Fully gauge-invariant maximally path-dependent gluon TMD: Coordinate representation

This content has been downloaded from IOPscience. Please scroll down to see the full text.
2016 J. Phys.: Conf. Ser. 678 012049
(http://iopscience.iop.org/1742-6596/678/1/012049)
View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 146.175.11.111
This content was downloaded on 25/05/2016 at 11:12

Please note that terms and conditions apply.
Fully gauge-invariant maximally path-dependent
gluon TMD: Coordinate representation

Igor O. Cherednikov

EDF - Departement Fysica, Universiteit Antwerpen, Belgium
E-mail: igor.cherednikov@uantwerpen.be

Abstract. We propose an approach to obtain gauge-invariant maximally path-dependent operator definition of the gluon transverse-momentum dependent distribution function (gTMD). We demonstrate that the evolution equations for the gTMD can be derived from the shape-variation integral-differential equations formulated in the coordinate space.

1. Introduction

Transverse-momentum dependent (unintegrated) parton density functions (TMD pdfs in what follows) extend the idea of collinear (integrated) pdfs, which absorb mass singularities in any order of perturbative expansion, making it possible to apply the QCD factorisation approach to the inclusive hadron processes (e+e− → hadrons, DIS) [1]. In more exclusive cross-sections, which one deals with in the semi-inclusive reactions (semi-inclusive DIS, Drell-Yan, Higgs, vector boson, heavy-flavour production) factorisation often implies intrinsic transverse-momentum dependence of non-perturbative pdfs, which must be constructed in such a way to absorb not only mass, but also rapidity singularities [2, 3, 4, 5, 6]. With TMD pdfs one gets access to the three-dimensional structure of the nucleon in the momentum space, which is actively investigated in ongoing and planned experiments at the LHC, JLab + JLab 12 GeV upgrade, RHIC, EIC etc., providing an incentive to the rapid theoretical progress in the field (see, e.g., [7, 8] and Refs. therein).

As compared to the quark TMD case, the gauge-invariant gluon TMD exhibits much more involved structure of the gauge links (Wilson lines), so that the path-dependence brings about extra complications to the understanding of (non-)universality of the TMD pdfs. On the other hand, this path-dependence allows us to manipulate freely with the gauge links entering the operator definition of the gTMD. In the present paper we develop a quantum-field theoretic approach to the definition of the gluon TMD (gTMD) making use of the arbitrariness of the trajectories of the Wilson lines to formulate evolution of the gTMD in terms of the equations of motion in the so-called generalized loop space, which elements are the hadronic matrix elements of the arbitrary Wilson loops.

2. Operator structure of TMD pdfs

Following the standard approach one derives the operator definitions of the various TMD pdfs starting with the factorisation of a given process in a convenient gauge, which yields an obviously gauge-dependent pdf, such as
that is a hadronic matrix element, which corresponds to the distribution of the gluons with momentum $k$ inside the hadron $h$ with momentum $P$ and spin $S$ [2, 9]. Summing up collinear gluon contributions yields a gauge-invariant pdf supplied with the Wilson lines, whose path $\gamma$ is determined by the factorisation scheme:

$$G_{g-\rho}^{\mu\nu}(k; P, S) = \int d^4 z \; e^{-ikz} \langle h| A^{\mu}(z) A^{\nu}(0) |h\rangle,$$  

(1)

where the field strength is defined in the adjoint representation

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - ig [A^{\mu}, A^{\nu}], \quad F_{\mu\nu} = F_{\mu\nu}^{a} T^{a}.$$  

(3)

Let us explore a possibility of an alternative approach to the definition of the TMD pdfs. We construct first a generic fully gauge-invariant (by construction) and maximally path-dependent object consisting of the Wilson lines/loops defined on the set of absolutely arbitrary trajectories $\gamma$:

$$G_{g-\rho}^{\mu\nu}(k; P, S) \sim \int dk^{-} \; G^{+i+j}(k; P, S) =$$  

$$\int dz^{-} d^2 z_{\perp} \; e^{-ikz} \langle h| F^{+i}(z) W_{\gamma} F^{+j}(0) |h\rangle,$$  

(2)

(4)

(5)

3. Equations of motion in the loop space

Shape variations (or evolution in the coordinate representation) of arbitrary Wilson loops can be consistently formulated in terms of the equations of motion in the loop space, that is the integral-differential equations which the elements of this space (Wilson loops) obey, if the underlying contours $\gamma_{i}$ on which the path-ordered exponentials of the gauge fields are defined experience certain infinitesimal local variations. The variations of the contours give rise to the variations of the exponentials themselves, the latter being described by the infinite set of the Makeenko-Migdal loop equations [10, 11, 12, 13, 14, 15]. More specifically, the elements of the loop space are the vacuum matrix elements of products of the Wilson loops, that is

$$\langle 0|W_{\gamma_{1}}\cdots W_{\gamma_{n}}|0\rangle = \langle 0|TW_{\gamma_{1}}\cdots W_{\gamma_{n}}|0\rangle.$$  

(7)

These fundamental gauge-invariant (by construction) degrees of freedom obey the Makeenko-Migdal loop equations (in the large-$N_{c}$ limit)

$$\partial^{\nu} \frac{\delta}{\delta g^{2}(x)} \langle 0|W_{\gamma_{1}}|0\rangle = N_{c} g^{2} \int_{\gamma} dz_{\perp} \delta^{(4)}(x-z) \langle 0|W_{\gamma_{x}}^{2}|0\rangle,$$  

(8)

where the area and path differential operators are defined as follows [10, 11, 12]:

$$W_{\gamma} = \mathcal{P}_{\gamma} \exp \int_{\gamma} d\zeta_{\mu} A^{\mu}(\zeta).$$  

(6)
• Area derivative describes the behaviour of the Wilson loop under the infinitesimal variations of the area $\delta \sigma_{\mu\nu}$ at a given point

$$\frac{\delta}{\delta \sigma_{\mu\nu}(z)} \langle 0 | W_\gamma | 0 \rangle = \lim_{{|\delta \sigma_{\mu\nu}(z)| \to 0}} \frac{\langle 0 | W_\gamma \delta \gamma_{\sigma} | 0 \rangle - \langle 0 | W_\gamma | 0 \rangle}{|\delta \sigma_{\mu\nu}(z)|} \langle 0 | W_\gamma | 0 \rangle$$

(9)

• Path derivative deals with the infinitesimal path extensions and contractions $\delta z_{-1}^\mu \delta z_{\mu}$ at a given point

$$\partial_\mu \langle 0 | W_\gamma | 0 \rangle = \lim_{{|\delta z_\mu| \to 0}} \frac{\langle 0 | W_{\delta z_{-1}^\mu \delta z_{\mu}} | 0 \rangle - \langle 0 | W_\gamma | 0 \rangle}{|\delta z_\mu|} \langle 0 | W_\gamma | 0 \rangle$$

(10)

These differential operators in the loop space determine the evolution of the Wilson loops in the coordinate representation. In other words, starting from a loop having a given shape, one can come to a loop with another shape by solving the above evolution equations. This is exactly what is needed to adjust a generic Wilson loop to some specific geometrical layout prescribed by a factorisation framework.

4. Stokes-Mandelstam gluon TMD

Let us show how this strategy can be practically implemented. Consider first the Wilson loops themselves, not their matrix elements. This is needed since we want to work with the hadronic matrix elements, not with the vacuum ones. Making use of non-Abelian Stokes’ theorem

$$P_\gamma \exp \oint_\gamma d \zeta A^\mu(\zeta) = P_\gamma P_\sigma \exp \int_\sigma d \sigma_{\mu\nu}(\zeta) F_{\mu\nu}(\zeta)$$

(11)

and the Mandelstam formula

$$\frac{\delta}{\delta \sigma_{\mu\nu}(x)} P_\gamma \exp \oint_\gamma d \zeta A^\mu(\zeta) = P_\gamma F_{\mu\nu}(x) \exp \oint_\gamma d \zeta A^\mu(\zeta),$$

(12)

we see that the generic correlation function (2) can be represented in the following form

$$\tilde{G}^{\mu\nu|\rho\sigma}(z; P, S) = \frac{\delta}{\delta \sigma_{\mu\nu}(z)} \frac{\delta}{\delta \sigma_{\rho\sigma}(0)} \langle h | W_\gamma[z, 0] | h \rangle$$

(13)

$$= \frac{\delta}{\delta \sigma_{\mu\nu}(z)} \frac{\delta}{\delta \sigma_{\rho\sigma}(0)} \sum_X \langle h | W_{\gamma}[z, X] \rangle \langle X | W_{\rho}[0] | h \rangle.$$  

(14)

We coin this representation the \textit{Stokes-Mandelstam gluon TMD} definition. The key feature of this definition (and of the entire approach) is that one first calculates the hadronic matrix element of an arbitrary Wilson loop

$$\langle h | W_\gamma[z, 0] | h \rangle,$$

choosing its shape the most convenient for the practical purposed. In particular, in the situations where the non-Abelian exponentiation is applicable

$$\langle h | W_\gamma[z, 0] | h \rangle = \exp \left[ \sum a_n W^{(n)} \right], \ W^{(n)} = \text{hadronic correlators},$$

(15)

one can even obtain an explicit expression for this matrix element and thereafter evaluate the area derivatives in terms of the fundamental hadronic correlation functions. The latter can be taken, for instance, from lattice simulations. Let us illustrate the use of this framework by a simple Abelian example.
4.1. Example: Abelian exponentiation

Wilson loops in the Abelian gauge theory are known to exponentiate

\[ \langle h | W_\gamma | h \rangle = \langle h | \mathcal{P} \exp \oint_\gamma d\zeta^\mu A^\mu(\zeta) | h \rangle \tag{16} \]

where the basic hadronic correlator reads

\[ D_{\mu\nu}(\zeta - \zeta') = \langle h | \mathcal{P} A_\mu(\zeta) A_\nu(\zeta') | h \rangle. \tag{18} \]

For the sake of simplicity, let us consider a spinless hadron, which entails the following parameterisation of the basic correlator (18)

\[ D_{\mu\nu}(z) = g_{\mu\nu} D_1(z, P) + \partial_\mu \partial_\nu D_2(z, P) + \{ P_\mu \partial_\nu \} D_3(z, P) + P_\mu P_\nu D_4(z, P). \tag{19} \]

Given this representation and the exponentiation, the area derivative can be evaluated straightforwardly

\[ \frac{\delta}{\delta \sigma_{\mu\nu}(z)} \langle h | W_\gamma | h \rangle = -\frac{g^2}{2} \left[ \frac{\delta}{\delta \sigma_{\mu\nu}(z)} \oint_\gamma d\zeta^\mu \oint_\gamma d\zeta'^\nu D_{\mu\nu}(\zeta - \zeta') \right] \langle h | W_\gamma | h \rangle \tag{20} \]

After taking the path derivative \( \partial_\nu \), the non-vanishing terms are

- the standard ‘Makeenko-Migdal’ term:
  \[ \oint_\gamma d\zeta^\nu \partial^2 D_1(z, P) \tag{21} \]

- the hadron momentum-dependent term, obviously absent in the loop space with vacuum matrix elements
  \[ \oint_\gamma d\zeta^\nu (P \partial)^2 D_1(z, P) \tag{22} \]

Therefore, the shape evolution equation for the hadronic Wilson loops in the Abelian gauge theory is given by

\[ \partial^2 \frac{\delta}{\delta \sigma_{\mu\nu}(z)} \langle h | W_\gamma | h \rangle = \]

\[ -\frac{g^2}{2} \left[ \oint_\gamma d\zeta^\nu \left( \partial^2 D_1(z, P) + (P \partial)^2 D_4(z, P) \right) \right] \langle h | W_\gamma | h \rangle \tag{24} \]

Taking into account that for the vacuum matrix elements

\[ \partial^2 D_1(z) = -\delta^{(4)}(z), \]

and that \( D_4 = 0 \) in vacuum, one easily re-obtains the Makeenko-Migdal equation in the leading order.

To conclude, we proposed a new definition of the generic gluon TMD, which allows us to work directly with the entirely gauge-invariant and maximally path-dependent hadronic matrix elements before introducing relevant gluon TMD. All necessary information is absorbed in the basic hadronic correlator (18) and the evolution equation (24) generalises the Makeenko-Migdal equation for the Abelian gauge theory to the case of the ‘hadronic’ loop space.

Further progress will be presented in a separate work [16].
Acknowledgments
The work is supported by the Belgian Federal Science Policy Office. Fruitful discussions with T. Mertens, P. Taels and F. Van der Veken are gratefully appreciated.

References
[1] Dokshitzer Y L, Diakonov D and Troian S I 1980 Phys. Rept. 58 269
[2] Collins J C and Soper D E 1981 Nucl. Phys. B 193 381; Nucl. Phys. B 194 (1982) 445
[3] Collins J 2011 Foundations of Perturbative QCD (Cambridge University Press)
[4] Collins J C 2003 Acta Phys. Polon. B 34 3103
[5] Cherednikov I O and Stefanis N G (2008) Phys. Rev. D 77 094001; Cherednikov I O and Stefanis N G (2008) Nucl. Phys. B 802 146; Cherednikov I O and Stefanis N G (2009) Phys. Rev. D 80 054008
[6] Cherednikov I O, Karanikas A I and Stefanis N G (2010) Nucl. Phys. B 840 379
[7] Boer D et al. 2011 Preprint arXiv:1108.1713 [nucl-th]
[8] Angeles-Martinez R et al. (2015) Preprint arXiv:1507.05267 [hep-ph]
[9] Mulders P J and Rodrigues J (2001) Phys. Rev. D 63 094021
[10] Makeenko Y M and Migdal A A 1980 Phys. Lett. B 88 135, [Erratum-ibid. B 89 437] Makeenko Y M and Migdal A A 1981 Nucl. Phys. B 188 269; Makeenko Y 2002 Methods of Contemporary Gauge Theory (Cambridge University Press)
[11] Brandt R A, Neri F and Sato M A 1981 Phys. Rev. D 24 879
[12] Brandt R A, Gocksch A, Neri F and Sato M A 1982 Phys. Rev. D 26 3611
[13] Tavares J N (1994) Int. J. Mod. Phys. A 9 4511
[14] Cherednikov I O, Mertens T and Van der Veken F F 2014 Wilson Lines in Quantum Field Theory (Berlin: De Gruyter)
[15] Cherednikov I O, Mertens T and Van der Veken F F 2012 Phys. Rev. D 86 085035; Cherednikov I O, Mertens T and Van der Veken F F 2013 Phys. Part. Nucl. 44 250
[16] Cherednikov I O, Mertens T and Taels P 2016 [in preparation]