Admission Control and Power Allocation for NOMA-Based Satellite Multi-Beam Network

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ABSTRACT This paper investigates the admission control problem on the satellite multi-beam networks with non-orthogonal multiple access (NOMA). The goal is to maximize the number of supported users on the premise of ensuring the quality of service (QoS) by optimizing the subchannel and power allocation. We provide the system model and then formulate the admission control problem as a mixed integer non-convex optimization problem. The non-convexity and existence of integer variable make the optimal solution difficult to get. Therefore, we propose a joint subchannel matching and power allocation algorithm to obtain the suboptimal solution so as to reduce the computation complexity. The proposed algorithm can be used for both NOMA and orthogonal frequency division multiplexing access (OFDMA). Specifically, the subchannel matching problem is solved by a two-stage matching process where users are accessed to subchannel dynamically. The power allocation problem is modeled as a super-modular game where the existence and uniqueness of Nash equilibrium (NE) are analyzed. Moreover, an iterative power allocation algorithm is proposed based on the NE searching method. Finally, simulation results are provided for demonstrating the effectiveness and feasibility of the proposed algorithm.

INDEX TERMS Admission control, matching theory, multi-beam satellite system, non-orthogonal multiple access, subchannel, power allocation.

I. INTRODUCTION
With the increasing of new services and the shortage of spectrum resource in the future satellite communication systems, multi-beam satellite system has been paid more attention due to its advantages of increasing the capacity and allowing frequency reuse. With an apply of the frequency reuse among beams and Ka frequency band, the signals received by the users on the beams suffer from the inter-beam interference and rain attenuation. To ensure the performance requirement, how to efficiently allocate the limited resources to manage the interference among beams is a hot topic now. In [1], the authors proposed a robust on-board beamforming scheme by considering the effect of both forward and return link. The performance of multiuser detection was investigated in the case of dealing with the strong co-channel interference in [2]. The secrecy performance based on network coding of multi-beam satellite system is analyzed in [3]. Due to reusing the same frequency band among different beams, co-channel interference from multiple beams always exists. Some works focus on resource allocation and interference management to improve the performance of the system. In [4], the authors proposed a power allocation scheme by using a Stackelberg game model and analyzed the effect of price on the performance. In [5], a two-stage approach aiming at optimizing multiple objects was studied to designing the power allocation algorithm.

However, the frequency reuse in multi-beam satellite system is only applied to different beams. The channel reuse for the users on the same beams is not taken into account. In recent years, non-orthogonal multiple access (NOMA) was proposed to share the channel with multiple users and successive interference cancellation (SIC) technique was adopted to remove partial interference [6]. Due to the superior...
performance of NOMA [7]–[10], many works have been investigated to optimize the application of NOMA. For the cases of satellite networks, The authors in [11] analyzed the advantage of NOMA in different satellite systems and provided some research direction in the future. Reference [12] provided the theoretical analysis of outage performance for the application of NOMA in satellite-terrestrial relay network and demonstrated that the superiority of NOMA than OMA. A new interference cancellation method and symmetrical coding scheme were proposed in [13] to improve the performance of NOMA and guarantee the practical application of NOMA. Some comprehensive literature focused on the user scheduling and power allocation on NOMA systems such as [14]–[18]. In [14], the authors explored the application of NOMA in multi-cell network and proposed a matched theory based user association scheme. In [15], the authors investigated an energy efficient resource allocation problem in heterogeneous network by taking into account the energy harvesting and using NOMA. A matching based user scheduling algorithm was proposed in [16] to maximize the secure energy efficiency and showed that the performance of NOMA is superior to the OFDMA in terms of secrecy energy efficiency. A novel deep reinforcement learning method was provided for solving the complex resource allocation problem in NOMA network in [17]. However, the work above paid more attention on static network. For a dynamic network, the authors in [18] studied the energy efficiency problem in NOMA systems from the point of time stability, and then they provided the suboptimal but stable resource allocation algorithms.

In recent years, with the number of users continuing to grow, supporting all users with the required quality of service (QoS) is difficult due to the resource constraints. Therefore, admission control to maximize the number of users becomes a necessary problem. In [19], the authors studied the admission control problem of OFDMA system. Meanwhile, the optimizations of spectral efficiency and energy efficiency were provided on the basis of ensuring the admission number of users, respectively. By using an alternating direction method of multipliers, a distributed admission control algorithm was proposed for multiple-input single-output systems in [20]. With an application of decomposition, a closed-form solution for admission control optimization problem was given for device-to-device system in [21]. The authors in [22] investigated the admission control problem of energy harvesting OFDMA system by combining fuzzy logic theory. However, the recent work paid more attention on the admission control problems of OFDMA systems. In terms of the admission control problem of NOMA systems, the work was few and was concluded as follows. In [23], the paper focused on the study of uplink NOMA system and proposed a maximum independent set searching based admission control scheme. In [24], the authors provided a greedy admission scheme for each cluster. The same authors proposed a new greedy admission scheme by considering the change of power on each cluster in [25]. The admission control problem was studied from the aspect of power allocation. But the subchannel matching and the interference among clusters were not taken into account. For the satellite multi-beam networks, the admission problem becomes more complicated due to the existence of co-channel interferences from both intra-beam and inter-beam.

Motivated by the analysis above, we investigate the application of NOMA in multi-beam satellite system and consider the admission control problem. Compared with the terrestrial networks, the investigated satellite multi-beam systems have the following features: i) The signals suffer from rain attenuation and large path loss due to the use of high frequency and the long-distance transmission. In addition, the intra-beam and inter-beam co-channel interference coexist with an application of NOMA and multi-beam technique. On account of severe interference and path loss, admission control for NOMA based satellite multi-beam systems becomes a considerable problem which is rarely studied. ii) The satellite can uniformly allocate all the resources that is different with the current heterogeneous network where the power resource is allocated by each small-cell base station [19]. iii) The processing ability of satellite system is lower than the terrestrial networks. Therefore, the designed resource allocation algorithms should be implementable and lower complexity. The focus of this paper is to optimize the subchannel matching and power allocation so that more number of users can be supported in the systems.

The main contributions of this paper are listed as follows:

- NOMA is applied to multi-beam satellite system to improve the spectral efficiency and the interference model of each user is analyzed. By considering the power constraint and the maximal accessed number of subchannel and users, the admission control problem in terms of ensuring the QoS of the accessed users is formulated as an optimization problem which is a mixed integer non-convex program.
- A joint subchannel matching and power allocation algorithm is proposed based on many-to-one matching theory. Compared with some existing works that set up the utility function as the channel gain value, we provide a dynamic update strategy for the utility function of users according to the current subchannel matching state.
- An iterative power allocation algorithm is proposed on the basis of NE searching. We formulate the user set selection in matching process as an optimized power allocation problem. Then, the power allocation problem of users on the same subchannel is modeled as a super-modular game frame and the existence and uniqueness of NE is analyzed. Moreover, the NE of game is proved to be equivalent to the optimal solution of the power allocation problem. The proposed algorithm is implementable and lower complexity which is suitable for the satellite systems.
- The performance analysis of the proposed algorithm is provided by comparing with the existing algorithms.
A is a set. Let the Cartesian product.

A conclusion is given in Section VI. The results show that the number of supported users is more than that with the other algorithms.

The rest of the paper is organized as follows. The model of NOMA based multi-beam satellite system is provided and the QoS based admission control problem is formulated in Section II. In Section III, a many-to-one subchannel matching algorithm is given. In Section IV, a super-modular game is introduced and the power allocation algorithm is proposed based on NE searching. Section V provides the simulation results to show the effectiveness of proposed algorithm. A conclusion is given in Section VI.

Notations: In this paper, the following notations are defined as follows if not specified. The bold font represents vector or matrice. \( ^\top \) represents the transpose of vector. \( \otimes \) represents the Cartesian product. \(|A|\) denotes the number of elements if \( A \) is a set. Let \( x \) and \( x' \) be two vectors. \( x > x' \) denotes that \( x_i > x'_i, i = 1, \ldots, N \) where \( N \) is length of vector and \( x_i \) is the \( i \)-th component of \( x \).

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. SYSTEM MODEL

In this paper, we consider a downlink multi-beam satellite communication system. As shown in Fig. 1, the coverage area of geostationary satellite is divided into \( K \) beams. All the beams share the same frequency band to communicate with the geostationary satellite. The frequency band is divided into \( N \) sub-bands called as subchannels to support communication between the users and the geostationary satellite. The user \( m \) of each beam suffers from the co-channel interference of both intra-beam and inter-beams. Let \( K = \{1, \ldots, K\} \) and \( N = \{1, \ldots, N\} \) be the sets of beams and subchannels, respectively. \( \mathcal{M}_k \) denotes the set of users of \( k \)-th beam. Then, \( \mathcal{M} = \sum_{k \in K} \mathcal{M}_k \) represents the set of all the users.

To access more users for the given number of subchannels, NOMA that allows multiple users to share the same subchannel is applied. The SIC technique is used to eliminate the partial interference, i.e., the users with better channel gain can eliminate the interference from the users with bad channel gain.

B. CHANNEL MODEL

The transmission process considered in this paper is that the geostationary satellite communicates with multiple users on ground by using Ka frequency bands (20GHz) where the transmitted signals have an attenuation through the atmospheric. The common empirical model is applied to model the effect of rain attenuation in [26], [27]. Then, the rain fading coefficient of \( n \)-th subchannel on the \( k \)-th beam can be given as follows:

\[
h_{k,n} = \frac{1}{\sqrt{\xi_{k,n}}}
\]

where \( \xi_{k,n} = 10^{\frac{\mu\sigma}{20}} \) and \( \xi_{k,n}^{dB} \) follows the lognormal distribution i.e., \( \ln\xi_{k,n}^{dB} \sim N(\mu, \sigma^2) \), with the mean \( \mu \) and the variance \( \sigma^2 \). The probability density function of \( \xi_{k,n}^{dB} \) can be provided as follows:

\[
f_{\xi_{k,n}^{dB}}(x) = \frac{1}{x \sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right)
\]

Let \( d_{k,m} \) denote the distance between satellite and \( m \)-th user of \( k \)-th beam and \( d_{k,m}^{0} \) denotes the distance between \( k \)-th beam’s center and \( m \)-th user of \( k \)-th beam. Then, the angle between \( k \)-th beam’s center and \( m \)-th user of \( k \)-th beam from the view of satellite can be approximated as \( \hat{\theta}_{k,m,k} = \arcsin d_{k,m,k}^{0}/d_{k,m} \). Define the 3-dB angle as \( \hat{\theta}_{k,m,k}^{3dB} = \arcsin r/D_k \) where \( r \) is the beam radius and \( D_k \) is the distance between satellite and the edge of the \( k \)-th beam. Then, combining Bessel function model with free space loss model, the beam gain can be written as

\[
G_{k,m,k} = G_{m,k}^{loss}G_{r}\left(\frac{B_1(\mu_{k,m,k})}{2\mu_{k,m,k}} + 36\frac{B_3(\mu_{k,m,k})}{\mu_{k,m,k}^3}\right)^2
\]

where \( \mu_{k,m,k} = 2.07123 \sin\hat{\theta}_{k,m,k}/\sin\hat{\theta}_{k,m,k}^{3dB} \), \( B_1(\cdot) \) and \( B_3(\cdot) \) are the Bessel functions of the first kind of order 1 and 3, respectively. \( G_{r} \) and \( G_{I} \) denote the transmission antenna gain and receiving antenna gain, respectively. \( G_{m,k}^{loss} \) represents the free space loss from satellite to \( k \)-th beam of \( j \)-th beam and is given as

\[
G_{m,k}^{loss} = \left(\frac{\lambda}{4\pi d_{m,k}}\right)^2
\]

where \( \lambda \) is the wavelength.

C. SIGNAL MODEL

Assume that \( \mathcal{M}_{k,n} \) is a set of the users multiplexing on the subchannel \( n \) on the \( k \)-th beam and \( \bigcup_{n \in \mathcal{N}} \mathcal{M}_{k,n} \subseteq \mathcal{M}_k \).

Let \( s_{k,m} \) denote the signal transmitted from satellite to \( m \)-th user of \( k \)-th beam. Then, the total signal transmitted by satellite to \( k \)-th beam on subchannel \( n \) can be written as

\[
x_{k,n} = \sum_{m \in \mathcal{M}_{k,n}} \sqrt{p_{k,m,n}}s_{k,m}
\]
where \( p_{k,m,n} \) is the transmitted power for the signal of \( m \)-th user of \( k \)-th beam on subchannel \( n \). Then, the received signal can be given as
\[
y_{k,m,n} = \sum_{i \in K} \sum_{j \in M_i} h_{k,n} \sqrt{G_{i,k,m}} p_{i,j,m,n} s_{i,j} + z_{k,n}
\]
\[
= h_{k,n} \sqrt{G_{i,k,m}} p_{k,j,m,n} s_{k,j} \]
\[
+ h_{k,n} \sum_{j \in M_i, j \neq m} \sqrt{G_{i,k,m}} p_{j,m,n} s_{j} + z_{k,n}
\]
\[
+ \sum_{i \in K} \sum_{j \in M_i, j \neq m} h_{k,n} \sqrt{G_{i,k,m}} p_{i,j,n} s_{i,j} + z_{k,n}
\]  

(6)

On the right-hand side of the equation above, the first term represents the desired signal, the second term denotes the interference from the users on the intra-beam, the third term represents the interference from the users on the inter-beam, the fourth term \( z_{k,n} \) denotes the additive white Gaussian noise (AWGN) with zero mean and variance \( \sigma^2 \). Based on the analysis above, the signal-to-interference-plus-noise ratio (SINR) of \( m \)-th user of \( k \)-th beam on subchannel \( n \) can be obtained as follows
\[
\text{SINR}_{k,m,n}^m = \frac{H_{k,m,n} p_{k,m,n}}{I_{\text{intra},k,m,n} + I_{\text{inter},k,m,n} + 1}
\]  

(7)

where \( I_{\text{intra},k,m,n} \) and \( I_{\text{inter},k,m,n} \) are intra-beam interference and inter-beam interference, respectively.
\[
I_{\text{intra},k,m,n} = \sum_{j \in M_i, j \neq m} H_{i,k,m} p_{j,m,n}
\]
\[
I_{\text{inter},k,m,n} = \sum_{i \in K, i \neq k} \sum_{j \in M_i} H_{i,k,m,n} p_{i,j,n}
\]  

(8)

where \( H_{i,k,m,n} = |h_{k,n}|^2 G_{i,k,m}/\sigma^2 \).

By using the SIC technique, the users can remove the interference from users with bad channel gain on the intra-beam. Without loss of generality, suppose that the order of channel gain has been sorted out as follows
\[
H_{k,m,n} < H_{k,\hat{m},n}, \forall m < \hat{m}, m, \hat{m} \in M_k.
\]  

(9)

Then, the revised intra-beam interference \( I_{\text{intra}}^{\text{intra}} \) can be given as
\[
I_{\text{intra}}^{\text{intra}} = \sum_{j \in M_i, j \neq m} H_{i,k,m,n} p_{j,m,n}
\]  

(10)

Accordingly, the modified SINR can be rewritten as
\[
\text{SINR}_{k,m,n}^m = \frac{H_{k,m,n} p_{k,m,n}}{I_{\text{intra}}^{\text{intra}} + I_{\text{inter},k,m,n} + 1}
\]  

(11)

According to the Shannon formula, the \( m \)-th user’s throughput of \( k \)-th beam on subchannel \( n \) is given as
\[
R_{k,m,n} = \log \left( 1 + \text{SINR}_{k,m,n}^m \right), \text{ (bps/Hz)}
\]  

(12)

D. PROBLEM FORMULATION
Define \( a_{k,m,n} \) as the binary admission control variable. If the throughput of user is higher than the threshold, \( a_{k,m,n} = 1 \), otherwise, \( a_{k,m,n} = 0 \). The total optimization objective is to maximize the supported number of users.
\[
\max \sum_{k \in K} \sum_{m \in M_k} \sum_{n \in N} a_{k,m,n}
\]  

(13)

To guarantee the QoS of the accessed users, the following constraint condition should be added in the optimization problem.
\[
R_{k,m,n} \geq a_{k,m,n} R_{\text{min}}, \quad \forall k \in K, \ m \in M_k, \ n \in N
\]  

(14)

where \( R_{\text{min}} \) is the minimum required throughput.

Due to the limited energy of satellite, the total power allocated to all the accessed users should not exceed maximum transmitted power.
\[
\sum_{k \in K} \sum_{m \in M_k} \sum_{n \in N} a_{k,m,n} p_{k,m,n} \leq P
\]  

(15)

where \( P \) is the maximum transmitted power of satellite.

By using NOMA scheme, multiple users can be multiplexing on the same subchannel. But the number of accessed users is limited. Then, the corresponding constraints can be written as follows.
\[
\sum_{m \in M_k} a_{k,m,n} \leq N_{\text{max}}, \quad \forall k \in K, \ n \in N
\]  

(16)

where \( N_{\text{max}} \) is the maximum number of accessed users.

For each user on the beams, one user only uses one subchannel to communicate with satellite. Correspondingly,
\[
\sum_{n \in N} a_{k,m,n} \leq 1, \quad \forall k \in K, \ m \in M_k
\]  

(17)

According to the analysis above, we can formulate the admission control problem as an optimization problem with admission control variable \( a = [a_{k,m,n}]_{K \times M \times N} \) and power variable \( p = [p_{k,m,n}]_{K \times M \times N} \).
\[
\max_{a,p} \sum_{k \in K} \sum_{m \in M_k} \sum_{n \in N} a_{k,m,n}
\]  

s.t. \( (14) - (17) \)
\[
a_{k,m,n} \in \{0, 1\}, \quad \forall k \in K, \ m \in M_k, \ n \in N
\]
\[
p_{k,m,n} > 0, \quad \forall k \in K, \ m \in M_k, \ n \in N
\]  

(18)

The proposed problem is a mixed integer non-convex optimization problem because the constraints (14) and (15) are both non-convex and the variable \( a \) is an integer variable. The current feasible method to get optimal solution is the exhaustive searching method which is not efficient due to its exponential complexity. Hence, the efficient algorithm is necessary to reduce the complexity.
III. SUBCHANNEL MATCHING SCHEME
In this section, we propose a low complexity algorithm to solve the matching scheme between subchannels and users based on match theory. The idea of proposed algorithm is to avoid the interference and dynamically access the users according to the whole network state. The proposed algorithm can be used for both NOMA and OFDMA.

A. PROBLEM ANALYSIS AND MATCHING THEORY
According to the NOMA principle, multiple users can share the same subchannel to achieve the communication. Although partial interference of intra-beam can be removed with the SIC technique, the co-channel interference from each beam still exists. This makes the problem analysis difficult due to the mixed interference. But the problem may be solved by analyzing the key resource constraints of the admission control problem, one is the limited power and another is the limited number of subchannels. To support more accessed users, the idea of optimization is to access the users with low power consuming for each subchannel so that the system can make the most of the resources.

According to the model described above, one user only occupies one subchannel but one subchannel can be reused by multiple users. Therefore, the many-to-one match model is suitable for the problem analysis. Before describing the efficient matching algorithm, we first introduce two related definitions.

Definition 1: For the given user $m$, beam $k$, and subchannels $n_1$ and $n_2$, define that the user $m$ prefers subchannel $n_1$ than $n_2$ if $\hat{Q}_{k,m}(n_1) > \hat{Q}_{k,m}(n_2)$, and denoted by $n_1 >_m n_2$ where $\hat{Q}_{k,m}(\cdot)$ is the utility function of user $m$ of $k$-th beam.

Definition 2: For the given subchannel $n$ and user sets $M_1$ and $M_2$, define that the subchannel $n$ prefers user $M_1$ than $M_2$ if $Q_n(M_1) > Q_n(M_2)$, and denoted by $M_1 >_n M_2$ where $Q_n(\cdot)$ is the utility function of subchannel $n$.

The main idea of applying NOMA is to access more users when the number of subchannels is not enough. For example, assume the number of subchannels and users are $N$ and $M$. For the case $M > N$, the network can access at most $N$ users by using OFDMA scheme. However, $M$ users can be possibly accessed by using NOMA scheme because more users are allowed to share the subchannel. Therefore, we aim to design a general algorithm that can be used in both OFDMA and NOMA. The design of utility functions is based on the idea of avoiding interference.

The utility function of user $m$ of $k$-th beam for subchannel $n$ is designed as follows:

\[
\hat{Q}_{k,m}(n) = \frac{H_{k,m,n}}{I_{k,m,n}^{\text{intra}} + I_{k,m,n}^{\text{inter}} + 1}
\]

\[
I_{k,m,n}^{\text{intra}} = \sum_{j \in M_{k,m}, j \neq m} H_{k,m,n,p_{k,j,n}}
\]

\[
I_{k,m,n}^{\text{inter}} = \sum_{i \in K, i \neq k} \sum_{j \in M_{i,n}} H_{i,k,m,n,p_{i,j,n}}
\]  

Then, the users can create the corresponding preference lists for the set of subchannels $\mathbf{PF} = \{PF_{k,m,n}\}_{k \times M \times N}$ according to the utility function.

The utility function of subchannel $n$ for users set $\mathcal{M}$ is designed as follows:

\[
Q_n(\mathcal{M}) = \begin{cases} 
|\mathcal{M}|, & \omega = 1 \\
- \sum_{(k,m) \notin \mathcal{M}} p_{k,m,n}, & \omega = 2 
\end{cases}
\]  

where $\omega$ denotes the priority. The subchannels decide the accessed users set according to the utility function.

B. ALGORITHM DESIGN
First, each user on the beams updates the preference list $\mathbf{PF}$. Then, the user with the maximum utility function value requests to access the subchannel corresponding to the first element of the preference list. The subchannel determines whether to allow the requested user to access according to their own situation.

If the maximum number of access allowed on this subchannel has not been reached yet, the subchannel directly agrees to the user’s request and accesses the user. Then, calculate the total power consumption increment by using the proposed Algorithm 2 in next section. If the residual power is enough to support the new user, update the residual power value. If not, the accessed user is rejected.

However, if the maximum number of requested users on this subchannel has been reached yet, there is a competition between the requested user and matched users. Set up the candidate set which is composed of the matched users and the accessed user on the subchannel. Then, calculate the utility function value of each subset of candidate set where the number of elements of subset is $N_{\text{max}}$.

Select the subset with the maximum utility function value as the new matching set and update the candidate set value. Unselected user is removed from the subchannel and removes the element corresponding to the subchannel from the preference list $\mathbf{PF}$. The algorithm iterates until no user requests.

Remark: The designed algorithm is a process of improving the utility function as defined in (20) iteratively. It can be seen that the utility function of each subchannel is improved in each request. Specifically, the user first requests to access the preferred subchannel. According to the definition of utility function, the first priority is the number of matched users on the subchannel. Therefore, the subchannel needs to check whether the maximum access number on this subchannel has not been reached yet, i.e., do the step 4 and 14. If yes, the user is accessed and the utility function has improved. If not, we need to check whether the utility function has room for improvement, i.e., the second priority. Then, the subchannel accesses the user as a candidate and selects a new users set with the maximum utility function. The algorithm iterates until the utility function can not be improved more.
Algorithm 1 Subchannel Matching Algorithm

1: Initialize the preference list $PF$. Let $\text{Matched}$ and $\text{Unmatched}$ be the sets of matched and unmatched users on the subchannels.
2: while $\{\text{Unmatched}\} \neq \emptyset$ do
3:   Select the user with the maximum value in $PF$. The user $m$ on the beam $k$ requests the corresponding subchannel $n$.
4:   if $|\text{Matched}(k, n)| < N_{\text{max}}$ then
5:     The subchannel $n$ accesses the user $m$ on the beam $k$. Calculate the increased power consumption by using Algorithm 2.
6:   if $|\Delta P| < P_{\text{residual}}$ then
7:     Update the residual power by
8:     
9:   else
10:     The subchannel $n$ rejects the requested user.
11:    end if
12:   Set $PF_{k,m,n} = 0$.
13:   Update $PF$ according to the formula (19) and update the sets $\text{Unmatched}$ and $\text{Matched}$.
14: end if
15: if $|\text{Matched}(k, n)| = N_{\text{max}}$ then
16:   a) Set up $S = \{\text{Matched}(k, n), m\}$ as the candidate set.
17:   b) Select user set with the maximum utility function value.
18:   
19:   c) Update the sets $\text{Unmatched}$ and $\text{Matched}$.
20:   d) Update $PF$ and the residual power according to the formula (21).
21: end if
22: for $(k, m) \in \text{Unmatched}$ do
23:   if $PF_{k,m,n} == 0$, $\forall n$ then
24:     Remove the user $m$ of $k$-th beam from $\text{Unmatched}$.
25: end if
26: end for
27: end while

C. CONVERGENCE AND COMPLEXITY ANALYSIS

In this subsection, we analyze the convergence and complexity of proposed Algorithm 1. The convergence of Algorithm 1 is obvious because each user removes the corresponding subchannel from preference list if it is rejected. The maximum length of the preference list is $N$. The while loop stops after at most $KMN$ times. Therefore, the algorithm converges.

The complexity of Algorithm 1 is analyzed in this subsection. In the initialization, the complexity of all the users sorting the preference list in descending order is $O(KMN \log N)$. Then, $N$ operations are needed to select the best user. The complexity of power allocation algorithm is $O(I_{\text{max}} K^2)$ where $I_{\text{max}}$ is the maximum number of iterations. Assume that the user has been accessed. All the users update the preference list with $KMN$ operations. In the worst case, the preference list of all the users is full when the algorithm stops. Finally, the total operations number is $O(KMN \log N + KMN(N + I_{\text{max}} B^2 + KMN))$. Therefore, the total complexity of proposed Algorithm 1 is $O(KMN(I_{\text{max}} K^2 + KMN))$.

IV. POWER ALLOCATION SCHEME

In the previous section, the subchannel matching scheme has been given based on matching theory where the power allocation scheme among users multiplexing on the same subchannel is a necessary step. In order to ensure the efficiency of the algorithm, we proposed a low complexity iterative power allocation algorithm on the basis of super-modular game in this section.

A. PROBLEM ANALYSIS

According to the subchannel matching scheme, the subchannel needs to select a users subset with the maximum utility function when a new user requests to access the subchannel if the maximum number of accessed users has been satisfied. Then, the utility function values of subchannel $n$ for each users subset can be given by solving the following problem.

$$
U^* = \arg \max_{S \subseteq S} Q_n(\hat{S})
$$

where

$$
\begin{align*}
\min_{p_{k,m,n}} & \quad \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}_k,n} a_{k,m,n} p_{k,m,n} \\
\text{s.t.} & \quad R_{k,m,n} \geq a_{k,m,n} R_{\text{min}}, \quad \forall k \in \mathcal{K}, m \in \mathcal{M}_k,n \\
& \quad p_{k,m,n} > 0, \quad \forall k \in \mathcal{K}, m \in \mathcal{M}_k,n \\
& \quad \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}_k,n} a_{k,m,n} (p_{k,m,n} - p_{k,m,n}^0) \leq P_{\text{residual}}
\end{align*}
$$

where $p_{k,m,n}^0$ is the initial power values without the new accessed user and $p_{k,m,n} = 0$ for the new accessed user. In the optimization problem above, the binary variable $a_{k,m,n}$ has been determined because the matched users have been known in the subchannel matching scheme. Therefore, the optimization problem (23) can be transformed to a linear programming problem through some basic transformation. Some existing methods are usually used to solve it such as simplex method and interior point method. The complexity of simplex method in the worst case is the exponential complexity. The complexity of interior point method is calculated according to [20]. There are $KM$ variables and $2KM + 1$ inequality constraints. Then, the complexity of interior point method is $O(K^3 M^3 \sqrt{2KM+1})$, i.e., $O(K^3 M^3 \sqrt{5})$. Therefore, a low complexity and efficient algorithm is designed based on super-modular game theory.

B. PRELIMINARY KNOWLEDGE OF GAME THEORY

A game frame can be denoted by $\mathcal{G} = \langle \mathcal{V}, \mathcal{S}, \mathcal{U} \rangle$ where $\mathcal{V}$ is the player set, $\mathcal{S}$ is the strategy space, and $\mathcal{U}$ is the utility function set. The strategy space is $\mathcal{S} = \bigotimes_{k \in \mathcal{K}} \bigotimes_{m \in \mathcal{M}_k,n} S_{k,m,n}$.
where each subspace $S_{k,m,n}$ can be represented as
\[ S_{k,m,n} = \{p_{k,m,n} | R_{k,m,n} \geq a_{k,m,n}^n, p_{k,m,n} > 0, \]
\[ \sum_{k \in \mathcal{M}_n} a_{k,m,n} (p_{k,m,n} - p_{k,m,n}^0) \leq P_{\text{residual}} \} \] (24)
The utility function set is \(U = \{U_{k,m,n}, k \in \mathcal{K}, m \in \mathcal{M}_n\}\) where
\[ U_{k,m,n} = -a_{k,m,n} p_{k,m,n} \] (25)

For a given game frame, a strategy point is called Nash equilibrium (NE) if all the players achieve the maximum benefit at this point. From the definition, it is also a local optimal point of the optimization problem. Therefore, the solution of proposed optimization problem can be obtained by searching the NE of corresponding game. The formulated definition of NE can be given as follows.

**Definition 3**: The power solution \(\mathbf{p}^* = (p_{k,m,n}^*)_{k \in \mathcal{K}, m \in \mathcal{M}_n}\) in the strategy space \(S\) is called as a NE of the game \(G\) when the following conditions are satisfied.
\[ U_{k,m,n}(p_{k,m,n}^*) \geq U_{k,m,n}(p_{k,m,n}p_{k,m,n}^{-1}), \forall k \in \mathcal{K}, m \in \mathcal{M}_n \] (26)
where \(p_{k,m,n}^{-1} = \mathbf{p}^{-1} p_{k,m,n}\) is the power strategy of all users except \(m\)-th user on \(k\)-th beam.

However, the feasible solution of problem (23) may not exist because the set of feasible solutions generated by the constraints is possible to be full. Accordingly, the strategy space may be full. The NE of the game \(G\) may not exist. To deal with it, we revised the problem (23) as
\[
\min_{p_{k,m,n}} \sum_{k \in \mathcal{K}, m \in \mathcal{M}_n} \left( p_{k,m,n} - b_{k,m,n} q_{k,m,n} \right)^2 \\
\text{s.t. } 0 < p_{k,m,n} - p_{k,m,n}^0 < P_{\text{residual}}, \forall \] (27)
\[ q_{k,m,n} = \sum_{j \in \mathcal{M}_n} p_{k,j,n} + \frac{1}{H_{k,m,n}} + \sum_{i \in \mathcal{K}, i \neq k} \sum_{j \in \mathcal{M}_n} H_{k,m,n} p_{i,j,n} \]
where \(b_{k,m,n} = 2a_{k,m,n}P_{\text{min}}\). In fact, the revised problem (27) is relaxation of the original problem (23). To show it, we provided a theorem as follows:

**Theorem 1**: If the point \(\mathbf{p}^* = (p_{k,m,n}^*)_{k \in \mathcal{K}, m \in \mathcal{M}_n}\) is an optimal solution of problem (23), then it is also an optimal solution of problem (27).

**Proof**: The proof of Theorem 1 is given in Appendix A.

Then, the corresponding game frame can be rewritten as \(G' = (\mathcal{V}, \mathcal{S}', \mathcal{U}')\) where \(\mathcal{S}' = \bigotimes_{k \in \mathcal{K}} \bigotimes_{m \in \mathcal{M}_n} S_{k,m,n}'\), \(Q_{k,m,n}' = \{p_{k,m,n}'^0, P_{\text{residual}} + p_{k,m,n}\} \) and \(\mathcal{U}' = \{U_{k,m,n}'^0, k \in \mathcal{K}, m \in \mathcal{M}_n\}\)
\[ U_{k,m,n}' = -\left( p_{k,m,n} - b_{k,m,n} q_{k,m,n} \right)^2, \forall k \in \mathcal{K}, m \in \mathcal{M}_n \] (28)

Now, we discuss the existence of NE of new game \(G'\) and design the algorithm to search NE. Sequentially, we can obtain the power allocation scheme on the basis of NE. Before designing the power allocation algorithm, we first introduce the definition of super-modular game and the related properties.

**Definition 4**: For a given game \(G' = (\mathcal{V}, \mathcal{S}', \mathcal{U}')\), it is super-modular game if the strategy space \(\mathcal{S}'\) and the utility function set \(\mathcal{U}'\) satisfy the following conditions, respectively.
- Strategy space’s conditions: for given \(p_{k,m,n}^0\)
  1) \(S_{k,m,n}'\) is not full.
  2) \(S_{k,m,n}'\) is compact sublattice.
- Utility function’ conditions:
  1) \(U_{k,m,n}'\) is twice differentiable.
  2) property: \(\frac{\partial^2 U_{k,m,n}'}{a_{k,m,n}^n p_{k,m,n}^{-1}} \geq 0\).

**Theorem 2**: If the game \(G' = (\mathcal{P}, \mathcal{Q}', \mathcal{U}')\) is super-modular game, then the following conditions are met.
- The existence of NE:
  1) The set of NE \(E\) is not full.
  2) If \(|E| \neq 1\), there are two elements \(p_0, p_0' \in E\) are the smallest NE and largest NE, respectively, i.e., \(p_0 < p < p_0' \forall p \in E\).
- The reachability of smallest (largest) NE: the smallest (largest) NE can be reached if two conditions are met.
  1) Each player \(v_{k,m,n}\) updates the power through the best response from the smallest (largest) element of strategy subspace \(Q_{k,m,n}'\).
  2) Each user’s best response is single-valued.

**Proof**: The proof can be referred to [28].

The formulated definition of the best response is given as follows:

**Definition 5**: For given players’ power strategy \(p_{k,m,n}^{-1}\) the best response of player \(v_{k,m,n}\) is
\[ \hat{p}_{k,m,n} = \arg\max_{p_{k,m,n}} U_{k,m,n}(p_{k,m,n}p_{k,m,n}^{-1}) \] (29)

**C. ALGORITHM DESIGN AND COMPLEXITY ANALYSIS**

From the analysis above, we can know that the NE of super-modular game exists and can be reached by using the best response. Therefore, we first show the proposed game frame \(G'\) is a super-modular game and then design a power allocation scheme through searching the NE.

**Theorem 3**: The proposed game \(G' = (\mathcal{V}, \mathcal{S}', \mathcal{U}')\) is super-modular game.

**Proof**: The proof of Theorem 3 is given in Appendix B.

According to Theorem 2, we can know that the NE of proposed game \(G'\) exists. To show the uniqueness of NE, we provide the following theorem.

**Theorem 4**: The NE of proposed game \(G'\) is unique.

**Proof**: The proof of Theorem 4 is given in Appendix C.
Algorithm 2 Power Allocation Scheme Based on NE Searching

1: Initialize \( p_{k,m,n} = p^{0}_{k,m,n} \) and set up the threshold \( \epsilon \), maximum number of iterations \( I_{\text{max}} \), and residual power value \( P_{\text{residual}} \).

2: for \( t = 0 \) to \( I_{\text{max}} \) do
   3: Update the power strategy \( p \) by solving
   \[
   \hat{p}_{k,m,n}(t + 1) = \arg \max_{p_{k,m,n}} U_{k,m,n}(p_{k,m,n}|p_{k,m,n}^{-1}). \tag{30}
   \]
   4: Calculate the total increasing power \( \Delta P \).
   \[
   \Delta P = \sum_{k \in K_{m}} \sum_{m \in M_{k,n}} (\hat{p}_{k,m,n}(t + 1) - \hat{p}_{k,m,n}(t)) \tag{31}\]
   5: if \( \Delta P > P_{\text{residual}} \) then
      6: Break the algorithm and output the power strategy \( p_{k,m,n} = P_{\text{residual}} \).
   7: else
   8: if \( |p(t + 1) - p(t)| < \epsilon \) then
      9: Break the algorithm and output the power strategy \( p(t + 1) \).
   10: else
      11: \( t = t + 1 \)
   12: end if
   13: end if
14: end for

response (29). Calculate the total increasing power between the current power and the original power. If the total increasing power is more than the residual power \( P_{\text{residual}} \), then set each player’s power strategy as \( P_{\text{residual}} \) and stop the algorithm. Otherwise, if the error is less than the threshold, then stop the algorithm and output the current power strategy, if not, the algorithm continues. The details of algorithm can refer to Algorithm 2.

Although the NE can be reached by using Algorithm 2, we take into account the practical power constraint in the algorithm design. When the residual power can not support all the accessed users, the algorithm is stopped to reduce computational complexity. The reason is that the obtained NE is not a feasible solution of the proposed problem because the power constraint is not satisfied. For the case that power constraint is met, the NE searching by Algorithm 2 is the optimal solution of problem (23).

**Theorem 5:** The NE \( p^* \) given by Algorithm 2 is the optimal solution of problem (23).

**Proof:** The proof of Theorem 5 is given in Appendix D. ■

The complexity of proposed iterative algorithm is analyzed as follow. The total number of players in Algorithm 2 is at most \( N_{\text{max}} \cdot K \). In each iteration, each player needs to update the power strategy according to \( N_{\text{max}} \cdot K - 1 \) other players’ power strategy. The number of multiplication and addition is \( N_{\text{max}} \cdot K \). Then, the total operations of \( N_{\text{max}} \cdot K \) players is \( N_{\text{max}}^{2} \cdot K^{2} \). Assume that the the maximum number of iterations is \( I_{\text{max}} \).

Therefore, the complexity of Algorithm 2 is \( O(I_{\text{max}} \cdot K^{2}) \). In appendix E, we show that \( I_{\text{max}} \) is a constant depended on the error threshold \( \epsilon \) and coefficients in iterative process.

### V. SIMULATION RESULTS

In this section, simulations are provided for evaluating the performance of the proposed algorithm on the basis of Monte Carlo method. The scenario considered in the simulation is downlink communication system which includes one satellite and multiple beams. The users are randomly located on the beams. The specific simulation parameters are listed in table 1.

We compare the proposed algorithm with the following algorithms to demonstrate the efficiency of the proposed algorithm. All the following algorithms used to compare with the proposed algorithm use the same power allocation scheme (Algorithm 2) proposed in this paper.

- Static matching scheme [29]: The preference lists of all users are set up according to the subchannel gain values which is fixed. The users always request the subchannel on basis of the fixed preference lists.
- Matching scheme without exchange step. This scheme is a simplified form of the proposed Algorithm 1 that the subchannel rejects the requested user if the maximum access number of subchannel has been achieved. It is marked as “Dynamic-Noexchange” in figures.
- Static matching scheme without exchange step. The step of users requesting and the step of subchannel rejecting are same as “Static matching scheme” and “Matching scheme without exchange step”, respectively. It is marked as “Static-Noexchange” in figures.
- OFDMA. One user only occupies one subchannel. The admission process is a special case of Algorithm 1 that the maximum access number is set up as 1.

Fig. 2 shows the effect of the increased number of users per beam on the number of supported users. The number of subchannels and beams is set to 15 and 3, respectively. The total power and the minimum throughput threshold are set to 500W and 2bps/Hz, respectively. With the number of users per beam increasing, the number of supported users is also increased. That is because there are more choices between users and subchannels with the increasing number of users. The performance of the proposed algorithm is better than the other compared algorithms. Even though the proposed

| Table 1. Simulation parameters. |
|--------------------------------|
| Parameters | Value       |
| Satellite height | 3600km      |
| Beam radius | 100km       |
| Satellite antenna gain | 52dBi      |
| User antenna gain | 41.7dBi    |
| Frequency band | 20GHz        |
| Rain attenuation parameter \( \mu \) | -3.125 |
| Rain attenuation parameter \( \sigma^{2} \) | 1.591 |
| Noise power spectral density | -174dBm |
| maximum number of accessed users \( N_{\text{max}} \) | 2 |
| Maximum number of iterations \( I_{\text{max}} \) | 10 |
| Minimum convergence threshold \( \epsilon \) | 0.01 |
algorithm without the exchange step, the performance is also superior to the algorithm in [29] when the number of users per beam is more. Compared with OFDMA scheme, the number of supported users with NOMA has a significant increasing by using proposed algorithm. But notice that the performance of “Static-Noexchange” is worse than OFDMA scheme. This shows that the efficient subchannel matching and power allocation scheme is necessary for the application of NOMA.

Fig.3 demonstrates the change of the number of supported users when the number of beams is increasing from 1 to 7. The total power and the minimum throughput threshold is set to 500 W and 2 bps/Hz, respectively. The number of users per beam and subchannels is assumed to 30 and 15, respectively. It can be seen that the number of supported users increases with the number of beams increasing. When the number of beams is few, the performances of all algorithms are almost same. The reason is that interference scheduling has not a significant effect on the performance of system because the interference among beams is low and the total power is enough. When the number of beams is increasing, the performance of the proposed algorithm is superior to other algorithms because the interference among beams is increasing. This shows that our proposed algorithm has better ability on interference scheduling than other compared algorithms.

In Fig. 4, the effect of the number of subchannels on the number of supported users has been given. The total power and the minimum throughput threshold is set to 500 W and 2 bps/Hz, respectively. The number of users per beam and beams is assumed to 30 and 3, respectively. It can be seen that the number of supported users increases linearly with the number of subchannels increasing. The reason is that the number of supported users on each subchannel is almost same because there is no interference among different subchannels. From the figure, the number of supported users with the proposed algorithm is more than it with the other algorithms. On other hand, the proposed algorithm and it without exchange step have almost same performance. But the performance of static algorithm has a huge difference compared with “Static-Noexchange”. This shows that the proposed algorithm doesn’t depend on the exchange step but the static algorithm does.

Fig. 5 shows that the effect of the total power of satellite on the number of supported users. The minimum throughput threshold is set to 2 bps/Hz. The number of users per beam, beams, and subchannels is assumed to 30, 3 and 15, respectively. The number of supported users increases slowly with
the total power of satellite increasing. When the total power increases, more users can be accessed because the required power of them can be satisfied. But it needs to take more power to access a new user when there are users supported on the subchannels because the accessed users later can not usually remove the co-channel interference by using the SIC technique. From the figure, it can be seen that the proposed algorithm is superior to the other algorithms.

In Fig. 6, the effect of the minimum throughput threshold on the number of supported users is provided. The total power is set to 500W. The number of users per beam, beams, and subchannels is set to 20, 3 and 10, respectively. From the figure, the number of supported users decreases with the minimum throughput threshold increasing. When the minimum throughput threshold is small, all the users can always be accessed due to the adequate resources no matter what algorithm we use. However, when the minimum throughput threshold is large, the performance of the proposed algorithm is better than the other algorithms. When the required minimum throughput threshold is extremely high, the proposed algorithm and it without exchange step also have almost same performance. The reason is that the payoff of supporting one more user is extremely expensive.

Fig. 7 shows the convergence of the proposed NE searching algorithm where the game player set consists of two users from different beams. According to the Theorem 2 and 3, the NE can be iteratively reached by using the best response with the initial value as the smallest value in strategy space. From the figure, the power strategy converges to NE after 3 iterations.

Fig. 8 shows the average power of supported user versus the total power. The parameter setting is the same as Fig. 6. The average power of supported user is set up as the proportion between the total power and the number of supported users. As shown in Fig. 5, the number of supported users increases with the growth of total power. But the increasing is slow. In Fig. 8, it can be seen that the average power of supported user is increasing when the total power grows. The reason is that more power needs to be taken when the new users are accessed due to the interference increasing.

VI. CONCLUSION

In this paper, we investigate the admission control problem in NOMA based multi-beam satellite system. The admission control problem is formulated as a mixed integer non-convex optimization problem which is NP-hard. Then, a low complexity joint subchannel matching and power allocation algorithm aiming to maximize the number of accessed users with QoS constraint is proposed. The proposed algorithm is suitable for both NOMA and OFDMA, i.e., OFMDA is used when the number of channels is enough to support all users, and NOMA is used in contrast. Simulation results demonstrated that the proposed algorithm outperforms the existing algorithms in terms of the number of supported users.

APPENDIXES

APPENDIX A PROOF OF THEOREM 1

Suppose that \( p \) is an optimal solution of problem (23), then the first constraint holds with equality. If not, for given \( k \in \mathcal{K}, m \in \mathcal{M}_{k,n}, R_{k,m,n}(p_{k,m,n}) > a_{k,m,n}R_{min} \), we can set the new power \( p'_{k,m,n} \) and keep the other power values invariant so that \( R_{k,m,n}(p'_{k,m,n}) = a_{k,m,n}R_{min} \). Then,

\[
p'_{k,m,n} < p_{k,m,n}
\]  

(32)

and for other users \( R_{k,m,n} \geq a_{k,m,n}R_{min} \) holds. That is because \( R_{i,n} \) is increasing with \( p_{k,m,n} \) increasing and \( R_{k,m,n} \) is decreasing with \( p_{k,m,n} \) increasing. However, the objective function is decreasing due to the inequality (32). This is a contradiction. Thus, the first constraint in problem (23) holds with equality. Then, the objective function of problem (27)
is 0 which is minimum value apparently. The constraints in problem (27) must be satisfied according to the third constraint in problem (23). Therefore, \( p \) is an optimal solution of problem (27).

**APPENDIX B PROOF OF THEOREM 3**

The power strategy space is not full according to expression. Then, define that
\[
\mathbf{x} \land \mathbf{y} = (\min(x_1, y_1), \min(x_2, y_2), \ldots, \min(x_N, y_N))
\]
\[
\mathbf{x} \lor \mathbf{y} = (\max(x_1, y_1), \max(x_2, y_2), \ldots, \max(x_N, y_N)).
\]
(33)

For any \( \mathbf{x}, \mathbf{y} \in \mathcal{P}' \), we know that \( x_i, y_i \in [p_0^i, P_{\text{residual}} + p_1^i], \forall i = 1, \ldots, N \) Then, \( \min(x_i, y_i) \in [p_0^i, P_{\text{residual}} + p_1^i] \) and \( \max(x_i, y_i) \in [p_1^i, P_{\text{residual}} + p_1^i] \) hold. Thus, \( \mathbf{x} \land \mathbf{y} \in \mathcal{P}' \) and \( \mathbf{x} \lor \mathbf{y} \in \mathcal{P}' \). According to the definition of sublattice, we can obtain that the power strategy space \( \mathcal{P}' \) is a compact sublattice.

The utility function \( U_{k,m,n} \) is twice differentiable because it is a polynomial function. The first-order differential of the utility function is calculated as follows:
\[
\frac{\partial U_{k,m,n}}{\partial p_{k,m,n}} = -2(p_{k,m,n} - b_{k,m,n}q_{k,m,n}).
\]
(34)

The second-order differential of the utility function is
\[
\frac{\partial^2 U_{k,m,n}}{\partial p_{k,m,n}\partial p_{i,j,n}} = 2b_{k,m,n}\frac{\partial q_{k,m,n}}{\partial p_{i,j,n}} = 2b_{k,m,n}\frac{H_{k,m,n}}{H_{k,k,m,n}} > 0.
\]
(35)

All the conditions of definition have been verified. Therefore, the game \( \mathcal{G}' = (\mathcal{V}', \mathcal{S}', \mathcal{U}') \) is a super-modular game.

**APPENDIX C PROOF OF THEOREM 4**

According to (31), we can obtain the iterated function as \( I(p) \) where
\[
I_{k,m,n}(p) = b_{k,m,n}\left( \sum_{j \in M_{k,n}, j < m} p_{k,j,n} + \frac{1}{H_{k,k,m,n}} + \sum_{i \in K} \sum_{j \in M_{i,n}} H_{i,k,m,n} p_{i,j,n} \right).
\]
(36)

The iterative process is \( p(k + 1) = I(p(k)) \). Then, to show the uniqueness of NE, we need to verify the properties of \( I(p) \) for \( p > 0 \).
- Positivity: It is obvious due to \( p > 0 \).
- Monotonicity: If \( p > p' \), then
\[
I_{k,m,n}(p) - I_{k,m,n}(p') = b_{j,k,n}\left( \sum_{j \in M_{k,n}, j > m} (p_{k,j,n} - p'_{k,j,n}) + \sum_{i \in K} \sum_{j \in M_{i,n}} H_{i,k,m,n} (p_{k,j,n} - p'_{k,j,n}) \right) > 0
\]
(37)

Therefore, we know \( I(p) > I(p') \).
- Scalability: For any \( \alpha > 1 \),
\[
aI_{k,m,n}(p) - I_{k,m,n}(\alpha p') = \frac{b_{k,m,n} (\alpha - 1)}{H_{k,k,m,n}} > 0
\]
(38)

Then, we can know \( aI(p) > I(\alpha p') \).

**APPENDIX D PROOF OF THEOREM 5**

Suppose that \( p_1 \) is the NE iterating by Algorithm 2, then the objective function of problem (27) is 0 by substituting \( p_1 \). If not, the users must adjust the power because the iterated function in Algorithm 2 is increasing. It is a contradiction with the definition of NE. Thus, the first constraint in problem (23) holds with equality. According to the equation (31) and the criterion \( \Delta P > P_{\text{residual}} \), we can know that the third power constraint in problem (23) is satisfied naturally. Thus, all the constraints of problem (23) are met and then the NE is a feasible solution of (23).

As we all know, the NE is a locally optimal solution of problem (27). However, it is also an optimal solution because problem (27) is a convex problem. According to Theorem 1, if the optimal solution of problem (23) exists, then it is an optimal solution of problem (27). Assume that the NE \( p_1 \) is not optimal solution of problem (23). Then, there must be an optimal solution \( p_2 \neq p_1 \) of problem (23). According to Theorem 1, \( p_2 \) is an optimal solution of problem (27). However, \( p_1 \) is also optimal solution of problem (27). This creates a contradiction because problem (27) is a strictly convex problem and the optimal solution is unique. To show the strict convexity of problem (27), we only need to show Hessian matrix is strictly positive. It can be easily proved because Hessian matrix is a diagonal matrix and all the diagonal elements are positive. The proof has been achieved.

**APPENDIX E PROOF OF MAXIMUM NUMBER OF ITERATIONS**

We first rewrite the equation (36) as a vector form as follows.
\[
p^{t+1} = Hp^t + b
\]
(39)

According to Theorem 2 and Theorem 3, the proposed algorithm must converge to an optimal solution. Then, let \( t \to \infty \), we have
\[
p^* = Hp^* + b
\]
(40)

where \( p^* \) is the optimal solution. Now, we can see the error vector as follows:
\[
e'(p) = p^* - p'
= Hp^* + b - (Hp^{t-1} + b)
= H(p^* - p')
= H'(p^* - p^t)
\]
(41)

Because the algorithm converges, we have
\[
\lim_{t \to \infty} e'(p) = 0
\]
(42)
Equivalently,
\[
\lim_{t \to \infty} \mathbf{H}' = 0
\]
(43)

For a given error threshold \( \varepsilon \), we can calculate the maximum number of iterations required in the following. Let
\[
\|e'^t\| = \|e'^t (p^* - p^0)\| \leq \|H\|^t \|p^* - p^0\| \leq \varepsilon
\]
(44)

Then, we can obtain
\[
H |\|p^* - p^0\| \leq \varepsilon \quad \frac{t \log (\|H\|)}{\log (\|p^* - p^0\|)} \leq \frac{\log (\frac{\varepsilon}{\|p^* - p^0\|})}{\log (\|H\|)}
\]
(45)

Therefore, we can know that maximum number of iterations \( I_{\text{max}} \) is a constant.

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