Fully interpretable deep learning model of transcriptional control

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Abstract

Motivation: The universal expressibility assumption of Deep Neural Networks (DNNs) is the key motivation behind recent works in the systems biology community to employ DNNs to solve important problems in functional genomics and molecular genetics. Typically, such investigations have taken a ‘black box’ approach in which the internal structure of the model used is set purely by machine learning considerations with little consideration of representing the internal structure of the biological system by the mathematical structure of the DNN. DNNs have not yet been applied to the detailed modeling of transcriptional control in which mRNA production is controlled by the binding of specific transcription factors to DNA, in part because such models are in part formulated in terms of specific chemical equations that appear different in form from those used in neural networks.

Results: In this paper, we give an example of a DNN which can model the detailed control of transcription in a precise and predictive manner. Its internal structure is fully interpretable and is faithful to underlying chemistry of transcription factor binding to DNA. We derive our DNN from a systems biology model that was not previously recognized as having a DNN structure. Although we apply our DNN to data from the early embryo of the fruit fly Drosophila, this system serves as a test bed for analysis of much larger datasets obtained by systems biology studies on a genomic scale.

Availability and implementation: The implementation and data for the models used in this paper are in a zip file in the supplementary material.

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Supplementary information: Supplementary data are available at Bioinformatics online.

1 Introduction

A central unsolved problem in biology is to understand how DNA specifies how genes turn on and off in multicellular organisms. The ‘universal expressibility’ of deep neural nets (DNNs), combined with standardized and effective optimization methods such as stochastic gradient descent (SGD), suggests that they might be a valuable tool in this undertaking. At the same time, the applicability and acceptance of DNNs in solving problems in natural science has been limited by the uninterpretability of their internal computations. We address both of these areas in this report by describing the reimplementation of a specific and highly predictive model of transcriptional control as a DNN. This work is a demonstration that DNN techniques can be applied to the apparently purely chemical problem of transcriptional control. Although applied here to a very specific problem in early embryo of Drosophila, we discuss below the reasons why the results here should generalize well to the study of transcription in other organisms including humans. The work presented here also provides an example of a DNN in which the internal structure is well understood and which may be of value to the machine learning field.

Deep learning has been widely deployed in genomics and systems biology over the last few years (Alipanahi et al., 2015; Assaf et al., 2019; Celesti et al., 2017; Cupers et al., 2017; Greenside et al., 2018; Koh et al., 2017; Libbrecht and Noble, 2015; Movva et al., 2019; Nair et al., 2019; Pouladi et al., 2015; Rui et al., 2007; Shen et al., 2018). Many of the developed tools have been highly successful in classification problems such as the identification of binding sites, open regions of chromatin and the location of enhancers. Deeper understanding requires more quantitative studies. One recent example that goes beyond classification concerns a fully quantitative and highly predictive DNN model of the role of untranslated RNA leader sequences in gene expression in yeast (Cupers et al., 2017). We believe that these studies have two sets of limitations. First, they take a universal expressibility approach without much understanding and interpretation of the underlying chemical and biological mechanisms giving rise to the phenomena under study. This limits the contributions DNNs can make to increasing human understanding of fundamental biological processes. Here, we consider a DNN in which each layer has a specific chemical or biological interpretation. Studies of regulatory DNA with DNNs have treated only the sequence itself, but in metazoan (multicellular
animals) different cell types have very different gene expression states although they contain the same DNA. For example, one recent study used a sequence based DNN classifier of open and closed chromatin regions to find binding sites for tissue specific transcription factors (TFs), but the actual binding state of these factors was not part of the model (Nair et al., 2019). Here, we consider a DNN in which state is described not only by the sequence, but also by the set of regulatory proteins present in each cell and the state of binding of each factor constitutes a specific layer of the model. We now describe the problem to be solved in both mathematical and biological terms.

The ‘expression’ of a gene is the rate at which it synthesizes its ultimate product, which may be an RNA or protein. Here, we focus on genes transcriptional by RNA polymerase II, which transcribes all genes other than those coding for ribosomal or transfer RNA. Although the expression of these genes can be affected at the level of RNA splicing or translation of protein, regulation of expression takes place chiefly at the level of transcriptional control. We thus consider the rate at which mRNA (the ‘transcript’) is synthesized from a complementary DNA template. This rate $\frac{d[mRNA]}{dt} = f(D, v)$ where $D = (D_i)$, where each $D_i \in \{A, C, G, T\}$ is a base in the sequence of regulatory DNA and $v = (v_1, \ldots, v_n)$, where each $v_i$ is the nuclear concentration of a regulatory DNA binding protein known as a ‘TF’. The machine learning task is to learn the function $f$ from a series of observations $(D, v_k)$ of the expression of sequence $j$ in cell type $k$, where $k \in \{1, \ldots, M\}$. The essence of the scientific problem is that each sequence $D_j$ must express correctly in each cell type, reflecting the fact that in a multicellular organism, different sets of genes are expressed in different cell types, but each cell type contains the same DNA.

Regulatory DNA is non-coding DNA which can be upstream (5'), downstream (3'), or within (intronic) the complementary mRNA template. TFs bind to DNA in its double stranded form. In metazoans, regulatory DNA is frequently much larger than the coding portion of the gene. The regulatory DNA contains segments of 500 to 1000 base pairs (bp) called ‘enhancers’, each of which directs expression in a particular domain or tissue type. In this study, we consider a gene called eve which acts in the early embryo of the fruit fly Drosophila melanogaster, at which time it forms a pattern of seven stripes as shown in Figure 1. The entire gene is 1.65 kb of DNA in length (Fuijoka et al., 1996), but the mRNA transcript is only 1.5 kb (Fig. 2). We consider the action of eve from 1 to 3 h after the start of embryonic development. At this time, the embryo is a hollow ellipsoid of cell nuclei that can be treated like a naturally grown gene chip in which $\frac{d[mRNA]}{dt}$. $D_{a}$ and $v_{a}$ are fully observable at a quantitative level. The embryo contains two orthogonal axes in the anterior–posterior (A-P) and dorsal–ventral (D-V) directions. In the central portion of the embryo, gene expression on the two axes is uncoupled, so cell type and gene expression can be visualized by plotting relevant state variables in one dimension.

We have chosen a specific dataset involving the expression driven by fused and unfused enhancers for eve stripes 2 and 3 in normal and reversed orientation (Fig. 2). Because the fused enhancers drive a novel pattern compared to the unfused enhancers, this dataset provides a stringent and highly informative test of capabilities of thermodynamic models. In particular, it is known that eve stripe 2 is repressed at its borders by TFs known as quenchers which operate over a range of 150 bp (Gray et al., 1994; Small et al., 1992; Stanojevic et al., 1991). Multiple quenchers must bind in order to elicit repression. It is known that Bicoid (Bcd) and D-STAT (Dst) are key activators of stripes 2 (Small et al., 1992) and 3 (Small et al., 1996), respectively, and that Bcd binding to DNA involves cooperativity (Burb and Hanes, 2001; Lebrecht et al., 2005). Moreover, it is known that Hunchback (Hb) acts as a repressor on stripe 3 (Small et al., 1996) and as an activator on stripe 2 (Small et al., 1992), and that it is converted from a quencher to an activator by nearby sites occupied by Bcd or Caudal (cad) (Small et al., 1993). As we explain below, this dense set of prior knowledge is not required for general modelling of transcription, but it provides a useful tool for selecting a particular thermodynamic model for reimplementation as a DNN.

In this paper, we show that DNNs can be used to generate a predictive model of gene expression. We chose a model for reimplementation as a DNN as follows. The ability to predict gene expression from DNA sequence is limited to so-called thermodynamic transcription models (Barr and Reinitz, 2017; Barr et al., 2017; Bertolino et al., 2016; Fakhouri et al., 2010; He et al., 2010; Janssens et al., 2006; Kazemian et al., 2010; Kim et al., 2013; Martinez et al., 2014; Reinitz et al., 2003; Samee and Sinha, 2014; Sayal et al., 2016; Segal et al., 2008). These models are defined by the fact that occupancy of DNA by TFs is calculated using thermodynamics, and phenomenological rules are used to calculate the transcription rate from the concentration of bound factors. Thus, all thermodynamic model compute the occupancy of TF binding sites from the concentration of TFs and DNA sequences, and are hence capable of predicting expression from DNA sequences not used for learning. Thermodynamic models can be further classified according to which chemical and regulatory mechanisms are included. Not all thermodynamic models consider the role of cooperativity in DNA binding (Bertolino et al., 2016; Fakhouri et al., 2010; He et al., 2010; Janssens et al., 2006; Kazemian et al., 2010; Martinez et al., 2014; Samee and Sinha, 2014; Sayal et al., 2016; Segal et al., 2008), but some do (Barr and Reinitz, 2017; Barr et al., 2017; Kim et al., 2013; Segal et al., 2008). It is our experience (Kim and Reinitz, unpublished data) that failure to consider the cooperativity of Bcd produces pattern defects in stripes 2 and 3 using the dataset considered here. In addition, thermodynamic models differ in terms of the processing steps performed after calculating the state of TF binding to DNA. The simplest sum activation and repression in a manner that does not take into account the limited range of action of quenchers (Bertolino et al., 2016; Kazemian et al., 2010; Segal et al., 2008). Such models cannot account for the narrow interstripes between the stripes of eve expression. Because eve stripes are known to be formed by repression by gap genes, the limited range of gap gene repression prevents repression at one enhancer from repressing

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**Fig. 1.** (a) Shows a Drosophila embryo about 3 h after fertilization which has been stained for Eve protein as described (Sukkova et al., 2008). Anterior is to the left and dorsal is up. The dark shades indicate the concentration of Eve protein and the stripes are numbered. The white box is the one dimensional region of interest used to generate the data (Janssens et al., 2005). (b) The TF concentrations found across the embryo (Sukkova et al., 2008). In the graph, 58 data points are shown, corresponding to 58 nuclei on the A-P axis. Each nucleus is 1% E.L. in size. The identity of TFs is shown in the key; the horizontal axis shows position in percent egg length (E.L.) and the vertical axis shows protein concentration.
expression at a nearby enhancer: thus, Hb bound at 3 enhancer is unable to repress expression from the stripe 2 enhancer. Many published models take into account the short range effect of quenchers at a nearby enhancer: thus, Hb bound at 3 enhancer is unable to repress expression from the stripe 2 enhancer. Many published models take into account the short range effect of quenchers (Fakhouri et al., 2010; He et al., 2010; Janssens et al., 2006; Martinez et al., 2014; Reinitz et al., 2003; Saval et al., 2016), but do not include in addition the ability of repressors to be transformed into activators by other bound factors (Barr et al., 2017; Kim et al., 2013). The ability to model coactivation is essential for eve because it is necessary for Hb to function as a coactivated activator for stripe 2. Indeed, the absence of coactivation is the probable reason why one effort to model many enhancers failed to generate any expression from stripe 2 (Segal et al., 2008). Among models of individual enhancers, we chose Kim et al. (2013) for reimplementation as an DNN because it has a sufficiently rich set of mechanisms with which to model stripes 2 and 3 and is thus a suitable test bed. It is the simplest model for such testing in that it does not contain additional mechanisms required to model a whole locus rather than an enhancer (Barr and Reinitz, 2017; Samee and Sinha, 2014).

Like DNNs, thermodynamic transcription models have a feed forward structure that can be described in layers. Unlike DNNs, thermodynamic models arise from chemistry and at least superficially have a different mathematical structure than DNNs. The form of the resulting equations makes use of backpropagation and hence stochastic gradient descent difficult or impossible because of the need to hand code complex partial derivatives, so these models are optimized by zero order methods such as Simulated Annealing or Genetic Algorithms. Despite this apparent mathematical distinction, we were able to translate the chemical model of transcription into a standard DNN form that could perform rapid learning by SGD.

Thermodynamic transcription models differ from existing DNNs used in genomics and systems biology because they compute the different levels of mRNA expression in different cell types by computing it from regulatory DNA sequence and TF concentration. They also differ from purely phenomenological models that do not use any sequence information; such models have been applied to the dynamics of a small network using a variant of a recurrent Hopfield neural net (Jaeger et al., 2004; Mana et al., 2009; Reinitz and Sharp, 1995) and to the expression of enhancers using logistic regression (Isley et al., 2013). Models in this class cannot predict gene expression from DNA sequence.

The resulting model is, to our knowledge, one of the first fully interpretable DNNs with an exact interpretation for each of the unknown parameters. It is also an example of a biologically validated DNN that is not perceptron-based. The parameters used to calculate the position and affinity of DNA binding sites for TFs (Section 2.1.1) are obtained from independent experiments. Activators activate transcription, quenchers suppress the action of activators over a limited range, and certain activators can also convert nearby quenchers into activators. In each of these regulatory mechanisms, multiple bound TFs are required to perform a regulatory action. We perform this calculation as follows. Binding site locations and affinities are determined from sequence. Together with TF concentrations, this information is used to calculate the occupancy of each binding site. We then calculate the effects of coactivation, followed by the effects of quenching. The total amount of remaining activation is then summed and passed through a thresholding function. Each step in this calculation corresponds to one or more of the layers shown in Figure 4. We describe these layers below.

2. Materials and methods

2.1 Computing fractional occupancy

2.1.1 Identifying the binding sites

An indicator representation of DNA sequence is used as input. The column index is the base pair position number in the sequence. The row index indicates which of the four bases (A, C, G, T) is observed. For example, if we have a sequence of ACITGTA, the corresponding matrix is

\[
\begin{pmatrix}
A & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
C & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
G & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
T & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
\end{pmatrix}
\]

The nine TFs of interest are Bicoid (Bcd), Caudal (Cad), Drosophila-STAT (Dst), Dichaete (Dic), Hunchback (Hb), Kruppel (Kr), Knirps (Kni), Giant (Gt) and Tailless (Tll) (Fig. 1b). The identification and affinity characterization of binding sites for TF a
requires a (PWM). Chemically, the PWM represents an additive model of binding in which the Gibbs free energy \( AG \) of binding is the sum of the free energies of binding to each nucleotide. Statistically, the PWM score can be regarded as the likelihood of finding a binding site at a given position, calculated using a variational approximation of the likelihood using only the marginal likelihood of each base. The physical size of the binding site, which is important for calculating competitive interactions between TFs, can usually be read from DNase I footprints (e.g. Small et al., 1992) and is larger than the set of nucleotides that are specifically recognized by the TF. We take this fact into account by padding the edges of the PWM with zeros to reflect their physical size. Most of our Position Weight Matrices (PWMs) were obtained from high quality SELEX (Orgawi and Biggin, 2012; Roulet et al., 2002) or bacterial one-hybrid data (Noyes et al., 2008); complete details are given elsewhere (Kim et al., 2013). Because these data were both reliable and independent, we chose not to adjust PWM values to fit the data, as has been done in other studies (Segal et al., 2008). In the context of DNNs, PWMs can be understood as a convolution kernel in which the number of columns represents the number of nucleotides of DNA in physical contact with a bound TF. Unlike many convolution matrices used in deep learning, PWMs make direct experimental predictions about DNA properties, a fact used in a previous study to experimentally confirm PWMs obtained by deep learning techniques (Cuperus et al., 2017).

The resulting score, denoted by \( S_{ij+m} \), is an affine transformation of the free energy \( \Delta G_{ij+m} \) of TF \( a \) binding to a site extending from base \( i \) to base \( i+m \). In many cases, including most of the TFs considered here, \( m \) can be read off directly from DNase I footprints (e.g. Small et al., 1992). TFs physically bind in the major groove of the DNA double helix but the PWM is convolved with only a single strand. We compensate for this fact by also scoring the complementary strand, in which bases are replaced by their complements (\( A \rightarrow T; C \rightarrow G; T \rightarrow A; G \rightarrow C \)), and orientation is reversed. Scoring each strand results in a \( 1 \times n \) array of scores \( S_{ns} \), where \( n \) is the sequence length. At each base \( i \), we set the score to be the larger of the two scores at that position.

We next calculate the equilibrium affinity \( K_{i+m} \) of each binding site. If \( \Delta G_{ij+m} \) were known in units of kcal/mole, then \( K_{i+m} = \exp(\Delta G_{ij+m}/RT) \), where \( R \) is Boltzmann’s constant and \( T \) the absolute temperature. \( S_{ij+m} \) is related to \( \Delta G_{ij+m} \) by an affine transformation \( a + b \), or more precisely, \( S - \text{Max} / \lambda \), in which \( \text{Max} \) is found from experiment and \( \lambda \) is learned by training as follows. Here, \( \text{Max} \) is the maximum possible score for any particular TF \( a \). \( \text{Max} \) is determined experimentally from the tightest possible binding sequence. We only consider binding to sites with scores greater than zero, so we introduce a parameter to assure that below threshold sites disappear, so that

\[
K_{i+m} = \exp \left( -\frac{(S_{ij+m} - \text{Max})}{\lambda} \right) \mathbf{1}(S_{ij+m} > 0), \tag{1}
\]

where \( \lambda \) is a learnable parameter for each TF \( a \). The indicators, one for each TF \( a \), give rise to what is essentially a ReLU and is hence denoted as ‘ReLU Exponential Activation’ in Figure 4. The \( K_{i+m} \), arranged according to positions on the DNA sequence, produce another \( 1 \times (n-m) \) vector \( K_{r} \). Adjusting for the size of \( m \), we further concatenate \( K_{r} \) for all TFs to produce \( K \), which we use for the calculation of fractional occupancy.

2.1.2 Belief propagation and partition function computation

Moving from affinity \( K_{i+m} \) to fractional occupancy requires the consideration of all possible states in which the TFs can bind on the DNA. The fractional occupancy \( f_{i+m} \) denotes the average occupancy of the site at equilibrium. Its calculation requires consideration of interactions between sites. Two overlapping sites cannot be occupied at the same time as illustrated in Figure 3a, and in some cases a TF bound at one site increases the binding affinity at a nearby site by a factor of \( u_{ij}^{\text{coop}} \). In the present application, cooperativity only occurs between pairs of bound Bcd <60 bp apart as shown in Figure 3a (Burz et al., 1998; Ma et al., 1996). The resulting mathematical structure is, in essence, a Markov random field with known conditional probabilities \( f_{i+m} \) at each position \( (i,i+m) \) for each TF \( a \). Using this interpretation, \( f_{i+m} \) denotes the marginal probability of finding the specific protein \( a \) at the site \( (i,i+m) \) at a given instant. The calculation of \( f_{i+m} \) is best performed by computing the partition function \( Z_{a} \), which can be calculated by a fast, recently discovered algorithm (Supplementary Appendix S1 in Barr and Reinitz, 2017). The algorithm is essentially a form of Belief propagation (Koller and Friedman, 2009) and is given in Algorithm 1, which

![Figure 3](image_url)

**Figure 3** (Burz et al., 1998; Ma et al., 1996). The resulting mathematical structure is, in essence, a Markov random field with known conditional probabilities \( f_{i+m} \) at each position \( (i,i+m) \) for each TF \( a \). Using this interpretation, \( f_{i+m} \) denotes the marginal probability of finding the specific protein \( a \) at the site \( (i,i+m) \) at a given instant. The calculation of \( f_{i+m} \) is best performed by computing the partition function \( Z_{a} \), which can be calculated by a fast, recently discovered algorithm (Supplementary Appendix S1 in Barr and Reinitz, 2017). The algorithm is essentially a form of Belief propagation (Koller and Friedman, 2009) and is given in Algorithm 1, which

**Algorithm 1** The Algorithm for Fractional Occupancy

Initial: \( Z_{N} = 1 \) and \( Z_{0} = 1 \)

for \( i \rightarrow 1 \to N \) do

\( q_{a} = [\text{TF}]_{a} K_{a} \)

for \( a \in \{\text{Transcription Factors}\} \) do

\( Z_{i}^{a} = q_{a} Z_{i-1}^{a} \)

\( Z_{N-i}^{a} = q_{a} Z_{N-i}^{a} \)

end for

end for

return \( Z_{N}^{a}, Z_{N}^{b}, Z_{N}^{c}, Z_{N}^{d} \)
2.2 TF–TF interactions

Interactions between bound TFs follow phenomenological rules. A central feature of the cis-regulatory DNA of metazoan genes is the fact that biological function is encoded in multiple binding sites (Small et al., 1992, 1996; Stanojevic et al., 1991). This fact is expressed mathematically in the phenomenological equations below. In the present application, bound TFs have specific roles derived from specific experimental results, although this approach also works if the roles are not known a priori (Bertolino et al., 2016). Here, Bcd, Cad, Dst and Dic are activators; Hb, Tll, Kni, Kr and Gt are quenchers; both Bcd and Cad are coactivators of Hb. We now describe the actions of each class of TF in the order which we compute them. The ultimate goal of this computation is to obtain the summed action of activators after their contribution has been increased by coactivation and diminished by quenching.

2.2.1 Coactivators

Coactivators turn a nearby quencher into an activator as schematically illustrated in Figure 3b. This action is described by the equation (Barr and Reinitz, 2017; Kim et al., 2013)

$$ f_i^{QC} = f_i^{QC} \prod_{j=i-k}^{i+k} (1 - d_{i,j} E_C^{QC} f_j^C), \quad (3) $$

where $f_i^{QC}$ is the portion of activator fractional occupancy created from the total fractional occupancy $f_i$; $d_{i,j}$ is a convolutional kernel describing the distance dependence of coactivation. $d_{i,j}$ is 1 if $|j - i| < 156$ bp. In this application, $k = 206$. As $|j - i|$ increases to 206 bp, $d_{i,j}$ decreases linearly to 0. $E_C^{QC} \in [0, 1]$ denotes the relative strength of the coactivators. $f_i^C$ simply refers to fractional occupancy of Bcd and Cad since they are the only coactivators in this setting.

It is easy to observe that Equation (3) is the first term of a Taylor expansion of

$$ f_i^{QC} \approx f_i^{QC} \prod_{j=i-k}^{i+k} (1 - E_C^{QC} f_j^C) d_{i,j}, \quad (4) $$

This can be turned into three convolutional linear activation units, so that

$$ y_i = \log(1 - E_C^{QC} f_i^C); \quad z_i = \exp \left( \sum_{j=i-k}^{i+k} d_{i,j} y_j \right); \quad f_i^{QC} = f_i^{QC} z_i. \quad (5) $$

This is schematically designated by the pink box labeled ‘co-activators’ in Figure 4.

2.2.2 Quenchers

As their name suggests, quenchers suppress activation by all activators, both native and those subject to coactivation, as schematically indicated in Figures 3b and 4. Their action is local and occurs only within around 100–150 base pairs (Hewitt et al., 1999). The basic mathematical formulation of the effect of nearby quenchers on the activator at position $i$ is given by Kim et al. (2013):

$$ Z_i = \frac{C_0}{C_i} \left( \frac{C_{i+1}}{C_i} \right)^{k+1} Z_{i+1}.$$
\[ f_i^A = f_{i,0} \prod_{j \neq i} (1 - d_q(j)E^{Q_j}_{A_j}), \]

where \( f_i^A \) is the fractional occupancy of activators whether coactivated or not, \( f_{i,0} \) is the fractional occupancy of activators after quenching and \( f^{Q_j}_{A_j} \) is the fractional occupancy of a quencher bound at position \( j \). \( E^{Q_j}_{A_j} \in (0,1) \) is the strength of quencher \( Q \) on activator \( A \) and \( d_q(j) \) is a convolutional Kernel representing the range of quenching on the DNA strand. \( k = 150 \) bp and \( d_q(j) = 1 \) when \( |j - i| \leq 100 \) and goes linearly down to 0 from 150 to 150 bp. Using a mathematical argument similar to that of the previous section, we can write

\[ f_i^A \approx \exp \left( \sum_{j \neq i} d_q(j) \log(1 - E^{Q_j}_{A_j}) \right) f_{i,0}^Q. \]

This gives three convolutional linear activation units.

\[ y_i = \log(1 - E^{Q_j}_{A_j} f_i^Q); \]
\[ z_i = \exp \left( \sum_{j \neq i} d_q(j) y_j \right); \]
\[ f_i^Q = f_{i,0}^Q z_i. \]

### 2.2.3 Activation

Last, we sum the fractional occupancies of all activators remaining after the previous two steps. We consider the activators to lower the energy barrier in a diffusion limited Arrhenius rate law (Barr and Reinitz, 2017), which has the mathematical form of a sigmoidal thresholding function. This results in a fully connected layer, shown schematically in Figure 4, which yields the mRNA synthesis rate. In the thresholding function. This results a fully connected layer, shown schematically in Figure 4, which yields the mRNA synthesis rate.

In the experimental system used, mRNA has a lifetime much shorter than the time required to change transcription rates, so that the mRNA concentration \( [\text{mRNA}] \), an experimentally observable quantity, is given by

\[ [\text{mRNA}] \propto \frac{d[\text{mRNA}]}{dt} = R \left( \frac{\exp(\sum_{j \neq i} (A_Q, E_i) \sum_{j \neq i} f_j^Q) - \theta}{1 + \exp(\sum_{j \neq i} (A_Q, E_i) \sum_{j \neq i} f_j^Q) - \theta} \right). \]

Here, \( E_i > 0 \) represents the activating strength of each activator and is obtained by training on the data, \( \theta \), also obtained by training, is the amount of activation in the absence of activator and \( R \) is the maximum synthesis rate.

### 3 Implementation, training and results

We train on the target

\[ [\text{mRNA}]_{\text{train}} = \frac{[\text{mRNA}]_{\text{mcell}}}{\sqrt{\sum_{\text{cells}}[\text{mRNA}]_{\text{mcell}}^2}}, \]

using an L2 norm, minimizing the loss function

\[ L = ||[\text{mRNA}]_{\text{train}} - [\text{mRNA}]_{\text{mcell}}||^2. \]

We implemented and trained this model in Keras with a TensorFlow backend (Abadi et al., 2015; Chollet et al., 2015). The resulting architecture is shown in Figure 4. The figure shows nine layers, essentially one for each class of box. The actual implementation in Keras, which performed the precise computation described above, was deemed by Keras to contain 223 layers. This large number of layers is a consequence of the fact that we did not use any of the conventional architecture (Jaderberg et al., 2015; Krizhevsky et al., 2012) and hence needed to use the Lamméa functions in Keras to represent some of the activation functions and PWM convolutions. Algorithm 1 was implemented as a special layer in Keras with two nn functions. Our implementation contains 52 unknown parameters.

The training data and PWMs were as previously described (Kim et al., 2013), although training data were limited to the fusion constructors M32, M3_2, M23 and M2_3. The model was trained using a single Intel Core i7-8700K CPU. The training data contain 232 unique observations of [mRNA]_{\text{train}}. Each of these observations is associated with the DNA sequence that drives the expression together with the concentration level of TFs that is characteristic of the position of the observed nucleus in the embryo. The model was trained using Adam (Kingma and Ba, 2014), for 200 epochs, with Nesterov momentum as implemented in Keras. Training took ~2 h. This compares with several days of serial simulated annealing before Algorithm 1 was devised (Kim et al., 2013), and is about equivalent to the time taken by code using Algorithm 1 running in parallel with the loss function for each construct computed on a separate core. However, optimization by simulated annealing requires several million evaluations of the loss function. While the implementation uses here required about 10 000. This is indicative of an algorithmic speedup on the order of 100. The earlier work used about 10 000 lines of compiled C++ while this work uses <1000 lines of Python.

However, the mismatch between algorithmic and wall-clock speedup indicates that there is considerable scope for improvement of the TensorFlow backend for this type of problem. In addition, this implementation is more modular than that described in Kim et al. (2013). The use of Keras enables us to break the model up into blocks represented by proteins and interactions, which were stacked on top of one another with minimal modification to the code. This will allow for greater flexibility when producing similar models for other organisms or for extending model to different biological functions, such as chromatin accessibility.

### 4 Results

The results of the training are shown in Figure 2. It is important to note that this model tends not to over-fit the data since the number of parameters is only 52 compared to the 232 nuclei we used as the training set. Nevertheless, the RMSE, an approximate measure of the average error between the behavior of the model and a quantitative observation, is 6.89, approximately the uncertainty level of the data itself, which is accurate to about 5–10%. This quality of fit is less than the RMSE of 2.3 reported in Kim et al. (2013), but this study also adjusted parameters controlling the range of co-activation and quenching, as well as the minimum PWM score to include a binding site, which was set to 0 here. Although the reduction in degrees of freedom resulted in a poorer fit, it was nevertheless within experimental error. Comparison to other studies with thermodynamic models is complicated by the diverse set genes selected for modeling. However, we note that a comparison to one prominent published study (Segal et al., 2008) is impossible because the model used failed to give any expression from stripe 2. Another study using comparable data (He et al., 2010) reported values of correlation coefficient \( R \) between 0.55 and 0.60, while the \( R \) value in our study is 0.91 (Fig. 5).

We tested the predictive power of our DNN by confronting it with set of enhancer sequences which the model has not previously seen. The test sequences must be capable of driving expression in \( D. melanogaster \) embryos, so that the same TF dataset used for training can be employed to calculate the predicted expression along the \( A-P \) axis. In Figure 6, we show the predicted expression of four enhancer sequences, each of which consists of 58 nuclei and hence 58 separate predictions. We chose these enhancers for presentation because they are exceptionally stringent tests in the sense that they involve enhancers with no DNA homology with those used in the training set, either because they are from different genes or distant species. Seventeen additional predictions are given in the Supplementary Material.

We comment in detail on the following predictions. When considering the accuracy of these predictions, it is important to note that the experimental data used for the comparison is in the form of non-quantitative images, which are referred to by specific figure in
the citation below. Because the comparison is made to non-quantitative data, it is not possible to assess the quantitative accuracy of the prediction, but only the spatial position. \textit{run\_str3\_7} is the enhancer of the \textit{run} gene of \textit{D. melanogaster} that drives \textit{run} stripes 3 and 7, each of which is about 2% E.L. anterior of the corresponding eve stripes. The positions of the stripes are accurate (Klingler et al., 1996; Fig. 3D and K in Klingler et al., 1996). eve\_S2E\(\text{(cyn)}\) and eve\_S2E\(\text{(spp)}\) are respectively the eve stripe 2 enhancers from \textit{Sepsis cynipsea} and a different but unidentified \textit{Sepsis} species. eve\_S37E\(\text{(pun)}\) is the stripe 3/7 enhancer from \textit{Sepsis punctum}. The \textit{Sepsis} genus is a member of the family Sepsidae, not Drosophilidae, and are so distant from \textit{D. melanogaster} that there is no sequence homology with the \textit{D. melanogaster} enhancer longer than a pair of overlapping binding sites (Hare et al., 2008a,b). Nevertheless, all \textit{Sepsis} enhancers that have been assayed in \textit{D. melanogaster} embryos drive expression of stripe 2 1–2% E.L. posterior to their native position and stripes 3 and 7 at the native position (Table 2 in Hare et al., 2008b). For eve\_S2E\(\text{(cyn)}\), the predicted stripe 2 is of the correct width and 2% E.L. more posterior than reported. In Sepsidae, stripe 2 enhancer also drives expression of stripe 7. For eve\_S2E\(\text{(cyn)}\), our prediction of stripe 7 expression is completely accurate. For eve\_S2E\(\text{(spp)}\), the predicted stripes are in the correct location, but stripe 2 is significantly wider than observed. For eve\_S37E\(\text{(pun)}\), our prediction for stripe 3 is about 2 nuclei posterior and twice as wide as observed, and for stripe 7, our prediction is completely correct. These predictions demonstrate the generalization capabilities of the DNN implementation of the model. The quality of prediction is comparable to but slightly poorer than the original implementation (in Fig. 4 and Supplementary Fig. S5 Kim et al., 2013).

5 Discussion

The work presented here constitutes a strong proof-of-concept for the proposition that DNN can be extremely useful for the construction of precise models of metazoan transcriptional control. Specialized properties of the early \textit{Drosophila} embryo made it uniquely advantageous for the study of the fundamental properties of metazoan transcription before the onset of genomics. For that reason, it is natural that thermodynamic models of metazoan transcription were developed in \textit{Drosophila} (Barr and Reinitz, 2017; Barr et al., 2017; Fakhouri et al., 2010; He et al., 2010; Janssens et al., 2006; Kazemian et al., 2010; Kim et al., 2013; Martinez et al., 2014; Reinitz et al., 2003; Samee and Sinha, 2014; Sayal et al., 2016; Segal et al., 2008). As explained in the introduction, we selected what is arguably the most accurate and predictive enhancer level thermodynamic model for reimplementation as a DNN. We have shown that a simpler reimplementation of this model performance almost as well as the original (Kim et al., 2013). The reimplementation is much simpler to code than the original model, amounting to about 600 lines of Keras compared to about 10,000 lines of C++. Implementation in Keras also provides a numerical advantage by permitting the use of backpropagation and stochastic gradient descent for optimization without the need to hand code partial derivatives. As reported above, the use of SGD provides an algorithmic speedup of about 100. Although some issues connected to the wall-clock speedup of the Keras implementation remain, these results suggest that models of this type could be scaled up to much larger datasets. A possible limitation of such generalization is the extensive prior knowledge of details of \textit{Drosophila} transcription that were used in this study.

There are ample reasons for believing that this lack of prior knowledge can compensated for by increased quantity and quality of computation. In the absence of prior knowledge of the functional roles of TFs, a model very similar to the one we reimplemented here applied to erythropoiesis in mouse (Bertolino et al., 2016) considered all possible combination of TF functional roles and selected the combination which minimized the loss function. The resulting model, while computationally expensive, proved quite predictive (Repele et al., 2019). This shows that extensive prior knowledge is not required.

Until recently, a more serious limitation was the absence of data-sets that contained not only information about the sequence but also the concentration of TFs and levels of reporter expression. Such datasets, on a genomic scale, have begun to be available for flies and mammals, including humans (Arnold et al., 2013; Liu et al., 2017; Patwardhan et al., 2009; Smith et al., 2013; Ulirsch et al., 2016). These genomics datasets are much larger than that used in the mouse study (Bertolino et al., 2016). However, the factor of 100 algorithmic speedups, if converted to wall-clock speedup, provides a way forward. The introduction of reinforcement learning techniques may provide further computational efficiency in determining the functional roles of TFs.

With respect to Deep Learning, our model constitutes an example of a fully interpretable DNN that is not merely biologically plausible but biologically validated (Kim et al., 2013). It is our hope that this example will provide insights into the interpretabilities of DNNs in general, a problem that has received wide attention in the community (Boger and Guterman, 1997; Castelvecchi, 2016; Garson, 1991; Li et al., 2015; Maaten and Hinton, 2008; Zeiler and Fergus, 2014).

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