Research Article

Design of a Data-Driven Control System based on Reference Model using Predicted Input/Output Responses

Yuki Nakatani, Takuya Kinoshita, Toru Yamamoto

Graduate School of Advanced Science and Engineering, Hiroshima University, 1-4-1 Kagamiyama, Higashi-Hiroshima, Higashi-Hiroshima, Hiroshima 739-8527, Japan

1. INTRODUCTION

In industrial systems, data-driven control schemes [1–4] have been actively studied to achieve the desired control performance for the controlled system with unknown structure and parameters. These schemes use only a set of experimental data to design a controller that satisfies the desired control performance specified by the reference model offline. Generally, the reference model is designed to focus on only the output response. However, both input/output responses should be considered in designing the control system because there is a limit of the actuator performance in actual machines. If the input response could be estimated in advance, it would be very useful in designing the reference model.

The Estimated Response Iterative Tuning (ERIT) scheme [5] has been proposed to predict the input/output response before applying the adjusted control parameters. However, this scheme can only be applied to Two-Degree-of-Freedom (2DOF) control systems. On the other hand, it is important to design One-Degree-of-Freedom (1DOF) controller because there are also many 1DOF control systems in industries.

In this paper, the new data-driven control scheme is proposed to design a 1DOF controller considering input response for unknown structure systems. The features of the proposed scheme are as follows:

(i) The 1DOF control system design that predicts the input response in advance, even for unknown system.

(ii) Adjusting the desired input/output responses with one parameter $\lambda$.

Finally, the effectiveness of the proposed scheme is verified in Section 4.

2. OVERVIEW OF THE PROPOSED SCHEME

Figure 1 shows an overview of the proposed data-driven control scheme. In the proposed scheme, the controller is designed by the following procedure:

(I) Obtaining the desired predicted data $\hat{y}(t)$ and $\hat{u}(t)$.

(II) Constructing the reference model $G_m(z^{-1})$ based on $\hat{y}(t)$ and $\hat{u}(t)$ obtained in (I).

(III) Designing a controller using the reference model $G_m(z^{-1})$ obtained in (II).

The detailed procedure is explained in the next section.

3. DESIGN OF THE CONTROLLER

3.1. Generating Predicted the Data using the ERIT Scheme

In this section, the ERIT scheme [5] is described to generate the predicted data. This scheme is theoretically only applicable to the 2DOF control system. In the proposed scheme, the ERIT scheme is utilized for only predicting the input/output data in Figure 2. Note that the controller is not designed by the ERIT scheme.
First, the initial output \( y_0(t) \) is obtained in a 1DOF control system. In the case of the feedforward controller \( C_{FF}(z^{-1}) = 0 \) in Figure 2, \( y_0(t) \) is given as follows:

\[
y_0(t) = \frac{G(z^{-1})C_{FF}(z^{-1})}{1 + G(z^{-1})C_{FF}(z^{-1})} r(t).
\]  

(1)

Then, extending to a 2DOF control system for predicting the data, the predicted output \( \hat{y}(t) \) is defined as:

\[
\hat{y}(t) = \frac{G(z^{-1})C_{FF}(z^{-1}) + G(z^{-1})C_{PB}(z^{-1})}{1 + G(z^{-1})C_{PB}(z^{-1})} r(t).
\]

(2)

Here, system identification is required to obtain an exactly predicted output \( \hat{y}(t) \) because Equation (2) contains a controlled system \( G(z^{-1}) \). Therefore, substituting Equation (1) into Equation (2) yields the following equation:

\[
\hat{y}(t) = \frac{C_{FF}(z^{-1})}{C_{PB}(z^{-1})} y_0(t) + y_0(t).
\]

(3)

Consequently, the predicted data \( \hat{y}(t) \) can be derived offline without \( G(z^{-1}) \) by using the initial data \( y_0(t) \) by tuning \( C_{PB}(z^{-1}) \). \( \hat{u}(t) \) can also be derived in the same procedure using \( u_0(t) \) as follows:

\[
\hat{u}(t) = \frac{C_{FF}(z^{-1})}{C_{PB}(z^{-1})} u_0(t) + u_0(t).
\]

(4)

From Equations (3) and (4), it is possible to obtain the predicted data \( \hat{y}(t) \) and \( \hat{u}(t) \) offline corresponding to the various \( C_{PB}(z^{-1}) \) by using the initial data \( y_0(t) \) and \( u_0(t) \). Note that the 2DOF control system was only implemented virtually offline to obtain the predicted data \( \hat{y}(t) \) and \( \hat{u}(t) \).

### 3.3. Design of \( G_m(z^{-1}) \) Based on the Desired Predicted Data \( \hat{y}(t) \)

In this section, \( G_m(z^{-1}) \) is designed based on the desired predicted data \( \hat{y}(t) \). The evaluation norm for designing \( G_m(z^{-1}) \) is defined as follows:

\[
J_{ns} = \frac{1}{N} \sum_{i=0}^{N-1} (\hat{y}(t) - y_0(t))^2
\]

(6)

\[
y_m(t) = G_m(z^{-1}) r(t).
\]

(7)

\( G_m(z^{-1}) \) is designed by minimizing \( J_{ns} \).

### 3.4. Design of a PID Controller based on the Extended Output \( \phi(t) \)

In this section, a 1DOF controller \( C(z^{-1}) \) is designed as an \( I-PD \) controller [7] by using the reference model \( G_r(z^{-1}) \) that considers the input response obtained in the previous section. It is possible to design the controller by using any data-driven approach. In this paper, the Proportional-Integral-Differential (PID) control scheme based on extended output [8] is used.

I-PD controller is given as follows:

\[
\Delta u(t) = K_e(t) - K_p \Delta y(t) - K_d \Delta^2 y(t),
\]

(8)

where \( K_e, K_p \), and \( K_d \) are the proportional gain, integral gain, and derivative gain, respectively. In this scheme, \( C(z^{-1}) \) is designed by minimizing the following evaluation norm:

\[
J = \frac{1}{N} \sum_{i=0}^{N-1} e(t)^2
\]

(9)
\[ e(t) = G_m(z^{-1})\phi(t) - y(t), \]

where \( e(t) \) denotes the difference between the output \( y(t) \) of the closed-loop transfer function and the output \( G_m(z^{-1})\phi(t) \) of the reference. \( C(z^{-1}) \) can be designed by minimizing \( J \).

For reasons of space, the details are omitted and refer to the PID control scheme based on extended output \( \phi(t) \) [8].

### 4. NUMERICAL EXAMPLE

#### 4.1. Controlled System

The effectiveness of the proposed scheme is verified using numerical simulation. The controlled system is the experimental thermal equipment for simulating bag-and-bound welding owned by our laboratory. The model of the system is given as follows:

\[
\begin{align*}
\dot{y}(t) &= \frac{R}{1 - e^{-\frac{R}{H}t}} z^{-1} u(t) \\
y(t) &= \dot{y}(t) + d(t) + \xi(t),
\end{align*}
\]

where \( R, H, d(t), \) and \( T_f \) are the thermal resistance, the heat capacity, the room temperature, and the sampling time, respectively. And \( \xi(t) \) is Gaussian white noise with mean 0 and variance 0.1. The coefficients \( R, H \) were derived by system identification. Table 1 shows the value of thermal resistance \( R \) and heat capacity \( H \).

The proposed scheme is applied to the system. The reference signal is \( r(t) = 100(\degree C) \), the sampling time is \( T_s = 0.04(s) \), and the room temperature is \( d(t) = 20(\degree C) \). In this numerical example, \( I-P \) controller was used, whose initial proportional and integral gains were set as follows:

\[ K_p^{ini} = 1.5, \quad K_i^{ini} = 2.0 \times 10^{-3}. \] (13)

Note that the initial PI gains of \( K_p^{ini} \) and \( K_i^{ini} \) are set to obtain the stable input/output data \( y_0(t), u_0(t) \). Additionally, the initial data \( y_0(t) \) and \( u_0(t) \) were filtered by using \( 1 / [1 + (4/3)s] \).

Next, the feedforward controller \( C_{ff}(z^{-1}) \) was calculated by using the proposed scheme as follows:

\[
C_{ff}(z^{-1}) = \frac{\alpha_1}{1 + \alpha_2 z^{-1} + \alpha_3 z^{-2}}.
\] (14)

The order of the denominator was set to second-order because the low order term significantly affects the output response of the controlled system. Furthermore, the value of \( \alpha_i \) \( (i = 1, 2, 3) \) was determined by minimizing \( J_{dss} \) in Equation (5), for example, using the Matlab/Simulink Ver.9.8.0.1396136(R2020a), Optimization Toolbox 'fminsearch.m'.

Furthermore, \( G_m(z^{-1}) \) was determined as follows:

\[
G_m(z^{-1}) = \frac{\theta_1 + \theta_2 z^{-1}}{1 + \theta_3 z^{-1} + \theta_4 z^{-2}},
\] (15)

where the value of \( \theta_i \) \( (i = 1, 2, 3, 4) \) is determined by using \( fminsearch.m \) to minimize \( J \) in Equation (6).

Finally, the tuned proportional gain \( K_p^{tuned} \) and integral gain \( K_i^{tuned} \) were designed by minimizing \( J \) in Equation (9).

In this numerical example, the control results are compared by changing \( \lambda \) in Equation (5), and Table 2 shows the feedforward controller \( C_{ff}(z^{-1}) \), the reference model \( G_m(z^{-1}) \), the tuned proportional gain \( K_p^{tuned} \) and the tuned integral gain \( K_i^{tuned} \) respectively in the cases of \( \lambda = 0.5 \) and 5.

#### 4.2. Control Result

Figures 3 and 4 show simulation results corresponding to the cases of \( \lambda = 0.5 \) and 5, where \( y_0(t) \) and \( u_0(t) \) denote initial I/O data, \( y^*(t) \) and \( u^*(t) \) the desired predicted I/O data, and \( y(t) \) and \( u(t) \) denote I/O data given by the proposed scheme. The predicted response can be adjusted with one parameter according to \( \lambda \) because the responsiveness is high because \( \lambda = 0.5 \) in Figure 3, and the responsiveness is low because \( \lambda = 5 \) in Figure 4.

| Table 2 Obtained parameters in the simulation | \( \lambda = 0.5 \) | \( \lambda = 5 \) |
|----------------------------------------------|----------------|----------------|
| \( \alpha_1 \)                              | 2.55 \times 10^{-2} | 3.64 \times 10^{-3} |
| \( \alpha_2 \)                              | -1.98           | -1.99          |
| \( \alpha_3 \)                              | 3.25 \times 10^{-1} | 1.77 \times 10^{-2} |
| \( \theta_1 \)                              | -4.08 \times 10^{-4} | -2.24 \times 10^{-3} |
| \( \theta_2 \)                              | 4.76 \times 10^{-4} | 2.39 \times 10^{-4} |
| \( \theta_3 \)                              | -4.08 \times 10^{-1} | -2.24 \times 10^{-3} |
| \( \theta_4 \)                              | 9.80 \times 10^{-1} | 9.93 \times 10^{-1} |
| \( K_p^{tuned} \)                           | 6.18            | 2.12           |
| \( K_i^{tuned} \)                           | 2.08 \times 10^{-2} | 4.77 \times 10^{-3} |

![Figure 3](image318x76to548x253)

Figure 3 Simulation results by applying the proposed scheme where \( \lambda = 0.5 \).
the actuator by predicting the input in advance. In the future, the scheme how to choose $\lambda$ and the controller design considering the input saturation will be studied.

CONFLICTS OF INTEREST

The authors declare they have no conflicts of interest.

REFERENCES

[1] T. Yamamoto, O. Kaneko, S. Wakitani, T. Kinoshita, Y. Ohnishi, H. Kugemoto, et al., Data-oriented PID control (in Japanese), Morikita Publish, Tokyo, 2020.
[2] O. Kaneko, Data-driven controller tuning: FRIT approach, IFAC Proc. Vol. 46 (2013), 326–336.
[3] H. Hjalmarsson, M. Gevers, S. Gunnarsson, O. Lequin, Iterative feedback tuning: theory and applications, IEEE Control Syst. Mag. 18 (1998), 26–41.
[4] M.C. Campi, A. Lecchini, S.M. Savaresi, Virtual reference feedback tuning: a direct method for the design of feedback controllers, Automatica 38 (2002), 1337–1346.
[5] O. Kaneko, T. Nakamura, T. Ikezaki, A new approach to updating feedforward controllers in two-degree-of-freedom control systems estimated response iterative tuning (ERIT) (in Japanese), Trans. Soc. Instrum. Control Eng. 54 (2018), 857–864.
[6] D. Graham, R.C. Lathrop, The synthesis of optimum response: criteria and standard forms, part 2, Trans. AIEE 72 (1953), 273–288.
[7] M. Ogawa, T. Katayama, A robust tuning method for I-PD controller incorporating a constraint on manipulated variable, Trans. Soc. Instrum. Control Eng. E-1 (2001), 265–273.
[8] A. Yoichiro, S. Wakitani, T. Yamamoto, Design of an augmented output-based multiloop self-tuning PID control system, Ind. Eng. Chem. Res. 58 (2019), 11474–11484.

5. CONCLUSION

In this paper, the new data-driven control scheme has been proposed to design a 1DOF by considering input response for unknown structure systems. The effectiveness has been numerically verified by the simulation example. It is possible to predict the appropriate amount of input response to correspond the performance of

AUTHORS INTRODUCTION

**Mr. Yuki Nakatani**

He received his B. Eng. from Hiroshima University in Japan in 2020. He is currently a Master course student in Hiroshima University in Japan.

**Dr. Takuya Kinoshita**

He received his B. Eng., M. Eng. and D. Eng. from Hiroshima University in Japan in 2013, 2015 and 2017, respectively. He was postdoctoral fellow of JSPS (Japan Society for the Promotion of Science) in 2017. He is currently an Assistant Professor with the Department of Graduate School of Advanced Science and Engineering, Hiroshima University, Japan. His research interests are performance-driven control.
Prof. Toru Yamamoto

He received the B. Eng. and M. Eng. degrees from Tokushima University, Tokushima, Japan, in 1984 and 1987, respectively, and the D. Eng. degree from Osaka University, Osaka, Japan, in 1994. He is currently a Professor with the Graduate School of Advanced Science and Engineering, Hiroshima University, Japan. He was a Visiting Researcher with the Department of Mathematical Engineering and Information Physics, University of Tokyo, Tokyo, Japan, in 1991. He was an Overseas Research Fellow of the Japan Society for Promotion of Science with the University of Alberta for 6 months in 2006. His current research interests are in the area of data-driven control, and process control. He was the recipient of the Commendation for Science and Technology by the Minister of Education, Culture, Sports and Technology in 2009.