$^3\eta$He nucleus modeling in the frame optical potential model

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Abstract

The conditions, at which quasi-bound $\eta-^3\text{He}$ state is possible, have been investigated and compared with the available findings about $\eta\text{N}$-scattering length and the information about $^3\text{He}$ nucleus from references. We conclude that the existence of $\eta-^3\text{He}$ quasi-bound state within the framework of the optical potential model, which doesn't contradict all collected findings, is not possible, but the observing anomaly of $\eta-^3\text{He}$-interaction at low energies is a virtual state.

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Quark-compound meson having the final lifetime can form short-living bound states with atom nuclei called mesic nuclei in case of it interaction of attraction with nucleons. Analyzing the result of the precise experiment [1], the authors of the work [2] have shown that the quasi-bound state existence of $\eta-^3\text{He}$ or the presence of virtual state of the system is possible. Considering that it has been $\eta$-nucleus of $\eta-^3\text{He}$ (quasi-bound state), we have investigated the conditions in the frame optical potential model, at which its formation is possible. The obtained conditions have been compared with the available findings about $\eta\text{N}$-scattering length and the information about $^3\text{He}$ nucleus, which is in the literature.

The optical potential for $\eta$-nucleus interaction is written in the following form:

$$2U(r)\mu = -4\pi(1 + \frac{m_\eta}{m_N})\rho(r)a_0$$ (1)

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where, $m_\eta$, $m_N$ are meson and nucleon masses, $\mu$ is reduced meson-nucleus mass, $a_0$ is $\eta N$-scattering length, $\rho(r)$ is the spherically symmetrical nucleon density of nucleus, which has been chosen in Fermi form:

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R_c}{a}\right)}$$

where $R_c$ is the half-density radius, $a$ is the diffusion thickness of nucleus, $\rho_0$ is the maximum nucleon density of nucleus. For $^3\text{He}$, $R_c$ and $a$ can be determined from the following conditions:

$$3 = \int_0^\infty r^2 \rho(r)dr, \quad \langle r^2 \rangle = \frac{1}{3} \int_0^\infty r^4 \rho(r)dr$$

where $\langle r^2 \rangle^{1/2}$ is the root-mean-square radius of $^3\text{He}$ nucleus determined during the experiment, which has been taken as being equal to 1.9 Fm, as the mean from some works, where this radius has been measured or determined. The obtained distributions of nucleon density of nucleus $^3\text{He}$ at different diffusion $a/R_c$ is shown in figure 1.

![Figure 1](image-url)  

Figure 1: The distribution of nucleon density of nucleus $^3\text{He}$ for different values of diffusion $a/R_c$. The curve parameters are given in table 1.
Table 1:

| \(N^0\) | \(a/R_c\) | \(R_c, \text{ Fm}\) | \(\rho_0, \text{nucleon/Fm}^3\) |
|-------|---------|-----------------|------------------|
| 1     | 0.10    | 2.210           | 0.060            |
| 2     | 0.15    | 1.991           | 0.074            |
| 3     | 0.20    | 1.770           | 0.093            |
| 4     | 0.25    | 1.571           | 0.114            |

As it is shown in the figure, for the diffusion, which is more than 0.2, the nucleon density distribution in nucleus \(^3\)He doesn’t have plateau in the middle, that, in our opinion, corresponds to the reality, so we have stopped at these values \(a/R_c\).

For quasi-bound state formation in the complex potential with own complex energy \(E = - (\epsilon + i\Gamma/2)\) where \(\epsilon\) is the binding energy, and \(\Gamma\) is the level width, the definite relation between the absolute values of imaginary and real parts of potential \([3,4]\) is required. We have calculated the formation boundary of \(\epsilon \approx 0\) of the discussed \(\eta\)-nucleus in the dependence of imaginary potential part on the real one at different nucleon densities of nucleus \(^3\)He. This dependence has been shown in the complex plane of free \(\eta\)N-scattering length (fig.2).

It is evident from this figure, that the quasi-bound state occurrence of system \(\eta-^3\)He with the low binding energy is possible at \(|Re(a_0)| \geq 0.75\ \text{Fm}\) for the values of nucleus \(^3\)He diffusion, \(a/R_c = 0.25\). It should be noticed that at the further diffusion increasing, \(a/R_c < 0.25\), the boundary of \(\eta\)-mesic nucleus formation practically doesn’t shift. The obtained limit for \(\eta\)-scattering length from the existence of \(^3\)He doesn’t contradict the available quantity values of \(|Re(a_0)|\) in set works, which are in the interval \((0.27 \div 1.0)\) Fm (see work \([5]\)). But their imaginary parts don’t allow the \(^3\)He formation to be considered as possible.

Thus, in the experiment \([1]\) the virtual state \(\eta-^3\)He, which is close by energy to the sum mass of free particles, has been revealed. Off-shell \(\eta\)N-scattering length use in potential (1), as the authors of work \([5]\) insist, couldn’t change our
Figure 2: Curves are the formation boundaries of quasi-bounded states for $\eta-^3\text{He}$, the darkened areas are the existence areas of quasi-bounded states for $\eta\text{N}$-scattering length with different parameters of $^3\text{He}$ nucleus diffuseness: $1-0.1$, $2-0.15$, $3-0.25$, in the complex plane; $\eta\text{N}$ scattering lengths of interaction have been taken from works: ■ $- [6]$, ● $- [7]$, ▲ $- [8]$. 

Conclusion, because the quantity decreasing of real and imaginary parts $a_0$ in this case must occur proportionally. At the same time our result agrees to the work [9] conclusion about the impossibility of the existence of quasi-bounded states of $\eta$-meson with three nucleons at different $\eta\text{N}$-scattering lengths, which are in the special literature, while the formation of virtual state $\eta-^3\text{He}$ is possible [9].

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