Stickiness of randomly rough surfaces with high fractal dimension: is there a fractal limit?

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Abstract

Two surfaces are "sticky" if breaking their mutual contact requires a finite tensile force. At low fractal dimensions $D$, there is consensus stickiness does not depend on the upper truncation frequency of roughness spectrum (or "magnification"). As debate is still open for the case at high $D$, we exploit BAM theory of Ciavarella and Persson-Tosatti theory, to derive criteria for all fractal dimensions. For high $D$, we show that stickiness is more influenced by short wavelength roughness with respect to the low $D$ case. BAM converges at high magnifications to a simple criterion which depends only on $D$, in agreement with theories that includes Lennard-Jones traction-gap law, while Persson-Tosatti disagrees because of its simplifying approximations.

Keywords: stickiness criterion, adhesion, Dalquist criterion, surface roughness
I. INTRODUCTION

The interplay of roughness and adhesion is certainly a problem of great interest. Various industries are interested in robust adhesion, like in tapes made of soft polymers [1–5], sometimes imitating Nature, where very elaborate strategies have evolved [6–10]. In general, roughness explains the "adhesion paradox" [11], namely that all surfaces should be strongly adhered to each other because of the strength of van der Vaals forces. With "stickiness", we denote the possibility of sustaining macroscopic tensile pressures or else non-zero contact area without load [12].

For smooth bulk solids without hierarchical structures, stickiness is generally reached only for small elastic Young modulus (smaller than about 1 MPa according to the 3M criterion of Dahlquist [4, 5]). However, real solids are characterized by surface roughness that could extend on several length scales: in principle, for example, we could span kilometers to nanometers and obtain 12 orders of magnitude — which is way beyond the computational capabilities of present numerical codes. Seven decades of almost pure power law roughness spectrum have been observed in ref. [13], limiting the measurement to nanometer amplitude of roughness. With the development of faster computers, simulations of contact mechanics with roughness have become possible [14–16]. However, due to mentioned high computational costs, only a very limited range of roughness wavelengths can be considered even in state-of-the-art technology as in the "contact challenge" recently setup by Martin Müser [17], spanning only about 3 orders of magnitude of roughness. In such sense, the study of adhesion between surfaces with broad roughness spectra is still a challenge. Fortunately, we have theoretical models that can predict the behaviour for much broader spectra, and here we attempt for the first time in the best of the authors’ knowledge, the discussion of the limit for infinitely broad spectra, in the case of arbitrary fractal dimension. The case of low fractal dimension is probably more common ([18]), but it is important to have a general understanding of the contact problem, also for reference to validate future computational and experimental efforts.

The very early models of rough contact mechanics used the concept of "asperities". Fuller and Tabor (FT) [19] involved a combination of factors depending on the long wavelength content of surface roughness, namely the root mean square (rms) roughness amplitude $h_{\text{rms}}$, and factors depending on the small wavelength component, namely the rms curvature $h''_{\text{rms}}$. 

2
More recently, Pastewka & Robbins (PR) \cite{PR20} performed Boundary Element Method (BEM) simulations of adhesive rough contact between fractal surfaces having roughness spectra of about 3 orders of magnitude in wavelengths (from nano to micrometer scale). From an extrapolation of their numerical results, they extrapolated tentatively that stickiness should depend \textit{only} on small scale features of surface roughness, i.e. rms slope $h'_{\text{rms}}$ and rms curvature $h''_{\text{rms}}$. The quantities $h'_{\text{rms}}$ and $h''_{\text{rms}}$ are strongly related to the smallest wavelength $\lambda_1 = 2\pi/q_1$ of roughness spectrum, which can be expressed in terms of its Power Spectral Density (PSD). With $q_1$ we have denoted the cut-off wavevector value. The ratio $\zeta = q_1/q_L = \lambda_L/\lambda_1$ is the so-called \textit{magnification}, being $q_L = 2\pi/\lambda_L$ the wavevector of roughness PSD that corresponds to the biggest wavelength $\lambda_L$.

Recently, Ciavarella \cite{C21} has compared three different adhesion theories of rough contact mechanics, namely Persson & Tosatti (PT) \cite{PT22}, Persson & Scaraggi (PS) \cite{PS23} as re-elaborated by Violano et al. \cite{V12}, and BAM \cite{BAM24}. In Refs. \cite{V12,C21} it is shown that for self-affine fractal surfaces with power-law PSD and low fractal dimensions ($D \lesssim 2.4$), stickiness is magnification-independent, depending mainly on $h_{\text{rms}}$ and $\lambda_L$, and therefore noticed that the PR observation was affected by limitations in the computer capabilities and the use of narrow spectra. This result is in agreement with the theoretical findings of ref. \cite{M25}, where a full Lennard-Jones traction-gap law is exploited to study the adhesive contact between randomly rough surfaces with broad roughness spectrum. In particular, it is shown that the probability distribution of gaps converges with $\zeta$. The nominal mean traction at the interface is computed by convolution of the probability distribution of gaps with the Lennard-Jones traction-gap law, showing that the effect of smaller roughness wavelength on interfacial tractions becomes negligible at high $\zeta$.

Moreover, Violano et al. \cite{V26} studied the adhesive rough contact between self-affine fractal surfaces with an advanced multiasperity model, where adhesion is implemented according to Derjaguin, Muller & Toporov (DMT) force-approach \cite{DMT27}. In their simulations, the pull-off force is almost independent on the value of $h'_{\text{rms}}$, again in agreement with most of the criteria introduced in refs. \cite{V12,C21}. Specifically, adhesion is rapidly destroyed by a little increase of $h_{\text{rms}}$.

The very recent experimental investigations of Tiwari et al. \cite{Ti28} are also consistent with the former results. In ref. \cite{Ti28}, normal contact adhesion experiments are carried out on rough samples with different $h_{\text{rms}}$ and similar rms slope $h'_{\text{rms}}$. Experiments confirmed that increase
in $h'_{\text{rms}}$ without any change in $h''_{\text{rms}}$ or $h''_{\text{rms}}$ "kills" adhesion, leading to vanishing pull-off force, confirming with definitive evidence that criteria like PT and PS as re-elaborated by Violano et al. [12], and BAM [21] seem correct (at low fractal dimensions). The former is a remarkable result, if one takes into account the intrinsic difficulty in defining a value of $h'_{\text{rms}}$, or $h''_{\text{rms}}$ which is related to the sensitivity of roughness measurement instrumentation [13, 29, 30].

The debate is still open for the case at high fractal dimensions $D$. In this work, we extend BAM and PT stickiness criteria to all fractal dimensions. We shall use sometimes instead of the fractal dimension the Hurst exponent $H = 3 - D$. We show that at low $H$, PT and BAM criteria may show a magnification-dependence. Moreover, the two criteria predict that stickiness could increase with $H$. This trend is in agreement with recent numerical [31, 32] and theoretical findings [33], according to which the pull-off force increases with $H$. The dependence with the Hurst exponent is much more evident for PT solution when high magnifications are considered.

But the "fractal limit" behaviour, i.e. the trend for $\zeta \to \infty$ is qualitatively different for PT and BAM, as we shall explore.

II. METHODS

In this paper, we use BAM theory [24] and the Persson and Tosatti (PT) theory [22], to derive stickiness criteria for arbitrarily wide power law spectra, with low Hurst exponent $H$ (i.e. high fractal dimension).

A. BAM stickiness criterion

BAM [24] is developed loosely speaking in the framework of the DMT theory [27], in the sense that the repulsive adhesiveless solution and the attractive adhesive one can be found separately. In particular, adhesion acts in an attractive area outside of the compressive contact area and does not deform the contact shape. BAM assumes a Maugis law of attraction [34], which permits an elegant and trivial estimate of the total force of attraction in closed form [24, 35]. In the simple case of spherical contact, the model returns exactly the DMT solution by estimating the area of attraction as the increase of the bearing area geometrical
intersection, when the indentation is increased by the Maugis range of attraction. In the case of frictionless contact between randomly rough surfaces, BAM exploits the repulsive adhesiveless Yang & Persson formulation \[36\] in its asymptotic version. The relation between the squeezing repulsive pressure \(p_{\text{rep}}\) and the mean separation \(u\) is

\[
p_{\text{rep}}(u) = \beta E^* \exp\left(-u/u_0\right)
\]

where \(u_0\) and \(\beta\) are given by Yang & Persson formulation \[35, 36\]. Fig. 1\text{a} shows that the normalized repulsive pressure \(\hat{p} = p_{\text{rep}}/(E^*q_Lh_{\text{rms}})\) rapidly converges with \(\zeta\) for \(H = 0.8\). Notice that, with \(E^* = E/(1 - \nu^2)\) we have denoted the plane strain elastic modulus, being \(\nu\) the Poisson’s ratio. The pressure \(\hat{p}(\zeta)\) still converges when very high magnifications are reached for \(H = 0.3\) (fig. 1\text{b}). We are interested in a range of normalized mean separation \(1 < \bar{u}/h_{\text{rms}} < 3\). For greater values one could have finite effects due to poor statistics of the surface. In the following, we will show BAM predictions for a normalized mean separation \(\bar{u}/h_{\text{rms}} = 2\).

Following ref. \[24, 35\], summing up repulsive and attractive contributions for the random gaussian surface roughness, BAM gives to a very elementary closed form solution

\[
\frac{\sigma(u)}{\sigma_0} \simeq \beta E^* \exp\left(-u/u_0\right) - \frac{1}{2} \left[ \text{Erfc}\left(\frac{u - \epsilon}{\sqrt{2}h_{\text{rms}}}\right) - \text{Erfc}\left(\frac{u}{\sqrt{2}h_{\text{rms}}}\right) \right]
\]

where \(\text{Erfc}\) is the complementary error function, \(\sigma(u)/\sigma_0\) is the ratio between the actual stress acting on the surface with respect to the theoretical stress of the material \(\sigma_0 = \Delta\gamma/\epsilon\), being \(\Delta\gamma\) the surface energy and \(\epsilon\) the range of attractive forces. As pointed out in Ref. \[21\], it is not possible to obtain a closed form solution for BAM stickiness criterion. However, one can predict the decay in the pull-off pressure \(\sigma_{\text{po}}\) with \(h_{\text{rms}}\) from eq. (2). Stickiness can be defined when abrupt decay of pull-off pressure by 5 orders of magnitude (for example) with respect to the theoretical stress is found \[21\].

Fig. 2\text{a} shows the prediction of BAM model of the stickiness threshold \((h_{\text{rms}}/\epsilon)_{\text{thresh}}\) as a function of the quantity \(l_a\lambda_L/\epsilon^2\), being \(l_a = \Delta\gamma/E^*\) a characteristic length scale for adhesion. Notice in particular that \(l_a/\epsilon \simeq 0.05\) for the classical Lennard-Jones force-separation law. This results in a BAM criterion, which generalize previous results of Ciavarella [21].
FIG. 1: The normalized repulsive pressure $\hat{p}$ as a function of $\zeta$, for mean interfacial separation $\bar{u}/h_{\text{rms}} = [1, 2, 3]$ and for $H = 0.8$ (a) and $H = 0.3$ (b). The quantity $\bar{u}/h_{\text{rms}}$ increases in the direction indicated by the arrow.

$$\frac{h_{\text{rms}}}{\epsilon} > \beta_{\text{BAM}}(H, \zeta) \sqrt{\frac{l_a}{\epsilon} \frac{\lambda_L}{\epsilon}}.$$

where $\beta_{\text{BAM}}(H, \zeta)$ is a function of both $H$ and $\zeta$. However, let us explore first the limit $\zeta \to \infty$. This "fractal limit" is well defined for BAM, since

$$\lim_{\zeta \to \infty} \beta_{\text{BAM}}(H, \zeta) = \beta_{\text{BAM}}^\infty (H)$$

exists and gives an effective stickiness boundary very close to that of the low fractal dimensions, confirming the finding of the theories based on more refined use of Lennard-Jones
FIG. 2: a) The $(\langle h_{\text{rms}}/\epsilon \rangle)_{\text{thresh}}$ vs. $\lambda_L l_a/\epsilon^2$ relation for BAM theory (red solid line). Black dashed lines represents a power-law fit. Results are shown for $H = [0.3, 0.4, 0.5, 0.6, 0.7, 0.8]$ and $\zeta = 10^7$. The Hurst exponent increases in the direction indicated by the arrow. b) The $\beta_{\text{BAM}}(H)$ parameter that should be used in eq. (5) for obtaining the power-law fit. The black dashed line denotes the linear fit $\beta_{\text{BAM}}(H) = 0.9H - 0.1$.

Results for $\beta_{\text{BAM}}^\infty(H)$ are shown for $H = [0.3, 0.4, 0.5, 0.6, 0.7, 0.8]$. and $\zeta = 10^7$ in fig. 2b. We will show in the section dedicated to results, that BAM solution has already converged for this value of the magnification, independently on the value of $H$. We can use a power-law fit (black dashed lines) for approximating the red curves in fig. 2b. This leads to the following new and universal general BAM stickiness criterion (but consider $H \in [0.3, 0.8]$ as outside this range results may show obviously some deviations)

\[
\frac{h_{\text{rms}}}{\epsilon} > (0.9H - 0.1) \sqrt{\frac{l_a \lambda_L}{\epsilon}} \tag{5}
\]

where a linear fit for $\beta_{\text{BAM}}^\infty(H) = \beta_{\text{BAM}}(H, \zeta > 10^7)$ has been used. The effect of fractal dimension $D$ i.e. of $H$ is therefore quite modest, contrary to the effect predicted by the PT theory, as we explore in the next paragraph.
B. Persson and Tosatti stickiness criterion

PT theory \cite{22} studies the adhesive contact between a randomly rough rigid surface and an elastic half-space. PT argue with a energy balance between the state of full contact and that of complete loss of contact that the effective energy available at pull-off is

\[ \Delta \gamma_{\text{eff}} = \frac{A}{A_0} \Delta \gamma - \frac{U_{\text{el}}}{A_0} \]  

where \( A \) is not the real contact area, but rather an area in full contact, increased with respect to the nominal one \( A_0 \), because of an effect of roughness-induced increase of contact area, so that \( \frac{A}{A_0} > 1 \). We shall neglect this effect in the following, as there is no consensus on the importance of the area-term \( \frac{A}{A_0} \), and in the interest of simplicity. \( \Delta \gamma \) is the surface energy corresponding to the smooth case. Finally, \( U_{\text{el}} \) is the elastic strain energy stored in the half-space having roughness with isotropic power spectrum \( C(q) \) when this is squeezed flat

\[ \frac{U_{\text{el}}(\zeta)}{A_0} = \frac{\pi E^*}{2} \int_{q_0}^{q_1} q^2 C(q) \, dq = E^* l(\zeta). \]  

In eq. (7), we have integrated over wavevectors in the range \( q_0, q_1 \). In the following we will assume \( q_0 = q_1 \). Notice that the intrinsic assumption made in computing the elastic energy is that the wavelengths in the integration range correspond to waves of roughness being squeezed completely flat, when in contact against the rigid countersurface. This is an approximation close to the JKR theory \cite{37}, which becomes questionable for very small scales, where we expect the actual range of attractive forces being important. Indeed, the JKR model retains the Signorini dichotomy between regions of contact and separation (see the review paper \cite{14}), but relaxes the requirement that contact tractions be non-tensile. In the single sinusoid case, JKR model may cause deviations from a more precise solution that includes the Lennard-Jones force-separation law, when the amplitude of the sine wave becomes of the order of the attractive range \cite{33}. Indeed, it is intuitive to expect that very small roughness will induce almost no change in the pressure distribution, rather than a strong effect as considered in eq. (7) when Hurst exponent is \( H < 0.5 \). We have introduced in (7) a length scale \( l(\zeta) \). In the fractal limit \( \zeta \to \infty \), the elastic energy \( U_{\text{el}}(\zeta) \) is unbounded for surfaces with fractal dimension \( D \geq 2.5 \) \( (H \leq 0.5) \), (see ref. \cite{24}). As a consequence, PT theory would lead to vanishing adhesion even for arbitrarily small rms height \( h_{\text{rms}} \). This
is, again, due to the JKR-like approximation.

For pure power law PSD \( C(q) = Zq^{-2H-2} \)

\[
l(\zeta) = \frac{\pi}{2} \int_{q_1}^{q_L} q^2 C(q) \, dq = \frac{\pi Z}{2} \int_{q_1}^{q_L} q^{-2H} \, dq = \frac{\pi}{2} \frac{h_0^2}{\lambda_L} f(H, \zeta)
\]

where

\[
f(H, \zeta) = H^{\zeta - 2H + 1} - 1
\]

is a function introduced in ref. [22] and \( h_0^2 = 2h_{\text{rms}}^2 \).

As stressed by PT, as long as \( \Delta \gamma_{\text{eff}} < 0 \), "a finite pull-off force will be necessary in order to separate the bodies". In such case, the stickiness limit is \( \Delta \gamma_{\text{eff}} = \Delta \gamma - \frac{U_{\lambda}}{\lambda_0} = 0 \), which leads to the following stickiness criterion

\[
\frac{h_{\text{rms}}}{\epsilon} > \beta_{\text{PT}}(H, \zeta) \sqrt{\frac{l_a \lambda_L}{\epsilon \epsilon}}
\]

with \( \beta_{\text{PT}} = \sqrt{\frac{f^{-1}(H, \zeta)}{\pi}} \). This is quativelively very close to the BAM criterion, except of course for the prefactor. For \( H = 0.8 \), rapid convergence is obtained to \( \beta_{\text{PT}} = 0.49 \), while for \( H = 0.3 \) there is no convergence (see fig. 3). This absence of convergence however is due to PT simple JKR-like assumption which eventually at large \( \zeta \) becomes questionable, when one considers the actual Lennard-Jones distribution of forces at small scales, as in refs. [25, 33]. This problem does not occur in the BAM model, because the repulsive adhesionless contact gives converging results [35], while the adhesive force estimate is based on an entirely different idea, purely geometrical one, which gives no dependence on the fractal dimension (or Hurst exponent).

III. RESULTS

Figs. 4a-b show a comparison of the stickiness criteria (BAM, PT) for \( H = 0.8 \) and \( H = 0.3 \), respectively. As already shown in Ref. [21], for \( H = 0.8 \), BAM and PT solutions return very close results and rapidly converge with \( \zeta \). As a result, stickiness thresholds collapse into a single curve, which is independent on the value of \( \zeta \). The same trend is observed in Ref. [12], where a stickiness criterion has been derived from PS theory [23].
However, for $H = 0.3$ (fig. 4b) and contrary to BAM, the PT criterion is strongly magnification-dependent. This is due to the assumption in its derivation, that we discussed. In particular, the stickiness threshold decreases with increasing $\zeta$ both in PT and BAM solution, but it converges for BAM only with increasing $\zeta$. This is in agreement with the Lennard-Jones based theory developed in refs. 25, 33, according to which the probability distribution of gaps converges when smaller and smaller roughness wavelengths are involved in the adhesive contact. We don’t report a precise comparison with the theoretical findings of refs. 25, 33, since for this theory stickiness criterion has not been derived, and we haven’t implemented the relatively complex recursive integration procedure involved.

Once again, to explain the convergence in the BAM model, we have shown in figs. 2a-b the convergence of the mean repulsive pressure $p_{rep}(u)$ with $\zeta$, for both $H = 0.3$ and $H = 0.8$, which corresponds to the convergence of BAM stickiness threshold in figs. 4a-b.

In order to understand the dependence with $\zeta$ at all fractal dimensions, we show in fig. 5a the stickiness threshold as a function of the Hurst exponent $H$ and for fixed values of the quantity $\frac{\Delta n}{\zeta} = 10^6$ and $\zeta = [10^2, 10^3, 10^5, 10^7, 10^9]$. Fig. 5a confirms that PT solution is very close to BAM at high Hurst exponent and at low magnifications, where the JKR approximation gives no problem to the strain energy calculation in the very simple and elegant PT criterion. In general, the two criteria suggest that stickiness increases with increasing $H$. However, this effect is much less noticeable for BAM criterion.
FIG. 4: The stickiness criteria according to BAM theory (red solid line) and PT theory (blue dashed line). The stickiness threshold divides the plot in two regions: “sticky” and ”not sticky”. Results are shown for $H = 0.8$ (a) and $H = 0.3$ (b) with magnifications $\zeta = [10^3, 10^4, 10^5]$. The magnification increases in the direction indicated by the arrow.

The discrepancies between BAM and PT are observed for low $H$, especially at high magnifications, where eventually the two criteria differ quantitatively and qualitatively, as discussed. As an example, fig. 5b shows the stickiness threshold as a function of $\zeta$, for $H = [0.3, 0.5]$ and 0.8 with $\frac{L}{\epsilon} \epsilon = 10^6$.

IV. DISCUSSION

The results of BAM stickiness criteria we have derived are in general semi-quantitative agreement with the Literature. In particular, in refs. [25, 33], a semi-analytical model for studying the adhesive contact between fractal surfaces is developed. In such model, the details of the Lennard-Jones force-separation law are considered, and neither DMT nor JKR type of approximations are made, as instead done in BAM or PT, respectively. In ref. [25], it is found that the gap distribution between adhesive surfaces converges with magnification, in contrast with the JKR type of assumption made by PT. Moreover, the effect of the fractal dimension is studied in ref. [33]. It is found that the pull-off force depends weakly on the
FIG. 5: a) The \((h_{\text{rms}}/\epsilon)_{\text{thresh}}\) vs. \(H\) relation for BAM theory (red solid line) and PT theory (blue dashed line). Results are shown for \(\zeta = [10^2, 10^3, 10^5, 10^7, 10^9]\) and \(\lambda L/\epsilon = 10^6\). The magnification increases in the direction indicated by the arrow. b) The \((h_{\text{rms}}/\epsilon)_{\text{thresh}}\) vs. \(\zeta\) relation for BAM theory (red solid line) and PT theory (blue dashed line). Results are shown for \(H = [0.3, 0.5, 0.8]\) and \(\lambda L/\epsilon = 10^6\).

fractal dimension but, for relatively high rms roughness amplitude, it increases with \(H\).

In Ref. [32], Li et al. used a BEM numerical code for predicting adhesion between rough elastic spheres. They found that a finite pull-off force can be detected for higher and higher surface roughness as \(H\) is increased. Fig. 6a shows the pull-off force \(F_{po}\) obtained in Ref. [32], normalized with respect to the JKR [37] pull-off force \(F_{JKR} = 1.5\pi\Delta\gamma R\) for a smooth sphere of radius \(R\). Results are collected for \(H = [0.1, 0.3, 0.5, 0.7]\) and for increasing normalized roughness parameter \(h/R\), which is proportional to \(h_{\text{rms}}\). In fig. 6b, we can select a different \((h/R)_{\text{thresh}}\) for each value of \(H\) that corresponds to vanishing pull-off force. Fig. 6b rearranges \((h/R)_{\text{thresh}}\) as a function of \(H\). As expected, \((h/R)_{\text{thresh}}\) increases with \(H\) and this is consistent with our findings in fig. 5a.

V. CONCLUSIONS

We have compared BAM and PT stickiness criteria in the case of large fractal dimensions, and we found that PT theory leads to a stickiness criterion which strongly depends
on the truncating large wavevector, showing less and less stickiness if fine scale details are added. Such dependence is due to the strong simplifying assumption in the theory which neglects the effective Lennard-Jones distribution of the forces at the contact, and corresponds to a JKR approximation which cannot hold at small scales. BAM doesn’t consider Lennard-Jones forces either, but results in a converged result because it estimates the effect of adhesion with a pure geometrical construction, while the repulsive pressure converges with the magnification. PT and BAM criteria return similar results at high $H$ (or low $D$) because the JKR approximation in that case does not affect the convergence of the elastic strain energy.

The BAM solution shows a very simple universal result for stickiness, which depends weakly on fractal dimension, and deserves of course numerical or experimental final confirmation, which explains why we left the problem open as in our title. At present, there are no experimental or detailed numerical investigations on the effect of $\zeta$ on stickiness at very large magnifications and at low $H$. However, BAM and Lennard-Jones based theories \cite{25, 33} seem to suggest the same theoretical limit. In other words, our tentative conclusion is that BAM is qualitatively correct.

FIG. 6: a) The pull-off force $F_{po}/F_{JKR}$ vs. the normalized surface roughness $h/R$. Results are shown for $H = [0.1, 0.3, 0.5, 0.7]$ and are taken from fig. 7A of Ref. \cite{32}. The black empty squares indicate the values of $(h/R)_{\text{thresh}}$ for which vanishing pull-off force is detected. b) The $(h/R)_{\text{thresh}}$ values as a function of $H$. 

\[ \zeta \]
We showed that, for fixed magnification and rms roughness amplitude, stickiness increases with $H$. This result is in agreement with the analytical findings of refs. [25, 33], according to which the pull-off force could enhance with $H$, for relatively high rms values. The same trend has been observed with numerical simulations in Refs. [31, 32].

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[1] Creton C., Leibler L., Journal of Polymer Science: Part B: Polymer Physics 1996, 34, 545
[2] Creton C., Ciccotti M. (2016). Fracture and adhesion of soft materials: a review. Reports on Progress in Physics, 79(4), 046601.
[3] Menga, N., Afferrante, L., Carbone, G. (2016). Adhesive and adhesiveless contact mechanics of elastic layers on slightly wavy rigid substrates. International Journal of Solids and Structures, 88, 101-109.
[4] Dahlquist, C. A. in Treatise on Adhesion and Adhesives, R. L. Patrick (ed.), Dekker, New York, 1969,2, 219.
[5] Dahlquist, C., Tack, in Adhesion Fundamentals and Practice. 1969, Gordon and Breach: New York. p. 143-151.
[6] Autumn, K., Sitti, M., Liang, Y. A., Peattie, A. M., Hansen, W. R., Sponberg, S., Kenny, T.W., Fearing, R., Israelachvili, J.N. & Full, R. J. (2002). Evidence for van der Waals adhesion in gecko setae. Proceedings of the National Academy of Sciences, 99(19), 12252-12256.
[7] Gao, H., Wang, X., Yao, H., Gorb, S., & Arzt, E. (2005). Mechanics of hierarchical adhesion structures of geckos. Mechanics of Materials, 37(2-3), 275-285.
[8] Gao, H., Ji, B., Jäger, I. L., Arzt, E., & Fratzl, P. (2003). Materials become insensitive to
flaws at nanoscale: lessons from nature. Proceedings of the national Academy of Sciences, 100(10), 5597-5600.

[9] Dening, K., Heepe, L., Afferrante, L., Carbone, G., Gorb, S. N. (2014). Adhesion control by inflation: implications from biology to artificial attachment device. Applied Physics A, 116(2), 567-573.

[10] Santos R., Aldred N., Gorb S., Flammang P., (2012). Biological and Biomimetic Adhesives: Challenges and Opportunities. RSCPublishing.

[11] Kendall K (2001) Molecular Adhesion and Its Applications: The Sticky Universe (Kluwer Academic, New York).

[12] Violano, G., Afferrante, L., Papangelo, A., & Ciavarella, M. (2019). On stickiness of multiscale randomly rough surfaces. The Journal of Adhesion, 1-19.

[13] Dalvi, S., Gujrati, A., Khanal, S. R., Pastewka, L., Dhinojwala, A., & Jacobs, T. D. (2019). Linking energy loss in soft adhesion to surface roughness. Proceedings of the National Academy of Sciences, 116(51), 25484-25490.

[14] Ciavarella, M., Joe, J., Papangelo, A., Barber, JR. (2019) The role of adhesion in contact mechanics , J. R. Soc. Interface, 16, 20180738

[15] Vakis A.I., et al., (2018), Modeling and simulation in tribology across scales: An overview. Tribology International, 125,169-199.

[16] Afferrante, L., Bottiglione, F., Putignano, C., Persson, B. N. J., & Carbone, G. (2018). Elastic Contact Mechanics of Randomly Rough Surfaces: An Assessment of Advanced Asperity Models and Persson’s Theory. Tribology Letters, 66(2), 75.

[17] Müser, M. H., Dapp, W. B., Bugnicourt, R., Sainsot, P., Lesaffre, N., Lubrecht, T. A., ... & Rohde, S. (2017). Meeting the contact-mechanics challenge. Tribology Letters, 65(4), 118.

[18] Persson, B. N. J. (2014). On the fractal dimension of rough surfaces. Tribology Letters, 54(1), 99-106.

[19] Fuller, K.N.G., Tabor, D., (1975), The effect of surface roughness on the adhesion of elastic solids. Proc. R. Soc. Lond. A, 345(1642), 327-342.

[20] Pastewka, L., & Robbins, M. O. (2014). Contact between rough surfaces and a criterion for macroscopic adhesion. Proceedings of the National Academy of Sciences, 111(9), 3298-3303.

[21] Ciavarella, M. (2020). Universal features in “stickiness” criteria for soft adhesion with rough surfaces. Tribology International, 146, 106031.
[22] Persson, B. N. J., & Tosatti, E. (2001). The effect of surface roughness on the adhesion of elastic solids. The Journal of Chemical Physics, 115(12), 5597-5610

[23] Persson, B. N., & Scaraggi, M. (2014). Theory of adhesion: rôle of surface roughness. The Journal of chemical physics, 141(12), 124701.

[24] Ciavarella, M. (2018). A very simple estimate of adhesion of hard solids with rough surfaces based on a bearing area model. Meccanica:1-10. http://dx.doi.org/10.1007/s11012-017-0701-6

[25] Joe, J., Scaraggi, M., & Barber, J. R. (2017). Effect of fine-scale roughness on the tractions between contacting bodies. Tribology International, 111, 52-56.

[26] Violano, G., Demelio, G., & Afferrante, L. (2019). A note on the effect of surface topography on adhesion of hard elastic rough bodies with low surface energy. Journal of the Mechanical Behavior of Materials, 28(1), 8-12.

[27] Muller, V. M., Derjaguin, B. V., & Toporov, Y. P. (1983). On two methods of calculation of the force of sticking of an elastic sphere to a rigid plane. Colloids and Surfaces, 7(3), 251-259.

[28] Tiwari, A., Wang, J., & Persson, B. N. J. (2020). Adhesion paradox: Why adhesion is usually not observed for macroscopic solids. Physical Review E, 2020, 102.4: 042803.

[29] Ciavarella, M. (2020). Comments on old and recent theories and experiments of adhesion of a soft solid to a rough hard surface. Tribology International, 106779.

[30] Ciavarella, M., & Papangelo, A. (2017). Discussion of “measuring and understanding contact area at the nanoscale: A review” (Jacobs, TDB, and Ashlie Martini, A., 2017, ASME Appl. Mech. Rev., 69 (6), p. 060802). Applied Mechanics Reviews, 69(6).

[31] Violano, G., Afferrante, L. (2019). On DMT methods to calculate adhesion in rough contacts. Tribology International, 130, 3642. https://doi.org/10.1016/j.triboint.2018.09.004

[32] Li, Q., Pohrt, R., & Popov, V. L. (2019). Adhesive Strength of Contacts of Rough Spheres. Frontiers in Mechanical Engineering, 5, 7.

[33] Joe, J., Thouless, M. D., Barber, J. R. (2018). Effect of roughness on the adhesive tractions between contacting bodies, Journal of the Mechanics and Physics of Solids, doi.org/10.1016/j.jmps.2018.06.005

[34] Maugis, D. (1992). Adhesion of spheres: the JKR-DMT transition using a Dugdale model. Journal of colloid and interface science, 150(1), 243-269.

[35] Ciavarella, M., & Papangelo, A. (2019). Extensions and comparisons of BAM (Bearing Area Model) for stickiness of hard multiscale randomly rough surfaces. Tribology International, 133,
263-270.

[36] Yang, C., Persson, B.N.J. (2008). Contact mechanics: contact area and interfacial separation from small contact to full contact. J. Phys.: Condens. Matter, 20, 215214.

[37] Johnson, K. L., Kendall, K., & Roberts, A. (1971). Surface energy and the contact of elastic solids. Proceedings of the royal society of London. A. mathematical and physical sciences, 324(1558), 301-313.