POSSIBLE GENERATION OF $\pi$-CONDENSATION IN A FREE SPACE BY COLLISIONS BETWEEN PHOTONS AND PROTONS

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The possible generation of $\pi$-condensation in a free space by photon-proton collisions is discussed.

Key words: Hadron Quantum electrodynamics, Stimulated $\pi^+$-emission in photon-proton collisions, $\pi$-condensation in a free space. Ring resonator.

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I. INTRODUCTION

Possible $\pi$-condensation in nuclear matter was first considered in seventies last century. To put the work on a better mathematic foundation we exactly solved the Dirac equation for the nucleon in a classical $\pi$-field. A theory of nuclear matter was therefore proposed in the form of relativistic mean field theory, in which mean $\pi$-field is taken into account. To minimize the energy per-nucleon, a nonzero value of mean $\pi$-field appears. It shows the possible existence of the $\pi$-condensation in nuclear matter.

The problem is that, until now we have not seen any direct experimental evidence for the existence of a $\pi$-condensation, neither in nuclear matter nor in free space. It makes us eager to generate a $\pi$-condensation experimentally, in free space first. Of course, to generate a $\pi$-condensation in free space itself would be interesting.

We have designed a way for generating a $\gamma$-photon condensation by use of the photon-electron collisions. It is because we found that, the angular distribution of the outgoing direction, we see that the $\gamma$-photon is sharply peaked. Since the energy of an emitted $\gamma$-photon is a definite function of its outgoing direction, we see that the $\gamma$-photons are emitted almost into one state. In this paper, we shall show that the same situation appears also for the $\pi^+$-mesons in the photon-proton collisions. The angular distribution of the emitted $\pi^+$-mesons is also sharply peaked. We may therefore design a similar way for generating a $\pi^+$-condensation in free space, by use of photon-proton collisions.

II. ANGULAR DISTRIBUTION OF THE EMITTED $\pi^+$-MESONS IN A HEAD ON PHOTON-PROTON COLLISION

Consider the reaction

$$\gamma + p \rightarrow \pi^+ + n,$$

in which a $\gamma$-photon and a proton p head on collide with each other, and transit into a $\pi^+$-meson and a neutron n. When the energy of the photon is not too high, the colour degrees of freedom in hadrons are not important. The problem may therefore be handled by hadron quantum electrodynamics. Denote the proton field, neutron field, charged meson field and photon field by $\Psi_p, \Psi_n, \Phi$ and $(A_\mu)$ respectively, the Lagrangian density for the system is

$$\mathcal{L} = \mathcal{L}_p + \mathcal{L}_n + \mathcal{L}_m + \mathcal{L}_\gamma + \mathcal{L}_s,$$

in which

$$\mathcal{L}_p = -\overline{\Psi}_p [\gamma_\mu (\partial_\mu - ieA_\mu) + m] \Psi_p,$$

$$\mathcal{L}_n = -\overline{\Psi}_n (\gamma_\mu \partial_\mu + m) \Psi_n,$$

$$\mathcal{L}_m = -\left[(\partial_\mu + ieA_\mu)\Phi^\dagger (\partial_\mu - ieA_\mu) \Phi + m_\pi^2 \Phi^* \Phi\right],$$

$$\mathcal{L}_\gamma = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu),$$

and

$$\mathcal{L}_s = i\sqrt{2}G(\overline{\Psi}_p \gamma_5 \Psi_n \Phi + \overline{\Psi}_n \gamma_5 \Psi_p \Phi^*)$$

are Lagrangian densities of protons with their electromagnetic interactions, neutrons, $\pi^\pm$-mesons with their electromagnetic interactions, photons, and the strong interactions between related hadrons, respectively. The nature unit system of $\hbar = c = 1$ is used. Symbols are defined in the usual way as given in standard text books, for examples in [10-11]. $\alpha$ is interaction constants for electromagnetic interaction and strong interaction respectively, with the corresponding values $\alpha = e^2/4\pi = 1/137.\cdots$ and $\alpha_s = G^2/4\pi = 14.6$.

For the reaction (1), a factor of electromagnetic interaction appears always with a factor of strong interaction. The constants $\alpha$ and $\alpha_s$ always appear together in the form of a product $\alpha \alpha_s$. Since $\alpha \alpha_s < 1$, a perturbation treatment for the reaction seems reasonable. The lowest order transition matrix element for the reaction is

$$\langle \sigma_n, q, \kappa | \mathcal{T} | k, e, p, \sigma_p \rangle = -Ge \prod_{\mu=0}^{3} \delta(p_\mu + k_\mu - q_\mu - \kappa_\mu)$$

$$\times \pi_{\sigma_n}(q) \gamma_5 \left[ \frac{1}{\gamma\mu(p_\mu + k_\mu) + m} i\gamma^\mu e_\mu + \frac{1}{(p_\mu - q_\mu)(p_\mu - q_\mu) + m_\pi^2} 2\kappa^\mu e_\mu \right] u_{\sigma_p}(p),$$

in which $[p_\mu], [q_\mu], [k_\mu], [\kappa_\mu]$, with $\mu = 0, 1, 2,$ and $3$, are energy-momentum four vectors of proton, neutron, photon, and $\pi^+$-meson respectively. $[e_\mu]$ with $\mu = 0, 1, 2,$ and $3$, is the polarization four vector of the photon. $m$ and $m_\pi$ are masses of the nucleon and the charged pion.
respectively. \( u_{\sigma_n}(q) \) is the Dirac spinor of the neutron with spin \( \sigma_n \) and momentum \( q \), and \( u_{\sigma_p}(p) \) is that of the proton with spin \( \sigma_p \) and momentum \( p \). \( \delta - \) functions in \( \mathbb{R}^3 \) show energy-momentum conservation in the reaction (1). They, together with the energy-momentum relations

\[
p_0 = \sqrt{p^2 + m_n^2}, \quad q_0 = \sqrt{q^2 + m_p^2}, \quad \kappa_0 = \sqrt{\kappa^2 + m_\pi^2}
\]

and 

\[
k_0 = k, \quad \text{give}
\]

\[
(A^2 - 1)\kappa^2 + 2AB\kappa + B^2 - m_\pi^2 = 0,
\]

\[
A = \frac{k - p}{p_0 + k} \cos \theta \quad \text{and} \quad B = \frac{m_n^2 + 2k(p_0 + p)}{2(p_0 + k)}.
\]

(9)

(10)

\( \theta \) is the angle between moving directions of the incident photon and the emitted pion. \( \kappa > 0 \) is the absolute value of the pion momentum, therefore should be the positive root of equation (9). It defines the energy \( \kappa_0 \) as a function of the moving direction \( \theta \) for the pion.

Take the Coulomb gauge, in which the contribution from longitudinal and temporal components of the electromagnetic field is collected in the coulomb energy between charged particles, and is negligible when space charge effect being unimportant. Only the contribution from the transverse components of the electromagnetic field will be considered in the following. Let \( e_i e^{i k \cdot x} \) with \( i = 1, 2 \) show the transverse plane wave, we have \( e_i = 0, e_i \cdot k = 0 \) for \( i = 1, 2 \), and \( e_i^* \cdot e_{i'} = \delta_{i i'} \).

For experiments without measuring spins, all transition probabilities and cross-sections have to be summed up over the final spin states and averaged over the initial spin states. Using the projection operator method, we obtain

\[
\frac{1}{4} \sum_{i=1,2} \sum_{\sigma_n=-1,1} \sum_{\sigma_p=-1,1} \left| \pi_{\sigma_n}(q) \gamma_5 \frac{1}{\gamma^\mu(p_\mu + k_\mu) + m} \gamma^\mu e_{i\mu} + \frac{1}{(p^\mu - q^\mu)(p_\mu - q_\mu) + m_\pi^2} 2\kappa^\mu e_{i\mu} \right| u_{\sigma_p}(p) = \frac{X}{p_0 q_0}. \tag{11}
\]

\[
X = \frac{1}{2} f^2 \left\{ [2k(p + p_0) - \kappa_0 (k + p_0) + \kappa (k - p) \cos \theta] \left[ k(2p + p_0) - p^2 \right] + 2pk(p_0 + p)(p - k + \kappa \cos \theta) + [p^2 + p_0(k - \kappa_0)] (p - k)^2 - (p^2 + p_0k)(p - k)(p - k + \kappa \cos \theta) \right\} \kappa^2 g(\theta) \sin^2 \theta \{ f k(p + p_0) + g(\theta) [p_0(k - \kappa_0) + p(k - \kappa \cos \theta)] \}, \tag{12}
\]

with \( f = 1/[2k(p_0 + p)] \), and \( g(\theta) = 1/[2k(\kappa \cos \theta - \kappa_0)] \).

The transition probability per-unit time for the pion goes into a differential solid angle \( d\Omega \) is

\[
\frac{dP}{dt} = 2\alpha_x \alpha \frac{X J \kappa^2}{k \kappa_0 q_0} d\Omega d\kappa \delta(E_f - E_i). \tag{13}
\]

\( J = 1/V \) is the incident photon current density in our unit system, and \( V \) is the volume of the reaction space. \( E_i = p_0 + k \) and \( E_f = q_0 + \kappa_0 \) are initial and final energies of the process respectively. Under fixed initial momenta \( p \) and \( k \),

\[
d\kappa = \frac{\kappa_0(p_0 + k - \kappa_0)}{(p_0 + k)\kappa + \kappa_0(p - k) \cos \theta} d(E_f - E_i). \tag{14}
\]

We therefore have

\[
\frac{dP}{dt} = 2\alpha_x \alpha X J Y \lambda_\pi^2 d\Omega. \tag{15}
\]

\( \lambda_\pi \) is the Compton wavelength of the charged pion, and

\[
Y = \frac{\kappa^2 m_\pi^2}{kp_0(p_0 + k)\kappa + \kappa_0(p - k) \cos \theta}. \tag{16}
\]

The differential cross-section for the photo-production of the charged pion on a proton is therefore

\[
\frac{d\sigma}{d\Omega} = 2\alpha_x X Y \lambda_\pi^2. \tag{17}
\]

Notice, \( X \) and \( Y \) are dimensionless.

An example of numerical results is shown in figures 1 and 2. The angular distribution shown in figure 1 is rather characteristic. It is sharply peaked, and is therefore favorable for emitting pions into the most probable state. However, the most probable state is not unique. In

FIG. 1: Relation between the differential cross-section in unit of barn for pion emission and the angle \( \theta \) in unit of \( \pi \), in a head on collision between the 1.4MeV photon and the 434GeV proton


FIG. 2: Relation between the emitted pion energy $E$ in unit of GeV and the angle $\theta$ in unit of $\pi$, in a head on collision between the 1.4MeV photon and the 434GeV proton.

the example shown in figure II the most probable emission directions distribute on the surface of a cone, each along a generatrix. The vertex of the cone is at the reaction point, and the axis is on the incident line of the proton. The angle between the generatrix and the axis is 0.006278$\pi$. In the following we shall show that a resonance mechanism makes almost all emitted pions go to one selected most probable state, therefore generates a $\pi^+$-condensation in free space.

III. STIMULATED EMISSION, RESONANCE, AND THE RING RESONATOR FOR THE $\pi$-CONDENSATION

The key ingredient for making a laser is the stimulated emission of radiation. This is also true for making a $\pi$-condensation in the free space. If there are already $N$ pions in a state, the transition probability for emitting one more pion into this state has to be multiplied by an extra factor $N+1$. The equation (15) is therefore generalized to

$$\frac{dP}{dt} = 2(N + 1)\alpha\alpha XYZ\lambda^2 d\Omega,$$

which includes contributions of both spontaneous and stimulated emissions of pions into a given state. Here we see a positive feedback between pion numbers of already in and emitted into a given state. The result is a collapse of pion population into the most probable states.

To make emitted pions condense in one state, we need a resonance mechanism. Figure 3 shows a schematic designation of a ring resonator for the stimulated emission of $\pi^+$-mesons in head on collisions between photons and protons. The straight line denotes the incident line, and are designed to collide with each other at the gaps. Elementary geometry tells us that angles between tangents of the circle at two gaps and the incident line equal each other, so that we may design these tangents along the most probable directions for pion emission at both gaps. $\pi^+$-mesons emitted on this most probable direction therefore enter the inner space of the ring. It is its central circular channel. They move along the channel under the interaction of an appropriate constant magnetic field perpendicular to the ring plane. A coincidence of colliding photons and protons together with the earlier emitted $\pi^+$-mesons at the gaps is designed, so that the stimulated emissions may happen. The already stored $\pi^+$-mesons stimulate the new $\pi^+$-meson emissions at the gap. The $\pi^+$-mesons are emitted along the most probable direction selected by the ring, and enter the storage ring at the gap under the interaction of the magnetic field. They are therefore prepared to stimulate the next $\pi^+$-meson emissions at the next gap. In this way a resonance is formed and a special most probable emission is selected at each gap.

In our example of head on collision between a 1.4MeV photon and a 434GeV proton, the energy of the most probable emitted $\pi^+$-meson is 6.9GeV, as shown in figure 4. In a magnetic field of $B = 1T$, they would move in a storage ring of radius 23m. Using the data shown at the end of last section, we see that two gaps on the ring in figure 3 open a central angle of 0.012556$\pi$ at the center of the ring. Therefore the length of the arc between these two gaps on the ring of radius 23m is 0.9m. One may therefore open many pairs of gaps on the ring to intensify the condensed pion beam many times in one circle. On the other hand, the half-life time of a 6.9GeV $\pi^+$-meson is 8.948 $\times$ 10$^{-7}$s. A half of $\pi^+$-mesons in the beam may move 268.33m before their decay. It is about twice of the ring circumference. It seems, we may generate a rather intense condensed pion beam in this way,
and store it in the ring. However, there are various interac-
tions between $\pi^+$-mesons. Among them, the long range elec-
 tromagnetic interaction may be important at not too high density of the meson beam. This is the so-called space charge problem. The electric force of the space charge, perpendicular to the meson trajectories, therefore makes nonzero probability for mesons to leave
from the resonance orbits, and limits the beam density. It offers a saturation mechanism for the $\pi^-$-condensation in our example. Besides, charged pions running in a ring may lose energy by Bremsstrahlung. But it may be easily compensated by usual acceleration techniques.

IV. CONCLUSIONS

The $\pi$-condensation may be generated in a way like that in the laser generation. First of all, we need a pion source. In our example proposed here, it is played by the hadronic reaction [1]. Various sharply peaked spectra of pion emissions appear. They make emissions concentrate to some specified pion states, and therefore are welcome. We then need a way for realizing the stimulated pion emissions to start the $\pi$-condensation, and need a resonance mechanism for selecting a special pion state to condense. These are designed in the ring resonator shown in the last section.

The reaction [1] was analysed by the hadron electrodynam-ics. Hadron-dynamics is not a fundamental theory, but an effective theory only. Therefore we should not rely on its quantitative results. We may obtain the quantitative results directly by experiments. However, some qualitative characters do not depend on the dynamical details. In the derivation and the numerical calculation we see, that the sharply peaked structures of the angular distribution for pion emissions are connected directly with the relativistic energy-momentum conservation relations [9] and [10]. It is a result of kinetics governing the reaction, and therefore is reliable. The designation of the ring resonator is based on fundamental electromagnetism, and is therefore reliable too. It makes us believe that our proposal is worthy to try experimentally.

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