The Low Energy Behavior of some Models with Dynamical Supersymmetry Breaking

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ABSTRACT

We study supersymmetric $SU(5)$ chiral gauge theories with 2 fields in the 10 representation, $2 + N_F$ fields in the 5 representation and $N_F$ fields in the $\bar{5}$ representation, for $N_F = 0, 1, 2$. With a suitable superpotential, supersymmetry is shown to be broken dynamically for each of these values of $N_F$. We analyze the calculable limit for the model with $N_F = 0$ in detail, and determine the low energy effective sigma model in this case. For $N_F = 1$ we find the quantum moduli space, and for $N_F = 2$ we construct the s–confining potential.
Introduction and Results

In this Letter we study the low energy dynamics of supersymmetric $SU(5)$ chiral gauge theories with two matter fields $T_{ij}^a$ in the 10 representation, $2 + N_F$ fields $\bar{F}_a^\alpha$ in the 5 representation and $N_F$ fields $F_{i\sigma}$ in the $\bar{5}$ representation, for $N_F = 0, 1$ and 2. Here $i, j = 1..5$ are $SU(5)$ indices and $\alpha, a$ and $\sigma$ are flavor indices. With this matter content, the theories are anomaly free. They are also asymptotically free, as the one loop coefficient of the beta function is given by $b_0 = 11 - N_F$. For each value of $N_F$, we describe the classical moduli space in terms of holomorphic gauge invariant polynomials \([1, 2, 3]\). The manner in which the quantum moduli space differs from the classical one depends on the value of $N_F$; for $N_F = 0$ the classical moduli space is completely lifted by a dynamically generated superpotential, for $N_F = 1$ one of the constraints among the gauge invariants is modified, and for $N_F = 2$ the quantum moduli space is identical to the classical one. The low energy behavior for $N_F = 0, 1$ and 2 is therefore similar to supersymmetric QCD with $N_C$ colors and $N_C - 1$, $N_C$ and $N_C + 1$ flavors, respectively \([4, 5, 6]\). However, in contrast to supersymmetric QCD, in the models we consider supersymmetry is broken dynamically when suitable tree level superpotentials are added. For each value of $N_F$, the physics below the dynamical scale of the gauge interactions can be described by an effective theory. The light degrees of freedom in the effective theory are adequately represented by the same gauge invariants that characterize the moduli space. We construct these effective theories with the objective to show that supersymmetry is broken. This method of analysis is consistent as long as the supersymmetry breaking scale is below the dynamical scale. In general, the Kähler potential is under much less control than the superpotential. (The calculable limit of the model for $N_F = 0$ is an exception.) However, when the low energy degrees of freedom have been identified correctly, the Kähler potential is supposedly well–behaved. Under this assumption, it is possible to draw qualitative conclusions from an analysis of the superpotential alone. Finally, we show how the models for different values of $N_F$ are related by holomorphic decoupling.

For $N_F = 0$ the well-known $SU(5)$ model \([7, 8]\) with calculable dynamical supersymmetry breaking is obtained. In this model the classical moduli space is completely lifted by a dynamically generated superpotential. Supersymmetry is broken when a tree level potential is introduced which prevents the vacuum from moving to infinity. When the coupling constant of the tree level superpotential is sufficiently small, the vacuum occurs at weak coupling, with expectation values of the fields which lie near the classical moduli space and which are much larger than the dynamical scale of the theory. The model is thus seen to be very similar to the $SU(3) \otimes SU(2)$ \([1, 4, 10]\) model, the paradigm of models with calculable dynamical supersymmetry breaking.

We discuss the calculable limit of the $N_F = 0$ model from three different angles. In Section 1 we review the model and we extend our previous numerical calculation \([8]\) of the mass spectrum to next to leading order in the ratio of the superpotential coupling and the gauge coupling. In Section 1.1 we construct the low energy effective sigma model in terms of the moduli fields, including both the Kähler potential and the superpotential. With this sigma model in hand, we recalculate the vacuum energy, vacuum expectation values and the mass spectrum. The global symmetries of the sigma model elucidate the origin of degeneracies in the mass spectrum which appeared accidental in the full theory \([8]\). In
Section 1.2 we give an explicit parametrization in terms of vacuum expectation values of the fundamental fields of a particular direction in the classical moduli space which includes the vacuum. This result provides yet another way to calculate quantities such as the mass spectrum, vacuum energy and expectation values.

In Section 2 we add a flavor to the model. For \( N_F = 1 \) the classical moduli space is modified by nonperturbative effects, and the classical singularities are removed. We add a suitable superpotential with a mass term for the additional flavor and a Yukawa term. With the additional assumption that the Kähler potential is non–singular, we then show that supersymmetry is broken. We also check the consistency of our results; when the additional flavor is integrated out by taking its mass much larger than the dynamical scale, the theory for \( N_F = 0 \) with the correct superpotential ensues.

In Section 3 we add yet another flavor to the theory. In the case \( N_F = 2 \) there are no modifications to the classical moduli space. However, the singularities of the moduli space are interpreted differently. At the classical level singularities in the moduli space indicate that the gauge symmetry is only partially broken, and as a consequence the spectrum contains additional massless gauge multiplets at these points. At the quantum level, the theory confines, and the singularities are associated with additional massless composite degrees of freedom. We determine the confining superpotential. When a tree level superpotential with mass terms for the two flavors and a Yukawa term is added to the model, supersymmetry is shown to be broken. We also show that when one of the flavors is integrated out, the theory with \( N_F = 1 \) with the correct quantum modified moduli space results.

1 \( N_F = 0 \)

In the absence of a superpotential the model with \( N_F = 0 \) has an \( SU(2)_T \otimes SU(2)_{\bar{F}} \otimes U(1)_{A'} \otimes U(1)_{R'} \) global symmetry, under which the fundamental fields transform as

\[
\begin{array}{|c|c|c|c|c|}
\hline
 & SU(2)_F & SU(2)_T & U(1)_{A'} & U(1)_{R'} \\
\hline
 F_\alpha & 2 & 1 & 3/5 & -4 \\
 T_a & 1 & 2 & -1/5 & 1 \\
\hline
\end{array}
\]

The classical moduli space is parametrized by the expectation values of the six holomorphic gauge invariant polynomials\[^1\]

\[
X_a = \epsilon_{\alpha\beta} \bar{F}_i^\alpha \bar{F}_j^\beta T_a^{ij},
\]

\[
J_a^\alpha = \epsilon_{ijklm} \bar{F}_n^{\alpha} T_a^{ij} T_b^{kl} T_c^{mn} \epsilon^{bc}.
\]

Under the global symmetry transformations these gauge invariants transform as

\[
\begin{array}{|c|c|c|c|c|}
\hline
 & SU(2)_F & SU(2)_T & U(1)_{A'} & U(1)_{R'} \\
\hline
 X_a & 1 & 2 & 1 & -7 \\
 J_a^\alpha & 2 & 2 & 0 & -1 \\
\hline
\end{array}
\]

\[^1\]These invariants are slightly different from the ones defined in Ref.\[^2\] in order to make their transformation properties explicit.
Holomorphy and symmetries uniquely determine the form of a possible dynamically generated superpotential\(^2\)

\[
W_{np} = \frac{9}{2} \Lambda_0^{11} J_\alpha^a J_\beta^b \epsilon_{\alpha \beta \epsilon^{ab}}. \tag{2}
\]

The scale \(\Lambda_0\) can be defined by relating it to \(\Lambda_0\), the scale at which the one–loop renormalization group equation for the gauge coupling diverges. In principle, this requires an instanton calculation, but we can make use of the fact that in some D–flat directions the model reduces to a simpler model for which such an instanton calculation has been performed already. In the D-flat direction with \(T_1^{45} = -T_1^{54} = F_1^1 = F_2^2 = a\), and all other components vanishing, the \(SU(5)\) gauge symmetry is broken to \(SU(3)\). Apart from two singlets, the theory below the scale \(a\) is supersymmetric QCD with three colors and two flavors. Expanding the superpotential (2) around this flat direction yields

\[
W_{np} = \frac{1}{64 a^4} \frac{\Lambda_0^{11}}{\det(Q^p Q^q)}. \tag{3}
\]

Here the indices \(p\) and \(q\) label the flavor of the quarks \(Q\). The dynamically generated superpotential for the \(SU(3)\) theory has been calculated \([13]\) to be

\[
W^{(3)} = \frac{\Lambda_0^{7}}{\det(Q^p Q^q)}, \tag{4}
\]

where \(\Lambda_0^{(3)}\) is the scale at which the \(SU(3)\) gauge coupling diverges. A comparison of the superpotentials (3) and (4) yields \(\Lambda_0^{11} = 64 a^4 \Lambda_0^{(3)}\). Matching the \(SU(3)\) and \(SU(5)\) gauge coupling at the scale \(a\) results in \(\Lambda_0^{11} = a^4 \Lambda_0^{(3)}\). As a consequence \(\Lambda_0 = 2^{6/11} \Lambda_0\).

At tree level a renormalizable superpotential can be added to the model, so that the full superpotential is

\[
W = W_{np} + \lambda X_1. \tag{5}
\]

The renormalizable part of the superpotential explicitly breaks the \(SU(2)_T\) flavor symmetry, but a non-anomalous \(SU(2)_F \otimes U(1)_A \otimes U(1)_R\) symmetry remains. Under this reduced symmetry group the fundamental and composite fields transform as

| Field | \(SU(2)_F\) | \(U(1)_A\) | \(U(1)_R\) |
|-------|-------------|------------|-------------|
| \(F^{\alpha}\) | 2           | \(3/5\)    | -4          |
| \(T_1\)   | 1           | -6/5       | 10          |
| \(T_2\)   | 1           | 4/5        | -8          |
| \(X_1\)   | 1           | 0          | 2           |
| \(X_2\)   | 1           | 2          | -16         |
| \(J_1^{\alpha}\) | 2          | -1        | 8           |
| \(J_2^{\alpha}\) | 2          | 1         | -10         |

For small values of \(\lambda\) the \(SU(5)\) gauge symmetry is completely broken by vacuum expectation values which are much larger than the dynamical scale of the gauge interactions. In

\(^2\)The sole purpose of the factor 9/2 here is to maintain consistency with Ref.\([8]\).
earlier work[8] the particle spectrum was calculated in this case by numerically minimizing the scalar potential of the model. The vacuum energy was also calculated and found to be non-vanishing. Hence it was verified that supersymmetry is broken in this model. In addition, it was determined that the global internal symmetries are broken to a single remaining $U(1)_Q$ with $Q = I_3^{(F)} + A/2$. When the value of $\lambda$ is increased, the vacuum expectation values of the fields approach the dynamical scale. The vacuum becomes strongly interacting and the Kähler potential is no longer under control. However, there appears to be no invariant distinction between a Higgs description and a confined description. Supersymmetry is therefore likely also broken for large values of $\lambda$, although the model is then not “calculable”. In addition, the moduli fields are the appropriate degrees of freedom for the low energy effective theory for both small and large values of $\lambda$.

The model has only two parameters, which, for example, can be chosen as the dynamical scale $\Lambda_0$ and the scale $v$ of the vacuum expectation values of the scalar fields. In order to quantify for what range of the parameters the model is perturbative, note that the coupling constant $\lambda$ at the scale $v$ varies as

$$\lambda(v) \sim \left( \frac{\Lambda_0}{v} \right)^{11},$$

whereas the gauge coupling $g$ scales as

$$g^2(v) \sim -\frac{8\pi^2}{11 \ln \frac{\Lambda_0}{v}}.$$ (7)

The model is calculable if the gauge coupling is in the perturbative range at the scale $v$. Taking this to mean that $g^2(v) < 8\pi^2$, it follows from Eq.(7) that $v > e^{\pi/2} \Lambda_0$. Eq.(7) then implies that $\lambda < e^{-1}$.

In the calculable limit $g(v)$ is much larger than $\lambda(v)$. As a consequence, a sequence of scales arises. In decreasing order there is a large mass scale $\sim gv$, the scale of the vacuum expectation values of the scalar components $v$, the dynamical scale $\Lambda_0$, the supersymmetry breaking scale $\sim \lambda^{1/2} v$, and a light mass scale $\lambda v$. The heavy sector of the spectrum contains the components of twenty four massive vector multiplets. The light spectrum contains the components of the six chiral multiplets which remain after the other chiral multiplets are absorbed by the vector multiplets through the Higgs mechanism. The light spectrum includes four massless scalars and two massless fermions. The massless scalars are the Goldstone bosons associated with the spontaneously broken global internal symmetries. One of the massless fermions is a Goldstino associated with the spontaneous breaking of global supersymmetry. The second massless fermion is charged and saturates the anomaly matching condition for the unbroken $U(1)_Q$ symmetry. The masses of the light particles can be expanded in powers of $\lambda/g$ as follows:

$$m = \lambda^{10/11} \Lambda_0 \left( \mu_0 + \frac{1}{2} r \left( \frac{\lambda}{g} \right)^2 + O \left( \frac{\lambda}{g} \right)^4 \right).$$ (8)

Previously[8], the masses of the light scalars and fermions were calculated in leading order in $\lambda/g$. In that calculation the full scalar potential of the model, including D– and F–terms, was
Table 1: The parameters $\mu_0$ and $r$ which determine the twelve scalar masses according to Eq.(8). Each scalar is classified according to its $SU(2)_D \otimes U(1)_{A_e}$ representation and $U(1)_Q$ charge.

| $SU(2)_D$ | $A_e$ | $Q$ | $\mu_0$ | $r$ |
|---------|-------|-----|--------|-----|
| 1       | 0     | 0   | 0      | 0   |
| 3       | 0     | $-1$ | 0      | $-44.9$ |
|         |       | $+1$ |        |      |
| 1       | $-1$  | $-1$ | 2.550  | $-12.3$ |
|         |       | $+1$ |        |      |
| 3       | 0     | $-1$ | 2.744  |      |
|         |       | $+1$ |        |      |
| 1       | 0     | 0   | 3.904  | $-66.4$ |
| 1       | 0     | 0   | 5.946  | $-2.3$ |
| 1       | 0     | 0   | 9.320  | $-45.7$ |

minimized numerically, and the masses of both the light and heavy particles were obtained by expanding the theory around the minimum. We have now extended this calculation to next to leading order in $\lambda/g$. The resulting values for the parameters $\mu_0$ and $r$ are listed in Table 1 for scalars and in Table 2 for fermions. As was already observed in Ref.[8], degeneracies in the values of $\mu_0$ indicate that to lowest order in $\lambda/g$ the spectrum displays an $SU(2)_D \otimes U(1)_{A_e}$ global symmetry. This group is not a subgroup of the global symmetry group of the model. Moreover, its transformations do not consist of combinations of transformations from the gauge group (with constant parameters) and the global symmetry group either. We will show how the moduli fields transform when we discuss the low energy effective sigma model in the next section. $SU(2)_D \otimes U(1)_{A_e}$ is broken to $U(1)_Q$ with $Q = I^D_3 + A_e$ in next to leading order in $\lambda/g$, as follows from the values of $r$.

1.1 The low energy effective sigma model

In the calculable limit the vacuum expectation values are large and in the vicinity of the D–flat directions. Below the dynamical scale $\Lambda_0$ the heavy vector multiplets can be integrated out. The interactions of the remaining light degrees of freedom are adequately described by a sigma model with the moduli fields $X_a$ and $J^a_\alpha$ as coordinates. The Kähler potential of this sigma model is the classical Kähler potential on the moduli space. This Kähler potential can be determined by the method of Affleck, Dine and Seiberg[1]. According to this method, the classical Kähler potential of the fundamental theory with the gauge interactions switched off is projected onto the moduli fields. The method is powerful because it is non–local. This means that it is not necessary to specify a specific point in the moduli space around which the theory is expanded. As a consequence it is not necessary to minimize the scalar potential of the fundamental theory to find the vacuum expectation values of the moduli
Table 2: The parameters $\mu_0$ and $r$ which determine the six fermion masses according to Eq.(8). Each fermion is classified according to its $SU(2)_D \otimes U(1)_{A_e}$ representation and $U(1)_Q$ charge.

| $SU(2)_D$ | $A_e$ | $Q$ | $\mu_0$ | $r$   |
|-----------|-------|-----|---------|------|
| 1         | 0     | 0   | 0       | 0    |
| 1         | +1    | +1  | 0       | 0    |
| 3         | 0     | 0   | 0.716   | −1.8 |
| 1         | 0     | 0   | 7.486   | −18.7|

fields. The construction of the sigma model is completed when the non–perturbative and tree level superpotentials in terms of the moduli fields are added.

The effective sigma model is supersymmetric and invariant under the full global symmetry group of the fundamental theory. However, when its scalar potential is minimized some of the moduli fields get expectation values. Supersymmetry and some of the global symmetries are then spontaneously broken. The sigma model is thus equivalent to the fundamental theory after the heavy degrees of freedom are integrated out.

Applying this method, the effective Kähler potential of the sigma model is invariant under all internal global symmetries and supersymmetry. Hence it takes the form

$$K_{\text{eff}} = K_{\text{eff}}(I_1, I_2, I_3, I_4),$$

with

$$\begin{align*}
I_1 &= X^a \dagger X_a \\
I_2 &= J^a \dagger_J^a \\
I_3 &= X^a \dagger_J^b X_b J^\beta a \\
I_4 &= J^a \dagger_J^\beta J^\beta_J^a.
\end{align*}$$

Following the Affleck, Dine and Seiberg procedure, the objects $T^a \dagger T_a$ and $\bar{F}_a^\dagger \bar{F}_a$ have to be related to $I_1, I_2, I_3$ and $I_4$ in the moduli space. Defining

$$\begin{align*}
A &= 125 I_1 \\
B &= 25 \left( \frac{1}{2}I_2 + \frac{1}{2}\sqrt{2I_4 - I_2^2} + \sqrt{\frac{1}{2}I_2 - \frac{1}{2}\sqrt{2I_4 - I_2^2}} \right),
\end{align*}$$

we proved that the relations

$$\begin{align*}
A &= (T^a \dagger T_a + 4\bar{F}_a \dagger \bar{F}_a)^2(\frac{1}{2}T^a \dagger T_a + 3\bar{F}_a \dagger \bar{F}_a) \\
B &= \frac{2}{3}(T^a \dagger T_a + 4\bar{F}_a \dagger \bar{F}_a)(T^a \dagger T_a - \bar{F}_a \dagger \bar{F}_a)
\end{align*}$$

(12)
are identically valid in the D–flat directions. The Kähler potential of the sigma model is determined by inverting these identities in order to write the Kähler potential of the fundamental theory in terms of the moduli fields. Defining \( p = T^a T_a + 4 \bar{F}^\dagger_a \bar{F}^a \) and \( q = T^a T_a - F^\dagger_a F^a \), the system of equations (12) is equivalent to

\[
p^3 - 3Bp - 2A = 0 \\
q = \frac{3B}{2p}.
\]  

(13)

Solving the cubic equation for \( p \) and choosing the appropriate root gives

\[
p = 2 \sqrt{B} \cos \left( \frac{1}{3} \arccos \frac{A}{B^\frac{3}{2}} \right).
\]  

(14)

The resulting effective Kähler potential in terms of \( I_1, I_2, I_3 \) and \( I_4 \) is

\[
K_{\text{eff}} = \left( \frac{1}{2} T^a T_a + \bar{F}^\dagger_a \bar{F}^a \right)|_{\text{flat}} = \frac{1}{10} (3p + 2q) = \frac{3}{10} \left( p + \frac{B}{p} \right).
\]  

(15)

Surprisingly, \( K_{\text{eff}} \) does not depend on \( I_3 \). As a consequence, the global symmetries of the effective Kähler potential extend those of the underlying fundamental theory. The complete global symmetry group of the effective Kähler potential is \( SU(2)_{\bar{F}} \otimes SU(2)_1 \otimes SU(2)_2 \otimes U(1)_{R'} \), under which the moduli fields transform as

| \( X_b \) | \( SU(2)_{\bar{F}} \) | \( SU(2)_1 \) | \( SU(2)_2 \) | \( U(1)_{R'} \) |
|---|---|---|---|---|
| 1 | 1 | 2 | -4 |

| \( J^\alpha_a \) | \( SU(2)_{\bar{F}} \) | \( SU(2)_1 \) | \( SU(2)_2 \) | \( U(1)_{R'} \) |
|---|---|---|---|---|
| 2 | 2 | 1 | 1 |

To keep track of the action of the various \( SU(2) \) symmetry transformations, note that \( a \) is an \( SU(2)_1 \) index, \( b \) is an \( SU(2)_2 \) index, and \( \alpha \) is an \( SU(2)_{\bar{F}} \) index as before. The original \( SU(2)_T \) group is the vectorial subgroup of \( SU(2)_1 \otimes SU(2)_2 \). In addition, observe that because the moduli fields are invariant under gauge transformations, \( SU(2)_1 \) and \( SU(2)_2 \) are not just combinations of gauge transformations with a constant parameter and \( SU(2)_T \) transformations. All information pertaining to the low energy limit of the \( N_F = 0 \) model is contained in the sigma model defined by the Kähler potential (15) and the superpotential (5). The tree level superpotential breaks some of the global symmetries explicitly. The remaining symmetry group is \( SU(2)_{\bar{F}} \otimes SU(2)_1 \otimes U(1)_{Ae} \otimes U(1)_{Re} \). Under this group the moduli fields transform as

| \( X_1 \) | \( SU(2)_{\bar{F}} \) | \( SU(2)_1 \) | \( U(1)_{Ae} \) | \( U(1)_{Re} \) |
|---|---|---|---|---|
| 1 | 1 | 0 | 2 |

| \( X_2 \) | \( SU(2)_{\bar{F}} \) | \( SU(2)_1 \) | \( U(1)_{Ae} \) | \( U(1)_{Re} \) |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 |

| \( J^\alpha_a \) | \( SU(2)_{\bar{F}} \) | \( SU(2)_1 \) | \( U(1)_{Ae} \) | \( U(1)_{Re} \) |
|---|---|---|---|---|
| 2 | 2 | 0 | -1 |

Taking into account the non-canonical form of the Kähler potential, the scalar potential of the sigma model was minimized. The resulting vacuum energy was found to be

\[
V = 2.807 \lambda^{18/11} \Lambda_0^4.
\]  

(16)
The vacuum expectation values of the moduli fields in the minimum were found to be

\[
\begin{align*}
X_1 &= 0.247\lambda^{-3/11}\Lambda_0^3 \\
X_2 &= 0 \\
J_1^1 &= -3.452\lambda^{-4/11}\Lambda_0^4 \\
J_1^2 &= 0 \\
J_2^2 &= 0 \\
J_2^2 &= -3.452\lambda^{-4/11}\Lambda_0^4,
\end{align*}
\]

(17)
in accordance with the calculation in the full theory [8]. The form of the vacuum expectation values is such that both the order parameter for \(U(1)_Q\) symmetry breaking, \(I_3^2 - I_1 I_2 I_3 + 1/2I_1^2 I_2^2 - 1/2I_2^2 I_4\), and the order parameter for \(SU(2)_D\) breaking, \(2I_4 - I_2^2\), vanish. This is not surprising, since points with extended symmetry are stationary points of the potential. The symmetry breaking pattern is thus \(SU(2)_F \otimes SU(2)_1 \otimes U(1)_Ae \otimes U(1)_Re \rightarrow SU(2)_D \otimes U(1)_Ae\), where \(SU(2)_D\) is a diagonal subgroup of \(SU(2)_F \otimes SU(2)_1\). In order to discuss the mass spectrum, it is useful to study fields that transform under irreducible representations of the unbroken subgroup. The six fields \(X_1, X_2, N = J_a^\alpha \delta^\alpha_a\) and \(S^i = J_a^i (\sigma^i)^\alpha_a\) with transformation properties

|       | \(SU(2)_D\) | \(U(1)_A\) |
|-------|-------------|-------------|
| \(X_1\) | 1           | 0           |
| \(X_2\) | 1           | 1           |
| \(N\)   | 1           | 0           |
| \(S^i\) | 3           | 0           |

form an equivalent coordinate system for the sigma model. All masses and interactions of the theory can be determined by expanding the Kähler potential and the superpotential around the vacuum expectation values (17). We calculated the scalar and fermion masses in the sigma model, and the results were in complete agreement with the light masses determined in the full theory to lowest order of \(\lambda/g\), as they can be gleaned from Eq.(8) and Tables 1 and 2. The Goldstino is a linear combination of the fermionic components of \(X_1\) and \(N\). The charged massless fermion forms the fermionic component of \(X_2\), while the scalar components of this field correspond to the massive complex charged scalar in the spectrum. The scalar components of \(S^i\) form a massless and a massive triplet of real scalars, and one linear combination of the scalar components of \(X_1\) and \(N\) corresponds to the neutral massless scalar.

### 1.2 Parametrization of the moduli space

The classical moduli space is described by twelve parameters, which can be conveniently chosen as the six complex vacuum expectation values of the gauge invariants (1). Alternatively, the moduli space can be described by the vacuum expectation values of the fundamental fields. The D–flat directions are given by the solutions to the equation

\[
T_{ij}^a T_{ik}^a - \bar{F}_i^\alpha F_j^\alpha \sim \delta^k_j.
\]

(18)
modulo a gauge transformation. The moduli space is determined by a four parameter solution to Eq. (18) which breaks all global symmetries, and the eight parameters of global $SU(2)_F \otimes SU(2)_T \otimes U(1)_{Ae} \otimes U(1)_{R'}$ transformations. We did construct a generic four parameter solution, and we used it to check the identities (12) in the previous section. However, as the minimum of the scalar potential occurs in a special direction of the moduli space in which the $SU(2)_D$ symmetry is not broken, we just provide a parametrization of this special direction here:

\[
T_1 = \begin{pmatrix}
0 & a & 0 & e & 0 \\
-a & 0 & p & 0 & 0 \\
0 & -p & 0 & q & 0 \\
-e & 0 & -q & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\quad T_2 = \begin{pmatrix}
0 & 0 & 0 & 0 & r \\
0 & 0 & 0 & b & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & -b & 0 & 0 & 0 \\
-r & 0 & -s & 0 & 0
\end{pmatrix}
\quad \tilde{F}^1 = \begin{pmatrix}
m \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\quad \tilde{F}^2 = \begin{pmatrix}
n \\
0 \\
0 \\
0 \\
0
\end{pmatrix}.
\]

The independent parameters of this solution are $a$ and $b$. The remaining parameters in terms of $a$ and $b$ are

\[
q = \sqrt{a^2 + ab} \\
e = (b + a)\sqrt{\frac{b - a}{a}} \\
r = \sqrt{b^2 + ab} \\
m = b\sqrt{\frac{b}{a}} \\
n = b\sqrt{\frac{b}{a}} \\
s = -\sqrt{b^2 - ab} \\
p = \sqrt{b^2 - a^2}.
\]

The expectation values of the gauge invariant polynomials in this flat direction are

\[
X_1 = 2\frac{b^3}{a^2}\sqrt{(b^2 - a^2)(a^2 + \sqrt{a^2b^2})} \\
X_2 = 0 \\
J^1_1 = -12b^4(1 + \frac{b}{a}) \\
J^1_2 = 0 \\
J^2_1 = 0 \\
J^2_2 = -12b^4(1 + \frac{b}{a}).
\]

The invariance of the vacuum under $SU(2)_D \otimes U(1)_{Ae}$ symmetry transformations is manifested by the fact that $J^1_1 = J^2_2$; invariance under $U(1)_Q$ only requires $X_2 = J^2_1 = J^1_2 = 0$, with $J^1_1$ and $J^2_2$ arbitrary. The scalar potential in this D-flat direction is

\[
\frac{V}{\Lambda_0^4 \Lambda_{1/2}^8} = \frac{1}{2^{12}} \left( \frac{\lambda^{-\frac{1}{4}} \Lambda_0}{b} \right)^{18} \frac{1}{(1 + b/a)^4} \left( 4 + \frac{8}{1 + b/a} \right)
\]
The minimum of the scalar potential is obtained for

\[
a = 0.5712 \lambda^{-1/11} \Lambda_0, \\
b = 0.6106 \lambda^{-1/11} \Lambda_0,
\]

and the vacuum energy in the minimum is given by

\[
V = 2.807 \lambda^{18/11} \Lambda_0^4.
\]

Both the expectation values and the vacuum energy agree with the results as calculated in the full theory \cite{8} to lowest order of \(\lambda/g\), and in the low energy effective sigma model as described in the previous section. The method proposed by Poppitz and Randall \cite{3} could, in principle, be used to calculate derivatives of the Kähler potential at this point in the moduli space, providing yet another method to calculate the interactions and spectrum in the low energy theory.

2 \( N_F = 1 \)

For \( N_F = 1 \) the global symmetry group of the model is \( SU(3)_F \otimes SU(2)_T \otimes U(1)_{A'} \otimes U(1)_{B'} \otimes U(1)_{R'} \), under which the fields transform as

|       | \( SU(3)_F \) | \( SU(2)_T \) | \( U(1)_{A'} \) | \( U(1)_{B'} \) | \( U(1)_{R'} \) |
|-------|---------------|---------------|----------------|----------------|----------------|
| \( F^\alpha \) | 3             | 1             | 2              | \( \frac{1}{3} \) | \( -\frac{2}{3} \) |
| \( F \)    | 1             | 1             | 0              | -1             | 1              |
| \( T_a \)  | 1             | 2             | -1             | 0              | 1              |

The classical moduli space is described by the eighteen basic gauge invariants

\[
X_{\alpha,a} = \epsilon_{\alpha\beta\gamma} \tilde{F}_i^\beta \tilde{F}_j^\gamma T_a^{ij}, \\
J_a^\alpha = \epsilon_{ijklm} \tilde{F}_n^\alpha T_a^{ij} T_b^{kl} T_c^{mn} \epsilon_{bc}, \\
M^\alpha = \tilde{F}_i^\alpha \tilde{F}^i, \\
B_{ab} = \epsilon_{ijklm} \tilde{F}_i^j T_a^{lk} T_b^{lm},
\]

subject to the two constraints

\[
\epsilon_{\alpha\beta\gamma} \epsilon^{ab} J_a^\alpha J_b^\beta M^\gamma - \frac{3}{2} \epsilon^{ca} \epsilon^{db} B_{cd} J_c^\alpha X_{\alpha,b} = 0 \quad (26) \\
\epsilon^{ab} J_a^\alpha X_{\alpha,b} = 0. \quad (27)
\]

The moduli fields transform under the global symmetry group as
where \( b_0 = 10 \) for \( N_F = 1 \), and \( \Lambda_1 \) is dynamical scale of the gauge interactions. This modification is consistent with the requirement that anomaly matching conditions for unbroken global symmetries are saturated at points of enhanced symmetry in the moduli space. Moreover, the modified constraint gives rise to the correct dynamical superpotential in the \( N_F = 0 \) model when the additional flavor is integrated out. To illustrate this holomorphic decoupling and to study the issue of supersymmetry breaking, a mass term and a Yukawa term are added to the superpotential:

\[
W_{\text{tree}} = m_1 M^3 + \lambda X_{3,1}.
\] (29)

The constraints can be incorporated into the superpotential by the introduction of the Lagrange multiplier fields \( L_1 \) and \( L_2 \). The complete superpotential therefore is

\[
W = W_{\text{tree}} + L_1 \left( \epsilon_{\alpha \beta \gamma} \epsilon^{ab} J^\alpha_a J^\beta_b M^\gamma - \frac{3}{2} \epsilon^{ca} \epsilon^{db} B_{cd} J^\alpha_a X_{\alpha b} - \Lambda^0_1 \right) + L_2 \epsilon^{ab} J^\alpha_a X_{\alpha b}.
\] (30)

In the large \( m_1 \) limit the additional flavor can be integrated out by imposing the equations of motion for the fields \( M^\alpha, J^\beta_a, X_{1,1}, X_{2,1} \) and \( B_{ab} \). With the matching condition \( 2 \Lambda^0_1 m_1 = 9 \Lambda^1_0 \), the theory for \( N_F = 0 \) with superpotential (3) ensues.

Supersymmetry is broken because the vacuum expectation values of the auxiliary \( F \) components do not all vanish simultaneously. The equations of motion for the \( F \) components are

\[
\begin{align*}
(M^\gamma)_{\alpha}^1 & = & m_1 \delta^\gamma_\gamma + L_1 J^\alpha_a J^\beta_b \epsilon^{ab} \epsilon_{\alpha \beta} \\
(B^\epsilon f)_{\alpha}^1 & = & -\frac{3}{2} L_1 J^\alpha_a X_{\alpha b} \left( \epsilon^{ca} \epsilon^{db} + \epsilon^{f a} \epsilon^{e b} \right) \\
(J^\alpha_a)_{\alpha}^1 & = & 2 L_1 J^\beta_b M^\gamma \epsilon^{eb} \epsilon_{\alpha \beta \gamma} - \frac{3}{2} L_1 B_{cd} X_{\alpha b} \epsilon^{ce} \epsilon^{eb} + L_2 X_{\alpha b} \epsilon^{eb} \\
(X^{\alpha,\epsilon})_{\alpha}^1 & = & \lambda \delta^\alpha_3 \delta^\epsilon_1 - \frac{3}{2} L_1 B_{cd} J^\alpha_a \epsilon^{ca} \epsilon^{de} + L_2 J^\alpha_a \epsilon^{ac}
\end{align*}
\] (31)

To show that some \( F \) fields obtain a vacuum expectation value, assume first that the vacuum expectation values of all \( F \) components vanish. Then consider

\[
J^\alpha_c (M^\alpha)_{\alpha}^1 = m_1 J^3.
\] (32)

If \( (M^\alpha)_{\alpha}^1 \geq 0 \), and \( m_1 \neq 0 \) then \( J^3 \geq 0 \). However, in this case

\[
< (X_{3,1})_{\alpha}^1 > = \lambda.
\] (33)

This is inconsistent with the assumption. Hence some \( F \) components have a vacuum expectation value, and therefore supersymmetry is broken.
For \( N_F = 2 \) the global symmetry group of the model is \( SU(4)_F \otimes SU(2)_F \otimes SU(2)_T \otimes U(1)_{A'} \otimes U(1)_{B'} \otimes U(1)_{R'} \), and the fields transform as

|       | \( SU(4)_F \) | \( SU(2)_F \) | \( SU(2)_T \) | \( U(1)_{A'} \) | \( U(1)_{B'} \) | \( U(1)_{R'} \) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( F^\alpha \) | 4              | 1              | 1              | \( \frac{4}{2} \) | \( \frac{1}{2} \) | \( -\frac{3}{2} \) |
| \( F \)    | 1              | 2              | 1              | 0              | -1             | 1              |
| \( T_a \)  | 1              | 1              | 2              | -1             | 0              | 1              |

The basic gauge invariants which parametrize the classical moduli space are

\[
X_{\alpha\beta,a} = \epsilon_{\alpha\beta\gamma\delta} \bar{F}^\gamma_i F^\delta F^a T_a
\]
\[
J^a = \epsilon_{ijklm} \bar{F}^a T_a T^k T^l T^m \epsilon^{bc}
\]
\[
M^\sigma = \bar{F}^a F^a
\]
\[
B_{\sigma,ab} = \epsilon_{ijklm} \bar{F}^a T_a T^b T^l T^m \epsilon^{\sigma\tau} \epsilon^{ab}
\]
\[
Y^\alpha = \epsilon_{ijklm} \bar{F}^a T_a T^b T^k T^l \epsilon^{\sigma\tau} \epsilon^{\sigma\tau}
\]

subject to various constraints. These gauge invariants transform under the global symmetry transformations as

|       | \( SU(4)_F \) | \( SU(2)_F \) | \( SU(2)_T \) | \( U(1)_{A'} \) | \( U(1)_{B'} \) | \( U(1)_{R'} \) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( X_{\alpha\beta,a} \) | 6              | 1              | 2              | -2             | -1             | 1              |
| \( J^a \)    | 4              | 1              | 2              | -3/2           | 1/2            | 3/2            |
| \( M^\sigma \) | 4              | 2              | 1              | -1/2           | -1/2           | 3/2            |
| \( B_{\sigma,ab} \) | 1              | 2              | 3              | 3              | 1              | 3              |
| \( Y^\alpha \) | 4              | 1              | 1              | -1/2           | 3/2            | 5/2            |

The model for \( N_F = 2 \) is part of a class of models \([4]\) whose infra–red behavior is commonly referred to as “s-confinement”. The quantum moduli space is identical to the classical one, but its singularities are interpreted differently. Where in the classical picture the singularities are associated with massless gauge multiplets, at the quantum level they are associated with additional massless composite fields. Close to the origin, all moduli fields \([34]\) are physical. They interact through the confining potential

\[
W = \frac{1}{\Lambda^2} \left( -JJMM + 3JYX + 3JBXM - \frac{9}{32} BBXX \right),
\]

where \( b_0 = 9 \) for \( N_F = 2 \), and

\[
JJMM = J^a a^b J^b J^c M^d \epsilon_{\alpha\beta\gamma\delta} \epsilon^{\sigma\tau} \epsilon^{ab}
\]
\[
JYX = J^a Y^b X_{\alpha\beta\gamma} \epsilon^{ab}
\]
\[
JBXM = J^a B_{\sigma,cd} X_{\alpha\beta\gamma} \epsilon_{\sigma\tau} \epsilon^{ab}
\]
\[
BBXX = B_{\sigma,ab} B_{\tau,cd} \epsilon^{\sigma\tau} \epsilon^{\sigma\tau} \epsilon_{\alpha\beta\gamma} \epsilon_{\sigma\tau} \epsilon^{be} \epsilon^{df}
\]

The coefficients of the terms in this potential are such that the F-flatness conditions reproduce the constraints of the classical moduli space. At the origin none of the global symmetries
are broken. All anomaly coefficients in the fundamental theory and the low energy effective theory match, providing a stringent test for this picture. In order to show that the model for $N_F = 2$ is related to the model for $N_F = 1$ by holomorphic decoupling and to study dynamical supersymmetry breaking, the superpotential

$$W_{\text{tree}} = m_1 M_1^3 + m_2 M_2^4 + \lambda X_{34,1}$$

(37)

is added. In the limit $m_2 \gg \Lambda_2$ the heavy degrees of freedom can be integrated out, and the theory for $N_F = 1$ results with the matching condition $m_2 \Lambda_2^3 = 2 \Lambda_1^{10}$. Supersymmetry is broken by the O’Raifeartaigh mechanism. The proof is by reductio ad absurdum. Assume that none of the auxiliary $F$ fields has an expectation value. Note that

$$J_a^\alpha (M_\alpha^\sigma)^\dagger_F - (Y_\alpha)^\dagger_F J_c^0 B_{r,da} \epsilon^{cd} \epsilon^{\tau \sigma} = m_1 J_a^3 S_1^\sigma + m_2 J_a^4 S_2^\sigma.$$  

(38)

Therefore, if $< (M_\alpha^\sigma)^\dagger_F >= 0$ and $< (Y_\alpha)^\dagger_F >= 0$, then $< J_a^3 >= 0$ and $< J_a^4 >= 0$ for $m_1 \neq 0$ and $m_2 \neq 0$. In addition,

$$(M_\alpha^\dagger)^\tau_F X_{\beta \gamma, a} \epsilon^{\alpha \beta \gamma \delta} - (Y_\alpha)^\dagger_F X_{\beta \gamma, c} B_{r,ab} \epsilon^{be} \epsilon^{\alpha \beta \gamma \delta} \epsilon^{\tau \gamma} + \frac{8}{3} (Y_\alpha)^\dagger_F J_a^\alpha M_\tau^4 \epsilon^{\tau 1} = -2m_1 X_{12,a}.$$  

(39)

As a consequence, $< X_{12,a} >= 0$ if $< (M_\alpha^\dagger)^\tau_F >= 0$ and $< (Y_\alpha)^\dagger_F >= 0$. However, in that case $< (X_{\alpha \beta}^\dagger,a)^\tau_F >= \lambda$, in contradiction with our assumption. Some $F$ components therefore have an expectation value, and supersymmetry is broken.

**Conclusions**

The classical moduli space for the model with $N_F = 0$ without a superpotential has a bigger symmetry group than the fundamental theory. In the present work this followed algebraically from the explicit calculation of the Kähler potential in the moduli space. However, it seems to us that the extended symmetry of the moduli space is probably a necessary consequence of supersymmetry and the specific representation of the matter fields under the gauge group. We surmise therefore that there must be a more elegant method based on representation theory and geometry to determine the symmetries of the classical moduli space in this particular model, and perhaps also in supersymmetric gauge theories in general. We may attempt to address this issue in a later paper.

The superpotential in the model with $N_F = 0$ is invariant under part of the extended symmetries of the effective Kähler potential in the moduli space. Even after spontaneous symmetry breaking in the effective low energy sigma model some of the extended symmetry remains. This remnant explains previously observed degeneracies in the mass spectrum. It is interesting to note that even though the symmetry breaking patterns as displayed in the fundamental theory and the effective low energy sigma model are different, the number of broken generators is identical in both cases. The number of Goldstone bosons is therefore also the same, which is, of course, required for consistency. All the features of this calculable model are now well understood. It can therefore be used as a controlled laboratory for dynamical supersymmetry breaking, just as the $SU(3) \otimes SU(2)$ model.
For the calculable limit of the $N_F = 0$ model the existence of a supersymmetric vacuum at the origin at strong coupling cannot strictly be excluded. In Ref. [7] a strong case was made that such behavior is implausible. Moreover, even if such a supersymmetric vacuum existed, the supersymmetry breaking vacuum at weak coupling would be at least meta–stable. As the $N_F = 1$ and $N_F = 2$ models are strongly interacting, the argument that supersymmetry is broken in these models hinges upon the assumed correct identification of the low energy degrees of freedom and the Kähler potential. Although the arguments for broken supersymmetry are quite convincing in each case, there always remains a loophole.

Further support for the hypothesis that supersymmetry is broken is provided by the following argument [10, 11]: The models we consider are related by holomorphic decoupling, i.e. a model with less flavors can be obtained by varying mass parameters of a model with more flavors. It was shown in Ref. [12] that if supersymmetry is broken for a range of the parameters, it is broken for generic values of those parameters, with the possible exception of isolated special points. It then follows that if supersymmetry is broken in one of the models under consideration, it is broken in all of them.

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