A Grill between Weak Forms of Faint Continuity

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Abstract
We are interested introducing new classes of faint continuity namely faint G-γ continuity, faint G-α-continuity and faint G-semi-continuity to obtain their characterizations and some of their properties. AMS Mathematics Subject Classification: (2000) 54D10, 54C10.

Keywords: G-γ-open sets; faint G-semi-continuity; Faintly G-γ-continuous; faint G-α-continuity

Introduction
The idea of grill topological space aimed at generalize the concept of topological space since it generates newtopology \( \tau_g \), which helps to measure the things that was difficult to measure. Choquet [1] in 1947 was the first author introduced the idea of grill. It has been explored that there is some of similarity between Choquet's concept and that ideals, nets and filters. In 2007, Roy and Mukherjee introduced the definition of \( \tau_g \) which is related to two operators \( \Phi \) and \( \Psi \). They have determined the relation between \( \tau \) and \( \tau_g \). A number of theories and characterization has been handled in previous studies [2-4] whether respect to sets or functions. Hatir and Jafari [5] have introduced the idea of grill. It has been explored that there is some of similarity between Choquet's concept and that ideals, nets and filters. In 2007, Roy and Mukherjee introduced the definition of \( \tau_g \) which helps to

Preliminaries

"In what follows, by a space \( Y \) we shall mean space \( Y \) which carries topology \( \tau \). For \( A \subseteq Y \), we shall adopt the usual notations \( \text{Cl}(A) \) and \( \text{Int}(A) \) to \( A \), respectively denote the closure and interior of \( A \) in \( Y \). Also, the power set of \( Y \) will be written as \( P(Y) \). A nonempty subset \( G \) of \( Y \) is called a grill [2] if

\[ \phi \notin G \]
\[ A \in G \text{ and } A \subseteq B \subseteq Y \text{ implies that } B \in G, \]
\[ \text{if } A:B \subseteq Y \text{ and } A \cup B \text{ G implies that } A \cup B \text{ G or } B \cup G. \]

Remark 2.1

(1) "The minimal grill is \( G=\{Y\} \) in any space \( Y \) which carries topology \( \tau \). (2) The maximal grill is \( G=P(Y)\backslash\{\phi\} \) in any topological space \( (Y; \tau) \).

Since the grill depends on the two mappings \( \Phi \) and \( \Psi \), which is generated a unique grill topological space finer than on space \( Y \) denoted by \( \tau_g \) on \( Y \) have been discussed [6].

Definition 2.2: "Let \( Y \) be a space which carries topology \( G \) be a grill on \( Y \). A subset \( A \) in \( Y \) is called [3]:

(1) G-open if \( A \subseteq \text{Int}(\Phi(A)) \);

(2) G-preopen if \( A \subseteq \text{Int}(\Psi(A)) \);

(3) G-semi-open if \( A \subseteq \Psi(\text{Int}(A)) \);

(4) G-open if \( A \subseteq \text{Int}(\Psi(\text{Int}(A))) \);

(5) G-preopen if \( A \subseteq \text{Int}(\Psi(A)) \cup \Psi(\text{Int}(A)) \).

"The family of all \( G \)-open (resp. \( G \)-preopen, \( G \)-semipopen) sets of \( (Y, \tau, G) \) is denoted by \( GSO(Y) \) (resp. \( GPO(Y), \text{GPO}(Y) \));

"The family of all \( G \)-open (resp. \( G \)-preopen, \( G \)-semipen) sets of \( (Y, \tau, G) \) containing a point \( y \in Y \) is denoted by \( GSO(y) \) (resp. \( GPO(y), \text{GPO}(y) \));

Definition 2.3: "The intersection of all \( G \)-closed (resp. \( G \)-preclosed, \( G \)-semiclosed) sets contained \( S \subseteq Y \) is called the \( G \)-closure (resp. \( G \)-preclosure, \( G \)-semiclosure) of \( S \) and is denoted by \( \gamma Cl(G)(S) \) (resp. \( Cl(G)(S), \gamma Cl(G)(S) \))".

Definition 2.4: "A function \( h: (Y; \tau; G) \to (Z; \sigma) \) is said to be faintly \( G \)-continuous at each point \( y \in Y \) if for each \( \theta \)-open set \( B \in Z \) containing \( h(y) \), there exists \( A \subseteq O(Y; y) \) such that \( h(A) \subseteq B \).

Definition 2.5: "A subset \( A \) of \( Y \) is said to be \( \theta \)-open [9] if for each \( y \in A \) there exists an open set \( U \) such that \( y \in U \subseteq Cl(U) \subseteq A \).

Definition 2.6: The \( \theta \)-closure of \( A \), denoted by \( Cl(\Theta)(A) \), is defined to be the set of all \( y \in Y \) such that for each \( y \in A \) there exists an open set \( U \in Cl(U) \) for every open neighborhood \( U \) of \( Y \).

Definition 2.7: "A function \( h: (Y; \tau; G) \to (Z; \sigma) \) is said to be faintly continuous [9] if \( h^{-1}(H) \) is open in \( Y \) for every \( \theta \)-open set \( H \) of \( Z \).

Definition 2.8: "A function \( h: (Y; \tau; G) \to (Z; \sigma) \) is said to be faintly G-precontinuous [6] at a point \( y \in Y \) if for each \( \theta \)-open set \( B \in Z \) containing \( h(y) \), there exists \( A \subseteq O(Y; y) \) such that \( h(A) \subseteq B \) and \( y \in A \).

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Received May 24, 2018; Accepted June 12, 2018; Published June 25, 2018

Citation: Azzam AA (2018) A Grill between Weak Forms of Faint Continuity. J Phys Math 9: 273. doi: 10.4172/2090-0902.1000273

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containing $h(y)$, there exists $A \in \text{GPO}(Y; y)$ such that $h(A) \subset B$. If $h$ has this property at each point of $Y$, then it is said to be faintly G-precontinuous.

### Some Fundamental Properties

In this section, we define some modern classes of functions called faint G-$\gamma$-continuity faint G-$\gamma$-continuity and faint G-semi-continuity. Depictions and essential properties of these concepts are studied.

**Definition 3.1:** A function $h$: $(Y; \tau; G) \rightarrow (Z; \sigma)$ is said to be faintly G-$\gamma$-continuous (resp. faintly G-$\alpha$-continuous, faintly G-semi-continuous) if for each $y \in Y$ and each $\theta$-open set $B$ of $Z$ containing $h(y)$, there exists $A \in G\alphaO(Y; y)$ (resp. $A \in G\gammaO(Y; y)$, $U \in GSO(Y; y)$) such that $h(A) \subset B$.

**Theorem 3.2**

For a function $h$: $(Y; \tau; G) \rightarrow (Z; \sigma)$, the following statements are equivalent:

1. $h$ is faintly G-$\gamma$-continuous;
2. $h^{-1}(H)$ is G-$\gamma$-open in $Y$ for every $\theta$-open set $H$ of $Z$;
3. $h^{-1}(F)$ is G-$\gamma$-closed in $Y$ for every $\theta$-closed set $F$ of $Z$;
4. $h$: $(Y; \tau; G) \rightarrow (Z; \sigma)$ is G-$\gamma$-continuous;
5. $y \in G\gammaO(Y; y)$, $U \in G\gammaO(Y; y)$ for every subset $W$ of $Z$;
6. $h^{-1}(\text{Int}_G(W)) \subseteq \text{Int}_G(h^{-1}(W))$ for every subset $W$ of $Z$.

**Proof:** The proof is similar to the proof of Theorem 3.2.

**Theorem 3.3**

For a function $h$: $(Y; \tau; G) \rightarrow (Z; \sigma)$, the following statements are equivalent:

1. $h$ is faintly G-$\alpha$-continuous;
2. $h^{-1}(H)$ is G-$\alpha$-open in Y for every $\theta$-open set $H$ of $Z$;
3. $h^{-1}(F)$ is G-$\alpha$-closed in $Y$ for every $\theta$-closed set $F$ of $Z$.

**Theorem 3.4**

For a function $h$: $(Y; \tau; G) \rightarrow (Z; \sigma)$, the following statements are equivalent:

1. $h$ is faintly G-semi-continuous;
2. $h^{-1}(H)$ is G-semi-open in $Y$ for every $\theta$-open set $H$ of $Z$;
3. $h^{-1}(F)$ is G-semi-closed in $Y$ for every $\theta$-closed set $F$ of $Z$.

**Proof:** The converse of Theorem 3.5. is not true in general as it can be seen in the following example.

**Example 3.9:** Let $Y=\{1, 2, 3\}$, $\sigma=\{\emptyset, Y, \{1\}, \{2\}\}$ and $G=\{Y, \{1\}, \{2\}, \{1, 2\}\}$. The identity function $h$: $(Y, \tau) \rightarrow (Y, \sigma)$ is faintly G-$\gamma$-continuous function but not faintly G-$\alpha$-continuous function.

**Remark 3.8**

1. (1) The following figure shows the relationship among these new classes of functions and other corresponding types.
2. (2) The converse are not true in general as shown by the following examples.

**Example 3.10:** Let $Y=\{1, 2, 3\}$, $\tau=\{\emptyset, Y, \{1\}, \{1, 2\}\}$ and $G=\{Y, \{1\}, \{2\}, \{1, 2\}\}$. The identity function $h$: $(Y, \tau; G) \rightarrow (Y, \sigma)$ is faintly G-$\gamma$-continuous function but not faintly G-$\alpha$-continuous function since $h^{-1}(1, 3)$ GPO(Y, $\tau$) where $\{1, 3\} \notin \sigma$.

**Example 3.11:** Let $Y=\{a, b, c\}$, $\tau=\{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$, $\sigma=\{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$.
Let \( h: (Y, \tau, G) \rightarrow (Z, \sigma) \) be faintly \( G \)-continuous and \( (Z, \sigma) \) is \( G \)-\( \gamma \)-continuous. This implies that \( h(A) \subseteq Y \times Z \) for every \( A \in \mathcal{G}(Y) \). Hence, \( h \) is faintly \( G \)-\( \gamma \)-continuous.

**Theorem 3.20**

Let \( h: (Y, \tau, G) \rightarrow (Z, \sigma) \) be faintly \( G \)-\( \gamma \)-continuous injection and \( Z \) be a \( \theta_{T_{1}} \)-space, then \( Y \) is a \( G \)-\( \gamma \)-\( \theta_{T_{1}} \)-space.

**Proof:** Using the same method as Theorem 3.17.

**Theorem 3.21**

Let \( h: (Y, \tau, G) \rightarrow (Z, \sigma) \) be a faintly \( G \)-\( \gamma \)-continuous-injection function and \( Z \) be a \( \theta_{T_{1}} \)-space, then \( Y \) is a \( G \)-\( \gamma \)-\( \theta_{T_{1}} \)-space.

**Proof:** Using the same method as Theorem 3.17.
Proving (2, 3) are Similar to (1).

Conclusion

The study of faintly grill topological spaces is very important. It is a generalization of faintly topological spaces. So we introduced neoteric classes of functions called faintly $G_\gamma$-continuous, faint $G_\alpha$-continuous and faint $G$-semi-continuous in grill topological spaces that helps us in many applications such as computer and information systems. Furthermore, relationships between different classes are introduced. Also, some of their basic properties of different types of functions between grill topological spaces are obtained.

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