Asteroseismology of the DBV star CBS 114

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Abstract

Asteroseismology is a unique and powerful tool to investigate the internal structure of stars. CBS 114 is the sixth known pulsating DBV star. It was observed by Handler, Metcalfe, & Wood at the South African Astronomical Observatory over three weeks in 2001. Then, it was observed by Metcalfe et al. for seven nights (2004 Feb. 19–25) on the 1.8 m telescope at the Bohyunsan Optical Astronomy Observatory and seven nights (2004 Feb. 21–27) on the 2.1 m telescope at the McDonald Observatory. Totally two triplets, four doublets, and five singlets were identified. The frequency splitting values are very different, from 5.2 \( \mu \)Hz to 11.9 \( \mu \)Hz, which may reflect differential rotations. We evolve grids of white dwarf models by \textsc{MESA}. Cores, added with He/C envelopes, of those white dwarf models are inserted into \textsc{WDEC} to evolve grids of DBV star models. With those DBV star models, we calculate eigenperiods. Those calculated periods are used to fit observed periods. A best-fitting model is selected. The parameters are \( T_{\text{eff}} = 25000 \) K, \( M^* = 0.740 \) \( M_\odot \), and \( \log(M_{\text{He}}/M^*) = -4.5 \). With the relatively large stellar mass, the effective temperature is close to the previous spectroscopic result. In addition, kinetic energy distributions are calculated for the best-fitting model. We find that the observed modes with large frequency splitting values are fitted by the calculated modes with a large amount of kinetic energy distributed in the C/O core. After preliminary analysis, we suggest that the C/O core may rotate at least two times faster than the helium layer for CBS 114.

Key words: stars: oscillations (including pulsations) — stars: individual (CBS 114) — white dwarfs

1 INTRODUCTION

White dwarfs are the final evolutionary stage of about 98% of stars (Fontaine et al. 2001). Research on white dwarfs is significant in many fields. About 80% of white dwarfs show a hydrogen atmosphere (DA type) and about 20% of them show a helium atmosphere (DB type) (Bischoff-Kim & Metcalfe 2011).

Basically, there are no thermonuclear reactions occurring in white dwarfs. They are cooling down by eradiation. Along the cooling track, there are instability strips corresponding to DOVs (around 75 000 K to 170 000 K), DBVs (around 22 000 K to 29 000 K) and DAVs (around 10 800 K to 12 270 K) (Winget & Kepler 2008). The pulsation of white dwarfs makes it possible for us to study their internal structure.

Observed eigenfrequencies carry information about the internal structure of a pulsating star. Asteroseismology can be used to detect this type of information about the internal structure. White dwarfs are \( g \)-mode pulsators. Radial order \( k \), spherical harmonic degree \( l \) and azimuthal number \( m \) are used to characterize an eigenmode. Tassoul (1980) published an asymptotic theory for \( g \)-modes described by

\[
\Delta \bar{\nu}(l) = \frac{2\pi^2}{\sqrt{l(l+1)}} \int_0^R \frac{|N|}{\tau} dr.
\]

In Equation (1), \( R \) is stellar radius. The Brunt-Väisälä frequency \( N \) should be calculated as an absolute value. The parameter \( k \) cannot be identified from observations. It shows the nodes of the standing wave inside a star. According to Equation (1), relative radial orders may be counted from observations. Star rotation can cause the phenomenon of frequency splitting. An approximate formula that describes the relationship between frequency splitting (\( \delta \nu_{k,l} \)) and rotational period (\( P_{\text{rot}} \)) was reported by Brickhill (1975) as

\[
m\delta \nu_{k,l} = \nu_{k,l,m} - \nu_{k,l,0} = \frac{m}{P_{\text{rot}}} \left[ 1 - \frac{1}{l(l+1)} \right].
\]

In Equation (2), the parameter \( m \) is taken to be an integer from \( -l \) to \( l \). For \( l = 1 \), modes with \( m = -1, 0, +1 \) form
a triplet. For $l = 2$, modes with $m = -2, -1, 0, +1, +2$ form a quintuplet. According to Equation (2), the faster a star rotates, the larger the frequency splits. The frequency splitting can be used to study the phenomenon of star rotation. The values of different frequency splittings can be used to study differential rotation. Assuming low $k$ modes are more sensitive to the interior than high $k$ modes, Winget et al. (1994) reported that the envelope of GD 358 rotated some 1.8 times faster than its core. Studying rotational splitting inversions of observational data for GD 358, Kawaler et al. (1999) reported that the core rotated faster than its envelope.

GD 358 is the prototype of the DBV class. DBV stars are considered to be powerful probes of the energy loss rate for plasma neutrinos (Winget et al. 2004). Kim et al. (2006) studied the energy loss rate for plasma neutrinos by measuring the rate of change for the period of hot DBV stars. In addition, research on DBV stars is helpful to check the theory of stellar evolution. Dehner & Kawaler (1995) tried to study evolutionary connections between PG 1159 stars and DBV stars. Diffusion has an important effect on the evolution. The evolutionary scenario requires a Very Late Thermal Pulse (VLTP) burning off the residual hydrogen in the envelope during post-asymptotic giant branch (post-AGB) evolution. Iben et al. (1983) reported that about 20% of post-AGB stars would experience a VLTP when they descend to the white dwarf cooling track. However, it is difficult to interpret the origin of the ‘DB gap’ (Liebert et al. 1986). After the VLTP process, little hydrogen ($\log(M_H/M_\ast) \sim -11$) is left (Herwig et al. 1999). When the effective temperature cools to 30000 K, helium convection dilutes the hydrogen that is left in the photosphere. This helium convection may be interpreted in terms of the ‘DB gap’. Hence, asteroseismological study of DBV stars is significant and meaningful.

CBS 114 is the sixth known DBV star discovered by Winget & Claver (1988, 1989). Handler et al. (2002) (HMW2002) described their 65 h of single-site time-resolved CCD photometry targeting CBS 114. Totally, seven independent modes were identified. Some of them might need corrections of daily alias 11.60 µHz, as they reported. Taking a C-core, O-core and C/O-core into account, they conducted an asteroseismological study of those seven independent modes (assuming $l = 1$). In order to obtain more modes to constrain fitting models for CBS 114, a dual-site campaign using 2 m class telescopes was organized by Metcalfe et al. (2005) in February 2004. The previous seven modes were recovered and four new ones were discovered. With a pure carbon core and uniform He/C envelope, Metcalfe et al. (2005) made grids of white dwarf models to fit those $11 l = 1$ modes. The identified components show different frequency splitting values, from 5.2 µHz to 11.9 µHz, which may correspond to the phenomenon of differential rotation.

Paxton et al. (2011) published a description of Modules for Experiments in Stellar Astrophysics (MESA), which can evolve stars from the pre-main sequence to the white dwarf stage. We try to insert the core and envelope compositions of white dwarf models evolved by MESA into an early program, the White Dwarf Evolution Code (WDEC) (Wood 1990). With this method, grids of DBV star models are evolved with an element diffusion effect. We conduct an asteroseismological study of CBS 114 and try to study its differential rotation.

In Section 2, we introduce the input physics and model calculations. Model fittings on CBS 114 are shown in Section 3. In Sections 3.1, 3.2 and 3.3, we study mode identifications, fitting results and differential rotations associated with CBS 114 respectively. Finally, we provide some discussions and summarize our conclusions in Section 4.

### 2 INPUT PHYSICS AND MODEL CALCULATIONS

MESA is a new program to simulate star evolutions, which can evolve stars from the pre-main sequence stage to the white dwarf stage. The cores of white dwarfs evolved by MESA are results of thermonuclear burning. The opacities are from Cassisi et al. (2007). The thermonuclear reaction

![Figure 1](image1.png)

**Fig. 1** Abundance of a 0.614 $M_\odot$ white dwarf model evolved from a 2.8 $M_\odot$ main sequence star.

![Figure 2](image2.png)

**Fig. 2** Diagram of abundance and Brunt-Väisälä frequency for a DBV star. The model parameters are $T_{\text{eff}} = 25000$ K, $M_\ast = 0.600 M_\odot$ and $\log(M_{\text{He}}/M_\ast) = -3.0$.
rates are from Caughlan & Fowler (1988) and Angulo et al. (1999), see Paxton et al. (2011) for more details. We downloaded and installed the 4298 version of MESA. A module named ‘make_co WD’ (mesa/star/test_suite/make_co_wd) was used to evolve stars from the main sequence stage to the white dwarf stage. With default input settings (mixing length parameter $\alpha = 2.0$ and metal abundance $Z = 0.02$), we evolved stellar mass for main sequence stars from 2.0 $M_\odot$ to 5.0 $M_\odot$, as shown in the first column in Table 1. On the Hertzsprung-Russell diagram, we can trace their luminosity over the course of the evolution. When those stars enter the white dwarf cooling track, we stop their evolutions. The corresponding white dwarf masses are shown in the second column in Table 1. Those white dwarf models evolved with ‘make_co_wd’ usually have a thick helium layer and hydrogen atmosphere.

In Figure 1, we plot the abundance of a 0.614 $M_\odot$ white dwarf model which evolved from a 2.8 $M_\odot$ main sequence star. We remove the C/O core ($M_{\text{core}} = 0.595 M_\odot$). With this method, those core masses are listed in the third column in Table 1.

In Figure 1, we can see that there is an He/C envelope above the C/O core. We take the location where the He abundance is 0.5 as being the boundary of envelope/core. We define $M_{\text{en}}$ as the mass of the He/C envelope. Log($M_{\text{en}}/M_{\text{core}}$) equals $-1.754$ for the model in Figure 1. In the envelope, $X_{C(\text{en})}$ is defined as carbon abundance and $X_{\text{He}(\text{en})}$ is helium abundance.

In Figure 1, $X_{\text{C}(\text{en})}$ equals 0.185 and $X_{\text{He}(\text{en})}$ equals 0.785. The other 3% are abundances of $^{16}\text{O}$, $^{18}\text{O}$, $^{20}\text{Ne}$, $^{22}\text{Ne}$, $^{24}\text{Mg}$ and $^{28}\text{Si}$. For the helium layer in Figure 1, we can also see that the He abundance is a little smaller than 1.00. However, we only calculate $^1\text{H}(\text{H})$, $^4\text{He}(\text{He})$, $^{12}\text{C}(\text{C})$ and $^{16}\text{O}(\text{O})$ by WDEC. For approximate calculations, we take $X_{\text{C}(\text{en})} = 0.185$ and $X_{\text{He}(\text{en})} = 0.815$ for the model, as shown in the fifth column of Table 1.

WDEC is a historical program to simulate white dwarf evolutions, which is convenient to evolve grids of white dwarf models. The cores of white dwarfs evolved by WDEC are artificially constructed previously, such as full C, full O, homogeneous C/O profiles and so on. Therefore, we try to insert the core profiles of white dwarfs calculated by MESA into WDEC to evolve grids of white dwarf models.

The equation of state is from Lamb (1974) and Saumon et al. (1995). The radiative opacities and conductive opacities are from Itoh et al. (1983). The mixing length theory is from Bohm & Cassinelli (1971) and Tassoul et al. (1990). The ratio of mixing length to pressure scale height (\(\alpha\)), for a DBV star, is usually adopted around 1.25 (Bergeron et al. 2011; Montgomery 2007; Koester 2010). We adopt $\alpha = 1.25$ for calculations.

With MESA evolutions, we obtain a grid of white dwarf models. The core mass, corresponding envelope mass, carbon abundance and approximate helium abundance are shown in Table 1. Each core is used to evolve a group of white dwarfs by WDEC. For example, the 0.595 $M_\odot$ core is used to evolve white dwarfs from 0.580 $M_\odot$ to 0.605 $M_\odot$. The mass range of white dwarf models evolved by WDEC is shown in the fourth column of Table 1.

After MESA evolutions, we take out the structure parameters, including mass, radius, luminosity, pressure, temperature, entropy and carbon abundance. Those parameters are inserted into the prototype models in WDEC. The oxygen abundance approximately equals 1.000 minus the carbon abundance. The envelope masses as well as C and He abundances in the envelope are constrained by the last column in Table 1. The envelope masses are different in Table 1. There is no envelope for white dwarfs evolved from 3.8 $M_\odot$ or 5.0 $M_\odot$ main sequence stars. Core compositions and envelope abundances of white dwarfs evolved by MESA are used as input physics for WDEC. WDEC evolves white dwarfs from above 100,000 K to the DBV instability strip. A process describing element diffusion derived by Thoul et al. (1994) is incorporated into WDEC by Su et al. (2014). We take 500,000 years as the evolution step.

Grids of white dwarf models are evolved by WDEC. The stellar mass $M_* \approx 0.550 M_\odot$ to $0.850 M_\odot$ in steps of 0.005 $M_\odot$. The effective temperature is from 21,000 K to 30,000 K in steps of 200 K. The envelope

| MS ($M_\odot$) | W(D(MESA)) ($M_\odot$) | $M_{\text{core}}$(MESA) ($M_\odot$) | W(D(WDEC)) ($M_\odot$) | log($M_{\text{en}}/M_{\text{core}}$) | ($X_{\text{C}(\text{en})}, X_{\text{He}(\text{en})}$) |
|----------------|------------------------|----------------------------------|------------------------|--------------------------------|----------------------------------|
| 2.0            | 0.580                  | 0.558                            | 0.550–0.575            | $-1.714$ (0.155, 0.845)       |                                  |
| 2.8            | 0.614                  | 0.595                            | 0.580–0.605            | $-1.754$ (0.185, 0.815)       |                                  |
| 3.0            | 0.633                  | 0.616                            | 0.610–0.635            | $-1.938$ (0.171, 0.829)       |                                  |
| 3.2            | 0.659                  | 0.644                            | 0.640–0.665            | $-2.085$ (0.145, 0.855)       |                                  |
| 3.4            | 0.689                  | 0.675                            | 0.670–0.695            | $-2.430$ (0.025, 0.975)       |                                  |
| 3.6            | 0.723                  | 0.713                            | 0.700–0.725            | $-2.328$ (0.046, 0.954)       |                                  |
| 3.8            | 0.751                  | 0.742                            | 0.730–0.755            |                                  |                                  |
| 4.0            | 0.782                  | 0.774                            | 0.760–0.785            | $-2.430$ (0.025, 0.975)       |                                  |
| 4.5            | 0.805                  | 0.799                            | 0.790–0.815            | $-2.592$ (0.055, 0.945)       |                                  |
| 5.0            | 0.832                  | 0.827                            | 0.820–0.850            |                                  |                                  |
mass fraction is shown in the last column of Table 1 for each group of white dwarfs. The helium layer mass fraction ($\log(M_{\text{He}}/M_\ast)$) is from $-6.0$ to $-3.0$ in steps of $0.5$.

In Figure 2, we show a diagram of abundance and Brunt-Väisälä frequency for an evolved DBV star. The model parameters are $T_{\text{eff}} = 25\,000$ K, $M_\ast = 0.600 M_\odot$ and $\log(M_{\text{He}}/M_\ast) = -3.0$. Its core compositions and envelope abundances are evolved from the white dwarf model in Figure 1. The composition gradients in the lower panel of Figure 2 cause ‘spikes’ in the upper panel. Those ‘spikes’ may cause a mode trapping effect (Winget et al. 1981; Brassard et al. 1992). In addition, there is an extremely thin convection zone on the surface of the star. With those grids of DBV star models, we numerically solve the full equations of linear and adiabatic oscillation. Each eigenmode will be scanned out. Those calculated modes will be used to fit the observed modes of CBS 114.

3 MODEL FITTINGS ON CBS 114

In this section, we analyze the previously observed modes for CBS 114 and then apply model fittings to CBS 114. According to different frequency splitting values and distributions of kinetic energy for corresponding fitting modes, we try to study the differential rotation effect for CBS 114.

3.1 Mode Identifications for CBS 114

In Table 2, we show mode identifications for CBS 114 by HMW2002 and Metcalfe et al. (2005). HMW2002 identified seven independent modes. They were recovered by mode identifications of Metcalfe et al. (2005). In the second column, $F_{\text{ref}}$ denotes the frequencies identified by Metcalfe et al. (2005). In the parentheses, $F_{\text{ref}}$ in HMW denotes the frequencies identified by HMW2002. The minus sign means that the negative daily alias ($f_i - 11.60$ µHz) may also be the correct eigenfrequency. The plus sign means that the positive daily alias ($f_i + 11.60$ µHz) may also be the correct eigenfrequency, as reported in Table 2 of HMW2002. Amp2001 is the amplitude of modes identified by HMW2002. Amp1998 is the amplitude of modes re-analyzed by HMW2002 based on the observations of Winget & Claver (1988, 1989). Metcalfe et al. (2005) treated $f_{10} + 11.60$ µHz as the eigenmode. In addition to those seven modes, Metcalfe et al. (2005) also identified four new ones, $f_1, f_2, f_3$ and $f_6$.

Metcalfe et al. (2005) identified two triplets ($f_4, f_5$) and two doublets ($f_9, f_{11}$), as shown in Table 2. In addition, we also treat modes $f_7$ and $f_8$ as doublets, which are identified by both HMW2002 and Metcalfe et al. (2005). The frequency splitting value is shown in the third column in Table 2. The mode $f_7$ identified by HMW2002 may be three values, 1824.04 µHz, 1835.64 µHz and 1847.24 µHz. Therefore, the frequency splitting for $f_7$ may be 5.56 µHz, 6.04 µHz and 17.64 µHz. Those frequency splitting values in Table 2 are scattered, which is very important for the study of differential rotation.

3.2 Fitting Results for CBS 114

We use the grids of DBV star models to fit the 11 identified $m = 0$ modes. A formula for fitting the residuals is expressed as

$$\sigma_{\text{RMS}} = \sqrt{\frac{\sum_{i=1}^{n_{\text{obs}}} (\text{Per}_{\text{mod}} - \text{Per}_{\text{obs}})^2}{n_{\text{obs}}}}.$$  

In Equation (3), $n_{\text{obs}}$ is the number of observed modes. In this paper, $n_{\text{obs}}$ is 11. Per$_{\text{mod}}$ is the calculated periods and Per$_{\text{obs}}$ is the observed periods. The model with the smallest $\sigma_{\text{RMS}}$ is considered to be the best-fitting one.

In Figure 3, we show the color residual diagram for the fitting results. The bluer the color, the smaller the residual. The smallest $\sigma_{\text{RMS}}$ is 2.65 s which corresponds to the best-fitting model. The model parameters are $T_{\text{eff}} = 25\,000$ K,
Table 2 Mode identifications by HMW2002 and Metcalfe et al. (2005). Fre. denotes the pulsating frequency in µHz. Per. is the corresponding period in second (s). Fre. Spl. is the frequency splitting value.

| ID | Fre. (Fre. in HMW) | Fre. Spl. | Per. (Per. in HMW) | Amp (Amp2001) | l | m |
|----|--------------------|-----------|-------------------|--------------|---|---|
|    | (µHz)              | (µHz)     | (s)               | (mmag)       |   |   |
| f₁ | 1189.9             | 840.38    | 3.1               | 1 or 2       | 0?|   |
| f₂ | 1252.4             | 798.46    | 4.3               | 1 or 2       | 0?|   |
| f₃ | 1383.0             | 723.06    | 3.0               | 1 or 2       | 0?|   |
| f₄ | 1509.0             | 662.67    | 7.5               | 1            | -1|   |
| f₅ | 1518.8 (1518.75⁻)  | 658.44 (658.43) | 11.8 (16–33) (33) | 1 | 0 |   |
| f₆ | 1530.7             | 653.30    | 9.0               | 1            | +1|   |
| f₇ | 1601.2             | 624.55    | 4.4               | 1            | -1|   |
| f₈ | 1612.1 (1613.12)   | 620.31 (619.91) | 15.4 (10–17) (15) | 1 | 0 |   |
| f₉ | 1622.7             | 616.27    | 13.3              | 1            | +1|   |
| f₁₀| 1729.8 (1729.73)   | 578.09 (578.13) | 7.0 (22–37) (18)  | 1 or 2       | 0?|   |
| f₁₁| 1829.6 (1835.64⁴⁻) | 546.56 (544.77) | 3.0 (<4.2) (13)  | 1            | 0?|   |
| f₁₂| 1963.2 (1969.60)   | 609.37 (507.71) | 5.5 (11–17) (<5) | 1            | 0?|   |
| f₁₃| 2086.5             | 479.27    | 4.8               | 1            | -1|   |
| f₁₄| 2093.6             | 477.65    | 4.9               | 1            | 0?|   |
| f₁₅| 2317.7 (2306.38⁺)  | 431.45 (433.58) | 8.9 (4–12) (13)  | 1 or 2       | 0?|   |
| f₁₆| 2510.0 (2509.89⁻)  | 398.41 (398.42) | 6.8 (4–10) (9)   | 1            | 0?|   |

Table 3 The best-fitting models for asteroseismological study by HMW2002, Metcalfe et al. (2005), us and spectroscopic study by Beauchamp et al. (1999) and Kleinman et al. (2013).

| T_{eff}(K) | log g   | M/M_☉ | log(M_{He}/M_*) |
|------------|---------|-------|-----------------|
| HMW2002(C-core) | 24 600  | 0.655 | -3.96          |
| HMW2002(O-core) | 25 800  | 0.640 | -3.96          |
| HMW2002(C/O core) | 21 000  | 0.730 | -6.66          |
| Metcalfe et al.(pure C) | 25 800  | 0.630 | -5.96          |
| This paper   | 25 000  | 8.287 | 0.740          |
| Beauchamp et al.(no H) | 26 200  | 8.00  | -4.5           |
| Beauchamp et al.(H) | 23 300  | 7.98  | -5.96          |
| Kleinman et al. | 25 754±368 | 7.95±0.037 | -3.96         |

M_*= 0.740 M_☉ and log(M_{He}/M_*) = -4.5. The model has a gravitational acceleration of log g = 8.287. In Table 1, we can see that the 0.740 M_☉ white dwarf is from the 3.8 M_☉ main sequence star, which has no He/C envelope.

In Figure 4, we show a diagram of abundance and Brunt-Väisälä frequency for the best-fitting model. In the lower panel, there is a C/O envelope above the C/O core, instead of a He/C envelope.

In Table 3, we show best-fitting models of the asteroseismological study by HMW2002, Metcalfe et al. (2005), us and the spectroscopic study by Beauchamp et al. (1999) and Kleinman et al. (2013). Fitting seven independent modes with C-core and O-core white dwarfs, HMW2002 obtained their similar best-fitting models. The parameters are T_{eff} = 24 600 K, M_*= 0.655 M_☉, log(M_{He}/M_*) = -3.96 and T_{eff} = 25 800 K, M_*= 0.640 M_☉, log(M_{He}/M_*) = -3.96 respectively.
Moreover, with C/O core white dwarfs, they obtained a best-fitting model of low effective temperature (21 000 K), large stellar mass (0.730 $M_\odot$) and a thin helium atmosphere ($\log(M_{\text{He}}/M_*) = -6.66$). Fitting 11 independent modes with pure carbon core white dwarfs, Metcalfe et al. (2005) obtained a best-fitting model of $T_{\text{eff}} = 25 800$ K, $M_\star = 0.630$ $M_\odot$ and $\log (M_{\text{He}}/M_*) = -5.96$. For spectroscopic work, Beauchamp et al. (1999) obtained a best-fitting model of low effective temperature (21 000 K), with pure carbon core white dwarfs. Metcalfe et al. (2005) obtained their best-fitting model with a small fitting error. However, the mode of $f_6$ can only be fitted by an $l = 2$ mode. Even when fitted by an $l = 2$ mode, the fitting error is also not small (–5.477 s). We doubt whether $f_6$ is an $m = 0$ component.

Assuming those 11 modes to be $l = 1$ and $m = 0$, Metcalfe et al. (2005) obtained their best-fitting model with $\sigma_{\text{RMS}} = 2.33$ s. They did not show detailed calculated modes or fitting errors. We do not know their fitting result for $f_6$. For those 11 modes, there is a good mean period spacing of 36.534 s. HMW2002 showed a mean period spacing of 37.1±0.7 s.

In Equation (4), $\rho_0$ is the local density and $R_{\text{He/C}}$ is the location of the He/C interface. For the best-fitting model, $R_{\text{He/C}}$ is located at $\log(1-M_*/M_\star) = -4.5$. $\xi_c(r)$ is the radial displacement and $\xi_h(r)$ is the horizontal displacement. The 11 observed modes are fitted by nine $l = 1$ modes and two $l = 2$ modes. The mode $f_{10}$ is fitted by 432.710 s $(l = 2, k = 17)$ in Table 4, which can also be fitted by the $l = 1$ mode 434.502 s $(l = 1, k = 9)$ with a small fitting error. However, the mode of $f_6$ can only be fitted by an $l = 2$ mode. Even when fitted by an $l = 2$ mode, the fitting error is also not small (–5.477 s). We doubt whether $f_6$ is an $m = 0$ component.

In Figure 5, the filled dots show the observed 11 modes. They present a good straight line versus the calculated radial orders. The open dots show the calculated $l = 1$ modes of the best-fitting model. On the whole, the open dots match the fitted dots well. In detail, $f_6$ is badly fitted. However, it is impossible to rule out an $l = 2$ mode in the $l = 1$ sequence. It is also impossible to rule out that $f_6$ is an $m \neq 0$ mode. Therefore, a relatively reliable identification of $f_6$ requires more observations.

The percentage of kinetic energy distributions for $l = 1$ modes is shown in the fourth column in Table 4. In the parenthesis of the second column, the frequency splitting values are displayed again. We notice that the observed mode $f_{11}$ with a frequency splitting of 5.2 $\mu$Hz corresponds to the calculated $l = 1$, $m = 8$ mode. This mode has 56% of the kinetic energy distributed in the He layer. The observed mode $f_8$ with a frequency splitting of 6.40 $\mu$Hz corresponds to the calculated $l = 1$, $m = 11$ mode. This mode has 48% of the kinetic energy distributed in the He layer.

### 3.3 Differential Rotations for CBS 114

In Table 4, we show observed periods, calculated periods, fitting errors and percentage of kinetic energy distributions. The percentage of kinetic energy distributed in the C/O core and He layer is calculated by the equation

$$\frac{K_{\text{In core}}}{K_{\text{In He}}} = \frac{4\pi \int_{0}^{R_{\text{He/C}}} \left[ (\xi_c(r))^2 + l(l+1)(\tilde{\xi}_h(r))^2 \right] \rho_0 r^2 dr}{4\pi \int_{0}^{R_{\text{He/C}}} \left[ (\xi_c(r))^2 + l(l+1)(\tilde{\xi}_h(r))^2 \right] \rho_0 r^2 dr}.$$  \hspace{1cm} (4)
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| ID | \( \text{Per}_{\text{obs}} (s) \) | \( \text{Per}_{\text{mod}} (s) \) | \( \text{Kin}_{\text{core}}, \text{Kin}_{\text{He}} \) | \( \text{Per}_{\text{obs}} - \text{Per}_{\text{mod}} (s) \) |
|----|-------------------------------|-------------------------------|---------------------------|-----------------------------|
| 1  | 398.41 (5.2)                  | 396.681 (1.8)                | (44%, 56%)                | 1.729                       |
| 2  | 431.45                        | 432.710 (2.17)               | (78%, 22%)                | -1.260                      |
| 3  | 477.65 (7.1)                  | 474.397 (1.10)               | (66%, 34%)                | 3.253                       |
| 4  | 509.37 (6.40)                 | 505.517 (1.11)               | (52%, 48%)                | 3.867                       |
| 5  | 546.56 (17.64?)               | 544.374 (1.12)               | (74%, 26%)                | 2.186                       |
| 6  | 578.09                        | 583.567 (2.24)               | (68%, 32%)                | -5.477                      |
| 7  | 620.31 (10.6,10.9)            | 617.807 (1.14)               | (67%, 33%)                | 2.503                       |
| 8  | 658.44 (11.9, 9.8)            | 659.540 (1.15)               | (68%, 32%)                | -1.100                      |
| 9  | 723.06                        | 724.714 (1.17)               | (76%, 24%)                | -1.654                      |
| 10 | 798.46                        | 800.004 (1.19)               | (60%, 40%)                | -1.544                      |
| 11 | 840.38                        | 840.123 (1.20)               | (73%, 27%)                | 0.257                       |

Table 4: Table of fitting results. Per\(_{\text{obs}}\) and Per\(_{\text{mod}}\) denote observed periods and calculated periods respectively. Kin\(_{\text{core}}\) and Kin\(_{\text{He}}\) denote percentage of kinetic energy distributed in the C/O core and He layer for calculated modes respectively.

In this paper, we evolve grids of white dwarf models by MESA. Metcalfe et al. (2005) made dual-site observations of CBS 114 and identified 11 independent modes. The \( m = 0 \) modes in two triplets show higher amplitudes. Therefore, the higher amplitude modes in doublets are identified as \( m = 0 \) modes. The singlets are identified as \( m = 0 \) modes. Triplets and doublets are identified as \( l = 1 \) and singlets are identified as \( l = 1 \) or \( 2 \) by us.

Fitting the 11 identified modes, we obtain a best-fitting model of \( T_{\text{eff}} = 25\,000 \, \text{K} \), \( M_\star = 0.740 \, M_\odot \) and \( \log(M_{\text{He}}/M_\star) = -4.5 \). The residual is 2.65 s and the gravitational acceleration is \( \log g = 8.287 \). The effective temperature is close to the previous spectroscopic result of 25754\pm 368 K. However, the gravitational acceleration is obviously larger than the spectroscopic result of 7.95\pm 0.037. In fact, our evolved DBV star models are contradictory to the ‘DB gap’ phenomenon. In future work, we will try to evolve DBV star models with a little hydrogen left in order to pass the ‘DB gap’. The effect of convective helium dilution will change those DA stars into DB stars when \( T_{\text{eff}} \) cools to 30\,000 K, as discussed in the Introduction. Kleiman et al. (2004) reported that there was a relative overabundance of DA stars inside the ‘DB gap’ according to SDSS DR1. The DBV star models with a thin hydrogen atmosphere may have a chance to solve the problem of large gravitational acceleration.

Finally, we study the frequency splitting values for observed modes and the kinetic energy distributions for best-fitting modes. The observed modes with large frequency splitting values correspond to the calculated modes with a large amount of kinetic energy distributed in the C/O core. The frequency splitting values of \( f_4 \) and \( f_5 \) are basically two times that of \( f_{11} \), as shown in Table 4. We suggest that the C/O core may rotate at least two times faster than the He layer for CBS 114.
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