Gravitational Decay Modes of the Standard Model Higgs Particle

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If the Einstein field equations are employed at the tree level, then the decay of the standard model Higgs particle into two gravitons is shown to be independent of the gravitational coupling strength $G$. The result follows from the physical equivalence between the Higgs induced “inertial mass” and the “gravitational mass” of general relativity. If the Higgs mass lies well between the mass of a bottom quark anti-quark pair and the mass of a top quark anti-quark pair, then the Higgs decay into two gravitons will dominate both the QED induced two photon decay and the QCD induced two jet decays.

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The last major notion of the standard electro-weak model [1–3] which has yet to receive experimental confirmation [4–9] is the prediction of the Higgs particle [10–14]. In part, the problem may be simply connected to our lack of knowledge of the value of the Higgs mass $M_H$ [15]. But other problems arise on a more conceptual level.

The Higgs field is thought to provide the mechanism for the existence of all inertial mass [16]. Yet the standard model does not relate this Higgs induced inertial mass to the important and presumably equivalent value of the gravitational mass. In what follows, we shall add to the standard electro-weak model Higgs notion of inertial mass [17], the Einstein notion of gravitational mass via the conventional curvature field equations of general relativity

$$R_{\mu\nu} - \left(\frac{1}{2}\right) g_{\mu\nu} R = \left(\frac{8\pi G}{c^4}\right) T_{\mu\nu}. \quad (1)$$

In particular, for the trace $T = g^{\mu\nu} T_{\mu\nu}$ we shall employ

$$T = -\left(\frac{c^4}{8\pi G}\right) R. \quad (2)$$

In the standard model, the Higgs field is entirely responsible for the possible existence of $T \neq 0$. Thus, the Higgs field is entirely responsible for the possible existence of a non-trivial scalar curvature $R \neq 0$ in general relativity. Nevertheless, the modes of the interaction between the Higgs field and conventional Einstein gravity have gone virtually unnoticed with regard to high energy laboratory quantum gravity experiments.

The reason for this sad state of affairs is that the value of the Planck mass $M_P$, defined by

$$\left(\frac{GM_P^2}{\hbar c}\right) = 1, \quad (3)$$

is thought to be much too large to allow for quantum gravity observations using conventional high energy beams. The weak interaction (Fermi coupling $G_F$) version of Eq.(3), i.e.

$$\left(\frac{\sqrt{2} G_F M_F^2}{\hbar c}\right) = 1, \quad (4)$$

sets the mass scale at the vacuum condensation value of the Higgs field

$$M_F = \left(\frac{\hbar \langle \phi \rangle}{c}\right). \quad (5)$$

The value of $M_F$ is thus known to be

$$M_F \approx 246 \text{ GeV/c}^2, \quad (6)$$

well within the present day technology of high energy beams. Let us return to the quantum gravity aspects of the problem.

For Higgs particle excitations one normally writes the total field

$$\phi = \langle \phi \rangle + \chi, \quad (7)$$

while the effective action employed for computing the decay of the Higgs particle is given by

$$S_{\text{eff}} = \left(\frac{1}{c \langle \phi \rangle}\right) \int \chi T d\Omega, \quad (8)$$

where $d\Omega = \sqrt{-g} d^4x$ is the space-time “volume” element. While $T$, the trace of the stress tensor, has many contributions, e.g. a term $(-mc^2 \bar{\psi} \psi)$ for each massive fermion species, the total sum over all the fields coupling into the Higgs particle is most simply described by the effective action in Eq.(8).

From Eqs.(2) and (8), it follows that the effective action depends on the scalar curvature

$$S_{\text{eff}} = -\left(\frac{c^3}{8\pi G \langle \phi \rangle}\right) \int \chi R d\Omega. \quad (9)$$

Eqs.(8) and (9) express the fact that the Higgs couples equivalently into inertial and gravitational mass, but in the latter case we can relate the result to the Lagrangian density $\mathcal{L}_g$ of the gravitational field; i.e.
The Higgs coupling to Lagrangian density of gravitons is then

\[ S_{\text{eff}} = - \left( \frac{2}{c^2} \beta \right) \int \chi \mathcal{L}_g \, d\Omega. \]  

One may now be aware that in the Higgs coupling to gravitons, the gravitational coupling strength \( G \) has very quietly slipped away. (The situation is reminiscent of the discussions between Bohr and Einstein on the completeness of the quantum mechanical view. Toward the end of these discussions, Bohr had to invoke general relativity to “save” the energy-time uncertainty principle. Notwithstanding the need for a finite gravitational coupling \( G \) in the intermediate stages of the argument, \( G \) dropped out of the final results.)

A general rule is that an oscillator Hamiltonian \( H = (1/2)\hbar \omega (a \dagger a + a a) \) corresponds to an oscillator Lagrangian \( L = (1/2)\hbar \omega (a^\dagger a^2 + a^2 a) \). For the problem at hand, \( \mathcal{L}_g \) may create or may destroy two gravitons. The rate at which a Higgs at rest will decay into two gravitons requires the matrix element \( \langle gg | S_{\text{eff}} | H \rangle \). In terms of quantum fields, one requires \( \langle 0 | \chi \rangle | H \rangle \) and \( \langle gg | \mathcal{L}_g | 0 \rangle \). From Eq.(5) and (11), one computes the rate

\[ \Gamma(H \rightarrow g + g) = \left( \frac{1}{16 \pi} \right) \left( \frac{M_H}{M_F} \right)^2 \left( \frac{M_H c^2}{\hbar} \right). \]  

In terms of the Fermi coupling strength Eq.(4), the Higgs into two gravitons has the decay rate

\[ \Gamma(H \rightarrow g + g) = \left( \frac{\sqrt{2}}{16 \pi} \right) \left( \frac{G_F M_H^2}{\hbar c} \right) \left( \frac{M_H c^2}{\hbar} \right), \]  

which is the central result of this work. The gravitational coupling strength does not appear in the final gravitational decay rate due to the physical equivalence between the inertial (Higgs induced) mass and the gravitational mass.

Our central Eq.(13) for the Higgs decay into two gravitons \( H \rightarrow g + g \) may in some ways be compared with the computation of Higgs decay into two photons \( H \rightarrow \gamma + \gamma \). A “scalar particle” decay into two photons begins with the Schwinger anomaly [19] for the trace of the stress tensor \( T_{\gamma \gamma} = (2\alpha/3\pi) \mathcal{L}_\gamma \), where \( \alpha \) is the quantum electromagnetic coupling strength, and \( \mathcal{L}_\gamma \) is the free photon Lagrangian density. The anomalous \( T_{\gamma \gamma} \) is a one loop process. The one loop Higgs decay rate into two photons is lower than the tree level Higgs decay into two gravitons by more than \( \sim 10^{-6} \). (In reality, one requires a one loop renormalized coupling strength \( \alpha \), and this further lowers the two photon decay rate relative to the two graviton decay rate.) In a similar fashion, the Higgs decay into two gravitons is seen to be much larger than the decay into two gluon jets. If the Higgs mass is much larger than twice the bottom quark mass but still much less than twice the top quark mass, then the decay of the Higgs into two gravitons dominates too the decay of the Higgs into quark anti-quark jet pairs. Lastly, if the mass of the Higgs is smaller than the mass of two \( Z \) Bosons or the mass of a \( W^+W^- \) pair, then the Higgs into two graviton decay rate will dominate the decay rate into all \( SU(2) \times U(1) \) channels; the heavy gauge Boson pairs are ruled out on the basis of the above kinematics.

Consider the analogy between Higgs decay \( H \rightarrow g + g \) and the well known weak interaction decay \( Z \rightarrow \nu + \bar{\nu} \) [2]. The latter \( (Z\text{-decay}) \) has been observed even though the neutrino anti-neutrino pair escapes direct detection other than by “missing” the four momenta. The full process is \( e^- + e^+ \rightarrow Z + \gamma \rightarrow \nu + \bar{\nu} + \gamma \). There is a burst of soft photon radiation indicating that the electron positron pair has been destroyed, and then there is “nothing”. Now suppose, as one specific example among many, the following analog event. One produces a Higgs from a proton anti-proton event \( p^+ + p^- \rightarrow H \rightarrow g + g \). Here too would be a soft photon radiation burst from the proton anti-proton destruction, and then nothing but the missing four momenta of the two hard final state gravitons. Such a process would occur resonantly at a square total four momentum \( s = -(P^+ + P^-)^2/c^2 = M_H^2 \).

The experimental search for the Higgs particle has been argued above to also be an experimental search for quantum gravity. Presently there is no direct experimental evidence for the Higgs particle nor for quantized gravitational waves (gravitons). It would however appear that the two are closely connected. Further progress on the source of inertial masses ultimately requires that gravity be added to the electroweak sector. It is hoped that the present exploration of these ideas will stimulate further work on these notions.

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