Monte-Carlo Simulation of Hard Probes in Heavy-Ion Collisions

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Abstract. Results from the Modular Algorithm for Relativistic Treatment of heavy Ion Interactions (MARTINI) are presented. This comprehensive event generator for the hard and penetrating probes in high energy nucleus-nucleus collisions employs a time evolution model for the soft background, PYTHIA 8.1 and the McGill-AMY parton evolution scheme including radiative as well as elastic processes. It generates full event configurations in the high $p_T$ region, taking into account thermal QCD and QED effects as well as effects of the evolving medium.

1. Introduction

High transverse momentum jets emerging from the central rapidity region in heavy-ion collisions provide important information on the produced hot quark-gluon plasma (QGP). In order to extract this information from the experimental data, it is important to have a good theoretical understanding of the interactions of hard partons with the medium.

Gluon bremsstrahlung including the Landau-Pomeranchuk-Migdal (LPM) [1] effect has been proposed as the dominant mechanism for energy loss and different theoretical formalisms have been developed and applied to describe it [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Also, binary elastic scattering off thermal partons is potentially important for the energy loss and momentum broadening of high-$p_T$ partons. In [13], elastic energy loss was combined with the AMY [10, 11, 12] radiative energy loss within the McGill-AMY evolution formalism, and in [14] the description of the collisional processes was further improved.

The main goal of these and other calculations is to create a quantitative basis for the “tomography” of the hot and dense nuclear medium created in heavy-ion collisions. Among the “tomographic variables” is the nuclear modification factor $R_{AA}$, which is defined as the ratio of the hadron yield in A+A collisions to that in binary-scaled p+p interactions. A variety of computations that differ significantly in the applied energy loss mechanism can reproduce the measured $R_{AA}$ in central Au+Au collisions at $\sqrt{s} = 200$ AGeV at the Relativistic Heavy-Ion Collider (RHIC), given the present experimental errors [15]. Hence, more differential observables, such as $R_{AA}$ as a function of both $p_T$ and azimuth in non-central collisions [16, 17, 18, 19], should be studied to improve this situation.

Further important observables in heavy-ion collisions are electromagnetic probes, such as photons and dileptons. Once produced, they usually escape the medium without further interaction because of the weakness of the electromagnetic coupling. Hence they provide undistorted information on the early stages of a heavy-ion collision. Photon production from...
nuclear collisions at RHIC has been calculated in [20, 21, 22]. Photons in nuclear collisions come from a variety of sources, namely direct photons, fragmentation photons, jet-plasma photons and thermal photons. Thermal photons have a negligible contribution at high \( p_T \) and thus are not relevant in this range. On the other hand, jet-plasma photons from photon radiation \( q \rightarrow q\gamma \) (bremsstrahlung photons) and jet-photon conversion via Compton and annihilation processes have been shown to be important for the understanding of experimental data for photon production in Au+Au collisions at RHIC [23, 20, 21, 22].

For a best possible comparison of the theoretical description with experimental data, the McGill-AMY formalism [10, 11, 12, 24, 13] for radiative energy loss as well as elastic processes [14] is incorporated into the event generator MARTINI (Modular Algorithm for Relativistic Treatment of heavy Ion Interactions) [25]. Its main ingredients are PYTHIA 8.1 [26, 27] to generate the hard partons from the individual nucleon-nucleon collisions and to take care of the final fragmentation into hadrons, the evolution of the background medium (e.g. from hydrodynamic models), and the McGill-AMY parton evolution formalism. This way it is possible to study hard observables in heavy-ion collisions theoretically on an event-by-event basis, keeping information on correlations.

Several Monte-Carlo simulations for heavy-ion collisions have been or are being developed [28, 29, 30, 31, 32, 33, 34, 35, 36]. The implementation of medium effects and use of approximations varies significantly between the different models. Most of these models include medium effects by modifying the vacuum shower directly, typically introducing a medium induced increase in virtuality that leads to increased radiation. In contrast, MARTINI and the simulation described in [30, 31] separate the vacuum and in-medium shower, assuming that when the medium evolution begins the partons are essentially on-shell. This is supported also by other Monte-Carlo simulations as described in [37], where it is found that in YaJEM [33, 34] the initial branching processes down from a highly virtual state happen on a time scale comparable to the thermalization time \( \tau_0 \).

MARTINI is the first Monte-Carlo simulation to include AMY bremsstrahlung combined with elastic processes and in-medium photon production. It is very flexible due to its ability to incorporate any soft background evolution that provides information on the temperature and flow velocities of the medium. Currently, hydrodynamic evolution calculations from five different groups [38, 39, 40, 41, 42, 43, 44] have been implemented.

After comparing results for a brick of QGP at constant temperature with those obtained by integrating the McGill-AMY rate equations, we show results for pion spectra, the azimuth averaged \( R_{AA} \) as a function of \( p_T \), and photon yields and photon \( R_{AA} \) for Au+Au collisions at \( \sqrt{s} = 200 \) GeV.

2. Monte-Carlo simulation
To outline the functionality of MARTINI, we follow the evolution of one event. First, the number of individual nucleon-nucleon collisions that produce partons with a certain minimal transverse momentum \( p_T^{\text{min}} \) is determined from the total inelastic cross-section, provided by PYTHIA. The initial transverse positions \( r_\perp \) of these collisions are determined by the initial jet density distribution \( P_{AB}(b, r_\perp) \), which for A+B collisions with impact parameter \( b \) is given by

\[
P_{AB}(b, r_\perp) = \frac{T_A(r_\perp + b/2)T_B(r_\perp - b/2)}{T_{AB}(b)}.
\]

We use a Woods-Saxon form for the nuclear density function, \( \rho(r_\perp, z) = \rho_0/[1 + \exp((z_0 - R)/d)] \), to evaluate the nuclear thickness function \( T_A(r_\perp) = \int dz \rho_A(r_\perp, z) \) and the overlap function of two nuclei \( T_{AB}(b) = \int d^2r_\perp T_A(r_\perp)T_B(r_\perp + b) \). The values of the parameters \( R = 6.38 \) fm and \( d = 0.535 \) fm are taken from [45]. The initial parton distribution functions can be selected with
the help of the Les Houches Accord PDF Interface (LHAPDF) [46]. We include nuclear effects on the parton distribution functions using the EKS98 [47] or EPS08 [48] parametrization, by user choice. In practise we sample the initial positions of nucleons in nucleus A and B, superimpose the transverse areas depending on the impact parameter and then divide the overlap region into circles with area $\sigma_{\text{inel}}$. In three dimensions these are tubes and we determine how many jet events with given $p_T^{\text{min}}$ occur within such a tube using the number of nucleons from A and B in the tube, and the probability for a jet event $\sigma_{\text{jet}}(p_T^{\text{min}})/\sigma_{\text{inel}}$ for each of their combinations.

The soft medium is described by hydrodynamics or other models, which provide information on the system’s local temperature and flow velocity $\beta$. Before this medium has formed, i.e., before the hydrodynamic evolution begins ($\tau < \tau_0$), the partons shower as in vacuum. To model the transition, one way is to include the complete vacuum shower, because there is no apparent reason why the vacuum splittings should end immediately once the medium has formed. Since most of the vacuum shower occurs before the medium has formed, this is a reasonable approximation. Another way is to stop the vacuum evolution at the virtuality scale $Q_{\text{min}} = \sqrt{p_T/\tau_0}$, determined by the time $\tau_0$ at which the medium evolution begins. This modifies the parton distribution such that the strong coupling constant has to be chosen approximately 10% larger to describe the pion $R_{\text{AA}}$. The change of the photon $R_{\text{AA}}$ when using different prescriptions is shown in Section 3. At this stage, we do not include final state radiation (FSR) of the partons that have left the medium - all vacuum showers take place before the medium evolution and there is no interference between the medium and vacuum showers. The improvement on this is under investigation. We are also working on implementing the rates computed in [49] that incorporate finite-size effects of the medium and will lead to a length dependence of the energy loss $\Delta E \sim L^2$ for small $L$.

Once $\tau > \tau_0$ for a certain parton, its in-medium evolution begins. The partons move through the background according to their velocity. To compute probabilities for the included in-medium processes, we perform a Lorentz-boost into the rest-frame of the fluid cell at the parton’s position and determine the transition rates according to the local temperature and the parton’s energy in this frame. For a detailed review of the included processes and on how to derive the radiative rates, see [25] and references therein. The probability for a parton to undergo any process during a time step of length $\Delta t$ is given by $\Delta t \Gamma(p, T)_{\text{total}}$, where $\Gamma(p, T)_{\text{total}} = \int d\omega d k \Gamma_{\text{total}}/d k$ for the radiative processes, and the integral over both $\omega$ and $k$ for the elastic processes. $d \Gamma_{\text{total}}/d k$ is the sum over all possible transition rates, which include those for photon production.

In case that some process occurs, we decide which one it is according to the relative weights of the different processes at the given temperature and parton energy. We sample the radiated or transferred energy from the transition rate of the occurring process using the rejection method. In case of an elastic process, we also sample the transferred transverse momentum, while for radiative processes we assume collinear emission for now, which is a good approximation in the weak coupling limit since the emission angle is of order $g$ [12]. After energy and momentum are transferred, we boost back into the laboratory frame, where the parton continues to move along its trajectory. Radiated partons are also further evolved if their momentum is above a certain threshold $p_{\text{min}}$, which can be set (typically we choose $p_{\text{min}} \simeq 3 \text{ GeV}$). This leads to a growing and broadening in-medium shower. The overall evolution of a parton stops once its energy in a fluid cell’s rest frame falls below the limit of $4T$, where $T$ is the local temperature. As both radiative and elastic transition rates were computed in the limit $E \gg T$, it would not make sense to attempt to learn anything about the evolution of partons with $E \simeq T$ within this formalism. For partons that stay above that threshold, the evolution ends once they enter the hadronic phase of the background medium. Depending on the employed hydrodynamic model there will be a mixed phase or a crossover period. In both cases processes occur only for the QGP fraction.

When all partons have left the QGP phase, hadronization is performed by PYTHIA, to
which the complete information on all final partons is passed. Because PYTHIA uses the Lund string fragmentation model [50, 51], it is essential to keep track of all color strings during the in-medium evolution. Fragmentation in heavy-ion collisions should be modified as compared to proton-proton collisions, since the color structure of the shower should be randomized by scatterings with the medium. Also recombination of jet and medium partons should be possible. We will be studying different hadronization mechanisms to learn more about the significance of these effects. The hadronization concludes the evolution of one event.

The concept of MARTINI is modular, such that we can turn on and off different processes independently, and use different hydrodynamic or other data inputs. It is also possible to extend MARTINI to use different energy loss formalisms, which makes it an important tool to perform detailed comparisons of theoretical models with each other and experimental data. The parameters are set using the same interface as PYTHIA and all options for PYTHIA can still be modified by the user.

3. Results

3.1. The QGP brick in MARTINI

For a single initial parton with fixed energy traversing a QGP of constant temperature the average over many MARTINI events by construction has to be the same as the result obtained from solving the rate equations. Fig. 1 demonstrates that this is the case for the example of a $T = 200$ MeV QGP brick of length $L = 2$ fm. Here, we show the final quark and gluon distributions after a quark of initial energy $E_{\text{ini}} = 20$ GeV passed through the brick, comparing to a direct solution of the rate equations (see e.g. [14]) and the result obtained after $10^5$ MARTINI runs, including both radiative and elastic processes.

3.2. Pion production and nuclear modification factor $\pi^0 R_{AA}(p_T)$

Next we show the MARTINI results for the spectrum of neutral pions in p+p collisions at RHIC energy ($\sqrt{s} = 200$ GeV) as well as in central (0-10%) Au+Au collisions compared to data by PHENIX [52, 17] in Fig. 2. The calculations were performed using CTEQ5L parton distribution functions [53] including nuclear shadowing effects using the EKS98 parametrization [47]. Au+Au calculations take into account both radiative and elastic processes in the medium described by either the 2+1 dimensional hydrodynamics of [39, 40, 41] or the 3+1 dimensional hydrodynamics of [42], using a coupling constant $\alpha_s = 0.33$ or $\alpha_s = 0.3$, respectively. $\alpha_s$ was adjusted to describe the experimental measurement of the neutral pion nuclear modification
Figure 2. Neutral pion spectrum in p+p and 0−10% central Au+Au collisions at RHIC energy compared to data by PHENIX [52, 17].

Figure 3. The neutral pion nuclear modification factor for mid-central (upper panel) and central (lower panel) collisions at RHIC with $\sqrt{s} = 200$ GeV. MARTINI results ($b = 2.4$ fm and $b = 7.5$ fm) using 3+1 dimensional [42] (left) and 2+1 dimensional hydrodynamic evolution [39, 40, 41] (right) with different $\alpha_s$ compared to PHENIX data from [54]. In the left figure dashed lines correspond to only radiative processes and solid lines to both radiative and elastic processes. The right figure shows radiative and elastic combined only.

factor $R_{AA}$ in most central collisions (see below). For both hydro evolutions $\tau_0 = 0.6$ fm/c. Note that the shown results are averages over typically $\sim 5 \cdot 10^8$ simulated events.

In Fig. 3 we present the results for the neutral pion nuclear modification factor, defined by

$$R_{AA} = \frac{1}{N_{\text{coll}}(b)} \frac{dN_{AA}(b)}{d^2p_Tdy} / \frac{dN_{pp}}{d^2p_Tdy},$$

in Au+Au collisions at RHIC measured at mid-rapidity in two different centrality classes (0-10%) and (20-30%), employing the corresponding average impact parameters, 2.4 fm and 7.5 fm.
We compare with the experimental measurements by PHENIX [54]. All parameters are the same as in the calculation shown in Fig. 2, i.e., hydrodynamic background evolution from [42] and $\alpha_s = 0.3$ for Fig. 3 (left) and hydro evolution from [39, 40, 41] and $\alpha_s = 0.33$ (right). The same value of $\alpha_s$ is used in all following calculations. Increasing $\alpha_s$ would lead to large suppression, hence smaller $R_{AA}$. We find very good agreement with the data for both centrality classes. The result for $R_{AA}(p_T)$ is flat, in agreement with the experimental data. On the theory side, this behavior has been investigated in [55] and shown to arise naturally when taking into account the evolution of the full parton distribution as opposed to taking into account only the mean energy loss. In MARTINI, we evolve every parton separately such that after averaging over many events, we obtain full modified parton distributions, leading to the correct $p_T$ dependence of $R_{AA}$.

The value for $\alpha_s$ for which the data is described if we only include radiative processes is $\alpha_s \simeq 0.39$, higher than for the calculations in [13], where it was 0.33. Differences in the treatment of vacuum showers and the fragmentation scheme are responsible for this difference. We have observed that when varying parameters in PYTHIA, such as the factorization or renormalization scale, or parameters in the fragmentation function, we need slightly different $\alpha_s$ to describe the data. This freedom leads to an approximately 10 - 20% uncertainty in $\alpha_s$.

The value of $\alpha_s$ is again closer to that used in [13] ($\alpha_s = 0.27$) when including elastic processes, because the elastic energy loss in our formulation is slightly larger than that implemented in [13] - see [14] for details on this. Like previous works (e.g. [13]) we find that elastic processes are important for the computation of $R_{AA}$.

### 3.3. Photon production

Photons in the high $p_T$ region produced in nuclear collisions are dominantly direct photons, fragmentation photons, and jet-plasma photons. Direct photons are included in PYTHIA. Apart from leading order direct photons, PYTHIA produces additional photons emitted during the vacuum showers. Parts of these overlap with effects that are found in NLO calculations [56, 57], but there is no simple theoretical way to identify the amount of overlap between the two. Also fragmentation photons are part of the photons produced in the showers in PYTHIA. We computed photon production within PYTHIA with and without photons from showers and found that the shower contribution leads to an effective $K$-factor of approximately 1.8 in the regarded $p_T$ range.

For heavy-ion reactions MARTINI adds the very relevant jet-medium photons from photon radiation $q \rightarrow q\gamma$ (bremsstrahlung photons) and jet-photon conversion via Compton and annihilation processes. We present results for photon production in p+p and Au+Au collisions at $\sqrt{s} = 200$ GeV compared to data by PHENIX [58, 59, 60] in Fig. 4.

Another observable that provides information on the effect of the nuclear medium on photon production is the photon nuclear modification factor

$$R_{AA}^\gamma = \frac{1}{N_{coll}(b)} \frac{dN_{\gamma AA}^\gamma(b)/d^2p_Tdy}{dN_{\gamma pp}^\gamma/d^2p_Tdy}.$$  

(3)

To study the effect of different transitions from vacuum to the medium, in one case we include the full vacuum shower, in another we end the vacuum shower at the virtuality $Q_{min} = \sqrt{p_T/\tau_0}$. As mentioned above, in the latter case we need a 10% larger $\alpha_s$ to describe the $\pi^0$-$R_{AA}$. The change in photon $R_{AA}$ for that $\alpha_s$ is shown in Fig. 5, where we plot $R_{AA}^\gamma$ as a function of $p_T$ for most central Au+Au collisions ($b = 2.4$ fm) at RHIC compared with $0 - 10%$ central PHENIX data.
4. Conclusions
We presented results obtained with the Modular Algorithm for Relativistic Treatment of heavy Ion Interactions (MARTINI). This hybrid approach describes the soft background medium using hydrodynamics or other medium models and simulates the hard event microscopically, using PYTHIA 8.1 to generate individual hard nucleon-nucleon collisions. Hard partons are evolved through the medium using the McGill-AMY evolution scheme including radiative and elastic processes. Fragmentation is performed employing PYTHIA 8.1 which uses the Lund string fragmentation model. Apart from parameters in PYTHIA which were fixed by matching the neutral pion and photon spectra in p+p collisions to experimental data, $\alpha_s$ is the only free parameter. Employing the 3+1 dimensional hydrodynamic evolution from [42], it was set to $\alpha_s = 0.3$ to match the neutral pion $R_{AA}$ measurement for central collisions. Using the same value for all other calculations, we were able to describe the neutral pion $R_{AA}$ in mid-central collisions as well as photon yields and photon $R_{AA}$. For photon production, photons from jet-
medium interactions are added to the prompt and shower photons from PYTHIA. Both photon spectra and $R_{AA}$ are described well within the same simulation used to describe pion production. Having established a baseline using one-body observables, MARTINI’s full potential can now be explored by studying many-body observables and using its results as input for full jet reconstruction algorithms.

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