Drag resistance of 2D electronic microemulsions

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Motivated by recent experiments of Pillai et al., Phys. Rev. Lett.\textbf{90}, 226801 (2003), we present a theory of drag in electronic double layers at low electron concentration. We show that the drag effect in such systems is anomalously large, it has unusual temperature and magnetic field dependences associated with the Pomeranchuk effect, and does not vanish at zero temperature.

Over the last decade a large number of dramatic phenomena have been discovered in studies of the two dimensional electron gas (2DEG) in MOSFETs and semiconductor heterojunction devices. Collectively, these phenomena are referred to under the euphonious title of the “2D apparent metal insulator transition.” They are associated with the behavior of the 2DEG in very clean (high mobility) devices with strong interactions, \textit{i.e.} when the ratio of the typical interaction strength to the Fermi energy, \( r_s = 1/\sqrt{na_F} \), is large, \( r_s \approx 10-40 \). (Here \( n \) is the electron density per unit area and \( a_F \) is the effective Bohr radius.) These observations make it clear that the physics of this simple, but conceptually vital system is much richer than was previously appreciated. Elsewhere, we have proposed that these phenomena are associated with a set of “electronic microemulsion phases” which occur as a consequence of Coulomb frustrated phase separation in the ideal (zero disorder) 2DEG at densities intermediate between the high densities, where the system is a Fermi liquid, and low densities, where it is a Wigner crystal.

In the present paper we concentrate on the drag resistance of 2D large \( r_s \) metals, with the goal of explaining the unexpectedly large value of the drag resistance and the unusual temperature and magnetic field dependences that have recently been reported in bilayer p-type GaAs heterojunction devices. In a bilayer device, in the presence of a dissipative current \( I_A \) in the first (active) layer, the inter-layer electron-electron interactions induce a momentum transfer between layers, which in turn produces a voltage, \( V_P \), across the second (passive) layer. In the linear regime this effect can be characterized by a drag resistance (per square)

\[ R_D = \frac{V_P}{I_A}. \] (1)

Measurements of \( R_D \) are a potentially useful probe of interesting correlations of an electron fluid. It is exceedingly small at low temperatures in a Fermi liquid while, as we shall see, it is parametrically larger in a “bubble fluid” phase. It is therefore an ideal diagnostic for the presence of such phases.

The existing theories of drag are mostly based on the Fermi liquid theory of the electron liquid in individual layers. A common feature is that, at small temperatures, \( R_D \) is small and vanishes as \( T \to 0 \) or as the spacing between layers, \( d \to \infty \):

\[ R_D \sim (\hbar/e^2)(k_F d)^{-\alpha_d}(T/E_F)^{-\alpha_T}. \] (2)

For a Fermi liquid, \( \alpha_T = 2 \) up to logarithmic corrections and \( \alpha_d = 2-4 \) depending on the value of \( k_F \ell \), where \( \ell \) is the elastic mean-free path and \( k_F \) is the Fermi momentum.

**Experimental Background:** The drag experiments that motivated this study are performed on high mobility samples whose conductances are significantly larger than \( e^2/\hbar \). Therefore, the putative electron localization length one would infer on the basis of localization theory is exponentially long, and hence the associated 2D localization phenomena can be ignored for present purposes.

In double layer devices with relatively small \( r_s \), measurements of the drag are in qualitative agreement with Fermi liquid theory. However, at large \( r_s \), experiments on p-GaAs double layers differ significantly from the predictions of this theory:

1. The drag resistance in these samples found to be 2-3 orders of magnitude larger than expected on the basis of Fermi liquid theory.

2. In the presence of a magnetic field, \( H_\parallel \), parallel to the film, the temperature dependence of \( R_D(T) \) is significantly suppressed. (Here, \( H_\parallel \) primarily couples to the spins of the electrons, rather than to their orbital motion.) Whereas in a Fermi liquid, \( R_D(T) \) is a quadratic function of \( T \), in large \( r_s \) devices, at \( H_\parallel = 0 \), the value of \( \alpha_T \) in Eq. \( \ref{eq:2} \) appears to be noticeably larger than \( 2 \): for example, \( \alpha_T = 2.7 \) in Refs. \textsuperscript{4} and \textsuperscript{5}. The value of \( \alpha_T(H_\parallel) \) then decreases with \( H_\parallel \) and saturates for \( H_\parallel > H^* \) at a value which is significantly smaller than \( 2 \); for example, in Refs. \textsuperscript{4} and \textsuperscript{5}, \( \alpha_T(H_\parallel > H^*) \sim 1.2 \). \( H^* \) is generally believed to be the field required to fully polarize the electron spins.

3. As a function of increasing \( H_\parallel \), \( R_D(H_\parallel) \) increases by a factor of 10-20 and saturates when \( H_\parallel > H^* \) (See Fig.1 in Ref.\textsuperscript{4}). In the framework of the Fermi liquid theory, one would expect a significant decrease of \( R_D(H_\parallel) \) in the spin polarized state because \( E_F \) is increased by a factor of \( 2 \) as \( H_\parallel \) increases from 0 to \( H_\parallel > H^* \), thus decreasing the electron-electron scattering rate.
4. The \( T \) and especially the \( H \parallel \) dependences of \( R_D(T, H \parallel) \) and the resistances of the individual layers \( R(H \parallel, T) \) look qualitatively similar to one another. (See Figs. 1, 2 in \[7\].) In fact, \( R(H \parallel, T) \), itself, appears highly anomalous from the viewpoint of Fermi liquid theory. Thus, although our primary focus will be on \( R_D \), we will also summarize the most salient anomalies in \( R(H \parallel, T) \). Since similar phenomena have been observed in studies of the 2DEG in MOSFETs and of the 2D hole gas (2DHG) in semiconductor heterojunction devices, we report observations from both systems:

5. An apparent continuous metal insulator transition as a function of \( n \) has been observed in Si MOSFET's \[1\] and to a lesser extend in GaAs heterojunctions \[4, 5\]. (No such transition is predicted on the basis of Fermi liquid theory.) For the most part, in this paper we will focus on the slightly higher density samples on the metallic side of the apparent transition.

6. The resistance of these samples increases significantly with increasing \( T \) by as much as a factor of 6 in Si MOSFET's \[1\] and as much as 3 in GaAs heterojunctions \[4, 5\].

7. The resistance of metallic samples at low \( T \) is significantly increased by non-zero \( H \parallel \), and becomes nearly \( H \parallel \) independent at high magnetic fields, \( H \parallel > H \ast \parallel \). Moreover, non-zero \( H \parallel \) strongly suppresses the \( T \) dependence of \( R \), so much so that \( R \) is nearly temperature independent for \( H \parallel > H \ast \parallel \). (To date, this latter effect has been documented only in Si MOSFET's \[2\], where \( dR/dT \) at low temperature is decreased for \( H \parallel > H \ast \parallel \) by as much as a factor of 100 relative to \( H \parallel = 0 \).)

**Theoretical Background:** The phase diagram of the bilayer 2DEG depends on many parameters. For simplicity, in this article we consider only the case when the distance between the layers, \( d \), is larger than the inter-electron distance so that the discrete nature of the electrons is irrelevant in the calculation of the interaction energy between electrons in different layers \[20\]. Moreover, we restrict ourselves to the case in which the “passive” layer has a large electron density, \( n_p \) (i.e. small \( r_s \)), while the “active” layer has a low electron density, \( n_A \) (i.e. \( r_s \sim r_c \)), where \( r_c \) is the critical value of \( r_s \) for the liquid-crystal transition — for a single band with an isotropic effective mass, \( r_c \) is estimated \[13\] to be \( r_c \approx 38 \). In this limit, the passive layer acts as a ground-plane, so the sequence of microemulsion phases in the active layer is the same as was derived previously \[7, 8\] for the 2DEG in a MOSFET. For zero quenched disorder, the FL to WC transition is always expected to be first order \[14\]. In the absence of long-range interactions, this would lead to a regime of density exhibiting two-phase coexistence; due to the screening by electrons in the passive layer, the electrons in the active layer interact via a dipolar interaction. It has been shown in \[7\] that in this case, first order transitions are forbidden because the energy of a long interface between the two phases is negative. As a result, there exists between the WC (for \( n_A < n_{WC}^c \)) and the FL (for \( n_A > n_{FL}^c \)), an intermediate range (\( n_{WC}^c < n_A < n_{FL}^c \)) of densities in which there is a set of new “microemulsion” phases which on the mean field level can be characterized as a mixture of microphase separated regions of liquid and crystal \[7, 8\]. The relative concentration, sizes, shapes, and organization of these regions is determined by thermodynamics. Even the full enumeration and classification of the distinct phases that result from Coulomb frustrated phase separation is not complete, much less a thorough investigation of their properties.

To be concrete, we will focus on the drag effect in the case when the active layer is in the bubble phase in which the majority phase is a FL, with a finite concentration of bubbles of WC. This phase can be viewed as a suspension of pieces of Wigner crystallites floating in an otherwise uniform Fermi fluid, like ice in a river. (Many of the same considerations apply to more general micro-emulsion phases, as we will discuss in a forthcoming paper \[17\].) As a function of decreasing, \( n_A \), the areal fraction of Wigner crystal, \( f_{WC} \), rises continuously from vanishingly small when \( n_A = n_{FL}^c \) to \( f_{WC} \) = 1 when \( n_A = n_{WC}^c \).

The WC bubble phase occurs when \( n_A \) is close to \( n_{FL}^c \) (See Figs. 1 and 2 in \[7\] and \[8\]). As \( n_A \to n_{WC}^c \), the bubble size approaches a constant value \( L_B = d \exp[\gamma] \), where \( \gamma \) is a (positive and order 1) dimensionless ratio of microscopic parameters which is proportional to the surface tension between the Wigner crystal and Fermi fluid phases. The spacing between bubbles \( [n_{FL}^c - n_A]^{-1/2} \) diverges as \( n_A \to n_{FL}^c \). At mean-field level, these bubbles, themselves, form a crystal of bubbles, but for sufficiently dilute bubbles, either quantum and classical fluctuations inevitably melt the bubble crystal. Nevertheless the charge inhomogeneities in the resulting bubble liquid are still very slowly fluctuating compared to the relaxation times of the Fermi liquid. The competition between the local tendency to phase separation and the Coulomb interaction (capacitive energy), results in a mean density difference between the WC and FL regions \( \Delta n \sim \sqrt{n_A}/d \).

The qualitative temperature and magnetic field dependence of \( f_{WC} \) is determined by the Pomeranchuk effect \[7, 8\]. Because the Wigner crystal has higher spin entropy density \( (S_{WC} \sim n \log(2)) \) than the Fermi liquid \( (S \sim n(T/E_F)) \), \( f_{WC} \) is an increasing function of temperature at low temperatures. This is a precise analogue of the Pomeranchuk effect in \( H \parallel \). Similarly, since the spins are substantially more polarizable in the Wigner crystal phase, \( f_{WC} \) is an increasing function of \( H \parallel \), but it saturates above a (temperature dependent) characteristic magnetic field strength, \( H^\ast \) at which the spins are fully polarized. Moreover, the two effects compete, so that when \( H \parallel > T \), the temperature dependence of \( f_{WC} \) is quenched. (Here, and henceforth, we will adopt units in which \( h = k_B = \) the Bohr magneton = 1.)

The strong dependence of \( f_{WC} \) on \( n_A, T \) and \( H \parallel \) is the origin of the strong dependences of all other properties of the system on these physical variables. To get a feeling
for the expected dependences, consider the case in which $T$ and $H_0$ are small enough that the Fermi liquid free energy can be well approximated by its zero temperature value, but large enough that we can ignore the (exponentially small) contributions in powers of $\sqrt{T}$ magnetic exchange energy in the Wigner crystal. In this case, and in the absence of disorder, the difference in free energy per unit area between the uniform WC and FL phases is

$$\Delta F(n_A, T, H_\parallel) = \Delta E(n_A) - n_AT \log[2 \cosh(H_\parallel/2T)].$$

(3)

Since $f_{WC}$ is determined by the competition between the local tendency to phase separation and the Coulomb interaction, it is a smooth function of $\Delta F$: we can obtain the $T$ dependence of $f_{WC}$ from the $T$ dependence of $\Delta F$ by the chain rule. For $H_\parallel = 0$, since $\Delta F$ is a linear function of $T$, it follows that $f_{WC}$ is as well.

**Drag in a dilute bubble liquid:** The reasons that $R_D$ is so small in a Fermi liquid are not hard to fathom. The fluctuations in the charge density in the active layer are small when $T/E_F$ is small, and are further reduced when averaged over a large length scale, $d \gg 1/\sqrt{n_A}$, hence the dependences exhibited in Eq. (3). In a microemulsion WC bubble phase, by contrast, there are large amplitude charge inhomogeneities on the length scale, $L_B$, which implies strong coupling to the electrons in the passive layer.

In the regime when the bubble concentration is small there are two types of the current carriers in the active layer: electrons and crystalline bubbles. At zero temperature the electron Fermi-liquid contribution to the drag resistance vanishes and the drag effect is entirely due to motion of Wigner crystallites. Each Wigner crystal bubble in the active layer casts an image potential in the passive layer. As bubbles in the active layer move with respect to the passive one, the electrons in the passive layer scatter on the moving image potential. To compute the drag resistance, we compute the mean force per unit area, $F_{PA}$, exerted on the electron fluid in the passive layer when a current $J$ is passed through the active layer. In the linear response regime, it is clear that $F_{PA} \propto n_B v_B$ where $n_B = f_{WC}/\pi L_B^2$ is the concentration of bubbles and $v_B$ is the mean drift velocity of the bubbles. Thus

$$R_D = \frac{1}{e^2} A_D \mu_B \frac{n_B}{n_A},$$

(4)

where $\mu_B \equiv v_B/\bar{v}$ is the relative mobility of the bubbles, $\bar{v} \equiv J/e n_A$ is the mean electron drift velocity in the active layer, and $A_D$ is a dimensionless constant (discussed below) which depends on the properties of the electron liquid in the passive layer.

In the absence of disorder, $\mu_B \rightarrow 1$. Moreover, as we will discuss below, $\mu_B$ remains non-zero in the presence of weak disorder so long as we are dealing with a bubble liquid phase. Thus, $R_D$ is similarly non-vanishing as $T \rightarrow 0$. Moreover, at low temperatures, the dominant $T$ and $H_\parallel$ dependences of $R_D$ are through the dependence of $f_{WC}$ on these variables - i.e. the Pomeranchuk effect discussed above.

The physics of the proportionality constant $A_D$ is in general, complicated - similar complexities occur as in the closely related problem of electro-migration in metals. To compute $A_D$ we need to assess the nature of the scattering potential induced in the passive layer by a bubble in the active layer. The long range tails of potential are screened away by the electrons in the passive layer. The magnitude of the potential, $V_B \sim ed\Delta n$, is determined by the magnitude of the charge deficit in the active layer, $\Delta n$, associated with the bubble. From the previously mentioned scaling of $\Delta n$ with $d$, it follows that the WKB barrier penetration factor, $W = \sqrt{meV_B L_B}$ is larger than 1, so the potential behaves like a hard object of radius $\sim L_B$ being dragged through the passive layer - electron tunnelling through the image potential is negligible. This is one of the key reasons that a bubble phase produces a large drag resistance.

It requires additional microscopic analysis to obtain an estimate of $A_D$, and there are numerous “regimes” depending on the relative values of $n_B^{1/2}$ (the mean spacing between bubbles), $L_B$, and the elastic and inelastic mean-free paths, $\ell_e$ and $\ell_{el}$, in the passive layer. The important general point is that $A_D$ is not generally small.

To give a concrete example, consider the case in which $n_B^{1/2} \gg \ell_e \gg L_B$ and $\ell_{el} \gg \ell_e$. In this case, each bubble acts as an uncorrelated scattering center with cross section proportional to $L_B$, and consequently

$$A_D = L_B n_B^{1/2},$$

(5)

In a future paper, we will report an analysis of $A_D$ in the various other regimes.

**Bubble mobility.** In the absence of quenched disorder, there are two distinct bubble phases - the bubble crystal and the bubble fluid. In the bubble liquid in the absence of pinning, the bubbles are swept along with the fluid, so that $\mu_B = 1$ and $R = R_D$. In the presence of weak disorder, we still expect that $\mu_B \approx 1$ at temperatures large compared to the pinning potential, although in this case, the additional channels for momentum relaxation in the active layer will generally result in $R > R_D$. The $T \rightarrow 0$ behavior of $\mu_B$ is a more subtle issue. At $\eta A d^2 \gg 1$ the bubbles are large compared to the spacing between electrons. To first order they can be treated as classical objects - at this level of approximation, they are pinned at favorable sites of the disorder potential. Quantum mechanically, however, the bubbles can tunnel from one pinning site to another. Moreover, the fact that the number of bubbles is only conserved on average affects the nature of this tunnelling process profoundly. Specifically, since the Fermi liquid degrees of freedom are "fast," we can integrate them out to obtain an effective action for the bubbles. (This can be done simply, using the method of Levitov and Shytov[18], as we will show in a forthcoming paper.) Among other interactions that
are generated by doing this is an effective bubble hopping matrix element, \( t(\vec{R}, \vec{R}') \sim |\vec{R} - \vec{R}'|^{-\beta} \) where \( \beta \sim 1/G \) and \( G \) is the conductance of the Fermi fluid in units of \( e^2 \). Physically, this derives from a process in which a bubble at pinning site \( \vec{R} \) virtually melts, the associated electron density deficit propagates through the Fermi fluid, and then recrystallizes at site \( \vec{R}' \). It is well known [10] that for \( \beta < D \), where \( D = 2 \) is the spatial dimension, Anderson localization does not occur. This establishes that, for clean enough systems, the bubbles, although large, are not localized. \( \mu_B \) is typically a fairly strongly increasing function of \( T \), but it approaches a non-zero value as \( T \to 0 \) if \( G \) is large enough!

**Qualitative comparison with experiment:** The theoretical model sketched above contains the ingredients necessary for a qualitative understanding of much concerning the listed experiments:

The drag resistance in the WC bubble phase \( R_D \) is not parametrically small, either in powers of \( 1/|k_F d| \) or of \( T/E_F \), as in the Fermi liquid, which accounts for point 1 in the above. According to Eq. 5 \( R_D \) is proportional to \( n_B \) and \( \mu_B \). Since \( L_0 \) is \( T \) and \( H_\parallel \) independent, \( n_B \propto f_{WC} \), so the general features of the \( T \) and \( H_\parallel \) dependances of \( R_D \) and \( R \), points 2, 3, 4, 6, and 7, follow directly from the Pomeranchuk effect, and the interplay between the \( T \) and \( H_\parallel \) dependance is readily understood from Eq. \[3\]. In particular, \( f_{WC} \) and, consequently, \( R_D \) are increasing function of \( H_\parallel \) which saturate at \( H_\parallel \sim \sim H^* \). When the spins are polarized by \( H_\parallel \), they no longer have any entropy, so the \( T \) dependence of \( f_{WC} \) and the corresponding contribution to the \( T \)-dependence of \( R_D(T) \) are quenched.

In the case of very small disorder \( \mu_B \) is close to unity and does not exhibit strong \( T \) and \( H_\parallel \) dependances. Then \( R(T,H_\parallel) \) is entirely determined by the aforementioned dependances of \( n_B \). The more detailed \( T \)-dependences of \( R_D(T,H_\parallel) \), points 2, cannot be discussed without a microscopic calculation of \( \mu_B(T,H_\parallel) \).

Since the dominant \( T \) and \( H_\parallel \) dependances of \( R_D \) and \( R \) derive from their implicit dependence of \( f_{WC} \), it is unsurprising that these dependances are similar.

It is an unavoidable consequence of the existence of microemulsion phases that, in the absence of quenched disorder, there is a sequence of *continuous* phase transitions (as opposed to a putative first order WC to FL transition) between different microemulsion phases and between these phases and the low and high density uniform phases. These phases become increasingly less conducting with decreasing \( n_A \), until the limiting WC phase is insulating. Thus, for weak enough disorder, an apparent metal insulator transition is inevitable, thus accounting for the observation that gave the subject its name, point 6. Since \( \mu_B \) does not vanish as \( T \to 0 \) the conceptually key conclusion of our analysis is that \( R_D \) does not vanish as \( T \to 0 \) in the WC bubble phase: \( R_D(T = 0) \neq 0 \). This fact has not been confirmed experimentally, perhaps because \( R_D(T = 0) \) is small.

Finally we would like to compare our approach with that of Ref. [21], to the experimental data on the drag resistance. The approach of Ref. [21] is supposed to work well at high electron concentration far from the regime of phase separation. Extending the results of [21] to the regime of low \( n \) one, in principle, can explain the large value of \( R_D \). On the other hand we suspect that the dramatic increase of \( R_D(H_\parallel) \) in the magnetic field and quenching it’s temperature dependence in low \( n \) samples can not be simply explained using that approach.

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