An Accelerating Cosmology Without Dark Energy

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The negative pressure accompanying gravitationally-induced particle creation can lead to a cold dark matter (CDM) dominated, accelerating Universe (Lima et al. 1996) without requiring the presence of dark energy or a cosmological constant. In a recent study Lima et al. 2008 (LSS) demonstrated that particle creation driven cosmological models are capable of accounting for the SNIa observations of the recent transition from a decelerating to an accelerating Universe. Here we test the evolution of such models at high redshift using the constraint on the redshift of the epoch of matter – radiation equality, provided by the WMAP constraints on the early Integrated Sachs-Wolfe effect (ISW). Since the contribution of baryons and radiation was ignored in the work of LSS, we include them in our study of this class of models. The parameters of these more realistic models with continuous creation of CDM is tested and constrained at widely-separated epochs in the evolution of the Universe. This comparison reveals a tension between the high redshift CMB constraint on the redshift of the epoch of matter – radiation equality and that which follows from the low redshift SNIa data, challenging the viability of this class of models.

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I. INTRODUCTION

A large amount of complementary cosmological data have established the surprising result that the Universe experienced a recent transition from a decelerating to an accelerating expansion. The current data are consistent with a flat, Friedman-Lemaitre cosmology whose dominant components at present consist of matter (cold dark matter plus baryons) and a cosmological constant ($\Lambda$) or, equivalently, the energy density of the vacuum; the $\Lambda$CDM concordance model. The matter and radiation dominate earlier in the evolution of the Universe, leading to its decelerating expansion, while the cosmological constant or vacuum energy component has come to dominate more recently, driving the accelerating expansion. While the concordance model provides the simplest, most economical explanation which is consistent with all extant data, provided that it is fine-tuned to fit that data, it is by no means the only candidate proposed to explain the accelerating expansion of the current Universe. Dozens (if not hundreds!) of alternate possibilities have been explored in the literature (see Lima et al. 2008 (LSS) for an extensive list of references) and many of them remain viable candidate models. Since the astronomical community is planning a variety of large observational projects intended to test and constrain the standard $\Lambda$CDM concordance model, as well as many of the proposed alternative models, it is timely and important to identify and explore a variety of physical mechanisms (or substances) which could also be responsible for the late-time acceleration of the Universe.

In principle, any homogeneously distributed “exotic” fluid endowed with a sufficiently negative pressure, $p < -\rho/3$, so-called dark energy, will suffice. The simplest candidates for the dark energy are a positive cosmological constant, $\Lambda$, or a non-zero vacuum energy density. In both cases the equation of state of the exotic fluid is $w = p/\rho = -1$. All current observational data are in good agreement with the cosmic concordance ($\Lambda$CDM) model, consisting of a cosmological constant or vacuum energy ($w_\Lambda = -1$) whose current density parameter is $\Omega_\Lambda \approx 0.74$, plus cold dark matter and baryons ($w_M = 0$) with $\Omega_M = \Omega_{CDM} + \Omega_B = 1 - \Omega_\Lambda \approx 0.26$. Nevertheless, the $\Lambda$CDM model encounters several challenges. For example, the value of the energy density of the vacuum required by the data ($\Omega_\Lambda \approx 0.26$) needs to be fine-tuned, exceeding the estimates from quantum field theories by some 50 – 120 orders of magnitude. If the vacuum energy (or $\Lambda$) differs from zero, why does it have the value needed for consistency with the data?

LSS noted recently that the key ingredient required to accelerate the expansion of the Universe is a sufficiently negative pressure, which occurs naturally as a consequence of cosmological particle creation driven by the gravitational
While completely different from it, there are some analogies between this class of models and the continuously accelerating, never decelerating, expansion of the steady-state cosmology driven, in the Hoyle-Narlikar \[5\] version, by C-field induced particle creation. The late-time evolution of the class of models explored by LSS is qualitatively very different from that of the standard, \(ΛCDM\) concordance model in that they lack a cosmological constant and, when baryons are included, the ratio of the dark matter to baryon densities evolves with time or redshift. Nonetheless, the LSS \[2\] analysis of the late-time evolution of such a CDM-dominated model with no cosmological constant or dark energy, lacking baryons and radiation, showed that it is quantitatively possible to account for the SNIa data which reveal a recent transition from decelerated to accelerated expansion. Indeed, while the flat \(ΛCDM\) concordance model provides a good fit to the SNIa data, the best fit actually corresponds to a positively curved (closed) \(ΛCDM\) model, so that for some choice of parameters the models considered by LSS may provide an even better fit to the supernovae data. For a subset of the models explored by LSS to be discussed here, the early-time, high redshift \((e.g., z \gtrsim 10)\) evolution is indistinguishable from that of the \(ΛCDM\) concordance model, so that the parameters controlling the early evolution of these models must agree with those of the concordance model. To set the stage for the constraints and tests discussed here, we note that for the \(ΛCDM\) concordance model with \(Ω_Λ = 1 - Ω_M = 0.76\), the redshift of the transition from decelerated to accelerated expansion is \(z_t = 0.79\), the redshift of the epoch of equal matter and radiation densities (related to the early, Integrated Sachs-Wolfe (ISW) effect) is \(z_{eq} = 3223\) and, the present age of the Universe in units of the Hubble age \((H_0^{-1})\) is \(H_0t_0 = 1.00\).

Since at present baryons are subdominant to CDM and the contribution of radiation is negligible, the neglect of baryons and radiation by LSS was not a bad approximation for their study of the recent evolution of the Universe. Here, we revisit the LSS model of dark matter creation driven acceleration in a more realistic model incorporating baryons and radiation. While it will be seen that there is a subset of models with baryons and radiation whose late time evolution \((i.e., H = H(z)\) for \(z \lesssim 5\)\) is sufficiently close to those studied by LSS and to the \(ΛCDM\) concordance model (and, therefore, they too will be consistent with the SNIa data), these models differ from those studied by LSS in their early evolution, in particular at the epochs of recombination and of equal matter and radiation densities. To further test these models we impose an additional, high-redshift constraint derived from the early Integrated Sachs-Wolfe (ISW) effect, provided by the WMAP \[4\] value of the redshift of matter – radiation equality, \(z_{eq} = 3141 \pm 157\).

In \(§\) II we describe the class of models under consideration and solve for the evolution of the Hubble parameter as a function of redshift \((H = H(z))\). In \(§\) III we compare the predictions of this class of models as a function of the two parameters related to the particle creation rate to the SNIa data \[2\] and to the constraint on the redshift of the epoch of equal matter and radiation densities provided by the WMAP observations of the CMB temperature anisotropy spectrum \[4\]. Our results are summarized and our conclusions presented in \(§\) IV.

**II. PARTICLE CREATION DRIVEN COSMOLOGY**

Consider a sufficiently large comoving volume \((V)\), representative on average of the Universe which, at some time, contains \(N = nV\) CDM particles \((\omega_{\text{CDM}} = 0)\). If CDM particles are being created at the expense of the gravitational field \[7\], then

\[
\frac{1}{N} \frac{dN}{dt} = \Gamma, \tag{1}
\]

where \(\Gamma\) is the creation rate (number of particles per unit time), assumed to be uniform throughout the Universe. In this case, since the mass density in the CDM is proportional to the number density,

\[
\frac{d[\ln(\rho_{\text{CDM}}V)]}{dt} = \Gamma, \tag{2}
\]

so that

\[
\rho_{\text{CDM}} = \rho_{\text{CDM},0}(1 + z)^3 \exp[-\int_{z}^{0} \Gamma dt]. \tag{3}
\]

In eq. 3, \(z\) is the redshift \((V = V_0(1 + z)^{-3})\) and the subscript 0 is for quantities evaluated at the present epoch \((z = 0)\).

The models we consider are defined by the choice for the particle creation rate, \(\Gamma\). The most natural choice would be a particle creation rate which favors no epoch in the evolution of the Universe, such as \(\Gamma = 3\beta H\), where \(H\) is the Hubble parameter and the free parameter \(\beta\) is positive. However, in the absence of baryons and radiation, it is easy to show (see LSS) that such a Universe will always decelerate if \(\beta < 1/3\) and will always accelerate if \(\beta > 1/3\). The inclusion of baryons and radiation, whose effect is to drive a decelerated expansion, opens the possibility of a transition
from an early, matter-dominated deceleration, to a late, CDM-driven acceleration. In this case the transition from decelerating to accelerating expansion is set by the baryon density and it occurs “naturally”, late in the evolution of the Universe at low redshift, without any need for fine-tuning. However, it will be seen that the early evolution of models with this form for the particle creation rate (\(\Gamma \propto H\)) are inconsistent with the early ISW data. This problem may be alleviated by adding a second creation term for the particle creation rate which is “tuned” to the current epoch, \(\Gamma \propto H_0\), where \(H_0\) is the present \(\langle t_0 \rangle\) value of the Hubble parameter. As a result, the class of particle-creation driven models to be studied here are defined by

\[
\Gamma = 3\gamma H_0 + 3\beta H,
\]

where \(0 \leq \{\gamma, \beta\} \leq 1\). In this case, the CDM mass/energy density evolves as,

\[
\rho_{\text{CDM}} = \rho_{\text{CDM},0}(1 + z)^{3(1 - \beta)}\exp[3\gamma(\tau - \tau_0)],
\]

where \(\tau \equiv H_0 t\) and \(\tau_0 \equiv H_0 t_0\) are the age and the present age of the Universe in units of the Hubble age \((H_0^{-1})\). For \(H_0 \equiv 100h\) km s\(^{-1}\)Mpc\(^{-1}\), \(H_0^{-1} = 9.78h^{-1}\) Gyr. In our analysis the HST Key project result \(h = 0.72 \pm 0.08\) is adopted, so that \(H_0^{-1} = 13.6 \pm 1.5\) Gyr.

For simplicity, as is the case for the cosmic concordance model, our considerations are limited to flat cosmologies, so that \(\Omega_M = \Omega_B + \Omega_{\text{CDM}} = 1\), neglecting the very small contribution from the radiation density at present, when \(\Omega_B\) and \(\Omega_{\text{CDM}}\) are evaluated. For flat models, the general Friedman equation reads,

\[
\left(\frac{H}{H_0}\right)^2 = \Omega_R(1 + z)^4 + \Omega_B(1 + z)^3 + (1 - \Omega_B)(1 + z)^3(1 - \beta)\exp[3\gamma(\tau - \tau_0)].
\]

In comparing the model predictions with the SNIa data, the radiation density term \((\Omega_R < 10^{-4})\) may be safely neglected.

At the time of Big Bang Nucleosynthesis (BBN) the Universe is radiation dominated and only the baryon density (along with the radiation density) plays an important role. BBN cares about neither the dark matter or the cosmological constant for the \(\Lambda\)CDM model, nor about the creation of dark matter for the \(\{\beta, \gamma\}\) model considered here. However, since the creation of dark matter is accompanied by a “creation pressure” \(p_c = -\Gamma\rho_{\text{DM}}/3H\) (see LSS), and the evolution of the dark matter density differs from the “usual” \((1 + z)^3\) evolution for conserved particles, the late time growth of perturbations in this model will likely depart from that in the concordance model. At very early times near the epochs of equal matter and radiation densities and recombination, \(w_c \equiv p_c/\rho_{\text{DM}} \rightarrow -\beta\). So if, as will be seen below, \(\beta = 0\) or \(\beta \ll 1\) are favored, the early growth of perturbations in the \(\{\beta, \gamma\}\) model will track that of the concordance model. As a result, for the second observational constraint we require that the model-predicted redshift of the epoch of equal matter and radiation densities, \(1 + z_{eq} \equiv \rho_\text{DM}/\rho_R\), agree with that determined by the WMAP observations of the early ISW effect [4]. Since the \(\Lambda\)CDM model is consistent with these observations, the early-time \((t \ll t_0)\) evolution of these \(\{\beta, \gamma\}\) models must track closely that of the standard \(\Lambda\)CDM model.

Before considering the general case, where \(\{\Omega_B, \beta, \gamma\} \neq 0\), it is instructive to examine several simplified cases where one or more of these parameters is set equal to zero.

### A. The LSS Model: \(\Omega_B = 0\)

Ignoring the contributions from baryons and radiation, this is the model studied by LSS [2]. In this case the Friedman equation simplifies to,

\[
\left(\frac{H}{H_0}\right)^2 = \left[\frac{1}{a}\frac{da}{d\tau}\right]^2 = (1 + z)^3(1 - \beta)\exp[3\gamma(\tau - \tau_0)],
\]

which has a solution for the scale factor \(a(t)\) (or the redshift, \(z\)) as a function of time,

\[
\frac{a(t)}{a_0} = \frac{1}{1 + z} = \left(\frac{1 - \gamma - \beta}{\gamma}\right)\left(e^{3\gamma\tau/2} - 1\right)^{1/(1 - \beta)}.
\]

Evaluating this at \(z = 0\), where \(\tau = \tau_0\) and \(a = a_0\), relates the present age, \(\tau_0\), to the \(\{\beta, \gamma\}\) parameters,

\[
\exp(3\gamma\tau_0/2) = \frac{1 - \beta}{1 - \gamma - \beta}.
\]
The present age of the Universe \( (t = t_0, \tau = \tau_0) \) is

\[
\tau_0 = H_0 t_0 = \frac{2}{3\gamma} \ln\left( \frac{1 - \beta}{1 - \gamma - \beta} \right).
\]  

(10)

Solving for the time – redshift relation

\[
\exp\left( \frac{3\gamma \tau}{2} \right) = 1 + \left( \frac{\gamma}{1 - \gamma - \beta} \right) (1 + z)^{-3(1-\beta)/2}.
\]  

(11)

Using the \( z \) versus \( \tau \) and \( \tau_0 \) relations above, the Friedman equation for \( H = H(z) \), reduces to the simple form

\[
\frac{H}{H_0} = (1 - \beta)^{-1}[\gamma + (1 - \gamma - \beta)(1 + z)^{3(1-\beta)/2}].
\]  

(12)

It is easy to confirm (see LSS for the details) that if \( \gamma = 0 \), the expansion of the Universe always decelerates for \( 0 \leq \beta < 1/3 \) and always accelerates for \( 1/3 < \beta \leq 1 \). In this case there is no transition from an early decelerating to a late accelerating Universe. However, with \( \gamma \neq 0 \) \( (0 \leq \gamma \leq 1) \) the redshift, \( z_t \), of the transition from early-time deceleration to late-time acceleration is given by

\[
1 + z_t = \left[ \frac{2\gamma}{(1 - 3\beta)(1 - \gamma - \beta)} \right]^{1/3(1-\beta)}.
\]  

(13)

As shown in LSS, there is a range of \( \{\gamma, \beta\} \) values whose fit to the SNIa data is at least as good as the fit provided by the parameters of the \( \Lambda \)CDM concordance model. In Figure 1 we illustrate this by comparing the relative distance modulus – redshift relations for several combinations of \( \{\gamma, \beta\} \) with that for the \( \Lambda \)CDM model and, for comparison with a model which leads to a poor fit, the Einstein-deSitter model. Note that for \( z \lesssim 1.5 \), many of the \( \{\beta, \gamma\} \) models, with and without baryons, are indistinguishable from the \( \Lambda \)CDM model. While the curves for the best fit values for \( \beta = 0 \) and \( \{\Omega_B, \gamma\} = \{0, 0.63\} \) and \( \{0.042, 0.66\} \) are very close to each other, the curve for \( \gamma = 0, \Omega_B = 0.042 \) and \( \beta = 0.61 \) deviates noticeably from them, especially at \( z \gtrsim 1 \). Furthermore, the curves for \( \beta = 0 \) and
FIG. 2: The 68% (dark blue) and 95% (light blue) contours in the $\beta - \gamma$ plane derived for $\Omega_0 = 0$ from the SNIa data [3], along with the 95% confidence band (black; see the inset which zooms in on this band) consistent with the early ISW constraint that $z_{eq} = 3141 \pm 157$ [4]. The SNIa data are best fit at $\beta = 0$ and $\gamma = 0.63$. The solid black curve, shown in more detail in the inset, is for the best fit relation between $\beta$ and $\gamma \equiv \gamma^0$ (see eqs. 16,17) consistent with the observed value of $z_{eq}$ [4].

FIG. 3: The likelihood functions for $\gamma \equiv \gamma^0$ inferred from the SNIa data (solid/gray) and from the $z_{eq}$ constraint (dashed/blue) for $\beta = 0$ and $\Omega_B = 0$.

$\gamma = 0.52$ (0.50), corresponding to the $z_{eq}$ constraint with (without) baryons, deviates significantly from those curves which provide the best fits to the SNIa data.

For the Union 2008 set of SNIa data [3] adopted here, the best fitting combination of parameters (for $\Omega_B = 0.042$) occurs for $\beta = 0$ and $\gamma = 0.63$, corresponding to $z_t = 1.26$. While this value for the transition redshift may seem surprisingly high compared to $z_t = 0.79$ for the $\Lambda$CDM concordance model with $\Omega_M = 1 - \Omega_\Lambda = 0.26$, as may be seen from Figure 1, the relative distance modulus – redshift relations for these two models are very similar at redshifts
The analysis of LSS [2] established that as an alternative to dark energy, a model with the creation of cold dark matter by the gravitational field is capable of accounting for the evolution of a Universe in which the early-time decelerated expansion is succeeded by a late-time accelerating phase, consistent with the SNIa data. This encourages us to explore a more realistic version of the LSS model including baryons (and radiation). While qualitatively different from the models explored by LSS, models with baryons are quantitatively very similar to them since for all intermediate and low redshifts $\rho_B \ll \rho_{CDM}$. As in LSS, the SNIa data [3] will be used to constrain the late-time evolution of this class of models. To further test them and to constrain their parameters, the observational early ISW effect constraint on the redshift of equal matter and radiation densities inferred from WMAP, $1 + z_{eq} = 3142(\pm 0.05)$ [4], is used.

For a successful model be consistent with the early ISW effect requires that the early, high redshift evolution of these models be nearly identical with that of the ACDM concordance model. However, this does not guarantee that the growth of structure in these models need be the same as in the concordance model. For the very early evolution of both models only baryons (for BBN) and radiation are important; $\rho_{CDM}$ and $\rho_\Lambda$ may be neglected. In particular, since the results of BBN depend only on the baryon and radiation densities and are independent of $\rho_{CDM}$ and $\rho_\Lambda$, as well as of any particle creation, it is best to choose $\Omega_B$ from BBN and not, for example, from observations of large

At high redshifts, including the contribution from radiation (but not yet from baryons), the Friedman equation is modified to

$$\left(\frac{H}{H_0}\right)^2 \approx \Omega_R(1 + z)^4 + \left(\frac{1 - \gamma - \beta}{1 - \beta}\right)^2 (1 + z)^3(1 - \beta),$$

so that the redshift of equal matter (CDM) and radiation densities in this class of models, without baryons ($\Omega_B = 0$), is given by

$$\Omega_R(1 + z_{eq})^{1+3\beta} = \left(\frac{1 - \gamma - \beta}{1 - \beta}\right)^2.$$

Fixing $z_{eq}$, along with the observed value of $\Omega_R$, provides a complementary constraint on the $\gamma - \beta$ relation to that from the SNIa data. For $\Omega_B = 0$, the $\gamma(\Omega_B = 0) = \gamma^0 - \beta$ relation is

$$\gamma^0 = (1 - \beta)\{1 - [\Omega_R(1 + z_{eq})]^{1/2}(1 + z_{eq})^{3/2}\}.$$

Accounting for the still relativistic neutrinos and for $\frac{h}{\Omega_R} = 0.72 \pm 0.08$, $\Omega_R = 8.065(1 \pm 0.22) \times 10^{-5}$. For the combination of parameters which best fits the SNIa data [3], $\beta = 0$ and $\gamma^0 = 0.63$, $z_{eq} = 1661$, a redshift which is much too small for consistency with the WMAP data which finds $z_{eq} = 3141 \pm 157$ [4].

For $1 + z_{eq} = 3142(\pm 0.05)$,

$$\gamma^0 = (1 - \beta)(1 - 0.503(1 \pm 0.114)(3142)^{3/2}).$$

The 68% and 95% constraints when $\Omega_B = 0$ for $\gamma^0$ versus $\beta$ derived from $z_{eq}$ are shown in Figure 2. For $\beta = 0$, the WMAP measurement of the early ISW effect [4] requires $\gamma^0 = 0.50 \pm 0.06$.

It is easy to understand this result by comparing the high redshift evolution of the $\{\beta, \gamma\}$ model (for $\beta = 0$) with that of the ACDM model. Consistency between them requires that $(1 - \gamma^0)^2 \approx \Omega_M = 0.26$, so that $\gamma^0 \approx 0.49$, in excellent agreement with the more direct result from the early ISW effect.

For each of the independent constraints from the SNIa data and the early ISW effect, the best fit occurs at $\beta = 0$ and, as may be seen from Figure 2 the parameters identified by these two constraints diverge from each other as $\beta$ increases. In Figure 3 are shown the probability distributions for $\gamma$ when $\beta = 0$ derived from the SNIa data [3] and from the early ISW effect [4].

The early ISW combination of parameters appears to be in some conflict with those identified by the SNIa data [3]. For example, for $\beta = 0$ and $\gamma^0_{SNIa} = 0.63 \pm 0.04$, leads to a prediction of $z_{eq}, (1 + z_{eq})_{SNIa} = 1661^{+544}_{-524}$ which is some $2.7\sigma$ away from the WMAP value [4]. We return to a more careful discussion of the tension between the SNIa and WMAP data for this class of models in the context of our discussion of the more realistic model including baryons. Nonetheless, the results presented here provide a useful background for the subsequent discussion of those cases for which $\Omega_B \neq 0$.

III. INCLUDING BARYONS

The analysis of LSS [2] established that as an alternative to dark energy, a model with the creation of cold dark matter by the gravitational field is capable of accounting for the evolution of a Universe in which the early-time decelerated expansion is succeeded by a late-time accelerating phase, consistent with the SNIa data. This encourages us to explore a more realistic version of the LSS model including baryons (and radiation). While qualitatively different from the models explored by LSS, models with baryons are quantitatively very similar to them since for all intermediate and low redshifts $\rho_B \ll \rho_{CDM}$. As in LSS, the SNIa data [3] will be used to constrain the late-time evolution of this class of models. To further test them and to constrain their parameters, the observational early ISW effect constraint on the redshift of equal matter and radiation densities inferred from WMAP, $1 + z_{eq} = 3142(\pm 0.05)$ [4], is used.

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scale structure such as those provided by galaxy correlation functions. In the standard model, with three flavors of light neutrinos, the predicted primordial abundances depend only on the baryon to photon ratio which is directly related to $\Omega_B h^2$. To constrain $\Omega_B$, deuterium is the baryometer of choice (see [10] for a recent review and further references). For BBN [11], using the latest observations of deuterium [12], $\Omega_B h^2 = 0.0218 \pm 0.0011$. For the HST Key Project value of $H_0$ [9], $\Omega_B = 0.042 \pm 0.010$. Since the models we consider are flat and contain no dark energy or a cosmological constant, $\Omega_{CDM} = 1 - \Omega_B = 0.958 \pm 0.010$. These are the values adopted in our analysis below.

A. $\Omega_B \neq 0, \beta \neq 0, \gamma = 0$

As noted earlier and demonstrated in LSS, in the absence of baryons and when $\gamma = 0$ the expansion of the Universe always decelerates for $0 \leq \beta < 1/3$ and always accelerates for $1/3 < \beta \leq 1$. However, with the inclusion of baryons, which dominate over the CDM at high redshifts, the early expansion of the Universe always decelerates. In this case, for a choice of $\beta$ in the range $\{1/3, 1\}$, the late time expansion will accelerate. To better understand this transition, consider the “total equation of state”, $w \equiv p_{TOT}/\rho_{TOT}$, where $p_{TOT} = p_B + p_{CDM} + p_c = p_c$, since $p_B = p_{CDM} = 0$. For $w > -1/3$, the expansion decelerates, while for $w < -1/3$, it accelerates. Including baryons, but ignoring the early-time contribution from radiation,

$$w = p_c/\left[\rho_B + \rho_{CDM}\right].$$

Since, for $\gamma = 0$, $p_c = -\beta \rho_{CDM}$,

$$w = -\beta \left[1 + \left(\frac{\rho_B}{\rho_{CDM}}\right)\right]^{-1} = -\beta \left[1 + \left(\frac{\Omega_B}{1 - \Omega_B}\right)(1 + z)^{3\beta}\right]^{-1}.$$  

(19)

At present, for $z = 0$, $w = -\beta(1 - \Omega_B)$, so that for $\Omega_B = 0.042$, the expansion is accelerating if $\beta > 1/3(1 - \Omega_B) = 0.348$. In the future ($a \rightarrow \infty$, $z \rightarrow -1$), $w \rightarrow -\beta$ (for the $\Lambda$CDM model, $w \rightarrow -1$).

![FIG. 4: The equation of state parameter $w$ versus redshift relations for $\Omega_B = 0.042$ and $\gamma = 0$ for several choices of $\beta$. Also shown for comparison are the $w(z)$ variations for the Einstein-deSitter model ($w = 0$) and for the $\Lambda$CDM model (solid curves). Notice that $z < 0$ corresponds to the future ($a > a_0$) evolution of the Universe: as $z \rightarrow -1$, $a \rightarrow \infty$.](image)

As may be seen from eq. 19, for $\gamma = 0$ and $\Omega_B \neq 0$ the transition from deceleration to acceleration ($w = -1/3$) occurs at redshift $z_t$ where,

$$1 + z_t = \left[\left(\frac{1 - \Omega_B}{\Omega_B}\right)(3\beta - 1)\right]^{1/3\beta}.$$  

(20)
The $z_t - \beta$ relation for $\gamma = 0$ and $\Omega_B = 0.042$ is shown in Figure 5. Without any fine-tuning the expansion of the Universe has a late-time, low-redshift ($0 \lesssim z_t \lesssim 4$) transition from decelerating to accelerating. The present epoch is “selected” in this model by the BBN-determined value of $\Omega_B \neq 0$.

In this case ($\gamma = 0$), as may be seen from Figure 7, the best (albeit not very good) fit (see Fig. 1) to the SNIa data is for $\beta = 0.61$, so that $z_t = 4.0$. For this model the baryons dominate over the CDM during the early evolution of the Universe, when $z > z_*$, where $\rho_B(z_*) = \rho_{\text{CDM}}(z_*)$,

$$1 + z_* \equiv \left( \frac{1 - \Omega_B}{\Omega_B} \right)^{1/3\beta}. \quad (21)$$

For the model with $\beta = 0.61$ and $\Omega_B = 0.042$ (and $\gamma = 0$), $z_* = 4.5$. At high redshifts, $z \gg 5$, $\rho_M \rightarrow \rho_B$, so that $1 + z_{eq} \rightarrow \Omega_B/\Omega_R = 522$, clearly in conflict with the constraint on $z_{eq}$ from the early ISW effect. So, although it is of some interest that the late time evolution of this model with $\gamma = 0$ and $\Omega_B \neq 0$ is capable of a transition from decelerated to accelerated expansion without any fine-tuning, the early evolution of this model is in conflict with the WMAP data.

B. $\Omega_B \neq 0, \gamma \neq 0, \beta = 0$

Motivated by the LSS result for $\Omega_B = 0$ that the best fit to the SNIa data occurs for $\beta = 0$, we next explore this case with $\Omega_B = 0.042$. Setting $\beta = 0$ but allowing for a non-zero value of the baryon density, the Friedman equation, including radiation, becomes

$$\left( \frac{H}{H_0} \right)^2 = \Omega_R(1 + z)^4 + (1 + z)^3\{\Omega_B + (1 - \Omega_B)e^{3\gamma\tau_0}\}.$$ \quad (22)

For the late-time evolution of $H = H(z)$ needed for comparison with the SNIa data, the contribution from the radiation term may be neglected. Since $\Omega_B \ll 1 - \Omega_B$ (and, it can be checked that $\Omega_B \ll (1 - \Omega_B)e^{-3\gamma\tau_0}$), it is a good approximation in solving for the $z$ versus $t$ relation to neglect the $\Omega_B$ term in eq. 19. If so, then at late-times when $z$ is not too large (and $\tau$ is not $\ll \tau_0$),

$$\left( \frac{H}{H_0} \right)^2 \approx \left[ \frac{1}{a} \left( \frac{da}{d\tau} \right) \right]^2 \approx (1 - \Omega_B)(1 + z)^3\exp[3\gamma(\tau - \tau_0)]. \quad (23)$$
In this approximation, accounting for the \((1 - \Omega_B)\) prefactor, eq. 21 is identical in form to eq. 7 for \(\Omega_B = 0\), so that the redshift – age relation is well approximated by

\[
a(t)/a_0 = (1 + z)^{-1} \approx (1 - \Omega_B)^{1/3} \gamma^{-2/3} e^{-\gamma \tau_0} (e^{\gamma \tau_0} - 1)^{2/3}.
\]  

(24)

Evaluating this expression at \(t = t_0\) (\(z = 0\)) establishes, in this approximation, the connection among \(\tau_0\), \(\gamma\), and \((1 - \Omega_B)\),

\[
\exp(-3\gamma \tau_0/2) \approx 1 - \gamma (1 - \Omega_B)^{-1/2} \equiv 1 - \gamma',
\]

(25)

where \(\gamma' \equiv \gamma (1 - \Omega_B)^{-1/2}\). The present age of the Universe \((t = t_0, \tau = \tau_0)\) is

\[
\tau_0 = H_0 t_0 = \frac{2}{3\gamma} \ln \left( \frac{1}{1 - \gamma'} \right).
\]

(26)

Solving for the age – redshift relation,

\[
\exp(3\gamma \tau/2) \approx 1 + \left( \frac{\gamma'}{1 - \gamma'} \right) (1 + z)^{-3/2}.
\]

(27)

Using this approximate \(\tau\) vs. \(z\) relation and neglecting for the moment the contribution from the radiation density, the Friedman equation for \(H = H(z)\) reduces to

\[
\left( \frac{H}{H_0} \right)^2 \approx \Omega_B(1 + z)^3 + (1 - \Omega_B)(1 + z)^3 e^{3\gamma(\tau - \tau_0)} \approx \Omega_B(1 + z)^3 + (1 - \Omega_B)[\gamma' + (1 - \gamma')(1 + z)^{3/2}]^2.
\]

(28)

Notice that for \(z = 0\), \(H = H_0\) and, for \(\Omega_B = 0\), this result agrees with that in eq. 12 for \(\beta = 0\). In the high redshift limit, when \(\tau \ll \tau_0\) and, including radiation,

\[
\left( \frac{H}{H_0} \right)^2 \rightarrow \Omega_R(1 + z)^4 + (\Omega_B + (1 - \Omega_B)(1 - \gamma')^2)(1 + z)^3.
\]

(29)

If the high redshift evolution of this model is to be consistent with that of the ΛCDM concordance model, \(\Omega_B + (1 - \Omega_B)(1 - \gamma')^2 = 0.042 + 0.958(1 - \gamma')^2 \approx \Omega_M(\Lambda\text{CDM}) = 0.26\). This suggest that \(\gamma' \approx 0.52\) and \(\gamma \approx 0.51\). If, instead, it is required that the redshift of equal matter and radiation densities agree with the WMAP determined value \([\Omega]\),

\[
\Omega_R(1 + z_{eq}) = \Omega_B + (1 - \Omega_B)(1 - \gamma')^2,
\]

(30)

we find \(\gamma = 0.52 \pm 0.06\). For \(\beta = 0\) and \(\gamma = 0.52\), \(\tau_0 = 0.97\) and the present age of the Universe in this model is \(t_0 = 13.2\) Gyr.

As a check on our approximation to the full Friedman equation, where \(\rho_B\) was neglected compared to \(\rho_{\text{CDM}}\) in order to find the \(z - \tau\) relation, we note that as \(z\) decreases, the ratio of \(\rho_B\) to \(\rho_{\text{CDM}}\) decreases from \(\Omega_B/(1 - \Omega_B)e^{-3\gamma \tau_0} \approx 0.20\) at high \(z\), to \(\Omega_B/(1 - \Omega_B) \approx 0.04\) at \(z = 0\). From early times to the present, the number of cold dark matter particles in a comoving volume increases by a factor of \(~4.5\). Notice that for \(\beta = 0\), the early ISW effect constraint on \(\gamma\) differs little without or with baryons, \(0.50 \pm 0.06\) versus \(0.52 \pm 0.06\), for \(\Omega_B = 0\) and \(\Omega_B = 0.042\) respectively.

A discussion of the SNIa constraint in this case \((\beta = 0)\) is included in the exploration of the more general case \((\beta \neq 0)\) considered next.

C. \(\Omega_B \neq 0, \gamma \neq 0, \beta \neq 0\)

Based on our analyses of the \(\Omega_B = 0\) case which allowed for non-zero values of both \(\beta\) and \(\gamma\), along with our analysis of the case where \(\Omega_B = 0.042\) and either \(\beta\) or \(\gamma = 0\), the redshift dependence of the full Friedman equation can be approximated as

\[
\left( \frac{H}{H_0} \right)^2 \approx \Omega_R(1 + z)^4 + \Omega_B(1 + z)^3 + \frac{1 - \Omega_B}{(1 - \beta)^2} \left[ \gamma' + (1 - \gamma' - \beta)(1 + z)^{3(1 - \beta)/2} \right]^2.
\]

(31)

For \(z = 0\), \(H = H_0\), and the age of the Universe is

\[
\tau_0 \equiv H_0 t_0 = \frac{2}{3\gamma} \ln \left( \frac{1 - \beta}{1 - \gamma' - \beta} \right).
\]

(32)
FIG. 6: The age of the Universe in units of the Hubble age, $\tau_0 \equiv H_0 t_0$, as a function of $\gamma$ for selected values of $\beta$, for models with and without baryons.

In Figure 6, are shown the $\tau_0 - \gamma$ relations for selected values of $\Omega_B$ and $\beta$. For $\gamma > 0.45$ and $\beta \geq 0$, $\tau_0 \gtrsim 0.9$. For $\gamma = 0$, $\tau_0 = 2/3(1 - \Omega_B)^{-1/2}(1 - \beta)^{-1}$. Notice that as $\gamma \to 0$ and $\beta \to 1$, $\tau_0 \to \infty$, as it should since this case is equivalent to the steady-state model where there is no beginning to the Universe.

FIG. 7: As Figure 2 now including baryons. The 68% (red) and 95% (green) contours in the $\beta - \gamma$ plane, derived for $\Omega_B = 0.042$ from the SNIIa data [3], are shown along with the 95% confidence band (black; see the inset which zooms in on this band) consistent with the early ISW constraint that $z_{eq} = 3141 \pm 157$ [4]. The SNIIa data are best fit at $\beta = 0$ and $\gamma = 0.66$. The solid black curve (see eq. 34), shown in more detail in the inset, is for the best fit relation between $\beta$ and $\gamma$ consistent with the observed value of $z_{eq}$.

The 68% (red) and 95% (green) contours in the $\beta - \gamma$ plane, derived for $\Omega_B = 0.042$ from the SNIIa data [3], are shown along with the 95% confidence band (black; see the inset which zooms in on this band) consistent with the early ISW constraint that $z_{eq} = 3141 \pm 157$ [4]. The SNIIa data are best fit at $\beta = 0$ and $\gamma = 0.66$. The solid black curve (see eq. 34), shown in more detail in the inset, is for the best fit relation between $\beta$ and $\gamma$ consistent with the observed value of $z_{eq}$. 
At high redshifts,
\[
\left( \frac{H}{H_0} \right)^2 \approx \Omega_R (1 + z)^4 + \Omega_B (1 + z)^3 + (1 - \Omega_B) \left( \frac{1 - \gamma' - \beta}{1 - \beta} \right)^2 (1 + z)^{3(1 - \beta)}. \tag{33}
\]
Equating \( \rho_R \) to \( \rho_M = \rho_B + \rho_{\text{CDM}} \) at \( z = z_{eq} \) leads to a constraint on the \( \gamma - \beta \) relation,
\[
\gamma = (1 - \beta) [(1 - \Omega_B)^{1/2} - (\Omega_R (1 + z_{eq}) - \Omega_B)^{1/2} (1 + z_{eq})^{3\beta/2}]. \tag{34}
\]

In Figure 8 are shown the 68% and 95% contours in the \( \beta - \gamma \) plane for \( \Omega_B = 0.042 \) from the SNIa data \( \text{[3]} \) and from the early ISW effect (eq. 34) constrained by the WMAP value of \( z_{eq} \) \( \text{[4]} \). For the SNIa data \( \text{[3]} \) the best fit occurs for \( \beta = 0 \) and \( \gamma = 0.66 \pm 0.04 \), in some conflict with the best fit to \( z_{eq} \) for \( \beta = 0 \), for which \( \gamma = 0.52 \pm 0.06 \). As may be seen from Figure 8 these two constraints diverge further from each other as \( \beta \) increases from zero. In Figure 8 are shown the probability distribution functions for \( \gamma \) derived from the SNIa data \( \text{[3]} \) and from \( z_{eq} \) \( \text{[4]} \) for \( \beta = 0 \) and \( \Omega_B = 0.042 \). Note that there is some tension between these two independent constraints. Indeed, if the best fit SNIa values of \( \beta \) and \( \gamma \) are used to predict the redshift of equal matter and radiation densities, \( (1 + z_{eq})_{\text{SNIa}} = 1798_{-552}^{+536} \), which is some 2.5\( \sigma \) away from the WMAP value \( \text{[4]} \).

IV. Summary and Conclusions

As an alternative to the standard, \( \Lambda \)CDM model, we have explored here a class of models whose late-time acceleration is driven by the creation of cold dark matter. In constrast to the \( \Lambda \)CDM model, the models considered here have no cosmological constant or vacuum energy. While the dark energy models have two or more adjustable parameters (the dark energy equation of state, \( w \), and its evolution with redshift, in addition to the choice of \( \Omega_{\text{DE}} \)), the flat \( \Lambda \)CDM model has only one free parameter, \( \Omega_\Lambda = 1 - \Omega_{\text{CDM}} \). Here we have considered a class of two-parameter (\( \{\beta, \gamma\} \)) models. We found that the supernovae data prefer \( \beta = 0 \) and that the early ISW effect is consistent with this choice. For this subset of creation-driven models, with only one free parameter (\( \gamma \)), the high-redshift evolution of the Universe is qualitatively indistinguishable from that of the \( \Lambda \)CDM model, while the recent evolution is sufficiently similar to it to allow consistency with the SNIa data. From the high redshift constraint on the WMAP determined value of \( z_{eq} \) \( \text{[4]} \) provided by the early ISW effect, we determined \( \gamma(\text{ISW}) = 0.52 \). For \( \beta = 0 \) and this value for \( \gamma \), \( H_0 t_0 = 0.97 \), consistent with the estimate \( H_0 t_0 = 1.00 \) from the \( \Lambda \)CDM model. This choice of \( \gamma \) corresponds to the WMAP determined value of \( z_{eq} = 3142 \), consistent, within the uncertainties, with the \( \Lambda \)CDM value of \( z_{eq} = 3223 \).
However, for \( \beta = 0 \), this choice of \( \gamma \) provides a poor fit to the SNIa data \cite{10}. In contrast, the SNIa data prefer \( \gamma = 0.66 \), which, for \( \beta = 0 \), corresponds to \( H_0 f_0 = 1.13 \), in good agreement with the \( \Lambda \)CDM value and with estimates of the age of the Universe. But, for \( \beta = 0 \) and \( \gamma = 0.66 \), the redshift of equal matter and radiation densities is \( z_{eq} = 1798 \), in conflict (at \( \approx 2.6\sigma \)) with the WMAP determined value.

While these models, which are consistent with the SNIa data, offer an intriguing alternative to the standard, \( \Lambda \)CDM concordance model, they face challenges. For example, although the one-parameter (\( \beta \neq 0, \gamma = 0 \)) model with baryons provides a natural solution to the observed, late-time acceleration of the Universe, without any need for fine-tuning, its early evolution is baryon-dominated and inconsistent with the early ISW effect as constrained by the CMB data. In general, there is a clear tension in this class of models between the SNIa data and the independent, high redshift constraint from the observed early ISW effect. It should also be noted that in these models the ratio of dark matter (CDM plus baryons) to baryons increases during the recent evolution of the Universe from \( \rho_{DM}/\rho_B \approx 6.0 \) at high redshifts (\( z \gtrsim 10 \)) to \( \rho_{DM}/\rho_\gamma \approx 24 \) at present (\( z = 0 \)). Correspondingly, the baryon fraction, \( f_B \equiv \rho_B/\rho_{DM} \) decreases from \( f_B \approx 0.17 \) at high redshifts to \( f_B \approx 0.04 \) at present. While this latter value appears to be in conflict with the x-ray cluster baryon fraction \cite{13}, if clusters were formed at sufficiently high redshifts, their baryon fraction may be representative of the earlier, higher value of \( f_B \). Due to the recent increase in the density of CDM, the late-time growth of structure and of the cluster baryon fraction in these models will differ from that of the \( \Lambda \)CDM concordance model. Therefore, before ruling out these models, it might be worthwhile to explore their late-time evolution more carefully, especially with regard to predictions for the CMB and for the growth of large scale structure.

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