Extended QED with CPT violation: clarifying some controversies.

Guy Bonneau*

Abstract

We rediscuss the controversy on a possible Chern-Simons like term generated through radiative corrections in QED with a CPT violating term. We analyse some consequences of the division of the Lagrangian density between “free part” and “interaction part”. We also emphasize the fact that any absence of an a priori divergence should be explained by some symmetry or some non-renormalisation theorem and show that the so-called “unambiguous result” based upon “maximal SO(3) residual symmetry” does not offer a solution.

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*Laboratoire de Physique Théorique et des Hautes Energies, Unité associée au CNRS UMR 7589, Université Paris 7, 2 Place Jussieu, 75251 Paris Cedex 05. Email: bonneau@lpthe.jussieu.fr
1 Introduction

In the last decade, the interesting issue of a possible spontaneous breaking of Lorentz invariance at low energy has been considered: this issue also led to CPT breaking [1, 2, 3]. In particular, the general Lorentz-violating extension of the minimal $SU(3) \times SU(2) \times U(1)$ standard model has been discussed: as many breaking terms are allowed, people look for possible constraints coming from experimental results as well as from renormalisability requirements, anomaly cancellation, microcausality and stability [4, 5, 6].

In that respect, there arose a controversy on a possible Chern-Simons like term generated through radiative corrections (first order in the Lorentz breakings) and on a possible mass term for the photon (second order in the Lorentz breaking) [2-19]. In this note, we intend to clarify the origin of the discrepancies. Among previous works on that subject, we particularly quote [15, 14, 17]. This phenomenon was extensively studied in QED, an abelian gauge theory, as a part of the standard model.

The Lagrangian density is:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}} + \mathcal{L}_1 + \mathcal{L}_2$$

a) $\mathcal{L}_0 = \bar{\psi} (i \slashed{\partial} - m) \psi - \frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\alpha} (\partial A)^2 + \frac{1}{2} \lambda^2 A_{\mu}^2$

where $\alpha$ is the gauge parameter and $\lambda$ an infra-red regulator photon mass,

b) $\mathcal{L}_{\text{int}} = -e \bar{\psi} A \psi$ where $e$ is the electron charge,

c) $\mathcal{L}_1(x) = -b^\mu \bar{\psi}(x) \gamma_\mu \gamma^5 \psi(x)$, where $b^\mu$ is a fixed vector,

d) $\mathcal{L}_2(x) = \frac{1}{2} c^\mu \epsilon_{\mu\nu\rho\sigma} F^{\nu\rho}(x) A^\sigma(x)$, where $c^\mu$ is a fixed vector.

Other breakings could be considered (see a discussion in the first paper of [2]), but we simplify and require charge conjugation invariance, which selects $\mathcal{L}_1(x)$ and $\mathcal{L}_2(x)$. Note for further reference that experiments on the absence of birefringence of light in vacuum put very restrictive limits on the value of $c^\mu$, typically for a timelike $c^0/m \leq 10^{-38}$ [2]. Then, it is really interesting to analyse the conjecture that, even if it vanishes at the tree level, a non zero $c^\mu$ might be generated through loop-corrections in presence of an $\mathcal{L}_1$ term. Note also that the Lagrangian density $\mathcal{L}_0 + \mathcal{L}_2$ would not lead to a coherent theory as an (infinite) counterterm $\mathcal{L}_1$ appears at the one-loop order [14].

In [14], the one-loop vacuum polarization tensor was computed within the consistent dimensional scheme [20, 21] and the complete, all-order theory was analysed in a perturbative spirit we shall comment later on. Some other authors use non standard regularisations for their one-loop calculations, whose consistent use in higher loop computations is rather unclear; let us recall that a renormalisation scheme (regularisation + subtraction algorithm + normalisation conditions) requires delicate proofs to be consistent to all orders (recall the technicalities of the BPHZ forest formula [22] and its specification in dimensional renormalisation in [21]).

However, in this note, to clarify the origin of some discrepancies, it will be sufficient to consider the one-loop photon vacuum polarization tensor $\Gamma_{\mu\nu}(p, -p)$.

- As is well known, to zero’th order in the Lorentz breaking parameters, power counting enforces a quadratic divergence, but gauge invariance $[p^\mu \Gamma_{\mu\nu}(p, -p) = 0]$ lowers the divergence to a logarithmic one (the usual charge renormalisation).

- Then, to first order in the dimension-one Lorentz breaking parameters, the polarization tensor should diverge linearly, but parity conservation and gauge invariance again allows
only a logarithmic one. However, all computations agree to give a finite contribution although everybody who learns renormalisation theory knows that the worst always happens - except if some extra symmetry forbids it (recall the chiral anomaly which is a finite quantity, thanks to gauge invariance ..). It is really surprising that in the thirty or so papers devoted to that subject, one could not find one line of argument to explain this “experimental” one-loop\(^1\) finiteness, except in the Coleman-Glashow analysis\([3]\) where the finiteness results from the gauge Ward identity on the unintegrated axial 3-point function :

\[
p_{\nu} < [\bar{\psi} \gamma^\mu \gamma^5 \psi](-p - q) A^\nu(p) A^\rho(q) > = \quad q_{\rho} < [\bar{\psi} \gamma^\mu \gamma^5 \psi](-p - q) A^\nu(p) A^\rho(q) > = \quad 0 , \quad (2)
\]

and in the review by Pérez-Victoria\([15]\) where the finiteness of the Chern-Simons like term is related to that of the standard triangle graph in ordinary QED : however, let us recall that, here also, the finiteness results from the gauge Ward identity on the unintegrated axial-vector-vector 3-point function (equ.(2)).

- Finally, to second order in the dimension-one Lorentz breaking parameters, the polarization tensor should diverge logarithmically, but standard gauge invariance argument forbids its appearance.

In our all-order analysis, the tool of local gauge invariance was used to prove a non-renormalisation theorem for the Chern-Simons like term\([14]\). More recently, a series of papers\([17, 19]\), using what is called “maximal SO(3) residual invariance after SO(1, 3) Lorentz breaking”, claimed that they offer for the first time unambiguous results for the \((\text{still unexplained})\) finite contributions, with a non-zero induced one-loop Chern-Simons contribution and a radiatively generated photon mass.

In this note we shall prove that the discrepancies among published results do not come from such kind of enforcement of a symmetry, but rather from two reasons :

- the main reason is the choice of the free theory : is it given by the bilinear part \(L_0\), such as considered by\([3, 14, 18]\) (we shall speak of a perturbative approach) or from the bilinear part of the complete lagrangian density, i.e. \(L_0 + L_1 + L_2\), such as considered by\([2, 12, 17]\) (we shall speak of a non-perturbative approach) ?

- the second one, among analyses choosing the non-perturbative approach, lies in the choice of regularisation and renormalisation scheme. First, these should not destroy the usual QED results (in particular the gauge invariance of the pure QED vacuum polarisation tensor) ; second, if gauge Ward identity holds true in the extended theory, one should prefer a regularisation that preserves gauge invariance such as Pauli-Vilars\([18]\) or dimensional regularisation\([14]\) or add finite quantum corrections to the Lagrangian density to restore the Ward identities : in the absence of some symmetry, the finiteness of the corrections remains unexplained and, moreover, there is no tool to fix the ambiguous regularisation dependent values (as Jackiw said, “When radiative corrections are finite but undetermined”\([7]\)).

So, in Sections 2 and 3 we successively discuss these alternatives and offer some remarks in the concluding Section.

\(^{1}\) If this finiteness was an “accidental” one (some authors remark a “miraculous” cancellation between two divergent quantities), it would have no reason to hold at higher-loop order!
2 The perturbative approach

The “perturbative” approach, with \( \mathcal{L}_0 \) as the free Lagrangian density, avoids the difficulties resulting from new poles in the propagators, and takes into account the smallness of the breakings to include them into the interaction Lagrangian density as super-renormalisable couplings. The free photon and fermionic fields and their corresponding asymptotic states are defined as usual. Moreover, the photon and electron masses are defined by the same normalisation conditions as in ordinary QED, e.g.

\[
\langle \psi(p)\bar{\psi}(-p) \rangle^{\text{prop.}} \bigg|_{\rho=0, \gamma=0, \phi=0} = 0, \quad \ldots
\]

According to standard results in renormalisation theory, the Lorentz invariance breaking adds new terms into the primitively divergent proper Green functions. By power counting, these are

\[
\Gamma_{\mu\nu}(p, -p), \quad \Sigma(p, -p), \quad \Gamma^{\rho}(p, q, -(p+q)) \quad \text{and} \quad \Gamma_{\mu\nu\rho\sigma}(p_1, p_2, p_3, -(p_1 + p_2 + p_3)),
\]

respectively the photon and electron 2-points proper Green functions, the photon-electron proper vertex function and the photon 4-point proper Green function. The corresponding overall divergences (sub-divergences being properly subtracted) are polynomial in the momenta and masses:\(^2\)

\[
\begin{align*}
\Gamma_{\mu\nu}(p, -p) &\bigg|_{\text{div}} = a_1 [g_{\mu\nu}p^2 - p_\mu p_\nu] + a_2 p_\mu p_\nu + [a_3 m^2 + a_4 \lambda^2]g_{\mu\nu} + \\
\quad &+ [a_5 b^\rho + a_6 c^\rho] \xi_{\mu\nu\rho\sigma} p^\sigma + a_7 b^\mu b^\nu, \\
\Sigma(p, -p) &\bigg|_{\text{div}} = a_8 \tilde{\phi} + a_9 m + [a_{10} b^\mu + a_{11} c^\rho] \gamma_\rho \gamma^5, \\
\Gamma^{\rho}(p, q, -(p+q)) &\bigg|_{\text{div}} = a_{12} \gamma^\rho : \text{no } b^\rho \text{ or } c^\rho \text{ dependance}, \\
\Gamma_{\mu\nu\rho\sigma}(p_1) &\bigg|_{\text{div}} = a_{13} [g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}] : \text{no } b^\rho \text{ or } c^\rho \text{ dependance}.
\end{align*}
\]

All parameters \( a_i \), positions and residues of the poles in propagators, couplings at zero momenta,... - but for the unphysical, non renormalised ones [such as the longitudinal photon propagator (gauge parameter \( \alpha \)) and the photon regulator mass \( \lambda^2 \) for unbroken QED] - require normalisation conditions, a point which has often been missed since the successes of minimal dimensional regularisation scheme [23] but is stressed in some reviews [15, 16]. In particular we shall require 2 new normalisation conditions to fix the breaking parameters \( b^\mu \) and \( c^\mu \):

\[
\begin{align*}
b^\mu &= -\frac{i}{4} \delta \Gamma \bigg|_{p=0}, \\
c^\mu &= \frac{1}{12} \xi^{\mu\nu\rho\sigma} \partial_{\rho} < A_\nu(p) A_\rho(-p) \rangle^{\text{prop.}} \bigg|_{p=0}.
\end{align*}
\]

Note that, contrary to \( \mathcal{L}_{\text{int}}(x) \) and \( \mathcal{L}_1(x) \), the \( \mathcal{L}_2(x) \) term also breaks the local gauge invariance of the Lagrangian density. But we emphasize the fact that - except for the unphysical part

\[
\int \left[ -\frac{1}{2\alpha} (\partial A)^2 + \frac{1}{2} \lambda^2 A_\mu^2 \right] - \text{the action } \Gamma = \int \mathcal{L} \text{ is invariant under local gauge transformations.}
\]

So a Ward identity may be written:

\[
\int d^4x \left\{ \frac{1}{e} \partial_\mu \Lambda(x) \frac{\delta \Gamma}{\delta A_\mu(x)} + i \Lambda(x) \left[ \tilde{\psi}(x) \frac{\delta}{\delta \psi(x)} - \frac{\delta \Gamma}{\delta \psi(x)} \psi(x) \right] \right\} = \ldots
\]

\(^2\) C invariance has been used. The Ward identity (5) will relate some of these parameters: \( a_2 = a_3 = a_4 = 0, \ a_{12} = e, \ a_8, \ a_{13} = 0.\)
\[ \int d^4 x \left\{ -\frac{1}{e\alpha} \partial_\mu A^\mu(x) \Box \Lambda(x) + \frac{\lambda^2}{e} A^\mu(x) \partial_\mu \Lambda(x) + \frac{1}{2e} \epsilon_{\alpha\beta\delta\mu} c^\alpha F^{\beta\delta}(x) \partial_\mu \Lambda(x) \right\} \]

\[ \Rightarrow \ W_x \Gamma \equiv \partial_\mu \frac{\delta \Gamma}{\delta A_\mu(x)} - ie[\bar{\psi}(x) \gamma^\mu \frac{\delta \Gamma}{\delta \psi(x)} - \frac{\partial}{\partial \psi(x)} \psi(x)] = \frac{1}{\alpha^2} \left[ e - \alpha \lambda^2 \right] \partial_\mu A^\mu(x). \quad (5) \]

We emphasize the fact that this equation is exactly the same as the one for ordinary QED.

As soon as we use a regularisation that respects the symmetries (gauge, Lorentz covariance and charge conjugation invariance), the perturbative proof of renormalisability reduces to the check that the \( O(\hbar) \) quantum corrections to the classical action \( \Gamma : \Gamma_1 = \Gamma_{\text{class}} + \hbar \Delta \), constrained by the Ward identity (5) may be reabsorbed into the classical action through suitable renormalisations of the fields and parameters of the theory. This has been proven in [14].

There, some local sources have been introduced to define the local operators \( L_1(x) \) and \( L_2(x) \). Although this is only a technical tool, it has been criticised\(^3\) and we shall discuss later on this point.

Then in [14], we have proven that, being linear in the quantum field, the variation of \( L_2(x) \) in a local gauge transformation is soft: no essential difference occurs between local gauge invariance of the action and the “softly” broken local gauge invariance of the Lagrangian density. As a consequence, the theory (1) is consistent (even with no \( L_2(x) \) term) and the CS term has been shown to be unrenormalised, to all orders of perturbation theory. So, its experimental “vanishing” offers no constraint on the other CPT breaking term \( L_1(x) \). To summarize, we have proven that:

- The local gauge invariance of the Lagrangian density is destroyed by a \( L_2 \) term (plus of course by the usual gauge fixing term): but, being bilinear in the gauge field, \( L_2(x) \) behaves as a minor modification of the gauge fixing term as \( \partial_\mu A^\mu \) remains a free field. As part of the “gauge term”, this \( L_2(x) \) is, as usual, not renormalised: so its all-order value is equal to its (arbitrarily chosen) classical one.

- In [18] we also check that gauge invariance is respected at second order in the breaking parameter \( b^\mu \).

- A theory with a vanishing tree level Chern-Simons like breaking term is consistent as soon as it is correctly defined: thanks to the gauge invariance of the action, we have proven that the normalisation condition \( c^\mu = 0 \) may be enforced to all orders of perturbation theory.

- The 2-photon Green function receives definite (as they are finite by power counting) radiative corrections [14]

\[ \simeq \frac{\hbar e^2}{12\pi^2} \frac{p^2}{m^2} \epsilon_{\mu\nu\rho\sigma} p^\rho b^\sigma + \cdots \]

Recall the case of the electric charge: physically measurable quantities occur only through the \( p^2 \) dependence of the photon self-energy (as the Lamb-shift is a measurable consequence of a non-measurable charge renormalisation). Unfortunately, as Coleman and Glashow explained, the absence of birefringence of light in vacuum, i.e. the vanishing of the parameter \( c^\mu \), gives no constraint on the value of the other one \( b^\mu \).

\(^3\) For example in page 3 of [16]: “Bonneau introduced external source fields for the axial vector current and the CS term, so the Ward identities he derived actually impose gauge invariance on Lagrangian density...” This assertion is wrong as the Lagrangian density is not gauge invariant (moreover it has been gauge-fixed..) but, as proven in our analysis [14], the breaking of local gauge invariance is a soft one and may be seen as a complementary part in the gauge fixing, then non-renormalised.
3 The non-perturbative approach

The second solution, the “non-perturbative approach”, introduces new poles in the fermion propagator and requires a thorough discussion about causality and stability. Many papers discuss that question [5, 6].

However, in the first paper in [2], Colladay and Kosteleck’y gave a direct analysis of the complete classical fermion Green function as defined by $L_0 + L_1$. In particular they check that the anticommutator of two fermionic fields vanishes for space-like separations, in agreement with microcausality (at least for a time-like breaking $b_\mu$). This confirms our analysis on the correctness of a theory with no classical CS term. Then, Adam and Klinkhamer show that the addition of a ( radiatively generated) CS term $L_2(x)$ with a time-like $c^\mu$ breaks microcausality [5]. As our non-renormalisation theorem ensures that, if absent at the classical level, the CS term will not appear in higher-loop order, microcausality will not be destroyed in higher-loop order.

According to [6], a vanishing Chern-Simons’s like parameter $c^\mu$ is required ; for other analyses, only a time-like $b^\mu$ is allowed and a space-like $c^\mu$ [6, 17]. But none of those papers were able to prove the finiteness of the one-loop corrections. On the contrary, note that if, as in the previous “perturbative” case, one introduces local sources to define the local operators $L_1(x)$ and $L_2(x)$, the one-loop finiteness will be obtained, but as we shall see on equ.(12), the calculation algorithm does not respect gauge invariance and finite $O(\hbar)$ terms have to be added in the Lagrangian density (see a discussion on another gauge invariance breaking algorithm in [18]).

Anyway, the analyticity argument of Coleman and Glashow is no longer at hand as the fermion propagator has new poles. Then, it is not surprising that a non-vanishing induced Chern-Simons like term appears.

To understand the discrepancies, let us discuss the electron propagator in both situations :

- I) in the “perturbative” approach, the fermion two point function is written as :

$$i[\not p - m - \not b \gamma^5]^{-1} = S_{F}^{(b)}(p) = \sum_{n=0}^{\infty} \frac{i}{\hat{p} - m} \left\{ -i \not b \gamma^5 \frac{i}{\hat{p} - m} \right\}^{n} ; \quad (6)$$

- II) in the “non-perturbative” approach, the fermion propagator is taken as a whole :

$$i[\not p - m - \not b \gamma^5]^{-1} = S_{F}^{(a)}(p) \equiv i \left[ \frac{p^2 - m^2 + b^2 + 2(b \not p + m \not b) \gamma^5}{(p^2 - m^2 + b^2)^2 - 4[(bp)^2 - m^2 b^2]} \right] (\not p + m + \not b \gamma^5) , \quad (7)$$

$$= S_{F}^{(b)}(p) \equiv i(\not p + m - \not b \gamma^5) \frac{p^2 - m^2 - b^2 + (\not p, \not b) \gamma^5}{(p^2 - m^2 + b^2)^2 - 4[(bp)^2 - m^2 b^2]} .$$

These two equations (7) illustrate two of the possible equivalent expressions for the complete fermionic propagator.

Of course, $b$ being a very small parameter, $S_{F}^{(a)}(p)$ may be expanded in power-series, and one recovers $S_{F}^{(b)}(p)$.

However, expanding in powers of $b$ from the very beginning, or at the end of the calculation of the Green function, makes some difference due to the question of regularisation (recall that when a Green function is primitively divergent, the Feynman integral should be regularised before any manipulation), and of the choice of the computational algorithm for the Green functions.

$^4$As the radiatively generated $c^\mu$ is proportional to $b^\mu$, if $c^\mu$ is absent at the classical level, it is hard to understand how a time-like $b^\mu$ and a space-like $c^\mu$ can be coherent ?
Let us consider dimensional regularisation with the unique consistent formulation in presence of the $\gamma^5$ matrix, the one of t’Hooft systematized by Breitenlohner and Maison [20, 21] (for a review see [23]):

$$\gamma^\mu = \hat{\gamma}^\mu + \check{\gamma}^\mu, \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad g^\mu = D, \quad \hat{g}^\mu = D - 4, \quad \check{g}^\mu = 4, \quad (8)$$

$$\{\hat{\gamma}^\mu, \hat{\gamma}^\nu\} = 0, \{\check{\gamma}^5, \check{\gamma}^\mu\} = 0, \quad [\gamma^5, \gamma^\mu] = 0, \quad \text{Trace}[\gamma^5 \hat{\gamma}^\rho \check{\gamma}^5 \check{\gamma}^\sigma] = 4e^{\mu\nu\rho\sigma}, \quad \text{e.t.c.}$$

One should not be surprised that extensions to $D$ dimensions of the different expressions [eqns. (6), (7a - b)] will lead to different results, differences being given by “evanescent operators” (which vanish when $D$ goes to 4) but giving finite contributions when inserted into a priori primitively divergent graphs. This illustrates the second reason for discrepancies among published results. To be more definite, one can easily show that 5:

$$S^1_F(p) = S^2_F(p) + 2 \hat{\gamma}^5 \frac{(\hat{p} + m) \hat{p}(\hat{p} + m)}{(p^2 - m^2)^2} + 2b^2 \frac{(\hat{p} + m) \hat{p}(\hat{p} + m)}{(p^2 - m^2)^3} + \mathcal{O}(b^3), \quad (9)$$

and

$$S^1_F(p) = S^2_F(p) + 2 \hat{\gamma}^5 \frac{(\hat{p} + m) \hat{p}}{(p^2 - m^2)^2} - 2b^2 \frac{(\hat{p} + m - 4 \check{p}) \hat{p}(\hat{p} + m)}{(p^2 - m^2)^3} + \mathcal{O}(b^3). \quad (10)$$

Notice that these evanescent terms do not come from taking the inverse “in $D$ dimensions” of the proper “non-perturbative” Green function $i[\slashed{p} - m - \slashed{b}\gamma^5]$ as claimed in [17], but rather from different continuations in $D-$dimensions of various $D = 4$ identical quantities obtained either from “non-perturbative” or “perturbative” approaches.

The complete one-loop calculation (up to third order in $b^\mu$) of the photon vacuum polarisation tensor with the “perturbative” (i.e. with $S^1_F(p)$ of equ.(6) as fermion propagator), gauge invariant approach and consistent dimensional regularisation or Pauli-Vilars method, may be found respectively in [14, 18] where it is shown that

$$\Pi^{\mu\nu}(p, -p) = \frac{\mu^2}{12\pi^2}[g^{\mu\nu}p^2 - p^\mu p^\nu] \left\{ \log \frac{4\pi \mu^2}{m^2} - p^2 \int_0^1 dz \frac{[1 - 2Z - 8Z^2]}{2\Delta} \right\} +$$

$$+ \frac{\mu^2}{\pi^2} \epsilon^{\mu\nu\alpha\beta} p_\alpha b_\beta \left\{ p^2 \int_0^1 dz \frac{Z}{\Delta} \right\} + \frac{\mu^2}{\pi^2} X^{\mu\nu} \int_0^1 dz \left[ \frac{Z}{\Delta} + \frac{Z^2}{(\Delta^2)^2} \right], \quad (11)$$

where:

- $\mu$ is the UV scale needed to renormalize the electric charge,
- $Z = z(1 - z)$ and $\Delta = m^2 - Zp^2$,
- $X^{\mu\nu}$ is the unique polynomial tensor of canonical dimension 4, quadratic in $b^\mu$ and transverse with respect to $p^\mu$ and $b^\nu$,

$$X^{\mu\nu} = b^2 (g^{\mu\nu} p^2 - p^\mu p^\nu) - g^{\mu\nu}(p.b)^2 - p^2 b^\mu b^\nu + (p.b)(p^\mu b^\nu - p^\nu b^\mu).$$

$^5$ Notice that, in a one-loop calculation, the physical $b$ parameter and the external photon momentum and component indices $\mu, \nu, \ldots$ stay in $D = 4$ dimensions : $b^\mu \equiv \hat{b}^\mu$, $\{\gamma^5, \hat{\gamma}\} = 0$. 

Lorentz and CPT violations in QED.
On the other hand, in [19], the calculation (still up to third order in $b^\mu$ and in the limit $p^2 \to 0$) with the fermion propagator $S_F^{-\alpha\beta}(p)$ of equ.(7) gives

$$\Pi^{\mu\nu}(p,-p) = i[g^{\mu\nu}p^2 - p^\mu p^\nu]\Pi_{\text{div}}(0) + i \frac{e^2}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} p_\alpha b_\beta + i \frac{e^2}{6\pi^2} \left[ g^{\mu\nu}b^2 + \frac{1}{m^2} \chi^{\mu\nu} \right].$$ (12)

As $p_\mu \Pi^{\mu\nu}(p,-p) \neq 0$, gauge invariance is lost. However, if one computes the extra contributions coming from the “evanescent” terms in equ.(9), one obtains (still in the limit $p^2 \to 0$):

$$\Delta \Pi^{\mu\nu}(p,-p) = -i \frac{e^2}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} p_\alpha b_\beta - i \frac{e^2}{6\pi^2} g^{\mu\nu}b^2,$$ (13)

which corresponds exactly to the difference between equ.(11) and equ.(12) (see also the remark in appendix B of [17]).

To summarize, the so-called “unambiguous” results in [17, 19] do not come from enforcing some “maximal residual symmetry” at the quantum level since we obtained the same result as theirs by using dimensional regularisation and the modified propagator (9). Moreover, if the approach of a physical cut-off in the three dimensional momentum space for fermions (as developed in [17]) is physically interesting, actually it plays no role in their computation.

Indeed, as we now explain, the sole ingredient of their calculation is a specific choice of regularisation.

• First, consider the computation in [[17], equ.(4.10)] of

$$\int \frac{d^4p}{(2\pi)^4} \frac{b_\rho (p^2 + 3m^2) - 4p_\rho(b,p)}{(p^2 - m^2 + i\epsilon)^3};$$ (14)

– in [17], a purely time-like $b \equiv (b_0, 0, 0, 0)$ is chosen and the authors firstly integrate on the variable $p_0$ (no need of a cutoff as the integration happens to be possible between $-\infty$ and $+\infty$). After that, the integration on the three dimensional momentum space variables also converges and gives the announced result $i/(2\pi^2)$, without any need to refer to an “SO(3)” residual symmetry;

– however, if one firstly integrates on the three dimensional momentum space variables, which also happens to be convergent, one finds a vanishing result, before the (convergent) $p_0$ integration;

– then, this proves (recall Fubini’s theorem) that the “four dimensions” integral (14) does not exist as a multidimensional one, even if it happens to be finite (which is still the main point to be understood!);

– finally, one easily checks that $D$ dimensional continuation of (14) and integration with average of $p_\rho p_\lambda \equiv p^2 g_{\rho\lambda}/D$ gives the same result $i/(2\pi^2)$.

• In the same manner, in Section 5 of the same paper, the authors in fact use dimensional regularisation and “$D$ dimensional spherical coordinates”: here again, their calculation does not rely upon the claimed “maximal residual symmetry”.

Remark: of course, as the authors remarked, if averaging is done in 4 dimensions - which destroys the gauge invariance of the dimensional regularisation scheme $^6$ (see [20]-p.196,[23]) -, a different result is obtained [7, 15]. This illustrates the second reason for discrepancies between published results.

$^6$ In particular, this will modify the ordinary QED vacuum polarisation tensor and give a quadratic divergence!
4 Discussion and concluding remarks

As a complement to the reviews [15, 16], let us now comment upon some points given in the literature:

• Use of local sources in the classical action

As said before, the use of local sources in [14] has been criticized. Of course, in ordinary QED, the axial current, being uncoupled, is absent from the Lagrangian density and so does not need to be defined as a quantum operator; no axial vertex being present, a fortiori there is no axial anomaly and no triangle graph to consider. However, as soon as PCAC is used to compute the decay $\Pi^0 \rightarrow \gamma \gamma$, this triangle graph has to be computed and gauge invariance of the unintegrated three-vertices function is required.

On the contrary, in CPT-broken QED of equ.(1), new axial insertions enter the game, but they are integrated ones $\int d^4x \mathcal{L}_1(x)$ and $\int d^4x \mathcal{L}_2(x)$. Then, some authors argue that introducing local sources for the Lagrangian density breaking means adding supplementary conditions on the theory. However, as explained in footnote 3, no local gauge invariance hypothesis results from the introduction of local sources for these insertions.

Suppose that one has only the Ward identity (5) at hand to constrain the possible ultraviolet divergences of equ.(3). This is not sufficient to prove that the breakings introduce no new infinities: in particular, the Chern Simons term is of the right canonical dimension and quantum numbers and satisfies the Ward-identity (5):

$$p^\mu \Gamma_{\mu\nu}(p,-p) = 0 \quad \text{in particular} \quad p^\mu < \int d^4x \mathcal{L}_1(x) A^\mu(p) A^\nu(-p) >= 0.$$  

So, first, we have no explanation of the fact that all one-loop calculations of the CPT breaking contribution to the photon self-energy give a finite result ($a_5 = a_6 = a_7 = 0$), second, being unconstrained, its finite part (renormalised value) has to be fixed by a normalisation condition (a different situation than a radiative correction such as the $(g-2)$ or the Lamb-shift for example). So, no prediction is possible and its value remains arbitrary, which is rather unsatisfactory.

• Use of the heat-kernel expansion

In [24], the one-loop calculation of the CS correction is done with the heat-kernel expansion and the Schwinger proper-time method, leading to a new finite result, claimed to be unambiguously determined. However,

- here again there is no explanation of the absence of infinities in the result: then the finite part is a priori ambiguous,
- other computations with the Schwinger proper-time method exist [25] and give a different result, proving at least that some “ambiguity” remains.

\[\text{Remember that in Fujikawa’s calculation of the axial anomaly, gauge invariance was implemented through the basis used to compute the fermionic Jacobian: he chose eigenvectors of the operator } i \, \not D - e \not A; \text{ another choice would allow the transfer of the axial anomaly to some vector anomaly (see also the discussion on the “minimal anomaly” in non-abelian gauge theory) [26].}\]
– some terms are lacking in this calculation: in particular a logarithmically divergent contribution to the CS term results from a thorough computation of the quantity given in equation (21) of [24] (in the absence of any precise criteria to substract infinite parts, this should not be a surprise).

In that work, we traced the main origin of the controversy on a possible Chern-Simons like term generated in PCT-broken QED (and on the $b^2$ contribution to the vacuum polarisation tensor) back to the delicate choice of the “unperturbed” Lagrangian density.

For us and other authors [5, 6], the non-perturbative choice suffers from delicate theoretical problems (microcausality, analyticity ...); moreover, in the absence of any Ward-identity 8 (we emphasized that, without introduction of local sources for the breaking terms, the gauge invariance of the complete action cannot be fully exploited), one is unable to explain the main phenomenon: the finiteness of all results, which allows for an unambiguous prediction. Moreover, as in that context the finiteness would appear as an “accidental one”, there would be no reason that such result holds to higher loop order.

References

[1] S. Caroll, G. Field and R. Jackiw, Phys. Rev. D 41 (1990) 1231.
[2] D. Colladay and V. A. Kostelecky, Phys. Rev. D55 (1997) 6760; Phys. Rev. D58 (1998) 116002, and references therein.
[3] S. Coleman and S. L. Glashow, Phys. Rev. D59 (1999) 116008.
[4] V. A. Kostelecky and R. Lehnert, Phys. Rev. D63 (2001) 065008, [hep-th/0012060].
[5] C. Adam and F.R. Klinkhamer, Nucl. Phys. B607 (2001) 247, [hep-ph/0101087].
[6] C. Adam and F.R. Klinkhamer, Phys. Lett. B513 (2001) 245, [hep-th/0105037].
[7] R. Jackiw, “When radiative corrections are finite but undetermined”, [hep-th/9903044].
[8] R. Jackiw and V. A. Kostelecky, Phys. Rev. Lett. 82 (1999) 3572.
[9] J.-M. Chung and P. Oh, Phys. Rev. D60 (1999) 067702.
[10] W. F. Chen, Phys. Rev. D60 (1999) 085007.
[11] J. M. Chung, Phys. Lett. B461 (1999) 138; M. Pérez-Victoria, Phys. Rev. Lett. 83 (1999) 2518.
[12] M. Pérez-Victoria, Phys. Rev. Lett. 83 (1999) 2518.
[13] W. F. Chen and G. Kunstatter, Phys. Rev. D62, 105029 (2000), [hep-ph/0002294].

8 In particular, we proved that the claimed “maximal residual symmetry” is in fact not used in actual computations, which then rely in some blind way upon regularisation (even on the order of the integration in multidimensional integrals ...).
Lorentz and CPT violations in QED.

[14] G. Bonneau, Nucl. Phys. B593 (2001) 398, [hep-th/0008210].

[15] M. Pérez-Victoria, J. H. E. P. 0104 (2001) 032, [hep-th/0102021].

[16] W. F. Chen, “Issues on radiatively induced Lorentz and CPT violation in quantum electrodynamics”, [hep-th/0106035].

[17] A. A. Andrianov, P. Giacconi and R. Soldati, J.H.E.P. 0202 (2002) 030.

[18] G. Bonneau, L. C. Costa and J. L. Tomazelli, “Vacuum polarization effects in the Lorentz and PCT violating Electrodynamics”, [hep-th/0510045].

[19] J. Alfaro, A. A. Andrianov, M. Cambiaso, P. Giacconi and R. Soldati, Phys. Lett. B639 (2006) 586.

[20] G. ’t Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189.

[21] P. Breitenlohner and D. Maison, Commun. Math. Phys. 52 (1977) 11.

[22] W. Zimmermann, “Local operator products and renormalisation in quantum field theory”, in 1970 Brandeis Lectures, vol. 1, p.395, eds. S. Deser et al. (M.I.T. Press, Cambridge, 1970).

[23] G. Bonneau, Int. J. of Mod. Phys. A 5 (1990) 3831.

[24] Yu. A. Sitenko, Phys. Lett. B515 (2001) 414, [hep-th/0103215].

[25] M. Chaichian, W. F. Chen and R. González Felipe, Phys. Lett. B503 (2001) 215, [hep-th/0010129] ; J.-M. Chung and B. K. Chung, Phys. Rev. D63 (2001) 105015, [hep-th/0101097].

[26] K. Fujikawa, Phys. Rev. Lett. 42 (1979) 1195 ; Phys. Rev. D21 (1980) 2848.