Spatial filtering in electrical impedance tomography

M Fernández-Corazza¹,²,³, N von Ellenrieder¹,³ and C H Muravchik¹,⁴

¹ Laboratorio de Electrónica Industrial, Control e Instrumentación (LEICI)
Facultad de Ingeniería, Universidad Nacional de La Plata (UNLP), Buenos Aires, Argentina.
² Departamento de Ciencias Básicas, Facultad de Ingeniería, UNLP, Buenos Aires, Argentina.
³ Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina.
⁴ Comisión de Investigaciones Científicas de la Provincia de Buenos Aires (CICpBA), Buenos Aires, Argentina.

E-mail: marianof.corazza@ing.unlp.edu.ar

Abstract. We propose a new method to localize electrical conductivity changes in the human head using Electrical Impedance Tomography (EIT). In EIT, a localized conductivity change produces a change in the electric potential distribution which is equivalent to a potential generated by a dipole at the same location. We propose to estimate the location of conductivity changes with the same techniques used to solve the Electroencephalography source localization problem. In particular, we show an adaptation of the Linear Constrain Minimum Variance Beamforming technique to perform this estimation. We simulate a localized 10% conductivity change in a realistic model of the human head to apply the method. Results show that depending on the noise level, the method can accurately localize the conductivity change.

1. Introduction

Conductivity changes in the human head are of interest, for example, in the study of epileptic seizures and brain function. Electrical Impedance Tomography (EIT) can image these conductivity changes when applied to the human head. In [1] it is shown that there is an equivalence between a localized conductivity change and a dipolar source. To our knowledge, the use of this equivalence to image or localize conductivity changes has not been fully explored yet. In this work, we study the possibility of applying the Beamforming technique used in the Electroencephalography (EEG) source localization problem to localize and image conductivity changes from EIT measurements.

EIT is a technique that can be used to estimate the electrical conductivity distribution of an object based on the electric potential measured on its surface when electric current is applied to it. Its clinical application is considered to have a great potential in diagnosis as EIT is a relatively low-cost and minimally invasive method [2]. While applied to the human head it can be used to estimate a relatively small number of parameters such as the electrical conductivity of the main tissues of the head [3, 4], or to image the internal conductivity distribution of the head, a problem known as EIT reconstruction [5]. The parametric approach is of interest for improving the electrical head models needed in the EEG source localization problem [6, 7] and in the EIT reconstruction problem. The conductivity changes that EIT can image are mostly produced by blood flow and volume changes known to occur during evoked activity and epileptic seizures [8, 2]. Several algorithms were designed for solving the EIT reconstruction problem, and a good review of them can be found in [1]. As the problem is ill-conditioned, most of the
algorithms are based on a regularized least squares minimization with different approaches for the regularization term.

Spatial Filtering, also known as Beamforming, was originally developed in the field of sensor-array signal processing, and its application in the field of electromagnetic brain imaging was first introduced by Robinson [9]. It applies a linear operator to the measurements of a sensor array such that it points to a particular location. The measurements are typically obtained from EEG and/or Magnetoencephalography (MEG) records, but in this work we introduce the application of Spatial Filtering to EIT measurements. The output of the filter would be an estimation of the conductivity at the pointing location. As the pointing location is controllable, the entire head (or a particular zone of interest) can be scanned producing a 3D map of the internal conductivity of the head. One of the main issues of Beamforming in brain imaging is that the filter output is less sensitive to deeper sources so that they are harder to detect leading to false positives in deep regions. To overcome this problem, normalization is required. The Linear Constrained Minimum Variance (LCMV) Beamformer introduced by Van Bein includes a possibility for normalization [10]. This Beamformer assumes the knowledge of the resulting potential on the electrodes for a dipolar current source of unit norm, oriented in the three canonical directions, and localized at each point of the region of interest. The output of this filter is an estimate of the amplitude and orientation of the sources [10]. The method can be improved if an orientation scanning is also performed which is called Synthetic Aperture Magnetometry (SAM) [11], but it increases the computational cost. An alternative is to estimate the dipole orientation at each spatial point and then to perform the volumetric scanning only for that orientation. This beamformer is known as the Minimum Contrast Beamformer (MCB) [12].

In EIT, a change of the electric potential produced by a conductivity change in a specific location is equivalent to a voltage distribution produced by a dipolar source localized at the same location and with its strength proportional to the conductivity change [1]. We propose the use of the Beamforming technique to estimate the conductivity change as used in EEG source localization problems. Two key issues particularly affect EIT beamforming: the noise (because the difference of the potential is of the order of the electric noise levels) and the combination of different measurements when several electrode pairs are used for the current injection. This work is mostly focused on these two issues.

2. Materials and Methods
2.1. Principle of equivalence
Let \( \sigma(\vec{x}) \) be the conductivity at each point of the space \( \vec{x} \) of a volume \( \Omega \). When an electric current is applied, an electric potential distribution \( \Phi(\vec{x}) \) is generated. If a conductivity change \( \delta \sigma(\vec{x}) \) is produced, the electric potential also changes \( \delta \Phi(\vec{x}) \). The equation that governs the physics of the problem is:

\[
\nabla \cdot (\sigma(\vec{x}) \nabla \Phi(\vec{x})) = 0,
\]

(1)
in all \( \vec{x} \) except from the current injection points. Relation (1) must also be satisfied when the conductivity change is produced:

\[
\nabla \cdot ((\sigma(\vec{x}) + \delta \sigma(\vec{x})) \nabla (\Phi(\vec{x}) + \delta \Phi(\vec{x}))) = 0.
\]

(2)

Applying properties of the gradient operator, (2) becomes

\[
\nabla \cdot (\sigma \nabla \Phi) + \nabla \cdot (\sigma \nabla \delta \Phi) + \nabla \cdot (\delta \sigma \nabla \Phi) + \nabla \cdot (\delta \sigma \nabla \delta \Phi) = 0,
\]

(3)

where the dependence with \( \vec{x} \) has been omitted for clarity. The first term of (3) is zero because of (1), and the last term of (3) can be neglected for small conductivity changes as it is \( O(\delta \sigma^2) \) [1]. Then, (2) can be approximated by

\[
\nabla \cdot (\sigma \nabla \delta \Phi) \approx -\nabla (\delta \sigma \nabla \Phi).
\]

(4)
This relation is equivalent to the equation

\[ \nabla \cdot (\sigma \nabla \Phi) = \nabla J_p, \tag{5} \]

which governs the EEG source localization problem with \( J_p \) being the primary current density of a dipolar source. Then, the EIT problem is equivalent to the EEG source localization problem where the difference of the potential is the known or measured data while the equivalent current density at a point of the space is the conductivity change multiplied by the gradient of the potential at that point. This enables the possibility of using the EEG/MEG source localization algorithms and tools to localize conductivity changes with EIT. To our knowledge, this is an interesting possibility that has not been explored yet. In this work we adapt the LCMV spatial filter to this new application, but some other well known algorithms such as LORETA, or min-norm could be studied as well.

2.2. Spatial Filter design

General concepts about the design of filters for Brain Imaging can be found in [9], while the LCMV and MCB filters are described in [10] and [12] respectively. In this section we focus on the design of the filter for its application in EIT.

We assume that two electrodes of a set of \( L \) electrodes are dedicated to the current injection, while the remaining \( L-2 \) electrodes are used to measure the potential. The measurement model is

\[ y = v_1 - v_2 + n_1 - n_2 = \delta v + \delta n, \tag{6} \]

where vectors \( v_1 \) and \( v_2 \) are the \( L-2 \) expected values of the electric potential measured at the electrodes with and without the conductivity change, and \( n_1 \) and \( n_2 \) account for the electronic noise due to the amplifiers and the electrode-skin impedance.

The Lead Field (LF) matrix \( LF(\vec{x}) \) is an \( L \times 3 \) matrix where each column has the expected potential at the measurement electrodes generated by a unit-magnitude dipole source localized at the point \( \vec{x} \) and directed by each of the three canonical directions (x, y, and z). As two electrodes are used for the current injection, the LF has the two rows corresponding to those electrodes removed. From the equivalence between (4) and (5), the expected \( \delta v \) is just the Lead Field matrix multiplied by the equivalent current density \( J_p \):

\[ y = -LF(\vec{x})\delta \sigma(\vec{x})\nabla \Phi(\vec{x}) + \delta n = -\delta \sigma(\vec{x})l_f(\vec{x}) + \delta n, \tag{7} \]

where \( l_f \) is the Lead Field vector which is the Lead Field matrix multiplied by the gradient field.

On EIT, the injection pair can be changed to another two electrodes and a new set of \( L-2 \) measurements can be obtained. Then, if \( M \) is the number of pairs used, there will be \( M \) sets of \( L-2 \) measurements. In this work, we assume that the electrodes used for the current injection are used to measure when the injection pair is changed. Then the LFs are different for each set. Assuming that there is no conductivity change between measurements with different sets, they can be arranged in a column resulting in an \( M(L-2) \) vector \( y_T = -\delta \sigma l_{fT} + \delta n_T \), where \( l_{fT} \) and \( \delta n_T \) are the arranged vectors with the expected values of the electric potential measured at the measurement electrodes generated by a unit-magnitude dipole source localized at the point \( \vec{x} \) and directed by each of the three canonical directions (x, y, and z). As two electrodes are used to measure the potential. The measurement model

The Beamforming method proposes a weight vector \( \hat{w} \) such that \( \hat{w}^T y_T \) is an estimator of the parameter of interest, in this case, \( \delta \sigma \). The weight vector for the unit-gain constraint LCMV Beamformer is

\[ \hat{w}^T = \left( l_{fT}^TC^{-1}l_{fT} \right)^{-1}l_{fT}^T, \tag{8} \]

where \( C \) is the covariance matrix of the signal [10, 9]. If the LCMP filter is used instead, the matrix \( C \) is replaced by the correlation matrix \( R \). In this work we used the LCMP filter since it performed better. Then, the output of the filter is an estimator of the conductivity change, i.e.

\[ \hat{\delta \sigma} = \hat{w}^T y_T = \left( l_{fT}^TR^{-1}l_{fT} \right)^{-1}l_{fT}^TR^{-1}y_T, \tag{9} \]
where
\[ R = (\delta \sigma_{lT})(\delta \sigma_{lT})^T + C_n, \] (10)
and \( C_n \) is the noise covariance matrix. As \( \delta \sigma \) is unknown, the estimator of the correlation matrix used in this work is
\[ \hat{R} = y_T y_T^T + C_n. \] (11)

We followed the normalization denominator used in [12], which is the variance of the filter output when only noise is being measured \((y_n)\). We define a Conductivity Change Index (CCI) as:
\[ \text{CCI} = \frac{(\hat{\delta} \sigma)^2}{\text{Var}(w_T y_n^T w)} = \frac{w_T y_T y_T^T w}{w_T C_n w}. \] (12)

Note that in (8) the inverse of \( C \) (\( R \) in our work) has to be computed. For a fast computation of the CCI, the matrix inversion lemma was used to decrease the size of the matrix to be inverted.

2.3. Simulation Setup
A 10% conductivity increase in a sphere of radius 1cm localized at the right side of the brain in the union of the parietal and frontal lobes was simulated. A three layer and realistically shaped head model was employed. An electrode cap with 64 electrodes was assumed, where the electrode positions correspond to the 10–10 EEG standard placement system. The assumption is the use of the same electrodes used in EEG where the surface of the electrodes is much lower than the total surface of the head and then, the electrodes can be modeled as a point with no surface [13]. The EIT forward problem was solved using the Finite Element Method (FEM), both for the simulation of the measurements and for the localization problem solution. However, a different tessellation (314798 tetrahedrons for the brain) was used for the localization problem where the tetrahedron distribution inside the brain is uniform. An amplitude of 100\( \mu \)A was adopted for the injected current strength as allowed by the IEC60601 standard [14, 15]. For the scalp and skull, anisotropic conductivity values were adopted with radial:tangential ratios of 1:1.5 (0.3 and 0.45 S/m) and 1:10 (0.0015 and 0.015 S/m) respectively [16, 17], whereas for the brain, an isotropic value of conductivity (0.4 S/m) was adopted.

The electronic noise due to the amplifiers and the electrode-skin contact impedance was modeled as white gaussian noise (WGN). Independence between channels was assumed. Another source of noise is the electrical activity of the brain. There are at least two possibilities to mitigate its effect: taking advantage of the knowledge of the waveform of the injected current [3, 18] or the use of frequencies greater than \( \sim 200\text{Hz} \) knowing that brain activity is mostly concentrated at low frequencies, but lower than \( \sim 1\text{kHz} \) so that the electromagnetic quasi-static approximation remains valid. We adopt \( C_n = 2\sigma_n^2 I_{(L-2)M} \), where \( \sigma_n \) is the standard deviation (SD) of the noise, the 2 factor is a consequence of the difference between \( n_1 \) and \( n_2 \), and \( I_{(L-2)M} \) is the identity matrix. Adopting \( 0.1\mu\text{V}/\sqrt{\text{Hz}} \) for the noise SD and assuming slow conductivity changes of bandwidth \( 1/\text{Hz} \), the noise SD would be 50nV. For 100 independent repetitions of the experiment the SD can be reduced up to 5nV. We performed simulations for three different values of \( \sigma_n \): \( 1 \times 10^{-7} \text{V} \), \( 1 \times 10^{-8} \text{V} \), and \( 1 \times 10^{-9} \text{V} \).

In order to study the influence of the number of sets used in the inverse problem, two injection patterns were simulated. The first one consisting of 63 pairs where the \( Cz \) electrode is fixed as one injection node and the other node varies between the other electrodes, and the second one consisting of \( M = 125 \) combinations where the fixed electrodes are the Pz and the Fz.

3. Results
The reconstruction problem was simulated for both patterns and for the three levels of noise. Figure 1 shows the normalized CCI \((CCI(\vec{x})/\max\{CCI\})\) for both patterns \((M = 63 \text{ and})\)
M = 125), and for different noise levels (1 × 10^{-7}V, 1 × 10^{-8}V, and 1 × 10^{-9}V of SD). Three cross sections of the head are shown and the colorbar indicates the normalized CCI level.

![Cross sections of the head depicting the CCI obtained with 63 (first two columns) and 125 (last three columns) electrode pairs for the current injection and with a noise SD of 1 × 10^{-7}V (third column), 1 × 10^{-8}V (first and fourth columns), and 1 × 10^{-9}V (second and last columns). White crosses indicate the true centre of the sphere.]

**Figure 1.** Cross sections of the head depicting the CCI obtained with 63 (first two columns) and 125 (last three columns) electrode pairs for the current injection and with a noise SD of 1 × 10^{-7}V (third column), 1 × 10^{-8}V (first and fourth columns), and 1 × 10^{-9}V (second and last columns). White crosses indicate the true centre of the sphere.

As an indicator of performance, the Localization Error (LE) was also computed. It is defined as: \( LE = ||\vec{x}_{\text{est}} - \vec{c}|| \), where \( \vec{x}_{\text{est}} \) is the location of the maximum CCI value and \( \vec{c} \) is the centre of the sphere. Twenty different noise realizations were simulated for each pattern and for each noise level. The LE is shown in Figure 2, where the simulations with 1 × 10^{-7} of noise SD were excluded for clarity.

![Localization Error for 63 and 125 electrode pairs used for the current injection with 1 × 10^{-8}V and 1 × 10^{-9}V for the noise SD of the simulated measurements. The central mark is the median, the edges of each box are the 25th and 75th percentiles, and the whiskers extend to the most extreme data points not considered outliers. The red crosses are considered outliers.]

**Figure 2.** Localization Error for 63 and 125 electrode pairs used for the current injection with 1 × 10^{-8}V and 1 × 10^{-9}V for the noise SD of the simulated measurements. The central mark is the median, the edges of each box are the 25th and 75th percentiles, and the whiskers extend to the most extreme data points not considered outliers. The red crosses are considered outliers.
4. Discussion and Conclusions

The equivalence between conductivity changes in EIT and dipolar sources in EEG appears to be promising as it enables the use of the techniques and algorithms of EEG and MEG source localization problems. Results of our work suggest that one of those techniques, the spatial filtering, could reveal the conductivity change location within the brain, based on EIT measurements. Other techniques should be studied and a comparison with the EIT linear reconstruction algorithms described in [1] must be performed. However, the Beamforming technique does not need the inversion of full Jacobian matrices and the CCI may be simpler to compute. Note that the reconstructed image indicates the location of conductivity changes that best fit with the difference of the measured potentials.

We believe that the noise effect is a major technical issue of the method presented here. However, it is an inherent limitation in EIT as the conductivity changes produced in the brain generate small changes in the potential measured at the scalp. Averaging, repeating experiments, a smart selection of electrode pairs for the current injection, and some prior information related to the problem under study may improve the results. Note that for equal noise levels, the use of 125 pairs instead of 63 pairs for the current injection does not appear to significantly improve the performance of the localization estimation. It can be a consequence of the sphere position as the fixed electrode for the pattern with 63 pairs (Cz) is nearer to the sphere than the fixed electrodes used for the injection pattern of 125 pairs (Fz and Pz). Further simulations where the conductivity change is localized at different positions are needed.

The introduction of Spatial Filtering to EIT to localize conductivity changes looks promising as it is a method known to have a excellent performance while applied to EEG/MEG source localization problems. The method should be validated with real measurements.

Acknowledgments

This work was supported by ANPCyT PICT 2011-0909, UNLP I166, CONICET and CICpBA.

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