Gluon distributions in nuclei at small $x$: guidance from different models

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The difference between the structure functions measured in nucleons and nuclei [1] is a very important and well known feature of the study of nuclear structure and nuclear collisions. At small values of the Bjorken variable $x (< 0.01$, shadowing region), the structure function $F_2$ per nucleon turns out to be smaller in nuclei than in a free nucleon. This shadowing corresponds to a shadowing of parton densities in nuclei. While at small $x$ valence quarks are of little importance and the behaviour of the sea is expected to follow that of $F_A^2$, the gluon distribution, which is not an observable quantity, is badly determined and represents one of the largest uncertainties in computation of cross sections both for moderate and large scales $Q^2$ in collinear factorization [2]. For example, the uncertainty in the determination of the glue for Pb at $Q^2 \sim 5 \text{ GeV}^2$ at LHC for $y = 0$, $x \sim m_T/\sqrt{s} \sim 10^{-4} \div 10^{-3}$, is a factor $\sim 3$ (see Fig. 1), which for the corresponding cross section in PbPb collisions results in a factor $\sim 9$.

In this situation and while waiting for new experimental data to come from lepton-ion [3-5] or pA colliders, the guidance from different theoretical models is of uttermost importance to perform safe extrapolations from the region where experimental data exist to those interesting for LHC studies. Two different approaches to the problem have been essayed: On the one hand, there exist models which try to explain the origin of shadowing, usually in terms of multiple scattering (in the frame where the nucleus is at rest) or parton interactions in the nucleus wave function (in the frame in which the nucleus is moving fast). On the other hand, other models parameterize parton densities inside the nucleus at some scale $Q^2_0$ large enough for perturbative QCD to be applied reliably, and then evolve these parton densities using the DGLAP [6-8] evolution equations; in this way, the origin of the differences of partons densities in nucleons with respect to nuclei is not addressed, but contained in the parameterization at $Q^2_0$ which is obtained from a fit to experimental data.

1. Multiple scattering and saturation models

The nature of shadowing is well understood qualitatively: In the rest frame of the nucleus, the incoming photon splits, at small enough $x$, into a $q\bar{q}$ pair long before reaching the nucleus, with a coherence length $l_c \propto 1/(m_N x)$ with $m_N$ the nucleon mass, which at small enough $x$ becomes of the order of or greater than the nuclear size. Thus this $q\bar{q}$ pair interacts coherently with the nucleus with typical hadronic cross sections, which results in absorption [9, 10, 11, 12, 13] (see [14] for a simple geometrical approach in this framework). In this way nuclear shadowing is a consequence of multiple scattering and is thus related with diffraction (see e.g. [15, 16]).

Multiple scattering is usually formulated in the dipole model [13-17], which is equivalent to $k_T$-factorization [18] at leading order. In this framework the $\gamma^*\text{nucleus}$ cross section is expressed through the convolution of the probability of the transversal or longitudinal $\gamma^*$ to split into a $q\bar{q}$ pair of transverse dimension $r$ times the cross section for scattering of such dipole with the nucleus. It is this dipole-nucleus cross section at fixed impact parameter which saturates (i.e. gets a maximum value allowed by unitarity), most frequently by multiple scattering in the Glauber-Gribov approach [19, 20, 21, 16]. This dipole-target cross section is related through a Bessel-Fourier transform to the so-called unintegrated gluon distribution $\varphi_A(x, k_T)$ in $k_T$-factorization [22, 19, 20], which in turn can be related to the usual collinear gluon density through

$$x G_A(x, Q^2) = \int_{\Lambda^2}^{Q^2} dk_T^2 \varphi_A(x, k_T)$$

(1)
located in a transverse area order of or larger than the nuclear size, leading to the overlap of gluon clouds from different nucleons (with $Q^2$ dipole-target cross section appears which leads to a geometrical scaling of proportional to the inverse QCD coupling constant, gluon correlations are absent and a form for the In the semiclassical framework the gluon field at saturation reaches a maximum value and becomes shadowing for gluons, defined by the ratio $xG_A/(xG_N)$ being $< 1$, are apparently different phenomena, i.e. saturation does not necessarily lead to shadowing for gluons [23, 24]. Indeed, in the framework of numerical studies of the non-linear equations for small $x$ evolution, the unintegrated gluon distribution turns out to be a universal function of just one variable $\tau = k_T^2/Q^2_s$ [30, 31, 32] (in [33] this universality is analytically shown to be fulfilled up to $k_T^2$ much larger than $Q^2_s$), vanishing quickly for $k_T^2 > Q^2_s$; this scaling appears also in the analysis of DIS experimental data on nucleons [29] and has been searched for in nuclear data [34]. In nuclei, $Q^2_s$ increases with increasing nuclear size, centrality and energy. So, through the relation with the collinear gluon given by Eq. (4), this scaling implies that the integral gives the same value (up to logarithmic corrections if a perturbative tail exists) with $Q^2 \gg Q^2_s$ the saturation momentum corresponding to a transverse length scale where the saturation of the dipole-target cross section occurs.

Other formulations of multiple scattering do not use the dipole formulation but relate shadowing with diffraction [15, 16] by Gribov theory. In this way diffraction in lepton scattering on nucleons is related to nuclear shadowing.

An equivalent explanation to multiple scattering in the frame in which the nucleus is moving fast, is that gluon recombination due to the overlap of the gluon clouds from different nucleons, makes gluon density in nucleus with mass number $A$ smaller than $A$ times that in a free nucleon [25, 26]: at small $x$ the interaction develops over longitudinal distances $z \sim 1/(m_N x)$ which become of the order of or larger than the nuclear size, leading to the overlap of gluon clouds from different nucleons located in a transverse area $\sim 1/Q^2$. These studies have received great theoretical impulse with the development of semiclassical ideas in QCD and the appearance of non-linear equations for evolution in $x$ in this framework (see [27, 28] and references therein), although saturation appears to be different from shadowing [23, 24] (i.e. the reduction in the number of gluons as defined in DIS is not apparent). In the semiclassical framework the gluon field at saturation reaches a maximum value and becomes proportional to the inverse QCD coupling constant, gluon correlations are absent and a form for the scaling has been found in small $x$ nucleon data [29], and also in analytical and numerical solutions of the non-linear equations [30, 31, 32, 33], but apparently the region where this scaling should be seen in lepton-nucleus collisions has not been reached yet in available experimental data [34]. Apart from eA colliders [3, 4, 5], LHC will be the place to look for non-linear effects. However, the consequences of such high density configurations may be masked in AB collisions by other effects, as those due to final state interactions. pA collisions would be, thus, essential for this type of studies (see e.g. [35]). Moreover, they would be required to fix the baseline for other studies in AB collisions as QGP search and characterisation. As a final comment, let us indicate that the non-linear terms in [25, 26] are of a higher-twist nature and in this case the low density limit recovers the DGLAP equations [6, 7, 8], while the non-linear equations for evolution in $x$ [27, 28] do not correspond to any definite twist and their linear limit correspond to the BFKL equation [36, 37].

Let us comment a little more on the difference between shadowing for gluons and saturation. In saturation models [27, 28] saturation, defined by a maximum value of the gluon field or by the scattering becoming black, and shadowing for gluons, defined by the ratio $xG_A/(xG_N)$ being $< 1$, are apparently different phenomena, i.e. saturation does not necessarily lead to shadowing for gluons [23, 24]. Indeed, in the framework of numerical studies of the non-linear equations for small $x$ evolution, the unintegrated gluon distribution turns out to be a universal function of just one variable $\tau = k_T^2/Q^2_s$ [30, 31, 32] (in [33] this universality is analytically shown to be fulfilled up to $k_T^2$ much larger than $Q^2_s$), vanishing quickly for $k_T^2 > Q^2_s$; this scaling appears also in the analysis of DIS experimental data on nucleons [29] and has been searched for in nuclear data [34]. In nuclei, $Q^2_s$ increases with increasing nuclear size, centrality and energy. So, through the relation with the collinear gluon given by Eq. (4), this scaling implies that the integral gives the same value (up to logarithmic corrections if a perturbative tail exists) with $Q^2 \gg Q^2_s$ the saturation momentum corresponding to a transverse length scale where the saturation of the dipole-target cross section occurs.

2. DGLAP evolution models
On the other hand, a different approach is taken in [38, 39, 40, 41]: parton densities inside the nucleus are parameterized at some scale $Q^2_0 \sim 1/5$ GeV$^2$ and then evolved using the DGLAP [6, 7, 8] evolution
equations. In this way, all nuclear effects on parton densities are included in the parameterization at $Q_0^2$, which is obtained from a fit to experimental data. The differences in the approaches come mainly from the sets of data used (e.g., the use of Drell-Yan data or not) to constrain the parton distributions. In these calculations the lack of experimental data makes the gluon to be badly constrained at very small $x$; experimental data on the evolution of $F_2$ with log $Q^2$ give direct constrains [42] through DGLAP evolution but only for $x \geq 0.01$. Please refer to [42] and references therein for more discussions on this kind of models.

3. Comparison between different approaches

The results from different models usually depend on additional semiphenomenological assumptions and often contradict each other. For example, concerning the $Q^2$-dependence of the effect, in [9] [10] [11] [12] [13] it is argued that $q\bar{q}$ configurations of a large dimension give the dominant contribution to the absorption, which results essentially independent of $Q^2$. On the other hand, in the gluon recombination approach of [25] [26] the absorption is obtained as a clear higher-twist effect dying out at large $Q^2$. Finally, in the models [38] [39] [40] [41] which use DGLAP, all $Q^2$-dependence comes from QCD evolution and is thus of a logarithmic, leading-twist nature. Predictions (particularly for the gluon density) on the $x$-evolution towards small $x$ turn out to be very different.

Let us compare between different approaches whose numerical results are available (when considering an approach based on the dipole model as in [19], one should keep in mind the difficulties to identify at small and moderate $Q^2$ the integral of the unintegrated gluon distribution with the ordinary gluon density, see Eq. (1) and comments below it). A comparison at $Q^2 = 5$ GeV$^2$ for the ratio of gluon densities in Pb over proton, can be found in Fig. 1. There it can be seen that the results of [19] [38] [39] [40] [21] at $x \approx 10^{-2}$ (relevant for RHIC) roughly coincide, while they are higher than those of [16] [43]; at $x \approx 10^{-4} \div 10^{-5}$ (accessible at LHC) the results of [19] become smaller than those of [38] [39] [40] [21], get close to those of [16] and are still larger than those of [43]. Apart from the constraints coming from existing DIS experimental data on nuclei which are very loose for the glue at small $x$, in [38] [39] [40] the saturation of gluon shadowing comes mainly from the initial condition for DGLAP evolution where this saturation has been imposed, while [19] [16] [21] are multiple scattering models. In [43] the behaviour of the glue, in the form of a $Q^2$-independent parameterization, has been fixed at $x \sim 10^{-2}$ in order to reproduce charged particle multiplicities in AuAu collisions at RHIC. In this approach the strongest gluon shadowing is obtained. However, it seems to be in disagreement with existing DIS data [42]. Additional caution has to be taken to compare results from multiple scattering models with those coming from DGLAP analysis [38] [39] [40]: the ratios for the glue at some moderate, fixed $Q^2$ and very small $x$ may result smaller (e.g. in [19]) than the ratios for $F_2$ at the same $x$, $Q^2$, which might lead to problems with leading-twist DGLAP evolution, see [42].

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Fig. 1: Ratios of gluon distribution functions from different models at $Q^2=5$ GeV$^2$; HKM refers to the results from [40], Sarcevic to those from [21], EKS98 to those from [38, 39], Frankfurt to those from [16], Armesto to those from [19, 20] and new HIJING to those from [43]. The bands represent the ranges of $x$ relevant for processes with a scale $Q^2 = 5$ GeV$^2$ at RHIC and LHC.
References

[1] M. Arneodo, Phys. Rept. 240 (1994) 301.
[2] J. C. Collins, D. E. Soper and G. Sterman, Adv. Ser. Direct. High Energy Phys. 5 (1988) 1.
[3] M. Arneodo, A. Bialas, M. W. Krasny, T. Sloan and M. Strikman, arXiv:hep-ph/9610423.
[4] H. Abramowicz et al. [TESLA-N Study Group Collaboration], DESY-01-011.
[5] *EIC White Paper*, preprint BNL-68933, Eds. A. Deshpande, R. Milner and R. Venugopalan.
[6] Y. L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641 [Zh. Eksp. Teor. Fiz. 73 (1977) 1216].
[7] V. N. Gribov and L. N. Lipatov, Yad. Fiz. 15 (1972) 781 [Sov. J. Nucl. Phys. 15 (1972) 438].
[8] G. Altarelli and G. Parisi, Nucl. Phys. B 126 (1977) 298.
[9] S. J. Brodsky and H. J. Lu, Phys. Rev. Lett. 64 (1990) 1342.
[10] V. Barone, M. Genovese, N. N. Nikolaev, E. Predazzi and B. G. Zakharov, Z. Phys. C 58 (1993) 541.
[11] B. Kopeliovich and B. Povh, Phys. Lett. B 367 (1996) 329 [arXiv:hep-ph/9509362].
[12] N. Armesto and M. A. Braun, Z. Phys. C 76 (1997) 81 [arXiv:hep-ph/9603360].
[13] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C 49 (1991) 607.
[14] N. Armesto and C. A. Salgado, Phys. Lett. B 520 (2001) 124 [arXiv:hep-ph/0011352].
[15] A. Capella, A. Kaidalov, C. Merino, D. Pertermann and J. Tran Thanh Van, Eur. Phys. J. C 5 (1998) 111 [arXiv:hep-ph/9707466].
[16] L. Frankfurt, V. Guzey, M. McDermott and M. Strikman, JHEP 0202 (2002) 027 [arXiv:hep-ph/0201230].
[17] A. H. Mueller and B. Patel, Nucl. Phys. B 425 (1994) 471 [arXiv:hep-ph/9403256].
[18] S. Catani, M. Ciafaloni and F. Hautmann, Nucl. Phys. B 366 (1991) 135.
[19] N. Armesto, Eur. Phys. J. C 26 (2002) 35 [arXiv:hep-ph/0206017].
[20] N. Armesto and M. A. Braun, Eur. Phys. J. C 22 (2001) 351 [arXiv:hep-ph/0107114].
[21] Z. Huang, H. J. Lu and I. Sarcevic, Nucl. Phys. A 637 (1998) 79 [arXiv:hep-ph/9705250].
[22] B. Andersson et al. [Small x Collaboration], Eur. Phys. J. C 25 (2002) 77 [arXiv:hep-ph/0204115].
[23] Y. V. Kovchegov and A. H. Mueller, Nucl. Phys. B 529 (1998) 451 [arXiv:hep-ph/9802440].
[24] A. H. Mueller, Nucl. Phys. B 558 (1999) 285 [arXiv:hep-ph/9904404].
[25] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. 100 (1983) 1.
[26] A. H. Mueller and J. W. Qiu, Nucl. Phys. B 268 (1986) 427.
[27] A. H. Mueller, arXiv:hep-ph/0208278.
[28] E. Iancu, A. Leonidov and L. McLerran, arXiv:hep-ph/0202270.
[29] A. M. Stasto, K. Golec-Biernat and J. Kwiecinski, Phys. Rev. Lett. 86 (2001) 596 [arXiv:hep-ph/0007192].

[30] M. Lublinsky, Eur. Phys. J. C 21 (2001) 513 [arXiv:hep-ph/0106112].

[31] K. Golec-Biernat, L. Motyka and A. M. Stasto, Phys. Rev. D 65 (2002) 074037 [arXiv:hep-ph/0110325].

[32] N. Armesto and M. A. Braun, Eur. Phys. J. C 20 (2001) 517 [arXiv:hep-ph/0104038].

[33] E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. A 708 (2002) 327 [arXiv:hep-ph/0203137].

[34] A. Freund, K. Rummukainen, H. Weigert and A. Schafer, arXiv:hep-ph/0210139.

[35] A. Dumitru and J. Jalilian-Marian, Phys. Rev. Lett. 89 (2002) 022301 [arXiv:hep-ph/0204028].

[36] V. S. Fadin, E. A. Kuraev and L. N. Lipatov, Phys. Lett. B 60 (1975) 50.

[37] I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822 [Yad. Fiz. 28 (1978) 1597].

[38] K. J. Eskola, V. J. Kolhinen and P. V. Ruuskanen, Nucl. Phys. B 535 (1998) 351 [arXiv:hep-ph/9802350].

[39] K. J. Eskola, V. J. Kolhinen and C. A. Salgado, Eur. Phys. J. C 9 (1999) 61 [arXiv:hep-ph/9807297].

[40] M. Hirai, S. Kumano and M. Miyama, Phys. Rev. D 64 (2001) 034003 [arXiv:hep-ph/0103208].

[41] D. Indumathi and W. Zhu, Z. Phys. C 74 (1997) 119 [arXiv:hep-ph/9605417].

[42] K. J. Eskola, H. Honkanen, V. J. Kolhinen and C. A. Salgado, Phys. Lett. B 532 (2002) 222 [arXiv:hep-ph/0201256].

[43] S. Y. Li and X. N. Wang, Phys. Lett. B 527 (2002) 85 [arXiv:nucl-th/0110075].