Superfluid density in disordered superconductors

Sudhansu S. Mandal$^{1,2}$ and T. V. Ramakrishnan$^3$

$^1$Department of Physics, Indian Institute of Technology, Kharagpur 721302, India
$^2$Centre for Theoretical Studies, Indian Institute of Technology, Kharagpur 721302, India
$^3$Department of Physics, Indian Institute of Science, Bangalore 560012, India

(Dated: April 1, 2020)

Abstract

Recent experiments on disordered superconductors find that the superfluid density $n_s(T)$ (or stiffness) decreases dramatically and characteristically with disorder. We describe here an approach which includes the effect of electron pair phase in a gauge invariant manner, and use it to explicitly obtain $n_s(T)$ in the presence of (static) disorder. We compare our results successfully with experiment.
I. INTRODUCTION

Superconductivity is identified with a nonzero complex gap function \( \psi(x) = \Delta(x) \exp[i\phi(x)] \) where \( x \) is a shorthand for spatial coordinate \( r \) and imaginary time \( \tau \), \( \Delta(x) \) is the amplitude of the gap and \( \phi(x) \) its phase. The microscopic theory of superconductivity\(^\text{1,2} \) identifies it with the statistical average of a Cooper pair of electrons. If the system has static nonmagnetic disorder which preserves time reversal invariance, the condition for pairing of electrons in time reversed states does not change provided the effective pairing attraction does not. This is Anderson’s theorem\(^\text{3} \), and consequently, in the BCS mean field theory the superconducting transition temperature \( T_c \) does not change, as broadly confirmed by experiment. Recent work on disordered superconductors shows however that both \( T_c \) and superfluid density \( n_s(T) \) decrease with disorder in a characteristic way\(^\text{4,5} \). The single particle density of states exhibits a pseudogap above \( T_c \), somewhat like what is seen in cuprate superconductors\(^\text{6–8} \). Moreover, there seems to be a new state of quantum matter, the failed superconductor state which occurs for relatively weak disorder or no disorder\(^\text{9} \). This state is most likely characterized by Cooper pairs whose phases are not mutually coherent. In all these systems, disorder is not large enough for effects of Anderson localization to be significant so that issues such as the superconductor-insulator transition due to disorder or the ‘superinsulator’ phase are not relevant. It seems quite likely that all these phenomena are connected with the effect of the phase \( \phi(x) \), affected by disorder or interactions.

While there is a theory of the electromagnetic properties of superconducting alloys\(^\text{10} \) this rather complex approach is restricted to \( T = 0 \). We outline here a simple general approach which directly expresses the Hamiltonian of the superconductor in terms of the electron pair phase (which may also depend on time in general). The Hamiltonian is obtained here as a function of the gauge invariant superfluid velocity \( v_s(x) = (1/2m)(\hbar \nabla \phi - (2e/c)A) \). The coefficient of the term quadratic in \( v_s \) is by definition \((1/2)\rho_s\), where \( \rho_s \) is the superfluid stiffness (Section II and see also Ref.\(^\text{11} \)) which is proportional to \( n_s \).

Using this result, we obtain in the subsequent section (Section III) results for \( n_s(T = 0) \) and \( n_s(T/T_c) \) as a function of static nonmagnetic disorder (characterized by a relaxation time \( \tau \)), using standard many body methods (for a many electron system with random impurities leading to a nonzero \( \tau^{-1} \), and with a zero range BCS pairing attraction). The disorder
dependence of $n_s(T = 0)$ is explicitly calculated in the relaxation time approximation; the London value of $n_s$ in the ground state ($T = 0$) is recovered in the clean limit ($\tau^{-1} \to 0$). We also obtain the dependence of $n_s(T/T_c)$ and show that the other limiting behaviour near $T_c$ (namely its going to zero) is also correctly obtained. In section III, we compare our results broadly with experiment on disordered superconductors, and obtain agreement both as to the trend of $n_s(T)$ as a function of disorder (it decreases) and the size. We show analytically that it decreases linearly with disorder for small disorder as is seen in experiment, and as predicted in Ref. 10. We obtain closed form expressions for all $T$ and for $\Delta \tau < 1$ as well as for $\Delta \tau > 1$ (but $\epsilon_F \tau << 1$). Some possible future directions including application to the failed superconductor situation (where $n_s(T = 0)$ vanishes at $T = 0$) are suggested in the concluding discussion section (Section IV).

II. FORMAL PRELIMINARIES

We consider electrons in a random potential $V(r)$ interacting via a zero range BCS attractive potential of strength $g$. The system Hamiltonian is

$$H = \int dr \sum_\sigma \left[ \frac{1}{2m} \left( \frac{\hbar}{i} \nabla - \frac{e}{c} A(r) \right)^2 + V(r) - \mu \right] \psi_\sigma(r) - g \int dr \psi_\uparrow^\dagger(r) \psi_\downarrow^\dagger(r) \psi_\downarrow(r) \psi_\uparrow(r).$$

In the standard manner, its partition function can be written as a functional integral over Grassmannian fields $\bar{\psi}_\sigma(x)$ and $\psi_\sigma(x)$ where $x \equiv (r, \tau)$ represents both the coordinate vector $r$ and imaginary time $\tau$. These formally substitute for the Fermi field creation and destruction operators $\psi_\sigma^\dagger(x)$ and $\psi_\sigma(x)$ respectively. The usual Hubbard Stratonovich transformation of the quartic self interaction term in (1) leads to a pair potential term $\Delta^*(x) \psi_\downarrow(x) \psi_\uparrow(x) + \Delta(x) \bar{\psi}_\uparrow \bar{\psi}_\downarrow$ with a Gaussian probability distribution $\exp[-|\Delta(x)|^2/g]$ acting on the electrons. If the Grassmannian fields are integrated out, one has the free energy as a functional of the complex field $\Delta(x)$. The latter is the pair field; the microscopically obtained free energy is a functional of the (Cooper) pair field only, namely we now have $F\{\Delta(x)\}$.

In practice, the above is not explicitly achievable in general. We describe here an approach which was proposed in Ref. 11. This approach uses a gauge transformation of the
Grassmannian field; the physically measured quantities do not depend on the gauge, and the transformed Hamiltonian is explicitly gauge invariant. For simplicity, we consider here the case where the order parameter does not depend on time. The approximation goes beyond the BCS mean field theory in which one essentially assumes that $\psi(x) = \Delta_0$, a real number whose equilibrium value (for a uniform superconductor) is determined selfconsistently. (In our language, this is done by the extremization (actually minimization) of $F_{BCS}(\Delta_0)$, namely by requiring that $(\partial F_{BCS}/\partial \Delta_0) = 0$). It is also valid at all temperatures below $T_c$ and thus goes beyond the Ginzburg Landau theory which proposes a phenomenological functional $F_{GL}(\psi)$ valid for small $\psi$ (i.e. near $T_c$) which varies smoothly. (As is well known, this was microscopically justified by Gor’kov who identified $\psi$ with the (Cooper) pair function $\Delta(x)$).

The complex pair function $\Delta(x)$ can always be written as

$$\Delta(x) = \Delta_0(x) \exp[i\phi(x)]$$

(2)

where $\Delta_0(x)$ is its real amplitude and $\phi(x)$ is its phase. The pair potential $\Delta(x)$ can be made real by absorbing the phase term $\phi(x)$ in a redefinition of the electron pair fields. This is not unique, and the mechanisms of how $\phi(x)$ can be connected in different ways with the members of the electron pair have been discussed by several authors. Here we use a symmetric gauge, namely transform the Grassmannian fields as $\psi_\sigma(x) = \tilde{\psi}_\sigma(x) \exp[i\phi(x)/2]$. This simplifies the free energy considerably. It can be described now as a functional of the real quantity $\Delta_0(x)$ and of the gauge invariant superfluid velocity $v_s(x) = (1/2m)(\hbar \nabla \phi - 2eA/c)$. For simplicity, we consider here the case where the pair phase $\phi$ as well as the amplitude depend only on $r$ and not on imaginary time $\tau$. On using the symmetric transformation, one can write the free energy $F$ as a sum of two terms, namely

$$F = F_0 + F_\phi$$

(3)

where $F_0$ corresponds to the partition function with $H_0$ given by

$$H_0 = \int dr \sum_\sigma \tilde{\psi}_\sigma(x) \left[ \frac{\partial}{\partial \tau} + \frac{p^2}{2m} + V(r) - \mu \right] \tilde{\psi}_\sigma(x)$$

$$+ \int dr \left[ \Delta_0(r) (\psi_\downarrow(x) \psi_\uparrow(x) + h.c.) - \frac{\Delta_0^2(x)}{g} \right]$$

(4)

and $F_\phi$ is the corresponding integral over $H_\phi$ which is given by

$$H_\phi = \frac{1}{4} \int dr \sum_\sigma \tilde{\psi}_\sigma(x) \left[ (p \cdot v_s + v_s \cdot p) + \frac{m}{2} v_s^2 \right] \tilde{\psi}_\sigma(x).$$

(5)
Equation (4) describes electrons moving in a random static potential $V(\mathbf{r})$ and a static pair potential $\Delta_0(\mathbf{r})$. We further assume that there are no spatial variations in $\Delta_0(\mathbf{r})$, i.e., take $\Delta_0(\mathbf{r})$ to be a uniform potential. In that case, $H_0$ is an exactly solvable quadratic Hamiltonian in the absence of the random potential, it is the BCS mean field Hamiltonian. The effect of the pair phase $\phi$ is determined via the Hamiltonian $H_\phi$. To second order in it, the free energy of the superconductor can be written as

$$F(\Delta_0, \nu_s) = F_0(\Delta_0) + \frac{1}{2} \rho_s(\Delta_0) \nu_s^2$$  \hspace{1cm} (6)

where the minimum of the first term gives the self consistent mean field BCS value of the gap in the presence of disorder. The superfluid stiffness $\rho_s$ is like the mass of the superfluid; $(1/2)\rho_s\nu_s^2$ is its kinetic energy. This approach gives us a direct route for obtaining the superfluid density in terms of the properties of the BCS superconducting state.

In Ref.[11] the above formulation, the first to describe the free energy of the electron system microscopically in terms of the pair phase degree of freedom, was used along with the exact eigenstates method of de Gennes.[17] It describes $\rho_s$, the coefficient of the second order terms above, in terms of measured conductivity of the system in the presence of the same disorder $V(\mathbf{r})$ but in the absence of pair interaction. The focus there was to investigate the effect of Anderson localization of electronic states on $\rho_s$.

Here we discuss the effect of $H_\phi$ explicitly using conventional perturbative many body theory methods and describe the effect of the random potential $V(\mathbf{r})$ in terms of an electron relaxation time $\tau$.

III. SUPERFLUID DENSITY

Since the Hamiltonian $H_0$ contains quadratic Grassmannian fields, it can be diagonalized to obtain a Green’s function $G(x, x’) = -\langle T \Psi(x) \bar{\Psi}(x’) \rangle$ where the Nambu spinor $\bar{\Psi}(x) = (\bar{\psi}_t, \bar{\psi}_l)$ and $\langle \cdots \rangle$ represents thermal average of the appropriate quantity with respect to $H_0$ in the absence of the disorder potential and $T$ represents time ordering operator. Within the Born approximation for the scattering caused by the disorder potential $V(\mathbf{r})$, one finds the Green’s function in Fourier space with the configuration average $\langle V(\mathbf{r})V(\mathbf{r’})\rangle_{\text{dis}} = \frac{1}{2\pi\nu\tau} \delta(\mathbf{r} - \mathbf{r’})$ for the disorder potential; $\tau$ is the relaxation time and $\nu$ is the density of states for
electrons of each spin at the Fermi energy. One has
\[
G(k, \omega_n) = \frac{i \tilde{\omega}_n \sigma_0 + \xi_k \sigma_3 + \tilde{\Delta}_0 \sigma_1}{(i \tilde{\omega}_n)^2 - \xi_k^2 - \Delta_0^2} \tag{7}
\]
with the following relation\footnote{10} between renormalized Matsubara frequency \( \tilde{\omega}_n \) and the BCS pair amplitude \( \tilde{\Delta}_0 \) and their respective counterparts in the absence of disorder:
\[
\frac{\tilde{\omega}_n}{\omega_n} = \frac{\tilde{\Delta}_0}{\Delta_0} = 1 + \frac{1}{2 \tau \sqrt{\omega_n^2 + \Delta_0^2}}. \tag{8}
\]
Here \( \sigma \)'s are the Pauli matrices and \( \xi_k = \frac{\hbar^2 k^2}{2m} - \mu \) is the kinetic energy of the electrons with respect to the chemical potential.

Up to the second order in \( v_s^2 \) for the free energy, one finds using \( H_\phi \),
\[
F_\phi = \frac{m}{2} \int d\tau v_s^2 \left[ -\text{Tr} G(x - x') \right]_{r'' = r'' = \tau = 0^+} + \frac{1}{m} \text{Tr} \langle p_\alpha G(x, x') p_\alpha G(x', x) \rangle_{\text{con}}. \tag{9}
\]
The first term in the above equation is the diamagnetic contribution (Fig 1a) which is the London term, the sole nonzero contribution at \( T = 0 \). The second or paramagnetic term describes the current-current correlation due to two particles of the same initial momentum and frequency moving in the uniform pair potential and the zero range random, ‘white noise’ potential. Because the random potential is of zero range, as is well known, vertex corrections (a possible process is shown in Fig. 1c), vanish and one can write the configuration averaged two particle Green’s function as the product of two configuration averaged one particle Green’s functions (Fig. 1b). We thus find an expression for the superfluid density as
\[
n_s(T) = n + \frac{\hbar^2}{3m} \beta \sum_{\omega_n} \frac{d^3 k}{(2\pi)^3} k^2 \text{Tr} [G(k, \omega_n)G(k, \omega_n)] \tag{10}
\]
where \( p = \hbar k \), the angular average of \( k_\alpha k_\alpha = k^2/3 \), and \( n \) is the electron density.

By explicit evaluation of \( n_s(T) \) in Eq. (10) with the use of Eqs. (7), we find
\[
n_s(T) = n \left[ 1 + \frac{1}{\beta} \sum_{\omega_n} \int d\xi_k \frac{\xi_k^2 + \Delta_0^2 - \tilde{\omega}_n^2}{(\xi_k^2 + \Delta_0^2 + \tilde{\omega}_n^2)^2} \right] \tag{11}
\]
whose zero temperature value can be expressed as
\[
n_s(T = 0) = n \left( 1 + \int \frac{d\omega}{2\pi} \int d\xi_k \frac{1}{(\xi_k^2 + \Delta_0^2 + \tilde{\omega})} - \frac{2\tilde{\omega}^2}{(\xi_k^2 + \Delta_0^2 + \tilde{\omega})^2} \right). \tag{12}
\]
Performing integration by parts for the first term in the above integral, the formal divergence factor can be removed\(^2\) to obtain

\[ n_s(T = 0) = n \left[ 1 + \int \frac{d\omega}{2\pi} \int d\xi_k \frac{2\bar{\omega}(\omega - \tilde{\omega})}{(\xi_k^2 + \Delta_0^2 + \tilde{\omega}^2)^2} \right] \]  

(13)

which further simplifies to

\[ n_s(T = 0) = n \left[ 1 - \frac{1}{4\tau} \int d\omega \frac{\omega^2}{(\Delta_0^2 + \omega^2)^2} \left( \sqrt{\Delta_0^2 + \omega^2 + 1/2\tau} \right)^2 \right] \]  

(14)

by performing the integration over \(\xi_k\) and using the relations in Eq.(8). The integration in Eq.(14) yields

\[
\begin{align*}
    n_s(T = 0) &= \begin{cases} 
    n(\pi \Delta_0 \tau) \left[ 1 - \left(8/\pi\right) \frac{\Delta_0 \tau}{\sqrt{1 - (2\Delta_0 \tau)^2}} \tanh^{-1} \left( \frac{\sqrt{1 - 2\Delta_0 \tau}}{1 + 2\Delta_0 \tau} \right) \right], & \text{for } 2\Delta_0 \tau < 1 \\
    n(\pi \Delta_0 \tau) \left[ 1 - \left(8/\pi\right) \frac{\Delta_0 \tau}{\sqrt{(2\Delta_0 \tau)^2 - 1}} \tan^{-1} \left( \frac{\sqrt{2\Delta_0 \tau - 1}}{2\Delta_0 \tau + 1} \right) \right], & \text{for } 2\Delta_0 \tau > 1
    \end{cases}
\end{align*}
\]  

(15)

In the pure limit (\(\Delta_0 \tau \gg 1\)), we recover the London limit for the superfluid density, i.e., \(n_s(T = 0) = n\) and in the extreme impure limit (\(\Delta_0 \tau \ll 1\)), superfluid density is linear in \(\tau\), i.e., \(n_s(T = 0) = n\pi \Delta_0 \tau\), as well known in this (dirty superconductor) limit. Figure 2 shows the variation of \(n_s(T = 0)\) with \(\Delta_0 \tau\) obtained using Eq.(15). We see that its rate of increase with \(\Delta_0 \tau\) gradually slows down from linear at small \(\Delta_0 \tau\) to exponentially small at large \(\Delta_0 \tau\), reaching the asymptotic London limit.

Since the experimental data of \(n_s(T = 0)\) are usually available as a function of conductivity, \(\sigma\), which is associated with Fermi energy, \(\epsilon_F\), rather than \(\Delta_0\), we show its variation in Fig. 3(a) with \(k_F \ell = (2\epsilon_F/\Delta_0)\Delta_0 \tau\) for two specific values of \(\Delta_0/\epsilon_F\) in the ball park of the experimental regime, where \(k_F\) and \(\ell\) are Fermi wave number and mean free path respectively for electrons. We note that the value of \(n_s\) is approximately within 1–10% of its London limit in the usual experimental range of \(k_F \ell\). The linear behavior at small \(k_F \ell\) satisfactorily agrees with the experimental data\(^3\) (Fig. 3(b)) which has been shown as the variation with \(\sigma\) which is proportional to \(k_F \ell\).
A. Superfluid density at nonzero temperature

The superfluid density at a nonzero temperature has a generalized form derivable using Eq. (14) as

\[
n_s(T) = n \left[ 1 + \frac{\pi}{2\tau} \int \frac{dz}{2\pi i} \frac{z^2}{(\Delta_0^2 - z^2) \left( \sqrt{\Delta_0^2 - z^2 + 1/2\tau} \right)^2 e^{\beta z} + 1} \right]
\]

where the integration is in the complex plane and \(1/(\exp[\beta z] + 1)\) is the Fermi function and the temperature dependent BCS gap \(\Delta_0(T)\). The associated integrand has poles at \(z = \pm \Delta\) and branch cuts for \(\Delta_0 < z\) and \(z < -\Delta_0\) along the appropriate contour. Calculating the residues at the poles and subtracting the contributions along above and below the branch cuts we find in terms of real integral,

\[
n_s(T) = n \left[ 1 + \pi \Delta_0 \tau \tanh \left( \frac{\Delta_0}{2T} \right) - \frac{\pi}{\tau^2} \int_{\Delta_0}^{\infty} \frac{d\epsilon}{2\pi} \frac{\epsilon^2}{\sqrt{\epsilon^2 - \Delta_0^2}} \left( \epsilon^2 - \Delta_0^2 + 1/(2\tau)^2 \right)^2 \tanh \left( \frac{\epsilon}{2T} \right) \right].
\]

As expected, the zero-temperature limit \((T \to 0)\) of Eq. (17) reproduces \(n_s\) as shown in Fig. 2. At the BCS critical temperature \(T_c\), \(n_s(T_c) = 0\).

It is convenient to expand Eq. (11) in the power series of \(\Delta_0^2(T)\) for the purpose of obtaining \(n_s(T)\) near \(T_c\) as \(\Delta_0^2(T) \sim (8\pi^2/\tau\zeta(3))(T_c - T)T_c\) is very small, where \(\zeta(3)\) is a Riemann zeta function. We thus find

\[
n_s(T \sim T_c) \approx n \Delta_0^2 \pi \sum_{\omega_n} \frac{1}{\omega_n(|\omega_n| + 1/(2\tau))}
\]

By employing the standard algebra for the series sum with Matsubara frequency, we find

\[
n_s(T \sim T_c) = n \left( \frac{\Delta_0}{2T_c} \right)^2 \left[ \frac{1}{(1/\pi)(4T_c \tau)} \Psi' \left( \frac{1}{2} \right) - (4T_c \tau)^2 \left\{ \Psi \left( \frac{1}{2} + \frac{1}{4\pi T_c \tau} \right) - \Psi \left( \frac{1}{2} \right) \right\} \right]
\]

where \(\Psi(x)\) is the digamma function and \(\Psi'(x)\) is its derivative. It is easy to check that for a clean superconductor \((T_c \tau \to \infty)\), \(n_s(T \sim T_c) = 2n(T_c - T)/T_c\) as known. The superfluid density is proportional to \(\Delta_0^2(T)\) near \(T_c\) and the proportionality constant decreases with the increase of disorder.
IV. DISCUSSION

We have described above a general approach to the superfluid density or stiffness $\rho_s$ of disordered superconductors as a function of disorder characterized by an electron relaxation time $\tau$. The approach is based on describing the energy as a function of the superfluid velocity, a gauge invariant quantity. The stiffness is the (half of the) coefficient of the square of the superfluid velocity. The superfluid stiffness is directly measured through the penetration depth. We have calculated $n_s$ both at $T = 0$ and $T \neq 0$. They have the expected pure (London) limit at $T = 0$ and vanish appropriately as $T$ approaches $T_c$. We have exhibited closed form expressions for $n_s(T)$ as a function for a wide range of $\tau$, from the very clean limit ($\Delta \tau > > 1$) to the normal disordered metal regime $\epsilon_F \tau > 1$, but $\Delta \tau > 1$ or $< 1$ (these results are available for the first time). We have compared our results with experiment.

The approach has the potential to explore many related questions, e.g. the effect of disorder on $T_c$; on the existence of a regime with nonzero mean pair field but vanishing pair stiffness (a non superconducting pseudogap(?)) regime); the nature of single particle states in this regime, in particular the single particle density of states (DOS). There are strong zero point fluctuation effects seen for example in vortex lattice melting at low temperatures\textsuperscript{18,19}. This is also related most likely with the failed superconductivity phenomenon reviewed in Ref.\textsuperscript{9}. We believe that including quantum (time dependent) phase fluctuation and going beyond the quadratic approximation in using the Hamiltonian Eq.5, are promising possibilities.

---

1 J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).
2 L.P. Gor’kov, Sov. Phys. JETP 36, 1364 (1959).
3 P. W. Anderson, J. Phys. Chem. Solids 11, 26 (1959).
4 M. Mondal A. Kamlapur, G. Saraswat, S. Kumar, J. Jesudasan, L. Benfatto, V. Tripathi, and P. Raychaudhuri, Phys. Rev. Lett. 106, 047001 (2011)
5 S. Mandal, S. Dutta, S. Basistha, I. Roy, J. Jesudasan, V. Bagwe, L. Benfatto, A. Thamizhavel, and P. Raychaudhuri, (Private Communication)
6 M. V. Sadovskii, Phys. Usp. 44, 515 (2001).
FIG. 1: Feynman diagrams for the contributions to superfluid density. While wavy lines represent the superfluid delocity, the solid lines represent the Fermionic Nambu Green’s functions. (a) The diamagnetic contribution. (b) The paramagnetic contribution without vertex correction. (c) The paramagnetic contribution for vertex correction due to one scattering (represented by dashed line) of Nambu quasiparticles and holes from the same scattering center; this however vanishes for the short-range disordered potential considered here.

7 P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. 78, 17 (2006).
8 P. W. Anderson, J. Phys. Conf. Series 449, 012001 (2013).
9 See recent review by A. Kapitulnik, S. A. Kivelson, and B. Spivak, Rev. of Mod. Phys. 91, 011002 (2019).
10 A. A. Abrikosov and L. P. Gor’kov, Soviet Phys. JETP 35 1090 (1959).
11 T. V. Ramakrishnan, Physica Scripta T72, 24 (1989).
12 V.L. Ginzburg and L.D. Landau, Zh. Eksp. Teor. Fiz. 20, 1064 (1950). English translation in: L. D. Landau, Collected papers (Oxford: Pergamon Press, 1965) p. 546.
13 J. R. Schrieffer and L.P. Gor’kov, Phys. Rev. Lett. 80, 3360 (1998).
14 P. W. Anderson, Arxiv:cond-mat/9812063.
15 M. Franz and Z. Tesanovich, Phys. Rev. Lett. 84, 554 (2000).
16 V. Ambegaokar, in Superconductivity, edited by R. D. Parks (Dekker, New York, 1969), vol 1.
17 P. G. De Gennes, Superconductivity of Metals and Alloys (Benjamin Inc, New York, 1966).
18 L. Li, J. G. Checkelsky, S. Komiyi, Y. Ando, and N.P. Ong, Nature Phys. 3, 311 (2007).
19 I. Roy, S. Dutta, A. N. Roy Choudhury, S. Basistha, I. Maccari, S. Mandal, J. Jesudasan, V. Bagwe, C. Castellani, L. Benfatto, and P. Raychaudhuri, Phys. Rev. Lett. 122, 047001 (2019).
FIG. 2: The superfluid density at zero temperature versus $\Delta_0\tau$ obtained using Eq. (15).

FIG. 3: (a) The superfluid density at zero temperature versus $k_F\ell$ for $\Delta_0/\epsilon_F = 0.001$ and 0.0005. (b) Experimental data of inverse square of penetration depth $\lambda^{-2}$ and the corresponding conductivity $\sigma$ extracted from Ref. [4]. The solid line is a guide to the eye.