Multiple Transitions in Vacuum Dark Energy and $H_0$ Tension

Hossein Mosshafi, Hassan Firouzjahi, and Alireza Talebian

School of Astronomy, Institute for Research in Fundamental Sciences (IPM) P.O. Box 19395-5531, Tehran, Iran; mosshafi@ipm.ir, firouz@ipm.ir, talebian@ipm.ir

Received 2022 August 16; revised 2022 October 10; accepted 2022 October 19; published 2022 November 29

Abstract

We study the effects of multiple transitions in the vacuum dark energy density on the $H_0$ tension problem. We consider a phenomenological model in which the vacuum energy density undergoes multiple transitions in the early as well as the late universe and compare the model’s predictions using the three sets of data from the cosmic microwave background, baryonic acoustic oscillations, and supernovae. The transient dark energy can be either positive (dS-like) or negative (AdS-like). We conclude that a transient late-time AdS-type vacuum energy typically yields the higher value of $H_0$, which can alleviate the $H_0$ tension. In addition, to obtain a value of $H_0$ comparable to the value obtained from the local cosmological measurements the spectral index $n_s$ moves toward its Harrison–Zel’dovich scale-invariant value.

Unified Astronomy Thesaurus concepts: Dark energy (351); Observational cosmology (1146); Hubble constant (758)

1. Introduction

The disagreement between the independent measurements of the Hubble constant, based on the early universe with the CDM model, and the value determined using direct observations of the local universe without assuming the CDM model is dubbed the Hubble tension (Riess 2019; Verde et al. 2019; Dainotti et al. 2021; Di Valentino et al. 2021a, 2021b; Perivolaropoulos & Skara 2022; Dainotti et al. 2022; Schöneberg et al. 2022). More precisely, the reported value of the Hubble parameter from the Planck satellite is $H_0 = 67.4 \pm 0.5 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ (Aghanim et al. 2020b), while the latest SH0ES Team (Riess et al. 2022) has reported $H_0 = 73.30 \pm 1.04 \text{ Km s}^{-1} \text{ Mpc}^{-1}$. This tension is an important open problem in cosmology. Ignoring the possible systematics origin of the tension (Freedman et al. 2019; Efstathiou 2020; Mortsell et al. 2022), it is natural to expect that either a modification to the cosmological model is required or there is a new physics behind the tension (Di Valentino et al. 2016; Mortsell & Dhawan 2018; Guo et al. 2019; Knox & Millea 2020; Vagnozzi 2020; Schöneberg et al. 2022).

Qualitatively speaking, there are two classes of models attempting to resolve the Hubble tension by introducing new physics. The first class of models is based on the modiﬁcations of dark components at late times (i.e., lower redshifts), e.g., by introducing a dynamical dark energy that alters the Hubble expansion. In the second class of models, the new physics aims to reduce the sound horizon by ~7% (Bernal et al. 2016; Lemos et al. 2019; Knox & Millea 2020) while keeping the baryon acoustic oscillation (BAO) and the uncalibrated supernovae (SNe) Ia data to be consistent with Planck measurements. The locations of the acoustic peaks in the cosmic microwave background (CMB) observations are among the most precisely measured quantities in cosmology. With an accuracy of 0.03% the Planck satellite (Aghanim et al. 2020b) has determined the angular size of the sound horizon at recombination, $\theta_s \equiv r_s/D_s$, on which the sound horizon $r_s$ is the comoving distance a sound wave could travel from the beginning of the universe to the time of recombination, and $D_s$ is the comoving integrated distance from now to the epoch of recombination. $D_s$ depends only on two parameters, $\Omega_m$ (the fractional matter energy density today) and the present value of Hubble expansion rate, $H_0$. Thus, given $r_s$ and an estimation of $\theta_s$, one can infer $H_0$ from the measurement of $\theta_s$. Using the Planck best-fit values of $\Omega_m$ and $r_s$ in the context of the CDM model (Aghanim et al. 2020b), $H_0$ is found to be significantly lower than that found from the more direct local measurements. If the value of the Hubble constant from SH0ES is considered, it would yield a much larger value of $\theta_s$ unless $r_s$ and/or $D_s$ were modified to preserve the observed CMB acoustic peak positions.

Pre-recombination early dark energy (EDE; Karwal & Kamionkowski 2016; Agrawal et al. 2019; Kaloper 2019; Lin et al. 2019; Poulin et al. 2019; Braglia et al. 2020; Lin et al. 2020; Niedermann & Sloth 2020; Sakstein & Trodden 2020; Smith et al. 2020; Ye & Piao 2020a; Gogoi et al. 2021; Seto & Toda 2021; Karwal et al. 2022; Niedermann & Sloth 2022) is one of the best-studied scenarios proposed as a solution to the Hubble tension. In this scenario, a dark energy like component is introduced. The energy injection before recombination boosts the Hubble expansion rate (by reducing the sound horizon). The EDE then decays rapidly in order not to spoil other observations. Various scenarios have been proposed for both the early energy injection (Agrawal et al. 2019; Kaloper 2019; Lin et al. 2019; Poulin et al. 2019; Ballesteros et al. 2020; Braglia et al. 2020; Lin et al. 2020; Ye & Piao 2020a; Sakstein & Trodden 2020; Smith et al. 2020; Zumalacárregui 2020; Braglia et al. 2021; Gogoi et al. 2021; Niedermann & Sloth 2021a; Nojiri et al. 2021; Seto & Toda 2021; Karwal et al. 2022) and the decaying processes (Poulin et al. 2019; Smith et al. 2020; Ye & Piao 2020a).

EDE models have important effects on primordial scalar perturbations and on large-scale-structure (LSS) physics. It has been found that the EDE models require a reinterpretation of the available data, resulting in higher values of $n_s$, all the way up to a scale-invariant Harrison–Zel’dovich spectrum of
Figure 1. A schematic view of the evolution of dark energy $\rho_x$ compared to total energy density $\rho$. Left: at early times $\rho_x \ll \rho$ and $H_{X}^{-1} \gg H^{-1}$ so the FLRW horizon is within one patch of the dS horizon associated to dark energy. Middle: around the time of transition, $\rho_x \sim \rho$ and $H_{X}^{-1} \sim H^{-1}$. This is the time when the effects of dark energy become important. Right: long after the transition, many patches of the dS horizon associated with $\rho_x$ enter the FLRW horizon and the effects of $\rho_x$ are represented by a phenomenological fluid with the equation of state $w$.

primordial scalar perturbation, i.e., $n_s = 1$ (Di Valentino et al. 2018; Ye et al. 2021a; Jiang & Piao 2022). The second effect of EDE models appears when one considers the galaxy clustering data (Hill et al. 2020; Ivanov et al. 2020; Krishnan et al. 2020; D’Amico et al. 2021). Although large EDE fractions $f_{\text{EDE}}$ are not ruled out by data sets (Murgia et al. 2021; Smith et al. 2021; Gómez-Valent 2022; Herold et al. 2022), increasing $f_{\text{EDE}}$ will increase the clustering amplitude $\sigma_8$ and the related value of $S_8$ (Di Valentino & Bridle 2018; Nunes & Vagnozzi 2021). In other words, the models with large fractions $f_{\text{EDE}}$ worsen the well-known “$S_8$ discrepancy.” Most EDE models proposed as a solution to the Hubble tension lead to an increase of the clustering amplitude ($S_8$) that worsens the fit to galaxy clustering data. Reeves et al. (2022) argued that freeing the total neutrino mass $M_\nu$ can suppress small-scale power and then improve EDE’s fit to galaxy clustering data.

Currently, the most precise large-scale CMB observation is from Planck, and alone it seems not to favor axion-like EDE models (Hill et al. 2020). However, the TT power spectrum of Planck data, especially in small scales, needs more consideration. Some inconsistencies between the $\ell_{\text{TT}} < 1000$ and $\ell_{\text{TT}} > 1000$ part of Planck’s TT power spectrum have been reported in Addison et al. (2016) and Aghanim et al. (2017). Moreover, the amplitude of CMB gravitational lensing in the Planck data is not consistent with what we expect from the ΛCDM model. The smoothing effect of gravitational lensing on acoustic peaks of the CMB power spectrum exceeds that expected in the ΛCDM model (Addison et al. 2016; Mootch & Hu 2020). However, small-scale ground-based CMB observations such as by ACT and SPT, providing precise measurements of the small-scale power spectrum, have not found this oversmoothing effect (Henning et al. 2018; Aiola et al. 2020; Dutcher et al. 2021). Recently, a combined analysis of Planck ($\ell_{\text{TT}} \lesssim 1000$) with ACT and SPT data for EDE models has also been performed, such as CMB+SPPol (Chudaykin et al. 2020, 2021; Jiang & Piao 2021), CMB+ACT DR4 (Hill et al. 2022; Poulin et al. 2021), and CMB+ACT DR4+SPT-3G (La Posta et al. 2022; Jiang & Piao 2022; Smith et al. 2022). Also, for CMB+LSS data see Hill et al. (2020); Ivanov et al. (2020); D’Amico et al. (2021); Ye et al. (2021b).

In this paper, we study a scenario with multiple transient phases of dark energy, both before and after the surface of last scattering, yielding a higher value of the current Hubble expansion rate compared to what is inferred from the ΛCDM model. This is a phenomenological model motivated by Firouzjahi (2022b), who studied the quantum vacuum zero-point energy in connection to the cosmological constant problem and the origin of dark energy. Alternatively, the current setup may be viewed as an independent phenomenological mechanism with some similarities to the EDE proposal. For other works involving phase transitions in dark energy and their impacts on $H_0$ tension, see also Klinkhamer & Volovik (2011); Banihashemi et al. (2020); Farhang & Khosravi (2021); Moshafi et al. (2021); Khosravi & Farhang (2022).

2. The Model

We consider a model that consists of some new energy density source $\rho_x$ arising from the quantum zero-point energy of the fields with the mass $m$. Depending on the type of the quantum field (boson or fermion) and the energy scale of interest, $\rho_x$ can be either positive (dS-like) or negative (AdS-like; Martin 2012; Firouzjahi 2022a, 2022b). For the sake of simplicity, here first we consider the case of a single quantum field, while the procedure for the multiple field is straightforward thanks to our simplifying assumption that the vacuum energies of the free fields do not exchange with each other.

Associated with the energy density $\rho_x$, we can define a horizon radius, denoted by $H^{-1}$, in which $3M_p^2 H_0^2 = [\rho_x]$ and $M_p$ is the reduced Planck mass. Let us also denote $H$ as the Hubble rate of the observable FLRW universe with $3M_p^2 H_0^2 = \rho$ in which $\rho$ is the total energy density. Depending on the mass scale $m$, the contribution of $\rho_x$ can be viewed as a source of dark energy in $\rho$. At early stages in the cosmic expansion history, the zero-point energy of the field appears as a small constant term in $\rho$. More specifically, $\rho_x = c \, m^4$ with $c$ as a constant. As illustrated in the left panel of Figure 1, in this regime $H^{-1} \ll H_{X}^{-1}$, so the FLRW horizon (at that time) is
within one patch of the vacuum energy. As time proceeds and $\rho$ decreases further, we can indicate a specific scale factor $a_c$ (or redshift $z_c$) when $\rho(a_c) \sim \rho_s \sim m^4$. At this stage in the cosmic epoch, $H^{-1} \sim H^{-1}$, as in the middle panel of Figure 1, and $\rho_s$ can be relevant as the source of dark energy at that time. After this phase, $\rho$ falls off rapidly and very soon we have $H^{-1} \gg H^{-1}$, as illustrated in right panel of Figure 1. It is argued in Firouzjahi (2022b) that, as many patches of vacuum horizon with the size $H^{-1}$ enters the FLRW horizon, the patches of zero-point energy develop inhomogeneities $\delta \rho_s > 1$.

The subsequent evolution of $\rho_s$ on the sub-Hubble scale is left as an open problem in Firouzjahi (2022b). The time of the transition in dark energy is determined by the mass of the quantum field, happening at the temperature $T_e \sim m$ (Park 2021a, 2021b). For example, for the simplest model in Standard Model (SM) with three massive neutrinos, we can have three transitions in dark energy in which the first transition may happen shortly after the time of CMB decoupling, while the other two transitions may happen much later in the cosmic expansion history. In addition, the current stage of dark energy may be associated with the zero-point energy of the lightest neutrino field with the mass $m_\nu \sim 10^{-3} eV$.

Motivated by the above discussions, we picture the effects of $\rho_s$ after the time when $a > a_c$, by a phenomenological dark fluid with the equation of state $w$. In this phenomenological view, $\rho_s$ is constant in the early universe $(a \ll a_c)$ and dilutes as $a^{-3(1+w)}$ at late times $(a \gg a_c)$. The requirement that the energy density of the dark fluid does not overclose the universe too early, we expect $\rho_s$ to fall off faster than radiation, so we impose $\frac{1}{3} < w < 1$. In addition, we keep the position of the transition free. In practice, motivated by EDE proposal, we take the position of the first step (the first transition) to be after the time of matter-radiation equality and before the CMB decoupling around the redshift $z_c \sim 2000–3000$, while the follow-up transitions can happen much later in the cosmic expansion history. Within the model of Firouzjahi (2022b) this corresponds to a field with the mass $m \sim eV$. As a field with this mass is not within the spectrum of the SM, one needs a new physics beyond the SM. For example, this may come from an e$V$-scale sterile neutrino, such as from short-baseline anomalies, but this is tightly constrained; see for example Hagstotz et al. (2021).

The simplest model satisfying our requirements may be realized by the following ansatz,

$$\rho_s(a) \propto \left[ 1 + \left( \frac{a}{a_c} \right)^{3(1+w)} \right]^{-1}. \quad (1)$$

Table 1

| Parameter | Priors Model 1 | Priors Model 2 | Priors Model 3 |
|-----------|----------------|----------------|----------------|
| $1 + z_c$ | 25             | 25             | 2500           |
| $f_1$     | 0.20           | 0.20           | 0.20           |
| $w_i$     | [0.5, 1]       | [0.5, 1]       | [0.5, 1]       |
| $n_i$     | [-1, 1.5]      | [-1, 1.5]      | [-1, 1.5]      |

Note. Note that we take the fraction of dark energy density $f_i$ for all cases to be the same, while we let $w_i$ and $n_i$ vary. The redshift of the step position is denoted by $1 + z_c$ and we test the model in three different step positions: before recombination, after recombination, and in the late-time era.

At early stages, $a \ll a_c$, in which $a_c$ is the transition scale factor and $\rho_s(a)$ is nearly constant but small, while long after the transition it falls off as $a^{-3(1+w)}$. However, to control the sharpness of the dark energy transition we introduce a transfer function $T(b, a - a_c)$ in which $b$ is a positive constant parameter measuring the sharpness of the transition. In our analysis we consider the following transfer function

$$T(b, a - a_c) = \frac{1}{2} \left[ 1 + \tanh \left( b(a - a_c) \right) \right]. \quad (2)$$

For $b|a - a_c| \gg 1$, the transfer function $T$ behaves like the Heaviside function $\Theta(a - a_c)$, indicating a sharp transition, while for $b|a - a_c| \ll 1$, $T$ describes a mild transition.

With the inclusion of the above transfer function, Equation (1) is modified as

$$\rho_s(a) = \rho_{s,c} \left[ T(b, a_c - a) + T(b, a - a_c) \left( \frac{a}{a_c} \right)^{3(1+w)} \right]^{-1}, \quad (3)$$

where $\rho_{s,c} = \rho_s(a = a_c)$. At the transition scale factor $a_c$, we introduce the fraction of dark energy density $f_X$ associated to the field (or dark fluid) as

$$f_X = \frac{\rho_{s,c}}{\rho(a_c)}, \quad (4)$$

where $\rho(a)$ is the total energy density in the Friedmann equation, $3M^2_H \dot{H}^2 = \rho(a)$, and is given by

$$\rho(a) = \rho_m a^{-3} + \rho_r a^{-4} + \rho_\Lambda + \rho_x(a), \quad (5)$$

where $\rho_m, \rho_r$, and $\rho_\Lambda$, respectively, are the present value ($a = 1$) of matter, radiation, and the dark energy density of the universe. At the transition scale $a_c$ we find

$$\rho(a_c) = \rho_m a_c^{-3} + \rho_r a_c^{-4} + \rho_\Lambda, \quad (6)$$

and then

$$\rho_{s,c} = \frac{f_X}{1 - f_X} (\rho_m a_c^{-3} + \rho_r a_c^{-4} + \rho_\Lambda). \quad (7)$$

Note that for dS-like (AdS-like) dark energy, $f_X > 0$ ($f_X < 0$).

Plugging the above value in Equation (5), the total energy density of our model is given by

$$\rho(a) = \rho_m a^{-3} + \rho_r a^{-4} + \rho_\Lambda + \frac{f_X}{1 - f_X} (\rho_m a_c^{-3} + \rho_r a_c^{-4} + \rho_\Lambda) \frac{a^{-3(1+w)}}{a_c^{-3(1+w)}}. \quad (8)$$

The fraction of the current energy density $\Omega_i$ for each component can easily be calculated. At present, $3M^2_H H_0^2 = \rho_0$, we have

$$\rho_0 = \rho_m + \rho_r + \rho_\Lambda + \rho_{s,0}\rho, \quad (9)$$
1 + z_{c,1} = 25
1 + z_{c,1} = 250
1 + z_{c,1} = 2500

Figure 2. One-dimensional likelihoods for One-step models for $H_0$ based on “CMB+BAO+SN” data. Note that the shaded area shows the measurement of $H_0$ done by the SH0ES team and its 1σ error (Riess et al. 2022).

Table 2
The 68% Limits for Parameters of One-step Models Compared to the ΛCDM model

| Parameter  | Best-fit ΛCDM | Best-fit Model 1 | Best-fit Model 2 | Best-fit Model 3 |
|------------|---------------|------------------|------------------|------------------|
| $n_1$      | ...           | >0.945           | >1.18            | >1.46            |
| $w_1$      | ...           | >0.880           | >0.923           | 0.808 ± 0.060    |
| $\Omega_m$ | 0.3092 ± 0.0070 | 0.3107 ± 0.0073 | 0.3040 ± 0.0068 | 0.2838 ± 0.0085 |
| $H_0$      | 67.70 ± 0.52  | 66.43 ± 0.54    | 67.80 ± 0.52    | 71.83 ± 0.67    |
| $S_8$      | 0.819 ± 0.014 | 0.793 ± 0.014   | 0.803 ± 0.013   | 0.890 ± 0.018   |
| $10^3A_s$  | 2.091 ± 0.027 | 2.069 ± 0.025   | 2.086 ± 0.027   | 2.052 ± 0.029   |
| $n_s$      | 0.9668 ± 0.0042 | 0.9795 ± 0.0044 | 0.9681 ± 0.0042 | 0.9932 ± 0.0044 |
| $\tau$     | 0.0541 ± 0.0059 | 0.0544 ± 0.0058 | 0.0551 ± 0.0059 | 0.0425 ± 0.0065 |
| $\Delta$AIC | 0.0           | 24.38           | 75.55            | 808.04           |

in which $\rho_{x,0} = \rho_x(a = 1)$. Correspondingly, for each component in cosmic fluid we have

$$\Omega_m = \frac{\rho_m}{\rho_0}, \quad \Omega_t = \frac{\rho_{t}}{\rho_0}, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_0}, \quad \Omega_x = \frac{\rho_{x,0}}{\rho_0}. \quad (10)$$

As a rough estimation of $\Omega_x$, using the simple form of $\rho_x$ from Equation (1), the fraction of dark energy density is found to be

$$\Omega_x = \frac{2f_x}{1 - f_x} a_c^{3w}\{\Omega_m + \Omega_{\Lambda} a_c^{-1} + \Omega_x a_c^2\}. \quad (11)$$

For the early time when $a_c \ll 1$, we approximately obtain

$$\Omega_x \simeq \frac{2f_x}{1 - f_x} a_c^{3w}\{\Omega_m + \Omega_{\Lambda} a_c^{-1}\}. \quad (12)$$
If we further assume that $a_c \ll a_{eq}$, i.e., the transition happens after the matter-radiation equality, then the above relation simplifies further to

$$\Omega_x \simeq \frac{2f_x}{1-f_x} a_c^{3w} \Omega_m.$$  \hspace{1cm} (13)

Note that the parameter $\Omega_x$ is what is defined in Karwal & Kamionkowski (2016) as $\Omega_{x,c}$. For $a_c \lesssim 10^{-3}$ and $f \sim 0.1$, with $w = 1$, we typically have $\Omega_x \sim 10^{-10}$ or so. For $a_c \sim 10^{-1}$ and $w = 1$, we obtain $\Omega_x \sim 10^{-5}$. So, in general, for $w > \frac{1}{3}$, $\Omega_x$ is smaller than $\Omega_c \sim 10^{-4}$.

For the $N$-field configuration, yielding to $N$ transitions in dark energy at $a_{c,i}$ with $i = 1, 2, \ldots, N$, one can extend Equation (8) to the following form

$$\rho(a) = \rho_{m,0} a^{-3} + \rho_{c,0} a^{-4} + \rho_h + \sum_{i=1}^{N} \frac{f_i}{1-f_i} (\rho_m a_{c,i}^{-3} + \rho_c a_{c,i}^{-4} + \rho_h)$$

$$+ \sum_{i=1}^{N} T(b_i, a_{c,i} - a) + T(b_i, a - a_{c,i}) \left( \frac{a}{a_{c,i}} \right)^{3(1+w_h)}.$$ \hspace{1cm} (14)

For a sharp phase transition in dark energy with $b_i a_{c,i} \gg 1$, we have $n_i > 0$, while for a mild transition $n_i < 0$.

In the following analysis, we also parameterize $b_i$ as

$$b_i \equiv \frac{10^{w_i}}{a_{c,i}}.$$ \hspace{1cm} (15)

In summary, the new parameters of the model are $\{f_i, a_{c,i}, n_i, w_i\}$ for each component $i = 1, 2, \ldots, N$ of dark fluid. The term $f_i$ measures the fraction of dark energy at the time of transition $a_{c,i}$ (or $z_{c,i}$), $n_i$ measures the sharpness of the transition, and $w_i$ represents the equation of state of the dark fluid after the transition $a(t) > a_{c,i}$.

Our model has some similarities to the EDE setup in that we consider an early stage of dark energy. However, we include the new parameter $n_i$ to control how sharply the transition has happened. Moreover, with our theoretical motivations in mind,
we allow for multiple transitions in dark energy during the universe’s evolution, $N\geq 1$. In addition, the contributions of the subsequent dark fluids ($1 < i \leq N$) can be either dS ($f_i > 0$) or AdS ($f_i < 0$). For earlier works concerning AdS vacua and $H_0$ tension or AdS-EDE, see Calderón et al. (2021), Ruchika et al. (2020), Dutta et al. (2020), Visinelli et al. (2019), Akarsu et al. (2020), Akarsu et al. (2021), Ye & Piao (2020a), Jiang & Piao (2021), Ye & Piao (2020b), and Sen et al. (2022).

There is an important comment in order. In the current analysis of the effects of multiple transitions in dark energy, we concentrate on the background evolution and do not study perturbations. In our setup the physical mechanism behind the dark energy transitions and the transfer of energy to the thermal bath are already complicated phenomena at the background level, and a full treatment of perturbations analysis is beyond the scope of the current analysis. While studying the background dynamics can shed some light on the $H_0$ tension problem, it is not fully consistent. This is an important limitation in our current analysis that should be improved in future studies considering the full dynamics of the background and the perturbations.

3. Observational Data and Statistical Methodology

In this section, we begin with a brief description of the main cosmological data sets used in this work. In our analysis we consider a combination of three types of data: “CMB + BAO + SN.”

1. **CMB**: We use the latest, most precise large-scale CMB temperature and polarization angular power spectra from the final release of “Planck 2018,” plikTTTEEE+lowl+lowE (Aghanim et al. 2020a, 2020b, 2020c). We use the full power spectrum and do not split it into high-$\ell$ and low-$\ell$ parts. We denote all of the Planck data (including temperature and polarization) by “CMB.”

2. **BAO**: We also take into account the various measurements of the baryon acoustic oscillations (BAO) from different galaxy surveys (Aghanim et al. 2020b): 6dFGS (Beutler et al. 2011), SDSS-MGS (Ross et al. 2015), and BOSS DR12 (Alam et al. 2017).

3. **SN**: We include the measurements of the 1048 SNe Type Ia luminosity distance in the redshift interval $z \in [0.01, 2.3]$ from the Pantheon sample (Scolnic et al. 2018). We show this catalog of supernovae as “SN.”
To analyze the data and extract the constraints on the cosmological parameters, we have modified the well-known cosmological Markov Chain Monte Carlo package CosmoMC (Lewis & Bridle 2002; Lewis 2013), which is publicly available. This package is equipped with a convergence diagnostic based on the Gelman and Rubin statistic (Gelman & Rubin 1992), assuming $R - 1 < 0.1$, and implements an efficient sampling of the posterior distribution using the fast/slow parameter decorrelations (Lewis 2013).

The parameter space for the ΛCDM model is:

$$ P_0 \equiv \{ \Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \text{ln}[10^{10}A_s] \}, $$

where $\tau$ is the reionization optical depth, $n_s$ is the scalar spectral index, $A_s$ is the amplitude of the scalar primordial power spectrum, and the $\theta_{MC}$ parameter is an approximation of $\theta_*$. 

Note. We choose the first step to occur before the recombination era $1 + z_{c,1} = 2500$ and let the second step occur after recombination but in three different eras.

The set of free parameters describing the One-step class of models (i.e., one transition in dark energy, $N = 1$) is given by

$$ P_1 \equiv \{ a_1, f_1, w_1, n_1, \Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \text{ln}[10^{10}A_s] \}. $$

Correspondingly, the parameter space of the Two-step class of models ($N = 2$) is given by

$$ P_2 \equiv \{ a_1, f_1, w_1, n_1; a_2, f_2, w_2, n_2; \Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \text{ln}[10^{10}A_s] \}. $$

### Table 5: Priors for Two-step Models with Different Second Step Positions $1 + z_{c,2}$

| Parameter | Priors Model 6 | Priors Model 7 | Priors Model 8 |
|-----------|----------------|----------------|----------------|
| $1 + z_{c,1}$ | 2500           | 2500           | 2500           |
| $f_1$     | 0.20           | 0.20           | 0.20           |
| $w_1$     | [0.5, 1]       | [0.5, 1]       | [0.5, 1]       |
| $n_1$     | $[-1, 1.5]$    | $[-1, 1.5]$    | $[-1, 1.5]$    |
| $1 + z_{c,2}$ | 833            | 83.3           | 8.3            |
| $f_2$     | 0.10           | 0.10           | 0.10           |
| $w_2$     | [0.5, 1]       | [0.5, 1]       | [0.5, 1]       |
| $n_2$     | $[-1, 1.5]$    | $[-1, 1.5]$    | $[-1, 1.5]$    |

### Table 6: The 68% Limits for Parameters of Two-step Models Based on CMB+BAO+SN Data

| Parameter | Best-fit Model 6 | Best-fit Model 7 | Best-fit Model 8 |
|-----------|------------------|------------------|------------------|
| $n_1$     | >1.47            | >1.42            | >1.46            |
| $w_1$     | >0.984           | 0.763±0.036      | 0.703±0.033      |
| $n_2$     | >0.363           | ...              | ...              |
| $w_2$     | >0.730           | <0.641           | <0.576           |
| $\Omega_m$ | 0.2801±0.0081    | 0.2811±0.0085    | 0.2832±0.0076    |
| $H_0$     | 71.92±0.39       | 71.55±0.27       | 70.11±0.59       |
| $S_8$     | 0.876±0.017      | 0.875±0.021      | 0.853±0.017      |
| $10^{10}A_s$ | 2.019±0.031    | 2.039±0.025      | 2.017±0.026      |
| $n_s$     | 0.9843±0.0048    | 0.9989±0.0059    | 1.0085±0.0052    |
| $\tau$    | 0.0426±0.0072    | 0.0434±0.0058    | 0.0448±0.0063    |
| $\Delta$AIC | 764.35          | 805.25           | 764.66           |

Note. Note that all the input parameters are the same and only the position of the second step $z_{c,2}$ is different (see Table 5). While $z_{c,2}$ changes by two orders of magnitude in these examples, $H_0$ does not change significantly.
Subsequently, the set of free parameters representing parameter space for the Three-step class of models \((N = 3)\) is given by

\[
\{a_1, f_1, w_1, n_1; a_2, f_2, w_2, n_2; a_3, f_3, w_3, n_3; \\
\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln[10^{10}A_s]\}.
\]

(19)

4. Results

To test our proposal we consider three main classes of models: One-step, Two-step, and Three-step models, in each of which we assume there is a transition in dark energy density. For simplicity, we begin with One-step models and continue to the Two-step and Three-step classes of models. We will test the effects of the step position \(z_{c,1}\), the strength of the fraction of total energy density \(f_i\), and also the effects of having dS \((f_i > 0)\) or AdS \((f_i < 0)\) phases of evolution in the following subsections.

We use two statistical criteria to quantify the tension between the cosmological inferred value and the SH0ES experiment. The first criterion is Akaike Information Criterion (AIC), which is a tool to measure the improvement of the fit.

We compute the AIC of the extended model \(M\) relative to that of \(\Lambda\)CDM, defined as

\[
\Delta\text{AIC} = \chi^2_{\text{min},M} - \chi^2_{\text{min},\Lambda\text{CDM}} + 2(N_M - N_{\Lambda\text{CDM}}),
\]

(20)

where \(N_M\) stands for the number of free parameters of the model and \(N_{\Lambda\text{CDM}}\) is the number of free parameters of the \(\Lambda\)CDM model (Akaike 1974).

Another way to quantify the tension on \(x = H_0\) is through the “rule of thumb difference in mean,” or Gaussian Tension (GT; Raveri & Hu 2019), defined as

\[
\Delta x = \frac{x_\text{D} - x_{\text{SH0ES}}}{(\sigma^2_\text{D} + \sigma^2_{\text{SH0ES}})^{1/2}},
\]

(21)

where \(x_i\) and \(\sigma_i\) are the mean and standard deviation of observation \(i\). When \(D\) includes (excludes) supernovae data, we quantify the tension on \(H_0\) using \(H_0 = 73.2 \pm 1.3\) km s\(^{-1}\) Mpc\(^{-1}\), with the uncertainties corresponding to the 68% confidence level (Schöneberg et al. 2022).
4.1 One-step Models

4.1.1 Effects of Step Position

In this subsection we consider One-step models that differ in the position \(z_{c,1}\) when the transition in dark energy takes place. In Table 1, we see our considered priors for this class of models. When we let the step position \(z_{c,1}\) and the fraction of energy density \(f_1\) both be free parameters, we had some computational problems. Therefore, we decided to test our hypothesis by keeping these two parameters fixed, but with different values in each run. So, in this and the rest of our analysis we assume the step positions and the fraction of total energy density to be fixed parameters but with different fixed values in each analysis.

In Table 2 we summarize observational constrains of this class of models by considering \(\Lambda \text{CDM}\) as an anchor. We point out that in all of the analysis of the models we use a combination of three types of data, i.e., “CMB+BAO+SN.”

Looking at Table 2, it is obvious that Model 3 shows the least tension with the SH0ES results. In this model, we assume the transition in dark energy occurs at \(z \simeq 2500\), which is long before the surface of last scattering and after the time of matter-radiation equality. Our conclusion for the One-step models is that an early phase of dark energy with \(f_1 > 0\), as in the EDE scenario, can reconcile the \(H_0\) tension. That said, One-step models with the transition happening after the surface of last scattering are not promising for this purpose.

The effects of considering an early step in the density evolution are shown in Figure 2. In this figure we see the likelihoods of different One-step models with different step positions. Also, contour plots for parameters \(H_0\) and \(\Omega_m\) can be seen in Figure 3.

![Figure 2](image-url)

**Figure 2.** Contour plots for Two-step models for \(H_0\) vs. \(\Omega_m\) based on CMB+BAO+SN data, comparing the effect of the second step’s position \(z_{c,2}\) with fixed \(f_1, f_2 > 0\). Note that the shaded area shows the measurement of \(H_0\) done by the SH0ES team and its 1σ error (Riess et al. 2022).

| Parameter | Best-fit Intervals | Best-fit Intervals |
|-----------|-------------------|-------------------|
| Parameter | Model 7            | Model 9            |
| \(n_1\)   | \(>1.42\)          | \(>1.46\)          |
| \(w_1\)   | \(0.763^{+0.010}_{-0.007}\) | \(0.827 \pm 0.061\) |
| \(n_2\)   | \(\ldots\)         | \(>0.783\)         |
| \(w_2\)   | \(<0.641\)         | \(>0.794\)         |
| \(\Omega_m\) | \(0.281^{+0.008}_{-0.007}\) | \(0.283^{+0.008}_{-0.007}\) |
| \(H_0\)   | \(71.5^{+0.7}_{-0.7}\) | \(72.1^{+0.5}_{-0.7}\) |
| \(S_8\)   | \(0.875 \pm 0.021\) | \(0.895^{+0.017}_{-0.013}\) |
| \(10^\Delta A_s\) | \(2.039^{+0.025}_{-0.036}\) | \(2.060 \pm 0.029\) |
| \(n_s\)   | \(0.998^{+0.009}_{-0.004}\) | \(0.9914 \pm 0.0046\) |
| \(\tau\)  | \(0.043^{+0.008}_{-0.007}\) | \(0.0427 \pm 0.0065\) |
| \(\Delta AIC\) | \(805.25\) | \(825.05\) |

**Table 8.** Best-fit Values and 68% Confidence Intervals for Parameters of Two-step Models

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Note. As in the standard EDE scenario, we choose the first step to be a dS type \(f_1 > 0\) while we let the second step be either of dS type \((f_2 > 0)\) or AdS type \((f_2 < 0)\). Other parameters are the same.
4.1.2. Effects of the Strength of the Fraction of Dark Energy Density

In this section we look at the effects of the fraction of dark energy density in the One-step class of models. We summarize our consideration of priors in this analysis in Table 3. According to our past analysis, we chose a fixed step position at $a_{c,1} = 0.0004 \times (1 + z_{c,1} = 2500)$ and we also fixed the fraction of dark energy density $f_1$, but with different values in each analysis. Both $w_1$ and $n_1$ are free parameters in all of our analysis.

The conclusion is that the higher the value of $f_1$, the larger the prediction for $H_0$. As we see in Table 4, Model 5, which has higher value for the fraction of energy density ($f_1 = 0.25$), shows more consistency with the value of $H_0$ from the SH0ES measurements. As we report in Table 13, the Gaussian tension in $H_0$ for Model 5 is just 0.47σ.

Figure 8. One-dimensional likelihoods for Two-step models for $H_0$ based on CMB+BAO+SN data, comparing the effect of the sign of $f_2$, dS ($f_2 > 0$) or AdS ($f_2 < 0$) in Models 7 and 9.

Table 9

| Parameter | Priors | Priors |
|-----------|--------|--------|
|           | Model 10 | Model 11 |
| $1 + z_{c,1}$ | 2500 | 2500 |
| $f_1$ | 0.25 | 0.25 |
| $w_1$ | [0.5, 1] | [0.5, 1] |
| $n_1$ | $[-1, 1.5]$ | $[-1, 1.5]$ |
| $1 + z_{c,2}$ | 83.3 | 83.3 |
| $f_2$ | 0.15 | -0.15 |
| $w_2$ | [0.5, 1] | [0.5, 1] |
| $n_2$ | $[-1, 1.5]$ | $[-1, 1.5]$ |

Note. This is similar to Table 7 for Models 7 and 9, but here we have increased both $f_1$ and $|f_2|$ compared to Table 7.

Table 10

| Parameter | Best-fit | Best-fit |
|-----------|----------|----------|
|           | Model 10 | Model 11 |
| $n_1$ | $>1.46$ | $>1.41$ |
| $w_1$ | $0.762 \pm 0.036$ | $0.824^{+0.087}_{-0.044}$ |
| $n_2$ | $<0.957$ | $>1.07$ |
| $w_2$ | $<0.534$ | $>0.786$ |
| $\Omega_m$ | $0.2666^{+0.0078}_{-0.0061}$ | $0.267^{+0.011}_{-0.0052}$ |
| $H_0$ | $73.18^{+0.48}_{-0.34}$ | $74.70^{+0.34}_{-0.34}$ |
| $S_8$ | $0.875^{+0.017}_{-0.013}$ | $0.904^{+0.025}_{-0.013}$ |
| $10^5 A_s$ | $2.021^{+0.026}_{-0.022}$ | $2.060 \pm 0.031$ |
| $n_s$ | $1.0223^{+0.0054}_{-0.0047}$ | $1.0077 \pm 0.0069$ |
| $\tau$ | $0.0414^{+0.0039}_{-0.0037}$ | $0.0420^{+0.0061}_{-0.0087}$ |
| $\Delta$AIC | 1355.95 | 1453.15 |

The conclusion is that the higher the value of $f_1$, the larger the prediction for $H_0$. As we see in Table 4, Model 5, which has higher value for the fraction of energy density ($f_1 = 0.25$), shows more consistency with the value of $H_0$ from the SH0ES measurements. As we report in Table 13, the Gaussian tension in $H_0$ for Model 5 is just 0.47σ.
In Figure 4 we present the likelihood probabilities for the One-step class of models with different values for $f_1$. Also, the two-dimensional contour plots for parameters $H_0$ versus $\Omega_m$ are shown in Figure 5. In both figures the shaded areas show the 1σ and 2σ allowed error regions based on SH0ES measurements for $H_0$ (Riess et al. 2022).

### 4.2. Two-step Models

Here, we extend our analysis to the Two-step class of models. In this type of models, the first transition in dark energy can occur at an earlier time, while the second transition can take place before the surface of last scattering or after that. Also, we have freedom to choose the second phase of dark energy to be of dS type or AdS type.

#### 4.2.1. Effect of Position of the Second Step

In this approach we assume the first step is fixed at the time $1+z_{c,1}=2500$ and we test different positions for the second step of the evolution. In Table 5, we present our assumptions for priors of parameters. Since we consider the first step to occur before the surface of last scattering, we assume the second step to occur after the surface of last scattering. In addition, in all of the models in this subsection we assume a dS phase for the second step ($f_2 > 0$).

Looking at Table 6, we see that the result for $H_0$ does not strongly depend on the position of the second step (as long as the sign of $f_2$ is fixed). For example, in the Models 6, 7, and 8, the values of $z_{c,2}$ change by two orders of magnitude while $H_0$ does not change drastically. Having said this, we notice that the value of $H_0$ in Model 6, where the second step occurs more closer to the CMB era, shows more consistency with the SH0ES measurements. By contrast, putting the second step closer to late times (Model 8), $H_0$ shows more tension.

In Figures 6 and 7 we present the likelihoods and contour plots for $H_0$ and $\Omega_m$ in Two-step models with different positions of the second step.

#### 4.2.2. Considering dS or AdS for the Second Step

As mentioned above, in the Two-step and Three-step models we have the freedom to choose the next phases to be dS-like or AdS-like. Hence, we want to test the effect of having a dS or an AdS in the second phase of the evolution. In Table 7 we have priors for two models, Model 7 and Model 9, that have similar priors but opposite signs for $f_2$. 

---

**Table 11**

| Parameter | Priors Priors Priors Priors |
|-----------|-----------------|-----------------|-----------------|-----------------|
|           | Priors Priors Priors Priors |
| $1 + z_{c,1}$ | 2500 2500 2500 2500 |
| $f_1$ | 0.20 0.20 0.20 0.20 |
| $w_1$ | [0.5, 1] [0.5, 1] [0.5, 1] [0.5, 1] |
| $n_1$ | [-1, 1.5] [-1, 1.5] [-1, 1.5] [-1, 1.5] |
| $1 + z_{c,2}$ | 833 833 833 833 |
| $f_2$ | +0.15 −0.15 +0.15 −0.15 |
| $w_2$ | [0.5, 1] [0.5, 1] [0.5, 1] [0.5, 1] |
| $n_2$ | [-1, 1.5] [-1, 1.5] [-1, 1.5] [-1, 1.5] |
| $1 + z_{c,3}$ | 83.3 83.3 83.3 83.3 |
| $f_3$ | +0.15 +0.15 −0.15 −0.15 |
| $w_3$ | [0.5, 1] [0.5, 1] [0.5, 1] [0.5, 1] |
| $n_3$ | [-1, 1.5] [-1, 1.5] [-1, 1.5] [-1, 1.5] |

**Note.** We fix the first step in the dS phase at $f_1 > 0$ and test different permutations of dS or AdS phases for the second and third steps.

In Figure 4 we present the likelihood probabilities for the One-step class of models with different values for $f_1$. Also, the two-dimensional contour plots for parameters $H_0$ versus $\Omega_m$ are shown in Figure 5. In both figures the shaded areas show the 1σ and 2σ allowed error regions based on SH0ES measurements for $H_0$ (Riess et al. 2022).
In Table 8 we summarize observational constraints for parameters of Model 7 and Model 9, while in Figures 8 and 9 we see likelihoods and contour plots for $H_0$ and $\Omega_m$ in these two models. Clearly, having the second phase be of AdS type leads to a higher value of $H_0$. Furthermore, in this numerical example when the second phase is of AdS type there is more consistency with the SH0ES measurements.

To confirm these conclusions, we have repeated this comparison for the Models 10 and 11, which have opposite signs of $f_2$, with the priors given in Table 9, while the results

In Table 12 we summarize observational constraints for parameters of Model 7 and Model 9, while in Figures 8 and 9 we see likelihoods and contour plots for $H_0$ and $\Omega_m$ in these two models. Clearly, having the second phase be of AdS type leads to a higher value of $H_0$. Furthermore, in this numerical example when the second phase is of AdS type there is more consistency with the SH0ES measurements.

To confirm these conclusions, we have repeated this comparison for the Models 10 and 11, which have opposite signs of $f_2$, with the priors given in Table 9, while the results

| Parameter | Best-fit Model 12 | Best-fit Model 13 | Best-fit Model 14 | Best-fit Model 15 |
|-----------|------------------|------------------|------------------|------------------|
| $n_1$     | $>$1.45          | $>$1.44          | $>$1.45          | $>$1.44          |
| $w_1$     | $>$0.988         | <0.511           | $>$0.992         | <0.517           |
| $n_2$     | $>$1.05          | $>$1.11          | $>$1.10          | $>$1.14          |
| $w_2$     | $>$0.938         | $>$0.842         | $>$0.797         | $>$0.938         |
| $n_3$     | <0.527           | <−0.609          | >0.514           | >0.916           |
| $w_3$     | <0.557           | <0.542           | >0.844           | >0.865           |
| $\Omega_m$ | 0.2743 ± 0.0068  | 0.2746 ± 0.0062  | 0.2743 ± 0.0076  | 0.2730 ± 0.0070  |
| $H_0$     | 71.35 ± 0.67     | 71.77 ± 0.59     | 72.81 ± 0.64     | 73.46 ± 0.66     |
| $\Delta_A$ | 0.845 ± 0.016   | 0.862 ± 0.017    | 0.874 ± 0.017    | 0.892 ± 0.018    |
| $10^3A_s$ | 1.977 ± 0.029    | 2.030 ± 0.025    | 2.006 ± 0.028    | 2.062 ± 0.028    |
| $r_\tau$  | 0.9942 ± 0.0046  | 1.0197 ± 0.0047  | 0.9810 ± 0.0043  | 1.0071 ± 0.0044  |
| $\tau$    | 0.0430 ± 0.0064  | 0.0419 ± 0.0059  | 0.0425 ± 0.0067  | 0.0422 ± 0.0063  |
| $\Delta$  | 744.79           | 836.76           | 774.85           | 908.96           |

Figure 10. One-dimensional likelihoods for Two-step models for $H_0$ based on CMB+BAO+SN data, comparing the effects of the sign of $f_2$ in Models 10 and 11. This plot is parallel to Figure 8 performed for Models 7 and 9, but now the value of $f_1$ is increased to $f_1 = 0.25$ and the values of $f_2$ for the dS and AdS cases are changed to $f_2 = +0.15$ and $f_2 = −0.15$, respectively.
are summarized in Table 10. In Figures 10 and 11, we see likelihoods and contour plots for $H_0$ and $\Omega_m$ in these two models. As expected, a second phase of AdS type yields a higher value of $H_0$.

### 4.3. Three-step Models

As a wider extension of our study, we extend the analysis to the models with three steps of transitions in dark energy. As before, we assume the first phase is of dS type, happening at early times at $z_{c,1} = 2500$, while the second and third steps can happen near or long after the surface of last scattering and be of either dS or AdS type. As described in Table 11, we have tested different permutations of the signs of the $f_2$ and $f_3$ parameters.

According to Tables 12 and 13, different cases of the Three-step class of models show consistency with the SH0ES results. However, as we see from Table 13, Model 14 and Model 15 show the least tension with the SH0ES measurements. In Model 12 all three steps are in the dS phase, but in Model 13 the second step is chosen to be in the AdS phase. The corresponding results are presented in Figure 12 for the likelihood probabilities, and also for contour plots of $H_0$ versus $\Omega_m$ in Figure 13. A general conclusion is that having the second and third phases both of AdS type, $f_2, f_3 < 0$, leads to higher values of $H_0$ compared to the cases where they are both in dS phases, $f_2, f_3 > 0$: compare Models 12 and 15. However, the competition is more nontrivial when a dS phase and an AdS phase are both present, i.e., the cases where $f_2 > 0, f_3 < 0$ compared to the case where $f_2 < 0, f_3 > 0$: see Models 13 and 14.

### 5. Summary and Discussions

We have studied a phenomenological model in which dark energy undergoes multiple transient stages. At early times prior to transition, the dark energy behaves like a small cosmological constant term. At the transition time, dark energy comprises a noticeable fraction of the total energy density and falls off rapidly afterwards. The equation of state of the fluid after transition is represented by the parameter $w$ with the requirement $\frac{1}{3} < w \leq 1$ in order not to modify the expansion.
history of universe drastically. While this is a phenomenological proposal that can mimic the EDE scenario, it may be realized theoretically as well. For example, this setup may be realized within the context of vacuum zero-point energy of quantum fields in connection to the cosmological constant problem. Alternatively, this proposal may emerge from theories beyond the SM of particle physics in which the energy densities of hidden sectors do not interact with the SM fields while they contribute to the expansion history of the universe.

We have studied various cases of a single transition, a double transition, and a triple transition in dark energy density. In the latter two cases we also allowed the second and/or third components of dark energy to be either dS-like ($\rho_2 > 0$) or AdS-like ($\rho_3 < 0$). As in the standard EDE setup, having a larger value of $f_1$ yields a larger value of $H_0$. In addition, AdS-like dark energy yields larger values of $H_0$. To solve the $H_0$ tension, as in the EDE scenario, the first transition is located sometime between the time of matter-radiation equality and the surface of last scattering, say at the redshift $z_{c,1} \sim 2500$. However, the second or third transitions can take place anytime after the CMB decoupling. We have considered the cases where these happen, say, at redshifts $z_{c,2} \sim 800$ and $z_{c,3} \sim 80$.

Our investigations show that the resulting values of $H_0$ are not sensitive to the locations of the second or third transitions ($z_{c,2}$ and $z_{c,3}$), but they are largely sensitive to the values and the signs of the fractions of dark energy, $f_2$ and $f_3$. Our analysis also shows that to obtain values of $H_0$ comparable to the value obtained from local measurements requires $n_s$ to move toward the Harrison–Zel’dovich scale-invariant value. In all of the examples that we studied so far, the least tension in the $H_0$ value occurs in a Two-step model in which both steps occur in dS-like phases ($f_i > 0$, $i = 1, 2$), and also in a Three-step model in which its first phase is dS-like and the evolution of the second and third steps occurs in AdS-like phases ($f_1 > 0, f_i < 0$, $i = 2, 3$; see Table 13).

As mentioned in Tables 2, 4, 6, 8, 10, and 12, our models produce a worse fit than the $\Lambda$CDM model, which we described by $\Delta$AIC criteria. In most of the EDE models, the energy density fraction $f$ is roughly 0.1–0.15 (Smith et al. 2020; Jiang & Piao 2021). Here, we have fixed the parameter $f$ to higher

![Figure 12. One-dimensional likelihoods for Three-step models for $H_0$ based on “CMB+BAO+SN” data, comparing the effects of the signs of the fraction of dark energy in the second and third phases ($f_2, f_3$) with a fixed $f_1 > 0$.](image-url)
values of 0.20 and 0.25 to resolve the Hubble tension. Obtaining a worse fit than the $\Lambda$CDM model may be a consequence of our assumptions to fix the parameters and also of ignoring the effects of perturbations in our analysis. To reconcile the $H_0$ tension we assumed higher values for the energy density fraction $f = 0.20$ and $0.25$, but it leads to higher values of $\chi^2$ compared to $\Lambda$CDM.

As we mentioned previously, in the current analysis we have concentrated only on the background evolution, as the physics behind the dark energy transition is already complicated at the background level. A fully consistent analysis of the effects of transient dark energies requires the perturbations to be included as well. While this is beyond the scope of the current analysis, it is an interesting question to extend the current analysis such that perturbations are included as well.

We are grateful to Sunny Vagnozzi and E. Di Valentino for insightful comments and discussions. H.F. and A.T. would like to thank the "Saramadan" federation of Iran for the partial supports. H.F. would like to thank YITP, Kyoto University for the hospitality during the workshop "Gravity: Current challenges in black hole physics and cosmology," where this work was in progress.

**ORCID iDs**

Hossein Mosha  
https://orcid.org/0000-0002-8914-6510

Hassan Firouzjahi  
https://orcid.org/0000-0002-1850-4392

Alireza Talebian  
https://orcid.org/0000-0002-2807-404X

**References**

Addison, G. E., Huang, Y., Watts, D. J., et al. 2016, ApJ, 818, 132

Aghanim, N., Akrami, Y., Ashdown, M., et al. 2017, A&A, 607, A95

Aghanim, N., Akrami, Y., Ashdown, M., et al. 2020a, A&A, 641, A5

Aghanim, N., Akrami, Y., Ashdown, M., et al. 2020b, A&A, 641, A6

Aghanim, N., Akrami, Y., Ashdown, M., et al. 2020c, A&A, 641, A8

Agrawal, P., Cyr-Racine, F.-Y., Pinner, D., & Randall, L. 2019, arXiv:1904.01016

Aiola, S., Calabrese, E., Maurin, L., et al. 2020, JCAP, 12, 047

Akaike, H. 1974, ITAC, 19, 716

Akarsu, O., Barrow, J. D., Escamilla, L. A., & Vazquez, J. A. 2020, PhRvD, 101, 063528

Akarsu, O., Kumar, S., Özlük, E., & Vazquez, J. A. 2021, PhRvD, 104, 123512

Alam, S., Ata, M., Bailey, S., et al. 2017, MNRAS, 470, 2617

Ballesteros, G., Notari, A., & Rompineve, F. 2020, JCAP, 11, 024

Banihashemi, A., Khosravi, N., & Shirazi, A. H. 2020, PhRvD, 101, 123521

Bernal, J. L., Verde, L., & Riess, A. G. 2016, JCAP, 10, 019

Beutler, F., Blake, C., Colless, M., et al. 2011, MNRAS, 416, 3017

Braglia, M., Ballardini, M., Finelli, F., & Koyama, K. 2021, PhRvD, 103, 043528

Braglia, M., Emond, W. T., Finelli, F., Gunnrukcuoglu, A. E., & Koyama, K. 2020, PhRvD, 102, 083513

Calderón, R., Gannouji, R., L’Huillier, B., & Polarski, D. 2021, PhRvD, 103, 023526

Chudaykin, A., Gorbunov, D., & Nedelko, N. 2020, JCAP, 08, 013

Chudaykin, A., Gorbunov, D., & Nedelko, N. 2021, PhRvD, 103, 043529

Dainotti, M. G., De Simone, B., Schiavone, T., et al. 2021, ApJ, 912, 150

Dainotti, M. G., De Simone, B., Schiavone, T., et al. 2022, Galax, 10, 24

D’Amico, G., Senatore, L., Zhang, P., & Zheng, H. 2021, JCAP, 05, 072

Di Valentino, E., & Bridle, S. 2018, Symm, 10, 585

Di Valentino, E., Anchordoqui, L. A., Akarsu, Ö., et al. 2021b, PhB, 131, 102605

Di Valentino, E., Melchiorri, A., Fantaye, Y., & Heavens, A. 2018, PhRvD, 98, 063508

Di Valentino, E., Melchiorri, A., & Silk, J. 2016, PhLB, 761, 242

Di Valentino, E., Mena, O., Pan, S., et al. 2021a, CQGra, 38, 153001

Dutcher, D., Balkenhol, L., Ade, P., et al. 2021, PhRvD, 104, 022003

Dutta, K., Ruchika, R. A., Sen, A. A., & Sheikh-Jabbari, M. M. 2020, GReGr, 52, 15

Efthatiou, G. 2020, arXiv:2007.10716

Farhang, M., & Khosravi, N. 2021, PhRvD, 103, 083523

Firouzjahi, H. 2022a, PhRvD, 106, 045015

Firouzjahi, H. 2022b, PhRvD, 106, 083510

**Figure 13.** Contour plots for Three-step models for $H_0$ vs. $\Omega_m$ based on “CMB+BAO+SN” data, comparing the effects of the signs of the fraction of dark energy in the second and third phases ($f_2, f_3$) with a fixed $f_1 > 0$. Note that the shaded area shows the measurement of $H_0$ done by the SH0ES team and its 1$\sigma$ error (Riess et al. 2022).
