A Tangent Bundle on Diffeological Spaces

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ABSTRACT. We define a subcategory of the category of diffeological spaces, which contains smooth manifolds, the diffeomorphism subgroups and its coadjoint orbits. In these spaces we construct a tangent bundle, vector fields and a de Rham cohomology.

1. Introduction

The category of diffeological spaces [2, 6] extends that of manifolds and allows many topological and geometrical constructions, including products, quotients, forms, homotopies, fibrations [4]. It has the property that Hom(X, Y) is an object in a canonical way whenever X and Y are, thereby allowing the study of infinite dimensional objects.

It is known that symplectic manifolds play a central role in the modelling of physical systems [1, 7], and in particular coadjoint orbits have a natural symplectic structure [5]. By using a covariant definition of forms, this result is extended by Souriau also to the category of diffeological spaces and even allows the construction of a prequantization [3, 6] whenever a certain cohomological obstruction is zero.

In this paper we introduce the concept of a smooth diffeological space (SDS). This is a subcategory that includes manifolds, but is general enough to include groups of diffeomorphisms and infinite-dimensional coadjoint orbits. This category allows a dynamical modelling: we prove that these objects possess canonical tangent bundles and that each of them are also SDS. This allows the construction of flows and Lie algebras of vector fields, and of de Rham cohomologies.

2. Smooth diffeologies

Given a set X, an n-plaque of X is a function p:U → X where U is an open subset of \( \mathbb{R}^n \) containing the origin 0. A diffeology on X [6] is a set P(X) of n-plaques for every n such that:

(i) The images of the plaques cover X.
(ii) If a set \( (p_i) \) of n-plaques admits a common extension, then the smallest such extension is also an n-plaque in P(X).

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(iii) For every $\psi \in C^\infty(U', U)$ where $U, U'$ are open subsets of $\mathbb{R}^m, \mathbb{R}^n$ respectively, and every plaque $p: U \to X, p \circ \psi$ is also in $P(X)$.

The set of $n$-plaques is denoted $P^n(X)$, and the set of $n$-plaques $p$ such that $p(0) = F \in X$ is denoted $P^n_F(X)$. It may be seen that every set $S$ of plaques on $X$ generates a smallest diffeology $P_S$ which is formed by plaques $p: U \to X$ such that for all $r \in U$ there exists $U_r \subset U$ open with $p|_{U_r} = f \circ \phi$ where $\phi: U_r \to V \subset \mathbb{R}^n$ is smooth and $f \in S$.

A map $f: X \to Y$ between two spaces with diffeologies $P(X)$ and $P(Y)$ is called differentiable if $p \in P(X)$ implies $f \circ p \in P(Y)$. The set of such maps is denoted by $D^\infty(X, Y)$.

We shall now introduce a particular class of diffeologies that we call smooth, in order to set the stage for infinite-dimensional tangent spaces in the diffeological category.

**Definition 2.1.** Let $k \in \mathbb{N}$, a triple $(X, P(X), \sim)$ is called a $C^k$-diffeology (or smooth diffeology when $k = \infty$) if:

(a) The pair $(X, P(X))$ is a diffeology.

(b) $\sim$ is a collection $\{ \sim^F_n : 1 \leq n \leq k, F \in X \}$, where $\sim^F_n$ is an equivalence relation on the set $P^n_F(X)$, that satisfy a consistency condition: $p_1 \circ \phi \sim^F_n p_2 \circ \phi$ whenever $p_1 \sim^F_n p_2$ and $\phi \in C^\infty(U', U)$, with $U' \subset \mathbb{R}^m$ and $p_1, p_2: U \to X$; and moreover, $p_1 \sim^F_n p_1$ whenever $p_1: U \to X$ and $V \subset U$ is an open neighborhood of $0$.

(c) the smooth diffeology is called linear if the set $V := \{ V^n_F : F \in X, n = 1, \ldots, k \}$ where $V^n_F = P^n_F(X)/\sim^F_n$, carries a vector space structure that satisfies the following consistency condition: whenever $p_{12} \in [p_1] + [p_2]$ with $p_i: U \subset \mathbb{R}^n \to X$ and $\phi: U' \subset \mathbb{R}^m \to U$ then $p_{12} \circ \phi \in [p_1 \circ \phi] + [p_2 \circ \phi]$.

The class of the plaque $p(t)$ at a point $F \in X$ will be denoted by $[p]$ or by $[p]_t$. Whenever there are several spaces, we may use the notation $[p]^X$ instead.

The linear structure is called continuous if given two $(n + m)$-plaques $p_i(r, s)$ with $i = 1, 2$ such that $p_1(r, 0) = p_2(r, 0)$ then there exists a plaque $p_{12}(r, s)$ such that

$$[p_{12}(r, s)]_s = [p_1(r, s)]_s + [p_2(r, s)]_s \quad \text{for all } r.$$ 

**Definition 2.2.** The set $T^n_F X := P^n_F(X)/\sim^F_n$ is called the $n$-th tangent space at $F$, and the disjoint union $T^n X := \bigsqcup_{F \in X} T^n_F X$ is called the $n$-th tangent bundle over $X$.

Notice that $T^n_F X$ need not carry a linear structure. For example, the union of two smooth curves that intersect transversally at $F$ is a smooth diffeological space, for which $T^n_F X$ is the union of two lines.

**Definition 2.3.** Let $(X, P(X), \sim), (Y, P(Y), \approx)$ be smooth diffeologies; a differentiable function $f: X \to Y$ is called smooth at $F$ if for all $n \in \mathbb{N}$, $f \circ p_1 \approx^F_n f \circ p_2$ whenever $p_1 \sim^F_n p_2$.

If the smooth diffeologies are linear, $f$ is called a smooth map if, in addition,

$$D^n_F f : T^n_F X \to T^n_{f(F)} Y : [p] \mapsto [f \circ p]$$

is linear for each $F \in X$. The set of smooth functions is denoted $C^\infty(X, Y)$. Notice that $C^\infty(X, Y) \subset D^\infty(X, Y)$. In particular, if $Y = \mathbb{R}$ with the smooth manifold diffeology that is described below, then the notation $C^\infty(X)$ will be used instead.
Notice that the set of smooth diffeological spaces as objects with the smooth maps as morphisms is a category, since if \( f : A \to B \) and \( g : B \to C \) are smooth then \( g \circ f \) is differentiable and preserves the equivalence relations. Moreover, the chain rule

\[
D^n_c(g \circ f)[p] = D^n_c(f)(D^n_c f[p])
\]

holds, and linearity is preserved by this composition; therefore \( g \circ f \) is a smooth map if \( f \) and \( g \) are smooth maps.

Let \((X, P, \sim)\) be a linear SDS (smooth diffeological space) and let \( Y \subset X \). The subspace diffeology is formed by the plaques \( p \) such that \( p(r) \in Y \) for all \( r \). Restrict \( \sim \) to \( P(Y) \). \( Y \) is called a sub-SDS if \( P^n(Y)/\sim^n_P \) is a subspace of \( P^n(X)/\sim^n_P \) for each \( F \in Y \).

Let \((X, P(X), \sim)\) and \((Y, P(Y), \equiv)\) be two (linear) SDS. Denote by \( P(X \times Y) \) the product diffeology on \( X \times Y \) formed by plaques of the form \( p(r) = (p_1(r), p_2(r)) \) where \( p_1 \in P(X) \) and \( p_2 \in P(Y) \). Define

\[
(p_1, p_2) \sim^n_{F_1} (p'_1, p'_2) \quad \text{if and only if} \quad p_1 \sim^n_{F_1} p'_1 \quad \text{and} \quad p_2 \equiv^n_{F_2} p'_2.
\]

Then \( V^n_{(F_1, F_2)} = V^n_{F_1} \times V^n_{F_2} \). The triple \((X \times Y, P(X \times Y), \sim)\) is an SDS.

Some examples of SDS are the following:

(1) Let \( M \) be a smooth manifold modelled on a locally convex vector space \( V \). Let \( P(M) \) be the manifold diffeology (formed by the smooth plaques). For each \( F \in M \), choose a chart \((U_F, \alpha_F)\) around \( F \), and define \( p_1 \sim^n_{F_1} p_2 \) if

\[
D^m(\alpha_F \circ p_1)(0) = D^m(\alpha_F \circ p_2)(0) \quad \text{for all} \quad m \leq n.
\]

The map

\[
\mathcal{T}_F M \to V : [p] \mapsto \left. \frac{d}{dt} (\alpha_F \circ p)(t) \right|_{t=0}
\]

is a bijection and defines \( V^1_F \). Other spaces are defined similarly. In this case \( C^\infty(M) \subset C^\infty(M) = D^\infty(M) \), where \( C^\infty(M) \) denotes the set of smooth functions with respect to the manifold structure. In particular, when \( M = \mathbb{R}^n \), we recover the standard differentiable structure on a finite-dimensional vector space.

(2) Let \( M \) be a smooth finite dimensional manifold and let \( X = \text{Diff}_c(M) \) (the infinite-dimensional group of diffeomorphisms of \( M \) with compact support). Define \( P(X) \) as the set of functions \( p : U \to X, U \subset \mathbb{R}^n \) open, such that

\[
\phi_p : U \times M \to M : (r, m) \mapsto p(r)(m)
\]

is smooth. Given \( g \in X \) and \( p_1, p_2 \in P^n(X) \) with \( p_1(0) = p_2(0) = g \), we define \( p_1 \sim^n_g p_2 \) if

\[
D^i p_1(t)(m)|_{t=0} = D^i p_2(t)(m)|_{t=0} \quad \text{for all} \quad m \in M, \ i \leq n.
\]

Then \( \mathcal{T}^1_g X = \Gamma_c(M) \), the space of vector fields with compact support on \( M \).
(3) There is a natural smooth diffeology on coadjoint orbits. Let $G$ be a subgroup of diffeomorphisms (of the group $X$ defined above) with Lie algebra $\mathcal{G}$ such that $\frac{d}{dt}p(t)|_{t=0} \in \mathcal{G}$ for each diffeotopy $p$ in $G$. This is a condition that holds for example if $G$ is the group $\text{Diff}_c(M)$ or the group of symplectic diffeomorphisms, or if $G$ is finite dimensional (it is an open question whether this holds for any closed subgroup). On any coadjoint orbit $\Omega_{F_0}$ define the diffeology $P(\Omega_{F_0})$ generated by the set of plaques of the form $b(r) = K(p(r))F$ where $F \in \Omega_{F_0}$ and $p \in P(G)$ (defined above from the subspace diffeology) and $K$ is coadjoint action. In $P(\Omega_{F_0})$ define $b_1 \sim_F^n b_2$ if

$$D^m b_1(t)(Y)|_{t=0} = D^m b_2(t)(Y)|_{t=0}$$

for each $Y$ in $\mathcal{G}$, and for each $m \leq n$. Let $b \in P(\Omega_{F_0})$, and let

$$b(t) =: K(p(t))F$$

where $p \in P(G)$. Define $\xi = (d/dt)p(t)|_{t=0}$. Then the map $[b] \mapsto dK(\xi)F$ is a bijection between $T_F \Omega_{F_0}$ and $\mathcal{G}/\mathcal{G}(F)$, and allows us to regard $T_F \Omega_{F_0}$ as a vector space by transport of structure. In this way $\Omega_{F_0}$ becomes a $C^1$ linear diffeological space.

We shall prove that $T^m X$ has a natural smooth diffeology. Consider the set of maps of the form

$$\tilde{p}(r_1, \ldots, r_n) := [p(r_1, \ldots, r_n, s_1, \ldots, s_m)]_s$$

where $p$ is a $(n+m)$-plaque. Let $P(T^m X)$ be the set of plaques generated by these plaques (via condition (ii) of the definition of diffeology).

Any $[\alpha] \in T^m X$ is a class of $m$-plaques at some point $F$. Let $\alpha: U \to X$, $U \subset \mathbb{R}^m$, $\alpha(0) = F$ be such a plaque, and let $\tilde{p}_1$, $\tilde{p}_2$ be two $n$-plaques on $T^m X$ such that $\tilde{p}_1(0) = \tilde{p}_2(0) = \alpha$; define

$$\tilde{p}_1 \sim_{[\alpha]}^n \tilde{p}_2 \iff p_1 \sim_{F}^{n+m} p_2.$$

**Proposition 2.1.** $(T^m X, P, \sim)$ is a smooth diffeology.

**Proof.** These plaques cover $T^m X$ since, in particular, if $[p(s_1, \ldots, s_m)] \in T^m X$, then $\tilde{q}(0) = [p]$, where $q(t, s_1, \ldots, s_m) := p(t, s_1, \ldots, s_m)$.

Now let $\psi: U' \to U$ where $U' \subset \mathbb{R}^k$, $U \subset \mathbb{R}^n$ are open. Then

$$\tilde{p}(\psi(r_1, \ldots, r_k)) = [p(\psi(r_1, \ldots, r_k), s_1, \ldots, s_m)]_s$$

$$= [(p \circ (\psi \otimes 1_m))(r_1, \ldots, r_k, s_1, \ldots, s_m)]_s,$$

where the map

$$(\psi \otimes 1_m)(r_1, \ldots, r_k, s_1, \ldots, s_m) := (\psi(r_1, \ldots, r_k), s_1, \ldots, s_m)$$

is smooth, $p \circ (\psi \otimes 1_m)$ is a $(k+m)$-plaque and $\tilde{p} \circ \psi$ is a $k$-plaque. Therefore $P(T^m X)$ is a diffeology.
Let \( \psi : U' \to U \) and assume \( \tilde{p}_1 \cong_{[\alpha]}^n \tilde{p}_2 \), then \( p_1 \sim_{F}^{n+m} p_2 \), therefore

\[
p_1 \circ (\psi \otimes 1_m) \sim_{F}^{n+m} p_2 \circ (\psi \otimes 1_m),
\]

thus

\[
p_1 \circ (\psi \otimes 1_m)^{\sim}_{[\alpha]} \sim_{[\alpha]}^{n} p_2 \circ (\psi \otimes 1_m)^{\sim}.
\]

It follows that \( \tilde{p}_1 \circ \psi \cong^n \tilde{p}_2 \circ \psi \).

The space \( T_{F}X \) is given the subspace diffeology of \( T^{n}X \). For example, for \( n = 1 \) it consists of plaques \( \tilde{p} \) of the form \( \tilde{p} : U \to T_{F}X \) such that \( \tilde{p}(r) = [p(r, t)]^{X}_{t} \) where \( p(r, 0) = F \) for each \( r \); then \( p(r, t) \) is a \( t \)-curve through \( F \) for each \( r \).

If \( X \) has a linear structure, then we can define a linear structure on \( T^{n}X \) in the following way: take a point \( (F, [\alpha]) \) at \( T^{n}X \) and two \( k \)-vectors at this point (these are classes of \( k \)-plaques \( \tilde{p}_i : U \subset \mathbb{R}^k \to T^{n}X \) such that \( s \mapsto p_i(0, s) \in [\alpha] \). Then define

\[
[\tilde{p}_1] + c[\tilde{p}_2] = [\tilde{p}_{12}],
\]

where \( p_{12} \in [\tilde{p}_1] + c[\tilde{p}_2] - c[\tilde{\alpha}] \), where \( \tilde{\alpha}(r, s) = \alpha \) for all \( r \) (notice that \( [\tilde{p}_{12}] \) is a \( k \)-plaque through \( (F, [\alpha]) \) and that if \( [\tilde{p}_1] = 0 \) then \( \tilde{p}_1 = \tilde{\alpha} \)). One can check that \( T_{F}^{n}X \) becomes also a linear SDS.

**Proposition 2.2.** Let \( f : X \to Y \) be smooth; then \( D^{m}f : T^{m}X \to T^{m}Y \) is smooth for each \( m \). Also, \( D^{m}_{F_{0}} : T_{F_{0}}^{m}X \to T_{f(F_{0})}^{m}Y \) is smooth for each \( m \) and each \( F_{0} \in X \).

**Proof.** First, we shall prove that \( D^{m}f \) is differentiable. Let \( \tilde{p} \) be an \( n \)-plaque on \( T^{m}X \), then \( \tilde{p}(r) = [p(r, s)]^{X}_{s} \), therefore \( f \circ \tilde{p} \) is an \((n + m)\)-plaque on \( Y \) and \([f \circ p(r, s)]^{Y}_{s} \in T^{m}Y \) for all \( r \), and it is equal to \((f \circ p)^{\sim}(r)\), which is an \( n \)-plaque on \( T^{m}Y \). Therefore

\[
(D^{m}f) \circ \tilde{p}(r) = [f \circ p(r, s)]^{Y}_{s} = (f \circ p)^{\sim}(r).
\]

This proves that \( D^{m}f \) is differentiable. It is also smooth: if \( \tilde{p}_1 \cong \tilde{p}_2 \) at \( (F, [\alpha]) \) then \( p_1 \sim p_2 \), therefore \( p_1 \sim f \circ p_2 \) and \((f \circ p_1)^{\sim} \sim (f \circ p_2)^{\sim} \) at \((F, [f \circ \alpha])\). Thus \( D^{m}f \circ \tilde{p}_1 \cong D^{m}f \circ \tilde{p}_2 \) at \((F, [f \circ \alpha])\).

If \( X \) has a linear structure, then one can check that the differential of these maps are also linear.

**Proposition 2.3.** The projection \( \pi : T^{n}X \to X \) defined by \( \pi[\alpha]_{F} := F \) is smooth.

**Proof.** Let \( \tilde{p} \) be a \( k \)-plaque of \( T^{n}X \) on \((F, [\alpha])\), \( \tilde{p}(r) = [p(r, t)]^{X}_{t} \). Then \((\pi \circ \tilde{p})(r) = p(r, 0) \) is a plaque on \( X \). Therefore \( \pi \) is differentiable. Assume \( \tilde{p}_1, \tilde{p}_2 \in P(T^{n}X) \). If \( \tilde{p}_1 \cong \tilde{p}_2 \) then \( p_1(r, t) \sim_{n+1} p_2(r, t) \), then \( p_1(r, 0) \sim^{n} p_2(r, 0) \); therefore \( \pi(\tilde{p}_1) \sim \pi(\tilde{p}_2) \). If \( X \) has a linear structure then the linearity of \( D\pi \) is easily proved from the definitions; hence \( \pi \) is smooth.

3. The de Rham cohomology

A vector field on \( X \) is a smooth section of the tangent bundle \( T^{1}X \). The set of vector fields is denoted \( \Gamma(X) \). Given a vector field \( \xi \) and \( f \) in \( C^\infty(X) \), define

\[
\xi(f) := \left. \frac{d}{dt} f(\xi(t)F) \right|_{t=0}.
\]
Lemma 3.1. \( \xi(f) \) is smooth and therefore \( \xi \) is a derivation on \( C^\infty(X) \).

Proof. The function \( \xi(f) \) is well defined since \( f \) is smooth. Let \( p \in P(X) \), then \( \xi \circ p \) is a plaque on \( T^1 X \), that is, \((\xi \circ p) r = [\bar{p}(r,t)]_t\). It follows that
\[
\xi(f) \circ p(r) = \xi(p(r)) f = \frac{d}{dt} f \circ p(r, t) \bigg|_{t=0}.
\]
This is a smooth function since \( f \) is differentiable, therefore \( \xi(f) \) is differentiable.

Assume \( p_1 \sim^p p_2 \), then since \( \xi \) is smooth we have that \((\xi \circ p_1)t \approx (\xi \circ p_2)t \) at \( \xi(F) \).

Let \( \bar{p}_1, \bar{p}_2 \) be such that \([\bar{p}_i(r,t)]_t = (\xi \circ p_i)(r)\); then \( \bar{p}_1(r,t) \sim \bar{p}_2(r,t) \) at \( F \).

Then
\[
D^n \frac{d}{dt} f(\bar{p}_1(r,t)) \bigg|_{t=0} = D^n \frac{d}{dt} f(\bar{p}_2(r,t)) \bigg|_{t=0}
\]
and so \( \xi(f)p_1 \sim^n \xi(f)p_2 \). If \( X \) is linear, the linearity of \( D^n\xi(f) \) follows from the linearity of \( D^n\xi \).

It is easy to prove that if \( X \) has a continuous linear structure, then \( \Gamma(X) \) is a \( C^\infty(X) \)-module.

A local flow is a map \( \phi: P(X) \to P(X) \) such that (a) if \( p \in P^n(X) \) then \( \phi(p) \in P^{n+1}(X) \); (b) \( \phi(p)(r,0) = p(r) \); and (c) if \( p_1(r) = p_2(s) \) then \([\phi(p_1)(r,t)]_t = [\phi(p_2)(s,t)]_t \).

Each local flow defines a unique vector field and each vector field defines a local flow \( \phi \). Indeed, for each vector field \( \xi \) define \( \phi_\xi(p) := \bar{p} \) (since \( \xi \) is smooth) where \( \xi(p(r)) = [\bar{p}(r,t)]_t \); and conversely, given \( \phi \), define \( \xi_\phi(F) := [\phi(p)]_t \), where \( p \) is any plaque such that \( p(r_0) = F \).

Let us assume that \( X \) has a continuous linear structure. Let \( \xi_1, \xi_2 \in \Gamma(X) \). Then \( B(\xi_1, \xi_2) \), defined by
\[
B(\xi_1, \xi_2)(f) := \xi_1 \xi_2 f - \xi_2 \xi_1 f,
\]
is an element of \( \text{Der}(C^\infty(X)) \). Now, let \( \mathcal{M} \) be a maximal subalgebra of \( \Gamma(X) \) with this product. For each such subalgebra, define an \( n \)-form as a section \( \omega \) of \( \Lambda^n(T(X)) \) such that \( \omega(\xi_1, \ldots, \xi_n)(F) := \omega(F)(\xi_1(F), \ldots, \xi_n(F)) \) is smooth whenever \( \xi_i \in \mathcal{M} \) for all \( i = 1, \ldots, n \).

The set of these, denoted by \( \Lambda^n(X) \), is a \( C^\infty(X) \)-module and an associative algebra with the operation:
\[
\omega \wedge \eta := \frac{(k + l)!}{k! l!} \text{Alt}(\omega \otimes \eta).
\]

An example of a subspace of \( \Lambda^n(X) \) is the set of forms \( \omega \) expressible as
\[
\omega = \sum_{I = \{i_1, \ldots, i_n\}} h_I df_{i_1} \wedge \cdots \wedge df_{i_n},
\]
where the \( f_i \) are smooth and \( \{h_I\} \) is a locally finite family of smooth functions.

Define \( d_n: \Lambda^n(X) \to \Lambda^{n+1}(X) \) by
\[
d_n \omega(\xi_1, \ldots, \xi_{n+1}) = \sum_{i=1}^{n+1} (-1)^{i+1} \xi_i \omega(\xi_1, \ldots, \hat{\xi}_i, \ldots, \xi_{n+1})
\]
\[
+ \sum_{i<j} (-1)^{i+j} \omega(B(\xi_i, \xi_j), \xi_1, \ldots, \hat{\xi}_i, \ldots, \hat{\xi}_j, \ldots, \xi_{n+1}).
\]
It is clear that $d_{n+1} \circ d_n = 0$. This allows the definition of $Z_n(X, \mathbb{R}) := \ker(d_n)$, $B_n(X, \mathbb{R}) := \operatorname{im}(d_{n-1})$ and

$$H^n_{\text{dR}}(X, \mathbb{R}) := Z_n/B_n.$$ 

This is the $n$-th de Rham cohomology group of $X$.

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References

[1] M. Adams, T. Ratiu and R. Schmid, A Lie group structure of diffeomorphism groups and invertible Fourier integral operators, with applications, in: V. Kac, ed., *Infinite Dimensional Groups with Applications*, MSRI Publications 4 (Springer, Berlin, 1985) 1–26.

[2] P. Donato, Géométrie des orbites coadjointes des groupes de difféomorphismes, in: C. Albert, ed., *Géométrie Symplectique et Mécanique*, Lecture Notes in Mathematics 1416 (Springer, Berlin, 1988) 84–104.

[3] P. Donato, Les difféomorphismes du cercle comme orbit symplectique dans les moments de Virasoro, Preprint CPT–92/P.2681, CNRS–Luminy, 1992.

[4] P. Donato and P. Iglesias, Cohomologie des formes dans les espaces diffeologiques, Preprint CPT–87/P.1986, CNRS–Luminy, 1987.

[5] A. A. Kirillov, *Elements of the Theory of Representations* (Springer, Berlin, 1976).

[6] J. M. Souriau, Un algoritme générateur de structures quantiques, Astérisque, hors série (1985) 341–399.

[7] N. Woodhouse, *Geometric Quantization* (Clarendon Press, Oxford, 1992).