GENTLE/4fan

A package of Fortran programs for the description of $e^+e^-$ annihilation into four fermions†

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Abstract

We describe the program package gentle/4fan, which consists of the Fortran codes gentle4fan.f, 4fan.f, and gentle nc qed.f and is devoted to the description of $e^+e^-$ annihilation into four fermions. The codes are based on the semi-analytical approach. Initial state QED corrections are taken into account. The program versions, which are described here were used in the 1995 workshop ‘Physics at LEP 2’.

† Contributions to the working groups on ‘Event Generators for WW Physics’ and ‘Event Generators for Discovery Physics’ of the 1995 Workshop on Physics at LEP 2, to appear in the proceedings (G. Altarelli, T. Sjöstrand and F. Zwirner (eds.), CERN 96–01)

$^1$Supported by European Union with grant INTAS–93–744.
1 Introduction

This note collects two slightly updated contributions to the working groups on ‘Event Generators for WW Physics’ [4] and on ‘Event Generators for Discovery Physics’ [3] of the Workshop on ‘Physics at LEP 2’ held at CERN in 1995. The underlying formulae may be found in references [3]–[9]. In this series of papers we advocate a semi-analytical approach to the reaction

\[ e^+e^- \rightarrow f_1 \bar{f}_1 f_2 \bar{f}_2 (n\gamma). \]  

(1)

The approach is characterized by the attempt to perform at least part of the eight-dimensional phase space integration analytically. As a result, the numerical integrations are smoothened and often faster than in a completely numerical approach. Further, the numerical accuracy is well under control. The analytical integrations are performed over (part of) the angular variables, while the boson invariant masses are left for numerical integration.

Four-fermion production processes may be very complex and proceed via a large variety of intermediate states. Doubly resonant processes are usually calles ‘basic processes’. The others are called background contributions. We repeat the classification of reference [5], with the number of Feynman diagrams in the charged current (CC) classes are shown in table 1 and for the final states corresponding to the neutral current (NC) classes in table 2.

|            | \( \bar{d}u \) | \( \bar{s}c \) | \( \bar{e}\nu_e \) | \( \bar{\mu}\nu_\mu \) | \( \bar{\tau}\nu_\tau \) |
|------------|----------------|----------------|----------------|----------------|----------------|
| \( d\bar{u} \) | 48             | 11             | 20             | 10             | 10             |
| \( e\bar{\nu}_e \) | 20             | 20             | 56             | 18             | 18             |
| \( \mu\bar{\nu}_\mu \) | 10             | 10             | 18             | 19             | 9              |

Table 1: Number of Feynman diagrams for class CC final states.

|            | \( \bar{d}d \) | \( \bar{u}u \) | \( \bar{e}e \) | \( \bar{\mu}\mu \) | \( \bar{\nu}_e\nu_e \) | \( \bar{\nu}_\mu\nu_\mu \) |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \bar{d}\bar{d} \) | 4·16           | 43             | 48             | 24             | 21             | 10             |
| \( \bar{s}s, \bar{b}b \) | 32             | 43             | 48             | 24             | 21             | 10             |
| \( \bar{u}\bar{u} \) | 43             | 4·16           | 48             | 24             | 21             | 10             |
| \( \bar{e}\bar{e} \) | 48             | 48             | 4·36           | 48             | 56             | 20             |
| \( \bar{\mu}\bar{\mu} \) | 24             | 24             | 48             | 4·12           | 19             | 19             |
| \( \bar{\tau}\bar{\tau} \) | 24             | 24             | 48             | 24             | 19             | 10             |
| \( \bar{\nu}_e\nu_e \) | 21             | 21             | 56             | 19             | 4·9            | 12             |
| \( \bar{\nu}_\mu\nu_\mu \) | 10             | 10             | 20             | 19             | 12             | 4·3            |
| \( \bar{\nu}_\tau\nu_\tau \) | 10             | 10             | 20             | 10             | 12             | 6              |

Table 2: Number of Feynman diagrams for class NC final states.
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\textsuperscript{a} Fortran code gentle\_4fan.f
\textsuperscript{b} Fortran code gentle\_nc\_qed.f

Description of the package

The GENTLE/4fan package is designed to compute selected total four-fermion production cross-sections and final-state fermion pair invariant mass distributions for charged current (CC) and neutral current (NC) mediated processes within the Standard Model (SM). For the CC\textsubscript{03} subprocess, the $W$ production angular distribution is also accessible. In the NC case, SM Higgs Production is included. The phase space integration is carried out by a semi-analytical technique, which is described below. The GENTLE/4fan package is written in Fortran. It consists of two branches. The basic branch gentle\_4fan.f contains all features of the package but complete initial-state radiation (ISR) to NC processes. The subroutine FOURFAN, which is called by gentle\_4fan.f performs the computation of NC cross-sections and is described in the next section. The independent branch gentle\_nc\_qed.f includes complete ISR to NC\textsubscript{02} and NC\textsubscript{08} and will be merged into gentle\_4fan.f.

Program features:

1. Method of integration:
The package is a semi-analytical one. Without (with) ISR, the phase space is parameterized by five (seven) angular variables and the final-state fermion pair invariant masses (plus the reduced center of mass energy squared). All angular phase space variables are integrated analytically. The resulting formulae are input to the package. Invariant masses are subsequently integrated numerically with a self-adaptive Simpson algorithm. Optionally, for the CC\textsubscript{03} subprocess, the $W$ production angle may also be numerically integrated. The method is numerically stable and, in most cases, very fast.

2. Possible final states:
The package may treat all four-fermion final states which do not contain identical particles, electrons, or electron neutrinos. This means that the package accesses all final states that are described by annihilation and conversion type Feynman diagrams:
Via flags, cross-sections for subsets of Feynman diagrams may be extracted.

3. Cuts
Cuts may be imposed on invariant masses of fermion pairs and on the invariant mass of the final-state four-fermion system. Using the structure function approach in gentle4fan.f, cuts on the electron/positron momentum fraction can be imposed. For the CC03 subprocess, cuts on the $W$ production angle are enabled.

4. Initial state radiation
ISR is implemented into the package. Universal ISR is present for all processes. In addition, the package includes complete, i.e. universal and non-universal ISR for the CC03, NC02, and NC08 processes. Non-universal ISR does not contribute to annihilation diagrams. It may be argued that non-universal ISR is very small, $O(10^{-3})$, for conversion-annihilation interferences. The speed of the package is reduced, if non-universal ISR is included, due to its complex analytical structure.

5. Final state radiation
Final state radiation is not implemented.

6. Treatment of final state decays
Final state decays are not accounted for.

7. Treatment of the Coulomb Singularity
The Coulomb singularity is included according to reference [4].

8. Treatment of the Anomalous Couplings
Anomalous couplings are not included.

9. Treatment of masses
In general, final-state masses are neglected in the matrix elements. Where needed, however, masses are retained in the phase space. In addition, masses of heavy particles coupling to the Higgs boson are taken into account where appropriate; see section 3.

10. Hadronization
No interface to hadronization is foreseen.
Input parameters

All input parameters are set inside the Fortran code. gentle_4fan.f uses the following flags, set in the subroutine WWIN00:

- **IBCKGR**: $CC03$ case (IBCKGR=0) or $CC11$ case (IBCKGR=1)
- **IBORNF**: Tree level (IBORNF=0) or ISR corrected (IBORNF=1) quantities
- **ICHNNL**: $CC03$ (ICHNNL=0), $CC11$ with specific final state \(l_1\nu_1l_2\nu_2(\text{ICHNNL} = 1), l\nu\bar{q}\) (ICHNNL = 2, 3), \(q_1\bar{q}_1q_2\bar{q}_2\) (ICHNNL = 4), and inclusive $CC11$ (ICHNNL=5)
- **ICOLMB**: Inclusion of Coulomb singularity (ICOLMB=1,...,5) or not (ICOLMB=0)
- **ICONVL**: Flux function (ICONVL=0) or structure function approach (ICONVL=1) for ISR. Recommended value: ICONVL=0
- **IGAMZS**: Constant Z width (IGAMZS=0) or s-dependent Z width (IGAMZS=1)
- **IINPT**: Input for tuned comparison (IINPT=0) or preferred input (IINPT=1)
- **IIQCD**: Naive inclusive QCD corrections are included (IIQCD=1) or not (IIQCD=0)
- **IMMIM**: Minimum number of a moment requested by IREGIM
- **IMMAX**: Maximum number of a moment requested by IREGIM
- **IONSHL**: On-shell (IONSHL=0) or off-shell heavy bosons (IONSHL=1)
- **IPROC**: $CC$ case (IPROC=1) or $NC$ case (IPROC=2, call to FOURFAN is initialized)
- **IQEDHS**: Determination of the universal ISR radiator:
  - \(\mathcal{O}(\alpha)\) exponentiated (IQEDHS=-1,0);
  - \(\mathcal{O}(\alpha)\) exponentiated plus different \(\mathcal{O}(\alpha^2)\) contributions (IQEDHS=1,...,4)
  Recommended value: IQEDHS=3
- **IREGIM**: Calculation of the total cross-section (IREGIM=0), the moments of the radiative loss of final-state four-fermion invariant mass (IREGIM=1), the moments of the radiative energy loss (IREGIM=2), the moments of the $W$ mass shift \(\sqrt{s}+\sqrt{s}−2M_W\) (IREGIM=3), and the first moments of \(\cos(n\theta_W)\), \(n = 1, ..., 4\) (IREGIM=4)
- **IRMAX**: Maximum value of IREGIM
- **IRSTP**: Step in a DO loop over IREGIM
- **ITVIRT**: Non-universal virtual ISR included (ITVIRT=1) or not (ITVIRT=0)
- **ITBREM**: Non-universal bremsstrahlung included (ITBREM=1) or not (ITBREM=0)
- **IZERO**: See equation (4.5) of [8]. Recommended value: IZERO=1
- **IZETTA**: See equation (4.21) of [8]. Recommended value: IZETTA=1

In the gentle_nc_qed.f branch, only the flags IBORNF, IONSHL, ITVIRT, ITBREM are used. The additional flag IBOSON in gentle_nc_qed.f distinguishes between the $NC02$ and the $NC08$ processes.

The center of mass energy squared is chosen by setting the variable IREG and the parameters ISMAXA or ISMAXB in the main program. The following input may be changed by the user:
GFER = \( G_{\mu} = 1.16639 \times 10^{-5} \text{ GeV}^{-2} \), the Fermi coupling constant

ALPW = \( \alpha(2M_W) = 1/128.07 \), the running fine structure constant at \( 2M_W \)

AME = \( m_e = 0.51099906 \times 10^{-3} \text{ GeV} \), the electron mass

AMZ = \( M_Z = 91.1888 \text{ GeV} \), the \( Z \) mass

AMW = \( M_W = 80.230 \text{ GeV} \), the \( W \) mass

GAMZ = \( \Gamma_Z = 2.4974 \text{ GeV} \), the \( Z \) width

ALPHS = \( \alpha_s(2M_W) = 0.12 \)

Output

The following derived quantities are computed in gentle_4fan.f and printed in the output:

\[
\text{GAMW} = \Gamma_W = \frac{9}{6\sqrt{2}\pi} G_{\mu} M_W^3 \left( 1 + \frac{2\alpha_s(2M_W)}{3\pi} \right)
\]

\[
\text{SIN2W} = \sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}
\]

\[
\text{GAE} = -\frac{e}{4s_W c_W} = -\frac{\sqrt{4\pi\alpha(2M_W)}}{4s_W c_W}
\]

\[
\text{GVE} = \text{GAE} \cdot (1 - 4s_W^2)
\]

\[
\text{GWF} = \frac{g}{2\sqrt{2}} = -\text{GAE} \cdot \sqrt{2} c_W
\]

\[
|\text{GWWG}| = \sqrt{4\pi\alpha(2M_W)}
\]

\[
|\text{GWWZ}| = |\text{GWWZ}| \cdot \frac{c_W}{s_W}
\]

\( \text{GVE} \) and \( \text{GAE} \) are the electron vector and axial vector couplings, \( \text{GWF} \) is the fermion-\( W \) coupling, and \( |\text{GWWG}| \) and \( |\text{GWWZ}| \) are the trilinear gauge boson couplings for the photon and the \( Z \) respectively. Further the output repeats the flag settings. After the cross-section calculation, the following output is printed:

\[
\text{SQS} = \sqrt{s}
\]

\[
\text{XSECO} = \sigma_{\text{tot}}(s) \quad \text{in nanobarns}
\]  

(2)

In addition, the calculated \textsc{MOMENTS} are printed. In the first column \textsc{IREGIM} is printed. The second column is arranged in blocks of three lines each. The first line contains the integer \( n \). The second line contains the \( n^{th} \) moment of the physical quantity indicated by \textsc{IREGIM}. The third line contains the dimensionless \( n^{th} \) moment obtained through division of the \( n^{th} \) moment by the proper power of \( \sqrt{s}/2 \).

Although variable names are slightly different, gentle.nc.qed.f uses the same derived quantities as gentle_4fan.f. For one run, gentle.nc.qed.f outputs the used flag values together with the fermion code numbers IFERM1/IFERM2, the color factors RNCOU1/RNCOU2, the masses AM1/AM2, and the invariant pair mass cuts CUTM12,CUTM34 for the final-state fermion pairs. In addition, the lower cut \textsc{CUTXPR} on the ratio of the four-fermion invariant mass squared over the
center of mass energy squared, $s'/s$ is output. The main output, however, is an array of center of mass energies and the corresponding total cross-sections.

**Availability**
The codes and this description are available from the authors upon E-Mail request or via WWW

- `gentle_4fan.f` from [http://www.ifh.de/~bardin/gentle_4fan.uu](http://www.ifh.de/~bardin/gentle_4fan.uu)
- `gentle_nc_qed.f` from [http://www.ifh.de/~lehner/gentle_nc_qed.uu](http://www.ifh.de/~lehner/gentle_nc_qed.uu)
- DESY-Zeuthen 96–05 from [http://www.ifh.de/theory/publist.html](http://www.ifh.de/theory/publist.html)
3 4fan version 1.3

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Description of the package

4fan is a semi-analytical program which calculates the process

\[ e^+ e^- \rightarrow f_1 \bar{f}_1 f_2 \bar{f}_2, \]  

where the three involved fermions \(e, f_1\) and \(f_2\) must be in different electroweak multiplets (the \(NC32\) class) \[6\]; see table 2. Optionally, Standard Model Higgs production can be included \[7\]. For calculations at the Born level, 4fan can be used as a stand-alone program. For the calculation of cross-sections including initial state radiation, the initial state radiation environment of the code gentle_4fan.f must be used which calls FOURFAN as a subroutine. For the description of gentle_4fan.f we refer to section 2. In the following, we describe the stand-alone program 4fan.f:

Six of the eight integrations of the four particle phase space were done analytically. The two remaining integrations over \(s_1 = [p(f_1) + p(\bar{f}_1)]^2\) and \(s_2 = [p(f_2) + p(\bar{f}_2)]^2\) are performed numerically allowing the inclusion of cuts for these variables.

Finite mass effects are taken into account using the following approximations:

- The phase space is treated exactly.
- In the Higgs contributions and the conversion diagrams \(e^+ e^- \rightarrow (\gamma\gamma) \rightarrow f_1 \bar{f}_1 f_2 \bar{f}_2\), the masses are treated up to order \(O[m^2(f_i)/s_i]\).
- Fermion masses are treated identically in traces and Higgs couplings.
- The Higgs width is calculated including the decays into \(b-, c-\) and \(\tau\)-pairs.

The numbers quoted in the tables of the report \[10\] are produced for zero fermion masses except in the Higgs couplings. The Higgs propagator is always connected with \(s_2\) by convention.

The initialization routine BBMMIN contains the input from the Particle Data Group \[11\]. In the subroutine DSDS752, the interferences between the three main subsets of the diagrams of the \(NC32\) class are calculated as well as those with the Higgs signal diagram. Their sum yields the double differential cross-section. Single interferences between these subsets are not printed.

The numerical integration is done by a twofold application of a one-dimensional self-adaptive Simpson algorithm with control over the relative and the absolute error. The singularities due
to resonant vector boson propagators are eliminated by transformations of the integration variables. To avoid numerical instabilities, the kinematical functions resulting from the six-fold analytical integration are replaced by Taylor expansions near the borders of the phase space. The shortest calculation time is achieved by a choice of the required absolute and relative errors in such a way that they give approximately equal contributions to the error of the output.

The calculation time of a Born cross-section is several seconds on an HP workstation, depending on the required accuracy and the cuts on $s_1$ and $s_2$. Ten times higher accuracy needs approximately two times longer calculational time.

Input and output are transferred through the arguments of the subroutine only.

Usage of the program:

```call fourfan(eps,abse,if1,if2,s,s1min,s1max,s2min,s2max,amh,iout,out)
```

**Input:**

- **EPS,ABSE:** The required relative and absolute error. If one of the two criteria is fulfilled, the calculation stops.
- **IF1,IF2:** Integers specifying the two final fermion pairs as in the Monte Carlo particle numbering scheme, see Particle Data Group [11], chapter 32.
- **S:** The c.m. energy squared of the $e^+e^-$ pair.
- **S1MIN,S1MAX:** The integration energy bounds of $s_1$.
- **S2MIN,S2MAX:** The integration energy bounds of $s_2$.
- **AMH:** The Higgs mass.
- **IOUT:** Integer, selecting the output. Currently IOUT=1, 2, 11 and 12 are implemented:
  - **IOUT=1:** Total cross-section $\sigma_t$ without Higgs.
  - **IOUT=2:** Differential cross-section $d\sigma/ds_2$ without Higgs.
  - **IOUT=11, 12:** The same as IOUT=1, 2 but with Higgs.

The units of the input (if required) are GeV$^2$ or GeV.

**Output:** **OUT** Depends on the value of **IOUT**. The output is given in $fb$ or in $fb/GeV$.

On HP workstations **4fan** must be compiled with the -K option.

**Availability**
The code and this description are available from the authors upon E-Mail request or via FTP or WWW

- **4fanv13.f** from [ftp://gluon.hep.physik.uni-muenchen.de](ftp://gluon.hep.physik.uni-muenchen.de)
- **4fanv13.f** from [http://www.ifh.de/theory/publist.html](http://www.ifh.de/theory/publist.html)
- **DESY-Zeuthen 96–05** from [http://www.ifh.de/theory/publist.html](http://www.ifh.de/theory/publist.html)
4 Concluding remarks

Not all of the final-state topologies of tables 1 and 2 have been treated by our approach so far. Those which have been treated belong to the classes \(CC11\) and \(NC32\) and are printed in \textbf{boldface} in the tables.

Presently we are studying the semi-leptonic and leptonic processes of the classes \(CC20, \ NC48, \ NC21\), which are printed in the tables in \textit{roman} \[12\].

For a complete treatment of the \(NC32\) class, the four jet production from \(e^+e^-\) annihilation into \(q\bar{q}gg\) has to be covered in addition to \(4f\) production \[13\].

Besides a verification of the Standard Model predictions for \(WW\) and \(ZZ\) pair production and in Higgs boson searches via \(ZH\) production, one is also interested in searches for anomalous triple gauge boson couplings. A description of \(CC03, \ CC11\) cross-sections with anomalous couplings is under development \[14\].

For a detailed study of the underlying physics, angular distributions are extremely helpful. Although in the semi-analytical approach one has a limited flexibility concerning distributions and cuts, we made several attempts to calculate some of them \[8, 14, 15\].

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