Triangular fuzzy graceful labeling on star graph

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Abstract. In this paper, we present an algorithm to find triangular fuzzy star graph $K_{1,n}$ atmost 1241 edges. We show that it admits triangular fuzzy labeling. Also we prove that it satisfies triangular fuzzy graceful labeling.

Keywords. Triangular Fuzzy Labeling, Triangular Fuzzy Graceful Labeling.

AMS Mathematics Subject Classification: 05C72, 05C78.

1. Introduction
In 1965, fuzzy sets were originated by Lofti.A. Zadeh[1] which had an outstanding improvement in mathematical design in case of ambiguity. A. Rahmani, F. Hosseinzadeh Loffi, M. Rostamy-Malkhalifeh, and T. Allahviranloo[2] discussed a new method for defuzzification and ranking of fuzzy numbers based on the statistical beta distribution.

Narsingh deo[3] developed the notion of graph theory which are associated between vertices and edges. A. Solairaju and S. Ambika [4] concluded some results on gracefulness of a new class of stars merged with trees. Fuzzy relations on fuzzy sets had been leading in an admirable way to compose a fuzzy graph model when there is an vagueness in vertices and edges.

Fuzzy graph model has been refined by K.R.Bhutani, J.N.Moderson and A.Rosenfield[5]. S.Mathew and M.Sunitha[6] extended the concepts of fuzzy graphs in to various graphs.

A. Nagoorgani, Muhammed Akram and D.Rajalakshmi(a)Subhasini [7,8] brought up the notion of labeling in fuzzy graph and proved some properties in fuzzy graph labeling. Existence of fuzzy edge vertex graceful labeling in some special graphs has been explained by S.Vimala and R.Jebesty Shajila [9]. Fuzzy graceful labeling technique has been demonstrated in an algorithmic approach for extended duplicate graph by S.Bala, M.L.MorslinLifin Lee, K.Thirusangu [10]. K.Ameenal Bibi, M.Devi[11] have been proceeded the results in fuzzy vertex graceful labeling on some special graphs.

In this paper, we present an algorithm to find triangular fuzzy star graph $K_{1,n}$ atmost 1241 edges. we show that it admits triangular fuzzy labeling. Also we prove that it satisfies triangular fuzzy graceful labelling.

2. Preliminaries

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Definition [1] 2.1. Let X be the space of points(objects), with a generic element of X denoted by x. Thus X={x}. A fuzzy set (class) A in X is characterized by a membership (characteristic) function \( \mu_A(x) \) which associates with each point in X a real number in the interval \([0, 1]\), with the value of \( \mu_A(x) \) at x representing the "grade of membership" of x in A.

Definition [2] 2.2. A fuzzy number is a fuzzy set in the form of \( \tilde{a} : R \rightarrow [0, 1] \) that satisfies the following conditions: (1) \( \tilde{a} \) is upper semi continuous, (2) \( \tilde{a}(x) \) is zero outside the interval \([l, u]\), (3) there exist real numbers \( m_1, m_2 \) such that \( l \leq m_1 \leq m_2 \leq u \) and (3.1) \( \tilde{a}(x) \) is increasing on \([l, m_1]\), (3.2) \( a(x) \) is decreasing on \([m_2, u] \), (3.3) \( \tilde{a}(x) = 1 \), \( m_1 \leq x \leq m_2 \). If \( m_1 = m_2 = m \) then the fuzzy number \( \tilde{a} = (l, m, u) \) is called the triangular fuzzy number and is defined as follows:

\[
\tilde{a}(x) = \begin{cases} 
\frac{x-l}{m-l}, & l \leq x \leq m \\
\frac{u-x}{u-m}, & m \leq x \leq u \\
0, & \text{otherwise}
\end{cases}
\]

Definition [3] 2.3. A Linear graph (or simply a graph) \( G = (V, E) \) consists of set of objects \( V=\{v_1, v_2, \ldots, v_n\} \) called vertices, and another set \( E=\{e_1, e_2, \ldots, e_m\} \) whose elements are called edges such that each edge \( e_k \) is identified with an unordered pair of vertices \( (v_i, v_j) \).

Definition [4] 2.4. The star graph \( S_n \) of order 'n' is a tree on 'n' nodes with one node having vertex degree (n-1) and the other (n-1) node having vertex degree 1. Star is a complete bi graph \( K_{1,n} \).

Definition [5] 2.5. Let U and V be two sets. Then \( \rho \) is said to be a fuzzy relation from U into V if \( \rho \) is a fuzzy set of \( U \times V \). A fuzzy graph \( G = (\sigma, \mu) \) is a pair of functions \( \sigma : V \rightarrow [0,1] \) and \( \mu : V \times V \rightarrow [0,1] \), where for all \( u, v \in V \), we have \( \mu(u,v) \leq \sigma(u) \land \sigma(v) \).

Definition [7] 2.6. A graph \( G = (\sigma, \mu) \) is said to be a fuzzy labeling graph if \( \sigma : V \rightarrow [0,1] \) and \( \mu : V \times V \rightarrow [0,1] \) is bijective such that the membership value of edges and vertices are distinct and \( \mu(u,v) < \sigma(u) \land \sigma(v) \) for all \( u, v \in V \).

Definition 2.7. A fuzzy labeling graph \( G = (\sigma, \mu) \) is said to be a fuzzy graceful graph if \( \sigma : V \rightarrow [0,1] \) such that \( \mu \ (u,v) = |\sigma(u) - \sigma(v)| \) and \( \mu \ (u,v) \) are distinct for all \( u, v \in V \).

Definition 2.8. A fuzzy labeling of a graph is said to be a triangular fuzzy labeling if it admits the triangular fuzzy number.

3. Main results
In this section, we present an algorithm to find triangular fuzzy star graph \( K_{1,n} \), at most 1241 edges. We have shown that it admits triangular fuzzy labeling. Also we prove that it satisfies triangular fuzzy graceful labeling.

3.1 Triangular fuzzy star graph \( K_{1,n} \)
In this section, we develop an algorithm for triangular fuzzy star graph \( K_{1,n} \), at most 1241 edges such as membership functions of the apex vertex \( \sigma(v_0) = (a_0, b_0, c_0) \), pendent vertices \( \sigma(v_i) = (a_i, b_i, c_{jk}) \) and pendent edges \( \mu (v_0, v_i) = (x_i, y_{ij}, z_{jk}) \).

Algorithm 3.1.1
Input: Star Graph
Procedure {Triangular fuzzy star graph \( (K_{1,n}) \), \( n < 1241 \)}
\( v_0 \leftarrow \text{Apex vertex of } K_{1,n} \)
\( \sigma(v_0) = (a_0, b_0, c_0) = [8,9,10] \)
for \( i = 1 \) to \( 15 \)
\{ \( a_i = \frac{10a_0-12+i}{10} \)
\{ for \( j = 1 \) to \( 16-i \)
\{ \( b_{ij} = \frac{10a_{1+j}}{10} \)
\{ for \( k = 1 \) to \( 32-(2i-1)-2j \)
\{ \( c_{ijk} = \frac{10b_{ij}+k}{10} \) \}
\( (v_{ij,k}) \leftarrow \text{Pendent vertices of } K_{1,n} \)
\( \sigma(v_{ij,k}) = (a_i, b_{ij}, c_{jk}) \)
\( (v_0, v_{ij,k}) \leftarrow \text{Internal edges of } K_{1,n} \)
\( \mu(v_0, v_{ij,k}) = (u_i, v_{ij}, w_{jk}) \)
\( u_i = |a_0 - c_{ijk}| \)
\( v_{ij} = |b_{ij} - b_0| \)
\( w_{jk} = |c_0 - a_i| \)
\} \}
end procedure.

### 3.1.2 ILLUSTRATIONS

| S.No. | Membership function of apex vertex of \( K_{1,n} \) \( \sigma(v_0) = (a_0, b_0, c_0) \) | Membership function of pendent vertices of \( K_{1,n} \) \( \sigma(v_{ij,k}) = (a_i, b_{ij}, c_{jk}) \) | No of. Edges with \( \mu(v_0, v_{ij,k}) = (x_i, y_{ij}, z_{jk}) \) | Total No. of edges |
|-------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|------------------|
| 1     | \( (a_0, b_0, c_0) \) = (8,9,10) \( a_i \) | \( (a_i, b_{ij}, c_{jk}) \) | \( (x_i, y_{ij}, z_{jk}) \) = \( (\frac{|a_0 - c_{ijk}|}{10},|b_{ij} - b_0|,|c_0 - a_i|) \) | 1 \( = 1 \) |
| 2     | \( (a_0, b_0, c_0) \) = (8,9,10) \( a_i \) | \( (a_i, b_{ij}, c_{jk}) \) | \( (x_i, y_{ij}, z_{jk}) \) = \( (\frac{|a_0 - c_{ijk}|}{10},|b_{ij} - b_0|,|c_0 - a_i|) \) | 3 \( = 4 \) |
| 3     | \( (a_0, b_0, c_0) \) = (8,9,10) \( a_i \) | \( (a_i, b_{ij}, c_{jk}) \) | \( (x_i, y_{ij}, z_{jk}) \) = \( (\frac{|a_0 - c_{ijk}|}{10},|b_{ij} - b_0|,|c_0 - a_i|) \) | 5 \( = 9 \) |
| 4     | \( (a_0, b_0, c_0) \) = (8,9,10) \( a_i \) | \( (a_i, b_{ij}, c_{jk}) \) | \( (x_i, y_{ij}, z_{jk}) \) = \( (\frac{|a_0 - c_{ijk}|}{10},|b_{ij} - b_0|,|c_0 - a_i|) \) | 7 \( = 16 \) |
|   | 8 | 9 | 10 | 8 | 8.2 | 8.3 to 8.7 | 5 |
|---|---|---|----|---|----|-----------|---|
|   | 8 | 9 | 10 | 8 | 8.3 | 8.4 to 8.6 | 3 |
|   | 8 | 9 | 10 | 8 | 8.4 | 8.5        | 1 |
| 5 | 8 | 9 | 10 | 7.9| 8   | 8.1 to 8.9 | 9 |
|   | 8 | 9 | 10 | 7.9| 8.1 | 8.2 to 8.8 | 7 |
|   | 8 | 9 | 10 | 7.9| 8.2 | 8.3 to 8.7 | 5 |
|   | 8 | 9 | 10 | 7.9| 8.3 | 8.4 to 8.6 | 3 |
|   | 8 | 9 | 10 | 7.9| 8.4 | 8.5        | 1 |
| 6 | 8 | 9 | 10 | 7.8| 7.9 | 8.0 to 9.0 | 11|
|   | 8 | 9 | 10 | 7.8| 8   | 8.1 to 8.9 | 9 |
|   | 8 | 9 | 10 | 7.8| 8.1 | 8.2 to 8.8 | 7 |
|   | 8 | 9 | 10 | 7.8| 8.2 | 8.3 to 8.7 | 5 |
|   | 8 | 9 | 10 | 7.8| 8.3 | 8.4 to 8.6 | 3 |
|   | 8 | 9 | 10 | 7.8| 8.4 | 8.5        | 1 |
| 7 | 8 | 9 | 10 | 7.7| 7.8 | 7.9 to 9.1 | 13|
|   | 8 | 9 | 10 | 7.7| 7.9 | 8.0 to 9.0 | 11|
|   | 8 | 9 | 10 | 7.7| 8   | 8.1 to 8.9 | 9 |
|   | 8 | 9 | 10 | 7.7| 8.1 | 8.2 to 8.8 | 7 |
|   | 8 | 9 | 10 | 7.7| 8.2 | 8.3 to 8.7 | 5 |
|   | 8 | 9 | 10 | 7.7| 8.3 | 8.4 to 8.6 | 3 |
|   | 8 | 9 | 10 | 7.7| 8.4 | 8.5        | 1 |
| 8 | 8 | 9 | 10 | 7.6| 7.7 | 7.8 to 9.2 | 15|
|   | 8 | 9 | 10 | 7.6| 7.8 | 7.9 to 9.1 | 13|
|   | 8 | 9 | 10 | 7.6| 8   | 8.0 to 9.0 | 11|
|   | 8 | 9 | 10 | 7.6| 8.1 | 8.1 to 8.9 | 9 |
|   | 8 | 9 | 10 | 7.6| 8.2 | 8.2 to 8.8 | 7 |
|   | 8 | 9 | 10 | 7.6| 8.3 | 8.3 to 8.7 | 5 |
|   | 8 | 9 | 10 | 7.6| 8.4 | 8.4 to 8.6 | 3 |
|   | 8 | 9 | 10 | 7.6| 8.4 | 8.5        | 1 |
| 9 | 8 | 9 | 10 | 7.5| 7.6 | 7.7 to 9.3 | 17|
|   | 8 | 9 | 10 | 7.6| 7.7 | 7.7 to 9.3 | 17|
|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 8 | 9 | 10 | 7.5 | 7.7 | 7.8 to 9.2 | 15 |
| 8 | 9 | 10 | 7.5 | 7.8 | 7.9 to 9.1 | 13 |
| 8 | 9 | 10 | 7.5 | 8  | 8.0 to 9.0 | 11 |
| 8 | 9 | 10 | 7.5 | 8  | 8.1 to 8.9 | 9  |
| 8 | 9 | 10 | 7.5 | 8.1 | 8.2 to 8.8 | 7  |
| 8 | 9 | 10 | 7.5 | 8.2 | 8.3 to 8.7 | 5  |
| 8 | 9 | 10 | 7.5 | 8.3 | 8.4 to 8.6 | 3  |
| 8 | 9 | 10 | 7.5 | 8.4 | 8.5     | 1  |

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|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 8 | 9 | 10 | 7.4 | 7.5 | 7.6 to 9.4 | 19 |
| 8 | 9 | 10 | 7.4 | 7.6 | 7.7 to 9.3 | 17 |
| 8 | 9 | 10 | 7.4 | 7.7 | 7.8 to 9.2 | 15 |
| 8 | 9 | 10 | 7.4 | 7.8 | 7.9 to 9.1 | 13 |
| 8 | 9 | 10 | 7.4 | 7.9 | 8.0 to 9.0 | 11 |
| 8 | 9 | 10 | 7.4 | 8  | 8.1 to 8.9 | 9  |
| 8 | 9 | 10 | 7.4 | 8.1 | 8.2 to 8.8 | 7  |
| 8 | 9 | 10 | 7.4 | 8.2 | 8.3 to 8.7 | 5  |
| 8 | 9 | 10 | 7.4 | 8.3 | 8.4 to 8.6 | 3  |
| 8 | 9 | 10 | 7.4 | 8.4 | 8.5     | 1  |

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|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 8 | 9 | 10 | 7.3 | 7.4 | 7.5 to 9.5 | 21 |
| 8 | 9 | 10 | 7.3 | 7.5 | 7.6 to 9.4 | 19 |
| 8 | 9 | 10 | 7.3 | 7.6 | 7.7 to 9.3 | 17 |
| 8 | 9 | 10 | 7.3 | 7.7 | 7.8 to 9.2 | 15 |
| 8 | 9 | 10 | 7.3 | 7.8 | 7.9 to 9.1 | 13 |
| 8 | 9 | 10 | 7.3 | 8  | 8.0 to 9.0 | 11 |
| 8 | 9 | 10 | 7.3 | 8.1 | 8.2 to 8.8 | 7  |
| 8 | 9 | 10 | 7.3 | 8.2 | 8.3 to 8.7 | 5  |
| 8 | 9 | 10 | 7.3 | 8.3 | 8.4 to 8.6 | 3  |
| 8 | 9 | 10 | 7.3 | 8.4 | 8.5     | 1  |

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|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 8 | 9 | 10 | 7.2 | 7.3 | 7.4 to 9.6 | 23 |

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|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 8 | 9 | 10 | 7.4 | 7.5 | 7.6 to 9.4 | 19 |
| 8 | 9 | 10 | 7.4 | 7.6 | 7.7 to 9.3 | 17 |
| 8 | 9 | 10 | 7.4 | 7.7 | 7.8 to 9.2 | 15 |
| 8 | 9 | 10 | 7.4 | 7.8 | 7.9 to 9.1 | 13 |
| 8 | 9 | 10 | 7.4 | 7.9 | 8.0 to 9.0 | 11 |
| 8 | 9 | 10 | 7.4 | 8  | 8.1 to 8.9 | 9  |
| 8 | 9 | 10 | 7.4 | 8.1 | 8.2 to 8.8 | 7  |
| 8 | 9 | 10 | 7.4 | 8.2 | 8.3 to 8.7 | 5  |
| 8 | 9 | 10 | 7.4 | 8.3 | 8.4 to 8.6 | 3  |
| 8 | 9 | 10 | 7.4 | 8.4 | 8.5     | 1  |

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|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 8 | 9 | 10 | 7.3 | 7.4 | 7.5 to 9.5 | 21 |
| 8 | 9 | 10 | 7.3 | 7.5 | 7.6 to 9.4 | 19 |
| 8 | 9 | 10 | 7.3 | 7.6 | 7.7 to 9.3 | 17 |
| 8 | 9 | 10 | 7.3 | 7.7 | 7.8 to 9.2 | 15 |
| 8 | 9 | 10 | 7.3 | 7.8 | 7.9 to 9.1 | 13 |
| 8 | 9 | 10 | 7.3 | 8  | 8.0 to 9.0 | 11 |
| 8 | 9 | 10 | 7.3 | 8.1 | 8.2 to 8.8 | 7  |
| 8 | 9 | 10 | 7.3 | 8.2 | 8.3 to 8.7 | 5  |
| 8 | 9 | 10 | 7.3 | 8.3 | 8.4 to 8.6 | 3  |
| 8 | 9 | 10 | 7.3 | 8.4 | 8.5     | 1  |

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| 8  | 9  | 10 | 7.2 | 7.4 | 7.5 to 9.5 | 21 |
|----|----|----|-----|-----|------------|----|
| 8  | 9  | 10 | 7.2 | 7.5 | 7.6 to 9.4 | 19 |
| 8  | 9  | 10 | 7.2 | 7.6 | 7.7 to 9.3 | 17 |
| 8  | 9  | 10 | 7.2 | 7.7 | 7.8 to 9.2 | 15 |
| 8  | 9  | 10 | 7.2 | 7.8 | 7.9 to 9.1 | 13 |
| 8  | 9  | 10 | 7.2 | 7.9 | 8.0 to 9.0 | 11 |
| 8  | 9  | 10 | 7.2 | 8   | 8.1 to 8.9 | 9  |
| 8  | 9  | 10 | 7.2 | 8.1 | 8.2 to 8.8 | 7  |
| 8  | 9  | 10 | 7.2 | 8.2 | 8.3 to 8.7 | 5  |
| 8  | 9  | 10 | 7.2 | 8.3 | 8.4 to 8.6 | 3  |
| 8  | 9  | 10 | 7.2 | 8.4 | 8.5       | 1  |

| 8  | 9  | 10 | 7.1 | 7.2 | 7.3 to 9.7 | 25 |
|----|----|----|-----|-----|------------|----|
| 8  | 9  | 10 | 7.1 | 7.3 | 7.4 to 9.6 | 23 |
| 8  | 9  | 10 | 7.1 | 7.4 | 7.5 to 9.5 | 21 |
| 8  | 9  | 10 | 7.1 | 7.5 | 7.6 to 9.4 | 19 |
| 8  | 9  | 10 | 7.1 | 7.6 | 7.7 to 9.3 | 17 |
| 8  | 9  | 10 | 7.1 | 7.7 | 7.8 to 9.2 | 15 |
| 8  | 9  | 10 | 7.1 | 7.8 | 7.9 to 9.1 | 13 |
| 8  | 9  | 10 | 7.1 | 8   | 8.0 to 9.0 | 11 |
| 8  | 9  | 10 | 7.1 | 8   | 8.1 to 8.9 | 9  |
| 8  | 9  | 10 | 7.1 | 8.1 | 8.2 to 8.8 | 7  |
| 8  | 9  | 10 | 7.1 | 8.2 | 8.3 to 8.7 | 5  |
| 8  | 9  | 10 | 7.1 | 8.3 | 8.4 to 8.6 | 3  |
| 8  | 9  | 10 | 7.1 | 8.4 | 8.5       | 1  |

| 8  | 9  | 10 | 7   | 7.1 | 7.2 to 9.8 | 27 |
|----|----|----|-----|-----|------------|----|
| 8  | 9  | 10 | 7   | 7.2 | 7.3 to 9.7 | 25 |
| 8  | 9  | 10 | 7   | 7.3 | 7.4 to 9.6 | 23 |
| 8  | 9  | 10 | 7   | 7.4 | 7.5 to 9.5 | 21 |
| 8  | 9  | 10 | 7   | 7.5 | 7.6 to 9.4 | 19 |
| 8  | 9  | 10 | 7   | 7.6 | 7.7 to 9.3 | 17 |
| 8  | 9  | 10 | 7   | 7.7 | 7.8 to 9.2 | 15 |
| 8  | 9  | 10 | 7   | 7.8 | 7.9 to 9.1 | 13 |

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### Example 3.1.3:

The triangular fuzzy star graph $K_{1,n}$ at most 1241 edges is shown in the following Figure (1), (2), and (3).

| $a_i$   | $b_{ij}$ | $c_{jk}$ | $E$     |
|---------|----------|----------|---------|
| 7.9     | 8.0 to 9.0 | 11       |
| 8       | 8.1 to 8.9 | 9        |
| 8       | 8.2 to 8.8 | 7        |
| 8       | 8.3 to 8.7 | 5        |
| 8       | 8.4 to 8.6 | 3        |
| 8       | 8.5       | 1        |
| 8       | 8.1 to 8.9 | 29       |
| 8       | 8.2 to 8.8 | 27       |
| 8       | 8.3 to 8.7 | 25       |
| 8       | 8.4 to 8.6 | 23       |
| 8       | 8.5       | 21       |
| 8       | 8.6 to 8.7 | 19       |
| 8       | 8.7 to 8.9 | 17       |
| 8       | 8.8 to 8.9 | 15       |
| 8       | 8.9       | 13       |
| 8       | 9.0 to 9.0 | 11       |
| 8       | 9.1 to 8.9 | 9        |
| 8       | 9.2 to 8.8 | 7        |
| 8       | 9.3 to 8.7 | 5        |
| 8       | 9.4 to 8.6 | 3        |

if $\sigma(v_{i,j,k}) = (a_i, b_{ij}, c_{jk})$ with $a_i = 6.9$, $b_{ij} = (7.0$ to $8.4)\quad c_{jk} = (7.1$ to $9.9$ to $8.4$ to $8.5$)

then $K_{1,n}$ has at most 225 edges which is shown in Figure (1).
if $\sigma(v_{i,k}) = (a_i, b_{i,j}, c_{i,k})$ with $a_i = 7.0$, $b_{i,j} = (7.1 \text{ to } 8.4)$ $c_{i,k} = (7.2 \text{ to } 9.8 \ldots 8.4 \text{ to } 8.5, 8.5)$
then $K_{1,n}$ has at most 196 edges which is shown in Figure (2).

if $\sigma(v_{i,k}) = (a_i, b_{i,j}, c_{i,k})$ with $a_i = (8.2)$, $b_{i,j} = (8.3, 8.4)$ $c_{i,k} = (8.4 \text{ to } 8.6, 8.5)$
then $K_{1,n}$ has at most 4 edges which is shown in Figure (3).

Figure(1), (2) and (3) Triangular Fuzzy star graph $K_{1,n}$.
Theorem 3.1.4:
The triangular fuzzy star graph $K_{1,n}$ with $(n < 1241)$ admits triangular fuzzy labeling.

Proof:
Let $K_{1,n}$ with $(n < 1241)$ be the star graph with 'n+1' vertices and 'n' edges.
To prove $K_{1,n}$ admits triangular fuzzy labeling. That is to prove that triangular fuzzy star graph satisfies
$$\mu(v_0, v_{i,j,k}) < \sigma(v_0) \quad \text{and} \quad \mu(v_0, v_{i,j,k}) < \sigma(v_{i,j,k}),$$
for $i = 1 \text{ to } 15, \quad j = 1 \text{ to } 16-i, \quad k = 1 \text{ to } 32-(2i-2)j$............................(1)

From Algorithm 3.1.1, membership function of the apex vertex $v_0$, the pendent vertices $v_i$ and the pendent edges $\mu(v_0, v_i)$ defined as follows.
$$\sigma : v_0 \rightarrow [0,1] \Rightarrow \sigma(v_0) = [a_0, b_0, c_0] = [8, 9, 10]$$
$$\sigma : v_{i,j,k} \rightarrow [0, 1] \Rightarrow \sigma(v_{i,j,k}) = [a_i, b_{i,j}, c_{j,k}], \quad i = 1 \text{ to } 15,$$
$$j = 1 \text{ to } 16-i, \quad k = 1 \text{ to } 32-(2i-1)-2j$$
where
$$a_i = \frac{10a_0-i+1}{10} \quad \text{..................................}(2)$$
$$b_{i,j} = \frac{10a_i+j}{10} \quad \text{..................................}(3)$$
$$c_{j,k} = \frac{10b_{ij}+k}{10} \quad \text{..................................}(4)$$

$$\mu : v_0 \times v_{i,j,k} \rightarrow [0,1] \Rightarrow \mu(v_0, v_{i,j,k}) = [u_i, v_{i,j}, w_{j,k}], \quad i = 1 \text{ to } 15,$$
$$j = 1 \text{ to } 16-i, \quad k = 1 \text{ to } 32-(2i-1)-2j$$
where
$$u_i = |a_o-c_{j,k}| \quad \text{..................................}(5)$$
$$v_{i,j} = |b_o-b_{i,j}| \quad \text{..................................}(6)$$
$$w_{j,k} = |c_o-a_i| \quad \text{..................................}(7)$$

Now,
$$u_i = |a_o-c_{j,k}|$$
$$= |a_o-\left[\frac{10b_{ij}+k}{10}\right]|$$
$$= |a_o-\left[\frac{10\left(\frac{10a_i+j}{10}\right)+k}{10}\right]|$$
\[
\begin{align*}
\mu_i &= \left\lfloor \frac{12 - i - j - k}{10} \right\rfloor \quad \text{(8)} \\
\nu_{i,j} &= \left\lfloor b_o - b_{i,j} \right\rfloor \\
&= \left\lfloor b_o - \left\lfloor \frac{10a_i + j}{10} \right\rfloor \right\rfloor \\
&= \left\lfloor b_o - \left\lfloor \frac{10a_0 - 12 + i}{10} + j \right\rfloor \right\rfloor \\
&= \left\lfloor b_o - \left\lfloor \frac{10a_0 - 12 + i + j}{10} \right\rfloor \right\rfloor \\
\nu_{i,j} &= \left\lfloor \frac{10b_0 - 10a_0 + 12 - i - j}{10} \right\rfloor \quad \text{(9)} \\
\omega_{j,k} &= \left\lfloor c_o - a_i \right\rfloor \\
&= \left\lfloor c_o - \left( \frac{10a_0 - 12 + i}{10} \right) \right\rfloor \\
\omega_{j,k} &= \left\lfloor \frac{10c_0 - 10a_0 + 12 - i}{10} \right\rfloor \quad \text{(10)}
\end{align*}
\]

Clearly, from (8), (9) and (10), we have \( \mu(v_0, v_{i,j,k}) < \sigma(v_0) \) and \( \mu(v_0, v_{i,j,k}) < \sigma(v_{i,j,k}) \), for all \( i, j \), and \( k \).

Hence the triangular fuzzy star graph \( K_{1,n} \) with \( n < 1241 \) admits triangular fuzzy labeling.

**Theorem 3.1.5:**
The triangular fuzzy star labeled graph \( K_{1,n} \) with \( n < 1241 \) satisfies triangular fuzzy graceful labeling.

**Proof:**
Let \( K_{1,n} \) with \( n < 1241 \) be the star graph with \( n+1 \) vertices and \( n \) edges.

To prove \( K_{1,n} \) admits triangular fuzzy graceful labeling, that is to prove that edges of triangular fuzzy labeled star graph has the distinct values.

Using theorem 3.1.4, we have
\[ \mu : v_0 \times v_{i,j,k} \rightarrow [0,1] \ni \mu(v_0, v_{i,j,k}) = \begin{bmatrix} u_i, v_{i,j}, w_{j,k} \end{bmatrix}, \quad i = 1 \text{ to } 15, \]
\[ j = 1 \text{ to } 16 - i, \]
\[ k = 1 \text{ to } 32 - (2i - 1) - 2j \]

where
\[ u_i = \left\lfloor \frac{12 - i - j - k}{10} \right\rfloor \]
\[ v_{i,j} = \left\lfloor \frac{10a_0 - 10a_0 + 12 - i - j}{10} \right\rfloor \]
\[ w_{j,k} = \left\lfloor \frac{10c_0 - 10a_0 + 12 - i}{10} \right\rfloor \]

Therefore, for any \( l, m, n, p, q \) and \( r \in i, j, k \) with \( (l,m,n) \neq (p,q,r) \)
\[ \mu(v_0, v_{l,m,n}) \neq \mu(v_0, v_{p,q,r}) \]

\[ \text{...........................................}(11) \]

Hence the triangular fuzzy star labeled graph \( K_{1,n} \) with \( n < 1241 \) satisfies triangular fuzzy graceful labelling.

4. Conclusion
We discussed triangular fuzzy star graph \( K_{1,n} \) at most 1241 edges. It has been shown that triangular fuzzy star graph admits triangular fuzzy labeling. Also proved that \( K_{1,n} \) satisfies triangular fuzzy graceful labeling. Further research work to be extended on some special graphs.

5. References
[1] Zadeh A 1965 Fuzzy sets Information and control (8)
[2] Rahmani A, Hosseinizadeh Lotfi F, Rostamy-Malkhalifeh M, and Allahviranloo T 2016 A New Method for Defuzzification and Ranking of Fuzzy Numbers Based on the Statistical Beta Distribution Advances in Fuzzy Systems 2016
[3] Narsingh Deo 1974 Graph Theory with applications to Engineering and Computer science Prentice-Hall, Inc. Englewood Cliffs, NJ
[4] Solairaju A and Ambika S 2012 Gracefulness of a New Class of Stars Merged with Trees, International Journal of Fuzzy Mathematics and Systems 2(1) 83–94
[5] Bhutani KR, Moderson J and Rosenfield A 2004 On degrees of end nodes and cut nodes in fuzzy graphs Iranian Journal of Fuzzy Systems (1) 57–64
[6] Mathew S and Sunita M S 2009 Types of arcs in fuzzy graph Information Sciences 179 1760–1768
[7] Nagoorgani A and Rajalakshmi(a)Subhasini D 2014 A note on fuzzy labeling International Journal of Fuzzy Mathematical Archive
[8] Nagoorgani A, Muhammed Akram and Rajalakshmi(a)Subhasini D 2014 Novel properties of fuzzy labeling graphs Research article Hindawi

[9] Vimala S and Jebestry Shajila R 2016 A note on edge vertex graceful labeling of star and Helmgraph, Advances in Theoretically and Applied Mathematics

[10] Bala S, MorslinLifin Lee M L and Thirusangu K 2018 Fuzzy graceful labeling for the extended duplicate graphs International journal of Mathematics Trends and Technology

[11] Ameenal Bibi K and Devi M 2017 A note on fuzzy vertex graceful labeling on special graphs, International Journal of Advanced Research in Computer Science 8(6)