A model of hard X-rays emission from free expanding Plasma-Focus discharges

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Abstract. A planar-piston model of the hard x-ray production in Plasma-Focus devices is presented. The model applies Von Karman approximations to represent the inner structure of the pinch. The hard x-ray emission is calculated assuming Bremsstrahlung from the collision on the anode base of an electron current running away from the pinch. The model was applied to analyse the experimental data of a small Plasma Focus without surrounding cathode, founding good agreement.

1. Introduction
Plasma-Focus (PF) devices produce hot ($\sim 0.3 - 1 \text{ keV}$) and dense ($\sim 10^{25} \text{ m}^{-3}$) dynamic pinches inducing ionization by means of high voltage discharges in coaxial geometries. Recently, small devices were developed aiming to applications of the pulsed radiation emissions of the pinch. Miniature devices were constructed following a special configuration derived from capillary discharges, where the surrounding cathode is replaced by a circular plate located at the base of the gun. This feature allows the current sheath to expand freely during the run down.

During the compression x-rays are produced by two mechanisms. A soft x-ray source inside the pinch is produced by line radiation, Coulomb interaction between charged particles and recombination. A hard x-ray source of Bremsstrahlung photons are produced by electrons that escape from the pinch and hit the anode base material. This work presents an extension of a lumped parameter code of free expanding PF [1], introducing a model of electron runaway and hard x-ray production. The model is validated with experimental data of a small PF device.

2. Model Description
A PF are basically two coaxial electrodes separated by an insulator material at one end (the anode at the interior). The interelectrode space is filled with low pressure gas and pulsed high voltage is applied between the electrodes. The discharge starts over the insulator from which the plasma sheath takes-off axially accelerated by the magnetic field generated by the current itself. After the current sheath runs over the open end of the central electrode, the plasma becomes rapidly compressed into a small column resulting in very hot and dense plasma (pinch). In cases where the cathode is a planar plate, the current sheath expands freely radially away from the axis. In the lumped parameter model, the current sheet is represented by planar pistons following a snow-plough approximation. Initially the sheet takes the shape of the insulator and
then expands in axial and radial direction, keeping the cylindrical symmetry and capturing gas with certain efficiency \[2\]. A constant linear mass density is assumed so that gas particles are re-distributed uniformly all over the sheet surface as they are captured, and this approximation is used to estimate the sheet thickness. For details of the evolution equations of the pistons see \[1\].

The pinch starts when the sheet collapses at the open end of the anode. The pinch is modeled as a plasma cylinder compressed by the Lorentz force and obeying the integral MHD equations. The inner structure of the plasma column is represented by means of Von Karman approximations of velocity and density profiles. The detailed set of equations of the pinch dynamics can be found in \[3\]. In the present version a current density profile depending on the radial coordinate \(r\) is included according to:

\[ J(r, t) = K(t) \left[ \frac{r}{R(t)} \right]^\alpha \]  

(1)

where \(R\) is the pinch radius, \(K\) is the current density at the external boundary, and \(\alpha\) is a shape parameter. This current profile brings about axial and radial components of electric field inside the pinch, which can be calculated using the Ohm’s law:

\[ \mathbf{E} + (\mathbf{V} \times \mathbf{B}) = \eta \mathbf{J} \]  

(2)

Where \(\mathbf{E}, \mathbf{V}, \mathbf{B}, \eta\) and \(\mathbf{J}\) represent the electric field, velocity, magnetic field, Spitzer’s resistivity and current density respectively. Considering a magnetic field with only an azimuthal component

\[ B_\theta(r,t) = \frac{1}{2} \frac{\mu_0 i(t)}{2\pi r} \]  

(3)

where \(\mu_0\) is the vacuum permeability and \(i(t)\) is the pinch current, then the electric field is given by:

\[ E_r(r,z,t) = E_r(r,z,t) \hat{e}_r + E_z(r,z,t) \hat{e}_z \]  

(4)

\[ E_z(r,z,t) = \frac{i(t)}{2\pi R^{\alpha+2} r^{\alpha}} \left[ \eta (\alpha + 2) - \frac{\alpha+1}{R^a V_{rad} \mu_0} \right] \]  

(5)

\[ E_r(r,z,t) = \frac{2Z}{H} \frac{\mu_0}{2\pi R^{\alpha+2}} i(t) V_{ax} r^{\alpha+1} \]  

(6)

where \(V_{ax}\) and \(V_{rad}\) are the axial and radial velocities of the cylindrical surfaces respectively. According to Dreicer theory, a fraction of the pinch electrons escape from the pinch accelerated by the axial field if the magnitude of the axial field is higher than the critical value:

\[ E_d = 3.9 \times 10^{-10} n \left[ \frac{1}{cm^7} \right] \frac{1}{K \eta eV} \]  

(7)

\(n\) being the particle pinch density.

Assuming that the energy of a single runaway electron, \(e\), is proportional to an effective voltage, gives

\[ e \sim q H (E_z - E_d) \]  

(8)

where \(H\) is the pinch length and \(q\) is the electron charge. Hence, using the appropriate conversion factors, the total energy produced by Bremsstrahlung in the anode base can be estimated as proportional to the square of the incident electron energy.
3. Results
In order to validate the model, the numerical results were compared with experimental measurements from a Plasma Focus with open cathode. Table 1 details the geometric and electrical parameters of the device [4]. Fig. 1 shows the measured dependence of the hard X-ray intensity on the charging voltage. The detectors were positioned at 90 degrees from the PF axis, and the Deuterium filling pressure was 16.4 mbar. The curve in Fig. 1 shows the numerical results obtained with the present model. The constant parameters used in the calculations are listed in Table 1. It can be seen that the model is able to follow the experimental trend. In particular, note that below 9 kV very low intensities were observed, which is in agreement with the absence of runaway due to the Dreicer condition. Another remarkable result is that a high exponent of the current density profile was needed to fit the experiments ($\alpha = 50$), indicating that the current is concentrated at the pinch border. The latter is in agreement with previous neutron production models where the current was assumed concentrated at the external boundary [5].

4. Conclusions
A lumped parameter model for estimating hard x-ray production in Plasma Focus discharges has been presented. Numerical calculations show good agreement with experimental data, particularly in predicting the dependence of the radiation intensity with the charging voltage. The model set out here offers a useful tool to calculate and design of open-cathode PF devices applied to production of pulsed beams of charged particles and hard x-rays.

References
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| Symbol | Parameter                          | Value       |
|--------|-----------------------------------|-------------|
| $C$    | Bank capacity                      | 0.8 $\mu F$|
| $V$    | Charging voltage                   | 18–28 kV    |
| $L_e$  | External inductance                | 65 nH     |
| $z$    | Anode length                       | 0.191 cm    |
| $R_a$  | Anode radius                       | 0.31 cm     |
| $p_0$  | Filling pressure                   | 16.4 mbars  |
| $\alpha_{sg}$ | Characteristic frequency of the spark gap | 350 ns$^{-1}$ |
| $t_0$  | Characteristic delay of the spark gap | 400 ns     |
| $\xi$  | Efficiency parameter               | 0.2         |
| $\alpha$ | Current shape parameter            | 50          |

Table 1. Parameters of the experiment and the model