Yukawa coupling unification in $SO(10)$ with positive $\mu$ and a heavier gluino

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Abstract

The $t - b - \tau$ unification with positive Higgs mass parameter $\mu$ in the minimal supersymmetric standard model prefers “just so” Higgs splitting and a light gluino $\lesssim 500$ GeV which appears to be ruled out by the recent LHC searches. We reanalyze constraints on soft supersymmetry breaking parameters in this scenario allowing independent splittings among squarks and Higgs doublets at the grand unification scale and show that it is possible to obtain $t - b - \tau$ unification and satisfy experimental constraints on gluino mass without raising supersymmetry breaking scale to very high value $\sim 20$ TeV. We discuss the origin of independent squark and Higgs splittings in realistic $SO(10)$ models. Just so Higgs splitting can be induced without significantly affecting the $t - b - \tau$ unification in $SO(10)$ models containing Higgs fields transforming as $10 + \overline{126} + 126 + 210$. This splitting arises in the presence of non-universal boundary conditions from mixing between 10 and other Higgs fields. Similarly, if additional matter fields are introduced then their mixing with the matter multiplet 16 is shown to generate the squark splitting required to raise the gluino mass within the $t - b - \tau$ unified models with positive $\mu$.
I. INTRODUCTION

Grand unified theories (GUTs) based on $SO(10)$ group not only unify the gauge interactions but also lead to a unified framework for the Yukawa couplings and hence fermion masses. In particular, $SO(10)$ model with a 10-plet of higgs coupling dominantly to the third generation implies an equality $y_t = y_b = y_\tau$ of the $t-b-\tau$ Yukawa couplings at the GUT scale. Quite independently, the renormalization group (RG) running of the Yukawa couplings in a softly broken minimal supersymmetric standard model (MSSM) can lead \cite{1-6} to the $t-b-\tau$ unification at the GUT scale making the supersymmetric $SO(10)$ broken to the MSSM at the GUT scale an attractive theory of unification.

$t-b-\tau$ unification at the GUT scale is however not the most generic property of the MSSM but follows only for a restricted set of boundary conditions for the soft supersymmetric breaking terms. These restrictions mainly arise due to the need of significant threshold corrections \cite{2, 3} to the $b$ quark mass required for the $t-b-\tau$ unification and difficulties in achieving the radiative electroweak symmetry breaking (REWSB) in the presence of large $b$ and $\tau$ Yukawa couplings \cite{3, 4}. Both of these depend on the soft breaking sector. It is realized \cite{2, 4} that $t-b-\tau$ unification generally requires departure from the universal boundary conditions assumed within the minimal supergravity (mSUGRA) framework. Universality of the gaugino masses is enforced by the $SO(10)$ invariance if it is assumed that supersymmetry (SUSY) is not broken at the GUT scale by a non-trivial representation contained in the symmetric product of two adjoints of $SO(10)$. In contrast, the soft masses $m_{16}$, $m_{10}$ for sfermions belonging to $16_M$ and the Higgs scalars belonging to the $10_H$ representations are allowed to be different and are also required to be so to obtain $t-b-\tau$ unification. In addition to this $SO(10)$ preserving non-universality, one also needs to introduce explicit $SO(10)$ breaking non-universality. Such non-universality can be induced spontaneously by a non-zero D-term (DT) which introduces splitting within $16_M$ of squarks and $10_H$ of the Higgs simultaneously \cite{6, 7}. In several situations, one also needs to assume that only the MSSM Higgs fields $H_u$, $H_d$ split at the GUT scale. This is termed as “just so” Higgs splitting (HS) \cite{8}.

Restrictions placed on soft parameters by the $t-b-\tau$ unification, the LEP and LHC bounds on the masses of the SUSY particles and other flavor violating observables have been worked out in detail in number of papers \cite{8, 13}. Two viable scenarios and their properties have been identified (see \cite{14} for details and references therein). These depend on the sign of the $\mu$ parameter of the MSSM. For example, one can achieve an exact $t-b-\tau$ unification in mSUGRA itself for negative $\mu$. But this needs very heavy SUSY spectrum with $m_0 \sim 5-12$ TeV and $m_{1/2} \sim (1.5 - 2)m_0$ \cite{10}. Also, perfect $t-b-\tau$ unification with relatively light SUSY spectrum ($\sim 2$ TeV) can be obtained with the introduction of DT or purely Higgs splitting. This appears to be the best and testable scenario as far as the $t-b-\tau$ unification is concerned. But the supersymmetric contribution to the muon $(g-2)$ is negative in this case. This adds to the existing discrepancy between theory \cite{15} and experiments \cite{16}. Scenario with positive $\mu$ proves better and allows the theoretical prediction for $(g-2)$ to agree with
experiments within $3\sigma$. In this case, achieving $t-b-\tau$ unification becomes considerably more difficult. The mSUGRA in this case at best allows unification at 65% level \[10\]. Even with non-universal boundary conditions, one needs specific relations between the soft parameters, $m_{10} \sim 1.2m_{16}$, $A_0 \sim -2m_{16}$ and $m_{1/2} \ll m_{16}$ together with $\tan\beta \sim 50$ in order to achieve $t-b-\tau$ unification \[9\]. The DT splitting in this case, allows unification at most 90% level but this requires $m_{16} \gtrsim 10$ TeV and a gluino mass $< 500$ GeV. Just so HS works much better than DT splitting and leads to the perfect Yukawa unification for $m_{16} \sim 10$ TeV but gluino is still light. In both these scenarios, the particle mass spectrum is characterized by lighter gluino which is within the reach of current LHC searches at $\sqrt{s} = 7$ TeV, whereas all scalar sparticles have masses beyond the TeV scale. The light gluino mainly decays through a three body channel $\tilde{g} \rightarrow b\bar{b}\tilde{\chi}_0$ leading to multijets plus missing transverse energy. The final states may also contain dileptons if $\tilde{t}$ is lighter than $\tilde{b}$. Recently, the ATLAS experiment with 2.05 fb$^{-1}$ data collected at $\sqrt{s} = 7$ TeV has excluded the light gluino masses below 620 GeV in $SO(10)+HS$ model \[17\]. As we will show later, this experimental limit on gluino mass rules out $t-b-\tau$ unification better than 90% for $m_{16} \sim 10$ TeV. The strong bound on the gluino mass follows from the need of suitable threshold corrections in bottom quark mass to achieve $t-b-\tau$ unification which requires the hierarchy $m_{1/2} \ll m_{16}$. One can thus raise the value of $m_{1/2}$ and hence the bound on the gluino mass by raising $m_{16}$. The gluino mass can be pushed in this case beyond the present experimental limit but at the cost of choosing $m_{16} \gtrsim 20$ TeV \[18\].

The best viable scenario with $\mu > 0$ corresponds to just so Higgs splitting and large $m_{16}$ and $m_{10}$. Theoretically, both these features are unsatisfactory. A large SUSY scale is unnatural and just so HS breaks $SO(10)$ explicitly. Just so HS can be indirectly introduced through the right handed neutrinos at the intermediate scale. Apart from causing problem with the gauge coupling unification, this case also does not do as well as the arbitrarily introduced just so HS, see \[12\]. We wish to discuss here possible ways to improve on both these aspects. Specifically, we show that just so HS arises naturally in realistic $SO(10)$ models containing additional Higgs fields, e.g. the one transforming as 126, $\overline{126}$, and 210 representations under $SO(10)$. Realistic fermion masses can be obtained if these fields are present and the $SU(2)_L$ doublets contained in them mix with each other. Moreover, all the angles involved in such mixing need not be small. We show that significant Higgs doublet mixing can generate just so HS in the presence of non-universal but $SO(10)$ preserving boundary conditions at the GUT scale without inducing any splitting between squarks or without significantly upsetting the $t-b-\tau$ Yukawa unification.

$SO(10)$ models also allow an interesting possibility of matter fields mixing among themselves \[19\]. This leads to just so squark/slepton splitting similar to the just so HS occurring due to Higgs mixing. Impact of such mixing was used earlier \[19\] to obtain departure from the $b-\tau$ unification that follows in $SU(5)$ or $SO(10)$ models. Here we discuss phenomenological implications of such mixing in the context of the $t-b-\tau$ unification. We discuss explicit example leading to independent squark and Higgs splitting and show that the presence of such splittings helps in raising the gluino mass prediction without raising the SUSY
parameters $m_{16}$ to high values around 20 TeV.

This paper is organized as follows. In the next section, we present a short review of the basic features of the $t - b - \tau$ unification and discuss the existing phenomenological results. We also update the existing results incorporating the recent limit on $B_s \to \mu^+\mu^-$ from LHCb [20]. The viability of $t - b - \tau$ unified solutions in the presence of independent Higgs splitting and squark splitting (SS) are discussed in Section III. In Section IV we discuss how HS and SS can arise in the realistic versions of $SO(10)$ models. The study is summarized in the last section.

II. $t - b - \tau$ UNIFICATION IN MSSM

In this section, we review numerical and analytic results presented in the literature [8–13] in the context of the $t - b - \tau$ unification. We have re derived several existing numerical results in a way which optimizes the rate of the $B_s \to \mu^+\mu^-$ to make it consistent with the recent more stringent experimental [20] bound without pushing the SUSY scale to a higher value. Before discussing this, we summarize aspects of the $t - b - \tau$ unification which allows understanding of the salient key features.

The hypothesis of the $t - b - \tau$ unification assumes that at the GUT scale

$$y_b \equiv \frac{m_b}{v \cos \beta} = y_\tau \equiv \frac{m_\tau}{v \cos \beta} = y_t \equiv \frac{m_t}{v \sin \beta},$$

where $v \approx 174$ GeV. This equation is motivated by a simple $SO(10)$ model containing only a single 10-plet Higgs. The appropriate choice of the free parameter $\tan \beta$ in Eq. (1) can always allow equality of $y_t$ with $y_b$ or $y_\tau$ at the GUT scale. However it is well known [13] that $y_b$ and $y_\tau$ and hence all of them derived from Eq. (1) using the experimental values for fermion masses extrapolated to the GUT scale do not unify for any value of $\tan \beta$. The degree of unification of three Yukawas or lack of it is usually measured by the parameter

$$R_{tbr} \equiv R = \frac{\text{Max.}(y_t, y_b, y_\tau)}{\text{Min.}(y_t, y_b, y_\tau)}.$$  

The parameter $R_{tbr}$ is defined at the GUT scale. Variation of $R_{tbr}$ with $\tan \beta$ obtained by using the tree level Yukawa couplings and fermion masses at $M_Z$ is shown in Fig. 1. For comparison we also show similar ratios $R_{br}$ and $R_{tr}$ defined using only two of the Yukawa couplings. The extrapolation from $M_Z$ to the GUT scale is done using the 1-loop RG equations which are independent of the details of the soft SUSY breaking. Fig. 1 explicitly shows that the three Yukawas do not meet for any $\tan \beta$ and the reason is that $b$ and $\tau$ Yukawa couplings never meet. The best value of $R_{tbr}$ seen from Fig. 1 is around 1.2. Thus any scheme which tries to achieve $t - b - \tau$ unification should do better than this tree level value.

It is known that the tree level Yukawa couplings, particularly that of the $b$ quark receive [2, 3] significantly large radiative corrections once the supersymmetry is broken. The corrected
FIG. 1. Tree level Yukawa unification as a function of $\tan \beta$. $R_{ij}$ defined in Eq. (2) measures the closeness of $y_i$ and $y_j$ at the GUT scale.

$y_b$ can be written as

$$y_b = y_b^{\text{tree}} \cos \beta (1 + \Delta y_b^g + \Delta y_b^\tilde{\chi} + \ldots) ,$$

where (...) contains the electroweak suppressed SUSY corrections, the standard model (SM) electroweak corrections and logarithmic corrections which are sub dominant. The dominant correction $\Delta y_b^g$ ($\Delta y_b^\tilde{\chi}$) induced by the gluino (chargino) exchange is approximately given by

$$\Delta y_b^g \approx \frac{2 g_3^2}{3 \pi} \mu \tan \beta \frac{m_{\tilde{g}}}{m_{b_2}^2} ,$$

$$\Delta y_b^\tilde{\chi} \approx \frac{y_t^2}{16 \pi^2} \mu \tan \beta \frac{A_t}{m_{\tilde{t}_2}^2} ,$$

where $m_{b_2}$ ($m_{\tilde{t}_2}$) is mass of the heaviest sbottom (stop).

The presence of $\tan \beta$ makes the radiative corrections significant. The corrections to top Yukawa is inversely proportional to $\tan \beta$ while corrections to tau Yukawa is proportional to $\tan \beta$ but electroweak suppressed. In order to achieve unification, one needs to reduce $R_{br}$ and hence $y_b$ compared to its tree level value by about 10-20%. This requires that $\Delta y_b^g + \Delta y_b^\tilde{\chi}$ in Eq. (3) should be negative. Since the gluino induced contribution dominates over most of the parameter space, one can make the radiative corrections negative by choosing a negative $\mu$. As a result, models with negative $\mu$ achieve $t - b - \tau$ unification more easily. For positive $\mu$, the chargino contribution has to dominate over gluino and it should be negative. This can be satisfied with a negative $A_0$ and light gluino with $|A_0|, m_{16} \gg m_{1/2}$. As a result, all the scenarios of $t - b - \tau$ unification with positive $\mu$ lead to a light gluino and very heavy SUSY spectrum as borne out by the detailed numerical analysis [8–12].

The requirement of a light gluino as argued above directly conflicts with the requirement of the REWSB unless an explicit HS is introduced. This can be seen as follows. In large
tan \beta \) limit, the REWSB can be achieved if

\[- m_{H_u}^2 > \frac{M_Z^2}{2}, \]
\[\Delta m_H^2 \equiv m_{H_d}^2 - m_{H_u}^2 > \frac{M_Z^2}{2}, \tag{6}\]

where \( m_{H_u,d} \) are soft scalar masses evaluated at the weak scale. Starting with a positive value at \( M_{\text{GUT}} \), \( m_{H_u}^2 \) gets driven to large negative values by a large \( y_t \) and the first equation gets satisfied. But the large \( y_b, y_\tau \) as required in the \( t - b - \tau \) unification drives \( m_{H_d}^2 \) even more negative and conflicts with the second requirement. In addition to the Yukawas, the gaugino and scalar mass terms also contribute to the \( \Delta m_H^2 \). The former contribution is positive while the latter is negative, see the semi analytic solution of the 1-loop RG equations presented for example in [4, 5]. In Fig. 2, we show the running \( \Delta m_H^2 \) for different values of \( m_{1/2} \).

![Graph](image)

**FIG. 2.** The solution of 1-loop RGE equation for \( \Delta m_H^2 = m_{H_d}^2 - m_{H_u}^2 \) in mSUGRA for \( m_0(= m_{10} = m_{16}) = 1 \) TeV, \( A_0 = 0 \) and \( \tan \beta = 50 \).

As can be seen from the figure, second of Eq. (6) can be satisfied by choosing \( m_{1/2} > m_0 \) such that the gaugino induced contribution in \( \Delta m_H^2 \) dominates. On the other hand, small \( m_{1/2} \) and hence light gluino around \( m_{\tilde{g}} \leq 500 \) GeV is required if significant corrections to \( y_b \) is to be obtained in case of \( \mu > 0 \). This corresponds to \( m_{1/2} \leq 200 \) GeV for which REWSB cannot be achieved unless one introduces splitting between Higgs fields at the GUT scale itself. This is clearly seen from Fig. 2. Moreover, the case in which only Higgs splitting is considered is more favorable than the D-term splitting. This follows [8] qualitatively from Eqs. (4, 5) which implies

\[
\left| \frac{\Delta y_\tilde{g}}{\Delta y_b^{\pm \pm}} \right| \approx 11\pi \frac{m_{\tilde{g}}}{|A_t|} \frac{m_{1/2}^2}{m_{b_2}^2}.
\tag{7}\]

One finds \( m_{\tilde{t}_2} \sim m_{b_2} \) for mSUGRA as well for just so HS. The D-term splitting introduces sfermion splitting together with Higgs splitting and leads to \( m_{\tilde{t}_2} > m_{b_2} \). As a result, one needs to choose even a lighter gluino or a larger \( |A_t| \) to suppress the gluino induced corrections in \( y_b \). As we will show later in this paper, the additional squark splitting can instead
reduce the ratio $m_{t_2}/m_{b_2}$ and make it less than one. This allows significantly higher gluino mass.

The above qualitative features are borne out by several numerical studies presented in a number of papers \cite{11-12}. A list of different scenarios proposed to achieve $t-b-\tau$ unification for $\mu > 0$ is given in Table (1) in Ref. \cite{12}. Among all the proposals, the $SO(10)$ model with just SO HS is found as the best scenario which leads to an exact $t-b-\tau$ unification corresponding to $R = 1$. It is shown \cite{10} that HS works particularly well for large $m_{16}$ and the Yukawa unification $R \lesssim 1.02$ can be achieved if $m_{16} \gtrsim 10$ TeV. We update this analysis for the following reasons. In \cite{10}, the $t-b-\tau$ unified solutions were obtained considering the experimental constraint $BF(B_s \to \mu^+\mu^-)_{\text{(exp)}} < 2.6 \times 10^{-6}$. The recent data collected by LHCb experiment has improved this bound significantly. The current limit $BF(B_s \to \mu^+\mu^-)_{\text{(exp)}} < 4.5 \times 10^{-9}$ is three order of magnitude stronger than old bound. As a result, all the solutions obtained in \cite{10} are found inconsistent with new limit on $B_s \to \mu^+\mu^-$ (see, Tabel (1) in \cite{10}). We repeat the old analysis considering the new limits on $B_s \to \mu^+\mu^-$ and $b \to s\gamma$. In addition, we also consider the present constraints on $B \to \tau\nu_{\tau}$ \cite{21} which was not considered in the old analysis.

We use the ISASUGRA subroutine of ISAJET 7.82 \cite{22} in our numerical analysis. For given boundary conditions (soft SUSY parameters at the GUT scale and $m_t$, $\tan\beta$ at the weak scale), ISASUGRA solves full 2-loop MSSM RG equations and incorporates 1-loop SUSY threshold corrections in all the MSSM sparticles and in the masses of third generation fermions. Moreover, it checks for (a) non-techyonic solutions and (b) consistent REWSB using the minimization of one-loop corrected effective MSSM Higgs potential. Once these conditions are satisfied, we calculate $R$ using Eq. (2). Then using CERN’s subroutine MINUIT, we minimize $R$. Finally, we calculate branching factor for $b \to s\gamma$, $B_s \to \mu^+\mu^-$ using the IsaTools package \cite{22} and $B \to \tau\nu_{\tau}$ using the expressions given in \cite{23}. For completeness, we also estimate the relic abundance of neutralino dark matter $\Omega_{CDM}h^2$ and the SUSY contribution to anomalous magnetic moment of muon $\Delta a_{\mu}$ (where $a_{\mu} = (g-2)/2$) using IsaTools. On the acquired solutions, we apply the following constraints obtained from experimental data:

$$BF(B_s \to \mu^+\mu^-) < 4.5 \times 10^{-9}$$

$$2.78 \times 10^{-4} \leq BF(b \to s\gamma) \leq 4.32 \times 10^{-4} \ (3\sigma)$$

$$0.62 \times 10^{-4} \leq BF(B \to \tau\nu_{\tau}) \leq 2.66 \times 10^{-4} \ (3\sigma)$$

Further, we impose the mass bounds given in PDG \cite{25} on all sparticles including the LEP bound on the mass of lightest Higgs ($m_h > 114.4$ GeV). We use $m_t = 172.9$ GeV in our analysis. The recent LHC limit on gluino mass is not considered here. We will discuss it in detail in the next section.

The results of our numerical analysis are displayed in Table (I). We study three different cases:

1. In case I, we do not impose constraint (8) and minimize $R$ for fixed $m_{16} = 10$ TeV. The
| Parameter          | Case I         | Case II        | Case III        | Case IV        |
|--------------------|----------------|----------------|-----------------|----------------|
| $m_{16}$           | 10000          | 10000          | 15000           | 20000          |
| $m_{1/2}$          | 34.05          | 43.0           | 51.07           | 62.65          |
| $A_0/m_{16}$       | $-2.29$        | $-2.26$        | $-2.29$         | $-2.45$        |
| $m_{10}/m_{16}$    | 1.08           | 1.11           | 1.08            | 0.94           |
| $\tan \beta$      | 51.39          | 51.62          | 51.39           | 54.81          |
| $\Delta m^2_H/m^2_{10}$ | 0.25           | 0.28           | 0.25            | 0.39           |
| $R$                | 1.01           | 1.04           | 1.03            | 1.02           |
| $m_{\tilde{g}}$   | 345.0          | 367.9          | 487.7           | 634.9          |
| $m_{\tilde{\chi}^0_{1,2}}$ | 49.6, 118.9   | 53.2, 127.1    | 73.6, 177.9     | 101.4, 241.7   |
| $m_{\tilde{\chi}^0_{3,4}}$ | 6658.5, 6658.6 | 6460.2, 6460.3 | 9966.9, 9966.9  | 17102, 17103   |
| $m_{\tilde{\chi}^+_{1,2}}$ | 119.6, 6650.6 | 126.5, 6452.6  | 183.7, 9953.2   | 242.9, 17097   |
| $m_{\tilde{u}_{L,R}}$ | 9984.3, 9891.6 | 9988.4, 9874.7 | 14988, 14850    | 20010, 19782   |
| $m_{\tilde{d}_{L,R}}$ | 9984.7, 10043 | 9988.8, 10052  | 14988, 15075    | 20010, 20126   |
| $m_{\tilde{e}_{L,R}}$ | 9926.0, 7374.4 | 9912.4, 7274.3 | 14893, 11193    | 19841, 14636   |
| $m_{\tilde{\ell}_{L,R}}$ | 9924.7, 10134 | 9911.2, 10159  | 14890, 15199    | 19838, 20312   |
| $m_{\tilde{t}_{1,2}}$ | 2554.9, 3181.6 | 2506.2, 3110.1 | 3957.5, 4859.7  | 5915.9, 6807.6 |
| $m_{\tilde{b}_{1,2}}$ | 2997.6, 3341.7 | 2902.4, 3226.2 | 4724.0, 5097.9  | 6538.7, 6984.1 |
| $m_{\tilde{\tau}_{1,2}}$ | 3904.9, 7365.2 | 3664.9, 7278.2 | 6329.0, 11192   | 7549.7, 14617  |
| $m_h$              | 126.0          | 125.8          | 127.8           | 125.0          |
| $m_H$              | 3463.6         | 4353.4         | 5664.2          | 8683.1         |
| $m_A$              | 3441.4         | 4325.4         | 5627.4          | 8626.5         |
| $m_{H^+}$          | 3465.5         | 4354.9         | 5665.3          | 8683.8         |
| BF$(b \to s \gamma)$ | $3.09 \times 10^{-4}$ | $3.07 \times 10^{-4}$ | $3.06 \times 10^{-4}$ | $3.06 \times 10^{-4}$ |
| BF$(B \to \tau \nu \tau)$ | $0.79 \times 10^{-4}$ | $0.79 \times 10^{-4}$ | $0.79 \times 10^{-4}$ | $0.79 \times 10^{-4}$ |
| BF$(B_s \to \mu^+ \mu^-)$ | $4.55 \times 10^{-9}$ | $4.22 \times 10^{-9}$ | $4.15 \times 10^{-9}$ | $3.96 \times 10^{-9}$ |
| $\Delta a_\mu$     | $0.024 \times 10^{-10}$ | $0.026 \times 10^{-10}$ | $0.008 \times 10^{-10}$ | $0.008 \times 10^{-11}$ |
| $\Omega_{CDM}h^2$  | 2737           | 719            | 39866           | 14115          |

TABLE I. The benchmark solutions obtained for $t-b-\tau$ Yukawa unification in SO(10)+HS model for positive $\mu$. Different columns correspond to different cases discussed in the text. All masses are in GeV units.

best unification found corresponding to $R = 1.01$. The solution predicts $\text{BF}(B_s \to \mu^+ \mu^-) = 4.55 \times 10^{-9}$ which is slightly above the upper bound $[8]$.  

2. The dominant contribution to $B_s \to \mu^+ \mu^-$ in MSSM is proportional to $m_A^{-4} [27]$ where $m_A$ is the mass of pseudo-scalar Higgs. Using this fact, we simultaneously maximize $m_A$ and minimize $R$ in case II for $m_{16} = 10$ TeV. As a result, we get a lower value of $\text{BF}(B_s \to \mu^+ \mu^-)$ which is consistent with experimental limit $[8]$. However one gets a
slight declination in Yukawa unification in this case.

3. Case III and IV correspond to an obvious way of decreasing SUSY contribution to flavor violation namely, uplifting the SUSY scale. We take $m_{16} = 15$ and 20 TeV which increase the masses of all SUSY spectrum including $m_A$. The unification achieved in these cases is 97-98%.

It is clear from the results of our analysis that a very good $t - b - \tau$ Yukawa unification can still be achieved with “low” $m_{16}$ in SO(10)+HS model without violating the present experimental constraint on $B_s \to \mu^+\mu^-$. The calculated values of $BF(b \to s\gamma)$, $BF(B \to \tau\nu\tau)$ and $\Delta a_\mu$ shown in Table (I) are almost similar to their standard model values. The SUSY contributions to these processes are negligible due to the heavy sparticle spectrum one typically gets in $t - b - \tau$ unified solutions for $\mu > 0$. Among the other well known features of $t - b - \tau$ unification with positive $\mu$ are relatively heavier Higgs $m_h \sim 125 - 130$ GeV arising due to large $m_{16}$ and the condition $A_0 \sim -2m_{16}$ [9, 28] and pure bino like lightest neutralino which leads to the over abundance of the neutralino dark matter [11]. One would need additional mechanism, e.g. tiny R parity violation [29] to reduce this abundance.

III. $t - b - \tau$ UNIFICATION AND HEAVIER GLUINO

As noted earlier, the $t - b - \tau$ unified solutions for positive $\mu$ generally require very light gluino mass $\lesssim 500$ GeV. The direct SUSY searches at the LHC has now excluded $m_{\tilde{g}} \lesssim 620$ GeV in SO(10)+HS model [17]. As a result, the solutions displayed in first three columns in Table (I) are ruled out. It is recently shown that consistent $t - b - \tau$ unification with heavier gluino can be obtained by increasing $m_{16}$ [18]. In fact one needs $m_{16} \gtrsim 20$ TeV to evade the present LHC bound on gluino mass e.g. case IV in Table (I). We propose here an alternate way to obtain heavier gluino in $t - b - \tau$ unified solution without increasing $m_{16}$. As mentioned in Section I the ratio $|\Delta y_{\tilde{g}}|/|\Delta y_{\tilde{b}}|$ in Eq. (7) can get an additional suppression if $m_{\tilde{t}_2} < m_{\tilde{b}_2}$. This allows $t - b - \tau$ unification with heavier gluinos. The required mass hierarchy $m_{\tilde{t}_2} < m_{\tilde{b}_2}$ can be obtained if the appropriate squark splitting is introduced at the GUT scale. For example, consider SU(5) invariant boundary conditions:

$$m_Q^2 = m_U^2 = m_E^2 \equiv m_{16}^2,$$
$$m_L^2 = m_D^2 \equiv m_{16}^2 + \Delta m_S^2.$$ (11)

The origin of such splitting in an SO(10) model will be discussed in the next section. As we will show later in Eq. (31, 32), $\Delta m_S^2$ is allowed to take any values greater than $-m_{16}^2$. The choice $\Delta m_S^2 > 0$ raises the mass of one eigenstate of sbottom squarks, i.e. $m_{\tilde{b}_1}$, compared to its value obtained with universal squark masses at the GUT scale. This leads to $m_{\tilde{t}_2} < m_{\tilde{b}_2}$ at weak scale which is the case of our interest.

To quantify the effect of squark splitting, we study the above case through detailed numerical analysis. Fixing $m_{16} = 10$ TeV we perform a random scan over all the remaining
soft parameters and $\tan \beta$. The analysis is performed for two scenarios: (1) only Higgs splitting, i.e. for $\Delta m^2_S = 0$ and (2) Higgs splitting + Squark splitting (HS+SS) with $\Delta m^2_H, \Delta m^2_S > 0$. We employ the same numerical technique and apply all the theoretical and experimental constraints discussed in Section II. The results of numerical analysis are shown in Fig. (3).

![Figure 3](image)

**FIG. 3.** Solutions of $t - b - \tau$ Yukawa unification as a function of gluino mass for $m_{16} = 10$ TeV. The figure in the left (right) panel shows the solutions obtained with only HS (HS+SS) at the GUT scale in $SO(10)$. The vertical line corresponds to the lower bound on gluino mass in $SO(10)+HS$ model derived from the recent ATLAS data [17]. All the points shown are consistent with various phenomenological constraints discussed in the text.

As can be seen from Fig. (3), the lower bound on gluino mass in $SO(10)+HS$ model rules out the Yukawa unification $R < 1.08$. In the presence of additional squark splitting, the unification up to 99% can be achieved without violating the present LHC limit on gluino mass. Also, a relatively heavier gluino up to 1.5 TeV is obtained assuming at most 10% deviation in Yukawa unification without uplifting $m_{16}$. It can be also seen that the number of valid solutions obtained with HS+SS are more compared to those obtained with only HS. In Fig. (4), we show the ratio $m_{\tilde{t}_2}/m_{\tilde{b}_2}$ in HS and HS+SS models. The ratio substantially decreases in HS+SS model compared to its value without squark splitting. As we mentioned earlier in this section, this allows a heavier gluino in the spectrum without uplifting the SUSY braking scale. Note that with $m_{16} = 10$ TeV, $m_{\tilde{g}} > 1.5$ TeV cannot be obtained for $R \lesssim 1.1$ even in the HS+SS model. This range of gluino mass is still in the reach of LHC and its future operations at $\sqrt{s} = 14$ TeV can significantly constraint the parameter space of HS+SS model if not rule it out completely.

### IV. $t - b - \tau$ UNIFICATION AND REALISTIC $SO(10)$

We consider two categories of $SO(10)$ models. One in which Higgs sector is extended to obtain realistic fermion masses and mixing and the other in which one introduces also
additional matter multiplet at $M_{GUT}$. The former class of models lead to just so HS and the latter also to an independent squark splitting. We discuss them in turn.

A. Just so HS in realistic $SO(10)$

$SO(10)$ model containing a 10-plet of Higgs field $10_H$ allows the following term in the superpotential:

$$Y_{10}16_F16_F10_H,$$  \hspace{1cm} (12)

where $16_F$ refers to the matter multiplet and $Y_{10}$ to the Yukawa coupling matrix in the generation space. The $t-b-\tau$ unification follows under two assumptions:

(A) Third generation of fermions obtain their masses only from Eq. (12).

(B) The MSSM fields $H_u$ and $H_d$ reside solely in $10_H$.

One however needs to violate both these assumptions in order to obtain correct masses for all fermions and non-trivial mixing among them. We estimate the effects of these violations on the $t-b-\tau$ unification. We will take as an example a popular minimal renormalizable $SO(10)$ model \[30\] which is found to explain fermion masses and mixing in a number of situations, for instance, see \[31\] and references therein. The model contains a $126_H$ field to generate neutrino mass and a $126_H$ to preserve supersymmetry at the GUT scale. In addition, it has a Higgs transforming as $210_H$ representation of $SO(10)$ which breaks $SO(10)$ to MSSM.

Due to the presence of additional Higgs fields particularly $210_H$, $SU(2)_{L}$ doublets residing in various Higgs fields mix with each other. This mixing plays an important role in generating the right type of the second generation masses \[30\]. But as we show below, this mixing also generates just so Higgs splitting required to obtain $t-b-\tau$ unification if the soft masses of the $10_H$, $126_H$ and $210_H$ fields are non-universal. On the negative side, the presence of $126_H$ and the Higgs mixing also lead to departures from the exact $t-b-\tau$ unification.
The $10_H$, $\overline{126}_H$, $126_H$, $210_H$ fields each contain four down-type and four up-type standard model doublets. They mix and produce four mass eigenstates denoted as $h_{u,d}^{a,b}$:

$$h_{u,d}^{a,b} = O_{ab}^{u,d} \phi_{u,d}^a,$$

(13)

where $\phi_{u,d}^{a,b}$ are components of bi-doublets in $10_H$, $\overline{126}_H$, $126_H$, $210_H$. One assumes that through fine tuning only $H_u \equiv h_1^u$ and $H_d \equiv h_1^d$ remain light. Consider now the soft mass terms of various $SO(10)$ Higgs prior to the $SO(10)$ breaking:

$$V_{\text{soft}} \ni m_{16}^2 16_F 16_F + m_{10}^2 10_H 10_H + m_{126}^2 (126_H 126_H + \overline{126}_H \overline{126}_H) + m_{210}^2 210_H 210_H.$$

(14)

Here we have assumed equal masses for the $126_H$ and $\overline{126}_H$ fields to avoid non-zero D-term. The masses of the other Higgs multiplets are taken non-universal. Substitution of Eq. (13) in Eq. (14) leads to

$$V_{\text{soft}} \ni m_{16}^2 (\tilde{Q}^\dagger \tilde{Q} + \tilde{U}^\dagger \tilde{U} + \tilde{D}^\dagger \tilde{D} + \tilde{L}^\dagger \tilde{L} + \tilde{E}^\dagger \tilde{E})$$

$$+ (m_{10}^2 |O_{11}^u|^2 + m_{126}^2 (|O_{21}^u|^2 + |O_{31}^u|^2) + m_{210}^2 |O_{41}^u|^2) H_u^\dagger H_u$$

$$+ (u \to d).$$

(15)

The above equation leads to the following boundary conditions at the GUT scale

$$m_{\tilde{Q}}^2 = m_{\tilde{U}}^2 = m_{\tilde{D}}^2 = m_{\tilde{L}}^2 = m_{\tilde{E}}^2 = m_{16}^2.$$

$$m_{H_{u,d}}^2 = m_{126}^2 + |O_{11}^{u,d}|^2 (m_{10}^2 - m_{126}^2) + |O_{41}^{u,d}|^2 (m_{210}^2 - m_{126}^2).$$

(16)

It is seen that the Higgs mixing generated through Eq. (13) has produced the desirable splitting only among $H_{u,d}$ masses without splitting squarks from each other unlike in case of the popular D-term splitting.

Let us now look at the impact of Higgs mixing on the $t - b - \tau$ unification. The presence of $\overline{126}_H$ field modifies Eq. (12) to

$$Y_{10} 16_F 16_F 10_H + Y_{126} 16_F 16_F \overline{126}_H,$$

(17)

where $Y_{126}$ additional Yukawa coupling matrix. By substituting Eq. (13) in Eq. (17) one arrives at the charged fermion mass matrices:

$$M_d = v_d (Y_{10} O_{11}^d + Y_{126} O_{21}^d),$$

(18)

$$M_l = v_d (Y_{10} O_{11}^d - 3 Y_{126} O_{21}^d),$$

(19)

$$M_u = v_u (Y_{10} O_{11}^u + Y_{126} O_{21}^u),$$

(20)

where $v_{u,d}$ denote the vacuum expectation values of the neutral component of $H_{u,d}$. We can go to a basis with $Y_{10}$ diagonal. Neglecting the contribution of $\overline{126}_H$ for the time being, one has

$$y_b = y_\tau = O_{11}^d,$$

(21)
Thus one source of the departure from the $t - b - \tau$ unification is the ratio $O_{11}^u/O_{11}^d$. The parametric form of the matrices $O^{u,d}$ is worked out \cite{32,33} in the model under consideration and we closely follow the notation in \cite{33}. This is based on the Higgs superpotential

$$W_H = M_{10} \ 10^3_H + M_{210} \ 210^2_H + M_{126} \ 126_H \ 1 \bar{126}_H + \lambda \ 210^3_H + \eta \ 210_H \ 126_H \ 1 \bar{126}_H + 210_H \ 10_H (\alpha \ 126_H + \bar{\alpha} \ 126_H). \quad (22)$$

The Higgs mass matrices and hence $O^{u,d}$ follow from the above superpotential after the $SO(10)$ breaking and in the most general situation with $SO(10)$ breaking to standard model one obtains (see, Eqs. (C18, C19) in \cite{33})

$$H_d = N_d \left( \frac{\phi_d^u}{x - 1} - \sqrt{6} \frac{\eta}{\eta} (2x - 1)(x + 1) \frac{p_5}{p_3} \phi_2^d - \sqrt{6} \frac{\tilde{\alpha}}{\eta} (3x - 1)(x^3 + 5x - 1) \phi_3^d + \tilde{\alpha} \frac{\sigma}{m_\phi} p_5^d \phi_4^d \right),$$

$$H_u = N_u \left( \frac{\phi_u^u}{x - 1} - \sqrt{6} \frac{\eta}{\eta} (2x - 1)(x + 1) \frac{p_5}{p_3} \phi_3^u - \sqrt{6} \frac{\tilde{\alpha}}{\eta} (3x - 1)(x^3 + 5x - 1) \phi_2^u - \tilde{\alpha} \frac{\sigma}{m_\phi} p_5^u \phi_4^u \right), \quad (23)$$

where $N_{u,d}$ are overall normalization constants. $x$ is an arbitrary parameter and $p_3, p_5, p_5^d$ are polynomial in $x$. The expressions of elements of $O^{u,d}$ can be read from the above equation. The ratio $O_{11}^u/O_{11}^d$ which measures deviation from $t - b - \tau$ unification can be exactly or nearly one in a number of situations. Obvious case is the limit $N_u = N_d \approx 1$ corresponding to the situation in which extra Higgs fields’ contribution to $H_{u,d}$ are sub-dominant. In general, various $\phi_u^u$ and $\phi_d^d$ are components of $SU(2)_L \times SU(2)_R$ bi-doublets residing in $10_H, \ 126_H, \ \bar{126}_H, \ 210_H$ representations. They are thus distinguished by the $SU(2)_R$ group. One therefore automatically has $O^u = O^d$ as long as $SU(2)_R$ is unbroken. This happens \cite{33} for $x = 0$. One can then show from Eq. (23) that $O^u = O^d$ in this case. Another interesting limit corresponds to choosing $\alpha = \tilde{\alpha}$ in Eq. (23). In this limit, $O^u \neq O^d$ but still $O_{11}^u = O_{11}^d$ thus one obtains exact $y_b = y_t$ in Eq. (21) even in the situations where $SU(2)_R$ is broken. This case also corresponds to $O_{41}^u = - O_{41}^d$ and thus HS also vanish in this limit as follows from Eq. (16). But mild deviation from the limit $\alpha = \tilde{\alpha}$ can generate sizable HS and approximate $t - b - \tau$ unification for large ranges in other parameter. This is illustrated in Fig. (14) where we show $\frac{N_u}{N_d}$ and $\left| \frac{O_{11}^d}{O_{11}^u} \right|$ as a function of $x$ for two specific values of $\epsilon = 1/2(\tilde{\alpha} - \alpha) = 0.1, \ 0.2$. The remaining unknown parameters appearing in Eq. (23) are equated to 1. It is seen that for most values of the unknown parameter $x$ one obtains almost exact $t - b - \tau$ unification, i.e. $\frac{N_u}{N_d} = 1$ and non-zero HS \cite{34} as given in Eq. (16).

Another threat to the $t - b - \tau$ unification comes due to the presence of the $Y_{126}$ Yukawa couplings in Eq. (18). This effect is somewhat model-dependent and we estimate it by specializing to the case of the second and third generations. We can write the charged fermion mass matrices as

$$M_d = v_d \begin{pmatrix} h_2 + x_2 & x & x \\ x & h_3 + x_3 \\ x \\ h_3 + x_3 \end{pmatrix}; \quad M_t = v_d \begin{pmatrix} h_2 - 3x_2 & -3x \\ -3x & h_3 - 3x_3 \end{pmatrix};$$

$$M_u = v_u \begin{pmatrix} O_{11}^u & O_{11}^d \\ h_2 + s x_2 & s x \\ s x & h_3 + s x_3 \end{pmatrix}, \quad (24)$$

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FIG. 5. The Higgs mixing parameters \( \frac{N_u}{N_d} \) (continuous lines) and \( \frac{|O'_{41}|}{|O''_{41}|} \) (dashed lines) as a function of \( x \in \) in the minimal SUSY SO(10) model. The blue (red) line corresponds to parameter \( \epsilon = 0.1 \) (0.2).

where \( s \equiv \frac{O_{21}^{d} O_{11}^{d}}{O_{21}^{u} O_{11}^{u}} \). Here \( h_{2,3} \) refer to elements of the diagonal \( Y_{10} O_{11}^{d} \) and \( x_2, x, x_3 \) to that of symmetric \( O_{21}^{d} Y_{26} \). Several of these can be determined from the known masses and mixing. Approximate \( t - b - \tau \) unification is obtained with the hierarchy \( x, x_2, x_3 \ll h_3 \). Assuming \( h_2 \ll x_2 \) then leads to the desirable mass relation \( 3m_s = m_\mu \). Given this hierarchy one finds for real parameters

\[
x_2 \sim \frac{m_s}{m_b}; \quad x \sim m_b V_{cb}(1 - s); \quad s \sim \frac{m_b m_c}{m_s m_t}
\]

The ratio \( x_2/x_3 \) remains undermined. If type II seesaw dominates and \( Y_{26} \) is to provide the neutrino masses and mixing then \( x_2 \sim x_3 \). In this case, one finds

\[
\begin{align*}
y_b &\approx h_3 \left( 1 + \frac{m_s}{m_b} \right) \left( 1 + O(V_{cb}^2) \right), \\
y_\tau &\approx h_3 \left( 1 - \frac{3m_s}{m_b} \right) \left( 1 + O(V_{cb}^2) \right), \\
y_t &\approx \frac{O_{11}^{u}}{O_{11}^{d}} h_3 \left( 1 + \frac{m_c}{m_t} \right),
\end{align*}
\]

where we have used Eq. (1). Using value for the mass ratios at the GUT scale for \( \tan \beta = 50 \), \( \frac{m_s}{m_b} \approx 0.016, \frac{m_t}{m_c} \approx 0.0023 \) already implies about 6% departure from \( b - \tau \) unification. The \( m_s, m_c \) dependent terms in Eq. (25) get suppressed if one assumes \( x_3 \ll x_2 \). In this case, one can still obtain \( b - \tau \) unification and reproduce the second generation masses and Cabibbo mixing. The Higgs mixing factor can be nearly one as argued before. It is thus quite plausible that one can obtain almost exact Yukawa unification not just in simplified but also in more realistic GUT based on SO(10).
B. Squark splitting and $t-b-\tau$ unification

The squark splitting can be induced by adding new matter fields having the same quantum numbers as some of the squarks and letting them mix with the normal squarks. The minimal possibility at the $SO(10)$ level is introduction of a $10_M$ field. This contains fields transforming under an $SU(5)$ subgroup of $SO(10)$ as $5'_M + \bar{5}'_M$. Of these, the $\bar{5}'_M$ can mix with $5_M$ contained in the matter multiplet $16_M$ of $SO(10)$. This mixing can generate the squark splitting. Let us discuss details within a specific model which has been studied extensively [37, 38] for several different reasons. The model contains three generations of $16_M$ and three copies of $10_M$. We will explicitly consider only the third generation and a $10_M$ in the following. The Higgs sector consists of the usual $10_H$ supplemented by $16_H + \bar{16}_H$ and $45_H + 54_H$. The $16_H$ is introduced to break the $B-L$ gauge symmetry and $45_H + 54_H$ are needed to complete the breakdown of $SO(10)$ to SM. This model serves as a good example in which (a) independent squark and Higgs splitting can be generated and (b) there exist ranges of parameters for which $t-b-\tau$ unification is approximately maintained. We shall discuss only a part of the superpotential relevant to describe Higgs and squark mixing, see [38] for a general discussion of the model. The superpotential of the model can be divided in two parts one describing matter-Higgs interaction and the other describing Higgs-Higgs interactions:

$$W_M = Y 16_M 16_M 10_H + F 16_M 10_M 16_H + M 10_M 10_M,$$
$$W_H = M_{16} \overline{16}_H 16_H + M_{10} 10_H 10_H + H 16_H 16_H 10_H.$$  \hspace{1cm} (26)

The above superpotential is designed to respect the matter parity under which all the matter (Higgs) fields are odd (even). This symmetry is essential for preventing renormalizable baryon and lepton number violating terms. Scalar components of none of the matter fields acquire vacuum expectation value (vev) and thus matter parity remains unbroken. Only fields appearing in the above superpotential and acquiring the GUT scale vev are $1_H + \overline{1}_H$ contained in $16_H + \overline{16}_H$ of $SO(10)$. Thus after the GUT scale breaking, above superpotential maintains invariance under the $SU(5)$ subgroup of $SO(10)$. As a result, the mixing between the following $SU(5)$ components is allowed and arise from Eq. (26):

$$\begin{pmatrix} \bar{5}'_M \\ 5'_M \end{pmatrix} = R(\theta) \begin{pmatrix} \bar{5}_M \\ 5_M \end{pmatrix};$$
$$\begin{pmatrix} \bar{5}'_H \\ 5'_H \end{pmatrix} = R(\gamma) \begin{pmatrix} \bar{5}_H \\ 5_H \end{pmatrix};$$
$$\begin{pmatrix} 5^l_M \\ 5^l_H \end{pmatrix} = R(\delta) \begin{pmatrix} 5_H \\ 5'_H \end{pmatrix},$$  \hspace{1cm} (27)

where

$$R(j) = \begin{pmatrix} \cos j & -\sin j \\ \sin j & \cos j \end{pmatrix}.$$  \hspace{1cm} (28)

The fields with (without) prime are component of the original $10 \ (16 + \overline{16})$ of $SO(10)$. It is assumed that fields labeled with superscript $l$ are kept light by fine tuning and those with the superscript $h$ pick up masses at the GUT scale. The mixing angles appearing above are
related to parameters in Eq. (26) and are explicitly given as
\[ \tan \theta = \frac{F v_{16}}{M}, \]
\[ \tan 2\gamma = \frac{2v_{16}(HM_{10} + H'M_{16})}{(M_{10}^2 - M_{16}^2) + v_{16}^2(H^2 - H'^2)}, \]
\[ \tan 2\delta = \frac{2v_{16}(H'M_{10} + HM_{16})}{(M_{10}^2 - M_{16}^2) - v_{16}^2(H^2 - H'^2)}, \] (28)

where \( v_{16} \) is a vev of SU(5) singlets residing in \( 16_H + \overline{16}_H \). The light fields transform under \( SU(5) \) as \( 5_H + \overline{5}_H \) of Higgs and \( 10_M + 1_M + \overline{5}_M \) which together makes \( 16_M \) and \( 10_H \) of \( SO(10) \). The effective Yukawa couplings of these fields can be worked out from Eqs. (26) and (27) in a straightforward way. One finds
\[ W_{eff} = Y s_\delta 10_M 10_M 5_H^l + (Y s_\gamma c_\theta + F s_\gamma c_\gamma) 10_M 5_M 5_H^l, \] (29)

where \( s_j = \sin j \) and \( c_j = \cos j \).

The Higgs mixing as given in Eq. (27) leads to both the squark and Higgs splitting through \( SO(10) \) invariant non-universal boundary conditions. Consider the following soft terms:
\[ V_{soft} \ni m_{16}^2 16_M^l 16_M + m_{10}^2 10_M^l 10_M + m_{16}^2 16_H^l 16_H + m_{10}^2 10_H^l 10_H \]
\[ = m_{16}^2 (10_M^l 10_M + 5_M^l 5_M + 1_M^l 1_M) + m_{10}^2 (5_M^l 5_M + 5_M^l 5_H) \]
\[ + m_{16}^2 (10_H^l 10_H + 5_H^l 5_H + 1_H^l 1_H) + m_{10}^2 (5_H^l 5_H + 5_H^l 5_H). \] (30)

Substitution of Eq. (27) in Eq. (30) leads to the following boundary conditions for the soft mass parameters of squarks and Higgs:
\[ m_Q^2 = m_U^2 = m_E^2 = m_{16}^2, \]
\[ m_L^2 = m_D^2 = m_{16}^2 + s_\theta^2 (m_{10}^2 - m_{16}^2), \]
\[ m_{H_d}^2 = m_{16}^2 s_\gamma^2 + m_{10}^2 s_\delta^2, \]
\[ m_{H_u}^2 = m_{16}^2 s_\delta^2 + m_{10}^2 s_\delta^2 \] (31)

resulting in the following squarks and Higgs splitting
\[ \Delta m_Q^2 \equiv m_Q^2 - m_D^2 = s_\delta^2 (m_{10}^2 - m_{16}^2), \]
\[ \Delta m_{H}^2 \equiv m_{H_d}^2 - m_{H_u}^2 = (s_\gamma^2 - s_\delta^2)(m_{10}^2 - m_{16}^2). \] (32)

Thus model under consideration simultaneously generates independent mixing among squarks and Higgses that result into squark splitting and Higgs splitting respectively. The mixing angles which generate these splitting also lead to departure from the exact \( t - b - \tau \) unification as before. But the exact \( b-\tau \) unification follows for arbitrary values of these mixing angles. Even \( t - b - \tau \) unification also holds approximately in limiting cases, e.g. \( F s_\delta \ll 1 \) and \( H = H' \). Another interesting limit corresponds to \( s_\delta \approx s_\gamma c_\theta \) and \( F s_\delta \ll 1 \). In this limit one gets \( y_b = y_t \approx y_t \) and simultaneously non-zero splittings, \( \Delta m_H^2 \approx s_\delta^2 \Delta m_5^2 \) for \( m_{10} = m_\prime_{10} \) and \( m_{16} = m_\prime_{16} \) in Eq. (32). This relation automatically implies \( \Delta m_H^2 > 0 \) as required for the REWSB when \( \Delta m_5^2 \) is chosen positive to raise the gluino mass limit.
V. SUMMARY

We have addressed two important issues in this paper in the context of the $t - b - \tau$ unification in $SO(10)$ broken to MSSM with a positive $\mu$ parameter. The existing analysis [9–12] have either assumed only HS with degenerate squarks at the GUT scale or a D-term splitting in which case squark splitting is correlated to the HS. This scenario appears to be ruled out save for very high SUSY scale $m_{16}$ around 20 TeV [18]. Detailed phenomenological analysis presented here shows that independent and positive squark splitting $\Delta m_{S}^{2}$, Eq. (11) can change the allowed gluino mass and scenario can be compatible with $t - b - \tau$ unification $R \sim 1.01$ and the recent ATLAS bound $m_{\tilde{g}} > 620$ GeV. Moreover, $R < 1.1$ requires $m_{\tilde{g}} < 1.5$ TeV. Thus viability of the $t - b - \tau$ unification can be tested in future at LHC with $\sqrt{s} = 14$ TeV. The other issue addressed here concerns the origin of just so HS. While just so HS is introduced as a phenomenological parameter in many works [9–12], its origin is not justified in most of the existing analysis, see however [6, 39]. We have taken here a concrete realistic model [30] used to understand fermion mass spectrum and shown within it that just so HS is an automatic consequence of the non-universal boundary conditions and Higgs mixing. It is also shown that one can obtain sizable just so HS and retain almost exact $t - b - \tau$ unification in this realistic model. Independent squark splitting required to relax the gluino mass bound is also shown to follow in an extended model in which squarks mix with additional matter multiplet. One may conclude based on the analysis presented here that $t - b - \tau$ unification with positive $\mu$ is still phenomenologically viable and theoretically well-founded.

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