Braided Topology and the Emergence of Matter

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Abstract. In recent years, the possibility that particles may be modelled by topological structures occurring in the discrete spacetime of certain quantum gravity theories has been investigated. We present an overview of this research program, based on a hypothetical substructure of quarks and leptons. Under appropriate dynamical rules, such topological structures are capable of interacting with each other, and we show how the ability to interact may be combined with a mapping to fermion and boson states of the Standard Model.

1. Introduction
During the 1970s and 1980s several models were proposed that treated quarks and leptons as composite fermions, formed from a common set of subcomponents or “preons” (after the nomenclature used in [1]). One particularly interesting example was proposed independently and simultaneously by Harari and Shupe [2, 3]. This model postulates two kinds of preons (called “rishons” in Harari’s terminology), one carrying an electric charge of $+e/3$ (where $e$ is the charge on the proton), the other neutral, that combine into triplets. Permutations of the triplets with a mix of neutral and charged rishons are interpreted as colour charge. The resulting list of possible triplets consists of a single neutral triplet (interpreted as the $\nu_e$), three states with charge $+e/3$ (the $\bar{d}$), three states with charge $+2e/3$ (the $u$), and one state with charge $+e$ (the $e^+$). The model also incorporates equivalent combinations formed from charged and neutral anti-rishons to create the $e^-$, $\bar{u}$, $\bar{d}$, and $\bar{\nu}_e$. This accounted for the precise ratios of lepton and quark electric charges, and the correspondence between colour charge and fractional electric charge. Gauge bosons were postulated to be formed form a mix of rishons and anti-rishons.

This report discusses a modification of the Harari/Shupe model, dubbed the Helon Model [4], originally described in 2005, along with subsequent work [5] which has developed the idea that topological structures in a discrete spacetime may play the role of preons, combining to form emergent fermions and bosons consistent with those of the Standard Model.

In the notation of the Helon Model, the positively-charged preons are denoted by $H_+$, the negatively-charged anti-preons by $H_-$, and the neutral preons by $H_0$. There are no distinct neutral anti-preons. The construction of the fermions of the first generation is then as shown in the table of Figure 1 (quarks at top, leptons in the bottom row). Notice immediately that without a neutral anti-preon there is no explicit construction of the anti-neutrino.

In Loop Quantum Gravity and related approaches to canonical quantum gravity, discrete spacetime structure may be represented by a network or graph, with nodes dual to volumes and links dual to areas. The simplest graphs to consider are trivalent. In the case of a non-zero cosmological constant, the links become framed (i.e. acquire a width) which allows them to carry twists. When the graph is embedded within a manifold, the links may braid around each
The fermions formed by adding zero, one, two or three charges (i.e. twists) to a neutral braid, where (3) denotes that there are three possible permutations of charge position. We are using a basic braid for illustrative purposes, however arbitrary braiding is possible, in principle. For a given charge magnitude there are twice as many charged states as neutral states.

Figure 1. The fermions formed by adding zero, one, two or three charges (i.e. twists) to a neutral braid, where (3) denotes that there are three possible permutations of charge position. We are using a basic braid for illustrative purposes, however arbitrary braiding is possible, in principle. For a given charge magnitude there are twice as many charged states as neutral states.

| $H_H^+H_0^+ (u_R)$ | $H_H^+H_0^+ (u_G)$ | $H_H^+H_0^+ (u_B)$ | $H_H^+H_0^+ (u_B)$ |
|----------------------|----------------------|----------------------|----------------------|
| $H_0^+H_0^+ (d_R)$ | $H_0^+H_0^+ (d_G)$ | $H_0^+H_0^+ (d_G)$ | $H_0^+H_0^+ (d_R)$ |
| $H_H^+H_0^+ (\pi_R)$ | $H_H^+H_0^+ (\pi_G)$ | $H_H^+H_0^+ (\pi_G)$ | $H_H^+H_0^+ (\pi_R)$ |
| $H_H^+H_0^+ (c_B)$ | $H_H^+H_0^+ (c_G)$ | $H_H^+H_0^+ (c_G)$ | $H_H^+H_0^+ (c_R)$ |
| $H_0^+H_0^+ (e^+)$ | $H_0^+H_0^+ (\nu_e)$ | $H_0^+H_0^+ (\nu_e)$ | $H_0^+H_0^+ (e^-)$ |

If a braid is connected to a larger network (“the Universe”) at only one end, which we may think of as the bottom, and the other end is attached only to the braid itself, then the top of the braid is simply a trivalent node. In this case it is possible to deform a braid on three strands to obtain a structure which carries only twisting. Such a braid is simply a series of crossings of the first (leftmost) leg over/under the second, and crossings of the second leg over/under the third (rightmost) leg. When a trivalent node is flipped over along the axis connecting the centre of the node with the crossing of two links, as in Fig. 2, the crossing is undone and a series of twists (through $\pm \pi$) are induced on the three links leading out of the node. In [6] it was shown that by undoing each crossing in a braid in sequence, one obtains a series of three uncrossed strands with twists $a$, $b$, and $c$ on the first, second, and third strands respectively ($a$, $b$, $c$ are multiples of $\pi$). The triple $[a, b, c]$ defines an equivalence class of framed braids. Any two braids which may be reduced to the same unbraided state fall within the same topological equivalence class. Therefore for braids within networks it is not different braids, but rather different equivalence classes of braids that correspond to the various fermions. This process of undoing crossings will only work in the trivalent case (flipping a node with four or more legs may undo the crossing of two strands but will induce a crossing on the others).

We can model the electroweak interactions between quarks and leptons in terms of braid products. The product of braids in the Artin braid [7] group is taken by joining the bottom of the first strand in the first braid to the top of the first strand in the second braid, and likewise each of the subsequent strands, to form a single larger braid. If the first braid consists of $n$ crossings and the second braid consists of $m$ crossings, their product consists of at most $n + m$ crossings (some crossings may cancel with each other, resulting in fewer crossings in the product). When we take the product of two framed braids, the twists on the joined...
Figure 2. Flipping a trivalent node over uncrosses the links emerging from it, but induces twists on the links.

Figure 3. Interactions between fermions and bosons (left) can be represented by taking the product of framed braids (right).

strands may combine together or cancel. To illustrate this, consider a braid composed of two $H_+s$ (strands with right-handed twists), and one $H_0$, and another braid composed of three $H_-s$. The first braid corresponds in charge to an up quark, the second corresponds to a $W^-$ boson. When the product of these braids is taken, the resultant braid is composed of a single $H_-$ and two $H_0$s. This corresponds to a down quark. This interaction is illustrated in Fig. 3. Weak interactions can not change the location of the “odd-man-out” helon within a braid (this would change quark colour) hence the substructure of the electroweak bosons must be braids with a trivial permutation of the strand ordering between the top and bottom. In general several braids including the identity braid (three strands with no crossings) may fulfill this requirement, with different braid structures corresponding to bosons with different helicities.

2. Interactions in networks

If the fermions and bosons of the SM truly emerge from an appropriate theory of quantum gravity, we must move from abstract braids to considering the interactions of braids embedded within networks. In [5] it was shown that under the standard evolution moves of LQG, braids in a network will not decay, but neither will multiple braids combine or a single braid split into several. It seems that a new evolution move is necessary to allow interactions to occur.

Reassuringly, an approach to interactions which does not require a new evolution move was described by Wan and Smolin in [8]. Here it was shown that tetravalent networks evolving under the dual Pachner moves [9] allow certain braided structures to interact in a manner reminiscent of the interactions of bosons and fermions. The Pachner moves allow one tetravalent node to become four nodes, and vice-versa, and allow a pair of tetravalent nodes connected by a single link to become three nodes arranged on the vertices of a triangular ring. These networks are realised as spherical nodes, connected by tubular links. This format can be made to correspond with the framed networks discussed throughout the rest of this paper by observing that if one slices such tubular networks in half, down opposite sides of each link, the resulting surfaces are tetravalent framed networks, and tetravalent nodes may be viewed as pairs of trivalent nodes.

It was shown in [8] that there exists a certain class of braids, which can be merged with other braids, by a process illustrated schematically in Fig. 4. The two braids are moved through the network until they are connected by a common link. The two nodes in between the pair of braids are then replaced by a ring, via a Pachner move. This ring is manipulated through the crossings of one braid, to form a tetrahedral structure of four nodes at one end of a new braid, composed of the crossings of the two original braids. The four nodes are then reduced to a single node via a Pachner move. Such interactions are, of course, reversible, and have a natural interpretation in terms of a boson being absorbed or emitted by a fermion.

While these results suggested that interactions may more naturally be achieved in the tetravalent case than the trivalent case, it was not clear whether there was a direct mapping of
tetravalent braids to fermion and boson states, as provided in the trivalent case by the Helon Model. Also, as noted above, the number of distinct braids is not reduced in the tetravalent case, as it was in the trivalent case. If we are to avoid having an unrealistically large list of braids (and hence fermion species), we must limit the type of allowed braids in the tetravalent case, or find a way to combine tetravalent braids into equivalence classes, as was done in the trivalent case. We now describe how the latter possibility can be achieved.

We start with a trivalent braid, and reduce it to its pure twist form. It is then possible to use node-flipping to eliminate all the twists on one strand (the rightmost, say). This introduces new twists and crossings on the other two strands, but that is not problematic. Once there is no twisting on one strand, one of the nodes at the “bottom” of the braid may be moved along this strand, and merged with the node at the “top” of the braid, to form a tetravalent node. The remaining two nodes at the bottom of the braid are likewise merged to form a tetravalent node, as in Fig 5. In this way we can associate a tetravalent braid to each trivalent braid. This allows us (as in the trivalent case) to use the Helon Model to map equivalence classes of braids to particle states, in the tetravalent setting where interactions have been shown to occur.

3. Discussion and Unresolved Issues
A more complete discussion of the Helon Model can be found in [4]. While there remain several issues the Helon Model has yet to address, including the asymmetry of the weak interaction, the origin of mass, and the nature of Cabbibo-mixing, the work presented here goes some way towards resolving similarly significant problems. It now appears that interactions between braids in networks can be achieved, without sacrificing the mapping between braid states and particle states, and without the need to arbitrarily invent new evolution moves, by the approach we have described, which combines the best features of the tetravalent and trivalent schemes. This brings the prospect of emergent, interacting matter from discrete spacetime a step closer.

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