The M-Theory Two-Brane in $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$

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Abstract
We construct the supermembrane action in an $AdS_4 \times S^7$ and $AdS_7 \times S^4$ background to all orders in anticommuting coordinates. The result is compared to and agrees completely with results obtained earlier for generic supergravity backgrounds through gauge completion at low orders in $\theta$. 

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1. Introduction

The two-brane (as well as its dual five-brane) plays a central role in M-theory \[1\]. Recently the interactions of this supermembrane \[2\] in a background described by the component fields of 11-dimensional supergravity \[3\] were written down in low orders of the anticommuting coordinates \(\theta\) \[4\]. In principle this offers the possibility for studying the two-brane in a number of interesting supergravity backgrounds with a high degree of supersymmetry. In view of the recent connection noted between the near-horizon D-brane solutions and certain superconformal field theories \[5\], it is of interest to study the supermembrane in backgrounds of an anti-de Sitter spacetime times a compact manifold. This program was recently carried out for the type-IIB superstring and the D3-brane in a IIB-supergravity background of this type \[6, 7, 8\]. In the context of 11-dimensional supergravity the \(AdS_4 \times S^7\) and \(AdS_7 \times S^4\) backgrounds stand out as they leave 32 supersymmetries invariant \[9, 10\]. These backgrounds are associated with the near-horizon geometries corresponding to two- and five-brane configurations and thus to possible conformal field theories in 3 and 6 spacetime dimensions with 16 supersymmetries, whose exact nature is not yet completely known.

The purpose of this paper is to discuss the supermembrane in these two backgrounds. As these spaces are local products of homogeneous spaces, their geometric information can be extracted from appropriate coset representatives leading to standard invariant one-forms corresponding to the vielbeine and spin-connections. Our approach differs from that of a recent paper \[11\] where one constructs the geometric information exploiting simultaneously the kappa symmetry of the supermembrane action, in that we determine the geometric information independent from the supermembrane action. As this construction holds to all orders in the fermionic coordinates \(\theta\), it provides valuable complementary information to the low-order \(\theta\) results obtained by gauge completion \[4\].

The antisymmetric four-rank field strength of M-theory induces the compactifications of the theory to \(AdS_4 \times S^7\) and \(AdS_7 \times S^4\), which leave the 32 supersymmetries intact. These two compactifications are thus governed by the Freund-Rubin field \(f\), defined by (in Pauli-Källén convention, so that we can leave the precise signature of the spacetime open),

\[
F_{\mu \nu \rho \sigma} = 6 f \varepsilon_{\mu \nu \rho \sigma},
\]

with \(\varepsilon\) the vierbein determinant. When \(f\) is purely imaginary we are dealing with an \(AdS_4 \times S^7\) background while for real \(f\) we have an \(AdS_7 \times S^4\) background. The non-vanishing curvature components corresponding to the 4- and 7-dimensional subspaces
are equal to

\[ R_{\mu \nu \rho \sigma} = -4f^2 (g_{\mu \rho} g_{\nu \sigma} - g_{\mu \sigma} g_{\nu \rho}) , \]
\[ R_{\mu' \nu' \rho' \sigma'} = f^2 (g_{\mu' \rho'} g_{\nu' \sigma'} - g_{\mu' \sigma'} g_{\nu' \rho'}) . \]  

(1.2)

Here \( \mu, \nu, \rho, \sigma \) and \( \mu', \nu', \rho', \sigma' \) are 4- and 7-dimensional world indices, respectively. We also use \( m_{4,7} \) for the inverse radii of the two subspaces, defined by \( |f|^2 = m_r^{-2} = \frac{1}{4} m_4^{-2} \). The Killing-spinor equations associated with the 32 supersymmetries in this background take the form

\[ (D_\mu - f \gamma_\mu \gamma_5 \otimes 1) \epsilon = (D_{\mu'} + \frac{1}{2} f 1 \otimes \gamma_{\mu'}) \epsilon = 0 , \]

(1.3)

where we make use of the familiar decomposition of the (hermitean) gamma matrices appropriate to the product space of a 4- and a 7-dimensional subspace\(^1\). Here \( D_\mu \) and \( D_{\mu'} \) denote the covariant derivatives containing the spin-connection fields corresponding to \( \text{SO}(3,1) \) or \( \text{SO}(4) \) and \( \text{SO}(7) \) or \( \text{SO}(6,1) \), respectively.

2. Structure of \( \mathfrak{osp}(8|4) \) and \( \mathfrak{osp}(6,2|4) \)

The algebra of isometries of the \( \text{AdS}_4 \times S^7 \) and \( \text{AdS}_7 \times S^4 \) backgrounds is given by \( \mathfrak{osp}(8|4) \) and \( \mathfrak{osp}(6,2|4) \). Their bosonic subalgebra consists of \( \mathfrak{so}(8) \oplus \mathfrak{sp}(4) \cong \mathfrak{so}(8) \oplus \mathfrak{so}(3,2) \) and \( \mathfrak{so}(6,2) \oplus \mathfrak{usp}(4) \cong \mathfrak{so}(6,2) \oplus \mathfrak{so}(5) \), respectively. The spinors transform in the \( (8, 4) \) of this algebra. Observe that the spinors transform in a chiral representation of \( \mathfrak{so}(8) \) or \( \mathfrak{so}(5) \).

We decompose the generators of \( \mathfrak{osp}(8|4) \) or \( \mathfrak{osp}(6,2|4) \) in terms of irreducible representations of the bosonic \( \mathfrak{so}(7) \oplus \mathfrak{so}(3,1) \) and \( \mathfrak{so}(6,1) \oplus \mathfrak{so}(4) \) subalgebras. In that way we get the bosonic (even) generators \( P_r, M_{rs} \), which generate \( \mathfrak{so}(3,2) \) or \( \mathfrak{so}(5) \), and \( P_{r'}, M_{r's'} \), which generate \( \mathfrak{so}(8) \) or \( \mathfrak{so}(6,2) \). All the bosonic generators are taken antihermitean (in the Pauli-Källén sense). The fermionic (odd) generators \( Q_{aa'} \)

\(^1\)The 11-dimensional gamma matrices decompose into the ones referring to the 4-space and the 7-space, denoted by \( \gamma^r \) and \( \gamma'_{r'} \), respectively. Hence the 4- and 7-dimensional tangent-space indices are denoted by \( r, s, \ldots \) and \( r', s', \ldots \); 11-dimensional tangent-space indices are distinguished by a caret. The 11-dimensional gamma matrices then take the form \( \Gamma_r = \gamma_r \otimes 1 \) and \( \Gamma_{r'} = \gamma_{r'} \otimes \gamma'_{r'} \). The charge-conjugation matrix decomposes into the ones referring to the 4- and 7-dimensional subspaces according to \( \mathcal{C} = \mathcal{C} \otimes \mathcal{C}' \). Observe that \( \mathcal{C} \) is antisymmetric and \( \mathcal{C}' \) is symmetric, so that \( \mathcal{C} \) is always antisymmetric. Furthermore we have \( C^{-1} \gamma_r C = -\gamma^T_r \) and \( C'^{-1} \gamma'_{r'} C' = -\gamma'^T_{r'} \). Also \( \gamma_5 = \frac{1}{24} \varepsilon_{rstu} \gamma_r \gamma_s \gamma_t \gamma_u \). For future use we also note that spinorial tangent-space indices are denoted by \( a, b, \ldots, a', b', \ldots \) and \( a, b, \ldots \) for 4-, 7- or 11-dimensional indices. In the gauge we are working in there will be no distinction between tangent and world spinor indices. Gamma matrices with multiple indices denote weighted antisymmetrized products of gamma matrices in the standard way.
are Majorana spinors. The commutation relations between even generators are

\[ [P_r, P_s] = -4f^2 M_{rs}, \quad [P_{r'}, P_{s'}] = f^2 M_{r's'}, \]
\[ [P_r, M_{st}] = \delta_{rs} P_t - \delta_{rt} P_s, \quad [P_{r'}, M_{s't'}] = \delta_{r's'} P_{t'} - \delta_{r't'} P_{s'}, \]
\[ [M_{rs}, M_{tu}] = \delta_{ru} M_{st} + \delta_{st} M_{ru} - \delta_{rt} M_{su} - \delta_{su} M_{rt}, \quad [M_{r's'}, M_{t'u'}] = \delta_{r'u'} M_{s't'} + \delta_{s't'} M_{r'u'} - \delta_{r't'} M_{s'u'} - \delta_{s'u'} M_{r't'}. \]

The odd-even commutators are given by

\[ [P_r, Q_{aa'}] = -f(\gamma_r \gamma_5)_{a}^{b} Q_{ba'}, \quad [P_{r'}, Q_{aa'}] = -\frac{1}{2}f(\gamma'_{r'})_{a'}^{b'} Q_{a'b'}, \]
\[ [M_{rs}, Q_{aa'}] = -\frac{1}{2}(\gamma_{rs})_{a}^{b} Q_{ba'}, \quad [M_{r's'}, Q_{aa'}] = -\frac{1}{2}(\gamma'_{r's'})_{a'}^{b'} Q_{a'b'}. \]

Finally, we have the odd-odd anti-commutators,

\[ \{Q_{aa'}, Q_{bb'}\} = -\gamma_5 C_{ab} \left(2(\gamma'_{r'})_{a'}^{b'} P_{r'} - f(\gamma'_{r's'})_{a'}^{b'} M_{r's'}\right) - C_{a'b'} \left(2(\gamma_r C)_{ab} P_{r} + 2f(\gamma_{rs} \gamma_5 C)_{ab} M_{rs}\right). \]

All other (anti)commutators vanish. The normalizations of the above algebra were determined by comparison with the supersymmetry algebra in the conventions of [4] in the appropriate backgrounds. In fact it is convenient for later application to recast the above results again in 11-dimensional form. The (anti)commutation relations that involve the supercharges then read as,

\[ [P_r, \bar{Q}] = \bar{Q} T_{\hat{r} \hat{s}\hat{t}\hat{u}\hat{v}} F_{\hat{s}\hat{t}\hat{u}\hat{v}}, \quad [M_{\hat{r}\hat{s}}, \bar{Q}] = \frac{1}{2} \bar{Q} \Gamma_{\hat{r}\hat{s}}, \]
\[ \{Q, \bar{Q}\} = -2\Gamma_{\hat{r}} P_{\hat{r}} + \frac{1}{144} \left[\Gamma_{\hat{r}\hat{s}\hat{t}\hat{u}\hat{v}} F_{\hat{t}\hat{u}\hat{v}\hat{w}} + 24 \Gamma_{\hat{r}\hat{s}} F_{\hat{r}\hat{s}\hat{t}\hat{u}}\right] M_{\hat{r}\hat{s}}, \]

where \( T \) is the following combination of \( \Gamma \)-matrices,

\[ T_{\hat{r}\hat{s}\hat{t}\hat{u}\hat{v}} = \frac{1}{288} \left(\Gamma_{\hat{r}\hat{s}\hat{t}\hat{u}\hat{v}} - 8 \delta_{\hat{r} \hat{s}} \Gamma_{\hat{t}\hat{u}\hat{v}}\right). \]

The above formulae are only applicable in the background where the field strength takes the form given in (1.1).

3. The coset space representations of \( \text{AdS}_4 \times S^7 \) and \( \text{AdS}_7 \times S^4 \)

Both backgrounds that we consider correspond to homogenous spaces and can thus be formulated as coset spaces [12]. In the case at hand these (reductive) coset spaces \( G/H \) are \( OSp(8\vert 4)/[SO(7) \times SO(3,1)] \) and \( OSp(6,2\vert 4)/[SO(6,1) \times SO(4)] \). To each element of the coset \( G/H \) we associate an element of \( G \), which we denote by \( L(Z) \). Here \( Z^A \) stands for the coset-space coordinates \( x^r, \theta^a \) (or, alternatively, \( x^r, y^{\nu'} \) and \( \theta^{\alpha' \nu'} \)). The coset representative \( L \) transforms from the left under constant \( G \)-transformations corresponding to the isometry group of the coset space and from the right under local \( H \)-transformations: \( L \rightarrow L' = g L h^{-1} \).
The vielbein and the torsion-free $H$-connection one-forms, $E$ and $\Omega$, are defined through
\[ dL + L \Omega = LE, \] (3.1)
where
\[ E = E^\hat{r} P_{\hat{r}} + \tilde{E} Q, \quad \Omega = \frac{1}{2} \Omega^{\hat{r}\hat{s}} M_{\hat{r}\hat{s}}. \] (3.2)
The integrability of (3.1) leads to the Maurer-Cartan equations,
\[ d\Omega - \Omega \wedge \Omega - \frac{1}{2} E^\hat{r} \wedge E^\hat{s} [P_{\hat{r}}, P_{\hat{s}}] - \frac{1}{288} E \left[ \Gamma^{\hat{r}\hat{s}\hat{u}\hat{v}\hat{w}} F_{\hat{t}\hat{u}\hat{v}\hat{w}} + 24 \Gamma_{\hat{t}\hat{u}} F^{\hat{r}\hat{s}\hat{t}} \right] E M_{\hat{r}\hat{s}} = 0, \]
\[ dE^\hat{r} - \Omega^\hat{r} \wedge E_{\hat{s}} - E \Gamma^\hat{r} \wedge E = 0, \]
\[ dE + E^\hat{r} \wedge T_{\hat{r}}^{\hat{t}\hat{u}\hat{v}} E F_{\hat{t}\hat{u}\hat{v}} - \frac{1}{4} \Omega^{\hat{r}\hat{s}} \wedge \Gamma_{\hat{r}\hat{s}} E = 0, \] (3.3)
where we suppressed the spinor indices on the anticommuting component $E^\hat{a}$. The first equation in a fermion-free background reproduces (1.2) upon using the commutation relations (2.1).

The purpose of this section is to determine the vielbeine and connections to all orders in $\theta$ for the spaces of interest. The choice of the coset representative amounts to a gauge choice that fixes the parametrization of the coset space. We will not insist on an explicit parametrization of the bosonic part of the space. It turns out to be advantageous to factorize $L(Z)$ into a group element of the bosonic part of $G$ corresponding to the bosonic coset space, whose parametrization we leave unspecified, and a fermion factor. Hence we write
\[ L(Z) = \ell(x) \hat{L}(\theta), \quad \text{with } \hat{L}(\theta) = \exp[\bar{\theta} Q]. \] (3.4)
Following [6, 7] and [13], we rescale the odd coordinates according to $\theta \rightarrow t \theta$, where $t$ is an auxiliary parameter that we will put to unity at the end. Taking the derivative with respect to $t$ of (3.1) then leads to a first-order differential equation for $E$ and $\Omega$ (in 11-dimensional notation),
\[ \dot{E} - \dot{\Omega} = d\bar{\theta} Q + (E - \Omega) \bar{\theta} Q - \bar{\theta} Q (E - \Omega) \] (3.5)
After expanding $E$ and $\Omega$ on the right-hand side in terms of the generators and using the (anti)commutation relations (2.4) we find the coupled differential equations,
\[ \dot{E}^\hat{a} = \left( d\bar{\theta} + E^\hat{r} T_{\hat{r}}^{\hat{t}\hat{u}\hat{v}\hat{w}} \bar{\theta} F_{\hat{t}\hat{u}\hat{v}\hat{w}} - \frac{1}{4} \Omega^{\hat{r}\hat{s}} \Gamma_{\hat{r}\hat{s}} \bar{\theta} \right)^\hat{a}, \]
\[ \dot{E}^\hat{r} = 2 \bar{\theta} \Gamma^\hat{r} E, \]
\[ \dot{\Omega}^{\hat{r}\hat{s}} = \frac{1}{12} \bar{\theta} \left[ \Gamma^{\hat{r}\hat{s}\hat{u}\hat{v}\hat{w}} F_{\hat{t}\hat{u}\hat{v}\hat{w}} + 24 \Gamma_{\hat{t}\hat{u}} F^{\hat{r}\hat{s}\hat{t}} \right] E. \] (3.6)
These equations can be solved straightforwardly \[\text{[7]}\]. For instance, we can determine the corresponding equations for multiple \(t\)-derivatives, after which we explicitly construct the solution by a formal Taylor expansion about \(t = 0\). Of course, this expansion will constitute only a finite series, as the \(n\)-th derivative will be proportional to the \(n\)-th power of \(\theta\). A crucial role is played by the initial conditions,

\[
\begin{align*}
E^\hat{a}|_{t=0} & = 0, \\
E^\hat{\nu}|_{t=0} & = e^{\hat{\nu}}(x) \equiv dx^\hat{\mu} e_\hat{\mu}^\hat{\nu}(x), \\
\Omega_{\hat{a}\hat{b}}^{\hat{c}}|_{t=0} & = \omega^{\hat{a}\hat{b}}(x) \equiv dx^\hat{\mu} \omega_\hat{a}\hat{b}(x),
\end{align*}
\]

where \(e_\hat{\mu}^\hat{\nu}\) and \(\omega_\hat{a}\hat{b}\) denote the vielbein and spin-connection components of the product of the \(AdS\) space and the sphere. From the initial conditions it follows that the only nonvanishing derivative at \(t = 0\) equals

\[
D\theta^\hat{a} \equiv E^\hat{a}|_{t=0} = (d\theta + e^{\hat{\nu}} T^{\hat{\nu}\hat{\mu}\hat{a}} \theta F_{\hat{\mu}\hat{a}\hat{b}} - \frac{1}{4} \omega^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}} \theta)^\hat{a}.
\]

We can now give the explicit solution (setting \(t = 1\)),

\[
\begin{align*}
E(x, \theta) & = \sum_{n=0}^{16} \frac{1}{(2n + 1)!} \mathcal{M}^{2n} D\theta, \\
E^\hat{\nu}(x, \theta) & = dx^\hat{\mu} e_\hat{\mu}^\hat{\nu} + 2 \sum_{n=0}^{15} \frac{1}{(2n + 2)!} \hat{\theta} \Gamma^\hat{\nu} \mathcal{M}^{2n} D\theta, \\
\Omega_{\hat{a}\hat{b}}^{\hat{c}}(x, \theta) & = dx^\hat{\mu} \omega_\hat{a}\hat{b}\hat{c} \\
& + \frac{1}{2} \sum_{n=0}^{15} \frac{1}{(2n + 2)!} \hat{\theta} \Gamma_{\hat{a}\hat{b}} \mathcal{M}^{2n} D\theta,
\end{align*}
\]

where the matrix \(\mathcal{M}^2\) equals \[\text{[7]}\],

\[
(\mathcal{M}^2)_{\hat{a}}^{\hat{b}} = 2 (T^{\hat{\nu}\hat{\mu}\hat{a}} \theta)^\hat{\nu} F_{\hat{\mu}\hat{a}\hat{b}} (\hat{\theta} \Gamma^\hat{\nu})_{\hat{b}} \\
- \frac{1}{288} (\Gamma_{\hat{a}\hat{b}} \theta)^\hat{a} (\hat{\theta} \Gamma_{\hat{a}\hat{b}} F_{\hat{a}\hat{b}\hat{c}} + 24 \Gamma_{\hat{a}\hat{b}} F_{\hat{a}\hat{b}\hat{c}})_{\hat{b}}.
\]

The lowest-order terms in these expansions are given by

\[
\begin{align*}
E^\hat{\nu} & = e^{\hat{\nu}} + \hat{\theta} \Gamma^\hat{\nu} d\theta + \hat{\theta} \Gamma^\hat{\nu} (e^m T^\hat{m} \hat{a}\hat{b} F_{\hat{a}\hat{b}\hat{c}} - \frac{1}{4} \omega^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}}) \theta + \mathcal{O}(\theta^4), \\
E & = d\theta + (e^{\hat{\nu}} T^{\hat{\nu}\hat{\mu}\hat{a}} F_{\hat{\mu}\hat{a}\hat{b}} - \frac{1}{4} \omega^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}}) \theta + \mathcal{O}(\theta^3), \\
\Omega_{\hat{a}\hat{b}}^{\hat{c}} & = \omega_{\hat{a}\hat{b}}^{\hat{c}} + \frac{1}{144} \hat{\theta} \Gamma_{\hat{a}\hat{b}} F_{\hat{a}\hat{b}\hat{c}} + 24 \Gamma_{\hat{a}\hat{b}} F_{\hat{a}\hat{b}\hat{c}} \theta + \mathcal{O}(\theta^4),
\end{align*}
\]

and agree completely with those obtained through gauge completion in \[\text{[4]}\] and, for the spin-connection field, in \[\text{[14]}\].
4. The four-form super field strength

The Wess-Zumino-Witten part of the action can be constructed by considering the most general ansatz for a four-form invariant under tangent-space transformations. Using the lowest-order expansions of the vielbeine (3.11) and comparing with (4) shows that only two terms can be present. Their relative coefficient is fixed by requiring that the four-form is closed, something that can be verified by making use of the Maurer-Cartan equations (3.3). The result takes the form

$$f^{(4)} = \frac{1}{24} \left[ e^\hat{r} \wedge e^\hat{s} \wedge e^\hat{t} \wedge e^\hat{u} F_{\hat{r}\hat{s}\hat{t}\hat{u}} - 12 \bar{e} \wedge \Gamma^\hat{r} e^\hat{s} \wedge e^\hat{t} \wedge e^\hat{u} \right].$$

To establish this result we also needed the well-known quartic-spinor identity in 11 dimensions. The overall factor in (4.1) is fixed by comparing to the normalization of the results given in [4].

Because \(f^{(4)}\) is closed, it can be written locally as \(f^{(4)} = dB\). To low orders in \(\theta\), the three-form \(B\) is given by [4]

$$B = \frac{1}{6} e^\hat{r} \wedge e^\hat{s} \wedge e^\hat{t} C_{\hat{r}\hat{s}\hat{t}} - \frac{1}{2} e^\hat{r} \wedge e^\hat{s} \wedge \bar{e} \bar{\Gamma}^\hat{r} d\theta + \mathcal{O}(\theta^4).$$

The general solution for \(B\) can be found by again exploiting the one-forms with rescaled \(\theta\) coordinates according to \(\theta \rightarrow t \theta\). Using (3.6) we then find

$$\frac{d}{dt}f^{(4)} = -d\left(\bar{e} \bar{\Gamma}^\hat{r} E^\hat{s} \wedge E^\hat{t} \wedge E^\hat{s}\right),$$

where again we made use of the quartic-spinor identity. This equation can directly be integrated so that we find for the three-form,

$$B = \frac{1}{6} e^\hat{r} \wedge e^\hat{s} \wedge e^\hat{t} C_{\hat{r}\hat{s}\hat{t}} - \int_0^1 dt \bar{e} \bar{\Gamma}^\hat{r} E^\hat{s} \wedge E^\hat{t} \wedge E^\hat{s},$$

which agrees to order \(\theta^2\) with the result above. Furthermore the flat-space result (obtained by setting \(F_{\hat{r}\hat{s}\hat{t}\hat{u}} = \omega^{\hat{r}\hat{s}} = 0\),

$$B = \frac{1}{6} e^\hat{r} \wedge e^\hat{s} \wedge e^\hat{t} C_{\hat{r}\hat{s}\hat{t}} - \frac{1}{2} \left[ e^\hat{r} \wedge e^\hat{s} \wedge \bar{e} \bar{\Gamma}^\hat{r} d\theta + \frac{1}{3} \bar{e} \bar{\Gamma}^\hat{r} d\theta \wedge \bar{e} \bar{\Gamma}^\hat{s} d\theta \right] \wedge \bar{e} \bar{\Gamma}^\hat{t} d\theta,$$

is correctly reproduced.

The supermembrane action is then written in terms of the superspace embedding coordinates \(Z^M(\zeta) = (X^\hat{\mu}(\zeta), \theta^\hat{a}(\zeta))\), which are functions of the world-volume coordinates \(\zeta^i (i = 0, 1, 2)\). To all orders in anticommuting coordinates in an \(AdS_4 \times S^7\) or \(AdS_7 \times S^4\) background it is thus given by

$$S = -\int d^3 \zeta \sqrt{-\det g_{ij}(Z(\zeta))} + \int_{M_3} B,$$

where the induced worldvolume metric equals \(g_{ij} = \Pi_i^\hat{r} \Pi_j^\hat{s} \delta_{\hat{r}\hat{s}}\) and \(\Pi_i = \partial Z^M / \partial \zeta^i\). The pullback of the supervielbein to the membrane worldvolume. This action is
invariant under local fermionic $\kappa$ transformations \[^2\] as well as under the superspace isometries corresponding to $osp(8|4)$ or $osp(6,2|4)$.

We have already emphasized that the choice of the coset representative amounts to adopting a certain gauge choice in superspace. The choice that we made in this paper connects directly to the generic 11-dimensional superspace results, written in a Wess-Zumino-type gauge, in which there is no distinction between spinorial world and tangent-space indices. In specific backgrounds, such as the ones discussed here, there are gauge choices possible which allow further simplifications. For instance, one could reparametrize the anticommuting coordinate $\theta$ with an $x$-dependent matrix, $\theta = \ell^{-1}(x) \theta'$, where $\ell(x)$ is the coset representative of the bosonic part of the coset space introduced earlier, but written in the spinor representation. In that case $\ell(x)$ satisfies

$$
d\ell^{-1} - \frac{1}{4} \omega_{\mathbf{r}\mathbf{s}} \Gamma_{\mathbf{r}\mathbf{s}} \ell^{-1} = -e^{\hat{r}} T_{\mathbf{r}\mathbf{s}\mathbf{u}\mathbf{v}} F_{\mathbf{s}\mathbf{u}\mathbf{v}} \ell^{-1}. \tag{4.7}$$

This equation can only be solved in a fully supersymmetric background. In this gauge (called the Killing gauge in \[^7\]) the expression for $D\theta$ is replaced by $d\theta'$, while $\mathcal{M}^2$ is appropriately redefined. This choice, combined with a suitable fixing of $\kappa$-symmetry, was reported to simplify the corresponding action for the superstring considerably \[^16\]. Observe that, in this gauge, the 32 fermionic Killing vectors are given by $\partial/\partial \theta$.

5. Discussion

In this paper we have constructed the superspace vielbein and three-form tensor gauge field for the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ solutions of 11-dimensional supergravity to all orders in anticommuting coordinates. Our results provide a strong independent check of the low-order $\theta$ results obtained previously by gauge completion for general backgrounds \[^4\] \[^14\]. A great amount of clarity was gained by expressing our results in 11-dimensional language, where we were able to cover both the $AdS_4 \times S^7$ and the $AdS_7 \times S^4$ solution in one go. Expressed in terms of the on-shell supergravity component fields of the vielbein and three-form tensor gauge field, our findings are in reassuring agreement with the gauge completion results obtained for general backgrounds. Note that in the particular background we consider here, the gravitino vanishes. We have no reasons to expect that the 11-dimensional form of our results will coincide with the expressions for a generic 11-dimensional superspace (with the gravitino set to zero) at arbitrary orders in $\theta$.

The obtained superspace geometry was then utilized to construct the complete M-theory two-brane action in $AdS_4 \times S^7$ and $AdS_7 \times S^4$ to all orders in $\theta$. This represents a further step in the program of finding the complete anti-de Sitter background actions for the superstring \[^3\] \[^4\] and the M2, D3 \[^3\] and M5-branes initiated for the bosonic part in \[^1, 3\]. Certainly our results for the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ superspace
geometries will be of use to construct the still missing M5-brane action in these backgrounds as well.

Moreover our results might prove to be a starting point for the construction of a matrix model description of M-theory in $AdS_4 \times S^7$ and $AdS_7 \times S^4$ along the lines of [17], an option that was already discussed in [4].

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References

1. P.K. Townsend, Phys. Lett. B350 (1995) 184, hep-th/9501068. Phys. Lett. B373 (1996) 68, hep-th/9512062. Four Lectures on M-theory, lectures given at the 1996 ICTP Summer School in High Energy Physics and Cosmology, Trieste, hep-th/9612121.

2. E. Bergshoeff, E. Sezgin and P.K. Townsend, Phys. Lett. 189B (1987) 75.

3. E. Cremmer, B. Julia and J. Scherk, Phys. Lett. 76B (1978) 409.

4. B. de Wit, K. Peeters and J.C. Plefka, Nucl. Phys. B, to appear, hep-th/9803209.

5. J. Maldacena, The large-N limit of superconformal theories and supergravity, hep-th/9711200.

S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B428 (1998) 105, hept-th/9802109;

E. Witten, Anti-de Sitter space and holography, hep-th/9802150.

6. R.R. Metsaev and A.A. Tseytlin, Type IIB Superstring Action in $AdS_5 \times S^5$ Background, hep-th/9805028.

7. R. Kallosh, J. Rahmfeld and A. Rajaraman, Near Horizon Superspace, hep-th/9805217.

8. R.R. Metsaev and A.A. Tseytlin, Type IIB Supersymmetric D3 brane action in $AdS_5 \times S^5$, hep-th/9806095.

9. M.J. Duff and C.N. Pope, in Supersymmetry and Supergravity ’82, eds. S. Ferrara, J.G. Taylor and P. van Nieuwenhuizen (World Scientific, 1983), 183;

B. Biran, F. Englert, B. de Wit and H. Nicolai, Phys. Lett. 124B (1983) 45; (E) 128B (1983) 461.

10. K. Pilch, P. van Nieuwenhuizen and P.K. Townsend, Nucl. Phys. B242 (1984) 377.

11. G. Dall’Agata, D. Fabbri, C. Fraser, P. Fré, P. Termonia and M. Trigiante, The $Osp(8|4)$ singleton action from the supermembrane, hep-th/9807115.
12. P. van Nieuwenhuizen, in Supersymmetry and Supergravity ’84, eds. B. de Wit, P. Fayet and P. van Nieuwenhuizen (World Scientific, 1984) 239;
   L. Castellani, R. D’Auria and P. Fré, Supergravity and Superstring Theory: a geometric perspective (World Scientific, 1990).
13. D.G. Boulware and L.S. Brown, Ann. Phys. 138 (1982) 392.
14. E. Cremmer and S. Ferrara, Phys. Lett. 91B (1980) 61.
15. P. Claus, R. Kallosh, J. Kumar, P. Townsend and A. Van Proeyen, JHEP 06 (1998) 004.
16. R. Kallosh, Superconformal actions in Killing gauge, [hep-th/9807206].
17. B. de Wit, J. Hoppe and H. Nicolai, Nucl. Phys. B305 [FS23] (1988) 545.