Mathematical model of the submerged jet for the cases of man-made spills

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Abstract. The paper considers a mathematical model of the spread of a multiphase submerged jet, the flow of which is associated with a well damage during the development of a field or pipeline during oil transport. The jet consists of oil droplets and gas bubbles, as well as water involved into the jet. The case is considered when the environmental conditions correspond to the conditions of a stable hydrate existence, because a hydrate shell begins to form and the gas bubble turns into a hydrate on the surface of the bubbles. The integral Lagrangian control volume method is used to describe the jet spread process. The thermo-physical characteristics of a multiphase submerged jet, depending on the vertical coordinate, as well as the trajectory of the jet, are obtained.

1. Introduction

Due to the increase in oil production in the deep-water fields of the world’s oceans, the probability of oil spills into the ocean increases. Often such spills are man-made, occurred as a result of well, oil pipelines and drilling rigs damage. The cause of the spill may be corrosion of the pipeline, mechanical damage to the oil production structure, fire, etc. Oil spills in the Gulf of Mexico on the Deepwater Horizon platform, Ixtoc-1, as well as in the Gulf of Alaska Exxon Valdez are the prime examples indicating the need to study the leaks eliminating methods \([1,2]\). Over the past decade, more than 1 billion gallons of oil have been spilled into the world's oceans as a result of accidents around the world, indicating the serious scale of the problem. \([3]\)

In this regard, there is a need to study the characteristics of the spread of the petroleum products that fall into the ocean as a result of accidental spills. Forecast migration of petroleum products will reduce the time of the leakage elimination \([4,5]\). To do this, it is important to study the features of the oil and associated gas spread, their interaction with the surrounding water. The study of the undercurrents effects on the oil spread in the environment is also important. To predict the spread of a multiphase submerged jet, the integral Lagrangian control volume method described in \([6-8]\) is used. According to this method, the jet is considered as a sequence of control volumes, each characterized by the temperature, speed, density and volume content of the components included in the control volume. Knowing these parameters of the control volume, you can get similar information about the jet.
Under the conditions of stable hydrate existence on the bubbles surface, a hydrate shell is formed, turning the gas bubble into a hydrate one. The process of hydrate formation should be given sufficient attention, as it can affect the dynamics of the jet. An important role is played by the flow of the environment, which acts on the submerged jet. Under the effects of this flow, the jet can deviate, changing its trajectory, radius and concentration of substances entering the jet.

2. Basic equations

Let us assume there is some source with radius \( r \) at the bottom of the reservoir from which a mixture of oil and gas flows with a known initial temperature \( (T^o) \) and volume flow rates \( (Q^o) \) and \( (Q^g) \). The temperature \( (T_w) \) and the velocity \( (\vec{V}_w) \) of the surrounding water are also known. Oil spreads in the form of droplets, and gas leaves in the form of bubbles, which at the initial moment are in the center of the jet. Because of the surrounding water flow, the jet can deviate, and the bubbles leave the jet.

The integral Lagrangian control volume method (ILCVM) proposed in [7,8] is used to simulate the flow of a submerged jet. According to this method the jet (figure 1) consists of a series of control volume (CV) cylinders, characterized by the height \( h \), radius \( b \) and spatial coordinates \((x, y, z)\), corresponding to the center of the axial section of the cylinder (hereinafter the center of the cylinder).

![Figure 1. Scheme of a submerged jet.](image)

The equation of mass conservation CV:

\[
\frac{dM}{dt} = \rho_w Q_w - \rho_{com} Q^f, \quad \rho = \sum_i \alpha_i \rho_i, \quad \sum_i \alpha_i = 1,
\]

here \( M = \sum_i M_i \) (i = o, g, h, w) is the CV mass; \( Q^f = Q^g + Q^h \) is the volume bubbles flow "leaving" the CV due to the jet curvature, \( \alpha_i \) is the volume contents of the corresponding components in the CV, \( \rho_{com} \) is the bubble density, that can be gaseous, and gaseous with a hydrate shell or completely hydrate. Here and further the subscripts o, g, h, w refer respectively to the parameters of oil, gas, hydrate and water. The initial values of the size of the control volume correspond to the radius of the well \( r \), where hydrocarbons come from: \( h_0 = r, b_0 = r \).

Due to the formation of hydrate shells on the gas bubbles surface, the gas, hydrate and water masses change as follows:

\[
\frac{dM_g}{dt} = -J_g - \rho_g Q^f, \quad \frac{dM_h}{dt} = J_h - \rho_{hh} Q^h,
\]

(2)
here $M_i$, $\rho_i$, $i = o$, $g$, $h$, $w$ are the mass and density of the corresponding components in the CV, $Q_w$ is the volume flow of the surrounding water involved in the jet, accordingly, $J_w$, $J_g$ are the intensity of water and gas consumption in the hydrate formation, $J_h$ is the intensity of gas hydrate formation.

Gas, water and hydrate flow intensities are related by the following relationships:

$$J_g = G \cdot J_h, J_w = (1 - G)J_h, J_h = N \frac{4\pi a_{gh}^2}{3} J_h,$$  

where $G$ is the hydrate number, $N$ is the number of bubbles in the CV, $a_{gh}$ is the radius of composite bubbles consisting of a gas core and a hydrate shell, $J_h$ is the intensity of hydrate formation related to the bubble surface area.

The equations of the momentum conservation for CV formulated taking into account the action on the jet of three-dimensional flow of the environment have the following form:

$$\frac{d}{dt}(Mu) = u_w \rho_w Q_w - u \rho_{com} Q',$$  

$$\frac{d}{dt}(Mv) = v_w \rho_w Q_w - v \rho_{com} Q',$$  

$$\frac{d}{dt}\left[(M_w + M_g)w + (M_g + M_h)\cdot(w + w_b)\right] = w_w \rho_w Q_w - (w + w_b) \rho_{com} Q' +$$  

$$+ (\rho_w - \rho)\pi b^2 h (\alpha_w + \alpha_e) g + (\rho_w - \rho_{com}) \pi b^2 h (\alpha_w + \alpha_h) g,$$

where $u$, $v$, $w$ are the CV velocity $\overrightarrow{V} = u\overrightarrow{i} + v\overrightarrow{j} + w\overrightarrow{k}$ components, $u_w$, $v_w$, $w_w$ are the components of the water flow velocity $\overrightarrow{V}_w = u_w\overrightarrow{i} + v_w\overrightarrow{j} + w_w\overrightarrow{k}$ of the ambient CV, $g$ is the gravity acceleration, $\rho = \rho_o \rho_w + \rho_o \rho_g$ is the density of liquid component in CV.

The CV coordinates are defined by the following expressions:

$$\frac{dz}{dt} = w, \quad \frac{dx}{dt} = u, \quad \frac{dy}{dt} = v.$$  

The equation of conservation of energy CV has the following form:

$$\frac{d}{dt}(cMT) = c_w T_u \rho_w Q_w + J_h L - J_w c_w T_h, \quad c = \sum c_i \chi_i,$$

where $c$, $c_w$, $T$, $T_w$ is the heat capacity and the temperature of CV and water, $L$ is the heat of the phase transitions. $c_i, \chi_i = \frac{M_i}{M} (i = o, g, h, w)$ are the heat capacity and the mass content of oil, gas, hydrate and water in CV.

3. Calculation results

To analyze the thermo-physical characteristics of the submerged jet, we solve equations numerically (1)-(5). Let us consider the initial parameters of the system corresponding to the conditions of stable hydrate existence. Here and further the calculations are given for the initial parameters of the system: $r = 0.25$ m, $Q_o = 0.1$ m$^3$/s, $Q_g = 0.1$ m$^3$/s, $T_w = 4$°C, $T_e = 80$°C, $p = 15$ MPa, $\rho_w = 1030$ kg/m$^3$, $\rho_o = 650$ kg/m$^3$, $\rho_g = 99.4$ kg/m$^3$, $\rho_h = 910$ kg/m$^3$, $\varphi = 90$°, $\theta = 0$°, $U_w = 1$ m/s, $U_o = 0$ m/s, $W_w = 0$ m/s, $c_o = 2090$ J/(kg K), $c_g = 2365$ J/(kg K), $c_h = 2100$ J/(kg K), $c_w = 4200$ J/(kg K), $\lambda_w = 0.58$ W/(m·K), $\mu_w = 1.57 \cdot 10^{-3}$ Pa·s, $L = 5 \cdot 10^{5}$ J/kg, $G = 0.12$, $D^* = 10^{-10}$ m$^3$/s, $s = 9.8$ m/s$^2$.

The trajectory of the jet is given in the figure 2a. In the graphs we see that the jet is strongly deflected under the influence of the flow of the environment. Because of the process of involving the surrounding water into the jet, the jet expands.

The dependence of the jet temperature on the vertical coordinate is given in the figure 2b. The jet temperature is calculated for two cases: the line 1 corresponds to the case when hydrate formation is
limited by heat exchange, the case when the hydrate formation is limited by the gas diffusion through the hydrate shell is represented by the line 2. The jet temperature decreases because of the involvement of cold ambient water into the jet and reaches the equilibrium temperature of hydrate formation $T_{hs} = 17^\circ C$ at the height $z = 0.5$ m. You can see also in the graph that the line 2 is located above the line 1. This is due to the exothermic process of hydrate formation, in which the heat releases heating additionally the jet.

Figure 2. Jet trajectory (a), the dependence of the jet temperature on the vertical coordinate (b).

The dependence of the jet density on the vertical coordinate is presented in the figure 3a. The jet consists only of oil droplets and gas bubbles at the initial stage. Then the surrounding water begins to flow into the jet, as a result of which the jet density increases.

Figure 3. The dependence of jet density (a) and jet velocity (b) on vertical coordinate.

The dependence of the CV velocity on the vertical coordinate $z$ for two cases is presented in the figure 3b. The line 1 corresponds to the case when the hydrate formation is limited by the heat exchange with the environment; the line 2 presents the limitation of the hydrate formation by gas diffusion. The jet velocity increases to a value of $z = 2.3$ m. This is due to the action of initial pulse on
the jet. Then the jet velocity decreases with coordinate increasing due to the capture of the surrounding water and, as a consequence, the jet weighting.

4. Conclusion
A mathematical model of the flow of a multiphase submerged jet is considered in this paper. An integral Lagrangian control volume method is developed for the study of multiphase submerged jets taking into account the hydrate formation and the action of three-dimensional flow of the environment. The trajectory of the jet, dependences of the jet temperature, velocity and density on the vertical coordinate are obtained. As a result of numerical calculations, the dependencies of the thermo-physical parameters of the submerged jet (temperature, velocity, density) on the vertical coordinate and the jet trajectory are obtained. The influence of the three-dimensional flow of the surrounding water on the submerged jet dynamics is also analyzed.

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