Behaviour of the Ice Cover under the Conditions of Flexural-Gravity Resonance with the Pair Movement of the Load

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Abstract. The nature of ice cover failure technology implemented with amphibian hovercrafts (AHC) is stated in works of Kozin [1, 2] and represents generation of a wave system in an ice cover under the effect of moving load on the ice cover. If gravity wave in water accompanies flexural wave in an ice plate, this combination of waves will be called flexural gravity wave (FGW), the speed of moving load \( v_c \) equal to the speed of FGW propagation will be called critical. However, during the implementation of ice breaking works by single (AHC), AHC’s parameters may appear insufficient for failure of the ice cover of specified thickness at specific ice conditions. In these cases, efficiency of ice breaking works may be increased by simultaneous use of several hovercrafts, i.e. by interference of FGWs generated by hovercrafts. This work is dedicated to research of the ice cover deformation during the motion of several (paired) loads over it. A comparison of the theoretical and experimental data obtained confirmed the operability of the selected criterion for the failure of the ice cover, which allows it to be used to develop practical recommendations for use FGWs generated by several loads to failure the ice cover. Original results obtained.

1. Introduction
The nature of ice cover failure technology implemented with amphibian hovercrafts (AHC) is stated in works of Kozin [1, 2] and represents generation of a wave system in an ice cover under the effect of moving load on the ice cover. If gravity wave in water accompanies flexural wave in an ice plate, this combination of waves will be called flexural gravity wave (FGW), the speed of moving load \( v_c \) equal to the speed of FGW propagation will be called critical.

Single load motion has been reviewed in known researches [3, 4]. In article [5] of Shishmarev is presented the results of the analytical and numerical analysis of the response of an ice cover to an oscillating load moving along a frozen rectangular channel is studied. The investigate the solvability of the problem of the moving load in the channel in the cases of elastic and viscoelastic ice plates is reviewed by Shishmarev et al. [6]. The solution of the linear hydroelastic problem of steady-state forced vibrations of a semi-infinite ice cover under the action of a localized external load is presented in work of Sturova [7]. In work [8] the spatial problem of steady-state forced oscillations of a fluid and
a semi-infinite ice cover under the action of a localized external load moving along its straight edge at a constant speed is considered. Work of Sturova and Tkacheva [9] is dedicated to three-dimensional problem for the wave disturbances induced by a pressure distribution moving with uniform speed along the rectilinear edge of semi-infinite ice sheet is presented. In work [10] the dependences of the plate deflection and the bending moment on the load frequency and the boundary conditions they studied. Linear progressive waves in a channel covered with ice sheet studied by Korobkin et al [11].

However, during the implementation of ice breaking works by single (AHC), AHC’s parameters may appear insufficient for failure of the ice cover of specified thickness at specific ice conditions. In these cases, efficiency of ice breaking works may be increased by simultaneous use of several hovercrafts, i.e. by interference of FGWs generated by hovercrafts. This work is dedicated to research of the ice cover deformation during the motion of several (paired) loads over it.

Natural ice cover can be simulated using various ice models. There are relevant similarity conditions for each of the models but no existing ice models comply with all the requirements to the full extent. Therefore, ice cover is usually simulated with partial compliance with the similarity conditions [12]. The scale \( \lambda_0 \) should be assumed based on dimensions of the ice channel. The channel walls shall not have any effect on wave pattern parameters. The specified similarity conditions [12] allow simulation of ice cover fracture by resonant FGWs.

During simulation experiments, ice models of various thickness were frozen on in order to determine maximum flexural strength of the ice model. U-shaped consoles with dimensions of \( b \times b = 0.15 \times 0.45 \text{ m} \) are cut out of the ice model to prefabricated template. Force used to break the console is determined with electronic dynamometer Mark – 10 (USA).

Kozin in his work proposed a geometrical criterion of complete ice fracture, obtained experimentally, for ice fracture by moving loads. The assumed criterion is a tangent inclination angle relative to the curved surface of the ice model corresponding to complete fracture stage of the plate.

\[
\alpha = \frac{2 \pi \omega}{l}
\]

The value of the slope of the tangent to the curved surface characterizes the curvature of the plate which defines the level of normal stresses. According to the model of experimental studies of ice breaking during motion of a submarine [13], for \( \alpha > 0.04 \), there are complete opening of main cracks and ice breaking. The results of measurements of the coefficient \( \alpha \) for ice breaking by AHCs (model and full-scale) [13] show that in all the cases of ice breaking, \( \alpha > 0.04 \).

2. Determination of ice breaking capacity of FGWs caused by moving single load

At initial stage of the research, a number of experiments were performed to determine effect of the load weight on parameters of FGWs generated in the ice model. Simplified model of AHC “Murena” (scale 1:60) is used for the experiments. The model is fabricated with 3D printer using layer-by-layer technique. The model length is \( l = 0.5 \text{ m} \). The model width is \( h = 0.216 \text{ m} \). During the research, the model weight varied within the range of \( m = 0.94 \div 1.6 \text{ kg} \) using ballast. The model speed is \( \nu = 1.26 \div 2.2 \text{ m/sec} \). Key experiment results are shown in Figure 1a,b.

Resonant speed is obtained by experiment using maximum deflections of ice cover, which are determined using FGW profiles recorded by motion sensor. According to the experiment, the highest amplitudes of FGWs generated in the ice cover are achieved at the load speed about \( \nu = 1.75 \text{ m/sec} \). For the minimum model weight of 0.94 kg, ice fracture was not observed even if the speed was close to the resonant speed. If the weight of the moving model is 1.27 kg, ice fracture occurred within the narrow speed range of 1.55÷1.85 m/sec. If the model weight is 1.6 kg, ice fracture occurred throughout the whole speed range examined, excluding supercritical speeds (Figure 2).

If the speed is low, main cracks are observed within the ice; fracture area is lowest (Figure 2a). As the speed approaches the critical value, intensity of ice fracture becomes higher, dense concentric cracks occur (Figure 2b), ice pieces become smaller, and load bearing capacity of the ice cover is lost.
completely. Maximum fracture is observed during flexural gravity resonance. At the higher speeds, fractured area becomes smaller again.

Figure 1. Results of experimental research ($h_m = 0.002$ m) with load speed curves depending on the load weight $m_m$: curve composed from rhombes 1 corresponds to weight 0.94 kg; curve composed from squares 2 corresponds to weight 1.27 kg; curve composed from triangles 3 corresponds to weight 1.6 kg: (a) ice model deflection, (b) coefficient $\alpha$.

The results allow determination of load weight initiating ice cover fracture by FGWs at resonant speed and speeds much less than critical speed.

3. **Objective laws of ice model failure by moving paired load**

The experiments performed in 1986 show that ice cover can be destroyed by FGWs with higher efficiency in case if two loads are moving side by side. In order to obtain the parameters of the FGWs generated by moving paired loads, a number of experiments are performed where AHCs are moving side by side.

Figure 2. Ice model ($h_m = 0.002$ m) fraction pattern caused by FGWs generated by a model ($m_m = 1.6$ kg): (a) $v_m = 1.43$ m/sec, (b) $v_m = 1.6$ m/sec.

The highest deflections are recorded for zero distance between the models. Ice was fractured efficiently at the speeds higher and lower than the resonant speed (Figure 3a). At the critical speed, dense cracks are observed, ice cover loses its load bearing capacity, ice pieces are chipped and
overturned. The ice was fractured over the significant area and directly under the models (Figure 3a). As the speed increases, the fracture efficiency decreases and ice cracking pattern changes. The cracks are caused by divergent waves (Figure 3b).

Figure 4. Results of experimental research ($h_m = 0.002$ m) with load speed curves for two loads moving side by side: curve composed from rhombes 1 corresponds to zero distance; curve composed from squares 2 corresponds to distance between models is $b_m$; curve composed from triangles 3 corresponds to distance between models is $2 \times b_m$: (a) ice model deflection, (b) coefficient $\alpha$.

The minimum model weight was selected, which did not cause ice cover fracture even at speeds close to resonant speeds. This weight is 0.94 kg. This is to find out if the ice fracture efficiency of the resonant method is higher. The model speed is $v_m = 1.26 \div 2.2$ m/sec. Key experiment results are shown in Figure 4. The distance between the models varied from zero to $2 \times b_m$.

4. Theoretical research

Theoretical research of effect of mutual location of AHCs on the parameters of FGW generated by them has been performed based on solution of differential equation of floating visco-elastic plate under effect of external load that may be written as follows [14]:

$$\frac{Gh^3}{3} \left(1 - \tau_\phi u \frac{\partial}{\partial x} \right) \nabla^4 w + \rho_f gw + \rho_1 hu u \frac{\partial^2 w}{\partial x^2} - \rho_f u \frac{\partial \Phi}{\partial x} = -q$$

(2)

Where: $G$ is modulus of ice elasticity during the shift; $G = 0.5E/(1 + \nu)$; $h$ is ice cover thickness; $\tau_\phi$ is time of deformation relaxation; $w$ is ice sagging; $\rho_f$ is ice plate density; $\rho_1$ is water density; $g$ is acceleration of gravity; $u$ is speed of load motion; $q$ is system of moving pressures; $\Phi$ is motion potential of liquid consistent with Laplace equation $\Delta \Phi = 0$.

Expression for $w$ is obtained in form of [15]:

$$w(x, y) = \frac{4q_0}{\pi^2 \cdot \rho_f \cdot u^2} \cdot \int_0^\infty \lambda^2 \cdot \tanh(\lambda H) \times \ldots \\
\times \cos(y\sqrt{\lambda^2 - \alpha^2}) \sin(L_n \frac{\sqrt{\lambda^2 - \alpha^2}}{2}) \sin(B_n \frac{\sqrt{\lambda^2 - \alpha^2}}{2}) \frac{\cos(\alpha \xi) \cdot \xi + \sin(\alpha \xi) \cdot \eta}{\alpha(\lambda^2 - \alpha^2)(\xi^2 + \eta^2)} \ d\alpha d\lambda$$

(3)

where
\[ \xi = -\frac{Gh^3 \lambda \tanh(\lambda H)}{3\rho_1 u^2} - \frac{g \lambda \tanh(\lambda H)}{u^2} + \frac{\rho_2 h \lambda \alpha^2 \tanh(\lambda H)}{\rho_1} + \alpha^2 \]  

(4)

\[ \eta = \frac{Gh^3 \lambda \tanh(\lambda H) \alpha \tau_{\phi}}{3\rho_1 u} \]  

(5)

Where: \( q_0 \) is load intensity; \( L_n \) is load length; \( W_n \) is load width; \( H \) is water depth.

In order to determine total theoretical sagging of an ice cover during the motion of two loads upon it, following dependences were used:

\[ w_f(x, y) = w(x) + w(x, y - L_y) \]  

(6)

Where: \( L_y \) is distance between loads during their motion abreast.

Calculations were made using following parameter values of model of AHC "Murena": \( l_m = 0.5 \text{ m} \); \( b_m = 0.216 \text{ m} \); \( m_m = 0.94 \div 1.6 \text{ kg} \); \( h = 0.002 \text{ m} \); \( H = 1 \text{ m} \); \( \rho_1 = 900 \text{ kg/m}^3 \); \( \rho_2 = 1000 \text{ kg/m}^3 \); \( E = 1 \times 10^9 \text{ N/m}^2 \); \( v = 0.33 \); \( \tau_{\phi} = 0.69 \text{ sec} \). Where: \( l_m \) is model length; \( b_m \) is model width; \( m_m \) is model weight; \( h_m \) is the ice thickness; \( H \) is the depth in the ice basin; \( \rho_1 \) is ice plate density; \( \rho_2 \) is water density; \( E \) is Young's modulus; \( v \) is Poisson's ratio; \( \tau_{\phi} \) is time of deformation relaxation. The relaxation time \( \tau_{\phi} \) was chosen in accordance with the results of Takizawa [16]. It was performed a series of experiments for defining critical speed at different model weight.

At the beginning of theoretical research were performed calculations ice cover deflection and coefficient \( \alpha \) depending on the speed of one load.

**Figure 5.** Results of theoretical research \((h_m = 0.002 \text{ m})\) with load speed curves depending on the load weight \( m_m \): curve composed from rhombes 1 corresponds to weight 0.94 kg; curve composed from squares 2 corresponds to weight 1.27 kg; curve composed from triangles 3 corresponds to weight 1.6 kg: (a) ice model deflection, (b) coefficient \( \alpha \).

We can see satisfactory agreement theoretical (Figure 5a) and experiment (Figure 1a) results.

Figure 5b shows the results of theoretical research the dependencies coefficient \( \alpha \) vs. the speed of movement of one load. Figure 5b shows that the coefficient \( \alpha \) exceed the value 0.04. During the experiments in the ice basin with the above-mentioned parameters of the model and ice conditions, complete failure of the ice cover occurred.

We can see satisfactory agreement theoretical (Figure 5b) and experiment (Figure 1b) results.

After that, the ice cover deflection and coefficient \( \alpha \) was calculated for two models loads for side by side movement. During the calculations, the models weight is \( m_m = 0.94 \text{ kg} \). The models speed is \( v_m = 1.26 \div 2.2 \text{ m/sec} \). The distance between the models varied from zero to \( 2 x b_m \).

The research performed allows to make the following conclusions:
• Use of AHCs moving abreast is promising because the possibility exists to considerably increase the efficiency of proposed method for ice cover destruction if this method will be implemented by group of more than two hovercrafts;
• An increase in the distance between loads leads to a decrease of the ice breaking possibility at side by side movement of two loads;
• Thus a comparison of the theoretical and experimental data obtained confirmed the operability of the selected criterion for the failure of the ice cover, which allows it to be used to develop practical recommendations for use FGWs generated by several loads to failure the ice cover
• The work performed shows reasonability of further research in area of improvement of proposed method for ice cover destruction that was proposed before and has proved its value.

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