Adaptive Multi-Point Temperature Control for Microwave Heating Process via Multi-Rate Sampling

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Abstract: Microwave heating has been gradually extended to industrial material process from domestic microwave ovens because of its substantial advantages such as high-efficiency, pollution free, and selective heating. Unfortunately, the drawback of the temperature non-uniformity, which may cause thermal runaway, becomes an obstruction for the development of microwave energy. Besides, a common problem associated with microwave heating systems is that the speed of microwave power transmission is faster than the temperature detection period. Thus, to ensure the global temperature uniformity and to enhance the system adaptivity for deviation of the temperature detecting position in the microwave heating system with input constraints, a multi-rate simple adaptive multi-point temperature control strategy based on almost strictly positive real conditions is proposed, where the use of multi-rate sampling and lifting technique is to solve the case that the system has less inputs than outputs. Finally, simulation results demonstrate the effectiveness of the proposed control strategy.

Key Words: microwave heating, temperature control, multi-rate sampling, adaptive control.

1. Introduction

As a novel heating method with a series of advantages, microwave heating has been extended to industrial applications rapidly in the last few decades, such as drying, sintering of ceramics, melting, and so on [1]. In the conventional heating process, absorbed thermal energy of the materials relies on the processes of conduction, convection, and radiation from the heating sources. This heating method consumes a greater deal of resource and requires relatively longer processing time. Different from conventional one, microwave can generate thermal energy directly due to internal friction of molecules under the effect of an electromagnetic field. Compared with conventional heating, microwave heating offers several advantages, including faster start-up speed, higher energy utilization, shorter processing time, and non-pollution [2]. Nevertheless, the non-uniform temperature distribution, which usually leads to some problems such as overheating and thermal runaway, is always a major drawback for microwave heating process [3],[4]. Coupled with the characteristics of nonlinearity, time variance, and uncertainty [5], it is important to control multi-point temperatures of materials and guarantee the uniformity under high efficiency.

Adaptive control can adjust its own characteristics automatically to conform to new circumstance so that the system can work in the optimal or sub-optimal state according to the control objectives [6]. In addition, it can also achieve good performance when the controlled plant is characterized by parameter uncertainties, nonlinearity, and time variance. Heretofore, the application of adaptive control has been extended from aviation and aerospace fields to industrial process, where also contains the microwave heating. Some relevant work for adaptive temperature control problem in microwave heating process is also reported, such as simple adaptive model-based controllers [7], adaptive neural fuzzy inference system [8], model reference adaptive control with expert control system [9], adaptive neural control [10],[11]. Thus, it is practical to apply adaptive control in the microwave heating process.

In the actual microwave heating systems, the speed of microwave power transmission is normally faster than the period of temperature detection. From the perspective of control, this phenomenon can be seen as that the execution cycle is faster than the feedback cycle in closed-loop control. Besides, to ensure the global temperature uniformity, multi-point temperature tracking control must be employed. However, the electric field strength is the only input of the microwave heating process; in other words, the number of outputs exceeds that of inputs, and the system is non-square. Thus, there exists a multi-rate problem in the microwave heating temperature control process, of which the above-mentioned controllers ignore it. In our previous work [12], a multi-rate adaptive control strategy is designed to address this problem. However, the adopted microwave heating model ignores the heat dissipation on the boundary of the material, which deteriorates the accuracy of the model. Thus, a microwave heating model with inhomogeneous boundary condition, which considers the heat dissipation, is employed in this paper. However, when considering the heat dissipation in the model, a disturbance can be generated if the previous adaptive control algorithm is adopted. Thus, to prevent the temperature tracking error, an improved multi-rate adaptive control strategy is designed to track the multi-point temperature of the improved model.

The rest of the paper is organized as follows: Section 2 is de-
voted to deriving a square lifted discrete-time system with the multi-rate sampling scheme and lifting technique based on the SIMO microwave heating one-dimensional temperature model. In Section 3, by introducing an error system and almost strictly positive real (ASPR) conditions, an improved adaptive controller is designed based on the augmented ASPR error system with parallel feedforward compensator (PFC). And the stability of the closed-loop system is also discussed. Section 4 validates the superiority of the proposed multi-rate adaptive control strategy by comparing the performances with the single-rate control strategy. Finally, conclusions are presented in Section 5.

2. Preliminaries

2.1 Model Description and Multi-Rate Discretization

The one-dimensional temperature distribution model of microwave heating, which belongs to typical partial differential equations (PDEs) in the expression, can be transformed into the traditional numerical models by many numerical methods, such as finite-different time-domain (FDTD), finite-volume time-domain (FVTD), transmission line matrix (TLM), and so on. These numerical models can well represent the change of physical fields, but it is not convenient to extract state characteristics or design a controller. For this reason, an effective field method, which can derive a state space representation from traditional numerical models by many numerical methods, such as the homogeneous PDE, is proposed in [13] for the microwave heating temperature model, which is employed in this paper. It considers the homogeneous Neumann boundary conditions, \( \frac{\partial f(z)}{\partial z} = f_2(T) \) and \( \frac{\partial f(z)}{\partial z} = f_2(T) \), where \( z \) denotes the distance from the origin along the \( Z \) axis, \( L \) is the total length in the direction of the \( Z \) axis, \( T = T(z,t) \) denotes the temperature at each time and \( z \) direction, and \( f_2(T) \) and \( f_2(T) \) denote the nonhomogeneous boundary conditions in different positions. The symbolic model for microwave heating can be seen in Fig. 1. The boundary conditions adopted here are different from our previous study [12]. The corresponding state-space representation for the plant is described by \( G_c \) [13]:

\[
\begin{align*}
\bar{T}(t) &= A_c \bar{T}(t) + b_c u(t) + d_c, \\
T(t) &= C_c \bar{T}(t) + f_c,
\end{align*}
\]

(1)

where \( \bar{T}(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^1, T(t) \in \mathbb{R}^m \) are the state vector, control input, and plant output, respectively. The control input is the square of initial electric field strength \( |E|_0^2 \), and the plant output is the temperature of the medium in \( m \) points. The parameter matrix and disturbance can be calculated as follows:

\[
A_c = \frac{\kappa}{\rho C_p} \text{diag}(\lambda_0, \lambda_1, \ldots, \lambda_{n-1}),
\]

(2)

\[
b_c = \frac{2}{\rho C_p L} \int_0^L \pi [\phi(z), \phi(z), \ldots, \phi_{n-1}(z)]^T dz,
\]

(3)

\[
C_c = [2c_i(z_1), c_i(z_2), \ldots, c_i(z_m)]^T,
\]

(4)

\[
d_c = \frac{2k_g}{\rho C_p L} \int_0^L \pi [\phi(z), \phi(z), \ldots, \phi_{n-1}(z)]^T dz,
\]

(5)

\[
f_c = [f(z_1), f(z_2), \ldots, f(z_m)]^T,
\]

(6)

where \(\lambda_i = \left(\frac{i \pi}{L}\right)^2 (i = 0, 1, 2, \ldots, n-1)\),

\[
c_i(z) = [2\phi_i(z), \phi_i(z), \ldots, \phi_{i-1}(z)]^T
\]

\[
\phi_0(z) = \frac{1}{2}, \phi(z) = \cos \frac{i \pi z}{L} (i = 1, 2, \ldots, n-1),
\]

\[
f(z) = \left[\frac{f_1(T) - f_2(T)}{2L} \pi_1 z + f_1(T)\right] (i = 1, 2, \ldots, m),
\]

where \( \rho, C_p, \) and \( \kappa \) are the material density, specific heat capacity, and thermal conductivity, respectively; \( f_i \) can be regarded as the disturbance relevant to the boundary condition, while \( d_i \) can be regarded as the disturbance related to material characteristics; \( g \) is a constant, \( \phi(z) \) denotes the eigenfunction of position, \( z_i \) is the position at the \( i \)th point.

In the above microwave heating system, the system input is the square of the initial electric field strength, which must be non-negative. Besides, to protect the equipment, the output power of magnetron typically has a saturation feature, and thus the initial electric field strength also has a saturation feature. Therefore, the input constraints of the microwave heating system can be given as

\[
u_c(t) = \begin{cases} 0, & u(t) \leq 0, \\ u(t), & 0 < u(t) \leq E_{\text{max}}^2, \\ u_{\text{sat}}, & u(t) > E_{\text{max}}^2. \end{cases}
\]

(7)

2.2 Discretization Using Multi-Rate Sampling and Lifting

It is obvious that the plant \( G_c \) in (1) is non-square when multi-point temperatures are chosen as the outputs. However, for the use of a simple adaptive control strategy, a critical necessary condition is to ensure that the system is square. Thus, a square discrete-time system will be derived by using multi-rate sampling and lifting technique in the microwave heating system.

The multi-rate sampled-data system is depicted in Fig. 2, where \( S_{R_e} \) is an ideal sampler with period \( T_0 \), and \( H_{R_0/m} \) is the zero-order hold operator with period \( T_0/m \). Therefore, the correspondingly discretized system \( G_d \) can be expressed as:

\[
G_d = S_{R_e} G_c H_{R_0/m}.
\]

Then, the lifting technique is adopted so that the input number is equal to the output number with the frame period \( T_0 \). Therefore, lifting system can be expressed as

\[
\begin{bmatrix}
\bar{u}(k) \\
\bar{u}_c(k) \\
\vdots \\
\bar{u}_{R_0/m}(k)
\end{bmatrix} =
\begin{bmatrix}
u_c(kT_0) \\
u_c(k + 1/mT_0) \\
\vdots \\
u_c(k + (m-1)/mT_0)
\end{bmatrix}
\]

(8)

Here, the lifted input \( \bar{u}(k) \) is defined as

\[
\bar{u}(k) = \begin{bmatrix}
\bar{u}_1(k) \\
\bar{u}_2(k) \\
\vdots \\
\bar{u}_{R_0/m}(k)
\end{bmatrix} =
\begin{bmatrix}
u_c(kT_0) \\
u_c(k + 1/mT_0) \\
\vdots \\
u_c(k + (m-1)/mT_0)
\end{bmatrix}
\]
Thus, \( \overline{u}(k) \) is an \( m \)-th order vector. Let \( L_m \) be an \( m \)-fold lifting operator which maps \( \overline{u}(k) \) into \( u(k) \). Then, there exists an inverse lifting operation \( L_m^{-1} \), and the lifted system \( \overline{G}_d \) with the period \( T_f \) can be derived as

\[
\overline{G}_d = G_d L_m^{-1} = S_{T_h} G_i H_{T_i/m} T_m^{-1}.
\]

(9)

It is obvious that \( \overline{G}_d \) is square. The state-space representation for \( \overline{G}_d \) is obtained by

\[
\begin{align*}
\overline{T}(k + 1) &= A_d \overline{T}(k) + B_d \overline{u}(k) + d_d, \\
T(k) &= C_d \overline{T}(k) + f_d,
\end{align*}
\]

(10)

where

\[
A_d = e^{A_T T_f}, \\
B_d = [e^{A_T (m-1)T_i/b_1}, \ldots, e^{A_T/m b_1}],
\]

\[
\hat{b} = \int_0^{T_i/m} e^{A_T \tau} d\tau, \quad C_d = C, \\
d_d = \int_0^{T_i} e^{A_T \tau} d\tau, \quad f_d = f.
\]

3. Design of Adaptive Controller with Input Constraints

3.1 Problem Statement

The objective in this paper is to design an adaptive control scheme such that the plant output \( T(k) \) tracks the reference output \( T_m(k) \) asymptotically for the system \( \overline{G}_d \) with bounded disturbances under the constraint of non-negative input.

First, the following assumptions are imposed.

Assumption 1. The relative degree of each subsystem \( G_{ij} \) is 1, where \( G_{ij} \) is the \( i \)-th row and the \( j \)-th column element of the square system \( A_d, B_d, C_d \), and \( i, j = 1, \ldots, m \).

Assumption 2. The square system \((A_d, B_d, C_d)\) is minimum phase.

Next, consider the reference output \( T_m(k) \) that satisfies the following conditions:

\[
\begin{align*}
M(z)[T_m(k)] &= 0, \\
\|T_m(k)\| &\leq \bar{m},
\end{align*}
\]

(11)

where the polynomial matrix \( M(z) \) is a diagonal matrix, and \( z \) is the forward-shift operator. In our applications, \( M(z) \) is selected to make the difference between the next moment and current reference temperature \( T_m(k) \) becomes 0 when time step trends to infinity, while the second condition makes \( T_m(k) \) bounded.

Since the MIMO square lifted system \((A_d, B_d, C_d)\) is derived by the SIMO system, similar to the method proposed in [16],[17], an internal model filter \( \overline{G}_{IM}(z) = N(z)M^{-1}(z) \) connected in front of the MIMO square lifted system shall be introduced, where the polynomial matrix \( N(z) \) is a stable diagonal matrix, and the order of each element is correspondingly equal to that of \( M(z) \).

The system with an internal model filter is shown in Fig. 3.

Based on the above conditions, the error system \( \overline{G}_e \) including the internal model filter with disturbances can be represented by

\[
\begin{align*}
\overline{T}_e(k + 1) &= A_e \overline{T}_e(k) + B_e \overline{u}_{IM}(k) + d_e, \\
e_T(k) &= C_e \overline{T}_e(k) + f_e,
\end{align*}
\]

(12)

where \( A_e, B_e, \) and \( C_e \) are the system matrices of \( \overline{G}_e \), while \( d_e \) and \( f_e \) can be regarded as the corresponding disturbances of the error system, which are related to \( d_i \) and \( f_i \), and \( e_T(k) = T(k) - T_m(k) \) is the output error.

Therefore, the objective in this paper is equivalent to designing an adaptive controller which stabilizes the error system \( \overline{G}_e \) (12) with the bounded disturbances \( \overline{u}_i(k) \geq 0 \), where \( \overline{u}_i(k) \) is the \( i \)-th input of \( \overline{u}(k) \).

Remark 3.1 The internal model filter aims to obtain an error system. The error system (12) is indispensable to analyze the stability of a closed-loop system after being implemented the adaptive controller, but its parameters, \( A_e, B_e, \) and \( C_e \) will not be used in the adaptive control rules, that is, it is not required to know an exact error system. In addition, the performance of the system can be improved through the internal model filter, for instance, an integrator is chosen as the internal model filter.

3.2 Augmented ASPR System and PFC

In order to apply an adaptive control scheme based on ASPR conditions, an ASPR controlled system should be obtained. The sufficient conditions for the discrete-time system to be ASPR [18],[19] are given as follows:

- The system has the same number of finite poles and finite zeros;
- All the zeros are stable, namely the system is minimum phase.

Evidently, the square error system \( \overline{G}_e \) is not ASPR due to the non-existence of direct feedthrough matrix.

Now, impose the following assumption:

Assumption 3. For the controllable and observable error system \( \overline{G}_e \) without the disturbances, there exists a known stable PFC

\[
\begin{align*}
\overline{T}_f(k + 1) &= A_f \overline{T}_f(k) + B_f \overline{u}_{IM}(k), \\
T_f(k) &= C_f \overline{T}_f(k) + D_f \overline{u}_{IM}(k),
\end{align*}
\]

(13)

where \((D_f + D_f^T) > 0\), such that the augmented system \( \overline{G}_a = \overline{G}_e + \overline{G}_f \) in (14) is ASPR.

\[
\begin{align*}
\overline{T}_a(k + 1) &= A_a \overline{T}_a(k) + B_a \overline{u}_{IM}(k), \\
e_a(k) &= C_a \overline{T}_a(k) + D_a \overline{u}_{IM}(k),
\end{align*}
\]

(14)

where \( \overline{T}_a(k) = \begin{bmatrix} \overline{T}_e(k) \\ \overline{T}_f(k) \end{bmatrix}, A_a = \begin{bmatrix} A_e & 0 \\ 0 & A_f \end{bmatrix}, B_a = \begin{bmatrix} B_e \\ B_f \end{bmatrix}, C_a = \begin{bmatrix} C_e & C_f \end{bmatrix}, D_a = D_f. \)

It should be noted that according to the definition of an ASPR system and Assumption 3, there exists a static output feedback

\[
\overline{u}_{IM}(k) = \overline{K}_e^* e_a(k) + \nu(k), \quad \overline{K}_e^* > 0,
\]

(15)

such that the resulting closed-loop system, after an input transformation \( \overline{v}(k) = (I + \overline{K}_e^* D_a)^{-1} \nu(k) \),

\[
\begin{align*}
\overline{T}_a(k + 1) &= A_a \overline{T}_a(k) + B_a \overline{v}(k), \\
e_a(k) &= C_a \overline{T}_a(k) + D_a \overline{v}(k),
\end{align*}
\]

(16)
is strictly positive real (SPR). In this case,\[ A_{ac} = A_0 - B_0 \hat{K}_r (I + D_0 \hat{K}_r)^{-1} C_0, \]
\[ C_{ac} = (I + D_0 \hat{K}_r)^{-1} C_0. \]
Further, since the system (16) is SPR, there exist positive symmetric matrices \( P = P^T \geq 0, \hat{Q} = \hat{Q}^T > 0 \) and appropriate matrices \( L, W \) such that based on the Kalman–Yakubovich Lemma, the following hold:\[ \begin{align*}
A_0^T P A_{ac} & = -LL^T - \hat{Q}, \\
A_0^T P B_0 & = C_0^T L W, \\
B_0^T P B_0 & = D_0 + D_0^T - W^T W.
\end{align*} \]

### 3.3 Adaptive Control Law with Input Constraints

Based on the ASPR conditions, the improved adaptive controller which could conform to the input constraints of the plant is designed by\[ \bar{u}_{IM}(k) = -H(k) K_c(k) \hat{e}_a(k), \] (18)
where \( \hat{e}_a(k) = C_l \bar{T}_c(k) = e_I(k) + C_l \bar{T}_f(k), \) and \( H(k) \in \mathbb{R}^{m \times m} \) is an appropriate shift matrix which renders the input \( \bar{u}_a(k) \geq 0. \) With the change of the output error, the feedback gain matrix \( K_c(k) \) in (18) is adaptively adjusted, and the adjusting law is\[ \begin{align*}
K_c(k) & = K_R(k) + K_I(k), \\
K_R(k) & = e_a(k) \hat{e}_a(k)^T \Gamma_p, \\
K_I(k) & = K_I(k-1) + e_a(k) \hat{e}_a(k)^T \Gamma_I - \sigma K_J(k),
\end{align*} \]
(19, 20, 21)
where \( \Gamma_p = \Gamma_p I > 0, \Gamma_I = \Gamma_I T > 0, \) and \( 0 < \sigma < 1. \)

It is noteworthy that the augmented error \( e_a(k) \) cannot be directly obtained by a simple algebraic operation because it consists of the control input. However, by using the available formulas, we have\[ \begin{align*}
e_a(k) & = \hat{e}_a(k) + D_0 \bar{u}_{IM}(k) \\
& = \hat{e}_a(k) - D_0 H(k) K_I(k) \hat{e}_a(k), \\
& = \hat{e}_a(k) - D_0 H(k) \left\{ \frac{K_{r(k-1)}}{1 + \sigma} + e_a(k) \hat{e}_a(k)^T \Gamma_I \right\} \hat{e}_a(k),
\end{align*} \]
(22)
where \( \Gamma_I = \Gamma_I T + (1/(1 + \sigma)) \Gamma_p, \) and \( e_a(k) \) can be derived by\[ e_a(k) = \left\{ \frac{I + \hat{e}_a(k)^T \Gamma_I \hat{e}_a(k) + D_0 H(k)}{1 + \sigma} \right\} \hat{e}_a(k), \] (23)
and the adaptive control system with the input constraints is illustrated in Fig. 4.

However, it is too difficult to choose an appropriate shift matrix \( H(k) \) directly which leads to \( 0 \leq \bar{u}_a(k) \leq u_M (i = 1, \ldots, m). \) So the following design steps are proposed to overcome this problem.

**Step 1** Assume that the shift matrix \( H(k) = I, \) where \( I \) is the \( m \)-dimensional identity matrix. In this case, calculate the error \( e_a(k), \) the feedback gain \( K_c(k), \) and the input \( \bar{u}_{IM}(k) \) successively.

**Step 2** According to the input \( \bar{u}_{IM}(k) \) deduced in Step 1 and the internal model filter \( \hat{u}_{IM}(k), \) obtain the input \( \bar{u}(k). \)

**Step 3** From Step 2, judge whether \( 0 \leq \bar{u}(k) \leq \bar{u}_{\text{max}} (i = 1, \ldots, m) \) or not. If yes, then \( \bar{u}_{IM}(k) = -K_c(k) \hat{e}_a(k); \) if not, then choose an appropriate \( \bar{u}_{IM}(k) \) to have \( 0 \leq \bar{u}(k) \leq \bar{u}_{\text{max}} (i = 1, \ldots, m) \) for all the \( i (i = 1, \ldots, m), \) where \( \bar{u}_{IM} \) satisfies the formula (24). With this appropriate \( \bar{u}_{IM}(k), \) recalculate \( e_a(k) \) and \( K_c(k). \) Thus, \( H(k) \) will be produced an infinite set of values, and it is not necessary to have an exact \( H(k) \) due to this design process.

\[ \text{rank} \begin{bmatrix} \bar{u}_{IM}(k) \\ K_c(k) \hat{e}_a(k) \end{bmatrix} = \text{rank} \begin{bmatrix} K_c(k) \hat{e}_a(k) \end{bmatrix}. \] (24)

### 3.4 Stability Analysis

The following theorem referring to the stability of the control system is concluded.

**Theorem 1** Consider the error system (12) with bounded disturbances, under Assumption 3, all the variables in the resulting closed-loop system with control input in (18) are uniformly bounded. That is, under Assumptions 1–3, the square MIMO plant (10) with the input constraints of both non-negativity and quantization can be stabilized by the adaptive scheme in (18–22).

**Proof:** For the system \( \bar{u}_a(k), \) considering an ideal control input \( \bar{u}_{IM}(k): \)
\[ \bar{u}_{IM}(k) = -K_r^* \hat{e}_a(k), \] (25)
\[ K_r^* = (I + \hat{K}_r D_0)^{-1} \hat{K}_r. \] (26)

Thus, the closed-loop system with input (18) can be represented as\[ \bar{u}_a(k + 1) = \hat{A}_{u} \bar{u}_a(k) + B_0 \bar{u}_{IM}(k) - d_a, \]
\[ e_a(k) = C_0 \bar{u}_a(k) + D_0 \bar{u}_{IM}(k) + (I - D_0 K_r^*) f_a, \] (27)
where\[ \Delta \bar{u}_{IM}(k) = \bar{u}_{IM}(k) - \bar{u}_{IM}(k), \]
\[ \bar{A}_u = A_0 - B_0 K_r^* C_0, \]
\[ C_0 = (I - D_0 K_r^*) C_0, \]
\[ d_a = B_0 K_r^* f_a - d_a, \] (28, 29, 30, 31)
From (26), we have \( I - D_0 K_r^* = (I + D_0 \hat{K}_r)^{-1}. \) According to (29) and (30), we can obtain \( \bar{A} = A_0, \bar{C} = C_0, \) so the closed-loop system is SPR, which satisfies the Kalman-Yakubovich Lemma.

Consider the following positive quadratic function \( V(k): \)
\[ V(k) = V_1(k) + \rho V_2(k), \]
\[ V_1(k) = \bar{T}_a^T \bar{T}_a(k), \]
\[ V_2(k) = \text{tr}(\Delta K_f(k-1) I_{\Gamma}^{-1} \Delta K_f^T(k-1)), \] (32–35)
where\[ \Delta K_f(k) = K_f(k) - K_f^*, \]
Define the difference of \( V(k), \) and we have\[ \Delta V(k) = \Delta V_1(k) + \rho \Delta V_2(k), \]
\[ \Delta V_1(k) = V_1(k + 1) - V_1(k), \]
\[ \Delta V_2(k) = V_2(k + 1) - V_2(k). \] (36–38)
In terms of \( \Delta V_1(k), \) we have\[ \Delta V_1(k) = \bar{T}_a^T(k + 1) \bar{T}_a(k + 1) - \bar{T}_a^T(k) \bar{T}_a(k). \] (39)
From (21) and (35) we obtain

\[ \frac{\partial V(k)}{\partial k} + \frac{\partial V(k)}{\partial y} \]

From (42), it follows that

\[ \delta \leq V(k) + \delta \Delta \]

Scaling the inequality and defining \( \delta \), we get

\[ (1 - \delta) \Delta V(k) \]

Select \( \delta \), \( d_l \), and \( d_u \) satisfying the following inequalities:

\[ 0 < \delta < 1, \quad d_l \geq \| f_k \|, \quad \text{and} \quad d_u \geq \| I - D_s K_s \| = \| (I + D_s K_s) \|^2 \].

From (33) and (39), we can get

\[ \Delta V(k) \leq -\rho(\Delta \max[P] - \delta \Delta \max[P]) \| T_a(k) \|^2 \]

Substituting (41) and (44) into (36), we have

\[ \Delta V(k) \leq -\rho(\Delta \min[P]) \| T_a(k) \|^2 \]

Since we have \( \Delta \max[P] = \Delta \max[P] \| T_a(k) \| \), we obtain

\[ \Delta \max[P] \| T_a(k) \|^2 \]

Substituting into (44), we have

\[ \Delta V(k) \leq -\rho(\Delta \min[P]) \| T_a(k) \|^2 \]

we have

\[ \Delta \max[P] \| T_a(k) \|^2 \]
Thus, we have

\[
\Delta V(k) \leq \psi(k) - \rho(\lambda_{\text{min}}[Q] - \delta I_{\text{max}}[P]) \| T_{\omega}(k) \|^2
\]

where

\[
\psi(k) = 2 \rho d_k \| (I - H(k)) K_\omega \| e_k(k)
\]

If \( H(k) = I \), then \( \psi(k) \leq 0 \); if \( H(k) \neq I \), we should only find the solution which can make \( \min(\psi(k)) \leq 0 \).

\[
\| e_k(k) \|^2 \leq \| C_{\omega} \| \cdot \| T_{\omega}(k) \|
\]

By using this inequality, we have

\[
2 d_k \| \Delta K_\omega \| e_k(k)
\]

for any \( \delta > 0 \). Similarly, we have

\[
2 d_k \| e_k(k) \|^2 \leq -2 \| e_k(k) \|^2 - 2 \| e_k(k) \|^2 \| \Delta K_\omega \|^2 + \delta \| T_{\omega}(k) \|^2
\]

By summarizing above, the following inequality is obtained:

\[
\Delta V(k) \leq -\rho \left( \alpha_1 - \frac{\lambda_{\text{max}}[\Gamma_f]}{2} \| e_k(k) \|^2 \| C_{\omega} \|^2 \right) \| T_{\omega}(k) \|^2
\]

where

\[
\alpha_1 = \lambda_{\text{min}}[Q] - \delta \lambda_{\text{max}}[P] - \delta_1 > 0,
\]

\[
\alpha_2 = \sigma \lambda_{\text{max}}[\Gamma_f^{-1}] > 0,
\]

\[
R_1 = \frac{1}{\delta} \lambda_{\text{max}}[P] \| d_k \|^2 + \rho \sigma \lambda_{\text{max}}[\Gamma_f^{-1}] \| K_\omega \|^2.
\]

Thus, according to (53), it can be concluded that \( T_{\omega}(k) \) and \( K_\omega(k) \) are uniformly bounded.

### 4. Simulation Result and Analysis

In this section, the proposed multi-rate adaptive controller will be validated by simulation. For the one-dimensional microwave heating temperature model, assume that the heated material is deionized water column with length \( l = 8 \) cm. Because the mass transfer and convection are not considered in the model, the big temperature difference at the same heating time is likely to happen. Thus, we only consider the temperature distribution of 2 cm at the right end of the medium which is most sensitive to microwave power variations. The temperatures of the four points, \( z_1 = 0.1 \) cm, \( z_2 = 0.7 \) cm, \( z_3 = 1.3 \) cm and \( z_4 = 1.9 \) cm, are chosen for the outputs of the controlled object. The system sampling period is set as \( T_s = 0.5 \) s, \( T_{\text{yi}} = 2 \) s (\( i = 1, 2, 3, 4 \)).

The range of initial electric field strength \( E \) (V/cm) is set as 0 to 25. All the reference outputs are given as

\[
T_{\text{om}}(t) = \begin{bmatrix} T_{\text{om}1}(t) & T_{\text{om}2}(t) & T_{\text{om}3}(t) & T_{\text{om}4}(t) \end{bmatrix}^T,
\]

where

\[
T_{\text{om}}(t) = \begin{bmatrix} R_t + T_{\text{ini}} & 0 \leq t \leq \frac{T_{\text{om}} - T_{\text{ini}}}{R} \,
\end{bmatrix}^T \quad (i = 1, 2, 3, 4).
\]

The temperature rising rate \( R = 0.25 \), the initial temperature \( T_{\text{ini}} = 20 \) °C and steady-state temperature is \( T_{\text{max}} = 80 \) °C. The internal model that satisfies a series of conditions is given by

\[
\Gamma_{\text{im}}(z) = \begin{bmatrix} \frac{1}{z - 1} & \frac{1}{z - 1} & \frac{1}{z - 1} & \frac{1}{z - 1} \end{bmatrix}
\]

If there exists \( K_{o,0} > 0 \) such that the system is rendered SPR for any \( K_\omega > K_{o,0} \). So there certainly exists an ideal feedback gain. Then we can choose a sufficiently large \( K_\omega \) so as to make \( d_k \) small enough while \( d_k \geq \| (I + D_k K_\omega^{-1}) \| \). So we can obtain

\[
\alpha_1 - \frac{\lambda_{\text{max}}[\Gamma_f]}{2} \| e_k(k) \|^2 > 0,
\]

\[
\alpha_2 - \frac{d_k^2 \| C_{\omega} \|^2}{\delta_1} > 0.
\]

So \( \Delta V(k) \) satisfies

\[
\Delta V(k) \leq -\rho \bar{V}(k) + R_1,
\]

where

\[
\bar{V} = \min\left( \alpha_1 - \frac{\lambda_{\text{max}}[\Gamma_f]}{2} \| e_k(k) \|^2 \left| \lambda_{\text{max}}[P] \right|, \alpha_2 \frac{d_k^2 \| C_{\omega} \|^2}{\delta_1} \right).
\]

The controller parameters are set as \( \Gamma_f = \text{diag}[10, 10, 10, 10], \Gamma_f = \text{diag}[40, 40, 40, 40] \), and \( \sigma = 0.05 \).
Based on the currently selected internal model, if there exists an \( i \) such that \( 0 \leq \bar{u}_i(k) \leq E_{\text{max}}^2 \), the appropriate \( \bar{u}_{IM}(k) \) will be set as

\[
\bar{u}_{IM}(k) = \begin{cases} 
-\bar{u}(k-1) & \bar{u}_i(k) < 0, \forall i (i = 1, 2, 3, 4), \\
E_{\text{max}}^2 - \bar{u}_i(k-1) & \bar{u}_i(k) > E_{\text{max}}^2, \forall i (i = 1, 2, 3, 4),
\end{cases}
\]

where \( \bar{u}_{IM}(k) (i = 1, 2, 3, 4) \) is the each component of \( \bar{u}_{IM}(k) \).

The simulation results are shown in Figs. 5 and 6.

In order to better validate the superiority of the proposed algorithm, we compare it with that of the single-rate sampling. Likewise, the sampling period is set as \( T = 2 \text{s} \). To prevent the thermal runaway, the maximum temperature of the four points \( T_1 \) is chosen as the output. The feed-forward compensator \( G_f = \Gamma_p = 0.005 \) and the controller parameters are designed as \( \Gamma_p = 40, \Gamma_I = 150, \) and \( \sigma = 0.05 \). Figures 7 and 8 show the simulation results that adopt the single-rate adaptive algorithm. Figures 9 and 10 give information about the comparison of global temperature distribution between the multi-rate and single-rate adaptive control algorithms.

From the comparison of the simulation results, we can find that

- Both two algorithms can achieve a good tracking performance for the maximum temperature \( T_1 \), while the tracking errors of single-rate control in \( T_2, T_3, \) and \( T_4 \) are bigger than multi-rate control. Besides, it can be found from Figs. 9 and 10 that the global temperature distribution with multi-rate control is relatively uniform. Overall, compared with the single-rate control system, the system that adopts the proposed multi-rate adaptive controller shows better performances, not only in the temperature uniformity but also in the steady-state errors and the maximum temperature. Thus, the proposed algorithm can improve the microwave heating uniformity.

- The frequency of the input oscillation with the multi-rate
sampling is higher than that with the single-rate sampling.

Finally, the simulation results demonstrate that the improved multi-rate simple adaptive algorithm that considers the input constraints can realize the temperature tracking control effectively; at the expense of the input oscillation frequency, the adaptive strategy using multi-rate sampling obtains better control performances than that of single-rate sampling, particularly in the problem of temperature uniformity.

5. Conclusion

In this paper, adaptive control of the microwave heating process is designed. The control objectives are to guarantee the multiple outputs perform tracking of the reference outputs and achieve good performance. Therefore, an improved adaptive control strategy using multi-rate sampling has been proposed in the microwave heating system with input constraints. And the stability of the closed-loop system is analyzed. Finally, this algorithm is validated by numerical simulation, and the results show that the proposed scheme has better performance than that of single-rate. Moreover, it provides guidance to avoid thermal runaway phenomenon in the microwave heating process.

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