Sivers effect in semi-inclusive DIS and in the Drell-Yan process

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Abstract

The Sivers parton distribution function has been predicted to obey a particular “universality relation”, namely to have opposite sign in semi-inclusive deeply inelastic scattering (SIDIS) and the Drell-Yan process. We discuss how, on the basis of present HERMES data, this remarkable prediction of the QCD factorization approach to the description of single spin asymmetries related to the Sivers effect could be checked experimentally in future experiments at PAX and COMPASS.

1 Introduction

It was understood early \cite{1} that single spin asymmetries (SSA) in hard processes, such as those observed in $p^{\uparrow}p \rightarrow \pi X$ \cite{2,3} or in SIDIS \cite{4,5,6,7,8,9}, cannot be explained by means of leading twist collinear QCD factorization. One of the non-perturbative effects which could account for such SSA considers a non-trivial correlation between the transverse component of the nucleon spin and intrinsic transverse parton momenta $p_T$ in the nucleon \cite{10}, and is quantified in terms of the so-called Sivers function $f_{1T}^\perp(x, p_T^2)$ \cite{11}. The effect is referred to as “(naively) T-odd”, since it is proportional, e.g., in the infinite momentum frame where the nucleon momentum $P_N \rightarrow \infty$, to the T-odd structure $(S_T \times p_T)P_N$. The Sivers effect was shown to be able to explain the SSA in $p^{\uparrow}p \rightarrow \pi X$ \cite{12}, though also other mechanisms exist which could contribute in this reaction \cite{13,14,15}.

The precise definition of $f_{1T}^\perp(x, p_T^2)$ in QCD was worked out only recently \cite{16,17,18}. A particularly interesting feature of the Sivers function concerns its universality property. This property ensures for usual parton distributions that one deals with, e.g., the same unpolarized parton distribution $f_1(x)$ in SIDIS and in the Drell-Yan process (DY): $f_1(x)_{\text{SIDIS}} = f_1(x)_{\text{DY}}$. In the case of the Sivers function (and other T-odd distributions) the universality property takes, however, a different form. On the basis of time-reversal arguments it was predicted \cite{17} that $f_{1T}^\perp$ in SIDIS and DY have opposite sign,

$$f_{1T}^\perp(x, p_T^2)_{\text{SIDIS}} = -f_{1T}^\perp(x, p_T^2)_{\text{DY}}.$$  \hspace{1cm} (1)

The experimental check of Eq. (1) would provide a thorough test of our understanding of the Sivers effect within QCD and, hence, our understanding of SSA. It would crucially test the factorization approach to the description of processes sensitive to transverse parton momenta \cite{19,20,21}.

In this work we shall discuss how the relation (1) could be checked experimentally in the Polarized Antiproton eXperiment (PAX) planned at GSI \cite{22,23}. A primary goal of this experiment will be to provide a polarized antiproton beam and to measure the transversity distribution $h_T^\perp(x)$, c.f. \cite{24}. However, PAX will also be well suited to access the Sivers function via SSA in $\bar{p}p \rightarrow \mu^+\mu^- X$ or $\bar{p}p \rightarrow \mu^+\mu^- X$ \cite{22,23}. In the COMPASS experiment at CERN \cite{25}, making use of a $\pi^-$ beam, one would also be able to study the Sivers function via SSA in $\pi^- p \rightarrow \mu^+\mu^- X$.

In order to estimate the magnitude of the Sivers effect in those experiments we will roughly parameterize $f_{1T}^\perp(x, p_T^2)_{\text{SIDIS}}$ from the (preliminary) HERMES data \cite{7} using as a guideline relations derived from the QCD limit of a large number of colours $N_c$ \cite{26}. Such large-$N_c$ relations are observed to hold in nature within their expected accuracy \cite{27} and, as a byproduct of our study, we shall observe that this is also the case here. On the basis of the obtained parameterization we estimate SSA in the Drell-Yan process for the PAX and COMPASS experiments. We also comment briefly on parameterizations of $f_{1T}^\perp$ reported previously in the literature and on model calculations.
2 The Sivers function

A definition of the unintegrated unpolarized distribution function \( f_1(x, p_T^2) \) and the Sivers function \( f_{1T}^q(x, p_T^2) \) can be given in terms of the light-cone correlator

\[
\Phi^q(x, p_T) = \int \frac{d\xi d^2\xi_T}{2(2\pi)^3} e^{ip\cdot \xi} \langle P, S_T | \psi_q(0) \gamma_\mu n_+^\mu \mathcal{W}[0, \xi; \text{process}] \psi_q(\xi) | P, S_T \rangle \bigg|_{\xi^+ = 0}
\]

\[
= f_q^q(x, p_T^2) + f_{1T}^q(x, p_T^2) \varepsilon_{\mu\nu\rho\sigma} n_+^\mu n_+^\nu p_T^\rho S_+^\sigma \frac{M_N}{c},
\]

(2)

where the dimensionless light-like vectors \( n_\pm \) are defined such that \( n_+ \cdot n_- = 1 \). (See Ref. [28] for a precise definition and the meaning of unintegrated distribution functions in QCD.)

The Wilson link \( \mathcal{W}[0, \xi; \text{process}] \) is defined in Fig. 1, c.f. Refs. [17, 18]. For observables integrated over \( p_T \), the process dependence of the gauge link usually cancels out. However, the situation is different for \( f_{1T}^q \). If one neglected the gauge link, under time-reversal the Sivers function would transform into its negative, i.e. it would vanish [14]. However, initial or final state interactions [16, 29], needed to obtain non-zero SSA [30], generate a Wilson link for the Sivers function in any gauge [17, 18]. Under time reversal the gauge link of SIDIS is transformed into the gauge link of DY, and vice versa [17]. (The gauge link structure in the more involved hadronic process \( p^5p \to \pi X \) was addressed in [31].) This yields the peculiar universality relation in Eq. (1).

Only little is known about the non-perturbative properties of the Sivers function. In Ref. [32] bounds were derived from the positivity of the spin density matrix, which constrain \( f_{1T}^q \) in terms of other transverse momentum dependent distributions including also so far experimentally unknown functions. Eliminating the unknown distribution, at the price of relaxing the bound, one obtains

\[
\frac{|p_T|}{M_N} |f_{1T}^q(x, p_T^2)| \leq f_1^q(x, p_T^2).
\]

(3)

The average parton transverse momentum defined as \( \langle p_T^2 \rangle = \int dx \int d^2 p_T \, p_T \Phi^q(x, p_T) \) was shown [33, 34] (with analogously defined gluon transverse momentum) to obey the relation

\[
\sum_{a=g,u,d,...} \langle p_T^2 \rangle = 0.
\]

(4)

Eq. (4) is more than trivial momentum conservation in the plane transverse to the hard momentum flow, since it connects transverse momenta due to final state interactions which the scattered quark experiences in incoherently summed scattering events [33, 34]. Inserting the definition of \( \Phi^q(x, p_T) \) into Eq. (4) and using the fact that (analogously for gluons)

\[
\int dx \int d^2 p_T \, \frac{p_T^2}{M_N} f_{1T}^q(x, p_T^2) = \delta^{kl} \int dx \ f_{1T}^{(1)q}(x), \quad \text{with} \quad f_{1T}^{(1)q}(x) \equiv \int d^2 p_T \, \frac{p_T^2}{2M_N} f_{1T}^q(x, p_T^2),
\]

(5)

one obtains

\[
\sum_{a=g,u,d,...} \int dx \ f_{1T}^{(1)q}(x) = 0.
\]

(6)

The sum rule (6) may prove a useful constraint for parameterizations of the Sivers function, in particular, because it is \( f_{1T}^{(1)q}(x) \) which enters in a model independent way cross sections properly weighted with transverse momenta [11].

Another property of the Sivers function, which will be used later on, is the relation derived in the limit of a large number of colours \( N_c \) in QCD [26], namely

\[
f_{1T}^{1q}(x, p_T^2) = -f_{1T}^{d\bar{d}}(x, p_T^2) \mod 1/N_c \text{ corrections}.
\]

(7)

It should be noted that in the large-\( N_c \) limit \( xN_c = \mathcal{O}(1) \), such that Eq. (7) can be expected to be satisfied in the valence region of not too small and not too large \( x \) to within an accuracy of \( \mathcal{O}(1/N_c) \) [27].

Neglecting strange and heavier quarks, which is a reasonable assumption in the case of the nucleon, one obtains from Eqs. (6, 7) that the Sivers gluon distribution is suppressed in the large-\( N_c \) limit with respect to the quark distribution functions. More precisely, it is of the same order of magnitude as the flavour singlet combination\(^1\), i.e., \( f_{1T}^{uu} + f_{1T}^{dd} \sim f_{1T}^{1g} \sim \mathcal{O}(N_c^0) \). Thus, in the large-\( N_c \) limit the gluon Sivers effect

\(^1\)What matters in the large-\( N_c \)-counting is the spin-flavour symmetry of the involved operator. In this respect the operator entering the flavour singlet and the gluon Sivers function have the same behaviour and thus the same large-\( N_c \) counting. We thank Pavel Pobylitsa for discussions on this point.
can be expected to be suppressed with respect to the non-singlet quark Sivers effect at not too small $x$, which is an interesting constraint for phenomenological studies. In order to obtain a feeling to which extent such large-$N_c$ relations may be expected to hold, it is interesting to mention that the helicity distribution function exhibits a similar behaviour in the large-$N_c$ limit, namely $|g_1^u-g_1^d(x)| \sim O(N_c^2)$ is larger than $|(g_1^n+g_1^q)(x)| \sim g_1^d(x) \sim O(N_c)$. This is – for quarks – roughly consistent with phenomenology, and predicts a suppression of the (presently poorly known) helicity gluon distribution with respect to the unpolarized gluon distribution function $g_1^n(x)/f_1^d(x) \sim 1/N_c$ at moderate values of $x$ [27].

The Sivers function was studied in several models, in which the gauge link was modeled explicitly by considering the perturbative effect of one-gluon exchange. Calculations based on the spectator model [16, 35, 36] and the bag model [37] yield a sizeable $f_{1T}^{1u}$ but a negligible $f_{1T}^{1d}$ and, thus, for the chosen parameter sets, do not respect the large $N_c$ counting rule in Eq. (7).

In a large class of chiral models, which are based on Goldstone boson and (effective) quark degrees of freedom, T-odd distributions are zero [38]. This can be understood by recalling that in such models there are no gluons, whose presence is crucial to generate T-odd effects via the gauge link structure. Combining the no-go-theorem (concerning modeling of $f_{1T}$ in chiral models) of Ref. [38] with notions from the instanton model of the QCD vacuum [39] one is lead to the suspicion [40] that T-odd distributions could be suppressed with respect to T-even distributions in the instanton vacuum model, which is supported by estimates [41]. For a discussion of possible instanton effects in the Drell-Yan process see [42] and references therein.

By assuming that the SSA in $p^+p \rightarrow \pi X$ [2] is dominated by the Sivers effect phenomenological parameterizations of the Sivers function were obtained [12, 43, 44] which, worthwhile stressing, approximately respect the large-$N_c$ pattern in Eq. (7). The assumption that this process is dominated by the Sivers effect seems to be reasonable in the light of recent studies [45, 46], which show that the contribution of the Collins effect in this process is small. However, one also has to keep in mind twist-3 effects [13, 15] which might be equally important.

Let us finally mention that a relation of $f_{1T}^{1g}$ to the generalized parton distribution $E^g(x, \xi, t)$ was proposed, namely the leading light-cone Fock component of the Sivers function may be represented as a convolution of the Wilson link and the same overlap integrals between light-cone wave functions differing by one unit of orbital angular momentum, which enter the description of $E^g(x, \xi, t)$ [16, 47]. This has been seen explicitly in a quark-diquark model calculation [48] and is compatible with the large $N_c$-limit in the sense that $E^g$ and $f_{1T}^{1g}$ have the same large-$N_c$ behaviour [49]. From these relations it was concluded that

$$\int dx \ f_{1T}^{(1)u}_{\text{SIDIS}}(x) < 0, \quad \int dx \ f_{1T}^{(1)d}_{\text{SIDIS}}(x) > 0 .$$

(8)

The above connection to the generalized parton distribution could further be exploited to draw conclusions on the large-$x$ behaviour of the Sivers function. Since $E^g(x, \xi, t) \propto (1-x)^5$ at large $x$ [50] one may conclude that also $f_{1T}^{(1)1g}(x) \propto (1-x)^5$. One must keep in mind, however, that the above assumes dimensional counting behaviour of the usual quark distribution functions, which yields $f_1^g(x) \propto (1-x)^3$ at large $x$ [51]. While for $f_1^u(x)$ parameterizations, e.g. [52, 53], are roughly compatible with this prediction, this is not the case for $f_1^d(x)$. However, if this were true also for $E^g(x, \xi, t)$ and $f_{1T}^{(1)g}(x)$, there need not to be a conflict with large-$N_c$ relations, such as Eq. (7), which apply only as long as $x N_c = O(1)$.
3 Sivers effect in SIDIS

Consider the process $lp' \to l'hX$ (see Fig. 2) where $a^{\uparrow T}$ denotes the transverse (with respect to the beam) target polarization. Let $P, l (l')$ and $P_h$ denote respectively the momentum of the target proton, incoming (outgoing) lepton and the produced hadron. The relevant kinematic variables are $s := (P + l)^2$, $q = l - l'$ with $Q^2 = -q^2$, $x = \frac{Q^2}{2s}$, $y = \frac{P^2}{s}$, $z = \frac{P_N^2}{s}$. The azimuthal SSA asymmetry is defined as

$$A_{UT}(x) = \frac{1}{2} \sum_i \sin(\phi_i - \phi_{S,i}) \frac{|P_{h,i}|}{M_N} \left\{ N_i^1(\phi_i, \phi_{S,i}) - N_i^1(\phi_i, \phi_{S,i} + \pi) \right\},$$

where $N_i^1(\phi_i, \phi_{S,i})$ are the event counts for the respective target polarization (corrected for depolarization effects). The z-axis is chosen in direction of the virtual photon (see Fig. 2), in agreement with the HERMES convention [4]. The angles $\phi_i$ and $\phi_{S,i}$ are the azimuthal angles of the produced hadron and the target spin. Neglecting power suppressed terms $\propto M_N^2/Q^2$ the SSA (9) is given by [11]

$$A_{UT,\pi}(x) = \frac{-2}{\text{cuts}} \int dz \frac{\alpha_s^2}{Q^4} \left(1 - y + \frac{y^2}{2}\right) \sum_a e_a^2 x f_{1T}^{1(a)}(x) \int dz \frac{\alpha_s^2}{Q^4} \left(1 - y + \frac{y^2}{2}\right) \sum_a e_a^2 x f_{1T}^{1}(x) D_i^{a/\pi}(z).$$

There are two reasons why we prefer to study the preliminary data for the asymmetries weighted with a power of $P_{h,\perp}$ instead of the final data for the asymmetries weighted without $P_{h,\perp}$ [8], in spite of the caveat that preliminary data can be subject to changes due to refined data analyses. Firstly, in the parton model approximation only asymmetries weighted with an appropriate power of transverse momentum (e.g., $P_{h,\perp}$ in the case of Sivers effect in SIDIS) allow a model independent disentanglement of transverse parton momenta in the target and in the fragmentation hadron [11]. An analysis of $A_{UT}^{\sin(\phi - \phi_S)}$ would inevitably be biased by our prejudice concerning the distribution of transverse parton momenta in the target and in the fragmentation process, while the use of the asymmetry (9, 10) allows to avoid this problem elegantly. Secondly, below we will be interested in discussing the Sivers effect in DY pair production in the COMPASS experiment at considerably higher energies. It has been argued that asymmetries weighted without transverse momentum could be subject to strong dilution due to Sudakov effects, while this effect could be minimized by weighting the SSA by an appropriate power of transverse momentum [55].

Using the fact that the unpolarized distribution $f_1^p(x)$ and fragmentation $D_1^q(z)$ functions are well known and parameterized (see, e.g., [52, 53, 56]), one could try to extract directly information on the Sivers function using the so-called purity method which is being pursued by the HERMES Collaboration [54]. Instead, we choose a different strategy here and fit the HERMES data [7] for which we employ the ansatz

$$f_{1T}^{(1)a}(x) = -x f_{1T}^{(1)d}(x) = A x B (1 - x)^5, \quad f_{1T}^{(1)q}(x) = 0.$$

Several comments are in order. Firstly, we assume that the Sivers function for antiquarks can be neglected in comparison to the one for quarks. Secondly, we imposed the condition (7) derived from the large-$N_c$ limit [26]. Both assumptions are severe constraints, but – given the size of the experimental error bars [7] – they can be expected to hold with sufficient accuracy for our purposes. In fact, we shall see that the present data are compatible with the ansatz. Of course, one should expect that future precision data may demand to relax these constraints. At the present stage, however, these assumptions are very helpful to reduce the number of unknown quantities to only one, namely the Sivers $u$-quark distribution. We also neglect strange quark effects.

\footnote{Note that throughout this paper we also neglect contributions from soft gluons [19, 20, 21].}
In order to illustrate to which extent the HERMES data allow to constrain the parameters in the ansatz (11) we performed two fitting procedures, one (I) with the parameter $B = 1$ fixed from the very beginning, and another one (II) where both parameters $A$ and $B$ were kept free. Using for $f_1^T(x)$ and $D_1^T(z)$ the parameterizations [52] (or [53] which yields a negligible difference) and [56] at $Q^2 = 2.5 \text{ GeV}^2$ we obtain the fits

\begin{align*}
\text{Fit I: } & \quad x f_{1T}^{(1)u}(x) = -0.4 x (1 - x)^5, \\
\text{Fit II: } & \quad x f_{1T}^{(1)u}(x) = -0.1 x^{0.3} (1 - x)^5, \quad (12)
\end{align*}

with a comparable $\chi^2$ per data point of about 0.4. The fitting functions are of different shape, see Fig. 3, but they describe the HERMES data [7] equally well, see Figs. 4a-c. The scale for the Sivers function in Eq. (12) corresponds to the average scale in the HERMES experiment of $(Q^2) = 2.5 \text{ GeV}^2$. We remark that the fits (12) are mainly constrained by the $\pi^+$_data. Leaving the $\pi^0$ (and/or $\pi^-$) data out of the fit does not affect the numbers in (12) significantly.

Thus we see that the experimental accuracy of the data does not allow one to constrain more sophisticated ansätze with more than two free parameters. Considering the discussion of the large-$x$ behaviour in the previous section, we have been guided to the ansatz (11). However, one should keep in mind that we use this ansatz only in the region $x < 0.4$ covered by HERMES, so the precise shape of $f_{1T}^{(1)}(x)$ in the limit $x \rightarrow 1$ is of no relevance for us.

Let us confront the results of our fit to the $z$-dependent data from [7]. Since the latter was not used to constrain the fit, the comparison in Figs. 4d-f can be viewed as a “cross check” of the fitting procedure. The expression for the asymmetry is given by Eq. (10) but with the average with respect to $x$ instead of $z$. The shape of the SSA is dictated by the parameterization for $D_1^T(z)$ from Ref. [56]. The asymmetry is linearly rising with $z$ for $\pi^0$ (where $D_1^T$ is the same for all $q = u, \bar{u}, d, \bar{d}$) and nearly so for $\pi^+$ (where favoured flavour approximation works well), but it has a peculiar shape for $\pi^-$ (where $1/N_c$-corrections to the Sivers function

![Figure 3: Sivers function according to Eqs. (11, 12) as obtained from a fit to the HERMES data [7], see Figs. 4a-c. The unpolarized quark distributions $x f_1^u(x)$ at $Q^2 = 2.5 \text{ GeV}^2$, rescaled by the factor $(-1)/10$, are shown for the sake of comparison.](image)

![Figure 4: (a,b,c) The azimuthal SSA $A_T^{\sin(\phi_h - \phi_p)} P_{h,M_N}$ as function of $x$. The preliminary data are from the HERMES experiment [7]. The curves are obtained from the large-$N_c$ constrained fits I and II (denoted as in Fig. 3) of the Sivers function. (d,e,f) $A_T^{\sin(\phi_h - \phi_p)} P_{h,M_N}$ as function of $z$, with the preliminary data from [7], and the theoretical curves from the fits I and II of the Sivers function. The $z$-dependent data were not used for the fit, and serve as a cross check of our results.](image)
would have the most impact). We conclude that the ansatz (11) and the fits (12) are well compatible with the z-dependence of the data, see Figs. 4d-f.

We observe that the obtained fit satisfies $|f_{1T}^{(1)\alpha/\pi}(x)| < \frac{1}{10} f_{1T}^a(x)$, see Fig. 3. Multiplying Eq. (3) by $|p_T|/(2MN)$ and integrating it over transverse momenta gives the inequality $|f_{1T}^{(1)\alpha/\pi}(x)| \leq \frac{(p_T)}{2MN} f_{1T}^a(x)$ which (defines a phenomenological mean parton transverse momentum inequality observed in Fig. 3, if we assume $\langle p_T \rangle \approx 0.8\text{GeV}$ (see Ref. [44]). In this sense, we note that our result is in agreement with the positivity bound in Eq. (3).

Let us also remark that the HERMES data [7] are compatible with the large-$N_c$ counting rule in Eq. (7) within their present statistical accuracy, which is proven by the fact that a fit with the ansatz (11) works. The sum rule (6) is satisfied by our parameterization (recall the suppression of the gluon Sivers function in the large-$N_c$ limit).

Experiments with the deuterium target are suppressed with respect to proton asymmetries by a power of $A_{\perp}$. The process $p^\uparrow h \to p^\uparrow \pi^- X$ (with $h = \bar{p}, \pi^-$ in the following) is characterized by the variables $s = (p_1 + p_2)^2$, the dilepton invariant mass $Q^2 = (k_1 + k_2)^2$ with $p_{1/2}$ and $k_{1/2}$ indicating the momenta of the incoming proton and hadron $h$ (and the outgoing lepton pair), and the rapidity

$$y = \frac{1}{2}\ln\frac{p_2(k_1 + k_2)}{p_1(k_1 + k_2)}.$$ (13)

Let us consider the azimuthal SSA which is weighted by $|q_T|$, the dilepton momentum transverse with respect to the collision axis, and defined as a sum over the events $i$ according to

$$A_{UT}^{\sin(\phi-\phi_S)}(y, Q^2) = \frac{\sum_i \sin(|\phi_i - \phi_S|) |q_T|}{\frac{1}{2} \sum_i \{N^\uparrow(\phi_i, \phi_S, i) + N^\downarrow(\phi_i, \phi_S, i) + \pi\}},$$ (14)

where $\uparrow$ ($\downarrow$) denote the transverse polarization of the proton. (See Fig. 2b for the definition of the kinematics.)

To leading order the SSA is given by

$$A_{UT}^{\sin(\phi-\phi_S)}(y, Q^2) = 2 \frac{\sum \alpha^2 x_1 f_{1T}^{(1)\alpha/\pi}(x_1) x_2 f_{1T}^{\pi/h}(x_2)}{\sum \alpha^2 x_1 f_{1T}^{\alpha/p}(x_1) x_2 f_{1T}^{\pi/h}(x_2)},$$ (15)

We neglect nuclear binding effects and assume isospin symmetry which is legitimate given the present level of accuracy. Parton distributions without a target label ($D =$ deuteron, $p =$ proton, $n =$ neutron) refer, as everywhere in this note, to the proton.

4 Sivers effect in the Drell-Yan process

The information on $f_{1T}$ deduced in Sec. 3 from the HERMES data [7] is rough, however, as we shall see, sufficient for our goal to predict the sign and to gain insight into the magnitude of the SSA in DY.

The process $p^\uparrow h \to p^\uparrow \mu^- X$ (with $h = \bar{p}, \pi^-$ in the following) is characterized by the variables $s = (p_1 + p_2)^2$, the dilepton invariant mass $Q^2 = (k_1 + k_2)^2$ with $p_{1/2}$ and $k_{1/2}$ indicating the momenta of the incoming proton and hadron $h$ (and the outgoing lepton pair), and the rapidity

$$y = \frac{1}{2}\ln\frac{p_2(k_1 + k_2)}{p_1(k_1 + k_2)}.$$ (13)

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where $\uparrow$ ($\downarrow$) denote the transverse polarization of the proton. (See Fig. 2b for the definition of the kinematics.)

To leading order the SSA is given by

$$A_{UT}^{\sin(\phi-\phi_S)}(y, Q^2) = 2 \frac{\sum \alpha^2 x_1 f_{1T}^{(1)\alpha/\pi}(x_1) x_2 f_{1T}^{\pi/h}(x_2)}{\sum \alpha^2 x_1 f_{1T}^{\alpha/p}(x_1) x_2 f_{1T}^{\pi/h}(x_2)},$$ (15)
where the parton momenta $x_{1/2}$ in Eq. (15) are fixed in terms of $s$, $Q^2$ and $y$,

$$x_{1/2} = \sqrt{\frac{Q^2}{s}} e^{y}. \quad (16)$$

The sums in Eq. (15) run over all quark and antiquark flavours, and we indicate explicitly to which hadron the distributions refer.

In the PAX experiment antiprotons with a beam energy of 25 GeV could be available, i.e. $s = 45 \text{GeV}^2$. In this kinematics one could explore the region around $Q^2 = 2.5 \text{GeV}^2$, which is below the region of $J/\psi$ production, and well above the region of dileptons from $\Phi(1020)$-decays. Taking into account the change of sign in the Sivers function in DY as compared to SIDIS, see Eq. (1), we obtain the result shown in Fig. 5a.\footnote{The DY asymmetry appears positive like the SIDIS asymmetry at HERMES, despite the change of sign of the Sivers function due to conventions: In DY we define the $z$-axis in the direction in which the polarized particle moves. In SIDIS at HERMES the $z$-axis is defined in the opposite direction, see Figs. 2a and 2b.}

We observe that the two fits I and II, which describe the HERMES data of SIDIS equally well, give clearly distinguishable results in DY. Considering depolarization, detector acceptance and other effects, it might be difficult to distinguish the effect of the different parameterizations in Eq. (12). However, the asymmetry is large enough to check unambiguously the QCD prediction of the different sign of the Sivers function in DY and SIDIS.

In the COMPASS experiment using a $\pi^-$ beam ($s = 400 \text{GeV}^2$) one could also measure the asymmetry (15). In Fig. 5b we show the asymmetry for $Q^2 = 20 \text{GeV}^2$ using for the pion the parameterization from Ref. [61]. Although $f_1^{\text{p}/\pi}(x)$ is far less constrained by data compared to $f_1^{\text{p}/p}(x)$ the result in Fig. 5b is rather insensitive to the choice of parameterization, and changes very little if we use the pion distributions of Ref. [62] (consistently in combination with the nucleon distributions from Ref. [53]). We observe a situation, which is qualitatively and quantitatively similar to the case of DY from $pp$-collisions. Note that we neglected evolution effects (from $Q_0^2 = 2.5 \text{GeV}^2$ in Eq. (12) to $Q^2 = 20 \text{GeV}^2$ in Fig. 5) for the Sivers function. However, the influence of evolution is presumably much smaller than other uncertainties in our study. Note that by using the $q_{\perp}$-weighted SSA we have avoided another serious problem in this context, namely Sudakov suppression [55], see the remarks in the previous section.

In order to extract quantitative information from the future COMPASS and PAX experiments it is necessary to go beyond the LO formalism, to consider effects of soft gluons and K-factors, and to study the role of possible higher twist effects. The corrections due to these effects cannot be expected to be negligible. However, they are unlikely to be able to change the sign of the asymmetry. Thus, both the COMPASS as well as the PAX experiment could provide a thorough experimental test of the QCD prediction in Eq. (1).

SSA in DY can also be studied at RHIC in $p^+ p \rightarrow \mu^+ \mu^- X$. Since only one proton needs to be polarized the counting rates would be somehow more sizeable than in the case of double spin asymmetries related to the transversity distribution $h_1^\perp(x)$ which are, however, small [63]. Moreover, in this case, one is sensitive to the Sivers antiquark distribution which is not constrained by the HERMES data. We remark that the RHIC experiment is well suited to learn, e.g., about the Sivers function from SSA in $p^+ p \rightarrow \pi X$ [3] or the gluon Sivers function [64, 65].
5 Conclusions

The recently reported HERMES data [7, 8] on SSA provide a theoretically unambiguous experimental evidence for the existence of T-odd distribution (and fragmentation) functions. We analyzed the HERMES data and demonstrated that they are consistent with predictions from the large-$N_c$ limit of QCD [26] for the Sivers functions, namely $f_{1T}^{u} = -f_{1T}^{d}$ modulo $1/N_c$ corrections. Imposing this large-$N_c$ result as an exact constraint we were able to obtain parameterizations of the Sivers quark distribution functions. The neglect of $1/N_c$ corrections (as well as antiquark effects) in a first approximation is reasonable, keeping in mind the large error bars of the present data which do not allow to constrain more sophisticated ansätze.

On the basis of the obtained parameterizations we estimated SSA in the Drell-Yan process for the PAX ($p^+ \bar{p} \rightarrow \mu^+ \mu^- X$) and COMPASS ($p^+ \pi^- \rightarrow \mu^+ \mu^- X$) experiment. According to the theoretical understanding of T-odd parton distributions in QCD the Sivers function should obey a particular universality relation, namely appear with opposite sign in DY and SIDIS [17]. Our estimates show that both experiments could be able to test this prediction, which would be a crucial check of the present understanding of T-odd distribution functions and the QCD factorization approach to the description of SSA.

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Note added: After this manuscript was completed the COMPASS collaboration has published final data on transverse target SSA from a deuteron target [66]. We stress that the large-$N_c$ prediction for the flavour dependence of the Sivers function [26] naturally explains the compatibility of the sizeable Sivers SSA from a proton target observed at HERMES [7, 8] and the small (consistent with zero within error bars) SSA from a deuteron target observed at COMPASS [9, 66]. The Sivers effect in the deuteron is sensitive to the flavour combination ($f_{1T}^{u} + f_{1T}^{d}$), and thus suppressed with respect to the Sivers effect in the proton by one order of $N_c$ in the large-$N_c$ expansion. In nature $N_c = 3$ is sufficiently large to explain the observations – considering the statistics of the first experiments [7, 8, 9, 66].

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