Phase Transitions in the Two-Dimensional Random Gauge XY Model

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The two-dimensional random gauge XY model, where the quenched random variables are magnetic bond angles uniformly distributed within \([-r\pi, r\pi]\) (0 \(\leq r \leq 1\)), is studied via Monte Carlo simulations. We investigate the phase diagram in the plane of the temperature \(T\) and the disorder strength \(r\), and infer, in contrast to a prevailing conclusion in many earlier studies, that the system is superconducting at any disorder strength \(r\) for sufficiently low \(T\). It is also argued that the superconducting-to-normal transition has different nature at weak disorder and strong disorder: termed Kosterlitz-Thouless (KT) type and non-KT type, respectively. The results are compared to earlier works.

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I. INTRODUCTION

The XY gauge glass model\textsuperscript{1} has attracted much interest in connection to the vortex glass phase of high-\(T_c\) superconductors.\textsuperscript{2,3} In three dimensions (3D) there is a general consensus that the XY gauge glass model exhibits a finite-temperature glass transition.\textsuperscript{4,5} However, in 2D there exist conflicting evidences: On the one hand, equilibrium studies of defect energy,\textsuperscript{6,7,8} Monte Carlo (MC) simulations of the root-mean-square current,\textsuperscript{9} and resistance calculation\textsuperscript{10} have suggested that no finite-\(T\) ordering exists. The glass order parameter has furthermore been analytically shown to vanish at any finite \(T\).\textsuperscript{11} On the other hand, the MC studies of the XY model without disorder,\textsuperscript{12,13} as well as dynamical simulations of the resistance\textsuperscript{14} and non-equilibrium relaxation\textsuperscript{11,15}, have indicated a possibility of a finite-\(T\) transition. Also the MC simulations of the helicity modulus in Ref.\textsuperscript{15} were interpreted as being compatible with a finite-\(T\) transition.

In this paper we study a generalization of the 2D XY gauge glass model—the random gauge XY model—where both the temperature \(T\) and disorder strength \(r\) can be varied. When \(r\) has the maximum value 1, it corresponds to the usual XY gauge glass model, while in the opposite limit of \(r = 0\), the standard XY model without disorder is recovered.\textsuperscript{16,17} It has been proposed that there is a Kosterlitz-Thouless (KT) like transition at a finite temperature \(T_c\) when the disorder strength is sufficiently small, and that as \(r\) is increased \(T_c\) becomes smaller until it vanishes as \(r\) reaches the critical disorder strength \(r_c\).\textsuperscript{16,17} However, even if the glass order parameter is zero and even if there is no finite-\(T\) KT transition for \(r > r_c\), the existence of a finite-\(T\) transition with a different character cannot \textit{a priori} be ruled out.

In the present paper we perform extensive MC simulations to study the phase transition of the random gauge XY model. It is found that the system is superconducting at any \(r\) and that the transition from normal to superconducting phase is consistent with a Kosterlitz-Thouless (KT) type at weak disorder and a non-KT type at strong disorder. In addition to this we suggest that there exist two different superconducting phases at sufficiently low temperatures separated by a non-KT phase transition (see Fig. 1). For the special case of \(r = 1\), which corresponds to the 2D XY gauge glass model, we also compute the root-mean-square current in the same way as in Ref.\textsuperscript{15} and, in contrast to Ref.\textsuperscript{15}, again consistently find a finite-temperature transition.
II. MODEL AND SIMULATIONS

The Hamiltonian of the 2D random gauge XY model on an \(L \times L\) square lattice under the fluctuating twist boundary condition (FTBC) is given by

\[
\hat{H} = -J \sum_{\langle ij \rangle} \cos \left( \phi_{ij} \equiv \theta_i - \theta_j - \frac{1}{L} \hat{r}_{ij} \cdot \Delta - A_{ij} \right),
\]

where \(J\) is the coupling strength (set to unity from now on), the sum is over nearest neighbor pairs, \(\hat{r}_{ij} \equiv \hat{x}(y)\) if \(j = i + \hat{x}(y)\). The phase angle \(\theta_i\) at the lattice point \(i\) satisfies the periodicity \(\theta_{i+L\hat{x}} = \theta_{i+L\hat{y}} = \theta_i\), and \(A_{ij} \in \mathbb{Z}[-\pi, \pi]\) is a uniform quenched random variable with the disorder strength \(0 \leq r \leq 1\). The twist variable \(\Delta = (\Delta_x, \Delta_y)\) corresponds to the global twist across the system, i.e., the summation of the gauge invariant phase difference \(\phi_{ij}\) along the \(x\) (\(y\)) direction equals \(\Delta_x\) (\(\Delta_y\)).

For a given disorder realization, we first compute the distribution \(P(\Delta)\), which is related to the free energy \(F\) by \(\partial F/\partial \Delta = -T(\partial \ln P/\partial \Delta).\) The twist variable \(\Delta_0\) which minimizes \(F\) (or maximizes \(P\)) is determined from \(P\) and then fixed when the helicity modulus \(\Upsilon \equiv \partial^2 F/\partial \Delta^2\) and the 4th order modulus \(\Upsilon_4 \equiv \partial^4 F/\partial \Delta^4\) are computed.

To ensure that the cooling is slow enough, we simultaneously cool two replicas \((\alpha, \beta)\) of the system and measure \(\Delta_0^{\alpha,\beta}\). For the first cooling at a new temperature we use 120000 update sweeps (for spin and twist variables respectively). Then we check that

\[
|\Delta_{x,0}^{\alpha} - \Delta_{x,0}^{\beta}| < \delta \text{ and } |\Delta_{y,0}^{\alpha} - \Delta_{y,0}^{\beta}| < \delta
\]

where \(\delta\) sets the precision of the cooling. The idea with the annealing condition Eq. \(2\) is to keep the system close to the lowest-energy state at a particular temperature (so the system does not freeze into a local minimum that bias \([\Upsilon]\) and \([\Upsilon_4]\)). Since the system moves more swiftly over the configuration space the higher the temperature is we can choose \(\delta\) increasing with temperature. A choice that proves good in practice is

\[
\delta = \begin{cases} 
0.02\pi & \text{for } T < 0.3 \\
0.15\pi & \text{for } 0.3 \leq T < 0.6 \\
\infty & \text{for } T \geq 0.6
\end{cases}
\]

If the annealing condition Eq. 2 fail we repeat the cooling with three times as many update sweeps; if it fails again we increase the number of cooling sweeps a factor three again, and so on until the condition is fulfilled. When \(\Delta_0\) is chosen, before cooling, we let the system run until Eq. 2 is fulfilled with \(\Delta_0\) replaced by \(\Delta\).

We repeat the above calculations for more than 500 different disorder realizations (the disorder average is denoted by \(\langle \cdot \rangle\) throughout this paper).

For the case of \(r = 1\), we also use the periodic boundary condition PBC, corresponding to \(\Delta = 0\) in Eq. 1, and compute the root-mean-square current defined by

\[
I_{\text{rms}} = \left( \frac{1}{L} \sum_{\langle ij \rangle} \sin \phi_{ij} \right)^{1/2},
\]

for comparisons with earlier works.

For \(I_{\text{rms}}\), for each disorder, we use 50000 update sweeps for thermalizations and 500000 sweeps for measurements where the actual measurement was performed every tenth sweep. We used this computer time saving strategy since we found that performing the measurement every sweep gave closely the same result.
where $[\Upsilon] \sim L^{-b}$ at the critical temperature $T_c$, and the critical exponent $\nu$ is related to the divergence of the coherence length. At $T_c$, the scaling function $f$ has the same value irrespective of $L$, implying that $L^b[\Upsilon]$ versus $T$ should have a unique crossing point at $T_c$ for various sizes. In Fig. 3(b), it is clearly shown that $[\Upsilon]$ has a scale-invariant behavior $[\Upsilon] \propto L^{-0.4}$ at a unique $T$, signaling a phase transition. The inset of Fig. 3(b) furthermore confirms that this scaling behavior is consistent with the standard form with $\nu \approx 1.1$.

Simple dimensional analysis for the non-disordered case, $r = 0$, gives for $[\Upsilon]$ the exponent $b = 0$ and from such a dimensional analysis one would likewise conclude that $b = 0$ also for the disordered case. Thus if $[\Upsilon]$ scales with $b \neq 0$ for the disordered case this is equivalent to the appearance of an anomalous dimension not accounted for by simple dimensional analysis. Our suggestion, based on the simulation results, is consequently that the disorder introduces such an anomalous dimension.

Figure 4 shows the root-mean-square current $I_{\text{rms}}$ for the PBC. We first note in Fig. 4(a) that the finite-size-scaling form for $T_c = 0$ used in Ref. 8, $I_{\text{rms}} = L^{-b/2}f(TL^{1/\nu})$ with $\nu = 2.2$, shows systematic deviations from the data collapse to a single scaling curve at lower temperatures, in contrast to what was concluded in Ref. 8, when more and better converged data are included. Furthermore a $T_c = 0$-collapse cannot be achieved with any value of $\nu$. On the other hand if we, in analogy with the finding for $[\Upsilon]$ above, use the scaling form

$$I_{\text{rms}} = L^{-b}f(L^{1/\nu}(T - T_c)),$$

which allows for the same anomalous dimension, one obtains the scaling plot in Fig. 4(b) with $b \approx 0.5$ and $T_c \approx 0.2$. From Figs. 4(a) and 4(b) we conclude that the 2D XY gauge glass ($r = 1$) exhibits a non-KT type superconducting-to-normal transition at $T_c \approx 0.2$ characterized by the existence of the anomalous dimension $b \approx 0.5$ and $\nu \approx 1.1$.

Next we investigate the phase transition with decreasing $r$. For $r = 0.9, 0.8, \cdots, 0.5$, we obtain the finite-size scaling plots of the same quality as in Fig. 3(b), and determine the phase boundary (the dashed line in Fig. 4). As $r$ is changed from 0.5 to 0.4 the exponents $b$ and $\nu$ exhibit quite rapid changes, $b$ from 0.27 to 0.06 and $\nu$ from 1.1 to 2.0. This, in our interpretation, reflects that near $r = 0.4$ the nature of the superconducting-normal transition changes from a non-KT type to a KT type.

For $r = 0$ the phase transition is of the KT nature and it has been suggested that this character should persist along the phase boundary up to some $r_c < 1$. The KT transition is characterized by that $[\Upsilon]$ jumps from a finite value $[\Upsilon]$ at $2T/\pi$ to zero as the phase line is crossed from the small $(T, r)$ region. For $r = 0$ we find that the KT transition is also characterized by the increase of the 4th order modulus $[|\Upsilon_4|] \propto L^c$ at $T_c$, with a positive exponent $c$, and $[\Upsilon_4]$ stays at a constant value below $T_c$ as $L$ is increased. Figure 4(a) shows $[\Upsilon_4]$ II.

III. RESULTS FROM SIMULATIONS

We first investigate the standard 2D XY gauge glass model, corresponding to the fully disordered case ($r = 1$), and show in Fig. 4(a) the helicity modulus $[\Upsilon]$ as a function of the system size $L$ for various temperatures. It is clearly shown that at high $T$, $[\Upsilon]$ goes to zero as the system size $L$ is increased. The crucial point is that, at low enough temperatures ($T \lesssim 0.2$), $[\Upsilon]$ changes its curvature in terms of $L$, and appears to saturate to a finite value as $L$ is increased. This behavior suggests a phase transition with a scale-invariant power law dependence at the critical temperature and a diverging characteristic length. Such a behavior can often be described by the finite-size scaling form

$$[\Upsilon] = L^{-b}f(L^{1/\nu}(T - T_c)),$$  \hspace{1cm} (5)

where $[\Upsilon] \sim L^{-b}$ at the critical temperature $T_c$, and the critical exponent $\nu$ is related to the divergence of the coherence length. At $T_c$, the scaling function $f$ has the same value irrespective of $L$, implying that $L^b[\Upsilon]$ versus $T$ should have a unique crossing point at $T_c$ for various sizes. In Fig. 3(b), it is clearly shown that $[\Upsilon]$ has a scale-invariant behavior $[\Upsilon] \propto L^{-0.4}$ at a unique $T$, signaling a phase transition. The inset of Fig. 3(b) furthermore confirms that this scaling behavior is consistent with the standard form with $\nu \approx 1.1$.

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$$I_{\text{rms}} = L^{-b}f(L^{1/\nu}(T - T_c)),$$  \hspace{1cm} (6)

which allows for the same anomalous dimension, one obtains the scaling plot in Fig. 4(b) with $b \approx 0.5$ and $T_c \approx 0.2$. From Figs. 4(a) and 4(b) we conclude that the 2D XY gauge glass ($r = 1$) exhibits a non-KT type superconducting-to-normal transition at $T_c \approx 0.2$ characterized by the existence of the anomalous dimension $b \approx 0.5$ and $\nu \approx 1.1$.

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for $T = 0.6$ as a function of $r$. The KT condition in the inset gives $r \approx 0.4$. This means that if the transition is of KT type then it would occur around $r \approx 0.4$. The onset of size dependence for $|\Theta_4|$ in Fig. 4(a) is consistent with a transition at around $r \lesssim 0.4$. Thus the transition is compatible with a KT-character, although a different character cannot be ruled out. The situation for $T = 0.1$ in Fig. 4(b) is quite different: the inset shows that the KT jump condition is not fulfilled and a KT transition can be ruled out. Yet there is a marked structure in $|\Theta|$ around $r \approx 0.4$. This structure corresponds to the onset of strong size dependence in $|\Theta_4|$. We interpret this onset as the reflection of a true divergence in $|\Theta_4|$ consistent with a phase transition. Thus we suggest that the whole boundary to the low- $T$ KT phase (solid line in Fig. 1) is reflected in a divergence of $|\Theta_4|$ and that this line ends at $(T = 0, r_c \approx 0.4)$. A phase line which ends at $(T = 0, r_c \approx 0.4)$ has been found in many earlier investigations (see e.g. Ref. 1 and references therein). The difference with earlier work is that in our case such a phase line is for lower temperatures between the two different superconducting phases SI and SII (as demonstrated by our direct calculation of $|\Theta|$), whereas earlier work have concluded that such a phases line does indeed exist but separates a superconducting phase from a normal phase all the way down to $T = 0$. Our suggested phase diagram is thus consistent with earlier work as to the existence of a phase line ending at $(T = 0, r_c \approx 0.4)$. For temperatures larger than the merging of the second line (above dashed line in Fig. 1) the transition is consistent with a KT transition although a different character cannot be ruled out. However, below the merging with this second line the transition is not a KT transition. It may be that there is a jump in $|\Theta|$ at this transition, but this is then between two non-vanishing values.

Based on the numerical evidences we suggest the structure of the phase diagram is sketched in Fig. 1. One striking feature is the finite-$T$ transition line between normal and superconducting phases, starting from the boundary of the low-$T$ KT-phase (solid line) and changing into the (dashed) line which ends at $T_c \approx 0.2$ at $r = 1$. The latter line (dashed line) is characterized by the appearance of an anomalous dimension $b$. Since the KT transition does not have such an anomalous dimension it follows that, if the transition along the boundary to the low-$T$ KT phase boundary has KT character, then $b$ should approach $b = 0$ when the two transition line merge. Thus the rapid drop from $b \approx 0.27$ to $b \approx 0.06$ as $r$ is changed from $r = 0.5$ to $0.4$ is consistent with a change over to a KT transition. Another interesting feature is that the phase line (solid line in Fig. 1), associated with the divergence of $|\Theta_4|$, continues even inside the superconducting region and ends at $r \approx 0.4$ for $T = 0$. Although the transition separating the KT phase and the normal phase (SI and N, respectively, in Fig. 1), may well be of the true KT type, manifested by a universal jump in the helicity modulus, the phase line separating SI and SII in Fig. 1 is not a KT transition and the helicity modulus has a non-zero value on both sides.

It is interesting to note that earlier works have found evidences for only two phases separated by a single phase line; either, in the more prevailing view, a phase line ending at a point $T = 0$ for a finite $r_c < 1$, or, in the less prevailing view, a phase line ending at a point $T > 0$ for $r = 1$. From our numerical simulations we instead suggest three distinct phases separated by two phase lines, which combines the two earlier proposed scenarios and provides a unified picture: On the one hand, in Ref. 1, the end point of the phase boundary to the low-$T$ KT-phase has been obtained to be $r_c \approx 0.37$ and $T = 0$, which is consistent with our cruder estimate $r \approx 0.4$. On the other hand, the phase transition point at $T \approx 0.22$ with $\nu = 1.1$, found in Refs. 12 and 13 for $r = 1$, is in very good agreement with the end point of our finite-$T$ phase line ($T \approx 0.2, \nu \approx 1.1$). However, the difference with the previous works is that according to our interpretation the phase line below $T \approx 0.2$ ending at $T = 0$ and $r_c \approx 0.4$ separates two distinct superconducting phases, while the whole phase line in Ref. 1 is for the superconducting to normal transition. A very hand-waving picture of the scenario of our phase diagram in terms of vortex motion.
is sketched in Fig. 11. In SI the vortex motion is suppressed by vortex pair binding, in N pair unbinding has occurred and free vortices exist which are not entirely pinned by the disorder, whereas in SII vortex pair unbinding has occurred but the vortices are pinned by the disorder.

IV. SUMMARY

One main conclusion from this phase diagram is that the XY gauge glass model (\( r = 1 \)) has a finite-\( T \) transition. How is this possible in view of earlier conflicting evidences? In Refs.5,6,7,8 it was concluded, on the basis of an analysis of data for the root-mean-square current for standard periodic boundary conditions, that no finite-\( T \) transition exists in 2D. We have found that, taking into account the possibility of an anomalous scaling dimension, \( I_{\text{rms}} \) displays a transition at a finite-temperature (see Fig. 13). Another puzzling evidence to the contrary is the \( T = 0 \) calculations of the size scaling of the defect energy.9,10,11. However, as discussed in Refs. 21 and 22, the local vorticity conservation must be properly taken into account when calculating the energy barriers. Thus the energy barrier for vortex dissipation increases with system size when taking the local vorticity conservation into account.23 This growing of the energy barrier for vortex dissipation with system size supports the possibility of a finite-\( T \) transition.

The appearance of a new superconducting phase for the XY random gauge model is intriguing. In particular, since it is neither a low \( T \) KT phase nor a phase with a finite glass order parameter. The true nature of this phase and the existence of similar phases in other related models are open questions.

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