EMBEDDINGS OF MAXIMAL TORI IN CLASSICAL GROUPS AND HASSE PRINCIPLES

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0. Introduction

The aim of this paper is to revisit a topic investigated in [PR 10], [GR 12], [Lee 14], [BLP 15], [BLP 16], [BLP 18], namely the question of embeddings of maximal tori in classical groups. As in the above references, this is reformulated in terms of embeddings of commutative étale algebras with involution in central simple algebras with involution.

If the ground field is a global field of characteristic $\neq 2$, a necessary and sufficient condition for the local-global principle to hold is given in [BLP 18], Theorem 4.6.1. This result is formulated in terms of an obstruction group. The first aim of the present paper is to give a simpler version of Theorem 4.6.1 of [BLP 18], in terms of a different description of the obstruction group, see Theorem 4.1.

We show that if the commutative étale algebra is a product of pairwise independent fields, then the obstruction group is trivial, and hence the local-global principle holds (see Corollary 5.5).

1. Definitions, notation and basic facts

Let $L$ be a field with $\text{char}(L) \neq 2$, and let $K$ be a subfield of $L$ such that either $K = L$, or $L$ is a quadratic extension of $K$.

Étale algebras with involution

Let $E$ be a commutative étale algebra of finite rank over $L$, and let $\sigma : E \rightarrow E$ be a $K$–linear involution. Set $F = \{ e \in E | \sigma(e) = e \}$, and $n = \dim_L(E)$. Assume that if $L = K$, then we have $\dim_K(F) = \lceil \frac{n+1}{2} \rceil$. Note that if $L \neq K$, then $\dim_K(F) = n$ (cf. [PR 10], Proposition 2.1.).

Central simple algebras with involution

Let $A$ be a central simple algebra over $L$. Let $\tau$ be an involution of $A$, and assume that $K$ is the fixed field of $\tau$ in $L$. Recall that $\tau$ is said to be of the first kind if $K = L$ and of the second kind if $K \neq L$; in this case, $L$ is a quadratic extension of $K$. After extension to a splitting field of $A$, any involution of the first kind is induced by a symmetric or by a skew-symmetric form. We say that the involution is of the orthogonal type in the first case, and of the

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symplectic type in the second case. An involution of the second kind is also called unitary involution.

Embeddings of algebras with involution

Let \((E, \sigma)\) and \((A, \tau)\) be as above, with \(n = \dim_L(E)\) and \(\dim_L(A) = n^2\); assume moreover that \(\sigma|_L = \tau|_L\).

An embedding of \((E, \sigma)\) in \((A, \tau)\) is by definition an injective homomorphism \(f : E \to A\) such that \(\tau(f(e)) = f(\sigma(e))\) for all \(e \in E\). It is well–known that embeddings of maximal tori into classical groups can be described in terms of embeddings of étale algebras with involution into central simple algebras with involution satisfying the above dimension hypothesis (see for instance [PR 10], Proposition 2.3).

Let \(\epsilon : E \to A\) be an \(L\)–embedding which may not respect the given involutions. There exists an involution \(\theta\) of \(A\) of the same type (orthogonal, symplectic or unitary) as \(\tau\) such that \(\epsilon(\sigma(e)) = \theta(\epsilon(e))\) for all \(e \in E\), in other words \(\epsilon : (E, \sigma) \to (A, \theta)\) is an \(L\)–embedding of algebras with involution (see [K 69], §2.5. or [PR 10], Proposition 3.1).

For all \(a \in F^\times\), let \(\theta_a : A \to A\) be the involution given by \(\theta_a = \theta \circ \Int(\epsilon(a))\). Note that \(\epsilon : (E, \sigma) \to (A, \theta_a)\) is an embedding of algebras with involution.

Proposition 1.1. The following conditions are equivalent :

\(a\) There exists an \(L\)–embedding \(\iota : (E, \sigma) \to (A, \tau)\) of algebras with involution.

\(b\) There exists an \(a \in F^\times\) such that \((A, \theta_a) \simeq (A, \tau)\) as algebras with involution.

Proof. See [PR 10] Theorem 3.2.

Oriented embeddings

An orthogonal involution \((A, \tau)\) is said to be split if \(A\) is a matrix algebra. Note that if \(n\) is odd, then every orthogonal involution is split. In the case of a nonsplit orthogonal involution, we need an additional notion, called orientation (see [BLP 18], §2).

Assume that \((A, \tau)\) is of orthogonal type and that \(n\) is even. We denote by \(C(A, \tau)\) its Clifford algebra (cf. [KMRT 98] Chap II. (8.7)), and by \(Z(A, \tau)\) the center of the algebra \(C(A, \tau)\). Then \(Z(A, \tau)\) is a quadratic étale algebra over \(K\).

Let \(\Delta(E)\) be the discriminant algebra of \(E\) (cf. [KMRT 98] Chapter V, §18, p. 290). An isomorphism of \(K\)–algebras

\[ \Delta(E) \to Z(A, \tau) \]

will be called an orientation (see [BLP 18], §2).

Let \(u : \Delta(E) \to Z(A, \theta)\) be the orientation of \((A, \theta)\) constructed in [BLP 18], 2.3, and for all \(a \in F^\times\) let \(u_a : \Delta(E) \to Z(A, \theta_a)\) be as in [BLP 18], 2.5. Recall the notion of oriented embedding, introduced in [BLP 18], definition 2.6.1:
Definition 1.2. Let \((A, \tau)\) be an orthogonal involution with \(A\) of even degree, and let \(\nu: \Delta(E) \to Z(A, \tau)\) be an orientation. An embedding \(\iota: (E, \sigma) \to (A, \tau)\) is called an oriented embedding with respect to \(\nu\) if there exist \(a \in F^*\) and \(\alpha \in A^*\) satisfying the following conditions:

(a) \(\text{Int}(\alpha): (A, \theta_a) \to (A, \tau)\) is an isomorphism of algebras with involution such that \(\text{Int}(\alpha) \circ \iota = \iota\).

(b) The induced automorphism \(c(\alpha): Z(A, \theta_a) \to Z(A, \tau)\) satisfies \(c(\alpha) \circ u_a = \nu\).

We say that there exists an oriented embedding of algebras with involution with respect to \(\nu\) if there exists \((\iota, a, \alpha)\) as above. The elements \((\iota, a, \alpha, \nu)\) are called parameters of the oriented embedding.

2. The obstruction groups

A general construction

Recall from [B 20] the following construction. Let \(I\) be a finite set, and let \(C(I)\) be the set of maps \(I \to \mathbb{Z}/2\mathbb{Z}\). Let \(\sim\) be an equivalence relation on \(I\). We denote by \(C_\sim(I)\) the set of maps that are constant on the equivalence classes. Note that \(C(I)\) and \(C_\sim(I)\) are finite elementary abelian 2-groups.

An example

This example will be used in §5. We say that two finite extensions \(K_1\) and \(K_2\) of a field \(K\) are independent if the tensor product \(K_1 \otimes_K K_2\) is a field. Let \(E = \prod_{i \in I} E_i\) be a product of finite field extensions \(E_i\) of \(K\), and let us consider the equivalence relation generated by

\[i \sim j \iff E_i\text{ and }E_j\text{ are independent over }K.\]

Let \(C_{\text{indep}}(E)\) be the quotient of \(C_\sim(I)\) by the constant maps; this is a finite elementary abelian 2-group.

Commutative étale algebras with involution

Let \((E, \sigma)\) be a commutative étale \(L\)-algebra with involution such as in §1. Note that \(E\) is a product of fields, some of which are stable by \(\sigma\), and the others come in pairs, exchanged by \(\sigma\). Let us write \(E = E' \times E''\), where \(E' = \prod_{i \in I} E_i\) with \(E_i\) a field stable by \(\sigma\) for all \(i \in I\), and where \(E''\) is a product of fields exchanged by \(\sigma\). With the notation of §1 we have \(F = F' \times F''\), with \(F' = \prod_{i \in I} F_i\), where \(F_i\) is the fixed field of \(\sigma\) in \(E_i\) for all \(i \in I\). Note that \(E'' = F'' \times F''\). For all \(i \in I\), let \(d_i \in F_i^*\) be such that \(E_i = F_i(\sqrt{d_i})\), and let \(d = (d_i)\).

The subsets \(V_{i,j}\)

Let \(V\) be a set, and for all \(i, j \in I\) let \(V_{i,j}\) be a subset of \(V\). We consider the equivalence relation on \(I\) generated by \(i \sim j \iff V_{i,j} \neq \emptyset\).

Global fields
Assume that $K$ is a global field, and let $V_K$ be the set of places of $K$. If $v \in V_K$, we denote by $K_v$ the completion of $K$ at $v$.

For all $i \in I$, let $V_i$ be the set of places $v \in V_K$ such that there exists a place of $F_i$ above $v$ that is inert or ramified in $E_i$. For all $i, j \in I$, set $V_{i,j} = V_i \cap V_j$, and let $\sim$ be the equivalence relation generated by $i \sim j \iff V_{i,j} \neq \emptyset$.

Let $C(E, \sigma)$ be the quotient of $C_\infty(I)$ by the constant maps; note that $C(E, \sigma)$ is a finite elementary abelian 2-group.

As a consequence of [BLP 18], Theorem 5.2.1, we show the following. Let $(A, \tau)$ be as in §1. For all $v \in V_K$, set $E_v = E \otimes_K K_v$ and $A_v = A \otimes_K K_v$.

**Theorem 2.1.** Assume that for all $v \in V_K$, there exists an oriented embedding $(E_v, \sigma) \to (A_v, \tau)$, and that $C(E, \sigma) = 0$. Then there exists an embedding $(E, \sigma) \to (A, \tau)$.

The proof is given in section §1 as a consequence of Theorem 4.1.

### 3. Embedding data

Let $K$ be a global field, and let $(E, \sigma)$ and $(A, \tau)$ be as above. The aim of this section is to recall some notions from [BLP 18] that we need in the following section.

We start by recalling from [BLP 18] the notion of embedding data. Assume that for all $v \in V_K$ there exists an embedding $(E_v, \sigma) \to (A_v, \tau)$. The set of $(a) = (a_v)$, with $a_v \in (F_v)^\times$, such that for all $v \in V_K$ we have $(A_v, \tau) \simeq (A_v, \theta_{a_v})$, is called a local embedding datum. This is sufficient if $(A, \tau)$ is unitary or split orthogonal; however, when $(A, \tau)$ is nonsplit orthogonal, we need the notion of oriented local embedding data, as follows.

Let us introduce some notation.

**Notation 3.1.** If $M$ is a field, let $\text{Br}(M)$ be the Brauer group of $M$. For $a, b \in M^\times$, we denote by $(a, b)$ the class of the quaternion algebra determined by $a$ and $b$ in $\text{Br}(M)$.

For $K$ and $F$ as above, and for $v \in V_K$, set $F_v = F \otimes_K K_v$, and we denote by $\text{cor}_{F_v/K_v} : \text{Br}(F_v) \to \text{Br}(K_v)$ the corestriction map. Recall that we have a homomorphism $\text{inv}_v : \text{Br}(K_v) \to \mathbb{Q}/\mathbb{Z}$.

**Oriented embedding data**

Assume that $(A, \tau)$ is nonsplit orthogonal, and let $\nu : \Delta(E) \to \mathbb{Z}(A, \tau)$ be an orientation. Suppose that for all $v \in V_K$ there exists an oriented embedding $(E_v, \sigma) \to (A_v, \tau)$. An oriented embedding datum will be an element $(a) = (a_v)$ with $a_v \in (F_v)^\times$ such that for all $v \in V_K$ there exists $\alpha_v \in (A_v)^\times$ such that $(\text{Int}(a) \circ \epsilon, a_v, \alpha_v, \nu)$ are parameters of an oriented embedding, and that moreover the following conditions are satisfied see [BLP 18, 4.1] :

- Let $V'$ be the set of places $v \in V_K$ such that $\Delta(E_v)$ is a quadratic extension of $K_v$. Then $\text{cor}_{F_v/K_v}(a_v, d) = 0$ for almost all $v \in V'_K$. 

We have
\[ \sum_{v \in V_K} \text{cor}_{F_v/K_v}(a^v, d) = 0. \]

We denote by \( \mathcal{L}(E, A) \) the set of oriented local embedding data (of course, the orientation is only required in the nonsplit orthogonal case - if \((A, \tau)\) is unitary or split orthogonal, then \( \mathcal{L}(E, A) \) is by definition the set of local embedding data).

4. A necessary and sufficient condition

Let \( K \) be a global field, and let \((E, \sigma)\) and \((A, \tau)\) be as in the previous sections. The aim of this section is to reformulate the necessary and sufficient condition of [BLP 18]; the only difference is a simpler description of the obstruction group. Suppose that for all \( v \in V_K \) there exists an oriented embedding \( (E^v, \sigma) \to (A^v, \tau) \), and let \((a) = (a^v) \in \mathcal{L}(E, A) \) be an oriented local embedding datum. For all \( i \in I \), recall that \( d_i \in F_i \times \) is such that \( E_i = F_i(\sqrt{d_i}) \).

Let \( C(E, \sigma) \) be the group defined in §2. We define a homomorphism \( \rho_a : C(E, \sigma) \to \mathbb{Q}/\mathbb{Z} \) by setting
\[ \rho_a(c) = \sum_{v \in V_K} c(i) \text{inv}_{v \text{cor}} F_v/K_v(a^v_i, d_i). \]

We have the following

**Theorem 4.1.**
(a) The homomorphism \( \rho \) is independent of the choice of \( (a) = (a^v) \in \mathcal{L}(E, A) \).
(b) There exists an embedding of algebras with involution \((E, \sigma) \to (A, \tau)\) if and only if \( \rho = 0 \).

**Proof.** As we will see, the theorem follows from [BLP 18], Theorems 4.4.1 and 4.6.1, and from the fact that the group \( C(E, \sigma) \) above and the group \( \text{III}(E', \sigma) \) of [BLP 18] are isomorphic, fact that we prove here. Note that part (a) of the theorem can also be deduced directly from [B 20], Proposition 12.4.

Let us recall the definition of \( \text{III}(E', \sigma) \) from [BLP 18], 5.1 and §3. Recall that \( E' = \prod_{i \in I} E_i \) with \( E_i \) a field stable by \( \sigma \), and \( F' = \prod_{i \in I} F_i \), where \( F_i \) is the fixed field of \( \sigma \) in \( E_i \) for all \( i \in I \). As in [BLP 18], §3, let \( \Sigma_i \) be the set of \( v \in V_K \) such that all the places of \( F_i \) above \( v \) split in \( E_i \); in other words, \( \Sigma_i \) is the complement of \( V_i \) in \( V_K \). Let \( m = |I| \). Given an \( m \)-tuple \( x = (x_1, ..., x_m) \in (\mathbb{Z}/2\mathbb{Z})^m \), set
\[ I_0 = I_0(x) = \{ i \mid x_i = 0 \}, \]
\[ I_1 = I_1(x) = \{ i \mid x_i = 1 \}. \]

Let \( S' \) be the set
\[ S' = \{(x_1, ..., x_m) \in (\mathbb{Z}/2\mathbb{Z})^m \mid (\cap_{i \in I_0} \Sigma_i) \cup (\cap_{j \in I_1} \Sigma_j) = V_K \}, \]
and let $S = S' \cup (0, \ldots, 0) \cup (1, \ldots, 1)$. Componentwise addition induces a group structure on $S$ (see [BLP 18], Lemma 3.1.1). We denote by $\text{III}(E', \sigma)$ the quotient of $S$ by the subgroup generated by $(1, \ldots, 1)$.

We next show that the groups $\text{III}(E', \sigma)$ and $C(E, \sigma)$ are isomorphic. The first remark is that with the above notation, we have

$$S' = \{(x_1, \ldots, x_m) \in (\mathbb{Z}/2\mathbb{Z})^m \mid (\bigcup_{i \in I_0} V_i) \cap (\bigcup_{j \in I_1} V_j) = \emptyset\}.$$ 

Let us consider the map $F : (\mathbb{Z}/2\mathbb{Z})^m \to C(I)$ sending $(x_i)$ to the map $c : I \to \mathbb{Z}/2\mathbb{Z}$ defined by $c(i) = x_i$. We have

$$S' = \{(x_1, \ldots, x_m) \in (\mathbb{Z}/2\mathbb{Z})^m \mid (\bigcup_{c(i)=0} V_i) \cap (\bigcup_{c(j)=1} V_j) = \emptyset\}.$$ 

Note that this shows that the following two properties are equivalent:

(a) $(x_i) \in S$

(b) If $i, j \in I$ are such that $V_i \cap V_j \neq \emptyset$, then we have $c(i) = c(j)$.

Recall from §2 the definition of the group $C(E, \sigma)$. We consider the equivalence relation on $I$ generated by $i \sim j \iff V_{ij} \neq \emptyset$, and we denote by $C_\sim(I)$ the set of $c \in C(I)$ that are constant on the equivalence classes.

Since $(a) \implies (b)$, $F$ sends $S$ to $C_\sim(I)$. Moreover, $F$ is clearly injective. Let us show that $F : S \to C_\sim(I)$ is surjective: this follows from the implication $(b) \implies (a)$. Hence we obtain an isomorphism of groups $S \to C_\sim(I)$, inducing an isomorphism of groups $\text{III}(E', \sigma) \to C(E, \sigma)$, as claimed.

The isomorphism $F : \text{III}(E', \sigma) \to C(E, \sigma)$ transforms $\overline{f} : \text{III}(E', \sigma) \to \mathbb{Q}/\mathbb{Z}$ defined in [BLP 18], 5.1 and 4.4 into the homomorphism $\rho : C(E, \sigma) \to \mathbb{Q}/\mathbb{Z}$ defined above. By [BLP 18], Theorem 4.4.1 (see also 5.1) the homomorphism $\overline{f}$ is independent of the choice of $(a) = (a'_i) \in \mathcal{L}(E, A)$, and this implies part (a) of the theorem. Applying Theorem 4.6.1 and Proposition 5.1.1, we obtain part (b).

**Proof of Theorem 2.1** This is an immediate consequence of Theorem 4.1.

5. **An application - independent extensions**

We keep the notation of the previous section; in particular, $K$ is a global field. We say that two finite extensions $K_1$ and $K_2$ of $K$ are independent if the tensor product $K_1 \otimes_K K_2$ is a field (if $K_1$ and $K_2$ are subfields of a field extension $\Omega$ of $K$, then this means that $K_1$ and $K_2$ are linearly disjoint). The first result of the section is the following:

**Theorem 5.1.** Assume that $E = E_1 \times E_2$, where $E_1$ and $E_2$ are independent field extensions of $K$, both stable by $\sigma$. Then $C(E, \sigma) = 0$.

**Proof.** Let us show that $V_1 \cap V_2 \neq \emptyset$. Let $\Omega/K$ be a Galois extension containing $E_1$ and $E_2$, and set $G = \text{Gal}(\Omega/L)$. Let $H_i \subset G_i$ be subgroups of $G$ such that for $i = 1, 2$, we have $E_i = \Omega^{H_i}$ and $F_i = \Omega^{G_i}$. Since $E_i$ is a quadratic extension of $F_i$, the subgroup $H_i$ is of index 2 in $G_i$. By hypothesis, $E_1$ and $E_2$ are linearly disjoint over $K$, therefore $[G : H_1 \cap H_2] = [G : H_1][G : H_2]$. 

Note that $F_1$ and $F_2$ are also linearly disjoint over $K$, hence $[G : G_1 \cap G_2] = [G : G_1][G : G_2]$. This implies that $[G_1 \cap G_2 : H_1 \cap H_2] = 4$, hence the quotient $G_1 \cap G_2 / H_1 \cap H_2$ is an elementary abelian group of order 4.

The field $\Omega$ contains the composite fields $F_1F_2$ and $E_1E_2$. By the above argument, the extension $E_1E_2/F_1F_2$ is biquadratic. Hence there exists a place of $F_1F_2$ that is inert in both $E_1F_2$ and $E_2F_1$. Therefore there exists a place $v$ of $K$ and places $w_i$ of $F_i$ above $v$ that are inert in $E_i$ for $i = 1, 2$.

Let $(A, \tau)$ be a central simple algebra as in the previous sections.

**Corollary 5.2.** Assume that $E = E_1 \times E_2$, where $E_1$ and $E_2$ are independent field extensions of $K$, both stable by $\sigma$, and suppose that for all $v \in V_K$ there exists an oriented embedding $(E_v, \sigma) \rightarrow (A_v, \tau)$. Then there exists an embedding of algebras with involution $(E, \sigma) \rightarrow (A, \tau)$.

**Proof.** This follows from Theorems 4.1 and 5.1.

To deal with the case where $E$ has more than two factors, we introduce a group $C_{\text{indep}}(E, \sigma)$. As in §2, we write $E = E' \times E''$, where $E' = \prod_{i \in I} E_i$ with $E_i$ a field stable by $\sigma$ for all $i \in I$, and where $E''$ is a product of fields exchanged by $\sigma$. Let $\approx$ be the equivalence relation on $I$ generated by

$$i \approx j \iff E_i \text{ and } E_j \text{ are independent over } K.$$

We denote by $C_{\text{indep}}(E, \sigma) = C_{\text{indep}}(E')$ the group constructed in §2 using this equivalence relation.

**Theorem 5.3.** Assume that $C_{\text{indep}}(E, \sigma) = 0$, and suppose that for all $v \in V_K$ there exists an oriented embedding $(E_v, \sigma) \rightarrow (A_v, \tau)$. Then there exists an embedding of algebras with involution $(E, \sigma) \rightarrow (A, \tau)$.

**Proof.** Recall that the group $C(E, \sigma)$ is constructed using the equivalence relation generated by $i \sim j \iff V_i \cap V_j \neq \emptyset$. Theorem 5.1 implies that if $E_i$ and $E_j$ are independent over $K$, then $V_i \cap V_j \neq \emptyset$, hence $i \equiv j \implies i \sim j$. By hypothesis, $C_{\text{indep}}(E, \sigma) = 0$, therefore $C(E, \sigma) = 0$; hence Theorem 2.1 implies the desired result.

**Corollary 5.4.** Assume that there exists $i \in I$ such that $E_i$ and $E_j$ are independent over $K$ for all $j \in I, j \neq i$. Suppose that for all $v \in V_K$ there exists an oriented embedding $(E_v, \sigma) \rightarrow (A_v, \tau)$. Then there exists an embedding of algebras with involution $(E, \sigma) \rightarrow (A, \tau)$.

**Proof.** Since there exists $i \in I$ such that $E_i$ and $E_j$ are independent over $K$ for all $j \in I, j \neq i$, the group $C_{\text{indep}}(E, \sigma)$ is trivial, hence the result follows from Theorem 5.3.

**Corollary 5.5.** Assume that the fields $E_i$ are pairwise independent over $K$. Suppose that for all $v \in V_K$ there exists an oriented embedding $(E_v, \sigma) \rightarrow (A_v, \tau)$. Then there exists an embedding of algebras with involution $(E, \sigma) \rightarrow (A, \tau)$.

**Proof.** This follows immediately from Corollary 5.4.
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