A Weighted Evidence Combination Method Based on the Pignistic Probability Distance and Deng Entropy

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ABSTRACT: The Dempster-Shafer (D-S) theory is widely applied in various fields involved with multi-sensor information fusion for radar target tracking, which offers a useful tool for decision-making. However, the application of D-S evidence theory has some limitations when evidences are conflicting. This paper proposed a new method combining the Pignistic probability distance and the Deng entropy to address the problem. First, the Pignistic probability distance is applied to measure the conflict degree of evidences. Then, the uncertain information is measured by introducing the Deng entropy. Finally, the evidence correction factor is calculated for modifying the bodies of evidence, and the Dempster's combination rule is adopted for evidence fusion. Simulation experiments illustrate the effectiveness of the proposed method dealing with conflicting evidences.

KEYWORDS: Multi-sensor information fusion; Conflicting evidence; D-S evidence theory; Pignistic probability distance; Deng entropy.

INTRODUCTION

In complex battlefield environments, modern combat systems need to identify air targets accurately and quickly. However, the information obtained by a single information source is likely to be inaccurate, incomplete and unreliable because it usually fails to meet the operational requirement (Xiao and Qin 2018; Song and Deng 2019). Multi-sensor systems can obtain more abundant, precise and reliable information through information fusion, and they may overcome the limitations of single sensor systems. Unfortunately, data collected from different sensors are often inaccurate and uncertain (Sarabi-Jamab and Araabi 2018; Song and Deng 2019). How to model and process uncertain information is still an open issue.

The Dempster-Shafer (D-S) theory can manage uncertain information and offer a useful fusion tool for decision-making (Chen et al. 2017; Li et al. 2017; Xiao 2019; Xiao 2019b). Dempster put forward the evidence theory in 1967 (Shafer 196), and then Shafer further studied the theory in 1976 (Dempster 1967). However, the quality of evidence combination is affected by the conflicting information especially when the sources of evidence are unreliable (Klein et al. 2016). In addition, the counter-intuitive results may be generated by the combination of Dempster’s rule, which is first highlighted by Zadeh (1986).
To solve the above problems, domestic and foreign scholars have proposed a number of improved methods, which are generally divided into two types (Han et al. 2011; An et al. 2019): modification to fusion rules and pre-processing for evidence sources. The former considers that the irrationality conclusion under high-level evidence is generated in the normalization step of the Dempster combination rule, and the key to solving this problem is how to redistribute the conflict between evidence, i.e., which focal elements of the conflict should be reallocated and how to determine the proportional coefficient of assignment. In an earlier study, Yager (1987) regarded all the information contained in the conflict as unknown and assigned all the conflict factors to the identification framework. Since Yager’s method is too conservative, it was further improved by Smets (1990). He believed that conflicts are caused by the incompleteness of the identification framework, and proposed a new composition rule in which conflicting items are allocated to empty set. Dubois and Prade (1988) suggested assigning the highly conflicting mass to the whole set or a particular set. However, when the evidences are highly conflict, the worse fusion results may be obtained. In fact, the good nature of the combination rules is often destroyed due to the modification. In addition, if the counter-intuitive result is caused by sensor failure, this modification is considered unreasonable. Therefore, to solve the combination problem of evidence conflict, researchers tend to adopt the second type of method (i.e., pre-process the subject of evidence).

These methods believe that the Dempster combination rule has a solid mathematical foundation and has no problem in itself. But it ignores the fact that each piece of evidence has different reliability. When dealing with high-conflict evidence, the conflict evidence should be pre-processed before using the Dempster combination rule. For example, Murphy (2000) proposed a method of averaging evidence, the idea is to modify the original evidence without changing the Dempster combination rule, i.e., to calculate the Basic Probability Assignment (BPA) of each evidence before evidence fusion. In this work, the same reliability is averaging for multiple sets of evidence, but the correlations between individual evidence were not considered. Han et al. (2004) showed that the Murphy’s method can be further improved by adding a distance function, which measures the degree of similarity between the evidence and the determined weight of evidences. Although this method has some improvement on the evidence of high degree of conflict, the distance formula cannot be used to describe the degree of mutual support between different evidence, thus the effect of the evidence itself was ignored in the target identification process. Zhang et al. (2014) applied the idea of distance-based evidence conflict analysis and proposed a new method based on the law of cosines to identify and represent conflict data. However, it ignores the influence of evidence on the correction coefficient, so the method in this paper introduces Deng entropy (Zhang and Deng 2019; Kang and Deng 2019; Gao and Deng 2020; Gao and Deng 2019) to improve the performance of information fusion.

In a word, the above improved methods based on redistribution of conflicting evidence do not fully take into account the fact that each piece of evidence has different reliability. To solve this problem, a new combination method for multi-sensor conflicting information is proposed in this paper. Compared with the Jousselme distance used by most scholars, the Pignistic distance can better judge the conflict between the evidences and has lower complexity (Liu 2006). Only the distance between the evidence is it not a good measure of the conflict of evidence. Deng entropy is used to quantify the uncertainty of different evidences, which not only can better measure evidence conflicts, but also solves the problem of non-convergence in calculation. This paper introduces Pignistic probability distance and Deng entropy to compute the evidence correction factor, and then the bodies of evidence are modified before using Dempster’s combination rule. By correcting the evidence, reasonable and effective fusion results can be obtained. The simulation results and analyses demonstrate that the proposed method can not only achieve accurate fusion results with low conflicting evidence, but also obtain reliable performance under high conflicting information compared with several existing methods. Hence, the novelty and practicability of the proposed method are verified. As the reliability and accuracy of information fusion are both solved, the effective application of multi-sensor systems is further guaranteed.
D-S EVIDENCE THEORY

As the generalization of the probability theory and Bayesian reasoning, the D-S evidence theory can obtain fusing results without the requirement of prior knowledge and conditional probability. Based on the accumulation of evidences, the effective and accurate multi-sensor fusion results can be obtained. The basic concepts are introduced as below.

FRAME OF DISCERNMENT

In D-S evidence theory, a sample space is called a frame of discernment, represented by Θ, which is composed of M objects. The objects are mutually exclusive and contain the entire object to be identified. The frame of discernment defined as

\[ Θ = \{A_1, A_2, \ldots, A_M\} \]  (1)

Accordingly, we can derive the power set \(A ≜ 2^θ\) of D-S evidence theory.

\[ 2^θ = \{∅, \{A_1\}, \{A_2\}, \ldots, \{A_M\}, \{A_1, A_2\}, \ldots, \{A_1, A_2, \ldots, A_M\}\} \]  (2)

where \(∅\) is the empty set. The power set \(2^θ\) is composed with \(2^M\) propositions from (2), and any proposition \(A ⊆ Θ\) satisfies \(A ∈ 2^θ\).

MASS FUNCTION

In D-S theory evidence, evidences are obtained through multi-sensor information. If the function \(m: 2^θ → [0, 1]\) satisfies equation (3) and (4), it is called the basic probability assignment (BPA, also called mass function). BPA reflects the degree of evidence support for propositions in the frame of discernment, namely \(m(A)\).

\[ m(∅) = 0 \]  (3)

\[ \sum_{A ⊆ Θ} m(A) = 1 \]  (4)

for \(A ⊆ Θ, A ∈ 2^θ\)

UNCERTAINTY REPRESENTATION

For a proposition \(A ⊆ Θ\), the sum of BPA corresponding to all subsets in Θ is called belief function. The belief function \(Bel: 2^θ → [0, 1]\) is defined as

\[ Bel(A) = \sum_{α ⊆ A} m(α) \]  (5)

\[ P(\bar{A}) = \sum_{A' \cup α = Θ} m(α) = 1 - Bel(\bar{A}) \]  (6)

where \(α\) is any subset of the set \(A\). The function \(Bel(A)\) and \(P(\bar{A})\) separately reflects the lower and upper bounds of limit function of proposition \(A\).
D-S COMBINATION RULE

D-S combination rule can synthesize multi-sensor information to obtain effective and accurate decision-making results. Assuming that the frame of a multi-sensor system is \( \Theta = \{ A_1, A_2, \ldots, A_N \} \), the D-S combination rule of two evidences \( m_1 \) and \( m_2 \) is defined as

\[
m(A) = \frac{1}{1-K} \sum_{A_i \cap A_j \neq \emptyset} m_1(A_i)m_2(A_j), A \neq 0
\]

\[
m(\emptyset) = 0
\]

where \( K \) is the conflicting factor that reflects the conflicting degree of evidences \( m_1 \) and \( m_2 \). \( \frac{1}{1-K} \) is the normalized factor that ensures the unity property of fused mass.

\[
K = \sum_{A_i \cap A_j \neq \emptyset} m_1(A_i)m_2(A_j)
\]

Obviously, the D-S combination rule satisfies both commutative law and associate law, which are shown separately as follows:

\[
m_1 \oplus m_2 = m_2 \oplus m_1
\]

\[
(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3)
\]

A NEW METHOD FOR MODIFYING COMBINATION RULES

In this paper, a new combination method based on the Pignistic distance and Deng entropy is proposed to deal with the evidence conflict problem. Compared with the Jousselme distance, the Pignistic distance can better measure the difference of evidence. Based on the Pignistic distance, the evidence support can be derived to describe the reliability of evidence. However, only using the Pignistic distance between the evidence is not necessarily effective for describe the conflict degree of evidences. Thus Deng entropy is introduced and used to measure the information volume of evidence, e.g., the greater the amount of information, the greater the uncertainty of the evidence. In other words, Deng entropy is adopted to quantify the uncertainty of different evidences. Based on the Pignistic distance and Deng entropy, the proposed method not only can better measure evidence conflicts but also solves the computational divergence problem.

PIGNISTIC PROBABILITY FUNCTION

The representative methods of measuring conflict evidence mainly include: Pignistic probability distance, Jousselme distance correlation coefficient, compatibility coefficient, etc. Recently, some new measurement methods have also appeared one after another. For example, Yu et al. (2015) proposed support probability distance, and Smets (2007) presented a similarity measure combining Pignistic probability distance and Tanimoto measure. Tessem (1993) proposed the Pignistic probability distance based on the Pignistic probability function, which is adopted in this paper and specifically defined as follows.

Let \( m \) be a BPA on \( \Theta \). Its associated Pignistic probability function \( BetP_m: \Theta \to [0, 1] \) is defined as:

\[
BetP_m(\omega) = \sum_{A \subseteq \Theta, \omega \in A} \frac{m(A)}{|A| \frac{1}{1-m(\emptyset)}, m(\emptyset) \neq 1}
\]

where \( \omega \in \Theta, |A| \) is the cardinality of subset \( A \).
Let $m_1$ and $m_2$ be two BPA on frame $\Theta$ and let $BetP_{m_1}$ and $BetP_{m_2}$ be the results of two Pignistic transformations from them respectively. Then:

$$\text{difBetP}_{m_1}^{m_2} = \max_{A \subseteq \Theta} \{|Bet_{m_1}(A) - Bet_{m_2}(A)|\}$$  

(12)

Obviously, $\text{difBetP}_{m_1}^{m_2} = 0$ when $m_1 = m_2$, i.e., when any two pieces of evidence are the same, their BPA’s betting commitment distance is always 0.

### Deng Entropy

Due to the complex natural environment and human interference, the information obtained by some detection sensors is disturbed and conflicts with other sensors (Lin et al. 2016; Wang et al. 2018; Sun et al. 2018; Gong et al. 2018). Therefore, effective analysis of the detection information is required. In thermodynamics, the entropy concept of the disordered state size of the system, Claude Shannon defined the information entropy in information theory, and estimated the redundancy or uncertainty of the information according to (Zhang et al. 2017; Jiang et al. 2017). But it is not applicable to evidence theory, because there is a multi-subset proposition in evidence theory.

A new type of belief entropy, known as the Deng entropy, was first proposed by Deng (2016). The basic concepts are introduced as follows:

$$E_s(m) = - \sum_{A \subseteq \Theta} m(A) \log_2 \frac{m(A)}{2^{\left|A\right|} - 1}$$  

(13)

where $m$ is a mass function defined on the frame of discernment $\Theta$, and $A$ is the focal element of $m$, $|A|$ is the cardinality of $A$, i.e., the number of elements in $A$.

Deng entropy reflects the amount of information contained in the evidence. The more Deng entropy of evidence is, the more information it contains (i.e., the more uncertainty it has); On the contrary, the less Deng entropy of evidence is, the less information it contains (i.e., the less uncertainty it has). However, only using Deng entropy to calculate the weight may increase the weight of interference evidence in fusion, which may lead to unreasonable results. In other words, Deng entropy cannot judge whether there is conflict between evidences. Fortunately, the pignistic distance of evidence is an effective tool to reflect the conflicted degree between evidences. Combination of Deng entropy and the pignistic distance can greatly improve the effect of evidence fusion and the recognition rate of evidence.

### Improved New Methods

D-S evidence theory is widely used in the field of multi-sensor target recognition, which can deal with the uncertain information fusion problem. However, the application of D-S evidence theory has some limitations when evidences are conflicting. Traditional evidence theory fusion rules usually may not distribute evidence conflicts reasonably, which makes the result of decision fusion is often contrary to the facts. Besides, most existed methods only redistribute the conflicted evidence without considering their credibility. In other words, they do not fully take into account the fact that each piece of evidence has different degrees of reliability. Thus this paper a weighted evidence combination method based on the Pignistic probability distance and Deng entropy, in which the uncertainty of evidence not only can be reflected and the conflict degree can be descried. It has not only the better identification performance and faster convergence speed, but also the less risk of decision-making. Even if there exist high conflicts between evidences, the proposed method can also make correct identification more rapidly than other approaches.

**Step 1:** Calculate the credibility of evidence.

1) Equations (11)-(13) are used to calculate the Pignistic probability distance between $n$ evidence pairs collected in $\text{difBetP}_{m_1}^{m_2}$.

2) Calculate the support and reliability of the evidence. Suppose the similarity degree $S_y$ between $m_i$ and $m_j$ is:

$$S_y = 1 - \text{difBetP}_y$$  

(14)
The greater the distance $\text{difbet}_P$ between the evidences, the greater the similarity $s_{ij}$ is. In other words, the higher evidence conflicts make them less similar. Based on the definition of the similarity degree, the support and credibility of the evidence are calculated separately

$$Sup_i = \sum_{j=1,j\neq i}^N S_{ij} \quad (15)$$

$$Cre_i = \frac{\sup_i}{\sum_{j=1}^N \sup_j} \quad (16)$$

Step 2: Calculate the information entropy of the evidence.

1) Set thresholds and select credible evidence. After verification and comparison of experimental data, the threshold rate is defined as 10%, and the threshold is calculated as:

$$\varphi = \sum_{i=1}^N Cre_i \times 10\% \quad (17)$$

2) Calculate the information entropy of credible evidence.

When the credibility of the evidence is higher than the threshold, they are regarded as credible evidences. Otherwise, they are incredible evidences.

Select all credible evidences $E_l (l=1,2,L,S)$ and calculate their respective Deng entropy through equation (14). When the focal element in the identification framework is monad set, equation (13) can be simplified as in [35]:

$$I_i' = \sum_{i=1}^M m_A(A) \log m_A(A) (t-1,2,L,M) \quad (18)$$

After the information entropy is normalized, it can be obtained:

$$I = \frac{I_i'}{\sum_{i=1}^N I_i'} \quad (19)$$

Step 3: calculate the correction coefficient of evidence.

1) The correction coefficient of the evidence in the selected evidence set is obtained, and the correction coefficient of the $t$-th evidence is expressed as:

$$\omega_j = (1-I)e^t \quad (20)$$

2) If the correction coefficients of untrustworthy evidence are replaced with credibility, the correction coefficients of all evidence are normalized to obtain:

$$\omega_j = \frac{\omega_j'}{\sum_{i=1}^N \omega_i'} \quad (21)$$

Step 4: Calculate the correction coefficient of each evidence according to the above algorithm, and perform weighted average on the basic probability distribution of all evidence. The Dempster rule is used to fuse the number of iterations (n-1) to obtain the fusion result (Fig. 1).
A Weighted Evidence Combination Method Based on the Pignistic Probability Distance and Deng Entropy

Step 1
Generate basic probability assignment (BPA)

Calculate the Pignistic distance between the evidence and figure out the reliability

Step 2
Is the credibility greater than the threshold

Y
Calculate the Deng entropy of this evidence
The reliability of discredited evidence as the correction factor

N
Calculating correction factors for credible evidence

Step 3

Step 4
Weighted average of evidence get average evidence
Use Dempster’s combination rule to get the fusion result

Figure 1. Flow chart of the proposed method.

NUMERICAL EXPERIMENTS AND ANALYSES

To verify the validity and superiority of the proposed D-S combination method, three numerical simulations are conducted. Two kinds of multi-sensor data are adopted respectively, where low and high conflicting information are included. The methods in D-S (Shafer 1976), Yager (1987) and Yuan et al. (2016) are compared with the presented method using D-S combination rule. It contains multi-sensor data with low conflict information and high conflict information, and the method in D-S, Yager and Yuan is compared with the method using d-s combination rule.

LOW CONFLICTING INFORMATION

Example 1. In the multi-sensor system, assume that there are 5 evidences in the framework \( \Theta=\{A,B,C\} \), and proposition \( A \) is the true; the low conflicting evidences are exhibited in Table 1.
Table 1. Mass assignments of low conflicting information.

| Sensors  | Targets |  | A   | B   | C   |
|----------|---------|---|-----|-----|-----|
| E₁:m₁(·) | 0.9     |   | 0   | 0.1 |
| E₂:m₂(·) | 0.88    |   | 0.01| 0.11|
| E₃:m₃(·) | 0.5     |   | 0.2 | 0.3 |
| E₄:m₄(·) | 0.98    |   | 0.01| 0.01|
| E₅:m₅(·) | 0.9     |   | 0.05| 0.05|

Table 2. Fusion results of different methods with low conflicting information.

| Methods   | Targets | E₁⊕E₂ | E₁⊕E₂⊕E₃ | E₁⊕E₂⊕E₃⊕E₄ | E₁⊕E₂⊕E₃⊕E₄⊕E₅ |
|-----------|---------|-------|-----------|--------------|-----------------|
| D-S       | A       | 0.9863| 0.9917    | 0.9999       | 1               |
|           | B       | 0     | 0         | 0            | 0               |
|           | C       | 0.0137| 0.0083    | 0.0001       | 0               |
|           | Θ       | 0     | 0         | 0            | 0               |
| Yager     | A       | 0.7920| 0.3960    | 0.3881       | 0.3493          |
|           | B       | 0     | 0         | 0            | 0               |
|           | C       | 0.011 | 0.0033    | 0            | 0               |
|           | Θ       | 0.1970| 0.6007    | 0.6119       | 0.6507          |
| Kaijuan Yuan | A   | 0.7695| 0.9533    | 0.9915       | 0.9984          |
|           | B       | 0.0804| 0.0104    | 0.0011       | 0.0002          |
|           | C       | 0.1501| 0.0363    | 0.0074       | 0.0014          |
|           | Θ       | 0     | 0         | 0            | 0               |
| The proposed method | A | 0.8356| 0.9793    | 0.9976       | 0.9997          |
|           | B       | 0.0571| 0.0046    | 0.0003       | 0               |
|           | C       | 0.1073| 0.0161    | 0.0021       | 0.0003          |
|           | Θ       | 0     | 0         | 0            | 0               |

Table 2 shows the fusion results of D-S, Yager, Yuan and the proposed method. Under a low conflicting condition, it indicates that the true proposition $A_1$ is identified by all methods. Based on these results, the following analyses can be obtained:

1) D-S evidence theory cannot effectively deal with high conflict evidence. Since $m(A_2)=0$, the proposition $A_2$ is completely negated. Even if there is more evidence to support the proposition $A_2$, the fusion result always shows that the support of the proposition $A_2$ is 0.

2) Our method and other three existing method identify the target by using two evidences correctly when fusing low conflicting information. It has almost the same identification performance as other method except Yager. This is because Yager assigns all evidence conflicts to unknown $m(Φ)$. As the evidence (support for object $A_1$) increases, we can see that the value of $m(Φ)$ increases (see Table 1).

3) The method in this paper can effectively deal with the case of interference evidence, and has a faster convergence speed. In addition, compared with other methods, the true proposition $A_1$ quality of this method is the largest, which verifies its validity and accuracy, as shown in Fig. 2.

From the above analysis, we can draw the conclusion that the proposed method can handle the conflicting situation more precisely and efficiently.
HIGHLY CONFLICTING INFORMATION.

In order to further test the performance of the proposed method under highly conflicting conditions, we set some highly conflicting evidences as Example 2 (see Table 3). Table 4 gives the fusing results of D-S, Yager, Yuan and the proposed method.

1) Evidence 2 has not support for \( A_1 \), even though the evidence supporting proposition \( A_1 \) increases, D-S evidence theory considers that proposition \( A_3 \) is a true, which is obvious contrary to the intuition. The D-S evidence theory has the wrong result, so it is unreliable in highly conflicting conditions.

2) Yager's results are similar to D-S evidence theory. No matter how much evidence is collected in the future, Yager totally denies the proposition \( A_1 \), and the value of unknown terms is always increasing. Therefore, Yager is not completely impacted in a highly conflicting situation. Yager supports true proposition \( A \) the most. It has a relatively complex method, so in practical application, it cannot quickly identify the target. Besides, Yager's method has worse performance than the method in this paper when fusing highly conflicting conditions. This method needs further refinement.

3) Comparing with other methods, our method assigns a bigger mass to true proposition \( A_3 \). Thus this method maintains the accurate fusion performance when combining highly conflicting conditions. It reduces the impact of conflicted evidences on the fusion results and strong anti-disturbance ability, as shown in Fig. 3.

To sum up, the method proposed in this paper keeps its superiority to obtain accurate and stable fusion. Apparently, along with the increasing conflicting degree among evidences, the proposed method always has the best combination effects.

![Figure 2. Comparison of evidence fusion results for various method with normal evidence, example 1.](image)

Table 3. Mass assignments of highly conflicting information.

| Sensors | Targets | \( A \) | \( B \) | \( C \) |
|---------|---------|--------|--------|--------|
| E₁;m₁(·) | 0.9 | 0 | 0.1 |
| E₂;m₂(·) | 0 | 0.01 | 0.99 |
| E₃;m₃(·) | 0.5 | 0.2 | 0.3 |
| E₄;m₄(·) | 0.98 | 0.01 | 0.01 |
| E₅;m₅(·) | 0.9 | 0.05 | 0.05 |
Table 4. Fusion results of different methods with highly conflicting information.

| Methods    | Targets | \(E_1 \oplus E_2\) | \(E_1 \oplus E_2 \oplus E_3\) | \(E_1 \oplus E_2 \oplus E_3 \oplus E_4\) | \(E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5\) |
|------------|---------|---------------------|---------------------|---------------------|---------------------|
| D-S        | A       | 0                   | 0                   | 0                   | 0                   |
|            | B       | 0                   | 0                   | 0                   | 0                   |
|            | C       | 1                   | 1                   | 1                   | 1                   |
|            | \(\Theta\) | 0   | 0                   | 0                   | 0                   |
| Yager      | A       | 0                   | 0                   | 0                   | 0                   |
|            | B       | 0                   | 0                   | 0                   | 0                   |
|            | C       | 0.0990              | 0.0297              | 0.0003              | 0                   |
|            | \(\Theta\) | 0.9010 | 0.9703              | 0.9997              | 1                   |
| Kaijuan Yuan | A       | 0.7052              | 0.9126              | 0.9770              | 0.9940              |
|            | B       | 0.1016              | 0.0189              | 0.0029              | 0.0004              |
|            | C       | 0.1932              | 0.0685              | 0.0201              | 0.0056              |
|            | \(\Theta\) | 0     | 0                   | 0                   | 0                   |
| The proposed method | A | 0.8685 | 0.9865 | 0.9986 | 0.9999 |
|            | B       | 0.0367              | 0.0018              | 0.0001              | 0                   |
|            | C       | 0.0948              | 0.0118              | 0.0013              | 0.0001              |
|            | \(\Theta\) | 0    | 0                   | 0                   | 0                   |

Figure 3. Comparison of evidence fusion results for various method with conflict evidence, example 2.

Example 3. In the multi-sensor system, assume that there are 5 evidences in the framework \(\Theta=\{A,B,C\}\), and proposition \(A\) is the true; the evidences are shown in Table 5.
Table 5. Mass assignments of multi-elements subsets.

| Sensors | A    | B    | C    | AC   |
|---------|------|------|------|------|
| E_1 m_1 | 0.41 | 0.29 | 0.3  | 0    |
| E_2 m_2 | 0    | 0.9  | 0.1  | 0    |
| E_3 m_3 | 0.58 | 0.07 | 0    | 0.35 |
| E_4 m_4 | 0.55 | 0.1  | 0    | 0.35 |
| E_5 m_5 | 0.6  | 0.1  | 0    | 0.3  |

Table 6 shows the fusion results of D-S, Yage, Yuan and the proposed method. Under a highly conflicting condition, it indicates that the true proposition A is identified by all methods. Based on these results, the following analyses can be obtained:

Table 6. Fusion results of different methods with multi-elements subsets.

| Methods (Low) | Targets | E_1 E_2 | E_1 E_2 E_3 | E_1 E_2 E_3 E_4 | E_1 E_2 E_3 E_4 E_5 |
|---------------|---------|---------|-------------|-----------------|---------------------|
| D-S           | A       | 0       | 0           | 0               | 0                   |
|               | B       | 0.8969  | 0.6575      | 0.3321          | 0.1422              |
|               | C       | 0.1031  | 0.3425      | 0.6679          | 0.8578              |
|               | Θ       | 0       | 0           | 0               | 0                   |
| Yager         | A       | 0       | 0.4112      | 0.6508          | 0.7732              |
|               | B       | 0.2610  | 0.0679      | 0.0330          | 0.0167              |
|               | C       | 0.0300  | 0.0105      | 0.0037          | 0.0011              |
|               | AC      | 0       | 0.2481      | 0.1786          | 0.0938              |
|               | Θ       | 0.7090  | 0.2622      | 0.1339          | 0.1152              |
| Kaijuan Yuan  | A       | 0.2849  | 0.8274      | 0.9596          | 0.9886              |
|               | B       | 0.5306  | 0.0609      | 0.0032          | 0.0002              |
|               | C       | 0.1845  | 0.0986      | 0.0267          | 0.0072              |
|               | AC      | 0       | 0.0131      | 0.0106          | 0.0039              |
|               | Θ       | 0.7090  | 0.2622      | 0.1339          | 0.1152              |
| The proposed method | A | 0.0055 | 0.6128 | 0.9985 | 0.9999 |
|               | B       | 0.9696  | 0.3702      | 0.0004          | 0                   |
|               | C       | 0.0249  | 0.0168      | 0.0009          | 0.0001              |
|               | AC      | 0       | 0.0002      | 0.0002          | 0                   |
|               | Θ       | 0       | 0           | 0               | 0                   |
1) Evidence 2 has not support for proposition $A_2$, even though the evidence supporting proposition proposition $A_1$ increases, D-S evidence theory comes to a wrong conclusion, so it is unreliable in highly conflicting conditions.

2) Yager's method assigns a bigger mass to true proposition $A_3$, which means it fails to identify the correct target, when there are only two pieces of evidence. With the increase of effective evidence, Yager's can make a correct decision, but it is obvious that the effect is not very good and the accuracy is not high.

3) It's clear that the proposed method is not only efficient but also reliable. Though both Yuan's method and the proposed method can identify the object is $A$, our method assigns a bigger mass to true proposition $A_3$, when there are only four evidences. Under the situation of five evidences, the proposed method improves the accuracy of identification to 0.9999, while Yuan's method only has 0.98864. Therefore the proposed method can deal with conflict and make decision effectually.

In this section, we choose two different kind of evidences with low and highly conflicting information. The simulation results prove that the method in this paper effectively solves the problem of conflict evidence combination.

CONCLUSION

Potential errors in sensor measurement, uncertainty in the unknown monitoring environment, and even possible human interference can lead to ambiguity and conflicts of information in multi-sensor systems. As the conflict information is common in multi-sensor systems, how to combine them becomes the core problem of achieving reliable and accurate fusion results.

D-S evidence theory is a widely used uncertainty management method in multi-sensor fusion systems. However, the conflict phenomenon usually occurs in the application of D-S theory, so its practical application has certain limitations. In order to solve the problem of evidence conflict, a new multi-sensor conflict information combination method is proposed in this paper. First, by introducing the Pignistic probability distance function and Deng entropy, the conflict degree and uncertainty information are put forward respectively to obtain the evidence correction coefficient. Then, the body of evidence is modified before using Dempster's combination rules. Finally, two simulation experiments are carried out, the results verify the effectiveness and accuracy of the proposed method in low-conflict evidence, and prove its stability and superiority in high-conflict evidence.

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