AN IMPROVED TECHNIQUE FOR MEASUREMENT OF COLD H\textsc{i} IN MOLECULAR CLOUD CORES

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ABSTRACT

The presence of atomic gas mixed with molecular species in a "molecular" cloud may significantly affect its chemistry, the excitation of some species, and can serve as probe of the cloud's evolution. Cold neutral atomic hydrogen (H\textsc{i}) in molecular clouds is revealed by its self absorption of background galactic H\textsc{i} 21 cm emission. The properties of this gas can be investigated quantitatively through observation of H\textsc{i} narrow self-absorption (HINSA). In this paper we present a new technique for measuring atomic gas physical parameters from HINSA observations that utilizes molecular tracers to guide the HINSA extraction. This technique offers a significant improvement in the precision with which H\textsc{i} column densities can be determined over previous methods, and it opens several new avenues of study of relevance to the field of star formation.

Subject headings: astrochemistry — ISM: clouds — ISM: evolution — line: profiles — molecular processes

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1. INTRODUCTION

Star formation occurs in molecular clouds which are thought to have evolved from diffuse atomic hydrogen (H\textsc{i}) regions to form dense, cold, well-shielded regions composed primarily of molecular hydrogen (H$_2$). Our quantitative understanding of cloud evolution and specifically the conversion process (from H\textsc{i} to H$_2$ gas) has been hindered by our inability to measure confidently the H\textsc{i} abundance in evolving clouds. In this paper we present a new technique for measuring H\textsc{i} column densities in dark clouds offering a significant improvement over previous methods. In the interests of brevity this paper includes the results from only a few clouds on which this technique has been applied in order to demonstrate the technique. The results of a much larger survey of observational data and analysis are to follow in a subsequent publication.

The ability to determine accurately the H\textsc{i} component of molecular clouds could have a variety of benefits. Measurement of H\textsc{i}/H$_2$ ratios in clouds, used in conjunction with astrochemical models, allows us to determine the chemical ages of individual clouds or entire molecular complexes (e.g., Taurus, Perseus, etc.). This will greatly expand our understanding by constraining star formation models thus yielding insights into the collapse process and the interplay of magnetic fields, ambipolar diffusion, turbulence, and various potential sources of cloud support. By studying age distributions in large-scale regions we can learn about the processes that may trigger the collapse of large complexes. In contrast to previous methods, our technique allows us to determine the H\textsc{i}/H$_2$ ratios for individual velocity components within a cloud, thus yielding unique information about cloud kinematics. Further, the technique allows for the absolute measurement of quantities such as H\textsc{i} column density, in contrast to many previous studies which were limited to comparative measurements.

H\textsc{i} is the dominant constituent of the diffuse ISM and its 21 cm emission line is prevalent everywhere throughout the sky, especially near the galactic plane. Typical H\textsc{i} emission spectra are composed of numerous superimposed velocity components. The emission line widths of the overall features are typically on the order of a few 10s of km s$^{-1}$. They include velocity variations owing to galactic rotation, as well as very significant peculiar velocities resulting from localized phenomena. It is rare to find a molecular region for which one can confidently claim that the H\textsc{i} emission observed along the line of sight to the cloud is associated with the cloud, and is not due to background or foreground sources. Owing to such velocity crowding it is difficult or impossible to disentangle H\textsc{i} emission originating from a particular cloud from the background galactic H\textsc{i} emission.

The situation for H\textsc{i} absorption is different. H\textsc{i} within a galactic cloud of any type can absorb the continuum emission from distant (galactic or extragalactic) radio sources. Because the H\textsc{i} optical depth varies inversely with the cloud temperature, the absorption by galactic H\textsc{i} is stronger in cold, galactic H\textsc{i} clouds. The integrated optical depth of all the clouds along the line of sight through the galactic disk can exceed unity as demonstrated in Kolpak et al. (2002) and Garwood & Dickey (1989).

These cold, interstellar H\textsc{i} clouds may also be identified through their absorption of warm background H\textsc{i} emission originating within the galaxy. Because the emission being absorbed by cold H\textsc{i} clouds in this case is galactic H\textsc{i} emission, the resultant spectral absorption features are referred to as H\textsc{i} self-absorption (HISA). There have been many surveys with HISA detections over the years, including Garzoli & Varsavsky (1966), Heiles (1969), Knapp (1975),
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Heiles & Gordon (1975), Wilson & Minn (1977), McCutcheon et al. (1978), Myers et al. (1978), Bowers et al. (1980), Batrla et al. (1981), Shuter et al. (1997), van der Werf et al. (1988), Feldt & Wendker (1993), Montgomery et al. (1995), Gibson et al. (2000), and Kavars et al. (2003) among others. It is important to emphasize that the term HISA refers to an observable spectral absorption feature rather than being a description of a specific physical process.

Molecular spectral emission lines provide an independent view of that subset of interstellar clouds that are cold and composed primarily of molecular species. These “molecular clouds” are expected to maintain a residual abundance of atomic hydrogen if for no other reason than the cosmic-ray disassociation of H\textsubscript{2}. This applies even when the chemical evolution of the cloud has reached equilibrium (Solomon & Werner 1971; Goldsmith & Li 2005). With the residual H\textsc{i} coexisting throughout the cloud with molecular species, the observed spatial and velocity (kinematic) structure of the molecular cloud will be similar whether observed in molecular emission or H\textsc{i} self-absorption lines. Thus, we expect to observe H\textsc{i} self-absorption features along the lines of sight of molecular clouds which share the spatial distribution and kinematics (nonthermal line width) of molecular emission lines. Such localized association between molecular emission and H\textsc{i} self-absorption is observed for many nearby clouds as reported by Li & Goldsmith (2003) and Goldsmith & Li (2005). The specific case in which the H\textsc{i} absorption features observed in the direction of a molecular cloud share the spatial and kinematic structure seen in the molecular lines is called H\textsc{i} narrow self-absorption (HINSA) as defined in Li & Goldsmith (2003). The term “narrow” arises from the typically small nonthermal line widths (on the order of 0.1 km s\textsuperscript{-1}) of HINSA features, very similar to the nonthermal line widths of molecular tracers along the same lines of sight. HINSA can be considered to be a subset of HISA, but it is a subset derived from an understanding of a specific, observable physical phenomenon in molecular clouds.

While both are simply acronyms for spectral absorption features, HISA can be caused by a variety of different conditions and processes, but H\textsc{i} narrow self-absorption (HINSA) is that subset of HISA in which the atomic H\textsc{i} absorption correlates well with molecular emission of certain tracers (most notably 13CO) in sky position, central velocity, and nonthermal line width. Based on our current understanding of cold molecular clouds, the most satisfactory picture is that HINSA features are a result of H\textsc{i} gas located within these cold, dense, well-shielded regions. Some early examples of HINSA studies that predate the use of this term are those of Wilson & Minn (1977), van der Werf et al. (1988), and Jackson et al. (2002). The technique which we describe in this paper pertains only to the extraction of H\textsc{i} data from HINSA features.

The general picture which emerges, as found in Li & Goldsmith (2003) and Goldsmith & Li (2005), is that the H\textsc{i} gas located within cold, quiescent cores of dark clouds produces HINSA absorption features. The well-defined center velocities and narrow line widths allow us to separate the H\textsc{i} gas associated with individual clouds from the galactic background. However, the complexity of the background emission spectra that are frequently encountered makes extracting accurate data (especially in terms of obtaining the cold H\textsc{i} column density) from the absorption features difficult. Several methods (discussed in §4) have been used previously, but all are recognized to introduce significant uncertainties in the results. We here present a new technique that aims to improve the situation by using the properties of molecular emission to characterize the region producing the HINSA features, and then employs the HINSA spectral features to derive H\textsc{i} column densities.

In §2 we present selected data and show the results of applying the new technique to them, and in §3 we contrast this with previous methods for analyzing HINSA data. In §§4 and 5 we describe the technique and the combination of molecular data with the HINSA spectra. In §§6 and 7 we verify the validity of our technique using simulated data and also examine its limitations.

2. RESULTS USING OBSERVATIONAL DATA

In this section we demonstrate the utility of our procedure by showing some results of its implementation based on observations of two molecular clouds. While a full discussion of the implications of these results is reserved for another publication, we hope that the results obtained encourage careful consideration of the technique described here.

An observational survey, whose complete results and analysis are presented elsewhere, was performed using the Green Bank Telescope (GBT) and the Five College Radio Observatory (FRCAO) to obtain H\textsc{i}, OH, 13CO, 12CO, and C18O maps of more than 30 dark clouds which exhibit HINSA features. Figure 1 shows sample H\textsc{i} spectra from two well known clouds, Lynds 134 (L134) and L1757. HINSA absorption features are prominent in both sources. The dipless lines represent the recovered background spectra after HINSA has been removed. This representation is reassuring in that the smooth, natural looking background spectra indicate that the HINSA removal process has not produced marked distortions in the background. While L134 represents a simple case of a single emission component, in L1757 our technique is able to discern two emission components closely spaced in velocity. Previous methods based solely on the H\textsc{i} line profile would have significantly overestimated the H\textsc{i} optical depth by assuming there was only a single emission component.
By utilizing the associated molecular data as described below, the possibility of a single emission component is largely eliminated.

The technique we use here assumes that the cold H\textsc{i} and molecular constituents are well correlated spatially, so that we can assume that the line center velocity and nonthermal line width for the cold atomic gas and the molecular gas in a particular direction are the same. This is justified by previous studies of HINSA and CO isotopologues, including Li & Goldsmith (2003) and Goldsmith & Li (2005). Our technique depends on use of these spectra to calculate the H\textsc{i} optical depths and column densities. In conjunction with observations of molecular tracers this allows us to obtain H\textsc{i} column densities in dark clouds with improved accuracy. Further, we are able to obtain unprecedented detail by examining individual velocity components within each cloud, even when their velocity separations are small enough to make individual components difficult to distinguish in H\textsc{i} spectra. Additional details are given in § 4.

Figures 2 and 3 and Tables 1 and 2 show measured H\textsc{i}, H\textsc{2}, and total proton column densities, and the H\textsc{i}/H\textsc{2} ratios for individual velocity components throughout numerous positions.
components have similar \( H_\text{i} \) abundances and exhibit very different properties. While in \( L134 \) all the velocity components have roughly the same \( H_2 \) density but very different \( H_\text{i} \) abundances, in \( L1757 \) the situation is quite different, with ratios varying systematically as a function of velocity and extending over a much greater range. The most likely explanation for this phenomenon is that \( L134 \) is a mature cloud in which the chemical evolution of all of the velocity components has proceeded to the point where \( H_\text{i} \) abundances have approached their equilibrium values. In contrast, \( L1757 \) represents a much younger cloud in which each component is evolving at a different rate governed by its total density. This information yields valuable insight into the dynamics and chemistry of dark clouds, but has never before been available. A complete discussion of the results for a large sample of clouds will be presented in a forthcoming paper as a proper analysis requires more space than is available here.

### 3. LIMITATIONS OF PREVIOUS \( H_\text{i} \) SELF-ABSORPTION ANALYSIS TECHNIQUES

Owing to the complexity of \( H_\text{i} \) background emission along most lines of sight, it is clearly quite arbitrary to fit a straight line across the absorption feature. It is equally problematic to use OFF source observations since the \( H_\text{i} \) background spectra change significantly over the angular size of the foreground cloud as sampled by the telescope beams that have been employed. Several methods for extracting \( H_\text{i} \) column densities from HISAG absorption have been tried in the past. These can be classified into two groups, each with its own set of limitations. The most commonly used methods rely on fitting simple mathematical functions such as Gaussians or polynomials over the observed spectra while masking the absorption features. As discussed in Li & Goldsmith (2003), these techniques result in significant uncertainty because the analysis is underconstrained. Arbitrary choices for the degree of the fitted polynomial and the masking ranges must be made, and these greatly affect the final results.

Recently, Kavars et al. (2005) implemented an automated algorithm for measuring HISAG which relies on spotting the characteristic HISAG features (the dip) in \( H_\text{i} \) spectra. However, this technique (as pointed out by its authors) suffers from several limitations. The Kavars et al. (2005) technique, as well as function fitting methods, are unable to determine the gas temperature and the optical depth without assuming a value for one of these important quantities. These methods are thus limited to comparative measurement of one quantity, such as \( H_\text{i} \) column density, over large regions recognizing that absolute values cannot be obtained.

Owing to the complexity of the galactic \( H_\text{i} \) background, and lacking additional information, it is often impossible to tell which dips in the spectra are genuine absorption features and which are produced simply by the superposition of two emission features. Both phenomena may be spatially extended with continuous velocity distributions. Molecular observations can be used to distinguish between these two scenarios. In molecular clouds there are often multiple components along the same line of sight with closely spaced central velocities. The lower mass of atomic hydrogen compared to molecular tracers (and consequently greater column density, over large regions recognizing that absolute values cannot be obtained).

### TABLE 2

**DERIVED COMPONENT PROPERTIES FOR L134 AND L1757 (CONTINUED)**

| Cloud       | Peak HINSA Optical Depth | Peak \(^{13}\)CO Optical Depth | \(^{12}\)CO \(T_{ex} \) (K) | Mean Continuum at 21 cm (K) |
|-------------|--------------------------|-------------------------------|-----------------------------|-----------------------------|
| L134 A      | 0.53                     | 0.84                          | 19                          | 3.1                         |
| L134 B      | 0.20                     | 0.67                          | 17                          |                             |
| L134 C      | 0.20                     | 0.24                          | 17                          |                             |
| L134 D      | 0.19                     | 0.22                          | 14                          |                             |
| L1757 A     | 0.20                     | 0.31                          | 16                          | 3.2                         |
| L1757 B     | 0.35                     | 0.29                          | 17                          |                             |
| L1757 C     | 0.36                     | 0.74                          | 19                          |                             |
| L1757 D     | 0.30                     | 0.35                          | 15                          |                             |

Notes.—The \(^{12}\)CO temperature, derived directly from the \(^{12}\)CO spectra, is used as an estimate of the \( H_\text{i} \) temperature represented by \( T_{HI} \) in eq. (1).
merge into a single H I absorption feature. Using the molecular gas as a template for the HINSA analysis has the potential to greatly enhance the accuracy of the H I/H₂ comparison since the ratio may be different in each velocity component.

Previous analysis techniques have been applied to both HISA and HINSA sources without distinguishing between the two. Our technique is limited to HINSA sources for which (by definition) there is molecular data available. Limiting our scope in this manner allows us to utilize observational data from various molecular tracers to assist in extracting meaningful results from HINSA features. As will be shown in what follows, to the extent that our assumption that H I is mixed with the molecular gas is valid, we are able to measure both the temperature and optical depth of the H I gas without any ambiguities regarding fitting functions. In this way we may treat each velocity component in a cloud separately.

4. THE TECHNIQUE

4.1. Analytic Representation

An idealized HINSA spectrum can be represented by the radiation transfer equation as

\[ T_d(v) = (T_b(v) + T_e)e^{-\tau(v)} + T_H(1 - e^{-\tau(v)}), \]  

(1)

where \( v \) corresponds to velocity, \( T_d(v) \) is the observed spectrum, \( T_b(v) \) is the background emission due H I emission clouds, \( T_e \) represents the continuum emission along the line of sight at 21 cm including the 2.7 K microwave background (\( \S \) 4.2), \( T_H \) is the gas temperature of the foreground absorbing cloud producing HINSA, and \( \tau(v) \) is the optical depth of the absorbing H I gas at 21 cm which we can describe as \( \tau(v) = \tau_0e^{-\left|v - v_H\right|/2\sigma^2_H} \), where \( \tau_0 \) is the peak optical depth, \( v_H \) is the central velocity of the HINSA component, and \( \sigma_H \) is the HINSA line width. In this particular representation we have assumed that there is no intervening H I emission originating between us and the absorbing HINSA cloud; this topic is further discussed in \( \S \) 4.3. When multiple absorption components overlap in velocity within the same line of sight, the transfer equation must be applied consecutively for each component from farthest to nearest.

HINSA absorption features are typically Gaussian-like dips with much smaller amplitudes and line widths than the characteristics of the emission spectra \( |T_b(v)| \) on which they reside. These features can be so small in comparison to the background emission that they can be difficult to distinguish by eye. However, by utilizing a fundamental property of Gaussians we can obtain a much better representation of the data in which the HINSA becomes more apparent.

The derivatives in velocity or frequency of a Gaussian function can be written as

\[ f(v) = f_0e^{-v^2/(2\sigma^2)} \]

\[ \frac{df(v)}{dv} = -\frac{v}{\sigma^2}f(v) \]

\[ \frac{d^2f(v)}{dv^2} = -\frac{1}{\sigma^2}f(v) + \frac{v^2}{\sigma^4}f(v), \]  

(2)

where it is apparent that higher order derivatives show a stronger dependence on line width. For conceptual purposes we can temporarily approximate the HINSA absorption and background emission features as Gaussians. Because the line widths for HINSA features (typically 1 km s\(^{-1}\)) are much narrower than the line widths for the background emission components, in the first and second derivative representations, even a low-amplitude absorption feature will become readily apparent in comparison to the bright background emission. This is illustrated in Figure 4. In the second derivative representation, HINSA becomes the dominant feature in the spectrum thus making it easier to isolate and quantify.

One could in principle use higher derivative representations to search for the best fit as the HINSA becomes progressively more prominent. However, with real data the noise also rises with higher derivatives. The second derivative representation has been found to be the optimum choice for our GBT and Arecibo observations. It would be possible to take the second derivative representation of any HINSA spectrum and attempt to fit a simple function to it to extract the absorption. However, HINSA spectra are rarely so simple as to lend themselves to this kind of fitting and we would have very limited information on the temperature of the H I gas. Furthermore, the large thermal line widths of H I spectra tend to blend all the velocity components along a single line of sight. Unless we are convinced that all the velocity components in a cloud can be characterized by the same H I/H₂ ratio, we must find a way to extract the HINSA using additional constraints based on physical parameters.

The primary benefit of limiting our technique to HINSA features is that we can use data from molecular tracers in order to constrain the analysis. If we can determine the values of all the other parameters (line width, center velocity, and temperature) for each velocity component then we can ideally limit ourselves to searching for that value of \( \tau_0 \) which produces the \( T_b \) emission spectrum whose second derivative best removes the HINSA feature. Minimizing the integrated squared sum of the second derivative background spectrum is sufficient to extract the HINSA feature (see \( \S \) 4.5).
There are various sources of continuum emission ($T_c$) in equation (1) throughout the galaxy that contribute to the sky brightness at 21 cm. The emission may originate from a variety of sources such as synchrotron emission within the galaxy and continuum emission from ionized gas such as that in the Galactic Halo. Continuum emission can also originate from individual discrete galactic and extragalactic sources, such as pulsars, H ii regions, AGNs, accretion disks, and quasars. These sources will typically produce emission whose intensity is constant across the observed H i band. Observing techniques such as frequency switching, as well as software baseline removal algorithms remove the continuum emission and the 2.7 K background along a particular line of sight from the reduced spectrum. The continuum emission component should ideally be included when calculating HINSA since the temperature of the cold gas clouds (10 K) is comparable to the antenna temperature produced by the continuum emission. Several galactic surveys have been performed which have measured the value of the 21 cm continuum including Reich et al. (1997), Uyaniker et al. (1999), and Reich & Reich (1986), among others. These data can be utilized to determine the expected continuum emission along any HINSA source. Typical values are on the order of a few K. According to equation (1), the value of $T_c$ while significant, becomes critical only when the HINSA optical depths are high and gas the temperature is low.

4.3. Foreground H i Emission

Usually there will be H i emission between the observer and the HINSA cloud due to intervening H i gas. To assess the importance of the effect this intervening gas might have on our observations we must expand equation (1) to include all three gas components, that of the H i emission behind the HINSA cloud, absorption due to the HINSA cloud itself, and emission between us and the HINSA cloud;

$$T_i(v) = T_c e^{-\tau_i(v)} + T_{ex,i}(1 - e^{-\tau_i(v)})$$  \hspace{1cm} (3)

represents the emission component due to the warm H i gas behind the HINSA cloud whose excitation temperature is $T_{ex,i}$ and optical depth is $\tau_i(v)$. At this stage, only the background continuum emission $T_c$ is attenuated.

The HINSA cloud then attenuates the background emission, while also adding its own contribution as in

$$T_{H}(v) = T_i(v) e^{-\tau_H(v)} + T_{ex,H_i}(1 - e^{-\tau_H(v)})$$  \hspace{1cm} (4)

where $\tau_H$ and $T_{ex,H_i}$ represent the optical depth and excitation temperature of the cold H i gas within a molecular cloud. Similarly, any foreground gas will attenuate the spectrum further, as well as adding its own emission to produce the final observed spectrum:

$$T_{H}(v) = T_{H_i}(v) = T_i(v) e^{-\tau_f(v)} + T_{ex,f}(1 - e^{-\tau_f(v)})$$  \hspace{1cm} (5)

where $\tau_f$ and $T_{ex,f}$ represent the optical depth and excitation temperature of the foreground H i gas between us and HINSA cloud.

We now have two new terms, $\tau_f$ and $T_{ex,f}$, describing emission due to H i gas between us and the HINSA cloud. There is no way to operationally disentangle which portion of the observed emission spectrum originates from in front or behind the molecular cloud, but clearly any foreground emission we observe will attenuate the HINSA features and we must somehow correct for this. To get a sense of the effect and scale of the attenuation, we can assume that both the background and foreground emission gas have the same excitation temperature $T_{ex,tot} = T_{ex,b} = T_{ex,f}$. We can further assume the H i gas is distributed uniformly throughout the galaxy along the line of sight so that a linear relationship can be established between $\tau_b$ and $\tau_f$ such that $\tau_b = p\tau_{tot}$ and $\tau_f = (1 - p)\tau_{tot}$ where $\tau_{tot}$ is the total optical depth of all the emitting H i gas behind and in front of the HINSA cloud and $p$ represents the fractional distribution between the two which we can estimate using

$$p(b) = \frac{1}{1 + \frac{D_{cloud} \sin(b)}{R_{disk}}} \cdot$$  \hspace{1cm} (6)

Here, $D_{cloud}$ is the distance to the HINSA cloud, $b$ is the galactic longitude along the given line of sight, and $R_{disk}$ represents the distance from Earth to the edge of the galactic disk as defined by its H i content. With a typical scale height of 100 pc near the Sun’s position within the Milky way we take this value to be 200 pc (Stahler & Palla 2004). For a more complete discussion see van der Werf et al. (1988) or Li & Goldsmith (2003). In this case we are using geometry of the local galactic disk, and the HINSA cloud’s position within the disk to estimate how much of the H i emission visible along a particular line of sight is in fact located in front of a HINSA source attenuating the HINSA feature.

This emission, if located at the same velocity as the HINSA feature, will have the effect of washing out the absorption feature and thus the apparent HINSA optical depth will be lower than the intrinsic value. In fact, it can be shown from the above formulation that for optically thin HINSA, $p(l, b) \propto \tau^{-1}$ as shown in van der Werf et al. (1988), where $\tau$ now represents the observed apparent optical depth of any HINSA feature. Thus to obtain the intrinsic HINSA optical depth of a cloud we must divide the apparent optical depth by $p(l, b)$. Due to the uncertainties in the H i distribution it is impractical to directly model the effects of foreground clouds when trying to extract HINSA profiles. Instead HINSA optical depths are computed on the assumption that there is no intervening emission and are subsequently adjusted using the factor $p(l, b)$ which itself depends only on the cloud’s position and is calculated independently of the observed emission spectrum.

The observed spectra thus give us an apparent optical depth which then must be adjusted by $p$ to yield an estimate of the intrinsic optical depth. For most nearby clouds this correction factor is in fact small in comparison to other sources or error. It must be true that $p(l, b)$ is not much less than unity if a significant HINSA feature is to be observed otherwise the foreground emission would obscure the HINSA absorption. For the two sample clouds in this paper, L134 and L1757 the correction factor has values of 0.5 and 0.7 accordingly.

4.4. Obtaining Constraints Using Molecular Parameters

It is possible to obtain data from molecular tracers which can provide us with reliable estimates of the HINSA feature properties for each velocity component by utilizing certain assumptions justified by previous work. Li & Goldsmith (2003) showed a very strong correlation between the line center velocity of each HINSA component and those of its molecular counterparts using the OH, $^{13}$CO, and C$^{18}$O molecules as tracers. Thus, once each molecular component has been identified and separated using traditional fitting we can assume that each velocity component visible in molecular emission has an accompanying HINSA component whose optical depth may yet turn out to be 0. We can
use the central velocity from any of the above molecules as an estimate of the number of HINSA components present and their center velocities ($v_{11}$).

Spectral line widths are composed of both thermal and nonthermal (turbulent) components. The two components add according to $\sigma_{\text{obs}} = (\sigma_{\text{th}}^2 + \sigma_{\text{nt}}^2)^{1/2}$, where $\sigma_{\text{obs}}, \sigma_{\text{th}},$ and $\sigma_{\text{nt}}$ are the observed line width and the thermal and nonthermal components, respectively. The thermal component arises from the random motions of individual particles within the gas and will be different for varying atomic or molecular tracers due to the different particle masses. The nonthermal component results from turbulence and bulk motions within the cloud. If two species of gas were uniformly mixed throughout a cloud, one would expect their spectra to share similar nonthermal line widths. Li & Goldsmith (2003) showed a good correlation between the nonthermal line widths of HINSA and the accompanying $^{13}$CO emission. We can use $^{13}$CO to estimate our expected H i line widths, but first we must disentangle the thermal components using information about the gas temperature within the cloud.

Under the assumption that the cold H i is in thermal equilibrium with the surrounding molecular gas we can use the kinetic temperature of a molecular tracer to estimate the H i gas temperature ($T_H$). The $J = 1\rightarrow 0$ transition of $^{12}$CO has been used in the past to estimate gas temperatures because it usually appears as optically thick in molecular clouds. Equation (1) is valid for molecular emission as well as H i absorption as it describes generic radiative transfer. In the case of $^{12}$CO there is generally no background or continuum emission so the equation reduces to

$$T_A(v) = T^{12}_{CO} \left(1 - e^{-\tau^{12}_{CO}(v)}\right),$$

where $T_A(v)$, $T^{12}_{CO}$, and $\tau^{12}_{CO}(v)$ describe the observed $^{12}$CO antenna temperature, the $^{12}$CO gas excitation temperature (which in high density regions can be taken to equal the kinetic temperature), and the optical depth. If the optical depth is taken as $\gg 1$, then the equation reduces simply to $T_A = T^{12}_{CO}$. Thus $^{12}$CO provides us with estimates of the H i gas temperature allowing us to estimate the HINSA optical depths, column densities, and thermal and nonthermal line widths. Some derived $^{12}$CO temperatures for L134 and L1757 are given in Table 2. Since $^{12}$CO emission has been shown to exist wherever there is $^{13}$CO emission and HINSA we can measure the $^{13}$CO temperature at each position and use it to estimate the $^{13}$CO and HINSA temperatures at each position in the cloud. Using these constraints we are left with only one free parameter, the peak optical depth of the cold H i gas.

Due to the uncertainties in the assumptions made the constraints parameters cannot be taken as exact values of the H i gas properties. Instead they can be used as close guess values to help us constrain the fit. By using $^{13}$CO and $^{12}$CO data we are also able to estimate H$_2$ column densities using fractional abundances of carbon monoxide that are well determined in many nearby dense clouds (Stahler & Palla 2004).  

4.5. The Fitting Procedure

As explained in § 4, any narrow HINSA feature will be the dominant feature in the second derivative representation of the observed spectrum. As such, once guess parameters for the center velocity, line width and temperature of the H i gas have been obtained from molecular tracers the remaining task is to find the value of $\tau_0$ for each velocity component which minimizes the resulting area under the curve of the second derivative of the recovered background emission spectrum after HINSA is removed in accordance with the discussion in § 4.1. This is done by first using the trial values of the HINSA line width, center velocity, temperature, and optical depth and using those on a particular observed spectrum [$T_A(v)$] to obtain the recovered background spectrum without HINSA [$T_B(v)$] as described in equation (1). According to § 4.1 the HINSA feature will be revealed as a large feature in the second derivative representation of any observed spectrum. To find the best trial $T_B(v)$ spectrum the algorithm must search for one whose integrated intensity in the second derivative representation is minimized. The second derivative of the recovered spectrum [$T_B(v)$] is calculated numerically through traditional methods. Since the second derivative representation will have negative values as well as positive ones, it is squared prior to integration. We thus minimize the value of $I$ as described by the function

$$I = \int \left(\frac{d^2T_B(v)}{dv^2}\right)^2 dv.$$  

which is simply the squared integrated intensity of the second derivative of the background spectrum ($T_B$) that is recovered after trial values of the HINSA optical depths and other parameters are removed from the observed spectrum. Finally, determining the smallest value of $I$ will give us the best-fit values of the HINSA parameters. It is possible to use a more sophisticated minimizing function, but the HINSA feature is so dominant in the second derivative representation that this has proven not to be necessary.

Previous studies have shown that the correlation between H i and $^{13}$CO center velocities is strong (Li & Goldsmith 2003). Hence, the $v_{11}$ parameter is held fixed for each velocity component. The $T_B$ estimates obtained from $^{12}$CO do not carry as much confidence (as discussed in the next section); however, the temperature is still kept as a fixed parameter during our fit. For optically thin spectral lines such as HINSA the temperature cannot sufficiently constrain the shape of the absorption feature to be allowed as a free parameter since a wide range of combinations of temperatures and optical depths would be able to fit the data. Thus the gas temperature must be specified in any fits.

The line widths obtained from $^{13}$CO observations can only be applied to the H i gas under the assumption that the two species are coexistent throughout the cloud. This assumption, while strongly suggested by the previously referenced studies, is clearly not exact. Hence the molecular line widths for each velocity component are used as guess values for the H i fit. The H i line widths are also allowed to vary in value during the fit near the initial guess values.

In practice, $\tau_0$ is left as a free parameter, and $\sigma_{nt}$ as a constrained parameter for each component. For every trial value of $\tau_0$, a $T_B$ spectrum is constructed and its second derivative is checked for the fitting parameter as specified by equation (8). All velocity components along the same line of sight must be fitted simultaneously to account for obscuration due to overlapping. There are a variety of fitting algorithms that can be used for the final fitting. In the present case a variation of the Levenberg-Marquardt method was implemented. In the interest of clarity, we give in the Appendix a step-by-step description of the entire process including the determination of molecular parameters and HINSA extraction.

5. COMING TO TERMS WITH VARIABLE BEAM SIZES AND UNRESOLVED SOURCES

Our technique combines observations from different telescopes with different frequencies and beam sizes. The H i data presented in this paper was obtained using the 100 m Greenbank Telescope (GBT) having a 9’ beam size, while the $^{12}$CO and $^{13}$CO data were...
obtained using the Five College Radio Observatory (FCRAO) 14 m telescope having a 45° beam size. Combining these data sets is nontrivial. Convolving the CO data directly to a 9° beam has the effect of smoothing out a great deal of structure, kinematically as well as spatially, which is neither necessary nor desirable.

By fitting the molecular data prior to convolution we preserve information regarding individual velocity components which may otherwise disappear or be distorted in an H I beam. It is not necessary for all molecular components to have accompanying HINSA features. Maintaining this information is crucial to studying the comparative composition of different velocity components within a cloud, as well as properly determining their H I/H₂ ratios. High-resolution molecular data should only be convolved to a beam size which results in sufficient signal-to-noise to produce good fits to the individual emission components. Following this argument and a series of numerical experiments, our FCRAO spectra were convolved to a 2° beam size. Parameters including center velocity, line width, temperature, and optical depth for each molecular emission velocity component are derived using conventional fitting techniques. Corresponding guess parameters for the H I gas are then constructed by convolving the molecular fit parameters to the H I beam. This approach allows us to measure the HINSA column densities for each individual molecular component, as well as to utilize the high spatial resolution information on the molecular constituents of the cloud.

Many sources are unresolved when observed with available single-dish H I beams. Previous studies (Li & Goldsmith 2003; Goldsmith & Li 2005) have shown strong spatial and line width correlations between HINSA and ¹³CO. With this information we may make the assumption that HINSA and ¹³CO are well mixed throughout a cloud. This affords us the opportunity to make estimates of the H I beam filling factors by assuming that HINSA is present wherever significant ¹³CO emission is observed. This assumption, while not precise, is useful. In the case of resolved sources the beam filling factor plays a minimal role. In unresolved sources, or the edges of larger clouds, the beam filling factor as implemented by the present technique, does not alter the calculated H I/H₂ ratio, but only alters the total column densities of both species.

6. EXAMPLE SOLUTIONS USING SIMULATED DATA

A controlled way to assess the validity of our technique is to apply it to synthetic yet realistic data and compare the derived results with the known input values. Simulated data does not allow us to test the assumptions on the relations between molecular and atomic gas that are used by this technique. It is possible, however, to assume those relations to be correct and to test the validity of the HINSA fitting technique by itself. Further it is possible to observe the effects on the HINSA fit of having incorrectly determined the fitting parameters based on molecular data.

6.1. A Simple Case

The simplest case consists of a single emission component obscured by a single HINSA absorption feature as in Figure 5. As expected, HINSA is the dominant feature in the second derivative representation. For the simulated run, the center velocity and temperature of the cold H I gas are provided as precisely known quantities. The line width, however, is given only as a guess parameter. The task then is to search for those values of the line width and the peak optical depth which minimize the squared difference between the original simulated background spectrum and the calculated optical depth of 0.198.

6.2. Effects of Faulty Parameter Determination from Molecular Data

It is clear from the previous simulation that in an idealized case where the properties (center velocity, line width, and temperature) of the H I gas are well known, the HINSA optical depths can readily be determined to a high degree of accuracy. However, in practice, the properties of the H I gas are not known but are instead surmised with some uncertainty from observations of molecular tracers. Hence it is necessary to examine the impact of incorrectly deriving the H I properties from molecular data. One concern is the gas temperature, which we obtain from ¹³CO. It is reasonable to assume that the H I excitation temperature is equal to the kinetic temperature of the gas. However, temperature gradients may exist resulting in the average temperature appropriate for analyzing the HINSA being different from that determined from the optically thick common isotopologue of carbon monoxide. Since HINSA features are typically optically thin, an incorrect estimate of the H I gas temperature would not strongly alter the shape of the fitted HINSA, but would only affect the
derived optical depths and column densities. Thus, incorrect temperature estimates would be difficult to detect through fitting alone.

Incorrect estimates of the H I line widths based on $^{13}$CO data are plausible since the relation between them is only approximate. Here we investigate the effect of such an error by forcing our fitting routine to find the best fit HINSA optical depth while being constrained to use an incorrect value for the H I line width. Li & Goldsmith (2003) showed a variance of roughly 20% in the observed relation between HINSA and $^{13}$CO nonthermal line widths. Figure 7 shows the resulting fit when forced to utilize a HINSA line width 20% larger than the actual value of the same simulated data as in the previous example. It is no longer possible for the algorithm to produce a smooth recovered background spectrum, regardless of the optical depth assumed. This is a key indicator of an error in the input information. Figure 8 shows that the new best fit HINSA optical depth is 0.245 rather than 0.20. Similarly inaccurate solutions result when a faulty center velocity is used.

6.3. Complex Examples

It is often the case that two emission components may overlap at the velocities at which HINSA may be expected, thus obscuring the absorption feature and increasing the uncertainty in the analysis. In such cases, previous techniques have had great difficulty in determining whether any HINSA is present at all. Our technique attempts to recover the smoothest, most Gaussian-like background spectrum while being constrained by the H I gas parameters.

Fig. 7.—Using a HINSA line width 20% greater than the correct value when fitting the same simulated HINSA spectrum as in Fig. 5 produces a distorted recovered background spectrum. The second derivative representation has a clear signature at the velocity of the HINSA absorption feature observed, and both its appearance and that of the recovered background spectrum are unlikely to be representative of a realistic system. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 8.—Squared sum residuals of the second derivative of the recovered background spectra for different trial values of line optical depth using the same data, and incorrect line width as in Fig. 7. This function $I$ represents the quantity to be minimized in our fitting procedure described in § 4.5. The minimum value of $I$ is considered to be the best fit, and in this case corresponds to an optical depth of 0.245, compared to the correct optical depth of 0.20. Realistic simulated statistical noise is included in this plot, and while the range in optical depth corresponding to a specified increase in the residuals may be significant, there are no localized minima to confuse the results. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 9.—Two relatively broad identical emission components combined with a HINSA feature located midway between them yield the simulated observed spectrum. The recovered background spectrum and accompanying second derivative representation illustrate the results after HINSA extraction. The derived optical depth is 0.208, which compares very well with the input optical depth of 0.20. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 10.—Top: Two unequal emission components together with a HINSA feature result in the simulated observed spectrum. The recovered HINSA optical depth is 0.215, and the HINSA extraction results in the recovered background spectrum, which is nearly identical to the input background spectrum. Bottom: A similar case where the HINSA feature has been placed to coincide with a narrow emission component whose shape is slightly distorted by the presence of another emission component. The derived HINSA optical depth of 0.232 is still close to the value 0.20 that was input to the calculation. The second derivative representation of the recovered background spectra (green) shows that there is still significant structure left after HINSA extraction. This structure is the result of the interaction between two emission components. [See the electronic edition of the Journal for a color version of this figure.]
derived from molecular observations. It is unlikely for emission components to appear over a HINSA features in such a way as to completely obscure the absorption.

As an example, Figure 9 shows that our technique is able to accurately extract HINSA features which on visual inspection might be ambiguous. In this simulation, the HINSA feature with an optical depth of 0.20 was placed in between two identical emission components. The recovered HINSA optical depth had a value of 0.208, which is well within the error limits of other uncertainties as discussed in the section. Figure 10 shows further examples with more complex geometry in which our technique is still able to pick out HINSA features with good precision.

6.4. Distorted Emission Components

The technique we present here is based on the assumption that the background emission is composed primarily of a superposition of Gaussian-like components. Significant deviations from this assumption can lead to incorrect results. Figure 11 shows the case in which the background spectrum is complex, due to the superposition of overlapping components. The presence of a background component of width only modestly greater than that of the HINSA feature, in combination with the broader emission feature, results in a noticeable error in the derived HINSA optical depth. On casual inspection, there may be little indication that the solution is problematic (as shown in Fig. 11, bottom panel). Similarly, the residuals shown in Figure 12 do not betray any hint that the solution in Figure 11 may be wrong or ambiguous. When dealing with real data, where the actual background spectrum is not known beforehand, there is no simple way to determine when such distortions may occur, or what their effect might be on our derived HINSA optical depths. Although instances such as this are likely to be rare, it is worthwhile to be especially wary of HINSA measurements when the absorption feature seems to lie in the trough between two emission components.

6.5. Confidence in Derived HINSA Optical Depths from Simulated Data

It may be tempting to take Figure 8 and use standard methods for determining the uncertainties in least-squares linear fitting problems to determine the uncertainties in our estimates of $\tau$ based on the width of the trough of the fit residuals. But our problem does not strictly allow such analyses. There are two components that contribute uncertainty to our estimates of $\tau$. The first is radiometric noise. The simulated data in this section all include realistic simulated noise for observations made with a large single-dish telescope such as Arecibo or the GBT. This source of uncertainty is in fact linear in nature and could be treated using traditional least-squares linear fitting analyses. However, as $\text{H}_i$ spectra are typically bright and easily obtainable with only a few seconds of integration time this source of uncertainty is insignificant compared to the other uncertainties. If we were to look directly at Figure 8, then the uncertainties due to radiometric noise would appear as small, narrow localized minima. However, in the case of this and all our simulations this contribution is smaller than even the thickness of the line in the plot.

The dominant source of uncertainty which produces the trough is nonlinear and somewhat systematic. The fundamental principle on which our technique depends, as discussed in § 4.1, requires that our observed spectra resemble the superposition of multiple (positive and negative) Gaussians. According to the radiative transfer equation, however, this is not strictly the case. When multiple emission and absorption components are superimposed, they attenuate one another in a nonlinear fashion. Hence our observed spectra will never strictly represent Gaussians, but will only resemble them. This introduces a somewhat systematic, “nonlinearity” error which is much greater than that introduced by radiometric noise.

As an example, if we were to rerun the simulation from Figure 8 numerous times with all the same parameters for emission and absorption components but different radiometric noise (of the same average amplitude), the measured optical depth would always be very close to 0.245, the only variation in the results would be due to radiometric noise which in these realistic cases is insignificant. This represents a 22.5% error (not an uncertainty)
relative to the original imputed value of 0.20 which is produced not by random local minima which would confuse the results, but by a systematic shifting of the entire residuals trough. This systematic error is produced when several emission and absorption components are superimposed in such a way that the final spectra no longer resemble simple superpositions of Gaussians. Throughout this section we have shown successively more complex superpositions of this nature in an attempt to test the limits of this systematic error. We have found that for the vast majority of observations likely to be encountered, this error will be only a few percent. In an attempt to find the limits, we constructed the most complex superpositions that would likely be observed in real data and found that in the very worst cases the systematic error reached only about 50%. Figures 9, 10, and 11 represent some of those complex cases. For the majority of observations, this nonlinearity error will be minor in comparison to other sources of error due to uncertainties in physical conditions in the cloud. In the most extreme cases, the nonlinearity error will be on a par with, but not dominant compared to these other sources of uncertainty, which are discussed in the following section.

These simulations cannot directly test the validity of our assumptions regarding the relations between atomic and molecular gas properties as those are inherently nonstatistical uncertainties whose properties are currently not well known. However, the simulations do show that the solutions are modestly sensitive to any such errors in that errors in our $^{13}$CO-based estimates of either the H$_i$ line widths or center velocities necessarily produce visible artifacts in the recovered background spectra after HINSA extraction, as shown in Figure 7. This allows us to identify faulty fits, where we can then go back to the molecular data and construct better templates for the HINSA extraction. Hence it is important to understand all the possible sources of uncertainty in our estimates of the HINSA properties based on molecular templates as discussed in the following section.

7. LIMITATIONS: SOURCES OF AMBIGUITY AND UNCERTAINTY

While it has been shown in the previous section that our technique is able to produce good fits to the H$_i$ data when the H$_i$ gas properties are correctly derived, there are several factors which limit the accuracy of our technique. These sources of error are nonstatistical in nature in that they arise from our reliance on the association between H$_i$ and molecular tracers. As such, they make the placement of simple error bars on derived H$_i$ optical depths impossible. While it is certainly possible to place uncertainty estimates numerically through standard least-squares fitting methods, and compound those with estimates of the uncertainties due to measurements of the molecular parameters and their relation to the H$_i$ gas, any such numbers would be meaningless and misleading. The best way to estimate the uncertainties in our results is to examine the spread in the results in Figures 2 and 3 and assume that in an ideal case each component would be represented by a single line (of certain shape) which would be governed by the physical processes within the cloud such as the H$_i$ to H$_2$ conversion process. This would give us an upper limit on our uncertainty in deriving the H$_i$/H$_2$ ratio. However, the present models of cloud evolution do not yet offer us sufficient confidence to make such estimates with certainty. More detailed observations and modeling of molecular cloud processes are necessary before such estimates can be made with confidence. It is our hope that the technique described in this paper will help in this regard. This development is likely to be an iterative process. It is important to note that the sources of uncertainty in previous techniques, while inherently somewhat different, are not smaller nor better determined than those in the method presented here. It is, however, of critical importance to quantify and to understand each of these sources of uncertainty in order to understand the level of confidence in the results obtained using this new technique.

7.1. Gas Temperature

Knowledge of the H$_i$ gas temperature is necessary for estimating the nonthermal line widths of $^{13}$CO, as well as the observed line widths of the H$_i$ gas, the measurement of the H$_i$ optical depth, and column densities both for H$_i$ and molecular tracers. As such, accurate temperature estimates are critical. At the present time the most practical method for estimating the H$_i$ gas temperature in these dark cloud cores is through obtaining the $^{12}$CO temperature. Typically, $^{12}$CO emission is optically thick, and thus if large-scale velocity gradients are absent, as is generally the case in dark clouds, this measurement represents only the surface temperature of the cloud and does not necessarily reflect the average H$_i$ gas temperature in the interior of the cloud. The cores of these clouds are shielded from the external UV radiation field and thus are likely to be at a lower temperature than the surface. While this bias introduces overestimates of the H$_i$ gas temperature by only a few degrees, typical core temperatures are on the order of 10 K thus the effect is significant. It may be possible to use alternate means to estimate the temperatures of other molecular tracers which share similar distributions as H$_i$ within a cloud. One promising candidate is the inversion transitions of ammonia. Detailed comparison of this or other tracers with $^{12}$CO would require a better understanding of the spatial distribution of cold H$_i$ necessitating interferometric maps of HINSA features.

7.2. Radiometric Noise and Baselines

Twenty-one centimeter spectra with high signal-to-noise ratios are relatively easy to obtain with short integration times. Radiometric noise should thus not significantly affect HINSA measurements. Obtaining good baselines on the H$_i$ spectra is important in that any artificial slopes can significantly alter the derived optical depths. Since H$_i$ emission is highly variable spatially, ON-OFF observations are impractical. Total power ON measurements are ideal if system stability permits this approach to be used, as reported by (Li & Goldsmith 2003). Frequency-switched observations can yield sufficiently good baselines, but only if system bandpass and calibration are sufficiently good. A very successful combination of these techniques is available with the recently installed Arecibo L-band Feed Array (ALFA) in conjunction with the Galactic-ALFA (GALFA) spectrometer at Arecibo. Least-squares frequency switching (LSFS) allows the RF and IF bandpasses to be separately determined so that total power observations produce sufficiently good baselines for HINSA observations (M. Krčo et al. 2008, in preparation).

7.3. Choice of Molecular Tracer

Li & Goldsmith (2003) showed good correlations between the central velocities, nonthermal line widths and spatial distributions of HINSA and both $^{13}$CO and C$^{18}$O. In many cases, $^{13}$CO shows a better spatial correlation, but C$^{18}$O shows a better line width correlation. This would not be expected if the relative abundances of H$_i$, $^{13}$CO, and C$^{18}$O were constant throughout the cloud. The variations may be a reflection of the higher critical density of C$^{18}$O, the density dependence of the H$_i$/H$_2$ conversion process, and possibly CO depletion onto dust grains at high
densities. Thus the choice of which molecule makes the best template for the HINSA line width is complex. Based on experience, using $^{13}$CO fits provides somewhat better results. As a practical matter, the $^{13}$CO lines are much brighter and easier to observe than those of C$^{18}$O. As a result, we have selected $^{13}$CO as the template molecule for HINSA extraction.

7.4. Distances and Foreground Gas

The presence of intervening H$\text{\textsc{i}}$ emission sources between the observer and the HINSA source will tend to wash out the absorption and introduce underestimates of the HINSA optical depth. Van der Werf et al. (1988) introduced an adjustment factor to compensate for this effect; however, its precise value is poorly known. This approach was followed by Li & Goldsmith (2003), but these authors found it to be relatively unimportant for the relatively nearby clouds in the Taurus Molecular Cloud Complex. A more detailed study of the relation between HINSA features and distance is required when dealing with more distant sources.

7.5. Component Order

Several velocity components are often observed in molecular clouds. It is assumed that there are correspondingly multiple HINSA components. Equation (1) indicates that when multiple overlapping HINSA components are present along the same line of sight with appreciable optical depths, there is significance in which component is placed in the background, and which is in the foreground. There is no simple way to determine the sequence of multiple components along the same line of sight. Since the typical optical depths for HINSA are on the order of 0.1, the resulting ambiguity for the H$\text{\textsc{i}}$ column densities is below 10%, smaller than the other sources of error in our technique.

7.6. Molecular Emission Fits and the Uniqueness Problem

The center velocities of an individual molecular component may vary significantly within a cloud. It is often the case that two components may be widely separated in velocity at one point in the cloud while being nearly indistinguishable in another region. This poses a problem when trying to make measurements on individual components and is especially pronounced when working with H$\text{\textsc{i}}$ spectra with high thermal line widths. Further, it is difficult to predict how a specific implementation of the technique proposed here might react when working with such spectra. When working with a sufficiently small number of spectra it is possible to recognize and dismiss the affected spectra by eye. However, in larger regions, where there may be many thousands of spectra, and where fully automated fitting algorithms must be used, the problem becomes more pronounced. Thus we have developed a method for partly alleviating the uniqueness problem and allowing for easier identification of overlapping emission spectra. This is a problem most often avoided in many projects involving complex emission spectra since usually only the total integrated intensity is used. For the technique presented here, however, the individual properties of each emission component are necessary.

Molecular emission spectra are typically characterized by (overlapping) Gaussian components. It is a property of Gaussians that taking any Gaussian function to the fourth power effectively reduces its line width by a factor of 2. Therefore, by taking all our molecular spectra to the fourth power prior to fitting, the individual emission components become significantly more apparent. However, doing this also greatly magnifies the noise in the spectra. Thus it is necessary to perform some filtering by removing the high-frequency Fourier components of each spectrum. The magnification and filtering processes can significantly alter the amplitude, shape, and line widths of the final spectra. However, the final center velocities remain the same. Thus, the filtered spectra may be used to identify the number and center velocity of the emission components within a spectrum. Subsequently, the known velocities can be used to constrain the fitting of the original data. While not yielding a unique solution, this method does allow more accurate fitting of molecular emission components using automated algorithms and dealing with large spectral data sets.

8. SUMMARY AND FUTURE WORK

The significance of accurately measuring the H$\text{\textsc{i}}$ content of dark clouds and star forming regions has long been recognized. HINSA features have been shown as a promising method to achieve this goal, but the difficulty in confidently disentangling HINSA from the galactic background emission has limited research efforts in this field. The technique described here builds on previous work to provide new opportunities. By utilizing a second derivative representation in which HINSA becomes the dominant feature in the spectrum, and using information from associated molecular tracers to constrain our fits, we are able to obtain H$\text{\textsc{i}}$ column densities with greater confidence than previously possible. This technique enables us to study individual velocity components within a cloud. Several uncertainties and sources of error still remain, most notably the errors inherent in temperature determination through $^{12}$CO, and the precise relations between the properties of cold H$\text{\textsc{i}}$ and molecular tracers such as $^{13}$CO. As shown in § 6 the purely statistical errors derived from our constructed data were only a few percent in simple cases which represent the majority of observed spectra. The nonlinearity error may be as large as 50% in the extreme cases.

While the scope of this paper is limited to a demonstration of the new technique, the improved confidence in the results have the potential to yield significant scientific advances in the field of molecular cloud and star formation studies which are deferred to a subsequent publication. The two astronomical sources discussed briefly here for demonstration purposes are part of a much larger survey of over 30 dense cloud cores. In conjunction with numerical modeling these data provide us with the chemical ages of the clouds and individual components therein, thus providing a significant constraint on theoretical models of star formation. Large maps have been collected with the Arecibo L-band Feed Array (ALFA) of the Taurus and Perseus star forming regions. These regions show plentiful HINSA features whose analysis may shed light on the dynamics of such complexes including the processes which may have triggered their formation.

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DESCRIPTION OF THE PROCEDURE

For definitiveness, we here describe the process of obtaining molecular parameters to be used as a template as well as the actual HINSA extraction in a step-by-step fashion. We model this description based on our specific observations and available data although the technique can be easily adapted to other circumstances. We obtained \(^{12}\)CO and \(^{13}\)CO data with the 14 m Five College Radio Observatory having a 45" to 50" FWHM beam size and the H\(_i\) data were obtained using the 100 m Green Bank Telescope having a 9' FWHM beam size.

1. Reduction and regridding.—It is assumed that all data have been properly reduced, calibrated, and regridded.

2. Convolution.—Initially the \(^{12}\)CO and \(^{13}\)CO spectra are convolved to a 2' FWHM beam size in order to improve the signal to noise ratio. A Gaussian convolving beam shape is used.

3. CO fitting (first pass).—While assuming a particular gas temperature (10 K), emission functions are fitted to all \(^{13}\)CO spectra to obtain their center velocities. These velocities are used to determine the excitation temperature of each \(^{13}\)CO velocity component, using the accompanying \(^{12}\)CO spectrum. The temperatures are derived under the assumption that the \(^{12}\)CO emission is in LTE and optically thick at line center. Then, for \(^{12}\)CO (Stahler & Palla 2004)

\[
f(T_{\text{kin}}) = f(T_{\text{ex}}) = \frac{T_{\text{Bu}}}{T_0} + f(T_{\text{bg}}),
\]

where the excitation temperature \(T_{\text{ex}}\) is taken to be equal to the kinetic temperature \(T_{\text{kin}}\), \(T_{\text{Bu}}\) is the observed brightness temperature at line center, \(T_0\) is the equivalent temperature of the transition (5.5 K for the \(J = 1 \rightarrow 0\) transition of \(^{12}\)CO), \(T_{\text{bg}}\) is the blackbody radiation temperature of the background field (2.7 K), and

\[
f(T) = \frac{1}{\exp(T_0/T) - 1}.
\]

4. CO fitting (second pass).—Line width, and optical depth are derived for each \(^{13}\)CO emission component using the previously obtained temperatures and center velocities. Subsequently the nonthermal line widths are obtained using

\[
\sigma_{\text{obs}}^2 = \sigma_{\text{nt}}^2 + \sigma_{\text{th}}^2,
\]

where \(\sigma_{\text{obs}}\) is the observed total line width, and \(\sigma_{\text{nt}}\) and \(\sigma_{\text{th}}\) are, respectively, the nonthermal and thermal components of the line width.

5. Molecular column densities.—Column densities for \(^{13}\)CO for each component along each line of sight are calculated according to the technique described in Stahler & Palla (2004). \(H_2\) column densities are estimated using a \(H_2/^{13}\)CO abundance ratio of \(7 \times 10^3\) for these galactic clouds.

6. Estimating atomic parameters.—The center velocity, nonthermal line width, and temperature for the H\(_i\) gas are determined for each velocity component along each line of sight by convolving the previously obtained \(^{13}\)CO parameters to the GBT 9' beam as weighted by the \(^{13}\)CO column densities according to

\[
X_{j,\text{comp}} = \frac{\sum_i f(\theta_{ij})X_{i,\text{comp}}N_{i,\text{comp}}^{^{13}\text{CO}}}{\sum_i f(\theta_{ij})N_{i,\text{comp}}^{^{13}\text{CO}}},
\]

where \(X_{j,\text{comp}}\) represents the temperature, nonthermal line width, or center velocity of a particular velocity component at sky position \(j\), \(i\) represents all nearby sky positions to be convolved, \(f(\theta_{ij})\) represents the beam response function for angular distance \(\theta\) between points \(i\) and \(j\), and \(N_{i,\text{comp}}^{^{13}\text{CO}}\) represents the corresponding \(^{13}\)CO column density for a particular component at position \(i\). For comparison with \(H\) column densities, the \(H_2\) column densities are similarly convolved without the mass weighting factor.

7. Having estimates of the temperature, central velocity, and line width for each HINSA component we can, for any value of the optical depth, remove the HINSA absorption to obtain the background emission using equation (1). Since their true ordering along the line of sight is not known, as a matter of definition velocity components with the highest positive velocities are taken to be the most distant.

8. Finally we search for those values of \(\tau_0\) and \(\sigma_{\text{th}}\) which minimize the value of equation (8) to arrive at our solutions.

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