Gravireggeons in Extra Dimensions and Interaction of Ultra-high Energy Cosmic Neutrinos with Nucleons

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Abstract
We present the results on non-perturbative quantum gravity effects related to extra dimensions which can be comparable, in some cases, with the SM contributions, e.g. in lepton-lepton or lepton-nucleon scattering. The case of cosmic neutrino gravitational interaction with atmospheric nucleons is considered in detail.

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1 Gravi-Reggeon effects in multidimensional scattering amplitudes

During last years there is a growing practical interest in models with compact extra spatial dimensions. Their compactification radius, $R_c$, varies from 1 fm to 1 mm, depending on a number of extra dimensions $d = D - 4$ [1]. The models predict massive Kaluza-Klein (KK) excitations of the graviton and KK modes of the SM fields (provided the latter are allowed to propagate in higher dimensions). If $D$-dimensional space-time has a flat metric, the coupling of the massive graviton modes with the SM particle is very weak and is defined by the Newton constant $G_N = 1/\tilde{M}_{Pl}^2$, where $\tilde{M}_{Pl}$ is the reduced Planck mass. Nevertheless, in the case when SM particles are confined to a 4-dimensional flat “brane”, summing up the KK graviton excitations results in a $D$-dimensional gravitational coupling $G_D \sim 1/M_{D}^{2+d}$, with a fundamental Planck scale $M_D$ of order 1 TeV [1].

Let us first consider the SM in $D$-dimensional flat space-time, $D > 4$, without gravity. Due to extra spatial dimensions, an effective “transverse interaction region” becomes larger than in four dimensions. One manifestation of this is a modification of the Froissart-Martin upper bound in a flat space-time with arbitrary $D$ dimensions [2]:

$$\sigma^D_{\text{tot}}(s) \leq \text{const}(D) \, R^D_{0}(s) \, R^{D-2}(s),$$

(1)

$\sqrt{s}$ being a collision energy. The “transverse radius” in (1) is given by $R_0(s) = N(D) \ln s/\sqrt{t_0}$, where $t_0$ denotes the nearest singularity in the $t$-channel, assumed non-zero, while $N(D)$ is some integer depending on $D$. It is interesting to see, what happens with scattering amplitudes when we replace infinite extra dimensions by compact ones?

In Ref. [3] the Froissart-Martin bound was generalized for scattering in $D$-dimensional space-time with compact extra dimensions. For one extra dimension with the compactification radius $R_c$, the upper bound is of the form:

$$\text{Im} \, T_D(s, 0) \leq \text{const}(D) \, s \, R_0^{D-2}(s) \, \Phi \left( \frac{R_0}{R_c}, D \right),$$

(2)

where $\text{Im} \, T(s, t)$ is the scattering amplitude, $t$ is a momentum transfer (in $D$ dimensions) and $\Phi(R_0/R_c, D)$ is a known function. At $R_c \ll R_0(s)$ the equality (2) results in [3]

$$\text{Im} \, T_D(s, 0) \leq \text{const}(D) \, s \, R_0^{D-3}(s) \, R_c,$$

(3)
while in the opposite limit, $R_c \gg R_0(s)$, the inequality (2) reproduces the upper bound (1).

Now let us allow for the gravity to come into play. As was argued in a number of papers [4]-[5], in the Minkowski space-time with $D$ dimensions ($D > 4$) the gravity becomes strong in a transplanckian region ($s \gg M_D$), since an effective gravitational coupling, $G_D s$, rises with energy.

In Refs [4] the eikonal representation for the scattering amplitude of the gravitons in the string theory was obtained:

$$A(s,t) = -2i s \int d^{D-2}b e^{i q b} \left[ e^{i \chi(s,b)} - 1 \right], \quad (4)$$

where $\chi(s,b) \approx \text{Im} \chi(s,b)$ is large at $b \lesssim b_1 = 2 \sqrt{\alpha'_g} \ln s$ ($\alpha'_g$ is a string tension). Thus, one gets asymptotically

$$\sigma^D_{\text{in}}(s) \simeq \text{const}(D) b_1^{D-2}(s). \quad (5)$$

Due to the absence of infrared divergences in the flat space-time with more than four dimensions, the gravitational cross section (5) appears to be finite and similar to the upper bound (1).

In what follows, we will first consider the scattering of two particles in the model with one compact extra dimensions ($D = 5$) in the transplanckian kinematical region:

$$\sqrt{s} \gg M_D, \quad s \gg -t, \quad (6)$$

t = -q^2 being four-dimensional momentum transfer. The generalization to $D > 5$ is straightforward and it will be done below. Thus, we start from the consideration of the scattering of bulk particles in four spatial dimensions, one of which is compactified with the large radius $R_c$.

In the eikonal approximation an elastic scattering amplitude in the transplanckian kinematical region (6) is given by the sum of reggeized gravitons in $t$-channel. So, we assume that both massless graviton and its KK massive excitations lie on linear Regge trajectories:

$$\alpha(t_D) = \alpha(0) + \alpha'_g t_D, \quad (7)$$

where $t_D$ denotes $D$-dimensional momentum transfer. Since the extra dimension is compact with the radius $R_c$, we come to splitting of the Regge trajectory (7) into a leading vacuum trajectory (6)

$$\alpha_0(t) \equiv \alpha_{\text{grav}}(t) = 2 + \alpha'_g t \quad (8)$$

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and infinite sequence of secondary, “KK-charged”, gravireggeons:

\[ \alpha_n(t) = 2 - \frac{\alpha'_g}{R_c^2} n^2 + \alpha'_g t, \quad n \geq 1. \]  

(9)

The string theory implies that the slope of the gravireggeon trajectory is universal for all \( s \), and \( \alpha'_g = 1/M_s^2 \), where \( M_s \) is a string scale.

If we assume that multidimensional theory at short distances is a string theory, than the scale \( M_D \) can be of order the fundamental string scale \( M_s = (\alpha')^{-1/2} \). For instance, in the type I theory of open and closed strings one has \[ M_s = \left( \frac{g_s^2}{4\pi} \right)^{2/(2+d)} M_D, \]  

(10)

where \( g_s \) is a gauge coupling at the string scale. This relation leads to \( D \)-dimensional Planck mass a bit higher than the string scale (for \( g_s^2/4\pi \simeq 0.1 \)).

Thus, instead of taking a “bare” graviton exchange, we calculate a contribution from the Pomeron trajectory to which this KK graviton mode belongs:

\[ -G_N \frac{1 + \exp(-i\pi \alpha_n(t))}{\sin \pi \alpha_n(t)} \alpha'_g \beta_n^2(t) \left( \frac{s}{s_0} \right)^{\alpha_n(t)}. \]  

(11)

The Born amplitude is, therefore, of the form

\[ A_B(s, t, n) = G_N (2\pi R_c) \left[ i - \cot \frac{\pi}{2} \alpha_n(t) \right] \alpha'_g \beta_n^2(t) \left( \frac{s}{s_0} \right)^{\alpha_n(t)} . \]  

(12)

In order to get an idea of possible \( t \)-dependence of Regge residues \( \beta_n^2(t) \), we consider scattering of \( D \)-dimensional gravitons. The corresponding amplitude has been calculated in Refs. \[ A_{string}^B(s, t_D) \sim G_D s^2 \left| t_D \right| \Gamma(1 - \alpha'_g t_D/2) \Gamma(1 + \alpha'_g t_D/2) \left( \alpha'_g s \right)^{\alpha'_g t_D}. \]  

(13)

The expression (13) is valid in the region \( \alpha'_g |t| < 1 \) in which it can be recast in the form:

\[ A_{string}^B(s, t_D) \sim G_D s^2 \left| t_D \right| e^{\gamma \alpha'_g t_D} \left( \alpha'_g s \right)^{\alpha'_g t_D}, \]  

(14)

where \( \gamma \simeq 0.58 \) is the Euler constant.
Thus, we have $A(s,t) \sim \exp(\alpha_g c t)$, where $c$ is of order of unity. Let us assume that Regge residues in (12) have an analogous $t$-dependence:

$$\beta_n^2(t) = \beta^2(0) e^{\alpha_g b_0 (t-n^2/R_c^2)}.$$ (15)

Since the coupling of all KK states to the SM fields is universal in ADD model, we expect that $\beta_n^2(t)$ depends on $n$ via $t_D = t - n^2/R_c^2$. Accounting for the fact that the product $\alpha' g b_0$ appears only in a combination with $\alpha' g \ln(s/s_0)$, we can neglect it in forthcoming calculations at large $s$ and put $\beta_n^2(t) \simeq \beta^2(0)$. At $t \to 0$, $n = 0$ expression (11) should reproduce singular term $G_N s / |t|$ related with the massless graviton, that results in the relation $2\beta^2(0)/\pi s_0^2 = 1$.

The expression for 5-dimensional eikonal amplitude looks like ($k$ being the exchanged KK quantum number)

$$A(s,t,k) = 2i R_c s \int d^2 b \, e^{i q_{\perp} b + i k \phi} \int d\phi \left[ 1 - e^{i \chi(s,b,\phi)} \right],$$ (16)

with the eikonal given by

$$\chi(s,b,\phi) = \frac{1}{4\pi s} \int_0^\infty q_{\perp} dq_{\perp} \, J_o(q_{\perp} b) \frac{1}{2\pi R_c} \sum_{n=-\infty}^{\infty} e^{-in\phi} A^B(s,-q_{\perp}^2,n).$$ (17)

The variable $\phi$ runs the region $-\pi \leq \phi \leq \pi$. These inequalities imply that $-\infty \leq y \leq \infty$ in the limit $R_c \to \infty$ (flat extra dimension), $y = R_c \phi$ being the 5-th component of the impact parameter.

One can easily obtain from (16) that at $k = 0$ and $s < 4/R_c^2$ only modes with $n = 0$ contribute and effectively $\chi(s,b,\phi) = \chi^0(s,b)$, corresponding to $n = 0$ contribution in the sum in Eq. (17). So, at low energy the scattering amplitude does not feel extra dimensions (the factor $R_c$ is trivial and is absent at proper normalization).

Let us consider first the imaginary part of the eikonal. From equations (17), (12) we obtain:

$$\text{Im} \chi(s,b,\phi) = G_N s \frac{\alpha_g}{8 R_g^2(s)} \exp \left[ -b^2/4 R_g^2(s) \right] \theta_3(v,q),$$ (18)

where

$$R_g(s) = \sqrt{\alpha_g' (\ln(s/s_0) + b_0)}.$$ (19)
is a gravitational slope. The quantity $\theta_3$ in (18) is one of Jacobi $\theta$-functions [8]:

$$\theta_3(v) = \theta_3(v,q) = 1 + 2 \sum_{n=1}^{\infty} \cos(2\pi n v) q^n.$$  \hfill (20)

In our case, it depends on variables

$$v = \frac{\phi}{2\pi},$$
$$q = \exp \left[ - \frac{R_c^2(s)/R_g^2(s)}{c^2} \right].$$  \hfill (21)

The function $\theta_3(v,q)$ is well-defined for all (complex) $v$ and all values of $q$ such as $|q| < 1$. It has a singularity at $q \to 1$ (see below). The $\theta$-functions are often defined in terms of variable $\tau$:

$$\theta(v) = \theta(v|\tau),$$  \hfill (22)

where

$$q = e^{i\pi \tau}.$$  \hfill (23)

Let us define the ratio:

$$a = \frac{R_c}{2R_g(s)}$$  \hfill (24)

(that is, $q = \exp(-1/4a^2)$). From the equality [1]

$$R_c = 2 \cdot 10^{31/d-17} \left( \frac{1 \text{TeV}}{M_D} \right)^{1+2/d} \text{cm}$$  \hfill (25)

we see that the compactification scale $R_c^{-1}$ varies from $10^{-3}$ eV for $d = 2$ to 10 MeV for $d = 6$. Since $R_c^{-1} \ll (2R_g(s))^{-1}$ even at ultra-high energies, we have $a \gg 1$ and, consequently, $(1 - q) \ll 1$.

The behavior of $\theta_3(v,q)$ at $q \to 1$ can be derived by using unimodular transformation of $\theta_3$-function (known also as Jacobi imaginary transformation) [3]:

$$\theta_3 \left( \frac{v}{\tau} - \frac{1}{\tau} \right) = (-i\tau)^{1/2} e^{i\pi v^2/\tau} \theta_3(v|\tau).$$  \hfill (26)

Here $(-i\tau)^{1/2}$ has a principal value which lies in the right half-plane. In variable $q$, equality (26) looks like

$$\theta_3(v,q) = \left( -\frac{\pi}{\ln q} \right)^{1/2} \sum_{n=-\infty}^{\infty} e^{(2\pi n - \phi)^2/4\ln q}.$$  \hfill (27)

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The series in the RHS of (27) converges very quickly at \( q \to 1 \), contrary to original series (20):

\[
\theta_3(v, q) = 2a\sqrt{\pi}\left\{e^{-\phi^2a^2} + \sum_{n=1}^{\infty} \left[e^{-\phi^2a^2} + e^{-(2\pi n - \phi)^2a^2} + e^{-(2\pi n + \phi)^2a^2}\right]\right\}.
\]

(28)

Notice that \( a^2 = -1/4 \ln q \).

From all said above, we get

\[
\text{Im} \chi(s, b, \phi) \simeq G_N s \frac{\alpha'_g R_c \pi^{1/2}}{8R_g^3(s)} \exp \left[-\left(\frac{b^2 + R_c^2 \phi^2}{4R_g^2(s)}\right)\right].
\]

(29)

The expression (18) is directly generalized for \( d \) extra dimensions (\( d \geq 1 \)):

\[
\text{Im} \chi(s, b, \phi_1, \ldots, \phi_d) = G_N s \frac{\alpha'_g R_c^d \pi^{d/2}}{8R_g^{d+2}(s)} \times \exp \left[-\left(\frac{b^2}{4R_g^2(s)}\right)\right] \prod_{i=1}^{d} \theta_3(v_i, q),
\]

(30)

where \( v_i = \phi_i/2\pi \). Correspondingly, we obtain

\[
\text{Im} \chi(s, b, \phi_1, \ldots, \phi_d) \simeq G_N s \frac{\alpha'_g R_c^d \pi^{d/2}}{8R_g^{d+2}(s)} \times \exp \left[-\left(\frac{b^2 + R_c^2 \phi_1^2 + \ldots + R_c^2 \phi_d^2}{4R_g^2(s)}\right)\right].
\]

(31)

We see from (31) that the imaginary part of the eikonal decreases exponentially in variables \( b, \phi_i \) outside the region:

\[
b^2 + (R_c \phi_1)^2 + \ldots + (R_c \phi_d)^2 \lesssim R_0^2(s),
\]

(32)

where

\[
R_0^2(s) \approx 4R_g^2(s) \ln(s/M_D^2)
\]

(33)

at high \( s \).

Let \( t_D = (t, -n_1^2/R_c^2, \ldots, -n_d^2/R_c^2) \) be a bulk momentum transfer. Then we get the following expression for multidimensional scattering amplitude:

\[
A_D(s, t, n_1, \ldots, n_d) = -2is R_c^d \int d^2b \ e^{i\mathbf{q} \cdot \mathbf{b}} \int_{-\pi}^{\pi} d\phi_1 \cdots \int_{-\pi}^{\pi} d\phi_d
\]

\[
\times \prod_{i=1}^{d} e^{i n_i \phi_i} \left[e^{i\chi(s, b, \phi_1, \ldots, \phi_d)} - 1\right].
\]

(34)
Correspondingly, the inelastic cross section in the space-time with \( d \) compact dimension is given by
\[
\sigma_D^{in}(s) = (2\pi R_c)^d \int d^2b \int_{-\pi}^{\pi} d\phi_1 \cdots \int_{-\pi}^{\pi} d\phi_d \left[ 1 - e^{-2\text{Im}\chi(s,b,\phi_1,\ldots,\phi_d)} \right].
\] (35)

As was already shown, the imaginary part of the eikonal is negligibly small outside region \((32)\). That results in the estimates:
\[
\sigma_D^{in}(s) \simeq \text{const}(D) \times \begin{cases} R_0^{2+d}(s), & R_c \gg R_0(s) \\ R_0^2(s) R_c^d, & R_c \ll R_0(s) \end{cases}
\] (36)

which remind Eqs. \((1)\) and \((3)\) obtained previously. As was mentioned above, \( R_0(s) \ll R_c \) for any reasonable \( s \). So, the size of the compact extra dimensions is irrelevant to the behavior of the inelastic cross section and \( \sigma_D^{in}(s) \sim (\alpha_g')^{D/2-1}(\ln s)^{D-2} \). Only at \( s \to \infty \), when the transverse interaction region \( R_0(s) \) becomes much larger than \( R_c \), we get \( \sigma_D^{in}(s) \sim \alpha_g' R_c^{D-4}(\ln s)^2 \).

## 2 Scattering of the SM fields in the presence of compact extra dimensions

Now we consider the case when the colliding particles are confined on the 4-dimensional brane, while the exchange quanta (KK gravitons) are allowed to propagate in the bulk. Thus, the collisions of the SM particles take place in a two-dimensional impact parameter space. In Refs. \([9,10]\) the scattering of two SM particles was calculated in the eikonal approximation by summing up only “bare” KK gravitons. The massive graviton modes originated from the extra dimensions change four-dimensional propagator by
\[
\frac{1}{-t} \to \sum_{n_1^2+\ldots+n_d^2 \geq 0} \frac{1}{-t + \sum_{i=1}^d \frac{n_i^2}{R_c^2}}.
\] (37)

Since a contribution from only non-reggeized KK excitations of the graviton has been taken into account in \([9,10]\), the eikonal has no imaginary parts in such an approach. As was shown in \([10]\), the \( D \)-dimensional brane amplitude
has a renormalized Born pole at $t = 0$ and an infinite phase. Notice, series \(37\) diverges and needs renormalization at $d \geq 2$.

In Ref. [11] it was shown that an amplitude of $M \to N$ transition observed in four dimensions, $A_{MN}$, is related to a corresponding $D$-dimensional amplitude $A_{MN}^D$ by the relation

$$A_{MN} = (2\pi R_c)^{d(1-(M+N)/2)} A_{MN}^D$$

(in our case, $M = N = 2$). Amplitudes $A_{MN}$ have non-zero limit at $R_c \to 0$, reproducing the usual 4-dimensional pseudoeuclidean case. Since the colliding particles are confined on the brane, their momenta lie in four-dimensional space. Therefore, the impact parameter belongs to the two-dimensional space and we have to put $\phi_i = 0, i = 1, \ldots, d$, in (31). With taking account of this, the expression for four-dimensional eikonal amplitude (in the presence of $d$ compact extra dimensions) looks like

$$A(s, t) = 2i s \int d^2 b \ e^{iq\perp b} [1 - e^{i\chi(s, b)}],$$

where $\chi(s, b) = \chi(s, b, \phi_1 = 0, \ldots, \phi_d = 0)$. Taking into account (31), we get the expression

$$\text{Im} \chi(s, b) = \frac{1}{\pi^{d/2-1}} \frac{s}{M_D^2} \left( \frac{M_s}{2M_D} \right)^d \left[ \ln \left( \frac{s}{s_0} \right) \right]^{-(1+d/2)} \times \exp[-b^2/4R_g^2(s)],$$

where the relation $M_P^2 = (2\pi R_c)^d M_D^{2+d}$ is used [11]. The detailed analysis of the real part of the eikonal will be given elsewhere. Here we only note that, contrary to (40), the real part of the eikonal (with the massless graviton term subtracted) decreases as a power of the impact parameter at large $b$.

The important features of the gravitational contribution to cross sections are its independence of types of colliding particles and a strong dependence on the collision energy. So, one can expect that at superplanckian energies gravity exchanges will dominate the SM electroweak interactions. That is why we now focus on leptonic and semileptonic collisions.

Let us first consider $e^+e^-$ annihilation. Unfortunately, future linear colliders will provide us with the c.m.s. energies $\sqrt{s}$ around $M_D (0.5 \div 2 \text{ TeV})$. In order to estimate $\sigma_{e^+e^-}$ numerically, we need to fix Regge free parameter $s_0$ in [11]. Since $s_0$ is related rather with a mass scale of exchange quanta than
with mass scales of colliding particles, we can treat the scattering amplitude of two graviton [13] instead of SM particle collision, and deduce that
\[ s_0 = (\alpha'_g)^{-1}. \] (41)

This relation is also motivated by the duality [13]. The results of our calculations of inelastic cross section \( \sigma_{e^+e^-} \) at \( \sqrt{s} = 1 \) TeV based on formulae (40), (41) are presented in the second row of Table 1.

Table 1: Cross sections of the processes induced by graviton exchanges in \( t \)-channel (second row) and \( s \)-channels (third row) at \( \sqrt{s} = 1 \)TeV for different numbers of extra dimensions \( d \) (in \( \text{pbarn} \)).

| \( d \) | \( e^+e^- \rightarrow e^+e^- + X \) | \( e^+e^- \rightarrow ff \) |
|-------|----------------|----------------|
| 2     | 1.06 \cdot 10^3 | 9.3            |
| 3     | 1.10 \cdot 10^2 | 3.7            |
| 4     | 1.78 \cdot 10^1 | 2.0            |
| 5     | 3.84            | 1.3            |
| 6     | 1.02            | 0.9            |

These cross sections are larger than the cross sections of the processes induced by massive graviton exchanges in \( s \)-channel (at least, for \( d \leq 6 \)).\(^2\) For definiteness, consider matrix element for fermion pair production \( e^+e^- \rightarrow ff \):
\[ \mathcal{M} = G_N T^e_{\mu\nu} P^{\mu\nu\alpha\beta} T^{f}_{\alpha\beta} \sum_{n_1^2+...+n_3^2 \geq 0} \frac{1}{s - \sum_{i=1}^{d} n_i^2 R_c^2}. \] (42)

Here \( P^{\mu\nu\alpha\beta} \) is a tensor part of a graviton propagator, while \( T^{e(f)}_{\mu\nu} \) is the energy-momentum tensor of field \( e(f) \) \(^{17,18}\). The sum in (42) diverges for \( d \geq 2 \). It can be estimated if one convert it into an integral and introduce an explicit ultraviolet cut-off \( M_s \). Then we get for \( d > 2 \):
\[ \sum_{n_1^2+...+n_3^2 \geq 0} \frac{1}{s - \sum_{i=1}^{d} n_i^2 R_c^2} \simeq \begin{cases} \frac{-2R_c^d}{(d-2)\Gamma(d/2)(4\pi)^{d/2}} M_s^{d-2}, & \sqrt{s} \ll M_s \\ \frac{R_c^d}{\Gamma(1+d/2)(4\pi)^{d/2}} \frac{M_s^d}{s}, & \sqrt{s} \gg M_s \end{cases} \] (43)

\(^1\)In hadronic physics, the phenomenological parameter \( s_0 \approx 1/\alpha'(0) \), where \( \alpha'(0) \approx 1 \) GeV\(^{-2} \) is the slope of hadronic Regge trajectories \(^{12}\).

\(^2\)It is worth to note that, generally, QFTs for \( d > 0 \) are not renormalizable. So, the following estimates are of illustrative character.
(an asymptotics at $\sqrt{s} \ll M_s$ was first found in [13]).

Taking into account that the sum in indices results in a factor proportional to $s^2$, we obtain from (41), (43):

$$\mathcal{M} \sim \lambda s^2 \left( \frac{M_s}{M_D} \right)^{2+d} \times \left\{ \begin{array}{ll}
1 & \sqrt{s} \ll M_s \\
\frac{1}{M_s^2} & \sqrt{s} \gg M_s
\end{array} \right.$$  

(44)

where $\lambda = O(1)$ has opposite sign for small and large $\sqrt{s}$. Two asymptotics are well-matched at $\sqrt{s} \simeq M_s$. Thus, at $\sqrt{s} \gtrsim M_s$ we arrive at the expression

$$\sigma(e^+e^- \rightarrow f\bar{f}) \simeq \lambda^2 \frac{N_c}{40\pi} \left( \frac{M_s}{M_D} \right)^{2+d} \frac{s}{M_s^4},$$

(45)

where $N_c$ represents the number of colors of the final state. The result of numerical calculations by using formula (45) is presented in the third row of Table 1.

To compare, hadronic SM background in $e^+e^-$ annihilation ($e^+e^- \rightarrow e^+e^- + \text{hadrons}$), including the effects due to the (anti)tagging of the electron and accounting for all available data on $\gamma\gamma$ collisions, was estimated to be [15]

$$\sigma_{e^+e^- \rightarrow \sum q\not=tq\bar{q}} (\sqrt{s} = 1\text{TeV}) \simeq (2.7 - 4.0) \cdot 10^4 \text{pb}. $$

(46)

The SM processes with different final states ($\sum_{q\not=t} q\bar{q}$, $W^+W^-$, $tt\bar{t}$, $\chi^+\chi^-$, $\mu_R\mu_R^*$, $Zh$, etc.) have cross sections which are less than 1 pb at $\sqrt{s} = 1$ TeV (see, for instance, Fig. 1.3.1 from Ref. [16]). The highest rate has the process $e^+e^- \rightarrow \sum q\not=tq\bar{q}$, its cross section is about 0.7 pb.

Our goal is to find the process in which gravity forces can dominate SM interactions. Such a process has to obey the following requirements: (i) colliding energy is much larger than $M_D \simeq 1$ TeV, (ii) SM cross section does not rise rapidly in $s$. The best candidate is the scattering of ultra-high energy (UHE) neutrinos off the nucleons. These neutrinos is a part of ultra-high energy cosmic rays (UHECR) with energy $E \gtrsim 10^{18}$ eV, which are dominated by extragalactic sources of protons [14]. It is a detection of UHE neutrinos that can help us to discriminate between different origin of UHECR. For instance, in cosmological (“bottom-up”) scenarios, neutrino fluxes are almost equal to gamma ray fluxes. In astrophysical (acceleration)
approach, the neutrino flux is only a fraction of the gamma ray flux and is modified due to a propagation of cosmic rays before they reach the Earth.

The cosmic neutrinos with extremely high energies $E \gtrsim 10^{20}$ eV are also believed to explain so-called Greisen-Zatsepin-Kuzmin (GZK) cut-off of UHECR spectrum [19] (see below). During UHECR propagation, the protons scatter off cosmic microwave background (CMB):

$$p + \gamma_{CMB} \rightarrow N + \pi.$$  \hspace{1cm} (47)

Taking into account that a typical CMB photon energies are $10^{-3}$ eV, one can obtain that the nucleon interaction length drops to about 6 Mpc at GZK bound of $E_{GZK} \approx 5 \cdot 10^{19}$ eV [19]. The observation of UHECR at $E > E_{GZK}$ is a serious problem for theories in which the origin of CR is based on acceleration of charges particles in astrophysical objects. Due to the energy losses (say, through process (47)), the UHECR particles cannot originate at distances larger than 60 Mpc from the Earth. On the other hand, all potential astrophysical sources of UHECR events are far beyond this distance.

At the same time the process (47) is an origin of so-called cosmogenic neutrinos due to a consequence decay of charged pion as $\pi^\pm \rightarrow \mu^\pm \nu_\mu$, $\mu^\pm \rightarrow e^\pm \nu_e \bar{\nu}_\mu$. The fraction of the proton energy carried by the neutrino is $E_\nu/E_p \approx 0.05$ and independent of $E_p$. The cosmogenic neutrino flux was first estimated in [21], [22]. More recent estimates can be found in Refs. [23]-[26]. The predicted fluxes depend on the evolution parameter $m$ and on the value of the redshift $z$, and lie in the range: $E_\nu^2 \Phi_\nu \approx (0.5 \cdot 10^{-9} - 10^{-8})$ GeV cm$^{-2}$ s$^{-1}$ sr$^{-1}$ at $E_\nu = 10^{20}$ eV ($\nu = \nu_\mu, \bar{\nu}_\mu, \nu_e$).\footnote{Below $E_{GZK}$, the dominant energy loss for the proton is due to the process $p \gamma_{CMB} \rightarrow pe^+e^-$, down to the threshold energy of $4.8 \cdot 10^{17}$ eV.}

There are, however, other possible origin of UHE neutrinos. It is usually anticipated that $\Phi_{\nu_e} \approx \Phi_{\bar{\nu}_\mu} \approx \Phi_{\nu_\mu}$. We present below the total flux of muonic neutrinos and antineutrinos in a number of models at $E_\nu = 10^{20}$ eV. In the active galactic nuclei (AGN), the dominant mechanism for neutrino creation is the accelerated proton energy loss due to $pp$ or $p\gamma$ interactions [27]. Note, AGN produce a large fraction of the gamma rays in the Universe, and their spectra agree with the prediction that gamma rays are produced by hadrons. In the AGN approach it was obtained that

\footnote{Some cosmic ray protons with energies above $10^{20}$ eV are converted into neutrons by pion photo-production. The neutrons decay again into protons during their propagation producing electronic anti-neutrinos. This mechanism is important at $E_\nu < 10^{17}$ eV.}
\[ E_\nu^2 \Phi_\nu \simeq 0.3 \cdot 10^{-8} \text{ GeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \] In Z-busts scenario, cosmic neutrinos with extremely high energies \( E_\nu > 4 \cdot 10^{21} \text{ (1eV}/m_\nu) \text{ eV} \) collide with relic neutrinos \[28\]. If the masses of the background neutrinos \( m_\nu \) are of several eV, the cosmic neutrinos initiate high energy particle cascades which can contribute 10% to the observed cosmic ray flux at energies above the GKZ cut-off (one of the main processes is a resonant \( \nu \nu \) collision via Z-bozon). The neutrino flux is \[ E_\nu^2 \Phi_\nu \simeq 0.3 \cdot 10^{-6} \text{ GeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \] \[28\]. In the so-called topological defect models \[29\], UHECR are produced via decays of supermassive \( X \)-particles related to a grand unification theory. The expected neutrino flux is about \[ E_\nu^2 \Phi_\nu \simeq 0.5 \cdot 10^{-6} \text{ GeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \] \[29\]. In gamma-ray bursts (GRB) model \[30\], the neutrino flux is strongly suppressed at \( E_\nu > 10^{19} \text{ eV} \), since the protons are not expected to be accelerated to energies much larger than \( 10^{20} \text{ eV} \).

It is worth to mention model-independent upper bounds on the intensity of high energy neutrinos produced by photo-meson interactions. If the size of cosmic ray source is not larger than photo-meson free path, the upper limit is (for evolving sources) \( 4.5 \cdot 10^{-8} \text{ GeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \) \[31\]. However, for optically thick pion photoproduction sources, the upper limit is less restrictive: \( 2.5 \cdot 10^{-6} \text{ GeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \) \[32\]. Note, the considerably higher flux of cosmogenic neutrinos was obtained in Ref. \[33\]. The cosmogenic flux is the most reliable, as it relies only on two assumptions: (i) the observed extremely high energy cosmic rays (EHECR) contain nucleons, (ii) EHECR are primarily extragalactic in origin.

One possible way to resolve the GZK puzzle\(^5\) is to assume that the primary UHECR particles are neutrinos which deposit a part of their energy to proton fragments in \( \nu N \)-interactions. Unfortunately, the SM neutrino-nucleon cross sections are not large enough to resolve the problem. Indeed, at \( 10^{16} \text{ eV} \lesssim E_\nu \lesssim 10^{21} \text{ eV} \) the conventional contributions from charged and neutral current \( \nu N \)-scattering can be parameterized by \[34\]

\[
\sigma_{\nu N}^{cc} \simeq 4.429 \cdot 10^3 \left( \frac{E_\nu}{10^8 \text{ GeV}} \right)^{0.363} \text{ pb}, \\
\sigma_{\nu N}^{nc} \simeq 1.844 \cdot 10^3 \left( \frac{E_\nu}{10^8 \text{ GeV}} \right)^{0.363} \text{ pb},
\]

\(^5\)Note, however, recent paper \[20\], in which it is argued that the data from Fly’s Eye, HiRes and Yakutsk cosmic ray experiments are consistent with the expected suppression of cosmic ray spectrum above \( 5 \cdot 10^{19} \text{ eV} \). The AGASA data show an excess in this region.
The total SM cross section for $\bar{\nu}N$-scattering has practically the same magnitude and energy dependence at energies under consideration \[3 4\]. Putting, $E_\nu = 10^{21} \text{ eV}$ in (48), we get an estimate $\sigma_{\nu N}^{\text{SM}} \simeq 3.55 \times 10^5 \text{ pb}$. Such a value of neutrino-nucleon cross section is two small to be relevant to the GZK problem.

So, interactions beyond the SM\footnote{There is, however, a possibility that SM instanton-induced processes may give a large neutrino-nucleon cross-section \[38\].} are needed in order to explain possible excess of the UHECR flux. One possibility is high-energy scattering mediated by gravitational forces in theories with compact extra dimensions \[35-39\]. In a number of papers \[39-46\] it was shown that in a model with extra dimensions the neutrino-nucleon cross section can be enhanced by a black hole production. The corresponding cross sections were estimated to be one order of magnitude or more above the SM predictions (48) at $E_\nu \gtrsim 10^{18} \text{ eV}$.

In papers \[40, 42\] the opportunities were considered to search for black hole signatures by using neutrino telescopes such as AMANDA/IceCube, Baikal, ANTARES or NESTOR. The expected black hole production cross section is around $10^6 \text{ pb}$ for $M_{BH}^{\text{min}} = M_D = 1 \text{ TeV}$, where $M_{BH}^{\text{min}}$ is a minimal mass of produced black hole.\footnote{The production rate of black holes depends on the number of extra dimensions and, essentially, on the ratio $M_{BH}^{\text{min}} / M_D$.}

Another possibility, which we will concentrate on, is an observation of air showers triggered by UHE neutrino interactions. The technique used for studying extensive air showers of UHECRs or UHE neutrino is the detection of shower particles by ground detectors, or the detection of fluorescence light produced by the shower. The first technique is used by one of the largest operating AGASA experiment, while the second one was developed for Fly’s Eye (HiRes) detector. The largest project under construction is the Pierre Auger Observatory \[47\]. It will consist of two sites, each having 1600 particle detectors overlooked by four fluorescence detectors. For a detail study of extensive air showers with energy above $10^{18} \text{ eV}$, 10% of the events will be detected by both ground array and fluorescence detectors.

It is worth also mentioned space-based experiments EUSO and OWL which will be sensitive to CRs with energies above $10^{19} \text{ eV}$. The future of the neutrino astronomy may be related with radio frequency detectors, such as RICE and ANITA.

The neutrino interaction length is given by (in units of km water equivalent...
lent, 1 km we $\equiv 10^5$ g cm$^{-2}$)

$$L_\nu(E_\nu) \simeq 1.7 \cdot 10^7 \left[ \frac{1 \text{pb}}{\sigma_{\nu N}(E_\nu)} \right] \text{km we.}$$

(49)

For typical black hole production cross section $\sigma_{\nu N} = 10^6$ pb, we get $L_\nu = 17$ km we. This interaction length is much larger than the vertical Earth’s atmospheric depth, which is equal to 0.01 km we. The atmospheric depth for neutrinos transverse (almost) horizontally is 36 times larger. That is why, it was proposed to search for uniformly produced quasi-horizontal showers at ground level [48].

The Fly’s Eye and AGASA Collaborations have searched for deeply penetrating quasi-horizontal air showers, with the depth $L_{sh} > 2500$ g cm$^{-2}$. The probability of cosmic protons and gamma rays initiating air showers deeper than 2500 g cm$^{-2}$ is about $10^{-9}$. Thus, any shower starting that deep in the atmosphere is a nice candidate for a neutrino event.

The non-observation of such events puts an upper limit on the product of the neutrino differential flux, $\Phi_\nu = (1/4\pi)dN_\nu/dE_\nu$, times neutrino-nucleon cross section. The Fly’s Eye Collaboration gives the bound which can be parametrized by [49]

$$\left( \Phi_\nu \sigma_{\nu N}(E_\nu) \right) \leq 3.74 \cdot 10^{-42} \left( \frac{E_\nu}{1 \text{ GeV}} \right)^{-1.48} \text{ GeV}^{-1} \text{s}^{-1} \text{sr}^{-1},$$

(50)

while the upper limit from Ref. [37] can be recast as follows

$$\left( \Phi_\nu \sigma_{\nu N}(E_\nu) \right) \leq 10^{-41} \bar{y}^{-1/2} \left( \frac{E_\nu}{1 \text{ GeV}} \right)^{-1.5} \text{ GeV}^{-1} \text{s}^{-1} \text{sr}^{-1},$$

(51)

where $\bar{y}$ is an average fraction of the neutrino’s energy deposited into the shower. The inequalities are valid in the range $10^8$ GeV $\leq E_\nu \leq 10^{11}$ GeV, provided $\sigma_{\nu N}(E_\nu) \leq 10 \mu b$.

Let us now estimate the neutrino-nucleon cross section in our approach. The neutrino scatters off the quarks and gluons distributed inside the nucleon.

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8At large zenith angles a background from hadronic cosmic rays is negligible, since the showers initiated by hadrons are high in the atmosphere due to very short interaction length of the proton. Around $10^{20}$ eV, the hadronic mean free path is only 40 g cm$^{-2}$, and gamma-rays of such energy have interactions lengths of 45-60 g cm$^{-2}$. 

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(see a comment after formula (53)). Then the cross section is presented by

\[ \sigma_{\nu N}^{\nu N}(s) = \int_{x_{\text{min}}}^{1} dx \sum_i f_i(x, \mu^2) \sigma_{\text{in}}(\hat{s}), \]  

(52)

where \( f_i(x, \mu^2) \) is a distribution of parton \( i \) in momentum fraction \( x \), and \( \hat{s} = x s \) is an invariant energy of a partonic subprocess. In our approach, the partonic cross section \( \sigma_{\text{in}}(\hat{s}) \) is defined via the eikonal (40). As it follows from (40), \( \chi(\hat{s}, b) \) is small at \( \hat{s} \lesssim M_D^2 \), and we can put \( x_{\text{min}} = M_D^2 / s \) in (52). At \( \hat{s} \gtrsim M_D^2 \), the main contribution comes from the region:

\[ b^2 \lesssim b_{\text{max}}^2(\sqrt{\hat{s}}) = 4R_g(\hat{s}) [\ln(\hat{s}/M_D^2) + 1]. \]  

(53)

We choose the neutrino energy \( E_\nu \) to be \( 10^{17} \) eV, \( 10^{18} \) eV, \( 10^{19} \) eV, \( 10^{20} \) eV, and \( 10^{21} \) eV. The invariant energy of \( \nu N \) collision is then \( 14 \) TeV, \( 43 \) TeV, \( 137 \) TeV, \( 433 \) TeV and \( 1370 \) TeV, respectively. Since \( b_{\text{max}}(\sqrt{s} = 1370 \) TeV) \( \simeq 3 \cdot 10^{-2} \) GeV\(^{-1} \) = 6 \cdot 10^{-3} \) fm (for \( 2 \leq d \leq 6 \)), our assumption that the neutrino interacts with the proton constituents is well justified.

We use the set of parton distribution functions (PDFs) from paper [50] based on an analysis of existing deep inelastic data in the next-to-leading order QCD approximation in the fixed-flavor-number scheme. The extraction of the PDFs is performed simultaneously with the value of the strong coupling and high-twist contributions to structure functions. The PDFs are available in the region \( 10^{-7} < x < 1 \), \( 2.5 \) GeV\(^2 \) < \( Q^2 < 5.6 \cdot 10^7 \) GeV\(^2 \) [50]. We take the mass scale in PDFs to be \( \mu = 1 / b_{\text{max}}(\sqrt{s}) \), with \( b_{\text{max}} \) defined by equation (53). The result of our calculations of \( \sigma_{\nu N}^{\nu N}(E_\nu) \) is presented in Table 2. These neutrino-nucleon cross sections do not violate experimental upper bounds [50, 51].

Note, the total SM cross sections for (\( \nu + \bar{\nu} \))-scattering defined by formula (18) are equal to \( 6.27 \cdot 10^3 \) pb, \( 1.45 \cdot 10^4 \) pb, \( 3.35 \cdot 10^4 \) pb, \( 7.72 \cdot 10^4 \) pb, and \( 1.78 \cdot 10^5 \) pb, respectively. Thus, the SM interactions become comparable with (larger than) gravity interaction for \( d = 3 \div 4 \) (for \( d \geq 4 \div 5 \)), depending on the neutrino energy \( E_\nu \).

The number of horizontal hadronic air showers with the energy \( E_{sh} \) larger than a threshold energy \( E_{th} \), initiated by the neutrino-nucleon interactions,

\[ \text{Note, the SM contributions to neutrino-nucleon cross sections are not included in Table 2.} \]
Table 2: Inelastic neutrino-nucleon cross section for the graviton induced scattering at fixed neutrino energy, $E_\nu$, for different numbers of extra dimensions $d$ (in pbarn).

| $d$ | 2 | 3 | 4 | 5 | 6 |
|-----|---|---|---|---|---|
| $E_\nu = 10^{17}$ eV | 8.63 · 10^{4} | 5.63 · 10^{4} | 5.53 · 10^{2} | 7.16 · 10^{1} | 1.13 · 10^{1} |
| $E_\nu = 10^{18}$ eV | 6.53 · 10^{5} | 3.39 · 10^{4} | 2.47 · 10^{3} | 2.26 · 10^{2} | 2.41 · 10^{1} |
| $E_\nu = 10^{19}$ eV | 4.20 · 10^{6} | 2.05 · 10^{5} | 1.21 · 10^{4} | 8.59 · 10^{2} | 6.99 · 10^{1} |
| $E_\nu = 10^{20}$ eV | 2.05 · 10^{7} | 1.32 · 10^{6} | 7.06 · 10^{4} | 4.29 · 10^{3} | 2.94 · 10^{2} |
| $E_\nu = 10^{21}$ eV | 8.74 · 10^{7} | 7.47 · 10^{6} | 4.56 · 10^{5} | 6.99 · 10^{4} | 1.52 · 10^{4} |

is given by

$$N_{sh}(E_{sh} \geq E_{th}) = T N_A \left[ \int dE_\nu \Phi_\nu(E_\nu) \sigma_{\nu N}^{\text{grav}}(E_\nu) A(E_\nu) \theta(E_\nu - E_{th}) \right.$$  

$$+ \sum_{i=e, \mu, \tau} \int dE_{\nu i} \Phi_{\nu i}(E_{\nu i}) \sigma_{\nu_{\nu i} N}^{\text{SM}}(E_{\nu i}) A(y_i E_{\nu i}) \theta(y_i E_{\nu i} - E_{th}) \right],$$  

(54)

where $N_A = 6.022 \cdot 10^{23}$ g$^{-1}$, $T$ is a time interval, and $A$ is a detector acceptance (in units of km$^3$ steradian water equivalent). The quantity $\Phi_{\nu i}(E_{\nu i})$ in (54) is a flux of the neutrino of type $i$, and $\Phi_{\nu i}(E_{\nu i}) = \sum_{i=e, \mu, \tau} \Phi_{\nu i}(E_{\nu i})$.\footnote{Both neutrino and antineutrino are everywhere included in the sum.}

The inelasticity $y_i$ defines a fraction of the neutrino energy deposited into the shower in the corresponding SM process (see below).

The AGASA acceptance for deeply penetrating quasi-horizontal air showers with zenith angles $\theta > 75^\circ$ can be found in the second paper of Refs.\cite{45}. It rises linearly in $E_{sh}$ in the interval $10^7$ GeV $< E_{sh} < 10^{10}$ GeV, while in the ultrahigh high-energy region the acceptance is constant and equal to $A(E_{sh} \geq 10^{10}$ GeV) $\approx 1.0$ km$^3$ we sr \cite{45}.

The neutrino acceptance of the Pierre Auger detector is roughly 30 times larger, taking into account the ratio between Auger and AGASA surface areas. The acceptance of the Auger ground surface array has been studied in details in Ref.\cite{51}, while the acceptance of fluorescence detector to neutrino-like air showers with the large zenith angles was calculated in Refs.\cite{53}, \cite{54}. The Auger observatory efficiency is high, since the low target density in the atmosphere is compensated by the very large surface area of the array.
(each side of it covers an area of 3000 km$^2$). The highest efficiency for quasi-horizontal shower detection is expected at $E_{sh} > 10^9$ GeV [51].

The number of extensive quasi-horizontal showers induced by the neutrinos with energy larger than some threshold energy $E_{th}$, which can be detected by the array of the southern site of the Pierre Auger observatory, is presented in Table 3. The neutrino-nucleon cross section $\sigma_{\nu N}$ in Eq. (54) describes the contributions from the reggeized KK gravitons. The cosmogenic neutrino flux is from Refs. [25], assuming a maximum energy of $E_{max} = 10^{21}$ eV for the UHECR. The acceptance of the Auger detector is taken from Ref. [51] (it is not assumed that shower axis falls certainly in the array).

**Table 3:** Yearly event rates for nearly horizontal neutrino induced showers with $\theta_{zenith} > 70^\circ$ and $E_{sh} \geq E_{th}$ for cosmogenic neutrino flux from [25] at three values of threshold energy $E_{th}$. Number of events corresponds to one side of the Auger ground array.

| $d$ | 2   | 3   | 4   | 5   | 6   |
|-----|-----|-----|-----|-----|-----|
| $E_{th} = 10^8$ GeV | 34.88 | 2.00 | 0.32 | 0.21 | 0.20 |
| $E_{th} = 10^9$ GeV | 30.21 | 1.66 | 0.21 | 0.12 | 0.12 |
| $E_{th} = 10^{10}$ GeV | 13.16 | 0.74 | 0.062 | 0.025 | 0.022 |

The neutrino-nucleon inelastic interactions induced by gravireggeons remind the SM neutral currents events. We assume that such events should result in hadronic dominated showers without leading lepton. That is why, we choose the inelasticity to be equal to unity$^{11}$ for the events induced by gravireggeon exchange (the first term in the RHS of Eq. (54)). We have also put $y_e = 1$ for the SM charged current interactions initiated by electronic neutrino, while for the SM neutral interactions initiated by $\nu_\mu$ and for $\nu_\mu/\nu_\tau$-events we have taken $y_e = y_\mu = y_\tau = 0.24$, following the calculations presented in Ref. [52].

The neutrino event rates are expected to be much higher for the neutrino fluxes obtained in “optimistic” scenarios considered in Refs. [33]. As an example, we have presented the yearly event rates for the Z-burst scenario in

$^{11}$The estimates from Ref. [39] are not applicable in our case, since in [39] an energy loss in elastic neutrino-nucleon cross section induced by “bare” gravitons was considered, while we deal with inelastic cross section in the gravireggeon model.
Table. 4. One can see from Table 4 that the main contribution to the shower rate comes from the region of extremely high neutrino energies ($E_\nu > 10^{10}$ GeV). It can be understood as follows: at UHEs, the neutrino flux times $E_\nu$ varies slowly in $E_\nu$ in the Z-burst model (up to $2.5 \cdot 10^{12}$ GeV), while both the acceptance of the Auger array and “gravitational” part of the neutrino-nucleon cross section rise with the neutrino energy (see Table 2).

Table 4: The same as in Table 3 but for the Z-burst neutrino flux from [28].

| d  | $E_{th} = 10^8$ GeV | $E_{th} = 10^9$ GeV | $E_{th} = 10^{10}$ GeV |
|----|--------------------|---------------------|------------------------|
| 2  | $12.60 \cdot 10^2$ | $12.59 \cdot 10^2$  | $12.55 \cdot 10^2$    |
| 3  | $11.53 \cdot 10^1$ | $11.53 \cdot 10^1$  | $11.51 \cdot 10^1$    |
| 4  | $9.26$             | $9.26$              | $9.20$                |
| 5  | $1.90$             | $1.90$              | $1.85$                |
| 6  | $1.50$             | $1.50$              | $1.44$                |

The calculations of the yearly event rates in the energy interval $10^8$ GeV $\leq E_{sh} \leq 10^{11}$ GeV in the Z-burst scenario result in 44, 2.7, 0.38, 0.26, and 0.25 for $d = 2, 3, 4, 5$ and 6, respectively. Remembering that combine results from AGASA and Fly’s Eye imply an upper bound of 3.5 at 90% CL from quasi-horizontal neutrino events [45], and taking into account that the AGASA acceptance is roughly 30 times smaller than the Auger acceptance, we can conclude that the Z-burst neutrinos do not violate bounds (50), (51) in our scheme for $d \geq 3$.\textsuperscript{12}

3 Conclusions

In the model with compact extra spacial dimensions, we have calculated the contribution of the KK gravireggeons into the inelastic cross section of the high energy scattering of both $D$-dimensional and four-dimensional SM particles. The usually adopted summing non-reggeized gravitons leads to a divergent sum in KK-number $n$ (for $D \geq 6$). In our approach, on the contrary, the contribution of gravireggeon with the KK-number $n$ to the eikonal is exponentially suppressed at large $n$. As a result, the corresponding sum in $n$ is finite, and it can be analytically calculated.

\textsuperscript{12}We do not discuss here cosmological bounds on the number of extra dimensions [1] (see also [55] and references therein).
In the case when the SM fields propagate in all $D$ dimensions, the dependence of the inelastic cross section on invariant energy $\sqrt{s}$ appeared to be similar to the upper limit for the total cross section obtained previously for the SM in the $D$-dimensional flat space-time without gravity. When, on the contrary, only gravity lives in extra dimensions, the imaginary part of the eikonal is derived in a closed form, which depends (except for $\sqrt{s}$ and the impact parameter $b$) on the number of extra dimensions $d = D - 4$ and their size $R_c$.

We have estimated the event rate for the quasi-horizontal air showers, induced by the interactions of UHE neutrinos with nucleons, which can be yearly detected by the ground array of the Pierre Auger observatory. It decreases rapidly if $d$ varies from 2 to 5. For $d = 4$, we expect 10 events per year for the neutrino flux predicted in the Z-burst model. For the cosmogenic neutrino flux, gravireggeon induced interactions do not increase the event rate significantly with respect to the number of the neutrino events calculated in the SM, except for the case $d \leq 3$, which is likely to be excluded by the cosmological data.
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