Baryon Regge Trajectories in the Light of the $1/N_c$ Expansion

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Abstract

We analyze Regge trajectories in the light of the $1/N_c$ expansion of QCD. Neglecting spin-orbit contributions to the large $N_c$ baryon mass operator, we consider the evolution of the spin-flavor singlet component of the masses with respect to $\ell$. We find two distinct and remarkably linear Regge trajectories for symmetric and for mixed symmetric spin-flavor multiplets.

1. Introduction

The ordering of hadronic states on approximately linear Regge trajectories in the Chew-Frautschi plot is one of the most remarkable features of the QCD spectrum. It manifests the underlying non-perturbative QCD dynamics, which at long distances becomes dominated by the string-like behavior that leads to confinement. In fact this picture has been the motivation for the development of string/flux tube models of hadrons [1], which contemporarily are described as effective theories in the so called AdS/QCD framework [2]. The latter is valid in the large $N_c$ limit, $N_c$ being the number of colors, and has been applied almost exclusively to mesons, while extensions to baryons are being explored [3]. Furthermore, it has been shown recently that flux tube model and large $N_c$ mass formulas are compatible [4]. Regge trajectories have also been recently considered in the context of the quark-diquark picture of baryons [5].

In this work we will analyze the baryon Regge trajectories in the light of the $1/N_c$ expansion, which is in principle an approach consistent with QCD. The $1/N_c$ expansion for baryons is based on the emergent $SU(6)$ spin-flavor symmetry (for three light flavors) in the large $N_c$ limit [5–7]. For excited baryons, a convenient scheme consists in organizing states into multiplets of the $SU(6) \times O(3)$ group. The justification for this approach relies on the small contributions to baryon masses by effects that break this symmetry at $O(N_c^0)$, as it occurs with spin-orbit interactions for baryons in the mixed-symmetric spin flavor representation. Following this approach various works [9] have shown that the $1/N_c$ expansion is a very useful tool for analyzing the baryon spectrum. Note that the analysis of the present work relies fundamentally in the use of the $SU(6) \times O(3)$ symmetry.

In the $1/N_c$ expansion, the mass operator for a given $SU(6) \times O(3)$ multiplet is expressed in terms of a series in effective operators [8,9] ordered in powers of $1/N_c$. The coefficients associated with the operators are obtained by fitting to the empirical masses. The various analyses have shown that these coefficients are of natural magnitude or smaller (dynam-
ically suppressed), lending support to the consistency of the framework. To a first approximation, it turns out that the main features of the spectrum can be captured by taking into account a few operators, namely the $O(N_c)$ spin-flavor singlet operator, one $O(1/N_c)$ hyperfine operator, and the strangeness operator of $O(N_c^0 m_s)$. For the finer aspects of the spectrum, more operators are of course needed. The coefficients of the three operators considered in this work are $O(N_c^0)$, and differ from multiplet to multiplet by amounts $O(1/N_c)$. The purpose of this work is to analyze the coefficients as a function of $\ell$. In particular we focus on the evolution of the coefficient of the leading spin-flavor singlet operator, which determines the Regge trajectories.

2. Analysis

We start by considering the $[56,\ell]$ and the $[70,\ell]$ multiplets of $SU(6) \times O(3)$, which correspond respectively to the symmetric (S) and mixed-symmetric (MS) spin-flavor multiplets at $N_c = 3$. We entirely disregard possible mixings between these multiplets [10], an approximation that seems to be consistent phenomenologically as shown by analyses of strong transition amplitudes [11] as well as electromagnetic transitions [12].

For the ground state baryons, which consist of the octet and decuplet in the $[56,0^+]$ multiplet, the mass formula reads:

$$\tilde{M}_{GS} = N_c \, c_1 \mathbb{1} + \frac{1}{N_c} c_{HF} \left( S^2 - \frac{3}{4} N_c \right) - c_S \tilde{S}, \quad (1)$$

where $\tilde{S}$ is the baryon spin operator and $\tilde{S}$ is the strangeness operator. Note that the hyperfine term has been defined such that in the limit of a non-relativistic quark picture it corresponds to the operator $\frac{1}{N_c} \sum_{i \neq j} S_i \cdot S_j$.

For excited baryons a similar set of mass operators are used. In particular, for baryons with $\ell > 0$, the hyperfine interaction of interest can be defined following the large $N_c$ Hartree picture of the baryon [13]: an excited quark carrying the orbital angular momentum and a core made out of the rest $N_c - 1$ quarks sitting in the ground state. This motivates the choice of hyperfine operator as the one that takes into account the hyperfine interactions between core quarks only. A second hyperfine operator involves the interaction between core quarks and the excited quark. In MS states one can separate these two hyperfine interactions explicitly; it was shown that the latter hyperfine effect is much weaker, and thus we neglect it here. Therefore, for excited baryons, we use the following form for the mass operator:

$$\tilde{M}' = N_c \, c_1 \mathbb{1} + \frac{c_{HF}}{N_c} \left( S^2 - \frac{3}{4} (N_c - 1) \right) - c_S \tilde{S}, \quad (2)$$

where $\tilde{S}$ is the spin operator of the core. Note that for $N_c = 3$ one can identify the core with a diquark. Note that the mass formulas generalize beyond the quark model, as they are entirely given in terms of generators of the spin-flavor group, and thus, only the spin-flavor nature of the states will matter. The coefficients $c_1$ and $c_{HF}$ and $c_S$ are determined by fitting to the masses of the corresponding multiplet. They are all $O(N_c^0)$, differing by $O(1/N_c)$ from multiplet to multiplet.

The matrix elements of the mass operators in the different cases for non-strange excited baryons are:

$$M_{S}(S) = N_c \, c_1 + \frac{N_c - 2}{N_c} c_{HF} \left( S(S + 1) - \frac{3}{4} N_c \right),$$

$$M_{MS}(S=I) = N_c \, c_1 + \frac{1}{N_c} c_{HF} \left( S(S + 1) - \frac{3}{4} N_c - \frac{1}{2} \right),$$

$$M_{MS}(S=I-1) = N_c \, c_1 + \frac{1}{N_c} c_{HF} \left( S + \frac{1}{2} \right) \left( S + \frac{3}{2} \right),$$

$$M_{MS}(S=I+1) = N_c \, c_1 + \frac{1}{N_c} c_{HF} \left( S^2 - \frac{1}{4} \right), \quad (3)$$

where the sublabel S and MS indicate symmetric and mixed symmetric spin flavor representations respectively. For $N_c = 3$ the mass formulas then read:

$$N_{GS} = 3 c_1 - \frac{1}{2} c_{HF}, \quad \Delta_{GS} = 3 c_1 + \frac{1}{2} c_{HF},$$

$$N_{S} = 3 c_1 - \frac{1}{6} c_{HF}, \quad \Delta_{S} = 3 c_1 + \frac{1}{6} c_{HF},$$

$$N_{MS} \left( S = \frac{1}{2} \right) = 3 c_1 - \frac{1}{6} c_{HF}, \quad (4)$$

$$N_{MS} \left( S = \frac{3}{2} \right) = \Delta_{MS} \left( S = \frac{1}{2} \right) = 3 c_1 + \frac{1}{6} c_{HF},$$

where we denote $N \equiv M_N$, etc. Note that for the MS states we need to specify the total quark spin $S$. The case of strange baryons is obvious, except for the SU(3) singlet $\Lambda$ states in the $[70]$-plets, where the mass formula becomes:

$$\Lambda_{MS} = 3 c_1 - \frac{1}{2} c_{HF} + c_S \quad (5)$$
Tables 1 and 2, for 56- and 70-plet baryons respectively, display the input masses, which include all states listed by the Particle Data Group [15] that can be identified with a reasonable level of confidence to belong to a definite SU(6) × O(3) multiplet. They also give the results for the coefficients $c_1$, $c_{\text{HF}}$ and $c_5$, and the theoretical masses resulting from the fits. We note here that in the MS states there is in general two mixing angles that correspond to the mixing of the octet states with $J = \frac{1}{2}$ and $\frac{3}{2}$. In the fit these mixings are neglected because they only originate in the presence of mass operators we have neglected. We have checked that this approximation does not affect in any significant way the conclusions of this work.

In the case of the GS baryons the large value of the $\chi^2$ is primarily due to subleading SU(3) breaking effects that have been disregarded. The higher order analysis shows improvement consistent with the $1/N_c$ expansion [14]. A similar situation occurs in the other well established multiplets where the $\chi^2$ is large. In the case of the $[56, 2^+]$ multiplet, the large $\chi^2$ is primarily due to the exclusion of an SU(3) breaking spin dependent operator, and in the case of the $[70, 1^-]$ multiplet the main contribution to the $\chi^2$ is consequence of the absence of the spin-orbit operator, which produces the splitting between the SU(3) singlet $\Lambda$ states. Note that the available information about the $[56, \ell = 4, 6]$ and the $[70, \ell = 2, 3, 5]$ states is somewhat limited; in each case, it is sufficient for determining the coefficient $c_1$ with significant accuracy for the purpose of our analysis, while the hyperfine and strangeness splittings are only roughly determined.

The focus of our study is the relation across multiplets of the leading order coefficient $c_1$. Figure 1 shows the plot $(N_c c_1)^2$ vs $\ell$. It displays two distinct Regge trajectories corresponding to the $[56, \ell]$ and the $[70, \ell]$ states. We note first that in large $N_c$ limit a plot linear or quadratic in $c_1$ would be equivalent, the reason being that the baryon masses are order $N_c$ while the splittings between multiplets are order $N_c^0$. In the real world they differ slightly, with the quadratic plot giving the best approximation to linear trajectories. A splitting between the S and MS trajectories would be expected on general grounds. In particular, in the Hartree picture a contribution to such splitting is due to the exchange interaction between the excited quark and the core; this interaction turns out to be different for S and MS representations, being order $N_c^0$ in the first case and order $1/N_c$ in the latter case. The linear fits to the trajectories give:

$$(3c_1([56, \ell]))^2 = (1.19 + 1.05 \ell) \text{ GeV}^2,$$

$$(3c_1([70, \ell]))^2 = (1.34 + 1.18 \ell) \text{ GeV}^2. \quad (6)$$

We note that the results for $c_1$ obtained with only non-strange baryons agree with those obtained including the strange ones. It is remarkable that the spin-flavor singlet piece of the squared masses fit so well on linear Regge trajectories; the spread observed in the Regge trajectories in terms of the physical masses have, therefore, to do with the non-singlet spin-flavor components of the masses, which are dominated by the hyperfine components. For the splitting between 56- and 70-plet, the following linear relation gives a fair approximation:

$$(c_1([56, \ell]) - c_1([70, \ell]))^2 = (5.3 + 4.4 \ell) \times 10^{-4} \text{ GeV}^2. \quad (7)$$

This corresponds to a mass splitting that increases with $\ell$, going from $\sim 70$ MeV at $\ell = 0$ intersect to $\sim 170$ MeV at $\ell = 6$. Since hyperfine terms have this magnitude or larger, the differentiation of the two trajectories can only be clearly seen upon removal of those terms as we have done here.

Note that the quantity with $\mathcal{O}(N_c^0)$ slope is $N_c c_1^2$ rather than the one we plotted. It is, therefore, somewhat of a coincidence that at $N_c = 3$ the Regge slopes of mesons and baryons are so similar.

It is interesting to note that the strength of the HF interaction tends to increase with $\ell$. It is clearly shown by the $[70, 1^-]$ and the $[56, 2^+]$ multiplets, where its strength is clearly larger than for the GS baryons. Unfortunately, for baryons with $\ell > 2$, $c_{\text{HF}}$ has large uncertainty and we cannot establish that trend. Although, according to the $1/N_c$ expansion, the value of $c_{\text{HF}}$ differs by $\mathcal{O}(1/N_c)$ across multiplets, in reality changes by a factor larger than two in going from the GS to the $\ell = 2$ baryons. This should be expalined by the fact that the hyperfine interaction is more sensitive to the effective size of the core than the other terms in our analysis. In particular, in the quark-diquark picture of the baryon, this sensitivity in the hyperfine effect indicates a reduction in the size of the diquark that is significant. Finally, and as one would expect, the strangeness coefficient remains, within errors, roughly constant across multiplets.
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Table 1
The coefficients $c_1$, $c_{HF}$ and $c_S$ and the theoretical masses (MeV) for the 56-plets. The experimental masses used for the fit are also presented.

| Multiplet | Multiplet | Baryon Name, status | Exp. (MeV) | Theo (MeV) | $c_1$ (MeV) | $c_{HF}$ (MeV) | $c_S$ (MeV) | $\chi^2_{ dof}^2$ |
|-----------|-----------|---------------------|------------|------------|-------------|----------------|-------------|----------------|
| [56, 0+]  | $N_{1/2}$ | N(939)***           | 939 ± 1    | 955 ± 2    | 363 ± 1     | 269 ± 2        | 156 ± 1     | 232           |
|           | $\Lambda_{1/2}$ | Λ(1116)***       | 1116 ± 1   | 1111 ± 2   |             |                |             |               |
|           | $^8\Sigma_{1/2}$ | Σ(1193)***       | 1192 ± 4   | 1111 ± 2   |             |                |             |               |
|           | $^8\Xi_{1/2}$ | Ξ(1318)***        | 1318 ± 3   | 1267 ± 2   |             |                |             |               |
|           | $\Delta_{1/2}$ | Δ(1232)***        | 1232 ± 1   | 1224 ± 2   |             |                |             |               |
|           | $^{10}\Sigma_{1/2}$ | Σ(1385)***     | 1383 ± 3   | 1380 ± 1   |             |                |             |               |
|           | $^{10}\Xi_{1/2}$ | Ξ(1530)***       | 1532 ± 1   | 1536 ± 2   |             |                |             |               |
|           | $\Omega_{1/2}$ | Ω(1672)***        | 1672 ± 2   | 1692 ± 3   |             |                |             |               |
| [56, 2+]  | $N_{3/2}$ | N(1720)****        | 1700 ± 50  | 1697 ± 18  | 603 ± 5     | 671 ± 58       | 126 ± 19    | 6.95          |
|           | $\Lambda_{3/2}$ | Λ(1890)****       | 1880 ± 30  | 1823 ± 12  |             |                |             |               |
|           | $N_{5/2}$ | N(1680)****        | 1683 ± 8   | 1697 ± 18  |             |                |             |               |
|           | $\Lambda_{5/2}$ | Λ(1820)****       | 1820 ± 5   | 1823 ± 12  |             |                |             |               |
|           | $^8\Sigma_{5/2}$ | Σ(1915)****      | 1918 ± 18  | 1823 ± 12  |             |                |             |               |
|           | $\Delta_{1/2}$ | Δ(1910)****       | 1895 ± 25  | 1921 ± 18  |             |                |             |               |
|           | $\Delta_{3/2}$ | Δ(1920)***        | 1935 ± 35  | 1921 ± 18  |             |                |             |               |
|           | $\Delta_{5/2}$ | Δ(1905)***        | 1895 ± 25  | 1921 ± 18  |             |                |             |               |
|           | $\Delta_{7/2}$ | Δ(1950)****       | 1950 ± 10  | 1921 ± 18  |             |                |             |               |
|           | $^{10}\Sigma_{7/2}$ | Σ(2030)****     | 2033 ± 8   | 2047 ± 18  |             |                |             |               |
| [56, 4+]  | $N_{9/2}$ | N(2220)****        | 2245 ± 65  | 2245 ± 92  | 770 ± 20    | 398 ± 372      | 110 ± 94    | 0.13          |
|           | $\Lambda_{9/2}$ | Λ(2350)***        | 2355 ± 15  | 2355 ± 21  |             |                |             |               |
|           | $\Delta_{7/2}$ | Δ(2390)*          | 2387 ± 88  | 2378 ± 84  |             |                |             |               |
|           | $\Lambda_{9/2}$ | Λ(2300)*          | 2318 ± 132 | 2378 ± 84  |             |                |             |               |
|           | $\Delta_{11/2}$ | Δ(2420)*          | 2400 ± 100 | 2378 ± 84  |             |                |             |               |
| [56, 6+]  | $N_{13/2}$ | N(2700)**          | 2806 ± 207 | 2806 ± 207 | 954 ± 40    | 342 ± 720      | 405 ± 122   |               |
|           | $\Delta_{15/2}$ | Δ(2950)**         | 2920 ± 122 | 2920 ± 122 |             |                |             |               |
Table 2
The coefficients $c_1$, $c_{HF}$ and $c_S$ and the theoretical masses (MeV) for the $70$-plets. The experimental masses used for the fit are also presented.

| Multiplet | Baryon Name, status | Exp. (MeV) | Theo (MeV) | $c_1$ (MeV) | $c_{HF}$ (MeV) | $c_S$ (MeV) | $\chi^2_{\text{ dof}}$ |
|-----------|---------------------|-----------|------------|-------------|---------------|-------------|----------------|
| [70, 1$^-$] | $N_{3/2}$ | $N(1535)^{****}$ | 1538 ± 18 | 1513 ± 14 | 529 ± 5 | 443 ± 19 | 148 ± 13 | 61 |
| | $^8\Lambda_{1/2}$ | $\Lambda(1670)^{****}$ | 1670 ± 10 | 1662 ± 6 | 1538 ± 18 | 1513 ± 14 | 148 ± 13 | 61 |
| | $N_{3/2}$ | $N(1520)^{****}$ | 1523 ± 8 | 1513 ± 14 | 1513 ± 14 | 148 ± 13 | 61 |
| | $^8\Lambda_{3/2}$ | $\Lambda(1690)^{****}$ | 1690 ± 5 | 1662 ± 6 | 1690 ± 5 | 1662 ± 6 | 148 ± 13 | 61 |
| | $^8\Sigma_{3/2}$ | $\Sigma(1670)^{****}$ | 1675 ± 10 | 1662 ± 6 | 1675 ± 10 | 1662 ± 6 | 148 ± 13 | 61 |
| | $^8\Xi_{3/2}$ | $\Xi(1820)^{****}$ | 1823 ± 5 | 1810 ± 15 | 1823 ± 5 | 1810 ± 15 | 148 ± 13 | 61 |
| | $N'_{1/2}$ | $N(1650)^{****}$ | 1660 ± 20 | 1661 ± 17 | 1660 ± 20 | 1661 ± 17 | 148 ± 13 | 61 |
| | $^8\Lambda'_{1/2}$ | $\Lambda(1800)^{****}$ | 1785 ± 65 | 1809 ± 12 | 1785 ± 65 | 1809 ± 12 | 148 ± 13 | 61 |
| | $^8\Sigma'_{1/2}$ | $\Sigma(1750)^{****}$ | 1765 ± 35 | 1809 ± 12 | 1765 ± 35 | 1809 ± 12 | 148 ± 13 | 61 |
| | $N'_{3/2}$ | $N(1700)^{****}$ | 1700 ± 50 | 1661 ± 17 | 1700 ± 50 | 1661 ± 17 | 148 ± 13 | 61 |
| | $N'_{5/2}$ | $N(1675)^{****}$ | 1678 ± 8 | 1661 ± 17 | 1678 ± 8 | 1661 ± 17 | 148 ± 13 | 61 |
| | $^8\Lambda'_{3/2}$ | $\Lambda(1830)^{****}$ | 1820 ± 10 | 1809 ± 12 | 1820 ± 10 | 1809 ± 12 | 148 ± 13 | 61 |
| | $^8\Sigma'_{5/2}$ | $\Sigma(1775)^{****}$ | 1775 ± 5 | 1809 ± 12 | 1775 ± 5 | 1809 ± 12 | 148 ± 13 | 61 |
| | $\Delta_{1/2}$ | $\Delta(1620)^{****}$ | 1645 ± 30 | 1661 ± 17 | 1645 ± 30 | 1661 ± 17 | 148 ± 13 | 61 |
| | $\Delta_{3/2}$ | $\Delta(1700)^{****}$ | 1720 ± 50 | 1661 ± 17 | 1720 ± 50 | 1661 ± 17 | 148 ± 13 | 61 |
| | $^1\Lambda_{1/2}$ | $\Lambda(1405)^{****}$ | 1407 ± 4 | 1514 ± 4 | 1407 ± 4 | 1514 ± 4 | 148 ± 13 | 61 |
| | $^1\Lambda_{3/2}$ | $\Lambda(1520)^{****}$ | 1520 ± 1 | 1514 ± 4 | 1520 ± 1 | 1514 ± 4 | 148 ± 13 | 61 |
| [70, 2$^+$] | $N'_{1/2}$ | $N(2100)^*$ | 1926 ± 26 | 1987 ± 50 | 1926 ± 26 | 1987 ± 50 | 148 ± 13 | 61 |
| | $N'_{5/2}$ | $N(2000)^{**}$ | 1981 ± 200 | 1987 ± 50 | 1981 ± 200 | 1987 ± 50 | 148 ± 13 | 61 |
| | $^8\Lambda'_{1/2}$ | $\Lambda(2110)^{****}$ | 2112 ± 40 | 2108 ± 71 | 2112 ± 40 | 2108 ± 71 | 148 ± 13 | 61 |
| | $N'_{7/2}$ | $N(1990)^{**}$ | 2016 ± 104 | 1987 ± 50 | 2016 ± 104 | 1987 ± 50 | 148 ± 13 | 61 |
| | $\Lambda'_{1/2}$ | $\Lambda(2020)^*$ | 2094 ± 78 | 2108 ± 71 | 2094 ± 78 | 2108 ± 71 | 148 ± 13 | 61 |
| | $\Delta_{1/2}$ | $\Delta(2000)^{**}$ | 1976 ± 237 | 1987 ± 50 | 1976 ± 237 | 1987 ± 50 | 148 ± 13 | 61 |
| [70, 3$^-$] | $N_{5/2}$ | $N(2200)^{**}$ | 2057 ± 180 | 2153 ± 67 | 2057 ± 180 | 2153 ± 67 | 148 ± 13 | 61 |
| | $N_{7/2}$ | $N(2190)^{****}$ | 2160 ± 49 | 2153 ± 67 | 2160 ± 49 | 2153 ± 67 | 148 ± 13 | 61 |
| | $N'_{7/2}$ | $N(2250)^{****}$ | 2239 ± 76 | 2236 ± 81 | 2239 ± 76 | 2236 ± 81 | 148 ± 13 | 61 |
| | $\Delta_{7/2}$ | $\Delta(2200)^*$ | 2232 ± 87 | 2236 ± 81 | 2232 ± 87 | 2236 ± 81 | 148 ± 13 | 61 |
| | $^1\Lambda_{7/2}$ | $\Lambda(2100)^{****}$ | 2100 ± 20 | 2100 ± 28 | 2100 ± 20 | 2100 ± 28 | 148 ± 13 | 61 |
| [70, 5$^-$] | $N_{11/2}$ | $N(2600)^{***}$ | 2638 ± 97 | 900 ± 20 (Est) | 2638 ± 97 | 900 ± 20 (Est) | 148 ± 13 | 61 |
Fig. 1. Values of the coefficient $(N_c c_1)^2$ vs $\ell$ for the 56-plets (+) and the 70-plets ($\times$). The solid line represents the Regge trajectory for the symmetric multiplets and the dashed line, the Regge trajectory for the mixed symmetric multiplets.