Presentations from DIS2002 in Krakow

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Both of my talks at DIS2002, on Generalised Parton Distributions and Nuclear Shadowing are presented. In an appendix I summarise some of the discussions which followed the talks.

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1. Introduction

This document includes both of the talks I gave at DIS2002 in Krakow in May 2002. Section 2 contains the proceedings contribution for my talk on Generalised Parton Distributions¹, given in the structure function working group. This presentation is closely related to that of Andreas Freund [1] concerning our NLO QCD analysis of Deeply Virtual Compton Scattering [2, 3] in the diffractive working group. Ruben Sandapen also gave a talk [4] on this process in the same session, looked at from the dipole model framework, which presented results from our joint paper [5]. Section 3 contains my proceedings contribution on nuclear shadowing and diffraction, given in the diffractive interactions working group and based on the analysis of [6]. The transparencies for both talks may be found on my homepage [7]. Finally, in the appendix of section 4, I comment on some of the issues raised at the meeting pertaining to these talks.

2. Generalized parton distributions at next-to-leading order

Abstract: This talk discusses generalized parton distributions (GPDs), which encode various types of non-perturbative information relevant to the QCD description of exclusive processes. Results on their next-to-leading order (NLO) QCD evolution are presented. We find that models for the

¹ In collaboration with Andreas Freund (Regensburg University)
input GPDs based on double distributions require some modification in order to reproduce the available data on deeply virtual Compton scattering.

Generalized parton distributions (GPDs) are required to calculate a wide variety of hard exclusive processes (e.g. diffractive electroproduction of vector mesons, or dijet photoproduction). The easiest and cleanest way to access GPDs is via the electroproduction of a real photon, i.e. Deeply Virtual Compton Scattering [1] (see figure 1, which also defines some kinematic variables). DVCS amplitudes have been proven to factorize [8], i.e. to involve convolutions of perturbatively calculable coefficient functions with GPDs. The ZEUS, H1, HERMES and CLAS experiments all have data available [9]. On the theoretical side the next-to-leading order (NLO) leading-twist analysis of DVCS is now complete and a great deal has been understood about the role of higher twist corrections (see e.g. [10] and references therein). We have completed a NLO numerical analysis of GPDs, DVCS amplitudes and observables [2] and present some of our results here. Most of our analysis code is available from the HEPDATA website [3].

GPDs are defined by Fourier transforms of twist two operators sandwiched between unequal momentum nucleon states. They encode a variety of non-perturbative information about the nucleon, including conventional parton distribution functions (PDFs), distributions amplitudes and form factors, and reproduce these in various limits. The essential feature of the two parton correlation function shown in figure 2 is the presence of a finite momentum transfer, $\Delta = P - P'$, in the $t$-channel. Hence the partonic structure of the hadron is tested at distinct momentum fractions, $x_1, x_2$.

On the light cone these matrix elements are parameterized by double distributions (DDs) which depend on two plus-momentum fractions with respect to two external momenta, on the four momentum transfer squared, $t = \Delta^2$, and on a four-momentum scale $\mu^2$. The external momenta can be selected...
in several ways (e.g., either the “symmetric” $(\Delta, \bar{P} = (P + P')/2)$, or “natural” $(\Delta, P)$ choices). Unfortunately this freedom has led to a proliferation of definitions and nomenclature in the literature (skewed, off-diagonal, non-diagonal, off-forward, · · ·) to describe essentially the same objects, which has led to considerable confusion. Hence the collective name generalized has been introduced to attempt to clarify the situation.

Radyushkin [11] introduced symmetric DDs, with plus momentum fractions, $x, y$ of the outgoing and returning partons defined as shown in the left hand plot of figure 2. They exist on the diamond-shaped domain shown to the right. For a given skewedness, $\xi = \zeta/(2 - \zeta)$, the outgoing parton lines of course only have a single plus momentum, so that Ji’s distributions $H(v, \xi)$ [12] are related to these DDs, via an integral involving $\delta(v - x - \xi y)$, along the off-vertical lines in the diamond ($v \in [-1, 1]$, and the dotted line corresponds to $v = \xi$). For our numerical solution of the renormalization group equations we prefer to work with the natural off-diagonal PDFs defined by Golec-Biernat and Martin [13], which have a momentum fraction $X \in [0, 1]$ of the incoming proton’s momentum. Their relationship to Ji’s functions is shown in figure 3. There are two distinct regions: the DGLAP region, $X > \zeta$, in which the GPDs obey a generalized form of the DGLAP equations for PDFs, and the ERBL region, $X < \zeta$, where the GPDs obey a generalized form of the ERBL equations for distributions amplitudes. In the ERBL region, due to the fermion symmetry, $F_q$ and $F_{\bar{q}}$ are not independent and this leads to an anti-symmetry of the unpolarised quark distributions about the point $\zeta/2$ (the gluon GPD is symmetric). Another formal property of the GPDs, which can be proved on general grounds, is that the N-moments of $H$ are polynomials of degree $\xi^N$: this is known as polynomiality. In addition, any input model for GPDs must reproduce the conventional PDFs for very small skewedness: $\lim_{\zeta \to 0} F_i(X, \zeta) \to f_i(X)$: the “forward limit”.

As a model for the GPDs, with the correct features, we use Radyushkin’s factorized ansatz [11] for the double distributions:
Fig. 3. Relation between $F$ and $H$

\[ F_{DD}(x,y) = \pi(x,y)f^i(x)A^i(t) \]  

where $A^i(t)$ is a form factor form for the factorized $t$-dependence, $f^i(x)$ is the forward PDF and

\[ \pi(x,y) = \frac{\Gamma(2b+1)\,[(1-|x|)^2-y^2]^b}{2^{2b+1}\Gamma^2(b+1)\,(1-|x|)^{2b+1}} \]  

is the profile function which introduces the dependence on skewedness (normalised such that $\int_{-1+|x|}^{1-|x|} dy \, \pi(x,y) = 1$). In the canonical model $b_q = 1$ and $b_q = 2$, $b = \infty$ corresponds to the forward case. By design this model automatically respects the forward limit. To respect polynomiality an additional term, the so-called D-term, is required in the ERBL region, for which we use the model of [14]. Numerical studies indicate that this term is significant only at large $\zeta$ (its influence drops below 1% in for $\zeta < 0.01$).

In the DGLAP region integration over $y$ of the DD leads to the following type of integral:

\[ \mathcal{F}_{q,a}(X,\zeta) = \frac{2}{\zeta} \int_{X-\zeta}^{X} dx' \pi^q(x',\frac{v-x'}{\xi})q^a(x') . \]  

This leads to a serious problem in this model: when $X \rightarrow \zeta$ the PDF is sampled down to zero, where it has not yet been measured. For non-singular distributions this presents no particular problems (although it does involve an extrapolation to $x' = 0$), however, for singular distributions the precise extrapolation is crucial and in general leads to a large enhancement.
of the GPD relative to the PDF in this region (for CTEQ6M the factor can be as large as five). When we compared this model to the H1 DVCS data it overshoots by a factor of approximately 4-6 because of this! To tame this rather unnatural enhancement we introduce a modification of such integrals, via a lower cutoff of the form $a \zeta$. This may be justified by examining the effect of imposing exact kinematics on the imaginary part of the DVCS amplitude which would be required to produce finite mass hadrons in the final state. Such reasoning indicates that $a \sim m_{\text{hadron}}^2/Q_0^2 \approx 1/2$ is a reasonable value. Introducing such a cutoff reduces the enhancement factor of the input GPD close to $X = \zeta$ considerably and allows the H1 data to be well described at both LO and NLO. Unfortunately, it leads to a mild violation of the polynomiality condition since it may introduce higher moments, or slightly alter the highest allowed moments.

Both the continuity of the GPD through the boundary point, $X = \zeta$, and the symmetries about the point $X = \zeta/2$ are preserved under evolution. The evolution equations, at NLO accuracy, are solved numerically on a grid for each value of $\zeta$. For example figure 4 shows the quark GPD at $\zeta = 0.1$ at the input scale and evolved to $Q = 5$ GeV for CTEQ6M (C6M) and MRST01 (M01) [15] input PDFs. This figure demonstrates that the antisymmetry about $\zeta/2$ is preserved under evolution.

The H1 data is already starting to constrain the allowed input GPDs. At present input models are based only on the formal mathematical properties of the GPDs (polynomiality, symmetries and the forward limit). As the data improves it will become necessary to fit the input distributions via minimization methods in a similar fashion to the inclusive case. Our analysis indicates that the cutoff parameter, $a$, and the profile function power, $b$, may
be good candidates for fit parameters, since both of them control the level of skewedness imposed at the input scale.

3. Diffraction and nuclear shadowing

Abstract: I present results from our recent leading twist QCD analysis of nuclear shadowing and contrast them with predictions using the eikonal model. By exploiting QCD factorization theorems, the leading twist approach employs diffractive parton distributions, extracted from diffractive DIS measurements at HERA, to calculate the nuclear shadowing correction on the parton level. Large nuclear shadowing effects are found for the gluon channel which are reflected in the predictions for $F^A_L$.

The most naive assumption about deep inelastic scattering (DIS) on a nucleus one can make is that the photon scatters independently of each nucleon, which gives for the nuclear structure function: $F^A_2 = AF^N_2$. For small $x \lesssim 0.05$ the main negative nuclear correction is nuclear shadowing, i.e. the coherent interaction of the photon with several nucleons at once, which leads to $F^A_2/AF^N_2 < 1$. For a low density nucleus, nuclear shadowing is closely related to diffraction off a nucleon. The leading twist QCD analysis of Frankfurt and Strikman [16] relates nuclear PDFs to diffractive parton distribution functions (DPDFs), by exploiting the QCD factorization theorem for inclusive diffraction [17]. In this talk I present some results from the detailed analysis of [6] which exploited the latest available DPDFs to make predictions for nuclear shadowing and hence nuclear PDFs. The leading twist approach has a sharply contrasting space-time picture and predictions to the popular eikonal approach to nuclear shadowing (which is closely related to the $q\bar{q}$-dipole model of diffraction).

Why should one be interested in DIS on a nucleus? Firstly, nuclear PDFs provide boundary conditions for novel process (e.g. the search for beyond the standard model effects, or for new matter states in QCD such as the quark gluon plasma) as well as for “standard” QCD processes in nuclear collisions. Secondly, a particularly interesting feature of eA is the access to a high parton density regime at much lower energies than in DIS on nucleons. Thirdly, because of the intimate connection between nuclear shadowing and diffraction on a nucleon, high statistics data on nuclear PDFs could be used to discriminate between competing models of diffraction. Lastly, the study of DIS on nuclei is timely since high energy electron nucleus collisions are currently being considered seriously for HERA after 2006 and there is an electron-ion collider (EIC) planned for the USA circa 2012.

2 In affiliation with V. Guzey (Adelaide University), L. Frankfurt (Tel Aviv University) and M. Strikman (Penn State University)
The starting point of the leading twist approach to nuclear shadowing is the application of the logic of Gribov [18] to DIS on the deuteron. The optical theorem relates the total cross section, \( \sigma_{\text{tot}}(\gamma^* D) \) to the imaginary part of the forward scattering amplitude. On the forward amplitude level the photon may interact elastically with either the proton or the neutron or diffractively with both. The latter case corresponds to the nuclear shadowing correction (see figure 5). Hence the nuclear shadowing correction, \( \delta F_2^D = F_2^D + F_2^n - F_2^D \), can be expressed in terms of the structure function for the diffractive scattering of the photon off a nucleon:

\[
\delta F_2^D(x, Q^2) = 2\frac{1 - \eta^2}{1 + \eta^2} \int dt dx F_2^{D(4)}(\beta, Q^2, x^p, t) F_D(4t + 4x^2 m_N^2) \tag{4}
\]

where the prefactor, involving \( \eta = \text{Re} A^D/\text{Im} A^D = \pi/2(\alpha_{\text{em}}(0) - 1) \) comes from the AGK cutting rules and \( F_D \) is the deuteron form factor.

The result for deuteron generalises easily to any pair of nucleons in a nucleus with \( A \) nucleons:

\[
\delta F_2^A(2) = \frac{A(A-1)}{2} 16 \pi \text{Re} \left[ \frac{(1-i\eta)^2}{1+\eta^2} \int d^2b \int_{-\infty}^{\infty} dz_1 \int_{z_1}^{\infty} dz_2 \int_{x^p,0}^{x^p} dx^p \times F_2^{D(4)}(\beta, Q^2, x^p, k_t^2) \right|_{k_t^2=0} \rho_A(b, z_1) \rho_A(b, z_2) e^{ix^p m_N(z_1-z_2)} \right] \tag{5}
\]

where \( \rho \) is the nucleon density, \( z, b \) are longitudinal position and impact parameter of the nucleon concerned. The interaction with more than two nucleons requires some modelling, and we invoke \( \sigma_{\text{eff}} \) for the rescattering cross section (calculated in the quasi-eikonal approximation).

Since inclusive and diffractive structure functions both factorize, and have the same coefficient functions, one can factor off the hard pieces to
relate the PDFs themselves. Hence, on the parton level

$$
\delta f_{j/A}(x, Q^2) = \frac{A(A-1)}{2} 16\pi R e \left[ \frac{(1-i\eta)^2}{1+\eta^2} \int d^2b \int_\infty^\infty d_{z_1} \int_\infty^\infty d_{z_2} \int x_{F,0} dx_{F} \times (6) \right]
$$

where \( f_{j/N}^D \) is the DPDF for a parton of flavour \( j \). The exponential factor in the last line calculates the rescattering. We used several models of DPDFs, tuned to the HERA data [19]. For the effective cross section for rescattering of octet configurations we found that it was necessary to introduce corrections to prevent unitarity being violated (so called saturation effects). Since gluon DPDFs are large we find a corresponding large nuclear shadowing for gluons (see figure 6). For a discussion of enhanced nuclear shadowing correction for central collisions, of the uncertainties associated with the unmeasured diffractive slope, and of the implementation of charm, we refer the reader to our paper [6].

In the eikonal model for nuclear shadowing [20] the \( q\bar{q} \) pair scatters elastically off many nucleons in the target (see figure 7). The fundamental interaction of the dipole with a nucleon is eikonalized and the formula is given by

$$
\delta F_2^A(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \frac{A(A-1)}{2} \left[ (1-i\eta)^2 \int d\alpha \int d^2d_t \sum_i |\psi(\alpha, Q^2, d_t^2)|^2 \times \right. 
\left. \int d^2b \int_\infty^\infty d_{z_1} \int_\infty^\infty d_{z_2} \left[ \sigma^{total}_{qqN}(x, d_t^2, m_i) \right]^2 \rho_A(b, z_1) \rho_A(b, z_2) \times \right.
\left. e^{2x m_N (z_1 - z_2)} e^{-(A/2)(1-i\eta)\sigma_{qqN}(x, d_t^2, m_i) \int z_2 dz \rho_A(z) \times} \right]
$$

where \( \psi \) is the light-cone wavefunction for \( \gamma^* \rightarrow q\bar{q} \), taken from QED (\( \alpha \) is the momentum fraction carried by the quark). Generally, the mixing with higher Fock states in the virtual photon is neglected (this implies that \( Q^2 \)-dependence is not consistent with DGLAP !). Hence a parton level description of nuclear shadowing in the eikonal model is impossible. To implement the eikonal model we employed the MFGS-dipole [21] for \( \sigma^{total}_{qqN} \), but we could also have used other dipole models. In the eikonal model nuclear shadowing is suppressed by colour transparency (since \( \sigma \propto d_t^2 \), at small transverse size \( d_t \)). This implies that nuclear shadowing of \( F_2^A \) decreases rapidly with increasing \( Q \) (see figure 8). This higher-twist nature of nuclear shadowing in the eikonal model is clearest for hard processes, which are most sensitive to small size configurations (e.g. \( F_L^A \) at large \( Q^2 \), see figure 9).
To conclude, the leading twist QCD analysis of [6] suggests that nuclear shadowing is a leading twist phenomena. It produces radically different predictions to the eikonal approach popular in the literature, (cf. the dipole model for diffraction) for which nuclear shadowing is a higher twist effect. A systematic measurement of nuclear PDFs (via $F_{2A}$, nuclear DVCS and Drell-Yan, which should be possible at HERA III and EIC), and hence of nuclear shadowing, can help establish the correct model for diffraction. It may well be possible to investigate non-linear QCD in DIS on a large
Fig. 8. Eikonal model predictions for $F_2^A$ for $Q = 2, 5, 10$ GeV (solid, dashed, dotted curves).

Fig. 9. Contrasting predictions from the eikonal model (black) and the leading twist model (blue and red) for $F_L^A$ for $Q = 2, 5, 10$ GeV (solid, dashed, dotted curves).

nucleus, but one needs to understand nuclear shadowing first.

4. Appendix: discussions

In the discussion session the question was raised whether the skewedness effect necessarily leads to an enhancement in general at NLO (as is the case at LO). Unfortunately, it is not possible to give a complete answer at this time. Within the framework of the double distribution input model
one sees a suppression of the GPD at the input scale for a falling forward distribution as $x \to 0$, and an enhancement for a rising distribution, relative to the forward case. It is clear however that skewed evolution produces an enhancement relative to forward evolution of a given input. So, even for the relatively clean DVCS process, whether one sees an enhancement or a suppression depends on what one chooses for the input GPD and how close one is to the input scale.

As J. Bartels pointed out correctly, to answer this question for a particular process, for example diffractive vector meson production of $J/\psi$, it is necessary to have a complete NLO calculation, which includes calculating the $q\bar{q}g$ component of the virtual photon and diffractively produced system. At present the NLO coefficient function are known only for DVCS [10]. It is clear that a full NLO analysis is required for each process.

A global analysis of high energy exclusive processes, i.e. DVCS and photoproduction and electroproduction of all vector meson states, within the framework of the dipole model, is also highly desirable. In order to proceed in a quantitative fashion, it would be optimal to place the semi-qualitative dipole model developed in [21] on a firmer footing by performing a fit to the available inclusive structure function data. Such a fit is currently under active study. One could then use this “fitted” dipole cross section in the global analysis of exclusive processes (remembering to take skewedness into account, as appropriate).

Following my talk on nuclear parton distributions a question was raised concerning a comparison to the available data. My collaborator V. Guzey is currently looking into this issue. He also plans to provide analytic forms for the nuclear parton distribution functions fitted to the numerical results obtained in our paper [6].

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REFERENCES

[1] A. Freund, DVCS vs. GPDs: What does DVCS in LO/NLO tell us about GPDs?, talk at DIS2002.
[2] A. Freund, M. McDermott, Phys. Rev. D65, 056012 (2002); Phys. Rev. D65, 074008 (2002); Phys. Rev. D65, 091901(R) (2002); Eur. Phys. J. C23, 651 (2002).
[3] http://durpdg.dur.ac.uk/hepdata/dvcs.html
[4] R. Sandapen, *Colour dipoles and virtual Compton scattering*, talk at DIS2002.
[5] M. McDermott, R. Sandapen, G. Shaw, *Eur. Phys. J. C* 22, 655 (2002).
[6] L. Frankfurt, V. Guzey, M. McDermott, M. Strikman, *J. High Energ. Phys.* 0202, 027 (2002).
[7] http://www.liv.ac.uk/~mfmcd
[8] J. C. Collins, A. Freund, *Phys. Rev. D* 59, 074009 (1999).
[9] P. R. Saull, for ZEUS Collab., hep-ex/0003030; C. Adloff et al., H1 collab., *Phys. Lett. B* 517, 47 (2001); A. Airapetian et al., HERMES Collab., *Phys. Rev. Lett.* 87, 182001 (2001); S. Stepanyan et al., CLAS Collab., *Phys. Rev. Lett.* 87, 182002 (2001).
[10] A. V. Belitsky, D. Mueller, L. Niedermeier, A. Schafer, *Phys. Lett. B* 474, 163 (2000); *Nucl. Phys. B* 593, 289 (2001); A. V. Belitsky, D. Mueller, A. Kirchner, *Nucl. Phys. B* 629, 323 (2002).
[11] A. V. Radyushkin, *Phys. Rev. D* 56, 5524 (1997); *Phys. Lett. B* 380, 417 (1996); *B* 385, 333 (1996); *B* 449, 81, (1999); I. V. Musatov, A. V. Radyushkin, *Phys. Rev. D* 61, 074027 (2000).
[12] X. Ji, *J. Phys. G* 24, 1181 (1998) 1181; *Phys. Rev. D* 55, 7114 (1997).
[13] K. Golec-Biernat, A. D. Martin, *Phys. Rev. D* 59, 014029 (1999).
[14] M. V. Polyakov, C. Weiss *Phys. Rev. D* 60, 114017 (1999).
[15] J. Pumplin et al., hep-ph/0201195; A. D. Martin, et al., *Eur. Phys. J. C* 23, 73 (2002).
[16] L. Frankfurt, M. Strikman, *Eur. Phys. J. A* 5, 293 (1999).
[17] J. Collins, *Phys. Rev. D* 57, 3051 (1998); (E) *D* 61, 019902 (2002).
[18] V. N. Gribov, *Sov. Phys. JETP* 29, 483 (1969).
[19] L. Alvero, J Collins, J Whitmore, J Terron, *Phys. Rev. D* 59, 074022 (1999); F. Hautmann, Z. Kunszt, D. Soper, *Nucl. Phys. B* 563, 153 (1999); H1 Collab., *Z. Phys. C* 76, 613 (1997).
[20] N. N. Nikolaev, B. G. Zakharov, *Phys. Lett. B* 260, 414, (1991); B. Kopeliovich et al., *Phys. Lett B* 367, 329 (1996); *Phys. Rev. C* 62, 035204 (2000); E. Gotsman et al. *Phys. Lett. B* 492, 47 (2000); *Nucl. Phys. A* 683, 383 (2001).
[21] M. McDermott, L. Frankfurt, V. Guzey, M. Strikman, *Eur. Phys. J. 16*, 641 (2000).