Impact parameter dependence of the scaling of anisotropic flows in intermediate energy HIC

YAN Ting-Zhi(1,1) LI Shan(2)

1 School of Energy Resources and Power Engineering, Northeast Dianli University, Jilin 132012, China
2 School of Science, Northeast Dianli University, Jilin 132012, China

Abstract: The scaling behaviors of anisotropic flows of light charged particles are studied for 25 MeV/nucleon $^{40}$Ca+$^{40}$Ca collisions at different impact parameters by the isospin-dependent quantum molecular dynamics model. The number of nucleons scaling of elliptic flow is existed and the scaling of the ratios of $v_4/v_2^2$ and $v_3/(v_1v_2)$ are applicable for collisions at almost all impact parameters except for peripheral collisions.

Key words: Impact parameter, anisotropic flows, scaling behaviors

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1 Introduction

Anisotropic flows are interesting subjects in theoretical and experimental investigations on nuclear reaction dynamics in both intermediate and high energy heavy ion collisions. Many studies of the first and second anisotropic flows (directed flow and elliptic flow, respectively) dependence on beam energies, mass number or quark number, isospin and impact parameter have been done and revealed much interesting physics about the properties and origin of the collective flow\[1–10\]. $^{197}$Au + $^{197}$Au collision experiments at RHIC energy demonstrated the number of constituent-quark (NCQ) scaling for the transverse momentum dependent elliptic flow for different mesons and baryons\[11, 12\], and a popular interpretation is assuming that the mesons and baryons are formed by the coalescence or recombination of the constituent quarks. Our work\[12\] found the similar elliptic scaling for light particles in intermediate energy heavy ion collisions and it may also be the outcome of coalescence mechanism but at nucleonic level. As we know, the flow value is strongly depended on the impact parameter, so the elliptic scaling is tested for light particles at different impact parameter of $^{40}$Ca + $^{40}$Ca collisions in intermediate energy in this paper, and $v_4/v_2^2$ and $v_3/(v_1v_2)$ scalings predicted in RHIC energy are also investigated.

Anisotropic flows are defined as different $n$th harmonic coefficients $v_n$ of the Fourier expansion for the particle invariant azimuthal distribution,

$$\frac{dN}{d\phi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi), \quad (1)$$

where $\phi$ is the azimuthal angle between the transverse momentum of the particle and the reaction plane.

The anisotropic flows $v_n$ can further be expressed in terms of single-particle averages,

$$v_1 = \langle \cos \phi \rangle = \frac{p_x}{p_t}, \quad (2)$$

$$v_2 = \langle \cos(2\phi) \rangle = \frac{p_x^2 - p_y^2}{p_t^2}, \quad (3)$$

$$v_3 = \langle \cos(3\phi) \rangle = \frac{3p_x^2 - 3p_xp_y^2}{p_t^3}, \quad (4)$$

$$v_4 = \langle \cos(4\phi) \rangle = \frac{p_x^4 - 6p_x^2p_y^2 + p_y^4}{p_t^4}, \quad (5)$$

where $p_x$ and $p_y$ are, respectively, the projections of particle transverse momentum parallel and perpendicular to the reaction plane, and $p_t$ is the transverse momentum ($p_t = \sqrt{p_x^2 + p_y^2}$).

2 Theoretical framework

The intermediate energy heavy-ion collision dynamics is complex since both mean field and nucleon-nucleon collisions play the competition role. Furthermore, the isospin-dependent role should be
also incorporated for asymmetric reaction systems. Isospin-dependent quantum molecular dynamics model (IQMD) has been affiliated with isospin degrees of freedom with mean field and nucleon-nucleon collisions\cite{14-19}. The IQMD model can explicitly represent the many-body state of the system and principally contains correlation effects to all orders and all fluctuations, and can well describe the time evolution of the colliding system. When the spatial distance $\Delta r$ is smaller than 3.5 fm and the momentum difference $\Delta p$ between two nucleons is smaller than 300 MeV/c, two nucleons can coalesce into a cluster\cite{14}. With this simple coalescence mechanism, higher degrees of freedom with mean field and nucleon-nucleon interactions model (IQMD) has been affiliated with isospin dependence of elliptic flow per nucleon for light charged particles from 25 MeV/A $^{40}$Ca + $^{40}$Ca collisions with different reduced impact parameters. Squares represent for protons, circles for fragments of $A=2$, up-triangles for $A=3$, down-triangles for $A=4$.

In the model the nuclear mean-field potential is parameterized as

$$ U(\rho, \tau_z) = \alpha \left( \frac{\rho_n}{\rho_0} \right) + \beta \left( \frac{\rho_p}{\rho_0} \right)^\gamma + \frac{1}{2} (1 - \tau_z) V_c + C_{sym} \frac{(\rho_n - \rho_p)}{\rho_0} \tau_z + U_{Yuk}, \quad (6) $$

where $\rho_0$ is the normal nuclear matter density (0.16 fm$^{-3}$), $\rho_n$, $\rho_p$ and $\rho$ are the neutron, proton and total densities, respectively; $\tau_z$ is the $z$th component of the isospin degree of freedom, which equals 1 or $-1$ for neutrons or protons, respectively. The coefficients $\alpha$, $\beta$ and $\gamma$ are the parameters for nuclear equation of state. $C_{sym}$ is the symmetry energy strength due to the density difference of neutrons and protons in nuclear medium, which is important for asymmetry nuclear matter ($C_{sym} = 32$ MeV is used). $V_c$ is the Coulomb potential and $U_{Yuk}$ is Yukawa (surface) potential. In the present work, we take $\alpha = 124$ MeV, $\beta = 70.5$ MeV and $\gamma = 2$ which corresponds to the so-called hard EOS with an incompressibility of $K = 380$ MeV.

3 Results and discussions

Now we move to the calculations. About 200,000 $^{40}$Ca + $^{40}$Ca collisions have been simulated with hard EOS at 25 MeV/nucleon. In this study, we extract the physical results at 200 fm/c for light charged particles when the system has been in freeze-out.

Figure 1 shows the transverse momentum dependence of elliptic flows (left column) and the transverse momentum per nucleon dependence of elliptic flow per nucleon (right column) for light charged particles from 25 MeV/A $^{40}$Ca + $^{40}$Ca collisions with different reduced impact parameters. The reduced impact parameter is defined as $b_{red} = b/b_{max}$ and $b_{max} = R_p + R_t$, where $R_p$ and $R_t$ are the radius of projectile and target respectively. The four rows are for four different reduced impact parameter bins of 0.1 < $b_{red}$ < 0.3, 0.3 < $b_{red}$ < 0.5, 0.5 < $b_{red}$ < 0.7 and 0.7 < $b_{red}$ < 0.9, respectively. Squares represent for protons, circles for fragments of $A=2$, up-triangles for $A=3$, down-triangles for $A=4$. From the figures of the transverse momentum dependence of elliptic flows (left panel), they show that the elliptic flow is positive, and it increases with the increasing $p_t$ and then becomes to decrease with the increasing $p_t$ at a certain $p_t$, and the heavier fragment has a greater $p_t$ of the inflection point. But in the figures of the transverse momentum per nucleon dependence of elliptic flow per nucleon (right panel), the curves for different particles overlap with each other.
This behavior is apparently similar to the number of constituent quarks scaling of elliptic flow versus transverse momentum per constituent quark \( (p_t/n) \) for mesons and baryons which was observed at RHIC [9]. We called it the number of nucleons scaling of elliptic flow [13], which reflects that the formation of the fragments during the reaction obeys the coalescence mechanism. Figure 1 also shows that the elliptic flow increases with impact parameter, which indicates that the fragments prefer more to be emitted in the reaction plane with greater eccentricity at larger impact parameter. But the number of nucleons scaling is broken for fragments with great \( p_t/A \) at large impact parameter of \( 0.7 < b_{red} < 0.9 \), which may indicates that the collective effect on lighter fragments are much stronger than heavier fragments, i.e., lighter fragments are emitted at a higher thermal pressure in the overlap zone.

show that \( v_{2,M} \approx \frac{1}{4} + \frac{1}{2} v_{2,M} \) and \( v_{2,B} \approx \frac{1}{3} + \frac{1}{3} v_{2,B} \), where \( v_{n,q} \) denotes the quark anisotropic flows. The meson and baryon anisotropic flows thus satisfy the scaling relations if the quark anisotropic flows also satisfy such relations, and this ratio is experimentally determined to be 1.2 [23]. In view of the above behaviors of the flows at RHIC energies, we display the anisotropic flows in intermediate energy. The left panel of Figure 2 show the transverse momentum dependence of \( v_2/v_4 \) for light particles from 25 MeV/A \( ^{40}\text{Ca} + ^{40}\text{Ca} \) collisions with different reduced impact parameters. It shows that the ratios of \( v_4/v_2 \) for different fragments are nearly a constant value 0.5 at all the impact parameter bins. If we assume the scaling laws of mesons and baryons are also valid for \( A = 2 \) and 3 nuclear clusters, respectively, then \( v_4/v_2 \) for \( A = 2 \) and 3 clusters indeed give the same value of 1/2 as nucleons(protons). Coincidently the predicted value of the ratio of \( v_4/v_2 \) for hadrons is also 1/2 if the matter produced in ultra-relativistic heavy ion collisions reaches to thermal equilibrium and its subsequent evolution follows the laws of ideal fluid dynamics [25]. It is interesting to note the same ratio was predicted in two different models at very different energies, which is of course worth to be further investigated in near future. In addition, Kolb et al. suggested another scaling relationship between \( v_3 \) and \( v_1 v_2 \) as also insinuated by the coalescence model. The right panel of Fig.2 display the \( p_t \) dependence of the ratio \( v_3/(v_1 v_2) \). It shows that the ratios of \( v_3/(v_1 v_2) \) for different LCP are also scaled and nearly a constant value 0.6. It may be another parameter that can reflect the thermalization of the matter in the overlap zone. It is also worth to be further studied theoretically and experimentally at both RHIC energy and intermediate energy.

The RHIC experimental data also demonstrated a scaling relationship between \( v_4 \) and \( (v_2^2) \) [21]. It has been shown that such scaling relation follows from a naive quark coalescence model [12, 22, 23] that only allows quarks with equal momentum to form a hadron. Denoting the meson anisotropic flows by \( v_{n,M}(p_t) \) and baryon anisotropic flows by \( v_{n,B}(p_t) \), Kolb [22] et al. found that if quarks have no higher-order anisotropic flows than the fourth term, one can...
are as expected almost a constant of 0.5 and 0.6 at different rapidity, respectively. But to the peripheral collisions of $0.7 < b_{red} < 0.9$, the heavier fragments have a little greater ratio value than the lighter ones. That may because the flow difference of the light particles at high $p_t/A$ as shown in the Figure 1. It may reflect that the overlap zone is more approaching to the thermal equilibrium at the moment the lighter fragments emitted for the peripheral collisions.

4 Summary

We have investigated the scaling behaviors of elliptic flows and the ratios of $v_4/v_2^2$ and $v_3/(v_1v_2)$ for light charged particles at different collision parameters for the simulations of $^{40}$Ca + $^{40}$Ca at 25 MeV/nucleon by the IQMD model. It is shown that the number of nucleons scaling of elliptic flow is existed at broad reduced impact parameter except for some deviation at the peripheral collisions $(0.7 < b_{red} < 0.9)$. The ratios of $v_4/v_2^2$ and $v_3/(v_1v_2)$, which may reflect the degree of thermalization of the matter produced in these heavy ion collisions, are almost constant of about 0.5 and 0.6 respectively, for all light fragments at different transverse momentum, rapidity and impact parameter except for some deviation at the peripheral collisions. It may reflect that at the peripheral collisions different fragments may emitted at different thermalization extent of the overlap zone because of the so large eccentricity. Further theoretical and experimental investigations are awaiting.

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