Positivity in the effective field theory of cosmological perturbations

Gen Ye\textsuperscript{1*} and Yun-Song Piao\textsuperscript{1,2†}

\textsuperscript{1} School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China and
\textsuperscript{2} Institute of Theoretical Physics, Chinese Academy of Sciences,
P.O. Box 2735, Beijing 100190, China

Abstract

Requiring the existence of a unitary, causal and local UV-completion places a set of positivity bounds on the corresponding effective field theories (EFTs). We apply this positivity argument to the EFT of cosmological perturbations. Taking a $c_T = 1$ beyond-Horndeski EFT as an illustrative example, we drive such bounds, which naturally incorporate the cosmological correction of order $H^2/\Lambda^2$, in which $\Lambda$ is the cutoff scale. Applications of our results are briefly discussed.

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I. INTRODUCTION

The effective field theory (EFT) of cosmological perturbations is a powerful tool to study perturbations around a given cosmological background. Since the EFT of inflation [1], the relevant idea has been also applied to other cosmological fields, such as the EFT of dark energy [2–5] which captures the physics of scalar-tensor theories [6–10] (for a review, see [11, 12]) at the cosmological scale. As another example, based on the EFT approach, it has been found that fully stable nonsingular cosmologies exist in theories beyond Horndeski [13–20].

The full UV-complete theory of gravity is yet unknown. Instead of starting top-down from a UV theory, usually one works directly with the EFT, which captures the physics of the underlying theory at certain scales. However, not all low-energy EFTs have a consistent UV theory, see e.g.[21, 22]. Assuming the UV-complete theory is causal, unitary and local Lorentz-invariant, one can derive dispersion relations relating the IR limit of a scattering amplitude with its UV behavior, which place a set of bounds on the properties of the corresponding low-energy EFTs, the so-called positivity bound [23–31]. The positivity bounds have been applied to various EFTs [32–39].

Recently, Ref.[40] derived positivity bounds on a covariant shift-symmetric Horndeski theory (which might explain the current accelerated expansion), and paired these bounds with a cosmological parameter estimation analysis. It is natural to ask what will happen if we incorporate the cosmological background evolution. It is convenient for our purpose to work with the EFT of cosmological perturbations. However, the background evolution is itself the biggest obstruction in applying the positivity arguments because it breaks time-translation symmetry and makes Lorentz-invariant scattering ill-defined. In the limit of a Lorentz-invariant background, the positivity bounds saturated by the UV completions of single-field inflation have been investigated in [41].

Theoretically, an EFT is obtained after one integrates out physics above the cutoff scale $\Lambda$. On the other hand, the curvature of a homogeneous FRW universe is proportional to $H^2, \dot{H}$, so one can use $H^2, \dot{H}$ as additional dimensionful parameters in constructing the EFT, e.g.[42].

In this paper, based on similar observation, we will regard the cosmological background as $O(H^2n/\Lambda^{2n}, \dot{H}^n/\Lambda^{2n})$ correction in the amplitude, as pointed out also in Ref.[41], and derive the corresponding positivity bounds incorporating the cosmological background.
This paper is structured as follows. In section-II, we briefly review the dispersion relation that relates the UV and IR in relativistic quantum field theories. The effective Goldstone Lagrangian is derived in section-III A. Then in section-III B, under some assumptions, we derive positivity bounds with leading cosmological correction. Applications of our bounds to the cosmological scenarios of interest are discussed in section-III C.

II. POSITIVITY BOUNDS

We briefly review the positivity argument, see e.g.[23] for details. In a Lorentz-invariant UV theory with locality and causality, the $2 \rightarrow 2$ scattering amplitude is expected to be an analytic function of the Mandelstam variables $(s, t, u)$ with poles and branch cuts. Unitarity is encoded as the polynomial boundedness of the amplitude [43, 44].

Consider a massive scalar field with mass $m$. The Mandelstam variables are not independent $s + t + u = 4m^2$. We denote the $2 \rightarrow 2$ amplitude by $A(s, t)$. For fixed $t$, it is an analytic function $A_t(s)$ of $s$. We assume $|t| < m^2$ so no poles are encountered when pushed to the forward limit $t \rightarrow 0$. $A_t(s)$ can be extended to the complex s-plane by crossing symmetry.
\( A_t(s) = A_t(4m^2 - t - s) \) and analytic continuation \( A_t(s) = A_t^*(s^*) \). Consider the following Cauchy integral at fixed \( t \)

\[
\sum \text{Res} \left( \frac{A_t(s)}{(s - M^2)^3} \right) = \frac{1}{2\pi i} \oint_C \frac{A_t(s)}{(s - M^2)^3} ds.
\]  

(1)

The analytic structure of \( \lim_{t \to 0} A_t(s) \) and the integration contour \( C \) are depicted in Fig.1. By the Froissart-Martin bound \( |A_t(s)| \sim \mathcal{O}(s \ln^2 s) \) as \( s \to \infty \) [43, 44], only integration along the branch cuts is not equal to 0 when the contour is pushed to infinity. To extract the positivity bound, we now pass to the forward limit \( t \to 0 \). Using the optical theorem

\[
\text{Im}[A(s)] = \sqrt{s(s - 4m^2)} \sigma^{2 \to \text{any}}(s),
\]  

(2)

we can write the integration along the cuts as

\[
\frac{1}{\pi} \int_{-\infty}^{m^2} + \int_{4m^2}^{+\infty} \text{Im}[A(s)] ds = \frac{1}{\pi} \int_{-\infty}^{m^2} + \int_{4m^2}^{+\infty} \sqrt{s(s - 4m^2)} \sigma^{2 \to \text{any}}(s) ds.
\]  

(3)

There are three poles included. When the EFT has only derivative interactions (the Goldstone action we are to consider later falls into this category), residues at \( s = m^2 \) and \( s = 3m^2 \), associated with propagators in the exchange diagrams, are proportional to powers of \( m^2 \) and thus vanish when \( m^2 \to 0 \). Then the final bound in the massless limit is

\[
A''(s = M^2) \simeq \frac{4}{\pi} \int_{0}^{\infty} \frac{\sigma(s)}{s^2} ds > 0,
\]  

(4)

up to \( \mathcal{O}(M^2/\Lambda^2) \) correction. Besides, combine the optical theorem with the partial wave expansion \( A(s, t) = 16\pi \sqrt{s/(s - 4m^2)} \sum_l (2l + 1) P_l(\cos \theta) A_l(s) \), one obtains another bound for \( s \geq 4m^2 \) [45] (for recent exploitation of this bound, see e.g.[27])

\[
\frac{\partial^n}{\partial \epsilon^n} \text{Im}[A(s + i\epsilon, 0)] \Bigr|_{\epsilon = 0} \geq 0.
\]  

(5)

Apply this bound to Eq.(1) and Eq.(3), one reaches the positivity of \( \partial_t^n A''(s = M^2, t) \Bigr|_{t = 0} \) with \( \mathcal{O}(M^2/\Lambda^2) \) correction.

### III. Positivity Bounds in Cosmology

We will apply the positivity argument to the EFT of cosmological perturbations. We want to focus on the Goldstone EFT, so we will work in the decoupling limit [1]. Such limit, however, becomes subtle in a \( c_T \neq 1 \) theory [46]. It is also noticed that the EFTs
of modified gravity at cosmological scales have been strictly constrained by GW170817 to $c_T = 1$ \cite{47-49}. Also, as mentioned, the stable nonsingular cosmological models can be implemented only in theories beyond Horndeski. Thus we are well-motivated to consider a $c_T = 1$ beyond-Horndeski EFT as an illustrative example.

The shift-symmetric $c_T = 1$ beyond-Horndeski theory can be written as \cite{9,48}

$$L = M_p^2 \Lambda^2 \left[ B(X) \frac{R}{\Lambda^2} + G_2(X) + G_3(X) \frac{\Box \phi}{M_p \Lambda^2} - \frac{4}{X} B_X \phi^\mu \phi^\nu \phi_{\mu \nu} - \frac{\phi^\mu \phi_{\mu \nu} \phi_{\lambda \nu}}{M_p^4 \Lambda^6} \right],$$

where $M_p$ is the reduced Planck mass and $\Lambda$ is the EFT cutoff. All coefficients $B(X)$, $G_{2,3}(X)$ and fields are dedimensionalised by $\phi \rightarrow \phi / M_p$, $\partial \rightarrow \partial / \Lambda$ and $X \equiv \phi^\mu \phi_{\mu} / (M_p^2 \Lambda^2)$. Subscript $X$ denotes partial derivatives with respect to $X$, for instance $B_X \equiv \frac{\partial}{\partial X} B$. The positivity argument in section-II only holds for a massive scalar field. In (6), we can add a small but nonvanishing $\phi$-dependent potential, which provides a small effective Goldstone mass (well outside the regime of validity of the EFT), and work in the vanishing-mass limit.

In the unitary gauge ($\delta \phi = 0$), the Lagrangian (6) is equivalent to \cite{48}

$$L = M_p^2 \Lambda^2 \left[ G_2(X) + Q(X) \frac{K}{\Lambda^2} + B(X) \frac{R^{(3)} + K^\mu_{\nu} K^\nu_{\mu} - K^2}{\Lambda^2} \right]$$

where $K^\nu_{\mu}$ and $R^{(3)}$ are the extrinsic curvature tensor and Ricci scalar of the spacelike uniform-$\phi$ hypersurface, respectively, and $Q(X) \equiv - \int \sqrt{-X} G_{3X}(X) dX$.

A. The effective Goldstone Lagrangian

An arbitrary time slicing $(t, \bm{x})$ is related to the unitary gauge time $(\tilde{t}, \tilde{\bm{x}})$ by $\tilde{t} = t + \pi(t, \bm{x})$ and $\bm{x} = \tilde{\bm{x}}$, then

$$g^{00}(\tilde{t}, \tilde{\bm{x}}) = \frac{\partial \tilde{t}}{\partial x^\mu} \frac{\partial \tilde{t}}{\partial x^\nu} g^{\mu\nu}(t, \bm{x}) = (1 + \dot{\pi})^2 g^{00} + 2(1 + \dot{\pi}) \partial_i \pi g^{0i} + g^{ij} \partial_i \pi \partial_j \pi,$$

where $\pi$ is a Goldstone field. Quantities in the unitary gauge are labeled with tildes. Due to the derivative coupling, the scattering process $\pi \pi \rightarrow \pi \pi$ is dominated by sub-Hubble contribution, where the Goldstone mode decouples from gravity \cite{1}. We are thus allowed to choose the standard FRW metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j$$

5
as the metric in the new coordinate system \((t, \mathbf{x})\). Relevant Stuckelberg tricks are given in Appendix-A.

Conventional EFTs (as well as the underlying UV theories) are Poincare invariant. The EFT of cosmological perturbations, on the other hand, are obtained by breaking the time-diffeomorphism invariance with a gauge choice (unitary gauge). When the time-translation symmetry is broken because of the evolving background, the notion of Mandelstam variables becomes ill-defined. However, since the Goldstone theory only contains derivative interactions, the scattering process are dominated by contribution near the cutoff scale \(\Lambda\). Symmetry breaking correction from the cosmological background is expected to be small. We assume that the cosmological evolution only manifests as \(\mathcal{O}(H/\Lambda, \dot{H}/\Lambda^2)\) correction to the coefficients and does NOT affect perturbative calculation of the scattering amplitudes.

It is convenient to define the dimensionless parameters

\[
\epsilon \equiv \frac{\dot{\phi}^2}{2 M_p^2 H^2}, \quad \epsilon_H \equiv -\frac{\dot{H}}{H^2},
\]

where \(\epsilon_H\) describes evolution of the universe. We will assume \(|\ddot{\phi}/(H\dot{\phi})| \ll 1\), so that we can safely neglect \(\ddot{\phi}\). Here, \(\epsilon < 1\) is not necessary\(^1\). In a \(\phi\)-dominating universe, \(\epsilon\) is related to \(\epsilon_H\) by the Friedman equation and the equation of motion of \(\phi\), in particular, \(\epsilon = \epsilon_H\) if \(\phi\) is canonical. We thus have the time derivative of a function \(f\) as

\[
\frac{df}{dt} = \left( -\epsilon_H H^2 \frac{\partial}{\partial H} + \epsilon_H^{(1)} \frac{\partial}{\partial \epsilon_H} \right) f,
\]

where \(\dot{\epsilon}_H \equiv H \epsilon_H^{(1)} \simeq 2H\epsilon_H^2\) if \(\phi\) dominates.

At the tree level, it is sufficient to consider \(S^{(2)}\), \(S^{(3)}\) and \(S^{(4)}\). To quadratic order, after some integration by parts, one has

\[
S^{(2)} = (M_p \Lambda)^2 \int dx^4 a^3 \left[ U \dot{\pi}^2 - V \left( \frac{\partial_i \pi}{a^2} \right)^2 \right],
\]

where

\[
U = \epsilon \left( \frac{H}{\Lambda} \right)^2 \left[ -2G_{2X} + (12B_X + 12\sqrt{2}\epsilon^{1/2}G_{3X} + 8\epsilon G_{2XX}) \left( \frac{H}{\Lambda} \right)^2 + \ldots \right],
\]

\(^1\) Since \(|\ddot{\phi}/(H\dot{\phi})| \ll 1\) here, the scalar field rolls at nearly constant speed. In one Hubble time, \(\Delta\phi \sim \sqrt{2\epsilon} M_p\). The swampland conjecture \([21, 50]\) implies \(\epsilon \lesssim \mathcal{O}(1)\).
\[ V = \epsilon \left( \frac{H}{\Lambda} \right)^2 \left[ -2G_{2X} + (4(7 - 4\epsilon H)B_X + 8\sqrt{2}\epsilon^{1/2}G_{3X}) \left( \frac{H}{\Lambda} \right)^2 + \ldots \right]. \]  

(13b)

Here, the ellipsis refers to the parts proportional to higher powers of \( H/\Lambda \) and the coefficients are evaluated on the background \( \bar{X} = -\dot{\phi}^2/(M_p^2 \Lambda^2) = 2(H/\Lambda)^2\epsilon \). Here and in the rest of this paper, the results are presented using \( \epsilon \) and \( \epsilon_H \) instead of \( \dot{\phi}^2 \) and \( \ddot{H} \). Truncation to leading order in \( H/\Lambda \) is valid if the correction is \( \lesssim \mathcal{O}(1) \). For unity \( B_X, \epsilon G_{2XX}, \epsilon^{1/2}G_{3X} \), it is sufficient to have \( H \lesssim 0.1 \Lambda \).

The sound speed squared is

\[ c_s^2 \equiv V/U = 1 + \frac{2}{G_{2X}} \left( 4(-1 + \epsilon_H)B_X + \sqrt{2}\epsilon^{1/2}G_{3X} + 2\epsilon G_{2X} \right) \left( \frac{H}{\Lambda} \right)^2 + \ldots, \]  

(14)

which is approximately constant since \( \frac{d}{dt}(\epsilon^2/\Lambda) \sim \epsilon_H H^3/\Lambda^3 \) is of higher order in \( H/\Lambda \).

Rescale the spacial coordinates \( x \to c_s x \) and define the canonically normalised field \( \pi_c = \sqrt{2c^3 U} M_p \Lambda \pi \), we have

\[ S^{(2)} = \int dx^4 a^3 \left[ \frac{1}{2} (\partial \pi_c)^2 \right], \]  

(15)

where \( (\partial \pi_c)^2 = \dot{\pi}^2 - (\partial_i \pi_c)^2 \).

The final Lagrangian for the UV scattering is (the subscript \( c \) is omitted)

\[
L = \frac{1}{2} (\partial \pi)^2 + \frac{1}{M_p \Lambda} \left[ \alpha_1 \dot{\pi}^3 + \alpha_2 \dot{\pi}(\partial \pi)^2 \right] + \frac{1}{M_p^2 \Lambda^2} \left[ \beta_1 \dot{\pi}^4 + \beta_2 \ddot{\pi}^2(\partial \pi)^2 + \beta_3 [(\partial \pi)^2]^2 \right]
+ \frac{\beta_4}{M_p^2 \Lambda^4} \left[ \dot{\pi} \partial_i \partial_j \pi \partial_l \dot{\pi} \partial_j \pi - \ddot{\pi} \partial_i \dot{\pi} \partial_l \pi \nabla^2 \pi \right],
\]  

(16)

see Appendix-B for the details of derivation. The explicit expressions of \( \alpha, \beta \) in terms of \( G_2(X), G_3(X) \) and \( B(X) \) in (6) are given in Appendix-C. The coefficients of the final Lagrangian (16), as expected, do not contain \( B(X) \) or \( Q(X) \), since in the covariant Lagrangian (6) the Ricci curvature \( R \) is invariant and only the \( X \) dependent part of \( G_3(X) \) is relevant, though they appear in coefficients before individual vertices (for example, eq.(B12) contains \( B(X) \)). This can serve as a quick consistency check of our result.

**B. Positivity bounds**

The tree level 2 \( \to 2 \) amplitude corresponding to (16) in the center of mass frame is

\[ A(s, t) = \left( -\frac{9}{4} \alpha_1^2 - 6\alpha_1 \alpha_2 - 4\alpha_2^2 + \frac{3}{2} \beta_1 + 2\beta_2 \right) \frac{s^2}{M_p^2 \Lambda^2} + 2\beta_3 \frac{s^2 + t^2 + u^2}{M_p^2 \Lambda^2} + \frac{1}{2} \beta_4 \frac{s t u}{M_p^2 \Lambda^2}. \]  

(17)
Detailed calculation of (17) but with $\beta_4 = 0$ has been presented in Ref.[41]. The Lagrangian (16) is actually non-relativistic, in that uncontracted time derivatives are present. However, if it indeed captures the physics of some UV-complete theory below the high energy cutoff $\Lambda$ and above the decoupling scale $E_{\text{mix}} \lesssim \epsilon_H^{1/2} H$ [1], positivity derived from locality and unitarity should be inherited. To the leading order in $H/\Lambda$, the positivity bound (4) reads (without loss of generality we set $G_{2X} = -1/2$ hereafter)

\[
(1 + \epsilon_H) B_X + \epsilon G_{2XX} \\
+ \left[ 4B_X^2(-61 - 34\epsilon_H + 34\epsilon_H^2 - 2\epsilon_H^{(1)}) + 4\epsilon(1 + 33\epsilon_H)B_XG_{2XX} \\
+ 2\sqrt{2}\epsilon^{1/2}(-23 + 5\epsilon_H)B_XG_{3X} + \epsilon(15 + \epsilon_H)B_{XX} + 4\epsilon^2(2G_{2XX}^2 - G_{2XX}) \\
- 2\epsilon G_{3X}^2 + \sqrt{2}\epsilon^{3/2}(4G_{2XX}G_{3X} - 3G_{3XX}) \right] \left( \frac{H}{\Lambda} \right)^2 \geq 0. \tag{18}
\]

Applying bound (5) yields ($n = 1$)

\[
-B_X + 4\left[ \epsilon B_{XX} + (34 - 28\epsilon_H)B_X^2 - 10\epsilon B_XG_{2XX} - \sqrt{2}\epsilon^{1/2}B_XG_{3X} \right] \left( \frac{H}{\Lambda} \right)^2 \geq 0. \tag{19}
\]

Recall in section-II, we commented that both bounds have $O(M^2/\Lambda^2)$ uncertainty with $M$ being the energy scale at which the bounds are evaluated. The EFT considered here has both UV cutoff $\Lambda$ and IR cutoff $E_{\text{mix}}$. Usually, the Hubble scale satisfies $\Lambda > H > E_{\text{mix}}$. If one evaluates the bounds at $M^2 \simeq H^2$, the uncertainty $O(M^2/\Lambda^2)$ of the bound itself is comparable to the background correction, since $M^2/\Lambda^2 \simeq H^2/\Lambda^2$ is of the same order as the leading order cosmological correction. Another interesting case is when $\epsilon_H \ll 1$, so we can have $M^2 \simeq E_{\text{mix}}^2 \ll H^2$. In such scenarios, the cosmological correction is much larger than the uncertainty $O(M^2/\Lambda^2)$, so one is allowed to consider the full corrected bounds.

In the calculation, we did not assume the correction is even powers of $H/\Lambda$. But it turns out the leading order correction is indeed $H^2/\Lambda^2$, as expected in the introduction section, so the bounds do not distinguish between contraction and expansion. As commented before, the cosmological evolution breaks time-translation symmetry of the EFT, so it is not surprising that the bound in Minkowski spacetime (or the limit $H/\Lambda \to 0$) may be violated for a relatively large $H^2/\Lambda^2$, even if $H^2/\Lambda^2 < 1$.

### C. Discussion

**In the limit $H/\Lambda \to 0$**
We can ignore the $H^2/\Lambda^2$ correction. Without the corrections, the bounds (18) and (19) are

$$(1 + \epsilon_H)B_X + \epsilon G_{2XX} \geq 0, \quad B_X \leq 0.$$  

(20)

For $1 + \epsilon_H > 0$, we have the positivity constraint on $B_X$ as

$$\epsilon G_{2XX} \geq -(1 + \epsilon_H)B_X \geq 0.$$  

(21)

In comparison, the Minkowski bound only gives $G_{2XX} \geq 0$ (see Appendix-D). Ref.[40] reports $G_{4X} \leq 0$ for a shift-symmetric Horndeski Lagrangian with only $G_2(X)$ and $G_4(X)$. The covariant theory ($c_T = 1$) studied here is essentially different from that in [40] ($c_T \neq 1$), in which the bound $G_{4X} \leq 0$ is actually equivalent to the subluminal condition $c_T \leq 1$. In fact, the $G_{4X} \leq 0$ bound in [40] is derived from $Y^{(2,1)} \geq 0$ in [27]. The second bound in Eq.(20) has similar origin to $Y^{(2,1)} \geq 0$ and it also implies $B_X \leq 0$. However, as $B_X$ also contributes in the tree level $\pi\pi \rightarrow \pi\pi$ scattering, we have an additional bound in Eq.(20) which further constrains $B_X$. As the last comment, one must be careful when applying the results of Ref.[27] where the bounds are evaluated at $s \sim \mathcal{O}(m^2)$, which is much smaller than the low energy cutoff $E_{mix}$ of the EFT considered here.

In addition, the subluminality of $c_s^2$ brings another bound

$$\sqrt{2} \epsilon^{1/2} G_{3X} \geq 4(1 - \epsilon_H)B_X - 2\epsilon G_{2XX}.$$  

(22)

This relation is automatically satisfied if $G_{3X} \geq 0$ and $\epsilon_H < 1$. However, if $G_{3X} = 0$ and $\epsilon_H > 1$, it requires $\epsilon G_{2XX} \geq -2(\epsilon_H - 1)B_X$, which is stronger than (21) for $\epsilon_H > 3$.

**With $H^2/\Lambda^2$ correction**

An interesting example is the slow-roll inflation, where $0 < \epsilon \simeq \epsilon_H \ll 1$, so the bounds (18) and (19) with $H^2/\Lambda^2$ correction can be simplified. To see this, setting $\epsilon = \epsilon_H = 0$ in (18) and (19), we have

$$B_X - 244B_X^2 \left( \frac{H}{\Lambda} \right)^2 \geq 0, \quad -B_X + 136B_X^2 \left( \frac{H}{\Lambda} \right)^2 \geq 0.$$  

(23)

This suggests $B_X = 0$ unless we consider the slow-roll suppressed parts. This implies that for the potential-driving dS inflation, the $c_T = 1$ beyond-Horndeski EFT (6) reduces to GR. However, for the $\dot{\phi}$-driving inflation, such as k-inflation [51, 52] and G-inflation [53], since $\epsilon \ll 1$ may be violated, the constraint on $B_X$ will be released.
FIG. 2: The shaded region plots parameter space where $G_{2XX} \geq 0$ and $c_s^2 \leq 1$ are satisfied. The $H^2/\Lambda^2$ correction in (24) is positive in the darker triangle.

Inspired by the previous example, we look at the GR limit, in which $\phi$ is not coupled to the Ricci scalar ($B \equiv 1/2$). The corresponding EFT is called Galileon [24]. Assuming $G_{2XXX} = G_{3XX} = 0$ for simplicity, we have

$$
\epsilon G_{2XX} + \left[8(\epsilon G_{2XX})^2 + 4(\epsilon G_{2XX})(\sqrt{2} \epsilon^{1/2} G_{3X}) - (\sqrt{2} \epsilon^{1/2} G_{3X})^2\right] \left(\frac{H}{\Lambda}\right)^2 \geq 0. \tag{24}
$$

Combining it with $c_s^2 \leq 1$, we plot the constrained parameter space for $\epsilon G_{2XX}, \sqrt{2} \epsilon^{1/2} G_{3X}$ in Fig.2. It is seen that the positivity bound at $H/\Lambda \to 0$ may be violated (the region shaded with light gray) when the cosmological background $H^2/\Lambda^2$ becomes important.

IV. CONCLUSION

We have investigated the application of positivity arguments in the EFT of cosmological perturbations. As an illustrative example, we considered a $c_T = 1$ beyond-Horndeski EFT (6). We explicitly showed by calculation that the leading cosmological correction to positivity bounds indeed comes at $H^2/\Lambda^2$ and $\dot{H}/\Lambda^2$ order, consistent with our observation in the
It is also observed that the positivity bounds found in the limit $H/\Lambda \to 0$ (or in Minkowski spacetime) might be violated when $H$ is not far smaller than the cutoff scale $\Lambda$, since the coefficient of $H^2/\Lambda^2$ correction is at $10^2$ order. Depending on the sign of coefficient, the bound with cosmological corrections is either weaker or stronger. We also discussed the applications of our bound. It is found that positivity favors a suppressed $B_X$ (comparable in size with $\epsilon G_{2XX}$ or $\sqrt{2} \epsilon^{1/2} G_{3X}$) for slow-roll inflation.

Lagrangian (6) can be used to implement fully stable cosmological bounce. Nonpathological bouncing models built in Ref.[15, 17–20] all have $G_{2XX}/G_{2X} < 0$ somewhere, which seems to be inconsistent with the bound (20) at first sight. There is no tension for now. Typically, to violate the null energy condition (NEC, see [54] for a review), one requires that the operator $X^2$ is not negligible (depending on the value of $\phi$) at the NEC-violating regime, while the beyond-Horndeski operator takes effect as a stabiliser that controls gradient stability. Usually, such models display obvious $\phi$-dependence and nonnegligible $\ddot{\phi}$ around the bounce point, which invalidates the assumptions we used to derive the bounds here. Thus our bounds cannot be directly applied to the bouncing models. It is possible to relax these assumptions in more complete study. We might come back to relevant issues in future works.

It is also interesting to integrate out the IR part of the RHS of (4) within the regime of validity of the EFT to give a more precise bound [36]. Recently, the positivity bounds with heavy spinning intermediate states have been studied in inflation but from a covariant point of view [55], and also Ref.[56] has explored positivity in the Higgs-Dilaton inflation model. It is also well-motivated to go beyond the decoupling limit and study the EFT with graviton and high-spin particles included [57, 58].

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Appendix A: The Stuekelberg trick

In the unitary guage

$$X \equiv \partial_\mu \phi \partial^\mu \phi = \dot{\phi}^2 g^{00},$$

(A1)
where $\tilde{g}^{00}$ is given in Eq. (8). To expand Lagrangian (7), we still need the expression of the extrinsic tensor. The normal of the uniform-$\phi$ hypersurface is

$$n_\mu = \frac{\partial_\mu \tilde{t}}{\sqrt{-\partial_\mu \tilde{t} \tilde{g}^{\mu\nu} \partial_\nu \tilde{t}}} = \delta_\mu^0 + \partial_\mu \pi \sqrt{-\tilde{g}^{00}}. \tag{A2}$$

Recall that we are allowed to raise and lower indices with the unperturbed FRW metric in the decoupling limit

$$n_\mu = \frac{-\delta_\mu^0 + \partial_\mu \pi \sqrt{-\tilde{g}^{00}}}{\tilde{g}^{00}}. \tag{A3}$$

The extrinsic curvature $K$ is

$$K = -\nabla_\mu n^\mu, \quad K^\mu_\nu K^\nu_\mu = \nabla_\mu n^\nu \nabla_\nu n^\mu, \tag{A4}$$

with the relevant expressions as follows

$$\nabla_0 n^0 = \partial_0 \left( \frac{-1 - \dot{\pi}}{\sqrt{-\tilde{g}^{00}}} \right), \tag{A5a}$$

$$\nabla_0 n^i = \partial_0 \left( \frac{\partial_i \pi / a^2}{\sqrt{-\tilde{g}^{00}}} \right) + \frac{H \partial_i \pi / a^2}{\sqrt{-\tilde{g}^{00}}}, \tag{A5b}$$

$$\nabla_i n^0 = \partial_i \left( \frac{-1 - \dot{\pi}}{\sqrt{-\tilde{g}^{00}}} \right) + \frac{H \partial_i \pi}{\sqrt{-\tilde{g}^{00}}}, \tag{A5c}$$

$$\nabla_i n^j = \partial_i \left( \frac{\partial_j \pi / a^2}{\sqrt{-\tilde{g}^{00}}} \right) - \frac{(1 + \dot{\pi}) H \delta^i_j}{\sqrt{-\tilde{g}^{00}}}. \tag{A5d}$$

According to the Gauss-Codazzi formula, the 3d Ricci scalar $R^{(3)}$ is

$$R^{(3)} = R + 2 R_\mu^\nu n^\mu n^\nu - K^2 + K^\mu_\nu K^\nu_\mu. \tag{A6}$$

On the RHS, the Ricci scalar $R$ is invariant and $R_\mu^\nu$ can be easily calculated using the spatially flat FRW metric

$$R_{00} = -3 \frac{\ddot{a}}{a}, \quad R_{ij} = (a \ddot{a} + 2 \dot{a}^2) \delta_{ij}. \tag{A7}$$

Now we are well-equipped to expand Lagrangian (7) in powers of $\pi$ and its derivatives.
Appendix B: Derivation of Goldstone Lagrangian (16)

The equation of motion (EoM) of the free Goldstone field is

$$\ddot{\pi} + 3H\dot{\pi} - c_s^2\nabla^2\pi = 0.$$  

(B1)

Each $\pi$ field can have at most two derivatives. In fact, only $K_{\mu\nu}$ contributes second derivatives of $\pi$. Thus the $n$-th order Lagrangian $L^{(n)}$ contains at most $(n + 2)$ derivatives. We first consider the part of $L^{(3)}$ with up to four derivatives

$$g_1\dot{\pi}^3 + g_2\frac{(\partial_\mu\pi)^2}{a^2} + g_3\dot{\pi}\partial_\mu\partial_\nu\pi + g_4\dot{\pi}\frac{(\partial_\mu\pi)^2}{a^2} + g_5\dot{\pi}^2 \nabla^2\pi + g_6\dot{\pi}^2,$$  

(B2)

which includes all possible three point interactions with at most four derivatives that may yield nonvanishing amplitudes in the center of mass (CM) frame. Note that in the CM frame, vertices with no time-derivatives (e.g: $\partial_i\partial_j\pi\partial_i\pi\partial_j\pi$) do not contribute in exchange diagrams since the exchanged virtue particle has vanishing 3-momentum. After some integration by parts, we have

$$S^{(3)} = M_p^2\Lambda^2 \int dx^4 a^3 \left\{ g_1\dot{\pi}^3 + (g_2 - D_1g_4)\frac{\dot{\pi}(\partial_\mu\pi)^2}{a^2} + \dot{\pi}^2 \left[ g_6\dot{\pi} - \frac{(g_3/2 - g_4 - g_5)}{a^2} \nabla^2\pi \right] \right\},$$  

(B3)

where $D_n \equiv nH + d/dt$ is defined, and $g_n$, $n = 1, 2, \ldots, 6$ are only dependent on time. Insert the EoM (B1) and switch to the rescaled coordinates and normalised field, we get

$$S^{(3)} = M_p^2\Lambda^2 \int dx^4 \sqrt{-g} \left\{ \left[ g_1 + \frac{g_2}{c_s^2} - \frac{D_1g_4}{c_s^2} - \frac{1}{3}D_3g_6 + (3H - D_3/3)\frac{g_3}{c_s^2} - \frac{g_4 + g_5}{c_s^2} \right] \ddot{\pi}_c^3 + \left( -\frac{g_2}{c_s^2} + \frac{D_1g_4}{c_s^2} \right) \dot{\pi}_c \frac{(\partial_\mu\pi)^2}{a^2} \right\} \frac{1}{(\sqrt{2c_s^3U} M_p\Lambda)^3} \equiv \int dx^4 \sqrt{-g} \left[ \alpha_1\ddot{\pi}_c^3 + \alpha_2\dot{\pi}_c \frac{(\partial_\mu\pi)^2}{a^2} \right] \frac{1}{M_p\Lambda}. $$  

(B4)

Now consider the part with higher-order derivatives

$$\frac{1}{a^4} \left[ g_7 \dot{\pi}(\nabla^2\pi)^2 - \dot{\pi}(\partial_\mu\partial_\nu\pi)^2 \right] + g_8 \dot{\pi}(\partial_\mu\partial_\nu\pi\nabla^2\pi - \partial_\mu\partial_\nu\pi\dot{\pi}\partial_\mu\partial_\nu\pi) \right]\ .$$  

(B5)

After some integration by parts, we find that the higher derivatives cancel out and

$$-\frac{1}{a^4}(-H + \frac{d}{dt})\frac{g_8}{2} \nabla^2\pi(\partial_\mu\pi)^2$$  

(B6)

remains, which can be safely neglected in the CM frame. The calculation of $S^{(4)}$ is similar but more involved. In particular, higher-order derivative operators may contribute in contact
diagrams. We first consider operators with at most five derivatives and then look at the higher derivative part. The part of $S^{(4)}$ with at most five derivatives is

\[
\begin{align*}
&h_1 \dot{\pi}^4 + h_2 \dot{\pi}^2 \left( \frac{\partial_i \pi}{a^2} \right)^2 + h_3 \left( \frac{\partial_i \pi}{a^4} \right)^4 + h_4 \frac{\partial_i \pi \partial_j \pi (\partial_i \pi)^2}{a^4} + h_5 \frac{\partial_i \pi \partial_j \pi \partial_k \pi}{a^4} \\
&+ h_6 \frac{\nabla^2 \pi (\partial_i \pi)^2}{a^4} + h_7 \frac{\nabla^2 \pi (\partial_i \pi)^2}{a^4} + h_8 \frac{\nabla^2 \pi (\partial_i \pi)^2}{a^4} + h_9 \frac{\nabla^2 \pi (\partial_i \pi)^2}{a^4} + h_{10} \nabla^2 \pi (\dot{\pi})^3
\end{align*}
\]

\[
\rightarrow \left[ h_1 + (3H - D_3/4) \left( \frac{h_9}{c_s^2} - \frac{h_8}{8c_s^2} + \frac{h_7}{6c_s^2} + \frac{h_6}{3c_s^2} + \frac{h_5}{6c_s^2} - \frac{D_3}{4h_{10}} \right) \dot{\pi}^4 \\
+ \left[ h_2 + (D_1/4 - 3H/2) \left( \frac{h_5}{c_s^2} - \frac{2h_6}{c_s^2} - \frac{D_1}{2c_s^2} \right) \dot{\pi}^4 \right] \frac{(\partial_i \pi)^4}{a^4} \\
+ \left[ h_3 - \frac{D_1}{8}(2h_4 - h_5) \right] \frac{(\partial_i \pi)^4}{a^4} \equiv A \dot{\pi}^4 + B \dot{\pi}^2 (\partial_i \pi)^2 + C (\partial_i \pi)^4
\end{align*}
\]

\[
\equiv \frac{1}{M_p^4 \Lambda^4} \left[ \beta_1 \dot{\pi}_c^4 + \beta_2 \dot{\pi}_c^2 (\partial_i \pi)^2 + \beta_3 (\partial_i \pi)^4 \right].
\]

The highest derivative part of $S^{(4)}$ divides into two sectors. The first sector consists solely of spacial derivatives

\[
\sim h_{11} \left[ (\partial_i \pi)^2 (\partial_i \partial_j \pi)^2 - (\partial_i \pi)^2 (\nabla^2 \pi)^2 \right] + h_{12} (\partial_i \pi \partial_j \pi \partial_j \pi \partial_k \pi \partial_k \pi - \partial_i \pi \partial_j \pi \partial_k \pi \partial_k \pi \nabla^2 \pi).
\]

\[(B8)\]

It is easy to check that both the $h_{11}$ and $h_{12}$ vertices cancel out in the CM frame. Vertices in the other sector are

\[
\begin{align*}
&h_{13} (\dot{\pi}^2 (\partial_i \partial_j \pi)^2 - \dot{\pi}^2 (\nabla^2 \pi)^2) + h_{14} (\dot{\pi} \partial_i \partial_j \pi \partial_j \pi - \dot{\pi} \partial_i \partial_j \pi \partial_j \pi) + h_{15} ((\partial_i \pi)^2 (\partial_j \pi)^2 - \dot{\pi} (\partial_i \pi)^2 \nabla^2 \pi) + h_{16} ((\partial_i \pi)^2 (\partial_j \pi)^2 - \dot{\pi} (\partial_i \pi)^2 \nabla^2 \pi) \\
&\rightarrow (h_{14} - 2h_{13}) (\dot{\pi} \partial_i \partial_j \pi \partial_j \pi) - \dot{\pi} \partial_i \partial_j \pi \nabla^2 \pi + h_{15} ((\partial_i \pi)^2 (\partial_j \pi)^2 - \dot{\pi} (\partial_i \pi)^2 \nabla^2 \pi) + h_{16} ((\partial_i \pi)^2 (\partial_j \pi)^2 - \dot{\pi} (\partial_i \pi)^2 \nabla^2 \pi)
\end{align*}
\]

\[(B9)\]

with the coefficients

\[
\begin{align*}
h_{13} &= 6B + 6 \left( \frac{\dot{\phi}}{M_p \Lambda} \right)^2 B_X + 4 \left( \frac{\dot{\phi}}{M_p \Lambda} \right)^4 B_{XX}, \\
h_{14} &= 24B + 16 \left( \frac{\dot{\phi}}{M_p \Lambda} \right)^2 B_X, \quad h_{15} = -h_{16} = 4B.
\end{align*}
\]

\[(B10)\]
Performing integration by part again to the terms proportional to $B(X)$, we have

$$
\left[ 4 \left( \frac{\dot{\phi}}{M_p \Lambda} \right)^2 B_X - 8 \left( \frac{\dot{\phi}}{M_p \Lambda} \right)^4 B_{XX} \right] \left( \dot{\pi} \partial_i \partial_j \pi \partial_i \partial_j \pi - \ddot{\pi} \partial_i \partial_j \pi \right) \\
+ \mathcal{D}_{-1}(6B) \dot{\pi} \nabla^2 \pi \partial_j \pi^2 - \frac{B}{2} \frac{d^2}{dt^2} [ (\partial_i \pi)^4 ].
$$

So in conclusion, the higher derivative operators introduce one new vertex $(\dot{\pi} \partial_i \partial_j \pi \partial_i \partial_j \pi - \ddot{\pi} \partial_i \partial_j \pi \nabla^2 \pi)$ and modify the coefficients

$$
h_3 \rightarrow h_3 - \frac{B}{2} (1 + \epsilon_H) H^2, \quad h_6 \rightarrow h_6 - 6BH.
$$

Putting together all the results in this Appendix, we get Lagrangian (16).

**Appendix C: Coefficients in (16)**

We list here the explicit expressions of the coefficients in (16), calculated to the leading order in $H/\Lambda$,

$$
\alpha_1 = \frac{1}{\epsilon^{1/2}(-G_{2X})^{3/2}} \left( \frac{H}{\Lambda} \right) \left[ (2 + 8\epsilon_H/3)B_X + \sqrt{2}\epsilon^{1/2}G_{3X} \right],
$$

$$
\alpha_2 = \frac{1}{\epsilon^{1/2}(-G_{2X})^{3/2}} \left( \frac{H}{\Lambda} \right) \left[ 2(-1 + \epsilon_H)B_X + \frac{1}{2} \sqrt{2}\epsilon^{1/2}G_{3X} + \epsilon G_{2XX} \right],
$$

$$
\beta_1 = \frac{1}{\epsilon G_{2X}^2} \left\{ \frac{2B_X(-3 + \epsilon_H)}{3} \\
+ \frac{1}{3G_{2X}} \left[ -12B_X^2 \left( 5\epsilon_H^2 - 36\epsilon_H + \epsilon^{(1)}_H + 35 \right) - 14B_X\epsilon(\epsilon_H - 9)G_{2XX} \right. \\
+ 3\sqrt{2}B_X\epsilon^{1/2}(3\epsilon_H + 5)G_{3X} + B_{XX}\epsilon(\epsilon_H - 27)G_{2X} \left( \frac{H}{\Lambda} \right)^2 \right\},
$$

$$
\beta_2 = \frac{1}{\epsilon G_{2X}^2} \left\{ B_X(3 - \epsilon_H) \\
+ \frac{1}{G_{2X}} \left[ 4B_X^2 \left( 10\epsilon_H^2 - 53\epsilon_H + \epsilon^{(1)}_H + 49 \right) + 4B_X\epsilon(4\epsilon_H - 15)G_{2XX} \\
- 2\sqrt{2}B_X\epsilon^{1/2}(\epsilon_H + 2)G_{3X} \right. \\
+ \epsilon \left( 12B_{XX}G_{2X} - \epsilon G_{2X}G_{2XXX} + 2\epsilon G_{2XX}^2 + \sqrt{2}\epsilon^{1/2}G_{2XX}G_{3X} \right) \left( \frac{H}{\Lambda} \right)^2 \right\}.
$$
\[ \beta_3 = \frac{1}{\epsilon G_{2X}} \left\{ \frac{B_X (3\epsilon_H - 5) + \epsilon G_{2XX}}{8} \right. \\
- \frac{1}{8G_{2X}} \left[ 4B_X^2 \left( 42\epsilon_H^2 - 121\epsilon_H + 85 \right) + 4B_X \epsilon (29\epsilon_H - 42)G_{2XX} \right. \\
+ 2\sqrt{2}B_X \epsilon^{1/2}(3\epsilon_H - 5)G_{3X} + 6B_{XX}\epsilon G_{2X} + \sqrt{2}\epsilon^{3/2}(3G_{2X}G_{3XX} + 2G_{2XX}G_{3X}) \right. \\
+ 20\epsilon^2 G_{2XX} \left\} \left( \frac{H}{\Lambda} \right)^2 \right\} (C1e) \\
\beta_4 = \frac{1}{\epsilon G_{2X}} \left[ \frac{B_X}{2} \left( \frac{H}{\Lambda} \right)^2 - \frac{\left( 34 - 28\epsilon_H \right) B_X^2 - B_X \left( 10\epsilon G_{2XX} + \sqrt{2}\epsilon^{1/2}G_{3X} \right) - 2\epsilon B_{XX}G_{2X}}{G_{2X}} \right] (C1f) \]

Appendix D: Positivity in the Minkowski spacetime

To derive positivity bounds in the Minkowski spacetime for (6), one should switch back to \( \dot{\phi}^2 \) and \( \dot{H} \) by inserting (10), and pass to the \( H, \dot{H} \to 0 \) limit. The extrinsic curvature \( K \) vanishes in the Minkowski spacetime. Thus positivity only constrains \( G_{2}(X) \) and its derivatives. The Minkowski bound as well as the sound speed are \( (G_{2X} = -1/2) \)

\[ G_{2XX} + \left( 4G_{2XX}^2 - 2G_{2XXX} \right) \left( \frac{\dot{\phi}}{M_p\Lambda} \right)^2 + \left( 40G_{2XX}^3 - 4G_{2XX}G_{2XXX} + \frac{G_{2XXXX}}{2} \right) \left( \frac{\dot{\phi}}{M_p\Lambda} \right)^4 + \cdots \geq 0, \]  

\[ c_{s}^2 = \frac{1}{1 + 4G_{2XX} \left( \frac{\dot{\phi}}{M_p\Lambda} \right)^2}. \]  

Absence of superluminality places the bound \( G_{2XX} \geq 0 \). An interesting case is that if \( G_{2}(X) \sim X^n \), in which \( X^n \) is the first nonnegligible higher-order derivative operator in the \( G_{2}(X) \) polynomial, the positivity bound (D1) implies \( G_{2XXX} \leq 0 \) if \( G_{2XX} = 0 \) and \( G_{2XXXX} \geq 0 \) if \( G_{2XX} = G_{2XXX} = 0 \), consistent with the result of Ref.[29].

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