Integrated Fault-Tolerant Control for Close Formation Flight

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This paper investigates the position-tracking and attitude-tracking control problem of close formation flight with vortex effects under simultaneous actuator and sensor faults. On the basis of the estimated state and fault information from unknown input observers and relative output information from neighbors, an integration of decentralized fault estimation and distributed fault-tolerant control is developed to deal with bidirectional interactions and to guarantee the asymptotic stability and $H_{\infty}$ performance of close formations.

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1. INTRODUCTION

Formation control of unmanned aerial vehicles (UAVs) has gained considerable attention in recent years. Close formation flight (CFF) is defined as formation geometry with a lateral spacing that is less than a wingspan in between UAVs. Multiple UAVs that fly in a CFF pattern can achieve a significant reduction in power demand, thereby improving cruise performances, extending mileage, and increasing payload via induced drag reduction [1], [2]. This drag reduction in CFF is due to beneficial wake vortex encounters. A Lead UAV generates vortices that induce an up-wash on the wing and a side-wash on the vertical tail behind the Wing UAVs [3], [4]. Thus, positive effects can be obtained by specifying the close formation concept.

Various control approaches, such as adaptive control [5], sliding-mode control [6], and receding horizon control [7], have been introduced by studies that focused on CFF problems. Most of the results are based on CFF models that either linearize nonlinear dynamics or disregard the vortex effects. Additional aerodynamic coupling effects, six-dimensional CFF equations, and proportional-integral formation-hold autopilots were established [8]. The literature indicates that research on nonlinear CFF has received little attention thus far. A decentralized robust control strategy was presented by using a high-gain observer for nonlinear CFF [9]. In addition, CFF problems only focused on the separated position-tracking [2], [10] or attitude-tracking issues [11]. Tracking control of roll dynamics in CFF was proposed by using $H_{\infty}$ techniques to withstand lateral aerodynamical perturbations [11].

However, faults may occur in one or more UAVs. Such faults can cause undesirable performances or even lead to catastrophic results in CFF systems. Therefore, fault-tolerant formation control (FTFC) is required to guarantee the stability and satisfactory properties of CFF systems. Three fault-types, namely, communication [12], [13], actuator [14]–[19], and sensor faults [20], [21], can be considered for CFF. An FTFC design for the formation flight of multiple UAVs was proposed to accommodate actuator faults on the basis of reference generator and finite-time convergence target [15]. Permanent and intermittent actuator faults were also dealt with by developing an FTFC scheme for UAV formation control [16]. On the one hand, most FTFC schemes in the literature are applied to counteract separated communication, actuator, or sensor faults, and the problem of coping with simultaneous actuator and sensor faults in UAV formation flight should be investigated. On the other hand, aside from the fact that previous works [15], [22]–[26] have only focused on separated fault estimation (FE) and fault-tolerant control (FTC) protocols, the presented control schemes have not considered the bidirectional interactions between FE and FTC systems and the direct application of estimated fault information from FE to FTC systems. Lan and Patton [27], [28] proposed integrated FE and FTC protocols for uncertain systems in the presence of Lipschitz nonlinearities, disturbances and simultaneous actuator, and sensor faults. That work simultaneously...
handled the effects of mutual couplings from the disturbances and nonlinearities between FE and FTC systems. Furthermore, their approaches [27], [28] used the so-called integrated FE/FTC design, which is known as a decentralized structure [20]. This integrated FE/FTC design cannot be applied in distributed formation control due to mechanical interconnections and restricted topology problem in practical applications. The distributed control designs [14], [17], [19], [29]–[32] were equipped with information interactions from their coupled neighbors and had the advantages of low cost and easy implementation. The consensus formation protocol in [17] only required the neighbors’ relative information, whereas the FTC protocol in [19] needed both its own information and the information of its neighbors to construct a local FTC approach in a fully distributed fashion. A distributed FTC strategy was previously proposed on the basis of adaptive approximations and local-state information for nonlinear uncertain interconnected systems [29]. Therefore, how to develop an integrated FE and FTC design with bidirectional interactions in a distributed fashion for CFF systems, in the presence of simultaneous actuator and sensor faults, is a challenging topic.

The major contributions of this paper can be summarized as follows.

1) The nonlinear CFF modeling with vortex effects and simultaneous actuator and sensor fault modeling are proposed. Feasible reference models in [15] and [24] are not required, instead, a strategic $H_{\infty}$ design is needed in CFF systems.

2) The proposed unknown input observers in the decentralized FE system are developed to estimate the faults and states without prior information requirements on the basis of previous works [27], [28]. The FE design does not require the bounds of the parametric system uncertainties [17], [24] and faults [32] nor any global knowledge about communication topology [17].

3) This paper considers the integration of decentralized FE and distributed FTC protocols compared with the globally decentralized FE/FTC structure [27], [28]. The bidirectional interactions between FE and FTC systems are also considered. Furthermore, unlike FTC designs based on local state information [29] or estimated fault information [17], [24], [31], the proposed FTC protocol is implemented in a fully distributed manner based on the estimated information in the FE system and on the output information of the neighbors.

The remainder of this paper is organized as follows. In Section II, the model description and system formulation are introduced. Section III is devoted to the decentralized FE design. Two types of distributed control schemes including the separated and integrated FE/FTC designs are presented in Section IV to achieve the good tracking of attitude and position commands. Simulation in Section V validates the efficiency of the proposed control algorithm. Finally, conclusions follow in Section VI.

Notations: The symbol $\dagger$ denotes the pseudoinverse, $\otimes$ denotes the Kronecker product, $He(X) = X + X^T$, and $\ast$ represents the symmetric part of the specific matrix.

Graph theory: An undirected graph $\mathcal{G}$ is a pair $(v, \zeta)$, where $v = \{v_1, \ldots, v_N\}$ is a nonempty finite set of nodes and $\zeta \subseteq v \times v$ is a set of edges. The edge $(v_i, v_j)$ is denoted as a pair of distinct nodes $(i, j)$. A graph is said to be undirected with the property $(v_i, v_j) \in \zeta$ that signifies $(v_j, v_i)$ for any $v_i, v_j \in v$. Node $j$ is called a neighbor of node $i$ if $(v_i, v_j) \in \zeta$. The set of neighbors of node $i$ is denoted as $N_i = \{j \mid (v_i, v_j) \in \zeta\}$. The adjacency matrix $\mathcal{A} = [a_{ij}]_{N \times N}$ is represented as the graph topology. $a_{ij}$ is the weight coefficient of the edge $(v_i, v_j)$ and $a_{ii} = 0, a_{ij} = 1$ if $(v_i, v_j) \in \zeta$, otherwise $a_{ij} = 0$. The Laplacian matrix $\mathcal{L} = [l_{ij}]_{N \times N}$ is defined as $l_{ij} = \sum_{i \neq j} a_{ij}$ and $l_{ii} = -a_{ij}, i \neq j$.

II. MODEL DESCRIPTION AND SYSTEM FORMULATION

In this section, the CFF modeling including formation-hold autopilots, kinematics, and aerodynamic coupling vortex effects are effectively established. The simultaneous actuator and sensor fault modeling in the longitudinal, lateral, and vertical directions are further introduced.

A. Close Formation Modeling

It is first envisaged that the Lead UAV is equipped with the Mach, heading, and altitude hold autopilots, respectively [8]

\[
\begin{aligned}
\dot{v}_0 &= - \frac{1}{\tau_v} v_0 + \frac{1}{\tau_v} v_{0c} \\
\dot{\psi}_0 &= - \left( \frac{1}{\tau_{\psi_a} + \frac{1}{\tau_{\psi_b}}} \right) \psi_0 - \frac{1}{\tau_{\psi_a} \tau_{\phi_b}} \psi_0 + \frac{1}{\tau_{\psi_a} \tau_{\phi_b}} \psi_{0c} \\
\dot{h}_0 &= - \left( \frac{1}{\tau_h_a + \frac{1}{\tau_h_b}} \right) h_0 - \frac{1}{\tau_h_a \tau_h_b} h_0 + \frac{1}{\tau_h_a \tau_h_b} h_{0c}
\end{aligned}
\]

(1)

where $v_0, \psi_0, h_0$ are the Lead’s velocity, heading angle, and altitude, respectively; $v_{0c}, \psi_{0c}, h_{0c}$ denote the reference inputs; and $\tau_v, \tau_{\psi_a}, \tau_{\psi_b}, \tau_h_a$, and $\tau_h_b$ are constant parameters.

According to the property of the Coriolis equation, the velocity of the Lead UAV in the Wing frame is described as

\[
v_{w}^W = v_{l}^W - \omega \times v_{w}^W - R_{w}^W v_{w}^W + \omega \times R_{w}^W
\]

(2)

where $v$, $\omega$, and $R$ denote the velocity, angular velocity, and position, respectively. The superscript $W$ represents the Wing frame and the subscripts $l$ and $w$ refer to the Lead and Wing UAVs, respectively. The vectors (2) in the longitudinal, lateral, and vertical directions are represented as

\[
\begin{bmatrix}
\omega_{w}^L \\
\psi_{w}^L
\end{bmatrix}, R_{w}^W = \begin{bmatrix}
x_{w}^W \\
y_{w}^W \\
z_{w}^W
\end{bmatrix}, v_{w}^W = \begin{bmatrix}
v_{w}^W \\
0 \\
0
\end{bmatrix}
\]

(3)
where \(x^W, y^W, \) and \(z^W\) denote the relative separations between the Lead and Wing UAVs in the Wing frame. Furthermore, the velocity of the Lead UAV in the Wing frame is given by \(v^W = C^{WL}v^L\) with the rotation matrix \(C^{WL}\) from the Lead frame to the Wing frame in the following form:

\[
C^{WL} = \begin{bmatrix}
\cos(\psi_l - \psi_w) & -\sin(\psi_l - \psi_w) & 0 \\
\sin(\psi_l - \psi_w) & \cos(\psi_l - \psi_w) & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

(4)

On substituting (3) and (4) into (2), the nonlinear kinematics in the three directions are given by

\[
\begin{align*}
\dot{x}^W &= v_l \cos(\psi_l - \psi_w) + \dot{\psi}_w y^W - w_w \\
\dot{y}^W &= v_l \sin(\psi_l - \psi_w) - \dot{\psi}_w x^W \\
\dot{z}^W &= 0.
\end{align*}
\]

(5)

The flight control of the Wing UAV is essential to be accommodated in close formation geometry to account for aerodynamic coupling vortices from up-washes and side-washes of the Lead UAV. Here, the stability derivatives \(\Delta C_{D_w}, \Delta C_{L_w}, \) and \(\Delta C_s\) in the Wing’s drag, lift, and side force are modeled in the following forms [8, 23]:

\[
\begin{align*}
\Delta C_{D_w} &= -\frac{2C_{L_w}C_{L_w}}{\pi^2 A_R} \ln \left( \frac{\bar{y}^2 + \bar{z}^2}{(\bar{y}-\pi/4)^2 + \bar{z}^2} \right) \\
\Delta C_{L_w} &= \frac{2a_u C_{L_w}}{\pi^2 A_R} \ln \left( \frac{\bar{y}^2 + \bar{z}^2}{(\bar{y}-\pi/4)^2 + \bar{z}^2} \right) \\
\Delta C_s &= \frac{\eta S_w a_\alpha C_{L_w}}{2\pi^2 A_R S} \times \ln \left( \frac{(\bar{y}-\bar{h}/8)^2 + \bar{z}^2}{(\bar{y}-\bar{h}/8)^2 + (\bar{z}+\bar{h}/2)^2} \right)
\end{align*}
\]

(6)

where \(\bar{y} = y^W/b\) and \(\bar{z} = z^W/b\), \(C_{L_w}\) and \(C_{L_w}\) denote the lift coefficients of the Lead and Wing UAVs, \(a_u\) and \(a_\alpha\) denote the lift curve slopes of the wing and vertical tail, \(A_R\) is the aspect ratio of the wing, \(\eta\) is the aerodynamic efficiency factor of the tail, \(S_w\) is the area of the vertical tail, \(b\) is the wing span, and \(h_z\) is the height of the vertical tail.

The existing vortices from the Lead UAV can reduce the induced drag and increase the lift of the Wing UAV in the close formation geometry, as shown in Fig. 1. To achieve the minimal drag and maximum lift for fuel saving, it is determined that the derivatives of the stabilities are given by

\[
\frac{d\Delta C_{D_w}}{d\bar{y}} = \frac{d\Delta C_{D_w}}{d\bar{z}} = 0, \quad \frac{d\Delta C_{L_w}}{d\bar{y}} = \frac{d\Delta C_{L_w}}{d\bar{z}} = 0
\]

(7)

and hence it can be shown that the optimal separations between the Lead and Wing UAVs are \(\bar{y} = \pm \pi/4\) and \(\bar{z} = 0\) in solving the equality constraints (7). To determine the changes in the drag, lift, and side force, linearization is performed on the basis of the optimal close formation geometry with the relative separations in the later and vertical directions as \(\bar{y} = \pi/4\) and \(\bar{z} = 0\):

\[
\begin{align*}
p_{D_{wy}} &= \frac{\partial \Delta C_{D_w}}{\partial \bar{y}} \bigg|_{\bar{y} = \pi/4, \bar{z} = 0} = -\frac{2C_{L_w}C_{L_w}}{\pi^4 A_R} \\
p_{L_{wy}} &= \frac{\partial \Delta C_{L_w}}{\partial \bar{y}} \bigg|_{\bar{y} = \pi/4, \bar{z} = 0} = \frac{2a_u C_{L_w}}{\pi^4 A_R} \\
p_{S_{wy}} &= \frac{\partial \Delta C_s}{\partial \bar{y}} \bigg|_{\bar{y} = \pi/4, \bar{z} = 0} = \frac{\eta S_w a_\alpha C_{L_w}}{\pi^4 A_R S} \\
p_{S_{wy}} &= \frac{\partial \Delta C_s}{\partial \bar{z}} \bigg|_{\bar{y} = \pi/4, \bar{z} = 0} = -\frac{\eta S_w a_\alpha C_{L_w}}{\pi^4 A_R S}
\end{align*}
\]

(8)

where

\[
\mu_1 = \frac{1}{\pi^2 (64) (\pi^2/64 + h_z^2/b^2)}
\]

\[
\mu_2 = \frac{3}{(9\pi^2/64) (9\pi^2/64 + h_z^2/b^2)}
\]

and \(\partial \Delta C_{D_w}/\partial \bar{z} = \partial \Delta C_{L_w}/\partial \bar{z} = \partial \Delta C_s/\partial \bar{z} = 0\). On the basis of the formation-hold autopilots of the Lead UAV (1) and the optimal stability derivatives (8), the formation-hold autopilots of the \(i\)th Wing UAV in line \((i = 1, \ldots, N)\), as shown in Fig. 1, are represented as

\[
\begin{align*}
\dot{\psi}_i &= -\frac{1}{\tau_v} v_i + \frac{1}{\tau_v} v_{ic} + \frac{qS}{m} \sum_{k=1}^{i} p_{D_{wy}} y_{k,k-1} \\
\dot{\psi}_i &= -\frac{1}{\tau_{\psi_a}} \psi_{i} + \frac{1}{\tau_{\psi_a}} \psi_{ic} + \frac{qS}{m} \sum_{k=1}^{i} (p_{S_{wy}} y_{k,k-1} + p_{S_{wy}} z_{k,k-1}) \\
\dot{h}_i &= -\frac{1}{\tau_{ha}} h_i + \frac{1}{\tau_{ha}} h_{ic} + \frac{qS}{m} \sum_{k=1}^{i} p_{L_{wy}} y_{k,k-1}
\end{align*}
\]

(9)

where \(y_{i,i-1}\) and \(z_{i,i-1}\) denote the relative separations between the \(i\)th UAV and the \((i-1)\)th UAV in the lateral
and vertical directions, \( q \) is the dynamic pressure, \( S \) is the surface area of the elliptical wing, and \( m \) is the total mass of each UAV.

Define the errors \( x_{ei} = x_e - x_{i,i-1}, y_{ei} = y_e - y_{i,i-1}, z_{ei} = z_e - z_{i,i-1}, v_{ei} = v_e - v_i, \) and \( \psi_{ei} = \psi_{i-1} - \psi_i, \) \( i = 1, \ldots, N, \) where \( x_e, y_e, \) and \( z_e \) denote the optimal separations, and \( x_{i,i-1} \) denotes the relative separation in the longitudinal direction. Furthermore, define \( \xi_{i,i-1} = z_{i,i-1} - h_i - h_{i-1}. \) On the basis of the optimal separations \( x_e, \) \( y_e, \) and trimming velocity \( v_e, \) the kinematics of the \( i \)-th UAV in line can be rewritten as

\[
\begin{align*}
\dot{x}_{i,i-1} &= v_c \cos \psi_{ei} + \psi_{i-1} y_c - v_i \\
y_{i,i-1} &= v_c \sin \psi_{ei} - \psi_{i-1} x_c \\
\dot{z}_{i,i-1} &= -\left( \frac{1}{\tau_{ha}} + \frac{1}{\tau_{hb}} \right) \xi_{i,i-1} - \frac{1}{\tau_{ha}} z_{i,i-1} \\
&+ \frac{1}{\tau_{ha} \tau_{hb}} (h_{ic} - h_{i-1,c}) + \frac{qS}{m} p_{Lao} y_{i,i-1}. 
\end{align*}
\]  

**REMARK 2.1** First, the vortex effects are difficult to measure and model. The strongly nonlinear and coupling characteristics in CFF can be represented by nonlinear but linearly parameterized functions [5], [8] or be treated as unknown functions in a nonparametric form [9]. Real-time accurate knowledge of the aerodynamic effects in CFF is generally unavailable; thus, linearization based on the optimal CFF geometry is used in this paper. Second, unlike the directed networks communicating with one preceding UAV through sensors [9], black and red arrows in Fig. 1 show the undirected data transmission amongst the neighboring Wing UAVs (e.g., interval position, velocity, heading angle, and angular velocity information).

B. Simultaneous Actuator and Sensor Fault Modeling

Define the input vectors \( u_iX, u_iY, \) and \( u_iZ, \) state vectors \( x_iX, x_iY, \) and \( x_iZ, \) output vectors \( y_iX, y_iY, \) and \( y_iZ, \) system uncertainties \( d_iX, d_iY, \) and \( d_iZ, \) and nonlinear item \( g(x_iY) \) for the Mach, heading, and altitude hold autopilots, i.e., \((X, Y, Z)\) channels in the longitudinal, lateral, and vertical directions.

\[
\begin{align*}
u_{iX} &= v_{ic}, \quad u_{iY} = \psi_{ic}, \quad u_{iZ} = h_{ic} \\
x_{iX} &= \left[x_{i,i-1} x_{el} / v_i \right] T \\
x_{iY} &= \left[y_{i,i-1} y_{el} / \psi_i \right] T \\
x_{iZ} &= \left[z_{i,i-1} \xi_{i,i-1} z_{el} / s \right] T \\
d_{iX} &= \left[ x_c v_{i-1} \sum_{k=1}^{i} y_{k,k-1} \psi_i \cos \psi_{el} \right] T \\
d_{iY} &= \left[ y_c \psi_{i-1} \sum_{k=1}^{i} z_{k,k-1} \sum_{k=1}^{i} y_{k,k-1} \right] T \\
d_{iZ} &= \left[ h_{i-1,c} z_{e,c} y_{i,i-1} \right] T \\
y_{iX} &= \left[ x_{i,i-1} v_i \right] T, \quad y_{iY} = \left[ y_{i,i-1} \psi_i \psi_{i} \right] T \\
y_{iZ} &= \left[ z_{i,i-1}, g(x_iY) = [v_c \sin \psi_{el} 0 0 0 0] \right] T.
\end{align*}
\]  

Assume that each UAV suffers from additive actuator and sensor faults, the dynamic models of the \( i \)-th UAV in line \((i = 1, \ldots, N)\) in the \( X, Y, \) and \( Z \) channels are described as

\[
\begin{align*}
\dot{x}_{iX} &= A_{Xx_iX} + B_X \left( u_{iX} + f_{i}^a \right) + D_X d_{iX} \\
y_{iX} &= C_{Xx_iX} + F_X f_{iX} \\
\dot{x}_{iY} &= A_{Yx_iY} + B_Y \left( u_{iY} + f_{i}^a \right) + D_Y d_{iY} + g(x_iY) \\
y_{iY} &= C_{Yx_iY} + F_Y f_{iY} \\
\dot{x}_{iZ} &= A_{Zx_iZ} + B_Z \left( u_{iZ} + f_{hi}^a + f_{hi(i-1)}^a \right) + D_Z d_{iZ} \\
y_{iZ} &= C_{Zx_iZ} + F_Z f_{iZ}
\end{align*}
\]  

where scalars \( f_{iX}^a, f_{iY}^a, \) and \( f_{iZ}^a \) denote the additive actuator faults of the \( i \)-th UAV in the \( X, Y, \) and \( Z \) input channels, \( f_{hi(i-1)}^a \) denotes the additive actuator fault of the \((i-1)\)th UAV in the \( Z \) input channel. \( f_{iX}^a \in R^{8N}, f_{iY}^a \in R^{4N}, \) and \( f_{iZ}^a \in R^{2N} \) denote the sensor faults in the output channels. Matrices \( A_{Xx}, B_X, D_X, A_{Yx}, B_Y, D_Y, A_Z, B_Z, \) and \( D_Z \) are appropriate gains under specific flight conditions in (9) and (10). Matrices \( C_{Xx}, C_{Yx}, C_{Zx}, F_X, F_Y, C_Z, \) and \( F_Z \) are of known and appropriate dimensions.

**REMARK 2.2** First, it is verified that the close formation conditions (12)–(14) are controllable and observable. Second, the physical meaning of actuator fault is the deviation of the reference control input. These actuator faults \( f_{iX}^a, f_{iY}^a, \) and \( f_{iZ}^a \) may cause additive disturbances in the low-level autopilot’s response to velocity, heading angle, and altitude commands \( v_{ic}, \psi_{ic}, \) and \( h_{ic} \) in CFF. The physical meaning of sensor fault refers to the deviation of the sensor output, such as the position errors \( x_{i,i-1}, y_{i,i-1}, \) and \( z_{i,i-1} \) by GPS measuring, the heading angle \( \psi_i, \) and heading angular acceleration \( \dot{\psi}_i \) errors. The control surface actuator faults and the sensor errors caused by hardware or cyber-attacks are not considered herein. Third, the nonlinear term \( g(x_iY) \) in the \( Y \) channel of the \( i \)-th UAV in line satisfies the Lipschitz constraint, i.e., \( \|g(x_iY) - g(x)\| \leq L_g \|x_iY - x\| \) where \( L_g = v_e \) is the Lipschitz constant. Moreover, the initial condition \( g(0) = 0 \) is assumed for simplicity.

**DEFINITION 2.1** (see [33]) Let \( \gamma > 0 \) and \( \epsilon > 0 \) be given constants, the closed-loop system can achieve a \( \mathbb{H}_\infty \) performance index no larger than \( \gamma \), i.e., \( \|G_{sd}\| < \gamma \) if the following form holds:

\[
\int_0^\infty z^T(t) z(t) dt \leq \gamma^2 \int_0^\infty d(t)^T d(t) dt + \epsilon.
\]

**LEMMA 2.1** (see [34]) There exists a zero eigenvalue for the Laplacian matrix \( L \) with \( 1_N \) as a corresponding right eigenvector and all nonzero eigenvalues have positive real parts in the undirected graph \( G \). Assume that \( \lambda_i \) denotes the \( i \)-th eigenvalue of \( L \), thus, \( 0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N \). Furthermore, if \( 1_N^T X = 0 \), then \( X^T L X \geq \lambda_2 X^T X \).

**Control objective:** This study aims to stabilize the dynamics of the \( i \)-th UAV in line (12)–(14) in the \( X, Y, \) and \( Z \) channels through an FE/FTC design involving first, the decentralized FE protocol to estimate the state and fault information, and second, the distributed FTC protocol based on...
on estimated information and relative output information of neighbors. Furthermore, the proposed controllers in CFF models are developed so that the Wing UAV’s velocity, heading angle to track with the relative signals of the Lead UAV, and separations in the longitudinal, lateral, and vertical directions are invariant while the Lead UAV is being maneuvered.

### III. DECENTRALIZED FE DESIGN

Define the extended states and system uncertainties as

$$\dot{x}_{ij} = A_j \dot{x}_{ij} + B_j u_{ij} + D_j \dot{a}_{ij}$$
$$y_{ij} = C_j \dot{x}_{ij}, \ j = X, Z$$

and augment the dynamics of the $i$th UAV (12)–(14) into

$$\dot{\bar{x}}_{ij} = \bar{A}_j \bar{x}_{ij} + \bar{B}_j u_{ij} + \bar{D}_j \dot{a}_{ij} + \bar{g}(A_0 \bar{x}_{ij})$$
$$\dot{y}_{ij} = \bar{C}_j \bar{x}_{ij}$$

where the gain matrices are described as

$$\bar{A}_X = \begin{bmatrix} \bar{A}_X & B_X & 0_{4 \times q_{ki}} \\ 0_{(1+q_{ki}) \times 4} & 0_{(1+q_{ki}) \times 1} & 0_{(1+q_{ki}) \times q_{ki}} \end{bmatrix}$$
$$\bar{A}_Y = \begin{bmatrix} \bar{A}_Y & B_Y & 0_{3 \times q_{ki}} \\ 0_{(1+q_{ki}) \times 3} & 0_{(1+q_{ki}) \times 1} & 0_{(1+q_{ki}) \times q_{ki}} \end{bmatrix}$$
$$\bar{A}_Z = \begin{bmatrix} \bar{A}_Z & B_Z & B_Z & 0_{3 \times q_{ki}} \\ 0_{2 \times q_{ki}} & 0_{2 \times q_{ki}} & 0_{2 \times q_{ki}} \end{bmatrix}$$

the $i$th estimation error dynamics are obtained as

$$\dot{\hat{e}}_{ij} = M_Y e_{ij} + \Gamma_Y \bar{D}_j \bar{d}_{ij} + \bar{g}(A_0 \hat{x}_{ij})$$

where $\Delta g_i = \bar{g}(A_0 \bar{x}_{ij}) - \bar{g}(A_0 \hat{x}_{ij})$.

The designed matrices $M_Y$, $G_Y$, and $J_Y$ can be obtained with the derived matrices $J_Y$ and $H_Y$. Furthermore, define $e_Y = [e_{1Y}^T, \ldots, e_{NY}^T]^T$, $\bar{d}_Y = [\bar{d}_{1Y}^T, \ldots, \bar{d}_{NY}^T]^T$, and $\Delta g = [\Delta g_1^T, \ldots, \Delta g_N^T]^T$ with $\hat{x}_Y = [\hat{x}_{1Y}^T, \ldots, \hat{x}_{NY}^T]^T$ and $\tilde{x}_Y = [\tilde{x}_{1Y}^T, \ldots, \tilde{x}_{NY}^T]^T$, and it follows that

$$\dot{\hat{e}}_Y = (I_N \otimes (\Gamma_Y \bar{A}_Y - J_Y \bar{C}_Y)) e_Y$$
$$+ (I_N \otimes \Gamma_Y \bar{D}_Y) \bar{d}_Y + (I_N \otimes \Gamma_Y) \Delta g$$.

Here, a sufficient condition for the existence of a robust unknown input observer (19) is given.

**THEOREM 3.1** There exists a robust unknown input observer (19) if the estimation error system (25) is robustly asymptotically stable with the constraints in (21)–(24).

**PROOF** With the definitions in (21)–(24), the estimation error system (25) is equivalent to the original estimation error dynamics (20). Hence, if (25) is robustly asymptotically stable, then (20) is also robustly asymptotically stable, indicating that $\lim_{t \to \infty} E_Y = 0$ in the presence of system uncertainties and nonlinear items. Furthermore, the objective of obtaining the unknown input observer is to design $H_Y$ and $J_Y$ such that (26) is robustly asymptotically stable.
Remark 3.1 First, the $i$th estimation error dynamics can be completely decoupled when the terms $\Gamma Y D Y \tilde{d}_Y = 0$ and $\Gamma Y \tilde{\Delta g}_i = 0$ are satisfied. The Hurwitz condition of the matrix $M_Y$ ensures that (26) is robustly asymptotically stable. However, (25) and (26) show that the FE performance is influenced by the system uncertainty $\tilde{d}_Y$ and the nonlinear error $\tilde{\Delta g}_i$. Second, the prior information of the nonlinear error $\tilde{\Delta g}_i$ and actuator and sensor faults in the system uncertainty $\tilde{d}_Y$ is not required in this paper. This positive effect is evident compared with the assumptions of bounded system uncertainties and nonlinearities [17], [24], [32]. Third, unlike the Luenberger observer, which generates residual signals and fault estimators to detect, isolate, and estimate the faults [20], unknown input observers are proposed in this paper. The system uncertainty $\tilde{d}_Y$ and the nonlinear error $\tilde{\Delta g}_i$ can be dealt with instead of being decoupled by the following separated and integrated FE/FTC strategies.

IV. DISTRIBUTED FTC DESIGN

In this section, the undirected topology $\mathcal{G}$ in Fig. 1 implies that each Wing UAV in line can receive the relative output information rather than the state information of its neighboring Wing UAVs. On the basis of the estimated information in the unknown input observers (19) and the relative output information of neighbors, two distributed protocols are put forward, namely, the separated FE/FTC and the integrated FE/FTC designs.

Consider that the $Y$ channel represents the general description, the distributed fault-tolerant controller for the $i$th UAV in the lateral direction is designed as

$$
    \dot{x}_{iY} = -K_{iY} \tilde{x}_{iY} - g_Y K_{iY} \sum_{j=1}^{N} a_{ij} \times \left( y_{ij} - F_{ij} \tilde{f}_{ij} + \gamma_{ij} + F_{ij} \tilde{f}_{ij} \right)
$$

where $K_{iY} = [K_{ij} 1_{0 \times q_1}]$, denotes the augmented gain with the state feedback gain $K_{iY} \in R_+^{2 \times 2}$, $a_{ij}$ denotes the $(i, j)$th entry of the adjacency matrix $A$, $K_{iY} \in R_+^{2 \times 2}$ denotes the distributed gain, and $g_Y$ is a positive scalar.

Then, the closed-loop system (13) is rewritten as

$$
    \dot{x}_{iY} = (A_Y - B_Y K_{iY}) x_{iY} + B_Y K_{iY} e_{iY} + g_Y \langle x_{iY} \rangle + D_Y \tilde{d}_Y - g_Y B_Y K_{iY} \sum_{j=1}^{N} a_{ij} \times \left( C_Y (x_{iY} - x_{jY}) + F_{ij} (e_{ij} - e_{jY}) \right).
$$

A. Separated FE and FTC Design

Note that the estimation errors in the decentralized FE system are not considered in the following separated FTC system, thus the corresponding FTC system (28) with the distributed controller (27) is derived as

$$
\begin{align*}
    \dot{x}_{Y} &= (I_N \otimes (A_Y - B_Y K_{iY}) - g_Y \mathcal{L} \otimes B_Y K_{iY} C_Y) x_Y + (I_N \otimes D_Y) \tilde{d}_Y + g_Y \langle x_Y \rangle \\
    \dot{z}_{Y1} &= C_{xy} x_Y
\end{align*}
$$

where $d_Y = [d_{1Y}^T, \ldots, d_{NY}^T]^T$, $x_Y = [x_{1Y}^T, \ldots, x_{NY}^T]^T$, and $g_Y = [g(x_{1Y})^T, \ldots, g(x_{NY})^T]^T$, $z_{Y1} \in R^{r_{1}}$ is a measurable output of a distributed observer corresponding to the distributed gain $K_{iY}$ to guarantee the robust stability of the separated FTC system (29).

Theorem 4.1 Given positive scalars $\gamma_1$ and $\varepsilon_{Y1}$, matrix $C_{Y0} \in R^{r_{1} \times 5}$, the separated FTC system (29) with the distributed controller (27) is stable with the $H_\infty$ performance $\|G_{z_{Y1}d_{Y}}\| < \gamma_1$, if there exists a positive definite matrix $Q_{Y0} \in R^{5 \times 5}$, and matrices $X_1 \in R^{1 \times 5}$ and $X_2 \in R^{1 \times 5}$ such that

$$
\begin{bmatrix}
    \Omega_1 I_N \otimes Q_{Y0} D_Y & I_N \otimes Q_{Y0} I_N \otimes C_{Y0}^T \\
    -\gamma_1^2 I_N & 0 & 0 \\
    -\varepsilon_{Y1} I_N & 0 & 0 \\
    -\varepsilon_{Y1} I_N & 0 & 0 \\
    -I_r N & 0 & 0 \\
\end{bmatrix} < 0
$$

with $\Omega_1 = I_N \otimes \text{He}(Q_{Y0} A_Y - B_Y K_{iY}) - g_Y \mathcal{L} \otimes \text{He}(g_Y B_Y C_{iY})$. Then, the state feedback gain is given by $K_{iY} = Q_{Y0}^{-1} X_1$, and the distributed gain is given by $K_{iY} = \hat{Q}_{Y0}^{-1} X_2$ with $Q_{Y0} B_Y = B_Y \hat{Q}_{Y0}$.

Proof Consider a Lyapunov function $V_{iY} = x_{iY}^T Q_{Y} x_{iY}$ with a symmetric positive definite matrix $Q_{Y}$, and the time derivative of $V_{iY}$ is obtained with a positive scalar $\varepsilon_{Y1}$.

$$
\dot{V}_{iY} \leq \varepsilon_{Y1}^T \text{He}(Q_{iY} (I_N \otimes (A_Y - B_Y K_{iY}) - g_Y \mathcal{L} \otimes B_Y K_{iY} C_{iY})) x_{iY} + \text{He}(x_{iY}^T Q_{iY} (I_N \otimes D_Y) \tilde{d}_Y) + \varepsilon_{Y1}^T Q_{iY} \tilde{x}_{iY} + \varepsilon_{Y1}^T x_{iY}^T g_Y \langle x_{iY} \rangle.
$$

On the basis of the condition $g(0) = 0$, then $\|g(x_{iY})\| \leq L_g \|x_{iY}\|$. According to Definition 2.1, the sufficient condition of achieving the $H_\infty$ performance $\|G_{z_{Y1}d_{Y}}\| < \gamma_1$ is $z_{Y1}^T \gamma_1 z_{Y1} - \gamma_1^2 d_{Y}^T d_{Y} + \dot{V}_i \leq 0$. Denote $C_{iY} = I_N \otimes C_{Y0}$ and $Q_{iY} = I_N \otimes Q_{Y0}$ with the symmetric positive definite matrix $Q_{Y0}$. According to the condition $Q_{Y0} B_Y = B_Y \hat{Q}_{Y0}$, $K_{iY} = \hat{Q}_{Y0}^{-1} X_1$, and $K_{iY} = \hat{Q}_{Y0}^{-1} X_2$, the Schur Lemma is used to obtain the linear matrix inequality (LMI) (30).

Furthermore, it is shown that the nonlinear error $\tilde{\Delta g}$ in the FTC system is not considered in the following separated FE system, thus the corresponding FE system (26) becomes

$$
\begin{align*}
    \dot{e}_{Y} &= \left( I_N \otimes (\Gamma Y \tilde{A}_{Y} - J_{1Y} \tilde{C}_{Y}) \right) e_{Y} + \left( I_N \otimes \Gamma Y \tilde{D}_{Y} \right) \tilde{d}_{Y} + \gamma_2 c_{t},
    z_{Y2} \in C_{XY} x_{Y}
\end{align*}
$$

where $z_{Y2} \in R^{r_{2} \times 5}$ is the measured output with $C_{XY} \in R^{r_{2} \times (5 + q_1) N}$. Hence, the objective of the proposed FE design is to devise the gains $H_Y$ and $J_{1Y}$ to guarantee the robust stability of the separated FE system (32).

Theorem 4.2 Given a positive scalar $\gamma_2$, matrices $C_{XY} \in R^{r_{2} \times 5}$, $C_{YY} \in R^{r_{2} \times 1}$, and $C_{XY} \in R^{r_{2} \times 4 q_1}$, the separated FE system (32) is stable with the $H_\infty$ performance $\|G_{z_{Y2}d_{Y}}\| < \gamma_2$, if there exist symmetric
positive-definite matrices $P_{Y1} \in \mathbb{R}^{5 \times 5}$, $P_{Y2} \in \mathbb{R}^{1 \times 1}$, and $P_{Y3} \in \mathbb{R}^{(\nu_y \times q_y)}$, and matrices $X_5 \in \mathbb{R}^{5 \times 1}$, $X_4 \in \mathbb{R}^{5 \times 1}$, $X_5 \in \mathbb{R}^{1 \times 1}$, $X_6 \in \mathbb{R}^{1 \times 3}$, $X_7 \in \mathbb{R}^{(\nu_y \times x)}$, and $X_8 \in \mathbb{R}^{(\nu_y \times x)}$ such that

$$\begin{bmatrix}
\Omega_{211} & \Omega_{212} & \Omega_{213} \\
\Omega_{311} & 0 & \Omega_{313} \\
0 & -X_3 F_{YF} & 0
\end{bmatrix} \cdot \begin{bmatrix}
I_N & 0 & 0 \\
0 & I_N & 0 \\
0 & 0 & I_N
\end{bmatrix} = 0 \quad (33)$$

with

$$\Omega_{211} = I_N \otimes \left[ \begin{array}{cccc}
\Omega_{211} & \Omega_{212} & \Omega_{213} \\
\Omega_{311} & 0 & \Omega_{313} \\
0 & -X_3 F_{YF} & 0
\end{array} \right]$$

$$\Omega_{212} = I_N \otimes \left[ \begin{array}{cccc}
\Omega_{211} & \Omega_{212} & \Omega_{213} \\
\Omega_{311} & 0 & \Omega_{313} \\
0 & -X_3 F_{YF} & 0
\end{array} \right]$$

$$\Omega_{213} = I_N \otimes \left[ \begin{array}{cccc}
\Omega_{211} & \Omega_{212} & \Omega_{213} \\
\Omega_{311} & 0 & \Omega_{313} \\
0 & -X_3 F_{YF} & 0
\end{array} \right]$$

Then, the unknown input observer gains are given by

$$H_{Y1} = P_{Y1}^{-1} X_3, \quad H_{Y2} = P_{Y2}^{-1} X_4, \quad H_{Y3} = P_{Y3}^{-1} X_6$$

**Remark 4.1** First, graph theory is adopted to describe undirected transformation networks from an arbitrary connected topology [10] to the CFF networks in this paper. Second, unlike the integration of fault detection and FTC mechanisms, which uses residuals between sensor measurements and desired values from monitors for detecting fault occurrence [14], [25], the proposed FE/FTC control scheme does not utilize any fault detection and isolation information to detect, identify, and isolate faults. As a result, online computation is minimized and the responsiveness of distributed controllers is expedited. Third, unlike the design based on local state information [29], FE information [17], [24], [31], or only output estimation errors [19], [21], the proposed FTC scheme (27) is constructed in a fully distributed fashion based on estimated information in FE and on the output information of neighbors.

**B. Integrated FE and FTC Design**

Note that the bidirectional interactions exist in both the FE and FTC systems. The estimation error $e_Y$ from the FE process influences the FTC performance and the nonlinear error $\Delta \tilde{g}$ from the FTC process influences the FE performance in turn. It follows that the integrated FE/FTC model is derived as

$$\dot{\tilde{x}}_Y = \left( I_N \otimes (\gamma_Y X_5 \gamma_Y X_5) \right) e_Y + \left( I_N \otimes (\gamma_Y X_5 \gamma_Y X_5) \right) e_Y$$

$$\dot{\gamma}_Y = \left( \gamma_Y \gamma_Y \right) e_Y + \left( \gamma_Y \gamma_Y \right) e_Y$$

$$\gamma_Y = \gamma_Y \gamma_Y + \gamma_Y \gamma_Y$$

where $Y = \gamma_Y X_5 \gamma_Y X_5$ and $S_2 = [I_4 0_{4 \times 1} 0_{4 \times q_y}]$, $z_Y = \gamma_Y X_5 \gamma_Y X_5$, is the accessible output vector in order to verify the integrated FE/FTC performance with the matrices $\gamma_Y \gamma_Y X_5 \gamma_Y X_5$, and $\gamma_Y \gamma_Y X_5 \gamma_Y X_5$. Hence, the objective of the proposed integrated FE/FTC design is to devise the state feedback gain $K_{Y1}$, the distributed gain $K_{Y2}$, and the unknown input observer gains $H_{Y1}$ and $H_{Y2}$ to guarantee the robust stability of the integrated structure system (35).

**THEOREM 4.3** Given positive scalars $\lambda_3$, $\gamma_2$, and $\gamma_3$, matrices $C_{\gamma} \gamma_0 \in \mathbb{R}^{(\nu_y \times 5)}$, $C_{\gamma} \gamma_0 \in \mathbb{R}^{(\nu_y \times 5)}$, and $C_{\gamma} \gamma_0 \in \mathbb{R}^{(\nu_y \times 5)}$, the integrated FE/FTC system (35) is stable with the $H_{\infty}$ performance $\|G_{Y1} \gamma_0 \| < \gamma_3$, if there exist symmetric positive-definite matrices $P_{Y1} \in \mathbb{R}^{5 \times 5}$, $P_{Y2} \in \mathbb{R}^{x \times x}$, and $P_{Y3} \in \mathbb{R}^{(\nu_y \times q_y)}$, and $Q_{Y0} \in \mathbb{R}^{(\nu_y \times 5)}$, and matrices $X_1 \in \mathbb{R}^{(\nu_y \times 5)}$, $X_2 \in \mathbb{R}^{(\nu_y \times 5)}$, $X_3 \in \mathbb{R}^{(\nu_y \times 5)}$, $X_4 \in \mathbb{R}^{(\nu_y \times 5)}$, $X_5 \in \mathbb{R}^{(\nu_y \times 5)}$, $X_6 \in \mathbb{R}^{(\nu_y \times 5)}$, $X_7 \in \mathbb{R}^{(\nu_y \times 5)}$, and $X_8 \in \mathbb{R}^{(\nu_y \times 5)}$ such that

$$\begin{bmatrix}
\Omega_{3211} & \Omega_{3212} & \Omega_{3213} \\
\Omega_{3221} & \Omega_{3222} & \Omega_{3223} \\
\Omega_{3231} & \Omega_{3232} & \Omega_{3233}
\end{bmatrix} \cdot \begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & 0 & 0 & 0 & \Omega_{15} \\
\Omega_{21} & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 & \Omega_{25} & \Omega_{26} & \Omega_{27} \\
\Omega_{31} & \Omega_{32} & \Omega_{33} & \Omega_{34} & 0 & 0 & 0 & 0 \\
0 & \Omega_{44} & \Omega_{45} & 0 & 0 & 0 & 0 & 0 \\
0 & \Omega_{54} & \Omega_{55} & 0 & 0 & 0 & 0 & 0 \\
0 & \Omega_{64} & \Omega_{65} & \Omega_{66} & 0 & 0 & 0 & 0 \\
0 & \Omega_{74} & \Omega_{75} & \Omega_{76} & \Omega_{77} & 0 & 0 & 0
\end{bmatrix} = 0 \quad (36)$$

with
The designed gains for the integrated system are given by $K_{Y} = \tilde{Q}^{-1}_{Y0}\bar{X}_1$, $K_{G} = \tilde{Q}^{-1}_{Y0}\bar{X}_2$, $H_{Y1} = \tilde{P}^{-1}_{Y1}\bar{X}_3$, $J_{Y1} = \tilde{P}^{-1}_{Y1}\bar{X}_4$, $H_{Y2} = \tilde{P}^{-1}_{Y1}\bar{X}_5$, $J_{Y2} = \tilde{P}^{-1}_{Y1}\bar{X}_6$, $H_{Y3} = \tilde{P}^{-1}_{Y1}\bar{X}_7$, and $J_{Y3} = \tilde{P}^{-1}_{Y1}\bar{X}_8$ with $Q_{Y0}B_{Y} = B_{Y}\tilde{Q}_{Y0}$.

**Proof.** Consider the respective Lyapunov functions $V_{xy} = x^T_{xy}\tilde{Q}_{Y}x_{xy}$ and $V_{ey} = e^T_{xy}\tilde{P}_{xy}e_{xy}$ with symmetric positive-definite matrices $\tilde{Q}_{Y}$ and $\tilde{P}_{xy}$. Then, the respective time derivatives of $V_{xy}$ and $V_{ey}$ are obtained with positive scalar $\varepsilon_{Y2}$ and $\varepsilon_{Y3}$:

$$
\dot{V}_{xy} \leq x^T_{xy}(\text{He}((I_{N} \otimes (A_{Y} - B_{Y}K_{Y}))) - g_{Y}\mathcal{L} \otimes B_{Y}K_{G}(C_{Y}))x_{Y} + \text{He}(x^T_{xy}\tilde{Q}_{Y}(I_{N} \otimes D_{Y}S_{2})\tilde{d}_{Y}) + \varepsilon_{Y2}x_{xy} + \varepsilon_{Y2}g_{Y}(x_{Y})g_{Y}(x_{Y}) + \text{He}(x^T_{xy}\tilde{Q}_{Y}(I_{N} \otimes B_{Y}K_{Y} - g_{Y}\mathcal{L} \otimes B_{Y}K_{G}F_{Y}S_{1})e_{Y}) \varepsilon_{Y3}.
$$

(37)

$$
\dot{V}_{ey} \leq e^T_{xy}(\text{He}(\tilde{P}_{xy}(I_{N} \otimes (\Gamma_{Y}\bar{A}_{Y} - J_{Y}\bar{C}_{Y})))) + \varepsilon_{Y3}L_{e}^{2}(I_{N} \otimes A_{e}^{2}A_{0}) + \varepsilon_{Y3}e_{xy} + \text{He}(e^T_{xy}\tilde{P}_{xy}(I_{N} \otimes \Gamma_{Y}\bar{A}_{Y})\tilde{d}_{Y}) e_{y}.
$$

(38)

Denote $P_{Y} = I_{N} \otimes \text{diag}(P_{Y1}, P_{Y2}, P_{Y3}, \tilde{Q}_{Y1} = I_{N} \otimes \tilde{Q}_{Y0}$, $\tilde{C}_{Y} = I_{N} \otimes \tilde{C}_{Y0}$, and $\tilde{C}_{ey} = I_{N} \otimes [\tilde{C}_{ey}, \tilde{C}_{ey0}, \tilde{C}_{ey1}]$ with symmetric positive-definite matrices $P_{Y1}, P_{Y2}, P_{Y3}$, and $\tilde{Q}_{Y0}$. Define matrices $H_{Y} = [H_{Y1}^T, H_{Y2}^T, H_{Y3}^T]$ and $J_{Y} = [J_{Y1}^T, J_{Y2}^T, J_{Y3}^T]^T$. According to Definition 2.1, the sufficient condition of achieving $\|G_{z_{xy}}\|_{V_{xy}} < \gamma_{xy}$ is $z_{xy}^Tz_{xy} - \gamma_{xy}^2d_{xy}^2 + d_{xy} + V_{xy} + V_{xy} < 0$. Thus, the Schur Lemma is applied and the proof of Theorem 4.3 is straightforward and is omitted here.

**Remark 4.2.** Note that the undirected graph plays a role in the description of the LMI formulations, i.e., $\text{He}(Q_{Y}(g_{Y}\mathcal{L} \otimes B_{Y}K_{G}C_{Y}))$ in (31), and $\text{He}(\tilde{Q}_{Y}(g_{Y}\mathcal{L} \otimes B_{Y}K_{G}C_{Y}))$ and $\text{He}(\tilde{Q}_{Y}(g_{Y}\mathcal{L} \otimes B_{Y}K_{G}F_{Y}S_{1}))$ in (37). Since the undirected graph $G_{u}$ is connected, it follows from Lemma 2.1 that $x^T_{xy}(\mathcal{L} \otimes B_{Y}K_{G}C_{Y})x_{Y} \geq \lambda_{2}x^T_{xy}(I_{N} \otimes B_{Y}K_{G}C_{Y})x_{Y}$ and $x^T_{xy}(\mathcal{L} \otimes B_{Y}K_{G}F_{Y}S_{1})x_{Y} \geq \lambda_{2}x^T_{xy}(I_{N} \otimes B_{Y}K_{G}F_{Y}S_{1})x_{Y}$, where $\lambda_{2}$ is the smallest nonzero eigenvalue of $\mathcal{L}$. In order to avoid the requirement of the global information of undirected graph, the following derivation is obtained:

$$
\hat{V}_{xy} \leq \varepsilon_{Y2}x_{Y}^{T}(\text{He}(\tilde{Q}_{Y}(I_{N} \otimes (A_{Y} - B_{Y}K_{Y})))) + \varepsilon_{Y2}x_{Y}^{T}(\text{He}(\tilde{Q}_{Y}(I_{N} \otimes D_{Y}S_{2})\tilde{d}_{Y})) + \varepsilon_{Y2}x_{Y}^{T}(\text{He}(\tilde{Q}_{Y}(I_{N} \otimes B_{Y}K_{Y} - \lambda_{2}g_{Y}B_{Y}K_{G}F_{Y}S_{1})e_{Y})) \varepsilon_{Y3}.
$$

(39)

**Remark 4.3.** The general dynamics (18) in the $Y$ channel are selected in Theorems 4.1–4.3. Furthermore, the integrated FE/FTC model based on the augmented dynamics (17) in the $X$ channel is considered with available output information

$$
\begin{align*}
\dot{x}_{x} &= (I_{N} \otimes (A_{X} - B_{X}K_{X} - g_{X}\mathcal{L} \otimes B_{X}K_{G}C_{X}))x_{X} + (I_{N} \otimes B_{X}K_{X} - g_{X}\mathcal{L} \otimes B_{X}K_{G}F_{X}S_{3})e_{X} + (I_{N} \otimes D_{X}S_{4})\tilde{d}_{X} \\
\dot{e}_{x} &= (I_{N} \otimes (\Gamma_{X}\bar{A}_{X} - J_{X}\bar{C}_{X}))e_{X} + (I_{N} \otimes \Gamma_{X}\bar{D}_{X})\tilde{d}_{X} \\
z_{X} &= \bar{C}_{ez}x_{X} + \bar{C}_{ez}e_{x}
\end{align*}
$$

(40)

where $S_{3} = [0_{q_{0} \times 4} 0_{q_{0} \times 1} I_{q_{0}}]$ and $S_{4} = [I_{z} 0_{x \times 1} I_{z \times q_{0}}]$. Note that only the estimation error $e_{x}$ from the FE process in the $X$ channel influences the FTC performance and the FTC process does not influence the FE performance in turn.

**Remark 4.4.** Note that the additive actuator fault $f_{a}^{u}$ occurs in the $i$th UAV and the actuator fault $f_{a}^{u(i-1)}$ occurs in the $(i-1)$th UAV in the $Z$ channel. The existing fault $f_{a}^{u(i-1)}$ makes the distributed FTC controller (27) not appropriate in the FTC process. Thus, the distributed fault-tolerant controller for the $i$th UAV in the vertical direction is designed as

$$
u_{i} = -K_{Z}\bar{z}_{i} + g_{Z}K_{Z} \sum_{j=1}^{N} a_{ij}(y_{i} - y_{j})
$$

(41)

where $K_{Z} = \bar{B}_{Z}\bar{A}_{Z}$ with $\bar{B}_{Z} = (\bar{B}_{Z}^{\dagger}\bar{B}_{Z})^{-1}\bar{B}_{Z}^{\dagger}$. Furthermore, the integrated FE/FTC model in the $Z$ channel is considered with the distributed fault-tolerant controller (41)

$$
\begin{align*}
\dot{x}_{z} &= (I_{N} \otimes \bar{A}_{Z})z_{Z} - (g_{Z}\mathcal{L} \otimes \bar{B}_{Z}K_{G}C_{Z})z_{Z} + (I_{N} \otimes \bar{D}_{Z})\tilde{d}_{Z} \\
\dot{e}_{z} &= (I_{N} \otimes (\Gamma_{Z}\bar{A}_{Z} - J_{Z}\bar{C}_{Z}))e_{Z} + (I_{N} \otimes \Gamma_{Z}\bar{D}_{Z})\tilde{d}_{Z} \\
z_{Z} &= \bar{C}_{ez}z_{Z} + \bar{C}_{ez}e_{Z}
\end{align*}
$$

(42)

Note that only the estimation error $e_{z}$ from the FE process in the $Z$ channel influences the FTC performance and the FTC process does not influence the FE performance in turn.

**V. SIMULATION RESULTS**

In this section, an application of integrated FE and FTC scheme for CFF models with simultaneous actuator and sensor faults is put forward to validate the effectiveness of the proposed control scheme. The parameters of CFF models in the $X$, $Y$, and $Z$ channels are characterized in Table I. The simulated parameters of the FE/FTC schemes in Theorems 4.1–4.3 are designed as $\varepsilon_{Y2} = \varepsilon_{Y3} = g_{Y} = 1, \gamma_{3} = 0.1, \bar{C}_{y0} = \bar{C}_{y1}$.
\[ \bar{C}_{eYx} = \begin{bmatrix} -1 & 1.2 & 0.5 & 0.1 & 0 \end{bmatrix}, \quad \bar{C}_{eYs} = \bar{C}_{eYf} = 0, \quad \text{and the sensor fault distribution matrices are satisfied with } F_{sX} = [0 1]^T, \quad F_{sY} = [1 0 1]^T, \quad \text{and } F_{sZ} = 1. \]

Then, the unknown input observer gains and the FE/FTC gains are derived as

\[
K_X = \begin{bmatrix} -382.5806 & 209.7101 & 350.8470 & 70.8220 \end{bmatrix},
\]

\[
K_Z = \begin{bmatrix} 34.4142 & 12.0474 & -28.7564 \end{bmatrix},
\]

\[
K_{sY} = \begin{bmatrix} -0.7936 & -5.2812 & -0.0943 & 3.6588 & 3.4755 \end{bmatrix},
\]

\[
K_{sY} = \begin{bmatrix} -0.6887 & -0.1054 & 3.1247 \end{bmatrix},
\]

\[
G_Y = \begin{bmatrix} 0 & 0.0266 & -0.0266 & 0 \\ 0 & -0.0565 & 0.0565 & 0 \\ 0 & -0.5661 & 0.5661 & 0 \\ 0 & 0.6785 & -0.6785 & 0 \\ 0 & -1.2357 & 0.2357 & 1 \end{bmatrix},
\]

\[
H_Y = \begin{bmatrix} 0.9811 & -0.1014 & -0.8797 \\ 0.0217 & 0.0875 & -0.1092 \\ -0.3889 & 0.4821 & -0.0932 \\ -0.0123 & -1.2979 & 1.3101 \\ 1.6721 & -0.2859 & -1.3862 \\ 0.3296 & 0.2527 & -0.5823 \\ 0.5279 & -0.1356 & 0.6635 \end{bmatrix},
\]

\[
J_Y = \begin{bmatrix} 0.9734 & 0.0266 & 0 & 0 & 0 \\ 0.0565 & -0.0565 & 0 & 0 & 0 \\ 0.5661 & 0.4339 & 0 & 0 & 0 \\ -0.6785 & 0 & 0.0882 & 0 \\ -0.1515 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
\Gamma_Y = \begin{bmatrix} 0.9734 & 0.0266 & 0 & 0 & 0 \\ 0.0565 & -0.0565 & 0 & 0 & 0 \\ 0.5661 & 0.4339 & 0 & 0 & 0 \\ -0.6785 & 0 & 0.0882 & 0 \\ -0.1515 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
\bar{P}_{sY} = \begin{bmatrix} 0.9734 & 0.0266 & 0 & 0 & 0 \\ 0.0565 & -0.0565 & 0 & 0 & 0 \\ 0.5661 & 0.4339 & 0 & 0 & 0 \\ -0.6785 & 0 & 0.0882 & 0 \\ -0.1515 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
\bar{P}_{sY} = 0.0266, \quad \bar{P}_{sY} = -0.0266, \quad \bar{P}_{sY} = 0.
\]

The maneuver step inputs and the actuator and sensor faults per combined maneuvering case are described in Table II. The table shows that the same velocity, heading angle, and altitude maneuvers are applied to two cases, in which the fault-free case is considered in Case 1, the actuator faults are injected into the X and Z channels and the simultaneous actuator and sensor faults are considered in the Y channel in Case 2.

As can be seen in Figs. 2 and 3 (Case 1), the velocities and altitudes of the Lead and Wing UAVs are always coincident while the Lead UAV is being maneuvered at the respective \( t = 10 \) s and \( t = 30 \) s in combined form. The separations in the X and Z channels vary and finally remain at rated values due to the lack of prophetic information of maneuvering, thus, resulting in the time delay of tracking. Fig. 2 (Case 1) shows that the heading angles of the Wing UAVs track that of the Lead UAV while the Lead UAV is being maneuvered at \( t = 20 \) s. The separations in the X and Y channels increase first and then decrease at rated values because the Wing UAVs on the outside need to fly farther away to track with the Lead UAV in order to keep the formation geometry.
Fig. 3. Case 1: The combined maneuvering without actuator/sensor faults.

Fig. 4. Case 2 with simultaneous actuator/sensor faults (X channel).

Fig. 5. Case 2 with simultaneous actuator/sensor faults (Y channel).

Fig. 6. Case 2 with simultaneous actuator/sensor faults (Z channel).

Fig. 7. Case 2: The respective estimated actuator and sensor faults in the Y channel with the integrated FE/FTC scheme and the control scheme in [23].

Fig. 8. Case 2: The respective estimated actuator and sensor faults in the Z channel with the integrated FE/FTC scheme and the control scheme in [23].

The curves in Figs. 7 and 8 (Case 2) simulated by both the approach [23] and the proposed integrated FE/FTC algorithm show the good tracking properties of rated and estimated actuator and sensor faults in the respective X, Y, and Z channels. Furthermore, there is a small oscillation on the separation in the Z channel due to the coupling item $qS_m p_{S_m}$ between the Y and Z channels.

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The curves in Figs. 7 and 8 (Case 2) simulated by both the approach [23] and the proposed integrated FE/FTC algorithm show the good tracking properties of rated and estimated actuator and sensor faults in the respective X, Y, and Z channels.
and Z channels. Compared with our previous study [23], although there exists a sharp peak in the proposed integrated FE/FTC scheme due to its rapid convergence, the integrated algorithm shows faster convergence and smaller amplitudes of the oscillations in estimated faults to an extent.

Figs. 9 and 10 show that the heading angles of the four Wing UAVs track with the rated angle of the Lead UAV, i.e., ($\psi_0 = -8^\circ$) at $t = 5$ s and ($\psi_0 = 0^\circ$) at $t = 35$ s. The separations amongst each UAV remain to the rated values, i.e., ($x_c = 18.9$ m) in the X channel in Fig. 11 and ($y_c = 7.42$ m) in the Y channel in Fig. 12. Due to the existence of the coupling item $\frac{1}{m}PD_y$ and the parameter selection of $C_{XX}$ and $C_{cX}$ in the separated design, more oscillation responses are shown in the separated FE/FTC in Fig. 11. Compared with the separated FE/FTC in Theorems 4.1 and 4.2, the integrated FE/FTC in Theorem 4.3 shows a smaller convergence amplitude of the heading angles and separations in the X and Y channels at each fault occurring time instant because the integrated FTC system contains more information from the FE process. Fig. 13 shows the position-space trajectories of each Wing UAVs for two different Lead UAV maneuvers (with actuator/sensor faults from Case 2). All the trajectories show some separation errors when the actuator or sensor faults occur, but quickly return to the rated values. Hence, the combined maneuvering cases with simultaneous actuator/sensor faults demonstrate the effectiveness of the proposed integrated FE/FTC control scheme for CFF systems, and the control objective is achieved.
VI. CONCLUSION

In this paper, an integrated FE and FTC design has been developed for CFF systems with simultaneous actuator and sensor faults to track the position and attitude of commanded motions of the Lead UAV. Compared with the separated design, integrated FE/FTC design considers the bidirectional interactions between FE and FTC systems and makes full use of the estimated fault information of FE system, so that the convergence amplitude in the integrated design is smaller. Integrated designs, namely, decentralized FE and distributed FTC protocols, are proposed to guarantee the excellent tracking of position and attitude while the Lead UAV is being maneuvered. Current investigations focus on extensions of the proposed method to unpredicted maneuvering, communication faults, mission completion, and formation reconfiguration.

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