Topological Defects in Contracting Universes

P.P. Avelino, C.J.A.P. Martins, C. Santos, and E.P.S. Shellard

1Centro de Astrofísica da Universidade do Porto, R. das Estrelas s/n, 4150-762 Porto, Portugal
2Departamento de Física da Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre 687, 4169-007, Porto, Portugal
3Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
4Institut d’Astrophysique de Paris, 98 bis Boulevard Arago, 75014 Paris, France
5Centro de Física da Universidade do Porto, Rua do Campo Alegre 687, 4169-007, Porto, Portugal

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We study the behaviour and consequences of cosmic string networks in contracting universes. They approximately behave during the collapse phase as a radiation fluids. Scaling solutions describing this are derived and tested against high-resolution numerical simulations. A string network in a contracting universe, together with the gravitational radiation it generates, can affect the dynamics of the universe both locally and globally, and be an important source of radiation, entropy and inhomogeneity. We discuss possible implications for bouncing and cyclic models.

INTRODUCTION

Cosmological scenarios involving oscillating or cyclic universes have been known for a long time [1]. Recent interest has been associated with a cyclic extension of the ekpyrotic scenario [2]. A related result was the realization [3, 4, 5] that the presence of a scalar field seems to be necessary to make cosmological scenarios with a bounce observationally realistic. And if scalar fields are present, we should contemplate the possibility of topological defect formation [6]. Here we study cosmic string evolution in a collapsing universe, and discuss some implications of the presence of cosmic strings for bouncing universes. In a bouncing universe scenario the properties of the universe in the expanding phase depend on physics happening in a previous collapsing phase. Hence, if defects do exist in these models, it is crucial to understand their evolution and consequences in both the expanding and collapsing phases. In particular, we expect that cosmic strings will become ultra-relativistic, behaving approximately like a radiation fluid. This means that a cosmic string network, both directly and through the gravitational radiation emitted by its loops, will soon become a significant source of entropy (and inhomogeneity), making it a further problem for cyclic universes if a suitable and efficient mechanism for diluting the entropy is not available. A more detailed analysis can be found in [7].

COSMIC STRING EVOLUTION

The world history of a cosmic string can be represented by a two-dimensional surface in space-time, obeying the usual Goto-Nambu action, from which it is easy [8] to derive the microscopic string equations of motion. Consider for a start the evolution of a circular cosmic string loop in a cyclic universe. Simple analytic arguments show that a loop whose initial radius is much smaller than the Hubble radius will oscillate freely with a constant invariant loop radius and an average velocity \( \bar{v} = 1/\sqrt{2} \). (Note that we are assuming units in which \( c = \hbar = 1 \).) On the other hand, once the collapse phase begins, we will eventually get to a stage in which the physical loop radius becomes comparable to the Hubble radius \( ar \sim H^{-1} \) and then gets above it. In this regime the loop velocity is typically driven towards unity \( v \to 1 \) and it straightforward to show that the invariant loop length (which is proportional to the energy of the loop) grows as \( R \propto a^{-1} \) and the Lorentz factor as \( \gamma \propto a^{-2} \). Despite its growing energy \( R \), the actual physical loop radius \( ar = R/\gamma \to a \), so the loop shrinks with the scale factor and inexorably follows the collapse into final big crunch singularity.

Importantly, this relativistic final state for a loop in a collapsing universe is generic and quite different to the initial condition usually assumed for super-horizon loops in the expanding phase. There, loops begin with a vanishing velocity which only becomes significant when the loop falls below the Hubble radius. Such evolution cannot be reproduced in reverse during the collapsing phase without fine-tuning the velocity as the loop crosses outside the Hubble radius. This simple fact introduces a fundamental time asymmetry for string evolution in a cyclic universe (and for all other defects). The analytic expectations for our circular loop solution can easily be confirmed by a numerical study [9].

Two complementary approaches are available to study the evolution of a cosmic string network: one can resort to large numerical simulations [8, 9, 10], or one can develop analytic tools [11, 12, 13, 14] which provide an averaged description of the basic properties of the network. We shall use the best motivated of these analytic models, the velocity-dependent one-scale (VOS) model [12, 14, 15, 16]. The VOS model describes the string dynamics in terms of two ‘thermodynamical’ parameters: the string RMS velocity, \( v_{\infty} \), and a single length scale—the string network is thus assumed to be a Brownian
random walk on large enough scales, with a correlation length $L$. Hence one can simply relate it with the energy density in long strings as $\rho_{\infty} = \mu/L^2$, where $\mu$ is the string mass per unit length. Note that the commonly used ‘correlation length’ $L$ is really a measure of the invariant string length or energy, rather than the typical curvature radius of the strings. By including the appropriate Lorentz factor $\gamma_\infty = (1 - v_\infty^2)^{-1/2}$ for the long strings, we can denote the physical correlation length by $L_{\text{phys}} = L\gamma_\infty$. With this assumption the VOS model has one phenomenological parameter $\tilde{c}$, commonly called the loop chopping efficiency, which describes the rate of energy transfer from the long-string network to loops. The evolution equations then take the following form:

$$2 \frac{dL}{dt} = 2(1 + v_\infty^2)HL + \tilde{c}v_\infty + 8\tilde{\Gamma}G\mu v_\infty^6,$$  

(1)

$$\frac{dv_\infty}{dt} = (1 - v_\infty^2) \left( \frac{k(v_\infty)}{L} - 2Hv_\infty \right).$$  

(2)

The final term in the evolution equation for the correlation length describes the effect of gravitational back-reaction. We are not including in either equation additional terms arising from friction due to particle scattering, which could conceivably be important during the final stages of collapse. We shall return to this point below. Here $k(v_\infty)$ is the momentum parameter, which is formally contracted (except for their rapidly growing velocities). However, there are several factors that must be considered which complicate this simple state of affairs.

When the contraction phase starts and the Hubble parameter becomes negative the velocity will tend to increase: as in the simple case of the circular loop, the string velocity will gradually tend towards unity. In this approximation, and neglecting for the moment the loop production and gravitational back-reaction terms, the evolution equation for the correlation length easily yields $L \propto a^2$. Note that this is the same overall scaling law for the string network as that in a radiation-dominated expanding universe; the string network effectively behaves like a radiation component. In terms of the physical correlation length, $L_{\text{phys}} \propto a$, as if strings were being conformally contracted (except for their rapidly growing velocities). However, there are several factors that must be considered which complicate this simple state of affairs.

First, there is the issue of loop production. Under the above assumptions on velocity, but putting the loop production term back in the correlation length equation, one finds the following approximate solution in the radiation and matter eras,

$$L_{\text{rad}} = \left( L_{\text{max}} - \frac{\tilde{c}}{2} \ln a \right) a^2$$  

(3)

$$L_{\text{mat}} = \left[ L_{\text{max}} + \frac{\tilde{c}}{2} \left( a^{-1/2} - 1 \right) \right] a^2$$  

(4)

where $L_{\text{max}}$ is the string correlation length at the time of maximal size of the universe, and the scale factor at that time was chosen to be unity (so the logarithm term in the first case is positive). Hence if $\tilde{c}$ remains constant (or is slowly varying), asymptotically the scale factor dependent terms will dominate, so that $L \propto a^2 \ln a$ in the radiation era, and $L \propto a^{3/2}$ in the matter era. The latter is also the scaling law for the correlation length in the matter-dominated, expanding universe. This highlights the different roles played by loop production in the scaling behaviour of a cosmic string network in the radiation and matter eras. A strong argument can be made, however, for a relativistic correction to the loop production term. In the simplest form of the VOS model there is an identification between the correlation length, $L$, and the physical distance $L_{\text{phys}}$ which a string segment is expected to travel before encountering another segment of the same size forming a loop in the process. However, taking into account the Lorentz factor in the physical correlation length, one would expect that $\tilde{c} \propto \gamma_\infty^{1/2}$ thus driving $\tilde{c}$ rapidly towards zero and asymptotically yielding our simple solution $L \propto a^2$ both in the radiation and matter eras. Of course, during re-collapse we expect that $\tilde{c}$ will depend on a number of other properties of the string network such as the enhanced build-up of small scale structure due to the contraction. Eventually, however, the Hubble radius will fall below even the length scale of wiggles on the string after which our asymptotic solution $L \propto a^2$ should be valid. In what follows, we shall consider the two well-motivated cases, first, $\tilde{c} = \text{const.} \neq 0$ initially and, secondly, $\tilde{c} = 0$ the probable asymptotic case. Further supporting evidence for this behaviour is discussed in [12].

Returning to our analytic solutions for the constant loop production case [15] and [11], we can use the velocity equation to find an approximate, implicit solution

$$1 - v^2 \propto a^4 \exp \left[ 2k(v) \frac{a^{1/2} \ln a}{L(a)} \right],$$  

(5)

where $\lambda = 1$ in the radiation era and $\lambda = 2$ in the matter era. Substituting [15] one respectively obtains

$$1 - v^2_{\text{rad}} \propto a^4 (- \ln a)^{4k(v)/\lambda},$$  

(6)

$$1 - v^2_{\text{mat}} \propto a^{4+2k(v)/\lambda}.$$  

(7)

Hence in the limit where $v \rightarrow 1$ and therefore $k \rightarrow 0$ the asymptotic solution would have the form $\gamma^{-2} \propto (1 - v^2) \propto a^4$. The momentum corrections, which phenomenologically account for the existence of small scale structures on the strings, imply that convergence will be slower than this. These solutions will still hold when one includes the gravitational back-reaction term [16]. As a final caveat, it is also worth emphasizing that the VOS model assumes that the long string network has a Brownian distribution on large enough scales, which may not be a realistic approximation in a closed, collapsing universe. This point clearly deserves further investigation.
As a test to the above solutions, we have performed a number of very high resolution Goto-Nambu simulations on the COSMOS supercomputer, using a modified version of the Allen-Shephard string code \[\text{[6]}.\] Further numerical details can be found in \[\text{[7]}.\] Our results are consistent with the existence of an attractor solution of the type described above. The result of two such simulations, for universes filled with radiation and matter, is shown in Fig. \[\text{[8]}.\] During the expanding phase we confirm the usual linear scaling regimes in the radiation and matter eras, respectively

\[ L_{\text{exp, rad}} \propto t \propto a^2, \quad v_\infty = \text{const.} \quad (8) \]

\[ L_{\text{exp, mat}} \propto t \propto a^{3/2}, \quad v_\infty = \text{const.} \quad (9) \]

Once the contraction starts, these are modified: the velocity starts increasing, and the scaling of the correlation length with the scale factor also drops, being approximately constant to begin with, and then rising slowly. One can identify a transient scaling phase, valid in the period \( \eta \sim 1.0 - 1.4 \), where one approximately has \( L_{\text{trans}} \propto a \) in the radiation-dominated case, and \( L_{\text{trans}} \propto a^{5/4} \) in the matter era. Unfortunately, the extremely demanding requirements in terms of resolution of the simulation do not currently allow us to run simulations with longer dynamic range to establish beyond reasonable doubt whether this scaling law approaches \( \beta = 2 \), as predicted above. However, there are strong indications that the networks are evolving towards this asymptotic regime, as shown by the relatively rapid climb of the exponent in Fig. \[\text{[9]}.\] It is clearly noticeable that the velocity rises much faster in the matter era than in the radiation era. It is also interesting to point out that during the collapse phase the loop and long string velocities are noticeably different, and this difference (which is more significant in the radiation than in the matter case) increases with time. The plot also shows an apparent difference in the expanding phase, but this is not significant: the initial lattice conditions tend to give different velocities to small loops than to long strings and they start evolving, and this difference is gradually erased.

We also notice that the network keeps chopping off loops throughout the simulation, and that there is a dramatic increase in the small scale structure of the network, particularly at later times. Visually, the string network develops large numbers of ‘knots’, highly convoluted strings regions where the wiggly long strings have collapsed inhomogeneously. These small scale features have proved to be difficult to evolve numerically, and this in fact turns out to be the main limiting factor at present preventing us from running the simulations closer to the big crunch. A comparison of our solutions \[\text{[6] \& [8]}\] and \[\text{[9]}\] with the numerical simulations described above, produces a very good matching—see \[\text{[9]}.\] Finally it is also worth keeping in mind that any discussion of the evolution of a cosmic string network with the present formalism is only applicable while one is well below the Hagedorn temperature, at which the strings would ‘dissolve’ in a reverse phase transition. Discussions of asymptotic regimes should be taken with some caution, since a cosmic string network will only survive the bounce intact if this happens before the Hagedorn temperature is reached.

**DISCUSSION: COSMOLOGICAL CONSEQUENCES**

The overall density of strings remains constant relative to the background density \( \bar{\rho} \) in both radiation and matter eras, \( \rho_\infty/\bar{\rho} = \sigma \sigma_t \), with \( \sigma_t \approx 400 \) and \( \sigma_m \approx 60 \) respectively \[\text{[9] \& [10]}\]. During curvature domination or accelerated expansion, the string density grows relative to the other matter as \( \rho_\infty/\rho_m \propto a \). For GUT-scale strings with \( G \mu \sim 10^{-20} \), this gives the interesting conclusion that today strings have a comparable energy density to the CMBR. However, a realistic cyclic model will continue to expand well beyond \( t_0 \), so the string density at maximum expansion will end up being much greater than the radiation density. In addition, the gravitational (or other) radiation produced through the continuous decay of the string network evolves as \( \rho_\text{gr} \propto a^{-4} \). It might appear that this contribution would become negligible during the matter era but in each Hubble time the strings
lose about half their energy into gravitational radiation, so this background always remains comparable to the string density $\rho_{gr} \sim \rho_\infty$. Now consider the collapsing phase in which the string network, like the gravitational waves they have produced, begins to behave like radiation. Globally, the density of both the strings and the gravitational waves will grow as $a^{-4}$ and, together with any other radiation components, they will eventually dominate over any nonrelativistic matter. In a realistic cyclic model, sufficiently massive strings and their decay products will have a greater density than the microwave and neutrino backgrounds. As the universe contracts, it will eventually reach a state in which the relativistic string network and/or their gravitational waves dominate the global dynamics of the universe! This would lead to a dramatically different universe after it emerges from the next bounce. Even lighter strings, which do not dominate the universe, would end up with a much greater density in the collapsing phase than they had previously during expansion. If the universe went through a bounce, the energy density in the cosmic strings and gravitational radiation produced by the network would be much greater after the bounce than before it. Bounds on the string mass per unit length may be severely modified, in addition to more general constraints on extra relativistic fluids [13].

Furthermore, unlike the uniform CMB, the energy density in both cosmic strings and gravitational radiation may be very inhomogeneous. In the collapsing regime, an increasingly small fraction of Hubble regions will have a string passing through them. Those that do will become string dominated since the string energy density in those regions will approximately evolve as $\rho_\infty/\dot{\rho} \propto \gamma_\infty \propto a^{-2}$, up to the corrections described above. For these regions the assumption of a FRW background will cease to be valid at late times, and the defects can make the universe very inhomogeneous [13] or anisotropic [19]. Even Hubble regions without strings will have large fluctuations in their gravitational radiation content. For sufficiently massive strings, both of these effects can survive the bounce to create large inhomogeneities in the next cycle.

A possible caveat to these solutions is dynamical friction (which we have neglected so far). In the $\gamma >> 1$ limit, one can estimate [3] that strings lose all their momentum due to this effects in one Hubble time when $G\mu \sim \gamma^{-2}$ (radiation era result) or $G\mu \sim \gamma^{-1}$ (matter era result). However, this assumes a homogeneous and isotropic background, so need not apply in our case. Moreover, the fact that a significant amount of momentum will be transferred from the strings to the background will in itself add to the anisotropies which naturally occur in our model. This will be developed further elsewhere [3].

We conclude that a cosmic string network will be a significant source of radiation, entropy and inhomogeneity which may be problematic in the cyclic context. Some of the results described in this paper should also be valid for other topological defects, in particular domain walls. Conversely, if direct evidence is found for the presence of topological defects in the early universe, their existence alone will impose constraints on the existence and characteristics of previous phases of cosmological collapse.

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\[ \rho_\infty \sim (\text{radiation era result}) \quad \text{or} \quad \rho_{gr} \sim \ldots \]

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