Instanton-induced charm contribution
to polarized deep-inelastic scattering

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Abstract

Recent data on B decays involving $\eta'$ may be explained if the singlet axial current of charmed quarks has a large matrix element to $\eta'$, and instantons were shown to be able to generate this effect. We study the magnitude of charm contributions to nucleon polarized structure functions generated in a similar way. Comparing the charm contribution, which is related to a dim(6) gluonic operator, to that of light quarks, which are related by the anomaly equation to $\tilde{G}\tilde{G}$, we found that $\Delta e/\Delta \Sigma = -(0.2-0.08)$. Future experiments like COMPASS at CERN identifying charm production in DIS can measure this “intrinsic polarized charm” component of the nucleon.
1. It was believed that non-perturbative phenomena in QCD can generate light quark (and gluon) “sea” in nucleon structure functions, while the admixture of heavy flavors such as charm is (i) very small due to their mass, and that (ii) it should be produced by perturbative effects alone. Although the first part of the argument (suppression by the power of charm mass squared) is correct, it can be counterbalanced by sufficiently strong non perturbative gluon fields. Small-size instantons present in the QCD vacuum have fields $G \sim 1\text{GeV}^2$, which is not much smaller than $m_c^2 \sim 2\text{GeV}^2$, and so in principle one may expect a sizable “intrinsic charm” of at least some hadrons (as anticipated by Brodsky et al. for a long time [1]).

Probably the first indication of that came from recent CLEO data on $B \to \eta'K$ and inclusive $B \to \eta' + \ldots$ decays [2], which can be explained [3] if $\eta'$ has such a charm component. Quantitative estimates of this effect – based on an instanton liquid model for the non perturbative vacuum – were made in [4], which concluded that indeed current information about instantons lead to surprisingly large charm content of the $\eta'$, comparable to what is needed to explain the CLEO data.

It was further speculated in [4] that other processes related to axial current of charm quarks may also show similar effects. One of them is the charm contribution to the polarized deep inelastic scattering (DIS), which we discuss in this work. Crude estimates given in [4] suggested that it may be even comparable to that of all light quarks together (which is, we remind, already strongly suppressed, as compared to the naive unit value for the “spin content” of the nucleon).

In the present paper we address the question of an intrinsic polarized charm component directly, by evaluating relevant three-point correlation functions in the vacuum. We use the same instanton liquid models as in ref. [4]. The results however are about an order of magnitude smaller than the estimates presented there. Nevertheless, they are still very different from the perturbative predictions, which imply that the charm
sea is mostly unpolarized.

2. Let us first recall few standard formulae for polarized DIS on a nucleon. The first moment of the polarized structure function

\[ \Gamma_{p1}(Q^2) = \int_0^1 dx g_1(x, Q^2) \]  

(1)

can be related to the matrix element of the axial current by the standard OPE treatment. The answer (including charm component \( \Delta c \)) can be written as

\[ \Gamma_{p1}(Q^2) = C_{NS}(Q^2) \left( \frac{g^3_A}{12} + \frac{g^8_A}{36} \right) + \frac{1}{9} C_S(Q^2) (\Sigma + 2\Delta c) \]  

(2)

where \( \Delta c \) is

\[ s_\mu \Delta c(\mu^2) = \langle p, s | (\bar{c}\gamma_\mu\gamma_5 c)_{\mu^2} | p, s \rangle \]  

(3)
at a relevant mass scale \( \mu^2 \). Here the coefficients \( C_{NS}(Q^2), C_S(Q^2) \) account for perturbative corrections of the form \( 1 - \alpha_s/\pi + \ldots \). The measureable singlet axial coupling constant can be defined as

\[ g_A^{(0)} = \Delta \Sigma + 2\Delta c \]  

(4)

where the first term is the contribution of all light quarks (u,d,s) together, defined as \( \Delta \Sigma = \Delta u + \Delta d + \Delta s \).

Both charm and light quark contributions can be calculated from the divergence of the axial current: for each flavor one has (in Euclidean space)

\[ -\partial_\mu (\bar{q}_E \gamma_\mu \gamma_5 q_E) = 2m_q \bar{q}_E \gamma_5 q_E + \frac{1}{16\pi^2} i (G_{\mu\nu} \tilde{G}_{\mu\nu})_E \]  

(5)

However, the role of these two terms is different for each flavor. For light quarks one can take the chiral limit \( m \to 0 \) and ignore the first term in the r.h.s. The OPE

\footnote{See e.g. \cite{5} for a recent review and nomenclature.}
expansion of the first term in $1/m$ was done by Halperin and Zhitnitsky\textsuperscript{3} and it shows that the anomaly term actually cancels, while the leading term in the heavy quark limit is

$$- \partial_\mu (\bar{c} E \gamma_5 c_E) = +i \frac{1}{16\pi^2 m_c^2} f^{abc} G_{\mu\nu}^a \tilde{G}_{\nu\alpha}^b G_{\alpha\nu}^c + O(G^4/m_c^4)$$

(6)

Note that it is indeed suppressed by $1/m_c^2$, as it should.

3. Using lattice gauge configurations (or a model which defines gauge fields in some way, as we do below), one can correlate these two gluonic operators with the nucleon and thus evaluate the matrix elements in question, $\Delta \Sigma$ and $\Delta c$. Information about them is contained in the three point correlation functions

$$\Pi_{\mu\nu}(x/2, x/2, y) = \Gamma_{\beta\alpha}^{(\nu)\pm} \langle 0 \mid T \{ \eta_{\alpha}(-x/2) j_{\mu 5}(y) \eta^{\dagger}_{\beta}(x/2) \} \mid 0 \rangle$$

(7)

where $\mu$ is the component of the axial current $j_{\mu 5}(y) = \bar{q} \gamma_\mu \gamma_5 q$ and $\{ \nu \}$ a multiple n-index for the projection operator $\Gamma_{\nu}^{(\nu)\pm} = \gamma_{\nu 1} \cdots \gamma_{\nu n} P_\pm$, $P_\pm = P_R \pm P_L$. The $\eta_{\alpha}(x), \eta^{\dagger}(x)$ are the so called Ioffe currents, local operators with the quantum numbers of the nucleon or in a simplified consideration the quarks.

Before we go for the evaluation of the three-point functions, let us qualitatively explain the signs and magnitude of the gluonic operators for an instanton (anti-instanton) configuration. Introducing the topological charge $Q_I = 1$ (-1) and substituting the known analytic solution for the instantons one finds

$$\frac{1}{16\pi^2} G_{\mu\nu} \tilde{G}_{\mu\nu} = Q_I \frac{1}{\pi^2} \frac{12\rho^4}{(\rho^2 + (y - z_I)^2)^4}$$

(8)

and

$$\frac{1}{16\pi^2 m_c^2} f^{abc} G_{\mu\nu}^a \tilde{G}_{\nu\alpha}^b G_{\alpha\nu}^c = -Q_I \frac{96}{\pi^2 m_c^2} \frac{\rho^6}{(\rho^2 + (y - z_I)^2)^6}$$

(9)

\textsuperscript{2}Since the latter do not contain charm quarks, the contribution of charm quarks is only via the disconnected, OZI violating, diagrams.
where $\rho$ is the size and the $z_I$ the position. The first important observation now is that each instanton/anti-instanton contributes to $\Delta \Sigma$ and $\Delta c$ with the opposite sign.

The second is that, by dimension, charm component relative to light quark one is $\sim <\rho^{-2}>/m_c^2$, where angular brackets mean averaging over instanton size distribution. Obviously this average favor smaller sizes than the usual r.m.s. one.

Of course, due to the factor of the topological charge $Q_I = \pm 1$ the average of both of those pseudo scalar operators in the vacuum vanishes. Non-zero results can only be obtained if it is correlated with another pseudo scalar observable, such as $\vec{\sigma} \vec{q}$, a product of spin vector and the momentum transfer. Another useful way to explain it is to note that a spin vector acts on the vacuum like a dipole operator, thereby polarizing (topological) charges and creating non-equal densities of instantons and anti-instantons along its vector.

4. Quantitative calculations are based on the three-point correlation functions indicated above. Instead of using the axial current itself, we are considering its divergence in eq. (7). Then for $x$ in Euclidean time direction, the spin asymmetry induced by the pseudoscalar gluonic operators in the direction $n$ of $y$ can be traced out with $\Gamma^n$. The correlator is then $\sim (y \cdot s)$ which translates into $\sim (q \cdot s)$ in momentum space. Since we plot the correlator in coordinate space as a function of $y$ we are not facing the problem of a vanishing forward matrix element of these operators.

For comparison, we use two instanton ensembles, the random instanton liquid (RILM) and the so called interacting instanton liquid model (IILM) (for review see [8]). The former is a simple model based on phenomenological parameters of the instantons like the average size, while the latter is a sophisticated model including the 't Hooft effective interaction to all orders. The ensembles have 256 instantons in a rectangular box of $(5.7\text{fm})^2 \times (2.8\text{fm})^2$. The instanton density is the same $n = 1 \text{ fm}^{-4}$ for all configurations.
Correlations functions with the nucleon propagator decay very fast with x due to large nucleon mass, which in practice strongly restricts the range of x,y at which the measurements can be made. Virtual charm pairs are rather well localized anyway, and so one may probably use a single “constituent” quark\(^3\) instead of three in the nucleon as a reasonable first approximation. That is what we did below. So we do not actually discuss nucleons, but some “generic” hadrons, made of loosely bound constituent quarks.

Our results for both correlators are shown in Fig. 1 and Fig. 2. The geometry of the x and y directions and their ratio we have adopted from previous three-point correlation function studies \([10]\): their particular choice is not important for the conclusions. The dashed lines are guiding analytic calculations based on the single instanton approximation, and serve only as a benchmark for the numerical data at smaller distances \((x, y \ll 0.5\text{fm})\).

The main finding is that both operators show very similar shape of the correlation functions. This is further illustrated in Fig.3, where we plot their ratio. The instanton ensemble used however matters: for IILM the charm-related operator shows a much weaker signal. This is related to different instanton size distributions: in RILM all of them have \(\rho = 1/3\text{fm}\) while in IILM the distribution is peaked at about 0.4fm. Another important reason for the reduction effect in IILM are positive (screening-type) correlations between instantons and anti-instantons induced by quark exchanges. We use the similarity in shape of both correlators as a sign that the form factors for both \(\Sigma(q^2)\) and \(\Delta c(q^2)\) are the same, and the ratio of the correlation functions shown in Fig.3 can be immediately translated to the ratio \(\frac{\Delta c}{\Sigma}\).

\(^3\)More complete analysis of structure functions of a constituent quark resulting from those measurements will be presented elsewhere \([9]\).
Finally our results are

\[ \frac{\Delta c}{\Sigma} \simeq -0.20 \pm 0.04 \ (RILM), \tag{10} \]

and

\[ \frac{\Delta c}{\Sigma} \simeq -0.08 \pm 0.006 \ (IILM). \tag{11} \]

Several things have to be remarked now. Using an approximate scheme for the non-zero mode propagator involved in the correlation function would give the analytic expression

\[ \frac{\Delta c}{\Sigma} = -\frac{12}{5N_f(m_{\rho})^2} \simeq -0.2 \] for the standard values of the RILM. This scheme is also shown in Fig.3 and actually very close to the full RILM simulation.

The positive sign of \( \Delta \Sigma \) here agrees with the experimental data and the negative sign for \( \Delta c \) is somehow in agreement with lattice calculations \[11\] of disconnected matrix element of the axial current itself, which give a negative sign for the three light quark flavors. Both \( \Delta \Sigma \) and \( \Delta c \) signs are opposite however to the scenario proposed by Halperin and Zhitnitsky \[6\], which is mainly based on an older low energy theorem by Kühn and Zakharov \[12\].

These numbers \[10,11\] are significantly smaller than the crude estimates for \( g_A^{(0)} \) based on Goldberger-Treiman-type relations for \( \eta' \) exchange used in \[4\]. Nevertheless the size of \( \Delta c \) is in fact surprisingly large. It seems to be only 3-6 times smaller than that of the strange quark sea. Hopefully the next generation of polarized DIS experiments \[13,14\] with charm-jet tagging will be able to observe it.

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Fig. 1: The quark three-point correlation function for the charm contribution $\simeq f^{abc} g^{a}(y) \tilde{G}^{b}(y) G^{c}(y)$ for the random configuration (RILM). The approximate (app) formula is also shown.
Fig. 2: The quark three-point correlation function for the charm contribution $\simeq f^{abc} G^a(y) \tilde{G}^b(y) G^c(y)$ for the interacting configuration (IILM).
Fig. 3: The ratio of the three-point function for $\frac{N_f}{16\pi^2}G\tilde{G}$ and $-\frac{N_f}{16\pi^2}f^{abc}G^a(y)\tilde{G}^b(y)G^c(y)$ for the RILM and IILM configuration.

The ratio of the three-point function for $\frac{N_f}{16\pi^2}G\tilde{G}$ and $-\frac{N_f}{16\pi^2}f^{abc}G^a(y)\tilde{G}^b(y)G^c(y)$ for the RILM and IILM configuration.