Thermal noise of a gram-scale cantilever flexure

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Abstract.

Measured thermal noise displacement spectra from low frequency to 3kHz of Niobium and Aluminium flexures are presented. With a simple thermal noise model dominated by structural and thermoelastic losses, the agreement between the theory and measurement has been robust. The thermal noise spectra were recorded up to an order of magnitude below and above the fundamental resonance, with the fundamental resonances for both Aluminium and Niobium flexures within the range of 50Hz to 300Hz

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1. Introduction

Thermal noise has become one of the fundamental sources of displacements in many experimental systems and of increasing interest to many research groups. The mitigation of thermal noise in high sensitive interferometers, such as a gravitational wave detector, is essential and constitutes one of the most challenging aspects for the design of the interferometer mirrors and suspensions [1], [2],[3].

A matured and commonly used procedure to determine dominant thermal losses is extracted from the quality factor through a series of ring down measurement. The mechanical system is excited at various resonant frequencies. The ringdown measurement is obtained and fitted with complex algorithm of models for different type of losses. Many attempts to measure and identify the thermal noise of different materials have been made in the last few decades [4], [5], [6], [7]. However, few direct measurements of the off-resonant thermal noise source have been reported, worth mentioning are coating mirror thermal noise for tatanla/silica (on resonance) [8], [9] and for Sapphire mirrors (off-resonance) [10], while those of suspension thermal noise were reported by [11], [12],[13].

This paper presents direct measurements of broadband, off-resonance, thermal noise displacement spectra of Niobium and Aluminium flexures. The frequency range of these spectra lay within the detection band of ground base gravitational wave detectors. The distinct difference in our experiment is that we used optical interferometry and PDH readout to retrieve the flexure displacements due to thermal noise source. The results were matched with fluctuation dissipation theory through a straightforward application of loss models. The recorded thermal noise spectra were up to an order of magnitude below and above the fundamental resonance, with the fundamental resonances for both Aluminium and Niobium flexures in the range of 50 Hz to 300 Hz.

Thermal noise is present in all harmonic oscillators and driven by $k_B T$ amount of energy in every mode. From the fluctuation dissipation theorem, the power spectrum of the thermal noise of a mechanical system is constructed from the temperature of the heat bath (for a particular mode) and the mechanical response of the oscillator [14],[15]. The general equation describing this power spectrum is given by:

$$\hat{x}_{th}^2 = \sum_{i=0}^{n} \frac{4k_B T \omega_i^2 \phi_i}{m_i \omega_i^2 ((\omega_i^2 - \omega^2)^2 + \omega_i^4 \phi_i^2)}$$

(1)

where $\omega_k$ and $m_k$ are the modal resonances and the effective masses of the mechanical oscillator; $k_B$ is the Boltzmann constant; $T$ is the temperature of the measured mode; $\phi_k$ represents the linear sum of all the modal losses in the system at $\omega_k$.

2. Experimental system

We designed a monolithic inverted pendulum made from Niobium and Aluminium as shown in the insert in figure 1 (box 1A - Niobium and 1B - Aluminium). The geometrical
simplicity of the mechanical oscillators was aimed to isolate the fundamental resonant frequencies from higher order modes. The mechanical oscillators were manufactured by using Electric - Discharge - Machining (EDM).

For the Niobium flexure, the membrane is 6.35 mm wide, 1 mm high and 72 \( \mu \text{m} \) thick. The Test Cavity rear mirror, 7 mm diameter and 1 mm thickness, was glued to the top of the Niobium structure. The effective mass of the flexure with the mirror is 0.7 g, resulting in a fundamental resonant frequency of 85 Hz. The quality factor Q was independently determined from a ringdown and fitted to be 44 000.

The second flexure, made from Aluminium, is more compact with membrane of 5 mm wide, 1 mm high and 120 \( \mu \text{m} \) thick. A 1/4” diameter and 2 mm thick mirror was glued at the top of the flexure. With an effective mass of 0.4 g, the fundamental resonant frequency is 271 Hz with a Q of 2 200, also independently determined using a ringdown measurement. The lower Q material was chosen in order to increase the off-resonant suspension thermal noise while reducing the optically driven mechanical ringing.

We used a Fabry-Perot cavity to measure the microscopic displacement of the Aluminium and Niobium flexures due to the thermal noise, operated at room temperature. A schematic layout of the experiment is shown in figure [fig:experiment]. The 12 mm long cavity comprised of a front mirror glued to a piezo-transducer (PZT) and a rear mirror mounted on the flexure. The cavity was kept on resonance using the Pound-Drever-Hall (PDH) locking technique [16],[17]. The cavity length fluctuations due to the thermal noise of the flexure were obtained via the error-signal of the PDH technique (labelled Readout in figure [fig:experiment]). The cavity finesse was 600 and 700 for the Aluminium and Niobium displacement experiments respectively.
The laser was locked to a high finesse (6000), 20 cm long Zerodur reference cavity to reduce the frequency noise, to below the equivalent thermal noise displacement. Due to the much longer reference cavity, we gain a factor of 17 in frequency stability. A ‘seismic wall’ at 10 Hz to 40 Hz is provided by a multi-stage vibration isolation system.

The locking bandwidth during the Niobium flexure experiment was about 850 Hz. As the flexure resonance was within the locking bandwidth, the displacement spectrum was compensated with the closed loop locking servo response.

In contrast, the servo unity gain frequency during the Aluminium flexure experiment was below 10 Hz, well below the fundamental resonant frequency, and the servo effect to the measured output was checked to be insignificant, hence no correction was required. The Aluminium flexure measurements have been recorded using a digital control system, while that of the Niobium flexure have been recorded using a SR785 spectrum analyser.

Due to the geometry of the flexures presented here, the thermal noise is dominated by structural loss ($\phi_{\text{struc}}$) and thermoelastic loss ($\phi_{\text{te}}(f)$):

$$\phi_{\text{tot},k}(f) = \phi_{\text{struc},k} + \phi_{\text{te},k}(f)$$

$\phi_{\text{tot},k}(f)$ is the total loss obtained from ringdown measurements. The structural damping loss is independent of frequency and related to the inverse of the quality factor of the mechanical oscillator $Q_{\text{struc}}$. On the other hand, thermoelastic loss is dependent on frequency, bulk material and geometry of the flexure. More specifically, the loss is described with a characteristic strength $\Delta$, and characteristic time $\tau$ \cite{10}, given by:

$$\phi_{\text{te}}(f) = \Delta \frac{\omega \tau}{1 + (\omega \tau)^2}$$

$$\Delta = \frac{\alpha^2 E_y T}{\rho C_v}$$

$$\tau = \frac{\rho C_v t^2}{\kappa \pi^2}$$

here $\alpha$ is the linear thermal expansion coefficient, $E_y$ the Young modulus, $T$ the temperature, $\rho$ the density, $C_v$ the specific heat, $\kappa$ the thermal conductivity of flexure material and $t$ the heat transferring distance across the membrane. The characteristic time defines the optimal time for heat transfer across the flexure thickness, and creates a maximum loss in the frequency response. Table \ref{tab:parameters} summarises the details of the parameters used to construct the theoretical thermal noise models.

The amplitude sum of the structural and thermoelastic losses over frequency contributes to the overall loss of the thermal noise power spectrum of the flexures. Shown in figure \ref{fig:losses} are the dominant losses of the Aluminium and Niobium flexures. For the Aluminium flexure, the thermoelastic loss is maximum at around 5.6 kHz while this happens at 7.2 kHz for the Niobium flexure. In the figure the fundamental flexure resonance frequency for each metal is indicated by the vertical dotted line. At frequencies below the fundamental resonance, structural loss is the dominant noise source, while at
Table 1: A list of parameters used in the theoretical models

| Parameters                              | Aluminium   | Niobium     | Units    |
|-----------------------------------------|-------------|-------------|----------|
| Resonant frequency                      | 271         | 85          | Hz       |
| Mirror diameter                         | $6.35 \times 10^{-3}$ | $6.35 \times 10^{-3}$ | m        |
| Mirror thickness                        | $2 \times 10^{-3}$ | $1 \times 10^{-3}$ | m        |
| Young’s modulus ($E_y$)                 | 71          | 105         | GPa      |
| Linear thermal expansion coefficient ($\alpha$) | $23 \times 10^{-6}$ | $7.3 \times 10^{-6}$ | K$^{-1}$ |
| Specific heat ($C_v$)                   | 904         | 265         | J(\text{kgK}$^{-1}$  |
| Thermal conductivity ($\kappa$)         | 138         | 54          | W(\text{mK}$^{-1}$  |
| Density ($\rho$)                        | 2820        | 8578        | kgm$^{-3}$ |
| Thermoelastic resonance ($f_{te}$)      | $5.6 \times 10^3$ | $7.2 \times 10^3$ | Hz       |

Figure 2: shows frequency responses of the structural, thermoelastic and sum of the two losses for Aluminium and Niobium flexures. The quality factors from the ringdown were used to scale the sum of the losses at the fundamental resonant frequencies. The thermoelastic losses were calculated using equation 3. This resulted to the structural loss angles using equation 2. Gas damping loss was insignificant in both cases and hence omitted.

frequencies above the fundamental resonance the loss is dominated by the thermoelastic loss. At much higher frequencies, beyond the inverse of the characteristic time (frequencies beyond the peak amplitude frequency), the thermoelastic noise will drop below the structural loss.

The thermoelastic loss at the flexure resonant frequency was calculated using equation 3, resulting in a loss of $2.1 \cdot 10^{-4}$ and $8.5 \cdot 10^{-6}$ for Aluminium and Niobium respectively. The total loss at the fundamental resonance for each flexure was obtained from the quality factor measured previously. In combination with equation 2, the structural loss is calculated to be $2.7 \cdot 10^{-4}$ for Aluminium and $1.5 \cdot 10^{-5}$ for Niobium.

For experiments investigating radiation pressure noise effect, structural losses above the mechanical resonance frequency are required. In such experiments careful design of
the mechanical system should be considered to mitigate the thermoelastic noise.

3. Results and Discussions

3.1. Aluminium flexure

The measured thermal noise displacement spectrum of the Aluminium flexure is shown in figure 3 as the mustard colour, trace (1). Also included are the frequency noise equivalent displacement noise, trace (2), and the electronics noise, trace (3). Trace (4) is the modeled thermal noise displacement spectrum as per equation 1 with the sum of the modal losses as discussed above. The red trace (5) is the quadratic sum of traces (2) and (3), and trace (4). No fitting was performed, and the ratio of trace (1) and (5) is calculated to indicate their agreement in figure 3(b).

Below 40 Hz, the measured displacement, trace (1), deviates from the predicted total noise, trace (5), due to residual seismic coupling into the final test cavity suspension stage and spurious interferometer scattering. Above about 1.5 kHz the measurement is limited by a combination of the residual equivalent frequency noise and the electronics noise contributions, added in quadrature. (b) A ratio between the measured PDH error signal and the predicted noise shows good overlapping between theoretical models and measurement.
noise. The displacement was calibrated providing an experimental uncertainty of 23%.

3.2. Niobium flexure

The measured Niobium thermal noise displacement spectrum is shown in figure 4 as trace (1). Also included are the dark noise, trace (2), and the equivalent frequency noise, trace (3). Trace (4) is the sum of the structural and thermoelastic loss models.

Trace (5) is the quadratic sum of the dark noise (2), equivalent frequency noise (3) and the flexure displacement model (4). From 10 Hz to 2 kHz trace (5) follows the measured thermal noise spectrum. Calibration lines where injected at 33 Hz and 723 Hz setting an experimental uncertainties of 17%.

Other types of damping mechanisms have been investigated, including gas damping, damping due to clamping, the amount of glue or the type of glue used. None of the tests showed pronounced changes to the off-resonant thermal noise region above the flexure resonant frequency, and hence being exempted from the plot presented here.

![Graph](image)

Figure 4: (a) The Niobium flexure displacement spectrum. 1- Measured PDH error signal equivalent flexure displacement. 2- Experimental electronic noise. 3- Gain limited frequency noise equivalent to displacement. 4- Sum of structural damping and thermoelastic damping models. 5- Sum of the theoretical models and noise budget. (b) The difference between the predicted noise (trace (5) in (a)) and the measured noise (trace (1) in (a))
4. Conclusions

This paper reported a successfully measured off-resonance thermal noise spectra of two metallic materials: Aluminium and Niobium down to 40 Hz. With the current system setup, both flexures were found to produce combined responses of frequency independence and dependence. Thermoelastic loss was the dominated contributor in both flexure spectra at higher frequencies above their fundamental resonances, while at the low frequency ends, some forms of frequency independence showed dominant effects. Over all the theoretical models were well predicted for both types of metals.

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References

[1] K. Agatsuma and et al. Thermal-noise-limited underground interferometer clio. Classical and Quantum Gravity, 27(8):084022, 2010.
[2] Cumming A. V. and et al. Silicon mirror suspensions for gravitational wave detectors. Class. Quantum Grav., 31:025017.
[3] G. D. Hammond and et al. Class. Quantum Grav.
[4] Duffy W. Jr. Acoustic quality factor of molybdenum and tungsten at low temperatures. Journal of Applied Physics, 72.
[5] L. Ju and et al. The quality factor of niobium flexure pendulums. Phys. Lett. A, 254.
[6] S. Reid and et al. Mechanical dissipation in silicon flexures. Physics Lett. A, 351.
[7] Pendulum mode thermal noise in advanced interferometers: a comparison of fused silica fibbers and ribbons in the presence of surface loss.
[8] E. D. Black and et al. Direct observation of broadband coating thermal noise in a suspended interferometer. Phys. Lett. A, 328:1–5, 19 July 2004.
[9] K. Numata and et al. Wide-band direct measurement of thermal fluctuations in an interferometer. Phys. Rev. Lett., 91:260602, Dec 2003.
[10] E. D. Black and et al. Thermoelastic-damping noise from sapphire mirrors in a fundamental-noise-limited interferometer. Phys. Rev. Lett., 93:241101, Dec 2004.
[11] G. I. González and P. Saulson. Brownian motion of a torsion pendulum with internal friction. Physics Letters A, 201(1):12 – 18, 1995.
[12] K. Agatsuma and et al. Direct measurement of thermal fluctuation of high-q pendulum. Phys. Rev. Lett., 104:040602, Jan 2010.
[13] A. R. Neben and et al. Structural thermal noise in gram-scale mirror oscillators. New Journal of Physics, 14(115008).
[14] R. F. Greene and H.B. Callen. On a theorem of irreversible thermodynamics. ii. Phys. Rev., 88:1387–1391, Dec 1952.
[15] P. Saulson. Thermal noise in mechanical experiments. Physical Review D, 42(8):2437 – 2445, 1990.
[16] Drever R. W. P. and et al. Laser phase and frequency stabilisation using an optical resonator. Appl. Phys. B: Photophys. Laser Chem., 31.
[17] E. D. Black. An introduction to pound drever hall laser frequency stabilisation. Am. J. Phys., 69(1):79–87, January 2001.
[18] M. Cerdonio, L. Conti, A. Heidmann, and M. Pinard. Thermoelastic effects at low temperatures and quantum limits in displacement measurements. Phys. Rev. D, 63:082003, Mar 2001.