An experimental study of forced convective heat transfer from smooth, solid spheres

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Forced convective heat transfer from smooth, solid and isothermal spheres of various diameters has been studied experimentally in air flows with various free-stream velocities. The average heat transfer coefficient has been determined from the steady measured power input to a heating element inside the spheres and the steady measured temperatures of the flowing air and of the surface of the spheres, employing corrections to account for heat transfer due to thermal radiation and due to natural convection. The current data for the average heat transfer coefficient, expressed as a relationship between the Nusselt number and the Reynolds number, complement data in literature with respect to the range of large Reynolds numbers that have been considered: here the Reynolds numbers were between $7.8 \times 10^3$ and $3.3 \times 10^5$. The experimental results show a sudden increase in the Nusselt number above a critical Reynolds number of approximately $2.9 \times 10^5$, analogous to the "drag crisis" for the drag force on the sphere. A correlation for the Nusselt number as a function of the Reynolds number has been formulated for air flows that describes these data well for Reynolds numbers below the critical Reynolds number.

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1. Introduction

Forced convective heat transfer from spherical objects, or objects that are modelled as spheres, is ubiquitous in many disciplines in engineering and science. The rate of heat transfer $Q_{\text{for}}$ due to forced convection is expressed through the average heat transfer coefficient $h$ by

$$Q_{\text{for}} = hA_s(T_f - T_s)$$

(1)

where $A_s$ is the surface area at which convective heat transfer occurs, $T_f$ is the surface temperature of the isothermal sphere and $T_s$ is the temperature of the free-stream fluid that is flowing at velocity $U_\infty$. The diameter of the sphere is denoted by $D$.

Dimensional analysis of the problem of forced convective heat transfer at low fluid speeds (i.e. at low Mach numbers) shows that the Nusselt number $Nu$ is dependent on Reynolds number $Re$ and Prandtl number $Pr$

$$Nu = f(Re, Pr)$$

(2)

These dimensionless numbers are defined by

Here $k$ is the thermal conductivity, $v$ is the kinematic viscosity and $\alpha = k/(\rho c_p)$ is the thermal diffusivity with $\rho$ and $c_p$ the density and the specific heat at constant pressure, respectively. All these fluid properties are based here on tabulated values given in Cengel and Ghajar [8]. These are evaluated here at the film temperature $T_f = (T_s + T_\infty)/2$ (as also done by [32,27,14]).

Many experimental studies have been performed of forced convective heat transfer from isothermal spheres, where the average convective heat transfer coefficient $h$ is given in terms of a correlation of the type in Eq. (2). Here an overview is given of the main experimental studies and correlations.

Kramers [20] performed (steady) experiments with Reynolds numbers in the range $0.4 < Re < 2100$ with air, water and an oil that have different Prandtl numbers $Pr$. Spheres with diameters of 1.26, 0.787 and 0.709 cm were employed. A correlation for the Nusselt number as a function of Reynolds number and Prandtl number was formulated

$$Nu_{\text{Kramers}} = 2 + 1.3Pr^{0.15} + 0.66P^{0.31}Re^{0.50}$$

$$0.4 < Re < 2100$$

$$0.71 < Pr < 380$$

(4)

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The reported uncertainty of this correlation is about 10% for 10 < Nu < 40.

Yuge [32] performed (steady as well as unsteady) experiments in air (with Pr = 0.715) using spheres of different diameters (2.2 and 6.0 cm for measurements at high Reynolds numbers) and different wind tunnels with different air velocities. In addition, combined heat transfer due to forced and natural convection (in cross, parallel and counterflow) was studied. For heat transfer by forced convection only, the results for air for the Nusselt number as a function of Reynolds number were correlated by

\[ Nu_{Yuge} = 2 + 0.493Re^{0.50} \quad 10 < Re < 1.8 \times 10^4 \]
\[ Nu_{Yuge} = 2 + 0.300Re^{0.57} \quad 1.8 \times 10^4 < Re < 1.5 \times 10^5 \] (5)

Vliet and Leppert [29] measured the (steady) rate of heat transfer between a sphere (with diameter of 2.2 cm) and flows of water in a (rather small) water tunnel under conditions where a considerable temperature difference exists between the surface of the sphere and the free-stream water (up to 65 K). The range of Reynolds numbers considered in their measurements is 50 < Re < 5 \times 10^4. As the variation with temperature of the dynamic viscosity of water is significant (in comparison to that of air), this was accounted for in their correlation for water

\[ Nu_{vliet} = \left[ 2.7 + 0.12R^0.66 \right] Pr^{0.5} \left( \frac{\mu_s}{\mu} \right)^{0.25} \quad 50 < Re < 5 \times 10^4 \] (6)

where \( \mu_s \) and \( \mu \) are the dynamic viscosity of water at the surface temperature and at the temperature of the free-stream, respectively.

Thomson and Eckert [27] carefully performed (steady) experiments in air where the Reynolds number was in the range 3.6 \times 10^4 < Re < 5.2 \times 10^4. Diameters of the spheres were 1.27, 2.54 and 5.08 cm. The influence of the position of the support on the rate of heat transfer was also investigated: with a cross-flow support it is about 10% higher than with a rear support. With an increase in turbulence level of the flow (up to about 5%), they observed that the rate of heat transfer increased by 7.5% at Re = 3.6 \times 10^4 and by 17.5% at Re = 5.2 \times 10^4. Their correlation for air describing these measurements with rear support and with a low turbulence intensity of 0.15% is

\[ Nu_{thomson} = 2 + 0.21Re^{0.61} \quad 3.6 \times 10^4 < Re < 5.2 \times 10^4 \] (7)

The maximum reported deviation between measurements and results of this correlation is 2%.

Eastop and Smith [14] developed a correlation for air that is based on theoretical considerations of laminar boundary layers for the front half of the sphere and data from literature for the heat transfer from the rear half of the sphere where the flow has separated. Their correlation for air for the Nusselt number as a function of the Reynolds number is given by

\[ Nu_{eastop} = 0.42Re^{0.50} + 0.0035Re^{0.92} \quad 3.0 \times 10^3 < Re < 1.0 \times 10^5 \] (8)

where the first and second term on the right-hand side correspond to the heat transfer from the front and rear halves of the sphere, respectively.

Based on experimental data from literature (by Kramers [20], Yuge [32] and Vliet and Leppert [29]), Whitaker [30] proposed a correlation that is given in many textbooks (for example [8,23])

\[ Nu_{whitaker} = 2 + 0.4Re^{0.50} + 0.06Re^{0.67} \quad Pr^{0.33} \left( \frac{\mu_s}{\mu} \right)^{0.25} \quad 3.5 < Re < 7.6 \times 10^4 \quad 0.71 < Pr < 380 \] (9)

where 1 < \( \mu_s/\mu \) < 3.2. The fluid properties are evaluated at the free-stream temperature \( T_{\infty} \), except that \( \mu_s \) is the dynamic viscosity at the surface temperature \( T_s \). The maximum deviation reported is 30%.

Ahmed and Yovanovich [4] developed an approximate analytical solution of the energy equation for the limit cases of \( Pr \rightarrow 0 \) and of \( Pr \rightarrow \infty \). These solutions were combined (using some empirical data), to yield a correlation stated to be valid for all Prandtl numbers

\[ Nu_{Ahmed} = 2 + 0.775Re^{0.50} \frac{Pr^{0.33}}{\sqrt{2\gamma + 1} \left[ 1 + \left( \frac{1}{(2\gamma - 1)Pr} \right)^{0.17} \right]} \quad 1.0 < Re < 1.0 \times 10^5 \] (10)

where \( \gamma = Re^{-0.25} \)

It is well-known that the turbulence level Tu of the free-stream influences the heat transfer characteristics from the sphere [6,22,27,16,25,19]. Ahmed et al. [5] incorporated this influence of the turbulence level Tu of the free-stream on the heat transfer characteristics from the sphere. For Tu → 0, their complex relation (represented by their Eqs. (53) and (54)) reduces to the correlation by Ahmed and Yovanovich [4], Eq. (10).

Measurements of local Nusselt numbers in turbulent air flow have been performed by Xenakis et al. [31] for various Reynolds numbers, and by Aufermuer and Joss [6], Galloway and Sage [16] and Hayward and Pei [17] for various Reynolds numbers and turbulence intensities. In the measurements by Aufermuer and Joss [6], Galloway and Sage [16] and Hayward and Pei [17] with low turbulence intensity the local Nusselt number in the laminar boundary layer decreases from the stagnation point towards the point of separation. In the wake region the local Nusselt number increases almost monotonically for the Reynolds numbers that have been considered. Measurements by Xenakis et al. [31] (with side support) have been made at very high Reynolds numbers (up to Re = 4.98 \times 10^5). From the distribution of the local Nusselt number over the sphere (as represented by Clift et al. [12] on their page 120) for Reynolds numbers of Re = 2.59 \times 10^5 and 4.98 \times 10^5 it seems (by the presence of a region with very high local Nusselt numbers) that a turbulent boundary layer is present over part of the rear half of the sphere. These conditions therefore correspond to supercritical values of the Reynolds number.

Experiments on the rate of heat transfer from spheres in the turbulent, outdoor environment have been performed by Kowalski and Mitchell [19], who found that under such conditions the rate of heat transfer is up to twice as large as in low-turbulence laboratory conditions. Experiments on the rate of heat transfer with a fluid with a low Prandtl number of 0.003 have been reported by Melisari and Argyropoulos [24]. Convective heat transfer from rotating spheres in various stationary fluids has been studied experimentally by Kreith et al. [21] and Eastop [15]. An analogy between drag and heat transfer has recently been discussed by Duan et al. [13].

The development of the flow field with changes in Reynolds number is described in detail by Clift et al. [12] in their Section 5.III.2. At the front part of the sphere the flow in the boundary layer is laminar. For Reynolds numbers larger than about 20 the flow at the rear part of the sphere has separated. At a Reynolds number of about 400, the flow becomes unsteady and asymmetric, with periodic vortex shedding. At a critical Reynolds number, Re_{crit} \approx 3 \times 10^5 for smooth spheres, boundary layer transition occurs. The turbulent boundary layer remains attached over part of the rear half of the sphere, resulting in a narrower wake with corresponding lower pressure drag. Therefore, the drag coefficient shows a pronounced drop, the so-called “drag crisis”, at the critical Reynolds number. Analogously, it is expected that there is an increase in the rate of heat transfer.

The experimental study of the rate of heat transfer from spheres has received less attention in the literature than that for cylinders in cross-flow. For cylinders, extensive experimental results for local and total heat transfer rates have been reported for Reynolds
numbers up to $4 \times 10^6$ (for instance [2]). Therefore, the objectives of this experimental study on the (average) rate of heat transfer from spheres are twofold:

- Considering the restricted range in Reynolds number, $0.4 < Re < 1.5 \times 10^5$, for which detailed experiments have been reported in the literature, the focus is on the range of high Reynolds numbers, up to $Re = 3.3 \times 10^5$.
- It is investigated whether an increase in the heat transfer coefficient is observed for Reynolds numbers larger than some critical value, in analogy to the drag crisis for the drag coefficient.

The outline of this study is as follows. The experimental test setup, experimental methods and materials are described in Section 2. The experimental results are presented and analysed in Section 3. Conclusions are formulated in Section 4.

2. Experiments

The employed test facility is described in Section 2.1. The measurement procedure is outlined in Section 2.2 and the data processing is explained in Section 2.3.

2.1. Test facility

The measurements have been performed in a closed-loop wind tunnel (see Fig. 1) with a semi-open test section inside an anechoic chamber. A radial fan with maximum power of 130 kW generated the air flow. A heat exchanger is used to keep the temperature of the air constant in the closed loop. Turning vanes are employed to smoothly change the direction of the flow in the elbows of the wind tunnel. Anti-turbulence screens are installed in the test loop upstream of the test section to reduce the turbulence level of the air flow. A contraction is present just upstream of the test section to limit the boundary layer thickness at the walls. The test section with a square cross-section is 0.9 m wide and 0.7 m high. The test section is shown schematically in Fig. 2.

Two aluminium spheres have been used, with diameters of $D = 60$ mm and $D = 100$ mm. These spheres are called the ‘small’ and the ‘large’ sphere, respectively. Each of the spheres has been assembled from two solid hemispheres that were afterwards glued together. Cavities have been made into the hemispheres in order to insert heating elements, thermocouples and wiring. Space left after insertion of the heating elements, thermocouples and wiring was filled with heat conducting paste. The spheres have been painted.

Fig. 1. Closed-loop wind tunnel: 1 Radial fan with 130 kW electric motor; 2 heat exchanger; 3 turning vanes; 4 anti-turbulence screens; 5 contraction 10:1; 6 test section (0.9 m wide, 0.7 m high); 7 anechoic chamber. Flow direction is clockwise; in the test section, the $x$ and $z$ directions correspond to the flow direction and the vertical direction, respectively.

Fig. 2. Schematic diagram of the test section: side view (left) and top view (right). All dimensions in mm. The wing with NACA 0012 profile can be positioned horizontally such that only a single sphere is present in the centre of the test section in each of the measurements. Configurations shown on the right in green and brown are for the measurements with the large sphere and the small sphere, respectively. In the test section, the $x$ and $z$ directions correspond to the flow direction and the vertical direction, respectively, while the $y$ direction is from side wall to side wall.
The roughness of the spheres is 4 μm, as determined with a VK 9700 confocal microscope (based on a surface of 1.4 mm by 1.1 mm).

The heating elements (high density cartridge heaters of type GC-cart by GC-heat) are of cylindrical shape. Their dimensions are: length of 40 mm, diameter of 6.5 mm for the small sphere and length of 50 mm, diameter of 8.0 mm for the large sphere. The maximum power of these heating elements is 160 W and 315 W for the small and large spheres, respectively. The input power has been measured with a power meter (by Voltcraft, type Energy Check 3000). The uncertainty of the measurement of the input power is 1% + 1 W.

The spheres are supported downstream by cylindrical fibreglass tubes to an aerodynamically-shaped ‘wing’ with symmetric (NACA 0012) profile that is located downstream of the spheres (see Fig. 2). The wing is attached (on the sides) to a frame that can be positioned such that in each measurement only a single sphere is located in the centre of the test section (the other sphere then is located near a wall of the test section). This frame is connected to the floor of the anechoic chamber and to the sides of the test section in order to prevent it from vibrating. The centres of the spheres were located 190 mm upstream from the outlet of the test section to measure the air temperature and through the supports, as justified in Appendix A). The rate of heat transfer due to natural convection is denoted by $Q_{\text{nat}}$. Hence the steady power balance reads

$$Q_{\text{tot}} = Q_{\text{nat}} + Q_{\text{rad}}$$

Wind tunnel off

The rate of heat transfer $Q_{\text{tot}}$ due to natural convection is estimated, based on a correlation for the Nusselt number $Ra$ and the Prandtl number $Pr$ [10]

$$Nu_{\text{tot}} = 2 + \frac{0.589Ra_{1/4}}{1 + (0.469/Pr)^{1/6}}$$

where $Ra = g\beta(T_s - T_{\infty})D^4/\nu^2Pr$, $g$ is the gravitational acceleration, $\beta$ is the thermal expansion coefficient at constant pressure of the air and $\nu$ is the kinematic viscosity of the air. According to Lienhard and Lienhard [23], this correlation has an estimated uncertainty of 5% for air.

Results for various values of the input power $Q_{\text{tot}}$ have been used to obtain the values of the emissivity: $\epsilon = 0.9$ for the small sphere and $\epsilon = 1.0$ for the large sphere. These values agree with the range of emissivities given by Cengel and Ghajar [8] for paints of $\epsilon = 0.8 – 1.0$.

For the measurements of the forced convection heat transfer coefficients, the power input of the heating element $Q_{\text{tot}}$ has been regulated as well as the free-stream velocity $U_\infty$ through the power of the fan (see Fig. 1). With a set power of the heating element and a set free-stream velocity, the resulting surface temperature of the sphere $T_s$ and the air temperature $T_{\infty}$ have been measured with the thermocouples. These have been monitored as a function of time until steady values were observed. Typically, measurements have been taken about two minutes after a steady state appeared to have been reached. The temperature difference $T_s - T_{\infty}$ varied in the range of 39–114 K.

To assess the reproducibility of the measurements, each measurement at specified free-stream air velocity $U_\infty$ and power input $Q_{\text{tot}}$ has been performed four times, on different days with (slightly) different atmospheric conditions.
2.3. Data processing

At lower air speeds heat transfer by natural convection may be of some significance. Under these conditions combined heat transfer occurs [32,11,8]. To correct for combined heat transfer due to forced convection and by natural convection (once again, heat conduction losses through the support of the spheres have been neglected), the approach by Churchill [11] is followed, where the rate of forced convective heat transfer \( Q_{\text{tot}} \) is determined by

\[
Q_{\text{tot}} - Q_{\text{rad}} = Q_{\text{emb}} = \left( Q_{\text{for}} + Q_{\text{nat}} \right)^{1/n}
\]

(14)

Here \( Q_{\text{emb}} \) is the combined rate of convective heat transfer due to forced and natural convection. In Eq. (14), heat transfer due to thermal radiation and due to convection have been considered as independent, noninteracting physical processes. This method of correcting for heat transfer due to natural convection and due to radiation has also been employed by Ahmed et al. [5], with \( n = 1 \). Here a value of \( n = 4 \) has been used, as recommended by Churchill [11] for heat transfer from spheres in cross-flow (with respect to the directions of forced and natural convective flows). Yuge [32] and Raithby and Eckert [27] corrected the measured total power \( Q_{\text{tot}} \) from the heating element for heat transfer by thermal radiation only. They used a value for the emissivity \( \epsilon \) that had been obtained from separate measurements.

The rate of forced convective heat transfer \( Q_{\text{for}} \) is determined according to Eq. (14) from the measured power input to heating element \( Q_{\text{emb}} \) and the measured temperatures of the sphere \( T_s \) and of the air \( T_a \), thus correcting for the rate of heat transfer due to thermal radiation \( Q_{\text{rad}} \) according to Eq. (11) and due to natural convection \( Q_{\text{nat}} \), using Eq. (13) to determine the natural convection heat transfer coefficient.

The (average, forced convection) heat transfer coefficient \( h \) is determined according to Eq. (1) from the measured temperature difference \( T_s - T_a \) between surface of the sphere and the free-stream and the rate of forced convection heat transfer \( Q_{\text{for}} \) obtained from Eq. (14); the total surface area of the sphere has been used. The measured average convective heat transfer coefficient \( h \) and the free-stream velocity \( U_{\infty} \) are converted to the dimensionless Nusselt number \( Nu \) and the Reynolds number \( Re \) defined in Eq. (3). Material properties are evaluated at the film temperature \( T_f \).

For the considered free-stream velocities \( U_{\infty} \) the maximum in the relative standard deviation in the measured average heat transfer coefficient \( h \) for the four reproducibility measurements is 1%. The uncertainty in the measurements of the Nusselt number \( Nu \) and the Reynolds number \( Re \) is determined according to Eq. (13) from the measured temperature difference \( T_s - T_a \). Also shown are the predictions by the correlations by Yuge [32], Raithby and Eckert [27] and Eastop and Smith [14], given by Eqs. (5), (7) and (8) respectively, that are based on measurements with cylinders, Achenbach [2] reports a decrease of the Nusselt number at the critical Reynolds number. The measurements of the distribution of local Nusselt numbers over the sphere by Xenakis et al. [31] (as represented by Clift et al. [12] on their page 120) indicate supercritical flow at a Reynolds number of \( Re = 2.59 \times 10^5 \).

3. Results and discussion

The experimental results for the dependence of the Nusselt number \( Nu \) on the Reynolds number \( Re \) are shown in Fig. 3 for the large and the small sphere. The Nusselt number is a rapidly increasing function of Reynolds number. It is clear that the results for the small and large spheres, expressed in dimensionless form, are essentially identical (the relative deviation between the Nusselt numbers for the large and the small sphere, at identical Reynolds number, is smaller than 1.4%), as expected from the dimensional analysis, Eq. (2). As discussed in Appendix A this corroborates the assumption that heat conduction losses through the supports were not important.

Between \( Re = 2.84 \times 10^5 \) and \( Re = 2.93 \times 10^5 \), a sudden increase (by 19%) in the Nusselt number is observed, from \( Nu = 591 \) to \( Nu = 703 \). This is analogous to the drag crisis for the drag coefficient where the drag coefficient suddenly decreases at a critical Reynolds number \( Re = 3.0 \times 10^5 \) for smooth spheres. The existence of such a critical Reynolds number for heat transfer has also been mentioned by Achenbach [3]. Note that for similar measurements with cylinders, Achenbach [2] reports a decrease of the Nusselt number at the critical Reynolds number. The measurements of the distribution of local Nusselt numbers over the sphere by Xenakis et al. [31] (as represented by Clift et al. [12] on their page 120) indicate supercritical flow at a Reynolds number of \( Re = 2.59 \times 10^5 \).

Also shown in Fig. 3 are the predictions by the correlations by Yuge [32], Raithby and Eckert [27] and Eastop and Smith [14], given by Eqs. (5), (7) and (8) respectively, that are based on measurements in air. These results are shown for the reported range of validity in Reynolds number. The current results for the Nusselt number are higher than those of Yuge [32] and Raithby and Eckert [27]. In comparison with the correlation by Raithby and Eckert [27], the current result is 8% higher at \( Re = 1.0 \times 10^6 \) and 14% higher at \( Re = 5.2 \times 10^6 \). For large values of the Reynolds number the current results agree well with those of Eastop and Smith [14]: the current result is 6% higher at \( Re = 1.0 \times 10^6 \) and 2% higher at \( Re = 5.2 \times 10^6 \).

It is well known [6,22,27,16,25,19] that the free-stream turbulence intensity can have a significant influence on the heat transfer characteristics. This could be a cause for the current Nusselt numbers being larger than those measured by Yuge [32] and Raithby and Eckert [27]. To investigate this possibility the theory of Ahmed et al. [5] is used, as it gives a prediction for the influence of the turbulence intensity on the heat transfer rate. The current experimental results are compared to predictions by their theory in Fig. 4 for the actual measured turbulence level, a turbulence level of 1% and of 2%. Fig. 4 shows that a difference in turbulence intensity does not constitute the origin of the differences between the measurements by Yuge [32] and Raithby and Eckert [27] and the current experimental results.
Differences in blockage ratio (ratio between cross-sectional areas of sphere and test section) could also lead to discrepancies between measurements [26]. Achenbach [1] systematically investigated the influence of the blockage ratio on the drag coefficient of a sphere. Based on his data, the increase in the drag coefficient for the current test tests would be smaller than 0.5%. Hence, wind tunnel blockage is not considered to be important here.

To assess the influence of the correction method for rate of heat transfer due to natural convection through Eq. (14), the Nusselt number has been determined in the following ways: without any correction, with \( n = 4 \) and with \( n = 1 \) in Eq. (14). The results are shown in Table 1. With \( n = 4 \), the correction has only a small influence. However, with \( n = 1 \) as used by Ahmed et al. [5] the Nusselt number becomes very low, also in comparison with measurements by Yuge [32] and Raithby and Eckert [27], for the experiment with Reynolds number \( Re = 10^4 \). Therefore, the correction with \( n = 4 \) has been used here, as recommended by Churchill [11].

Based on the combined results for the small and the large sphere, a correlation has been formulated for the relation between the Reynolds number \( Re \) and the Nusselt number \( Nu \), for air:

\[
N_u = 2 + ARe^{1/2} + BRe
\]

\[
A = 0.493 \pm 0.015 \quad B = 0.0011 \times (1 \pm 0.035)
\]

(15)

for Reynolds numbers in the range \( 7.8 \times 10^3 \leq Re \leq 2.9 \times 10^5 \).

In expression (15) it has been taken into account that in the fully-conductive limit, \( Re \to 0 \), the surface-averaged Nusselt number over the sphere \( Nu \to 2 \) and that for low values of the Reynolds number \( Re \) the Nusselt number \( Nu \) scales with the square root of the Reynolds number, corresponding to heat transfer in laminar boundary layers at the front part of the sphere. A least-squares method (weighted using the estimated total uncertainties) has been employed, with (subcritical) data for which \( Re \leq 2.9 \times 10^5 \), to determine the coefficients \( A \) and \( B \). The resulting fitted \( A \) and \( B \) are given in Eq. (15), together with their confidence 95% interval. Note that the value of the fitted coefficient \( A \) agrees well with the corresponding coefficient in the correlation by Yuge [32], Eq. (5), for low Reynolds numbers. The mean relative deviation between the fit and the combined data for the large and the small sphere is 1.6%.

Alternatively, a description as \( Nu = 2 + CRe^{2/3} \), where the exponent of 2/3 has been suggested by Richardson [28] for separated flows (see also Eqs. (6) and (9)) based on an analogy between heat transfer characteristics for natural convection and those for forced convection, also leads to a good fit (with \( C = 0.12 \) and a mean relative deviation between the fit and the combined data for the large and the small sphere of 2.2%). Including a term \( Re^{1/2} \) into such a fit to account for the heat transfer in laminar boundary layers at the front half of the sphere with a positive weight factor does not lead to a satisfactory fit.

The correlation by Churchill and Bernstein [9] for heat transfer from cylinders that is given in many textbooks can also be very well represented by an expression \( Nu = K + LEre^{1/2} + MRe \) that is analogous to that in Eq. (15).

The fit of the current data, Eq. (15), involves an exponent of 1 that differs from the exponent of 2/3 suggested by Richardson [28] for separated flows, as present at the rear half of the sphere. To further investigate this difference, data on local Nusselt numbers from literature have been analysed. Aufdermauer and Joss [6] and Galloway and Sage [16] report measurements of local Nusselt numbers for Reynolds numbers in the range \( Re = 4.1 \times 10^3 \) to \( 6.8 \times 10^5 \). Based on their tabulated values for the local Nusselt numbers, surface-averaged Nusselt numbers have been computed for the front and rear parts of the sphere by integration of the spline-interpolant to their data points. The results are shown in Fig. 5, where data by Aufdermauer and Joss [6] without turbulence grid and by Galloway and Sage [16] with turbulence intensities smaller than 2% have been used. The dependence of the surface-averaged Nusselt numbers over the front and rear parts of the sphere on the Reynolds number \( Re \) has been fitted to power-law expressions of the expression \( Nu - 1 = ERRe^2 \). In this expression it has been taken into account that in the fully-conductive limit, \( Re \to 0 \), the surface-averaged Nusselt number over half of the sphere \( Nu \to 1 \). The exponent resulting from the fit to the data represented in Fig. 5 for the

\[
Nu = \frac{1}{100} \left[ 1 + 0.003 \times \frac{Re}{10^3} \right]^2
\]

(16)

for Reynolds numbers in the range \( 1 \times 10^3 \leq Re \leq 7 \times 10^5 \), with a mean relative deviation between the fit and the combined data for the large and the small sphere of 3.5%.

In Eq. (15), the correction with \( n = 4 \) as recommended by Churchill [11] has been used for the fully-conductive limit, \( Re \to 0 \), with \( n = 1 \) in Eq. (14).

Table 1

| Method       | \( Re \times 10^3 \) | \( Re \times 10^4 \) |
|--------------|----------------------|----------------------|
| None         | 62.3                 | 266.6                |
| \( n = 4 \)  | 62.2                 | 256.6                |
| \( n = 1 \)  | 46.9                 | 252.5                |

Fig. 4. Results for the (average, forced convection) heat transfer coefficient \( h \) from the experiments with the small and the large sphere in terms of the relationship between the Reynolds number \( Re \) and the Nusselt number \( Nu \). Also shown are the predictions according to the theory of Ahmed et al. [5] for turbulence levels Tu of 2%, 1% and the actual measured turbulence intensity.

Fig. 5. Results for the surface-averaged Nusselt numbers over front and rear parts of sphere, based on data from Aufdermauer and Joss [6] and Galloway and Sage [16].
Nusselt number for the front part is \( p = 0.52 \), while that for the rear part of the sphere is \( p = 0.88 \). The power \( p = 0.52 \) for the front part agrees well with that expected for heat transfer in laminar boundary layers (\( p = 1/2 \)), while the fitted exponent \( p = 0.88 \) for the rear part is closer to the exponent \( p = 1 \) present in Eqs. (8) and (15) than to an exponent \( p = 2/3 \) that has been suggested by Richardson \[28\]. Thus, this detailed analysis of local Nusselt number for heat transfer due to thermal radiation and due to natural convection. The total uncertainty in the measurements of the Nusselt number for the rear part is closer to the exponent \( p = 58 \) than to an exponent \( p = 2/3 \) that has been suggested by Richardson \[28\]. Thus, this detailed analysis of local Nusselt numbers shows that the correlation of the current data given by Eq. (15) is consistent with the data reported by Aufdermauer and Joss \[6\] and Galloway and Sage \[16\].

4. Conclusions

Forced convective heat transfer from smooth, solid and isothermal spheres in air flows has been studied experimentally for a wide range of free-stream velocities. From steady measurements of the air and surface temperatures and of the power input to heating elements inside the two considered (small and large) spheres, the (average, forced convection) heat transfer coefficients have been determined. Corrections have been performed to account for heat transfer due to thermal radiation and due to natural convection. The total uncertainty in the measurements of the Nusselt number is estimated to be 4.0% on average for Reynolds numbers in the range \( 7.8 \times 10^3 < Re < 2.9 \times 10^5 \); for larger \( Re \) it is in the range 4–8%. The experimental results for the small and the large sphere agree, as expected, in terms of the relation between Nusselt and Reynolds numbers. In comparison to previous studies, the range of considered Reynolds numbers is larger: \( 7.8 \times 10^3 < Re < 3.3 \times 10^5 \). The experimental results show a sudden increase in the heat transfer coefficient above a critical Reynolds number of \( 2.9 \times 10^5 \), analogously to the “drag crisis” for the drag force on the sphere. A correlation, Eq. (15), has been formulated that well describes the relation between the Nusselt and Reynolds number \( (below the critical Reynolds number) \) for flows of air.

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Appendix A. Heat conduction losses

Auxiliary heating elements in the support of the sphere have been used by Yuge \[32\] and Raithby and Eckert \[27\] to eliminate heat conduction losses through the support. Such heating elements have not been employed here, instead the supports were made of insulating fibreglass material. In this appendix the heat conduction losses are considered in more detail.

In the worst-case situation where the temperature and the heat transfer coefficient at the surface of the support are the same as those at the sphere, this leads to an increase (in comparison to the case where there are no heat conduction losses) in the rate of heat transfer, and hence of the apparent heat transfer coefficient for the sphere, by a factor \((1 + 0.58)/(1 + 0.22) \approx 1.3 \) than that of the large sphere. Such a discrepancy is clearly not observed in Fig. 3.

In a more realistic analysis of the heat conduction losses the support can be considered as a cooling fin (see also Yuge \[32\]). Expressions for the fin efficiency, i.e. the ratio between the actual rate of heat transfer from the fin over the rate of heat transfer of a fin whose temperature is equal to that of the sphere, are given by Cengel and Ghajar \[8\], for example. The case that is relevant here is where the temperature of the tip of the fin is equal to the air temperature, as the support is attached to an airfoil-type structure made of well-conducting material and with a large surface area. For this case the fin efficiency \( \eta_{\text{fin}} \) is given by

\[
\eta_{\text{fin}} = \frac{Q_{\text{fin}}}{h_p L (T_S - T_a)} = \frac{\cosh ml}{ml \sinh ml} \left( \frac{m^2}{2} \right) = \frac{h_p p}{k_f A_f} \quad (16)
\]

where \( h_p \) is the heat transfer coefficient at the fin surface, \( p \) is the perimeter of the cylindrical fin, \( L \) is its length, \( A_f \) is the cross-sectional area of the conducting part of the fin and \( k_f \) is the thermal conductivity of the fin material. Assumptions implicit in this expression are discussed by Cengel and Ghajar \[8\].

With the geometrical characteristics of the supporting tubes as given in Section 2.1 and the heat transfer coefficient at the fin surface \( h_p \) taken as equal to the heat transfer coefficient at the sphere surface, the heat conduction losses are estimated to be smaller than 1% of the power input.

Appendix B. Temperature distribution inside sphere

For the experiments the temperature inside the sphere is considered to be uniform due to the high thermal conductivity of the aluminium \( k_{alu} \), as the Biot number, \( Bi = hR/k_{alu} \) with \( R \) the sphere radius, is small: for the large sphere the maximum value the Biot number over the range of considered Reynolds numbers is 0.06.

Here the variation of the temperature distribution inside the sphere is analysed, theoretically and numerically, in more detail. This temperature distribution is described by the steady, three-dimensional heat conduction equation. By assuming that the heating element is spherical in shape and that the heat transfer coefficient is constant in circumferential direction (but may vary in other directions) the heat conduction equation in cylindrical coordinates \((r, \phi, z)\) is given by

\[
\frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial \phi} \left( \frac{\partial T}{\partial \phi} \right) = 0 \quad (17)
\]

The thermal conductivity \( k_{alu} \) has been taken uniform. The \( z \)-axis is in the flow direction and the point \((r = 0, z = 0)\) is located at the centre of the sphere.

Boundary conditions for this heat conduction equation are a convective boundary condition at the surface of the sphere and a uniform heat flux at the outer location of the (assumed spherical) heating element.

If the heat transfer coefficient is constant over the surface of the sphere, the local temperature is solely a function of the distance from the centre of the sphere, given by \( \sqrt{r^2 + z^2} \). This implies that the surface temperature is uniform in this case. The temperature distribution then is given by

\[
T(r, z) - T_a = \frac{hR}{k_{alu} \sqrt{r^2 + z^2} - 1} \quad (18)
\]

This equation confirms that the temperature inside the sphere can be considered to be uniform if the Biot number is small.

At the front stagnation point the local heat transfer coefficient will be higher than elsewhere at the surface of the sphere. To assess the influence of the variation of the heat transfer coefficient
on the distribution of the temperature over the surface of the sphere, the heat conduction Eq. (17) has been solved numerically using a finite element method. With a heat transfer coefficient that is constant over the surface of the sphere, the numerical solution for this case agrees (with excellent accuracy) with the analytical solution given by Eq. (18).

For the large sphere, the variation of the heat transfer coefficient over the surface of the sphere according to measurements of Aufdermauer and Joss [6] has been considered for their largest Reynolds number Re = 66,000. This variation of the local Nusselt number Nu is shown in Fig. 6 (left). At the inner boundary a uniform heat flux has been prescribed. With these boundary conditions, the heat conduction Eq. (17) has been solved numerically using a finite element method, employing a mesh with about 100,000 nodes. The resulting distribution of the (outer) surface temperature is shown in Fig. 6 (right). The difference between the local surface temperature Ts and the surface-averaged temperature Ts is smaller than 0.5% in this case. At the location of the thermocouple, z = 150°, the difference is about 0.25%. As expected, the temperature is high at locations where the heat transfer coefficient is low. This more detailed analysis confirms that the temperature is practically constant over the surface of the sphere.

**Appendix C. Uncertainty of measurements**

Here the uncertainty of the measurements is estimated (conservatively), following the methodology described by Beckwith et al. [7], for instance. The bias error of the measurement of the surface and the air temperature by the thermocouples is estimated at 0.5 K. The bias error due to the non-uniformity of the surface temperature is estimated at 0.25%, see Appendix B. The bias error of the measurement of the power input by the heating element is estimated at 1% + 1 W, while the bias error due to heat conduction losses through the supports is estimated at 1%, see Appendix A.

The precision error of the measurements of the temperature difference Ts − Tw and of the power input Qtot has been propagated to the total uncertainty for the Nusselt number Nu. For Reynolds numbers 7.8 × 10³ ≤ Re ≤ 2.9 × 10⁷ the total uncertainty is smaller than 5.0%; the average uncertainty in this range is 4%. For Re > 2.9 × 10⁷ the total uncertainty is in the range 4–8%. For these high, supercritical Reynolds numbers the rate of heat transfer is higher, leading to lower temperature differences (due to input power limitations of the heating elements) and larger corresponding (relative) bias errors. In addition, the precision errors in this range were also larger. In all cases, the uncertainty in the Reynolds number Re is much smaller than the uncertainty in the Nusselt number Nu.

**Appendix D. Supplementary material**

Supplementary data associated with this article can be found, in the online version, at [http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.02.018](http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.02.018).

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