Analysis of student’s mathematical connection and communication in algebra: the exponential equations

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Abstract. Mathematical connection, an ability to connecting prior knowledge with the new one to get profound meaning, with mathematical communication, both are considered as key success in learning algebra. In this article, we report result of a study that aims to analyzing student’s mathematical connection and communication on solving algebraic problems. An individual written test on exponential equation tasks was administrated based on mathematical connection and communication aspect, then followed by interview to gain further information and finally, we analyzing student solution strategies in the light of mathematical connection and communication. This study took a sample of 30 grade X (ten) students (15-16 years old) on a senior high school in West Java. The result of this research shows that student’s mathematical connection and communication is still low. We found that most of the students still have problems in building connections with the materials that have been studied and in expressing correct mathematical concept. This findings indicate that the students still need intensive guidance in mathematical connection and communication in order to improve their ability in solving the mathematics problem. For further investigation, we consider to promote the teaching method or strategy to enhance both mathematical connection and communication as well.

1. Introduction

Algebra is one of the topics that exists in high school mathematics curricula. Algebra is core topic in secondary school mathematics that should be mastered by students for advanced study or professional work [1]. However, students find that algebra is difficult subject to master. Based on the several study showed that algebra is widely recognized as one of the most difficult topics and students achievement was significantly below the average student performance in other Southeast Asian countries [2, 3, 4]. The meaningfulness teaching strategy is needed to minimize these students’ difficulties [5].

Taking this into account, we carried out a study on analyzing students’ difficulties on algebra in terms of their mathematics abilities to provide appropriate treatment to students. There are five mathematics abilities according to NCTM [6], namely problem solving ability, reasoning ability, communication ability, connection making ability, and representation ability. This study aims to analyze students’ mathematical connection and communication in solving mathematics problems. Mathematical connection and mathematical communication are considered to be in line with the nature of mathematics that it is a systematic, structured science and an important symbol language that should be learned [7, 8]. These abilities improvement are also in line with the goals of mathematics teaching-learning in Indonesian curriculum [9].

The students’ mathematical connection ability is related to connecting the prior knowledge that students have with new knowledge so that the material can have profound meaning. Three aspects of
mathematical connection ability, namely: (1) write down the problems of everyday life in the form of mathematical models, (2) write down the mathematical concept that underlies the answer. (3) Write the relationship between objects and mathematical concepts [10].

Someone displays mathematical communication in the form of written communication if s/he can: (1) describe the problem situation and state the problem solution using images, tables, charts, algebraically, (2) declare results in written form, (3) using a thorough representation to express a mathematical concept and its solution, (4) create a mathematical situation by providing ideas and information in written form, (5) use the language and mathematical symbols appropriately [10].

2. Method
The research method used is a qualitative description by conducting qualitative analysis on students' work result when doing problem in algebraic material and interview to obtain more information. The observation took place in a senior high school in West Java, Indonesia. Subject in this study were 30 grade ten students in one class. Object in this study was result of student answer in solving problem related to algebraic material to know students’ mathematical connection and communication ability.

To collect the data, first, make good observations of learning in the classroom as well as observation of student activities. Next, we developed three algebra tasks on exponential equations based on the aspect of mathematical connection and communication for high school algebra (see Table 1). Then, we conducted interviews to further deepen the students’ intent in solving the problem. Finally, we analyzed student solution strategies in solving the problem.

Table 1. Mathematical connection and communication task on solving exponential equations

| No. | Mathematical connection aspects | Mathematical communication aspects | Task |
|-----|---------------------------------|-----------------------------------|------|
| 1.  | Write down the mathematical concept that underlies the answer | All mathematical communication aspects | $3^{x-5} = 1$ |
| 2.  | Write the relationship between objects and mathematical concepts | All mathematical communication aspects | $2^{2x} - 8 \cdot 2^x + 16 = 0$ |
| 3.  | Write down the problems of everyday life in the form of mathematical models | All mathematical communication aspects | At the time of its founding in 2015, a social organization has 300 members. If the number of members doubled every 2 years, how many members of the organization at the end of 2013. |

Note: Task 2 and 3 are adapted from Indonesian mathematics textbook series [11, 12].

3. Result and Discussion
Students work in solving exponential equations task that showed their mathematical connection and communication is summarized in Table 2. For example, 50% of students showed their mathematical connection and 30% of students showed their mathematical communication for solving Task 1.

Table 2. Students work in terms of their mathematical connection and communication

| Task | Students work | %Mathematical connection aspects | %Mathematical communication aspects |
|------|---------------|---------------------------------|-----------------------------------|
| 1    |               | 50%                             | 30%                               |
| 2    |               | 53%                             | 3%                                |
| 3    |               | 80%                             | 66%                               |

For the Task 1, we found that half of the student solve the task appropriately. Concerning solution strategies to solve the equation $3^{x-5} = 1$ by recognizing the property of exponents as $a^0 = 1$, where
$a \neq 0$. Someone can solve this equations in the following manner: $3^{x-5} = 1 \iff 3^{x-5} = 3^0 \iff x - 5 = 0 \iff x = 5$. Figure 1(a) shows an example that student can write down the mathematical concept that underlies the answer by recognizing $3^{x-5} = 0 \iff x - 5 = 0 \iff x = 5$, and Figure 1(b) shows an example of error representation to express mathematical concept by expressing $3^{x-5} = 1 \iff 3^x = 1 + 5$ and $3^x = 6 \iff x = \frac{6}{3}$. Based on the result of the student work, it is known that most of the students were able to show the mathematical connection aspects that we expected, but only few of them can show their written communication aspect since they expressed error representation for mathematical concept and use mathematical symbol inappropriately, they also declare result in written form even if the result was incorrect.

Task 2, solving the equation $2^{2x} - 8 \cdot 2^x + 16 = 0$. This is a good task for investigating high school students’ skills dealing with equations related to quadratic one. The relationship between mathematical concepts shown by using the substitution $y = 2^x$ to simplify the equation into $y^2 - 8y + 16 = 0$, to factorize it into $(y - 4)^2 = 0$, and to obtain $y = 4$. We found that there is only one student solves the problem correctly. Most of the students recognized that the equation can be simplified to quadratic one, but mistake happens since they make calculate step and wrong represent mathematical concept. From Figure 2(a), we can see an example of student work that shows both mathematics connection and communication as well, and Figure 2(b) shows an example of error representation to express
mathematical concept. The student has tried to relate that equation to quadratic one, but calculation steps that are trying to build still encounter error. That result showed the lack of student communication aspect especially in create a mathematical situation by providing ideas and information in written form and use mathematical symbols appropriately.

Contextual problems related to the exponential equations is administered on Task 3. In terms of mathematical connections, we found that most of students can write down the problems of everyday life in the form of mathematical models and solve the problem as well, but they use sequences concept instead of exponential equation (see Figure 3(a)). We interviewed five students who can solve the problem correctly and from that we found that it was too complicated for them to solve the problem by using the exponential equations so they solved the problem by modeling the problem as a sequence. Even if the result was correct, still students have to state the problem with exponential equation because exponential equation was the main topics in this test. We can modeling the problem on Task 3 into exponential equation \( y = y_0(1 + a)^t \), where \( y_0 \) express initial quantity, \( a \) express grow factor and \( t \) express number of time. By substitute all of variables value, we obtain \( y = 300(2)^4 = 4800 \). In terms of mathematical communication, student can describe and state problem situation algebraically and tried to pour the idea by create a mathematical situation by providing ideas and information in written form, but same mistake happened since the calculation steps that are trying to build still encounter error and declare incorrect result. (see Figure 3(b)).

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\begin{align*}
\text{Jumlah anggota suatu organisasi sosial saat awal didirikan pada tahun 2005 adalah 300 orang. Jika jumlah anggota menjadi 2 kali lipat setiap 2 tahun, berapakah jumlah anggota organisasi tersebut pada akhir tahun 2013.} \\
\end{align*}
\]

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\begin{align*}
\text{Dik : 2005 = 300 orang} \\
\text{Dikelu: 2 per 2 tahun} \\
2006 : 300 \\
2007 : 600 \\
2009 : 1200 \\
2011 : 2400 \\
2013 : 4800 \\
\text{Pada tahun 2012 : 4800 orang} \\
\end{align*}
\]

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\begin{align*}
\text{Jumlah anggota suatu organisasi sosial saat awal didirikan pada tahun 2005 adalah 300 orang. Jika jumlah anggota menjadi 2 kali lipat setiap 2 tahun, berapakah jumlah anggota organisasi tersebut pada akhir tahun 2013.} \\
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\text{Dik : 2005 = 300 anggota} \\
\text{2 kali lipat setiap 2 tahun} \\
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concept. This findings indicate that the students still need intensive guidance in mathematical connection and communication through the learning process in order to improve their ability in solving the mathematics problem particularly in exponential equations. For further investigation, we wonder that whether we can implement the teaching method that can enhance students’ mathematical connection and communication as well.

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