MCSANCee generator with one-loop electroweak corrections for processes with polarized $e^+e^-$ beams

R R Sadykov$^1$, A B Arbuzov$^{2,3}$, S G Bondarenko$^2$, Ya V Dydyshka$^1$, L V Kalinovskaya$^1$, I I Novikov$^1$, V L Yermolchyk$^1$, L A Rumyantsev$^{1,4}$

$^1$ Dzhelepov Laboratory of Nuclear Problems, JINR, Joliot-Curie str. 6, Dubna, 141980, Russia
$^2$ Bogoliubov Laboratory of Theoretical Physics, JINR, Joliot-Curie str. 6, Dubna, 141980, Russia
$^3$ Dubna State University, Universitetskaya str. 19, Dubna, 141982, Russia
$^4$ Institute of Physics, Southern Federal University, Rostov-on-Don, 344090 Russia

E-mail: sadykov@cern.ch

Abstract. A new Monte Carlo event generator MCSANCee for simulations of processes at future $e^+e^-$ colliders is presented. Complete one-loop electroweak radiative corrections and polarization of the initial beams are taken into account. The present generator includes the following processes: $e^+e^+ ightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-, ZH$. Numerical results for the $e^+e^- \rightarrow ZH$ process are shown. Plans for the further extension of the MCSANCee generator are discussed.

1. Introduction

Radiative corrections with effects due to polarization of the initial particles will play an important role in the high-precision program at the future $e^+e^-$ colliders. MCSANCee is a Monte Carlo generator of unweighted events for polarized $e^+e^-$ scattering and annihilation processes with complete one-loop electroweak (EW) corrections. The generator is based on the SANC computer system.

The scheme of the SANC framework is shown in Figure 1. Analytical expressions for form-factors and amplitudes of generalized processes $ffff \rightarrow 0$, $ffbb \rightarrow 0$ and $bbbb \rightarrow 0$ stored as FORM language expressions that are translated to Fortran modules for differential cross sections. The modules utilize Looptools and SANClib packages for evaluation of the loop integrals. The generator uses the adaptive Monte Carlo algorithm mFOAM [1], which is a part of the ROOT [2] framework.

The SANC computer system is capable to calculate cross-sections of general Standard Model (SM) processes with up to three final state particles [3,4]. By using the SANC system, we calculated electroweak radiative corrections at the one-loop level to the polarized Bhabha scattering [5,6] which is the basic normalization process at $e^+e^-$ colliders. For processes

$$e^+e^- \rightarrow \mu^-\mu^+, \tau^-\tau^+, ZH$$  \hspace{1cm} (1)

we made a few upgrades of the standard procedures in the SANC system. We investigated the effect of the polarization degrees of initial particles to the differential cross-sections. We found that the EW corrections to the total cross-section range from $-18$ percent to $+69$ percent when the centre-of-mass energy $\sqrt{s}$ varies in the set 250 GeV, 500 GeV and 1 TeV.
2. Cross-section structure
The cross-section of a generic $2 \rightarrow 2(\gamma)$ process $e^+e^- \rightarrow X_3X_4(\gamma)$ (where $X_3 X_4 = e^−e^+, \mu^−\mu^+, \tau^−\tau^+, ZH$) reads
\[
\sigma_{p^−p^+} = \frac{1}{4} \sum_{\chi_1,\chi_2} (1 + \chi_1 P^−)(1 + \chi_2 P^+) \sigma_{\chi_1\chi_2},
\]
where $\chi_i = -1(+1)$ corresponds to a lepton with left (right) helicity state.

The cross-section at the one-loop level can be divided into four parts:
\[
\sigma^\text{one-loop} = \sigma^\text{Born} + \sigma^\text{virt}(\lambda) + \sigma^\text{soft}(\lambda, \omega) + \sigma^\text{hard}(\omega),
\]
where $\sigma^\text{Born}$ is the Born level cross-section, $\sigma^\text{virt}$ is the virtual (loop) contribution, $\sigma^\text{soft}$ is due to soft photon emission, $\sigma^\text{hard}$ is due to hard photon emission (with energy $E_\gamma > \omega$). Auxiliary parameters $\lambda$ (“photon mass”) and $\omega$ cancel out after summation.

We treat all contributions using the helicity amplitudes (HA) approach:
\[
\sigma^\text{Part}_{\chi_1\chi_2} = \frac{1}{2s} \sum_{\chi_1,\chi_2 \geq 3} |\mathcal{H}^\text{Part}_{\chi_1\chi_2\chi_3\ldots}|^2 d\text{LIPS}, \tag{2}\]
where Part $\in \{\text{Born, virt, hard}\}$, and $d\text{LIPS}$ is a volume element of the Lorentz-invariant phase space.

The soft photon contribution is factorized in front of the Born-level cross-section:
\[
d\sigma^\text{soft}_{\chi_1\chi_2} = d\sigma^\text{Born}_{\chi_1\chi_2} \cdot \frac{\alpha}{2\pi} K^\text{soft}(\omega, \lambda).
\]

3. Numerical results and comparison
The following input parameters are used for numerical estimates and comparisons below
\[
\alpha^{-1}(0) = 137.03599976, \\
M_W = 80.4514958 \text{ GeV}, \\
M_Z = 91.1867 \text{ GeV}, \\
\Gamma_Z = 2.49977 \text{ GeV}, \\
m_e = 0.51099907 \text{ MeV}, \\
m_\mu = 0.10565839 \text{ GeV}, \\
m_\tau = 1.7705 \text{ GeV}, \\
m_d = 0.083 \text{ GeV}, \\
m_s = 0.215 \text{ GeV}, \\
m_b = 4.7 \text{ GeV}, \\
m_u = 0.062 \text{ GeV}, \\
m_c = 1.5 \text{ GeV}, \\
m_t = 173.8 \text{ GeV}.
\]
For comparison of the real photon emission we apply the cut on the photon energy $E_\gamma > 1$ GeV. To calculate one-loop EW RC we use the soft-hard separator $\omega \ll \sqrt{s}$.

Tuned comparison of our results for polarized Born and hard Bremsstrahlung processes with the results of WHIZARD [7] and CalcHEP [8] programs shows an agreement within statistical errors. Unpolarized soft + virtual contributions agree with the results of [9] for $e^+e^- \to \mu^+\mu^- (\tau^+\tau^-)$ and with the ones of the GRACE system [10]. For $e^+e^- \to ZH$ we found agreement with the results of the GRACE system [10] and with the ones given in paper [11].

The integrated cross-sections of the $e^+e^- \to ZH$ process and the relative corrections $\delta$ are given in Table 1 for various energies and beam polarization degrees. In this Table we summarize the estimation of the Hard, Born and one-loop cross-sections in fb and the relative corrections $\delta$ in percent for the set (0, 0; -0.8, 0; -0.6; -0.8, +0.6) of longitudinal polarizations $P_{e^-}$ and $P_{e^+}$ of the electron and positron beams, respectively. The energy values 250, 500, and 1000 GeV were taken. The relative correction $\delta$ is defined as

$$\delta = \frac{\sigma_{\text{one-loop}} - \sigma_{\text{Born}}}{\sigma_{\text{Born}}} \cdot 100\%.$$  \hspace{1cm} (3)

Fig. 2 shows the distributions of the left-right asymmetry $A_{LR}$ in $\cos \vartheta_\mu$ and $\cos \vartheta_Z$ for Born and one-loop level for $\sqrt{s} = 250, 500, 1000$ GeV where $A_{LR}$ is defined as

$$A_{LR} = \frac{\sigma(-1,1) - \sigma(1,-1)}{\sigma(-1,1) + \sigma(1,-1)}.$$  \hspace{1cm} (4)

![Figure 2](image.png)

**Figure 2.** Distribution of the left-right asymmetry $A_{LR}$ in $\cos \vartheta_Z$ for Born and one-loop level for $\sqrt{s} = 250, 500, 1000$ GeV for the $e^+e^- \to ZH$ process.
4. Conclusion

As can be seen from Table 1 the difference between values $\delta$ for polarization degrees of initial particles $(0, 0)$ and $(-0.8, 0; -0.8, -0.6; -0.8, +0.6)$ amounts to a significant value: 6-20%.

In assessing theoretical uncertainties for future $e^+e^-$ colliders, it is necessary to achieve the accuracy of approximately $10^{-4}$ for many observables. Estimating the value $\delta$ at different degrees of polarization of the initial states, we see that taking into account beam polarization is crucial.

Further development of the process library of the Monte-Carlo generator MCSAnGee involves $e^+e^{\gamma} \rightarrow Z\gamma$, $e^+e^\gamma \rightarrow \gamma\gamma$, $\gamma\gamma \rightarrow e^+e^-$ and $\gamma\gamma \rightarrow \gamma\gamma$ processes. For 4-fermion processes we have started the implementation of higher-order corrections through the $\Delta\rho$ parameter as well as the implementation of multiphoton emission contributions.

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Table 1. Hard ($E_\gamma > 1$ GeV), Born and one-loop cross sections in fb and relative correction $\delta$ in % for various energies and polarizations of the initial particles in the $e^+e^- \rightarrow ZH$ process.