A New Reconstruction Approach in Diffraction Ultrasound Tomography: Combine Transmission Mode with Reflection Mode Tomography

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Abstract

Diffraction Ultrasound Tomography based on Fourier scattering theorem is studied. In this paper, we reveal the relation between the transmission mode (TMDT) and the reflection mode diffraction tomography (RMDT). Different from the transmission mode ultrasound tomography which lose the details of the picture, we combine the forward scatter field data and the backward data with the higher frequency information. The iterative reconstruction method with forward non-uniform fast Fourier transform (NUFFT) is used in the proposed algorithm. The simulation result indicates that the algorithm proposed in this paper is obviously improve the reconstruction quality than the TMDT or the RMDT.

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1. Introduction

Since it has enabled doctor to view internal organs with high precision and safety to the patients [1], the computed tomography has been used widely. Due to the hazard of the radiation, X-ray based techniques are not suitable to used frequently. Ultrasound tomography with diffracting source is an important type of acoustic imaging.

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Since the object inhomogeneities are large compared to the wavelengths, energy propagation is influenced by diffraction significant.

The Fourier diffraction projection theorem [1] is valid when the inhomogeneities in the object are only weakly scattering. In the transmission mode diffraction tomography, the forward scattered field carries the low frequency message of the scattered field. While the reflection mode diffraction tomography, the backwards scattered field carries the band-pass frequency message. We combine the TMDT and RMDT together in the frequency domain.

There are two main approach in the diffraction tomography reconstruction: interpolation in the frequency domain (gridding [2]) or interpolation in the space domain like NUFFT. Fessler and Sutton has recently proposed a fast and accurate implementation of the forward non-uniform Fourier transform [3]. Inverse NUFFT used in the diffraction ultrasound tomography is achieved iteratively by M.M Bronstein [4]. We adopt this approach for iterative reconstruction in RMDT and TMDT.

In the next section we introduce the principles of diffraction tomography briefly; in section 3 we describe NUFFT with its iterative reconstruction algorithm, and then we derive a new reconstruction approach which combined the TMDT and RMDT. The last section is the computer simulation and the conclusion.

2. The Fourier Diffraction Projection Theorem

In Fig.1 a 2-D object is shown being illuminate by a plane wave propagating along a unit propagation vector \( s_0 \). The Fourier Diffraction Theorem relates the Fourier transform of the measured scattered field projection with the Fourier transform of the object.

**Fourier Diffraction Theorem for Backscattered Waves (RMDT):**

By Fourier-transforming Equation and only considering the backscattered projections, the Fourier diffraction theorem for reflection mode diffraction tomography is obtained as follows.

Given a projection \( F_i(\Omega) \) of the backward scattered field \( u_i(\vec{r}) \) obtained by illuminating an object \( \Omega(\vec{r}) \) with a plane wave, the following equation holds:

\[
F_{1D}(\Omega)|\omega\rangle = F_{2D}(\Omega(\vec{r}))(k_x, k_y)
\]  
(1)

Where

\[
k_x(\omega) = \omega \sin \theta - \sqrt{k_0^2 - \omega^2 - k_0^2} \cos \theta
\]  
(2)

\[
k_y(\omega) = \omega \sin \theta + \sqrt{k_0^2 - \omega^2 - k_0^2} \cos \theta
\]  
(3)

\( F_{1D} \) and \( F_{2D} \) denote respectively one- and two-dimensional Fourier transforms. This theorem forms the mathematic basis of the RMDT, which establish relationship between the 1D Fourier transform of the measured scattered projection and the 2D Fourier transform of the object. In other words, the 1D Fourier transform of a backward projection gives value of the 2D Fourier transform of the object on an outward semi-circle (Ewald semicircle) in the frequency domain, as depicted in Fig.1.
Fourier Diffraction Theorem for Forward scattered Waves (TMDT):

In the same way, we get the projection $F_1$ of the forward scattered field $u_x(\rho)$ obtained by illuminating an object $o(\rho)$ with a plane wave, the following equation holds:

$$F_{1D}(F_1(\omega)) = F_{1D}(o(\rho)(k_x, k_y))$$

(4)

$$k_x(\omega) = \omega \cos \theta - \left( \sqrt{k_x^2 - \omega^2} - k_z \right) \sin \theta$$

(5)

$$k_y(\omega) = \omega \sin \theta + \left( \sqrt{k_x^2 - \omega^2} - k_0 \right) \cos \theta$$

(6)

Different from the Fourier Diffraction theorem for backscattered waves, the Fourier transform of the forward projection gives the values of the 2D Fourier transform of the object along a semicircular arc in the spatial frequency domain, as depicted in Fig.2.

In the next section we introduce an iterative reconstruction method which allows the reconstruction of the image from non-uniform samples in the frequency domain without using gridding.

3. Nonuniform Fourier Transform (NUFT) based Iterative Algorithm

The heart of iterative image reconstruction from non-uniform frequency samples is the forward NUFFT. At first, we give out a simple introduce of the non-uniform Fourier transform in the 1D case.

$$F(\omega_{lm}) = \sum_{k,l} f_k e^{-i\omega_{lm}}$$

(7)
Where $\omega_{nm} \in [0, M - 1]$ be a vector of nonuniformly distributed frequencies and $\mathbf{f} = (f_0, \ldots, f_{M-1})$. $\mathbf{f} \in \mathbb{C}^N$ be a vector representing a discrete signal in the space domain. The process of NUFT can be considered as an operator $\Psi: \mathbb{C}^N \rightarrow \mathbb{C}^M$, defined as

$$\mathbf{F}^o = \Psi \mathbf{f} \tag{8}$$

$\Psi$ is a full column rank matrix containing $M$ discrete basis function in its rows: $\Psi = (\psi_1, \ldots, \psi_M)^T$, $\psi_m = e^{-i \omega_m m}$. A fast approximation $\mathbf{F} = U_s \Phi$ of the NUFT operator can be achieved in two steps. The first is to project the signal $\mathbf{f}$ to some oversampled uniform Fourier grids using standard FFT, the second is to carry out efficient interpolation.

When the projection operation accomplished, spectrum on projection circular arcs is known. $\mathbf{f}$ is given by the Moore-Penrose pseudoinverse [4,5]:

$$\mathbf{F}^o = \Phi^o \mathbf{F}^o \tag{9}$$

which is not unique when the size of $\mathbf{f}$ is large. Alternatively, equation (9) can be reformulated as a least square problem:

$$g(\mathbf{f}) = \min_{\mathbf{F}^o} \| \Phi \mathbf{F} - \mathbf{F}^o \|^2 \tag{10}$$

The optimization problem in equation (10) can be solved iteratively using conjugate gradients (CG) method, which requires efficient computation of the cost function $g(\mathbf{f})$ and its gradient. Computation of the gradient of the $g(\mathbf{f})$ can be carried out by using the fast forward operator and its adjoint $\mathbf{F}^*$ according to the following equation:

$$\nabla g(\mathbf{f}) = 2 \mathbf{F}^* (\Phi \mathbf{F} - \mathbf{F}^o) \tag{11}$$

The adjoint operator $\mathbf{F}^* = \Phi^o \Phi^o$, which is crucial for iterative optimization algorithm, can be precomputed.

Based on what have been discussed above, our iterative image reconstruction algorithm is composed of the following steps:

Step 1. Precompute the $\Phi$ and the adjoint operator $\Phi^*$ according to [3].
Step 2. Set the initial estimate of the reconstruction image. In order to reduce the number of iteration and save convergence time, we get the initial image using the frequency domain methods, like gridding.
Step 3. An updated version of the image is generated each time round of iteration is performed, until $\frac{\| \Delta \mathbf{f}^{(i)} \|_2}{\| \mathbf{f}^{(i)} \|_2} < \delta$ satisfy the following criterion:

$$\frac{\| \Delta \mathbf{f}^{(i)} \|_2}{\| \mathbf{f}^{(i)} \|_2} < \delta \tag{12}$$

Where $i$ is the number of iteration, $\delta$ a threshold value for termination of iterative operations.

4. Combine the Forward and the Backwards scattered Data

As was shown before, the transform corresponds to upper Ewald semicircle when operating in transmission mode, while the lower Ewald semicircle refers to reflection mode. When incident direction changes from 0 to $2\pi$, the projection in frequency domain of transmission mode covers the circle of diameter $\sqrt{T_{\text{Max}}} \text{centered in origin}$, while the transform value of projection of reflection mode covers the annulus with inner diameter and outer diameter are $\sqrt{\text{T}_{\text{Max}}} - 2\text{k}_{\text{Max}}$, respectively. The reconstructed object is a low-pass version of the original in the transmission mode diffraction tomography. While the reflection mode is a band-pass version, who can complement some message lost in the TMDT.

Two main reconstruction strategies in diffraction tomography are interpolation in the frequency domain (analogous to the direct Fourier inversion in straight ray tomography) and interpolation in the space domain (analogous to the filtered back-projection), usually termed filtered-back-propagation algorithm. However, unlike straight ray tomography, interpolation in the space domain is computationally extensive, so the majority of efficient algorithm are based on frequency-domain interpolation.
A common method of image reconstruction in the frequency domain is the gridding algorithm [2]. The non-uniform data is interpolated to a uniform Cartesian grid using, for example, bilinear interpolation. Afterward, the inverse Fourier transform is efficiently computed using 2D FFT. However, this approach is liable to introduce inaccuracies and is sensitive to the configuration of the sample point.

\[ \text{TMDT} + \text{RMDT} = \]

Fig.3 The Distribution of the Fourier transform of the Projections

5. Simulation Results and conclusion

We used the Shepp-Logan phantom [6] which has a simple analytical expression in our computer simulation to avoid projection errors. We assume weak scattering, we can calculate the value of the diffracted projection. In the
Fig. 4 (b) reconstructed by the combined data is clearly than (d) by the transmission data only.

Fig. 4 (a) Original phantom (b) Reconstruction with the transmission and reflection data (c) Reconstruction with reflection data (d) Reconstruction with transmission data

In this paper we have discussed diffraction tomography with plane wave illumination. The Fourier diffraction projection theorem in the RMDT and TMDT is analysed. We combine the advantage of the forward and the backward scatter field data. The image quality was improved in our way.

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