Gravitational Faraday Rotation in Binary Pulsar Systems

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ABSTRACT

We study the gravitational Faraday rotation, on linearly polarized light rays emitted by a pulsar, orbiting another compact object. We relate the rotation angle to the orbital phase of the emitting pulsar, as well as to other parameters describing its orbit and the orientation of the angular momentum of the binary companion. We give numerical estimates of the effect for the double-pulsar system PSR J0737-3039, and we note that the expected magnitude is exceedingly small, making the effect unlikely to be observed with present technology. It is however interesting per se, since in this phenomenon, gravito-magnetism plays a leading role, unlike what happens, for instance, when studying light bending or gravitational time delay, where it appears as a correction to the gravito-electric contribution. Also, we envisage the possibility that this effect could be relevant, at least in principle, for a pulsar orbiting a non charged black-hole.

Key words: gravitation, relativity, polarization, pulsars: general, pulsars: individual (PSR J0737-3039)

1 INTRODUCTION

The physical properties of the recently discovered double-pulsar system PSR J0737-3039 (Burgay et al. (2003), Lyne et al. (2004)) make this system a rare laboratory for testing relativistic gravity. Namely, the system is composed of a 22-ms pulsar, PSR J0737-3039 A, in a slightly eccentric ($e \simeq 0.088$) $2.4 h$ orbit, together with PSR J0737-3039 B, whose period is $2.7 s$: thanks to the high mean orbital speed ($v \simeq 0.001c$) this system is highly relativistic, and allows the detection of four post-Keplerian parameters, thus accurately constraining the masses of the two stars and providing also stringent tests of General Relativity (GR) (Kramer et al. (2005)).

Actually, excluding cosmological issues, GR has passed all observational tests with excellent results (Will (2005)). However, we must remember that most of the tests of GR come from Solar System experiments, where the gravitational field is in the "weak" regime. On the other hand, it is expected that deviations from GR can occur in the "strong" field regime: hence, the Solar System experiments are inadequate to this end. On the contrary, the strong gravitational field is best tested by means of pulsars. Among the post-Newtonian effects that should be relevant in strong field regime (Will (1993)), there are the so called gravito-magnetic (GM) effects, which arise from the rotation of the sources of the gravitational field: these effects are generally small, since the gravitational coupling with the angular momentum of the source is much weaker than the coupling with mass alone (the so called gravito-electric interaction), and this makes their detection very difficult (see Mashhoon et al. (2001); Ruggiero & Tartaglia (2002)).

In previous papers (Tartaglia & Ruggiero (2005); Ruggiero & Tartaglia (2005)), we pointed out that binary pulsar systems in general and, in particular, the double-pulsar PSR J0737-3039, could be very interesting for testing the GM effects. The relevance of GM effects in binary pulsar systems was investigated in the past and also more recently by many authors, who also pointed out the difficulties that the detection of these effects must face (Doroshenko & Kopeikin (1995); Laguna & Wolszczan (1997); Wex & Kopeikin (1999); Kopeikin & Mashhoon (2002); Rafikov & Lai (2006, 2007)), mainly due to their smallness, which is overwhelmed by other dominant effects.

Here, we would like to study a physical phenomenon where the gravito-magnetic interaction has a leading role, and which can be tested, at least in principle, in binary pulsar systems, i.e. systems where a pulsar is orbiting a compact object, such as another pulsar, as in PSR J0737-3039, or, hopefully, a black hole. The phenomenon we are going to deal with is the gravitational Faraday rotation.

The polarization vector of an electromagnetic (e.m.) wave propagating in a gravitational field undergoes a change of direction, as a result of the deflection of e.m. signals, and, in addition a rotation around the propagation vector. As
The phenomena we try and study here could be particularly relevant in the case of X-ray emission from accretion disks or other thick material envelopes surrounding rotating magnetized compact objects and specifically black holes. Indeed the intrinsic birefringence of the plasma in the magnetic field, together with the general relativistic effects due to the curvature in proximity of horizons, can be the primary source of polarization and of the rotation of the polarization plane of the emitted short wavelength radiation \cite{connors1980, broderick2004, nouri-zonoz1999, sereno2003}.

Under the hypothesis that, to a reasonable order of approximation, the space-time around a rotating compact object (i.e. a pulsar, or a black hole) can be described by the Kerr metric, here we investigate the gravitational Faraday rotation induced on the light rays emitted by a pulsar orbiting around it. We study the effect and give estimates of its magnitude, and evaluate the possibility of detecting it.

The propagation of e.m. signals in a gravitational field, can be studied by writing the Maxwell equations in the Lorentz gauge; on following \cite{fayos1982, nouri-zonoz1999}, we may write:

\begin{align}
\nabla_\alpha A^\alpha &= 0, \\
\nabla_\alpha \nabla_\alpha A^\alpha &= 0.
\end{align}

This is done considering two basic approximations: (i) the electromagnetic field is weak enough not to affect the gravitational field; (ii) the geometric optics limit is adopted, where it is assumed that the wavelength $\lambda$ of the e.m. waves is much smaller than $L$, where $L = \text{min}(L, R)$, $L$ being a typical distance where amplitude, polarization and wavelength vary appreciably and $R$ is the order of magnitude of the (spatial) curvature radius. Then, we may introduce the wave four-vector $k^\alpha$ and polarization four-vector $f^\alpha$, which fulfill the following relations

\begin{equation}
k'^0 = 0, \quad k'^f = 0, \quad f^0 = 1.
\end{equation}

Light rays follow null geodesics, whose tangent vector is $k^\alpha$; furthermore, the polarization four vector undergoes parallel transport along these geodesics. Consequently, we may write

\begin{equation}
k'^\alpha \nabla_\alpha k^\beta = 0, \quad k'^\alpha \nabla_\alpha f^\beta = 0.
\end{equation}

The changes induced by the gravitational field on the wave and polarization four-vectors, are then obtained integrating the equations \cite{fayos1982, nouri-zonoz1999}. Actually, this problem can be suitably studied in the 1+3 projection formalism, applied to the stationary space-time describing the gravitational field in which waves propagate. This approach is outlined by \cite{fayos1982, nouri-zonoz1999}; here, we briefly recall its foundations.

Let $\mathcal{M}$ be the manifold which is the model of our space-time, endowed with the metric $g_{\alpha\beta}$, together with a metric connection $\Gamma$. Since the space-time is stationary, there exists a time-like Killing vector field $\xi$. Then, one can show that, for each $p \in \mathcal{M}$, there is a three-dimensional manifold $\Sigma_3$, defined by the smooth map

\begin{equation}
\Psi : \mathcal{M} \rightarrow \Sigma_3,
\end{equation}

$\Psi = \Psi(p)$ being the orbit of the Killing field $\xi$ passing through $p$. We can introduce a coordinate system adapted to the congruence $\xi_i = \partial_t$, and let $\gamma_{ij}$ be the projected three-dimensional metric of $\Sigma_3$. Suitable differential operators can be also defined on $\Sigma_3$, starting from $\gamma_{ij}$, in such

\footnote{Greek indices run from 0 to 3, Latin indices run from 1 to 3; the space-time metric has signature $(1, -1, -1, -1)$, and we use units such that $G=c=1$; boldface letters refer to three dimensional vectors.}
a way that a covariant derivative $\nabla$ for the three vectors belonging to $\Sigma_3$ is defined (see, for details Fayos & Llosa (1982); Nouri-Zonoz (1999); Landau & Lifshitz (1971) and references therein).

If we write the metric of a stationary space-time in the form
\[
ds^2 = h \left(dx^0 - A_i dx^i\right)^2 - dl^2,
\]
where
\[
A_i = -\frac{g_{0i}}{g_{00}}, \quad h = g_{00},
\]
and
\[
dl^2 = \left(-g_{ij} + \frac{g_{0i}g_{0j}}{g_{00}}\right) dx^i dx^j = \gamma_{ij} dx^i dx^j,
\]
it is possible to introduce the gravito-electric and gravitomagnetic fields, given respectively by
\[
E_g = -\nabla h^{1/2},
\]
\[
B_g = \nabla \times A,
\]

In this context, on the basis of an orthogonal decomposition in adapted coordinates, the three-vectors $k$ and $f$, representing the projection of $k^a, f^a$ on $\Sigma_3$, can be taken to be equivalent to the contravariant components of $k^a$ and $f^a$, respectively. Then, from equations (12) and (13), it is possible to write the three-dimensional equations
\[
\nabla_k k = L \times k + (E_g \cdot k) k
\]
\[
\nabla_k f = L \times f
\]
where $L = L(E_g, B_g, k, f)$ (see Fayos & Llosa (1982); Nouri-Zonoz (1999)).

Eqs. (12, 13) show, in particular, that the polarization vector rotates with angular velocity $L$ along the projected geodesic. By carrying out explicit calculations, the angle of rotation around $k$ which expresses the gravitational Faraday rotation is given by
\[
\alpha_g = -\frac{1}{2} \int_{\text{sou}}^{\text{obs}} \sqrt{h} B_g \cdot dl,
\]
where the integral is evaluated from the source (emission point) to the observer, along the null geodesics whose tangent (spatial) vector is $\hat{k}$, such that $dl = k dl$.

It is useful to recall here the expression of the electromagnetic Faraday rotation (Lorimer & Kramer 2003):
\[
\alpha_{\text{Faraday}} = \frac{4\pi c^3}{m_e^2 c^2 \omega^2} \int_{\text{sou}}^{\text{obs}} n B || dl,
\]
where we have the physical quantities that describe the properties of the plasma where the wave propagates ($n$ is the refraction index), the angular frequency $\omega$ of the radiation and $B ||$, which is the component of the magnetic field along the line of sight. On comparing (14) and (15) we see that both depend on the component of the gravitomagnetic or magnetic field along the line of sight. There is a formal analogy, even though the effects are physically different since the gravitational Faraday rotation is a purely geometric effect, while the traditional Faraday rotation depends on the frequency of the light rays.

Formula (14) shows also that the gravitational Faraday rotation is an effect due to mass-current, i.e. to the rotation of the source of the gravitational field, so it is a purely gravito-magnetic phenomenon, and it is null in space-times around non rotating sources.

As we said before, this effect was investigated in the past by many authors, in suitable weak field approximations. We are interested in the effect on light rays emitted by a pulsar orbiting a compact object; hence, we cannot apply here calculations outlined in Plebanski (1960), since their level of approximation is too low, and gives no rotation of the polarization vector unless light rays penetrate into rotating distributions of matter.

A better level of approximation was considered by Ishihara et al. (1983); Nouri-Zonoz (1999); Serend (2002), who studied propagation of light rays outside the horizon of a rotating Kerr black hole. These authors agree on the right order of approximation needed, however there is some disagreement on numerical values. In particular Serend (2002) obtained for the gravitational rotation angle the expression
\[
\alpha_g = -\mu \frac{\pi M J ||}{4 \xi^3}
\]
where $\mu$ is a parameter that quantifies the contribution of angular momentum to space-time curvature ($\mu = 1$ in GR), $M$ is the mass of the source of gravitational field and $J ||$ is the component of its angular momentum along the line of sight, $\xi$ is the projection in the plane of sight of the distance between the emission point and the source of the gravitational field.

In the following Section, under the hypothesis that, to a reasonable order of approximation, the space-time around a rotating compact object (i.e. a pulsar, or a black hole) can be described by the Kerr metric, we apply (14) in order to study the gravitational Faraday rotation on the light rays emitted by a pulsar orbiting the compact object.

Before going on, we notice that a better (or more realistic) approximation for describing the space-time around a rotating compact object such as a pulsar, is given by the metric obtained by Hartle & Thorne (1968). Since a neutron star’s moment of inertia is larger than the one of a black hole of similar mass, the Hartle-Thorne metric could lead to a slightly larger gravitational Faraday rotation, however we do not expect that this can significantly change the order of magnitude of the effect.

### 3 Gravitational Faraday Rotation in Binary Pulsar Systems

We use the notation of Figure 1 for the description of the pulsar orbit around the compact binary companion (see Straumann (2004)). We choose a first set of Cartesian coordinates $\{x, y, z\}$, with origin in the center of mass of the binary system, and such that the line of sight is parallel to the $z$ axis. Then, we introduce another set of Cartesian coordinates $\{X, Y, Z\}$, with the same origin: the $X$ axis is directed along the ascending node, the $Z$ axis is perpendicular to the orbital plane. The angle between the $x$ and $X$
axes is Ω, the longitude of the ascending node, while the angle between the z and Z axes is i, the inclination of the orbital plane.

Let \( \vec{x}_p \) be the vector describing the orbit of the pulsar (here and henceforth, the suffix p refers to the emitting pulsar, while c refers to its binary companion): it is described by
\[
\hat{x}_p = \cos(\omega + \varphi) \hat{X} + \sin(\omega + \varphi) \hat{Y},
\]
(17)
in terms of the argument of the periastron, \( \omega \), and the true anomaly, \( \varphi \). Let us pose
\[
\theta = \omega + \varphi,
\]
(18)
for the sake of simplicity. Then, we use the notation
\[
\vec{r} = \vec{x}_p - \vec{x}_c
\]
(19)
to describe the position of the pulsar with respect to its companion, and we remember that we have, for the Keplerian problem
\[
r = \frac{a(1 - e^2)}{1 + e \cos \varphi},
\]
(20)
where \( a \) is the semi-major axis of the relative motion and \( e \) is the eccentricity. The astronomical elements \( \Omega, i, \omega, a, e \) represent the Keplerian parameters.

Then, let \( \vec{J} \) be the angular momentum of the binary companion: its direction is determined by the angle \( \lambda_c \) and \( \eta_c \) between the projection of the \( \hat{J} \) onto the plane of the sky and the ascending node of the binary orbit (see Figure 1).

As a consequence, we may write
\[
\vec{J} = \sin \lambda_c \cos (\Omega + \eta_c) \hat{x} + \sin \lambda_c \sin (\Omega + \eta_c) \hat{y} + \cos \lambda_c \hat{z}.
\]
(21)

**Figure 1.** Notation used for describing the pulsar orbit.

Let \( \vec{r} = r_p \hat{x}_p \) be the vector describing the orbit of the pulsar (here and henceforth, the suffix p refers to the emitting pulsar, while c refers to its binary companion): it is described by
\[
\hat{x}_p = \cos(\omega + \varphi) \hat{X} + \sin(\omega + \varphi) \hat{Y},
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As a consequence, we may write
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\vec{J} = \sin \lambda_c \cos (\Omega + \eta_c) \hat{x} + \sin \lambda_c \sin (\Omega + \eta_c) \hat{y} + \cos \lambda_c \hat{z}.
\]
(21)

**Figure 2.** Notation and conventions for describing the pulsar orbit and the spin axis of the binary companion. The spin axis \( \hat{J} \) orientation is determined by the angles \( \lambda_c, \eta_c \). The system \( \{x, y, z\} \) is such that \( xy \) is the plane of sight, the \( \{X, Y, Z\} \) system is such that \( XY \) is the orbital plane.

The unit vector along the line of sight is \( \hat{n} = -\hat{z} \), so that we have
\[
\hat{n} = \hat{Z} - \sin \theta \hat{Y},
\]
(23)
we have
\[
\xi = r \sqrt{\cos^2 \theta + \sin^2 \theta \cos^2 \varphi},
\]
(24)
Consequently, the rotation angle of the polarization vector \( \alpha_g \), setting \( \mu = 1 \) as in GR, becomes
\[
\alpha_g = \frac{\pi M|\vec{J}|}{4 \cos^2 \theta + \sin^2 \theta \cos^2 \varphi} (\cos^2 \theta + \sin^2 \varphi)^{3/2},
\]
(25)
or, explicitly writing the equation of the orbit \( r \), setting \( \mu = 1 \) as in GR, becomes
\[
\alpha_g = \frac{\pi G^2}{4 c^5} \frac{M|\vec{J}|}{a^3(1 - e^2)^3} (\cos^2 \theta + \sin^2 \varphi + \sin^2 \theta \cos^2 \varphi)^{3/2},
\]
(26)
in physical units.

We see that the rotation angle depends on the orbital phase: this fact, could lead, in principle, to a way of detecting it.

**4 DISCUSSION**

In order to quantitatively evaluate the magnitude of the gravitational Faraday rotation for light rays emitted by a pulsar in a binary system, it is useful to write \( \alpha_g \) in the form
\[
\alpha_g = AF(\varphi),
\]
(27)
We may say that, provided that $J_\parallel \neq 0$, the effect is bigger for small orbits with great eccentricity and inclination. Actually, even though the peculiar dependence on the orbital phase may give a chance to measure the rotation angle, in very favorable conditions, the overall effect remains very small, and furthermore, it is overwhelmed by the electromagnetic Faraday rotation (see, for instance, Demorest et al. [2004], and references therein).

However, it is interesting to point out that since the electromagnetic Faraday rotation is proportional to the square of the wavelength of the radiation (inversely proportional to $\omega^2$), the situation could be different in the case of short wavelength emission, such as X-rays.

To give a quantitative evaluation, we recall that the polarization position angle (PPA) can be written in the form (see Lorimer & Kramer (2005))

$$\alpha_{PPA}(\lambda) = \frac{\alpha_{Faraday}}{2} \equiv \lambda^2 \times RM,$$

(31)

where $\lambda$ is the wavelength, and the rotation measure $RM$ is defined by

$$RM = \frac{e^3}{2\pi mc^2} \int_{sou} nB_\parallel dl.$$

(32)

The rotation measures of the two pulsars A and B in the double system PSR J0737-3039 have been recently measured by Demorest et al. [2004], and the corresponding values are $-112.3 \pm 1.5 \, \text{rad m}^{-2}$ and $-118 \pm 12 \, \text{rad m}^{-2}$, respectively. On using these values, we may estimate the PPA of the X-ray emission from PSR J0737-3039 A. On the ground of the recent observations of the X-ray emissions from the double pulsar performed by McLaughlin et al. [2004] and Campana et al. [2004], we consider emission at 1 keV ($\lambda = 1.24 \times 10^{-9} \text{m}$), which gives

$$\alpha_{PPA}(\lambda = 1.24 \times 10^{-9} \text{m}) \approx 1.72 \times 10^{-16} \text{rad}.$$  

(33)

In other words, if the other physical conditions are the same, the electromagnetic Faraday effect for X-rays is some 17 orders of magnitude smaller than for radio wavelengths: in these conditions, the purely geometrical frequency independent rotation, i.e. the gravitational Faraday rotation, can become relevant.

5 CONCLUSIONS

We have studied the gravitational Faraday rotation on light rays emitted by a pulsar, orbiting another compact object, such as a pulsar or a black-hole, whose gravitational field, to a reasonable order of approximation, can be described by the Kerr metric, of which we have considered the suitable weak-field limit.

In this phenomenon, gravito-magnetism plays a leading role, unlike what happens, for instance, when studying light bending or gravitational time delay, where it appears as a correction to the gravito-electric contribution.

The effect is a purely geometric one, contrary to the electromagnetic Faraday rotation, which depends on the frequency of the light rays; furthermore, it depends on the component of the angular momentum of the source of the gravitational field along the line of sight. So, when the latter is not null, a net rotation of the polarization plane is expected.

We have obtained a formula which relates the rotation
angle to the orbital phase of the emitting pulsar, as well as to the parameters describing its orbit and to the orientation of the angular momentum of the binary companion.

The numerical estimates of the magnitude of this effect, in the context of the double-pulsar system PSR J0737-3039, show that at radio wavelengths it is overwhelmingly dominated by the electromagnetic Faraday rotation. Nevertheless, as a result of its achromatic nature, gravitational Faraday rotation will dominate electromagnetic Faraday rotation at X-ray energies. However, in practice, due to its exceedingly small nature, it is unlikely to be observed in pulsar systems.

Anyway, this effect could be of some relevance for pulsar-black hole binaries, or accreting black holes (Connors et al. 1980). In such systems, peculiar modulations of polarization orientation during the orbital motion may be evidence that an unseen companion is a black hole.

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