GENERALIZED QCD$_2$ VIA THE BILOCAL METHOD

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Abstract
We use the bilocal method to derive the large $N$ solution of the most general QCD$_2$.

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\textsuperscript{2}Work supported by the EU grant ERB4001GT930141.
1 Introduction

One of the most interesting and still not fully understood problem of Quantum Field Theory concerns the calculation of the mass of relativistic bound states. This problem is even more complicated to solve in the case of QCD, the $SU(3)$ gauge theory of quarks and gluons, that describes the strong interaction physics where the basic constituents are confined and perturbation theory cannot be used. The expectation is that the low energy spectrum consists of colorless mesons and baryons, but up to now the only available method seems to be based on numerical calculation using the lattice theory formulation of the theory.

The large-$N$ expansion technique proposed by 't Hooft [1, 2] several years ago seems to be the most promising approach to obtain analytic results on the hadron spectrum. It is well known that in the limit in which the number $N$ of colours becomes very large the theory becomes much simpler, in the sense that only planar Feynman graphs survive. Furthermore, when $N \to \infty$ the theory only contains colorless, stable and noninteracting mesons with two-body decay and scattering amplitudes proportional to $\frac{1}{\sqrt{N}}$ and $\frac{1}{N}$, respectively (there is also a way of describing baryons as solitons of the effective Lagrangian in the large $N$ limit, but this will not interest us in this context).

Unfortunately, all efforts to try to solve four-dimensional large-$N$ QCD have failed up to now. The main problem is to find the semi-classical configuration, namely the master field [3], from which the action is dominated in the large-$N$ limit and whose fluctuations around the vacuum should give the particle spectrum of the theory.

The main reason of this failure is due to the fact that no method has yet been found to solve matrix models for a space time dimension $D > 2$. On the other side it is well known that the large $N$ expansion can be explicitly performed in vector models as for instance the $O(N)$ vector model (see for instance ref. [4] and references therein), the two-dimensional $CP^{N-1}$ model [5] and also in $QCD_2$ [4, 6] and [7, 8, 9] with matter in the fundamental representation of the gauge group Some results can be obtained with both fermionic and bosonic adjoint matter [11] though the model is a real 2D matrix model.

In particular this set of models are solved by means of two slightly different methods. The vector-like models are solved in the large $N$ expansion by
introducing a local "colourless" composite field and by explicitly integrating over the fundamental fields (See for instance Refs. [4] and [5]): with the help of the composite local field we can extract the explicit N dependence and apply the saddle point technique.

The $QCD_2$-like models are characterized by the fact that the number of components of the non abelian gauge field goes to infinity in the large N expansion and from this viewpoint they are matrix models. The gauge field in two dimensions has no physical degrees of freedom and therefore can be eliminated by using its classical equation of motion. In this way one gets a non local Coulomb interaction that is quartic in terms of the vector-like matter fields. The theory can then be solved in the large N limit by introducing a bilocal composite field as discussed in Refs. [7, 8, 9]. We can rephrase this by saying that the kernel of the solubility lies in the fact that the matrices of the theory (the gauge fields) are expressible as tensor product of vectors. The same happens in the generalized QCD [11], even if the theory seems a 2 matrices model, it is actually a nonlocal vector model.

In this paper we present a solution of the most general case of generalized $QCD_2$ using the bilocal method (see [13] for an extensive application of the method to the other known cases).
2 The master field of the general QCD\(_2\): the simplest case.

The usual QCD\(_2\) action can be written in the first order formalism as \[\]

\[
S = \int d^2 x \left\{ \frac{N}{8\pi} tr_c(E\tilde{F}) - \frac{Ng^2}{4\pi} tr_c(E^2) + \bar{\psi}^a(i\bar{\nabla} - m^i \mathbb{1})\psi^a \right\} \tag{2.1}
\]

where \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]\), \(\tilde{F} = F_{\mu\nu}e^{\mu\nu}\), \(i\bar{\nabla}_{AB} = \gamma_\mu(i\partial_\mu)_{AB} - A_\mu^a T^a_{AB}\), \(a, b = 1...N^2\) are the indices of the adjoint representation of the colour group, \(A, B = 1...N\) run over the fermionic representation of the colour \(U(N)\) and \(a, b...\) are flavour indices, and \(tr_c\) is the color trace.

From the previous form it can be directly generalized to (\[\]

\[
S = \int d^2 x \left\{ \frac{N}{8\pi} tr_c(E\tilde{F}) - \frac{Ng^2}{4\pi} \sum_{n=2}^\infty f_n tr_c(E^n) + \bar{\psi}^a(i\bar{\nabla} - m^i \mathbb{1})\psi^a \right\} \tag{2.2}
\]

To solve this theory we choose the usual gauge \(A^\bar{a}_- = 0\). and we can rewrite the previous expression as

\[
S = \int d^2 x \left\{ \frac{N}{4\pi} tr_c(E\partial_- A_+) - \frac{Ng^2}{4\pi} \sum_{n=2}^\infty f_n tr_c(E^n) + \bar{\psi}^a(i\bar{\nabla} - m_i \mathbb{1})\psi^a - A^\bar{a}_+(\bar{\psi}^a \gamma_- T^a \psi^a) \right\} \tag{2.3}
\]

\(^1\)Conventions.

\(x^\pm = x^0 \mp \frac{i}{\sqrt{2}}(x^1 \pm x^2)\) \hspace{1cm} \(A^\mu B_\mu = A_0 B_0 - A_1 B_1 = A_+ B_- + A_- B_+\) \hspace{1cm} \(e^{\mu} = -e^{\nu} = 1\)

\[
\begin{align*}
\gamma_+ &= \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix} \quad \gamma_- &= \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix} \quad \gamma_0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma_1 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\
\gamma_5 &= -\gamma_0 \gamma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad P_{R,L} = \frac{1 \pm \gamma_5}{2}
\end{align*}
\]

\[
\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad \bar{\psi} = \begin{pmatrix} \bar{\psi}_- & \bar{\psi}_+ \end{pmatrix} \quad \chi = -\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \bar{\psi} P_R \chi & \bar{\psi} \gamma_5 \chi \\ \psi \gamma_5 \chi & \sqrt{2} \bar{\psi} P_L \chi \end{pmatrix}
\]

\[
\int_x = \int d^2 x \quad \int_p = \int \frac{d^2 p}{(2\pi)^2}
\]

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where \( tr_a \) and \( tr_\alpha \) are the traces over the flavour and spin indices.

If we integrate over \( A_{+}^a \), we get the constraint
\[
\partial_x - \left( \frac{N}{4\pi} \bar{E}^a \right) + \bar{\psi}^a \gamma_- T^a \psi^a = 0
\]
(2.4)
in the form of a delta function in the path integral. Since \( E \) is in the adjoint representation of \( U(N) \), the constraint (2.4) can be rewritten as
\[
\partial_x - \left( \frac{N}{4\pi} \bar{E}^{AB} \right) + \bar{\psi}^B \gamma_- \psi^A = 0
\]
(2.5)
Now we can integrate easily over \( E^{AB} \) getting the effective action for the \( \psi \) fields
\[
S_{\text{eff}} = \int d^2x \left\{ \bar{\psi}^A (i\partial^x - m_a) \psi^A - \frac{Ng^2}{4\pi} \sum_n \left( \frac{4\pi}{N} \right)^n f_n (\bar{\psi}^{-1}_n A_{-a} \gamma_- \psi^{A_1} a_1) \cdots (\bar{\psi}^{-1}_n A_{-a} \gamma_- \psi^{A_n} a_n) \right\}
\]
(2.6)

The interaction term suggests to introduce the (composite) bilocal field
\[
U_{a\alpha,b\beta}(x,y) = UPQ = -\frac{\sqrt{2}}{N} \text{tr}(\bar{\psi}^a_\alpha (x) \psi^b_\beta (y)) = \begin{pmatrix} \sigma_R^{ab}(x,y) & \rho^{ab}_R(x,y) \\ \rho^{ab}_L(x,y) & \sigma_L^{ab}(x,y) \end{pmatrix}
\]
(2.7)
where \( P = (xa\alpha) \), \( Q = (yb\beta) \), and
\[
P_{PQ}(u) = P_{xa\alpha,yb\beta}(u) = \left( -\frac{4\pi}{\sqrt{2}} \right) \delta^2(x-y) \partial_{-1}^{-1}(u,y) \delta^{ab} \gamma_- \alpha \beta
\]
(2.8)
The effective action can then be rewritten as
\[
\frac{1}{N} S_{\text{eff}} = \int_{x,y} d^2 x \frac{1}{\sqrt{2}} \delta^2(x-y) \text{tr}_{a\alpha} \left( (i\partial^x - m_a) U(x,y) \right) + \frac{g^2}{4\pi} \sum_n f_n \int_x Tr (P(x) U)^n
\]
(2.9)
where \( Tr = tr_P = tr_x tr_a \), and
\[
D = \| D_{PQ} \| = \begin{pmatrix} -\frac{1}{\sqrt{2}} m_a \delta^{ab} \delta^2(x-y) & i \delta^{ab} \partial_x \delta^2(x-y) \\ i \delta^{ab} \partial_x \delta^2(x-y) & -\frac{1}{\sqrt{2}} m_a \delta^{ab} \delta^2(x-y) \end{pmatrix}
\]
(2.10)
In order to get the effective action for the field $U$, we have to compute the jacobian of the transformation $J[U]$ from $\psi$ to $U$. This can be accomplished as in Ref.s (8, 12). The result is

$$J[U] \propto \int [dM] \exp \left\{ N[i \text{Tr}(MU) + Tr \log M] \right\} \quad (2.11)$$

The final form for the effective action is given as a functional of the $U$ and $M$ fields, and it is

$$\frac{1}{N} S_{\text{eff}} = Tr \left( DU + \frac{g^2}{4\pi} \sum_n f_n \int_x [P(x)U]^n + M U - i \log M \right) \quad (2.12)$$

From this action we can immediately compute the saddle point equations:

$$\begin{cases}
U_0 \mathcal{M} - i \mathbb{1} = 0 \\
D + \frac{g^2}{4\pi} \sum_n n f_n \int_x [P(x)U]^{n-1} P(x) + \mathcal{M} = 0
\end{cases} \quad (2.13)$$

that yield

$$D U_0 + \frac{g^2}{4\pi} \sum_n n f_n \int_x (P(x)U)^n = -i \mathbb{1} \quad (2.14)$$

Following the approach used in the usual case (8), we introduce the self-energy $\Sigma = \frac{1}{2} \left( \begin{array}{cc} \Gamma_L & \Gamma_- \\ \Gamma_+ & \Gamma_R \end{array} \right)$ through the definition

$$U_0 = -i(D + \Sigma)^{-1} \quad (2.15)$$

Inserting it in eq. (2.14) we get the following eq. for $\Sigma$

$$\Sigma = \frac{g^2}{4\pi} \sum_n n f_n \int_x \left( P(x) \frac{-i}{D + \Sigma} \right)^{n-1} P(x) \quad (2.16)$$

that yields immediately $\Gamma_L = \Gamma_- = \Gamma_R = 0$ because of the $\gamma_-$ included in $P(x)$. Assuming translational invariance for the ground state as in Ref. (8), we can write the master field as

$$U_{0PQ} = \int_{p,q} e^{ipx+iqy} U_{0\alpha\beta}^{ab}(p,q)$$

$$U_{0\alpha\beta}^{ab}(p,q) = (2\pi)^2 \delta(p+q) \frac{i \delta^{ab}}{p^2 - m_a^2 - p_- \Gamma_+(p) + i\epsilon} \left( \begin{array}{cc} -\sqrt{2} m_a & 2p_- \\ 2p_+ - \Gamma_+(p) & -\sqrt{2} m_a \end{array} \right)_{\alpha\beta} \quad (2.17)$$
Using the previous eq. (2.17) the eq. (2.16) can be easily rewritten in the momentum space as

\[ \Gamma_{\pm}^a(p) = 2g^2 \sum_n (-i)^n n f_n I_n^a(p, p) \]  

(2.18)

where

\[
\delta_{ab} I_n^a(p, q) = (4\pi)^{n-1} \int_{p_1\ldots p_{n-1}} \frac{1}{(p - p_1) - \rho^{k_1}_-(p_1)} \frac{1}{(p_1 - p_2) - \rho^{k_2}_-(p_2)} \ldots \frac{1}{(p_{n-1} - q) - \rho^{k_{n-1}}_-(p_{n-1})} \]  

(2.19)

where \( \rho_- \) is defined in eq. (2.7).

Using the principal value regularization for \( \frac{1}{k_-} \) as in Ref. ([11]) and symmetric \( p_+ \) integration done by 't Hooft ([2], we get the result

\[ I_n^a(p, p') = \int_{-\infty}^{\infty} dk_1 \ldots dk_{n-1} \]  

\[ \frac{\mathcal{P}}{p - k_1} sgn(k_1) \frac{\mathcal{P}}{k_1 - k_2} sgn(k_2) \ldots \frac{\mathcal{P}}{k_{n-2} - k_{n-1}} sgn(k_{n-1}) \frac{\mathcal{P}}{k_{n-1} - p'} \]  

(2.20)

Notice the useful property: \( I_n(\alpha p, \alpha p') = \frac{1}{\alpha} I_n(p, p') \), that will turn out to be useful while computing the full \( q\bar{q} \) amplitude in the next section.

### 3 The full quark-antiquark amplitude

In order to compute the full \( q\bar{q} \) amplitude we add a source \( J \) coupled to the composite field \( U \) to the action. The partition function in presence of an external source \( J \) is given by:

\[ Z[J] = Z_J = \int [dU][dM] e^{iNS_J} \]  

(3.1)

where

\[ S_J = S_{eff} - i Tr(JU) \]  

(3.2)

Differentiating wrt \( J \) the logarithm of the partition function, we find the connected Green functions (from now on all the capital roman indices will
be used to indicate collection of indices \( (xaα) \)

\[
G_{1AB} = \langle U_{AB} \rangle_c = \frac{1}{N} \left. \frac{\delta \log Z_J}{\delta J_{BA}} \right|_{J=0} 
\]

\[
G_{AB;CD} = \langle U_{AB} U_{CD} \rangle_c = \frac{1}{N^2} \frac{\delta^2 \log Z_J}{\delta J_{BA} \delta J_{DC}} 
\]

To the leading order in \( N \) we have obviously that

\[
\log(Z_J^{(0)}) = i NS_J[U] 
\]

\[
\langle U_{AB} \rangle_c^{(0)} = U_{0AB} 
\]

\[
\langle U_{AB} U_{CD} \rangle_c^{(0)} = \frac{1}{N^2} U_{AB;CD} 
\]

where \( U_J \) is the saddle point value in presence of the source, i.e.

\[
U_{JAB} = U_{0AB} + U_{1AB;PQ} J_{QP} + \ldots 
\]

and it is given by the solution of the equation generalizing \((2.14)\) in the presence of \( J \)

\[
D + i U^{-1}_J + \frac{g^2}{4\pi} \sum_n n f_n \int_x (P(x)U)_J^{n-1} P(x) + J = 0 
\]

This equation cannot be solved exactly for an arbitrary \( J \), but it can be solved perturbatively in \( J \). Inserting \((3.8)\) in the previous equation and using

\[
U^{-1}_J \approx \left( U^{-1}_0 - U^{-1}_0 U_{1PQ} U^{-1}_0 J_{QP} \right)_{J_{AB}} 
\]

we find

\[
D + i U^{-1}_0 + \frac{g^2}{4\pi} \sum_n n f_n \int_x (P(x)U)^{n-1}_0 P(x) = 0 
\]

\[
\text{and} \\
- i U^{-1}_0 U_{1PQ} U^{-1}_0 - i X_{QP} + \\
+ \frac{g^2}{4\pi} \sum_n n f_n \sum_{m=0}^{m=n-2} \int_x \left( (P(x)U)^m_0 P(x) \right) U_{1PQ} \left( (P(x)U)^{n-m-2}_0 P(x) \right) = 0 
\]
where $X_{AB,PQ} = \delta_{AQ}\delta_{BP}$ The first of these equations has as solution the expression in eq. (2.17), while the other can be formally solved as follows:

$$U_1 = - \left( \mathbb{I} + \frac{i g^2}{4\pi} \sum n f_n \sum_{m=-2}^{m=-1} \int_x \left[ (P(x)U_0)^{m+1} \otimes (P(x)U_0)^{n-m-1} \right] \right)^{-1} U_0 \otimes U_1 X \tag{3.13}$$

If we define

$$i U_0 \otimes U_1 X = \frac{g^2}{4\pi} \sum n f_n \sum_{m=0}^{m=-1} \int_x \left[ (P(x)U_0)^{m+1} \otimes (P(x)U_0)^{n-m-1} \right] \tag{3.14}$$

the previous equation for $U_1$ can be rewritten in two different revealing forms as

$$U_1 = -(U_0 \otimes U_1 X) - (U_0 \otimes U_1 X) \ i G \ U_1 \tag{3.15}$$

The first recursive form reveals clearly that the role of the usual gluonic interaction is now played by $G$. Moreover the second form shows that

$$T = G - i X U_0 \otimes U_1 G^{-1} \tag{3.16}$$

is the full quark-antiquark truncated amplitude and it is the generalization of the full $q\bar{q}$ amplitude of Callan, Curtis and Gross $T_{CCG}$ ([3]), as it is transparent from the following recursion relation, that it satisfies

$$T = G - i X (U_0 \otimes U_1) T \tag{3.17}$$

Notice however that the recursion relation for $T$ has a minus sign in front of the second term on rhs of difference wrt the CCG one: this has no consequences and it is due to the fact that we are dealing with an effective theory.

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\^2 We use the convention that all the matrix operations written in capital letter refer to the mesonic indices, i.e. couples $(AB)$, while the others to the quark ones, i.e. $A, B, \ldots$. Explicitly $(A^T)_{AB,CD} = A_{CD,AB}$ is the transposition wrt to the mesonic indices while $(X^t)_{AB} = X_{BA}$ is wrt to the quark ones.

We use also $(X \otimes Y)_{AB,CD} = X_{AC}Y_{BD}$ and $(A_{PQ} \otimes Y)_{AB,CD} = A_{AC,PQ}Y_{BD}$ and $(A_{PQ}X)_{AB} = A_{AC,PQ}X_{CB}$. 

where the gluonic degrees of freedom are integrated away. This implies the appearance of quartic fermionic vertices and hence the substitution of every loop quark-gluon-quark-gluon in the original theory by a fermionic loop due to the elimination of the gluons. Since every fermionic loop carries a minus sign, this explains the origin of the difference.

Now we go to the momentum space and we write the "gluonic" interaction $G$ and the full $q\bar{q}$ amplitude explicitly as

$$G_{AB;CD} \equiv G^{ab,cd}_{\alpha\beta\gamma\delta}[p, q, q', p'] = (2\pi)^2 \delta^2(p + p' + q + q') \delta^{bc} \delta^{da} (\gamma_-)^{\beta\gamma} (\gamma_-)^{\delta\alpha} G^{ab}(p, p'; r = p + q)$$  \hspace{1cm} (3.18)

$$G^{ab}[p, q, q', p'] = 2\pi g^2 \sum_{n=2} (-i)^n n f_n \sum_{m=1}^{m=n-1} I^b_m(q, -q') I^a_{n-m-1}(p', -p)$$  \hspace{1cm} (3.19)

and

$$T_{AB;CD} \equiv T^{ab,cd}_{\alpha\beta\gamma\delta}[p, q, q', p'] = (2\pi)^2 \delta^2(p + p' + q + q') \delta^{bc} \delta^{da} (\gamma_-)^{\beta\gamma} (\gamma_-)^{\delta\alpha} T^{ab}(p, p'; r)$$  \hspace{1cm} (3.20)

where all the momenta are incoming. The recursion relation (3.17) reads now as

$$T^{ab}(p, p'; r) = G^{ab}(p, p'; r) - 2i \int \rho^a_{-m}(r - m) T^{ab}(m, p'; r)$$  \hspace{1cm} (3.21)

Defining as usual the $x$ and $x'$ variables as $p_- = x r_-, p'_- = -x' r_-$ (the minus is due to the fact that our momenta are all incoming and we want to make contact with the usual conventions) and

$$\phi(x, x'; r) = \frac{i}{2} \int dp_+ \rho^a_{-}(p - p) T^{ab}(p, p'; r)$$  \hspace{1cm} (3.22)

the previous equation (3.21) yields $T$ as a functional of $\phi$

$$T^{ab}(p, p'; r) = \frac{1}{r_-} G^{ab}(x, -x'; 1) - \frac{1}{\pi r_-} \int dy G^{ab}(x, y; 1) \phi^{ab}(y, -x'; r)$$  \hspace{1cm} (3.23)

Defining the eigenfunctions $\phi^{ab}_n(x)$ through the equation

$$M^2_{(ab)n} \phi^{ab}_n(x, x'; r) = \left[ \frac{m^2_a + x \Gamma_+(x)}{x} + \frac{m^2_b + (1 - x) \Gamma_+(1 - x)}{1 - x} \right] \phi^{ab}_n(x, x'; r) - \int_0^1 dy \frac{2}{\pi} G^{ab}(x, -y; 1) \phi^{ab}_n(y, x'; r)$$  \hspace{1cm} (3.24)
and supposing that they form a complete system in $x \in [0, 1]$, we can write
the full amplitude $T$ as

$$T^{ab}(p, p'; r) = \frac{1}{r^2} G^{ab}(x, -x'; 1)$$

$$- \frac{4}{\pi^2 r^2} \sum_n \frac{1}{r^2 - M_{(ab)n}} \int_0^1 dy \ G^{ab}(x, -y; 1) \phi^{ab}_n(y, r) \int_0^1 dy' \ G^{ab}(y', -x'; 1) \phi^{abc}_n(y'; r)$$

(3.25)

Specializing the previous equation to the normal case, we notice that $T_{CCG} = 2i T$; the factor $i$ can be traced back to the definition (3.16) and the factor 2 is due to the definition of $U$ (2.7) (see also (3.7)).

4 The general case.

The most general expression for the generalized QCD is given by the action

$$S = \int d^2 x \left\{ N \left[ \frac{1}{8\pi} \text{tr}(E\tilde{F}) - \frac{Ng^2}{4\pi} \sum_{\{n_i\}} f_{\{n_i\}} \prod_{n \in \{n_i\}} \text{tr}(E^n) + \bar{\psi}^a(i\gamma^\mu - m^i) \psi^a \right] \right\}$$

where $\{n_i\}$ is a whatever sequence of natural number satisfying the condition $n_i \leq n_{i+1}$. Repeating the previous steps we find the effective action

$$\frac{1}{N} S_{eff} = Tr \left( D U + \frac{g^2}{4\pi} \sum_{\{n_i\}} f_{\{n_i\}} \int_x \prod_{n \in \{n_i\}} (P(x)U)^n + M U - i \log M \right)$$

(4.1)

and the saddle point condition

$$D U_0 + \frac{g^2}{4\pi} \sum_{\{n_i\}} f_{\{n_i\}} \sum_{n \in \{n_i\}} \int_x (P(x)U)_0^n \prod_{m \in \{n_i\} \backslash \{n\}} Tr(P(x)U)^m = -i \mathbb{1}$$

(4.2)

Using the translational invariance of the saddle point, i.e. of the ground state, it turns out that

$$T_n = Tr(P(x)U)^n_0 = 4\pi \left( -\frac{\sqrt{2}}{4\pi i} \right)^n \int_p \sum_a I^n_a(p, p) \rho^{aa}_n(p)$$

(4.3)

is independent of $x$, this means that the whole effect of introducing the most general potential amounts in generating the effective coupling constants $\tilde{f}_n$
given by
\[ \bar{f}_n = \frac{1}{T_n} \sum_{\{n_i\}} f_{\{n_i\}} d_n(\{n_i\}) \prod_{m \in \{n_i\}} T_m \] (4.5)

where \(d_n(\{n_i\})\) is the degeneration of \(n\) in the sequence \(\{n_i\}\). As it can be shown easily all the formula of the previous section are still valid with the substitution \(f_n \rightarrow \bar{f}_n\) (4.5).

We notice however that the theory is independent of \(\bar{f}_1\) because \(\int_x P(x) = 0\) as a consequence of \(\int_x P(x) \propto a_+ \gamma_-\) and of the absence of a constant vector \(a_+\) in the theory.

Moreover we have
\[ T_{2n+1} = 0 \] (4.6)

and this implies that it is impossible to induce terms with odd powers starting from terms with even total powers. The demonstration of (4.4) goes as follows:
\[ T_{n+1} \propto \int_{p,p_1...p_n} \frac{1}{(p-p_1)_-} \rho^{a_1}_-(p_1) \frac{1}{(p_1-p_2)_-} \cdots \rho^{a_n}_-(p_{n-1}) \frac{1}{(p_{n-1}-p)_-} \rho^{b_1}_-(p) \] (4.7)

then we rename the integration variables as \(p \leftrightarrow p_n, p_1 \rightarrow p_{n-1}, p_2 \rightarrow p_{n-2} \cdots\) and we use \(\rho_-(p) = -\rho_-(p)\) (that follows from (2.18, 2.17)), and we find
\[ T_n = (-)^n T_n, \] that proves what asserted, independently of the regularization scheme.

## 5 Conclusions.

In this work we have solved the most general case of minimally coupled generalized QCD\(_2\) (\([11]\)) using the bilocal method (\([12, 8]\)). This is achieved in a straightforward and transparent way, thus showing the effectiveness of the method, when it is applied to vector-like models coupled to matrix gluon fields, which in two dimensions are not dynamical (see also \([13]\]): the main point that guarantees the successful application of the large \(N\) techniques is the possibility of defining colourless fields (see eq. (2.17)) that allows one to extract the \(N\) dependence both in the action and in the measure of integration (the mesonic bilocal \(U(x, y)\) is a global colour singlet, as it is the auxiliary bilocal field \(M(x, y)\)).
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