Spin waves in the magnetized plasma of a supernova and its excitation by neutrino fluxes

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Abstract

The spin effects on electromagnetic waves in a strongly magnetized plasma with rare collisions is considered with the help of relativistic kinetic equations, which takes into account the electron spin dynamics in the selfconsistent electric and magnetic fields. It is shown that for electromagnetic waves propagating almost perpendicular to ambient magnetic field the spin effects become essential in the vicinity of electron gyrofrequency and the corresponding wave dispersion and growth rate of the electromagnetic spin waves in the presence of intense quasi monoenergetic fluxes of neutrino is determined.

Key words: Elementary particles (neutrino) Plasma waves Supernova

We consider the spin wave propagation in a dense plasma with the strong magnetic field \( \mathbf{B}_0 = (0, 0, B_0) \) and find in the case of the quasi-perpendicular wave propagation in the cold magnetized electron gas, \( k_z \ll k_\perp \), a new eigen mode with the spectrum given by the paramagnetic spin resonance of electrons, \( \omega \approx \Omega_e \). The excitation of spin waves in solids (ferromagnets) by the electron beam is well-known in literature [1]. It should be kept in mind that we consider here the Fermi gas of free electrons in contrast to the quasi-particle approach in a condensed matter and neglect also the exchange interaction of electrons since the long-range forces are dominant in a plasma. The suitable object for the appearance of spin waves in plasma would be a polarized electron gas of a magnetized supernova (SN). The powerful neutrino flux can excite spin waves there analogously to the possible excitation of plasma waves in an isotropic SN plasma [2].
Let us derive the spectrum of spin waves in a magnetized plasma. The system of the self-consistent Relativistic Kinetic Equations (RKE) for the electron number density distribution \( f^{(e)}(p, x, t) \) and the electron spin density distribution \( S^{(e)}(p, x, t) \), has the form

\[
\left( \frac{\partial}{\partial t} + (v \nabla) \right) f^{(e)} = e \left( E + \frac{1}{c} [v, B] \right) \frac{\partial f^{(e)}}{\partial p} + St \left( f^{(e)} \right), \tag{1}
\]

\[
\left( \frac{\partial}{\partial t} + (v \nabla) \right) S^{(e)} = \left[ e \left( E + \frac{1}{c} [v, B] \right) \frac{\partial}{\partial p} \right] S^{(e)} + \\
\frac{2\mu_B}{\hbar} \left[ \frac{S^{(e)} \cdot B}{\gamma} + \frac{E (v S^{(e)}) - v \left( E S^{(e)} \right)}{(1 + \gamma) c} \right] + St \left( S^{(e)} \right), \tag{2}
\]

For our purpose the exact forms of collisional terms are not essential, or considering small perturbations of distribution functions with respect to equilibrium ones, it is possible to present them as

\[
St \left( f \right) = -\nu_e (f - f_0), \quad St \left( S^{(e)} \right) = -\nu_s \left( S^{(e)} - S_0^{(e)} \right) \tag{3}
\]

with some effective collisional frequencies \( \nu_e, \nu_s \).

Equations (1) and (2) are completed by the Maxwell equations

\[
\frac{\partial B}{c \partial t} = [\nabla, E], \quad c [\nabla, B] + \frac{\partial E}{\partial t} = 4\pi j, \tag{4}
\]

where the total electron current \( j(x, t) = j^{\text{conv}}(x, t) + j^{\text{mag}}(x, t) \) consists of the convection current of electrons \( j^{\text{conv}}(x, t) = -e \int d^3 p v f^{(e)}(p, x, t) \), and the magnetization current \[ j^{\text{mag}}(x, t) = \mu_B \int d^3 p_e \frac{\left[ \nabla, S^{(e)} \right]}{\gamma} - \frac{(v \nabla) \left[ v, S^{(e)} \right] / c^2}{1 + \gamma}. \]

(5)

Here \( e = | e | \) is the electric (proton) charge; \( \gamma = \varepsilon_p/m_e c^2 \) is the electron gamma-factor; \( \mu_B = e \hbar / 2m_e c = 5.79 \times 10^{-5} \text{ eV} T^{-1} \) is the Bohr magneton; \( B \to B_0 + B \) is the total magnetic field.

Note that only in the non-relativistic (NR) limit \( v/c \to 0, \gamma = 1 \), the magnetization current takes the usual form \( j^{\text{mag}}(x, t) = c [\nabla, M(x, t)] \), where the
magnetization $M(x, t)$ is given by

$$M(x, t) = \mu_B \int \frac{d^3p}{(2\pi)^3} S^{(e)}(p, x, t)$$  \hspace{1cm} (6)

For the thermodynamical equilibrium in the external magnetic field $B_0$ there appears the mean spin polarization

$$S^{(e)}(\varepsilon_p) = S^{(e)}_0(\varepsilon_p) = -\frac{\mu_B}{\gamma} \frac{df^{(e)}_0(\varepsilon_p)}{d\varepsilon_p} B_0$$  \hspace{1cm} (7)

and the corresponding mean magnetization $M = M_0 = \chi_0 B_0$, where the Fermi distribution $f_0(\varepsilon_p)$ and the electron density $n_e$ are given by

$$f^{(e)}_0(\varepsilon_p) = \frac{1}{\hbar^3 \exp[(\varepsilon_e(p_e) - \varepsilon_F)/T_e] + 1}, \quad n_{e0} = \int \frac{d^2p_e}{(2\pi)^3} f^{(e)}_0(\varepsilon_p),$$

correspondingly, and the static susceptibility of the polarized non-relativistic (NR) degenerate electron gas

$$\chi_0 = -\mu_B^2 \int \frac{d^3p_e}{(2\pi)^3} \frac{df_0(\varepsilon_p)}{d\varepsilon_p}$$  \hspace{1cm} (8)

is small, $\chi_0 = \alpha v_{Fe}/4\pi^2c \ll 1$. This is the reason why the static magnetic induction $B_0 = (1 + 4\pi\chi_0) H_0$ and the magnetic field strength $H_0$ practically coincide there.

Taking into account a small value of the static susceptibility for NR plasma (8) one can expect that the electron spin influence the electromagnetic waves should be important near the electron gyrofrequency $\Omega_e$ only, where such influence takes the resonance character. On the other hand, the electromagnetic field itself influences the electron trajectory in the resonant way near the gyrofrequency $\Omega_e$, and, in general, this influence is more stronger than the electron spin-magnetic resonance.

However, there is an exception. The ordinary electromagnetic wave which propagates strictly across the magnetic field, $k = (k_x, 0, 0)$, $k_x = k_\perp$, has the linear polarization $E = (0, 0, E_z)$, $B = (0, B_y, 0)$ due to which the perturbation of the Larmour rotation of an electron is minimized and one can expect an appearance of the spin effects. Really, let us consider the case of the transversal wave propagation in NR magnetized plasma separating the mean polarization (7) and perturbations of electromagnetic fields $E$, $B$,

$$S^{(e)}(p, x, t) = S^{(e)}_0(\varepsilon_p) + \delta S^{(e)}(p, x, t), \quad B \to B_0 + B.$$
In the Fourier representation, \( \frac{\partial}{\partial t} \rightarrow -i\omega \), \( \nabla \rightarrow i\mathbf{k} \), from the Maxwell equation (4) rewritten as

\[
\left( \frac{k_\perp^2 c^2}{\omega^2} - \varepsilon_{zz} \right) E_z = 4\pi \frac{ck_\perp}{\omega} \Delta M_y, \tag{9}
\]

after the substitution the magnetization component \( \Delta M_y \) given as it follows from (2) by

\[
\Delta M_y = \mu_B \int \frac{d^3p}{(2\pi)^3} \frac{\delta S^{(e)}_+ (p, \mathbf{k}, \omega) - \delta S^{(e)}_- (p, \mathbf{k}, \omega)}{2i} = 4\pi c^2 k_\perp \omega \delta M_y,
\]

we obtain the dispersion equation for the cold magnetized plasma

\[
\frac{k_\perp^2 c^2}{\omega^2} - 1 + \frac{\omega_p^2}{\omega (\omega + i\nu_e)} = 2\pi \mu_B^2 \frac{c^2 k_\perp^2 n_{e0}}{\omega^2 \varepsilon_F} - \frac{\Omega_e}{\omega - \Omega_e}, \tag{10}
\]

where the r.h.s. represents the electron spin-magnetic resonance at the gyrofrequency \( \omega \sim \Omega_e \); \( \omega_p = \sqrt{4\pi e^2 n_{e0}/m_e} \) is the plasma frequency. Obviously, the l.h.s. of this dispersion equation gives the standard ordinary wave spectrum accounting for the electron collisions in plasma if one neglects the electron spin in the r.h.s. in (10). However, if the ordinary wave frequency is far from \( \Omega_e \), \( \omega_p^2 + k_\perp^2 c^2 \neq \Omega_e^2 \), there appears the new branch: the electron spin wave with the frequency

\[
\omega \simeq \Omega_e \left( 1 + \frac{2\pi \mu_B^2 n_0}{\varepsilon_F} \frac{k_\perp^2 c^2}{k^2 c^2 + \omega_p^2 \left( 1 - \frac{\nu_e}{\omega} \right) - \Omega_e^2} \right) - i\nu_s, \tag{11}
\]

where: (i) for a strong magnetic field one can neglect the electron collisions, \( \nu_e, \nu_s \ll \Omega_e \), and (ii) even for a slightly relativistic dense electron gas, \( n_0 = (p_F/h)^3/3\pi^2 \simeq \lambda_e^{-3} = (m_e c/h)^3 \), \( \varepsilon_F \sim m_e c^2 \), the correction in parentheses is small, \( \sim \alpha \).

Note, that at \( \mu_B^2 B_0^2 / \varepsilon_F mc^2 \ll 1 \) the main dissipation of the spin mode,

\[
Im \omega = \nu_s + \Omega_e \frac{2\pi \mu_B^2 n_0}{\varepsilon_F} \frac{\nu_e \omega_p^2 k_\perp^2 c^2}{(k^2 c^2 + \omega_p^2 - \Omega_e^2)^2} \simeq
\]

\[
\simeq \nu_s + \nu_e \frac{\omega_p^4 k_\perp^2 c^2 \mu_B^2 B_0^2}{\varepsilon_F mc^2} \frac{\Omega_e^2 (k_\perp^2 c^2 + \omega_p^2 - \Omega_e^2)^2}{\Omega_e^2 (k_\perp^2 c^2 + \omega_p^2 - \Omega_e^2)^2} \tag{12}
\]
is due to the collisional relaxation of electron spins \( \sim \nu_s \). Special case \( k_z^2 c^2 + \omega_p^2 = \Omega_e^2 \), which corresponds to the coupling of spin and electromagnetic (with \( \omega \approx k_z^2 c^2 + \omega_p^2 \)) modes, is not considered here. It is reasonable to suppose, that \( \nu_s \) and \( \nu_e \) are of the same order and in a plasma of SN shells are determined by Coulomb collisions. Thus, we can estimate the collision frequency \( \nu_s \) according to

\[
\nu_s \sim \nu_e \sim n_e v_e \pi \left( \frac{e^2}{\varepsilon_e} \right)^2 \ln \Lambda ,
\]

where \( v_e \) is the typical velocity and \( \varepsilon_e \) is the typical energy of colliding electrons, \( \ln \Lambda \sim 10 - 20 \) is the Coulomb logarithm. For the electron density \( n_e \geq 10^{31} \text{ cm}^{-3} \) plasma is degenerated \( (\varepsilon_F \geq m_e c^2 \geq T_e) \) and an estimation of \( \nu_s \) in (13) must be multiplied by the factor \( \sim (T_e/\varepsilon_F)^2 \). In such case putting \( v_e \sim c \) and \( \varepsilon_e \sim \varepsilon_F \) one gets

\[
\nu_e \sim 7.5 \times 10^{16} \left( \frac{n_e}{10^{31}} \right) \left( \frac{m_e c^2}{\varepsilon_F} \right)^2 \left( \frac{T_e}{\varepsilon_F} \right)^2 \ln \Lambda / 10 \text{ sec}^{-1}
\]

and \( \omega_p = 1.7835 \times 10^{20} \text{ sec}^{-1} \), \( \nu_s/\omega_p \sim 4.2 \times 10^{-4} \). At lower electron density \( n_e = 10^{29} \text{ cm}^{-3} \) in plasma with \( T_e \sim m_e c^2 \gg \varepsilon_F \) in the similar way we find the next estimation

\[
\nu_s \sim 7.5 \times 10^{15} \left( \frac{n_e}{10^{29}} \right) \min \left[ 1, \left( \frac{T_e}{m_e c^2} \right)^{1/2} \right] \left( \frac{\varepsilon_e}{\varepsilon_F} \right)^2 \ln \Lambda / 10 \sim 10^{16} \text{ sec}^{-1}
\]

and \( \nu_s/\omega_p \sim 5.6 \times 10^{-4} \). In the more general case of the quasi-perpendicular wave propagation \( k_z \neq 0, k_z \ll k_{\perp} \), one can find the bounds on the spin wave existence in a polarized electron gas. Substituting the electric field component \( E_x \neq 0 \) expressed through \( E_z \neq 0 \) one obtains from the Maxwell equation (4) the dispersion equation

\[
\frac{k_{\perp}^2 c^2}{\omega^2} - \varepsilon_{zz} - \left( \frac{\varepsilon_{xx}}{k_{\perp}^2 c^2} + \varepsilon_{zz} \right) \frac{k_{\perp} k_z c^2}{\omega^2} = 4\pi \frac{c k_{\perp} M_B}{\omega} \frac{\mu_B}{E_z},
\]

where in the l.h.s. the permittivity tensor components in the third term (both \( \varepsilon_{xx} \) and \( \varepsilon_{zz} \)) contain the resonance terms \( \sim (\omega - \Omega_e)^{-1} \). However, in the vicinity \( \Omega_e \) this term is the correction of the order \( \sim (k_z/k_{\perp})^2 \ll 1 \) comparing with the spin resonance term in the r.h.s. since \( \varepsilon_{xx} \ll \varepsilon_{zz} \). There appears also the resonance term \( \sim (\omega - \Omega_e)^{-1} \) in the diagonal component \( \varepsilon_{zz} \) connected, in particular, with the cyclotron damping at the Doppler resonance \( k_z v_z = \omega - n\Omega_e, n = \pm 1, \ldots \). Combining all these corrections one finds the new dispersion equation

\[
\frac{k_{\perp}^2 c^2 + \omega_p^2}{\omega^2} - \varepsilon_{zz} + \frac{\omega_p^2 \omega}{\left( \Omega_e - \omega \right) A_1} - \frac{k_{\perp} k_z \varepsilon_F A_1 k_{\perp} k_z c^2}{2 \left( \omega - \Omega_e \right) m_e \Omega_e A_2} = 2\pi \mu_B^2 \frac{c^2 k_{\perp}^2 n_0}{\varepsilon_F} \frac{\Omega_e}{\omega - \Omega_e},
\]

(15)
where additional resonance terms in the l.h.s. while competing with the spin resonance contribution in the r.h.s. are given by the integrals

\[ A_1 = -\frac{1}{n_0} \int J_1^2(k_{\perp}n_L) m_ee^2 dp_\perp d\pi p_\perp^2 \frac{df}{d\varepsilon}, \]

\[ A_2 = -\frac{\varepsilon_F}{n_0} \int J_1^2(k_{\perp}n_L) dp_\perp d\pi p_\perp^2 \frac{df^{(e)}(\varepsilon_p)}{d\varepsilon_p}. \]

If the ordinary wave frequency is far from the electron gyrofrequency \( \Omega_e \), \( \omega_p^2 + k_{\perp}^2 c^2 \neq \Omega_e^2 \), we find finally from eq. (15) the spin wave spectrum

\[ \omega - \Omega_e = \frac{\Omega_e c^2 k_{\perp}^2}{k_{\perp}^2 c^2 + \omega_p^2 - \Omega_e^2} \left[ 2\pi \mu_B n_0 \frac{\varepsilon_F}{\omega_p} + \frac{k_{\perp}^2 \varepsilon_F A_1}{2m_e \Omega_e^2 A_2} - \omega_p^2 A_1 - \omega_p^2 \varepsilon F A_2 \right]. \] (16)

While the second term within brackets is small for the quasi-perpendicular wave propagation \( k_z \ll k_{\perp} \), the third one coming from the tensor component \( \varepsilon_{zz} \) would be small too for the strong magnetic fields obeying the inequality

\[ \frac{p_F}{m_e c} \ll \frac{\mu_B B_0}{2(m_e c^2)} = \frac{1}{4} \left( \frac{B_0}{B_c} \right). \] (17)

One can easily see from the last inequality that for a slightly relativistic electron gas \( p_F \leq m_e c \) spin waves exist if the magnetic field \( B_0 \) is stronger than the Schwinger field \( B_c = 4.41 \times 10^{13} \) Gauss, \( B_0 \gg B_c \), otherwise, for a moderate field \( B_0 \leq B_c \) the electron density should be deluted in the NR plasma, \( p_F \ll m_e c \). In the Standard Model of electroweak interactions (SM) the system of the corresponding RKE’s including the neutrino one is derived by the Bogolyubov method analogously to the case of an isotropic lepton plasma [4]. For the case of NR plasma the linearized electron spin RKE (2) is completed in SM by the main weak interaction term coming from the \( \nu e \)-scattering amplitude [3]

\[ \frac{G_F \sqrt{2} c_A f_0^{(e)}(\varepsilon_p)}{m_e} \int \frac{d^3 q}{(2\pi)^3} \nabla \delta f^{(\nu)}(q, x, t), \] (18)

where \( G_F \) is the Fermi constant; \( c_A = \mp 0.5 \) is the axial weak coupling with upper (lower) sign for electron (muon or tau) neutrinos correspondingly. Let us consider the monoenergetic neutrino beam \( f_0^{(\nu)}(q) = (2\pi)^3 n_{\nu 0} \delta^{(3)}(q - q_0) \), which propagates in a polarized electron gas across the magnetic field \( B_0 = (0, 0, B_0) \), namely along \( n_0 = q_0 / q_0 = (1, 0, 0) \). Hence we should find the solution of the linearized neutrino RKE for the total (Lorentz-invariant) number...
density distribution \( f^{(\nu)}(q, x, t) = f^{(\nu)}_0(q) + \delta f^{(\nu)}(q, x, t) \),

\[
\frac{\partial \delta f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial t} + \vec{n} \frac{\partial \delta f^{(\nu)}(\vec{q}, \vec{x}, t)}{\partial \vec{x}^2} + F^{(\nu)}_{j\mu} \left( \vec{x}, t \right) \frac{q^{\mu}}{\epsilon_q} \frac{\partial f^{(\nu)}_0(q)}{\partial q_j} + F^{(A)}_{j\mu} \left( \vec{x}, t \right) \frac{q^{\mu}}{\epsilon_q} \frac{\partial f^{(\nu)}_0(q)}{\partial q_j} = 0,
\]

which obeys the neutrino current conservation \( \frac{\partial j^{(\nu)}_\mu(x, t)}{\partial x_\mu} = 0 \) automatically since the weak interaction terms have the Lorentz structure and are given by the antisymmetric tensors \( F^{(\nu; A)}_{j\mu}(\vec{x}, t) \),

\[
F^{(\nu)}_{j0}(\vec{x}, t)/G_F\sqrt{2}c_V = -\nabla_j \delta n^{(e)}(\vec{x}, t) - \frac{\partial \delta J^{(e)}_{j\nu}(\vec{x}, t)}{\partial t},
\]

\[
F^{(\nu)}_{jk}(\vec{x}, t)/G_F\sqrt{2}c_V = \epsilon_{jkl}(\nabla \times \delta J^{(e)}_{l\nu}(\vec{x}, t))_t,
\]

\[
\sqrt{2}F^{(A)}_{j0}(\vec{x}, t)/G_Fc_A = -\nabla_j \delta A^{(e)}_{0\nu}(\vec{x}, t) - \frac{\partial \delta A^{(e)}_{j\nu}(\vec{x}, t)}{\partial t},
\]

\[
\sqrt{2}F^{(A)}_{jk}(\vec{x}, t)/G_Fc_A = \epsilon_{jkl}(\nabla \times \delta A^{(e)}_{l\nu}(\vec{x}, t))_t.
\]

Here \( j^{(\nu)}_\mu(x, t) = \int (q_\mu/\epsilon_q) f^{(\nu)}(q, x, t) d^3q/(2\pi)^3 \) and \( \delta j^{(e)}_\mu(x, t) = \int (p_\mu/\epsilon_p) \delta f^{(\nu)}(p, x, t) d^3p/(2\pi)^3 \) are four-vectors of the neutrino current density and the electron current density perturbation correspondingly; \( \delta A^{(e)}_\mu(x, t) = m_e \int (d^3p/(2\pi)^3) \delta a^{(e)}_\mu(p, x, t) \) is the axial four-vector of the spin density perturbation where the axial four-vector \( \delta a^{(e)}_\mu(p, x, t) \) has the components

\[
\delta a^{(e)}_\mu(p, x, t) = \left[ \frac{p\delta S^{(e)}(p, x, t)}{m_e}; \delta S^{(e)}(p, x, t) + \frac{p(\delta S^{(e)}(p, x, t))}{m_e(\epsilon_e + m_e)} \right].
\]

The latter is the statistical generalization of the Pauli-Lubański four-vector \( a_\mu \), [5]

\[
a_\mu(p) = \left[ \frac{p\gamma_\mu}{m_e}; \gamma + \frac{p(\gamma_\mu)}{m_e(\epsilon_e + m_e)} \right].
\]

Note that the neutrino RKE (19) differs from the result of [2] by the last term which is stipulated by the parity violation through the axial vector currents contribution to weak interactions. Substituting the solution of the neutrino RKE (19) into eq. (18) and properly solving the modified spin RKE (2) one finds the dispersion equation for spin waves enhanced in NR plasma by the neutrino beam,

\[
\left( (\omega + i\nu_s)^2 - \Omega_e^2 \right) \left( \omega + i\nu_s - \frac{c_A^2 A^{(\nu)}_z}{2} k_z \right) - \frac{(\omega + i\nu_s)^2}{4} c_A^2 A^{(\nu)}_z \left( A^{(\nu)}_+ k_+ + A^{(\nu)}_- k_- \right) = 0,
\]

(22)
where the spin collision frequency $\nu_s$ is estimated in (13); the dimensionless parameter $\Delta^{(\nu)} = 2G_F^2n_e n_\nu / m_e q_0$ is given by the mean densities of the electrons and neutrinos $n_e, n_\nu$, and the vector $A_i^{(\nu)}$ is given by

$$A_i^{(\nu)} = \int \frac{d^3q}{(2\pi)^3} \hat{f}_0^{(\nu)}(q) \left( \frac{[(kn)^2 - k^2]n_l(q)}{(\omega - kn)^2} + \frac{(kn)n_l - k_l}{\omega - kn} \right).$$

(23)

From the dispersion equation (22) we find the increment of the neutrino driven streaming instability for the spin waves which arises due to the Čerenkov resonance $\omega = kn_0 c + i\delta = \Omega_e + i\delta$ and has the form

$$\delta = \delta_{\nu_s=0} \left( \frac{\delta_{\nu_s=0}}{\nu_s} \right)^{1/2},$$

(24)

where the increment in the absence of collisions $\delta_{\nu_s=0}$ is given by [3]

$$\delta_{\nu_s=0} = \Omega_e \sqrt{\frac{3}{4}} (\Delta^{(\nu)})^{1/3} (\sqrt{2} | c_A \sin \theta_{q_0} |)^{2/3}. $$

(25)

Here $\theta_{q_0}$ is the angle between the neutrino beam direction $n_0$ and the wave vector $k$. We neglected here the vector current terms ($\sim c_V$) entering the neutrino RKE (19) which give a negligible contribution ($\sim c_A c_V$) in the spin RKE for long wave lengths exceeding the Compton one, $2\pi/k \gg \hbar/m_e c$, that is the reasonable approximation in NR plasma.

For the mean neutrino energy in a magnetized supernova (SN) $q_0 \sim 10$MeV and for the NR plasma outside the neutrinosphere and behind the shock with the densities $n_e \approx 10^{29}\text{cm}^{-3}, n_\nu \approx 10^{32}\text{cm}^{-3}$ the parameter is very small, $\Delta^{(\nu)} \sim 10^{-25}$, while the increment (25) would be big enough, e.g. for the strong magnetic field $B_0 = 10^{12}$ Gauss and the corresponding gyrofrequency $\Omega_e = 1.7 \times 10^7 B_0 = 1.7 \times 10^{19} \text{ sec}^{-1}$ it reaches the value $\delta_{\nu_s=0} \sim 10^{10} \text{ sec}^{-1}$. One can see that the spin wave amplitude increases faster than a neutrino passes through SN envelope ($\sim 10^{-3}\text{ sec}$). However, the value $\delta_{\nu_s=0}$ is less than the spin wave damping due to collisions $\sim \nu_s \sim 10^{16} \text{ sec}^{-1}$ resulting in a decrease of the neutrino driven instability growth rate, $\sim \delta_{\nu_s=0} (\delta_{\nu_s=0}/\nu_s)^{1/2} \sim 10^7 \text{ sec}^{-1}$.

Nevertheless, the energy exchange between plasma and neutrino fluxes by their interaction with spin waves probably could lead to the shock revival during the neutrino burst ($\sim 10 - 20$ sec) through the heating of the surrounding plasma. Really these spin waves are generally coupled to the magneto-sonic ones analogously to the spin waves in ferromagnets [1], or their energy can be transferred to the electromagnetic and plasma waves at the cross of spectra.

Note that in a strong magnetic field, $\Omega_e \lesssim \omega_{pe}$, the increment (25) is less suppressed for the small angles $\theta_{q_0} \leq \arccos (<v>/c)$ than the corresponding
one for plasma waves in the isotropic plasma [2],

$$\delta_{\nu_s=0} = \frac{\omega_p}{2} \sqrt{3} (\Delta^{(\nu)})^{1/3} \left( \sqrt{2} c_V \frac{\sin^3 \theta_{\theta_0}}{\cos^2 \theta_{\theta_0}} \right)^{2/3},$$

that allows to decrease the inevitable Landau damping of collective modes excited by the neutrino beam when spin waves propagate through a relativistic plasma with the mean electron velocity $<v> \sim c$.

There is the second advantage of spin waves enhanced via the weak axial vector currents ($\sim c_A = \mp 0.5$) instead of the case of plasma waves excited via the weak vector currents with the small vector coupling in the case of muon and tau neutrinos (choosing the lower sign in $c_V = 2\xi \pm 0.5$ where $\xi \simeq 0.23$ is the Weinberg parameter). This is the reason why authors [2] considered the case of electron neutrinos only and put for them $c_V \rightarrow 1$. Note that during the main neutrino burst in SN all neutrino species are produced in the hot SN core via the pair annihilation $e^+e^- \rightarrow \nu_\alpha \bar{\nu}_\alpha, \alpha = e, \mu, \tau$.

Thus, we conclude that spin waves (16) exist in a magnetized plasma with the strong magnetic field obeying the condition (17) and can be efficiently excited, e.g. in a magnetized SN by a powerful neutrino flux that could be an effective collective mechanism to revive the shock with the following blust of the SN envelope.

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