Exotic Quarkonia from Lattice QCD

T. Manke for the CP-PACS Collaboration †
Center for Computational Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan

Abstract
We present non-perturbative results for the spectrum of heavy quarkonia. Using an anisotropic formulation of Lattice QCD we achieved an unprecedented control over statistical and systematic errors. We also study relativistic corrections to the leading order predictions for heavy hybrids and conventional bound states.

† CP-PACS Collaboration: A. Ali Khan, S. Aoki, R. Burkhalter, S. Ejiri, M. Fukugita, S. Hashimoto, N. Ishizuka, Y. Iwasaki, K. Kanaya, T. Kaneko, Y. Kuramashi, T. M., K. Nagai, M. Okawa, H.P. Shanahan, A. Ukawa and T. Yoshié

1. INTRODUCTION
Lattice studies play an important role for our theoretical understanding of QCD. However, the conventional approach is not well suited to accommodate physical systems with widely separate scales. When studying heavy quarkonia on isotropic lattices several approximations have to be made to render the numerical simulations tractable. In this context a non-relativistic approach (NRQCD) has lead to very precise calculations of the low lying spectrum in heavy quarkonia [1, 2]. New complications arise if high energetic excitations are to be resolved, for which the temporal discretisation is often too coarse and the correlator of such heavy states cannot be measured accurately for long times [3, 4, 5]. More recently anisotropic lattices were demonstrated to circumvent this problem by giving the lattice a fine temporal resolution whilst maintaining a coarse discretisation in the spatial direction [6]. This approach has resulted in very encouraging results for the spectrum of glueballs and hybrid states, which are of particular interest as they are non-perturbative revelations of the gluon degrees of freedom [7, 8]. Here we extend those methods to investigate also other excitations and the spin structure in heavy quarkonia more carefully.

2. RESULTS
In order to study excited quarkonia with small statistical errors we employ an anisotropic and spatially coarse gluon action and the NRQCD approach for the forward propagation of the heavy quarks. The latter is improved to contain spin-dependent terms up to $O(mv^6)$. From the implementation details given in [9] we expect discretisation errors of $O(a^4, a^2 t)$. Radiative corrections to this classical result are reduced by imposing a mean-field improvement on all gauge links. In Fig. 1 we present our results for the excitations in Charmonium and Bottomonium as normalised to the $1S-1P$ splitting. In each case we have studied bound states with several different quantum numbers, including the exotic hybrid, $^3B_{1+}^-$. The possibility to resolve all these states reliably is clearly due to the fine temporal resolution of our lattices. Using statistical ensembles of up to a few thousand independent measurements, we could rely on very simple-minded hadron operators to obtain the excitation energies from correlated multi-exponential fits.

Within the NRQCD approach it is paramount to establish a scaling region for physical quantities already at finite lattice spacing. Our analysis demonstrates the existence of such a window for the excitations in Charmonium and Bottomonium [7]. Moreover, the excellent agreement of the lowest lying charmonium hybrid with a relativistic calculation on isotropic lattices should be considered a combined success of anisotropic lattices and the NRQCD approach. Owing to the efficiency of this new approach we were able to test our results against finite size effects. For the lowest lying $c\bar{c}g$ hybrid we could not resolve any change for volumes larger than 1.2 fm and we presently continue this analysis for all other excitations. For the Bottomonium hybrid we have also consistent results from two different anisotropies, which confirms our initial assumption of small temporal lattice spacings artefacts.

The inclusion of relativistic corrections is a significant improvement over previous NRQCD
calculations of hybrid states, which were restricted to only leading order in the velocity expansion. At this level there are no spin-dependent operators and we have a strict degeneracy of all singlet and triplet states. Here we also include spin terms to break this degeneracy. In particular, we could directly observe the exotic hybrid, which is the state of greatest phenomenological interest. Our results for the spin structure in Bottomonium are shown in Fig. 2. Similar results for Charmonium are presented elsewhere [9]. We notice a clear reduction of the fine structure in D-states when compared to that of P-states. Similarly, the hyperfine splitting, $^3X - ^1X$, is suppressed as the orbital angular momentum is increased. This is in accordance with expectations from potential models. Our data also indicates that the fine structure in hybrid states is enlarged as the result of the gluon angular momentum to which the spin can couple.

Finally one should notice that the $^3S_1 - ^1S_0$ splitting does not scale on the lattices considered here. It is apparent that one needs a better improvement description to account for lattice spacing artefacts in such UV-sensitive quantities.

In conclusion, we find that coarse and anisotropic lattices are extremely useful for precision measurements of higher excited states. This is due to an improved resolution in the temporal direction and the possibility to generate large ensembles of gauge field configurations at small computational cost. It has lead to an unprecedented control over statistical and systematic errors in lattice studies of heavy quarkonia. We could observe a clear hierarchy in the spin structure, depending on the orbital angular momentum. The remaining systematic error for all our predictions is an uncertainty in the scale as the result of the quenched approximation. This is not yet controlled and we find a variation of 10-20%, depending on which experimental quantity is used to set the scale.

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