Integrating Holographic Vector Dominance to Hidden Local Symmetry for the Nucleon Form Factor

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Abstract

We derive a two-parameter formula for the electromagnetic form factors of the nucleon described as an instanton by “integrating out” all KK modes other than the lowest mesons from the infinite-tower of vector mesons in holographic QCD while preserving hidden local symmetry for the resultant vector fields. With only two parameters, the proton Sachs form factors can be fit surprisingly well to the available experimental data for momentum transfers $Q^2 \lesssim 0.5 \text{ GeV}^2$ with $\chi^2/\text{dof} \lesssim 2$. We interpret this agreement as indicating the importance of an infinite tower in the soliton structure of the nucleon. The prediction of the Sakai-Sugimoto holographic dual model is checked against the fit values to assess its accuracy in describing the proton structure. We find that the structure of the “core” of roughly 1/3 in the proton size indicated in experiments and commonly associated with an intrinsic quark-gluon structure in QCD is “hidden” in the infinite tower in the holographic model.
1 Introduction

The celebrated Sakurai vector dominance (sVD for short) model for the EM form factors \cite{1} works surprisingly well for the mesons \cite{2} but it famously fails for the nucleon \cite{3, 4, 5, 6}. The failure has been interpreted as an indication that the nucleon has a “core” which is not present in the pion structure \cite{3}. The “core” has been attributed – among a variety of possibilities – to a compact microscopic structure of QCD variables, such as for instance a little chiral bag with quarks confined within \cite{4}.

The recent development of holographic dual QCD (hQCD for short), specially, the Sakai-Sugimoto string theory model \cite{7} that implements correctly chiral symmetry of QCD, indicates a dramatic return of the notion of vector dominance for both mesons and baryons \cite{8, 9, 10, 11}. What characterizes the baryon structure in the hQCD model is that the baryon emerges as a soliton in the presence of an infinite tower of vector mesons. It is the infinite tower that renders the VD description applicable to both mesons and baryons. In fact, the isovector component of the EM form factor has the “universal” form

\begin{equation}
F_V^h = \sum_{k=0}^{\infty} \frac{g_{v(k)} g_{v(k)hh}}{Q^2 + m_{2k+1}^2}
\end{equation}

where \(g_{v(k)}\) is the photon-\(v^{(k)}\) coupling and \(g_{v(k)hh}\) is the \(v^{(k)}-hh\) coupling for \(h = \pi, N\). Here \(v^{(k)}\) is the \(k\)-th isosvector vector mesons \(\rho, \rho', \ldots\) in the tower. What distinguishes the hadron probed is the vector-meson coupling to the hadron. This form – which is almost completely saturated by the first four vector mesons – works surprisingly well for both mesons and baryons at low-momentum transfers, say, \(Q^2 \lesssim 0.5 \text{ GeV}^2\) as we shall show below.

Three questions arise from these results.

The first is what makes the difference between the “good” sVD for the pion and the “bad” sVD for the nucleon disappear when the infinite tower is present? For this issue, it is perhaps important to note that even for the pion form factor, the sVD is quite fragile under certain external conditions. For instance, in hidden local symmetry theory \cite{12, 13} that will be the main tool of this paper, the sVD of the pion form factor is shown to break down in thermal background \cite{14} and is expected to break down even more precociously in dense matter.

The second is the problem of the “core.” While it is reasonable to view the pion as point-like when probed at long wave length, the nucleon is an extended object, for which a local field approximation must break down at some not too high momentum scale. There are indications from high-energy proton-proton scattering, experiments on high mass muon pairs and also in deep inelastic scattering off nucleon that the nucleon has a core of \(\sim 0.2 - 0.3\) fm in size \cite{15}. It is this class of observations that led to the notion of the “Little Bag” \cite{16}.

1As a measure of “goodness” and “badness” of the sVD, the \(\chi^2/\text{dof}\) for the electric form factor up to momentum transfer \(Q^2 \sim 1 \text{ GeV}^2\) is 4.3 and 1116, respectively for the pion and the nucleon.

2As for the nucleon form factor, we are referring here specifically to the isovector component of the charge form factor. However the same discussion applies to both the Sachs electric and magnetic form factors measured in experiments as we will see later.
and to the hybrid structure of the nucleon with a core made up of a quark bag surrounded by a meson cloud \[4\]. The question is whether the core is lodged in the infinite tower and if so, in what form? Closely tied to this question is: Does the photon “see” the size of the instanton – which is a skyrmion in the infinite tower of vector mesons – which goes as \(\sim 1/\sqrt{\lambda}\) where \(\lambda = N_c g^2_{YM}\) is the ’t Hooft constant? For a large value of \(\lambda\), the soliton (instanton) size is small. However this size cannot be a physical quantity. The physical size should be independent of \(\lambda\). The instanton size therefore must be akin to the bag size which is also unphysical according to the Cheshire Cat principle. So the last question is: Is the “core” a physical observable?

The objective of this paper is to address the above three questions.

2 Holographic Form Factors

Let us first define the notations that we shall use and then give a concise summary of the hQCD calculation of the form factors as described in \([8, 9, 10]\). We shall follow the notations of \([8, 9]\).

The nucleon form factors are defined from the matrix elements of the external currents, \(\langle p'| J^\mu(x) | p \rangle\),

\[
\langle p'| J^\mu(x) | p \rangle = e^{i q x} \bar{u}(p') O^\mu(p, p') u(p) ,
\]

(2)

where \(q = p' - p\) and \(u(p)\) is the nucleon spinor of momentum \(p\). From the Lorentz invariance and the current conservation, with the assumption of CP invariance, the operator \(O^\mu\) takes the form

\[
O^\mu(p, p') = \gamma^\mu \frac{1}{2} \left[ F^s_1(Q^2) + F^a_1(Q^2)\tau^a \right] + i \frac{\sigma^{\mu\nu}}{2m_N} q_{\nu} \frac{1}{2} \left[ F^s_2(Q^2) + F^a_2(Q^2)\tau^a \right] ,
\]

(3)

where \(m_N \approx 940\) MeV is the nucleon mass, \(F^s_1\) and \(F^s_2\) are the Dirac and Pauli form factors for isoscalar current respectively, and \(F^a_1, F^a_2\) are for isovector currents.

As matrix elements, the form factors contain all one-particle irreducible diagrams for two nucleons and one external current \(A_\mu\) given as

\[
\langle p'| J^\mu(x) | p \rangle = \langle p'| \frac{\delta}{\delta A_\mu} e^{i S_{eff}[A]} | p \rangle .
\]

(4)

Thus they are very difficult to calculate in QCD. It turns out, however, that the anti-de Sitter/conformal field theory (AdS/CFT) correspondence, or gravity/gauge theory correspondence, found in certain types of string theory, enables one to compute such non-perturbative quantities as hadron form factors within certain approximations.

According to the AdS/CFT correspondence, the low energy effective action of the gravity dual of QCD becomes the generating functional for the correlators of an operator \(O\) in QCD in the large \(N_c\) limit, i.e., \(e^{i S_{ eff}[\phi(z, x)]} = \langle \exp \left[ i \int_x \phi_0 O \right] \rangle_{\text{QCD}}\), where \(\phi(z, x)\) is a bulk field, acting as a source for \(O\) when evaluated at the UV boundary \(z = \epsilon\). Now the normalizable
modes of the bulk field are identified as the physical states in QCD, created by the operator \( \mathcal{O} \).

The model we shall use is the gravity dual of low energy QCD with massless flavors in the large \( N_c \) (or quenched) approximation constructed by Sakai and Sugimoto (SS) 7. The holographic dual of spin-\( \frac{1}{2} \) baryons, or nucleons, in this model with two flavors was constructed in [8] by introducing a bulk baryon field, whose effective action is given in the “conformal coordinate” \((x, w)\) as

\[
S_{\text{eff}}^{5D} = \int_{x,w} \left[ -i \mathcal{B} \gamma^m D_m \mathcal{B} - im_b(w) \mathcal{B} \mathcal{B} + \kappa(w) \mathcal{B} \gamma^{mn} F_{mn}^{SU(2)} \mathcal{B} + \cdots \right] + S_{\text{meson}},
\]

where \( \mathcal{B} \) is the 5D bulk baryon field, \( D_m \) is the gauge covariant derivative with \( m = 1, 2, 3, 4, 5 \), \( \gamma^{mn} \) is defined as \( \gamma^{mn} = (1/2) [\gamma^m, \gamma^n] \) and \( S_{\text{meson}} \) is the effective action for the mesons. \( \kappa(w) \) is an effective warped constant that depends on the holographic coordinate \( w \). There is an additional parity-odd term called Chern-Simons term, which is not specified since it is not needed for our discussion. Using the instanton nature of baryon, the coefficients \( m_b(w) \) and \( \kappa(0) \) can be reliably calculated in string theory. In (5) the ellipsis stands for higher derivative terms – higher in \( \alpha' \) or in \( 1/\lambda \) – that are expected to be suppressed at low energy, \( E < M_{KK} \), where the KK mass sets the cut-off scale. Note that the magnetic coupling involves only the non-abelian part of the flavor symmetry \( SU(2)_I \) – with abelian \( U(1)_B \) being absent – due to the non-abelian nature of instanton-baryons.

In computing the electromagnetic (EM) form factors for the nucleons using the AdS/CFT correspondence, we need to identify the dual bulk field of the external EM current, which is the bulk photon field. Since the electric charge operator is the sum of the isospin and the baryon charge, \( Q_{\text{em}} = I_3 + \frac{1}{2} B \), we have to identify a combination of \( A_3^\mu \) and \( A_B^\mu \), the third component of the isospin gauge field and the \( U(1)_B \) gauge field, respectively, as the photon field. Then all baryon bilinear operators in the effective action that couple to either \( U(1)_B \) gauge fields or \( SU(2)_I \) gauge fields will contribute to the EM form factors.

The (nonnormalizable) photon field is written as

\[
A_\mu(x, w) = \int_q A_\mu(q) A(q, w) e^{iqx},
\]

with boundary conditions that \( A(q, w) = 1 \) and \( \partial_w A(q, w) = 0 \) at the UV boundary, \( w = \pm w_{\text{max}} \) and the (normalizable) bulk baryon field as

\[
\mathcal{B}(w, x) = \int_p [f_L(w) u_L(p) + f_R(w) u_R(p)] e^{ipx}.
\]

These 5D wave functions, \( A(q, w) \) and \( f_{L,R}(w) \), are determined by solving the equation of motion from our action [5]. Then, using the AdS/CFT correspondence, one can read off the

\[\text{\textsuperscript{3}}\]We shall use the notation and numerical values of Ref.[8]. Bulk baryon fields are introduced to represent the soliton configurations but the description with them is equivalent to collective-quantizing solitons as is done in [10].
form factors. The Dirac form factor $F_1(Q^2) = F_{1\text{min}} Q_{\text{em}} + F_{1\text{mag}} I_3$ with $(Q^2 \equiv -q^2)$ is of the form

$$F_{1\text{min}}(Q^2) = \int_{-w_{\text{max}}}^{w_{\text{max}}} dw \left| f_L(w) \right|^2 A(q, w),$$

(8)

$$F_{1\text{mag}}(Q^2) = 2 \int_{-w_{\text{max}}}^{w_{\text{max}}} dw \kappa(w) \left| f_L(w) \right|^2 \partial_w A(q, w),$$

(9)

where $F_{1\text{min}}$ is from the minimal coupling, and $F_{1\text{mag}}$ the magnetic coupling. Similarly the Pauli form factor is given as

$$F_2(Q^2) = F_3^2(Q^2) I_3$$

with

$$F_3^2(Q^2) = 4 m_N \int_{-w_{\text{max}}}^{w_{\text{max}}} dw \kappa(w) f_L^*(w) f_R(w) A(q, w),$$

(10)

which comes solely from the magnetic coupling. The form factors (8), (9) and (10) receive corrections from the higher order operators in the effective action (5), but they are expected to be suppressed by powers of $E/M_{KK}$ at low energy. Note however that our result contains full quantum effects in the large $N_c$ limit.

As shown in [8], one can replace the form factors by an infinite sum of vector-meson exchanges [8], if we expand the nonnormalizable photon field in terms of the normalizable vector meson $\psi_{2k+1}$ of mass $m_{2k+1}$ as

$$A(q, w) = \sum_{k=0}^{\infty} \frac{g_{v(k)} \psi_{2k+1}(w)}{Q^2 + m_{2k+1}^2},$$

(11)

where the decay constant of the $k$-th vector mesons is given as $g_{v(k)} = m_{2k+1}^2 \zeta_k$ with

$$\zeta_k = \frac{\lambda N_c}{108 \pi^3} M_{KK} \int_{-w_{\text{max}}}^{w_{\text{max}}} dw \frac{U(w)}{U_{KK}} \psi_{2k+1}(w),$$

(12)

where $U_{KK}$ is a parameter of the SS model and

$$dw = \frac{3 U_{KK}^{1/2}}{2 M_{KK} \sqrt{U^3 - U_{KK}^4}} dU.$$  

(13)

The resulting EM form factors then take the form

$$F_1(Q^2) = F_{1\text{min}} Q_{\text{em}} + F_{1\text{mag}} \tau^3 \frac{\zeta_k m_{2k+1}^2}{2 Q^2 + m_{2k+1}^2},$$

(14)

$$F_2(Q^2) = F_3^2(Q^2) \frac{\tau^3}{2} \sum_{k=0}^{\infty} \frac{g_{v(k)} \zeta_k m_{2k+1}^2}{Q^2 + m_{2k+1}^2},$$

(15)
where

\[ g_{V,min}^{(k)} = \int_{-w_{max}}^{w_{max}} dw \, |f_L(w)|^2 \psi_{(2k+1)}(w) \tag{16} \]

\[ g_{V,mag}^{(k)} = 2 \int_{-w_{max}}^{w_{max}} dw \, \kappa(w) \, |f_L(w)|^2 \partial_w \psi_{(2k+1)}(w) , \tag{17} \]

\[ g_2^{(k)} = 4m_N \int_{-w_{max}}^{w_{max}} dw \, \kappa(w) f_L^*(w) f_R(w) \psi_{(2k+1)}(w) . \tag{18} \]

This is the set of formulas that we shall use for our analysis that follows.

3 Integrating Out the Tower

In order to focus on the role that the lowest vector mesons \( V^{(0)} \equiv (\rho, \omega) \) play as in the sVD, we would like to integrate out all other vector mesons than the \( V^{(0)} \). The resulting form factors will be given in terms of the properties of the \( V^{(0)} \) arranged in power series of the momentum transfer involved, \( Q^2 \). Since the resulting hidden local theory is endowed with a chiral invariance (lightly broken by the quark masses) as discussed in [13], we would like to do this to the next-to-leading order (NLO) in the expansion. This means that the form factors will involve terms up to \( O(p^4) \) in the HLS Lagrangian. We will follow [17, 18] in deriving the formulae for the proton. We will restrict to the number of flavors to be 2 and focus on the vector channel only, i.e., \( U(2) \) symmetric \( \rho \) (isovector) and \( \omega \) (isoscalar). Unless required otherwise, we will show only the \( \rho \) contribution with the appropriate \( \omega \) contribution understood.

An aspect which appears to play an important role in understanding the infinite-tower structure of the form factors is that the local vector meson fields that figure in the tower in the bulk sector are degrees of freedom in a warped space with certain geometry. What we will do is to integrate out all the vector degrees of freedom lying above the \( M_{KK} \) scale, which leaves only the \( V^{(0)} \equiv (\rho, \omega) \). We suppose that this corresponds in some sense to integrating out the \( \rho' (1450), \rho'' (1700), \ldots \) and \( \omega' (1420), \omega'' (1650), \ldots \) in the dual gauge sector. This procedure may be considered as a part of the renormalization group flow in the radial direction (namely the fifth direction \( z \)) in the bulk sector which may be associated with the renormalization group flow of the boundary (gauge) theory. Because of the warped geometry in the bulk sector, it is not clear that one can simply map the \( v^{(k)} \)'s that are integrated out in the bulk sector to the \( \rho^k \)'s integrated out in the gauge sector. What we will do below is to integrate out the \( v^{(k)} \)'s for \( k > 0 \) at the classical level but in a certain warped geometrical background and it is possible that this does not necessarily map one-to-one to integrating out all \( \rho \)'s in the gauge sector lying higher than \( \Lambda_\chi \sim 1 \) GeV. Thus the two-parameter formula derived here may not be directly compared with the generalized vector dominance models employed in the literature to accurately fit the nucleon form factor data. This aspect will be discussed in Section 5.3.
3.1 Hidden local symmetry (HLS)

Following the strategy proposed in [18] for the pion form factor, we wish to integrate out all KK modes other than \( V^{(0)} = (\rho, \omega) \) such that the resulting action is hidden local symmetric in \( V^{(0)} \). The reason for resorting to HLS is to keep track of chiral symmetry in making power expansion [13]. Now in doing this, the equations of motion for the higher KK modes are used to replace them in favor of the \( V^{(0)} \) to \( \mathcal{O}(p^4) \) in the derivative expansion. Going to higher order may not be justified unless one incorporates higher order terms in \( \alpha' \) of the DBI action (i.e., higher \( 1/\lambda \) terms) and loop corrections (i.e., higher \( 1/N_c \) terms) in the bulk sector, the task of which seems at present out of reach of systematic treatments. We are essentially “integrating out” the tower at the tree level with higher tower effects lodged in the action given to \( \mathcal{O}(p^4) \). It is in this sense that we are exposing the infinite tower effect as corrections to the lowest KK mode. The validity of such a procedure is clearly limited to low momentum transfer. We are thus limiting our consideration to \( Q^2 \lesssim 0.5 \text{ GeV}^2 \). We should also note that confining to \( N_f = 2 \), we are leaving out certain other degrees of freedom (such as \( \phi \) that enters in the isoscalar channel in the gauge sector).

We start with a brief recap of the method used in [18] for the pion form factor which is immediately applicable to the nucleon form factors as described in Appendix.

Consider the meson action \( S_{\text{meson}} \) in (5) of the Sakai-Sugimoto hQCD model [7] in the form compactified in [8]:

\[
S_{\text{meson}} = -\int d^4x dw \frac{1}{2 e^2(w)} \text{tr} F_{mn} F^{mn}
\]  

(21)

where \( m = (\mu, z) = (1, 2, 3, 4, 5 = z) \). Taking the \( A_5 = 0 \) gauge, the 5D gauge field \( A_\mu \) is expanded as [1]

\[
A_\mu(x, w) = \alpha_{\mu\parallel}(x) + \hat{\alpha}_{\perp\mu}(x) \psi_0(w) + \sum_{m=1}^{\infty} A^{(m)}_\mu(x) \psi_{(2m)}(w) - \sum_{k=0}^{\infty} \hat{\alpha}^{(k)}_{\mu\parallel}(x) \zeta_k \psi_{(2k+1)}(w),
\]  

(22)

where

\[
\hat{\alpha}^{(k)}_{\mu\parallel}(x) = \alpha_{\mu\parallel}(x) - V^{(k)}_\mu(x)
\]  

(23)

\(^4\)Our convention is

\[
\text{tr} [T_a T_b] = \frac{1}{2} \delta_{ab}, \quad (a, b) = 0, 1, 2, \ldots, N_f^2 - 1,
\]  

(19)

with

\[
A_\mu = A_\mu^a T_a.
\]  

(20)

\(^5\)Here we follow the normalization of the wave function \( \psi_{2k+1}(w) \) adopted in Ref. [8], which is different from the one in Ref. [18]. Note that the vector meson fields in Ref. [8], say \( \rho^{(k)}_\mu \), is related to the fields \( V^{(k)}_\mu \) as \( V^{(k)}_\mu = g_{2k+1} \rho^{(k)}_\mu \) with \( g_{2k+1} \) being the HLS gauge couplings. This \( g_{2k+1} \) is related to the parameter \( \zeta_k \) given in Eq. (12) as \( \zeta_k = 1/g_{2k+1} \).
\[ \alpha_{\mu\parallel}(x) = \frac{1}{2i} \left[ \partial_\mu \xi_R \xi_R^\dagger + i \xi_R \mathcal{R}_\mu \xi_R^\dagger + \partial_\mu \xi_L \xi_L^\dagger + i \xi_L \mathcal{L}_\mu \xi_L^\dagger \right] , \]
\[ \hat{\alpha}_{\perp \mu}(x) = \alpha_{\perp \mu}(x) = \frac{1}{2i} \left[ \partial_\mu \xi_R \xi_R^\dagger + i \xi_R \mathcal{R}_\mu \xi_R^\dagger - (\partial_\mu \xi_L \xi_L^\dagger + i \xi_L \mathcal{L}_\mu \xi_L^\dagger) \right] \]

Here \( A_\mu^{(n)} \) and \( V_\mu^{(n)} \) are, respectively, 4D axial vector and vector fields at \( n \)-th tower. An important observation exploited in [18] is that in the \( A_5 = 0 \) gauge, there is the residual gauge symmetry
\[ A_\mu(x, z) \rightarrow h(x)(i \partial_\mu + A_\mu(x, z))h^\dagger(x) \]

with \( h \in SU(2)_F \). This means that when KK-reduced from 5D to 4D, \( A_\mu \) will be given as a stack of infinite tower hidden local gauge fields [7]. With the constraint
\[ \phi^R(z) + \phi^L(z) - \sum_{k=0}^{\infty} \zeta_k \psi_{(2k+1)}(z) = 1 \]

the transformations are
\[ V_\mu^{(k)}(x) \rightarrow h(x)(i \partial_\mu + V_\mu^{(k)}(x))h^\dagger(x) \]
\[ A_\mu^{(m)}(x) \rightarrow h(x)(A_\mu^{(m)}(x))h^\dagger(x) \]
\[ \alpha_{\parallel \mu}(x) \rightarrow h(x)(i \partial_\mu + \alpha_{\parallel \mu}(x))h^\dagger(x) \]
\[ \alpha_{\perp \mu}(x) \rightarrow h(x)(\alpha_{\perp \mu}(x))h^\dagger(x) . \]

We would like to integrate out all axial-vector fields as well as the tower of the vector fields except the lowest, \( V^{(0)} \), such that the hidden gauge invariance for the \( V^{(0)} \) be preserved to the NLO in the action (21). This can be done by setting
\[ A_\mu^{(m)}(x) = 0 \text{ for all } m , \]
\[ \alpha_{\parallel \mu}(x) - V_\mu^{(k)}(x) = 0 \text{ for } k > 0 . \]

These are the equations of motion for the fields to be integrated out to the leading chiral order that we will be considering. Note that the kinetic energy terms are of higher order so they do not figure. Now when the resulting gauge field
\[ A_\mu(x, w) = \alpha_{\parallel \mu}(x) + \alpha_{\perp \mu}(x) \psi_0(w) - \hat{\alpha}_{\parallel \mu}(x) \zeta_0 \psi_1(w) , \]

is substituted into the action (21), one obtains the standard HLS Lagrangian in 4D,
\[ \mathcal{L}_{HLS} = F_\pi^2 \text{tr}[\hat{\alpha}_{\perp \mu} \hat{\alpha}^{\mu\dagger}] + a F_\pi^2 \text{tr}[\hat{\alpha}_{\parallel \mu} \hat{\alpha}^{\mu\dagger}] - \frac{1}{2g^2} \text{tr}[V_{\mu\nu} V^{\mu\nu}] + \mathcal{L}_4 \]

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with
\[ \hat{\alpha}_\mu(x) = \alpha_\mu(x) - V^{(0)}_\mu(x). \] (36)

Here the parameters \( F_\pi \) and \( a \) are given in terms of the parameters of the holographic model, namely, \( N_c, \lambda = g^2_{YM}N_c \) and \( M_{KK} \). \( \mathcal{L}_4 \) is the \( \mathcal{O}(p^4) \) Lagrangian in the chiral expansion in the leading order in \( N_c \) that consists of 26 terms with the external vector and axial vector (electroweak) fields included. We won’t write them down here, referring to \([17, 18, 19]\) for details. We note that they are all determined by the three parameters– \( N_c, \lambda, M_{KK} \)– and the wave functions \( \psi_0 \) and \( \psi_1 \) for the pion and \( V^{(0)} \) respectively. Once the meson sector is fixed, there are no free parameters for the baryon sector.

Calculating in tree order – which is the best we can do given that loops cannot be computed in the bulk sector –, the pion EM form factor will have the standard form (1 – \( a^2/2 \)) + \( a^2m^2/2Q^2 + ZQ^2/m^2 + \cdots \) plus the contribution from the \( \mathcal{L}_4 \) term. Thus
\[ F_\pi^V = (1 - \tilde{a}/2) + \tilde{a}m^2/2Q^2 + \tilde{z}Q^2/m^2 + \cdots \] (37)

with
\[ \tilde{a} = a + \delta a \] (38)

where \( \delta a \) and \( \tilde{z} \) are contributions from the \( \mathcal{L}_4 \) term. They are entirely determined once the three parameters are fixed. The ellipsis stands for terms of higher order in \( Q^2 \).

It is important to note that this is a generic formula that would arise from any holographic model that gives 5D YM action in a warped space. Thus it can come from top-down as from the SS model or bottom-up as in dimensionally deconstructed models \([20]\). Physics will be encoded in the parameters that will figure in the action. For instance, the standard Sakurai vector dominance would correspond to \( \tilde{a} = 2 \) and \( \tilde{z} = 0 \). This point will be important for later discussions. In HLS theory as an effective theory of QCD, \( \delta a \) and \( \tilde{z} \) should receive contributions from one-loop terms as well as from the \( \mathcal{O}(p^4) \) counter terms. To the leading order, we would have \( \delta a = \tilde{z} = 0 \), so for the mass formula \( m^2 = 2F_\pi^2g^2 \) with \( a = 2 \), we would have the Sakurai VD formula with the “contact” term vanishing. In this holographic model, the \( \mathcal{O}(p^4) \) contributions come, in some RG flow sense but involving no loops, from the high tower at the classical level.

Given that the formula (37) is generic, it makes sense to consider the \( \tilde{a} \) and \( \tilde{z} \) as free parameters and fit to low-momentum data. Such a fit was made in \([19]\) to momentum transfers \( Q^2 \sim 1 \text{ GeV}^2 \), finding the best fit parameter \( \tilde{a} = 2.44 \) and \( \tilde{z} = 0.08 \) with \( \chi^2/\text{dof} = 1.6 \). Compare this to the Sakurai VD with \( \tilde{a} = 2 \) and \( \tilde{z} = 0 \) that gives \( \chi^2/\text{dof} = 4.3 \). Although \( Q^2 \sim 1 \text{ GeV}^2 \) is a bit too high a momentum transfer for the validity of the expansion, it indicates that the Sakurai VD has some room for improvement even for the pion although the deviation is not significant.

The key observation that we will exploit for the baryon is that the formula (37) can be derived directly from the infinite-tower formula (1). It was shown in \([19]\) that using the sum
rules satisfied by $F_V^\pi(0)$ and $\frac{dF_V^\pi(Q^2)}{dQ^2}|_{Q^2=0}$, the infinite-tower formula (37) yields

$$\tilde{a} = \frac{g_v g_{\rho \pi \pi}}{m^2_{\rho}}, \quad (39)$$

$$\tilde{z} = -\sum_{k=1}^{\infty} \frac{g_v^{(k)} g_{v^{(k)} \pi \pi}}{m_{2k+1}^4}, \quad (40)$$

These quantities are given in terms of the known wave functions $\psi_0$, $\psi_1$ and their eigenvalues. From the Sakai-Sugimoto work [7], it comes out that $\tilde{a} \simeq 2.62$ and $\tilde{z} \simeq 0.08$ with the $\chi^2/dof = 2.8$. It is surprising that the $\chi^2$ comes out better with the SS model than with the Sakurai VD.

### 3.2 Two-parameter formulae for the nucleon

As alluded above, the nucleon form factors can also be given in the same two-parameter form (37). In deriving it, there are two different options in dealing with the baryons. One option is to first integrate out all vector mesons in the tower except for the $V^{(0)}$ from the SS action obtaining the HLS action – which is just (35), construct the soliton baryon from that reduced action and then compute the form factors. This will be equivalent to the construction of the holographic baryons discussed in [21]. We will comment on the structure of the form factors obtained in this way in Section 5.2. The option we will take in this work is to first construct the instanton baryon in 5D, and then compute the form factors with the vector mesons and pions coupled to the instanton baryon. The physics is captured by the action (5), which is completely equivalent to what is obtained by the collective quantization of Hashimoto et al. [10].

We shall now derive the two parameter formula from by integrating out the tower of vectors except for $V^{(0)}$ and all axial vectors from the infinite-tower formulae (14) and (15) for the nucleon. To see how the integrating-out procedure gives rise to an identical formula for both the pion and the nucleon, it is instructive to see how the generic structure of the vector dominance (1) arises for both of them with the only difference being in the $v^{(k)} h h$ coupling $g_{v^{(k)} h h}$. It also illustrates the universal nature of the role of the degrees of freedom that are integrated out. To illustrate this, we use a slightly different, simpler and transparent notation since the axial fields are not involved. From [7], one can write down the direct $\gamma \pi \pi$ coupling in the form

$$\mathcal{L}_{\gamma \pi \pi} \sim \left(1 - \sum_{k=0}^{\infty} \frac{g_{v^{(k)} g_{v^{(k)} \pi \pi}}}{m_{2k+1}^2} \right) \text{tr}\left(\pi, \partial^\mu \pi \right) \mathcal{V}_\mu$$

(41)

---

6The power of this approach was shown also in few-nucleon systems where many-body nuclear interactions are involved: The structure of few-nucleon systems comes out to be much closer to experiments than the other method [22].
where $\mathcal{V}_\mu$ is the photon field and with \[8\], the direct $\gamma NN$ coupling in the form

$$\mathcal{L}_{\gamma NN} \sim \left(1 - \sum_{k=0}^{\infty} \frac{g_{\nu(k)} g_{\nu(k)} NN}{m_{2k+1}^2} \right) \bar{B}_\gamma \gamma \nu \mu B \nu \mu.$$  \hspace{1cm} (42)

Now the charge sum rule gives for the pion and the proton,

$$\sum_{k=0}^{\infty} \frac{g_{\nu(k)} g_{\nu(k)} NN}{m_{2k+1}^2} = 1,$$  \hspace{1cm} (43)

$$\sum_{k=0}^{\infty} \frac{g_{\nu(k)} g_{\nu(k)} \pi \pi}{m_{2k+1}^2} = 1.$$  \hspace{1cm} (44)

Thus the direct photon coupling vanishes, i.e., $\mathcal{L}_{\gamma \pi \pi} = \mathcal{L}_{\gamma NN} = 0$. This implies that when the tower of the vector mesons is integrated out leaving only the $\rho$ meson, a direct photon coupling to the pion and the baryon of the same form will reappear in the form factor from the degrees of freedom that are integrated out. This would signal a deviation from the Sakurai VD.

It will be shown in Appendix how the resulting two-parameter formulae showing the direct photon coupling can be derived starting from the action (5).

For comparison with experiments and for physical interpretation, it is more convenient to work with the Sachs form factors given in terms of the Dirac and Pauli form factors given above as

$$G_E^p(Q^2) = F_1^p(Q^2) - \frac{Q^2}{4m_N^2} F_2^p(Q^2),$$  \hspace{1cm} (45)

$$G_M^p(Q^2) = F_1^p(Q^2) + F_2^p(Q^2).$$  \hspace{1cm} (46)

The hQCD predictions \[8, 9, 10\] are

$$F_1^p(Q^2) = F_{1, \min}(Q^2) + \frac{1}{2} F_{1, \mag}(Q^2),$$

$$F_2^p(Q^2) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{g_{\nu(k)}^2 g_{\nu(k)} \nu \nu}{Q^2 + m_{2k+1}^2}.$$  \hspace{1cm} (47)

with

$$F_{1, \min}(Q^2) = \sum_{k=0}^{\infty} \frac{g_{\nu, \min} g_{\nu(k)} \nu \nu}{Q^2 + m_{2k+1}^2},$$

$$F_{1, \mag}(Q^2) = \sum_{k=0}^{\infty} \frac{g_{\nu, \mag} g_{\nu(k)} \nu \nu}{Q^2 + m_{2k+1}^2}.$$  \hspace{1cm} (48)

The Sachs form factors have simple physical interpretations as the spatial Fourier transforms of the charge and magnetization distributions of the nucleon in the Breit frame. Note that
since we are dealing with the EM current, we have replaced $\zeta_k m_{2k+1}^2$ by the photon-$\nu^{(k)}$ coupling $g_{\nu^{(k)}}$, i.e.,

$$g_{\nu^{(k)}} = \zeta_k m_{2k+1}^2.$$  \hfill (49)

Now following the procedure worked out for the pion form factor, when higher KK modes other than the lowest one, $V^{(0)}$, are integrated out, we can immediately write down the resultant Sachs form factors in the generic form

$$G^p_E(Q^2) = \left(1 - \frac{a_E}{2}\right) + z_E \frac{Q^2}{m_V^2} + \frac{a_E}{2} \frac{m_V^2}{m_V^2 + Q^2},$$  \hfill (50)

$$G^p_M(Q^2)/\mu_p = \left(1 - \frac{a_M}{2}\right) + z_M \frac{Q^2}{m_V^2} + \frac{a_M}{2} \frac{m_V^2}{m_V^2 + Q^2},$$  \hfill (51)

where $a_E$, $z_E$, $a_M$ and $z_M$ are parameters to be determined, and we have written $m_1 = m_V$ with $V$ standing for $V^{(0)}$ and assumed that $m_\rho = m_\omega = m_V$.\footnote{Hereafter we will write $V$ for $V^{(0)}$ unless otherwise noted.} Now formally integrating out higher modes is equivalent to expanding (50) and (51) in $Q^2/m_{2k+1}^2$ for $k \geq 1$ up to $O(Q^2)$ while keeping the $\rho$ ($\omega$) meson ($k = 0$ mode) propagator as it is.

In what follows we show explicitly the expressions for the electric form factor but the same procedure holds for the magnetic form factor.

The result for the electric form factor takes the form

$$G^p_E(Q^2) = \left(g_{V,\text{min}}^{(0)} + \frac{1}{2} g_{V,\text{mag}}^{(0)}\right) \frac{\zeta_0 m_V^2}{Q^2 + m_V^2}$$

$$+ \sum_{k=1}^{\infty} \left(g_{V,\text{min}}^{(k)} + \frac{1}{2} g_{V,\text{mag}}^{(k)}\right) \frac{\zeta_k m_{2k+1}^2}{m_{2k+1}^2} \left[1 - \frac{Q^2}{m_{2k+1}^2}\right]$$

$$- \frac{Q^2}{8m_N^2} \sum_{k=0}^{\infty} \frac{g_2^{(k)} \zeta_k m_{2k+1}^2}{m_{2k+1}^2}.$$  \hfill (52)

Now thanks to the sum rules

$$\sum_{k=0}^{\infty} \left(g_{V,\text{min}}^{(k)} + \frac{1}{2} g_{V,\text{mag}}^{(k)}\right) \zeta = 1,$$  

$$\sum_{k=0}^{\infty} g_2^{(k)} \zeta_k = g_2,$$  \hfill (53)

the form factor is reduced to

$$G^p_E(Q^2) = \left(g_{V,\text{min}}^{(0)} + \frac{1}{2} g_{V,\text{mag}}^{(0)}\right) \frac{\zeta_0 m_V^2}{Q^2 + m_V^2}$$

$$+ \left[1 - \left(g_{V,\text{min}}^{(0)} + \frac{1}{2} g_{V,\text{mag}}^{(0)}\right) \zeta_0\right] - \frac{Q^2}{m_V^2} \sum_{k=1}^{\infty} \left(g_{V,\text{min}}^{(k)} + \frac{1}{2} g_{V,\text{mag}}^{(k)}\right) \frac{\zeta_k m_V^2}{m_{2k+1}^2}$$

$$- \frac{Q^2}{8m_N^2} g_2.$$  \hfill (54)
Comparing (50) and (54), we find the $a$ and $z$ parameters for the electric form factor as
\begin{equation}
\alpha^{(\text{hQCD})}_E = 2 \left( g_{V,\text{min}}^{(0)} + \frac{1}{2} g_{V,\text{mag}}^{(0)} \right) \zeta_0 , \tag{55}\end{equation}
\begin{equation}
\beta^{(\text{hQCD})}_E = - \sum_{k=1}^{\infty} \left( g_{V,\text{min}}^{(k)} + \frac{1}{2} g_{V,\text{mag}}^{(k)} \right) \zeta_k \frac{m_V^2}{m_{2k+1}^2} - \frac{m_V^2}{8m_N^2} g_2 . \tag{56}\end{equation}

Making a similar expansion for the magnetic form factor, one finds from (51)
\begin{equation}
\alpha^{(\text{hQCD})}_M = \left[ 2 \left( g_{V,\text{min}}^{(0)} + \frac{1}{2} g_{V,\text{mag}}^{(0)} \right) \zeta_0 + g_2^{(0)} \zeta_0 \right] / \mu_p , \tag{57}\end{equation}
\begin{equation}
\beta^{(\text{hQCD})}_M = - \sum_{k=1}^{\infty} \left[ \left( g_{V,\text{min}}^{(k)} + \frac{1}{2} g_{V,\text{mag}}^{(k)} \right) \zeta_k + \frac{1}{2} g_2^{(k)} \zeta_k \right] \frac{m_V^2}{m_{2k+1}^2} / \mu_p , \tag{58}\end{equation}
with $\mu_p = 1 + (1/2)g_2$.

4 Numerical Analysis

The form factor we will analyze has the generic form characterized by two parameters $a$ and $z$
\begin{equation}
G(Q^2)/\beta = \left( 1 - \frac{a}{2} \right) + z \frac{Q^2}{m_V^2} + \frac{a}{2} \frac{m_V^2}{m_N^2} + Q^2 . \tag{59}\end{equation}
This form holds for the pion form factor with $\beta = 1$ and the nucleon form factors with $\beta = 1$ for the electric and $\mu_p$ for the magnetic form factor (assuming $m_\rho = m_\omega$). Hadron structure enters in the $a$ and $z$ parameters.

4.1 Best Fit

We first consider seriously the structure of the form factor (59) given at low momentum transfer, say, $Q^2 \ll M_{KK}^2$ or (in QCD) $Q^2 \ll \Lambda^2$. We could then subject the two-parameter formula to experimental data. In this spirit, we best-fit (59) to the accurate experimental data given, say, in Ref. [23]. Given that the approximation is valid for low momentum transfers, we limit to $Q^2 \leq 0.5$ GeV$^2$. Throughout we use the values
\begin{equation}
m_V = 0.775 \text{ GeV} , m_N = 0.938 \text{ GeV} . \tag{60}\end{equation}

The best $\chi^2$ fit is given in red (labeled as (a)) in Fig 1 for the form factors and in Fig 2 for the same divided by the dipole form factor
\begin{equation}
G_D(Q^2) = \left( \frac{1}{1 + \frac{Q^2}{0.71 \text{ GeV}^2}} \right)^2 . \tag{61}\end{equation}
Figure 1: (Color online) $G^p_E$ (left panel) and $G^p_M$ (right panel) vs. $Q^2$. Vertical axis shows the value of $G^p_{E,M}$, while the horizontal axis shows the value of $Q^2$ in unit of GeV$^2$. The red curve (a) is the “best fit,” the green curve (b) the sVD prediction, the blue curve (c) the hVD prediction and the black curve (d) the Bijker-Iachello two-component model prediction described below.

The best fit parameters and $\chi^2$’s are:

$$G^p_E: \quad a_E^{(best)} = 4.55, \quad z_E^{(best)} = 0.45; \quad \chi^2_E/dof = 1.5 \quad (62)$$

$$G^p_M: \quad a_M^{(best)} = 4.31, \quad z_M^{(best)} = 0.40; \quad \chi^2_M/dof = 1.1. \quad (63)$$

It is surprising that the fit is so good with so few parameters. A more stringent test is the ratio $\mu_p G^p_E/G^p_M$ for which fairly accurate data are available at low momentum transfers [24]. The predicted ratios are plotted and compared with the experiments in Fig. 3. The agreement of the fit parameters (62) and (63) with the experiments is surprising, considerably better than other models discussed below.

Since the fit parameters are close to each other with similar $\chi^2$ – although we are aware of no reason why they should be, it is tempting to take $a_E = a_M$ and $z_E = z_M$ and make the best-fit. The result is: $a^{(best)} = 4.42$ and $z^{(best)} = 0.42$ with $\chi^2/dof = 1.90$. It is interesting to compare the best-fit for the nucleon to the best-fit for the pion obtained in [19] using the “universal” formula (59): $a^{(best)}_\pi = 2.44$ and $z^{(best)}_\pi = 0.08$ with $\chi^2/dof = 2.44$ while the sVD with $a = 2$ and $z = 0$ gives $\chi^2/dof = 4.3$. The low-momentum data are more accurate for the nucleon than the pion and that accounts for the better $\chi^2$ of the nucleon form factors. It is interesting to note that in the case of the pion, the deviation in $\chi^2$ of the best-fit from the sVD is relatively small accounting for the general acceptance of the sVD.

4.2 sVD and hVD predictions

As noted, it is the parameters $(a, z)$ that carry information on the nucleon structure. We now examine how the vector dominance models, Sakurai vector dominance (sVD) and holographic vector dominance (hVD), fare in predicting these parameters and in fitting the data for the nucleon.
Figure 2: (Color online) $G_{E}^{p}/G_{D}$ (left panel) and $G_{M}^{p}/\mu_{D}G_{D}$ (right panel) vs. $Q^2$. Vertical axis shows the value of $G_{E,M}^{p}/G_{D}$, while the horizontal axis shows the value of $Q^2$ in unit of GeV$^2$. The red (a), green (b), blue (c) and black (d) curves are for, respectively, the “best fit,” the sVD prediction, the hVD prediction and the Bijker-Iachello two-component model fit.

Figure 3: (Color online) $\mu_{p}G_{E}^{p}/G_{M}^{p}$ vs. $Q^2$ given by the “best fit” (a, red), sVD (b, green), hVD (c, blue) and BI two-component model (d, black) compared with the experiments of [24].
First we consider the sVD. It has been known that the sVD does not work at all contrary to the case for mesons. The sVD corresponds to taking
\[ a^V_D = 2, ~ z^V_D = -m^2_p/(4m^2_N) \simeq -0.171, \]
\[ a^M_D = 2, ~ z^M_D = 0. \]

The result is shown in green (labeled as (b)) in Figs. 1 and 2. The $\chi^2$/dof comes out to be 187 and 852, respectively, for $G^p_E$ and for $G^p_M$. This reconfirms the well-known story that sVD simply fails for the nucleon.

Now turning to hVD, we will use the results of [8] given in Table 1 reproduced from [8]. We are limiting to the lowest four states since it is found numerically that the charge k
\begin{align*}
  m^2_{V_{min}} & \quad g^{(k)}_{V_{min}} \quad g^{(k)}_{V_{mag}} \quad g^{(k)}_{V_{min}} \zeta_k \quad g^{(k)}_{V_{mag}} \zeta_k \\
  0 & \quad 0.67 & \quad 0.272 & \quad 5.933 & \quad -0.816 & \quad 1.615 & \quad -0.222 & \quad 3.323 \\
  1 & \quad 2.87 & \quad -0.274 & \quad 3.224 & \quad -1.988 & \quad -0.882 & \quad 0.544 & \quad -1.918 \\
  2 & \quad 6.59 & \quad 0.272 & \quad 1.261 & \quad -1.932 & \quad 0.343 & \quad -0.526 & \quad 0.828 \\
  3 & \quad 11.8 & \quad -0.271 & \quad 0.311 & \quad -0.969 & \quad -0.084 & \quad 0.262 & \quad -0.243 \\
  \text{sum} & \quad - & \quad - & \quad - & \quad - & \quad - & \quad 0.992 & \quad 0.058 & \quad 1.989 (g_2 = 2.028)
\end{align*}

Table 1: Numerical results for vector meson couplings for the lowest four excitations in the case $\lambda N_c = 50$ (taken from [8]). Sum rules hold to a high precision. Our convention for the vector meson fields differ by sign from that of Sakai and Sugimoto for odd $k$. The vector meson mass squared is in the unit of $M^2_{KK}$.

and magnetic sum rules are almost completely saturated by them [9]. One should however be careful in using this observation for form factors since the four states may not saturate momentum-dependent observables as fully as the static quantities.

By using the values listed in Table 1 the parameter $a$ comes out to be
\[ a^{(hQCD)}_E = 3.01, \]
\[ a^{(hQCD)}_M = 3.14 \]

As for $z_{E,M}$, there are no known sum rules for the sums in Eqs. (56) and (58). We shall simply take the values for $k = 1, 2, 3$ from Table 1. We find
\begin{align}
  z^{(hQCD)}_E & \simeq - \sum_{k=1}^3 \left( g^{(k)}_{V_{min}} + \frac{1}{2} g^{(k)}_{V_{mag}} \right) \zeta_k m^2_V m^2_{2k+1} - \frac{m^2_V}{8m^2_N} g_2 \\
  & = -0.042, \\
  z^{(hQCD)}_M & \simeq - \sum_{k=1}^3 \left[ \left( g^{(k)}_{V_{min}} + \frac{1}{2} g^{(k)}_{V_{mag}} \right) \zeta_k + \frac{1}{2} g^{(k)}_{V_{mag}} \zeta_k \right] \frac{m^2_V}{m^2_{2k+1}/\mu_p} \\
  & = 0.16. \end{align}

Note that unlike $z^M_D$, $z^V_D$ differs from 0 because of the second term in the expression for $G_E$, eq. (65).
The form factors predicted parameter-free in hQCD given by

\[
(a_E, z_E) = (3.01, -0.042) \\
(a_M, z_M) = (3.14, 0.16),
\]

are plotted in blue (labeled as (c)) in Figs. 1 and 2. The $\chi^2$ comes out to be 20.2 for $G_E^p$ and 133 for $G_M^p$. While $G_E$ comes out to be rather reasonable, $G_M$ is less so, although vastly better than with sVD. We will speculate how this comes about in the discussion section.

It is important for later discussions to note that in both the best fit and the SS holographic model, the direct photon coupling represented by $D = 1 - a/2$ is negative. This is in a stark contrast to the result $D \geq 0$ obtained in all QCD-motivated models as will be elaborated below.

5 Comparison with Other Models

It is interesting to compare what we have found in the holographic model with those found in other descriptions. In doing so, we discover that there is a basic difference from the results of chiral perturbation theory, generalized vector dominance model (GVDM) and chiral bag model (CBM). This points to a puzzle on the role of the “core” in the proton structure mentioned in Introduction.

The puzzle we face will surely be resolved by future lattice QCD calculations. At present, it is not. There are lattice QCD results at momentum transfers up to $Q^2 \sim 1.4 \text{ GeV}^2$. With the minimum quark mass reached at $m_\pi \sim 300 \text{ MeV}$, however, the form factors are found to scale less slowly than the experimental and their fit is markedly less good than hQCD [25].

At the next level of fundamental approach, there have been a large number of calculations in baryon chiral perturbation theory involving nucleons and pions, i.e., baryon chiral perturbation theory (BChPT). Calculations to $\mathcal{O}(p^4)$ in BChPT provide a decent description up to only $Q^2 \sim 0.1 \text{ GeV}^2$ and fail for higher $Q^2$. For a reasonable description, it seems indispensable to go to $\mathcal{O}(p^5)$ involving two-loops or alternatively implement explicit vector-meson degrees of freedom. With the vector mesons suitably introduced, it has been possible to extend the treatment with certain success to momentum transfers $Q^2 \sim 0.4 \text{ GeV}^2$ [26]. Since chiral perturbation theories, with or without vector mesons, deal with the local baryon field, there is always non-vanishing point photon coupling, and the numerical importance of vector mesons can only be accidental.

Much more pertinent to our considerations are the GVDM and CBM to which we turn.

5.1 GVDM

As mentioned in Introduction, the failure of sVD in describing the nucleon form factors has led to numerous efforts to modify the vector dominance structure. The most successful approach to improve the fits is purely phenomenological in nature. It consists of bringing in more massive vector mesons with widths, if available, and coupling to continuum with
the asymptotic $Q^2$ behavior of perturbative QCD, thereby enabling one to go to higher momentum transfers $Q^2 > 1 \text{GeV}^2$. One such sophisticated model is the Lomon model that consists of $\rho$ (with its width), $\rho'(1.45)$, $\omega$ and $\omega'(1419)$ and hadron form factors including the logarithmic momentum transfer behavior of asymptotic freedom [27]. This model is found to be highly successful in representing the existing high-quality data up to $Q^2 \sim 10 \text{GeV}^2$.

For our purpose, it suffices to consider the two-component model of Bijker and Iachello [5, 6] (BI for short). It captures the essential features that we are interested in. This model with six parameters – which improves on the Iachello-Jackson-Landau model [3] – illustrates both long- and short-distance structures we would like to unravel. In the form used therein, it attempts, by implementing the widths and perturbative QCD effects, a global fit to nucleon form factors up to $Q^2 \sim 10 \text{GeV}^2$. The BI fit is plotted in black (labeled as (d)) in Figs 1 and 2. The corresponding $\chi^2$ up to $Q^2 = 0.5 \text{GeV}^2$ comes to be $\chi^2/\text{dof} = 2.0$ and 6.1 respectively for $G_{E,M}^p$. The fit is quite good, but despite the larger number of parameters involved, it is not as good as the two-parameter fit with (62) and (63).

This two-component model allows one to give a meaning to, and make a simple discussion on, the “core.” The “core” size obtained in [6] is $\sim (0.3 - 0.4) \text{fm}$, comparable to what is observed in nature. To see how this comes about in this model, we will drastically simplify the BI parametrization. For this we ignore $\phi$ which contributes to the isoscalar form factor and the widths of the vector mesons, both of which are absent in the hQCD formulation. The first is because we are working with two flavors and the second because they are higher order in $1/N_c$. We also assume the flavor $U(2)$ symmetry so $m_\rho = m_\omega$. Then $G_{E}^p$ of [6] can be written in the form (the same argument holds for $G_{M}^p$)

$$G_{E}^p(Q^2) \approx g(Q^2) \left(1 - \frac{a_{BI}}{2}\right) + z_{BI} \frac{Q^2}{m_\rho^2} + \frac{a_{BI}}{2} \frac{m_\rho^2}{m_\rho^2 + Q^2}.$$  

(70)

Here $g(Q^2)$ stands for the asymptotic $Q^2$ behavior of perturbative QCD parameterized in [5, 6] as $g(Q^2) = (1 + \gamma Q^2)^{-2}$. The fit parameters of BI that yield the BI results in Figs 1 and 2 give

$$a_{BI} \approx 1.64, \quad z_{BI} \approx -0.23$$  

(71)

with $\gamma = 0.515 \text{GeV}^{-2}$.

In the two-component model of [6], the core contribution to the form factor is identified to be the deviation from the vector dominance, namely, $g(Q^2) \left(1 - \frac{a_{BI}}{2}\right) + z_{BI} \frac{Q^2}{m_\rho^2}$ in (70). This gives the size

$$\langle r^2 \rangle_{\text{core}} = -\frac{6}{m_\rho} \left(z_{BI} - 2\gamma \left(1 - \frac{a_{BI}}{2}\right)m_\rho^2\right) \approx (0.36 \text{fm})^2.$$  

(72)

$^9$Since (70) is simplified and approximated from the multi-parameter BI formula by retaining only three parameters, one cannot expect it to do as well as the six-parameter fit. Indeed the $\chi^2/\text{dof}$ comes out 17.6 which is comparable to that of the SS model.
This reproduces the core size obtained in [6], which is to be compared with the core measured in the experiments [15], i.e., \( \sim (30-40)\% \) of the proton charge radius \( \sqrt{\langle r^2 \rangle_{E,p}} \approx (0.875 \text{ fm}) \). This feature with a varying core size is generally reproduced in all sophisticated phenomenological models of the Lomon type [27]. As we will discuss below, the relevant quantity for the BI model is the sign of \( D \equiv 1 - a_{BI}/2 \) which controls the sign and magnitude of the core size. It will turn out that the two-parameter formula we derived from the infinite tower structure of hQCD exhibits a qualitatively different feature.

### 5.2 CBM

One way closer to the quark/gluon degrees of freedom of QCD that removes the difficulty of the sVD for the nucleon form factors was suggested in [4] in terms of the chiral bag model of the nucleon. It anchors on the role of pion in the baryon structure dictated by the spontaneously broken chiral symmetry of QCD. The basic idea is that a small confined region of quarks and gluons, namely, “bag,” is surrounded by the meson cloud of pions and the photon couples both directly to the bag, i.e., an intrinsic core, and through the meson cloud. This idea is realized by the chiral bag that consists of a quark-bag coupled to meson cloud via chiral boundary condition. The simplest picture suggested in [4] is the “little bag” coupled at the bag surface to the chiral field at the magic chiral angle \( \theta = \pi/2 \) at which half of the baryon charge is lodged inside the bag and the other half outside. Imagine the photon coupling to the proton via a “bag” with a coefficient \( a \) and through a vector meson with a coefficient \( b \). Now treat the bag as a point-like object. The charge conservation requires of course that \( a + b = 1 \). On the other hand, the bag does not contribute to the anomalous magnetic moment whereas the coupling through the \( \rho \) vector meson contributes, via the tensor \( \rho^{-NN} \) coupling, \( eb \frac{\kappa}{2m_N} \). So the anomalous magnetic moment of the proton will be given by

\[
\kappa_V = b \kappa_{\rho}.
\]  

(73)

Thus the chiral bag at the magic angle \( a = b = 1/2 \) will give

\[
\kappa_{\rho} \approx 2 \kappa_V.
\]  

(74)

Experimentally \( \kappa_{\rho} = 6.6 \pm 0.6 \) and \( \kappa_V = \mu_p - \mu_n = 3.71 \), so (74) is more or less satisfied. The magic angle is just an idealization and it is possible that \( b \) could be greater than 1/2.

This idealized half-and-half model works fairly well for the proton charge form factor up to \( Q^2 \sim 0.4 \text{ GeV}^2 \) as one can see in Fig. 4 of [4]. In this picture, there is a core provided by the bag carrying half of the proton charge resembling the two-component model described above with a comparable size. A similar description arises when the bag is replaced by a Skyrme soliton. Suppose that the skyrmion has a size of the baryon as in the standard skyrmion (i.e., the Skyrme soliton) or the skyrmion in the presence of the \( \rho \) meson in the hQCD Lagrangian where all other mesons than the \( \rho \) have been integrated out from the tower as in [21]. Such a skyrmion will carry all the charge of the proton and hence should describe – modulo small
contributions from the fluctuating pion field – most of the form factor structure. It turns out that such a description does not fare well in comparison with nature and in fact, demands incorporating fluctuating vector meson (ρ and ω fields) coupled to the soliton for better description. It has in fact been recognized since sometime that the vector mesons could play an important role in the electromagnetic structure of the skyrmion model for the nucleon [28]. Indeed the resulting structure is analogous to the chiral bag description described above. It has been shown that this “π, ρ, ω” model, with mild adjustment of parameters, can actually describe very well all nucleon form factors to $Q^2 \sim 10$ GeV$^2$ [29]. In some sense there is a manifestation here of the Cheshire Cat property [30] where the bag and soliton structures could be traded in.

The bottom line of all these structures in the models anchored on the premise of QCD is that they all predict the existence of a “core” to which photon couples point-like.

5.3 The problem of the “core”

The spatial Fourier transform of the form factors $G_p^E(Q^2)\ (G_p^M(Q^2))$ describes the charge (magnetic moment) distribution of the proton, so the two-parameter formula should reflect how the charge (and magnetic moment) is distributed spatially. The question we ask is: Can one say about the “core” from the two-parameter best-fit result and the SS model?

To see what it is all about, we need to specify what we mean by “core.” As mentioned above, in the two-component model of [6] and also the chiral bag model of [4], one may define the core of, say, the proton to be the (extended) component of the proton that is not accounted for by the vector-meson cloud. In the BI model, what is significant is that the constant term in (70) is not only positive, $g(Q^2)\left(1 - \frac{a_{BI}}{2}\right) > 0$ but also $1 - \frac{a_{BI}}{2} \approx \frac{1}{2}$. This feature is shared by the chiral bag model at the magic chiral angle $\pi/2$ with a half-and-half sharing of the baryon charge.

Now the best-fit case with (62) (and also with (63)) is drastically different. Here because of the fact that the lowest-vector-meson cloud carries a charge that exceeds the proton charge, the quantity $1 - a/2$ is negative and hence cannot be naively associated with the “core” size. In the SS model, this is seen in that the next-lying vector meson $\rho'$ mediates, with a big coefficient, the photon-nucleon coupling with the sign opposite to that of the $\rho$. This feature which is generic in the SS model applying both to the pion and to the baryon, seems to be supported by the recent experiment of the decay $\tau^- \to K^- \pi^- K^+ \nu_\tau$ [31]. It should be noted, however, that it is at odds with the GVDM of the Lomon type [27] where the $\rho'$ contributes negligibly and with positive sign.

Since the formula (51) does give correct charge and correct radius, there seems to be nothing unphysical about the structure at least at low momentum transfer: It gives as good a description of the proton structure as (or even superior to) the QCD-motivated models like the one of [6]. So the question is where in the holographic description is the core “seen” in QCD-motivated models and also in experiments? Answering this questions will require a more sophisticated analysis that we relegate to a future publication.

It is interesting to note that the infinite-tower form factor calculated in [8, 9] gives also
a consistent value of the $\rho NN$ coupling that in the past served as a support for the chiral bag model,

$$\frac{\kappa_\rho}{\kappa_V} \approx 1.67$$

(close to the empirical value $\frac{\kappa_\rho}{\kappa_V}^{\text{exp}} \approx 1.78$. This suggests that the two-component model such as the chiral bag at the magic angle is not necessarily the only compelling mechanism for the anomalous $\rho NN$ tensor coupling.

6 Further Comments

While the infinite tower structure of hQCD model -- with $(a,z) \simeq (2.44, 0.08)$ -- gives the pion form factor that deviates little from the sVD (Sakurai VD) with $(a,z) = (2, 0)$, the nucleon form factor deviates markedly from the sVD with the fit value giving $(a,z) \simeq (4.4, 0.4)$. This implies, in the way the form factor is parameterized, that the $V^{(0)} = \rho, \omega$ contribution to the nucleon form factor is greater by a factor of two than that to the pion, so the conventional VD with the “universality” does not hold at all. This feature suggested by Nature is reproduced, semi-quantitatively, in the hQCD model of Sakai and Sugimoto in that for instance, the charge contributed from the lowest lying vector meson overshoots the one unit of charge for the proton, which then is largely compensated by the negative charge contribution coming from the first excited $\rho$ and $\omega$ mesons. Understanding what this means will be one of the main tasks of our future work.

Our main finding is that our two-parameter formulae inferred from the infinite tower structure of hQCD work surprisingly well for $G_{E,M}^{P}$ for momentum transfers $Q^2 \ll 0.5$ GeV$^2$. They are much simpler than other QCD-motivated models and provide better $\chi^2$’s. However translated into a two-component structure, namely, “core” plus vector-dominance, they have a very different form. Given that the parameters $(a,z)$ determined in the fit seem to represent Nature, the challenge for the theorists is to calculate the parameters from a theory that reproduce the fit parameters.

Two further remarks are in order: One on hidden local symmetry in general and the other on the infinite tower structure in the baryon and nuclear sector.

Hidden local symmetry on which we heavily rely is a not-yet fully elucidated notion. It is a flavor gauge symmetry tied to emergent gauge symmetries in theories with a weakly coupled dual description and pervades in other areas of physics, namely in the dynamics of supersymmetric QCD [32] and strongly-correlated condensed matter systems [33]. Applied to dense baryonic matter for which no model-independent theoretical tools are available with lattice QCD incapable of handling large chemical potential, it makes an unexpected prediction that the famous short-range repulsion between two nucleons effective in matter-free space should get strongly suppressed as the system approaches density-driven chiral restoration [34]. If confirmed, this mechanism will have a drastic consequence on the equation of states of neutron stars and other forms of compact stars.
We have noted that the infinite-tower structure in the nucleon form factors is somewhat less successful in the magnetic response than in the electric response. The reason for the apparent defect of the hQCD for $G_M$ is most likely in the importance of $1/N_c$ corrections in the Pauli form factor $F_2$, specially the isoscalar part. This is reflected in the magnetic moment in that $\mu_I=0 \sim \mathcal{O}(1/N_c)$ while $\mu_I=1 \sim \mathcal{O}(N_c)$. Unfortunately it is not known how to systematically calculate $1/N_c$ corrections in hQCD models.

The other point to make is the potential importance of the infinite tower structure in hadronic physics in general, particularly in multi-baryon systems. In addition to the role that they play in the nucleon form factors, the infinite tower of vector mesons in the guise of stack of hidden local symmetry fields are found to make the nucleon structure as a soliton qualitatively different from the Skyrme model with pion fields only [8]. Perhaps more importantly, they are found to bring a big improvement in describing multi-nucleon systems, i.e., nuclei, in terms of topological solitons [22]. In particular, it seems highly likely that the tower with their hidden local symmetry plays an important role in dense baryonic matter relevant to the physics of compact stars for which reliable model-independent theoretical tools are still lacking.

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Appendix

In this Appendix, we derive the formulae (50) and (51) from the action (5) together with the expansion of the 5D gauge field in Eq. (22). In the following we normalize the wave functions $\psi_n$ as

$$\int dw \frac{1}{e^2(w)} \psi_n(w) \psi_{n'}(w) = \delta_{nn'} \quad \text{(A1)}$$

and introduce the mass parameter as

$$m_n^2 \equiv -\int dw \frac{1}{e^2(w)} \left( \dot{\psi}_n(w) \right)^2. \quad \text{(A2)}$$

Following Ref. [8], we expand the baryon field as

$$\mathcal{B} = \left( f_L(w) \frac{1+\gamma_5}{2} + f_R(w) \frac{1-\gamma_5}{2} \right) B, \quad \text{(A3)}$$
with \( f_L(w) = f_R(-w) \).

We start with the equations of motion for the flavor non-singlet vector fields \( \rho^{(k)a}_\mu \equiv \zeta_k \dot{V}_{\mu}^{(k)a} \) \( (k \geq 1, a = 1, 2, \ldots, N_f^2 - 1) \) given by

\[
0 = -\int dw \frac{1}{2e^2(w)} \, [T_a F^\mu_\nu \dot{\psi}_{2k+1} \, (A4) \\
+ \int \frac{1}{e^2(w)} \, [T_a \{\partial_\nu F^\nu_\mu \} \psi_{(2k+1)}] \, (A5) \\
- \int \, d\bar{w} \, B \gamma^\mu T_a B \psi_{(2k+1)} \, (A6) \\
+ 2 \int \, d\kappa(w) \bar{B} \gamma^5 T_a B \dot{\psi}_{(2k+1)} \, (A7) \\
- 2 \int \, d\kappa(w) \psi_{(2k+1)} \, \{\partial_\nu \bar{B} \gamma^\mu T_a B - i\bar{B} \gamma^\mu [T_a, A_\nu] B\} \, . \, (A8)
\]

Dropping the second line (A5) and the second term in the fifth line (A8), both of which are of higher order in the chiral counting, we obtain

\[
0 = -m^2_{2k+1} \left( \rho^{(k)}_\mu - \zeta_k \epsilon_{\mu}^\parallel \right) - \left( g^{(k)}_{V,min} + g^{(k)}_{V,mag} \right) \bar{B} \gamma_5 T_a B \\
- 2 \frac{g^{(k)}_2}{4mN} \partial_\nu (\bar{B} \gamma^\mu T_a B) \, . \, (A9)
\]

The quantities \( g^{(k)}_{V,min}, g^{(k)}_{V,mag} \) and \( g^{(k)}_2 \) are given as \( w \) integrals over the functions \( f_{L,R} \) and \( \psi_{2k+1} \) as defined in (16)-(18). Similarly, the EoM for flavor-singlet vector is reduced to

\[
0 = -m^2_{2k+1} \left( \rho^{(k)a=0}_\mu - \zeta_k \epsilon_{\mu}^{a=0} \right) - g^{(k)}_{V,min} \bar{B} \gamma^\mu T_0 B \, . \, (A10)
\]

The EoM for axial-vector mesons \( (m \geq 1) \) becomes

\[
0 = m^2_{2m} A^{(k)a}_\mu \left( g^{(m)}_{A,min} + g^{(m)}_{A,mag} \right) \bar{B} \gamma_5 T_a B \, . \, (A11)
\]
By substituting the above EoMs, the 5D gauge field after integrating out becomes

\[
A_{\mu}^{\text{integ}}(x, w)
= \alpha_{||\mu}(x) - \hat{\alpha}_{\perp\mu}(x) \psi(0)(w)
+ \sum_{k=1}^{\infty} \left( \frac{g_{A,\text{min}}^{(k)} + g_{A,\text{mag}}^{(k)}}{m_{2k}^2} \right) \bar{B} \gamma^{\mu} \gamma^{5} T_{a} B \psi(2m)(w) T_{a}
+ \left( \rho_{\mu}^{(0)} - \zeta_{0} \alpha_{||\mu} \right) \psi(1)(w)
- \sum_{k=1}^{\infty} \sum_{a=0}^{3} \frac{1}{m_{2k+1}^2} \left[ \tilde{g}_{V}^{(k)} \bar{B} \gamma_{\mu} T_{a} B + 2 \frac{g_{2}^{(k)}}{4m_{N}} \partial^\nu \left( \bar{B} \gamma_{\nu \mu} T_{a} B \right) \right] \psi(2k+1)(w) T_{a}
- \sum_{k=1}^{\infty} \tilde{g}_{V,\text{min}}^{(k)} \tilde{B} \gamma_{\mu} T_{0} B \partial_{w} \psi(2k+1)(w) T_{0} ,
\]

where

\[
\tilde{g}_{V}^{(n)} \equiv g_{V,\text{min}}^{(n)} + g_{V,\text{mag}}^{(n)} .
\]

Using this \(A_{\mu}^{\text{integ}}\), the field strengths are obtained as

\[
F_{5\mu}^{\text{integ}}(x, w)
= -\hat{\alpha}_{\perp\mu} \partial_{w} \psi(0) + \left( \rho_{\mu}^{(0)} - \zeta_{0} \alpha_{||\mu} \right) \partial_{w} \psi(1)
- \sum_{k=1}^{\infty} \sum_{a=0}^{3} \frac{1}{m_{2k+1}^2} \left[ \tilde{g}_{V}^{(k)} \bar{B} \gamma_{\mu} T_{a} B + 2 \frac{g_{2}^{(k)}}{4m_{N}} \partial^\nu \left( \bar{B} \gamma_{\nu \mu} T_{a} B \right) \right] \partial_{w} \psi(2k+1) T_{a}
- \sum_{k=1}^{\infty} \tilde{g}_{V,\text{min}}^{(k)} \tilde{B} \gamma_{\mu} T_{0} B \partial_{w} \psi(2k+1)(w) T_{0} ,
\]

\[
F_{\mu\nu}^{\text{integ}}(x, w)
= \partial_{\mu} \alpha_{||\nu} - \partial_{\nu} \hat{\alpha}_{\perp\mu} \psi
+ \left( \rho_{\nu}^{(0)} - \zeta_{0} \alpha_{||\nu} \right) \psi(1)
- \sum_{k=1}^{\infty} \sum_{a=0}^{3} \frac{1}{m_{2k+1}^2} \partial_{\mu} \left[ \tilde{g}_{V}^{(k)} \bar{B} \gamma_{\nu} T_{a} B + 2 \frac{g_{2}^{(k)}}{4m_{N}} \partial^\sigma \left( \bar{B} \gamma_{\sigma \nu} T_{a} B \right) \right] \psi(2k+1)(w) T_{a}
- \sum_{k=1}^{\infty} \tilde{g}_{V,\text{min}}^{(k)} \partial_{\mu} \tilde{B} \gamma_{\nu} T_{0} B \psi(2k+1)(w) T_{0}
- \left( \mu \leftrightarrow \nu \right) \cdots .
\]

\(^{10}\)The axial vector fields are not needed for our purpose but they are kept for completeness.
From this, the relevant terms of action become

\[
S = \int d^4 x dw \left[ -\overline{B} \gamma^\mu A_{\mu}^{\text{integ}} B + 2\kappa(w) \overline{B} \gamma^\mu F_{5\mu}^{\text{integ};SU(2)} B + \kappa(w) \overline{B} \gamma^\mu F_{\mu\nu}^{\text{integ};SU(2)} B \right. \\
- \left. \frac{1}{2e^2(w)} \left\{ 2\text{tr} \left( F_{5\mu}^{\text{integ}} F_{5\nu}^{\text{integ}} \right) + \text{tr} \left( F_{\mu\nu}^{\text{integ}} F_{\mu\nu}^{\text{integ}} \right) \right\} \right] \\
= \int d^4 x \left[ -\overline{B} \gamma^\mu \left\{ \alpha_{\mu\nu} + g_{V,\text{min}}(0) \rho^{(0)}_\nu - \zeta_0 \alpha_{\mu\nu} \right\} B - g_{V,\text{mag}}(0) \overline{B} \gamma^\mu \left( \rho^{(0)}_\nu - \zeta_0 \alpha_{\mu\nu} \right) B \right. \\
+ 2\overline{B} \gamma^\mu \left\{ \frac{g_2}{2m_N} \partial_\mu \alpha_{\mu\nu}^{SU(2)}(x) + \frac{g_2^{(n)}}{4m_N} \partial_\mu \left( \rho^{(n)}_\nu - \zeta_0 \alpha_{\mu\nu} \right) \right\} \right] \\
- 2 \int d^4 x \text{tr} \left\{ \partial_\nu \alpha_{\mu\nu}(x) - \partial_\mu \alpha_{\mu\nu}(x) \right\} \\
\times \left\{ -\partial_\nu \sum_{n=1}^{\infty} \frac{\zeta_n}{m_{2n+1}^2} \left[ g_{V,\text{min}}^{(n)} B \gamma^\mu T_a B + \frac{g_2^{(n)}}{4m_N} \partial_\sigma \left( \overline{B} \gamma^\sigma \sigma \mu T_a B \right) \right] T_a \\
- \partial_\nu \sum_{n=1}^{\infty} \frac{\zeta_n}{m_{2n+1}^2} g_{V,\text{min}}^{(n)} B \gamma^\mu T_0 B \right\} + \cdots . \tag{A16} \right.
\]

The field \( \alpha_{\mu\nu} \) is expanded as

\[
\alpha_{\mu\nu} = \tilde{A}_\mu Q + \cdots , \tag{A17} \]

where \( \tilde{A}_\mu \) is the photon field and \( Q = \text{diag.(1, 0)} \) is the charge matrix. Substituting this, we obtain

\[
S = \int d^4 x \left[ -\overline{B} \gamma^\mu \tilde{A}_\mu \left\{ Q \left( 1 - g_{V,\text{min}}^{(0)} \zeta_0 \right) + \frac{\tau_3}{2} g_{V,\text{mag}}^{(0)} \zeta_0 \right\} B \right. \\
- \tilde{B} \gamma^\mu \left\{ g_{V,\text{min}}^{(0)} \rho_{\mu} + g_{V,\text{mag}}^{(0)} \rho_{\mu}^{SU(2)} \right\} B \\
+ \frac{1}{2m_N} \tilde{B} \gamma^\mu \frac{\tau_3}{2} \partial_\nu \tilde{A}_\nu \left( g_2 - g_2^{(0)} \zeta_0 \right) - g_2^{(0)} \tilde{B} \gamma^\mu \partial_\nu \rho_{\mu}^{SU(2)} B \\
- \int d^4 x \tilde{B} \sum_{k=1}^{\infty} \frac{\zeta_k}{m_{2k+1}^2} \gamma^\mu \partial_\sigma \partial^\sigma \tilde{A}_\mu \left( g_{V}^{(k)} Q + g_{V,\text{min}}^{(k)} T_0 \right) \\
- \sum_{k=1}^{\infty} \frac{\zeta_k}{m_{2k+1}^2} g_{2k+1}^{(k)} \gamma^\mu \partial_\sigma \partial^\sigma \partial_\nu \tilde{A} Q \right\} B . \tag{A18} \right.
\]
Now we can read off the Dirac and Pauli form factors from Eq. (A18):

\[
F_1^p(Q^2) = \left(g_{V,\text{min}}^{(0)} + \frac{1}{2} g_{V,\text{mag}}^{(0)}\right) \frac{\zeta_0 m_V^2}{Q^2 + m_V^2} \\
+ \sum_{k=1}^{\infty} \left(g_{V,\text{min}}^{(k)} + \frac{1}{2} g_{V,\text{mag}}^{(k)}\right) \zeta_k \left[1 - \frac{Q^2}{m_{2k+1}^2}\right],
\]

\[
F_2^p(Q^2) = \frac{1}{2} g_2^{(0)} \zeta_0 m_V^2 + \frac{1}{2} \left(g_2 - g_2^{(0)}\right) \zeta_0 - \frac{1}{2} \sum_{k=1}^{\infty} g_2^{(k)} \zeta_k \frac{Q^2}{m_{2k+1}^2}. \tag{A19}
\]

The Sachs form factors follow from the definitions (45) and (46).

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