Crystal plasticity in small volumes has been investigated in the last two decades through the compression of micro and nanopillars. In these small volumes, the material strength is size-dependent due to strain gradients generated due to the absence of typical gradient-free dislocation motion and multiplication mechanisms. Furthermore, macroscopic work hardening is replaced by a wealth of abrupt plastic events that originate in both the presence of dislocation correlations, as well as the dramatic small volume effect of mobile dislocations forming geometric steps on free pillar surfaces. Abrupt plastic events are common in avalanche phenomena of various disordered non-equilibrium systems across length scales. Elastic interface depinning phenomena, with which crystals share similar, but not identical, avalanche statistics. However, in typical crystal depinning modeling attempts, avalanches are caused by a direct competition of elastic loading and long-range elastic interactions with quenched disorder, without temporal bursts in the number of elastic degrees of freedom. In contrast, dislocations in nano-crystals can also nucleate, multiply and deposit on free boundaries, naturally causing additional frustration that may influence the statistical avalanche behavior. Here, in the context of an explicit dislocation dynamics model, we show that the competition between two different ways to mediate plastic slip — dislocation nucleation and over-damped dislocation mobility (i.e. dislocation drag) — leads to a distinct rate effect on the avalanche statistics that becomes more pronounced for stress-controlled loading conditions. We interpret the phenomenon in terms of a spatial integration of avalanche behaviors across slip planes. This is a generic mechanism in bifurcation processes such as the Frank-Read nucleation of a single dislocation, and thus we argue that the proposed effect should extend to 3D Discrete Dislocation Dynamics studies have observed experimentally in diameter-D micro and nano pillar compression studies where power law statistics for the sizes of the form \( P(S) = S^{-\tau} \) has been established, where \( \tau \) is a power law exponent that ranges from 1.2 to 1.8, and \( \sim 0 \) resembles an exponential cutoff function. Two dimensional models of atomic displacements and dislocation avalanches simulations have established that \( \tau \sim 1.5 \) or lower, regardless of loading protocols, even if there are still various issues on how the statistics is estimated. Recent 3D Discrete Dislocation Dynamics studies showed that avalanche statistics strongly depend on the loading protocol, where power law statistics with \( \tau \sim 1.5 \) only exhibited in stress-controlled (SC) loading. In addition, recent experiments and continuum modeling have been suggesting that \( \tau \) may take much larger values, with possible reasoning focused on internal disorder of thermal relaxation mechanisms such as cross-slip.

The effect of loading protocols on the statistical behavior of nanopillar compression response has been studied recently, even though the connection between stress rate \( \dot{\sigma} \) (in SC) and strain rate \( \dot{\epsilon} \) (in displacement-controlled loading (DC)) has been lacking at small rates.
In contrast, at large loading rates ($> 10^3/s$) and in the macroscale, it is well known that crystals exhibit a sharp increase of the flow stress due to viscoplastic dislocation drag effects when strain rate is higher than $\sim 5000/s$ [63-70]. This fact has been well verified in DDD simulations [77-79] and originates in the natural competition between the timescale for a dislocation to move a Burgers vector distance at terminal speed and the timescale for dislocation “nucleation” at a source (for example, at a pinned bulk segment – Frank-Read source) [80]. How does this competition translate to the statistical behavior of plasticity avalanches in small volumes at rates smaller than $10^2/s$?

In this paper, we consider a minimal model of crystal plasticity for uniaxial compression in small volumes. By “minimal”, we imply a model that respects: i) the energetics of room temperature crystal deformation being mediated by dislocations gliding along slip planes of at least one slip system ii) the fact that small-volume crystalline plastic deformation originates in nucleation, iii) open boundaries absorb dislocations. In order to maximize statistical sampling and computational efficiency, we perform simulations of 2D samples using a benchmarked dislocation dynamics model [81, 82] that displays the basic phenomenology of nanocrystalline compression: Size effects in the material yield strength and emergent cracking noise. For pure elasticity, SC and DC loading modes can be compared by using $\dot{\sigma} = \nu \dot{\epsilon}$, where $\nu$ is the equivalent modulus for plane strain applications and $\nu$ is the Poison’s ratio. The loading strain-rate $\dot{\epsilon}$ is varied from $10^2/s$ to $10^3/s$. The model crystal is initially stress and mobile-dislocation free. This is analogous to a well-annealed sample, yet with pinned dislocation segments that can act either as dislocation sources (e.g. Frank Read sources) or as obstacles. Dislocations are generated from randomly distributed point sources when the resolved shear stress crosses a random threshold for a finite time $10 ns$ [91]. The nucleated dislocation pair is placed at a distance $L_{nuc} = E/(4\pi(1 - \nu^2))/b/\tau_{nuc}$ and for our system parameters, it is $35 nm$ on average [92]. Randomly distributed point obstacles account for precipitates and forest dislocations on out-of-plane slip systems. Microstructural parameters are chosen based on a previous study [81] that matches various experimental facts.

The timescale competition in this model is generic and present not only in all dislocation dynamics models, but also in generic non-equilibrium processes [83]. Its basic origin can be distilled by considering an imperfect pitchfork bifurcation: $d\sigma/dt = \sigma + \mu \dot{\epsilon} - \dot{\epsilon}^2$, where $\sigma, \mu$ are scalars resembling strain and stress variables, and $\mu$ is a mobility parameter. Neglecting dislocation interactions, on a slip plane without sources but a mobile dislocation, $\mu = \mu_{\text{drift}}$ is negative and the relaxation timescale for every incremental step of $\sigma$ is $\tau_{\text{drift}} = |\mu_{\text{drift}}|^{-1}$. However, if a dislocation source is present without any mobile dislocations, then $\mu = \mu_{\text{nuc}} > 0$ due to the existence of the two states with and without a dislocation pair, and the relaxation timescale during dislocation increments is typically $\tau_{\text{nuc}} = \mu_{\text{nuc}}^{-1}$. Typically, $\tau_{\text{nuc}} \gg \tau_{\text{drift}}$, so increments of $\sigma$ will typically be accommodated by nucleation events. However, if a system of such possible bifurcations interact (if multiple dislocation sources are present), then mutual dislocation interactions may cause a frustrating situation where the disparity of relaxation times may cause a complexity in the evolution dynamics. In our DDD model, the two timescales are concerned with the nucleation and propagation of single dislocations, where the timescale for a dislocation to move by a Burgers vector distance when the applied stress is near the dislocation nucleation stress $B/\tau_{\text{nuc}} \sim 2 \times 10^{-3} ns$ where $B$ is the linear drag coefficient.

Driven by local stress-induced forces [80], dislocations follow athermal dynamics with mobility $\mu_\sigma$. Sample lateral surfaces are free for dislocations to escape from the surfaces. Samples (aspect ratio $h/w = 4$) are assumed to carry single slip plasticity oriented at $30^\circ$ (cf. Fig. 1(a)). Dislocation sources (red dots) and obstacles (blue dots) are located on slip planes, spaced $10 b$ apart, with $b = 0.25nm$ the Burgers vector’s length. The Young’s modulus is assumed $E = 70 GPa$ and $\nu = 0.33$. As it may be seen in Fig. 1(b), a significant difference between two loading rates (SC) can be seen through strain patterning at the same final strain (5%).: plasticity is localized (Fig. 1(c)) for small loading rates while it is relatively uniform for a high loading rate (Fig. 1(d)).

As expected and shown in Fig. 1(a), SC leads to hardening while DC to softening, with the discrepancy becoming dramatic as system size decreases to sub-micron dimensions. Typical size effects ($\gamma \sim w^{-0.4 \pm 0.0}$) are seen for both loading protocols (cf. Fig. 1(b)), despite the fact that dislocation density at 2% strain, shown in the inset, increases with increasing $w$ in different ways depending on the loading conditions. In addition, the flow stress shows a rate dependence for both loading conditions (see Fig. 2(c)), even though DC shows weaker dependence. The dislocation density and flow stress dependences on the rate suggest that SC rates statistically resemble larger DC rates. This conclusion is also supplemented by avalanche statistics (cf. Fig. 2(d)): In SC, event size is defined as $S = \sum_{i} \delta \epsilon; \delta \sigma > \sigma_{\text{threshold}}$; in DC, an event is characterized by stress drops $\delta \sigma$ which lead to temporary displacement overshoots – thus, in order to compare the two loading conditions, a DC strain burst event size is defined as $S = \sum_{i} \delta \epsilon; \delta \sigma > \sigma_{\text{threshold}}$. In this model, dislocation plasticity is loading rate dependent as there are two intrinsic time scales [77]: First, the dislocation nucleation time $\tau_{\text{nuc}}$, which is chosen as $10 ns$ and can be associated to the dislocation multiplication timescale in other models. Second, the ratio between dislocation mobility and material Young’s modulus $B/E$ which is chosen as $10^{-9}$ ns. These parameters
are consistent with recent single-crystal thin film experiments [51–54]. Phenomenology in metallurgy [55–57], suggests that at low rates the flow stress is controlled by dislocation nucleation while above a certain strain rate (≈ 1000 – 5000/s), it is mainly controlled by dislocation drag. (b) Sample stress strain curves of compression at high (10^5/s) and low (10^2/s) stress rates. (c) strain pattern after deformation at low σ∗, (d) strain pattern after deformation at high σ∗.

FIG. 1: The model. (a) The pillar has width w and aspect ratio h/w=4. Single slip system which oriented at 30° relative to y axis is used. Distance between planes is 106 where b = 0.25mm is the magnitude of the Burgers vector. Red dots stand for dislocation sources while blue dots represent dislocation obstacles. (b) Sample stress strain curves of compression at high (10^5/s) and low (10^2/s) stress rates. (c) strain pattern after deformation at low σ∗, (d) strain pattern after deformation at high σ∗.

The avalanche size distribution exponent discrepancy between SC and DC disappears at high stress loadings. The avalanche size distribution exponent clearly changes from -3.5 to -2.1. Increasing σ∗ from 60 MPa to 150 MPa (purple curve) to 150 MPa (blue curve) leads to the exponent changing from -2.5 to -2.1. Increasing B from 10^3 Pa.s to 10^4 Pa.s results in the change of exponent from -2.5 to -2.2.

FIG. 3: SC Rate Effect Crossover. (a): Event statistics for different σ∗ using SC. The effective τ changes from ~ 3.5 for σ = E∗ * 10/s to ~ 1.5 for σ = E∗ * 10^4/s. (b): Effect of dislocation source density ρnuc and mobility B on power law exponent: when σ = E∗ * 10^4/s, changing ρnuc from 60μm^2 (purple curve) to 15μm^2 (blue curve) leads to the exponent changing from -2.5 to -2.1. Increasing B from 10^3 Pa.s to 10^4 Pa.s results in the change of exponent from -2.5 to -2.2.
indicates that there is an intrinsic connection between event statistics and dislocation drag. In order to verify the connection, in Fig. 3(b) red curve, we increase the dislocation mobility $B$ by which the drag effect is enhanced, it is seen that the exponent changes from -2.5 to -2.2. Dislocation drag effect will also magnify when dislocation nucleation effect is weakened due possibly to dislocation cross-slip and other mechanisms (since the main source of plasticity will be the moving of dislocations instead of nucleations of new dislocations). This can be seen in Fig. 3(b) blue curve, when lower dislocation source density is used, the exponent changes from -2.5 to -2.1.

Power law avalanche behavior in the elastic response of disordered systems has been well established [29, 34–36, 38]. In the context of nanopillars, the dislocation ensemble should be the homogeneously disordered elastic system and in this case, the spatial distribution of events on all slip planes should be on average flat or display relatively small fractal exponents [39] in the absence of localization. However, crystal plasticity is known to be unstable to strain localization [40]. In Fig. 4(a) and (b), we plot events spatial distribution along all slip planes for the whole loading process (from small $\epsilon$ to large $\epsilon$ which is represented by the color map from purple to yellow). Fig. 4(a) shows the event spatial distribution which is represented by the color map from purple to yellow. Fig. 4(b) shows the event spatial distribution along all slip planes. The color changes from dark purple to yellow with increasing strain. The color changes from dark purple to yellow with increasing strain. However, crystal plasticity is known to be unstable to strain localization [40].

The onset of quasi-periodic response at small rates, in the absence of overall weakening in this model, is the outcome of the interplay between a timescale competition (as in other elasticity models [27]) and a distinct feature of small volumes: i.e. Free boundaries that may absorb propagating dislocations. Due to this property, it is natural to expect an integration of avalanche behaviors, dependent on the resetting behavior that emerges from absorption and re-nucleation of dislocations at various slip planes. The overall effect can be thought of as originating within a relaxation process (nucleation) that contributes to slip, in addition to mobile dislocation motion. This is the type of coarse-grained dislocation modeling that was pursued in Ref. [41] and its analysis leads to critical power law exponents that are higher than typical ones ($\sim 3/2$). Local heterogeneity biases the integration of the size probability distribution of the conventional depinning models. In this paper, through dislocation dynamics simulations, we connect plasticity local heterogeneity to strain rate effect: the lower loading rate results in the higher heterogeneity which leads to a higher power law exponent. If $P(S, k) \sim S^{-\tau_0} e^{-kS}$, with $k$ a cutoff parameter then this spatiotemporal integration leads to an effective probability distribution:

$$P_{int}(S) = \int_0^{\infty} g(k') P(S, k') dk'$$  \hspace{1cm} (1)$$

where $g(k')$ is the weight factor that characterizes the contribution of various sub-critical, quasi-localized spatial contributions to slip events and depends on the applied loading rate. This weight factor $g(k')$ contains a natural $k' \rightarrow 0$ limit, due to the quasi-periodic resetting, which in many cases takes the form of a power-law [41], thus identifying a novel exponent $g(k') \sim k^\alpha$. Thus, for the critical aspect of $P_{int}(S) \sim S^{-\tau_0-\alpha-1}$, with the ultimate avalanche size exponent being,

$$\tau = \tau_0 + \alpha + 1$$  \hspace{1cm} (2)$$

For the current model, by the analysis of Figs. 4(a, b), we can estimate $\alpha$: If we assume that each 3 nearby slip planes are locally independent from the rest of the system, then the max event size in that area can provide an estimate of the cutoff scale ($k_0 \sim 1/S_0$). Then, the distribution of $k_0$’s provides the exponent. We find that $\alpha \simeq 1$ by plotting the histogram of events that considering $\tau_0 \simeq 1.5$. However, the statistics has not been exhaustive enough to justify a precise identification of these binding energies.
In conclusion, we provided strong evidence through an explicit model of crystal plasticity for nanopillar compression, that the statistical behavior of nanocrystal plasticity forms a novel universality class that is distinct from other plasticity behaviors such as amorphous BMGs and granular systems. We find that the free nanoscale boundaries and the competition between dislocation nucleation and drag conspire to cause the emergence of unconventional quasi-periodic avalanche bursts and higher critical exponents as strain rate decreases. While the investigated strain-rates and the associated transition emerges at relatively high loading rates, the experimentally relevant quasi-static response may be controlled by the same qualitative behavior, or more timescales might be in competition. Plasticity is locally heterogeneous, both spatially and temporally, and this reason lies behind the rate dependence of the avalanche distribution exponent.

We would like to thank I. Groma, P. Ispanovity, R. Maass, L. Ponson for encouraging and insightful comments. This work is supported through awards DOC - No. 1007294R (SP) and DOE-BES de-sc0014109. This work benefited greatly from the facilities and staff of the Super Computing System (Spruce Knob) at West Virginia University.

References

[1] J. R. Greer and J. T. M. De Hosson, Progress in Materials Science 56, 654 (2011).
[2] M. D. Uchic, P. A. Shade, and D. M. Dimiduk, Annual Review of Materials Research 39, 361 (2009).
[3] J. R. Greer, W. C. Oliver, and W. D. Nix, Acta Materialia 53, 1821 (2005).
[4] J. R. Greer and W. D. Nix, Physical Review B 73, 245410 (2006).
[5] I. Ryu, W. Cai, W. D. Nix, and H. Gao, Acta Materialia 95, 176 (2015).
[6] J. Krebs, S. Rao, S. Verheyden, C. Miko, R. Goodall, W. Curtin, and A. Mortensen, Nature materials 16 (2017).
[7] M. F. Doerner and W. D. Nix, Journal of Materials research 1, 601 (1986).
[8] W. D. Nix and H. Gao, Journal of the Mechanics and Physics of Solids 46, 411 (1998).
[9] J. J. Vlassak and W. Nix, Philosophical Magazine A 67, 1045 (1993).
[10] M. R. Begley and J. W. Hutchinson, Journal of the Mechanics and Physics of Solids 46, 2049 (1998).
[11] Y. Wei and J. W. Hutchinson, Journal of the Mechanics and Physics of Solids 51, 2037 (2003).
[12] J. W. Hutchinson, International Journal of Solids and Structures 37, 225 (2000).
[13] M. D. Uchic, D. M. Dimiduk, J. N. Florando, and W. D. Nix, Science 305, 986 (2004).
[14] M. D. Uchic, D. M. Dimiduk, J. N. Florando, and W. D. Nix, MRS Online Proceedings Library Archive 753 (2002).
[15] N. Fleck and J. Hutchinson, Journal of the Mechanics and Physics of Solids 49, 2245 (2001).
[16] E. C. Aifantis, Journal of Engineering Materials and technology 106, 326 (1984).
[17] E. C. Aifantis, Strain gradient interpretation of size effects (Springer, 1999), pp. 299–314.
[18] P. M. Anderson, J. P. Hirth, and J. Lothe, Theory of dislocations (Cambridge University Press, 2017), ISBN 0521864364.
[19] B. Devincre, T. Hoc, and L. Kubin, Science 320, 1745 (2008).
[20] F. F. Csikor, C. Motz, D. Weygand, M. Zaiser, and S. Zapperi, Science 318, 251 (2007).
[21] D. M. Dimiduk, C. Woodward, R. LeSar, and M. D. Uchic, Science 312, 1188 (2006).
[22] M.-C. Miguel, A. Vespignani, S. Zapperi, J. Weiss, and J.-R. Grasso, Nature 410, 667 (2001).
[23] R. Maaß, P. M. Derlet, and J. R. Greer, Scripta Materialia 69, 586 (2013).
[24] M. Kosloski, R. LeSar, and R. Thomson, Physical review letters 93, 125502 (2004).
[25] P. Sammonds, Nature materials 4, 425 (2005).
[26] C. R. Weinberger and W. Cai, Proceedings of the National Academy of Sciences (2008).
[27] J. A. El-Awady, S. I. Rao, C. Woodward, D. M. Dimiduk, and M. D. Uchic, International Journal of Plasticity 27, 372 (2011).
[28] J. T. Uhl, S. Pathak, D. Schorlemmer, X. Liu, R. Swindeman, B. A. Brinkman, M. LeBlanc, G. Tsekenis, N. Friedman, R. Behringer, et al., Scientific reports 5, 16493 (2015).
[29] D. S. Fisher, K. Dahmen, S. Ramanathan, and Y. Ben-Zion, Physical review letters 78, 4885 (1997).
[30] Y. Ben-Zion, Reviews of Geophysics 46 (2008).
[31] A. L. Demirel and S. Granick, Physical review letters 77, 4330 (1996).
[32] S. Papanikolaou, Y. Cui, and N. Ghoniem, Modelling and Simulation in Materials Science and Engineering 26, 013001 (2017).
[33] P. D. Ispanovity, L. Laurson, M. Zaiser, S. Zapperi, and M. J. Alava, Physical review letters 112, 235501 (2014).
[34] M. Zaiser and P. Moretti, Journal of Statistical Mechanics: Theory and Experiment 2005, P08004 (2005).
[35] M. Zaiser, Advances in physics 55, 185 (2006).
[36] K. A. Dahmen, Y. Ben-Zion, and J. T. Uhl, Physical review letters 102, 175501 (2009).
[37] E. Jagla and A. Kolton, Journal of Geophysical Research: Solid Earth 115 (2010).
[38] P. Moretti, M.-C. Miguel, M. Zaiser, and S. Zapperi, Physical Review B 69, 214103 (2004).
[39] Z. Shan, R. K. Mishra, S. S. Asif, O. L. Warren, and A. M. Minor, Nature materials 7, 115 (2008).
[40] V. Bulatov and W. Cai, Computer simulations of dislocations, vol. 3 (Oxford University Press on Demand, 2006), ISBN 0198526148.
[41] S. Papanikolaou, D. M. Dimiduk, W. Choi, J. P. Sethna, M. D. Uchic, C. F. Woodward, and S. Zapperi, Nature 490, 517 (2012).
[42] E. Jagla, Physical Review E 81, 046117 (2010).
[43] E. Jagla, Physical Review E 76, 046119 (2007).
[44] Y. Cui, G. Po, and N. Ghoniem, Physical review letters 117, 155502 (2016).
[45] J. Weiss, F. Laheie, and J. R. Grasso, Journal of Geo-
physical Research: Solid Earth 105, 433 (2000).

[46] C. Fressengeas, A. Beaudoin, D. Entemeyer, T. Lebedkina, M. Lebyodkin, and V. Taupin, Physical Review B 79, 014108 (2009).

[47] J. Weiss, T. Richeton, F. Louchet, F. Chmelik, P. Dobron, D. Entemeyer, M. Lebyodkin, T. Lebedkina, C. Fressengeas, and R. J. McDonald, Physical review B 76, 224110 (2007).

[48] N. Friedman, A. T. Jennings, G. Tsekenis, J.-Y. Kim, M. Tao, J. T. Uhl, J. R. Greer, and K. A. Dahmen, Physical review letters 109, 095507 (2012).

[49] P. D. Ispánovity, I. Groma, G. Györgyi, F. F. Csikor, and D. Weygand, Physical review letters 105, 085503 (2010).

[50] M.-C. Miguel, A. Vespignani, M. Zaiser, and S. Zapperi, Physical review letters 89, 165501 (2002).

[51] M.-C. Miguel, A. Vespignani, S. Zapperi, J. Weiss, and J.-R. Grasso, Materials Science and Engineering: A 309, 324 (2001).

[52] P. Moretti, M.-C. Miguel, M. Zaiser, and S. Zapperi, Physical Review B 69, 214103 (2004).

[53] S. Zapperi and M. Zaiser, Materials Science and Engineering: A 309, 348 (2001).

[54] L. Laurson and M. J. Alava, Physical Review E 74, 066106 (2006).

[55] L. Laurson, M.-C. Miguel, and M. J. Alava, Physical review letters 105, 015501 (2010).

[56] M. Ovaska, L. Laurson, and M. J. Alava, Scientific reports 5, 10580 (2015).

[57] I. Groma, G. Györgyi, and P. Ispanovity, Physical review letters 108, 269601 (2012).

[58] G. Tsekenis, N. Goldenfeld, and K. A. Dahmen, Physical review letters 106, 105501 (2011).

[59] O. U. Salman and L. Truskinovsky, Physical review letters 106, 175503 (2011).

[60] O. U. Salman and L. Truskinovsky, International Journal of Engineering Science 59, 219 (2012).

[61] S. Kale and M. Ostoja-Starzewski, Physical review letters 112, 045503 (2014).

[62] P. D. Ispánovity, Á. Hegyi, I. Groma, G. Györgyi, K. Rat- ter, and D. Weygand, Acta Materialia 61, 6234 (2013).

[63] T. A. Parthasarathy, S. I. Rao, D. M. Dimiduk, M. D. Uchic, and D. R. Trinkle, Scripta Materialia 56, 313 (2007).

[64] S. I. Rao, D. Dimiduk, T. A. Parthasarathy, M. Uchic, M. Tang, and C. Woodward, Acta Materialia 56, 3245 (2008).

[65] M. Stricker and D. Weygand, Acta Materialia 99, 130 (2015).

[66] T. Crosby, G. Po, C. Erel, and N. Ghoniem, Acta Materialia 89, 123 (2015).

[67] H. Swan, W. Choi, S. Papanikolaou, M. Bierbaum, Y. S. Chen, and J. P. Sethna, Materials Theory 2, 5 (2018), URL https://doi.org/10.1186/s41313-018-0012-x.

[68] G. Tsekenis, J. T. Uhl, N. Goldenfeld, and K. A. Dahmen, EPL (Europhysics Letters) 101, 36003 (2013).

[69] J. Weiss, W. B. Rhouma, T. Richeton, S. Dechanel, F. Louchet, and L. Truskinovsky, Physical review letters 114, 105504 (2015).

[70] P. Zhang, O. U. Salman, J.-Y. Zhang, G. Liu, J. Weiss, L. Truskinovsky, and J. Sun, Acta Materialia 128, 351 (2017).

[71] R. Maas, M. Wraith, J. Uhl, J. Greer, and K. Dahmen, Physical Review E 91, 042403 (2015).

[72] G. Sparks and R. Maas, Acta Materialia 152, 86 (2018).

[73] R. Armstrong and S. Walley, International Materials Reviews 53, 105 (2008).

[74] W. Murphy, A. Higginbotham, G. Kimmia, B. Barbrel, E. Brings, J. Hawrelia, R. Kodama, M. Koenig, W. McBarron, M. Meyers, et al., Journal of Physics: Condensed Matter 22, 065404 (2010).

[75] W. Tong, R. J. Clifton, and S. Huang, Journal of the Mechanics and Physics of Solids 40, 1251 (1992).

[76] R. J. Clifton, International Journal of Solids and Structures 37, 105 (2000).

[77] P. K. Agnihotri and E. Van der Giessen, Mechanics of Materials 90, 37 (2015).

[78] H. Song, V. Deshpande, and E. Van der Giessen, Proc. R. Soc. A 472, 20150877 (2016).

[79] J. Hu, Z. Liu, E. Van der Giessen, and Z. Zhuang, Extreme Mechanics Letters 17, 33 (2017).

[80] J. P. Hirth and J. Lothe (1982).

[81] S. Papanikolaou, H. Song, and E. Van der Giessen, Journal of the Mechanics and Physics of Solids 102, 17 (2017).

[82] E. Van der Giessen and A. Needleman, Modelling and Simulation in Materials Science and Engineering 3, 689 (1995).

[83] P. S. Sahni, D. J. Srolovitz, G. S. Grest, M. P. Anderson, and S. Safran, Physical Review B 28, 2705 (1983).

[84] Y. Xiang and J. Vlassak, Acta Materialia 54, 5449 (2006).

[85] L. Nicola, Y. Xiang, J. Vlassak, E. Van der Giessen, and A. Needleman, Journal of the Mechanics and Physics of Solids 54, 2089 (2006).

[86] P. Follansbee and U. Kocks, Acta Metallurgica 36, 81 (1988).

[87] R. Clifton, Applied mechanics reviews 43, S9 (1990).

[88] K. A. Dahmen, Y. Ben-Zion, and J. T. Uhl, Nature Physics 7, 554 (2011).

[89] D. S. Fisher, Physics reports 301, 113 (1998).

[90] R. Asaro and V. Lubarda, Mechanics of solids and materials (Cambridge University Press, 2006), ISBN 1139448994.

[91] this process mimics the physical process of Frank Read source multiplication

[92] thus, the plastic strain generate by a nucleated dipole (which will be put at 35nm apart along a 60 degree slip plane) is \( b \ast \sin 60^\circ / h \ast 35nm \ast \cos 60^\circ / w \approx 10^{-6} \)