Experimental Implementation of Dense Coding Using Nuclear Magnetic Resonance

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Abstract

Quantum dense coding has been demonstrated experimentally in terms of quantum logic gates and circuits in quantum computation and NMR technique. Two bits of information have been transmitted through manipulating one of the maximally entangled two-state quantum pair, which is completely consistent with the original ideal of the Bennett-Wiesner proposal. Although information transmission happens between spins over inter-atomic distance, the scheme of entanglement transformation and measurement can be used in other processes of quantum information and quantum computing.

Key words: dense coding, entangled states, quantum computation

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Introduction

While the miraculous properties of entangled states of quantum systems are revealed and understood, more attention is paid to the use of such entangled states for quantum information transmission and processing. Among the major application schemes investigated are cryptography based on Bell’s theorem, teleportation and dense coding. As sending more than one bit of information classically requires manipulation of more than two-state particle, dense coding proposed by Bennett and Wiesner\cite{1} in 1992, which can transmit two bits message by manipulating one of the entangled two-state pair,
is seemingly striking. Several alternatives for realizing dense coding have been suggested [2, 3, 4, 5, 6, 7]. Quite recently, Shimizu et al. [8] proposed a scheme, with which the capacity can be enhanced to 3 bits by the local operation of a frequency-dependent phase shift on one of polarization entangled twin photons. However, only one experiment of dense coding has been reported [5] so far. In Ref. [5], the polarization-entangled photon pair was exploited for transmitting 3 messages per two-state photon, i.e. 1 'trit' ≈ 1.58 bit.

One of the reasons why the quantum computer can outperform the classical counterpart is that multi-particle entanglement is widely used for information processing in quantum computing. It is not surprising that there is much in common between quantum computation and dense coding as the latter is actually a process of generation, manipulation and measurement of entangled states for two-particle systems. Many schemes for realizing quantum computation have been proposed, some of them, e.g. trapped ultracold ions [9], cavity-QED [10] and nuclear magnetic resonance (NMR) [11, 12], have been experimentally demonstrated for a single quantum logic gate. However, the real experiments [13, 14, 15, 16] of executing a quantum algorithm have been solely performed with NMR due to its physical advantages and technical maturity. While the ensemble averaging over nuclear spin-1/2 molecules in a bulk solution sample has been chosen as a two-state quantum system, its quantum behavior is equivalent to that of any real two-state micro-particle. Furthermore, principles and techniques developed are able to be used for constructing the logic gate and quantum circuits in quantum computation conveniently. As an interesting example of quantum information transmission, quantum teleportation was proposed to achieve in terms of primitive operations [17] in quantum computation and experimentally demonstrated [18] by using technique of NMR quantum computing with molecules of trichloroethylene over inter-atomic distances. Therefore, it is natural to attempt to do another type of quantum communication, dense coding, in much the same way. Here we report our experiment of quantum dense coding with the help of logic gates in quantum computation and NMR technique, transmitting 4 messages, i.e. 2 bits, by treating one of the entangled quantum pair, exactly as the original meaning of the Bennett-Wiesner proposal, over angstroms distance.

The concept of dense coding is briefly discussed here (see Fig. 1). Alice, the information receiver, prepares a pair of Einstein-Podolsky-Rosen (EPR) particles and sends one of them to Bob, the information sender, while re-
taining the other at her hand. When Bob wants to communicate with Alice, he can encode his message by manipulating the particle sent by Alice with one of the four transformation of \( I, \sigma_z, \sigma_x \) and \( i\sigma_y \), where \( \sigma_x \), \( \sigma_y \) and \( \sigma_z \) are the Pauli matrices and \( I \) is the unit matrix, then transmit the encoded particle to Alice. Alice decodes her received message by measuring the two particles at her hand and learns which transformation Bob applied.

1 The network of quantum dense coding

The network that we use for realizing quantum dense coding in terms of the logical gates and circuits in quantum computation is shown in Fig. 2, where \( a \) and \( b \) denote two quantum systems with two states \(|0>\) and \(|1>\). It consists of three parts separated by the dotted lines. Our strategy includes three steps.

1.1 Preparation of the entangled state (EPR pair)

The EPR pair \( \frac{1}{\sqrt{2}}(|00>-|11>\) can be prepared by the operations of the NOT gate \( N_b \), the Walsh-Hadamard gate \( H_b \) and the controlled-NOT (C-NOT) gate \( CN_{ba} \). Supposing that the input state is \(|0>_b|0>_a\), we have

\[
\begin{align*}
|0>_b|0>_a & \xrightarrow{N_b}\frac{1}{\sqrt{2}}|1>_b|0>_a \\
& \xrightarrow{H_b}\frac{1}{\sqrt{2}}(|0>-|1>_b|0>_a \\
& \xrightarrow{CN_{ba}}\frac{1}{\sqrt{2}}(|00>-|11>)
\end{align*}
\]

It is easily seen that the other three Bell base states \( \frac{1}{\sqrt{2}}(|00> + |11>\), \( \frac{1}{\sqrt{2}}(|01> - |10>\) and \( \frac{1}{\sqrt{2}}(|01> + |10>)\) can be produced by replacing \( N_b \) with \( I \) (the identity transformation, i.e. nothing being done), \( N_bN_a \) and \( N_a \), respectively.

1.2 Transformation of the entangled states

Through operating one member of the EPR pair with four different transformation \( U_{ai}=I, \sigma_z, \sigma_x \) and \( i\sigma_y \) (for \( i=1, 2, 3 \) and 4), one obtains four Bell base states respectively, thus achieving message encoding. The processes are represented as
\[
\frac{1}{\sqrt{2}} (|00 > - |11 >) \rightarrow \frac{U_{a1}}{\sqrt{2}} (|00 > - |11 >)
\]
\[
\frac{U_{a2}}{\sqrt{2}} (|00 > + |11 >)
\]
\[
\frac{U_{a1}}{\sqrt{2}} (|01 > - |10 >)
\]
\[
U_{a1} - \frac{1}{\sqrt{2}} (|01 > + |10 >)
\]

(2)

1.3 Measurement of the entangled states

It is accomplished by the action of the C-NOT gate \(CN_{ba}\) and the gate \(H_b\). After these measurements that can distinguish different Bell base states, four states, \(|y_{bi} > |x_{ai} >\equiv |yx_i <=|10 >, |00 >, |11 > and -|01 > for i = 1, 2, 3 and 4 respectively are read out and the messages are decoded.

It can be clearly seen that four messages have been transmitted from Bob to Alice via encoding transformation on one member of the entangled pair and decoding measurement of the whole system in terms of the network shown in Fig. 2, which is consistent with the original idea of the Bennett-Wiesner proposal. If quantum system \(a\) (e.g. photon) could move far away from \(b\) in the processes from step 1 to step 2 and step 2 to step 3, which means Alice locating a long distance away from Bob, then one fulfills long-distance communication. If a (e. g. atom or nuclear spin) would stay in the vicinity of \(b\) in the whole process, which means that Alice and Bob are close to each other, then close communication is brought about in this case. No matter which of the four Bell states the process begins from, we always get definite outputs associated with different transformation operations. The relationship between the encoding transformation \(U_{ai}\) and the decoding outputs \(|yx_i >\), under various starting EPR states is listed in table 1.

2 The experimental procedure and result

The network stated above can be put into practice with NMR techniques. We chose \(^1H\) and \(^{13}C\) in the molecule of carbon-13 labeled chloroform \(^{13}CHCl_3\) (Cambridge Isotope Laboratories,Inc.) as the two-spin system in the experiments and d6-acetone as the solvent, the solute/solvent ratio being 1:1 (v/v). The sample was flame sealed in a standard 5-mm NMR tube. Spectra were recorded on a Bruker ARX500 spectrometer with a probe tuned at 500.13MHz for \(^1H\) (denoted by a), and 125.77MHz for \(^{13}C\) (denoted by b).
The NOT gate was realized by applying a pulse of $X(\pi)$, the $H_b$ gate was implemented by using pulses of $X(\pi)Y\left(-\frac{\pi}{2}\right)$, and the C-NOT gate was realized by the pulse sequence shown in Fig. 3. Each pair of $X(\pi)$ pulses applied on the spin took opposed phases in order to reduce the error accumulation caused by imperfect calibration of the $\pi$ pulses. The $U_{ai}$ transformations correspond to pulses through $U_{a1} \sim I$, $U_{a2} \sim Z_a\left(\frac{\pi}{2}\right)$, $U_{a3} \sim X_a(\pi)$ and $U_{a4} \sim Y_a(\pi)$.

The experiments were carried out in a procedure as follows. Firstly, the effective pure state $|00\rangle$ was prepared by temporal averaging\cite{13}. Then the operations applied according to the network shown in Fig. 2 were executed by a series of NMR pulses mentioned above. Finally, the outputs of experiments, described by the density matrices $\rho_{outi} = |yx\rangle\langle ii|$, corresponding to four operations of $U_{ai}$, were reconstructed by the technique of state tomography\cite{13}.

The matrices reconstructed by fitting the measured data from the recorded spectra are shown in Fig. 4. The axes of the horizontal plane denote the locations of the elements in the matrix, its values 0, 1, 2 and 3 correspond to the states $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$, respectively, and the vertical axis represents the module of the theoretical and experimental matrices, we can find that they are in good agreement. Experimental errors were primarily due to the inhomogeneity of the RF field and static magnetic field, inaccurate calibration of pulses and signal decay during the experiment. The largest error is about 10%. These results show that the network actually does the quantum dense coding communication.

3 Conclusion

We have experimentally implemented quantum dense coding by using NMR quantum logic and circuits in quantum computation. With the help of non-classical effects inherent in quantum systems, four messages, i.e., 2 bits of information (instead of three messages or 1.58 bit as in Ref.\cite{5}) have been transmitted via treating one of the entangled two spin-$\frac{1}{2}$ systems, which demonstrated physically the original idea of the Bennett-Wiesner proposal. Principally, the dense coding experiment with maxially entangled states for $n>2$ particles can be performed by utilizing an appropriate NMR sample with $n$ spins and a generalized quantum network. As the nuclear spins in a molecule are coupled through chemical bond, the communication by dense
coding via NMR technique actually conducts between spins in angstrom distance. Nevertheless, the concept and the method of transformation and measurement of the maximally entangled states in terms of the quantum computation network should be useful in other processes of quantum information processing and quantum computing.

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Captions of the Figures and Table

Fig. 1. The schematic diagram of dense coding.
Fig. 2. The network for quantum dense coding.
Fig. 3. The pulses of controlled-NOT gate.
Fig. 4. The distribution of the density matrices of the quantum system. The a, b, c and d are the experimental results, and e, f, g and h are the theoretical ones corresponding to the operations I, $\sigma_z$, $\sigma_x$ and $i\sigma_y$ respectively.

Table 1 Correspondence between the starting states and output states.
| U_a1 | 10> | 00> | 11> | 01> |
|------|------|------|------|------|
| U_a2 | 00> | 10> | 01> | 11> |
| U_a3 | 11> | 01> | 10> | 00> |
| U_a4 | 01> | -11> | 00> | 10> |

Table 1
\[
\frac{1}{\sqrt{2}}(|00> - |11>) \quad \frac{1}{\sqrt{2}}(|00> - |11>) \quad \frac{1}{\sqrt{2}}(|00> - |11>) \quad \frac{1}{\sqrt{2}}(|00> - |11>)
\]
References

[1] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. 69(16)(1992)2881-2884

[2] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wooters, Phys. Rev. Lett. 70(13)(1993)1895-1899

[3] S. M. Barnett, R. London, D. T. Pegg, and J. D. Phoenix, J. Mod. Opt. 41(1994)2351-2373

[4] A. Zeilinger, H. J. Bernstein, and M. A. Horne, J. Mod. Opt. 41 (1994) 2375-2384

[5] K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. 76 (25)(1996)4656-4659

[6] C. H. Bennett, Phys. Today 48 October (1995)24-30

[7] M. Michler, K. Mattle, H. Weinfurter, and A. Zeilinger, Phys. Rev. A 53 (1996)R1209-1212

[8] K. Shimizu, N. Imoto, and T. Mukai, Phys. Rev. 59, (1999)1092-1101

[9] C. Monroe, D. Meekhof, B. King, W. Itano, and D. Wineland, Phys. Rev. Lett. 62(1995)2124-2127

[10] L. Davidovich, A. Maali, M. Brune, J. Rainmond, and S. Haroche, Phys. Rev. Lett. 71(15)(1993)2360-2363

[11] D. Cory, A. Fahmy, and T. Havel, Proc. of the 4th Workshop on Phys. and Comp. PhysComp96, Boston University, 22-24 November, New England Complex System Institute(1996)87-91

[12] N. A. Gershenfeld and I. L. Chuang, Science 275(17)(1997)350-356.

[13] I. L. Chuang, N. Gershenfeld, and M. Kubinec, Phys. Rev. Lett. 80(15) (1998) 3408-3411

[14] I. L. Chuang, L. M. K. Vandersypen, Xinlan Zhou, D. W. Leung, and S. Lloyd, Nature 393(14)(1998) 143-146

[15] J. A. Jones, M. Mosca, and R. H. Hansen, Nature 393(28)(1998)344-346
[16] M. Pravia, et al, LANL e-print-ph/9905061

[17] G. Brassard S. L. Braunstein, and R. Cleve, Phys. D 120(1998)43-47

[18] M. A. Nielsen, E. Knill, and R. Laflamme, Nature396(5)(1998)52-55
fig4 d
fig4 f
fig4  g
Fig. 1
Fig. 3