On the survival of poor peasants

Andrea C. Levi · Ubaldo Garibaldi

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Abstract. Previously, in underdeveloped countries, people tried to keep the prices of food products artificially low, in order to help the poor to buy their food. But it became soon clear that such system, although helpful for the city poor, was disastrous for the peasants (who usually are even poorer), so that hunger increased, instead of decreasing. More recently, thus, higher prices have been imposed. But a high-price system does not solve the problems. It helps, indeed, a peasant to buy in the city non-edible products, but not to buy (more expensive) food products from other peasants. The question is discussed here in more detail starting from the simplest conceivable case of two peasants producing each a different food product (bread and cheese, say), then generalizing to several food items and to any number of peasants producing a given food item j. Like in every economic system which wants to be sustainable, or able to reproduce itself in a stationary state at least, prices are determined by the necessity of exchanging “means of production” among “industries”, except that here “industries” are replaced by working peasants and “means of production” are replaced by food. It is found that prices must obey certain inequalities related to the minimal amount of each food item necessary for survival. Inequalities may be rewritten as equations and, in an important special case, such equations give rise to a simple version of the matrix equation used by famous authors to describe the economy.

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A.C. Levi
Department of Physics, University of Genoa, Italy

U. Garibaldi
Department of Physics, University of Genoa, Via Dodecaneso 33, 16146 Italy
tel: 010-3536554
fax: 010-314218
E-mail: garibaldi@fisica.unige.it
1 Introduction

Previously, in underdeveloped countries, the governments tried to keep food prices artificially low, in order to help the poor to buy their food. But it became soon clear that such system, although helpful for the city poor, was disastrous for the peasants (who usually are even poorer), so that hunger increased, rather than decreasing. More recently, thus, higher prices have been imposed. But food prices non-uniformly higher, besides the possible disadvantage of the city poor, do not solve the problems. They help, indeed, a peasant to buy in the city non-edible products, but does not help him at all to buy food products from some other peasants, if such products have become more expensive.

In general exogenously fixed prices\[1\] must satisfy some constraints to allow the economy to stay alive. Further, if they are far from the “natural prices” (to be defined below)\[II\], they induce unfairness among individuals, and instability among sectors, possibly to be suppressed by authority. The question will be discussed in more detail starting with simple examples.

2 Two peasants

Let us suppose, to begin with, two food products to be needed in order to survive, bread and cheese, say. Let 1 be the farmer producing the cereals from which bread is made, 2 the shepherd raising the sheep from whose milk cheese is made.

The monetary income of 1, \( R_1 \), must be spent to buy cheese. The inequality

\[ R_1 \geq p_2 F_2 \]  

holds, where \( p_2 \) is the price of cheese and \( F_2 \) is the minimal amount of cheese needed for survival.

On the other hand, how does 1 obtain his income \( R_1 \)? Selling bread. If he produces the amount \( Q_1 \) of bread, he eats at least \( F_1 \) with his family and sells \( V_1 \):

\[ Q_1 \geq F_1 + V_1 \]  

and, if \( p_1 \) is the price of bread, his income is

\[ R_1 = p_1 V_1 \]  

Therefore

\[ R_1 \leq p_1 (Q_1 - F_1) \]  

From \[I\] and \[II\] the inequality

\[1\] Suppose e.g a government agency buying from producers and selling them what they need at fixed prices
\[ p_2 F_2 \leq p_1 (Q_1 - F_1) \] (5)

gains, i.e.

\[ p_2 / p_1 \leq (Q_1 - F_1) / F_2 \] (6)

(survival of the farmer).

But a similar reasoning, considering the shepherd instead of the farmer, yields the inequality

\[ p_1 / p_2 \leq (Q_2 - F_2) / F_1 \] (7)

or

\[ p_2 / p_1 \geq F_1 / (Q_2 - F_2) \] (8)

(survival of the shepherd). (6) and (8) are compatible (i.e. a price system exists under which both farmer and shepherd can live) only if

\[ F_1 / (Q_2 - F_2) \leq (Q_1 - F_1) / F_2 \] (9)

which implies

\[ F_1 / Q_1 + F_2 / Q_2 \leq 1. \] (10)

This seems to be an interesting conclusion; in fact, it is rather trivial. Both \( Q_1 / F_1 \) and \( Q_2 / F_2 \) must always be larger than (or equal to) 2. If the size of the population is 2,

\[ Q_i \geq 2F_i, i = 1, 2. \] (11)

A sizable production, thus, is required in each sector. E.g. if the farmer produces the amount of bread needed for the survival of both, then the shepherd must do the same for cheese. But if e.g. both have an overproduction of 50\% only, there is no hope: with any price system, one of the two succumbs.

The really interesting conclusions are (6) and (8), which may be written together:

\[ F_1 / (Q_2 - F_2) \leq p_2 / p_1 \leq (Q_1 - F_1) / F_2 \] (12)

yielding the admissible range of prices.

If e.g. \( Q_2 = 2F_2 \) and \( Q_1 = 3F_1 \), (12) implies \( 1 \leq p_2 F_2 / p_1 F_1 \leq 2 \). In this interval both survive. On the contrary if the price of the cheese dose falls below that of the bread dose, the shepherd must draw on his savings (if and as long as possible) to buy bread, or reduce the self-consumption \( F_2 \) or \( F_1 \); both choices are unsustainable.

Moreover, assuming the limiting value \( p_2 F_2 / p_1 F_1 = 1 \), the shepherd can just survive, while the farmer can save (or extra consume) a dose of bread, or spend the saved value \( p_1 F_1 \) elsewhere. An exogenous price policy might produce deep inequalities in the population.

(12) imply also
\[p_1 F_1 + p_2 F_2 \leq p_1 Q_1\] (13)

(and similarly for \(p_2 Q_2\)). The meaning of (13) is commonsensical: the final revenue has not to be lower than the costs of production. It will become clearer below (see discussion after eq. (19)).

3 Many peasants, several sectors

It is now easy to generalize to more than two food products (but the food products necessary for survival are in all cases very few). It is also convenient to deal with sectors rather than with single peasants: this language shift is clearly allowed.

Let \(N_i\) the number of peasants of the \(i\)-sector (i.e., producing the food product \(i\)), while \(N = \sum_i N_i\) will be the total number of peasants of all sectors, the fraction of \(i\)-peasants being \(n_i = N_i/N\). Then (1) is replaced by

\[R_i \geq N_i \sum_{j \neq i} p_j F_j\] (14)

and (2) and (4) are replaced respectively by

\[Q_i \geq N_i F_i + V_i\] (15)

and by

\[R_i \leq p_i (Q_i - N_i F_i)\] (16)

Taking together (14) with (16)

\[\sum_{j \neq i} (p_j/p_i) F_j \leq Q_i/(N_i - F_i)\] (17)

i.e.

\[\sum_j (p_j/p_i) F_j \leq Q_i/N_i\] (18)

or

\[N_i \sum_j p_j F_j \leq p_i Q_i\] (19)

The inequality (19) establishes an acceptable price system, since

\[\sum_j p_j F_j = M\] (20)

denotes the cost of production per worker. Thus (19) says that the cost of production of any sector cannot overcome its final revenue. We can normalize
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the common factor of prices by posing e.g. \( p_i = 1 \), or in this special case (wherein the cost of production per worker \( M \) is the same for all sectors) posing \( M = 1 \), so that all prices would be relative to the "numeraire" arbitrary chosen. But we postpone this kind of choice, and we maintain not normalized prices.

We have two basic inequalities for each \( i \). First of all the "technical" condition

\[
Q_i \geq NF_i
\]

must hold, implying that any food product \( i \) is produced sufficiently for everybody, assuming all people to have the same needs. (This equation generalizes (11)). The second basic inequalities is, from (19), (20),

\[
p_i \geq MN_i / Q_i
\]

4 Equalities

The inequalities (21) and (19) may be written as equalities. Assuming that there is a surplus \( Y_i = NF_i s_i \geq 0 \) of food item \( j \), we rewrite (21) as

\[
Q_i = NF_i + Y_i = NF_i (1 + s_i)
\]

Similarly, we rewrite (19) as

\[
\frac{N_i}{Q_j} \left( \sum_i p_i F_i \right) \leq p_j.
\]

that is, posing

\[
A_{ij} = F_i N_j / Q_j,
\]

\[
\sum_i p_i A_{ij} \leq p_j
\]

follows, and finally, introducing \( r_j \geq 0 \),

\[
\sum_i p_i A_{ij} (1 + r_j) = p_j
\]

Equation (26), except for the presence of different \( r_j \)'s, has a form similar to equations well known in the economic literature[1,2]. Typically, such equations describe the production of industries of each sector using inputs from industries of other sectors; in this case \( r_j \) is the "rate of profit" of the \( j \)-sector. Although in the present case of poor peasants the word "profit" sounds improper, or even ironical, it is true that the well-being of the different sectors depends precisely on the value of their \( r_j \), and that, if for the \( j \)-sector \( r_j \) happens to become negative, then the \( j \)-sector is destroyed, the peasants of that sector can no longer survive, and the whole system is at risk. In the case of exogenous prices, provided they are consistent with inequalities (19),
the set of equations (26) can be solved for the $r_i$s, which are different among sectors. Given that the prices are exogenous, (26) can be solved for all $r_i$, and the solution is

$$r_i = \frac{p_i - \frac{N_i}{Q_i} M}{\frac{N_i}{Q_i} M},$$

(27)

where $\frac{N_i}{Q_i}$ is the cost of production per $j$–unit of food.

A general formula relates the rates of profit $r_i$ with the surplus production realized in the different sectors:

$$\sum_i n_i \frac{1 + r_i}{1 + s_i} = 1.$$  

(28)

Proof of (28). Using (25), eq. (29) may be written

$$\frac{N_i}{Q_i} (1 + r_i) \sum_j p_j F_j = p_i.$$  

(29)

But $\sum_j p_j F_j = M$, and $Q_i = N F_i (1 + s_i)$, hence (using $n_i = N_i / N$)

$$\frac{n_i}{F_i} \frac{1 + r_i}{1 + s_i} M = p_i.$$  

(30)

Multiplying by $F_i$ and summing over $i$, we find again $\sum_j p_j F_j = M$ at the right-hand side, and (28) is found. Equation (28), which can be rewritten usefully also as

$$\sum_i \frac{N_i F_i}{Q_i} (1 + r_i) = 1$$

(31)

is a pure macroeconomical constraint, which connects physical magnitudes and economical decisions. In fact, given (27), to fix exogenous rates of profit is tantamount to fix exogenous prices. From (28), given the non-negativity of $r_i$ and the trivial $\sum_i n_i = 1$, we see that in the limiting case of all $s_i = 0$ (that is a purely self-reproducible system, no surplus) all $r_i$ are null, and then, via (27), the sole possible prices are fixed, i.e. $p_i = \frac{N_i}{Q_i} M$. In words, if the surplus vanishes, and the whole gross product is necessary to the reproduction of the system, there is no room for any policy of exogenous prices if the system has to stay alive. Note that only in the case of two sectors (see (12)) relative prices have both a lower and an upper bound, fixed by the actual physical production. In the case of many sectors the disequations (24) and (22) furnish only a lower bound for each price. The importance of (28) or (31) is due to the implicit double bound which prices have to respect for the system staying alive.

A possible exogenous price fixing, suggested by the simple form (25), is to put $r_i = s_i$. This position would reward sectors endowed by a large surplus, and would punish less productive ones. This policy cannot but be a temporary
measure, as it would produce a bias for peasants abandoning sectors worth of a larger production towards sectors which have no need to increase. For example, take \( n_1 = 2/3, n_2 = 1/3, s_1 = 1/2, s_2 = 1/4 \). Then, according to Section 1, the inequalities were \( 4/11 \leq p_2F_2/(p_1F_1) \leq 5/4 \). The prices induced by (28) with \( r_1 = s_i \) are \( p_2F_2/(p_1F_1) = 1/2 \).

Another limit situation (see the next Section) is that of a policy that would impose a uniform rates of profit for all sectors, but this would be indistinguishable from a system left to itself and so "naturally" driven to equalize different rates of profit. In this situation, with the above parameters, \( p2F2/(p1F1) = 3/5 \)

5 A single rate of profit

Economic theory [1,2,4] involves equations describing how industries of a given sector use, for their production processes, commodities obtained from industries of other sectors. In some cases such equations have the general form

\[ \sum_i p_i A_{ij} (1 + r) = p_j. \]

(32)

They are a system of linear equations whose unknowns are the \( n \) prices (the number of sectors) and the (unique) rate of profit, and \( A_{ij} \) are known production coefficients. This is a homogeneous system, and prices are determined up to an arbitrary common factor. Actually, the system is usually more complicated than this, because it involves explicitly the competition for the resources between capital and labour, which coincide in our simple pure labour economy [3]. In our case of a simple pure labour economy, the means of production are nothing else than the survival feeding of workers.

Our equation (32) seems to belong to this class, although the matrix \( A_{ij} \) (see 25) is very simple, each entry being the product of \( F_i \) (the \( i \)-dose of food) times \( N_j/Q_j \), the labour for a unit of product of the \( j \)-sector. Or more materialistically: \( N_jF_i \) is the quantity of \( i \)-commodity necessary to produce \( Q_j \). Given that the price (of a commodity) is always implicitly referred to the unit of that commodity, \( p_iA_{ij} = p_iF_i N_j/Q_j \) is the contribution of the value of the \( i \)-commodity to the cost of a unity of the \( j \)-commodity.

\[ A_{ij} = F_i a_j, \]

(33)

\[ a_j = N_j/Q_j, \]  

(34)

where \( a_j \) is the coefficient of labour of the \( j \)-sector, that is the quantity of labour per \( j \)-unit (its reciprocal \( h_j \) is the \( j \)-productivity). Note that the role of prices is quite different here: in (32) they are endogenous, and can

\[ ^2 \text{It is well known that an expanding economy the rate of growth is large as the minimum surplus rate of its basic sectors. See[2].} \]
be obtained (up to a scale factor) together with the (single) rate of profit \( r \), while in (20) we had different rates of profit \( r_j \) for the different sectors. If prices are fixed exogenously, and the rates of profit in the different sectors are considerably different, the temptation for the individuals to pass to sectors with high rate of profit is strong, but it is constrained by the vinculum (23). In words, sectors with low profit are compelled to be populated, in order to keep alive the economic system; a sort of serfdom. Instead if prices are endogenous and they are obtained by (32) together with the single rate of profit, there is no reason for change of sector, and the economy would be in a "natural" equilibrium.

Due to the simplicity of our matrix, (32) can be solved exactly for the \( n \) prices and the rate of profit \( r \), as the actual variables are only \( n-1 \) relative prices and \( r \). Considering (32) with \( A_{ij} \) from (33) we have

\[
\frac{N_j}{Q_j} \left( \sum_i p_i F_i \right) (1 + r) = p_j, \ j = 1, \ldots, n
\]

(35)

which implies

\[
p_j = V \frac{N_j}{Q_j} = V a_j
\]

(36)

If we put the constant \( V = 1 \), i.e

\[
p_j = \frac{N_j}{Q_j}
\]

(37)

prices are measured as the values produced by the work of the peasants in a given time \( t \) (for example a day, a year). If \( Q_i \) is the amount of food of type \( j \) produced in time \( t \), and \( \frac{N_i}{Q_i} \) is the amount of labour needed to produce a \( j \)-unit, then prices are equal to "values" (in the sense of the classical school of economics).

Substituting (37) in equations (35) we obtain

\[
\frac{1}{1 + r} = \sum_i p_i F_i = \sum_i \frac{N_i}{Q_i} F_i
\]

(39)

Note that, due to (21), \( \sum_i \frac{N_i}{Q_i} F_i \leq \sum_i \frac{N_i}{N} = 1 \), and \( r \geq 0 \)

Hence

\[
r = \frac{1}{\sum_i a_i F_i} - 1.
\]

(40)

\[^3\] From (23)

\[
\frac{N_i F_i}{Q_i} = \frac{n_i}{1 + s_i}
\]

(38)

It is interesting and correct (although at first sight unexpected) the fact that the price of the \( i \)-dose is proportional to \( n_i = N_i/N \): peasants belonging to big sectors must obtain a relatively high price for food doses produced by them. This is due to the fact that they must first of all feed themselves and their families and, if they are many, the few doses left to be sold must be exchanged with the many doses of the other commodities they need to survive.
Prices are proportional to \( \frac{N_j}{Q_j} = a_j \), i.e. the labour, whereas up to here peasants appeared only via the food they need to survive. This food appears as capital to be anticipated, and the surplus is shared among sectors proportionally to the capital. But in a pure labour economy the capital is the "slave wage" only, which is proportional to the number of workers. In the following we will assume endogenous ("natural") prices and accordingly a single rate of profit \( r \). A more formal approach to "natural prices economy" is given in the following Chapter.

6 Matrix approach

In order to introduce the matrix approach, we turn to the mathematical form of eq. (32), which is attractive from a theoretical point of view. Eq. (32) is an eigenvalue equation, to which all the corresponding theorems can be applied. It can be written in symbolic form as

\[
p \cdot A(1 + r) = p.
\]

where \( p = (p_1, \ldots, p_n) \) is a row vector with as many components as the number of commodities; \( A \) is a \( n \times n \) matrix whose entries are the technical coefficients (quantity of \( i \) needed for a unit of \( j \)); and \( r \geq 0 \) is the rate of profit. In (41) \( A \) is given, so that we have \( n + 1 \) unknowns. But due to homogeneity, a price can be fixed at will, say \( p_1 = 1 \) (the numeraire, so that all the other prices become relative), and the system has a unique solution: the "natural price" \( p \), is the left eigenvector of \( A \) corresponding to the eigenvalue \( \lambda = 1/(1 + r) \). Actually, by the Perron-Frobenius theorem (see Appendix of [2]), \( \lambda \) is the maximum eigenvalue, it is not greater than 1, and it is the sole whose eigenvector has all elements positive. Further in our simple case the matrix factorizes, and it can be made even simpler taking as units the individual survival doses of each commodity. In this case we introduce \( X_i = Q_i/F_i \), whose the meaning is the number of individual doses, and \( A_{ij} = \frac{N_j}{X_j} \), the reciprocal of the product number per capita \( \frac{X_i}{X_j} \). Thus

\[
A = \begin{pmatrix} \frac{N_1}{X_1} & \frac{N_2}{X_1} & \ldots & \frac{N_n}{X_1} \\
\frac{N_1}{X_2} & \frac{N_2}{X_2} & \ldots & \frac{N_n}{X_2} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{N_1}{X_n} & \frac{N_2}{X_n} & \ldots & \frac{N_n}{X_n} 
\end{pmatrix}
\]

all rows being equal (hence its determinant vanishes, but this is irrelevant in the present treatment). The maximum eigenvalue is (the capital \( R \) indicates the maximum value of \( r \))

\[
\lambda = \sum_i a_i = \sum_i N_i/X_i \leq 1
\]

(see Sect. 5), where \( a_i = N_i/X_i \) is the coefficient of labour of the \( j \)-sector, that is the quantity of labour per \( j \)-unit (its reciprocal is a measure of the
productivity). The corresponding left eigenvalue $p$ is proportional to $a = (a_1, \ldots, a_n)$, and the maximum profit rate is

$$R = \frac{1}{\sum a_i} - 1.$$ (43)

We have commented in Sect. 5 that the fact that the "natural price" is proportional to the labour necessary to produce one unit (one dose, from now on) looks strange in a scheme where all surplus is attributed to profit and nothing to labour. In facts the complete form of (41) would be:

$$p \cdot A (1 + r) + aw = p.$$ (44)

where $a$ is the labour coefficient row vector, and $w$ is the price of a unit of labour (the "human part" of labour, which exceeds pure subsistence). Given that $p \cdot A$ is the value of the means of production, the value of the surplus $p - p \cdot A$ is divided into a capital part $(p \cdot A) r$ and a labour part $aw$. The opposite of (41) is

$$p \cdot A + aw = p.$$ (45)

where all surplus is allocated to labour. The formal solution of (45) is

$$p = a \cdot (I - A)^{-1} w.$$ (46)

(46) says that prices are not simply proportional to $a$, but they are proportional to the vertically integrated labour coefficients $a \cdot (I - A)^{-1}$, a notion whose detailed description we can now afford to omit. This because fortunately the two notions are in turn proportional if $a$ is a left eigenvector of $A$, which is the basic property of a pure labour economy, hence $p = Va$. We can conclude that in a pure labor economy prices are independent of the distribution of the net product, and that sector shares of the surplus are the same, either proportional to the invested "capital" or to labour.

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4 This notion is fundamental in many works of L. Pasinetti (see e.g. [5], pag.75). The symbolic formulation of the system of quantities is $A \cdot Q + Y = Q$, where $Q$ and $Y$ are column vectors representing the $n$ quantities and surpluses. Then $(I - A) \cdot Q = Y$, and $Q = (I - A)^{-1} \cdot Y$.

The elements of the non-negative matrix $(I - A)^{-1}$ (which exists being $I - A$ non singular) represent the physical quantities i needed, directly or indirectly, to obtain a unit of $j$-commodity as surplus. This can be realized using the identity $(I - A)^{-1} = I + A + A^2 + \ldots + A^k + \ldots$, where each term adds the mean of production necessary to the previous stage.

Then $a \cdot (I - A)^{-1}$ represents the quantity of labour used, directly or indirectly, to obtain a physical unit of any commodity.

5 This remains true at any level of $r$ and $w$ in the general form (44). In the whole treatise we suppose that sectors have no master, i.e. the final product belongs to workers, who collectively anticipate the means of production and share the net product. If so there is no difference is they "think" themselves as capitalists or workers or any mixture of two. If there were masters, though prices would not change, peasants' welfare would be proportional to $w$. 
7 Exchanges with the city

So far we have supposed that the countryside economy is a pure labour economy, where there is no room for non-edible goods. If one wants to introduce “luxury goods” as consumptions available to country people, one has to introduce the sector to produce them. Coming back to the simple example of Chapter 2, besides the farmer and the shepherd we introduce a carpet seller, who, to produce $X_3$ carpets, needs bread, cheese and a fraction of his product. The first observation is that the countryside can have access to “luxury goods” only if the country surplus is enough to sustain one more worker, i.e. $X_i \geq 3$, see (11). To avoid inessential complications we consider the added value all given to profit (we have treated the peasants as little entrepreneurs, having used for them the reduced form (41). The new system of linear equations is the following, with $r = R$

$$\begin{cases} (p_1 + p_2)(1 + R) = p_1X_1 \\ (p_1 + p_2)(1 + R) = p_2X_2 \\ (p_1 + p_2 + p_3)(1 + R) = p_3X_3 \end{cases}$$

(47)

The different role of the third equation with respect to the previous two is apparent. The first two equations are unchanged with respect to the old closed system, they can be solved analytically, and the solution is just the old one. If we put the system (47) in matrix form,

$$A = \begin{pmatrix} \frac{1}{X_1} & \frac{1}{X_2} & \frac{1}{X_3} \\ 0 & 0 & \frac{1}{X_3} \end{pmatrix},$$

(48)

we see that the matrix is reducible, and the main submatrix (describing the basic-commodities) is identical to the closed system. Posing $p_1 = 1$, i.e. measuring values in terms of a dose of bread, we get $p_2 = \frac{X_1}{X_2}$. (See the note to the previous chapter). The rate of profit $R$ is

$$R = \frac{1}{X_1^{-1} + X_2^{-1}} - 1$$

(49)

where $X_1^{-1} + X_2^{-1}$ is the maximum eigenvalue of the main submatrix (see(??)).

All these variables being determined in the the main submatrix (describing the basic-commodities), turning to the third equation we find

$$p_3 = \frac{X_1}{X_3 - 1 - R},$$

(50)

and the system is completely solved. Note that (50) must be taken with care: we see that $X_3$, or rather $X_3 - 1$, must be large, i.e. the production which exceeds its means, must be larger than $R$, to avoid strange behaviour of $p_3$.

We can consider the effect on the countrymen of enlarging the economy to comprehend the carpet seller. Neither the prices did change, nor the income of each sector, that is $RM$, i.e. the maximum rate of profit times the value of the
individual means of production. The sole advantage is given by the possibility of owning carpets, besides bread and cheese, as commodities to be consumed. For city workers there is the possibility of being sustained by the country food, which they exchange with carpets.

Instead for the city sector also the price depends explicitly on the rate of profit, and this can be critical, due to the denominator of (50).

Suppose that $X_3 - R$ is small. The city sector is fragile, ruled by $(p_1 + p_2 + p_3)(1 + R) = p_3X_3$. Its fragility is due to the fact that, at odds with basic-commodities cases, $p_1, p_2$ and $R$ are given exogenously from the point of view of the city. Otherways $p_3$ has no limit, as it does not affect any other price. For all basic equations $p_iX_i > p_i(1 + R)$ holds by definition, because $R$ is endogenously derived. But in case of a non-basic commodity, it may happen that $p_3X_3 < p_3(1 + R)$, being $R$ exogenous. If $X_3 - 1 - R$ is small and positive, i.e. the production is small, the price can grow without limit to support the exchange with country commodities, to pay wages and retain a "fair" profit. But in case of $p_3X_3 < p_3(1 + R)$, even an infinite price is unable to satisfy the equation, as the total revenue is less than the sole "fair" profit.

7.0.1 Productivity changes induced by exchanges with the city.

The contact with the city is not only a chance for new types of expenditures: it may induce the desire to improve one's technical ability, or to better cultivation methods, like a farmer buying a plough. Starting from equations (44), we do not introduce new means of production: we simply suppose that the farmer increases its productivity, while the shepherd maintains its technical coefficients.

A first way to express the improved productivity is that of supposing the farmer producing simply more bread, *ceteris paribus*. Instead of (52) we can write.

$$\begin{align*}
(p_1 + p_2)(1 + R) &= \gamma p_1X_1, 1 \leq \gamma \\
(p_1 + p_2)(1 + R) &= p_2X_2.
\end{align*}$$

(51)

The augmented production of bread changes the technical matrix, its eigen-system and $R$, but in a simple way. $p_2 = \frac{\gamma X_1}{X_2}$ grows linearly with $\gamma$, and the total income too, i.e. $\gamma X_1 - 2 + \frac{\gamma X_1}{X_2}(X_2 - 2) = 2\gamma(X_1 - \frac{X_1}{X_2}) - 2$, and it is partitioned into two equal parts, since the means of productions are equal in the two sectors. The technological improvement of the farmer betters (economically at least) both sectors, and the increasing income can support some other development.

In the second case we suppose that the farmer produces the same previous amount working less (due to his increased productivity), i.e., using the complete form derived from (44)...

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6 This example is discussed at some length in [1].
\[(p_1 + p_2)(1 + r) + gw = p_1X_1, \quad 0 \leq g \leq 1\]
\[(p_1 + p_2)(1 + r) + w = p_2X_2\]  
(52)

The submatrix (48) is unchanged, thus eigenvalues and eigenvectors do not change, the maximum rate of profit is still (19). What changes is the labour coefficient vector, which is not a left eigenvector of the technical matrix unless \(g = 1\). Hence we expect that natural prices depend on \(r\). But if we consider \(r = R\) and \(w = 0\) the dependence on \(g\) disappears, and all economical values are the same. In this case the farmer can benefit of the improved quality of life without any economical loss.

A third case of interaction with the city can be represented by the enlargement of the two country sectors to comprehend a sector which produces iron objects (like ploughs, spades,..), useful for improving the technique of the farmer. At odds with luxury goods, let us suppose that the new technique of the farmer needs a dose of iron, useless to the shepherd, while the iron worker needs bread, cheese and one dose of iron. We suppose that the production of bread increases, and that of cheese is the same. The equations are:

\[
\begin{align*}
(p_1 + p_2 + p_3)(1 + R) &= p_1X_1 \\
(p_1 + p_2)(1 + R) &= p_2X_2 \\
(p_1 + p_2 + p_3)(1 + R) &= p_3X_3
\end{align*}
\]  
(53)

Different from the case of luxury goods (17), now the third price is essential for the price of bread, so that the countryside is not independent from the city sector. In this elementary case we can solve the system:

\[
\begin{align*}
p_1 &= 1 \\
p_3 &= X_1/X_3 \\
p_2 &= X_1(X_3-1-R)/X_2X_3
\end{align*}
\]  
(54)

The analytic solution for \(R\) is omitted to avoid complications. The farmer and the smith share the same part of the net product. If \(X = (8, 3, 2)\), i.e. the output of bread is doubled, that of cheese is invariant, and the iron output is just enough for maintaining the economic flow, \(R = .37\), and the new prices are:

\[p_1 = 1, p_2 = \frac{8 \times (2 - 1 - .37)}{4 - 2} = 0.84, p_3 = \frac{8}{2} = 4.0;\]  
the net product is \(Y = (5, 0, 0)\), and its value is \(p.Y = 5.0\).

The means of productions of both bread and iron amount to \(p_1 + p_2 + p_3 = 1 + .84 + 4.0 = 5.84\), and the income of the two sectors is \(5.84 \times .37 = 2.16\). The income of the shepherd boils down to \(1.84 \times .37 = 0.68\), and the sum of the three incomes is just \(0.68 + 2 \times 2.16 = 5.0 = p.Y\). The introduction of iron objects has improved the income of the farmer (from 5/3 = 1.67 to 2.16 doses of bread), while it has diminished the income of the shepherd (from 5/3 = 1.67 to 0.68).

Unexpectedly, the fortune of the shepherd would rise abruptly if the smith should produce one more dose. If \(X = (8, 3, 3), R = .64, Y = (5, 0, 1), p.Y = 7.67,\) farmer’s and smith’s income becomes 3.12, while shepherd’s one rises to 1.41.
References

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