\( \mathcal{O}(1/m_Q) \) Order Corrections to Masses of Excited Heavy Mesons
From QCD Sum Rules

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Abstract

The \( \mathcal{O}(1/m_Q) \) corrections to masses of excited heavy mesons are studied with sum rules in the heavy quark effective theory. Numerical results for the matrix elements of the heavy quark kinetic energy operator \( \mathcal{K} \) and chromomagnetic interaction operator \( \mathcal{S} \) are obtained for the lowest excited doublets \((0^+, 1^+)\) and \((1^+, 2^+)\).

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I. INTRODUCTION

In recent years there has been a continuous interest in the study of excited heavy mesons composed of a heavy quark and a light anti-quark. This interest arises from several reasons. Some of these excited states have been observed in experiments. They will be objects of further study in future experiments with B-factories. In particular, they are useful for tagging in CP experiments \cite{1}. Theoretically, the relative simplicity of the dynamical condition makes it of interest for exploring the internal dynamics of systems containing a light quark.

Important progresses in the theoretical description of such systems have been achieved with the development of the Heavy Quark Effective Theory (HQET) \cite{2}. Based on the spin-flavor symmetry of QCD, valid in the limit of infinite heavy quark mass $m_Q$, this framework provides a systematic expansion of heavy hadron spectra and transition amplitudes in terms of the leading contribution, plus corrections decreasing as powers of $1/m_Q$. However, to obtain some detailed predictions one needs to combine it with some non-perturbative methods. The spectra and decay widths of heavy meson excited states have been studied with the $1/m_Q$ expansion in the relativistic Bethe-Salpeter equations in \cite{3,4}. They can also be studied with the QCD sum rules in HQET which have been used both for ground states of heavy mesons \cite{5-7} and for lowest excited heavy meson doublets ($0^+, 1^+$) and ($1^+, 2^+$) at leading order of the $1/m_Q$ expansion \cite{8}. For related results, see \cite{9}.

The effective Lagrangian of the HQET, up to order $1/m_Q$, can be written as

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \mathcal{K} + \frac{1}{2m_Q} \mathcal{S} + \mathcal{O}(1/m_Q^2), \tag{1}$$

where $h_v(x)$ is the velocity-dependent field related to the original heavy-quark field $Q(x)$ by

$$h_v(x) = e^{i m_Q v \cdot x} \frac{1 + \gamma_5}{2} Q(x), \tag{2}$$

$\mathcal{K}$ is the operator of nonrelativistic kinetic energy with a negative sign defined as

$$\mathcal{K} = \bar{h}_v (i D_\perp)^2 h_v, \tag{3}$$

where $D^\mu_\perp = D^\mu - (v \cdot D) v^\mu$, with $D^\mu = \partial^\mu - ig A^\mu$ is the gauge-covariant derivative, and $\mathcal{S}$ is the Pauli term, describing the chromomagnetic interaction:
\[ S = \frac{g}{2} C_{mag}(m_Q/\mu) \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v, \]  
(4)

where \( C_{mag} = \left( \frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right)^{3/\beta_0}, \) \( \beta_0 = 11 - 2n_f/3. \) Apart from leading contribution, the Lagrangian density contains to \( \mathcal{O}(1/m_Q) \) accuracy two additional operators \( \mathcal{K} \) and \( \mathcal{S}. \) The matrix elements of these two operators over heavy meson states are fundamental parameters in the \( 1/m_Q \) expansion with HQET that should be either determined phenomenologically from experimental data or estimated using a nonperturbative theoretical approach. The matrix element of the chromomagnetic interaction operator is the leading contribution to the mass splitting among the states belonging to the same heavy meson doublet.

In our previous work \[8\], the masses of lowest excited heavy meson doublets \((0^+,1^+)\) and \((1^+,2^+)\) are calculated at the leading order of \( 1/m_Q \) expansion by using the QCD sum rule approach in HQET. As an extension of this work, in the present article we shall use the QCD sum rules in HQET to obtain quantitative estimates of the matrix elements of the kinetic energy and chromomagnetic interaction operators, which are responsible for the kinetic energy and the chromomagnetic mass corrections.

The remainder of this paper is organized as follows. In Section II we begin with a brief review on the interpolating currents for excited heavy mesons and some important properties of these currents. Some results of two-point functions for the doublets \((0^+,1^+)\) and \((1^+,2^+)\) at the leading order of the \( 1/m_Q \) expansion are outlined. Section III is devoted to \( \mathcal{O}(1/m_Q) \) corrections, the sum rules for the relevant matrix elements of kinetic energy and chromomagnetic interaction operator are derived. Finally, numerical results are presented in Section IV.

II. INTERPOLATING CURRENTS FOR HEAVY MESONS OF ARBITRARY SPIN AND PARITY AND TWO-POINT CORRELATION FUNCTION

A basic element in the application of QCD sum rules to excited heavy mesons is to choose a set of appropriate interpolating currents in terms of quark fields which create (annihilate) definite states of excited heavy mesons. For excited heavy mesons with arbitrary spin and parity this problem has been studied in our previous paper \[8\]. Here we briefly outline the results of this analysis which will be used in this article.
From the general form of the leading order B-S wave function obtained in \cite{10}, we can write the general expression of the interpolating current creating a excited heavy meson with arbitrary spin \( j \) and parity \( P \) as

\[
J_{j,P,i}^{\alpha_1 \cdots \alpha_j}(x) = \bar{h}_v(x) \Gamma_{j,P,i}^{\alpha_1 \cdots \alpha_j}(D_{xt}) q(x) ,
\]  

(5a)

or

\[
J_{j,P,i}^{\ast \alpha_1 \cdots \ast \alpha_j}(x) = \bar{h}_v(x) \Gamma_{j,P,i}^{\alpha_1 \cdots \alpha_j}(D_{xt})(-i) \mathcal{D}_{xt} q(x) ,
\]  

(5b)

where \( i = 1, 2 \) correspond to two series of doublets of the spin-parity \([j^{(-1)} j^{+1}),(j + 1)^{(-1)} j^{+1}\) and \([j^{(-1)}),(j + 1)^{(-1)}\] respectively, \( D_{xt} \) is the covariant derivative \( \frac{\partial}{\partial x_t} - igA_t(x) \), subscript \( t \) denotes the transverse component of a 4-vector and

\[
\Gamma^{\alpha_1 \cdots \alpha_j}(D_{xt}) = \text{Symmetrize} \left\{ \Gamma^{\alpha_1 \cdots \alpha_j}(D_{xt}) - \frac{1}{3} g_{\alpha_1 \alpha_2} g_{\alpha_3 \alpha_4} \Gamma_{j,P,i}^{\alpha_1 \alpha_2 \alpha_3 \alpha_4}(D_{xt}) \right\}
\]

with \( g_{\alpha_1 \alpha_2} = g^{\alpha_1 \alpha_2} - v^{\alpha_1} v^{\alpha_2} \). For the doublets of the spin-parity \([j^{(-1)} j^{+1}),(j + 1)^{(-1)} j^{+1}\) and \([j^{(-1)}),(j + 1)^{(-1)}\] the expressions for \( \Gamma^{\alpha_1 \cdots \alpha_j}(D_{xt}) \) have been explicitly given in \cite{8,10} as

\[
\Gamma(D_{xt}) = \begin{cases} 
\sqrt{2j + 1} \gamma^5 (i)^j D_{xt}^{\alpha_2} \cdots D_{xt}^{\alpha_j} (D_{xt} - \frac{j}{2j + 1} \gamma^{\alpha_1} \mathcal{P}_{xt}) & \text{for } j^{(-1)} j^{+1}, \\
\frac{1}{\sqrt{2}} \gamma^{\alpha_1} (i)^j D_{xt}^{\alpha_2} \cdots D_{xt}^{\alpha_j + 1} & \text{for } (j + 1)^{(-1)} j^{+1},
\end{cases}
\]

(6)

\[
\Gamma(D_{xt}) = \begin{cases} 
\frac{1}{\sqrt{2}} \gamma^5 \gamma^{\alpha_1} (i)^j D_{xt}^{\alpha_2} \cdots D_{xt}^{\alpha_j + 1} & \text{for } (j + 1)^{(-1)} j^{+1}, \\
\sqrt{\frac{2j + 1}{2j + 2}} (i)^j D_{xt}^{\alpha_2} \cdots D_{xt}^{\alpha_j + 1} (D_{xt} - \frac{j}{2j + 1} \gamma^{\alpha_1} \mathcal{P}_{xt}) & \text{for } j^{(-1)} j^{+1}.
\end{cases}
\]

(7)

The currents hermitian conjugate to (5a) and (5b)

\[
J_{j,P,i}^{\ast \alpha_1 \cdots \ast \alpha_j}(x) = \bar{q}(x) \Gamma_{j,P,i}^{\alpha_1 \cdots \alpha_j}(\mathcal{D}_{xt}) h_v(x) ,
\]

(8a)

\[
J_{j,P,i}^{\alpha_1 \cdots \alpha_j}(x) = \bar{q}(x) (i \mathcal{D}_{xt}) \Gamma_{j,P,i}^{\alpha_1 \cdots \alpha_j}(\mathcal{D}_{xt}) h_v(x) ,
\]

(8b)

where \( \bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0 \), correspond to the annihilation of the same meson.

The currents defined in (5) and (8) have nice properties. We had shown in \cite{8} that two currents in the sets (5a) and (5b) with not identical values of \( j, P, i \) never mix in the \( m_Q \to \infty \) limit either in the heavy meson or in the quark-gluon level and heavy quark symmetry is explicit with these currents. This verifies that they are the appropriate
interpolating currents for heavy meson states with definite $j$, $P$ and the light quark angular momentum $j_l$ which is conserved in QCD in $m_Q \to \infty$ limit.

The properties for currents mentioned above are important for applications to QCD sum rules for excited heavy mesons. If one use other currents or study the sum rules in full QCD, there are in general contributions from two nearby poles corresponding to states of the same $j$, $P$ which have different values of the total angular momentum of the light component $j_l = j \pm \frac{1}{2}$ in the $m_Q \to \infty$ limit. Their contributions may not be separated correctly. Furthermore, the mixing of such two states can be calculated within our formalism by introducing the $\mathcal{O}(1/m_Q)$ terms in the Lagrangian in HQET.

Let $|j, P, i\rangle$ be a heavy meson state with the quantum numbers $j$, $P$, $i$ in the $m_Q \to \infty$ limit, together with the corresponding interpolating current, we have [8]

$$\langle 0|J^{\alpha_1,\ldots,\alpha_j}_{j,P,i}(0)|j', P', i'\rangle = f_{P j} \delta_{jj'} \delta_{PP'} \delta_{ii'} \eta^{\alpha_1,\ldots,\alpha_j},$$

where the decay constant $f_{jP} = f_{P j}$ has the same value for the two states in the same doublet and $\eta^{\alpha_1,\ldots,\alpha_j}$ is the transverse, symmetric and traceless polarization tensor.

In the framework of QCD sum rules the decay constant can be obtained from the two-point correlator:

$$\Pi_{j,P,i}^{\alpha_1,\ldots,\alpha_j,\beta_1,\ldots,\beta_j}(k) = i \int d^4x e^{ik \cdot x} \langle 0|T \left(J^{\alpha_1,\ldots,\alpha_j}_{j,P,i}(x)J^{\beta_1,\ldots,\beta_j}_{j,P,i}(0)\right)|0\rangle,$$

where $k$ is the residual momentum, corresponding to the decomposition $p^\mu = m_Q v^\mu + k^\mu$ of the heavy meson momentum. This expression has the following form at the leading order [8]

$$\Pi_{j,P,i}^{\alpha_1,\ldots,\alpha_j,\beta_1,\ldots,\beta_j}(k) = (-1)^j S g_{\alpha_1\beta_1} \cdots g_{\alpha_j\beta_j} \Pi_{j,P,i}(\omega),$$

where $\omega = 2v \cdot k$ is twice the external off-shell energy and $S$ denotes symmetrization and subtracting the trace terms in the sets $(\alpha_1 \cdots \alpha_j)$ and $(\beta_1 \cdots \beta_j)$, and $\Pi_{j,P,i}(\omega) = \Pi_{P,j,i}(\omega)$ is independent of $j$ and satisfies dispersion relations of the form:

$$\Pi_{j,P,i}(\omega) = \int d\nu \frac{\rho_{j,P,i}(\nu)}{\nu - \omega} + \text{subtractions},$$

where $\rho_{j,P,i}(\nu) = \frac{1}{\pi} \text{Im}\Pi_{j,P,i}(\nu)$ represents the spectral density.

We shall confine our study to the doublets $(0^+, 1^+)$ and $(1^+, 2^+)$ here. According to (5a), (5b) and (7), there are two possible choices for currents creating $0^+$ and $1^+$ of the doublet $(0^+, 1^+)$, either
\[ J_{0,+,2}^\dagger = \frac{1}{\sqrt{2}} \bar{h}_v q , \]
\[ J_{1,+,2}^{1\alpha} = \frac{1}{\sqrt{2}} \bar{h}_v \gamma^5 \gamma^\alpha q , \]

or
\[ J_{0,+,2}^{1\dagger} = \frac{1}{\sqrt{2}} \bar{h}_v (-i) \mathcal{D}_t q , \]
\[ J_{1,+,2}^{1\dagger} = \frac{1}{\sqrt{2}} \bar{h}_v \gamma^5 \gamma^\alpha (-i) \mathcal{D}_t q . \]

Similarly, there are two possible choices for the currents creating 1\textsuperscript{+} and 2\textsuperscript{+} of the doublet (1\textsuperscript{+}, 2\textsuperscript{+}). One is
\[ J_{1,+,1}^{1\alpha} = \sqrt{\frac{3}{4}} \bar{h}_v \gamma^5 (-i) \left( \mathcal{D}_t^\alpha - \frac{1}{3} \gamma^\alpha \mathcal{D}_t \right) q , \]
\[ J_{2,+,1}^{1\alpha_1\alpha_2} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma^5 (-i) \left( \gamma_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} + \gamma_t^{\alpha_2} \mathcal{D}_t^{\alpha_1} - \frac{2}{3} g_t^{\alpha_1\alpha_2} \mathcal{D}_t \right) q . \]

Another choice is obtained by adding a factor \(-i\mathcal{D}_t\) to (17) and (18). Note that, without the last term in the bracket in (17) the current would couple also to the 1\textsuperscript{+} state in the doublet (0\textsuperscript{+}, 1\textsuperscript{+}) even in the limit of infinite \(m_Q\).

Usually, the currents with the least number of derivatives are used in the QCD sum rule approach. The sum rules with them have better convergence in the high energy region and often have better stability. However, there is a motivation for using the currents (13), (14) for the doublet (0\textsuperscript{+}, 1\textsuperscript{+}). In the non-relativistic quark model, which usually gives correct ordering of energy levels of hadron states, the doublets (0\textsuperscript{+}, 1\textsuperscript{+}) and (1\textsuperscript{+}, 2\textsuperscript{+}) are orbital p-wave states which correspond to one derivative in the space wave functions. In fact, as proved in [8], the coupling constant for the currents (13), (16) and that for (17), (18) are proportional to the large components of the B-S wave functions for the doublet (0\textsuperscript{+}, 1\textsuperscript{+}) and (1\textsuperscript{+}, 2\textsuperscript{+}) respectively in the non-relativistic approximation. Therefore, we shall consider both the currents (13), (14) and (17), (18) for the doublet (0\textsuperscript{+}, 1\textsuperscript{+}).

For the doublet (0\textsuperscript{+}, 1\textsuperscript{+}), when the currents \(J'_{0,+,2}, J'_{1,+,2}\) in (15), (16) are used the sum rule (same for the two states) after the Borel transformation is found to be

\[ \text{There is a typo in the coefficient of the last term of Eq. (25) in ref. [8], the coefficient 2 should be 2/3.} \]
\[ f^2 e^{-2\tilde{\Lambda}/T} = \frac{3}{2^{6/2}} \int_0^{\omega_c} \omega^4 e^{-\omega/T} d\omega - \frac{1}{2^4} m_0^2 \langle \bar{q}q \rangle. \quad (19) \]

The corresponding formula when the current \( J_{0,+2} \) and \( J_{1,+2} \) in (13) and (14) are used instead of \( J'_{0,+2} \) and \( J'_{1,+2} \) is the following

\[ f^2 e^{-2\tilde{\Lambda}/T} = \frac{3}{16\pi^2} \int_0^{\omega_c} \omega^4 e^{-\omega/T} d\omega + \frac{1}{2} \langle \bar{q}q \rangle - \frac{1}{8T^2} m_0^2 \langle \bar{q}q \rangle. \quad (20) \]

When the currents (17) and (18) are used the sum rule for the \((1^+, 2^+)\) doublet is found to be

\[ f^2 e^{-2\tilde{\Lambda}/T} = \frac{1}{2^6\pi^2} \int_0^{\omega_c} \omega^4 e^{-\omega/T} d\omega - \frac{1}{12} m_0^2 \langle \bar{q}q \rangle - \frac{1}{2^5} \frac{\alpha_s}{\pi} G^2 T. \quad (21) \]

Here \( m_0^2 \langle \bar{q}q \rangle = \langle \bar{q}g\sigma_{\mu\nu}G^{\mu\nu}q \rangle \). In the above derivations, we have confined us to terms of the lowest order in perturbation and operators of dimension less than six. Above results shall be used in the next Section.

### III. THE SUM RULES AT THE \( O(1/M_Q) \) ORDER

From Lorentz covariance, the correlator (10) still has the form (11) beyond the leading order of \( 1/m_Q \) expansion if we confine \( k \) to be a longitudinal vector. But now \( \Pi_{j,P,j}(\omega) \) depends on \( j \) due to the chromomagnetic interaction operator \( \mathcal{S} \). Inserting the heavy meson eigen-state of the Hamiltonian up to the order \( O(1/m_Q) \), the pole term on the hadron side becomes

\[ \Pi(\omega)_{pole} = \frac{(f + \delta f)^2}{2(\Lambda + \delta m) - \omega} = \frac{f^2}{2\Lambda - \omega} - \frac{2\delta m f^2}{(2\Lambda - \omega)^2} + \frac{2f\delta f}{2\Lambda - \omega}, \quad (22) \]

where \( \delta m \) and \( \delta f \) are of the order \( O(1/m_Q) \).

To extract \( \delta m \) in (22) we follow the approach of [6] and consider the three-point correlation functions

\[ \delta O \Pi_{j,P,i}^{\alpha_{1}\cdots\alpha_{j},\beta_{1}\cdots\beta_{j}}(\omega, \omega') = i^2 \int d^4x d^4y e^{ikx - ik'y} \langle 0 | T \left( J_{j,P,i}^{\alpha_{1}\cdots\alpha_{j}}(x) O(0) J_{j,P,i}^{\beta_{1}\cdots\beta_{j}}(y) \right) | 0 \rangle, \quad (23) \]

where \( O = \mathcal{K} \) or \( \mathcal{S} \). The sum \( \delta \mathcal{K} \Pi + \delta \mathcal{S} \Pi \) is equal to the correlator (10) in the order \( O(1/m_Q) \) when \( \omega = \omega' \). The scalar function corresponding to (23) can be represented as the double dispersion integral

\[ \delta O \Pi(\omega, \omega') = \frac{1}{\pi^2} \int \frac{\rho_{o}(s,s')dsds'}{(s - \omega)(s' - \omega')}. \quad (24) \]
The pole parts for them are

\[\delta_K \Pi(\omega, \omega')_{pole} = \frac{f^2 K}{(2\Lambda - \omega)(2\Lambda - \omega')} + \frac{f G_K(\omega')}{2\Lambda - \omega} + \frac{f G_K(\omega)}{2\Lambda - \omega}, \tag{25}\]

\[\delta_S \Pi(\omega, \omega')_{pole} = \frac{d_M f^2 \Sigma}{(2\Lambda - \omega)(2\Lambda - \omega')} + d_M f \left[ \frac{G_S(\omega)}{2\Lambda - \omega} + \frac{G_S(\omega)}{2\Lambda - \omega} \right], \tag{26}\]

where

\[K_{j,P,j_l} = \langle j, P, j_l | \bar{h}_v (iD_{\perp})^2 h_v | j, P, j_l \rangle, \tag{27}\]

\[2d_M \Sigma_{j,P,j_l} = \langle j, P, j_l | \bar{h}_v g_{\mu\nu} G^{\mu\nu} h_v | j, P, j_l \rangle, \tag{28}\]

\[d_M = d_{j,j_l}, \quad d_{j_l-1/2,j_l} = 2j_l + 2, \quad d_{j_l+1/2,j_l} = -2j_l. \tag{29}\]

Let \(\omega = \omega'\) in (25) and (26), and compare it with (22) one obtains

\[\delta m = -\frac{1}{4m_Q} (K + d_M C_{mag} \Sigma). \tag{30}\]

The simple pole term in (25) and (26) comes from the region in which \(s(s') = 2\bar{\Lambda}\) and \(s(s')\) is at the pole for a radical excited state or in the continuum. These terms are suppressed by making double Borel transformation for both \(\omega\) and \(\omega'\). One obtains thus the sum rules for \(K\) and \(\Sigma\) as

\[f^2 K e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} \int_0^{\omega_c} d\omega d\omega' e^{-(\omega + \omega')/2T} \rho_K(\omega, \omega'), \tag{31}\]

\[f^2 \Sigma e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} \int_0^{\omega_c} d\omega d\omega' e^{-(\omega + \omega')/2T} \rho_S(\omega, \omega'), \tag{32}\]

where the spectral densities are obtained from straightforward calculations in HQET.

Confining us to the leading order of perturbation and the operators with dimension \(D \leq 5\) in OPE, the relevant Feynman diagrams, which contribute to \(\delta_K \Pi\) and \(\delta_S \Pi\) are shown in Fig. 1 and Fig. 2 respectively. The graphs missing there turn out to be vanishing in the fixed point gauge \(x^\mu A_\mu(x) = 0\). The spectra functions \(\rho_K\) and \(\rho_S\) can be calculated from these diagrams. For the \(j_l^P = \frac{1}{2}^+\) doublet, we find

\[f^2 K e^{-2\bar{\Lambda}/T} = \frac{3}{2^7\pi^2} \int_0^{\omega_c} \omega^6 e^{-\omega/T} d\omega + \frac{3}{2^4\pi} \langle \alpha_s G G \rangle T^3, \tag{33a}\]

\[f^2 \Sigma e^{-2\bar{\Lambda}/T} = \frac{1}{48\pi} \langle \alpha_s G G \rangle T^3, \tag{33b}\]

when the currents (13) and (13) are used and
\[ f^2 K e^{-2\Lambda/T} = - \frac{3}{2^{5/2}} \pi^2 \int_0^{\omega_c} \omega^4 e^{-\omega/T} d\omega - \frac{1}{2^{5/2}} \langle \alpha_s GG \rangle T - \frac{3}{8} m_0^2 \langle \bar{q}q \rangle, \tag{34a} \]
\[ f^2 \Sigma e^{-2\Lambda/T} = \frac{1}{24 \pi} \langle \alpha_s GG \rangle T + \frac{1}{48} m_0^2 \langle \bar{q}q \rangle \tag{34b} \]
when the currents (13) and (14) with no extra derivative are used. In contrast to (33b), there is a nonvanishing mixing condensate term in (34b). It arises from the diagram depicted in Fig. 2(b). For \( j^P = \frac{3}{2} \) doublet, a straightforward calculation yields
\[ f^2 K e^{-2\Lambda/T} = - \frac{1}{2^{7/2}} \pi^2 \int_0^{\omega_c} \omega^6 e^{-\omega/T} d\omega + \frac{7}{3} \times \frac{2^{5/2}}{2^{7/2}} \langle \alpha_s GG \rangle T^3, \tag{35a} \]
\[ f^2 \Sigma e^{-2\Lambda/T} = \frac{1}{72 \pi} \langle \alpha_s GG \rangle T^3. \tag{35b} \]

Combining (33), (34) and (35) with (19), (20) and (21) we can obtain sum rules for \( K \) and \( \Sigma \) in the three cases.

The above results are not the whole story. The spin-symmetry violating term \( S \) not only causes splitting of masses in the same doublet, but also causes mixing of states with the same \( j, P \) but different \( j_l \). This mixing is characterized by the matrix element
\[ \langle j, P, j_l = \frac{3}{2} | \bar{h} \sigma_{\mu\nu} G^{\mu\nu} h_v | j, P, j_l = -\frac{1}{2} \rangle = -2 m_{+\mp}(j^P). \tag{36} \]

This quantity can be extracted from the correlator
\[ i^2 \int d^4 x d^4 y e^{ikx - ik'y} \langle 0 | T \left( J_{j, P, j_l = \frac{3}{2}}(x) \bar{h}_{\mu} g^{\sigma_{\mu\nu}} G^{\mu\nu} h_{\nu}(0) J_{j, P, j_l = -\frac{1}{2}}(y) \right) | 0 \rangle, \tag{37} \]
the double pole term of which is
\[ - \frac{2m_{+\mp}(j^P) f_{P,j_l=\frac{3}{2}} f_{P,j_l=-\frac{1}{2}}}{(2\Lambda_{+\mp,j_l=\frac{3}{2}} - \omega)(2\Lambda_{+\mp,j_l=-\frac{1}{2}} - \omega')}. \tag{38} \]
With the methods similar to that used above one can obtain the sum rule of \( m_{+\mp}(j^P) \).

For two \( 1^+ \) states we find
\[ m_{\frac{3}{2}, \frac{3}{2}}(1^+) \left[ f_{\frac{1}{2}, \frac{1}{2}} f_{\frac{3}{2}, \frac{1}{2}} e^{-\Lambda_{+\mp,\frac{3}{2}}/T} e^{-\Lambda_{+\mp, \frac{1}{2}}/T} \right] = \frac{\sqrt{6}}{72 \pi} \langle \alpha_s GG \rangle T^3. \tag{39} \]

As an example, here we have used the currents (16) and (17) to do the analysis.

The masses of two \( 1^+ \) states are obtained by diagonalizing the mass matrix
\[
\begin{pmatrix}
-\Lambda_{+\mp, \frac{3}{2}} - \frac{1}{4m_Q} (K_{+\mp, \frac{3}{2}} - C_{mag} \Sigma_{+\mp, \frac{3}{2}}) & \frac{1}{4m_Q} C_{mag} m_{\frac{3}{2}, \frac{3}{2}}(1^+) \\
\frac{1}{4m_Q} C_{mag} m_{\frac{3}{2}, \frac{3}{2}}(1^+) & -\Lambda_{+\mp, \frac{3}{2}} - \frac{1}{4m_Q} (K_{+\mp, \frac{3}{2}} + 5 C_{mag} \Sigma_{+\mp, \frac{3}{2}})
\end{pmatrix}.
\tag{40}
\]
The correction to the masses due to the mixing is formally of the order of \(0(1/m_Q^2)\). However, due to the smallness of the difference between the two diagonal elements there is some possibility that this correction is numerically comparable to the \(\mathcal{O}(1/m_Q)\) corrections. Therefore we shall calculate it also.

### IV. NUMERICAL RESULTS AND DISCUSSIONS

We now turn to the numerical evaluation of these sum rules. We can eliminate the explicit dependence of the sum rules obtained in Section III on \(f\) and \(\bar{\Lambda}\) by means of the two-point function sum rules outlined in Section II. Dividing the sum rules in (33), (34) and (35) by the sum rules in (19), (20) and (21) respectively, we obtain relevant expressions for the \(K\) and \(\Sigma\) as functions of the Borel parameter \(T\) and the continuum threshold \(\omega_c\). For the QCD parameters entering the theoretical expressions, we use the following standard values

\[
\langle \bar{q}q \rangle = -0.24 \text{ GeV}^3, \quad \langle \alpha_s GG \rangle = 0.038 \text{ GeV}^4, \quad m_0^2 = 0.8 \text{ GeV}^2.
\] (41)

The results for \(K\) as a function of the Borel variable \(T\), and for different values of the continuum threshold \(\omega_c\), are shown in Fig. 3. In particular, in Fig. 3a and 3b we depict the results of two kinds of sum rules for doublet \((0^+, 1^+)\), which correspond to using two kinds of currents \((13)\), \((14)\) and \((15)\), \((16)\) respectively. In Fig. 3c, we display the result of sum rule associated with doublet \((1^+, 2^+)\).

Following the QCD sum rule procedure, we should check the existence of stable regions for the results of \(K\) in the Borel variable \(T\) and the continuum threshold \(\omega_c\). As one can see from Fig. 3, stability starts at values of Borel parameter \(T\) slightly larger than 0.7, and stretches practically to \(T \to \infty\). The familiar criterion that both the high-order power corrections and the contribution of the continuum model should not be too large, generally speaking, less than 30\%, rather strongly restricts the ‘working’ region. Thus, the stability at large \(T\) values is not useful, since in this region the sum rules is strongly contaminated by higher resonance states. This continuum model contamination problem is quite severe also in the case of sum rules analysis of ground state heavy meson to \(1/m_Q\).
It originates from the high power dependence of the spectral densities. According to [3], it is better to determine the working window by considering the stability of two-point sum rules, and then to use the same values in the evaluation of the three-point sum rules. The choice of so-called stability window does not necessarily coincide with the stability plateau for the three-point sum rules. In the case of two-point function sum rules one finds that the working windows are $\omega_c = 2.5 - 2.9$ GeV, $T = 0.7 - 0.9$ GeV for the doublet $(0^+,1^+)$ and $\omega_c = 2.7 - 3.2$ GeV, $T = 0.7 - 0.9$ GeV for the doublet $(1^+,2^+)$ [3].

From Fig. 3, taking the working regions of the sum rules as that for the two-point function for the three cases, we obtain the set of values for $K$ as following:

$$K = -1.85 \pm 0.30 \text{ GeV}^2,$$

for the doublet $(0^+,1^+)$ when the currents (13), (14) without the derivative are used,

$$K = -1.90 \pm 0.40 \text{ GeV}^2,$$

for the same doublet when the currents (15), (16) with the derivative are used, and

$$K = -2.00 \pm 0.40 \text{ GeV}^2,$$

for the doublet $(1^+,2^+)$. The errors reflect the variations with the Borel parameter $T$ and the continuum threshold $\omega_c$ within the working windows. As can be seen when comparing Fig. 3a and Fig. 3b with each other, $K$ value in Fig. 3b is more sensitive to the value of $\omega_c$ than that in Fig. 3a owing to the higher dimension of spectral density for the using of current with extra derivative.

In Fig. 4 we show the numerical results of the sum rules for $\Sigma$, corresponding to the three chosen values of the continuum threshold $\omega_c$. From these figures, we see that there is no stability plateau for the sum rules of $\Sigma$. This is related to the absence of perturbation terms for $\Sigma$ in our approximation. However, due to the strategy mentioned above we can take the working window as that of two-point function sum rules like the case of $K$. The results for $\Sigma$ for the doublet $(0^+,1^+)$ in the working regions of the sum rules are

$$\Sigma = 0.023 \pm 0.003 \text{ GeV}^2,$$

when the currents (13), (14) without the derivative are used and
\[ \Sigma = 0.014 \pm 0.003 \text{ GeV}^2, \]  

when the currents (15), (16) with the derivative are used. For the doublet \((1^+, 2^+)\), the value of \(\Sigma\) is given by

\[ \Sigma = 0.020 \pm 0.003 \text{ GeV}^2. \]  

Dividing the sum rule in (39) by the sum rules in (20) and (21), we also obtain the expression for \(m_{1^+, 2^+}(1^+)\). In Fig. 5 we plot the result of this sum rules as a function of \(T\), and for three values of \(\omega_c\). Using the working windows as \(\omega_c = 2.6 - 3.0\) GeV and \(T = 0.7 - 0.9\) GeV, we obtain

\[ m_{1^+, 2^+}(1^+) = 0.032 \pm 0.004 \text{ GeV}^2. \]  

After combining with (40), we found that the correction to the masses due to the mixing is negligibly small.

In conclusion, we have calculated the \(1/m_Q\) correction to the masses of lowest excited heavy mesons from sum rules within the framework of the HQET. This study refines the leading order analysis [8]. From (42)-(47), together with (30), we can obtain the corrections to the masses of excited \(D\) mesons. For the doublet \((0^+, 1^+)\), our final results read

\[ \frac{1}{4} (m_{D_0^*} + 3m_{D_1'}) = m_c + \bar{\Lambda} + \frac{1}{m_c} [(0.46 \pm 0.08) \text{ GeV}^2], \]  

\[ m_{D_1'} - m_{D_0^*} = \frac{1}{m_c} [(0.023 \pm 0.003) \text{ GeV}^2], \]  

when the currents (13) and (14) are used and

\[ \frac{1}{4} (m_{D_0^0} + 3m_{D_1'}) = m_c + \bar{\Lambda} + \frac{1}{m_c} [(0.48 \pm 0.10) \text{ GeV}^2], \]  

\[ m_{D_1'} - m_{D_0^0} = \frac{1}{m_c} [(0.023 \pm 0.003) \text{ GeV}^2], \]  

when the currents (15) and (16) are used. As for the doublet \((1^+, 2^+)\), the result is

\[ \frac{1}{8} (3m_{D_1} + 5m_{D_2^*}) = m_c + \bar{\Lambda} + \frac{1}{m_c} [(0.50 \pm 0.10) \text{ GeV}^2], \]  

\[ m_{D_2^*} - m_{D_1} = \frac{1}{m_c} [(0.040 \pm 0.006) \text{ GeV}^2], \]  

where the values of \(\bar{\Lambda}\) in three cases are given in [8]. Note that we have reasonably neglected the renormalization coefficient \(C_{\text{mag}}\) of the chromomagnetic operator for charmed...
meson in the discussion. We should also note that the given errors present only the uncertainties of $T$ and $\omega_c$ in the working regions.

The above results for the spin splittings are not theoretically reliable because we have not included the Feynman diagrams which are of the one loop order in the perturbation theory and may be of values comparable to that of the gluon condensate term leading in our approximation. Nevertheless, they are in approximate agreement with the experimental values for the mass splitting between $D_1$ and $D^*_2$ \cite{12} if we take $m_c \simeq 1.3$ GeV.

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Figure Captions

Fig. 1. Feynman diagrams contributing to the sum rules for $K$, in the coordinate gauge. The gray square corresponds to the insertion of kinetic energy operator at $\mathcal{O}(1/M_Q)$ in the HQET Lagrangian. The right vertex is put to the origin in coordinate space. There are no figures (c) (d) (h) when the currents $[13]$ and $[14]$ are used.

Fig. 2. Feynman diagrams contributing to the sum rules for $\Sigma$, in the coordinate gauge. The gray square corresponds to the insertion of chromomagnetic interaction operator at $\mathcal{O}(1/M_Q)$ in the HQET Lagrangian.

Fig. 3. The sum rules for $K$ as a functions of the Borel parameter $T$ for different values of the continuum threshold $\omega_c$. (a) (b) for $(0^+, 1^+)$ doublet with the currents $[13]$ $[14]$ and $[15]$ $[16]$ used respectively. From top to bottom the curves correspond to $\omega_c = 2.9, 2.7, 2.5$ GeV respectively. (c) for $(1^+, 2^+)$ doublet. From top to bottom the curves correspond to $\omega_c = 3.1, 2.9, 2.7$ GeV respectively.

Fig. 4. The sum rules for $\Sigma$ as a functions of the Borel parameter $T$ for different values of the continuum threshold $\omega_c$. (a) (b) for $(0^+, 1^+)$ doublet with the currents $[13]$ $[14]$ and $[15]$ $[16]$ used respectively. From top to bottom the curves correspond to $\omega_c = 2.9, 2.7, 2.5$ GeV respectively. (c) for $(1^+, 2^+)$ doublet. From top to bottom the curves correspond to $\omega_c = 3.1, 2.9, 2.7$ GeV respectively.

Fig. 5. The sum rules for $m_{1/2, 3/2}(1^+)$ as a functions of the Borel parameter $T$ for different values of the continuum threshold $\omega_c$ (from top to bottom: $\omega_c = 3.0, 2.8, 2.6$).
Fig 1

(a)  (b)  (c)  (d)  (e)

(f)  (g)  (h)  (i)  (j)

Fig 2

(a)  (b)
Fig 3
Fig 4
Fig 5