Making carbon in stars

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The triple alpha (3\alpha) process plays an important role in the production of $^{12}\text{C}$ in stars. Its rate is known with an accuracy of about 12\%. We examine the corresponding uncertainties introduced in the description of pre-supernova stars, of nucleosynthesis in a core-collapse SN explosion, and of the production of $^{12}\text{C}$ during the third dredge-up in asymptotic giant branch (AGB) stars. For the AGB case we consider also the effects of uncertainties in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rate. We conclude that the present accuracy of the 3\alpha rate is inadequate and describe new experiments that will lead to a more accurate value.

1. INTRODUCTION

Although the 3\alpha reaction plays the central role in the production of $^{12}\text{C}$ in stars, little attention has been paid to the effect of uncertainties in its rate. The emphasis has been on the following reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. During core helium burning, these two reactions are in competition, and the ratio $R$ of their rates determines the relative amounts of $^{12}\text{C}$ and $^{16}\text{O}$ produced. The C/O ratio in turn affects the later evolution of the star. In the case of the pre-supernova evolution of a massive star, for example, it is found that the size of the Fe core at the onset of core collapse depends on $R$, as does the composition of the material later ejected into the interstellar medium. In the context of a parametrized explosion model, the composition of the nucleosynthesized material constrains $R$ to within about 10\%. Given the required $\pm 10\%$ precision in $R$ the present $\pm 12\%$ precision of the 3\alpha rate is inadequate.

The 3\alpha process also plays an important role in asymptotic giant branch (AGB) stars, stars burning hydrogen and helium in shells around a degenerate core containing mainly carbon and oxygen. We have investigated $^{12}\text{C}$ production in these stars and have found that uncertainties in the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate have little effect, but that uncertainties in the 3\alpha and $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rates are important.

In the following sections we will discuss these phenomena in more detail and will then discuss measurements that promise to improve the accuracy of the 3\alpha reaction rate to about $\pm 6\%$.

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2. CORE-COLLAPSE SUPERNOVAE

The dependence of the synthesis of A=16-40 nuclei on R is well documented. Calculations were performed [2] for a 25$M_{\odot}$ star using a range of values of the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction rate, $r_{\alpha,12}$, and assuming a standard value of the 3$\alpha$ rate. For values of $r_{\alpha,12}$ of 1.2(±10%) times the rate suggested by Buchmann [3], the production factors for these nuclei are almost independent of A, allowing a simultaneous reproduction of their abundances. Less well known is the dependence of the size of the pre-collapse iron core on R. Fig. 1 shows this dependence and the relevant range of R as determined from the nucleosynthesis constraints discussed above. The core mass varies significantly as a function of R, by about 0.2$M_{\odot}$ in the relevant range. This uncertainty is likely to be important for the behavior of the SN explosion; it takes $3 \times 10^{51}$ ergs to dissociate 0.2$M_{\odot}$ into nucleons, similar to (or greater than?) the energy released in a supernova explosion.

![Figure 1](image.png)

**Figure 1.** The Fe core size obtained [2] for a 25$M_{\odot}$ pre-supernova star for various values of the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction rate (and hence of R). The unit rate is that recommended by Buchmann [3]. The shaded bar corresponds to the range of R that gives a nearly constant production factor for A = 16 – 40 nuclei.

This is an important issue. Even if R were determined with 10% accuracy, and that will not be easy, there would be an intrinsic uncertainty of 0.2$M_{\odot}$ in the Fe core mass for this star. At least two issues need to be examined. First, whether the value of R obtained from nuclear experiments is consistent with that obtained from SN models of nucleosynthesis. That is, do the SN models reflect what really happens in stars? And second, whether rapid core mass changes with R occur for a range of stellar masses.
3. AGB STARS

Following completion of core hydrogen and helium burning in intermediate mass stars, hydrogen and helium are burned in shells surrounding a degenerate carbon-oxygen core. Eventually, burning in the helium shell becomes unstable and thermal flashes induce convective behavior and production of $^{12}\text{C}$ in the inter-shell region. Later, the surface convection zone moves into the inter-shell region; carbon-rich material enters the convective envelope of the star and is carried to the stellar surface in a process known as the third dredge-up. The surface of the star becomes carbon rich and much of this material is blown into the interstellar medium. Such behavior is observed for light stars, $2-3 \, M_{\odot}$ in the Magellanic clouds. However, it has not, so far, been possible to reproduce this behavior theoretically. The flash behavior found in most calculations is too weak to produce enough $^{12}\text{C}$, although certain approximations involved in these calculations prevent firm conclusions. We discuss here whether uncertainties in the reaction rates involved might also have important effects. It is known that stronger flashes lead to more efficient dredge-up and that weaker hydrogen burning leads to stronger flashes, but these phenomena have not been studied in a systematic fashion and related to changes in the underlying nuclear reaction rates. We describe here calculations made to study the sensitivity of the AGB-thermal flash process to the rates for the $3\alpha$, $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, and $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reactions. Additional details can be found in Ref. [1].

Initially we used the NACRE [4] reaction rates and their uncertainties, as shown in the right-hand area of Fig. 2. We found that carbon production was insensitive to the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate. However, changes in the $3\alpha$ and $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rates produced large

Figure 2. Values of the reaction rates at which calculations have been done, compared to the NACRE values. On the right the circles are at the NACRE values $\pm$ their errors; on the left are the new values of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rates. From F. Herwig.
effects. To estimate these effects quantitatively, one rate at a time was changed to its upper or lower error limit. The calculations correspond to points 2 (recommended values), 5, 6, 8, and 9 in Fig. 2.

![Figure 3. Values of the C/O ratio at the stellar surface. Time increases to the right. The values reached at the end of the AGB phase are at the far right.](image)

The carbon enrichment of the stellar surface and the production of $^{12}\text{C}$ increased by about a factor of two when either the $3\alpha$ rate was increased by its uncertainty (case 6), or when the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rate was decreased by its uncertainty (case 8). The opposite changes (cases 5 and 9) produced much smaller effects. These results are shown in more detail in Fig. 3. The values for the two highest lying curves are characteristic of carbon stars. As noted previously they correspond to cases 6 and 8 in Fig. 2.

Of course these results by themselves do not constitute a resolution of the carbon star problem. Other poorly understood phenomena might produce opposite changes \[\text{[1]}\], or the true rates might not be at the limiting values that lead to large effects. But they do demonstrate that the nuclear reaction rates must be better understood to reach a reliable theoretical conclusion.

Fortunately, improved values of the S factor $S_{1,14}$ for $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction have been obtained from new experimental results from Duke, LUNA, Texas A and M, and Tokyo; the resulting reaction rate is about half that recommended by NACRE. Work of the LUNA group was presented at this Symposium \[\text{[5]}\]. A detailed summary of the various experiments is given in \[\text{[1]}\]. Based on our evaluation of these results we have chosen to use an unweighted average: $S_{1,14} = 1.7 \pm 0.25$ keV b. This value is smaller than the lower limit of the NACRE values, and makes it more probable that a large $^{12}\text{C}$ production will
result from the AGB process.

We have undertaken an extended set of calculations to investigate this in more detail. Preliminary results have been obtained for the left-hand points in Fig. 2 where we have considered also simultaneous variations of the two rates. The calculation with the smaller $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rate indicated by the new data (case 13) yields still larger $^{12}\text{C}$ production. And a simultaneous low $^{14}\text{N}(p, \gamma)^{15}\text{O}$ and high $3\alpha$ rate (cases 14 and 15) yields a further $^{12}\text{C}$ enhancement. We have not yet combined low $^{14}\text{N}(p, \gamma)^{15}\text{O}$ AND low $3\alpha$ rates to find the result of this combination. It is clear, however, that the AGB physics provides a further incentive for obtaining accurate values of the $3\alpha$ rate.

4. IMPROVING THE $3\alpha$ RATE

For reference, the nuclei involved in the $3\alpha$ process are shown in Fig 4. As can be seen there, the process will be strongest if the two resonances involved, the ground state of $^{8}\text{Be}$ and 7.65 MeV state of $^{12}\text{C}$, lie in the Gamow window. For temperatures in the range encountered in the present scenarios this is so, the process is resonant, and the rate of the $3\alpha$ reaction has a simple form:

$$r_{3\alpha} \propto \Gamma_{\text{rad}} \exp(-Q/kT).$$

In this circumstance, only two quantities need to be determined: $Q$ and $\Gamma_{\text{rad}}$. $Q = E_r - 3M_\alpha c^2$, where $E_r \sim 7.6$ MeV and is known to $\pm 0.2$ keV from measurements of $E_r$ [3]. We note that the error quoted in many tabulations of $^{12}\text{C}$ levels is incorrect; it was taken from the wrong table in [6]. The uncertainty due to this factor is negligible, 1.2% for $T_h = 2$ and decreasing at higher $T$. $\Gamma_{\text{rad}}$ is the radiative width of the 7.65 MeV state. At higher and lower $T$ the situation is more complex [4]. New experiments [7,8] provide no evidence for a 9.2 MeV $2^+$ state that is predicted theoretically [4] to have a significant effect on $r_{3\alpha}$ at $T_h > 2$. Instead there is evidence for a $0^+$ state at 11.2 MeV that will interfere with the 7.6 MeV $0^+$ state and modify $r_{3\alpha}$ at low temperatures [9]; details of these effects are not yet published.

Essentially all the uncertainty in $r_{3\alpha}$ at the temperatures relevant to the present considerations is due to the uncertainty in $\Gamma_{rad}$ and is given by the product of three quantities as shown in Fig. 5. All these quantities have been measured several times with generally consistent results. This is a tribute to the experimenters involved, since all these measurements are difficult. Nevertheless, two quantities, the pair width and the pair-decay branch of the 7.6 MeV state, need to be measured better.

The pair width $\Gamma_\pi$ is determined from the transition charge density for inelastic electron scattering to the 7.6 MeV state. There is a new, as yet unpublished result [10], based on a compendium of extant measurements over a large momentum transfer range, that has a quoted accuracy of $\pm 2.7\%$. It is difficult to imagine that a more accurate value of $\Gamma_\pi$ can be obtained. On the other hand, this value is not quite consistent with the earlier values of $\Gamma_\pi$.

The pair branch $\Gamma_\pi/\Gamma$ is the least well known quantity, primarily because it is so small, about $6 \times 10^{-6}$. A new experiment [11], a Western Michigan University (WMU), Michigan State University (MSU) collaboration, is underway using the Tandem accelerator at WMU. A schematic of the apparatus is shown in Fig. 6. This detector is an improved version of that used by Robertson et al. [12]. The 7.6 MeV state in $^{12}\text{C}$ is excited by inelastic
Figure 4. Nuclei involved in the $3\alpha$ process. The first step (I) of this process is the fusion of two $\alpha$ particles to form an equilibrium concentration of $^8\text{Be}$. The subsequent (II) capture of an alpha particle forms an equilibrium concentration of $^{12}\text{C}$ in its 7.65 MeV state. Occasionally $^{12}\text{C}$ is formed by a leak, via a $\gamma$ cascade or pair emission, to the ground state of $^{12}\text{C}$.

proton scattering, taking advantage of a strong resonance at an excitation energy of 10.6 MeV and a scattering angle of 135 degrees in the lab. In order to reduce gamma ray backgrounds, a coincidence is required between a thin cylinder and a large plastic scintillator surrounding it; this should little affect the number of detected pairs, but will strongly discriminate against $\gamma$ rays—they have only small probability of interacting in the thin cylinder. The pair branch is given simply by the ratio of the number of positron-electron pairs detected in the plastic scintillators to the number of counts in the 7.65 MeV peak in the proton spectrum. An examination of the systematic uncertainties in the similar Robertson experiment leads us to estimate that an accuracy of 5% is achievable.

5. SUMMARY AND COMMENTS

An examination of the effects of the rate for the $3\alpha$ process on the production of $^{12}\text{C}$ in core-collapse supernovae (Type II) and in low mass AGB stars shows that this rate is inadequately known. Variations within the ±12% errors cause a change of $0.2M_{\odot}$ in the core mass of a $25M_{\odot}$ pre-supernova star and a factor of two change in the surface abundance of $^{12}\text{C}$ in light AGB stars. In the supernova case, it is the ratio of $r_{3\alpha}$ and
\[ \Gamma_{rad} - \Gamma_{\gamma} + \Gamma_{\alpha} - \frac{\Gamma_{\gamma} + \Gamma_{\pi}}{\Gamma} \geq \frac{\Gamma}{\Gamma_{\alpha}} \quad Q_{3\alpha} = E_{r} - 3M_{\alpha}^{2} \]

\% RATE uncertainty

\begin{align*}
2.7 & \uparrow \uparrow \uparrow \\
9.2 & \uparrow \uparrow \uparrow \\
6.4 & \uparrow \uparrow \\
1.2 & (T_{8} = 2)
\end{align*}

Figure 5. Expressions for $\Gamma_{rad}$ and $Q$. The uncertainties in $r_{3\alpha}$ from the various terms are also shown.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure6}
\caption{Schematic diagram of an apparatus for observing the pair branch of the 7.65 MeV state in $^{12}$C.}
\end{figure}

$r_{1,14}$ that is important at the 10% level. To reach this level, both these rates must be significantly better known. In the AGB case, only the $3\alpha$ rate is important, and significantly better accuracy is required.

It is likely that the uncertainty in the $3\alpha$ rate can be halved by ongoing experiments and analyses. Further improvements beyond this would be extremely difficult.

A final comment concerns the nature of the core mass in pre-supernova stars. It appears that rather strong mass enhancements can occur over a small range of $R$. Thus it is possible that the core collapse simulations use a mass that, for a 25 $M_{\odot}$ star, is too large (or too small) by perhaps 0.2$M_{\odot}$. Such a difference might be large enough to cause an explosion to fail or succeed just because the wrong model and core mass were used. It seems germane to ask whether sufficient attention is being paid to the detailed nature of
the pre-supernova stars employed in supernova calculations.

REFERENCES

1. F. Herwig and Sam M. Austin, Astrophys. J. 613, L73 (2004).
2. S. E. Woosley, A. Heger, T. Rauscher, and R.D. Hoffman, Nucl. Phys. A718, 3c (2003).
3. L. Buchmann, Astrophys. J. Lett. 468,127 (1996).
4. C. Angulo, et al., Nucl. Phys. A690, 765(1999).
5. H. Costantini, for LUNA Collaboration, Abstract F04.
6. J. A. Nolen, Jr., and S. M. Austin, Phys. Rev. C 13, 1773 (1976).
7. H. O. U. Fynbo, et al., Phys. Rev. Lett. 91, 082502 (2003).
8. H. O. U. Fynbo, Private communication, 2004.
9. C. A. Diget, unpublished.
10. H. Crannell, et al., Abstract F027, This Symposium.
11. A. Wuosmaa, K. Starosta, and Sam M. Austin, unpublished.
12. R. G. H. Robertson, R. A. Warner, and S. M. Austin, Phys. Rev. C 15, 1072 (1977).