Tests of CPT Invariance at Neutrino Factories

Samoil M. Bilenky‡ Martin Freund† Manfred Lindner‡ Tommy Ohlsson‡ and Walter Winter¶

Institut für Theoretische Physik, Physik-Department, Technische Universität München, James-Franck-Straße, 85748 Garching bei München, Germany
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We investigate possible tests of CPT invariance on the level of event rates at neutrino factories. We do not assume any specific model but phenomenological differences in the neutrino-antineutrino masses and mixing angles in a Lorentz invariance preserving context, such as it could be induced by physics beyond the Standard Model. We especially focus on the muon neutrino and antineutrino disappearance channels in order to obtain constraints on the neutrino-antineutrino mass and mixing angle differences; we found, for example, that the sensitivity $|m_3 - m_\bar{3}| \lesssim 1.9 \cdot 10^{-4} \text{ eV}$ could be achieved.

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I. INTRODUCTION

The CPT theorem [1] is one of the milestones of local quantum field theory. It is based on such general principles as Lorentz invariance, the connection of spin and statistics, and the locality and hermiticity of the Lagrangian. The SU(3) × SU(2) × U(1) Standard Model of Elementary Particle Physics (SM), for which the CPT theorem is valid, is in very good agreement with all existing experimental data. Beyond the SM, like in string theory models or in models involving extra dimensions, CPT invariance could be violated [2, 3]. Thus, the search for possible effects of CPT violation is connected to the search for physics beyond the SM. Many different tests of CPT invariance have been carried out. So far, no CPT violation has been found and rather strong bounds on the corresponding parameters have been obtained [4].

One of the basic consequences of the CPT theorem is the equality between the masses of particles and their corresponding antiparticles. A strong bound on a possible violation of CPT invariance has been obtained from the $K^0$-$\bar{K}^0$ system. This violation is characterized by the parameter

$$
\Delta \equiv \frac{\mathcal{H}_{K^0,\bar{K}^0} - \mathcal{H}_{\bar{K}^0,K^0}}{2(\lambda_L - \lambda_S)}, \tag{1}
$$

which can be related to measurable quantities [5]. In Eq. (1), $\lambda_{L,S} \equiv m_{L,S} - \frac{1}{2}\Gamma_{L,S}$, $m_{L,S}$ and $\Gamma_{L,S}$ are the masses and the total decay widths of the $K^0_L$ and $\bar{K}^0_S$ mesons, respectively, and $\mathcal{H}$ is the effective non-Hermitian Hamiltonian of the $K^0$-$\bar{K}^0$ system in the representation $|K^0\rangle$ and $|\bar{K}^0\rangle$, which are eigenstates of the Hamiltonian of strong and electromagnetic interactions. For the complex diagonal matrix elements, we have $\mathcal{H}_{K^0,\bar{K}^0} = m_{K^0} - i\Gamma_{K^0}$ and $\mathcal{H}_{K^0,\bar{K}^0} = m_{\bar{K}^0} - i\Gamma_{\bar{K}^0}$, where $m_{K^0}$, $\Gamma_{K^0}$ and $m_{\bar{K}^0}$ and $\Gamma_{\bar{K}^0}$ are the bare masses and the total decay widths of the $K^0$ and $\bar{K}^0$ mesons, respectively, with corrections due to weak interactions. The CPLEAR experiment obtained [6]

$$
|m_{K^0} - m_{\bar{K}^0}| = (1.5 \pm 2) \cdot 10^{-18} \text{ GeV},
$$

Using all relevant data on the $K^0$-$\bar{K}^0$ system, it follows that [7]

$$
\frac{|m_{K^0} - m_{\bar{K}^0}|}{m_{\text{average}}} \lesssim 10^{-18},
$$

where $m_{\text{average}} \equiv (m_{K^0} + m_{\bar{K}^0})/2$. Recently, also an upper bound on the mass difference between the $B^0_d$ and $\bar{B}^0_d$ mesons has been obtained [8]

$$
\frac{|m_{B^0_d} - m_{\bar{B}^0_d}|}{m_{B^0_d}} \lesssim 1.6 \cdot 10^{-14}.
$$

Here, we will consider possible CPT invariance tests that can be performed in future high-precision experiments with neutrinos from neutrino factories, which are now under active investigation [8] [9] [10] [11] [12] [13] [14] [15]. They have mainly been proposed to study neutrino oscillations in detail. In addition, in the framework of Lorentz non-invariant models, possible CPT invariance tests with neutrino experiments have been discussed (see, e.g., Refs. [16] [17]).

Compelling evidence for neutrino oscillations has been found by atmospheric [18] and solar [19] [20] [21] neutrino experiments. The following best-fit value for the atmospheric mass squared difference $\Delta m^2_{\text{atm}}$, has been obtained [18]:

$$
\Delta m^2_{\text{atm}} \approx 2.5 \cdot 10^{-3} \text{ eV}^2.
$$

From the global analysis of all solar neutrino data, several allowed regions in the neutrino oscillation parameter space have been found. For the preferred so-called large mixing angle (LMA) solution in Ref. [22], the solar mass squared difference has been determined to be

$$
\Delta m^2_{\odot} \approx 4.5 \cdot 10^{-5} \text{ eV}^2.
$$
Furthermore, there are at present indications for neutrino oscillations with an even larger mass squared difference, which were found by the LSND experiment. From the analysis of the data of the LSND experiment, the best-fit value of the neutrino mass squared difference was found.

\[ \Delta m_{\text{LSND}}^2 \simeq 0.24 \text{eV}^2 \]

The strongest kinematical bound on the absolute neutrino mass scale \( m_1 \) is obtained from the endpoint of the \( \beta \)-spectrum of \( ^3\text{H} \). The latest measurements yielded \( m_1 \lessapprox 2.2 \text{eV} \). From neutrinoless double-\( \beta \)-decay there exists also a strong bound \( |\langle m \rangle| = |\sum_i U_{ei}^2 m_i| \lessapprox (0.2 - 0.6) \text{eV} \) for Majorana masses (for an overview see, e.g., Ref. [21]). Here \( U_{ei} \) are matrix elements of the neutrino mixing matrix \( U \) and \( m_i \) are the masses of the neutrino mass eigenstates. Furthermore, somewhat weaker but similar bounds emerge from astrophysics and cosmology. It nevertheless follows from the existing neutrino data that neutrino masses are not equal to zero and that they are much smaller than the masses of all other fundamental fermions (leptons and quarks). From empirical lepton and quark mass patterns a hierarchical (or inverse hierarchical) mass pattern seems to be rather plausible.

It is a general belief that the smallness of the neutrino masses requires some new mechanism beyond the SM. The classical mechanism of neutrino mass generation is the see-saw mechanism, which connects the smallness of the neutrino masses with the violation of lepton numbers at an energy scale much higher than the electroweak scale. In this case, massive neutrinos have to be Majorana particles and the neutrino masses have to satisfy a hierarchy relation. The see-saw mechanism is based on local quantum field theory, and therefore, violation of CPT invariance cannot be expected.

Furthermore, it has recently been suggested that the smallness of the neutrino masses could have a natural explanation in models with large extra spatial dimensions. In such models, the smallness of the Dirac neutrino masses follows from the suppression of Yukawa interactions of the left-handed neutrino fields, localized on a three-dimensional brane, and the singlet right-handed neutrino fields propagating together with the gravitational field in a bulk. In models with \( n \) extra dimensions, the neutrino masses are proportional to

\[ \sqrt{\frac{1}{M^n V_n}} = \frac{M}{M_G} \simeq 10^{-16} \frac{M}{\text{TeV}}, \]

where \( V_n \) is the volume of the extra space, \( M_G \simeq 1.2 \cdot 10^{19} \text{GeV} \) is the Planck mass, and \( M \simeq 1 \text{TeV} \) is the Planck mass in the 4 + \( n \) dimensional space. Moreover, there are other approaches to the generation of small Dirac or Majorana neutrino masses in models with extra dimensions (see, e.g., Refs. [22, 23]). Since the symmetries of the SM are violated in the bulk, neutrino mass generation in extra dimension models is a plausible candidate for the violation of CPT invariance.

In order to accommodate all existing neutrino oscillation data, including the data of the LSND experiment, it is necessary to have three independent mass squared differences. Thus, we need to assume that there exist (at least) four massive mixed neutrinos, i.e., in addition to the three active flavors \( \nu_e, \nu_\mu, \) and \( \nu_\tau \) at least one sterile neutrino has to exist.

In Refs. [24, 25], it was assumed that CPT violation in the neutrino sector can be so strong that the mass spectra of neutrinos \( \nu_i \) and antineutrinos \( \bar{\nu}_i \) are completely different. In this case, it is possible to describe atmospheric, solar, and LSND neutrino data with a framework of three massive neutrinos and three massive antineutrinos (assuming that \( \Delta m_{\text{LSND}}^2 \) belongs to the antineutrino spectrum). Such an extreme picture can, in principle, be tested by the future MiniBooNE [26], KamLAND [27], and other similar neutrino experiments.

In Ref. [1], the effect of a term in the neutrino Hamiltonian violating CPT and Lorentz invariance has been considered and the \( \nu_\mu \rightarrow \nu_\mu \) and \( \bar{\nu}_\mu \rightarrow \bar{\nu}_\mu \) transition probabilities with the \( \nu_\mu \) and \( \bar{\nu}_\mu \) coming from neutrino factories have been calculated. It was demonstrated that in such a model the effects of CPT violation could be rather large in a wide range of the corresponding parameter values.

**II. BASIC FORMALISM**

In this paper, we will assume Lorentz invariance and consider possible violation of CPT invariance by the mechanism of neutrino mass generation. In the case of the usual neutrino mixing, we have

\[ \nu_{\alpha L} = \sum_i U_{\alpha i} \nu_i L, \]  

where \( U \) is a unitary mixing matrix and \( \nu_i \) are the neutrino fields (Dirac or Majorana) with masses \( m_i \). The neutrino flavor state \( |\nu_\alpha \rangle \) is given by

\[ |\nu_\alpha \rangle = \sum_i U_{\alpha i}^* |\nu_i; m_i, L\rangle, \]

where \( |\nu_i; m_i, L\rangle \) are the neutrino states with masses \( m_i \), negative helicity \( L \), 3-momentum \( p \), and energy

\[ E_i = \sqrt{m_i^2 + p^2} \simeq p + \frac{m_i^2}{2p} \]

in the ultra-relativistic limit. For the antineutrino flavor state \( |\bar{\nu}_\alpha \rangle \) we have

\[ |\bar{\nu}_\alpha \rangle = \sum_i U_{\bar{\alpha i}} |\bar{\nu}_i; m_i, R\rangle \]

in the case of Dirac neutrinos and

\[ |\bar{\nu}_\alpha \rangle = \sum_i U_{\bar{\alpha i}} |\bar{\nu}_i; m_i, R\rangle, \]
in the case of Majorana neutrinos. In these relations, \( |\nu_i; m_i, R\rangle \) and \( |\nu_i; m_i, R\rangle \) are the right-handed antineutrino and Majorana neutrino states, respectively, which also have the 3-momentum \( p \) and the energy \( E_\nu \).

Assuming the usual Lorentz invariant propagation of neutrino states for the neutrino and antineutrino transition probabilities in vacuum, we find the expressions

\[
P(\nu_\alpha \rightarrow \nu_{\alpha'}") = \left| \sum_i U_{\alpha'i} e^{-i\Delta m_i^2 \frac{E_\nu}{2} \alpha_i} U_{\alpha'i}^* \right|^2
\]

and

\[
P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}) = \left| \sum_i U_{\alpha'i} e^{-i\Delta m_i^2 \frac{E_\nu}{2} \alpha_i} \right|^2,
\]

which automatically satisfy the relation

\[
P(\nu_\alpha \rightarrow \nu_{\alpha'}) = P(\bar{\nu}_{\alpha'} \rightarrow \bar{\nu}_\alpha).
\]

In Eqs. (7) and (8), \( \Delta m_i^2 \equiv m_i^2 - m_j^2 \) is the mass squared difference, \( L \approx t \) is the distance between the source and detector, and \( E_\nu \) is the neutrino energy. Note that Eq. (9) is a consequence of CPT invariance inherent to standard neutrino mixing and oscillations.

If the generation mechanism of neutrino masses and mixings violates CPT invariance, then the relations for antineutrino flavor states will differ from Eqs. (7) and (8). In the case of massive Dirac neutrinos, the antineutrino masses \( m_i \) will be different from the neutrino masses \( m_i \), and the mixing matrices will, in general, not be connected by complex conjugation. Thus, for the antineutrino flavor states we have

\[
|\bar{\nu}_\alpha \rangle = \sum_i U_{\alpha'i} |\bar{\nu}_i; m_i, R\rangle.
\]

In the case of massive Majorana neutrinos, neutrinos and antineutrinos are identical. For the right-handed antineutrino flavor states, we therefore have

\[
|\bar{\nu}_\alpha \rangle = \sum_i U_{\alpha'i} |\nu_i; m_i, R\rangle.
\]

Further on, we will assume that there is no violation of Lorentz invariance in the propagation of massive neutrinos and antineutrinos.

### III. CPT Tests at Neutrino Factories

In this section, we will investigate the sensitivity of future high-precision neutrino oscillation experiments at neutrino factories to neutrino-antineutrino mass and mixing angle differences. Neutrino factories \([10, 11]\) will allow to investigate the phenomenon of neutrino oscillations, which has been observed by the atmospheric and solar neutrino experiments, with unprecedented accuracy. It will be possible to determine the leading neutrino oscillation parameters \( \Delta m_{23}^2 \) and \( \sin^2 2\theta_{23} \) governing the \( \nu_\mu \rightarrow \nu_e \) oscillations in the atmospheric region very well. Depending on their values, it will also be possible to limit or to measure the mixing angle \( \theta_{13} \) to search for the connected matter effects and to discriminate between a hierarchical neutrino mass spectrum and a mass spectrum with reversed hierarchy. In the most likely LMA case, the effects of CP violation in the lepton sector can be studied. Details of neutrino factory phenomenology can be found in Refs. \([10, 11, 12, 13, 14, 15]\). As we will show below, because of the high precision of neutrino factories, we can estimate the sensitivity of experiments to the presumably small violations of CPT invariance in the neutrino sector, being an unambiguous sign of new physics.

At neutrino factories neutrinos will be produced in muon decays \( \mu^+ \rightarrow e^+ \nu_\mu \bar{\nu}_e \) (or \( \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \)). The straightforward way to test CPT invariance at neutrino factories would be to check the appearance relation \( P(\nu_\mu \rightarrow \nu_\mu) = P(\nu_\mu \rightarrow \nu_\mu) \) (or \( P(\nu_\mu \rightarrow \nu_\mu) = P(\nu_\mu \rightarrow \nu_\mu) \)) with neutrinos from \( \mu^+ \) (\( \mu^- \)) decays. However, such tests would require to measure the sign of the charge of the produced lepton. The sign of a muon charge can be determined very reliably, but measuring the sign of an electron (or positron) charge is a rather challenging problem. The possibility to measure the electron (or positron) charge with moderate efficiency with liquid argon detectors would not be precise enough. Therefore, we consider a CPT invariance test in the \( \nu_\mu \) and \( \bar{\nu}_\mu \) disappearance channels by checking the equality

\[
P(\nu_\mu \rightarrow \nu_\mu) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu).
\]

The \( \nu_\mu \) and \( \bar{\nu}_\mu \) disappearance channels have several advantages:

1. The effect of neutrino oscillations in the atmospheric mass squared difference region is large.
2. The matter effects are small.
3. There is no relevant background from the \( \nu_e \)'s (\( \nu_e \)'s), which are accompanying the \( \nu_\mu \)'s (\( \bar{\nu}_e \)'s) in the decays of the \( \mu^- \) (\( \mu^- \))'s.
4. The event rates are high for obtaining good statistical information.

We will only consider the possible violation of CPT invariance in the \( \nu_\mu \rightarrow \nu_\mu \) and \( \bar{\nu}_\mu \rightarrow \bar{\nu}_\mu \) oscillations. If CPT invariance is violated, then these oscillations will be characterized by the leading parameters \( \Delta m_{23}^2 \), \( \sin^2 2\theta_{23} \), and \( \Delta m_{32}^2 \), \( \sin^2 2\theta_{23} \), respectively.

In Ref. \([11]\), a comprehensive study of the accuracy of the measurement of neutrino oscillation parameters in neutrino factory experiments was performed. Our calculations will be based on this study. Since matter effects give only small contributions to the \( \nu_\mu \) and \( \bar{\nu}_\mu \) survival probabilities, uncertainties in the Earth matter density profile are of little importance for the parameter measurements. In Ref. \([11]\), Fig. 3, the relative statistical
errors of the parameters $\delta \Delta m^2_{32}$ and $\delta \theta_{23}$, determined by a general analysis including correlations, are plotted as functions of the luminosity

$$L = 2 N_\mu m_{kt},$$

where $N_\mu$ is the number of stored muons per year and $m_{kt}$ is the mass of the detector in kilotons.

Violation of CPT invariance in neutrino oscillations can be characterized by the following parameters:

$$\delta \equiv |\Delta m^2_{32} - \bar{\Delta} m^2_{32}|, \quad \epsilon \equiv |\sin^2 2\theta_{23} - \sin^2 2\bar{\theta}_{23}|. \quad (12)$$

If the minimal neutrino mass $m_1$ and the CPT violating effects are small ($m_1 \ll \sqrt{\Delta m^2_{03}}, |m_3 - \bar{m}_3| \ll (m_3)_{\text{average}}$), then we find for the hierarchical neutrino mass spectrum or the spectrum with reversed hierarchy that

$$\delta \simeq 2a_{\text{CPT}} \Delta m^2_{32}, \quad (14)$$

where

$$a_{\text{CPT}} \equiv \frac{|m_3 - \bar{m}_3|}{(m_3)_{\text{average}}}. \quad (15)$$

is a dimensionless parameter which characterizes the violation of CPT invariance. We can also write $\epsilon$ as

$$\epsilon \simeq 2b_{\text{CPT}} \sin^2 2\theta_{23} \sqrt{1 - \sin^2 2\theta_{23}} \arcsin \sqrt{\sin^2 2\theta_{23}} = 2b_{\text{CPT}} \theta_{23} \sin \theta_{23}, \quad (16)$$

where

$$b_{\text{CPT}} \equiv \frac{\theta_{23} - \bar{\theta}_{23}}{(\theta_{23})_{\text{average}}}. \quad (17)$$

The experimental sensitivity to the possible CPT violation is given by the accuracy with which the parameters $a_{\text{CPT}}$ and/or $b_{\text{CPT}}$ can be measured. In order to estimate the sensitivity we will treat the neutrino and antineutrino channels as different experiments which are not combined to fit a common $\Delta m^2_{32}$ and $\theta_{23}$. In order to establish an effect we therefore need to compare the values of the parameters $a_{\text{CPT}}$ and $b_{\text{CPT}}$, which are describing the asymmetry between these two experiments, with the corresponding statistical errors of the neutrino oscillation parameters determined in Ref. [15], Fig. 3. Only if the mass squared or mixing angle difference between neutrinos and antineutrinos is larger than the respective relative statistical error $\delta \Delta m^2_{32}$ or $\delta \theta_{23}$ of the measurement of $\Delta m^2_{32}$ or $\theta_{23}$, CPT violation will be detectable on the respective confidence level of the statistical evaluation, i.e., the sensitivities $\delta a_{\text{CPT}}$ and $\delta b_{\text{CPT}}$ to the CPT violating parameters $a_{\text{CPT}}$ and $b_{\text{CPT}}$ are given by:

$$\delta a_{\text{CPT}} \sim \frac{\delta \Delta m^2_{32}}{2}, \quad (18a)$$

$$\delta b_{\text{CPT}} \sim \delta \theta_{23}, \quad (18b)$$

where $a_{\text{CPT}} \leq \delta a_{\text{CPT}}$ and $b_{\text{CPT}} \leq \delta b_{\text{CPT}}$. The factor of two in the first relation comes from the translation from mass squared differences to masses for a hierarchical (or inverse hierarchical) mass spectrum in Eq. (14). As an example, a statistical error of 7% in the determination of $\Delta m^2_{32}$ would correspond to a mass asymmetry sensitivity between neutrinos and antineutrinos of 3.5%. The sensitivities described by Eqs. (18a) and (18b) are plotted in Fig. 1, where the sensitivity $\delta a_{\text{CPT}}$ to the asymmetry

FIG. 1: The sensitivities $\delta a_{\text{CPT}}$ and $\delta b_{\text{CPT}}$ of an estimate of the asymmetries $a_{\text{CPT}}$ and $b_{\text{CPT}}$ at a neutrino factory as functions of the luminosity $L$. The solid curve refers to the mass asymmetry $a_{\text{CPT}}$ (hierarchical or inverse hierarchical mass spectrum only) and the dashed curve to the mixing angle asymmetry $b_{\text{CPT}}$. The underlying calculations in Ref. [15], Fig. 3, were performed with 50 GeV muon energy and baselines of 7000 km ($\theta_{23}$) and 3000 km ($\Delta m^2_{32}$).

IV. SUMMARY AND CONCLUSIONS

CPT is a fundamental symmetry preserved in any Lorentz invariant local quantum field theory. Especially, the SM is a CPT invariant theory. However, CPT
variance can be violated in models beyond the SM, like models with extra dimensions or string theory models. It is important to note that the expected effects of CPT violation depend on the assumed model. If the Planck mass is close to the TeV scale, such as it is for models with large extra dimensions, these effects could be observable in future experiments. We especially addressed the question of CPT violation by small neutrino mass or mixing angle differences between neutrinos and antineutrinos, which could, most plausibly, be generated by a mechanism beyond the SM. Furthermore, we investigated the sensitivity of future neutrino factory experiments to the presumably small mass and mixing angle differences in the $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance channels. Finally, we have shown that for the neutrino-antineutrino mass difference in a hierarchical (or inverse hierarchical) neutrino mass spectrum, the upper bound
$$|m_3 - \bar{m}_3| \lesssim 1.9 \cdot 10^{-4} \text{ eV}$$
can be obtained.

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[38] In Eq. (2), the mixing matrix elements $U_{\alpha i}$ are defined for neutrino fields $\nu_{\alpha L}$ in coordinate space. Neutrino flavor states $|\nu_{\alpha}\rangle$ (in momentum space) are, however, created from the vacuum by creation operators of definite momentum. The transition from coordinate space to momentum space explains why the $U_{\alpha i}^*$’s show up in Eq. (3).