Suppression of the $n - \bar{n}$ Oscillation in Nuclei

M.I. Krivoruchenko
Institute for Theoretical and Experimental Physics,
B. Cheremushkinskaya, 25, 117259 Moscow, Russia
(e-mail: mikhail@vxitep.itep.ru)

Abstract

The problem of nuclear decays occurring due to the neutron-antineutron oscillation is analyzed in the optical potential model and within the field-theoretic S-matrix approach. The result of the optical potential model for the nuclear decay width is rederived within the field-theoretic S-matrix approach. The $n-\bar{n}$ oscillation in nuclei is suppressed drastically. We discuss an example for the spin precession of an atom, in which analogous suppression takes place due to transparent physical reasons.

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The possible occurrence of the baryon-number-violating processes predicted by the grand unified theories (GUT’s) stimulated searches for the neutron-antineutron oscillation \(^1\textsuperscript{–}^8\). The limits for the neutron oscillation time in the vacuum, \(\tau = 1/\epsilon > 1\) yr, are derived from stability of nuclei \(^1\textsuperscript{–}^9\) and experiments with free neutrons \(^10\).

The n-\(\bar{n}\) oscillation is described by the Hamiltonian

\[
V = \frac{1}{2} \epsilon \int d\mathbf{x} \bar{\Psi}(\mathbf{x}) \sigma_1 \Psi(\mathbf{x})
\]

where \(\sigma_1\) is the Pauli matrix and

\[
\Psi(\mathbf{x}) = \begin{pmatrix} \psi_n(\mathbf{x}) \\ \psi_{\bar{n}}(\mathbf{x}) \end{pmatrix}
\]

is a doublet of the neutron and antineutron fields. The \(V\) is Hermitian, since \(\psi_{\bar{n}}(\mathbf{x}) = (\psi_n(\mathbf{x})^c = \psi_n(\mathbf{x})^+\) (in the Majorana representation).

For nuclei with atomic numbers \(A >> 1\), the decay width due to the n-\(\bar{n}\) oscillation takes the form \(^1\textsuperscript{–}^8\)

\[
\Gamma = \epsilon^2 N \frac{\Gamma_A}{(M - \bar{M})^2 + \Gamma_A^2/4}.
\]

Here \(N = A - Z\) is the number of neutrons in the nucleus, \(Z\) the number of protons, \(M\) the nuclear mass, and \(\bar{M}\) the average mass of the state obtained from the nucleus by the replacement of a neutron by an antineutron. The value \(\Gamma_A\) is a hadron-scale antineutron annihilation width of the intermediate state. The nuclear decay rate per neutron, \(\Gamma/N = \epsilon \xi\), contains an additional small parameter

\[
\xi = \frac{\epsilon \Gamma_A}{(M - \bar{M})^2 + \Gamma_A^2/4} < 10^{-31}
\]
as compared to the rate \(\epsilon\) of the vacuum n-\(\bar{n}\) oscillation.

In recent paper \(^11\) the finite-time S-matrix approach is developed for evaluation of the nuclear decay rate due to the neutron oscillation. It is claimed that the optical potential model is in disagreement with the field-theoretic S-matrix approach. We analyze more carefully the optical potential model, derive the Eq.(3) within the infinite- and finite-time S-matrix approaches, and clarify physical nature of the mechanism responsible for occurrence of the small parameter \(\xi\). The potential model is shown to be in agreement with the S-matrix theory.

We shall be interested in solutions to the evolution problem with the Hamiltonian of the form
\[ H = \epsilon \sigma_1 + U_n P_+ + U_\bar{n} P_- - i \Gamma(t)/2P_- \]  

(5)

where \( P_\pm = (1 \pm \sigma_3)/2 \) are projection operators to the neutron and antineutron states, \( \Gamma(t) \) is a time dependent width. The real potentials, \( U_n \) and \( U_\bar{n} \), describe the neutron and antineutron rescattering effects on the surrounding nucleons. The potential model for the in-medium neutron oscillation is described by the Hamiltonian (5) with constant width \( \Gamma(t) = \Gamma_A \). The time dependent width \( \Gamma(t) \) is considered for further applications (see below).

The Hamiltonian (5) can be split into two parts, \( H = H_0 + V \) with \( H_0 = U_n P_+ + U_\bar{n} P_- - i \Gamma(t)/2P_- \) and \( V = \epsilon \sigma_1 \). In the interaction representation, the potential \( V \) takes the form

\[ V_I(t) = U^{-1}(t,0)VU(t,0) \]  

(6)

with \( U(t,0) = \exp(-i \int H_0(t')dt') = e^{-i U_n t} P_+ + e^{-i U_\bar{n} t-\gamma(t)} P_- \) and \( \gamma(t) = \frac{1}{2} \int \Gamma(t')dt' \). The potential \( V_I(t) \) can be found to be

\[ V_I(t) = \epsilon(e^{-i \Delta U t-\gamma(t)} \sigma_+ + e^{+i \Delta U t+\gamma(t)} \sigma_-) \]  

(7)

where \( \Delta U = U_\bar{n} - U_n = \bar{M} - M \) and \( \sigma_\pm = (\sigma_1 \pm i \sigma_2)/2 \).

The transition amplitudes are determined by the S-matrix elements in the Heisenberg representation: \( S(t,0) = U(t,0)T \exp(-i \int V_I(t')dt') \). To the second order in the \( \epsilon \), we get

\[ S(t,0) = e^{-i U_n t}(P_+ + e^{-i \Delta U t-\gamma(t)} P_-) - \epsilon(F_-(t) \sigma_+ + e^{-i \Delta U t-\gamma(t)} F_+(t) \sigma_-) \]  

\[ -e^2(G_-(t) P_+ + e^{-i \Delta U t-\gamma(t)} G_+(t) P_-) + ...) \]  

(8)

where

\[ F_\pm(t) = \int_0^t e^{\pm i \Delta U t' \pm \gamma(t')} dt', \]  

\[ G_\pm(t) = \int_0^t e^{\pm i \Delta U t' \pm \gamma(t')} F_\pm(t) dt'. \]  

(9)

The disappearance of the neutrons is described by a projection \( S(t,0) P_+ \) of the S-matrix to the neutron component of the initial wavefunction (2). Using Eqs.(8) and (9), we obtain for \( \Gamma(t) = \Gamma_A \) and \( t >> 1/\Gamma_A \)

\[ S(t,0) P_+ = e^{-i U_n t}(P_+ - i \epsilon/\chi \sigma_- - \epsilon^2 t/\chi P_- + ...) \]  

(10)
where $\chi = \Gamma_A/2 + i(M - M)$. The decay probability is determined by absolute square of the S-matrix element $|\langle \psi_n | S(t, 0) | \psi_n \rangle|^2 = 1 - \epsilon \xi t + ..., where the use is made of the relation $\epsilon \text{Re}\{2/\chi\} = \xi$. Comparison with the decay law for quasistationary states, $|\langle Q | S(t, 0) | Q \rangle|^2 = \exp(-\Gamma_Q t) = 1 - \Gamma_Q t + ...$ at $t << 1/\Gamma_Q$ allows to fix the nuclear decay width per neutron, $\Gamma/N = \epsilon \xi$. In this way we reproduce Eq.(3).

The potential model operating with the non-Hermitian Hamiltonian (5) gives the phenomenological description for the nuclear decays. The results of the potential model can be justified within the field-theoretic S-matrix approach.

Let $|N, Z\rangle$ be a discrete energy eigenstate of the strong Hamiltonian $H_0$, associated to a nucleus of mass $M$, charge $Z$, atomic number $A = N + Z$, and zero momentum. With respect to the total Hamiltonian $H = H_0 + V$ (in the field-theoretic S-matrix approach, the $H_0$ and $V$ are Hermitian), the state $|N, Z\rangle$ is no longer an energy eigenstate. In what follows, the center-of-mass variables are factored out, so the normalization condition reads simply $\langle N, Z | N, Z \rangle = 1$. The decay probability of the state $|N, Z\rangle$ to a channel $X$ in the first order to the $V$ is determined by the matrix element

$$\langle X | V | N, Z \rangle = \epsilon \sqrt{N} g_X A(M_X).$$

(11)

The state $|X\rangle$ consists of $N - 2$ neutrons, $Z$ protons, and a number of mesons. It belongs to the continuum energy spectrum of the $H_0$. The factor $\sqrt{N}$ takes into account structure of the operator $V$. The function $A(M_X)$ has the Lorentz form (cf. Ref.12, Ch.27)

$$A^2(M_X) = \frac{\Gamma_A}{(M - M)^2 + \Gamma_A^2/4}.$$  

(12)

The state $V|N, Z\rangle$ has indefinite energy and definite (zero) momentum, since $[H_0, V] \neq 0$ and $[P, V] = 0$. The value $\bar{M}$ is the average mass of this state. The scattering states $|X\rangle$ are grouped around the invariant mass $M_X \approx \bar{M}$. The function $A(M_X)$ reflects spread in the $M_X$ of the scattering states $X$ entering a superposition to form the localized wave packet $V|N, Z\rangle$. The state $V|N, Z\rangle$ decays with a strong width $\Gamma_A$. The value $g_X$ is a coupling constant to the channel $X$. This is a smooth function of the $M_X$.

In the nonrelativistic approximation, the state $V|N, Z\rangle$ consists of the fixed number of particles. Its normalization condition reads $\langle N, Z | V^2 | N, Z \rangle = \epsilon^2 N$. Inserting in this equation complete set of the states $|X\rangle$, we obtain

$$\int \frac{dM'}{2\pi} A^2(M') e(M') = 1$$

(13)
where

\[
ed(M) = \sum_X g_X^2 (2\pi)^4 \delta^4(P_X - P)d\tau_X, \tag{14}\]

\[P' = (M', 0), \text{ and } d\tau_X \text{ is element of the phase space.}\]

The expression for the nuclear width,

\[
\Gamma = \sum_X \int |<X|V|N, Z>|^2 (2\pi)^4 \delta^4(P_X - P)d\tau_X \tag{15}\]

where \(P = (M, 0)\) with the use of Eqs.(11) and (14) takes the form \(\Gamma = \epsilon^2 N A^2(M) e(M)\). For heavy nuclei \(\Gamma_A << \bar{M}\) and \(\bar{M} - M << \bar{M}\). Substituting in Eq.(13) \(A^2(M_X) \approx (2\pi)\delta(M_X - \bar{M})\) one obtains \(e(M) \approx e(\bar{M}) \approx 1\). In agreement with Eq.(3), we get \(\Gamma \approx \epsilon^2 NA^2(M)\).

It is instructive to analyze the decay process in the time-dependent S-matrix approach. In the interaction representation with respect to the strong Hamiltonian \(H_0\), the state \(|N, Z >\) evolves according to the law \(|N, Z, t > = S_I(t, 0)|N, Z, 0 >\) where \(|N, Z, 0 > = |N, Z >\) and \(S_I(t, 0) = T\exp(-i \int V_I(t')dt')\). The value \(V_I\) is defined by Eq.(6) with \(U(t, 0) = \exp(-iH_0t)\). To the second order in the \(V\), the decay amplitude takes the form

\[
<N, Z, 0|N, Z, t> = 1 - \int_0^t dt_1 \int_0^{t_1} dt_2 <N, Z|V e^{-i(H_0-M)t_1}V|N, Z >. \tag{16}\]

Inserting complete set of the states \(X\) to the matrix element in the right hand side of this expression, we get

\[
<N, Z|V e^{-i(H_0-M)t}V|N, Z> = \epsilon^2 N \int \frac{dM'}{2\pi} A^2(M') e^{-i(M' - M)t} e(M'). \tag{17}\]

Here the use is made of Eqs.(11) and (14). The values of the \(M'\) are grouped around the \(\bar{M}\) by the Lorentz function \(A^2(M')\). Setting \(e(M') \approx e(\bar{M}) \approx 1\) and performing the integration, we obtain

\[
<N, Z|V e^{-i(H_0-M)t}V|N, Z > \approx \epsilon^2 N e^{-\Gamma_A/2 + i(\bar{M} - M)t}. \tag{18}\]

The decay amplitude (16) at \(t >> 1/\Gamma_A\) takes the form

\[
<N, Z, 0|N, Z, t > \approx 1 - \epsilon^2 N t/\chi. \tag{19}\]
The term $2Re\{\varepsilon^2 N/\chi\} = \varepsilon N\xi$ appearing in the absolute square of this amplitude can be recognized as the nuclear decay width $\Gamma$. We derived therefore Eq.(3).

The possibility of reproducing the potential model results within the field-theoretic S-matrix approach has recently been questioned\textsuperscript{11}. We show that the results of Ref.11 are erroneous due to a misinterpretation of the optical theorem.

The S-matrix determined by the Hamiltonian $H = H_0 + V$ can be expanded into a sum of the unit operator and the T-matrix: $S = 1 - iT$. The unitarity relation $SS^+ = 1$ implies $2ImT = -TT^+$.

The complete orthogonal basis $|X\rangle = \{|X_b\rangle, |X_{sc}\rangle\}$ can be constructed from eigenstates of the Hamiltonian $H_0$. The states $|X_b\rangle$ are the finite-norm bound states (nuclei), while the states $|X_{sc}\rangle$ are the infinite-norm scattering ones. The normalization conditions are $<X_b|X_b> = \delta_{bb}', <X_{sc}|Y_{sc}> = \delta(X - Y)$. The expansion of the unit takes the form

$$1 = \sum_b |X_b><X_b| + \int d\mu(X_{sc})|X_{sc}><X_{sc}|. \quad (20)$$

The value $<X_b|TT^+|X_b>$ with the help of Eq.(20) can be represented in the form

$$<X_b|TT^+|X_b> = W_b + |<X_b|(-iT)|X_b>|^2 \quad (21)$$

where

$$W_b = \sum_{b' \neq b} |<X_b|S|X_{b'}>|^2 + \int d\mu(X_{sc})|<X_b|S|X_{sc}>|^2. \quad (22)$$

Here the orthogonality is taken into account: $<X_b|(-iT)|X_{b'}> = <X_b|S|X_{b'}>$ for $b' \neq b$ and $<X_b|(-iT)|X_{sc}>=<X_b|S|X_{sc}>$. The value $W_b$ is the total decay probability of the state $X_b$. The second term in Eq.(21) can be transformed as $|<X_b|(-iT)|X_b>|^2 = |<X_b|S|X_{b}>|^2 - 2Re <X_b|S|X_b> + 1$. The value $|<X_b|S|X_{b}>|^2 = 1 - W_b$ is the probability of finding the system t sec after its preparation in the initial state $|X_b>$. Eq.(21) becomes

$$<X_b|TT^+|X_b> = 2(1 - Re <X_b|S|X_b>). \quad (23)$$

For the finite-norm states, the quantity $<X_b|TT^+|X_b>$ has no probability meaning, along with the imaginary part of the T-matrix element $-2Im <X_b|T|X_b>$.

The value $<X_{sc}|TT^+|X_{sc}>$ with the help of Eq.(20) is represented by

$$<X_{sc}|TT^+|X_{sc}> = W_{sc} \quad (24)$$
where
\[ W_{sc} = \sum_b | <X_{sc}|S|Y_b>|^2 + \int d\mu(Y_{sc}) | <X_{sc}|S|Y_{sc}>|^2. \] (25)

The term analogous to the second term in Eq.(21) does not occur because of zero measure of the infinite-norm scattering states. The value \(W_{sc}\) is the total transition probability to the states \(Y \neq X_{sc}\). Eq.(24) is the optical theorem.

The excited nuclei correspond to quasistationary states. These states represent localized wave packets and have finite norm like the bound states. One can show that for quasistationary states the relation (23) holds true with the substitution \(|X_b> \rightarrow |Q>\). There is a deep analogy between properties of bound states and quasistationary states (see Ref.12, Ch.34). The parallel can also be drawn towards absence of the probability meaning for imaginary parts of the T-matrix diagonal amplitudes. The optical theorem takes place for the infinite-norm scattering states only. Notice that nuclei are bound states with respect to the strong Hamiltonian \(H_0\) and quasistationary ones with respect to the total Hamiltonian \(H\) including the baryon-number-violating part.

It is worthwhile to make three remarks. (i) The decay amplitude of a quasistationary state is given by the S-matrix element \(<Q|S|Q>\). Up to a phase factor, \(<Q|S|Q> = e^{-\Gamma_Q t/2}\), in which case \(<Q|TT^+|Q> = 2(1 - e^{-\Gamma_Q t/2})\). In Ref.10, the value \(<Q|TT^+|Q>\) is misinterpreted as the decay probability and an expression \(<Q|TT^+|Q> = 1 - e^{-\Gamma_Q t}\) is erroneously suggested. It is equivalent to the decay law \(<Q|S|Q> = (1 + e^{-\Gamma_Q t})/2\) which has nothing in common with the time behavior of the decay amplitude for quasistationary states. In particular, at \(t \rightarrow \infty\) the quarter of the state \(|Q>\) survives. So, conclusions following Eq.(16) of Ref.11 are erroneous. (ii) It is claimed also that the infinite-time field-theoretic S-matrix approach is infrared divergent. In Fig.1 of Ref.11, the neutron line of the Green’s function belongs to the nuclear Bethe-Salpeter wave function, and so the neutron is out of the mass shell. The decay amplitude therefore is not singular. Indeed, we showed that the infinite-time S-matrix approach (Eqs.(11) to (15)) reproduces the result of the potential model (Eqs.(5) to (10)) and the result of the finite-time S-matrix approach (Eqs.(16) to (19)). (iii) The exponential decay law for quasistationary states takes place at \(1/E < t < \log(E/\Gamma)/\Gamma\) where \(E\) is energy release in the process (Ref.12, Ch.30). In our case \(E \approx 2\ GeV\) and the value \(\Gamma\) is given by Eq.(3). The time interval for the exponential decay can be evaluated to be \(10^{-25} < t < 10^{38}\ sec\) for \(\tau = 1\ yr, \Gamma_A = 100\ MeV,\) and \(N = 100\). At an interval \(1/E << t << 1/\Gamma,\) the decay probability is linear in time \((e^{-\Gamma_Q t} = 1-\Gamma_Q t+...).\)

The quadratic time dependence11 of the nuclear decay rate is in contradiction with the exponential decay law for quasistationary states.
To clarify physical nature of the decay rate suppression, we wish to discuss gedanken-experiment with deuterium ($^2H$) atoms in the spin $j = 1/2$ state, which exhibits transparent mechanisms for the spin-precession-rate suppression.

Let us consider sequence of the Stern-Gerlach magnets (SG) which measure the spin-$z$ projection and merge the outgoing atomic beams together (for a description such magnets see Ref.13). The SG magnets are placed along the $x$ axis at equal distances $l_0$ one from another, so that the atoms propagating along the straight line pass successively all SG magnets. The atoms entering the first magnet are polarized along the $z$ axis. The existence of the magnetic field $\mathbf{H} = (H_x, 0, 0)$ with a small component $H_x$ along the atomic beam (version I) is assumed outside the magnets. The atom spin precession is described by the Hamiltonian $H = -\mu \sigma H$ with $\mu$ being the atomic magnetic moment ($\mu \approx \mu_B/3$ where $\mu_B$ is the Bohr magneton). The atom has spin up at a time $t$ with probability

$$w_I(t) = \cos^2(\omega_x t)$$

(26)

where $\omega_x = -\mu H_x$. The neutron oscillation is described by the same equation with the substitution $\xi \leftrightarrow \omega_x$.

In the next experiment, the existence of the magnetic field $\mathbf{H} = (H_x, 0, H_z)$ with $H_x << H_z$ (version II) is assumed outside the magnets. The SG magnets are supplied also by detectors for the spin-down atoms. The time evolution of the spin-up atoms with the switched off detectors is given by equation

$$w_{II}(t) = 1 - \frac{\omega_x^2}{\omega^2} \sin^2(\omega t)$$

(27)

where $\omega = -\mu \mathbf{H}, \omega = |\omega|$. If detectors are switched on, the evolution law (27) is no longer valid. The operating detector installed at the k-th SG magnet can produce no signal when a deuterium atom passes through it. In such a case, we know with certainty that the atom has its spin up. The passage of the atom is accompanied by the wavefunction collapse. Afterwards, the atomic wavefunction up to a normalization factor coincides with the initial one, and the oscillation process starts from the beginning. The probability that the first $N$ detectors all give no signals is given by

$$w_{II}(t)_{on} = \left(1 - \frac{\omega_x^2}{\omega^2} \sin^2(\omega t)\right)^N$$

(28)

where $N = t/t_0 >> 1$, $t_0 = l_0/v$ is the time needed to pass from one magnet to the next, and $v$ is the atom velocity. The time evolution of atoms in between the SG magnets is described quantum mechanically, whereas at $t >> t_0$ due to the measurement
procedure, the classical rules should be applied for calculation of the probability. Each instant of time $t = kt_0$ the atom is either detected with a probability $1 - w_{II}(t_0)$ or continue its propagation with a probability $w_{II}(t_0)$. To derive Eq.(28), one should take $N$ times the product of the elementary probabilities $w_{II}(t_0)$. The value $w_{II}(t_0)$ is close to unity, since $\omega_x << \omega$, so the probability to survive in the spin-up state follows the exponential decay law

$$w_{II}(t)_{on} \approx \exp(-\omega_x \xi' t)$$

(29)

where $\omega_x > 0$ is assumed and

$$\xi' = \omega_x t_0 \frac{\sin^2(\omega_z t_0)}{\omega_z t_0^2}$$

(30)
to the first order in the $\omega_x$.

In the first experiment, the value $\omega_x$ determines precession rate of the spin-up atoms. In the second experiment, new dimensionless parameter $\xi' << 1$ occurs. The decay rate for the spin-up atoms turns out to be $\omega_x \xi' << \omega_x$.

The apparent analogy between the neutron oscillation and the atomic spin precession is summarized below.

| neutrons | $\leftrightarrow$ | spin-up atoms |
| antineutrons | $\leftrightarrow$ | spin-down atoms |
| vacuum n-\bar{n} oscillation | $\leftrightarrow$ | atom spin precession (v.I) |
| vacuum oscillation rate $\epsilon$ of neutrons | $\leftrightarrow$ | spin precession rate $\omega_x$ of atoms (v. I) |
| $\Delta U = M - M$ | $\leftrightarrow$ | $\omega_z$ (v. II) |
| $\bar{n}$ length free path in nuclear matter | $\leftrightarrow$ | distance $l_0$ between SG magnets (v.II) |
| antineutron annihilation | $\leftrightarrow$ | detecting spin-down atoms (v.II) |
| suppression parameter $\xi$ | $\leftrightarrow$ | suppression parameter $\xi'$ (v.II) |
| nuclear decay rate per neutron $\epsilon \xi$ | $\leftrightarrow$ | atom spin-up decay rate $\omega_x \xi'$ (v.II) |

Reasons, according to which physical results in these two problems are identical, can be explained by considering the atomic spin dynamics described by the Hamiltonian (5) with the time dependent width

$$\Gamma(t) = \sum_k \alpha \delta(t - kt_0).$$

(31)

Here the k's are integer numbers. In the intervals $kt_0 < t < (k + 1)t_0$ the atom is out of the SG magnets and its spin precession is governed by the magnetic field
\[ H = (H_x, 0, H_z) \]. At the time \( t = kt_0 \) the act of the measurement takes place and the spin-down component of the wavefunction vanishes. In order to set the lower component equal to zero, we must pass to the limit \( \alpha \to \infty \). The upper component at \( t = kt_0 \) remains continuous. The probability to find the deuterium atom in the spin-up state can be computed according to the usual rule as the modulus squared of the upper component of the atomic wavefunction. Using such a prescription, the results of Eqs.(28) and (29) can easily be reproduced.

One can show that the S-matrix projection \( S(t, 0)P_+ \) to the spin-up initial state, being averaged over an interval \( (t - T, t + T) \) at \( T >> t_0 \),

\[
S(t, 0)P_+ = \frac{1}{2T} \int_{-T}^{T} dt' S(t', 0)
\]  

(32)

coincides with the S-matrix projection for the constant width \( \Gamma_A \). It means that projection \( S(t, 0)P_+ \) responsible for disappearance of atoms from the beam can be reproduced in a model identical to the potential model for description of the n-\( \bar{n} \) oscillation in nuclei.

In the limit \( \alpha \to \infty \), the functions entering Eq.(8) can easily be computed. After averaging over the time, we get the following expression

\[
S(t, 0)P_+ = e^{i\omega_z t}(P_+ - i\omega_x/\chi' - \omega_x^2 t/\chi' P_+ + ...)
\]  

(33)

where \( 1/\chi' = (\sin^2(\omega_z t_0) - i((\omega_z t_0) - \sin(\omega_z t_0)\cos(\omega_z t_0)))/(2\omega_z^2 t_0) \). Notice that \( \omega_x \text{Re} \{2/\chi'\} = \xi' \). Eq.(33) is identical to Eq.(10) for \( \epsilon = \omega_x \) and \( \chi = \chi' \). Given that \( \Gamma_A \) and \( \Delta U \) are known, one can find \( t_0 \) and \( \omega_z \). To the second order in the \( \omega_x \) (\( \epsilon \)), the evolution laws in these two problems are identical in the average sense.

The qualitative explanation for the neutron-decay-rate suppression can be given in the following way. In the case \( \Delta U >> \Gamma_A \) one can start from Eq.(31) that gives average admixture \( \approx \omega_z^2/\omega_x^2 \) of the spin-down atoms in the beam. The rate of disappearance of the atoms can be estimated to be \( \approx \omega_x^2/(\omega_z^2 t_0) << \omega_x \). The equation \( \chi = \chi' \) gives \( \omega_z = \Delta U/2, \ t_0 \approx 1/\Gamma_A >> 2/\Delta U \). Using the above analogy, we derive the corresponding neutron decay rate \( \approx \epsilon^2 \Gamma_A/\Delta U^2 = \epsilon \xi << \epsilon \).

The inequality \( \Delta U \neq 0 \) is not unique reason for suppression of the decay rate. Let \( \Delta U = 0 \). The equation \( \chi = \chi' \) gives \( \omega_z = 0 \) and \( t_0 = 4/\Gamma_A \). The decay rate of the spin-up atoms becomes \( \omega_x \xi' = \omega_x^2 t_0 << \omega_x \). The detection of the spin-down atoms suppresses the spin precession process by itself. The neutron propagation in nuclei is accompanied by interactions with surrounding nucleons. The meson exchanges take place each \( t_0 = 4/\Gamma_A \approx 10^{-24} \text{ sec} \). They can be interpreted as acts of measurements...
of the lower antineutron component of the wavefunction (2). These interactions are followed by collapse of the wavefunction to the pure neutron state. The wavefunction collapse in turn reduces the nuclear decay rate down to $4\varepsilon^2/\Gamma_A = \epsilon \xi \ll \epsilon$.

The analogous suppression mechanism exists for ultracold neutrons in a trap. The absorption probability for antineutrons in collisions with the copper boundary is estimated to be $\approx 1/5$ for the tangent velocities $\approx 4\, \text{m/sec}$. The suppression of the precession rate occurs due to the wavefunction collapse that takes place with certainty under those conditions one time per $\approx 5$ neutron collisions with the trap boundary. The suppression factor is about $5\epsilon t_0 < 10^{-6}$, where $t_0$ ($\approx \text{seconds}$) is a typical time between two collisions.

In conclusion, the field-theoretic S-matrix approach allows to justify the results of the potential model for heavy nuclei. The suppression of the nuclear decay rate is the result of the energy gap $\Delta U$ and the antineutron annihilation width $\Gamma_A$. The suppression of the neutron oscillation process is not specific for nuclear physics. In the SG-type experiment described above, the atom spin precession is suppressed due to the similar physical reasons. For light nuclei, the procedure of solving the inhomogeneous Schrödinger equation for antineutron wavefunction in the optical potential is more adequate than the direct use of Eq.(3). Being applied for heavy nuclei, such a procedure reproduces Eq.(3). The models account to a various degree of accuracy for dynamics of the antineutron creation and annihilation in nuclei. Further refinement of parameters of the antineutron optical potential entering Eq.(3) would be desirable. It is unlikely, however, that the corresponding numerical estimates are changed more than ten by an order of magnitude.

After submitting this work our attention has been called to a recent paper [14] in which results of Ref.11 obtained with the use of the field-theoretic methods are criticized on the basis the optical potential model. The authors give two new derivations for expression for the nuclear decay width within the optical potential model (Eqs.(1) to (5) and Eqs.(11) to (14)). We argued that the imaginary part of the T-matrix has no probability meaning for bound and quasistationary states. It is not clear then why the value $W_n$ in Eq.(3) of Ref.14 is not a probability whereas the value $W$ in Eq.(5) does. These values both are defined, respectively, as imaginary parts of strong and baryon-number violating (GUT’s) T-matrix amplitudes. We discussed the T-matrix properties for Hermitian Hamiltonians. The optical model Hamiltonian $H = -i\Gamma/2$ used in Ref.14 is not Hermitian, and so it is necessary to reanalyze the problem on slightly more general grounds.

Notice that Eq.(16) of Ref.11 which represents a finite-time analog of Eq.(4) of Ref.3 is correct and it remains valid for non-Hermitian Hamiltonians also, since hermiticity
of the Hamiltonian is not assumed in deriving this equation. It is allowed therefore to substitute here $H = -i\Gamma/2$. In this way, purely imaginary T-matrix occurs: $T = -iW/2$ with $W = \epsilon\xi t$ for $t >> 1/\Gamma_A$. The S-matrix becomes $S = 1 - iT = 1 - W/2$. The probability of finding the system in the initial state $t$ sec after its preparation becomes $|S|^2 = 1 + 2\text{Re}(-iT) + TT^+ \approx 1 + 2ImT = 1 - W$. The quantity $W = -2ImT$ thus really receives, to the first order in $T = O(t)$, a meaning of the decay probability. We can expand the value $W_n$ in Eq.(3) of Ref.14 in power series in $t(= t_{\alpha} - t_{\beta})$ and verify, in agreement with that conclusion, that the lowest order term gives the decay probability $\Gamma_A t << 1$. The reason stems from the fact of neglecting the small term $TT^+ = O(t^2)$ in the expression for the $|S|^2$. At high times, however, $W_n \to 2$. This is no longer a surprise, since $TT^+$ becomes high as $t$ increases, and so it should be taken into account in the $|S|^2$. The value $W$ defined by Eq.(1) in Ref.14 is calculated to the first order in $t$. This is a reason for it gives the correct result for the probability. The next order term in $t$, however, should already be wrong, if the value $TT^+$ is disregarded. The instability of quasistationary states implies $<Q|S|Q> \to 0$ when $t$ increases. Our Eq.(23) gives then $W_n \to 2$ at $t >> 1/\Gamma_A$ and $W \to 2$ at $t >> 1/\Gamma$.

Therefore, the value $W_n$ in Eq.(3) of Ref.14 is not a probability, when it is computed to all orders in $t$. To the first order in $t$ it acquires the probability meaning. The value $W$ in Eq.(5) is not a probability also. It coincides with the probability to the first order in $t$ only.

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