Illuminating the nucleon spin

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Nucleon spin sum rule in QCD

• Spin of the proton given in terms of quark and gluon angular momentum (AM) distributions
• Different ways to split the AM terms: I will take the Jaffe-Manohar sum rule

What if we include QED corrections? (i.e. How?)
Evolution equations in QCD

\[ \frac{d}{d \ln Q^2} \begin{pmatrix} \Delta \Sigma(Q^2) \\ \Delta G(Q^2) \\ L_q(Q^2) \\ L_g(Q^2) \end{pmatrix} = \int_0^1 dx \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta P(z, Q^2) & \Delta \hat{P}(z, Q^2) \\ \Omega \hat{P}(z, Q^2) & \Omega P(z, Q^2) \end{pmatrix} \begin{pmatrix} \Delta \Sigma(x, Q^2) \\ \Delta G(x, Q^2) \\ L_q(x, Q^2) \\ L_g(x, Q^2) \end{pmatrix} \]

\[ \Delta P(z, Q^2) = \begin{pmatrix} \Delta P_{qq}(z, Q^2) \\ \Delta P_{gq}(z, Q^2) \\ \Delta P_{qg}(z, Q^2) \\ \Delta P_{gg}(z, Q^2) \end{pmatrix} \]

- Coupled evolution equations
- Evolution of helicity distributions driven only by themselves: \( \Delta \hat{P}(z, Q^2) = 0 \)
**Evolution equations in QCD**

- Take the evolution of quark helicity as an example:

$$\frac{d}{d\ln Q^2} \left( \frac{1}{2} \Delta \Sigma \right) = \int_0^1 dx \int_x^1 \frac{dz}{z} \left[ \Delta P_{qq}(z, Q^2) \Delta \Sigma + \Delta P_{qg}(z, Q^2) \Delta G + \Delta \hat{P}_{qq}(z, Q^2) L_q + \Delta \hat{P}_{qg}(z, Q^2) L_g \right]$$

- Need perturbative expansions:

$$\Delta \Sigma(z, Q^2) = 2 n_f \delta(1 - z) + \sum_{n=1} \left( \frac{\alpha_s(Q)}{2\pi} \right)^n \Delta \Sigma^{(n)}(z, Q^2)$$

$$\Delta G(z, Q^2) = \delta(1 - z) + \sum_{n=1} \left( \frac{\alpha_s(Q)}{2\pi} \right)^n \Delta G^{(n)}(z, Q^2)$$

$$\Delta P_{qq}(z, Q^2) = \sum_{n=1} \left( \frac{\alpha_s(Q)}{2\pi} \right)^n \Delta P_{qq}^{(n)}(z, Q^2)$$

- **q/g helicity kernels known up to NNLO**
- **q/g OAM kernels known at LO**

- At LO we get:

$$\frac{d}{d\ln Q^2} \left( \frac{1}{2} \Delta \Sigma \right) = \frac{\alpha_s}{2\pi} \left( \frac{1}{2} \Delta P_{qq}^{(1)} \right) (2n_f \delta_{jq} + \delta_{jg}) + O(\alpha_s^2)$$

Integrated kernels!
Spin sum rule at LO in QCD

\[ \Delta P_{qq}^{(1)} = C_F \int_0^1 dx \left( \frac{1 + x^2}{(1 - x)_+} + \frac{3}{2} \delta(1 - x) \right) = 0 \]

\[ \Delta P_{qg}^{(1)} = 2 n_f T_R \int_0^1 dx (2x - 1) = 0 \]

\[ \Delta P_{gq}^{(1)} = C_F \int_0^1 dx (2 - x) = \frac{3}{2} C_F \]

\[ \Delta P_{gg}^{(1)} = 2 C_A \int_0^1 dx \left( \frac{1}{(1 - x)_+} - 2x + 1 \right) + \frac{\beta_1}{2} \delta(x - 1) = \frac{\beta_1}{2} \]

\[ \Delta \hat{P}^{(1)}(z, Q^2) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \]

\[ \Omega \hat{P}_{qq}^{(1)} = C_F \int_0^1 dx \left( \frac{x(1 + x^2)}{(1 - x)_+} + \frac{3}{2} \delta(1 - x) \right) = -\frac{4}{3} C_F \]

\[ \Omega \hat{P}_{qg}^{(1)} = 2 n_f T_R \int_0^1 dx x(2x + (1 - x)^2) = \frac{2}{3} n_f T_R \]

\[ \Omega \hat{P}_{gq}^{(1)} = C_F \int_0^1 dx (1 + (1 - x)^2) = \frac{4}{3} C_F \]

\[ \Omega \hat{P}_{gg}^{(1)} = 2 C_A \int_0^1 dx \left( \frac{x^2 - x + 1}{(1 - x)_+} + \frac{\beta_1}{2} \delta(x - 1) \right) = \frac{\beta_1}{2} - \frac{11}{6} C_A \]

As expected!!

\[ \frac{d}{d \ln Q^2} \left( \frac{1}{2} \Delta \Sigma + \Delta G + L_q + L_g \right) = \frac{\alpha_s}{2\pi} \sum_{i,j=q,g} \left( \Delta P_{ij}^{(1)} \left( \frac{1}{2} \delta_{iq} + \delta_{ig} \right) + \Omega \hat{P}_{ij}^{(1)} + \Omega \hat{P}_{ij}^{(1)} \right) \left( 2n_f \delta_{jq} + \delta_{jg} \right) + O(\alpha_s^2) = 0 \]
Inclusion of QED corrections

• Need to consider QED corrections in interactions and also modify (some of the) operators

• Take quark helicity as an example:

\[ \Delta q_f(x, Q^2) = \int \frac{dy^-}{2\pi} e^{-iy^-xP^+} \langle PS| \left[ \bar{\psi}_f W \hat{W}_f \right] (y^-) \frac{\gamma^+ \gamma_5}{2} \left[ \hat{W}_f^\dagger W^\dagger \psi_f \right] (0) |PS\rangle \]

\[ W(x) = \bar{P} \exp \left[ ig_s \int_{-\infty}^{0} ds A^+(x + s\bar{n}) \right] \rightarrow \hat{W}_f(x) = \exp \left[ ieQ_f \int_{-\infty}^{0} ds B^+(x + s\bar{n}) \right] \]

• Quark OAM distribution also acquires photon Wilson lines

• Gluon operators don’t
Nucleon spin sum rule in QCDxQED

- Need to include also **lepton and photon distributions**!!

\[
\frac{1}{2} \Delta \Sigma + \Delta G + \frac{1}{2} \Delta l + \Delta \gamma + L_q + L_g + L_l + L_\gamma = \frac{1}{2}
\]

\[\Delta l(Q^2) = \sum_f \int_0^1 dx \ (\Delta l_f(x, Q^2) + \Delta \bar{l}_f(x, Q^2))\]

\[\Delta \gamma(Q^2) = \int_0^1 dx \ \Delta \gamma(x, Q^2)\]

\[L_l(Q^2) = \sum_f \int_0^1 dx \ (L_f(x, Q^2) + \bar{L}_f(x, Q^2))\]

\[L_\gamma(Q^2) = \int_0^1 dx \ L_\gamma(x, Q^2)\]

\[
\frac{d}{d \ln Q^2} \left( \frac{1}{2} \Delta \Sigma + \Delta G + \frac{1}{2} \Delta l + \Delta \gamma + L_q + L_g + L_l + L_\gamma \right) = 0
\]
**Evolution equations in QCDxQED**

- QED is sensitive to **electric charge**: we need to **distinguish U-type** and **D-type** quark distributions:

\[
\frac{d}{d\ln Q^2} \begin{pmatrix}
\Delta \Sigma_U(Q^2) \\
\Delta \Sigma_D(Q^2) \\
\Delta G(Q^2) \\
\Delta l(Q^2) \\
\Delta \gamma(Q^2) \\
L_U(Q^2) \\
L_D(Q^2) \\
L_g(Q^2) \\
L_l(Q^2) \\
L_\gamma(Q^2)
\end{pmatrix} = \int_0^1 dx \int_x^1 \frac{dz}{z} \begin{pmatrix}
\Delta P(z, Q^2) & \Delta \hat{P}(z, Q^2) \\
\Omega \hat{P}(z, Q^2) & \Omega P(z, Q^2)
\end{pmatrix}
\]

5x5 matrices

100 kernels!

- Perturbative expansions generalized as:

\[
\Delta \Sigma(x, Q^2) = \Delta \Sigma_U(x, Q^2) + \Delta \Sigma_D(x, Q^2)
\]

\[
\Delta \Sigma_{U(D)}(x, Q^2) = n_f \delta(1 - x) + \sum_{n, m=0} \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^n \left(\frac{\alpha(Q^2)}{2\pi}\right)^m \Delta \Sigma^{(n,m)}_{U(D)}(x, Q^2)
\]
**Evolution kernels at LO in QCD**

|     | U     | D     | g     | l     | γ     |
|-----|-------|-------|-------|-------|-------|
| U   | Δ\(P_{qq}^{(1,0)}\) | 0     | \(\frac{1}{2} Δ\(P_{qq}^{(1,0)}\) | 0     | 0     |
| D   | 0     | Δ\(P_{qq}^{(1,0)}\) | Δ\(P_{qq}^{(1,0)}\) | \(\frac{1}{2} Δ\(P_{qq}^{(1,0)}\) | 0     |
| g   | Δ\(P_{gg}^{(1,0)}\) | Δ\(P_{gg}^{(1,0)}\) | Δ\(P_{gg}^{(1,0)}\) | 0     | 0     |
| l   | 0     | 0     | 0     | 0     | 0     |
| γ   | 0     | 0     | 0     | 0     | 0     |
| U   | Ω\(\hat{P}_{qq}^{(1,0)}\) | 0     | \(\frac{1}{2} Ω\(\hat{P}_{qq}^{(1,0)}\) | 0     | 0     |
| D   | 0     | Ω\(\hat{P}_{qq}^{(1,0)}\) | Ω\(\hat{P}_{qq}^{(1,0)}\) | \(\frac{1}{2} Ω\(\hat{P}_{qq}^{(1,0)}\) | 0     |
| g   | Ω\(\hat{P}_{gg}^{(1,0)}\) | Ω\(\hat{P}_{gg}^{(1,0)}\) | Ω\(\hat{P}_{gg}^{(1,0)}\) | 0     | 0     |
| l   | 0     | 0     | 0     | 0     | 0     |
| γ   | 0     | 0     | 0     | 0     | 0     |

\[ Δ\hat{P}(z, Q^2) = 0 \]

Kernels for lepton & photon distributions at LO in QCD are all zero!!
**Evolution kernels at LO in QED: recipe**

- We can obtain the QED kernels from their QCD analogues.
- The **abelianization** recipe is simple at LO:

\[
\begin{align*}
C_F & \to Q_i^2 \\
C_A & \to 0 \\
T_R & \to 1 \\
2n_f & \to \sum_{i=q,\bar{q},l,\bar{l}} N_{C,i}Q_i^2 = 2N_C \frac{n_f}{2} (Q_U^2 + Q_D^2) + 2n_l
\end{align*}
\]

- For example:

\[
\beta^{(1,0)} = \frac{11}{3} C_A - \frac{4}{3} T_R n_f \\
\hat{\beta}^{(0,1)} = -\frac{4}{3} \left( N_C \frac{n_f}{2} (Q_U^2 + Q_D^2) + n_l \right)
\]
Evolution kernels at LO in QED: results (1/4)

- All these are new (zeros too!!)
- Can be computed carefully from the analogous ones in QCD
- Towards the precision test of saturation physics (CGC) at RHIC and LHC. Key!

\[ \Delta \hat{P}(z, Q^2) = 0 \]
Evolution kernels at LO in QED: results (2/4)

- Evolution of helicity distributions driven by helicity distributions

\[
\begin{align*}
\Delta P_{qq}^{(0,1)} &= Q_i^2 \int_0^1 dx \left( \frac{1 + x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) = 0 \\
\Delta P_{gg}^{(0,1)} &= 2 n_f T_R \int_0^1 dx \ (2x - 1) = 0 \\
\Delta P_{gg}^{(1)} &= C_F \int_0^1 dx \ (2 - x) = \frac{3}{2} C_F \\
\Delta P_{gg}^{(1)} &= 2 C_A \int_0^1 dx \left( \frac{1}{(1-x)_+} - 2x + 1 \right) + \frac{\beta^{(1)}}{2} \delta(x - 1) = \frac{\beta^{(1)}}{2}
\end{align*}
\]

\[
\begin{align*}
\Delta P_{ii}^{(0,1)} &= Q_i^2 \int_0^1 dx \left( \frac{1 + x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) = 0 \\
\Delta P_{i\gamma}^{(0,1)} &= n_f N_C Q_i^2 \int_0^1 dx \ (2x - 1) = 0 \\
\Delta P_{\gamma i}^{(0,1)} &= Q_i^2 \int_0^1 dx \ (2 - x) = \frac{3}{2} Q_i^2 \\
\Delta P_{\gamma \gamma}^{(0,1)} &= \int_0^1 dx \ \frac{\hat{\beta}^{(0,1)}}{2} \delta(x - 1) = \frac{\hat{\beta}^{(0,1)}}{2} \\
\Delta P_{li}^{(0,1)} &= Q_i^2 \int_0^1 dx \left( \frac{1 + x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) = 0 \\
\Delta P_{l\gamma}^{(0,1)} &= 2 n_l Q_i^2 \int_0^1 dx \ (2x - 1) = 0 \\
\Delta P_{\gamma l}^{(0,1)} &= Q_i^2 \int_0^1 dx \ (2 - x) = \frac{3}{2} Q_i^2
\end{align*}
\]
Evolution kernels at LO in QED: results (3/4)

- Evolution of OAM distributions driven by helicity distributions

\[
\Omega \hat{P}^{(1)}_{qq} = C_F \int_0^1 dx \ (x^2 - 1) = -\frac{2}{3} C_F
\]
\[
\Omega \hat{P}^{(1)}_{gg} = 2 \ n_f \ T_R \int_0^1 dx \ (1-x)(1-2x+2x^2) = \frac{2}{3} n_f \ T_R
\]
\[
\Omega \hat{P}^{(1)}_{gq} = C_F \int_0^1 dx \ (x-1)(-x + 2) = -\frac{5}{6} C_F
\]
\[
\Omega \hat{P}^{(1)}_{gg} = 2 \ C_A \int_0^1 dx \ (x-1)(x^2 - x + 2) = -\frac{11}{6} C_A
\]
\[
\Omega \hat{P}^{(0,1)}_{ii} = Q_i^2 \int_0^1 dx \ (x^2 - 1) = -\frac{2}{3} Q_i^2
\]
\[
\Omega \hat{P}^{(0,1)}_{i\gamma} = Q_i^2 \int_0^1 dx \ (1-x)(1-2x+2x^2) = \frac{1}{3} n_f \ N_C Q_i^2
\]
\[
\Omega \hat{P}^{(0,1)}_{\gamma\gamma} = 0
\]
\[
\Omega \hat{P}^{(0,1)}_{il} = Q_l^2 \int_0^1 dx \ (x^2 - 1) = -\frac{2}{3} Q_l^2
\]
\[
\Omega \hat{P}^{(0,1)}_{i\gamma} = 2 n_l Q_i^2 \int_0^1 dx \ (1-x)(1-2x+2x^2) = \frac{2}{3} n_l
\]
\[
\Omega \hat{P}^{(0,1)}_{\gamma l} = Q_l^2 \int_0^1 dx \ (x-1)(-x + 2) = -\frac{5}{6} Q_l^2
\]
Evolution kernels at LO in QED: results (4/4)

- Evolution of OAM distributions driven by OAM distributions

\[
\begin{align*}
\Omega P_{qg}^{(1)} &= C_F \int_0^1 dx \frac{(x(1 + x^2) + 2 \delta(1 - x))}{(1 - x)_+} = -\frac{4}{3} C_F \\
\Omega P_{gg}^{(1)} &= 2 n_f T_R \int_0^1 dx x(x^2 + (1 - x)^2) = \frac{2}{3} n_f T_R \\
\Omega P_{gq}^{(1)} &= C_F \int_0^1 dx (1 + (1 - x)^2) = \frac{4}{3} C_F \\
\Omega P_{gg}^{(1)} &= 2 C_A \int_0^1 dx \frac{(x^2 - x + 1)^2}{(1 - x)_+} + \frac{\beta^{(1)}}{2} \delta(x - 1) = \frac{\beta^{(1)}}{2} - \frac{11}{6} C_A
\end{align*}
\]

\[
\begin{align*}
\Omega P_{\gamma i}^{(0,1)} &= Q_i^2 \int_0^1 dx \left( \frac{x(1 + x^2)}{(1 - x)_+} + \frac{3}{2} \delta(1 - x) \right) = -\frac{4}{3} Q_i^2 \\
\Omega P_{\gamma \gamma}^{(0,1)} &= n_f N C \int_0^1 dx x(x^2 + (1 - x)^2) = \frac{1}{3} n_f N C Q_i^2 \\
\Omega P_{\gamma i}^{(0,1)} &= Q_i^2 \int_0^1 dx (1 + (1 - x)^2) = \frac{4}{3} Q_i^2 \\
\Omega P_{\gamma \gamma}^{(0,1)} &= \int_0^1 dx \frac{\hat{\beta}^{(0,1)}}{2} \delta(x - 1) = \frac{\hat{\beta}^{(0,1)}}{2} \\
\Omega P_{ll}^{(0,1)} &= Q_i^2 \int_0^1 dx \left( \frac{x(1 + x^2)}{(1 - x)_+} + \frac{3}{2} \delta(1 - x) \right) = -\frac{4}{3} Q_i^2 \\
\Omega P_{\gamma l}^{(0,1)} &= Q_i^2 \int_0^1 dx (1 + (1 - x)^2) = \frac{4}{3} Q_i^2 \\
\Omega P_{\gamma l}^{(0,1)} &= 2 n_l Q_i^2 \int_0^1 dx x(x^2 + (1 - x)^2) = \frac{2}{3} n_l
\end{align*}
\]
Scale independence of the nucleon spin in QCDxQED

\[
\frac{1}{2} \Delta \Sigma(Q^2) + \Delta G(Q^2) + \frac{1}{2} \Delta l(Q^2) + \Delta \gamma(Q^2) + L_q(Q^2) + L_g(Q^2) + L_l(Q^2) + L_\gamma(Q^2) = \frac{1}{2}
\]

\[
\frac{d}{d \ln Q^2} \left( \frac{1}{2} \Delta \Sigma(Q^2) + \Delta G(Q^2) + \frac{1}{2} \Delta l(Q^2) + \Delta \gamma(Q^2) + L_q(Q^2) + L_g(Q^2) + L_l(Q^2) + L_\gamma(Q^2) \right) = \\
\left[ \frac{\alpha_s(Q^2)}{2\pi} \sum_{i,j=U,D,g,\gamma,l} (\Delta P_{ij}^{(1,0)} \left( \frac{1}{2} \delta_i U + \frac{1}{2} \delta_i D + \delta_i g + \frac{1}{2} \delta_i l + \delta_i \gamma \right) + \Omega \hat{P}_{ij}^{(1,0)} + \Omega P_{ij}^{(1,0)}) \right] + \frac{\alpha(Q^2)}{2\pi} \sum_{i,j=U,D,g,\gamma,l} (\Delta P_{ij}^{(0,1)} \left( \frac{1}{2} \delta_i U + \frac{1}{2} \delta_i D + \delta_i g + \frac{1}{2} \delta_i l + \delta_i \gamma \right) + \Omega \hat{P}_{ij}^{(0,1)} + \Omega P_{ij}^{(0,1)})
\times \left( n_f \delta_j U + n_f \delta_j D + \delta_j g + 2n_f \delta_j l + \delta_j \gamma \right) + O(\alpha^2, \alpha_s^2, \alpha \alpha_s) = 0
\]

Scale independent!!
(at LO)

This is a (perturbative) check of the extended sum rule and the new computed kernels
Conclusions

• Extended the nucleon spin sum rule to QCDxQED: QED corrections + new distributions
• Computed QED corrections at LO for all q/g/lepton/photon distributions
• Computed QCD corrections at LO for the newly introduced lepton/photon distributions
• Checked explicitly the scale-invariance of the spin sum rule at LO in both QCD and QED
• These results will allow in the future for a more precise pheno studies of the spin sum rule