Electroproduction of $D$- and $B$-mesons in high-multiplicity $ep$ collisions

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In this paper we study the electroproduction of open heavy flavor $D$- and $B$-mesons in the kinematics of future $ep$ colliders, such as the Electron Ion Collider (EIC), the Large Hadron electron Collider (LHeC) and the Future Circular Collider (FCC-he). We study in detail the dependence of the cross-sections on multiplicity of co-produced hadrons, in view of its possible sensitivity to contributions from multipomeron contributions, and discuss different observables which might be used for its study. According to our theoretical expectations, in $ep$ collisions the multipomeron contributions are small in the EIC kinematics, although they might be sizable at LHeC and FCC-he. We also provide theoretical predictions for the production cross-sections of heavy mesons in the kinematics of all the above-mentioned $ep$ colliders.

I. INTRODUCTION

Due to the high luminosity of the forthcoming LHC upgrade (HL-LHC) and future electron-proton colliders, many rare processes recently got renewed theoretical interest. One of the directions which might benefit from the outstanding luminosity is the production of different hadrons in high-multiplicity events. The development of theoretical framework for the study of such events was initiated more than forty years ago in [10]. However, for a long time the experimental study of such processes was limited by the insufficient luminosity of existing high-energy experiments (see however the discussion in [7,12] related to HERA). At RHIC and LHC, thanks to the very large luminosity, the multiplicity dependence of hadroproduction processes has been studied in great detail, and various elaborate observables have been measured experimentally, extending our understanding of the mechanisms of these processes. For example, the experimental study of yields of light charged hadrons co-produced together with heavier mesons [13–18] revealed that the multiplicity dependence is faster than in the absence of heavy mesons, and, as was suggested in [19,23], might be explained by contributions of higher twist multipomeron mechanisms. This finding is important, because it gives possibility to understand better the onset of saturation in high-energy collisions.

It is expected that the future Electron Ion Collider (EIC) [24,25], the Large Hadron electron Collider (LHeC) [26] and the Future Circular Collider (FCC-he) [27–29] also will have very large luminosities, which will make possible a study of physics at the intensity frontier in electroproduction processes. The measurement of the multiplicity dependencies at these new colliders might be used for better understanding of the underlying microscopic mechanisms of different electroproduction processes. In what follows we will focus on the production of heavy flavor $D$- and $B$-mesons, as well as non-prompt $J/\psi$ mesons. These states might be described approximately in the heavy quark mass limit [30,31], and for this reason have been used since the early days of QCD as a probe for testing the predictions of perturbative Quantum Chromodynamics (QCD) (see e.g. [32–40] for an overview). In what follows we will focus on the kinematics of photoproduction, where most of the heavy mesons are produced from quasi-real photons with virtuality $Q^2 \approx 0$. In this kinematics the typical values of Bjorken $x_B$ are small, $x_B \ll 1$, and the gluon densities significantly exceed the sea quark contributions. In the proton rest frame the interaction might be viewed as a scattering of the color dipole, formed from the photon, in the proton gluonic field. The appropriate description of such process is the color dipole framework (also known as CGC/Sat) [41–49]. This approach has been successfully applied to the phenomenological description of both hadron-hadron and lepton-hadron collisions [9,11,50,55], and allows a straightforward extension for the description of high-multiplicity events [10,50,56,61]. The color dipole approach is not valid for larger values of $x_B \gtrsim 0.1$, due to possible contributions of intrinsic quarks (e.g. intrinsic charm). For this reason in what follows we will consider only the variables which do not get significant contributions from that region. We also will analyze explicitly the role of the multipomeron mechanisms, which are usually omitted as higher twist effects. Since such contributions have more pronounced dependence on multiplicity, their presence could be straightforwardly deduced from experimental data on multiplicity dependence.

The paper is structured as follows. In Section [II] we discuss the framework used for the open-heavy meson production evaluation, taking into account the contributions of the single- and double-pomeron mechanisms, compare the theoretical expectations with experimental data and make predictions for the kinematics of the future electron-proton colliders. In Section [IV] we suggest observables which might help to measure the multiplicity dependence, and make theoretical predictions for them in the dipole framework. Finally, in Section [V] we draw conclusions.
II. PRODUCTION OF OPEN HEAVY FLAVOR MESONS

The cross-section of open heavy-flavor meson production via the fragmentation mechanism is given by 34, 35, 39, 40, 62, 64:

\[
\frac{d\sigma_{ep\rightarrow M+X}}{dxdydn\sigma_{pp}} = \frac{d\sigma_{ep\rightarrow Q,Q_i+X}}{dxdydn\sigma_{pp}}
\]

where we use standard DIS notations \( Q^2, x_B, y \) for the virtuality of intermediate photon, Bjorken variable \( x_B \) and elasticity (fraction of electron energy which passes to the photon in the proton rest frame); while \( \eta \) and \( p_T \) are the rapidity and the transverse momentum of the produced heavy meson. The fragmentation function \( D_i(z) \) describes the probability of fragmentation of the parton \( i \) into a heavy meson. For \( D_1 \) and \( B \)-mesons production, as well as for non-prompt \( J/\psi \) production, the corresponding fragmentation functions are known from the literature 34, 35, 65.

While in 1 there is a sum over all parton flavors, the dominant contribution to all the mentioned states stems from the heavy \( c \)- and \( b \)-quarks. This implies that the cross-section \( d\sigma_{ep\rightarrow Q,Q_i+X}/d\eta d^2p_T \), for heavy quark production might be evaluated in the heavy quark mass limit. It is convenient to separate explicitly the leptonic and hadronic parts of the cross-section, and rewrite it as 66:

\[
\frac{d\sigma_{ep\rightarrow Q,Q_i+X}}{d\eta d^2p_T} = \frac{\alpha_{em} Q^2}{(s_{ep} - m_i^2)} \left[ (1 - y) \frac{d\sigma_L}{d\eta d^2p_T} + \left( 1 - y + \frac{y^2}{2} \right) \frac{d\sigma_T}{d\eta d^2p_T} \right],
\]

where \( d\sigma_L \) and \( d\sigma_T \) in the right-hand side of the equation (2) correspond to the cross-sections of heavy quark production by a longitudinally and transversely polarized photon respectively. In the literature the results for leptonic processes were frequently discussed in terms of these photon-proton cross-sections \( d\sigma_{L,T} \), which have simpler structure. In the dipole approach the cross-sections \( d\sigma_{L,T} \) are given by

\[
\frac{d\sigma_{a}}{d\eta d^2p_T} = \int_0^1 dz \int \frac{d^2r_1}{4\pi} \int \frac{d^2r_2}{4\pi} e^{i(r_1-r_2)\cdot k_T} \times \\
\Psi^T_a(r_2, z) \Psi^T_a(r_1, z) N_M (x_2(y); r_1, r_2), \quad a = L, T
\]

where \( \eta \) and \( p_T \) are the rapidity and transverse momenta of the produced heavy meson; \( \Psi^T_a(r, z) = \Omega Q \) component of the light-cone wave function of the photon; \( r_{1,2} \) are the transverse separation between quarks in the amplitude and its conjugate; while \( z \) is the light-cone fraction of the photon momentum carried by the quark. For \( \Psi^T_a \), in the heavy quark mass limit we may use the standard perturbative expressions 67, 68:

\[
\Psi^T_a(r_2, z, Q^2) = \frac{e^{if}}{2\pi^2} \left\{ \begin{array}{l}
K_0(\epsilon r_1) K_0(\epsilon r_2) [e^{iy_{12} z^2} + e^{-iy_{12} (1-z)^2}]
\end{array} \right.
\]

\[
\Psi^T_L(r_2, z, Q^2) = \frac{e^{if}}{2\pi^2} \left\{ 4Q^2 z(1-z) K_0(\epsilon r_1) K_0(\epsilon r_2) \right\},
\]

where \( \theta_{12} \) is the azimuthal angle between vectors \( r_1 \) and \( r_2 \), \( m_f \) is the mass of the quark of flavor \( f \), and we used standard shorthand notations

\[
e^{if} = \left( 1 - z \right) Q^2 + m_f^2,
\]

\[
\left| \Psi_{L}^{(f)} (r, z, Q^2) \right|^2 = \left| \Psi_{T}^{(f)} (r, z, Q^2) \right|^2 + \left| \Psi_{L}^{(f)} (r, z, Q^2) \right|^2.
\]

The meson production amplitude \( N_M \) depends on the mechanism of the \( QQ \) pair formation. For the case of production on a single-pomeron (see the left panel of the Figure 1), in leading order it is given by 10, 18:

\[
N_M^{(1)} (x, \bar{r}_1, \bar{r}_2) =
\]

\[
= - \frac{1}{2} N (x, \bar{r}_1 - \bar{r}_2) - \frac{1}{16} [N (x, \bar{r}_1) + N (x, \bar{r}_2)] - \frac{9}{8} N (x, \bar{z} (\bar{r}_1 - \bar{r}_2))
\]

\[
+ \frac{9}{16} [N (x, \bar{z} \bar{r}_1 - \bar{r}_2) + N (x, \bar{z} \bar{r}_2 - \bar{r}_1) + N (x, \bar{z} \bar{r}_1) + N (x, \bar{z} \bar{r}_2)],
\]
where \( N(x, r) \) is the amplitude of the color singlet dipole scattering. The amplitude (8) has a structure similar to the leading twist result for the hadroproduction of heavy quarks; however, this similarity is no longer valid for higher twist amplitudes. For numerical estimates of this contribution, we need to fix a parametrization of the amplitude \( N(x, r) \).

In what follows, for the sake of definiteness we will use the CGC parametrization of the dipole amplitude, which was proposed in [69] (see also [70–72] for more recent phenomenological analyses). Since we are interested in the \( p_T \) dependence, we will use the impact parameter dependent fit, taken from [70]. As we can see from Figure 2, the single-pomeron contribution provides a very reasonable description of the available data from HERA. In Figures 3, 4 we have shown the theoretical expectations for the cross-sections of \( D^\pm \), \( B^\pm \) and non-prompt \( J/\psi \) meson production, in the kinematics of the future accelerators EIC (\( \sqrt{s_{ep}} \) up to 141 GeV), LHeC (\( \sqrt{s_{ep}} \approx 1.3 \) TeV) and FCC-he (\( \sqrt{s_{ep}} \approx 3.5 \) TeV) [24–29].

It is also interesting to understand the role of the multipomeron mechanisms in electroproduction. While sometimes it is assumed that all such contributions are taken into account by the universal dipole cross-section, in reality the situation is more complicated. The CGC parametrization [70–72], used for our numerical estimates, does not take into account such corrections. Another widely used parametrization of the dipole cross-section, the so-called b-Sat [66, 73], takes into account such corrections, making additional simplifying assumptions. For this reason our goal is to perform a microscopic evaluation using the CGC model. We understand that a systematic evaluation of all such corrections in high-multiplicity events presents a challenging problem, and for this reason we will focus on the contribution of two-pomeron mechanisms, which are shown in the central and right panels of the Figure 1. Formally such contributions are expected to be small, because they are of higher twist. However, it is desired to reassess them for electroproduction, because earlier studies [29] revealed that for hadroproduction such corrections might be pronounced in the charm sector.
and in small-$p_T$ kinematics, especially for high-multiplicity events. In what follows we will refer to the diagrams shown in the central and right panels of the Figure 1 as genuine and interference corrections (in view of the clear interference nature of the latter). For both types of contributions the corresponding cross-section has the familiar structure (3), so these corrections might be rewritten as an additional contribution to the amplitude $N_M$ given by

$$N_M^{(2)}(x, \vec{r}_1, \vec{r}_2) = N_M^{(\text{genuine})}(x, z, \vec{r}_1, \vec{r}_2) + N_M^{(\text{int})}(x, z, \vec{r}_1, \vec{r}_2)$$  \hspace{1cm} (9)
where

\[ N_M^{(\text{genuine})} (x, z, \vec{r}_1, \vec{r}_2) = \frac{1}{8} \left[ N_\pm^2 (x, z, \vec{r}_1, \vec{r}_2) \left( \frac{3N_c^2}{8} \right) + N_\pm^2 (x, \vec{r}_1, \vec{r}_2) \left( \frac{(43N_c^4 - 320N_c^2 + 720)}{72N_c^2} \right) \right. \]
\[ + \left. \frac{1}{2} \left( N_\pm^2 - 4 \right) N_\pm (x, z, \vec{r}_1, \vec{r}_2) N_\mp (x, \vec{r}_1, \vec{r}_2) \right], \]

\[ N_M^{(\text{int})} (x, z, \vec{r}_1, \vec{r}_2) = -\frac{3}{16} \left[ 2N_\pm (x, z, \vec{r}_1, \vec{r}_2) \bar{N}_\pm (x, z, \vec{r}_2) \left( \frac{3N_c^2}{8} \right) + \right. \]
\[ - N_\pm (z, \vec{r}_1, \vec{r}_2) \bar{N}_\mp (x, \vec{r}_2) \left( \frac{(43N_c^4 - 320N_c^2 + 720)}{72N_c^2} \right) \]
\[ + \left. \frac{1}{2} \left( N_\pm^2 - 4 \right) \left( N_\pm (z, \vec{r}_1, \vec{r}_2) \bar{N}_\mp (x, \vec{r}_2) + \bar{N}_\pm (x, \vec{r}_2) N_\mp (z, \vec{r}_1, \vec{r}_2) \right) \right] \]

and we introduced the shorthand notations

\[ N_\mp (x, \vec{r}_1, \vec{r}_2) \equiv -\frac{1}{2} \left[ N(x, \vec{r}_2 - \vec{r}_1) - N(x, \vec{r}_1 - \vec{r}_2) \right] \]
\[ N_\pm (x, z, \vec{r}_1, \vec{r}_2) \equiv -\frac{1}{2} \left[ N(x, \vec{r}_2 - \vec{r}_1) + N(x, \vec{r}_1 + \vec{r}_2) \right] + N(x, z\vec{r}_1 - \vec{r}_2) + N(x, z\vec{r}_2) \]

The derivation of these expressions is straightforward and follows the procedures described in \[23, 40, 48, 49\]. Both functions \( N_\pm (z, \vec{r}_1, \vec{r}_2) \) are invariant with respect to the permutation \( r_1 \leftrightarrow r_2 \). The \( p_T \)-integrated cross-sections get contributions only from \( \vec{r}_1 = \vec{r}_2 = \vec{r} \), so the cross-sections \( \bar{N}_\pm \) simplify to

\[ \bar{N}_\mp (x, \vec{r}) \equiv N_\mp (x, \vec{r}, \vec{r}) = N(x, \vec{r}) \]
\[ \bar{N}_\pm (x, z, \vec{r}) \equiv N_\pm (x, z, \vec{r}, \vec{r}) = 2N(x, z\vec{r}) + 2N(x, z\vec{r}) - N(x, \vec{r}) \]

In Figure (4) we show the ratio of cross-sections, where the numerator and denominator were evaluated using the two-pomeron contribution \( (9) \) and the single-pomeron contribution \( (8) \) respectively,

\[ R(x_B) = \frac{d\sigma^{(2)}_{ep \to DX}/dx_B}{d\sigma^{(1)}_{ep \to DX}/dx_B}. \]
As we can see from Figure [4], in the kinematics of EIC the ratio is quite small. However, the situation is different in the kinematics of the future LHeC and FCC-he colliders, which might probe significantly smaller values of $x_B$. In that kinematics the values of the two-pomeron contributions might reach up to 40 per cent of the total result in the charm sector.

### III. MULTIPLICITY DEPENDENCE

The theoretical study of multiplicity dependence in high energy collisions was initiated long ago in [1–6] in the framework of the Regge approach. Relying on very general properties of particle-reggeon vertices, which are largely independent of the underlying quantum field theory, it was suggested that the enhanced multiplicity of high energy final states could be considered as one of the manifestations of the multiple pomeron contributions. Later it was demonstrated in [7–12] that all these findings are also valid in the context of QCD, and thus could be confirmed by experimental evidence. The dependence on multiplicity differs from the dependencies on other kinematic variables, which are sometimes used for the extraction of dipole amplitudes, fragmentation functions or parton distributions from experimental data. This implies that the multiplicity dependence might be used as a litmus test to probe the role of multipomeron contributions.

The probability distribution $P(N_{ch}, \langle N_{ch} \rangle)$ of high-multiplicity fluctuations inside each pomeron decreases rapidly as a function of number of produced charged particles $N_{ch}$, as was found both at $ep$ and $pp$ collisions [77–78]. The theoretical modeling of the essentially nonperturbative probability distribution $P(N_{ch}, \langle N_{ch} \rangle)$ is very challenging. In order to exclude this common suppression factor, it is convenient to analyze the multiplicity dependence of the theoretical modeling of the essentially nonperturbative probability distribution as a function of number of produced charged particles.

Figure 5. The ratio of the two-pomeron and single-pomeron contributions, defined in (16) in the kinematics of future accelerators. For the sake of definiteness we consider $D^+$ and $B^+$-mesons; for other $D$- or $B$-mesons as well as for non-prompt $J/\psi$ the ratio has a very similar shape. The region $x_B \lesssim 5 \times 10^{-6}$ is kinematically forbidden for EIC energy, and for this reason the solid curve abruptly vanishes there.

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The description of high-multiplicity events in the CGC/Sat framework has been discussed in detail in [10, 39, 56, 61]. It is expected that at high multiplicities the dipole amplitude should satisfy the same Balitsky-Kovchegov equation...
Figure 6. The ratio (16) of the 2-pomeron to single-pomeron contributions, as a function of $x_B$ (left diagram) and $p_T$ (right diagram). The upper row corresponds to $D^{\pm}$-mesons, the lower row is for $B^{\pm}$-mesons. The variable $n \equiv N_{ch}/\langle N_{ch} \rangle$ is the relative enhancement of multiplicity.

(and thus maintain its form), although the saturation scale $Q_s(x, b)$, which contributes to the dipole amplitude, should be modified as

$$Q_s^2(x, b; n) = n Q_s^2(x, b). \tag{17}$$

For multipomeron configurations, we should take into account that multiplicity fluctuations occur independently in each pomeron, and the observed multiplicity $n$ might be shared in all possible ways between all cut pomerons in a given rapidity window. However, as was discussed in detail in [19, 20, 80], with good precision we may assume that the observed multiplicity $n$ is shared equally between all pomerons which participate in $e p$ process. Using this assumption, as well as certain convolution properties of $P(N_{ch}, \langle N_{ch} \rangle)$, it is possible to show that for the ratio of different cross-sections the probability distribution $P(N_{ch}, \langle N_{ch} \rangle)$ cancels altogether. Thus for the evaluation of the cross-sections in a given multiplicity class, we may use a simple prescription (17), properly adjusting the parameter $n$ in each pomeron to take into account equal sharing of total multiplicity.

As we can see from the Figure 6, the theoretical estimates suggest that in high-multiplicity events the role of the multipomeron contributions increases. Numerically, in EIC kinematics this contribution becomes pronounced at $n \gtrsim 5$ for $D$-mesons, although still remains relatively small for $B$-mesons. This difference in the size of multipomeron terms suggests that we can study experimentally the multiplicity dependence of the ratio of $D$- and $B$-meson cross-sections in order to estimate unambiguously the role of the two-pomeron contribution in $D$-meson production. In order to avoid the effects related to the $x_B$-dependence, we suggest to study the double ratio of cross-sections

$$R_{D/B}^{D/B}(x_B, n) = \frac{d\sigma_{D''}(x_B, n)/d\sigma_{B''}(x_B, n = 1)}{d\sigma_{B''}(x_B, n)/d\sigma_{B''}(x_B, 1)}. \tag{18}$$
This ratio equals one in the heavy quark mass limit, yet for finite values of \( n \) deviates from this value due to more
pronounced higher twist corrections for \( D \)-meson (numerator of \( \text{(18)} \)). In the left panel of Figure \( 8 \) we show the
dependence of the ratio \( \text{(18)} \) on \( n \). The dependence on \( n \) exists even for the leading twist, due to higher twist
corrections, but becomes more pronounced when the multipomeron contributions are taken into account. The growth
of the ratio as a function of \( n \) agrees with the elevated contribution of multipomeron mechanisms in the large-\( n \)
kinematics. However, we expect that for asymptotically large values of \( n \) the ratio should saturate, because the
multipomeron contributions will also become important in the denominator. In the right panel of the same Figure \( 8 \)
we show a similar self-normalized double ratio \( \text{(18)} \), in which we replaced \( B \)-mesons with non-prompt \( D \)-mesons.
Since the latter mechanism is dominated by \( b \)-meson decays, we can see that qualitatively the ratio has a similar dependence
on \( n \). Comparison of the left and right panels of Figure \( 8 \) clearly illustrates that the enhancement of the ratio \( \text{(18)} \)
is not related to differences of the \( D \)- and \( B \)-meson fragmentation functions. We expect that non-prompt charmonia
should demonstrate a similar behavior.

Another observable which might be easily measured experimentally is the dependence of the average momentum
\( \langle p_T \rangle \) of heavy mesons on the multiplicity \( n \). This observable has been extensively studied in the context of \( pp \)
collisions. In Figure \( 8 \) we show the dependence of \( \langle p_T \rangle \) on \( n \), for electroproduction of both \( D \)- and \( B \)-mesons. Since
multipomeron contributions are suppressed at large momenta \( p_T \), we can see that their inclusion decreases the average
\( \langle p_T \rangle \), compared to what is expected from single-pomeron. Although the expected effect is not very large, we believe
that it might be seen in experimental data, since \( \langle p_T \rangle \) might be measured with very good precision.

To summarize, we believe that the multiplicity dependence might reveal information about the contribution of the
multipomeron mechanisms. However, in EIC kinematics we do not expect drastic enhancement of the multiplicity
dependence, as was observed in \( pp \) collisions. This happens because in general multipomeron contributions are small
at EIC energies. The situation might be different in the kinematics of future accelerators like LHeC and FCC-he,
where the role of the multipomeron contributions is more pronounced. In our analysis we took into account only the
first multipomeron correction, namely the production on two pomerons. We could see that its relative contribution is
small in EIC kinematics, in agreement with general expectations based on twist counting, and for this reason we do
not consider the corrections of even higher order. However, at very small values of \( x_B \) (significantly smaller than \( 10^{-7} \))
we approach the deeply saturated regime, where the expectations based on twist expansion are not reliable, and thus
the inclusion of all higher twist might be required.

The mechanism of multiplicity generation suggested in this section introduces dependence on the multiplicity of
soft produced particles, and is quite different from other approaches, such as the percolation approach \cite{81} or the
modification of the slope of the elastic amplitude \cite{82}, suggested earlier in the context of \( pp \) studies. We expect that the
experimental confirmation of the predicted multiplicity dependence could help to understand better the mechanisms
of multiplicity enhancement in high energy collisions.
Figure 8. The multiplicity dependence of average $\langle p_T \rangle$ of produced $D$-mesons (left column) and $B$-mesons (right column). The upper row corresponds to the invariant photon energy $W_{\gamma p} \approx 100$ GeV (EIC kinematics), whereas the lower row corresponds to higher energy $W_{\gamma p} \approx 1000$ GeV, achievable at both LHeC and FCC-he. The dashed curve with label “Leading twist” in all plots, stands for the leading twist (single pomeron) contribution.

IV. CONCLUSIONS

In this paper we analyzed the mechanisms of open-heavy flavor meson electroproduction. Motivated by earlier findings in $pp$ collisions, we also analyzed the relative contribution of the first subleading multipomeron correction. We found that for electroproduction this correction is relatively small for EIC kinematics, although it grows with energy and becomes relevant for charm production at LHeC and FCC-he, especially in the small-$p_T$ kinematics. This correction is less important for $B$-mesons and non-prompt charmonia production, and does not exceed ten per cent even at LHeC and FCC-he. The dependence of the correction on $p_T$ agrees with general expectations based on large-$p_T$ and heavy quark mass limit. Our evaluation is largely parameter-free and describes very well the data from HERA, as well as provides plausible predictions for EIC, LHeC and FCC-he.

We also analyzed the multiplicity dependence, which might be studied experimentally in detail at future EIC, LHeC and FCC-he, due to its outstanding luminosity. The high-multiplicity events present special interest for theoretical studies, because they allow to get better understanding of the production mechanisms at high gluon densities. Since the probability of rare high-multiplicity events is exponentially suppressed, for the analysis of their dynamics it is important to study properly the designed variables. We analyzed in detail the dependence on multiplicity for the average momentum of heavy meson $\langle p_T \rangle$ and the double ratio defined in [18]. The first variable is easier to measure, although it is less sensitive to higher twist effects, due to the smallness of subleading contributions. The double ratio [18] is more interesting, because its deviations from unity allow to quantify directly the size of the higher twist corrections, including multipomeron contributions. Due to the smallness of multipomeron contributions, we do not expect a significant relative enhancement of the cross-sections at large multiplicity in EIC kinematics, and only mild...
enhancement in the kinematics of LHeC and FCC-he. This expectation differs significantly from what was found experimentally in pp collisions at LHC [13]. We expect that the experimental confirmation of these findings could help to understand better the mechanisms of multiplicity generation in high energy collisions.

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