Quasi-singular methods for determining the limiting properties of the spacecraft entry trajectories into the planet's atmosphere

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Abstract. The problem of the limiting dynamic properties determining of the spacecraft (SC) trajectories with non-zero aerodynamic quality entered the atmosphere of a planet (Earth, Venus, Mars etc.) often has singularities. First of all, the singularity of the optimal control and of the SC dynamic characteristics is appeared after the interplanetary flight or after the SC maneuver in deep space. The target landing point is either the regular landing site on the Earth, or is the selected region with the specified geological characteristics on the surface of the planet (for example, Venus). The bulk and very laborious numerical experiments have shown that the optimal in terms of maximum coverage of the planet's surface by virtual trajectories of the descending spacecraft often presented as the trajectory with non-monotonic altitude changes (up to secondary spacecraft departure from the atmosphere and secondary entrance) and with the compact atmospheric regions of the spacecraft velocity break points. As it turned out, at the same time, the reachability areas on the surface of the planet obtained by optimization methods, have non-trivial geometric details. One of the most interesting nuances is the non-smooth beake-like border of the reverse side of the reachable area. In the certain sense, we can talk about generalized SC ricochets on the planet's atmosphere, which allow us to expand the reachability of the SC (both in terms of range and lateral range). The paper presents a phenomenological semi-analytical model for the phase beams constructing of the quasi-singular SC descent in the planet's atmosphere, which allows us to synthesize the main nuances of the reachability areas, instead the long-time big data optimization. As the result, the time of the preliminary mission ballistic analysis can be significantly reduced.

1. Introduction
Modern methods of the ballistic design of space flights (entry of a spacecraft with non-zero aerodynamic quality into the planet's atmosphere, carrying out gravity assist maneuvers around planets [1, 2, 3, 4] are associated with the need to calculate a lot of trajectories (i.e. of the phase beams). For their effective use and determination of dynamically admissible reachability regions, it is necessary to identify and study the structure of accompanying singularities of the phase parameters and to construct the corresponding adequate and adaptive models.

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2. The universal indicatrices approach

The representation of the trajectory of a mechanical system as an extremal of the variational principle in its extended phase space has been known since the time of Hamilton. Accordingly, it becomes possible to draw an analogy between the movement of an object of Newtonian mechanics and the propagation of a light beam in geometric optics in an optical medium with a suitably constructed refractive index \([1, 2, 5]\). In the latter case, the role of the variational principle is played by the principle of the smallest optical path (Fermat's principle). At the same time, we can rely on well-studied problems of geometric optics (as well as geometric acoustics and geometric electrodynamics). Using the indicatrices technique for the phase beams representation is well known and is the one of interesting approach for the physical systems obtaining with quasi-singularities \([1, 2]\). The indicatrices technique was appearing initially in the geometric optic during the light wave fronts studying \([5, 6]\).

The optic–mechanical analogy relates the trajectories of a mechanical system with the light trajectories in an anisotropic medium \([5, 6]\). For that reason, we may consider the indicatrix as the surface of all possible velocities with which the spacecraft leaves the reflection “point” \([1, 2, 5]\).

3. Quasi-singular indicatrix approach for the problem of spacecraft entry into the atmosphere

The toolkit of the quasi-singular indicatrix approach of spacecraft entry into the atmosphere (QIAE) is briefly as follows.

Consider the motion of the ricocheting spacecraft in the planet’s gravity field with the atmosphere areas described by a system of differential equations, where \( \mathbf{x} = f(x, t) \) is the phase state vector of the system and \( t \) is the undimensional time. Let’s introduce the reflection hypersurface \( \mathbf{x} = \mathbf{x}_R \) (usually it is determined by the level of the conditional dense altitude \( h \) of the planet’s atmosphere: \( h = h_0 \))

1. The movement of the ricocheting mechanical system (RM-system) (for example, a spacecraft during its ricochet entry in the planet atmosphere) can be described as a multiple chain of two-element couplings: the smooth ballistic trajectory sections BTA (ballistic trajectory arcs) and the local distortion zones with the strong atmosphere, DZ (distortion zone).

2. The smooth ballistic trajectory sections correspond to conservative motion (when a spacecraft moves near a planet in the thin atmosphere cloud), while break zones DZ correspond to strong non-conservative effects (the aerodynamic drag during the spacecraft enters the atmosphere).

3. The change in speed at the each current BTA occurs according to the ballistic law of motion, so that the vector of the "entry" velocity in the BTA \( v_{in,k}, k = 1,...,N_{BTA} \) is equal magnitude to the vector of the "exit" from the BTA, and their trajectory angles \( \vartheta_{in}, \vartheta_{out} \) will coincide up to a sign (Figure 1).
The length of the i-th ballistic section \( l_i \) depends on the parameters \( V_i \), - SC velocity speed and the angle of inclination of the trajectory \( \vartheta_i \) at the exit with the i-th zone maneuver. The \( l_i(V_i, \vartheta_i) \) can be determine according formula [2]:

\[
l_i(V_i, \vartheta_i) = 2(R_p + h_0) \arctan(V_i^2 \cos \vartheta_i \sin \vartheta_i (\mu/(R + h_0) - V_i^2 \cos \vartheta_i)^{-1})
\]

where \( R_p \) is the planetary radius (usually Earth mean radius) and \( \mu \) is the planetary gravitational constant.

4. The change in the velocity vector in the distortion zone DZ occurs almost singularly.

5. As a consequence of the previous point, the length of the BTA sections significantly exceeds the length of the break zones, and only these forms the areas of reachability of the ricochet path.

6. To describe the law of the singular change in the spacecraft velocity vector in the distortion zone DZ \( l_i(V_i, \vartheta_i) \), one can introduce an operator (by analogy with the light-front propagation indicatrix) an autonomous DZ indicatrix \( \mathbf{i}_{DZ}(\mathbf{x}, \mathbf{u}_{DZ}) \) for converting the input velocity vector \( \mathbf{v}_i \) into the output velocity vector \( \mathbf{v}_{out} \), where there are all kinds of virtual control functions in the DZ section, so that

\[
\mathbf{v}_{out} = (\mathbf{i}_{DZ}) \mathbf{v}_i
\]

7. It is possible to calculate “virtual” \( \mathbf{i}_{DZ} \) by precise previously solving in the DZ a set of optimization problems on the , varying \( \mathbf{u}_{DZ} \) (cumbersome, see Figure 2), or by using advanced modifications of the Monte Carlo method [7] with exact numerical integration (also cumbersome).

8. With the help of the choice of the previously calculated “virtual” \( \mathbf{u}_{DZ} \), one can change the initial data for calculating the next element of the chain (the current Cauchy problem generation) in order to maximize the final reachable region or some other special functionals. The cumbersomely mass calculations are remaining “out of the board” during the quick ballistic design.

9. To identify the main structural properties of the RM-systems, we can use also the phenomenological types of indicatrices \( \mathbf{i}_{DZ}(\mathbf{x}, \mathbf{u}_{DZ}) \) (Figure 3) with the carrying out mass modelling.

The synthesis of trajectory rays is carried out by choosing a large set of fixed vectors \( \mathbf{i}_{DZ}(\mathbf{x}, \mathbf{u}_{DZ}^j), j = 1,...,N_{mod} \) and performing the corresponding calculations of the Cauchy problems using the advanced Monte Carlo methods [1, 7]. We can describe the modelling results: cuts of the indicatrix borders and the borders of the attainability region in projection onto the reflecting plane in coordinates D,B (longitudinal range and lateral deviation).

4. The model problem of the ricocheting pebble on the surface of the water

The phenomenological model in the “problem of a ricochet pebble” helps to recognize the main structural properties of the spacecraft entry into the planet’s atmosphere in the first approximation. Consider a pebble ricocheting with attenuation along the surface of the water, and set the model problem of determining all points on this surface that it can hit at a fixed vector of the initial velocity \( \mathbf{V}_i \). For simplicity, the surface will be considered a horizontal plane. Obviously, the type of the sought-for area will be determined by the nature of the reflection of the pebble at each contact with water, which, in turn, depends on the current speed of the pebble, its rotation and orientation in space. A cumbersome optimization problem in a complex environment - the construction of the region of attainability of the specified motion, becomes solvable if we use the following model assumptions.

1. The increments of the functional of the longitudinal range D and lateral deviation B in the areas of direct reflection from the water are small compared to the areas of free flight.

2. Air resistance in free flight areas can be neglected. We can see that QIAE approach includes these positions.
Figure 1. The entry velocity vector in the BTA (altitude $h = h_0$) is equal magnitude to the exit vector volume, trajectory angles will coincide up to a sign.

Figure 2. The attainability region of the spacecraft entry into the Earth atmosphere, which was accomplished by the precise numerical optimisation [2]. D,B – the longitudinal range and lateral deviation, dashed lines – projections of the SC’s optimal trajectories on the reflection plane.

We can see that the attainability region is simply connected. The purpose is in possibilities of pebble dipping every ricochet.

Figure 3. The ricocheting pebble indicatrices

Figure 4. The ricocheting pebble range region

5. Spacecraft entry into the planet’s atmosphere
The main points of the presented QIAE approach and its tools were nurtured precisely for this interesting and urgent task. This approach made it possible to explain the structure of the spacecraft entry trajectories into the planet's atmosphere and to describe its main features. One of the most interesting nuances is the non-smooth beake-like border of the reverse side of the reachable area. Main details are presented in [1, 2, 3]. The structure of the indicatrices and the attainability region are presented below (Figure 5, Figure 6). One of the most interesting nuances is the synthesis of non-smooth beake-like border of the reverse side of the reachable area (let's compare with the Figure 2). The important and interesting details in [1, 2,] are explained.
Conclusion

The problem of the limiting dynamic properties determining of the spacecraft (SC) trajectories with non-zero aerodynamic quality entered the atmosphere of a planet (Earth, Venus, Mars etc.) often has singularities. First of all, the singularity of the optimal control and of the SC’ dynamic characteristics is appeared after the interplanetary flight or after the SC maneuver in deep space. The target landing point is either the regular landing site on the Earth, or is the selected region with the specified geological characteristics on the surface of the planet (for example, Venus). The bulk and very laborious numerical experiments have shown that the optimal in terms of maximum coverage of the planet’s surface by virtual trajectories of the descending spacecraft often presented as the trajectory with non-monotonic altitude changes (up to secondary spacecraft departure from the atmosphere and secondary entrance) and with the compact atmospheric regions of the spacecraft velocity break points. As it turned out, at the same time, the reachability areas on the surface of the planet obtained by optimization methods, have non-trivial geometric details. One of the most interesting nuances is the non-smooth beake-like border of the reverse side of the reachable area. In the certain sense, we can talk about generalized SC ricochets on the planet's atmosphere, which allow us to expand the reachability of the SC (both in terms of range and lateral range). The paper presents a quasi-singular indicatrix approach of spacecraft entry into the planet’s atmosphere (QIAE), which allows us to synthesize the main nuances of the reachability areas, instead the long-time big data optimization. As the result, the time of the preliminary mission ballistic analysis can be significantly reduced.

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