Universal scaling of Hall resistivity in clean and moderately clean limits for Hg- and Tl-based superconductors

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The mixed-state Hall resistivity ρ_{xy} and the longitudinal resistivity ρ_{xx} in HgBa$_2$CaCu$_2$O$_{8+δ}$, HgBa$_2$Ca$_2$Cu$_3$O$_{8+δ}$, and Tl$_2$Ba$_2$CaCu$_2$O$_{8+δ}$ thin films have been investigated as functions of the magnetic field (H) up to 18 T. We observe the universal scaling behavior between ρ_{xy} and ρ_{xx} in the regions of the clean and the moderately clean limit. The scaling exponent β in ρ_{xy} = Aρ_{xx}^β is 1.9±0.1 in the clean limit at high H and low temperature (T) whereas β is 1±0.1 in the moderately clean limit at low H and high T, consistent with a theory based on the midgap states in the vortex core. This finding implies that the Hall conductivity σ_{xy} is also universal in Hg- and Tl-based superconductors.

PACS number:74.60.Ge, 74.25.Fy, 74.72.Gr, 74.76.-w

When a type II superconductor is cooled down from a normal state into a superconducting state, the Hall effect shows very unusual features, which have been a long-standing problem and have remained as unresolved issues for more than three decades. The sign reversal of the Hall effect below T_c is one of the most interesting phenomena in the flux dynamics for high-Tc superconductors (HTS) and has attracted both experimental and theoretical interest. Furthermore, a scaling behavior between ρ_{xy} and ρ_{xx} has been found in most HTS. The puzzling scaling relation, ρ_{xy} = Aρ_{xx}^β, with β ~ 2 has been observed for Bi$_2$Sr$_2$CaCu$_2$O$_{8}$ (Bi-2212) crystals and Tl$_2$Ba$_2$Ca$_2$Cu$_3$O$_{10}$ (Tl-2223) films. Other similar studies have found β = 1.5 ~ 2.0 for YBa$_2$Cu$_3$O$_y$ (YBCO) films, YBCO crystals, and HgBa$_2$Ca$_2$Cu$_2$O$_{6}$ (Hg-1201) films. Even more interestingly, β ~ 1 was observed in the Hg-1212 thin films after heavy-ion irradiations.

To interpret this scaling behavior, a number of theories have been proposed. The first theoretical attempt was presented by Dorsey and Fisher. They showed that ρ_{xy} and ρ_{xx} could be scaled with an exponent β = 1.7, and they explained the experimental results of Luo et al. for YBCO films. A phenomenological model was put forward by Vinokur et al. They claimed that in the thermally assisted flux-flow (TAFF) region, β should be 2 and independent of the pinning strength. Their result was consistent with the observed exponent in Bi-2212 crystals and Tl-2223 films only for high H. Another phenomenological model was proposed by Wang et al. They showed that β could change from 2 to 1.5 as the pinning strength increased, which agreed with the results reported for YBCO crystals and Hg-1212 films. However, all these theories fail to explain the wide range of β from 1 to 2 observed in ion-irradiated Hg-1212 films.

Recently, a more detailed theory based on localized states in vortex cores was developed by Kopnin and Lopatin (KL) for the clean limit (CL) and the moderately clean limit (MCL). Due to the short coherence lengths, HTS change from the MCL to the CL as T decreases from T_c. KL showed that σ_{xy} = ρ_{xy}/ρ_{xx}^β was universal in the CL whereas the tangent of the Hall angle was universal in the MCL. This implies that β can change from 1 to 2 with decreasing T, which is consistent with previous work on ion-irradiated Hg-1212 films. This theory also well describes the recent observation of a triple Hall sign reversal in Hg-1212 thin films containing a high density of columnar defects. Localized core states have been observed in HTS by using various experimental setups, such as far-infrared spectroscopy and scanning tunneling spectroscopy, and they are in good agreement with the theoretical predictions. Furthermore, in the MCL case, Kopnin and Volovik showed that σ_{xy} for d-wave superconductors was very similar to the results for s-wave superconductors. On the other hand, Frantz and Tesanovic claimed that a bound state does not exist in the vortex core of a d-wave superconductor.

In this Letter, we report the first demonstration of the universal scaling behavior of the Hall resistivity in the CL and the MCL regions for Tl- and Hg-Based Superconductors, and the results can be well described by the recent KL theory. In the present study, by using a low-noise preamplifier prior to a nanovoltmeter, we were able to expand the sensitivity of the Hall voltage up to one order of magnitude compared to the sensitivities in previous works, and we were able to confirm the universality of the Hall scaling behavior for an extended H range up to 18 T.

The transport properties and fabrication process of Hg-based superconducting thin films are described in de-
Since films before and after irradiations are shown in Fig. 4. plotted in Fig. 3. The corresponding data for Tl-2212 and Hg-1223 films for various H up to 18 T are extracted from the slope of the solid lines, as shown in Figs. 3 and 4. Hall scaling is observed over roughly two decades of \( \rho_{xy} \), and even four decades in high H. This scaling relation is valid in the TAFF region. Note that the TAFF region expands to lower T at high H due to a huge resistive broadening in H for these materials. Thus, we can investigate the Hall behavior in the clean limit by applying high H [23]. On the other hand, the low-H data correspond to the MCL since the TAFF region in this case is limited to near \( T_c \).

H dependence of the Hall scaling is clearly demonstrated by the above data, and the results, including previously observed data for irradiated Hg-1212 films, are summarized in Fig. 5. As H increases, \( \beta \) changes from 1.4 to 1.9 for Hg-1212, from 1.3 to 1.9 for Hg-1223, from 1.0 to 1.9 for the pristine TI-2212, from 1.0 to 1.9 for the irradiated TI-2212, and from 1.0 to 1.9 for the irradiated Hg-1212 [8]. Note that at higher H, \( H \geq 8 \) T, the scaling exponent \( \beta = 1.9 \pm 0.1 \) shows a universal behavior, regardless of H, the number of CuO\(_2\) layers, the types of defects, and even the types of compounds. More strikingly, at low H, \( \beta = 1 \pm 0.1 \) also appears as a universal number although the observed H range is rather limited. The scaling exponent is independent of H below \( H = 0 \). 3 T for pristine TI-2212 and below \( H = 1.2 \) T for irradiated TI-2212 and Hg-1212 films. This universal behavior of the scaling is our principal finding, and this observation has serious implications for the physics of the Hall behavior, as discussed below.

With short coherence lengths and large energy gaps in HTS, the discrete nature of the energy levels, \( \omega_a \), in the vortex cores has been observed experimentally [12][13] and has been interpreted theoretically [1]. Considering these localized states and an additional force induced by the kinetic effects of charge imbalance relaxation, KL [10] calculated the Hall and the longitudinal conductivities, \( \sigma_L \), in the CL and the MCL regions. According to this theory, the Hall conductivity can be described by three terms: \( \sigma_H = \sigma_H^{(L)} + \sigma_H^{(D)} + \sigma_H^{(A)} \), where \( \sigma_H^{(L)}, \sigma_H^{(D)}, \) and \( \sigma_H^{(A)} \) are the contributions from localized excitations, delocalized excitations, and an additional force, respectively. The additional force is determined by the energy derivative of the density of states at the Fermi surface. Since \( \sigma_H^{(A)} \) dominates over \( \sigma_H^{(L)} \) and \( \sigma_H^{(D)} \) near \( T_c \), the Hall anomaly can take place, as observed in most HTS. \( \sigma_H^{(D)} \) originates from the density of quasiparticles outside the vortex core; thus, \( \sigma_H^{(D)} \) is comparable to the normal-state Hall conductivity very near \( T_c \), but is very small at low T compared to \( \sigma_H^{(L)} \). Due to this, we can neglect \( \sigma_H^{(A)} \) and \( \sigma_H^{(D)} \) in the low-T region. Note that the Hall scaling behavior is observed in the TAFF regions, which correspond to T regions below the positive peaks in the \( \rho_{xy} - T \) curves. In the TAFF regions, therefore, \( \sigma_H \) and \( \sigma_L \) can be expressed by [10].
\[ \sigma_H \sim \frac{N_{e}}{B} \frac{\omega_{\tau}^2}{1 + (\omega_{\tau})^2}, \quad (1) \]
\[ \sigma_L \sim \frac{N_{e}}{B} \frac{\omega_{\tau}}{1 + (\omega_{\tau})^2}, \quad (2) \]

where \( N \) is the density of charge carriers and \( \tau \) is the relaxation time. It has been found \[10,21\] that the tangent of the Hall angle, \( \tan \Theta = \sigma_H/\sigma_L \sim \omega_{\tau} \), is very small (\(< 1\)) in the dirty region near \( T_c \) while it is very large (\( > 1 \)) in the superconducting region \( T \ll T_c \). According to Eqs. (1) and (2), there are two distinct scaling regions. For the low-\( T \) (CL) region with \( (\omega_{\tau})^2 \ll 1 \) in the superconducting region \( T \ll T_c \), it has been found \[10,21\] that the tangent of the Hall angle, \( \tan \Theta = \sigma_H/\sigma_L \sim \omega_{\tau} \), is very small (\(< 1\)) in the dirty region near \( T_c \) while it is very large (\( > 1 \)) in the superconducting region \( T \ll T_c \).

In the case of YBCO, however, the Hall scaling \[1,4\] could be different from those observed for Hg- and Tl-based superconductors. Since the Hall scaling in YBCO takes place for \( \rho_{xy} < 0 \), where \( \sigma_{H}^{(A)} \) is comparable to \( \sigma_{H}^{(L)} \), the Hall scaling can be modified by \( T \) dependence of \( \sigma_{H}^{(A)} \). Furthermore, because \( \sigma_{H}^{(A)} \) is more pronounced with increasing \( T \), the scaling range of \( \rho_{xy} \) is narrower than those observed in Tl- and Hg-based superconductors. This is a possible explanation why \( \beta = 1 \) has not been observed in YBCO.

In summary, the universal Hall scaling behaviors between \( \rho_{xy} \) and \( \rho_{xx} \) in Hg-1212, Hg-1223, and Tl-2212 thin films were investigated as functions of \( H \). We found the universal behavior of the Hall scaling for the CL and the MCL regions. Within the context of a recent theory \[10\] based on the localized states in vortex cores, this universal behavior was explicitly understood. However, this behavior is valid only if \( \sigma_{H}^{(L)} \) is the main contribution to the Hall effect, which is not the case for YBCO.

This work is supported by the Creative Research Initiative of the Korean Ministry of Science and Technology.

\[ \begin{align*}
[1] & \text{J. Luo et al., Phys. Rev. Lett. 68, 690 (1992).} \\
[2] & \text{A. V. Samoilov, Phys. Rev. Lett. 71, 617 (1993).} \\
[3] & \text{R. C. Budhani, S. H. Liou, and Z. X. Cai, Phys. Rev. Lett. 71, 621 (1993).} \\
[4] & \text{W. N. Kang et al., Phys. Rev. Lett. 76, 2993 (1996).} \\
[5] & \text{W. N. Kang et al., Phys. Rev. B55, 621 (1997).} \\
[6] & \text{W. N. Kang et al., Phys. Rev. B59, R9031 (1999).} \\
[7] & \text{A. T. Dorsey and M. P. A. Fisher, Phys. Rev. Lett. 68, 694 (1992).} \\
[8] & \text{V. M. Vinokur et al., Phys. Rev. Lett. 71, 1242 (1993).} \\
[9] & \text{Z. D. Wang, J. Dong, and C. S. Ting, Phys. Rev. Lett. 72, 3875 (1994).} \\
[10] & \text{N. B. Kopnin and A. V. Lopatin, Phys. Rev. B51, 15291 (1995); N. B. Kopnin, ibid B54, 9475 (1996).} \\
[11] & \text{W. N. Kang et al., e-print cond-mat/9903427.} \\
[12] & \text{K. Karrai et al., Phys. Rev. Lett. 69, 152 (1992).} \\
[13] & \text{I. Maggio-Aprile et al., Phys. Rev. Lett. 75, 2754 (1995).} \\
[14] & \text{Y. Morita, M. Kohmoto, and K. Maki, Phys. Rev. Lett. 78, 4841 (1997), and references therein.} \\
[15] & \text{N. B. Kopnin and G. E. Volovik, Phys. Rev. Lett. 79, 1377 (1998).} \\
[16] & \text{M. Franz and Z. Tesanovic, Phys. Rev. Lett. 80, 4763 (1998).} \\
[17] & \text{W. N. Kang, R. L. Meng, and C. W. Chu, Appl. Phys. Lett. 73, 381 (1998).} \\
[18] & \text{W. N. Kang, Sung-Ik Lee, and C. W. Chu, Physica C315, 223 (1999).} \\
[19] & \text{Superconductor Technologies Inc., 460 Word Drive, Santa Barbara, CA 93111-2310, USA} \\
[20] & \text{K. A. Lokshin et al., Physica C300, 71 (1998).} \\
[21] & \text{J. M. Harris et al., Phys. Rev. Lett. 73, 1711 (1994).} \\
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Fig. 3. Kang et al.
