Analysis of the statistical errors of the emissivity measurement of a laser heated surface by acousto-optical tunable filter

Yu V Mantrova, P V Zinin, A A Bykov and K M Bulatov

Scientific-Technological Center of Unique Instrumentation of the Russian Academy of Sciences, Butlerova 15, Moscow 117342, Russia

E-mail: mantrovayv@gmail.com

Abstract. The aim of this report is to conduct a rigorous data analysis of the emissivity and temperature measurements on the surface of a solid specimen during laser heating (LH). Measurements of the emissivity and temperature is based on the use of a tandem acousto-optic filter (tandem acousto-optical tunable filter or TAOTF) connected with a high-resolution video camera. The set of spectral images obtained with the help of TOTF in the range of 750-800 nm makes it possible to calculate the dependence of the radiation intensity of the heated surface on the wavelength at each point of the surface. The distribution of the temperature and the emissivity of the heated body surface is obtained by fitting the experimental spectral dependence of the radiation intensity at each point of the heated object to the Planck distribution using the least squares method (LSM). A development of the mathematical analysis of the statistical errors of the emissivity determination by the LSM is described.

1. Introduction

The emissivity of a material describes the ability of a material's surface to emit radiation at a specific wavelength relative to a black body radiation. Measurement of emissivity at high temperatures is necessary for understanding many physical phenomena, including heat exchange in the center of the Earth, processes in a diffusion flame, obtaining solar cells with low emissivity, insulation and energy saving of buildings, etc. The laser-heating is the only experimental tool able to create extreme temperatures and is widely used in research on materials science and geophysics [1]. Non-contact radiometric methods are based on comparison surface radiation of the studied materials with a calibration source (absolutely black body) under the same temperature, geometric and spectral conditions [2]. Since the measurement of the emissivity is closely related to the measurement of temperature, the measurement methods differ in the methods of heating the samples. Measurement of emissivity is a fundamental constant, and also plays an important role in high-pressure physics, when it is a task to establish heat balance, for example, in the Earth's crust. Previously, the measurement of the emissivity was difficult because it was impossible to obtain the temperature distribution. The possibility of using acousto-optic filters made it possible to measure the temperature distribution in the laser beam [3]. Unfortunately, distribution of the emissivity during laser heating was studied. In this paper, we will conduct (a) a rigorous data analysis of the emissivity and temperature measurements of laser heated areas of the tungsten plate and (b) will provide an estimate of measurement errors of the emissivity.
2. Experimental Method

Description of the laser heating system combined with TAOTF can be found elsewhere [3]. Briefly, the system consists of four components: (1) 100 W fiber laser LH (1064 nm, IPG Photonics), which is designed to allow precise control of the total power in the range from 2 to 100 W by changing the diode current, for heating samples; (2) TAOTF; (3) motorized sample stage; and (4) high magnification imaging system based on long working distance infinity-corrected objective. This system is unique and allows: (a) measurement of 2-D temperature distribution (TD) with a resolution less than 2 µm in a sample under high temperature using Planck’s law; and (b) measurement of 2-D intensity distribution of an IR laser beam on the surface of specimen [4].

An experimental LH-TAOTF setup operating in the reflection configuration is shown in figure 1. The fiber laser beam is guided to the sample position with mirrors (M5, M6), and a narrow line mirror (M4). A pi-shaper is located between mirrors M5 and M4 and designed to allow control of the IR laser spot shape (e.g., gauss, flat-top, donut) and size (8-100 µm) [5]). The M4 separates the laser beams and visible radiation. The thermal radiation, coming from the sample during LH, is directed toward the TAOTF by mirrors M1 and M2. The image of the heated specimen at a selected wavelength is taken by a camera after a long focal distance lens.

![Figure 1. The sketch of the LH-TAOTF system. OB1: geoHEAT, 40NIR, AldOptica, GmbH; TAOTF: tunable acousto-optic filter, A-Optic, Russia; Beam splitter; lens, f = 400 mm, AC254-400-B Thorlabs; M: mirrors on kinematic mounts; M4: narrow line mirror (1064 nm); pi-Shaper; Camera: monochrome camera, Allied Vision Mako G-030B, which runs 309 frames per second at resolution 644×484 pixels; CW laser: continuous wave laser (IPG Photonics, YLR-100-AC-Y11).](image)

Temperature and emissivity can be calculated from the radiation emitted by a specimen using Planck’s equation (for details see [6, 7]):

\[
I(\lambda, \varepsilon, T) = \varepsilon g(\lambda, T), \quad g(\lambda, T) = \frac{c_1}{\lambda^5 \left[ \exp\left(\frac{c_2}{\lambda T}\right) - 1\right]},
\]

where \(I(\lambda, \varepsilon, T)\) is the spectral intensity, \(\varepsilon\) is the sample emissivity, \(\lambda\) is the wavelength, \(T\) is the temperature, and \(c_1\) and \(c_2\) are physical constants (\(c_1 = 2\pi c^2\), \(c_2 = h \cdot c / k = 0.01432 \text{ m-K}\)). To calibrate the
optical system, a lamp with a known temperature (1700 K) is used, which is placed in the
measurement area. In our experimental set-up calibration is based on the imaging of a certified lamp at
a temperature of 1700 K [8]. A radiometric power supply is used to provide a very precise and
reproducible lamp current, giving a constant radiance. The lamp is placed at the position normally
occupied by the sample place so that the spectral intensity \( I(\lambda)_{\text{optics}} \) of the lamp is acquired through the
optical pathways. To determine the spectral radiance of the heated specimen, \( I(\lambda)_{\text{corrected}} \), the following
equation is used

\[
I(\lambda)_{\text{corrected}} = I(\lambda)_{\text{measured}} \cdot I(\lambda)_{\text{standard}} I(\lambda)_{\text{optics}} \tag{2}
\]

where \( I(\lambda)_{\text{measured}} \) is the spectral intensity of the specimen during heating [3].

3. Least squares method

The standard method of the temperature and emissivity determination from the thermal radiation data,
\( I_{\text{corrected}}(\lambda) \), is to find such values of \( T_0 \) and \( \varepsilon_0 \) at which the sum of squares or S function,

\[
S(\varepsilon, T) = \sum_{i=1}^{N} \left[ I(\lambda_i) - \varepsilon g(\lambda_i, T) \right] ^2, \tag{3}
\]

has a minimum (2D non-linear least square fitting) [9-11]. To reduce the problem of the 2D non-linear least square fitting (3) to the one dimensional (1D) least-squares fitting we used a method proposed in [12]. It is based on the fact that emissivity \( \varepsilon \) in (1) is a linear parameter. It allows us to obtain an analytical expression for \( \varepsilon_0 \) using the following procedure. If the function \( S(T, \varepsilon) \) has a minimum at \( T_0 \) and \( \varepsilon_0 \), then the following conditions should be satisfied: 1) \( \partial S/\partial \varepsilon |_{\varepsilon=\varepsilon_0, T=T_0} = 0 \); 2) \( \partial S/\partial T |_{\varepsilon=\varepsilon_0, T=T_0} = 0 \). The first equation gives the expression for the \( \varepsilon_0 \):

\[
\varepsilon_0 = \frac{\sum_{i=1}^{N} [ I(\lambda_i) g(\lambda_i, T) ]}{\sum_{i=1}^{N} [ g^2(\lambda_i, T) ]}. \tag{4}
\]

The second equation can be written as

\[
\frac{\partial S}{\partial T} = -\frac{2\varepsilon_0 g_0}{T^2} \sum_{i=1}^{N} \frac{1}{\lambda_i} \left[ I(\lambda_i) g(\lambda_i, T) - \varepsilon_0 \right] = 0. \tag{5}
\]

The 1D least square fitting strategy is following. The temperature is varied and the (4) is used to
determine \( \varepsilon \) for a given \( T \). Then \( S(T, \varepsilon) \) is calculated and values of \( T_0 \) and \( \varepsilon_0 \), at which the \( S \) function has a minimum, are determined.

Because the emissivity \( \varepsilon \) is a linear parameter in (1) it is also possible to obtain an analytical
expression for the standard deviation for \( \varepsilon_0 \) following the method described elsewhere [13]. To use this
method we rewrite (4) in the form

\[
\varepsilon_0 = \frac{\sum_{i=1}^{N} [ I(\lambda_i) G(\lambda_i, T_0) ]}{\sum_{i=1}^{N} [ g^2(\lambda_i, T_0) ]}, \quad \text{where} \quad G(\lambda_i, T_0) = \frac{g(\lambda_i, T_0)}{\sum_{i=1}^{N} [ g^2(\lambda_i, T_0) ]}. \tag{6}
\]

If we assume that all the measurement of \( I(\lambda_i) \) are independent then the standard deviation of the
emissivity can be written as

\[
\sigma^2_\varepsilon = \left[ \sum_{i=1}^{N} \left( \frac{\partial \varepsilon(\lambda_i)}{\partial T} \sigma_T \right)^2 \right]^{1/2}, \quad \text{where} \quad \sigma_T = \sqrt{\frac{\sum_{i=1}^{N} [ I(\lambda_i) - \varepsilon_0 g(\lambda_i, T_0) ]^2}{N-2}}. \tag{7}
\]
The derivative in (6) can be taken: Then the expression for the standard deviation of the emissivity \( \varepsilon_0 \) can be written

\[
\sigma_{\varepsilon} = \frac{\sigma_f}{\sum_{i=1}^{N} [g^2(\lambda_i, T_o)]}.
\]  

(8)

We will use this expression to estimate the error of the emissivity measurement.

4. Results

To conduct the laser heating experiments we used a tungsten lamp, which is a standard test object in the pyrometry. The tungsten ribbon of the lamp was heated by the IR laser with a fixed power of 8 W. Figure 2 shows the temperature (figure 2 (a)) and emissivity (figure 2 (b)) distributions simulated by the least square method, (3). Evaluations of the statistical errors of temperature and emissivity were conducted at three points with different temperatures: in the center of the heating spot (area I), where the heating temperature was highest, 2640 K (figure 2 (a)); and in two areas (area II and area III) along its edge, where the heating temperature was 2451 K (area II, figure 4 (a)); and 2135 K (area III, figure 6 (a)). At these points, emissivity was also measured (figures 2 (b), 4 (b) and 6 (b)). For all the points, the thermal radiation measurements were made in the wavelength range from 750 to 800 nm.

![Figure 2](image)

The confidence interval of the \( T_o \) and \( \varepsilon_0 \) was evaluated using the F-test developed for the non-linear regression technique [14], where the confidence level is derived from the expression,

\[
S(T_o^*, \varepsilon) = S(T_o, \varepsilon) \{ 1 + \frac{P}{(N-P)} F(N,P,\alpha) \}.
\]  

(9)

where \( N \) is the number of measurements, \( P \) is the number of variables, \( F(P,N,1-\alpha) \), is the Fisher coefficient, and the coefficient \( (1-\alpha) \) is the level of confidence. For the current experiment \( N = 51, P = 2 \). Expression (9) provides the confidence interval for temperature and emissivity measurements. For the level of confidence of 0.95, it is shown in figures 3-5 as a red line.
Figure 3. Behavior of $S$ as a function of (a) of the temperature, and (b) of the emissivity simulated at the point in the center of the heating spot, area I (figure 2 (a) and (b)).

Figure 3 shows that the $S$ function has a minimum, $S_0 = 0.1843$, at $\varepsilon_0 = 0.3548$ and $T_0 = 2640$ K. The confidence interval for the emissivity is 0.084 indicating that the relative error is pretty high 23%. Similarly, we calculate the confidence interval for the temperature. It is found to be 89 K indicating that the relative error is around 3.4%. Now, we can compare the confidence interval obtained with the F-test with that obtained with (8). The value of the $\sigma_\varepsilon$ was measured to be 0.268. The confidence interval can be obtained using the formula,

$$
\Delta \varepsilon = \frac{\sigma_\varepsilon \cdot t}{\sqrt{N}},
$$

where $t$ is the Student coefficient equal to 2.006. Then the confidence interval should be 0.075 for the emissivity (relative error is around 21%) which pretty close to the value obtained by the F-test.

Figure 4 shows the temperature distribution (figure 4 (a)) and emissivity (figure 4 (b)) for the area II. Figure 5 shows the behavior of the $S$ function as a function of the temperature (figure 5 (a)) and emissivity (figure 5 (b)).

Figure 4. Temperature and emissivity distributions of the laser heated area II: (a) temperature distribution; (b) emissivity distribution. Exposure time 1/4 s.
Figure 5. Behavior of the $S$ as function (a) of the temperature, and (b) of the emissivity simulated at the point of the area II (Figure 4a and 4b).

The value of the $S$ function at the minimum, $\varepsilon_0 = 0.4274$ and $T_0 = 2451$ K, is $S_0 = 0.14934$. The F-test confidence interval for the emissivity is 0.06 (the relative error is 14%) and is 66 K for temperature (the relative error is 2.7%). The confidence interval using (10) is 0.0066 for the emissivity indicating that the relative error of the emissivity measurement is around 15%. It is in a good agreement with the value obtained by the F-test.

Simulations of the temperature and temperature distributions at the third point in the area III are shown in figure 6. Figure 7 shows the behavior of the $S$ function for temperature (figure 7 (a)) and emissivity (figure 7 (b)). The $S$ function has its minimum ($S_0 = 0.08123$) at $\varepsilon_0 = 0.4478$ and $T_0 = 2135$ K. The F-test confidence interval for emissivity is 0.035 (the relative error is 7.9%) and for the temperature is 43 K (the relative error is 2.0%). The confidence interval for emissivity obtained using (8) is 0.041 (the relative error is 9%), and is close to that obtained by the F-test.

Our results demonstrate show the higher the temperature the lower the value of emissivity in the wavelength range from 750-800 nm which is in a good agreement with published experimental data [15]. We note that emissivity values measured by TAOTF system are also in a good agreement (within experimental errors) with those given in a handbook [15]: (a) $\varepsilon_0 = 0.355 \pm 0.084$ measured at $T_0 = 2640$ K is close to the value given in [15], $\varepsilon = 0.394$ obtained at $T = 2600$ K; (b) $\varepsilon_0 = 0.427 \pm 0.066$ measured at $T_0 = 2451$ K is within experimental error with the value $\varepsilon = 0.398$ obtained at $T = 2400$ K; (c) $\varepsilon_0 = 0.448 \pm 0.041$ measured at $T_0 = 2135$ K is close to that $\varepsilon = 0.403$ obtained at $T = 2200$ K.

Figure 6. Temperature and emissivity of the laser heated area III: (a) temperature distribution; (b) emissivity distribution. Exposure time 1/4 s.
5. Conclusion

Analysis of the experimental data demonstrates that the application of the TAOFT allows us to measure emissivity as well as temperature with relative errors of 23% and 4% respectively. Relative statistical errors of the emissivity appeared to be nearly seven times higher as that for the temperature. We also found the statistical error of the emissivity increased with temperature. The possible reason of this phenomenon might be related to the strong dependence of the emissivity value on the wavelength at high temperature: $\varepsilon_0(\lambda)$. In future work, we plan to determined dependence of the emissivity on the wavelength from thermal radiation measurements in the wavelength range from 600 to 850 nm.

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