ISOSPIN SPLITTING IN THE BARYON OCTET AND DECUPLET

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Abstract

Baryon mass splittings are analyzed in terms of a simple model with general pairwise interactions. At present, the \( \Delta \) masses are poorly known from experiments. Improvement of these data would provide an opportunity to make a significant test of our understanding of electromagnetic and quark-mass contributions to hadronic masses. The problem of determining resonance masses from scattering and production data is discussed.

The isospin splittings of the masses and coupling constants of baryons and mesons arise from the mass differences of the up and down quarks and the electromagnetic interactions between them. These splittings provide a good way to test our knowledge of the internal wave functions of these particles. At present there is no evidence for isospin splitting of coupling constants, at least at the 3\% level, although further work might show an effect\[1\]. However, mass splittings in many isospin multiplets are known, and there has been extensive theoretical discussion.

I shall outline here a simple way to describe baryon splittings that embodies the main features of more detailed calculations made by others\[2, 3\]. The explicit models often contain several adjustable parameters that may not have a transparent meaning. One advantage of rewriting the mass perturbation effects is that it provides a clearer picture of how our theoretical understanding is affected by the experimental situation. A surprising aspect of this is that the \( \Delta \) masses are the ones that are in most need of further study.

To introduce the approach that is used here, and also to obtain some parameters, I first consider the SU(6) splittings induced by hyperfine interactions and by the difference \( m_s - m_n \) of the strange quark mass and the average of the natural quark masses. The model is based on the one introduced by De Rujula, Georgi, and Glashow\[4\]. I assume the energy is the sum of one-body and two-body quark effects, in which the third quark is an inert spectator. The one-body effects (simple mass and kinetic energy terms) are included in the two-body effects, with half of each single-quark term ascribed to each of two pairs. There are five different two-body terms distinguished by their quark content, triplet terms \( T_{nn}, T_{ns}, \) and \( T_{ss} \), along with singlet terms \( S_{nn} \) and \( S_{ns} \). In the simple \( S \)-wave model, decuplet states contain 3 triplet pairs, while octet states contain \( 3/2 \) triplet pairs and \( 3/2 \) singlet pairs. Some generalizations of this picture are discussed later.

In fitting to the experimental quantities, I use a formulation in which a “model error” \( t \) is added in quadrature to the experimental errors\[5\]. This allows a consistent way to judge the general goodness of fit in a situation in which some data are known much more precisely than others. If the model is not good enough to fit all data within their errors, the model error is defined to be the value of \( t \) required to give a value for \( \chi^2 \) equal to the
Table 1: Multiplet Masses

|      | $T_{nn}$ | $T_{ns}$ | $T_{ss}$ | $S_{nn}$ | $S_{ns}$ | total     | data     | ±       | $\chi^2_2$ |
|------|----------|----------|----------|----------|----------|-----------|----------|---------|------------|
| $N$  | 618.4    | 0.0      | 0.0      | 318.7    | 0.0      | 937.0     | 938.9    | 0.00    | 0.05       |
| $\Lambda$ | 0.0      | 730.2    | 0.0      | 212.5    | 175.7    | 1118.4    | 1115.6   | 0.50    | 0.10       |
| $\Sigma$ | 412.2    | 243.4    | 0.0      | 0.0      | 527.2    | 1182.8    | 1193.2   | 0.04    | 1.35       |
| $\Xi$  | 0.0      | 243.4    | 556.8    | 0.0      | 527.2    | 1327.4    | 1318.0   | 0.16    | 1.12       |
| $\Delta$ | 1236.7   | 0.0      | 0.0      | 0.0      | 0.0      | 1236.7    | 1232.8   | 0.26    | 0.20       |
| $\Sigma^*$ | 412.2    | 973.6    | 0.0      | 0.0      | 0.0      | 1385.8    | 1384.7   | 0.30    | 0.02       |
| $\Xi^*$ | 0.0      | 973.6    | 556.8    | 0.0      | 0.0      | 1530.4    | 1533.4   | 0.36    | 0.11       |
| $\Omega$ | 0.0      | 0.0      | 1670.4   | 0.0      | 0.0      | 1670.4    | 1672.5   | 0.30    | 0.06       |

Table 2: Pair Energies

|      | $nn$ | $ns$ | $ss$ |
|------|------|------|------|
| $T$  | 412.25 | 486.80 | 556.79 |
| $S$  | 212.46 | 351.47 |

number of degrees of freedom.

The best fit to the central masses of isospin multiplets, which has a model error of 8.9 MeV, is given in table 1. (These central masses are obtained by fitting to isospin splittings.) It is seen that this simple pair model can fit the splittings to within 5%. The $\Sigma$ and $\Xi$ masses are the most discrepant. The pair term energies that give this fit, shown in table 2, will be used later to estimate parameters for a more explicit model. In the context of this model, it is possible to interpret the triplet contributions as containing contributions of tensor forces, to the extent that these do not lead to non-spectator effects. It is possible to add plausible non-spectator terms that reduce the model error, but this is not useful here.

To fit the masses of the individual isospin components, besides the central mass values, there are four distinct isospin-splitting pair terms to be used. These are $T^1$, $T^2$, $T^1_s$, and $S^1_s$, where the superscript $I$ denotes the isospin tensorial rank and a subscript $s$ indicates that the pair contains one strange quark. Note that the Coleman-Glashow relation among octet masses is automatically satisfied by parametrization with the three independent $I = 1$ pair terms, independently of any specific model of the origin of the isospin-splitting terms. In addition, however, the pair model implies that relations exist among the octet and decuplet mass splittings. Use of all available data from the PDG compilation except the $\Delta^+$ mass gives a fit with a model error of 0.14 MeV, which is also about 5% of typical splittings. This fit is the main result reported here, and provides a starting point for further discussion. The numerical values, identified here as fit A, are shown in table 3 and in the figures.

The results in table 3 show that the $\Delta$ masses are the hardest to fit. The data used here are an average of the values obtained by Koch and Pietarinen and Abaev (which are very similar). The VPI group has recently obtained the preliminary value $M(\Delta^0) - M(\Delta^{++}) = 1.1 \pm 0.1$ MeV, which is more compatible with the model. The unused discrepant $\Delta^+$ datum was obtained from photoproduction. Crawford has reported a value (1231.6 MeV) that is more consistent with the prediction, but did not
Table 3: Contributions to particle masses in fit A

|        | $M_0$   | $T^1$ | $T^2$ | $T^1_s$ | $S^1_s$ | total  | data   | ±   | $\chi^2$ |
|--------|---------|-------|-------|---------|---------|--------|--------|-----|----------|
| $N^+$  | 938.92  | −0.74 | 0.00  | 0.00    | 0.00    | 938.18 | 938.27 | 0.00 | 0.39     |
| $N^0$  | 938.92  | 0.74  | 0.00  | 0.00    | 0.00    | 939.66 | 939.57 | 0.00 | 0.39     |
| $\Sigma^+$ | 1193.15 | −0.74 | 0.24  | −0.39   | −2.94   | 1189.32| 1189.37| 0.07 | 0.09     |
| $\Sigma^0$ | 1193.15 | 0.00  | −0.47 | 0.00    | 0.00    | 1192.68| 1192.55| 0.10 | 0.50     |
| $\Sigma^-$ | 1193.15 | 0.74  | 0.24  | 0.39    | 2.94    | 1197.45| 1197.50| 0.05 | 0.11     |
| $\Xi^0$ | 1318.02 | 0.00  | 0.00  | −0.39   | −2.94   | 1314.69| 1314.80| 0.80 | 0.02     |
| $\Xi^-$ | 1318.02 | 0.00  | 0.00  | 0.39    | 2.94    | 1321.35| 1321.34| 0.14 | 0.00     |

|        | $\Delta^{++}$ | −2.21 | 0.71  | 0.00    | 0.00    | 1231.25| 1231.00| 0.30 | 0.58     |
|        | $\Delta^+$    | 1232.76| −0.74 | −0.71   | 0.00    | 1231.31| 1234.90| 1.40 | *        |
|        | $\Delta^0$    | 1232.76| 0.74  | −0.71   | 0.00    | 1232.78| 1233.40| 0.50 | 1.41     |
|        | $\Delta^-$    | 1232.76| 2.21  | 0.71    | 0.00    | 1235.68| ?      |     | *        |
| $\Sigma^{*+}$ | 1384.72 | −0.74 | 0.24  | −1.54   | 0.00    | 1382.68| 1382.80| 0.40 | 0.08     |
| $\Sigma^{*0}$ | 1384.72 | 0.00  | −0.47 | 0.00    | 0.00    | 1384.25| 1383.70| 1.00 | 0.29     |
| $\Sigma^{*-}$ | 1384.72 | 0.74  | 0.24  | 1.54    | 0.00    | 1387.24| 1387.20| 0.50 | 0.01     |
| $\Xi^{*0}$   | 1533.38 | 0.00  | 0.00  | −1.54   | 0.00    | 1531.84| 1531.78| 0.34 | 0.02     |
| $\Xi^{*-}$   | 1533.38 | 0.00  | 0.00  | 1.54    | 0.00    | 1534.93| 1535.20| 0.80 | 0.11     |

$* = $ omitted from fit

Table 4: Predicted $\Delta$ masses from fit B

|        | $M_0$   | $T^1$ | $T^2$ | $B$     | $A$     | data   | ±   | $\chi^2$ |
|--------|---------|-------|-------|---------|---------|--------|-----|----------|
| $\Delta^{++}$ | 1232.48| −1.94 | 0.89  | 1231.43| 1231.25| 1231.0 | 0.3 |          |
| $\Delta^+$   | 1232.48| −0.65 | −0.89 | 1230.95| 1231.31| 1234.9 | 1.4 |          |
| $\Delta^0$   | 1232.48| 0.65  | −0.89 | 1232.24| 1232.78| 1233.4 | 0.5 |          |
| $\Delta^-$   | 1232.48| 1.94  | 0.89  | 1235.31| 1235.68| ?      |     |          |

Pedroni, et al., have measured the total cross sections for scattering of $\pi^\pm$ mesons from deuterium\[13\]. In the impulse approximation, this can determine a value for the mass combination $D = M(\Delta^-) - M(\Delta^{++}) + \frac{1}{3}(M(\Delta^0) - M(\Delta^+))$. Their result, after numerous and sizeable theoretical corrections, was $D = 4.6 \pm 0.2$ MeV, which corresponds to $-T^1(\Delta) = 1.38 \pm 0.06$ MeV. The uncertainty here represents only the statistical errors. This value, obtained by experimentation involving only $\Delta$ states, is intermediate between the values obtained from the more global fits $A$ and $B$.

Although the ease of fitting with the model was represented by a “model error”, it was actually the experimental $\Delta$ masses that were hard to accommodate. These experimentally-derived values may also be subject to unrecognized model-dependent systematic errors — this is a separate question. The model error was used here as a device to allow the $N$ and $\Sigma$ masses to relax somewhat from their precisely-known values in a

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* = omitted from fit
Figure 1: Octet isospin splittings, in MeV. The crosses give the experimental data and errors, and the circles show the fitted values. The radius is given by the model error.

global fit. Even so, it was the $\Delta$ masses that contributed the most to $\chi^2$, and when the $\Delta$ masses were ignored, the other mass values did not need to relax.

In a simple $S$-wave picture of the states, using first-order perturbation theory, the four pair energies can be expressed as linear combinations of four distinct contributions. The first of these is the single-particle mass term $\delta = \kappa (m_d - m_u)$, where $\kappa$ is a reduction factor arising from the momentum of the quarks. The “constituent” masses are interpreted here as “magnetic moment” masses. A simple standard fit to magnetic moments gives $m_n \sim 344$ MeV and $m_s \sim 533$ MeV, and $x = m_n/m_s = 0.65$. A second effect of the mass differences arises from the color-hyperfine interaction, parametrized by a coefficient $d$. There is also a Coulomb term $C$, proportional to $\langle 1/r \rangle$, and a magnetic interaction term with a coefficient $b$. Both $b$ and $d$ are proportional to $\psi(0)^2$, and inversely proportional to the product of masses. Taking into account charge and spin factors, the pair energies can be expressed as

$$
T^1 = \frac{1}{3}C - \frac{1}{3}b - \delta + 2d,
T^2 = C - b,
T^1_s = -\frac{1}{3}C + \frac{1}{3}xb - \frac{1}{2}\delta + xd,
S^1_s = -\frac{1}{3}C - xb - \frac{1}{2}\delta - 3xd.
$$

(1)

Stevenson, et al., (SMG) have pointed out that electromagnetic box and penguin
graphs can contribute additional effects. However, they consider these only within the general class of pair terms, and these graphs do not alter the fact that there can be only four such independent terms. Rather, they give additional contributions to the right hand side of Eq. (1), and it would not be possible to determine them independently from the data. These effects call attention to the fact that the effective mass differences of quarks subjected to confinement have themselves been influenced by electromagnetic contributions. In other words, a clean separation of electromagnetic from quark-mass effects is not really possible. In particular, this affects the interpretation of the quark mass parameters $\delta$ and $d$. The meaning ascribed to the quantities $\langle 1/r \rangle$ and $\psi(0)^2$ could also be changed. The additional contributions introduced by SMG, as well as the four simple effects included in Eq. (1), may also depend on the environment provided by the third quark and thus also contribute to non-spectator effects.

In the simplified model, it is not possible to determine the four parameters from Eq. (1) because the coefficient matrix is singular. In this model, the pair energies should satisfy the constraint

$$2xT^1 - 2xT^2 - (3x + 1)T_s^1 + (1 - x)S_s^1 = 0.$$  

(2)

Using $x = \frac{2}{3}$ and the covariance matrix from fit A gives for the LHS of Eq. (2) the value $-0.54 \pm 1.07$ MeV. The constraint is therefore acceptable, and there is no need for additional terms. To eliminate $d$ as a free parameter, let $d = \tau \delta$ and use the same model to determine $\tau$ from the pair energies given in table 2. The mass-dependent hyperfine effect can be isolated in the quantity

$$d_s = \frac{1}{4} (T_{nn} - S_{nn} - T_{ns} + S_{ns}) = 16.1 \pm 2.1 \text{ MeV}.$$  

(3)

The pair energies also provide estimated values $x = 0.68$ and $\delta_s = \kappa_s (m_s - m_n) = 181$ MeV, which are compatible with the values from magnetic moments. The estimate $\kappa_s / \kappa \sim 1.5$ then gives

$$\tau = \frac{\kappa_s d_s}{x \kappa \delta_s} = 0.2.$$  

(4)

Fitting this constrained model to the pair energies from fit A gives an acceptable $\chi^2$ and the results shown in table 5. The numerical values of the parameters are similar to those determined by Isgur[4]. Note that there is considerable cancelation, especially in $T^1$, and except in $S_s^1$. This helps to explain why isospin splittings for hyperons are much larger than for the $N$ and $\Delta$. It also suggests that the latter may be more sensitive to non-spectator effects, because the pair effects tend to cancel, and the corrections might not.
Figure 2: Decuplet isospin splittings, in MeV. The small filled circles give the predicted \( \Delta \) masses, when the \( \Delta \) data are omitted.

Table 6: Comparison of triplet pair energies

|       | A   | B   | I   | C   |
|-------|-----|-----|-----|-----|
| \( -T^1 \) | 1.48| 1.29| 2.3 | 1.9 |
| \( T^2 \)   | 1.42| 1.78| 1.6 | 1.7 |

\( I = \text{Isgur}[2], \ C = \text{Capstick}[3] \)

A comparison of the fitted values of \( T^1 \) and \( T^2 \) with the numbers obtained from recent explicit baryon models is shown in table 6. The Isgur[2] and Capstick[3] values for \( T^1 \) and \( T^2 \) listed in this table were obtained from their predictions for \( \Delta \) masses. They both adjusted some parameters to fit the \( n - p \) mass difference \( -T^1(N) = 1.3 \) MeV. The difference between their \( T^1 \) values for the \( N \) and \( \Delta \) is a measure of the non-spectator effects in their models.

The results in table 6 show that the \( \Delta \) masses provide a sensitive test of models. These masses also provide the best opportunity for improvement in the experimental data. At the same time, it should be possible to study isospin breaking effects in their partial widths.

A different view, that improved \( \Sigma^* \) mass values would provide the most improvement in our understanding, was expressed by SMG[14]. The \( \Sigma^* \) and \( \Xi^* \) masses certainly have large uncertainties, as can be seen from Fig. 2. It is clear that reduction of these uncertainties would contribute greatly to our understanding of the structure of decuplet states. However, the apparent sensitivity of the undetermined SMG parameters to the input \( \Sigma^* \) data also depends in part on the method of fitting they employed. Moreover, it
has been shown here that the reported $\Delta$ masses already show an inconsistency with pair-interaction models that do incorporate the terms suggested by SMG. There are also two $\Delta$ charge states for which reliable masses are not yet available. Examination of the $\Delta$ mass determinations suggests that, in addition to the statistical errors, these may at present be subject to model-dependent systematic errors amounting to perhaps $\frac{1}{2}$ MeV. This should be resolved by new, independent analyses. Similar effects would also be present in the $\Sigma^*$ and $\Xi^*$ masses, but in these cases the existing statistical errors dominate.

The explicit calculations$[2, 3]$ suggest that decuplet masses may be sensitive to model parameters that influence non-spectator contributions. Independently, it might be expected that pair models would fail to describe splittings in baryons that contain heavy quarks. The structure of states that contain charm or bottom quarks might differ in important respects from baryons with three lighter quarks. Further experimentation with heavy quark systems could give useful information about this important point.

Improved experimentation and analysis for the reactions $\gamma p \rightarrow \pi^0 p (\pi^+ n)$ could lead to significantly better mass values for the $\Delta^+$. In addition, further experimentation with deuteron targets could provide valuable information about all the $\Delta$ states, but would also require difficult and careful analysis. Quasi-free scattering of $\pi^-$ from the neutron could give information about the $\Delta^-$. A set of experiments comparing quasi-free $\pi^\pm$ scattering from the proton and the neutron in the deuteron would provide a set of interlocking comparisons in which some of the systematic errors in the determination of the free-nucleon cross sections might cancel out. Elastic differential $\pi^\pm$ scattering from deuterons would be more difficult to analyse, but would give another way to compare amplitudes, especially for the $\Delta^{++}$ and the $\Delta^-$. Similar analyses could be applied to photoproduction from the deuteron, and give a direct comparison of $\Delta^+$ with $\Delta^0$.

Comparison of the masses and couplings of states with different flavors requires use of a common set of conventions and definitions. This is especially important when data from different kinds of experiments are used. Consider the $S$-matrix for an isolated narrow multichannel resonance, which may be written as $S(E) = \tilde{S}_B S_R(E) S_B$, where $S_R$ is a simple Breit-Wigner resonance factor (with energy-dependent partial widths) and $S_B$ is a slowly varying background factor. In analysis of production experiments, the initial factor $S_B$ would be replaced by other factors depending on energy. In general, the resonance energy appearing in the factor $S_R$ provides a satisfactory initial estimate for the mass of the excited resonant state. However, removal of the background amplitude may introduce some model dependence. One way to proceed would be to look for the resonance pole, and then use the explicit energy dependence in $S_R$ to go back to the real axis.

At first sight, the problem of accounting for electromagnetic corrections to scattering amplitudes is relatively straightforward, but it involves several distinct aspects. The first aspect, that of correctly calculating the contribution of the initial and final state electromagnetic interactions to the $\pi p$ scattering amplitudes, can be treated by the method of Tromborg, et al.$[15]$. However, the best correction to be applied to determine the effective energy at which hypothetical chargeless particles would interact has not been firmly established. Furthermore, the models considered here involve idealized states with three quarks. The coupling to baryon-meson states introduces certain mass shifts. What we are concerned with here is the part of the difference in these shifts that originates in the electromagnetic interactions and the mass differences in the coupled channels, and it may be fruitful to focus attention on this restricted problem. These questions also arise in discussions of the rest of the baryon decuplet, and should be given a uniform treatment.
Acknowledgements: I wish to thank S. Capstick for information about isospin breaking models, G. Höhler for communications about the problem of extracting resonance parameters from data, and V. Abaev and R. Arndt for discussions of their analyses. I thank a referee for calling my attention to the relevance of the paper by Stevenson, et al. This work was supported by the U. S. Dept. of Energy under contract No. DE-AC02-76ER-03066.

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