ENERGY RELEASE DUE TO ANTINEUTRINO UNTRAPPING FROM HOT QUARK STARS *

D. N. AGUILERA, †
Fachbereich Physik, Universität Rostock, Universitätsplatz 1, 18051 Rostock, Germany
Instituto de Física Rosario, Bv. 27 de febrero 210 bis, 2000 Rosario, Argentina
E-mail: deborah@darss.mpg.uni-rostock.de

D. BLASCHKE
Fachbereich Physik, Universität Rostock, Universitätsplatz 1, 18051 Rostock, Germany
Bogoliubov Laboratory for Theoretical Physics, JINR, 141980 Dubna, Russia
email: david@thsun1.jinr.ru

H. GRIGORIAN
Fachbereich Physik, Universität Rostock, Universitätsplatz 1, 18051 Rostock, Germany
Department of Physics, Yerevan State University, Alex Manoogian Str. 1, 375025 Yerevan, Armenia
email: hovik@darss.mpg.uni-rostock.de

An equation of state for 2-flavor quark matter (QM) with diquark condensation under the conditions for compact stars -\(\beta\)-equilibrium, charge and color neutrality- is presented. Trapped antineutrinos prevent the formation of the diquark condensate at moderate densities above a critical value of the antineutrino chemical potential \(\mu_{\bar{\nu}_e}\). The following consequences are presented: 1) The star develops a 2-phase structure \(\mu_{\bar{\nu}_e} \geq \mu_{\bar{\nu}_e}^{\text{c}}\): a color superconducting QM core and a normal QM shell. 2) During the cooling, when the temperature is small enough \(T < 1 \text{ MeV}\) the antineutrino mean free path becomes larger than the thickness of the normal QM shell and the antineutrinos get untrapped in a sudden burst. The energy release is estimated as \(\sim 10^{52}\) erg and an antineutrino pulse is expected to be observed.

*Conference Proceedings of the KIAS-APCTP International Symposium in Astro-Hadron Physics Compact Stars: Quest for New States of Dense Matter, Nov. 10-14, 2003, Seoul, Korea.
†Partially supported by Landesgraduiertenförderung von M V Germany and by the Virtual Institute of the Helmholtz Association “Dense Hadronic Matter and QCD Phase Transition” under grant No. VH-VI-041.
1. Introduction

It has been proposed that cold dense quark matter should be in a superconducting state with the formation of a diquark condensate\(^1\),\(^2\). The consequences of the diquark condensation for the configuration and the cooling behaviour of compact stars have been broadly studied \(^3\),\(^4\),\(^5\),\(^6\) and the question if this phase can be detected by the signatures still remains \(^7\).

Also the engine for the most energetic phenomena in the universe like supernova explosions and gamma ray burst does not have a satisfactory explanation yet \(^8\) and it has been proposed that the energy involved could be related with the occurrence of the color superconductivity phase \(^9\),\(^10\).

Since the pairing energy gap in quark matter is of the order of the Fermi energy, the diquark condensation gives a considerable contribution to the equation of state (EoS) that is estimated of the order of \((\Delta/\mu)^2\).

Disregarding relativistic effects, the total binding energy release in the core of a cooling protoneutron star has been estimated as \((\Delta/\mu)^2 M_{\text{core}} \simeq 10^{52}\) erg. But, if relativistic effects are considered, the gravitational mass defect of the cooling star decreases when diquark condensation is included and there is no explosive process \(^6\) possible since the color superconductivity transition is second order.

In this work a new mechanism of releasing the energy in an explosive way is presented (for the original idea see \(^11\)). During the collapse of a protoneutron star antineutrinos are produced by the \(\beta\)-processes and remain trapped due to the small mean free path. This increases the asymmetry in the system and therefore the diquark condensation is inhibited at moderate densities. So, a two-phase structure develops in the star: a superconducting interior and a surrounding shell of normal quark matter, the latter being opaque to antineutrinos for \(T \geq 1\) MeV \(^12\). In the cooling process the antineutrino mean free path increases above the size of this normal matter shell and an outburst of neutrinos occurs releasing an energy of about \(10^{51} - 10^{52}\) erg. This first order phase transition leads to an explosive phenomenon in which a pulse of antineutrinos could be observed.

1.1. Equation of state for 2-flavour quark matter

A nonlocal chiral quark model for 2-flavour \(\{u,d\}\) and three color \(\{r,b,g\}\) superconducting (2SC) quark matter in the mean field approximation is used, for details see \(^6\),\(^13\). The order parameters are the mass gap \(\phi_f\) and the diquark gap \(\Delta\) for the chiral and superconducting phase transitions respectively. As in \(^13\), the following chemical potentials are introduced:
\[ \mu_q = (\mu_u + \mu_d)/2 \]
for quark number, \( \mu_I = (\mu_u - \mu_d)/2 \)
for isospin asymmetry and \( \mu_8 \) for color charge asymmetry. The deviation in the color space is considered \( \mu_8 \ll \mu_q \), so the effect of considering \( \mu_8 \) is neglected.

The quark thermodynamic potential is expressed as

\[
\Omega_q(\phi, \Delta; \mu_q, \mu_I, T) + \Omega_{\text{vac}} = \frac{\phi^2}{4G_1} + \frac{\Delta^2}{4G_2} - \frac{2}{2\pi^2} \int_0^\infty dq q^2 (N_c - 2) \{2E_\phi + \omega [E_\phi - \mu_q - \mu_I, T] \\
+ \omega [E_\phi - \mu_q + \mu_I, T] + \omega [E_\phi + \mu_q - \mu_I, T] + \omega [E_\phi + \mu_q + \mu_I, T]\} \\
- \frac{4}{2\pi^2} \int_0^\infty dq q^2 \{E_+ + E_- + \omega [E_\phi^+ - \mu_I, T] \\
+ \omega [E_+ + \mu_I, T] + \omega [E_\phi^+ - \mu_I, T] + \omega [E_\phi^+ + \mu_I, T]\} \\
= \omega[E, T] = T \ln [1 + \exp (-E/T)] .
\]

The dispersion relations for the quarks of unpaired and paired colors are respectively,

\[
E_\phi^2 = q^2 + (m + F^2(q)\phi)^2 \\
E_\phi^\pm = (E_\phi^\pm + \mu)^2 + F^4(q)\Delta^2
\]

The interaction between the quarks is implemented via a Gaussian form-factor function \( F(q) \) in the momentum space (Gaussian types give stable hybrid configurations) as \( F(q) = \exp(-q^2/\Lambda^2) \).

The parameters \( \Lambda = 1.025 \text{ GeV}, G_1 = 3.761 \text{ } \Lambda^2 \) and \( m_u = m_d = m = 2.41 \text{ MeV} \) are fixed by the pion mass, pion decay constant and the constituent quark mass at \( T = \mu = 0 \). The constant \( G_2 \) is a free parameter of the approach and is fixed as \( G_2 = 0.86 G_1 \).

### 1.1.1. Stellar matter conditions

The stellar matter in the quark core of compact stars is considered to consists of \( u \) and \( d \) quarks and leptons (electrons \( e^- \) and antineutrinos \( \bar{\nu}_e \)) under the following conditions

- \( \beta \)-equilibrium \( \quad d \leftrightarrow u + e^- + \bar{\nu}_e, \quad \mu_e + \mu_{\bar{\nu}_e} = -2\mu_I \)
- Charge neutrality \( \quad 2\frac{2}{3}n_u - \frac{2}{3}n_d - n_e = 0, \quad n_B + n_I - 2n_e = 0 \)
- Color neutrality \( \quad n_8 = 0, \quad 2n_q - n_q^b = 0 \)
where \( n_j = \frac{\partial \Omega}{\partial \mu_j} \bigg|_{T, \phi = \phi_0, \Delta = \Delta_0} \) are the number densities corresponding to the chemical potential \( \mu_j \) defined above.

The lepton contributions \((l = e, \bar{\nu}_e)\) as ideal Fermi gases

\[
\Omega_l(\mu, T) = -\frac{1}{12\pi^2}\mu^4 - \frac{1}{6}\mu^2T^2 - \frac{7}{180}\pi^2T^4
\]

are added to the quark thermodynamical potential

\[
\Omega(\phi, \Delta; \mu_q, \mu_I, \mu_e, T) = \Omega_q(\phi, \Delta; \mu_q, \mu_I, T) + \Omega_e(\mu_e, T) + \Omega_{\bar{\nu}_e}(\mu_{\bar{\nu}_e}, T).
\]

The baryon chemical potential \( \mu_B = 3\mu_q - \mu_I \) is introduced as the conjugate of the baryon number density \( n_B \).

The \( \Omega \) function can have several minima in the \( \phi, \Delta \) plane, an example is shown in Fig. 1. The global minimum represents the stable equilibrium of the system and the minima search is performed solving the gap equations

\[
\frac{\partial \Omega}{\partial \phi} \bigg|_{\phi = \phi_0; \Delta = \Delta_0} = 0 = \frac{\partial \Omega}{\partial \Delta} \bigg|_{\phi = \phi_0; \Delta = \Delta_0}
\]

under the conditions that are mentioned above for the stellar interior.

The thermodynamics of the system, e.g. pressure \( P \), energy density \( \epsilon \), number density \( n \) and entropy density \( s \), is defined via this global minimum

\[
\Omega(\phi_0, \Delta_0; \mu_B, \mu_I, T) = \epsilon - Ts - \mu_Bn_B - \mu_In_I = -P.
\]

To fulfill the charge neutrality condition (see Fig. 2, right) a mixed phase between a subphase with diquark condensation (subscript \( \Delta > 0 \)) and normal quark matter subphase (subscript \( \Delta = 0 \)) is defined via the Glendenning construction. The Gibbs condition for equilibrium at fixed \( T \) and \( \mu_B \) is that the pressure of the subphases should be the same

\[
P = P^{\Delta > 0}(\mu_B, \mu_I, \mu_e, T) = P^{\Delta = 0}(\mu_B, \mu_I, \mu_e, T).
\]

The volume fraction \( \chi \) that is occupied by the subphase with diquark condensation is defined by the charge \( Q \) in the subphases,

\[
\chi = Q_{\Delta > 0}/(Q_{\Delta > 0} - Q_{\Delta = 0})
\]

and is plotted on the right panel in Fig. 2 for different antineutrino chemical potentials as a function of \( \mu_B \). In the same way, the number
Equation 11 and 12 express the densities for the different particle species \( j \) and the energy density are given by

\[
n_j = \chi n_{j\Delta > 0} + (1 - \chi)n_{j\Delta = 0},
\]

\[
\epsilon = \chi \epsilon_{\Delta > 0} + (1 - \chi)\epsilon_{\Delta = 0}.
\]

1.1.2. Equation of state with trapped antineutrinos

Increasing the antineutrino chemical potential \( \mu_{\bar{\nu}_e} \) increases the asymmetry in the system and this shifts the onset of the superconducting phase transition to higher densities.

Above a critical value of \( \mu_{\bar{\nu}_e} \geq \mu_{\bar{\nu}_e}^c \approx 30 \text{ MeV} \) (see critical value in Fig.}
2, on the left) first a normal quark matter phase occurs and then the phase transition to superconducting matter takes place, see Fig. 3, on the left.

The consequences for the equation of state can be seen on the right of the Fig. 3: the onset of the superconductivity in quark matter is shifted to higher densities and the equation of state becomes harder.

1.2. Quark stars and antineutrino trapping

The configurations for the quark stars are obtained by solving the Tolman-Oppenheimer-Volkoff equations for a set of central quark number densities $n_q$ for which the stars are stable.

In Fig. 4 the configurations for different antineutrino chemical potentials are shown. The equations of state with trapped antineutrinos are softer and therefore this allows more compact configurations. The presence of antineutrinos tends to increase the mass of the star for a given central
Figure 3. **Left:** Solutions of the gap equations and $\mu_I$ as a function of $\mu_B$. Increasing the antineutrino chemical potential increases the asymmetry in the system and the superconducting phase is inhibited at moderate densities. **Right:** Equation of state for different values of $\mu_{\bar{\nu}_e}$ of trapped antineutrinos. As $\mu_{\bar{\nu}_e}$ increases the equation of state becomes harder.

density.

A reference configuration with total baryon number $N_B = 1.51 N_\odot$ (where $N_\odot$ is the total baryon number of the sun) is chosen and the case with (configurations A and B in Fig. 4) and without antineutrinos ($f$ in Fig. 4) are compared. A mass defect can be calculated between the configurations with and without trapped antineutrinos at constant total baryon number and the result is shown on the right panel of Fig. 4). The mass defect could be interpreted as an energy release if the configurations A, B with antineutrinos are initial states and the configuration $f$ without them is the final state of a protoneutron star evolution.

1.2.1. **Protoneutron star evolution with antineutrino trapping**

After the collapse of a protoneutron star the star cools down by surface emission of photons and antineutrinos. Antineutrinos are trapped because they were generated by the direct $\beta$-process in the hot and dense matter and could not escape due to their small mean free path. The region of the star where the temperature falls below the density dependent critical value for diquark condensation, will transform to the color superconducting state.
which is almost transparent to (anti)neutrinos. But nevertheless due to
the trapped antineutrinos there is a dilute normal quark matter shell which
prevents neutrino escape from the superconducting bulk of the star, see Fig.
5 and Fig. 6. The criterion for the neutrino untrapping transition is to cool
the star below a temperature of about 1 MeV when the mean free path of
neutrinos becomes larger than the shell radius. If at this temperature the
antineutrino chemical potential is still large then the neutrinos can escape
in a sudden outburst. If it is small then there will be only a gradual increase
in the luminosity. An estimate for the possible release of energy within the
outburst scenario can be given via the mass defect defined in the previous
subsection between an initial configuration with trapped neutrinos (state
A or B) and a final configuration without neutrinos (state f).
Figure 5. Left graph: Quark star cooling by antineutrino and photon emission from the surface. Middle graph: Two-phase structure develops due to the trapped antineutrinos: a normal quark matter shell and a superconducting interior. Right graph: Antineutrino untrapping and burst-type release of energy.

Figure 6. Star evolution corresponding to Fig. 5 plotted in the phase diagram

2. Conclusions

The effects of trapped antineutrinos on the diquark condensates in quark star configurations are investigated. At fixed baryon number the energy release in the antineutrino untrapping transition is of the order of $10^{52}$ erg. This is a first order transition and leads to an explosive release of energy that could help to explain energetic phenomena in the universe like gamma ray bursts or supernova explosions.
Acknowledgments

D.N.A and D.B. thank the organizers of the KIAS-APCTP International Symposium in Astro-Hadron Physics of Compact Stars for their invitation and in particular D.K. Hong and the staff of the Department of Physics at Pusan National University for their hospitality and interest in this work. D.N.A acknowledges the DAAD-HOST programm D/03/31497 and the local Department of Physics for the financial support of the visit to Pusan University. The authors enjoyed the wonderful atmosphere and lively discussions with all colleagues during the conference.

References
1. M. Alford, J. A. Bowers and K. Rajagopal, J. Phys. G 27 (2001) 541 [Lect. Notes Phys. 578 (2001) 235] [arXiv:hep-ph/0009357].
2. D. Blaschke, N. K. Glendenning and A. Sedrakian, “Physics Of Neutron Star Interiors.” Springer Lecture Notes in Physics 578 (2001). Prepared for ECT* International Workshop on Physics of Neutron Star Interiors (NSI00), Trento, Italy, 19 Jun - 7 Jul 2000
3. D. Blaschke, T. Klahn and D. N. Voskresensky, Astrophys. J. 533 (2000) 406 [arXiv:astro-ph/9908334].
4. D. Page, M. Prakash, J. M. Lattimer and A. Steiner, Phys. Rev. Lett. 85 (2000) 2048 [arXiv:hep-ph/0005094].
5. D. Blaschke, H. Grigorian and D. N. Voskresensky, Astron. Astrophys. 368 (2001) 561 [arXiv:astro-ph/0009120].
6. D. Blaschke, S. Fredriksson, H. Grigorian and A. M. Oztas, [arXiv:nucl-th/0301002].
7. D. Blaschke, H. Grigorian, D. N. Aguilera, S. Yasui and H. Toki, AIP Conf. Proc. 660 (2003) 209 [arXiv:hep-ph/0301087].
8. T. Piran and E. Nakar, arXiv:astro-ph/0202403.
9. R. Ouyed and F. Sannino, Astron. Astrophys. 387 (2000) 725 [arXiv:astro-ph/0103022].
10. D. K. Hong, S. D. H. Hsu and F. Sannino, Phys. Lett. B 516, (2001) 362 [arXiv:hep-ph/0107017].
11. D. Aguilera, D. Blaschke and H. Grigorian, [arXiv:astro-ph/0212237].
12. S. Reddy, M. Prakash and J. M. Lattimer, Phys. Rev. D 58 (1998) 013009 [arXiv:astro-ph/9710115].
13. H. Grigorian, D. Blaschke and D. N. Aguilera, arXiv:astro-ph/0303518.
14. O. Kiriyama, S. Yasui and H. Toki, Int. J. Mod. Phys. E 10 (2001) 501 [arXiv:hep-ph/0105170].
15. S. M. Schmidt, D. Blaschke and Y. L. Kalinovsky, Phys. Rev. C 50 (1994) 435.
16. M. Prakash, J. M. Lattimer, R. F. Sawyer and R. R. Volkas, Ann. Rev. Nucl. Part. Sci. 51 (2001) 295 [arXiv:astro-ph/0103095].