INTERMEDIATE REGIME BETWEEN THE FERMI GLASS AND THE
MOTT INSULATOR IN ONE DIMENSION

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We consider the ground state reorganization driven by an increasing nearest neighbor repulsion \( U \) for spinless fermions in a strongly disordered ring. When \( U \to 0 \), the electrons form a glass with Anderson localized states. At half filling, a regular array of charges (Mott insulator) is pinned by the random substrate when \( U \to \infty \). Between those two insulating limits, we show that there is an intermediate regime where the electron glass becomes more liquid before crystallizing. The liquid-like behavior of the density-density correlation function is accompanied by an enhancement of the persistent current.

1 Introduction

The interplay between the quantum interferences due to a random substrate and the correlations induced by charge repulsion is a central problem of mesoscopic physics. Its understanding seems to be a necessary step towards explaining the recently discovered metallic phase in two dimensions\textsuperscript{6} and the large value of the persistent currents observed in mesoscopic rings\textsuperscript{7}. In one dimension (1d), assuming a Luttinger liquid in the clean limit and using renormalization group arguments for including elastic scattering, one finds\textsuperscript{3} that repulsive interactions cannot delocalize spinless fermions. However, two particles in a disordered chain can be (de)localized on a length \( L_2 \) larger than the one-particle localization length \( L_1 \) when a short range interaction is present. This is valid for two particle excitations of sufficient energy while a ground state built from two one-particle states localized far away from each other remains obviously unchanged.

Interesting effects for the ground state require reasonably high filling factors. This led us to study the ground state of half filled strongly disordered rings and to find that the conclusions of Ref.\textsuperscript{3} are no longer valid when \( L_1 \) becomes of the order of the separation between the carriers. In this case, charge repulsion gives rise to successive reorganizations of the ground state, accompanied by a substantial delocalization of the particles and a large enhancement of the persistent current\textsuperscript{5}. The interactions \( U_c \) for which the charge density is reorganized fluctuate from sample to sample. Thus, the delocalization effect becomes essentially negligible, though visible after ensemble averaging. But it becomes very striking if one studies the orbital response of individual mesoscopic samples as a function of \( U \).

At a finite density and when \( U \to 0 \), the electrons form a glass with Anderson localized states filled up to the Fermi energy. This Fermi glass is characterized by an inhomogeneous charge configuration imposed by the random substrate. When \( U \to \infty \), at half filling for a nearest neighbor repulsion \( U \), the electrons form a Mott insulator (or a pinned Wigner crystal for long range Coulomb repulsion and lower filling factors). It is characterized by large crystalline domains, possibly separated by some remaining dislocations which vanish as \( U \to \infty \). The crossover between those two different insulators yields successive spatial reorganizations of the ground state\textsuperscript{5} which correspond to weakly avoided crossings between the ground state and the first excitation. At those level crossings, the system becomes more sensitive to any external perturbation as a twist in the boundary conditions or a flux through a ring, and this enhances the persistent currents.
2 Model and method

We consider \( N \) spinless fermions on a chain of \( L \) sites with nearest neighbor (NN) interaction

\[
H = -t \sum_{i=1}^{L} (c_i^\dagger c_{i-1} + c_{i-1}^\dagger c_i) + \sum_{i=1}^{L} V_i n_i + U \sum_{i=1}^{L} n_i n_{i-1}
\]

and twisted boundary conditions, \( c_0 = \exp(i\Phi)c_L \). The operators \( c_i \) (\( c_i^\dagger \)) destroy (create) a particle on site \( i \) and \( n_i = c_i^\dagger c_i \) is the occupation operator. The on-site random energies \( V_i \) are drawn from a box distribution of width \( W \). The strength of the disorder \( W \) and the interaction \( U \) are measured in units of the kinetic energy scale \( (t = 1) \). We study interaction between NN at half filling, where the ground state will be a periodic array of charges located on the even or odd sites of the chain when \( U \to \infty \). The numerical results are obtained using the density matrix renormalization group (DMRG) algorithm. Ground state properties in disordered 1d systems can be calculated with an accuracy comparable to exact diagonalization, but for much larger systems.

3 Crossover glass-liquid-crystal for increasing repulsions

The reorganization of the ground state induced by the NN repulsion is shown in Fig. 1, where the density \( \rho \) (expectation value of \( n_i \)) is plotted as a function of \( U \) and site index \( i \). To favor the inhomogeneous configuration, the disorder is taken large \( (W = 9) \) and \( L_1 \approx 100/W^2 \) is order of the mean spacing \( k_F^{-1} = 2 \) between the charges. For \( U \approx 0 \), one can see a strongly inhomogeneous and sample dependent density, while for large \( U \) a periodic array of charges sets in. These two limits are separated by a sample dependent crossover regime.

\[ C(r) = \frac{1}{N} \sum_{i=1}^{L} \rho_i \rho_{i+r} \]
for values $0 \leq r \leq L/2$. The parameter $\gamma = \max_r \{C(r)\} - \min_r \{C(r)\}$ is used to distinguish between the electron liquid with constant density ($\gamma = 0$) and the regular crystalline array of charges ($\gamma = 1$). If one includes the translation $r = 0$ in the definition of $\gamma$, one gets $\gamma \neq 0$ for the electron glass. So $\gamma$ measures charge crystallization from an electron liquid as well as the melting of the glassy state towards a more liquid ground state.

Fig. 1 (right) shows the dependence of $\gamma$ on the interaction strength $U$ for four individual samples. For certain impurity configurations, like in sample (a), the periodic array is obtained at a weak repulsive interaction, while one needs a strong interaction for other samples like (b) and (d). Typically, $\gamma$ assumes a minimum for a small repulsive interaction of the order of the kinetic energy scale $t$. This means that the charge distribution is closest to a liquid there and suggests a maximum of the mobility of the charge carriers. This is an indication for a delocalization of the ground state by repulsive interactions. In addition, most of the samples show small steps in the interval $0 \leq U \leq 2t$, caused by instabilities between different configurations of similar structure. The formation of the regular array of charges imposed by strong repulsive interactions occurs only at an interaction strength $U \approx U_W$ with $U_W \propto W$. It can be inferred from the step-like increase of $\gamma$ in individual samples that the regular array is established abruptly at an interaction value which depends on the disorder realization of the given sample. Therefore, the jumps of $\gamma$ are smeared out when the average over the ensemble is calculated.

Nevertheless, it can be clearly observed that the charge density does not cross over smoothly from the disordered Fermi glass to the regular Mott insulator. There is a regime at weak repulsive interaction $U \approx U_F \approx t$, where the interaction delocalizes the particles and counteracts the disorder before the tendency towards the Mott insulator starts to dominate at stronger interaction strength. This is similar to the behavior observed in 2d systems, where an intermediate phase has been found at moderate Coulomb repulsion and an abrupt transition to a regular Wigner crystal when the interaction strength increases.

### 4 Enhanced persistent currents for intermediate repulsions

To measure the delocalization effect associated to the change of charge configuration, we study the phase sensitivity of the ground state. The energy difference between periodic ($\Phi = 0$) and anti-periodic ($\Phi = \pi$) boundary conditions, $\Delta E = (-)^N (E(0) - E(\pi))$ conveys similar information, in the localized regime, as other measures of the response of the ground state to an infinitesimal flux threading the ring: the Kohn curvature (charge stiffness) $\propto E'(\Phi = 0)$ and the persistent current $I \propto -E'(\Phi = 0)$. For strictly 1d systems, the sign of $E(0) - E(\pi)$ simply depends on the parity of $N$, and the factor $(-)^N$ makes $\Delta E$ positive. The phase sensitivity $D(U) = (L/2)\Delta E$ is shown in Fig. 2 for four samples at half filling with $W = 9$. Both for $U \approx 0$ and $U \gg 1$, $D(U)$ is very small, but sharp peaks appear at sample dependent values $U_c$, where $D(U_c)$ in certain samples can be 4 orders of magnitude larger than for free fermions. Remarkably, the curves for each sample do not present any singularity at $U = 0$ which could have allowed to locate the free fermion case. Peaks can be seen at different sample dependent values of $U$ (positive or negative). A comparison of the peak values with the step positions in Fig. 1 shows that the sharp peaks are in all cases accompanied by a change of the ground state structure.

For small repulsive interactions, the system is an Anderson insulator delocalized by $U$, and $D(U)$ increases as a function of $U$, consistent with the decrease of $\gamma$ seen in Fig. 1. This decrease means that one has first a progressive melting of the electron glass when one turns on the interaction. At the maximum of $D(U)$ occurring at the largest value of the interaction, the regular array of charges is established, and thereafter it becomes more and more rigid (pinned by the random lattice); thus $D(U)$ decreases as a function of $U$.

The thresholds $U_c$ are strongly sample dependent giving rise to a very wide distribution of $D(U)$: the ensemble average at a given $U$ mixes very different behaviors and provides very
incomplete information. As shown in Fig. 2, $\langle \log D(U) \rangle$ decreases for repulsive interactions, except for a small interval around $U_{F} \approx t$ (inset) where a local maximum is obtained. This is reflected in the behavior of the ensemble average $\langle \gamma \rangle$ (Fig. 1), which increases smoothly at large interaction except for a maximum at $U_{F}$.

This delocalization effect of the ground state for $U \approx U_{F}$ only occurs at strong disorder. For weak disorder ($W = 2, L_{1} \approx L$) we recover the behavior expected starting from the clean limit, using bosonization and renormalization group arguments: a repulsive interaction reinforces localization, in contrast to a (not too strong) attractive interaction which delocalizes. However, the conclusion that a repulsive interaction favors localization is no longer valid at strong disorder. Similar conclusions have been drawn from a study of the conductance of one- and two-dimensional systems at half filling.

5 Avoided level crossings

The sharp changes in the ground state structure and the peaks observed in the phase sensitivity are the consequences of avoided crossings between the ground state and the first excitation, when $U$ increases. While the ground state at weak interaction is well adapted to the disordered potential, another state with a different structure being better adapted to repulsive interactions, becomes the ground state at stronger interaction. In the case of large disorder, with a one-particle localization length $L_{1}$ which is of the order of the mean distance between the particles, the overlap matrix elements between the different noninteracting eigenstates due to the interaction are very small and the levels almost cross. There is only a very small interaction range where a significant mixing of two states is present. This is exactly where the peaks of the phase sensitivity appear.

This scenario is confirmed by Fig. 2 (right), where the phase sensitivity and the energy level spacing $\Delta$ between the ground state and the first excited state are shown for sample (d). A minimum of $\Delta$ appears always at the same interaction value as a peak in the phase sensitivity. A gap of increasing size opens between the ground state and the first excitation after the last avoided crossing at interactions $U > U_{W}$, when the Mott insulator is established. A study of many other samples leads to the same conclusions. It is interesting to investigate the statistics of the first excitation energy. In 2d with Coulomb repulsion, this has recently been addressed giving rise to intermediate statistics at the opening of the quantum Coulomb gap.
6 Conclusions

We summarize our main conclusions: (i) Observables for individual samples convey a clearer information than the (log)-averages over the ensemble; (ii) Spinless noninteracting fermions in 1d form an insulating Fermi glass at strong disorder, which becomes a Mott insulator when $U \approx U_W \propto W$; (iii) At an intermediate repulsion $U \approx U_F \approx t < U_W$, the charge density is closest to a liquid and the mobility of the charge carriers becomes maximum, indicating the existence of a regime where the interaction delocalizes between two insulating limits. In small 2d clusters with Coulomb interaction, a rather similar behavior applies. From a study of the longitudinal and transverse currents driven by a flux threading the system around its longitudinal axis, an intermediate phase was found, characterized by the suppression of the transverse currents and the persistence or the enhancement of the longitudinal currents. According to transport measurements, this new phase might correspond to a new kind of 2d metal. In one dimension, the study of $D(U)$ when the size $L$ increases at a given filling factor allows us to exclude the existence of a 1d metal. We can neither characterize a change in the ground state by the suppression of the transverse current as in 2d. However, from a study of the density-density correlation functions for the ground state, we have shown that also in 1d, the ordered array of charges is established at an interaction strength $U_W$ larger than the value $U_F$ where one has a maximum interaction induced delocalization. We believe that the regime $U \approx U_F$ where the system is nor a Fermi glass neither a Mott insulator in 1d can be seen as a 1d precursor of the probable metallic phase appearing in 2d.

Acknowledgments

We gratefully acknowledge financial support from the DAAD and the A.P.A.P.E. through the PROCOPE program and from the European Union through the TMR program.

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