Newtonian noise cancellation in tensor gravitational wave detector

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Abstract. Terrestrial gravity noise produced by ambient seismic and infrasound fields poses one of the main sensitivity limitations in low-frequency ground-based gravitational-wave (GW) detectors. This noise needs to be suppressed by 3–5 orders of magnitude in the frequency band 10 mHz to 1 Hz, which is extremely challenging. We present a new approach that greatly facilitates cancellation of gravity noise in full-tensor GW detectors. It makes explicit use of the direction of propagation of a GW, and can therefore either be implemented in directional searches for GWs or in observations of known sources. We show that suppression of the Newtonian-noise foreground is greatly facilitated using the extra strain channels in full-tensor GW detectors. Only a modest number of auxiliary, high-sensitivity environmental sensors is required to achieve noise suppression by a few orders of magnitude.

1. Introduction

The Newtonian noise (NN) generated by moving local masses poses a formidable challenge to approaching the detector noise limit in gravitational-wave (GW) detectors at frequencies below 1 Hz. At low frequencies, the NN is dominated by Rayleigh waves and infrasound waves. For a laser interferometer to overcome this noise below 1 Hz, the ground and air motion within tens of kilometers from the detector must be measured with a large number of seismometers and microphones with sufficient accuracy, and then the induced NN computed and subtracted from the detector output. The NN from Rayleigh waves could be canceled up to one part in \(10^3\) by using this method [1]. But for infrasound waves, cancellation works only for waves coming in certain favorable directions.

In contrast, by using its tensor nature, SOGRO can mitigate the NN from both Rayleigh waves and infrasound waves to one part in \(10^3\) for all incident angles. A detailed analysis of NN mitigation for SOGRO has been published elsewhere [2]. In this paper, we summarize the result and discuss the possibility of using mini-SOGROs to mitigate NN for advanced laser interferometer GW detectors.

2. Mitigation of NN on SOGRO

Assuming that the interferometer is underground at depth \(z < 0\), the gravitational perturbation of a single test mass due to a Rayleigh wave incident at an angle \(\psi\) with respect to the sensitive axis \(x\) of the test mass and an infrasound wave incident in direction \((\psi, \delta)\) is given [1] by

\[
X(\omega) = -2\pi i \cos \psi G \rho_0 y_R \frac{\xi(\omega)}{\omega^3} \exp \left( \frac{\omega z}{c_R} \right) - 4\pi i G \sin \delta \cos \psi \frac{\delta \rho(\omega) c_{IS}}{\omega^3} \exp \left( \frac{\omega z}{c_{IS}} \sin \delta \right),
\]

(1)
where $\xi(\omega)$ and $\delta \psi(\omega)$ are the vertical ground displacement and atmospheric density fluctuation directly above the test mass. $\gamma_a \approx 0.83$ is a factor that accounts for the partial cancellation for the Rayleigh NN from surface displacement by the sub-surface compressional wave content of the wave field. $c_s \approx 3.5$ km/s and $c_p \approx 330$ m/s are the speed of the Rayleigh waves underground and the infrasound waves, respectively, and $\rho_0$ is the mean mass density of the ground.

The metric perturbation tensor in the detector coordinates can be shown to be

$$h_\gamma(\omega) = [a(\omega)\xi(\omega) + b(\omega,\vartheta)\rho(\omega)] \begin{pmatrix} \cos^2\psi & \cos\psi\sin\psi & -i\cos\psi \\ \cos\psi\sin\psi & \sin^2\psi & -i\sin\psi \\ -i\cos\psi & -i\sin\psi & -1 \end{pmatrix},$$

(2)

where

$$a(\omega) = \frac{2\pi G \rho_0 \gamma_a}{\omega c_R} \exp\left(\frac{\omega}{c_R} z\right), \quad b(\omega,\vartheta) = \frac{4\pi G}{\omega^2} \sin^2\vartheta \exp\left(\frac{\omega}{c_{IS}} z \sin \vartheta\right).$$

(3)

Consider a GW coming from ($\theta, \phi$) direction in the presence of multiple Rayleigh and infrasound waves. The full strain tensor in the GW coordinates has the form:

$$h'(\omega) = \begin{pmatrix} h_\gamma(\omega) + h'_{N11}(\omega) & h_\gamma(\omega) + h'_{N12}(\omega) & h'_{N13}(\omega) \\ h_\gamma(\omega) + h'_{N22}(\omega) & -h_\gamma(\omega) + h'_{N22}(\omega) & h'_{N33}(\omega) \\ h'_{N11}(\omega) & h'_{N23}(\omega) & h'_{N33}(\omega) \end{pmatrix},$$

(4)

with

$$h'_{N11}(\omega) = \sum_i [a(\omega)\xi_i(\omega) + b(\omega,\vartheta_i)\rho_i(\omega)]\cos(\psi_i - \phi)\cos \theta + i\sin \theta]^2,$$

(5a)

$$h'_{N22}(\omega) = \sum_i [a(\omega)\xi_i(\omega) + b(\omega,\vartheta_i)\rho_i(\omega)]\sin^2(\psi_i - \phi),$$

(5b)

$$h'_{N33}(\omega) = \sum_i [a(\omega)\xi_i(\omega) + b(\omega,\vartheta_i)\rho_i(\omega)]\cos(\psi_i - \phi)\sin \theta - i\cos \theta]^2,$$

(5c)

$$h'_{N12}(\omega) = \sum_i [a(\omega)\xi_i(\omega) + b(\omega,\vartheta_i)\rho_i(\omega)]\sin(\psi_i - \phi)\cos(\psi_i - \phi)\cos \theta + i\sin \theta],$$

(5d)

$$h'_{N23}(\omega) = \sum_i [a(\omega)\xi_i(\omega) + b(\omega,\vartheta_i)\rho_i(\omega)]\sin(\psi_i - \phi)\cos(\psi_i - \phi)\sin \theta - i\cos \theta],$$

(5e)

$$h'_{N13}(\omega) = \sum_i [a(\omega)\xi_i(\omega) + b(\omega,\vartheta_i)\rho_i(\omega)]\cos(\psi_i - \phi)\cos \theta + i\sin \theta]\cos(\psi_i - \phi)\sin \theta - i\cos \theta].$$

(5f)

Due to the transverse nature of the GW, $h'_{13}$, $h'_{23}$ and $h'_{33}$ contain only the NN components. To recover $h_\gamma(\omega)$ and $h_\gamma(\omega)$, the NN could be removed from $h'_{11}$ and $h'_{12}$ by correlating them with $h'_{13}$, $h'_{23}$ and $h'_{33}$, and possibly also with some CM channels and subtracting the correlated parts.

Figure 1 shows the residual NN achieved for Rayleigh waves in the absence of infrasound waves by using $h'_{13}$, $h'_{23}$, $h'_{33}$ and $a$, plus seven seismometers with signal-to-noise ratio (SNR) of $10^4$ at the radius of 5 km as the input of the Wiener filter. The NN has been removed to about $10^{-2}$ with environmental sensors (seismometers) alone. The local channels of SOGRO improve the noise significantly only near $\theta = \pi/2$, where the noise of the DM and CM channels drop out.

Figure 2 is the residual NN achieved for infrasound waves in the absence of Rayleigh waves by using $h'_{13}$, $h'_{23}$, $h'_{33}$ and 15 microphones of SNR of $10^3$, one at the detector, seven each at the radius of 600 m and 1 km around the detector. With the environmental sensors (microphones) alone, the NN cannot be mitigated except at $\theta = 0, \pi/2$ and $\pi$. This is because the infrasound waves come from a half space and the microphones deployed over a surface is insufficient to measure the effect of 3D density variations of the atmosphere. Thus mitigation of infrasound NN constitutes a formidable challenge for laser interferometers. In SOGRO, the vertical strain component $h'_{33}$ largely makes up for this deficiency. With the aid of the local strain channels, the NN has been rejected to $10^{-2}$ for all $\theta$. 
3. Mitigation of NN on interferometers with the aid of SOGROS

Since SOGRO is a very sensitive gravity strain gauge, one may be able to employ scaled-down SOGROs with arm-length $\ell \ll L$, in place of a large array of seismometers, to directly measure and remove the NN affecting the interferometer test masses. We restrict our discussion to underground detectors like KAGRA [3] or Einstein Telescope (ET) [4].

The Rayleigh waves are expected to dominate the NN for an underground detector [1]. In the presence of a GW and Rayleigh waves, the arm-length along the x axis is modulated by

$$\Delta L = hL + X_R(x_2) - X_R(x_1),$$

where $X_R(x_i)$ is the first term of equation (1) summed over multiple waves for the $i$-th test mass on the x axis. At 10 Hz, the Rayleigh wave length becomes $\lambda_R \sim 350 \text{ m} \ll L$, causing $X_R(x_1)$ and $X_R(x_2)$ to be uncorrelated. Hence we need to measure $X_R(x_i)$ for each test mass by using a separate SOGRO co-located with it, as shown in figure 3.

From equations (1) and (2), we find that the 13-component of the SOGRO output is given by

$$\eta_{13}(x_i) = h_{13} + (i\omega/c_R)X_R(x_i).$$

We solve equation (7) for $X_R(x_i)$ and substitute it into equation (6) to obtain

$$h = \frac{\Delta L}{L} - \frac{c_R}{\omega L} \left[ \eta_{13}(x_2) - \eta_{13}(x_1) \right].$$

The sensitivity required for mini-SOGRO to recover $h$ is then given by

$$\eta = \frac{\Delta L}{\ell} = \frac{1}{\sqrt{2}} \frac{\omega L}{c_R} h.$$
Figure 4 shows the sensitivity goals of advanced LIGO (aLIGO) [5] and ET [4]. The shaded region represents the parameter space dominated by the NN. A worthy goal would be rejecting the NN by an order of magnitude to $h \sim 10^{-22}$ Hz$^{-1/2}$ at 3 Hz and to $10^{-23}$ Hz$^{-1/2}$ at 10 Hz. For ET with $L = 10$ km, equation (9) yields $\eta \approx 4 \times 10^{-22}$ Hz$^{-1/2}$ at 3 Hz and $1.3 \times 10^{-21}$ Hz$^{-1/2}$ at 10 Hz. The NN between SOGRO test masses must be highly correlated. According to Beker et al. [6],

$$S = \frac{1}{\sqrt{1 - C_{SN}^2}} \leq \frac{c_R}{\omega \ell},$$

where $S$ is the mitigation factor and $C_{SN}$ is the correlation between the test masses. To obtain $S = 10$ at $10$ Hz, we need $C_{SN} = 0.995$ and $\ell \leq c_R / \omega S = 5.6$ m. Mitigating the NN for ground detectors is much more challenging since the low speed of the Rayleigh waves on the surface, $c_R \approx 250$ m/s, reduces $\ell$ to $\leq 0.4$ m. Such a small SOGRO would hardly have enough sensitivity.

Figure 5 shows the instrument noise spectral density for SOGRO with $\ell = 5$ m, $M = 1$ ton (each test mass), and $Q = 5 \times 10^8$ cooled to $0.1$ K and coupled to a dc SQUID with $2h$ noise. The expected sensitivity of SOGRO comes to within a factor of 2 from that required for $S = 10$. The same SOGRO with $Q = 10^9$ coupled to a $1h$ SQUID would meet the sensitivity requirement, provided all the other noise could be reduced to below its intrinsic noise limit. Should these sensitivities be achieved, SOGRO could make it possible to construct ET less deep.

It is interesting to see how a mini-SOGRO two orders of magnitude less sensitive to GWs can help ET mitigate the NN by an order of magnitude. This is because a SOGRO with $\ell = 5$ m is quite efficient to detect the Rayleigh NN with $\lambda_R / 2\pi = 56$ m and SOGRO employs a highly sensitive superconducting displacement sensor. Although achieving a test mass $Q$ of $10^8$ and reaching the quantum limit for the SQUID noise is very challenging, it is worth investigating the SOGRO option since it has intrinsic advantages over seismometers in that it detects the NN directly and can monitor the local gravity gradient environment with high sensitivity.

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