A new 2-D multi-stable chaotic attractor and its MultiSim electronic circuit design

Sundarapandian Vaidyanathan¹, Aceng Sambas², Mohamad Afendee Mohamed³, Mustafa Mamat³, W. S. Mada Sanjaya⁴, Sudarno⁵
¹Research and Development Centre, Vel Tech University, Chennai, Tamil Nadu, India
²Department of Mechanical Engineering, Universitas Muhammadiyah Tasikmalaya, Indonesia
³Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, KualaTerengganu, Malaysia
⁴Department of Physics, UIN Sunan Gunung Djati Bandung, Indonesia
⁵Faculty of Engineering, Universitas Muhammadiyah Ponorogo, Indonesia

ABSTRACT

A new multi-stable system with a double-scroll chaotic attractor is developed in this paper. Signal plots are simulated using MATLAB and multi-stability is established by showing two different coexisting double-scroll chaotic attractors for different states and same set of parameters. Using integral sliding control, synchronized chaotic attractors are achieved between drive-response chaotic attractors. A MultiSim circuit is designed for the new chaotic attractor, which is useful for practical engineering realizations.

Keywords:
Chaos
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Corresponding Author:
Mohamad Afendee Mohamed
Faculty of Informatics and Computing
Universiti Sultan Zainal Abidin
KualaTerengganu, Malaysia
Email: mafendee@unisza.edu.my

1. INTRODUCTION

Chaotic dynamical models with double-scroll attractors have been analyzed in the science literature [1], [2]. These attractors resemble like butterfly wings due to their double-scroll shape. Especially, the dynamical plants exhibiting multi-stability and cohapening chaotic attractors have been studied [3], [4]. Engineering fields have many utilizations of chaotic attractors [2]. Some common utilizations are enlisted such as oscillations [5], [6], vibrations [7], neuron models [8], [9], control and memristor models [10]-[12], mechanical attractors [13], [14].

In the control literature, there are many control techniques available for the control and synchronization of chaotic systems [2]. Bahoo and Poria [13] used active control method for food chain model. Mustafa et al. [14] used chaos-enhanced cuckoo search for economic dispatch with valve point effects. Vaidyanathan and Rasappan [15] used active control for the hybrid synchronization of hyperchaotic Qi and Lü systems. Vaidyanathan [16] used active control for stabilizing the state trajectories of a new hyperchaotic system with three quadratic nonlinearities. Medhaffar et al. [17] investigated the stabilization of unstable periodic orbits of continuous time chaotic systems using adaptive fuzzy controllers. Boubellouta and
Boulkroune [18] investigated the problem of chaos synchronization based on fractional-order intelligent sliding-mode control approach for a class of fractional-order chaotic optical systems with unknown dynamics and disturbances. Vaidyanathan [19] studied the global chaos synchronization of Tokamak chaotic systems with symmetric and magnetically confined plasma. Khan and Kumar [20] studied the T–S fuzzy observed based design and synchronization of chaotic and hyper-chaotic dynamical systems.

The novelty of this work is the modelling a new double-scroll chaotic attractor with interesting dynamic properties. The signal plots, dynamical properties and multi-stability with co-happening chaotic attractors are reported for the new chaotic attractor. For practical realizations, an electronic circuit is immensely useful after the modelling of a new chaotic attractor [21–26]. A MultiSim electronic circuit model of the new chaotic attractor is carried out and a good match between the MultiSim circuit outputs and the MATLAB signal plots has been found.

2. A NEW DOUBLE-SCROLL MULTI-STABLE CHAOTIC ATTRACTOR

We first give the dynamics of a new system described as follows:

\[
\begin{align*}
\dot{p}_1 &= \alpha (p_2 - p_1) + p_2 p_3, \\
\dot{p}_2 &= \beta p_2 - p_3, \\
\dot{p}_3 &= p_1 p_2 - \gamma p_1 + \delta |p_2|.
\end{align*}
\]

We note that \(\Lambda = (\alpha, \beta, \gamma, \delta)\) is the parameter and \(P = (p_1, p_2, p_3)\) is the phase vector. Using Wolf’s approach [27], we will show that the model (1) will exhibit a chaotic attractor for \(\Lambda = (\alpha, \beta, \gamma, \delta) = (40, 26, 5, 0.2)\).

For MATLAB simulations, the initial phase vector is chosen as \(P(0) = (0.1, 0.3, 0.2)\). Then the Lyapunov indices of (1) are estimated using Wolf’s approach [27] as follows:

\[
LE_1 = 3.9125, \quad LE_2 = 0, \quad LE_3 = -22.9125
\]

Using (3), it is concluded that the model (1) has chaoticity and dissipativity. The double-scroll attractor of the model (1) is simulated in various planes in Figure 1.

The balance points of the new double-scroll attractor (1) for \((\alpha, \beta, \gamma, \delta) = (40, 26, 5, 0.2)\) are calculated as below:

\[
P_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad P_1 = \begin{bmatrix} 11.3022 \\ 7.8017 \\ 17.9473 \end{bmatrix}, \quad P_2 = \begin{bmatrix} -11.3022 \\ -7.8017 \\ 17.9473 \end{bmatrix}
\]

By finding spectral values of the linearization matrices of the double-scroll system (1), it can be ascertained that the balance point \(P_0\) is a saddle point, and \(P_1, P_2\) are saddle-foci.

We next demonstrate that the new double-scroll system (1) has co-happening chaotic attractors.

When selecting \((\alpha, \beta, \gamma, \delta) = (40, 26, 5, 0.2)\), and the initial phase vectors \(P_0 = (0.1, 0.3, 0.2)\) (blue) and \(Q_0 = (-0.5, -0.3, -0.5)\) (red), the new double-scroll chaotic attractor (1) depicts co-happening chaotic attractor (blue) and chaotic attractor (red) as plotted in Figure 2.
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Figure 1. MATLAB phase plots showing double-scroll chaotic attractor of the model (1)

Figure 2. Multi-stability of the new double-scroll attractor (1): Cohappening chaotic attractors

3. INTEGRAL SLIDING CONTROL DESIGN FOR THE PHASE SYNCHRONIZATION OF DOUBLE-SCROLL CHAOTIC ATTRACTORS

For the phase synchronization of double-scroll chaotic attractor, we consider a pair of drive-response chaotic attractors listed as follows.

\[
\begin{align*}
\dot{p}_1 &= \alpha (p_2 - p_1) + p_2 p_3 \\
\dot{p}_2 &= \beta p_2 - p_1 p_3 \\
\dot{p}_3 &= p_1 p_2 - \gamma p_3 + \delta |p_2|
\end{align*}
\]

\[(5)\]
\begin{align*}
\dot{q}_1 &= \alpha(q_2 - q_1) + q_2q_3 + v_i \\
\dot{q}_2 &= \beta q_2 - q_3q_2 + v_2 \\
\dot{q}_3 &= q_2q_3 - \gamma q_3 + \delta \big| q_2 \big| + v_3 \\
\end{align*} 
\tag{6}

The phase synchronization error between (5) and (6) can be defined as below:

\begin{align*}
\varepsilon_1 &= q_1 - p_1 \\
\varepsilon_2 &= q_2 - p_2 \\
\varepsilon_3 &= q_3 - p_3 \\
\end{align*} 
\tag{7}

A simple calculation pinpoints the dynamics of the phase synchronization error as below:

\begin{align*}
\dot{\varepsilon}_1 &= \alpha(\varepsilon_2 - \varepsilon_1) + q_2q_3 - p_2p_3 + v_1 \\
\dot{\varepsilon}_2 &= \beta \varepsilon_2 - q_3q_2 + p_1p_3 + v_2 \\
\dot{\varepsilon}_3 &= -\gamma \varepsilon_3 + q_2q_3 - p_1p_2 + \delta \big( \big| q_2 \big| - \big| p_2 \big| \big) + v_3 \\
\end{align*} 
\tag{8}

We define the integral sliding surface associated with each error variable as below:

\begin{align*}
\dot{z}_1 &= \left[ \frac{d}{dt} + \psi_1 \right] \int_0^t \dot{\varepsilon}_1(r) \, dr = \varepsilon_1 + \psi_1 \int_0^t \dot{z}_1(r) \, dr \\
\dot{z}_2 &= \left[ \frac{d}{dt} + \psi_2 \right] \int_0^t \dot{\varepsilon}_2(r) \, dr = \varepsilon_2 + \psi_2 \int_0^t \dot{z}_2(r) \, dr \\
\dot{z}_3 &= \left[ \frac{d}{dt} + \psi_3 \right] \int_0^t \dot{\varepsilon}_3(r) \, dr = \varepsilon_3 + \psi_3 \int_0^t \dot{z}_3(r) \, dr \\
\end{align*} 
\tag{9}

Taking time-derivative of all the of (9), we obtain as below:

\begin{align*}
\dot{\dot{z}}_1 &= \dot{\varepsilon}_1 + \psi_1 \dot{\varepsilon}_1 \\
\dot{\dot{z}}_2 &= \dot{\varepsilon}_2 + \psi_2 \dot{\varepsilon}_2 \\
\dot{\dot{z}}_3 &= \dot{\varepsilon}_3 + \psi_3 \dot{\varepsilon}_3 \\
\end{align*} 
\tag{10}

We take \( \psi_1, \psi_2, \psi_3 \) as positive constants.

Next, we set the dynamics of the sliding variables as follows:

\begin{align*}
\dot{z}_1 &= -\theta_1 \text{sgn}(z_1) - \mu_1 z_1 \\
\dot{z}_2 &= -\theta_2 \text{sgn}(z_2) - \mu_2 z_2 \\
\dot{z}_3 &= -\theta_3 \text{sgn}(z_3) - \mu_3 z_3 \\
\end{align*} 
\tag{11}

From (10) and (11), we deduce the following:

\begin{align*}
\dot{\varepsilon}_1 + \lambda_1 \varepsilon_1 &= -\phi_1 \text{sgn}(s_1) - k_1 s_1 \\
\dot{\varepsilon}_2 + \lambda_2 \varepsilon_2 &= -\phi_2 \text{sgn}(s_2) - k_2 s_2 \\
\dot{\varepsilon}_3 + \lambda_3 \varepsilon_3 &= -\phi_3 \text{sgn}(s_3) - k_3 s_3 \\
\end{align*} 
\tag{12}

Combining (8) and (12), we obtain the following:

\begin{align*}
\alpha(\varepsilon_2 - \varepsilon_1) + q_2q_3 - p_2p_3 + v_1 + \psi_1 \varepsilon_1 &= -\theta_1 \text{sgn}(z_1) - \mu_1 z_1 \\
\beta \varepsilon_2 - q_3q_2 + p_1p_3 + v_2 + \psi_2 \varepsilon_2 &= -\theta_2 \text{sgn}(z_2) - \mu_2 z_2 \\
-\gamma \varepsilon_3 + q_2q_3 - p_1p_2 + \delta \big( \big| q_2 \big| - \big| p_2 \big| \big) + v_3 + \psi_3 \varepsilon_3 &= -\theta_3 \text{sgn}(z_3) - \mu_3 z_3 \\
\end{align*} 
\tag{13}
The integral sliding controls are deduced from (13) as below:
\[
\begin{align*}
    v_1 &= -\alpha(e_1 - e_1) - q_2 q_3 + p_1 p_3 - \psi_1 e_1 - \theta_1 \text{sgn}(z_1) - \mu_1 z_1 \\
    v_2 &= -\beta e_2 + q_1 q_3 - p_1 p_3 - \psi_2 e_2 - \theta_2 \text{sgn}(z_2) - \mu_2 z_2 \\
    v_3 &= \gamma e_3 - q_1 q_2 + p_1 p_2 - \delta(\|q_2\| + |p_2|) - \psi_3 e_3 - \theta_3 \text{sgn}(z_3) - \mu_3 z_3
\end{align*}
\] (14)

**Theorem 1.** The integral sliding control law defined by (14) achieves the global phase chaos synchronization between the new double-scroll attractors (5) and (6), where the constants \(\psi_i, \theta_i, \mu_i\) \((i = 1, 2, 3)\) are all positive.

**Proof.** First, as a positive definite Liapunov function candidate, we choose the function
\[
W(z_1, z_2, z_3) = 0.5\left(z_1^2 + z_2^2 + z_3^2\right)
\] (15)

We calculate the time-derivative of \(W\) as below:
\[
\dot{W} = z_1 \dot{z}_1 + z_2 \dot{z}_2 + z_3 \dot{z}_3
\] (16)

Combining (11) and (16), we get
\[
\dot{W} = z_1[-\theta_1 \text{sgn}(z_1) - \mu_1 z_1] + z_2[-\theta_2 \text{sgn}(z_2) - \mu_2 z_2] + z_3[-\theta_3 \text{sgn}(z_3) - \mu_3 z_3]
\] (17)

Simplifying (17), we obtain
\[
\dot{W} = -\theta_1 z_1^2 - \mu_1 z_1^2 - \theta_2 z_2^2 - \mu_2 z_2^2 - \theta_3 z_3^2 - \mu_3 z_3^2
\] (18)

Since \(\theta_1, \theta_2, \theta_3 > 0\) and \(\mu_1, \mu_2, \mu_3 > 0\), we see that \(\dot{W}\) is a negative definite function.

From Liapunov stability theory \([28]\), we find \((z_1(t), z_2(t), z_3(t)) \to (0, 0, 0)\) as \(t \to \infty\).

Hence, we observe that \((e_1(t), e_2(t), e_3(t)) \to (0, 0, 0)\) as \(t \to \infty\).

For MATLAB simulations, we assume the parameter vector as in the chaotic case, viz. \((\alpha, \beta, \gamma, \delta) = (40, 26, 5, 0.2)\). We also assume the gains as \(\psi_i = 0.1, \theta_i = 0.1, \) and \(\mu_i = 20\) for \(i = 1, 2, 3\).

The initial state of the drive system (5) is picked as \((p_1(0), p_2(0), p_3(0)) = (3, -0.5, 2)\) and the initial state of the response system (6) is picked as \((q_1(0), q_2(0), q_3(0)) = (1.5, 0.9, 4.2)\).

Figure 3 shows the phase synchronization error between systems (5) and (6).

![Figure 3. The phase synchronization error between the systems (5) and (6)](image-url)
4. MULTISIM CIRCUIT DESIGN OF THE NEW DOUBLE-SCROLL ATTRACTOR

The MultiSim electronic circuit of the new double-scroll attractor (1) is realized by using off-the-shelf components such as resistors, capacitors, operational amplifiers and analog multipliers. The phases $p_1$, $p_2$, $p_3$ of the double-scroll attractor (1) are the voltages across the capacitors $C_1$, $C_2$ and $C_3$, respectively. The electronic circuit of the new double-scroll attractor is realized in MultiSim by 19 resistors, 8 operational amplifiers (TL082CD), 3 multipliers (AD633JN), 2 diodes (1N4148) and 3 capacitors. By the use of Kirchhoff’s circuit laws into the circuit in Figure 4, its circuital equations are obtained as (19):

$$
\begin{align*}
\dot{p}_1 &= \frac{1}{C_1 R_1} p_2 - \frac{1}{C_1 R_2} p_1 + \frac{1}{10 C_1 R_3} p_2 p_3 \\
\dot{p}_2 &= \frac{1}{C_2 R_4} p_2 - \frac{1}{10 C_2 R_5} p_1 p_3 \\
\dot{p}_3 &= \frac{1}{10 C_3 R_6} p_1 p_2 - \frac{1}{C_3 R_7} p_3 + \frac{1}{C_3 R_8} p_2
\end{align*}
$$

We selected $R_1 = R_2 = 10 \, k\Omega$, $R_3 = R_5 = 40 \, k\Omega$, $R_4 = 15.384 \, k\Omega$, $R_7 = 80 \, k\Omega$, $R_8 = 2 \, M\Omega$, $R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = 100 \, k\Omega$, $C_1 = C_2 = C_3 = 3.2 \, nF$.

Figures 5-7 with MultiSim outputs of the double-scroll chaotic attractor (19) exhibit a good match with the MATLAB outputs of the double-scroll chaotic attractor (1) shown in Figure 1.

![Figure 4. The circuit schematic of the double-scroll chaotic attractor (19) (Note: $p_1, p_2, p_3 = x_1, x_2, x_3$)](image_url)
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5. CONCLUSION
In this paper, a new multi-stable system with a double-scroll chaotic attractor is developed and detailed. Signal plots were simulated using MATLAB and multi-stability was established by showing two different coexisting double-scroll chaotic attractors for different states and same set of parameters. Using integral sliding control, synchronized chaotic attractors are achieved between drive-response chaotic attractors. A MultiSim electronic circuit was designed for the new double-scroll attractor, which is useful for practical engineering realizations.

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