A Simple Model for Mixing and Cooling in Cloud–Wind Interactions

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Abstract

We introduce a simple entropy-based formalism to characterize the role of mixing in pressure-balanced multiphase clouds and demonstrate example applications using ENZO-E (magneto)hydrodynamic simulations. Under this formalism, the high-dimensional description of the system's state at a given time is simplified to the joint distribution of mass over pressure ($P$) and entropy ($K = P/\rho^\gamma$). As a result, this approach provides a way to (empirically and analytically) quantify the impact of different initial conditions and sets of physics on the system evolution. We find that mixing predominantly alters the distribution along the $K$ direction and illustrate how the formalism can be used to model mixing and cooling for fluid elements originating in the cloud. We further confirm and generalize a previously suggested criterion for cloud growth in the presence of radiative cooling and demonstrate that the shape of the cooling curve, particularly at the low-temperature end, can play an important role in controlling condensation. Moreover, we discuss the capacity of our approach to generalize such a criterion to apply to additional sets of physics and to build intuition for the impact of subtle higher-order effects not directly addressed by the criterion.

Supporting material: animations

1. Introduction

Stellar-feedback-driven galactic outflows play a critical role in galaxy formation and evolution. It is important for regulating star formation and transporting metals out of galaxies (White & Rees 1978; Dekel & Silk 1986; White & Frenk 1991). To reproduce the observed galaxy properties, large-scale cosmological simulations often assume that these galactic winds effectively accelerate cool gas, with mass-loading factors of $\sim 1-100$ (e.g., Pillepich et al. 2018; Davé et al. 2019). Moreover, observations of outflows serve as direct evidence for the existence of stellar feedback. A robust test of simulations is their ability to reproduce these observations (Somerville & Davé 2015).

Observations have revealed that these winds are inherently multiphase (for reviews of observations, see Veilleux et al. 2005; Rupke 2018); there is cool gas comoving with hot gas. This multiphase nature presents a challenge to the conventional model that the outflows are driven by hot ($\gtrsim 10^6$ K) winds produced by supernovae. There has been a considerable effort to determine whether ram pressure acceleration of cool ($\sim 10^4$ K) ISM, by these winds, can produce a comoving multiphase flow (e.g., Klein et al. 1994; Cooper et al. 2009; Scannapieco & Brüggen 2015; Schneider & Robertson 2017; Sparre et al. 2019). Difficulties arise because hydrodynamical instabilities (e.g., Kelvin–Helmholtz and Rayleigh–Taylor) grow from the initial velocity difference between the cloud and wind and drive mixing between the phases; the phases can be homogenized before the cloud is entrained.

Klein et al. (1994) showed that this destruction of cool clouds is roughly characterized by the cloud-crushing timescale. For a nonradiative cloud with density $\rho_{\text{cl}}$ and radius $R_{\text{cl}}$, initially at rest with respect to a hot wind, with density $\rho_w = \rho_{\text{cl}}/\chi$ ($\chi \sim 100–1000$) and velocity $v_w$, this timescale is given by

$$t_{\text{cc}} = \frac{\chi^{1/2} R_{\text{cl}}}{v_w}.$$

Because the cloud is destroyed within a few $t_{\text{cc}}$, and $t_{\text{cc}}$ is a factor of $\sim \chi^{1/2}$ smaller than the ram pressure acceleration timescale, it is challenging for hot winds to entrain the cool gas before it is destroyed.

Subsequent studies have modeled additional physical effects in attempts to delay cloud disruption long enough for them to be embedded within the wind. The most common additional set of physics is radiative cooling (e.g., Cooper et al. 2009; Scannapieco & Brüggen 2015; Schneider & Robertson 2017). However, the general consensus was that the cloud’s lifetime is not prolonged enough for it to be fully entrained in the wind. Moreover, Zhang et al. (2017) compellingly showed (with semianalytic methods) that real observations cannot be reproduced by ram-pressure-accelerated cold clouds in the interval of time before they are destroyed.

Another approach for extending the cloud’s lifetime has been the inclusion of magnetic fields. Certain configurations can inhibit mixing and provide an additional tension force that resists destruction. McCourt et al. (2015) demonstrated that the presence of a tangled magnetic field in the cloud gave promising results for $\chi = 50$ (with the inclusion of radiative cooling). Unless magnetic pressure dominates ($\beta < 1$) in the wind, however, magnetic fields alone do not appear to inhibit the disruption of higher-density contrast clouds ($\chi \sim 100–1000$) enough to allow their entrainment (Gronke & Oh 2020a).
Other models have also been proposed to produce multi-phase outflows without requiring thermal supernova winds to entrain clouds. Alternatives include the acceleration of outflowing gas by nonthermal feedback like radiation pressure (e.g., Zhang et al. 2018) and cosmic rays (e.g., Wiener et al. 2019) or in situ cloud formation within a cooling outflow (e.g., Thompson et al. 2015; Schneider et al. 2018; Lochhaas et al. 2021). These alternatives have achieved varying degrees of success, but no single model appears to apply in all cases. Recent work (Armillotta et al. 2016; Gronke & Oh 2018) has shed new light on cloud acceleration by a hot wind in the limit of rapid cooling. Gronke & Oh (2018, 2020a) showed that if mixed gas cools sufficiently fast, then it becomes part of the colder cloud phase before it is further homogenized with the wind. We hereafter refer to this process as turbulent radiative mixing layer (TRML; which we pronounce as “turbanmoil”) entrainment. This process not only inhibits the depletion of the cloud mass but also transfers mass and momentum to it from the wind (similar to an inelastic collision).

Gronke & Oh (2018) argued that this process occurs when the mixing timescale, $\tau_{\text{mix}}$, exceeds the cooling timescale of the mixing layer, $\tau_{\text{cool,mix}}$; the mixing layer has a temperature $T_{\text{mix}} \sim \sqrt{T_{\odot} T_{\text{w}}}$ and number density $n_{\text{mix}} \sim \sqrt{n_{\odot} n_{\text{w}}}$. They recast this criterion, $\tau_{\text{cc}} > \tau_{\text{cool,mix}}$, as a radius requirement for a spherical cloud. Clouds should survive when their radius exceeds

$$R_{\text{cl,ent}} \sim \frac{V_{\text{w}} T_{\text{cool,mix}}}{\chi^{1/2}} \approx 2 \text{ pc} \frac{T_{\odot}^{5/2} M_{\text{w}}}{P_{3} \Lambda_{\text{mix},-21.4} 100},$$

where $T_{\text{cl,d}} \equiv T_{\odot}/(10^{4} \text{ K})$, $M_{\text{w}}$ is the Mach number of the wind, $P_{3} \equiv n_{\text{w}}/(10^{3} \text{ cm}^{-3})$, and $\Lambda_{\text{mix},-21.4} \equiv \Lambda(T_{\text{mix}})/(10^{21.4} \text{ erg cm}^{-3} \text{ s}^{-1})$.3

This picture has been bolstered by recent related studies of individual shear layers (Ji et al. 2019; Fielding et al. 2020; Tan et al. 2021). These simulations lack the overall cloud geometry but are able to reach significantly higher resolution. The key finding from these studies is that the crucial parameter that determines the rate of cooling and the rate at which wind material is advected into the cloud is the ratio of the cooling time to the eddy turnover time. The eddy turnover time is comparable to the cloud-crushing time.

Recently, Li et al. (2020) found a different survival criterion, based on the cooling time of the wind, $\tau_{\text{cool,w}}$. They argue that clouds survive, when $\tau_{\text{cool,w}}/\tau_{\text{cc}} < 10^{f}$, while $f$ is an order-unity term with weak power-law dependence (exponents range from $\sim 0.3$ to $\sim 0.6$) on $R_{\text{cl,d}}$, $n_{\text{w}}$, and $V_{\text{w}}$. Sparre et al. (2020) reached a similar result (with additional $M_{\text{w}}$ dependence), while Kanjilal et al. (2021) found results in support of the Gronke & Oh (2018) criterion. The differences in the criteria seem to largely arise from differences in how cloud destruction is defined (Kanjilal et al. 2021; Sparre et al. 2020). However, some debate remains over the dominant physical effects.

Prior works have clearly assembled models for how various circumstances and physical effects modify the cloud–wind interaction. Unfortunately, simple characterizations of how different effects influence the interaction are often incompatible with one other. The complex multidimensional nature of this process is a barrier to making comparable and composable characterizations. We, therefore, explore higher-order characterization of the mixing and cooling processes in these systems in order to isolate the competing effects.

In this paper, we present a simple entropy-based formalism for characterizing how different physical effects affect mixing (and other destruction processes). This mixing model builds on the premise that changes in the pressure–entropy ($p−K$) phase distribution broadly capture the cloud–wind interaction’s evolution. Our approach is particularly conducive to comparing the impact of radiative cooling against mixing. Furthermore, it naturally complements the Gronke & Oh (2018, 2020a) physical model for TRML entrainment.

Our paper is organized as follows: In Section 2 we motivate and describe the formalism, and in Section 3 we describe the numerical methods used to test it. Videos of our simulations can be found at http://matthewabruzzo.com/visualizations/. Subsequently, in Section 4 and Section 5, we describe our results from two example applications that demonstrate how the formalism both (i) broadly captures the system’s evolution and (ii) can quantitatively characterize how different conditions and physical effects modify the system’s evolution. These applications include a nonradiative parameter study (Section 4) and a more detailed study involving radiative cooling (Section 5). Finally, we discuss the significance of our results in Section 6 and summarize our conclusions in Section 7.

2. Model Overview

2.1. $p−K$ Representation

The cloud–wind interaction is fundamentally an interaction between a finite pool of colder, dense gas (the cloud) and a large reservoir of hotter, more diffuse gas (the wind). We are interested in understanding how different physical conditions affect the interaction’s outcome. The two outcomes are (i) the homogenization of the colder phase within the more abundant hotter phase (cloud destruction) or (ii) the long-term survival and coexistence of both phases (entrainment). It is instructive to specify the system’s state purely in terms of this thermodynamic description.

Figure 1 helps illustrate this premise for a nonradiative wind interaction. In this section, the reader should just understand that the figure depicts a wind-tunnel simulation of a cloud–wind interaction, where gas is initialized in pressure equilibrium (more details about this $R_{\text{cl}}/\Delta = 64$ NR×X100 simulation are provided later in Section 3.1). The left column illustrates snapshots of the system’s morphological evolution while the center-left column shows the $n−T$ phase-space evolution. We note that the “jet”-like structure in the morphological evolution is a consequence of the symmetric initial conditions. The initial properties of each gas phase are denoted by black circles in the phase diagrams. At early times, the initial shock introduces pressure perturbations and slightly elevates the entropy of some fluid elements originating in the colder phase. As the interaction progresses, adiabatic mixing drives gas from the colder, dense phase toward the hotter,

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3 Following the arguments from Begelman & Fabian (1990), (when $\mu$ has $\rho$ and $P$ dependence) we find that $T_{\text{mix}} \sim \sqrt{\rho_{\text{mix}} T_{\odot}}$ and $n_{\text{mix}} \sim \sqrt{n_{\odot} n_{\text{w}}}$, and $R_{\text{cl,ent}}$ scales with $\mu_{\odot}^{5/2}$. Using CRACKLE (Smith et al. 2017), in tabulated mode, we find slightly different characteristic values for solar metallicity, $\chi = 100$, $M_{\odot} = 1$, $T_{\odot} = 1$, and $P_{3} = 1$ Under these conditions, $\mu_{\odot} = 0.827$ and $R_{\text{cl,ent}} \sim 6$ pc ($\Lambda_{\text{mix}}$ is unchanged).
diffuse phase. At a given snapshot, the phase distribution clearly encodes information about the system’s state.

Although number density, $n$, and temperature, $T$, are familiar thermodynamic quantities, we choose to base our mixing model on $p-K$ phase space (center-right column of Figure 1). The quasi-isobaric nature of the problem makes pressure, $p$, an intuitive choice for a phase space axis. Although the supersonic wind seeds small transient pressure perturbations, the second phase-space dimension effectively indexes the continuum of properties gas can have between the two initial states. Following convention, we pair $p$ with an entropy-like quantity, $K = p\rho^{-\gamma}$ (hereafter, we refer to $K$ as entropy), and we take $\gamma = 5/3$ throughout this work. The dashed (dotted) lines in the center column of Figure 1 denote contours of constant $p$ ($K$) that increase by a factor of 10.

The choice of $p-K$ space over $p-T$ space is somewhat discretionary. Because $T$ directly characterizes thermal energy, it more directly governs heat flow (e.g., cooling). However, it is easier to characterize scale-free, dynamical effects in terms of $K$; relating $T$ to hydrodynamic quantities requires scale-dependent knowledge about the mean molecular mass, $\mu$. Additionally, the entropy of a fluid element is unchanged by compression or expansion, which explains why the spread in gas at a given pressure lies along $K$ contours in the center column of Figure 1. Only irreversible processes change entropy: shocks and mixing increase it while radiative cooling.

Figure 1. Illustration of the cloud–wind interaction until $8.5t_{\text{cc}}$ for an initially spherical cloud, a wind with Mach number $M_w = 1.5$ and density contrast $\chi = 100$ at a resolution of $R_{cl}/\Delta x = 64$. The left column illustrates the cloud surface density. We note that the simulated domain is far longer than these panels depict (the shock remains in the domain over the entire simulation) and the axis labels do not describe the position in the domain. The center columns depict the mass-weighted $n-T$ (center left) and $p-K$ (center right) phase diagrams of all gas in the simulation. The $n-T$ diagrams are computed assuming a fixed mean molecular mass of $\mu = 0.6$. The black circles near the top (bottom) of the panels denote the initial location of fluid elements originating in the wind (cloud). The dashed (dotted) gray lines in the $n-T$ diagram denote logarithmically spaced lines of constant $p$ ($K$). The rightmost column depicts the one-dimensional entropy distribution. The red histogram traces all gas in the domain, while the blue traces just the fluid elements initialized in the cloud. Each row of the figure shows panels from a single frame of the animation. The cloud appears to move upstream in the animation because the density panels follow the center of mass of gas originating in the cloud. In reality, the cloud moves downstream.

(An animation of this figure is available.)
decreases it. Moreover, fluid elements have continuous trajectories through \( p-K \) space in the absence of shocks because mixing and cooling modify \( K \) smoothly.

We now take a more careful look at the \( p-K \) representation and evolution for the nonradiative, hydrodynamic cloud–wind interaction. At initialization, all gas has a single pressure \( p \), the colder phase lies at \( K_{cl} = p \rho_{cl}^{-5/3} \), and the hotter phase lies at \( K_{h} = \chi^{3/4}K_{cl} \). As the system evolves, hydrodynamical instabilities drive the mixing of the two phases. Mixing increases the entropy of the colder-phase gas and initially decreases the entropy of the hotter-phase gas. By the time the cloud is destroyed, all gas has an entropy of \( K \sim K_{h} \). The gas pressure remains relatively constant throughout this process.

The precise \( p-K \) evolution depends on the initial conditions and the simulated physics. Changes to either may modify how the distribution evolves. Herein lies the true value of this thermodynamic description of the system’s state: it provides a low-dimensional domain for characterizing mixing that is well suited for comparing different physical processes.

### 2.2. Mixing Model

Consider the motion of fluid elements originating in the colder phase through \( p-K \) space; this is illustrated by the red arrow in Figure 1. We anticipate factors that inhibit mixing, like strong magnetic fields, to decelerate the rate that fluid elements increase in entropy. Conversely, we expect factors that hasten mixing, such as larger values of \( M_{p} \), to accelerate that rate. Thus, we can empirically model mixing by characterizing the fluid elements’ motion through \( p-K \) space.

For simplicity, we largely ignore pressure perturbations in the context of mixing. Motion along the pressure dimension may be particularly relevant in cases with strong sources of nonthermal pressure support. The right column of Figure 1 illustrates just the entropy evolution of fluid elements.

In this work, we focus on the motion of fluid elements that originate in the cloud. Because we trace these fluid elements with a passive scalar, we refer to their total mass as \( M_{ps} \). We defer analysis of the \( p-K \) evolution for fluid elements originating in the hot phase to a future work. These fluid elements encode information that is most relevant at times and locations in phase space where the motion of fluid element from the clouds are not representative of all fluid elements at that location. We briefly revisit this point in Section 3.2.

Under these assumptions, the cloud–wind interaction at a time \( t \) can be quantitatively described by its initial pressure \( (p_{0}) \), the distribution of the initial cloud fluid elements with respect to entropy, \( (dM_{ps}/dK)(K, t) \), and \( (K, t) \), the ensemble-averaged Lagrangian derivative of \( K \) for all initial cloud fluid elements with a given value of \( K \). The form of \( (K, t) \) dictates the outcome of the interaction. It is dependent on the initial conditions and modeled physics (in full generality, the notation resembles \( K(K, t; p_{0}, \gamma, \chi, R_{cl}, M_{ps}, \beta, \ldots) \)).

For purely nonradiative (magnetohydrodynamic) interactions, the only sources of entropy are the initial shock and mixing. Because we expect mixing to dominate outside of highly supersonic flows, we refer to \( (K, t) \) as \( K_{mix}(K, t) \) for these simulations. Because of the scale-free nature of such an interaction, the value of \( K_{mix}(aK_{cl}, bK_{cl})/(K_{cl}t_{cc}^{-1}) \), where \( a \) and \( b \) are arbitrary positive values, is constant for any choice of \( \rho_{cl}, p_{0}, \) and \( R_{cl} \) (as long as \( \gamma, \chi, M_{ps}, \beta, \) and the initial geometry remain unchanged).

When \( K_{mix}(K, t) \) is known, it can be used to predict the outcome of interactions involving radiative cooling through comparisons against expected contributions from cooling, \( K_{cool}(p, K) \). For optically thin gas, \( K_{cool} = K_{e} = -K/t_{cool} \) and Figure 2 illustrates \( t_{cool}(p, K) \). When \( K_{mix}(K, t) + \dot{K}_{cool}(p_{0}, K) \gg 0 \) for \( K \in [K_{cl}, K_{h}] \), we generally expect the colder phase to be destroyed. Conversely, the existence of a large subinterval over \([K_{cl}, K_{h}]\), where the sum is far less than zero, suggests the long-term survival of a cool phase. The outcome is ambiguous when the sum is close to zero because the sum does not directly give the total \( K(K, t) \) for an interaction with cooling (hereafter \( K_{cool}(K, t) \)).

This reasoning is reminiscent of the comparisons between \( t_{cc} \) and \( t_{cool,mix} \) that underlie the Gronke & Oh (2018) survival condition. In fact, we can apply analogous arguments in the context of our mixing model to derive a criterion comparable to \( t_{cc} \) \( \gg t_{cool,mix} \). For simplicity, suppose \( K_{mix}(K_{mix}) \) gives the characteristic mixing rate for a nonradiative interaction at entropy \( K \) at times when \( dM_{ps}/dK(K, t) > 0 \) (in Section 4.3 we confirm that \( K_{mix}(K_{mix}) \) is indeed a well-conceived quantity). Then, following their logic, we expect TRML entrainment to occur when \( K_{mix}(K_{mix}) < |K_{cool}(p_{0}, K_{mix})| \), where the entropy at the mixing layer is \( K_{mix} \sim \sqrt{K_{cl}K_{h}} \).

### 3. Methods

#### 3.1. Simulations

To run magnetohydrodynamical (MHD) simulations for this work, we make use of the ENZO-E4 code. This code is a rewrite of ENZO (Bryan et al. 2014) that targets exascale computing and is built on the distributed, scalable, adaptive mesh refinement framework CELLO (Bordner & Norman 2012, 2018).
Although ENZO-E is still under active development, it has matured enough that it can be used for basic scientific studies.

For this work, we implemented the second-order accurate unsplit VL + CT (van Leer + Constrained Transport) algorithm presented by Stone & Gardiner (2009). This is a predictor–corrector scheme that employs the Constrained Transport (CT) method (Evans & Hawley 1988). Each simulation uses second-order reconstruction and the HLLD Approximate Riemann Solver (Miyoshi & Kusano 2005). We provide a brief assessment of how the choice of integrator affects the evolution of the cloud–wind interaction in Appendix A.

In our cloud–wind simulations, we solve the ideal, adiabatic MHD equations on a fixed, uniform three-dimensional Cartesian grid. We initialize each simulation with a spherical cloud of radius $R_c$ embedded in a steady wind and a mass density that is a factor of $\chi$ lower than the cloud. The cloud initially has no bulk velocity and is in pressure equilibrium with the wind. We also initialize a passively advected scalar that traces the gas initially confined to the cloud. Most of our nonradiative simulations use $M_* = 1.5$, but we consider a subset with varying $M_*$ for exploratory purposes. In another subset, we also initialize magnetic fields transverse to the wind velocity that are constant throughout the entire domain.

Our runs including radiative cooling employ the GRACKLE chemistry and cooling library (Smith et al. 2017), assume solar metallicity, and do not use the self-shielding approximations. All of our cooling runs employ GRACKLE’s tabulated mode, which models the heating and cooling rates of H and He similarly to those of metals (i.e., the rates are interpolated from precomputed tables). Our main cooling runs use tables that are shipped with GRACKLE and were constructed under the assumptions of ionization equilibrium and incident radiation given by the Haardt & Madau (2012) UV background model at $z = 0$. In Section 5.3 we also consider cooling runs with tables that store custom broken power-law cooling curves.

For simplicity, we restrict cooling to only occur between $\sim T_{cl}$ and $\sim 0.6 T_w$. We primarily introduced the floor to ease comparisons with prior work (e.g., Gronke & Oh 2018). To restrict cooling, we have modified both the tables of heating and cooling rates such that $\Lambda/m_i^2 = \Gamma/m_i^2 = 10^{-99} \text{erg cm}^3 \text{s}^{-1} \text{g}^{-2}$ in the restricted regions. We assess the consequences of restricted cooling in Appendix A.

The simulation domain extends $100R_e$ downwind of the cloud’s initial location and $12R_e$ along each transverse dimension. Gas with wind properties (including magnetic fields, if $\beta$ is finite) flows into the domain $20R_e$ upwind of the cloud’s initial center of mass. We enforce outflow conditions for the other boundaries. Table 1 provides a summary of our simulation properties. The primary simulations use to assess the cloud–wind interactions including radiative cooling have $R_e/\Delta x = 64$. We briefly investigate the impact of resolution in Appendices A and C.

Finally, we note that our simulations employ a reference frame tracking scheme. Unfortunately, an implementation bug caused our $R_e/\Delta x < 64$ simulations to effectively have no frame tracking (compared to $v_w$, the frame velocity is near-zero). However, our $R_e/\Delta x = 64$ simulations used an improved version of the code, in which every 0.0625$c_{\text{esc}}$, the scheme correctly modifies the frame velocity such that the minimum velocity of cells with passive scalar densities $\geq 0.1\Delta x$/1000 is zero postupdate. However, if an update would cause the frame velocity to decrease, it is held constant instead. This strategy was selected to ensure that the bow shock remained in the simulation domain. In Appendix B we further discuss the differences between the two versions of the code used for this work.

### 3.2. $\dot{K}$ Calculation

As a postprocessing step, we estimate $\dot{K}$ as a function of $K$ and $t$ for each of our simulations. Recall that $\dot{K}(K, t)$ is the Lagrangian entropy derivative averaged over all fluid elements originating in the cloud with entropy $K$. $\dot{K}$ is simply $\dot{K}$ averaged over some time interval $\Delta t$.

Because mass (or in this case, passive scalar mass) is conserved, we can write an analog to the continuity equation:

$$\frac{\partial}{\partial t} (\frac{m_{\text{ps}}}{K}) + \frac{\partial}{\partial K} (\frac{Km_{\text{ps}}}{dK}) = 0. \quad (3)$$

This describes the changes in passive scalar mass profile as a function of $K$ for a pair of snapshots measured at $t^n$ and $t^{n+1}$.

Consider a set of discrete $K$ bins where the $i$th bin has center $K_i$, width $\Delta K_i$, and encloses a passive scalar mass of $m_{\text{ps},i}$. Integrating Equation (3) in time from $t^n$ to $t^{n+1}$ and over the $i$th $K$ bin (from $K_{i-1/2} = K_i - 0.5\Delta K_i$ to $K_{i+1/2} = K_i + 0.5\Delta K_i$) yields

$$\frac{m_{\text{ps},i}^{n+1} - m_{\text{ps},i}^n}{\Delta K_i} = \left(\frac{K_{\text{ps}}}{dK} \right)^{n+1/2} - \left(\frac{K_{\text{ps}}}{dK} \right)^{n+1/2}, \quad (4)$$

where $(K_{\text{ps}}/dK)^{n+1/2}$ is time-averaged between $t^n$ and $t^{n+1}$. By selecting a minimum bin, $K_1$, such that $K_{1/2} = 0$ at all $t$, we can compute $(K_{\text{ps}}/dK)^{n+1/2}$ at all bin interfaces from changes in the measured profiles.

Finally, if we assume that $(dK_{\text{ps}}/dK)_{i+1/2}$ is near constant between $t^n$ and $t^{n+1}$, then

$$\dot{K}_{i+1/2} \approx \left(\frac{K_{\text{ps}}}{dK} \right)^{n+1/2} \left(\frac{dK_{\text{ps}}}{dK} \right)_{i+1/2}. \quad (5)$$

We approximate $(dK_{\text{ps}}/dK)_{i+1/2}^n$ via linear interpolation of the average $dK_{\text{ps}}/dK$ from adjacent bins and then average the values from $t^n$ and $t^{n+1}$ to estimate $(dK_{\text{ps}}/dK)_{i+1/2}^n$.

If $K_{i+1/2}$ is a well-behaved function of time, and $dK_{\text{ps}}/dK_{i+1/2}$ has linear temporal dependence (between $t^n$ and $t^{n+1}$), one can show that the magnitude of the relative error for our $\dot{K}_{i+1/2}$ measurement is proportional to $f(f+2)$, where $f$ is
Table 1: Table of Simulations (Additional Simulations are Discussed in the Appendices)

| Name             | $\chi$ | $M_\infty$ | $c^b$ | $R_d$ (pc) | $T_\text{d}$ (K) | $R_d/\Delta x$ | Cooling$^d$ | $t_{\text{cool, max}}/t_{\text{cool, cl}}$ | $R_d/t_{\text{cool}}$ | $t_{\text{cool, max}}/t_{\text{cool, cl}}$ |
|------------------|--------|------------|-------|-----------|----------------|----------------|-------------|---------------------------------|----------------------|---------------------------------|
| NR-X100          | 100    | 1.5        | $\infty$ | 1         | $4 \times 10^4$ | 8, 64          | N           |                                 |                      |                                 |
| NR-X300          | 300    | 1.5        | $\infty$ | 1         | $4 \times 10^4$ | 8, N           | N           |                                 |                      |                                 |
| NR-X1000         | 1000   | 1.5        | $\infty$ | 1         | $4 \times 10^4$ | 8, N           | N           |                                 |                      |                                 |
| NR-X100-M0.75    | 100    | 0.75       | $\infty$ | 1         | $4 \times 10^4$ | 8, N           | N           |                                 |                      |                                 |
| NR-X100-M3       | 100    | 3          | $\infty$ | 1         | $4 \times 10^4$ | 8, N           | N           |                                 |                      |                                 |
| NR-X100-M4.5     | 100    | 4.5        | $\infty$ | 1         | $4 \times 10^4$ | 8, N           | N           |                                 |                      |                                 |
| NR-X100-B10      | 100    | 1.5        | 10      | 1         | $4 \times 10^4$ | 8, N           | N           |                                 |                      |                                 |
| NR-X100-B100     | 100    | 1.5        | 100     | 1         | $4 \times 10^4$ | 8, N           | N           |                                 |                      |                                 |
| SlowRC-T1e4      | 100    | 1.5        | $\infty$ | 1         | $10^4$       | 64, Y          | 8.982, 0.668 | 1.044                           |                      |                                 |
| FastRC-T4e4      | 100    | 1.5        | $\infty$ | 5050      | $4 \times 10^4$ | 64, Y          | 0.2165, 388, 63.06 |                      |                      |                                 |
| FastRC-T1e4      | 100    | 1.5        | $\infty$ | 57        | $10^4$       | 64, Y          | 0.1576, 38.1, 1.044 |                      |                      |                                 |
| BPLawRC-1        | 100    | 1.5        | $\infty$ | 66.51     | $10^4$       | 8, BPL         | 0.1576, 9.52, 1 |                      |                      |                                 |
| BPLawRC-2        | 100    | 1.5        | $\infty$ | 66.51     | $10^4$       | 8, BPL         | 0.1576, 19.0, 2  |                      |                      |                                 |
| BPLawRC-6        | 100    | 1.5        | $\infty$ | 66.51     | $10^4$       | 8, BPL         | 0.1576, 57.1, 6  |                      |                      |                                 |
| BPLawRC-60       | 100    | 1.5        | $\infty$ | 66.51     | $10^4$       | 8, BPL         | 0.1576, 571, 60 |                      |                      |                                 |

Notes. All simulations were initialized with an initial thermal pressure of $p/k_B = 10^3$ cm$^{-3}$ K.
$^a$ Sonic Mach number of the wind.
$^b$ Plasma beta (thermal pressure divided by magnetic pressure). All cells (in both the wind and cloud) are initialized with this value, and it is also the value for the inflowing wind fluid over the course of the simulation.
$^c$ For nonradiative simulations, $T_d$ is computed assuming $\mu = 0.6$.
$^d$ Indicates whether radiative cooling is included. Simulations with the value “BPL” used a custom broken power-law cooling curve with shape given by the specified $\eta_{\text{mix, cl}} = t_{\text{cool, max}}/t_{\text{cool, cl}}$ and Equation (7) (and a constant $\mu$ of 0.6).
$^e$ $\xi_{\text{cool}}$ is the “cooling length,” $t_{\text{cool}} = \min(t_{\text{cool, cl}})$ (McCourt et al. 2018).

given by

$$f = \frac{\min((d M_{ps}/d K)^{n+1}_{i+1/2}, (d M_{ps}/d K)^{n+1}_{i-1/2}) - (d M_{ps}/d K)^{n}_{i+1/2})}{(d M_{ps}/d K)^{n+1}_{i+1/2}}.$$ (6)

To minimize the impact of this error, our analysis focuses on $\frac{\Delta K}{K}$ measurements where $f$ is less than 0.25, and we omit measurements altogether where $f > 2$. While our measurements of $\frac{\Delta K}{K}$ may be somewhat biased, the overall dependence on $K$ and $t$ is still useful, particularly when the dependence is stable in time.

In practice, we estimate $\frac{\Delta K}{K}$ from pairs of snapshots satisfying $\frac{\Delta n}{n} = t^{c^b} + t_{\text{cool}}/2$. If the passive scalar advects into or out of the simulation domain, then Equation (3) should include an additional source or sink term. Thus, we only consider snapshots at times before 1% of the initial passive scalar mass escapes the domain. We note that vorticity near the transverse outflow boundaries can introduce artificial passive scalar inflow. In practice, this is only an issue for NR-X100-M0.75, and we conservatively discard all data from that run measured after $T_{\text{cool}}$.

Appendix C includes a brief study on how resolution affects measurements of the passive scalar mass profile and our measurements of $\frac{\Delta K}{K}$ ($K$, $t$). Because the profile’s evolution is most sensitive to resolution for $K$ bins holding under 1% of the total passive scalar mass $M_{ps}$, our subsequent analysis primarily focuses on $\frac{\Delta K}{K}$ measurements where $m_{i-1}^n$, $m_i^n$, $m_{i+1}^n$, and $m_{i+2}^n$ are all $\geq 0.01 M_{ps}$.

For simulations with no cooling or inefficient cooling, the dependence of our $\frac{\Delta K}{K}$ measurement on $K$ and $t$ is remarkably robust with respect to resolution. At the same time, measurements for simulations with rapid cooling are less robust. In the appendix, we argue that this is not surprising given our measurement method and that the measurements are adequate for conveying the utility of our mixing model. In these simulations, mixing would probably be better characterized by measurements of the average Lagrangian entropy derivative for all fluid elements in the system (rather than just those originating in the cloud).

4. Nonradiative Parameter Study Results

4.1. $p$–$K$ Phase Space

In this section, we apply our formalism to a suite of (magneto)hydrodynamic nonradiative simulations that probe a wide variety of properties.

Figure 3 illustrates the time evolution of the passive-scalar weighted $p$–$K$ distribution for three hydrodynamic simulations with varying $\chi$ (NR-X100, NR-X300, NR-X1000). Unlike the panels in the center-right column of Figure 1, these only show phase distributions for fluid elements originating in the cloud, depict lower-resolution simulations, and have rescaled entropy axes. Throughout this work, we plot entropy as $\log_\gamma (K/K_{cl})$ to remove most of its $\gamma$ dependence. With this definition, it spans values from 0 through $\gamma = 5/3$.

The figure indicates that the initial shock does not significantly alter the entropy of the cloud. Instead, mixing is the primary source of entropy and gradually moves the fluid elements up from $K_{cl}$ to $K_{ps}$ (denoted by black circles). Fluid elements that have already exited the domain should generally have $K$ comparable to or larger than the remaining ones because they mixed faster. We avoid comparisons involving panels for times when a significant fraction ($\geq 1\%$) of the fluid elements have exited. The higher $\chi$ simulations lose fluid elements more quickly because they have larger $v_{cc}$. The dotted black line denotes $\mu_{cl}/3$, a density threshold commonly used to identify cloud mass (e.g., Scannapieco & Brüggen 2015; Schneider & Robertson 2017; Gronke & Oh 2018). The figure shows that the vast majority of fluid elements cross this threshold by $\sim 4.5\Delta t_{\text{cool}}$ (as discussed in Appendix C there is some resolution dependence), which is consistent with prior work (e.g., Sparre et al. 2019). The log$_\gamma$
scaling clearly removes most of the $\chi$ dependence in the entropy evolution.

Each simulation’s $p$ follows a common evolution, largely independent of $\chi$. In each case, the shock initially produces large $p$ perturbations. By $2.5t_{cc}$, the motions giving rise to the under-pressured gas have been slightly damped. While not shown, the distribution’s extent is stable between $\sim 1.5t_{cc}$ and $\sim 3.5t_{cc}$, albeit with minor fluctuations in the minimum. The low-pressure gas at these times is presumably supported by the vorticity produced by the initial shock, the postshock flow in the shearing layer (at the cloud boundary), and the formation of the vortex rings (Klein et al. 1994). Between $3.5t_{cc}$ and $4.5t_{cc}$ the minimum pressure drops once more and subsequently all pressure perturbations damp away.

The $\chi$ dependence manifests in the $p$ distribution in two main ways. First, the minimum pressure at $4.5t_{cc}$ is larger for $\chi = 100$ than it is in the other cases. While it is unclear how robust this difference is, we note that Klein et al. (1994) reported that the postshock flow was the primary generator of vorticity in their $\chi = 10$, $M_w = 0.9$, 2D ellipsoidal cloud simulation and argued that it scales with $\sim \chi^{1/2}$. However, we note that the minimum pressure at $2.5t_{cc}$ is slightly larger for $\chi = 1000$ than in the other cases.

The other difference is that the mode of the $\chi = 1000$ pressure distribution has a positive offset at $8.5t_{cc}$\textsuperscript{10} This is an unexpected artifact caused by the reflection of waves and discontinuities off of the transverse outflow boundaries. We reran these simulations with three times larger transverse widths and found that this artifact is first noticeable at $\sim 6t_{cc}$. Furthermore, while all three simulations were affected, the magnitude strongly scaled with $\chi$. We do not expect this effect to significantly influence our results because subsequent analysis of NR-X300 and NR-X1000 (our only $\chi > 100$ simulations) ignores data at $t/t_{cc} > 5.5$ and 4.5 (more than 1% of the passive scalar leaves the domain by these times).

Having explored the impact of $\chi$ on the gas distribution, we will now briefly consider the impact of the Mach number $M_w$. While not shown, we find that the entropy distribution is broadly unchanged with $M_w$, while the size of the pressure perturbation increases with $M_w$. In particular, the NR-X100–M0.75 fluid elements extend over a narrower $p$ interval and evolve in the $K$ direction slightly more quickly than those of NR-X100, with more general morphological differences in the 2D distribution forming by $2.5t_{cc}$. In contrast, NR-X100–M3 and NR-X100–M4.5 have larger $p$ perturbations than even NR-X300 or NR-X1000 and those perturbations damp more slowly. We note that the stronger initial shock also has a greater impact on the cloud’s entropy and the fluid elements from the cloud can overshoot $K_w$. We defer future consideration of the $M_w$ dependence of the $p-K$ distribution to the future and largely focus on $M_w = 1.5$ simulations for the remainder of this work.

Next, we consider the effect of magnetic fields. Figure 4 illustrates the phase evolution of the fluid elements originating
in the cloud for three simulations with transverse magnetic fields of different strengths (NR-X100-B10, NR-X100-B100, and NR-X100-B1000). We remind the reader that magnetic fields are initially uniform throughout the domain. In contrast to the pure hydro cases, cloud destruction proceeds far more slowly when $\beta = 10$. It takes longer for fluid elements to cross the $\rho_3/3$ threshold, and at $t = 8.5t_{cc}$, a much larger fraction of the fluid elements have $K < K_w$ (and lie within the simulation domain).

As $\beta$ increases, the clouds are more readily destroyed and the phase distributions bear greater resemblance to those of the pure hydro simulations. The minimum pressure appears to correlate with the initial $\beta$ at early times. This suggests that it is supported by magnetic stress. Because the magnetic fields are initially transverse, we expect the shock that propagates through the cloud to transfer energy to the magnetic field, thereby elevating the magnetic pressure. We expect the pressure to be less supported by vorticity (than in the purely hydrodynamical case) because magnetic fields impede its growth. We defer an assessment of how variations in the initial magnetic field configuration affect mixing to future work.

Both figures clearly illustrate that much of the interesting evolution of the cloud-crushing problem occurs over the $K$ dimension. While there is variation in the pressure, it is a transient effect that largely fades away at late times and is smaller than the variation in $K$. However, these same pressure variations appear to be more prominent in our $M_w > 1.5$ simulations.

4.2. Passive Scalar Mass Distribution over $K$

Having qualitatively established that the bulk motion of cloud fluid elements through $p-K$ space both occurs primarily along $K$ and reflects the initial conditions and modeled physics, we now consider a more quantitative parameterization of mixing.

If we assume that pressure perturbations are broadly unimportant for the system’s evolution, we can integrate over the phase distribution’s pressure dependence to get $dM_{ps}/dK$. We effectively trade the information encoded in the pressure perturbations for a dimensionality reduction. Recall that $(dM_{ps}/dK) dK$ specifies the mass of all fluid elements with entropy between $K$ and $K+dK$. Figure 5 illustrates the time evolution of $(dM_{ps}/dK) dK$ for a selection of times for each nonradiative simulation listed in Table 1 with a resolution of $R_{cl} / \Delta x = 8$.

At $t = 0.5t_{cc}$, each simulation’s profile has a peak at $K_w$ and a long tail extending to $K_c$. Over time, mixing increases the entropy of the fluid elements near the lower edge of the histogram, $K_{min}$. By $t = 3.5t_{cc}$, $K_{min}$ starts to increase, indicating that all of the fluid elements from the cloud have started mixing. We largely ignore differences in the profiles at intermediate and late times that are depicted by dashed lines because different fractions of passive scalar remain in the simulation domain when those are measured.

For the hydrodynamical simulations (top two rows of Figure 5), the narrow histograms near $K_w$ at $t = 8.5t_{cc}$ reflect how the initial cloud fluid elements have largely homogenized with the wind. The figure shows that the scaling of our $K$ bins and $t$ in terms of $\log_2(K/K_w)$ and $t_{cc}$ almost entirely captures the evolution’s $\chi$ dependence. This $t$ scaling also largely removes the $M_w$ dependence for supersonic simulations. However, the histograms’ upper edges do scale weakly with $M_w$; this may be a consequence of the entropy from the initial shock.

The subsonic run, NR-X100-M0.75, has the most unique evolution. In this case, mixing appears to more rapidly increase...
entropy below $\log (K/K_{cl}) \sim 0.7$, and the intermediate distributions develop a more prominent central peak. We defer further examination of the subsonic cloud–wind interaction to future work.

Finally, we turn to the MHD simulations (bottom row of Figure 5). At early times ($t \lesssim 2.5t_{cc}$) the profile evolution is largely the same as before, but by $t = 3.5t_{cc}$ the suppression of mixing by the magnetic fields causes the evolutionary paths to diverge. Because stronger fields (in a given configuration) more strongly suppress mixing, the rate at which $K_{\text{min}}$ increases scales with increasing $\beta$.

### 4.3. Mixing Rate Estimation

In this section, we use this distribution evolution to estimate the rate at which fluid elements from the cloud mix. Figure 6 illustrates our measurements of $\dot{K}(K,t)$, averaged over $\Delta t = t_{cc}/2$, as functions of $K$ for each of our simulations. See Section 3.2 for explanations of the calculation and how we identify the best measurements. As discussed in Section 2.2, we refer to these measurements as $\dot{K}_{\text{mixing}}(K,t)$ because mixing is the dominant entropy generation mechanism.

In each hydro simulation, $\dot{K}_{\text{mixing}}/(K_{cl}^{-1}t_{cc}^{-1})$ broadly has a power-law relationship in terms of $K/K_{cl}$, with a near-unity slope for $0.3 \lesssim \log (K/K_{cl}) \lesssim 1.4$. The section below $\log (K/K_{cl}) \sim 0.8$ may be slightly steeper ($\sim 1.1$ for NR-X100) while the upper section’s slope may be slightly shallower ($\sim 0.9$ for NR-X100) and could be time dependent. Note that the top row of Figure 25 from Appendix C suggests that these trends are robust to resolution effects.

The higher $\chi$ simulations appear to have slightly steeper slopes than NR-X100, but the dearth of high-quality measurements makes this comparison tenuous, especially at high $K$. The higher $M_w$ simulations have greater variance in their measurements than NR-X100 (possibly due to their stronger initial shocks) but are otherwise broadly consistent. Note that for the higher $M_w$ simulations, $\dot{K}_{\text{mixing}}(K,t)$ may include nonnegligible entropy changes from the initial shock. Without better measurements, we are unable to make any comparisons with NR-X100-$M0.75$.

Next, we consider the MHD simulations. As in Figure 5, the $\beta \leq 100 \dot{K}_{\text{mixing}}(K,t)$ measurements only start diverging from the NR-X100 measurements at $\sim 3.25t_{cc}$. The suppression of mixing gives $\dot{K}_{\text{mixing}}(K)$ a shallower slope. As the initial field...
strength decreases, the measurements more closely resemble those from NR-X100.

Figure 7 illustrates $K/\dot{K}_{\text{mixing}}(K, t)$, which is the time that mixing (and the initial shock, at high $\mathcal{M}_w$) takes to double $K$, in units of $t_{cc}$. It makes the slope variations in $\dot{K}_{\text{mixing}}$, above and below $\log_\chi K/K_{cl} \sim 0.8$, more apparent. Additionally, $K/\dot{K}_{\text{mixing}}(K, t)$ is generally within a factor of $\sim 2.5$ of $t_{cc}$ in each hydro simulation. This implies that $\dot{K}_{\text{mixing}}(K, t)$ and $t_{cc}^{-1}$ share similar $\mathcal{M}_w$ and $\chi$ dependence.

The results in the section broadly indicate that $\dot{K}_{\text{mixing}}(K, t)$ robustly characterizes cloud destruction. For idealized conditions, our results further suggest that $\dot{K}_{\text{mixing}}(K, t)$ has a strong $t$ dependence, and we can approximate $\dot{K}_{\text{mixing}}(K, t)$ with a time-independent function, $K_{\text{mixing, char}}(K)$. In the presence of additional physical effects (e.g., the presence of magnetic fields), $\dot{K}_{\text{mixing}}(K, t)$ shows stronger time dependence and improves on the description of cloud destruction offered by $t_{cc}$. This is conveyed in Figures 6 and 7 for NR-X100-B10; $\dot{K}_{\text{mixing}}(K, t)$ clearly captures the decreasing destruction rate, presumably caused by the tangling of magnetic fields.

5. Results with Cooling

5.1. $p-K$ Phase Evolution

Next, we apply our mixing model to hydrodynamic simulations with radiative cooling. We consider two main regimes of cooling: slow, $t_{\text{cool, mix}} \sim 10t_{cc}$, and fast, $t_{\text{cool, mix}} \sim 0.2t_{cc}$. Per Gronke & Oh (2018), the cloud should be destroyed in the former case and survive in the latter. For the fast-cooling regime, we consider two separate initial cloud temperatures: $T_{cl} = 10^4$ K (FastRC-T1e4) and $T_{cl} = 4 \times 10^4$ K (FastRC-T4e4). Figure 8 depicts the latter case. However, we only present one slow-cooling simulation with $T_{cl} = 10^4$ K (SlowRC-T1e4) because $T_{cl}$ has minimal impact in this regime. We compare these simulations against the non-radiative simulation NR-X100.

Figure 9 depicts how radiative cooling modifies the $p-K$ phase-space distribution for fluid elements originating in the cloud. The key takeaway is that cooling slows the spread of cloud material into the background high-entropy phase. This suppression is stronger for higher cooling rates, which reflects the fact that intermediate entropy material cools to low entropy prior to mixing with high-entropy material.
Unsurprisingly, the evolution of our slow-cooling case is minimally changed from the nonradiative case; the rate at which gas migrates to the high-entropy phase is slower. Rapid cooling more significantly modifies the distribution. Consistent with our expectation of entrainment, a reservoir of gas is always present at \((p_0, K_{cl})\) throughout the system’s evolution. Interestingly, after an early transient phase, which has a large scatter in \(p\), the distribution approaches a near steady state. In this state, the conditional pressure distributions have reduced scatter and a mode that lies mostly along the \(p_0\) isobar but has a decrement near \(\log_\epsilon(K/K_{cl}) \sim 0.2\).

The decrement in the conditional pressure distribution’s mode is likely an artifact of underresolved cooling (Fielding et al. 2020; Tan et al. 2021). This is supported by the fact that the decrement is almost nonexistent in FastRC-T1e4, where \(\ell_{cool}\) (see Appendix C) is actually resolved. Moreover, the first and third rows from the top of Figure 20 (from Appendix A) illustrate that the decrement is far more exaggerated in lower-resolution versions of FastRC-T1e4 and FastRC-T4e4.\(^{11}\)

The overdense diagonal line, in Figure 9, intersecting \((p_0, K_{cl})\) lies along the isotherm corresponding to the cooling curve’s temperature floor. In the absence of this floor, the gas would cool to lower \(K\). This is shown in Appendix A.

Figure 10 illustrates the bulk motion of the fluid elements originating in the cloud along \(K\). For both fast-cooling simulations, it shows a bistable medium with long-lived cold and hot gas. During the early stages of the interaction, the colder phase loses mass to the hotter phases, but after some time this reverses. While the exact timescale depends on \(T_{ch}\), cooling gradually becomes more effective at opposing cloud destruction. Eventually, it is effective enough that it not only prevents the loss of additional mass but also recaptures the lost mass. This behavior manifests over a notably shorter timescale for FastRC-T4e4.

5.2. Turbulent Radiative Mixing Layer Entrainment

Figure 11 shows how the different cooling regimes affect the bulk property evolution of the colder, denser phase (gas with \(\rho > \rho_{cl}/3\)). The top panel illustrates the total mass evolution and confirms that the Gronke & Oh (2018) criterion accurately predicts the cloud’s fate. The cloud is destroyed in both the nonradiative and slow-cooling cases, although cooling slows the...
destruction rate. On the other hand (as noted in Section 5.1), in the fast-cooling cases, the cloud not only survives but also starts to rapidly grow in mass.

The bottom and middle panels depict the evolution of the velocity and purity fraction (i.e., the cold-phase mass fraction of fluid elements initialized in the cloud). The correlation in the evolution of velocity and purity fraction reflects an inelastic collision in the fast-cooling limit; this is expected for TRML entrainment (Gronke & Oh 2018; Schneider et al. 2020, Tonnesen & Bryan 2021). The sustained high-purity fraction signals that a different process, probably ram pressure, dominates acceleration in the nonradiative and weak-cooling regime. Note that the minor offset in the velocity and purity fraction evolution suggests that ram pressure could play a subdominant role in the fast-cooling limit.

Interestingly, Figure 11 also indicates that the $t_{\text{cool,mix}}/t_{\text{cc}}$ criterion alone does not fully specify the cloud evolution. In the fast-cooling limit, the rate of cloud growth depends on $T_c$; FastRC-T4e4 takes at least twice as long as FastRC-T4e4 to show growth despite having nearly identical $t_{\text{cool,mix}}/t_{\text{cc}}$ ratios.

Figure 8. The same as Figure 1 except this shows FastRC-T4e4, which shows significant cloud growth. In this simulation, $\mu$ is a function of $n_H$ and $T$. As in Figure 1, mixing initially drives material from the colder phase to the hotter phase, but rapid cooling slows the transfer rate. Shortly before $4.5t_{\text{cc}}$, cooling causes this transfer to reverse: the cool phase starts to accrete mass. The stable phase diagrams are a manifestation of this growth because there is a limitless supply of hot phase gas. The growth is even more obvious in the right column; by $8.5t_{\text{cc}}$, the mass of the gas with $K < 3 \times 10^{31}$ cm$^2$ s$^{-2}$ has more than doubled. The figure’s rows correspond to individual frames from the animation. Note that the density projection panels try to capture as much of the cool phase as possible and thus do not reflect how the cloud is accelerated downstream during the entire simulation. The abrupt downstream motion near the end of the animation occurs when the cool phase gas starts to leave the simulation domain.

(An animation of this figure is available.)
This depressed cloud growth in FastRC-T1e4 is accompanied by a delay in the time at which the cloud is entrained.

Figure 12 underscores the significance of this difference in growth. The green and brown curves show the mass growth of simulations that are, respectively, identical to FastRC-T4e4 and FastRC-T1e4, except that they have $t_{cool,mix}/t_{cc} = 1 \pm 0.12$. The green curve shows nearly identical growth to FastRC-T1e4 (shown in cyan), despite the difference in $t_{cool,mix}/t_{cc}$. As we will conclude below, this difference in growth arises from differences in $t_{cool}/t_{cc}$ between $T_{cl}$ and $T_{mix}$ (for reference the minimum $t_{cool}/t_{cc}$ for the green curve is half of that for FastRC-T1e4). Moreover, the fact that the brown curve goes to zero, despite having a comparable $t_{cool,mix}/t_{cc}$ to the green curve, illustrates that this difference can even modify the cloud survival criterion.

To interpret these results, we consider them in terms of our mixing model. For each cooling case, Figure 13 compares standalone nonradiative $\dot{K}_{\text{mixing}}(K, t)$ measurements and the $\dot{K}_{\text{cool}}(K)$ prediction against the $\dot{K}_{\text{total}}(K, t)$ measurements from the simulations with radiative cooling.

Given our result from Section 4.3 that $K/\dot{K}_{\text{mixing}}(K_{\text{mix}}, t) \sim t_{cc}$ for most times when $(dM_{ps}/dK)(K_{\text{mix}}) > 0$, the survival criterion $t_{cc} > t_{cool,mix}$ can be directly visualized in terms of this plot. TRML entrainment is expected when $-\dot{K}_{\text{cool}}$ exceeds the characteristic value of $\dot{K}_{\text{mix}} = \chi_{\text{max}}/K$. Given $\dot{K}_{\text{mixing, char}}(K) \sim K/t_{cc}$ and $|\dot{K}_{\text{cool}}|$’s mostly inverse dependence on $K$ for realistic ISM conditions, satisfaction of the survival criterion basically guarantees that $K_{\text{mixing, char}}(K) < K_{\text{cool}}(p_{0}, K)$ for $K \in [K_{\text{CL}}, K_{\text{mix}}]$. Thus, we predict a negative $\dot{K}_{\text{total}}(K, t)$ over that interval and, by extension, entrainment.

The outcome of the slow-cooling case ($t_{cool,mix}/t_{cc} \sim 10$) is clear cut. Because $|\dot{K}_{\text{mixing, char}}(K)| > |\dot{K}_{\text{cool}}(p_{0}, K)|$, $\dot{K}_{\text{total}}(K, t)$ resembles $\dot{K}_{\text{mixing}}(K, t)$ and the cloud is destroyed. Likewise, the ultimate fates in the fast-cooling cases are also predictable. However, the detailed shape and temporal evolution of $\dot{K}_{\text{total}}(K, t)$ are less straightforward.

The $\dot{K}_{\text{total}}(K, t)$ evolution for the cool cases directly reflects the discussion from the previous section (Section 5.1). In the earliest stages of the interaction, $\dot{K}_{\text{total}}(K, t)$ is positive because radiative cooling is unable to prevent the initial mixing of the phases. As the process continues, $\dot{K}_{\text{total}}(K, t)$ gradually decreases with time, which indicates that cooling becomes more effective at combating mixing. Eventually, the opposition of cooling to mixing becomes so effective that it reverses the transfer of gas between phases, which causes $\dot{K}_{\text{total}}(K, t)$ to become negative.

It is around this time that our measurements of $\dot{K}_{\text{total}}(K, t)$ lose meaning, because an increasing fraction of the colder

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**Figure 9.** Mass-weighted pressure–entropy evolution for fluid elements originating in the cloud. This is similar to Figure 3 except that each simulation has $\chi = 100$ and $R_{\text{cl}}/\Delta x = 64$. Instead, the radiative cooling effectiveness differs between rows. The top row has no cooling, the second row has slow cooling, and the bottom row has fast cooling. At $t \geq 2.5t_{cc}$, these distributions are qualitatively similar to the phase distributions that include all gas in the domain at low and intermediate $K$. This difference in $t_{cool}$ exceeds the characteristic value of $\dot{K}_{\text{mix}} = \chi_{\text{max}}/K$. Given $\dot{K}_{\text{mixing, char}}(K) \sim K/t_{cc}$ and $|\dot{K}_{\text{cool}}|$’s mostly inverse dependence on $K$ for realistic ISM conditions, satisfaction of the survival criterion basically guarantees that $K_{\text{mixing, char}}(K) < K_{\text{cool}}(p_{0}, K)$ for $K \in [K_{\text{CL}}, K_{\text{mix}}]$. Thus, we predict a negative $\dot{K}_{\text{total}}(K, t)$ over that interval and, by extension, entrainment.

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This difference in $t_{cc}$ is achieved by reducing $R_{\text{cl}}$ by a factor of 5.

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phase is composed of gas originating in the hot phase. This is discussed further in Appendix C. Nevertheless, we have included the measurements because they illustrate, if imprecisely, the expected behavior.

We now consider why \texttt{FastRC-T4e4} begins rapid growth in roughly half the time as \texttt{FastRC-T4e4}. Figure 13 suggests that this difference derives from the local shape of the cooling curve. The major difference is that \texttt{FastRC-T4e4} includes the measurements because they illustrate, if imprecisely, the expected behavior.

To approximately match the realistic cooling curve but use constant \( \mu \) and employ broken power-law cooling functions with \( \eta_{\text{mix, } cl} \in \{1, 2, 6, 60\} \) (\texttt{BPLawRC-1, BPLawRC-2, BPLawRC-6, BPLawRC-60}). For consistency with the previous work, \texttt{FastRC-T4e4} for \( T = 10^{4.65} \), they share the same \( \mu(T) \) at \( T = 10^{4.65} \) and have comparable power-law slopes from there to \( T = 10^{6.75} \).

The value of \( \alpha \) is 0.138259 and is set by the intersection of the middle and upper segments. For simplicity, we define \( \Lambda(T) \) such that the above equation is satisfied at all pressures and for constant \( \mu \).

The one-dimensional entropy distribution as in Figure 5 except that some of the simulations include radiative cooling, and they all have \( \chi = 100 \), \( \mu_{\text{eq}} = 1.5 \), and \( R_{\Delta }/\Delta x = 64 \). The simulations are in the same order as for Figure 9.

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The value of \( \alpha \) is 0.138259 and is set by the intersection of the middle and upper segments. For simplicity, we define \( \Lambda(T) \) such that the above equation is satisfied at all pressures and for constant \( \mu \).

We run four simulations that resemble the fast-cooling simulations but use constant \( \mu \) and employ broken power-law cooling functions with \( \eta_{\text{mix, } cl} \in \{1, 2, 6, 60\} \) (\texttt{BPLawRC-1, BPLawRC-2, BPLawRC-6, BPLawRC-60}). For consistency with the previous work, \texttt{FastRC-T4e4} for \( T = 10^{4.65} \), they share the same \( \mu(T) \) at \( T = 10^{4.65} \) and have comparable power-law slopes from there to \( T = 10^{6.75} \).

Ordinarily, \( \mu \) drops by \( \sim 21\% \) between \( 10^4 \) and \( 10^6 \). Above that, it only drops by \( \sim 4\% \). With a modified version of \texttt{FastRC-T4e4}, we confirmed that these \( \mu \) variations have negligible impact on mass evolution. This simulation used \( R_{\Delta }/\Delta x = 8 \), a fixed \( \mu \), a rescaled \( \Lambda(T, \mu) \) such that \( t_{\text{cool}}(p, T) \)’s shape is unchanged, and an \( R_{\Delta } \) that maintained \( t_{\text{cool, } cl}/\Delta x = 0.1576 \).
for the initial pressure, and the shaded regions show variations from
curves are shown measured with and without radiative cooling for three sets of physical
measurements involving both mixing and cooling. The red dashed line shows
Figures 12 and cyan curves are measured from lower-resolution versions of FastRC-T4e4 and FastRC-Tle4, while the green and brown curves have radii of 1000 pc and 9 pc. Note, we have verified that the brown curve goes to zero (the y-axis
starts at 0.1). This demonstrates that the $t_{\text{cool,max}}/t_{cc}$ criterion alone does not fully specify the cloud evolution.

with earlier simulations, the cooling functions are truncated below $T_{\text{cl}}$ and above 0.6$T_{\text{mix}}$. Figure 14 illustrates the shapes of the cooling curves normalized by the properties at the mixing layer.

Figure 15 shows the dependence of the late-time passive-scalar mass–entropy profiles on $\eta_{\text{mix,cl}}$. The high-$\eta_{\text{mix,cl}}$ simulations resemble FastRC-T4e4, and as $\eta_{\text{mix,cl}}$ decreases, they begin to more closely resemble FastRC-Tle4. The temporal stability of these profiles indicates that each simulation has a bistable medium. However, BPLawRC-1’s cold phase may not yet be stable; its peak near $K_{\text{cl}}$ decreases (increases) by a factor of ~2 between 11$T_{\text{cc}}$ and 14$T_{\text{cc}}$. Note that FastRC-T4e4’s profiles in Figure 10 suggest that for high-$\eta_{\text{mix,cl}}$ cases, there may be more variability at intermediate $K$ at higher resolutions.

Figure 16 establishes a clear trend: as $\eta_{\text{mix,cl}}$ increases and the break in the cooling curve moves to lower $T$, rapid cloud growth sets in more quickly. This figure also supports our expectation that BPLawRC-1 is just starting to grow in mass between 11$T_{\text{cc}}$ and 14$T_{\text{cc}}$. The overtaking of the mass and velocity growth in BPLawRC-60 by BPLawRC-6 may be a resolution effect. Figure 23 in Appendix C shows that velocity evolution is particularly sensitive to resolution.

Our results clearly demonstrate that while $t_{\text{cool,max}}$ is important for identifying the conditions under which cooling occurs, the shape of the cooling curve below $T_{\text{mix}}$ is important for determining when rapid growth commences.

To gain some intuition for why these different cooling times matter, we examine the $K_{\text{cool}}(K, t)$ measurements in Figure 17. Unsurprisingly, these measurements resemble the fast-cooling simulations; it is most obvious when considering measurements
from simulations of comparable resolution (see Figure 25 from Appendix C). As expected, the \( \eta_{mix, cl} \leq 2 \) cases (when the power-law break is at high \( T \)) resemble FastRC-T1e4, while the \( \eta_{mix, cl} \geq 6 \) cases resemble FastRC-T4e4. Comparing the relative magnitudes of \( K_{mixing}(K, t) \) and \( K_{cool}(P_0, K) \) in the middle left two panels provides some intuition for why there is such a big difference between \( \eta_{mix, cl} = 2 \) and \( \eta_{mix, cl} = 6 \).

However, the precise property of the cooling curve on the interval \( T_{cl} \lesssim T \lesssim T_{mix} \) that controls the growth rate remains somewhat ambiguous. We speculate that the crucial quantity is some kind of (weighted) average over the interval, possibly related to (although not exactly equal to) \( t_{cool, mix}/t_{cc} \) or \( t_{cool} \), and hereafter refer to it as the characteristic cooling time of the cold phase \( t_{cool, cl} \). Regardless of \( t_{cool, cl} \)'s true nature, Figure 2 clearly illustrates that it must be considerably smaller for FastRC-T4e4 than it is for FastRC-T1e4. Therefore, rapid growth commences more quickly in FastRC-T4e4.

The onset of rapid growth may coincide with the transition between the “tail growth” and “entrained phases” of the cloud’s areal growth (Gronke & Oh 2020a). Because areal growth is more rapid in the earlier phase, this transition likely corresponds to the point when the system is able to reach equilibrium and cooling is able to balance the destructive mixing effects (Fielding et al. 2020). If true, then the delayed
transition may imply that a larger area is required when \( t_{\text{mix,cl}} \) is smaller. Such an interpretation would be consistent with the mixing layer being linked to both \( t_{\text{cool,cl}} \) and \( t_{\text{cool,mix}} \), rather than just the latter.

6. Discussion

6.1. What Does Our Model Offer?

Our entropy evolution mixing model provides three main benefits. First, it offers a condition-agnostic method for the characterization and quantification of the cloud–wind interaction’s evolution. Our nonradiative parameter study showed that the model meaningfully captures the processes of cloud destruction. For idealized, hydrodynamic interactions, it reproduces the well-known destruction timescale \( t_c \). However, the independence of these characterizations with respect to the interaction’s physical conditions warrants emphasis.

Our model offers a robust approach for describing cloud destruction in circumstances where the \( t_c \) description breaks down. We have already demonstrated that it quantitatively captures the well-documented effects that magnetic draping has on extending the cloud’s lifetime (e.g. Dursi & Pfommer 2008; McCourt et al. 2015; Banda-Barragán et al. 2018; Gronke & Oh 2020a). Another interesting application might be characterizing the evolution of networks of small clouds where the idea of having a monolithic cloud with a well-defined \( R_\text{cl} \) does not really apply.

Second, our model facilitates comparisons between the effects of radiative cooling and empirical characterizations of other effects that affect the system’s evolution. Figure 18 helps illustrate this capability for different scenarios. The effects of radiative cooling, \( K_{\text{cool}}(p_0, K) \), are shown in red and the time-independent effects of nonradiative mixing, \( K_{\text{mixing,cl}}(K_{\text{mix}}) \), are approximated by the blue curves. We describe a complementary relationship to the \( t_{\text{cool}} < t_{\text{cool,mix}} \) survival criterion from Gronke & Oh (2018) at length in Section 2.2 and Section 5.2. We liken the relationship to that of a distribution function and point estimation; there is a trade-off between information content and computational convenience.

The timescale comparison is equivalent to comparing just the red and blue points in each panel of Figure 18. Panels a and b demonstrate how in most cases, the timescale comparison is sufficient for predicting the system’s fate. Panels c and d illustrate that our model provides insight when the timescale comparison is ambiguous. These panels respectively correspond to the simulation conditions for which the brown and green curves, in Figure 12, show the cold-phase mass evolution. Our model suggests that the differences in the ratio, \( |K_{\text{cool}}(p_0, K)| / K_{\text{mixing,cl}}(K_{\text{mix}}) \), for \( K < K_{\text{mix}} \) is responsible for explaining the difference in the cloud’s fate.

Our case study of interactions that included radiative cooling further exemplified this relationship. While the timescale criterion accurately predicted whether the clouds survived, it did not predict the delay in both entrainment and onset of rapid growth in the fast cooling \( T_d = 10^4 K \) case. However, our mixing model revealed that these factors are sensitive to the characteristic cooling time of the colder phase \( t_{\text{cool,cl}} \). We defer discussions of this finding’s significance to Section 6.3. Future work should develop a simple criterion encoding this information that either supplements or improves upon the existing survival criterion.

Finally, the model’s simplicity makes it extendable. Given the largely unimodal distribution of \( p \) at each value of \( K \), the model is conducive to layering additional quantities atop \( p - K \) space; one could imagine constructing manifolds in higher-dimensional space. For example, one could supplement the \( p - K \) space with the wind-aligned velocity (similar to Schneider & Robertson 2017, Kanjilal et al. 2021). This would also connect our model to the established relation between a fluid element’s wind-aligned velocity and the fraction of its mass that originated in the hot phase (e.g., Melso et al. 2019; Schneider et al. 2020, Tonnesen & Bryan 2021). As mentioned earlier, it would also be useful to consider the entropy flow for fluid elements originating in the hotter phase in addition to the fluid elements from the colder phase.

6.2. Limitations and Missing Physics

The omission of geometric information may be a limitation of our model. Consider a nonradiative hydrodynamic simulation with an initially turbulent cloud. Because turbulent clouds are destroyed faster than spherical clouds (e.g., Schneider & Robertson 2017), one might expect to measure larger \( K_{\text{mixing,cl}}(K) \) and thus predict stricter conditions for TRM entrainment. In reality, Gronke & Oh (2020a) showed that turbulent clouds not only survive under the same conditions as spherical clouds, but initially grow faster because they have larger surface areas. Additionally, it is unclear how well the model captures the evolution of a system in which each phase has different levels of nonthermal pressure support.

We note that these proposed limitations are entirely hypothetical. Simulations are needed to assess whether there are actually issues in these scenarios. Regardless, we are unaware of any alternative models with similar predictive power that are devoid of these issues.

Like other models (Gronke & Oh 2018; Li et al. 2020; Sparre et al. 2020, e.g.), our mixing model facilitates predictions about the fate of cloud–wind interactions involving radiative cooling by comparing the effects of cooling on isolated gas against characterizations of adiabatic mixing. An interesting question is how the mixing process itself is affected by additional physical processes. For example, it is not clear how radiative cooling modifies the properties of turbulence. We plan to investigate this in future work.

Our model’s effectiveness is also somewhat unclear for cloud–wind interaction cases with high \( M_\text{w} \). The method for measuring \( K_{\text{mixing}}(K) \) implicitly assumed that entropy changes from the initial shock were minimal. Our higher-\( M_\text{w} \), nonradiative simulations showed signs that this might not be a great assumption for \( M_\text{w} \sim 3 \) or 4.5. Because measurements of \( K_{\text{mixing}}(K) \) in cases with large \( M_\text{w} \) still characterize the total destruction of the cloud, it is not obvious that our model will work any more poorly than other existing models. Further simulations are clearly necessary to investigate this.

This work entirely ignored relevant physical effects like viscosity, conduction, and cosmic rays. It also did not consider magnetic fields at the same time as radiative cooling. Additionally, we artificially prevented cooling below \( T_{cb} \), which appears to have a large impact on entrainment (see Appendix A; Gronke & Oh 2018). Moreover, we only considered idealized initial conditions. The influence of metallicity variations, different magnetic field configurations, and the presence of turbulence warrant attention in future work. However, we emphasize that our model is well equipped for
characterizing how each of these conditions modifies the conditions for TRML entrainment.

6.3. Turbulent Radiative Mixing Layer Entrainment

Our mixing model is conducive to applications related to cloud survival through TRML entrainment. It naturally provides a general condition under which this entrainment mechanism is expected (i.e., \( \frac{K_{\text{mix}}}{t_{cc}} + \frac{K_{\text{cool}}(P_0, K)}{t_{cc}} \ll 0 \) for a subinterval of \( K \in [K_{\text{cl}}, K_{\text{w}}] \); see Section 2.2) that is useful for building intuition about the process. However, this condition does not replace the more analytic form of survival criteria presented by Gronke & Oh (2018) and Li et al. (2020). Whereas such survival criteria facilitate isolated predictions about cloud survival, our mixing model currently requires empirical measurements of \( K_{\text{mix}}(K, t) \) from nonradiative simulations to predict the interaction’s fate. Therefore, our mixing model complements such criteria and can be used to help improve them.

6.3.1. Relevant Timescale

Our results in Sections 5.2–5.3 suggest that the most important cooling timescales for TRML entrainment are at temperatures ranging from \( T_{\text{cl}} \) through \( T_{\text{mix}} \). While the Gronke & Oh (2018) survival criterion, which is based on \( t_{\text{cool},\text{mix}} \), appears to accurately predict cloud survival, it does not fully specify the interaction’s evolution. Specifically, the characteristic cooling time of the colder phase, \( t_{\text{cool},\text{cl}} \), affects how quickly rapid cloud growth commences (the delay from the start of the simulation appears correlated with \( t_{\text{cool},\text{cl}}/t_{\text{cool},\text{mix}} \)).

The delay is significant because it provides additional opportunities for other processes (e.g., externally driven turbulence in the wind) to destroy the cloud. This raises a
broader point. Although the distinction between cloud survival and destruction is of primary interest, knowing how close a surviving cloud comes to being destroyed (or how quickly rapid growth commences) would be insightful. We discuss how the delay in rapid growth may affect the prevalence of TRML entrainment in Section 6.3.3.

There is also direct evidence that $t_{\text{cool,w}}$, which underlies the Li et al. (2020) and Sparre et al. (2020) criteria, is not the dominant cooling timescale. Figure 3 of Gronke & Oh (2018) and Figure 21 in Appendix A show that switching cooling on and off above $\sim 0.6T_w$ has minimal effect on the mass evolution for FastRC-T4e4 and FastRC-T1e4, respectively. The main consequence of wind cooling is that the system’s equilibrium pressure drops (by $\lesssim 30\%$ for FastRC-T4e4 and FastRC-T1e4), which does not appear to be significant. While this does affect the cooling function, we do not expect it to be significant in most cases (see Appendix A for further discussion).

The fact that the difference in $t_{\text{cool,w}}$ between FastRC-T1e4 and FastRC-T4e4 so efficiently accounts for the variations in mass growth rates reinforces our conclusion that $t_{\text{cool,w}}$ is not the dominant timescale. If $t_{\text{cool,w}}$ were dominant, we would expect it to explain the difference.

Finally, we address Sparre et al.’s (2020) proposed explanation for why $t_{\text{cool,w}}$ could be important to TRML entrainment: they suggest that a fluid element’s temperature evolution from $T_w$ to $T_{\text{cl}}$ is rate limited by an initial cooling phase near $T_{\text{mix}}$ set by $t_{\text{cool,w}}$. While we acknowledge that initial cooling of the wind could possibly make entrainment easier, we expect this to be a high-order effect. In fact, the small impact that switching cooling on and off above $0.6T_w$ has on the cold-phase mass growth suggests that the temperature change at high $T$ is dominated by mixing.

### 6.3.2. Comparison with Prior Work

The results of our simulations are largely consistent with Gronke & Oh (2018, 2020a), Kanjilal et al. (2021), and Sparre et al. (2020). However, our simulation results are in slight tension with the survival criterion put forth by Li et al. (2020). Li et al.’s (2020) criterion gives minimal survival radii of 79 pc and 6163 pc for clouds with properties matching FastRC-T1e4 and FastRC-T4e4, while the clouds in our simulation survive with 20%–30% smaller radii. We are largely unconcerned with this minor disagreement; Kanjilal et al. (2021) suggest that the difference between the Li et al. (2020) and Gronke & Oh (2018) criteria primarily arise from differences in how long the simulation is run before the cloud’s fate is assessed and the precise definition of cloud destruction.

We do note that our results generally favor the physical arguments presented by Gronke & Oh (2018) over those of Li et al. (2020) and Sparre et al. (2020). Specifically, Figure 21 in Appendix A shows that switching cooling on and off above $\sim 0.6T_w$ has minimal effect on the mass evolution for FastRC-T4e4 and FastRC-T1e4. In other words, $t_{\text{cool, mix}}$ seems to be more relevant than $t_{\text{cool,w}}$ for determining cloud survival, at least for $\chi = 100$. However, it remains plausible that $t_{\text{cool,w}}$ could be more important for higher-$\chi$ simulations (like those studied in Sparre et al. 2020) or simulations that include additional physics, like thermal conduction (as in Li et al. 2020).

### 6.3.3. Prevalence

Our result that rapid cloud growth takes longer to commence at larger $t_{\text{cool, cl}}/t_{\text{cool, mix}}$ has important implications for the prevalence of TRML entrainment. This effect is most pertinent for clouds at or near thermal equilibrium, where $t_{\text{cool, cl}}$ is largest. To give a concrete example with a realistic cooling curve, consider a system at $p = 10^3 k_B$ K cm$^{-3}$ in which $\mathcal{M}_w = 1.5$, $T_{\text{cl}} \sim 6 \times 10^3$ K and $\chi \gtrsim 180$.16

For such clouds, the delay in growth provides additional opportunities for destructive processes (like mixing) to destroy the cloud. Therefore, these clouds require a minimum survival radius that is somewhat larger than the Gronke & Oh (2018) criterion predicts. Figure 12 illustrates this effect for an idealized, initially laminar wind. However, this delay may be even more relevant for clouds embedded in winds with turbulence driven by external processes (e.g., supernovae) because mixing may more efficiently destroy clouds.17 In this scenario, one might predict that turbulent diffusion prevents the formation of a near-continuous tail. Because the tail makes up a large fraction of the cloud’s surface area, its accretion rate would be reduced (Gronke & Oh 2020a).

In their high-resolution starburst-driven galactic wind simulation, Schneider et al. (2020) cited external turbulence as a potential explanation for the lack of cloud growth at large radii. Furthermore, their cooling curve’s shape and 10$^3$ K floor (Schneider & Robertson 2018) make the delayed growth and entrainment, from large $t_{\text{cool, cl}}/t_{\text{cool, mix}}$ relevant (as in FastRC-T1e4). We expect that the combination of external turbulence and delay potentially impedes growth near the galaxy. At larger radii, the hot phase’s $\sim 10$ times larger pressure than the cold phase (in the simulation) may further exacerbate the effect. If the intermediate phase also has a somewhat elevated pressure, then $t_{\text{cool, cl}}/t_{\text{cool, mix}}$ should be larger because $t_{\text{cool}}$ has an inverse dependence on pressure for photoionized gas. Although Gronke & Oh (2020a) showed rapid growth in expanding winds, their model explicitly assumes that the cold and hot phases are in sonic contact.

### 7. Conclusion

We have presented an entropy-based formalism for interpreting the cloud–wind interaction’s evolution. The basic premise of the approach is that information about the system’s state is encoded in the evolution of its thermodynamic phase space. We consider the $p–K$ phase space to take advantage of the system’s quasi-isobaric nature and the conservation of a fluid element’s specific entropy in the absence of irreversible processes (like shocks, mixing, and heating/cooling). Thus, in the adiabatic limit, the gas distribution along $K$ is primarily governed by the history of mixing, the dominant cloud destruction process.

16 We conservatively chose this lower bound on $\chi$ to ensure that the equilibrium pressure drop from cooling above 0.6$T_w$ has no more influence on cloud growth than it does on FastRC-T1e4 (see Appendix A).

17 Note, magnetic fields might directly mitigate this. Banda-Barragán et al. (2018) showed that magnetic fields have a stabilizing effect on turbulent clouds in a laminar wind. They might plausibly have a similar impact in a wind with externally driven turbulence.
We leverage the fact that mixing occurs in $K$ space to introduce an empirical mixing model. We characterize mixing with the average rate of change in the entropy of the fluid elements originating in the cloud, $K_{\text{mixing}}(K, t)$. From this knowledge, we can define a mixing timescale, $K/K_{\text{mixing}}(K, t)$, as a function of $K$ and $t$. Additionally, the model provides the capability for making predictions about how radiative cooling will modify adiabatic mixing by facilitating comparisons of $K_{\text{cool}}(p_0, K) = K/\ell_{\text{cool}}(p_0, K)$ with measurements of $K_{\text{mixing}}(K, t)$.

We have considered two example applications, using ENZO-E simulations, to demonstrate that this mixing model works as expected and provides useful insight. We enumerate our four main results below:

1. The timescale of cloud destruction from adiabatic mixing is well characterized by $K/K_{\text{mixing}}(K, t)$ for most entropy values ranging from the initial value in the cloud, $K_{\text{cl}}$, to the initial value in the wind. In fact, $K/K_{\text{mixing}}(K, t)$ is comparable to $\ell_{\text{cc}}$ for idealized, nonradiative, hydrodynamical interactions.

2. In addition, the model can characterize the change in destruction rate due to other physical processes. For example, we have demonstrated that the model reflects the reduction in the cloud destruction rate from the tangling of initially transverse magnetic fields.

3. These characterizations are well suited for comparisons against the effects of radiative cooling. An analogous form of the Gronke & Oh (2018) survival criterion, $\ell_{\text{cc}} > \ell_{\text{cool}, \text{mix}}$, can be formulated in terms of this model.

4. We used our model to show that the local shape of the cooling curve can influence the process of cloud entrainment via the rapid cooling of gas that has mixed with the wind. Independent of the cooling time at the mixing layer, variations in the characteristic cooling time of the cold phase can more than double the elapsed time required for clouds to commence rapid growth and become entrained.

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Software: numpy (Harris et al. 2020), matplotlib (Hunter 2007), yt (Turk et al. 2011), scipy (Virtanen et al. 2020), pandas (McKinney 2010), Launcher Utility (Wilson & Fonner 2014), GRACKLE (Smith et al. 2017), ENZO-E (http://cello-project.org).

### Appendix A

#### Simulation Robustness

_A.1. Cooling Curve Restrictions_

Figure 19 illustrates how switching cooling on and off at temperatures above $\sim 0.6T_w$ and below $T_{\text{cl}}$ affects the cold-phase mass evolution for FastRC-T4e4 and FastRC-T1e4. Switching cooling on and off above $\sim 0.6T_w$ is relatively insignificant. However, when cooling is allowed below $T_{\text{cl}}$, the initial period of mass loss is reduced and is followed by a period of slower growth. These results are consistent with Gronke & Oh (2018), who argue that the cooling below $T_{\text{cl}}$ affects mass evolution because it causes the cloud to contract.

Figure 20 shows how the difference between the fiducial restricted cooling curves (used in the bulk of this work) and the unrestricted cooling curves impact the $p-K$ evolution for all gas in FastRC-T4e4 and FastRC-T1e4. There are two main differences. First, cooling below $T_{\text{cl}}$ drives mass to much lower $K$ (or equivalently, $T$). Second, cooling of the wind slightly decreases (by $\lesssim 30\%$) the system’s equilibrium pressure.

These figures also convey that relaxation of cooling restrictions affects FastRC-T4e4 and FastRC-T1e4, when the cooling curves are unrestricted. Because $\ell_{\text{cool}, \text{cl}}/\ell_{\text{cc}}$ for FastRC-T1e4 is $\sim 46$ and is $\sim 3.3$ times smaller than for FastRC-T4e4, allowing cooling above $\sim 0.6T_w$ causes a slightly larger pressure drop and has slightly more effect on mass growth for FastRC-T1e4. At the same time, FastRC-T4e4 has a significantly larger density increase than FastRC-T1e4; by $\ell_{\text{cc}}/\ell_{\text{cool}, \text{cl}} = 2.5$ the maximum densities have increased to $\sim 120\rho_{\text{cl}}$ and $\sim 35\rho_{\text{cl}}$, respectively. This makes sense given that the former’s $\ell_{\text{cc}}$ is both $\sim 39$ times larger than the latter’s $\ell_{\text{cc}}$ and $\geq 100$ times larger than the time needed for the gas to cool (whether the cooling proceeds isobarically or isochorically) between the respective $T_{\text{cl}}$.

Interestingly, when cooling is unrestricted, FastRC-T4e4 has faster cold-phase growth than FastRC-T1e4, even though its gas contracts more. Although increased resolution may affect the growth rate, this suggests that the early-time value of $\ell_{\text{cool}, \text{cl}}$ (see Section 5) may be more important for setting the properties of cloud growth. Future work must investigate cloud evolution using the full cooling curve.

#### A.2. Influence of Hydrodynamical Integrator

Figure 21 illustrates how differences in the hydrodynamical integrator modify the cold-phase mass evolution. Specifically,

![Figure 19. Comparison of how cooling curve restrictions affect the mass evolution of the cold phase (gas with $p > \rho_0/3$) for low-resolution versions ($R_0/\Delta x = 8$) of both FastRC-T4e4 (left) and FastRC-T1e4 (right).](http://cello-project.org)
we compare the VL+CT integrator (which is used in the rest of this paper) with the ppm integrator, which was previously ported from ENZO (Bryan et al. 2014). The defining differences are that the ppm solver is dimensionally split and employs third-order spatial reconstruction. Additionally, while the VL+CT integrator employs a predictor–corrector scheme, the ppm solver updates the grid in a single pass. More minor differences include implementation choices for the dual-energy formalism and the choice of the HLLD (Two-Shock) Riemann solver for our simulations with VL+CT (ppm).

We primarily consider how the different integrators affect the mass growth in FastRC-T1e4 and FastRC-T4e4 when using our standard restricted cooling curves (bottom row of Figure 21). The simulations using the PPM integrator each show elevated mass growth rates, compared to the VL+CT simulations. However, we find solace in the way that the simulations using the PPM integrator appear to trend toward the converged curves (discussed in Appendix C) as we increase resolution.

We also compare how the difference in integrators affects the mass growth when cooling is unrestricted (top row of Figure 21). Interestingly, the FastRC-T4e4 mass evolution has slower growth when using the PPM curve. Nevertheless, simulations with both integrators indicate that FastRC-T4e4 shows faster growth than FastRC-T1e4. Our results suggest that while the precise values measured in simulations with different integrators may differ, the trends between the values measured in different simulations (with a single integrator) are fairly robust.

Figure 20. Comparison of how restricting cooling affects the mass-weighted pressure–entropy evolution. The top row and second row from the bottom are similar to the bottom two rows of Figure 9. The two differences are that this figure shows the distribution of all fluid elements in the simulation domain (not just the fluid elements originating in the cloud) and the simulation resolution is lower ($R_{cl}/\Delta x = 8$). The other rows show simulations with the same initial conditions but have unrestricted cooling. The gray region indicates wherever $t_{cool} > 10^5 t_{cc}$ or heating dominates.

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18 When magnetic fields are zero the HLLD solver reduces to an HLLC solver.
Figure 21. Comparison of how different integrators affect the mass evolution for the cold phase (gas with $\rho > \rho_{cl}/3$) for lower-resolution versions of both FastRC-T4e4 (left column) and FastRC-T1e4 (right column). The top (bottom) row further shows how the mass evolution in each integrator is affected by cooling curve restrictions (differences in resolution). All simulations in the top row have a fixed resolution of $R_{cl}/\Delta x = 8$. We note that all the solid orange and blue curves are identical to the curves in Figure 19.
Appendix B
Frame-tracking Scheme Comparison

Two different developmental versions of ENZO-E were employed in this work. The main difference between them is in the reference frame tracking scheme. The earlier version (used for simulations with $R_{cl}/\Delta x < 64$) has a bug that only allows the frame velocity to be updated once, immediately after the very first update cycle. Thus, the frame velocity remains near zero for the duration of the simulation.

As we mentioned at the end of Section 3.1, this bug is fixed in the later version (used for simulations with $R_{cl}/\Delta x = 64$). The later version of the code also features a more efficient Riemann Solver implementation.

Figure 22 illustrates the difference in the frame tracking schemes for the two versions of the code. The figure compares the evolution of two simulations using a single set of initial conditions but with different code versions. We used initial conditions matching FastRC-Tle4 but with a resolution of $R_{cl}/\Delta x = 16$. The figure’s top panel illustrates that in the later version of the code, the frame velocity correctly updates over time. At the same time, the bottom panel demonstrates that the cool-phase mass evolution is largely consistent, which is expected because none of the cool phase gas has left the domain during the displayed times. While not shown here, we have also verified that both versions of the code produce consistent results for the phase evolution (again, at times before the cool phase can leave the domain).

Figure 22. Comparison of the evolution of the frame velocity and the cold phase for a cloud–wind simulation that is run with two versions of ENZO-E. These simulations’ initial conditions are the same as those for the bottom row of Figure 9, except that $R_{cl}/\Delta x = 16$. The simulation corresponding to the dashed lines (like all other simulations with $R_{cl}/\Delta x < 64$) effectively has no frame tracking; while the other simulation has frame tracking (and a slightly refactored Riemann Solver).

Appendix C
Resolution Study

In this appendix we briefly assess how resolution affects our measurements of the cloud–wind interaction. To do this we consider the primary four initial conditions discussed in Section 5 (i.e., NR-X100, SlowRC-T1e4, FastRC-T4e4, FastRC-Tle4) at the resolutions $R_{cl}/\Delta x = [8, 16, 32, 64]$.

C.1. Relevance of Shattering

A relevant length scale for our convergence study is the so-called “cooling length,” $\ell_{cool} \sim \min(C_{d}/\chi_{cool})$ (McCourt et al. 2018), McCourt et al. (2018) first showed in 2D simulations that large clouds with sizes exceeding $\ell_{cool}$ are prone to fragmenting into a swarm of cloudlets of size $\sim \ell_{cool}$. Thus, simulations where clouds “shatter” may not have well-converged properties when $\ell_{cool}$ is not converged. For reference, Table 1 lists each simulation’s $\ell_{cool}$. The length scale is well resolved at all resolutions of SlowRC-Tle4 and barely resolved ($\ell_{cool} = 1.7\Delta x$) for FastRC-Tle4 when $R_{cl}/\Delta x = 64$. However, it is not resolved in any other simulations with cooling.

More recently, Gronke & Oh (2020b) considered cloud shattering in 3D simulations and linked the shattering of clouds to cloud growth through cooling. They demonstrated that all clouds with $R_{cl} > \ell_{cool}$ that are overpressurized compared to the ambient medium and have density inhomogeneities undergo some degree of shattering. However, the cloud’s fate depends on how the density contrast (when the cloud is overpressurized) compares to $\chi_{crit} \sim 300$. When the contrast exceeds $\chi_{crit}$, the cloud breaks apart. Otherwise, the cloud recoagulates, and has the opportunity to accrete material from the cooling ambient medium.

Although Gronke & Oh (2020b) only studied simulations in which the thermal instability made clouds overpressurized (the contraction leads to overshooting pressure equilibrium), they argued that similar conditions arise from the shock that supersonic winds drive through clouds. Because they predict that clouds should only shatter when $\mathcal{M}_{w} > 1.6$ (if $\chi = 100$ and gas cannot cool below $T_{cl}$), we do not expect the clouds in our simulations to shatter. Nevertheless, the resolution of $\ell_{cool}$ could be important for convergence of simulation properties because of its link to growth.

C.2. Measurement Sensitivity to Resolution

Figure 23 illustrates how the evolution of the cloud’s bulk properties (survival fraction, purity fraction, and bulk velocity) vary with resolution. The figure illustrates a remarkable level of convergence that seems to imply that the net effects of mixing generally have only a weak dependence on resolution.

Although the net effect of mixing does not change significantly, the microscopic details can and do change with resolution. While we might not expect the average time derivative of $K$ for all fluid elements to vary much with resolution, the derivative for individual fluid elements can vary wildly. For this work, we have measured the time-averaged $\bar{K}(K, t)$ just for fluid elements originating in the cloud. Therefore, we expect the $\bar{K}(K, t)$ measurements to be fairly robust for $K$-bins in which the majority of the fluid elements originated in the cloud. However, when a large fraction of the fluid elements in a bin originated in the wind (and the fluid elements originating from the cloud are no longer representative
Figure 23. Comparison of how resolution affects the evolution of total mass, purity fraction, and average velocity. The evolution clearly converges at high resolution.

Figure 24. Comparison of $\frac{dM_{\text{cl}}}{dx}$ evolution with respect to resolution. Different rows illustrate different simulations and resolution varies between columns.
of all fluid elements in the bin, $\tilde{K}(K, t)$ should be treated with care.

Thus, we expect our $\tilde{K}(K, t)$ measurements for low to intermediate $K$ to be fairly robust for NR-X100 (and nonradiative simulations in general) and SlowRC-Tle4 because the purity fraction remains high. However, for the fast-cooling cases where the purity fraction drops, we expect more variation in $\tilde{K}(K, t)$ at increasing $K$ and over a larger range in $K$ at later times. These are generally reflected in the convergence properties of $dM_{ps}/dK$ and $\tilde{K}$, which are illustrated in Figures 24 and 25.

We note that $dM_{ps}/dK(K, t)$ appears to have the most variability in bins with under 1% of the initial mass. Thus, we focus our assessment of $\tilde{K}(K, t)$ throughout this work on values computed from $K$ bins that include at least 1% of the cloud’s initial mass. Because the measurements that do not satisfy this condition can still be instructive (particularly for simulations with rapid cooling), we still show the other measurements in our figure, as translucent lines.

We further note that calculation of the average $\tilde{K}$ of all fluid elements in the system would improve substantially upon the reliability of our measurements. While doing this is possible, we consider our current measurements to be adequate for the purposes of conveying the premise of our mixing model.

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