Correction of ‘The Kellogg property and boundary regularity for \( p \)-harmonic functions with respect to the Mazurkiewicz boundary and other compactifications’

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**ABSTRACT**

We fill in a gap in the proofs of Theorems 1.1–1.4 in ‘The Kellogg property and boundary regularity for \( p \)-harmonic functions with respect to the Mazurkiewicz boundary and other compactifications’, to appear in *Complex Var. Elliptic Equ.*, doi:10.1080/17476933.2017.1410799.

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It has come to my attention that there is a gap in the argument showing that every boundary point \( x_0 \in \partial \Omega \) splits nicely with respect to the Mazurkiewicz boundary if \( \Omega \) is as in Theorems 1.1–1.4 in [1] (i.e. \( \Omega \subset X \) is a bounded domain which is finitely connected at the boundary, where \( X \) is a complete metric space equipped with a doubling measure supporting a \( p \)-Poincaré inequality, \( 1 < p < \infty \)). This fact is mentioned after Definition 6.1, tacitly assuming that \( \Phi^{-1}(x_0) \) is at most countable. However, \( \Phi^{-1}(x_0) \) can be uncountable even under the assumptions in Theorems 1.1–1.4 in [1], see Example 7.5 in Björn et al. [2]. Nevertheless, \( x_0 \) does split nicely in this case.

To see this, let \( \hat{x} \in \Phi^{-1}(x_0) \) and let \( V \) be a Mazurkiewicz neighbourhood of \( \hat{x} \). (The Mazurkiewicz metric is always defined with respect to \( \Omega \).) Then there is \( r > 0 \) so that \( V \supset B^M(\hat{x}, 3r) := \{ x \in \overline{\Omega}^M : d_M(x, \hat{x}) < 3r \} \). Let \( G \) be the component of \( \Omega \cap B(x, r) \) which has \( \hat{x} \) in its Mazurkiewicz closure, and let

\[
U = \overline{G}^M \setminus \Phi^{-1}(\{ x \in \overline{\Omega} : d(x, x_0) = r \}),
\]

which is an open Mazurkiewicz neighbourhood of \( \hat{x} \). As \( G \) is connected, we see that \( d_M(x, y) \leq \text{diam}G \leq 2r \) whenever \( x, y \in G \). Thus

\[
B^M(\hat{x}, r) \subset U \subset B^M(\hat{x}, 3r) \subset V.
\]
Moreover, if $x' \in \overline{U}^M \setminus U$, then $d(\Phi(x'), x_0) = r$ and in particular $\Phi(x') \neq x_0$. Hence $x_0$ splits nicely.

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