Quality Management of Building Materials and Building Productions Based on the Using of Real Estimates of Measurement Accuracy

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Abstract. The most important direction of quality improvement in construction is to increase the accuracy of measurements of parameters of building products and construction products, as well as the parameters of the technological process of construction and of production of building materials. Currently used standard estimates of measurement accuracy provide guaranteed but underestimated accuracy estimates. Although in fact, with the correct measurement methods and the use of a posteriori information, it is possible to build more realistic estimates of accuracy. The accuracy of such estimates is several times higher than the accuracy of standard estimates. The article investigated the problem of increasing the accuracy of measurements through the use of a posteriori information. An example of the application of the Bayesian approach in the conditions of the possibility of obtaining additional information and its use in order to build effective estimates of measurement accuracy is presented. It is assumed that the measurement result is the sum of the actual value of the parameter being measured and the measurement error. For the case of a uniform distribution of the real value of the measured parameter and measurement error, explicit analytical formulas were obtained for the differential distribution function of the measurement result and the conditional distribution of the real value for a known measurement result. These analytical dependencies, obtained as a result of integrating fractional rational expressions, are composite functions, which in different parts of the argument are specified using different analytical formulas. The analysis of the obtained analytical dependencies. Using numerical integration, we plotted the dependence of the square of the standard deviation of the real value of the measured parameter on the observed value of the random variable. It is shown that in the case when the standard deviation of the measuring device is less than three times or more than the standard deviation of the actual value of the parameter, the use of a posteriori information allows us to construct more realistic estimates of accuracy. The accuracy of the measurements can be obtained several times higher. It is noted that in the case of normal and exponential distribution of the real value and measurement error, the use of a posteriori information does not lead to an increase in measurement accuracy. The results obtained can be applied in the development of measurement methods, in the development of methods for verification and calibration of measurement instruments.

1. Introduction
Quality management is important problem for construction [1,2], machine building [3,4], mining and other activities [5,6]. Reliability, durability and ease of use of construction sites are directly related to both the quality of materials used in construction and the use of modern building technologies. The
accuracy of building products and materials largely determines their level of quality, reliability, durability and directly affects competitiveness. Improving the accuracy of measurements in the manufacture of building products and materials is the most important problem. The pace of scientific and technological progress is directly dependent on the solution of this problem.

Accuracy in general is formed and ensured at all stages of the life cycle, including design, production, control, operation and disposal. This makes the problem of accuracy extremely capacious, which can be solved with the involvement of researchers, designers, production workers, both to develop high-quality product designs and to create an effective technological environment to ensure high-quality production, including reliable quality control of products. To solve the problem of accuracy, all professionals involved in the chain of building products and materials should be guided by common scientific principles to ensure accuracy [2].

In the case of direct measurements, the measurement result is the sum of two random variables [5]: the actual value of the measured parameter and the measurement error. These random variables may have different distribution laws.

Examples of real situations that lead to the need to consider uniformly distributed random variables can be: an error when taking readings from measuring instruments, in the case when rounding to the nearest integer division is performed; description of the errors of the analog - digital conversion; measuring harmonic oscillations with a random phase; quantization errors in digital devices. Therefore, consideration of the case of uniformly distributed random variables is of practical interest. Note also that the uniform distribution is often used as a “first approximation” in the description of the a priori distribution of the analyzed parameters.

2. Formulation and study the problem

Let the result of measurement is equal $Z = X + Y$, where $Y$ - the actual value of the measured value, $X$ - the measurement error. The measurement error will be estimated by the standard deviation $\sigma$ [6,7].

In the case when it is possible to observe and analyze the measurement result $Z$, additional information appears that can be used for a posteriori assessment (reassessment) of measurement error and increase in measurement accuracy. The task of improving the accuracy of measurements through the use of additional information will be solved on the basis of the Bayesian approach [6,7].

Let the actual value of the random variable and the measurement error have a uniform distribution on the segments $[-d,d]$ and $[-a,a]$, respectively, and $a < d$. Uniform distribution is not stable with respect to the summation operation. The sum of the actual value of the measured quantity and the error will have a trapezoidal distribution [6,7]:

$$f(z) = f(x + y) = \begin{cases} 
0, & x + y < -a - d \\
\frac{x + y + a + d}{4ad}, & -a - d \leq x + y < a - d \\
\frac{1}{2d}, & a - d \leq x + y < d - a \\
-\frac{x + y + a + d}{4ad}, & d - a \leq x + y < a + d \\
0, & a + d \leq x + y 
\end{cases}$$

The conditional distribution function of a quantity, provided that it is calculated by the formulas [6,7]:

$$f_y(z) = \frac{f(z)}{f_1(y)}, \quad f_1(y) = \int_{-\infty}^{\infty} f(x, y)dx,$$
has the form:

\[ f_y(z) = \begin{cases} 
\frac{2(z+a+d)}{4a^2-(y+d)^2+4a(y+d)} & , \quad (y,z) \in H_1 \\
\frac{4a}{4a^2-(y+d)^2+4a(y+d)} & , \quad (y,z) \in H_2 \\
\frac{1}{2a} & , \quad (y,z) \in H_3 \
\frac{4a}{4a^2-(d-y)^2+4a(d-y)} & , \quad (y,z) \in H_4 \\
\frac{2(-z+a+d)}{4a^2-(d-y)^2+4a(d-y)} & , \quad (y,z) \in H_5 \\
0 & , \quad (y,z) \notin \bigcup_{j=1}^{5} H_j
\end{cases} \]

Where \( H_1 = \{y \geq -d; \quad y-a \leq z \leq a-d\} \); \( H_2 = \{y \leq -d+2a; \quad a-d < z \leq y+a\} \); \( H_3 = \{d+2a < y \leq d-2a; \quad y-a \leq z \leq y+a\} \); \( H_4 = \{y > d-2a; \quad y-a \leq z \leq -a+d\} \); \( H_5 = \{y \leq d, \quad -a+d < z \leq y+a\} \).

The function \( f_1(y) \) can be found by the formula:

\[ f_1(y) = \begin{cases} 
0 & , \quad y < d \\
\frac{a}{8ad} & , \quad -d \leq y < -d+2a \\
\frac{d}{8ad} & , \quad -d+2a \leq y < d-2a \
\frac{a}{8ad} & , \quad d-2a \leq y < d \\
0 & , \quad d \leq y
\end{cases} \]

The function \( f_1(y) \) is a continuous convex upward function, and on the interval \(-d+2a \leq y < d-2a\) takes a constant value (figure 1).

\[ \text{Figure 1. The characteristic form of the normalizing function.} \]

Here and below, to illustrate the theoretical position, is taken \( a = 3, \quad d = 10 \).
The differential function of the conditional distribution of a random variable \( Y \) under the condition \( Z \) calculated by the formula [5]:

\[
f_z(y) = \frac{f(y) \cdot f_z(y)}{\int_{-\infty}^{\infty} f(y)f_{y}(z) dy}
\]

has the appearance

\[
f_z(y) = \begin{cases} 
\frac{4\sqrt{2}a}{4a^2 - (y + d)^2 + 4a(y + d)} & \frac{1}{\ln \left( \frac{(2\sqrt{2} - 1)a + z + d}{(2\sqrt{2} + 1)a - z - d} \cdot \sqrt{\frac{2}{2} + 1} \right)} , \quad (y, z) \in \Omega_1 \\
\frac{4a}{4a^2 - (y + d)^2 + 4a(y + d)} & \frac{1}{\ln \left( \frac{(2\sqrt{2} + 3)a - z - d}{(2\sqrt{2} - 3)a + z + d} + \frac{z + d - a}{2a} \right)} , \quad (y, z) \in \Omega_2 \\
\frac{1}{2a} & \frac{1}{\ln \left( \frac{(2\sqrt{2} + 3)a - z - d}{(2\sqrt{2} - 3)a + z + d} + \frac{z + d - a}{2a} \right)} , \quad (y, z) \in \Omega_3 \\
\frac{2a}{\sqrt{2}} & \frac{1}{\ln \left( \frac{(2\sqrt{2} + 3)a + z - d}{(2\sqrt{2} - 3)a + z + d} + \frac{d - a - z}{2a} \right)} , \quad (y, z) \in \Omega_4 \\
\frac{4a}{4a^2 - (d - y)^2 + 4a(d - y)} & \frac{1}{\ln \left( \frac{(2\sqrt{2} + 3)a + z - d}{(2\sqrt{2} - 3)a - z + d} + \frac{d - a - z}{2a} \right)} , \quad (y, z) \in \Omega_5 \\
\frac{4\sqrt{2}a}{4a^2 - (d - y)^2 + 4a(d - y)} & \frac{1}{\ln \left( \frac{(2\sqrt{2} - 1)a - z + d}{(2\sqrt{2} + 1)a + z - d} \cdot \sqrt{\frac{2}{2} + 1} \right)} , \quad (y, z) \in \Omega_6 \\
0 & \quad (y, z) \in \bigcup_{i=1}^{7} \Omega_i 
\end{cases}
\]

Figure 2 shows the graphs of the function \( f_z(y) \) for next values \( z = \pm 11; \pm 10; \pm 9; \pm 7; \pm 6; \pm 5; \pm 4 \) (numbering from top to bottom). The left branches of the function correspond to negative values of \( z \), the right branches of the function correspond to positive values of \( z \).
Figure 2. Function $f_z(y)$ for different values of $z$.

Note that when $-d + 3a \leq z \leq d - 3a$, the function $f_z(y)$ equal to $1/2a$, for all values $y$. For the indicated values of $z$, the conditional distribution function of the random variable $Y$ coincides with the unconditional distribution function of this random variable. In this area can be considered random variables $Y$ and $Z$ as independent.

If $d \leq 3a$, then the corresponding area of independence does not exist, and the values $Y$ and $Z$ cannot be considered as independent random variables. The areas of definition of functions $f(x+y)$, $f_z(y)$ and $f_z(z)$, which are compound functions, (in different parts of the argument are specified using different analytical formulas), are presented in figure 3a), b), c), respectively.

Figure 3. Areas of functions $f(x+y)$, $f_z(z)$, $f_z(y)$.

The domain of each function is divided into subdomains, in which a compound function is specified using a separate analytical formula. Analytical formulas are obtained in the present work by calculating the corresponding integrals of fractional-linear expressions.
The dependence of the conditional standard deviation $\sigma_z = \sqrt{D_z(Y)}$ is calculated by the formula
\[ \sigma_z = \sqrt{D_z(Y)} , \text{ where } D_z(y) = \int_{-\infty}^{\infty} f_z(y) \cdot y^2 dy . \]
The corresponding dependence of the square standard deviation of the conditional distribution from the measured value (the sum of the real value and the error) is presented in figure 4.

![Figure 4](https://example.com/figure4.png)

**Figure 4.** The dependence of the square of the standard deviation of the conditional law of distribution from the result of measuring the magnitude

It is seen that rather small values of the standard deviation are achieved in cases when the measured value is close to zero or when the measured value is close in magnitude to the maximum possible value. Thus, high measurement accuracy is achieved with measurements of values close to the nominal and measurements close to the maximum possible values.

Note that the square of the standard deviation of an uniformly distributed random variable $Y$ is $\sigma^2 = \frac{1}{3} d^2$. In the above example $\sigma^2 = 33, (3)$. Standard deviation when using a posteriori information 1.6 - 8 times less than unconditional (a priori) standard deviation.

3. **Main results. Discussions. Conclusions**

The article proposes a method for constructing estimates of the accuracy of single measurements based on the use of a posteriori information and presents the results of its use in the case of a uniform distribution of the actual value of the measured value and measurement error.

The results show that by taking into account additional information that can be obtained as a result or the measurement process, it is possible to build more reliable estimates of the measurement accuracy. A prerequisite for this is the establishment of the fact of the dependence of the sum of the real value and the error on the real value of the measured parameter.

Investigations of other distribution laws of random variable $X$ and $Y$ have shown that:
- if the random variables $X$ and $Y$ are independent and these variables have a normal distribution laws, then the random variables $Y$ and $Z$ can be considered as independent normally distributed random variables;
- if the random variables $X$ and $Y$ are independent and these variables have an exponential distribution laws, then the random variables $Y$ and $Z$ can be considered as independent exponentially distributed random variables.
Thus, in cases of normal and exponential distribution laws of random variables $X$ and $Y$ then the use of a posteriori information does not lead to a decrease in standard deviation (increase in measurement accuracy).

Note that in most practical problems, the form of the distribution function of the measured quantity is, as a rule, unknown. The task of establishing the type of empirical distribution function and establishing the fact of dependence (independence) of these quantities can be an independent task. In these practical problems, after building an empirical distribution function, the approach described in the article to constructing an estimate of the measurement accuracy can also be effectively applied.

The results obtained in the article can be used for:
- development of State standards for the production of building products and materials.
- in the development of measurement techniques for testing and calibration of measuring instruments [8].

The method presented in the article is advisable to use both to development the method of estimation of the accuracy of one-time measurements and to development the method of estimation of the accuracy of measurements of rapidly changing processes in which the repeating an experiment (a test) is not impossible or impractical.

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