A surface with canonical map of degree 24

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Abstract
We construct a complex algebraic surface with geometric genus \( p_g = 3 \), irregularity \( q = 0 \), self-intersection of the canonical divisor \( K^2 = 24 \) and canonical map of degree 24 onto \( \mathbb{P}^2 \).

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1 Introduction
Let \( S \) be a smooth minimal surface of general type with geometric genus \( p_g \geq 3 \). Denote by \( \phi : S \dashrightarrow \mathbb{P}^{p_g-1} \) the canonical map and let \( d := \deg(\phi) \). The following Beauville’s result is well-known.

Theorem 1 (Beauville). If the canonical image \( \Sigma := \phi(S) \) is a surface, then either:
(i) \( p_g(\Sigma) = 0 \), or
(ii) \( \Sigma \) is a canonical surface (in particular \( p_g(\Sigma) = p_g(S) \)).
Moreover, in case (i) \( d \leq 36 \) and in case (ii) \( d \leq 9 \).

Beauville has also constructed families of examples with \( \chi(\mathcal{O}_S) \) arbitrarily large for \( d = 2, 4, 6, 8 \) and \( p_g(\Sigma) = 0 \). Despite being a classical problem, for \( d > 8 \) the number of known examples drops drastically. Tan’s example [Ta , §5] with \( d = 9 \) and Persson’s example [Pe] with \( d = 16, q = 0 \) are well known. Du and Gao [DG] show that if the canonical map is an abelian cover of \( \mathbb{P}^2 \), then these are the only possibilities for \( d > 8 \). More recently the author has given examples with \( d = 12 \) [Ri2] and \( d = 16, q = 2 \) [Ri3].

In this paper we construct a surface \( S \) with \( p_g = 3, q = 0 \) and \( d = 24 \), obtained as a \( \mathbb{Z}_4^2 \)-covering of \( \mathbb{P}^2 \). The canonical map of \( S \) factors through a \( \mathbb{Z}_2^2 \)-covering of a surface with \( p_g = 3, q = 0 \) and \( K^2 = 6 \) having 24 nodes, which in turn is a double covering of a Kummer surface.

Notation
We work over the complex numbers. All varieties are assumed to be projective algebraic. A \((-n)\)-curve on a surface is a curve isomorphic to \( \mathbb{P}^1 \) with self-intersection \(-n\). Linear equivalence of divisors is denoted by \( \equiv \). The rest of the notation is standard in Algebraic Geometry.

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2 \( \mathbb{Z}_2^r \)-coverings

The following is taken from [Ca], the standard reference is [Pa].

**Proposition 2.** A normal finite \( G \cong \mathbb{Z}_2^r \)-covering \( Y \to X \) of a smooth variety \( X \) is completely determined by the datum of

1. reduced effective divisors \( D_\sigma, \forall \sigma \in G \), with no common components;
2. divisor classes \( L_1, \ldots, L_r \), for \( \chi_1, \ldots, \chi_r \) a basis of the dual group of characters \( G^\vee \), such that

\[
2L_i \equiv \sum_{\chi_\sigma = -1} D_\sigma.
\]

Conversely, given 1. and 2., one obtains a normal scheme \( Y \) with a finite \( G \cong \mathbb{Z}_2^r \)-covering \( Y \to X \), with branch curves the divisors \( D_\sigma \).

The covering \( Y \to X \) is embedded in the total space of the direct sum of the line bundles whose sheaves of sections are the \( \mathcal{O}_X(L_i) \), and is there defined by equations

\[
u_{\chi_i}u_{\chi_j} = u_{\chi_i}\chi_j \prod_{\chi_\sigma = -1} x_\sigma,
\]

where \( x_\sigma \) is a section such that \( \text{div}(x_\sigma) = D_\sigma \).

The scheme \( Y \) can be seen as the normalization of the Galois covering given by the equations

\[
u_{\chi_i} = \prod_{\chi_\sigma = -1} x_\sigma,
\]

and \( Y \) is irreducible if \( \{ \sigma | D_\sigma > 0 \} \) generates \( G \).

For a covering \( \pi : Y \to X \) with ramification divisor \( R \), the Hurwitz formula \( K_Y = \pi^*(K_X) + R \) holds. Let us describe the canonical system for the case where \( \pi \) is a \( \mathbb{Z}_2^r \)-covering with smooth branch divisor. We have branch curves \( D_1, D_2, D_3 \) and relations \( 2L_i \equiv D_j + D_k \), for all permutations \( (i,j,k) \) of \( \{1,2,3\} \). The covering \( \pi \) factors as

\[\phi : Y \to W_i, \quad \varphi : W_i \to X,\]

where \( \varphi \) is the double covering corresponding to \( L_i \). Let \( R_i \) be the ramification divisor of \( \phi \). One has

\[K_Y \equiv \phi^*(K_{W_i}) + R_i \quad \text{and} \quad K_{W_i} \equiv \varphi^*(K_X + L_i),\]

which gives

\[K_Y \equiv \pi^*(K_X + L_i) + \frac{1}{2}\pi^*(D_i), \quad i = 1, 2, 3.\]

Finally we notice that taking the quotient by a subgroup \( H \) of the Galois group of the covering corresponds to considering the subalgebra generated by the line bundles \( L_{\chi^{-1}} \), where \( \chi \) ranges over the characters orthogonal to \( H \).
3 The construction

We show in the Appendix the existence of reduced plane curves $C_6$ of degree 6 and $C_7$ of degree 7 through points $p_0, \ldots, p_5$ such that:

- $C_7$ has a triple point at $p_0$ and tacnodes at $p_1, \ldots, p_5$;
- $C_6$ is smooth at $p_5$, has a node at $p_0$ and tacnodes at $p_1, \ldots, p_4$;
- the branches of the tacnode of $C_j$ at $p_i$ are tangent to the line $T_i$ through $p_0, p_i$, $j = 1, 2, i = 1, \ldots, 4$;
- the branches of the tacnode of $C_7$ at $p_5$ are tangent to $C_6$;
- the singularities of $C_6 + C_7$ are resolved via one blow-up at $p_0$ and two blow-ups at each of $p_1, \ldots, p_5$.

**Step 1 (Construction)**

Consider the map

$$\mu : X \rightarrow \mathbb{P}^2$$

which resolves the singularities of the curve $C_7$. Then $\mu$ is given by blow-ups at $p_0, p_1, p_1', \ldots, p_5, p_5'$, where $p_i'$ is infinitely near to $p_i$. Let $E_0, E_1, E_1', \ldots, E_5, E_5'$ be the corresponding exceptional divisors (with self-intersection $-1$).

Let $x, y, z, w$ be generators of the group $\mathbb{Z}_2^4$ and

$$\psi : Y \rightarrow X$$

be the $\mathbb{Z}_2^4$-covering defined by

$$D_x := \tilde{T}_1 - E_0 - 2E_1',$$
$$D_y := \tilde{T}_2 - E_0 - 2E_2',$$
$$D_z := \tilde{C}_6 - 2E_0 - \sum_{i=1}^4 (2E_i + 2E_i') - 2E_5',$$
$$D_w := \tilde{C}_7 + \tilde{T}_4 - 4E_0 - \sum_{i=1}^3 (2E_i + 2E_i') - (2E_4 + 4E_4') - (2E_5 + 2E_5'),$$
$$D_{xy} := \tilde{T}_3 - E_0 - 2E_3',$$
$$D_{xz} := \cdots := D_{zw} := 0,$$

where the notation $\tilde{\cdot}$ stands for the total transform $\mu^*(\cdot)$.

We note that each of the divisors $D_x, D_y, D_{xy}$ and $T_4 - E_0 - 2E_4'$ (contained in $D_w$) is a disjoint union of two $(-2)$-curves.

For $i, j, k, l \in \{-1, 1\}$, let $\chi_{ijkl}$ denote the character which takes the value $i, j, k, l$ on $x, y, z, w$, respectively. There exist divisors $L_{ijkl}$ such that

$$2L_{ijkl} \equiv \sum_{\chi_{ijkl}(\sigma)=-1} D_\sigma, \quad (1)$$
thus the covering \( \psi \) is well defined. Since there is no 2-torsion in the Picard group of \( X \), then \( \psi \) is uniquely determined. The surface \( Y \) is smooth because the curves \( D_x, \ldots, D_{xy} \) are smooth and disjoint. Division of the equations (14) by 2 gives that the \( L_{ijkl} \) are according to the following table. For instance \( L_{-1111} \equiv \tilde{T} - E_0 - E'_1 - E'_3 \).

| \( L_{-1111} \)  | \( \tilde{T} \) | \( E_0 \) | \( E_1 \) | \( E'_1 \) | \( E_2 \) | \( E'_2 \) | \( E_3 \) | \( E'_3 \) | \( E_4 \) | \( E'_4 \) | \( E_5 \) | \( E'_5 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( L_{-1111} \)  | 1               | -1             | 0              | -1             | 0              | 0              | -1             | 0              | 0              | 0              | 0              | 0              |
| \( L_{-1111} \)  | 1               | -1             | 0              | -1             | 0              | 0              | -1             | 0              | 0              | 0              | 0              | 0              |
| \( L_{111} \)     | 3               | -1             | -1             | -1             | -1             | -1             | -1             | -1             | -1             | 0              | -1             |
| \( L_{111} \)     | 4               | -2             | -1             | -2             | -1             | -1             | -2             | -1             | 1              | 0              | -1             |
| \( L_{-111} \)    | 4               | -2             | -1             | -2             | -1             | -2             | -1             | -1             | 0              | -1             |
| \( L_{-111} \)    | 4               | -2             | -1             | -2             | -1             | -2             | -1             | -1             | 0              | -1             |
| \( L_{111} \)     | 4               | -2             | -1             | -1             | -1             | -1             | -2             | -1             | -1             |
| \( L_{111} \)     | 5               | -3             | -1             | -2             | -1             | -1             | -2             | -1             | -2             | -1             |
| \( L_{-111} \)    | 5               | -3             | -1             | -1             | -2             | -1             | -2             | -1             | -2             | -1             |
| \( L_{111} \)     | 5               | -3             | -1             | -2             | -1             | -2             | -1             | -1             |
| \( L_{-111} \)    | 7               | -3             | -2             | -2             | -2             | -2             | -3             | -1             |
| \( L_{-111} \)    | 8               | -4             | -2             | -3             | -2             | -2             | -3             | -1             |
| \( L_{111} \)     | 8               | -4             | -2             | -2             | -3             | -2             | -3             | -1             |
| \( L_{-111} \)    | 8               | -4             | -2             | -2             | -3             | -2             | -3             | -1             |

**Step 2 (Invariants)**

Since

\[ K_X \equiv -3\tilde{T} + E_0 + \sum_1^5 (E_i + E'_i), \]

then

\[ \chi(O_Y) = 16\chi(O_X) + \frac{1}{2} \sum (L_{ijkl}^2 + K_X L_{ijkl}) = -16 - 1 - 1 - 1 + 0 - 1 - 1 - 1 + 0 - 1 - 1 - 1 - 1 = 4. \]

For the computation of

\[ p_g(Y) = p_g(X) + \sum h^0(X, O_X(K_X + L_{ijkl})), \]

let

\[ T_1 := (\tilde{T} - E_0 - 2E'_4 + E_5 - E'_5), \]

\[ T_2 := (\tilde{T} + \tilde{T}_3 + \tilde{T}_4 - 3E_0 - \sum_2^4 2E'_i + E_5 - E'_5), \]

\[ L_1 := \left| 3\tilde{T} - E_0 - \sum_1^3 (E_i + E'_i) - E_4 - E_5 \right| \]

and

\[ L_2 := \left| 2\tilde{T} - (E_1 + E'_1) - E_2 - E_3 - E_4 - E_5 \right|. \]

Each of \( T_1, T_2 \) is a disjoint union of \((-2)\)-curves intersecting negatively \( K_X + L_{111-1-1} \), \( K_X + L_{-1-1-1-1} \), respectively, thus we have

\[ \left| K_X + L_{111-1-1} \right| = T_1 + L_1 \]
and

\[ |K_X + L_{1-1-1-1}| = \mathcal{T}_2 + \mathcal{L}_2. \]

We show in the Appendix that \( \mathcal{L}_1 \) has only one element and \( \mathcal{L}_2 \) is empty. Hence

\[ h^0(X, \mathcal{O}_X(K_X + L_{11-1-1})) = 1 \]

and

\[ h^0(X, \mathcal{O}_X(K_X + L_{1-1-1-1})) = 0. \]

Analogously

\[ h^0(X, \mathcal{O}_X(K_X + L_{-11-1-1})) = h^0(X, \mathcal{O}_X(K_X + L_{-1-1-1-1})) = 0. \]

It is easy to see that

\[ h^0(X, \mathcal{O}_X(K_X + L_{11-11})) = h^0(X, \mathcal{O}_X(K_X + L_{111-1})) = 1 \]

and

\[ h^0(X, \mathcal{O}_X(K_X + L_{ijkl})) = 0 \]

for the remaining cases. We conclude that

\[ p_g(Y) = 0 + 1 + 1 + 1 = 3. \]

Now we compute the self-intersection of the canonical divisor for the minimal model \( S \) of \( Y \). The divisor

\[ \xi_1 := \frac{1}{2} \psi^* \left( \sum_{1}^{3} (\tilde{T}_1 - E_0 - 2E'_1) \right) \]

is a disjoint union of \( 8 \times 6 = 48 \) \((-1)\)-curves and the divisor

\[ \xi_2 := \frac{1}{2} \psi^* \left( \tilde{T}_4 - E_0 - 2E'_4 + E_5 - E'_5 \right) \]

is a disjoint union of \( 8 \times 3 = 24 \) \((-1)\)-curves.

The covering \( \psi \) factors through the double covering \( \varphi : W \to X \) with branch locus \( D_z + D_w \). We have \( K_W \equiv \varphi^*(K_X + L_{11-1-1}) \), hence the Hurwitz formula gives

\[ K_Y \equiv \xi_1 + \psi^*(K_X + L_{11-1-1}). \]

Thus one of the canonical curves of \( Y \) is

\[ \xi_1 + 2\xi_2 + \psi^*(\mathcal{C}), \]

where \( \mathcal{C} \) is the unique element in the linear system \( \mathcal{L}_1 \) defined above. From \( \xi_1 \xi_2 = \xi_1 \psi^*(\mathcal{C}) = \psi^*(\mathcal{C})^2 = 0 \) and \( \xi_2 \psi^*(\mathcal{C}) = 24 \), we get \( K_Y^2 = -48 \). We show in the Appendix that the curve \( \mathcal{C} \) is irreducible, therefore \( \psi^*(\mathcal{C}) \) is nef and then \( K_Y^2 = 24 \).

**Step 3** (The canonical map)
The divisors

\[ D_z, D_w, D_{zw} \]
define a $\mathbb{Z}_2^2$-covering $\rho : U \to X$.

We have

$$\chi(O_U) = 4\chi(O_X) + \frac{1}{2} \sum (L_{11}^2 + K_X L_{11}) = 4 + 0 + 0 + 0 = 4$$

and

$$p_g(U) = p_g(X) + \sum h^0(X, O_X(K_X + L_{11})) = 0 + 1 + 1 + 1 = 3.$$

The surface $U$ is the quotient of $Y$ by the subgroup $H$ generated by $x, y$. The group $H$ acts on the minimal model $S$ of $Y$ with only isolated fixed points, so $S/H$ is the canonical model $\overline{U}$ of $U$ and then

$$K^2_{\overline{U}} = 6.$$

Finally we want to show that the canonical map of $U$ is of degree 6 onto $\mathbb{P}^2$. It suffices to verify that the canonical system has no base component nor base points. The canonical system of $U$ is generated by the divisors

$$K_1 := \frac{1}{2} \rho^*(D_z) + \rho^*(K_X + L_{111})$$
$$K_2 := \frac{1}{2} \rho^*(D_w) + \rho^*(K_X + L_{11})$$
$$K_3 := \frac{1}{2} \rho^*(D_{zw}) + \rho^*(K_X + L_{11-1}).$$

Denote by $\vartheta_1, \ldots, \vartheta_4$ the four $(-1)$-curves

$$\frac{1}{2} \rho^*(\overline{T}_4 - E_0 - 2E_4)$$

and by $\vartheta_5, \vartheta_6$ the two $(-1)$-curves

$$\frac{1}{2} \rho^*(E_5 - E_5').$$

Let

$$\pi : U \to U'$$

be the contraction to the minimal model and $q_1, \ldots, q_6 \in U'$ be the points obtained by contraction of $\vartheta_1, \ldots, \vartheta_6$. If $\kappa$ is an effective canonical divisor of $U'$, then

$$H := \pi^*(\kappa) + \vartheta_1 + \cdots + \vartheta_6$$

is a canonical curve of $U$. So, the multiplicity of a curve $\vartheta_i$ in $H$ is 1 if and only if the curve $\kappa$ does not contain the point $q_i$.

Since the multiplicity of $\vartheta_5 + \vartheta_6$ in $K_1$ is 1, the points $q_5, q_6$ are not base points of the canonical system of $U'$. The multiplicity of $\vartheta_1 + \cdots + \vartheta_4$ in $K_2$ is 1, so also the points $q_1, \ldots, q_4$ are not base points of the canonical system of $U'$. Now to conclude the non-existence of other base points, it suffices to show that the plane curves

$$\mu \circ \rho(K_i), \ i = 1, 2, 3,$$
have common intersection \( \{p_0, p_1, \ldots, p_5\} \) and their singularities are no worse than stated. This is done in the Appendix. Here we just note that these curves are

\[ T_4 + C_3, \quad T_4 + C_7, \quad T_4 + C_6, \]

where \( C_3 \) is the plane cubic corresponding to the unique element in the linear system \( L_1 \), defined in Step 2 above.

**Step 4** (Conclusion)
The \( \mathbb{Z}_2^2 \)-covering \( \psi : Y \to X \) factors as

\[ Y \xrightarrow{4:1} U \xrightarrow{4:1} X. \]

Since \( p_g(Y) = p_g(U) = 3 \) and the canonical map of \( U \) is of degree 6, then the canonical map of \( Y \) is of degree 24.

**Remark 3.** Consider the intermediate double covering \( \epsilon : Q \to X \) of \( \rho \) with branch locus \( D_z \). Then \( Q \) is a Kummer surface: each divisor \( \epsilon^* \left( \tilde{T} - E_0 - 2E_i' \right) \) is a disjoint union of four \(-2\)-curves. The surface \( U \) contains 24 disjoint \(-2\)-curves \( A_1, \ldots, A_{24} \), the pullback of \( \sum_1^3 \epsilon^* \left( \tilde{T} - E_0 - 2E_i' \right) \), such that the covering \( Y \to U \) is a \( \mathbb{Z}_2^2 \)-Galois covering ramified over the divisors

\[ A_1 + \cdots + A_8, \quad A_9 + \cdots + A_{16}, \quad A_{17} + \cdots + A_{24}. \]

**Appendix**

The following code is implemented with the Computational Algebra System Magma [BCH], version V2.21-8.

First we compute the curves \( C_6 \) and \( C_7 \) referred in Section 3. We choose the points \( p_0, \ldots, p_5 \) with a symmetry axis and compute the curves using the Magma function LinSys given in [Ri1].

```plaintext
A<x,y>:=AffineSpace(Rationals(),2);
P:=[A![0,0],A![2,2],A![-2,2],A![3,1],A![-3,1],A![0,5]];
M1:=[[2],[2,2],[2,2],[2,2],[2,2],[1,1]];
M2:=[[3],[2,2],[2,2],[2,2],[2,2],[2,2]];
T:=[[1],[1,1],[[-1,1]],[[3,1]],[[1,0]]];
J6:=LinSys(LinearSystem(A,6),P,M1,T);
J7:=LinSys(LinearSystem(A,7),P,M2,T);
C6:=Curve(A,Sections(J6)[1]);
C7:=Curve(A,Sections(J7)[1]);
```

We consider the projective closure of the curves and verify that they are irreducible and the singularities are exactly as stated.

```plaintext
P2<x,y,z>:=ProjectiveClosure(A);
C6:=ProjectiveClosure(C6);
C7:=ProjectiveClosure(C7);
IsAbsolutelyIrreducible(C6);
IsAbsolutelyIrreducible(C7);
```
SingularPoints(C6 join C7);
HasSingularPointsOverExtension(C6 join C7);
[ResolutionGraph(C6,P[i]): i in [1..#P-1]];
[ResolutionGraph(C7,P[i]): i in [1..#P]];
[ResolutionGraph(C6 join C7,P[i]): i in [1..#P]];

To clarify the situation at the origin, we use:

d:=DefiningEquation(TangentCone(C7,A![0,0]));
d eq y*(x^2 + 40585383/1587545*y^2);

thus the singularity is ordinary.

The defining polynomials of $C_6$ and $C_7$ are:

$$
289*x^6+754326*x^4*y^2+2610657*x^2*y^4+1906344*y^6-2013848*x^4*y*z
-17946576*x^2*y^3*z-22212504*y^5*z+1336400*x^4*z^2
+35856160*x^2*y^2*z^2+89326224*y^4*z^2-22270208*x^2*y*z^3
-146421504*y^3*z^3+295936*x^2*z^4+84049920*y^2*z^4
$$

and

$$
8683464*x^6*y-494984955*x^4*y^3-1064093674*x^2*y^5-558251235*y^7
-11358312*x^6*z+253333746*x^4*y^2*z+8340957732*x^2*y^4*z
+7286240034*y^4*z^2-17394911410*x^2*y^3*z^2
-32292299971*y^5*z^3+179839940*x^4*z^3+11716330200*x^2*y^2*z^3
+55580514660*y^4*z^3-32468306400*y^2*z^4
$$

Now we show that the linear system $L_1$, defined in Step 2 above, has exactly one element. Let $L_1$ be the corresponding linear system of plane cubics. By parameter counting, $\dim(L_1) \geq 0$. If $\dim(L_1) \geq 1$, then one of its curves contains the line $T_3$, because

$$
\left( \tilde{T}_3 - E_3 - E_5 \right) \left(3\tilde{T} - E_0 - \sum_{1}^{3}(E_i + E'_i) - E_4 - E_5 \right) = 0.
$$

The other component of this curve is a conic, but one can verify that the conic through $p_4$ tangent to the lines $T_1, T_2$ at $p_1, p_2$, which is given by the equation

$$
x^2 - 9y^2 + 32y - 32 = 0,
$$

does not contain the point $p_5$. We compute the unique plane cubic $C_3$ in $L_1$ and show that it is irreducible:

M:=[[1],[1,1],[1,1],[1,1],[1,0],[1,0]];
J3:=LinSys(LinearSystem(A,3),P,M,T);
#Sections(J3) eq 1;
C3:=ProjectiveClosure(Curve(A,Sections(J3)[1]));
IsAbsolutelyIrreducible(C3);

The defining polynomial of $C_3$ is:

$$
17*x^3-924*x^2*y-153*x*y^2-996*y^3+1164*x^2*z
+544*x*y*z+6516*y^2*z-544*x*z^2-7680*y*z^2
$$
To conclude that the linear system $L_2$, defined in Step 2, is empty, it suffices to note that the conic $C$ through $p_1, \ldots, p_5$ is not tangent to the line $T_1$ at the point $p_1$. An equation for $C$ is

$$-12x^2 + 11y^2 - 93y + 190 = 0.$$ 

Finally we verify that the curves

$$T_4 + C_6, \ C_7, \ T_4 + C_3,$$

referred in the end of Section 3 have intersection $\{p_0, p_1, \ldots, p_5\}$:

$$T4:=\text{Curve}(P2, x+3*y);$$
$$\text{PointsOverSplittingField}((T4 \text{ join } C6) \text{ meet } C7 \text{ meet } (T4 \text{ join } C3));$$

and the singularities are no worse than stated:

$$[\text{ResolutionGraph}(T4 \text{ join } C3 \text{ join } C6 \text{ join } C7,p):p \text{ in } P];$$

To clarify the situation at the origin, we use:

$$TC:=\text{TangentCone}(T4 \text{ join } C3 \text{ join } C6 \text{ join } C7,P2![0,0,1]);$$
$$\text{DefiningEquation}(TC) \text{ eq } y*(x+3*y)*(x + 240/17*y)*$$
$$*(x^2 + 82080/289*y^2)*(x^2 + 40585383/1587545*y^2);$$

thus the singularity is ordinary.

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