Scalability and Fragility in Bounded-Degree Consensus Networks

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Abstract: We investigate the performance of linear consensus algorithms subject to a scaling of the underlying network size. Specifically, we model networked systems with \( n \)th order integrator dynamics over families of undirected, weighted graphs with bounded nodal degrees. In such networks, the algebraic connectivity affects convergence rates, sensitivity, and, for high-order consensus \( (n \geq 3) \), stability properties. This connectivity scales unfavorably in network size, except in expander families, where consensus performs well regardless of network size. We show, however, that consensus over expander families is fragile to a grounding of the network (resulting in leader-follower consensus). We show that grounding may deteriorate system performance by orders of magnitude in large networks, or cause instability in high-order consensus. Our results, which we illustrate through simulations, also point to a fundamental limitation to the scalability of consensus networks with leaders, which does not apply to leaderless networks.

Keywords: Distributed control; Large-scale systems; Robustness

1. INTRODUCTION

Starting with early works on consensus problems over networks, there has been a great deal of interest in dynamic systems properties of classes of networked dynamic systems. Fundamental questions have been posed related to convergence (Olfati-Saber and Murray, 2004), controllability (Ölsheisky, 2014; Pasqualetti et al., 2014), and performance (Bamieh et al., 2012; Siami and Motee, 2014). In these cases, there have been several situations where poor dynamic behaviors can be observed in large networks.

A particular component of poor behavior in families of consensus networks can be described as scale fragility (or just fragility) wherein stability properties are lost for large-scale networks in the family. Here, Stäydl et al. (2017) provide an analysis of certain classes of cyclic networks, and Tegling et al. (2019) point to such scale fragilities in networks with high-order dynamics. Even if stability can be maintained for all elements of a family of networks, it is desirable that the behavior be scalable, that is, that performance (such as time constants and sensitivity properties) be uniform with respect to network size within the family. See, e.g., discussions in Lestas and Vinnicombe (2006); Bamieh et al. (2012) and Tegling et al. (2019b).

In both cases, fragility and scalability, it turns out that the algebraic connectivity, as defined by certain eigenvalue properties of the network graph Laplacian, plays a crucial role. For example, in first order consensus problems, the algebraic connectivity is directly related to the slowest mode in the exponential convergence to consensus (Olfati-Saber and Murray, 2004). Sensitivity and lack of network coherence can also be attributed to the algebraic connectivity approaching zero as the network size grows (Siami and Motee, 2014; Tegling et al., 2019b). It is therefore of interest to consider families of networks where the algebraic connectivity may be bounded away from zero, independent of the network size. At the same time, many applications require communications overheads to be modest. It is therefore relevant to enforce a uniform (with respect to network size) upper bound on the nodal degrees. This is also a key assumption in our present work.

The two objectives – bounded nodal degrees yet well-behaved algebraic connectivity – are reconciled only in so-called expander families. Expander families are typically characterized through combinatorial conditions ensuring that the networks are sufficiently interconnected (Alon, 1986). As one of this paper’s results, we complement those conditions with a tractable algebraic characterization of non-expander families (of undirected, weighted graphs).

Not surprisingly, the fact that consensus algorithms perform well over expander networks has been observed in earlier consensus literature. In particular, Olfati-Saber has showcased the fast convergence properties of consensus in small-world networks (2005), and Ramanujan graphs (graphs that maximize the algebraic connectivity) (2007). Kar et al. (2008) show that Ramanujan graphs optimize the convergence speed of distributed inference problems, and Li et al. (2009) discuss quantized consensus over expanders. To the best of our knowledge, however, an issue that has not been observed in the literature is the fragility of these results towards a grounding of the network. This is the focus of the present paper.

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Grounding a network implies that the state at one of the nodes is fixed, and made independent of neighboring nodes. The terminology originates from electrical networks; in the context of consensus one often speaks of leader-follower consensus since the grounded node acts as a leader for the remaining network. Leader-follower consensus is natural in many contexts, like platooning after a lead vehicle (Seiler et al., 2004), slack bus control in DC networks (Andreasson et al., 2017), and pinning control (Chen et al., 2007). It may, however, also arise inadvertently, if a local controller ceases to function, if a node one-sidedly disconnects from its neighbors, or through a malicious attack. Either way, the good performance that was achievable in expanders networks is inevitably lost.

The dynamics of grounded networked systems are described by a grounded graph Laplacian. Therefore, performance aspects, which in standard consensus depend on the algebraic connectivity, now instead depend on the slowest mode of the grounded Laplacian (here termed grounded eigenvalue). While the algebraic connectivity can stay bounded away from zero in bounded-degree networks, the grounded eigenvalue is shown to always decrease in network size (in undirected graphs). This is a fundamental difference between the two types of consensus dynamics — and one we wish to pinpoint here as an important fragility.

The scalability and fragility properties we discuss apply to consensus algorithms of various orders. In Section 2, we introduce family of grounded networks, results in a leader-follower consensus algorithm. Provided the consensus algorithm converges, it does so to a leader-follower consensus algorithm: 

Without loss of generality, assume that node 1 is grounded and let its state be $x_1 = \dot{x}_1 = \ldots = x_1^n = 0$. The closed-loop dynamics for the remaining nodes can be written as

$$\frac{d}{dt} \xi = \begin{bmatrix} 0 & I_N & 0 & \cdots & 0 \\ 0 & 0 & I_N & \cdots & \vdots \\ 0 & 0 & 0 & \ddots & \vdots \\ -a_0L - a_1L - a_2L - \cdots - a_{n-1}L \\ \end{bmatrix} \xi,$$ 

where the graph Laplacian $L$ was defined in Section 2.1 and $I_N$ denotes the $N \times N$ identity matrix. This model adheres to the one considered in, e.g., Ren et al. (2007) and is a straightforward extension of the better known first- and second-order algorithms.

2.3 Leader-follower consensus in grounded networks

Grounding the network, by fixing the state at one of the nodes, results in a leader-follower consensus algorithm. Provided the consensus algorithm converges, it does so to the state at the grounded node (the leader).

Without loss of generality, assume that node 1 is grounded and let its state be $x_1 = \dot{x}_1 = \ldots = x_1^n = 0$. The closed-loop dynamics for the remaining nodes can be written as

$$\frac{d}{dt} \xi = \begin{bmatrix} 0 & I_{N-1} & 0 & \cdots & 0 \\ 0 & 0 & I_{N-1} & \cdots & \vdots \\ 0 & 0 & 0 & \ddots & \vdots \\ -a_0L - a_1L - a_2L - \cdots - a_{n-1}L \\ \end{bmatrix} \bar{\xi},$$

where $L$ is the grounded Laplacian obtained by deleting the first row and column of $L$ and $\xi$ is obtained by removing the states at node 1. The eigenvalues of $L$ are denoted $\lambda_1$ (or $\lambda_1(G)$) and are numbered as $0 < \lambda_1 \leq \ldots \leq \lambda_{N-1}$. We will be particularly interested in the smallest eigenvalue, $\lambda_1$, which we will refer to as the grounded eigenvalue.

2.4 Underlying assumptions

In the upcoming sections, we will discuss properties of the system (2) pertaining to its performance and robustness.
subject to a scaling of the network size. The following underlying assumptions on the system will be important for that discussion.

**Assumption 1.** (Bounded neighborhoods). Each local controller can receive measurements from at most \( q \) neighbors, where the number \( q \) is fixed and independent of \( N \). That is, \( |N_i| \leq q \) for all \( i \in V \).

**Assumption 2.** (Bounded edge weights). The graph’s edge weights are bounded, i.e., \( 0 < w_{\text{min}} \leq w_{ij} \leq w_{\text{max}} < \infty \) for all \( (i, j) \in E \). These bounds hold for every graph in a family \( \{G_N\} \).

**Assumption 3.** (Fixed and bounded gains). The system’s gains are bounded, i.e., \( a_k \leq a_{\text{max}} < \infty \) for all \( k = 0, 1, \ldots, n \). They are also fixed, meaning that they do not change if the underlying network graph changes. In particular, they are independent of \( N \).

Together, Assumptions 1 and 2 imply that the graph’s nodal degrees remain bounded, even if the number of nodes increases. Assumption 3 implies that the local controller tunings are not affected by such an increase.

### 3. CONNECTIVITY SCALING AND EXPANDERS

The connectivity of the network graph plays an important role for the performance of the consensus algorithm. Here, we will focus on the **algebraic connectivity**, quantified through the smallest non-zero eigenvalue, \( \lambda_2 \), of the graph Laplacian. In this section, we review some – both well and lesser known – results on its role in consensus problems of different orders. We also discuss the scaling of \( \lambda_2 \) as networks grow and focus on expander families, which have particularly good connectivity properties.

#### 3.1 The role of \( \lambda_2 \)

**Convergence rate** Consider a first order consensus algorithm (\( n = 1 \)). The rate of convergence is determined by the algebraic connectivity according to

\[
||x(t) - x^{\text{avg}}|| \leq ||x(0) - x^{\text{avg}}||e^{-\frac{2}{\lambda_2}t},
\]

(4)

(Olfati-Saber and Murray, 2004), where \( x^{\text{avg}} = (\sum_{i=1}^{N} x_i)/N \) (an invariant quantity). This implies that the speed at which a state of consensus is reached is inversely related to the size of \( \lambda_2 \).

**Sensitivity** Assume that the system in (2) is subject to a disturbance input:

\[
\dot{\xi} = A\xi + d,
\]

where \( d \in \mathbb{R}^N \), and let us consider the deviation from the consensus subspace \( y_i = x_i - x^{\text{avg}} \) as a measure of the algorithm’s performance. Now, denote by \( G \) the input-output system from disturbance \( d \) to the performance output \( y \). Then, for first-order consensus (\( n = 1 \)) it holds

\[
||G||_{\infty} = \frac{1}{a_0\lambda_2}
\]

(Siami and Motee, 2014). The \( H_\infty \) norm of a system has several interpretations. The interpretation as an induced norm (or \( L_2 \) gain) \( ||G||_{\infty} = \sup_{\|d\|_{2}} ||y||_{2}/||d||_{2} \) is particularly useful to characterize sensitivity. The relation (5) thus implies that the system may amplify certain disturbance signals by a gain that is inversely proportional to \( \lambda_2 \).

For second-order consensus (\( n = 2 \)), the expression in (5) instead gives a tighter lower bound on the \( H_\infty \) norm (follows from Theorem 2 in Pirani et al. (2017)).

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**Fig. 1.** Partitioning of graph for Lemma 3.1. The set \( X_2 \) is a **bottleneck** if it stays small compared to both \( X_1 \) and \( X_3 \) as the network grows. In this case, the algebraic connectivity decreases towards zero.

**Stability** Now, consider the consensus algorithm (1) with \( n \geq 3 \). A necessary (but not sufficient) condition for system stability, i.e., convergence to consensus, is

\[
\lambda_2 > \frac{a_{n-3}}{a_{n-1}a_{n-2}}
\]

(Tegling et al., 2019, Theorem 3). The condition implies that if the algebraic connectivity \( \lambda_2(G_N) \) tends to zero as \( N \to \infty \) in a graph family, then stability cannot be upheld beyond a certain network size (note, the \( a_k \) are fixed by Assumption 3). High order (\( n \geq 3 \)) consensus therefore has a scale fragility in such families of graphs.

**Grounded networks** If the network is grounded, the above properties depend on the grounded eigenvalue \( \lambda_1 \) instead of on \( \lambda_2 \). That is, for leader-follower consensus with \( n = 1 \), the convergence rate is given by \( \lambda_1 \) and the \( H_\infty \) norm from a disturbance to control error is \( ||G||_{\infty} = 1/(a_0\lambda_1) \) (Pirani et al., 2017). For leader-follower consensus with \( n \geq 3 \) a necessary stability condition reads

\[
\lambda_1 > \frac{a_{n-3}}{(a_{n-1}a_{n-2})}
\]

(Tegling et al., 2019, Theorem 5).

#### 3.2. Scaling of connectivity

The algebraic connectivity tends not to scale well with network size in bounded-degree networks, leading to a **lack of scalability** of the consensus algorithm. More precisely, for families of graphs \( \{G_N\} \) that do not satisfy certain expansion properties it holds \( \lambda_2(G_N) \to 0 \) as \( N \to \infty \). Before discussing those properties, we will provide a more tractable algebraic description of graph families in which indeed \( \lambda_2(G_N) \to 0 \).

For this purpose, partition a graph’s vertex set into three disjoint sets \( X_1, X_2, X_3 \) so that \( X_1 \cup X_2 \cup X_3 = V \) and \( |X_1| = N_1, |X_2| = N_2, |X_3| = N_3 \) as illustrated in Fig. 1. Each node in \( X_2 \) is connected to at least one node in both \( X_1 \) and \( X_3 \), but no edges connect \( X_1 \) and \( X_3 \) directly. In other words, \( X_2 \) is the boundary set of both \( X_1 \) and \( X_3 \). This partitioning is always possible, unless the graph is complete (note that \( X_1, X_2, X_3 \) need not be connected subgraphs).

By re-numbering the nodes, the graph Laplacian becomes

\[
L = \begin{bmatrix}
L_{11} & L_{12} & 0_{N_1,N_1} \\
L_{12} & L_{22} & L_{12} \\
0_{N_3 \times N_1} & L_{12} & L_{32}
\end{bmatrix}
\]

(7)

If \( N_2 \) can be made small in relation to both \( N_1 \) and \( N_3 \), we say that the graph has a **bottleneck**. The following lemma shows that if the bottleneck remains as the network grows, then \( \lambda_2(G_N) \to 0 \).

**Lemma 3.1.** Consider a graph family \( \{G_N\} \) and let Assumptions 1–2 hold. If every graph \( G_N \) in the family can be partitioned as outlined above in such a way that \( N_2/N_1 \to 0 \) and \( N_2/N_3 \to 0 \) as \( N \to \infty \), then \( \lambda_2(G_N) \to 0 \) as \( N \to \infty \).
In the previous section, we saw that maintaining good performance in growing consensus networks requires the algebraic connectivity of the graph to be large. Loosely speaking, a large Cheeger constant means that the graph has no bottlenecks; a partitioning as in (7) can be done with a relatively large set $X_1$. A precise definition of expander families follows:

**Definition 1.** (Expander family) Let $\{G_N\}$ be a graph family in which $N \to \infty$. If the sequence $\{h(G_N)\}$ is bounded away from zero, $\{G_N\}$ is an expander family.

The following result is central to our discussion:

**Result 3.2.** The graph family $\{G_N\}$ is an expander family if and only if the sequence $\{\lambda_2(G_N)\}$ is bounded away from zero as $N \to \infty$.

(See e.g. Chung (1997); Krebs and Shaheen (2011) for a proof.) Result 3.2 implies that the algebraic connectivity can indeed be prevented from decreasing towards zero in bounded-degree networks, making the consensus algorithm scalable. The formal requirement is that the Cheeger constant in (10) does not decrease towards zero.

**Explicit constructions and random graphs** It is known that there exist $k$-regular expander families for every $k \geq 3$ (Pinsker, 1973), but constructing them is non-trivial. Research into explicit construction rules has been ongoing since the 1970’s. We will not review such rules here, but refer interested readers to, e.g., Krebs and Shaheen (2011).

Interestingly though, random graphs will almost surely be expander families for sufficiently large $N$ (Friedman, 1991). Here, we mean graphs that are random in the sense that edges are selected through equally likely permutations of the node set. The good expansion properties of certain random graphs are exploited in Olfati-Saber (2007); Kar et al. (2008), as well as in our upcoming simulations.

For networked control problems, this implies that a way to achieve good connectivity, and thus scalability, properties, is to assign neighbors randomly across the network. Bandwidth can stay limited, since the number of neighbors remains bounded, but an ability to communicate across the scale of the network would be required. A more detailed study on the construction of expander families from a practical perspective is part of ongoing work.

Fig. 3 illustrates some of this section’s results. Here, we have generated a sequence of random graphs of increasing size using the algorithm proposed by Kim and Vu (2006). An example of one member of the sequence is given in Fig. 3. Fig. 2 compares their algebraic connectivity to that of 2-dimensional lattice graphs. In both cases, the degree of each node is 4, but their connectivities scale differently in $N$. Fig. 2 also displays the drastic discrepancy between $\lambda_2$ of the random graphs, and the corresponding grounded eigenvalue $\lambda_1$. This is the topic of the next section.

4. **Fragility Towards Network Grounding**

In the previous section, we saw that maintaining good performance in growing consensus networks requires the...
underlying graphs to constitute an expander family. This

However, if the network is grounded, the grounded eigen-

value \( \lambda_1 \) inevitably decreases towards zero as the network

grows. Consider the following lemma:

**Lemma 4.1.** Consider a graph family \( \mathcal{G}_N \) and let Assump-
tions 1–2 hold. The smallest eigenvalue \( \lambda_1(\mathcal{G}_N) \) of the
grounded Laplacian \( \bar{L}(\mathcal{G}_N) \) then satisfies

\[
\bar{\lambda}_1(\mathcal{G}_N) \leq \frac{q}{N - 1} w_{\text{max}}. \tag{11}
\]

**Proof:** By the Rayleigh-Ritz theorem it holds

\[
\bar{\lambda}_1 \leq \frac{v^T \bar{L} v}{v^T v}, \quad \forall v \neq 0.
\]

This implies in particular that

\[
\bar{\lambda}_1 \leq \frac{1}{\bar{L}} \frac{1}{1 - N - 1} = \sum_{k \in \mathcal{N}_1} \frac{w_{11k}}{N - 1} \leq \frac{q w_{\text{max}}}{N - 1},
\]

where \( \sum_{k \in \mathcal{N}_1} w_{11k} \) is the total weight of the edges leading to

the grounded node 1. The equality holds since each row \( k \) of \( \bar{L} \) sums to zero if the corresponding node \( k \) has no

connection to the leader, and otherwise if \( w_{11k} \leq w_{\text{max}}. \)

Lemma 4.1 says that, under the given assumptions, \( \bar{\lambda}_1 \to 0 \)
as \( N \to \infty \). Therefore, the performance of the leader-

follower consensus algorithm never scales well in grounded

bounded-degree networks. We next discuss some implica-
tions in more detail.

4.1 Implications

**Performance degradation** Lemma 4.1 implies that the

consensus algorithm can be fragile to grounding of the

network. Consider a scenario where a large bounded-

degree network has been carefully designed to avoid the

bottlenecks from Lemma 3.1 to ensure \( \lambda_2 \) is large. For

example, the network in Fig. 3. Assume first that the

consensus algorithm (1) with \( n = 1 \) is run over this network.

If a single node (say, number 1) turns off its controller

so that \( u_1 = 0 \), the system instead obeys the leader-

follower dynamics (3). Since we may have \( \bar{\lambda}_1 << \bar{\lambda}_2 \), the

convergence time and sensitivity can increase radically.

High-order \( (n \geq 3) \) consensus is yet more fragile. In

this case, the breakdown of one controller, or the active

decision of one agent to disconnect from its neighbors,

would cause a grounding of the network. If the network

is sufficiently large, the fact that \( \bar{\lambda}_1 << \bar{\lambda}_2 \) leads to a loss of

stability. This scenario is simulated in Fig. 5.

**Lack of scalability** Lemma 4.1 and Result 3.2 together

imply that there is an important difference between stan-
dard consensus and leader-follower consensus algorithms

in their scalability properties. It is possible to achieve

good scalability (in terms of the properties discussed in

Section 3.1) in standard consensus over bounded-degree

networks, but it is fundamentally impossible in leader-

follower consensus.

The scalability of consensus in bounded-degree networks is

therefore, in a sense, fragile to the assumption that the net-

work has no leader. This has implications for, e.g., vehicle

platooning problems. Here, one may wish to add commu-
nication links in an optimal way to increase connectivity

and thereby improve performance (see, e.g., Darbha et al.

(2019)). When each vehicle has a bounded number of links,
Fig. 5. Position trajectories (relative to average) in simulation of 3rd order consensus over the network in Fig. 3. After the network has been grounded at t = 30 s, the system is no longer stable and diverges when subjected to a disturbance—a fragility.

6. CONCLUSIONS

In this paper, we have considered classes of symmetric, high order consensus problems. Looking at networks with uniformly bounded node degrees, we examined issues related to scalability. Specifically, the possibility of maintaining algebraic connectivity bounded away from zero as networks grow in size. We have seen that the presence of ‘bottlenecks’ in the network, or grounding (which creates leader-follower networks) guarantee that the algebraic connectivity (or the grounded eigenvalue, which plays a similar role) approaches zero as the network size increases.

There are bounded-degree networks, ‘expander families’, where algebraic connectivity scales well. Such families can (almost surely) be constructed through random graphs. However, their performance is highly fragile to network grounding, particularly in large networks, because the grounded eigenvalue is much smaller than the algebraic connectivity.

Ongoing and future work include studies of, on one hand, design rules to achieve scalability in practical networked systems. On the other, strategies to mitigate the identified fragilities towards grounding.

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