Semileptonic and nonleptonic decays of the axial-vector tetraquark $T_{bb,cd}^-$

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The semileptonic and nonleptonic decays of the double-beauty axial-vector tetraquark $T_{bb,cd}^-$ to a state $T_{bc,cd}^0$ (hereafter $T_{bb}$ and $T_{bc}$, respectively) are investigated in the context of the QCD sum rule method. The final-state tetraquark $T_{bc}^0$ is treated as an axial-vector particle built of a heavy axial-vector diquark $b^T\gamma_qCq$ and light scalar antidiquark $\bar{c}C\bar{q}d$. Its spectroscopic parameters are calculated using the two-points sum rules by taking into account contributions of quark, gluon and mixed condensates up to dimension 10. We study the dominant semileptonic $T_{bc}^0 \rightarrow \bar{c}C\bar{q}d\ell\nu\ell$ and nonleptonic decays $T_{bc}^0 \rightarrow \bar{c}C\bar{q}dM$, where $M$ is one of the pseudoscalar mesons $\pi^-$, $K^-$, $D^-$ and $D_s^-$. The partial widths of these processes are computed in terms of weak form factors $G_i(q^2)$, $i = 1, 2, 3, 4$, extracted by employing the QCD three-point sum rule approach. Predictions obtained for partial widths of considered decays are used to improve accuracy of theoretical predictions for full width and lifetime of the tetraquark $T_{bb}^-$, which are important for experimental exploration of this exotic meson.

I. INTRODUCTION

The four-quark exotic mesons containing a few heavy $Q = b, c$ quarks are particles, investigation of which attracted interest of scientists more than thirty years ago.⁴ Among these particles most interesting are states composed of heavy diquarks $QQ'$ and light $\bar{q}q'$ antiquarks, because they are real candidates to stable exotic mesons. First qualitative results concerning a stability of the compounds $QQ'\bar{q}q'$ against strong decays were obtained already in Refs. ⁶,⁷. Exotic mesons $QQ'\bar{q}q'$ may decay through strong interaction to mesons $Q\bar{q}$ and $Q'\bar{q}'$ or to $Q\bar{q}'$ and $Q'\bar{q}$ if the mass of the master particle exceeds masses of final-state conventional mesons. It was demonstrated that such four-quark states would be stable provided that the ratio $m_{Q'/m_q}$ is sufficiently large. A well known particle from this range is the axial-vector tetraquark $T_{bb}^-$. Calculations carried out in the context of different models proved that this state is below the $BB^*$ threshold, and $T_{bb}^-$ is the particle stable against strong decays.

During the last decade properties of tetraquarks composed of heavy $bb$ and $bc$ diquarks were investigated in numerous articles using various methods (see, for example Refs. ⁹,¹² and references therein). Recent analysis of the heavy-light particles $QQ'\bar{q}q'$ confirmed stable nature of the tetraquark $T_{bb}^-$. In our work the axial-vector particle $T_{bb}^-$ was studied by means of the QCD sum rule method. In accordance with our result, the mass of $T_{bb}^-$ is equal to $m = (10035 \pm 260)$ MeV which is below the $B^-\bar{B}^0$ and $B^-\bar{B}^0\gamma$ thresholds. In other words, this particle is stable against the strong and radiative decays. Hence, it dissociates to conventional mesons via weak processes considered also in Ref. ¹³. We explored the semileptonic decays $T_{bb}^- \rightarrow Z_{bc}^0\ell\nu\ell$, where the final-state tetraquark $Z_{bc}^0 = [bc][\bar{q}d]$ was treated as a scalar particle. By computing partial widths of these decays, we estimated the width $\Gamma$ and mean lifetime $\tau$ of the axial-vector tetraquark $T_{bb}^-$. Predictions obtained for these parameters $\Gamma = (7.17 \pm 1.23) \times 10^{-8}$ MeV and $\tau = 9.18^{+1.99}_{-1.34}$ fs may be useful for experimental investigation of double-heavy exotic mesons. Problems connected with calculation of parameters and weak decay channels of $T_{bb}^-$ were addressed in Ref. ¹⁶, as well.

Investigations showed that not only the tetraquark $T_{bb}^-$, but also other double-beauty states may be strong and electromagnetic interactions stable particles. Thus, the scalar counterparts of $T_{bb}^-$, i.e., the tetraquark $T_{bd}^-$, the scalar and axial-vector four-quark mesons $T_{bb,cd}^-$ are stable against strong and radiative decays. The spectroscopic parameters, widths and lifetimes of these exotic mesons were computed in Refs. ¹⁷,¹⁸.

As we have mentioned above, tetraquarks containing a diquark $bc$ are also interesting object for studies, because some of them may be stable particles. Thus, the mass of the scalar exotic meson $Z_{bc}^0$ equals to $m_Z = (6660 \pm 150)$ MeV, which is below thresholds for strong and radiative decays of $Z_{bc}^0$ to conventional mesons.¹² As a result, $Z_{bc}^0$ is the strong- and electromagnetic-interaction stable compound weak decays of which were studied in Ref. ²⁰. Predictions for width and lifetime of $Z_{bc}^0$ obtained there provide valuable information on features of this particle. The spectroscopic parameters and possible strong and weak decay channels of the axial-vector tetraquark $T_{bc}^0 = [bc][\bar{q}d]$ was studied, as well.

In the present article, we extend our analysis of the tetraquark $T_{bb}^-$ by considering its new weak decay channels $T_{bb}^--\tilde{T}_{bc}^0\ell\nu\ell$ and $T_{bb}^--\tilde{T}_{bc}^0M$, where $\tilde{T}_{bc}^0$ is an axial-vector state. This investigation will allow us to improve estimates for the full width and lifetime of $T_{bb}^-$. 

\[ \text{Eq. 1} \]
We treat $\tilde{T}_{bc}^0$ as a tetraquark composed of color-antitriplet heavy axial-vector diquark $bc$ and light color-triplet scalar antidiquark $\bar{u}\bar{d}$. It is worth noting that quark contents and quantum numbers of the tetraquarks $T_{bc}^0$ and $\tilde{T}_{bc}^0$ are the same, and both of them have the antisymmetric color structure $[3_c]_{bc} \otimes [3_c]_{\bar{u}\bar{d}}$. But $T_{bc}^0$ and $\tilde{T}_{bc}^0$ differ from each another due to their internal organizations. In fact, the heavy diquark in $T_{bc}^0$ is a scalar state, whereas the tetraquark $\tilde{T}_{bc}^0$ is made of an axial-vector heavy diquark $bc$. The reason for such choice of the final-state tetraquark $\tilde{T}_{bc}$ will be explained later.

To compute partial widths of the initial $T_{bb^-}$ particle’s weak decays, apart from its mass and current coupling, one needs also spectral parameters of the tetraquark $T_{bc}^0$. The mass $\tilde{m}$ and coupling $\tilde{J}$ of the state $T_{bc}^0$ are calculated using the QCD sum rule method [22, 23], which is a powerful tool to calculate parameters of conventional mesons and baryons. It can be successfully applied to analyze multiquark hadrons, as well [24, 25]. We calculate the mass and current coupling of the tetraquark $T_{bc}^0$ using relevant interpolating current by taking into account various quark, gluon, and mixed condensates up to dimension 10. Spectral parameters of $T_{bc}^0$ extracted from such analysis are also of particular interest to explore the family of tetraquarks $bc\bar{u}\bar{d}$.

There are different weak decays of $T_{bb^-}$, but dominant ones are processes triggered by a subprocess $b \rightarrow W^- c$ responsible for transformation of $T_{bb^-}$ to the final axial-vector state $\tilde{T}_{bc}^0$. In semileptonic decays $T_{bb^-} \rightarrow \tilde{T}_{bc}^0\nu$, the tetraquark $\tilde{T}_{bc}^0$ is accompanied by a lepton (\tilde{e}), whereas in nonleptonic processes $T_{bb^-} \rightarrow \tilde{T}_{bc}^0 c \bar{b}$ there is an additional ordinary meson $M$ in the final phase of the process. We consider decays in which $M$ is one of the conventional pseudoscalar mesons $\pi^-$, $K^-$, $D^-$ and $D_s^-$. To evaluate partial widths of weak decays one has to determine form factors $G_i(q^2)$, $i = 1, 2, 3, 4$ which govern weak transitions: They enter to differential rate $d\Gamma/dq^2$ of semileptonic and partial width of nonleptonic processes. To this end, we employ the QCD three-point sum rule approach, and extract $G_i(q^2)$ at $q^2$ accessible for sum rule calculations. As usual, these $q^2$ do not cover a full region $m^2 \leq q^2 \leq (m - \tilde{m})^2$ necessary to integrate the differential rates $d\Gamma/dq^2$ of semileptonic decays. Therefore, one has to introduce model functions $G_i(q^2)$ that coincide with the sum rule predictions when they are accessible, and can be easily extrapolated to all $q^2$. Usage of $G_i(q^2)$ in calculations solves these technical problems.

This article is structured in the following way: In Sec. VII we evaluate the mass and current coupling of the tetraquark $T_{bc}^0$ by employing the QCD two-point sum rule method. Calculations of the weak form factors $G_i(q^2)$ in the framework of the three-point sum rule approach are performed in section VIII. Here, we determine the model functions $G_i(q^2)$ and also find partial widths of the semileptonic decays $T_{bb^-} \rightarrow \tilde{T}_{bc}^0\nu$. Section IX is devoted to analysis of the nonleptonic weak transformations of the tetraquark $T_{bc}^0$. In section X we calculate the full width and lifetime of $T_{bc}^0$, and discuss obtained results. This section contains also our concluding notes.

II. MASS AND CURRENT COUPLING OF THE AXIAL-VECTOR TETRAQUARK $\tilde{T}_{bc}^0$

The mass $\tilde{m}$, and coupling $\tilde{J}$ of the tetraquark $\tilde{T}_{bc}^0$ are important parameters of the problem under consideration: they are required to find partial widths of the weak processes $T_{bb^-} \rightarrow \tilde{T}_{bc}^0\nu$ and $T_{bb^-} \rightarrow \tilde{T}_{bc}^0 M$. Besides, the axial-vector tetraquark $\tilde{T}_{bc}^0$, as its partner state $T_{bc}^0$, may be strong- and/or electromagnetic-interaction stable particle, which is interesting in itself.

The sum rules to extract spectroscopic parameters of $\tilde{T}_{bc}^0$ can be derived from analysis of the two-point correlation function $\Pi_{\mu\nu}(p)$ given by the expression

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0 | T\{\tilde{J}_\mu(x)\tilde{J}_\nu(0)\} | 0 \rangle,$$

where $\tilde{J}_\mu(x)$ is the interpolating current to the axial-vector tetraquark $\tilde{T}_{bc}^0$. The structure of this current is determined, in some respects, by the organization of the initial particle $T_{bb^-}$. It is instructive to consider the structure and interpolating current $J_\mu(x)$ of the tetraquark $T_{bb^-}$

$$J_\mu(x) = b_\mu^a(x)\gamma_\mu Cb_b(x)\sigma_{a\beta}(x)C\gamma_5 d^T_{\beta}(x),$$

where $a$ and $b$ are color indices and $C$ is charge-conjugation operator. The current $J_\mu(x)$ was used in Ref. 13 to study the exotic mesons $T_{bb^-}$. As seen, $T_{bb^-}$ is built of the axial-vector diquark $b^T \gamma_5 Cb$ and light scalar antidiquark $\bar{u}\bar{d}C\gamma_5 T$. It is also clear that these diquarks are symmetric in color indices and $J_\mu$ belongs to $[6]_{bb} \otimes [\bar{6}]_{\bar{u}\bar{d}}$ representation of the color group $SU_c(3)$. In fact, the diquark field in Eq. (2) is symmetric under exchange of quarks flavors and color indices. The antidiquark field in a full color-symmetric form is given by the expression $\bar{u}_{\mu}C\gamma_5 \bar{d}_{\mu}^T + \bar{u}_{\mu}C\gamma_5 \bar{d}_{\mu}^T$. But, because these two fields generate equal currents, we keep in $J_\mu$ one of them.

Weak decays of $T_{bb^-}$ run through transition of $b$-quark $b \rightarrow c$, therefore expected organization of the final diquark field is $b^T \gamma_5 Cc_b$, whereas the antidiquark field preserves its quark content and scalar nature. Then, at the final state we get the axial-vector tetraquark, which is described by the current

$$\tilde{J}_\mu(x) = b_\mu^a(x)\gamma_\mu Cc_b(x)\left[\bar{u}_{\mu}(x)C\gamma_5 \bar{d}_{\mu}^T(x) - \bar{u}_{\mu}(x)\gamma_\mu C\gamma_5 \bar{d}_{\mu}^T(x) \right].$$

The current $\tilde{J}_\mu$ is antisymmetric in color indices and has color-triplet structure $[3_c]_{bc} \otimes [3_c]_{\bar{u}\bar{d}}$. It is known that scalar and axial vector triplet diquarks are most favorable two-quark states to construct tetraquarks with
\[ J^P = 1^+ \quad \text{[26]} \] The current \( \bar{J}_\mu \) corresponds to lower lying tetraquark with structure \( \gamma_a C \otimes C \gamma_b \) and spin-parity \( J^P = 1^+ \). Of course, there is an alternative choice for \( \bar{J}_\mu \) composed of a heavy scalar diquark and an axial-vector light antidiquark. Properties of such state \( T_{bc}^0 \), with a composition \( C \gamma_5 \otimes \gamma_5 C \), its weak and strong decays were investigated in Ref. [21]. To model \( T_{bc}^0 \) we choose \( \bar{J}_\mu \) given by Eq. (3) as a current stemming naturally from organization of the master particle \( T_{bc}^0 \).

To find the sum rules for the mass \( \tilde{m} \) and coupling \( \tilde{f} \) of the tetraquark \( T_{bc}^0 \), we write down the correlation function \( \Pi_{\mu \nu}^{\text{phys}}(p) \) using physical parameters of \( T_{bc}^0 \). We treat \( T_{bc}^0 \) as a ground-state particle, and separate its contribution to \( \Pi_{\mu \nu}^{\text{phys}}(p) \) from other terms

\[
\Pi_{\mu \nu}^{\text{phys}}(p) = \frac{\langle 0 | \bar{J}_\mu | T_{bc}^0(p) \rangle \langle T_{bc}^0(p) | \bar{J}_\nu | 0 \rangle}{\bar{m}^2 - p^2} + \cdots . \tag{4}
\]

Effects of higher resonances and continuum states are denoted in \( \Pi_{\mu \nu}^{\text{phys}}(p) \) by dots. The expression (14) is derived by saturating the correlation function with a complete set of \( J^P = 1^+ \) states with required quark content and performing integration in \( \Pi_{\mu \nu}(p) \) over \( x \).

The correlator \( \Pi_{\mu \nu}^{\text{phys}}(p) \) can be simplified by introducing the matrix element \( \langle 0 | \bar{J}_\mu | T_{bc}^0(p) \rangle \)

\[
\langle 0 | \bar{J}_\mu | T_{bc}^0(p) \rangle = \bar{m} f \epsilon_\mu , \tag{5}
\]

where \( \epsilon_\mu \) is the polarization vector of the tetraquark \( T_{bc}^0 \).

In terms of the mass \( \tilde{m} \) and coupling \( \tilde{f} \) the function \( \Pi_{\mu \nu}^{\text{phys}}(p) \) takes the form

\[
\Pi_{\mu \nu}^{\text{phys}}(p) = \frac{\bar{m}^2 \tilde{f}^2}{\bar{m}^2 - p^2} \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{\bar{m}^2} \right) + \cdots . \tag{6}
\]

The sum rules require computation of \( \Pi_{\mu \nu}(p) \) in terms of quark propagators, as well. To this end, one needs to substitute \( \bar{J}_\mu(x) \) into the correlation function (11) and contract relevant light and heavy quark fields. These manipulations yields

\[
P_{\mu \nu}^{\text{OPE}}(p) = i \int d^4 x e^{ipx} \text{Tr} \left[ \gamma_\mu \gamma_b^{a'}(x) \gamma_\nu S_{bb'}(x) \right] \times \left\{ \frac{1}{2} \left[ \text{Tr} \left[ \gamma_5 \gamma_5^{b'}(0) \gamma_5 S_{ab'}(0) \right] \right] \right. \\
\left. - \frac{1}{2} \left[ \text{Tr} \left[ \gamma_5 \gamma_5^{b'}(0) \gamma_5 S_{ab}(0) \right] \right] \right. \\
\left. + \frac{1}{2} \left[ \text{Tr} \left[ \gamma_5 \gamma_5^{b'}(0) \gamma_5 S_{ba}(0) \right] \right] \right. \\
\left. - \frac{1}{2} \left[ \text{Tr} \left[ \gamma_5 \gamma_5^{b'}(0) \gamma_5 S_{ab'}(0) \right] \right] \right\} , \tag{7}
\]

where \( S_{ab}(x) \) and \( S_{ab}(x) \) are the heavy \( Q = b(c) \) and light \( q = d(u) \) quark propagators, respectively. Here, we have introduced also the notation

\[
\bar{S}_{Q(q)}(x) = CS_{Q(q)} T(x) C . \tag{8}
\]

In the present work, we use the light quark propagator given by the expression [27]

\[
S_{q(\bar{q})}(x) = i \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \left( \frac{\delta_{ab}}{k^2 - m_q^2} \frac{\langle \bar{q} q \rangle}{k^2 - m_Q^2} \right) \right. \\
\left. + \frac{g_s^2 G^2}{12 \delta_{ab} m_Q} \frac{k^2 + m_q \bar{k}}{(k^2 - m_Q^2)^2} + \frac{g_3^2 Q^2}{48 \delta_{ab} (k^2 - m_Q^2)} \right. \\
\left. \left[ \bar{k} (k^2 - 3m_Q^2) + 2m_Q (2k^2 - m_Q^2) \right] \right) \frac{\langle \bar{q} q \rangle}{k^2 - m_Q^2} + \cdots . \tag{9}
\]

The propagator of the heavy quarks \( Q \) is determined by the formula

\[
S_{Q(\bar{Q})}(x) = i \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \left( \frac{\delta_{ab}}{k^2 - m_q^2} \frac{\langle \bar{q} q \rangle}{k^2 - m_Q^2} \right) \right. \\
\left. + \frac{g_s^2 G^2}{12 \delta_{ab} m_Q} \frac{k^2 + m_q \bar{k}}{(k^2 - m_Q^2)^2} + \frac{g_3^2 Q^2}{48 \delta_{ab} (k^2 - m_Q^2)} \right. \\
\left. \left[ \bar{k} (k^2 - 3m_Q^2) + 2m_Q (2k^2 - m_Q^2) \right] \right) \frac{\langle \bar{q} q \rangle}{k^2 - m_Q^2} + \cdots . \tag{10}
\]

In expressions [9] and [10]

\[
G_{ab} = G_{ab}^A t_A^a , \\
G^2 = G_{ab}^A G^A_{ab} , \\
G^3 = f^{ABC} G_{ab}^A G_{bc}^B G_{cd}^C , \tag{11}
\]

where \( a, b = 1, 2, 3 \) are color indices and \( A, B, C = 1, 2 \cdots 8 \). Here \( t^A = \lambda^A/2 \), where \( \lambda^A \) are the Gell-Mann matrices. The gluon field strength tensor is fixed at \( x = 0 \), i.e., \( G_{ab}^A = G_{ab}^A(0) \).

To proceed one should choose invariant amplitudes corresponding to the same Lorentz structures from both \( \Pi_{\mu \nu}^{\text{phys}}(p) \) and \( \Pi_{\mu \nu}^{\text{OPE}}(p) \). There are two Lorentz structures proportional to \( g_{\mu\nu} \) and \( p_\mu p_\nu \) in these correlation functions. Because invariant amplitudes \( \Pi_{\mu \nu}^{\text{phys}}(p^2) \) and \( \Pi_{\mu \nu}^{\text{OPE}}(p^2) \) corresponding to terms \( g_{\mu\nu} \) do not contain contributions of scalar particles, we work with these functions. The sum rules for \( \bar{m} \) and \( \tilde{f} \) can be obtained by equating these invariant amplitudes and performing standard prescriptions of the sum rule method. As the first step, one applies the Borel transformation to both sides of obtained equality, which is necessary to suppress contributions due to higher resonances and continuum states. Afterwards, these contributions should be subtracted from the extracted side of this equality by employing the hypothesis on quark-hadron duality. After these manipulations, a final expression becomes a function of the Borel \( M^2 \) and continuum threshold \( s_0 \) parameters. The second expression required to determine sum rules for \( \bar{m} \) and \( \tilde{f} \) can be obtained from the first equality by acting on it by the operator \( d/d(-1/M^2) \). As a result, for
Spectroscopic parameters of these tetraquarks have been deduced out computations. The vacuum condensates and masses of the auxiliary parameters which have to be specified in order to carry out Π(M², s₀) is the Borel transformed and continuum subtracted invariant amplitude Π^{OPE}(p²), and Π(2M², s₀) = d/d(−1/M²)Π(M², s₀).

The function Π(M², s₀) has the following form

\[ Π(M², s₀) = \int_{M²} dsρ^{OPE}(s)e^{−s/M²} + Π(M²), \]  

(14)

where \( M = m_b + m_c \). Here, \( ρ^{OPE}(s) \) is the two-point spectral density computed as an imaginary part of the correlation function. The second term in Eq. (14) contains nonperturbative contributions computed directly from \( Π^{OPE}(p) \). In the present work, we calculate Π(M², s₀) by taking into account nonperturbative terms up to dimension 10. The explicit expression of the function Π(M², s₀) is rather lengthy, therefore we do not provide it here.

The obtained sum rules contain numerous input parameters, which have to be specified in order to carry out computations. The vacuum condensates and masses of b, and c quarks are universal parameters and do not depend on the problem under analysis: Their values are listed below

\[ \langle \overline{q}q \rangle = −(0.24 ± 0.01)^3 \text{ GeV}^3, \]
\[ \langle \overline{gg}_s\sigma Gq \rangle = m_0^2\langle \overline{q}q \rangle, \quad m_0^2 = (0.8 ± 0.1) \text{ GeV}^2, \]
\[ \frac{α_sG^2}{π} = (0.012 ± 0.004) \text{ GeV}^4, \]
\[ G_s^2G^2 = (0.57 ± 0.29) \text{ GeV}^6, \]
\[ m_c = 1.27 ± 0.2 \text{ GeV}, \quad m_b = 4.18^{+0.03}_{−0.02} \text{ GeV}. \]  

(15)

The mass and coupling of the tetraquark \( T_{bc}^0 \) depend on the auxiliary parameters \( M² \) and \( s₀ \), and their correct choice is one of important problems of our studies. We fix the upper allowed value of the Borel parameter from a restriction \( PC > 0.2 \), where \( PC \) is a pole contribution to the sum rules. The lower bound is found from convergence of the sum rules. Additionally, quantities extracted from Eqs. (12) and (13) should be as stable as possible against variations of \( M² \). The continuum threshold parameter \( s₀ \) divides the ground-state contribution and effects of higher resonances and continuum states. Therefore, \( s₀ \) should be below the first excited state of the tetraquark \( T_{bc}^0 \), and obey \( √s₀ − \tilde{m} ≈ 600 \text{ MeV} \), which can be considered as a reasonable restriction for heavy tetraquarks.

Performed numerical analyses demonstrate that regions

\[ M² ∈ [5.5, 7] \text{ GeV}², \quad s₀ ∈ [58, 60] \text{ GeV}² \]  

(16)

satisfy all aforementioned constraints on \( M² \) and \( s₀ \). Indeed, at \( M² = 7 \text{ GeV}² \) the pole contribution is 79%, whereas at \( M² = 5.5 \text{ GeV}² \) amounts to 37% of the whole result. These values of \( M² \) fix the boundaries of the region in which the Borel parameter can be varied. At the minimum of \( M² = 5.5 \text{ GeV}² \) contributions of last three terms to \( Π(2M², s₀) \) do not exceed 1% of its value.

For \( \tilde{m} \) and \( \tilde{f} \) we find

\[ \tilde{m} = (7050 ± 125) \text{ MeV}, \]
\[ \tilde{f} = (8.3 ± 1.3) × 10^{-3} \text{ GeV}^4. \]  

(17)

In Fig. 1 we plot the sum rule’s prediction for \( \tilde{m} \), where its dependence on the Borel \( M² \) and continuum threshold parameter \( s₀ \) is seen explicitly. Theoretical errors in the case of \( \tilde{m} \) amount to ±1.8%, which confirms a nice accuracy of performed computations. The ambiguities in deriving of the coupling \( \tilde{f} \) are equal to ±16% of the central value: they are larger than that for \( \tilde{m} \), but still within limits accepted in sum rule computations. Reasons behind of these effects are quite clear. Indeed, the sum rule for the mass \( \tilde{m} \) is given by the ratio (12) which smooths the dependence of \( \tilde{m} \) on the parameter \( M² \), whereas the sum rule for \( \tilde{f} \) (13) contains only the correlator Π(M², s₀).

It is interesting to compare the result obtained for the mass of the axial-vector tetraquark \( T_{bc}^0 \) with the mass of \( T_{bc}^0 \). Let us remind that the latter has the same quark content and quantum numbers, but is composed of the heavy scalar diquark \( b^TCγ_5c \) and light axial-vector antidiquark \( πCγ_µ\gamma^µ \). This particle has the mass \( (7105 ± 155) \text{ MeV} \) and is \( Δm ≈ 50 \text{ MeV} \) “heavier” than \( T_{bc}^0 \). Our estimate for the mass splitting between \( T_{bc}^0 \) and \( T_{bc}^0 \) is obtained using central values of their masses. Spectroscopic parameters of these tetraquarks have been computed in the context of the QCD sum rule method, predictions of which contain theoretical errors. It is evident that the mass difference \( Δm \) is smaller than uncertainties of this analysis and does not allow one to distinguish these particles from each other reliably. Real exotic mesons may be superpositions of these tetraquarks.
III. SEMILEPTONIC DECAY $T_{bb}^− \rightarrow \overline{T}_{bc}^0 l \ell_i$

Weak decays of $T_{bb}^−$ can be triggered by subprocesses $b \rightarrow W^- c$ and $b \rightarrow W^- u$. The processes generated by the transition $b \rightarrow W^- c$ are dominant decay modes of $T_{bb}^−$. The reason is that the subprocess $b \rightarrow W^- u$ leads to decays suppressed relative to dominant ones by a factor $|V_{bu}|^2 / |V_{bc}|^2 \approx 0.01$, where $V_{qi,q2}$ are the Cabibbo-Kobayasi-Maskawa (CKM) matrix elements. In the present work, we consider only dominant weak decays of $T_{bb}^−$, which consist of the semileptonic $T_{bb}^− \rightarrow \overline{T}_{bc}^0 l\ell_i$ and nonleptonic $T_{bb}^− \rightarrow \overline{T}_{bc}^0 M$ processes.

In this section, we concentrate on semileptonic decays $T_{bb}^− \rightarrow \overline{T}_{bc}^0 l \ell_i$ of the tetraquark $T_{bb}^−$. The analysis carried out in the section II has allowed us to calculate the spectroscopic parameters of the tetraquark $T_{bc}^0$, which are input information to investigate decays of the initial particle $T_{bb}^−$. A big mass gap between the initial and final-state tetraquarks makes kinematically possible semileptonoc decays with all lepton spieces $l = e, \mu$, and $\tau$.

The effective Hamiltonian to study processes $b \rightarrow W^- c$ at the tree-level is determined by the expression

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{bc} \overline{\tau} \gamma_\alpha (1 - \gamma_5) b \gamma^\alpha (1 - \gamma_5) \nu_l,$$

where $G_F$ and $V_{bc}$ are the Fermi coupling constant and CKM matrix element, respectively. The matrix element of $\mathcal{H}_{\text{eff}}$ placed between the initial and final tetraquarks contains the leptonic and hadronic factors

$$\langle \overline{T}_{bc}^0 (p') \rangle \mathcal{H}_{\text{eff}} \langle T_{bb}^− (p) \rangle = L^\alpha H_\alpha.$$

We are interested in calculation of $H_\alpha$, because the leptonic part of the matrix element $L^\alpha$ is universal for all semileptonic decays and does not contain information on tetraquarks. Then, $H_\alpha$ is nothing more than the matrix element of the current

$$J^\text{tr}_{\alpha} = \overline{\tau} \gamma_\alpha (1 - \gamma_5) b,$$

sandwiched between the initial and final particles. It can be modeled in terms of the form factors $G_i(q^2)$ which parametrize the long-distance dynamics of the weak transition

$$\langle \overline{T}_{bc}^0 (p', \epsilon' \ell') | J^\text{tr}_{\alpha} | T_{bb}^− (p, \epsilon) \rangle = \epsilon^{\mu} \epsilon'^{\nu} \left[G_1(q^2) g_{\mu\nu} P_\alpha + G_2(q^2) (q_\mu g_{\alpha\nu} - q_\nu g_{\alpha\mu}) - \frac{G_3(q^2)}{2 m^2} q_\mu q_\nu P_\alpha \right] + G_4(q^2) \epsilon_{\alpha\mu\nu\rho} \epsilon^{\mu} \epsilon^{\rho} P_\nu,$$

where $(p, \epsilon)$ and $(p', \epsilon')$ are momenta and polarization vectors of $T_{bb}^−$ and $\overline{T}_{bc}^0$, respectively. Here, we also use $P = p + p'$ and $q = p - p'$. The momentum transfer in the weak process $q^2$ changes within the limits $m_l^2 \leq q^2 \leq (m - \bar{m})^2$, where $m_l$ is the mass of the lepton $l$.

The weak form factors $G_i(q^2)$ are key ingredients of our investigations. They should be determined from the QCD three-point sum rules, which can be derived using the correlation function

$$\Pi_{\mu\nu}(p, p') = i^2 \int d^4 x d^4 y e^{i(p'y - px)} \langle 0 | T \{ \overline{J}_\nu (y) J_{\alpha\mu}^+(0) \} J^\mu_\rho (x) \} | 0 \rangle.$$

The standard methods of the sum rule analysis require calculation of the correlation function $\Pi_{\mu\nu}(p, p')$ using the physical parameters of the tetraquarks and, by this way, to find the physical side of the sum rules. At the next phase of studies, one has to determine $\Pi_{\mu\nu}(p, p')$ by employing the quark propagators, and express $\Pi_{\mu\nu}^{\text{OPE}}(p, p')$ in terms of quark, gluon and mixed vacuum condensates. By equating obtained results and using the assumption about the quark-hadron duality,

\[ ... \]
it is possible to derive sum rules and compute the form factors of interest.

The physical side of the sum rules $\Pi_{\mu\nu}^{\text{phys}}(p, p')$ can be written down in the following form

$$\Pi_{\mu\nu}^{\text{phys}}(p, p') = \frac{\langle 0 | \bar{T}_\mu^{(0)}(p', \epsilon') \gamma_\mu T_{bb}(p, \epsilon) | 0 \rangle}{(p^2 - m^2)(p'^2 - m^2)},$$

where the contribution of the ground-state particles is shown explicitly, whereas other terms are denoted by dots.

Calculation of $\Pi_{\mu\nu}^{\text{phys}}(p, p')$ can be finished by taking into account Eq. (5), the explicit expression of the matrix element $(T_{bc}^{(0)}(p', \epsilon') | J_{\mu}^\nu | T_{bb}(p, \epsilon))$, and the formula

$$\langle T_{bb}^-(p, \epsilon) | J_{\mu}^\nu | 0 \rangle = f m e_{\mu \nu}^\ast,$$

where $f$ is the coupling of the state $T_{bb}^-$. Having substituted the relevant matrix elements into Eq. (23), we find the final expression for $\Pi_{\mu\nu}^{\text{phys}}(p, p', q^2)$

$$\Pi_{\mu\nu}^{\text{phys}}(p, p') = \frac{f m f m}{(p^2 - m^2)(p'^2 - m^2)} \left\{ G_1(q^2) p_\alpha g_{\mu\nu} \right. + G_2(q^2) \left[ 1 - \frac{m^2 - \tilde{m}^2}{2m^2} \right] p_\mu g_{\rho\nu} - \frac{G_3(q^2)}{2m^2} p_\alpha p_\rho p_\mu \left. + G_4(q^2) e_{\mu\nu\rho\sigma} p_\rho \right\} + \cdots.$$  

The dots in $\Pi_{\mu\nu}^{\text{phys}}(p, p')$ stand for not only effects due to excited and continuum states, but also for contributions of structures which will not be used in following analysis.

The QCD side of the sum rules can be derived from Eq. (22) by using the interpolating currents and contracting relevant quark fields. The result of these computations is given by the following formula

$$\Pi_{\mu\nu}^{\text{QCD}}(p, p') = i^2 \int d^4 x d^4 y e^{i(p'y - px)} \left\{ \text{Tr} \left[ \gamma_\delta \bar{S}_d^b (x-y) \right] \right. - \text{Tr} \left[ \gamma_\delta \bar{S}_d^{b\prime} (x-y) \gamma_\gamma \gamma_\gamma \bar{S}_u^{a\prime} (x-y) \right] \left. \times \left[ \text{Tr} \left[ \gamma_\mu \bar{S}_b^{a\prime} (y-x) \gamma_\mu \gamma_\gamma \bar{S}_u^{b\prime} (y-x) \right] + \text{Tr} \left[ \gamma_\mu \bar{S}_b^{a\prime} (y-x) \gamma_\gamma \gamma_\gamma \bar{S}_u^{b\prime} (y-x) \right] \right\},$$

The correlation function $\Pi_{\mu\nu}^{\text{QCD}}(p, p')$ contains the same Lorentz structures as its counterpart $\Pi_{\mu\nu}^{\text{phys}}(p, p')$. We use corresponding invariant amplitudes to obtain the required sum rules for the form factors $G_i(q^2)$. But before this final operation, we make double Borel transformation over variables $p^2$ and $p'^2$ to suppress contributions of the higher excited and continuum states, and perform continuum subtraction. These rather routine manipulations give the sum rules for the form factors $G_i(q^2)$. For $G_i(q^2), i = 1$ and 4 we get the similar sum rules

$$G_i(M^2, s_0, q^2) = \frac{1}{f m f m} \int_{M^2}^{s_0} \int_{M^2}^{s_0'} ds \ ds' \times \rho_i(s, s', q^2) e^{(m^2 - s)/M^2} e^{(\tilde{m}^2 - s)/M^2},$$

where $M^2_1, M^2_2$ and $s_0, s_0'$ are the Borel and continuum threshold parameters, respectively. The pair of parameters $(M^2_1, s_0)$ corresponds to a channel of the initial tetraquark $T_{bb}$, whereas $(M^2_2, s_0')$ describe the final-state particle $T_{bc}^0$. The remaining two sum rules read:

$$G_2(M^2, s_0, q^2) = \frac{2m}{f m f m} \int_{M^2}^{s_0} \int_{M^2}^{s_0'} ds \ ds' \rho_2(s, s', q^2) e^{(m^2 - s)/M^2} e^{(\tilde{m}^2 - s)/M^2},$$

and

$$G_3(M^2, s_0, q^2) = \frac{2m}{f m f m} \int_{M^2}^{s_0} \int_{M^2}^{s_0'} ds \ ds' \rho_3(s, s', q^2) e^{(m^2 - s)/M^2} e^{(\tilde{m}^2 - s)/M^2}.$$  

As is seen the sum rules are written down using the spectral densities $\rho_i(s, s', q^2)$ which are proportional to the imaginary part of the corresponding invariant amplitudes in $\Pi_{\mu\nu}^{\text{QCD}}(p, p')$. All of them contain both the perturbative and nonperturbative contributions and are calculated with dimension-5 accuracy. Explicit expressions of $\rho_i(s, s', q^2)$ are cumbersome, therefore we refrain from providing them here.

The differential rate of the semileptonic decay $T_{bb} \to T_{bc}^0 T_l$ is determined by the weak form factors $G_i(q^2)$ and is given by the expression

$$\frac{d\Gamma}{d^2 q^2} = \frac{G_2^2 |V_{cb}|^2}{3 \cdot 2^{2 \cdot 6 \cdot 3 \cdot m^2}} \left( q^2 - m^2 \right) \lambda \left( m^2, \tilde{m}^2, q^2 \right) \times \left[ \sum_{i=1}^{i=4} G_i^2(q^2) A_i(q^2) + G_1(q^2) G_2(q^2) A_{12}(q^2) + G_1(q^2) G_3(q^2) A_{13}(q^2) + G_2(q^2) G_3(q^2) A_{23}(q^2) \right],$$

where

$$\lambda \left( m^2, \tilde{m}^2, q^2 \right) = \left[ m^4 + \tilde{m}^4 + q^4 - 2(m^2 \tilde{m}^2 + m^2 q^2 + \tilde{m}^2 q^2) \right]^{1/2}. $$

The decay rate $d\Gamma/d^2 q^2$ depends also on functions $A_i(q^2)$ and $A_{ij}(q^2)$ which can be found in Ref. 28.

Sum rules for $G_i(q^2)$ are necessary to find corresponding fit functions $G_i(q^2)$ and calculate the width of the semileptonic decays. Technical details of numerical computations to extract weak form factors are well known.
The spectroscopic parameters of the tetraquark $\bar{T}_{bc}^{0}$ have been found in the present work. We need also to fix Borel and continuum threshold parameters to carry out numerical analysis. In the initial particle channel ($M_{1}^{2}, s_{0}$) are chosen as in Ref. [15], in which the mass and coupling of $T_{bb}^{-}$ were calculated

$$ M^{2} \in [9, 13] \text{ GeV}^{2}, \quad s_{0} \in [115, 120] \text{ GeV}^{2}. \quad (33) $$

For the next pair ($M_{2}^{2}, s_{0}$) we use parameters from Eq. (10).

The sum rules give reliable results for $G_{i}(q^{2})$ in the region $m_{i}^{2} \leq q^{2} \leq 7 \text{ GeV}^{2}$, which is not enough to find the partial width of the process $T_{bb}^{-} \to \bar{T}_{bc}^{0} \pi_{l}$ under consideration. To calculate the width of the semileptonic decay $d\Gamma/dq^{2}$ must be integrated over $q^{2}$ in the boundaries $m_{l}^{2} \leq q^{2} \leq (m - \bar{m})^{2}$, i.e., in the limits $m_{l}^{2} \leq q^{2} \leq 8.9 \text{ GeV}^{2}$. But this region is wider than the one where the sum rules lead to strong results. This problem can be solved by introducing extrapolating (fit) functions $G_{i}(q^{2})$. At the momentum transfers $q^{2}$ accessible for the sum rule computations they must coincide with $G_{i}(q^{2})$, but have analytic forms suitable to carry out integrations over $q^{2}$.

![Graph](image)

**FIG. 2:** Sum rule predictions for the weak form factors $G_{1}(q^{2})$ (the upper blue circles) and $|G_{2}(q^{2})|$ (the lower red squares). The lines denote the fit functions $G_{1}(q^{2})$ and $|G_{2}(q^{2})|$, respectively.

To this end, we use the functions

$$ G_{i}(q^{2}) = G_{i}^{0} \exp \left[ g_{1}^{i} \frac{q^{2}}{m^{2}} + g_{2}^{i} \left( \frac{q^{2}}{m^{2}} \right)^{2} \right], \quad (34) $$

where parameters $G_{i}^{0}$, $g_{1}^{i}$, and $g_{2}^{i}$ should be fitted to satisfy sum rules’ predictions. The parameters of the functions $G_{i}(q^{2})$ extracted from numerical analysis are collected in Table I. As an example, the functions $G_{1}(q^{2})$ and $|G_{2}(q^{2})|$ are depicted in Fig. 2, in which we see a quite nice agreement between the sum rule predictions and fit functions.

To calculate the partial widths of the semileptonic decays, apart from the form factors, we also use the parameters

$$ G_{F} = 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \quad |V_{bc}| = (42.2 \pm 0.08) \times 10^{-3}. \quad (35) $$

Our results for the partial widths of the semileptonic decay channels are presented below

$$ \Gamma(T_{bb}^{-} \to \bar{T}_{bc}^{0} e^{-} \nu_{e}) = (2.02 \pm 0.39) \times 10^{-9} \text{ MeV}, $$
$$ \Gamma(T_{bb}^{-} \to \bar{T}_{bc}^{0} \mu^{-} \nu_{\mu}) = (1.96 \pm 0.37) \times 10^{-9} \text{ MeV}, $$
$$ \Gamma(T_{bb}^{-} \to \bar{T}_{bc}^{0} \tau^{-} \nu_{\tau}) = (1.03 \pm 0.19) \times 10^{-10} \text{ MeV}. \quad (36) $$

Obtained information on partial widths of semileptonic channels can be used to improve predictions for the full width and lifetime of the exotic meson $T_{bb}$.  

**IV. NONLEPTONIC PROCESSES $T_{bb}^{-} \to \bar{T}_{bc}^{0} M$**

Dominant nonleptonic decays of $T_{bb}^{-}$ are triggered by the subprocess $b \to W^{-} c$, whereas the transition $b \to W^{-} u$ leads to decays suppressed relative to main ones, as it has been explained in the previous section. Therefore, in this section we consider nonleptonic decays $T_{bb}^{-} \to \bar{T}_{bc}^{0} M$ of the tetraquark $T_{bb}^{-}$. In these processes $M$ is one of the pseudoscalar mesons $\pi^{-}$, $K^{-}$, $D^{-}$, and $D_{s}^{-}$. At the final state of the process they appear due to decays of $W^{-}$ to pairs of quark-antiquark $d\bar{c}$, $s\bar{d}$, $c\bar{s}$, and $s\bar{s}$, respectively. The masses and decay constants of the mesons $\pi^{-}$, $K^{-}$, $D^{-}$, and $D_{s}^{-}$ are presented in Table III. It is not difficult to see, that the mass of $T_{bb}^{-}$ obeys a requirement $m > \bar{m} + m_{M}$ for all of these mesons, and these decays are kinematically allowed processes.

| $G_{i}(q^{2})$ | $G_{i}^{0}$ | $g_{1}^{i}$ | $g_{2}^{i}$ |
|----------------|------------|------------|------------|
| $G_{1}(q^{2})$ | 2.13       | 3.28       | -4.33      |
| $G_{2}(q^{2})$ | -0.72      | 6.26       | -17.13     |
| $G_{3}(q^{2})$ | 334.57     | 3.31       | -5.71      |
| $G_{4}(q^{2})$ | -1.33      | 3.36       | 0.35       |

**TABLE I:** Parameters of the extrapolating functions $G_{i}(q^{2})$. 

To our knowledge, because we need $m > \bar{m} + m_{M}$.
We describe production of mesons \( M \) by employing the effective Hamiltonian, and introducing relevant effective weak vertices. To study the nonleptonic decays \( T_{bb} \rightarrow \bar{T}_{bc}^0 \pi^- \), we use also the QCD factorization approach. This method is fruitful for studying of ordinary mesons’ nonleptonic decays \([30,31]\), but can be applied to investigate weak decays of tetraquarks as well \([18,21,29]\).

We present in a detailed form the decay \( T_{bb} \rightarrow \bar{T}_{bc}^0 \pi^- \), and write down final results for other channels. The effective Hamiltonian \( \mathcal{H}^{\text{eff}} \) for this decay at the tree-level is given by the expression

\[
\mathcal{H}^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{bc} V_{ud}^* c_1(\mu) Q_1 + c_2(\mu) Q_2, \tag{37}
\]

where

\[
Q_1 = \langle \bar{u}_i u_j \rangle_{\bar{v} = 0} \langle \bar{c}_i b_j \rangle_{\bar{v} = 0},
\]

\[
Q_2 = \langle \bar{u}_i u_j \rangle_{\bar{v} = 0} \langle \bar{c}_i b_j \rangle_{\bar{v} = 0}, \tag{38}
\]

and \( i, j \) are the color indices. In Eq. (37) the abbreviation \( \langle \bar{v}_1 q_2 \rangle_{\bar{v} = 0} \) means

\[
(\bar{v}_1 q_2)_{\bar{v} = 0} = \bar{v}_1 \gamma_\mu (1 - \gamma_5) q_2. \tag{39}
\]

Let us note that, we do not include into Eq. (37) current-current operators appearing due the QCD penguin and electroweak-penguin diagrams. The short-distance Wilson coefficients \( c_1(\mu) \) and \( c_2(\mu) \) are given at the factorization scale \( \mu \).

The amplitude of the decay \( T_{bb} \rightarrow \bar{T}_{bc}^0 \pi^- \) is determined by the following expression

\[
\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{bc} V_{ud}^* a_1(\mu) \langle \pi^- (q) \rangle \langle \bar{u}_i u_j \rangle_{\bar{v} = 0} |0\rangle \times \langle \bar{T}_{bc}^0 (p') \rangle \langle \bar{c}_i b_j \rangle_{\bar{v} = 0} |T_{bb}^- (p)\rangle, \tag{40}
\]

where

\[
a_1(\mu) = c_1(\mu) + \frac{1}{N_c} c_2(\mu), \tag{41}
\]

with \( N_c = 3 \) being the number of quark colors.

The matrix element \( \langle \bar{T}_{bc}^0 (p') \rangle \langle \bar{c}_i b_j \rangle_{\bar{v} = 0} |T_{bb}^- (p)\rangle \) in terms of the weak form factors is given by Eq. (21). The matrix element \( \langle \pi^- (q) \rangle \langle \bar{u}_i u_j \rangle_{\bar{v} = 0} |0\rangle \) in \( \mathcal{A} \) can be written down in the following form

\[
\langle \pi^- (q) \rangle \langle \bar{u}_i u_j \rangle_{\bar{v} = 0} |0\rangle = i f_\pi q_\mu, \tag{42}
\]

where \( f_\pi \) is the decay constant of the pion. Then, the amplitude \( \mathcal{A} \) of the nonleptonic weak decay is determined by the expression

\[
\mathcal{A} = i \frac{G_F}{\sqrt{2}} f_\pi V_{bc} V_{ud}^* a_1(\mu) \left\{ P q \left[ G_1(q^2) \epsilon \cdot c' \right] - \frac{G_3(q^2)}{2m^2} q \cdot c \cdot c' \right\} + G_4(q^2) \delta_{\alpha\beta} \epsilon^\alpha \epsilon^\beta P^\mu P_{\mu} q^\alpha \right\}, \tag{43}
\]

For completeness we provide below the partial width of this process

\[
\Gamma(T_{bb}^0 \rightarrow \bar{T}_{bc}^0 \pi^-) = \frac{|A|^2}{48\pi m_0^3} \lambda (m^2, \bar{m}_Z^2, q^2), \tag{44}
\]

where

\[
|A|^2 = \frac{G_F^2}{2} f_\pi^2 V_{bc}^2 V_{ud}^2 a_1(\mu) \left\{ G_1(q^2) (m^2 - \bar{m}_Z^2)^2 \right\} \times \left\{ m^4 + (\bar{m}_Z^2 - q^2)^2 - 2m^2 (5\bar{m}_Z^2 - q^2) \right\} \nonumber
\]

\[
+ G_3(q^2) (m^2 - \bar{m}_Z^2)^2 \nonumber
\]

\[
+ G_3(q^2) (2m^2 \bar{m}_Z^2 + 2G_4(q^2) (m^2 - \bar{m}_Z^2)^2 \nonumber
\]

\[
- 2m^2 (\bar{m}_Z^2 + q^2)^2 \right\} + G_1(q^2) G_3(q^2) \left( m^2 - \bar{m}_Z^2 \right)^2 \nonumber
\]

\[
\times \left\{ m^6 + (\bar{m}_Z^2 - q^2)^3 - m^4 (\bar{m}_Z^2 + 3q^2) \right\} \nonumber
\]

\[
- m^2 (\bar{m}_Z^2 + 2m^2 - 3q^2) \right\}. \tag{45}
\]

In Eqs. (41) and (44) the weak form factors \( G_i(q^2) \) and \( \lambda (m^2, \bar{m}_Z^2, q^2) \) are computed at \( q^2 = m^2 \). The decay modes \( T_{bb} \rightarrow \bar{T}_{bc}^0 K^- (D^-, D^+) \) can be studied in a similar way. For these purposes, in expressions above one should replace \( (m_\pi, f_\pi) \) by the masses and decay constants of the mesons \( K^-, D^- \), and \( D^+ \), make the substitutions \( V_{ud} \rightarrow V_{us}, V_{cd}, \) and \( V_{cs} \), and fix the form factors \( G_i \) and \( \lambda \) at \( q^2 = m^2 \).

Input parameters required for numerical computations are collected in Table III. This table contains the masses and decay constants of the final state mesons, and relevant CKM matrix elements. The coefficients \( c_1(m_b) \), and \( c_2(m_b) \) with next-to-leading order QCD corrections are borrowed from Refs. [32,33]

\[
c_1(m_b) = 1.117, \ c_2(m_b) = -0.257. \tag{46}
\]

For the decay \( T_{bb}^- \rightarrow \bar{T}_{bc}^0 \pi^- \) calculations yield

\[
\Gamma(T_{bb}^- \rightarrow \bar{T}_{bc}^0 \pi^-) = (5.84 \pm 1.11) \times 10^{-10} \text{ MeV}. \tag{47}
\]

Partial widths of other nonleptonic decays of the tetraquark \( T_{bb}^- \) are

\[
\Gamma(T_{bb}^- \rightarrow \bar{T}_{bc}^0 K^-) = (6.43 \pm 1.32) \times 10^{-11} \text{ MeV}; \\
\Gamma(T_{bb}^- \rightarrow \bar{T}_{bc}^0 D^-) = (3.01 \pm 0.64) \times 10^{-11} \text{ MeV}; \\
\Gamma(T_{bb}^- \rightarrow \bar{T}_{bc}^0 D^+) = (7.80 \pm 1.54) \times 10^{-10} \text{ MeV}. \tag{48}
\]

Results of this section are second part of required information.
TABLE II: Masses and decay constants of the final state pseudoscalar mesons. The CKM matrix elements are also included.

| Quantity | Value |
|----------|-------|
| $m_{\pi}$ | 139.570 MeV |
| $m_K$ | $(493.677 \pm 0.016)$ MeV |
| $m_D$ | $(1869.61 \pm 0.10)$ MeV |
| $m_{D_0}$ | $(1968.30 \pm 0.11)$ MeV |
| $f_\pi$ | 131 MeV |
| $f_K$ | $(155.72 \pm 0.51)$ MeV |
| $f_D$ | $(203.7 \pm 4.7)$ MeV |
| $f_{D_0}$ | $(257.8 \pm 4.1)$ MeV |
| $|V_{us}|$ | 0.97420 ± 0.00021 |
| $|V_{cb}|$ | 0.2243 ± 0.0005 |
| $|V_{ud}|$ | 0.218 ± 0.004 |
| $|V_{cs}|$ | 0.997 ± 0.017 |

V. DISCUSSION AND CONCLUDING NOTES

Results of the previous two sections allow us to improve predictions for the full width and mean lifetime of the tetraquark $T_{bb}^-$. The semileptonic decays of this particle $T_{bb}^- \rightarrow Z_{bc}^0 l \nu_l$ were analyzed in Ref. [13]. Using partial widths of these processes and widths of the semileptonic and nonleptonic decays $T_{bb}^- \rightarrow Z_{bc}^0 \nu_l$ and $T_{bb}^- \rightarrow Z_{bc} M$, it is not difficult to evaluate relevant parameters. Thus, for the full width of $T_{bb}^-$, we get

$$\tilde{\Gamma} = (7.72 \pm 1.23) \times 10^{-8} \text{ MeV}. \quad (49)$$

The lifetime of $T_{bb}^-$ is estimated in the range

$$\bar{\tau} = 8.53^{+1.57}_{-1.18} \text{ fs}. \quad (50)$$

Comparing improved estimates for $\tilde{\Gamma}$ and $\bar{\tau}$ with previous results

$$\Gamma = (7.17 \pm 1.23) \times 10^{-8} \text{ MeV},$$
$$\tau = 9.18^{+1.90}_{-1.34} \text{ fs}, \quad (51)$$

one can see that semileptonic processes $T_{bb}^- \rightarrow Z_{bc}^0 l \nu_l$ are dominant decay channels of the tetraquark $T_{bb}^-$. In fact, a difference $\Delta = \tilde{\Gamma} - \Gamma = 0.55 \times 10^{-8} \text{ MeV}$ is equal to 8% of $\Gamma$. In other words, 7 new decay modes considered in the present work constitute approximately 8% part of the full width $\Gamma$. The branching ratios of different weak decay modes of $T_{bb}^-$ are presented in Table IIII excluding two nonleptonic decays $BR$s of which are negligible.

We have explored the weak decays of $T_{bb}^-$, where the final-state tetraquark $T_{bc}^0 = [bc] \bar{\nu}l$ has been considered as an axial-vector particle. By computing partial widths of these decays, we have calculated the full width $\tilde{\Gamma}$ and mean lifetime $\bar{\tau}$ of the axial-vector tetraquark $T_{bb}^-$ and improved existing estimations for them. Up to now experimental collaborations did not discover weak decays of tetraquarks. But some of active experiments, such as LHC, have a certain potential to observe weak decay channels of tetraquarks $T_{bb}^-$ [33]. In Ref. [33] the authors demonstrated that during 1 – 4 runs of LHC one may expect $O(10^8)$ events with $T_{bb}^-$. Such potential will have also a Tera-Z factory [36]. Predictions obtained for parameters of the tetraquark $T_{bb}^-$ may be useful for analysis of these processes.

| Channels | $BR$ (%) |
|----------|----------|
| $T_{bb}^- \rightarrow Z_{bc}^0 e^- \nu_e$ | 34.3 |
| $T_{bb}^- \rightarrow Z_{bc}^0 \mu^- \nu_\mu$ | 34.2 |
| $T_{bb}^- \rightarrow Z_{bc}^0 \tau^- \nu_\tau$ | 24.4 |
| $T_{bb}^- \rightarrow \bar{T}_{bc}^0 e^+ e^-$ | 2.6 |
| $T_{bb}^- \rightarrow \bar{T}_{bc}^0 \mu^+ \mu^-$ | 2.5 |
| $T_{bb}^- \rightarrow \bar{T}_{bc}^0 \tau^+ \tau^-$ | 0.13 |
| $T_{bb}^- \rightarrow T_{bc}^0 \pi^-$ | 0.76 |
| $T_{bb}^- \rightarrow T_{bc}^0 D^+_s$ | 1.01 |

TABLE III: Branching ratios of the weak decay channels of the tetraquark $T_{bb}^-$.  

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