Study on Heat Transfer Agent Models of Transmission Line and Transformer

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Abstract. When using heat transfer simulation to study the dynamic overload of transmission line and transformer, it needs to establish the mathematical expression of heat transfer. However, the formula is a nonlinear differential equation or equation set and it is not easy to get general solutions. Aiming at this problem, some different temperature change processes caused by different initial conditions are calculated by differential equation and equation set. New agent models are developed according to the characteristics of different temperature change processes. The results show that the agent models have high precision and can solve the problem that the original equation cannot be directly applied in some practical engineers.

1. Introduction
With economic development, the demand for electricity is continuous increasing. The rated capacities of existing transmission line and transformer do not meet the increase of demand for electricity. Meanwhile, the actual capacity of the existing power transmission and transformation equipment is often not fully utilized when it is in normal operation, and there are still hidden capacity available, so it has become the best way to meet the power demand that the equipment is scientifically and safely dynamic overload [1-3].

Temperature of transmission line and insulation temperature inside the transformer are the key factors that directly affect their load capacity. At present, direct temperature measurement and indirect temperature measurement can get the transmission line temperature and transformer hot spot temperature. In the direct temperature measurement, sensors were directly installed on the transmission line and the transformer and monitored their temperature in real time [4, 5]. Besides, using the relationship between the wire temperature and its impedance, the temperature was calculated by directly measuring the line current and voltage [6, 7]. Indirect temperature measurement mainly includes heat transfer simulation [4, 8], numerical calculation [9, 10] and intelligent calculation [11, 12] methods. However, this mathematical expression of heat transfer simulation is a nonlinear differential equation or equation set, and it is not easy to obtain its general solution directly. Based on the original
theoretical model, the new heat transfer agent models of transmission line and transformer are derived in this paper. The change process of the new models object approximately equal to that of the original models under the conditions that the initial conditions of both original models and agent models are the same, which solves the problem that the original models cannot be directly applied in some practical engineers.

2. Heat transfer characteristics of transmission line

2.1. Heat transfer transient differential equation of transmission line

The maximum allowable temperature of a transmission line is an important factor which limits the power carried by the transmission line. The temperature depends on various factors, including the current level, sunshine intensity, wind speed and ambient temperature. For heat transfer of transmission line, firstly solar radiation and current flows through the resistor make the transmission line generate thermal energy. Secondly some of the thermal energy is stored inside the transmission line, causing it to rise in temperature. The other part is dissipated by the convection of the wind and its radiation to the surrounding environment.

Transmission line current and the external environment are constantly changing, the heat transfer transient differential equation can be expressed as follows in equation (1) [8].

\[
\frac{dT_s}{dt} = \frac{P + P_{st} - P_{conv}}{MC}
\]  

Where \( P = I^2 \cdot R_T \), \( P_{st} = \gamma \cdot D \cdot S_i \), \( P_{conv} = 9.92 \cdot (T_e - T_0) \cdot (V \cdot D)^{0.485} \) and \( P_{rad} = s \cdot \pi \cdot D \cdot k_e \cdot (T_e^4 - T_0^4) \) respectively, and \( M \) is the mass per unit length of the transmission line [kg/m]; \( C \) is the specific heat capacity of the wire [J/(kg·℃)]; \( P \) is the power of thermal energy production due to the current per unit length of the transmission line [W/m]; \( P_{st} \) is the power of sunshine heat absorption per unit length of the transmission line [W/m]. \( P_{conv} \) is the power of convection cooling per unit length of the transmission line [W/m]; \( P_{rad} \) is the power of heat dissipation to the surroundings per unit length of the transmission line [W/m]; \( I \) is the current carried by the transmission line [A]; \( R_T \) is the AC resistance per unit length of the transmission line [Ω]; \( \gamma \) is the surface heat absorption coefficient; \( D \) is the diameter of the line [m]; \( S_i \) is the sunshine intensity [W/m²]; \( T_e \) is the temperature of the transmission line [K]; \( t \) is the time [s]; \( T_0 \) is the ambient temperature [K]; \( V \) is the wind speed [m/s]; \( s \) is the Stephen-Boltzmann constant (5.67×10⁻⁸); \( k_e \) is the radiation coefficient, and the bright new line is 0.23 ~ 0.46, while the dark degenerated line is 0.9 ~ 0.95.

2.2. Agent model of heat transfer in transmission line

Equation (1) is a first-order nonlinear differential equation, not easy to find its general solution, but a particular transmission line can be used numerical solution given approximate solution. For ACSR LGJ400/35 (steel wire wrapped in aluminum wire), the aluminum cross sectional area is 400mm², and the steel cross sectional area is 35mm². The \( M \) is 1.349kg/m; The \( C \) is 880J/(kg·℃); The \( R_T \) is 0.000073892Ω; The \( T_0 \) is 40℃; The \( V \) is 0.5m/s; the \( S_i \) is 1000W/m²; The \( k_e \) is 0.9; The \( \gamma \) is 0.9; The outer diameter is 0.02682m; The line voltage is 138kV. Several different initial conditions are given, which are different initial temperature and load power. Their temperature change process is shown in figure 1.
In figure 1, the first "*" from the left of each curve is the node at which it changes from the initial temperature to a quarter of the difference from the initial temperature to steady-state temperature. The abscissas of first "*" of each curve are almost the same. The second "*" from the left of each curve is the node at which it changes from the initial temperature to a half of the difference from the initial temperature to steady-state temperature. The abscissas of second "*" of each curve are also almost the same. The n-th "*" from the left of each curve is the node at which it changes from the initial temperature to 1-1/(2n-1) of the difference from the initial temperature to steady-state temperature. Meanwhile, the abscissas of n-th "*" of each curve are almost the same (figure 1). According to this feature, a linear piecewise function can be constructed as its agent model. The coordinates of the segment nodes are shown in table 1.

**Table 1. Point of subsection of approximate function of heat transfer of transmission line**

| X-axis | node1 | node2 | node3 | node4 | node5 | node6 | node7 |
|--------|-------|-------|-------|-------|-------|-------|-------|
| 125    | 375   | 750   | 1250  | 1625  | 2000  | 2775  |

Where, \( \Delta T = T_s - T_p \), \( T_s \) is the steady-state temperature, \( T_p \) is the initial temperature. The equation (1) is assumed to equal to 0, and the steady-state temperature can be obtained. The steady-state temperature is calculated per 1MW at load power from 1MW to 200MW. According to the obtained scattered values, the function have fitted with the binomial function, which is as shown in equation (2).

\[
T_s = a \cdot P_i^2 + b \cdot P_i + c
\]  

(2)

Where, \( a = 0.0006 \), \( b = 0.0043 \), \( c = 325.4 \).

In this way, it is possible to construct an agent model as shown in equation (3), and at a given initial temperature and the load power, an approximate function of the temperature at any time can be obtained.

\[
T_s = f(t, T_p, P_i) = \begin{cases} 
T_p + [0.25 \cdot (T_s - T_p)] \frac{t - 0}{t_1 - 0}, & t \in [0, t_1) \\
[T_p + 0.25 \cdot (T_s - T_p)] + [(0.5^i - 0.25) \cdot (T_s - T_p)] \frac{t - t_i}{t_2 - t_i}, & t \in [t_i, t_2) \\
\ldots & \\
[T_p + (1 - 0.5^i) \cdot (T_s - T_p)] + [(0.5^i - 0.5^j) \cdot (T_s - T_p)] \frac{t - t_j}{t_6 - t_6}, & t \in [t_6, t_7) \\
T_s, & t \in [t_7, +\infty) 
\end{cases}
\]  

(3)

3. Heat transfer characteristics of a transformer

3.1. Heat transfer transient differential equations of transformer
The internal heat transfer process of the transformer is not the same with transmission line, and the temperature of the iron cores and windings, the transformer oil and the housing are quite different, as shown in figure 2.

First, the iron cores and windings generate thermal energy due to power loss. Some of the heat, \( \Phi_1 \), is transferred to the oil due to the convection heat transfer. Some of the heat in the oil, \( \Phi_2 \), is dissipated by the convection heat transfer to the transformer housing. The heat in the housing, \( \Phi_3 \), is delivered to the surrounding air due to convection and thermal radiation. From the above heat transfer process, the heat transfer differential equations of transformer can be written as showing in equation set (4) - (6).

\[
M_I \cdot C_I \cdot \frac{dT_1}{dt} = P - A_1 \cdot h_1 \cdot (T_1 - T_2) \\
M_o \cdot C_o \cdot \frac{dT_2}{dt} = [A_1 \cdot h_1 \cdot (T_1 - T_2) - A_2 \cdot h_2 \cdot (T_2 - T_3)] \\
M_s \cdot C_s \cdot \frac{dT_3}{dt} = [A_2 \cdot h_2 \cdot (T_2 - T_3) - A_2 \cdot h_2 \cdot (T_3 - T_4) - s \cdot A_2 \cdot k_s \cdot (T_4^3 - T_4^4)]
\]

where \( C_I \) is the specific heat capacity of the iron cores and windings [J/(kg·°C)]; \( C_o \) is the specific heat capacity of the oil [J/(kg·°C)]; \( C_s \) is the specific heat capacity of the housing [J/(kg·°C)]; \( M_I \) the mass of the iron cores and windings [kg]; \( M_o \) is the mass of the oil [kg]; \( M_s \) is the mass of the housing [kg]; \( A_1 \) is the effective heat dissipation area of the iron cores and windings [m\(^2\)]; \( A_2 \) is the effective heat dissipation area of the housing [m\(^2\)]; \( h_1 \) is the convection heat transfer coefficient of the iron cores and windings [W/(m\(^2\)·°C)]; \( h_2 \) is the convection heat transfer coefficient of the housing [W/(m\(^2\)·°C)]; \( P \) is the power loss [W]; \( T_1 \) is the temperature of the iron cores and windings [K]; \( T_2 \) is the temperature of the oil [K]; \( T_3 \) is the temperature of the housing [K]; \( T_4 \) is the ambient temperature [K].

3.2. Agent model of heat transfer in the transformer

Equation set (4) - (6) are first-order nonlinear differential equation set. It is more difficult to solve than equation (1), but it can also be solved by numerical solution under the given initial conditions.
S11-M-type transformer, its iron core is silicon steel sheet; the windings are oxygen-free copper; the windings interlayer insulating material is rhombus adhesive tape; the rated capacity is 400kVA; The maximum iron cores and windings temperature is 15°C higher than the average temperature of the transformer; The maximum temperature should not exceed 95°C. To ensure long-term operation, the average temperature should not exceed 80°C. The \( M_1 \) is 295kg; The \( C_1 \) is 450J/(kg·°C); The \( P \) is 7500(0.0125×600×10³)kW; The \( A_1 \) is 0.7m²; The \( h_1 \) is 500 W/(m²·°C); The \( M_6 \) is 280 kg; The \( C_6 \) is 2060 J/(kg·°C); The \( A \) is 5m²; The \( h_1 \) is 500 W/(m²·°C); The \( M_6 \) is 600kg; The \( C_6 \) is 400J/(kg·°C); The \( h_2 \) is 50W/(m²·°C).

Under five different initial conditions, their temperature changing curves are shown in figure 3. Similar to the transmission line, an agent model is developed as shown in equation (3). The coordinates of the segment nodes are shown in table 2.

| X-axis | node1 | node2 | node3 | node4 | node5 | node6 | node7 |
|--------|-------|-------|-------|-------|-------|-------|-------|
|        | 300   | 1225  | 3650  | 6131  | 8568  | 10970 | 13410 |

The equation set (4) - (6) are assumed to equal to 0, the steady-state temperature of the transformer can be obtained. The steady-state temperature is calculated per 1kW at load power from 1kW to 600kW. The steady-state temperature function \( T_s \) is fitted with a linear function according to the obtained scattered values, which is as shown in equation (7).

\[
T_s = 0.08433P + 313
\]  

(7)

In this way, the linear piecewise function can be constructed as an agent model of the temperature change process of the iron cores and windings. Given the initial temperature of iron cores and windings and the load power of the transformer, the temperature at any time can be calculated by equation (3), Where \( t_1, t_2, ..., t_7 \) nodes are shown in table 2, \( T_s \) is as shown in equation (7).

For the S11-M transformer in this paper, in order to verify the accuracy of the agent model (3), two different initial conditions are selected to draw the theoretical temperature changing curve and the approximate temperature changing curve as shown in figure 4. For the two kind of initial conditions (the 1st one and the 2nd one in figure 4), a series of time nodes are selected. The theoretical and approximate temperature values for the 1st one are compared as shown in table 3. From figure 4 and table 3, it can be seen that the errors of agent model at different time in table 3 are less than 0.1%, indicating that the agent model has higher accuracy. The error is calculated by ‘(proximate temperature-theoretical temperature)/theoretical temperature’.
Figure 3. Temperature variation process of iron cores and windings of S11-M transformer under several different initial conditions.

Table 3. Theoretical temperature and approximate temperature of iron cores and windings for the 1st one

| Time (s) | Theoretical temperature (K) | Proximate temperature (K) | Relative temperature (K) | Error |
|---------|-----------------------------|---------------------------|--------------------------|-------|
| 500     | 347.071                     | 346.747                   | -0.325                   | -0.094% |
| 1000    | 348.812                     | 348.546                   | -0.266                   | -0.076% |
| 2000    | 350.730                     | 350.418                   | -0.312                   | -0.089% |
| 3000    | 352.051                     | 351.791                   | -0.260                   | -0.074% |
| 5000    | 353.827                     | 353.588                   | -0.240                   | -0.068% |
| 10000   | 355.577                     | 355.426                   | -0.151                   | -0.042% |
| 15000   | 356.002                     | 355.822                   | -0.180                   | -0.051% |

4. Conclusions
In this paper, explicit agent function models of heat transfer of transmission line and transformer have been established respectively. After an initial temperature and load power are given, the temperature at any time can be calculated. The results show that the error between the agent model and the theoretical model is very small (in table 3 and figure 4), which can be applied to dynamically confirm capacity.

Acknowledgments
The work described in this paper was supported by key project of the National Natural Science Foundation (51537010).

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