The spatial distribution of nearby galaxy clusters in the northern and southern galactic hemispheres

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Abstract. We compare the spatial distributions of galaxy clusters in the northern and southern galactic hemispheres, and the Abell and ACO clusters distributions. We perform a statistical (correlation and cluster) analysis of a sample of Abell and ACO galaxy clusters in the southern galactic hemisphere ($b_{II} \leq -40^\circ$ and $m_{10} \leq 16.5$). We compare these results with a symmetric sample ($b_{II} \geq +40^\circ$ and $m_{10} \leq 16.5$) at northern galactic latitude taken from Postman et al. (1992). For the northern sample, we substantially confirm the results of Postman et al. We find that the two-point spatial correlation function $\xi_{cc}(r)$ of northern and southern clusters is comparable, with mean correlation length $r_0 \sim 19.6 \, h^{-1} \text{ Mpc}$ and slope $\gamma \sim -1.8$. Moreover, ACO and Abell clusters show similar spatial correlations. $\xi_{cc}(r)$ is positive up to $\sim 45 \, h^{-1} \text{ Mpc}$ in all our samples, and it is systematically negative in the range $50 \lesssim r \lesssim 100 \, h^{-1} \text{ Mpc}$. Percolation properties are remarkably similar in the northern and southern cluster samples. We give also a catalog of superclusters. In the south galactic hemisphere the main feature is a very rich, extended supercluster (spreading over $\sim 65 \, h^{-1} \text{ Mpc}$) in the Horologium region at a redshift $z \sim 0.06$, near to a large void.

Key words: galaxies: clustering – large-scale structure of the Universe

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1. Introduction

Galaxy cluster catalogs offer us the possibility to study the large-scale structure of the Universe in much larger volumes than presently available galaxy catalogs, reaching depths beyond $z \sim 0.2$ (see Huchra et al. 1990). Abell himself (1958) observed that clusters were not distributed uniformly on the sky, indicating a second-order clustering. Hauser & Peebles (1973) calculated the two-point angular correlation function $w_{cc}(\theta)$ of Abell clusters, finding a strong correlation. Bahcall & Soneira (1983) and Klypin & Kopylov (1983) calculated the two-point spatial correlation function $\xi_{cc}(r)$ for cluster samples with measured redshifts, finding an amplitude $\sim 18$ times higher than that of galaxies. Postman et al. (1986) recalculated $\xi_{cc}(r)$, considering the effect of the Corona Borealis supercluster, and while finding a somewhat lower amplitude confirmed the strong clustering of Abell (and Zwicky) clusters.

To have an all-sky catalog, galaxy clusters with $\delta \leq -17^\circ$ have been independently selected on deeper and more sensitive J plates (Abell, Corwin, Olowin, 1989). The two-point angular correlation function $w_{cc}(\theta)$ of ACO clusters has already been determined by Bahcall et al. (1988), Couchman et al. (1988), Batuski et al. (1989; hereafter BBOB). Also the three-point angular correlation function has been estimated (Tóth et al. 1989, Jing & Zhang 1989). Moreover, McGill & Couchman have calculated the spatial correlation function of $RC \geq 1$ ACO clusters by inverting $w_{cc}(\theta)$ – assuming a model for $\xi_{cc}(r)$ –, a technique subject to quite large uncertainties.

The above authors generally find comparable results for Abell and ACO clusters for what concerns $w_{cc}(\theta)$ taken at its face value –i.e., without correction for projection effects–.

Given the importance of the existence of structures for the theories of galaxy formation, and given in particular the difficulty of the CDM theory to account for the power at large scales shown by the observed cluster distribution, it was natural to ask if clusters are reliable indicators of the large-scale structure.

Indeed there is presently a debate about the reliability of Abell and ACO cluster catalogs. Projection effects might be important (see Lucey 1983, Sutherland 1988, Dekel et al. 1989, Olivier et al. 1990, Sutherland & Efstathiou 1991): they can artificially increase the amplitude of the 2-point correlation function. However, Szalay et al. (1989) don’t find such strong projection effects, and X-ray selected clusters (Lahav et al. 1989), and cD
clusters (West & van den Bergh 1991), which don’t rely on the same selection criterion of Abell clusters, show a strong correlation. Struble & Rood (1991a) examined a sample of clusters with measured redshift and found contamination only in 3% of the 1682 Abell clusters in the statistical sample. Moreover, Jing et al. (1992) find evidence that real clustering and not contaminations can be the origin of the positive redshift correlations at large redshift and small angular separations.

It is usually assumed that this problem will be clarified by the availability of catalogs of clusters derived from an automated search, like the recent APM survey of galaxy clusters. Nevertheless, it will be necessary to assess how much these automated catalogs are better than the classical, Abell and ACO catalogs, and which kind of systems they are sampling—for example, the APM survey contains only poor clusters, and this could affect their correlations (Bahcall & West, 1992).

Very recently, not only $\xi (r)$ has been subject to some criticism, but also the result of Postman et al. (1992; hereafter PHG92), who find $r_0 = 20.6 \, h^{-1} \, \text{Mpc}$ for a sample of 208 clusters with $|b| \geq 30^\circ$, has been questioned by Efstathiou et al. (1991; hereafter EDSM), who find for the same data $r_0 = 17.4 \, h^{-1} \, \text{Mpc}$, and, after correcting for projection effects, $r_0 = 13 \, h^{-1} \, \text{Mpc}$. This discrepancy between PHG92 and EDSM for $r_0$ of clusters is important because, quoting EDSM themselves, “the differences in the respective estimates for $\xi (r)$ in the region where $\xi \sim 1$ are similar to the changes caused by the corrections for anisotropies”.

In this paper we will calculate and compare the “observed” $\xi (r)$ of Abell and ACO clusters in two symmetric samples. In this way we explore different regions of the sky; this is an important test, because the presence of structures like Corona Borealis can affect the statistical analysis (Postman et al. 1986). We have also the opportunity of estimating the spatial $\xi (r)$ for a sample made only of ACO southern clusters. This directly estimated ACO $\xi (r)$ can be compared with that of Abell clusters.

Moreover, in order to check if the spatial distributions of clusters in the northern and southern galactic hemispheres are statistically comparable we perform also cluster analysis (Einasto et al. 1984, Tago, Einasto, Saar 1984), and we search for southern superclusters, as an extension of the PHG92 catalog.

In section 2, we describe our chosen samples and the reasons of this choice. In section 3 we deal with their correlation functions; in section 4 we analyse their percolation properties and we give a catalog of superclusters, with three different contrasts. Our conclusions are in section 5.
In what follows, we assumed $H_0 = 100h\ \text{km sec}^{-1}\ \text{Mpc}^{-1}$ and $q_0 = 0.1$, which are standard values used by other authors.

2. The samples

Up to recent times, a statistical analysis of clusters in the southern galactic hemisphere has been impossible because of the $\delta$ limit ($\delta \geq -27^\circ$) of Abell catalog. The Abell, Corwin & Olowin (ACO, 1989) catalog of southern clusters has offered the opportunity to extend our knowledge of the distribution of rich galaxy clusters.

In order to determine distances of clusters and to study their internal dynamics, a large quantity of cluster redshifts has been measured in the last years, in both the northern and southern hemispheres.

We collected published redshifts for Abell clusters from Struble & Rood (1991b) and PHG92; for ACO clusters from Muriel et al. (1990, 1991), Vettolani et al. (1989), Cappi et al. (1991), and again PHG92 (who give redshifts of 15 ACO clusters).

We selected all clusters with $m_{10} \leq 16.5$, $|b_{II}| \geq 40^\circ$, and richness class $RC \geq 0$.

The choice of $|b_{II}| \geq 40^\circ$ allows us to have two complete, equal-volume samples in the northern and southern galactic hemispheres. Moreover, this symmetry minimizes any effect due to the galactic latitude selection function.

The magnitude limit allows to avoid problems of contamination from foreground / background galaxies, and to dispose both of a northern sample where all clusters have measured redshifts, and a southern sample where 90% of clusters have measured redshifts. Of course, a magnitude limit corresponds to a flux limit; however, up to $z \sim 0.08$ cluster density is approximately constant, so we will limit our samples at that redshift (see discussion in PHG92).

Our southern subsample (S40) includes 130 clusters with $b_{II} \leq -40^\circ$ and $m_{10} \leq 16.5$; only 14 of them don’t have any measured redshift. For these 14 clusters we used the $\log(cz) - m_{10}$ relation, as given by Scaramella et al. (1991, hereafter SZVC; see their equation 1). In this sample there are 60 Abell clusters and 70 ACO clusters.

In an analogous way we extract from the PHG92 sample all clusters with $b_{II} \geq +40^\circ$ and $m_{10} \leq 16.5$, with 103 clusters (N40). This represents a totally symmetric northern sample.

We consider also a southern subsample (ACO) limited at $b_{II} \leq -40^\circ$, $\delta \leq -27^\circ$, which includes 66 (all ACO) clusters.

Finally we reanalyse the PHG92 statistical sample (NST), with 208 clusters ($|b_{II}| \geq 30^\circ$, $\delta \geq -27^\circ.5$, $m_{10} \leq 16.5$), for which PHG92 and EDSM find discrepant results.
Therefore we have decided to make another, independent estimation of $\xi_{cc}(r)$ for this sample.

**Table 1.** Parameters of the samples ($z_{lim} \leq 0.08$, $m_{10} \leq 16.5$)

|        | N40 | S40 | ACO | NST |
|--------|-----|-----|-----|-----|
| $N_c$  | 103 | 130 | 66  | 208 |
| $z_{mean}$ | 0.0558 | 0.0566 | 0.0547 | 0.0566 |
| $n_c$ ($h^3$ Mpc$^{-3}$) | 1.1E-05 | 1.4E-05 | 1.6E-05 | 1.1E-05 |
| $\gamma$ | -1.90 | -1.71 | -1.71 | -2.02 |
| (+/−)  | 0.42 | 0.33 | 0.53 | 0.37 |
| $r_0$  | 19.4 | 19.7 | 20.1 | 19.2 |
| (+/−)  | 5.1/4.2 | 3.3/2.8 | 6.6/5.1 | 3.5/3.0 |

In figures 1 and 2 we show the projected distributions of N40 and S40 clusters, while in fig. 3 we show their redshift distribution: large structures are apparent.

We report in table 1 the main characteristics of each subsample. Note the difference in cluster density of the northern and southern subsamples, corresponding to a $3\sigma$ poissonian deviation. It has already been noticed from an analysis of clusters with estimated distance $D \leq 300$ $h^{-1}$ Mpc that ACO clusters have a higher density (BBOB, SZVC). It is therefore interesting to verify if there are other differences which can be revealed by statistical analysis.

3. Correlation Analysis

3.1. Description of the method

We calculated positions converting redshifts to distances in Mpc using the Mattig formula for $q_0 > 0$ (Mattig, 1958; Weinberg 1972):

$$r = \frac{c}{q_0 H_0} \left[1 - q_0 + q_0 z + (q_0 - 1) \sqrt{2q_0 z + 1}\right]/(1 + z)$$  \hspace{1cm} (1)

where we assumed, as we noted previously, $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$ and $q_0 = 0.1$.

Clusters don’t have high peculiar motions ($\leq 1000$ km/s, Huchra et al. 1990). Cluster redshifts can be considered reasonably accurate when at least $\sim 4$ cluster galaxies have measured redshifts; however, even in the case that only one galaxy has a measured redshift, this is usually the brightest member, probably at the potential bottom of the cluster.
We used as estimator of $\xi_{cc}(r)$ the formula:

$$\xi_{cc}(r) = 2\frac{N_{\text{ran}} N_{cc}(r)}{N_{\text{clu}} N_{cr}(r)} - 1$$

(2)

where $N_{cc}(r)$ is the number of cluster-cluster pairs at a distance $r$, $N_{cr}(r)$ is the number of cluster-random point pairs (this is the best way to avoid edge effects due to the limited size of tested volume), $N_{\text{ran}}$ is the total number of random points and $N_{\text{clu}}$ is the total number of objects. Equation 2 is the standard estimator used to calculate the two-point correlation function for both galaxies and clusters. We assumed $N_{\text{ran}} = N_{\text{clu}}$, and we generated 100 random catalogs each of them with the same number of objects as that in the real sample, then averaging, in order to have a good estimate of $N_{cr}(r)$. Random objects were distributed uniformly taking into account the non-euclidean geometry. Observed redshifts were assigned to random points with a gaussian smoothing ($\sigma = 3000$ km/s) to avoid possible effects of incompleteness in our estimate of the correlation function. The result is of course a lower amplitude than in the case we had assumed no selection effect; while if we did not have smoothed redshifts, some small-scale clustering would have been reproduced in random catalogs, thus lowering the amplitude. But the particular choice of $\sigma$ is not critical, as noted by PHG92, if $3000 \leq \sigma \leq 6000$ km/s.

We adopted a logarithmic step ($\Delta \log(r)$ between 0.1 and 0.2 depending on samples). As a check, we recalculated $\xi_{cc}(r)$ with a linear step (5 $h^{-1}$ Mpc up to $r = 60$ $h^{-1}$ Mpc, then doubling the step to increase the signal to noise ratio).

Given the high cut in galactic latitude, the use of the selection function

$$P(b) = \text{dex} \left[ C (1 - \csc |b|) \right]$$

(3)

where $C = 0.3$ for Abell clusters and $C = 0.2$ for ACO clusters, has negligible effects. Anyway we estimated $\xi_{cc}(r)$ taking $P(b)$ into account.

We took into account also the $\delta$ selection function $P(\delta)$, using the expression given by BBOB respectively for Abell clusters:

$$P_{\text{Ab}}(\delta) = \begin{cases} 1 & 24^\circ \leq \delta \leq 90^\circ \\ 0.675 + 0.01125\delta & -27^\circ < \delta < +24^\circ \end{cases}$$

(4)

and for ACO clusters:

$$P_{\text{ACO}}(\delta) = \text{dex} \left[ 0.6(\cos |\delta| - 1) \right] \quad -75^\circ > \delta > -20^\circ$$

(5)

BBOB note that this effect may be partly due to real superclustering, partly to a spur of galactic obscuration, or probably to higher air mass at low declinations (if this is the
correct explanation, then SZVC show that it can be described as a function of zenithal distance –see their equation 6–).

As in the case of \( P(b) \), \( P(\delta) \) does not change significantly the results: the two above selection functions represent second-order corrections.

We estimated errors through the bootstrap resampling method (Ling, Frenk & Barrow 1986); these are the errors we show in graphs and used for best-fitting. The optimal way of determining errors can be matter of discussion, but it is known that bootstrap errors are more realistic than poissonian errors. We checked that best-fit parameters of \( \xi_{cc}(r) \) do not change significantly by using poissonian errors.

We fitted points with a least-square method, without imposing any slope. The choice of the range is important: values of \( r_0 \) and \( \gamma \) depend on it. The correlation function of our samples drops beyond \( 40 \ h^{-1} \) Mpc and below \( \sim 5 \ h^{-1} \) Mpc, deviating from the expected power-law. At small scales, below about \( 5 \ h^{-1} \) Mpc, we have very few clusters at small separations –their small number is attested also by the large error bars–. Given that clusters have linear Abell diametres of \( 3 \ h^{-1} \) Mpc, below that scale the correlation function is not meaningful.

It is also clear that we cannot significantly sample pairs at scales comparable to the size of the sample, i.e. typically beyond \( \sim V^{1/3}/2 \), where \( V \) is the total volume.

We have chosen to fit data in the range \( 5 < r < 40 \ h^{-1} \) Mpc, where \( \xi_{cc}(r) \) has a significant signal –in that range it is always positive, being the first negative value at \( 50 \ h^{-1} \) Mpc–.

In figures 4, 5 and 6, we report \( \xi_{cc}(r) \) for our four samples, compared with the power-law \( \xi_{cc}(r) = (r/20)^{-2} \) (as EDSM did). We assumed a logarithmic step \( \Delta \log(r) = 0.12 \) for all samples except for the NST sample, for which \( \Delta \log(r) = 0.1 \) (we will discuss below the consequences of adopting a different step). Uncertainties on \( \gamma \) are derived from the fit, while \( (1 \sigma) \) errors on \( r_0 \) are derived directly from the same pseudo-data catalogs used for calculating bootstrap errors on the correlation function.

3.2. Discussion of results

Our best-fit values for \( \gamma \) and \( r_0 \) are shown in table 3. In our fixed range slopes vary from -1.7 to -2.0 depending on samples. They are all very similar (needless to say that the equal slopes of the ACO and S40 correlations are a coincidence).

We find \( r_0 \sim 19.4 \ h^{-1} \) Mpc for the N40 sample and \( r_0 \sim 19.7 \ h^{-1} \) Mpc for the S40 sample. ACO clusters appear to have a correlation radius slightly larger (\( r_0 \sim 20.1 \ h^{-1} \) Mpc) and a flatter slope than Abell clusters; anyway this small difference is less than
$1\sigma$, and the presence of a very rich supercluster (see next section) gives a significant contribution to the amplitude of $\xi_{cc}(r)$ in this small sample.

To check the dependence of our results on the richness class, we recalculated $\xi_{cc}(r)$ of clusters with $RC \geq 1$. Of course, having only 49 northern in the N40 sample and 56 southern clusters in the S40 sample, $\xi_{cc}(r)$ is more noisy; for that reason we have chosen a logarithmic step $\Delta \log(r) = 0.2$. We obtain results which are fully consistent with those obtained for the larger samples (fig.7). Amplitude and slopes are very similar to the total samples. Therefore the southern sample S40 is a further confirmation of PHG92 result: there is no clear richness effect in the distribution of nearby clusters, at least between $RC = 0$ and $RC \geq 1$ clusters.

It is striking the similarity of the correlation functions in the two opposite galactic hemispheres, and of ACO and Abell clusters. $r_0$ is much larger than that of galaxies, and consistent with other published values of northern samples. It is anyway smaller than the value of $30 \, h^{-1}$ Mpc resulting from the indirect estimate of McGill & Couchman.

We detect in all subsamples a positive signal only up to $45 \, h^{-1}$ Mpc (for the estimate in linear step; up to $\sim 40$ for the logarithmic step): no superclustering is detected beyond that scale.

We note that all the above samples show an anticorrelation between 50 and 100-120 $h^{-1}$ Mpc, where $\xi_{cc}(r) \sim -0.1$. For all the 4 samples -and also for the $R \geq 1$ N40 and S40 samples- the first negative point is at $r \sim 50 \, h^{-1}$ Mpc.

In order to visualize this effect of anticorrelation at large scales we plot $1 + \xi_{cc}(r)$ for our 4 samples (fig.8). We do not report errors to avoid confusion: it should already be clear to the reader that all points are well within $1\sigma$ error bars.

The strongest anticorrelation is shown by the N40 and NST samples in the bin centered at $r \sim 60 \, h^{-1}$ Mpc (respectively $\xi_{cc}(r) \sim -0.3$ and -0.14); by the S40 and ACO samples in the next bin, centered at $r \sim 80 \, h^{-1}$ Mpc (respectively $\xi_{cc}(r) \sim -0.18$ and -0.22). The effect is slightly more than $1\sigma$, but the same behaviour is shown also by the N40 and ACO samples, which are completely independent, and have different boundaries. Calculating $\xi_{cc}(r)$ for the NST sample with a cut in redshift of $z = 0.06$ and $z = 0.07$ we continue to find the same effect with the strongest anticorrelation at the same value of $r$ ($\sim 60 \, h^{-1}$ Mpc).

This effect has been previously detected in the angular correlations: Bahcall et al. (1988) and BBOB have shown that for two samples of respectively Abell and ACO clusters limited at an estimated distance $D \leq 300 \, h^{-1}$ Mpc, $\omega_{cc}(\theta)$ becomes negative ($\sim -0.18$) at $\theta \sim 12^\circ$, corresponding to $\sim 50 \, h^{-1}$ Mpc. Their deeper samples do not show any
anticorrelation. They have suggested that this anticorrelation of nearby clusters might be due to the presence of underdense regions between more clustered regions, and as an example they indicate the region at \( l_{II} \sim 230^\circ \) and \( b_{II} \sim -50^\circ \). Alternatively, we note that it might be a spurious effect. We do not know the “universal” mean density of clusters, therefore it is assumed \( n = N_c/V_T \), where \( N_c \) is the total number of clusters and \( V_T \) is the volume of the sample. The mean number of clusters at a distance \( r \) from a randomly chosen cluster is

\[
<N> = nV + n \int_0^r \xi_{cc}(r) dV
\]

where \( n \) is the mean density (see Peebles, 1980). When \( r \) includes the whole sample, the number of neighbors is \( N_c - 1 \), while \( nV = N_c \) with our choice of \( n \). Then the above integral constraint implies that \( \xi_{cc}(r) \), being positive at small scales, is forced to be negative at large scales.

Now we will investigate the reasons of the discrepancy between EDSM and PHG92. In fig. 3 we have shown \( \xi_{cc}(r) \) for the PHG92 statistical sample: we agree well with the results of PHG92. Indeed, we do not find positive points between 50 and 100 \( h^{-1} \) Mpc, where PHG92 find some positive signal, e.g. at \( \sim 70 h^{-1} \) Mpc. This difference can be easily explained. Our bins are equal to those of PHG92 (0.1), but we have a different zero point. If we shift our zero point of one half step we find a positive point at \( r \sim 70 h^{-1} \) Mpc (see figure 3). We verified that using \( N_{rr}(r) \) –as PHG92– instead of \( N_{cr}(r) \) there are no significant differences: \( \xi_{cc}(r) \) is only slightly higher especially at larger scales, where edge effects become stronger (at the point at \( \sim 70 h^{-1} \) Mpc \( \xi_{cc}(r) \) reaches a level of \( \sim 0.1 \)). Our \( r_0 \) (19.2 \( h^{-1} \) Mpc) is smaller than that found by PHG92 (20.6 \( h^{-1} \) Mpc) because they took into account all points up to 75 \( h^{-1} \) Mpc. For example, fitting in a larger range, \( 1.5 < r_0 < 45 h^{-1} \) Mpc, we find \( r_0 = 19.4 h^{-1} \) Mpc and \( \gamma = -1.89 \), which are more similar to the values found by PHG92.

Why then did EDSM find \( r = 17.4 h^{-1} \) Mpc and the last positive point at \( r \sim 30 h^{-1} \) Mpc using a very similar estimator?

We are left with only two differences between PHG92 (and our) method and that of EDSM: the redshift selection function and the logarithmic step. The redshift selection function is however very similar, and it cannot generate a significant difference. On the contrary, we can show that here again the discrepancy is only apparent, being due to a different logarithmic step.

We have recalculated \( \xi_{cc}(r) \) trying to reproduce the positions and step \( \sim 0.2 \), of EDSM (see their fig.4): we have found \( r_0 = 17.7 h^{-1} \) Mpc, or an amplitude \( 10^{2.32\pm0.19} \) and a
slope $\gamma = -1.86 \pm 0.17$, i.e. we obtained their actual result with the same (poissonian) uncertainties. The use of bootstrap errors does not change the fit but increases substantially the uncertainties. This is also visualized in fig. (asterisks connected by a solid line). Notice that the last positive point is at $\sim 31 \, h^{-1} \text{ Mpc}$, because the following is already at $r$ higher than $50 \, h^{-1} \text{ Mpc}$. There is no intermediate point, therefore it is not possible to appreciate the fact that correlation becomes negative at $\sim 45 \, h^{-1} \text{ Mpc}$, not at $30 \, h^{-1} \text{ Mpc}$. We believe that with that step there is a real loss of information with respect to the smaller step. Returning to fig. it is possible to appreciate that the best resolution is given by the $5 \, h^{-1} \text{ Mpc}$ linear step. In that case, a fit of $\xi_{cc}(r)$ gives $r_0 = 20.7 \, h^{-1} \text{ Mpc}$.

As a useful exercise, we have recalculated $\xi_{cc}(r)$ of the S40 sample with a logarithmic step $\Delta \log(r) = 0.1$, instead of 0.12, then we fitted points in the same range. We find $r_0 = 19.1 \, h^{-1} \text{ Mpc}$ and $\gamma = -1.94$, to be compared to our previous values $r_0 = 19.4 \, h^{-1} \text{ Mpc}$ and $\gamma = -1.71$.

These variations of the parameters induced by the chosen range and step indicate clearly that values of $r_0$ and $\gamma$ as reported in table can only be rough estimates; what is really important is the range of values, which is centered around $r_0 \sim 19.6$ and $\gamma \sim -1.8$.

Moreover the above discussion makes clear that an accurate determination of $\xi_{cc}(r)$ at scales $\gtrsim 40 \, h^{-1} \text{ Mpc}$ is not possible for the moment. The problem is worse for deeper samples: for example, Olivier et al. (1990) have found a discrepancy at large scales between angular correlations of Abell and ACO clusters of distance classes 5 and 6. Their corrected $w_{cc}(\theta)$ indicates a null correlation for ACO clusters at large scales.

### 3.3. Cluster pair elongations

High peculiar motions and/or projection effects can cause an elongation of cluster pairs in the redshift direction. To check for the presence of elongations, we can study the distribution of values of the angle $\beta$ between the line which connects a pair and the line-of-sight direction—or the angle $\alpha = 90^\circ - \beta$ formed with the plane of the sky—as a function of the spatial separation of clusters. This test, proposed by Sargent & Turner (1977), has been used by PHG92, who didn’t detect significant elongations in their statistical sample. We measure $\beta$ not at the midpoint between the pair, as in Sargent & Turner, but at the midpoint of the projected separation, therefore we have:

$$\tan(\beta) = \tan \left( \frac{\theta}{2} \right) \frac{D_1 + D_2}{D_1 - D_2}$$

(7)
where $\theta$ is the angular separation of the two clusters, and $D_1$ and $D_2$ are their respective distances ($D_1 > D_2$); because of geometrical constraints, $\beta$ is always $> \theta/2$. It is important to eliminate all pairs whose distance from the limits of the sample is less than their separation, otherwise the distribution would be biased in the redshift direction. In fig. 3a-d we report the histograms of the distribution of $\cos(\beta)$ for the N40 and S40 samples corresponding to different spatial separations.

All the distributions are similar (the distribution of $\cos(\beta)$ for clusters within $15 \, h^{-1}$ Mpc is comparable to that for clusters with larger separations) thus indicating that there are no significant elongations in the samples. Moreover, the mean value of $\alpha$ is very similar at all separations in both samples, $\sim 35^\circ$, to be compared to the value $32.7^\circ$ expected for an isotropic distribution.

4. Cluster analysis

4.1. Percolation analysis

In order to verify the similarity of Abell and ACO cluster spatial distributions, we applied cluster analysis (Einasto et al., 1984; Tago, Einasto, Saar, 1984).

We searched for all clusters with separation less than a fraction $s$ (the percolation parameter) of the mean inter-cluster distance $r_m = n^{-1/3}$. A set of clusters where each cluster is at a separation less than $s$ from another cluster constitutes a supercluster. Superclusters with at least three members give an additional, higher-order information relatively to $\xi_{cc}(r)$.

Dekel & West (1985) showed that percolation has some problems: they can be reduced by using volume-limited samples confined to the same volume, and this is substantially the case for our N40 and S40 samples, to which we will limit our analysis below.

Figure 10 shows that percolation of the two samples is similar. We plot $l_p = L_{\text{max}}/L_s$, where $L_{\text{max}}$ is a measure of the largest supercluster (taken as the maximum distance between two clusters in the same supercluster) and $L_s$ is the size of the sample (defined as $V^{1/3}$; this is not a critical definition, because the two samples have identical volumes), as a function of $s$. We report again bootstrap errors (for sake of clarity, only errors for S40 sample are displayed; errors for N40 are comparable), computed from 100 pseudo-data catalogs.

In figures 11, 12 and 13 we visualize the parameters $\alpha$, $\beta$ and $\gamma$: they represent the fraction of clusters in small, intermediate and large superclusters. They are found by dividing the maximum multiplicity $m_{\text{max}} = \log_2(N_c)$, where $N_c$ is the total number
of clusters in the sample, into three equal parts. α includes isolated clusters, so that
$$\alpha + \beta + \gamma = 1.$$ 

At first sight, no main difference is apparent. The two sample show an identical
behaviour up to $$s = 0.6$$ (corresponding to a scale of $$\sim 25 \, h^{-1} \, \text{Mpc}$$ for the southern
sample and $$\sim 31 \, h^{-1} \, \text{Mpc}$$ for the northern sample). Beyond $$s = 0.6$$ there is indeed some
difference. The S40 sample percolates before the N40 sample (at $$s = 0.95$$ vs. $$s = 1.09$$);
moreover, it has a lower fraction of clusters in small superclusters ($$\alpha$$ curve) and a higher
fraction in rich superclusters ($$\beta$$ and $$\gamma$$ curves). However, the difference is at the $$1\sigma$$ level.
The fact that it appears beyond $$s = 0.6$$ means that we are examining large structures at
a very low contrast. It is probably due to the presence of a rich supercluster in the south
galactic hemisphere, which we will describe in the next section.

4.2. Superclusters

We searched for superclusters selected at different contrast, mainly as a “complement”
of PHG92, in its turn an extension of Bahcall & Soneira catalog (1984; see also Batuski
& Burns, 1985b). We do not have the limit $$\delta = -27^\circ30'$', but a higher limit in $$b_{II}$$. We
consider separately the two galactic hemispheres, using the N40 and S40 samples. For the
lowest contrast, we have fixed $$s = 0.5$$; given the different densities, it corresponds to a
length $$r_s = 20.6 \, h^{-1} \, \text{Mpc}$$ for SA and $$r_s = 22.3 \, h^{-1} \, \text{Mpc}$$ for NA and, from the formula
$$\frac{\delta n}{n} = \left( \frac{4}{3} \frac{\pi r_s^3 n}{n} \right)^{-1} - 1 \tag{8}$$
to a contrast of $$\sim 0.9$$ or a space density enhancement $$f = 1 + \delta n / n \sim 1.9$$. We
have considered also the enhancements $$f = 5$$ and $$f = 10$$ (corresponding to $$s = 0.363$$
and $$s = 0.288$$) to allow a direct comparison with PHG92 results. In tables 3 and 8
we give a catalog of these superclusters for each of the chosen enhancements. Of course,
many superclusters are common to those found by PHG92 (see their table 5), mainly the
northern ones. We missed some superclusters because of the higher $$b_{II}$$ limit. We have
one more supercluster, N6, simply because our enhancement is 1.9 and not 2.0 –with the
higher enhancement the cluster A1709 would not be connected to the system–. The most
relevant features in this north galactic cap are N9, which corresponds to the well known
Hercules cluster – A2197/A2199 region and the Corona Borealis supercluster (N10) (see
Giovanelli & Haynes, 1991). In the south galactic cap we find 4 more superclusters, S10,
S11, S12, S13. S11 is particularly interesting: it is made of 15 clusters (7 with $$RC = 0$$)
at the lowest contrast, and it is broken into two subsystems at the higher contrasts. Its
characteristic size, defined as the largest separation between two members, is $$65 \, h^{-1}$$
Mpc ($f = 1.9$), and its mean redshift is $\sim 0.06$. Moreover, it could extend beyond our limits in $b_{II}$ and $z$. We cannot define its precise extension also because 6 of its 15 members happen to have an estimated distance (4 are $RC = 0$ clusters). Anyway, this structure is surely real, as it was demonstrated by Lucey et al. (1983) mainly on the basis of the galaxy distribution, and it is prominent in the cone diagram that we show in fig. [4], where it appears as a filament perpendicular to the line of sight, in front of a concentration at higher redshift; these two parts are separated at high contrast (respectively S11b and S11a in table [3]). Lucey et al. called it the Horologium-Reticulum supercluster; it is in a region characterized by a higher density of galaxies with $v \sim 18000$ km/s (see also Giovanelli & Haynes, 1991). This large structure traces partially the edge of a large void. S9 is the central part of the Pisces-Cetus supercluster candidate of Batuski & Burns (1985b). Here again our catalog does not include some systems because of their low galactic latitude.

It is again remarkable the similarity between the two hemispheres. At the lowest contrast there are 19 superclusters in N40 and 20 in S40, taking into account also binary systems. Largest superclusters are $63\ h^{-1}\ Mpc$ in N40 and $65\ h^{-1}\ Mpc$ in S40. The numbers of superclusters with more members are similar. The southern and northern richest superclusters have respectively 15 and 10 members. A total of 69 clusters are in systems of 2 or more clusters in N40; 84 clusters are in corresponding systems in S40. This means that 67% of northern clusters and 65% of southern clusters are in systems with 2 or more members; these percentages become respectively 51% and 54% if we count only superclusters with at least 3 members.

If we choose a higher density enhancement, evidently we find less superclusters with less members. However, even for an enhancement of 10, we continue to find 17 and 16 superclusters with at least 2 members respectively in the northern and southern hemispheres; the richest ones have 4 clusters. These are concentrations of clusters, which at lower contrast are connected to other clusters. At the higher contrasts there is a larger number of rich superclusters in the S40 sample. This is partly due to the presence of two systems, S1 (9 members at $f = 1.9$) and the already discussed S11, which are broken into two smaller systems at higher contrasts.

The characteristic sizes of superclusters (including binary systems, not reported in the tables) are visualized on the histograms in fig. [15] and [16], fixing a space density enhancement $f = 1.9$. The dashed line represents the expected distribution for a random sample with the same number of objects and the same volume as in the real sample (we made 50 random catalogs and averaged the results). A comparison with the corresponding random samples is needed because of the difference in density between N40 and S40.
It appears that the N40 and S40 distributions are similar. The most common size of superclusters is in the range $20 - 30 \, h^{-1} \text{Mpc}$, while for random catalogs we would expect it in the range $10 - 20 \, h^{-1} \text{Mpc}$; on the contrary, in this second range real samples have less superclusters than the poissonian expectation. It is well apparent the excess of large structures, up to $\sim 60 \, h^{-1} \text{Mpc}$, and of rich systems (figures 17 and 18), which are not present in random samples.

We come to the conclusion that the two samples have very similar distributions from a statistical point of view: the main difference is their density. Moreover the southern polar cap presents a particularly rich supercluster.

As a final observation, we note that our samples do not include the so-called Shapley concentration (Raychaudhury 1989, Scaramella et al. 1989) made (with one exception) by ACO clusters at northern galactic latitude but below our limit $b_{II} \geq +40^\circ$.

5. Conclusions

We compared the clustering properties of two complete, high galactic latitude cluster samples symmetric to the galactic plane and we made a direct estimate of the two-point spatial correlation function of ACO clusters in the southern galactic hemisphere. From the analysis of these samples we can draw the following conclusions.

- Correlation functions are all compatible with the same power-law $\xi_{cc}(r) = (\frac{r}{r_0})^{-\gamma}$.
  In the fixed range $5 < r < 40 \, h^{-1} \text{Mpc}$ we find consistent values of $r_0$, $19.3 \leq r_0 \leq 20.1$ for all subsamples. The largest value corresponds to the ACO clusters, but the difference is not statistically significant. From the northern and southern symmetric sample we derive a mean value of $19.6 \, h^{-1} \text{Mpc}$ for $r_0$.

- In the same range of $r$ slopes have values from -1.7 to -2.0. The northern samples have a steeper slope, but here again, the difference is not statistically significant (1 $\sigma$). The mean slope of the northern and southern clusters is $\gamma \sim -1.8$.

- For the statistical sample NST we find $r_0 = 19.2$ and $\gamma = -2.0$. These values are a little different from those given by PHG92 because of the different fit range. We have shown that EDSM have found different results mainly because of their larger step. We have shown that it is important to use a sufficiently small step to determine where $\xi_{cc}(r)$ becomes negative.

- We find for $r > 40 \, h^{-1} \text{Mpc}$ a quite steep cutoff for both northern and southern clusters; the first negative value is at $r \sim 50 \, h^{-1} \text{Mpc}$. We find a small (significant at the 1$\sigma$ level) but systematic (it is present in all samples) anticorrelation between 50
**Table 2.** Northern superclusters $b_H \geq +40^\circ$, $z \leq 0.08$

| ID  | No. | Members (enhancement $f = 1.9$)                  | PHG ID |
|-----|-----|-------------------------------------------------|--------|
| N1  | 4   | 999, 1016, 1139, 1142                           | 2      |
| N2  | 3   | 1149, 1171, 1238                                 | 3      |
| N3  | 5   | 1177, 1185, 1228, 1257, 1267                     | 4      |
| N4  | 3   | 1216, 1308, 1334                                 | 5      |
| N5  | 8   | 1270, 1291, 1318, 1377, 1383, 1436, 1452, 1507  | 6      |
| N6  | 3   | 1631, 1644, 1709                                 |        |
| N7  | 4   | 1775, 1800, 1831, 1873                           | 7      |
| N8  | 3   | 1781, 1795, 1825                                 | 8      |
| N9  | 10  | 2052, 2063, 2107, 2147, 2148, 2151, 2152, 2162, 2197, 2199 | 9      |
| N10 | 7   | 2061, 2065, 2067, 2079, 2089, 2092, 2124        | 10     |
| N11 | 3   | 2168, 2169, 2184                                 | 11     |

| ID  | No. | Members (enhancement $f = 5$)                  | PHG ID |
|-----|-----|-------------------------------------------------|--------|
| N3  | 5   | 1177, 1185, 1228, 1257, 1267                     | 4      |
| N5  | 6   | 1291, 1318, 1377, 1383, 1436, 1452               | 6      |
| N7  | 4   | 1775, 1800, 1831, 1873                           | 7      |
| N9  | 5   | 2107, 2147, 2148, 2151, 2152                     | 9      |
| N10 | 4   | 2061, 2065, 2067, 2089                           | 10     |

| ID  | No. | Members (enhancement $f = 10$)                  | PHG ID |
|-----|-----|-------------------------------------------------|--------|
| N3  | 3   | 1177, 1185, 1267                                 | 4      |
| N5  | 4   | 1291, 1318, 1377, 1383                           | 6      |
| N7  | 4   | 1775, 1800, 1831, 1873                           | 7      |
| N9  | 3   | 2147, 2151, 2152                                 | 9      |
Table 3. Southern superclusters ($b_{II} \leq -40^\circ$, $z \leq 0.08$)

| ID | No. | Members (enhancement $f = 1.9$) | PHG ID |
|----|-----|---------------------------------|--------|
| S1 | 9   | 14, 27, 74, 86, 114 133, 2716, 2800, 2824 | 13     |
| S2 | 3   | 85, 117, 151                      | 14     |
| S3 | 3   | 102, 116, 134                     | 15     |
| S4 | 6   | 119, 147, 160, 168, 193, 195      | 16     |
| S5 | 3   | 154, 158, 171                     | 17     |
| S6 | 4   | 419, 428, 3094, 3095              | 18     |
| S7 | 3   | 2366, 2399, 2415                  | 20     |
| S8 | 3   | 2459, 2462, 2492                  | 21     |
| S9 | 4   | 2589, 2592, 2593, 2657,          | 22     |
| S10| 6   | 2731, 2806, 2860, 2870, 2877, 2911 |        |
| S11| 15  | 3089, 3093, 3100, 3104, 3108, 3111, 3112, 3122, 3123, 3125, 3128, 3133, 3135, 3158, 3164 |        |
| S12| 4   | 3144, 3193, 3202, 3225            |        |
| S13| 7   | 3771, 3785, 3806, 3822, 3825, 3826, 3886 |        |

| ID  | No. | Members (enhancement $f = 5$) | PHG ID |
|-----|-----|--------------------------------|--------|
| S1a | 3   | 74, 86, 2800                   | 13     |
| S1b | 3   | 114, 133, 2824                 | 13     |
| S4  | 3   | 119, 147, 168                  | 16     |
| S6  | 3   | 419, 3094, 3095                | 18     |
| S9  | 3   | 2589, 2592, 2593               | 22     |
| S10 | 5   | 2731, 2806, 2860, 2870, 2877   |        |
| S11a| 7   | 3093, 3100, 3104, 3108, 3111, 3112, 3133 |        |
| S11b| 4   | 3125, 3128, 3158, 3164         |        |
| S12 | 4   | 3144, 3193, 3202, 3225         |        |
| S13 | 4   | 3806, 3822, 3825, 3826         |        |

| ID  | No. | Members (enhancement $f = 10$) | PHG ID |
|-----|-----|--------------------------------|--------|
| S1a | 3   | 74, 86, 2800                   | 13     |
| S1b | 3   | 114, 133, 2824                 | 13     |
| S4  | 3   | 119, 147, 168                  | 16     |
| S10 | 3   | 2860, 2870, 2877               |        |
| S11a| 3   | 3104, 3111, 3112              |        |
| S11b| 3   | 3125, 3128, 3158              |        |
and $100 - 120 \ h^{-1} \ Mpc$, with minima between $-0.14$ and $-0.3$ depending on samples; this effect had been previously detected by Bahcall et al. (1988) and BBOB analysing the angular correlation functions.

- Percolation properties are similar in the northern and southern hemispheres, but the S40 sample has richer structures for values of the percolation parameter $s > 0.6$.

- We have given catalogs of superclusters respectively for the northern and southern galactic hemispheres, mainly as a supplement to the PHG92 catalog for the southern galactic hemisphere. The main feature at high latitude in the southern galactic hemisphere is a large supercluster (15 members) in the Horologium region, at a mean redshift $z \sim 0.06$, with a linear size $D \sim 65 \ h^{-1} \ Mpc$, near to a large void (see Lucey et al., 1983).

The above results do not depend on the galactic latitude or delta selection functions $P(b)$ and $P(\delta)$; exclusion of richness 0 clusters does not change our results. Therefore our general conclusion is that nearby Abell and ACO clusters both in the north and south galactic hemispheres have a much higher correlation function than galaxies; their $\xi_{cc}(r)$ becomes negative beyond $\sim 45 \ h^{-1} \ Mpc$. Correlations of clusters in the northern and southern galactic hemispheres and their clustering properties don’t show any difference which can be claimed statistically significant. Of course, this implies that ACO southern clusters have the same distribution as Abell clusters: if there are important projection effects, then both of them are affected in the same way. This is confirmed by the spatial correlation function of the small sample of ACO clusters.

Finally we remark that other valuable information on cluster distribution can be obtained from the scaling properties of the Void Probability Function (Jing, 1990; Cappi, Maurogordato, Lachièze-Rey 1991) and from counts in cells (Cappi & Maurogordato, 1991). The application of these statistics to the available cluster samples will be the object of a following paper.

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Fig. 1. N40 sample (outer contour: $b_{II} = +40^\circ$)

Fig. 2. S40 sample (outer contour: $b_{II} = -40^\circ$)

Fig. 3. Redshift histogram (solid line: SA clusters; dashed line: NA clusters)

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**Fig. 4.** The spatial correlation function $\xi_{cc}(r)$ for the N40 sample (asterisks); S40 sample (circles); dashed line: $\xi_{cc}(r) = (r/20)^{-2}$

**Fig. 5.** $\xi_{cc}(r)$ for the ACO sample; dashed line as in fig. 4

**Fig. 6.** $\xi_{cc}(r)$ for the NST sample; asterisks: $\Delta \log(r)=0.2$; circles: $\Delta r=5 \, h^{-1} \, \text{Mpc}$; crosses with error bars: $\Delta \log(r)=0.1$; dashed line as in fig. 4

**Fig. 7.** $\xi_{cc}(r)$ for the N40 (asterisks) and S40 (circles) rich clusters ($R \geq 1$); dashed line as in fig. 4

**Fig. 8.** $1 + \xi_{cc}(r)$: asterisks: N40; circles: S40; crosses: ACO; squares: NST; dashed line as in fig. 4

**Fig. 9.** Histograms of $\cos(\beta)$ distribution. a N40 sample. Solid line: pairs with separation $D_{sep} < 15 h^{-1} \, \text{Mpc}$; dashed line: pairs with separation $15 \leq D_{sep} < 30 h^{-1} \, \text{Mpc}$; b N40 sample. Solid line: pairs with separation $30 \leq D_{sep} < 45 h^{-1} \, \text{Mpc}$; dashed line: pairs with separation $45 \leq D_{sep} < 60 h^{-1} \, \text{Mpc}$; c S40 sample. Solid line: pairs with separation $D_{sep} < 15 h^{-1} \, \text{Mpc}$; dashed line: pairs with separation $30 \leq D_{sep} < 45 h^{-1} \, \text{Mpc}$; d S40 sample. Solid line: pairs with separation $30 \leq D_{sep} < 45 h^{-1} \, \text{Mpc}$; dashed line: pairs with separation $45 \leq D_{sep} < 60 h^{-1} \, \text{Mpc}$

**Fig. 10.** $l_p$ as a function of $s$. Solid line: S40; dashed line: N40

**Fig. 11.** $\alpha$ as a function of $s$. Solid line: S40; dashed line: N40

**Fig. 12.** $\beta$ as a function of $s$. Solid line: S40; dashed line: N40

**Fig. 13.** $\gamma$ as a function of $s$. Solid line: S40; dashed line: N40

**Fig. 14.** Slice of the S40 sample: $210^\circ \leq l_{II} \leq 300^\circ, -75^\circ \geq b_{II} - 40^\circ, z \leq 0.08$

**Fig. 15.** Size distribution of superclusters (solid line: N40 sample; dashed line: random sample)

**Fig. 16.** Size distribution of superclusters (solid line: S40 sample; dashed line: random sample)

**Fig. 17.** Richness distribution of superclusters (solid line: N40 sample; dashed line: random sample)

**Fig. 18.** Richness distribution of superclusters (solid line: S40 sample; dashed line: random sample)