Determination of temperature dependence in Modified-Mohr-Coulomb failure model for process simulation of shear cutting

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Abstract. Shear cutting is a proven process for chip-less separation of metals and used as a cost-effective production method. The process design of shear cutting is usually based on time and cost intensive experimental tests. Therefore, numerical depiction of the process offers great potential to reduce these practical tests. An important aspect hereby is the representation of temperature dependency of the material. However, the classical Modified-Mohr-Coulomb (MMC) failure model does not depict this effect directly. Therefore, the objective of this paper is to investigate the temperature dependency of MMC failure model for simulating shear cutting. The required data is obtained from tensile tests on miniaturised specimens under variation of temperature on dual-phase steel DP1000, using an additional optical measurement system. To determine stress triaxiality, Lode angle parameter and plastic strain at failure, the experimental tests are simulated using the Finite-Element simulation program ABAQUS. The MMC failure surfaces are fitted using the least squares method in Matlab. With this approach, temperature dependent MMC failure model was built to be used for shear cutting simulation of DP1000 steel.

1. State of the Art
Increasing quality demand on sheet metal components using shear cutting requires high precision, which is achieved with a suitable process design [1]. Standard simulations based on Finite-Elemente (FE) method use simplified calculation algorithms to describe stress dependent material failure [2]. Due to the strong temperature increase in shear cutting [3], it is necessary to develop and integrate a temperature dependent failure model, so that the numerical representation of these processes can accurately predict the temperature influence on material failure [4]. The classical Mohr-Coulomb (MC) model predicts material failure at a certain combination of normal stresses $\sigma_1$, $\sigma_2$ and $\sigma_3$ as shown in [5]:

$$c_2 = \max \left\{ \frac{1}{2} |\sigma_1 - \sigma_2| + c_1 (\sigma_1 + \sigma_2), \frac{1}{2} |\sigma_2 - \sigma_3| + c_1 (\sigma_2 + \sigma_3), \frac{1}{2} |\sigma_3 - \sigma_1| + c_1 (\sigma_3 + \sigma_1) \right\}$$  \hspace{0.5cm} (1)

The constant $c_1$ is referred to as coefficient of friction and $c_2$ as shear resistance in MPa. Through geometric relations in the deviator plane, $\sigma_1$, $\sigma_2$ and $\sigma_3$ can be given as functions of mean stress $\sigma_m$, stress triaxiality $\eta$ and Lode angle parameter $\theta$ [5]:

$$\sigma_1 = \left( 1 + \frac{2 \cos(\theta)}{3\eta} \right) \sigma_m, \sigma_2 = \left( 1 + \frac{2 \cos\left(\frac{\pi}{3} - \theta\right)}{3\eta} \right) \sigma_m, \sigma_3 = \left( 1 + \frac{2 \cos\left(\frac{\pi}{3} - \theta\right)}{3\eta} \right) \sigma_m$$  \hspace{0.5cm} (2)

In [5] the MC criterion was represented as a function of the equivalent stress $\bar{\sigma}$:
\[ \bar{\sigma} = c_2 \left( \frac{1 + c_3^2}{3} \cos \left( \frac{\pi}{6} - \theta \right) + c_4 \left( \eta + \frac{1}{3} \sin \left( \frac{\pi}{6} - \theta \right) \right) \right)^{-1} \]  

Furthermore, Lode angle parameter can be reduced to a range of \(-1 \leq \bar{\theta} \leq 1\) by normalizing it \([5]\):

\[ \bar{\theta} = 1 - \frac{6\theta}{\pi} \]  

However, in \([5]\) it was also shown that ductile fracture detection improves by taking the strain into account. Thus, by transforming the MC fracture criterion from stress based into stress and strain based space under condition of monotonic loading, the Modified-Mohr-Coulomb (MMC) failure model was built in terms of equivalent plastic strain at fracture \(\bar{\varepsilon}_p\):

\[ \bar{\varepsilon}_p = \left\{ \frac{A}{c_2} \left[ c_3 + \frac{\sqrt{3}}{2 - \sqrt{3}} \left( 1 - c_4 \right) \left( \sec \left( \frac{\bar{\theta} \pi}{6} \right) - 1 \right) \right] \left[ \frac{1 + c_3^2}{3} \cos \left( \frac{\bar{\theta} \pi}{6} \right) + c_4 \left( \eta + \frac{1}{3} \sin \left( \frac{\bar{\theta} \pi}{6} \right) \right) \right] \right\}^{-\frac{1}{n}} \]  

with \(A\) and \(n\) as strength and work hardening exponent in Swift hardening law, respectively, as well as \(c_3\) as material constant \([6]\). Various authors have successfully implemented the MMC failure model for aluminum \([6]\) as well as for steel \([7]\). Xiao et al. in \([8]\) extended the MMC failure model to integrate the strain rate and temperature dependency for aluminum according to the Johnson-Cook failure model. However, the data shown only portrayed results made under constant temperature.

Based on the literature it is clear that the failure models are affected by material temperature dependency. Therefore, this paper conducts a solution approach using MMC failure model on DP1000 to improve the calculation of ductile material failure by taking the temperature into account.

2. Experimental tests

2.1. Tensile tests

The tensile tests were carried out on a quenching and forming dilatometer DIL 805A/D+T from TA Instruments, shown in figure 1 (a), at a quasi-static strain rate of 0.001 s\(^{-1}\). The length change of the specimen was recorded by two quartz push rods. A more detailed description of dilatometer can be found in \([9]\) and \([10]\). Uniaxial specimens were used to evaluate flow behaviour in uniaxial tension conditions.

![Figure 1. Schematic setup of dilatometer (a), sample geometries and dimensions (b).](image-url)
and notched specimens were performed to generate different stress states, shown in figure 1 (b). In shear tensile tests, triaxialities of $\eta = 0$ were to be induced. Notched tensile specimens with radius $R = 3$ mm and $R = 10$ mm were to produce higher triaxiality values than uniaxial tensile test. Since in [2] and [4] the determined stress triaxiality during shear cutting approximated the range of $-0.2 \geq \eta \geq 0.5$, it was decided to use such specimen geometries to ideally cover the region of stress triaxialities in shear cutting. The specimens were cut along the rolling direction by means of a water jet cutter of a 1 mm thick DP1000 sheet. The tests were repeated three times for each temperature and each specimen geometry.

A calculation of the flow curves determined from dilatometer tests was required for the simulation, in which a maximal forming degree of $\varepsilon = 0.06$ was reached. Since this was not sufficient for the numerical simulation, an extrapolation was done using a combined approach of the Swift [11] and Voce [12] (eq. 6) hardening law in equation 7. These methods contained the lowest values in the least squares method to the flow curves obtained from dilatometer tests compared to Ludwig and Hockett-Sherry approaches.

$$k_{\text{Swift}}(\varepsilon) = A(\varepsilon_0 + \varepsilon)^n, \quad k_{\text{Voce}}(\varepsilon) = \sigma_0 + (\sigma_0 - \sigma_S)e^{\frac{\varepsilon}{\varepsilon_0}}$$

$$k_{\text{Comb}}(\varepsilon) = A(\varepsilon_0 + \varepsilon)^n + (1 - \alpha)(\sigma_S + (\sigma_0 - \sigma_S))e^{\frac{\varepsilon}{\varepsilon_0}}$$

$k$ stands for flow stress, $\varepsilon_0$ for starting plastic strain, $\sigma_0$ for yield stress, $A, n, \sigma_S, E$ for material parameters and $\alpha$ for a weighting factor. For each temperature considered, material parameters were obtained from dilatometer tensile tests. To validate the accuracy of the extrapolation methods, a tensile testing machine Dynamesh S100/ZD with a mechanical extensometer and GOM optical measurement system ARAMIS was used. Hereby the change in length and width of tensile samples in $0^\circ$, $45^\circ$ and $90^\circ$ rolling direction was recorded. The digital extensometer of the optical measurement system allowed to obtain the flow curve beyond the uniform strain. Therefore, flow curve of a uniaxial specimen at room temperature was built up to a plastic strain of $\varepsilon = 0.2$ at a quasi-static strain rate. Moreover, using tensile specimens in different rolling directions, the anisotropic parameters $r_0$, $r_{45}$, $r_{90}$ were determined.

2.2. Design of the simulation model

Specimen models were created in ABAQUS 6.14 from dilatometer samples presented in chapter 2.1. To numerically investigate material anisotropy, ABAQUS parameters $R_{11}, R_{22}, R_{33}, R_{12}, R_{13}, R_{23}$ were determined from anisotropy parameters $r_0$, $r_{45}$, $r_{90}$, together with Hill’48 coefficients [13]. The Hill’48 model was prior used to describe the forming and failure behavior of DP980 sheet in literature [14, 15].

Solid eight-node 3D elements C3D8R were used for the implicit static simulation. Direct method was used as a solver with a full Newton solution technique. The specimens were meshed with an element edge length of 0.1 mm. In the forming area Arbitrary Lagrangian Eulerian (ALE) adaptive mesh domain was activated for the steps of displacement. No mass nor time scaling was implemented. Symmetries were used for the uniaxial and both notched specimens, therefore only quarter models were simulated. Due to the geometry of the shear specimen only the symmetry relative to the central plane of the sheet was exploited. A time-length change curve was applied as a boundary condition for axial displacement to the upper end of each specimen model, while the lower end was fixed. To identify the length change in ABAQUS, the distance between the upper and the lower shoulder of each specimen was measured, shown in figure 1 (b). When this area reached the length change at fracture from dilatometer experiments, the simulation was declared finished, thus indicating fracture. Of the three curves of time-length change at fracture, which resulted from the test repetition, the intermediate one was taken as the specimen fracture condition. Such technique was used simulating each specimen. With this approach, $\varepsilon_p^L, \eta$ and $\dot{\theta}$ were determined. While $\varepsilon_p^L$ was the equivalent of the deformation degree at maximal length change, the stress state varied during the deformation, thus impacting $\eta$ and $\dot{\theta}$ [16]. For this reason, [16] suggested to use average values of $\eta_{av}$ and $\dot{\theta}_{av}$, which are defined in equation 8.

$$\eta_{av} = \frac{1}{\varepsilon_p^L} \int_0^{\varepsilon_p^L} \eta(\varepsilon)d\varepsilon_p, \quad \dot{\theta}_{av} = \frac{1}{\varepsilon_p^L} \int_0^{\varepsilon_p^L} \dot{\theta}(\varepsilon_p)d\varepsilon_p$$


3. Results and Discussion

3.1. Results of the tensile tests

Results depicted from dilatometer tests on the uniaxial specimen are shown as a force-length change curve in figure 2 (a) for various temperatures and as a length change at fracture-temperature diagram for various specimen geometries in figure 2 (b). In literature, steels are assumed to have decreasing strength with increasing temperature. The so-called blue brittleness is considered to be an exception. If the steel material contains a certain mass of dissolved foreign atoms such as carbon and nitrogen, they diffuse to the dislocations at particular temperatures and block them. This means the tensile strength increases [17]. In figure 2 (a) a decrease of force was noticed from 25 °C to 100 °C, caused by a thermal softening effect. When considering the length change versus temperature of the uniaxial specimen in figure 2 (b), a blue brittleness range was identified, in which the length change decreases at 100 °C and 200 °C compared to 20 °C, whilst the force stays nearly constant in figure 2 (a). Moreover, a blue brittleness range was identified between 200 °C and 300 °C at which the force remains almost constant as well.

![Figure 2](image-url)

Figure 2. Force-length change of the uniaxial specimen at various temperatures (a) and length change at fracture-temperature diagram for each specimen geometry (b).

Figure 3 (a) shows flow curves of uniaxial specimen at various temperatures from dilatometer testing. Figure 3 (b) compares the flow curves of dilatometer and Dynamess tests as well as the extrapolation approaches of Swift, Voce and the combined method at 20 °C, shown in equations 6 and 7.

![Figure 3](image-url)

Figure 3. Flow curves of the uniaxial specimen at various temperatures (a), flow curves of Swift, Voce and combined extrapolation model compared to experimental flow curves at 20 °C (b).

The value of $\alpha = 0.7$ was fitted as the weighting factor. It can be depicted, that the combined extrapolation approach was the most suitable. The same weighting factor $\alpha$ was used for extrapolation of the flow curves for all further temperatures in figure 3 (a), since the parameters of Swift and Voce hardening law were determined for each temperature separately. The parameters are shown in table 1.
Table 1. Swift and Voce hardening law parameters for each temperature.

| T in °C | A in MPa | $\varepsilon_0$ | n | $\sigma_S$ in MPa | $\sigma_0$ in MPa | E  |
|--------|----------|-----------------|---|------------------|------------------|----|
| 20     | 1450     | 0.0002          | 0.0773 | 1128          | 850              | 93.8 |
| 100    | 1410     | 0.0008          | 0.0772 | 1083          | 850              | 92.3 |
| 200    | 1480     | 0               | 0.0933 | 1083          | 748              | 111.8 |
| 300    | 1460     | 0               | 0.0951 | 1098          | 760              | 70.9 |
| 500    | 640      | 0               | 0.0473 | 511           | 407              | 471.1 |

Table 2 shows the anisotropy parameters $r_0$, $r_{45}$, $r_{90}$, determined from Dynamess tensile tests on uniaxial specimens, together with ABAQUS parameters $R_{11}$, $R_{22}$, $R_{33}$, $R_{12}$, $R_{13}$, $R_{23}$ and the Hill’48 coefficients, which were used to portray material anisotropy in the simulations.

Table 2. Anisotropy parameters and Hill’48 coefficients for input in ABAQUS.

| $r_0$ | $r_{45}$ | $r_{90}$ | $F$ | $G$ | $H$ | $N$ | $R_{11}$ | $R_{22}$ | $R_{33}$ | $R_{12}$ | $R_{13}$ | $R_{23}$ |
|-------|----------|----------|-----|-----|-----|-----|---------|---------|---------|---------|---------|---------|
| 0.86  | 1.27     | 1.13     | 0.41| 0.54| 0.46| 1.68| 1.00    | 1.07    | 1.03    | 0.95    | 1.00    | 1.00    |

3.2. Results of the simulations

The combined extrapolation approach, described in section 2.1, was evaluated by comparison of the force-length change diagrams from simulations with the experimentally determined results. Figure 4 (a-d) shows the findings for uniaxial, shear and both notched specimens, correspondingly.

Figure 4. Force-length change curve of uniaxial (a), shear (b), notched R = 3 mm (c) and R = 10 mm (d) specimen from experiment (solid line) and simulation (dashed line) at temperatures. Plastic strain-stress triaxiality (e) and plastic strain-normalized Lode angle parameter (f) at 20 °C for all specimens.
Overall, good agreement was observed between experiment and simulation results, in particular at temperatures up to 300 °C. At 500 °C the force-length change curves held the largest deviation between numerical and experimental results for all specimens in that the force was overestimated in the simulation. Since at 500 °C a strong thermal softening effect occurred in the experimental testing for all specimens, which lead to a partial halving of tensile strength compared to lower temperatures, the temperature-dependent weighting factor $\alpha$ should be considered for subsequent simulations to depict this effect correctly. Figures 4 (e) and (f) show exemplary curves of plastic strain-triaxiality and plastic strain-normalized Lode angle parameter for all specimens at 20 °C, respectively. The values of the stress triaxiality and normalized Lode angle change over the plastic strain. While the stress triaxialities show values close to each other, the normalized Lode angles cover a wider range of stress states.

3.3. Generation of failure surfaces
As described above, only the average values over the degree of deformation $\varepsilon_p^f$ were calculated for the stress values $\eta$ and $\bar{\theta}$ to build the MMC failure model using equation 8, which are summarized in table 3 for all specimen geometries at various temperatures. As expected, the average triaxiality of the shear specimen was at about $\eta_{av} = 0$. The notched specimens showed slightly higher values regarding the average stress triaxiality, but lower plastic strain at fracture compared to uniaxial and shear specimens, with an exception at 500 °C of the notched specimen with $R = 10$ mm.

| T in °C | Uniaxial | Shear | R = 3 mm | R = 10 mm |
|--------|----------|-------|----------|-----------|
|        | $\eta_{av}$ | $\bar{\theta}_{av}$ | $\varepsilon_p^f$ | $\eta_{av}$ | $\bar{\theta}_{av}$ | $\varepsilon_p^f$ | $\eta_{av}$ | $\bar{\theta}_{av}$ | $\varepsilon_p^f$ |
| 20     | 0.53     | 0.72  | 0.78     | 0.01     | 0.12     | 1.08    | 0.57    | 0.48     | 0.33     | 0.54     | 0.63     | 0.56     |
| 100    | 0.43     | 0.84  | 0.43     | 0.01     | 0.06     | 0.97    | 0.59    | 0.47     | 0.32     | 0.50     | 0.68     | 0.41     |
| 200    | 0.46     | 0.80  | 0.59     | 0.01     | 0.05     | 0.97    | 0.57    | 0.49     | 0.36     | 0.54     | 0.64     | 0.59     |
| 300    | 0.50     | 0.76  | 0.72     | 0.02     | 0.09     | 1.16    | 0.63    | 0.45     | 0.57     | 0.57     | 0.63     | 0.68     |
| 500    | 0.64     | 0.63  | 1.07     | 0.07     | 0.25     | 1.68    | 0.77    | 0.35     | 0.96     | 1.01     | 0.36     | 1.97     |

The determination of the material parameters $c_1$, $c_2$ and $c_3$ in the MMC model was carried out with the curve fitting tool in Matlab using the least squares method. For each temperature the determined parameters are shown in table 4.

| T in °C | $c_1$ | $c_2$ in MPa | $c_3$ |
|--------|-------|--------------|-------|
| 20     | 0.08  | 775          | 0.92  |
| 100    | 0.12  | 735          | 0.90  |
| 200    | 0.11  | 800          | 0.95  |
| 300    | 0.09  | 795          | 0.92  |
| 500    | 0.05  | 340          | 0.89  |

Figure 5 (a-e) shows the failure surfaces for temperatures 20 °C, 100 °C, 200 °C, 300 °C and 500 °C, correspondingly. All failure surfaces held the lowest plastic strain at fracture for stress triaxiality $\eta = 0.8$. A minimal influence of stress triaxiality can be depicted with respect to normalized Lode angle parameter at values of $\eta \approx 0.8$, but an increasing influence at $\eta \approx 0$. For the stress state parameters $\eta = 0$ and $\bar{\theta} = -1$, the plastic strain at fracture was at its highest for all temperatures. At 100 °C the stress state seemed to have less influence on the plastic strain at fracture, which made the MMC surface to appear flatter. When compared to 100 °C, the curvature of the surface at 200 °C increased and thus the influence
of the stress state to the plastic strain at fracture. The failure surfaces at 200 °C showed a higher level than at 100 °C due to the increasing forming capacity. Same behaviour was observed for failure surfaces at 300 °C and 500 °C. Since the plastic strain at fracture was well above the maximal axis value of $\varepsilon_p^f = 3.0$ at $\eta \approx 0$ and $\beta = -1$ at 500 °C, the failure surface was cut off from the axis in figure 5 (e).

![Failure surfaces at varying temperatures](image)

**Figure 5.** MMC failure surfaces at (a) 20 °C, (b) 100 °C, (c) 200 °C, (d) 300 °C and (e) 500 °C.

The failure surfaces exhibited different behaviour at varying temperatures, thus implying that a temperature dependent adjustment of the curvature is necessary. It can be therefore concluded that a simple multiplicative scaling of the failure surface represented at room temperature to obtain the MMC damage model at elevated temperatures is not useful for the correct representation of the plastic strain at fracture.

4. Conclusion

Due to the temperature increase in shear cutting, the aim of this paper was to investigate the temperature effect on MMC failure model, which is to be implemented into a numerical simulation model of shear cutting. The first step hereby was to characterize the temperature dependency of a DP1000 sheet material by the use of miniaturized tensile specimens on a dilatometer. An abnormal behavior caused by blue brittleness was depicted. The data from tensile tests was successfully imported into ABAQUS simulation using a combined extrapolation method of Swift and Voce to obtain the required values for the determination of MMC failure model surfaces. It was successfully shown, that the temperature dependent material behavior highly influences the MMC damage model. Therefore, future investigations should be carried out considering and extension and parametrization of the damage model to incorporate the temperature dependency directly into the equation, thus increasing the accuracy when simulating the shear cutting process. In further publications, the failure model will be applied for simulating shear cutting process.
Acknowledgement
The authors are much obliged to the DFG (German Research Foundation) for the financial support of the project “Improved FE simulation of the shear cutting process using a temperature and strain rate-dependent extension of the MMC model” (project number: 199808648).

References
[1] Schneider M, Peshekhodov I A, Bouguecha A and Behrens B-A 2016 A new approach for user-independent determination of formability of a steel sheet sheared edge J. Prod. Eng. Res. Devel. 10 241-252
[2] Gutknecht F, Steinbach F, Hammer T, Clausmeyer T, Volk W and Tekkaya A E 2016 Analysis of shear cutting of dual phase steel by application of an advanced damage model Procedia Struct. Integr. 2 1700–1707
[3] Behrens B-A, Bouguecha A, Vucetic M, Krimm R, Hasselbusch T and Bonk C 2014 Numerical and Experimental Determination of Cut-edge after Blanking of Thin Steel Sheet of DP1000 within Use of Stress based Damage Model Procedia Eng. 81 1096 – 1101
[4] Behrens B-A, Bouguecha A, Peshekhodov I and Bonk I 2013 Comparison between the Lemaitre and a modified Lemaitre damage model in sheet steel blanking IDDRG 2013 Zurich 201–206
[5] Bai Y and Wierzbicki T 2009 Application of extended Mohr–Coulomb criterion to ductile fracture Int. J. Fract. 161 1-20
[6] Mirnia M J and Shamsari M 2017 Numerical prediction of failure in single point incremental forming using a phenomenological ductile fracture criterion J. Mater. Process. Technol. 244 17–43
[7] Cheloe D A, Kadkhodapour J, Pourkamali A A, Khoshbin M, Alaie A and Schmauder S 2021 Micromechanical modeling of damage mechanisms in dual-phase steel under different stress states Eng. Fract. Mech. 243 107520
[8] Xiao X, Pan H, Bai Y, Lou Y and Chen L 2019 Application of the Modified Mohr-Coulomb fracture criterion in predicting the ballistic resistance of 2024-T351 aluminum alloy plates impacted by blunt projectiles Int. J. Impact Eng. 123 26–37
[9] Behrens B-A, Dröder K, Hürkamp A, Droß M, Wester H and Stockburger E 2021 Finite Element and Finite Volume Modelling of Friction Drilling HSLA Steel under Experimental Comparison J. Mater. 14 5997
[10] Behrens B-A, Rosenbusch D, Wester H and Stockburger E 2022 Material Characterization and Modeling for Finite Element Simulation of Press Hardening with AISI 420C J. Mater. Eng. Perform. 31 825–832
[11] Swift H W 1952 Plastic instability under plane stress J. Mech. Phys. Solids 1 1–18
[12] Voce E The relationship between stress and strain for homogeneous deformations J. Inst. Met. 74 537–62
[13] Dassault Systèmes Simulia Corporation 2017 Abaqus Analysis User's Manual: Anisotropic yield/creep https://abaqus-docs.mit.edu/2017/English/SIMACAEIMATRefMap/simamat-e-anisoyield.htm
[14] Li S, He J, Gu B, Zeng D, Xia Z C, Zhao Y and Lin Z 2018 Anisotropic fracture of advanced high strength steel sheets: Experiment and theory Int. J. of Plast. 103 95–118
[15] Gu B, He J, Li S and Lin Z 2020 Anisotropic fracture modeling of sheet metals: From in-plane to out-of-plane Int. J. Solids Struct. 182-183 112–140
[16] Bai Y and Wierzbicki T 2008 A new model of metal plasticity and fracture with pressure and Lode dependence Int. J. Plast. 24 1071–1096
[17] Liu W and Lian J 2021 Stress-state dependence of dynamic strain aging: Thermal hardening and blue brittleness Int. J. Miner. Metall. 28 854