Finding viable Models in SUSY Parameter Spaces with Signal Specific Discovery Potential

Thomas Burgess\textsuperscript{a}  Jan Øye Lindroos\textsuperscript{a}  Anna Lipniacka\textsuperscript{a}  Heidi Sandaker\textsuperscript{a}

\textsuperscript{a}Department of Physics and Technology, University of Bergen, Norway

E-mail: thomas.burgess@ift.uib.no, jan.lindroos@ift.uib.no, heidi.sandaker@ift.uib.no, anna.lipniacka@ift.uib.no

Abstract: Recent results from ATLAS gives a Higgs mass of 125.5 GeV, further constrain already highly constrained supersymmetric models such as pMSSM or CMSSM/mSUGRA. Finding potentially discoverable and non-excluded regions of model parameter space is becoming increasingly difficult. Several groups have invested large effort in studying the consequences of Higgs mass bounds, upper limits on rare $B$-meson decays, and limits on relic dark matter density on constrained models, aiming at predicting superpartner masses, and establishing likelihood of SUSY models compared to that of the Standard Model vis-à-vis experimental data. In this paper a framework for efficient search for discoverable, non-excluded regions of different SUSY spaces giving specific experimental signature of interest is presented. The method employs an improved Markov Chain Monte Carlo (MCMC) scheme exploiting an iteratively updated likelihood function to guide search for viable models. Existing experimental and theoretical bounds as well as the LHC discovery potential are taken into account. This includes recent bounds on relic dark matter density, the Higgs sector and rare $B$-mesons decays. A clustering algorithm is applied to classify selected models according to expected phenomenology enabling automated choice of experimental benchmarks and regions to be used for optimizing searches. The aim is to provide experimentalist with a viable tool helping to target experimental signatures to search for, once a class of models of interest is established. As an example a search for viable CMSSM models with $\tau$-lepton signatures observable with the 2012 LHC data set is presented. In the search 105209 unique models were probed. From these, ten reference benchmark points covering different ranges of phenomenological observables at the LHC were selected.

Keywords: SUSY, CMSSM, MCMC, LHC
1 Introduction

Supersymmetry (SUSY) may alleviate many of the problems associated with the Standard Model of particle physics (SM) if the mass of the superpartners lies close to the TeV-scale [1], [2]. Furthermore, it provides a natural dark matter candidate in the form of the Lightest Supersymmetric Particle (LSP), if $R$-parity is conserved [3]. However, even the simplest SUSY extension of the SM, the so called Minimal Supersymmetric Standard
Model (MSSM), introduces over 100 new free parameters making them very difficult to experimentally constrain. On the other hand a large part of the MSSM parameter space is already ruled out, as it would lead to unobserved phenomena like non-conservation of lepton numbers, flavour changing neutral currents or large CP violation [4]. It is therefore common practice to look at constrained models that assume a partial unification of parameters at some high energy scale and where the dynamics of the high energy theory ensures more viable phenomenologies [5]. The minimal SUper GRAvity model (mSUGRA) [6] is an example of such a constrained model where the SUSY parameters are assumed to unify at the GUT scale into five universal parameters, a common scalar mass $m_0$, a common gaugino mass $m_{1/2}$, the ratio between the SUSY Higgs vacuum expectation values $\tan \beta$, a common trilinear Higgs-sfermion coupling $A_0$, and the sign of the Higgsino mass parameter $\mu$. In the “lighter” version of it, the so called constrained MSSM (CMSSM) [7–9] gravitino mass is not forced to unify at the same scale as other gaugino masses. In NUHM (Non-Universal Higgs Masses) [10, 11] models, masses of Higgs bosons do not unify with sfermions to the common $m_0$.

ATLAS and CMS experimental searches for SUSY usually present results only in two-dimensional slices of the parameter space of some simplified model assuming fixed values for other parameters [12, 13]. Due to complicated dependence of physical masses and thus experimental signatures on all the model parameters, it is easy to leave specific corners of the model space unexplored in such an approach, leaving out regions where experimental search may have large discovery potential. This has led several theoretical groups [14, 15] to reinterpret experimental searches in different regions of parameter space with help of simplified simulators of detector response like DELPHES [16] or PGS [17]. This approach can be relatively reliable for moderately simple experimental signatures involving jets and missing transverse energy ($E_T^\text{miss}$), but it cannot be trusted for more difficult experimental objects like photons or tau leptons.

MCMC based parameter inference has been successfully employed to find the most viable region of the full parameter spaces, based on requirements that the models should be in accordance with recent experimental constraints [14, 15], including these on the Higgs boson mass and rare $B$-mesons decays. While such scans provide a more complete picture of the still allowed regions of parameter space they do not consider whether these parameter space regions are within experimental reach. This poses difficulties for experimentalists when trying to make direct use of the results.

In this paper, a MCMC-based framework for determining the part of non-excluded model parameter space where a given experimental signature can be observed is presented. By adding a signature specific discoverability parameter to the set of current experimental constraints, the interesting regions of the parameter space are found. Models from these regions are then partitioned according to phenomenology using a clustering algorithm to enable an automatized construction of reference points for optimizing experimental searches. This step distinguishes our approach from existing similar frameworks, for example [18].
The procedure is applicable to a wide range of signatures and models, and is intended as a tool for experimentalist to extend limits to more interesting regions of parameter space. It is important to note that we do not intend to find the true maximal likelihood regions, as the discoverability measure does not reflect any existing constraint. In order to provide a proof of concept, a concrete example defining a non-excluded part of CMSSM parameter space which could be discoverable with $\tau$-leptons in the 2012 LHC data is outlined.

The paper is structured as follows: section 2 discusses the publicly available software tools used to calculate low energy CMSSM observables, scan and clustering algorithms, as well as the specific constraints and phenomenological parameters used. Section 3 describes the results of the scan and the phenomenological reference points constructed. Appendix A explains the details of the algorithm implementation and presents cross-checks of the effects of experimental constraints with other existing results. In Section 4, a summary and comments on the procedure are provided.

2 Algorithms and Tools

Experimental constraints on dark matter relic density $\Omega h^2$ as well as on rare processes such as $B_s \rightarrow \mu\mu$ and $b \rightarrow s + \gamma$ set strict bounds on the parameter space of CMSSM (see for example [15]). Furthermore, the Higgs boson mass of 125.5 GeV as measured by ATLAS [19] is hard to accommodate in CMSSM, making the fraction of viable models within current experimental reach extremely small. This renders simple uniform scans highly inefficient. A rough random scan made to explore the parameter dependence in CMSSM, gave a fraction of $10^{-5}$ models in accordance with current experimental constraints. Therefore, more advanced techniques need to be employed to get a representative picture of the discoverable and non-excluded regions of parameter space in an efficient way.

The approach used in this paper is to employ a likelihood distribution $P$, that reflects how well models fit the data and their discovery potential, to perform a guided random walk through parameter space using Markov Chain Monte Carlo [20]. This increases the search efficiency as the parameter space is sampled according to the distribution $P$ thus less time is spent sampling low likelihood regions. In this work an adaptive MCMC is implemented, where the likelihood map is based on the compatibility of low energy properties of CMSSM models with experimental and theoretical constraints, and discovery potential. These properties are calculated using several publicly available software tools.

2.1 Software Tools

A series of publicly available software tools is used to calculate the low energy parameters needed to check experimental constraints on the SUSY models, and to construct the like-
likelihood map used in the MCMC scan. Parameters are passed between the different tools using the SLHA-interface \[21\]. The tools are called in sequence starting with the least computationally costly, and after each step the likelihood is updated based on the available parameters. Each component \((i)\) of the likelihood is constructed to have a maximal value of \(P_i=1\) so that the likelihood always decreases as the chain progresses. This makes it possible to check for rejection after every step in the tool sequence, and enables early termination of the calculations for a large fraction of low likelihood models.

In the first step, \textsc{IsaJet} with \textsc{IsaRed} \[22\] is used to run the GUT scale universal parameters down to the electroweak scale, calculate \(Br(B_s \to \mu\mu)\), and to check whether the models are allowed by several theoretical constraints, including requirements of a \(\tilde{\chi}_1^0\) LSP and correct electroweak symmetry. In the next step, \textsc{FeynHiggs} \[23\] and \textsc{HiggsBounds} \[24\] are used to recalculate and check if the model fulfills experimental constraints on the Higgs sector. Afterward, the dark matter relic density, \(\Omega h^2\), and \(Br(b \to s + \gamma)\) is calculated using \textsc{darkSUSY} \[25\], which also checks against experimental constraints on sparticle masses from LEP \(\Delta \rho\) and Z-width (see for example \[26, 27\]). Finally, 1000 pp signal events at \(\sqrt{s} = 8\) TeV are generated using \textsc{Pythia} \[28\] in order to get a leading order estimate of the SUSY cross-section, \(\sigma_{LO}\), and to calculate the fraction of events \((Br_\tau, Br_{jet} \ldots)\) containing respectively at least one \(\tau, e, \mu,\) jet with pseudorapidity in the central part of the detector, \(|\eta| < 2.5\), and sufficiently large momentum in the plane perpendicular to the beam axis, \(p_T > 20\) GeV, and the average number of these objects per SUSY event \((n_\tau, n_{jet} \ldots)\). For each of these objects, the average \(p_T\) is calculated for the two with the highest transverse momentum. The average missing transverse energy per event, \(E_T\), is also calculated.

The software tools and their employment are summarized in Table \ref{tab:software}.

| Tool                  | Information used                                           |
|-----------------------|------------------------------------------------------------|
| \textsc{IsaJet} 7.83 & \textsc{IsaRed} | SUSY masses, \(Br(B_s \to \mu\mu)\)                           |
| \textsc{FeynHiggs} 2.9.4 & \textsc{HiggsBounds} 3.8.1 | Higgs sector                                               |
| \textsc{darkSUSY} 5.1.1 | \(\Omega h^2, Br(b \to s + \gamma)\)                         |
| \textsc{Pythia} 8.175 | \(\sigma_{LO}, Br(n_{\ell}, p_T)\) for \(\ell = \tau, e, \mu, \text{jet}\) |

\[2.2\] Likelihood Map and Experimental Constraints

The likelihood map \(P\) used to explore CMSSM parameter space is constructed by combining a likelihood \(P_{\text{exp}}\) based on experimental and theoretical constraints with an ad-hoc likelihood related to the expected number of events with tau leptons, \(P_\tau\). Here \(P_\tau\) is based on the probability of producing observable \(\tau\)-leptons with \(21/fb\) of the LHC data collected.
in 2012. \( P_\tau \) can be easily replaced by another likelihood function related to observability of any signal of interest. The likelihoods are normalized so that each individual contribution \( P_i \) has a maximal value \( \max(P_i) = 1 \). Thus, the full likelihood becomes:

\[
P_{\text{tot}} = P_{\text{exp}} \cdot P_\tau \quad \text{and} \quad P_{\text{exp}} = \prod_i P_i,
\]

where \( P_i \) are the likelihoods related to experimental limits and theoretical constraints. Some of \( P_i \) are either 0 or 1 as specified in table 2. These include most of theoretical constraints, limits checked internally by the software tools used. For other experimentally measured quantities Gaussian errors are assumed and the resulting likelihoods are continuous. Gaussian distributions around the central experimental values are used for \( \text{Br}(b \to s + \gamma) \), \( \text{Br}(B_s \to \mu\mu) \), and the Higgs mass, while for the relic density a uniform distribution is chosen with a Gaussian tail above the best observational value. The latter accepts models with the relic density lower than the recent Planck result [29], allowing for other unknown sources except of CMSSM neutralinos to contribute to the relic density. The central values and standard deviations used are \( \Omega h^2 = 0.1199 \pm 0.0027 \) for the relic density [29], \( \text{Br}(b \to s + \gamma) = (3.55 \pm 0.42) \cdot 10^{-4} \) [30] for the charmless \( b \)-quark decay, with a theoretical uncertainty \( \sigma_{th} = \pm 0.33 \cdot 10^{-4} \) [31], and \( \text{Br}(B_s \to \mu\mu) = (3.2 \pm 1.5) \cdot 10^{-9} \) [32]. For the Higgs mass, the combined ATLAS best fit from the \( H \to \gamma\gamma, 4l \) channels is used [19], with a theoretical uncertainty \( \sigma_{th} = \pm 1.5 \text{GeV} \) is assumed [33], giving \( m_{h0} = (125.5 \pm 1.7) \text{ GeV} \). The experimental and theoretical constraints are summarized in table 2. The 2011 and 2012 ATLAS and CMS results of direct searches for SUSY in \( R \)-parity conserving channels are not included in the present work. The reason for it is two-fold. Firstly, the high Higgs mass translates in CMSSM into rather high sparticle masses, on the border of the present direct searches sensitivity. Secondly, our aim is to propose precise regions, where this sensitivity should be checked, and not to exclude them from our scans. Results obtained by ATLAS and CMS experimenters using dedicated detector response simulations to translate the present limits into other regions of parameter space should be more reliable than ones employing only approximate modelling of detectors response. We thus prefer to use this opportunity to provide tools to experimenters so that they can choose somewhat more interesting regions of SUSY parameter space to present their results.

The discoverability likelihood \( P_\tau \) is chosen as a Poissonian discoverability measure constructed from the sum of likelihoods for observing a given number of tau events, \( N_\tau \geq 1 \), given the expected number of events containing at least one \( \tau \), \( \langle N_\tau \rangle = \text{Br}_\tau \cdot L \cdot \sigma_{\text{LO}} \).

\[
P_\tau = \sum_{N_\tau = N_{\tau \min}} P(N_\tau | \langle N_\tau \rangle) , \quad P(N_\tau | \langle N_\tau \rangle) = \frac{\langle N_\tau \rangle^{N_\tau} \exp \left[ -\langle N_\tau \rangle \right]}{N_\tau!}.
\]
Table 2: Experimental and theoretical constraints used and the associated likelihoods

| Constraints                          | Likelihoods $P_i$ Values          |
|--------------------------------------|-----------------------------------|
| $\tilde{\chi}_1^0$, LSP, Correct EWSB, No tachyons ... | ISAJET 7.81 |
| Sparticle masses, $\Delta \rho$, Z-width | darkSUSY 5.0.5 |
| OK Higgs sector                      | HiggsBounds 3.7.0                 |
| $\text{Br}(B_s \rightarrow \mu \mu)$ | $\exp \left[ \frac{[\text{Br}_{B_s \rightarrow \mu \mu} - \mu_{B_s}]^2}{2 \sigma_{B_s}^2} \right]$ |
| $\Omega \chi^2$                      | $\exp \left[ \frac{[\text{Br}_{B_s \rightarrow \mu \mu} - \mu_{B_s}]^2}{2 \sigma_{B_s}^2} \right]$ |
| $\text{Br}(b \rightarrow s + \gamma)$ | $\exp \left[ \frac{[m_{B_s} - m_{B_s}^0]^2}{2 \sigma_{B_s}^2} \right]$ |
| $m_{h_0}$                            | $\exp \left[ \frac{[m_{h_0} - m_{h_0}^0]^2}{2 \sigma_{h_0}^2} \right]$ |

luminosity of 21/ fb and the fraction of events containing at least one $\tau$, $\text{Br}_\tau$, as found from Pythia, are used to calculate $\langle N_\tau \rangle$ above.

Experimental selection in search of specific signal has broadly speaking two steps.

The first step ensures that a specific experimental signature characterizing the signal is observed in the detector. In our case this signature consists of taus, jets and missing transverse energy. In order for this step to be fulfilled one needs to make sure that these experimental objects are within fiducial volume of the detector and have transverse momentum above a given threshold. The number of signal events passing this first step can be predicted with relatively good accuracy without using any sophisticated detector description.

In the second step of the selection, specific cuts in order to reject the backgrounds are performed, and some measure of sensitivity is used in order to optimize background rejection while keeping as much of the signal as possible. The number of expected signal events after such a selection can vary orders of magnitude depending on specific strategy chosen.

One example of this is the ATLAS $\tau$ search as presented in [34], where two different strategies are considered, one for events where exactly 1 $\tau$ is selected and another for events with 2 or more $\tau$ leptons. This difference in selection strategy gives large differences in signal selection efficiency. It is clear that precise choice of strategy needs to be done with precise tools using reliable detector simulation, background estimation and cutflow optimization, and this can be done only withing experimental collaborations.

The aim of the scan presented here however is to find interesting regions for $\tau$ searches in which this second step of the selection can be performed, because there is enough events passing the first step.

This is why the constructed measure $P_\tau$ is based on the number of $\tau$ events observable in the detector rather then a more precise estimate valid for a specific analysis strategy. The
main uncertainty in $P_\tau$ comes from neglecting NLO corrections to the cross section and theoretical uncertainties in the LO cross section. The NLO corrections for CMSSM can be relatively large, with k-factors of the order of 3 [35].

The LO cross section uncertainty estimate is sensitive to variations in renormalisation/factorization scale, parton distribution function and the strong coupling $\alpha_s$, and can lead to uncertainties of order 100%, compared to $\sim 20\%$ at NLO [36]. In comparison Pythia MC uncertainties are negligible with relative errors of order $\sigma_{MC} \lesssim 0.1$ for reasonable branching fractions $Br_\tau \gtrsim 0.1$. The combined uncertainty in $P_\tau$ is also much smaller than the uncertainty associated with experimental selection.

Including NLO calculations for each point increases computational time by a factor of five or more, and given the uncertainties related to experimental selection outlined above, the computational gain outweighs the loss in precision. As our ambition is merely to identify which regions of mSUGRA parameter space are more interesting than others, NLO corrections matter only if they vary a lot across the parameter space. The option for including Prospino NLO calculations [37] is implemented in the package, and is used to get more precise estimates for the proposed benchmark points. Indeed the corrections are quite comparable across proposed the benchmark points.

The search range in CMSSM parameter space follows the suggestion in [38]. The ranges for $m_0$, $m_{1/2}$, $A_0$ and $\tan \beta$ are presented in table 3. The anomalous muon magnetic moment, $\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$, is not taken into account in this scan, other than as a reason for choosing sign($\mu$) > 0, which is required to give positive SUSY contributions. This is because the value of $\delta a_\mu$ is generally incompatible with other constraints [15], leading to maximal likelihood regions in agreement with neither. In addition the actual value of $\delta a_\mu$ seems to be open for debate due to uncertainties in both LO and NLO hadronic contributions to $a_\mu^{\text{SM}}$ [15, 39]

A fixed value for $m_{\text{top}} = 173$ GeV was taken.

**Table 3**: Search ranges used for the CMSSM MCMC scans.

| Parameter | Range          |
|-----------|----------------|
| $m_0$     | [60,3000] GeV  |
| $m_{1/2}$ | [60,3000] GeV  |
| $A_0$     | [-5000,5000] GeV|
| $\tan \beta$ | [2,60]        |
| sign($\mu$) | +1            |

The lower bounds on the universal masses and on $\tan \beta$ in table 3 stem from LEP [27, 40] bounds, while the upper limits are chosen on the basis of naturalness for the masses $m_0$.
and $m_{1/2}$ and perturbativity of the Yukawa couplings for $\tan \beta$. The range for the trilinear coupling $A_0$ is extended compared to [38], since the Higgs mass has a quadratic dependence on $A_0$ [41] allowing for higher $m_{h_0}$ at large values of $|A_0|$.

It is important to note that the more realistic the likelihood, the more computationally efficient is the search for interesting models. However, the set of models found does not depend on fine details of the likelihood function used, as we accept points in a range of likelihood. Our goal is to find a set of interesting models fulfilling latest experimental constraints, in order to guide further experimental searches. We do not intend to make any *statistically quantified decision* on which of the selected models are more likely than others.

### 2.3 MCMC Algorithm

The MCMC method used here is a Metropolis-Hastings algorithm [42] where, given a point $x = \{x_1, x_2, \ldots, x_D\}$ in a D-dimensional parameter space, a proposal distribution, $Q(y|x)$, is used to sample a new point $y$. The proposal distribution is related to the likelihood $P$ of the new point $y$ being “interesting” from the point of view of requirements described in section 2.2. The new point is accepted randomly with a probability given by

$$\alpha(y|x) = \min \left(1, \frac{P(y)Q(x|y)}{P(x)Q(y|x)} \right),$$

(2.3)

If $y$ is accepted, it is added to the chain and the next point is sampled starting from $y$. If it is not accepted, the chain remains at $x$ and the process is repeated as illustrated in figure 1. The asymptotic distribution of likelihoods calculated for the resulting chain of points is the desired likelihood distribution $P$.

In order to efficiently map possible high likelihood regions of the parameter space which are separated by large regions of low likelihood a regional adaptive MCMC algorithm similar to [43] has been implemented. The algorithm approximates the target likelihood distribution $P$ as a mixture of normalized multivariate Gaussian distributions and uses this approximation as a basis for a proposal $Q(y|x)$, as explained in the following section 2.3.1. This proposal is used to guide multiple MCMC search chains in parallel and it is iteratively updated according to the resulting selected sample of points.

#### 2.3.1 Adaptive Multi Chain Monte Carlo

An initial estimate for the proposal was constructed by uniformly sampling the space such that all separated regions where the likelihood $P$ is high are covered. The idea is similar to
A proposal distribution $Q(y|x)$ used to sample new points $(y_1, y_2)$ from a point $x$. These points are either accepted ($y_2$) or rejected ($y_1$) depending on the ratio between the underlying likelihood $P$ and $Q$ as given by $\alpha$ (2.3).

**Figure 1**: Illustrations of the standard MCMC sampling method, 1a, and a typical MCMC random walk, 1b.

The bank sampling introduced in [44], where prior knowledge about the local maxima of the likelihood distribution is incorporated into the proposal to increase efficiency of sampling the distributions where these maxima are separated by large regions of low likelihood. The points of CMSSM parameter space chosen for the initial sample were required to pass all discrete cuts and to give experimentally measured physical variables within a reasonable range of the experimentally preferred values, see table 2 for details. Sampled points in CMSSM parameter space were weighted according to their likelihood and clustered using $k$-means algorithm, to be defined in 2.4. The "shape" of each cluster was estimated by calculating the weighted mean $\mu$ vector and covariance matrix $\Sigma$. The number of clusters corresponded to the number of normalized Gaussian distributions (normal mixture) that was to be used to approximate the likelihood distribution $P$, as explained below.

A small fraction of large jumps [38, 43, 45] was added to the standard small jumps illustrated in figure 1a in order to increase sampling efficiency. To achieve this a global proposal term $q_G(y)$, was added to the standard local one, $q_L(y|x)$, giving the full proposal distribution:

$$Q(y|x) = \beta q_L(y|x) + (1 - \beta)q_G(y) ,$$

(2.4)

Here $\beta$ is a mixing parameter relating the global and the local proposal terms, explained
The global proposal $q_G(y)$ was taken as a set of $m$ multivariate normal distributions, $N$, as in [46]. The set was large enough to describe the main features of the target likelihood. Each multivariate distribution was multiplied by a weight factor $w_i$, defined in the formula below.

$$
q_G(y) = \sum_{i=1}^{m} w_i N(y|\mu_i, \Sigma_{G,i}), \quad N(y|\mu, \Sigma) = \frac{\exp \left[ -\frac{1}{2}(y - \mu)^T \Sigma^{-1}(y - \mu) \right]}{\sqrt{(2\pi)^D |\Sigma|}}, \quad (2.5)
$$

$$
w_i = \frac{\sum_{x_i} P(x_i)}{\sum_{x} P(x)}
$$

Each weight factor was estimated by summing up the total likelihood over CMSSM parameter space points in a cluster $i$. Here $\mu_i$ is the vector of the means of the $i$'th normal distribution, while $\Sigma_{G,i}$ is the covariance matrix of the $i$'th component of the mixture. The local proposal $q_L(y|x)$ was taken as a normal distribution with mean, $x$, and the covariance, $\Sigma_{L,i}$, characterizing the closest cluster in the parameter space. An euclidean distance measure was used and each parameter was scaled so that the search ranges defined in table 3 varied from 0 to 1. The local proposal covariance was chosen so that: $\Sigma_{L,i} = \alpha_i \Sigma_{G,i}$, $i \in \{1, 2, \ldots m\}$, where $\alpha_i$ was a parameter adapted such that the local acceptance rate for points in the parameter space region within the cluster $i$ was between 0.05 and 0.15. The rather low acceptance rate was chosen because the hierarchical nature of the likelihood calculation yields higher computational speed for low acceptance rates. Thus our optimal acceptance rate is probably lower than that of 0.23 found in [47]. The acceptance probability for stepping from a given point $x$ to a new point $y$ was then given as:

$$
\alpha(y|x) = \min \left( 1, \frac{P(y) \left[ \beta q_L(x|y) + (1 - \beta)q_G(x) \right]}{P(x) \left[ \beta q_L(y|x) + (1 - \beta)q_G(y) \right]} \right). \quad (2.6)
$$

The search chains were started from random CMSSM parameter space points in the weighted sample and followed independently. After a given number of steps data were re-clustered and the proposals were updated, taking into account the new sampled parameter space points, where the new points were weighted according to the estimated likelihood. A certain likelihood threshold $P_{\text{min}}$ was required for the first relevant point in each chain, since we are interested in high likelihood regions. The implementation details are described in the Appendix A.
2.4 Clustering Algorithm

A modified $k$-means \cite{48} algorithm, to be defined below, has been devised in order to cluster likelihood-weighted points. The role of clustering is two-fold. Firstly, clusters in CMSSM parameter space were needed to calculate the approximate Gaussian distributions used in the proposal described in section 2.3.1. Secondly, sets of high-likelihood model-points in CMSSM parameter space were clustered according to the different experimental signatures they were expected to exhibit in the detectors at the LHC.

The $k$-means algorithm defines clusters in the parameter space by assigning each point to the closest centroid, $(C)$. The algorithm was initialized by choosing at random $k$ points in the parameter space as cluster centers, $C$. Next, each $C_i$ was refined as the average of the points near to it and points were reassigned to the new $C$, and the procedure was repeated until it converged to a set of stable $C$s. To increase the speed of the algorithm, a maximum number of iterations and a minimum improvement between iterations was set for the centroids positions refinement.

In order to define a closest centroid, a distance measure is required. An euclidean distance measure in CMSSM space was employed, scaling each parameter such that the search ranges defined in table 3 varied from 0 to 1. Using another distance measure (for example a log scale) only alters the proposal distribution and thus only affects the efficiency of the algorithm, not the results. The $k$-means algorithm, described above, with predetermined number of clusters was used to cluster points in CMSSM space.

In order to define reference points in phenomenological space the arbitrary choice of the number of such points has to be avoided. To this end $k$-means formed the basis for a $g$-means algorithm, with which the number of clusters was determined automatically, as explained further. In both instances the random first guess of cluster centers (centroids) positions was improved as suggested in \cite{49}, ($k$-means++ algorithm). In this method probability of picking a new point as a cluster center was weighted by the square distance to the closest already picked point. This guess reduced as well the average number of iterations to achieve convergence.

To determine the number of clusters $k$ in the space of phenomenological observables, a $g$-means algorithm \cite{50} was employed. It started with applying $k$-means for $k = 2$, thus dividing the parameter space into two sub-clusters. Then a statistical test to verify if the likelihood distributions of both clusters could be described by a single Gaussian was performed. For each phenomenological observable, $x_i$, the standard deviation, $\sigma_i$, and the mean, $\bar{x}_i$, were calculated and each observable transformed $x'_i = (x_i - \bar{x}_i)/\sigma_i$ to facilitate defining the distance for clustering purposes. If the test rejected the single Gaussian distribution hypothesis, the procedure was repeated recursively for each of the new clusters, otherwise recursion was terminated. The statistical test was performed by means of the one
dimensional Anderson-Darling normality measure [51]. The distances between the points and a plane that separated clusters and was perpendicular to the vector between the two centroids were subjected to the measure above. As the clustering could be sensitive both to outliers and to the random positioning of initial centroids, the whole splitting procedure was iterated $n_{\text{avg}}$ times and the average number of clusters $\langle k \rangle$ was noted. Next, the clustering outcome with $k$ closest to $\langle k \rangle$ was picked. If there were several clustering outcomes giving the same number of clusters $k$, one of these was picked at random. In the final step the obtained centroids were subjected to the $k$-means clustering one more time to ensure that a stable configuration has been found.

3 CMSSM with $\tau$ Signatures

3.1 Results

All viable models found have large negative values of $A_0$ in common, but otherwise span a relatively large range of sparticle masses and values of $\tan \beta$. The ranges for the mean $E_T$ and $p_T$ for leading jet and $\tau$ are shown in the CMSSM mass planes $m_0 - m_{1/2}$ and $A_0 - \tan \beta$ in figure 2 and two dimensional likelihood distributions are shown in figure 3. The distributions are constructed by binning the models into $N_{\text{bins}} = 50$ bins along each dimension, where the likelihood of each bin is approximated by the number of models contained. The effects of different constraints separately are discussed in the Appendix A.

![Figure 2](image.png)

**Figure 2**: Average value per bin for mean $E_T$ and $p_T$ for leading jet and $\tau$ shown in the CMSSM mass plane $m_0 - m_{1/2}$ (above) and $A_0 - \tan \beta$ plane (below)
Figure 3: Marginalized likelihood maps for different planes in CMSSM space.

The *discoverability* likelihood constrains the SUSY $\tau$ production cross-section to not be too small. This sets upper bounds on how large the gaugino and scalar masses can be since the production cross-section falls sharply as the masses of colored sparticles grow. The
cross-section has as well a slight $A_0$ dependence which allows for higher masses at higher negative values of $A_0$.

![Figure 4: Average value per bin for the mean number of $\tau$s and jets per SUSY event shown in the $m_0 - m_{1/2}$ plane (above) and $A_0 - \tan \beta$ plane (below)](image)

**Figure 4:** Average value per bin for the mean number of $\tau$s and jets per SUSY event shown in the $m_0 - m_{1/2}$ plane (above) and $A_0 - \tan \beta$ plane (below)

### 3.2 Phenomenology and Reference Points

The relatively wide range of values for SUSY masses and values of $\tan \beta$ found leads to a wide range of values of phenomenological properties such as average $\not{E}_T$, the average missing energy per SUSY event, $p_T(\tau_1)$, $p_T(\text{jet}_1)$, the average $p_T$ of the leading $\tau$ and the leading jet see figure 2, and $n_\tau$, $n_{\text{jet}}$, the average number of $\tau$’s/jets per SUSY event, see figure 4.

The $p_T$ values for the leading jet and $\tau$ lepton obviously tend to be higher for high sparticle masses, since higher masses in CMSSM lead to higher mass splittings between the sfermions and the LSP. The $p_T$-s also become larger with $A_0$ closer to 0 and for high $\tan \beta$ values. The missing energy on the other hand tends to become larger at smaller scalar masses.
Table 4: The range and best fit value for CMSSM parameters, relic density, $m_{h0}$, expected number of events with taus, $\langle N_\tau \rangle$ and $\text{Br}_{B_s \to \mu \mu}$.

| min  | $bf$ | max  | min  | $bf$ | max  | min  | $bf$ | max  | min  | $bf$ | max  |
|------|------|------|------|------|------|------|------|------|------|------|------|
| $m_0$ [GeV] | $m_{1/2}$ [GeV] | $A_0$ [GeV] | $\tan \beta$ |
| 123.7 | 448.8 | 2739  | 371.3 | 961.1 | 1881 | $-4998$ | $-2673$ | 52.75 | 4.598 | 15.80 | 59.44 |
| $\Omega h^2$ | $m_{h0}$ [GeV] | $\langle N_\tau \rangle$ | $\text{Br}_{B_s \to \mu \mu}$ $[10^{-9}]$ |
| 0.01 | 0.1164 | 0.1296 | 119.2 | 125.0 | 126.2 | 0.01 | 10.64 | 6784 | 3.840 | 3.907 | 8.494 |

and increasing gaugino mass. The increase in $E_T$ with $m_{1/2}$ is likely due to increasing neutralino mass. These dependencies are illustrated in figure 2.

The average number of $\tau$s per SUSY events is mostly due to the branching fraction into $\tau$s as it can be seen comparing figure 4 and 5. One tau with high $p_T$ per event is produced on average. The SUSY branching fraction to $\tau$s is largest at low values of $\tan \beta$ and $m_0$. At least one high $p_T$ jet is expected in almost every event. The average numbers of jets increases with $m_0$, $\tan \beta$ and $|A_0|$.

In order to construct reference benchmark models that cover these different phenomenological properties the sample was clustered according to the phenomenological observables: $E_T$, $n_{\text{jet}}$, $p_T(\text{jet}_1)$, $n_{\tau}$, $p_T(\tau_1)$, details are described in the Appendix A.

With these, ten phenomenological clusters shown in table 5 were found. The SUSY and model related parameters for these clusters are shown in table 6 and 7. The centroids of the clusters can be regarded as reference (benchmark) points. additional k-factors were calculated for these benchmark points to give a more precise estimate for the expected number of $\tau$’s. As it can be seen in table 6, NLO corrections are small ranging from 1.1 to 1.7, and around 125 events with high energetic taus in the central part of the detector are expected for the highest cross-section reference points, which might be enough to detect the signal using the $21/\text{fb}$ of data gathered in 2012. Exploration of the other reference points might have to wait until the LHC 13 TeV operations.

Various graphical projections of the clusters are shown in figures 6 and 7. It is clear from figures 7 that one finds viable models lying in the tails of clusters, far away from centroid positions. These models exhibit either low jet activity in the central part of the detector, but produce high $p_T$ tau leptons, or have low number of high $p_T$ jets (monojets) and low momentum taus. We have not investigated these models further yet, but it is clear that standard LHC SUSY searches assuming presence of high $p_T$ jets for triggering purpose
Expected number of $\tau$ events with $\int L dt = 21 fb^{-1}$ SUSY cross section

Figure 5: Average value per bin for expected number of $\tau$ events and SUSY cross-section in the mass plane $m_0 - m_{1/2}$ (above) and $A_0 - \tan \beta$ (below)

might fail for such models.

We have investigated decay branching fractions of the lightest Higgs boson to $\gamma \gamma$, $ZZ$, $WW$, $\tau \tau$ and $\mu \mu$ for the ten reference points, compared to these of the SM Higgs of the same mass. These branching fractions are typically somewhat higher, alas they do not differ by more than 5% from the SM values.

4 Conclusions

This work presents a new method for finding and classifying SUSY models that can be potentially discovered in an accelerator experiment, here LHC experiments. The method uses an adaptive MCMC algorithm to find interesting models and uses a clustering algorithm to classify the models according to phenomenology. The likelihood map is constructed using
Table 5: Phenomenological parameters of clusters found. The first two columns are the cluster index, id, (matches id in table 6) and number of model-points in the cluster, n. For each parameter the cluster centroid value, cent, is listed along with the minimum, min, and maximum, max, for the cluster. Centroid values can be regarded as reference values characterizing given experimental phenomenology.

| id | n   | $E_T$ [GeV] | $n_{jet}$ | Jet$_1$ ($p_T$) [GeV] | $n_{\tau}$ | $\tau_1$ ($p_T$) [GeV] |
|----|-----|-------------|-----------|------------------------|------------|-------------------------|
|    |     | min  cent   max           | min  cent   max           | min  cent   max           | min  cent   max           | min  cent   max           |
| 1  | 16811 | 266.7 323.9 439.9 | 1.5 2.7 3.8 | 206.4 285.7 487.4 | 0.1 0.3 0.5 | 48.3 164.1 274.2 |
| 2  | 6905  | 167.1 273.9 332.5 | 0.1 1.7 2.8 | 31.1 317.8 487.8 | 0.2 0.5 0.8 | 121.8 225.9 303.2 |
| 3  | 6830  | 67.3  252.8 316.0 | 1.1 3.1 4.4 | 56.9 238.8 344.1 | 0.1 0.3 0.6 | 55.2 152.9 239.8 |
| 4  | 11881 | 290.6 383.1 467.2 | 0.9 2.1 3.1 | 316.1 493.1 666.5 | 0.2 0.6 1.0 | 66.5 132.5 208.6 |
| 5  | 9786  | 285.7 339.4 451.8 | 0.8 1.7 3.0 | 243.9 432.5 556.4 | 0.2 0.4 0.7 | 146.2 225.0 316.5 |
| 6  | 10255 | 111.0 267.4 331.1 | 0.0 0.5 1.5 | 376.5 561.4 821.5 | 0.5 0.6 1.5 | 131.9 266.1 367.1 |
| 7  | 11653 | 279.1 339.9 389.1 | 0.0 0.5 1.2 | 473.8 632.7 1127 | 0.5 0.6 1.1 | 191.9 280.9 367.3 |
| 8  | 10744 | 300.9 365.2 456.6 | 0.7 1.2 2.0 | 446.2 591.4 728.2 | 0.2 0.5 0.8 | 121.1 234.7 298.5 |
| 9  | 8999  | 259.9 338.0 477.2 | 0.0 0.5 2.5 | 0.0 361.4 534.5 | 0.5 0.6 1.2 | 221.9 307.7 476.2 |
| 10 | 11345 | 0.4  228.3 302.6 | 0.0 0.4 2.7 | 151.3 356.2 577.5 | 0.5 0.7 2.0 | 56.4 273.8 385.6 |

Figure 6: Projections of clusters. Colors indicate to which cluster a given model belongs.
Table 6: CMSSM parameters, relict density, 8 TeV CMS LHC production LO cross-section, total NLO k-factors and the number of expected events with $\tau$ leptons for the centroids of clusters. All models have $\text{sign} \mu > 0$ and $m_{\text{top}} = 173$ GeV.

| id | $m_0$ [GeV] | $m_{1/2}$ [GeV] | $A_0$ [GeV] | $\tan \beta$ | $\langle N_\tau \rangle_{\text{LO}}$ | $\langle N_\tau \rangle_{\text{NLO}}$ | $\ln P$ | $\Omega h^2$ | $\sigma_{\text{LO}}$ [fb] | $k_{\text{NLO}}$ |
|----|-------------|----------------|-------------|-------------|------------------|-----------------|-------|-------------|----------------|-----------|
| 1  | 821.7       | 937.4          | -2995.0     | 28.4        | 3.6              | 4.9             | -1.0  | 0.1         | 0.8             | 1.33      |
| 2  | 1150.0      | 854.5          | -3318.0     | 37.2        | 19.7             | 28.2            | -1.5  | 0.1         | 2.0             | 1.43      |
| 3  | 678.4       | 747.7          | -2807.0     | 25.7        | 76.5             | 125.5           | -1.9  | 0.1         | 14.1            | 1.64      |
| 4  | 278.0       | 740.4          | -1974.0     | 12.3        | 64.1             | 85.9            | -1.3  | 0.1         | 7.1             | 1.34      |
| 5  | 669.4       | 814.6          | -2685.0     | 25.8        | 29.9             | 43.1            | -1.1  | 0.1         | 3.5             | 1.44      |
| 6  | 1045.0      | 961.4          | -2774.0     | 37.9        | 7.5              | 8.7             | -1.2  | 0.1         | 0.6             | 1.16      |
| 7  | 749.6       | 986.2          | -2450.0     | 29.7        | 6.6              | 7.6             | -1.1  | 0.1         | 0.5             | 1.15      |
| 8  | 481.5       | 824.0          | -2149.0     | 21.5        | 26.4             | 33.5            | -1.1  | 0.1         | 2.6             | 1.27      |
| 9  | 1483.0      | 1070.0         | -3806.0     | 41.0        | 3.5              | 4.1             | -1.1  | 0.1         | 0.3             | 1.17      |
| 10 | 1813.0      | 985.9          | -4153.0     | 46.5        | 6.6              | 7.7             | -1.5  | 0.1         | 0.5             | 1.17      |

Table 7: Higgs and sparticles masses for the centroids of clusters.

| id | $m_{h^0}$ [GeV] | $m_{t^1}$ [GeV] | $m_{\tilde{g}}$ [GeV] | $m_{\tilde{\chi}_1^0}$ [GeV] | $m_{\tilde{\tau}_1}$ [GeV] |
|----|----------------|----------------|------------------------|-----------------------------|-----------------------------|
| 1  | 125.3          | 997.4          | 2082                   | 404.4                       | 406.9                       |
| 2  | 125.3          | 874.2          | 1932                   | 369.7                       | 375.8                       |
| 3  | 124            | 628.5          | 1690                   | 319.6                       | 323.5                       |
| 4  | 123.7          | 828.3          | 1658                   | 313.5                       | 314.6                       |
| 5  | 125            | 824.3          | 1827                   | 348.8                       | 354                          |
| 6  | 124.5          | 1201           | 2143                   | 416                         | 421.5                       |
| 7  | 124.3          | 1229           | 2177                   | 424.9                       | 425.3                       |
| 8  | 124.2          | 969.2          | 1837                   | 351.6                       | 353.1                       |
| 9  | 125.6          | 1246           | 2384                   | 467.9                       | 469                          |
| 10 | 125.6          | 1185           | 2233                   | 432.8                       | 437.8                       |

an extendible tool chain that incorporates recent limits from multiple sources through the SLHA interface. As the method employs the SLHA interface to communicate between the different tools, it is easily be extendible to other parameter spaces and experimental signatures. For example one can look for interesting regions of GMSB for a two lepton
analyses. This amounts to creating a steering file and specify the model, the parameter range of interest, the event topologies counted by Pythia and what constraints to take into account, e.g. count Pythia events containing $\ell = e, \mu$, and constrain on $\langle N_{2\ell} \rangle$. At the moment the actual code is only restricted by limitations to external software tools such
as isasugra (allowed parameter spaces) and DarkSUSY (regions with neutralino LSP). In addition the code includes options for doing gridded or uniform random scans in addition to MCMC based methods. The plan is to make the code publicly available in the near future including more parameter spaces and different LSP.

As an example and test of the method the highly constrained CMSSM parameter space has been searched for models that could potentially be discovered using 2012 LHC with τ-lepton based signatures.

Although simplified models like CMSSM are severely constrained, we are still able to find regions fulfilling recent LHC bounds on Higgs mass and rare B-meson decays, and giving relic density in agreement with WMAP results. All models we found have Higgs BR very close to those of SM. Thus if two sigma excess of Higgs to gamma gamma BR appears to be real then CMSSM is clearly disfavored as observed by [52]. Ten reference (bench-mark) points exhibiting different phenomenologies were found with use of a g-means clustering algorithm. These reference points can be used to optimize searches. This makes the method very attractive from an experimental point of view, and applying the method to other models and different signatures would be a natural extension of this work. In fact the region found in this search is already part of an effort within the ATLAS astroparticle forum to provide additional model grids taking into account constraints from astro- and astroparticle physics.

The method has proven successful in finding and classifying SUSY models, but could still benefit from several extensions and improvements. More constraints could be added to the likelihoods and more advanced statistical analysis of the simulated data could be incorporated. Another interesting prospect would be to include detector simulations using PGS [17] or DELPHES [16] to get somewhat more realistic estimates for the expected signal, although we are skeptical about realism of such simulations. The MCMC algorithm constructed could still benefit from improvements to increase stability and efficiency, in addition to rigorous numerical testing. Finally a better distance measure for the clustering could allow for precise predictions of expected discovery potential.

On the more experimental side we find some viable models lying in the tails of clusters formed based of phenomenological observables. These models exhibit either low jet activity in the central part of the detector, but produce high $p_T$ tau leptons, or have low number of high $p_T$ jets (monojets) and low momentum taus. We have not investigated these models further yet, but it is clear that standard LHC SUSY searches assuming presence of high $p_T$ jets for triggering purpose might fail for such models.
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Appendix A  Scan Implementation, clustering and cross-checks

The implementation of the scan was written in Python. It was capable of running multiple MCMC chains in parallel. The scan was initiated using a random sample containing roughly 100 points to find an estimate for the proposal distribution. The initial points were required to pass all discrete constraints, be within $2\sigma$ of the best fit values for $\text{Br}(b \rightarrow s + \gamma)$, $\text{Br}(B_s \rightarrow \mu \mu)$, $m_{h_0}$, while having $\Omega h^2$ of the right order of magnitude and an expected number of produced $\tau$ leptons, $\langle N_{\tau} \rangle > 1$. The values are summarized in table 8, together with the distributions used for initializing the proposal distribution.

Table 8: Experimental constraints used in the initial sampling.

| Constraints | Range | Distribution |
|-------------|-------|--------------|
| $\Omega h^2$ | $(0, 0.2)$ | Flat |
| $\text{Br}(b \rightarrow s + \gamma)$ | $[2.71, 4.39] \cdot 10^{-4}$ | Gaussian |
| $\text{Br}(B_s \rightarrow \mu \mu)$ | $[0.2, 6.2] \cdot 10^{-9}$ | Gaussian |
| $m_{h_0}$ | $[122.3, 128.7]$ GeV | Gaussian |
| $\langle N_{\tau} \rangle$ | $[1, \infty)$ | Flat |

Five clusters were established from the initial parameter space points with the $k$-means algorithm. This number was found to be sufficient to give a reasonable approximation of the likelihood distribution of the sample. From the initial sample, ten chains were initiated with two chains starting from each cluster. Before sampling started, each chain was required to reach a minimum likelihood to be included in the sample. This was chosen to be $2\sigma$ away from the central value for $\text{Br}(b \rightarrow s + \gamma)$, $\text{Br}(B_s \rightarrow \mu \mu)$, $m_{h_0}$, $\Omega h^2$, in addition to $\langle N_{\tau} \rangle \geq 1$, corresponding to the likelihood, $\ln P_{\text{min}} \sim -4 \cdot \frac{8}{2} - 0.5 = -8.5$. 

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The proposal distribution was updated at intervals $\Delta N = 1000$ steps by adding the new sample points and recalculating cluster means and covariances. The new points were added without weights since they were already a product of weighted sampling. For practical purposes we end the optimization after 10 000 steps, which was found to be sufficient to give a good proposal estimate. By fixing the proposal after a certain number of steps the algorithm also satisfy the necessary conditions to ensure asymptotic convergence toward the true likelihood distribution, since the algorithm becomes equivalent to running a set of independent Metropolis-Hastings chains. The search chains were run in parallel on twenty cores for roughly 200 hours, resulting in a sample size of $N = 2\,076\,133$, corresponding to 105 209 unique models. From this sample 848 outliers (corresponding to 66 unique models) with log-likelihood $\ln P < -8.5$ were removed.

In order to illustrate the effects of the different experimental and theoretical constraints, low energy properties were calculated for 300 000 models sampled uniformly within the search range. The computationally expensive Pythia simulations where not done for these models and a looser relic density constraint compared to the one used for MCMC initialization was used to get sufficient data to describe the qualitative features of the constraint.

As a cross-check with the vast literature on the subject (see for example [38, 53, 54]) we briefly describe the effects of the most important constraints by visualizing how the initial selections affect the model density. The effects are illustrated in figure 8. We observe, in agreement with results of [15, 53] that:

- **Theoretical constraints**
  Theoretical constraints remove the low $m_0$ and $m_{1/2}$ regions primarily avoiding a $\tilde{\tau}_1$-LSP and tachyonic sparticles. The excluded regions becomes larger at large $\tan \beta$ and $|A_0|$, and for large values of $A_0$ a considerable part of the low mass regions, $m_0, m_{1/2} \lesssim 1000$ GeV gives tachyons.

- **Higgs mass $m_{h0} \in [122, 128]$**
  Requiring a 125.5 GeV Higgs mass, with positive $\mu$ excludes all positive values of $A_0$ within the selected $m_0, m_{1/2}$-range. For large negative values of $A_0$ however, the $\tilde{t}$-loop corrections to the Higgs mass become large. This is the main reason for the asymmetry in $A_0$ seen in 3. At large values of $m_{1/2}$ and $m_0$ the FeynHiggs calculations of the Higgs mass corrections become inaccurate and thus we excluded these regions.

- **Relic Density $\Omega h^2 < 1$**
  As is well known, the relic density Dark Matter in CMSSM is generally orders of magnitude larger than allowed by WMAP and PLANCK results [54–56], apart from special regions where the relic density is suppressed by resonant neutralino annihilation or co-annihilation cross-sections. The low $m_{1/2}$ region where $m_{\tilde{\chi}^0_1} \lesssim 10$, the relic density is mainly suppressed through $\chi$-annihilation to fermions through sfermion exchange (low $m_0$), and to $W, Z$ pairs (high $m_0$). This region is excluded primarily by
Higgs mass requirements. Along the $\tilde{\chi}_1^0$-LSP boundary the relic density is reduced by $\chi-\tilde{\tau}$-coannihilation, since the coannihilation cross-section is significantly enhanced due to mass degeneracy between the lightest stau and the lightest neutralino. The middle region in the mass plane, the well known Higgs funnel [57], corresponds to high $\tan\beta$ models with $m_{\tilde{\chi}_1^0} \sim 1/2 m_H, A^0$, giving an increase in $\chi-\chi$-annihilation through heavy neutral higgs bosons, $(H^0, A^0)$. The preference for $A_0 \sim 0$ arises mainly from the fact that large parts of the low $m_{1/2}$ regions exhibit charged LSP or tachyonic particles for large values of $|A_0|$, as off-diagonal terms in the third generation sfermion mass matrices grow with $|A_0|$. The preference for high $\tan\beta$ is in part due to the additional relic density suppression through the Higgs channel $\chi$-annihilation.

- Rare Decays $\text{Br}(B_s \rightarrow \mu\mu) < 4.5 \cdot 10^{-9}$, $\text{Br}(b \rightarrow s + \gamma) \in [3, 4] \cdot 10^{-4}$

Of the constraints on decays, $B_s \rightarrow \mu\mu$ poses the most stringent one, as the SUSY contribution grows like $\tan\beta^6$. This branching fraction tends to get too large at low values of $m_0$ and $m_{1/2}$. The size of the excluded area in the mass plane increases with increasing $\tan\beta$ and decreasing $|A_0|$. $\text{Br}(b \rightarrow s + \gamma)$ is generally too low compared to the central experimental value of $3.55 \cdot 10^{-4}$ and excludes large parts of the low $m_{1/2} \lesssim 500$ range, stretching as far as $m_0 \sim 2000$ for high values of $\tan\beta$ and low $|A_0|$. Too high $\text{Br}(B_s \rightarrow \mu\mu)$ and too low $\text{Br}(b \rightarrow s + \gamma)$, together with the requirement of non-tachyonic sparticles constrains the lowest allowed values of $m_{1/2}$.

![Figure 8: 2D-histograms in $m_0, m_{1/2}$ and $A_0, \tan\beta$-planes showing the effects of different constraints. The constraints used corresponds to requirements chosen for the initial sample given in table 8](image)

The properties of selected high likelihood models are presented in the results section, 3.1. The relatively wide range of values for SUSY masses and values of $\tan\beta$ for the selected models lead to a wide range of values of phenomenological properties such as average $\bar{E}_T$, the average missing energy per SUSY event, $p_T(\tau_1), p_T(\text{jet}_1)$, the average $p_T$ of the
leading $\tau$ and the leading jet, and $n_\tau, n_{\text{jet}}$, the average number of $\tau$’s/jets per SUSY event, see figures 2 and 4. In order to construct reference models that cover these different phenomenological properties the sample was clustered according to the phenomenological observables listed above. In order to avoid bias from the scale of the different variables, each variable $x$ is first transformed as $x' = (x_i - \bar{x})/\sigma_x$ so that the mean $\bar{x}' = 0$ and variance $\sigma_x'^2 = 1$. Because the non-Gaussian nature of clusters the $g$-means algorithm often fail and split too often. To remedy this the constraining parameters mentioned in section 2.4 are used. By setting an approximate maximum number of possible clusters $n_{\text{max}}$, one gets $\min_P = \lceil N_{\text{OK}}/n_{\text{max}} \rceil$ for the minimum number of points in a cluster and $\min_s = \lceil \log_2 n_{\text{max}} \rceil$ for the maximal splitting depth. The maximal number of iterations per split attempt was set to $\max_i = 20$. Here the number is chosen to be well above the final number of clusters but low enough, for this case $n_{\text{max}} = 100$ was found to be appropriate. The minimal cluster distance parameter was set to $\min_d = 1.3$. The optimization was run $n_{\text{avg}} = 7$ times and an average of 9.8 clusters were found. Thus, one of the results with 10 clusters was picked at random. The properties of models at the centroids of these clusters, which can be seen as reference models for search optimization, are presented in the results section 3.1.

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