Dynamic research of elementary differential gear with elastic links

T M Slobodyanik* and E E Balakhnina
National Research Technology University NUST «MISIS», Leninsky Ave., 4, 119049, Moscow, Russia
*tsloodyanik@gmail.com

Abstract. Many research works are devoted to study of static and dynamic parameters of planetary gears of various designs. A number of dynamic problems can be solve accurately enough without taking into account the elasticity of loaded elements. In calculations of mining machines transmissions dynamic processes must be considered gear flexibility as is much more than flexibility of large diameter and small length shafts. In this article are considered possibilities and features of research of dynamic phenomena that take a place in planetary mechanisms of mining machine transmissions. For this purpose planetary mechanisms are presented as elastic mechanical systems. For research the dynamic model of elementary differential mechanism with taking into account the link elasticity is used. For this purpose the differential gear presented as equivalent planetary row with inertialess planetary carrier. Main links of such mechanism have inertia moments determined by known formulas. The equivalent dynamic model of elementary differential gear is presented as three-mass branched system. Moments of elasticity, occurring in mechanical constraints of system, are determined. Equations of movement for each of the masses give the linear homogeneous differential equation system or free oscillations equation system of mechanism. This system is representable as linear algebraic equation system. Particular solutions of inhomogeneous differential equation system are of practical interest, as they permit to make the analysis of the effect of external loads, acting on the mechanism. Operator solution method application will give particular solutions of linear inhomogeneous differential equations. Received systems will allow solving many dynamic tasks of elastic mechanical systems: character change and maximum values of mechanism links dynamic loads, periods and frequencies of oscillations, conditions of the resonant state of the mechanical system.

1. Introduction
Ensuring profitable activity of the various industry enterprises increase in efficiency of equipment using is very important. Economic aspects of rational design of enterprises equipment are considered in research papers [1 - 7].

Planetary gears are widely used in various industries. In mining the most reliable easy to manufacture and install single stage transmission 2W-C is applied [1]. At the same time planetary gearboxes are infrequently used in the mining machine drives owing to mining machine heavy duty and very small durability of planetary wheel rolling bearings.
Many research papers [8-13] are devoted to study of static and dynamic parameters of planetary gears of various designs. A number of dynamic problems can be solve accurately enough without taking into account the elasticity of loaded elements [9].

In calculations of dynamic processes of mining machine transmissions must be considered gear flexibility as is much more than flexibility of large diameter and small length shafts.

In this article are considered possibilities and features of research of dynamic phenomena that take a place in planetary mechanisms of mining machines transmissions. Planetary mechanisms are presented as elastic mechanical systems. Dynamic tasks of elastic systems are: character of change and maximum values of mechanism links dynamic loads, periods and frequencies of oscillations, conditions of the resonant state of the system.

In this paper the dynamic model of elementary differential mechanism with taking into account the elasticity of links is proposed. For this purpose the differential gear is presented as equivalent planetary row with inertialess planetary carrier [9]. Main links of such mechanism have inertia moments determined by known formulas.

In figure 1 an elementary differential gear (elementary row) with three main links 1, 2, 3 and satellite group 4 is presented. 3’ – constructive planetary carrier; $C_{33}$ – rigidity of corresponding to index constraint.

Figure 1. Kinematic scheme of elementary differential mechanism

The elementary planetary row main links have equivalent inertia moments, determined by formulas [9]:

$$I_{1}^{2} = I_{1} + \chi_{1} k l_{4}; I_{2}^{2} = I_{2} + \chi_{2} k l_{4}; I_{3}^{2} = k m_{4} r_{3}^{2} + \chi_{3} k l_{4}. \tag{1}$$

where $I_{1}$, $I_{2}$, $I_{4}$ – inertia moments of central wheels and planetary wheels (satellites) relative to its own axis of rotation; $m_{4}$ – planetary wheels (satellites) mass; $k$ – planetary wheel (satellite) number in the elementary row; $r_{3}$ – the installation radius of the satellite axes in the carrier; $\chi_{1},\chi_{2},\chi_{3}$ – coefficients of canonical form
\[
\begin{align*}
\chi_1 & = i_{41}^2 [i_{32}^2]^{-1} [i_{43}^2 i_{32}^2 - i_{42}^2 i_{31}^2]; \\
\chi_2 & = i_{42}^1 [i_{31}^2]^{-1} [i_{42}^1 i_{31}^2 - i_{41}^1 i_{32}^1]; \\
\chi_3 & = i_{41}^2 [i_{42}^1] [i_{31}^2 i_{32}^1],
\end{align*}
\]

where \( i_{ij}^o \) – gear ratio from link \( i \) to link \( j \) relative link \( o \).

Equivalent dynamic model of elementary differential gear is presented as three-mass branched system (figure 2) \[9\]. In figure 2 \( M_1, M_2, M_3 \) – external moments, acting on the corresponding links; \( C_{jo} \) – reduced rigidity of the corresponding constraint.

\[\text{Figure 2. Dynamic model of elementary differential mechanism}\]

Dynamic processes in machine drives substantially depend on motor type that is from its speed-torque characteristic. Asynchronous slip-ring motors are often used in mining machine drives. The working section of their speed-torque characteristic can be represented by the straight line. The moment value on motor shaft can be determined according to angular velocity from the following formula \[9\]:

\[
M_1 = M_h \frac{\omega_o - \omega}{\omega_o - \omega_n},
\]

where \( M_h, \omega_n \) – nominal values of moment and angular velocity on motor shaft; \( \omega_o \) – angular velocity with no load.

It is also assumed, that the remain external moments are either constant values or are functions of time. Really, for correctly calculated brakes, braking torque varies slightly during braking, and it can be considered constant. And resistance moment depends on the nature of the operations provided for a particular machine.

Using the method of Kozhevenikov \[11\], will define the moments of elasticity (not mass rotation angles), arising in the system constraints. Movement equations for each of the masses will write in the following form:

\[
\begin{align*}
I_1 \varphi_1'' & = M_h \frac{\omega_o - \omega}{\omega_o - \omega_n} - M_{10}; \\
I_2 \varphi_2'' & = M_2(t) - M_{20}; \\
I_3 \varphi_3'' & = M_3(t) - M_{30},
\end{align*}
\]

where \( \varphi_j'' \) – angular acceleration of the corresponding mass; \( M_{jo} \) – moment, developed by elastic forces in corresponding constraint; \( j = 1, 2, 3. \)
When compiling the equation system (4) it was assumed, that all external moments, acting on system, have one sign and coincide with the sign of corresponding mass angular acceleration.

As external moments, acting on system, are functions of various variables, the equation system (4) represents the system of differential equations with heterogeneous variables. Will bring the equation system (4) to the form, when all external moments are single variable functions – a time. For this purpose the shaft motor angular velocity must be excluded from the first equation.

Differentiating in time the first and the second equations of the system (4), and dividing each of them by the corresponding inertia moment, will subtract the second equation from the first equation

\[
\phi_1'''' - \phi_2'''' = -\frac{M_n}{I_1(\omega_o-\omega_n)}\phi_1'' - \frac{M_{10}}{I_1} - \frac{M_2}{I_2} + \frac{M_{20}}{I_2}.
\]

Introducing into the resulting expression (5) two new addends \( \pm \frac{R}{I_1} \phi_2'' \), there \( R = \frac{M_n}{\omega_o-\omega_n} \), and replacing in one of them \( \phi_2'' \) value, found from second equation of system (4), receive:

\[
(\phi_1'''' - \phi_0''') + \frac{R}{I_1} (\phi_1'''' - \phi_2''') + \frac{M_{10}}{I_1} - \frac{M_{20}}{I_2} = \frac{M_1}{I_2} - \frac{R}{I_1 I_2} M_2 + \frac{R}{I_1 I_2} M_{20}.
\]

Introducing pairwise two new addends \( \pm \phi_0''' \) and \( \pm \frac{R}{I_1} \phi_0'' \) into the resulting expression (6), where \( \phi_0''' \) – angular acceleration of the section “O”, will receive

\[
(\phi_1'''' - \phi_0''') - (\phi_2'''' - \phi_0''') + \frac{R}{I_1} (\phi_1'''' - \phi_0''') - \frac{R}{I_1} (\phi_2'''' - \phi_0'') + \frac{M_{10}}{I_1} - \frac{M_{20}}{I_2} - \frac{R}{I_1 I_2} M_2 = - \frac{M_2}{I_2} - \frac{R}{I_1 I_2} M_{20}.
\]

And finally, taking into account \( M_{j0} = C_{j0}(\phi_j - \phi_0) \), we obtain

\[
\frac{M_{10}'''}{C_{10}} - \frac{M_{20}'''}{C_{20}} + \frac{RM_{10}'''}{I_1 C_{10}} - \frac{RM_{20}'''}{I_2 C_{20}} + \frac{M_{10}}{I_1} - \frac{M_{20}}{I_2} - \frac{R}{I_1 I_2} = - \frac{M_1}{I_2} - \frac{R}{I_1 I_2} M_2.
\]

Noticing that external moments \( M_2 \) and \( M_3 \) are functions of time or constant values, can be similar above to compile the difference of angular accelerations of corresponding masses and branch section. Then, after multiplying by corresponding rigidity of the constraint receive the equation, which will include constraint elastic forces moments and their derivatives.

As a result, we obtain the following system of differential equations

\[
\begin{align*}
\frac{M_{10}'''}{C_{10}} - \frac{M_{20}'''}{C_{20}} + \frac{RM_{10}'''}{I_1 C_{10}} - \frac{RM_{20}'''}{I_2 C_{20}} + \frac{M_{10}}{I_1} - \frac{M_{20}}{I_2} - \frac{R}{I_1 I_2} M_2 &= - \frac{M_1}{I_2} - \frac{R}{I_1 I_2} M_2; \\
\frac{M_{20}'''}{C_{20}} - \frac{M_{30}'''}{C_{30}} + \frac{M_{20}}{I_2} + \frac{M_{30}}{I_3} &= \frac{M_2}{I_2} - \frac{M_3}{I_3},
\end{align*}
\]

where \( C_{j0} \) – reduced rigidity of the corresponding constraint.

To define unknown \( M_{j0} \) it is necessary to compile another equation, which can be written as zero of constraint elastic forces moments in branch section “O”, those

\[
\sum_1^i M_{j0} = 0.
\]
So, the system of independent linear homogeneous differential equations with three required moments $M_{f0}$ of constraint elastic forces will take the following form

\[
\begin{align*}
&M_{10}'' + \frac{R}{l_1}M_{10}' + \frac{M_{10}}{l_1} = -M_{20}'' + \frac{R}{l_1}M_{20}' + \frac{M_{20}}{l_1} + \frac{R}{l_1}M_{30}' = \frac{M_{10}}{l_2} - \frac{R}{l_1}M_{20}; \\
&M_{20}'' + \frac{M_{20}}{l_2} = -M_{30}'' + \frac{M_{30}}{l_3} = \frac{M_{10}}{l_1} - \frac{M_{10}}{l_1}M_{20}' = 0; \\
&M_{10} + M_{20} + M_{30} = 0.
\end{align*}
\]  

(11)

Equating the right parts of the equations of system (11) to zero, will receive homogeneous differential equation system or mechanism free oscillations equation system.

\[
\begin{align*}
&M_{10}'' + \frac{R}{l_1}M_{10}' + \frac{M_{10}}{l_1} = -M_{20}'' + \frac{R}{l_1}M_{20}' + \frac{M_{20}}{l_1} = 0; \\
&M_{20}'' + \frac{M_{20}}{l_2} = -M_{30}'' + \frac{M_{30}}{l_3} = 0; \\
&M_{10} + M_{20} + M_{30} = 0.
\end{align*}
\]  

(12)

Differential equation system (12) is representable as algebraic equation system. In dynamic research of machine drives particular solutions of inhomogeneous differential equation system are of practical interest, as they permit to make the analysis of the effect of external loads, acting on the mechanism. Application of operator solution method will give particular solutions of linear inhomogeneous differential equations.

**Summary**

For known equivalent dynamic model of elementary differential mechanism with elastic links independent linear inhomogeneous differential equation system for elastic force moments, arising in the mechanism constraint, is received.

**References**

[1] Slobodyanik T M 2017 *Scientific and Practical Conference*, (Ekaterineburg) pp142-145

[2] Zarubina E M, Nikitina O A and Slobodyanik T M 2019 *Advances in Economics, Business and Management Research* **79** (“Far East Con” ISCFEC) pp 132-134

[3] Nikitina ÖA, Slobodyanik TM and Melikhova YuM 2016 *Guide to the Entrepreneur* № 31 pp 114-120

[4] Nikitina O A, Litovskaya Yu V, Savinkova T A, Zinoveva E G and Ponomareva OS *Espacios* **V38** № 33 pp 17-26

[5] Nikitina O A, Litovskaya Yu V and Ponomareva O S 2018 *Academy of Strategic Management Journal* **V17** Issue 5 pp 1-7

[6] Gorbatyuk S M, Gerasimova A A and Belkina N N 2016 *Materials Science Forum*, **870**, pp 564-567

[7] Gerasimova A A, Radyuk A G and Titlyanov A E 2015 *Steel in Translation* **45(3)** pp 185-187

[8] Komarov M C 1969 *Dynamic of Mechanisms and Machines* (Moscow: Mashinostroenie)

[9] Weitz V L, Kochura A E and Martynenko A M 1971 *Dynamic Calculations of Machines Drives* (Leningrad: Mashinostroenie)

[10] Artobolevsky I I 1967 *Theory of Mechanisms* (Leningrad: Main publ, physics-techn. liter)

[11] Kozhevnikov S N 1966 *Dynamic of Machines with Elastic Links* (Kiev: AN of SU)

[12] Kudryavtsev V N 1966 *Planetary Gears* (Moscow: Mashinostroenie)

[13] Timofeev G A, Frolov K V, Popov S A and Musatov A K 2012 *Theory of Mechanisms and Mechanics of Machines* (Moscow: MGTU by N E Bauman)