Supra semi-compactness via supra topological spaces

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ABSTRACT
In this paper, we utilize a supra semi-open sets notion to introduce and study the concepts of supra semi-compact (supra semi-Lindelöf) spaces, almost supra semi-compact (almost supra semi-Lindelöf) spaces and mildly supra semi-compact (mildly supra semi-Lindelöf) spaces in supra topological spaces. We investigate some properties of supra semi-closed and supra semi-clopen subsets of these spaces and we give the equivalent conditions for the concepts of supra semi-compact (supra semi-Lindelöf) spaces and mildly supra semi-compact (mildly supra semi-Lindelöf) spaces. With the help of examples, we illustrate the relationships among these concepts. We also derive some results which associate these spaces with some mappings.

1. Introduction
In 1963, Levine [1] introduced and studied a notion of semi-open sets in topological spaces. Mildly and almost compact spaces [2, 3] were introduced in 1974 and 1975, respectively. Dorsett [4] in 1980, presented a concept of semi-compact spaces and Mashhour et al. [5] in 1983, formulated a supra topological spaces concept and investigated some of its properties. In 2006, Min [6] introduced a notion of p-supracompactness by using the convergence of ultrapastakes and investigated some of its properties, and in 2013, Mustafa [7] introduced the concepts of supra b-compact and supra b-Lindelöf spaces. Al-shami [8] initiated some results concerning supra topologies and presented some types of supra compact spaces. He [9] formulated six new kinds of supra compact spaces by utilizing a supra α-open sets notion. In [10, 11], he investigated some concepts related to supra semi-open sets in supra topological spaces and supra topological ordered spaces, respectively.

The main purpose of this work is to introduce and study the concepts of supra semi-compact (supra semi-Lindelöf) spaces, almost supra semi-compact (almost supra semi-Lindelöf) spaces and mildly supra semi-compact (mildly supra semi-Lindelöf) spaces in supra topological spaces by using a notion of supra semi-open sets. The equivalent conditions for the concepts of supra semi-compact (supra semi-Lindelöf) spaces and mildly supra semi-compact (mildly supra semi-Lindelöf) spaces are given and investigated. Some results about the image of these spaces under some maps are discussed and some illustrative examples are supplied to show the relationships among these concepts.

2. Preliminaries
Here are some definitions and results required in the sequel.

Definition 2.1 ([5]): A collection μ of subsets of 2^X is said to be a supra topology on X if μ contains X and the union of an arbitrary family of sets in μ belongs to μ. A pair (X, μ) is called a supra topological space. Every member of μ is said to be supra open and its complement is said to be supra closed.

Remark 2.1: Throughout this work, (X, τ) and (X, μ) denote to a topological space and a supra topological space, respectively, and the notations N, Z, Q and R stand for the set of natural numbers, the set of integer numbers, the set of rational numbers and the set of real numbers, respectively.

Definition 2.2 ([5]): Let E be a subset of (X, μ). Then the supra closure of E, denoted by cl^μ(E), is the intersection of all supra closed sets containing E and the supra interior of E, denoted by int^μ(E), is the union of all supra open sets contained in E.

Definition 2.3 ([10]): A subset E of (X, μ) is called supra semi-open if E ⊆ cl^μ(int^μ(E)). The complement of a supra semi-open set is called supra semi-closed.
Definition 2.4 ([10]): Let $E$ be a subset of $(X, \mu)$. Then the supra semi-closure of $E$, denoted by $\text{cl}_S^\mu(E)$, is the intersection of all supra semi-closed sets containing $E$ and the supra semi-interior of $E$, denoted by $\text{int}_S^\mu(E)$, is the union of all supra semi-open sets contained in $E$.

Definition 2.5 ([10]): A map $g : X \to Y$ is said to be:

(i) Supra semi-continuous (resp. Supra semi-irresolute) if the inverse image of each open (resp. supra open) subset of $Y$ is a supra semi-open subset of $X$.
(ii) Supra semi-open (resp. Supra semi-closed) if the image of each open (resp. closed) subset of $X$ is a supra semi-open (resp. supra semi-closed) subset of $Y$.

Theorem 2.6 ([10]): For a map $g : X \to Y$, we have the following two results:

(i) $g$ is supra semi-irresolute if and only if $\text{cl}_S^\mu(g^{-1}(G)) \subseteq g^{-1}(\text{cl}_S^\mu(G))$, for each $G \subseteq Y$.
(ii) $g$ is supra semi-closed if and only if $\text{cl}_S^\mu(g(H)) \subseteq g(\text{cl}_S^\mu(H))$, for each $H \subseteq X$.

Definition 2.7 ([8]): A supra topological spaces $(X, \mu)$ is said to be:

(i) Supra compact (resp. Supra Lindelöf) if every supra open cover of $X$ has a finite (resp. countable) subcover.
(ii) Almost supra compact (resp. Almost supra Lindelöf) if every supra open cover of $X$ has a finite (resp. countable) sub-collection, the supra closure of whose members cover $X$.
(iii) Mildly supra compact (resp. Mildly supra Lindelöf) if every supra clopen cover of $X$ has a finite (resp. countable) subcover.

Definition 2.8: A collection $\Lambda$ of sets is said to have the finite intersection property (resp. countable intersection property) if every finite (resp. countable) sub-collection of $\Lambda$ has a non-empty intersection.

3. Supra semi-compact spaces

In this section, we introduce the concepts of supra semi-compact and supra semi-Lindelöf spaces and study the equivalent conditions for them.

Definition 3.1: A collection $\{G_i : i \in I\}$ of supra semi-open subsets of $(X, \mu)$ is called a supra semi-open cover of a subset $E$ of $X$ provided that $E \subseteq \bigcup_{i \in I} G_i$.

Definition 3.2: A supra topological spaces $(X, \mu)$ is called supra semi-compact (resp. supra semi-Lindelöf) provided that every supra semi-open cover of $X$ has a finite (resp. countable) subcover.

Now, we present two examples, the first one satisfies a concept of supra semi-compactness and the second one does not satisfy.

Example 3.3: Let $\mu = \{\emptyset, G \subseteq Z \text{ such that } G^c \text{ is finite}\}$ be a supra topology on $Z$. In this supra topological space, we observe that a subset of $Z$ is supra semi-open if it is supra open. Then $(Z, \mu)$ is a supra semi-compact space.

Example 3.4: Let $\mu = \{\emptyset, Q, G \subseteq Q : 1 \in G \text{ or } 2 \in G : 3 \in G^c\}$ be a supra topology on $Q$. A collection $\Lambda = \{\{1, x\} : x \in Q\}$ forms a supra semi-open cover of $Q$. Since $\Lambda$ has not a finite subcover of $Q$, then $(Q, \mu)$ is not a supra semi-compact space.

The proofs of the following two propositions are straightforward and so will be omitted.

Proposition 3.5: Every supra semi-compact space is supra semi-Lindelöf.

Proposition 3.6: Every supra semi-compact (resp. supra semi-Lindelöf) space is supra compact (resp. supra Lindelöf).

It can be seen from Example 3.4 that the converse of Proposition 3.5 fails. Also, if we replace $Q$ by $\mathbb{R}$ in Example 3.4, we obtain that $(\mathbb{R}, \mu)$ is a supra compact space, whereas it is not supra semi-Lindelöf. So the converse of Proposition 3.6 fails as well.

Proposition 3.7: Any finite (resp. countable) supra topological space $(X, \mu)$ is supra semi-compact (resp. supra semi-Lindelöf).

Proof: It is well known that the largest cover of any set $X$ consists of the singleton subsets of $X$. So if $X$ is finite (resp. countable), then the largest cover of $X$ is finite (resp. countable). Hence the desired result is proved.

Definition 3.8: A subset $E$ of $(X, \mu)$ is said to be supra semi-compact (resp. supra semi-Lindelöf) relative to $X$ if every supra semi-open cover of $E$ is reducible to a finite (resp. countable) subcover.

Proposition 3.9: A finite (resp. countable) union of supra semi-compact (resp. supra semi-Lindelöf) subsets of $(X, \mu)$ is supra semi-compact (resp. supra semi-Lindelöf).

Proof: Straightforward.

Proposition 3.10: Every supra semi-closed subset of a supra semi-compact (resp. supra semi-Lindelöf) space $(X, \mu)$ is supra semi-compact (resp. supra semi-Lindelöf).
Proof: Let \( \{G_i : i \in I\} \) be a supra semi-open cover of a supra semi-closed subset \( F \) of \( X \). Then \( F^c \) is a supra semi-open set and \( F \subseteq \bigcup_{i \in I} G_i \). Therefore \( X = \bigcup_{i \in I} G_i \cup F^c \). Since \( (X, \mu) \) is supra semi-compact, then \( X = \bigcup_{i \in I} G_i \cup F^c \). So \( F \subseteq \bigcup_{i \in I} G_i \). Hence \( F \) is a supra semi-compact set.

The proof is similar in case of a supra semi-Lindelöf space.

It can be seen from Example 3.4 that \( \{1, 2\} \) is a supra semi-compact set. But it is not a supra semi-closed set. So the converse of Proposition 3.10 fails.

**Theorem 3.11:** A supra topological space \( (X, \mu) \) is supra semi-compact (resp. supra semi-Lindelöf) if and only if every collection of supra semi-closed subsets of \( X \) satisfies the finite (resp. countable) intersection property, has, itself, a non-empty intersection.

**Proof:** We prove the theorem in case of supra semi-compactness and the case between parentheses made similarly.

Necessity: Let \( \Lambda = \{F_i : i \in I\} \) be a collection of supra semi-closed subsets of \( X \) which has the finite intersection property. Assume that \( \bigcap_{i \in I} F_i = \emptyset \). Then \( F = \bigcup_{i \in I} F_i^c \). Since \( X \) is supra semi-compact, then \( X = \bigcup_{i \in I} F_i^c \). Therefore \( \bigcap_{i \in I} F_i = \emptyset \). But this contradicts that \( \Lambda \) has the finite intersection property. Thus \( \Lambda \) has, itself, a non-empty intersection.

Sufficiency: Let \( \{G_i : i \in I\} \) be a supra semi-open cover of \( X \). Suppose, to the contrary, that \( \{G_i : i \in I\} \) has no finite sub-cover. Then \( X \setminus \bigcup_{i=1}^{n} G_i \neq \emptyset \), for any \( n \in \mathbb{N} \). Now, \( \bigcap_{i=1}^{n} G_i^c = \emptyset \). This implies that \( \{G_i^c : i \in I\} \) is a collection of supra semi-closed subsets of \( X \) which has the finite intersection property. Therefore \( \bigcap_{i \in I} G_i^c \neq \emptyset \). Thus \( X \neq \bigcup_{i \in I} G_i \). But this contradicts that \( \{G_i : i \in I\} \) is a cover of \( X \). Hence \( (X, \mu) \) is a supra semi-compact space.

**Proposition 3.12:** If \( A \) is a supra semi-compact (resp. supra semi-Lindelöf) subset of \( X \) and \( B \) is a supra semi-closed subset of \( X \), then \( A \cap B \) is supra semi-compact (resp. supra semi-Lindelöf).

**Proof:** Let \( \Lambda = \{G_i : i \in I\} \) be a supra semi-open cover of \( A \cap B \). Then \( A \subseteq \bigcup_{i \in I} G_i \cup B^c \). Since \( A \) is supra semi-Lindelöf, then there exists a countable set \( S \) such that \( A \subseteq \bigcup_{i \in S} G_i \cup B^c \). Therefore \( A \cap B \subseteq \bigcup_{i \in I \cap S} G_i \). Thus \( A \cap B \) is a supra semi-Lindelöf set.

Similarly, one can prove the proposition in case of a supra semi-compact space.

**Theorem 3.13:** The supra semi-irresolute image of a supra semi-compact (resp. supra semi-Lindelöf) set is supra semi-compact (resp. supra semi-Lindelöf).

**Proof:** Let \( g : X \to Y \) be a supra semi-irresolute map and let \( A \) be a supra semi-compact subset of \( X \).

Suppose that \( \{G_i : i \in I\} \) is a supra semi-open cover of \( g(A) \). This automatically implies that \( A \subseteq \bigcup_{i \in I} g^{-1}(G_i) \). Since \( g \) is supra semi-irresolute, then \( g^{-1}(G_i) \) is a supra semi-open set, for each \( i \in I \).

By hypotheses, \( A \) is supra semi-compact, then \( A \subseteq \bigcup_{i=1}^{n} g^{-1}(G_i) \). So \( g(A) \subseteq \bigcup_{i=1}^{n} G_i \). Hence \( g(A) \) is supra semi-compact.

A similar proof can be given for the case between parentheses.

**Theorem 3.14:** If \( g : X \to Y \) is a bijective supra semi-open map and \( \Lambda \) is a supra semi-compact (resp. supra semi-Lindelöf) space, then \( \Lambda \) is compact (resp. Lindelöf).

**Proof:** Let \( \{G_i : i \in I\} \) be an open cover of \( X \). Then \( g(X) = g(\bigcup_{i \in I} G_i) \). Therefore \( Y = \bigcup_{i \in I} g(G_i) \). Since \( g(G_i) \) is a supra semi-open set, for each \( i \in I \) and \( Y \) is a supra semi-compact space, then \( Y = \bigcup_{i=1}^{n} g(G_i) \). Since \( g \) is injective, then \( X = \bigcup_{i=1}^{n} G_i \). Thus \( X \) is compact.

A similar proof can be given for the case between parentheses.

**Corollary 3.15:** If \( g : X \to Y \) is an injective supra semi-open map and \( g(X) \) is a supra semi-compact (resp. supra semi-Lindelöf) space, then \( X \) is compact (resp. Lindelöf).

4. Almost supra semi-compact spaces

In this section, the concepts of almost supra semi-compact and almost supra semi-Lindelöf spaces are formulated and their properties are investigated with the help of examples.

**Definition 4.1:** A supra topological spaces \( (X, \mu) \) is called almost supra semi-compact (resp. almost supra semi-Lindelöf) provided that every supra semi-open cover of \( X \) has a finite (resp. countable) sub-collection, the supra semi-closure of whose members cover \( X \).

In what follows, we give two examples, the first one satisfies a concept of almost supra semi-compactness and the second one does not satisfy.

**Example 4.2:** Let \( \mu = \{\emptyset, G_n = \{1, 2, \ldots, n\} : n \in \mathbb{N}\} \) be a supra topology on \( N \). In this supra topology, any supra open set is supra dense. We know that a set \( E \) is supra semi-open if and only if there exists a supra open set \( G \) such that \( G \subseteq E \subseteq \text{Cl}^\mu(G) \). Then every supra semi open is supra dense as well. So \( (N, \mu) \) is an almost supra semi-compact space. On the other hand, \( \Lambda = \{\{1, x\} : x \in N\} \) is a semi-open cover of \( N \). Since \( \Lambda \) has not a finite subcover, then \( (N, \mu) \) is not supra semi-compact.

**Example 4.3:** Let \( \mu = \{\emptyset, G \subset R \text{ such that } 1 \in G \text{ or } 2 \in G\} \) be a supra topology on \( R \). Obviously,
\( \Lambda = \{1, x : x \neq 2 \in R\} \cup \{2\} \) forms a supra semi-open cover of \( R \). Since \( cl_s^\mu((1, x)) = \{1, x\} \), for all \( x \neq 2 \in R \) and \( cl_s^\mu(\{2\}) = \{2\} \), then \( (R, \mu) \) is not an almost supra semi-compact space.

**Proposition 4.4:** Every almost supra semi-compact space is almost supra Lindelöf.

**Proof:** The proof is obtained immediately from Definition 4.1.

In Example 3.4, we note that \( \Lambda = \{11, x : 2 \neq x \in Q\} \cup \{2\} \) forms a supra semi-open cover of \( Q \). Since \( \Lambda \) has not a finite sub-cover, the supra semi-closure of whose members cover \( X \), then \( (Q, \mu) \) is not almost supra semi-compact. On the other hand, it is almost supra semi-Lindelöf. Hence the converse of the above proposition need not be true in general.

The proofs of the following two propositions are easy and so will be omitted.

**Proposition 4.5:** A finite (resp. countable) union of almost supra semi-compact (resp. almost supra semi-Lindelöf) subsets of \((X, \mu)\) is almost supra semi-compact (resp. almost supra semi-Lindelöf).

**Proposition 4.6:** Every supra semi-compact (resp. supra semi-Lindelöf) space is almost supra semi-compact (resp. almost supra semi-Lindelöf).

**Corollary 4.7:** Any finite (resp. countable) supra topological space \((X, \mu)\) is almost supra semi-compact (resp. almost supra semi-Lindelöf).

The converse of Proposition 4.6 is not always true as illustrated in the following example.

**Example 4.8:** Let \( \mu = \{\emptyset, G_\alpha = (-\infty, a) : a \in R\} \) be a supra topology on \( R \). Obviously, \((R, \mu)\) is an almost supra semi-compact space but it is not supra semi-Lindelöf.

**Definition 4.9:** A subset \( E \) of \((X, \mu)\) is said to be supra semi-clopen provided that it is supra semi-open and supra semi-closed.

**Proposition 4.10:** Every supra semi-clopen subset of an almost supra semi-compact (resp. almost supra semi-Lindelöf) space is almost supra semi-compact (resp. almost supra semi-Lindelöf).

**Proof:** Let us prove the proposition in case of an almost supra semi-compact space and the other can be made similarly.

Let \( \{G_i : i \in I\} \) be a supra semi-open cover of a supra semi-clopen subset \( F \) of \((X, \mu)\). Then \( F^c \) is a supra semi-open set and \( F \subseteq \bigcup_{i \in I} G_i \). Therefore \( X = \bigcup_{i \in I} G_i \cup F^c \). Since \((X, \mu)\) is almost supra semi-compact, then \( X = \bigcup_{i=n}^{\infty} cl_s^\mu(G_i) \cup F^c \). Thus \( F \subseteq \bigcup_{i=n}^{\infty} cl_s^\mu(G_i) \). Hence \( F \) is an almost supra semi-compact set.

It can be seen from Example 4.3 that \( \{1, 2\} \) is an almost supra semi-compact set. But it is not a supra semi-clopen set. So the converse of Proposition 4.10 fails.

**Proposition 4.11:** If \( A \) is an almost supra semi-compact (resp. almost supra semi-Lindelöf) subset of \( X \) and \( B \) is a supra semi-clopen subset of \( X \), then \( A \cap B \) is almost supra semi-compact (resp. almost supra semi-Lindelöf).

**Proof:** Let \( \Lambda = \{G_i : i \in I\} \) be a supra semi-open cover of \( A \cap B \). Then \( A \subseteq \bigcup_{i \in I_1} G_i \cap B \). Since \( A \) is almost supra semi-compact, then \( A \subseteq \bigcup_{i=n}^{\infty} cl_s^\mu(G_i) \cap B \). Therefore \( A \cap B \subseteq \bigcup_{i=n}^{\infty} cl_s^\mu(G_i) \). Thus \( A \cap B \) is an almost supra semi-compact set.

A similar proof can be given for the case between parentheses.

**Theorem 4.12:** The supra semi-irresolute image of an almost supra semi-compact (resp. almost supra semi-Lindelöf) set is almost supra semi-compact (resp. almost supra semi-Lindelöf).

**Proof:** Let \( g : X \rightarrow Y \) be a supra semi-irresolute map and let \( A \) be an almost supra semi-compact subset of \( X \). Suppose that \( \{G_i : i \in I\} \) be a supra semi-open cover of \( g(A) \). This automatically implies that \( A \subseteq \bigcup_{i \in I_1} g^{-1}(G_i) \). Since \( g \) is supra semi-irresolute, then \( g^{-1}(G_i) \) is a supra semi-open set, for each \( i \in I \). By hypotheses, \( A \) is almost supra semi-compact, then \( A \subseteq \bigcup_{i=n}^{\infty} cl_s^\mu(g^{-1}(G_i)) \). It follows, by Theorem 2.6, that \( cl_s^\mu(g^{-1}(G_i)) \subseteq g^{-1}(cl_s^\mu(G_i)) \), for each \( G \subseteq Y \). So \( (g(A) \subseteq \bigcup_{i=n}^{\infty} g^{-1}(cl_s^\mu(G_i))) \subseteq \bigcup_{i=n}^{\infty} cl_s^\mu(G_i) \). Hence \((g(A)\) is an almost supra semi-compact set. A similar proof can be given for the case between parentheses.

**Corollary 4.13:** The supra semi-continuous image of an almost supra semi-compact (resp. almost supra semi-Lindelöf) set is almost compact (resp. almost Lindelöf).

**Theorem 4.14:** If \( g : X \rightarrow Y \) is a bijective supra semi-open map and \( Y \) is almost supra semi-compact (resp. almost supra semi-Lindelöf), then \( X \) is almost compact (resp. almost Lindelöf).

**Proof:** Let \( \{G_i : i \in I\} \) be an open cover of \( X \). Then \( g(X) = g(\bigcup_{i \in I} G_i) \). Therefore \( Y = \bigcup_{i \in I} g(G_i) \). Since \( g(G_i) \) is a supra semi-open set, for each \( i \in I \) and \( Y \) is almost supra semi-compact, then \( Y = \bigcup_{i=n}^{\infty} cl_s^\mu(g(G_i)) \). Since \( g \) is bijective supra semi-open, then \( g \) is supra semi-closed. Therefore by Theorem 2.6, we obtain
that \( cl^n_s(g(G_i)) \subseteq g(cl(G_i)) \). Thus \( X = \bigcup_{i=1}^{n} cl(G_i) \). Hence \( X \) is almost compact.

A similar proof can be given for the case between parentheses.

**Theorem 4.15:** If every collection of supra semi-closed subsets of \((X, \mu)\) satisfies the finite (resp. countable) intersection property, has, itself, a non-empty intersection, then \((X, \mu)\) is almost supra semi-compact (resp. almost supra semi-Lindelöf).

**Proof:** This is easily obtained from Theorem 3.11 and Proposition 4.6.

Assume that \((N, \mu)\) is the same as in Example 4.2. A collection \( A_n = \{n + 1, n + 2, \ldots\} \) consists of supra closed subsets of \( N \) and has a finite intersection property. Whereas \( \bigcap_{i=1}^{n} A_n = \emptyset \). So the converse of the above theorem is not always true.

### 5. Mildly supra semi-compact spaces

We present in this section the notions of mildly supra semi-compact and mildly supra semi-Lindelöf spaces. Also, we investigate the equivalent conditions for them and show the relationships between them with the help of examples.

**Definition 5.1:** A supra topological space \((X, \mu)\) is called mildly supra semi-compact (resp. mildly supra semi-Lindelöf) provided that every supra semi-clopen cover of \( X \) has a finite (resp. countable) subcover.

It can be proved the following two propositions easily, so their proofs will be omitted.

**Proposition 5.2:** Every mildly supra semi-compact (resp. mildly supra semi-Lindelöf) space is mildly supra compact (resp. mildly supra Lindelöf).

**Proposition 5.3:** Every mildly supra semi-compact space is mildly supra semi-Lindelöf.

The two examples below illustrate that the converse of the above propositions is not always true.

**Example 5.4:** Let \((R, \mu)\) be a supra topological space, where \( \mu \) is the usual topology. From the fact it is connected, we conclude that \( R \) and \( \emptyset \) are the only clopen subsets of \((R, \mu)\). So it is mildly supra compact. On the other hand, a collection \( \Lambda = \{(i, i + 1); i \in \mathbb{Q}^+\} \) forms a supra semi-open cover of \( R \). Obviously, it is not a countable subcover, hence \((R, \mu)\) is not mildly supra semi-Lindelöf.

**Example 5.5:** We replace \( R \) by \( Q \) in Example 4.3. Then \((Q, \mu)\) is mildly supra semi-Lindelöf. On the other hand, \( \Lambda = \{(1, x); 2 \neq x \in R\} \cup \{2\} \) forms a supra semi-clopen cover of \( Q \). Since \( \Lambda \) has not a finite subcover, then \((Q, \mu)\) is not mildly supra semi-compact.

**Proposition 5.6:** Every almost supra semi-compact (resp. almost supra semi-Lindelöf) space is mildly supra semi-compact (resp. mildly supra semi-Lindelöf).

**Proof:** Let \( \Lambda = \{H_i; i \in I\} \) be a supra semi-clopen cover of \((X, \mu)\). Since \((X, \mu)\) is almost supra semi-compact, then \( X = \bigcup_{i=1}^{n} cl^n_s(H_i) \). For each \( i \in I \), we have \( cl^n_s(H_i) = H_i \), hence \((X, \mu)\) is mildly supra semi-compact. A similar proof can be given for the case between parentheses.

**Corollary 5.7:** Every supra semi-compact (resp. supra semi-Lindelöf) space is mildly supra semi-compact (resp. mildly supra semi-Lindelöf).

The converse of the above corollary need not be true in general as the next example shows.

**Example 5.8:** Let \( \mu = \{\emptyset, R, [1]\} \) be a supra topology on \( R \). Since the only supra clopen subsets of \((R, \mu)\) are \( \emptyset \) and \( R \), then \((R, \mu)\) is mildly supra semi-compact. On the other hand, a semi-open cover \( \{\{1, x\}; x \in R\} \) of \( R \) has not a finite sub-cover. So \((R, \mu)\) is not supra semi-Lindelöf.

**Proposition 5.9:** Every supra semi-clopen subset of a mildly supra semi-compact (resp. mildly supra semi-Lindelöf) space \((X, \mu)\) is mildly supra compact (resp. mildly supra Lindelöf).

**Proof:** Let \( \{G_i; i \in I\} \) be a supra semi-clopen cover of a supra semi-clopen subset \( F \) of \( X \). Then \( F^s \) is a supra semi-clopen set and \( F \subseteq \bigcup_{i \in I} G_i \). Therefore \( X = \bigcup_{i \in I} G_i \cup F^s \). Since \( X \) is mildly supra semi-compact, then \( X = \bigcup_{i=1}^{n} G_i \cup F^s \). Thus \( F \subseteq \bigcup_{i=1}^{n} G_i \). Hence \( F \) is a mildly supra semi-compact set. The proof is similar in case of a mildly supra semi-Lindelöf space.

**Proposition 5.10:** If \( A \) is a mildly supra semi-compact (resp. mildly supra semi-Lindelöf) subset of \( X \) and \( B \) is a supra semi-clopen subset of \( X \), then \( A \cap B \) is mildly supra semi-compact (resp. mildly supra semi-Lindelöf).

**Proof:** Let \( \Lambda = \{G_i; i \in I\} \) be a supra semi-clopen cover of \( A \cap B \). Then \( A \subseteq \bigcup_{i \in I} G_i \cup B^s \). Since \( A \) is mildly supra semi-compact, then \( A \subseteq \bigcup_{i=1}^{n} G_i \cup B^s \). Therefore \( A \cap B \subseteq \bigcup_{i=1}^{n} G_i \). Thus \( A \cap B \) is a mildly supra semi-compact set. The proof is similar in case of a mildly supra semi-Lindelöf space.
Theorem 5.11: A supra topological space $(X, \mu)$ is mildly supra semi-compact (resp. mildly supra semi-Lindelöf) if and only if every collection of supra semi-clopen subsets of $X$, satisfies the finite (resp. countable) intersection property, has, itself, a non-empty intersection.

Proof: We only prove the theorem when $(X, \mu)$ is mildly supra semi-compact, the other case can be made similarly.

Let $\Lambda = \{F_i : i \in I\}$ be a collection of supra semi-clopen subsets of $X$ which has the finite intersection property. Assume that $\bigcap_{i \in I} F_i = \emptyset$. Then $X = \bigcup_{i \in I} F_i$. Since $X$ is mildly supra semi-compact, then $X = \bigcup_{i \in I} F_i$. Therefore $\bigcap_{i \in I} F_i = \emptyset$. But this contradicts that $\Lambda$ has the finite intersection property. Thus $\Lambda$ has, itself, a non-empty intersection.

Conversely, Let $\{G_i : i \in I\}$ be a supra semi-clopen cover of $X$. Suppose, to the contrary, that $\{G_i : i \in I\}$ has no finite sub-cover. Then $X \setminus \bigcup_{i = 1}^{n} G_i \neq \emptyset$, for any $n \in N$. Now, $\bigcap_{i = 1}^{n} G_i \neq \emptyset$. This implies that $\{G_i : i \in I\}$ is a collection of supra semi-clopen subsets of $X$ which has the finite intersection property. Therefore $\bigcap_{i \in I} G_i \neq \emptyset$. Thus $X \neq \bigcup_{i \in I} G_i$. But this contradicts that $\{G_i : i \in I\}$ is a cover of $X$. Hence $(X, \mu)$ is a mildly supra semi-compact space.

Proposition 5.12: If $g : X \rightarrow Y$ is a bijective supra semi-open map and $Y$ is mildly supra semi-compact, then $X$ is mildly compact.

Proof: Let $\{G_i : i \in I\}$ be a clopen cover of $X$. Then $g(X) = g(\bigcup_{i \in I} G_i)$. Therefore $Y = \bigcup_{i \in I} g(G_i)$. Now, $Y$ is mildly supra semi-compact, then $Y = \bigcup_{i = 1}^{n} g(G_i)$. Since $g$ is bijective supra semi-open, then $X = \bigcup_{i = 1}^{n} G_i$. Hence $X$ is mildly compact.

6. Conclusion

The concept of compactness is considered as one of fundamental concepts in topological spaces for its contribution in the study of many of topological problems. In this work, we present and study the concepts of supra semi-compact (supra semi-Lindelöf) spaces, almost (mildly) supra semi-compact spaces and almost (mildly) supra semi-Lindelöf spaces depending on a notion of supra semi-open sets. Also, we show the relationships among them with the help of examples. We give the equivalent conditions for the concepts of supra semi-compact (supra semi-Lindelöf) spaces and mildly supra semi-compact (mildly supra semi-Lindelöf) spaces. We verify that the semi-irresolute image of a supra semi-compact (an almost supra semi-compact) set is supra semi-compact (almost supra semi-compact). In an upcoming work, we plan to use a notion of somewhere dense sets [12] to study these concepts in topological spaces. In the end, we hope that the results obtained in this work will help research teams promote further study in supra topological spaces to carry out a general frame work for the practical applications.

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