Soliton tunneling transistor

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(Dated: October 24, 2018)

Abstract

We report on a macroscopic version of the single-electron transistor (SET), which we call the soliton tunneling transistor (STT). The STT consists of a gate capacitor coupled to a NbSe$_3$ crystal with a charge density wave (CDW). We find that the current-voltage characteristic of an STT is periodically modulated by the gate voltage, as in the SET, except that the measured periodicity corresponds to a macroscopic displacement charge. These results appear to be consistent with time-correlated quantum nucleation of solitons and antisolitons [see Phys. Rev. Lett. 84, 1555 (2000)]. We discuss how the microscopic degrees of freedom within the condensate might enable quantum behavior at high temperatures, and report on preliminary modeling studies using a coupled-phase Hamiltonian to interpret our results.
I. BACKGROUND AND MOTIVATION

A wide class of nonperturbative phenomena in field theory can be understood in terms of quantum tunneling. A well-known example is the quantum decay of the false vacuum, which has been of broad scientific interest in cosmology and other fields for over two decades. In three dimensions, the boundary between the bubble of true vacuum and the surrounding false vacuum is a type of topological defect known as a domain wall. A variety of topological defects in condensed matter systems have been proposed to nucleate via quantum or thermal fluctuations. These include vortex-antivortex pairs and vortex rings in superconductors, superfluids and Bose-Einstein condensates, dislocation pairs in Wigner crystals and vortex lattices, phase-slip vortex rings in charge density waves (CDWs), charge (or flux) soliton-antisoliton pairs in density waves (or Josephson junctions); and soliton-like domain walls surrounding cigar-shaped bubbles of true vacuum in three-dimensional CDWs. The quantum decay of the metastable false vacuum of a scalar field \( \phi \), accompanied by the creation of solitons and antisolitons in (1+1) dimensions (or soliton domain walls in (3+1) dimensions), has been studied extensively in quantum field theory.

A CDW is a condensate in which the electronic charge density in a quasi-1-D metal is modulated, \( \rho(x, t) = \rho_0(x, t) + \rho_1 \cos[2k_F x - \phi(x, t)] \). Here \( \rho_0(x, t) \) contains the background charge of the condensed electrons, and an excess or deficiency of charge proportional to \( \pm \partial \phi / \partial x \). Weakly pinned density waves (DWs) and weakly-coupled Josephson junctions (JJs) can be approximately described by a phase, \( \phi(x, t) \), in a sine-Gordon (s-G) potential. They are dual in that the roles of charge and flux are interchanged, as well as those of current and voltage. The current in a density wave is \( I = (Q_0 / 2\pi) \partial \phi / \partial t \), where \( Q_0 \sim 2N_{ch}e \) and \( N_{ch} \) is the number of parallel chains, whereas the voltage across a JJ is \( V = (\Phi_0 / 2\pi) \partial \phi / \partial t \), where \( \Phi_0 = h/2e \). Charge (flux) solitons in a density wave (JJ) carry a charge (flux) of \( \pm Q_0 (\pm \Phi_0) \) (see Table I). The width of a Josephson vortex is roughly the Josephson penetration length, \( \lambda_J \propto J_c^{-1/2} \) whereas, in a DW, the soliton width is \( \lambda_0 = c_0 / \omega_0 \), where \( c_0 \) is the phason velocity and \( \omega_0 \) is the pinning frequency. Thus, \( \lambda_0 \) will increase with decreasing impurity concentration (as \( \omega_0 \) decreases), and may approach the distance between contacts in extremely pure samples. This is equivalent to approaching the short junction limit, \( L < \lambda_J \), in a JJ, where the \( V-I \) curves become significantly less rounded.

Krive and Rozhavsky (KR) point out the existence of a Coulomb blockade threshold,
due to the electrostatic energy of the charged solitons and antisolitons, for the creation of soliton-antisoliton ($S - S'$) pairs in density waves. A more recent paper\cite{25} proposes an analogy to time-correlated single electron tunneling to explain the observed lack of DW polarization below threshold, and to interpret key features of DW dynamics, such as coherent oscillations and narrow-band noise, above threshold. Observations of Aharonov-Bohm (A-B) oscillations\cite{25} in the magneto-conductance of CDWs in NbSe$_3$ crystals with columnar defects provide compelling evidence for quantum transport. Additional evidence for quantum behavior is found in rf experiments\cite{27,28,29} that show good agreement with photon-assisted tunneling theory, and the observed Zener-like $I$-$V$ characteristics.\cite{30} This Zener-like behavior is expected in the quantum picture when $L \gg \lambda_0$ and the $I$-$V$ curves are rounded. The linear $I$-$V$ curves seen in the extremely pure NbSe$_3$ samples suggest a possible Coulomb blockade mechanism, the dual of which is also suggested by the linear $V$-$I$ curves seen in cuprate JJs in the short junction limit ($L < \lambda_f$).

Evidence against classical depinning includes bias-independent rf and microwave responses below threshold in CDWs\cite{27,31} and Wigner crystals\cite{32} and the observed small phase displacements of charge\cite{33,34} and spin\cite{35} density waves below threshold. These experiments strongly suggest that, as the electric field is increased, $S - S'$ or dislocation pairs are created long before the classical depinning field is attained.\cite{36} Moreover, attempts\cite{37} using a scanning tunneling microscope (STM) to directly observe either displacement of the CDW below threshold or sliding above threshold have been unsuccessful. The apparent lack of sliding seen in STM experiments, the jerky dynamics revealed by NMR experiments\cite{33} and the mode locking observed at high drift velocities\cite{38} all suggest that the CDW spends most of its time in the pinned state even above threshold.

The quantum interpretation of the threshold field, as a pair-creation threshold due to Coulomb blockade, is motivated by Coleman’s paper\cite{39} on soliton pair-creation in the massive Schwinger model. A pair of $S$ and $S'$ domain walls with charges $\pm Q_0$ produce an internal field of magnitude $E^* = Q_0/\varepsilon A$, as shown in Fig. 1, where $A$ is the cross-sectional area. When a field $E$ is applied, the difference in electrostatic energies of a state with a pair of separation $l$ and of the “vacuum” is $\Delta U = \frac{1}{2} \varepsilon Al[(E \pm E^*)^2 - E^2] = Q_0[\frac{1}{2}E^* \pm E]$, which is positive when $|E| < \frac{1}{2} E^*$. Conservation of energy thus forbids pair production for fields less than a quantum threshold, $E_T \equiv \frac{1}{2} E^* = Q_0/2\varepsilon A$, which appears to be about two orders of magnitude smaller than the classical depinning field in NbSe$_3$.\cite{25} The observed universality
relation, \( \varepsilon E_T \sim \varepsilon N_{ch}/A \) thus arises quite elegantly in this model.\[^{12}\]

A density wave between two contacts behaves as a capacitor with an enormous dielectric constant (Fig. 1). The initial charging energy is \( Q^2/2C \), where \( Q \) is the displacement charge and \( C = \varepsilon A/L \). We define \( \theta \equiv 2\pi Q/Q_0 = 2\pi E/E^* = \pi E/E_T \) and note that a displacement \( \phi \) near the middle creates a non-topological kink-antikink pair with charges \( \pm(\phi/2\pi)Q_0 \), if \( \phi = 0 \) at the contacts. The washboard pinning and quadratic charging energies can then be written as:\[^{12}\]

\[
U[\phi] = \int_0^L dx \left\{ u_p [1 - \cos \phi(x)] + u_c [\theta - \phi(x)]^2 \right\}
\]  

(1)

where the first and second terms represent the pinning and electrostatic charging energies, respectively, and where \( u_p \gg u_c \) for NbSe\(_3\).\[^{12}\] If the system starts out in its ground state, conservation of energy will prevent tunneling when the applied field is below threshold, \( \theta < \pi \) \( (E < E_T) \), as illustrated in Fig. 2. However, when \( \theta \) exceeds \( \pi \) what was formerly the true vacuum becomes the unstable false vacuum. A bubble of true vacuum, with soliton domain walls at its surface, then nucleates and expands rapidly (Fig. 1).\[^{14}\] After \( n \) solitons of charge \( Q_0 \) (and antisolitons of charge \(-Q_0\)) have reached the contacts, the charging energy becomes

\[
\frac{(Q - nQ_0^2)}{2C} = \frac{Q_0^2}{8\pi^2 C} (\theta - 2\pi n)^2
\]

(2)

This series of piecewise parabolas is similar to the charging energy of a single-electron tunnel junction, except that \( Q_0 \) now represents a macroscopic charge.

A single-electron transistor (SET)\[^{12}\] consists of a gate capacitor \( C_g \) coupled to an island electrode between two small capacitance tunnel junctions in series. The gate voltage modulates the \( I-V \) curves between the source and drain electrodes, with a period \( e \) in displacement charge, \( Q_g = C_g V_g \). The displacement charges \( Q_{1,2} \) across the two tunnel junctions are related as \( Q_2 = Q_1 + Q_g + q_0 \), where \( q_0 \) is a phenomenological offset charge induced during cooling.\[^{13}\] The SET is related by charge-flux duality to the dc SQUID. The critical voltage across an SET is a periodic function of \( Q_g \), whereas the critical current across a SQUID is periodically modulated (with period \( \Phi_0 \)) by the flux \( \Phi \).
The model discussed above suggests that it may be possible to demonstrate a macroscopic version of the SET, by attaching a gate capacitor to an island electrode near the center of a quasi-1-D crystal with a density wave. The displacement charge induced by the gate electrode would then periodically modulate the total critical voltage between the source and drain electrodes. Ideally, in the absence of any shunt conductance, the periodicity of the gate displacement charge might be expected to be $\sim Q_0$. However, screening by the normal, uncondensed electrons will tend to reduce the effectiveness of the gate which, unlike the source and drain contacts, cannot be driven by a current source. The displacement charges across the two segments of the crystal will be related as $Q_2 = Q_1 + \beta(Q_g + q_0)$, where $\beta \sim \exp(-L_{eff}/L_s) \ll 1$ and $L_s \sim 2\pi/k_s$ is a screening length. The total charging energy of the two segments, in this idealized model, will then be

$$\frac{(Q_1 - n_1Q_0)^2}{2C_1} + \frac{(Q_2 - n_2Q_0)^2}{2C_2} = \frac{Q_0^2}{8\pi^2} \left\{ \frac{(\theta_1 - 2\pi n_1)^2}{2C_1} + \frac{(\theta_2 - 2\pi n_2)^2}{2C_2} \right\}$$

(3)

where $C_1$ and $C_2$ are the capacitances of the two segments separated by the island electrode. The analogy to the SET suggests that a gate voltage might modulate the $I-V$ curves between source and drain contacts, with a periodicity $\Delta V_g \sim Q_0/\beta C_g$. The gate capacitance $C_g$, and hence the attainable displacement charge $Q_g$, may have been too small to observe non-monotonic behavior in previous experiments, in which a gate electrode was fabricated directly on the crystal to form a MOSFET-like structure. The soliton tunneling transistor (STT), discussed in the next section, employs a much larger, 1-$\mu$F, gate capacitor coupled to an NbSe$_3$ crystal, and exhibits non-monotonic behavior.

II. EXPERIMENTAL MEASUREMENTS

Single crystals of NbSe$_3$ were employed in the experiments reported here. This material forms two independent CDWs, at Peierls transition temperatures of 145 K and 59 K, respectively. The Peierls gap opens up over most of the Fermi surface (FS) below the lower transition, but leaves a small portion of the FS intact, so that a significant concentration ($\sim 6 \times 10^{-18}$ cm$^{-3}$) of normal, uncondensed carriers remain down to low temperatures. The geometry used in our experiment is illustrated in the inset to Figure 3, where the width of the crystal is exaggerated for clarity. The NbSe$_3$ crystal was placed onto an alumina
substrate with a series of evaporated, 25-µm wide gold contacts. The substrate was thermally anchored to a cold-finger in the vacuum shroud of an open cycle helium flow cryostat and the temperature was controlled using a Lake Shore temperature controller attached to a heater coil wrapped around the cold-finger. A Keithley programmable dc current source injected the current between two contacts, which were bonded to the crystal near the ends using silver paint. The “source-to-drain” voltage was measured between two additional gold contacts, as illustrated, and a 1-µF gate capacitor was attached to the center gold contact using silver paint. The spacing between contacts along the crystal was 500 µm center-to-center, and the gate capacitor was kept inside the cryostat.

We found that substantially smaller gate capacitors (as well as gate capacitors with longer leads) were unable to induce a periodic modulation of the I-V characteristic. Moreover, dc I-V (rather than differential dV/dI) measurements were necessary to avoid inducing a displacement current through the gate capacitor. A programmable voltage source was coupled to the gate capacitor via a 10-kΩ resistor, which limited the current flowing through the crystal during changes in gate voltage when the gate capacitor either partially charged or discharged. The cryostat was kept inside an electromagnetically shielded enclosure, and \( V_{sd} \) was measured with a nanovoltmeter.

The measurements were primarily carried out at 35 K. Previous “field effect transistor” experiments showed the greatest modulation at around 30 K, where the threshold field is near its minimum. We also observed the largest modulation, using our geometry, at comparable temperatures. We attained the best temperature stability (better than ±0.01 K) when the temperature was set to 35 K, and thus chose this temperature for most of the measurements reported here.

Figure 3 shows several plots of CDW current as a function of source-to-drain voltage, \( I_{cdw} \) vs. \( V_{sd} \), in a NbSe\(_3\) crystal at 35 K, for different values of gate voltage \( V_g \). The gate voltage is seen to modulate the threshold voltage in the I-V curves of Fig. 3. Figure 4 displays plots of source-to-drain voltage \( V_{sd} \) vs. \( V_g \) for three values of total bias current above threshold. The plots exhibit roughly periodic behavior, similar to that observed in SETs. However, in our system, the measured periodicity \( \Delta V_g \sim 10 \) V is consistent with a macroscopic displacement charge \( \Delta Q = C_g \Delta V_g \sim 6 \times 10^{13} \) e, comparable to the charge of the conducting electrons between the contacts.

The behavior shown in Fig. 4 is quite extraordinary, and appears to be consistent with
the soliton tunneling hypothesis. The number of parallel CDW chains, $N_{ch}$, is about $10^8$. Thus, one might estimate the screening parameter as follows: $\beta \sim Q_0/\Delta Q \sim 2N_{ch}e/\Delta Q \sim 3 \times 10^{-6}$. Assuming that $\beta \sim \exp(-L_{eff}/L_s) \sim 3 \times 10^{-6}$, and taking $L_{eff}$ to be about the distance between contacts ($\sim 500 \mu m$), yields an effective screening length $L_s$ of about 40 $\mu m$. However, an alternative interpretation for the observed periodicity might be that all of the normal electrons between the contacts participate in screening out the displacement charge $Q_0$ of the CDW. Thus, the observation that $\Delta Q/e \sim 6 \times 10^{13}$ is roughly the number of conducting electrons between contacts may not be a coincidence. Further work is needed to better understand the effects of screening by the normal, uncondensed electrons.

III. MODELING STUDIES

We believe that phase-coherent, Josephson-like tunneling of many microscopic degrees of freedom enables quantum transport to dominate at all temperatures below the Peierls transition temperature, $T_p$. On the one hand, the observation of complete mode locking with an ac source in high quality crystals shows that the phase is coherent throughout macroscopic regions within the crystal. On the other hand, magneto-transport experiments on NbSe$_3$ crystals with columnar defects demonstrate Aharonov-Bohm (AB) oscillations with a periodicity of $h/2e$, and not $h/2Ne$ as predicted theoretically, where $N$ is the number of coupled chains. These apparently contradictory results can be reconciled by treating a density wave as a condensate containing many quantum degrees of freedom within a phase-coherent volume.

This suggests writing down a Hamiltonian in terms of the phases of individual standing waves created by the dressed electrons, coupled to the $2k_F$ phonons, in the CDW condensate:

$$\hat{H} = \sum_n \hat{H}_n + \sum_{n,n'} U_{n,n'}[1 - \cos(\phi_n - \phi_{n'})],$$

where $\phi_n \equiv \frac{1}{2}(\phi_{n\uparrow} + \phi_{n\downarrow})$, $U_{n,n'}$ is related to the coupling between nearby chains (or transverse wavevectors), and

$$\hat{H}_n = E_F \sum_{\sigma=\uparrow,\downarrow} \left\{ \frac{\Pi_\sigma^2}{2\mu} + E'_p[1 - \cos(\phi_{n\sigma})] + E'_c[\phi_{n\sigma} - \theta]^2 \right\} + U'[1 - \cos(\phi_{n\uparrow} - \phi_{n\downarrow})].$$
Here $\phi_{n\uparrow}(\phi_{n\downarrow})$ is the phase (near the mid-point between the kink and antikink) of the standing wave created by a delocalized spin-up (spin-down) dressed electron in the condensate, $\Pi_{n\sigma} = -i\partial / \partial \phi_{n\sigma}$ is the momentum operator, and $E_F$ is the Fermi energy. The parameter $\mu = M_F/m$ is the Fröhlich mass ratio, while $E_p' = E_p/E_F$, $E_c' = E_c/E_F$, and $U' = U/E_F$ represent normalized pinning, charging, and coupling energies, respectively. Note that, in a spin density wave, the spin-up and spin-down CDWs are out-of-phase, so the last term in Eq. (6) would be $U'[1 + \cos(\phi_{n\uparrow} - \phi_{n\downarrow})]$ in an SDW. The effective charge of $2e$ observed in the AB experiments suggests that the coupling $U$ between the two spin components is strong compared to the other coupling terms, i.e. $U >> U_{nn'}$. We thus take $U'$ to be nonzero in our calculations, and consider the case where $U' >> E_p' >> E_c'$. Alternatively, if the phases $\phi_{n\uparrow}$ and $\phi_{n\downarrow}$ are taken to be identical (i.e., if $U \to \infty$), our model can be treated as one in which pairs of chains are coupled.

In the variational approach, the energy is minimized to estimate the ground-state wavefunctions and energies for different values of $\theta$. Because a CDW is highly dissipative, the system will tend to quickly relax to its lowest energy state. In the variational method, the energy expectation value

$$E = \langle \Psi | \hat{H} | \Psi \rangle$$

is minimized by setting $\delta E = 0$ to estimate the ground state energy.

The wavefunctions of the system $\chi(\phi_{n\sigma})$ are approximated as superpositions of sharply peaked Gaussians centered at the pinning potential minima. The state of the system, incorporating the two spin components, is described as a trial function with properties that depend on the parameters $b_m$ and $\alpha$,

$$\chi(\phi_{n\sigma}) = \sum_{m=-N}^{N} b_m \exp[-\alpha(\phi_{n\sigma} - 2\pi m)^2]$$

$$\Psi[\phi_{n\sigma}] = \prod_{n\sigma} \chi(\phi_{n\sigma}).$$

The coefficients $b_m$ asymptotically approach zero as $m$ approaches infinity. We set $N = 2$. 
in Eq. (7) to render the problem computationally tractable. For each quantum degree of freedom $\phi_{n\sigma}$, we choose the coefficients to satisfy the following normalization condition:

$$\sum_{m=-2}^{2} b_m^2 = 1.$$ 

(9)

We have studied the problem using a variety of parameter values in the Hamiltonian of Eq. (5), with qualitatively similar results. For the results illustrated here, we take $\mu = 354$, $E_p/E_F = 10^{-5}$, $E_c/E_F = 10^{-6}$, and $U/E_F = 5 \times 10^{-3}$. Fig. 5 shows a plot of the minimized energy $E(\theta)$. The divergence for $|\theta| > 4\pi$ is an artifact of our having limited the number of coefficients $b_m$ to 5, i.e. taking $N = 2$, in Eq. (7). An exact calculation would yield a plot in which $E$ would be a periodic function of $\theta$. In Fig. 5, the energy is reduced slightly at the crossing points, $\theta = n\pi$, as compared to piecewise parabolas, but Eq. (2) provides a reasonable approximation to $E(\theta)$:

$$E(\theta) \sim (\theta - 2\pi n)^2.$$ 

(10)

As in the SET, the voltage across each segment of the STT is related to the charging energy as $V_{1,2} = dE/dQ_{1,2} \propto dE/d\theta_{1,2}$. If we use the approximation given by Eq. (10), this yields a sawtooth function, which can be expanded as a Fourier series:

$$V_{1,2} = -V_0 \text{saw}(\theta_{1,2}) = \frac{V_0}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(n\theta_{1,2}).$$ 

(11)

Fig. 6 shows a comparison between the simple sawtooth function and the numerical result obtained using the parameters listed above. This model has also been used to model the dynamics of density waves and to calculate the narrow-band noise spectra, with excellent agreement with experiment. These results will be reported in some detail in a separate publication.

Eq. (11) is related by charge-flux duality to the current-phase relation of a Josephson junction. The periodic behavior of the source-to-drain voltage vs. gate voltage of an STT can readily be understood by exploiting the duality with a dc SQUID. When each JJ in a dc SQUID has an ideal, sinusoidal current-phase relation and the critical currents $I_0$ are identical, the total current is $I = I_0[\sin \varphi_1 + \sin \varphi_2]$, where $\varphi_2 = \varphi_1 + 2\pi \Phi/\Phi_0$. This yields
a total critical current, \( I_c = 2I_0|\cos(2\pi\Phi/\Phi_0)| \), which is a periodic function of the flux \( \Phi \). Similarly, the total source-to-drain voltage of an ideal STT, \( V = V_1 + V_2 \), where \( V_{1,2} \) are given by Eq. (11), yields a critical voltage that is a periodic function of gate voltage \( V_g = Q_g/C_g \), as shown in Fig. 7. Here we note that \( \theta_2 = \theta_1 + \theta_g + \theta_0 \), where \( \theta_g = 2\pi\beta Q_g/Q_0 \), \( \theta_0 = 2\pi\beta q_0/Q_0 \), \( Q_g \) is the displacement charge across the gate capacitor, \( q_0 \) is the offset charge, and \( \beta \) is the screening parameter due to the normal electrons. Fig. 7, although rather simplistic, provides at least a preliminary, qualitative interpretation of our experimental results.

IV. CONCLUSION

We have carried experiments which, combined with our modeling studies, provide further evidence for a quantum mechanism of CDW transport. The use of wave functions, in Eq. (7) and (8), is based on the approximation that the \( \phi_{n\sigma} \)'s are roughly uniform sufficiently far from the kink and antikink. An extension of the model would employ wave functionals that incorporated spatial variations of the phases \( \phi_{n\sigma}(x) \). A soliton domain wall could then be viewed as a condensate of a microscopic quantum solitons. The wave functional approach to quantum field theory, together with a non perturbative treatment of the functional Schrödinger equation, has previously been discussed in the context of QCD scalar 2-D QED, and (1+1)-D quantum gravity.

Further refinements might include the addition of disorder to the s-G pinning potential, which would enable a description of metastability, hysteresis, and related phenomena. Krive and Rozhavskii have studied the effective Lagrangian for macroscopic quantum tunneling of a CDW in a random medium, and have found that disorder significantly enhances the tunneling rate. A quantum version of a modified Fukuyama-Lee-Rice (FLR) model which included the electrostatic charging energy due to spatial variations of the phase, would provide a more realistic description of many aspects of the problem. However, we believe it is also important to incorporate the numerous microscopic degrees of freedom when calculating the effective action relevant for tunneling. We are thus exploring a generalization of the tunneling Hamiltonian to the case of matrix elements between wave functionals of scalar fields representing these degrees of freedom.

The quantum decay of the false vacuum, accompanied by the nucleation of topological defects, is a far-reaching problem, which could impact many areas of physics. For example,
topological defects, such as flux vortices, play an important role in the cuprates and other type-II superconductors. Magnetic relaxation rates that depend weakly on temperature up to 20 K or even decrease with temperature suggest that Abrikosov vortices may tunnel over a wide temperature range. Moreover, the consistently low $I_cR_n$ products of cuprate Josephson devices suggest that Josephson vortex-antivortex pair creation may occur when the current is much smaller than the “classical” critical current $I_0 \sim \Delta/R_N e$. In cosmology, the existence of many matter fields may facilitate quantum nucleation of a universe even when the total action is large, as suggested by Hawking et al. Finally, the extraordinary rapidity of first-order phase transitions, such as the palpably visible nucleation of ice in supercooled water, suggest a possible similarity to the decay of the false vacuum.

The authors gratefully acknowledge the valuable contributions of Lei-Ming Xie, James Claycomb, and Carlos Ordóñez. This work was supported, in part, by the State of Texas through the Texas Center for Superconductivity and the Texas Higher Education Coordinating Board Advanced Research Program (ARP), and by the Robert A. Welch Foundation (E-1221). AGP and WM gratefully acknowledge the World Laboratory Center for Pan-American Collaboration in Science and Technology for generous support.

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However, the polarization below threshold and dramatic $I-V$ characteristic with a relatively high threshold field, observed in blue bronze when the normal carriers are frozen out, suggest that the classical depinning field may be attained in this case. See Ref. 25 for additional remarks.

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Screening by the normal carriers may enhance $\varepsilon$. If $\varepsilon$ scales as $n_n/n_i^2$, where $n_i$ is the concentration of pinning impurities, then $E_T$ will vary as $n_i^2/n_n$.

Actually, many small “bubbles” will nucleate, followed by annihilation of colliding $S$ and $S'$ domain walls, which causes the phase to advance by $2\pi$ between the contacts.

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FIG. 1: (a) CDW phase vs. position, illustrating the production of a soliton-antisoliton domain wall pair. (b) Model of a CDW capacitor, showing the nucleated domain walls moving towards the contacts. The electric field between the domain walls is reduced by the internal field $E^*$. The distances $l$, $\lambda_0$, and the crystal thickness are greatly exaggerated for clarity.

FIG. 2: Potential energy vs. $\phi$ for two different values of $\theta$. Tunneling is prevented by conservation of energy when $\theta < \pi$. The small energy barrier for each microscopic degree of freedom within the condensate enables tunneling when $\theta > \pi$, while the Peierls condensation energy prevents thermal hopping.

FIG. 3: CDW current vs. source-to-drain voltage for several values of gate voltage at 35 K. (The shunt current of the normal electrons has been subtracted for clarity.)

FIG. 4: $V_{sd}$ vs. $V_g$ for fixed values of total bias current at 35 K.

FIG. 5: Computed ground state energy $E$ vs. $\theta$, showing the periodic, piecewise parabolic form. The apparent divergence for $\theta > 4\pi$ is an artifact of the finite number of coefficients used in the trial wave function.

FIG. 6: Voltage as a function of $\theta$, similar to the dual of the Josephson current-phase relation. The sawtooth expansion with $10^4$ Fourier components (dashed line) is compared to the numerical results obtained by minimizing the energy using the trial function discussed in the text.

FIG. 7: Predicted critical voltage vs. normalized gate voltage $\theta_g$ for several values of $\theta_0$ using the idealized model discussed in Section III, showing the periodic behavior.
TABLE I: Charge-flux duality between density waves and Josephson junctions.

|                             | Density Wave                  | Josephson junction              |
|-----------------------------|-------------------------------|---------------------------------|
| Soliton or antisoliton      | kink w/ charge $\pm Q_0$      | Josephson vortex w/ flux $\pm \Phi_0$ |
| Type of threshold           | Threshold field $E_T$         | Threshold current $I_T$         |
| Transport characteristic    | $I$ vs. $V$, $I = 2\pi Q_0 \partial \phi / \partial t$ | $V$ vs. $I$, $V = 2\pi \Phi_0 \partial \phi / \partial t$ |
\[ \theta < \pi \quad \theta > \pi \]
$I_{\text{CDW}}$ (mA)

$V_{sd}$ (mV)

$V_g$

$10 \text{ k}\Omega$

$1 \mu\text{F}$

$V_{sd}$

$I_{sd}$

NbSe$_3$

$T = 35 \text{ K}$
$\text{NbSe}_3$

$T = 35 \text{ K}$

$V_{sd} (\text{mV})$

$V_{\text{gate}} (\text{V})$

0.66 mA
0.72 mA
0.60 mA
