No Maxwell Electromagnetic Wave Field Excited In Cloaked Concealment

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The GL electromagnetic (EM) modeling is used to simulate the 3D EM full wave field propagation through cloaks. The 3D GL simulation of the EM wave field excited by a point source outside of the cloaks has been done. The simulation of the EM wave field from the point source excitation inside of the cloak device is presented in this paper. By using the GL modeling simulation, we found a phenomenon that there is no Maxwell EM wave field which is excited by nonzero local sources inside of the single layer cloaked concealment. The theoretical proof of the phenomenon by GL method is proposed in this paper. The GL method is fully different from the conventional methods. The GL method has double abilities of the theoretical analysis and numerical simulations to research the physical process and cloak metamaterial properties that is exhibited in this paper.

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I. INTRODUCTION

For the point source located outside of the cloak, the 3D Global and Local field (GL) EM modeling method [1-3] has been used to simulate the 3D full EM wave field propagation through the cloaks. Our simulations and theoretical analysis in [1] verify the ideal cloak functions [4]. The EM wave field propagation outside of cloak does not penetrate into the concealment and never be disturbed by the cloaks. There are several papers to simulate the plane wave propagation through the cloak from outside of the cloak [5-7]. The plane wave is excited by plane source which can not be located inside of the cloak or concealment. The cloak simulation of the point sources and the local sources inside of the cloak is absent. In the most published papers. The GL modeling simulations of the EM wave field through cloak and excited by the nonzero local sources inside of the cloak is presented in this paper. Moreover, by GL EM method simulation of the nonzero local sources inside of the concealment, we found a phenomenon that there exists no Maxwell EM wave field excited by nonzero local sources inside of the single layer cloaked concealment. The phenomenon is proved by the GL method theoretical analysis and the EM integral equation [1-3]. In paper [9] and [10], authors studied the effect on invisibility of active devices inside the cloak region. Our statement is that there exists no Maxwell EM wave field excited by nonzero local sources inside of the single layer cloaked concealment. The detailed proof of the statement and 3D GL simulations are presented in this paper.

Our GL method is fully different from conventional methods for cloak and physical and science simulations. It has advantages over the conventional methods. The GL method consistent combines the theoretical analytical and numerical method together. In the GL modeling, there is no big matrix equation to solve and no absorption condition on artificial boundary to truncate infinite domain. The method is a significant physical scattering process. The finite inhomogeneous domain is divided into a set of small sub domains. The interaction between the global field and anomalous material polarization field in the sub domain causes a local scattering wave field. The local scattering wave field updates the global wave field by an integral equation. Once all sub domains are scattered, the wave field in the inhomogeneous anomalous materials will be obtained. Therefore, the GL method can be used to both of theoretical analysis and numerical simulation for physical and chemical phenomena and process.

The description arrangement of the paper is as follows. The introduction is described in Section 1. The EM integral equation is proposed in Section 2. The Global and Local EM field modeling is proposed in Section 3. The phenomenon that there is no the Maxwell EM wave field excited by the local sources inside of the cloak is proved in Section 4. The GL modeling simulations of the EM wave field propagation excited by the local source inside of the cloak are presented in Section 5. We conclude this paper in Section 6.

II. 3D ELECTROMAGNETIC INTEGRAL EQUATION

We have proposed the 3D EM integral equation in frequency domain in papers [1] and [2]. In this section, we propose the EM integral equation in time domain as follows:
\[
\begin{bmatrix}
E(r, t) \\
H(r, t)
\end{bmatrix} =
\begin{bmatrix}
E_0(r, t) \\
H_0(r, t)
\end{bmatrix}
+ \int \frac{G_{E,H}^{J,M}(r', r, t)}{\Omega} \ast_1 \delta[D(r')] \begin{bmatrix}
E_0(r', t) \\
H_b(r', t)
\end{bmatrix} dr',
\]
(1)

where
\[
G_{E,H}^{J,M}(r', r, t) = \begin{bmatrix}
E^J(r', r, t) & H^J(r', r, t) \\
E^M(r', r, t) & H^M(r', r, t)
\end{bmatrix},
\]
(2)

\(E(r, t)\) is the electric field, \(H(r, t)\) is the magnetic field, \(E_0(r, t)\) and \(H_0(r, t)\) is the incident electric and magnetic field in the background medium, \(E^J(r', r, t)\) is electric Green’s tensor, \(H^J(r', r, t)\) is magnetic Green’s tensor, they are excited by the point impulse current source, \(E^M(r', r, t)\) and \(H^M(r', r, t)\) are electric and magnetic Green’s tensor, respectively, they are excited by the point impulse magnetic moment source, \(\ast_1\) is convolution with respect to \(t\), \(\delta[D]\) is the electromagnetic material parameter variation matrix,
\[
\delta[D] = \begin{bmatrix}
\delta D_{11} & 0 \\
0 & \delta D_{22}
\end{bmatrix},
\]
(3)

\(\delta D_{11}\) and \(\delta D_{22}\) are a \(3 \times 3\) symmetry, inhomogeneous diagonal matrix for the isotropic material, for anisotropic material, they are an inhomogeneous diagonal or full matrix, \(I\) is a \(3 \times 3\) unit matrix, \(\sigma(r)\) is the conductivity tensor, \(\varepsilon(r)\) is the dielectric tensor, \(\mu(r)\) is susceptibility tensor which can be dispersive parameters depend on the angular frequency \(\omega\), \(\varepsilon_0\) is the permittivity, \(\mu_0\) is the permeability in the background free space, \(\Omega\) is the finite domain in which the parameter variation matrix \(\delta[D] \neq 0\), the \((\varepsilon(r) - \varepsilon_0 I)\)\(E\) is the electric polarization, and \((\mu(r) - \mu_0 I)\)\(H\) is the magnetization.

### III. 3D GL EM MODELING

We propose the GL EM modeling based on the EM integral equation (1) in the time space domain.

(3.1) The domain \(\Omega\) is divided into a set of \(N\) sub domains, \(\{\Omega_k\}\), such that \(\Omega = \bigcup_{k=1}^{N} \Omega_k\). The division can be mesh or meshless.

(3.2) When \(k = 0\), let \(E_0(r, t)\) and \(H_0(r, t)\) are the analytical global field, \(E_0^J(r', r, t)\), \(H_0^J(r', r, t)\), \(E_0^M(r', r, t)\), and \(H_0^M(r', r, t)\) are the analytical global Green’s tensor in the background medium. By induction, suppose that \(E_{k-1}(r, t), H_{k-1}(r, t), E_{k-1}^J(r', r, t), H_{k-1}^J(r', r, t), E_{k-1}^M(r', r, t),\) and \(H_{k-1}^M(r', r, t)\) are calculated in the \((k - 1)\)th step in the subdomain \(\Omega_{k-1}\).

(3.3) In \(\{\Omega_k\}\), upon substituting \(E_{k-1}(r, t), H_{k-1}(r, t), E_{k-1}^J(r', r, t), H_{k-1}^J(r', r, t), E_{k-1}^M(r', r, t),\) and \(H_{k-1}^M(r', r, t)\) into the integral equation (1), the EM Green’s tensor integral equation (1) in \(\Omega_k\) is reduced into \(6 \times 6\) matrix equations. By solving the \(6 \times 6\) matrix equations, we obtain the Green’s tensor field \(E_k^J(r', r, t), H_k^J(r', r, t), E_k^M(r', r, t),\) and \(H_k^M(r', r, t)\).

(3.4) According to the integral equation (1), the electromagnetic field \(E_k(r, t)\) and \(H_k(r, t)\) are updated by the interaction scattering field between the Green’s tensor and local polarization and magnetization in the subdomain \(\Omega_k\) as follows,
\[
\begin{bmatrix}
E_k(r, t) \\
H_k(r, t)
\end{bmatrix} =
\begin{bmatrix}
E_{k-1}(r, t) \\
H_{k-1}(r, t)
\end{bmatrix}
+ \int \frac{\{E_k^J(r', r, t) & H_k^J(r', r, t) \\
E_k^M(r', r, t) & H_k^M(r', r, t)\}}{\Omega_k} \delta[D(r')] \begin{bmatrix}
E_{k-1}(r', t) \\
H_{k-1}(r', t)
\end{bmatrix} dr',
\]
(4)

(3.5) The steps (3.2) and (3.4) form a finite iteration, \(k = 1, 2, \ldots, N\), the \(E_N(r, t)\) and \(H_N(r, t)\) are the electromagnetic field of the GL modeling method. The GL electromagnetic field modeling in the space frequency domain is short named as GLT method.

The GL EM modeling in the space frequency domain is proposed in the paper [2], we call the GL modeling in frequency domain as GLF method.

### IV. NO MAXWELL ELECTROMAGNETIC WAVE FIELD EXCITED IN CLOAKED CONCEALMENT

**Theorem:** Suppose that a 3D anisotropic inhomogeneous closed strip cloak domain separates the whole 3D space into three sub domains, one is the cloak domain \(\Omega_c\) with the cloak material; the second one is the cloak concealment domain \(\Omega_d\) with normal EM materials; other one is the free space outside of the cloak. If the Maxwell EM wave field excited by a point source outside of the concealment to be vanished in inside of the concealment, then there is no Maxwell EM wave field excited by the local sources inside of the cloak concealment.

The Maxwell EM wave field is the EM wave field which satisfies the Maxwell equation and continuous interface boundary conditions. We call the Maxwell EM wave field as the EM wave field and use inverse process to prove the theorem as follows: Suppose that there exists Maxwell EM wave field excited by the local sources inside the concealment with the normal materials, the wave field satisfies the Maxwell equation in the 3D whole space \(R^3\) which includes the anisotropic inhomogeneous cloak domain \(\Omega_c\) and concealment \(\Omega_d\), and satisfies the continuous interface conditions on the inner boundary surface \(S_1\) and outer boundary surface \(S_2\) of the cloak domain \(\Omega_c\).

Let \(R_c = R^3 - \Omega_c \cup \Omega_d\), \(R_d = R^3 - \Omega_d\), and by the EM
integral equation (1), the EM wave field satisfies

\[
\begin{bmatrix}
E(r, t) \\
H(r, t)
\end{bmatrix} = \begin{bmatrix}
E_b(r, t) \\
H_b(r, t)
\end{bmatrix} + \int_{\Omega_c \cup \Omega_d} G_{E,H}^{J,M}(r', r, t) \ast_t \delta[D] \begin{bmatrix}
E_b(r', t) \\
H_b(r', t)
\end{bmatrix} \, dr',
\]

(5)

where \(G_{E,H}^{J,M}(r', r, t)\) is the EM Green’s tensor, its components \(E^J, H^J, E^M,\) and \(H^M(r', r, t)\) are the EM Green’s function on \(\Omega_c \cup \Omega_d \cup R_c\), excited by the point impulse sources outside of the concealment, \(r \in R_d\). By the assumptions, \(G_{E,H}^{J,M}(r', r, t)\) exists on \(\Omega_c \cup \Omega_d \cup R_c\) and \(G_{E,H}^{J,M}(r', r, t) = 0\), when \(r' \in \Omega_d\). The integral equation (5) becomes to

\[
\begin{bmatrix}
E(r, t) \\
H(r, t)
\end{bmatrix} = \begin{bmatrix}
E_b(r, t) \\
H_b(r, t)
\end{bmatrix} + \int_{\Omega_c} G_{E,H}^{J,M}(r', r, t) \ast_t \delta[D] \begin{bmatrix}
E_b(r', t) \\
H_b(r', t)
\end{bmatrix} \, dr'.
\]

(6)

We consider the Maxwell equation in \(R_d\), the virtual source located \(r, r \in R_d\) and the point source located \(r_s, r_s \in \Omega_d\) and \(r_s \notin R_d\), we have

\[
-\nabla \times \nabla \times \begin{bmatrix}
E_b(r', t) \\
H_b(r', t)
\end{bmatrix} = [D] G_{E,H}^{J,M}(r', r, t) + I\delta(r', r)\delta(t),
\]

(7)

and

\[
-\nabla \times \nabla \times \begin{bmatrix}
E_b(r', r_s, t) \\
H_b(r', r_s, t)
\end{bmatrix} = [D_b] \begin{bmatrix}
E_b(r', r_s, t) \\
H_b(r', r_s, t)
\end{bmatrix},
\]

(8)

By using \([E_b(r, t), H_b(r, t)]\) to convolute (7), and \(G_{E,H}^{J,M}(r', r, t)\) to convolute (8), to subtract the second result equation from the first result equation and make their integral in \(\Omega_c \cup R_c\), and make integral by part and some manipulations, we can prove

\[
\begin{bmatrix}
E_b(r, t) \\
H_b(r, t)
\end{bmatrix} + \int_{\Omega_c} G_{E,H}^{J,M}(r', r, t) \ast_t \delta[D] \begin{bmatrix}
E_b(r', t) \\
H_b(r', t)
\end{bmatrix} \, dr' = -\int_{S_1} G_{E,H}^{J,M}(r', r, t) \times \begin{bmatrix}
E_b(r', t) \\
H_b(r', t)
\end{bmatrix} \, dS.
\]

(9)

Because \(G_{E,H}^{J,M}(r', r, t) = 0\), \(r' \in \Omega_d\), by continuous interface conditions of \(G_{E,H}^{J,M}(r', r, t)\), the term in right hand side of (9) is vanished, we have

\[
\begin{bmatrix}
E_b(r, t) \\
H_b(r, t)
\end{bmatrix} + \int_{\Omega_c} G_{E,H}^{J,M}(r', r, t) \ast_t \delta[D] \begin{bmatrix}
E(r', t) \\
H(r', t)
\end{bmatrix} \, dr' = 0.
\]

(10)

Upon substituting integral equation (10) into the integral equation (6), we have

\[
\begin{bmatrix}
E(r, t) \\
H(r, t)
\end{bmatrix} = 0.
\]

(11)

From the continuous property of the EM wave field, we obtain the following over vanish boundary condition on the boundary \(S_1\) of the concealment \(\Omega_d\), we have

\[
\begin{bmatrix}
E(r, t) \\
H(r, t)
\end{bmatrix} \bigg|_{S_1} = 0.
\]

(12)

Because the EM wave field is excited by local sources inside of the concealment domain \(\Omega_d\), it satisfies the following Maxwell equation,

\[
\begin{bmatrix}
-\nabla \times \nabla \times \\
-\nabla \times \nabla \times
\end{bmatrix} \begin{bmatrix}
E(r', r_s, t) \\
H(r', r_s, t)
\end{bmatrix} = [D] \begin{bmatrix}
E(r', r_s, t) \\
H(r', r_s, t)
\end{bmatrix} + Q(r', r_s, t).
\]

(13)

The equation (13) and its over vanish boundary condition (12) form contradiction equations on the concealment \(\Omega_d\), where \([D] = \text{diag} [\varepsilon \mu] (\partial/\partial t)\) with the normal EM material parameters \(\varepsilon, \mu, r_s \in \Omega_d\) is the source location, \(Q(r', r_s, t)\) is the nonzero local source inside of \(\Omega_d\). There is no any Maxwell EM wave field to satisfy the contradiction equations (13) in \(\Omega_d\) with over vanish boundary condition (12) on the boundary \(S_1\). Therefore, we proved that there is no Maxwell EM wave field excited by the nonzero local sources inside of the cloaked concealment.

In paper [1], we have used the simulations and theoretical analysis by the GL method to prove that the excited EM wave field by the point source outside of the sphere cloak [4] through the cloak and never propagate enter the concealment and never be disturbed by the cloak. The assumptions and conditions in the theorem in this paper are validated, therefore, we stress that there is no Maxwell EM wave field excited by the nonzero local sources inside of the center sphere concealment cloaked by sphere cloak and the arbitrary geometry closed strip cloak. In practical cloak metamaterial fabrication and experiments, the phenomenon is presented in this paper should be received attentions, because the EM wave field excited in the concealment may be an irregular EM chaos propagation in this region, it may interfere the EM devices and equipments working inside of the concealment, the high frequency irregular EM chaos radiation may hurt the health of the human.

V. SIMULATION OF THE EM WAVE FIELD THROUGH THE CLOAK BY GL METHOD

The simulation model: the 3D domain is \([-0.5m, 0.5m] \times [-0.5m, 0.5m] \times [-0.5m, 0.5m]\), the
The electric current point source is defined as
\[ \delta (r - r_s) \delta (t) \hat{e}, \] (14)
where the \( r_s \) denotes the location of the point source, the unit vector \( \hat{e} \) is the polarization direction, the time step \( dt = 0.3333 \times 10^{-10} \) second, the largest frequency \( f = 10GHz \), the shortest wave length is 0.03m. The EM cloak \( \Omega_c \) is the spherical annular with the center in the origin and internal radius \( R_1 = 0.2m \) and exterior radius \( R_2 = 0.3m \). The cloak is divided into \( 50 \times 50 \times 50 \) cells, the antenna subdomain \( \ominus \), \( \Omega_s \), is divided into 63 cells. The spherical coordinate is used in the sphere \( r \leq R_2 \), the Cartesian rectangular coordinate is used in other where to mesh the domain.

For a point source located outside of the cloak, the 3D GL EM modeling has been used to simulate the EM wave field through the sphere, ellipsoid, cylinder, and arbitrary closed strip complex geometry cloaks, the simulations and theorems of the single and multiple sphere cloaks are proposed in paper [1]. The cloak simulations in papers [5-7] are proposed for the outside plane wave through cloak. The plane source to excite the plane wave can not be located inside of the cloak and concealment. Simulation for the EM wave field excited by the point source, in particular, the source located inside of cloak, \( r_s \in \Omega_c \), or inside of concealment \( r_s \in \Omega_t \) is lack. The Figure 1 shows that the electric wave field \( E_{xx} \) is excited by the current point source in the direction \( \hat{e} = \hat{x} \) and located at \((1.1m,0,0,0)\) on the \( X \) axis where is outside of the cloak. At the time step \( 110dt \), the electric intensity wave field \( E_{xx} \) is propagating through the sphere annular cloak and around the sphere concealment, it does disperse and split into the two phases around the sphere concealment, the front phase speed exceeds the light speed; the back phase is slower than the light speed. The wave front outside of the cloak is the same as the exact \( E_{xx} \) propagation in free space, the \( E_{xx} \) wave field outside of the cloak never been disturbed by the cloak and never penetrate enter the centre sphere concealment with the antenna \( \ominus \). The simulations of the EM wave field through the cloak excited by the point source inside of the cloak are presented in this paper. The electric wave field \( E_{xx} \) excited by the current point source in the direction \( \hat{e} = \hat{x} \) and located at the point \((0.25m,0,0)\) inside of the cloak is propagating at the time step \( 15dt \) that is shown in Figure 2. The Figure 3 shows that the electric wave field \( E_{xx} \) propagates around the cloaked concealment at the time step \( 18dt \), but does not penetrate into it and its right wave front has been outside of the cloak. At the time step \( 48dt \), the electric wave field \( E_{xx} \) is propagating outside of the cloak that is shown in Figure 4, however, to compare an \( E_{xx} \) excited by the same source in free space, the electric wave field \( E_{xx} \) propagation outside of the cloak is disturbed by the cloak. Figures 2-4 show that the wave field \( E_{xx} \) propagates through the cloak and never enter the concealment. The concealment is complete concealed by the cloak from the EM wave field excited by the point sources inside of the cloak and in free space outside of the cloak. We did use the GL modeling to simulate many cases of the EM wave field excited by the point sources inside of the concealment. However, all simulations are unstable and chaos. When the EM wave field is propagating to arrive the interface boundary \( S_1 \) between the concealment and cloak, the GL simulation become unstable and chaos. The GL modeling

FIG. 1: (color online) By the point source excitation in \( x = 1.1m \) on X axis outside of the cloak, at the time step \( 110dt \), the electric wave field \( E_{xx} \) propagation does disperse and split into the two phases around the concealment with an antenna \( \ominus \), and does not penetrate into it.

FIG. 2: (color online) The electric wave field \( E_{xx} \) is excited by the point source inside of the cloak at \( x=0.25m \) on X axis, at the time step \( 15dt \), the \( E_{xx} \) propagates around the concealment and does not penetrate into it.
Simulation experiments remind us to think may there is no any Maxwell EM wave field excited by the point source inside of the concealment. This is the motivation and idea of our theorem proposed in this paper. After the rigorous proof by the GL method, we obtained the theorem that there is no Maxwell EM wave field excited by the point sources located inside of the concealment.

![Electric Wave Exx Propagation in Sphere EM Cloak Excited by Point Source in Cloak Using 3D GL Modeling in GLGEO](image)

**FIG. 3:** (color online) At the time step $18dt$, the $Exx$ propagates around the concealment and does not penetrate into it, its right wave front is outside of the cloak.

VI. CONCLUSIONS

The GL method is used to simulate the invisibility of the sphere and arbitrary cloaks and theoretically and rigorously proved theorem that there is no Maxwell EM wave field excited by the nonzero local sources inside of the concealment which is cloaked by the sphere cloak and arbitrary closed strip cloak. A least square or regularizing chaos propagation of the EM wave field excited in the concealment will be modeling and inversion by the GL metre carlo method [8] in next paper. However, any field excited in the cloaked concealment cannot be propagation outside of the concealment.

The GL EM modeling is fully different from FEM and FD and Born Approximation methods and overcome their difficulties. There is no big matrix equation to solve in GL method. Moreover, it does not need artificial boundary and absorption condition to truncate the infinite domain. The GL EM method consistent combines the analytical and numerical approaches together. The GL method has double abilities of the theoretical analysis and numerical simulations that is shown in this paper.

The 3D GL simulations of the EM wave field through the single and multiple sphere, cylinder, ellipsoid, and arbitrary geometry cloaks show that the GLT and GLF EM modeling are accurate, stable and fast. It saves more storages than the conventional methods and needs 10 to 50 minute to run the 3D EM wave field through the cloaks with 64 to 128 frequencies in the PC. The high performance GL parallel algorithm in PC cluster and super parallel computer is very fast and powerful to simulate complex and large scale physical and chemical process.

The 3D and 2D GL parallel software is made and patented by GLGEO. The GL modeling can be extended to its inversion [8] and GL EM quantum field modeling to solve quantization scattering problem of the electromagnetic field in the dispersive and loss metamaterials, cloaks and more wide anisotropic materials.

Acknowledgments

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