Increasing two-photon entangled dimensions by shaping input beam profiles

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Photon pair entangled in high dimensional orbital angular momentum (OAM) degree of freedom (DOF) has been widely regarded as a possible source in improving the capacity of quantum information processing. The need for the generation of a high dimensional maximally entangled state in the OAM DOF is therefore much desired. In this work, we demonstrate a simple method to generate a broader and flatter OAM spectrum, i.e., a larger spiral bandwidth (SB), of entangled photon pairs generated through spontaneous parametric down-conversion by modifying the pump beam profile. By investigating both experimentally and theoretically, we have found that an exponential pump profile that is roughly the inverse of the mode profiles of the single-mode fibers used for OAM detection will provide a much larger SB when compared to a Gaussian shaped pump.

Introduction. Two-photon high dimensional entangled state (HD-ES), $\sum_{n=0}^{d-1} c_n |n_A \rangle |n_B \rangle$, has been widely regarded as useful in increasing capacity for quantum information processing. From the fundamental physic standpoint, such states imply a larger violation of local-realism theories and a lower fidelity bound in quantum state tomography $[3, 6]$, increasing dimensions of the Bell state in dense coding, entanglement swapping or teleportation $[7, 9]$, and improving the quality of imaging or quantum sensors $[10, 12]$. 

In photonic systems, one could construct an HD-ES in many of the photon’s degrees of freedom (DOF) $[13, 14]$. For example, in orbital angular momentum (OAM) $[15–21]$, in paths $[22, 24]$, frequency $[25]$, photon number $[26]$, or temporal modes $[27, 28]$. HD-ES in OAM has been gaining more attention due to their easy scalability in dimension; for example, 100 × 100 dimensional entanglement has been created via employing both the OAM and radial DOFs of entangled photon pairs $[29]$. 

The most common method in generating OAM entangled photon pairs is via the process of spontaneous parametric down-conversion (SPDC) $[15]$. According to OAM conservation, the sum of OAM from the signal and idler photons must equal to that of the pump photon i.e., $\ell_p = \ell_s + \ell_i$. When $\ell_p = 0$, the output two-photon state of SPDC can be Schmidt decomposed into $\sum_{\ell} C_{\ell, \ell} | -\ell \rangle | \ell \rangle$. Here, $C_{\ell, \ell}$ is the probability amplitude($\sum |C_{\ell, \ell}|^2 = 1$) of finding the signal photon with OAM $-\ell$ and the idler photon with OAM $\ell$ in coincidence. The width of the OAM spectrum is often known as the spiral bandwidth (SB) $[30]$. The Schmidt number $K = 1/\sum C_{\ell, \ell}^2$ is also often defined to evaluate the entanglement dimensions $[31, 33]$: a larger value of $K$ depicts a larger dimensions of entanglement. For a maximally entangled state (MES) of $|C_{\ell, \ell}| = 1/\sqrt{d}$, the Schmidt number is $d$. Entangled photons with a larger Schmidt number could be beneficial to implementing higher dimensional quantum protocols like cryptography, computation, imaging and metrology.

For SPDC, the coincidence amplitudes $C_{\ell, \ell}$ for the OAM DOF can be calculated via the overlap integral between the input pump mode and both of the signal and idler modes in the Laguerre-Gaussian (LG) basis $[32, 33]$. Previous works have shown several ways to increase the SB of OAM entanglement. Firstly, one can adjust the beam waist ratio between the pump and the measured LG modes $\gamma = w_p/w_s(i)$ $[30, 32]$: the SB increases with increasing $\gamma$. Second is by changing the down-conversion angle through adjusting the SPDC phase matching $[33, 35]$: the SB changes based on the down-conversion angle between the SPDC photons. Lastly, is to engineer a crystal with spatially varying phase matching $[36, 38]$: there has been little experimental progress in this avenue due to the complex fabrication technology. There have also been attempts to prepare HD-MES in OAM through some complex engineering of the pump beam profile $[21, 39, 42]$. In $[41, 42]$, a three dimensional maximally entangled state $1/\sqrt{3}(|-1\rangle |0\rangle + |0\rangle |1\rangle |1\rangle)$ has been generated by shaping the pump into a superposition of several LG modes. However, the generation of higher dimensional MES remains difficult due to the crosstalk between OAM modes.

Here, we proposed an ingenious method to increase the SB via some simple shaping of the pump beam profile. We show both theoretically and experimentally that a pump with an exponential profile that is roughly the inverse of the combined profile of the single-mode optical fibers (SMF) used for detection will significantly flatten the OAM spectrum and extend the SB. We then performed high dimensional quantum state tomography in
a three- and five-dimensional subspace using the optimized exponential pump; the corresponding fidelities are 90.74% and 81.46% for three- and five-dimensional MES, respectively. Traditionally, when the input pump beam profile is a Gaussian function, the distribution of coincidence amplitude with OAM is strongly mode-dependent [32-34]. Therefore, mode post-selection has to be performed in order to generate a HD-MES. Our method demonstrates a simple way to broaden and flatten the SB, which could allow future quantum protocols using the OAM DOF to access higher dimensional MES without requiring mode filtering.

Theory. SPDC photons entangled in arbitrary superpositions of OAM are often described in terms of the LG modes. In our analysis, we are only interested in the modes. In the thin crystal limit, the phase matching function of the SPDC process can be approximated to unity and the coincidence amplitudes $C_{\ell_1, \ell_2}$ can be calculated from the overlap integral [32-34, 35]

$$C_{\ell_1, \ell_2} = \int \Phi(x)[LG_{\ell_1}(x)]^* [LG_{\ell_2}(x)]^* G^2(x) dx,$$

(1)

where $\Phi(x)$ is the mode function of the pump and $G(x)$ is the Gaussian mode of the SMF used for detection. $LG_{\ell}(x)$ is the LG mode [36]. When the pump profile is a Gaussian and by choosing the LG mode size of the signal and idler beams to be equal $w_s = w_i = w_{si}$, the coincidence probability can be evaluated as [32-34, 35]

$$C_{-\ell, \ell} \propto \left( \frac{2 \gamma^2}{2 \gamma^2 + 2 \eta^2 + 1} \right)^{\ell}.$$

(2)

Here $\gamma = w_p/w_{si}$ is the beam waist ratio between that of the pump and the LG modes of the signal and idler photons measured at the nonlinear crystal plane. $\eta = w_p/w_f$ is the beam waist ratio between the pump beam and the mode size of the SMFs. Based on the Eq. (2), the OAM spectrum always peaks at $\ell = 0$ and rapidly decreases with increasing $\ell$.

Looking at Eq. (1), we see that if the combined profile (CP) of $\Phi(x)$ and $G^2(x)$ is a constant, the overlap integral should be a constant with respect to $\ell$ resulting in a HD-MES. [34] classically simulated this using the Klyshko’s advanced-wave representation [36]. It was found that the SB can be expanded significantly when this CP is flat. We shall look at a more general situation that uses an adjustable exponentially shaped pump beam whose beam profile is given by

$$\Phi(r) = \exp \left( \frac{a r^2}{w_p^2} \right) * H(-r + w_p).$$

(3)

Here $a$ is a parameter that determines the width and curvature of the exponential function and $H(x)$ is the heaviside step function which limits the width of the beam to same. After evaluating the overlap integral in Eq. (1), $C_{-\ell, \ell}$ is determined to be

$$C_{-\ell, \ell} \propto \left( \frac{2 \gamma^2}{2 \gamma^2 + 2 \eta^2 + a} \right)^{\ell} \times \left[ 1 - \frac{1}{|\ell|!} \Gamma(1 + |\ell|, 2 \gamma^2 + 2 \eta^2 + a) \right],$$

(4)

with $\Gamma(n, z) = \int_z^{\infty} t^{n-1} e^{-t} dt$ being the incomplete gamma function.

Eq. (4) is similar to Eq. (2) albeit the extra gamma function. Some interesting behaviour can be observed when $a$ is varied:

i) When $a < 2 \eta^2$, the CP of the pump and the SMFs is still a Gaussian. The OAM spectrum is essentially the same as that in Eq. (2) (with some small deviations.
coming from the incomplete gamma function at larger $\ell$) where it peaks at $|\ell| = 0$ and decreases rapidly with larger $|\ell|$ values (Fig. 1(a1)). The SB broadens as $a$ approaches $2\eta^2$.

ii) When $a = 2\eta^2$, the CP will be a flat-top. The first term in Eq. (1) becomes a constant resulting in a flat OAM spectrum, however, the incomplete gamma function will suppress $|C_{-\ell,\ell}|^2$ for larger $|\ell|$ values (Fig. 1(a2)). To further broaden the OAM spectrum, one can increase $\gamma$ as seen in Fig. 1(d1).

iii) When $a > 2\eta^2$, the CP is an exponential. The denominator in the first term of Eq. (1) is now smaller than its numerator, so the term will grow with increasing $|\ell|$. $|C_{-\ell,\ell}|^2$ is still suppressed by the incomplete gamma function at larger $|\ell|$ values. This results in an OAM spectrum that peaks at some non-zero $|\ell|$ value (Fig. 1(a3)).

In Fig. 1(d2), it can be seen that for larger $\gamma$ values, the Schmidt number is a maximum when $a \approx 2\eta^2$. However, when $\gamma$ is small, the maximum Schmidt number occurs at $a > 2\eta^2$. This is a result of a larger contribution from the incomplete gamma function when $\gamma$ is small therefore suppressing $C_{-\ell,\ell}$ at smaller $\ell$ values.

Results To verify the theoretical prediction in Eq. (1), we measured two-photon OAM correlations using different input beam profiles (parameter $a$). The corresponding experimental setup is shown in Fig. 2 which includes three parts: pump beam shaping - Fig. 2(a), state generations - Fig. 2(b), and projection measurements - Fig. 2(c). First, the pump beam is shaped using either a SLM (Path2) or a $\pi$-shaper (Path1) from a Gaussian into the desired beam shape (details on beam shaping can be found in the supplementary material). Then, a 10mm long nonlinear crystal (PPKTP) is pumped by the shaped beam to generate OAM entangled photon pairs. The SPDC photons at the nonlinear crystal plane were then imaged to the surface of another SLM for mode decomposition and then coupled into single-mode fibres connected to a superconducting nanowire single-photon detector for coincidence measurements. In our setup, the beam width ratio between the pump and SMF $\eta$ is 0.31. It should be noted that though a SLM have more versatility in the beam shaping it can perform, however, it could not support high pump intensities and have lower conversion efficiencies compared to a commercial $\pi$-shaper.

In situations where high pump power (350mW) is required to increase the SPDC photon production and reduce data acquisition time, the $\pi$-shaper had to be used.

Fig. 3(a) shows the OAM spectrum generated with different pump beam profiles ($a = -1, 0$, and 0.8), where the beam width ratio $\gamma$ is 1.25. From the results in Fig. 3(a), one can see that the OAM spectrum broadens as $a$ increases, just as theoretically predicted from Eq. (1). Also to note here is that since $\gamma$ is small, the beam shape parameter $a$ which gives the largest SB (or $K$) is actually larger than $2\eta^2$ as seen in Fig. 1(d2). The theoretical OAM spectrum for these three cases can be found in the supplementary material.

In addition to the beam profile parameter $a$, the beam width ratio $\gamma$ is another important parameter that affects the dimension of entanglement. For the following results in Fig. 3(b), (c) and Fig. 4, the $\pi$-shaper was used to shape the pump in order to increase the production rate of entangled photons. In Fig. 3(b), we show the measured coincidence rate for various $\ell$ values as a function of $\gamma$. One can see that for larger value of $\gamma$ (blue area in Fig. 3(b) $\gamma > 2.5$), the difference between the coincidence rate for the various $\ell$ values are relatively small which indicates a broader SB. This is in agreement with our theoretical results shown in Fig. 1(d1).

In Fig. 3(c) we show the theoretical and experimental OAM spectrum from $\ell = -12$ to 12 for $\gamma = 2.4$. A significantly broader OAM spectrum can be observed when compared to a Gaussian pump ($a = -1$) with the same $\gamma$. The experimental azimuthal Schmidt number $K$ is determined to be 21.9, which is in good agreement with the theoretical prediction of 20.7, for a Gaussian pump $K$ is 15. For quantifying the crosstalk between two neighboring OAM values, we measured the crosstalk-visibility $(1 - \sum_{i,j = \pm1} C_{i,j}^2 / \sum_{i = -12}^{12} C_{i}^2)$ and obtained a value of 93.91%.

From Fig. 3(c), one can see that the HD-MES is pre-
pared at least in a five-dimensional subspace. We re-
structured the density matrices for the cases of dimension
$\ell = 3$ (Fig. 3(a), (b)) and $\ell = 5$ (Fig. 3(c), (d)) through
dimensional quantum state tomography [42, 47, 48].

The measured fidelity, $F = |\text{Tr}\sqrt{\rho\rho_{\text{exp}}}|^2$, was
$0.9071\pm0.005$ for $\ell=3$ and $0.8146\pm0.0014$ for $\ell=5$ with
the uncertainty obtained through statistical simulations
that assumed the coincidence events follow a Poissonian
distribution. The fidelity for both $\ell = 3$ and $\ell = 5$
etangled states exceeded the dimensional threshold of
$\ell - 1)/\ell$, signifying that the density matrix cannot be
decomposed into an ensemble of pure states with low
Schmidt number [19, 21]. From the density matrix we can
also calculate the linear entropy, $S_{\text{ent}} = 1 - \text{Tr}(\rho^2)$, giving
$S_{\text{ent}} = 0.1043\pm0.009$ and $0.2851\pm0.0093$ for $\ell = 3$
and $\ell = 5$ cases respectively. Furthermore, the CGLMP
Bell inequality [2] was determined to be $2.85 \pm 0.03$ and
$2.40 \pm 0.01$ for the $\ell = 3$ and $\ell = 5$ entangled states re-
spectively. The lower values for the $\ell = 5$ case is mainly
attributed to imperfect mode overlap between the SPDC
photons and the measurement SLM. These values are
listed in the table below Fig. 4 for clarify.

**Discussion**

In this work, a simple technique of shaping the pump
beam profiles to increase the two-photon OAM entangle-
dimentions in a SPDC processes is demonstrated.
Theoretically and experimentally, we found that the coin-
cidence amplitude will become mostly mode independent
when the pump profile is an exponential that roughly
cancels the Gaussian profile of the SMFs used for photon
detection. This adds a new way of expanding the SPDC
OAM spectrum and can be used concurrently with pre-
viously suggested techniques such as increasing $\gamma$
and adjusting the down conversion angle between the signal
and idler photons. Through employing the beam shaping
method, we achieved an OAM MES of up to five-
dimensions without needing to use mode post-selection
and entanglement concentration techniques. The ability
to generate such HD-MES will be of great importance in
quantum communication, quantum sensing and also in
fundamental physics research.

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