An investigation of the $K_F$-type Lorentz-Symmetry Breaking
Gauge Models with Vortex-like Configurations.

H. Belich $^a$, F.J.L. Leal$^b$, H.L.C. Louzada$^a$, M.T.D. Orlando$^a$

$^a$Universidade Federal do Espírito Santo (UFES),
Departamento de Física, Av. Fernando Ferrari 514,
Vitória, ES, CEP 29060-900, Brasil; and

$^b$Instituto Federal de Educação, Ciência e Tecnologia do
Estado do Espírito Santo (IFES) - Campus Linhares,
Av. Filogônio Peixoto S/N, Bairro Aviso,
Linhares - ES, CEP 29901-291, Brasil

(Dated: May 11, 2014)

For the CPT-even case of the minimal Standard Model Extension, the spin-projector method is adopted to account for the breaking (tensor) $K_{\mu\nu\kappa\lambda}$ term. We adopt a particular decomposition of this term in fourvectors, and carry out a detailed analysis of causality and unitarity. From this study, we are able to impose conditions on the decomposition of the $K_{\mu\nu\kappa\lambda}$ and vortex formation is also investigated in different situations.

I. INTRODUCTION

Studies about symmetry breaking are well-known in nonrelativistic quantum systems involving phase transitions such as ferromagnetic systems, where the rotation symmetry is broken when the system is under the influence of a magnetic field. Similarly, in superconductors the spontaneous violation of a gauge symmetry shields electromagnetic interaction, but in type II superconductors the magnetic field penetrates, as determined by Abrikosov [1], and form a 2D vortices lattice. The two-dimensional electronic system is one of the most studied systems. Especially, a great amount of effort has been done to investigate a 2D electron system under a strong magnetic field to understand the quantum Hall effect.

*Electronic address: belichjr@gmail.com, nandojll@ig.com.br, haofisica@bol.com.br, mtdorlando@gmail.com
The Chern-Simons term for the gauge field yields a change in statistics of the vortex in these systems. Vortex configurations in planar models can be induced by a Chern-Simons term. Such a type of solution presents an interesting property to have electric charge. The Chern-Simons vortices were studied with nonminimal coupling, and remain being a topic of intensive investigation with recent developments.

For relativistic systems, the study of symmetry breaking can be extended by considering a background set up by tensors with rank $n \geq 1$. The background fields, in this situation, break the symmetry $SO(1, 3)$ instead of the symmetry $SO(3)$. This line of research is known in the literature as the spontaneous violation of the Lorentz symmetry. This new possibility of spontaneous violation was first suggested in 1989 in a work of Kostelecky and Samuel, indicating that, in the string field theory, the spontaneous violation of symmetry by a scalar field could be extended to tensor fields. This extension has as an immediate consequence: a spontaneous breaking of the Lorentz symmetry. In the electroweak theory, an $SU(2)$-doublet scalar field acquires a nonzero vacuum expectation value which yields mass to the $SU(2)$ gauge bosons (Higgs Mechanism). Similarly, in a string scenario a tensor field may trigger symmetry breaking. Nowadays, these theories are encompassed in the framework of the Extended Standard Model (SME) as a possible extension of the minimal Standard Model of the fundamental interactions. For instance, the violation of the Lorentz symmetry is implemented in the fermion section of the Extended Standard Model by two CPT-odd terms: $v_\mu \bar{\psi} \gamma^\mu \psi$ and $b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi$, where $v_\mu$ and $b_\mu$ correspond to the Lorentz-violating backgrounds.

An extension to the Chern-Simons form is implemented by $v_\mu$ background field in the form,

$$\Sigma_{CS} = -\frac{1}{4} \int dx^4 \epsilon^{\mu\nu\alpha\beta} v_\mu A_\nu F_{\alpha\beta}. \quad (1)$$

Such a term appears in the gauge sector of the SME and corresponds to a CPT-odd sector (the Carroll-Field-Jackiw term). A possibility to find an effect of the violation of this symmetry would be by the analysis of the defects that could be formed after Lorentz-symmetry breaking has been realized. Taking this point of view, we examine vortex solution in this scenario.

The vortex solution of the Maxwell-Chern-Simons with Lorentz violation was first studied in, and a dimensional reduction was adopted to study planar vortex solution. As expected, this vortex solution presents an electric charge. Also interference effects in topo-
logical defects with Lorentz violation have been investigated\cite{16}. This line of investigation contemplates the possible phases that can be generated, and by means of interference processes, we could detect the violation spontaneous of Lorentz symmetry. As the expectation of spontaneous Lorentz symmetry breaking by a background is beyond of the dynamics of the Standard Model, by experimental measurements we expect to set up stringent bounds on the parameters of the breaking (\(v_\mu, b_\mu\)).

Besides the CPT-odd terms, in the gauge sector, we have the CPT-even sector, which is represented by a tensor \(K^{\mu\nu\alpha\beta}\) with the same symmetries as the Riemann tensor, as well as an additional double-traceless condition \cite{17}. In this set-up, we present two possibilities of constructing a supersymmetric version for the \(K\)-type models.

We actually propose to carry out the supersymmetric extension to the bosonic action below:

\[
S = -\frac{1}{4} \int d^4x \, K_{\mu\nu\kappa\lambda} F^{\mu\nu} F^{\kappa\lambda}.
\]  

(2)

The CPT-even gauge sector of the SME has been studied since 2002, after the pioneering contributions by Kostelecky & Mewes\cite{18,19}, and there is extensive literature dealing with the extension of the standard model in the even sector of (SSM) by this term \cite{20}. We propose to work with a decomposition vectorial above because we can access to new physical properties where CPT violation does not occur.

The “tensor” \(K_{\mu\nu\kappa\lambda}\) is CPT even, i.e., it does not violate the CPT-symmetry. Though CPT violation implies violation of Lorentz invariance\cite{21}, the reverse is not necessarily true. The action above is Lorentz-violating in the sense that the “tensor” \(K_{\mu\nu\kappa\lambda}\) has a non-zero vacuum expectation value. That “tensor” presents the following symmetries:

\[
K_{\mu\nu\kappa\lambda} = K_{[\mu\nu][\kappa\lambda]}, \quad K_{\mu\nu\kappa\lambda} = K_{\kappa\lambda\mu\nu}, \quad K^{\mu\nu} \,_{\mu\nu} = 0 = 0,
\]

we can reduce the degrees of freedom and take into account the ansätze \cite{17}:

\[
K_{\mu\nu\kappa\lambda} = \frac{1}{2} (\eta_{\mu\kappa} \tilde{K}_{\nu\lambda} - \eta_{\mu\lambda} \tilde{K}_{\nu\kappa} + \eta_{\nu\lambda} \tilde{K}_{\mu\kappa} - \eta_{\nu\kappa} \tilde{K}_{\mu\lambda}),
\]

(4)

\[
\tilde{K}_{\mu\nu} = \kappa (\xi_{\mu} \xi_{\nu} - \eta_{\mu\nu} \xi^\alpha \xi_\alpha / 4),
\]

(5)
\[ \kappa = \frac{4}{3} \tilde{\kappa}^{\mu\nu} \xi_\mu \xi_\nu, \]  

(6)

where \( \tilde{\kappa}^{\mu\nu} \) is a traceless "tensor". Using the restrictions \((4), (5)\), in expression \((2)\), we obtain,

\[ S = \frac{\kappa}{4} \int d^4x \left\{ \frac{1}{2} \xi_\mu \xi_\nu F^{\mu\lambda} F^{\kappa\lambda} + \frac{1}{8} \xi_\lambda \xi_\kappa F^{\mu\nu} F^{\mu\nu} \right\}. \]  

(7)

An interesting topic of research is the discussion on the violation of supersymmetry (susy) \[22\]. We have investigated the possibility that SUSY and Lorentz symmetry are broken down at the same time. This study has already been carried out for the odd sector, and we are proposing here the extension to the even sector. This work is the beginning of a study of consistency to this decomposition suggested in the sector even, and we are also starting up an investigation of susy violation in this sector \[23\].

In this work, we analyze the possibility of having a consistent quantization of an Abelian theory which incorporates the Lorentz violating term of equation \((2)\), whenever gauge spontaneous symmetry breaking (SSB) takes place. Using the decomposition \((4, 5)\) the analysis is carried out by pursuing the investigation of unitarity and causality as read off from the gauge-field propagators. We therefore propose a discussion at tree-approximation, without going through the canonical quantization procedure for field operators. In this investigation, we concentrate on the analysis of the residue matrices at each pole of the propagators. Basically, we check the positivity of the eigenvalues of the residue matrix associated to a given simple pole in order that unitarity is respected at a semi-classical level.

In order to deepen our comprehension of the physics presented by this model, we also study vortex-like configurations, by analyzing the influence of the direction selected by \( K_{\mu\nu\kappa\lambda} \) in space-time. The decomposition of the \( K_{\mu\nu\kappa\lambda} \) tensor produces interesting modifications on the equations of motion that may yield vortex formation.

This work is outlined as follows: in Section 2, we study the SSB and present our method to derive the gauge-field propagators. In Section 3, we set our discussion on the poles and residues of the propagators. We study the formation of vortices in Section 4, and, finally, in Section 5, we present our Concluding Comments.
II. THE GAUGE-HIGGS MODEL

We propose to carry out our analysis by starting off from the action

\[ \Sigma = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^* D^\mu \varphi - V(\varphi) + \mathcal{L}_\kappa \right\}, \]

where \( \mathcal{L}_\kappa \) is the Lorentz violation term

\[ \mathcal{L}_\kappa = -\frac{1}{4} (K^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}). \]

Taking into account the ansatz presented in the introduction, we obtain:

\[ K^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = 2\kappa (g^{\mu\rho} (\xi^\nu \xi^\sigma - g^{\nu\sigma} \xi^\varepsilon \xi^\varepsilon/4)) F_{\mu\nu} F_{\rho\sigma}. \]

The potential, \( V \), given by

\[ V(\varphi) = m^2 |\varphi|^2 + \lambda |\varphi|^4 \]

is the most general Higgs-like potential in 4D. Setting suitably the parameters such that the \( \varphi \)-field acquires a non-vanishing vacuum expectation value (v.e.v.), namely, \( \lambda > 0 \) and \( m^2 < 0 \), the mass spectrum of the photon can be read off after the spontaneous breaking of local gauge symmetry and the \( \varphi \)-field has been shifted by its v.e.v. . The Higgs field is minimally coupled to the electromagnetic by means of its covariant derivative under U(1)-local gauge symmetry, namely

\[ D_\mu \varphi = \partial_\mu \varphi + ieA_\mu \varphi. \]

This symmetry is spontaneously broken, and the new vacuum is given by

\[ \langle 0 | \varphi | 0 \rangle = a, \]

where

\[ a = \left( -\frac{m^2}{2\lambda} \right)^{1/2}; \quad m^2 < 0. \]

As usually, we adopt the polar parametrization

\[ \varphi = \left( a + \frac{\sigma}{\sqrt{2}} \right) e^{i\rho/\sqrt{2a}}, \]

where \( \sigma \), and \( \rho \) are the scalar quantum fluctuations. Since we are actually interested in the analysis of the excitation spectrum, we choose to work in the unitary gauge, which is realized actually by setting \( \rho = 0 \). Then, the bilinear gauge action is given as below:

\[ \Sigma_g = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} 2\kappa ((g^{\mu\rho} (\xi^\nu \xi^\sigma - g^{\nu\sigma} \xi^\varepsilon \xi^\varepsilon/4)) F_{\mu\nu} F_{\rho\sigma} + \frac{M^2}{2} A_\mu A^\mu \right\}. \]
where \( M^2 = 2e^2a^2 \).

We can express this action as a bilinear as follows below:

\[
\Sigma_g = \int d^4x \frac{1}{2} A^\mu \{ O_{\mu\nu} \} A^\nu
\]  

(17)

where \( O_{\mu\nu} \) is the wave operator. The wave operator can be formulated in terms of spin-projection operators as follows, where \( \theta_{\mu\nu} \) and \( \omega_{\mu\nu} \) are respectively the transverse and longitudinal projector operators:

\[
\theta_{\mu\nu} = g_{\mu\nu} - \partial_\mu \partial_\nu \Box , \quad \omega_{\mu\nu} = \partial_\mu \partial_\nu \Box.
\]  

(18)

In order to invert the wave operator, one needs to add up other two new operators, since the ones above do not form a closed algebra, as the expression below indicates:

\[
\Sigma_{\mu\nu} = \xi_\mu \partial_\nu , \quad \lambda \equiv \Sigma_{\mu} = \xi_\mu \partial_\mu , \quad \Lambda_{\mu\nu} = \xi_\mu \xi_\nu.
\]  

(19)

We can express \( O_{\mu\nu} \) as,

\[
O_{\mu\nu} = ( (1 - \kappa \xi^\mu \xi_\nu /2) \Box + \kappa \lambda^2 + M^2 ) \theta_{\mu\nu} + ( \kappa \lambda^2 + M^2 ) ( \omega_{\mu\nu} ) + \kappa \Box \xi_\mu \xi_\nu - \kappa \lambda ( \xi_\mu \partial_\nu + \partial_\mu \xi_\nu ) .
\]

The propagator is given by

\[
\langle 0 \mid T [ A_\mu (x) A_\nu (y) ] \mid 0 \rangle = - i \left( O^{-1} \right)_{\mu\nu} \delta^4 (x - y) .
\]  

(20)

These results indicate that two new operators, namely, \( \Sigma \) and \( \Lambda \), must be included in order to have an operator algebra with closed multiplicative rule. The operator algebra is displayed in Table 1.

|       | \( \theta^\alpha_{\mu} \) | \( \omega^\alpha_{\nu} \) | \( \Lambda^\alpha_{\nu} \) | \( \Sigma^\alpha_{\nu} \) | \( \Sigma^\beta_{\nu} \) |
|-------|------------------|------------------|------------------|------------------|------------------|
| \( \theta_{\mu\alpha} \) | \( \theta_{\mu\nu} \) | 0                | \( \Lambda_{\mu\nu} - \frac{1}{2} \Sigma_{\nu\mu} \) | \( \Sigma_{\mu\nu} - \lambda \omega_{\mu\nu} \) | 0                |
| \( \omega_{\mu\alpha} \) | 0                | \( \omega_{\nu\mu} \) | \( \frac{1}{2} \Sigma_{\nu\mu} \) | \( \lambda \omega_{\mu\nu} \) | \( \Sigma_{\nu\mu} \) |
| \( \Lambda_{\mu\alpha} \) | \( \Lambda_{\mu\nu} - \frac{1}{2} \Sigma_{\mu\nu} \) | \( \frac{1}{2} \Sigma_{\mu\nu} \) | \( \xi^2 \Lambda_{\mu\nu} \) | \( \xi_2 \Sigma_{\mu\nu} \) | \( \Lambda \Lambda_{\mu\nu} \) |
| \( \Sigma_{\mu\alpha} \) | 0                | \( \Sigma_{\mu\nu} \) | \( \lambda \Lambda_{\mu\nu} \) | \( \lambda \Sigma_{\mu\nu} \) | \( \Lambda_{\mu\nu} \Box \) |
| \( \Sigma_{\alpha\nu} \) | \( \Sigma_{\nu\mu} - \lambda \omega_{\mu\nu} \) | \( \lambda \omega_{\mu\nu} \) | \( \xi^2 \Sigma_{\nu\mu} \) | \( \xi^2 \Box \omega_{\mu\nu} \) | \( \lambda \Sigma_{\nu\mu} \) |

Table 1: Multiplicative table fulfilled by \( \theta, \omega, S, \Lambda \) and \( \Sigma \). The products are supposed to obey the order "column times row".
Using the spin-projector algebra displayed in Table 1, the propagator may be obtained after a number of algebraic manipulations. Its explicit form in momentum space can be written down upon use of the equation:

$$O_{\mu\alpha}(O^{-1})^{\alpha}_{\nu} = \theta_{\mu\nu} + \omega_{\mu\nu}.$$ 

The expressions containing the poles of the propagator are cast below:

$$D = (1 - \kappa \xi^2/2) \Box + \kappa \lambda^2 + M^2, \quad (21)$$
$$E = (1 + \kappa \xi^2/2) \Box + M^2. \quad (22)$$

The final form of the propagator is

$$\langle A_\mu A_\nu \rangle = \frac{i}{D} \left\{ \theta_{\mu\nu} + \left( \frac{1}{M^2} \left( \frac{(1 - \kappa \xi^2/2) \Box + M^2)^2 + \kappa \lambda^2 (1 + \kappa \xi^2/2) \Box}{E} \right) \right\} \omega_{\mu\nu}$$
$$- \left( \frac{\kappa \Box}{E} \right) \Lambda_{\mu\nu} + \left( \frac{\lambda \kappa}{E} \right) \Sigma_{\mu\nu} + \left( \frac{\lambda \kappa}{E} \right) \Sigma_{\nu\mu} \right\}. \quad (23)$$

The expression above enables us to set up our discussion on the nature of the excitations, which can be read off as pole propagators, present in the spectrum. At a first sight, the denominator $E$ appearing in connection with the operators $\omega$, $\Lambda$, $\Sigma$, once multiplying the overall denominator $D$, could be the origin for dangerous multiple poles that plague the quantum spectrum with ghosts. For this reason, a careful study of this question is worthwhile. With this purpose, it is advisable to split our discussion into 2 cases: time-like, and space-like $\xi_{\mu}$. In the next section, we carefully analyze these possibilities.

### III. Dispersion Relations, Stability and Causality

In this section, we analyze the causality from a classical perspective (the tree-level), which is based on the positivity of the poles of the propagators in the variable $p^2$, which is the associated momentum. The starting point of our analysis is the propagator, whose poles are associated with the dispersion relations (DR), which provide information on the stability and causality of the model. The analysis of causality is related to the signal of the poles propagator, given in terms of $p^2$, so that we should have $p^2 \geq 0$ to preserve causality (avoid
tachyons). From the viewpoint of second quantization, the stability is related to the states of positive energy in the Fock space for any moment. Here, stability is directly associated with positive energy for each mode that is coming from to the DR. The propagators of the fields, given by expressions. \((D, E)\) present two families of poles in \(p^2\):

\[
(i) D = (1 - \kappa \xi^2/2) (-p_\mu p^\mu) + \kappa \lambda^2 + M^2, \\
(ii) E = (1 + \kappa \xi^2/2) (-p_\mu p^\mu) + M^2.
\]

For \(\xi^\mu = (1; 0, 0, 0)\), when we analyze the dispersion relations: \((i)\) gives us poles, \(p^0 = \pm \sqrt{(2-\kappa)|\vec{p}|^2 + 2M^2}/(2+\kappa)\), and to the \((ii)\) we have \(p^0 = \pm \sqrt{|\vec{p}|^2 + M^2/(1+\kappa/2)}\). We note that if we make the substitution to \(\vec{p} \rightarrow -\vec{p}\), the dispersion relations keep the same. This behavior implies that we do not have birefringence. To avoid tachyonic modes we have \(\kappa \epsilon(-2, 2)\).

The case \(\xi^\mu = (0; 0, 0, 1)\), \((i)\) gives us poles \(p^0 = \pm \sqrt{2M^2 + |\vec{p}|^2(2-\kappa)}/(2+\kappa)\), and, for \((ii)\), we have \(p^0 = \pm \sqrt{|p^3|^2 + M^2/(1-\kappa/2)}\).

From these (DR) we see that the condition \(\kappa \epsilon(-2, 2)\) avoid presence of tachyons. Then upon control of the value of \(\kappa\), we are able to preserve causality. As the (DR) above do not exhibit a linear dependence on the component \(p^0\), the theory is not birefringent.

**IV. ANALYSIS OF UNITARITY**

For the analysis of the model at the classical level, we adopt the method of saturating the propagator with external currents. The fact that our model has two sectors (the scalar, and "gauge") implies that we saturate the scalar propagator and "gauge" separately. Thus, we write the propagators saturated as:

\[
SP_{\langle A_{\mu} \rangle} = J^{*\mu} \langle A_{\mu}(k) A_{\nu}(k) \rangle J^{\nu}.
\]

The continuity equation, \(\partial_\mu J^\mu = 0\), in the space of momenta takes the form of: \(k_\mu J^\mu = 0\).

To infer on the physical nature of the simple pole, have to calculate the eigenvalues of the matrix of residues at each of the poles. This will be done in the sequel. We must, without loss of generality, set the external vector as \((\xi^\mu)\) time-like, and space-like. We shall carry out an analysis of the residues by taking \(p^\mu = (p^0; 0, 0, p^3)\) as the linear momentum. The current conservation law also reduces to two the number of terms of the photon propagator which contribute to the calculation of the saturated propagator:
\[ B_{\mu\nu}(k) = -\frac{i}{D} \left\{ g_{\mu\nu} - \left( \frac{\kappa \Box}{E} \right) \Lambda_{\mu\nu} \right\}, \quad (26) \]

\[ SP_{(A_\mu A_\nu)} = J_\mu^\ast(k) \left\{ -\frac{i}{D} \left( g_{\mu\nu} - \left( \frac{\kappa \Box}{E} \right) \Lambda_{\mu\nu} \right) \right\} J_\nu(k), \quad (27) \]

Writing this expression in the space of momenta, we obtain:

\[ SP_{(A_\mu A_\nu)} = J_\mu^\ast(k) \left\{ iB_{\mu\nu} \right\} J_\nu(k), \quad (28) \]

Our present task consists in checking the character of the poles presented in different configurations of \( \xi^\mu \). We pursue our analysis of the residues by taking \( p^\mu = (p^0, 0, 0, p^3) \). To infer about the physical nature of the simple poles, we have to calculate the eigenvalues of the residue matrix for each of these poles. This is done in the sequel.

With \( \xi^\mu = (0; 0, 0, 1) \) space-like, and in the pole : \( p_0 = m_s \), and taking into account the current conservation, we have to study the residues matrix of the form,

\[ res_{p^\rho = m_s} B_{ij}(k) = \frac{i}{m_s} \left( \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & (2 + \kappa) & 0 & 0 \\ 0 & 0 & (2 + \kappa) & 0 \\ 0 & 0 & 0 & (2 + \kappa) + 2 \left( \frac{\kappa \Box}{E} \right) \end{array} \right), \quad (29) \]

and we observe that the only ambiguity in the sign of the matrix terms is in the \( res_{p^\rho = m_s} B_{33} \). Then we have to study the dependence of the sign with \( \kappa \). We call \( W = res_{p^\rho = m_s} B_{33} \) and,

\[ W = (2 + \kappa) + \frac{\kappa \Box}{E} = (2 + \kappa) + \frac{\kappa (p^3)^2 - M^2}{(1 - \kappa/2) (p^3)^2 + M^2}. \quad (30) \]

To make our analysis independent of momentum, we take the limit \( M \to 0 \). Essentially, we are going to the limit in which the condensate begins to show up. The expression of \( W \) in this limit is,

\[ W = \frac{4 - \kappa^2 + 4\kappa}{(2 - \kappa)}, \quad (31) \]

and the interval in which \( W > 0 \) is \( \kappa\epsilon(2 - 2\sqrt{2}, 2) \) or \( \kappa\epsilon(2 + 2\sqrt{2}, +\infty) \). To avoid tachyonic modes, we impose \( \kappa\epsilon(-2, 2) \). Then we select only the interval \( \kappa\epsilon(2 - 2\sqrt{2}, 2) \).
With $\xi^\mu = (1; 0, 0, 0)$ time-like, and in the pole: $p_0^2 = m_\tau^2$, we have,

$$W = (2 + \kappa) + \frac{\kappa (p^3)^2 - M^2}{M^2 + (1 - \kappa/2) (p^3)^2}.$$  \hspace{1cm} (32)

In the limit $M \to 0$, we have

$$W = \frac{4 - \kappa^2 + 2\kappa}{2 - \kappa},$$

and the valid interval in which $W > 0$ is $\kappa \epsilon (1 - \sqrt{3}, 2)$. We have to make $(1 - \sqrt{3}, 2) \cap (2 - 2\sqrt{2}, 2) = (2 - 2\sqrt{2}, 2)$. Then the interval of validity, for a while, is $\kappa \epsilon (2 - 2\sqrt{2}, 2)$.

This preliminary study done in this section establishes the domain of validity of $\kappa$, avoiding ghosts and tachyons. In the next section, we shall study whether this model can provide vortex solutions obeying the restriction of the $\kappa \epsilon (2 - 2\sqrt{2}, 2)$.

V. A DISCUSSION ON VORTEX-LIKE CONFIGURATIONS

Once our discussion on the consistency of the quantum-mechanical properties of the model has been settled down, we would like to address to an issue of a classical orientation, namely, the reassessment of vortex-like configurations in the presence of Lorentz-breaking term as the one we tackle here.

In our case, with the $K_{\mu\nu\kappa\lambda}$ term included, we get, from the action (8), the equations of motion

$$D^\mu D_\mu \varphi = -m^2 \varphi - 2\lambda \varphi |\varphi|^2,$$  \hspace{1cm} (33)

and

$$\kappa \xi^\nu \xi^\sigma \partial^\rho F_{\rho\sigma} - \kappa g^{\mu\rho} \xi^\mu \xi^\sigma \partial_\mu F_{\rho\sigma} + ie (\varphi \partial^\nu \varphi^* - \varphi^* \partial^\nu \varphi) + 2e^2 A^\nu \varphi^* \varphi = - (1 - \kappa \xi^2/2) (\partial_\mu F^{\mu\nu}),$$  \hspace{1cm} (34)

so that we can explicitly derive the modified Maxwell equations,

$$- \left[ \left( 1 - \kappa \xi^2/2 - \kappa \xi^0 \xi^0 \right) \nabla \cdot - \kappa \lambda \xi^2 \right] E = \kappa \xi^0 \left( \partial^0 \bar{\xi} \cdot \mathbf{E} + \bar{\xi} \cdot \nabla \times \mathbf{B} \right) +$$

$$+ ie \left( \varphi \partial^0 \varphi^* - \varphi^* \partial^0 \varphi \right) + 2e^2 \varphi^* \varphi \Phi,$$  \hspace{1cm} (35)

$$\nabla \times E = - \frac{\partial \mathbf{B}}{\partial t},$$  \hspace{1cm} (36)
and

$$\nabla \cdot \mathbf{B} = 0, \quad (37)$$

$$- (1 - \kappa \xi^2 / 2) (-\partial_0 \mathbf{E} + \nabla \times \mathbf{B}) = \kappa \xi^0 \left( \xi \cdot \nabla \right) \mathbf{E} - \kappa \left( \xi + \kappa \lambda \right) \left( \xi \cdot \nabla \times \mathbf{B} \right)$$

$$- ie (\varphi \nabla \varphi^* - \varphi^* \nabla \varphi) + 2e^2 \mathbf{A} \varphi^* \varphi. \quad (38)$$

We would like to handle the modified Maxwell equations above (eqs. (35)-(38)), before going on to analyze vortex configurations. We need to understand the anisotropy generated by the kind of Lorentz violation we are considering. For this purpose, we remove the charged scalar field and see that the modified Maxwell equations presents the contribution of the fourvector $\xi^\mu$ decomposition. The modified Gauss law is, in the stationary regime,

$$\left( 1 - \kappa \xi^2 / 2 - \kappa (\xi^0)^2 \right) \nabla \cdot E = \kappa \xi^0 \left( \xi \cdot \nabla \times \mathbf{B} \right) + 2e^2 \varphi^* \varphi. \quad (39)$$

To search for the vortex-type solutions, we consider a scalar field in 2-dimensional space

$$\varphi = \chi (r) e^{in\theta}. \quad (40)$$

The asymptotic solution is proposed to be a circle ($S^1$)

$$\varphi = a e^{in\theta}, \quad (r \to \infty). \quad (41)$$

Asymptotically, the magnetic field is screened and we have

$$\left( 1 - \kappa \xi^2 / 2 - \kappa (\xi^0)^2 \right) \frac{d}{dr} \left( \frac{d}{dr} \Phi \right) - \kappa \lambda |\xi_r| \frac{d}{dr} \Phi - 2e^2 a^2 \Phi = 0, \quad (42)$$

the differential equation do not present a independent term in relation of $\Phi$. Then this equation admit the trivial solution $\Phi = 0$, and this imply that the vortex solution is not charged.

To seek the vortex-type solutions, we assume that the gauge field takes over the form

$$\mathbf{A} = \frac{1}{e} \nabla (n\theta); \quad (r \to \infty), \quad (43)$$

or, in term of its components:

$$A_r \to 0, \quad A_\theta \to -\frac{n}{er}; \quad (r \to \infty). \quad (44)$$
Studying the modified Ampère-Maxwell equation (38) in the stationary regime, and as our vortex solution does not present electrical charge ($E = 0$), we have,

$$-(1 - \kappa \xi^2/2) (\nabla \times \mathbf{B}) = \kappa \xi \left( \left( \nabla \cdot \left( \xi \times \mathbf{B} \right) \right) \right) - \kappa \left( \left( \xi \times \left( \xi \cdot \nabla \right) \mathbf{B} \right) \right) +$$

$$+ie \left( \varphi \nabla \varphi^* - \varphi^* \nabla \varphi \right) + 2e^2 A \varphi^* \varphi. \quad (45)$$

In the case $\xi^\mu = (1; 0, 0, 0)$,

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (rA) \right] - 2e \frac{\chi^2}{(1 - \kappa/2)} \left( \frac{n}{r} + eA \right) = 0, \quad (46)$$

in the approximation $\lim_{r \to \infty} \chi(r) = a$, we obtain the solution,

$$A(r) = -\frac{n}{er} + \frac{c}{e} CK_1 \left( |e| \frac{a}{\sqrt{1 - \kappa/2}} r \right). \quad (47)$$

In the asymptotic limit,

$$\lim_{r \to \infty} A(r) = -\frac{n}{er} + \frac{c}{e} \left( \frac{\pi}{2 |e|} \frac{a}{\sqrt{1 - \kappa/2}} r \right)^{1/2} \exp \left( - |e| \frac{a}{\sqrt{1 - \kappa/2}} r \right) \quad (48)$$

we have the asymptotic solution behavior is governed by $\frac{a}{\sqrt{1 - \kappa/2}} = a'(\kappa)$. In the interval $\kappa \epsilon(2 - 2\sqrt{2}, 2)$, $a'$ is positive, and, in the $\lim_{\kappa \to 2^-} a'(\kappa) = +\infty$. What happens in this limit? As $\kappa$ tends to $2^-$ the value of $a'(\kappa)$ increases, i.e., the condensate is enhanced obliging the vortex to be more confined until it disappears.

To the case $\xi^\mu = (0; 0, 0, 1)$,

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (rA) \right] - 2e \frac{\chi^2}{(1 + \kappa/2)} \left( \frac{n}{r} + eA \right) = 0. \quad (49)$$

The solution is in the asymptotic limit,

$$\lim_{r \to \infty} A(r) = -\frac{n}{er} + \frac{c}{e} \left( \frac{\pi}{2 |e|} \frac{a}{\sqrt{1 + \kappa/2}} r \right)^{1/2} \exp \left( - |e| \frac{a}{\sqrt{1 + \kappa/2}} r \right) \quad (50)$$

$$\kappa \epsilon(2 - 2\sqrt{2}, 2) \quad (51)$$

In the interval $\kappa \epsilon(2 - 2\sqrt{2}, 2)$, $a' = \frac{2a}{\sqrt{2 + \kappa}}$ is positive. When $\kappa$ starts from the value $(2 - 2\sqrt{2})$, and goes up towards 2, we observe that the value of $a'$ decrease. Then, the
vortex penetration increases, since the condensate looses intensity, but the vortex is not completely suppressed by the condensate.

To study the stability of the vortex solution, we calculate the energy associated with this setup and we get the expression:

\[ E = \frac{1}{8} B^2 (4 - \kappa) + \frac{M^2}{2} (A(r))^2 \]  \hspace{1cm} (52)

the domain which we are considering, taking into account the criteria of consistency, is \( \kappa \epsilon (2 - 2\sqrt{2}, 2) \). In such range the energy is positive, then we have a stable solution in the stationary regime.

**VI. CONCLUDING COMMENTS**

Our work primarily makes the study of the quantization consistency of an Abelian model with violation of Lorentz symmetry by the \( K_{\mu\nu\kappa\lambda} \) "tensor" (decomposed in fourvectors \( \xi^\mu \)) contemporarily with the spontaneous breaking of gauge symmetry. Then, in this work we access the Standard Model Extension in the even sector. The analysis carried out with the help of the propagators, derived thanks to an algebra of extended spin operators, reveals that unitarity is preserved if \( \kappa \epsilon (2 - 2\sqrt{2}, 2) \). In this regime we have the foton with three degrees of freedom.

Can such a phase transition produce topological defects? To answer that, we have taken into account the classical vortex-like configurations. The analysis of this defect shows interesting aspects: the interval of \( \kappa \) from the analysis of consistence gives a stable solution. Taking into account the solution \([48]\), as \( \kappa \) tends to \( 2^- \) the value of \( a'(\kappa) \) increases, i. e., the condensate is enhanced by having the vortex be more, and more confined until it fades off. On the other hand, taking the solution \([50]\), as \( \kappa \) tends to \( 2^- \) the value of \( a'(\kappa) \) increases, the vortex becomes more confined, but it does not disappear.

**Acknowledgments**

The authors are grateful to M. M. Ferreira Jr. for very clarifying and detailed discussions.
They also express their gratitude to CNPq for the invaluable financial help.

[1] A. Abrikosov, Sov. Phys. JETP **32**, 1442 (1957); H. Nielsen, P. Olesen, Nucl. Phys. **B 61**, 45 (1973).

[2] Z.F. Ezawa, Quantum Hall Effects, World Scientific (2000).

[3] Quantum Field Theory in Strongly Correlated Electronic Systems (Theoretical and Mathematical Physics), ISBN 3-540-65537-9 Springer-Verlag Berlin Heidelberg New York (1999).

[4] S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (NY) **140**, 372 (1982); Gerald V. Dunne, arXiv:hep-th/9902115.

[5] R. Jackiw and E. J. Weinberg, Phys. Rev. Lett. **64**, 2234 (1990); R. Jackiw, K. Lee, and E.J. Weinberg, Phys. Rev. D **42**, 3488 (1990); J. Hong, Y. Kim, and P.Y. Pac, Phys. Rev. Lett. **64**, 2230 (1990); G.V. Dunne, Self-Dual Chern-Simons Theories (Springer, Heidelberg, 1995).

[6] P.K. Ghosh, Phys. Rev. D **49**, 5458 (1994); T. Lee and H. Ming, Phys. Rev. D **50**, 7738 (1994).

[7] N. Sakai and D. Tong, J. High Energy Phys. 03 (2005) 019; G. S. Lozano, D. Marques, E. F. Moreno, and F. A. Schaposnik, Phys. Lett. B **654**, 27 (2007).

[8] S. Bolognesi and S.B. Gudnason, Nucl. Phys. B **805**, 104 (2008).

[9] V. A. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989).

[10] D. Mattingly, Living Rev. Relativity 8, 5 (2005).

[11] V. A. Kostelecky, CPT and Lorentz Symmetry (World Scientific, Singapore, 2011).

[12] D. Colladay and V. A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); D. Colladay and V. A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).

[13] S.M. Carroll, G.B. Field and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).

[14] A. P. B. Scarpelli, H. Belich, J. L. Boldo, L. P. Colatto, J. A. Helayel-Neto, and A. L. M. A. Nogueira, Nucl. Phys. B, Proc. Suppl. 127, 105 (2004); H. Belich, T. Costa-Soares, M. M. Ferreira, Jr., J. A. Helayel-Neto, and M. T. D. Orlando, Int. J. Mod. Phys. A 21, 2415 (2006); H. Belich, E. O. Silva, M. M. Ferreira, Jr., and M. T. D. Orlando, Phys. Rev. D **83**, 125025 (2011); C. Miller, R. Casana, M. M. Ferreira Jr., and E. da Hora, Phys. Rev. D **86**, 065011 (2012); R. Casana, M. M. Ferreira, E. da Hora, and C. Miller, Phys. Lett. B **718**, 620 (2012).

[15] H. Belich, T. Costa-Soares, M.M. Ferreira Jr, J.A. Helayel-Neto, M.T.D. Orlando, Int. J. Mod.
Phys. A 21, 2415 (2006).

[16] K. Bakke, H. Belich and E. O. Silva, Ann. Phys. (Berlin) 523, 910 (2011); K. Bakke, H. Belich and E. O. Silva, J. Math. Phys. 52, 063505 (2011); K Bakke and H Belich 2012 J. Phys. G: Nucl. Part. Phys. 39 085001.

[17] Q. G. Bailey, V.A. Kosteleck´, Phys. Rev. D 70, 076006 (2004); G. Betschart, E. Kant, and F. R. Klinkhamer, Nucl. Phys. B, 815, 198-214 (2009).

[18] V. A. Kostelecky and M. Mewes, Phys. Rev. Lett. 87, 251304 (2001).

[19] V. A. Kostelecky and M. Mewes, Phys. Rev. D 66, 056005 (2002).

[20] R. Casana, et al., Phys.Rev.D82:125006 (2010); R. Casana, et al., Phys.Rev.D84:045008 (2011).

[21] O.W. Greenberg, Phys. Rev. Lett. 89, 231602 (2002).

[22] H. Belich, J. L. Boldo, L. P. Colatto, J.A. Helayel-Neto, A.L.M.A. Nogueira, Phys.Rev. D 68, 065030 (2003); H. Belich, T. Costa-Soares, J.A. Helayel-Neto, M.T.D. Orlando, R.C. Paschoal, Phys.Lett. A 370,126-130 (2007).

[23] H. Belich, G. S. Dias, J.A. Helayël-Neto, F.J.L. Leal, W. Spalenza, PoS ICFI2010 (2010) 032.