Eccentric connectivity polynomial [ECP] of some standard graphs

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Abstract. Eccentric connectivity index of a graph $G$ denoted by $\xi_c(G)$, can be defined as $\xi_c(G) = \sum_{v \in V(G)} \text{deg}_G(v) \cdot \text{ecc}(v)$ where $\text{deg}_G(v)$ represents the vertex degree of $v$ in $G$ and $\text{ecc}(v) = \max\{x, v\} / x \in V(G)$ eccentricity of $v$, is the distance to a farthest vertex from $v$ and it is denoted by $\xi_c(G)$. Also corresponding Eccentric connectivity polynomial [ECP] is denoted by $ECP(G, x)$ is given by, $ECP(G, x) = \sum_{v \in V(G)} \text{deg}_G(v) \cdot x^{\text{ecc}(v)}$. This paper consists of Eccentric Connectivity polynomial [ECP] of some standard graphs and for some class of graphs.

1. Introduction
Chemical compounds can be modeled by molecular graphs where the atoms represents vertices and covalent bonds represents edges. This model plays an important role as a tool to predict, pharmacological, physicochemical, and toxicological properties of a compound directly. As a result, critical step is taken in pharmaceutical drug design continues, which is useful to identify and optimize the compounds in cost effective way. Analysis of this kind is called as quantitative structure activity relationship ([1]-[4]). Graph models of related properties are being studied, these are giving numerical graph invariants ([12]-[14]). Parameters are derived from this graph-theoretic model of a chemical structure. These parameters are being used in studies of quantitative structure activity relationship pertaining to molecular design and assessment of chemicals in environmental hazard.

Topological indices of different kind are found in the field of biochemistry nanotechnology and also in chemistry, which are useful in isomer discrimination, structure-property relationship, structure-activity relationship, and pharmaceutical drug design. In chemical literature, various topological indices have been defined and various applications and mathematical properties of these indices also have been considered. In recent research, many other indices have been defined, like eccentric distance sum, and the adjacency-cum distance-based eccentric connectivity index have been considered. Topological models of these kind with high degree of predictability of pharmaceutical properties leads for the development of safe and potent compounds.

In this paper, we consider one of such indices called the Eccentric connectivity polynomial [ECP] ([5], [8]-[10]). We define terms required as follows.
Section 1.1

**Definition 1.1** For the graph $G$ Eccentric connectivity index $\xi_c(G)$ is defined as in [6] and is given by $\xi_c(G) = \sum_{v \in V(G)} \deg_G(v).ecc(v)$, here $\deg_G(v)$ represents the vertex degree of $v$ in $G$ and $ecc(v) = \max((x,v)/x \in V(G))$ eccentricity of $v$, is the distance to a farthest vertex from $v$ and it is denoted by $\xi_c(G)$.

**Definition 1.2** $[ECP]$ of a graph $G$, which is called Eccentric connectivity polynomial denoted by $ECP[G,x]$ is defined as in [7] as $ECP[G,x] = \sum_{v \in V(G)} \deg_G(v).x^{ecc(v)}$

Here $x$ is just a symbol, not a variable.

**Note:** First order derivative of $ECP [G, x]$ found at $x = 1$ gives Eccentric connectivity index of a graph.

**Example 1.1** ECP for the graph shown in Figure 1, can be written as $6x^3 + 6x^2$.

![Figure 1. ECP for the graph.](image)

**ECP of some Standard Graphs:** This section includes Eccentric connectivity polynomial (ECP) of some standard graphs, which is given by, for a,

1. $p$ - regular graph $G$, ECP can be written, and is given by, $ECP[G,x] = p\sum_{v \in V(G)} x^{ecc(v)}$

2. Complete graph of order $k$, the ECP can be written as $ECP[G,x] = (k-1)\sum_{v \in V(G)} x^1 = k(k-1)x$, for $k > 1$

3. Self centered graphs $G$ of radius $r$ the ECP is given as $ECP[G,x] = 2q.x^r$

4. Similarly $[ECP]$ for a complete bipartite graph $K_{m,n}$ is $ECP[K_{m,n}] = 2(mn)x^2$

5. The ECP for a tree (T) can be written by using following two cases as:
   
   **Case(i):** Number of pendant vertices is 2, then $T$ is a path and hence, ECP for number of pendent vertices equal to 2, can be written as, $ECP[P_{2n},x] = 2x^r(\frac{1-x^r}{1-x})$
   $ECP[P_{2n+1},x] = 4x^r(\frac{1-x^r}{4-x})$
   Where $r$ is the diameter of a tree.

   **Case(ii):** Number of pendant vertices > 2, say $k > 2$. 


The ECP for Broom graph

Theorem 2.1: We find the ECP of Broom graph

ECP for number of pendant vertices > 2, can be written, using following two sub cases,

(a) ECP for tree on one center:

\[ ECP[T, x] = kx^d + (k + 2)x^{d-1} + \ldots \ldots + (\text{no. of vertices adjacent to central vertex})(x^r) \]

\[ ECP[T, x] = [k(x^d) + k(x^{d-1}) + k(x^{d-2}) + \ldots \ldots] + [2(x^{d-1}) + 4(x^{d-2}) + \ldots \ldots] + \]

( no. of vertices adjacent to central vertex + 2)(x^r).

Where \( d \) is the diameter and \( r \) is the radius of tree.

(b) ECP for tree on two centers:

\[ ECP[T, x] = k(x^d) + (x^{d-1}) + (x^{d-2}) + \ldots \ldots] + [2(x^{d-1}) + 4(x^{d-2}) + \ldots \ldots] + \]

( no. of vertices adjacent to central vertex + 2)(x^r).

Where \( d \) is the diameter and \( r \) is the radius of tree.

Section 2.1

This section, we find the ECP for class of graphs, Broom graph, Lollipop graph and Volcano graphs given in [14], which can be defined as follows:

Definition 2.1: Broom graph is the graph, obtained from path \( p_d \) together with \( (n - d) \) end vertices all adjacent to the same end vertex of \( p_d \) and it is denoted by \( B_{n,d} \)

Example 2.1: Broom graph \( B_{9,4} \) is shown in the following Figure 2.

![Figure 2. Broom graph B_{9,4}](image)

We find the ECP of Broom graph \( B_{n,d} \) and can be written as:

Theorem 2.1: The ECP for Broom graph \( B_{n,d} \) is given by,

\[ ECP[B_{n,d}] = [(n - d + 1)x^d + (n - d + 3)x^{d-1}] + [(d - 2)x^{d-2} + (d - 4)x^{d-3}] + \ldots \ldots \]

when \( d \) is even and

\[ ECP[B_{n,d}] = [(d - 1)x^{d-2} + (d - 3)x^{d-3}] + \ldots \ldots \] when \( d \) is odd.

proof: Consider a Broom graph \( B_{n,d} \). Here two cases arise.

(i) For a path on one center:

ECP \[ B_{n,d} = [(n - d + 1)x^d + (n - d + 3)x^{d-1}] + [(d - 2)x^{d-2} + (d - 4)x^{d-3}] + \ldots \ldots + 2(x^r) \]

when \( d \) is even

(ii) For path contains two centers:

ECP \[ B_{n,d} = [(n - d + 1)x^d + (n - d + 3)x^{d-1}] + [(d - 1)x^{d-2} + (d - 3)x^{d-3}] + \ldots \ldots \]
+[(s + 1)x^r + 2(x^r) when d is odd
where r = \frac{d}{2} if d is even, and \frac{d+1}{2} if d is odd
Hence the proof.

Similarly to find eccentric connectivity polynomial for Lollipop graph \( L_{n,d} \), we define lollipop graph as follows:

**Definition 2.2** Lollipop graph is the graph, obtained from a complete graph \( K_{n-d} \) and path \( P_d \) by joining one of the vertices \( P_d \) to all the vertices of \( K_{n-d} \) and it is denoted by \( L_{n,d} \).

**Example 2.1** Following Figure 3 is lollipop graph \( L_{9,4} \)

![Figure 3. Lollipop graph, L_{9,4}](image)

Next, we find the ECP for Lollipop graph and can be written as:

**Theorem 2.2** The ECP for Lollipop graph is given as,

ECP \[ L_{n,d} \] = \[(n - d)^2 + 1\]x^d + \[(n - d + 3)x^{d-1}\] + \[(d - 2)x^{d-2} + (d - 4)x^{d-3}\] + ........... + 2x^r
when d is even and
ECP \[ L_{n,d} \] = \[(d - 1)x^{d-3}\] + \[(d - 3)x^{d-5}\] + ........... + \[(s + 1)x^r\] + 2x^r when d is odd.

Next result gives the ECP of Volcano graph \( V_{n,d} \), which can be defined as follows:

**Definition 2.3** Volcano graph is the graph obtained from a path \( P_{d+1} \) and a set of \( (n - d - 1) \) vertices, by joining each vertex in S to a central vertex of \( P_{d+1} \) and it is denoted by \( V_{n,d} \)

**Example 3.1** Figure 4 is Volcano graph \( V_{11,6} \)

![Figure 4. Volcano graph, V_{11,6}](image)

The ECP for Volcano graph \( V_{n,d} \) can be written as follows:

**Theorem 2.3** The ECP for Volcano graph \( V_{n,d} \) is given as,

ECP \[ V_{n,d} \] = \[2[x^d + 2(2x^{d-1}) + 2(2x^{d-2}) + .......... + 2(x^{r+1})] + s(x^{r+1}) + 2(x^r) + (s + 2)(x^r)\]
if path contain one center
ECP \[ V_{n,d} \] = \[2[x^d + (2x^{d-1}) + (2x^{d-2}) + .......... + (2x^{r+1})] + s(x^{r+1}) + (s + 2)(x^r)\]
if path contains two centers
where r = \frac{d+1}{2} if path length is odd, \frac{d}{2} if path length is even
where s = is the set of number of pendant vertices to central vertex.

2. Conclusions
Although many results can be obtained on eccentric connectivity polynomials of graphs, in this paper we were able to find only for certain class of graphs. We wish to continue in this regard and to find many general results.
Figure 4. Volcano graph, $V_{11,6}$

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