Kinetically Modified Non-Minimal Chaotic Inflation

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ABSTRACT: We consider supersymmetric (SUSY) and non-SUSY models of chaotic inflation based on the $\phi^n$ potential with $2 \leq n \leq 6$. We show that the existence of a nonminimal coupling to gravity $f_R = 1 + c_R \phi^{n/2}$ with a kinetic mixing of the form $f_K = c_K f_R^n$ can accommodate values of the spectral index, $n_s$, and the tensor-to-scalar ratio, $r$, favored by the BICEP2/Keck Array and Planck results for $0 \leq m \leq 4$ and $2.5 \cdot 10^{-4} \leq r_{IK} = c_K/c_K^{1/4} \leq 1$. Inflation can be attained for subplanckian inflaton values with the corresponding effective theories retaining the perturbative unitarity up to the Planck scale.

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INTRODUCTION – It is well-known [1–3] that the presence of a non-minimal coupling function

$$f_R(\phi) = 1 + c_R \phi^{n/2}, \quad (1)$$

between the inflaton $\phi$ and the Ricci scalar $R$, considered in conjunction with a monomial potential of the type

$$V_{CI}(\phi) = \lambda^2 \phi^n/2n^n, \quad (2)$$

provides, at the strong $c_R$ limit with $\phi < 1$ (in the reduced Planck units with $m_p = 1$) an attractor [3] towards the values

$$n_s \approx 1 - 2/\tilde{N}_s = 0.965 \quad \text{and} \quad r \approx 12/\tilde{N}_s^2 = 0.0036, \quad (3)$$

for $\tilde{N}_s = 55$ e-foldings with negligible $n_s$ running, $a_s$. Although perfectly consistent with the present combined BICEP2/Keck Array and Planck results [4, 5].

$$n_s = 0.968 \pm 0.0045 \quad \text{and} \quad r = 0.048^{+0.035}_{-0.032}, \quad (4)$$

$r$ in Eq. (3) lies well below its central value in Eq. (4) and the sensitivity of the present experiments searching for primordial gravity waves [6, 7]. Nonetheless, this model – called henceforth non-minimal Chaotic Inflation (nMCI) – exhibits also a weak $c_R$ regime, with $\phi > 1$ and $c_R$-dependent observables [3] approaching for decreasing $c_R$'s their values within minimal chaotic inflation (MCI) [8]. Focusing on this regime, we would like to emphasize that solutions covering nicely the 1-$\sigma$ domain of the present data in Eq. (4) can be achieved, even for $\phi < 1$, by introducing a suitable non-canonical kinetic mixing $f_K(\phi)$. For this reason we can call this type of nMCI kinetically modified. Although a new parameter $c_K$, included in $f_K$, may take relatively high values within this scheme, no problem with the perturbative unitarity arises.

NON-SUSY FRAMEWORK – nMCI is formulated in the Jordan frame (JF) where the action of $\phi$ is given by

$$S = \int d^4x \left(-\frac{1}{2} \hat{g} \hat{R} + \frac{f_K}{2} g^{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - V_{CI}(\hat{\phi})\right). \quad (5)$$

Here $\hat{g}$ is the determinant of the background Friedmann-Robertson-Walker metric, $g^{\mu\nu}$ with signature (+, -, -, -) and we allow for a kinetic mixing through the function $f_K(\phi).$ By performing a conformal transformation [2] according to which we define the Einstein frame (EF) metric $\hat{g}_{\mu\nu} = f_R g_{\mu\nu},$ we can write $S$ in the EF as follows

$$S = \int d^4x \left(-\frac{1}{2} \hat{R} + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - \hat{V}_{CI}(\hat{\phi})\right), \quad (6a)$$

where hat is used to denote quantities defined in the EF. We also introduce the EF canonically normalized field, $\hat{\phi}$, and potential, $\hat{V}_{CI}$, defined as follows:

$$\frac{d\hat{\phi}}{d\phi} = J = \sqrt{\frac{f_K}{f_R} + \frac{3}{2} \left(\frac{f_K}{f_R}\right)^2} \quad \text{and} \quad \hat{V}_{CI} = \frac{\hat{V}_{CI}}{f_R^2}, \quad (6b)$$

$r$ and $\phi$ asymptote their values in Eq. (3).

Inspired by Ref. [9, 10], where non-canonical kinetic terms assist in obtaining inflationary solutions for $\phi < 1$, we liberate $c_R$ from its first role above implementing it by a kinetic function of the form

$$f_K(\phi) = c_K f_R^m \quad \text{with} \quad c_K = (c_R/r_{IK})^{4/n}. \quad (7)$$

Plugging Eqs. (7) into Eq. (6b) we obtain

$$j^2 = \frac{c_K}{f_R^{1-m}} + \frac{3n^2 c_K^2 \phi^{n-2}}{8 f_R f_K^{1-m}} \simeq \frac{c_K}{f_R^{1-m}} \quad \text{and} \quad \hat{V}_{CI} = \frac{\lambda^2 \phi^n}{2n^2 f_R^2} \quad (8)$$

assuming $c_K \gg c_R$. In contrast to Ref. [10] the presence of both $f_K$ and $f_R$ plays a crucial role within our proposal.

SUPERGRAVITY EMBEDDING – The supersymmetrization of the above models requires the use of two gauge singlet chiral superfields, i.e., $z^{\alpha} = \Phi, S$, with $\Phi$, $\Phi = (\alpha = 1)$ and $S$ ($\alpha = 2$) being the inflaton and a “stabilized” field respectively. The EF action for $z^{\alpha}$'s within Supergravity (SUGRA) [11] can be written as

$$S = \int d^4x \left(-\frac{1}{2} \hat{R} + K_{\alpha\beta} \hat{g}^{\mu\nu} \partial_\mu z^\alpha \partial_\nu z^\beta - \hat{V} \right), \quad (9a)$$

where summation is taken over the scalar fields $z^{\alpha}, K$ is the Kähler potential with $K_{\alpha\beta} = K_{\alpha\beta} = K_{z^\alpha z^\beta}$ and $K_{\alpha\beta} K_{\beta\gamma} = \delta^\gamma_\alpha$. 

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Also $\hat{V}$ is the EF F–term SUGRA potential given by
\[
\hat{V} = e^K \left( K^{\alpha \beta} (D_{\alpha} W)(D_{\beta} W^*) - 3 |W|^2 \right),
\] (9b)
where $D_{\alpha} W = W,_{\alpha} + K,_{\alpha} W$ with $W$ being the superpotential. Along the inflationary track determined by the constraints
\[
S = 0 \quad \text{and} \quad \Phi = \Phi^*, \quad \text{or} \quad s = \bar{s} = \theta = 0
\] (10)
if we express $\Phi$ and $S$ according to the parametrization
\[
\Phi = \phi e^{i\theta}/\sqrt{2} \quad \text{and} \quad S = (s + i\bar{s})/\sqrt{2},
\] (11)
$V_{CI}$ in Eq. (2) can be produced, in the flat limit, by
\[
W = \lambda S \Phi^{n/2}.
\] (12)
The form of $W$ can be uniquely determined if we impose two symmetries: (i) an $R$ symmetry under which $S$ and $\Phi$ have charges 1 and 0; (ii), a global $U(1)$ symmetry with assigned charges $-1$ and 2/n for $S$ and $\Phi$.

On the other hand, the derivation of $\hat{V}_{CI}$ in Eq. (8) via Eq. (9b) requires a judiciously chosen $K$. Namely, along the track in Eq. (10) the only surviving term in Eq. (9b) is
\[
\hat{V}_{CI} = \hat{V}(\theta = s = \bar{s} = 0) = e^K K^{SS*} |W, S|^2.
\] (13)
The incorporation $f_{IR}$ in Eq. (1) and $f_K$ in Eq. (7) dictates the adoption of a logarithmic $K$ [11] including the functions
\[
F_{IR}(\Phi) = 1 + 2\pi^2 \Phi^2 c_{IR} \quad \text{and} \quad F_K = (\Phi - \Phi^*)^2.
\] (14a)
Here $F_{IR}$ is an holomorphic function reducing to $f_{IR}$, along the path in Eq. (10), and $F_K$ is a real function which assists us to incorporate the non-canonical kinetic mixing generating by $f_K$ in Eq. (7). Indeed, $F_K$ lets intact $\hat{V}_{CI}$, since it vanishes along the trough in Eq. (10), but it contributes to the normalization of $\Phi$ – contrary to the naive kinetic term $|\Phi|^2/3$ [11] which influences both $J$ and $\hat{V}_{CI}$ in Eq. (6b). Although $F_K$ is employed in Ref. [3] too, its importance in implementing non-minimal kinetic terms within $nMCI$ has not been emphasized so far. We also include in $K$ the typical kinetic term for $S$,
\[
F_S = |S|^2/3 - k_S |S|^4/3,
\] (14b)
considering the next-to-minimal term for stability reasons [11] – see below. Taking for consistency all the possible terms up to fourth order, $K$ is written as
\[
K = -3 \ln \left( \frac{F_K}{2m_0^2} (F_{IR} + F_K)^n F_K \right) + \frac{1}{2} (F_{IR} + F_K^*) - F_S + \frac{k_{SS}}{6} F_K^2 - \frac{k_{SS}}{3} F_K |S|^2
\] (14c)
Our models are completely natural in the ’t Hooft sense because, in the limits $c_{IR} \to 0$ and $\lambda \to 0$, the theory enjoys the following enhanced symmetries – cf. Ref. [12]:
\[
\Phi \to \Phi^*, \quad \Phi \to \Phi + c \quad \text{and} \quad S \to e^{i\alpha S},
\] (15)

| TABLE I: Mass spectrum along the path in Eq. (10) |
|-----------------|-----------|-----------------|
| **FIELDS** | **EINGESTATES** | **MASS SQUARED** |
| 1 real scalar | $\theta$ | $\hat{m}_{\theta}^2 = 4V_{CI}/3 \approx 4H_{CI}^2$ |
| 2 real scalars | $s, \bar{s}$ | $\hat{m}_{s, \bar{s}}^2 = 2(6k_s f_s - 1)\hat{V}_{CI}/3$ |
| 2 Weyl spinors ($\hat{\psi}_s + \hat{\psi}_{\bar{s}})/\sqrt{2}$ | $\hat{m}_{\hat{\psi}_{s, \bar{s}}}^2 = n^2\hat{V}_{CI}/2c_k\delta^2f_{\text{IR}}^{m-i}$ |

where $c$ is a real number. Therefore, the terms proportional to $c_{IR}$ can be regarded as a gravity-induced violation of the symmetries above.

To verify the appropriateness of $K$ in Eq. (14c), we can first remark that, along the trajectory in Eq. (10), it is diagonal with non-vanishing elements $K_{\Phi,\Phi^*} = J^2$, where $J$ is given by Eq. (8), and $K_{SSS^*} = 1/f_{IR}$. Upon substitution of $K_{SSS^*} = f_{IR}$ and $\exp K = f_{IR}^{3}$ into Eq. (13) we easily deduce that $\hat{V}_{CI}$ in Eq. (8) is recovered. If we perform the inverse of the conformal transformation described in Eqs. (6a) and (5) with frame function $\Omega/3 = - \exp (-K/3)$ we end up with the JF potential $V_{CI} = 1/2^2\hat{V}_{CI}/9$ in Eq. (2). Moreover, the conventional Einstein gravity at the SUSY vacuum, $(S) = (\Phi) = 0$, is recovered since $-\langle S \rangle/3 = 1$.

Defining the canonically normalized fields via the relations
\[
d\hat{\phi}/d\phi = \sqrt{K_{\Phi,\Phi^*}} = J, \quad \hat{\theta} = J\theta, \quad \hat{\phi} = \sqrt{K_{SSS^*}}(s, \bar{s})
\] (16)
and $(\hat{\phi}, \hat{\theta}, \hat{\phi})$ we can verify that the configuration in Eq. (10) is stable w.r.t the excitations of the non-inflaton fields. Taking the limit $c_{IR} \gg c_{IR}$ we find the expressions of the masses squared $\hat{m}_{\phi, \bar{\phi}}^2$ (with $\phi^2 = \theta$ and $\bar{\phi}^2 = \bar{\theta}$) arranged in Table I, which approach rather well the quite lengthy, exact expressions taken into account in our numerical computation. These expressions assist us to appreciate the role of $k_S > 0$ in retaining positive $\hat{m}_{\phi}^2$. Also we confirm that $\hat{m}_{\phi, \bar{\phi}}^2 \gg \hat{H}_{CI}^2 = \hat{V}_{CI}/3$ for $\phi \leq \phi \leq \phi^*$. In Table I we display the masses $\hat{m}_{\phi, \bar{\phi}}^2$ of the corresponding fermions too. We define $\hat{\psi}_s = \sqrt{K_{SSS^*}}\psi_s$ and $\hat{\psi}_{\bar{s}} = \sqrt{K_{SSS^*}}\psi_{\bar{s}}$ where $\psi_s$ and $\psi_{\bar{s}}$ are the Weyl spinors associated with $S$ and $\Phi$ respectively.

Inserting the derived mass spectrum in the well-known Coleman-Weinberg formula, we can find the one-loop radiative corrections, $\Delta \hat{V}_{CI}$ to $\hat{V}_{CI}$. It can be verified that our results are immune from $\Delta \hat{V}_{CI}$, provided that the renormalization group mass scale $\Lambda$, is determined by requiring $\Delta \hat{V}_{CI}(\phi_s) = 0$ or $\Delta \hat{V}_{CI}(\phi_{\bar{s}}) = 0$. The possible dependence of our results on the choice of $\Lambda$ can be totally avoided if we confine ourselves to $k_{SS} \sim 1$ and $k_S \sim (0.5 - 1.5)$ resulting to $\Lambda \simeq (1 - 5) \cdot 10^{14}$ GeV – cf. Ref. [2, 13]. Under these circumstances, our results in the SUGRA set-up can be exclusively reproduced by using $\hat{V}_{CI}$ in Eq. (8).

**INFLATION ANALYSIS** – The period of slow-roll $nMCI$ is determined in the EF by the condition
\[
\max(|\dot{\epsilon}(\phi)|, |\ddot{\eta}(\phi)|) \leq 1,
\] (17a)
where the slow-roll parameters $\dot{\epsilon}$ and $\ddot{\eta}$ read
\[
\dot{\epsilon} = \left( \frac{\dot{V}_{CI, \dot{\phi}}}{\sqrt{2}\hat{V}_{CI}} \right)^2 \quad \text{and} \quad \ddot{\eta} = \frac{\ddot{V}_{CI, \ddot{\phi}}}{\hat{V}_{CI}}
\] (17b)
and can be derived employing \( J \) in Eq. (6b), without express explicitly \( \hat{V}_{\text{CI}} \) in terms of \( \dot{\varphi} \). Our results are

\[
\tilde{c} = \frac{n^2}{2\phi^2 c_K f_{1/m}^2}; \quad \tilde{n} = 2 \left( \frac{1}{n} - \frac{n(1+m)}{2n} \right) c_R \phi^\frac{n}{2};
\]

(18)

Given that \( \phi \ll 1 \) and so \( f_R \simeq 1 \), Eq. (17a) is saturated at the maximal \( \phi \) value, \( \phi_t \), from the following two values

\[
\phi_{1t} \simeq n/\sqrt{2c_K} \quad \text{and} \quad \phi_{2t} \simeq \sqrt{(n-1)n/c_K},
\]

(19)

where \( \phi_{1t} \) and \( \phi_{2t} \) are such that \( \tilde{c}(\phi_{1t}) \simeq 1 \) and \( \tilde{n}(\phi_{2t}) \simeq 1 \).

The number of e-foldings \( \hat{N}_s \) that the scale \( k_s = 0.05/\text{Mpc} \) experiences during this nMCI and the amplitude \( A_s \) of the power spectrum of the curvature perturbations generated by \( \dot{\varphi} \) can be computed using the standard formula

\[
\hat{N}_s = \int_{\phi_i}^{\phi_s} d\tilde{\varphi} \left( \frac{\tilde{V}_{\text{CI}}}{\tilde{V}_{\text{CI},\phi}} \right) \quad \text{and} \quad A_s^{1/2} = \frac{1}{2\sqrt{5\pi}} \left( \frac{\tilde{V}_{\text{CI}}^{3/2}(\phi_s)}{\tilde{V}_{\text{CI}}(\phi_i)} \right),
\]

(20)

where \( \phi_s \) are the values of \( \tilde{\varphi} \) when \( k_s \) crosses the inflationary horizon. Since \( \phi_s \gg \phi_t \), from Eq. (20) we find

\[
\hat{N}_s = \frac{c_K \phi^2}{2n} 2 F_1 \left( -m, 4/n; 1 + 4/n; -c_R \phi^2/n^2 \right),
\]

(21)

where \( 2 F_1 \) is the Gauss hypergeometric function [14] which reduces to unity for \( m = 0 \) (and any \( n \)) or to the factor \( (f_{1/m}^2 - 1)/c_R (1 + m) \) for \( n = 4 \) (and any \( m \)). Concentrating on these cases, we solve Eq. (21) w.r.t \( \phi_s \) with result

\[
\phi_s \simeq \begin{cases} 
\sqrt{2n\hat{N}_s/c_K} & \text{for } n = 0, \\
\sqrt{m - 1} / \sqrt{r_{\text{RR}} c_K} & \text{for } n = 4, 
\end{cases}
\]

(22)

where \( f_{1/m}^2 = 1 + 8(m + 1)r_{\text{RR}} \hat{N}_s \). In both cases there is a lower bound on \( c_K \), above which \( \phi_s < 1 \) and so, our proposal can be stabilized against corrections from higher order terms. From Eq. (20) we can also derive a constraint on \( \lambda \) and \( c_K \) i.e.

\[
\lambda = \sqrt{3A_s \pi} \left( \frac{c_K/n\hat{N}_s}{2} \right)^{\frac{3}{4}} \left( 2n f_{m*}/\hat{N}_s \right)^{\frac{3}{4}} \left( f_{1/m} - 1 \right)^{\frac{1}{2}} \left( f_{m*} \right)^{\frac{3}{4}} \quad \text{for } m = 0, \\
16c_K r_{\text{RR}}^2 \left( f_{m*} - 1 \right)^{\frac{1}{2}} \left( f_{m*} \right)^{\frac{3}{4}} \quad \text{for } n = 4.
\]

(23)

where the variables with subscript \( s \) are evaluated at \( \phi = \phi_s \) and \( \xi = \hat{V}_{\text{CI},\phi} \hat{V}_{\text{CI},\phi\phi} / \hat{V}_{\text{CI}} \). For \( m = 0 \) we find

\[
n_s = 1 - (4 + n + n/f_{m*}) / 4\hat{N}_s, \quad r = 4n/f_{m*} \hat{N}_s,
\]

(25a)

\[
a_s = (n^2 - n + 4) f_{m*} - 4(n + 4) f_{m*}^2 / 16 f_{m*}^2 \hat{N}_s^2.
\]

(25b)

In the limit \( r_{\text{RR}} \to 0 \) or \( f_{m*} \to 1 \) the results of the simplest power-law MCI, Eq. (2), are recovered – cf. Ref. [8]. The formulas above are also valid for the original nMCI [3], for \( c_K = 1 \) and \( r_{\text{RR}} = c_R \) lower than the one needed to reach the attractor’s values in Eq. (3). In this limit our results are in agreement with those displayed in Ref. [15] for \( n = 4 \). Furthermore, for \( n = 4 \) (and any \( m \)) we obtain

\[
n_s = 1 - 8r_{\text{RR}}^{2} m - 1 - (m + 2) f_{m*},
\]

(26a)

\[
r = \frac{128r_{\text{RR}}^{2/3} f_{m*}^{1+m} (1 + m)}{(f_{m*} - 1)^{2} f_{m*}^{4+1+m}}.
\]

(26b)

\[
f_{m*}^{2} \left( f_{m*} - 1 \right)^{2} f_{m*}^{4+1+m} \left( f_{m*} - 1 \right)^{2} f_{m*}^{4+1+m}.
\]

(26b)

For \( n = 4 \) and \( m = 1, 2, 4 \) the outputs of Eqs. (25a)-(26b) are specified in Table II after expanding the relevant formulas for \( 1/\hat{N}_s \ll 1 \). We can clearly infer that increasing \( m \) for fixed \( r_{\text{RR}} \), both \( n_s \) and \( r \) increase. Note that this formulae, based on Eq. (22), is valid only for \( r_{\text{RR}} \gg 0 \) (and \( m \neq 0 \)).

These conclusions can be verified and extended to other \( n \)'s and \( m \)'s numerically. In particular, confronting the quantities in Eq. (20) with the observational requirements [4]

\[
\hat{N}_s \simeq 55 \quad \text{and} \quad A_s^{1/2} \simeq 4.627 \cdot 10^{-5},
\]

(27)

we can restrict \( \lambda \) and \( \phi \) and compute the model predictions via Eqs. (24a) and (24b), for any selected \( m, n, c_K, \) and \( r_{\text{RR}} \). The outputs, encoded as lines in the \( n_s-r \) plane, are compared against the observational data [4, 5] in Fig. 1 for \( m = 0, 1, 2, \) and 4 and \( n = 2 \) (dashed lines), \( n = 4 \) (solid lines), and \( n = 6 \) (dot-dashed lines). The variation of \( r_{\text{RR}} \) is shown along each line. To obtain an accurate comparison, we compute \( r_{0.002} = \tilde{c}(\phi_{0.002}) \) where \( \phi_{0.002} \) is the value of \( \phi \) when the scale \( k = 0.002/\text{Mpc} \), which undergoes \( \hat{N}_{0.002} = \hat{N}_s + 3.22 \) e-foldings during nMCI, crosses the horizon of nMCI. For low enough \( r_{\text{RR}} \)'s – e.g. \( r_{\text{RR}} = 10^{-7}, 10^{-4} \), and 0.001 for \( n = 6, 4, \) and 2 the various lines converge to the \( (n_s, r) \)'s obtained within MCI. At the other end, these lines terminate for \( r_{\text{RR}} = 1 \), beyond which the theory ceases to be unitarity safe – see below. For \( m = 0 \) we reveal the results of Ref. [3],

\[
\begin{array}{c|c|c}
\text{Table II: Inflationary predictions for } n = 4 \text{ and } m = 1, 2, 4.
\end{array}
\]
i.e., the displayed lines are almost parallel for \( r \geq 0.01 \) and converge even at \( r_{\text{RK}} = 1 \). For \( m > 0 \) the curves move to the right and span more densely the 1-\( \sigma \) ranges in Eq. (4) for quite natural \( r_{\text{RK}} \)'s – e.g. \( 0.005 \lesssim r_{\text{RK}} \lesssim 0.1 \) for \( m = 1 \) and \( n = 4 \). It is worth emphasizing that the requirement \( r_{\text{RK}} \lesssim 1 \) provides a lower bound on \( r \), which ranges from 0.0032 (for \( m = 0 \) and \( n = 4 \)) to 0.019 (for \( m = 4 \) and \( n = 2 \)). Note, finally, that our estimations in Eqs. (25a)–(25b) are in agreement with the numerical results for any \( r_{\text{RK}} \) and \( n = 2 \) or \( r_{\text{RK}} \lesssim 0.002 \) [0.05] and \( n = 6 \) [4]. For \( m > 0 \) (and \( n = 4 \)) our findings in Eqs. (26a)–(26b) (and Table II) approximate fairly the numerical outputs for 0.003 \( \lesssim r_{\text{RK}} \lesssim 1 \).

**The Effective Cut-Off Scale** – The selected \( f_K \) in Eq. (7) not only reconciles nMCI with the 1-\( \sigma \) ranges in Eq. (4) but also assures that the corresponding effective forms for \( r_{\text{RK}} \) and \( f_{\text{RK}} \) are bounded, see Ref. [10, 13, 18].

To clarify further this point we determine the **ultraviolet** (UV) cut-off scale \( \Lambda_{\text{UV}} \) of our models analyzing their small-field behavior in the EE. We focus on the second term in the right-hand side of Eq. (9a) for \( \mu = \nu = 0 \) and expand it about \( \langle \phi \rangle = 0 \) in terms of \( \hat{\phi} \) – see Eq. (6b). Our result for \( m = 0 \) and \( n = 2, 4, \) and \( 6 \) can be written as

\[
J^2 \dot{\phi}^2 = \left( 1 - r_{\text{RK}} \ddot{\phi} + \frac{3n^2}{8} r_{\text{RK}}^2 \dot{\phi}^{n-2} + r_{\text{RK}}^2 \dot{\phi}^n \cdots \right) \dot{\phi}^2.
\]

(28)

Similar expressions can be obtained for the other \( m \)'s too. Expanding similarly \( \tilde{V}_{\text{CI}} \), see Eq. (8), in terms of \( \hat{\phi} \) we have

\[
\tilde{V}_{\text{CI}} = \frac{\lambda^2 \dot{\phi}^n}{2^{n/2}} \left( 1 - 2r_{\text{RK}} \ddot{\phi} + 3r_{\text{RK}}^2 \dot{\phi}^{n-2} - 4r_{\text{RK}}^2 \ddot{\phi}^n + \cdots \right)
\]

(29)

independently of \( m \). From the expressions above we conclude that \( \Lambda_{\text{UV}} = r_{\text{RK}}^{-2/n} m \) and therefore our models do not face any problem with the perturbative unitarity for \( r_{\text{RK}} \leq 1 \).
**Conclusions** – Prompted by the recent joint analysis of BiCER2/Keck Array and Planck which, although does not exclude inflationary models with negligible $r$’s, seems to favor those with $r$’s of order 0.01 we proposed a variant of nMCI which can safely accommodate $r$’s of this level. The main novelty of our proposal is the consideration of the non-canonical kinetic mixing in Eq. (7) – involving the parameters $m$ and $c_K$ – apart from the non-minimal coupling to gravity in Eq. (1) which is associated with the potential in Eq. (2). This setting can be elegantly implemented in SUGRA too, employing the super- and Kähler potentials given in Eqs. (12) and (14c). Prominent in this realization is the role of a shift-symmetric quadratic function $F_K$ in Eq. (14a) which remains invisible in the SUGRA scalar potential while dominates the canonical normalization of the inflaton. Using $m \geq 0$ and confining $r_{RK}$ to the range $(2.5 \cdot 10^{-4} - 1)$, we achieved observational predictions which may be tested in the near future and converge towards the “sweet” spot of the present data – the improvement compared to the trivial ($m = 0$) case, especially for $n = 4$ and 6, is evident from Fig. 1. These solutions can be attained even with subplanckian values of the inflaton requiring large $c_K$’s and without causing any problem with the perturbative unitarity. It is gratifying, finally, that a sizable fraction of the allowed parameter space of our model (with $n = 4$) can be studied analytically and rather accurately.

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