Sensitivity analysis and optimization method for the fabrication of one-dimensional beam-splitting phase gratings

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Abstract: A method to design one-dimensional beam-splitting phase gratings with low sensitivity to fabrication errors is described. The method optimizes the phase function of a grating by minimizing the integrated variance of the energy of each output beam over a range of fabrication errors. Numerical results for three 1x9 beam splitting phase gratings are given. Two optimized gratings with low sensitivity to fabrication errors were compared with a grating designed for optimal efficiency. These three gratings were fabricated using gray-scale photolithography. The standard deviation of the 9 outgoing beam energies in the optimized gratings were 2.3 and 3.4 times lower than the optimal efficiency grating.

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1. Introduction

It is often desirable to split one laser beam into multiple beams with equal energy for applications such as laser machining and material processing in parallel, sensor systems, interferometry, communication systems, and image processing and gathering systems [1]. The authors require multiple beams with equal energy for a scanning array confocal microscope [2]. There are several methods for beam-splitting. For example, conventional beamsplitters, diffractive optics, and pupil division have been shown to accomplish amplitude beam-splitting of a laser [3]. Binary phase gratings [4–6], continuous phase gratings [6–8], and refractive lenslet arrays [9] also accomplish phase beam-splitting. Continuous phase gratings are of interest, since they yield the highest efficiencies out of all beam-splitting methods [10].

Using calculus of variations, Romero and Dickey found analytical expressions of the optimal continuous phase functions of one- and two-dimensional gratings that maximize energy into \( N \) outgoing beams, where \( N \) is the desired number of output beams [11,12]. While these gratings theoretically provide the optimal efficiency into \( N \) desired output beams, fabrication errors can significantly degrade their performance. For example, Y. Miklyaev et al. demonstrated fabrication of the grating profiles described by Romero and Dickey, but fabrication errors prevented equal energy outgoing beams [13].

Fabrication errors degrade the performance of any beam-splitting method. In the presence of fabrication errors, multiple output beams typically no longer have designed energy ratios. For gratings, the grating profile determines the amount by which the energies vary [14]. A grating design with a high sensitivity to fabrication errors may result in an unacceptable variation of the outgoing beam energy ratios. If the sensitivity of the design is low enough, then a lower precision fabrication method could be chosen over a costly, high precision fabrication method. This paper describes the challenges associated with fabricating continuous phase diffraction gratings and describes a design technique that minimizes sensitivity of continuous phase gratings to fabrication errors. With this technique, continuous phase gratings can be fabricated with good performance, even in the presence of significant fabrication errors.

2. Theory

Assume a lossless, one-dimensional grating is characterized by the periodic height function \( h(x) \). In the absence of fabrication errors and using a thin grating approximation, the transmitted phase in air of a beam at position \( x \) is changed by an amount

\[
\phi(x) = h(x) \frac{2\pi (n - 1)}{\lambda},
\]

where \( n \) is the grating material refractive index and \( \lambda \) is the wavelength of light. If there are multiplicative fabrication errors \( \Delta \), the accumulated phase has a total phase transmission \( (1 + \Delta) \phi(x) \). In this paper, \( \Delta \) is a constant, although in principle any functional form of \( \Delta(x) \) can be used. For a grating illuminated by a unit-amplitude plane wave, the output beams, ignoring Fresnel losses, are characterized by the grating’s Fourier coefficients:

\[
a_p = \frac{1}{T} \int_{-T/2}^{T/2} e^{i(1 + \Delta)\phi(x)} e^{-i2\pi px/T} \, dx \quad \text{for } p = 0, \pm 1, \pm 2, \ldots
\]

where \( T \) is the period of the grating. The Fourier coefficients determine energies and phases of the output beams [15].
The efficiency of each beam is defined as $|a_p|^2$. A $1 \times N$ vector $\eta[\Delta, \phi(x)]$ is defined that contains the efficiency of each desired output beam, where

$$\eta_p[\Delta, \phi(x)] = |a_p|^2. \quad (3)$$

The efficiency of each output beam is dependent on the amount of fabrication error and the phase function of the grating. Total efficiency $E$ of the grating is

$$E[\phi(x)] = \sum_{p=1}^{N} |a_p|^2, \quad (4)$$

where $N$ sums over the number of desired output beams. Note that there may be significant energy in orders outside the desired range, so $E$ might be less than unity. For a $1 \times 9$ beam-splitting grating, the total efficiency is the sum of $N = 9$ desired output beams.

Fourier coefficients of an error-free uniform beam-splitting grating are subject to the constraint

$$|a_p|^2 = E / N, \quad (5)$$

which ensures that energies in all desired output beams are equal. If there are fabrication errors, output beam energies in the desired range of $p$ are not equal.

Since the point spread function (PSF) of the writing instrument significantly degrades the performance of the beam-splitting gratings, a phase function that accounts for the PSF of the writing instrument is calculated. If a phase function $\phi(x)$ is desired and the PSF of the writing instrument is given by $h_{\text{inst}}(x)$, the phase function taking the PSF of the writing instrument into account is given by

$$\phi(x) = \phi_u(x) * h_{\text{inst}}(x), \quad (6)$$

where $\phi_u(x)$ is the instrument input function needed to obtain the desired grating phase function. The necessary instrument input function is found by a deconvolution operation. In Fourier-space, the convolution reduces to

$$\Phi(\xi) = \Phi_{\text{in}}(\xi) H_{\text{inst}}(\xi). \quad (7)$$

$\Phi_{\text{in}}(\xi)$ is found by dividing $\Phi(\xi)$ by $H_{\text{inst}}(\xi)$. However, if $H_{\text{inst}}(\xi)$ contains zeroes, there is significant noise amplification. A rect function is multiplied in Fourier-space to act as a low-pass filter and prevent division by zero in the Fourier domain. Since the low-pass filter cuts off higher spatial frequencies, a perfect reconstruction of the desired phase function cannot be computed. The low-pass filtered instrument input function is given by

$$\Phi_{\text{in}}(\xi) = \left( \frac{\Phi(\xi)}{H_{\text{inst}}(\xi)} \right) \text{rect} \left( \frac{\xi}{\Delta \xi} \right), \quad (8)$$

where $\Delta \xi$ is the width of the low-pass filter. The input phase function in the spatial domain is given by the inverse Fourier transform of $\Phi_{\text{in}}(\xi)$. By rescaling the phase function with a constant $A$, another parameter for optimization is added to compensate for the loss of higher spatial frequencies. The approximate instrument input function that compensates for the PSF is given by

$$\phi_{\text{in}}(x) = A \mathcal{F}^{-1} \left( \frac{\Phi(\xi)}{H_{\text{inst}}(\xi)} \right) \text{rect} \left( \frac{\xi}{\Delta \xi} \right), \quad (9)$$

where $\mathcal{F}^{-1}$ is the inverse Fourier transform and $A$ is a scaling factor.

The two main fabrication errors that significantly degrade the performance of beam-splitting diffraction gratings are: (1) fabrication errors in the heights of the gratings $\Delta$ and (2)
the PSF of the writing instrument $k_{\text{inst}}(x)$. A method to optimize gratings in the presence of fabrication errors in the height and errors caused by PSF of the writing instrument is discussed in the following sections.

3. Optimization for fabrication errors $\Delta$ in height and fabrication results

3.1 Optimization

In this section, the instrument PSF $k_{\text{inst}}(x)$ is assumed to be narrow compared to the width of the spatial features in $\phi(x)$, so that it can be ignored. The method for optimizing the design of a beam-splitting phase grating that splits a beam into $N$ equal energy beams with less sensitivity to $\Delta$ is as follows: Find a function $\phi(x)$, where the energy for each of the $N$ desired beams is equal, assuming there is no fabrication error ($\Delta = 0$). For a range of fabrication errors, integrate the variance of the outgoing beam efficiencies. Find the phase function that minimizes the integrated variance for the specified range of fabrication errors. The problem reduces to finding a function $\phi(x)$ that minimizes the cost function

$$C[\phi(x)] = \int_{\Delta}^{\Delta} \sigma^2 \{\eta[\Delta, \phi(x)]\} d\Delta,$$

where $\sigma^2$ is the variance of the vector $\eta[\Delta, \phi(x)]$ and $\phi(x)$ is subject to the constraint $|\alpha_j|^2 = E/N$ when $\Delta = 0$. Since the efficiency is unitless, both the variance and the integrated variance are also unitless.

This method is tested on a 1x9 beam-splitting phase grating. The form of the phase function used to design beam-splitting phase gratings into odd numbers of beams is described analytically by [11]:

$$\phi_{\text{design}}(x) = \tan^{-1}\left(\frac{\alpha_j + \mu_j}{\alpha_j - \mu_j}\right),$$

$$P(x, \alpha, \mu) = 1 + 2 \sum_{j=1}^{M} \mu_j \cos(\alpha_j) \cos(jx),$$

$$Q(x, \alpha, \mu) = 2 \sum_{j=1}^{M} \mu_j \sin(\alpha_j) \cos(jx).$$

Note that the phase function is parameterized by the phase and amplitude of the corresponding sinusoids.

Due to the large size of the parameter space for a 1x9 grating, only a small subset of possible phase functions is randomly sampled with $M = 4$. Using a constrained nonlinear optimization solver in MATLAB [16], phase functions that satisfy the necessary constraints are calculated. Initial phase functions are generated by randomly choosing $\alpha$ and $\mu$ over $[0, 2\pi]$ and $[0, 4]$, respectively. The solver then minimizes the cost function, while satisfying equal energy constraints for the $\Delta = 0$ grating design. Since fabrication errors of 10% are typical, the phase function is optimized over $\Delta = [-0.1, 0.1]$. In order to sample a large enough portion of the parameter space and prevent a search of only local minima, this program is repeated with new initial conditions until 1,000 valid designs are calculated.

Two phase functions to test fabrication are chosen from the 1,000 designs found by the custom optimization program. The first design, grating 1, is the phase function with the minimum value for the integrated variance over the fabrication error range. This design has a relatively low $\Delta = 0$ grating efficiency of $E = 0.81$. Although the entire parameter space is not explored, this phase function should be sufficiently resistant to fabrication errors. The second design, grating 2, has a slightly worse value for the integrated variance, but the $\Delta = 0$ grating efficiency is $E = 0.95$. This design exhibited the best tradeoff between grating efficiency and insensitivity to fabrication errors.
These two designs are compared to the grating with the optimal efficiency as described by Romero and Dickey [11], which is referred to as grating 3. A plot of the variance of $\eta(\Delta, \phi(x))$ for each grating design is plotted as a function $\Delta$ in Fig. 1. Note that a slight error in the height of grating 3 significantly degrades the performance, and the other two grating profiles are significantly less sensitive to $\Delta$. The integrated variances for grating designs 1-3 are $C[\phi_{\text{design}}(x)]x10^6 = 5.71, 28.8,$ and $110$, respectively. While grating 3 provides the optimal efficiency $E_{\text{design}} = 0.99,$ the sensitivity to fabrication errors is approximately 19 times and 4 times more sensitive than gratings 1 and 2, respectively. The specifications for each of these gratings are listed in Table 1. The peak to valley phase difference for each grating is given by $\phi_{pv}$. $E_{\text{design}}$ is the designed efficiency for each grating.

Table 1. Grating Specifications

| $\alpha$               | $\phi_{pv}$ (rad) | $E_{\text{design}}$ | $C[\phi(x)]x10^6$ |
|------------------------|-------------------|----------------------|-------------------|
| Grating 1 (4.57, 1.91, 4.61, 5.33) | (1.39, 1.53, 1.14, 0.75) | 5.86 | 0.812 | 5.71 |
| Grating 2 (5.78, 3.56, 4.58, 1.39) | (1.03, 1.36, 1.23, 1.57) | 5.12 | 0.945 | 28.8 |
| Grating 3 (0.72, 5.57, 3.03, 1.41) | (0.971, 0.963, 0.943, 1.03) | 6.47 | 0.993 | 110 |

Fig. 1. Sensitivity plot for the three ideal 1x9 grating designs: grating 1 (red dotted line), grating 2 (green dashed line), and grating 3 (blue solid line). The variance of the energy $\sigma^2(\eta(\Delta, \phi_{\text{design}}(x)))$ is plotted as a function of the fabrication error $\Delta$. The cost function for each ideal grating evaluates to $C[\phi_{\text{design}}(x)]x10^6 = 5.71, 28.8,$ and $110$, respectively. Horizontal and vertical error bars correspond to $\sigma^2[\eta(\Delta_{\text{fab}}, \phi_{\text{fab}}(x))]$ for the measured range of $\Delta_{\text{fab}}$. The midpoint of each bar is the measured variance.

3.2 Fabrication and profile measurement

The gratings are fabricated using the University of Arizona maskless photolithography tool [17]. The grating period for each of the gratings is 147 μm, and the pixel spacing in the gray-scale photolithographic process is 2.1 μm. The gratings are designed for a wavelength of 488 nm. Height profiles for each fabricated grating are measured using a Bruker Nano NT 9800 optical profiler. The designed height profiles and the average measured height profiles with standard deviations from the measurements at 20 different positions are shown in Fig. 2(a). The difference between the average measured height profile and the designed height profile $\Delta h$ is shown in Fig. 2(a). The average peak-to-valley heights for each fabricated grating are $695 \pm 24$ nm, $624 \pm 16$ nm, and $765 \pm 24$ nm, as shown in Table 2. The designed heights for each grating are 740 nm, 646 nm, and 817 nm. The fabrication error for each fabricated grating is calculated from the height measurements using

$$\Delta_{\text{fab}} = \frac{h_{\text{fab}} - h_{\text{design}}}{h_{\text{design}}},$$

and is listed in Table 2.
Each grating is tested by illuminating with a collimated laser beam and placing a detector at the focal plane of a 150 mm lens. A profile of the diffraction pattern is measured using a BladeCam-XHR beam profiling camera. The fabricated grating is about 100 times larger than the incident laser beam, so diffraction patterns are measured when the incident beam is placed at 20 different locations on the grating. The average diffraction patterns are plotted in Fig. 2(b). The average measured diffraction patterns are normalized by scaling the mean peak irradiances to unity. Error bars on the peak irradiance of each outgoing beam are the standard deviations of the peak irradiances for all 20 measurements. After normalization, the standard deviation of the measured peak irradiances for each 20 measurements are calculated. Standard deviations $\sigma_{\text{fab}}$ of the normalized peak irradiances are $0.156 \pm 0.027$, $0.195 \pm 0.031$, and $0.201 \pm 0.047$ for gratings 1-3, respectively, as shown in Table 2. The uncertainty is the standard deviation of the measured variance of the peak irradiances from the 20 measurements for each grating. Differences between the minimum and maximum scaled peak irradiances $\delta_{\text{fab}}$ are calculated for each of the 20 locations and are $0.384 \pm 0.080$, $0.541 \pm 0.097$, and $0.597 \pm 0.140$ for gratings 1-3, respectively. The minimum, mean and maximum scaled peak irradiances for each grating are shown as solid lines in Fig. 2(b). To compare measured performance of the fabricated gratings to theoretical performance, variance is calculated after scaling the mean peak irradiances to the designed $E/N$. The measured variances after scaling to the designed $E/N$ and range of fabrication errors for each grating are indicated in Fig. 1. There is a significant disagreement with the theoretical variance curves, which is due to not accounting for $l_{\text{cut}}(x)$ in the design. Even though there are significant fabrication errors in grating 1, the $\delta_{\text{fab}}$ performance of grating 1 is nearly a factor of two better than grating 2 or 3.

The absolute diffraction efficiency of each grating is measured by calculating the ratio of energy in the desired outgoing beams to the energy in the incident laser beam. The measured absolute diffraction efficiency is reported in Table 2 as $E_{\text{fab}}$, which is lower than the design diffraction efficiency for several reasons. The first reason for a reduced $E_{\text{fab}}$ is due to Fresnel losses and absorption. A measurement resulted in $18 \pm 2\%$ of the incident light being lost due to Fresnel losses and absorption. Also, in the presence of fabrication errors in the grating profile, energy is transferred from the desired 9 outgoing beams into beams outside the desired range. A third reason is due to tool signature in the direction perpendicular to the grating profile, which causes slight fluctuations in the height of the grating in the perpendicular direction. Due to the tool signature, light diffracts in a direction perpendicular to the grating profile. This diffraction can result in multiple low amplitude copies of the diffraction pattern from the grating in the perpendicular direction, reducing the diffraction efficiency into the desired 9 outgoing beams. While $E_{\text{fab}}$ is significantly lower than $E_{\text{design}}$, the measured diffraction efficiency is ordered in the same way as the designed case. That is, grating 3 has the highest efficiency and grating 1 has the lowest. A summary of the performance for each grating is shown in Table 2.

| Grating  | $E_{\text{design}}$ | $E_{\text{fab}}$ | $h_{\text{design}}$ (nm) | $h_{\text{fab}}$ (nm) | $\delta_{\text{fab}}$ | $\sigma_{\text{fab}}$ | $\delta_{\text{fab}}$ |
|----------|----------------------|------------------|-------------------------|---------------------|-----------------|----------------|----------------|
| Grating 1 | 0.812                | 0.591 $\pm$ 0.011 | 740                     | 695 $\pm$ 24        | -0.062 $\pm$ 0.031 | 0.156 $\pm$ 0.027 | 0.384 $\pm$ 0.080 |
| Grating 2 | 0.945                | 0.679 $\pm$ 0.017 | 646                     | 624 $\pm$ 16        | -0.034 $\pm$ 0.024 | 0.195 $\pm$ 0.031 | 0.541 $\pm$ 0.097 |
| Grating 3 | 0.993                | 0.753 $\pm$ 0.021 | 817                     | 765 $\pm$ 24        | -0.063 $\pm$ 0.030 | 0.201 $\pm$ 0.047 | 0.597 $\pm$ 0.140 |
4. Effect of the point spread function during the writing process

The effect of $h_{\text{inst}}(x)$ is most evident if the theoretical and measured grating profiles of grating 1 are compared in Fig. 2(a). The sharp falloff of the central portion of the designed grating profile is significantly smoothed out in the measured profile. We assume $h_{\text{inst}}(x)$ is approximately $h_{\text{inst}}(x) = e^{-x^2/\sigma^2}$, due to the design properties of the writing instrument.

In order to study the effect that the finite beam width of the laser has on the performance of the beam-splitting gratings from Section 3, the designed grating profiles are convolved
with a beam having a Gaussian shape. Based off the measurements of the fabricated grating profiles, the beam width is estimated to be 0.0205 \( T \), where \( T \) is the period of the grating. Using these convolved grating profiles, the sensitivity is considered again. A plot of the variance of outgoing beams \( \sigma^2 \{ \eta[\Delta, \phi_{\text{conv}}(x)] \} \) for each theoretical convolved grating design is plotted as a function of the fabrication error \( \Delta \) in Fig. 3. The integrated variances for gratings 1-3 are \( C[\phi_{\text{conv}}(x)] \times 10^6 = 20.6, 60.9, \) and 126, respectively, which are significantly higher than the originally designed gratings. Note that the measured variance and measured fabrication error now agree very well with the theoretical values for the variance, which indicates that the point spread function of the laser beam during the photolithographic process has a significant impact on the performance of the diffraction gratings. Also, note that it is now impossible for \( \sigma^2 = 0 \), which indicates it is impossible to achieve equal energy in the outgoing beams with these designs. Therefore, a significant performance improvement can be realized if the gratings are optimized in order to account for the PSF of the writing instrument.

5. Optimization for point spread function and fabrication results

5.1 Optimization

The steps for optimizing the design of a phase grating that splits a beam into \( N \) equal energy beams with less sensitivity to the PSF of the writing instrument and \( \Delta \) are as follows: Define a phase function \( \phi(x) \) by the parameters \( \alpha \) and \( \mu \). Deconvolve \( \phi(x) \) to obtain \( \phi_{\text{in}}(x) \). Find a function \( \phi_{\text{out}}(x) \) when convolved with the estimated PSF, \( e^{-x^2/w_{\text{est}}^2} \), where \( w_{\text{est}} \) is the estimated beam width, yields \( N \) equal energy outgoing beams, assuming there is no height fabrication error (\( \Delta = 0 \)). Since there may be an error in the estimation of the beam width \( w \), the grating is optimized for the \( w_{\text{est}} \) variable. There are also fabrication errors in the height of the gratings. Therefore, the grating is also optimized to minimize the sensitivity to errors in the height \( \Delta \). Variance of the outgoing beam efficiencies is integrated over a range of beam widths and fabrication errors in height. Find a phase function that, when deconvolved, minimizes the integrated variance for the specified range of fabrication errors. The problem reduces to finding a function \( \phi(x) \) that minimizes the cost function

\[
C[\phi(x, \alpha, \mu, A)] = \int_{w_{\text{est}}}^{w_{\text{est}}} \int_{-0.1}^{0.1} \sigma^2 \{ \eta[\Delta, \phi_{\text{in}}(x, \alpha, \mu, A) \ast e^{-x^2/w_{\text{est}}^2}] \} \, d\Delta \, dw
\]

subject to the constraint \( \int_{-0.1}^{0.1} [\alpha, \mu] = E/N \) when \( \Delta = 0 \) and \( w = w_{\text{est}} \). This constraint ensures equal energy output beams assuming the PSF was estimated correctly and there are no fabrication errors in the height. A workflow diagram for the algorithm used to minimize the cost function is shown in Fig. 4.

For the optimization, initial phase functions are generated by randomly choosing \( \alpha \) and \( \mu \) over \([0, 2\pi]\) and \([0, 4]\), respectively. The scaling factor \( A \) is initially set to unity. The estimated beam width is determined by minimizing the difference between the measured grating profiles from Section 3 and a simulated profile when the designed profile is convolved with \( h_{\text{out}}(x) \). The beam width that minimizes this difference is \( w_{\text{est}} = 0.0205 \, T \), where \( T \) is the period of the grating. The phase function is optimized over \( \Delta = [-0.1, 0.1] \), and the range of beam widths considered in the optimization is \( w = [0.015 \, T, 0.026 \, T] \). The range of beam widths is chosen based on knowledge of the expected fluctuations of the PSF of the writing instrument. The cutoff frequency for the low-pass filter used in the deconvolution is half the Nyquist frequency. The solver minimizes the cost function while satisfying the equal energy constraint when \( \Delta = 0 \) and \( w = w_{\text{est}} \). In order to sample a large enough portion of the parameter space and prevent a search of only local minima, this program is repeated with new initial conditions until 1,000 valid designs are calculated.
Two phase functions to test fabrication are chosen from the 1,000 calculated designs found by the custom optimization program. The specifications for these gratings are listed in Table 3. The first design, grating 1, is the phase function with the minimum value for the integrated variance over the fabrication error range. This design has a relatively low $\Delta = 0$, $w = w_{ext}$ grating efficiency of $E = 0.741$. Although the entire parameter space is not explored, this phase function should be sufficiently resistant to fabrication errors. The second design, grating 2, has a slightly worse value for the integrated variance, but the $\Delta = 0$, $w = w_{ext}$ grating efficiency is $E = 0.945$. This design exhibited the best tradeoff between grating efficiency and insensitivity to fabrication errors. The third design is the optimal efficiency design re-optimized to account for the effect of $h_{in}(x)$ and has a $\Delta = 0$, $w = w_{ext}$ grating efficiency of $E = 0.993$.

Since the optimization algorithm is optimizing over two parameters, a plot of the variance of the output beams is now three-dimensional. It depends on the beam width of the PSF and $\Delta$. A three-dimensional plot of the variance of the output beams is displayed in Fig. 5(a). Like the case for optimization with $\Delta$ only, grating 3 is significantly more sensitive to fabrication errors than grating 1 or 2. A density plot of the variance of the output beams is shown in Fig. 5(b). The integrated variances for gratings 1-3 are $C[\phi_{design}(x)]10^6 = 15.3$, 47.0 and 157, respectively. Grating 3 is approximately ten times and three times more sensitive than gratings 1 and 2, respectively. The designs are not as sensitive to $w$ as compared to $\Delta$ over the ranges chosen.

| Table 3. Optimized Grating Parameters for Estimated Beam Width $w_{ext} = 0.0205$ $\mu m$ |
|---|---|---|---|---|---|
| Grating 1 | $5.31, 4.48, 4.99, 1.16$ | $1.16, 1.24, 0.88, 0.55$ | 6.77 | 0.741 | 15.3 |
| Grating 2 | $5.78, 3.56, 4.58, 1.40$ | $1.03, 1.35, 1.23, 1.57$ | 5.09 | 0.945 | 47.0 |
| Grating 3 | $0.72, 5.56, 3.03, 1.41$ | $0.973, 0.963, 0.945, 1.03$ | 6.46 | 0.993 | 157 |

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5.2 Fabrication and profile measurement

All three gratings are fabricated using maskless lithography with the same parameters as the gratings in Section 3. The designed height profiles after convolution with the estimated PSF and the average measured height profiles with standard deviations measured at 20 different positions are shown in Fig. 6(a). The average peak-to-valley heights for each fabricated grating are 780 ± 24 nm, 650 ± 12 nm, and 807 ± 23 nm. The designed heights for each grating are 854 nm, 643 nm, and 815 nm, respectively. The difference between the average measured height profiles and the designed height profile Δh is also shown in Fig. 6(a). From the measured grating plots, the ringing in the grating profiles is indicative that the beam width of the PSF was estimated incorrectly. By analyzing the measured grating profiles, the beam width during the writing process was determined to be w_{fab} = 0.017 \mu m, instead of the assumed value w_{est} = 0.0205 \mu m used during the optimization. The incorrect beam width estimation results in the ringing seen in gratings 1 and 2. Also, note the significant errors in the fabrication of grating 2, where the fabricated profile is steeper than the theoretical profile. Slices of the plots in Fig. 5 at the fabricated beam width w_{fab} = 0.017 \mu m are shown in Fig. 7.

5.3 Testing

The diffraction pattern for each grating is tested using the same methods as described in Section 3. The diffraction patterns for each grating are shown in Fig. 6(b). The standard deviations σ_{fab} are 0.088 ± 0.018, 0.058 ± 0.009, and 0.140 ± 0.064 for gratings 1-3, respectively. The measured variance for each grating calculated after scaling the mean peak irradiances to the designed E/N is plotted in Fig. 7 at the measured Δ_{fab}. Note that the diffraction pattern for grating 2 is very close to equal energy, as shown in the zoomed-in portion of the variance of grating 2 in Fig. 7. The differences δ_{fab} between the minimum and maximum scaled peak irradiances is calculated for each of the 20 locations and are 0.252 ± 0.042, 0.168 ± 0.030, and 0.396 ± 0.170 for gratings 1-3, respectively. The measured absolute diffraction efficiencies for gratings 1-3 are 0.628 ± 0.020, 0.729 ± 0.049, and 0.729 ± 0.013, respectively. In the presence of fabrication errors, grating 3 has the same measured diffraction efficiency as grating 2. These diffraction efficiencies are lower than the theoretical efficiencies for the same reasons listed in Section 3. A summary of the performance for each grating is shown in Table 4.

| Grating  | E_{design} | E_{fab} | h_{design} (nm) | h_{fab} (nm) | Δ_{fab} | σ_{fab} | δ_{fab} |
|----------|------------|---------|----------------|-------------|---------|---------|---------|
| Grating 1 | 0.741      | 0.628 ± 0.020 | 854 | 780 ± 24 | −0.086 ± 0.012 | 0.088 ± 0.018 | 0.252 ± 0.042 |
| Grating 2 | 0.945      | 0.729 ± 0.049 | 643 | 650 ± 12 | 0.010 ± 0.003 | 0.058 ± 0.009 | 0.168 ± 0.030 |
| Grating 3 | 0.993      | 0.729 ± 0.013 | 815 | 807 ± 23 | −0.010 ± 0.019 | 0.140 ± 0.064 | 0.396 ± 0.170 |

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Fig. 6. (a) Plot of the design grating profile after convolution with the estimated PSF (solid blue line) and measured average fabricated grating profile (dashed red line). Above each grating profile is the difference between the two profiles $\Delta h$ in nanometers. (b) Measured diffraction pattern for each fabricated diffraction grating is scaled so the mean peak irradiance is equal to the designed beam efficiency. For comparison, the red dotted, green dashed, and blue solid horizontal lines are the minimum, mean, and maximum peak irradiances for each grating.

Fig. 7. Sensitivity plot for the three theoretical convolved gratings with estimated beam width of 0.017 $T$; grating 1 (red dotted line), grating 2 (green dashed line), and grating 3 (blue solid line). The variance of the energy of the outgoing beams is plotted as a function of the fabrication error $\Delta$ as a beam travels through a theoretical convolved grating. Horizontal and vertical error bars correspond to $\sigma^2[\Delta_{\text{fab}}, \phi_{\text{fab}}]$ for the measured range of $\Delta_{\text{fab}}$. The box on the left graph is expanded on the right graph, where the measured variance and range of $\Delta$ is shown for grating 2.

By taking $\delta_{\text{design}}(x)$ into account during optimization, the performance of all three gratings is improved, even in the presence of a 20% error in the estimation of the beam width of the PSF. The values for $\sigma_{\text{fab}}$ in Table 4 are smaller than the values in Table 2 by a factor of 0.56, 0.30, and 0.70 for gratings 1-3, respectively. The values for $\delta_{\text{fab}}$ are smaller by a factor of 0.66, 0.31, and 0.66 for gratings 1-3, respectively. Grating 2 has the best performance and the best improvement with values that are smaller by a factor of 0.3. Compared to grating 3 in Table 2, the performance of the optimized grating 2 is improved by a factor of 3.4 and grating 1 is improved by a factor of 2.3. By looking at the measured profile for grating 3 in Fig. 6(a), it can be seen that the profile is very close to the theoretical profile, but the diffraction pattern is very far from equal energy output beams. If grating 2 is considered, there are significant fabrication errors, notably the ringing in the middle portion of the grating and the incorrect steepness of the edge of the grating; however, the performance of this grating is very close to...
equal energy output beams. For grating 1, there is a very large error in the height of the grating profile, yet it still performs better than grating 3. Therefore, the optimization algorithm developed in this paper creates grating designs that are robust to fabrication errors in height and can be used to account for the PSF during the writing process. While grating 3 theoretically provides the optimal efficiency of light into the desired 9 outgoing beams, this paper demonstrates that this design is extremely sensitive to fabrication errors and may be too difficult to fabricate. Therefore, a design that has a slightly lower efficiency, but is more robust to fabrication errors is more desirable.

6. Conclusion

In conclusion, two significant factors that degrade the performance of continuous phase gratings are identified, which are errors in the height of the grating profiles and errors caused by the finite beam width of the PSF during the writing process. A method for optimizing the grating design in the presence of these fabrication errors is developed. These optimized grating designs sacrifice total efficiency, but they provide lower sensitivity to fabrication errors. Using the numerical results for a 1x9 beam-splitting grating, three different gratings are fabricated. Two gratings are optimized for fabrication and are compared to the grating optimized for maximum efficiency.

An optimization algorithm that only reduces sensitivity to fabrication errors in the height of the gratings is developed. It is shown by fabricating three gratings that the finite beam width of the PSF during the photolithographic writing process significantly decreases the performance of the gratings. A second optimization algorithm that reduces sensitivity to fabrication errors in the height and sensitivity to the PSF of the writing instrument is developed. Three gratings are fabricated using the results from this optimization algorithm. Once again, both gratings using the developed algorithm perform better than the optimal efficiency grating. Measured performance of the best grating fabricated using the developed algorithm is 3.4 times better than the optimal efficiency grating, with only a slight reduction (0.945 compared to 0.993) in the theoretical efficiency. The measurement of $\sigma_{\text{fab}}$ is 0.058 ± 0.009 for the best grating fabricated, whereas $\sigma_{\text{fab}}$ is 0.201 ± 0.047 for the optimal efficiency grating. Therefore, in the presence of fabrication errors, the optimal efficiency design may not be desirable since the high sensitivity to fabrication errors may yield too large of a deviation from equal energy output beams. Depending on the accuracy of the writing instrument, a less efficient grating design with less sensitivity to fabrication errors may be desirable. This paper demonstrates an effective algorithm to design gratings with less sensitivity to fabrication errors.

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