups, like the one in the Innsbruck group, where, at variance with the Raman scheme in NIST, a quadrupolar two-level transition is directly and strongly excited. This interest is well founded since recently F. Schmidt-Kaler et al. have realized a 2-qubit gate through an off-resonant coupling of a laser field to a motional sideband, producing a phase shift conditioned to the motional state and showing that a selective mechanism might be realistically designed and implemented.

Let us consider the initial pure atom-motion state

$$|g\rangle \otimes \sum_n c_n |n\rangle$$

(13)

and tune our system to be selectively resonant within the subspace $\{ |g, N_0\rangle \leftrightarrow |e, N_0 + 1\rangle \}$. Then, we let it evolve for a time equivalent to a $\pi$-pulses, $\Omega_{\text{eff}} \sqrt{N_0 + 1} = \pi$, in
containing a finite population of the motional state
involving many atoms, like $|g, N_0\rangle$, we showed that after a pulse, $N_0 + 1$ out of any initial state containing a finite population of the motional state $|N_0\rangle$.

Similar results are obtained if the initial state is a thermal state or any statistical mixture, as we will see. For example, let us consider the ion initially in the ground state $|g\rangle$ and a precooled motional state, so as to have a finite contribution of Fock state $|1\rangle$,

$$|e\rangle\langle e| \otimes \sum_n p_n |n\rangle\langle n|.$$ (15)

If we tune to resonance the subspace $\{ |g, 0\rangle, |e, 1\rangle \}$ the evolution corresponding to a $\pi$-pulse yields

$$|g\rangle\langle g| \sum_n p_n |n\rangle\langle n| + |e\rangle\langle e| |N_0 + 1\rangle\langle N_0 + 1|.$$ (16)

Here, by measuring the internal state $|g\rangle$, see Fig. 2, we produce a "single-shot" cooling onto the motional ground state $|0\rangle$. This unconventional cooling mechanism requires a finite population of the Fock state $|1\rangle$ in the initial motional state in order to happen. From that point of view, it could be used for resetting the ground state in a previously cooled ion or ion chain.

Possible extensions of the notion of selectivity to other subspaces, like $\{ |g, N_0\rangle, |e, N_0 + k\rangle \}$, or to subspaces involving many atoms, like $\{ |gg, N_0\rangle, |ee, N_0 + k\rangle \}$, and/or different collective vibrational modes, are naturally expected but not developed here. They would allow us to engineer arbitrary atom-motion superposition states in a simplified manner.

When we tuned to resonance the subspace $\{ |g, N_0\rangle, |e, N_0 + 1\rangle \}$, under the selectivity condition $S \gg 1$, we showed that after a $\pi$-pulse

$$P_e = |c_{N_0}|^2 \equiv P_{N_0}. $$ (17)

That implies that by measuring the population of the internal excited state, $P_e$, we measure directly the population of a preselected Fock state $|N_0\rangle$ of an arbitrary and initially unknown motional state. To our knowledge, it is the first proposal for measuring directly a given motional population without requiring the complete state reconstruction. If the selectivity parameter $S$ is not large enough, we could repeat the selective procedure and set the experimental parameters to put in resonance the subspace $\{ |g, N_0 + 1\rangle \rightarrow |e, N_0 + 2\rangle \}$. The probability of finding again the excited state $|e\rangle$ becomes closer to $P_{N_0}$, yielding a continuously convergent measurement technique. Combined with the possibility of displacing the motional state [29], this method permits a full reconstruction of the associated Wigner function [30].

$$W(\alpha) = 2 \sum_n (-1)^n P_n(-\alpha), $$ (18)

where $P_n(\alpha) = \langle n| D(\alpha) p D^{-1}(\alpha) |n\rangle$ and $D(\alpha)$ is a displacement operator. As known, the Wigner function contains the same information as the density operator, and both are related through a Fourier transform [31].

Selective addressing of subspaces and conditional dynamics are interrelated concepts [27], so it should not be a surprise that selectivity finds a natural environment in tailored quantum state engineering, as well as in quantum-logic devices. As an example, we propose an implementation of a controlled-phase gate (CPG) [13] in the internal states of two ions at arbitrary positions in a row of $N$. Nevertheless, it is also possible to produce other gates, like swap gates, with a reduced number of steps.

Let us consider the two internal levels of ion $j$, $\{ |g_j\rangle, |e_j\rangle \}$, as the control qubit and the two internal levels of ion $k$, $\{ |g_k\rangle, |e_k\rangle \}$, as the target qubit. The protocol involves three steps where the first and third consist in mapping forth and back, respectively, the control qubit into the $\{ |0\rangle, |1\rangle \}$ motional states, and the second step consists in realizing a selective CPG between the mapped motional state and the target qubit. For the sake of simplicity, although its applicability is general, we will illustrate the protocol assuming the initial pure state

$$(\alpha|g_j\rangle|g_k\rangle + \beta|g_j\rangle|e_k\rangle + \gamma|e_j\rangle|g_k\rangle + \delta|e_j\rangle|e_k\rangle)|0\rangle $$ (19)

as a possible intermediate state in a certain computation.

The first step, that is the mapping of the target qubit $j$ onto the motion, can be realized with a $\pi$-pulse of a selective interaction in the subspace $\{ |g_j, 1\rangle, |e_j, 0\rangle \}$

$$|g_j\rangle(\alpha|0\rangle|g_k\rangle + \beta|0\rangle|e_k\rangle + \gamma|1\rangle|g_k\rangle + \delta|1\rangle|e_k\rangle). $$ (20)

The second step, that is the selective CPG between the motional state and ion $k$, can be realized by changing the sign of the state $|g_k, 1\rangle$ through a $2\pi$-pulse in the preselected subspace $\{ |e_k, 1\rangle, |g_k, 2\rangle \}$

$$|g_j\rangle(\alpha|0\rangle|g_k\rangle + \beta|0\rangle|e_k\rangle + \gamma|1\rangle|g_k\rangle - \delta|1\rangle|e_k\rangle). $$ (21)
The final third step consists in mapping back the motional state onto ion $j$ through a similar procedure
\[
(\alpha |g_j\rangle|g_k\rangle + \beta |g_j\rangle|e_k\rangle + \gamma |e_j\rangle|g_k\rangle - \delta |e_j\rangle|e_k\rangle)|0\rangle. \tag{22}
\]
This last equation reflects the implementation of a selective CPG between the qubits in ions $j$ and $k$. It is noteworthy to mention that this protocol is still valid if the initial motional state is any superposition state $a|0\rangle+b|1\rangle$. Unfortunately, we have not succeeded in finding a protocol robust to any initial motional state.

We have considered the realistic implementation of selective interactions in trapped ion systems. We have discussed a method for generating Fock states, large or not, in the CM motion of a single trapped ion or in a collective mode of an ion chain. We showed that this scheme could be used as a *sui generis* cooling device in precooled systems when the chosen Fock state is the motional ground state. We demonstrated that selectivity offers us the possibility of measuring distinctively the motional state population and also, if required, its Wigner function in a straightforward manner. We sketched that selective interactions, when not enough accurate ($\mathcal{S}$ not so large), can be applied subsequently, yielding each time more accurate measurements. A wide family of nonclassical states, linear superpositions, entangled states, and quantum information devices could be engineered by selectively tailoring the Hilbert space in this new context. Possible implementations of quantum blockade and turnstile devices in CQED and trapped ion systems are under current research. We envisage also further investigation and generalization of selective schemes in other physical systems like optical lattices and atomic clouds.

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