Modeling of wear processes by a regularized damage-plasticity model based on the emulated RVE

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Wear is defined as damage to the surface of a solid body, involving progressive material removal, due to relative motion and frictional contact with another surface. This process is usually slow but is considered as one of the major factors causing damage and consequently failure of component parts during the lifetime of tools or machines in different applications such as tunneling.

The implementation of conventional local material models in finite element simulations involving softening behavior (e.g., softening plasticity or damage) tends toward an ill-posed boundary value problem after the onset of softening due to non-convex and non-coercive energy functions and suffers from strongly mesh-dependent results. Therefore, different regularization strategies are developed to overcome the mentioned problem, such as integral or gradient enhancement, introducing an internal length scale and subsequently increasing computation effort.

In this study, we present a variational approach for regularization of damage material model coupled with local plasticity based on emulated representative volume element (ERVE). This model shall be applied for the numerical investigation of wear processes, where a fine resolution of the involved constituents (cut sheet and abrasive particles in the soil) are required. We start with the theoretical derivation of material model, briefly present the numerical treatment, and prove the efficiency of this new approach via some numerical examples. Furthermore, results to the simulation of different wear modes in single scratch tests are presented.

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1 Material Model

The model is specifically based on the mechanical part of Helmholtz free energy function for isotropic damage and plasticity, thus

\[ \Psi_m = \frac{1}{2} f(d)(\varepsilon - \varepsilon_p) : \varepsilon : (\varepsilon - \varepsilon_p) + w(\alpha_p) = f(d)\Psi_0 + w(\alpha_p). \] (1)

In the current material model, we investigate the linear isotropic hardening with the plastic potential \( w(\alpha_p) = \frac{1}{2}K_H\alpha_p^2 \), and an isotropic damage with the function \( f(d) = (1 - d)^2 \) with the internal damage variable \( d \in [0; 1] \).

In this study for classical rate-independent plasticity, we chose the dissipation function as \( D_p = r_p|\varepsilon_p| \), which is a homogeneous function of order one. Utilization of the damage function \( f(d) \) leads to a softening behavior and causes the ill-posedness and numerically instability problems. This problem can be better understood and consequently remedied in a time incremental setting. Therefore, we introduce the damage dissipation distance in the form

\[ \Delta D_d(d_0, d_1) = \begin{cases} r_\alpha(d_1 - d_0) & \text{for } d_1 \geq d_0 \\ \infty & \text{for } d_1 < d_0 \end{cases} \] (2)

Applying the principle of the minimum of the dissipation potential (PMDP), the evolution equations are obtained for plastic internal variables as

\[ \dot{\varepsilon}_p = \frac{|\varepsilon_p|}{r_p + K_H\alpha_p} \text{dev} \sigma, \quad \alpha_p = |\varepsilon_p| = \Delta \rho \left( r_p + K_H\alpha_p \right) \] (3)

and for damage variable \( d \) as

\[ d_1 = \arg \min \left\{ \Psi_m(\varepsilon, \varepsilon_p, \alpha_p, d_1) + \Delta D_d(d_0, d_1) \right\} \] (4)

Introducing a dissipation distance within a time increment \( \Delta t \) results in a point-wise minimization problem including condensed energy. A straightforward way to achieve coercivity of the condensed energy is by rate-limitation of the damage evolution. Although the convexity of the energy is now ensured, the stability of the results will be achieved by calculation of the quasiconvex envelope of the potential. Therefore, the problem will be limited to find the optimal fluctuation by minimization defined on a representative volume element \( \Omega_{rep} \) of unit volume. Considering the strain field and the damage variable as

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the two unknown in each time increment leads to a high-dimensional global optimization problem with a numerically expensive solving procedure. Therefore, a specific damage distribution is assumed inside the \( \Omega_{\text{rep}} \), i.e., what’s happening inside \( \Omega_{\text{rep}} \) is not calculated directly, rather it is modeled. This procedure is called *Emulated Representative Volume Element (ERVE)*.

To describe the damage distribution inside \( \Omega_{\text{rep}} \), the RVE is divided into a specific number of subdomains of equal volume, which results in the mentioned Emulated Representative Volume Element (ERVE). Each subdomain possesses a constant strain \( \varepsilon_i \) as well as constant damage \( d_i \). Instead of minimizing with respect to \( u \), we will do the minimization with respect to all \( \varepsilon_i \) yielding a given overall strain, resulting in

\[
\Psi^{\text{rel}}(\varepsilon, \varepsilon_p, \{d_i\}) = \inf \left\{ \frac{1}{n} \sum_{i=1}^{n} f(d_i) \Psi_0(\varepsilon_i, \varepsilon_p) \mid \varepsilon = \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i \right\} = f \Psi_0, \quad \text{with} \quad f = n \left[ \sum_{i=1}^{n} \frac{1}{f(d_i)} \right]^{-1}
\]  

This minimization results in damage evolution equation in the form of

\[
d_i - d_{0i} = \begin{cases} 
k \Delta t & \text{if} \quad q_{d,i} \geq \frac{r_d}{n} \\ 0 & \text{else} \end{cases}
\]  

with \( q_{d,i} = -\frac{\partial \Psi}{\partial d_i} \) being the damage driving force in each subdomain.

## 2 Numerical Results

A three-dimensional scratch process is modeled and simulated using the developed coupled damage-plasticity material model, implemented as a user subroutine (UMAT) in Abaqus. The model consists of a rigid tip with a predefined radius and a flat deformable specimen. The specimen is fixed and the tip slides over the surface such that a groove forms. In order to investigate the influence of the particle size on the cross sectional geometry of the groove, simulations are repeated for particles with radii of \( R = 25, 50 \) and \( 100 \) \( \mu \)m. The cross-sectional geometry of the groove caused by a tip with a radius of \( 100 \) \( \mu \)m reveals a completely plastic deformation with large shoulders and no material removal. Contrary, in the scratch cross-section built by the tip with a radius of \( 25 \) \( \mu \)m, no shoulders are almost observable, and large wear volume is recorded.

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### References

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