SUPERSONIC TURBULENCE IN THE INTERSTELLAR MEDIUM: STELLAR EXTINCTION DETERMINATIONS AS PROBES OF THE STRUCTURE AND DYNAMICS OF DARK CLOUDS

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ABSTRACT

Lada et al. have described a method for studying the distribution of dust in dark clouds using infrared imaging surveys. In particular, they show that the method provides some information about the structure of the gas (dust) on scales smaller than their resolution.

In the present work we clarify the nature of the information provided by their method. We show that:

1. The three-dimensional density field of the gas is well described by a lognormal distribution down to very small scales.
2. The power spectrum and the standard deviation of the three-dimensional density field can be constrained.
3. Such a structure of the density field is likely to be the effect of random supersonic motions present in the gas.

In fact, we find a qualitative and quantitative agreement between the predictions based on recent numerical simulations of randomly forced supersonic flows by Nordlund & Padoan and by Padoan, Nordlund, & Jones and the constraints given by the infrared dust extinction measurements.

Subject heading: dust, extinction — ISM: kinematics and dynamics — turbulence

1. INTRODUCTION

In a recent paper, Lada et al. (1994) have illustrated the method of mapping the distribution of dust, and therefore gas, in dark clouds by using stellar extinction measurements in the near-infrared. The method is based on the use of multichannel array cameras, which allow the simultaneous determinations of the colors of hundreds to thousands of stars through a molecular cloud. The infrared color excess is proportional to the dust column density, and the dust-to-stars through a molecular cloud. The infrared color excess is determined by et al. and previously by Lada (1994), Dobashi et al. (1992). Millimeter-wave data for the same cloud have been kindly provided to us by the authors. The measurements are taken through the dark cloud IC 5146 in Cygnus, which is the equivalent of Figure 7 in et al. Figure 1, Lada (1994), shows the $A_V$ map and the CS map.

The second method involves plotting the mean extinction, $A_V$, and its standard deviation, $\sigma$, measured at any position. Lada et al. (1994) found that the dispersion grows with the average extinction, and realized that this behavior contains information about the structure of the extinction (and therefore of the gas mass distribution) in the cloud, on scales smaller than the resolution of the extinction map. They give examples of mass distributions that would generate or not generate such a plot, but their interpretation of the plot does not go very far.

In this work we focus on the second method of using the extinction data, that is, on the meaning of the $\sigma$-$A_V$ plot as a tracer of structure on scales below the resolution of the map.

In the first method of analysis the data are spatially binned, as in stellar-count and millimeter-wave observations. At any position, a few stars are found so that an average extinction $A_V$ can be measured. The result is an extinction map that compares well with the stellar-count map and the CS map.

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In §§ 3 and 4 we show that the \( \sigma-A_V \) plot is due to the “intermittent” distribution of the dust (that is, of the gas density field in the cloud), and we show how to constrain such a distribution using randomly generated fields with given statistics and power spectra. Before giving such details, though, we present in § 2 the results of recent numerical simulations concerning the density field in isothermal random supersonic flows. It will be clear, in § 4 and in the following discussion, that random supersonic flows are in fact excellent candidates for interpretation of the extinction data and explanation of the origin of the distribution of dust in dark clouds.

2. RANDOMLY FORCED SUPERSONIC FLOWS

Nordlund & Padoan (1996) and Padoan, Nordlund, & Jones (1996) have recently discussed the importance of supersonic flows in shaping the density distribution in the cold interstellar medium (ISM).

They have run numerical simulations of isothermal flows randomly forced to high Mach numbers. Their experiments are meant to represent a fraction of a giant molecular cloud: \( \approx 10 \) pc in size and \( 10^3-10^4 \, M_\odot \) in mass. These simulated random supersonic motions are, in fact, observed in molecular clouds.

It is found that most of the mass concentrates in a small fraction of the total volume of the simulation, with a very intermittent distribution. The statistic of the density field is well approximated by a lognormal distribution:

\[
P(\ln x) d\ln x = \frac{1}{(2\pi \sigma_{ln x}^2)^{1/2}} \exp \left[ -\frac{1}{2} \left( \frac{\ln x - \ln \overline{x}}{\sigma_{ln x}} \right)^2 \right] d\ln x ,
\]

where \( x \) is the relative number density,

\[
x = n/n ,
\]

and the standard deviation \( \sigma_{ln x} \) and the mean \( \overline{x} \) are functions of the rms Mach number of the flow, \( \mathcal{M} \):

\[
\overline{x} = -\frac{\sigma_{ln x}^2}{2}
\]

and

\[
\sigma_{ln x}^2 = \ln(1 + \beta^2, \mathcal{M}^2) ,
\]

or, for the linear density,

\[
\sigma_x = \beta \mathcal{M} ,
\]

where \( \beta \approx 0.5 \). Therefore, the standard deviation grows linearly with the rms Mach number of the flow.

It is also found that the power spectrum, \( P(k) \), of the density distribution is consistent with a power law:

\[
P(k) \sim k^{-2.6} ,
\]

where \( k \) is the wavenumber.

In the following sections, we will show that the extinction data are consistent with these theoretical predictions.

3. NUMERICAL GENERATION OF EXTINCTION DETERMINATIONS

In order to interpret the extinction data, we have generated random three-dimensional density distributions with given statistics and power spectra, projected them in two dimensions, and sampled them randomly, as happens when stars are found through the cloud. The stars are assumed to be uniformly distributed in space. Then a grid has been created on the distribution, and the mean extinction \( A_V \) and its dispersion \( \sigma \) have been measured in every bin using the position selected by the few “stars” found in the bin.

In Figure 2 we show the case of a Gaussian distribution, to be compared with the case of a lognormal distribution, shown in Figure 3. It is only in the case of the lognormal distribution that the plot \( \sigma-A_V \) is similar to the observational one (Fig. 1). Clearly, some sort of intermittent tail is needed in order to produce the growth in the dispersion with the growth in mean extinction.

Intermittence is a natural explanation of the plot, and it is also the main feature of the density distribution in random supersonic flows. Note that having only high-density clumps (e.g., steep power spectrum) is not enough to generate the plot, as already shown by Lada et al. (1994).

We have studied the sensitivity of the plot to different power spectra and standard deviations in the three-dimensional density distribution.

![Fig. 1.—Dispersion vs. the mean extinction for every bin in the regular grid, superposed to the observed region. A bin contains on average about five stars. The data are the original ones from Lada et al. (1994)](image1)

![Fig. 2.—Same plot as in Fig. 1, but obtained numerically starting from a three-dimensional random distribution with a Gaussian statistic. The Gaussian statistic is clearly unable to reproduce the observed growth of dispersion with mean extinction.)](image2)
Figure 3.—Same as in Fig. 2, but from a lognormal distribution: now the observational trend is reproduced.

Figure 4 shows a $\sigma-A_V$ plot from a random distribution with standard deviation larger than in the case of Figure 3: the slope of the plot increases together with the standard deviation of the density field. This behavior is easily understood. In fact, the plot is related to the structure of the density field on a scale below the resolution of the extinction map: if there were no structure on such small scale, $\sigma$ would be close to zero. The larger the fluctuations on small scale, the larger $\sigma$ becomes.

4. STATISTICS AND POWER SPECTRUM OF THE ISM DENSITY FIELD

Figure 1 shows the $\sigma-A_V$ observational plot. A linear regression analysis gives

$$\sigma = \text{const} + (0.35 \pm 0.01)A_V,$$  (11)

where the value of the constant is irrelevant in the present work, because the numerical version of the plot can be freely translated along $A_V$. Note that the values $\sigma = 0.0$, due to the presence of only one star in some of the bins, are also used. The elimination of those values would give the linear regression coefficient found by Lada et al. (1994).

We want to understand how the linear regression is affected by the errors in the color excess. Lada et al. (1994) estimated a maximum error in the color excess of $\pm 0.15$ mag, which translates into an error of 2.5 mag in $A_V$.

In order to study the effect of the color excess errors we have randomly added such errors to the original data, both with normal and with uniform distributions. This can be done many times, until any correlation $\sigma-A_V$ is completely lost. By applying the errors once, we find on the average a coefficient of $(0.33 \pm 0.02)$. This is an encouraging result, because it means that even after the addition of the errors, the uncertainty in the coefficient is still low. Of course the coefficient has decreased a bit, because the correlation between $\sigma$ and $A_V$ is diminished every time errors are added.

The $\sigma-A_V$ relation to be compared with the numerical ones is therefore

$$\sigma = \text{const} + (0.36 \pm 0.02)A_V.$$  (12)

4.1. Statistics

The slope of the numerical plot could depend on both the power spectrum index, $\alpha$, and the standard deviation of the three-dimensional density distribution, $\sigma_{x,3D}$. In Figure 5, we draw lines of constant $\sigma-A_V$ linear regression coefficient $C_r$ on the plane $\alpha$-$\sigma_{x,3D}$.

Since the value of $C_r$ is known observationally with very small uncertainty (see eq. [12]), its contours on the numerical plane $\alpha$-$\sigma_{x,3D}$ may in principle be used to constrain the power index of the three-dimensional density field when its standard deviation is known, or vice versa.

In fact, Figure 5 shows that the lines of constant $C_r$ are almost lines of constant $\sigma_{x,3D}$, which means that the plane $\alpha$-$\sigma_{x,3D}$ constrains the value of $\sigma_{x,3D}$ but not that of $\alpha$. Given the observational value of $C_r$ and the value of $\alpha$ determined below, one gets

$$\sigma_{x,3D} = 5.0 \pm 0.5.$$  (13)

This is the value of the standard deviation of the three-dimensional density distribution in the cloud IC 5146, as inferred from stellar extinction measurements.

4.2. Power Spectrum

The $\alpha$-$\sigma_{x,3D}$ plane does not constrain directly the index of the (power-law) power spectrum of the three-dimensional

Fig. 5.—Contours of constant value of the slope of the $\sigma-A_V$ plot, $C_r$; $\alpha$ is the spectral index of the three-dimensional density distribution (the power spectrum is assumed to be a power law), and $\sigma_{x,3D}$ is the standard deviation of the same distribution. Around the observed value $C_r = 0.36$, the plane gives a good constraint for the three-dimensional standard deviation, $\sigma_{x,3D}$. 

...
density distribution. Nevertheless, it is indirectly useful because it gives the three-dimensional standard deviation, which can be compared with the observed two-dimensional standard deviation, in order to constrain the spectral index. In fact the projection into two dimensions of the three-dimensional distribution is such that the two standard deviations are related in a way that depends on the value of the spectral index. This can be shown using the numerically generated random distributions. In Figure 6, lines of constant spectral index are plotted in the plane \( \sigma_{x,2D} - \sigma_{x,3D} \). For a fixed three-dimensional standard deviation, the value of the projected two-dimensional standard deviation decreases toward steeper spectra.

The two-dimensional standard deviation, \( \sigma_{x,2D} \), is measured in the extinction map, on a regular grid that contains on average about five stars per bin. Its standard deviation is

\[
\sigma_{x,2D} = 0.7 \pm 0.1 .
\]

Entering the plane \( \sigma_{x,2D} - \sigma_{x,3D} \) with this value and with the previously determined value of \( \sigma_{x,3D} \), one gets

\[
\alpha = -2.6 \pm 0.5 .
\]

This is the value of the spectral index of the three-dimensional density distribution in the cloud IC 5146.

5. DISCUSSION

We have seen that the origin of the \( \sigma - A_V \) plot is the intermittence in the three-dimensional density distribution of the dark cloud; i.e., the occurrence of very strong density fluctuations with a finite probability. We were inspired toward this explanation of the plot by recent results emerging from our numerical experiments of highly supersonic random flows. The numerical experiments showed that most of the mass concentrates in a small fraction of the total volume, very large (orders of magnitude) density contrasts appear in the flow, the distribution of mass density is well described by a lognormal, and the standard deviation of the statistics grows linearly with the rms Mach number of the flow.

Nevertheless, the extinction map itself shows that the distribution of the projected density has an intermittent tail that resembles the projection of a lognormal, and even the distribution based on the sampling of column density star by star, which is a random sampling, is qualitatively the same. In other words, the intermittence of the density distribution in the cloud IC 5146 is an observed property, and not a consequence of our interpretation of the dynamical origin of the density field.

We have shown how the generation of random fields with given statistics and power spectra (assumed to be power laws) leads to the plot of constant-value contours of the slope of the \( \alpha - A_V \) relation on the plane \( \sigma_{x,3D} \), and how one may extract both the standard deviation and the spectral index of the original three-dimensional field of the dark cloud.

Clearly the contour plot of \( C_r \) on the plane \( \sigma - \sigma_{x,3D} \), obtained with numerically generated random distributions, is a powerful tool to investigate the three-dimensional structure of a dark cloud, down to scales smaller than the resolution of the extinction map, when stellar extinction determinations through the cloud are available.

It should be noted that the method of using dust extinction measurements in order to constrain the three-dimensional density field of dark clouds has several advantages, when compared with the traditional method of using maps of molecular emission lines.

First of all, it is a much smaller observational effort.

Second, the translation of the flux at one given line of one given molecule into column density through the whole cloud is far more complex than the transformation of stellar color excess into column density.

Finally the random sampling of points in space (random locations of single stars behind the cloud) allows for the extraction of information from scales smaller than the resolution of the extinction map, based on averaging at any position a few of the randomly selected points. The information is extracted by using the observational \( \sigma - A_V \) plot, together with the numerical prediction of the slope of that plot in the \( \alpha - \sigma_{x,3D} \) plane.

The information one gets can be summarized in the following points:

1. There must be structure on scales at least 10 times smaller than the resolution of the extinction map, that is, down to about 0.03 pc, for the cloud IC 5146.

2. The density distribution is consistent with a lognormal.

3. The standard deviation and the spectral index are measured.

The present work shows that the dust extinction measurements are consistent with a scenario where the complex density field in dark clouds is caused primarily by random supersonic motions. Such a scenario is appealing, for the simple reason that supersonic motions have indeed been observed and measured in dark clouds for the last 20 years!

The connection between the observations and supersonic turbulence is not only qualitative in nature (given the lognormal shape of the three-dimensional density distribution), but also quantitative.

In fact, in our numerical experiments (Nordlund & Padoan 1996) we have determined the relation between the standard deviation of the three-dimensional density field, \( \sigma_{x,3D} \), and the rms Mach number of the flow, \( \mathcal{M} \) (see eq. [9]). The same may be done observationally, since the cloud IC 5146 has been studied in millimeter wavelength by Lada et al. (1994) and by Dobashi et al. (1992). Dobashi et al. obtain measurements of temperature and velocity disper-
7. Power spectrum in 1283 simulations of supersonic turbulence
(Nordlund & Padoan 1996). The spectrum is consistent with the index α inferred from the observations.

The values of the standard deviation and of the spectral index of the three-dimensional density field of dark clouds are very useful for modeling the origin of the mass distribution of protostars. This has in fact been done with considerable success (Padoan, Nordlund, & Jones 1996).

6. CONCLUSIONS

In the present work we have reinterpreted the observational results obtained by Lada et al. (1994), i.e., the fact that the mean stellar extinction at any given position in space increases together with the dispersion of the extinction, where the averages are taken among the stars found at that position in space.

The authors were able to conclude the following:

1. Structure must be present down to scales smaller than the extinction map resolution.
2. Generic models for the cloud structure (e.g., uniform or in clumps) do not easily reproduce the σ-AV plot.
3. The σ-AV plot is a basic test for any model of the dynamics and structure of the cold interstellar medium.

We have simulated the observations by generating random density distributions. In this way we have been able to better define the information contained in the σ-AV plot. We can therefore add the following:

1. The statistics of the three-dimensional density field in the dark cloud is certainly very intermittent; in particular, it is consistent with a lognormal distribution.
2. The standard deviation of the statistics is σx,3D = 5.0 ± 0.5.
3. The index of the power spectrum (assumed to be a power law) of the three-dimensional density field is α = 2.6 ± 0.5.
4. The relation between the rms Mach number of the flow and the standard deviation of the three-dimensional density field is about σx,3D ≈ 0.5M.

We therefore conclude that the scenario for star formation and for the dynamics of dark clouds proposed by Padoan (1995) and by Padoan, Nordlund, & Jones (1996) is fully consistent with the dust extinction measurements in the cloud IC 5146 by Lada et al. (1994).

In that scenario the dynamics of dark clouds is characterized by supersonic random motions, which are responsible for fragmenting the mass distribution, and for the origin of a Miller-Scalo stellar mass function. Here we have shown that the shape, standard deviation, and spectral index of the density distribution, predicted with numerical simulations of randomly forced supersonic flows, and used in the theoretical model for the stellar IMF, are consistent with the observations of stellar extinction in the dark cloud IC 5146.

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