An analysis of the big-bang theory according to classical physics

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This paper collects a consistent body of information on the observable Universe, from which an estimate of the total mass of the Universe is calculated as a function of the angle whose vertex is at the center of the Universe, and whose extremities stand on the Earth and on the limits of the horizon of visibility. This result leads to an analysis of the dynamics of the Big-Bang, taking into account the limitations imposed by the Schwarzschild radius, $R_S$. Where if $R_0$ is the radius of the incipient Universe when the formation of elementary particles has just finished, the value of the quotient $R_0/R_S$ determines its subsequent evolution. An important conclusion from this concerns the expansion of the Universe; all signs point to its being destined to expand indefinitely.

I. PRELIMINARY CONSIDERATIONS

This paper uses the $(e, m_e, c)$ system of units in which the basic nits are the quantum of electric charge, electron mass and the speed of light.

According to the theory of the Big-Bang, the history of the Universe begins with an unimaginably great outburst of photons of very high energy around a point $\omega$, after which time begins to elapse. The formation of heavy particles at once begins, and ends when the temperature of the Universe has fallen to below the required level, and the radius of the Universe has attained the dimension $R = R_0$.

![FIG. 1:](image)

The dynamics of the Big-Bang thus appears to be the result of the joint action of the kinetic energies of the particles which emerge at a velocity close to $c$ at a distance $R_0$ from $\omega$, and the attraction of the mass of the Universe $M_0$, which is exerted from $\omega$, the center of its mass, reasonably assuming a uniform expansion, causing matter to be distributed as spherical surfaces of radius $R$. Figure 1 shows a diametric section on which Point $A$ indicates our observatory on Earth, Point $B$ represents the limit of the visibility from $A$, so that the velocity of growth of $\overline{AB}$ is $c$, and Point $\omega$ is the centre of the Universe, perhaps the starting point of the Big-Bang, perhaps the center of what was emerged from the Big-Bang.

The relation between the volume of the observable Universe, $V_\phi$, and that of the totality of the Universe, $V_0$, is given by the relation between the area of the sphere and the spherical segment $\overline{BAB}'$. In other words:

$$\frac{V_0}{V_\phi} = \frac{4\pi R^2}{2\pi R^2(1 - \cos \phi)} = \frac{2}{1 - \cos \phi}.$$  

This relation holds for $\phi \leq \pi$; for $\pi < \phi < 2\pi$, it would be $V_0/V_\phi = 1/(2 + \cos \phi)$.

Just after the Big-Bang, the radius of the Universe was growing at speed $c$. As the speed of the growth of $R$ decreased, the value of $\phi$ must have gone on increasing, so that the speed at which $B$ moves away from $A$ would continue to be $c$.

II. CALCULATION OF THE MASS OF THE UNIVERSE

The information used in this paper comes mostly from the third edition of “Universe”, published in 2000. This book contains (Fig. 26-22) a sky map which is said to show approximately 10 % of the visible Universe, and which has permitted an estimate to be made of the number of galaxies. Roughly $2 \times 10^6$ galaxies have thus been detected. We can therefore guess that the number of galaxies in the observable Universe could be of the order of $2 \times 10^7$, though this is only a preliminary estimate.

Also these galaxies are distributed as follows:

- 77 % spiral galaxies (S) and (SB) of between $10^9$ and $4 \times 10^{11}M_\odot$.
- 20 % elliptical galaxies (E) of between $10^5$ and $10^{13}M_\odot$.
- 3% irregular galaxies (Irr) of between $10^7$ and $10^9M_\odot$.

Where $M_\odot$ is the mass of the Sun (Universe, p. 646).

It is estimated that dispersed interstellar matter (elementary particles, atoms, molecules and “dust”) is insignificant in elliptical galaxies and represents between 2.5% and 25% of the mass of stars and planets in the spiral and irregular galaxies.
Keeping in mind the fact that the immense majority of elliptical galaxies are dwarfs and that the giant elliptical galaxies are rare, an estimate has been made of the average mass of the group, assuming that the probability distribution of the logarithms of their masses follows a Poisson distribution. This produces an average mass of $10^7 M_\odot$ (any future more exact study would start from the statistical analysis of the available data—not too awkward a task).

For galaxies (S) and (Irr) a normal distribution has been assumed, with the result that for $20 \times 10^9$ galaxies in question, we obtain this preliminary estimate:

| Type       | $10^{18} M_\odot$ |
|------------|-------------------|
| Spirals    | $3.550855 \times 10^{18} M_\odot$ |
| Ellipticals| $4 \times 10^{13} M_\odot$ |
| Irregular  | $3.48 \times 10^{14} M_\odot$ |
|           | $3.551243 \times 10^{18} M_\odot$ |

It may seem absurd to go as far as 7 significant figures in a preliminary approximation to the mass of spiral galaxies, this has been done in order to highlight the great preponderance of these galaxies within the present estimate.

To this sum, $3.5512 \times 10^{18} M_\odot$, we must add the intergalactic “dark matter”, scattered through the heart of the clouds of galaxies, and whose mass is thought to be of the same order of magnitude as that of the galaxies in the clouds (Universe, pág. 659). That would mean that this figure should be doubled, giving the preliminary result:

$$M_\varphi = 7.1025 \times 10^{18} \odot \left(1.99 \times 10^{33} g/\odot\right) \times 1.097 \times 10^{27} m_e/g = 1.55 \times 10^{79} m_e,$$

(1)

where $M_\varphi$ is the mass of the “visible Universe”, $\varphi$ is the angle $A\varphi B$ between the Earth, $\omega$ (origin of the Big-Bang) and $B$, which is the most distant observed object and shows a redshift $z = 5.34$ (Universe, p. 597); $z = \lambda - \lambda_0$, where $\lambda$ is the wavelength of a spectral line observed in a cosmic object, and $\lambda_0$ is the wavelength of the corresponding line within the light emitted by an object at rest on Earth.

The equation $v = \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1}$ allows us the infer that the said remotest cosmic object is moving away from us at $v = 0.95 c$.

The fact that there exists a “limit of visibility” which is moving away at the speed of light shows that the Universe is expanding. Returning to Fig. 1, and remembering that the length of the arc $AB$ is equal to $R \times \varphi$ ($\varphi$ expressed in radians), it is clear that the speed at which the length of that arc increases is $\frac{dAB}{dt} = R \frac{d\varphi}{dt} + \varphi \frac{dR}{dt} = 1 \cdot c$

Table I shows as functions of certain values of $\varphi$, the values of the speed of increase of $R$, $V_R$; the kinetic energy of an electron at rest at $A$, viewed from a reference system at rest at $\omega$ and whose axis $OX$ coincides with $\omega A$, and the relation $M_0/M_\varphi$ between the mass of the Universe and that of the observable Universe as estimated above.

This is not the place for deep discussion about the nature of space in our Universe; the simplest hypotheses are that is either a three-dimensional Euclidean space, or a three-dimensional spherical surface. Against the first of these hypotheses, it can be argued that if it were true, the lines of sight taken along $\omega A$ would imply distances to the “frontier of the Universe”, which are much shorter than the lines of sight taken at right angles with $\omega A$ (see Fig. 2). This has not been observed, and would be not the case if the second hypothesis were true. According to the second hypothesis the light would move along geodesics within the three-dimensional surface $x^2 + y^2 + z^2 + u^2 = R^2$, which are great circles of radius $R$, and since we are living inside it, we would not be in a situation where we could observe any 4th dimension, just as the imaginary “flatlanders” living on the two-dimensional surface $x^2 + y^2 + z^2 = R^2$ could not perceive the existence of a 3rd dimension.

It has been estimated (Universe, p. 677) that the value of the Hubble constant, $H_U$, lies between 60 and 90 km/s. per megaparsec, i.e., between 18.4 and 27.6 km/s. for every $10^6$ light years. For the lower of these values the distance to the horizon of visibility would be $1.629 \times 10^{10}$ light years, which would thus be the length of the arc $AB$ in Fig. 1. For a value of $H_U = 27.6$ km/s. per $10^6$ light years, the length of the same arc would be only $1.086 \times 10^{10}$ light years. Table II shows the lengths of $AB$ and $R$ for various values of $H_U$ and $\varphi$ (in radians).

Nowhere have we mentioned the possibility that $\varphi$
might be greater than $\pi$ radians. If that were so, we could, after viewing a cosmic object near to the horizon of visibility, turn out our telescope in the diametrically opposite direction and obtain a closer view, but in the other side of that cosmic object. We must study this possibility by analysis of the information available from cosmic cartography. Figure 3 illustrates this possibility, while Table III gives preliminary values for $v_2/v_1$ which correspond to images $(\pi - \varphi)$ of other such objects with $z_1 > 3.5$. We must take into account the fact that such images are much more recent, all the more recent as $\varphi$ becomes larger. It would be necessary to find many $z_2 < z_1$ correspondencies, in diametrically opposite views, in order to support the hypothesis that the horizon of visibility is at an angle of $\varphi$ greater than $\pi$ radians.

III. DYNAMICS OF THE LAST PARTICLE OF MASS $m_e$ TO EMERGE FROM THE BIG-BANG AT A DISTANCE $R_0$ FROM $\omega$

Using the ($e, m_e, c$) system, the mass of the Universe is given by $M_0 m_e$ and the numerical coefficient of the gravitational constant is $G_c = 2.4 \times 10^{-43}$. When the last particle of matter was formed as a result of the processes following the Big-Bang, the formation of pairs of greater mass had ended relatively long ago. The final addition to the mass derived from the said processes must have been a pair $e^-; e^+$. This electron of mass $m_e$, formed at a distance $R_0$ from the center of mass of the Universe,

![FIG. 2: Lines of sight to the frontiers of the Universe in an Euclidean space. If both $AC'$ and $AC$ are greater than the distance from $A$ to the “horizon of visibility” no difference would be detected.](image)

![FIG. 3: First line of sight $AAB = \pi + \varphi$. Second line of sight $AB = \pi - \varphi$.](image)

| $\varphi$ (radians) | $v_2/v_1$ | Values of $z_1$ and of $v_1/c$ |
|----------------------|-----------|-------------------------------|
| $\pi/2$              | 1/3       | 0.3933 0.3900 0.3860 0.3809 0.3744 0.3657 |

| $H_U$ | $\overline{AB}$ | $R$ | $\overline{AB}/\varphi$ | $\overline{AB}/\varphi$ |
|-------|------------------|-----|------------------------|------------------------|
|       |                  |     | $\varphi = 1.25$       | $\varphi = 1.50$       |
|       | $\overline{AB}$ |     | $\varphi = 2.00$       | $\varphi = 2.50$       |
| 60    | 1.62             | 1.30| 1.08                   | 0.81                   |
| 65    | 1.50             | 1.20| 1.00                   | 0.75                   |
| 70    | 1.39             | 1.11| 0.93                   | 0.70                   |
| 75    | 1.30             | 1.04| 0.87                   | 0.65                   |
| 80    | 1.22             | 0.98| 0.81                   | 0.61                   |
| 85    | 1.14             | 0.91| 0.76                   | 0.57                   |
| 90    | 1.08             | 0.86| 0.72                   | 0.54                   |
\( \omega \), and moving away from it at a speed very close to \( c \), must have had a kinetic energy in \((R_0, t_0)\) very close to \( \frac{1}{2} m_e c^2 \), being \( t_0 \) the time elapsed between the Big-Bang and the formation of that final pair. After a period \( t \) had elapse since \( t_0 \), its distance from \( \omega \) must have been \( R_0 + R_t \), its velocity \( v_t < c \), and its kinetic energy \( E_t = \frac{1}{2} m_e (v_t)^2 < \frac{1}{2} m_e c^2 \), since the particle in question was at all times subjected to the attraction of the mass of the Universe, \( M_0 m_e \), situated at a distance \( R_0 + R_t = R \) from it. We are trying to determine:

- \( E_t \) as a function of \( R_t \).
- The value of \( R_t \), when after a determined lapse \( t_{\Omega} \), both \( E_t \) and \( v_t \) would become null and the particle begins a journey towards \( \omega \).
- \( v_t \) as a function of \( R_t \).
- \( R_t \) as a function of \( t \).

A. Analysis of the evolution of the kinetic energy of the particle

The evolution of \( E_t \) is given by

\[
E_t = \frac{1}{2} m_e c^2 - \int_{R_0}^{R} \frac{(M_0 m_e) m_e dR}{(R e)^2} \times 2.4 \times 10^{-43} \frac{e^3}{m_e e^2},
\]

where \( l_e = e^2/(m_e c^2) \) and \( t_e = e^2/(m_e c^3) \) being the units of length and time in the \((e, m_e, c)\) system of units, and \( R = (R_0 + R_t) \). Obviously \( l_e / t_e = c \).

By multiplying out and taking \( K_0 = M_0 2.4 \times 10^{-43} \) we obtain

\[
\frac{E_t}{m_e c^2} = \frac{1}{2} - \int_{R_0}^{R_{0} + R_{t}} \frac{K_0 dR}{R^2},
\]

which integrates to

\[
\frac{E_t}{m_e c^2} = \frac{1}{2} - \left[ \frac{K_0}{R} \right]_{R_0}^{R_{0} + R_{t}} = \frac{1}{2} - \frac{K_0}{R_0} + \frac{K_0}{R_0 + R_t}. \tag{2}
\]

The condition for \( E_t \) being null is

\[
\frac{1}{2} - \frac{K_0 R_t}{R_0 (R_0 + R_t)} = 0.
\]

Remembering that the Schwarzschild radius \( R_S \) for the mass \( M_0 m_e \) is

\[
R_S = \frac{2(M_0 m_e) G e}{c^2} = \frac{2M_0 m_e}{c^2} \times 2.4 \times 10^{-43} l_e c^2 \frac{m_e}{m_e} = 4.8 \times 10^{-43} M_0 l_e,
\]

that is \( 2K_0 l_e \), the condition (2) can be also written as

\[
\frac{1}{2} - \frac{1}{2} \frac{R_S R_t}{R_0 (R_0 + R_t)} = 0;
\]

whence:

\[
R_t = \frac{R_S^2}{R_0 - R_0} \geq 0. \tag{3}
\]

- When \( R_0 = R_S \), \( E_t \) is null for \( R_t \to \infty \), and we have an unlimited expansion with a kinetic energy tending towards zero, and therefore a velocity also tending towards zero.

- When \( 0 < R_0 < R_S \), \( R_t \) increases towards a maximum value \( R_t = \frac{R_S^2}{R_0 - R_0} \), and when it reaches this value, it begins to decrease, which means that the particle returns towards \( \omega \).

- When \( R_0 > R_S \), \( E_t \) can never get null; \( \frac{K_0}{R_0} \) in (2) is always less than \( 1/2 \), and \( R_S < R_0 \). The particle under consideration continues to recede from \( \omega \), but its kinetic energy does not tend towards zero as \( t \) increases indefinitely. It tends towards \( \frac{1}{2} \left( 1 - \frac{R_S}{R_0} \right) m_e c^2 \), to which corresponds the velocity \( v = \left( 1 - \frac{R_S}{R_0} \right)^{1/2} c \).

B. Analysis of the evolution of the velocity of the particle

The evolution of the velocity of the particle, \( v_t \), as a function of time starts from \( R_0, t_0 \); where and when \( v_t \cong c \), i.e. \( 1 \cdot c \) in the system \((e, m_e, c)\). From then on, its velocity decreases, because the particle of mass \( m_e \) is subjected to the attraction of the mass \( M_0 m_e \), which is distant \( R_l = (R_0 + R_t)l_e \) from it, and which exerts on it the force

\[
f(t) = \frac{K_0}{(R_0 + R_t)^2 (t_e)^2} \frac{m_e l_e}{(2(R_0 + R_t)^2 (t_e)^2)}
\]

This force, applied to the particle of mass \( m_e \), determines that this particle, emerging at \((t_0, R_0)\), and moving away from \( \omega \) at a velocity very close to \( c \), suffers a deceleration of:

\[
-a(t) = \frac{-R_S}{2(R_0 + R_t)^2} \frac{l_e}{t_e^2},
\]

for every \( dt \). We can therefore write:

\[
\frac{v(t)}{dt} = c - \int_{t_0}^{t} \frac{R_S}{2(R_0 + R_t)^2} dt. \tag{4}
\]

From this we derive:

\[
\frac{d^2 R}{(dt)^2} = -\frac{R_S}{2(R_0 + R_t)^2}
\]
By next multiplying both these terms by $2dR$ we obtain:

$$2dR \cdot \frac{d^2 R}{dt^2} = \frac{d}{dt} \left( \frac{dR}{dt} \right)^2 = (-1) \cdot \frac{R_S \cdot 2dR}{2R^2},$$

because $R_0 + R_t = R$.

By integrating we have $\left( \frac{dR}{dt} \right)^2 = (-1) \cdot \frac{R_S}{R} + K_0$,

whence:

$$\frac{dR}{dt} = \left( 1 - \frac{R_S}{R_0} + \frac{R_S}{R_0 + R_t} \right)^{1/2}.$$  \hspace{1cm} (5)

The condition for $\frac{dR}{dt}$ being zero is:

$$1 - \frac{R_S}{R_0} + \frac{R_S}{R_0 + R_t} = 0,$$

whence

$$R_t = \frac{(R_0)^2}{R_S - R_0} = \frac{R_0}{R_0 - 1},$$

which is identical to (3).

The relation (5) can be written $\frac{dR}{dt} = \left( 1 - \frac{R_S}{R_0} + \frac{R_S}{R} \right)^{1/2}$. Therefore $1 - \frac{R_S}{R_0} + \frac{R_S}{R} \geq 0$

- For $R_S/R_0 \geq 1$; $R_S/R_0 = 1 + x$; $R_S/R \geq x$. If this condition is fulfilled, $R$ grows up to

$$R = R_0 + \frac{R_0}{R_S/R_0 - 1} = \frac{R_S}{R_0/R_0 - 1},$$

and thereafter decreases towards $\omega$. When by decreasing reaches $R_0$, $dR/dt$ will be equal to $-c$, which is a boundary that, out of black hole conditions, can not be trespassed.

- For $R_S/R_0 \leq 1$, the condition for the annulation of $dR/dt$ cannot be fulfilled because the condition (3) would lead to negative values of $R_t$. When $R_S/R_0 = 1 - x$, $x \to 0$, the value of $dR/dt \to 0$ for $t \to \infty$, but never can be equal to zero, as in the aforementioned case when $R_S/R_0 = 1 + x$, $x \to 0$. In all the other cases $R_S/R_0 < 1$ means that, when both $t$ and $R \to \infty$

$$\frac{dR}{dt} \to \left( 1 - \frac{R_S}{R_0} \right)^{1/2}.$$  \hspace{1cm} (6)

C. Analysis of the evolution of $R$

In the last analysis we obtained at (5)

$$\frac{dR}{dt} = \left( 1 - \frac{R_S}{R_0} + \frac{R_S}{R_0 + R_t} \right)^{1/2} = \left( 1 - \frac{R_S}{R_0} + \frac{R_S}{R} \right)^{1/2},$$

whence we obtain:

$$\frac{dR}{\left( 1 - \frac{R_S}{R_0} + \frac{R_S}{R} \right)^{1/2}} = dt. \hspace{1cm} (7)$$

Integral (171), on page 409 of the 18th edition of the ‘CRC Mathematical Tables’, states that

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = \frac{x}{2} (x^2 + a^2)^{1/2} + \frac{a^2}{2} \log[x + (x^2 + a^2)^{1/2}].$$

If we make $R = x^2$, $\frac{R_S}{(1 - R_S/R_0)} = a^2$, equation (7) becomes

$$\frac{2}{(1 - R_S/R_0)^{1/2}} \cdot \frac{x^2 dx}{(x^2 + a^2)^{1/2}} = dt$$

If we apply solution (8) to the first term of this equation we obtain

$$\frac{1}{(1 - R_S/R_0)^{1/2}} \left\{ R^{1/2} \left( R + \frac{R_S}{1 - R_S/R_0} \right)^{1/2} - \frac{R_S}{1 - R_S/R_0} \log \left[ R^{1/2} + \left( R + \frac{R_S}{1 - R_S/R_0} \right)^{1/2} \right] \right\}$$

$$= t + K_2.$$  \hspace{1cm} (8)

For $t = 0$; $R_t = 0$ and $\frac{dR}{dt}$ must equal 1, which is in fact what happens in (5). From the last equation we obtain

$$K_2 = \frac{R_0}{1 - \frac{R_S}{R_0}} - \frac{R_S}{1 - \frac{R_S}{R_0}} \log \left[ R_0^{1/2} + \frac{1}{(1 - \frac{R_S}{R_0})^{1/2}} \right].$$

By introducing this value of $K_2$ into the same equation we obtain

$$\frac{1}{(1 - \frac{R_S}{R_0})^{1/2}} \left\{ R^{1/2} \left( R + \frac{R_S}{1 - \frac{R_S}{R_0}} \right)^{1/2} - \frac{R_S}{1 - \frac{R_S}{R_0}} \right\}$$

$$\times \log \left[ R_0^{1/2} + \left( R + \frac{R_S}{1 - \frac{R_S}{R_0}} + \frac{R_S}{1 - \frac{R_S}{R_0}} \right)^{1/2} \right]$$

$$= t + \frac{R_0}{1 - \frac{R_S}{R_0}}.$$  \hspace{1cm} (9)
By making $A = (1 - R_S/R_0)$ this equation may be written

$$\frac{dR}{dt} = \left(1 - \frac{R_S}{R_0} + \frac{R_S}{R}\right)^{1/2}$$

diminishes and when both variables tend to $\infty$, $dR/dt$ tends to $(1 - R_S/R_0)^{1/2}$. In this case (10) can be written

$$A^{1/2}(R - A^{1/2}t) = \frac{R}{A^{1/2}} \log \left(\frac{R}{R_0}\right)^{1/2} + \frac{2A^{1/2}}{A^{1/2} + 1} + R_0$$

Therefore we have a parabolic branch parallel to $R = A^{1/2}t$. Knowing this and the value of $dR/dt$ for the starting point $t = 0$, $R = R_0$, we can easily draw the curve for (10). Only the values for $t > 0$, $R > R_0$ are valid (see Fig. 4).

For $R_S/R_0 = 1 - x$; $x \to 0$, $A \to x$ and (10) can be written

$$R \left(\frac{x + R_S}{R}\right)^{1/2} - \frac{R_S}{x^{1/2}} \times \log \frac{x^{1/2}(R/R_0)^{1/2}}{x^{2} + 1}$$

When $x \to 0$

$$\lim_{x \to 0} \frac{R_S}{x^{1/2}} \log \frac{x^{1/2}(R/R_0)^{1/2}}{x^{2} + 1} = \lim_{x \to 0} R_S \cdot \log(1 - x)^{1/2} = 0.$$

Therefore, for $x \to 0$ we have $R^{1/2} = \frac{x t}{R_S^{1/2}} + R_S^{1/2}$.

For each value of $x$ we have a function $R(t, R_S)$. We know that $x$ is very little; it tends to zero, but it is not equal to zero, and for each value of $t$ we have a function like that of Fig. 4, with a value of $dR/dt$ given by $dR/dt = (x + R_S/R)^{1/2}$, which tends to zero when $t \to \infty$ and $R \to \infty$, in agreement with the analyses developed in 3.1 and 3.2.

In line with (10), it turns out that the value of $dR/dt$, which starts from being 1, diminishes when both $t$ and $R$ grow greater, and tends to $dR/dt = (1 - R_S/R_0)^{1/2}$, when these variables tend to $\infty$. Therefore the value of

$$R = \int_{R_0}^{R} \frac{dR}{dt} \cdot dt,$$

must be of the same order of magnitude as $t$, and when $t \to \infty$, $R \to \infty$ but $R < t$.

* * *

We must now consider the measurement $R_0$ of the radius of the Universe at the moment of completion of the formation of matter. The Big-Bang might either have happened exactly at the point $\omega$, or have burst out as a "sphere of light" of radius $R_e$, without making any difference to the eventual evolution of the Universe. What is really important is that the speed of formation of the matter existing after the elapse of $t$ since the happening of the Big-Bang is such that $R_e = R_x + c \cdot t > 2M_t \cdot G/c^2$, which in the $(c, m_e, \epsilon)$ system can be written as $R_x + c \cdot t > 4.8 \times 10^{-43} M_t \cdot \epsilon$, where $M_t$ is the mass of the Universe (a number of $m_e$), after the elapse of $t$
(a number of \(t_e\)), and where \(R_x + t\) is the radius of the Universe at that time, a number of \(t_e\).

In accordance with the first of these alternatives, \(R_x = 0\), and the mass of the Universe after \(t\) has elapsed since the start of time must fulfill the condition

\[ M_t < \frac{m_e}{4.8 \times 10^{43}} \frac{ct}{l_e} \]

For \(t = 1\) s, \(1.063871 \times 10^{23} l_e\), the radius of the Universe would measure \(R_1 = 1.063871 \times 10^{23} l_e\), for which value the “mass of Schwarzschild” would be \(2.216398 \times 10^{65} m_e\). A greater speed of transformation into matter of photons of very high energy, would result in the formation of a black hole. Therefore, the process of formation of matter, up to a total estimated at not less than \(1.55 \times 10^{79} m_e\) (see (1)), must have lasted for at least

\[ \frac{1.55 \times 10^{79} m_e}{2.216398 \times 10^{65} m_e/s} = 6.993 \times 10^{13} \text{s} \]

\[ = 2.216 \times 10^6 \text{ years.} \]

This value is a very strick bottom limit, since we must suppose that the speed of matter formation decreases as the density of energy decreases, together with the increase in the radius of the Universe and the disappearance of the photons of very high energy, which had already been consumed during the previous formation of matter. This limit obviously corresponds to the Schwarzschild radius for \(1.55 \times 10^{79} m_e\).

As for the second alternative, in which the Big-Bang starts with the sudden outburst of a “sphere of light” of radius \(R_x\), the radius of the Universe at the moment of completion of matter formation, would still be greater than \(2.216 \times 10^6\) light years, but the time-lapse needed to pass this limit would be

\[ \frac{1}{c} (2.216 \times 10^6 \text{ light years} - R_x \text{ light years}) \]

\[ = (2.216 \times 10^6 - R_x) \text{ years,} \]

and the speed of matter formation could have been much greater than \(2.216 \times 10^{65} m_e/s\).

According to Universe (p. 736):

“Thus, as soon as the temperature of the radiation field falls below \(1.09 \times 10^{13} K^0\), photon-antiproton pairs can no longer be created. Each kind of particle has its own threshold temperature. For example, electrons and positrons have masses 2000 times smaller than protons and antiprotons. Thus the threshold temperature for the creation of electron-positron pairs is about \(1/2000\) that for proton-antiproton pairs. Consequently, when the radiation temperature falls below \(5.93 \times 10^9 K^0\), the reaction \(\gamma + \gamma = e^+ + e^-\) that creates electron-positron pairs can no longer take place”.

From what immediately follows this lines from Universe, it can be inferred that the current hypotheses on the evolution of the Universe includes the supposition that the formation of matter was completed before 2 seconds has passed since the start of time, which implies that the Big-Bang happened as a sudden outburst of a “sphere of light”, of radius greater that \(2.216 \times 10^6\) light years.

This reasonings that have been developed for \(R_x = 2.216 \times 10^6\) light years, which correspond to \(\varphi = \pi\) radians, \(M_0 = M_\varphi\), would be equally repeated for the any value of \(M_0 = \frac{2}{1 - \cos \varphi};\ 1 < \varphi < \pi\). On Table I we can see that \(M_0\) may variate between \(1.00M_\varphi\) and \(4.351M_\varphi\).

* * *

Let \(R_0\) be the radius of the Universe at the completion of formation of matter in \(t = t_0\), and \(R_1\) and \(R_2\) the distances to \(\omega\) of 2 particles which were formed during \(t < t_0\). If the order of magnitude of \(t_0\) is \(2s. = 2.127742 \times 10^{23} l_e\) and that of \(R_0\) is greater than \(7.439663 \times 10^{46} l_e\), the values of \(R_1\) and \(R_2\) would be expressed for the whole radius \(\omega R_0\) in very huge numbers, while the time elapsed between \(t = 0\) and \(t = t_0\) will also produce huge figures for the practical whole of that time.

![FIG. 5: Positions of \((m_e)\), and \((m_e)2\) at \(t = t_0\), \(R = R_0\).](image-url)

The particle \((m_e)\) formed at distance \(R_1\) from \(\omega\) in \(t = t_1\), is only affected by the attraction of those particles whose distance from \(\omega\) is less that \(R_1\). Also, the density of the matter formed between \(t = 0\) and \(t = t_1\) must be uniform over the whole Universe if, as seems logical, we suppose that the “sphere of light” which arose in the Big-Bang had an equal density of energy in all its parts. If so, the mass which would attract \((m_e)\) from \(\omega\) would be equal to \(\frac{4}{3} \pi (R_1)^3 \delta\), and the mass which would attract \((m_e)2\) would be \(\frac{4}{3} \pi (R_2)^3 \delta\). Thus, there corresponds to
position $R_x$ the Schwarzschild radius:

$$R_{S_x} \cdot l_e = \frac{8\pi}{3} \delta(R_x)^3 2.4 \times 10^{-43} l_e = 6.4 \cdot 10^{-43} \pi \delta(R_x)^3 l_e$$

By writing $R_x = x R_0$, $x < 1$, we obtain

$$R_{S_x} = 6.4 \cdot 10^{-43} \pi \delta(R_0)^3 x^3$$

For $R_0$ we have $R_{S_0} = 6.4 \cdot 10^{-43} \pi \delta(R_0)^3$. Therefore $R_{S_x} = R_{S_0} \cdot x^3$, whence:

$$\frac{R_x}{R_{S_x}} = \frac{x R_0}{x^3 R_{S_0}} = \frac{1}{x^2} \frac{R_0}{R_{S_0}}.$$ 

This implies that if $\frac{R_0}{R_{S_0}} > 1; \frac{R_x}{R_{S_x}} \gg 1$. In the contrary case, even if the Universe had not begun as a black hole, there would always have been a black hole around $\omega$.

Assuming that particle $(m_\omega)_1$ will not “overtake” others which were initially located further from $\omega$, its velocity for very large values of $R$ compared with $R_1$, though not necessarily when compared with $R_0$, will be given by:

$$\frac{dR}{dt}_1 = \left(1 - \frac{R_{S_1}}{R_1}\right)^{1/2} = \left[1 - (x_1)^2 \frac{R_{S_0}}{R_0}\right]^{1/2}.$$ 

For particle $(m_\omega)_2$ we would have:

$$\frac{dR}{dt}_2 = \left(1 - \frac{R_{S_2}}{R_2}\right)^{1/2} = \left[1 - (x_2)^2 \frac{R_{S_0}}{R_0}\right]^{1/2}.$$ 

Since $x_1 < x_2$, $(dR/dt)_1 > (dR/dt)_2$. This means that the velocity of the particle which was formed more closed to $\omega$ would always be greater than that of the other one. Notwithstanding this particle would never collide with the other one formed at the same time $t$, but on $x_2 R_0 > x_1 R_0$. Obviously its speed when it reaches $x_2$ will be less than $1 \cdot c$, whilst the velocity of a particle $(m_\omega)_2$ formed at $x_2$ just before $(m_\omega)_1$ reaches $x_2$ would be $1 \cdot c$. Therefore $(m_\omega)_1$ never could collide with $(m_\omega)_2$. On the other hand, $(m_\omega)_2$ never could collide with $(m_\omega)_1$ formed also at $x_2$ but before $t'$, and in consequence $(m_\omega)_1$ never could collide with $(m_\omega)_2$. This is important because “no overtaking” means the elimination of the turbulence which would have complicated the dynamics of the particles emerging from the Big-Bang, and which would certainly have caused a far from homogeneous distribution.

IV. ON THE INCREASING IN THE RADIUS $R$ OF THE UNIVERSE AND OF THE ARC $AB$ BETWEEN THE EARTH AND THE LIMITS OF THE OBSERVABLE SPACE

Returning to Fig. 1, we find

$$\overline{AB} = R \cdot \phi \quad (\phi \text{ in radians})$$

$$\frac{d\overline{AB}}{dt} = c = \frac{dR}{dt} + R \frac{d\phi}{dt} \quad (11)$$

$$\frac{d^2 \overline{AB}}{(dt)^2} = 0 = \frac{d^2 R}{(dt)^2} + R \frac{d^2 \phi}{(dt)^2} + 2 \frac{dR}{dt} \frac{d\phi}{dt} \quad (12)$$

We also know that the speed of increase of $R$, which at first was very close to $1c$, has been slowing down because of the gravitational pull of the mass of the Universe $M_\otimes$, with its centre of mass in $\omega$ (supposing an expansion in no preferred direction, i.e. as spherical surfaces with centre at $\omega$, starting at the Big-Bang). This means that $\frac{d^2 R}{(dt)^2} \leq 0$ and that $\frac{d\phi}{dt} > 0$, so that an increase in $\phi$ compensates for the decrease of $\frac{dR}{dt}$ in (11) and keeps $\frac{d\overline{AB}}{dt} = 1 \cdot c$.

Equation (11) implies that $R \phi = t$ (13). By substituting $t/\phi$ for $R$ in (10) we obtain

$$\phi^2 (At + \{ \}) - t^2 A - t \phi R_S = 0; \quad (14)$$

where

$$\{ \} = R_0 + \frac{R_0}{A^{1/2}} \log \left(\frac{t}{R_0 \phi} \right) - \frac{1}{2} A^{1/2} + 1 + \frac{(R_S - R_0)}{R_0^2}.$$ 

In this curve, for $t = 0$, $\phi = 0$, and its origin is a double point with tangents defined by $\phi^2 \{ \}^2 - t^2 A - t \phi R_S$, one has positive gradient, $\frac{\phi}{t} = \frac{1}{R_0}$; the other negative, $\frac{\phi}{t} = \frac{(R_S - R_0)}{R_0^2}$.

The curve has a double asymptote parallel to the axis $t = 0$, given by $(At + \{ \})^2 = 0$, and two asymptotes parallel to the axis $\phi = 0$ given by $\phi^2 A^2 - A = 0; \phi = +A^{-1/2}; \phi = -A^{-1/2}$.

FIG. 6: Path of curve $\phi^2 (At + \{ \})^2 - \phi t R_S - t^2 A = 0$.

The valid part of this representation (the continuous line which is drawn on the region $t \geq 0; \phi \geq 0$) shows that for $t = 0$, $\phi = 0$, (we must remember that $t = 0$, $R = R_0$), and that $\phi$ increases very slowly until after
a very long time, it becomes indistinguishable from the asymptote \( \varphi = \left(1 - \frac{R_S}{R_0}\right)^{-1/2} \).

The equations (12) and (16) would help the studies over the dynamics of the elementary particles which have not suffered any interaction after the Big-Bang up to the present time. With their help we can be sure that right now \( \frac{d\varphi}{dt} \equiv 0; \frac{dR}{dt} \equiv \left(1 - \frac{R_S}{R_0}\right)^{1/2} \) and \( \varphi \cong \left(1 - \frac{R_S}{R_0}\right)^{-1/2} \).

V. RECAPITULATION OF DEFINITIONS AND RELATIONS AND EXPLOITATION OF THE MODEL

A. Definitions and relations

- \( M_\varphi \) = Mass of the Observable Universe.
- \( M_0 \) = Mass of the Universe. For \( \varphi \leq \pi \), \( M_0 = \frac{2M_\varphi}{1 - \cos \varphi} \), for \( \pi < \varphi \leq 2\pi \), \( M_0 = \frac{M_\varphi}{2 + \cos \varphi} \).
- \( R_S \) = Schwarzschild radius for \( M_0 = \frac{2M_0G}{c^2} \).
- \( R_0 \) = Radius of the Universe, the moment at which the formation of elementary particles has concluded, assumed to be after 2s. have passed since the Big-Bang.
- \( t_U \) = Age of the Universe. For \( H_U = 90 \text{ Km/Mpsec-s.} \), \( t_U = 1.086 \times 10^{10} \text{ years} \); for \( H_U = 60 \text{ Km/Mpsec-s.} \), \( t_U = 1.630 \times 10^{10} \text{ years} \).

\( \overrightarrow{AB} \): The definition of \( \overrightarrow{AB} \) as the distance to the limit of the Observable Universe implies that the length of \( \overrightarrow{AB} \) in light years is equal to the age of the Universe in years, since in Fig. 6 the valid alternative for the representation of \( \varphi \), starts from \( t = 0, \varphi = 0 \).

- \( \overrightarrow{AB} = R \cdot \varphi \).
- \( \frac{d\overrightarrow{AB}}{dt} = 1 = \varphi \frac{dR}{dt} + R \cdot \frac{d\varphi}{dt} \), whence \( R\varphi = t \).
- \( \frac{dR}{dt} = \left(1 - \frac{R_S}{R_0} + \frac{R_S}{R}\right)^{1/2} \); see (7) in p. 10; \( \frac{dR}{dt} \) for \( t = t_U \) is the present speed of increase of \( R \), which when \( R \to \infty \) tends to \( \left(1 - \frac{R_S}{R_0}\right)^{1/2} \), the direction of the parabolic branch in Fig. 4.
- The present value of \( \varphi \). Fig. 6 shows that \( \varphi \) began as zero, at \( t = 0 \), \( R = R_0 \), and has evolved by asymptotic approach to \( \varphi = \left(1 - \frac{R_S}{R_0}\right)^{-1/2} \), which must be approximately equal to its present value, which is given exactly by \( \varphi = \frac{\overrightarrow{AB}}{R} = \frac{t_U}{R} \).

- The value of \( \frac{d\varphi}{dt} \), at present very close to zero, is obtained by substituting, in \( \varphi \frac{dR}{dt} + R \frac{d\varphi}{dt} = 1 \), the value of \( \frac{dR}{dt} = \left(1 - \frac{R_S}{R_0} + \frac{R_S}{R}\right)^{1/2} \) and is:

\[
\frac{d\varphi}{dt} = \frac{1}{R} \left\{ 1 - \frac{t_U}{R} \left(1 - \frac{R_S}{R_0} + \frac{R_S}{R}\right)^{1/2} \right\}
\]

B. Exploitation of the model

The evaluation of the length of \( \overrightarrow{AB} \) derives from: A) Measurements of the redshifts of the furthest cosmic objects. B) Evaluations of \( H_U \), through the observation of redshifts and other characteristics of many cosmic objects. These measurements and evaluations lead to values of \( H_U \) between 60 and 90 km/Mpsec-s. Finally, the value of \( M_\varphi \) has been obtained from a preliminary evaluation which must be improved.

The relation \( t_U = \frac{\overrightarrow{AB}}{H_U} \) introduces into the evaluation of the age of the Universe the uncertainty implicit in the evaluation of \( H_U \), which means that \( 1.096 \times 10^{10} \text{ years} < t_U < 1.626 \times 10^{10} \text{ years} \).

It is reasonable to suppose that \( 1 < \varphi < \pi \) radians, and for each value of \( \varphi \) we have a value of \( R \) given by \( R = t_U/\varphi \), because the measure of \( \overrightarrow{AB} \) in light years is the same as the measure of \( t_U \) in years. After recalling all these facts, we can use equations (12) and (16) to obtain sets of coherent values of \( R, R_S, R_0, dR/dt, \varphi \) and \( d\varphi/dt \) for different values of both these unrelated magnitudes \( H_U \) and \( \varphi \), whose fields of variation we have just settled within relatively strict limits. For each pair of values of \( H_U \) and \( R_0/R_S \), there is only one value of \( R_U \) that satisfies (12), but the values of \( \varphi; R_S; R_0 \) and \( dR/dt \) depend only on \( R_0/R_S \) and on \( M_0 \). Present conditions depend on what was the value of \( R_0/R_S \) when the Universe was born. We must state here that \( R_U \) is the radius which sets the boundaries for the Universe of matter, subject to the attraction of \( M_0 \) placed in \( \omega \). Beyond \( R_0 \), we must consider \( R_L = t_U \cdot c \), whose measurement in light years is the same as \( t_U \) in years and which corresponds to the distance travelled, since the Big-Bang by the light not transformed in matter.

As it can be seen in Tables IV and V, the very low values of \( R_0/R_S \) always correspond to \( \varphi \approx \pi \), and the high values of this quotient correspond to \( \varphi \) tending to 1 radian. No value is given for \( \varphi > \pi \) radians. The value \( R_0/R_S = 1.00 \) correspond to \( \varphi \to \infty \), which is inadmissible.

Equation (11), therefore, implies the rejection of
its solution for \( \frac{R_0}{R_S} = 1 \), and the non-existence of any solutions for \( \frac{R_0}{R_S} < 1 \), because its term \( \frac{R_S}{A^{1/2}} \cdot \left( \frac{R}{R_0} \right)^{1/2} A^{1/2} + (A + R_S/R) \) would include square roots of negative numbers when \( \frac{R_S}{R_0} > 1 \), because \( A = \left( 1 - \frac{R_S}{R_0} \right) \).

However if ignoring (10), we consider only the evolution of the kinetic energy given by (2), and that of \( \frac{dR}{dt} \) given by (5) we could draw a curve which crosses the axis \( t = 0 \) at \( R = R_0 \), being there tangent to \( \frac{dR}{dt} = 1 \). When \( t \) grows, \( \frac{dR}{dt} \) diminishes down to zero at \( R_t = \frac{R_0}{R_0} \frac{R_S}{R_0} - 1 \), and from here the particle returns towards \( \omega \); when \( \frac{dR}{dt} \) is negative, and reaches its theoretical maximum negative value at \( R = R_0 \), \( t = \frac{2R_0}{(R_S/R_0) - 1} \) (which is positive being \( R_S > R_0 \)).

We must say here that, if \( R = R_0 \),

\[
\log \left( \frac{R}{R_0} \right)^{1/2} A^{1/2} + \left( A + \frac{R_S}{R} \right)^{1/2} A^{1/2} + 1 = \log 1 = 0
\]

Figure 7 shows the hypothetical path of \( R \) as a function of \( t \), when \( \frac{R_S}{R_0} > 1 \)

The point \((0, R_0)\) where \( \frac{dR}{dt} = 1 \) is always a point in the curve (10), because the value of

\[
\log \left( \frac{R}{R_0} \right)^{1/2} A^{1/2} + \left( A + \frac{R_S}{R} \right)^{1/2} A^{1/2} + 1
\]

is zero when \( R = R_0 \). The other point where \( R = R_0 \) corresponds for \( t = \frac{2R_0}{(R_S/R_0) - 1} \) and is also situated on curve (10). No other values of \( t \) give real values of \( R \) when introduced into (10), but we know that the maximum \( R_{\text{max}} = \frac{R_S}{(R_S/R_0) - 1} \) corresponds to \( t = \frac{R_0}{(R_S/R_0) - 1} \) which agrees with (3) and that \( \frac{dR}{dt} = -1 \) for \( t = \frac{2R_0}{(R_S/R_0) - 1} \), \( R = R_0 \).

Knowing these three points and the values of \( \frac{dR}{dt} \) in each one of them we can easily draw the path of \( R \) which can be seen on Fig. 7. The only problem is, that after reaching point \( P_3 \) our particle \((m_e)\) would go on towards \( \omega \) at velocities greater than \( c \). But \((m_e)\) would be then subject to black hole conditions, and its behaviour might be different.

The representation of (10) is that of \( R \) as function of \( t \), when \( 1 - \frac{R_S}{R_0} > 0 \). It is not valid for \( 1 - \frac{R_S}{R_0} < 0 \); only the points \( t = 0 \), \( R = R_0 \), \( t = \frac{2R_0}{(R_S/R_0) - 1} \), \( R = R_0 \), belong to the representation of (10) when \( 1 - \frac{R_S}{R_0} < 0 \). For any other value of \( t \), the equation (10) gives imaginary values of \( R \). But the fact that we can calculate easily the coordinates and the values of \( \frac{dR}{dt} \) for points \( P_1 \), \( P_2 \) and \( P_3 \) (see Fig. 7) leads us to believe that there would be a real representation of \( R \) as function of \( t \) when \( 1 - \frac{R_S}{R_0} < 0 \), as shown in Fig. 7. It would be convenient to know something about the possible implications of this, and it is possible to get some useful knowledge without knowing any equation relating \( R \) to \( t \) when \( 1 - \frac{R_S}{R_0} < 0 \).

The present value of \( R_U \), given by \( R_U = t_U/\varphi \), can be seen in Table IV to be more than 1000 times the radius of Schwarzschild which corresponds to \( \varphi = 1 \) radian in

### Table IV: Values of \( \varphi \), \( R_0 \), \( R_S \) and \( dR/dt \) for different values of \( R_0/R_S \).

| \( R_0/R_S \) | \( \varphi \) radians | \( R_S \) \(10^6 \) l.y. | \( R_0 \) \(10^6 \) l.y. | \( dR/dt \) c |
|----------------|-----------------|-----------------|-----------------|----------|
| 1.112745       | \( \pi \)       | 2.216           | 2.465842        | 0.3190   |
| 1.20           | 2.449           | 2.504           | 3.004906        | 0.4087   |
| 1.35           | 1.964           | 3.204           | 4.325892        | 0.5096   |
| 1.50           | 1.732           | 3.819           | 5.728286        | 0.5777   |
| 1.75           | 1.528           | 4.632           | 8.106675        | 0.6550   |
| 2.00           | 1.414           | 5.251           | 10.501669       | 0.7074   |
| 2.00           | 1.225           | 6.707           | 20.120639       | 0.8168   |
| 5.00           | 1.118           | 7.878           | 39.392139       | 0.8947   |
| 10.00          | 1.054           | 8.759           | 87.591861       | 0.9490   |
| 20.00          | 1.026           | 9.200           | 184.000595      | 0.9750   |
Table I, i.e. $M_0 = 4.351 M_\odot$. Moreover, this value is still increasing. On the other hand, it must be smaller than

$$R_{\text{max}} = \frac{R_S}{(R_S/R_0) - 1};$$

we are still seeing the very remote cosmic objects with a redshift and not with a blueshift.

The number $\left(1 - \frac{R_S}{R_0} + \frac{R_S}{R}\right)$ must be positive, to obtain real values for $\frac{dR}{dt} = \left(1 - \frac{R_S}{R_0} + \frac{R_S}{R}\right)^{1/2}$. We have

$$\left(1 - \frac{R_S}{R_0}\right) < 0; \quad \frac{R_S}{R} < \frac{R_S}{R_0};$$

because if $R < R_0$, we would be beyond point $P_3$, and the furthest cosmic objects would show us a blueshift instead of a redshift.

There is only one possibility left; $1 - \frac{R_S}{R_0} = -x$, being $x$ very small and $\frac{R_S}{R} > x$. Otherwise the values of $R_S$ would be greater than those of $R$, leading to values of $M_0$ absurdly greater than those which have been calculated in Table II. For instance, if $x = 1, M_0$ must be greater than $4.351 \times 10^3 M_\odot$, which seems to make no sense.

If $x \leq 10^{-3}$ we could have values of $R_S/R$ which make $1 - \frac{R_S}{R_0} + \frac{R_S}{R} > 0$, allowing reasonable estimates for $R_S$, $M_0$ and $R_0$.

The range of variation of $x$ leads to a very short range of variation of $R_S$. In effect $1 < \frac{R_S}{R_0} < 1.001$ is so short that it would mean that the birth of the Universe was exactly determined and would exclude any hypothesis based on chance.

**References**

[1] W. J. Kaufmann and R. A. Freedman (2000): *Universe* (Freeman, New York).

[2] P. Appell (1942) *Precis de Mecanique Rationnelle* (Gauthier Villars, Paris).

[3] E. R. Harrison (1986) *Cosmology* (Cambridge University Press, Cambridge).

[4] Ch. W. Misner and K. S. Thorne (1973): *Gravitation* (Freedman, New York)