Attenuation of Magnetic Effects in Soft Amorphous Ribbons

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Magnetization reversals taking place in ferromagnetic materials can be detected using pickup coils wrapped around the sample, the so-called Barkhausen noise (BN). The voltage peaks induced in the coils depend both on the value $\Delta M$ of each magnetization reversal and on the attenuation $V(x)$ of the inductive signal after traveling a distance $x$ inside the material. Here, we present a procedure for obtaining $V(x)$ starting from the BN induced in two pickup coils separated by a distance variable between 2 and 40 mm. This procedure starts from the experimental values of $R$, the ratio of the signals induced in coincidence by the same magnetization reversal in the two coils, measured in a thin amorphous ribbon of Fe$_{63}$B$_{4}$Si$_{8}$Ni$_{15}$. Working with the signal ratio allows us to get rid of the dependency on $\Delta M$. A mathematical model for extracting $V(x)$ from the histogram of the measured values of $R$ is presented.

Index Terms—Electromagnetic induction, ferromagnetic materials, magnetic noise.

I. INTRODUCTION

THE macroscopic properties of ferromagnetic materials are represented by the usual hysteresis loop which gives a thermodynamic description of the evolution of magnetization under the effect of an external field. On a microscopic level, however, the smooth and reproducible behavior of the hysteresis loop breaks down and strong discontinuities are observed. This jumpy nature of magnetism is well known since the original experiment of Barkhausen [1] who observed that magnetization reversal takes place through a sequence of sharp jumps instead of being a continuous process as the smooth aspect of the hysteresis loop might suggest. Ever since Barkhausen noise (BN) has been widely investigated and in recent years the interest in this phenomenon increased, both for its relevance in statistical physics [2] and for the applications in the characterization of magnetic metals in industrial applications [3].

Actually, the complexity of the magnetization process is overwhelming and it is unlikely that an accurate modeling will be available soon [4]. On the other hand, the available experimental techniques allow to collect large amounts of high-quality data suitable for different kinds of statistical analysis. One of the most prominent features of BN is its fractal nature. The probability distribution of the amplitude $\Delta M$ of the jumps taking place during magnetization reversal has been largely investigated and there is a general consensus on the fact that this distribution is a power law: $P(\Delta M) \propto \Delta M^{-\alpha}$. The term $\alpha$ is called critical exponent.

Another relevant issue related to the magnetization process is the propagation of magnetic effects through the material [5]. Here, we use the term “propagation” with the following meaning. Under the effect of the increasing external field, magnetization jumps take place inside the material. During each jump, a magnetic wall, which is initially pinned, is suddenly stripped off from its position and moves across the material up to its successive pinned configuration. In this way, a certain portion of the sample undergoes a magnetization reversal. Even if this process involves only a small portion of the sample, the associated sudden variation of the magnetic flux induces an electromotive force that can be detected far away from the region where the reversal is taking place. Improving our understanding of the attenuation of these signals inside the ferromagnetic material is a relevant issue in connection with the diagnostic applications of the BN.

In a typical Barkhausen experiment, a pickup coil is placed close to the sample and the BN is measured as a series of well-separated peaks. Each peak originates from a single magnetization jump which gives rise to an induced voltage signal in the pickup coil. The amplitude of this peak depends both on the amplitude of the magnetization variation inside the sample ($\Delta M$) and on the distance from the pickup coil of the reversion region.

In this paper, we present a statistical analysis of a set of data acquired with an experimental technique [6] based on the idea of using two pickup coils instead of one for measuring the BN. The use of two coils allows us to analyze the effect of the propagation of magnetic effects, independent of the amplitude of the originating magnetic reversal since only the relative amplitudes of the signals in the two coils are evaluated. However, this requires an analytical study of the relative amplitudes, which is developed here through a mathematical model that, starting from the experimental data, allows us to derive the attenuation function. To our knowledge, such a mathematical model is innovative and together with the two-coils concept offers a general tool for the analysis of magnetic effects attenuation.

II. EXPERIMENTAL APPARATUS

Our experimental apparatus is described in detail in [6] and schematically represented in the lower part of Fig. 1.

The sample used in this investigation is a ribbon of amorphous 100 mm long, 3 mm large, and 20 $\mu$m thick Fe$_{63}$B$_{4}$Si$_{8}$Ni$_{15}$ prepared by rapid quenching of the molten material. This material was chosen as representative of
this class of amorphous alloys that exhibit good magnetic properties, such as high remanence and low coercive field, together with high amorphous-forming ability that allows for the production of ribbons using common single-roller melt-spinning method [7]. By changing the alloy composition, these materials can demonstrate a giant magnetoimpedance effect with potential applications in magnetic field sensing applications [8], [9].

To ensure mechanical stability the ribbon has been fixed with a few drops of glue on a stiff plastic support. This assembly has been placed within a magnetizing coil made of 1000 turns of copper capable of generating a magnetic field up to 1000 Oe. The two pickup coils used for detecting the induced BN can be moved along the sample length. In Fig. 2 are shown the voltage peaks induced in the two coils. To ensure adequate sensitivity each coil is made of 1000 turns of a shielded copper wire (diameter 0.1 mm); the thickness of each coil is 0.5 mm. The bandpass of each coil is around 33 kHz. The whole apparatus is enclosed in a metallic box to shield the electromagnetic noise. The coils can be moved one with respect to the other using a manipulator to change their relative distance without opening the shielding box.

To avoid possible effects due to the proximity of the ribbon edges, during our measurements the intermediate position of the two coils has been always kept in the center of the ribbon. Another test we performed, to verify the homogeneity of the sample and the magnetization process, was to measure the hysteresis loops in different positions, using both coils spaced by 40 mm or more. As explained below, beyond this distance the Barkhausen signals from the two coils are practically uncorrelated and they are independently sampling different regions of the ribbon. A typical cycle is shown in Fig. 3. The hysteresis loops are repeatable except for the Barkhausen jumps which are clearly visible but have a statistical nature and therefore change from cycle to cycle.

The acquisition is controlled by a PC which provides the generation of the magnetic field and the simultaneous detection of the two signals coming from the coils. In a typical acquisition, the field performs a complete hysteresis cycle in 20 s. The signal has been sampled at a frequency of 50 kHz to obtain the best signal-to-noise ratio by taking into account the 33 kHz bandwidth of the coils. In this way, it is possible to sample nearly 50 000 peaks on each channel within one loop. All the data presented in the following come from a single hysteresis loop since the number of peaks is sufficient for the statistical accuracy needed.

Let us consider Fig. 2 where two small portions of the signals induced in the coils are shown. In the upper panel of Fig. 2, the distance \( d \) between the coils was set at its minimum value \( (d = 2 \text{~mm}) \). Clearly, the two signals are very similar and every single peak in one coil matches with an equivalent peak in the other coil. In the lower panel, we see the case of \( d = 20 \text{~mm} \). In this case, the correlation is lower. In fact, in the figure, there are peaks detected only by one of the coils (such as peaks A, C, and F). For the peaks detected simultaneously by the two coils, the amplitude is not the same due to the different attenuation.
For each peak generated within the coils, it is possible to define the amplitude which is simply the area below the peak. The area of an individual peak depends on two distinct factors: the true value of the magnetization reversal (\(\Delta M\)) and the distance between the reversal location and the coil. We will assume that the size of the sample portion involved in these reversions is small with respect to the distance between the coils also in the smallest case of \(d = 2\) mm. This assumption is justified by the large number of peaks in one loop (25 000 peaks in each branch of the loop) which corresponds to an average area of 10,000 \(\mu m^2\), to which corresponds a size of the order of 100 \(\mu m^2\). In this assumption, it is possible to unambiguously define the distance \(x\) between the reversal place and the coil.

III. Experimental Results

In our experiment, two pickup coils are wrapped around the sample at a distance \(d\), as shown in Fig. 1. The \(x\) coordinate travels along the sample length and the origin is placed midway between the coils. A magnetization reversal taking place in a location \(x\) and having amplitude \(\Delta M\), in general, will produce two peaks in the pickup coils. Let us indicate as \(V_1\) and \(V_2\) the amplitudes of these two peaks, respectively. Each of them will be proportional to the flux variation following the reversal, which, in turn, is proportional to \(\Delta M\). Furthermore, it is necessary to consider the attenuation of each signal. This attenuation is described by a function \(V(x)\) which will be a decreasing function of the distance between the reversal point and the coil. We have therefore: \(V_1\) proportional to \(\Delta M \times V(x + d/2)\) and \(V_2\) to \(\Delta M \times V(x - d/2)\). It can be assumed without loss of generality that the function \(V(x)\) is normalized to 1, i.e., that \(V(0) = 1\). It is also, for obvious symmetry reasons, \(V(x) = V(-x)\). Let us define \(R(x)\) as the ratio between the pulses generated in the two coils by the same magnetization reversal \(\Delta M\)

\[
R(x) = \frac{V_1(x)}{V_2(x)} = \frac{V(x + \frac{d}{2})}{V(x - \frac{d}{2})}. \tag{1}
\]

We see that \(R\) does not depend on \(\Delta M\). Furthermore, given the experimental geometry \(R(-x) = 1/R(x)\). In fact, a reversal taking place at a distance \(x\) from the origin has the same probability of occurring at the same reversal at a distance \(-x\). On the other hand, the ratio \(R\) in position \(x\) will become \(1/R\) in position \(-x\). In the following, we will show how to extract information on \(V(x)\) from the ensemble of the measured values of \(R\).

The first step is to evaluate the area of the various peaks in the two streams of data coming from the coils. This task is quite straightforward and it will not be discussed in detail. The second step is to identify the “corresponding” peaks in the two streams. Corresponding here means that we observe two peaks in the coils which both originated from the same magnetization reversal. The criterion adopted is that two peaks are considered to be coincidental if they fall within a certain time window. Also, in this case, the procedure can be simply accomplished by the analysis program and its details will be not discussed. The only relevant point to note is that the results we obtain do not critically depend on the details of these two operations and that the adopted algorithms are quite robust.

The output of this analysis is an ensemble of values of \(R\), the peak area ratio. Since the number of coincident peaks is sufficiently high it is possible to plot the histogram of these numbers which, by definition, is the probability distribution \(P(R)\) of obtaining a particular value of \(R\). A series of these distributions, measured for different values of the distance between the coils, is shown in Fig. 4. In the horizontal axis, it is shown that the logarithm of \(R\). The first point to consider is the total number of points in each histogram. When the coils are very close to each other the number of points is high. Actually, in each loop, the number of peaks is much higher, in the order of 50 000. However, to avoid artifacts, we set a high threshold for accepting the peaks and for deciding if they are coincident or not. The purpose of this threshold is to avoid the appearance of spurious correlations due to the random coincidence of electronic noise peaks. A criterion for choosing its value is therefore that it is substantially greater than the rms value of the measurement noise. In our case, this choice was made empirically, taking it as 5–6 times the peak value of the electronic noise obtained during the measurements. The validity of this choice can be verified by slightly varying the threshold value and verifying that the correlation function does not change. This selection lowers the number of events to 6573 for the case of \(d = 2\) mm. The same selection criteria have been adopted also in the other cases shown in Fig. 4.
Consider the distribution at distances in the order of 40 mm. For this reason, we decided to study the distribution by separate events. The distance above which this occurs indicates that the coincident peaks observed are not due to a single magnetization reversal but, instead, are produced by a semi-log plot, it is advisable to operate with the logarithms of the two functions $V(x)$ and $R(x)$

\[ w(x) = \ln[V(x)] \]

\[ z(x) = \ln[R(x)] \]

\[ = \ln \left[ V \left( x + \frac{d}{2} \right) \right] - \ln \left[ V \left( x - \frac{d}{2} \right) \right] \]

\[ = w \left( x + \frac{d}{2} \right) - w \left( x - \frac{d}{2} \right). \tag{4} \]

The function $w(x)$ can be expanded in power series as

\[ w(x) = \sum_{n=0}^{\infty} a_n x^n. \tag{5} \]

According to the definition of (3) and the condition $V(0) = 1$, it must be $w(0) = 0$ and therefore $a_0 = 0$. Furthermore, the function $w(x)$ has the same symmetry as $V(x)$, that is, $w(-x) = w(x)$. Therefore, only the terms with an even index are present in the summation

\[ w(x) = \sum_{k=1}^{\infty} a_{2k} x^{2k} = a_2 x^2 + a_4 x^4 + a_6 x^6 + \cdots. \tag{6} \]

It is now possible to expand also the function $z(x)$ by inserting the variables $x + d/2$ and $x - d/2$ in (6)

\[ z(x) = w \left( x + \frac{d}{2} \right) - w \left( x - \frac{d}{2} \right) \]

\[ = \sum_{k=1}^{\infty} a_{2k} \left( x + \frac{d}{2} \right)^{2k} - \sum_{k=1}^{\infty} a_{2k} \left( x - \frac{d}{2} \right)^{2k} \]

\[ = \sum_{k=1}^{\infty} a_{2k} \left[ \left( x + \frac{d}{2} \right)^{2k} - \left( x - \frac{d}{2} \right)^{2k} \right]. \tag{7} \]

In Fig. 4, the corrected probability distributions of $R$ are shown in a semi-log plot, and, in this representation, they appear to be symmetrical. This symmetry reflects the fact that, as discussed above, $R(-x) = 1/R(x)$.

Another interesting observation is that the probability distribution of $R$ is nearly Gaussian in a semi-log plot. The width of the distribution increases by increasing $d$ and the corresponding values of the FWHM are indicated with $\sigma$ in each panel.
It is easy to verify that while the function $w(x)$ contains only the even powers and, therefore, is even, $z(x)$ resulting from expression (7) contains only the odd powers of $x$ and, therefore, is odd: $z(-x) = -z(x)$, as required from its definition (3) and given that $R(-x) = 1/R(x)$.

From (7), the function $z(x)$ is expressed by a series of powers that will be completely identified once the coefficients $a_{2k}$ are obtained. To find them, one must resort to the experimental data which gives the probability density $p(z)$ of the variable $z$, and assuming the probability density $p(x)$ that a jump occurs at the $x$ coordinate. The probability of obtaining a certain $z$ is related to the probability that the jump occurs at the $x$ coordinate by the relationship

$$p(z)dz = p(x)dx. \quad (8)$$

The two terms of this equation are integrated into their respective domains. Given the symmetry of the $z$ function, it is sufficient to integrate only on the right half of the $x$-axis

$$\int_0^z p(z)dz = -\int_0^x p(x)dx. \quad (9)$$

The minus appears because after (1) in the region $x \geq 0$, the amplitude of $z$ decreases as $x$ increases. Once the functions $p(z)$ and $p(x)$ are known, these integrals provide the function $z(x)$ which will, in turn, be expanded in series

$$z(x) = \sum_{n=1}^{\infty} C_n x^n. \quad (10)$$

This time, the coefficients $C_n$ are known because the function $z(x)$ obtained from the integral of (9) is known. The series of (10) is compared term by term with the series of (7), so that from the known coefficients $C_n$, we can derive the unknown coefficients $a_{2k}$. From the latter, we build the function $w(x)$ through the series of (5) and finally the desired function $V(x) = \exp(w(x))$. If the convergence of the series is rapid, it will be sufficient to consider a limited number of terms and the calculation can be done analytically. However, nothing stands in the way of implementing a numerical calculation and therefore considering any number of terms. In the experimental results described in this article, the probability density is Gaussian: normalizing to the total number of jumps, we have

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{z^2}{2\sigma^2}\right). \quad (11)$$

As for the probability density $p(x)$, it can be assumed to be uniform. The part of the strip that is measured by the spaced coils has a length equal to the sum $d + d_0$, where $d$ is the distance between the coils and $d_0$ is the extension of the region around each coil such that the events taking place inside it are detected by the coil (see Fig. 1 for visualizing its meaning). In the following calculations, we assumed $d_0 = 40 \text{ mm}$ but it must be emphasized that, as far as the determination of the attenuation function $V(x)$ is concerned, the exact value assigned to this parameter is not particularly relevant. Therefore,

$$p(x) = \frac{1}{d + d_0}. \quad (12)$$

Equation (8) thus becomes

$$\int_0^z \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{z^2}{2\sigma^2}\right) dz = -\int_0^x \frac{1}{d + d_0} dx. \quad (13)$$

By changing variable $z^2 = 2\sigma^2 t$, it transforms as

$$\frac{2}{\sqrt{\pi}} \int_0^\sqrt{z^2} e^{-t^2} dt = -\frac{2x}{d + d_0}. \quad (15)$$

The expression on the left is nothing but the error function

$$\text{erf}(\frac{z}{2\sigma}) = -\frac{2x}{d + d_0}. \quad (16)$$

and therefore

$$z = -2\sigma \text{erf}^{-1}\left(\frac{2x}{d + d_0}\right)$$

where $\text{erf}^{-1}$ is the inverse function of the error function. The series expansion of this function is known [10]

$$\text{erf}^{-1}(y) = \sum_{n=1}^{\infty} C_n y^{2n-1}$$

$$= C_1 y + C_2 y^3 + C_3 y^5 + C_4 y^7 + \cdots. \quad (17)$$

The first 200 $C_n$ coefficients are tabulated in [10] and the first four orders are reported in Table I.

| Order | Coefficient |
|-------|-------------|
| 1     | 0.8862      |
| 2     | 0.2320      |
| 3     | 0.1276      |
| 4     | 0.0866      |

Equation (8) can now be numerically evaluated since all its terms are known: the $C_n$ coefficients, the experimental parameters $d$ and $d_0$, the experimentally determined standard deviation of the various distributions shown in Fig. 4. These values have been obtained by fitting the experimental data and the values obtained can be considered robust, as the fitting program yielded small relative uncertainties for the fit standard deviations. By comparing term by term this series with (7), we obtain $a_{2k}$ as a function of $d$ and $d_0$ and then we can calculate the series expression of $w(x)$ [see (6)] and finally $V(x)$ [see (3)]. The calculation is not reported here for simplicity. Only the final result is reported in graphical form in Fig. 5, that is, the form of the function $V(x)$ obtained from the experimental data statistics at the various coil distances. As can be seen, despite the distance between the coils varies from 2 to 20 mm and the distribution variance shown in Fig. 4 changes from 1.00 to 2.46, the shape of the $V(x)$ function remains almost unchanged, as expected.
where the sample width is \( d \) and the resistivity is \( \rho \). We can assume that the field \( B \) inside the sample undergoes a variation \( \Delta B = 2B_{\text{sat}} \). This variation, in turn, induces a damping field \( B_{\text{ind}} \) that, with elementary arguments, can be expressed as

\[
\frac{B_{\text{ind}}}{\Delta B} = \frac{\mu_0 d^2}{4 \rho} \frac{1}{\Delta t}
\]

(19)

where \( \Delta t \) is the time duration of the reversal. In our experiment, the sample width is \( d = 30 \) mm and the measured resistivity is \( \rho = 140 \times 10^{-8} \) \( \Omega \)m. By assuming a conservative value of the time of reversal \( \Delta t \approx 10 \) \( \mu s \), the induced field is in the order of \( 10^{-5} \) times the one present inside the sample. Therefore, the damping effect due to the eddy currents can be ruled out as a possible source of attenuation of the signal.

Having excluded the possible role of eddy currents, the problem of giving a physical explanation to the phenomenon of attenuation remains open.

VI. CONCLUSION

In this article, we presented a model for interpreting Barkhausen effect data acquired using two coils. For each magnetization reversal taking place inside the sample, the relevant information is represented by the ratio \( R \) of the voltage peaks induced in the coils. This method removes the effect of the amplitude of the magnetic reversals and permits us to analyze the decay of the field with distance for a wide range of reversal sizes. A mathematical model for deriving the attenuation function \( V(x) \) for these signals has been presented. The function \( V(x) \) obtained is demonstrated to remain almost the same for very different values of the distance between the coils, thus becoming a candidate for a general law.

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