Strong and Weak Interactions in $B \to \pi^+\pi^-K$ Decays

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To describe the weak three-body decays $B \to \pi^+\pi^-K$, we recently derived amplitudes based on two-body QCD factorization followed by $\pi^+\pi^-$ final state interactions in isoscalar $S$- and isovector $P$-waves. We study here the sensitivity of the results to the values of the $B$ to $f_0(980)$ transition form factor and to the effective decay constant of the $f_0(980)$.

1. INTRODUCTION

It is important to understand charmless three-body $B$ decays to probe the standard model. These decays are sensitive to $CP$ violation and supply information on strong interactions. To interpret in a reliable way weak decay observables it is important to take into account final state interactions between produced meson pairs. In the weak decays $B \to \pi^+\pi^-K$ $^{[1, 2]}$ one sees maxima around the $\pi\pi$ effective mass distributions in the $\rho(770)^0$ and $f_0(980)$ resonance regions.

For $\pi\pi$ effective mass $m_{\pi\pi}$ up to 1.2 GeV, the contribution of the isospin-zero $S$-wave ($\pi^+\pi^-)_S$ final state interactions was described in Ref. $^{[3]}$ and that of the isospin-one $P$ wave ($\pi^+\pi^-)_P$ was included in Ref. $^{[4]}$. The amplitudes, based on the QCD factorization approach without the inclusion of hard-spectator and annihilation terms, underestimate the $B$ to $\rho(770)^0K$ and $f_0(980)K$ branching fractions. Therefore, phenomenological amplitudes arising from enhanced $c\bar{c}$ loop diagrams (charming penguin terms $^{[5]}$) were added. Our presentation at the conference was based on the work described in Ref. $^{[4]}$. Here we show the sensitivity of the model $^{[4]}$ to two inputs of the $S$-wave amplitude: the $B$ to $f_0(980)$ transition form factor and the effective decay constant of the $f_0(980)$.

2. WEAK DECAY AMPLITUDES FOR $B \to \pi^+\pi^-K$

The amplitudes for the weak decays $B \to (\pi^+\pi^-)_{S(P)}K$ are derived $^{[3, 4]}$ in the QCD factorization framework $^{[6, 7]}$. As a first approximation, corrections arising from annihilation topologies and hard gluon scattering with the spectator quark are not included. These also contain several phenomenological parameters (see for instance $^{[7]}$).

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For the $B \rightarrow (\pi^+\pi^-)_S K$ decay amplitudes, we consider the three-body $\pi^+\pi^- K$ final state as arising from a quasi-two-body one with the produced $\pi^+\pi^-$ pair being in an isospin-0 $S$-wave state $R_S$ of mass $m_{\pi\pi}$. For $m_{\pi\pi} = m_f$ (mass of the $f_0(980)$) this $R_S$ state is the $f_0(980)$ resonance. Our amplitudes have a weak two-body decay part based on operator product expansion, heavy quark limit and QCD factorization \cite{6} followed by the strong decay of $R_S$ into $(\pi^+\pi^-)_S$ with inclusion of rescattering. One has \cite{3, 4},

$$
\langle (\pi^+\pi^-)_S K^- | H | B^- \rangle = M^S_K(m_{\pi\pi}) + M^S_{R_S}(m_{\pi\pi}) ,
$$

(1)

$$
M^S_K(m_{\pi\pi}) = \frac{G_F}{\sqrt{2}} f_K (M_B^2 - m_{\pi\pi}^2) F^B_{0\rightarrow R_S}(M_K^2) \left\{ \lambda_u [a_1 + a_4^u - a_4^c + (a_6^c - a_6^u)] r - S_u \right\}
+ \lambda_t (a_6^c r - a_4^c - S_t) \right\} \sqrt{\frac{2}{3}} \frac{\Gamma^m_{R_S\pi\pi}(m_{\pi\pi})}{\Gamma_{R_S\pi\pi}(m_{\pi\pi})} ,
$$

(2)

$$
M^S_{R_S}(m_{\pi\pi}) = \frac{G_F}{\sqrt{2}} (M_B^2 - M_K^2) \left\{ \langle R_S | \bar{s}s | 0 \rangle \frac{2F^B_{0\rightarrow R_S}(m_{\pi\pi})}{m_b - m_s} \left[ \lambda_u (a_6^c - a_6^u) + \lambda_t a_6^c \right] \right\}
- N_K (\lambda_u S_u + \lambda_t S_t) \right\} \sqrt{\frac{2}{3}} \frac{\Gamma^s_{R_S\pi\pi}(m_{\pi\pi})}{\Gamma_{R_S\pi\pi}(m_{\pi\pi})} .
$$

(3)

In Eqs. (2) and (3), $G_F$ is the Fermi constant, $f_K$ the kaon decay constant and $M_B$, $M_K$, $m_b$ and $m_s$ the $B$-meson, kaon, $b$- and $s$-quark masses, respectively. The functions $F^B_{0\rightarrow R_S}(m_{\pi\pi}^2)$ and $F^B_{0\rightarrow K}(m_{\pi\pi}^2)$ are the $B^-$ to $R_S$ or $K^-$ transition form factors. The $\lambda_{u,t}$ are products of the CKM matrix elements, $\lambda_u = V_{ub}V_{us}^*$ and $\lambda_t = V_{tb}V_{ts}^*$. The $a_i$ are the scale dependent effective coefficients built from the Wilson coefficients and including next-to-leading-order QCD corrections \cite{5, 7}. The chiral factor $r = 2M_K^2/[(m_b + m_u)(m_s + m_u)]$, $m_u$ being the $u$-quark mass. The phenomenological charming penguin parameters $S_u$ and $S_t$ are added to the QCD factorization terms. In Eq. (3) the weight factor $N_K$ is chosen to be $N_K = f_K F^B_{0\rightarrow R_S}(M_K^2)$ as in Eq. (2).
The amplitude $M_K^S(m_{\pi\pi})$ matches the topology of Figure 1 (a) with the production of a $K^-$ meson from the vacuum plus a $B^-$ transition into an $R_S(m_{\pi\pi})$ state. The amplitude $M_{R_S}^S(m_{\pi\pi})$ corresponds to the topology of Figure 1(b) with the emission of an $R_S(m_{\pi\pi})$ state from the vacuum plus a $B^-$ transition into a $K^-$ meson.

In Eqs. (2) and (3), the non-strange $\Gamma_{R_S}^n(m_{\pi\pi})$ and strange $\Gamma_{R_S}^S(m_{\pi\pi})$ vertex functions describe the strong decay of the state $R_S(m_{\pi\pi})$ into two pions and include $\pi^+\pi^-$ rescattering [3]. One can write

$$
\langle (\pi\pi)s|\bar{s}s|0\rangle = \Gamma_{R_S^{\pi\pi}}^S(m_{\pi\pi})\langle R_S|\bar{s}s|0\rangle = \sqrt{2}B_0\Gamma_1^S(m_{\pi\pi})
$$

where $\Gamma_1^S(m_{\pi\pi}) = \langle 0|ss|0\rangle/(\sqrt{2}B_0)$ is the strange scalar form factor and the normalization constant $B_0 = -\langle 0|\bar{q}q|0\rangle/f^2_\pi$, $f_\pi$ being the pion decay constant. Replacing $\bar{s}s$ by $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$ in Eq. (4) gives the non-strange vertex function $\Gamma_{R_S^{\pi\pi}}^n(m_{\pi\pi})$ in terms of the non-strange scalar form factor $\Gamma_1^n(m_{\pi\pi})$. Defining a scalar decay constant $f_{R_S}$ by

$$
\langle R_S|\bar{s}s|0\rangle = m_{R_S}f_{R_S}^n,
$$

one then obtains from Eq. (4),

$$
\Gamma_{R_S^{\pi\pi}}^n(m_{\pi\pi}) = \chi\Gamma_1^{n*}(m_{\pi\pi}) \text{ with } \chi = \sqrt{2}B_0/(m_{R_S}f_{R_S}^n).
$$

If we assume $\Gamma_{R_S^{\pi\pi}}^n(m_{\pi\pi}) = \chi\Gamma_1^{n*}(m_{\pi\pi})$ and identify $R_S(m_{f_0})$ with $f_0(980)$, we normalize $\chi$ by [3]

$$
\chi = g_{f_0\pi\pi}/[m_{f_0}\Gamma_{tot}(f_0)|\Gamma_{R_S^n}(m_{f_0})|],
$$

where $\Gamma_{tot}(f_0)$ is the total $f_0(980)$ width.

Note that replacing $\sqrt{2}/3\Gamma_{R_{SS}^{\pi\pi}}^n(m_{\pi\pi})$ by 1 in Eqs. (2) and (3) leads to a two-body $B^- \to R_SK^-$ decay amplitude.

In the $B \to (\pi^+\pi^-)_P K^-$ decay amplitudes, the produced $\pi^+\pi^-$ pair is in an isovector $P$-wave $(\pi^+\pi^-)_P$ state $R_P(m_{\pi\pi})$ identified as the $\rho(770)^0$ resonance. The explicit expression of the $B^- \to (\pi^+\pi^-)_P K^-$ amplitude is given in Ref. [4]. As the amplitudes underestimate the $B$ to $\rho(770)^0K$ branching fraction, we also introduce a phenomenological charming penguin term depending on two complex parameters proportional to $\lambda_u$ and $\lambda_t$.

The complete $B^- \to (\pi^+\pi^-)_{SS}\, K^-$ amplitude is obtained by adding the $S$-wave amplitude of Eq. (4) to that of the $P$-wave [4]. Replacing $\lambda_u$, $\lambda_t$ by $\lambda_u^*$, $\lambda_t^*$ gives the $B^+ \to (\pi^+\pi^-)_S K^+$ amplitude. The expressions for the neutral $B$-decay and $B^+ \to (\pi^+\pi^-)_P K^+$ amplitudes can be found in [3] and [4]. In the results shown below we use the same input parameters as in Ref. [4] unless otherwise stated.

3. FIT, RESULTS, DISCUSSIONS AND CONCLUSIONS

In Ref. [4], we use $F_{B^\to (\pi\pi)s}(M_K^2) = 0.46$ although recent calculations [8, 9, 10] give a value close to 0.25. Furthermore, the $\Gamma_{1}^{n*}(m_{\pi\pi})$ depend on some poorly determined low energy constants of chiral perturbation theory. Using their latest determinations [11], the moduli of the $\Gamma_{1}^{n*}(m_{\pi\pi})$ are larger by a factor of 1.25 than those of [4] in the $f_0(980)$ range. However, the constant value of $\chi|\Gamma_{0}^{n}(m_{f_0})|$ (see Eq. (7)) limits the
sensitivity to this variation. With these new inputs for the $S$-wave, the fit to the Belle and BaBar collaboration data (see references in [4]) is similar to that of Ref. [4]. The $\chi^2/d.o.f. = 346/(222-8) = 1.62$ in model [4] and 1.65 here. In this new fit, the resulting charming penguin parameters are modified and in particular, to compensate the decrease of $F_{B \to \pi \pi}^{B \to (\pi \pi)S}(M^2_{KK})$ from 0.46 to 0.25, the modulus of $S_u$ increases.

![Figure 2](image)

Figure 2. The $m_{\pi\pi}$ distributions in $B^\pm \to \pi^+\pi^-K^\pm$ decays (data from Ref. [1]). The solid line results from the fit with $F_{B \to (\pi \pi)S}(M^2_{KK}) = 0.25$ and with the $\Gamma_{n,s}^1(m_{\pi\pi})$ of larger moduli in the $f_0(980)$ range (see text). The vertical lines delimit the region of the fit.

In Fig. 2 the $m_{\pi\pi}$ branching fraction distributions are compared to the Belle data [1]. One sees an asymmetry in the number of events between the $B^- \to \pi^+\pi^-K^-$ and $B^+ \to \pi^+\pi^-K^+$ decays for the $\rho(770)^0$ and $f_0(980)$ regions. This results in a large direct $CP$ asymmetry for $B^\pm \to \rho(770)^0K^\pm$ decays of 0.32±0.03 to be compared with 0.30±0.14 and 0.32±0.16 from Belle and BaBar collaborations, Refs. [1] and [4] in [4], respectively.

It was found in Ref. [8] that the experimental average branching fractions of two-body $B^\pm \to f_0(980)K^\pm$ decays, $\mathcal{B}_{f_0}$, could be reproduced without charming penguin terms. However, we have important differences with Ref. [8] in the $S$-wave inputs.

First, concerning the $M^S_{RS}(m_{\pi\pi})$ amplitude, we have shown in section 2 that, with $R_S \equiv f_0$, we use an effective scalar decay constant $f_{f_0}^s = \sqrt{2}\mathcal{B}_0/(m_{f_0}\chi) = 94$ MeV with the input parameters of Ref. [4] or 117.5 MeV with the new $\Gamma_{n,s}^1$ just considered above. These values are to be compared with those of 370 MeV determined from QCD sum rules in Ref. [8] or of 245 MeV obtained in Ref. [12] applying the Dyson-Schwinger equations, which respect dynamical chiral symmetry breaking in modeling scalar mesons.

With $F_{B \to (\pi \pi)S}(M^2_{KK}) = 0.46$, if we set $S_u = S_t = 0$ and $\Gamma_{n,s}^1(m_{\pi\pi}) = 0$ (equivalent to an effective $f_{f_0}^s = 0$), $\mathcal{B}_{f_0} = 2.19 (0.67) \times 10^{-6}$ when integrating the $m_{\pi\pi}$ average distribution from 0.900 to 1.060 GeV. Here and below, the cited number in parenthesis corresponds to
the fit with $F_0^{B \to (\pi\pi)S}(M_K^2) = 0.25$ and with the $\Gamma_{1}^{s,0}(m_{\pi\pi})$ of larger moduli in the $f_0(980)$ range. With the addition of $M_{K^0_S}(m_{\pi\pi})$ of [4] (effective $f_{f_0}^0 = 94$ (117.5) MeV), $B_{f_0} = 2.66 (1.09) \times 10^{-6}$. If we multiply $\Gamma_{1}^{s,0}(m_{\pi\pi})$ by 4 (3.15) (effective $f_{f_0}^0 = 376$ (370) MeV), $B_{f_0} = 4.50 (2.49) \times 10^{-6}$. Remind that the corresponding Belle value is $6.06 \pm 1.08 \times 10^{-6}$.

In our case, if we add the contributions of $S_u$ and $S_t$ of Ref. [4], then $B_{f_0}$ increases from 2.66 (1.09) $\times 10^{-6}$ to 6.93 (6.59) $\times 10^{-6}$.

Secondly, the fit to experimental data of Ref. [8] includes hard spectator scattering terms with the parameters $X_A$ (plus annihilation terms with the parameters $X_B$). As stated in Ref. [13], the $a_1(f_0K)$ receives a large contribution from hard spectator interaction which will enhance the $M_{K^0_S}(m_{f_0}$) of Ref. [8].

In summary, uncertainties in $F_0^{B \to (\pi\pi)S}(M_K^2)$ and in the $S$-wave scalar form form factors lead to variations of charming penguin parameters, in particular of $S_u$. The scalar form factors that we use give low values for the effective $f_{f_0}^0$ decay constant equal to 94 or 117.5 MeV, to be compared to 370 MeV [8] or 245 MeV [12]. Despite these uncertainties, our conclusions [4] are unchanged. Our theoretical model is based on quasi two-body QCD factorization followed by $S$- and $P$-wave final state interactions between the produced $\pi\pi$ pairs. These interactions are constrained by other experiments, unitarity and chiral symmetry. Our model gives a good fit of the three-body $B \to \pi^+\pi^-K$ decay data. In particular it describes well the interference between the $f_0(980)$ and $\rho(770)^0$ resonances.

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