Wakefield decay in a radially bounded plasma due to formation of electron halo

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Abstract
There is a new effect that can limit the lifetime of a weakly non-linear wakefield in a radially bounded plasma. If the drive beam is narrow, some of the plasma electrons fall out of the collective motion and leave the plasma radially, forming a negatively charged halo around it. These electrons repeatedly return to the plasma under the action of the charge separation field, interact with the plasma wave and cause its damping. The lowest-energy halo electrons take the energy from the wave more efficiently, because their trajectories are bent by the plasma wave towards the regions of the strongest acceleration. For correct accounting of the wave damping in simulations, it is necessary to take the simulation window at least twice as wide as the plasma.

Keywords: plasma wakefield acceleration, numerical simulations, halo electrons, wavebreaking

1. Introduction
Plasma wakefield acceleration is a novel technique that promises to increase the electron energy available to research laboratories [1–4]. The wave that accelerates particles must be excited in the plasma by a driver: a laser, an electron or a proton beam. At present, only proton beams have sufficient energy to boost electrons to (sub)TeV energies in a single accelerating stage [5, 6], and there is a path to reach this aim in the foreseeable future [7].

Proton beams in modern high-energy synchrotrons are much longer than the plasma wavelength and can efficiently drive the wave only after micro-bunching, that is, splitting into a train of short bunches either before [8] or inside the plasma [9–11]. The micro-bunches then resonantly excite the plasma oscillations over many periods. During growth, the wave remains linear or weakly non-linear in the sense that the relative change of the wakefield period is small. Therefore, it is important to know how long the wakefield can remain regular and what physical effects limit its lifetime.

There are several effects that can destroy a plasma wave of moderate amplitude: non-linear elongation of the wave period [12–17], ion motion [18–22], and transverse plasma non-uniformity [23–27]. In this paper, we describe one more effect that occurs in a radially bounded plasma. If the drive beam is narrow, the wavefronts of the excited wakefield quickly become curved, and trajectories of some plasma electrons intersect. These electrons fall out of the collective motion and leave the plasma radially, which by itself does not lead to a catastrophic wave damping. However, these electrons form a negatively charged halo around the plasma and repeatedly return to the wakefield region under the action of the charge separation field [28]. The returning electrons interact with the plasma wave and cause its damping, if there are many of them. The net damping appears due to the asymmetry in the number of accelerated and decelerated electrons, which arises because of bending electron trajectories by the plasma wave towards acceleration regions.

The effect was noticed while simulating the AWAKE experiment at CERN [29–31]. In AWAKE, a 400 GeV proton beam self-modulates in the plasma [32–34] and excites the wakefield [35], which then accelerates the externally injected witness electrons [36, 37]. Agreement between simulations and measurements is only achieved for wide simulation windows that correctly accounts for the halo electrons [38, 39]. In a narrow window, the backflow of halo electrons is suppressed, the wakefield persists much longer than in a wide window, and simulation results disagree with experimental observations.

We study the details of plasma wave suppression by halo electrons numerically using the quasistatic axisymmetric code...
Table 1. Parameters for numerical simulations.

| Parameter, notation | Value |
|---------------------|-------|
| Plasma density, $n_0$ | $1.965 \times 10^{14}$ cm$^{-3}$ |
| Plasma skin depth, $c/\omega_P$ | 380 µm |
| Plasma radius, $R_p$ | 1.5 mm ($\approx 4c/\omega_P$) |
| Plasma ion-to-electron mass ratio, $M_i$ | 157 000 |
| Plasma initial temperature, $T_e$ | 0 eV |
| Beam population, $N_b$ | $2.86 \times 10^{11}$ |
| Beam energy, $W_b$ | 400 GeV |
| Beam energy spread, $\delta W_b$ | 0.35% |
| Beam length, $\sigma_z$ | 6.57 cm ($\approx 170 c/\omega_P$) |
| Beam radius, $\sigma_r$ | 200 µm ($\approx 0.5 c/\omega_P$) |
| Normalized beam emittance, $\varepsilon_b$ | 3.6 mm mrad |
| Seed pulse position, $\xi_s$ | 3.75 cm ($\approx 100 c/\omega_P$) |

LCODE [40, 41] and focus on the same set of parameters that was analyzed in [38] as the baseline case (table 1). The self-modulation is seeded by a laser pulse co-propagating with the proton beam at the distance $\xi_s \approx 0.6 \sigma_z$ ahead of its center. The difference between the speeds of proton and laser beams and the speed of light $c$ is insignificant at the considered propagation lengths. The plasma has a sharp boundary in the radial direction [42, 43], which does not change during the beam-plasma interaction, and is surrounded by neutral rubidium vapor, which we simulate as a vacuum region.

The simulation window encloses the entire proton beam and moves with the speed of light in the positive $z$ direction. It has a size of $r_w = 100c/\omega_P$ in the radius and up to $720c/\omega_P$ in the longitudinal coordinate $\xi = z - ct$, where $\omega_P = \sqrt{4\pi n_0 e^2/m}$ is the plasma frequency, $n_0$ is the plasma density, $e$ is the elementary charge and $m$ is the electron mass. The coordinate $\xi = 0$ corresponds to the seed laser pulse position. The grid size is 0.01$c/\omega_P$ in the $r$ and $\xi$ directions, and there are 10 or more (for drawing electron trajectories) radially weighted plasma particles of each sort per cell. We simulate the beam propagation to complete micro-bunching (at $z = 4$ m) with steps $\Delta z = 200c/\omega_P \approx 7.6$ cm and then study the wakefield excited by this beam. We characterize the wakefield strength by the wakefield potential $\Phi$, which shows both radial and longitudinal forces acting on an axially moving unit charge:

$$E_r - B_\phi = -\frac{\partial \Phi}{\partial r}, \quad E_\xi = -\frac{\partial \Phi}{\partial \xi}. \quad (1)$$

In section 2, we describe how the electron halo appears and what are its properties. In section 3, we analyze the energy exchange between the halo and the plasma wave and show that this is the main cause of wave damping. Then, in section 4, we consider the interaction of individual halo electrons with the wave and show that the energy preferably flows from the wave to halo because of bending of electron trajectories by the wave. Finally, in section 5, we summarize the main findings.

2. Motion of halo electrons

Plasma electrons drop out of the wave if the trajectories of inner and outer electrons intersect at a high ridge of electron density [44] (figure 1(a)). The former inner electron continues its radial motion and eventually becomes a halo electron. Other intersections of trajectories, which are also possible in the wave, do not lead to the emission of electrons (figure 1(b)).

The halo electrons appear in certain phases of the wakefield, and then their trajectories diverge, like a jet from a water-hose, and return to the plasma in a wide interval of $\xi$ (figure 2(a)). The distance traveled by an electron outside the plasma depends on the energy $\Delta W_{wb}$ acquired during the wavebreaking and further passing through the plasma column (figure 2(b)). There is no phase correlation between successive points of exit from and entry into the plasma.

Halo electrons can intersect the plasma several times, each time changing the energy by $\Delta W_{ee}$ (figure 2(b)). If the electron crosses the accelerating phase of the wave, it takes the energy from the wave ($\Delta W_{ee} > 0$), and returns it otherwise ($\Delta W_{ee} < 0$). The radial electric field around the plasma column grows as the number of halo electrons increases (figure 3). For this reason, the electrons return to the plasma with a higher energy than when they leave (figure 2(b)).

3. Energy balance

Damping of a plasma wave is always associated with a rapid loss of its energy. To understand the role of halo electrons in this process, we quantitatively characterize the energy exchange between them and the plasma wave.
Let us correlate the amount of energy $\Delta W$ received or returned by a halo electron in a single act of energy exchange and the coordinate $\xi$, at which the electron crosses the axis (for $\Delta W = \Delta W_{ee}$) or where the electron radius has the last local minimum before leaving the plasma (for $\Delta W = \Delta W_{wb}$) (figure 4). These energy changes can then be sequentially summed up, starting from the beam head, to obtain the total energies directly transferred from the wave to the halo:

$$\Psi_{wb}(\xi) = \sum_{\xi_r < \xi} \Delta W_{wb}(\xi_r), \quad \Psi_{ee}(\xi) = \sum_{\xi_r < \xi} \Delta W_{ee}(\xi_r),$$  \hspace{1cm} (2)$$

$$\Psi_{\Sigma} = \Psi_{wb} + \Psi_{ee}. \hspace{1cm} (3)$$

The summations in the formula (2) are carried out over electrons that enter the simulation window per unit of time, so the quantities $\Psi_{wb}$, $\Psi_{ee}$, and $\Psi_{\Sigma}$ have the dimension of the energy flux. We use

$$\Psi_0 = \frac{m^2 c^5}{4 \pi e^2}, \hspace{1cm} (4)$$

as the unit for them (figure 4). Note that $\Psi_{\Sigma}$ does not represent the actual energy content of the halo, as the halo electrons also change their energy while interacting with the electric field around the plasma (figure 2(b)).

In numerical simulations, the behavior of $\Psi_{\Sigma}(\xi)$ depends on the radius of the simulation window (figure 5). The outer boundary of the LCODE simulation window acts as a perfectly conducting wall that reflects plasma electrons and absorbs their energy. If this wall is located close to the plasma, it suppresses halo formation and, as a consequence, the wave damping [38]. Choosing a very distant wall, like in real experiments, is undesirable because of the large computational resources required. A compromise is possible if the simulation window is about twice wider than the plasma. In this case, the boundary of the simulation window still disturbs the trajectories of some halo electrons, but the overall effect of the halo on the wave is almost the same as in a very wide window. We discuss the reasons for this in section 4. Thus, the simulation window width of $100c/\omega_p$ is chosen with a large margin.

To understand how the plasma wave loses its energy, we introduce the energy fluxes in the co-moving window [22, 45]. The total energy flux $\Psi(t_m, \xi)$ is the sum of the electromagnetic part

$$\Psi_{\text{em}} = \int_{t_m}^{t_m+\Delta t} \int \left( \frac{c}{8\pi} (E^2 + B^2) - \frac{c}{4\pi} \left[ E \times B \right]_{z}^z \right) r dr dl, \hspace{1cm} (5)$$

and the kinetic energy part

$$\Psi_{\text{k}} = \sum_j (\gamma_j - 1) m_j c^2, \hspace{1cm} (6)$$
where the summation is over plasma particles (having mass \( m_j \) and relativistic factor \( \gamma_j \)) that intersect the circle of radius \( r_m \) in unit time. One can imagine that a continuous stream of plasma particles enters the simulation window, acquires some energy in some places and transfers it to others, forming energy flows. The electromagnetic field created by the plasma and beam particles also transfers energy and contributes to the energy flux. Plasma particles that reach the upper boundary of the simulation window transfer their energy to it. This energy, integrated from the beam head to some \( \xi \), forms the energy flux to the ‘wall’ \( \Psi_{\text{wall}}(\xi) \). The sum

\[
\Psi_{\Sigma} = \Psi_{\text{wall}} + \Psi(r_w, \xi),
\]

where the total flux \( \Psi \) is calculated over the entire simulation window, characterizes the total energy taken from the driver beam. It grows as the beam excites the wave and remains constant in the intervals of \( \xi \) where there is no energy exchange between the beam and the plasma (figure 6).

The derivative

\[
\Psi'(r, \xi) = \frac{\partial \Psi(r, \xi)}{\partial r},
\]

allows us to visualize areas of the simulation window that contain energy (figure 7). We see the energy of the plasma wave (feature 1), the energy of the return current that appears near the plasma boundary to compensate the current of halo electrons (feature 2), and the energy of halo electrons that penetrate everywhere after the wave begins to break (feature 3). Since the group velocity of the plasma wave is very low, the wave energy remains where it was released by the beam. To calculate this energy as accurately as possible, we define the radius \( r_{\text{ref}} = 2c/\omega_p \) of the wakefield region using the figure 7. However, the total flux

\[
\Psi_{\text{total}} = \Psi(r_{\text{ref}}, \xi),
\]

overestimates the wave energy, since the halo electrons that cross the wakefield region contribute to the energy flux being not a part of the plasma wave. We can improve the expression for the wave energy by noting that in a linear plasma wave, the average energy is equally distributed between the kinetic energy of particle motion and the electric field. The doubled flux of the electromagnetic energy gives us the best expression for the energy contained in the plasma wave:

\[
\Psi_{\text{wf}} = 2\Psi_{\text{em}}(r_{\text{ref}}, \xi).
\]

The difference between \( \Psi_{\text{total}} \) and \( \Psi_{\text{wf}} \) appears when the halo electrons return to the plasma wave region (figure 6); before this, the lines go alike.

The difference

\[
\Psi_{\text{loss}} = \Psi_{\Sigma} - \Psi_{\text{wf}},
\]
characterizes the energy losses from the plasma wave through all possible channels. It coincides with the energy $\gamma$, transferred to the halo electrons (figure 6), proving this to be the dominating channel of wakefield energy loss and the main cause of wave damping. The halo electrons then share this energy with the electromagnetic field around the plasma and with the directionally moving plasma electrons.

4. Lensing by the wakefield

When crossing the axis, the halo electrons take the energy from the wave on average (figure 4). Amounts of energy transferred to or taken from a single electron are comparable, but the number of accelerated electrons is greater, at least when the electrons cross the axis for the first time (at $\xi \sim -200\omega_p/c$ in figure 4). Figure 2(a) shows that the return points of halo electrons to the plasma are smoothly distributed over $\xi$. Consequently, the asymmetry between accelerated and decelerated electrons arises when they interact with the plasma wave. To understand the reasons for the asymmetry, we consider the trajectories of halo electrons that enter the plasma at almost the same angles near the point where $\gamma$ becomes nonzero (figure 8). Initially parallel trajectories deviate from straight lines under the action of the wakefield. The force arises mainly from the electric field, since the magnetic field in a weakly non-linear Langmuir wave is small. The magnetic field of the proton beam in the considered region is also small, because this is the field of individual micro-bunches, while the electric field is created by coherent contributions from tens of micro-bunches. Consequently, the wakefield potential is almost the same as the electric potential. Regions of positive potential (red areas in figure 8) are potential ‘wells’ for electrons, which attract them and bend their trajectories towards regions of strongest acceleration located at the highest potential gradients. And vice versa, the blue areas of negative potential deflect the electrons away from deceleration regions. In the laboratory frame, the halo electrons move predominantly forward (in the positive $z$ direction) [28], so their interaction with the wave is long and efficient. We can envisage this process as if properly phased witness bunches are taking energy from the wave.

The question now arises why the asymmetry of energy exchange is the strongest when the electrons just begin to return to the plasma (figure 4). To answer, let us note the correlation between the asymmetry and the angle $\alpha$ at which the electron enters the plasma in the co-moving frame (figure 9(a)). The smaller the angle, the stronger the asymmetry. Almost all electrons entering the plasma at $\alpha < 20^\circ$ gain energy. The entry angle, in turn, correlates with the electron energy (figure 9(b)): higher-energy electrons enter at larger angles. This is because the entry angle is determined by the size of the arc along which the electron moves outside the plasma (figure 10). A larger arc obviously means that the electron left the plasma with a larger radial momentum, which became even larger on return because of the electric field around the plasma, which increases with increasing $|\xi|$ (figure 3). High-energy electrons propagate in the plasma along almost straight lines without being significantly deflected by the wakefield (figure 11(a)), with the number of accelerated and decelerated electrons being approximately the same.

![Figure 8](image_url) Trajectories of electrons entering the plasma within two wave periods at approximately the same angle $\alpha$, plotted against the background of the wakefield potential $\Phi$. The bottom picture is an enlarged fragment of the top picture, marked with a green rectangle. The black horizontal line in the top picture is the plasma boundary. The red and blue dots on the trajectories at the points of their entry into the plasma indicate the sign (respectively, positive or negative) of the energy change when electrons pass through the plasma. The color of the trajectories shows the change of the electron relativistic factor $\Delta \gamma$ with respect to its value at the plasma boundary.

![Figure 9](image_url) Correlation between the angle $\alpha$ at which the halo electrons enter the plasma, the entry coordinate $\xi$, and (a) the energy gain (red points) or loss (blue points) when passing through the plasma, and (b) the relativistic factor of electrons $\gamma$ (color) at the entry point.
The contributions of these electrons cancel out when averaged over the plasma period, and we observe the constancy of $Y_r$ at $\xi < -350c/\omega_p$ in figure 4. The trajectories of low-energy electrons are strongly bent by the wakefield and cross the axis exclusively in the accelerating phase of the wave (figure 11(b)). These electrons are the earliest to return to the plasma after the wavebreaking (figure 2(a)), and their contribution to wave damping is the strongest. Since the low-energy electrons do not travel too far from the plasma, correct accounting for their contribution does not require a very wide simulation window (figure 5).

It is interesting to note that electrons change both longitudinal and transverse momentum when crossing the plasma wave. Let us consider a ‘red’ trajectory of an accelerated electron from figure 11(a) as an example. The electron first falls into the potential well from the side and increases its radial momentum. Then the electron climbs the potential hump, gaining longitudinal momentum. Finally, it goes down from the potential hump and gains additional radial momentum. For decelerated electrons, all changes in momentum have the opposite sign.

5. Summary

To sum up, halo electrons that appear around a radially bounded plasma as a result of wavebreaking can cause a quick decay of the wakefield. Passing through the plasma, these electrons act like witness beams, taking away the energy of the plasma wave. The most efficient energy exchange occurs with the lowest-energy halo electrons, because their trajectories are bent by the plasma wave towards the regions of strongest acceleration. The low-energy halo electrons do not travel too far from the plasma boundary. Therefore, for correct accounting of the wave damping in AWAKE simulations, it is sufficient to take the simulation window twice wider than the plasma. In this paper, we described the mechanisms of halo formation and wave decay due to the acceleration of halo electrons and left the study of the parameters at which this mechanism dominates for future works.

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References

[1] Esarey E, Schroeder C B and Leemans W P 2009 Rev. Mod. Phys. 81 1229
[2] Downer M C, Zgadzaj R, Debus A, Schramm U and Kaluza M C 2018 Rev. Mod. Phys. 90 035002
[3] Hogan M J 2016 Rev. Accel. Sci. Technol. 9 63
[4] Adli E and Muggli P 2016 Rev. Accel. Sci. Technol. 9 85
[5] Caldwell A, Lotov K, Pukhov A and Simon F 2009 Nat. Phys. 5 363
[6] Caldwell A and Lotov K V 2011 Phys. Plasmas 18 103101
[7] AWAKE Collaboration 2021 CERN Report CERN-SPSC-2021-005/SPSC-SR-285 (Geneva: CERN) (http://cds.cern.ch/record/2749515/files/SPSC-SR-285.pdf)
[8] Sheinman I and Petrenko A 2017 Proc. RuPAC2016 (St. Petersburg, Russia) pp 303–6
[9] Kumar N, Pukhov A and Lotov K 2010 Phys. Rev. Lett. 104 255003
[10] Lotov K V 2015 Phys. Plasmas 22 103110
[11] Lotov K V 2015 Phys. Plasmas 22 123107
[12] Akhiezer A I and Polovin R V 1956 Sov. Phys. JETP 3 696 (http://www.jetp.ac.ru/cgi-bin/ch/6_003_05_0696.pdf)
[13] Dawson J M 1959 Phys. Rev. 113 383
[14] Rosenbluth M N and Liu C S 1972 Phys. Rev. Lett. 29 701

Figure 10. The family of electron trajectories entering the plasma at $\xi \approx -260c/\omega_p$ (green lines), plotted against the background of the radial electric field $E_r$ (color map). The black horizontal line is the plasma boundary.

Figure 11. Trajectories of high-energy (a) and low-energy (b) halo electrons entering the plasma at approximately the same angles, plotted against the background of the radial electric field $E_r$ and the potential $e\Phi/mc^2$. The black horizontal lines show the plasma boundary. The red and blue dots on the trajectories at the points of their entry into the plasma indicate the sign of the energy change when electrons pass through the plasma. The color of the trajectories shows the change of the electron relativistic factor $\Delta \gamma$ with respect to its value at the plasma boundary. The axis ratio in fragment (b) is 1:4 to make the trajectories better visible.
[15] McKinstrie C J and Forslund D W 1987 Phys. Fluids 30 904
[16] Khachatryan A G 1999 Phys. Rev. E 60 6210
[17] Lotov K V 2013 Phys. Plasmas 20 083119
[18] Gorbunov L M, Mora P and Solodov A A 2001 Phys. Rev. Lett. 86 3332
[19] Gorbunov L M, Mora P and Solodov A A 2003 Phys. Plasmas 10 1124
[20] Vieira J, Fonseca R A, Mori W B and Silva L O 2012 Phys. Rev. Lett. 109 145005
[21] Vieira J, Fonseca R A, Mori W B and Silva L O 2014 Phys. Plasmas 21 056705
[22] Spitsyn R I, Timofeev I V, Sosedkin A P and Lotov K V 2018 Phys. Plasmas 25 103103
[23] Andreev N E, Gorbunov L M, Kirsanov V I, Nakajima K and Ogata A 1997 Phys. Plasmas 4 1145
[24] Andreev N E, Chizhonkov E V, Frolov A A and Gorbunov L M 1998 Nucl. Instrum. Methods A 410 469
[25] Marques J R et al 1998 Phys. Plasmas 5 1162
[26] Shvets G and Li X 1999 Phys. Plasmas 6 591
[27] Golian Y, Aslaninejad M and Dorranian D 2016 Phys. Plasmas 23 013109
[28] Lotov K V, Sosedkin A P and Petrenko A V 2014 Phys. Rev. Lett. 112 194801
[29] Caldwell A et al (The AWAKE Collaboration) 2016 Nucl. Instrum. Methods A 829 3
[30] Gschwendtner E et al (The AWAKE Collaboration) 2016 Nucl. Instrum. Methods A 829 76
[31] Muggli P et al (The AWAKE Collaboration) 2018 Plasma Phys. Control. Fusion 60 014046
[32] Turner M et al (The AWAKE Collaboration) 2019 Phys. Rev. Lett. 122 054801
[33] Adli E et al (The AWAKE Collaboration) 2019 Phys. Rev. Lett. 122 054802
[34] Braumuller F et al (The AWAKE Collaboration) 2020 Phys. Rev. Lett. 125 264801
[35] Turner M et al (The AWAKE Collaboration) 2020 Phys. Rev. Accel. Beams 23 081302
[36] Adli E et al (The AWAKE Collaboration) 2018 Nature 561 363
[37] Gschwendtner E et al (The AWAKE Collaboration) 2019 Phil. Trans. R. Soc. A 377 20180418
[38] Gorn A A et al (The AWAKE Collaboration) 2020 Plasma Phys. Control. Fusion 62 125023
[39] Chappell J et al (The AWAKE Collaboration) 2021 Phys. Rev. Accel. Beams 24 011301
[40] Lotov K V 2003 Phys. Rev. ST Accel. Beams 6 061301
[41] Sosedkin A P and Lotov K V 2016 Nucl. Instrum. Methods A 829 350
[42] Oz E and Muggli P 2014 Nucl. Instrum. Methods A 740 197
[43] Demeter G 2019 Phys. Rev. A 99 063423
[44] Gorn A A 2021 Trajectory crossing in low density plasmas due to interaction with a long ultrarelativistic proton beam (in preparation)
[45] Lotov K V 2004 Phys. Rev. E 69 046405
[46] Irkutsk Supercomputer Center of SB RAS 2021 (available at: http://hpc.icc.ru)