Submillimetre-wave gravitational lenses and cosmology

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ABSTRACT

One of the most direct routes for investigating the geometry of the Universe is provided by the numbers of strongly magnified gravitationally lensed galaxies as compared with those that are either weakly magnified or de-magnified. In the submillimetre waveband the relative abundance of strongly lensed galaxies is expected to be larger as compared with the optical or radio wavebands, both in the field and in clusters of galaxies. The predicted numbers depend on the properties of the population of faint galaxies in the submillimetre waveband, which was formerly very uncertain; however, recent observations of lensing clusters have reduced this uncertainty significantly and confirm that a large sample of galaxy–galaxy lenses could be detected and investigated using forthcoming facilities, including the FIRST and Planck Surveyor space missions and a large ground-based millimetre/submillimetre-wave interferometer array (MIA). We discuss how this sample could be used to impose limits to the values of cosmological parameters and the total density and form of evolution of the mass distribution of bound structures, even in the absence of detailed lens modeling for individual members of the sample. The effects of different world models on the form of the magnification bias expected in sensitive submillimetre-wave observations of clusters are also discussed, because an MIA could resolve and investigate images in clusters in detail.

Key words: galaxies: evolution – cosmology: observations – cosmology: theory – gravitational lensing – large-scale structure of universe – radio continuum: galaxies

1 INTRODUCTION

The importance of the statistical properties of gravitational lenses for investigating the geometry of the Universe has been discussed by Gott, Park & Lee (1989) and Fukugita et al. (1992). A particularly promising route to constraining the value of the cosmological constant $\Omega_\Lambda$ (Carroll, Press & Turner 1992; Kochanek 1996) would be offered by a determination of the relative abundance of lensed and unlensed galaxies and quasars. Important developments are expected in this field when the next generation of submillimetre-wave telescopes becomes available, because the abundance of lensed images, formed by both intervening galaxies and clusters, is expected to be significantly larger in this waveband as compared with other wavebands (Blain 1996a,b, 1997b).

A large sample, of perhaps several hundred gravitational lenses (Blain 1997a,c), could be compiled by combining the results of large-area extragalactic surveys using the Planck Surveyor (Bersanelli et al. 1996) and FIRST (Beckwith et al. 1993) space-borne telescopes with sub-arcsec-resolution imaging of the detected sources using a large ground-based millimetre/submillimetre-wave interferometer array (MIA; Brown 1996; Downes 1996). The probability of lensing, and hence the size of the sample, depends on the world model and the normalisation and form of evolution of the mass distribution of lensing galaxies. About 100 fainter lenses could also be detected in a small-area survey using an MIA alone (Blain 1996a,b). Here the prospects for using such large samples of galaxy–galaxy lenses to investigate the values of cosmological parameters and the evolution of large-scale structures are discussed. We also consider whether the magnification bias of distant galaxies in the field of a cluster of galaxies (Broadhurst, Taylor & Peacock 1995; Blain 1997b), which also depends on the world model, could be used to impose similar constraints.

Existing observations in the optical waveband have allowed constraints to be imposed on the values of cosmological parameters. The observed numbers of lensed quasars appear to rule out world models with large values of $\Omega_\Lambda$; Kochanek (1996) derives $\Omega_\Lambda < 0.66$ at a confidence level of 95% in a flat world model. Observations of lensed images in clusters appear to favour a flat world model with a value of $\Omega_\Lambda \approx 0.7$ (Fort, Mellier & Dantel-Fort 1997; Wu & Mao 1996).

The assumptions about world models and the form of evolution of distant galaxies in the present study are discussed in Section 2. The potential of submillimetre-wave observations of lensing by galaxies and clusters for inves-
tigating cosmology are discussed in Sections 3 and 4 respectively. In the sections of the paper that involve lensing by galaxies rather than clusters, galaxies that are multiply imaged and significantly magnified by lensing are described as ‘lensed’, those that are not significantly magnified are described as ‘unlensed’. A value of Hubble’s constant \( H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \) is assumed.

2 THE UNDERLYING MODELS

2.1 Faint galaxies in the submillimetre waveband

The form of the counts of galaxies in the submillimetre waveband is uncertain at present. Predictions can be made by extrapolating the luminosity function of galaxies detected by the IRAS satellite at a wavelength of 60 \( \mu \text{m} \) (Saunders et al. 1990) out to large redshifts (Blain & Longair 1993a); however, detailed knowledge will only be provided by the results of the first blank-field surveys in the submillimetre waveband (Blain & Longair 1996). Evidence from observations in a range of wavebands indicates that the populations of star-forming and active galaxies can be adequately described by pure luminosity evolution of the local galaxy population with an approximate form \( (1 + z)^3 \) out to a redshift \( z \approx 2 \) (Dunlop & Peacock 1990; Oliver, Rowan-Robinson & Saunders 1992; Hewett, Foltz & Chaffee 1993; Lilly et al. 1996). In this paper the population of IRAS galaxies is assumed to undergo this form of evolution out to \( z = 2 \), and then to retain its enhanced luminosity to a cutoff redshift \( z = 5 \). The same form of evolution was used in models 3, A and I2 in Blain & Longair (1996), Blain (1997a) and Blain (1997b) respectively. In observations carried out at a wavelength of 850 \( \mu \text{m} \) after this paper was submitted, Smail, Ivison & Blain (1996) confirmed that very strong evolution of the population of distant galaxies is taking place in the submillimetre waveband. A surface density of galaxies about 5 times larger than that predicted using the above model appears to be most consistent with their preliminary conclusions.

2.2 World models

The Friedmann equation for the expansion rate takes the form,

\[
\dot{R}^2 = H_0^2 \left[ \frac{\Omega_m}{R} + \Omega_\Lambda R^2 \right] - A^2 c^2,
\]

in the notation used here. The scale factor is represented by \( R \), the curvature parameter \( A = (H_0/c) \sqrt{\Omega_m + \Omega_\Lambda - 1} \) and the density parameter of the universe at the present epoch is represented by \( \Omega_0 \). In a flat world model \( \Omega_0 + \Omega_\Lambda = 1 \). The world models used here are listed in Table 1; four flat models, three open models with \( \Omega_\Lambda = 0 \), and one closed model with a large value of \( \Omega_\Lambda \).

Table 1: A summary of the world models used in this paper. \( \Omega_0 \) and \( \Omega_\Lambda \) represent the age of the Universe and the density parameters of metals (Section 2.3) respectively at the present epoch.

| \( \Omega_0 \) | \( \Omega_\Lambda \) | \( \Omega_m/10^{-4} \) | Geometry |
|---|---|---|---|
| 1.0 | 0.0 | 13.1 | Flat |
| 0.7 | 0.3 | 14.7 | Flat |
| 0.3 | 0.7 | 18.9 | Flat |
| 0.16 | 0.84 | 22.3 | Flat |
| 0.7 | 0.0 | 14.0 | Open |
| 0.3 | 0.0 | 15.8 | Open |
| 0.16 | 0.0 | 16.9 | Open |
| 0.16 | 1.2 | 28.5 | Closed |

Figure 1. The intensity of diffuse background radiation predicted by the model of galaxy evolution employed here (Section 2.2) in each of the eight world models listed in Table 1 (smooth curves). The background intensities and limits derived from observations using the FIRAS instrument on the COBE satellite (Fixsen et al. 1994) by Puget et al. (1996) and Fixsen et al. (1996) are represented by solid-edged boxes and points with error bars respectively. The residuals (Table 4; Fixsen et al. 1996) describe the signal that remains after models of both the cosmic microwave background radiation intensity and galactic emission are subtracted from the all-sky FIRAS data.

2.3 Comparison with observations

The intensity of diffuse extragalactic background radiation and the abundance of heavy elements at the present epoch, \( \Omega_m \), expected in each of the world models listed in Table 1 are compared in Fig. 1 and the final column of Table 1 respectively; for details of the calculations see Blain & Longair (1993a,b). The abundances were calculated assuming that all of the energy emitted by galaxies appears in the far-infrared waveband and is generated by the transmutation of hydrogen into heavy elements in massive stars. The efficiency of the conversion of rest mass of consumed hydrogen into energy in this process is assumed to be 0.7%. These calculations should yield a conservatively large estimate of \( \Omega_m \), as the contributions made to dust heating, and hence to the far-infrared luminosities of galaxies, by both active galactic nuclei and non-helium-burning stars are not considered. In a recent review of the properties of luminous infrared galaxies Sanders & Mirabel (1996) reported that about 15% contain active nuclei, while Calzetti et al. (1995) noted that about 30% of the luminosity of star-forming galaxies in the far-infrared waveband can be generated by the non-ionizing radiation from lower-mass stars.

The predicted background radiation intensities in Fig. 1 are all broadly consistent with the results of Puget et al.
The lensing optical depth then the values of $\Omega_m$ at the present epoch is about 0.05 (Bristow & Phillipps 1994), in the closed model. If the density parameter in baryons at (1996), although a generally larger flux density is predicted in stars by the present epoch. This range of values does not seem unreasonable, as the mass of carbon and heavier elements in the sun is approximately 2.4% of the mass of hydrogen (Savage & Sembach 1996). Hence, the predictions of neither background intensities nor metal abundances can be used to discriminate strongly between the different world models.

3 GALAXY–GALAXY LENSING

3.1 Introduction and formalism

Predictions of the counts of galaxy–galaxy gravitational lenses in the submillimetre waveband were made by Blain (1996b) in an Einstein–de Sitter world model, based on existing formalism (Peacock 1982; Pei 1995). More recently, world models including $\Omega_m \neq 0$ (Fukugita et al. 1992) have been considered (Blain 1997a); however, apart from noting that the surface density of lensed galaxies is expected to increase in such a world model, the properties of the resulting population of lensed galaxies were not discussed in any detail.

The probability that a galaxy at redshift $z$ is lensed into multiple images by a galaxy at a smaller redshift and magnified by a factor between $A$ and $A + dA$ is $a(z) A^{-3} dA$. The function $a$ incorporates all the details of the world model, which determines the relative distances between observer, lens and source, any evolution of the mass distribution of the population of lensing galaxies and the density parameter in compact lensing objects at the present epoch $\Omega_L$.

The lensing optical depth $\tau \approx a/8$. The mass distribution of lensing galaxies is assumed either to remain fixed with the form it takes at the present epoch or to evolve according to the prescription of the Press–Schechter formalism (Press & Schechter 1974), in which the mass distribution of collapsed objects is derived by assuming that initial gaussian density fluctuations evolve according to the theory of linear perturbation growth in an expanding universe (Peebles 1993). The comoving space density and mean mass of lensing galaxies are expected to increase and decrease respectively with increasing redshift in the Press–Schechter model, and so the probability of lensing is predicted to be smaller as compared with that derived in a non-evolving model (Blain 1996b).

Estimates of the form of $a$ in each of the world models listed in Table 1 are presented in Fig. 2(a) & (b), assuming evolving and non-evolving mass distributions respectively. $a$ is normalised to match the predictions of Pei (1993), which were derived in an Einstein–de Sitter world model with a non-evolving mass distribution. These predictions are similar to those made by a range of different authors. The predictions for each world model are smaller in the evolving model. The redshift dependence of $a$ is determined both by the values of $\Omega_0$ and $\Omega_L$ and by the form of evolution of the mass distribution, while its absolute normalisation is determined by the value of $\Omega_L$. The prediction plotted in Fig. 2 due to Pei (1995), for which the lensing normalisation is about a factor of 2 larger than that assumed here, corresponds to $\Omega_L = 0.16$ in lensing objects, distributed as $\Omega_L = 0.01, 0.05$ and 0.10 in compact objects, galaxies and clusters respectively. The galaxies and clusters are modeled as singular isothermal spheres. Recent work by Müller, Flores & Primack (1997) indicates that the probability of galaxy–galaxy lensing could be larger than that predicted by a model of singular isothermal spheres if the contribution of a disk component in galaxies is included. Hence, the probability of lensing assumed here corresponds to a density of singular isothermal spheres $\Omega_L = 0.08$. If this is an overestimate, then the numbers of strongly lensed images predicted in this section will be too large, but probably not by a factor greater than about 2.

3.2 Predicted counts of lensed galaxies

The counts of lensed galaxies can be predicted by combining the probabilities of galaxy–galaxy lensing calculated above
with models of the population of unlensed background galaxies (Blain 1996b). The counts expected in each of the world models listed in Table 1 are compared in Fig. 3, assuming an evolving population of lensing galaxies; in Fig. 4 the corresponding ratios between the counts of lensed and unlensed galaxies are compared, both with and without assuming an evolving population of lensing galaxies.

The magnitude of the ratio of counts of lensed and unlensed galaxies is expected to be modified significantly by changing the values of $\Omega_0$ and $\Omega_{\Lambda}$, particularly by increasing the value of $\Omega_{\Lambda}$, and by including evolution of the lensing galaxies. However, the shapes of all 16 curves presented in Figs 4(a) & (b) are very similar. The ratio of the counts of lensed and unlensed galaxies always has a maximum at the flux density at which the unlensed counts steepen significantly as compared with the counts expected in a Euclidean model (Blain 1996b). The predicted effect on the counts of increasing $\Omega_{\Lambda}$ is similar to that expected due to either reducing the strength of evolution of lensing galaxies or increasing the value of $\Omega_{\Lambda}$. Hence, if a statistical sample of lensed galaxies can be compiled in the submillimetre waveband, their counts compared with unlensed galaxies can be determined, then a joint limit to the values of $\Omega_0$, $\Omega_{\Lambda}$, and $\Omega_{\Lambda}$ to the evolution of the mass function of lensing galaxies can be imposed.

Additional information would be required to distinguish between the effects of these factors. The shape of the unlensed counts could be used to provide information about both the form of evolution of distant galaxies (Blain & Longair 1996) and the world model (Fig. 3). The redshifts and detailed image structures of individual lenses, derived from follow-up observations in the near-infrared, optical and submillimetre wavebands, could be combined with lens modeling (for example Williams & Lewis 1996) to impose additional independent limits to cosmological parameters, subject to knowledge of the mass distribution within the lensing galaxies.

### 3.3 Observing submillimetre-wave lenses

Considerable progress is being made in observational cosmology in the submillimetre waveband. The counts of galaxies in the submillimetre waveband with 850-µm flux densities of several mJy will soon be determined for the first time in a blank-field survey using the new SCUBA bolometer array detector (Cunningham et al. 1994) at the James Clerk Maxwell Telescope (JCMT), which operates at wavelengths of 450 and 850 µm (Blain & Longair 1996). Smail, Ivison & Blain (1997) have recently determined an estimate of these counts in the fields of lensing clusters. However, because the detection of order $10^2$ galaxies is expected in a long-term blank-field SCUBA survey, and the fraction of lenses at a flux density of a few mJy is not expected to be larger than about $10^{-2}$, it is unlikely that more than a handful of lensed galaxies will be detected in a deep blank-field SCUBA survey; the 6-arcsec angular resolution of the JCMT will also not resolve the different components of multiply-imaged lenses (Blain 1996b).

The next generation of submillimetre-wave telescopes – MIAs, other ground-based telescopes, FIRST and Planck Surveyor – will make the detection and investigation of lensed galaxies a key part of submillimetre-wave astronomy. A more detailed discussion of the strategy for detection and identification of lensed galaxies in future submillimetre-wave surveys is provided by Blain (1997a,c). First, the strategy relies on the detection of galaxies with flux densities corresponding to the peak of the curves in Fig. 4 in large-area surveys using the Planck Surveyor and FIRST telescopes. This ensures that the fraction of lensed galaxies in the total sample is as large as possible. Secondly, a large MIA would be used for high-resolution follow-up observations to identify the galaxies that show signs of arcs and multiple images. Thirdly, the MIA-selected candidates would also be observed in the near-infrared, optical and radio wavebands in order to determine the detailed properties of the lens and source.

In a large-area survey using the FIRST telescope, and an all-sky survey using Planck Surveyor, catalogues containing of order $10^4$ and $10^5$ galaxies and AGN with flux densities greater than about ten and several tens of mJy respectively are expected (Blain 1997a,c). These large catalogues will yield accurate submillimetre-wave source counts. The catalogues will not be affected significantly by source confusion (Blain, Ivison & Smail 1997) – a typical separation of about 10 arcmin is expected for sources brighter than 10 mJy (Fig. 3). However, because the resolution of these telescopes is coarse as compared with ground-based submillimetre-wave telescopes, the sources will not be resolved. All of the detected galaxies, which are selected solely because of their large flux densities in the submillimetre waveband, are candidates for further investigation as potential lenses; however, an optimized selection could be made if submillimetre/far-infrared colours were determining for these galaxies in a FIRST survey (Blain 1997a).

The sub-arcsecond angular resolution and excellent sensitivity of an MIA will allow the rapid resolution and identification of any sources containing structures that resemble arcs or multiple images in the FIRST and Planck Surveyor catalogues. Most catalogued sources are expected to be distant active or vigorous star-forming galaxies, and so are un-
likely to be extended or to contain significant substructure that could be misinterpreted as a signature of lensing. The far-infrared luminosity of merging luminous starburst galaxies at small redshifts is dominated by the emission from a single compact core region, which is typically only several hundred parsecs across (Solomon et al. 1997). Hence, the detection of arcs or multiple images in an MIA follow-up observation would provide a strong indication that a lensed galaxy had been found. Based on the size of the observation, this would provide a strong indication that a lensed galaxy had been found. Based on the size of the FIRST and Planck Surveyor catalogues and the estimated fractions of lensed galaxies shown in Fig. 4, a catalogue containing several hundreds of galaxy–galaxy lenses across a large area of sky could be compiled.

The selection method ensures that the flux densities of the candidates at a wavelength of 850 \(\mu\)m will be at least 10 mJy, corresponding to a bolometric luminosity of order \(10^{39} L_{\odot}\). Hence, follow-up observations of these very luminous sources in other wavebands should be relatively easy. The candidates are expected to have properties similar to that of the archetypal lensed ultraluminous galaxy IRAS F10214+4724 (Rowan-Robinson et al. 1991), which has a B-band magnitude of about 22. Lensed structures were clearly detected in near-infrared images of F10214 using a 4 m-class telescope in a 90-minute integration (Close et al. 1995). The flux densities of the candidate sources are expected to be sufficiently large for many of them to appear in the IRAS faint source catalogue; however, they will constitute only a very small proportion of the total number of IRAS sources.

The accurate counts of lensed and unlensed galaxies derived from this programme of observations could be compared with model predictions, such as those shown in Figs 3 and 4, in order to investigate the form of the world model and the growth of cosmic structure. As discussed by Blain (1997a,c), this programme would require a large, but not impractical, amount of observing time; several months of dedicated observations with an MIA in order to identify lensed structures, and many nights of spectroscopic observations using large telescopes in the near-infrared and optical wavebands in order to determine the redshifts of the detected sources and lensing galaxies.

4 LENSING BY CLUSTERS

The effects of lensing by a cluster on the population of background galaxies can be considered in terms of a magnification bias, which modifies the counts of lensed images as compared with background galaxies (Borgeest et al. 1991; Broadhurst, Taylor & Peacock 1995). The bias is introduced because both the flux densities and mean separations of the images of background galaxies are increased by a lens, and its magnitude is determined by the steepness of the slope of the counts of background galaxies. In general these counts are expected to be steeper in the submillimetre waveband as compared with the optical waveband (Blain 1997b), and so the bias is expected to be relatively large in the submillimetre waveband. The angular size of the region within which the bias is expected to be significant depends on the relative size of the observer–lens and lens–source distance. The forms of both the counts of background galaxies (Fig. 3; Blain 1997b) and these distances are expected to depend on the form of the world model, and so detailed observations of the properties of lensed images in clusters could be used to constrain the values of cosmological parameters.

The predicted surface density and magnification bias of lensed images in a rich cluster are compared in Fig. 5 as a function of flux density and position in the cluster for four world models selected from the list in Table 1. The expected number of detectable images within each radius are also shown. The details of the calculations are described in Blain (1997b). The form of the magnification bias is predicted to differ considerably between world models, particularly at flux densities of about 100 mJy and radii of about 30 arcsec, for which the bias factor is expected to exceed 100. However, the surface density of images is not expected to be sufficiently large for any images to be detected in this region, as shown by the dot-dashed contours in Fig. 5. Only at fainter flux densities of between about 0.1 and 1 mJy, for which the magnification biases are predicted to be less dramatic, would a reasonable number of detectable images be expected. Note that Smail, Ivison & Blain (1997) detected six sources within 1.1 arcmin of the core of two clusters at a 850-\(\mu\)m flux density limit of about 2 mJy using a 30-hour integration at the JCMT.
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Figure 5. The expected lensing properties of a cluster similar to Abell 2218 at $z = 0.171$ as a function of the flux density of the images and their angular distances from the core of a lensing cluster $\theta_1$. A spherical approximation to the mass distribution of Abell 2218 is used (Natarajan & Kneib 1996); this reproduces the total mass and approximate density profile of the cluster. The counts of lensed images of distant dusty galaxies (dashed lines), the magnification bias (solid lines for positive bias; dotted lines for negative bias) and the number of images expected within a radius $\theta_1$ (dot-dashed lines) are compared in four different world models (Table 1).

A large MIA should be able to map a field 1 arcmin in radius at a 5-$\sigma$ flux density limit of 0.1 mJy at a wavelength of 850 $\mu$m in about 5 hours (Brown 1996). The number of images expected to be detected in such an observation, with and without the effects of lensing, are listed in Table 2 for all the world models listed in Table 1. Hence, sensitive, but practical, submillimetre-wave observations of clusters could probably be used to impose limits to the values of cosmological parameters. Note, however, that the properties of the population of lensed images are expected to depend more sensitively on the form of galaxy evolution, as discussed by Blain (1997b), as compared with the world model. Their properties are also expected to depend on the lensing behaviour of the cluster (Natarajan & Kneib 1996). Hence, the world model could only be investigated in detail after both the form of evolution of distant galaxies, and the lensing properties of the cluster of interest had been determined accurately.

5 SUMMARY

The fraction of detectable gravitationally-lensed galaxies is expected to be significantly larger in the submillimetre waveband as compared with the optical and radio wavebands. Detecting lenses will be an important goal for the next generation of submillimetre-wave telescopes. Both ground-based interferometer arrays (MIAs) and the space-borne telescopes FIRST and Planck Surveyor will be able to detect large samples of galaxy–galaxy lenses, in both large-area (Blain 1997b; Brown 1996).

| $\Omega_0$ | $\Omega_\Lambda$ | Geometry | $N(>0.1 \text{ mJy})$ Lensed | $N(>1 \text{ mJy})$ Lensed |
|----------|----------------|----------|------------------|------------------|
| 1.0      | 0.0            | Flat     | 60 (110)         | 7 (7)             |
| 0.7      | 0.3            | Flat     | 76 (140)         | 7 (6)             |
| 0.3      | 0.7            | Flat     | 130 (220)        | 8 (5)             |
| 0.16     | 0.84           | Flat     | 180 (290)        | 8 (5)             |
| 0.7      | 0.0            | Open     | 74 (130)         | 7 (6)             |
| 0.3      | 0.0            | Open     | 110 (180)        | 7 (5)             |
| 0.16     | 0.0            | Open     | 130 (200)        | 7 (4)             |
| 0.16     | 1.2            | Closed   | 230 (400)        | 14 (8)            |
and very sensitive small-area (Blain 1996b) surveys. An MIA can also observe lensing by clusters in great detail (Blain 1997b). An analysis of the properties of a large sample of lenses selected in the submillimetre waveband could be used to investigate the form of evolution of distant dusty star-forming galaxies, the values of cosmological parameters, the total density in lensing objects and the form of evolution of structure in the Universe.

(i) Several hundred lensed galaxies and several hundred thousand unlensed galaxies are expected in a whole-sky survey using the Planck Surveyor satellite, a large-area survey using FIRST and surveys of smaller fields using a MIA, providing that the population of distant dusty galaxies evolves in a similar manner to that of quasars. The relative counts of lensed and unlensed galaxies can be used to impose joint constraints on the geometry of the universe, the total mass of lensing galaxies and the evolution of cosmic structure.

(ii) The effects of each of these factors could be distinguished using follow-up observations of a subsample of these lenses, using both an MIA to resolve their spatial structure and telescopes operating in optical and near-infrared waveband to determine the redshifts of both the lens and images. Detailed lens modeling, as demonstrated for observations in the optical and radio wavebands, could then be used to investigate the world model alone.

(iii) The surface and flux density distributions of lensed images in the fields of clusters are expected to depend on both the intrinsic properties of the background galaxies and the form of the world model. Observations using an MIA are potentially very useful for detecting these images. If the form of evolution of the population of distant star-forming galaxies is determined accurately in blank-field submillimetre-wave surveys then these observations can be used to impose direct constraints on the values of cosmological parameters.

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