Natural explanation for discrete R-symmetry in successful inflation in N=1 supergravity

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Abstract

Recently it was shown that discrete R-invariance in superpotential can lead to a successful flat potential in inflation. We suggest that this discrete R-symmetry arises from an underlying supersymmetric gauge theory, which gives rise to a scalar inflaton as a composite field. This discrete R-symmetry is common to the dynamical breaking scenarios in global supersymmetry studied several years ago. Some extension of the R-invariance method is also shown.
1 Introduction

Although supersymmetric theories seem very attractive for grand unification[1], there are some problems. It seems that low energy theories are well explained by N=1 supergravity theories, but there are potential cosmological problems. One of them is the difficulty to construct the inflation scenarios of the universe.

Many models are proposed to solve these problems. Recently Kumekawa et.al[2]. have proposed a successful inflation model imposing $Z_n$ symmetry on superpotential. This model has a very flat potential for inflation, and reheating temperature $T_R$, and gravitino mass $m_{3/2}$ are reasonable if we choose $Z_4$.

Using a composite field for the inflaton field, we suggest that the origin of the $Z_n$ symmetric inflation may be explained naturally by field condensation. We also consider what happens if we introduce another matter field for the hidden sector.

2 Review of discrete R-symmetric model

In this section we briefly review the idea of ref.[2]. We define a discrete $Z_n$ R-transformation on the inflaton field $\phi(x, \theta)$ as

$$\phi(x, \theta) \rightarrow e^{-i\alpha} \phi(x, e^{i\alpha} \theta), \quad \alpha = \frac{2\pi k}{n}, \quad (k = 0, \pm1, \pm2, ...). \quad (2.1)$$

General form for a Kähler potential $K(\phi, \phi^*)$ and a superpotential $W(\phi)$ for the inflaton are given by

$$K(\phi, \phi^*) = \sum_{m=1}^{\infty} a_m (\phi \phi^*)^m \quad (2.2)$$

$$W(\phi) = \phi \sum_{n=0}^{\infty} b_n \phi^n. \quad (2.3)$$

From now on, we take the minimum Kähler potential ($a_1 = 1, a_i = 0$ for $i \neq 1$). To see the shape of the potential near the origin, we write superpotential as

$$W(\phi) = \left(\frac{\lambda}{v^{n-2}}\right) \left(v^n \phi - \frac{1}{n+1} \phi^{n+1}\right) + \cdots. \quad (2.4)$$

Here $\lambda$ is a coupling constant and $v$ is a scale factor. A scalar potential can be derived using the relation
\[
V(\phi) = \exp \left( \frac{K(\phi, \phi^* )}{M^2} \right) \left\{ (K^{-1})^\phi (D\phi W)(D\phi W)^* - \frac{3|W|^2}{M^2} \right\} \tag{2.5}
\]

and taking the approximation \( \phi \approx 0 \), we get

\[
V(\phi) \simeq \left( \frac{\lambda}{v^{n-2}} \right)^2 \left\{ v^{2n} \left( |\phi|^2 \right)^2 - v^n (\phi^n + \phi^* n) \right\}. \tag{2.6}
\]

Where \( M \) is the gravitational mass \( M = \frac{M_{\text{pl}}}{\sqrt{8\pi}} \approx 2.4 \times 10^{18}\text{GeV} \). When \( n = 2 \), this potential does not have a flat potential. For \( n = 3 \) and more, we can neglect the \( \left( \frac{|\phi|^2}{M^2} \right)^2 \) term as ref.[2]. Setting the phase of \( \phi \) to vanish, we get

\[
V(\phi) \simeq \tilde{\lambda} \tilde{v}^4 \left\{ 1 - 2\left( \frac{\phi}{\tilde{v}} \right)^n \right\}. \tag{2.7}
\]

Here we set

\[
\phi \equiv \sqrt{2} \text{Re}\phi \tag{2.8}
\]
\[
\tilde{\lambda} \equiv \frac{1}{2} \lambda \tag{2.9}
\]
\[
\tilde{v} \equiv \sqrt{2} v. \tag{2.10}
\]

This potential is very flat near the origin. Using this potential, we can calculate Hubble constant \( H \), e-folding factor \( N \), density fluctuation \( \delta_\rho / \rho \), gravitino mass \( m_3 \), and inflaton mass \( m_\phi \), etc. In this case, \( \Lambda_{\text{cos}} \) should be fine tuned introducing a D-term.

Naively, decay width of the inflaton field \( \Gamma_\phi \) is so small that reheating is not enough. But this problem can be avoided introducing another singlet chiral super multiplet with a half \( Z_n \) charge [2].

This model can explain successful inflation, particularly for \( Z_4 \) case.

### 3 Natural explanation for \( Z_n \) symmetry

Let us consider supersymmetric Yang Mills (SYM) case, with SU(N) gauge symmetry. In this section, we follow the notations of ref.[3].

\[
L_{\text{SYM}} = \frac{1}{4} \int d^2 \theta \ W^a W^a + \frac{1}{4} \int d^2 \tilde{\theta} \tilde{W}^a \tilde{W}^a \tag{3.1}
\]
In this Lagrangian, there exists global R-symmetry $U(1)_R$ [3] for the gaugino field $\lambda$ as

$$\lambda \rightarrow e^{i\alpha}\lambda \quad (3.2)$$

This symmetry has anomaly, and is broken at the quantum level. But its subgroup $Z_{2N}(\alpha \equiv \frac{2\pi k}{2N}, k = 1, \ldots, 2N)$ still unbroken[3].

Considering gaugino condensation, we introduce a composite field.

$$S = \frac{g^2}{32\pi^2}W^aW^a, \quad (3.3)$$

with $< S > \neq 0$. This field has desired discrete R-symmetry $Z_N$.

Taking this composite field as inflaton field, we can get a flat potential as is described above.

Here we should mention the anomaly matching condition relating to the global R-symmetry(3.2). If we consider this condition, we get a logarithmic potential [2].

$$W(S) \sim S\log S^N. \quad (3.4)$$

This superpotential leads to the scalar potential which has a logarithmic singularity at the origin $S = 0$. This singularity stems from the fact that near the origin $S \simeq 0$, the anomaly cannot be simply computed in terms of the field $S$ but should be calculated by taking into account the dynamical degrees of freedom contained in the theory before the condensation, such as $\lambda$. Therefore, denoting the typical energy scale of the gaugino condensation as $\Lambda$, when $S \sim \Lambda$, we cannot describe the theory in terms of the composite field $S$ alone, and the anomaly matching condition for the superpotential breaks down. Even in such regions(i.e., $S \sim \Lambda$), when the condensation is almost completed, the behavior of the order parameter $S$ will be still described by the effective potential which is regular near the origin. This potential should be $Z_N$ symmetric since $Z_N$ symmetry always exists regardless whether we implement the anomaly matching condition in the scalar freedom or not, and the potential,which is analogous to the free energy, should not have singularity at the origin. This assumption seems reasonable except for the choice of applicable domain of the effective scalar potential, which should be assumed by hand for the successful inflational scenario. For example, in order to have successful inflation, potential should satisfy above conditions ($Z_N$ symmetry and no singularity near the origin) at least in the
The lower bound in (3.5) arises from the initial fluctuation and the upper bound is required for sufficient inflation [2]. When S is smaller than (3.5), the fermionic freedom $\lambda$ would dominate, and when $S$ is much larger than (3.5), the potential would be well described by (3.4).

Assuming that the flatness can be satisfied on the basis of a physical picture suggested above, the supersymmetry breaking required for phenomenological analysis is not quite satisfied in these simple models because the potential far from the origin is different from that in ref.[2]. The supersymmetry breaking by gaugino condensation have been studied in detail in[5](see also [6] and [7]), and we can apply this $Z_n$ mechanism to more complicated models, but this is beyond the scope of this paper. This problem will be studied elsewhere.

Related to this problem, the constraints to the parameter are different from these in ref.[2]. In our model, $v$ is not the vacuum expectation value of $S$ at the potential minimum. Constraints arising from successful inflation to $v$ are the same but those from supersymmetry breaking are very different from ref.[2].

We now comment on some related problems. We can add matter fields. For simplicity, here we consider supersymmetric QCD (SQCD)[3]. Lagrangian is given by

\[
L_{SQCD} = \frac{1}{4} \int d^2 \theta W^a W^a + \frac{1}{4} \int d^2 \theta \overline{W}^a W^a + \int d^4 \theta \left[ \Phi^i + i e g V^a T^a [N] \Phi_i + \tilde{\Phi}^i + i e g V^a T^a [\tilde{N}] \tilde{\Phi}_i \right] + \int d^2 \theta m_{ij} \Phi_i \tilde{\Phi}^j + \int d^2 \theta m_{ij} \overline{\Phi}_i \overline{\Phi}^j.
\]  

In this case, global R-symmetry becomes (in terms of composite fields)

\[
S \rightarrow e^{-2i\alpha} S(x, e^{i\alpha} \theta) \quad (3.7)
\]
\[
T \rightarrow e^{-2i\alpha} T(x, e^{i\alpha} \theta). \quad (3.8)
\]

This R-symmetry is also broken by anomaly. Here we used the definition

\[
T^j_i \equiv \Phi_i \tilde{\Phi}^j. \quad (3.9)
\]
There is another global symmetry, called X-symmetry.

\[ S \rightarrow e^{-2iM\alpha} S(x, e^{iM\theta}) \quad (3.10) \]

\[ T \rightarrow e^{2i(N-M)\alpha} T(x, e^{iM\theta}) \quad (3.11) \]

Here \( M \) means number of flavor. This X-symmetry is a combination of axial and R-symmetry, and anomaly-free. Now we can construct superpotential for SQCD. For example,

\[ W(S, T) = S \sum_{l=0}^{\infty} c_l(S^{N-M}|T|^l) \quad (3.12) \]

Notice that this potential has large Yukawa couplings. This means, even though we could make a successful flat potential for inflation, almost all energy runs away to the hidden sector. The reheating of hidden sector results in dark matter or nothing. Of course, reheating in the observable sector is extremely suppressed.

4 Conclusion

We have suggested that the gaugino condensation in the hidden sector can lead to the successful inflation, if one assumes that the behavior of the order parameter \( S \) from the gaugino condensation is controlled by the effective potential(which is singularity-free) in the vicinity of the condensation energy scale. This is because the basic underlying theory respects the \( Z_n \) symmetry as is required for the successful inflation in ref.[2]. We also analyzed some extension to include mater fields.

There are some problems to be resolved. To consider inflation, we are interested in the shape of the potential near the origin, which in turn means that we should consider the critical behavior of the condensation. Validity of our basic picture described in this note depends on the detailed dynamics of the condensation, and it’s analysis is beyond the scope of our discussion.

Our analysis in this paper should be regarded as a proposal of a possible physical picture behind the \( Z_n \) symmetry.

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