Polarized photoproduction of large-\(p_T\) hadron pairs as a probe of the polarized gluon distribution

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**ABSTRACT**

Production of large-\(p_T\) hadron pairs by polarized photon on a longitudinally polarized proton towards probing the polarized gluon distribution is studied. Resolved photon contributions and the effect of changing the scales are taken into account, and predictions are presented. A very recent experimental result at c.m. energy 7.18 GeV is compared to our predictions extended down to this energy. A proper combination of cross sections is also considered.

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I. INTRODUCTION

The determination of the size and shape of the polarized gluon distribution $\Delta g$ remains a major problem in Spin Physics. Clearly, the way to proceed is to study theoretically and experimentally polarized reactions dominated by subprocesses with gluons in the initial state. To this effect, experiments on charm production by polarized photons on longitudinally polarized protons (polarized photoproduction) [1], large-$p_T$ direct photon and jet production in polarized $p$-$p$ collisions [2], etc. will be carried [3,4].

As a reaction leading to useful information, Ref. [5] has proposed the production of large-$p_T$ hadron pairs $H_1$, $H_2$ in polarized photoproduction:

\[
\gamma^* + p \rightarrow H_1 + H_2 + X
\]  

(1.1)

An experiment could well be carried in COMPASS.

In view of this, we have undertaken an independent study of the reaction (1.1). Working, as in [5], at Born level (leading order (LO) in $\alpha_s$) we differ in the following from [5]:

(i) We take into account the resolved photon contributions, which are left out in [5].

(ii) In general, reaction (1.1) is dominated by the subprocesses

\[
\begin{align*}
(a) & \quad \gamma^* \gamma \rightarrow q\bar{q} , \\
(b) & \quad q\bar{q} \gamma \rightarrow qg 
\end{align*}
\]  

(1.2)

In [5], the first is well taken into account, but the second is treated in a rather unclear way. Here the subprocess (b) is treated on equal footing with (a).

(iii) We consider the effect of changing the renormalization and factorization scales; in [5] this effect has also been left out.

(iv) In [5] the fragmentation of the final partons to hadrons is treated via Monte-Carlo methods, which somewhat obscure the procedure. Here we use the conventional QCD approach with recent fragmentation functions [6].

(v) Very recently Ref. [7] presented an experimental result on (1.1); this is discussed and compared to our predictions.

(vi) We show that a proper combination of cross sections for certain choices of $H_1$ and $H_2$ will make a more clean probe. The combination, however, involves four cross sections, and the experiment will be more difficult.

Furthermore, apart from the cross section calculated in [5] and in relation with [7] ($\Delta d\sigma/d\phi_1 dx$, for the definition of $\phi_1$ and $x$ see Sect. II), we present also results for the transverse momentum distribution $\Delta d\sigma/d\phi_1 dx_T$. 
With the COMPASS experiment in mind (polarized muon-proton scattering), we take into account that the (initial) photons are in general quasi-real ($\gamma^*$).

Sect. II presents our general formalism for the cross section $\Delta d\sigma/d\phi_1 dx$ and Sect. III for $\Delta d\sigma/d\phi_1 dx_T$. Sect. IV presents results for $\Delta d\sigma/d\phi_1 dx$ and the corresponding asymmetries. Sect. V presents results for $\Delta d\sigma/d\phi_1 dx_T$. Sect. VI presents the above mentioned combination of cross sections as well as our results. Finally, Sect. VII presents our concluding remarks.

II. GENERAL FORMALISM FOR $\Delta d\sigma/d\phi_1 dx$

The reaction (1.1) has, to some extent, been studied in Ref. [8], and here we avoid repetition as much as possible. Consider the contribution of the subprocess

$$\bar{d} (p_1) + \bar{b} (p_2) \rightarrow c_1 (p_3) + c_2; \quad (2.1)$$

where the quantities in parentheses denote 4-momenta, and let

$$s = (p_1 + p_2)^2, \quad t = (p_3 - p_1)^2, \quad u = (p_3 - p_2)^2 \quad (2.2)$$

($s + t + u = 0$). Neglecting intrinsic transverse momenta, the hadrons $H_i$, ($i = 1, 2$) are produced in opposite hemispheres with transverse momenta $k_{iT}$ and c.m. pseudorapidities $\eta_i$ with respect to the photon. Denoting by $\sqrt{S}$ the total c.m. energy and by $\phi_1$ the azimuthal angle of $H_1$ and introducing

$$x_{iT} = 2k_{iT}/\sqrt{S},$$

it follows that the cross section for (1.1) is formally given by [8,9]

$$\Delta d\sigma_{d\phi_1 dx_T dx_{2T} d\eta_1 d\eta_2} = \frac{S}{4} \int dx_b \Delta F_{b/\gamma} (x_b) \Delta \sigma (S, x_b, x_{1T}, x_{2T}, \eta_1, \eta_2), \quad (2.3)$$

where $\Delta F_{b/\gamma}$ the polarized momentum distribution of parton $b$ inside the photon and

$$\Delta \sigma = \frac{1}{\pi} \Delta F_{a/p} (x_a) \frac{d\sigma}{dt} D_{H_1/c_1} (z_1) D_{H_2/c_2} (z_2); \quad (2.4)$$

the limits of integration in (2.3) are specified later. In (2.4), $\Delta d\sigma/dt$ is the cross section for the subprocess (2.1), $D_{H_i/c_i} (z_i)$ is the fragmentation function for $c_i \rightarrow H_i$ and

$$x_a = x_b \exp (-\eta_1 - \eta_2) \quad (2.5)$$

$$z_i = x_{iT} (\exp (\eta_1) + \exp (\eta_2))/2x_b \quad (2.6)$$
Eq. (2.4) expresses the contribution to the physical cross section from both direct and resolved $\gamma$, the former corresponding to $\Delta F_{b/\gamma}(x) = \delta(1-x)$. The cross sections for the subprocesses (1.2) are:

$$\Delta \frac{d\sigma_{g\gamma}}{dt} = -\frac{\pi a a_s e_q^2 s^2}{u t} \ , \quad \Delta \frac{d\sigma_{q\gamma}}{dt} = 8\pi a a_s e_q^2 s^2 - t^2$$

(2.7)

The corresponding cross sections for the resolved $\gamma$ contributions are taken from [10] with $t \leftrightarrow u$ (see also [8]).

Here we define the variable

$$x = \exp (-\eta_1 - \eta_2)$$

(2.8)

and determine first $\Delta d\sigma/d\phi_1 dx$. Also introduce

$$h = \exp (\eta_2)$$

(2.9)

Taking, as in [8], the $x$-$z$ plane to be defined by $\vec{p}_1^2$ and $\vec{k}_1$ (i.e. $\phi_1 = 0$) we may write still in a formal way

$$\Delta \frac{d\sigma}{d\phi_1 dx} (S, \phi_1 = 0, x) = \frac{S}{4x} \int dx_1 \int dx_2 \int dx_b \Delta F_{b/\gamma}(x_b) \int \frac{dh}{h} \Delta \sigma (S, x, x, x_b, x_1, x_2, h)$$

(2.10)

The physical meaning of the variable $x$ is clear from Eq. (2.5): it is $x = x_a$ for $x_b = 1$ (direct $\gamma$).

The limits on $h$ are specified by the condition $z_i \leq 1$, which, in view of (2.6), (2.8) and (2.9), implies:

$$h + x^{-1}h^{-1} \leq \lambda x_b,$$

where $\lambda \equiv \min(2/x_{1T}, 2/x_{2T})$. We find:

$$h_- \leq h \leq h_+,$$

where

$$h_\pm \equiv \frac{1}{2} \left[ \lambda x_b \pm \left( \lambda^2 x_b^2 - \frac{4}{x} \right)^{1/2} \right], \quad h_- h_+ = 1/x$$

(2.11)

Clearly, we must have

$$\lambda x_b \geq 2/\sqrt{x}$$

(2.12)

Denoting the lower limit of $x_{1T}$, $x_{2T}$ integrations by $x_T^{(0)} = 2k_T^{(0)}/\sqrt{S}$, $k_T^{(0)}$ to be fixed by experiment) we write:

$$\int_{x_{1T}}^{x_{1T}} dx_{2T} = \int_{x_T^{(0)}}^{x_{1T}} dx_{2T} + \int_{x_{2T, max}}^{x_{2T, max}} dx_{2T}$$

(2.13)
So we find:

\[
\Delta \frac{d\sigma}{d\phi_1 dx} (S, 0, x) = \frac{S}{4} \int_{x_T^{(0)}}^{x_T} \int_{x_T^{(0)}}^{x_T} \int_{h_{-}}^{h_{+}} \frac{dh}{h} \Delta F_{b/\gamma} (x_b) \Delta \sigma (S, x, x_b, x_{1T}, x_{2T}, h) \]

(2.14)

where

\[
I_1 = \int_{x_T^{(0)}}^{x_b \sqrt{\frac{x_b}{x_T}}} \int_{x_T^{(0)}}^{x_{1T}} \int_{x_T^{(0)}}^{x_{2T}} \int_{h_{-}}^{h_{+}} \frac{dh}{h} \Delta F_{b/\gamma} (x_b) \Delta \sigma (S, x, x_b, x_{1T}, x_{2T}, h)
\]

(2.15)

with

\[
h_{\pm} = x_{1T}^{-1} x_b \pm \left( x_{1T}^{-2} x_b^2 - 1/x \right)^{1/2},
\]

(2.16)

and

\[
I_2 = \int_{x_T^{(0)}}^{x_b \sqrt{\frac{x_b}{x_T}}} \int_{x_T^{(0)}}^{x_{2T}} \int_{x_T^{(0)}}^{x_{1T}} \int_{h_{-}}^{h_{+}} \frac{dh}{h} \Delta F_{b/\gamma} (x_b) \Delta \sigma (S, x, x_b, x_{1T}, x_{2T}, h)
\]

(2.17)

with

\[
h_{\pm} = x_{2T}^{-1} x_b \pm \left( x_{2T}^{-2} x_b^2 - 1/x \right)^{1/2}
\]

(2.18)

Given \(x_T^{(0)}\), the condition (2.12) determines the minimum value of \(x\) allowable. Clearly

\[x \geq \left( \frac{x_T^{(0)}}{x_b} \right)^2\]

(2.19)

and since \(x_b \leq 1\):

\[x_{\text{min}} = \left( \frac{x_T^{(0)}}{x_b} \right)^2\]

(2.20)

### III. GENERAL FORMALISM FOR \(\Delta d\sigma/d\phi_1 dx_T\)

We start again from Eqs. (2.3), (2.4) and change variables:

\[x_{2T} \to x_T = \frac{1}{2} (x_{1T} + x_{2T}), \quad \eta_i \to h_i = e^{\eta_i}, \quad i = 1, 2\]

(3.1)

so

\[
\frac{\Delta d\sigma}{d\phi_1 dx_T dx_{1T} dh_1 dh_2} = \frac{S}{2h_1 h_2} \int dx_b \Delta F_{b/\gamma} (x_b) \Delta \sigma (S, x_T, x_b, x_{1T}, h_1, h_2)
\]

(3.2)

and \(\Delta \sigma\) given by (2.4). Now

\[x_a = x_b / h_1 h_2\]

(3.3)
and
\[ z_1 = \frac{x_{1T}}{2x_b} (h_1 + h_2), \quad z_2 = \frac{2x_T - x_{1T}}{2x_b} (h_1 + h_2) \]  
(3.4)

The conditions \( z_i \leq 1 \) and \( x_a \leq 1 \) imply
\[ h_2 + x_b h_2^{-1} \leq \lambda x_b \]  
(3.5)

where \( \lambda \equiv \min \left( \frac{2}{x_{1T}}, \frac{2}{2x_T - x_{1T}} \right) \). As in Sect. II:
\[ h_- \leq h_2 \leq h_+ \]  
(3.6)

where now
\[ h_\pm \equiv \frac{1}{2} \left[ \lambda x_b \pm \left( \lambda^2 x_b^2 - 4x_b \right)^{1/2} \right] \, , \quad h_- h_+ = x_b \]  
(3.7)

Here we have the condition
\[ x_b \geq 4/\lambda^2 \]  
(3.8)

Clearly, for \( x_{1T} < x_T \): \( \lambda = 2/(2x_T - x_{1T}) \), whereas for \( x_{1T} > x_T \): \( \lambda = 2/x_{1T} \). So in the present case we write
\[
\int_{x_{1T,min}}^{x_{1T,max}} dx_{1T} = \int_{x_{1T,min}}^{x_T} dx_{1T} + \int_{x_T}^{x_{1T,max}} dx_{1T}
\]  
(3.9)

The final result is
\[
\Delta \frac{d\sigma}{d\phi_1 dx_T} (S, 0, x_T) = \frac{S}{2} (J_1 + J_2)
\]  
(3.10)

where
\[
J_1 = \int_{x_{1T,min}}^{x_T} dx_{1T} \int_{x_{2T}}^{1} dx_b \int_{h_-}^{h_+} \frac{dh_2}{h_2} \int_{h_2 / h}^{h_{1,max}} \frac{dh_1}{h_1} \Delta F_{b/\gamma} (x_b) \Delta \sigma
\]  
(3.11)

with \( x_{2T} \equiv 2x_T - x_{1T} \) and
\[
x_{1T,min} = \max \left( x_T^{(0)}, 2x_T - 1 \right), \quad h_{1,max} = 2x_b / x_{2T} - h_2
\]  
(3.12)

\[ h_\pm = \frac{x_b}{x_{2T}} \pm \left( \frac{x_b^2}{x_{2T}^2} - x_b \right)^{1/2} \]  
(3.13)

and
\[
J_2 = \int_{x_T}^{x_{1T,max}} dx_{1T} \int_{x_{2T}}^{1} dx_b \int_{h_-}^{h_+} \frac{dh_2}{h_2} \int_{h_2 / h}^{h_{1,max}} \frac{dh_1}{h_1} \Delta F_{b/\gamma} (x_b) \Delta \sigma
\]  
(3.14)
with

\[ x_{1T,max} = \min \left( 2x_T - x_T^{(0)}, 1 \right), \quad h_{1,\max} = 2x_b/x_{1T} - h_2 \]  

(3.15)

\[ h_\pm = \frac{x_b}{x_{1T}} \pm \left( \frac{x_b^2}{x_{1T}^2} - x_b \right)^{1/2} \]  

(3.16)

In determining \( x_{1T,\min} \) we took into account that \( x_{1T} = 2x_T - x_{2T} \geq 2x_T - 1 \), and in determining \( x_{1T,\max} \) that \( x_{1T} = 2x_T - x_{2T} \leq 2x_T - x_T^{(0)} \).

IV. RESULTS FOR \( \Delta d\sigma/d\phi_1 dx \) AND THE CORRESPONDING ASYMMETRIES

We present results for the three sets A,B,C of LO polarized distributions of [11], which can be roughly characterized as follows in terms of \( \Delta g(x) \equiv \Delta F_{g/p}(x, Q_0) \):

Set A: \( \Delta g(x) > 0 \) and relatively large

Set B: \( \Delta g(x) > 0 \) and small

Set C: \( \Delta g(x) \) changing sign; \( \Delta g(x) < 0 \) for \( x > 0.1 \).

The fragmentation functions \( D_{H_i/c_i} \) are taken from [6] (LO sets). In \( a_s(Q) \) we use \( \Lambda = 0.2 \) GeV and 4 flavors. The renormalization and factorization scales are taken equal and with a central value \( Q = Q_c \equiv k_{1T} + k_{2T} \). We first present results at a typical COMPASS energy \( \sqrt{S_{\gamma p}} \equiv \sqrt{S} = 12 \) GeV and for \( k_T^{(0)} = 1.4 \) GeV [5].

Regarding the resolved \( \gamma \) contributions, we have used the maximal and minimal saturation sets of the polarized photon distribution functions of [12,13]. We have also carried calculations with the distribution functions of [14], belonging to the class of the so-called asymptotic solutions, and we simply report the results.

To account for the fact that the photons are quasi-real we multiply (2.4) by the Weiszaecker-Williams factor:

\[ \Delta f(y) = \frac{\alpha}{2\pi} \Delta P_{\gamma l}(y) \ln \frac{Q_{\max}^2 (1 - y)}{m_\mu^2 y^2} \]  

(4.1)

where \( \Delta P_{\gamma l}(y) = \left[ 1 - (1 - y)^2 \right] / y \), \( m_\mu = \) muon mass, and we take a typical value \( Q_{\max}^2 = 4 \) GeV\(^2 \); for incident lepton c.m. energy \( \sqrt{S_{\gamma p}} \) (corresponding to \( E_l = 200 \) GeV [5]): \( y = S/S_{\gamma p} \).

Fig. 1a presents \( \Delta d\sigma/d\phi_1 dx \) for direct and resolved \( \gamma^* \) contributions with \( H_i = \pi^+ \) or \( \pi^- \) (fragmentation functions (A.4)-(A.8) of [6]). The presented resolved contributions correspond to the maximal saturation set of [12,13]; those of the minimal are somewhat smaller. So, in general, the resolved contributions are much smaller than the direct. However, in particular
for the set A of [11] and in the range $0.15 \leq x \leq 0.2$, where the direct contributions change sign, the resolved are not insignificant. The asymptotic solution of [14] gives even larger resolved contributions.

Notice that the differential cross sections for the direct $\gamma$ contributions change sign at some $x \leq 0.2$; this is due to the two competing subprocesses of Eq. (1.2). At the lower $x$, (a) dominates, whereas at higher $x$, (b) takes over. Hence the place to obtain information about $\Delta G$ is at the lower $x$, as first was pointed out in [5].

Fig. 1b presents the asymmetries

$$A = \frac{\Delta d\sigma/d\phi_1 dx}{d\sigma/d\phi_1 dx}$$

(4.2)

for the sum direct+resolved and again $H_i = \pi^+$ or $\pi^-$. For the unpolarized $d\sigma/d\phi_1 dx$ we use the CTEQ distributions [15] and the photon distribution functions of [16], LO sets. Here, to account for quasi-real photons, in (4.1) we replace $\Delta P_{\gamma l}$ by $P_{\gamma l}(y) = \left[1 + (1 - y)^2\right]/y$; hence the asymmetry is reduced. For each of the sets A, B and C the strong line corresponds to the central value $Q_c = k_{1T} + k_{2T}$. For the sets A and C, Fig. 1b presents also the effect of changing the scales in the range $Q_c/2 \leq Q \leq 2Q_c$.

Fig. 1b also presents an estimate of experimental errors using the expression

$$\delta A_{\gamma^* p} = \frac{1}{P_B P_T \sqrt{L \sigma_{\gamma^* p} \epsilon}}$$

(4.3)

We take beam polarization $P_B = 80\%$, target polarization $P_T = 25\%$, pion-kaon detection efficiency $\epsilon = 1$ and integrated luminosity $L = 2 f b^{-1}$ [5]; in (4.3) $\sigma_{\gamma^* p}$ is the unpolarized cross section for quasi-real photon-proton scattering integrated over a bin $\Delta x = 0.17$.

On the basis of Fig. 1b we conclude the following on the experiment: First, the sets A and B cannot be distinguished. Second, the sets A and C can barely be distinguished in the small range $0.15 \leq x \leq 0.2$. We note that at smaller $x$ the cross sections $\gamma^* p$ become much smaller and $\delta A_{\gamma^* p}$ much larger.

Now we turn to kaon production, and Fig. 2a presents $\Delta d\sigma/d\phi_1 dx$ for $H_i = K^+$ or $K^-$ (fragmentation functions (A.19)-(A.23) of [6]) and $Q = Q_c$. The presented resolved contributions are as in Fig. 1a; now for both sets A and B of [11], at $x \geq 0.25$, they are important. Fig. 2b presents the corresponding asymmetries together with the effect of changing the scales and an estimate of the experimental errors, as for Fig. 1b. Now the latter are significantly larger (smaller cross sections), making very difficult the distinction even between sets A and C.

In [5], apart from kaons, the production of charged hadron pairs is considered. The unpolarized cross sections for the production of charged hadrons are, of course, greater than those of charged pions only. Thus the estimated errors will be somewhat smaller.

As it has been stated, very recently an experimental result was presented for (1.1). Its energy is low, $\sqrt{s_l} = 7.18 \text{ GeV}$, and therefore $k_{iT}$ limited; also, the way one reaches the final
result is somewhat unclear. Nevertheless, in view of its importance and of the fact that it is the first experimental result, it is perhaps of interest to extend the calculation of Sects II and IV down to $\sqrt{S} = 7.18$ GeV. This experiment selects events containing at least one positively charged hadron and at least one negatively charged hadron. Hence the fragmentation functions should be separated to those for $\pi^+$ ($K^+$) and for $\pi^-$ ($K^-$). For the separation we use the subsequently presented expressions (6.3)-(6.6). The fact that the experiment was carried at a low energy necessitates the choice $k_{T(0)} = 1.1$ GeV. Taking into account also the virtual photon depolarization factor $D = 0.93$ [7], the predicted asymmetries together with the experimental result are shown in Fig 3; clearly, Set A (or B) is favored.

It is interesting also to note that the effect on the asymmetry of changing the scales is small.

The values $\sqrt{S} = 12$ GeV and $k_{T(0)} = 1.4$ GeV imply certain limits on the rapidities $\eta_i$ and the invariant mass of the hadron pairs $m(H_1H_2)$, which amount to acceptance cuts. Taking the variable $x$ in the range $0.055 \leq x \leq 0.8$, the integration limits for the variable $h$ in Eqs (2.15) and (2.17) combined with Eqs. (2.9) and (2.8) imply the following limits on the rapidities: $-1.9 \leq \eta_1 \leq 2.0$ and $-1.7 \leq \eta_2 \leq 2.1$. These limits imply $m(H_1H_2) \geq 2.81$ GeV, which is in accord with [5].

It should be remarked that, instead of $Q_c = k_{1T} + k_{2T}$, the choice $Q_c = (k_{1T} + k_{2T})/2$ is also reasonable. Then varying the scales in the range $Q_c/2 \leq Q \leq 2Q_c$, near the lower limit, with $k_{T(0)} = 1.4$ GeV, we enter a region where perturbative QCD is uncontrollable. One cannot take $k_{T(0)}$ much larger because $\Delta d\sigma/d\phi_1 dx$ becomes too small.

V. RESULTS FOR $\Delta d\sigma/d\phi_1 dx_T$

The transverse momentum distributions $\Delta d\sigma/d\phi_1 dx_T$ and $d\sigma/d\phi_1 dx_T$ are calculated for the same distributions and fragmentation functions as Sect. IV, as well as for $\sqrt{S} = 12$ GeV and $k_{T(0)} = 1.4$ GeV. We present results only for $Q = Q_c \equiv k_{1T} + k_{2T}$ as functions of $x_T = (k_{1T} + k_{2T})/\sqrt{S}$.

The indicated errors have been estimated on the basis of Eq. (4.3) with the unpolarized cross section integrated over a bin in $x_T$ corresponding to $\Delta p_T = 1$ GeV.

Fig. 4a presents asymmetries for $H_i = \pi^+$ or $\pi^-$. Clearly, even without accounting for the variation of the scales, sets A and C, as well, are hard to distinguish.

Fig. 4b presents asymmetries for $H_i = K^+$ or $K^-$. The conclusions are the same as for Fig. 4a.

Again, as in Sect. IV, taking the variable $x_T$ in the range $0.25 \leq x_T \leq 0.8$ we obtain the following limits: $-1.9 \leq \eta_1 \leq 2.1$, $-2.1 \leq \eta_2 \leq 2.1$ and $m(H_1H_2) \geq 2.79$ GeV.
VI. THE COMBINATIONS OF CROSS SECTIONS

Denote, for simplicity, \( \sigma (H_1 H_2) \) either of the cross sections \( \Delta d\sigma/d\phi_1 dx \) and \( \Delta d\sigma/d\phi_1 dx_T \) for \( \gamma^+ + \gamma^- \rightarrow H_1 + H_2 + X \). As it is discussed in Refs. [17] and [8], neglecting the resolved \( \gamma \) contributions, the combinations

\[
\Delta (\pi) = \sigma (\pi^+ \pi^-) + \sigma (\pi^- \pi^+) - \sigma (\pi^+ \pi^+) - \sigma (\pi^- \pi^-)
\]  
\[
\Delta (K) = \sigma (K^+ K^-) + \sigma (K^- K^+) - \sigma (K^+ K^+) - \sigma (K^- K^-)
\]  

isolate the contribution of the subprocess \( \gamma^+ \gamma^- \rightarrow q \bar{q} \).

When the resolved \( \gamma \) contributions, calculated via the polarized distribution functions of [12,13] or [14] are taken into account, the contribution of \( \gamma^+ \gamma^- \rightarrow q \bar{q} \) is not completely eliminated, but we find that for \( x \leq 0.4 \) it is smaller by \( \sim 2 \) orders of magnitude than the contribution of \( \gamma^+ \gamma^- \rightarrow q \bar{q} \). Hence the difference in \( \Delta g \) between the sets A, B and C is displayed much better. Below we present results for the corresponding asymmetries and for \( Q = Q_c \equiv k_{1T} + k_{2T} \).

The calculation of (6.1) requires the separation of the fragmentation functions for \( \pi^+ \) and \( \pi^- \). To this purpose, as in [8], we use:

\[
D_{\pi^+/u} (z) / D_{\pi^-/u} (z) = D_{\pi^-/d} (z) / D_{\pi^+/d} (z) = \frac{1+z}{1-z} \]  
\[
D_{\pi^+/g} (z) / D_{\pi^-/g} (z) = D_{\pi^+/s} (z) / D_{\pi^-/s} (z) = 1
\]

For the calculation of (6.2) we use [8]:

\[
D_{K^+/u} (z) / D_{K^-/u} (z) = D_{K^+ /\pi} (z) / D_{K^- /\pi} (z) = \frac{1+z}{1-z}
\]

\[
D_{K^+/g} (z) / D_{K^-/g} (z) = D_{K^+/d} (z) / D_{K^-/d} (z) = 1
\]

The effect of changing the scales is very similar to that of Figs 1b and 2b.

The indicated errors have been estimated as follows: First, for each of the cross sections \( \sigma (H_1 H_2) \) we have determined an error (via Eq. (4.3)) and then we have taken the square root of the sum of the squares of these errors (assuming independent measurements of \( \sigma (H_1 H_2) \)).

Fig. 5 displays asymmetries corresponding to cross sections \( \Delta d\sigma/d\phi_1 dx \); Fig. 5a refers to \( \Delta (\pi) \) and 5b to \( \Delta (K) \). Now the differences between the sets A, B and in particular C are larger and over a wider range of \( x \) than in Figs. 1b and 2b. The errors are larger, but \( \Delta (\pi) \)
appears to be useful in distinguishing between sets A and C. As for $\Delta (K)$, the errors are too large to be of any use.

Fig. 6 displays asymmetries corresponding to cross sections $\Delta d\sigma/d\phi_1 dx_T$; Fig. 6a refers to $\Delta (\pi)$ and 6b to $\Delta (K)$. Again, the differences between sets A, B and in particular C are larger. Fig. 6a seems to show that $\Delta (\pi)$ does distinguish between sets A and C at $0.25 \leq x \leq 0.3$. Again, for $\Delta (K)$ the errors are too large.

Of course, as in Sects IV and V, we present predictions for $\pi$ and $K$ separately. In an experiment detecting $\pi + K$ the errors will be somewhat smaller.

**VII. CONCLUDING REMARKS**

On the basis of Fig 1b we have concluded that the sets A and C of polarized parton disrtibutions can barely be distinguished and only in the small range $0.15 \leq x \leq 0.2$. The sets A and B cannot be distinguished.

Nevertheless, as we stated, in an experiment detecting all charged particles, the errors will be smaller. Furthermore, one way to increase the asymmetry (4.2) is by increasing $\sqrt S = \sqrt {S_{\gamma p}}$; this will somewhat reduce the implications of producing quasi-real instead of real photons. Alternatively, one may decrease the incident lepton c.m. energy $\sqrt {S_{lp}}$. On the other hand, producing quasi-real photons with $\sqrt S$ very near $\sqrt {S_{lp}}$ might make the experiment more difficult [8].

Polarized real photons at energy comparable to that of the present paper $\sqrt S \approx 10\, GeV$ are available at SLAC.

A very recent experiment at $\sqrt S = 7.18\, GeV$ [7] favors set A or B (Fig. 3). However, the fact that this energy is low and the way the final result is obtained makes necessary the repetition of the experiment at a higher energy [1] as well as experiments at even higher energies with different reactions involving polarized initial particles [2].

A somewhat better probe of $\Delta g$ appears at first sight to be the combination $\Delta (\pi)$ of cross sections corresponding to $\Delta d\sigma/d\phi_1 dx$ (Fig. 5a) and even better the combination $\Delta (\pi)$ corresponding to $\Delta d\sigma/d\phi_1 dx_T$ (Fig. 6a). However, in our estimate of errors, only statistical ones are taken into account. The systematic errors in an experiment measuring four cross sections may be significant; this holds even more if experiments at different places are involved.

In this work (and in [5]) the effect of next-to-leading order corrections (NLO) has not been considered. A number of other cases suggests that their effect on the asymmetries will be less important than that on the cross sections; a partial understanding can be found in Ref. [18]. With NLO, the effect on the cross sections of changing the scales is, in general, reduced. Whether (and how much) this affect will be reduced on the asymmetries is unclear. Unclear also is to what extend NLO affect the combinations $\Delta (\pi)$ and $\Delta (K)$, which at LO isolate the subprocess $g\gamma \rightarrow q\bar{q}$. Anyway, the interest in reaction (1.1) as a possible probe of $\Delta g$ makes imperative the determination of NLO.
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FIGURE CAPTIONS

Fig. 1. Results when each of the final hadrons $H_i, i = 1, 2,$ is $\pi^+$ or $\pi^-$. (a) Differential cross sections $\Delta d\sigma/d\phi_1 dx$ for direct and resolved $\gamma$ contributions for $Q = Q_c = k_{1T} + k_{2T}$. A, B and C refer to the parton distributions of Ref. [9]. (b) Asymmetries $A = (\Delta d\sigma/d\phi_1 dx) / (d\sigma/d\phi_1 dx)$ and their variation with changing the scales in the range $Q_c/2 \leq Q \leq 2Q_c$. Strong lines correspond to the scale $Q = Q_c$. The bands with forward and backward slanted hatches show this variation for sets A and C correspondingly. For set B the variation is not shown.

Fig. 2. The same as in Fig. 1 when each of the final hadrons $H_i$ is $K^+$ or $K^-$. 

Fig. 3. The predicted asymmetries at $\sqrt{S} = 7.18$ GeV together with the recent experimental result of Hermes collaboration [7].

Fig. 4. Asymmetries $A = (\Delta d\sigma/d\phi_1 dx_T) / (d\sigma/d\phi_1 dx_T)$. (a) When each of the hadrons $H_i$ is $\pi^+$ or $\pi^-$. (b) When each of $H_i$ is $K^+$ or $K^-$. 

Fig. 5. Asymmetries for the combinations (6.1) and (6.2) for cross sections $\Delta d\sigma/d\phi_1 dx$. (a) For the combination (6.1) of pions (b) For the combination (6.2) of kaons

Fig. 6. The same as in Fig. 4, but referring to the cross sections $\Delta d\sigma/d\phi_1 dx_T$
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