QED Penguin Contributions To Isospin Splittings of Heavy-Light Quark Systems

L. S. Kisslinger
Department of Physics, Carnegie Mellon University
Pittsburgh, PA 15213

T. Goldman
Theoretical Division, Los Alamos National Laboratory
Los Alamos, NM 87545

Z. Li
Department of Physics, Carnegie Mellon University
Pittsburgh, PA 15213

January 25, 2022

Abstract

Recent experiments show that the isospin-violating mass splitting of the B mesons is very small, but the best fits with a QCD sum rule analysis give a splitting of at least 1.0 MeV. The isospin-violating mass splittings of the charmed mesons, on the other hand, are in agreement with experiment. In this letter we show that the inclusion of 2\textsuperscript{nd} kind QED penguin diagrams can account for this discrepancy within the errors in the QCD sum rule method.

PACS numbers: 12.38.Lg, 13.40.-f, 14.40.Jz
1. Introduction

The development of a gauge-invariant method for QED corrections to composite systems[1] has led to a consistent treatment within the QCD sum rule method of the three sources of isospin mass splittings: the current quark mass differences in the QCD Lagrangian, the nonperturbative QCD isospin violations which arise from u-d flavor dependence of vacuum condensates, and electromagnetic effects. The application of the method of Ref.[1] removed serious difficulties in earlier work on the isospin splittings of heavy-light mesons[2]. In a recent application[3] to the charm and bottom pseudoscalar and vector mesons, satisfactory agreement with the experimental mass splittings was obtained within the expected accuracy of the method. However, although the D and D∗ isospin mass splittings were found consistent with experimental data, the theoretical result for the B⁺-B⁰ mass difference was about -1.2 MeV, while the experimental value is 0.35 ± 0.29 MeV[5]. Since the nonperturbative QCD effects tend to be quite small for the B mesons, this difference between theory and experiment seems to be somewhat larger than expected.

Several years ago it was pointed out[6] that there is a novel electromagnetic effect which modifies the quark-gluon vertex in a manner analogous to the penguin mechanism which leads to a weak correction to the quark-gluon vertex. This vertex modification is referred to in Ref.[6] as an “electromagnetic penguin of the second kind”, because the term “electromagnetic penguin” was previously applied to the weak corrections to the quark-photon vertex. From the analytic form of the vertex modification[6], it is evident the the effect become increasingly important with increasing quark mass. We explore here the possibility that the neglect of this (second) QED penguin mechanism is the source of the discrepancy in the B isospin splitting found in Ref.[3].
The main results of the present paper are to point out that it is straightforward to find the largest nonperturbative QCD/QED effect as well as perturbative effects arising from this QED penguin mechanism. We estimate the contribution of nonperturbative processes to the isospin-violating mass splittings of the D and B mesons and find a contribution of approximately 0.5 MeV to the pseudoscalar B-meson isospin mass splitting, while the penguins give a negligible contribution to the D mesons. The perturbative contributions are somewhat larger. The result is a 1.2-2.0 MeV mass splitting for the B and about 0.1 MeV for the D pseudoscalar mesons. From this we conclude that the QED penguins are of the correct magnitude to provide an explanation for the earlier discrepancy.

2. Nonperturbative QCD/QED Penguin Isospin-Violating Mass Splitting

Since the only isospin symmetry violating mechanism being considered in the present work is the (second) QED penguin vertex, it is convenient to take as the starting point of the QCD sum rules the correlators used for the heavy-light quark masses [without isospin violations]. For the pseudoscalar current \( J_5(x) =: \bar{q}(x)i\gamma_5Q(x) : \) the pseudoscalar current correlator is

\[
\Pi_5(p) = i \int d^4xe^{ipx} \langle 0|T(J_5(x)J_5^+(0))|0\rangle,
\]

and

\[
\Pi_{\mu\nu}(p) = i \int d^4xe^{ipx} \langle 0|T(V_\mu(x)V_\nu(0))|0\rangle
\]

\[
= (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi^{(1)}(q^2) + q_\mu q_\nu\Pi^{(0)}(q^2)
\]

is the correlator for the vector current \( V_\mu(x) =: \bar{q}(x)\gamma_\muQ(x) : \). In the QCD sum rule method the correlator \( \Pi(p^2) \) is evaluated in two ways: 1) it is calculated starting from
QCD using the operator product expansion, and 2) it is treated phenomenologically
by a dispersion relation.

In the QCD calculation, the QED penguin isospin violations are obtained by
including the quark-gluon vertex modification, depicted in Fig. 1,

\[ g_{pen} = V_q g, \] (3)

at appropriate places in the QCD evaluation. In Eq. 3, \( g \) is the quark-gluon coupling
constant and \( V_q \), derived in Ref.[6], is the penguin vertex modification.

Keeping terms up to dimension \( D = 5 \), the microscopic nonperturbative QCD
evaluation of \( \Pi(p^2) \) can be written as

\[ \Pi(p^2) = C_I I + C_3 \langle \bar{q}q \rangle + C_4 \langle \alpha_s G^2 \rangle + C_5 \langle \bar{q}(\sigma \cdot G)q \rangle \] (4)

where the coefficient \( C_I \) comes from the short distance correlation calculated using
perturbative quark propagators, \( \langle \bar{q}q \rangle \) is the quark condensate, \( \langle \alpha_s G^2 \rangle \) is the gluon
condensate, \( \langle \bar{q}(\sigma \cdot G)q \rangle \) is the mixed condensate and \( C_n \) are the Wilson coefficients.
\( C_I \) is calculated from the process shown in Fig. 2a plus other two-loop diagrams.
Since the graphs associated with the heavy-quark penguin vertices do not contribute
to the mass splittings of interest in the present work, \( C_I \) can be obtained by a simple
modification of previous calculations[7]. Introducing the variable \( \omega \)

\[ \omega = \frac{q^2 - M_Q^2}{2M_Q} \] (5)

one can show that the coefficients \( C_3 \) and \( C_4 \) are of the form

\[ C_3 \sim \frac{1}{\omega}, \] (6)

\[ C_4 \sim \frac{1}{\omega}. \]
Since we eliminate these D=3 and D=4 processes in our sum rules, as we show below, we do not give the detailed results for $C_3$ and $C_4$. The coefficient $C_5$ from the process of Fig. 2b is

$$C_{5a} = V_Q \frac{1}{8M_Q \omega^2}$$  \hspace{1cm} (7)

for the pseudoscalar currents, and vanishes for the vector current. For the process of Fig. 2c the D=5 mixed condensate coefficient is

$$C_{5b} = -V_q \frac{1}{16\omega^3}$$  \hspace{1cm} (8)

for both the pseudoscalar and vector currents.

Defining the Borel transformation by

$$Bf(\omega) = \lim_{\omega \to \infty, n \to \infty} \left[ \frac{\omega^{n+1}}{n!} \left( -\frac{d}{d\omega} \right)^n \right] f(\omega),$$  \hspace{1cm} (9)

and taking a derivative with respect to $\omega_B$ one obtains for the nonperturbative penguin graphs through D=5

$$\frac{d\Pi^\text{pen}_{np}}{d\omega_B} = -\frac{\langle q(\sigma \cdot G)q \rangle}{16\omega_B^3} \left( V_q + \frac{2V_Q \omega_B}{M_Q} \right)$$  \hspace{1cm} (10)

for the pseudoscalar and

$$\frac{d\Pi^\text{pen}_{np}}{d\omega_B} = -\frac{V_q \langle q(\sigma \cdot G)q \rangle}{16\omega_B^3}$$  \hspace{1cm} (11)

for the vector current. The Borel transformation of the D=3 and D=4 contributions [see Eq.6] are independent of $\omega_B$ and therefore do not contribute. Since we are only considering isospin mass splittings in the present paper, the heavy quark vertex modification, $V_Q$, does not contribute, and we can use the zero-mass vertex modification from Ref.[6] for $V_q$. The vertex modification for $V_q$ in the limit of characteristic momenta larger than the $\Lambda_{QCD}$ is[6]

$$V_q = -\frac{\alpha}{4\pi} e_q^2 \left( 1 + \frac{\pi^2}{3} + ln b + ln b^2 \right),$$  \hspace{1cm} (12)
with \(b = \frac{\Lambda_{QCD}}{M_Q^2}\). (Note that \(\Lambda_{QCD}\) is simply the relevant hadronic scale independent of quark masses and is not necessarily directly related to standard QCD quantities such as \(\Lambda_{MS}\).) The characteristic momentum of the heavy-light quark systems has been taken as the mass of the heavy quark, \(M_Q\).

After the Borel transformation [Eq. (9)], the phenomenological forms of the correlators for the pseudoscalar and vector currents are

\[
\Pi_{phys}^{ps} = f_p^2 \frac{M_p^4}{2M_Q^2} e^{-\frac{\Lambda_p}{\omega_p}} \tag{13}
\]

and

\[
\Pi_{phys}^{v} = f_v^2 \frac{M_v^2}{2M_Q} e^{-\frac{\Lambda_v}{\omega_v}}, \tag{14}
\]

respectively, where \(\Lambda = \frac{M^2 - M_Q^2}{2M_Q}\), \(M\) is the mass of the meson states, and \(\omega_p\) and \(\omega_v\) are the pseudoscalar and vector Borel parameters. Defining \(\Pi^{(0)}\) as the correlator without the penguin processes, our sum rule for the mass shift due to the penguins, \(\Delta M\), is obtained by taking \(d(\Pi - \Pi^{(0)})/d\omega_B\),

\[
\frac{2f_p^2M_p^3\Delta M_p}{M_Q^2\omega_p^2} e^{-\frac{\Lambda_p}{\omega_p}} (\Lambda_p + \frac{M_p^2}{4M_Q}) = \frac{d\Pi^{pen}_p}{d\omega_p} \tag{15}
\]

for the pseudoscalar and

\[
\frac{2f_v^2M_v^2\Delta M_v}{M_Q\omega_v^2} e^{-\frac{\Lambda_v}{\omega_v}} (\Lambda_v + \frac{M_v^2}{2M_Q}) = \frac{d\Pi^{pen}_v}{d\omega_v} \tag{16}
\]

for the vector meson. The penguin expressions on the right-hand side of Eqs. (15) and (16) consist of the sum of the D=5 nonperturbative terms given in Eqs. (10) and (11) and the perturbative contributions given by

\[
\frac{d\Pi^{pen}_{P,T}}{d\omega_B} = -\frac{1}{\pi} \int_{M_Q^2}^{s_0 + M_Q^2} ds e^{-\frac{s}{\omega_B}} \frac{\omega}{2M_Q\omega_B^2} ImC_1(s), \tag{17}
\]

where \(\omega\) is given by Eq. (3) with \(s = q^2\), and \(s_0\) is the threshold parameter for the continuum. The expression for the perturbative correlator, \(C_1\), can be obtained from
the results given in Ref[4] and are found in Ref[2].

\begin{equation}
Im C_I(s) = \frac{\alpha_s V_s}{2 \pi^2} s(1-x)^2 \left[ \frac{9}{4} + 2l(x) + \ln(x) \ln((1-x) \right. \\
\left. + \left( \frac{5}{2} - x - \frac{1}{1-x} \right) \ln(x) - \left( \frac{5}{2} - x \right) \ln(1-x) \right],
\end{equation}

where \( x = \frac{M_Q^2}{s} \) and \( l(x) \) is the Spence function[6].

We use the results of the sum rule analysis without the penguins[4] to fix the parameters needed for our estimate of the mass shifts due to the QED penguins. For the D meson: \( M_D = 1.867, M_Q = 1.5, F_D = 0.13 \) and \( \omega_D = 0.7 \), all in GeV. For the B meson: \( M_B = 5.279, M_Q = 5.0, f_B = 0.095 \) and \( \omega_B = 0.63 \). Neglecting the perturbative contributions and keeping the nonperturbative contributions up to \( D=5 \), we obtain our results for the nonperturbative QED penguin mass shifts from Eqs.[10,12,15,17]:

\begin{align*}
[\Delta M(D^+) - \Delta M(D^0)]_{np} &= 0.07\text{MeV} \\
[\Delta M(B^+) - \Delta M(B^0)]_{np} &= 0.5\text{MeV}.
\end{align*}

The perturbative contributions are sensitive to the choice of the threshold parameter, \( s_0 \). The values of the threshold parameter which we use are taken from a study of the continuum in Ref[4]. The are 1.8-2.1 GeV\(^2 \) for the D and 5.0-6.3 GeV\(^2 \) for the B systems. We find for the perturbative contributions:

\begin{align*}
[\Delta M(D^+) - \Delta M(D^0)]_{pt} &= 0.1 - 0.2\text{MeV} \\
[\Delta M(B^+) - \Delta M(B^0)]_{pt} &= 0.7 - 1.5\text{MeV}
\end{align*}

for this range of the \( s_0 \) values. Note that the largest nonperturbative QED penguin contributions are negligible for the D isospin mass splittings, while they are of the order of the discrepancy in Ref[3] for the B isospin mass splittings. The net B mass splitting resulting from combining the nonpenguin result, \( M(B^+)-M(B^0)=-1.2\text{MeV} \)
from Ref.\[3\], with the penguin results of Eqs.\[19\] and \[20\] is

\[
\left[\Delta M(B^+) - \Delta M(B^0)\right] = 0.0 - 0.8\text{MeV}, \quad (21)
\]

in agreement with experiment\[5\].

3. Conclusions

In the present analysis we have shown that the QED penguins are of the proper magnitude to account for the experimental result that there is essentially no isospin mass splitting in the pseudoscalar B meson systems. From Eq.\[12\], it is evident that the QED penquin contributions are much larger for the B than for the D systems, and that the good results of Ref.\[3\] for the D system are not modified. We conclude that the QCD sum rule method can account for the heavy-light meson isospin mass splitting with the parameters that have been used for light quark systems.

This work is supported in part by National Science Foundation grant PHY-9319641 and in part by the Department of Energy.

References

[1] L. S. Kisslinger and Z. Li, Phys Rev.Lett \textbf{74}, 2168 (1995).

[2] L. S. Kisslinger and Z. Li, Chinese J. Phys. \textbf{32}, 1213 (1994).

[3] L. S. Kisslinger and Z. Li, Carnegie Mellon preprint (1996). This reference contain references to earlier work on the use of QCD sum rules for isospin splitting.

[4] L. S. Kisslinger and Z. Li, Nucl. Phys. \textbf{A570}, 167c (1994).

[5] Particle Data Group, Phys. Rev \textbf{54},1 (1996).
[6] T. Goldman, K. R. Maltman and G. J. Stephenson Jr., Phys. Lett. 228, 396 (1989).

[7] S. C. Generalis, J. Phys. C: Nucl. Part. Phys. 16, 785 (1990); D. J. Broadhurst, Phys. Lett. B101, 423 (1981).

Figure Captions

1. The QED penguin vertex. The oscillating curve represents a photon and the corkscrew curve represents a gluon. The straight line represents a quark.

2. a) The perturbative QED penguin diagram; b), c) the D=5 nonperturbative penguin diagrams. The heavy line represents the massive quark (c or b) and the light line represents the light quark (u or d).
Figure 1
Figure 2a
Figure 2b
Figure 2c