Supersymmetry without a Light Higgs Boson

Riccardo Barbieri\textsuperscript{a}, Lawrence J. Hall\textsuperscript{b,c}, Yasunori Nomura\textsuperscript{b,c}, Vyacheslav S. Rychkov\textsuperscript{a}

\textsuperscript{a} Scuola Normale Superiore and INFN, Piazza dei Cavalieri 7, I-56126 Pisa, Italy
\textsuperscript{b} Department of Physics, University of California, Berkeley, CA 94720, USA
\textsuperscript{c} Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

Abstract

Motivated by the absence, so far, of any direct signal of conventional low-energy supersymmetry, we explore the consequences of making the lightest Higgs boson in supersymmetry relatively heavy, up to about 300 GeV, in the most straightforward way, i.e. via the introduction of a chiral singlet $S$ with a superpotential interaction with the Higgs doublets, $\lambda S H_1 H_2$. The coupling $\lambda$ dominates over all the other couplings and, to maintain the successful perturbative analysis of the ElectroWeak Precision Tests, is only restricted to remain perturbative up to about 10 TeV. The general features of this “$\lambda$SUSY” framework, which deviates significantly from the MSSM or the standard NMSSM, are analyzed in different areas: ElectroWeak Precision Tests, Dark Matter, naturalness bounds on superparticle masses, and LHC signals. There is a rich Higgs/Higgsino sector in the (200 – 700) GeV mass region, which may include LSP Higgsino dark matter. All other superpartners, apart from the top squarks, may naturally be heavier than 1–2 TeV. This picture can be made consistent with gauge coupling unification.
1 Introduction

The naturalness problem of the Fermi scale in the Standard Model (SM) amounts to understand the lightness of the Higgs boson relative to any mass scale in the theory, whatever it may be, that completes the SM itself at high energies, or in the Ultra-Violet (UV). If the relative Higgs lightness is not accidental, this gives the best hope for seeing new physics at the LHC, hence the focus on the naturalness problem. As is well known, the constraints on a positive solution to this problem largely come from the success of the SM in accounting for the results of the ElectroWeak Precision Tests (EWPT). In fact, the difficulty to solve it is exacerbated by the apparent lightness of the Higgs boson, as indirectly implied by the EWPT [1]. If, for one reason or another, we were misled in interpreting the data of the EWPT in terms of a light Higgs mass and the Higgs boson were heavier, the upper bound on the naturalness cutoff of the theory of the electroweak interactions might be relaxed by a non-negligible amount [2, 3].

This simple observation has a counterpart in supersymmetry, which is in many respects the most natural UV completion of the Standard Model (SM). In supersymmetric extensions of the SM, like the Minimal Supersymmetric Standard Model (MSSM), there is no problem in letting the cutoff become arbitrarily high, while the lightest Higgs boson is actually predicted to be light. So light in fact that to make its mass consistent with the experimental lower bound (about 114 GeV in most of the parameter space [4]) requires a large radiative correction due to a heavy stop, which in turn brings back at least a few percent fine-tuning in the Z boson mass. Accommodating a heavier Higgs boson would therefore ease the naturalness problem in supersymmetry too, while allowing heavier superpartners. This has motivated several attempts in the literature to increase the lightest Higgs boson mass [5 – 16].

Here we explore the possibility of making the lightest Higgs boson in supersymmetry relatively heavy in the most straightforward way, i.e. via the introduction of a chiral singlet $S$ with a large superpotential interaction $\lambda S H_1 H_2$, where $H_i$ are the two Higgs doublet multiplets, leading to a large quartic interaction of the Higgs doublets. Unlike the usual treatment of the Next to Minimal Supersymmetric Standard Model (NMSSM) [17], $\lambda$ is not perturbative up to the unification scale. We insist only that consistency with the EWPT can be analyzed in a fully perturbative manner, which we ensure by requiring that $\lambda$ remains perturbative up to $\approx 10$ TeV. At this scale a change of regime of the theory should intervene, which we leave unspecified. It is an open question whether this will allow a successful unification of the gauge couplings. A positive existence proof of this is given in Refs. [10, 11, 12].

With perturbativity preserved up to $\approx 10$ TeV, contributions to the EWPT from unknown UV physics can be sufficiently small. There may, however, still be calculable “infrared” contributions which make the theory incompatible with the EWPT. In particular, one might think that contributions from a lightest Higgs boson of mass $(200 \sim 300)$ GeV, as well as those from the other Higgs sector particles which grows like the fourth power of $\lambda$, are dangerous for the EWPT. Quite on the contrary, our general analysis of the contributions from the Higgs boson sector and, even more importantly from the Higgsino sector, shows that the theory is perfectly consistent with the EWPT in a significant range of the parameter space. Deviations from the “EWPT ellipse” due to a relatively heavy lightest Higgs boson are entirely canceled by the contributions from the other
Higgs bosons and the Higgsinos, in almost all the parameter space in which electroweak symmetry breaking occurs naturally.

In Section 2 we define our theory, minimize the scalar potential for large \( \lambda \), and give both the scalar and fermion spectrum. Many significant features of the theory are independent of the specific forms for the superpotential and soft operators, but depend critically on \( \lambda \) being large. Hence we call this framework “\( \lambda \)SUSY.” In Section 3 we consider the EWPT from both scalar and fermion sectors. In Section 4 we consider the candidate for Dark Matter (DM) in the special case of heavy weak gauginos, as allowed by naturalness. In Section 5 we obtain the naturalness bounds on the various particles in the limiting case in which the scalar \( S \) is taken heavy. The characteristic manifestations of the model at the LHC are described in Section 6. Summary and Conclusions are given in Section 7.

2 The \( \lambda \)SUSY Model

We consider the most general supersymmetric theory with \( SU(3)_C \times SU(2)_L \times U(1)_Y \) gauge invariance that contains a singlet chiral superfield \( S \) in addition to the fields of the MSSM. The theory possesses the superpotential interaction

\[
W_\lambda = \lambda S H_1 H_2, \tag{1}
\]

where \( H_1 \) and \( H_2 \) are the usual Higgs doublets, coupled respectively to the down-type and up-type quarks. We increase the mass of the lightest Higgs boson, and therefore the naturalness of the theory, by taking \( \lambda \) large. We call this the \( \lambda \)SUSY model, to stress the key role of the large coupling \( \lambda \). Such a large coupling may arise from compositeness of (some of) the Higgs states [10, 11, 12, 15].

How large can \( \lambda \) be? To keep perturbativity up to at least 10 TeV, we require \( \lambda^2 (\Lambda = 10 \text{ TeV}) \) be less than about \( 4\pi \). Given the Renormalization Group Equation (RGE) satisfied by \( \lambda (t = \ln E) \)

\[
\frac{d\lambda^2}{dt} = \frac{\lambda^4}{2\pi^2}, \tag{2}
\]

this implies that, at the low-energy scale of about 500 GeV, \( \lambda \) is less than about 2. Throughout the paper, unless differently stated, we take \( \lambda(500 \text{ GeV}) = 2 \). With this value of \( \lambda \) the Landau pole is typically a few tens of TeV.

The superpotential of our model takes the general form

\[
W = \mu(S) H_1 H_2 + f(S), \tag{3}
\]

and, neglecting the gauge terms, the scalar potential can be written in the form

\[
V = \mu_1^2(S)|H_1|^2 + \mu_2^2(S)|H_2|^2 - (\mu_3^2(S) H_1 H_2 + \text{h.c.}) + \lambda^2 |H_1 H_2|^2 + V(S). \tag{4}
\]

The mass parameters \( \mu_1^2(s), \mu_2^2(s), \mu_3^2(s), \mu(s) \) and

\[
M(s) \equiv \frac{d^2 f(s)}{ds^2}, \tag{5}
\]
calculated at the background expectation value \( s \) of the field \( S \), characterize many of the properties of the theory. We generally denote them as \( \mu_1^2, \mu_2^2, \mu_3^2, \mu, \) and \( M \), by leaving understood their argument \( s \). For simplicity we assume \( CP \) invariance of \( V \) and \( W \). This formulation of the \( \lambda \)SUSY theory is convenient because many equations take a similar form to the familiar MSSM case.

The stability of the potential (4) requires

\[
\mu_1^2, \mu_2^2 > |\mu|^2,
\]

(6)

for all the values of \( s \), whereas the condition for electroweak symmetry breaking is

\[
|\mu_3^2| > \mu_1 \mu_2,
\]

(7)

at the minimum of the effective potential for \( s \), obtained by replacing \( H_1 \) and \( H_2 \) by their \( s \)-dependent expectation values. Under the condition (6) electromagnetism is unbroken. To be more precise, once the gauge contribution to the potential is introduced, the potential is always stable, and the condition (6), with an additional small term on the right-hand-side, becomes the condition for unbroken electromagnetism. For our purposes, this difference has negligible impact.

In general we neglect in this section all the relatively small effects of the \( SU(2)_L \times U(1)_Y \) \( D \)-term contributions. We shall also neglect the possibility of spontaneous \( CP \) violation, i.e. we shall consider all the mass parameters real.\(^1\)

By minimizing the potential it is straightforward to find

\[
\tan \beta \equiv \frac{v_2}{v_1} = \frac{\mu_1}{\mu_2},
\]

(8)

\[
\lambda^2 v^2 = \frac{2\mu_3^2}{\sin 2\beta} - \mu_1^2 - \mu_2^2,
\]

(9)

where \( \lambda = \partial\mu(S)/\partial S \), and \( v_1, v_2, v \equiv (v_1^2 + v_2^2)^{1/2} \) are the usual Vacuum Expectation Values (VEVs) of the Higgs fields. One also finds that the mass of the charged Higgs bosons, \( H^\pm \), is

\[
m_{H^\pm}^2 = \mu_1^2 + \mu_2^2,
\]

(10)

where \( \mu_1^2 \) and \( \mu_2^2 \) should be evaluated at the minimum of \( s \). Unlike the mass of \( H^\pm \), the masses of the neutral scalars \( h, H \) and \( A \) can be expressed in terms of \( \mu_1, \mu_2 \) and \( \mu_3 \) only if their mixing with the scalar \( S \) can be neglected. In this case, the mass of the pseudoscalar Higgs state is given by

\[
m_A^2 = \frac{2\mu_3^2}{\sin 2\beta},
\]

(11)

while the two \( CP \)-even Higgs states \( h_i \equiv \sqrt{2}(\text{Re}H_i^0 - v_i) \) mix with the following mass matrix:

\[
\begin{pmatrix}
\frac{m_A^2 \sin^2 \beta}{\lambda^2 v^2 - \frac{1}{2} m_A^2} & \frac{(\lambda^2 v^2 - \frac{1}{2} m_A^2) \sin 2\beta}{m_A^2 \cos^2 \beta} \\
\frac{(\lambda^2 v^2 - \frac{1}{2} m_A^2) \sin 2\beta}{m_A^2 \cos^2 \beta} & m_A^2 \cos^2 \beta
\end{pmatrix},
\]

(12)

\( ^1 \)A physical phase in the Higgsino mass matrix (see below) could have relevant consequences.
where \( \mu_3^2 \) in Eq. (11) should be evaluated at the minimum of \( s \). The masses and composition of the mass eigenstates \( H \) and \( h \) are given by

\[
m^2_{H,h} = \frac{1}{2}(m_A^2 \pm X), \quad X^2 = m_A^4 - 4\lambda^2 v^2 m_{H^\pm}^2 \sin^2 2\beta, \tag{13}
\]

and

\[
H = \cos \alpha h_1 + \sin \alpha h_2, \tag{14}
\]

\[
h = -\sin \alpha h_1 + \cos \alpha h_2, \tag{15}
\]

respectively, where

\[
\tan \alpha = \frac{m_A^2 \cos 2\beta + X}{(\lambda^2 v^2 - m_{H^\pm}^2) \sin 2\beta}. \tag{16}
\]

We find that the following inequalities hold:

\[
m_h \leq \sin \beta \lambda v, \quad m_{H^\pm} \leq m_H < m_A. \tag{17}
\]

In the fermion sector, the \( SU(2)_L \times U(1)_Y \) gauginos are mixed with the other fermions only by the relatively small gauge terms. Since there is also no significant naturalness upper bound on their masses, it is relevant to consider the case in which these gauginos are decoupled from the other fermions. In this case the charged Higgsino has a mass \( \mu \) and the mass matrix of the neutral fermions

\[
N_1 = \frac{1}{\sqrt{2}}(\tilde{H}_1 - \tilde{H}_2), \quad N_2 = \frac{1}{\sqrt{2}}(\tilde{H}_1 + \tilde{H}_2), \quad N_3 = \tilde{S}, \tag{18}
\]

is given in terms of \( \mu \) and \( M \) by

\[
\mathcal{M} = \begin{pmatrix}
\mu & 0 & \frac{\lambda m_{H^\pm}}{\sqrt{2}} \\
0 & -\mu & -\frac{\lambda^2 v^2}{\sqrt{2}} \\
\frac{\lambda m_{H^\pm}}{\sqrt{2}} & -\frac{\lambda^2 v^2}{\sqrt{2}} & M
\end{pmatrix}, \tag{19}
\]

where \( \mu \) and \( M \) are, again, evaluated at the minimum of \( s \). Only the relative sign of \( \mu \) and \( M \) is physical, and we fix our convention to be \( \mu > 0 \) for definiteness. Note that the condition (6) implies the inequality

\[
\mu \leq \cos \beta m_{H^\pm}. \tag{20}
\]

The \( N_i \)'s will be related to the mass eigenstates (neutralinos) \( \chi_i \) by a non-trivial \( 3 \times 3 \) mixing matrix \( V \):

\[
N_i = V_{ij} \chi_j, \quad V^TV = 1, \quad V^T\mathcal{M}V = \text{diag}(m_1, m_2, m_3). \tag{21}
\]

The lightest neutralino \( \chi \) has a mass satisfying

\[
|m_\chi| \leq \mu, \tag{22}
\]
and the other two eigenvalues are above \( \mu \) and below \(-\mu\), respectively. Moreover, the determinant of the mass matrix, and thus \( m_\chi \), vanishes for a particular value of \( M \):

\[
m_\chi = 0 \quad \text{for} \quad M = -\frac{\lambda^2 v^2}{\mu} \sin 2\beta.
\]  

(23)

Thus there is a region of parameter space with a naturally light neutralino, which is an admixture of \( \tilde{S} \) and the Higgsinos. In fact, the lightest neutralino is much lighter than the characteristic scales for \( \mu \) and \( M \) in a wide region around the point satisfying Eq. (23).

The mixing matrix and the spectrum are difficult to determine analytically, except in the limiting case of \( \tan \beta = 1 \), which leads to

\[
m_1 = \mu, \quad m_{2,3} = \frac{1}{2}(M - \mu \pm Y), \quad Y^2 = (M + \mu)^2 + 4\lambda^2 v^2.
\]

(25)

In this case \( \chi_1 = N_1 \), while \( \chi_{2,3} \) are mixtures of \( N_{2,3} \) with a mixing angle \( \gamma \), where \( \tan \gamma = -(M + \mu + Y)/2\lambda v \). The lightest state is either \( \chi_1 \) or \( \chi_2 \) depending on whether \( M \) is above or below a critical value \( M_c = \mu - \frac{\lambda^2 v^2}{2\mu} \).

For sufficiently small \( \tan \beta - 1 \), one can approximately diagonalize the matrix (19) expanding in \( M_{13} \). The mixing between \( N_1 \) and \( N_{2,3} \) will be small everywhere except near \( M = M_c \). As a result, the coupling of the lightest neutralino \( \chi \) to the Z boson will be generically suppressed. This may play an important role in the interpretation of DM, as discussed in Section 4.

When \( \mu_1 = \mu_2 \), giving \( \tan \beta = 1 \), an \( SU(2) \) custodial symmetry survives, and plays an important role. In this limit, in the scalar sector either \( h \) or \( H \), depending on the sign of \( 2m^2_{H^\pm} - m^2_\lambda \), behaves like the SM Higgs boson, of mass \( m_h = \lambda v/\sqrt{2} \), whereas the other scalar with \( H^\pm \) forms a degenerate \( SU(2) \)-triplet. Similarly, in the fermion sector, the chargino and \( N_1 \) form a degenerate triplet of mass \( \mu \).

### 3 ElectroWeak Precision Tests

We perform the analysis of the EWPT in the usual \( S-T \) plane, with the experimental contours taken from [18]. With respect to the SM, the \( \lambda \)SUSY model has: 1) a modified contribution to \( S \) and \( T \) from the scalar Higgs sector [10, 16]; 2) new and relevant contributions from the stop-sbottom and from the Higgsinos (we consider heavy gauginos). The contributions of 1) and 2) can be analyzed separately.

To locate our model in the \( S-T \) plane before the stop-sbottom and Higgsino contributions are added, we subtract the one-loop SM Higgs contributions

\[
T_{\text{Higgs}}(m_h) = -3 \left[ A(m_h, m_W) - A(m_h, m_Z) \right],
\]

(26)

\[
S_{\text{Higgs}}(m_h) = F(m_h, m_Z) + m^2_Z G(m_h, m_Z),
\]

(27)
Figure 1: The black (darker) curve shows the SM results with a Higgs mass $m_h = 100 - 350$ GeV in 50 GeV increments. The ellipses show the regions of the $S$-$T$ plane allowed by EWPT at 1σ and 2σ. The red (lighter) curves give the predictions from the Higgs scalar sector in $\lambda$SUSY, as described in the text, with values of $\tan \beta$ in the interval $\tan \beta = 1 \sim 5$ as indicated and $m_{H^\pm} = 350, 500, 700$ GeV.

from the SM values of $S$ and $T$ (see Appendix A for the definitions), and then add the one-loop 2 Higgs-Doublet Model (2HDM) contributions (computed under the assumption of no doublet-singlet mixing) [19, 20]

$$T_{2\text{HDM}} = \sin^2(\beta - \alpha) \left[ T_{\text{Higgs}}(m_h) + A(m_{H^\pm}, m_H) - A(m_A, m_H) \right]$$

$$+ \cos^2(\beta - \alpha) \left[ T_{\text{Higgs}}(m_H) + A(m_{H^\pm}, m_h) - A(m_A, m_h) \right]$$

$$+ A(m_{H^\pm}, m_A),$$

$$S_{2\text{HDM}} = \sin^2(\beta - \alpha) \left[ S_{\text{Higgs}}(m_h) + F(m_A, m_H) \right]$$

$$+ \cos^2(\beta - \alpha) \left[ S_{\text{Higgs}}(m_H) + F(m_A, m_h) \right]$$

$$- F(m_{H^\pm}, m_{H^\pm}).$$

(28)

Here, $m_h$ in Eqs. (26, 27) represents a reference value of the Higgs boson mass in the SM, while that in Eqs. (28, 29) is the mass of the lightest $CP$-even Higgs boson in the $\lambda$SUSY model.

In the $\lambda$SUSY model, the 6 parameters appearing in Eqs. (28, 29), i.e. $m_h$, $m_H$, $m_A$, $m_{H^\pm}$, $\alpha$ and $\beta$, depend only on $\tan \beta$, $m_{H^\pm}$ and $\lambda$, which we set equal to 2. In Fig. 1 we show the result for the EWPT for several values of $\tan \beta$ and $m_{H^\pm}$ ($\tan \beta = 1 \sim 5$ and $m_{H^\pm} = 350, 500, 700$ GeV). Two features are manifest from this figure: i) the role of the custodial symmetry for $\tan \beta$ approaching unity, thus suppressing the corrections to $T$; ii) the fact that the positive $T$-correction brings most of the points of the $\lambda$SUSY model inside the region preferred by experiments, at least as long as $\tan \beta$ is not too large.
The expected stop-sbottom contribution to the $T$ parameter is constrained by naturalness to lie above the curve.

The *stop-sbottom* contributions in the zero left-right mixing limit are given by

\[
T_{\text{st-sb}} = 6 A(m_{\tilde{t}_L}, m_{\tilde{b}_L}) \approx \frac{m_{\tilde{t}}^4}{32\pi^2 a_{\text{em}} v^2 m_{\tilde{t}_L}^2} \approx 0.05 \left( \frac{500 \text{ GeV}}{m_{\tilde{t}_L}} \right)^2,
\]

\[
S_{\text{st-sb}} = F(m_{\tilde{b}_L}, m_{\tilde{b}_L}) - F(m_{\tilde{t}_L}, m_{\tilde{t}_L}) \approx -\frac{1}{12\pi} m_{\tilde{t}}^2 \approx -0.003 \left( \frac{500 \text{ GeV}}{m_{\tilde{t}_L}} \right)^2,
\]

where the approximate expressions follow from $m_{\tilde{t}_L}^2 - m_{\tilde{b}_L}^2 = m_{\tilde{t}}^2$ in the large $m_{\tilde{t}_L}^2$ limit. While the contribution to $S$ is always negligibly small, this is not the case, as is well known, for the contribution to $T$. Anticipating a $\tan\beta$-dependent upper bound on the stop masses from naturalness considerations (see Section 5), we show in Fig. 2 the minimum value of $T_{\text{st-sb}}$ when the stop masses are taken at this boundary.\(^2\) This strongly reinforces the conclusion that $\tan\beta$ cannot be too large. If the Higgs and stop-sbottom sectors were the only contributions to $S$ and $T$, any value of $\tan\beta$ above 5 or so would be almost excluded.

We finally consider the contributions from the Higgsino sector described in the previous Section. The couplings to the gauge bosons are

\[
\mathcal{L}_{\text{int}} = \frac{g}{2} W^+_{\mu} \left( -\bar{\Psi} \gamma^\mu N_1 + \bar{\Psi} \gamma^\mu \gamma^5 N_2 \right) + \text{h.c.}
\]

\[
+ \frac{g'}{2} W^3_{\mu} \left( \bar{\Psi} \gamma^\mu \Psi + \bar{N}_1 \gamma^\mu \gamma^5 N_2 \right) + \frac{g}{2} B_{\mu} \left( \bar{\Psi} \gamma^\mu \Psi - \bar{N}_1 \gamma^\mu \gamma^5 N_2 \right),
\]

where $\Psi$ represents the chargino. The contributions of Higgsinos to $T$ and $S$ can be written in

\(^2\)Taking into account the left-right mixing may help reduce $T_{\text{st-sb}}$, although not dramatically, because the corresponding $A$-term parameter, $A_t$, will be subject to the same naturalness bound as $m_{\tilde{Q}}$ and $m_{\tilde{t}_R}$. 
terms of the mixing matrix $V$ as follows:

$$
T_{\text{Higgsinos}} = \sum_{a=1}^{3} (V_{1a})^2 \tilde{A}(\mu, m_a) + (V_{2a})^2 \tilde{A}(\mu, -m_a)$$
$$- \frac{1}{2} \sum_{a,b=1}^{3} (V_{1a}V_{2b} + V_{1b}V_{2a})^2 \tilde{A}(m_a, -m_b),$$

(33)

$$S_{\text{Higgsinos}} = \frac{1}{2} \sum_{a,b=1}^{3} (V_{1a}V_{2b} + V_{1b}V_{2a})^2 \tilde{F}(m_a, -m_b) - \tilde{F}(\mu, \mu),$$

(34)

(see Appendix A for the definitions of the functions $\tilde{A}$ and $\tilde{F}$). These contributions are shown in Fig. 3 in the $\mu$-$M$ plane. Also shown in the plot is the region (shaded) in which the lightest neutralino mass is less than half the $Z$ boson mass: $m_\chi < m_Z/2$. Even though its coupling to the $Z$ boson is suppressed, this mass range is inconsistent with the negative searches from the LEP. In view of Figs. 1 and 2, as well as the mostly positive contributions to $S$ and $T$ in the plots of Fig. 3, we conclude that most of the $\mu$-$M$ plane is allowed for $\tan \beta \lesssim 3$, except for a $\tan \beta$-dependent strip around $M = 0$ where $T$ becomes too large.

The dependence of $T_{\text{Higgsinos}}$ on $\tan \beta$ can be approximated by

$$T_{\text{Higgsinos}} \approx F(\tan \beta) \tilde{T}(\mu, M), \quad F(t) = \left(\frac{t^2 - 1}{t^2 + 1}\right)^2,$$

(35)

where $\tilde{T}(\mu, M)$ is some function of $\mu$ and $M$. To make this scaling evident, the values for the contours of $T$ in each panel of Fig. 3 are chosen such that they are proportional to the contours of the $\tan \beta = 1.5$ panel, scaled by a factor $F(\tan \beta)/F(1.5)$. Even for $\tan \beta > 3$ there remain two strips around $M \approx 600$ GeV and $M \approx -800$ GeV where $T_{\text{Higgsinos}}$ is sufficiently small to be allowed (especially combined with the positive $S$ parameter). However, taking into account the contribution $T_{\text{st-sb}}$, which starts being increasingly problematic for $\tan \beta \gtrsim (4 \sim 5)$, as well as the fact the allowed region for $\mu$ from naturalness rapidly shrinks for $\tan \beta \gtrsim (4 \sim 5)$, we conclude that the region

$$\tan \beta \lesssim 3,$$

(36)

is preferred in the $\lambda$SUSY model.

## 4 Dark Matter for Heavy Gauginos

There are several possibilities in $\lambda$SUSY for the Lightest Supersymmetric Particle (LSP), corresponding to differing LHC signals and DM candidates. In this section we study the particular case that the LSP is dominantly Higgsino, having only small gaugino components. In the previous section we have shown that decoupling the gauginos leads to a large region of parameter space

---

\(^3\)This relation can be understood easily in two limiting cases: 1) for $\tan \beta \approx 1$, in which case we can compute $T$ perturbatively expanding in the $M_{13}$ entry of the Higgsino mass matrix. 2) in the decoupling limit of heavy Higgsinos, in which case integrating them out produces a dimension 6 operator $|H_1^* D_\mu H_1 - H_2^* D_\mu H_2|^2$. However, Eq. (35) apparently holds with reasonable accuracy even in the general case of light Higgsinos and/or $\tan \beta - 1$ being not so small.
Figure 3: The contributions from the Higgsinos to the $T$ parameter (red color; contour values not framed) and the $S$ parameter (blue color; contour values framed). Vertical dashed lines represent an upper limit on $\mu$ coming from naturalness. Shaded regions indicate the regions in which the lightest neutralino is lighter than $m_Z/2 \simeq 45$ GeV.
consistent with EWPT, and in this section we find that the presence of the singlet Higgsino allows the interesting possibility of Higgsino DM, whereas in the MSSM it is excluded.

We now compute the thermal relic abundance of the lightest Higgsino $\chi$. We use the standard formalism from Ref. [21]. The freeze-out point is given in terms of the scaled inverse temperature $x = m_\chi/T$:

$$x_f = \ln \frac{0.038 g_\chi m_{Pl} m_\chi \langle \sigma v_{\text{rel}} \rangle}{g_\ast^{1/2} x_f^{1/2}},$$

where $m_{Pl} \approx 1.22 \times 10^{19}$ GeV, $g_\chi = 2$, $g_\ast$ is the number of SM degrees of freedom relativistic at the time of freeze-out ($g_\ast = 86.25$ for $m_b \ll T_f \approx m_W$), and $\langle \sigma v_{\text{rel}} \rangle$ is the thermal averaged annihilation cross section ($\chi + \chi \to \text{all}$), which enters the Boltzmann equation for the $\chi$ number density $n$:

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v_{\text{rel}} \rangle (n^2 - n_{eq}^2).$$

At freeze-out, $\chi$ is nonrelativistic and the cross section can be expanded in $v$:

$$\sigma v_{\text{rel}} = a + bv_{\text{rel}}^2, \quad \langle \sigma v_{\text{rel}} \rangle = a + 6b/x.$$  

Defining the annihilation integral

$$J(x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma v_{\text{rel}} \rangle}{x^2} dx \approx \frac{a}{x_f} + \frac{3b}{x_f^2},$$

the present-day mass density of $\chi$ is given by

$$\Omega_\chi h^2 \approx 1.07 \times 10^9 \text{ GeV}^{-1} \times \frac{g_\ast^{1/2} m_{Pl} J(x_f)}{m_\chi^{1/2} x_f^{1/2}}.$$  

There are several caveats in the analysis described above. 1) We ignore possible coannihilations. This will be valid if all the other neutralinos and charginos have masses at least a few $\times T_f$ higher than $m_\chi$. This treatment is indeed justified in the parameter region of interest to us, but may not be in general. 2) We do not give a careful treatment of near-threshold situations. Indeed, there are important thresholds associated with the $W$ and $Z$ bosons. Near these thresholds, our calculation may be subject to larger errors, and a more careful treatment should proceed along the lines of Ref. [22].

Let us now calculate various annihilation processes in turn. We first calculate $\chi + \chi \to f \bar{f}$ via s-channel $Z$ exchange. The $\chi$ coupling to $Z$ is given by

$$\frac{g}{2c_w} \frac{\kappa_\chi}{2} Z_{\mu \bar{\chi}} \gamma^\mu \gamma^5 \chi, \quad \kappa_\chi = 2V_{1\chi} V_{2\chi}.$$  

The resulting annihilation cross section in the approximation of massless final states is $\sigma_{f \bar{f}} v_{\text{rel}} = b_{f \bar{f}} v_{\text{rel}}^2$:

$$b_{f \bar{f}} = 7.31 \kappa_\chi^2 \frac{g^4}{32\pi c_w^4} \frac{m_\chi^2}{(4m_\chi^2 - m_Z^2)^2}.$$
where 7.31 is the sum of \( g_2^2 + g_3^2 \) over all SM quarks and leptons except for the top quark. The coupling \( \kappa_\chi \) is generically suppressed, especially in the lower right corner of the \( \mu-M \) plane. For this reason, large values of \( \Omega_\chi h^2 \) are attained for modest values of \( m_\chi \).

Above the \( W \) thresholds, we have to take into account the annihilation into \( W^+W^- \) proceeding via \( s \)-channel \( Z \) exchange and \( t,u \)-channel chargino exchange. The amplitudes for these processes are given by

\[
\frac{g^2}{2} \kappa_\chi \left( \bar{v}_2 \gamma_\lambda \gamma_5 \upsilon_{u_1} \right) \frac{i}{s - m_Z^2} \times \left[ \eta^{\mu\nu}(k_- - k_+)^\lambda - \eta^{\lambda\nu}(k_+ + 2k_-)^\mu + \eta^{\lambda\mu}(2k_+ + k_-)^\nu \right] \varepsilon_\mu^*(k_+) \varepsilon_\nu^*(k_-),
\]

and

\[
\frac{g^2}{4} \left[ \bar{v}_2 \gamma_\mu P \frac{i(k_+ - \mu_1 + \mu)}{(k_+ - p_1)^2 - m_\chi^2} \gamma_\mu P u_1 - (1 \leftrightarrow 2) \right] \varepsilon_\mu^*(k_+) \varepsilon_\nu^*(k_-), \quad P = V_{1\chi} - V_{2\chi} \gamma_5;
\]

respectively, where \( u_{1,2} \) are the initial state spinors with momenta \( p_{1,2} \), \( \bar{v}_i \equiv u_i C \) with \( C \) being the charge conjugation matrix. Since the \( Z \)-exchange amplitude vanishes at rest, only the latter amplitude contributes to the \( a \) coefficient of the resulting cross section \( \sigma_{WW} v_{rel} = a_{WW} + b_{WW} v_{rel}^2 \):

\[
a_{WW} = \frac{g^4}{32\pi} \left( 1 - \frac{m_W^2}{m_\chi^2} \right)^{3/2} \frac{(V_{1\chi}^2 + V_{2\chi}^2) m_\chi^2}{(m_\chi^2 + m_\chi^2 - m_W^2)^2}.
\]

However, \( b_{WW} \) needs to be included since it turns out that in this case typically \( b \gg a \) and the correction to the cross section at freeze-out is of order 1. Both the \( Z \)-exchange and chargino-exchange amplitudes contribute to \( b_{WW} \). The resulting expression is very long, and we do not present it here.

Finally, above the \( Z \) thresholds, we have to take into account the annihilation into \( ZZ \) proceeding via \( t,u \)-channel exchanges of all the three neutralinos \( \chi_a (\chi \equiv \chi_1) \) with the amplitude:

\[
\frac{g^2}{4c_w^2} \sum_{a=1}^3 \kappa_a^2 \left[ \bar{v}_2 \gamma_\mu \gamma_5 \frac{i(k_1 - \mu_1 + m_a)}{(k_1 - p_1)^2 - m_a^2} \gamma_\mu \gamma_5 u_1 - (1 \leftrightarrow 2) \right] \varepsilon_\mu^*(k_1) \varepsilon_\nu^*(k_2), \quad \kappa_a = V_{1\chi} V_{2a} + V_{1a} V_{2\chi}.
\]

The \( a \) coefficient of the resulting cross section \( \sigma_{ZZ} \) can be given explicitly:

\[
a_{ZZ} = \frac{g^4}{64\pi c_w^4} \left( 1 - \frac{m_Z^2}{m_\chi^2} \right)^{3/2} \left[ \sum_{a=1}^3 \frac{m_a^2}{m_\chi^2 + m_a^2 - m_Z^2} \right]^2.
\]

However, \( b_{ZZ} \) is again needed, which we do not present here.

In our computation, we do not include the \( t \)-channel sfermion exchange contributions to the \( f \bar{f} \) final states or the \( s \)-channel Higgs-boson exchange contributions to the \( WW \) and \( ZZ \) final states, which are generically suppressed and depend on free parameters other than \( \mu, M, \lambda \) and \( \tan \beta \). We also do not consider final states involving the Higgs boson(s), whose thresholds are higher than the ones discussed above.
Figure 4: The lightest neutralino relic density $\Omega h^2$ for four values of $\tan \beta$. The contours for $\Omega h^2 = 0.01, 0.05$ and 0.2 are denoted by the blue (thin solid) lines, while the blue (darkest) shading indicates regions with $0.09 \lesssim \Omega h^2 \lesssim 0.13$, corresponding to the 95\% CL region from WMAP [23]. Gray shading indicates regions where $m_\chi < m_{Z/2}$. The contours for the mass of the LSP, $m_\chi = 80, 150$ and 200 GeV, are also superimposed by dashed lines.
We now report the results of our numerical computations for the DM abundance. In Fig. 4 we show plots for the DM relic abundance in the $\mu - M$ plane for 4 different values of $\tan \beta \equiv t = 1.3, 1.4, 1.5$ and 1.6. In most of the parameter space for $t \approx (1 \sim 3)$, which we are interested in, the thermal relic abundance is much smaller than the observed DM abundance ($< 10\%$). Around $t = 1.4$, there are regions where the abundance has the observed value. The plots show the situation for 4 representative values of $t$ around 1.4. Going to smaller $t$ is disfavored by the EWPT, while for larger $t$ the DM region gets pushed towards larger values of $\mu$, which are disfavored by naturalness. We conclude that the new possibility for DM discussed here is compatible with the EWPT and naturalness for $\tan \beta < \sim 1.7$.

In the MSSM with decoupled gauginos, the lightest neutralino is a pseudo-Dirac Higgsino of mass $\mu$. Such a Higgsino has a large coupling to the $Z$ and, since naturalness places a bound on how large $\mu$ can be, the Higgsinos annihilate easily in the early universe contributing little to the DM. How does the LSP Higgsino of SUSY avoid this? There are two possibilities. In the DM regions with low $|M|$, visible for the low values of $\tan \beta$ in Fig. 4, $\chi$ is mainly singlet and this reduces the $Z$ coupling. On the other hand, in the high $|M|$ DM regions, the $H_1$ and $H_2$ components of the Majorana $\chi$ lead to a partial cancellation of the $Z$ coupling, again allowing substantial DM.

We finally consider the direct detection cross section of $\chi$. The $\chi$ coupling to the lightest Higgs is given by

$$\frac{\lambda \delta}{\sqrt{2}} \bar{\chi} \chi h, \quad \delta = V_{3\chi} (V_{1\chi} \cos(\alpha - \pi/4) + V_{2\chi} \cos(\alpha + \pi/4)), \quad (49)$$

while the coupling of $h$ to the quarks is given by

$$\frac{1}{\sqrt{2}v} h \left( \frac{\cos \alpha}{\sin \beta} m_u \bar{q}_u q_u - \frac{\sin \alpha}{\cos \beta} m_d \bar{q}_d q_d \right). \quad (50)$$

The nucleon cross section is then given by

$$\sigma_h(\chi N \rightarrow \chi N) = \frac{m_r^2}{\sqrt{2}v} \frac{\lambda \delta}{m_h^2} \left( X_u \frac{\cos \alpha}{\sin \beta} - X_d \frac{\sin \alpha}{\cos \beta} \right)^2 m_N^2; \quad (51)$$

where $m_r \equiv m_\chi m_N / (m_\chi + m_N)$ is the reduced mass, and $X_u$ and $X_d$ are certain linear combinations of nucleon matrix elements, which we conservatively take as $X_u \simeq 0.14$ and $X_d \simeq 0.24$ [24]. The cross section on a nucleus, normalized to a nucleon, is (for $\lambda = 2$):

$$\sigma_h(\chi p \rightarrow \chi p) \equiv 10^{-44} \text{ cm}^2 G, \quad G \simeq 200 \times \delta^2 \left( X_u \frac{\cos \alpha}{\sin \beta} - X_d \frac{\sin \alpha}{\cos \beta} \right)^2 \left( \frac{300 \text{ GeV}}{m_h} \right)^4. \quad (52)$$

Recall that for a DM mass in the $50 - 200$ GeV range of interest to us, the limit on the spin-independent direct detection cross section is $(2 \sim 3) \times 10^{-43} \text{ cm}^2$ [25]. We have studied numerical values of the factor $G$ in the region of $\tan \beta = 1.3 - 3$ and found that it never exceeds $\simeq (20 \sim 30)$. Thus the cross section is not in contradiction with the existing limits, even without taking into account the fact that in most of the parameter space the LSP relic abundance is much below the observed DM density. We have further studied the factor $G$ for the regions of parameter space with
Figure 5: Dark matter detection cross section $\sigma_h$ in units of $10^{-44}$ cm$^2$ (i.e. the factor $G$ in the text) in the $\mu$-$M$ plane for $\tan \beta = 1.3, 1.5$ and $m_{H^\pm} = 400, 700$ GeV. The blue (darkest) regions indicate the regions for $0.09 \lesssim \Omega h^2 \lesssim 0.13$ (see Fig. 4), while the gray shaded regions indicate the ones where $m_\chi < m_Z/2$. 
\[ \tan \beta = 1.3 - 1.6 \] where we get the observed relic density, i.e. the blue (darkest) regions in Fig. 4. We find that \( G \) ranges from about 1 to 5, as depicted in Fig. 5 for the case of \( m_{H^\pm} = 400, 700 \text{ GeV} \) for \( \tan \beta = 1.3, 1.5 \). This puts the cross section (52) well within reach of experiments currently under way.

## 5 Naturalness Constraints on Superparticle Masses

As mentioned in the Introduction, making the lightest Higgs boson heavier helps in relaxing the naturalness constraints on the model, while allowing heavier superpartners. It is of interest therefore to establish on quantitative grounds the upper bounds that can be set on physical superparticle masses. This is of direct relevance also to understand the precise significance of the results obtained in the previous two sections.

For this purpose, we consider an explicit simplified version of the \( \lambda \text{SUSY} \) model, defined by the following superpotential and the soft supersymmetry-breaking Lagrangian

\[ W = \lambda S H_1 H_2 + mH_1 H_2 + \frac{1}{2} MS^2, \quad (53) \]

\[ L_{\text{soft}} = -m_S^2 |S|^2 - \sum_{i=1,2} m_i^2 |H_i|^2 + (BmH_1 H_2 + \text{h.c.}), \quad (54) \]

and we will study especially the case in which the soft breaking mass \( m_S \) is taken large, consistently with naturalness. (See Ref. [26] for an earlier analysis.) We do not expect the bounds obtained in the generic \( \lambda \text{SUSY} \) model to be significantly different, or in any case more restrictive, than the ones valid in this specific case.

The scalar potential takes the form of Eq. (4) with

\[ \mu_1^2 = m_1^2 + \mu^2, \quad \mu_2^2 = m_2^2 + \mu^2, \quad \mu_3^2 = Bm - \lambda Ms, \quad (55) \]

\[ V(S) = (M^2 + m_S^2)|S|^2 \equiv \mu_S^2 |S|^2, \quad (56) \]

and the chargino mass parameter defined in Eq. (3) is \( \mu = m + \lambda s \), where \( s \) is the background expectation value of \( S \). In this model, the stability condition (6) implies also the conservation of \( CP \) at the vacuum.

### 5.1 Minimizing the potential

To understand the dependence of the minimum of the potential on various parameters, it is best to minimize first with respect to \( v_1 \) and \( v_2 \) for fixed \( s \). We then obtain

\[ V = \mu_S^2 s^2 - \frac{1}{\lambda^2} \left( \mu_3^2(s) - \mu_1(s) \mu_2(s) \right)^2. \quad (57) \]

This potential can in turn be minimized with respect to \( s \), giving

\[ \lambda s = -\gamma (\mu_3^2 - \mu_1 \mu_2) = -\gamma \frac{\lambda^2 v^2}{t + t^{-1}}, \quad (58) \]

\[ \gamma = \frac{M + m(t + t^{-1})}{\mu_S^2 + \lambda^2 v^2}, \quad (59) \]
Figure 6: The region with $F > 1/2$ in the $\mu$-$M$ plane, for $t = 1.5$ (between red solid lines) and for $t = 3$ (between blue dashed lines).

where $t \equiv \tan \beta$. Plugging this expression for $s$ into Eq. (55) and then into (9), we can obtain the expression for $v$ in terms of the original parameters of the model. Assuming that $s$ is small relative to $v$, which is the case for sufficiently large $\mu_S$, we find

$$\lambda^2 v^2 \simeq Bm \left( t + \frac{1}{t} \right) - (m_1^2 + m^2) - (m_2^2 + m^2) + \lambda^2 v^2 \left( M + \frac{4m}{t + 1/t} \right) \gamma,$$

which can be rewritten in the form

$$\lambda^2 v^2 \simeq F^{-1} \left[ Bm \left( t + \frac{1}{t} \right) - (m_1^2 + m^2) - (m_2^2 + m^2) \right],$$

$$F = 1 - \left( M + \frac{4m}{t + 1/t} \right) \gamma.$$  

This gives, together with $t = \mu_1/\mu_2$, the explicit dependence of $v$ on the original parameters of the model, which in turn allows us to determine the naturalness constraints. Note that the expression in the square bracket of Eq. (61) is the one which we would obtain in the pure 2HDM without the singlet field $S$. By taking $m_S$ sufficiently large, consistently with naturalness (see below), and restricting the range of other parameters, we shall always require that the factor $F$ be sufficiently close to unity so that we can neglect its presence. This is consistent with our treatment of the EWPT in Section 3, where the mixings between the singlet and doublet scalars were neglected. $F > 1/2$ is the numerical condition that we shall take to constrain the general space of the parameters. The regions $F > 1/2$ are depicted in Fig. 6 in the $\mu$-$M$ plane for $t = 1.5$ and 3. Here, the values of $m_{H^\pm}$ and $m_S^2$ are taken to saturate the naturalness upper bounds discussed in the next subsection (see Eqs. (64, 66, 72)), although using smaller values of $m_{H^\pm}$ leads to only negligible variations of the regions.
5.2 Naturalness bounds

We are interested in the naturalness constraints on $\mu_3$, which sets the bound on the Higgs sector through (11) and on the chargino mass through (20). We also want to know the bound on the stop masses and on the mass of the scalar $S$, which both affect, via one loop corrections, the masses $m_3^2$ and $m_2^2$, and therefore the masses $\mu_1^2$ and $\mu_2^2$. As customary, we shall require that the logarithmic derivatives of $v^2$ with respect to $\mu_1^2$, $\mu_2^2$, $m_{\tilde{Q}}^2$, $m_{\tilde{t}_R}^2$ and $m_S^2$ be less than $\Delta$.

One has, taking into account also the variation of $\tan \beta$, 

\[ \lambda^2 \delta v^2 = \delta \mu_3^2 (t + t^{-1}) - \delta \mu_1^2 (2F_1 - F_2) - \delta \mu_2^2 F_2, \]  

(63)

where

\[ F_1 = 1 + \frac{(t - t^{-1})^2}{4} \left( \frac{\lambda^2 v^2}{m_{H^\pm}^2} + 1 \right), \quad F_2 = 1 + \frac{t^2 - 1}{2} \left( \frac{\lambda^2 v^2}{m_{H^\pm}^2} + 1 \right). \]  

(64)

Here, we have used the approximation $\mu \approx m$, $\mu_3^2 \approx Bm$ and $F \approx 1$, valid for small $s$.

From the variation on $\mu_3$, upon use of Eq. (11), one obtains, taking $\lambda = 2$ and normalizing to a fine-tuning $\Delta^{-1} = 20\%$,

\[ m_A \lesssim 800 \text{ GeV} \left( \Delta/5 \right)^{1/2}. \]  

(65)

This therefore sets, through (10, 20), the limits on the charged Higgs-boson and chargino masses (for $\Delta = 5$):

\[ m_{H^\pm} \lesssim 700 \text{ GeV}, \quad \mu \lesssim \cos \beta m_{H^\pm}. \]  

(66)

The stop masses, $m_{\tilde{Q}}$ and $m_{\tilde{t}_R}$, and the soft breaking mass for the scalar $S$, $m_S$, affect $m_1^2$ and $m_2^2$ through the one loop RGEs

\[ \frac{d m_1^2}{d t} = \frac{\lambda^2 m_S^2}{8\pi^2} + \cdots, \]  

(67)

\[ \frac{d m_2^2}{d t} = \frac{\lambda^2 m_S^2}{8\pi^2} + \frac{3}{8\pi^2} \lambda_i^2 (m_{\tilde{Q}}^2 + m_{\tilde{t}_R}^2) + \cdots. \]  

(68)

There is no particular problem in integrating these equations up to the messenger scale $\Lambda_{\text{mess}}$ to obtain the standard stop contribution

\[ \delta m_2^2 \approx - \frac{3}{8\pi^2} \lambda_i^2 (m_{\tilde{Q}}^2 + m_{\tilde{t}_R}^2) \ln \frac{\Lambda_{\text{mess}}}{1 \text{ TeV}}, \]  

(69)

with an RGE improvement needed if $\Lambda_{\text{mess}}$ gets far above 100 TeV or so. From the sensitivity of $v^2$ to $\mu_2^2$ in Eq. (63), taking $\Lambda_{\text{mess}} = 100$ TeV as a reference point, one obtains

\[ m_{\tilde{t}_R}, m_{\tilde{Q}} \lesssim 1.3 \text{ TeV} \sin \beta \left( \Delta/5 \right)^{1/2} / \sqrt{F_2}. \]  

(70)

The contributions of $m_S^2$ to $m_2^2$ are, on the other hand, not equally well defined due to a rapid increase of $\lambda$ with energy. By integrating the RGEs up to $\Lambda \approx 10$ TeV, where perturbation theory is still valid, one obtains

\[ \delta m_2^2(\Lambda) \approx - \frac{m_S^2}{4} \ln \left( 1 + \frac{\lambda_0^2}{2 \pi^2} \ln \frac{\Lambda}{500} \right) \approx -0.3 m_S^2, \]  

(71)

\[ ^4 \text{The direct naturalness limit on } \mu \text{ from its contribution to } \mu_1^2 \text{ and } \mu_2^2 \text{ is less stringent.} \]
where $\lambda_0$ represents the coupling $\lambda$ at $\Lambda$, and, consequently,

$$m_S \lesssim 1 \text{ TeV} (\Delta/5)^{1/2}/\sqrt{F_1}.$$  \hfill (72)

Finally, it is of interest to know the naturalness bound on the gluino mass $m_{\tilde{g}}$, which contributes to $m_5^2$ via a two loop effect. Similarly to the bound on the stop masses, again for $\Lambda_{m_{\text{ess}}} = 100 \text{ TeV}$, one obtains

$$m_{\tilde{g}} \lesssim 2.8 \text{ TeV} \sin \beta (\Delta/5)^{1/2}/\sqrt{F_2}.$$  \hfill (73)

Note that all these bounds are proportional to $\lambda$, which is taken to be equal to 2.

Figure 7 summarizes the knowledge of the spectrum for different values of $\tan \beta$ and for two reference values of $m_{H^\pm} = 700, 400 \text{ GeV}$. The former corresponds to the highest value compatible with the naturalness bound of Eq. (66), and both values are consistent with the constraint from $b \rightarrow s\gamma$ without a destructive contribution from a stop-chargino loop [27]. The top and bottom panels show, respectively, the values of the Higgs boson masses and the upper bounds from naturalness on $\mu$, $m_S$, $m_{Q, t_R}$, and $m_{\tilde{g}}$, allowed by a 20% tuning. The bound on the $S$-fermion mass $M$ is not explicitly given, since it depends, via $F > 1/2$, on many other parameters.

We finally remark that the naturalness bounds derived from variations of $\tan \beta$ alone are less restrictive than the above bounds derived from the variations of $v^2$.

6 LHC Phenomenology

How will LHC probe $\lambda$SUSY — the theory defined by (3) and (4)? Clearly we must consider signals that result from the spectrum and mixings of the Higgs and Higgsinos, paying particular attention to the consequences of a large value of $\lambda$, and to differences with the MSSM. A general and detailed treatment is far beyond the scope of this paper; we will be content with discussing a few illustrative examples, especially as they relate to a collider probe of DM.

Throughout we assume that the $S$ scalar is heavy and its mixing with the doublet scalars can be ignored. In this case, the parameters relevant for the doublet scalars are $\mu_{1,2,3}^2$ and $\lambda$, which are reduced to 3 by the minimization condition for the VEV $v$. Including both neutral and charged Higgsinos, in the limit that the gauginos are sufficiently heavy, there are an additional 2 parameters $\mu$ and $M$, giving a total of 5 free parameters. It might appear that there are 2 extra parameters in this sector compared to the MSSM, $\lambda$ and $M$, but this is not the case — in the MSSM one needs an enhanced quartic coupling that brings in the top squark sector. Hence there is a single extra parameter, $M$, that describes the mass and mixings of an extra neutralino, so that the theory is highly constrained. Furthermore, the addition of the singlino allows for the possibility of Higgsino DM.

The four doublet scalars themselves show important differences with the MSSM. First and foremost the lighter Higgs $h$ is much heavier than in the MSSM. Second, the ordering of the spectrum is fixed: $h, H^\pm, H$ with $A$ heaviest.\footnote{Parameter choices allow $A^\pm$ to be lighter than $h$, but this must be consistent with the limit on $m_{H^\pm}$ from $b \rightarrow s\gamma$.} In contrast, for the MSSM the ordering of $H^\pm, A$ and $H$ is not fixed, except $A$ is lighter than $H$. Finally, $\tan \beta$ less than about 3 is excluded in
Figure 7: The spectrum of the Higgs bosons (top) and the naturalness bounds on parameters (bottom) as functions of $\tan \beta$, for two cases of $m_{H^\pm} = 700$ GeV (solid lines) and 400 GeV (dashed lines). In the top panel, four lines represent $m_A$, $m_H$, $m_{H^\pm}$ and $m_h$ from above, for each case of $m_{H^\pm} = 700, 400$ GeV. In the bottom panel, four lines represent $m_{\tilde{g}}^{\text{max}}$, $m_{\tilde{S}}^{\text{max}}$, $m_{\tilde{Q}, \tilde{t}_R}^{\text{max}}$ and $\mu^{\text{max}}$ from above, for each case of $m_{H^\pm} = 700, 400$ GeV.
the MSSM, while it is strongly preferred in λSUSY, leading to important changes in the various decay rates for these scalars. The three neutral scalars $h, H, A$ are each copiously produced at LHC from gluon-gluon fusion with cross sections in the range $0.1 - 10$ pb, decreasing with mass. While these rates can be computed in terms of the Higgs masses and mixings, $h, H$ and $A$ may also be produced in cascade decays of top and bottom squarks (and from other squarks, if light enough to be produced) so that we will not consider the rates themselves for determining the 5 free parameters, but will turn to the masses and widths of the three neutral scalars.

With a branching ratio of about $10^{-3}$ in much of parameter space, $h$ and $H$ will be visible in “gold-plated” events where the scalar decays to $ZZ$, and each $Z$ decays to $l\bar{l}$, where $l$ is an electron or muon. This will allow accurate measurements of $m_h$ and $m_H$. Similarly, the decay $A \rightarrow Zh$ will allow a measurement of $m_A$ from events of the form $l\bar{l}l\bar{l}jj$ where each $l\bar{l}$ reconstructs to a $Z$, as does the two jet system $jj$. The measurements of $m_{h,H,A}$, together with the constraint from the electroweak VEV, will allow all 4 parameters of the Higgs sector to be determined. This includes both the quartic coupling $\lambda$, so that one can evaluate the scale at which non-perturbative physics sets in, and $\tan \beta = \mu_1/\mu_2$. Unlike the MSSM, the ratio of VEVs can be determined relatively easily. Notice that $m_{h,H,A}$, and hence the extraction of the 4 fundamental parameters, do not depend on any assumption about the superpartner spectrum.

To extract $\mu$ and $M$, we specialize to the case that the gauginos are sufficiently heavier than the Higgsinos that we can ignore the Higgsino/gaugino mixing. In practice the accuracy with which the parameters can be extracted will depend on the size of the gaugino components of the lighter neutralinos and charginos. Over much of the parameter space, the decay of $H, A$ and $h$ to Higgsino pairs is kinematically allowed. Since it is induced at tree-level by the large coupling $\lambda$, it will be competitive with the $WW$ mode, generically having a significant effect on the total widths $\Gamma_{H,A,h}$. Note that the $WW, ZZ$ and $t\bar{t}$ modes depend on the Higgs mixing parameters and on $\tan \beta$ (needed for example to compute the top Yukawa coupling). However, they can be accurately computed, since these parameters will have been determined by the mass measurements. The widths to Higgsinos depend on both the Higgs mixings, already determined by the Higgs masses, and by the Higgsino mixings and masses, which are determined by $\mu$ and $M$. Can the total widths $\Gamma_{H,A,h}$ be measured sufficiently accurately, by reconstructing the invariant mass from gold-plated events, to extract $\mu$ and $M$? For $A \rightarrow Zh$ there will be insufficient rate to rely on the $6l$ events, and the $4ljj$ events are unlikely to be reconstructed with sufficient accuracy. Hence we consider the possibility of measuring $\mu$ and $M$ via the contributions of the Higgsino pair mode to $\Gamma_H$ and $\Gamma_h$, reconstructing these total widths using the $4l$ final states.

The partial widths to Higgsinos for $H$ and $h$ are shown as contours in Fig. 8 for $\lambda = 2$, $\tan \beta = 1.3, 1.5$ and $m_{H^\pm} = 400, 700$ GeV in the $\mu-M$ plane. For each of the four panels we give the dominant non-Higgsino widths for comparison: $\Gamma_0(h) = \Gamma(h \rightarrow WW + ZZ)$ and $\Gamma_0(H) = \Gamma(H \rightarrow WW + ZZ + t\bar{t} + hh)$. Consider for example the panel with $\tan \beta = 1.3$ and $m_{H^\pm} = 400$ GeV, where $\Gamma_0(h) \approx 8$ GeV and $\Gamma_0(H) \approx 6$ GeV. From inspection of the contours, we find that the $\tan \beta$ starts deviating from 1.

---

6As $m_H$ increases and approaches its naturalness limit of $700$ GeV the decoupling regime is reached, so that the decays $H \rightarrow ZZ$ and $A \rightarrow Zh$ become suppressed relative to the $t\bar{t}$ mode. Since the production cross section is also reduced, mass measurements from the gold-plated decays may be difficult in this region. However, this region is not the most natural expectation. Note that while the couplings of either $h$ or $H$ to vector boson pairs vanish in λSUSY as $\tan \beta \rightarrow 1$, this suppression becomes rapidly ineffective as $\tan \beta$ starts deviating from 1.
Figure 8: Contours for the partial widths $H \to \text{Higgsinos}$ (dashed red curves) and $h \to \text{Higgsinos}$ (solid black curves), labeled in GeV, in the $\mu$-$M$ plane for $\lambda = 2$. The four panels are for $\tan \beta = 1.3, 1.5$ and $m_{H^\pm} = 400, 700$ GeV. For each of the four panels we give the dominant non-Higgsino widths, $\Gamma_0$, for comparison. The gray shaded regions have $m_\chi < m_Z/2$, while the dark (blue) shaded region corresponds to $0.09 \lesssim \Omega_\chi h^2 \lesssim 0.13$. 

21
Higgsino widths for both $h$ and $H$ are in the range of $(10 \sim 100)\%$ of the corresponding non-Higgsino widths, allowing a determination of $\mu$ and $M$ over much of the parameter space. The accuracy of the determination will depend on the location in the $\mu$-$M$ plane. For much of the DM region, shown shaded in Fig. 8, the percentage changes in the widths induced by the Higgsino mode is roughly in the region of 30%. Increasing $m_{H\pm}$ to 700 GeV leads to similar percentage changes in the widths, so again measurements of $\mu$ and $M$ are possible. The number of gold-plated events from $H$ decay decreases as both $m_H$ and $\tan \beta$ are increased, making the measurement more difficult as the regions disfavored by naturalness and EWPT are approached. It will, of course, be highly significant if measurements at the LHC indicate values of $\mu$ and $M$ in the DM region, determining whether the DM Higgsino is mainly singlino or has comparable doublet and singlet components, and allowing predictions for the direct detection rate and the LSP mass. As $\tan \beta$ is increased from 1.3 to 1.5 the signals in $\Gamma_{h,H}$ are comparable, but the dominantly singlino region is essentially absent, so it will be an easier task to verify consistency with the remaining DM region.

In Fig. 9 we show the contours for $\Gamma(H,h \rightarrow \text{Higgsinos})$ for larger values of $\tan \beta$ that do not allow thermal relic Higgsinos to be DM. Nevertheless, the change in the total widths induced by the Higgsino contributions is again a significant percentage, so that $\mu$ and $M$ can be measured. Determining parameters from studying the decay modes of Higgsinos may also be possible, but it looks to be very difficult. Consider pair production of top squarks, followed by decays to $t + \text{Higgsino}$. The heavier neutral Higgsinos cascade to the LSP and a Higgs boson, so that the decay chain is $\tilde{t} \rightarrow t\tilde{\chi}_{2,3}$ followed by $\tilde{\chi}_{2,3} \rightarrow \tilde{\chi}_1 H$ or $\tilde{\chi}_{2,3} \rightarrow \tilde{\chi}_1 h$, leading to signals $t\tilde{t}l\bar{l}l\bar{l}$ with the four leptons reconstructing to $m_H$ or $m_h$. The ratio of these two types of events is another probe of the parameter space, since it is independent of the mass and production rate of the top squarks. However, this could be contaminated by events with bottom squarks decaying to top quarks and the charged Higgsino, which itself cascades to $H$ or $h$.

### 7 Overview and Conclusions

One year from now the Large Hadron Collider will start directly exploring for the first time the energy range well above the Fermi scale. Many speculations of the last three decades on ElectroWeak Symmetry Breaking (EWSB) will be rendered irrelevant, while some, perhaps, may emerge as part of physical reality. Therefore, the time is right to briefly reconsider the status of these speculations, including the one presented in this work.

There are two main sets of considerations that have played a dominant role to orient ideas and specific work on the subject. One rotates around the naturalness problem of the Fermi scale. Why is the Higgs boson light relative to any mass scale appearing in the UV completion of the SM, whatever it may be? More quantitatively, as illustrated by all known examples that attack the problem, why is the Higgs mass not at least as large as the SM contribution with a cutoff scale where new physics sets in, $\Lambda_{\text{NP}}$? The second set of considerations relates to the significance of the EWPT to the EWSB problem. If new physics occurs at $\Lambda_{\text{NP}}$, would it not be manifest, even indirectly, in EWPT?

In principle these two kinds of considerations do not conflict with each other: they might have actually merged into a coherent picture for the physics that underlies EWSB. So far, it is fair to
Figure 9: As for Fig. 8, except for $\tan \beta = 2, 2.5$. There are no regions giving sufficient thermal relic Higgsino DM. Note that a larger region of $M$ is shown than in Fig. 8, but smaller regions in $\mu$, corresponding to the more stringent naturalness constraints at larger $\tan \beta$. 
say that this has not happened. On the contrary, the impressive success of the SM in providing an accurate perturbative description of the EWPT casts doubts on the physical relevance of many ideas put forward to attack the naturalness problem of the Fermi scale. This issue, the “little hierarchy problem,” is a quantitative one [28]: for $\Lambda_{NP}$ as low as implied in various attempts to attack the naturalness problem, why is the new physics not manifest in the EWPT? At first sight the supersymmetric attempts seem exempt from this difficulty, as they cope easily with the EWPT. In their simplest versions, however, they also have a problem accommodating the direct bound on the Higgs mass. To satisfy the bound apparently requires either a lack of naturalness or a complication of the theory.

It would neither be possible nor appropriate to recall here all the attempts to get around this difficulty. Nevertheless one aspect is worth emphasizing, as trivial as it may be. While ignoring the naturalness problem is not in contradiction with experiment, it has however a striking practical consequence, other than being theoretically unsatisfactory: it weakens the case for finding new physics at the LHC. Needless to say, what will be found at the LHC does not depend on our choice of the physically relevant problems. However, insisting on the naturalness problem, as we do in this paper, has at least the advantage that the proposals to solve it will be scrutinized when the LHC will be successfully turned on.

With this motivation in mind, the framework analyzed in this paper originates from the observation that the little hierarchy problem might have (part of) its source in a misinterpretation of the EWPT, especially related to the inferred strong bound on the Higgs mass. Maybe the results of the EWPT, although consistent with the SM, do hide after all some new physics. This might be especially the case if the corresponding new theory still allows a successful perturbative description of the EWPT in a large portion of its parameter space. At first sight, this does not seem to be of relevance to supersymmetry, since, as observed above, standard supersymmetry has no special problem with the EWPT. However supersymmetry in its minimal version has a theoretical bound on the Higgs mass which is even stronger than the indirect experimental bound from the EWPT, causing its own naturalness problems. Hence the proposal that we make here of relaxing as much as possible the upper bound on the Higgs mass by going to $\lambda_{SUSY}$, while still retaining the successful perturbative description of the EWPT. In our view, this “bottom-up” approach to the little hierarchy problem deserves attention.

We can summarize this bottom-up approach to supersymmetry as follows: the Higgs sector is pushed up to the (200 – 700) GeV range, the superpartners may then be heavier than 700 GeV, while most of the parameter space with successful EWSB naturally yields the observed values for the $S$ and $T$ parameters.

Needless to say $\lambda_{SUSY}$ has two related prices to pay, which have certainly restrained most people from considering it seriously up to now. The first is that it is not a UV-complete theory in the same sense that the MSSM or the standard NMSSM are: at an energy scale above 10 TeV or so, some change of regime of the theory should intervene that prevents the coupling $\lambda$ from exploding. This is not uncommon in most attempts to address the little hierarchy problem. The other price is that $\lambda_{SUSY}$ is not manifestly consistent with gauge coupling unification. To determine this would require specifying the form of the necessary change of regime at high energy; it is nevertheless remarkable that a successful solution already exists in the literature [10, 11, 12].
Acknowledgments

The work of R.B. and V.S.R. was supported by the EU under RTN contract MRTN-CT-2004-503369. R.B. was also supported in part by MIUR and by a Humboldt Research Award. R.B. thanks the Institute for Theoretical Physics of the Heidelberg University for hospitality while this work was completed. The work of L.J.H. and Y.N. was supported in part by the US Department of Energy under Contracts DE-AC03-76SF00098. L.J.H. was also supported in part by the National Science Foundation under grants PHY-00-98840 and PHY-04-57315, and Y.N. by the National Science Foundation under grant PHY-0403380, by a DOE Outstanding Junior Investigator award, and by an Alfred P. Sloan Research Fellowship.

A One-loop Contributions of New Particles to $S$ and $T$

Recall that the $T$ and $S$ parameters are given by

$$ T = \frac{A_{33}(0) - A_{WW}(0)}{\alpha_{em} M_W^2}, \quad (74) $$

$$ S = \frac{4 s_w c_w}{\alpha_{em}} F_{30}(M_Z^2) \approx \frac{4 s_w c_w}{\alpha_{em}} F_{30}(0), \quad (75) $$

where $A_{ij}$ and $F_{ij}$ are extracted from the gauge-boson pair vacuum polarization amplitudes:

$$ \Pi^{ij}_{\mu\nu}(q) = -ig^{\mu\nu}[A^{ij}(0) + q^2 F^{ij}(q^2)] + q_{\mu}q_{\nu} \text{terms}, \quad (76) $$

with $i, j = W, \gamma, Z$, or $i, j = 0, 3$ for $B_{\mu}$ and $W_{3,\mu}$, $\alpha_{em}$ is the structure constant, and $s_w \equiv \sin \theta_W$ and $c_w \equiv \cos \theta_W$ with $\theta_W$ the Weinberg angle.

We record here one-loop expressions for $A(0)$ [29] and $F(0)$ produced by bosons and fermions coupled to a generic gauge boson $W_{\mu}$ with unit strength. For a boson loop with internal masses $m_1$ and $m_2$ and coupling $iW_{\mu} \phi_1 \overset{\leftrightarrow}{\partial_{\mu}} \phi_2$ we have

$$ A(0) = \frac{1}{16\pi^2} \left[ \frac{m_1^2 + m_2^2}{2} - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2} \right] \equiv 2\alpha_{em} v^2 A(m_1, m_2), \quad (77) $$

$$ F(0) = \frac{1}{96\pi^2} \left[ -\ln \frac{\Lambda^4}{m_1^2 m_2^2} + \frac{4m_1^2 m_2^2}{(m_1^2 - m_2^2)^2} \right. $$

$$ \left. + \frac{m_1^6 + m_2^6 - 3m_1^2 m_2^2 (m_1^2 + m_2^2)}{(m_1^2 - m_2^2)^2} \ln \frac{m_1^2}{m_2^2} \right] \equiv \frac{1}{4\pi} F(m_1, m_2). \quad (78) $$

There is an additional diagram contributing to $S$ for a Higgs boson, in which the gauge boson $W_{\mu}$ and the Higgs boson $\phi$ propagate in the loop, giving $\delta S = m_W^2 G(m_\phi, m_W)$ where

$$ G(m_1, m_2) \equiv \frac{1}{2\pi} \left[ \frac{2m_1^2 m_2^2}{(m_1^2 - m_2^2)^3} \ln \frac{m_1^2}{m_2^2} - \frac{m_1^2 + m_2^2}{(m_1^2 - m_2^2)^2} \right]. \quad (79) $$
For a fermion loop with internal masses $m_1$ and $m_2$ and a vector coupling $W_\mu \bar{\psi}_1 \gamma^\mu \psi_2$, we have

\[
A(0) = \frac{1}{16\pi^2} \left[ (m_1 - m_2)^2 \ln \frac{\Lambda^4}{m_1^2 m_2} - 2m_1 m_2 + \frac{2m_1 m_2 (m_1^2 + m_2^2) - m_1^4 - m_2^4}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2} \right] \equiv 2\alpha_{em} v^2 \tilde{A}(m_1, m_2), \quad (80)
\]

\[
F(0) = \frac{1}{24\pi^2} \left[ -\ln \frac{\Lambda^4}{m_1^2 m_2^2} - \frac{m_1 m_2 (3m_1^2 - 4m_1 m_2 + 3m_2^2)}{(m_1^2 - m_2^2)^2} + \frac{m_1^6 + m_2^6 - 3m_1^2 m_2^2 (m_1^2 + m_2^2) + 6m_1^3 m_2^3}{(m_1^2 - m_2^2)^3} \ln \frac{m_1^2}{m_2^2} \right] \equiv \frac{1}{4\pi} \tilde{F}(m_1, m_2). \quad (81)
\]

For an axial coupling, the result can be obtained by letting $m_1 \to -m_1$. These expressions are valid for both Dirac and Majorana fermions, with an extra factor of 2 in the case of identical Majorana fermions.

References

[1] The LEP Collaborations, the LEP Electroweak Working Group, and the SLD Electroweak and Heavy Flavour Groups, arXiv:hep-ex/0509008, as updated on http://www.cern.ch/LEPEWWG

[2] R. Barbieri and L. J. Hall, arXiv:hep-ph/0510243.

[3] R. Barbieri, L. J. Hall and V. S. Rychkov, arXiv:hep-ph/0603188.

[4] R. Barate et al. [ALEPH Collaboration], Phys. Lett. B 565, 61 (2003) [arXiv:hep-ex/0306033]; LEP Higgs Working Group Collaboration, arXiv:hep-ex/0107030.

[5] J. R. Espinosa and M. Quiros, Phys. Rev. Lett. 81, 516 (1998) [arXiv:hep-ph/9804235].

[6] N. Polonsky and S. Su, Phys. Lett. B 508, 103 (2001) [arXiv:hep-ph/0010113].

[7] K. Tobe and J. D. Wells, Phys. Rev. D 66, 013010 (2002) [arXiv:hep-ph/0204196].

[8] P. Batra, A. Delgado, D. E. Kaplan and T. M. P. Tait, JHEP 0402, 043 (2004) [arXiv:hep-ph/0309149].

[9] J. A. Casas, J. R. Espinosa and I. Hidalgo, JHEP 0401, 008 (2004) [arXiv:hep-ph/0310137]; A. Brignole, J. A. Casas, J. R. Espinosa and I. Navarro, Nucl. Phys. B 666, 105 (2003) [arXiv:hep-ph/0301121].

[10] R. Harnik, G. D. Kribs, D. T. Larson and H. Murayama, Phys. Rev. D 70, 015002 (2004) [arXiv:hep-ph/0311349].

[11] S. Chang, C. Kilic and R. Mahbubani, Phys. Rev. D 71, 015003 (2005) [arXiv:hep-ph/0405267].

[12] A. Birkedal, Z. Chacko and Y. Nomura, Phys. Rev. D 71, 015006 (2005) [arXiv:hep-ph/0408329].

[13] A. Maloney, A. Pierce and J. G. Wacker, arXiv:hep-ph/0409127.
[14] K. S. Babu, I. Gogoladze and C. Kolda, arXiv:hep-ph/0410085.
[15] A. Delgado and T. M. P. Tait, JHEP 0507, 023 (2005) [arXiv:hep-ph/0504224].
[16] B. Gripaios and S. M. West, arXiv:hep-ph/0603229.
[17] M. Bastero-Gil, C. Hugonie, S. F. King, D. P. Roy and S. Vempati, Phys. Lett. B 489, 359 (2000) [arXiv:hep-ph/0006198].
[18] http://lepewwg.web.cern.ch/LEPEWWG/plots/summer2005/s05_stu_contours.eps
[19] P. H. Chankowski, T. Farris, B. Grzadkowski, J. F. Gunion, J. Kalinowski and M. Krawczyk, Phys. Lett. B 496, 195 (2000) [arXiv:hep-ph/0009271].
[20] P. H. Chankowski, M. Krawczyk and J. Zochowski, Eur. Phys. J. C 11, 661 (1999) [arXiv:hep-ph/9905436].
[21] E. W. Kolb and M. S. Turner, “The Early Universe,” Addison-Wesley (1990).
[22] K. Griest and D. Seckel, Phys. Rev. D 43, 3191 (1991).
[23] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003) [arXiv:astro-ph/0302209]; C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003) [arXiv:astro-ph/0302207].
[24] M. Drees and M. Nojiri, Phys. Rev. D 48, 3483 (1993) [arXiv:hep-ph/9307208]; R. Barbieri, M. Frigeni and G. F. Giudice, Nucl. Phys. B 313 (1989) 725.
[25] D. S. Akerib et al. [CDMS Collaboration], Phys. Rev. Lett. 96, 011302 (2006) [arXiv:astro-ph/0509259].
[26] Y. Nomura, D. Poland and B. Tweedie, Phys. Lett. B 633, 573 (2006) [arXiv:hep-ph/0509244].
[27] See, e.g., P. Gambino and M. Misiak, Nucl. Phys. B 611, 338 (2001) [arXiv:hep-ph/0104034]; M. Neubert, Eur. Phys. J. C 40, 165 (2005) [arXiv:hep-ph/0408179].
[28] R. Barbieri and A. Strumia, arXiv:hep-ph/0007265.
[29] R. Barbieri and L. Maiani, Nucl. Phys. B 224, 32 (1983).