Non-BPS D0-brane instanton effects in type I string theory

Yoji Michishita *

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

December 4, 1999

Abstract

We investigate the instanton effects of non-BPS D0-brane in type I string theory. We argue the general properties of these instanton effects and consider these on $R^9 \times S^1$ as a simple example using the effective action of D0-brane.

*michishi@gauge.scphys.kyoto-u.ac.jp
1 Introduction

D-branes give us various nonperturbative informations of string theories. One of them is the D-brane instanton effect \cite{1}. Euclidean D-branes wound around nontrivial cycles which are the solutions of equation of motion of worldvolume action contribute largely to the path integral (or its counterpart of string theory), and give corrections to amplitudes or low energy effective action. The way to calculate instanton effects by using effective action is discussed in \cite{1, 2} and stringy calculation is presented in \cite{3}. So far BPS D-branes are used to calculate instanton effects. However, various non-BPS stable D-branes are discovered recently. (For reviews see \cite{4}.) Since these are stable they give large contributions to the path integral similarly to BPS D-branes.

In this paper we argue non-BPS D0-brane instanton effects in type I string theory as an example of such effects. Non-BPS D0-brane has \(SO(32)\) spinor charge and is a counterpart of perturbative massive \(SO(32)\) spinor states in heterotic \(SO(32)\) theory \cite{5}. This brane forms a long multiplet of supersymmetry and therefore is non-BPS. However, \(SO(32)\) spinor charge protect it from decay. Its tension is \(\sqrt{2}\) times larger than that of BPS D-branes in the weak coupling regime, and is modified as the coupling constant grows. Therefore instanton action is a complicated function of dilaton. Since we do not know this function we restrict ourselves to the weak coupling regime.

Non-BPS D-branes have more fermion massless modes living on them than BPS D-branes and we need more vertex operators to saturate their zero modes as will be seen in a concrete example. Hence non-BPS D-brane instanton corrections are higher order effects than that of BPS D-branes.

This paper is organized as follows. In section 2, we briefly review the non-BPS D0-brane and discuss general properties of its instanton effects. In section 3, we review the superfield formalism of type I string theory and construct the effective action and fermion vertex operators following \cite{6}. In section 4, we consider \(S^1\) compactification as a simple example and calculate some simple amplitudes by using the effective action constructed in section 3. Section 5 contains some discussions.
Non-BPS D0-brane instanton

Non-BPS stable D0-brane in type I string theory can be constructed from D1-D1 system through the tachyon condensation mechanism \cite{5} and has no RR charge. Therefore its boundary state has NSNS part only and the overall factor is $\sqrt{2}$ times that of BPS D-brane \cite{7, 8}:

$$|D0\rangle = \sqrt{2} |\text{NS + NS}^+\rangle .$$

This factor $\sqrt{2}$ means that the tension of non-BPS D0-brane is $\sqrt{2}$ times larger than that of BPS D-brane:

$$T_{D0} = \frac{1}{2} \cdot \sqrt{2} \cdot \frac{1}{\sqrt{\alpha' g_s}} .$$

Here the factor $\frac{1}{2}$ comes from the orientifold projection and $g_s$ is the string coupling constant. This value is valid in the weak string coupling regime and is corrected as the coupling constant grows, unlike BPS D-branes. In the strong coupling regime the D0-brane corresponds to the perturbative massive states with $SO(32)$ spinor charge of dual heterotic theory. Their tension (mass) is $\frac{2}{\sqrt{\alpha'_{het} g_s}} = \sqrt{\frac{2}{\alpha' g_s}}$, where the right hand side is written in terms of the quantities of Type I theory. Thus the exact tension is expressed in terms of an unknown function $f(g_s)$:

$$T_{D0} = \frac{1}{\sqrt{2\alpha'}} f(g_s),$$

$$f(g_s) \rightarrow \frac{1}{g_s} \quad \text{as} \quad g_s \rightarrow 0, \quad f(g_s) \rightarrow \frac{2}{\sqrt{g_s}} \quad \text{as} \quad g_s \rightarrow \infty.$$

Since we do not know what $f(g_s)$ is, we restrict ourselves to the weak coupling regime.

The massless modes living on the D0-brane are scalars $X^m$ corresponding to ten spacetime directions, one Majorana-Weyl spinor $\theta^\alpha$ as a superpartner of $X^m$, and a fermion $\xi^I$ with a $SO(32)$ vector index $I$ \cite{7}. $X^m$ and $\theta^\alpha$ come from D0-D0 strings and $\xi^I$ from D0-D9 strings. Quantization of zero modes of $\xi^I$ gives $SO(32)$ spinor charge.

Non-BPS D0-brane is stable only when it is alone. When there are two D0-branes, they attract each other since there is no RR repulsive force and are pair-annihilated. This can be seen from the fact that the corresponding K-theory group is $\mathbb{Z}_2$ \cite{9}. The presence of branes of other type also destabilizes the configuration for the same reason as above. (NSNS force derived from cylinder amplitude vanishes only when the number of directions with Neumann boundary condition on the one end and Dirichlet on the other is four. (See e.g. \cite{10}.)

\footnote{There are some exceptions when some spacetime directions are compactified. For example, when two D0-branes are placed on the antipodal points of $S^1$ respectively, this configuration is stable.}
there are no branes which can make such configurations with a D0-brane in type I string theory.)

If a direction transverse to a D0-brane is compactified with the radius $R$, it decays to a D1-D1 system for $R < \sqrt{\frac{\alpha'}{2}}$. This can be understood by keeping track of the construction given in [5] and also by the following argument.

If there is some tachyon mode on a brane, then it is unstable. We can see whether there are any tachyon mode or not by computing the cylinder and Möbius amplitude. Their sum $A$ is

$$A = V \int_{0}^{\infty} \frac{dt}{2t} (8\pi^{2} \alpha')^{-\frac{1}{2}} \left( \sum_{w \in \mathbb{Z}} e^{\frac{-2i\pi}{\alpha'} R^{2}w^{2}t} \left[ \frac{f_{3}(e^{-\pi t})}{f_{1}(e^{-\pi t})} - \frac{f_{2}(e^{-\pi t})}{f_{1}(e^{-\pi t})} \right] + 16e^{-i\pi} \frac{f_{3}(ie^{-\pi t})}{f_{2}(ie^{-\pi t})} f_{1}(ie^{-\pi t}) - 16e^{i\pi} \frac{f_{4}(ie^{-\pi t})}{f_{2}(ie^{-\pi t})} f_{3}(ie^{-\pi t}) \right)$$

$$= V \int_{0}^{\infty} \frac{dt}{2t} (8\pi^{2} \alpha')^{-\frac{1}{2}} \left( \sum_{w \neq 0} e^{\frac{-2i\pi}{\alpha'} R^{2}w^{2}t} \left[ e^{\pi t} - O(e^{-\pi t}) \right] + 2 + O(e^{-\pi t}) \right), \tag{4}$$

where

$$f_{1}(q) = q^{\frac{1}{\alpha'}} \Pi_{n=1}^{\infty} (1 - q^{2n}), \quad f_{2}(q) = \sqrt{2} q^{\frac{1}{\alpha'}} \Pi_{n=1}^{\infty} (1 + q^{2n}),$$

$$f_{3}(q) = q^{-\frac{1}{\alpha'}} \Pi_{n=1}^{\infty} (1 + q^{2n-1}), \quad f_{4}(q) = q^{\frac{1}{\alpha'}} \Pi_{n=1}^{\infty} (1 - q^{2n-1}). \tag{5}$$

If the exponent of $e^{\frac{-2i\pi}{\alpha'} R^{2}w^{2}t + \pi t}$ in eq. (4) is positive, then this factor is divergent as $t \to \infty$ and represents the contribution of tachyon. Therefore for $R < \sqrt{\frac{\alpha'}{2}}$ the D0-brane is unstable.

Since D0-brane is stable for sufficiently large transverse radius, it gives large contribution to the path integral as a kind of instanton backgrounds. In order to give finite contribution, it must be Euclidean and wrapped around some compactified direction. For example, a D0-brane wrapped once around $S^{1}$ with radius $R$ gives the instanton action $\exp(-T_{D0} \cdot 2\pi R) = \exp(-\frac{2\pi R}{\sqrt{2\alpha' g_{s}}})$ at the weak coupling regime. The sum of its cylinder and Möbius amplitude $A'$ is

$$A' = \int_{0}^{\infty} \frac{dt}{2t} \sum_{n \in \mathbb{Z}} e^{\frac{-\pi}{\alpha'} \frac{1}{4}(n^{2})} \left[ \frac{f_{3}(e^{-\pi t})}{f_{1}(e^{-\pi t})} - \frac{f_{2}(e^{-\pi t})}{f_{1}(e^{-\pi t})} \right] + 16e^{-i\pi} \frac{f_{3}(ie^{-\pi t})}{f_{2}(ie^{-\pi t})} f_{1}(ie^{-\pi t}) - 16e^{i\pi} \frac{f_{4}(ie^{-\pi t})}{f_{2}(ie^{-\pi t})} f_{3}(ie^{-\pi t}) \right]$$

$$= \int_{0}^{\infty} \frac{dt}{2t} \sum_{n \in \mathbb{Z}} e^{\frac{-\pi}{\alpha'} \frac{1}{4}(n^{2})} \left[ 2 + O(e^{-\pi t}) \right], \tag{6}$$

3
This shows that there is no tachyon mode and the winding D0-brane is stable. As for multi-instanton configurations, though these are unstable as explained above, the D0-branes are infinitely massive in the weak coupling regime and stable approximately. Therefore these configurations can give large contributions to the path integral in the weak coupling regime.

D0-brane instanton effects give corrections to the low energy effective action of type I string theory. D0-brane has more fermion massless modes than BPS branes and therefore more fermion zero modes. We have to insert more vertex operators in order to soak up these zero modes. Hence its leading contribution appears in higher derivative terms in the bulk effective action than that of BPS branes. In type I string theory the BPS D-brane with the lowest worldvolume dimension is D1-brane and its instanton effects appear when at least two dimensions are compactified. But the D0-brane instanton effects emerge even when only one dimension is compactified. In the following sections we use the effective action of D0-brane to calculate the instanton effects.

3 The effective action and vertex operators for non-BPS D0-brane

In this section we briefly review the superfield formalism of type I string theory and construct the effective action and vertex operators of D0-brane following [6]. The results of this section will be used in the next section. We use the notation and results of [11].

To describe type I string theory we need supervielbein $E_M^A$, superconnection $\Omega_M^B$, RR 2-form $B_{MN}$, and $SO(32)$ gauge field $A_M$. Supertorsion $T$, supercurvature $R$, and field strengths $H \equiv dB$ and $F \equiv dA$ satisfy the Bianchi identities:

$$DT^A = E^B R_B^A,$$

$$DR_A^B = 0,$$

$$dH = c_1 \text{tr} F^2,$$

$$DF = 0.$$ (7-10)

Here $c_1$ is a certain constant. We impose the following constraints:

$$T_{a\beta}^a = 2(\Gamma^a)_{a\beta}, \quad T_{aa}^b = 0,$$

\[\text{Except for the cases mentioned in the previous footnote.}\]
\[ T_{\alpha\beta} = (\Gamma_\alpha \Psi)^{\alpha\beta}, \quad T_{\alpha\beta} = 0 , \quad T_{\alpha\beta} = 0 , \quad (11) \]
\[ H_{\alpha\beta\gamma} = 0 , \quad (12) \]
\[ F_{\alpha\beta} = 0 , \quad (13) \]

where \( \Psi^{\alpha\beta} \) is defined by the above equation. The solutions of the Bianchi identities are

\[ \Psi^{\alpha\beta} = -\frac{1}{24} T_{abc} (\Gamma^{abc})^{\alpha\beta} , \quad (14) \]
\[ H_{abc} = -\frac{3}{2} \Phi T_{abc} + \frac{c_1}{4} (\Gamma_{abc})_{\alpha\beta} \text{tr} [\chi^\alpha \chi^\beta] , \quad (15) \]
\[ H_{ab\alpha} = -\frac{1}{2} (\Gamma_{ab})^{\beta} \lambda^\beta , \quad (16) \]
\[ H^{ab\alpha} = \Phi (\Gamma_a)_{\alpha\beta} , \quad (17) \]
\[ F_{ab} = (\Gamma_a)_{\alpha\beta} \chi^\beta , \quad (18) \]

where \( \Phi \) and \( \chi^\alpha \) are defined by equations (17) and (18) respectively, and \( \lambda^\alpha \equiv D^\alpha \Phi \). We can set the components of \( E_M^A, \Omega_{MA}^B, B_{MN} \) and \( A_M \) as follows using the local symmetries:

\[ E_m^a |_{\theta=0} \equiv e_m^a, \quad E_m^a |_{\theta=0} \equiv \psi_m^a, \quad E_m^a |_{\theta=0} = 0, \quad E_m^a |_{\theta=0} = \delta_m^a, \quad (19) \]
\[ \Omega_{ma}^b |_{\theta=0} \equiv \omega_{ma}^b, \quad \Omega_{ma}^b |_{\theta=0} = 0, \quad (20) \]
\[ B_{mn} |_{\theta=0} \equiv B_{mn}, \quad B_{mn} |_{\theta=0} = 0, \quad B_{\mu\nu} |_{\theta=0} = 0, \quad (21) \]
\[ A_m |_{\theta=0} \equiv A_m, \quad A_m |_{\theta=0} = 0, \quad (22) \]

and we set

\[ T_{abc} |_{\theta=0} \equiv T_{abc}, \quad \Phi |_{\theta=0} \equiv \Phi, \quad \lambda_{\alpha} |_{\theta=0} \equiv \lambda_{\alpha}, \quad \chi^\alpha |_{\theta=0} \equiv \chi^\alpha. \quad (23) \]

We can determine the lower components of \( E_M^A, \Omega_{MA}^B, B_{MN} \) and \( A_M \) by comparing the solutions of Bianchi identities and the definitions of \( T, R, H \) and \( F \) in the appropriate gauge fixing conditions:

\[ E_m^a = e_m^a - 2\theta^\alpha (\Gamma_a)_{\alpha\beta} \psi_m^\beta + \frac{1}{2} \theta^\alpha (\Gamma^{abc})_{\alpha\beta} \omega_{mbc} - \frac{1}{24} \theta^\alpha (\Gamma^{bcd})_{\alpha\beta} \theta^\beta e_m^a T_{bcd} + \frac{1}{8} \theta^\alpha (\Gamma_{abc})_{\alpha\beta} \theta^\beta e_m^d T_{bcd} + \frac{1}{8} \theta^\alpha (\Gamma_{m\beta})_{\alpha\beta} \theta^\beta T_{bc} + O(\theta^2) , \quad (24) \]
\[ E_m^\alpha = \psi_m^\alpha + \theta^\beta \omega_{m\beta}^\alpha + \frac{1}{2} \theta^\beta (\Gamma_m \Gamma^{abc})_\beta T_{abc} + O(\theta^2) , \quad (25) \]
\[ E_\mu^\alpha = (\Gamma^\alpha)_{\mu\alpha} + O(\theta^2) , \quad (26) \]
\[ E_\mu^\alpha = \delta^\alpha_\mu + O(\theta^2) , \quad (27) \]
\[ \Omega_{ma}^b = \omega_{ma}^b + O(\theta), \]  
(28)

\[ \Omega_{\mu a}^b = \frac{1}{12} (\Gamma_a^{b c de})_{\mu a} \theta^\alpha T_{cde} + \frac{3}{2} (\Gamma_c)_{\mu a} \theta^\alpha T_{ca}^b + O(\theta^2), \]  
(29)

\[ B_{mn} = B_{mn} - \frac{1}{2} \theta^\alpha (\Gamma_{mn})_{\alpha \beta} \lambda_\beta \] 
\[ + 2 \theta^\alpha (\Gamma_{[m])_{\alpha \beta} \psi_\beta [\Gamma_b] + 2 c_1 \theta^\alpha (\Gamma_{[m})_{\alpha \beta} \text{tr}[A_{n]} \chi^\beta] + O(\theta^2), \]  
(30)

\[ B_{\mu \nu} = -\frac{1}{2} (\Gamma_m)_{\nu a} \theta^\alpha \Phi + O(\theta^2), \]  
(31)

\[ B_{\mu \nu} = 0 + O(\theta^2), \]  
(32)

\[ A_m = A_m - \theta^\alpha (\Gamma_m)_{\alpha \beta} \chi^\beta + O(\theta^2), \]  
(33)

\[ A_\mu = 0 + O(\theta^2), \]  
(34)

\[ \Phi = \Phi + \theta^\alpha \lambda_\alpha + \frac{1}{12} \theta^\alpha (\Gamma^{abc})_{\alpha \beta} \theta^\beta \Phi \left( T_{abc} + \frac{3}{8} c_1 (\Gamma_{abc})_{\gamma \delta} \text{tr}[\chi^\gamma \chi^\delta] \right) + O(\theta^3), \]  
(35)

where

\[ T_{abc} = -\frac{2}{3} \Phi^{-1} \widetilde{H}_{abc} + 2 \psi_{[a}^\alpha (\Gamma_{b})_{a \beta} \psi_c^\beta \] 
\[ - \psi_{[a}^\alpha (\Gamma_{bc})_{\alpha \lambda} \lambda_\beta \Phi^{-1} + \frac{1}{6} c_1 (\Gamma_{abc})_{\alpha \beta} \text{tr}[\chi^\alpha \chi^\beta] \Phi^{-1}, \]  
(36)

\[ \widetilde{H}_{abc} = 3 \partial_{[m} B_{np]} + c_1 \text{tr}[6 A_{[m} \partial_{n} A_{p]} - 4 A_{[m} A_{n} A_{p]}], \]  
(37)

\[ \omega_{abc} \equiv e_a^m \omega_{mbc} \] 
\[ = \omega_{abc}^{(0)} + \frac{1}{2} T_{abc} - \psi_{(a}^\alpha (\Gamma_{b})_{[a \beta} \psi_{c]}^\beta - \psi_{c [a}^\alpha (\Gamma_{a \beta})_{\alpha \beta} \psi_{b]}^\alpha + \psi_{(a}^\alpha (\Gamma_{b})_{a \beta} \psi_{c]}^\beta, \]  
(38)

\[ \omega_{abc}^{(0)} = e_a^n e_b^m \partial_{[m} c_{n]} + e_c^n e_a^m \partial_{[m} c_{n]} - e_b^n e_c^m \partial_{[m} c_{n]} a. \]  
(39)

On the flat background we can determine all the components:

\[ E_m^a = \delta_m^a, \quad E_m^a = 0, \quad E_\mu^a = (\Gamma^a)_{\mu a} \theta^\alpha, \quad E_\mu^a = \delta_\mu^a, \]  
(40)

\[ \Omega_{MA}^B = 0, \]  
(41)

\[ B_{mn} = 0, \quad B_{\mu \nu} = -\frac{1}{2} (\Gamma_m)_{\nu a} \theta^a \Phi, \quad B_{\mu \nu} = 0, \]  
(42)

\[ A_M = 0, \]  
(43)

\[ \Phi = \Phi = \text{constant}, \]  
(44)

\[ T_{abc} = 0. \]  
(45)
To relate $e_m^a$ to the string metric we need the following field (re)definition.

\[ \Phi = e^{-\frac{2}{3}\phi}, \quad (46) \]
\[ e_m^a \rightarrow e^{-\frac{2}{3}\phi} e_m^a, \quad (47) \]

where $\phi$ is the dilaton.\footnote{To relate $\gamma_\alpha$, $\lambda_\alpha$ and $\chi_\alpha$ to the standard gravitino, dilatino and gaugino respectively, we need the redefinition something like $\psi_\alpha \rightarrow a_1 \epsilon^{a_2 a_3} \psi_\alpha + a_3 \epsilon^{a_4 a_5} (\Gamma_m)^{a_3} \lambda_\beta$, $\lambda_\alpha \rightarrow a_5 \epsilon^{a_6 a_7} \lambda_\alpha + a_7 \epsilon^{a_8 a_9} (\Gamma_m)^{a_7} \psi_\beta$ and $\chi_\alpha \rightarrow a_9 \epsilon^{a_10} \chi_\alpha$, where $a_1, \cdots, a_{10}$ are constants.}

Now we construct the effective action of D0-brane following [6]. When we do not take account of $\xi^I$, the action has only the Born-Infeld part and does not have the Chern-Simons term. Therefore the spacetime supersymmetry is spontaneously broken completely. Worldsheet diffeomorphism is the only local symmetry which $S_{BI}$ has. The number of physical modes of $\theta^a$ is twice that of BPS D-branes because of the absence of $\kappa$-symmetry.

Now we take account of $\xi^I$. In addition to $S_{BI}$, we need the following term.

\[ S_\xi = \frac{1}{2} \int d\tau \frac{1}{2} i \xi^I (\delta_{IJ} \partial_\tau - \partial_\tau Z^M (A_M)_{IJ}) \xi^J. \quad (49) \]

This is determined by requiring invariance under worldsheet diffeomorphism and $SO(32)$ gauge transformation. The total action $S$ is the sum of $S_{BI}$ and $S_\xi$. Substituting eq. (24), (25), (26), (27), (33), (34), and (35) into $S$ we get

\[
S_{BI} = -\frac{1}{\sqrt{2\alpha'}} \int d\tau e^{-\frac{2}{3}\phi} \sqrt{g_{mn} \partial_\tau X^m \partial_\tau X^n} \\
\times \left[ 1 - \frac{5}{4} e^{\frac{2}{3}\phi} \theta^\alpha \lambda_\alpha + \frac{15}{384} c_1 \theta^\alpha (\Gamma^{abc})_{\alpha\beta}(\Gamma_{abc})_{\gamma\delta} \epsilon^{\gamma\delta} \right] \\
+ (g_{kl} \partial_\tau X^k \partial_\tau X^l)^{-1} \left[ -2 \partial_\tau X^m \partial_\tau X^n \theta^\alpha (\Gamma_m)^{\alpha\beta} \psi_\beta \right. \\
+ \left. (g_{mn} \partial_\tau X^m \partial_\tau X^n \theta^\alpha (\Gamma^b)^{\alpha\beta} T_{abc} + \frac{1}{4} \partial_\tau X^m \partial_\tau X^n e^a \theta^\alpha (\Gamma_{abc})_{\alpha\beta} \theta^\beta T_{a b c}) + O(\theta^2, \text{fermion}^2) \right] \\
+ O(\theta^2, \text{fermion}^2), \quad (50) \]

\[
S_\xi = \int d\tau \frac{1}{2} i \xi^I (\delta_{IJ} \partial_\tau - \partial_\tau Z^M (A_M)_{IJ} + \partial_\tau X^m \theta^\alpha (\Gamma_m)^{\alpha\beta} (\chi^\beta)_{IJ}) \xi^J \\
+ O(\theta^2, \text{fermion}^2), \quad (51) \]
where $g_{mn} = e_{m}^{a} e_{na}$ and

$$D_{\tau} \theta^{\alpha} = \partial_{\tau} \theta^{\alpha} + \frac{1}{4} \partial_{\tau} X^{m} \omega_{mab} \theta^{\beta} (\Gamma_{ab})_{\beta}^{\alpha}.$$  

(52)

From this action we can construct the vertex operators of $\psi_{m}^{\alpha}$, $\lambda_{\alpha}$, $\chi^{\alpha}$ and $A_{m}$:

$$\int d\tau (V_{\psi})_{\alpha}^{m} \psi_{m}^{\alpha} = \frac{1}{\sqrt{2\alpha'}} \int d\tau 2 e^{-\frac{\phi}{3\sqrt{2}}} \frac{\partial_{\tau} X^{m} \partial_{\tau} X^{n}}{\sqrt{g_{kl} \partial_{\tau} X^{k} \partial_{\tau} X^{l}}} \theta^{\beta} (\Gamma_{m})_{\beta}^{\alpha} \psi_{m}^{\alpha} + O(\theta^{2}),$$  

(53)

$$\int d\tau (V_{\lambda})_{\alpha}^{\lambda} = -\frac{1}{\sqrt{2\alpha'}} \int d\tau \frac{5}{4} e^{-\frac{\phi}{6}} \sqrt{g_{kl} \partial_{\tau} X^{k} \partial_{\tau} X^{l}} \theta^{\alpha} \lambda_{\alpha} + O(\theta^{2}),$$  

(54)

$$\int d\tau (V_{\chi})_{I}^{J} \chi_{IJ}^{\alpha} = \int d\tau \frac{1}{2} i \xi^{I} \xi^{J} \partial_{\tau} X^{m} \theta^{\beta} (\Gamma_{m})_{\beta}^{\alpha} \chi_{IJ}^{\alpha} + O(\theta^{2}),$$  

(55)

$$\int d\tau (V_{A})_{m}^{m} A_{m} = \int d\tau (-i) \frac{1}{2} \xi^{I} \xi^{J} \partial_{\tau} X^{m} A_{mIJ} + O(\theta^{2}).$$  

(56)

Similarly we can construct the vertex operators of graviton, dilaton, and RR 2-form. These vertex operators are used for calculating instanton effects following [1, 2].

4 A simple example

In this section we consider $S^{1}$ compactification and some amplitudes as a simple example of non-BPS D0-brane instanton effects using the prescription given in [1, 2]. We discuss the low energy and large radius regime in order to calculate instanton effects by the effective action, as explained in [2]. We do not keep track of the overall normalization of the amplitudes since it cannot be determined by this method.

Let us take the background geometry $S^{1} \times R^{9}$ ($g_{mn}$ is flat, $\phi$ is a constant and other fields are zero). We choose $X^{9}$ as the coordinate of $S^{1}$ and set the radius $R$. Substituting eq.(40), (43) and (44) to eq.(48) and (49) we get the D0-brane action in this background:

$$S = -\frac{1}{\sqrt{2\alpha'}} \int d\tau e^{-\phi} \sqrt{(\partial_{\tau} X^{m} - \partial_{\tau} \theta^{\alpha} (\Gamma_{m})_{\alpha}^{\beta} \theta^{\beta}) (\partial_{\tau} X^{m} - \partial_{\tau} \theta^{\gamma} (\Gamma_{m})_{\gamma}^{\delta} \theta^{\delta})} + \int d\tau \frac{1}{2} i \xi^{I} \partial_{\tau} \xi^{I}.$$  

(58)

Here we are using the string metric.

Let us consider an Euclidean D0-brane winding around $S^{1}$. If we choose the static gauge, then

$$X^{9} = R \tau, \quad 0 \leq \tau \leq 2\pi.$$  

(59)
The action up to the quadratic order becomes
\[ S = -\frac{2\pi R}{\sqrt{2\alpha'} g_s} - \frac{1}{\sqrt{2\alpha'} g_s R} \int d\tau \frac{1}{2} (\partial_\tau X^i)^2 + \frac{1}{\sqrt{2\alpha'} g_s} \int d\tau \partial_\tau \theta^\alpha (\Gamma^9)_{\alpha\beta} \theta^\beta + \int d\tau \frac{1}{2} i \xi^I \partial_\tau \xi^I, \]  
with \( i = 0, 1, \ldots, 8 \).

The zero modes are the constant modes of \( X^i, \theta^\alpha \) and \( \xi^I \). The zero modes of \( X^i \) represent the position of the D0-brane in \( R^9 \). To soak up the fermion zero modes we must insert 16 \( \theta \)'s and 32 \( \xi \)'s.

In the flat background the vertex operators of \( \psi_9^\alpha (X^i), \lambda_\alpha (X^i), \chi^\alpha (X^i) \) and \( A_9 (X^i) \) become
\[
\begin{align*}
(V_\psi)_9^\alpha & \sim (\Gamma^9)_{\alpha\beta} \theta^\beta, \\
(V_\lambda)^\alpha & \sim \theta^\alpha, \\
(V_\chi)^I_J & \sim (\Gamma_9)_{\alpha\beta} \theta^\beta \xi^I \xi^J, \\
(V_A)^{9I,J} & \sim \xi^I \xi^J,
\end{align*}
\]
where we ignored the coefficients.

The simplest amplitudes saturating the fermion zero modes are as follows.

- 16 \( V_\chi \) insertions
  This gives the following correction to the effective action.
  \[
  \epsilon^{\alpha_1 \cdots \alpha_{16}} \epsilon^{I_1 \cdots I_{32}} (\Gamma^9)_{\alpha_1 \beta_1} \chi^I_{I_1 I_2} \cdots (\Gamma^9)_{\alpha_{16} \beta_{16}} \chi^I_{I_{31} I_{32}} \exp\left(-\frac{2\pi R}{\sqrt{2\alpha'} g_s}\right)
  \]

- 16 \( (V_\lambda)^9 \), \( n \) \( V_\lambda \) and \( 16 - n \) \( (V_\psi)^9 \) insertions
  This gives the following correction to the effective action.
  \[
  \epsilon^{\alpha_1 \cdots \alpha_{16}} \epsilon^{I_1 \cdots I_{32}} (A_9)_{I_1 I_2} \cdots (A_9)_{I_{31} I_{32}} \lambda_{\alpha_1} \cdots \lambda_{\alpha_n} \\
  \times (\Gamma^9)_{\alpha_{n+1} \beta_{n+1}} \psi^I_{9} \beta_{n+1} \cdots (\Gamma^9)_{\alpha_{16} \beta_{16}} \psi^I_{9} \beta_{16} \exp\left(-\frac{2\pi R}{\sqrt{2\alpha'} g_s}\right)
  \]

We can consider the compactification on higher dimensional torus similarly to the above analysis. In this case the D0-brane is unstable if the radius of the direction transverse to it is sufficiently small, as explained in section 2. But we do not have to consider this fact since our discussion is restricted to the large radius \( (R \gg \alpha') \) regime.
5 Discussions

We calculated the instanton effects by the effective action. Of course we can consider stringy calculation by using the rule given in [7]. In this case we must be careful with the instability of the D0-brane caused by sufficiently small radius of the direction transverse to it.

The D0-brane corresponds to the perturbative massive modes of heterotic $SO(32)$ theory. It is interesting to calculate the effect corresponding to the D0-brane instanton effects in the heterotic side, though quantitative comparison with the results in type I side is difficult since these are not protected from the string coupling correction.

Type I string theory has non-BPS stable D($-1$)-brane besides D0-brane [3]. Its instanton effects exist even in flat ten dimensional spacetime. The properties of D($-1$)-brane are similar to the D0-brane. It is interesting to calculate these effects.

Instanton effects correct the low energy effective action. It may be possible to determine the exact tensions of D0 and D($-1$)-brane by completing the action supersymmetrically and identifying the instanton actions.

Acknowledgments

I would like to thank H. Hata for careful reading of the manuscript.

References

[1] K. Becker, M. Becker and A. Strominger, “Fivebranes, membranes, and nonperturbative string theory”, hep-th/9507158, Nucl. Phys. B456 (1995) 130

[2] J. A. Harvey and G. Moore, “Superpotentials and Membrane Instantons”, hep-th/9907026

[3] M. B. Green and M. Gutperle, “Effects of D-instantons”, hep-th/9701093, Nucl. Phys. B498 (1997) 195

[4] A. Sen, “Non-BPS States and Branes in String Theory”, hep-th/9904207 ; A. Lerda and R. Russo, “Stable Non-BPS States in String Theory: a Pedagogical Review”, hep-th/9905006 ; J. H. Schwarz, “TASI Lectures on Non-BPS D-brane Systems”, hep-th/9908144
[5] A. Sen, “SO(32) Spinors of Type I and Other Solitons on Brane-Antibrane Pair”, hep-th/9808141, JHEP. 9809 (1998) 023

[6] A. Sen, “Supersymmetric Worldvolume Action for Non-BPS D-branes”, hep-th/9909062, JHEP. 9910 (1999) 008

[7] A. Sen, “Type I D-particle and its Interactions”, hep-th/9809111, JHEP. 9810 (1998) 021

[8] M. Frau, L. Gallot, A. Lerda and P. Strigazzi, “Stable Non-BPS D-Branes in Type I String Theory”, hep-th/9903123

[9] E. Witten, “D-Branes and K-Theory”, hep-th/9810188, JHEP. 9812 (1998) 019

[10] M. Billó, P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, “Microscopic string analysis of the D0-D8 brane system and dual R-R states”, hep-th/9802088, Nucl. Phys. B526 (1998) 199

[11] J. J. Atick, A. Dhar and B. Ratra, “Superspace formulation of ten-dimensional $N = 1$ supergravity coupled to $N = 1$ super Yang-Mills theory”, Phys. Rev. D33 (1986) 2824