A numerical solution to simultaneous heat and mass transfer of convective drying of food

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Abstract. Typical theoretical method to solve simultaneous heat and mass transfer on surface area of drying of food can be divided into two different approaches, they are analytical method and numerical method. In the numerical method the water activity parameter is needed. The water activity is an individual parameter of each food and determines how strong the connections between food structure and water are. This parameter is developed by using experimental data. In this work, a simple numerical method which is coupled with water activity is proposed. A numerical method has been developed to solve one-dimensional governing equations. The method has been tested in convective drying of mango slice. The result reveals that experimental and numerical result show good agreement. The drying characteristics, effect of temperature and velocity of drying air are explored. The main conclusion can be drawn here is that the proposed method can be used to solve simultaneous heat and mass transfer problem in food drying.

1. Introduction
Drying has been known as a method that can be used to preserve food. By reducing the water content of the food, it prevents the changes in the material of food due to microorganism activities. In fact, drying is a transient complex problem where the heat transfer, mass transfer and vapor evaporation occur at the same time. The vapor transferred from the inside of the object to the surface and it must be carried out from the surface. To provide effective drying of food, heat and flow must be present on the surface. Thus, drying consumes significant energy in the form of heat and also mechanical energy. The mechanical energy is applied to provide stream of the drying medium. In order to design an effective and efficient drying method and equipment, the solution to heat and mass transfer process of convective drying must be available. This motivates researchers to investigate numerical solutions of simultaneous heat and mass transfer of convective drying. Thus, study on numerical solution to simultaneous heat and mass transfer of convective drying of food has come under scrutiny.

Cordova-Quiroz et al. [1] reported a study on numerical solution to convective drying by proposing a model with interfacial resistance to mass transfer to reproduce the experimental behavior of moisture evolution during carrot slab drying. In their study, the heat transfer equation was not solved. Hernandez et al. [2] proposed analytical solution of heat and mass transfer equation which the shrinkage is taken into account for modeling food-drying kinetics. In the model a constant average water diffusivity was proposed. The objective was to show and to validate a numerical solution of mass transfer equation in convective drying which shrinkage was taken into account. The results show that numerical and
experimental agree well. Ruiz-Lopez and Garcia-Alvarado [3] reported an analytical solution of a mass transfer equation for food-drying considering both shrinkage and variable diffusivity. The solution has been tested by comparing to the experiment drying kinetics of mango slabs at different air temperature. The results revealed that proposed model is able to predict experimental drying kinetics. Barati and Esfahani [4] proposed a new analytical solution for simultaneous heat and mass transfer during convective drying of food. It was claimed that the new solution fills the gap in analytical modelling of coupled heat and mass transfer during convective drying process. The main feature of the solution is the procedure to predict temperature and moisture content. The proposed solution has been tested to temperature and moisture history of a mango slab during convective drying. The results showed agreement between experimental and numerical simulation. The above solutions are focused in the one dimensional-problem which the velocity vector of drying air is not taken into account.

In several numerical solutions, the velocity above the solid-fluid domain is taken into account. Aversa et al. [5] solved all of the governing equations in two-dimensional case by using commercial Computational Fluid Dynamic (CFD) code. The vapor concentration on the surface is modeled using water activity coefficient. The results showed that the model can predict the vapor and fluid transfer where the evaporation and condensation occur. De Bonis and Roucco [6] solved all of the governing equations on the two-dimensional domain to model convective drying of food. Commercial CFD code was employed to perform the simulation. In the model, mass transfer of fluid and vapor within the solid domain were taken into account. Thus, the evaporation not only on the surface, but could be present in the solid domain. However, the comparison shows that the proposed model can not be used to simulate the evaporation and condensation process as reported by Aversa et al. [5]. In addition, Kadem et al. [7] employ the same technique [6] to model the convective drying of wood in three-dimensional domain using commercial code. Murugesan et al. [8] proposed a solution by solving all of the governing equations to simulate convective drying of brick. In the solid domain, the diffusion equation of fluid and vapor are separated. In the fluid domain, buoyancy effect resulted by temperature difference and concentration difference was considered in the analysis. The model proposed by Murugesan et al. [8] is adopted by Amanifard and Haghi [9] and Suresh et al. [10].

The above reported studies show that numerical solution to simultaneous heat and mass transfer of convective drying plays an important role in the research related to drying of foods. Recently, Castro et al. [11] reported a comprehensive review on mathematical modeling of convective drying of fruits. The results reveal that the challenges for convective drying of fruit lead to overcoming the dependence on empirical models for drying parameters determination, the lack of shrinkage inclusion and 3D modeling. As a note, our research group is developing a continuous solar drying of foods [12]. In order to provide sufficient information on process optimization and development of dryer design, a rigorous numerical solution to simultaneous heat and mass transfer of convective drying is needed. In this work, we propose the simplest numerical solution to simultaneous heat and mass transfer of drying. The objective is to provide an alternative numerical solution to convective drying of food.

2. Method
In this study a methodology to solve the simultaneous heat and mass transfer is proposed. In order to develop the governing equation, the schematic diagram of the drying object and drying air are depicted in Fig 1. The moist object is assumed to be a solid domain. The fluid is then flowed around the object. The water will be transferred to the surface and evaporated to the drying medium. In this study, the simplest model is proposed by promoting several assumptions as follows. In the moist object, the mass transfer is governed by diffusion in one-dimensional and the heat transfer is governed by conduction in one-dimension also. The shrinkage of the deformation of the object during drying is negligible. The thermophysical properties of the moist object are constant. The convective and mass transfer occur on the fluid-solid interface.
By employing those assumptions, the governing equations for temperature \( T \) distributions and moisture content \( X \) are given as follow.

\[
\frac{\partial (\rho c_p T)}{\partial t} = \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) \tag{1}
\]

\[
\frac{\partial (X)}{\partial t} = \frac{\partial}{\partial z} \left( D_w \frac{\partial X}{\partial z} \right) \tag{2}
\]

where, \( \rho \) [kg/m\(^3\)], \( c_p \) [kJ/kgK], \( D_w \) [m\(^2\)/s] and \( k \) [W/mK] are density, specific heat capacity, effective diffusivity and thermal conductivity of the food, respectively. In addition, \( t \) and \( z \) are time and space, respectively.

2.1. Simultaneous Heat and Mass Transfer

One of the key points in this research is the method to coupled heat and mass transfer on the solid-fluid surface area. The heat and mass transfer on the solid-fluid surface are is depicted in the Fig 2. The figure shows that the moist will move to the surface driven by concentration difference. Evaporation on the surface is caused by low vapor concentration in the drying air.

The mass transfer coefficient \( \bar{h}_m \) on the fluid-solid interface is calculated using the correlation proposed by Janjai et al. [13].

\[
\bar{h}_m = \frac{D_{air}}{L} \left( 2 + 0.522 \text{Re}^{0.5} \text{Sc}^{-0.33} \right) \tag{3}
\]

where \( D_{air} \) and \( L \) are effective diffusivity of vapor into air and length of the object dried, respectively. The Reynolds number \( \text{Re} \) and Schmidt number \( \text{Sc} \) are given by equations below.
In the above equations \( U \) [m/s] is defined as velocity of drying air and \( \nu \) [m\(^2\)/s] is kinematic viscosity. The heat transfer on the solid-fluid surface can be calculated by employing conservation of energy and formulated as follows

\[
-k \left. \frac{\partial T}{\partial z} \right|_{i} = h(T_{a} - T_{i}) + \dot{m}_{e}h_{fg}
\]

where, \( m_{e} \) [kg/s], \( h \) [W/m\(^2\)K], and \( h_{fg} \) [J/kg] are evaporation rate, convective heat transfer coefficient, and heat evaporation of the vapor, respectively. Convective heat transfer coefficient can be calculated using empirical correlation available in literature. In this study, since the moist object is assumed to be a convective heat transfer on horizontal plate, the correlation proposed by Perry and Green [14].

\[
h = 0.664 \left( \frac{k_{aw}}{L} \right) Re^{0.5} Pr^{1/3}
\]

where, \( k_{aw} \) [W/mK], \( L \) [m] and \( Pr \) are thermal conductivity of air, length of the product and Prandtl number of the drying air.

As a note, the last term of equation (6) represents the total of heat needed for evaporation of the water from liquid into vapor. The main driven of the evaporation is the concentration difference of the vapor in the air and in the surface of the solid object. The concentration of the vapor on the surface can be calculated by the following equation [15].

\[
C_{s} = 0.622 \frac{a_{u}}{P_{w} (p - P_{w} a_{u})}
\]

where \( P_{w} \) and \( a_{u} \) are the vapor pressure of water at the interface temperature and the water activity on the external surface of the food, respectively. The water activity is a property that very specific for a product and it can be formulated using experimental data. In this work, for the simulation purpose, the water activity for a Mango proposed by Hernandez et al. [16] is used.

\[
a_{u} = 1 - \exp\left(- \exp\left(0.914 + 0.5639 \ln X \right) \right)
\]

where, \( X \) [kg water/kg dry mango] is food moisture content.

### 2.2. Solution Method

In this study, a simplest numerical solution to simultaneous heat and mass transfer is proposed. In the method, the governing equations (equation (1) and equation (2)) are transformed into linear equation system by using forward time step marching technique. The resulted linear equations are coupled with all of the boundary conditions. The needed parameters to close the equations are calculated using the above equations. In order to solve all of the equations and boundary conditions, a Fortran code program is written.

### 3. Results and Discussions

The developed numerical method is employed to simulate the drying characteristics of food. The food that is dried in this study is mango slice. Dimension of the mango slice is 0.04 m in length, 0.04 m in wide and 0.0004 m in thickness. The mango slice is dried from the top and bottom surfaces. By assuming the symmetrical process, only a half of the slice is taken into account. The thermal properties of slice mango are as follows. The specific heat, density and thermal conductivity of the slice mango are 3240 J/kgK, 1359 kg/m\(^3\) and 0.5 W/mK, respectively. In addition, the effective diffusivity and heat of vaporization of water are \(8.5 \times 10^{-10} \) m\(^2\)/s\(^{-1}\) and 2400 kJ/kg, respectively. The slice mango is dried at initial temperature and moisture content of 28°C and 3.6511 kg water/kg dry mango, respectively.
The results and discussions are presented in four subsections. In the first section the numerical method will be validated using experimental data. In the last second subsection the drying characteristics are discussed and in the last two subsections effect of temperature and velocity of the drying air will be explored.

3.1. Numerical Validation
The result of the present numerical method is compared with experimental results [17]. The present numerical method is employed to simulate temperature history during the drying process. The results comparison is shown in Fig 3. In the experiment, temperature histories of the mango slice at different drying air temperature are shown. They are at temperature 50°C, 60°C, and 70°C, respectively. The similar conditions are simulated using the developed model. It can be seen that the experimental and numerical results show a good agreement. Thus, the present numerical method can be used to predict drying process of the food.

![Figure 3. Comparison of numerical and experimental results](image)

3.2. Drying Characteristics
Drying characteristics are explored using the developed model. Figure 4 shows the history of temperature, non-dimensional MR, and temperature during the drying process. The analysis is carried out for drying air at temperature and velocity of 60°C and 1 m/s, respectively. As a note, the initial temperature is 28°C. In the beginning, temperature of the mango slice increases significantly from initial temperature of 34°C. This is because the drying rate is very low and the heat transfer from the drying air is mainly used to heat the slice mango. In the addition, temperature of the food is still low. This drives the heat transfer rate significantly. From the temperature of 34°C to about 40°C the temperature of the food increase slowly. This is because the drying rate is relatively high. After 1 hour of drying, the temperature increases slowly due low mass transfer and high temperature of the food. The history of temperature affects the drying rate. In the figure the drying rate shown by solid blue line. The drying rate increases significantly in the beginning and reaches a maximum at time 20 minute. After reaching the maximum value the drying rate decrease gradually with increasing time. This is because, in the beginning, the object still full of water and the increasing temperature drives the evaporation rate. After reaching the maximum value, the evaporation rate mainly drives by the mass transfer in the solid domain of the mango slice. The non-dimensional MR of the mango slice during drying is shown in the figure by black dashed line. When the drying starts, the non-dimensional MR is one. It decreases with increasing time due to decreasing of the water content in the food.
3.3. Effect of Temperature
In this study, effect of the drying air temperature to the drying characteristics is explored. Figure 5 shows the non-dimensional MR histories at different drying temperature. They are at 50°C, 60°C, and 70°C which are shown by blue dashed line, black dot lines and solid red lines, respectively. In this case drying air velocity is fixed at 1 m/s. It can be seen clearly that temperature affects the drying rate. The higher drying air temperature results in the higher drying rate. This is because at high drying air temperature, the temperature of the mango slice will be higher. This fact reveals that high drying air temperature results in high drying rate.
3.4. Effect of Velocity

In order to explore effect of the drying air velocity to the drying characteristics a drying simulation at different drying air velocity has been carried out. Figure 6 shows the history of non-dimensional MR at four different drying air velocities. They are at 0.5 m/s, 1.0 m/s, 2.0 m/s and 3.0 m/s, respectively. On the other hand, the temperature of the drying air is fixed at 60°C. The figure shows that there is a significant effect of drying air velocity to the drying rate. The higher drying air velocity results in higher drying rate. This is because at higher drying air velocity results in a higher heat transfer coefficient. At the same time the higher heat transfer coefficient results in higher mass transfer coefficient. This will result the higher drying rate. Even though a higher drying air velocity results in higher drying rate, however, the drying rate at drying velocity of 2.0 m/s and 3.0 m/s shows only small different. This suggest that, the drying air velocity will have an optimal drying air velocity. This information suggests that there is no need to increase drying air velocity to a very high velocity. In the other words, there is a space for optimum drying air velocity.

\[ \text{Figure 6. Effect of velocity of drying air} \]

4. Conclusions

In this work a simple numerical method to solve simultaneous heat and mass transfer on surface area of drying of food has been proposed. In the method a water activity which is developed from experimental data has been employed to solve the mass and heat transfer on the surface of food. The conclusion of the present work are as follows. The main conclusion is that the proposed method can be used to solve simultaneous heat and mass transfer problem in food drying. It is suggest to employ the following method to solve numerical solution for drying food [18,19] and also drying clothes [20 – 22].

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