Research Article

Multiuser Detection Using Adaptive Multistage Matrix Wiener Filtering Schemes with Stage-Selection Criteria in DS-UWB

Chia-Chang Hu and Hsuan-Yu Lin

1 Department of Communications Engineering, National Chung Cheng University, Min-Hsiung, Chia-Yi 621, Taiwan
2 Telecom Technology Division, Telecom Technology Center, Lujiu, Kaohsiung 821, Taiwan

Correspondence should be addressed to Chia-Chang Hu, ieechh@ccu.edu.tw

Received 13 November 2007; Revised 11 June 2008; Accepted 10 September 2008

1. INTRODUCTION

Ultra-wideband (UWB) systems have drawn considerable attention as an indoor short-range high-data-rate transmission in wireless communications over the past few years. Equalization of the UWB signals [1, 2] based on the conventional RAKE receiver technique has been addressed for both additive white Gaussian noise (AWGN) and multipath rich channels [3–11]. However, the RAKE reception suffers from its multiple-access interference (MAI) suppression capability. It is well known that the linear minimum mean-square error (MMSE) criterion, is capable to suppress the MAI efficiently. In [13, 14], the MMSE-based detectors are proposed for direct-sequence (DS) UWB communication systems. Moreover, it is shown that the MMSE decision-feedback detection (DFD) receiver is able to provide a better performance than the MMSE receiver alone even when the error propagation occurs [15]. The MMSE-DFD usually consists of one MMSE receiver in the forward path and one feedback filter in structure. Unfortunately, the computation of the MMSE-based filter weights starts with the calculation of the inverse of the input signal autocorrelation matrix, which involves an expensive computational cost. This requirement is even more exacerbated when the MMSE-based receiver operates in a nonstationary environment. To alleviate computational complexity, the authors in [16–20] propose a considerably lower complexity version of the MMSE receiver that utilizes the reduced-rank multistage vector Wiener filter (MVWF). This MVWF technique obviates the necessity of either a covariance matrix inversion or an eigen-decomposition. Additionally, there exist other iterative matrix inversion techniques, among which the conjugate gradient (CG) [21] scheme is able to provide fast initial convergence of the iterative procedure. It can also be shown that the CG scheme as well as the MVWF technique produces an MMSE approximation in the same Krylov subspace [22].

In this paper, an adaptive fuzzy-inference (FI) multistage matrix Wiener filtering (MMWF) technique, based on the MMSE performance criterion, is proposed to detect DS-UWB signals. A reduced-rank DFD scheme based on the MMWF is also considered. The MMWF, which can be compared analogously to the MVWF, is introduced to implement the MMWF-DFD receiver without a direct matrix inversion or eigen-decomposition. The feedforward and feedback filters of the MMWF-DFD receiver are capable of sharing the same calculation basis to alleviate the computational...
burden without affecting system performance. Moreover, the reduced-rank MMWF-based receivers [23] provide a significant performance gain and rapid adaptive convergence, relative to the conventional full-rank MMSE-based receivers, when observation-data support is limited [24]. In addition, the matrix conjugate gradient (MCG) algorithm [25] is developed for a robust implementation of the full-rank MMWF. It should be pointed out that the filter-stage selection of the MMSE-based detectors governs the steady-state performance and the convergence characteristic. In general, a small-stage leads to rapid convergence but results in large steady-state MSE. The opposite phenomena occur when a large stage is chosen. To achieve better convergence/tracking capability and steady-state MSE performance of the MMWF-based receivers, we propose a fuzzy-inference controlled stage-selection mechanism in this paper. It can be shown that the fuzzy-inference system (FIS) [26] offers an effective and robust means to monitor instantaneous fluctuations of a dense multipath channel and thus is able to assist the MMWF-based receivers in selecting a proper time-varying filter stage \( M \).

The rest of the paper is organized as follows. Section 2 describes the channel and system model. Sections 3 and 4 present the reduced-rank MMWF and the MMWF-DDF schemes, respectively. The reduced-rank MCG scheme is developed in Section 5. The details of the fuzzy-inference controlled filter-stage selection mechanism are given in Section 6. Section 7 analyzes the computational complexity of the proposed mechanism. Section 8 describes three existing filter stage-selection criteria. Numerical results and conclusions are presented in Sections 9 and 10, respectively.

Symbols for matrices (vectors) are denoted by boldface upper/lower case letters. The subscripts \( (\cdot)_{[x]} \) and \( (\cdot)_{[x/y]} \) represent the integer floor of \( x \) and the integer floor division remainder operation of \( x/y \), respectively. The superscripts \((\cdot)^T\) and \((\cdot)^H\) stand for transposition and Hermitian transposition, respectively. \( E[\cdot] \) denotes the expected-value operator. \( |\cdot| \) and \( \|\cdot\| \) indicate, respectively, the absolute value and the matrix/vector Frobenius norm. \( I \) is the identity matrix. \( \text{sgn} \) denotes the sign operator. \( \text{tr}\{\cdot\} \) is the trace of a matrix. \( \text{Re}\{\cdot\} \) denotes the real part. Finally, \( \text{round}[\cdot] \) indicates rounding to the nearest integer.

2. SIGNAL AND SYSTEM MODEL

In a \( K \)-user DS-UWB communication system with the use of BPSK modulation, the transmitted signal from user \( k \) can be expressed as follows [27–30]:

\[
x_k(t) = \sum_{n = -\infty}^{+\infty} \sqrt{E_k} b_k[n/N_c] c_k[n/N_c] p(t - nT_s),
\]

where \( E_k \) denotes the \( k \)-th user’s energy per pulse at the transmitter end and \( p(t) \) is the short-duration UWB pulse with unit energy [1]. \( b_k[n/N_c] \in \{\pm 1\} \) denotes the \( n/N_c \)-th BPSK modulated data symbol of duration \( T_s \). Each symbol interval consists of \( N_c \) transmission chips of duration \( T_s \), that is, \( T_s = N_c T_c \). The pseudorandom code of length \( N_c \), \( \{c_k[n/N_c]\} \), denotes the normalized spreading code sequence of the \( k \)-th user, where \( c_k[n/N_c] \) takes the value of \(-1/\sqrt{N_c}\) or \(+1/\sqrt{N_c}\) with equal probability.

The UWB multipath channel of user \( k \) can be described by its complex impulse response [6, 31–34]:

\[
h_k(t) = \sum_{j = 0}^{h_k - 1} a_{kj} \delta(t - \tau_{kj}),
\]

where \( h_k \) is the number of resolvable multipaths of user \( k \). \( a_{kj} \) indicates the complex multipath gain coefficient and \( \tau_{kj} \) is the propagation delay, which are associated with the \( j \)-th path of user \( k \). The probability distribution of \( a_{kj} \) is given by \( N(0, (1/2) \sigma_{kj}^2) + jN(0, (1/2) \sigma_{kj}^2) \), where \( N(0, (1/2) \sigma_{kj}^2) \) is a zero-mean Gaussian random variable with variance \((1/2) \sigma_{kj}^2\), \( j = 0, 1, \ldots, J_k - 1 \). The energy of the \( j \)-th channel path of user \( k \), \( \sigma_{kj}^2 \), is given by

\[
\sigma_{kj}^2 = \sigma_w^2 e^{-k^2 T_r^2 / \text{RMS}^2},
\]

where \( \sigma_w^2 \) is chosen to ensure that the average received energy is unity and \( T_r \text{RMS} \) denotes the RMS delay spread. In addition, a chip-synchronous DS-UWB system is considered with \( \tau_{kj} = l_k T_c \), where \( l_k \in [0, J_k - 1] \) is selected randomly. In this paper, the parameters of CM4 [35] are used to generate the energy of each channel tap for the non-line-of-sight (NLOS) multipath channel.

After multipath fading channel “processing,” the total received signal at the receiver is a superposition of propagated signals from all \( K \) users and the background channel noise. The received signal \( r(t) \) can be written as

\[
r(t) = \sum_{k = 1}^{K} \sqrt{E_k} \sum_{j = 0}^{h_k - 1} a_{kj} \sum_{n = -\infty}^{+\infty} b_k[n/N_c] c_k[n/N_c] p(t - nT_s - \tau_{kj}) + n(t),
\]

where \( n(t) \) indicates an AWGN.

3. REDUCED-RANK MMWF SCHEME

The received signal \( r(t) \) in (4) is passed through the chip-matched filter and is then sampled at the chip-rate over the multipath extended \((N_c + J_k - 1)\)-chip period [36]. For simplicity of notation, let \( N \) stand for the number of \((N_c + J_k - 1)\) in what follows. Denote by

\[
r(i) = [r_1(i), r_2(i), \ldots, r_N(i)]^T
\]

the column \( N \)-vector of the discrete-time received samples corresponding to the \( i \)-th information symbol interval. For the purpose of analysis, the desired users, Users 1–\( J \), are assumed to be perfectly synchronized at the receiver [36]. Let \( \mathbf{b}(i) \) be the desired data \( J \)-vector and \( \mathbf{R}_{cb} \triangleq E[\mathbf{b}(i) \mathbf{b}^H(i)] \) denote the corresponding steering matrix. The MMSE receiver is the \( N \times J \) matrix \( \mathbf{W} \), which is chosen to minimize the MSE, that is, \( \text{MSE}(\mathbf{W}) \triangleq E[\|\mathbf{b}(i) - \mathbf{W}^H \mathbf{r}(i)\|^2] \). The weight matrix \( \mathbf{W} \) is given by

\[
\mathbf{W}_{\text{MMSE}} = \arg \min_{\mathbf{W}} \text{MSE}(\mathbf{W}) = \mathbf{R}^{-1}_{rr} \mathbf{R}_{cb},
\]
where $R_{rr} \triangleq E\{r(i)r^H(i)\}$. Evidently, the computation of matrix $W_{\text{MMSE}}$ in (6) requires the inversion of matrix $R_{rr}$. To avoid the computation of $R_{rr}^{-1}$, the MMWF is used to perform decompositions of the observation vector by utilizing a series of orthogonal projections. Define the nonsingular linear transformation $T_1$ with the structure [37]

$$T_1 = \begin{bmatrix} U_1^H \\ B_1 \end{bmatrix} = \begin{bmatrix} R_{rb}^{-H} \\ B_1^T \end{bmatrix},$$

(7)

where $U_1 = R_{rb}$ is an $N \times J$ matrix and $B_1$ is an $(N - J) \times N$ blocking matrix with $B_1U_1 = 0$. Hence, the transformation of the vector $r(i)$ by the operator $T_1$ in (7) yields a vector $z_1(i)$ in the form

$$z_1(i) = T_1r(i) = \begin{bmatrix} U_1^H & B_1 \end{bmatrix} \begin{bmatrix} b_1(i) \\ r_1(i) \end{bmatrix},$$

(8)

where $b_1(i) = U_1^H r(i)$ and $r_1(i) = B_1 r(i)$. Subsequently, the correlation matrix of $z_1(i)$, $R_{z_1z_1}$, and its inverse $R_{z_1z_1}^{-1}$ can be computed as

$$R_{z_1z_1} = T_1R_{rr}T_1^H = \begin{bmatrix} R_{rb} & R_{rb}^H \\ R_{rb}^T & R_{rr} \end{bmatrix},$$

$$R_{z_1z_1}^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & R_{rr}^{-1} \end{bmatrix} + \begin{bmatrix} I & -R_{rb}R_{rr}^{-1} \\ -R_{rb}^H & I - R_{rb}R_{rr}^{-1}R_{rb} \end{bmatrix},$$

(9)

where the $J \times J$ covariance matrix $\Sigma_1$ of error, $e_1 = b_1(i) - (R_{rb}R_{rr}^{-1}R_{rb}^H)r_1(i)$, is given by

$$\Sigma_1 = E\{e_1e_1^H\} = R_{rb} - R_{rb}R_{rr}^{-1}R_{rb}^H.$$

(10)

Consequently, the linear MMSE receiver of (6) can be re-expressed in the form

$$W_{\text{MMSE}} = T_1^H (T_1R_{rr}T_1^H)^{-1} (T_1R_{rb})$$

$$= (U_1 - B_1^H R_{rb})Y_1,$$

(11)

$$= (U_1 - B_1^H R_{rb})Y_1,$$

(12)

where $\Delta_1 = U_1^H U_1$, $W_1 = R_{rb}^{-1} R_{rb}$, and $Y_1 = \Sigma_1^{-1} \Delta_1$. The first-stage ($M = 1$) orthogonal decomposition process of the MMWF receiver in (12) is illustrated in Figure 1. Subsequently, the decomposition procedure applied to $W_{\text{MMSE}}$ in (11) is used to $W_1$ and continued until the minimum dimension of both the data vector and the corresponding Wiener filter are achieved. Evidently, the maximum number of stages in the MMWF receiver is defined by $M_{\text{MAX}} = \lfloor N/J \rfloor$. This results in a set of recursion equations with the number of stages $M$, as shown in Algorithm 1. Rank reduction is realized by truncating the multistage decomposition process at the $M$th stage, where $MJ \ll N$ (full rank). Thus, the $M$th output, denoted by $W_{\text{MMWF,M}}(r(i))$, can be obtained by the following equation:

$$W_{\text{MMWF,M}}(r(i)) = Y_1(b_1 - \cdots - Y_{M-1}(b_{M-1} - Y_{M}b_{M}))$$

$$= Y_1b_1 - \cdots + (-1)^{(M-1)} \left( \sum_{j=1}^{M} Y_1 \right) b_{M}.$$

(13)

**Algorithm 1**: Recursion equations for the $M$-stage MMWF/MMWF-DFD schemes.

4. **REDUCED-RANK MMWF-DFD SCHEME**

The MMSE-DFD receiver is known to be able to outperform a linear MMSE detector. The MMSE-DFD usually consists of one MMSE receiver in the forward path and one feedback filter. The former is used for MAI suppression and the latter is for self-interference cancellation. Here, the parallel decision feedback detector (P-DFD) [38] based on the MMSE criterion is considered for multipuser detection in the DS-UWB communication systems. The feedforward filter of the P-DFD consists of a linear MMSE filter followed by an error estimation filter, as shown in Figure 2. Following the
derivation in [38], the feedforward and feedback filters of the P-DFD receiver can be expressed, respectively, as

\[ F = W_{\text{MMSE}}(I + B), \]
\[ B = Q^{-1}D^{-1} - I. \]  

(14)

Here,

\[ Q \triangleq E\{ (b(i) - W_{\text{MMSE}}^H r(i))(b(i) - W_{\text{MMSE}}^H r(i))^H \} = I - R_{rb}^H R_{rb}^{-1} R_{rb} \]

(15)

defines the \( J \times J \) error covariance matrix and the \( J \times J \) matrix \( D = \text{diag} \cdot \{(Q^{-1})_{11}, \ldots, (Q^{-1})_{JJ}\} \) is adopted to normalize the matrix \( Q^{-1} \). Note that the MMSE receiver in the forward path, \( W_{\text{MMSE}} \), can be computed by the MMWF in a reduced-rank form with the use of \( U_j = R_{rb} \). Fortunately, the feedback filter \( B = (I - R_{rb}^H R_{rb}^{-1} R_{rb})^{-1}D^{-1} - I \) can be computed efficiently by sharing the information from the MMWF. Specifically, by applying \( T_i \) to both sides of \( R_{rb}^{-1} \) at the first stage of decomposition, we have

\[ R_{rb}^{-1} R_{rb} = R_{rb}^{-1} (T_i R_{rb} R_{rb}^{-1})^{-1} T_i R_{rb} = \Delta_i^H \Sigma_i^{-1} \Delta_i, \]

(16)

where

\[ T_i R_{rb} = \begin{bmatrix} \Delta_i \\ 0 \end{bmatrix}. \]

(17)

Consecutively, a sequence of \( T_2, \ldots, T_M \) is applied to perform successive orthogonal decompositions of \( R_{rb}^{-1} R_{rb}, \ldots, R_{rb}^{-1} R_{rb} \) and neglecting the term of \( R_{rb}^H R_{rb}^{-1} R_{rb} \), \( \Sigma_i \) becomes [39, 40]

\[ \Sigma_i = R_{rb} - R_{rb}^{-1} T_i R_{rb} - \Delta_i^H \Sigma_i^{-1} \Delta_i \]
\[ = R_{rb} - \Delta_i^H \Sigma_i^{-1} \Delta_i \]
\[ \approx R_{rb} - \Delta_i^H \Sigma_i^{-1} \Delta_i. \]

(18)

Therefore, the matrix \( \Sigma_i \) in (18) can be utilized to estimate \( Q \) in (15) as follows: \( Q \approx I - \Delta_i^H \Sigma_i^{-1} \Delta_i \). Note that the MMWF-DFD scheme eliminates the need for a large matrix inversion, that is, \( R_{rb}^{-1} \), thus a substantial reduction of the computational cost can be achieved from the MMSE-DFD receiver. The set of recursion equations of the MMWF-DFD scheme and the estimate of the desired data vector are summarized in Algorithm 1.

5. REDUCED-RANK MCG SCHEME

The MCG algorithm can be applied to the common problem that we encounter in adaptive transversal filters. In other words, this algorithm is ideally suitable for deriving the solution of linear equations of a system, such as

\[ R_{tt} W = R_{tb}. \]

(19)

It is indirectly minimizing a cost function \( \xi \) defined as

\[ \xi(W) = \text{tr}[R_{rb} - 2\text{Re}(R_{rb}^H W) + W^H R_{rb} W]. \]

(20)

Note that the method of CG is simply the method of conjugate directions [41] where the search directions are constructed by conjugation of the residuals. In addition, it is worth to emphasize that the CG scheme cures the problem that the steepest descent (SD) method often finds itself taking steps in the same direction as earlier steps. In the CG algorithm, a set of \( R_{tt} \)-orthogonal, or conjugate, search directions are picked and exactly only one step is taken in each search direction. Moreover, the difficulty is overcome by the CG method with using the Gram-Schmidt conjugation in the method of conjugate directions that all the old search vectors need to be kept in memory to construct each new one.

It is readily shown that the minimum MSE can be written as

\[ \xi(W_{\text{MMSE}}) = \text{tr}[R_{rb} - R_{rb}^H R_{rb} R_{tb}]. \]

(21)

The MCG algorithm for implementing the MMWF starts with the initial matrix \( W_{\text{MCG},0} \) the initial search direction matrix \( D_0 \), and the initial residual matrix \( G_0 = D_0 = R_{rb} - R_{rb} W_{\text{MCG},0} \). The MCG algorithm updates the filter matrix at the \( (j+1) \)th iteration as follows:

\[ W_{\text{MCG},j+1} = W_{\text{MCG},j} + D_j V_j, \]

(22)

where the step matrix is given by

\[ V_j = \left(D_j^H R_{tt} D_j \right)^{-1} G_j^H G_j. \]

(23)

The residual matrix is calculated according to the equation given by

\[ G_{j+1} = G_j - R_{tt} D_j V_j. \]

(24)

The \( R_{tt} \)-conjugate direction matrix is updated as follows:

\[ D_{j+1} = G_{j+1} + D_j (G_j^H G_j)^{-1} G_j^H G_{j+1}. \]

(25)

To sum up, the MCG algorithm is an iteration method for solving the Wiener-Hopf equation in a finite number of iterations. It can be shown that both the MCG and the MMWF schemes produce an MMSE approximation in the
FIS are transformed to the respective degrees to which briefly as follows: the function of each procedure in the FIS is introduced (algorithm is guaranteed to converge in same Krylov subspace [22]. Additionally, both algorithms are utilized for the variable MBFs with centroids of the VL, L, M, and S, respectively, and the squared error variation (\(\Delta e^2\)) is used to compute the area under the aggregated MBF, and \(\min(\cdot)\) operation is generally employed to truncate the defuzzification output \(M\) in order to improve convergence/tracking capability of the receiver. The crisp, physical control command is computed by the centroid-defuzzification method. The centroid-defuzzification output \(M\) is calculated by [43] 

\[
M(i + 1) = \frac{\sum_{l=1}^{Y} M^{(l)}(i) \cdot m^{(l)}(M^{(l)}(i))}{\sum_{l=1}^{Y} m^{(l)}(M^{(l)}(i))},
\]

where \(Y\) is the number of discrete samples of the output MBF, \(M^{(l)}(i)\) is the value at the location used in approximating the area under the aggregated MBF, and \(m^{(l)}(M^{(l)}(i)) \in [0, 1]\) indicates the MBF value at location \(M^{(l)}(i)\). To reduce the computational load in the centroid calculation, fewer points \(Y\) must be used. The calculation of \(M(i + 1)\) in (27) returns the center of the area under the aggregated MBFs.

### 7. COMPUTATIONAL COMPLEXITY ANALYSIS

For the real-time applicability, a computationally efficient version of the \(M\)-stage MMWF scheme is derived and summarized in Algorithm 3 with the use of the blocking matrix \(B_l = I - U_l U_l^H\) and the estimated cross-correlation matrix \(\hat{R}_{\theta, b}\), \(\theta = (1 - \mu)\hat{R}_{\theta} + \mu b b^H\). The quantity of \(\mu \in (0, 1]\) is referred to as the forgetting factor. The heavily computational operations of null(\cdot) and \(E(\cdot)\) can be avoided successfully. Thus, it can be easily evaluated from Algorithm 3 that the \(M\)-stage MMWF receiver costs
Initialization: $b_0 = b(i), r_0 = r(i), U_1 = \hat{R}_{bh}$.

**Forward recursion**

For $j = 1, 2, \ldots, M - 1$

$b_j = U_j^T r_j$

$r_{j+1} = U_j y_j - U_j b_j$

$\Delta_j = U_j U_j^T$

$U_j = \hat{R}_{j-1} b_j$

$U_{j+1} = (1 - \mu) \hat{R}_{j-1} b_{j-1} + r_j b_j$

End

**Backward recursion**

$c_M = b_M = U_M^T r_{M-1}$

$\Sigma_M = \hat{R}_{M-1} b_M$

$Y_M = \Sigma_M^T \Delta_j$

$c_j = b_j - Y_M c_{j+1}$

$\Sigma_j = \hat{R}_{j-1} - \Delta_j \Sigma_{j+1}$

$Y_j = \Sigma_j^T \Delta_j$

End

**Algorithm 3:** Recursion equations for the simplified $M$-stage MMWF scheme

A complexity of $O(J^2 MN)$. Here, the big $O(\cdot)$ (order of) notation is used to indicate that complexity in number of operations is proportional to the argument. The complexity of the feedback filter of the MMWF-DFD scheme is at most $O(J^2)$ (i.e., the computation of matrix $\Delta_j^T \Sigma_j \Delta_j$), which is relatively small while compared to that of the MMWF scheme. Consequently, the computational complexity of the MMWF/MMWF-DFD systems is reduced substantially from $O(N^3)$ to $O(J^2 MN)$ for each computing cycle of clock time, where $J^2 M \ll N^2$.

The primary complexity cost of the $M$-iteration MCG algorithm in Algorithm 2 is the calculation of the step matrix $V_j$, which involves $O(JN^2) + O(J^2) + O(J^2 N) \approx O(JN^2)$ of complexity per iteration. The computational complexities of the $W_{MCG, j+1}, G_{j+1}, F_{j+1}$, and $D_{j+1}$, in terms of multiplications, can be easily shown to be equal to $O(J^2 N), O(JN^2), O(J^2) + O(JN^2) \approx O(JN^2)$, and $O(J^2 N)$ per iteration, respectively. Hence, the $M$-iteration MCG algorithm costs roughly $O(JMN^2)$ of complexity.

The additional computational load introduced by the (2-to-1)-FIS, in terms of multiplications, is $I + J + 3$ at each sample time, in which the preparation of $e^2(i)$ requires $J + 2$ multiplications and the centroid-defuzzification output process costs $I + 1$ multiplications. Furthermore, some special instructions (with a total of 44 lookups + 32 compares + 32 I MAX operations) are required to perform the FIS, which come primarily from the fuzzification of two input variables (12 lookups), fuzzy OR operations (32 compares), fuzzy minimum implication (32 lookups), and aggregation of the output (32F MAX operations). Fortunately, these operations can be done very efficiently in the latest range of DSPs, which provide single cycle multiply and add, table lookups, and comparison instructions [44, 45].

8. **EXISTING STAGE-SELECTION CRITERIA**

In this section, three filter-stage adaptation schemes used in [22, 24] are briefly reviewed. The first stage-selection method is introduced originally in [46] for the rank-selection of an auxiliary-vector (AV) estimator. The time-varying stage-$M$ of the AV filter is determined by the stopping rule, given by

$$M(i) = \text{max}\left\{ n : \frac{|P_x(v_n, a)|}{|v_n|} > \eta \right\},$$

where $P_x(x)$ is the orthogonal projection of the vector $x$ onto the subspace $S$ and the small positive constant $\eta$ is computed by (37) in [24]. Note that the subspace $S_n$ denotes the Krylov space spanned by the basis vectors $v_1, v_2, \ldots, v_M$, where $v_i = \text{Vec}(R_{x_{r(i)}}^T R_{x_{r(i)}})$.

The second stage-selection technique for determining the filter stage is based on minimizing the cumulative exponentially-weighted squared error $\xi$, which is also known as the a posteriori LS method, given by

$$\xi_M(i) = \sum_{m=1}^{i} \mu^{|m-1|M} \|b(m) - W_M(r(m))\|_2^2,$$

where $(\cdot)_M$ denotes the dynamic filter-stage at time $i$. For each $i$, the value of $M$ is chosen to minimize $\xi_M(i)$ defined in (29).

The third stage-selection scheme is the well-known white noise gain constraint (WNGC) [22] technique where the filter-vector norm $\|w\|$ is utilized as a rank-selection tool. The criterion utilized for the rank selection of the WNGC is $10 \log \|w\|^2 \leq 1 \text{ dB in this paper}$.  

9. **NUMERICAL RESULTS**

A DS-UWB communication system with $K = 20$ is considered in multipath fading channels. Parameters $N_e = 310$ and $J_e = 100$ are used in computer simulations. In simulations, users 1 to 5 are the users of interest to be acquired, that is, $J = 5$. Additionally, the $(\epsilon^2, \Delta^2)$-FIS system with the $(8,4)$-partitioned regions to the fuzzy I/O domains [26] is employed due to its superior performance. The threshold level of the WNGC is selected as $1 \text{ dB}$ in simulations. All experimental curves are obtained using $10^3$ independent trials with the use of $\mu = 0.99$ and $\eta = 0.01$. Figure 3 compares the convergence rate of various reduced-rank MMWF-based algorithms with the use of training symbols for SNR = 20 dB. Results of dynamic-stage MMWF algorithms using adaptation criteria of (28) and (29) (i.e., [24, equations (73) and (75)]) and WNGC are provided and compared. It is demonstrated in the figure that with the use of a small-stage ($M = 2$), the MMWF algorithm produces a faster convergence rate, while using a large-stage ($M = 8$) accomplishes a lower steady-state MSE. Thus, the proposed FI-MMWF algorithm, which performs the fuzzy-logic filter-stage selection over the range of [2,8], takes advantage of both small and large stages in convergence and steady-state characteristic. Note that the extra computational load incurred by both stage-selection criteria in [24] is heavy, especially in the a posteriori LS method.

Figure 4 evaluates the convergence behavior of various blind-mode reduced-rank MMWF-based algorithms. The blind FI-MMWF-based algorithms can be obtained by...
simply substituting $\mathbf{R}_{tb}$ by $\hat{\mathbf{R}}_{tb}$ ("spreading" code matrix of the desired users). The filter-stage selection of the FI-MMWF-based algorithms is conducted over the set of $[2, 5]$. Experimental results in Figure 4 are similar to those of in Figure 3. It should be pointed out that the convergence rate of the low-stage MMWF is much faster than that of the high-stage MMWF-based in the blind version. Consequently, the advantage of fuzzy-stage selection MMWF-based algorithms in blind version is quite impressive.

Simulation results in Figure 5 show the convergence behavior of the blind-mode FI-MCG algorithm in terms of the number of iterations. Other parameters used in Figure 5 are set as in Figure 4. Evidently, the FI-MCG algorithm produces better convergence/tracking capability and steady-state MSE performance than MMWF schemes with a fixed stage. Additionally, the results in Figure 5 demonstrate that an improvement in MSE performance over the MCG scheme is achieved by the FI-MCG algorithm, presumably because of the use of a fuzzy variable stage in response to the time-varying fading channels. Also, these results show that the FI-MCG algorithm is able to accomplish a similar performance as the FI-MMWF-based approaches. Results in Figure 5 provide the convergence behavior of the MMWF-based algorithms using linear interpolation (LI) filter-stage selection criterion as well. With the use of the linear interpolation technique, the filter-stage update can be described by the following equations:

$$\mathcal{M}(i + 1) = \begin{cases} M_L, & e^2(i) < e^2_{L}, \\ M_L + \frac{(e^2(i) - e^2_L)}{(e^2_H - e^2_L)} (M_H - M_L), & \text{ow,} \\ M_H, & e^2(i) \geq e^2_H, \end{cases}$$

$$M(i + 1) = \text{round}(\mathcal{M}(i + 1)),$$

where $M_L$ and $M_H$ denote the minimum and, respectively, the maximum values allowed for the filter-stage $M(i + 1)$. Values of $e^2_{L}$ and $e^2_{H}$ define the lower and upper values used for the $e^2(i)$. In what follows, $M_L = 2$, $M_H = 5$, $e^2_{L} = 0.01$, and $e^2_{H} = 0.5$ are employed. It should be emphasized that the increased complexity incurred by the linear interpolation scheme is very little. It costs only 1 multiplication, 2 additions, and 2 compares per update if the calculation of $e^2(i)$, $e^2_{L}$, and $e^2_{H}$ is performed beforehand. Evidently, results in Figure 5 demonstrate that the FI-MMWF-based algorithms
achieve better convergence/tracking capability and steady-state MSE performance over the LI-MMWF algorithm due to making full use of the 2-to-1 fuzzy-inference-based filter-stage adaptation criterion.

10. CONCLUSIONS

The reduced-rank FI-MMWF-based receivers are proposed for data demodulation in the DS-UWB communication systems. The computational complexity of the forward path of the MMSE-DFD receiver is reduced by introducing the reduced-rank MMWF scheme. With the computation-basis sharing in the forward and backward filters of the MMWFD-DF receiver, the extra complexity incurred by the decision feedback mechanism is alleviated. Moreover, the MMWF-DFD receiver is able to achieve an improvement in convergence rate and offer an additional gain in performance for the MMWF receiver. In addition, the FI-MMWF-based receivers provide convergence/tracking and MSE performance benefits in multipath fading channels. Notably, the fuzzy-based MCG receiver is able to provide performance similar to those of the FI-based MMWF, and MMWF-DFD receivers. Furthermore, it is also noticed that the LI-MMWF algorithm does not outperform the FI-MMWF-based approaches, but does provide a lower complexity cost. As a consequence, these merits make the FI-based MMWF, MMWF-DFD, and MCG receivers well suitable for applications in the UWB wireless communications.

ACKNOWLEDGMENTS

This work was supported by Taiwan National Science Council under Grant no. NSC:95-2221-E-194-013. This work was presented in part at the IEEE International Conference on Communications (ICC2007), Glasgow, Scotland, UK, 24–28 June 2007.

REFERENCES

[1] A. Ridolfi and M. Z. Win, “Ultrawide bandwidth signals as shot noise: a unifying approach,” IEEE Journal on Selected Areas in Communications, vol. 24, no. 4, pp. 899–903, 2006.
[2] M. Z. Win, “A unified spectral analysis of generalized time-hopping spread-spectrum signals in the presence of timing jitter,” IEEE Journal on Selected Areas in Communications, vol. 20, no. 9, pp. 1664–1676, 2002.
[3] D. C. Laney, G. M. Maggio, F. Lehmann, and L. Larson, “Multiple access for UWB impulse radio with pseudochaotic time hopping,” IEEE Journal on Selected Areas in Communications, vol. 20, no. 9, pp. 1692–1700, 2002.
[4] M. Z. Win and R. A. Scholtz, “Ultra-wideband time-hopping spread-spectrum impulse radio for wireless multiple-access communications,” IEEE Transactions on Communications, vol. 48, no. 4, pp. 679–691, 2000.
[5] D. Cassioli, M. Z. Win, F. Vatalaro, and A. F. Molisch, “Low complexity Rake receivers in ultra-wideband channels,” IEEE Transactions on Wireless Communications, vol. 6, no. 4, pp. 1265–1275, 2007.
[6] A. Rajeswaran, V. S. Somayazulu, and J. R. Foerster, “Rake performance for a pulse based UWB system in a realistic UWB indoor channel,” in Proceedings of IEEE International Conference on Communications (ICC’03), 2003.
[7] M. Eslami and X. Dong, “Rake-MMSE-equalizer performance for UWB,” IEEE Communications Letters, vol. 9, no. 6, pp. 502–504, 2005.
[8] M. Z. Win and Z. A. Kostić, “Impact of spreading bandwidth on Rake reception in dense multipath channels,” IEEE Journal on Selected Areas in Communications, vol. 17, no. 10, pp. 1794–1806, 1999.
[9] M. Z. Win, G. Chrisek, and N. R. Sollenberger, “Performance of Rake reception in dense multipath channels: implications of spreading bandwidth and selection diversity order,” IEEE Journal on Selected Areas in Communications, vol. 18, no. 8, pp. 1516–1525, 2000.
[10] T. Q. S. Quek and M. Z. Win, “Analysis of UWB transmitted-reference communication systems in dense multipath channels,” IEEE Journal on Selected Areas in Communications, vol. 23, no. 9, pp. 1863–1874, 2005.
[11] T. Q. S. Quek, M. Z. Win, and D. Dardari, “Unified analysis of UWB transmitted-reference schemes in the presence of narrowband interference,” IEEE Transactions on Wireless Communications, vol. 6, no. 6, pp. 2126–2138, 2007.
[12] Z. Xie, R. T. Short, and C. K. Rushforth, “A family of suboptimum detectors for coherent multiuser communications,” IEEE Journal on Selected Areas in Communications, vol. 8, no. 4, pp. 683–690, 1990.
[13] Q. Li and L. A. Rusch, “Multiuser detection for DS-CDMA UWB in the home environment,” IEEE Journal on Selected Areas in Communications, vol. 20, no. 9, pp. 1701–1711, 2002.
[14] L. Yang and G. B. Giannakis, “Multistage block-spreading for impulse radio multiple access through ISI channels,” IEEE Journal on Selected Areas in Communications, vol. 20, no. 9, pp. 1767–1777, 2002.
[15] W. G. Phoe and M. L. Honig, “Transmitter diversity for DS-CDMA with MMSE decision feedback,” in Proceedings of IEEE
Global Telecommunications Conference (GLOBECOM '00), vol. 1, pp. 133–137, San Francisco, Calif, USA, November-December 2000.

[16] J. S. Goldstein, I. S. Reed, and L. L. Scharf, “A multistage representation of the Wiener filter based on orthogonal projections,” IEEE Transactions on Information Theory, vol. 44, no. 7, pp. 2943–2959, 1998.

[17] J. S. Goldstein and I. S. Reed, “Reduced-rank adaptive filtering,” IEEE Transactions on Signal Processing, vol. 45, no. 2, pp. 492–496, 1997.

[18] J. S. Goldstein and I. S. Reed, “Theory of partially adaptive radar,” IEEE Transactions on Aerospace and Electronic Systems, vol. 33, no. 4, pp. 1309–1325, 1997.

[19] J. S. Goldstein, I. S. Reed, and P. A. Zulch, “Multistage partially adaptive STAR CFAR detection algorithm,” IEEE Transactions on Aerospace and Electronic Systems, vol. 35, no. 2, pp. 645–661, 1999.

[20] J. R. Guerci, J. S. Goldstein, and I. S. Reed, “Optimal and adaptive reduced-rank STAP,” IEEE Transactions on Aerospace and Electronic Systems, vol. 36, no. 2, pp. 647–663, 2000.

[21] G. H. Golub and C. F. Van Loan, Matrix Computations, The Johns Hopkins University Press, Baltimore, Md, USA, 3rd edition, 1996.

[22] J. D. Hiemstra, Robust implementations of the multistage Wiener filter, Ph.D. Thesis, Virginia Polytechnic Institute and State University, Blacksburg, Va, USA, 2003.

[23] P. Thanyasrisung, I. S. Reed, and X. Yu, “Reduced-rank MMSE multiuser receiver for synchronous CDMA,” in Proceedings of the 21st Century Military Communications Conference (MILCOM '00), vol. 1, pp. 569–573, Los Angeles, Calif, USA, October 2000.

[24] M. L. Honig and J. S. Goldstein, “Adaptive reduced-rank interference suppression based on the multistage Wiener filter,” IEEE Transactions on Communications, vol. 50, no. 6, pp. 896–994, 2002.

[25] H. Ge, L. L. Scharf, and M. Lundberg, “Reduced-rank multiuser detectors based on vector and matrix conjugate gradient Wiener filters,” in Proceedings of the 5th IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC '04), pp. 189–193, Lisbon, Portugal, July 2004.

[26] H.-Y. Lin, C.-C. Hu, Y.-F. Chen, and J.-H. Wen, “An adaptive robust LMS employing fuzzy step size and partial update,” IEEE Signal Processing Letters, vol. 12, no. 8, pp. 545–548, 2005.

[27] R. J.-M. Cramer, R. A. Scholtz, and M. Z. Win, “Evaluation of an ultra-wideband channel,” IEEE Transactions on Antennas and Propagation, vol. 50, no. 5, pp. 561–570, 2002.

[28] D. Cassioli, M. Z. Win, and A. F. Molisch, “The ultra-wide bandwidth indoor channel: from statistical model to simulations,” IEEE Journal on Selected Areas in Communications, vol. 20, no. 6, pp. 1247–1257, 2002.

[29] M. Z. Win and R. A. Scholtz, “Characterization of ultra-wide bandwidth wireless indoor channels: a communication-theoretic view,” IEEE Journal on Selected Areas in Communications, vol. 20, no. 9, pp. 1613–1627, 2002.

[30] A. F. Molisch, D. Cassioli, C.-C. Chong, et al., “A comprehensive standardized model for ultrawideband propagation channels,” IEEE Transactions on Antennas and Propagation, vol. 54, no. 11, part 1, pp. 3151–3166, 2006.

[31] S. R. Aedudodla, S. Vijayakumaran, and T. F. Wong, “Timing acquisition in ultra-wideband communication systems,” IEEE Transactions on Vehicular Technology, vol. 54, no. 5, pp. 1570–1583, 2005.

[32] W. Suwansantisuk, M. Z. Win, and L. A. Shepp, “On the performance of wide-bandwidth signal acquisition in dense multipath channels,” IEEE Transactions on Vehicular Technology, vol. 54, no. 5, pp. 1584–1594, 2005.

[33] W. Suwansantisuk and M. Z. Win, “Multipath aided rapid acquisition: optimal search strategies,” IEEE Transactions on Information Theory, vol. 53, no. 1, pp. 174–193, 2007.

[34] W. Suwansantisuk, M. Chiani, and M. Z. Win, “Frame synchronization for variable-length packets,” IEEE Journal on Selected Areas in Communications, vol. 26, no. 1, pp. 52–69, 2008.

[35] J. Foerster, “Channel modeling sub-committee report final,” IEEE802.15-02/490, November 2003.

[36] E. Fishler and H. V. Poor, “Low-complexity multiuser detectors for time-hopping impulse-radio systems,” IEEE Transactions on Signal Processing, vol. 52, no. 9, pp. 2561–2571, 2004.

[37] P. Cifuentes, W. L. Myrick, S. Sud, J. S. Goldstein, and M. D. Zoltowski, “Reduced rank matrix multistage Wiener filter with applications in MMSE joint multiuser detection for DS-CDMA,” in Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '02), vol. 3, pp. 2605–2608, Orlando, Fla, USA, May 2002.

[38] G. Woodward, R. Ratasuk, M. L. Honig, and P. B. Rajapak, “Minimum mean-squared error multiuser decision-feedback detectors for DS-CDMA,” IEEE Transactions on Communications, vol. 50, no. 12, pp. 2104–2112, 2002.

[39] C.-C. Hu and I. S. Reed, “Space-time adaptive reduced-rank multistage Wiener filtering for asynchronous DS-CDMA,” IEEE Transactions on Signal Processing, vol. 52, no. 7, pp. 1862–1877, 2004.

[40] Y.-H. Chan and X. Yu, “A reduced-rank MMSE-DFE receiver for space-time coded DS-CDMA systems,” in Proceedings of the 60th IEEE Vehicular Technology Conference (VTC '04), vol. 5, pp. 3659–3663, Los Angeles, Calif, USA, September 2004.

[41] J. R. Shewchuk, An Introduction to the Conjugate Gradient Method without the Agonizing Pain, Carnegie Mellon University, Pittsburgh, Pa, USA, 1994.

[42] L. A. Zadeh, “Fuzzy sets,” Information and Control, vol. 8, no. 3, pp. 338–353, 1965.

[43] V. Kecman, Learning and Soft Computing: Support Vector Machines, Neural Networks, and Fuzzy Logic Models, MIT Press, Cambridge, Mass, USA, 2001.

[44] M. J. Patryra, J. L. Grantner, and K. Koster, “Digital fuzzy logic controller: design and implementation,” IEEE Transactions on Fuzzy Systems, vol. 4, no. 4, pp. 439–459, 1996.

[45] A. Costa, A. De Gloria, P. Faraboschi, A. Pagni, and G. Rizzotto, “Hardware solutions for fuzzy control,” Proceedings of the IEEE, vol. 83, no. 3, pp. 422–434, 1995.

[46] H. Qian and S. N. Batalama, “Data-record-based criteria for the selection of an auxiliary-vector estimator of the MVDR filter,” in Proceedings of the 34th Asilomar Conference on Signals, Systems and Computers (ACSSC '00), vol. 1, pp. 802–807, Pacific Grove, Calif, USA, October-November 2000.