The exploration of higher order thinking skills: students' difficulties and scaffolding in solving mathematical problems based on PISA

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Abstract. Understanding as well as planning to solve mathematical problems require a high-order thinking since it resembles the ability to think after the level of basic and critical thinking. This research aims to produce a theoretical review of students’ ways of thinking and difficulties in solving PISA-based and scaffolding-based mathematical problems properly. This research employed a qualitative method with inductive data analysis. The subjects of this research were two Junior High School students with high-level mathematical ability. The data validity was conducted through triangulation method by comparing data from the result of the test, interview, and documentation method. The analysis technique was carried out within three workflows; data reduction, data presentation, data verification and conclusion. The research result shows that the students experienced difficulties in solving PISA problems on the stage of devising a plan, and carrying out the plan. As for the scaffolding, the measures taken by the researcher to solve the difficulties were; asking students to read and understand the questions, asking directional questions, giving opportunities for the students to analyse to double prepare the answer plan properly, interpreting the identified inappropriateness in order to make a solving plan.

1. Introduction
Mathematics learning taught at schools include algebra, geometry, trigonometry and arithmetic. In studying the material math students often find it difficult. Teachers who dominate the conversation and interaction in the classroom, the material explanation only refers to the completeness curriculum that makes students experience difficulties in solving mathematical problems [1].

Learning disabilities can be defined as a condition in the learning process characterized by the presence of certain obstacles to achieve learning outcomes. Such obstacles can be psychological, sociological, and physiological in the overall learning process. In the process of learning, there are a lot of barriers experienced by students which can be influenced by internal factors on students as well as external factors in which those influence the level of difficulty experienced by students in solving mathematical problems. The learning difficulties usually occurs due to improper use of language so that an error occurred, especially in the cognitive understanding.

Research relevant to the students’ difficulties have been carried out by experts. Student difficulties in translating mathematical symbol can be judged from two dimensions, namely: student-centered and representation [2]. Students continuously demonstrate the difficulty in translating between numeric,
graphic and algebraic representations related to mathematics. There are mistakes in the student's reasoning applied to all cases in resolving the problem [3]. There are several factors that cause difficulties as an impact on the difficulty of solving mathematical problems, such as the students' cognitive factors, environmental factors and psychological factors [4].

Students are considered having difficulty as they are unable to solve mathematical problems, often have errors in the settlement process, and do not achieve the desired goal. The students who seemed to have difficulty are seen from the failure in achieving the learning objectives [5]. The difficulty in solving mathematical problems flagged an error in resolving mathematical problems [6]. To overcome these difficulties, the parts of the difficulties experienced by students should be identified [7]. By knowing the difficulties experienced by the students, it will soon be identified the appropriate help to overcome their difficulties.

The problems among the students in solving mathematical problems started from how to read and understand mathematics. In other words, students still do not understand the problem so that there is difficulty in solving mathematical problems. Levels of difficulty in mathematical translations are: mathematical errors, instructional experiences, interpretive and translation activities, and the use of transitional representations [2]. The existence of different levels of difficulty in each student allow for the different assistance to overcome these difficulties. Thus, a student needs their understanding of mathematical concepts and multiply math exercises so that students are able to apply math in the form of mathematical sentences which are easy to understand and be able to find solutions to these problems. The findings from the interviews with Grade 2-5 teachers were tabulated and compared. The interview guide to describe any difficulties that students have when working review their mathematical word problems. Most teachers reported more than one reason for reviewing their students’ difficulties. Almost half of teachers' responses (45%) indicated resources that solving math word problems is difficult, for students because students struggle with reading and understanding the problems [8].

The problem in this research is based on a math problem about PISA (Programme for International Student Assessment). PISA problem-solving requires a high reasoning power, understanding and creative planning. In understanding and making plans requires an adequate student thinking ability because the ability of high-level thinking skills comes after basic and critical thinking. Detailed analysis of students' thinking in solving PISA is needed to minimize the error so that the students’ reasoning becomes more critical. Moreover, it has an impact on mathematics achievement that could be improved.

It is needed a help for the students’ learning difficulty, especially through the proper scaffolding. The scaffolding is a kind of the proper help to do during learning interaction to solve mathematical problems or other tasks. There are three categories of student achievements in the effort to solve the problem, namely; students excellently achieve success, students achieve success with help, students fail to achieve success. Scaffolding can be interpreted as an attempt learners to guide students in achieving success. Scaffolding provided by teacher is essential to the achievement of the students to a higher level.

Scaffolding is not only done spontaneously. A teacher must know the character and the problems faced by students. So the scaffolding can address problems experienced by students. Scaffolding can foster students' creative and divergent thinking skills, and enhancing their independence, sense-making and self-confidence in mathematics [9]. Scaffolding can enhance student creativity and students’ thinking skills. In other words, learning to use scaffolding greatly assists students in developing creativity in solving mathematical problems. Metacognitive scaffolding cooperative learning can improve students' skills in understanding mathematical concepts as well as on mathematical procedures [10].

There are three levels of provision of scaffolding, namely; Environmental provisions, namely structuring the learning environment that enables progress without direct intervention from the teacher, explain; reviewing and restructuring, the scaffolding is done through the explanation; review
and restructuring, developing conceptual thinking, namely scaffolding to build conceptual thinking [11].

2. Method
This study used qualitative method and inductive data analysis. Subjects in this study were two ninth grade junior high students with high mathematics skills. Both subjects were divided into male and female. The test instrument were taken from a matter of PISA released mathematics items. Validity of the data was done by triangulation method, by comparing the data from the results of the test method, interviews, and documentation. The analysis technique was performed with three grooves activities, namely; data reduction, this stage corrected test result answers of students to find indicators of creative thinking and recorded the interview, the presentation of the data, the data collection process of the research that has been structured and organized, verification of the data and drawing conclusions; a process of meaning formulation derived from research result by doing a closer look to the truth in conclusion.

3. Results and Discussion
This study describes the difficulties of students in solving mathematical problems and efforts to resolve them using scaffolding. Mathematical problem solving refers to the concept of solving problems initiated [12]. Scaffolding is an effort to overcome the difficulties experienced by students referring to the shape and characteristics of the scaffolding. The level of scaffolding are; level of Environmental provisions (classroom organization; artefacts such as blocks, Explaining, reviewing and restructuring, developing conceptual thinking using scaffolding [11]. Scaffolding is intended to help students solve mathematical difficulties that consists of questions taken from the PISA questions. Scaffolding is done by identifying the trouble to find out which part the difficulties of students in solving PISA and choose the scaffolding in accordance with the level of difficulties.

There are five keys of scaffolding to work well, namely; Students explain and justify their solutions, teachers continuously assess students 'understanding, teachers take into consideration students' perspectives, scaffolding tailors to the needs of students and students take up or use the scaffolding. Finally, teachers need to reconceptualise and reviews their role as facilitators in the development of the students' mathematical constructions rather than the sole source of mathematical knowledge while employing scaffolding in the classrooms [13].

Before doing scaffolding teacher must know what is not understood by students, teachers give students the chance to uncover the problem. Teachers consider appropriate forms of scaffolding. Teachers provide opportunities for students to construct mathematical knowledge of students at the time of scaffolding. The application of scaffolding which works well helps students overcome the difficulties experienced, but the scaffolding that fails will make students dependent on assistance provided by the teacher, not creative, and difficult to think critically. A good scaffolding is done from starting point so that children are able to reason to get answers. When the answers are generated by the students, the teacher interprets the students’ mismatched answers and confirms them to the students [14].

There are some errors that the subjects made when doing PISA questions. The mistakes can be seen as difficulties experienced by the subjects. The possible factors causing errors in the work were mainly because the subject had difficulties in understanding the problems, making a plan and carrying out the settlement plan. Through the difficulties, the researcher conducted interview to determine the characteristics of the difficulties experienced and implemented scaffolding as an attempt to overcome the difficulties experienced by the subjects.

The context of understanding the problem in this issue is the student's ability to identify and formulate what is known and what is being asked. The inability of students to formulate what is known and what is being asked occurs because of the students’ carelessness in reading the questions. However, in terms of the difficulties experienced by subjects in understanding the problem without
scaffolding, the subjects have already read the issue repeatedly, so that they began to understand what is meant in questions.

Viewed from the four-step problem-solving process by Polya, the inability of subjects is classified in step completion stage of understanding the problem. The inability is included in “the levels of difficulty in mathematical translation of instructional experience” [2]. The subjects’ inability is to formulate what is known and what is being asked, then the subjects were included in the dimension of extracting all information from the question [15]. An understanding of the questions is an important component of solving mathematical problems, due to the inability of students to formulate what is known and what is being asked in the problem. As results, the students were unable to continue the settlement of the problems correctly.

Regarding the observation and interview of researchers with the subject, the researchers concluded that scaffolding is the way to overcome the difficulty in defining what is known and what is being asked. The researcher then gave opportunity to the subjects to reason according to his understanding, and being asked to formulate what is known and what is being asked. If students still have difficulties in interpreting the discrepancy, the tutor elaborated the mismatched answers and confirmed to the students so that he/she can understand the intent of a problem. Thus, the scaffolding conducted by researchers was included the second level scaffolding, i.e. "reviewing, explaining and restructuring" [11]. At this level scaffolding made the students reread the given problem. The figure show that the subjects have error in calculation.

Figure 1. The results of the work in solving PISA subjects, the figure show that the result the work of subject have an error in calculation

Figure 2. The results of the work in solving PISA subjects after give scaffolding

There are several difficulties experienced by students when working on PISA questions. Students do not understand what is meant by the question. This is proved by the inability of students to make
plans to solve problems. For example students do not understand the comparison between apartments A, B and C in the question. To overcome these difficulties the assistance provided is in the form of asking students to read the questions again until students are able to understand the questions well.

In addition, the difficulties experienced by students when completing the PISA problem are occurring at the stage of carrying out a plan. Students have not been able to do calculations well. This is indicated by the results of student calculations that to determine the area of apartment B students work by dividing the area of apartment B by the number of apartments A and C then multiplying by 300,000. To overcome these difficulties by asking students to wait for recalculation by paying attention to all the information on the question. Thus to find the area of apartment B by dividing the overall price of the apartment by the total area of the entire apartment which is 300000: (95 + 85 + 70) so that the result of 1200 is obtained.

Students made mistake in understanding the problem and carrying out a plan so the effect is in solving the problem. Students who were not able to make plan for settlement in accordance with the information known and asked in the question refers to the incompetence at the stage of planning (devising a plan) in solving the problem [12]. Students should be able to devise steps to resolve in accordance with the known information and asked to solve the problems as the first step. Thus, the subjects included on the dimensions of problem solving the use of transitional representations [15].

Scaffolding for subjects who were unable to make the settlement plan in accordance with the prior knowledge and the information known to ask the question comprises as follows: asking students to reflect on the answers that have been made so as to find the mistakes made, asking students to collect all the information on the matter, asking the students to try to associate the information known to the first step as the settlement plan, asking students to refine his work adapted to the information known to the questions, and the tutor interprets the discrepancy answer and confirms that the child can understand the intent of a problem. The scaffolding is classified at the level of developing conceptual thinking, explaining and restructuring [11].

The ability to use mathematical knowledge and to connect previously is essential for solving mathematical problems. In this study, subjects did not apply mathematical knowledge correctly. While on the given issue did not use a mathematical concept that they have on the calculation process.

When the problem occurred in the inability of the subject in applying mathematical knowledge possessed on the process of resolving problems and errors in the calculation process, the subjects were classified to the students who were not able to carry out plan [12]. If levels of difficulty is in mathematical translations, the subject will be classified at the level of mathematical error. The subject is categorized in mathematics dimension concepts, mathematization and reasoning in the process of resolving the problem [15].

Scaffolding conducted by researchers to assist the subject in the application of knowledge possessed in the calculation process in solving PISA was asking the subjects to count and giving the opportunity to reason how to find the answers. If students still have difficulty, the tutor will interpret the student’s mismatch answers and confirms that he/she can understand the intent of a problem. The other measures are asking the subject to rework the job, asking students to divide the image into three parts, and asking students to look for the shortest route in each section. The scaffolding is categorized at the level of “developing conceptual thinking, explaining and restructuring [11]. Checking back on the calculation is the last step to check the truth of the answers obtained. Mathematics problem is the one that requires a confirmed answer with what is known and what is asked in the problem.

Referring to the troubleshooting steps of Polya, the subject did not recheck the answers. All subjects were in the problem-solving process including the first and second to the fourth dimension computational skill standards and carefulness in carrying out computations [15]. The levels of difficulty in mathematical translations was included in the use of transitional level representations.

Scaffolding committed to all of the subjects is confirming the truth of the results obtained as proposed, connecting, and developing conceptual thinking [11]. This fourth scaffolding level was done by asking students to compare the answers that have been made by the students and to look for alternative answers other.
Flow analysis of the difficulties and scaffolding given to students in solving PISA can be seen in the following figure 3.

![Figure 3. Image Flow Problem Resolution Process PISA and Scaffolding](image.png)

### Table 1. The Description of The Figure 3

| No. | Term                                      | Code |
|-----|-------------------------------------------|------|
| 1   | Problem (about PISA)                      | P    |
| 2   | Understanding the problem                 | up   |
| 3   | Large of apartment A                      | I1   |
| 4   | Large of apartment B                      | I2   |
| 5   | Large of apartment C                      | I3   |
| 6   | Large of apartment A, B, C                | I4   |
| 7   | Understand what is being asked            | uq   |
| 8   | What is the payment of apartment B?       | Iq   |
| 9   | Make a plan (divising a plan)             | Dp   |
| 10  | The first difficulty                      | D1   |
| 11  | Ask students to read and understand the problem, give directives question | Scf1 |
| 12  | Carry out planning (carrying out the plan)| Co   |
| 13  | Looking back                              | lb   |
| 14  | the second and third difficulty           | D2&3 |
| 15  | Give directives question, provide an opportunity for students to reason to reconstitute the draft the right answer, | Scf2 &3 |
| 16  | The problem is solved                     | F    |

### 4. Conclusion

The results of the study show that students' difficulties at the stage of devising a plan were; unable to plan the completion step, unable to write mathematical formulas that match what is known. To give more details, in the stage of carrying out the plan stage covered; not using mathematical concepts well in the calculation process. The activities of scaffolding at the stage of making plan were; asking students to mention the formula that fits with what was known and what was asked, asking a landing question so that students were able to mention mathematical concepts that were in accordance with...
what was known and asked, giving students the opportunity to reason the right answer, interpreting the discrepancies that were known and confirmed so that students were able to make a settlement plan. The activities in the stage of carrying out the plan stage were; asking students to reflect on the answers they have made so they can find errors that were experienced, asking students to do the calculation process properly according to what was known and asked, interpreting the students’ incompatibility and confirming so that they can solve the math problems.

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