Generation and characterization of a source of wavelength division multiplexing quantum key distribution

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Abstract

Using spontaneous parametric down-conversion, photon pairs entangled in frequency and polarization were generated. After frequency resolving the photon pairs, the polarization correlations were measured on several polarization basis, and it was confirmed that the frequency resolved photon pairs were entangled in polarization, indicating the photon pairs can be used as a source of wavelength division multiplexing quantum key distribution.

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INTRODUCTION

Photon pairs, generated by spontaneous parametric down-conversion (SPDC), are entangled in various parameters. The polarization entanglement of these SPDC photon pairs has been used in varieties of quantum experiments to demonstrate quantum teleportation, quantum key distribution, Bell-inequality violations, and others [1].

The wave-vector entanglement has also been used in various experiments, like quantum imaging [2, 3], photonic de Broglie wavelength measurement [4, 5, 6], quantum interference [7, 8], and quantum lithography [9, 10, 11]. In case of quantum imaging, the information about the shape of a spatial filter is transferred by entangled photon pairs. The wave vector is a continuous parameter, therefore its entanglement can send much more information than the polarization entanglement. Actually, a protocol for quantum key distribution was proposed based on a system whose dimension is higher than 2 in Ref. [12].

Photon pairs entangled in frequency were used for nonlocal pulse shaping [13] and spectroscopy [14]. In Ref. [14], signal and idler photons of SPDC photon pairs were separated from each other by a polarizing beam splitter, which destroys the polarization entanglement. Replacing the polarizing beam splitter with a non-polarizing beam splitter, the photon pairs are got to be entangled not only in frequency but also in polarization. In this case, the photon pairs are entangled in polarization even after they are resolved by their frequency, therefore they can be used as a source of wavelength-division multiplexing quantum key distribution (WDM-QKD).

In this article, the photon pairs entangled polarization even after being resolved by their frequency were generated and their characterization was done. The result of the measurement shows that they are applicable as a source of WDM-QKD.

EXPERIMENT

The schematic drawing of our experimental setup is shown in Fig. 1. Frequency-nondegenerate photon pairs were generated by spontaneous parametric down-conversion (SPDC) in a 1-mm-thick type II BBO (\(\beta\)-BaB\(_2\)O\(_4\)) crystal pumped by the second harmonic light at 859.4 nm (1.5 mW) of a cw Ti:sapphire laser. The signal and idler photons of the SPDC pairs were emitted conically from the focal waist where the pump light was focused.
on the BBO crystal by a lens of 1000-mm focal length (L1). An iris diaphragms (IRIS) was placed at the 100 mm behind the BBO crystal in order to select spatially the crossing point of the signal light cone and the idler light cone where the SPDC photon pairs are entangled. These pairs passed through the iris were collimated by an off-axis parabolic mirror (M) of 25.4-mm focal length. A prism (P1) was used to eliminate the remainder of the pump light, which can be a noise source in the experiments. Another prism (P2) was used to compensate frequency dependent angular dispersion induced by the prism P1. When the light beam passes through the prism pairs, the beam height was lowered by a mirror (M3), and only the SPDC pairs were picked out by another mirror (M2). The signal and idler photons were separated from each other by a non-polarizing beam splitter (BS). A linear polarizer (POL1) was placed in the light path of idler photons (photons transmitted the BS). The polarizer was on a motorized continuous rotation stage, and its angle was computer-controlled. Signal photons (photons reflected by the BS) were diffracted by a grating (G) (1400 grooves/mm). Another linear polarizer (POL2) was placed in the signal light path, and its angle was also computer-controlled.

FIG. 1: Experimental setup.
THEORY

For simplification, the state of the SPDC photon pair of signal \((j = s)\) and idler \((j = i)\) photons is assumed to be a pure biphoton state in this article as follows

\[
|\Psi\rangle = \frac{1}{\sqrt{1 + f^2}} \left( |H\rangle_s |V\rangle_i + f e^{i\alpha} |V\rangle_s |H\rangle_i \right),
\]

where \(|H\rangle_j\) and \(|V\rangle_j\) are single-photon states with horizontal and vertical polarizations, respectively. The electric fields which transmit the polarizer POL1 and POL2 are expressed as

\[
E_s^{(+)} = a_{s,H} \sin \theta_s + a_{s,V} \cos \theta_s,
\]

\[
E_i^{(+)} = a_{i,H} \sin \theta_i + a_{i,V} \cos \theta_i.
\]

Here \(a_{j,H}\) and \(a_{j,V}\) are annihilation operators of horizontally polarized photons and vertically polarized photons, respectively, and \(\theta_j\) is the angle between the polarizer axis and the vertical axis.

The coincidence counting rate of SPDC photon pairs \(R_c(\alpha, f, \theta_s, \theta_i)\) can be written as

\[
R_c(\alpha, f, \theta_s, \theta_i) \propto \langle \psi | E_s^{(-)} E_i^{(-)} E_s^{(+)} E_i^{(+)} |\psi \rangle = \left| \langle 0 | E_s^{(+)} E_i^{(+)} |\psi \rangle \right|^2.
\]

Substituting Eqs. (1) (2) into Eq. (3), we get

\[
R_c(\alpha, f, \theta_s, \theta_i) \propto \frac{1}{4} \left( 1 + f^2 + 2 f \cos \alpha \right) \sin^2 (\theta_s + \theta_i) + \frac{1}{4} \left( 1 + f^2 - 2 f \cos \alpha \right) \sin^2 (\theta_s - \theta_i) + \frac{1}{2} \sin (\theta_s + \theta_i) \sin (\theta_s - \theta_i)
\]

Results calculated with sets of parameters are shown in Fig. 2.

In this article, \(\Theta_i(\theta_s)\) is defined as the angle \(\theta_i\) at which the coincidence counting rate is maximized. When \(f = 1\), Fig. 2 shows \(\Theta_i(45^\circ)\) and \(\Theta_i(135^\circ)\) are rotated by 45\(^\circ\) clockwise and anti-clockwise, respectively, from \(\Theta_i(0^\circ)\). However, when \(f \neq 1\), the angular rotation is smaller than 45\(^\circ\). The shift is calculated to be 30\(^\circ\) in case of \(f = 1.73\).

In case of \(\alpha = 0^\circ\) or 180\(^\circ\), the visibility of the polarization correlations in Fig. 2 is unity. However it is smaller than one in other cases, like \(\alpha = 60^\circ\). In case of \(f = 1\) and \(\alpha =\)
FIG. 2: Simulated results of the polarization correlations using of Eq. (4) for several configurations. (a) $f = 1, \alpha = 0^\circ$. (b) $f = 1, \alpha = 180^\circ$. (c) $f = 1, \alpha = 60^\circ$. (d) $f = 1.73, \alpha = 0^\circ$.

When the state of the SPDC photon pair is a mixed state as follows
$$|\Psi\rangle = (|H\rangle_s + |V\rangle_s) \cdot (|H\rangle_i + |V\rangle_i),$$
the coincidence counting rate of SPDC photon pairs $R_c(\theta_s, \theta_i)$ can be calculated as follows
$$R_c(\theta_s, \theta_i) \propto \sin^2(\theta_i + 45^\circ) \sin^2(\theta_s + 45^\circ),$$
by substituting Eq. (5) into Eq. (3). It shows $\Theta_i(\theta_s)$, the angle $\theta_i$ at which the coincidence counting rate is maximized, is independent of $\theta_s$, and $\Theta_i(0^\circ) = \Theta_i(45^\circ) = \Theta_i(135^\circ)$.

RESULTS AND DISCUSSION

As is mentioned before, an iris makes SPDC photon pairs to be entangled in their polarizations. It was confirmed under the condition when the detector of signal photons was pointed to the zeroth-order diffraction light from the grating, by measuring the polarization correlation of the SPDC photon pairs when the diameter of the iris was 12 mm or 1 mm (see Fig. 3). The coincidence counts were measured by rotating the angle of the polarizer (POL1) for several angles ($0^\circ, 45^\circ, 90^\circ$ and $135^\circ$) of the polarizer (POL2). The zero degree of the angle was defined by the hardware origin of the motorized continuous rotation stage, on which the polarizer was fixed. When the angle was zero degree, the polarizer axis was nearly
parallel to the vertical axis. The counted data were fitted by a function, $c (1 + v \cos(\theta + \theta_0))$, with fitting parameters of $c$, $v$ and $\theta_0$.

![Graphs showing measured coincidence counts of polarization correlated photon pairs. Marks in the figure show the results measured by rotating the angle of the polarizer POL2, when the polarizer POL1 was fixed to $0^\circ$ (filled circle), $45^\circ$ (open circle), $90^\circ$ (filled diamond) and $135^\circ$ (open diamond), respectively. Curves show the results fitted by a function, $c (1 + v \cos(\theta + \theta_0))$. (a) the diameter of the iris was 12 mm. (b) the diameter of the iris was set to be 1 mm.](image_url)

FIG. 3: Measured coincidence counts of polarization correlated photon pairs. Marks in the figure show the results measured by rotating the angle of the polarizer POL2, when the polarizer POL1 was fixed to $0^\circ$ (filled circle), $45^\circ$ (open circle), $90^\circ$ (filled diamond) and $135^\circ$ (open diamond), respectively. Curves show the results fitted by a function, $c (1 + v \cos(\theta + \theta_0))$. (a) the diameter of the iris was 12 mm. (b) the diameter of the iris was set to be 1 mm.

The result shows that the SPDC photon pairs were not entangled when the diameter of the iris was 12 mm, and that the pairs were entangled when the diameter of the iris was 1 mm. It means that SPDC photon pairs were entangled by spatially selecting the crossing point of the signal light cone and the idler light cone.

The main point of this article is to show that the SPDC photon pairs are entangled after being resolved by their frequencies, by measuring the polarization correlations of frequency resolved SPDC photon pairs. To compare with the theoretical simulations, the parameter $f$ in Eq. (1) must be measured. The parameter can be calculated from the coincidence counting rates of $|H\rangle_s|V\rangle_i$ and $|V\rangle_s|H\rangle_i$. The signal wavelength dependency of the coincidence counting rate of $|H\rangle_s|V\rangle_i$ was measured by rotating the grating around the vertical axis crossing the incident point of the signal beam, setting the angles of the polarizers POL1 and POL2 were set to transmit vertically polarized idler photons and horizontally polarized signal photons, respectively. Then, the idler wavelength dependency of the coincidence counting rate of $|V\rangle_s|H\rangle_i$ was measured by rotating the grating around the vertical axis crossing the incident point of the idler beam. This was performed by letting horizontally polarized idler photons and vertically polarized signal photons transmit through POL1 and
POL2 by selecting their angles, respectively (see Fig. 4). The results show that the coincidence counting rate of $|H\rangle_s |V\rangle_i$ was about three times larger than that of $|V\rangle_s |H\rangle_i$ when the signal wavelength was 866 nm, and both rates were nearly balanced when the signal wavelength was 870 nm.

Experiment was performed under two conditions as follows. First, the angle of the grating was set to diffract 866 nm signal photons to the detector, and the diameter of the iris was set to be 1 mm. Figure 5 shows the measured polarization correlation. The fact that $\Theta_i(45^o)$ and $\Theta_i(135^o)$ were shifted from $\Theta_i(0^o)$ by $10^o (< 45^o)$, indicates that even though the SPDC photon pairs were entangled but they were not maximally entangled. As is mentioned in the section of theory, the angular rotation was less than $45^o$ if $f \neq 1$. Actually, Fig. 4 shows that $f \approx 1.73$ when the signal wavelength was 866 nm.

Second, the angle of the grating was set to couple 870 nm signal photons to the detector, and the diameter of the iris was set to 1 mm. Figure 6 shows the measured result of the polarization correlation. The shift of $\Theta_i$ was larger than the case when the detector was coupled to 866 nm signal photons. It is because $f \approx 1$ in this case. Actually, the coincidence counting rate of $|H\rangle_s |V\rangle_i$ was nearly equal to the rate of $|V\rangle_s |H\rangle_i$ (see Fig. 4). The shift of $\Theta_i$ has increased from about $10^o$ to $30^o$. However the shift was still smaller than $45^o$, and the visibility of the polarization correlation was less than one. It means the polarization entanglement was not maximized. It can be improved by compensating walk-off effect and
FIG. 5: Measured coincidence counts of polarization correlated photon pairs when the coincidence counts of $|H\rangle_s|V\rangle_i$ and $|V\rangle_s|H\rangle_i$ were not balanced. Marks in the figure show the results measured by rotating the angle of the polarizer POL2, when the polarizer POL1 was fixed to $0^\circ$ (filled circles), $45^\circ$ (open circles), $90^\circ$ (filled diamonds) and $135^\circ$ (open diamonds), respectively. Curves show the results fitted by a function, $c(1 + v \cos(\theta + \theta_0))$ with fitting parameters of $c$, $v$ and $\theta_0$.

FIG. 6: Measured coincidence counts of polarization correlated photon pairs when the coincidence counts of $|H\rangle_s|V\rangle_i$ and $|V\rangle_s|H\rangle_i$ were balanced. Marks in the figure show the results measured by rotating the angle of the polarizer POL2, when the polarizer POL1 was fixed to $0^\circ$ (filled circles), $45^\circ$ (open circles), $90^\circ$ (filled diamonds) and $135^\circ$ (open diamonds), respectively. Curves show the results fitted by a function, $c(1 + v \cos(\theta + \theta_0))$ with fitting parameters of $c$, $v$ and $\theta_0$. 
group velocity dispersion in the non-linear crystal. The compensation can be performed by inserting a half-wave plate and a non-linear crystal of which thickness is half of the crystal which was used to generate SPDC photon pairs [15].

There are two points which are necessary to implement an ideal source of WDM-QKD. One point is that the photon pairs should be broadband in the spectrum and entangled in frequency and polarization. The generation of the broadband frequency-entangled photon pairs has been succeeded in [14], and the photon pairs can be entangled in polarization with a small modification in the setup as mentioned in [14]. The other point is that the coincidence counts of $|H\rangle_s|V\rangle_i$ and $|V\rangle_s|H\rangle_i$ are balanced in all over the spectral range of the SPDC photon pairs. It can be accomplished by using optical components whose efficiencies have no polarization dependency. Actually, the transmittance/reflectance of the real beam splitter and the diffraction efficiency of the grating have polarization dependency in the real experimental setup used in the present study, resulting in the unbalanced coincidence counts of $|H\rangle_s|V\rangle_i$ and $|V\rangle_s|H\rangle_i$. Proper improvement of these two points, the SPDC photon pairs will be performed in the implementation of WDM-QKD, which can send more information than the normal QKD, because each spectral channel in WDM-QKD can send same amount of information which the normal QKD can transfer.

CONCLUSION

In conclusion, it was verified that the frequency resolved SPDC photon pairs were polarization-entangled in appropriate configurations. When the iris transmits only photons passing through the crossing point of the signal light cone and the idler light cone, the SPDC photon pairs were entangled in polarization. The shift of $\Theta_i$ was $10^\circ$ when the coincidence counts of $|H\rangle_s|V\rangle_i$ and $|V\rangle_s|H\rangle_i$ were unbalanced, and the shift has increased to $30^\circ$ when they were balanced. However the observed shift was smaller than $45^\circ$. It means that the polarization entanglement of the photon pairs was not maximized, because of the walk-off effect and group velocity dispersion in the non-linear crystal. These can be improved by utilizing a half-wave plate and a half-thickness non-linear crystal. Requirements for the real applications of WDM-QKD have been discussed. We thank Drs. Haibo Wang and Yongmin Li and Mr. Tomoyuki Horikiri for their valuable discussion.
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