Time-dependent supergravity solutions in null dilaton background

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Abstract

A class of time dependent pp-waves with NS-NS flux in type IIA string theory is considered. The background preserves 1/4 supersymmetry and may provide a toy model of Big Bang cosmology with non trivial flux. At the Big Bang singularity in early past, the string theory is strongly coupled and Matrix string model can be used to describe the dynamics. We also construct some time dependent supergravity solutions for D-branes and analyze their supersymmetry properties.

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1 Introduction

The study of time dependent backgrounds in string theory is a challenging problem. Until now quite a few time dependent solutions in string theory/supergravity are known. Recently a simple cosmological background with a null dilaton has been proposed [1] as a toy model of Big Bang cosmology. The background preserves half of the thirty-two supersymmetries and one expects that there is a good control over the singularity at early times. The matrix degrees of freedom, rather than point particles or the perturbative string states describe the correct physics near the singularity. In discrete light cone type of quantization, a matrix string [2–4] with a time dependent coupling constant is used to study the dynamics at the Big Bang. The success of the model has to be tested by the further investigation of the relevant matrix model and its cosmological predictions. Some steps have already been taken in this direction [5–27] and the future studies may seed the revolution for the string cosmology. See [28] for a recent review of the subject.

A non-trivial extension of the background is to include the RR and NS-NS fluxes and some time dependent pp-wave solutions of the string theory with a null dilaton are considered in the literature [11] and the matrix string theory of a class of pp-wave background has been analyzed [9,18] (see also [29,30] for some early studies of time dependent pp-waves). The AdS/CFT correspondence has also been considered for a time dependent type IIB background [17,19–21] and the dual gauge theory has been realized as a time dependent supersymmetric Yang-Mills theory living on the boundary. In AdS/CFT correspondence, supersymmetry plays a very important role and the existence of supersymmetry gives us a better control over the nonperturbative behavior of the dual quantum field theory. In this sense, the time dependent backgrounds preserving some supersymmetry are useful tools for the study of the quantum gravity and dual field theories. The study of open strings in time dependent backgrounds is also an interesting problem. In this context, some D-brane solutions in a light-like linear dilaton background has also been discussed in [31] by taking a suitable Penrose limit of an intersecting brane solution in supergravity.

In this paper, we study the type IIA pp-waves with NS-NS three form flux in a light-like linear dilaton background. This can be considered as the simplest extension of the model of [1] with non-trivial flux. We study the supersymmetry of the background and show that it preserves 1/4 of type IIA supersymmetry. At early times, the string theory is strongly coupled and the matrix string description may be used. We also construct some time dependent supergravity p-brane solutions preserving some fraction of supersymmetry in this background. The solutions have singularities at early times similar to the Big Bang model but perhaps the matrix degrees of freedom may be used to resolve the singularity.

The rest of the paper is organized as follows. In section-2 we study the pp-waves
with three form NS-NS flux as a toy model of Big Bang cosmology. We analyze the
geodesic equations in this background and also give the matrix string description of
this class of solutions. We find that the background preserves 1/4 of the type II
supersymmetry. In section-3, we present classical solutions of D-branes and analyze
the supersymmetry properties of these solutions. We show that they preserve 1/8
supersymmetry. Finally in section-4, we present our conclusions.

2 The type II pp-waves with null dilaton

We begin with the following ansatz for the background metric and other fields,
\[
\begin{align*}
    ds^2 &= -2dx^+dx^- - \mu^2(x^+) \sum_i (x^i)^2 dx^+ dx^i + dx^i dx^i + dy^a dy^a, \\
    \Phi &= -Qx^+, \quad H_{+2} = H_{+34} = 2f(x^+).
\end{align*}
\]
(1)

Note that the dilaton is linear in light cone time and the indices run as \(i = 1, ..., 4, \ a = 5, ..., 8\). Only non zero component of Ricci curvature computed from the above metric
is,
\[
R_{++} = 4\mu^2(x^+)
\]
(2)

and to be a solution of type IIA supergravity equations of motion, one should satisfy
\[
\mu^2 = -\frac{1}{2} \ddot{\Phi} + f^2 = f^2,
\]
(3)

where dots denotes derivative with respect to \(x^+\). At the Big Bang, \(x^+ \to -\infty\) the
dilaton diverges and string theory is strongly coupled and a DLCQ type of description
can be given (see below). To study the nature of singularity at early times let us
consider the geodesic motion of a point particle in our background. For this we change
our metric to the Einstein frame metric,
\[
\begin{align*}
    ds^2_E &= e^{Qx^+/2} \left[ -2dx^+dx^- - \mu^2(x^+)(x^i)^2 dx^+ dx^i + dx^i dx^i + dy^a dy^a \right].
\end{align*}
\]
(4)

At \(x^+ \to -\infty\), the metric components shrink to zero, which corresponds to Big Bang
singularity. We would like to study the geodesic equation for a test particle near the
singularity. Null geodesic in the spacetime at constant \(X^-\), \(x^i\) and at \(X^a = 0\) is given by
\[
\frac{d^2X^+}{d\sigma^2} + \Gamma^+_{\alpha\beta} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} = 0.
\]
(5)

which, for our metric, can be written as
\[
\frac{d^2X^+}{d\sigma^2} + \frac{Q}{2} \left( \frac{dx^+}{d\sigma} \right)^2 = 0.
\]
(6)
Solving the above equation, one gets the affine parameter

$$\sigma = \exp(Qx^+/2),$$

(7)

upto a reparameterization. Therefore the singularity at $x^+ \to -\infty$ correspond to $\sigma = 0$, and it has finite affine distance to all points interior of the spacetime and the spacetime is geodesically incomplete. One can try to compute the Riemann tensors and can show that there is a curvature singularity at $\sigma = 0$ and gives a divergent tidal force felt by the inertial observer. At late times, $x^+ \to \infty$ the affine parameter diverges and it corresponds to the asymptotic region of spacetime.

**Supersymmetry:** The supersymmetry variation of dilatino and gravitino in string frame is given by

\[
\delta \lambda = \frac{1}{2}(\Gamma^\mu \partial_\mu \phi - \frac{1}{12}\Gamma^{\mu\nu\rho}H_{\mu\nu\rho}\Gamma_{11})\epsilon + \cdots \tag{8}
\]

\[
\delta \Psi_\mu = \left[\partial_\mu + \frac{1}{4}(w_{\mu\tilde{a}b} - \frac{1}{2}H_{\mu\tilde{a}b}\Gamma_{11})\Gamma_{\tilde{a}\tilde{b}}\right]\epsilon + \cdots \tag{9}
\]

where the dots stand for the terms coming for the R-R charges, and we have used $(\mu, \nu, \rho)$ to describe the ten dimensional space-time indices, and hated indices represent the flat tangent space indices.

Veilbeins and spin connection for the above metric are,

\[
e^+ = 1, \quad \hat{e}^- = 1, \quad e^+_i = \frac{1}{2}\mu^2 x^2_i, \quad e^i = \delta^i_j
\]

\[
e_a^b = \delta_a^b, \quad \omega^i = \mu^2 x^i \tag{10}
\]

For our metric and linear null dilaton background, putting the supersymmetry variations of dilatino and gravitino equal zero to gives,

\[
\delta \lambda \equiv \left(-Q\Gamma^+ - \frac{1}{12}\Gamma^{ij}H_{ij}\Gamma_{11}\right)\epsilon = 0,
\]

\[
\delta \Psi_+ \equiv \left(\partial_+ - \frac{1}{2}\mu^2 x^2\Gamma^{i}\Gamma^i - \frac{1}{2}\mu(x^+)\Gamma^i\Gamma_{ij}\right)\epsilon = 0,
\]

\[
\delta \Psi_- \equiv \partial_\epsilon = 0, \quad \delta \Psi_a = \partial_a \epsilon = 0,
\]

\[
\delta \Psi_i \equiv \left(\partial_i - \frac{1}{8}H_{ij}\Gamma^{ij}\Gamma_{11}\right)\epsilon = 0 \tag{11}
\]

The dilatino variation is solved by imposing

\[
\Gamma^+ \epsilon = 0, \tag{12}
\]
which breaks half of the supersymmetry. The $\delta \Psi_+ = 0$ equation further requires the condition

$$ (\Gamma^{12} + \Gamma^{33}) \epsilon = 0 $$

which breaks another half of the supersymmetry. Thus the background preserve $1/4$ supersymmetry.

Matrix string description: Near the big bang singularity, the dilaton diverges and the string theory is strongly coupled. However one can give a discrete light cone type of quantization (DLCQ) at the early times. For the DLCQ of the background, we pick up the direction $y^8$ and make the following identifications,

$$(x^+, x^-, y^8) \sim (x^+, x^- + R, y^8 + \epsilon R).$$

Making a Lorentz transformation as in [1]

$$ x^+ = \epsilon X^+, \quad x^- = \frac{X^+}{2\epsilon} + \frac{X^-}{\epsilon} + \frac{Y^5}{\epsilon} $$

and applying the T duality along $Y^8$ followed by an S duality we get the IIB background,

$$ ds^2 = r e^{QX^+} (-2dX^+dX^- - \mu^2(X^+)\epsilon^2(X^i)^2dX^+dX^- + dX^i dX^i + dY^a dY^a) $$

$$ \Phi = Q\epsilon X^+ + \log r, \quad F_{+12} = F_{+34} = 2f(X^+), \quad i = 1, \ldots, 4, \quad a = 5, \ldots, 8 $$

where $r = \frac{4R}{2\pi l_s}$ and $Y^8 \sim Y^8 + \frac{2\pi l_s}{r}$.

Let us focus on the DBI action of a single D1-brane in this background,

$$ S_{D1} = -\frac{1}{2\pi l_s^2} \int d\tau d\sigma e^{-\Phi} \sqrt{-det \left( \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu} + 2\pi l_s^2 F_{\alpha\beta} \right)} $$

Let us take $F_{\alpha\beta} = 0$, $Y^8 = \frac{\sigma}{r}$ and $X^+$ and $X^-$ depending on $\tau$ only. The equations of motion derived from the above action require, $\partial_\tau X^+ = \partial_\tau X^-$ and a classical solution is given by,

$$ X^+ = \frac{\tau}{r \sqrt{2}}, \quad X^- = \frac{\tau}{r \sqrt{2}}, \quad Y^8 = \frac{\sigma}{r}, \quad Y^{5,6,7} = 0, \quad X^i = 0. $$

Choosing the gauge, $X^+ = \frac{1}{r \sqrt{2}}$, $Y^8 = \frac{\sigma}{r}$ and defining the new variable $Z$ as, $X^+ = \frac{1}{r \sqrt{2}} + \sqrt{2}Z$, and expanding the action around the classical solution upto quadratic terms in the fluctuations, we get,

$$ S_{D1} = \frac{1}{2\pi l_s^2} \int d\tau d\sigma \left( -\frac{1}{r^2} + \frac{1}{2} \left( \frac{\mu^2}{r^2} \right)^2 (X^i)^2 + \frac{1}{2} \left( \partial_\tau X^i \right)^2 + (\partial_\sigma Z)^2 + (\partial_\tau X^i)^2 - (\partial_\sigma X^i)^2 \right) $$
+ (\partial_\tau Y^{5,6,7})^2 - (\partial_\sigma Y^{5,6,7})^2 \right) + 2\pi^2 l_s^4 \exp \left( -\frac{\sqrt{2}Q_\tau r}{r} \right) F_{\tau 0}^2 \right).}

This is the action for a single D1 branes that follows from the matrix string action [2, 3] for N D1 branes,

\[ S_{D1} = \frac{1}{2\pi l_s^2} \int \text{tr} \left( \frac{1}{2} (D_\mu X^i)^2 + \frac{1}{2} (D_\mu Y^a)^2 + \frac{1}{2} (\mu c)^2 (X^i)^2 + g_s^2 \frac{4}{x^2} F_{\mu \nu}^2 \right) \]

\[- \frac{1}{4\pi^2 g_s^2 l_s^4} [X^i, X^j]^2 - \frac{1}{4\pi^2 g_s^2 l_s^4} [Y^a, Y^b]^2 - \frac{1}{4\pi^2 g_s^2 l_s^4} [X^i, Y^a]^2 + \ldots), \]

where the dots stand for fermionic terms.

### 3 D-branes solutions

In general it is a difficult task to find p-brane solutions of supergravity in time dependent backgrounds. However in the linear dilaton background the following factorized ansatz solve the type II supergravity equations of motion.

\[ ds^2 = e^{\frac{4}{3} H^{-\frac{1}{2}}} \left[ -2 dx^+ dx^- - \mu^2 (x^+) x^2_i (dx^+)^2 + d\bar{y}_a^2 \right] + e^{-\frac{4}{3} H^{\frac{1}{2}}} d\bar{x}_a^2 \]

\[ e^{2\Phi} = e^{-\frac{(7-p)}{2} H} H^{\frac{(3-p)}{2}}, \quad H_{+12} = H_{+34} = 2\mu (x^+) \]

\[ F_{+a...a} = e^{2f} \partial_a H^{-1}, \quad \text{for } p \leq 3 \]

\[ F_{a_1...a_n} = \epsilon_{a_1...a_n} \partial_a H, \quad \text{for } p \geq 3 \]

where \( i = x^1, \ldots, x^4 \), and , and \( x^a \) and \( x^\alpha \) correspond to the directions parallel and transverse to the Dp-brane respectively. \( f = Q x^+ \), is a function of \( x^+ \) only and \( H = 1 + (\frac{1}{7-p})^{d} \), (where \( d = 7 - p \) in ten dimensions), is the harmonic function in transverse space and is independent of \( x^+ \).

However, asymptotically the above solutions do not go to the pp-wave background with null linear dilaton discussed in the previous section. To remedy the situation one has to give up the factorized ansatz and replace the harmonic function in the transverse space by \( H \to 1 + e^{-Q x^+} (\frac{1}{7-p})^{d} \) and modify and rewrite the field strengths accordingly. Explicitly one can check that the following solutions obey the supergravity equations of motion,

\[ ds^2 = \tilde{H}^{-\frac{1}{2}} \left[ -2 dx^+ dx^- - \mu^2 (x^+) x^2_i (dx^+)^2 + d\bar{y}_a^2 \right] + \tilde{H}^{\frac{1}{2}} d\bar{x}_a^2 \]
\[ e^{2\Phi} = e^{-2f \tilde{H}^{(3-p)/2}} , \quad H_{+12} = H_{+34} = 2\mu(x^+) \]

\[ F_{+\alpha\ldots\alpha} = e^f \partial_\alpha \tilde{H}^{-1}, \quad (\text{for } p \leq 3) \]

\[ F_{\alpha_1 \ldots \alpha_n} = \epsilon_{\alpha_1 \ldots \alpha_n} e^f \partial_\alpha \tilde{H}, \quad (\text{for } p \geq 3) \quad (22) \]

where \( f = Q x^+ \) as before and \( \tilde{H} = 1 + e^{-Q x^+} (\mu x^+) \).

We would like to study the geodesic equations of a point particle in this background. To see the nature of the trajectory, we pass on to the Einstein frame and the metric for the Dp-brane is given by,

\[ ds_E^2 = \frac{e^{f/2}}{\tilde{H}^{\frac{7-p}{8}}} \left[ -2dx^+dx^- - \mu^2(x^+)x^2(dx^+)^2 + d\vec{y}_a^2 \right] + e^{f/2} \tilde{H}^{\frac{1-p}{8}} d\vec{a}^2. \quad (23) \]

We would like to examine the trajectories near the singularity at constant \( X^- , y^a \) and at \( X^a = 0 \). Namely we check the trajectories moving along the \( X^+ \), which is given by

\[ \frac{d^2X^+}{d\sigma^2} + \left( \frac{j}{2} + \frac{p - 7}{8} \frac{\dot{\tilde{H}}}{\tilde{H}} \right) \frac{dx^+ dx^+}{d\sigma d\sigma} = 0. \quad (24) \]

Near the Big bang singularity (at \( x^+ \to -\infty \)), the above equation can be written as,

\[ \frac{d^2X^+}{d\sigma^2} + c \frac{dx^+ dx^+}{d\sigma d\sigma} = 0. \quad (25) \]

where, \( f = Q x^+ \) and \( c = ((11 - p)/8)Q \). Hence the affine parameter is

\[ \sigma = e^{cx^+} \quad (26) \]

upto an affine reparameterization invariance. The singularity at \( x^+ \to -\infty \) correspond to \( \sigma = 0 \), and can be approached in a finite affine time and the spacetime is geodesically incomplete. On the other hand, at \( x^+ \to \infty \), the string coupling \( g_s \) goes to zero, and the spacetime theory is free at late times. So the geodesic behavior is qualitatively similar to that of our background in the previous section.

**Supersymmetry:** Next we would like to analyze the supersymmetry of the Dp-brane solutions presented above. The supersymmetry variation of the gravitino and dilatino fields in type IIA supergravity in string frame is given by [32, 33],

\[ \delta \lambda = \frac{1}{2} (\Gamma^\lambda \partial_\lambda \Phi - \frac{1}{12} \Gamma^{\mu\nu\rho} H_{\mu\nu\rho} \Gamma_{11}) \epsilon + \frac{1}{8} e^\Phi (5 F^0 - \frac{3}{2} \Gamma^{\mu\nu} F_{\mu\nu}^{(2)} \Gamma_{11} + \frac{1}{4} \Gamma^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}^{(4)}) \epsilon, \quad (27) \]
\[ \delta \Psi_\mu = \left[ \partial_\mu + \frac{1}{8} \omega_{\mu \hat{a} \hat{b}} \Gamma^{\hat{a} \hat{b}} - \frac{1}{8} H_{\mu \hat{a} \hat{b}} \Gamma^{\hat{a} \hat{b}} \Gamma_{11} \right] \epsilon + \frac{e^\Phi}{8} \left[ F^{(0)} - \frac{1}{2!} \Gamma^{\mu \nu} F_{\mu \nu}^{(2)} \Gamma_{11} + \frac{1}{4!} \Gamma^{\mu \nu \rho \sigma} F_{\mu \nu \rho \sigma} \right] \Gamma_\mu \epsilon, \]

where we have used \((\mu, \nu, \rho)\) to describe the ten dimensional space-time indices, and the hated indices are the corresponding tangent space indices. Solving the dilatino variations for the Dp-brane solutions presented in (22) gives the following conditions on the spinor,

\[ \Gamma^{\hat{i}} \epsilon = 0, \]

and

\[ \Gamma^{\hat{a}} \epsilon - \Gamma^{\hat{i} \hat{1} \ldots \hat{i} \hat{p} \hat{a}} \epsilon = 0 \quad \text{(for } p \leq 3) \]

\[ \Gamma^{\hat{a}} \epsilon + \frac{1}{(n + 1)!} \epsilon^{\hat{a}_1 \ldots \hat{a}_n \hat{a}} \Gamma^{\hat{a}_1 \hat{a}_2 \ldots \hat{a}_n \hat{a}} \epsilon = 0 \quad \text{(for } p \geq 3). \]

We would like to emphasize that we need to impose both the conditions, (29) and (30), for the dilatino variation to be satisfied. Next we would like analyze the gravitino variation,

\[ \delta \Psi_+ \equiv \left( \partial_+ - \frac{1}{8} \frac{\partial_+ \bar{H}}{H} - \frac{\mu}{2} \bar{H}^{-\frac{1}{2}} (\Gamma^{\hat{1} \hat{2}} + \Gamma^{\hat{3} \hat{4}}) \Gamma_{11} \right) \epsilon = 0, \]

\[ \delta \Psi_- \equiv \partial_- \epsilon = 0, \quad \delta \Psi_\alpha \equiv \partial_\alpha \epsilon = 0, \]

\[ \delta \Psi_\alpha \equiv \left( \partial_\alpha - \frac{1}{8} \frac{\partial_\alpha \bar{H}}{H} \right) \epsilon = 0, \]

where in writing the above variations we have used the condition (29) and the brane supersymmetry condition (30). Now using the condition,

\[ (\Gamma^{\hat{1} \hat{2}} + \Gamma^{\hat{3} \hat{4}}) \epsilon = 0, \]

we are left with the following equations to be solved

\[ \left( \partial_+ - \frac{1}{8} \frac{\partial_+ \bar{H}}{H} \right) \epsilon = 0, \quad \left( \partial_\alpha - \frac{1}{8} \frac{\partial_\alpha \bar{H}}{H} \right) \epsilon = 0. \]

The above equations are solved by the spinor of the form \( \epsilon = \bar{H}^{\frac{1}{2}} \epsilon_0 \), with \( \epsilon_0 \) being a constant spinor. Now putting together the conditions, (29) (30), and (32), we conclude that our solutions (22) preserves 1/8 supersymmetry.
4 Conclusions

In this paper, we discussed a class of time dependent pp-waves with NS-NS three form flux in null linear dilaton background. It may serve as a toy model of Big Bang cosmology with non-trivial flux. At the beginning of the time, the dilaton diverges and the singularity may be interpreted as Big Bang singularity at the origin of the time. The background preserves 1/4 supersymmetry as opposed to 1/2 supersymmetry of the background in the absence of the flux. The geodesic equation near the singularity has been analyzed. Near the singularity, the perturbative string theory fails as dilaton is divergent, however one can use a discrete light cone type of quantization and the corresponding matrix string description is given.

We have also constructed a class of D-brane solutions in this background, by solving the type II field equations explicitly. Though we restrict ourselves to the type IIA brane solutions in this paper, the general expression is also valid for p-branes in type IIB theory as well. We have also worked out the geodesic equations, and the nature of singularity at early times. The supersymmetry variation revealed that this class of branes preserve 1/8 of the full type IIA spacetime supersymmetry.

There are several directions for future work. The lift of the background to eleven dimensional M-theory is straightforward. One can extend the discussion with RR three form flux and corresponding S-dual type IIB description can be written down. The list of D-brane solutions given by us, is based on a clever ansatz and perhaps not exhaustive. One may try to find more time dependent solutions and particularly some intersecting branes in this background. A matrix string description can also be given to brane solutions presented in this paper.

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References

[1] B. Craps, S. Sethi and E. P. Verlinde, JHEP 0510 (2005) 005 [arXiv:hep-th/0506180].

[2] T. Banks and N. Seiberg, Nucl. Phys. B 497 (1997) 41 [arXiv:hep-th/9702187].

[3] R. Dijkgraaf, E. P. Verlinde and H. L. Verlinde, Nucl. Phys. B 500 (1997) 43 [arXiv:hep-th/9703030].
[4] G. Bonelli, JHEP **0208**, 022 (2002) [arXiv:hep-th/0205213]; Nucl. Phys. B **649**, 130 (2003) [arXiv:hep-th/0209225].

[5] M. Li, Phys. Lett. B **626**, 202 (2005) [arXiv:hep-th/0506260].

[6] M. Berkooz, Z. Komargodski, D. Reichmann and V. Shpitalnik, JHEP **0512** (2005) 018 [arXiv:hep-th/0507067].

[7] S. Kawai, E. Keski-Vakkuri, R. G. Leigh and S. Nowling, Phys. Rev. Lett. **96**, 031301 (2006) [arXiv:hep-th/0507163].

[8] M. Li and W. Song, JHEP **0510**, 073 (2005) [arXiv:hep-th/0507185].

[9] S. R. Das and J. Michelson, Phys. Rev. D **72** (2005) 086005 [arXiv:hep-th/0508068].

[10] B. Chen, Phys. Lett. B **632**, 393 (2006) [arXiv:hep-th/0508191].

[11] B. Chen, Y. I. He and P. Zhang, Nucl. Phys. B **741**, 269 (2006) [arXiv:hep-th/0509113].

[12] T. Ishino, H. Kodama and N. Ohta, Phys. Lett. B **631**, 68 (2005) [arXiv:hep-th/0509173].

[13] D. Robbins and S. Sethi, JHEP **0602**, 052 (2006) [arXiv:hep-th/0509204].

[14] M. Li and W. Song, arXiv:hep-th/0512335.

[15] T. S. Tai, arXiv:hep-th/0601039.

[16] B. Craps, A. Rajaraman and S. Sethi, arXiv:hep-th/0601062.

[17] C. S. Chu and P. M. Ho, arXiv:hep-th/0602054.

[18] S. R. Das and J. Michelson, arXiv:hep-th/0602099.

[19] S. R. Das, J. Michelson, K. Narayan and S. P. Trivedi, arXiv:hep-th/0602107.

[20] F. L. Lin and W. Y. Wen, arXiv:hep-th/0602124.

[21] E. J. Martinec, D. Robbins and S. Sethi, arXiv:hep-th/0603104.

[22] H. Z. Chen and B. Chen, arXiv:hep-th/0603147.

[23] T. Ishino and N. Ohta, arXiv:hep-th/0603215.

[24] H. Kodama and N. Ohta, arXiv:hep-th/0605179.
[25] Y. Hikida, R. R. Nayak and K. L. Panigrahi, JHEP 0509, 023 (2005) [arXiv:hep-th/0508003].

[26] S. Kalyana Rama, arXiv:hep-th/0510008.

[27] J. H. She, JHEP 0601, 002 (2006) [arXiv:hep-th/0509067]; arXiv:hep-th/0512299.

[28] B. Craps, arXiv:hep-th/0605199.

[29] G. Papadopoulos, J. G. Russo and A. A. Tseytlin, Class. Quant. Grav. 20 (2003) 969 [arXiv:hep-th/0211289].

[30] M. Blau, M. O’Loughlin, G. Papadopoulos and A. A. Tseytlin, Nucl. Phys. B 673 (2003) 57 [arXiv:hep-th/0304198].

[31] R. R. Nayak and K. L. Panigrahi, arXiv:hep-th/0604172.

[32] J. H. Schwarz, Nucl. Phys. B 226, 269 (1983).

[33] S. F. Hassan, Nucl. Phys. B 568, 145 (2000) [arXiv:hep-th/9907152].