Orientation of Small UAV Images with Constraint

Xingxing Liu¹,²*, Guoqing Zhou¹*, Man Yuan¹,²
¹Guangxi Key Laboratory of Spatial Information and Geomatics, Guilin University of Technology, Guilin, Guangxi, 541004, China
²College of Geomatics and Geoinformation, Guilin University of Technology, Guilin, Guangxi, 541004, China
*Corresponding author’s e-mail: 1808319688@qq.com; gzhou@glut.edu.cn

Abstract. Unmanned aerial vehicle (UAV) platform, especially in image acquisition, has the advantages of low cost and convenience compared with other sensors platform. Due to the limitation of platform load and cost, small UAV integrated positioning and orientation system (POS) has low accuracy, which makes it difficult to accurately reconstruct the geometric position and attitude of the sensor at the imaging time. For this reason, this paper proposes an angle consistent constraint algorithm to improve the accuracy of exterior orientation parameters. In this paper, the small UAV images measured in the field are used for experiments, and the results show that the constraint algorithm can achieve high accuracy when the number of control points is small and the accuracy of initial values is low, which greatly expands the application space of images.

1. Introduction
UAV has the advantages of low cost and convenience, which is widely used in military and civil fields [1]. Such as homeland security, hazards and disaster risk monitoring, emergency mapping, and earth science research [2-3]. The object can be located in the world coordinate by using the pixel coordinates of the object in the image and the position and attitude of the sensor [4]. Therefore, the exterior orientation parameters are the basis of geometric processing of the image. Due to the limitation of platform load and cost, the accuracy of POS integrated by small UAV is low [5], which makes the exterior orientation parameters of the original image have great error. And the exterior orientation parameters have a high correlation, which makes it very difficult to solve the orientation parameters of the image. Zhou [6] established a direct positioning model of UAV video flow, and solved the video camera’s interior orientation parameters and the exterior orientation parameters of each video frame. With analysis above, this paper proposes the angle consistent constraint algorithm, which decouples the exterior orientation parameters by using the constraint equation, and constructs the orientation model with additional constraint. The method proposed in this paper can solve the exterior orientation parameters rightly and effectively, and the accuracy of the parameters solved using our method is increased, which provides an accurate orientation model for the subsequent geometric processing of the image, and improves the application potential of the image.

2. Orientation model with constraint
2.1. Rigorous sensor model
For frame sensor image, the collinear equation is expressed as
\[x_a - x_0 = -f \left( a_1(X_G - X_S) + b_1(Y_G - Y_S) + c_1(Z_G - Z_S) \right) / a_3(X_G - X_S) + b_3(Y_G - Y_S) + c_3(Z_G - Z_S) \]
\[y_a - y_0 = -f \left( a_2(X_G - X_S) + b_2(Y_G - Y_S) + c_2(Z_G - Z_S) \right) / a_3(X_G - X_S) + b_3(Y_G - Y_S) + c_3(Z_G - Z_S) \]

(1)

Where \(x_0, y_0, f\) are the interior orientation parameters of the camera. \((x_G, y_G)\) are the image coordinates corresponding to the ground point G. \((X_S, Y_S, Z_S)\) are the spatial coordinates of sensors in the ground coordinate system. \(a, b, c (i = 1, 2, 3)\) are the elements of the rotation matrix, which is related to the rotation angles \(\phi, \omega, \kappa\). \((X_G, Y_G, Z_G)\) are the coordinates of the ground point G in the ground coordinate system.

The equation (1) can be written:
\[V = A\hat{x} - l\]

(2)

Where \(A\) is the coefficient matrix, \(V = (v_{g_1}, v_{g_2}, ..., v_{g_m})^T\), \(l = (l_{g_1}, l_{g_2}, ..., l_{g_m})^T\), \(\hat{x} = (\Delta X_S, \Delta Y_S, \Delta Z_S, \Delta \phi, \Delta \omega, \Delta \kappa)^T\), when \(\tilde{Z} = a_3(X_G - X_S) + b_3(Y_G - Y_S) + c_3(Z_G - Z_S)\).

\[
A = \begin{bmatrix}
\frac{1}{Z}(a,f + a,x) & \frac{1}{Z}(a,f + a,x) \\
\frac{1}{Z}(b,f + b,x) & \frac{1}{Z}(b,f + b,x) \\
\frac{1}{Z}(c,f + c,x) & \frac{1}{Z}(c,f + c,x) \\
\frac{y}{f}\sin(\omega) - \left( \frac{x}{f}\cos(\omega) - \frac{y}{f}\sin(\omega) \right) & -\frac{x}{f}\cos(\omega) - \frac{y}{f}\sin(\omega) \\
\frac{y}{f}\sin(\omega) - \left( \frac{x}{f}\cos(\omega) - \frac{y}{f}\sin(\omega) \right) & -\frac{x}{f}\cos(\omega) - \frac{y}{f}\sin(\omega) \\
\end{bmatrix}
\]

2.2. Angle consistent constraint
Assuming that \(SA\) and \(SB\) are two imaging rays, the spatial coordinates of center of projection S and ground point A, B are respectively \((X_S, Y_S, Z_S), (X_A, Y_A, Z_A), (X_B, Y_B, Z_B)\), \(a\) and \(b\) are the image points corresponding to ground points A and B, whose image coordinates are respectively \((x_a, y_a)\) and \((x_b, y_b)\).

Since the position of sensor probe is fixed, the angle \(\angle aSB\) between the vector \(Sa\) and vector \(Sb\) formed by the connecting line between the center of projection and center of each sensor probe and the angle \(\angle ASB\) between the vector \(SA\) and vector \(SB\) formed by the center of projection and the ground points remain unchanged, i.e., \(\angle aSB = \angle ASB\). This is independent of the attitude angle of the sensor (see Figure. 1).
Figure 1. Angle consistent constraint.

When \( \overline{Sa} = (x_a, y_a, z_a) = (x, y, f) \), \( \overline{SA} = (X_A, Y_A, Z_A) = (X_A - X_S, Y_A - Y_S, Z_A - Z_S) \),

\( \overline{Sb} = (x_b, y_b, z_b) = (x, y, f) \), \( \overline{SB} = (X_B, Y_B, Z_B) = (X_B - X_S, Y_B - Y_S, Z_B - Z_S) \),

the constraint equation can be expressed as:

\[
\frac{x_y x_2 + y_1 y_2 + z_1 z_2}{\sqrt{(x_1^2 + y_1^2 + z_1^2)} \cdot \sqrt{(x_2^2 + y_2^2 + z_2^2)}} = \frac{X_1 X_2 + Y_1 Y_2 + Z_1 Z_2}{\sqrt{(X_1^2 + Y_1^2 + Z_1^2)} \cdot \sqrt{(X_2^2 + Y_2^2 + Z_2^2)}}
\]

(3)

Linear equation for solving Equation (3) can be written:

\[
C_1 \hat{x} + W_1 = 0
\]

(4)

Where \( C_1 \) is the coefficient matrix, \( \hat{x} \) is the unknown parameters, \( \hat{x} = (\Delta X_S, \Delta Y_S, \Delta Z_S)^T \), \( W_1 \) is the constant term.

When \( B_1 = \sqrt{(X_1^2 + Y_1^2 + Z_1^2)} \), \( B_2 = \sqrt{(X_2^2 + Y_2^2 + Z_2^2)} \):

\[
W_1 = \frac{(X_A - X_S)(X_B - X_S) + (Y_A - Y_S)(Y_B - Y_S) + (Z_A - Z_S)(Z_B - Z_S)}{\sqrt{(X_A - X_S)^2 + (Y_A - Y_S)^2 + (Z_A - Z_S)^2} \cdot \sqrt{(X_B - X_S)^2 + (Y_B - Y_S)^2 + (Z_B - Z_S)^2}}
\]

\[
\left(\frac{x_y x_2 + y_1 y_2 + z_1 z_2}{\sqrt{(x_1^2 + y_1^2 + z_1^2)} \cdot \sqrt{(x_2^2 + y_2^2 + z_2^2)}}\right)^2 - \frac{x_y x_2 + y_1 y_2 + z_1 z_2}{\sqrt{(x_1^2 + y_1^2 + z_1^2)} \cdot \sqrt{(x_2^2 + y_2^2 + z_2^2)}}
\]
From equations (2) and (4) we get the system of linear equations:

\[
\begin{align*}
V &= A\hat{x} - l \\
C_1\hat{x} + W_i &= 0
\end{align*}
\]  

(5)

Where \( A \) is the coefficient matrix, \( l \) is the constant term, and \( \hat{x} = (\Delta\phi, \Delta\omega, \Delta\kappa, \Delta X_g, \Delta Y_g, \Delta Z_g)^T \).

According to the least square principle, the Lagrange multiplier method is used to solve the equations (5):

\[
\begin{align*}
\Phi &= V^TPV + 2K_S^T(C_1\hat{x} + W_i) \\
P &= I
\end{align*}
\]  

(6)

Where \( K_S \) is the Lagrange multiplier, and the normal equation can be expressed as

\[
\begin{align*}
A^T PA\hat{x} + C_1^T K_S - A^T Pl &= 0 \\
C_1^T\hat{x} + W_i &= 0
\end{align*}
\]  

(7)

Where \( N_{AA} = A^T PA, \ W = A^T Pl \), then equation (7) can be expressed as:

\[
\begin{bmatrix}
N_{AA} & C_1^T \\
C_1 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{x} \\
W_i
\end{bmatrix} +
\begin{bmatrix}
-W \\
0
\end{bmatrix} = 0
\]  

(8)

Where \( d_{11} = N_{AA}^{-1} - N_{AA}^{-1}C_1^T(C_1N_{AA}^{-1}C_1^T)^{-1}C_1N_{AA}^{-1} \), \( d_{12} = N_{AA}^{-1}C_1^T(C_1N_{AA}^{-1}C_1^T)^{-1} \), \( d_{21} = (C_1N_{AA}^{-1}C_1^T)^{-1}C_1N_{AA}^{-1} \), \( d_{22} = -(C_1N_{AA}^{-1}C_1^T)^{-1} \).

Then, the inverse matrix of the coefficient matrix can be expressed as:

\[
\begin{bmatrix}
N_{AA} & C_1^T \\
C_1 & 0
\end{bmatrix}^{-1} =
\begin{bmatrix}
d_{11} & d_{12} \\
d_{21} & d_{22}
\end{bmatrix}
\]  

(9)

The solution of the unknown parameters can be expressed as:

\[
\begin{align*}
\hat{x} &= d_{11}W - d_{12}W_i \\
K_S &= d_{21}W - d_{22}W_i
\end{align*}
\]  

(10)

3. Experiments and analysis

In order to verify the validity and feasibility of constraint model proposed in this paper, the small UAV image covering the Guilin university of technology, China, is used to evaluate our method. 34 ground points is collected from the image (see Figure 2.) and the geographic coordinates of ground points are obtained by the GNSS receiver. The parameters of camera are shown in Table 1.
Table 1. The parameters of camera.

| The Size of Pixel (μm) | Nominal Focal Length (mm) | Calibrated Focal Length (pixel) | Image Center (pixel) | Principal Point Offset (pixel) |
|------------------------|---------------------------|---------------------------------|----------------------|-------------------------------|
| 2.4                    | 8.8                       | $f_x$                           | $f_y$                | $x_0$ | $y_0$ | $dx$ | $dy$ |
|                        |                           | 3668.41                         | 3661.6               | 2736  | 1824  | -26.33 | 13.56 |

Some of the 34 known points are selected as ground control points (GCPS) to solve the exterior orientation parameters, and the other points are used as checkpoints (CPS) to evaluate the solving accuracy of exterior orientation parameters. The experimental results are shown in Table 2.

Table 2. The accuracy comparison of the two methods.

| Methods            | GCPs | $X_S$(m) | $Y_S$(m) | $Z_S$(m) | $\varphi$(rad) | $\omega$(rad) | $\kappa$(rad) | RMS(Pixel) |
|--------------------|------|----------|----------|----------|----------------|---------------|--------------|------------|
| Traditional Model  | 6    | 2005428. | 54220    | 268597   | -              | -             | 0.35379     | 3.36       |
|                    | 15   | 2005398. | 54220    | 268591   | 5.6226         | -             | 0.12360     | 1.12       |
| Constraint Model   | 6    | 2005395. | 54220    | 268591   | 5.5626         | -             | 0.05952     | 1.27       |
|                    | 15   | 2005397. | 54220    | 268591   | 5.3597         | -             | 0.09940     | 0.98       |

As observed from Table 2, it can be found that:
In the unconstrained case, the accuracy of exterior orientation parameters can be improved with the increasing number of ground control points.

In the constrained case, the exterior orientation parameters can achieve higher accuracy even if the number of ground control points is small.

In the case of constraint, the number of ground control points has little influence on the solving accuracy of exterior orientation parameters.

4. Conclusion
The solving accuracy of position parameters obtained by our method with angle consistent constraint is improved. This method decouples the exterior orientation parameters by using the constraint equation, so as to improve the accuracy of attitude parameters. And the number of ground control points has little influence on the solving accuracy of exterior orientation parameters in this paper. Therefore, when there are relatively few ground control points, the solving accuracy of parameters is also relatively high, which saves manpower and material resources in filed.

Acknowledgment
This paper is financially supported by the National Natural Science of China (the grant #: 41961065 and 41431179), Guangxi Science and Technology Base and Talent Project (the grant #: Guike AD19254002); the Guangxi Innovative Development Grand Program (the grant #: GuikeAA18118038 and GuikeAA18242048); Guangxi Natural Science Foundation for Innovation Research Team (the grant #: 2019GXNSFGA245001), Guilin Research and Development Plan Program (the grant #: 201902102), the National Key Research and Development Program of China (the grant #: 2016YFB0502501) and the BaGuiScholars program of Guangxi.

References
[1] Kunii, Y. (2018) Development of UAV photogrammetry method by using small number of vertical images. In: ISPRS TC II Mid-term Symposium "Towards Photogrammetry 2020". Riva del Garda, Italy. pp.169-175.
[2] Goncalves, J. A., Henriques, R. (2015) UAV photogrammetry for topographic monitoring of coastal areas. ISPRS J. Photogramm Remote Sens,104:101-111.
[3] Gomez, C., Purdie, H. (2016) UAV-based photogrammetry and geocomputing for hazards and disaster risk monitoring—a review. Geoenviron. Disasters., 3:23.
[4] Barber, D. Blake, Joshua D. Redding, Timothy W. McLain, Randal W. Beard, Clark N. Taylor. (2006) Vision-based target geo-location using a fixed-wing miniature air vehicle. Journal of Intelligent and Robotic Systems, 47: 361-382.
[5] Zhou, G. (2009) Geo-referencing of video flow from small low-cost civilian UAV. IEEE Transactions on automation science and engineering, 7:156-166.
[6] Zhou, G. (2009) Near real-time orthorectification and mosaic of small UAV video flow for time-critical event response. IEEE Transactions on Geoscience and Remote Sensing, 47: 739-747.