Abstract

Model-based reinforcement learning (RL) methods can be broadly categorized as global model methods, which depend on learning models that provide sensible predictions in a wide range of states, or local model methods, which iteratively refit simple models that are used for policy improvement. While predicting future states that will result from the current actions is difficult, local model methods only attempt to understand system dynamics in the neighborhood of the current policy, making it possible to produce local improvements without ever learning to predict accurately far into the future. The main idea in this paper is that we can learn representations that make it easy to retrospectively infer simple dynamics given the data from the current policy, thus enabling local models to be used for policy learning in complex systems. To that end, we focus on learning representations with probabilistic graphical model (PGM) structure, which allows us to devise an efficient local model method that infers dynamics from real-world rollouts with the PGM as a global prior. We compare our method to other model-based and model-free RL methods on a suite of robotics tasks, including manipulation tasks on a real Sawyer robotic arm directly from camera images.

1 Introduction

Model-based reinforcement learning (RL) methods use learned models in a variety of ways, such as planning \cite{24} and generating synthetic experience \cite{38}. We can categorize model-based algorithms as either global model methods, where models are used for planning and trained to give accurate predictions for a wide range of states, or local model methods, where simple models provide gradient directions that are used for policy improvement. On simple, low-dimensional tasks, model-based approaches have demonstrated remarkable data efficiency, learning policies for systems like cart-pole swing-up with under 30 seconds of experience \cite{10} \cite{28}. However, for more complex systems, one of the main difficulties in applying model-based methods is model bias: local models will often underfit complex systems, but may still be preferred over global models which tend to overfit in the low-data regime and may be difficult to incorporate into control methods.

In this work, we propose a method that uses representation learning to mitigate model bias for local model methods. Recent results in representation learning have shown that policy learning algorithms, including model-based RL, can benefit from utilizing learned latent features of complex systems \cite{23}, e.g., condensing image observations into lower dimensional spaces. However, representation learning

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Figure 1: (a) A pictoral depiction of a trajectory for a one-dimensional system. (b) Global models may be used for prediction or planning forward through time, as depicted in red, but this can suffer from trajectory drift for complex systems. (c) Local linear models are fit to trajectories and do not suffer from drift, but may fit the system poorly for complicated interactions such as contacts, as illustrated by the poor model fit circled in gray. (d) Our method finds an embedding of observed trajectories into a latent space where local linear models produce a better fit.

for dynamical systems is typically aimed at improving reconstruction or forward prediction [18, 2, 41, 43], which largely benefits model-free RL and global model methods. Instead, in this work, we are specifically trying to learn representations that enable local model methods, in order to maximally combat model bias and provide for general, efficient model-based RL algorithms.

Most global model methods use the model to make forward predictions and then backpropagate through those predictions. However, this places a heavy burden on the dynamics model, and forward prediction often suffers from significant drift over longer trajectories. In contrast, local models are typically only used to provide gradient directions for local policy improvement [24], and thus a common choice for local model methods is to use linear models, which can themselves be interpreted as gradients. As illustrated in Figure 1 in our work, we present a method that automatically encourages learning representations where linear models better fit the data. From this, we devise an efficient local model method based on the linear-quadratic regulator (LQR) [8, 40, 24] that utilizes linear models for gradient directions for policy improvement. Our motivation is similar to that of prior works [43, 11]: however, as discussed in Section 5, our representation learning method specifically allows us to construct a local model method that performs inference in the latent space in order to improve the policy, rather than focusing on forward prediction and planning.

Our main contribution is a representation learning and model-based RL procedure, which we term stochastic optimal control with latent representations (SOLAR), which jointly optimizes a latent representation and model such that inference produces local linear models that provide good gradient directions for policy improvement. We demonstrate empirically in Section 6 that SOLAR is able to learn policies directly from raw, high-dimensional observations in several robotic environments including a simulated nonholonomic car, a simulated two degree-of-freedom (DoF) arm, and a real 7-DoF Sawyer arm, all of which are learned directly from image pixels. We compare to existing state-of-the-art RL methods and show that SOLAR, while significantly more data efficient than model-free methods, exhibits superior performance compared to other model-based methods.

2 Preliminaries

We first formalize our problem setting as a Markov decision process (MDP) $M = (S, A, p, C, \rho, T)$, where the state space $S$, action space $A$, and horizon $T$ are known, but the dynamics function $p(s_{t+1}|s_t, a_t)$, cost function $C(s_t, a_t)$, and initial state distribution $\rho(s_0)$ are unknown. The goal of reinforcement learning is to optimize a policy $\pi(a_t|s_t)$ to minimize the expected sum of costs $\eta[\pi] = \mathbb{E}_{\pi, p, \rho} \left[ \sum_{t=0}^{T} C(s_t, a_t) \right]$ under the distribution induced by the initial state distribution, dynamics function, and policy. Model-based methods decompose this problem into policy and model optimization subproblems, and we discuss each subproblem as it relates to our approach.

2.1 Model-based policy search

Policy search methods directly optimize parameterized policies with respect to $\eta(\theta) \triangleq \eta[\pi(\theta)]$ where the parameters $\theta$ may be, for example, weights in a neural network or matrices for a linear policy. Model-based policy search methods typically build models $\left(\hat{\rho}, \hat{\rho}, \hat{C}\right)$ of the unknown quantities and
compute the gradient of \( \tilde{\eta}(\theta) \triangleq \mathbb{E}_{\pi_{\theta}, \tilde{\rho}, \tilde{\gamma}} \left[ \sum_{t=0}^{T} \tilde{C}(s_t, a_t) \right] \) with this model. One particularly tractable model is the linear-quadratic system (LQS), which models the initial state distribution as Gaussian, the dynamics as time-varying linear-Gaussian (TVLG), and the cost as quadratic, i.e.,

\[
\tilde{\rho}(s_{t+1} | s_t, a_t) = \mathcal{N} \left( s_{t+1} \mid F_t \begin{bmatrix} s_t \\ a_t \end{bmatrix}, \Sigma_t \right), \quad \tilde{C}(s_t, a_t) = \frac{1}{2} s_t^\top C_s s_t + c^\top s_t.
\]

Any deterministic policy operating in an environment with smooth dynamics can be locally modeled with a time-varying LQS [6], while low-entropy stochastic policies are modeled approximately. This makes the time-varying LQS a reasonable local model even for complex dynamical systems. Furthermore, the optimal policy at any time step given the model is a linear function of the state and the optimal maximum-entropy policy is linear-Gaussian [39, 25]. As shown in prior work [16, 40], these optimal policies can be computed in closed form using dynamic programming by computing the first and second derivatives of the Q (cost-to-go) and value functions:

\[
Q_{\tilde{s}, t} = c_{\tilde{s}, t} + F_{\tilde{s}, t}^\top V_{s,t+1}, \quad Q_{s\tilde{s}, t} = C_{s\tilde{s}, t} + F_{s\tilde{s}, t}^\top V_{ss,t+1} F_{s\tilde{s}, t},
\]

\[
V_s, t = Q_{s, t} - Q_{sa, t} Q_{aa, t}^{-1} Q_a, t \quad V_{ss, t} = Q_{ss, t} - Q_{sa, t} Q_{aa, t}^{-1} Q_{as, t}.
\]

Here, similar to prior work [39, 24], we use subscripts to denote derivatives, and we use \( \tilde{s} \) to abbreviate \( s \). Once these values are computed, the optimal maximum-entropy policy is TVLG, i.e.,

\[
\pi_0(a_t | s_t) = \mathcal{N}(K_t s_t + k_t, S_t), \quad \text{where} \quad K_t = -Q_{aa,t}^{-1} Q_{as,t}, \quad k_t = -Q_{aa,t}^{-1} Q_{a,t}, \quad S_t = -Q_{aa,t}^{-1}.
\]

We refer the reader to Section [1] of the appendix and previous work for further details [24].

### 2.2 Learning latent dynamics models

The local model-based method described above requires us to learn both a quadratic cost function as well as a linear dynamical system (LDS). While in principle we can do this for any environment, complex real-world systems are often not accurately explained by the LQS model, resulting in poor policy performance. To help mitigate this, we utilize the Bayesian LDS model, which is given by

\[
\mu_{\tilde{\rho}}, \Sigma_{\tilde{\rho}} \sim \mathcal{N}\mathcal{I}W(\Psi, \nu, \mu_0, \kappa), \quad F, \Sigma \sim \mathcal{M}\mathcal{N}\mathcal{I}W(\Psi, \nu, M_0, V)
\]

\[
s_0 | \mu_{\tilde{\rho}}, \Sigma_{\tilde{\rho}} \sim \mathcal{N}(\mu_{\tilde{\rho}}, \Sigma_{\tilde{\rho}}), \quad s_{t+1} | s_t, a_t \sim \mathcal{N} \left( F \begin{bmatrix} s_t \\ a_t \end{bmatrix}, \Sigma \right) \quad \text{for} \ t \in [0, \ldots, T - 1],
\]

Where \( \mathcal{N}\mathcal{I}W \) is the normal-inverse-Wishart distribution and \( \mathcal{M}\mathcal{N}\mathcal{I}W \) is the matrix normal-inverse-Wishart (MNIW) distribution. This probabilistic graphical model (PGM) allows for tractable approximate inference, i.e., Bayesian linear regression, and also captures uncertainty in the form of a posterior distribution over the initial state and dynamics. However, for dynamical systems with complex non-linear dynamics, this model still suffers from significant bias.

Even when the system is poorly modeled by an LDS in the state space, we might be able to find a latent embedding and model the system as approximately linear in that latent space, which may allow us to find a better-performing policy that operates in the learned latent space. This shifts our problem setting to that of a partially observed MDP, as we do not observe the latent state. We can jointly train an embedding and model using the SVAE framework [17], which allows us to combine arbitrary embedding functions, such as neural networks, with PGMs. The model we build off of is given by

\[
\mu_{\tilde{\rho}}, \Sigma_{\tilde{\rho}} \sim \mathcal{N}\mathcal{I}W(\Psi, \nu, \mu_0, \kappa), \quad F, \Sigma \sim \mathcal{M}\mathcal{N}\mathcal{I}W(\Psi, \nu, M_0, V),
\]

\[
z_0 | \mu_{\tilde{\rho}}, \Sigma_{\tilde{\rho}} \sim \mathcal{N}(\mu_{\tilde{\rho}}, \Sigma_{\tilde{\rho}}), \quad z_{t+1} | z_t, a_t \sim \mathcal{N} \left( F \begin{bmatrix} z_t \\ a_t \end{bmatrix}, \Sigma \right) \quad \text{for} \ t \in [0, \ldots, T - 1],
\]

\[
s_t | z_t \sim f_\gamma(z_t) \quad \text{for} \ t \in [0, \ldots, T],
\]

Where \( f_\gamma(z) \) is an observation model, parameterized by neural network weights \( \gamma \), that outputs a distribution over \( s \), e.g., Gaussian or Bernoulli, depending on the nature of the data. This is very similar to the Bayesian LDS, except we are learning the PGM in the latent space.

Though this model does not admit the same efficient approximate inference algorithms when \( f_\gamma \) is non-linear, an efficient variational inference algorithm has previously been derived for the LDS SVAE [17, 44]. We describe the relevant aspects of this algorithm in the next section.
3 Learning and Modeling the Latent Space

In this section, we describe how we extend the LDS SVAE for model-based RL, such that we learn an action-conditioned LQS model in the latent space. This then enables a local model method that can leverage the LQS to infer the dynamics of sampled trajectories. In this way, our model-based RL algorithm circumvents the need for forward prediction, in contrast to model-based RL methods that use model-based rollouts or planning [43, 30, 10]. In Section 4, we describe how these components are combined through an alternating optimization into our final method, SOLAR.

Our goal with this model is to learn a latent representation of the state and a prior over the dynamics in this latent representation that is suitable for fitting local dynamics models via posterior inference. Specifically, we are interested in the setting where we have access to trajectories of the form \([s_0, a_0, c_0, s_1, a_1, c_1, \ldots, s_{T-1}, a_{T-1}, c_{T-1}, s_T]\), sampled from the system using our current policy and set of previous policies. Our aim is to infer local linear dynamics in the neighborhood of these trajectories, and we learn a model that makes this fitting process more accurate for the observed trajectories, thus enabling our local model method to find good directions for policy improvement.

We build upon the variational inference algorithm presented in [17], such that we are maximizing, with respect to both the PGM and neural network parameters, the variational lower bound (ELBO) of our observed data. This algorithm requires variational factors of the form

\[ q(z_t | s_t) = \mathcal{N}(e_\phi(s_t)), \quad q(F_t, \Sigma_t) = \mathcal{MNIV}(\Psi_t', \nu_t', M_0', V_t') \text{ for } t \in [0, \ldots, T - 1]. \]

\(e_\phi(s)\) is a recognition model, parameterized by neural network weights \(\phi\), that outputs the mean and diagonal covariance of a Gaussian distribution over \(z\). This recognition model is identical to that used in other prior works [20, 34, 12], however, as with prior work in the LDS SVAE, we also have variational factors of the form \(q(F_t, \Sigma_t)\), which represent our posterior belief about the system dynamics after observing the collected data. We also model this distribution as MNIW but with updated parameters compared to the prior from Equation 2. Given this, we can formulate the variational lower bound, which is presented in Section 2 of the appendix. This objective is then optimized using stochastic gradient updates for the neural network parameters \(\gamma\) and \(\phi\), specifically using the Adam optimization method [19], whereas the graphical model parameters are optimized using natural gradient updates and the variational message passing (VMP) framework [44].

Figure 2 details the graphical model presented in Equations 2-4 along with the variational family described above. Since we are interested in control and RL, there is the added notion of observed costs from the environment, and there are many ways we could model these additional observations. A natural choice is to model costs as a quadratic function of the latent state and action, such that we arrive at the LQS presented in Equation 1 except in the learned latent space. Specifically, we choose to fit \(C\) and \(c\) to the data collected by the current policy by encoding the observed states from our policy and regressing the latent states and actions to their observed costs. We refer readers to Section 2 of the appendix for more details about the model training and cost fitting procedures.

4 Policy Learning in the Latent Space

While we could use a variety of model-based policy learning methods in the learned latent space, the ability to infer local time-varying linear dynamics lends itself naturally to the particular analytic
which the closed form solutions are given in Section 3 of the appendix – as we linearize our policy given the updated model (line 6), we perform inference within our model to as discussed in the following sections, we can use the PGM in the previous section to formulate local policy in closed form. However, doing so is typically undesirable as the resulting policy will overfit to uncertainty and obtain dynamics estimates for policy improvement. As described in Section 3, our model provides us with non-linear neural network embedding and the TVLG policy.

Our overall algorithm, SOLAR, is presented in Algorithm 1. At every iteration, we collect N rollouts from the real world (line 4). Then, we update our model using data from the last B iterations (line 5), we linearize our policy given the updated model (line 6), we perform inference within our model to get the dynamics estimates (line 7), and we update our policy using the rollouts from our current iteration and our updated model (line 8). The following subsections detail the modules of our method that are involved in policy learning and improvement.

### 4.1 Dynamics inference under the model

To obtain a TVLG dynamics model, we could directly use linear regression to fit $F_t$ and $\Sigma_t$ to the observed latent trajectories $t = [z_0, a_0, \ldots, z_{T-1}, a_{T-1}, z_T]$. However, this may be poorly conditioned in the low-data regime. Instead, we can perform inference within our model to obtain dynamics estimates for policy improvement. As described in Section 3, our model provides us with variational approximations to the posterior over dynamics models, i.e., $\{q(F_t, \Sigma_t)_{t=0}^{T-1}\}$, which are MNIW. We can use these as a prior and condition on the data to obtain new variational posteriors $\{q(F_t, \Sigma_t | \{\tau\}_{i=0}^{N})_{t=0}^{T-1}\}$, which are also MNIW. Writing the parameters of these posteriors – for which the closed form solutions are given in Section 3 of the appendix – as $\{\Psi_t, M_{0t}, V_t, \nu_t\}_t$, we compute a maximum a posteriori estimate of the dynamics parameters, which gives us

$$F_t = M_{0t}, \Sigma_t = \frac{\Psi_t}{\nu_t},$$ \text{ for } t \in [0, \ldots, T - 1].$$

This inference procedure corresponds to Bayesian linear regression and can be interpreted as resolving the uncertainty in the global dynamics model conditioned on a real-world rollout. In essence, $\{q(F_t, \Sigma_t)_{t=0}^{T-1}\}$ captures uncertainty over the latent system dynamics by acting as a global model over all observed data, but in order to accurately model the system within the local region around the current policy, we condition on trajectories collected from the policy in order to resolve the uncertainty and obtain dynamics estimates $\{F_t, \Sigma_t\}_{t=0}^{T-1}$ that allow us to improve the policy.

### 4.2 Policy update

As described in Section 2.1 once we have our TVLG dynamics estimates $\{F_t, \Sigma_t\}_t$ and quadratic cost fit $C, c$, we can use dynamic programming on the Q and value functions to compute the optimal policy in closed form. However, doing so is typically undesirable as the resulting policy will overfit to

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Algorithm 1 SOLAR

1: Hyperparameters: # iterations $K$, # trajectories $N$, model training buffer size $B$
2: Initialize policy $\pi^{(0)}_\theta$, model $M^{(0)}$
3: for iteration $k \in \{1, \ldots, K\}$ do
4: Collect rollouts from the real world $D^{(k)} = \{ (s_0, a_0, \ldots, s_i) \}_{i=1}^N$
5: $M^{(k)} \leftarrow$ MODEL\_UPDATE($M^{(k-1)}$, $\{D^{(i)}\}_{i=k-B}$) (Section 3)
6: $\tilde{\pi}^{(k-1)}_\theta \leftarrow$ LINEARIZE\_POLICY($D^{(k)}, M^{(k)}$) (Section 4 of the appendix)
7: $\{F_t^{(k)}, \Sigma_t^{(k)}\}_t \leftarrow$ INFERENCE\_DYNAMICS($D^{(k)}, M^{(k)}$) (Section 4.1)
8: $\pi^{(k)}_\theta \leftarrow$ POLICY\_UPDATE($\tilde{\pi}^{(k-1)}_\theta$, $\{F_t^{(k)}, \Sigma_t^{(k)}\}_t, M^{(k)}$) (Section 4.2)
9: end for

local solution to the policy described in Section 2.1. This approach yields a policy that is TVLG in the latent space, which in general corresponds to a class of nonlinear policies in the original space formed by the composition of the nonlinear neural network embedding and the TVLG policy.
the model and likely will not perform well in the real environment. To mitigate this issue, prior work imposes a KL-divergence constraint on the policy update such that the shift in the induced trajectory distributions before and after the update, which we denote as \( \tilde{p}(\tau) \) and \( p(\tau) \), respectively, is bounded by a step size \( \epsilon \) \[24\]. This leads to a constrained optimization of the form

\[
\max_\theta \tilde{q}(\theta) \quad \text{s.t.} \quad D_{\text{KL}}(p(\tau)||\tilde{p}(\tau)) \leq \epsilon.
\]

We compute \( p(\tau) = \tilde{p}(z_0) \prod_{t=0}^{T-1} \pi_\theta(a_t|z_t)\tilde{p}(z_{t+1}|z_t, a_t) \), and analogously for \( \tilde{p}(\tau) \) with the previous policy. As shown in prior work \[24\], this constrained optimization can be solved by augmenting the cost function to penalize the deviation from the previous policy, such that the new cost function is given by \( \tilde{C}(z_t, a_t) = \frac{1}{\lambda}C(z_t, a_t) - \log \pi_\theta(a_t|z_t) \), where \( \pi_\theta \) denotes the previous policy. Note that this augmented cost function is still quadratic, since the policy is TVLG. \( \lambda \) is a dual variable that trades off between optimizing the cost function and staying close in distribution to the previous policy, and the weight of this term can be determined through a dual gradient descent procedure. Combined with the model learning from Section 3, we arrive at the complete SOLAR algorithm.

5 Related Work

Model-based RL methods have achieved significant efficiency benefits compared to model-free RL methods \[9, 30\] \[10\]. Many of these prior methods learn global models of the system that are then used for planning, generating synthetic experience, or policy search \[3, 33\]. These methods require an accurate and reliable model and will typically suffer from modeling bias, hence these models are still limited to short horizon prediction in more complex domains \[27, 30, 13, 42\]. Another class of model-based methods rely only on local system models to compute the gradient for a policy update \[1, 24, 14, 24, 5\]. These methods do not use models for long-term forward prediction, allowing for the use of simple models that enable policy improvement \[20, 26\]. As we show in Section 6, modeling bias for prior methods can be severely limiting in systems with complex observations such as images, whereas we are able to learn representations that mitigate the effects of modeling bias.

Utilizing representation learning within model-based RL has been studied in a number of previous works \[23\], including using embeddings for state aggregation \[56\], dimensionality reduction \[51\], self-organizing maps \[37\], value prediction \[32\], and deep auto-encoders \[22, 11, 43, 15\]. Within these works, deep spatial auto-encoders (DSAE) \[11\] and embed to control (E2C) \[43, 4\] are the most closely related to our work in that they consider local model methods combined with representation learning. The key difference in our work is that, rather than using a learning objective for reconstruction and forward prediction, we formulate a Bayesian latent variable model such that inference corresponds to fitting local models within the learned representation. As such, our objective enables local model methods by directly encouraging learning representations where fitting local models accurately explains the observed data. We also do not assume a known cost function, goal state, or access to the underlying system state as in DSAE and E2C, thus SOLAR is applicable even when the underlying states and cost function are unknown. We find that our approach tends to produce better results on a number of complex image-based tasks, as we discuss in the next section.

6 Experiments

We aim to answer the following questions through our experiments: (1) How does SOLAR compare to state-of-the-art model-based RL algorithms, in terms of sample efficiency, and state-of-the-art model-free algorithms, in terms of final policy performance? (2) How do local and global model methods compare to each other when operating in our learned representations? (3) Can we transfer learned representations from one task to learning new tasks? To answer these questions, we set up simulated image-based robotic domains for a nonholonomic car and 2-DoF arm, where the goal is to reach a specified target from raw image observations, as shown in Figure 3a. We also learn tasks directly from camera images on a real Sawyer robotic arm, as shown in Figure 3b. We compare our model to trust region policy optimization (TRPO) \[35\], a model-free method; LQR with fitted linear models (LQR-FLM) \[24\], a local model method; and a state-of-the-art global model-based method \[30\], which we refer to as model-predictive control with neural networks (MPC-NN). We

\[1\] In principle, these methods can be extended to unknown underlying states and cost functions, though the authors do not experiment with this and it is unclear how well these approaches would generalize.
Figure 3: (a) Top: Visualizing a trajectory in the car navigation environment, with the target denoted by the black dot, and the corresponding image observation. Bottom: An illustration of the 2-DoF arm environment, with the target denoted by the red dot, and the corresponding image observation. Note that we use sliding windows of past observations when learning both tasks. (b) Top: Illustration of the architecture we use for learning Lego block stacking. Bottom: Example trajectory from our learned policy stacking the yellow Lego block on top of the blue block.

also study ablations of our method where we learn models using robust locally-linear controllable embedding (RCE) [4], an improved version of E2C [43], and where we use our learned model as a global model and perform forward prediction for MPC in the latent space. We refer to these ablations as the “E2C-like ablation” and the “global model ablation”, respectively. Training hyperparameters, model architectures, and other details regarding experimental setup are provided in Section 5 of the appendix, and videos of the policies are available on the project website [2].

6.1 Tasks and Comparisons

**Image-based 2D navigation.** As an initial example, and to provide a point of comparison to other methods, we consider a simulated 2-dimensional navigation task where the goal changes from episode to episode. Observations consist of two 32-by-32 images indicating the position of the agent and the goal. We first compare our method to running LQR-FLM directly in the image observation space. We also evaluate the E2C-like ablation of our method [43], where we use RCE to train our model and then use LQR-FLM to learn a policy in the latent space. For our method, we use a spatial softmax [11] in our recognition model, and we found this to be important for efficiently learning a good policy.

**Nonholonomic car.** The nonholonomic car starts in the bottom right of the 2-dimensional space and controls its forward acceleration and steering velocity in order to reach the target in the top left. We evaluate our method using image observations, where we use a sliding window of four 64-by-64 images to capture velocity information. We compare to the global model ablation of our method in which we replace the local linear dynamics in our model with a neural network dynamics function and use this model for forward prediction and MPC in the latent space.

**Simulated reacher.** We experiment with the reacher environment from OpenAI Gym [7], where a 2-DoF arm has to reach a target denoted by a red dot, which we specify to be in the bottom left. For observations, we directly use 64-by-64-by-3 images of the rendered environment, which provides a top-down view of the reacher and target, and we use a sliding window of four images to encode velocity information. In this domain, we evaluate our method against TRPO.

**Sawyer reaching.** We test our method on an image-based reaching task on a real 7-DoF Sawyer robotic arm, where the controller only receives images as the observation, without joint angles or other information. In this task, we encode the image of the goal arm configuration using the learned representation and use the distance from this latent goal state as the reward function for the policy learning. The observations used are raw 84-by-84-by-3 images from a camera pointed at the robot.

[4] https://sites.google.com/view/solar-iclips
Figure 4: (a) Our method solves 2D navigation from images consistently across random seeds, whereas LQR-FLM and our E2C-like ablation are unable to make progress. (b) On the car from images, both our method and the global model ablation are able to reach the goal, however, the performance of our method is more consistent, and we encode prior information into the global model ablation by biasing the control to select positive actions. Without this bias, the global model ablation fails to make progress. (c) For reacher from images, we perform comparably to TRPO while needing about an order of magnitude fewer episodes to learn. Here we plot reward, so higher is better.

**Sawyer Lego block stacking.** To demonstrate an even more challenging task in the real world, we use our method to learn Lego block stacking with the Sawyer arm, as depicted in Figure 3b. Similar to Sawyer reaching, we use 84-by-84-by-3 images as our only observation.

### 6.2 Simulation Results

Figure 4 details our results on the simulated image-based experimental domains, where each method is tested on three random seeds and the mean and standard deviation of the performance is reported. For 2D navigation from images, we plot the average final distance to the goal as a function of the number of episodes, so lower is better. Our method is able to learn this domain very quickly, converging to a high-performing policy within 200 episodes. Note that the majority of that data is randomly collected at the start to train our model, and only a small amount of data is needed for each subsequent iteration. LQR-FLM struggles to learn the task, likely because the images are too complex for local linear model fitting, and LQR-FLM is unlikely to improve even with more data.

Despite using code directly from the authors of RCE, we were unable to get our E2C-like ablation to learn a good model for this task, and thus the learned policy was also very poor and made no improvement at all over the initial policy. One possible explanation for this difficulty is that this task is deceptively harder than the image-based 2D navigation task considered in E2C and RCE, where the target is fixed to be in the bottom right of the image rather than changing with each episode. To test this, we ran our E2C-like ablation on this simplified version of our 2D navigation domain, and we were indeed able to train a more successful policy. We present this result in Section 6 of the appendix.

On the image-based nonholonomic car, our method is able to learn a good policy with about 2000 episodes of experience. We compare to a global model ablation of our method, where we learn a neural network dynamics model, rather than a LDS model, jointly with a neural network cost model. We then use the MPC-NN procedure for control in the latent space, where we utilize the learned cost model in place of the true cost function. This approach is competitive with our method, however, we obtained this result by biasing the mean of the MPC random action selection to be positive, effectively encoding prior information that the car should move forward. We also noticed that, even with more data, the variance of the control performance remained higher than the policy learned by our method. These observations seem to indicate that forward prediction using the learned global models may be inaccurate, leading to inconsistent control performance. In contrast, our method does not heavily rely on an accurate model and can achieve more consistently good behavior on this task. We present further comparisons to global model methods in Section 6 of the appendix.

Finally, on the image-based reacher domain, we compare our method to TRPO and we plot the reward function as defined in Gym. Note that the x-axis of the plot is on a log scale: though our method achieves comparable final policy performance to TRPO, we do so with about an order of magnitude fewer episodes than TRPO, i.e., we use 1200 episodes whereas TRPO uses tens of thousands.
Figure 5: (a)-(c): Performance of our method on the real-world Sawyer end-effector reaching task. Note that we use the same pretrained representation from learning the first goal to warmstart learning on the second and third goals, demonstrating transferrability of our learned representations. (d): Performance on the real-world Sawyer block stacking task.

Gain in data efficiency compared to model-free methods is typical of model-based methods, however, SOLAR is able to handle this domain directly from raw image observations, which is challenging for other model-based methods. Thus, SOLAR retains much of the sample efficiency of model-based methods while approaching model-free methods in its ability to solve complicated tasks.

6.3 Real Robot Results

Figures 5a-5d detail our method’s performance on the Sawyer reaching and Lego block stacking tasks in terms of the average final distance in meters to the goal, where each method is tested on five random seeds and the mean and standard deviation of the performance is reported. For Lego block stacking, we define the goal position of the end effector such that reaching the goal leads to successful stacking of the block. Not only is our method able to solve both tasks directly from raw, high-dimensional camera images within 100-250 episodes, corresponding to under an hour of interaction time, our method is also successful at handling the complex, contact-rich dynamics of block stacking, which poses a significant challenge compared to the other contact-free tasks.

To test whether the learned representations can generalize and be transferred to other tasks, we warm-start the second and third goal-reaching experiments – Figures 5b-5c – using the pretrained representation learned from the first goal-reaching experiment. As can be seen the same representation can be used to successfully learn these different goals. In the case of the second goal, we also see substantially faster learning – despite one poorly performing random seed, the learned policy generally is successful after only 20 episodes. Videos of the learned policies, as well as a summary video of the learning process, are available on the project website.

7 Discussion and Future Work

We presented SOLAR, a model-based RL algorithm that is capable of learning policies in a data-efficient manner directly from raw high-dimensional observations. The key insights in SOLAR involve learning latent representations where simple models are more accurate and utilizing PGM structure to infer dynamics from data conditioned on entire real-world trajectories. Our experimental results demonstrate that SOLAR is competitive in sample efficiency, while exhibiting superior final policy performance, compared to other model-based methods. Furthermore, SOLAR is significantly more data-efficient compared to state-of-the-art model-free RL methods.

There are several interesting directions for future work. First, as we have shown, the ability to learn representations lends itself naturally to multi-task and transfer settings, where new tasks could potentially be learned much more quickly by starting from a latent embedding that has been learned from previous tasks. We can also in principle share dynamics models, where the PGM we learn from solving previous tasks can be used as a global prior when inferring local dynamics fits for a new task. This likely would require more sophisticated PGM structure in order to handle complex phenomena – e.g., switching models to deal with discontinuities and recurrence to handle partial observability – and extending our PGM is an exciting line of work to pursue.

[https://sites.google.com/view/solar-iclips](https://sites.google.com/view/solar-iclips)
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Appendix

1 Policy Learning Details

Given a TVLG dynamics model and quadratic cost approximation, we can approximate our Q and value functions to second order with the following dynamic programming updates, which proceed from the last time step $t = T$ to the first step $t = 1$:

$$Q_{s,t} = c_{s,t} + F_{s,t}^T V_{s,t+1}; \quad Q_{ss,t} = c_{ss,t} + F_{s,t}^T V_{ss,t+1} F_{s,t},$$

$$Q_{a,t} = c_{a,t} + F_{a,t}^T V_{a,t+1}; \quad Q_{aa,t} = c_{aa,t} + F_{a,t}^T V_{aa,t+1} F_{a,t},$$

$$Q_{sn,t} = c_{sn,t} + F_{s,t}^T V_{sn,t+1} F_{a,t},$$

$$V_{s,t} = Q_{s,t} - Q_{aa,t} Q_{nn,t}^{-1} Q_{a,t},$$

$$V_{ss,t} = Q_{ss,t} - Q_{sn,t} Q_{nn,t}^{-1} Q_{as,t}.$$

It can be shown (e.g., in [39]) that the action $a_t$ that minimizes the second-order approximation of the Q-function at every time step $t$ is given by

$$a_t = -Q_{aa,t}^{-1} Q_{as,t} s_t - Q_{aa,t}^{-1} Q_{a,t}.$$

This action is a linear function of the state $s_t$, thus we can construct an optimal linear policy by setting $K_t = -Q_{aa,t}^{-1} Q_{as,t}$ and $k_t = -Q_{aa,t}^{-1} Q_{a,t}$. We can also show that the maximum-entropy policy that minimizes the approximate Q-function is given by

$$\pi(a_t | s_t) = \mathcal{N}(K_t s_t + k_t, Q_{aa,t}).$$

Furthermore, as in [24], we can impose a constraint on the total KL-divergence between the old and new trajectory distributions induced by the policies through an augmented cost function $c(s_t, a_t) = \frac{1}{2} c(s_t, a_t) - \log \pi^{(t-1)}(a_t | s_t)$, where solving for $\lambda$ via dual gradient descent can yield an exact solution to a KL-constrained LQR problem.

2 Model Learning Details

To derive the variational lower bound (ELBO) of the observed data for our model, we first formulate our full variational posterior, given the variational factors presented in the paper, as

$$q(\{F_t, \Sigma_t\}_{t=0}^{T-1}, \{z_t\}_{t=0}^{T-1}, \{s_t\}_{t=0}^{T}) = \prod_{t=0}^{T-1} q(F_t, \Sigma_t) \prod_{t=0}^{T-1} q(z_t | s_t) \cdot q(s_0 | z_0),$$

Where we use $\lambda$ to denote the matrix normal-inverse-Wishart (MNIW) parameters of the variational factors on $\{F_t, \Sigma_t\}_t$. Thus, the ELBO is given by

$$\mathcal{L} = \mathbb{E}_q \left[ \log \frac{p(\{F_t, \Sigma_t\}_{t=0}^{T-1}, \{s_t\}_{t=0}^{T-1}, \{z_t\}_{t=0}^{T-1} | \{a_t\}_{t=0}^{T-1}, \{z_t\}_{t=0}^{T})}{q(\{F_t, \Sigma_t\}_{t=0}^{T-1}, \{z_t\}_{t=0}^{T-1} | \{s_t\}_{t=0}^{T})} \right]$$

$$= \mathbb{E}_q \left[ \log \left( \frac{\prod_{t=0}^{T} p_\phi(s_t | z_t)}{\prod_{t=0}^{T-1} q_\phi(z_t | s_t)} \right) \right]$$

$$- \sum_{t=0}^{T-1} \text{KL}(q_\phi(F_t, \Sigma_t) || p(F, \Sigma)) - \sum_{t=1}^{T} \mathbb{E}_q \left[ \text{KL}(q_\phi(z_t | s_t) || p(z_t | s_{t-1}, a_{t-1}, F_t, \Sigma_t)) \right].$$

Prior work has shown that, for conjugate exponential models such as the Bayesian LDS, the parameters can be updated using natural gradients, which can be computed in closed form using the variational
message passing framework [44]. Specifically, for the parameters of the MNIW \( \lambda \), the natural gradient update is

\[
\tilde{\nabla}_\lambda \mathcal{L} = \lambda^0 + B \mathbb{E}_q [t_{F, \Sigma}(F, \Sigma)] - \lambda,
\]

(6)

Where \( B \) is the number of minibatches in the dataset, \( \lambda^0 \) is the parameter for the prior distribution \( p(F, \Sigma) \), and \( t_{F, \Sigma}(F, \Sigma) \) is the sufficient statistic function for \( p(F, \Sigma) \). Thus, we can use this equation to compute the natural gradient update for \( \lambda \), whereas for \( \gamma \) and \( \phi \) we use stochastic gradient updates on Monte Carlo estimates of the ELBO, specifically using the Adam optimization scheme [19]. This leads to two simultaneous optimizations for the PGM parameters and the neural network parameters, and their respective learning rates are treated as separate hyperparameters. We have found \( 10^{-3} \) to be generally suitable for the natural gradient updates and \( 10^{-4} \) to be a good default value for Adam.

To perform our policy update, we fit a quadratic cost function to observed data in the learned latent space as follows. Given trajectories of the form \([s_0, a_0, c_0, \ldots, s_{T-1}, a_{T-1}, c_{T-1}, s_T]\), we first embed the observations \( \{s_t\} \) using the mean of our recognition model \( \mu(e, \phi(s)) \) to obtain a set of latent states \( \{z_t\} \). Then, we form the vector consisting of the latent state, action, and all cross terms \([\{z_t\}_1, \ldots, \{z_t\}_m, (a_t)_1, \ldots, (a_t)_n, \{z_t\}_1(a_t)_1, \ldots, \{z_t\}_m(a_t)_1, \ldots, \{z_t\}_m(a_t)_n]\)

Where we use \( m \) and \( n \) to denote the dimensionality of the latent state and action, respectively. Linear regression from these vectors onto the cost samples \( \{c_t\} \) will yield all coefficients of the quadratic cost matrix \( (C)_{ij} \) and linear cost vector \( (c)_i \), which we can assemble into our final cost function. One choice to make is whether we fit a time-varying cost function, i.e., perform separate regressions for the observed data at each time step, or if we aggregate all of the data together to fit a single time-invariant cost function. The former option allows us more flexibility to model complicated cost functions, whereas the latter is more data efficient. The domains we consider have generally simple cost functions, and we use limited data for all of our experiments, thus we default to fitting time-invariant cost functions for our tasks.

3  Dynamics Inference

Here we provide the closed form parameter computations for the posteriors of our dynamics given observed trajectories, as described in Section 4.1 of the main paper. Given variational factors from our model of the form

\[
q(F_t, \Sigma_t) = MNZW(\Psi_t, \nu_t, M_{0t}^T, V_t')
\]

We can condition on observed trajectories \( \tau \) to obtain new variational posteriors \( \{q(F_t, \Sigma_t | \{\tau\})_{i=0}^{N} \}_{t=0}^{T-1} \). These posteriors are also MNIW, and the parameters of these posteriors can be computed in closed form as

\[
\Psi_t = \Psi'_t + M_{0t}^T V_{t'}^{-1} M_{0t} + \sum_{i=1}^{N} z_{t+1}(i) a_{t+1}(i)^T - M_{0t} V_{t'}^{-1} M_{0t}^T, \quad \kappa_t = \kappa_t + N,
\]

\[
M_{0t} = \begin{pmatrix} M_{0t}^T V_{t'}^{-1} + \sum_{i=1}^{N} z_{t+1}(i) a_{t+1}(i)^T V_t, \quad V_t = \begin{pmatrix} V_{t'}^{-1} + \sum_{i=1}^{N} z_{t+1}(i) a_{t+1}(i)^T \end{pmatrix}^{-1} \end{pmatrix}.
\]

Then, a maximum a posteriori estimate gives us the TVLG dynamics parameters as described in the main paper.

4  Policy Linearization

The policy update described in Section 4.2 of the main paper requires us to compute the KL-divergence between the trajectory distributions before and after the policy update, denoted as \( p(\bar{\tau}) \) and \( p(\tau) \), respectively. We compute \( p(\tau) = \hat{p}(z_0) \prod_{t=0}^{T-1} \hat{p}(a_t | z_t) \hat{p}(z_{t+1} | a_t, z_t) \) and analogously for \( p(\bar{\tau}) \) with the previous policy, and we are able to compute these analytically because the policies and dynamics model are TVLG, thus the induced trajectory distributions are also Gaussian. However, this operates under the assumption that \( z \) is fixed, which does not hold since the model update changes the latent representation. Since our overall policy is a combination of the model embedding, given
by \(e_\phi(s)\), and the TVLG policy \(\pi_\theta(a_t|z_t)\), training \(e_\phi(s)\) will change the behavior of the policy even if \(\pi_\theta(a_t|z_t)\) stays fixed. In some cases, this may lead to a policy with worse performance, and constraining against this policy for the policy update may lead to poor results. In fact, what we want to do is to account for the model update by changing \(\pi_\theta(a_t|z_t)\) accordingly, so that the overall policy does not change in its distribution. Thus, using \((s_t, a_t)\) pairs from the previous data collection phase, we embed \(z_t = \mu(e_\phi(s_t))\) with our updated model and use linear regression to find the TVLG policy \(\tilde{\pi}_\theta(a_t|z_t)\) that best explains the data collected from the policy. This is line 6 of the SOLAR algorithm presented in the main paper, and after this, we can perform the policy update constrained against the trajectory distribution induced by \(\tilde{\pi}_\theta(a_t|z_t)\).

5 Experiment Setup

**Image-based 2D navigation.** Our recognition model architecture for the 2D navigation domain consists of two convolution layers with 2-by-2 filters and 32 channels each, with no pooling layers and ReLU non-linearities, followed by another convolution with 2-by-2 filters and 2 channels. The output of the last convolution layer is fed into a spatial softmax layer [11], which then outputs a Gaussian distribution with a fixed diagonal covariance of \(10^{-4}\) for the latent distribution. Our observation model consists of two fully-connected (FC) hidden layers with 256 ReLU activations, and the last layer outputs a categorical distribution over pixels. We initially collect 200 episodes which we use to train our model, and for every subsequent iteration we collect 20 episodes to fine tune our model.

**Image-based nonholonomic car.** The image-based car domain consists of 64-by-64 image observations. We include a window of the 3 previous 64-by-64 images in our observation to preserve velocity information. Our recognition model is a convolutional neural network that operates on each image in the sliding window independently. Its architecture is four convolutional layers with 4-by-4 filters with 4 channels each, and the first two convolution layers are followed by a ReLU non-linearity. The output of the last convolutional layer is fed into three FC ReLU layers of width 2048, 512, and 128, respectively. Our final layer outputs a Gaussian distribution with dimension 8. This leads to a final latent dimension of 32. Our observation model consists of four FC ReLU layers of width 256, 512, 1024, and 2048, respectively, followed by a Bernoulli distribution layer that models the image. Like the recognition model, the observation model only operates on each section of the latent information. Our recognition model only operates on each section of the latent representation corresponding to the image window independently. For this domain, we collect 100 episodes initially to train our model, and we collect 100 episodes per iteration after this.

**Reacher.** The reacher domain consists of 64-by-64-by-3 image observations. Similar to the car, we include a window of the 3 previous 64-by-64-by-3 images in our observation. Our recognition model is a convolutional neural network that again operates on each image in the sliding window independently. Its architecture is three convolutional layers with 2-by-2 filters with 64, 32 and 16 channels respectively. Each layer has a ReLU non-linearity followed by a 2-by-2 max-pooling. The output of the last convolutional layer is fed into an FC ReLU layer of width 200, followed by another FC ReLU layer of width 200. Our final layer outputs a Gaussian distribution with dimension 10, leading to a final latent dimension of 40. Our observation model consists of three FC ReLU layers of width 256, followed by a Bernoulli distribution layer that separately models each image. We collect 200 episodes initially to train our model, and we collect 100 episodes per iteration after this.

**Sawyer reaching.** The image-based Sawyer reacher domain consists of 84-by-84-by-3 image observations. Our recognition model is a convolutional neural network with the following architecture: a 5-by-5 filter convolutional layer with 16 channels followed by two convolutional layers using 5-by-5 filters with 32 channels each. The first two convolutional layers are followed by ReLU activations and the last by a FC ReLU layer of width 256 leading to a 16 dimensional Gaussian distribution layer. Our observation model consists of a FC ReLU layer of width 128 feeding into three deconvolutional layers, the first with 5-by-5 filters with 32 channels and the last two of 6-by-6 filters with 16 and 3 channels respectively. These are followed by a final Bernoulli distribution layer. For this domain, we collect 50 episodes initially to train our model, and 20 episodes per iteration thereafter.

**Sawyer Lego block stacking.** The image-based Sawyer block-stacking domain consists of 84-by-84-by-3 image observations. Our recognition and observation models are the same as those used in the Sawyer reaching domain. We collect 50 episodes initially to train our model, 20 episodes per iteration for the first 5 iterations, then 10 episodes per iteration for the remainder.
6 Additional Experiments

6.1 E2C-like ablation on simplified 2D navigation

As mentioned in Section 6, our E2C-like ablation was unable to make progress for the 2D navigation task, though we were able to get more successful results by fixing the position of the goal to the bottom right as is done in the image-based 2D navigation task considered in E2C [43] and RCE [4]. Figure 6 details this experiment, which we ran for three random seeds and report the mean and standard deviation of the average final distance to the goal as a function of the number of training episodes. It is clear that the policy is improving, and two of the seeds are able to make substantial progress, though the final seed is less successful and significantly worsens the average performance of the method. This indicates that the latent representation learned through RCE is less suitable for local model fitting, as accurate local model fitting is not explicitly encouraged by their representation learning objective.

6.2 Model-based comparisons on state-based nonholonomic car

To provide a point of comparison to model-based RL methods, we consider the car domain where the underlying state is observed. The states for the car domain include the position of the center of mass, orientation, forward and angular velocity of the car, and the position of the target, making for a 9-dimensional system. Since this observation is already quite simple, we use a single linear layer for our recognition and observation models that output Gaussian distributions, and we use the same dimensionality for our latent representation as the state dimensionality.

We plot the performances of our method, LQR-FLM [24], and MPC-NN [30], again based on the average final distance to the target, in Figure 7. In this setting, our method is competitive with LQR-FLM, learning a policy with similar performance in 200 episodes. MPC-NN performs the best for this task, learning a policy that consistently reaches the target in just 20 episodes, though it is given the true cost function whereas our method and LQR-FLM are not. For this simple setup where modeling bias is not an issue, we expect model-based methods to perform very well and learn efficiently. However, when we make the problem more challenging by using image observations, model-based methods will fail quickly: LQR-FLM is unable to fit complex pixel transitions using local linear models, as shown through the 2D navigation experiment, and MPC-NN has never been used with images, as forward video prediction and defining a cost function on images are both very difficult. We extend MPC-NN to the image-based task, and we term this the “global model ablation” of our method – as shown in the paper, this approach is able to make progress toward the goal, though our method is still significantly better at solving this difficult task.

Figure 6: On 2D navigation with the goal fixed to the bottom right, our E2C-like ablation is able to make progress toward the goal.

Figure 7: On the car from states, our method is competitive with LQR-FLM, demonstrating that we maintain the sample efficiency of model-based methods for simple tasks.