Continuous-Aperture MIMO for Electromagnetic Information Theory

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Abstract—In recent years, the concept of continuous-aperture MIMO (CAP-MIMO) is reinvestigated to achieve improved communication performance with limited antenna apertures. Unlike the classical MIMO composed of discrete antennas, CAP-MIMO has a continuous antenna surface, which is expected to generate any current distribution (i.e., pattern) and induce controllable spatial electromagnetic waves. In this way, the information can be modulated on the electromagnetic waves, which makes it promising to approach the ultimate capacity of finite apertures. The pattern design is the key factor to determine the system performance of CAP-MIMO, but it has not been well studied in the literature. In this paper, we propose the pattern-division multiplexing to design the patterns for CAP-MIMO. Specifically, we first derive the system model of a typical CAP-MIMO system, which allows us to formulate the capacity maximization problem. Then we propose a general pattern-division multiplexing technique to transform the design of continuous pattern functions to the design of their projection lengths on finite orthogonal bases, which is able to overcome the design challenge of continuous functions. Based on this technique, we further propose an alternating optimization based pattern design scheme to solve the formulated capacity maximization problem. Simulation results show that, the capacity achieved by the proposed scheme is about 260% higher than that achieved by the benchmark scheme, which demonstrates the effectiveness of the proposed pattern-division multiplexing for CAP-MIMO.

Index Terms—Continuous-aperture MIMO (CAP-MIMO), electromagnetic information theory, pattern-division multiplexing, capacity.

I. INTRODUCTION

From 3G to 5G, the system performance of wireless communications has been greatly improved by the use of multiple-input multiple-output (MIMO) [1]–[3]. Equipped with multiple discrete antennas with half-wavelength spacing, MIMO is capable of enhancing the wireless transmissions by exploiting spatial multiplexing and diversity [4]–[6]. Many experimental measurements have verified that, the transmission rate of the real-world MIMO systems is able to closely approach its theoretical performance limit, which is mainly determined by the area of MIMO antenna aperture [7]–[9]. In recent years, some advances in electromagnetic propagation theory have shown that, deploying sub-wavelength antennas more densely in a limited antenna aperture is able to achieve higher channel capacity [10]–[12], which is promising to break the performance limit of conventional MIMO with limited antenna apertures [12]. Thereby, as an ultimate MIMO structure with infinitely dense antennas, the concept of continuous-aperture MIMO (CAP-MIMO), which is also called as holographic MIMO [13]–[16] or large intelligent surface (LIS) [17]–[20], is reinvestigated for wireless communications in recent years.

Unlike the classical MIMO composed of multiple discrete antennas with half-wavelength spacing, CAP-MIMO takes the form of spatially-continuous electromagnetic surfaces [12]. Thus, CAP-MIMO can be viewed as an aperture-constrained MIMO consisting of an infinite number of infinitely small antennas [18]. Besides, different from the conventional MIMO which only controls the amplitudes and phase shifts of their transmitted signals [4]–[6], an ideal CAP-MIMO has a full control freedom of generating any current distribution on its spatially-continuous surface [14], so that its radiated electromagnetic waves can be artificially configured in a desired manner [16]. In this way, the information for receivers can be directly modulated on the spatial electromagnetic waves and radiated to the physical space [11], which is exactly the research focus of electromagnetic information theory [21]. Relying on this new information transmission mechanism, the physical properties of spatial electromagnetic waves can be sufficiently exploited. Thus, CAP-MIMO becomes a promising technology to achieve improved communication performance with limited apertures [10]–[12], which is expected to satisfy many challenging requirements of future wireless networks, such as the wide in-building coverage, high-speed uplink transmission, and high-accuracy localization [12].

A. Prior works

Realizing an ideal CAP-MIMO has a long history in the research field of micro-wave, dating back to Harold Wheeler’s work in 1965 [22] and David Staiman’s work in 1968 [23],
respectively. The recent works on CAP-MIMO include antenna design [24], physical model [14], performance analysis [15], channel estimation [13], and so on. For example, some techniques have been recently proposed to realize CAP-MIMO in practice, such as the current sheet made of tightly coupled dipole array and the monolayer metallic made of magnetic particles [24]. Then, to mathematically characterize the propagation process of the electromagnetic waves radiated by CAP-MIMO, the authors in [14] modeled the electromagnetic wireless channels between CAP-MIMO transceivers as Gaussian random fields. Subsequently, to reveal the potential performance of CAP-MIMO, the author in [17] derived the analytical expressions of the spatial degrees of freedom (DoFs) of CAP-MIMO, and the authors in [15] further analyzed its near-field DoF by providing a heuristic proof. By adopting Mercer expansion, the authors in [25] investigated the capacity between a couple of CAP-MIMO with infinitely-length linear aperture. Moreover, the authors in [13] proposed an overhead-reduced channel estimation scheme for CAP-MIMO, which exploits the array geometry to identify a subspace of reduced rank, thus it overcomes the issue of high overhead for channel estimation introduced by the high-dimensional channels of CAP-MIMO.

The pattern, i.e., the current distribution on the continuous aperture of CAP-MIMO transmitter, is the key factor determining the system performance of CAP-MIMO [14]. To support the coherent transmission of multiple data streams, it is necessary for CAP-MIMO transmitter to adopt a series of distinguishable patterns to carry different symbols [15], [16]. Specifically, the authors in [15] considered a near-field line-of-sight scenario, where one linear-aperture CAP-MIMO transmitter serves one linear-aperture electromagnetic-wave receiver. By adopting a series of square-wave functions to generate the patterns for multiple data streams, the electromagnetic waves carrying different symbols are radiated towards different spatial angles. Then, these electromagnetic waves are focused at the different positions of receiver apertures, thus the multi-stream transmissions can be achieved. Furthermore, the authors in [16] considered a similar near-field line-of-sight scenario with a couple of linear-aperture CAP-MIMO transceivers. Particularly, a wavenumber-division multiplexing scheme was proposed to directly generate the patterns by a series of Fourier basis functions. In this way, the transmitted symbols belonging to different streams are modulated on different spatial wavenumbers of radiated electromagnetic waves, thus these symbols become distinguishable, which is similar to the frequency-division multiplexing in conventional communications.

From the above discussions, we can find that, most existing works have directly adopted the patterns generated by given special functions to realize coherent CAP-MIMO transmissions [15], [16]. Despite these existing schemes can improve the performance of CAP-MIMO to some extent, they can only be applied to some special communication scenarios, such as single receiver, near field, and line-of-sight transmissions. To support CAP-MIMO in general communication scenarios with complex propagation environment and multiple distributed receivers, it is essential to design and adjust the patterns flexibly according to the real-time channel state information. Unfortunately, to the best of our knowledge, the research on such a flexible and general pattern design scheme for CAP-MIMO is still blank in the literature. One possible reason maybe the mathematical challenge introduced by the design of continuous patterns for CAP-MIMO, since it is actually a complex problem of non-convex density functional optimization [26], which is difficult to be solved by the classical discrete signal processing technique for conventional discrete MIMO systems.

### B. Our contributions

To fill in this gap, in this paper, we propose a general pattern-division multiplexing technique to flexibly design the patterns for CAP-MIMO according to the real-time channel state information\(^1\). Specifically, our contributions are summarized as follows.

- Starting from the electromagnetic information theory, we derive the system model including the expressions of capacity and power constraint for a typical CAP-MIMO system, where one CAP-MIMO transmitter with planar aperture serves multiple electromagnetic-wave receivers coherently. This allows us to formulate the capacity maximization problem to optimize the patterns (i.e., the current distributions) for CAP-MIMO, and it also provides a general framework for other technical problems in electromagnetic information theory, such as the analyses of channel DoFs and asymptotic capacity.

- We propose a general pattern-division multiplexing technique to flexibly design the patterns for CAP-MIMO according to the real-time channel state information. The key idea is to use series expansion to project the continuous functions of the patterns to be designed onto an orthogonal basis space, thus the design of continuous pattern functions is transformed to the design of their projection lengths on finite orthogonal bases. In this way, the challenge introduced by the optimization of continuous pattern functions for CAP-MIMO can be addressed.

- Based on the proposed pattern-division multiplexing technique, we further propose an alternating optimization based pattern design scheme to solve the formulated capacity maximization problem for CAP-MIMO. Simulation results show that, the patterns designed by the proposed pattern design scheme are almost mutually orthogonal for multiple electromagnetic-wave receivers. Particularly, the capacity achieved by the proposed scheme is about 260% higher than that achieved by the existing wavenumber-division multiplexing scheme [16].

### C. Organization and notation

**Organization:** The rest of this paper is organized as follows. Section II introduces the system model of CAP-MIMO and the problem formulation for capacity maximization. The general pattern-division multiplexing technique to flexibly design

\(^1\)Simulation codes will be provided to reproduce the results presented in this article: [http://oa.ee.tsinghua.edu.cn/dailinglong/publications/publications.html](http://oa.ee.tsinghua.edu.cn/dailinglong/publications/publications.html).
patterns, as well as the pattern design scheme to solve the capacity maximization problem, are proposed in Section III. Simulation results are provided in Section IV to validate the effectiveness of the proposed pattern design scheme and evaluate the capacity performance of CAP-MIMO. Finally, conclusions are drawn and future works are discussed in Section V.

**Notation:** \( \mathbb{C}, \mathbb{R}, \) and \( \mathbb{R}_+ \) denote the set of complex, real, and positive real numbers, respectively; \( [\cdot]^{-1}, [\cdot]^*, [\cdot]^T, \) and \( [\cdot]^H \) denote the inverse, conjugate, transpose, and conjugate-transpose operations, respectively; \( \|\cdot\| \) denotes the Euclidean norm of its argument; \( \det[\cdot] \) denotes the determinant of its argument; \( \mathbb{E}_x[\cdot] \) is the expectation operator with respect to the random vector \( z; \mathcal{R}_T[\cdot] \) denotes the real part of its argument; \( \ln(\cdot) \) denotes natural logarithm; \( \nabla_x \) denotes the first-order partial derivative operator with respect to \( z; j = \sqrt{-1} \) denotes the imaginary unit; Surfaces are indicated with calligraphic letters \( \mathcal{S}_T \), where \( A_T = |\mathcal{S}_T| \) is their Lebesgue measure; Finally, \( I_L \) denotes an \( L \times L \) identity matrix.

**II. SYSTEM MODEL AND PROBLEM FORMULATION FOR CAP-MIMO**

In this section, we study the capacity of a typical CAP-MIMO based communication system, where one CAP-MIMO transmitter with planar aperture simultaneously serves \( K \) electromagnetic-wave receivers in the downlink. Specifically, we first introduce the system model of a CAP-MIMO transmitter in Subsection II-A. Then, the electromagnetic channels between the transmitter and receivers are illustrated in Subsection II-B. Next, the electromagnetic waves at the receiver are modeled in Subsection II-C. Finally, the capacity maximization problem for CAP-MIMO is formulated in Subsection II-D.

**A. CAP-MIMO transmitter**

As shown in Fig. 2 (a), we consider a CAP-MIMO transmitter with surface \( \mathcal{S}_T \) of area \( A_T = |\mathcal{S}_T| \) working in a 3-D infinite and homogeneous medium\(^2\). In the ideal case, CAP-MIMO has a spatially-continuous antenna aperture, which is able to generate any current distribution on its continuous surface for wireless communications [11]–[17]. Let \( \mathbf{j}(s,t) \in \mathbb{R}^3 \) denote the monochromatic current density at a generic location \( s := (s_x, s_y, s_z) \in \mathbb{R}^3 \) and time \( t \). The ideally controllable current distribution at the CAP-MIMO transmitter can be written as

\[
\mathbf{j}(s,t) = \Re \left\{ \mathbf{j}(s)e^{-j2\pi f t} \right\}, \ s \in \mathcal{S}_T, \tag{1}
\]

where \( f \) is the current frequency. For simplicity but without loss of generality, we assume that the communication system works in a narrowband scenario, which allows us to ignore the time-related component \( e^{-j2\pi f t} \) and focus on the time-independent current density \( \mathbf{j}(s) \) so as to simplify the analysis. Note that this assumption is exactly the well-known and widely-used time-harmonic assumption in electromagnetic field analysis [21].

Consider that the CAP-MIMO transmitter simultaneously serves \( K \) electromagnetic-wave receivers in the downlink. Let \( \mathbf{x} \triangleq [x_1, \ldots, x_K]^T \in \mathbb{C}^K \) denote \( K \) symbols transmitted to \( K \) receivers, respectively. We assume that these symbols have normalized power, i.e., \( \mathbb{E}_{\mathbf{x}} \{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_K \). Then, similar to the conventional MIMO beamforming [27], the symbols to be transmitted are modulated on \( K \) different CAP-MIMO patterns, which aims to make these symbols orthogonal at different receivers as much as possible, and thus high channel capacity can be achieved. For simplicity, we assume that

\(^2\)In this paper, we only consider the CAP-MIMO with 2-D antenna aperture since it is more practical for realization [24], while it can also be easily extended to the 3-D case.
CAP-MIMO employs linear superposition to combine multiple information-carrying patterns, thus the combined current distribution $j(s)$ on the CAP-MIMO aperture can be modeled as

$$ j(s) = \sum_{k=1}^{K} \theta_k(s) x_k, \quad s \in S_T, \quad (2) $$

where the pattern $\theta_k(s) \in \mathbb{C}^3$ is the component of surface current density that carries symbol $x_k$.

### B. Electromagnetic channels

To model the radiated information-carrying electromagnetic waves in space, we define $e(r) \in \mathbb{C}^3$ as the electric field at point $r := (x, y, z) \in \mathbb{R}^3$, which is induced by the current distribution $j(s)$ on the CAP-MIMO aperture. According to Maxwell’s equations, the current distribution $j(r')$ and the electric field $e(r')$ satisfy the following inhomogeneous Helmholtz wave equation [17]:

$$ \nabla \times \nabla \times e(r') - \kappa^2 e(r') = j\kappa Z_0 j(r'), \quad (3) $$

where $r' \in \mathbb{R}^3$ is any arbitrary point in space; $\kappa$ is the spatial wavenumber; and $Z_0$ is the intrinsic impedance of spatial medium, which is 376.73 $\Omega$ in free space.

Then, to explicitly express the relationship between the current distribution $j(s)$ at the transmitter and the electric field $e(r)$ at the receiver, Green’s method [21] is utilized to solve (3), which can be viewed as the system impulse response. By introducing Green function $G(r, s) \in \mathbb{C}^{3 \times 3}$, the electric field $e(r)$ at point $r$ of the receiver can be induced from (3) as

$$ e(r) = \int_{S_T} G(r, s) j(s) ds, \quad (4) $$

where the Green function $G(r, s)$ is similar to the classical definition of wireless channels. In particular, $G(r, s)$ is determined by the specific transmission environment. For example, in ideal unbounded and homogeneous mediums, $G(r, s)$ is

$$ G(r, s) = \frac{j\kappa Z_0 \exp[-j\kappa \|r-s\|]}{4\pi \|r-s\|^2} \left( I_3 + \frac{\nabla r \nabla s^H}{\kappa^2} \right), \quad (5) $$

while $G(r, s)$ is usually modeled as a stochastic process in complex scattering environments [11].

### C. Electromagnetic-wave receivers

As shown in Fig. 2 (b), we assume that all $K$ receivers are located in the far-field region, and each receiver is equipped with an ideal isotropic antenna with effective aperture area $A_R = \frac{\lambda^2}{4\pi}$, which is much smaller than the transmitter aperture area $A_T$ [18]. In this case, similar to the discrete antenna of conventional MIMO, each electromagnetic-wave receiver can be reasonably approximated by a point in 3-D space.

Let $r_k \in \mathbb{R}^3$ denote the 3-D location of the $k$-th electromagnetic-wave receiver. In the ideal case, the receiver $k$ is expected to ideally sense and capture the whole information of the electromagnetic waves reaching point $r_k$. According to (2) and (4), the electromagnetic wave captured by receiver $k$ can be expressed as

$$ y_k = e_k + n_k $$

\[ \begin{aligned} 
&= x_k \int_{S_T} G_k(s) \theta_k(s) ds + \sum_{j=1, j \neq k}^{K} x_j \int_{S_T} G_k(s) \theta_j(s) ds + n_k, \quad \text{Noise} \\
&= x_k \underbrace{\int_{S_T} G_k(s) \theta_k(s) ds}_{\text{Desired signal to user } k} + \sum_{j=1, j \neq k}^{K} x_j \int_{S_T} G_k(s) \theta_j(s) ds + n_k. \end{aligned} \]

where $e_k := e(r_k)$, $G_k(s) := G(r_k, s)$, and $n_k$ is the electromagnetic noise at receiver $k$, which is produced by all incoming electromagnetic waves that are not generated by the transmitter [21]. Here we assume that, $n_k$ is additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2 I_3$, and $n_k$ for all $k \in \{1, \cdots, K\}$ are mutually independent.

### D. Capacity Maximization Problem formulation

Based on the above system model, in this subsection, we formulate the capacity maximization problem for CAP-MIMO. Firstly, by calculating the amount of mutual information, we derive the CAP-MIMO channel capacity in the following theorem.

**Theorem 1 (CAP-MIMO channel capacity):** The CAP-MIMO channel capacity, i.e., the sum-rate of $K$ receivers, can be written as

$$ R_{\text{sum}} = \sum_{k=1}^{K} \log_2 \det \left( I_3 + \alpha_k \alpha_k^H J_k^{-1} \right), \quad (7) $$

where $\alpha_k$ and $J_k$ are given by

$$ \alpha_k = \int_{S_T} G_k(s) \theta_k(s) ds, \quad (8) $$

$$ J_k = \sum_{j=1, j \neq k}^{K} \int_{S_T} G_k(s) \theta_j(s) ds \left( \int_{S_T} G_k(s') \theta_j(s') ds' \right)^H + \sigma^2 I_3. $$

**Proof:** See Appendix A.

In practical systems, we are interested in investigating the CAP-MIMO channel capacity under a given power constraint. By integrating the radial component of the Poynting vector over a sphere with infinite-length radius [21], we provide the following theorem to upper-bound the total transmit power of CAP-MIMO in the sense of expectation.

**Theorem 2 (Transmit power constraint of CAP-MIMO):** The physical transmit power of CAP-MIMO based communication systems can be upper-bounded by the following inequality:

$$ \sum_{k=1}^{K} \int_{S_T} \|\theta_k(s)\|^2 ds \leq P_T, \quad (9) $$

where $P_T$ can be viewed as the allowable maximum “transmit power” of CAP-MIMO, which is implicitly associated with the physical energy [21] and measured in $\lambda^2$ (or $\text{m}^2$).

**Proof:** See Appendix B.
By combing Theorem 1 and Theorem 2, the original problem of CAP-MIMO capacity maximization subject to the transmit power constraint can be formulated as

$$\mathcal{P}_o : \max_{\theta(s)} R_{\text{sum}} = \sum_{k=1}^{K} \log_2 \det \left[ I_3 + \alpha_k \alpha_k^T J_k^{-1} \right],$$  \hspace{1cm} (10a)

subject to

$$\sum_{k=1}^{K} \int_{S_T} ||\theta_k(s)||^2 ds \leq P_T,$$  \hspace{1cm} (10b)

where \(\theta(s)\) denotes the set of functions \(\theta_k(s)\) for all \(k \in \{1, \cdots, K\}\). Our goal is to maximize the channel capacity (10a) by appropriately designing the continuous pattern functions \(\theta_k(s) (k \in \{1, \cdots, K\})\), i.e., the current distributions on the continuous aperture \(S_T\) of CAP-MIMO.

Remark 1: Note that the capacity maximization problem \(\mathcal{P}_o\) in (10) is difficult to solve. The reason is that, the continuous functions \(\theta(s)\) within the integral terms exist in both optimization objective and constraint in (10), which is a complex non-convex density functional optimization problem [26]. In this case, since the explicit extreme points of functions \(\theta(s)\) are difficult to obtain, the classical discrete signal processing techniques for MIMO, such as gradient descent and fractional programming (FP) [28], are hard to be adopted. Such kind of functional optimization problems are common in the micro-wave area, where they are usually addressed by using commercial electromagnetic simulation software such as high frequency structure simulator (HFSS), which results in high time and space complexity [21].

Remark 2: Another intuitive way to solve the problem \(\mathcal{P}_o\) in (10) is to densely sample the continuous functions \(G_k(s)\) and \(\theta(s)\) in (10), so that this problem becomes discretized and is expected to be solved by the classical discrete signal processing algorithms such as the weighted mean-square error minimization (WMMSE) [29]. Unfortunately, this way is usually impractical, since the computational complexity of discrete signal processing methods for multi-receiver beamforming is usually unaffordably high. For example, the WMMSE complexity [29] of updating the MIMO beamforming vectors in each iteration is higher than \(O(K^2M^3)\), where \(M\) is the antenna number of transmitter. In this case, if we have \(K = 4\) receivers and the sampling number of the continuous CAP-MIMO aperture for each spatial dimension is 100 (i.e., \(M = 100 \times 100 = 10000\)), then more than \(1.6 \times 10^{13}\) Flops are required in each iteration of WMMSE to solve (10), which is unacceptable for computing devices in practice.

III. PROPOSED PATTERN-DIVISION MULTIPLEXING FOR CAP-MIMO

To overcome the challenge introduced by the optimization of continuous pattern functions as shown in problem \(\mathcal{P}_o\) in (10), in this section, we propose a general pattern-division multiplexing technique to realize flexible pattern design for CAP-MIMO. The key idea is to use series expansion to expand the continuous functions in an orthogonal basis space. In this way, these continuous functions are projected to several orthogonal bases, and the design of these functions is transformed to the design of the projection lengths on their expansion bases, which makes the optimization of continuous functions feasible. Specifically, we first introduce the proposed pattern-division multiplexing technique in Subsection III-A. Then, to show how to design CAP-MIMO patterns by using this technique, as a typical example, we propose an alternating optimization based pattern design scheme to solve the capacity maximization problem \(\mathcal{P}_o\) in (10) in Subsection III-B. Finally, the performance of the proposed pattern design scheme is analyzed in Subsection III-C.

A. A general pattern-division multiplexing technique

Different from existing works which directly adopt the patterns generated by given special functions for coherent transmissions [15], [16], we propose the pattern-division multiplexing technique to flexibly design the patterns for CAP-MIMO according to the real-time channel state information. Specifically, the proposed pattern-division multiplexing aims to make the information-carrying electromagnetic waves reaching different receivers as much orthogonal as possible. In this way, higher channel capacity can be achieved, which is similar to the space-division multiplexing in classical MIMO systems [29].

To efficiently optimize the continuous functions of the patterns \(\theta_k(s) (k \in \{1, \cdots, K\})\) for CAP-MIMO, an intuitive idea is to use series expansion to project these continuous functions onto an orthogonal space. Since Fourier series expansion has good generality, in this paper, we use Fourier bases to expand the continuous functions, while it can be extended to other feasible series expansions without difficulty.

According to the uniform convergence theorem of Fourier series expansion, we first introduce the following lemma.

Lemma 1 (Fourier series expansion): For any arbitrary continuous function \(f(t) \in \mathbb{C}\) defined in interval \(t \in [a, b] \in \mathbb{R}\), if \(f(t)\) is absolutely integrable in \(t \in [a, b]\), \(f(t)\) can be equivalently rewritten as

$$f(t) = \sum_{n=-\infty}^{\infty} F_n \Psi_n(t),$$  \hspace{1cm} (11)

where the projection lengths \(F_n \in \mathbb{C}\) and the Fourier bases \(\Psi_n(t) \in \mathbb{C}\) are respectively given by

$$F_n = \frac{1}{\sqrt{b-a}} \int_{a}^{b} f(t) e^{-j2\pi nt} dt,$$  \hspace{1cm} (12a)

$$\Psi_n(t) = \frac{1}{\sqrt{b-a}} e^{j2\pi nt}.  \hspace{1cm} (12b)$$

Particularly, note that the Fourier bases \(\Psi_n(t)\) satisfy

$$\int_{a}^{b} \Psi_n(t) \Psi^*_m(t) dt = \begin{cases} 1, & m = n, \\ 0, & m \neq n, \end{cases}  \hspace{1cm} (13)$$

which guarantees the orthogonality among the basis functions.

By extending the above Lemma 1 to 3-D scenario and applying it to our investigated pattern design problem, the continuous function patterns \(\theta_k(s)\) to be designed can be equivalently rewritten as

$$\theta_k(s) = \sum_{n} w_{k,n} \Psi_n(s), \quad s \in S_T,$$  \hspace{1cm} (14)
where $w_{k,n} \in \mathbb{C}^3$ is the projection length of $\theta_k(s)$ on the Fourier basis $\Psi_n(s) \in \mathbb{C}$, and here we define $n := (n_x, n_y, n_z)$ to distinguish different expansion items and $\sum_{n} := \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty}$ for expression simplification. Note that, in this 3-D scenario, Fourier basis function $\Psi_n(s)$ becomes

$$\Psi_n(s) = \frac{1}{\sqrt{AT}} e^{ij2\pi \left( \frac{nx}{L_x} s_x + \frac{ny}{L_y} s_y + \frac{n_z}{L_z} s_z \right)},$$

(15)

where $s := (s_x, s_y, s_z) \in S_T$ is the generic location on the CAP-MIMO aperture, while $L_x$, $L_y$, and $L_z$ denote the maximum projection lengths of CAP-MIMO aperture $S_T$ on the $x$-, $y$-, and $z$-axis of 3-D coordinate system, respectively. Similarly, we have

$$\int_{\pi \in S_T} \psi_n(s) \psi_m(s) ds = \begin{cases} 1, & m = n, \\ 0, & m \neq n, \end{cases}$$

(16)

which guarantees the orthogonality among the basis functions.

Relying on this transformation operation, we obtain the following two theorems.

**Theorem 3 (Continuous-discrete transformation for electromagnetic waves):** By adopting Fourier series expansion, the coupled Green function $G_k$ and pattern function $\theta_j$ in integral $\int_{S_T} ds$ for all $k, j \in \{1, \ldots, K\}$, i.e., the electric field, can be equivalently rewritten as

$$\int_{S_T} G_k(s) \theta_j(s) ds = \sum_{n} \Omega_{k,n} w_{j,n},$$

(17)

where

$$\Omega_{k,n} = \int_{S_T} G_k(s) \psi_n(s) ds,$$

(18)

and $\Omega_{k,n} \in \mathbb{C}^{3 \times 3}$ is exactly the Fourier transform of Green function $G_k(s)$ on the CAP-MIMO aperture $S_T$ at the spatial frequency of $\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$.

**Proof:** The proof completes by substituting the expansion (14) into (17).

**Theorem 4 (Continuous-discrete transformation for power constraint):** According to Parseval’s theorem, the square $L^2$ norm of current density $\theta_k$ in integral expressions, i.e., the transmit power, can be equivalently rewritten as

$$\int_{S_T} \|\theta_k(s)\|^2 ds = \sum_{n} \|w_{k,n}\|^2.$$  

(19)

**Proof:** The proof completes by substituting the expansion (14) into (10b).

Exploiting the above *Theorem 3* and *Theorem 4*, the design of the continuous pattern functions $\theta_k(s)$ becomes the design of the projection lengths $w_{k,n}$, and thus the density functional optimization [26] can be equivalently transformed as a common digital signal processing problem. However, since the number of expansion items is infinite as shown in (17) and (19), the projection lengths $w_{k,n}$ are still hard to optimize.

Fortunately, according to the inherent mathematical structure of Green function $G_k(s)$, we notice that, in most cases, the power of $G_k(s)$ in Fourier space is mainly distributed in the low-frequency band, while that in the high-frequency band is very low (see [16, Fig. 4] [25, Fig. 2 and Fig. 3]). In other words, when the number of expansion items $n$ is large enough, the value of $\Omega_{k,n}$ tends to be negligible. Some existing works on channel modeling also pointed out that, the continuous channel function of CAP-MIMO can be approximated by the sum of its finite Fourier plane-wave representations (see [14, Eq. (39)]). This fact inspires us to approximate the original continuous function with finite low-frequency Fourier expansion items, and thus we obtain the following theorem.

**Theorem 5 (Finite-item approximation of Fourier series):** The electric field in (17) and the transmit power in (19) can be approximated by

$$\int_{S_T} G_k(s) \theta_j(s) ds \approx \sum_{n} \Omega_{k,n} w_{j,n},$$

(20a)

$$\int_{S_T} \|\theta_k(s)\|^2 ds \approx \sum_{n} \|w_{k,n}\|^2,$$

(20b)

where we have $N := (N_x, N_y, N_z)$ with $N_x$, $N_y$, and $N_z$ being the numbers of reserved items, and we have $\sum_{n} := \sum_{n_x=0}^{N_x/2} \sum_{n_y=0}^{N_y/2} \sum_{n_z=0}^{N_z/2}$ for clarity.

**Proof:** The proof completes by employing a truncation operation on (17) and (19), respectively.

**Remark 3:** From the above theorem, we notice that, the numbers of expansion items $N_x$, $N_y$, and $N_z$ can be set to the values acceptable for the practical computing devices. In this way, it becomes feasible to optimize the continuous pattern functions $\theta_k(s)$ through designing the finite projection lengths $w_{k,n}$, which serves as a general technique to achieve flexible pattern design for CAP-MIMO systems.

### B. Proposed pattern design scheme for capacity maximization

Based on the general pattern-division multiplexing technique proposed in Subsection III-A, in this subsection, we propose an alternating optimization based pattern design scheme to solve the capacity maximization problem $P_o$ in (10).

Firstly, we consider to decouple the continuous functions of patterns $\theta_k(s)$ by adopting an equivalent transformation for capacity maximization problem, which is proposed in [29, *Theorem 1*], and we obtain the following lemma.

**Lemma 2 (Equivalent problem for capacity maximization):** By introducing an auxiliary variable $\rho = [\rho_k, \cdots, \rho_K]^T \in \mathbb{R}^K$ and the combining vectors $\psi = [\psi_1, \cdots, \psi_K] \in \mathbb{C}^{3 \times K}$ for all receivers, the original capacity maximization problem $P_o$ in (10) can be equivalently reformulated as

$$P_1: \max_{\rho, \psi, \theta(s)} R'_{\text{sum}} = \sum_{k=1}^{K} \log_2 \rho_k - \frac{1}{\ln 2} \sum_{k=1}^{K} \rho_k E_k + \frac{K}{\ln 2},$$

(21a)

subject to

$$\sum_{k=1}^{K} \int_{S_T} \|\theta_k(s)\|^2 ds \leq P_T.$$  

(21b)
Algorithm 1 Proposed pattern design scheme for capacity maximization.

Input: Green functions $G_k(s)$ with respect to $s \in S_T$ for all receivers $k \in \{1, \cdots, K\}$.

Output: Optimized channel capacity $R_{\text{sum}}$; optimized combiners $\psi$ at receivers; optimized patterns $\theta(s)$ on the aperture of the CAP-MIMO transmitter.

1: Initialize $\psi$ and $\theta(s)$;
2: while No convergence of $R_{\text{sum}}$ do
3: Update $\rho$ by (23);
4: Update $\psi$ by (25);
5: Update $w$ by (31) and (32);
6: Update $\theta(s)$ by (33);
7: Update $R_{\text{sum}}$ by (10a);
8: end while
9: return Optimized $R_{\text{sum}}^\text{opt}$, $\psi^\text{opt}$, and $\theta^\text{opt}(s)$.

Note that, $\psi_k$ is the combiner at receiver $k$ as shown in Fig. 2 (b), and $E_k$ is the mean-square error (MSE) of the decoded symbol $\hat{x}_k = \psi_k^H y_k$, defined as

$$E_k = \text{E}_{x,n} \left\{ |\hat{x}_k - x_k|^2 \right\} = |1 - \int_{S_T} \psi_k^H G_k(s) \theta_k(s) \, ds|^2 + \sum_{j=1, j \neq k} \int_{S_T} \psi_j^H G_k(s) \theta_j(s) \, ds|^2 + \sigma^2 \|\psi_k\|^2. \quad (22)$$

which is also related to the pattern functions $\theta(s)$.

To solve the equivalent problem $P_1$ in (21), similar to MIMO beamforming schemes, a scheme of pattern design can be established by optimizing variables $\rho$, combiners $\psi$, and continuous functions $\theta(s)$ alternately until the convergence of channel capacity $R_{\text{sum}}$. For clarity, we summarize the whole process of this pattern design scheme in Algorithm 1, where the update steps of $\rho$, $\psi$, and $\theta(s)$ will be introduced in the following three parts, respectively.

1) Fix $\psi$ and $\theta(s)$, then optimize $\rho$: While fixing the receiver combiners $\psi$ and the CAP-MIMO pattern functions $\theta(s)$ of CAP-MIMO, the optimal solution to the auxiliary variables $\rho$ can be obtained by setting $\frac{\partial P_2}{\partial \rho_k}$ to zero for all $k \in \{1, \cdots, K\}$, given by

$$\rho_k^\text{opt} = E_k^{-1}, \quad k \in \{1, \cdots, K\}. \quad (23)$$

2) Fix $\rho$ and $\theta(s)$, then optimize $\psi$: While fixing the auxiliary variables $\rho$ and the pattern functions $\theta(s)$ of CAP-MIMO, after removing the unrelated components of problem $P_1$ in (21), the subproblem of optimizing the receiver combiners $\psi$ can be reformulated as

$$P_2: \max_{\psi} - K \sum_{k=1}^{K} \psi_k^H A_k \psi_k + 2 \sum_{k=1}^{K} \Re \{ \psi_k^H \alpha_k \}, \quad (24)$$

where $A_k$ and $\alpha_k$ are defined as

$$A_k = \rho_k \int_{S_T} G_k(s) \theta_j(s) \, ds \left( \int_{S_T} G_k(s') \theta_j(s') \, ds' \right)^H + \rho_k \sigma^2 I_3,$$

$$\alpha_k = \rho_k \int_{S_T} G_k(s) \theta_k(s) \, ds.$$ 

Note that, subproblem $P_2$ in (24) is a standard convex quadratic program (QP), thus the optimal solution to $\psi$ can be easily calculated as $[30]$

$$\psi_k^{\text{opt}} = A_k^{-1} \alpha_k, \quad k \in \{1, \cdots, K\}. \quad (25)$$

3) Fix $\rho$ and $\psi$, then optimize $\theta(s)$: Given fixed auxiliary variables $\rho$ and the receiver combiners $\psi$, after removing the unrelated components of problem $P_1$ in (21), the subproblem of optimizing the continuous pattern functions $\theta(s)$ can be reformulated as

$$P_3: \max_{\theta(s)} \sum_{k=1}^{K} \rho_k g_k(\theta(s)), \quad (26a)$$

s.t. $\sum_{k=1}^{K} \int_{S_T} \|\theta_k(s)\|^2 \, ds \leq P_T, \quad (26b)$

where the function $g_k(\theta(s))$ is

$$g_k(\theta(s)) = \sum_{j=1}^{K} \int_{S_T} \psi_j^H G_k(s) \theta_j(s) \, ds \int_{S_T} \psi_k^H G_k(s) \theta_k(s) \, ds - 2 \Re \left\{ \int_{S_T} \psi_j^H G_k(s) \theta_k(s) \, ds \right\}. \quad (27)$$

Then, to address the challenging issue of density functional optimization as shown in (26), we consider to apply the continuous-discrete transformations in Theorem 3 and Theorem 4, as well as the truncation operation in Theorem 5, to reformulate problem $P_3$ in (26). In this way, the reformulated problem can be written as

$$P_4: \max_{w} \sum_{k=1}^{K} \rho_k \hat{g}_k(w), \quad (28a)$$

s.t. $\sum_{k=1}^{K} \sum_{n=1}^{N} \|w_{k,n}\|^2 \leq P_T, \quad (28b)$

where we have defined $w$ as the set of $w_{k,n}$ and

$$\hat{g}_k(w) = \sum_{j=1}^{K} \sum_{n=1}^{N} h_{k,n}^H w_{j,n} \, w_{j,n} - 2 \Re \left\{ \sum_{n=1}^{N} h_{k,n}^H w_{k,n} \right\}. \quad (29)$$

in which $h_{k,n} := \Omega_{k,n}^H \psi_k$.

To further simplify the expression, we define $h_k$ and $w_k$ as the vectorized sets of $h_{k,n}$ and $w_{k,n}$ for all $n = [n_x, n_y, n_z] \in \{-N_x, \cdots, N_x\}, \{-N_y, \cdots, N_y\}, \{-N_z, \cdots, N_z\}$, respectively. Accordingly, the optimization problem $P_4$ in (28) can be equivalently reorganized as

$$P_5: \max_{w} \sum_{k=1}^{K} \rho_k \left( \sum_{j=1}^{K} |h_k^H w_j|^2 - 2 \Re \left\{ h_k^H w_k \right\} \right), \quad (30a)$$

s.t. $\sum_{k=1}^{K} \|w_k\|^2 \leq P_T, \quad (30b)$
which is a standard quadratically constrained quadratic program (QCQP). By adopting Lagrange multiplier method [31], the optimal solution to problem $P_5$ in (30) is given by

$$w_k^{\text{opt}} = \rho_k \left( \rho_k \sum_{j=1}^{K} h_j h_j^H + \zeta I_{N_F} \right)^{-1} h_k, \quad \forall k \in \{1, \ldots, K\},$$

(31)

where $N_F := N_x N_y N_z$ is the total number of Fourier expansion items. Note that, $\zeta$ is the Lagrange multiplier, which should be chosen such that the complementarity slackness condition of the power constraint (30b) is satisfied, i.e.,

$$\zeta^{\text{opt}} = \min \left\{ \zeta : \sum_{k=1}^{K} \|w_k\|^2 \leq P_T \right\}.$$  

(32)

One-dimensional binary search can be an efficient way to solve (32) and obtain the optimal multiplier $\zeta^{\text{opt}}$ [29].

After calculating the optimal projection lengths $w_k^{\text{opt}}$, according to Lemma 1, the final solution to the patterns on CAP-MIMO aperture can be obtained by

$$\theta_k^{\text{opt}}(s) = \sum_{n} w_{k, n}^{\text{opt}} \psi_n(s), \quad s \in S_T,$$

(33)

which completes the proposed pattern design scheme.

C. Performance analysis

In this subsection, we analyze the convergence and the complexity of the proposed pattern design scheme, respectively.

1) Convergence: The alternating optimization based beamforming schemes for conventional MIMO, such as WMMSE [29] and FP [32], usually have strict convergence, since their iteration steps are all monotonous. By contrast, the convergence of the proposed pattern design scheme is asymptotic. Specifically, the updates of auxiliary variable $\rho$ in (23) and receiver combiner $\psi$ in (25) are monotonous. However, a performance gap exists between problem $P_3$ in (26) and problem $P_4$ in (28), which is caused by the truncation operation in (20). Fortunately, thanks to the uniform convergence theorem of Fourier series expansion, this performance gap tends to be zero when the number of expansion items $N_F$ is sufficiently large. In later simulations, we will show that, even for a small value of $N_F = 16$, the proposed pattern design scheme has still a good convergence with each iteration being monotonous.

2) Complexity: The overall computational complexity of the proposed pattern design scheme is mainly introduced by the updates of the variables $\rho$, $\psi$, $w$, and continuous pattern functions $\theta(s)$, as shown in (23), (25), (31), and (33), respectively. To make it clear, we summarize the computational complexity of the main variables and functions in Table I, where $I_s$ denotes the sampling number of the continuous integral operation $\int_{S_T} ds$, and $I_\zeta$ is the search number required by the one-dimensional binary searching for optimal $\zeta^{\text{opt}}$. In general, since the aperture of CAP-MIMO is continuous, it is reasonable to assume $I_s >> K$ and $N_F >> K$. Thus, the overall computational complexity of the proposed pattern design scheme can be approximated by $O \left( I_s (22K^2 I_s + N_F^2 I_\zeta) \right)$, wherein $I_s$ is the iteration number required by the convergence of capacity $R_{\text{sum}}$. It is worth noting that, if the WMMSE scheme is directly applied to solve problem $P_o$ in (10), the complexity will be $O \left( I_o K^2 I_s^3 \right)$. Due to the large component of $I_s^3$, the complexity of WMMSE is much higher than that of the proposed scheme and thus becomes unacceptable when the continuous antenna aperture is large.

| Variable | Computational complexity $O(\cdot)$ |
|----------|----------------------------------|
| $\rho$   | $12K^2 I_s + K^2 + 4K$           |
| $\psi$   | $10K^2 I_s + K^2 + 21K$          |
| $w$      | $I_\zeta (N_F^2 + 2KN_F^2 + 2K)$|
| $\theta(s)$ | $3I_s N_F$                      |

Table I. Computational Complexity of Each Iteration Step.

IV. Simulation Results

In this section, we provided extensive simulation results under different conditions to validate the performance of the proposed pattern-division multiplexing technique.

A. Simulation setup

1) Simulation scenario: For the simulation scenario, we consider a 3-D scenario with the topology shown in Fig. 3, where one CAP-MIMO transmitter simultaneously serves eight electromagnetic-wave receivers by the proposed pattern-division multiplexing. Fig. 3. An illustration of the simulation scenario, where one CAP-MIMO transmitter simultaneously serves eight electromagnetic-wave receivers by the proposed pattern-division multiplexing.
region, where four electromagnetic-wave receivers are located at $(\pm 1 \text{ m}, \pm 1 \text{ m}, 30 \text{ m})$, and the other four receivers are located at $(\pm 5 \text{ m}, \pm 5 \text{ m}, 30 \text{ m})$, respectively.

2) **Simulation parameters:** For simulation parameters, the frequency of information-carrying current $j(s)$ and electric field $e(r)$ is set to $f = 2.4 \text{ GHz}$, and the intrinsic impedance is set to $Z_0 = 376.73 \Omega$ [21]. Unless specially specified, the maximum transmit power of CAP-MIMO is set to $P_T = 10^2 \text{ mW}$, and the noise power is set to $\sigma^2 = 5.6 \times 10^{-3} \text{ V}^2/\text{m}^2$.

The Green functions $G_k(s)$ are generated by the free-space Green model (5). Similar to the conventional MIMO beamforming [27], we assume that the perfect state information of Green functions is known at the CAP-MIMO transmitter. To ensure the continuity of CAP-MIMO, the sampling number of the continuous integral operation $\int_{S_T} ds$ is set to $I_s = 1000$. The numbers of the reserved Fourier expansion items are set to $N_x = 4$, $N_y = 4$, and $N_z = 1$, respectively, i.e., $N_F = 16$. All pattern functions $\theta_k(s)$ and receiver combiners $\psi_k$ are randomly initialized by the standard complex-Gaussian stochastic processes and variables, respectively.

3) **Simulation benchmarks:** As for the benchmark scheme, we consider the wavenumber-division multiplexing recently proposed in [16] for comparison. Specifically, the key idea of the existing wavenumber-division multiplexing is to directly adopt multiple different orthogonal Fourier basis functions to generate the patterns for different receivers, respectively. In this way, the symbols for different receivers are actually modulated on different wavenumbers of the radiated electromagnetic waves, thus achieving multi-stream transmissions [16], which is similar to the frequency-division multiplexing for conventional MIMO. By contrast, the proposed pattern-division multiplexing designs the patterns for multiple receivers according to the channel state information, which resembles the space-division multiplexing for conventional MIMO. Furthermore, to compare the interference elimination ability of different schemes, similar to the ideal assumption in conventional MIMO systems, we consider the interference-free capacity as the upper bound for comparison, which can be realized by assuming all inter-receiver interferences can be ideally eliminated and then employing the proposed pattern design scheme.

### B. Capacity against the aperture area $A_T$

To show the impact of aperture area of CAP-MIMO $A_T$ on the system performance, we first plot the channel capacity against the aperture area $A_T$ in Fig. 4, where the aperture shape of CAP-MIMO always remains a square, i.e., $L_x = L_y$. From this figure, we have the following two observations.

Firstly, for all considered schemes, the capacity quickly improves as the aperture area $A_T$ increases. For example, as $A_T$ increases from $0.1 \text{ m}^2$ to $1 \text{ m}^2$, the capacity achieved by the proposed pattern-division multiplexing rises from $3.21 \text{ bps/Hz}$ to $16.45 \text{ bps/Hz}$ by about four times. The straightforward reason is that, larger CAP-MIMO aperture is able to provide higher spatial DoF [15], which allows the CAP-MIMO to generate the electromagnetic waves with higher spatial resolution and stronger orthogonality. In this way, the electromagnetic-wave receivers can decode their symbols with higher signal-to-noise ratio (SNR), thus the channel capacity can be improved.

Secondly, compared with the existing wavenumber-division multiplexing, the proposed pattern-division multiplexing achieves a higher capacity. For example, when $A_T = 1 \text{ m}^2$, the capacity achieved by the proposed scheme is 16.46 bps/Hz, which is about 260% higher than 4.53 bps/Hz achieved by the wavenumber-division multiplexing. The reason is that, the wavenumber-division multiplexing scheme directly uses $K$ different orthogonal Fourier bases to generate the patterns for $K$ different receivers. Despite this scheme can make the symbols orthogonal at the transmitter, their orthogonality cannot be guaranteed at the receivers. Particularly, after passing through the electromagnetic channels, the orthogonality among different symbols may be destroyed, which will result in high inter-receiver interference or even make the radiated electromagnetic wave unable to reach the receivers accurately.

By contrast, the proposed pattern-division multiplexing jointly designs the patterns for $K$ receivers according to the specific channel state information. During the process of pattern design, the proposed scheme actually makes a trade-off between the amplification of desired signals and the elimination of inter-receiver interferences, and it can also realize the power allocation among different receivers. In this way, the electromagnetic waves carrying symbols can be focused on the receivers accurately with stronger orthogonality, and thus higher capacity can be achieved.

It is also worth noting that, despite enlarging the aperture area $A_T$ can efficiently improve the capacity, it will also result in higher computational complexity. The reason is that, the overall complexity of the proposed pattern design scheme is proportional to the sampling number of the continuous integral operation $\int_{S_T} ds$, i.e., $I_s$, and it is also proportional to the third power of the number of Fourier expansion items $N_F$, as discussed in Subsection III-C. When the aperture area $A_T$ is too large, it requires a large $I_s$ to guarantee the continuity of the CAP-MIMO surface and a large $N_F$ to guarantee the
performance, which will reduce the computational efficiency. Therefore, in practical CAP-MIMO systems, it is essential to carefully choose the aperture size to make a balance between the performance and computational complexity.

C. Capacity against the transmit power $P_T$

To show the impact of transmit power on the system performance, we plot the channel capacity against the maximum transmit power $P_T$ in Fig. 5. From this figures, we have the following two observations.

Firstly, for all considered schemes, the achievable channel capacity increases quickly as the transmit power becomes higher, which is similar to the result in Fig. 4. We note that, the proposed pattern-division multiplexing always outperforms the existing wavenumber-division multiplexing in the considered range of transmit power. Particularly, when the transmit power $P_T$ is up to $10^{-3}$ A$^2$, the capacity achieved by the wavenumber-division multiplexing is 5.94 bps/Hz, while that achieved by the proposed pattern-division multiplexing is about 14.62 bps/Hz, which achieves an improvement of about 146%.

Secondly, we note that, the performance gaps, including the gap between the upper bound and the proposed scheme and the gap between the proposed scheme and wavenumber-division multiplexing scheme, become larger and larger as the allowable transmit power $P_T$ becomes higher. For example, when $P_T$ is lower than 10 mA$^2$, the proposed pattern-division multiplexing scheme achieves nearly the same performance as the ideal upper bound. However, when $P_T$ is higher than $10^3$ mA$^2$, the performance gap between them is larger than 15 bps/Hz. The reason behind this phenomenon is that, the performance gaps among these three schemes highly depend on the inter-receiver interferences, and the channel capacity is simultaneously influenced by the inter-receiver interferences and the electromagnetic noise. When the transmit power is low, the inter-receiver interferences at receivers are accordingly low, thus the noise dominates, which makes the performance gaps among these schemes small. By contrast, when the transmit power of CAP-MIMO becomes high, the inter-receiver interferences will be more serious, and finally dominates in the undesired factors for capacity improvement.

D. Patterns $\theta(s)$ of CAP-MIMO

To show the pattern functions optimized by the proposed pattern-division multiplexing scheme, we present the normalized amplitude of the x-component of the optimized patterns $\theta_k(s)$ for the former four receivers 1-4 in Fig. 6, and their phase in Fig. 7, respectively.

From these two figures, it is interesting to observe that, after designing the patterns (i.e., the current distributions) via the proposed pattern-division multiplexing, the patterns $\theta_k(s)$ for different receivers are nearly orthogonal. In particular, as shown in Fig. 6, the power of the patterns that carry different
symbols are distributed in non-overlapping regions, in this way, the inter-receiver interferences at each electromagnetic-wave receiver can be well eliminated. Besides, from Fig. 7 one can notice that, the phase of the patterns carrying different symbols are symmetrically distributed, which means that the electromagnetic-waves for four receivers are radiated towards four different spatial directions and focused on the four receivers respectively, which agrees with the simulation scenario shown in Fig. 3. This interesting phenomenon is similar to the design result of MIMO beamforming, which aims to generate orthogonal beams towards multiple receivers. It also intuitively shows why the proposed pattern design scheme can improve the capacity. We can conclude that, both of these two figures have provided intuitive explanations for the capacity improvement, which has further demonstrated the effectiveness of the proposed pattern-division multiplexing technique.

E. Convergence of the proposed pattern design scheme

To evaluate the convergence performance of the proposed pattern design scheme, in Fig. 8, we plot the channel capacity against the iteration number. From this figure, we can find that, the channel capacity achieved by the proposed pattern design scheme is nearly zero when \( I_0 = 1 \). It indicates that, multiple electromagnetic waves induced by the random patterns will mutually cancel, thus it has negligible contribution to capacity improvement. Fortunately, after several iterations, the capacity achieved by the proposed pattern design scheme gradually increases monotonously. For example, when \( I_0 = 2 \), the capacity achieved by the proposed scheme is 5.79 bps/Hz, which is more than three times larger than that of 1.77 bps/Hz achieved by the existing wavenumber-division multiplexing scheme. particularly, when the iteration number \( I_0 \) is five, the proposed scheme finally converges to about \( R_{\text{sum}} = 1.56 \) bps/Hz. We can conclude that, despite the use of truncation operation in (20), the proposed scheme still has a good convergence performance.

F. Coverage performance of the CAP-MIMO based system

To evaluate the coverage performance of the CAP-MIMO based communication system with the proposed pattern-division multiplexing, in this subsection, we assume that eight receivers are located at \((\pm 1\text{m}, \pm 1\text{m}, D)\) and \((\pm 5\text{m}, \pm 5\text{m}, D)\), respectively, wherein \( D > 0 \) is the vertical distance between the transmitter and the receivers. Then, we plot the channel capacity against the vertical distance \( D \) in Fig. 9. From this figure, we have the following two observations.

Firstly, we notice that the capacity performances of all three considered schemes decrease as the distance \( D \) becomes larger. This fact originates from two reasons. One reason is that, due to the large-scale fading of wireless channels, more transmitted power is lost into space as the receivers get far away from the transmitter. From Green function in (5), we can find that the electromagnetic-wave power reaching the receiver is roughly inversely proportional to the square of the distance. The other reason is that, the spatial angle resolution of CAP-MIMO is determined by the aperture area [19]. As the receivers get far, limited by the spatial angle resolution, it is increasingly difficult for CAP-MIMO to accurately focus the desired electromagnetic waves on the corresponding receivers.

Secondly, similar to the results in Fig. 5, we observe that, the performance gaps, including the gap between the upper bound and the proposed pattern-division multiplexing and that between the proposed scheme and wavenumber-division multiplexing, become larger as the vertical distance \( D \) becomes smaller. The reason is that, the undesired factors influencing capacity include the inter-receiver interferences and the noise. When the receiver is far from the transmitter, due to the large-scale fading, the inter-receiver interferences at receivers are low thus the noise dominates, which makes the performance gaps among these schemes small. By contrast, when the receiver is close to the transmitter, the power of the received electromagnetic-waves by the receivers become high, which results in strong inter-receiver interferences and makes
them dominate. This phenomenon has also agreed with the phenomenon in Fig. 5.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, we have tried to bridge the gap between the classical discrete MIMO and CAP-MIMO by proposing the pattern-division multiplexing to provide a flexible pattern design scheme for CAP-MIMO. Our derived system model of CAP-MIMO may provide a possible investigation framework for some open problems in electromagnetic information theory, such as the analyses of channel DoFs [25] and asymptotic capacity [33]. The proposed pattern-division multiplexing is able to transform the design of continuous pattern functions to the design of their projection lengths on finite orthogonal bases, so it could serve as a general signal processing framework for some important problems in electromagnetic information theory, such as the channel estimation [12] and energy efficiency optimization [34]. Thanks to the proposed pattern-division multiplexing, the CAP-MIMO capacity achieved by the proposed pattern design scheme is about 260% higher than that achieved by the existing wavenumber-division multiplexing scheme. In the future, the pattern design with lower complexity for CAP-MIMO is an important topic, and some other tools like deep learning can be leveraged for the pattern design of CAP-MIMO in complex electromagnetic channels.

APPENDIX A

PROOF OF THEOREM 1

For an arbitrary user \( k \), let \( I(x_k;y_k) \) denote the mutual information between the received electromagnetic waves \( y_k \) and the desired symbol \( x_k \). According to Shannon information theory, the achievable rate of user \( k \) is bounded by

\[
I(x_k;y_k) = H(y_k) - H(i_k + n_k),
\]

where \( H(\cdot) \) denotes the entropy of its argument, and \( i_k \) is the interference from other receivers, which is given by

\[
i_k = \sum_{j=1, j \neq k}^{K} x_j \int_{S_T} G_k(s) \theta_j(s) \, ds.
\]

Note that, the entropy \( H(y_k) \) and \( H(i_k + n_k) \) can be derived by the covariance matrices of their arguments, i.e.,

\[
H(y_k) = \log_2 \left( \pi e \det |K_{y_k}| \right),
\]

where \( K_{y_k} \) is the covariance matrix of its subscript argument.

Then, by applying expectation operation \( \mathbb{E}_{x_i n_i} \) to \( y_k y_k^H \), \( i_k i_k^H \), and \( n_k n_k^H \), respectively, we can calculate these covariance matrices as

\[
K_{y_k} = \sum_{j=1}^{K} \int_{S_T} G_k(s) \theta_j(s) \, ds \left( \int_{S_T} G_k(s') \theta_j(s') \, ds' \right)^H + \sigma^2 I_3,
\]

\[
K_{i_k} = \sum_{j=1, j \neq k}^{K} \int_{S_T} G_k(s) \theta_j(s) \, ds \left( \int_{S_T} G_k(s') \theta_j(s') \, ds' \right)^H
\]

\[
K_{n_k} = \sigma^2 I_3.
\]

By substituting (37) into (35), the maximum achievable rate of user \( k \) can be derived as

\[
I(x_k;y_k) = \log_2 \left( \frac{\det |K_{y_k}|}{\det |K_{i_k} + K_{n_k}|} \right) = \log_2 \det \left( I_3 + \alpha_k \sigma^2 J_k^{-1} \right),
\]

wherein \( \alpha_k \) and \( J_k \) are defined in (8).

Finally, since there is no information exchange among the receivers, the capacity of this CAP-MIMO based system can be obtained by adding up the mutual information of all receivers, i.e.,

\[
R_{\text{sum}} = \sum_{k=1}^{K} I(x_k;y_k),
\]

which completes the proof.

APPENDIX B

PROOF OF THEOREM 2

By integrating the radial component of the Poynting vector over a sphere with infinite-length radius, the authors in [16] have proved that, for a deterministic source in a given surface \( S_T \), the physical radiated power can be upper-bounded by the integral of the \( L^2 \) norm of \( j(s) \) over \( s \in S_T \). Here we extend this proof to a case in the sense of expectation, where the current distribution \( j(s) \) is a stochastic process consisting of multiple patterns that carry symbols \( x \), as shown in (2).

We first calculate the Poynting vector \( S(r) \) by its definition:

\[
S(r) = \frac{1}{2Z_0} \mathbf{e}(r) \mathbf{e}^H(r)
\]

\[
= \frac{1}{2Z_0} \left\| \sum_{k=1}^{K} x_k \int_{S_T} G(r,s) \theta_k(s) \, ds \right\|^2
\]

\[
= \frac{1}{2Z_0} \sum_{k=1}^{K} |x_k|^2 \left\| \int_{S_T} G(r,s) \theta_k(s) \, ds \right\|^2 + \frac{1}{2Z_0} \sum_{j \neq j'} \sum_{k=1}^{K} |x_j|^2 \left\| \int_{S_T} G(r,s) \theta_j(s) \, ds \right\|^2 \left\| \int_{S_T} G(r,s) \theta_j'(s) \, ds \right\|^2.
\]

where \( \sum_{j \neq j'} = \sum_{j=1}^{K} \sum_{j=1, j \neq j'}^{K} \). The physical radiated power of CAP-MIMO \( P_{\text{rad}} \) in the sense of expectation can be obtained by

\[
P_{\text{rad}} = \lim_{r \to \infty} \mathbb{E}_x \left\{ \int_{\Omega} S(r,r^2 \, d\omega) \right\}
\]

\[
= \lim_{r \to \infty} \frac{1}{2Z_0} \sum_{k=1}^{K} \int_{\Omega} \left\| \int_{S_T} G(r,s) \theta_k(s) \, ds \right\|^2 \, r^2 \, d\omega,
\]

where \( r = \|r\| \) and \( \omega \in \Omega \) is the solid angle of \( 4\pi \) steradians. Note that, when \( r \) is large enough, the Green function \( G(r,s) \) can be approximated by

\[
G(r,s) = \frac{j\kappa Z_0}{4\pi} \frac{e^{j\kappa r}}{r^2} \left( I_3 - \frac{pp^H}{r^2} \right) e^{-j\kappa T(\phi,\varphi)s},
\]

where \( \phi \in [0, \pi) \) and \( \varphi \in [-\pi, \pi) \) are the elevation angle and azimuth angle respectively, which is associated with the points \( r \) and \( s \). Plane-wave wave vector \( \kappa (\phi, \varphi) \) takes the form of

\[
\kappa(\phi, \varphi) = \frac{2\pi}{\lambda} \begin{bmatrix} \cos \phi \sin \phi, \sin \phi \sin \phi, \cos \phi \end{bmatrix}^T.
\]
By substituting (43) into (42) and letting $r \to \infty$, we obtain

$$ P_{\text{rad}} = \lim_{r \to \infty} \frac{\kappa^2 Z_0}{2\pi r^2} \sum_{k=1}^{K} \left\| \mathbf{I}_{3} - \frac{\mathbf{r}_{\perp}^{H}}{r^2} \mathbf{e}^{-j\mathbf{r}_{\perp}^{H}(\phi,\psi)_{\sigma}^{s} d\omega} \mathbf{S}_{\sigma} \right\|^{2} $n\right\|^{2} \left( \mathbf{I}_{3} - \frac{\mathbf{r}_{\perp}^{H}}{r^2} \right) e^{-j\mathbf{r}_{\perp}^{H}(\phi,\psi)_{\sigma}^{s} d\omega} \mathbf{S}_{\sigma} \right\|^{2} ds \\
\leq \lim_{r \to \infty} \frac{\kappa^2 Z_0}{32\pi r^2} \sum_{k=1}^{K} \int_{S_{\sigma}} \left\| \mathbf{I}_{3} - \frac{\mathbf{r}_{\perp}^{H}}{r^2} \right\|^{2} ds \times \left\| \mathbf{I}_{3} - \frac{\mathbf{r}_{\perp}^{H}}{r^2} \right\|^{2} ds \left( \mathbf{I}_{3} - \frac{\mathbf{r}_{\perp}^{H}}{r^2} \right) e^{-j\mathbf{r}_{\perp}^{H}(\phi,\psi)_{\sigma}^{s} d\omega} \mathbf{S}_{\sigma} \right\|^{2} ds \\
\leq \lim_{r \to \infty} \frac{\kappa^2 Z_0}{32\pi r^2} \sum_{k=1}^{K} \int_{S_{\sigma}} \left\| \mathbf{I}_{3} - \frac{\mathbf{r}_{\perp}^{H}}{r^2} \right\|^{2} ds \times \left\| \mathbf{I}_{3} - \frac{\mathbf{r}_{\perp}^{H}}{r^2} \right\|^{2} ds \left( \mathbf{I}_{3} - \frac{\mathbf{r}_{\perp}^{H}}{r^2} \right) e^{-j\mathbf{r}_{\perp}^{H}(\phi,\psi)_{\sigma}^{s} d\omega} \mathbf{S}_{\sigma} \right\|^{2} ds$