Global analysis for determining fragmentation functions and their uncertainties in light hadrons

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Fragmentation functions are determined for the pion, kaon, and proton by analyzing charged-hadron production data in electron-positron annihilation. It is important that uncertainties of the determined fragmentation functions are estimated in this analysis. Analysis results indicate that gluon and light-quark functions have large uncertainties especially at small $Q^2$.

We find that next-to-leading-order (NLO) uncertainties are significantly reduced in comparison with leading-order (LO) ones in the pion and kaon. The fragmentation functions are very different in various analysis groups. However, all the recent functions are roughly within the estimated uncertainties, which indicates that they are consistent with each other. We provide a code for calculating the fragmentation functions and their uncertainties at a given kinematical point of $z$ and $Q^2$ by a user.

Keywords: Fragmentation function, Quark, Gluon, QCD, Electron-positron annihilation

1 Introduction

A fragmentation function describes a hadronization process from a parent quark, antiquark, or gluon to a hadron. Hadron-production processes are often used for investigating important physics such as the origin of nucleon spin and properties of quark-hadron matters. Fragmentation functions are needed for describing such processes, so that precise functions should be obtained for discussing any physics outcome. Nevertheless, it is known that there are large differences in the parametrized fragmentation functions, for example, between the ones by Kniehl, Kramer, and Pötter (KKP)\(^1\) and Kretzer\(^2\). Recently updated functions by Albino, Kniehl, and Kramer (AKK)\(^3\) are also much different from these functions. This fact suggests that the fragmentation functions are not determined accurately; therefore, it is important to show reliable regions in discussing any hadron-production data.

Such error analyses have been investigated recently in the studies of unpolarized parton distribution functions (PDFs), polarized and nuclear PDFs\(^4\). It is straightforward to apply the technique for the fragmentation functions. We determine the fragmentation functions and their uncertainties by analyzing the data for charged-hadron production in electron-positron annihilation, $e^+ + e^- \rightarrow h + X$. The analyses are done in leading order (LO) and next-to-leading order (NLO) of the running coupling constant $\alpha_s$. Because accurate SLD data in 2004 are
included in our analysis, whereas they are not used in KKP, AKK, and Kretzer’s analyses, we expect to have improvements. Therefore, important points of our analysis are

- improvement due to addition of accurate SLD data,
- roles of NLO terms on the determination, namely on the uncertainties,
- comparison with other analysis results by considering the uncertainties.

Our analysis method is explained in section 2, results are explained in section 3, and they are summarized in section 4.

### 2 Analysis method

The fragmentation function is defined by the ratio of hadron-production cross section to the total hadronic cross section:

\[
F^h(z, Q^2) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz},
\]

where \(Q^2\) is given by the center-of-mass energy squared \((Q^2 = s)\), and \(z\) is defined by the ratio \(z = E_h/\sqrt{s}/2 = 2E_h/\sqrt{Q^2}\) with the hadron energy \(E_h\). Since the fragmentation occurs from primary quarks, antiquarks, and gluons, the fragmentation function is expressed by the their sum:

\[
F^h(z, Q^2) = \sum_i \int_z^1 \frac{dy}{y} C_i(y, \alpha_s) D^h_i(z/y, Q^2).
\]

Here, \(C_i(z, \alpha_s)\) is a coefficient function which is calculated in perturbative QCD, and \(D^h_i(z, Q^2)\) is the fragmentation function of the hadron \(h\) from a parton \(i\). The function \(D^h_i(z, Q^2)\) is associated with a non-perturbative aspect, and it cannot be theoretically calculated in a reliably way. It is the purpose of this work to obtain the optimum fragmentation functions for the pion, kaon, and proton by analyzing the experimental data for \(e^+ + e^- \rightarrow h + X\).

In order to determine the functions from the data, we express them in terms of parameters at a fixed scale \(Q^2_0\) (=1 GeV²):

\[
D^h_i(z, Q^2_0) = N^h_i z^{\alpha^h_i} (1 - z)^{\beta^h_i},
\]

where \(N^h_i, \alpha^h_i, \) and \(\beta^h_i\) are the parameters to be determined by a \(\chi^2\) analysis of the data. Because there is a sum rule due to the energy conservation: \(\sum_h M^h_i = \sum_h \int_0^1 dz z D^h_i(z, Q^2) = 1\), it is more convenient to choose the parameter \(M^h_i\) instead of \(N^h_i\). They are related by \(N^h_i = M^h_i/B(\alpha^h_i + 2, \beta^h_i + 1)\), where \(B(\alpha^h_i + 2, \beta^h_i + 1)\) is the beta function. In general, a common function is assumed for favored functions and different ones are used for disfavored functions. The favored indicates a fragmentation from a quark or antiquark which exists in the hadron as a constituent in a simple quark model. The disfavored means a fragmentation from other quark or antiquark. The details of the formalism are explained in Ref. 5. The optimum parameters are determined by minimizing the total \(\chi^2\) given by \(\chi^2 = \sum_j (F^\text{data}_j - F^\text{theo}_j)^2/(\sigma^\text{data}_j)^2\), where \(F^\text{data}_j\) and \(F^\text{theo}_j\) are experimental and theoretical fragmentation functions, respectively, and \(\sigma^\text{data}_j\) is an experimental error. Uncertainties of the determined fragmentation functions are estimated by the Hessian method:

\[
[\delta D^h_i(z)]^2 = \Delta \chi^2 \sum_{j,k} \left( \frac{\partial D^h_i(z, \xi_j)}{\partial \xi_j} \right) H^{-1}_{jk} \left( \frac{\partial D^h_i(z, \xi_k)}{\partial \xi_k} \right),
\]

where \(H_{jk}\) is the Hessian matrix, \(\xi_j\) is a parameter, \(\hat{\xi}\) indicates the optimum parameter set, and the \(\Delta \chi^2\) value is chosen so that the error becomes the one-\(\sigma\) range in the multiparameter space. The detailed explanations for the uncertainties are found in Refs. 4 and 5.
We explain analysis results. First, determined fragmentation functions are compared with charged-pion production data in Fig. 1. The curve indicates theoretical NLO results which are calculated by using determined parameters in the $\chi^2$ analysis, and the uncertainties are shown by the shaded band. The comparison suggests that the fit is successful in reproducing the data in four orders of magnitude.

Determined functions are shown at the initial scales ($Q^2=1 \text{ GeV}^2$, $m_c^2$, and $m_b^2$) and also at an evolved scale $Q^2 = M^2_Z$ in Fig. 2. The LO and NLO functions and their uncertainties are shown. We notice that the uncertainties are generally large at small $Q^2$, especially in the LO. The gluon and light-quark functions have especially large uncertainties. However, it is interesting to note that the situation is much improved in the NLO because the uncertainties become significantly smaller. The uncertainty bands are smaller at large $Q^2$ ($= M^2_Z$). Since the fragmentation functions are used at small $Q^2$ ($\sim 1 \text{ GeV}^2$), for example, in HERMES, RHIC-Spin, and RHIC heavy-ion experiments, one should be careful about the reliability of employed functions in one’s analysis.

Next, the determined functions are compared with other analysis results for $(\pi^+ + \pi^-)/2$, $(K^+ + K^-)/2$, and $(p + \bar{p})/2$ in Fig. 3. Our parametrization is denoted HKNS (Hirai, Kumano, Nagai, Sudoh). The determined functions in NLO and their uncertainties are shown by the solid curves and shaded bands. They are compared with other functions by KKP, AKK, and Kretzer at $Q^2 = 2$, 10, and 100 $\text{GeV}^2$. As mentioned earlier, there are much differences between the analysis groups. For example, the gluon and $s$-quark functions have large variations in the pion. However, almost all the curves are roughly within the estimated uncertainty bands. It suggests that all the analyses should be consistent with each other and that accurate functions cannot be determined by the current $e^+e^-$ data. After our paper, there appeared another analysis by de Florian, Sassot, and Stratmann. Although there are some differences from our functions,
they are also within the uncertainty bands in Fig. [3]

The determined fragmentation functions can be calculated by using a code at our web site [7] by supplying a kinematical condition for $z$ and $Q^2$ and a hadron species. It is noteworthy that the uncertainties can be also calculated by using the code.

4 Summary

The optimum fragmentation functions and their uncertainties have been obtained for the pion, kaon, and proton in both LO and NLO of $\alpha_s$ by the $\chi^2$ analyses of charged-hadron production data in electron-positron annihilation. It is the first analysis to show the uncertainties in the fragmentation functions. The uncertainties were estimated by the Hessian method. We found large uncertainties especially at small $Q^2$, so that they need to be taken into account for using the functions in the small $p_T$ regions of hadron-production measurements in lepton-proton, proton-proton, and heavy-ion reactions. We also found that the functions are determined more accurately in the NLO than the LO ones particularly in the pion by considering LO and NLO uncertainties. There are large differences between previous parametrizations of KKP, AKK, and Kretzer, but they are consistent with each other and with our results because they are within the uncertainty bands.

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