BARYOGENESIS, 30 YEARS AFTER.

(Lectures given at the 25th ITEP Winter School)

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A review of the basic principles of baryogenesis is given. Baryogenesis in heavy particle decays as well as electroweak, SUSY-condensate, and spontaneous baryogenesis are discussed. The models of abundant creation of antimatter in the universe are briefly reviewed.

1 Introduction

This year is the anniversary of two great dates: 25th anniversary of ITEP Winter Schools and 30th anniversary of the seminal paper by A.D. Sakharov on baryon asymmetry of the universe. In this paper a mechanism was proposed, which explains the dominance of particles over antiparticles in the universe by a dynamical evolution of an initially charge symmetric or even arbitrary state. The necessary conditions for the generation of the asymmetry, as formulated by Sakharov, are the following:

1. Different interactions of particles and antiparticles, or in other words, a violation of C and CP symmetries.
2. Nonconservation of baryonic charge $B$.
3. Deviation from thermal equilibrium in the early universe.

We will discuss these conditions in more detail below and now return to the history of the problem. The work by Sakharov remained unnoticed for several years, except for a paper by Kuzmin in 1970, where a somewhat different realization of the Sakharov’s ideas was considered. Still the hypothesis of baryonic charge nonconservation, which is necessary for the generation of an asymmetry between baryons and antibaryons, was not accepted by the establishment till 1974, when the models of Grand Unification were put forward. After it was understood that nonconservation of baryons might be quite a natural
and general phenomenon, the attitude to the possibility of a dynamical generation of baryon asymmetry of the universe has been drastically changed and two almost simultaneous papers by Ignatyev et al 5 and by Yoshimura 6 in 1978 stimulated the flood of papers which remains unabated for 20 years. For the review of the earlier stage of the theory of baryogenesis one may address the papers 7,8. Different scenarios of baryogenesis at that early period were mostly based on the decay of heavy particles in different versions of Grand Unification Models. They all have a general feature that the characteristic energy scale for the processes with B-nonconservation is extremely high, close to the Planck scale, $M_{GUT} = 10^{16}$ GeV. Since then many other scenarios of baryogenesis have been developed with baryon nonconservation at much lower energies, sometimes almost "at hand". For a review of these models one can address the paper 9. The most fashionable for already several years remains electroweak baryogenesis 10 where essential physical processes take place at the energy scale around 100 GeV. The dominant majority of papers are devoted to different realizations of this particular model. Reviews of activity in this field can be found in refs. 9,11,12. By necessity they are all incomplete because new papers on the subject constantly appear with a non-decreasing production rate.

In these lectures I will give an introduction to the theory of baryogenesis for non-experts paying most attention to general features of the phenomenon. I will not be able to cover very recent development in the field. This would demand much more time, space, and skill. The content of the lectures is the following. In the next four Sections the general picture and the three Sakharov’s conditions are discussed. In Sec. 6 the baryogenesis through decays of massive particles is considered. In Sec. 7 the other major baryogenesis scenarios are enumerated and three of them (electroweak, SUSY-condensate, and spontaneous) are discussed in some detail. In the last section some models leading to a large amount of antimatter are reviewed.

2 Generalities.

We know that for any known particle there exists the antiparticle with exactly (within experimental accuracy) the same mass, $m = \bar{m}$ and decay width, $\Gamma = \bar{\Gamma}$, and opposite signs of all the charges associated with this particle, $Q_j = -\bar{Q}_j$. In spite of this striking symmetry which would naturally imply equal number densities of particles and antiparticles in the universe, $n = \bar{n}$, the observed picture is quite different: the universe (at least in our neighborhood is predominantly populated by particles: protons, neutrons, and electrons with a very small fraction of antiparticles observed in cosmic rays, which all can be
explained by secondary origin in energetic particle collisions. It is not excluded of course that the dominance of matter over antimatter is only local and is realized inside a finite volume characterized by the linear size $l_B$, while further away the picture is reversed, and so on and so forth. In this way the universe could be globally symmetric with respect to particles and antiparticles. Even if this is true, it could be achieved only with baryon nonconservation because the separation of matter and antimatter on an astronomically large scale does not seem possible. The size of our matter domain $l_B$ is known to be quite large, roughly speaking $l_B > 10 \text{ Mpc}$ (in the recent paper much more restrictive bounds are presented). For a smaller $l_B$ one would expect a too large flux of energetic $\gamma$-rays coming from the reaction of annihilation of $p\bar{p}$ into $\pi$-mesons with the subsequent decay $\pi^0 \rightarrow 2\gamma$, which would take place in the boundary area between the world and anti-world. Another signature of domains of antimatter would be a distortion of spectrum and isotropy of cosmic microwave radiation. The value of $l_B$ permitted by such observations would be smaller if matter and antimatter domains are separated by voids which may appear because of an excessive pressure produced by the annihilation at earlier stages of the evolution of the universe or because of low density of matter and antimatter in the boundary regions, if the baryon asymmetry changes sign gradually so that in the intermediate region the asymmetry vanishes or is very small.

The convenient dimensionless number which characterizes the magnitude of the baryon asymmetry of the universe is the ratio of the baryonic charge density ($n_B - n_{\bar{B}}$) to the number density of photons in cosmic microwave radiation,

$$\beta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 3 \cdot 10^{-10}$$

(1)

Here $n_\gamma = 411.4(T_\gamma/2.736^\circ \text{K})^3 \text{cm}^{-3}$ and $T_\gamma = 2.736^\circ \text{K}$ is the temperature of the radiation. Direct observations of baryonic matter in the universe give for $\beta$, roughly speaking, one third of the above number, in particular because a large amount of baryons may be invisible (the recent discovery by the Hubble Space Telescope of very faint galaxies gives a larger fraction of directly observed baryonic matter). A more accurate estimate can be obtained from primordial nucleosynthesis, from which the result (1) is inferred. The abundances of light elements ($^4\text{He}, ^3\text{He}, ^7\text{Li}$, and especially $^2\text{H}$) produced during the "first three minutes" are sensitive indicators of the baryonic number density at that epoch (for the review given at an ITEP Winter School see ref. and for a more up-to-date analysis see e.g. the paper).

We believe that the magnitude of $\beta$ at nucleosynthesis was the same as it is now. If baryonic charge was conserved during the period from the primordial
nucleosynthesis till the present day, the difference \((n_B - n_{\bar{B}})\) remains constant in the comoving volume, i.e. in the volume which expands together with the universe, \(V \sim a^3(t)\), where \(a(t)\) is the scale factor describing the universe expansion (the separation, \(l(t)\), of two distant objects changes with time as \(l(t) \sim a(t)l_0\)). The number density of noninteracting photons also remains constant in the comoving volume, \(T_\gamma \sim 1/a(t)\) and \(n_\gamma \sim 1/a^3(t)\). In this approximation the ratio \(\beta\) would not change in the course of expansion. In fact \(a^3n_\gamma\) was not constant in the early universe. Indeed the annihilation of massive particles, when the temperature dropped below their masses, heated up the primeval plasma and increased the photon number density. However there is a conserved quantity, namely the entropy density,

\[
s = (p + \rho)/T = \frac{2\pi^2}{45}g_s(T)T^3,\tag{2}
\]

which stays constant in the comoving volume if thermal equilibrium is maintained. This is usually true with a very good accuracy during most of the universe history. In equation (2) \(\rho\) and \(p\) are respectively the energy and pressure densities in the primordial plasma and the factor \(g_s(T)\) is roughly speaking equal to the number of particle species with \(m < T\). Because of that it is more convenient to introduce the quantity \(\beta_s = (n_B - n_{\bar{B}})/s\) which does not change in the course of expansion in a thermal equilibrium state. In the standard physical model without new long-lived particles and unknown interactions, there is no entropy production after e\(^+\)e\(^-\)-annihilation so that \(n_\gamma a^3\) remains constant. Below \(T = m_e\) this new quantity \(\beta_s\) practically coincides with \(\beta\) while at higher temperatures they may differ by one-two orders of magnitude due to contribution from heavier particles. The original baryon asymmetry would be diluted by the same amount. Another source of entropy and of the corresponding dilution of the baryon asymmetry are possible phase transitions in the early universe of the first and even of the second order. They could also considerably diminish the previously generated asymmetry.

At higher temperatures when \(T \geq m_B = O(\text{GeV})\) baryons and antibaryons in the plasma were practically equally abundant \((n_B - n_{\bar{B}})/(n_B + n_{\bar{B}}) \approx 10^{-9}\), while presently \(n_{\bar{B}} \ll n_B\) and this ratio is very close to 1. In other words the universe which is 100% asymmetric now was almost charge symmetric during early stages, so we need to explain a very small breaking of this symmetry. Still this very small breaking resulted in a huge amplification of the present-day baryonic number density. In the case of a symmetric universe baryons would efficiently disappear at small temperatures due to mutual annihilation and the number density of survived baryons can be evaluated as

\[
n_B = n_B \approx n_\gamma/(\sigma_{\text{ann}}m_Bm_P) \approx 10^{-19}n_\gamma,\tag{3}
\]
where $\sigma_{ann}$ is the cross-section of nucleon annihilation, $m_B \approx 1\text{GeV}$ is their mass and $m_{Pl} = 1.2 \cdot 10^{19}\text{GeV}$ is the Planck mass. So in a baryo-symmetric universe the number density of baryons would be $9(!)$ orders of magnitude smaller than what is observed in reality. If this were true then there would not be enough building material for formation of celestial bodies and of course life would not be possible in such almost empty universe. One may invoke the anthropic principle for an explanation of the baryon asymmetry: in a symmetric universe life would not be possible and only in an asymmetric world there could be an observer to put the question why the universe is asymmetric. It is interesting to estimate the magnitude of the asymmetry which would allow formation of celestial bodies and ultimately creation of life, but we have a better option. The asymmetry can be generated dynamically and its magnitude can be expressed through (possibly not yet known) fundamental parameters of particle physics.

3 Rise and Fall of Discrete Symmetries.

Less than a half of a century ago (in fact before 1956) it was firmly believed that physical laws are symmetric with respect to mirror reflection, P, charge conjugation (transition from particles to antiparticles), C, and time inversion, T. Though none of these symmetries followed from any fundamental principle the belief was quite strong and it was a great shock when it was found that space parity is not conserved so that a mirror reflected process could be physically impossible. It was assumed immediately that simultaneous mirror reflection and charge conjugation, CP, restore the symmetry so that for each process with particles the mirror reflected process with antiparticles, and otherwise the same, is possible. However the general attitude to the discrete symmetries was changed towards ”everything which is not forbidden is permitted” and the search for CP-violation was discussed in the literature. In 1964 it was found that CP is also broken and after this discovery life in the universe became possible (of course with nonconservation of baryons).

The only discrete symmetry which survived to the present day is the combination of all three transformations, CPT. It has a rather good reason to exist: the so called CPT-theorem can be proven, which states that any Lorentz-invariant theory with positive definite energy and the normal relation between spin and statistics is invariant with respect to CPT-transformation. As a result of this symmetry masses of particles and antiparticles and their total decay widths must be exactly equal. However the probabilities of specific channels should be different for charged conjugated processes if both C and CP are broken. One comment is in order here. In the lowest order in pertur-
bation theory the amplitude of the charged conjugated process is equal to the complex conjugate of the original amplitude because of the hermicity of the Lagrangian. Thus in this approximation the probabilities of these processes are equal even if C and CP are violated. To break this equality higher order corrections are necessary. These corrections come from inelastic rescattering of the decay products. The corresponding imaginary part can be calculated from the unitarity condition for the scattering matrix, \( SS^+ = 1 \). Introducing the amplitude matrix in the usual way, \( S = 1 + iT \), one gets

\[ i(T - T^+) = -TT^+ \] (4)

Thus there are two sources of imaginary parts in amplitudes: imaginary parts in coupling constants related to C(CP)-nonconservation and dynamical imaginary parts due to on-mass-shell rescattering corrections. Therefore the effects of C(CP)-violation are always suppressed by an extra power of coupling constants. It can be seen that elastic rescattering of the decay products produce the same phase correction to particles and antiparticles and another channel is necessary to get a nontrivial phase correction and to break equality of the absolute values of the amplitudes of charge conjugated processes.

Experimentally the first Sakharov's condition is well justified, CP-violation is directly observed in the decays of \( K^0 \)-mesons, but it is not yet known what mechanism is responsible for it. A knowledge of that is very important for baryogenesis because baryogenesis took place at a much larger energy scale where the data on the kaon decays cannot be simply applied. Anyway it is known in principle that antiparticles are not just mirror reflections of particles, they have essentially different interactions and are produced with different probabilities in charge conjugated processes.

4 Nonconservation of Baryonic Charge.

In contrast to breaking of C and CP invariance, nonconservation of baryonic charge has not yet been observed in direct experiments. The only "experimental" evidence of baryonic charge nonconservation is the existence of our baryo-asymmetric universe. A rather strong argument in favor of nonconservation of baryons comes from the necessity of inflation. We do not see today any alternative way to create our smooth and flat universe without a relatively long inflationary stage. During inflation the universe expands in accordance with the law

\[ a(t) \sim \exp(Ht) \] (5)

where the Hubble parameter \( H \) is (approximately) constant. The necessary duration of the inflationary stage for the solution of the flatness and horizon
problems is about 60-70 Hubble times, $Ht = 60 - 70$. The Hubble parameter is related to the total energy density in the universe as $H = \sqrt{8\pi \rho_{\text{tot}} / 3m_p^2}$. To keep $H$ constant we need to have a constant energy density which is naturally realized by a scalar field, the inflaton.

Let us assume now that baryonic charge is strictly conserved. The density of baryonic charge at the present day with respect to the number density of cosmic microwave photons is about $10^{-9} - 10^{-10}$ (see eq. (1)). It means that at higher temperatures the energy density associated with baryonic charge with respect to the total energy density would be at least that large and would remain more or less constant during the radiation domination stage when the universe expanded as $a(t) \sim \sqrt{t}$. Now if we go deeper back in time to the inflationary stage, the energy density of matter would be in the form of an inflaton field and would remain constant except for the energy density associated with the baryonic charge, $\rho_B$. Since the charge is conserved, $\rho_B$ cannot be constant and evolves as $\rho_B \sim 1/a^4 \sim \exp(-4Ht)$ for relativistic baryons or $\rho_B \sim 1/a^3 \sim \exp(-3Ht)$ for nonrelativistic baryons (the latter is rather improbable). One can quite easily check that the total energy density cannot remain (approximately) constant for more than 6-7 Hubble times. It makes long inflation impossible. Note that this is not simply the statement that to have the observed baryon asymmetry today the preinflationary (initial) value of the baryonic charge density should be unnaturally huge but that the presence of conserved charge does not permit to have an approximately constant energy density and correspondingly does not let inflationary expansion last sufficiently long time.

Theoretically it is rather natural to expect that baryonic charge is not conserved. Normally, as we understand it, charge conservation is associated with a local (gauge) symmetry like $U(1)$ in electrodynamics. Such a symmetry usually implies the existence of massless gauge bosons ("photons") which produce long-range forces. Existence of such Coulomb-like forces would break the equivalence principle for elements with a different mass-to-baryonic-charge ratio. No such violation has been observed and this gives the upper limit $\alpha_B < 10^{-44}$ (compare with the electromagnetic coupling constant, $\alpha = 1/137$). Such a smallness of the coupling probably means that there is no conserved current related to baryonic charge. Moreover there are quite a few theoretical models which predict that baryonic charge is indeed nonconserved. To start with, there are grand unification theories which put quarks and leptons on equal footing (into the same particle multiplet). Thus there should be be transitions between quarks and leptons which break both baryon and lepton number conservation. This would give rise to proton decay or to neutron-antineutron oscillations,
unfortunately not yet discovered by experiment. A plethora of supersymmetric models also predict baryonic charge nonconservation (for a review see e.g. ref. {\textsuperscript{29}}) which could occur at the energies below the grand unification scale, \( m_{\text{GUT}} = 10^{15} - 10^{16} \) GeV and potentially be more dangerous for proton decay. Moreover the standard electroweak theory predicts nonconservation of baryonic charge through quantum corrections\textsuperscript{30}. This nonconservation is negligibly small at low energies but could be very much enhanced at high temperatures comparable with the electroweak scale\textsuperscript{13} (see section 7).

Thus we can conclude that baryonic charge is most probably nonconserved. Manifestation of its nonconservation are strongly suppressed at low energies but at high energies or temperatures, which existed in the early universe, the processes with a change in \( B \) might be efficient and produce an excess of baryons over antibaryons.

\section{5 Thermodynamics of Baryogenesis.}

For a gas or plasma in thermal equilibrium state with temperature \( T \) the particle occupation numbers are given by the function:

\[
    f(p) = \frac{1}{\exp\left[\frac{(\sqrt{p^2 + m^2} - \mu)}{T}\right] \pm 1}. \tag{6}
\]

The chemical potential \( \mu \) is nonzero in equilibrium only if the particles in question possess a conserved charge \( Q \) and the corresponding charge density is non-vanishing. Indeed the chemical potentials of particles and antiparticles in equilibrium are related by the condition \( \mu + \bar{\mu} = 0 \) and in the case that fermions bear the charge \( Q \) (in what follows we will take \( Q = B \) and talk about baryon asymmetry), the charge density is given by

\[
    n_Q = n - \bar{n} = g_s \int \frac{d^3p/(2\pi)^3}{\exp\left[\frac{(\sqrt{p^2 + m^2} - \mu)}{T}\right] + 1} - g_s \int \frac{d^3p/(2\pi)^3}{\exp\left[\frac{(\sqrt{p^2 + m^2} + \mu)}{T}\right] + 1}, \tag{7}
\]

where \( g_s \) is the number of spin states. In the case of nonconserved charge the corresponding chemical potential vanishes in equilibrium due to charge nonconserving reaction and thus \( n = \bar{n} \) and the asymmetry \( n_Q \) is zero. Recall that in thermal equilibrium the sum of chemical potentials of particles in the initial state is equal to that in the final state, \( \sum \mu_i = \sum \mu_f \). Making the conclusion about the vanishing of \( n_Q \) we have implicitly used CPT-theorem,
by which masses of particles and antiparticle are equal. Otherwise there would be an asymmetry even in equilibrium:

$$n_B \approx \frac{g_s q_B (m^2 - \bar{m}^2)}{4\pi^2} T^3 \int_1^\infty dy \exp(-my/T) \sqrt{y^2 - 1}, \quad (8)$$

where $q_B$ is the baryonic charge of particles in question.

Typically the rate of expansion is small in comparison with the reaction rates. The former is given by the Hubble parameter which is related to the total energy density in the universe by the Einstein equations:

$$H = \sqrt{\frac{8\pi\rho}{3m_{Pl}^2}} = \sqrt{\frac{8\pi^3 g_\ast T^2}{90}} m_{Pl}. \quad (9)$$

Here we have substituted the equilibrium expression for the energy density, $\rho = \pi^2 g_\ast T^4/30$, where $g_\ast$ counts the number of relativistic particle species in the primeval plasma, one for each bosonic spin state and $7/8$ for each fermionic spin state. Since $H$ is suppressed by the inverse big Planck mass, the expansion is normally slower than the reaction rates which are either $\Gamma_d \sim \alpha m$ for decays of particles with mass $m$ or $\Gamma_r \sim a^2 T$ for reactions. Still equilibrium is always somewhat broken for massive particles. To see this let us consider the kinetic equation in the expanding universe:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} - H_p \frac{\partial f}{\partial p} = I[f], \quad (10)$$

where we used the relation $dp/dt = -H_p$ (red shift of momenta in the expanding universe). The r.h.s. of this equation is a collision integral to be specified below. Here we need only to know that $I[f]$ is annihilated by the equilibrium distributions $\Xi$. Substituting them into the l.h.s. we get:

$$\text{l.h.s.} = \frac{\exp[(E - \mu)/T]}{[\exp((E - \mu)/T) + 1]^2} \left( \frac{\mu T}{T^2} - \frac{\dot{\mu}}{T} - \frac{E T}{T^2} - \frac{H_p^2}{T E} \right). \quad (11)$$

This expression can vanish for arbitrary $p$ only if the particles in question are massless, so that $p = E$, and if the following conditions are fulfilled: $\dot{\mu}/\mu = \dot{T}/T$ and $T/T = -H$. So we can estimate the deviation from equilibrium as

$$\frac{\delta f}{f_{eq}} \approx \frac{(H m^2 / T E \Gamma)}{}, \quad (12)$$

where $\Gamma$ is the characteristic rate of the reactions given by the collision integral in the r.h.s. One can roughly estimate this ratio at $T \approx m$ as $\delta f/f \approx$
$10m/\alpha^k m_{Pl}$, where $\alpha$ if the characteristic coupling constant (at the grand unification scale $\alpha \approx 1/50$) $k = 1$ or 2, depending upon whether decay or two-body reactions are essential, and we take $g_* \approx 100$. The deviation from equilibrium are noticeable either for very heavy particles with the mass around $10^{16} - 10^{15}$ GeV or for very weakly coupled ones with the coupling constant much smaller than $\alpha$. For electroweak interactions at the electroweak energy scale, $T = O$(TeV), the ratio (12) is close to $10^{-15}$.

Now let us have a closer look at the collision integral. It has the following form:

$$I[f] = \int \Pi' \frac{d^3 p_i}{(2\pi)^3 2E_i} \frac{d^3 p_f}{(2\pi)^3 2E_f} \frac{(2\pi)^4 \delta}{2 \prod_{i} \prod_{f}} \left[ |A_{if}|^2 \Pi_f \Pi_i (f_f \pm 1) - |A_{fi}|^2 \Pi_i \Pi_f (f_i \pm 1) \right] ,$$

where the integration is made over all final momenta and all initial ones, except for the particle under scrutiny, which distribution function enters the l.h.s. of equation (10). It is usually assumed that T-invariance is true so that the amplitudes of direct and inverse processes are equal up to a trivial change of signs of momenta, $|A_{if}|^2 = |A_{fi}|^2$. This is the very well known detailed balance condition. One can check that if this condition is true the collision integral vanishes on the equilibrium functions (1). The product $\Pi_f \Pi_i (f_f \pm 1)$ is equal to the product with $i$ and $f$ interchanged because of the conservation of energy, $\sum E_i = \sum E_f$, and chemical potentials, $\sum \mu_i = \sum \mu_f$, in equilibrium. We know however that CP-invariance is broken, so by CPT-theorem T-invariance also cannot be true, and the detailed balance condition is violated. One might worry if breaking of CP would simultaneously break normal equilibrium statistics. This is not the case however because, as we have already noted, CP- and correspondingly T-violation can be observable only when there are several different processes giving rise to inelastic rescattering in the final state. If these additional (and necessary) processes are taken into account, the total contribution of all of them ensures vanishing of the collision integral even if detailed balance is violated. Equilibrium statistics is more general than the detailed balance condition. In fact the validity of the canonical equilibrium distributions follows either from the unitarity of S-matrix or from CPT-theorem and conservation of probability. One of those is sufficient to maintain the so called cyclic balance (instead of the detailed one) when a cycle of several processes ensures thermal equilibrium. This problem is considered in detail in ref. [31].
6 Baryogenesis in Massive Particle Decays.

This is the simplest and historically first model of baryogenesis. It has a natural theoretical framework of grand unified models. As we have already mentioned baryonic charge is not conserved in these models and, since the gauge and Higgs bosons in these theories have very large masses, \( m = 10^{15} - 10^{16} \text{ GeV} \), one may expect that their number density could be essentially above the equilibrium one so that their B-nonconserving and charge asymmetric decay could produce a noticeable asymmetry between baryons and antibaryons. As a simple model let us assume that there is a charge symmetric collection of \( X \) and \( \bar{X} \) gauge bosons of grand unification, \( n_X = n_{\bar{X}} \), and no other particles. These bosons are known to have the following decay modes:

\[
X \to qq, \quad X \to \bar{q}l, \quad (14)
\]
\[
\bar{X} \to \bar{q}\bar{q}, \quad X \to \bar{ql}. \quad (15)
\]

where \( q \) and \( l \) are correspondingly quarks and leptons. It is clear that baryonic charge is not conserved in these reactions because the same initial state decays into particles with different baryonic charges. If C and CP are not conserved the widths of charge conjugated decay channels may be different:

\[
\Gamma_{X \to qq} = (1 + \Delta_q)\Gamma_q, \quad \Gamma_{X \to q\bar{l}} = (1 - \Delta_l)\Gamma_l, \\
\Gamma_{\bar{X} \to \bar{q}\bar{q}} = (1 - \Delta_q)\Gamma_q, \quad \Gamma_{\bar{X} \to \bar{ql}} = (1 + \Delta_l)\Gamma_l. \quad (16)
\]

Here \( \Delta_{q,l} \) are nonzero due to breaking of charge symmetries. If these decay modes are the only ones, then by CPT-theorem the total decay widths of \( X \) and \( \bar{X} \) should be the same and so \( \Delta_q\Gamma_q = \Delta_l\Gamma_l \).

If the energy of fermions produced by these decays quickly drops down due to expansion or thermalization by rescattering, so that baryonic charge becomes effectively conserved in their collisions, then the baryon asymmetry would be roughly equal to

\[
\beta \approx \frac{4\Delta_q\Gamma_q - 2\Delta_l\Gamma_l}{\Gamma_{tot}} \frac{n_X}{n_0}, \quad (17)
\]

where \( n_X \) is the number density of the initial \( X \)-bosons and \( n_0 \) is the number (or entropy) density of the produced light particles. The latter may be considerably larger than just \( 2n_X \) because of a possible increase of the number density of light particles in the process of thermalization. Neglecting the universe expansion we can estimate the number density of light particles as \( n_0 \approx \rho_X^{3/4} \approx n_X (m_X^{3/4}/n_X^{1/4}) \) (the last equality is true for nonrelativistic \( X \)-bosons).
If the processes of thermalization and of the cooling by expansion are not fast enough, the produced quarks and leptons would be quite energetic so that baryonic charge would be nonconserved in their reactions. Such reactions could wash out the baryon asymmetry produced by the decays of X-bosons. It is often stated in the literature that the asymmetry is erased by inverse decays, $qq \rightarrow X$, $q\bar{l} \rightarrow X$, etc. However one can see that this is incorrect. Indeed CPT-invariance permits one to express the probabilities of the inverse decays through the direct ones

$$
\Gamma_{\bar{q}\bar{q} \rightarrow \bar{X}} = (1 + \Delta_q)^{\frac{1}{2}} \Gamma_q, \quad \Gamma_{q\bar{l} \rightarrow \bar{X}} = (1 - \Delta_l)^{\frac{1}{2}} \Gamma_l, \\
\Gamma_{qq \rightarrow X} = (1 - \Delta_q)^{\frac{1}{2}} \Gamma_q, \quad \Gamma_{q\bar{l} \rightarrow X} = (1 + \Delta_l)^{\frac{1}{2}} \Gamma_l.
$$

(18)

So direct and inverse decays produce the same sign of baryon asymmetry and not the opposite one as is necessary for compensation. On the other hand we know that in thermal equilibrium no asymmetry is generated and the excess of baryons produced in the decays is erased by some other processes. These processes are B-nonconserving 2 → 2-scattering of quarks and leptons with X-boson exchange.

The mechanism outlined here would be operative if a nonequilibrium number density of X-bosons was created. Usually massive particles are in equilibrium at high temperatures, $T \gg m$, and their number density exceeds the equilibrium one when $T$ becomes comparable to $m$. If they are unstable, they sooner or later come back to equilibrium because the decay rate $\Gamma$ remains constant while the expansion rate goes down. However their number density at this stage is Boltzmann suppressed, $n \sim \exp(-m/T)$, and is negligibly small. Therefore the most favorable period for the generation of asymmetry is when $m/T = O(1)$. If $X$ are gauge (or Higgs) bosons of grand unification the situation is somewhat more complicated because they might never be in equilibrium at early stage of the universe evolution. First, even if the temperature of the primeval plasma was higher than the grand unification scale, $m_{GUT} = 10^{16} - 10^{15}$ GeV, the rate of their production would be smaller than that of the expansion and the number density of these bosons could always be smaller than the equilibrium one. To see it one needs to compare the Hubble parameter given by eq. (9) to the decay or two-body reaction rates. The bosons might be out of equilibrium both in unbroken symmetry phase (when $m_X = 0$, except for temperature corrections) and in the broken one. Second, the universe temperature could always be smaller than $m_{GUT}$ and correspondingly thermally produced X-bosons would never have been abundant and their role in the baryogenesis would be negligible. Another possibility is that the inflaton field is predominantly coupled to these bosons and though the temperature of the universe after thermalization was much smaller than their mass,
they might be abundantly produced by the inflaton decay so that their initial number density was well above the equilibrium one. In such a case the decays of the intermediate bosons of grand unification could produce the baryon asymmetry of the universe.

If we consider particles of smaller mass, the deviation from equilibrium is generically rather small (see eq. (12)). One possible way to break the equilibrium is to assume a weaker-than-gauge coupling. Another possibility is a late generation of mass\textsuperscript{34, 35}, when particles acquire their mass as a result of phase transition with $T < m$, while their number density remains those of massless particles, $n \sim T^3$, and is not Boltzmann suppressed.

There is one more problem usually associated with the GUT-scale baryogenesis, namely the problem of relic gravitino\textsuperscript{36, 37}. The gravitino is a spin-(3/2) particle with the interaction strength inversely proportional to the Planck mass which appears in supergravity theories as a superpartner of graviton. The cross-section of their production/annihilation is roughly $\sigma_{3/2} \approx 1/m_P^2$. Correspondingly their number density relative to entropy density should be equal to

$$n_{3/2}/s \approx 10^{-2}T_{reh}/m_P,$$

where $T_{reh}$ is the temperature of the reheated universe after inflation. The decay width of gravitino is

$$\Gamma_{3/2} = m_{3/2}^3/m_P^2 \approx (10^5 \text{sec})^{-1} (m_{3/2}/\text{TeV})^3$$

If $T_{reh} \sim T_{GUT}$, gravitini can be abundant at nucleosynthesis and destroy the good agreement of the theory with observations. However if the initial state produced by the decay of the inflaton was considerably out of equilibrium with abundant X-bosons but with the reheating temperature much below the GUT-scale, the gravitini would not be created in a dangerous amount.

To conclude, it is probably too early to abandon GUT-baryogenesis. It naturally has the necessary features: baryonic charge nonconservation (compatible with the observed proton stability and absence of neutron-antineutron oscillations), deviation from thermal equilibrium and may have a sufficiently strong CP-violation.

7 Other Models.

There are quite many scenarios of baryogenesis which are much more sophisticated than the simple ones based on heavy particle decays. They all can take place at much smaller temperatures or energies than $T_{GUT}$. This energy range may even be accessible in the acting accelerators. The (possibly incomplete) list of scenarios includes:
1. Electroweak baryogenesis.

2. Baryogenesis by supersymmetric baryonic charge condensate.

3. Spontaneous baryogenesis.

4. Baryo-through-lepto-genesis.

5. Baryogenesis in black hole evaporation.

6. Baryogenesis by topological defects (domain walls, cosmic strings, magnetic monopoles).

Below we will briefly discuss only the first three ones. A more detailed discussion of them, as well as of the remaining, and a list of references can be found in the review paper.

7.1. Electroweak baryogenesis

This model is the most fashionable now and a majority of the recent papers on the subject deals with different versions of the electroweak scenario. Surprisingly the standard model of electroweak interactions have all the necessary ingredients for successful baryogenesis. It is known from experiment that C and CP symmetries, are broken. Theoretically introduction of CP-nonconservation into the standard model is easily done either by a complex quark mass matrix with at least three generations or, what is essentially the same, by complex coupling constants of the Higgs fields. What is more surprising is that baryons are not conserved by the usual electroweak interactions. This is a rather complicated phenomenon and is related to the so called quantum chiral anomaly. The classical electroweak Lagrangian conserves baryonic charge. Quarks always enter in bilinear combinations $\bar{q}q$, so that a quark can disappear only in collision with an antiquark. In other words the classical baryonic current is conserved:

$$\partial_\mu J^B_\mu = \sum_j \partial_\mu(\bar{q}_j\gamma_\mu q_j) = 0.$$  \hspace{1cm} (21)

However quantum corrections destroy this conservation law and instead of zero in the r.h.s. one gets

$$\partial_\mu J^B_\mu = \frac{g^2 C}{16\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu},$$  \hspace{1cm} (22)
where $C$ is a numerical constant, $G_{\mu\nu} = G_{\alpha\beta} \epsilon_{\mu\nu\alpha\beta}/2$, and the gauge field strength, $G_{\mu\nu}$, is given by the expression

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu A_\nu].$$

(23)

An important fact is that the anomalous current nonconservation is proportional to the total derivative of a vector operator: $G_{\mu\nu} \tilde{G}_{\mu\nu} = \partial_\mu K_\mu$ where the anomalous current $K_\mu$ is

$$K_\mu = 2\epsilon_{\mu\nu\alpha\beta} \left( A_\nu \partial_\alpha A_\beta + \frac{2}{3} g A_\nu A_\alpha A_\beta \right).$$

(24)

The last term in this expression does not vanish only for non-Abelian gauge theories because the antisymmetric product of three vector potentials $A_\nu$ can be nonzero due to different group indices (e.g. for the electroweak group it should contain the product of $W^+, W^-$ and the isospin one part of $Z^0$).

Usually a total derivative is unobservable because one can get rid of it integrating by parts. However, this may be not true for $K_\mu$ (24). Indeed the gauge field strength $G_{\mu\nu}$ should vanish at infinity but the potential $A_\mu$ does not necessarily vanish. Thus different vacuum states, which all have $G_{\mu\nu} = 0$, differ by the value of $K_0$ (in fact by the integral from that over the space volume). Since the difference $J_B^0 - K_\mu$ is conserved, transition from one vacuum state to another leads to a change in baryonic charge. The path from one vacuum to another is separated by a potential barrier where $G_{\mu\nu} \neq 0$. As we know from quantum mechanics the barrier penetration at small energies is exponentially suppressed, that is why the probability of processes with $\Delta B \neq 0$ contains the very small factor, $\exp(-16\pi^2/g^2) \approx 10^{-160}$. However at high energies or temperatures (comparable or above the barrier height) the transition between different vacua can be achieved by a classical motion over the barrier. The hight of the barrier, as calculated in ref. 41, is around several TeV. In fact the barrier disappears at high temperatures together with the $W$ or $Z$ masses according to the law: $m_W^2(T) = m_W^2(0)(1 - T^2/T_c^2)$. This also occurs in the same TeV region. So one may expect that at high temperatures baryon nonconservation is not suppressed. It has been argued that above the electroweak phase transition the processes with $\Delta B \neq 0$ are much faster than the universe expansion, so that any preexisting baryon asymmetry would be washed out. To be more precise electroweak interactions (even with the chiral anomaly) conserve the difference between baryonic and leptonic charges, $(B - L)$. Thus at high temperatures only $(B + L)$ may be erased while a preexisting $(B - L)$ is conserved.

If this picture is correct, electroweak interactions at high temperatures play the role of a terminator of asymmetry and not of a creator. If the electroweak
phase transition is second order, then everything goes smoothly, thermal equilibrium is not disturbed, and asymmetry is not generated even below the phase transition. The type of the phase transition depends upon the mass of the Higgs boson (or bosons in extended models). For high mass the transition is second order while for a low mass it is first order. The boundary value of the mass is not well known even in the minimal standard model and different estimates give the values somewhere between 50 and 100 GeV. If the Higgs mass is below this boundary value, then phase transition is first order and the deviation from thermal equilibrium can be significant. In this case there would be both phases coexisting in the primeval plasma. In the symmetric (high temperature) phase the processes with $\Delta(B + L) \neq 0$ are fast and the asymmetry is washed out. In the broken symmetry (low temperature) phase everything is conserved and the asymmetry remains constant. Thus baryogenesis could proceed only in the boundary region between the phases.

We see that the standard model in principle has all the necessary properties for creation of the baryon asymmetry. However, now a consensus is reached that in the minimal standard model the effect is by far too small to create the observed asymmetry, because CP-violation is extremely weak. There are chances for success in extended models with several Higgs fields like e.g. low energy SUSY. Still the asymmetry could be washed out even in the broken phase because the preexponential factor can be quite large.\[4\]

However, all of the above may be wrong by the following reason. Processes of quark and lepton transformation with a nonzero change of baryonic (and leptonic) charge at high temperatures are accompanied by a change in the structure of the gauge and Higgs fields. Roughly speaking the classical field configuration should be present in the course of the transition, the so called sphaleron:\[4\]:

\[
\begin{align*}
A^{\text{sph}}_k &= \frac{i\epsilon_{klm}x^l x^m}{r^2} f_A(\xi), \\
\phi^{\text{sph}} &= \frac{i\eta}{\sqrt{2}} \frac{r^i x_i}{r} (0, 1) f_\phi(\xi),
\end{align*}
\]

where $\xi = g\eta r$, $\eta$ is the vacuum expectation value of the Higgs field, and the functions $f$ have the properties $f(0) = 1$ and $f(\infty) = 0$. The size of this object is much larger than its Compton wave length, that is why it is called a classical field configuration. It is assumed that sphalerons are in thermal equilibrium, so that their number density is determined by the Boltzmann exponent, $\exp(-F/T)$, where $F$ is its free energy. In the broken symmetry phase $F = O(\text{TeV})$, while in the symmetric phase $F \sim T$. If this is true, the processes with baryon nonconservation are not suppressed at high temperature. How-

\[16\]
ever the rate of production of classical field states in the collision of elementary particles is not known and we cannot say if they are in equilibrium or not. An analogy with magnetic monopoles in non-Abelian gauge theories\textsuperscript{43,44} (which are also classical states), shows that production of such states in two-body or few-body collision is exponentially suppressed. Nothing is known about the probability of production of monopoles or sphalerons in, say, a hundred-particle collision. Presumably to create a pair of monopole-antimonopole or a sphaleron, one needs to create a special coherent field configuration which is almost improbable in the primeval plasma. If this is true then electroweak processes do not produce or destroy baryons in significant amount. No analytical way to solve this problem is known at the present stage. The processes, which should be considered, are essentially non perturbative and multi-particle ones. The only available approach to the calculation of the sphaleron transition rates is a numerical lattice simulation. Unfortunately the results by different groups are conflicting and vary from zero\textsuperscript{45,46} to the non suppressed one\textsuperscript{47}, $\Gamma \sim (\alpha_w T)^4$.

### 7.2. Baryonic charge condensate\textsuperscript{48}

Supersymmetric theories opened new possibilities for baryogenesis, first, because in many of them baryonic charge is not conserved even at the energies below GUT scale and, second, because there exist scalar fields with nonzero baryonic charge: superpartners of quarks (they are denoted $\chi$ in what follows). The potential for these fields may have the so called flat directions along which the field can evolve without changing energy or, in other words, the field in this direction is massless. A massless scalar field is known to be infrared unstable in the de Sitter background\textsuperscript{49} and its quantum fluctuations rise as $\langle \phi^2 \rangle = H^4 t/(2\pi)^2$ where $H$ is the Hubble parameter describing the universe expansion, $a(t) \sim \exp(HT)$. For nonzero mass, but in the case that $H \gg m$, the rise of the field is stopped when its potential energy becomes equal to the kinetic one, $U(\phi) \sim H^4$. The calculations\textsuperscript{50,51} give $\langle \phi^2 \rangle = 3H^4/8\pi^2 m^2$. The wave lengths of quantum fluctuations are exponentially stretched up together with the expansion and during inflationary stage "classical" condensates of light scalar fields can be developed. This condensate may store baryonic charge (if the field, like e.g. $\chi$, possesses it) and when inflation is over, the decay of $\chi$ would produce a nonzero baryon asymmetry. The picture is slightly more complicated by the following reasons. First, the field $\chi$ should not possess any conserved quantum numbers. The current conservation condition

$$D_\mu j^\mu = \partial_\mu j^\mu + 3H j_0 = 0$$ (26)
results in vanishing of any conserved current density as $j_0 \sim \exp(-3Ht)$. So the interesting candidates for baryogenesis are the fields which are electrically neutral and colorless. Second, the contribution of $\chi$ into baryonic current is given by the expression:

$$j_\mu^{(\chi)} = i \left[ \chi^* \partial_\mu \chi - (\partial_\mu \chi^*) \chi \right] = -2|\chi|^2 \partial_\mu \theta,$$

where $\theta$ is the phase of the field $\chi$. The baryonic charge density can be visualized as the angular momentum of the mechanical rotation of a point-like body in the two-dimensional plane $(\text{Re}\chi, \text{Im}\chi)$ in the potential field $U(\chi)$. If this potential is spherically symmetric, i.e. $U = U(|\chi|)$, then the angular momentum or, in other words, baryonic charge is conserved. A potential which does not conserve $B$ and has flat directions for $m = 0$ and $|\lambda_2| = \lambda_1$ can e.g. be written as

$$U(\chi) = m^2|\chi|^2 + \frac{1}{2} \lambda_1 |\chi|^4 + \frac{1}{4} \lambda_2 (\chi^4 + \chi^4).$$

(28)

In this toy model the field $\chi$ would evolve as follows. During inflationary stage it would travel along the valley far from the origin. When inflation is over the field would relax down to $\chi = 0$. If the phase of the field tended to a constant value during inflation, so that $\dot{\theta} = 0$ and, moreover, the field itself was exactly on the bottom of the valley, then it would come down to origin without rotation i.e. with zero baryonic charge. No asymmetry would be generated in this case. However because of quantum fluctuations during inflation (or, what is the same, by production of quanta of the field $\chi$ by the background curved space-time) there should be some motion and displacement orthogonal to the direction of the valley. The energies of such motion and displacement are $|\chi|^2 (\partial_\mu \theta)^2 \sim \lambda |\chi|^4 \sim H^4$. These angular excitations would give rise to a rotation when $|\chi|$ evolves down to zero to the region where the potential is approximately spherically symmetric. The subsequent decay of $\chi$ into fermions may produce quite a large baryon asymmetry of the universe even if the decay goes with conservation of baryonic charge. The asymmetry may be close to one. While in other scenarios the asymmetry is generically smaller than the observed value and attempts should be taken to get it as large as possible, here it is another way around: the asymmetry is too large and one should invent a way to make it smaller. A strong suppression of the angular motion and respectively of the asymmetry can be achieved through particle production by the oscillation of $\chi$ in the direction orthogonal to the valley.\footnote{13}

An unusual feature of this model is that baryon asymmetry can be generated without explicit C and CP breaking in the Lagrangian. The sign of the asymmetry is determined by the direction of the rotation of $\chi$ and the latter is
chaotically distributed over the initial χ configuration created after inflation. It means that in such a scenario the universe as a whole is charge symmetric and there should be equally abundant domains of matter and antimatter (see below, Sec. 8). The characteristic size of these domains is about

\[ l_B \approx H^{-1}_I \exp \left(-1/\sqrt{\lambda}\right). \]  

In another version of this model an explicit CP-violation has been introduced, so that the direction of the valley has been shifted away from the center. Due to this misplacement the field χ comes down to the origin with already a nonzero angular momentum and the asymmetry has a definite sign determined by the direction of the valley.

7.3. Spontaneous baryogenesis.

If a theory possesses a U(1)-symmetry which is either generated by baryonic charge or by a charge which is not orthogonal to the baryonic one, then spontaneous breaking of this symmetry results in nonconservation of baryonic charge of physical particles (as we see below and is well known, the total charge, i.e. the charge of particles plus vacuum is conserved). As a result a baryon asymmetry may be generated in the broken symmetry phase. This kind of phenomenon may take place in some electroweak scenarios with several Higgs fields where their relative phase plays the role of Goldstone boson which appears after a spontaneous symmetry breaking.

Generically the model of spontaneous breaking of a global symmetry is described by a scalar field φ with the potential

\[ U(\phi) = \lambda(|\phi|^2 - \eta^2)^2, \]  

where η is a constant c-number. In the lowest energy state in this potential (vacuum) the field φ is non-vanishing, \( \phi = \eta \exp(i\theta) \). A particular choice of the vacuum state among many degenerate ones, corresponding to different values of θ, results in the spontaneous symmetry breaking. The field θ(x) is called the Goldstone boson. If there is no explicit symmetry breaking but only the spontaneous one, the theory is invariant with respect to the transformation

\[ \theta(x) \to \theta(x) + \text{const} \]  

This means that the field θ is massless. In other words the curve where the potential \( U(\phi) \) reaches its minimum, is flat and θ can evolve along this curve without changing energy. If the bottom of the potential is tilted, so that
the degeneracy in the potential energy of $\theta$ disappears, we speak about an explicit symmetry breaking (as e.g. in the axion case). In this case the $\theta$-field typically acquires a nonzero mass and becomes a pseudo-goldstone boson.

Let us consider the following toy model with the scalar field $\phi$ and two fermionic fields "quarks" $Q$ and "leptons" $L$. The theory is supposed to be invariant with respect to the "baryonic" $U(1)$-symmetry: $\phi \rightarrow \exp(i\alpha)\phi$, $Q \rightarrow \exp(i\alpha)Q$, and $L \rightarrow L$, where $\alpha$ is a constant phase. The corresponding Lagrangian has the form:

$$L = (\partial \phi)^2 - U(\phi) + i\bar{Q}\gamma_{\mu}\partial_{\mu}Q + i\bar{L}\gamma_{\mu}\partial_{\mu}L + (g\phi\bar{Q}l + \text{h.c.}).$$  \hspace{1cm} (32)

where $U(\phi)$ is given by eq. (30) and h.c. means hermitian conjugate. In the spontaneously broken phase when $\phi = \eta \exp(i\theta)$, the Lagrangian can be rewritten as:

$$L = \eta^2(\partial \theta)^2 - V(\theta) + i\bar{Q}\gamma_{\mu}\partial_{\mu}Q + i\bar{L}\gamma_{\mu}\partial_{\mu}L + \left[g\eta \exp(i\theta)\bar{Q}L + \text{h.c.}\right] + ...$$ \hspace{1cm} (33)

where the potential $V(\theta)$ describes a possible explicit symmetry breaking, which is not present in the original Lagrangian (32), and the radial degrees of freedom are supposed to be very heavy and are neglected.

Another representation of this Lagrangian may be useful, namely if we introduce the new quark field by rotation $Q \rightarrow \exp(i\theta)Q$, then we get

$$L = \eta^2(\partial \theta)^2 + \partial_{\mu}\theta j^B_{\mu} - V(\theta) + i\bar{Q}\gamma_{\mu}\partial_{\mu}Q + i\bar{L}\gamma_{\mu}\partial_{\mu}L + (g\eta\bar{Q}L + \text{h.c.}),$$ \hspace{1cm} (34)

where $j^B_{\mu} = \bar{Q}\gamma_{\mu}Q$ is the baryonic currents of quarks. In this expression the interaction of $\theta$ with matter fields enters only linearly. It is imperative that the current $j^B_{\mu}$ is not conserved, otherwise the interaction term

$$L_{\text{int}} = \partial_{\mu}\theta j^B_{\mu}$$ \hspace{1cm} (35)

can be integrated away. This current is indeed nonconserved. Combining the equations of motion for $Q$ and $L$ one sees that $\partial_{\mu}j^B_{\mu} = i\eta(\bar{Q}L - QL)$.

For the case of a homogeneous, only time-dependent field $\theta$, the expression (33) can be written as $\dot{\theta}n_B$ where $n_B$ is the density of baryonic charge. Therefore one is tempted to identify $\dot{\theta}$ with the baryonic chemical potential $\dot{\theta}$.

If this were so the baryonic charge density would be nonzero even in thermal equilibrium, when the reaction rates are fast, while $\theta$ is not relaxed down to the dynamical equilibrium point at the minimum of the potential, where $\dot{\theta} = 0$. The charge density for small $\dot{\theta}$ would be equal to

$$n_B = B_Q\dot{\theta}T^2/6,$$ \hspace{1cm} (36)
where $B_Q$ is the baryonic charge of the quarks $Q$. It is not the case, however, as can be seen immediately from the equation of motion for the $\theta$-field:

$$2\eta^2 \dddot{\theta} = -\partial_\mu J^B_\mu \tag{37}$$

In fact this equation is just the law of the total current conservation, $\partial_\mu J^{tot}_\mu = 0$, where $J^{tot}_\mu$ is the total baryonic current including the contribution from the scalar field $\phi$. Though the symmetry is spontaneously broken the theory still "remembers" that it was symmetric. In the case of space-point independent $\theta = \theta(t)$ the equation (37) is reduced to $2\eta^2 \ddot{\theta} = -\dot{n}_B$. It can be easily integrated giving:

$$\Delta n_B = -\eta^2 \Delta \dot{\theta} \tag{38}$$

which is evidently incompatible with eq. (36). One should definitely trust eq. (38) because this is simply the condition of total current conservation which is not disturbed by thermal corrections. Below we will discuss in some detail why $\dot{\theta}$ cannot be interpreted as the baryonic chemical potential, and thus why eq. (38) is incorrect, but let us first consider the generation of baryon asymmetry both in the pure goldstone and pseudo-goldstone cases. We have just seen that in the goldstone case the baryonic charge density is given by eq. (38). The initial value of $\dot{\theta}$ is determined by inflation and depends on whether the symmetry was broken prior to the end of inflation or after that. We assume that the former is true, then the kinetic energy of the $\theta$-field is given by $\eta^2 (\dot{\theta})^2 \sim H^4$. So $\dot{\theta} \sim H^2 / \eta$ where the Hubble parameter during inflation, $H_I$, can be found by matching the energy of the inflaton $\rho_{\text{inf}} \sim H_I^2 m_{\text{Pl}}^2$ and the thermal energy after reheating $\rho_{\text{reh}} \sim T_{\text{reh}}^4$. Comparing these expressions we find that

$$\beta \sim \frac{n_B}{T^3} \approx \frac{\eta T_{\text{reh}}}{m_{\text{Pl}}^2}. \tag{39}$$

If the scale of the symmetry breaking, $\eta$, and the reheating temperature are not far from the Planck scale the asymmetry would be large enough to explain the observed value, $\beta \approx 3 \cdot 10^{-10}$. However a serious problem emerges in this scenario. It is known that all regular classical motions during inflation are exponentially red-shifted down to zero. The initial non-vanishing $\dot{\theta}$ came from quantum fluctuations at the inflationary stage. The characteristic size of the region with a definite sign of $\dot{\theta}$ is microscopically small, $l_B^{\text{inj}} \sim H^{-1}$, and even after the red-shift $z_{\text{reh}} + 1 = T_{\text{reh}} / 3^o K$ it remains much smaller than the size of baryonic domains now, $l_B > 10 \text{ Mpc}$.

Let us turn now to the pseudo-goldstone case when $\theta$ has a nonzero potential $V(\theta) = \Lambda^4 \cos \theta$. If $\theta$ is close to the minimum of this potential it can be approximated by the mass term, $V(\theta) \approx -1 + m^2 \eta^2 (\theta - \pi)^2 / 2$ with
$m^2 = \Lambda^4/\eta^2$. The equation of motion for $\theta$ now acquires an extra term related to the potential force:

$$\eta^2 \ddot{\theta} + 3H \dot{\theta} + V'(\theta) = \partial_\mu j_\mu^B. \quad (40)$$

We have also taken into account the Hubble friction term connected to the expansion of the universe. We assume that initially $\theta$ is away from its equilibrium value at $\theta_{eq} = \pi$. It is natural to assume that $\theta$ can be found anywhere in the interval $(0, 2\pi)$ with equal probability. During inflation when $H \gg m$ the magnitude of $\theta$ remains practically constant due to the large friction term, $3H \dot{\theta}$. The region with a constant $\theta$ is exponentially inflated, $l_B \sim l_i \exp(HT)$ and may be large enough to be bigger than the lower limit to the size of baryonic domain today. When inflation is over and the Hubble parameter falls below $m$ we can neglect the Hubble friction and the field $\theta$ starts to oscillate in accordance with the equation

$$\ddot{\theta} + m^2 \theta = -\partial_\mu j_\mu^B/\eta^2. \quad (41)$$

The oscillating $\theta$ would produce both baryons and antibaryons but with different number densities because the current $j_\mu^B$ is not conserved. To calculate the asymmetry in this case the following arguments have been used in the literature. The equation of motion for $\theta$ with the back reaction of the produced particles was assumed to be

$$\dot{\theta} + m^2 \theta + \Gamma \dot{\theta} = 0 \quad (42)$$

This equation has a solution correctly describing the decrease of the amplitude of $\theta$ due to production of particles, namely

$$\theta = \theta_i \exp(-\Gamma t/2) \cos(mt + \delta) \quad (43)$$

Comparing eqs. (41) and (42) one may conclude that

$$\partial_\mu j_\mu^B = \eta^2 \Gamma \dot{\theta}. \quad (44)$$

However this identification is not correct. It can easily be seen, that if eq. (44) was true, then the energy of the produced particles would be larger than the energy of the parent field $\theta$. This of course cannot be true. Indeed if the expression (44) was correct then the energy density of the produced baryons could be estimated as follows. The energy of each quark produced by the field oscillating with the frequency $m$ is equal to $m/2$. The total number density of the produced quarks, $n_Q + n_{\bar{Q}}$, is larger than the density of the baryonic
charge, \( n_B = n_Q - n_{\bar{Q}} \). So the energy density of the produced baryons is larger than \( mn_B \). From eq. (14) follows that \( n_B \) is linear in \( \theta \) while the energy density of the field \( \theta \) is quadratic in \( \theta \). Thus in the limit of small \( \theta \) the energy of the produced particles would be bigger than the energy of the field-creator. This is of course impossible and it proves that the identification made above is wrong. In fact the correct solution of the equation does not necessarily mean that the equation itself is correct. For example one can describe the decaying field by the equation

\[
\dot{\theta} + (m - i\Gamma/2)^2 \theta = 0.
\] (45)

This equation has the same solution (13) but does not permit to make the identification (14).

In the paper 55 we have derived in one loop approximation the equation of motion for \( \theta \) with the account of the back reaction of the produced fermions. It is a nonlocal nonlinear equation which in the limit of a small amplitude of \( \theta \) has the same solution as equations (42) and (45) but does not permit to make a wrong identification (14). The direct calculation of the particle production by the time-dependent field (43) gives the result 57

\[
n_B \sim \eta^2 \Gamma \Delta \theta (\Delta \theta)^3,
\] (46)

where \( \Gamma \) is the width of \( \theta \)-decay with nonconservation of baryonic charge and \( \Delta \theta \) is the difference between the initial and final values of \( \theta \). The asymmetry in this case can roughly be estimated as

\[
\beta = g^2 (\Delta \theta)^3 \eta^2 m T^3.
\] (47)

The size \( l_B \) in this scenario depends upon the model parameters and can be either larger than the present-day horizon or much smaller, inside our visibility.

Let us turn now to the possibility of the interpretation of \( \dot{\theta} \) as the baryonic chemical potential. It enters the Lagrangian as \( L_{\theta} = \dot{\theta} n_B \), in exactly the same way as a chemical potential should enter the Hamiltonian. However from the relation between \( \mathcal{L} \) and \( \mathcal{H} \)

\[
\mathcal{H} = \frac{\partial \mathcal{L}_{\phi}}{\partial \dot{\phi}} - \mathcal{L},
\] (48)

follows that the contribution from \( L_{\theta} \) into the Hamiltonian formally vanishes. The Hamiltonian depends upon \( n_B \) through the canonical momentum, \( P = \partial\mathcal{L}/\partial\dot{\theta} = 2\eta^2 \dot{\theta} + n_B \). So from the kinetic term in the Lagrangian, \( \eta^2 (\dot{\theta}^2) \), one gets \( \mathcal{H} = (P - n_B)^2 / 4\eta^2 \). If however the field theta is an external one (let us denote it now as capital \( \Theta \)), so that the Lagrangian does not contain its kinetic term, and \( \Theta \) only comes there as \( \Theta n_B \), then we do not have any equation of
motion for $\Theta$, it is an external "constant" variable. In this case the Hamiltonian would be $\mathcal{H} = -\Theta n_B$ and this $\Theta$ is the baryonic chemical potential. In this case for sufficiently fast reactions the baryonic charge density would be given by expression (36).

However for our dynamical field $\theta$ the equation of motion, which governs its behavior does not permit $\theta$ to be an adiabatic variable which can change slowly with respect to the reactions with $\Delta B \neq 0$. The change of baryonic charge implies the similar change in $\theta$ so equilibrium is never reached. In the pure Goldstone situation this is seen of course from the equation of motion (37). For the pseudo-goldstone case the situation is slightly more complicated but still the result is the same. Let us consider the Dirac equation for quarks in the presence of theta-field:

$$\left(i\gamma_\mu \partial_\mu - \dot{\theta}\right)Q = -g\eta L$$

We neglected here a possible mass term which is not essential. In perturbation theory one is tempted to neglect the r.h.s. of this equation because it is proportional to the small coupling constant $g$ and to study the spectrum of the Dirac equation with zero r.h.s. The dispersion relation for this equation is $E = p \pm \dot{\theta}$ where signs "+" and "-" stand respectively for quarks and antiquarks. Thus energy levels of particles and antiparticles are shifted by $2\dot{\theta}$ and in equilibrium their number densities should be different. The point is, however, that the change in the population numbers proceeds with the same speed as the change in $\theta$ or, in other words, the current nonconservation which can create a difference between $Q$ and $\bar{Q}$ is proportional to the same coupling constant $g$ which enters the equation of motion (37) and governs the behavior of $\theta(t)$ in the Goldstone case. In the pseudo-goldstone case the variation of $\theta$ can be dominated by the potential term (40). Hence it may change (oscillate) faster than just in the limit of zero potential (Goldstone limit) and one has even less ground to suppose that $\theta(t)$ is an adiabatic variable. In this case the situation is worse than in the Goldstone case because the rate of variation of baryonic charge is much slower than the variation of $\theta$ and the system is even further from equilibrium.

It may be instructive to see how different fermion/antifermion levels are populated in the presence of the theta-field in the "rotated" fermion representation, $Q \rightarrow \exp(i\theta)Q$, when the Dirac equation has the form

$$i\gamma_\mu \partial_\mu Q = -g\eta L \exp(-i\theta).$$

This equation, in the limit of $g = 0$, has the same spectrum for particles and antiparticles, $E = p$, but the levels would be differently populated because the
interaction term (in the r.h.s.) does not conserve energy. Assuming that $\theta(t)$ is a slowly varying function of time we can write $\theta(t) \approx \dot{\theta} t$. Thus in the reactions with quarks their energy is increased by $\dot{\theta}$ in comparison with the energy of the participating particles, while the energy of antiquarks would decrease by the same amount. One sees from this example that the energies of particles and antiparticles are indeed getting different but the process of differentiation is proportional to the coupling $g$. The arguments presented above, concerning the possibility of the interpretation of $\dot{\theta}$ as the baryonic chemical potential, are based on discussions with K. Freese, R. Rangarajan, and M. Srednicki.

8 Antimatter in the Universe.

The scale $l_B$ which characterizes the size of the domain with the dominance of matter is not known, neither from theory nor from observations. In fact different theoretical models give certain predictions about $l_B$ but they are strongly parameter dependent and, even worse, we do not know which model is a true one. In the standard GUT-baryogenesis the magnitude of baryon asymmetry $\beta$ is constant over all the universe so $l_B$ is either infinitely large (in an open universe) or is equal to the universe size (in a closed universe). In this case there would be no place in the world with a substantial amount of antimatter. This is not obligatory however, and many scenarios of baryogenesis do not possess this property. In particular it is possible to create a universe which is charge symmetric as a whole with domains of matter alternating with domains of antimatter. Even in this case it is not excluded of course that $l_B$, though finite, is larger than the present-day horizon. If so, one cannot distinguish observationally the two possibilities of a charge symmetric universe and a completely asymmetric one. However, if we are lucky, domains of antimatter may not be so far away and we may have a chance to see them, in particular by observation of antinuclei in cosmic rays. The present-day experimental situation and prospects for the future search of antimatter is reviewed at this School by V.Plyaskin (see also the recent paper [1]). Theoretical models predicting an abundant amount of antimatter inside our visibility region are reviewed in refs. [58], [59].

The general conditions for cosmological creation of both matter and antimatter with sufficiently small scale parameter $l_B$ are:

1. Different signs of C and CP-violation in different space points.

2. Inflationary (but moderate) blow-up of regions with different signs of charge symmetry breaking.
The first condition can be realized in models with spontaneous breaking of charge symmetry. One can see that domains with opposite signs of C(CP)-odd phase are indeed formed through this mechanism. In these domains an excess of either matter or antimatter is generated by baryogenesis, depending upon the sign of CP-odd amplitude.

These models encounter two serious problems. First, the average size of the domains is too small. If they are formed in a second order phase transition, their size at the moment of formation is determined by the so-called Ginzburg temperature and is approximately equal to $l_i = 1/(\lambda T_c)$ where $T_c$ is the critical temperature at which the phase transition takes place and $\lambda$ is the self-interaction coupling constant. In this case different domains would expand together with the universe and now their size would reach $l_0 = l_i(T_c/T_0) = 1/(\lambda T_0^2)$ where $T_0 = 2.7$K is the present day temperature of the background radiation. If the phase transition is first order then the bubbles of the broken phase are formed in the symmetric background. In this case different bubbles are not initially in contact with each other, typically the distance between them is much larger than their size, and their walls may expand faster than the universe, even as fast as the speed of light. Thus at the moment when the phase transition is completed the typical size of the bubbles may be as large as the horizon, $l_f \approx T_f \approx m_P/T_f^2$. After that they are stretched out by the factor $T_f/T_0$ due to the universe expansion. To make the present day size around (or larger than) 10 Mpc we need $T_f \sim 100$ eV. It is difficult (if possible) to arrange that without distorting successful results of the standard cosmology. Thus to make an observationally acceptable size of the matter-antimatter domains, a super-luminous cosmological expansion seems necessary. This solution was proposed in ref. where exponential (inflationary) expansion was assumed. With this expansion law it is quite easy to over-fulfill the plan and to inflate the domains beyond the present day horizon. Effectively it would mean a return to the old charge asymmetric universe without any visible antimatter. So some fine-tuning is necessary which would permit to make the domain size above 10 Mpc and below 10 Gpc. As we have already mentioned the lower bound on $l_B$ presented in ref. are much more restrictive. However they may be not applicable to some particular models (especially with isocurvature fluctuations) and there still may be some room for antimatter inside the present-day horizon.

The second cosmological problem which may arise in these models is a very high energy density and/or large inhomogeneity created by the domain walls. This can be resolved if domain walls were destroyed at a later stage by the symmetry restoration at low temperature or by some other mechanism. However there could be scenarios of baryogenesis in which domains of matter-
antimatter may be created without domain walls. The basic idea of these scenarios is that baryogenesis proceeds when the (scalar) field which creates C(CP)-breaking or stores baryonic charge is not in the dynamically equilibrium state, as it takes place e.g. in the spontaneous baryogenesis scenario or in the model of baryogenesis with SUSY baryonic charge condensate (see Sec. 7). In these cases charge asymmetry is created by asymmetric initial conditions which in turn are created by quantum fluctuations during inflationary stage. In such scenarios one does not need an explicit C or CP violation for baryogenesis. Moreover domains of matter or antimatter which could be created in these models do not have any domain walls with a high energy density so this problem is avoided.

There is quite a rich spectrum of possibilities for objects made of antimatter, which is open by different models of this kind. There may be just simple regions like our neighborhood either with matter or antimatter with sufficiently large sizes. A more curious possibility has been considered in ref. There is a mechanism has been proposed there which could create regions with relatively small sizes and a very high value of the asymmetry, \( \beta = 0.01 - 1 \). The sign of the asymmetry with almost equal probability may be positive or negative. Such regions would mostly form primordial black holes and, if so, it would be impossible to distinguish whether they are formed by baryonic matter or antimatter. But on the tail of the distribution there might be antistars or clouds of antimatter, enriched by heavier elements (because of large \( \beta \) primordial nucleosynthesis would give larger primordial abundances of heavy elements). Another exotic possibility is a quasiperiodic universe filled with alternating baryonic and anti-baryonic layers. For more detail one may address refs. These models are of course very speculative and there are neither theoretical nor experimental arguments in favor of their necessity. Still they are permitted and, as we know, everything which is not forbidden has a right to exist. Second, the picture of a charge symmetric universe is more attractive than asymmetric one. And last, but not the least, a search for antimatter will not necessarily be successful but still something interesting may be found in the way.

9 Conclusion.

We see that cosmology provides really strong arguments in favor of nonconservation of baryonic charge. Though the Minimal Standard Model (MSM) in particle physics predicts that baryons are indeed nonconserved, this model seems to be unable to produce enough baryons for an explanation of the observed asymmetry. The necessity for baryogenesis is a strong indication that
there should be a new physics beyond the Standard Model. We do not know if
this is just the low energy supersymmetric extension of MSM or baryogenesis
demands something new at higher energies. Possibly the next generation of
accelerators will be able to resolve this very important issue. A very essential
for the understanding of the dynamics of baryogenesis would be an observation
of cosmic antimatter. However such observation (or non-observation) will be
able only to confirm baryosymmetric cosmology but not to reject it.

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