On the mechanisms of heavy-quarkonium hadroproduction

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Abstract. We discuss the various mechanisms potentially at work in hadroproduction of heavy quarkonia in the light of comparisons of higher-order QCD corrections both in the Colour-Singlet (CS) and Colour-Octet (CO) channels and the inclusion of the contribution arising from the $s$-channel cut in the CS channel. We also discuss new observables meant to better discriminate between these different mechanisms.

1 Introduction

Heavy quarkonia are among the most studied bound quark systems. It is however fair to say that, for the time being, there is no consensus concerning which mechanisms are effectively at work in their production in $pp$ collisions, that is at the Tevatron and RHIC. By extension, available theoretical predictions for the production rates at the LHC are rather uncertain. For recent reviews, the reader is guided to\cite{1,2,3} along with some perspectives for the LHC\cite{4}.

To what concerns the $T$ states, the latest NLO predictions\cite{5} in the QCD-based approach of the Colour-Singlet Model (CSM)\cite{6} including for the first time some of the important NNLO $a_T^2$ corrections show a satisfactory agreement with the data coming from the Tevatron\cite{16,17}.

On the other hand, to what concerns the charmonium family, it is well-known that the first measurements by the CDF Collaboration of the direct production of $J/\psi$ and $\psi'$ at $\sqrt{s}=1.8$ TeV\cite{18,19} brought to light a striking puzzle. They indeed found much larger rates than the prediction of the CSM.

Since then, other approaches were proposed (e.g. the Colour-Octet Mechanism (COM)\cite{11} or revived (e.g. the Colour-Evaporation Model (CEM)\cite{12}). Those are unfortunately not able to reproduce in a consistent way experimental studies of both cross-section and polarisation measurements for charmonia at the Tevatron\cite{20,21,22} along with the cross sections measured by PHENIX at RHIC\cite{23}. For instance, the seemingly solid prediction of the COM for a transverse polarisation of $\psi$'s produced at high transverse momentum is clearly challenged by the experimental measurements. The most natural interpretation of such a failure of NRQCD is that the charmonium system is too light for relativistic effects to be neglected and that the quark-velocity expansion ($v$) of NRQCD\cite{11} may not be applicable for the rather “light” $c\bar{c}$ system. This might be indeed the case in view of the aforementioned agreement between theory and the available experimental data on production in $pp$ (and inclusive decays) of the significantly heavier $T$. In this case, relativistic corrections are expected to be less important and the leading state in the Fock expansion, i.e. the heavy-quark pair in a colour singlet $^{3}S_{1}$, to be dominant. This would in turn explain why a computation incorporating the sole CS channel (LO contribution in the $v$-expansion of NRQCD) is sufficient—when $P_{T}^{-4}$ contributions are considered though.

Recently, many new theoretical results became available. Some completed our knowledge of quarkonium production like the up-to-date proof\cite{16} of NRQCD factorisation holding true at any order in $v$ in the gluon-fragmentation channel—therefore relevant at large $P_{T}$ where this channel should dominate—and the QCD corrections which we shall discuss in the next section. Others mainly questioned our understanding of the mechanisms at work in heavy-quarkonium production. Firstly, a complete survey of fixed-target measurements\cite{17} showed that the universality of the Long Distance Matrix Elements (LDMEs) of NRQCD\cite{11} cannot certainly be extended to the description of low-$P_{T}$ data. Secondly, NRQCD factorisation was shown to require modifications in fragmentation regions where 3 heavy quarks have similar momenta\cite{18}. Thirdly, the $c$-quark fragmentation approximation was shown\cite{19} to be only valid at much higher $P_{T}$ than expected from the pioneering works\cite{20}.

Parallel to that, we investigated in\cite{21,22} on the cut contributions due to off-shell and non-static quarks. In particular, we questioned the assumption of the CSM that takes the heavy quarks forming the quarkonium ($Q$) as being on-shell\cite{6}. If they are not, the usual s-channel cut contributes to the imaginary part of the amplitude\cite{1,2,3,4,11,12,13,14,15,16,17,18,19,20,21,22}. Let us remember that the universality of the LDMEs is clearly challenged by HERA data on photoproduction of $J/\psi$\cite{3}.

\textsuperscript{1} Note on the way that this assumption is also present in NRQCD.

\textsuperscript{a} Present address
and need to be considered on the same footing as the CSM cut. A first evaluation \cite{21} of the latter incorporating constraints for the low- and large-$P_T$ (the scaling limit) region gives rates significantly larger than the usual CSM cut. Moreover, low-$P_T$ data from RHIC are very well described without need of re-summing initial-gluon contributions. However, as expected \cite{21}, this approach underestimates the cross-section at large values of $P_T$ and other mechanisms have to be considered in this region.

In section 2, we present the latest results available on QCD corrections to hadroproduction of $J/\psi$, $\psi'$ and $\Upsilon(nS)$. In section 3, we discuss how the $s$-channel cut contribution to the CS channel can be evaluated and we present a comparison with data. In section 4, we briefly review other recent theoretical results. In section 5, we show how the study of the production of quarkonia in association with a heavy-quark pair of the same flavour may be used to disentangle between the different mechanisms proposed to explain quarkonium production. Finally, we present our conclusions and outlooks.

2 QCD corrections

More than ten years ago now, the very first NLO calculation on quarkonium production to date became available. It was centred on unpolarised photoproduction of $\psi$ \cite{24} via a colour-singlet (CS) transition. Later on, NLO corrections were computed for direct $\gamma\gamma$ collisions \cite{24,25} for which it had been previously shown \cite{26} that the LO CS contribution alone was not able to correctly reproduce the measured rates by DELPHI \cite{27}. NLO corrections have also recently been computed for the integrated cross section of two $J/\psi$-production observables at the $B$-factories: $J/\psi + c\bar{c}$ \cite{25} and $J/\psi + \eta_c$ \cite{27}. As of today, only the full colour-octet (CO) contributions to direct $\gamma\gamma$ collisions have been evaluated at NLO for $P_T > 0$ \cite{21,25}.

At the LHC and the Tevatron, $\psi$ and $\Upsilon$ production proceeds most uniquely via gluon-fusion processes. The corresponding cross section at NLO ($\alpha_S^3$ for hadroproduction processes) are significantly more complicated to compute than the former ones and became only available one year ago \cite{20,19}. We shall discuss them in the next section.

The common feature of all these calculations is the significant size of the NLO corrections, in particular for large transverse momenta $P_T$ of the quarkonia for the computations of differential cross sections in $P_T$. In $\gamma\gamma$ an $pp$ collisions, QCD corrections to the CS production indeed open new channels with a different behaviour in $P_T$ which raise substantially the cross section in the large-$P_T$ region.

Let us discuss this shortly for the gluon-fusion processes which dominate the yield in $pp$. If we only take into account the CS transition to $^3S_1$ quarkonium, it is well known that the differential cross section at LO as a function of $P_T$ scale like $P_T^{-6}$ \cite{6}. This is expected from contributions coming from the typical “box” graphs of Fig. 2 (a). At NLO \cite{20,19}, we can distinguish three noticeable classes of contributions. First, we have the loop contributions as shown on Fig. 2 (b), which are UV divergent but as far their $P_T$ scaling is concerned, they would still scale like $P_T^{-8}$. Then we have the $t$-channel gluon exchange graphs like on Fig. 2 (c). They scale like $P_T^{-6}$. For sufficiently large $P_T$, their smoother $P_T$ behaviour can easily compensate their $\alpha_S$ suppression compared to the LO ($\alpha_S^3$) contributions. They are therefore expected to dominate over the whole set of diagrams up to $\alpha_S^3$. To be complete, we should not forget the $\alpha_S^3$ contributions from $Q + Q\bar{Q}$ (where $Q$ is of the same flavour as the quarks in $Q$). Indeed, one subset of graphs for $Q + Q\bar{Q}$ is fragmentation-like (see Fig. 2 (d)) and scales like $P_T^{-4}$. Such contributions are therefore expected to dominate at large $P_T$, where the smoother decrease in $P_T$ is enough to compensate the suppression in $\alpha_S$ and the one due to the production of 4 heavy quarks. As mentioned above, in practice \cite{19}, this happens at larger $P_T$ than as expected before \cite{20}. We shall come back to this channel later. In the next sections, we shall discuss the impact of the NLO corrections to the CS channels and then a first computation including the a priori dominant $\alpha_S^3$ contributions i.e. topologies illustrated by Fig. 2 (e) and (f).

![Fig. 1](image-url) Representative diagrams contributing to $^3S_1$ hadroproduction via Colour-Singlet channels at orders $\alpha_S^3$ (a), $\alpha_S^5$ (b,c,d), $\alpha_S^3$ (e,f) and via Colour-Octet channels at orders $\alpha_S^3$ (g,h). The quark and antiquark attached to the ellipsis are taken as on-shell and their relative velocity $v$ is set to zero.

To what concerns the CO contributions, the effects of NLO (here $\alpha_S^3$) contributions are expected to be milder.

\footnote{These divergences can be treated as usual using dimensional regularisation, see e.g. \cite{29}.}
Indeed, the contributions from CO transitions are a priori suppressed by $\alpha^4$ and the reason why they can still appear significant comes from a lower power in $\alpha_S$ for similar topologies (and thus similar $P_T$ scaling). For instance, compare Fig. 1 (g) to Fig. 1 (e) and Fig. 1 (h) to Fig. 1 (c). At LO, $P_T^{-6}$ and $P_T^{-8}$ scaling are therefore already present. As a result, including $\alpha_S^4$ contributions will not open any new channels and NLO corrections are expected to be described by a roughly constant $K$ factor. As we shall see, this is indeed the trend seen in the results of Ref in which the NLO corrections to $1S_0^{[8]}$ and $3S_1^{[8]}$ colour octets going to $J/\psi$ were considered.

### 2.1 NLO corrections for Colour-Singlet channels

Let us first present a comparison between the measurements by the CDF collaboration and the result for the $J/\psi$ obtained following the procedure explained in Ref. 38. It is worth noting that the $\chi_c$ cross sections are not available for now at NLO accuracy. This would be necessary if we wanted to predict at this accuracy prompt-$J/\psi$ production cross section, in order to compare with the most recent measurements of RUN II. These focused only on the prompt yield. As a makeshift, we have multiplied those data by the averaged fraction of direct $J/\psi$ measured during RUN I for the rather similar beam energy 1.8 TeV and a similar range in $P_T$ and rapidity: $\langle P^{\text{direct}}_T \rangle = 64 \pm 6\%$.

In our calculation, we set $m_c = 1.5 \pm 0.1$ GeV and $m_b = 4.75 \pm 0.25$ GeV. We used the PDF set CTEQ6L1 (resp. CTEQ6M) for LO (resp. NLO) cross sections, and always kept the factorisation scale equal to the renormalisation scale: $\mu_f = \mu_R$. Except for the associated production channel where we took $\mu_0 = \frac{\sqrt{2m_Q^2 + P_T^2}}{2}$, the central scale is fixed at $\mu_0 = \sqrt{m_Q^2 + P_T^2}$ and then was varied by a factor of 2. To what concerns the non-perturbative inputs, we used the values related to the BT potential [34]: $\langle O^{J/\psi}(3S_1^{[1]}) \rangle = 1.16$ GeV$^3$ and $\langle O^f(3S_1^{[1]}) \rangle = 9.28$ GeV$^3$. For $\Upsilon(1S)$ production, we considered the prompt measurement at $\sqrt{s} = 1.8$ TeV in Ref. 7, multiplied by the averaged direct fraction obtained in Ref. 38: $\langle P^{\text{direct}}_T \rangle = 50 \pm 12\%$.

In both case, we have an illustration of the previous discussion. The differential cross section for the LO contribution, i.e. $gg \to J/\psi g$, has the steepest slope and is already an order of magnitude smaller than the NLO contribution at $P_T \approx 10$ GeV. The differential cross section for $Q \bar{Q}$ has the smoothest slope. In the case of $J/\psi$, it starts to be significant for $P_T > 20$ GeV. For $\Upsilon$, the suppression due to the production of a $b$ quark is stronger and this yield remains negligible in the accessible value of $P_T$. The bands denoted NLO refer to all the contributions up to order $\alpha_S^2$.

Those results were recently confirmed in Refs. 36,37. In the latter papers, the polarisation information was kept and the observable $\alpha$ was also computed. However, it is important to stress that for $\psi$ and $\Upsilon$ production the CS yields predicted at the NLO accuracy are still clearly below the experimental data especially at large $P_T$. In this respect, the predictions for the polarisation at this order cannot be usefully compared to the data.

The conclusion is that, in general, the inclusion of NLO contributions bring the CS predictions considerably closer to the data, although agreement is only reached at NLO in the photoproduction case Ref. 23.

### 2.2 NLO corrections for Colour-Octet channels

As aforementioned, NRQCD has reached a certain success by explaining the main features of charmonium and bottomonium hadroproduction via the introduction of the Colour-Octet (CO) mechanism. It indeed provides a good description of the $P_T$-differential cross-section for the direct $J/\psi$ and $\psi'$ for $P_T \gtrsim 5$ GeV as measured by CDF in pp Ref. 9,10. A reasonable agreement was also obtained with the first PHENIX measurements in pp at $\sqrt{s} = 200$ GeV Ref. 38,39. In both cases, the cross-section is dominated by the gluon fragmentation into a colour-octet $S$-wave state. Following the heavy-quark spin symmetry Ref. 111 of
NRQCD, the latter mechanism leads to transversally polarised \(J/\psi\) and \(\psi'\), the parent fragmenting gluon being mostly on-shell and thus transversally polarised at high \(P_T\).

However, \(J/\psi\) and \(\psi'\) are not seen to be transverse by the CDF experiment \[13\]. It measured a slight longitudinal polarisation for both the prompt \(J/\psi\) and direct \(\psi'\) yield. It is worth noting here that the feed-down from \(\chi_c\) can influence significantly the polarisation of the prompt \(J/\psi\) yield – this was taken into account in the NRQCD-based predictions \[10\]. Moreover, the recent preliminary result from PHENIX \[11\] indicates a polarisation compatible with zero for the total \(J/\psi\) production at forward rapidity (1.2 < \(|y|< 2.2\), but with large uncertainties.

Very recently, CO contributions from \(S\) waves \((^1S_0^{(8)}\) and \(^3S_1^{(8)}\) have become available \[31\] for hadroproduction. A complete phenomenological study is not yet available though. Anyhow this confirms that NLO corrections do not affect significantly the \(P_T\) dependence as expected from the introductory discussion of this section.

Let us define \(K\) factors as the ratios of NLO to LO cross section for a given CO channel. For the Tevatron, they are about 1.2 for the \(^1S_0^{(8)}\) state and 1.1 for the \(^3S_1^{(8)}\) (at the LHC, they are both about 0.8). Consequently, the value of the CO Long Distance Matrix Elements (LDMEs) fit to the Tevatron data at LO \((O^{J/\psi}(^1S_0^{(8)})) \approx 0.0012\) GeV\(^3\) and \((O^{J/\psi}(^3S_1^{(8)})) \approx 0.0045\) GeV\(^3\) \[33\] would be at most reduced by 15%. In this respect, the NLO corrections to the octets do not improve the universality of the matrix elements when the idea of the dominance of the CO transitions is confronted to the data on photoproduction from HERA.

According to the author of \[31\], it is not possible to obtain a satisfactory \(P_T\) distribution in terms of a unique \(O_f^{(8)}\) value when considering the whole range in \(P_T\) analysed by CDF. More precisely, they did not consider the experimental data with \(P_T < 6\) GeV, for which it seems that other mechanisms have to be at work if we believe that the COM is responsible for the major part of the cross section at large \(P_T\).

This in any case emphasises the need for more work dedicated to the description of the low-\(P_T\) region and maybe the relevance of the study of \(s\)-channel cut contributions, which we discuss later. Last but not least, the polarisation from CO transitions appears not to be modified at NLO with respect to LO results. Overall, this recent first study of CO contributions at NLO in hadroproduction at \(P_T > 0\) sounds like a confirmation of the flagrant discrepancy between the NRQCD predictions for the polarisation of the \(J/\psi\) and the experimental measurements from the CDF collaboration \[13\].

![Fig. 3. Full computation at NLO for (Left) \(\Upsilon(1S) + X\) (dashed line) vs. \(\Upsilon(1S) + 2\) light partons with a cut on \(s_{ij}^{\min}\) (grey band), (Right) \(\psi(2S) + X\) (dashed line) vs. \(\psi(2S) + 2\) light partons with a cut on \(s_{ij}^{\min}\) (grey curves)
](image)

2.3 QCD corrections up to \(\alpha_S^5\)

As noted above, the discrepancy between the NLO computations for the CSM and the experimental data, both for \(\psi\) and \(\Upsilon\) still grows with \(P_T\). If we parallel that to the existence of new \(P_T^{-4}\) channel at order \(\alpha_S^5\), it is reasonable to wonder what their size are effectively.

In fact, their contributions can be evaluated in a relatively “simple” \[4\] and reliable way by computing the \(\alpha_S^5\) contributions consisting in the production of a \(Q\) with 3 light partons (noted \(j\) thereafter). Among them are the topologies of Fig. \[1\] (d) (gluon fragmentation) and Fig. \[1\] (e) (“high-energy enhanced”), these close the list of kinematical enhancements from higher-order QCD corrections. This \(\alpha_S^5\) subset being the LO for a physical process \((pp \rightarrow Q + jjj)\), its contribution is finite except for soft and collinear divergences.

To avoid such divergences, we impose a lower bound on the invariant-mass of any light partons \((s_{ij})\). For the new channels opening up at \(\alpha_S^5\), and which specifically interest us, the dependence on this cut is to get smaller for large \(P_T\) since no collinear or soft divergences can appear there.

\(^4\) “simple” compared to a full –out-of-reach– NNLO computation and thanks to the automated generator of matrix elements MadOnia \[42\].
For other channels, whose LO contribution is at $\alpha_S^2$ or $\alpha_S^3$, the cut would produce logarithms of $s_{ij}/s_{ij}^{\min}$. Those can be large. Nevertheless, they can be factorised over their corresponding LO contribution, which scales at most as $P_T^{-6}$. The sensitivity on $s_{ij}^{\min}$ is thus expected to come to nothing at large $P_T$.

Thanks to the exact NLO computation of [30], such a procedure can be tested for the process $pp \to Q + jj$. For instance, the differential cross section for the real $\alpha_4$ corrections, $T(1S) + jj$ production, is displayed in Fig. 3 (Left). The grey band illustrates the sensitivity to the invariant-mass cut $s_{ij}^{\min}$ between any pairs of light partons when it is varied from $0.5m_J^2$ to $2m_J^2$. The yield becomes insensitive to the value of $s_{ij}^{\min}$ as $P_T$ increases, and it reproduces very accurately the differential cross section at NLO accuracy. In the charmonium case, the similar contributions from $pp \to \psi' + jj$ matches even better, for lower $P_T$ and with a smaller dependence of $s_{ij}^{\min}$ the full NLO computation, as seen on Fig. 3 (Right).

We now turn to the results concerning the real contribution at $\alpha_S^2$, which we refer to as NNLO*. We used the approach described in Ref. [12], which allows the automatic generation of both the subprocesses and the corresponding scattering amplitudes. The differential cross-sections for $T(1S)$ and $\psi(2S)$ are shown in Fig. 4. The red band (referred to as NNLO*) corresponds to the sum of the NLO yield and the $Q + jj$ contributions. In the $T$ case, the contribution from $T$ with three light partons fills the gap between the data and the NLO calculation, while for the $\psi(2S)$ there seems to remain a small gap between the NNLO* band and the preliminary CDF data. In both cases, the $\alpha_S^2$ contribution is very sensitive to the choice of the renormalisation scale, $\mu_r$. This is expected: for moderate values of the $P_T$, the missing virtual part might be important, whereas at large $P_T$, the yield is dominated by Born-level $\alpha_S^3$-channels from which we expect a large dependence on $\mu_r$. Even though the uncertainty on the normalisation is rather large, the prediction of the $P_T$ shape is quite stable and agrees well with the behaviour found in the data [7,8,43].

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Concerning the polarisation, the direct yield is predicted to be mostly longitudinal, see Fig. 5 (a). However, existing experimental data for $T$ are centred on the prompt yield [7,44]. In order to draw further conclusions, we would need first to gain some insights on NLO corrections to $P$-wave production at $P_T > 0$. Yet, since the yield from $P$-wave feed-down is likely to give transversely polarised $T$, the trend is more than encouraging. To what concerns
ψ(2S), one should be very careful before any comparison with experimental measurements since the yield is not exactly reproduced. Having this in mind, one sees in Fig. 5 (b) that the trend for longitudinally polarised ψ(2S) is reproduced but more marked. At very large PT where the contribution from ψ(2S) + c̄c becomes more and more significant the polarisation gets slightly less negative. In any case, further investigations are needed to draw any conclusions.

### 3 s-channel cut contribution

In this section, we briefly review our first evaluation of the s-channel cut contribution in hadroproduction of J/ψ and let us define the 3-point function

\[ \Gamma^{(3)}(p, P) = \Gamma(p, P)\gamma_5 \]  

where the (crossing-symmetric) function \( \Gamma^{(3)} \) is the (unphysical) value of the function \( F = F(c_1, c_2, q) \).

Nonetheless, the function \( F(c_1, c_2, q) \) must be chosen so that the current (3) satisfies crossing symmetry (i.e., symmetry under the exchange \( c_1 \leftrightarrow c_2 \)) and is free of singularities. The latter constraint implies \( F = F_0 \) at either pole position, i.e., when \( (c_2 + q)^2 = m^2 \) or \( (c_1 - q)^2 = m^2 \), where the constant \( F_0 \) is the (unphysical) value of the momentum distribution \( F(p, P) \) when all three legs of the vertex are on their respective mass shells. In principle, employing gauge invariance as the only constraint, we may take \( F = F_0 \) everywhere. This corresponds to the minimal substitution discussed by Drell and Lee [49] (for a complete derivation see [50]) who pointed out, however, that this does not provide the correct scaling properties at large energies, which means within the present context that \( F = F_0 \) would not lead to the expected \( P_T \) scaling of the amplitude. See [51] for a numerical comparison with data.

In order to obtain a correct scaling at large \( P_T \) and a behaviour close to the minimal substitution at low \( P_T \), we have chosen [21][51]

\[ F(c_1, c_2, q) = F_0 - h(c_1 \cdot c_2) \left( \frac{(I_0 - I_1)(I_0 - I_2)}{I_0} \right), \]  

where the (crossing-symmetric) function \( h(c_1 \cdot c_2) \) rises to become unity for large relative momentum. The phenomenological choice for the interpolating function \( h \) used in our calculations is

\[ h(c_1 \cdot c_2) = 1 - \frac{a}{\kappa^2 - (c_1 \cdot c_2 + m^2)} \]  

with two parameters, \( a \) and \( \kappa \).

Figure 7(a) shows our results for \( \sqrt{s} = 1.8 \) TeV in the pseudorapidity range \( |\eta| < 0.6 \) with parameter values \( a = 4 \) and \( \kappa = 4.5 \) GeV fixed to reproduce, up to \( P_T \approx 10 \) GeV, the cross-section measurement of direct J/ψ by CDF [10], the usual LO CSM from \( gg \rightarrow J/ψ q \bar{q} \) and LO CSM from \( gg \rightarrow J/ψ c \bar{c} \) [10]. Our results fit well the CDF data up to about \( P_T \approx 10 \) GeV. At higher \( P_T \), our curve falls below the data as expected from the genuine

\[ \sqrt{s} \]

We have taken \( c_2^2 = c_3^2 = m^2 \) and \( P_2^2 = M^2 \) with \( m \) and \( M \) being the masses of the quark and the J/ψ.
1/$P_T^8$ scaling of a LO box diagram (see the discussions of the previous section).

It is interesting to note the different $P_T$ behaviours of $\sigma_T$ and $\sigma_L$ leading to a dominance of the latter at large $P_T$ and a negative value for the polarisation $\alpha$ [21] at mid and large $P_T$. Figure 7(b) shows our results at $\sqrt{s} = 200$ GeV, still with $a = 4$ and $\kappa = 4.5$ GeV, compared with the PHENIX data [15].

Through this first evaluation of the $s$-channel cut contribution to the imaginary part of the production amplitude, incorporating low- and large-energy constraints as well as gauge invariance, we have shown that this cut can be significant. It is even possible to obtain a very good fit of the data from CDF at mid $P_T$ by proper choices of the parameters of our 4-point function. With the same parameters, we obtained an excellent description of the data taken at RHIC and down to very low $P_T$ without resuming initial-state gluon contributions. The $s$-channel cut indeed has a threshold at low $\hat{s}$ (thus low $P_T$) which corresponds to the energy needed to put the two $c$-quark on-shell.

Now that we have seen that the $s$-channel cut matters at low- and mid-$P_T$, it is necessary to have in the future a first evaluation of the contribution of the real part itself. On the other hand, we can start testing our parametrisation of the 4-point function, in photoproduction for instance, or in any other process involving a final state gluon.

4 Other theoretical advances

Beside the theoretical advances concerning QCD corrections and the inclusion of the $s$-channel cut contribution discussed in the previous sections, several interesting theoretical results have been obtained in the recent years. Let us review some of the most significant ones briefly.

On the side of NRQCD, Nayak, Qiu and Sterman provided an up-to-date proof [16] of NRQCD factorisation holding true at any order in $v$ in the gluon-fragmentation channel. They showed that new definitions of NRQCD matrix elements incorporating QCD Wilson lines were to be used, but that this was not to affect the existing phenomenological studies.

Last year, Collins and Qiu [52] showed that in general the $k_T$-factorisation theorem does not hold in production of high-transverse-momentum particles in hadron-collision processes, and therefore also for $\psi$ and $\Upsilon$. This is unfortunate since many studies [53,54,55,56,57,58,59,60,61,62], predicting mostly longitudinal yields and smaller CO LDMEs, in better agreement with the idea of LDME universality, were based on the hypothesis of such a factorisation in hadroproduction.

Besides, the $c$- and $b$-fragmentation approximation was shown to fail for the $P_T$ ranges accessible in experiments for quarkonium hadroproduction. By studying the entire set of diagrams contributing to $\psi$ and $\Upsilon$ production in association with a heavy-quark pair of the same flavour, we have shown [19] that the full contribution was significantly above (typically of a factor of 3) that obtained in the fragmentation approximation. A precision of 10% accuracy, say, can only be obtained at very large $P_T > 60$ GeV for $\psi$ and $P_T > 100$ GeV for $\Upsilon$. Note that the same observation was previously made for the process $\gamma \gamma \rightarrow J/\psi cc$ [63], and also for the $B_c^*$ hadroproduction, for which it was noticed that the fragmentation approximation was not reliable at the Tevatron [64,65].

Moreover, still in double-heavy-quark-pair production, the notion of colour-transfer enhancement was introduced by Nayak, Qiu and Sterman [18]. If three out of the four heavy quarks are produced with similar velocities, then there is the possibility that colour exchanges within this 3-quark system could turn CO configurations into CS ones, thus could effectively increase the rate of production of CS pairs. They finally discussed the introduction of specific new 3-quark operators –beyond the usual ones of NRQCD– necessary to deal with such an issue. A study of the colour-transfer effects in hadroproduction is still awaited for.
5 Associated production channels

As previously discussed, the results of QCD corrections for \( \Upsilon \) production seem to indicate that the CS transitions are dominant. Eventarily, this should put an end to the controversy related to \( \Upsilon \) production. Contrariwise, the situation remains unclear for the charmonium case. NLO corrections complemented by some dominant \( \alpha_S^2 \) contributions are large and seem to bring the prediction for the CS transitions very close to the data in the \( \psi' \) case for instance (see Fig. 4 (b)). Yet, theoretical uncertainties remain large and there seems to be some space left for CO transitions. Careful comparisons are still therefore due with polarisation observables. In this case, the theoretical uncertainties would certainly be competitive with experimental ones, for instance on prompt \( J/\psi \) yield \[^{[68]}\] . This requires however some knowledge on the QCD corrections to the \( P \)-wave CS yield. For the time being, nothing is known on this side.

It is therefore vital in order to progress in the understanding of the mechanisms responsible for heavy quarkonium production to introduce, compute and measure new observables. One of those is the hadronic activity around the quarkonium \[^{[72]}\] . Historically, UA1 compared their charged-track distributions with Monte Carlo simulations for a \( J/\psi \) coming from a \( B \) and a \( J/\psi \) coming from a \( \chi_c \). \[^{[68,69]}\] . At that time \( \chi_c \) feed-down was still expected to be the major source of prompt \( J/\psi \). Following either the idea of CO transitions or of CS transitions at higher-orders, we however expect now more complex distributions even for the prompt yield. It is therefore not clear if such methods are suitable to size up the \( B \)-feeddown otherwise than with the measurements of a displaced vertex typical of a \( B \) decay.

We therefore urgently need observables rather easy to predict and likely to test the many production models available \[^{[112]}\] . We argue here that the study of associated production channels, first in \( p\bar{p} \) collisions, then in \( pA \) and \( AA \), fills both these requirements. By associated production channels, we refer to \( \psi + c\bar{c} \) and \( T + b\bar{b} \).

A further motivation for such studies is that similar studies carried at \( B \)-factories showed an amazingly large fraction of \( J/\psi \) produced in association with another \( c\bar{c} \) pair. Indeed, the Belle collaboration first found \[^{[10]}\]  
\[ \frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi + X)} \approx 0.59^{+0.13}_{-0.15} \pm 0.12. \] 
After that, the analysis was improved and they obtained \[^{[71]}\]  
\[ \frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi + X)} = 0.82 \pm 0.15 \pm 0.14, \] 
\[ > 0.48 \text{ at 95\% CL.} \]  

Whether or not such a high fraction holds for hadroproduction as well, is a question which remains unanswered. Analyses at the Tevatron (CDF and D0) and at RHIC (PHENIX and STAR) are already possible. As computed in \[^{[19]}\]  for the RUN2 at the Tevatron at \( \sqrt{s} = 1.96 \text{ TeV} \), the integrated cross-section are significant :

\[ \sigma(J/\psi + c\bar{c}) \times B(\ell^+\ell^-) \simeq 1 \text{ nb} \]  
\[ \sigma(T + b\bar{b}) \times B(\ell^+\ell^-) \simeq 1 \text{ pb} \]  

(7)

As an illustration of the potentialities at RHIC, we chose to display in Fig. 8 the differential cross section for \( pp \rightarrow J/\psi + c\bar{c} \) computed for the STAR kinematics. Such studies could for instance be carried out by STAR in the next run with an integrated luminosities of around 50 \( \text{pb}^{-1} \) if dedicated triggers are available \[^{[72]}\] .

Without taking into account the likely reduction of the CO LDMEs induced by the QCD corrections mentioned in the previous sections, the integrated cross sections were found \[^{[73]}\]  to be dominated by the CS part, similarly to the differential cross section in \( pT \) up to at least 5 \( \text{GeV} \) for \( \psi \) and 10 \( \text{GeV} \) for \( T \). In other words, such observables can be thought of as a test of the CS contribution, for the first time since the introduction of the idea that CO transitions would be the dominant mechanism responsible for quarkonium production at high transverse momentum. If the effect of CO transitions is confirmed to be negligible for the \( T \), the \( T \) produced in association with a \( b\bar{b} \) pair are predicted to be strictly unpolarised, for any \( p_T \) (see Fig. 9) for the LHC.

Beside the property of discriminating between the CO and the CS transitions, the yield of \( \psi \) in association with \( c\bar{c} \) should show an \textit{a priori} completely different sensitivity to the \( \chi_c \) feed-down than the inclusive yield. The same holds for \( T \) with \( b\bar{b} \) with the \( \chi_b \) feed-down. To what concerns CS transitions, the \( P \)-wave yield is expected to be smaller than the \( S \)-wave one, since they are being suppressed by powers of the relative velocity \( v \) and here there is no extra gluon needed to be attached to the heavy-quark loop to produce \( \psi \) (or \( T \)) compared to \( P \)-waves as it is the case in the inclusive case.

For the CO transitions, associated production \( \chi_c + c\bar{c} \) can occur via the process \( gg \rightarrow gg \) for which the two final-
state gluons split into a $c\bar{c}$ pair, one of them hadronising into a $\chi_c$ via the CO mechanism. This contribution is certainly suppressed up to $P_T \sim 20$ GeV. For larger $P_T$, a dedicated calculation is needed. However, this mechanism would be very easily disentangled from the CS contributions since both $c$ quarks are necessarily emitted back to back to the $\chi_c$ and thus to the $J/\psi$.

Concerning the non-prompt signal, it would originate as usual from $gg \to b\bar{b}$, where one $b$ quark hadronises in $\psi$. Usually, this hadronisation of the $b$ produces the $\psi$ with light quarks only. This means that we have one single $c$ quark in the event. It is produced from the decay of the recoiling $b$ quark and is therefore back to back to the $\psi$. The non-prompt signal would then be simply cut down by searching for a $D$ meson near the $\psi$. Now it can happen that the hadronisation of the $b$ produces the $\psi$ and a $D$ meson. In this case, kinematical cuts would not help to suppress the non-prompt yield. Fortunately, this is a priori suppressed compared to the first case and even more than the direct yield since there is here no gain in the $P_T$ dependence since both $gg \to b\bar{b}$ and $gg \to \psi + c\bar{c}$ cross sections scale like $P_T^{-4}$. A cross check by sizing up the non-prompt yield with a displaced vertex measurement would be anyhow surely instructive.

Let us also mention that associated production has also been studied in direct $\gamma\gamma$ collisions in Ultra-peripheral collision (UPC) [44]. At least for direct $\gamma\gamma$ collisions, associated production is the dominant contribution to the inclusive rate for $P_T \geq 2$ GeV/c.

To conclude, studies can be carried on by detecting either the “near” or “away” heavy-quark with respect to the quarkonia. There are of course different way to detect the $D$, $B$, or a $b$-jet, ranging from the use of a displaced vertex to the detection of their decay in $e$ or $\mu$. As discussed above, this has to be considered by also taking into account the different backgrounds. The forthcoming Quarkonium-event-generator Madonia 2 [75] will surely be of a great help to achieve this task. In any case, we hope that such measurements would provide with clear information on the mechanisms at work in quarkonium production.\footnote{Note also that NLO QCD corrections have recently been computed for the production of a $J/\psi$ and $\Upsilon$ in association with a photon [76]. An experimental study of such process could be interesting as well.}

6 Conclusion

Recently significant progresses have been made in the evaluation of the QCD corrections to quarkonium production. The situation sounds now rather clear for the bottomonia where an agreement has been eventually obtained using only CS channels when dominant $\alpha_s^3$ contributions are incorporated. The polarisation predictions for the latter cases seem also quite encouraging considering CDF [47] and D0 [48] measurements. Yet, confirmations are awaited from the LHC.

On the other hand, those $\alpha_s^5$ contributions could be still unable to bring agreement with the measured $P_T$-differential cross-section of the direct charmonia. Dedicated further studies are however needed especially to what concerns the feed-down from $P$-waves which is not known at NLO accuracy. In the charmonium case, we have also seen that s-channel cut can bring a significant contribution to the cross section at low $P_T$ and hence a first evaluation of the real part of the production amplitude is needed.

Additional tests are now undoubtedly needed beyond the mere measurements of inclusive cross section and polarisation at the LHC. For instance, the hadroproduction of $J/\psi$ or $\Upsilon$ with a heavy-quark pair [19,73] appears to be a new valuable tool to separately probe the CS contribution, at least dominant at low-$P_T$ (below 15 GeV), as well as the study of the hadronic activity around the quarkonium.

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