The Edge Currents in IQHE

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Abstract
It is shown that an observed length in the potential drops across IQHE samples is a universal length for a given magnetic field strength which has the magnitude equal to the reciprocal magnitude of magnetic length and which results from the quantum mechanical uncertainty relation in presence of magnetic field. The analytic solution of Ohm’s equation for the potential in Corbino sample in IQHE is also given.
We recently showed that the microscopic theory of IQHE \[1\] can be given by the canonical quantization of a semi-classical theory of the usual ”classical” Hall-effect CHE \[2\].

The action functional for this is the semi-classical Schroedinger-Chern-Simons one for a 2-D non-interacting carrier system with the usual minimal electromagnetic coupling on a 2+1-dimensional manifold $M = \Sigma \times \mathbb{R}$ with spatial boundary which results in the Ohm’s equations as the equations of motion of the coupled electromagnetic potential. We showed also that the constraints of the theory under typical conditions of IQHE \[3\], i. e. with small carrier concentration and higher magnetic field, forces the coupled electromagnetic potential to be an almost pure gauge potential and it forces the potential to exist only close to the boundary of $\Sigma$ \[2\]. Accordingly, the edge currents are the preferred currents under the IQHE conditions, in view of the mentioned constraints of the theory.

Here we show that the recent results on the potential drops across IQHE samples near the edges \[4\] follow the universal uncertainty relations of quantum mechanics as it should be to expect in view of the universality of the QHE.

Let us first explain from the more fundamental point of view of quantum mechanics.

For charged systems, e. g. electrons in magnetic fields, the energy uncertainty is given by the minimum amount of the energy, i. e. the ground state energy. This amount of energy is proportional to the applied magnetic field strength. On the other hand, an energy uncertainty is correlated with a position uncertainty for the circulating electrons in magnetic field. Thus, quantum mechanically in presence of magnetic fields there is always an uncertainty of position of the electronic currents which is related with the width of the electron orbit. Therefore, if we consider the uncertainty of momentum equal to $(2m\Delta E)^{\frac{1}{2}}$ with $\Delta E = E_{n+1} - E_n = \frac{\hbar \omega_c}{2}$ and $\omega_c = \frac{eB}{M_e}$, then the mentioned uncertainty is given by $\Delta X = (\frac{\hbar}{eB})^{\frac{1}{2}}$ which is the magnetic length $l_B$. Since, the edge current is defined as the current which flows, in the ideal case, close to the edge within the length scale of the magnetic length \[3\]. This means that one should expect that according to Ohm’s equation for QHE, in the ideal case, also the potential distribution on the sample should be close to the boundary of sample within a distance which is proportional to the magnetic length. In view of the relations between the magnetic field strength $B$, magnetic length and the global density of electrons $n$ with the filling factor $\nu$, i. e. $l_B^2 = \frac{\hbar}{eB} = \frac{\nu}{2\pi n}$, it is obvious that a variation of only one of these factors changes the magnetic length and so it changes.
also the current position and the potential distribution on the sample. On the other hand, obviously if \( B \) or \( \nu/n \) remain the same for various IQHE samples the magnetic length should be invariant for all these samples under the IQHE conditions independent of their geometries and other factors. This is the quantum theoretical basics of what is observed in the mentioned experiments for the potential drops \[4\], where the authors report that they observed potential drops across the IQHE-samples over a length of 100\( \mu m \) from the edge of samples. We show that this length which has the magnitude of \(|l_B^{-1}|\) for the given data in Ref. \[4\] is indeed a universal quantity for a given \( B \) or for a given \( \nu/n \) \[5\].

There is however one basic point with respect to the electromagnetic potential distribution which must be taken into account, namely that a potential is itself no observable in view of its gauge dependence. The observables related with the potential or those related with its field strength are phase angle given by the circle integral of potential and the surface integral of field strength which are observable by the quantum mechanical interference patterns. Equivalently, a constant potential multiplied by a proper length, e. g. by the circumference of mentioned line integral is also observable. For example according to the definition of magnetic length \( l_B^2 = \frac{\hbar}{eB} \) we have (see also below):

\[
l_B^2 B = l_B A = \frac{\hbar}{e} ,
\]

which is equivalent to the definition of magnetic flux quantum through \( \oint \oint B = \oint A = \frac{\hbar}{e} \), where \( A = B \cdot \mathbf{e}_{mn} \) is the relevant component of electromagnetic gauge potential in the \( A_m = B \cdot \mathbf{e}_{mn} \) gauge.

Moreover as a general result let us mention that, if one considers the relation (3) in form \( 2\pi l_B A = 2\pi l_B^2 B = \frac{\hbar}{e} \) as given according to the flux quantization for electrons moving in the IQHE edge current on a ring with radius and width both equal to \( l_B \). Then one obtains with the given \( l_B \) according to the data in Ref. \[4\] for \( A = \frac{\hbar}{e} l_B^{-1} \) a value about 100\( \mu m \) for \( A \), which is the mentioned observed length for potential drops \[4\] \[6\].

This result show that in view of the definition of magnetic length the measured value of 100\( \mu m \) is a fundamental value for IQHE experiments on those samples \[4\] independent of other sample parameters.
but the filling factor is $\nu = 4$, one observed a potential penetration of $\approx 70\mu m$ is in good agreement with our theoretical results. Since for the $\nu = 4$ filling factor one obtains according the given data in Ref. [7] a magnetic length $l_B' \approx 1.4l_B \approx 1.4 \cdot 10^{-2} \mu m$, where $l_B \approx 10^{-2} \mu m$ is the magnetic length of samples in Ref. [4]. Thus, the theoretical value of $A = \frac{\hbar}{e}(l_B')^{-1}$ becomes $\approx 70\mu m$ which is indeed the measured value according to the Ref. [6] [7].

This circumstance explains why one observes such a distance from the boundary or edges in experiments concerning the IQHE [4] [7].

Therefore, one should claim that the measured penetration length of electromagnetic potential on IQHE samples should depend, according to the theoretical value of $A = \frac{\hbar}{e}(l_B)^{-1}$, only on the related value of $l_B^{-1}$.

However, we shall mention further that the generality of this result requires a more subtle and theoretically more fundamental origin for this fact. Such an origin should be given, as it is mentioned already, by the quantum mechanical uncertainty-principle and relation, where a charged particle in presence of magnetic fields acquires a position uncertainty $\Delta X = l_B$. Thus, considering $\Delta P = \Delta A = eA$, we are given under quantum mechanical conditions which apply to the QHE, the uncertainty relation $eA \cdot l_B = \hbar$ which is the same relation as already mentioned. Here $\frac{\hbar}{e}$ plays the same role in the quantum electrodynamical uncertainty as that played by $\hbar$ in the quantum mechanical uncertainty.

Therefore, in view of the fact that the value of $\frac{\hbar}{e}$ is a fixed quantity $\hbar'$, the value of potential (drop) under IQHE conditions is always given by $A = \frac{\hbar'}{l_B}$, no matter what other relevant quantities are.

Thus, in any IQHE sample one should measure for the potential drops on the edges the related value of $A = \frac{\hbar'}{l_B}$ according to the value of $l_B$ from the experimental data of sample, as it is confirmed by the results in Ref. [4] and [5] (see also [6]).

In view of the fact that this is a result from the uncertainty principle and as such it is an invariant result, it depends only on the basics of ”magnetic” quantization, i.e. on the uncertainty principle in quantum electrodynamics.

Furthermore, it is expected that the observed length of the potential drop should be related with parameters of samples. This is indeed true, if one recalls that the concentration of charge carriers is indeed
the main parameter of the sample and also the magnetic length depends on it.

In conclusion let us mention that such a penetration length is also comparable with London’s penetration length in superconductivity [8].

Footnotes and references

References

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[4] W. Dietsche, K. v. Klitzing and K. Ploog, Potential Drops Across Quantum Hall Effect Samples—In the Bulk or Near the Edges? MPI fuer Festkoerperforschung Stuttgart-preprint 1995;

[5] According to the data about the IQHE samples in Ref. [4] the global concentration \( n = 3.7 \cdot 10^{11} \text{cm}^2 \) and \( \nu = 2 \). Thus, one obtains \( l_B \approx 10^{-2} \mu m \).

The measured penetration length is given to be about 100 \( \mu m \) which is almost exactly \( |l_B^{-1}| \mu m \).

[6] Recall that the measured length should be considered according to the dimensional structure where \( \hbar \) contains \( L^2 \) dimensions according to its definition.

[7] P. F. Fontein, et al., Phys. Rev. B., 43, 12090 (1991). The given data in this report which are relevant for our calculation are \( n = 5.0 \cdot 10^{15} \text{m}^{-2} \) and \( \nu = 4 \).

[8] It is well known that superconducting effects can be considered as to be related with the QHE: see Ref. [1f]; R. B. Laughlin: in Ref. [1g]; and A. Karlhede, et al.: in Ref. [1e]; See further for empirical confirmations: D. Jerome, in J. G. Bednorz, K. A. Mueller (Eds), Superconductivity, (Springer-Verlag, Berlin 1990).