Does the Berry phase in a quantum optical system originate from the rotating wave approximation?

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The Berry phase (BP) in a quantized light field demonstrated more than a decade ago (Phys. Rev. Lett. 89, 220404) has attracted considerable attentions, since it plays an important role in the cavity quantum electrodynamics. However, it is argued in a recent paper (Phys. Rev. Lett. 108, 033601) that such a BP is just due to the rotating wave approximation (RWA) and the relevant BP should vanish beyond this approximation. Based on a consistent analysis we conclude in this letter that the BP in a generic Rabi model actually exists, no matter whether the RWA is applied. The existence of BP is also generalized to a three-level atom in the quantized cavity field.

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I. INTRODUCTION

Thirty years ago Berry discovered that, when a quantum system varies slowly around a closed loop in a certain parameter space the eigenstate of Hamiltonian will acquire a geometrical phase in addition to the usual dynamic phase. Shortly after, this phase, called usually Berry phase (BP), was generalized to various versions. Now, it is found that geometric phases are related to many physical problems such as Aharonov-Bohm effect, quantum Hall effect, and Born-Oppenheimer approximation, etc. Hopefully, geometric phases play an important role in fault-tolerant quantum computing.

The most BPs investigated previously are in the semiclassical context, namely, the quantum systems are driven by a classical field. In 2002, Fuentes-Guridi et al. generalized the original Berry model of a spin-1/2 particle in a time varying magnetic field to a full quantum counterpart, wherein the classical driving field was replaced by a quantized field. A photon-dependent BP is then found in the usual Jaynes-Cummings (JC) model. Interestingly, such a BP still exists even when the field is at vacuum. The existence of the BP in the two-level atom system has been successfully extended to various models, including multi-atom Dicke model and a multilevel atom in a quantized field.

Surprisingly in 2012 Larson claimed that, the BP in quantum optical system is just a result of the rotating wave approximation (RWA) and should vanish beyond this approximation. Note that this argument is obtained based on a semiclassical Rabi model, wherein the field operator is simply replaced by a complex C-number, and thus the relevant interpretation is practically not related to the quantized light field. Alternatively we present, in this letter, a universal formulation of the BPs for the Rabi model and show that the non-zero BP always exists, no matter whether the RWA is applied.

The general description of the BP in a Rabi model is given in Sec.II. Then the validity of such a BP is verified by investigating the semiclassical counterpart of Rabi model in Sec. III. In Sec. IV we generalize our results to a three-level atom in the quantized light field beyond the RWA. Finally we present the conclusion and discussion in Sec.V.

II. GENERAL FORMULATION OF BP IN RABI MODEL

The interaction of a two-level atom with a single-mode quantized field can be generally described by the Hamiltonian

\[
H_0 = \omega a^\dagger a + \frac{\nu}{2} \sigma_z + \lambda (\sigma_+ a + \sigma_- a^\dagger) + \lambda_{NR} (\sigma_+ a^\dagger + \sigma_- a),
\]

in which \(\sigma^\pm = (\sigma_x \pm i\sigma_y)/2\) are called the pseudospin operators for the two-level atom of the eigenfrequency \(\nu\). \(a^\dagger (a)\) denotes bosonic creation (annihilation) operator of the single-mode quantized field with the frequency \(\omega\). Obviously, when \(\lambda_{NR} = 0\), namely under the usual RWA, the Hamiltonian reduces to that of the JC model. Correspondingly, when \(\lambda_{NR} = \lambda\) it becomes the standard Rabi-model Hamiltonian.

Following Fuentes-Guridi et al. the BP can be obtained in terms of an unitary transformation

\[
U(\varphi) = \exp(-i\varphi a^\dagger a)
\]

applied to the Hamiltonian \(H_0\) such that

\[
H(\varphi) = U(\varphi) H_0 U^\dagger(\varphi) = \omega a^\dagger a^\prime + \frac{\nu}{2} \sigma_z + \lambda (\sigma_+ a^\prime + \sigma_- a^\dagger) + \lambda_{NR} (\sigma_+ a^\dagger + \sigma_- a^\prime),
\]

with \(a^\prime(\varphi) = a \exp(i\varphi)\). The eigenstates \(|\psi_n(\varphi)\rangle\) of the Hamiltonian \(H(\varphi)\) are obtained as \(|\psi_n(\varphi)\rangle = U(\varphi) |\psi_n\rangle\) with \(|\psi_n\rangle\) being the eigenstates of \(H_0\). When the angle variable \(\varphi\) slowly varies from 0 to \(2\pi\) a BP given by
\[ \gamma_n = i \int_0^\pi d\varphi \langle \psi_n(\varphi) \rangle \frac{d}{d\varphi} \langle \psi_n(\varphi) \rangle = 2\pi \langle \psi_n \rangle a^\dagger a \langle \psi_n \rangle \]  

is generated for the eigenstates \( |\psi_n(\varphi)\rangle \). When \( \lambda_{NR} = 0 \), \( H_0 \) reduces to the JC model Hamiltonian with the ground state denoted by \( |\psi_0\rangle = 0 \otimes |g\rangle \). Here, \( |n\rangle \) is the Fock state of \( n \) photons for the field, and \( |g\rangle (|e\rangle) \) is the atomic ground (excited) state. It is seen from Eq. (4) that the BP is zero for the ground state \( |\psi_0\rangle \), but non-zero for any excited state even if the filed is at the vacuum. One can easily prove that the non-zero BP is practically the half of the solid angle subtended by the traversed loop of the eigenstate \( |\psi_n(\varphi)\rangle \) on the relevant Bloch sphere in the basis of \( |n+1\rangle \otimes |g\rangle \) and \( |n\rangle \otimes |e\rangle \). Here, the half solid angle is \( \pi(1 - \cos \theta_n) \), with \( \theta_n \) being the angle between the eigenstate vector and the north axis. Note that \( \theta_n \) is also associated with the coefficient of the eigenstate, and thus the above solid angle can be expressed as \( 2\pi \langle |\psi_n\rangle a^\dagger a |\psi_n\rangle - n \rangle \).

On the other hand, when \( \lambda_{NR} = \lambda \) i.e. the case of Rabi model, any eigenstate can no longer be written as the form of \( |0\rangle \otimes (C_g |g\rangle + C_e |e\rangle) \) due to the existence of the counter rotating wave terms. This implies that, the average photon number of any eigenstate of the Rabi model should not be zero. As a consequence, the BP of any eigenstate induced by the unitary transformation \( U(\varphi) \) is always non-zero according to the Eq. (4). In fact the non-zero BPs have been found for various eigenstates in the Rabi model [12].

### III. APPARENT CONTROVERSY ON BP IN THE SEMICLASSICAL THEORY OF RABI MODEL AND ITS RESOLUTION

In this section we first of all briefly analyze how the claim of vanishing BP [8] comes out for the Rabi model, and then show that it is actually non-zero if the proper semiclassical approximation is made.

#### A. Vanishing BPs as a result of improper semiclassical approximation

Following Ref. [8], we begin with the "semiclassical approximation" of the Hamiltonian Eq. (3) (quotation mark here means improper) i.e.,

\[ H_C(\varphi) = \omega |a|^2 + \frac{\nu}{2} \sigma_z + \lambda (\alpha e^{i\varphi} \sigma_x + \alpha^* e^{-i\varphi} \sigma_-) + \lambda_{NR} (\alpha^* e^{-i\varphi} \sigma_x + \alpha e^{i\varphi} \sigma_-), \]  

which is obtained by directly replacing the bosonic operators \( a \) and \( a^\dagger \) with the complex C-numbers \( \alpha, \alpha^* \) respectively. It can be further written as

\[ H_C(\varphi) = \omega |a|^2 + |\alpha| \cos \phi (\lambda + \lambda_{NR}) \sigma_x + |\alpha| \sin \phi (\lambda_{NR} - \lambda) \sigma_y + \frac{\nu}{2} \sigma_z, \]  

with \( \alpha = |\alpha| e^{i\varphi} \) and \( \phi = \varphi + \varphi' \). Obviously this Hamiltonian is formally equivalent to that of a spin-1/2 particle driven by a magnetic field: \( \vec{B} = (|\alpha| \cos \phi (\lambda + \lambda_{NR}), |\alpha| \sin \phi (\lambda_{NR} - \lambda), \nu/2) \). The eigenstates of the effective Hamiltonian can be found as a spin coherent states \( \pm |\tilde{n}\rangle \), where \( \vec{n} \cdot \vec{\sigma} \pm |\tilde{n}\rangle = \pm |\tilde{n}\rangle \) with \( \tilde{n} \) being the unit vector along the \( \vec{B} \)-direction. Therefore the slow variation of the parameter \( \varphi \) from 0 to \( 2\pi \) corresponds to the eigenstates \( \pm |\tilde{n}\rangle \) traversing a loop on the Bloch sphere and then the eigenstates will acquire BPs given by \( \gamma_\pm = \pm \Omega/2 \), where \( \Omega \) is the enclosed solid angle by the loop. On the other hand if the eigenstates \( \pm |\tilde{n}\rangle \) traverse just an arc but not a closed loop, the relevant geometric phase is zero [3][10].

Specifically when \( \lambda_{NR} = 0 \) the Hamiltonian \( H_C(\varphi) \) becomes the "semiclassical approximation" of the JC model. Its eigenstates are found as

\[ |L_+\rangle = \left( \begin{array}{c} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{-i\phi} \end{array} \right), \quad |L_-\rangle = \left( \begin{array}{c} -\sin \frac{\theta}{2} \\ \cos \frac{\theta}{2} e^{i\phi} \end{array} \right) \]  

where \( \tan \theta = 2 |\alpha| \lambda/\Delta, \Delta = \nu - \omega \). It is seen that, when \( \varphi \) varies slowly from 0 to \( 2\pi \) the eigenstates \( |L_\pm\rangle \) acquire the BPs \( \gamma_\pm = \pm \Omega/2 \) with the solid angle \( \Omega = 2\pi(1 - \cos \theta) \). While for the "semiclassical approximation" of the Rabi model when \( \lambda_{NR} = \lambda \), the ground state becomes

\[ |R_-\rangle = \frac{1}{\sqrt{E_- (2E_- - \nu)}} \left( \begin{array}{c} 2 |\alpha| \cos \phi \\ E_- - \nu/2 \end{array} \right), \]  

with \( E_- = \omega |\alpha|^2 - \sqrt{\nu^2/4 + 4 |\alpha|^2 \cos^2 \phi} \). Since the elements \( 2 |\alpha| \cos \phi \) and \( E_- - \nu/2 \neq 0 \) are both real, the ground state \( |R_-\rangle \) traverses only an arc giving the zero BP when \( \varphi \) varies from 0 to \( 2\pi \). Thus the conclusion of vanishing BP in the Rabi model [8] is recovered and it seems that the non-zero BP in the JC model is just a result of the RWA.

However, this apparent controversy comes entirely from the improper semiclassical approximation used in Ref. [8]. The Hamiltonian \( H_C(\varphi) \) by directly replacing the creation (annihilation) operator \( a^\dagger (a) \) with a complex C-numbers \( \alpha^* (\alpha) \) does not correspond to the original one \( H(\varphi) \) obtained with the unitary transformation. The correct way to achieve the Hamiltonian \( H_C(\varphi) \) is by the sub-space average of \( H(\varphi) \) such that \( H_C(\varphi) = \langle \alpha | H(\varphi) | \alpha \rangle \), where \( |\alpha\rangle \) is the optical coherent state with the usual definition \( a |\alpha\rangle = |\alpha\rangle |\alpha\rangle \). Then one is able to obtain the semiclassical ground state of \( H_C(\varphi) \) with the standard variational method, in which the coherent state \( |\alpha\rangle \) acts as a trial wave function. Once doing so the BP emerges in both the Rabi and JC models independent of the RWA as it should be.

#### B. Variational ground-state of the Rabi model and nonvanishing BP

The proper semiclassical approximation begins with the average of the original Rabi-mode Hamiltonian \( H_0 \) (for \( \lambda_{NR} = \lambda \)
\( \lambda \) in the coherent state \( |\alpha\rangle \), which leads to the following effective spin Hamiltonian
\[
H_e(\alpha) = \langle \alpha | H_0 | \alpha \rangle = \omega |\alpha|^2 + \frac{\nu}{2} \sigma_z + \lambda (\alpha + \alpha^* )\sigma_x.
\] (9)

Its eigenvalues can be obtained by solving the energy eigenvalue equation,
\[
H_e(\alpha) |\psi\rangle = E(\alpha) |\psi\rangle.
\] (10)
The average energy
\[
E_\pm(\alpha) = \langle \psi_\pm | H_e(\alpha) |\psi_\pm \rangle = \omega |\alpha|^2 \pm \sqrt{\frac{\nu^2}{4} + \lambda^2 (\alpha + \alpha^*)^2}
\] (11)
is a function of \( \alpha \), which is considered as a variational parameter to be determined by the variation procedure, here \( |\psi_\pm\rangle \) denotes the two eigenstates of the effective spin Hamiltonian \( H_e(\alpha) \). Consequently the variational ground-state energy can be obtained by minimizing the energy function \( E_\pm(\alpha) \). It is easy to find that only \( E_-(\alpha) \) leads to the true ground state and the extremum condition is
\[
\frac{\partial E_-(\alpha)}{\partial \alpha} = 0,
\] (12)
which gives rise to the average photon number of the semiclassical ground-state
\[
\alpha_{gs} = \begin{cases} 
0, & \lambda \leq \lambda_c \\
\sqrt{\frac{\lambda^2}{\omega^2} - \nu^2/16\lambda^2}, & \lambda > \lambda_c
\end{cases}
\] (13)
and the average energy
\[
E_{gs} = \begin{cases} 
\frac{-\nu}{2}, & \lambda \leq \lambda_c \\
\frac{\lambda^2}{2\omega} - \frac{\nu^2}{16\lambda^2}, & \lambda > \lambda_c
\end{cases}
\] (14)
where
\[
\lambda_c = \frac{1}{2} \sqrt{\omega \nu}
\]
is the well known critical value of the quantum phase transition from the normal to the superradiant phases in the \( N \)-atom Dicke model [6, 9]. Therefore the desired semiclassical ground-state reads
\[
|\chi\rangle = |\alpha_{gs}\rangle |\psi_\perp\rangle = \begin{cases} 
|0\rangle |g\rangle, & \lambda \leq \lambda_c \\
|\alpha_{gs}\rangle (C_- |g\rangle + C_+ |e\rangle), & \lambda > \lambda_c
\end{cases}
\] (15)
with \( C_\pm = \sqrt{2\lambda^2 + \omega \nu}/2\lambda \), and the relevant BP is calculated as
\[
\gamma_{gs} = \begin{cases} 
0, & \lambda \leq \lambda_c \\
2\pi |\alpha_{gs}|^2, & \lambda > \lambda_c
\end{cases}
\] (16)
Obviously the BP of the semiclassical ground-state in Rabi model is non-zero when \( \lambda > \lambda_c \). For a JC model the results are the same except a possible shift of the critical point \( \lambda_c \). The variational result can be verified in a more general sense by numerical diagonalization of the Hamiltonian \( H_0 \) with \( \lambda_{NR} = \lambda \). The numerical values of BP as a function of the dimensionless coupling constant \( g = \lambda/\omega \) are plotted in Fig.1. It may be worthwhile to remark that the average photon-number \( \alpha_{gs} \) obtained from the variational treatment with the coherent state \( |\alpha\rangle \) may have a certain amount of deviation with respect to the accurate results of full quantum mechanical formulation [4, 5], and also to the numerical diagonalization shown in Fig.1. After all, the variational result in the coherent state is just a semiclassical approximation.

![FIG. 1: BP of the ground state in the Rabi model versus the dimensionless coupling parameter \( g = \lambda/\omega \) for different atomic eigenfrequency \( \nu \) and field frequency \( \omega \).](image)

**IV. GROUND-STATE BP OF A THREE-LEVEL_ATOM IN THE QUANTIZED LIGHT-FIELD**

We now generalize the above BP formulation of two-level atom to a three-level atom in the quantized cavity field. Without loss of the generality, we consider the \( \Lambda \)-type atom with the energy eigenvalues \( E_i = \hbar \omega_i \), for \( i = 1, 2, 3 \). The allowed dipole-transitions are assumed to be \( |3\rangle \leftrightarrow |2\rangle \) and \( |3\rangle \leftrightarrow |1\rangle \) but not for \( |1\rangle \leftrightarrow |2\rangle \) [11]. Beyond the RWA, the Hamiltonian of this system reads
\[
H^\Lambda = H_0^\Lambda + \eta (b + b^\dagger) (|1\rangle \langle 3| + |3\rangle \langle 1| + |2\rangle \langle 3| + |3\rangle \langle 2|).
\] (17)

Here,
\[
H_0^\Lambda = \omega_0 b^\dagger b + \omega_1 |1\rangle \langle 1| + \omega_2 |2\rangle \langle 2| + \omega_3 |3\rangle \langle 3|,
\]
and \( \eta \) is the coupling constant between atom and the field of frequency \( \omega_0 \), \( b \) and \( b^\dagger \) denote the corresponding field operators.

Similarly, the average of Hamiltonian \( H^\Lambda \) in the field coherent-state \( |\beta\rangle \langle \beta| \beta \rangle = \beta |\beta\rangle \) becomes an effective
Hamiltonian of atomic operator only

\[ H^\Lambda_e = \langle \beta | H^\Lambda | \beta \rangle \]

\[ = \omega_0 |\beta|^2 + \sum_l \omega_l |l\rangle \langle l| + 2\eta |1\rangle \langle 3| + |3\rangle \langle 1| + |2\rangle \langle 3| + |3\rangle \langle 2|, \]

with the complex number \( \beta = u + iv \) considered as a variational parameter to be determined. For simplicity, let us assume \( \omega_1 = \omega_2 \), and then the energy eigenvalue equation

\[ H^\Lambda |\psi^\Lambda(\beta)\rangle = E^\Lambda(\beta) |\psi^\Lambda(\beta)\rangle, \]

can be solved with the results given by

\[ E^\Lambda_0(\beta) = \omega_0 |\beta|^2 + \omega_1, \]

and

\[ E^\Lambda_{\pm}(\beta) = \omega_0 |\beta|^2 + \frac{\omega_1 + \omega_3}{2} \pm \sqrt{\frac{(\omega_1 - \omega_3)^2}{4} + 8\eta^2 u^2}. \]

One can easily check that the energy function \( E^\Lambda_{\pm}(\beta) \) is the lowest one i.e. \( E^\Lambda_{\pm}(\beta) \leq E^\Lambda_0(\beta), E^\Lambda_0(\beta) \). Therefore, the ground state energy can be determined from the extremum condition \( \partial E^\Lambda(\beta)/\partial \beta = 0 \) and the result is

\[ E^\Lambda_{gs} = \begin{cases} \omega_1, & \eta \leq F \\ \frac{-2\eta^2}{\omega_0} - \frac{\omega_0 (\omega_1 - \omega_3)^2}{32\eta^2} + \frac{\omega_1 + \omega_3}{2}, & \eta > F \end{cases} \]

with the relevant variational parameter found as

\[ \beta_{gs} = \begin{cases} 0, & \eta \leq F \\ \frac{2\eta^2}{\omega_0} - \frac{(\omega_1 - \omega_3)^2}{32\eta^2}, & \eta > F \end{cases} \]

where \( F = \sqrt{\omega_0 (\omega_1 - \omega_3)} / 8 \). Consequently, we apply a unitary transformation \( U(\tau) = \exp(-i\tau b^b) \) to the Hamiltonian \( H^\Lambda \) and obtain

\[ H^\Lambda(\tau) = U(\tau) H^\Lambda U^\dagger(\tau) \]

\[ = H^\Lambda_0 + \eta (b e^{i\tau} + b^b e^{-i\tau}) (|1\rangle \langle 3| + |3\rangle \langle 1| + |2\rangle \langle 3| + |3\rangle \langle 2|). \]

Thus, when \( \tau \) varies slowly from 0 to \( 2\pi \), the semiclassical ground state \( |\psi^\Lambda_{gs}(\tau)\rangle = U(\tau) |\psi^\Lambda_{gs}\rangle \) will acquire a BP evaluated as

\[ \gamma^\Lambda_{gs} = \frac{i}{\hbar} \int_0^\infty d\tau \langle \psi^\Lambda_{gs}(\tau)| \frac{d}{d\tau} |\psi^\Lambda_{gs}(\tau)\rangle \]

\[ = 2\pi \langle \psi^\Lambda_{gs}| b^b b^b |\psi^\Lambda_{gs}\rangle, \]

which leads to the result

\[ \gamma^\Lambda_{gs} = \begin{cases} 0, & \eta \leq F \\ \frac{2\eta^2}{\omega_0} - \frac{(\omega_1 - \omega_3)^2}{32\eta^2}, & \eta > F \end{cases} \]

Again, the BP depends on the coupling constant \( \eta \) as that in the two-level case. As a comparison the BP values are also evaluated by the numerical diagonalization of the Hamiltonian \( H^\Lambda \). Fig. 2 shows the plots of ground-state BPs versus the dimensionless coupling constant \( g' = \eta / \omega_0 \). It is seen that the numerical results are qualitatively in agreement with our semiclassical analysis.

![Ground-state BP in the three-level atom system versus the dimensionless atom-field coupling constant g'](image)

**V. CONCLUSION AND DISCUSSION**

We present in this letter a general formulation of the BPs for both the JC model (with RWA) and the Rabi model (without the RWA). In the semiclassical approximation the variational ground-state and the related BP for both Rabi and JC models are obtained analytically in a consistent manner. We argue that the vanishing BP of the Rabi model \([8]\) is due to the improper semiclassical approximation with simply replacing the bosonic operator by a complex C-number since the resulted Hamiltonian does not correspond to the original one in the semiclassical level. The valid effective spin Hamiltonian \( H_e(\alpha) \) in the semiclassical approximation is achieved by the average in the field coherent state and thus the macroscopic (semiclassical) quantum state should be obtained by means of the standard variational method. We show in this letter that the BP for a generic Rabi model is indeed non-zero. This observation is also been generalized to a three-level atom in the quantized cavity field.

The BP in the variational ground-state, which is a displaced vacuum (i.e. coherent state), may possess a simple interpretation as suggested by one Referee of the paper that the action of the unitary transformation Eq. (2) is to take this coherent state around the origin in phase space and thus the emerging BP is just the one of a harmonic oscillator. Following the Ref. \([7]\), the BPs beyond the RWA, obtained in the present work, should be also verified experimentally, in principle, with the usual cavity quantum electrodynamic system. The analogous experiment to measure the BP has been also designed in a single solid-state spin-quin [21].
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