Analysis of problem-solving skills in material probability in Kanisius Pakem vocational high school

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Abstract. The purpose of this study was to describe the learning process by using the Problem Based Learning model on the topic of probability for Kanisius Pakem Vocational High School students, as well as to describe the problem-solving skills of Kanisius Pakem Vocational High School students after the implementation of learning trajectories on the topic of probability. The type of research used is design research. Data collection techniques are used, namely observation of the learning process. Then the data is analyzed by reducing data, presenting data, and concluding the results of the study. The learning trajectory was tested on 24 students of class X Kanisius Pakem Vocational High School. The research instruments used were field notes and student test sheets. The learning process consists of five-stage, namely the orientation of the students to problem regarding probability, organizing students to learn related to the material probability, guiding individual/group experiences on the learning process material probability, developing and presenting the work obtained from group discussions, and conclude the results of the learning process that has been done. From the results of tests obtained by students after the learning process, four aspects of problem-solving ability will be reviewed. Viewed from the aspects of understanding the problem, the students can describe things that are known and asked in the tests given. Viewed from the aspects of planning problem-solving, students can develop plans based on the information provided. Viewed from the aspect of implementing the plan, the students can translate the problem given in the form of mathematical sentences and students can make decisions in answering the problem. Viewed from the aspect of interpreting the results of the answers, the students have not all been able to verify the answers given.

Keywords: Learning Process, Problem-Based Learning, Problem-Solving, Probability Concepts, Design Research

1. Introduction

Based on the interview along with observations on students, then the learning process obtained that students have several characteristics in following the learning process. The characteristics of these students include, among others students who seriously follow the learning process, some students are less serious in participating in the learning process so that it has an impact on their understanding of the concepts given in the learning process. Besides, students can be said to be difficult to understand a problem given by the teacher because students are not accustomed to learning by being given a problem.

To overcome this, it is important for students to carry out the learning process that uses problems as the starting point in learning. Besides, problem-solving becomes an important thing for the future because people need individuals who can solve problems systematically. Therefore, through Problem Based Learning (PBL), the learning process will always begin and center on the problems. Through PBL, students will work in their groups to identify what they know and what they don't know and they...
must solve a given problem. In Julie, Anderson state learning was not only a transferring knowledge activity but an activity that encouraged students to build or construct their knowledge. Meaningful learning presented the knowledge and cognitive processes that students need to solve problems [8].

Angkotasan found that the Problem Based Learning (PBL) model was effective in terms of mathematical reflective thinking skills and student's mathematical problem-solving abilities [1]. Therefore, in this study, the purpose of his research was to create a design learning based on Problem Based Learning on the topic of probability for vocational students and analyze the problem-solving abilities of vocational students on the topic of probability.

Problem Based Learning (PBL) is an innovation in the learning process because through PBL, the students thinking ability will be optimized through the process of working in groups so that students can empower, sharpen, test, and develop their thinking skills on an ongoing basis. Rusman state that in the problem-based learning process, there are several stages used, namely as follows [2]:

| Table 1. Syntax of Problem Based Learning |
|------------------------------------------|
| **Stage Performed** | **Educator Activity** |
| Stage 1: Orientation of students to the problem | Explain the purpose of learning, explain the logistics needed, and motivate students to be involved in problem-solving activities |
| Stage 2: Organize students to learn | Helping students to define and organize learning tasks related to the problem |
| Stage 3: Guiding individual/group experience | Encourage students to gather appropriate information, carry out experiments to get explanations and problem solving |
| Stage 4: Develop and present the work | Helping students in planning and preparing suitable works such as reports, and helping them with various tasks with their friends |
| Stage 5: Analyze and evaluate the problem-solving process | Helping students to reflect or evaluate their investigations and the processes they use |

Problem-solving is a process used to solve the problem. NCTM states that solving problems is not only a learning goal of mathematics but also as a primary tool for learning. Therefore, in all levels of education, from elementary school to tertiary education, problem-solving skills are the focus of learning [3]. Due to the importance of problem-solving aspects in learning mathematics, NCTM states that learning programs from pre-kindergarten to 12th grade should enable all students to be able to: (1) build new mathematical knowledge through problem solving, (2) solve problems that arise in mathematics and in other contexts, (3) implement and adopt various strategies that are suitable for solving problems, and (4) monitor and reflect on mathematical problem solving processes [3]. Various kinds of problems need to be given to students in stages. Every educator must understand that orientation in education is students. Therefore, students must be trained to solve various types of problems [4]. In this study, problem-solving will be carried out according to the stages of solving the Polya problem with indicators from NCTM. Polya suggested that four stages must be done in the problem-solving process, namely: (1) understanding the problem, (2) devising the plan, (3) implementing it according to plan (carrying out the plan), (4) checking back (looking back) [5]. The following are the steps in solving the Polya problem with indicator from NCTM [6]:

| Table 2. Stages of Polya Problem Solving with NCTM Indicators |
|---------------------------------------------------------------|
| **Polya Problem Solving Stage** | **NCTM Indicator** |
| Understanding the problem | 1. Writing down things or information that is known |
2. Method
The research method used in this research is design research. Prahmana states that design research is an appropriate research method to develop a solution to complex problems in educational practice and to develop or to validate a theory about the learning process, as well as a learning environment [7]. Prahmana suggests there are several stages in design research. The stage is the Preliminary Design, Design Experiment, and Retrospective Analysis [7]. At the preliminary design stage, the researcher makes an HLT that will be used in the learning process. To design this HLT, the researcher made observations to the school to see the problems experienced by students when studying material probability. Also, the researcher obtained data based on interviews with teachers. It aims to design the HLT prototype. Next is conducting a limited trial for HLT that has been designed to see the response of students to the learning process that has been designed.

At the design experiment stage, what is done is to test the HLT has been previously designed. Based on the HLT has been made, the learning process will be conducted in 2 meetings. The first meeting is to calculate the probability of a single event and at the second meeting related to stating compound events. At the retrospective analysis stage, what is done is reviewing the results obtained when testing the HLT at the previous stage. At this stage, the aim is to answer the problem formulation that has been designed. In Julie, Gravemeijer and Cobb state there were three phases in the design research, namely: (1) preparation of trial design, (2) trial design, and (3) a retrospective analysis [9]. Data collection techniques are used, namely observation on the learning process, giving tests to students, and documenting the learning process. Then the data is analyzed by reducing data, presenting data, and concluding the results of the study. The learning trajectory was tested on 24 students of class X Kanisius Pakem Vocational High School. The study was conducted in March 2019-April 2019.

3. Results and Discussion

3.1. Preliminary Design Phase
In this stage, the researcher designs learning that will be used to teach material probability using the Problem Based Learning (PBL) model. The development of the Hypothetical Learning Trajectory (HLT) in each learning activity is the most important part of designing the learning activities of students. Learning design is a concept map that will be passed by students during the learning process. Learning activities and student thinking results are hypothesized in HLT. In HLT it is planned that learning will take place in two meetings, in which the first meeting is related to the topic of calculating the chance of a single event and at the second meeting related to starting a compound event used in the probability.

3.2. Design Experiment Phase

3.2.1. Stage of Orientation of Students To Problems
In meeting 1, the educator gives an attempt to throw a coin once. Then educators ask students about the sample room, sample point, and the number of members of the sample room in the experiment. The following are discussions between students and educators:

Educator: I have a coin, I will throw the coin once, then how about the sample room, sample point, and the number of members of the sample room in the experiment?
Student: \{Numbers, pictures\} for the sample space, numbers and pictures for the sample point. For the number of members of the sample space, there are 2, denoted by \( n(S) = 2 \).
Educator: Then how is it related to the chance of numbers appearing in an attempt to throw a coin as much as one time?
Student: In this case, "A" is the occurrence of numbers in the attempt to throw coins 1 time. To determine the probability, namely:
\[
P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}.
\]

In meeting 2, the educator asks the students about the learning process at the previous meeting. It is due to build on initial knowledge of the students before beginning the learning process. Also, educators give questions to students about intersection and union notations. Then one of the students writes an intersection and union notation.

The following is one of the students who provide illustrations of intersection and union:

![Figure 1. Student gives an illustration about intersection and union](image)

Then the educator gives an example, namely "Ani likes to read and write". Furthermore, the educator gives an example, namely "A as Ani likes to read and B as Ani likes to write". If the educator gives a statement "Ani likes to read and likes to write", then the educator asks the students to ask questions
about the union operation or the right intersection to the statement. Students provide answers, namely $A \cap B$.

3.2.2. Stage of Organizing Students To Learn
In meeting 1, the educator divides students into 6 groups, with each group consisting of 4 people. In each group, educators give problems. The problem given to students are as follows:

**Figure 2.** Problems are given

In this stage, discussions that occur between educators and students, namely as follows:

**Educator:** Here are some of the problems associated with the experiment that is related to probability. On this occasion, then please discuss with group members to resolve the issue. The following will also be given some media that can help to solve these problems. The media is in the form of dice and balls, please use it as well as possible.

**Students:** OK Sir. Should the sample room, sample point, and the number of sample room members also be written, sir?

**Educator:** If the information is needed in the process of solving problems, then please write.

**Students:** OK Sir.

In the worksheet, then on the number 1 student is asked to determine the chance of the occurrence of the roll of the dice of prime numbers in an attempt to throw a 6-by-side dice once.

In meeting 2, educators divide students into 6 groups with each group consisting of 4 people. In each of these groups, the educator presents a problem. The following problems are given to each group as follows:

**Figure 3.** Problems are given
In question number 2, the problem is that in a box there are 12 red balls and 15 blue balls. In the event of a random retrieval of one ball. Make sure of the blue ball.

3.2.3. Stage Guide Individual/Group Experience

In meeting 1, the educator acts as a facilitator that helps students to organize an understanding of the concept of probability based on the problems given. The following is a discussion process of students in the group:

![Figure 4. Group discussions](image)

To help organize the understanding of students, the educator instructs that the props in the form of dice and the balls were given to each group can be used as well as possible. In the process of guiding the learning experience, the following is a discussion that occurs between educators and students, namely as follows:

*Educator: In number 1, what information is contained in the problem?*
*Students: There is an attempt to throw a 6-sided dice 1 time, sir.*
*Educator: Then what information is known?*
*Students: The sides are 6, so there are 6 members in the sample room, sir.*
*Educator: Then at number 1, what must be solved for the problem?*
*Students: What must be completed is to determine the chance of occurrence of the appearance of prime numbers of dice Sir.*
*Educator: Then what are the prime numbers?*
*Learners: The prime numbers are 2, 3, 5, 7, 11, and so on Sir. Wait a minute sir, isn't that right Sir?*
*Educator: What do you think?*
*Students: It's correct, Sir.*
*Educator: Then in the dice, what are the prime numbers? Try to see the dice that has been given.*
*Students: 2, 3, 5 only Sir.*
*Educator: Is that enough?*
*Students: It seems like you have it, Sir.*
*Educator: Then please determine the probability for the emergence of prime number dice in the experiment.*
*Students: OK Sir, we will try it first.*
*Educator: Then in number 2, what information can be found?*
*Students: There are 12 red balls and 15 blue balls in a box Sir.*
*Educator: Then what should be looked for in the problem?*
*Students: Here to look for probability for blue balls to appear Sir?*
*Educator: What can be done?*
*Students: Looking for the total number of balls in advance Sir, to look for the number of members of the sample room Sir. Then just looking for the probability by stating in advance the occurrence of the blue ball as "A" Sir. Then the probability for the emergence of the blue ball in the experiment was only sought, Sir.*
*Educator: OK, Please search first.*
*Students: OK Sir.*
The following are the answers raised by students:

**Figure 5.** Answers are given by students

In meeting 2, educators guide each group to solve the problems given. At this stage, the educator also gives some balls related to the problem given. In this stage, the discussion that occurs between educators and students is as follows:

**Educator:** What information is obtained from the problem?

**Students:** From these problems, there are 2 types of books, namely textbooks and reading books.

**Educator:** Then from this, then what happens that might happen to students when they want to borrow the book.

**Students:** Possible events that occur are borrowing textbooks and reading books and events to borrow textbooks or reading, Sir.

**Educator:** Then what kind of conjunctions are appropriate for the events mentioned above?

**Students:** For events that have conjunctions "and", it can be used "∩" Sir. For events that have a conjunction "or", it can be used "∪" Sir.

The following are the answers given by the group in solving problems, namely as follows:
3.2.4. Stage of Developing and Presenting The Work

In meeting 1, the educators ask each group to write the answer on the answer sheet provided. Then educators ask the group that has different answers than the other groups. At this stage, the students stated that the chance for the occurrence of prime numbers appeared, namely \( \frac{3}{6} \). This is obtained by determining the number of members of the sample room first, namely \( n(S) = 6 \). Then declare "A" as the occurrence of the emergence of prime numbers. Then the students state that the dice are prime numbers, which is 2, 3, 5 so that \( n(A) = 3 \).

Then for problem number 2, to determine the chance of the emergence of a blue ball, what can be done is to determine the number of members of the sample space first, namely \( n(S) = 17 \). Then declare the occurrence of the blue ball, which is \( \frac{15}{27} \). The following is the presentation of the results of the discussion of students in the group:

![Figure 6. Answers are given by students](image)

In meeting 2, the educator asks several groups to present the work results of the group. At this stage, educators ask for participation from other groups to provide input to the group that is presenting. At this stage, the aim is for students to suggest that at number 1 2 events can occur, namely "the occurrence of Reni borrowing textbooks or reading books", and "the events of Reni borrowing textbooks or reading books". In number 2, then events that might be occur, ie "Ana takes a white ball or blue ball", "Ana takes a white ball and a blue ball", "Ana takes a white ball and a yellow ball", "Ana takes a white ball or a yellow ball, "Ana takes a blue ball and a yellow ball", and "Ana takes a blue ball or a yellow ball".

![Figure 7. Students present the results of group discussions](image)
3.2.5. Stage of Analyzing and Evaluating The Problem Solving Process

In meeting 1, the educator helps students to evaluate the learning process, such as what they understand about the material probability in single events and compound event material. From this stage, students suggest that to determine the chance of an event, if requires many members of an event and many members of the sample room at that event. Therefore, the chance of an event can be written as $P(A) = \frac{n(A)}{n(S)}$, where $n(A)$ is the number of members for event “A”, while $n(S)$ is the number of members in the sample room.

In meeting 2, the educator gives questions to the students related to the compound events that have been learned. In the process of learning about compound material, students suggest that multiple events can be connected using intersection notation ($\cap$) and can use union notation ($\cup$). This is based on the observations of students when doing the learning process associated with intersection and union.

3.3. Retrospective Analysis Phase

The following is a problem given to find out the problem-solving abilities of students related to the probability of single events and multiple events. In this problem, the students are asked to take steps in the problem-solving process that is done when solving the problem.

**Figure 8. Students present the results of group discussions**

**Figure 9. A Problem is given**

Based on the learning outcomes obtained by each student, then the following is a description of the problem-solving abilities of students, namely as follows:
Based on the answers from the students, the problem-solving abilities of the participants were as follows:

- For problem number 1a, students can state information that is known, and students can state what is asked in the problem. Then in the process of planning the solution of these problems, students have a plan by stating many yellow marbles, many red marbles, and many blue marbles in a diagram. Furthermore, students can solve the problem by stating that the chance for yellow marbles is taken $P(K) = \frac{n(K)}{n(S)} = \frac{8}{15}$. Furthermore, students do not make conclusions related to the answers given.

- For problem number 1b, overall students can state the information that is known, and students can state what is asked in the problem. Then in the process of planning the solution of the problem, the students stated many yellow marbles, many red marbles, and many blue marbles. Furthermore, students can state how to determine the probability of red marbles. However, students do not make conclusions related to the answers given.

- For problem number 2, students can state information that is known, and what is asked about the problem. In terms of planning the solution of problems, the students state each known information into mathematical notation related to the number of members of the sample room, and the number of members to the occurrence of the problem. Then in solving the problem, the students can state how to determine the chances of the letter A being chosen in Yogyakarta. However, students do not make conclusions about the answers.
For problem number 3, students can state information that is known and what is to be solved in the problem. In this number, the students make a plan by stating that A is a male visitor event, B is the incidence of female visitors, C is the occurrence of visitors less than 35 years old, and D is the occurrence of visitors over 35 years old. In solving problems, students state events that might occur in mathematical notation. However, students do not make conclusions about the answers previously stated.

4. Conclusion
Based on the results of research on class X Kanisius Pakem Vocational High School, the following conclusions are obtained:

- The learning process consists of five stages, namely the orientation stage of students to problems regarding probability, organizing learners to learn related to probability material, guiding individual/group experiences of learning process material probability, developing and presenting the work obtained from discussions group, and conclude the results of the learning process that has been done.
- Viewed from the aspect of understanding the problem, students can describe things that are known and asked in the test givens. Viewed from the aspect of planning problem solving, students can develop plans based on the information provided. Judging from the aspect of implementing the plan, the students can translate the problem given in the form of mathematical sentences and students can make decisions in answering the problem. Judging from the aspect of interpreting the results of the answers, the students have not all been able to verify the answers given.

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