Bose-Einstein condensation of trapped atoms with dipole interactions

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The path integral Monte Carlo method is used to simulate dilute trapped Bose gases and to investigate the equilibrium properties at finite temperatures. The quantum particles have a long-range dipole-dipole interaction and a short-range $s$-wave interaction. Using an anisotropic pseudopotential for the long-range dipolar interaction and a hard-sphere potential for the short-range $s$-wave interaction, we calculate the energetics and structural properties as a function of temperature and the number of particles. Also, in order to determine the effects of dipole-dipole forces and the influence of the trapping field on the dipolar condensate, we use two cylindrically symmetric harmonic confinements (a cigar-shaped trap and a disk-shaped trap). We find that the net effect of dipole-dipole interactions is governed by the trapping geometry. For a disk-shaped trap, the net contribution of dipolar interactions is attractive and the shrinking of the density profiles is observed. For a disk-shaped trap, the net effect of long-range dipolar forces is repulsive and the density profiles expand.

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I. INTRODUCTION

The experimental observation of Bose-Einstein condensation (BEC) in dilute trapped and supercooled atomic vapors \cite{1} has stimulated enormous experimental and theoretical interest in the physics of the weakly interacting Bose gas (see \cite{2} for a review). Especially intriguing is the role of dimensionality and two-body interactions. Lower dimensional atomic gases have been realized with strong quantum confinement in one or more directions. These systems exhibit very peculiar properties, such as the exact mapping between interacting bosons and noninteracting fermions (Tonks-Girardeau gas) \cite{8}, because the role of quantum fluctuations and strong correlations is enhanced. Although these gases are very dilute, BEC of trapped atomic gases is strongly influenced by two-body interactions. Within a mean-field approximation, the strength of the interatomic interactions can be typically characterized by a single parameter, the $s$-wave scattering length $a_s$, at very low temperatures. This van der Waals type interaction is isotropic and short-range in comparison to the average interparticle separation. Accordingly, Bose-Einstein condensates created thus far have been very accurately described by the Gross-Pitaevskii theory (see \cite{2} for a review), a mean-field approach. Furthermore, it has been possible to tune the $s$-wave scattering length $a_s$ to essentially any value, positive or negative, by utilizing a magnetic atom-atom Feshbach resonance \cite{4}.

Studies of the influence of the interparticle interactions on the properties of BEC have focused mainly on the short-range interactions \cite{2}. However, understanding new kinds of systems will require the investigation of the effects of additional interactions, since Bose-Einstein condensation of molecules has now been observed using new techniques for creating ultracold molecules \cite{5}. In addition, Ling et al. \cite{6} proposed an efficient method to convert atomic condensates into stable molecules using a generalized Raman adiabatic passage scheme. Very recently, the generation of a BEC in a gas of $^{52}$Cr atoms was reported \cite{7}. As molecules can potentially have large dipole moments and a chromium atom has a very high magnetic moment of $\mu_B$ (Bohr magneton) in its ground state, BEC with dipole-dipole interactions in ultracold gases has attracted considerable theoretical attention over the past few years \cite{8,9,10,11,12,13}. Compared to the short-range interaction, the dipole-dipole interactions are long-range, anisotropic, and partially attractive. As a result, nonlinear effects due to long-range dipole-dipole interactions lead to new interesting properties; different quantum phases such as superfluid, supersolid, Mott insulator, checkerboard, and collapse phases \cite{12}, a self-bound Bose condensate in the field of a traveling wave \cite{13}, a roton minimum in the excitation spectrum \cite{13,14}, and promising candidates for the implementation of fast and robust quantum-computing schemes \cite{10}. First of all, Yi and You \cite{8} suggested an effective pseudopotential for anisotropic dipolar interactions in the Born approximation and recently Derevianko \cite{10} derived a more rigorous velocity-dependent pseudopotential for dipole-dipole interactions. Santos et al. \cite{11} showed a strong dependence of the stability of trapped dipolar BEC on the trapping geometry. O’Dell et al. \cite{15} derived exact results in the Thomas-Fermi regime for the statics and dynamics of a trapped dipolar BEC. However, in all the theoretical studies so far the $T=0$ K situation has been investigated. In this paper we address the properties of trapped condensates with dipolar interactions at finite temperatures. Several sources for dipolar BEC are atoms polarized by an electric field or atoms with large magnetic moments in addition to polar molecules. The magnitude and sign of dipole-dipole interactions can be changed by rapidly rotating an external field \cite{16}. Thus, by varying the strength of the dipolar interactions relative to the $s$-wave scattering (tuning the dipolar interaction or changing the...
s-wave scattering length $a_s$) it is possible to study the effects of the dipolar interaction on a condensate in detail.

In this paper we attempt to understand Bose-Einstein condensation of trapped dipolar gases, where the inter-particle interaction consists of a long-range dipole-dipole interaction in addition to the usual short-range s-wave interaction. We as-

cles have both a long-range dipole-dipole interaction and condensation in three dimensions. The quantum parti-
system to consider a harmonically trapped Bose-Einstein simulation results. Finally, we conclude in Sec. IV.

Our paper is structured as follow. Section II introduces the description of the system and method. Our paper briefly is also reviewed in Sec. II. Section III presents our approach. The path integral Monte Carlo technique describes anisotropic dipolar interactions in the Born approximati-
tion. The path integral Monte Carlo technique briefly is also reviewed in Sec. II. Section III presents our simulation results. Finally, we conclude in Sec. IV.

II. DESCRIPTION OF THE SYSTEM AND SIMULATION METHOD

We wish to study the problem of a quantum $N$-particle system to consider a harmonically trapped Bose-Einstein condensation in three dimensions. The quantum particles have both a long-range dipole-dipole interaction and the usual short-range hard-sphere interaction. We assume that all atomic dipole moments are equal and orien-
ted along the $z$-axis (trap axis). Within the Born approxima-
tion, the dipole-dipole interaction potential between two dipoles separated by $r_{ij}$ is given by

$$v^d(r_{ij}) = \frac{\mu^2(1 - 3 \cos^2 \theta_{ij})}{r_{ij}^3}. \quad (1)$$

where $\mu$ characterizes the dipole moment, $r_{ij} = r_i - r_j$ is the vector between the dipoles $i$ and $j$, and $\theta_{ij}$ the angle between the vector $r_{ij}$ and the direction of the dipoles ($z$). To date, this pseudopotential has worked reasonably well for the long-range dipole-dipole interaction. The corresponding Hamiltonian is then

$$H = H_0 + \sum_{i>j}^N v^d(r_{ij}) + \sum_{i>j}^N v(r_{ij}) \quad (2)$$

$$H_0 = -\frac{\hbar^2}{2m} \sum_{i=1}^N \nabla_i^2 + \frac{1}{2}m \sum_{i=1}^N (\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2)$$

where $H_0$ is the Hamiltonian for trapped ideal boson gases and $v(r)$ is the hard-sphere potential defined by

$$v(r) = +\infty \quad (r < a_s)$$

In order to investigate the dependence of the sign and the value of the dipole-dipole interaction energy on the trapping geometry, we consider Bose gases under a cylindrically harmonic confinement ($\omega_x = \omega_y = \lambda \omega_z$ and $\omega_z$), where $\lambda$ is the aspect ratio.

Consider a system of $N$ dipolar hard spheres. The statistical mechanics of a quantum system at an inverse temperature $\beta$, where $\beta = (k_B T)^{-1}$ and $k_B$ denotes the Boltzmann constant, is governed by the density matrix. The Bose-symmetrized density matrix is given by

$$\rho_B(\mathcal{R}, \mathcal{R}'; \beta) = \frac{1}{N!} \sum_{\mathcal{P}} \rho(\mathcal{R}, \mathcal{P}\mathcal{R}' \beta), \quad (4)$$

where $\mathcal{R}$ and $\mathcal{R}'$ are two configurations of $N$ dipolar hard spheres, which denote $3N$ dimensional vectors, $\mathcal{R} = (R_1, \ldots, R_N)$. $\mathcal{P}$ denotes a permutation of particle labels among hard spheres and $\mathcal{P}$ is one such permutation. The inclusion of particle permutations is crucial and superfluidity is an immediate consequence of Bose symmetry. However, evaluating the density matrix for interacting systems at very low temperatures is complicated by the fact that the kinetic and potential terms in the exponent of the density matrix do not commute. In order to avoid this problem, we insert $M-1$ intermediate configurations into Eq. 4:

$$\rho(\mathcal{R}, \mathcal{P}\mathcal{R}' \beta, \tau) = \int \cdots \int d\mathcal{R}_1 d\mathcal{R}_2 \cdots d\mathcal{R}_{M-1} \times \rho(\mathcal{R}, \mathcal{R}_1; \tau) \cdots \rho(\mathcal{R}_{M-1}, \mathcal{P}\mathcal{R}' \beta, \tau), \quad (5)$$

where $\tau = \beta/M$ is the imaginary time step and $M$ is referred to as the number of time slices. Eq. 5 is the path-integral formulation of the density matrix, which contains the discrete sum over permutations and multiple integrations over density matrices at a higher temperature ($k_B \tau)^{-1}$. Eq. 5 can be evaluated in the path integral Monte Carlo technique by a stochastic sampling of the discrete paths $\{\mathcal{R}, \mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_{M-1}, \mathcal{P}\mathcal{R}' \beta, \tau\}$ using multilevel Monte Carlo sampling [16, 21], which accounts for permutations. In order to use Monte Carlo sampling, we must first provide a pair-product form of the exact two-body density matrices. For the short-range hard-
sphere potential, we used the high-temperature approximation for the hard-sphere propagator derived by Cao and Berne [21, 22]. Unfortunately, it is impractical to de-

velop an analytic pair-product form for the anisotropic dipole-dipole interaction potential. Thus the primitive approximation, which separates the density matrix into kinetic and potential energy terms, was used. In order to choose the value of the number of time slices $M$, we performed consistency checks by varying $M$ to see that the results had converged. We used up to 160 time slices ($M = 160$) depending on the temperature and the number of particles. We employed the canonical ensemble, i.e. in each simulation we fixed the temperature $T$, the number of particles $N$, the diameter of a hard sphere.
and the strength of the dipolar interactions $\mu$. The thermal expectation value of an operator $O$ is given by $\langle O \rangle = \text{Tr}(\rho B O)/\text{Tr}(\rho B)$. Expectation values are calculated at the same time the discrete paths are sampled. Typically $20,000 - 30,000$ MC steps are required for equilibration and statistical averages are collected from $150,000 - 180,000$ MC steps after this. In each MC step, we attempted $60 - 1,250$ trial moves at each time slice. Multiple runs were performed for each set of parameters so that statistical error bars could be determined. We performed these calculations partly on IBM Power4 and partly on Athlon processors at the University of Georgia. For $N = 125$ dipolar particles and $\tau = 0.00446$, the simulation (thermalization + computing averages) at a single temperature took about 2100 IBM Power4 CPU hours. The density profile $n(R)$, which indicates the likelihood of finding particles at a given distance $R$ from the center of the trap, is defined as

$$n(R) = \frac{1}{M} \sum_{\kappa=0}^{M-1} \sum_{i=1}^{N} \delta(R_{i,\kappa} - R/R^2),$$  \hspace{1cm} (6)

where $R_{i,\kappa}$ denotes the distance between the position of the $i$th particle and the center of the trap at time slice $\kappa$ and the symbol $\langle \cdot \rangle$ denotes an ensemble average. The superfluid fraction along the axis of rotation $n$ is given by

$$\rho_s/\rho = \frac{4m^2}{\beta\hbar^2 T \langle A_n^2 \rangle},$$  \hspace{1cm} (7)

where $A_n = A \cdot n$ denotes the projected area,

$$A = \frac{1}{2} \sum_{\kappa=0}^{M-1} \sum_{i=1}^{N} (R_{i,\kappa} \times R_{i,\kappa+1}),$$  \hspace{1cm} (8)

and $I^0$ the classical moment of inertia,

$$I^0 = \frac{1}{M} \sum_{\kappa=0}^{M-1} \sum_{i=1}^{N} \left( n \times R_{i,\kappa} \right) \cdot \left( n \times R_{i,\kappa+1} \right).$$  \hspace{1cm} (9)

### III. SIMULATION RESULTS AND DISCUSSION

The path integral Monte Carlo (PIMC) method allows one to calculate accurate quantum mechanical expectation values of many-body systems, the only input being the many-body potential. Here, we use PIMC to simulate a system composed of dipolar hard spheres for various values of the number of hard spheres $N$ and the trap aspect ratio $\lambda$ at two fixed dipolar moments $\mu$ as a function of temperature $T$ and to obtain simulation results for a Bose-Einstein condensation with magnetic dipole-dipole forces. This section discusses the energetics and structural properties of trapped dipolar hard spheres obtained from our PIMC simulations. For simplicity, throughout this paper, we use a cylindrically symmetric harmonic trap with an aspect ratio $\lambda$, $\lambda = \omega_x/\omega_z$ and $\omega_x = \omega_y$. The approximate critical temperature $T^{3D}_c$ for an ideal gas of $N$ atoms in a trap is given by

$$1 = \left( \frac{T^{3D}_c}{T^0} \right)^3 + \frac{3\omega_0 g(2)}{2\omega_0 g(3)} \left( \frac{T^{3D}_c}{T^0} \right)^2 N^{-1/3}.$$  \hspace{1cm} (10)

$$T^{3D}_0 = \frac{\hbar \omega_0}{k_B} \left( \frac{N}{g(3)} \right)^{1/3},$$

where $T^{3D}_0$ is the critical temperature for an ideal Bose gas in a trap in the thermodynamic limit, $g(n)$ is the Riemann zeta function, and $\omega = (\omega_x + \omega_y + \omega_z)/3$ and $\omega_0 = (\omega_x \omega_y \omega_z)^{1/3}$ are the arithmetic average and the geometric average of the trap's frequencies, respectively.
The effects of dipolar interactions can be made visible, respectively. A measure of the strength of the dipolar interaction relative to the s-wave scattering in general is given by
\[ \varepsilon_{dd} = \frac{\mu^2 m}{3\hbar^2 a_s}. \] (11)

For chromium, the more common bosonic isotope, \(^{52}\text{Cr}\), has \(\varepsilon_{dd} \approx 0.089\) and the less common bosonic isotope, \(^{50}\text{Cr}\), has \(\varepsilon_{dd} \approx 0.36\) \([24]\). In this study, in order that the effects of dipolar interactions can be made visible, we used \(m = m(^{87}\text{Rb})\), \(\omega_z (\omega_z) = 2\pi \times 77.78\, \text{Hz}\), \(\omega_x = 10\, (\omega_y = 10\, \omega_z)\), the s-wave scattering length \(a_s = 10\, a_{\text{Rb}} = 0.0433\, a_z (a_x)\), and the dipole moment \(\mu = 0.2\) \((\hbar\omega_z a_x^3)^{1/2} ((\hbar\omega_x a_z^3)^{1/2})\) for a cigar-shaped trap (a disk-shaped trap) as an example for numerical calculations, giving \(\varepsilon_{dd} \approx 0.31\). Here, \(a_{\text{Rb}} = 100\, a_0\) (Bohr radii) is the s-wave scattering length of \(^{87}\text{Rb}\), \(a_z = \sqrt{\hbar/m\omega_z}\), and \(a_x = \sqrt{\hbar/m\omega_x}\). For these parameters, the short-range interactions and the dipole-dipole ones have a comparable strength, and we can see the interplay between short-range and dipole-dipole interactions and the deviation between the case with and without dipole-dipole interactions. The results are scalable. Moreover, the magnitude and sign of the values of \(a_s\) and \(\mu\) can be easily tuned by Feshbach resonance \([3]\) and by an external field \([13]\), respectively.

### A cigar-shaped trap

We first consider a 3D dipolar Bose gas in a cigar-shaped trap with \(\lambda = \omega_z/\omega_x = 10\) and \(\omega_x = \omega_y\) by increasing the confinement in the transverse direction. The length unit is \(a_z = \sqrt{\hbar/m\omega_z}\) and energies are measured in units of \(\hbar\omega_z\). Figure 1(a) shows the calculated density profiles \(n(\rho)\) as a function of \(\rho\), normalized such that \(\int_0^\infty n(\rho)d\rho = 1\) and \(\rho = \sqrt{x^2 + y^2}\), at \(T = 0.4\, T_c^{3D}\) for three different numbers of particles, \(N = 27\) (circles), 64 (diamonds), and 125 (squares). For comparison, we used two values \(\mu = 0\) and \(\mu = 0.2\). Filled symbols indicate the density profiles \(n(\rho)\) for \(\mu = 0.2\) and open symbols the density profiles \(n(\rho)\) for \(\mu = 0\). In all figures, when statistical errors cannot be seen on the scale of the figure, the error bars are smaller than the symbol sizes. Qualitatively, the density profiles for \(\mu = 0\) and for \(\mu = 0.2\) are similar in shape, as predicted. However, the density profiles \(n(\rho)\) for \(\mu = 0\) are broader than those for \(\mu = 0.2\).
The density profiles in contrast to the density profile for different numbers of particles are also shown in Fig. 1(b). A thick solid line in (b) is the density profile of particles growing. In addition, a thick solid line in Fig. 2 shows the total energy per particle $E/N$ at $T = 0.4 T_c^{3D}$ for $\mu = 0$ (open circles), $\mu = 0.2$ (filled circles), and an ideal Bose gas (open square).

The difference with respect to the density profile $n(\rho)$ without dipolar interactions is clearly seen as the number of particles increases. The density profiles $n(z)$ along the axial direction as a function of $z$, normalized such that $\int_{-\infty}^{\infty} n(z)dz = 1$, at $T = 0.4 T_c^{3D}$ for three different numbers of particles are also shown in Fig. 1(b). In contrast to the density profile $n(\rho)$, the difference in the density profiles $n(z)$ between for $\mu = 0$ (open symbols) and for $\mu = 0.2$ (filled symbols) is small as the number of particles grows. In addition, a thick solid line in Fig. 1(a) shows the density profile $n(\rho)$ for an ideal Bose gas for comparison. The density profile $n(\rho)$ for the interacting gas approaches that for an ideal gas as the number of particles $N$ decreases. Correspondingly the excitations in the longitudinal direction are largely frozen out and the system behaves quasi-one dimensionally. Our density profile studies support the facts that i) as we add more particles into the gas, the density profiles expand; the half-width of the density profile increases with the number of particles $N$, as shown in the inset of Fig 1(a) and ii) the net effect of dipole-dipole interactions in the cigar-shaped trap is attractive, i.e. shrinking of the density profiles is observed. To further characterize the behavior of particles with dipolar interactions at $T = 0.4 T_c^{3D}$, Fig. 2 shows the total energy per particle $E/N$ of the system as a function of the number of particles $N$. The total energy per particle is strongly affected by the particle-particle interaction and the overall dipole-dipole interaction is attractive in the cigar-shaped trap, because the total energy per particle for $\mu = 0$ is smaller than that for $\mu = 0$. From Fig. 2 we can clearly see the dependence on the number of particles $N$, and numerical calculations have previously shown that the finite-size effects are significant for rather small values of $N$.

The difference in the total energy per particle $E/N$ between for an ideal Bose gas and for an interacting gas increases with increasing $N$, which is a consequence of the expansion of the density profiles with interaction, as shown in Fig. 1, although the ideal bosonic gas energy per particle $E/N$ changes considerably less as the number of particles $N$ increases. Accordingly, the dependence of the total energy per particle $E/N$ on the number of particles $N$ reflects essentially the interaction energy alone, as the kinetic energy depends weakly on $N$. Figure 3 shows the dependence of the density profiles $n(\rho)$ on the temperature $T$ for $N = 64$. As the temperature increases, the effect of interactions is vanishingly small. At $T = 2.0 T_c^{3D}$, the density profiles $n(\rho)$ for $\mu = 0$ and for $\mu = 0.2$ are indistinguishable and the density profiles have the Gaussian profiles of a pure thermal distribution. Finally,
we calculate the superfluid fraction $\rho_s/\rho$ along the axis of rotation $z$ and in Fig. 4, we plot the superfluid fraction $\rho_s/\rho$ as a function of the number of particles $N$ at $T = 0.4 \, T_{c}^{3D}$ for $\mu = 0$ (open circles), $\mu = 0.2$ (filled circles), and an ideal Bose gas (open squares).

In addition, Fig. 4 shows the superfluid fraction $\rho_s/\rho$ for an ideal Boson gas. In general, the superfluid fraction is essentially unity at very low temperatures and decreases gradually with increasing temperature. For the ideal gas, the superfluid fractions show at most a very weak dependence on the number of particles considered. However, there are significant finite-size effects for the interacting gas. As the presence of repulsive interactions has the effect of lowering the value of the critical temperature, the superfluid fraction for the interacting gas is smaller than that for the ideal gas at the same temperature $T/T_{c}^{3D}$, as shown in Fig. 4. The dependence of the superfluid fraction $\rho_s/\rho$ on the number of particles increases with the number of particles $N$. It is not easy, however, to see the difference between the superfluid fractions $\rho_s/\rho$ for $\mu = 0.2$ (filled circles) and those for $\mu = 0$ (open circles) because of the size of the errorbars but the difference must clearly be small, if any.

### B. A disk-shaped trap

For the disk-shaped trap with the asymmetry factor $\lambda = \omega_z/\omega_x = 0.1$, i.e. increasing the confinement in the axial direction, the length unit is $a_x = \sqrt{\hbar/m\omega_x}$ and the energy unit $\hbar\omega_x$. In Fig. 5, we plotted the density profiles $n(\rho)$ and $n(z)$ as a function of $\rho$ and $z$, respectively, at $T = 0.4 \, T_{c}^{3D}$ for three different numbers of particles, $N = 27$ (circles), 64 (diamonds), and 125 (squares) using $\mu = 0$ (open symbols) and $\mu = 0.2$ (filled symbols). It is compared to corresponding density profiles $n(\rho)$ and $n(z)$ for a Bose gas with short-range interactions only and for an ideal bosonic gas. Compared to the density profiles for $\lambda = 10$, they are similar in shape. As $\omega_z$ is larger than $\omega_x$, the density profile $n(z)$ for an interacting gas approaches that for an ideal gas (see Fig 5(b)), indicating the excitations in the axial direction are largely frozen out and the system behaves quasi-two dimensionally. In a disk-shaped trap, the density profiles expand along both the $z$-axis and the $\rho$-axis as we increase $\mu$, in agreement with those found by other authors. Correspondingly, the net contribution of long-range dipolar forces is repulsive, in contrast to the cigar-shaped trap. Simulations for other values of $\lambda$ show that BEC in a disk-shaped trap is stable, even in very asymmetric traps, because the dipole-dipole interaction energy always remains positive. Figure 6 shows the total energy per particle $E/N$ as a function of the number of particles $N$ both for interacting atoms with $\mu = 0$ (open circles) and $\mu = 0.2$ (filled circles) and for noninteracting particles (open squares). The total energy per particle $E/N$ for $\mu = 0.2$ is larger than that for $\mu = 0$ due to the increasing repulsive interactions. The total energy per particle $E/N$ for an interacting Bose gas increases more rapidly than that for an ideal bosonic gas due to the finite-size effect, as $N$ increases. Therefore, in the Thomas-Fermi limit the kinetic energy of the particles in the trap is negligible in comparison to the interparticle interaction energy and the trapping potential. In addition, Fig. 7 shows the superfluid fraction $\rho_s/\rho$ along the axis of rotation $z$ as a function of the number of particles $N$ at $T = 0.4 \, T_{c}^{3D}$. The superfluid fraction $\rho_s/\rho$ for an noninteracting gas is also shown in the figure. The superfluid fraction $\rho_s/\rho$ for $\mu = 0.2$ (filled circles) is clearly smaller than that for $\mu = 0$ (open circles). The superfluid fraction $\rho_s/\rho$ shows no dependence on the number of particles $N$, whereas the interparticle interaction plays a significant role in decreasing of the superfluid fractions $\rho_s/\rho$. Interestingly, when we compare the superfluid fraction for $\lambda = 0.1$ to that for $\lambda = 10$, we find that the superfluid fraction in the disk-shaped trap is significantly smaller than that in the cigar-shaped trap because geometric arguments imply that more excited longitudinal modes are occupied for $\lambda = 0.1$ than for $\lambda = 10$.

## IV. CONCLUSION

In this paper, Bose-Einstein condensation of trapped dipolar Bose gases has been studied using a finite-temperature path integral Monte Carlo technique. The quantum particles have a dipole-dipole interaction and a $s$-wave scattering interaction. The dipolar potential energy is long-range, anisotropic, and partially attractive, compared to the short-range and purely repulsive $s$-wave scattering interaction. Using pseudopotential forms to describe the interatomic interactions, we have calculated the equilibrium properties, such as the energetics and structural properties, of a system composed of $N$ dipolar hard spheres. We find that for a cigar-shaped trap,
the net effect of dipolar forces is attractive and shrinking of the density profiles is observed; and for a disk-shaped trap, the net contribution of dipole-dipole interactions is repulsive and the density profiles expand. Accordingly, the net contribution of dipolar interactions depends on the trapping aspect ratio $\lambda$.

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