Density of states anomalies in multichannel quantum wires

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We reformulate the Tomonaga–Luttinger liquid theory for quasi-one-dimensional Fermion systems with many subbands across the Fermi energy. Our theory enables us to obtain a rigorous expression of the local density of states (LDOS) for general multichannel quantum wires, describing how the power-law anomalies of LDOS depend on inter- and intra-subbands couplings as well as the Fermi velocity of each band. The resulting formula for the exponents is valid in the case of both bulk contact and edge contact, and thus plays a fundamental role in the physical properties of multicomponent Tomonaga–Luttinger liquid systems.

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I. INTRODUCTION

Bosonization is one of the most powerful techniques for describing the properties of one-dimensional (1D) interacting electron systems. In 1D systems, even a slight interaction between electrons strongly affects the quantum nature, resulting in the occurrence of Tomonaga–Luttinger liquid (TLL) states. TLL states exhibit power-law anomalies in physical quantities, as predicted by the bosonization theory. A prominent example is the power-law singularity of the single-particle density of states \( D(E,T) \) near the Fermi energy, \( E_F \), represented by \( D(E,F,T) \propto |E-E_F|^\lambda \) and \( D(E,F,T) \propto T^\lambda \), with \( E \) and \( T \) being the energy and the temperature, respectively. The value of \( \lambda \), called the TLL exponent, is dependent on the interaction strength and other parameters characterizing the 1D system. Recently, it has been suggested that a continuous variation in \( \lambda \) can be produced by an external field, implying artificial control of the transport properties of quasi-1D conductors, since \( \lambda \) governs the power-law behaviors of the differential tunneling conductance \( dI/dV \propto |V|^\lambda \) at high bias voltages \((eV \gg k_BT)\) and the temperature-dependent conductance \( G(T) \propto T^\lambda \) at low voltages \((eV \ll k_BT)\).

Experimental realizations of TLL states encompass various systems showing highly anisotropic conductivity: metallic \(^{17,18}\) semiconducting \(^{19-25}\) and organic nanowires \(^{26-30}\) and carbon nanotubes \(^{31-34}\) are a few examples. These actual quasi-1D conductors possess a finite cross-section, thus exhibiting a finite number of transmission channels in the transverse direction (except for a limited case in which \( E_F \) is small enough for only the lowest subband to be involved). The presence of multiple channels at \( E_F \) causes inter-subband scatterings. Furthermore, different channels can have different Fermi velocities, i.e., the slope of the dispersion curve at \( E_F \) (see Fig. 1), and thus contributions from each channel to the TLL exponent differ from each other. Theories of multichannel TLL have been developed for the Hubbard model in the presence of an external magnetic field \(^{35-37}\), where the discrepancies in the Fermi velocity between up- and down-spins are taken into account. A similar issue was also discussed in the study of quasi-1D Bose gases.\(^{38}\) The effect of inter-subband scattering on the TLL exponent has been investigated in connection with the TLL behavior of multiwall carbon nanotubes.\(^{39}\) However, to the best of our knowledge, singular behavior in the density of states \( D(E,T) \) remains unresolved for multichannel TLL systems with the coexistence of inter-subband scatterings and Fermi-velocity variations. Hence, the rigorous expression of \( D(E,T) \) in multichannel TLL systems is desirable for describing the transport properties and photo-emission spectra those will be experimentally observed in actual quasi-1D conductors.

In this paper, we reformulate the multichannel TLL theory in order to derive the anomalous energy- and temperature-dependences of the local density of states of quasi-1D Fermion systems. Cases of locations both far from the boundary and close to it, which correspond to bulk contact and end contact of the transport properties, respectively, are discussed. We demonstrate clearly how the TLL exponents of multichannel systems depend on mutual interaction and Fermi velocities. The resulting formula for the exponents, as well as the theoretical framework we have established, will provide clues to exploiting the effects of subband couplings and Fermi-velocity variations on the nature of TLLs in real 1D systems.

The paper is organized as follows. In Section II, the multichannel TLL theory is developed for \( N \)-channel quasi-1D Fermionic systems with different Fermi velocities. The local densities of states far from and close to the boundary are calculated in Section III. As a simple example, in Section IV, the case of spinless Fermions is discussed and the \( N \)-dependences of the exponents are obtained for long-range interaction limits. The paper closes with a summary in Section V. In the following, the unit \( \hbar = k_B = 1 \) is used, unless explicitly stated otherwise.
of which satisfies the commutation relation

\[ k \cdot q = \frac{\pi}{L} \]

where the one-particle energy and the wavenumber \( k \) are measured from \( E_F \) and \( pk_{F\nu} \), respectively. The symbol \( p = + \) \((-\)\) indicates a one-particle state moving toward the right (left).

FIG. 1: Sketch of the energy dispersion of the present system, with \( N \) energy bands cross the Fermi energy, \( E_F \). The Fermi velocity and the Fermi wavenumber of the \( \nu \)-th band \((\nu = 1, \cdots, N)\) are denoted by \( v_{F\nu} \) and \( k_{F\nu} \), respectively, and the one-particle state moving to the right (left) are indicated by \( E \) \( \nu \) respectively. The symbol \( F \) \((\nu, \nu) \) is schematically shown in Fig. 1. Here, the Fermi velocity and \( F \) \( \nu \) are indicated by \( \nu \) \( \nu \) and \( \nu \) \( \nu \) respectively. The symbol \( \nu \) \( \nu \) is the length of the system. The most general form of the Hamiltonian, \( \mathcal{H}_k \), is expressed by

\[
\mathcal{H}_k = \sum_{\nu=1}^{N} \sum_{p=q=\pm} \sum_{k} \rho_{p,\nu}(q) \epsilon_{\nu,p,\nu} \cdot c_{\nu,p,\nu}^\dagger \cdot c_{\nu,p,\nu},
\]

where the one-particle energy and the wavenumber \( k \) are measured from \( E_F \) and \( pk_{F\nu} \), respectively. In Eq. (1), \( c_{\nu,p,\nu}^\dagger \) denotes the creation operator of the Fermion with wavenumber \( k \), branch \( p \), and band index \( \nu \).

Let us introduce the density operator of the \( \nu \)-th branch of the \( \nu \)-th band, defined as

\[
\rho_{p,\nu}(q) = \begin{cases} \sum_{k} \epsilon_{\nu,p,\nu}^\dagger \cdot c_{\nu,p,\nu} \cdot \cdots \; q \neq 0, \\ N_{p,\nu} = \sum_{k} \epsilon_{\nu,p,\nu}, \quad \cdots \; q = 0, \end{cases}
\]

which satisfies the commutation relation

\[
[\rho_{p,\nu}(q), \rho_{p',\nu}(q')] = \delta_{pp'} \delta_{\nu,\nu'} \delta_{qq'} p q L / (2\pi).
\]

In terms of \( \rho_{p,\nu}(q) \), \( \mathcal{H}_k \) is expressed by

\[
\mathcal{H}_k = \sum_{\nu=1}^{N} \frac{\pi v_{F\nu}}{L} \sum_{p,q} \rho_{p,\nu}(q) \rho_{p,\nu}(-q),
\]

where \( L \) is the length of the system. The most general form of the mutual interaction between Fermions leading to the \( N \)-component TLL is written as

\[
\mathcal{H}_{\text{int}} = \frac{1}{2L} \sum_{\nu,\nu'=1}^{N} \sum_{p,q} \left\{ \tilde{g}_2(\nu, \nu') \rho_{p,\nu}(q) \rho_{-p,\nu'}(-q) + \tilde{g}_3(\nu, \nu') \rho_{p,\nu}(q) \rho_{p,\nu'}(-q) \right\},
\]

The matrix elements \( \tilde{g}_2(\nu, \nu') \) and \( \tilde{g}_3(\nu, \nu') \) depend on the details of the model we consider. Specifically, the case with \( \tilde{g}_2 \equiv \tilde{g}_3 \) corresponds to the model for multiwall carbon nanotubes considered in Ref. 39. As an example, we will discuss the case of the spinless Fermion in Section II.

We introduce the phase variables \( \theta_{\nu}(x) \) and \( \phi_{\nu}(x) \) \((\nu = 1, \cdots, N)\), defined as

\[
\theta_{\nu}(x) = -\frac{1}{\sqrt{2}} \sum_{p} Q_{p,\nu} - \frac{2\pi p x}{L} N_{p,\nu},
\]

\[
\phi_{\nu}(x) = -\frac{1}{\sqrt{2}} \sum_{p} \left\{ Q_{p,\nu} - \frac{2\pi p x}{L} N_{p,\nu} \right\},
\]

where \([Q_{p,\nu}, N_{p',\nu'}] = i\delta_{pp'} \delta_{\nu,\nu'}\). In the summation in terms of \( q \), the ultraviolet cutoff \( \exp(-\alpha |q| / 2) \) is implicitly included. The phase variables satisfy the commutation relation, \( [\theta_{\nu}(x), \phi_{\nu}(x')] = i2\pi \delta_{\nu,\nu'} \theta(x - x') \) for \( L \to \infty \). In terms of the above phase variables, the Hamiltonian is written as

\[
\mathcal{H} = \frac{1}{2} \sum_{\nu,\nu'=1}^{N} \int x \left\{ \Pi_{\nu}(K^{-1})_{\nu,\nu'} \Pi_{\nu'} + \partial_x \theta_{\nu} V_{\nu,\nu'} \partial_x \theta_{\nu'} \right\},
\]

where \( \Pi_{\nu} = -\partial_x \phi_{\nu}/(2\pi) \). This is the general form of the phase Hamiltonian expressing the \( N \)-component TLL. The symmetric matrices \( K \) and \( V \) are defined as follows:

\[
(K^{-1})_{\nu,\nu'} = 2\pi v_{F\nu} \delta_{\nu,\nu'} + \tilde{g}_4(\nu, \nu'),
\]

\[
V_{\nu,\nu'} = \frac{v_{F\nu}}{2\pi} \delta_{\nu,\nu'} + \tilde{g}_4(\nu, \nu') - \frac{\tilde{g}_2(\nu, \nu')}{4\pi^2}.
\]

The Fermion operator, defined by \( \psi_{p,\nu}(x) = 1/\sqrt{L} \sum_{k} e^{i k x} c_{\nu,p,\nu} \), is related to the phase variables as

\[
\psi_{p,\nu}(x) = \frac{\eta_{\nu}}{\sqrt{2\pi}} \exp \left\{ i \frac{p}{\sqrt{2}} \left[ \theta_{\nu}(x) + p \phi_{\nu}(x) \right] \right\},
\]

where \( \eta_{\nu} \) expresses the Majorana Fermion operator satisfying \( \eta_{\nu} = \eta_{\nu}^\dagger \) and \( \{ \eta_{\nu}, \eta_{\nu'} \} = 2\delta_{\nu,\nu'} \).

B. Diagonalization

The Hamiltonian given by Eq. (7) has a bilinear form with respect to \( \partial_x \phi_{\nu} \) and \( \partial_x \theta_{\nu} \), and thus can be diagonalized by the standard unitary transformation, as shown below.
The equations of motion of the phase variables derived from Eq. (7) read as

\[ \frac{\partial}{\partial t} \Pi = V \frac{\partial^2}{\partial x^2} \Theta, \quad \frac{\partial}{\partial t} \Theta = K^{-1} \Pi, \tag{11} \]

with \( \Theta = (\theta_1, \theta_2, \ldots, \theta_N)^T \) and \( \Pi = (\Pi_1, \Pi_2, \ldots, \Pi_N)^T \).

The energy eigenvalue, \( \omega = v|k| \), and the eigenvector \( \Theta \) corresponding to it are determined by
\[ (v^2 K - V) \Theta = 0, \tag{13} \]
whose solutions are denoted by \( v_j \) and \( X_j (j = 1, 2, \ldots, N) \).

To obtain a concise representation of \( \mathcal{H} \), we define the unitary transformation as

\begin{align*}
\Theta &= X \Theta', \\
\Pi &= K X \Xi,
\end{align*}

where \( \Theta' = (\Theta_1, \Theta_2, \ldots, \Theta_N)^T \) and \( \Xi = (\Xi_1, \Xi_2, \ldots, \Xi_N)^T \).

The local density of states is given by the summand of \( \mathcal{H} \), and the contributions from each band:
\[ \sum_{\nu=1}^{N} D_{\nu}(\omega, T, x). \tag{18} \]

The contribution from the \( \nu \)-th band, \( D_{\nu}(\omega, T, x) \), is given by
\[ D_{\nu}(\omega, T, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{e}^{\text{i} \omega t} \left\{ \psi_{\nu}^\dagger(x, 0), \psi_{\nu}(x, t) \right\}, \tag{19} \]

where \( \psi_{\nu}(x, t) = \text{e}^{\text{i} k_{\nu} x} \psi_{\nu}(x, t) + \text{e}^{\text{-i} k_{\nu} x} \psi_{\nu}^e(x, t) \).

The derivations of Eqs. (19)–(22) are shown in Appendix A.

III. DENSITY OF STATES

In this section, we discuss the local density of states \( D(\omega, T, x) \) with \( \omega \equiv E - E_F \) for \( \omega \ll E_F \), where \( x \) denotes the position along the 1D direction. As noted earlier, the TLL exponent that characterizes the singularity of \( D(\omega, T, x) \) near \( E_F \) is \( x \) dependent. From a practical view, it is specifically interesting to study the semi-infinite system with its end at the origin and discuss the cases with \( x \rightarrow 0 \) and \( x \rightarrow \infty \), which correspond to the end contact and the bulk contact, respectively. In the following argument, we therefore derive the TLL exponent for both cases, as well as the explicit forms of \( D(\omega, T, x) \) as functions of \( \omega \) and \( T \).

The local density of states is given by the summation of the contribution from each band: \( D(\omega, T, x) = \sum_{\nu=1}^{N} D_{\nu}(\omega, T, x) \).

\[ \sum_{\nu=1}^{N} D_{\nu}(\omega, T, x). \tag{23} \]

Equation (23) implies that the \( \omega \)- and \( T \)-dependencies of
The Umklapp scattering are ignored. In fact, by parameterizing

\[ \lambda^{(b)}(\nu) = \sum_{j=1}^{N} \nu_{\nu,j} - 1 \]

for the bulk position, and

\[ \lambda^{(e)}(\nu) = \sum_{j=1}^{N} \nu_{\nu,j} - 1 \]

for the edge. Equations (26) and (27) are the main findings of

Eqs. (26) and (27) lead to the familiar forms

\[ \lambda^{(b)}(1) = \lambda^{(b)}(2) = \frac{1}{4} (K_{\rho} + K_{\rho}^{-1} + K_{\sigma} + K_{\sigma}^{-1}) - 1, \]

\[ \lambda^{(e)}(1) = \lambda^{(e)}(2) = \frac{1}{2} (K_{\rho}^{-1} + K_{\sigma}^{-1}) - 1, \]

with

\[ K_{\rho} = \sqrt{\frac{2\pi \nu_{F} + g_{4\|} + g_{4\perp} - g_{2\|} - g_{2\perp} + g_{1\|}}{2\pi \nu_{F} + g_{4\|} + g_{4\perp} + 2g_{2\|} + g_{2\perp} - g_{1\|}}, \]

\[ K_{\sigma} = \sqrt{\frac{2\pi \nu_{F} + g_{4\|} - g_{4\perp} - g_{2\|} + g_{2\perp} + g_{1\|}}{2\pi \nu_{F} + g_{4\|} - g_{4\perp} + 2g_{2\|} - g_{2\perp} - g_{1\|}}. \]

\[ \text{IV. N-CHANNEL SPINLESS FERMIONS} \]

In this section, we derive the matrix elements of the mutual interactions, which are included in the matrices \( K \) and \( V \) in Eqs. (8) and (9). As a simple example, we consider a quasi-

1D spinless Fermion system where \( N \) 1D energy bands cross the Fermi energy, \( E_{F} \). In addition, in order to clarify the effects of the number of channels on the exponents, those for the case of long-range interaction are derived.

The mutual interaction \( H_{\text{int}} \) of the spinless Fermion can be expressed generally as

\[ H_{\text{int}} = \frac{1}{2} \iint x x' \psi(x) \psi^\dagger(x') V(|x - x'|) \psi(x') \psi(x), \]

with \( \psi(x) \) being the annihilation operator of the spinless Fermion. Since we are discussing low-energy physics, the interaction processes among the particles close to \( E_{F} \) are necessary. In order to obtain such interaction processes, the operator \( \psi(x) \) is expanded, using the eigenfunctions of the states across \( E_{F} \), \( \phi_{\nu,K}(x) \), as

\[ \psi(x) = \sum_{\nu=1}^{N} \sum_{K} \alpha_{\nu,K} \phi_{\nu,K}(x), \]

where \( \alpha_{\nu,K} \) is the operator of the spinless Fermion with the eigenstate \((\nu,K)\). By inserting Eq. (37) into Eq. (36), \( H_{\text{int}} \) is expressed as

\[ H_{\text{int}} = \frac{1}{2} \sum_{\nu_{1},\nu_{2},\nu_{3},\nu_{4}K_{1},K_{2},K_{3},K_{4}} V_{\nu_{1}K_{1},\nu_{2}K_{2};\nu_{3}K_{3},\nu_{4}K_{4}} \times a_{\nu_{1}K_{1}} \dagger a_{\nu_{2}K_{2}} \phi_{\nu_{3}K_{3}} \phi_{\nu_{4}K_{4}}, \]

where the matrix element of the mutual interaction is written as

\[ V_{\nu_{1}K_{1},\nu_{2}K_{2};\nu_{3}K_{3},\nu_{4}K_{4}} = \iint dx dy V(|x - y|) \times \phi_{\nu_{1}K_{1}}^\dagger(x) \phi_{\nu_{2}K_{2}}(y) \phi_{\nu_{3}K_{3}}(y) \phi_{\nu_{4}K_{4}}(x). \]

We note that as a result of momentum conservation, the relation \( K_{1} + K_{2} - K_{3} - K_{4} = nG \) holds, where \( G \) is the reciprocal lattice vector and \( n \) is an integer. In the following, we discuss the case where the filling of each band is incommensurate. Then, only the normal processes satisfying \( n = 0 \) are taken into account as

\[ V_{\nu_{1}K_{1},\nu_{2}K_{2};\nu_{3}K_{3},\nu_{4}K_{4}} = \delta_{K_{1}+K_{2},K_{3}+K_{4}} \times V_{\nu_{1},\nu_{2};\nu_{3},\nu_{4}}(K_{1},K_{2};K_{3},K_{4}). \]
In this case, $\mathcal{H}_{\text{int}}$ is expressed by

$$\mathcal{H}_{\text{int}} = \frac{1}{2} \sum_{\nu_1, \nu_2, \nu_3, \nu_4} \sum_{p_1, p_2, p_3, p_4} \sum_{k_1, k_2, k_3, k_4} \times \phi_{p_1 k_{F\nu_1} + p_2 k_{F\nu_2} + p_3 k_{F\nu_3} + p_4 k_{F\nu_4}} \delta_{k_1 + k_2, k_3 + k_4} \times V_{p_1, p_2, p_3, p_4} (p_1 k_{F\nu_1} p_2 k_{F\nu_2} p_3 k_{F\nu_3} p_4 k_{F\nu_4}) \times c_{k_1, \nu_1}^\dagger c_{k_2, \nu_2} c_{k_3, \nu_3} c_{k_4, \nu_4}, \quad (41)$$

where $c_{p, k\nu} = a_{\nu, p} k_{F\nu} + k$. Here, the relations $K_i = p_i k_{F\nu_i} + k_i$ and $|k_i| \ll k_{F\nu_i}$ ($i = 1, \cdots, N$) are used. Assuming $k_{F\nu} \neq k_{F\nu'}$ for $\nu \neq \nu'$, Eq. $(41)$ is written as $\mathcal{H}_{\text{int}} = \mathcal{H}_{\text{int}, 1} + \mathcal{H}_{\text{int}, 2} + \mathcal{H}_{\text{int}, 4}$, where

$$\mathcal{H}_{\text{int}, 1} = \frac{1}{2} \sum_{k, k', q, \rho = \pm} \sum_{\nu, \nu' = 1} \sum_{1}^{N} V_{\nu, \nu', \nu, \nu'} (p k_{F\nu} - p k_{F\nu'}; p k_{F\nu} - p k_{F\nu'}) \times c_{k+q, \nu} c_{k', -q, \nu'} c_{k', \nu'} c_{k, -\nu}, \quad (42)$$

$$\mathcal{H}_{\text{int}, 2} = \frac{1}{2} \sum_{k, k', q, \rho = \pm} \sum_{\nu, \nu' = 1} \sum_{1}^{N} V_{\nu, \nu', \nu, \nu'} (p k_{F\nu} - p k_{F\nu'}; -p k_{F\nu} - p k_{F\nu'}) \times c_{k+q, \nu} c_{k', -q, \nu'} c_{k', \nu'} c_{k, -\nu}, \quad (43)$$

$$\mathcal{H}_{\text{int}, 4} = \frac{1}{2} \sum_{k, k', q, \rho = \pm} \sum_{\nu, \nu' = 1} \sum_{1}^{N} V_{\nu, \nu', \nu, \nu'} (p k_{F\nu} - p k_{F\nu'}; p k_{F\nu} - p k_{F\nu'}) \times c_{k+q, \nu} c_{k', -q, \nu'} c_{k', \nu'} c_{k, -\nu}$$

$$+ \frac{1}{2} \sum_{k, k', q, \rho = \pm} \sum_{\nu, \nu' = 1} \sum_{1}^{N} \{ V_{\nu, \nu', \nu, \nu'} (p k_{F\nu} - p k_{F\nu'}; p k_{F\nu} - p k_{F\nu'}) \times c_{k+q, \nu} c_{k', -q, \nu'} c_{k', \nu'} c_{k, -\nu} \} + V_{\nu, \nu', \nu, \nu'} (p k_{F\nu} - p k_{F\nu'}; p k_{F\nu} - p k_{F\nu'}) \times c_{k+q, \nu} c_{k', -q, \nu'} c_{k', \nu'} c_{k, -\nu}, \quad (44)$$

Here, $\mathcal{H}_{\text{int}, 1}$ represents the backward scattering, $\mathcal{H}_{\text{int}, 2}$ denotes the forward scattering among the different branches, and $\mathcal{H}_{\text{int}, 4}$ expresses the forward scattering between the same branches. It should be noted that we neglect accidental situations in the momentum conservation, for example, $k_{F\nu_1} - k_{F\nu_2} = k_{F\nu_3} + k_{F\nu_4}$ with $k_{F\nu_1} \neq k_{F\nu_4}$ and $k_{F\nu_2} \neq k_{F\nu_3}$, in the forward scattering among different branches. Equations $(42)$, $(43)$, and $(44)$ are reduced to

$$\mathcal{H}_{\text{int}} = \frac{1}{2L} \sum_{k, k', q, \rho = \pm} \sum_{\nu, \nu' = 1} \sum_{1}^{N} g_1 (\nu, \nu') c_{k+q, \nu} c_{k', -q, \nu'} c_{k', \nu'} c_{k, -\nu}$$

$$+ g_2 (\nu, \nu') c_{k+q, \nu} c_{k', -q, \nu'} c_{k', \nu'} c_{k, -\nu}$$

$$+ g_4 (\nu, \nu') c_{k+q, \nu} c_{k', -q, \nu'} c_{k', \nu'} c_{k, -\nu}, \quad (45)$$

where

$$g_1 (\nu, \nu') = LV_{\nu, \nu'; \nu, \nu'} (k_{F\nu}, -k_{F\nu'}; k_{F\nu}, -k_{F\nu'}), \quad (46)$$

$$g_2 (\nu, \nu') = LV_{\nu, \nu'; \nu, \nu'} (k_{F\nu}, -k_{F\nu'}; -k_{F\nu}, k_{F\nu'}), \quad (47)$$

$$g_4 (\nu, \nu') = L \{ V_{\nu, \nu', \nu, \nu'} (k_{F\nu}, k_{F\nu'}; k_{F\nu}, k_{F\nu'}) \delta_{\nu, \nu'}$$

$$+ [V_{\nu, \nu', \nu, \nu'} (k_{F\nu}, k_{F\nu'}; k_{F\nu}, k_{F\nu'})$$

$$- V_{\nu, \nu', \nu, \nu'} (k_{F\nu}, k_{F\nu'}; k_{F\nu}, k_{F\nu'})) (1 - \delta_{\nu, \nu'}) \} \}. \quad (48)$$

Here, the relation $\phi_{\nu, k} (x) = \phi_{-\nu \cdot K} (x)$, which is a result of time-reversal symmetry, is used. Note that $g_1 (\nu, \nu'), g_2 (\nu, \nu')$, and $g_4 (\nu, \nu')$ are the real symmetric matrices. By comparing Eq. $(45)$ with Eq. $(4)$, we obtain $\tilde{g}_2 (\nu, \nu') = g_2 (\nu, \nu') - g_1 (\nu, \nu')$ and $\tilde{g}_4 (\nu, \nu') = g_4 (\nu, \nu')$.

Here, we consider the case where the Fermi velocities of all the channels are equal to each other, i.e., $v_{F\nu} = v_F$. In addition, we assume that the matrix elements are given by $g_2 (\nu, \nu') = g_4 (\nu, \nu') = g$ and $g_1 (\nu, \nu') = 0$, which is a simple but effective approximation for long-range interactions. Here, the velocities of the excitations are obtained as $v_1 = v_F \sqrt{1 + N g / (\pi v_F^2)}$ and $v_j = v_F (j = 2, \cdots, N)$. The eigenvector corresponding to $v = v_1$ is written as $X_1 = \sqrt{2} \nu_{F} N (1, \cdots, 1)^T$. As a result, the exponents are obtained as follows:

$$\lambda^{(b)} (\nu) = \frac{1}{N} \left\{ \frac{1}{2} \left( \frac{v_{F}}{v_1} + \frac{v_1}{v_F} \right) - 1 \right\}, \quad (49)$$

$$\lambda^{(c)} (\nu) = \frac{1}{N} \left( \frac{v_1}{v_1 - 1} \right). \quad (50)$$

Thus, the TLL exponents for both locations decrease with increasing $N$, and are proportional to $N^{-1/2}$ for $N \gg 1$.

V. CONCLUSION

In the present paper, we reformulated the TLL theory for multichannel 1D Fermion systems. The theory obtained enables derivation of rigorous expressions for the local density of states and the corresponding TLL exponents, $\lambda^{(b/c)} (\nu)$, with respect to the $\nu$-th band. The strategy for evaluating $\lambda^{(b/c)} (\nu)$ is summarized as follows:

1. Define the functional forms of the 1D eigenfunction $\phi_{\nu, K} (x)$ and the interaction $V (|x - y|)$ appropriate for the system being considered.

2. Calculate $V_{\nu_1, K_1, \nu_2, K_2; \nu_3, K_3, \nu_4, K_4}$ using Eq. $(39)$.

3. Using the above result, set the mutual interaction terms, $\tilde{g}_i (\nu, \nu')$, that are necessary to define the interaction $\mathcal{H}_{\text{int}}$ given by Eq. $(4)$. Particularly when considering spinless Fermions, we can obtain $\tilde{g}_i (\nu, \nu')$ for $i = 1, 2, 4$ by substituting the results of step 2 into Eqs. $(46)$–$(48)$.

4. Set $(K^{-1})_{\nu, \nu'}$ and $V_{\nu, \nu'}$ according to Eqs. $(8)$ and $(9)$. 
5. Solve the eigenvalue problem \([12]\) to obtain \(v_j\) and \(X_j\) for \(j = 1, \ldots, N\).

6. Evaluate \(Y^{(b)}_{\nu,j}\) and \(Y^{(e)}_{\nu,j}\) from Eqs. \((21)\) and \((22)\).

7. Finally, we obtain the exponents \(\lambda^{(b)}(\nu)\) and \(\lambda^{(e)}(\nu)\) from Eqs. \((26)\) and \((27)\).

By applying the strategy for \(N\)-channel spinless Fermions with long-range interactions, we have revealed that both TLL and the energy-independent density of states, which is a manifestation of the realization of Fermi liquids, emerges for \(N \to \infty\).

Before closing, we remark that the present theory began with the electronic Hamiltonian of the mutual interaction in Eq. \((4)\), which leads to the bosonic Hamiltonian Eq. \((7)\) where no nonlinear terms exist. Even in the presence of interaction terms leading to nonlinear terms, the present theory can be useful if the the nonlinear terms are renormalized to zero. In such cases, it is necessary to take account of the renormalization of the parameters \(K\) and \(V\) by the diminishing nonlinear terms.

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Appendix A: Derivation of Eqs. \((19)\)–\((22)\)

We discuss a semi-infinite system with its end at the origin. For convenience, we scale the bosonic fields as

\[
\tilde{\Theta}_j(x, t) = \sqrt{2} \theta_j(x, t),
\]

\[
\tilde{\Phi}_j(x, t) = \frac{1}{2\pi \sqrt{v_j}} \Phi_j(x, t),
\]

where \([\tilde{\Theta}_j(x, t), \tilde{\Phi}_j(y, t)] = i \delta_{jj'} \theta(x - y)\). By using field operators, the Hamiltonian is written as

\[
\mathcal{H} = \sum_{j=1}^{N} \frac{v_j}{2} \int_0^\infty x \left\{ \tilde{\Xi}_j^2 + (\partial_x \tilde{\Theta}_j)^2 \right\},
\]

where \(\tilde{\Xi}_j = -\partial_x \tilde{\Phi}_j\). The boundary condition at the origin requires the Fermion field for the \(\nu\)-th subband \(\psi_\nu(0) = 0\), i.e., \(\psi_{-\nu}(0) = -\psi_{+\nu}(0)\). This condition leads to

\[
\frac{1}{\sqrt{2}} \sum_{j=1}^{N} \frac{X_{\nu,j}}{\sqrt{v_j}} \Theta_j(0, t) = \left( n + \frac{1}{2} \right) \pi,
\]

with \(n\) being an arbitrary integer.

The mode expansion, together with the canonical quantization, leads to

\[
\\\tilde{\Theta}_j(x, t) = C_j + \tilde{\Theta}_j(x, t),
\]

\[
\tilde{\Phi}_j(x, t) = \frac{1}{\sqrt{2}} \int_0^\infty q \sin qx \sqrt{q} \left\{ -ie^{-iqy} b_j(q) + ie^{iqy} b_j^\dagger(q) \right\},
\]

\[
\tilde{\Phi}_j(x, t) = -\frac{1}{\sqrt{2}} \int_0^\infty q \cos qx \sqrt{q} \left\{ e^{-iqy} b_j(q) + e^{iqy} b_j^\dagger(q) \right\},
\]

where \(C_j\) is the c-number satisfying \((1/\sqrt{2}) \sum_{j=1}^{N} (X_{\nu,j}/\sqrt{v_j}) C_j = \left( n + (1/2) \right) \pi\), and \(b_j(q)\) is the bosonic operator with \([b_j(q), b_j^\dagger(q')] = \pi \delta_{jj'} \delta(q - q')\). The ultraviolet cutoff \(\exp(-\alpha q/2)\) is inserted if necessary in the \(q\)-integral in Eqs. \((A5)\) and \((A6)\). Note that \(\partial_y \tilde{\Theta}_j(x, t) = -v_j \partial_x \tilde{\Phi}_j(x, t)\). As a result of Eqs. \((A5)\)–\((A7)\), the Fermion field of the \(\nu\)-th band satisfies \(\psi_{-\nu}(x, t) = -\psi_{+\nu}(-x, t)\). The Hamiltonian is written as

\[
\mathcal{H} = \sum_{j=1}^{N} \frac{1}{\pi} \int_0^\infty q v_j q b_j^\dagger(q) b_j(q).
\]

The quantity \(\left\{ \psi_{+\nu}^\dagger(x, 0), \psi_{-\nu}(x, t) \right\}\) is calculated as follows:

\[
\langle \{ \psi_{+\nu}^\dagger(x, 0), \psi_{-\nu}(x, t) \} \rangle \equiv \langle \{ \psi_{+\nu}^\dagger(x, 0), \psi_{+\nu}(x, t) \} \rangle + (x \to -x)
\]

\[
= \frac{1}{2\pi \alpha} \int_0^\infty \prod_{j=1}^{N} G_{\nu,j}(x, t) \mathcal{H}_{\nu,j}(x, t) + \int_0^\infty \prod_{j=1}^{N} G_{\nu,j}(x, t) \mathcal{H}_{\nu,j}^\dagger(x, t)
\]

\[
+ (x \to -x),
\]

where

\[
G_{\nu,j}(x, t) = \exp \left\{ -\frac{1}{\sqrt{2}} (f_{\nu,j}(x, 0) - f_{\nu,j}(x, t))^2 \right\}
\]

\[
= \exp \left\{ -\frac{1}{4} (f_{\nu,j}(x, 0) - f_{\nu,j}(x, t))^2 \right\},
\]

\[
H_{\nu,j}(x, t) = \exp \left\{ \frac{1}{4} [f_{\nu,j}(x, 0), f_{\nu,j}(x, t)] \right\},
\]

with \(f_{\nu,j}(x, t) = X_{\nu,j}/\sqrt{v_j} \tilde{\Theta}_j(x, t) + 2\pi \sqrt{\gamma} (K X)_{\nu,j} \tilde{\Phi}_j(x, t)\). Here, we ignore the rapidly oscillating terms proportional to \(\exp(\pm i2k_{F\nu} x)\) because these contributions can probably not be observed directly due to averaging over several lattice sites in the experiments.
From Eqs. (A6) and (A7), together with (A8),

\[ G_{\nu,j}(x,t) = \exp \left\{ -\frac{A_{\nu,j}}{2} \int_0^\infty q^2 \sin^2 q x \frac{q}{q} \times \left( 1 - e^{-i\nu_j q t} \right) \left( 1 + 2g(\nu_j q) \right) \right\} \]

\[ = \frac{B_{\nu,j}}{2} \int_0^\infty q^2 \cos^2 q x \frac{q}{q} \times \left( 1 - e^{-i\nu_j q t} \right) \left( 1 + 2g(\nu_j q) \right) \right\}, \quad (A12) \]

\[ H_{\nu,j}(x,t) = \exp \left\{ \frac{A_{\nu,j}}{2} \int_0^\infty q^2 \sin^2 q x \frac{q}{q} \left( e^{i\nu_j q t} - e^{-i\nu_j q t} \right) + \frac{B_{\nu,j}}{2} \int_0^\infty q^2 \cos^2 q x \frac{q}{q} \left( e^{i\nu_j q t} - e^{-i\nu_j q t} \right) \right\}, \quad (A13) \]

where

\[ A_{\nu,j} = \frac{(X_{\nu,j})^2}{2\pi v_j}, \quad B_{\nu,j} = 2\pi v_j \left[ (KX)_{\nu,j} \right]^2, \quad (A14) \]

and \( g(e) = (e^{e/T} - 1)^{-1} \) is the Bose distribution function. As a result,

\[ \langle \{ \psi_\nu^\dagger(x,0), \psi_\nu(x,t) \} \rangle = \frac{1}{\pi} \exp \left\{ -\sum_{j=1}^N \left[ C_+ I_j(0,t) + C_- I_j(x,t) \right] \right\} \]

\[ \times \exp \left\{ -\sum_{j=1}^N \left[ C_+ J_j(0,t) + C_- J_j(x,t) \right] \right\} \]

\[ + (t \to -t) \right\}, \quad (A15) \]

where \( C_{\pm} \equiv (B_{\nu,j} \pm A_{\nu,j})/2 \) and

\[ I_j(x,t) = \int_0^\infty q^2 \cos q x \frac{q}{q} \left( 1 - e^{-i\nu_j q t} \right) (1 - e^{i\nu_j q t}) g(\nu_j q) \]

\[ = \frac{1}{2} \sum_{n=1}^\infty \left\{ \log \left[ 1 + \frac{(\nu_j t + 2x)^2}{(\nu_j T)^2} \right] + \log \left[ 1 + \frac{(\nu_j t - 2x)^2}{(\nu_j T)^2} \right] \right\} \]

\[ = \frac{1}{2} \left\{ \sinh \pi T(t + 2x/v_j) \frac{\pi T x/v_j}{\sinh 2\pi T x/v_j} \right. \]

\[ + \left. \sinh \pi T(t - 2x/v_j) \frac{2\pi T x/v_j}{\sinh 2\pi T x/v_j} \right\}, \quad (A16) \]

\[ J_j(x,t) = \int_0^\infty q^2 \cos q x \frac{q}{q} (1 - e^{i\nu_j q t}) \]

\[ = \frac{1}{2} \left\{ \log \frac{\alpha - i(\nu_j t + 2x)}{\alpha + i2x} + \log \frac{\alpha - i(\nu_j t - 2x)}{\alpha + i2x} \right\}. \quad (A17) \]

Since \( I_j(\infty,t) = J_j(\infty,t) = 0 \), these results lead to Eqs. (19-22).
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