Non-conservative mass transfer in stellar evolution and the case of V404 Cyg/GS 2023+338

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ABSTRACT
We consider donor evolution and mass transfer in the microquasar V404 Cyg/GS 2023+338. Based on X-ray observations of its two outbursts, its average mass accretion rate is substantially lower than the theoretical mass-loss rate from its low-mass giant donor. A likely solution to this discrepancy is that a large fraction of the mass flowing from the donor leaves the binary in the form of accretion-associated outflows. We derive an analytical formula describing changes of the donor Roche lobe radius due to its mass change for non-conservative mass transfer, which we parameterize by the fractions of the mass and angular momentum leaving the system. We perform evolutionary calculations for V 404 Cyg. Given our estimated average accretion rate, >70 per cent of the mass lost from the donor has to leave the binary. The allowed solution for the actual donor mass loss rate is parameterized by our two outflow parameters. Our results are in agreement with the observed outflows from the outer disc as well as with the variable near-Eddington accretion observed during the outbursts, compatible with outflows from the vicinity of the black hole. In the latter case, the outflowing matter has a negligible angular momentum.

Key words: binaries: general – stars: evolution – stars: individual: V404 Cyg – stars: low mass – X-rays: binaries – X-rays: individual: GS 2023+338.

1 INTRODUCTION
GS 2023+338 was discovered by *Ginga* in May 1989 as a new X-ray transient (Makino 1989). Its optical counterpart was immediately identified as a variable star V404 Cyg (Wagner et al. 1989). V404 Cyg was originally found as the optical counterpart of Nova Cyg 1938, which was classified as a classical nova. However, Charles et al. (1989) found that it was neither classical nor recurrent nova but a low mass X-ray binary. Casares, Charles & Naylor (1992) discovered absorption lines of an early K star and determined the orbital period as 6.47 ± 0.001 d. The radial velocity amplitude gave the mass function of 6.08 ± 0.06M⊙, implying the presence of a black hole in the system (Casares & Charles 1994). Those authors also obtained a relatively precise estimate of the rotational broadening of the absorption lines of the donor of v sin i = 39.1 ± 1.2 km s⁻¹, which implies the mass ratio of q = M2/M1 ≃ 0.060±0.005, where M1 and M2 are the masses of the black hole and the donor, respectively. Miller-Jones et al. (2009) measured the radio parallax of the system and found the distance of D = 2.39 ± 0.14 kpc. Khargharia, Froning & Robinson (2010) presented near-infrared spectroscopy of the donor component. They established its spectral type as K3 III with the uncertainty of one subtype, i.e., within K2–K4 III, i.e., the star is a giant, confirming King (1993). Then they modeled the H-band light curve and on the basis of their fit determined the inclination of the system as i ≃ 67°±3°. Using this value, the mass function and the mass ratio, the mass of the black-hole and donor component is M1 ≃ 9.0+0.2−0.1M⊙ and M2 ≃ 0.54 ± 0.05M⊙, respectively.

The above observational data permit us to estimate relatively precisely the radius and the luminosity of the donor. Using the expression for the Roche lobe radius (Paczyński 1967) together with the third Kepler law, we obtain the donor radius of R2 = 5.50±0.17R⊙. The luminosity can be estimated from the donor spectral type. The calibration of Cox (2000) for K2–III (Khargharia et al. 2010) implies the effective temperature of T2 = 4274±161 K. With those R2 and T2, we find the donor luminosity of L2 = 8.9±2.5L⊙.

In July 2015, V 404 Cyg underwent renewed X-ray and optical activity after 26 years of dormancy (Barthelmé et al. 2015). Activity was strong (approaching Eddington limit) but lasted only about two weeks (Rodriguez et al. 2015; Kimura et al. 2016; Motta et al. 2016;a,b; Sánchez-Fernández et al. 2017). After several months of quiescence, the system erupted in December 2015, again for about two weeks but at much lower level of activity (Marti, Luque-Escamilla and García-Hernández 2016; Motta et al. 2016).

The average accretion rate implied by the X-ray luminosity observed during the 2015 outburst is difficult to estimate due to strong absorption. Fig. 4 of Kimura et al. (2016) gives the bolometric light curve based on hard X-ray observations by *Neil Gehrels Swift* BAT and *INTEGRAL/ISGRI* detectors, where their...
15–50 keV and 25–60 keV luminosities (presumed to be weakly affected by absorption) are multiplied by the rather large estimated bolometric-correction factors of 7 and 10, respectively. Even with those corrections, the light curve only occasionally exceeds the Eddington limit, $L_{\text{Edd}}$, by a factor $\lesssim 2$, and the average luminosity during the two-week outburst is $\sim 0.1L_{\text{Edd}}$ or so. Assuming a 10 per cent efficiency and averaging over the 26 years interval between the two most recent outbursts, we obtain the average accretion rate of $\langle \dot{M}_1 \rangle \approx 3.5 \times 10^{-11}M_\odot \text{ yr}^{-1}$.

However, it is in principle possible that the strong variability observed in V404 Cyg was due to almost full obscuration of the accretion flow during the low-flux periods (Motta et al. 2017a). This would increase the average luminosity during the 2015 outburst to $\sim 15–50 \text{ keV}$ and $25–60 \text{ keV}$ luminosities (presumed to be weakly absorbed), yielding $\langle \dot{M}_1 \rangle \approx 4.0 \times 10^{-10}M_\odot \text{ yr}^{-1}$ when averaged over 33 yr between 1989 and the previous outburst in 1956 (Chen, Shrader & Livio 1997 and references therein). Chen et al. (1997) also estimated the total radiated energy by integrating over an exponential rise and a decay of the light curve normalized to the peak luminosity for the 1989 outburst. At 10 per cent efficiency, they obtained the accreted mass of $\approx 2.7 \times 10^{32} \text{ g}$ (for $D = 2.39 \text{ kpc}$), which implies $\langle \dot{M}_1 \rangle \approx 4.0 \times 10^{-10}M_\odot \text{ yr}^{-1}$ when averaged over 33 yr between 1989 and the previous outburst in 1956 (Chen, Shrader & Livio 1997 and references therein). Chen et al. (1997) also estimated the total radiated energy by integrating over an exponential rise and a decay of the light curve normalized to the peak luminosity for the 1989 outburst. At 10 per cent efficiency, they obtained the accreted mass of $\approx 2.7 \times 10^{32} \text{ g}$ (for $D = 2.39 \text{ kpc}$), implying an almost the same average ($\dot{M}_1$) as that of Życki et al. (1999). Taking into account all of the above estimates, $4.0 \times 10^{-10}M_\odot \text{ yr}^{-1}$ appears to be a likely upper limit on ($\dot{M}_1$).

On the other hand, all of those estimates are lower than the value of the donor mass loss rate implied by considering evolution of giants. In particular, an approximate formula of (Webbink, Rappaport & Savonije 1983) given by their equation (25a) yields $\dot{M}_2 \approx 1.1 \times 10^{-9}M_\odot \text{ yr}^{-1}$ for the case of V404 Cyg. We confirm their estimate by our updated evolutionary model.

This discrepancy can be accounted for by assuming that a fraction of the mass lost from the donor can leave the binary in the form of an outflow. In the case of V404 Cyg, strong outflows from outer parts of the accretion disc have been observed (Mínuzz-Darias et al. 2016). Also, outflows from inner disc parts are expected at accretion rates corresponding to $L < L_{\text{Edd}}$ (e.g., Poutanen et al. 2007; Sałkowski & Narajan 2015). This possibility in V404 Cyg has been studied by Motta et al. (2017b). Strong outflows are also possible at low accretion rates, e.g., Yuan & Narayan (2014).

In this Letter, we consider evolution of a binary system in the presence of an outflow. We first derive a formula describing changes of the Roche lobe radius in the presence of a non-conservative mass transfer. We then present an evolutionary model of the donor in V404 Cyg, taking into account that effect.

2 EVOLUTION OF THE ROCHE LOBE IN THE PRESENCE OF AN OUTFLOW

Equation (12) of Webbink et al. (1983) describes evolution of the Roche lobe of the donor during conservative mass transfer, i.e., one conserving both the total mass of the binary and its angular momentum. It is in the form of the derivative $d \ln R_{2,1}/d \ln M_2$, where $R_{2,1}$ is the radius of the donor Roche lobe. As discussed in Webbink et al. (1983), this derivative has to equal that describing the evolution of the donor due to both nuclear burning and its mass loss. In consequence, this equality implies the rate of the mass loss from the donor, $\dot{M}_2$. For the case of the evolution of a giant with conservative mass transfer and negligible dependence on the donor total mass, that $\dot{M}_2$ is given by equation (15) of Webbink et al. (1983). Here, we derive a formula describing the evolution of the Roche lobe in the case of non-conservative mass transfer.

The total orbital angular momentum, $J$, of a binary system (in the approximation of two point masses, i.e., neglecting the spins) is given by

$$J = \frac{G a}{M},$$

where $M = M_1 + M_2$, $a$ is the orbital separation, and $G$ is the gravitational constant. The orbital angular momentum of the donor, $J_2$, and its specific angular momentum, $j_2$, are

$$J_2 = J M_1 / M_2,$$

$$j_2 = J / M_2,$$

respectively. A non-conservative mass transfer, i.e., with an outflow of a fraction of the transferred mass from the system, can be described by two parameters, which we can choose as $f_1$ giving the fraction of the mass lost by the donor that leaves the system, and $f_2$ describing the specific angular momentum of the lost mass,

$$f_1 = \frac{dM_{\text{esc}}}{dM} = \frac{dM}{M_1} = 1 + \frac{M_1}{M_2},$$

$$f_2 = \frac{dJ_{\text{esc}}}{dJ} = j_{\text{esc}} / j_2,$$

where $0 \leq (f_1, f_2) < 1$ with $f_1 = 0$ for conservative transfer, and $M_{\text{esc}}, J_{\text{esc}}$ and $j_{\text{esc}}$ are the mass of the outflowing matter, its angular momentum and its specific orbital angular momentum, respectively.

The radius of the Roche lobe for $M_2 \lesssim 0.6M_\odot$ is given by (Paczyński 1967)

$$R_2 \approx \frac{2a}{3^{2/3}} \left( \frac{M_1}{M} \right)^{1/3},$$

which implies

$$d \ln R_2 = d \ln a + \frac{1}{3} d \ln M_2 - \frac{1}{3} d \ln M.$$

On the other hand, equation (1) implies

$$d \ln a = 2d \ln J - 2d \ln M_2 - 2d \ln M + d \ln M.$$

According to the definitions in equations (3–4), the rate of the loss of the angular momentum of the binary due to the outflow is given by

$$dJ = j_{\text{esc}} dM_{\text{esc}} = f_1 f_2 j_2 dM_2,$$

Since equation (3) implies $dM = f_1 dM_2$, we finally have

$$d \ln R_2 = \frac{5}{3} - 2(f_1 - 1) M_2 / M_1 + \frac{2}{3} f_1 f_2 M_1 / M - 2f_1 f_2 M_1 / M.$$

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This formula represents a generalization of equation (12) of Webbink et al. (1983), and it reduces to it for \( j_1 = 0 \).

We can immediately estimate the value of \( f_2 \) if the outflow is from a vicinity of the accretor. A particle flowing from the donor to a radius \( r \ll a \) around the accretor has to lose its angular momentum (which is transferred back to the binary motion), and its remaining specific angular momentum becomes almost the same as of the accretor,

\[
    j_1 = \mathcal{J} \frac{M_2}{M_1} = j_2 \left( \frac{M_2}{M_1} \right)^2.
\]

If that particle then escapes the binary, the lost specific angular momentum is \( j_{\text{esc}} \approx j_1 \), which implies

\[
    f_2(r \ll a) \approx \left( \frac{M_2}{M_1} \right)^2 \ll 1
\]

(\( = 0.0036 \) for V404 Cyg). Thus, if the outflow is from an inner part of the accretion disc (as in the case of the luminosity comparable to the Eddington one), it carries a negligible fraction of its original angular momentum and \( f_2 \approx 0 \). On the other hand, if the outflow is from an outer part of the disc, e.g., close to its outer radius, the value of \( f_2 \) can be found by calculating the specific angular momentum in the inertial frame with respect to the centre of mass of the binary for a Keplerian motion (in the co-rotating frame) at a radius \( r \) and an azimuthal angle \( \phi \) around the accretor, and then averaging over \( \phi \).

We note that the effect of the mass and the associated angular momentum loss has been taken into account numerically in many computer simulations, in particular it is included in the Modules for Experiments in Stellar Astrophysics (MESA) suite of libraries for computational astrophysics (Paxton et al. 2011). However, to our knowledge, no analytical formula describing their effect on the Roche lobe evolution has been derived previously.

### 3 THE EVOLUTIONARY MODEL

In order to calculate evolution of stripped giants, we use the Warsaw stellar-evolution code described in Ziolkowski (2005). In Zdziarski et al. (2016), the code was calibrated to reproduce the Sun at the solar age. This calibration resulted in the chemical composition of the H mass fraction of \( X = 0.74 \), the metallicity of \( Z = 0.014 \), and the mixing length parameter of \( \alpha = 1.55 \).

Our program calculates evolutionary sequences of stripped giants for an assumed constant total mass, \( M_2 \), and a varying (growing) mass of the He core. The program calculates not only the radius and the luminosity of each model but also its entire internal structure. This structure is essential for calculating the reaction of the star to mass transfer. In the calculations, we followed the approach used to calculate models of GRS1915+105/V1487 Aql (Ziolkowski & Zdziarski 2017) and IGR J17451–3022 (Zdziarski et al. 2016). The situation is now simpler than in those cases since V404 Cyg has the reliable estimate of the donor mass, see Section 1. Therefore, we need to calculate the sequences of models for only limited range of the values of \( M_2 \).

#### 3.1 The core mass–radius and radius–luminosity planes

The results of our evolutionary calculations are presented in Fig. 1, which shows the evolutionary tracks for the stellar masses of 0.49, 0.54 and 0.59\( M_\odot \) of stripped giants in the core mass, \( M_\star \) vs. \( R_2 \) plane. The stars evolve at constant mass and the driving mechanism is the progress of the H-burning shell moving outwards. The radii of the partially stripped giants generally increase with \( M_\star \) except for the shrinking when the masses of their envelopes become very low (not shown). From crossings of these tracks with the corresponding lines showing the radii of the donor Roche lobe (Section 1), we find the core mass of \( M_\star = 0.195 \pm 0.001 M_\odot \).

Our evolutionary model predicts also the luminosity, \( L_2 \), as a function of \( M_\star \), resulting from H burning in the shell. We compare the range predicted for the determined range of \( M_\star \) with the range allowed observationally (see Section 1). A comparison for \( L_2(R_2) \) is given in Fig. 2. We see that the agreement of the predicted luminosity as a function of the radius with the constraints derived from the stellar spectral type is very good. The fact that we reproduce quite precisely the luminosity of the donor gives support to both the correctness of our model and to the accuracy of the observational determination of the mass (implying the radius) and the temperature of the star.

### 4 PROPERTIES OF THE MASS TRANSFER IN V404 CYG

With the above results, we can calculate the rate of the mass transfer under different assumptions about the outflow. We assume our model for the best-fit determination of the donor mass, 0.54\( M_\odot \). Due to the growing mass of the core, the star will expand at the rate \( dR_2/dt \), which will lead to the mass flow through the inner Lagrangian point. The rate of this flow will depend on the reaction of the radius of the Roche lobe around the donor to the mass transfer, and the process will be self-adjusting assuring that changes of the donor radius follow the changes of the radius of the Roche lobe. For a given rate of the mass outflow, there will be a dependence of the radius on the mass, \( dR_2/dM_1 = (dR_2/dt)/M_2 \), which we need to equal to the corresponding dependence for the Roche lobe evolution, given by equation (9).

For low rates of the mass outflow, the evolutionary expan-
sion dominates and the effect of the mass removal from the outer layers of the star is minor. Then \( dR_2/dM_2 < 0 \), i.e., the star expands. However, for sufficiently high outflow rates, the fast mass removal from the outer layers leads to a non-equilibrium configuration, and the star starts to shrink, \( dR_2/dM_2 > 0 \). The logarithmic derivative \( d\ln R_2/d\ln M_2 = (M_2/R_2)dR_2/dM_2 \) for our model with \( M_2 = 0.54M_\odot \) is shown by the black solid curve in Fig. 3. We see that the star starts to shrink at \( M_2 \gtrsim 2 \times 10^{-4}M_\odot \) yr\(^{-1} \).

We can compare our results with those of Webbink et al. (1983), who took into account only the evolutionary growth of the star (see their equation 13); thus their value of \( dR_2/dt \) is independent of \( M_2 \). Then, their time derivative for the considered model depends only on the core mass and the rate of the core mass growth (given by the luminosity and the H content in the H-burning shell at the present moment, which, according to our evolutionary model, is \( X = 0.54 \)), see their equation (14). That dependence is plotted in the magenta dotted curve in Fig. 3, and it is given by (normalized to the value for \( f_2 = 0 \), see below).

\[
\frac{d\ln R_2}{d\ln M_2} = 1.61 \times 10^{-4}M_\odot \text{yr}^{-1}.
\]

We see that the dependence of Webbink et al. (1983) is somewhat lower than ours. Also, it predicts the radius derivative to be always negative, while the effect of mass removal from the outer layers of the star becomes important at high values of \( -M_2 \), and the derivative becomes positive, as found in our solution.

The evolutionary change of the radius equals that of the Roche lobe change, given by equation (9). The latter depends on the outflow parameters \( f_1 \) and \( f_2 \), with \( f_1 = |1 - M_2/M_2| \) according to the definition of equation (3). Following our review of the estimates of the average accretion rate in V404 Cyg in Section 1, we adopt a fiducial value of \( M_2 = \langle M_1 \rangle = 4 \times 10^{-10}M_\odot \text{yr}^{-1} \). While this value remains relatively uncertain, it represents, most likely, a firm upper limit to \( M_1 \), see Section 1. We plot in Fig. 3 \( d\ln R_2/d\ln M_2(1 + M_1/M_2, f_2) \) as a function of \( -M_2 \) for the values of \( f_2 = 0, 0.5 \) and 0.9.

The self-consistent solution has to lie on the intersection of the curve describing our evolutionary model with that for the Roche lobe for a given choice of the \( f_1 \) parameter. The value of \( f_1 \) follows then from the value of \( -M_2 \) at that intersection (\( f_1 = 1 - M_2/M_2 \)).

Analysing Fig. 3, we find that at \( f_2 = 0 \), \( -M_2 \approx 1.2 \times 10^{-9}M_\odot \text{yr}^{-1} \), implying \( f_1 \approx 0.67 \) for \( f_2 \geq 0 \), i.e., the mass transfer has to be highly non-conservative. For \( f_2 = 0.5 \), \( -M_2 \approx 2.2 \times 10^{-9}M_\odot \text{yr}^{-1} \) and \( f_1 \approx 0.8 \). For \( f_2 = 0.9 \), \( -M_2 \approx 4.8 \times 10^{-9}M_\odot \text{yr}^{-1} \) and \( f_1 \approx 0.99 \), i.e., almost all transferred mass has to leave the binary. Note that this solution, with the donor shrinking, could not be found with the approximation of Webbink et al. (1983), Solutions for other values of \( f_2 \) can be readily found by plotting further dependencies of \( d\ln R_2/d\ln M_2(f_1, f_2) \) on Fig. 3 and determining their intersection with the black solid curve.

As discussed in the Section 2, outflows from the vicinity of the accretor (in the form of strongly ionized winds and jets) have \( f_2 \approx 0 \), which corresponds to our first solution above. On the other hand, outflows from outer parts of the accretion disc (in the form of neutral winds; Múnoz-Darias et al. 2016), in particular those from close to its outer edge, will have an intermediate value of \( f_2 \), and may roughly correspond to our solution with \( f_2 = 0.5 \). We note that Múnoz-Darias et al. (2016) have apparently been unable to estimate the range of radii of the neutral outflows they found. Still, since matter flowing from the donor to the accretor have to lose a sizeable fraction of its specific angular momentum, solutions with \( f_2 \geq 0.9 \) appear unphysical for V404 Cyg.

The main caveat to our results concerns the estimate of the mass accreted during the outbursts on the observed X-ray light curves. This requires the knowledge of the absorption (appearing to be strong) during outbursts and the radiative efficiency, with both being uncertain. Still, our adopted value most likely represents a firm upper limit. Furthermore, it appears that only a small fraction of the total disc mass is accreted during an outburst (Zycki et al. 1999), and then it is uncertain whether the mass accreted during an
outburst is directly related to that supplied to the disc during the preceding period of quiescence. Still, an equality of the two masses is expected when averaged over many outbursts.

5 CONCLUSIONS

Our main results can be summarized as follows.

We have derived an analytical formula describing reaction of the Roche lobe radius of a donor star to its mass loss due to the mass transfer within the binary taking into account its possible non-conservative character, as parameterized by the fractions of the mass and angular momentum leaving the system.

We have estimated the mass accretion rate in V404 Cyg averaged over intervals between its outbursts. While its value remains relatively uncertain, the best available upper limit appears to be $\langle \dot{M}_1 \rangle = 4 \times 10^{-10} M_\odot \text{yr}^{-1}$. This value can be compared to that estimated theoretically given that the donor is a giant with the well-established mass and spectral type (which parameters yield the luminosity in good agreement with the theoretical value from our evolutionary model). From those estimates, there is a significant discrepancy of the $\langle \dot{M}_1 \rangle$ with the donor mass loss rate determined based on evolutionary models, $-\dot{M}_2$, with the former being at least three times lower.

This discrepancy can be resolved taking into account the likely presence of outflows during the outbursts. Using our derived formalism, we find that at least 70 per cent of the mass lost from the donor has to leave the binary. The allowed solution for the actual donor mass loss rate is parameterized by our two outflow parameters. Our results are in agreement with the observed outflows from the outer disc (Múnoz-Darias et al. 2016) as well as with the character of accretion observed during the 2015 outburst (Motta et al. 2017b), compatible with outflows in the vicinity of the black hole.

In the latter case, the matter reaching that region had lost most of its angular momentum before outflowing, which corresponds approximately to our solution with $f_2 = 0$. In the former case, there is no available estimate of the disc radius from which outflows originate, and an intermediate value of $f_2$ appears appropriate, which we illustrate by the case with $f_2 = 0.5$.

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