Fast Magnetic Reconnection: “Ideal” Tearing and the Hall Effect

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Abstract

One of the main questions in magnetic reconnection is the origin of triggering behavior with on/off properties that, once it is activated, accounts for the fast magnetic energy conversion to kinetic and thermal energies at the heart of explosive events in astrophysical and laboratory plasmas. Over the past decade, progress has been made on the initiation of fast reconnection via the plasmoid instability and what has been called “ideal” tearing, which sets in once current sheets thin to a critical inverse aspect ratio \( (a/L) \). As shown by Pucci & Velli, at \( (a/L) \sim S^{-1/3} \), the timescale for the instability to develop becomes of the order of the Alfvén time and independent of the Lundquist number (here defined in terms of current sheet length \( L \)). However, given the large values of \( S \) in natural plasmas, this transition might occur for thicknesses of the inner resistive singular layer that are comparable to the ion inertial length \( \delta_i \). When this occurs, Hall currents produce a three-dimensional quadrupole structure of the magnetic field, and the dispersive waves introduced by the Hall effect accelerate the instability. Here we present a linear study showing how the “ideal” tearing mode critical aspect ratio is modified when Hall effects are taken into account, including more general scaling laws of the growth rates in terms of sheet inverse aspect ratio: the critical inverse aspect ratio is amended to \( a/L \sim (di/L)^{0.29}(1/S)^{0.19} \), at which point the instability growth rate becomes Alfvénic and does not depend on either of the (small) parameters \( di/L \), \( 1/S \). We discuss the implications of this generalized triggering aspect ratio for recently developed phase diagrams of magnetic reconnection.

Key words: magnetic reconnection – magnetohydrodynamics (MHD) – plasmas

1. Introduction

Magnetic reconnection is generally believed to be the mechanism responsible for explosive events in astrophysical, space, and laboratory plasmas (Yamada et al. 2010; Gonzalez & Parker 2016). Indeed, this process allows the magnetic field to access new topologies, dissipating magnetic energy into heat and accelerated particles. However, in order for the energy to be initially stored in the field—a requirement for bursty, intermittent energy release to be possible—reconnection cannot be occurring all the time: as measured with a clock based on the characteristic ideal dynamical timescale, reconnection and resistive instabilities must have an off-on character. A complete understanding of magnetic reconnection therefore requires explaining how energy accumulates in the magnetic field, how the configuration suddenly become unstable, and how magnetic energy release occurs on very fast timescales; this implies a study of dynamics over multiple length- and time- scales. Since natural plasmas are characterized by Lundquist numbers \( (S = Lv_m/\eta \) where \( \eta \) is the magnetic diffusivity) that can vary by many orders of magnitude—e.g., from \( 10^9 \) in tokamaks to \( 10^{13} \) in the solar corona—the dissipation scales can often approach kinetic (ion inertial or gyroradius) scales, so a lot of work has been devoted to the role of kinetic effects in explaining fast magnetic reconnection (Hosseinpur et al. 2009).

It has recently been shown (Pucci & Velli 2014) that, in high Lundquist number plasmas (and, in particular, for the ideal limit \( S \rightarrow \infty \)), resistive instabilities survive and can become ideal—i.e., grow on the Alfvén timescale—once an inverse aspect ratio \( a/L \sim S^{-1/3} \) is reached. For such current sheets, which are thicker than the Sweet-Parker (SP) sheets \( (a/L \sim S^{−1/2}) \), the maximum growth rate does not depend on \( S \), which means that the trigger condition is, in some sense, “independent of the environment” (“ideal” tearing; IT). The numerical simulations of this process by Landi et al. (2015) that started from the critical aspect ratio to understand the instability development, and even more so those of Tenerani et al. (2015) that followed the instability development via numerical simulations of a collapsing sheet, are illuminating. In particular, in Tenerani et al. (2015), where a collapsing current sheet at high Lundquist numbers was simulated, it was shown that, once the critical aspect ratio is reached, the instability indeed takes place on ideal timescales developing multiple plasmoids from which a hierarchy of secondary IT instabilities develops. In Tenerani et al. (2015), a modified version of the analytical fractal reconnection scenario developed by Shibata & Tanuma (2001) was shown to describe the subsequent evolution of self-similar collapsing sheets, providing a fairly complete description of the evolution in two dimensions. Uzdensky & Loureiro (2016) and Comisso et al. (2016) also discussed the initiation of tearing in collapsing sheets from an analytical point of view, finding results broadly consistent with those of Pucci & Velli (2014). In particular, Comisso et al. (2016), using the same collapsing sheet model of Tenerani et al. (2015), found logarithmic corrections in \( S \) to the critical aspect ratio for substantial mode development, but the analytical theory presented breaks down well before the observed evolution in the simulations of Tenerani et al. (2015). The proliferation of x-points has been shown to arise (Wan et al. 2013) in fully turbulent regimes, where the current layers are embedded in an evolving magnetic field with flows that cannot be directly described by the IT plasmoid instability as illustrated above. However, a specific study devoted to understanding the relationship between the two regimes, as well as extensions
to fully three-dimensional cases of collapsing sheets (Huang & Bhattacharjee 2016), would clearly be of great interest.

Here the focus is instead on the small scales in an “ideally” unstable current sheet. Indeed, even if the inverse aspect ratio of a collapsing current sheet never reaches the Sweet–Parker thickness, kinetic effects may become important at large enough Lundquist numbers when the typical collisional resistive scales become smaller than either the ion inertial length or the thermal ion gyroradii (Cassak et al. 2006; Malakit et al. 2009). When characteristic scales approach the ion inertial length, the Hall effect starts to play a role, and a quadrupolar magnetic field emerges, affecting magnetic reconnection dynamics. Indeed, there is now copious evidence, both from magnetospheric observations (Mozer et al. 2002; Cattell et al. 2005; Eastwood et al. 2007; Frank et al. 2016) at the dayside magnetopause (Vaivads et al. 2004) and in the near-Earth magnetotail (Vaivads et al. 2004; Borg et al. 2005; Nakamura et al. 2006) and from laboratory experiments (Cothran et al. 2005; Ren et al. 2005; Yamada et al. 2006; Tharp et al. 2013; Kaminou et al. 2016), that Hall reconnection occurs.

It is important to recognize that the Hall effect intervenes well before the overall current sheet thickness approaches $d_i$, because the Hall term will already affect the dynamics when the inner, tearing mode resistive singular layer thickness approaches ion scales. As we shall see in the following sections, this leads to results that remain consistent with previous works by reason of the properties of IT. In this paper, we consider the case of a planar configuration without a guide field. The effect of a finite ion inertial length in the framework of the tearing instability was first carried out by Terasawa (1983), where a schematic illustration of the effect of a quadrupole magnetic field on the planar configuration was shown.

Terasawa (1983) demonstrated that the Hall effect produced a growth rate enhancement at high Lundquist numbers, but the time for the instability to develop was still very slow because the current sheet thickness was assumed to be macroscopic. By contrast, when the ion inertial length $d_i$ becomes of the order of the length scale that characterizes the equilibrium magnetic field gradient $a$, ions and electrons decouple and whistler waves form, so two-fluid effects should be taken into account and the growth rate should be scaled using typical whistler frequencies. Starting from these considerations, the regime where the inner resistive layer thickness becomes of the same order of the ion inertial length is investigated here, in sheets where the inverse aspect ratio scales as powers both of the inverse Lundquist number $a/L \sim S^{-n}$ and of the normalized ion inertial length $d_i/L$. The idea is to recover a specific scaling for which the growth rate becomes independent of both the inverse Lundquist number and the ion inertial length, i.e., intrinsically fast, independent of the dominant small parameter driving the instability.

The plan of the paper is as follows. In Section 2, we introduce the Hall effect and summarize the scaling relations of the fastest-growing modes for current sheets with fixed aspect ratios. Section 3 generalizes the IT instability criterion, discussing how the corresponding scaling relations are affected by a finite ion inertial length and confirming them with numerical solutions of the eigenvalue equations. In Sections 4 and 5, we discuss these results and place them in context.

2. The Tearing Instability in the Presence of the Hall Effect

In this section, we summarize the classical stability problem of an equilibrium magnetic field configuration aligned along the x-axis ($\hat{i}$) and dependent on the perpendicular coordinate $y$ ($\hat{j}$) in the form of a Harris current sheet,

$$B(y) = B(y)\hat{i} = B_0 F(y/a)\hat{i},$$

where $F(y/a) = \tanh(y/a)$. If there is no out-of-plane (guide) field, a corresponding equilibrium pressure profile $p(y) = p_0 - B^2(y)/8\pi$ is required to guarantee equilibrium. Quantities are assumed to be uniform in the direction perpendicular to the plane of the equilibrium magnetic field, i.e., $\partial/\partial\zeta = 0$, but the perturbed fields are 3D, so the reconnection region may develop a spatial 3D structure. That the velocity and magnetic field perturbation along the $y$ and $z$ directions may be seen by writing out the generalized Ohm’s law including the Hall term

$$E + \frac{1}{c}(v \times B) = \frac{4\pi}{c^2} \eta_m J + \frac{1}{en_c} (J \times B),$$

where $E$ is the electric field, $v$ is the velocity, $J$ is the current, $\eta_m$ is the magnetic diffusivity, and $n$ is the electron and ion density. In this form, Ohm’s law includes the Hall effect and collisional resistivity but neglects electron pressure and electron inertia; the effect of these terms on “ideal” reconnection regimes has been discussed in Del Sarto et al. (2016).

Upon linearization, the equations are nondimensionalized using the magnetic field intensity $B_0$ (the perturbed magnetic field $b = b/B_0$), the magnetic field gradient scale $a$ (essentially the Harris sheet thickness), and the Alfvén time $\tau_A = a/v_A$, where the Alfvén speed $v_A = B_0/\sqrt{4\pi \rho}$. Wavenumbers $k$ along $x$ are also scaled with $a$, and we introduce the nondimensional displacement $\xi_y = iv_y/(\gamma a)$. The growth rate is normalized to $\tau_A$; consequently, the Lundquist number is defined as $S = a\nu_A/\eta$. Denoting derivatives with respect to the nondimensional coordinate $y = y/a$ with $'$, the tearing mode equations become

$$\left(\xi''_y - k^2 \xi_y\right) = -\frac{1}{\gamma^2} \left[F(b''_y - k^2 b_y) - F'' b_y\right],$$

$$\xi_z = -\frac{1}{\gamma^2} F k b_z,$$

$$b_y = k F \xi_y + \frac{1}{\gamma} \left[(b''_y - k^2 b_y) - \frac{h}{\gamma} F k^2 b_z\right],$$

$$b_z = k F \xi_z + \frac{1}{\gamma} \left[b'_y - k^2 b_y\right] - \frac{h}{\gamma} \left[F(b''_y - k^2 b_y) - b_y F''\right].$$

Here $h = d_i/a$ identifies the Hall coefficient that couples the $z$ and $y$ components of the perturbed magnetic field, and, when $h = 0$, the classic tearing mode equations (FKR, Furth et al. 1963) are recovered. As mentioned above, we expect the Hall term to become important when $d_i$ becomes of the same order as the thickness of the internal singular layer describing the resistive tearing mode. Since, for the classic resistive tearing mode, such thickness $\delta$ (also normalized to the shear length $a$)
scales as $\delta \sim S^{-1/4}$ for the fastest-growing mode, we define

$$P_h = \frac{h}{\delta} \sim h^{S^{1/4}}$$

so that $P_h \sim 1$ means that the Hall effect is no longer negligible.

The system of Equations (3)–(6) is a sixth-order two-point eigenvalue problem (for given $k$, $h$, $S$) in which the solutions develop large gradients in $y$ around the $x$-axis with increasing $h$, $S$. As mentioned above, the problem was first studied by Terasawa (1983), who showed that the tearing mode develops three characteristic layers: in addition to the sheet thickness $a$ and the inner singular layer, familiar from the purely resistive tearing mode (Furth et al. 1963), an intermediate layer arises in which the Hall current effect is also essential. This intermediate layer complicates the asymptotic analysis of the problem, introducing a dependence of the parameter $\Delta'$, defined below, on $h$. It is therefore best to resolve the eigenvalue problem numerically using the Lentini–Pereyra method (Lentini & Pereyra 1974) and compare the results to analytic estimations derived from a heuristic generalization of the classic tearing asymptotic matching (in the vein of Terasawa 1983).

We first illustrate the two-layer development with finite $h$ by examining the behavior of the eigenfunctions $\xi_y$ and $b_y$.

Recalling that $F$ is an odd function of $y$, it is easy to see that the ideal, marginal form of Equations (3)–(6) yields solutions for $b_y, \xi_y, b_z, \xi_z$ that are respectively even, odd, odd, even. Also, at great distances from the $x$-axis, solutions decay exponentially. As proxies for the intermediate, Hall layer, and inner singular layer around the $x$-axis, we use the distance between two peaks of the displacement $\xi_z$ and of the Hall-generated field $b_z$, plotted in Figure 1. The left panel shows the eigenfunction $\xi_y$ for a fixed Lundquist number $S = 10^4$ and values $P_h = 1, 10$, and $40$ in red, blue, and green, respectively. It can be seen how the profile widens with increasing $h$. Plotted in the right panel is the eigenfunction for $b_z$, which displays a different behavior, with a peak-to-peak central thickness that decreases with increasing $P_h$. It is not that $\xi_y$ does not display signatures of the internal singular layer, it is just that it is less apparent, coming as it does in the form of an abrupt change in its gradient, rather than the more visible maximum/minimum that $b_z$ displays.

The thicknesses of both the intermediate layer and the inner resistive layer are plotted as a function of the parameter $P_h$ in Figure 2. For very small $P_h$, one expects to recover the thickness of the resistive layer $\delta_y \sim S^{-1/4} \approx 0.01$; for increasing $P_h$, the intermediate layer thickness increases (almost linearly), $\delta_h \sim P_h^{0.94}$; and the resistive singular layer is found to decrease with $P_h$ as $\delta_y \sim P_h^{-1/2}$. Note that the two curves in the figure do not appear to intersect at small $P_h$ because the thicknesses are measured from different eigenfunctions: as seen in Equation (6), $b_z$ is proportional to $\xi_y^2$ at small $h$, so there is an offset in the maxima of $\xi_y$ and $b_z$.

To understand the thinning of the inner resistive layer, let us generalize the classical tearing mode asymptotic matching theory heuristically, taking into account the effects of $h (P_h)$. In the classic, resistive tearing mode without Hall effect, the maximum growth rate may be obtained by matching the scalings obtained in the so-called large $\Delta'$ and small $\Delta'$ regimes. We can define $\Delta'$ by searching for the solution of the linearized momentum equation neglecting the growth rate, i.e., assuming marginal stability:

$$F(b''_y - k^2 b_y) - F''b_y = 0.$$  

Because the tearing mode is center FKR where the current peaks and the equilibrium magnetic field changes sign, the correct solution vanishes at large $y$. It is easy to see that the solution has a discontinuity in derivative as $y \to 0$, and

$$\Delta' = \lim_{y \to 0} \frac{b'_y(y) - b'_y(-y)}{b_y(0)}.$$  

which, for the particular equilibrium at hand, gives $\Delta' = 2(1/k - k)$. The two regimes of the tearing mode are determined by whether the product $\Delta'\delta \gg 1$ (the large $\Delta'$ regime, also known as the resistive kink regime) or $\Delta'\delta \ll 1$ (small $\Delta'$, or the classical tearing mode regime); depending on the regime, the second derivative of $b_y$ within the inner singular layer scales as either $b'^2_y \sim b_y/\delta^2$ (large $\Delta'$) or $b'^2_y \sim b_y/\delta$ (small $\Delta'$). As a result, the relationships between $\xi_y, b_y$ obtained by matching the inner to outer layer together with $\delta, \gamma$ may be summarized as follows:

$$\begin{align*}
\text{large } \Delta' & \quad \text{small } \Delta' \\
\gamma^2 \xi_y/\delta^2 & \sim k b_y/\delta & \gamma^2 \xi_y/\delta^2 & \sim k \Delta' b_y/\delta \\
b_y & \sim k \xi_y & b_y & \sim \Delta' b_y/(S \gamma \delta) \\
b_y & \sim b_y/(S \gamma \delta^2) & b_y & \sim \Delta' b_y/(S \gamma \delta) \\
\end{align*}$$

Equation (8) comes from the ideal terms (outer region) of the induction equation, Equation (9) from the inner resistive layer of the induction equation, and Equation (10) comes from the momentum equation estimated within the inner resistive layer. From these equations, one immediately finds the growth rate scalings for the large and small $\Delta'$ regimes, $\gamma \sim k^2/3 \delta^{-1/3}$ and $\gamma \sim \Delta' \delta^{-1/3} S^{-1/2}$, respectively. With increasing $S$ at any fixed $k$ modes transition from large to small $\Delta'$, and the growth rate of the fastest-growing mode at $k(S), S$ can be found by matching the two dispersion relations, taking into account that, in the large $\Delta'$ regime, one has $\Delta' \sim 1/k$: $k_m \sim S^{-1/4}, \gamma \sim S^{-1/2}$.

Extending this reasoning to include Hall terms requires analyzing the equation for the Hall field, though one must be careful in how to approximate $\xi_y$ and $b_z$ at the edge of the inner resistive layer of thickness $\delta_y$. Because $b_z$ has the same symmetry as $\xi_y$, the dimensional estimate for the second derivative is $b''_y \sim -b_y/\delta^2$. A dimensionless estimate from Equations (3)–(6) shows, following the strategy of Equations (8)–(10), that the Hall term in the induction equation,
Figure 2. Intermediate Hall layer (yellow circles) and inner resistive layer (red squares) as a function of the parameter $P_h$ for $S = 10^8$. The intermediate thickness increases as $P_h$ grows, while the inner resistive layer decreases.

In the last term in Equation (5), has a magnitude

$$h^2 k^2 \gamma^2 \left[ 1 + \left(S \gamma \delta_\eta^2 + k^2 \delta_\eta^2 / \gamma \right) \right]$$

the regime, one has

$$h^2 k^2 \gamma^2 \left[ 1 + \left(S \gamma \delta_\eta^2 + k^2 \delta_\eta^2 / \gamma \right) \right]$$

In the denominators above, the third term is always negligible, while the $S \gamma \delta_\eta^2$ term is either $\ll 1$ in the small $\Delta'$ regime or $O(1)$ in the large $\Delta'$ regime. In both cases, once $P_h \gg 1$, the contribution of this Hall term to Equation (5) dominates compared to the ideal magnetohydrodynamics (MHD) contribution (convective term, first one on the right-hand side) leading to dispersion relations of the form

$$\gamma \sim (hk)$$

and

$$\gamma \sim (hk)^2 \Delta' S^{-1/2}$$

in the large and small $\Delta'$ regimes, respectively. Again matching the two to obtain the scaling of $k$ for the maximum growth rate, taking into account that, in the large $\Delta'$ regime, one has $\Delta' \sim 1/k$, one finds

$$k_m \sim h^{-1} S^{-1/3}$$

showing both that the growth rate of the instability is enhanced and that the resistive internal layer width decreases with increasing $h$. The above scalings have been obtained neglecting the influence of $h$ on $\Delta'$, which, as already observed by Terasawa (1983), is too strong an approximation. Indeed, the direct numerical resolution of the eigenvalue problem shows that the growth rate does not quite follow the derived scalings. To fit the numerical results, we use the approximation for maximum growth rate,

$$\gamma_{\max} \sim \gamma_0 S^{-1/2} \left(1 + \gamma_0 P_h \right)$$

where the well-known scaling in the IT regime when Hall is negligible implies that $\gamma_0 \approx 0.62$. Numerical results for the maximum value of the growth rate as a function of $P_h$ with four different fixed values of the Lundquist number (i.e., variable $h$) are shown in Figure 3. Dashed lines join points with the same value of the Hall coefficient $h$; note that the dashed lines in the regimes of small $P_h$ and large $P_h$, while separately parallel, are not parallel across the transition at $P_h \approx 1$. This is because the Hall term influences the scaling with $S$ via $P_h$ rather than simply $h$. Fitting the curve in Figure 3, for $P_h \gg 1$ we obtain

$$\gamma_{\max} \sim \gamma_0 S^{-1/2} P_h$$

where $\gamma_0 = 0.41 \pm 0.01$, (14)

3. Fast Resistive Magnetic Reconnection in the Presence of the Hall Effect

We come now to the question of how the critical resistive IT aspect ratio is modified once the resistive layer thickness becomes comparable to the ion inertial length. We will consider current sheets with a macroscopic length $L$, which will therefore be used to define the normalized ion inertial length $h \equiv d_i / L$. Let us first consider the resistive IT mode at the critical current sheet aspect ratio, scaling as $a/L \sim S^{-1/3}$. It was shown that, in this case, the resistive inner layer scales as $\delta_\eta \sim S^{-1/2}$ (Pucci & Velli 2014). It therefore seems logical to generalize the parameter defining the relevance of the Hall effect to

$$P_h \equiv h S^{1/2}. \quad \text{(15)}$$
To confirm this, we first consider a sequence of equilibria at various aspect ratios at large $S$ and scaling as $a/L \sim S^{-1/3}$ but fixing the value of $h = 10^{-6}$. As $S$ increases, we expect the maximum growth rate to be constant until $P_h \sim 1$, at which point the Hall effect acceleration should lead to an increase of the growth rate. This is because once $P_h \sim 1$ is passed, the scaling $a/L \sim S^{-1/3}$ thins the current sheet too much, leading to the same paradox of the plasmoid instability on SP sheets (Loureiro et al. 2007), namely, a growth rate that diverges with increasing $S$. This is confirmed by the numerical solution of the eigenvalue equations shown in Figure 4, where at first (dark blue through dark green lines) one sees that the maximum growth rate is the same, independent of $S$, yet once $P_h > 1$ is surpassed, the maximum growth rate rises again (light green through pink lines), rapidly increasing with increasing $S$. What this means is that once the Hall effect becomes important, the critical aspect ratio should no longer depend only on $S$ but also on the parameter $h$ or $P_h$, as there are now two asymptotic parameters at work.

We therefore generalize to an inverse aspect ratio that varies in the parameter space $(S, h)$, scaling as

$$\frac{a}{L} \sim S^{-\alpha} P_h^\beta,$$

and search for exponents such that the time for the instability to develop becomes independent of the parameters of the system. Starting from Equation (14), we must now renormalize all quantities to $L$ rather than $a$:

$$\gamma_{\text{max}} \frac{a}{L} \simeq S^{-1/2} \left( \frac{a}{L} \right)^{-1/2} \left( S^{1/4} h \left( \frac{a}{L} \right)^{-3/4} \right)^\zeta,$$

where $\zeta$ was determined in the previous section numerically. From the definition in Equation (15), inserting the explicit aspect ratio dependence from Equation (16), we obtain

$$\gamma_{\text{max}} \frac{a}{L} \simeq S^{-1/2(1+\zeta/2)} P_h^{-3/2(1+\zeta/2)}$$

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We now search for the values of $\alpha$, $\beta$ that cancel the growth rate dependence on $S, P_h$. First, notice that the growth rate is independent of the Lundquist number for $\alpha = 1/3$, independent of the value of $\zeta$. Then, for $\beta$, we may write

$$\beta = \frac{2}{3} \frac{\zeta}{(1 + \zeta/2)},$$

which allows us to calculate the exponents for which $\gamma_{\text{max}} \gamma_{\text{A}} \approx 1$,

$$\alpha = 1/3, \quad \beta = 0.29,$$

and the error coming from the numerical determination of $\zeta$ is then $\Delta \beta = 3 \Delta \zeta = 0.02$.

4. Results

The correctness of the prediction for the scaling relation of the inverse aspect ratio (Equation (20)) was verified numerically. First, in Figure 5, we show solutions for the growth rate as a function of wavenumber for different values of $S$ and a constant value $P_h \gg 1 (P_h = 10)$. The asymptotic value of the growth rate is $\gamma_{\text{max}} \gamma_{\text{A}} \approx 0.41 = \gamma_{\text{A}}$, as expected for a growth rate independent of the parameters. Second, we show the values of the growth rate, again as a function of $k$, at constant values of $S$ but different values of $P_h \gg 1$ (obtained by varying $h$) in Figure 6. Notice how $\gamma_{\text{max}} \gamma_{\text{A}}$ remains constant, $\gamma_{\text{max}} \gamma_{\text{A}} = \gamma_{\text{A}}$, and there is no shift in wavenumber $k$ of the maximum growth rate with changes in $h$. The fact that the coefficient $\gamma_{\text{A}} \approx 0.41 < 0.62$ does not have direct physical significance, stemming as it does from our definitions of aspect ratio scaling; as one may immediately verify, one could change the specific values of these coefficients by redefining the aspect ratio scaling in an arbitrary additive constant. What is important, of course, is the value of the scaling exponents $\alpha$, $\beta$ that determine the critical aspect ratios beyond which current sheets become so strongly unstable that they will never form.

Our choice for the normalization time, $t_{\text{A}}$, is valid only if $a/L \gg d_j/L$ so that $t_{\text{A}} < t_{\text{A}}$ (the whistler timescale). We want to verify that this hypothesis is satisfied by our critical aspect
Table 1

| Plasma             | Solar Chromosphere | Solar Corona | Solar Wind | Magnetotail | MRX |
|--------------------|--------------------|--------------|------------|-------------|-----|
| \( n \)            | \( 10^{14} \)     |              |            |              |     |
| \( L \)            | \( 5 \times 10^{8} \) | \( 10^9 \)   | \( 10^{13} \) | \( 10^{9} \times 10^{10} \) | (2-6) \times 10^{13} |
| \( B \)            | 50-200            | 100          | \( 10^{4} \) | \( 10^{-4} \) | 6-20 |
| \( T \)            | \( 10^5 \)        | \( 10^6 \)   | \( 10^6 \) | \( 10^{-5} \) | \( 10^{-5} \) |
| \( S \)            | \( 0.6-3 \times 10^8 \) | \( 5 \times 10^{13} \) | \( 10^{24} \) | \( 4 \times 10^{3} \times 10^{-5} \) | \( 10^{-6} \) |
| \( d_i/L \)        | \( 5 \times 10^{-6} \) | \( 4 \times 10^{-7} \) | \( 10^{-5} \) | \( 7 \times 10^{-3} \times 10^{-2} \) | \( 0.3-2 \times 10^{-3} \) |
| \( d/L \)          | \( 10^{-10} \)    | \( 8 \times 10^{-9} \) | \( 3 \times 10^{-8} \) | \( 2 \times 10^{-4} \times 10^{-3} \) | \( 0.3-2 \times 10^{-2} \) |
| \( a/L \)          | \( (2-3) \times 10^{-3} \) | \( 4 \times 10^{-5} \) | \( 2 \times 10^{-5} \) | \( (0.4-1) \times 10^{-3} \) | 0.2-0.5 |
| \( \delta/L \)     | \( (0.6-1) \times 10^{-4} \) | \( 10^{-7} \) | \( 10^{-7} \) | \( (0.5-2) \times 10^{-7} \) | \( (0.3-2) \times 10^{-1} \) |

**Note.** Physical quantities are expressed in cgs units. Bold quantities are measured (or empirically derived), while \( S \) is the Spitzer resistivity. We assume \( T = T_e \sim T_i \) and \( n = n_e \sim n_i \). The inner resistive layer is estimated as \( \delta/L \sim S^{-1/2} \), while the inverse aspect ratio is estimated as \( a/L \sim S^{-1/3} \) where \( d_i/b = p_h \ll 1 \) and as \( a/L \sim S^{-1/3}/p_h^{2/3} \) where \( d_i/b = p_h \gg 1 \). For values in the solar corona and the chromosphere, we refer to Aschwanden (2013), typical conditions in the plasma sheet during a substorm growth phase have been considered. For the MRX experiment, see Yamada et al. (2014). Red numbers mean \( a \ll d_i \), which means that the growth rate should be normalized with the whistler time.

![Figure 6](image)

**Figure 6.** Maximum growth rate in Alfvén time units as a function of the parameter \( k \) for \( S = 10^6 \) for different values of \( p_h \). Notice that the maximum growth rate is constant.

![Figure 7](image)

**Figure 7.** Revision of the phase diagram in Ji & Daughton (2011) on the basis of the results in Pucci & Velli (2014), Del Sarto et al. (2016), and this paper.

### 5. Comments and Conclusions

We have studied the linear resistive tearing instability for current layers whose thickness approaches the ion inertial length, in which the instability growth rate and parameters are modified by the Hall effect. We have generalized the IT criterion, taking into account a finite ion inertial length, and ended up with a trigger relation for the aspect ratio that varies in the parameter space \((S, p_h)\) depending on the specific plasma parameters. The result is a couple of values \( \alpha \) and \( \beta \) defining a critical aspect ratio scaling as \( a/L \sim S^{\alpha}p_h^{1/2} \), below which the reconnection process becomes explosive. Recently, phase diagrams involving a Lundquist number and the macroscopic system size in units of the ion inertial length (or ion sound gyroradius, if a guide field is present) have been created that summarize the essential dynamics of the plasma for a wide range of parameters (Ji & Daughton 2011). Figure 7 summarizes the results obtained using the IT criterion in the absence of a guide field (in the figure, \( \lambda = 1/h \)). The \((S, \lambda)\) parameter space has an extension on the left due to the fact that we also take into account the effect of finite electron skin depth...
in the reduced MHD case discussed in Del Sarto et al. (2016), where $d_e = \sqrt{m_i/m_e} d_r \sim 4d_r$. Indeed, the Hall effect on its own cannot break the frozen-in conditions; i.e., collisionless reconnection must be triggered by other effects, and the inertial terms in Ohm’s law are proportional to $d_r$. As in Ji & Daughton (2011), we have a region (blue shaded area) where the single X-line reconnection occurs, i.e., for Lundquist numbers smaller than the critical one determined by inflows and outflows; see, e.g., Tenerani et al. (2016b) and Shi et al. (2016). We have investigated the purple shaded region in Pucci & Velli (2014) and the white shaded region within this work. While the orange shaded region has been investigated in Del Sarto et al. (2016) (even if the Hall effect is negligible in the adopted frame), the green shaded region still has to be explored and, with that, a possible critical value of $d_e$ for which collisionless reconnection can present single or multiple x-points. These are useful to understand which reconnection regime dominates in astrophysical as well as in laboratory plasmas, and in the latter case to visualize the parameter space of an experimental facility, and in particular can be applied to the multipoint observations of the Magnetospheric Multiscale Mission (MMS), to compare theoretical predictions the spatial structure of the Hall magnetic and electric fields surrounding the diffusion region. The next step for linear studies is to include the 3D structures that naturally arise in the presence of a mean magnetic field in the direction orthogonal to the plane where magnetic reconnection occurs. In this case, other effects may occur due to the dependence on the third direction, and the system of equations would be more complicated, involving the complex component of the eigenfunctions. Then, electron pressure terms should be included. Such terms will introduce the dependence of the critical aspect ratio on the thermal ion gyroradius. From the nonlinear evolution point of view, it would be of great interest to follow a collapsing current sheet (initially with zero guide field) into the Hall regime. The resistive internal singular layer, which in two dimensions becomes the thickness of secondary current sheets in nonlinear evolution (Tenerani et al. 2016a) and where the Hall magnetic field is different from zero, tends to shrink for larger values of the Hall parameter, making it a challenging problem that we hope to address in the near future.

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