THE EFFECTS OF ROTATION ON THE MAIN-SEQUENCE TURNOFF OF INTERMEDIATE-AGE MASSIVE STAR CLUSTERS

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ABSTRACT

The double or extended main-sequence turnoffs (MSTOs) in the color–magnitude diagram (CMD) of intermediate-age massive star clusters in the Large Magellanic Cloud are generally interpreted as age spreads of a few hundred Myr. However, such age spreads do not exist in younger clusters (i.e., 40–300 Myr), which challenges this interpretation. The effects of rotation on the MSTOs of star clusters have been studied in previous works, but the results obtained are conflicting. Compared with previous works, we consider the effects of rotation on the main-sequence lifetime of stars. Our calculations show that rotating models have a fainter and redder MSTO with respect to non-rotating counterparts with ages between about 0.8 and 2.2 Gyr, but have a brighter and bluer MSTO when age is larger than 2.4 Gyr. The spread of the MSTO caused by a typical rotation rate is equivalent to the effect of an age spread of about 200 Myr. Rotation could lead to the double or extended MSTOs in the CMD of the star clusters with ages between about 0.8 and 2.2 Gyr. However, the extension is not significant, and it does not even exist in younger clusters. If the efficiency of the mixing were high enough, the effects of the mixing would counteract the effect of the centrifugal support in the late stage of evolution, and the rotationally induced extension would disappear in the old intermediate-age star clusters, but younger clusters would have an extended MSTO. Moreover, the effects of rotation might aid in understanding the formation of some “multiple populations” in globular clusters.

Key words: globular clusters: general – stars: evolution – stars: rotation

1. INTRODUCTION

Since Mackey & Broby Nielsen (2007) and Mackey et al. (2008) discovered the double main-sequence turnoffs (MSTOs) in the color–magnitude diagram (CMD) of intermediate-age star clusters, such as NGC 1846, 1806, and 1783 in the Large Magellanic Cloud (LMC), the phenomenon of the double or extended MSTOs has been discovered in more and more star clusters (Mackey et al. 2008; Glatt et al. 2008; Milone et al. 2009; Girardi et al. 2009; Goudfrooij et al. 2009, 2011b; Keller et al. 2012; Piatti 2013). Some star clusters, such as NGC 1751 in the LMC and NGC 419 (Milone et al. 2009; Girardi et al. 2009; Rubele et al. 2011) in the Small Magellanic Cloud have not only the double or extended MSTOs, but also two distinct red clumps (RCs). Recently, Girardi et al. (2013) found that NGC 411 in the Small Magellanic Cloud also has an extended MSTO. Moreover, \( \omega \) Centauri exhibits a main-sequence (MS) bifurcation (Piotto et al. 2005), and NGC 2808 (Piotto et al. 2007) and NGC 6752 (Milone et al. 2013b) possess a triple MS split. These observations are not consistent with the classical hypothesis that a star cluster is composed of stars belonging to a simple, single stellar population with a uniform age and chemical composition.

The double or extended MSTOs have generally been interpreted as suggesting that the population of star clusters have bimodal age distributions (Mackey & Broby Nielsen 2007; Mackey et al. 2008) or age spreads of about 100–500 Myr (Milone et al. 2009; Girardi et al. 2009; Rubele et al. 2010, 2011; Keller et al. 2012; Piatti 2013). However, Platais et al. (2012) noted that this long period of star formation seems to be at odds with the fact that none of the younger clusters are known to have such a trait. Bastian & Silva-Villa (2013) found that young massive clusters NGC 1856 and 1866 in the LMC do not have such large age spreads, which strongly questions the above interpretation of the age spreads. The age is about 280 Myr for NGC 1856 (Kerber et al. 2007) and \( \sim 160–250 \) Myr for NGC 1866 (Brocato et al. 2003). Furthermore, the age variation cannot be responsible for the MS spread in the young cluster NGC 1844 with an age of about 150 Myr (Milone et al. 2013a). Moreover, an age spread of about 460 Myr is required to explain the dual RCs of NGC 1751 (Rubele et al. 2011), which is two times longer than \( \sim 200 \) Myr for explaining the double MSTOs of NGC 1751 (Milone et al. 2009). The contributions of interactive binary stars to the double MSTOs and dual RCs were studied by Yang et al. (2011). They found that binary interactions and merging can reproduce the dual RCs and extended MSTO in the CMD of an intermediate-age star cluster. However, the fraction of the interactive binary systems is too low to explain the observed properties. In addition, the binary interactions make the stars appear to have a younger age than non-interactive systems, i.e., young stars are in the minority. This seems to be contrary to the findings of Milone et al. (2009) that about 70% of stars belong to the blue and bright (young) MSTO and around 30% to the red and faint (old) MSTO. The prediction of the interactive binaries seems to be contrary to this finding.

The effects of rotation on the structure and evolution of MS stars are mainly of three types: (1) the effect of the centrifugal support leads to a decrease in the effective temperature and luminosity by decreasing the effective gravity. (2) The mixing of elements can result in a decrease in the stellar radius and an increase in the effective temperature by redistributing chemical elements, compared to a non-rotating model at the same age. However, the mixing can also lead to an increase in the stellar radius and a decrease in the effective temperature by enlarging the convective core of stars, compared to the non-rotating model at the same evolutionary state (for example, at the end of the MS). (3) The Von Zeipel effect and the mixing of elements can affect the instability of convection by changing the radiative and
adiabatic temperature gradients. If the convective core increases, 
the lifetime of the MS will be prolonged, and vice versa (Maeder 
1987; Talon et al. 1997; Maeder & Meynet 2000; Yang et al. 
2013a).

The effects of rotation on the MSTO of intermediate-age star 
clusters were first studied by Bastian & de Mink (2009), based 
on stellar models computed with the code described by Heger 
et al. (2000) and Brott et al. (2011a). They found that rotating 
stars have a lower effective temperature and luminosity than non-rotating counterparts and concluded that stellar rotation can 
mimic the effect of a double population. However, Girardi et al. 
(2011) calculated the evolution of rotating models by using 
the Eggenberger et al. (2010) code and obtained that rotating 
models have a slightly hotter and brighter turnover with respect to 
non-rotating ones. They concluded that the rotational effect 
could not explain the presence of the extended MSTO. In the 
Bastian & de Mink (2009) models, the influence of rotation on 
the age of stars was neglected and the effect of the rotationally 
induced mixing may be weaker than that in the Girardi et al. 
(2011) models. This could lead to the fact that their rotating 
models have a lower temperature and luminosity than non-rotating ones in the whole stage of the MS. Moreover, Bastian 
& de Mink (2009) only calculated the evolution of a star with 
the mass of 1.5 $M_\odot$, assuming that the results for the star can be 
applied to others. In fact, the effects of rotation on the structure 
and evolution of stars depend on the mass of stars for a given 
rotation rate (Maeder & Meynet 2000; Yang et al. 2013a). On 
the contrary, however, in the models of Girardi et al. (2011), 
the effects of the mixing may be more efficient, which results 
in the fact that their rotating models have a higher effective 
temperature than non-rotating ones in the late stage of the MS for 
stars of any mass. In the early stage of the MS of the Eggenberger 
et al. (2010) models, the rotating models also exhibit a lower 
effective temperature and luminosity, which mainly results from 
the effect of the centrifugal support. The latest evolutionary 
tracks calculated by Georgy et al. (2013) also show that the 
rotating models have a lower effective temperature compared to 
the non-rotating ones in the early stage of the MS (see their 
Figure 11). If the extent of the effects of mixing in the Girardi 
et al. (2011) models were different, the results might also be 
different. Thus, the effects of rotation on the MSTO need to be 
carefully rechecked.

Furthermore, the observed characteristics of $\omega$ Centauri and 
NGC 2808 can be interpreted as suggesting that a fraction of 
globular cluster stars have sizable helium enhancements over the 
primordial value (Piotto et al. 2005, 2007). In addition, there are 
abundance anomalies in other globular clusters (Gratton et al. 
2004; Milone et al. 2010, 2013a; Keller et al. 2011). In order to 
understand the self-enrichment of globular cluster stars, Ventura 
et al. (2001) and D’Antona et al. (2002) suggested that the low-
mass stars may have been polluted at the surface by accretion 
from the gas that was lost from the evolving intermediate-mass 
asympotic giant branch stars, which requires a timescale of a 
few hundred Myr. However, mass-shedding models from fast-
rotating massive stars (Decressin et al. 2007) or massive binaries 
(De Mink et al. 2009) can enrich the cluster on a timescale of a 
few Myr. The rotational mixing in massive stars (Hunter et al. 
2008; Brott et al. 2011b) and solar models (Yang & Bi 2007; 
Bi et al. 2011) is far from settled. Other mixing processes may 
exist in the Sun and stars (Basu & Antia 2008; Hunter et al. 
2008; Brott et al. 2011b), such as magnetic fields, gravity waves, 
and so on.

The evolutionary tracks of rotating models with different 
rotation rates have been published by several groups (Brott et al. 
2011a, 2011b; Georgy et al. 2013). However, their calculations 
did not cover the mass range between 1.3 and 1.7 $M_\odot$. In this 
paper, we also focus on the effects of rotation on the MSTO of 
intermediate-age massive star clusters. The paper is organized 
as follows: we simply show our stellar evolutionary tracks in 
Section 2, we present the results in Section 3, and we discuss 
and summarize the results in Section 4.

2. EVOLUTIONARY TRACKS

2.1. Evolution Code and Assumptions

We used the Yale Rotation Evolution Code (Pinsonneault 
et al. 1989; Yang & Bi 2007) to compute the evolution of stellar 
models with and without rotation. The input physics and some 
initial parameters are the same as those used in Yang & Bi (2007) 
and Yang et al. (2013a, 2013b). The meridional circulation, the 
Goldreich–Schubert–Fricke instability, and the secular shear 
instability are considered (Endal & Sofia 1978; Pinsonneault 
et al. 1989) in all rotating models. The inhibiting effect of 
chemical gradients on the efficiency of rotational mixing (Mestel 
1953) is also considered in our models and is regulated by the 
parameter $f_\mu$, as described in Pinsonneault et al. (1989). The 
angular momentum transport and the mixing of elements caused 
by magnetic fields (Yang & Bi 2006) and magnetic braking 
(Kawaler 1988; Chaboyer et al. 1995) are only considered in 
stars with a mass less than 1.3 $M_\odot$. Angular momentum loss due 
to magnetic braking and mass loss can be considered negligible 
in A-type stars (MacGregor & Charbonneau 1994; Wolff & Simon 
1997; Zorec & Royer 2012). Thus, in stars with mass larger than 
1.3 $M_\odot$, the angular momentum loss is not taken into account; i.e., 
the total angular momentum is assumed to be conserved in these 
stars. The transport of angular momentum and the mixing of elements are treated as a diffusion process in 
our models (Pinsonneault et al. 1989; Yang & Bi 2006). The 
efficiency of rotational mixing is described by the parameter $f_\epsilon$, 
which is less than unity, and that parameter is used to account 
for the fact that the instabilities mix material less efficiently 
than they transport angular momentum. Usually, the value of $f_\epsilon$ 
is about 0.02–0.03 (Chaboyer & Zahn 1992; Yang & Bi 2006; 
Hunter et al. 2008; Brott et al. 2011b). The value of $f_\epsilon$ and $f_\mu$ 
is 0.0228 and 0.1, respectively, in the Bastian & de Mink (2009) 
models (see Brott et al. 2011b). In this work, the values of $f_\epsilon$ 
and $f_\mu$ are both enhanced, i.e., 0.2 for $f_\epsilon$ and 1.0 for $f_\mu$.

Centrifugal forces reduce the local effective gravity at any 
point not on the axis of rotation, and they are generally not 
parallel to the force of gravity. This leads to the facts that 
equipotential surfaces of rotating stars are no longer spheres and 
the radiative flux is not constant on an equipotential surface. 
These effects are taken directly into account in the equations 
of hydrostatic equilibrium and radiative equilibrium according 
to Kippenhahn & Thomas (1970) and Endal & Sofia (1976), 
as described in Endal & Sofia (1976) and Pinsonneault et al. 
(1989).

In our models, the initial metallicity $Z$ and hydrogen abun-
dance $X$ were fixed at 0.008 and 0.743, respectively. For a given 
mass, the rotating and non-rotating models share the same initial 
parameters except for the rotation rate. The rotating models were 
evolved from the zero-age MS (ZAMS) to the end of the MS, 
assuming the initially uniform rotation of $1.0 \times 10^{-4}$, $1.5 \times 10^{-4}$, 
and $2.0 \times 10^{-4}$ rad s$^{-1}$, which corresponds to an initial period 
($P_0$) of about 0.73, 0.49, and 0.37 days or an initial rotation rate 
($\omega_{\text{init}}$) of about 0.2, 0.3, and 0.4 times the Keplerian rotation rate 
on ZAMS, respectively. The initial period of 0.49 days produces
Figure 1. Evolutionary tracks of rotating and non-rotating models in the Hertzsprung–Russell (H–R) diagram. The value of $P_i$ ($i = 1, 2, 3$) is the initial period on the ZAMS. The notation “◦” shows a position of the models with a given age.

Figure 2. Same as Figure 1. Rotating models with the same initial period (0.49 days) but with different efficiencies of element mixing ($f_c$). The evolution of Rot B with $f_c = 0.2$ was computed from the Wolff ZAMS (Yang et al. 2013b) to the end of the MS.

2.2. Evolutionary Tracks

The evolutionary tracks of rotating and non-rotating models with $M = 1.15, 1.4, 1.5,$ and $2.2 M_\odot$ are shown in Figure 1, where the positions of models with a given age are labeled by the notation “◦.” For the star with $M = 1.15 M_\odot$, magnetic braking leads to the effect that surface rotation velocity decreases rapidly in a few hundred Myr. In this model, the effect of the centrifugal support after the strong magnetic braking is not significant. Thus, the effects of rotation on this star are mainly dominated by the rotational mixing. The rotating model mainly exhibits a higher effective temperature than the non-rotating one at the same age. The effects of rotation on the evolution of stars with mass less than $1.3 M_\odot$ are similar to those on the evolution of $1.15 M_\odot$ stars, and they are more sensitive to the efficiency of the mixing than to the rotation rate (see Figures 1 and 2). For the star with $M = 2.2 M_\odot$, in the early stage of the MS phase the effect of centrifugal support dominates the influences of rotation on the structure and evolution of the star. Therefore, the rotating model has a lower effective temperature and luminosity than the non-rotating one. As the evolution proceeds, however, efficient mixing leads to the fact that the outer edge of the convective core becomes slightly extended, which enhances the effects of mixing (Maeder & Meynet 2000; Yang et al. 2013a). The efficient mixing prolongs the lifetime of the MS by feeding fresh hydrogen fuel into the hydrogen-burning region and makes the rotating models exhibit a higher effective temperature than the non-rotating one at the same age (see Figure 1). However, at the end of the MS, the rotating model exhibits a lower effective temperature and a higher luminosity because the rotating models

surface velocities between about 130 and 180 km s$^{-1}$ for stars with masses between 1.3 and $3.0 M_\odot$ on ZAMS, whose typical velocity is about $150$ km s$^{-1}$ (Royer et al. 2007). Compared with the initial velocity on the ZAMS of $150$ km s$^{-1}$ in the Girardi et al. (2011) models, our initial velocity varies with mass. In the simulation of Bastian & de Mink (2009), the mean of the initial rotation rates is around 0.4 times the Keplerian rotation rate on the ZAMS.
consumed more hydrogen compared to the non-rotating ones. These are consistent with the results of Maeder (1987) and Eggenberger et al. (2010).

For the stars with $M = 1.4$ and $1.5 M_\odot$, in the early stage of the MS, due to the fast rotation and the fact that the gradients of chemical compositions are small, the effect of rotational mixing is not significant, and the effect of the centrifugal support plays a dominant role. Thus, the rotating models have a lower luminosity and effective temperature than their non-rotating counterparts. However, as the evolution proceeds, the rotating models can exhibit a higher luminosity and a lower effective temperature than the non-rotating one at the same age. The effective temperature of the rotating models also can be higher than that of the non-rotating one, depending on the age and rotation rate of the stars.

The effect of the centrifugal support leads to a decrease in the effective temperature and gravity of stars. The decrease of the gravity acts as an effective decrease in the mass of the stars. The mass of the convective core of MS stars decreases with decreasing stellar mass. Therefore, the centrifugal effect results in a decrease in the convective core. The simulation of Julien (1996) also shows that convective penetration may be hindered in rotating stars. Hydrogen is ignited at the same temperature. The smaller the convective core, the less hydrogen can be consumed in the core, and the shorter the lifetime of the MS. Hence, the effect of the centrifugal support leads to an acceleration of the evolution of stars. For example, the rotating model with $M = 1.5 M_\odot$ and $P = 0.49$ days has evolved to the MS hook at the age of 1.5 Gyr. However, the non-rotating counterpart is still on MS with a higher effective temperature and a lower luminosity (see Figure 1), which leads to the effect that the rotating model seems older than the non-rotating one. In fact, however, they have the same age.

The rotational mixing can bring hydrogen fuel into the core from outer layers and transport the products of H burning outward. The helium in the convective core is brought into the radiative region, which leads to an increase in the density $\rho$ at the bottom of the radiative envelope. The adiabatic gradient $\nabla_{ad}$ is proportional to $1/\rho$; thus, the convective core can be slightly increased by the mixing. The increase brings more hydrogen fuel into the core and enhances the effect of mixing. This prolongs the lifetime of the MS of stars and leads to an increase in the effective temperature by increasing the mean density of the stars as compared to the non-rotating model at the same age.

The effects of rotation on the structure and evolution of stars are a result of the competition between the effect of the centrifugal support and that of rotational mixing. For stars with mass larger than $1.3 M_\odot$, due to the absence of magnetic braking, the effect of the centrifugal support plays a dominant role in the early stage of the MS. In the late stage, however, although the effect of the centrifugal support plays an important role, the effects of mixing partly counteract the influences of the centrifugal effect and become even dominant. In the case of the evolution of the rotating model with $M = 1.5 M_\odot$ and $P = 0.37$ days, the effects of mixing counteract the effect of the centrifugal support in the late stage of the MS. Thus, the luminosity and effective temperature of the rotating model approximate those of the non-rotating one at the age of 1.5 Gyr. But before the age of about 1.2 Gyr, the rotating model has an obviously lower effective temperature compared to its non-rotating counterpart. Our calculations show that all stars with $Z = 0.008$ and masses between around 1.3 and $2.0 M_\odot$ have similar characteristics.

We also calculated the evolutionary tracks of rotating models with the initial rotation period of 0.49 days and a normal efficiency of mixing ($f_c = 0.03$ and $f_\mu = 0.1$). Figure 2 shows the evolutionary tracks of models with $M = 1.15$ and $1.5 M_\odot$. By way of comparison, the track of the rotating model without mixing ($f_c = 0$) of elements with other effects of rotation, such as the effect of the centrifugal acceleration, is also plotted in the panel of the star with $M = 1.5 M_\odot$. The lower the $f_c$, the smaller the effects of the mixing. Thus, the rotating models with a small $f_c$ have lower effective temperatures compared to the rotating models with a large $f_c$.

2.3. Comparison with Georgy’s Models

In panel (a) of Figure 3, we compare the equatorial velocity and He abundance of our models with $M = 1.7 M_\odot$ to the velocities of the Georgy et al. (2013) models. The angular momentum loss is not taken into account in our models with mass greater than $1.3 M_\odot$. However, in the models of Georgy et al. (2013), a substantial angular momentum was lost in the early stage of the MS. Thus, the equatorial velocity of our model with $\omega_{ini} = 0.3$ is obviously higher than that of the Georgy et al. (2013) model with $\omega_{ini} = 0.5$, with the result that the effect of the centrifugal support appears to be more significant in our model than in the Georgy et al. (2013) models with the same initial rotation rate.

There is no obvious justification for adopting $f_c = 2$. The change in the mass fraction of the surface He caused by rotational mixing is approximately equal to that of the surface hydrogen in our models, which is consistent with Georgy et al.’s (2013) calculation. The efficient mixing could lead to the surface He enrichment. Panel (b) of Figure 3 shows the evolution of surface He abundance of our models and that of Georgy et al.
The mass fraction of surface He increases from the initial 0.249 to 0.258 at the end of the MS in our rotating models with $f_c = 0.2$. The value of 0.258 is compatible with the value of 0.257 in Georgy’s model. The surface He abundance of rotating models with $f_c = 0.03$ and $f_\mu = 0.1$ is almost unchanged as the evolution proceeds (see panel (b) of Figure 3).

3. ISOCHRONES AND SYNTHETIC RESULTS

3.1. Isochrones

The rotating models of stars with masses between about 1.3 and 2.0 $M_\odot$ have a lower effective temperature than non-rotating ones, which could lead to a spread in color of intermediate-age massive star clusters. Thus, we calculated a grid of evolutionary tracks\(^3\) of rotating and non-rotating models with masses between 1.0 and 3.0 $M_\odot$. The mass interval $\delta M$ is between 0.01 and 0.02 $M_\odot$ for the stars with masses between 1.15 and 2.0 $M_\odot$ but is about 0.1 $M_\odot$ for others. The metallicity $Z$ of evolutionary models was first converted into $[\text{Fe}/\text{H}]$. Then the theoretical properties ($[\text{Fe}/\text{H}]$, $T_{\text{eff}}$, $\log g$, $\log L$) were transformed into colors and magnitudes using the color transformation tables of Lejeune et al. (1998).

Figure 4 shows the CMDs of different isochrones obtained from our evolutionary models. All the rotating models have the same initial rotation period of 0.49 days. The isochrone shown by the dotted (blue) line has been chosen by comparing the MSTOs of a set of isochrones with an age interval of 50 Myr to those of the isochrones of rotating models. The MSTO of rotating models with age $= 1.4$ Gyr is nearly coincident with that of the non-rotating models with age $= 1.6$ Gyr. This indicates that the rotationally induced spread in color is equivalent to the effect of an age spread of about 200 Myr. With the increase or decrease in age, the extension becomes narrower and narrower. For example, when the age increases to 1.7 or decreases to 0.9 Gyr, the extension is equivalent to the effect of an age spread of about 100 Myr. This is because the effect of the centrifugal acceleration is partly counteracted by the effects of mixing. When the age is located between 0.8 and 1.9 Gyr, rotating models exhibit a redder and fainter turnoff with respect to their non-rotating counterparts, which leads to the effect that rotating populations appear to be 100–200 Myr older than non-rotating counterparts in the CMDs. In fact, however, they have the same age. The evolution of rotating models with masses between about 1.3 and 2.0 $M_\odot$ is faster than that of non-rotating ones. Thus, the mass of the turnoff stars of the isochrone of rotating models is slightly less than that of non-rotating ones. Therefore, the MSTO of rotating models is fainter than that of non-rotating ones.

When the age of a star cluster is older than 2.4 Gyr, the rotating models exhibit a bluer and brighter MSTO with respect to their non-rotating counterparts. The rotationally induced spread is equivalent to the effect of an age spread of about 200–400 Myr. When the age of star clusters is younger than 0.6 Gyr, the rotating models also exhibit a bluer and brighter MSTO with respect to their non-rotating counterparts. For example, in a star cluster with an age of 0.4 Gyr, the spread caused by the rotation is similar to the effect of an age spread of about 50 Myr. Bastian & Silva-Villa (2013) found that in NGC 1856 and NGC 1866 the age spread is actually less than 35 Myr. When the mass of stars is larger than about 2 $M_\odot$ or less than about 1.3 $M_\odot$, our rotating models mainly exhibit a higher effective temperature than non-rotating ones at the same age. Thus, the rotating models have a bluer and brighter turnoff compared to their non-rotating counterparts.

\(^3\) The tracks can be obtained via e-mail from Wuming Yang.
The isochrones of rotating models with an initial period of about 0.73 days are shown in Figure 5. When the age of star clusters is less than 0.7 Gyr, the MSTO of isochrones of rotating models does not obviously deviate from those of non-rotating counterparts. When the age of star clusters is located between 0.9 and 2.2 Gyr, the rotating models have a redder and fainter MSTO compared to the non-rotating counterparts. The spreads caused by the rotation are equivalent to the effect of an age spread of about 100–200 Myr. Compared to the extensions caused by the rotation with $P_i = 0.49$ days, these spreads appear in older intermediate-age star clusters, but the results for the star clusters with age larger than 2.6 Gyr are the same.

Figure 6 shows the results obtained from the rotating models with an initial period of 0.37 days. The interesting scenarios are that the spread caused by rotation almost disappears in star clusters with ages between 1.4 and 2.0 Gyr, but a spread similar to the effect of an age spread of about 100–200 Myr still exists in the star clusters with ages between 0.8 and 1.3 Gyr. When the age is less than 0.7 Gyr, the rotating models have a bluer and brighter MSTO compared to non-rotating ones, which is equivalent to the effect of an age spread of about 100 Myr. This is because it is easier for the effects of mixing to play a dominant role in the late stage of the MS of stars for a high rotation rate (see Figure 1). These results are similar to those of Girardi et al. (2011), except for star clusters with ages between 0.9 and 1.2 Gyr. The MSTO of star clusters with an age larger than 2.6 Gyr is similar to that of star clusters with a lower rotation rate.

Comparing the isochrones obtained from rotating models with $f_c = 0.2$ but with different rotation rates, one can find that the largest extension is almost the same, i.e., it is approximately equivalent to the effect of an age spread of about 200 Myr. For the intermediate-age star clusters, with the increase in rotation rate the extension of the MSTO disappears in star clusters with an older age (i.e., about 1.5–2.0 Gyr) due to the fact that the effects of mixing counteract the effects of the centrifugal support. For the star clusters with age larger than 2.4 Gyr, the blue and bright extension of the MSTO is almost not affected by the initial rotation rates because magnetic braking dominates the rotation of stars with mass less than about 1.3 $M_\odot$. For young-age star clusters with ages less than 0.6 Gyr, a blue and bright extension of the MSTO can be produced by high rotation.

The observed extension of the MSTO of intermediate-age star clusters can be equivalent to the effect of an age spread of about 100–500 Myr, which is broader than the extension caused by rotating models with $f_c = 0.2$. Figure 2 shows that the effects of rotation on the evolution of stars are sensitive to the value of $f_c$. Thus, we calculated the rotating models with different rotation rates and a lower efficiency of mixing ($f_c = 0.03$ and $f_\mu = 0.1$). The results of this calculation for $P_i = 0.49$ days are shown in Figure 7. When the age of star clusters is located between 1.4 and 1.7 Gyr, the extension of MSTO caused by rotation is equivalent to the effect of an age spread of about 400 Myr. The extension decreases with increasing or decreasing age. However, the CMD of star clusters with age less than 0.6 Gyr is almost not affected by rotation at all. When the age is larger than 2.6 Gyr, rotating models exhibit a bluer and brighter MSTO compared to non-rotating counterparts, which can be equivalent to the effect of an age spread of about 350 Myr in the cluster with an age of 3.0 Gyr.

The extension of MSTO caused by rotation is mimicked by the effect of an age spread of star clusters, as is shown in Figure 8. The positive spread shows that the isochrone of rotating models has a redder MSTO, compared to that of non-rotating models.
with the same age. The extension changes with the age of star clusters, the initial rotation rate of stars, and the value of $f_c$, and it is more significant in star clusters with age between about 1–2 Gyr than in star clusters with age less than 0.6 Gyr. For $f_c = 0.03$, due to the low efficiency of mixing, the effect of mixing cannot compete with the effect of the centrifugal support in stars with mass between about 1.3 and 2.0 $M_\odot$. Thus, the extension caused by rotation with $f_c = 0.03$ is more significant in star clusters with age between about 1 and 2 Gyr, compared with that caused by rotation with $f_c = 0.2$. In addition, the
extension increases with increasing initial rotation rate because the effect of the centrifugal support increases with the increase in rotation rate (see panels (b), (d), and (f) of Figure 8).

3.2. Monte Carlo Simulation

In order to understand whether or not rotation can produce extended or double MSTOs, we performed a stellar population synthesis by way of Monte Carlo simulations for a total mass of about $3 \times 10^4 M_\odot$ following the lognormal initial mass function of Chabrier (2001). (The mass range of the intermediate-age massive star clusters is in the range of about $(1–20) \times 10^4 M_\odot$; Goudfrooij et al. 2011a.) In order to match the results of Milone et al. (2009), in which about 70% of stars belong to the blue and bright MSTO and around 30% to the red and faint MSTO, we assumed that 70% of stars do not rotate, but that 30% of stars rotate with the same initial period of 0.49 days, though this assumption may not represent the real situation. In our synthesized population, we included observational errors taken to be a Gaussian distribution with a standard deviation of 0.01 and 0.015 in magnitude and color as in Bastian & de Mink (2009), respectively. The deviation of 0.015 in $V-I$ corresponds to a deviation of about 90 K in effective temperature in our models.

The CMDs of the synthesized star clusters with different rotation rates are shown in Figure 9. The double MSTOs can be clearly seen in the CMDs of the star clusters with age = 1.2 and 1.5 Gyr, in which the hot and bright MSTO is dominant. A continuously extended MSTO exists in the star cluster with age = 0.9 Gyr, where the hot and bright MSTO is also dominant. Compared with the extension of the MSTO of star clusters with ages between 0.9 and 1.6 Gyr, that of the star clusters with age = 0.4 and 1.7 Gyr is not obvious. Moreover, there is almost no extension of the MSTO of the star cluster with age = 2.0 Gyr. For the star clusters with age = 2.6 and 3.0 Gyr, their MSTO is also extended by the effects of rotation, but the cool and faint MSTO is dominant. The difference in the $V-I$ between the isochrone of rotating models and that of non-rotating ones is less than 0.01 for the star cluster with the age of 0.4 Gyr. Thus, the extension of the MSTO in this cluster is dominated by the deviation of 0.015 in color. However, the difference is about 0.04 for the star cluster with an age of 1.2 Gyr. The spreads in star clusters of ages between 1.2 and 1.5 Gyr are mainly due to rotation.

The CMDs of the synthesized star clusters with different rotation rates and $f_c$ are shown in Figures 10, 11, and 12, where the double or extended MSTOs can be clearly seen in the CMD of star clusters with age between about 0.8 and 2.2 Gyr. The CMD of NGC 1806 has double MSTOs, and its age is about 1400–1600 Myr (Milone et al. 2009). The initial rotation period of stars in a star cluster should be different. For simplicity, we assumed that the population of a star cluster is composed of 70% non-rotating stars and 30% rotating stars, which is obtained by interpolating between our rotating models with
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Figure 9. CMDs of the synthetic star clusters at different ages. The total mass is assumed to be of about $3 \times 10^4 M_\odot$, following a Chabrier (2001) lognormal initial mass function. We assumed 30% of stars rotating with $P_0 = 0.49$ days and $f_c = 0.2$. The solid (green) line shows the isochrone of non-rotating models. The position of the model with $M = 1.15 M_\odot$ is marked by a slash. The typical errors in magnitude and color are 0.01 and 0.015, respectively.

different rotation rates, assuming that the initial rotation rate has a Gaussian distribution with a peak at about 0.3 times the Keplerian rotation rate on ZAMS and a standard deviation of 0.06. However, this assumption may not represent the real distribution. For example, the distributions of rotation rate of A0- to A1-type stars are flatter (Royer et al. 2007). A distance modulus of 18.45 and $E(m_{F435} - m_{F814})$ of 0.14 are adopted in this simulation, and the simulated results are shown in Figure 13, where the double MSTOs can be seen. The simulated rotation population reproduces well the extension of the MSTO of NGC 1806 observed by Milone et al. (2009). The age of the theoretical population is 1.4 Gyr, and the value of $f_c$ is 0.2.

4. DISCUSSION AND SUMMARY

4.1. Discussion

According to the studies of Wolff et al. (2004), Zorec & Royer (2012), and Yang et al. (2013b), the ZAMS models of A-type stars should be a Wolff ZAMS model, which is required for the study of the evolution of surface velocity and the transport of angular momentum (Yang et al. 2013b). For simplicity in computation, we adopted the ZAMS model with uniform rotation. The rotating models computed in accord with the Wolff ZAMS model also exhibit a lower effective temperature and higher luminosity with respect to non-rotating ones in the late stage of the MS of stars with masses between 1.3 and 2.0 $M_\odot$. This is consistent with the results obtained from the evolution of the ZAMS model with a uniform rotation (see Figure 2). Thus, our results about the effects of rotation on the CMDs cannot be significantly changed by the ZAMS model.

Our ZAMS models with mass larger than 2.0 $M_\odot$ have had a slight gradient of interior elements as compared with lower mass stars, in which the gradient can be completely neglected. Thus, the mixing of elements takes place at the beginning of evolution for the stars with mass larger than 2.0 $M_\odot$, but it acts in a later time for stars with mass less than 2.0 $M_\odot$. This leads to the conclusion that the early evolution of the MS of stars with mass less than 2.0 $M_\odot$ is readily dominated by the effect of centrifugal acceleration. Moreover, for a given rotation rate, the coefficient of the diffusion caused by rotation increases with increasing mass, i.e., the efficiency of the mixing increases with increasing mass. These lead to the result that there is a critical
mass $M_c$ for the effects of rotation on the structure and evolution of the stars. For stars with a mass larger than $M_c$, the effect of the mixing soon exceeds the centrifugal effect in the early stage of the MS. For stars with masses between about 1.3 $M_\odot$ and $M_c$, however, the effect of the centrifugal acceleration is dominant for a long time during the MS stage. In our models, the critical mass is about 2.0 $M_\odot$, which is close to the critical mass of the He flash of Yang et al. (2012), and it could be affected by the mixing processes. Thus, the value of about 2.0 $M_\odot$ is debatable.

In rotating stars, meridional circulation is an advection process. The rotational mixing could be much more complex than that described by a diffusion process. The value of 0.2 for $f_c$ in our models is obviously higher than that calibrated against the nitrogen abundance in massive stars (0.0228) (Hunter et al. 2008; Brott et al. 2011b) or against the solar model (0.03) (Yang & Bi 2006). There may be other mixing mechanisms undescribed by the present model in stars and sun (Hunter et al. 2008; Basu & Antia 2008), such as gravity waves, which are efficient at transporting angular momentum (Zahn et al. 1997; Talon & Charbonnel 2005). The value of 0.2 for $f_c$ corresponds to a high efficient mixing in stars, which results in a higher mean density for stars with mass less than 2.0 $M_\odot$ when they approach the end of MS, i.e., these models with a high efficient mixing have a lower radius. This helps explain the evolution of surface velocity of the stars with mass less than 2.0 $M_\odot$ (Zorec & Royer 2012; Yang et al. 2013b). However, if the mixing is too efficient, this would introduce an amount of hydrogen into the burning region of the star, with a result that the extension of the MSTO caused by rotation would disappear. Our current understanding of the mixing processes could be limited. The mixing in stars is debatable.

The rotationally induced extension of MSTO does not exist in the simulated clusters with age between 1.4 and 2.0 Gyr when the initial rotation period is 0.37 days, which is similar to the results of Girardi et al. (2011). In addition, the extension also disappears in the young clusters with age less than 0.7 Gyr and $P_0 = 0.73$ days. If we took a fixed rotation velocity $V_0$ as the initial rotation parameter for all models, the high-mass stars would have a lower rotation rate, but the low-mass stars would have a higher rotation rate. In that scenario, the result might have been that rotation does not obviously affect the MSTO of star clusters. Moreover, the efficiency of mixing would be different between our models and the Girardi et al. (2011) models because there is no adjustable parameter $f_c$ in the Geneva stellar evolution code (Maeder & Meynet 2000). The different initial rotation parameter and efficiency of mixing would lead to differences.

Figure 10. Same as Figure 9, but the initial period of rotating population here is 0.73 days.
between our results and those of Girardi et al. (2011). In addition, if Girardi et al. (2011) considered the angular momentum loss as Georgy et al. (2013) did for stars with mass larger than 1.3 $M_\odot$, the effect of mixing would be easier to exhibit in their models than the effect of the centrifugal support, i.e., their models would readily exhibit a higher effective temperature compared with the non-rotating counterparts.

Compared, moreover, with the Bastian & de Mink (2009) models, we calculated a dense grid of evolutionary tracks of rotating models and considered the effects of rotation on the lifetime of the MS in our isochrones and simulations. Our results are similar to those of Bastian & de Mink (2009), i.e., the MSTO of star clusters with ages between about 0.8 and 2.2 Gyr can be extended by rotation. However, in our models, the extension is dependent on rotation rate, the efficiency of mixing, and the age of star clusters.

In the 16 star clusters studied by Milone et al. (2009; see their Table 3), the star clusters with ages between 950 and 1400 Myr have an extended MSTO of about 150–250 Myr, but the star clusters with age larger than 1550 Myr have a smaller extension ($\Delta$age $< 100$ Myr). This seems to be similar to our result that rotationally induced extension decreases with the increase or decrease in the age of star clusters. It seems, moreover, to be more similar to the results obtained from high-efficiency models than to those obtained from low-efficiency models.

The extension of the MSTO caused by rotation is significantly affected by the efficiency of mixing and decreases with increasing efficiency. The star cluster SL 529, with an age of about 2.0 Gyr, has an apparent age spread of about 500 Myr (Piatti 2013). The rotationally induced extension of the MSTO of a star cluster with an age of 2.0 Gyr is similar to the effect of an age spread of about 350 Myr in our models with $f_c = 0.03$ and $P_\per = 0.37$ day. While the age of star clusters decreases to 1.7 Gyr, the extension is equivalent to the effect of an age spread of about 450 Myr (see panel (f) of Figure 8). The spread of star cluster SL 529 seems not to be conflicting with our results.

Stars with different rotational parameters and masses could evolve to the stage of the core helium burning at the same time. Thus, the effects of rotation should contribute to the formation of the dual RCs. Our current rotating models cannot be computed to the core-He-burning stage. Thus, we do not obtain the effects of rotation on the RC stars.

In addition, the MSTO of old-age star clusters also can be extended and the surface He abundance of stars can be changed by the effects of rotation. Thus, the effects of rotation might have

**Figure 11.** Same as Figure 9, but the initial period of rotating population here is 0.37 days.
some contributions to the formation of “multiple populations” in globular clusters.

### 4.2. Summary

For stars with masses between about 1.3 and 2.0 $M_\odot$, the effect of the centrifugal support plays a dominant role in the early stage of evolution, which leads to the fact that rotating models have a lower effective temperature and evolve faster than non-rotating ones. As the evolution proceeds, however, the effect of rotational mixing partly counteracts the influence of the centrifugal acceleration and even dominates the effects of rotation on the structure and evolution of stars. As a consequence, the MSTO of intermediate-age star clusters with ages between 0.8 and 2.2 Gyr can extend redward, but the extension decreases with increasing or decreasing age. In rotating stars with mass less than 1.3 $M_\odot$, the effect of rotational mixing plays a dominant role during the MS stage, which results in the fact that rotating models have a higher effective temperature than non-rotating ones at the same age. Thus, the MSTO of the older star clusters extends blueward. The evolution of rotating stars with mass larger than 2.0 $M_\odot$ is mainly affected by the effect of rotational mixing, which depends on rotation rate and the efficiency of mixing. Therefore, the MSTO of young star clusters can extend slightly blueward or cannot extend at all, dependent, likewise, on the rotation rate and efficiency of the mixing.

In this work, we calculated a grid of evolutionary tracks of rotating and non-rotating stars with $Z = 0.008$ and masses between 1.0 and 3.0 $M_\odot$. For a given rotation rate, the effects of rotation on the structure and evolution of the stars are dependent on their mass and age. For the initial rotation period of 0.49 days and the efficiency of mixing of 0.2, rotating models have a fainter and redder turnoff with respect to their non-rotating counterparts when age is located between about 0.8 and 1.9 Gyr. When the age is larger than 2.4 Gyr, rotating models have a brighter and bluer turnoff with respect to the non-rotating ones. However, when the age is less than 0.6 Gyr, the rotation hardly affects the MSTO of the star clusters. The extension of the MSTO caused by the rotation can be equivalent to an age spread of about 200 Myr when the age of the star clusters is located between about 1.2 and 1.6 Gyr. The extension decreases with a decrease or increase in age (see panel (c) of Figure 8). When the initial rotation period decreases to about 0.37 days, the extension of the MSTO caused by the rotation disappears in star clusters with ages between about 1.4 and 2.0 Gyr (see panel (e) of Figure 8).
Young star clusters with age less than 0.7 Gyr have a blueward extended MSTO, but star clusters with ages between 0.8 and 1.3 Gyr still have a redward extended MSTO. When the initial rotation period increases to about 0.73 days, the MSTO of star clusters with age less than 0.7 Gyr cannot be affected by rotation, but the MSTO of star clusters with ages between 0.8 and 2.2 Gyr or larger than 2.4 Gyr. The rotation seems not to be able to result in the extension of the MSTO of star clusters with age less than 0.6 Gyr. At least, the extension of the MSTO of these star clusters is not significant, as compared to that of intermediate-age star clusters. Considering the difference in the initial rotation rates and the efficiency of mixing, the redward extended MSTO may be able to be more easily observed in star clusters with ages between about 0.9 and 1.5 Gyr.

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Figure 13. CMDs of the star cluster NGC 1806. The red points indicate the observed data (Milone et al. 2009). The green points show the simulated data. A distance modulus of 18.45 is adopted. The value of 0.14 is adopted for $E(m_{F435} - m_{F814})$, which is larger than the 0.09 given by Milone et al. (2009). The blue and green lines in the top panel show the isochrones of non-rotating and rotating models, respectively, with an age of 1.4 Gyr. The jump of the green line at $m_{F435} \approx 22.1$ is not significant in $(V-I)$ (see Figure 4) and is related to the absence of magnetic braking in stars with $M > 1.3 M_{\odot}$. The simulated population shown in the middle panel does not include rotating models, but those shown in the bottom panel include a 30% rotating population obtained by interpolating between rotating models with different rotation rates.
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