Modelling of the flux penetration into a superconducting strip with magnetic sheath

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Abstract. The behaviour of a long superconducting strip, placed in a perpendicular external magnetic field, and surrounded with a (ferro)magnetic sheath in different geometrical arrangements is investigated. The numerical method of magnetic energy minimization, based on the critical state model, is used, and the commercial finite element software FEMLAB is utilized in the calculations. Our calculation method enables to insert arbitrary dependence of the permeability of the magnetic sheath material on the local magnetic field, may it be linear or non-linear function. The results of the flux penetration into the superconducting strip in an increasing external filed, together with the curves of initial magnetization are presented, for a few geometrical arrangements and different magnetic properties of the sheath.

1. Introduction
Analysing the behaviour of a superconducting wire in external magnetic field is a very complicated task and certain simplifying assumptions have to be made to obtain usable results. Very often, the superconductor is assumed to be very (infinitely) long in one direction and exposed to a homogenous external magnetic field that is perpendicular to this direction. The problem becomes two-dimensional in this situation and only the flux penetration in the superconductor’s cross-section lying in the plane perpendicular to the translational symmetry (long dimension) axis has to be considered. When the aim is to model the behaviour of a more or less flat superconductor, like the filaments in a multifilamentary BSCCO tape, the core, or eventually filaments, in an MgB$_2$ wire, or the superconducting layer of a YBCO-coated conductor, the cross-section is chosen to be of rectangular shape, with high aspect ratio in most cases.

For this configuration, analytical results were obtained by Brandt [1], when assuming the superconducting bar to be infinitely thin in the direction of the applied magnetic field and describing the superconductor by the critical state model [2]. However, if it is necessary to consider a cross-section of finite dimensions, one of the available numerical computing methods has to be used. Two approaches can be followed in the numerical calculations. It is the approach first utilized in the Brandt’s method [3], when the superconducting material is described by its non-linear current-voltage characteristic $E(j) = \rho(j)j$, most often considered in the form of the power function $E(j) = E_c (j/j_c)^n$, and the approach of the method of minimal magnetic energy variation (MMEV) [4], where the superconductor is described by the critical state model ($j = \pm j_c$ or 0).
In the case of the MgB$_2$ wires and YBCO-coated conductors, the superconductor is very often surrounded with a magnetic material, which further complicates the analysis. To examine the effect of the mutual interaction of the superconductor and the magnetic sheath, we developed a simple computational method.

2. Our method

Our computational method represents an approach to account for the change of the field distribution in the presence of the magnetic material and its influence on the flux penetration into the superconductor. It is based on the idea of MMEV method and thus on the critical state model.

The results presented here were calculated for a configuration, where a very long superconducting strip of a rectangular cross-section with the aspect ratio 1:10 is covered on its wide faces with two parallel strips from magnetic material, that represent the magnetic sheath. This tree-strip formation is placed inside a calculation box, which is much bigger than the dimensions of the strips. Inside the box a homogenous applied magnetic field $B_a$ is present, which is perpendicular to the wide face of the strips. The situation is shown in the figure 1. The superconducting strip is divided into a matrix of $10 \times 30$ rectangular current elements. In the current element, an electrical current with the critical current density $j_c$ can flow, in the $+z$ or $-z$ direction. The elements in which the current is flowing are called filled elements in the following.

The idea of the method is to find, at given applied field, the distribution of filled current elements that leads to the lowest magnetic energy $E_m$ of the box, among all the others possible distributions. The magnetic energy $E_m$ (per unit length) is calculated as

$$E_m = \frac{1}{2\mu_0} \int_{\text{box}} \vec{B}^2 \, dS$$

where $\vec{B} \equiv \vec{B}(x, y)$ is the magnetic field distribution inside the calculation box and the integration takes place in the $xy$ plane over the cross-section of the box.

The algorithm of the method works as follows. To an existing distribution of filled current elements two new pairs of filled elements are added. The new elements are added in pairs, where in one element the current density $+j_c$ and in the other $-j_c$ is present, because the total current flowing through the superconductor has to be zero at every moment. The elements in the pair form a current loop closed at the faraway ends of the strip (at the infinity). Two pairs of new elements can be added at once, due to the symmetry in the investigated geometry, which every distribution of the filled elements also has to reflect. The two mutually perpendicular mirror planes divide the strip’s cross-section into four quarters, and when an element in one quarter is filled, the corresponding elements in the other quarters, i.e. the counterpart to form the closed
loop and another symmetrical loop, are filled automatically. Thanks to the symmetry, also the direction of the current in the elements is fixed ahead.

In the process of adding new elements to an existing distribution all the elements lying in the testing range are added and for every intermediate distribution built in this way the magnetic field distribution $\vec{B}(x, y)$ and the energy $E_m$ are calculated. We use the commercial software FEMLAB [5], that employs the finite element method, for these calculations. The testing range from which the new filled elements are selected is restricted by a physical principle, according to which the magnetic flux penetrates into a superconducting body from its surface to the centre. The testing range is thus generated from the unfilled elements lying at the surface of the strip and in the neighbourhood of already filled elements.

From all the tested couples of pairs of newly filled elements the one that leads to the lowest value of $E_m$ is selected and firmly added to the original distribution. The whole process then continues in the same manner, starting from this new distribution. When adding any new couple of pairs does not lead to the decrease of the magnetic energy anymore, the process stops.

3. Results

Utilization of the finite element software, FEMLAB, enabled us to include in the calculations the properties of the magnetic strips, covering the superconducting one, very easily. To model the behaviour of the magnetic materials, we inserted the dependence of the relative magnetic permeability $\mu_r$ on the local magnitude of the magnetic field $B$ in the form

$$\mu_r = \frac{\mu_{\text{max}}}{1 + (B/B_c)^2} + 1$$

(2)

with the parameter values $\mu_{\text{max}} = 10^3$ and $B_c = 10^{-1}$ T.

In the calculations, five different geometrical arrangements of the magnetic sheath were considered (see figure 2). Two series of calculations were performed, each with different value of the critical current density, $j_c$, of the superconductor. The values $j_c = 10^5$ A/m² and $10^9$ A/m² were considered. In the fully penetrated state of the superconducting strip without magnetic sheath they lead to the order of magnitude of local magnetic fields $10^{-5}$ T and $10^{-1}$ T, respectively, which, in the first case, is well below, and, in the second one, is comparable to the characteristic field of the magnetic material $B_c$ from the equation (2).

The distributions of the filled current elements, calculated assuming the value $j_c = 10^5$ A/m², are shown in the figure 2, for all the considered configurations of the magnetic sheath that differ in the width of the magnetic strips. The dimensions of one current element are $0.1 \times 0.03$ mm². Depicted are the distributions at the applied magnetic field $B_a = 1.3 \cdot 10^{-5}$ T and they clearly demonstrate the straightening of the penetrating flux front in the area covered by the magnetic strips. It is because the magnetic flux lines inside the magnetic strips are practically parallel to their surface and concentrate there, and leave the strips only at edges, parallel to the direction.
of the applied field. The applied magnetic field is effectively diverted from the volume of the superconducting strip. This is in accordance with the results presented in [6] where similar computational method was used and a constant $\mu_r$ was considered.

In the configuration named “yoke”, the magnetic sheath does not consist of two separate strips, but completely surrounds the superconducting strip, having the form of a rectangular tube, infinitely long in the $z$ direction. In such closed shape of the magnetic sheath the diversion of the magnetic flux is much stronger and its penetration inside the superconducting strip starts at much higher applied fields. Hence the distribution at $B_a = 130 \cdot 10^{-5} \text{T}$ is shown in the figure 2 for this case.

The figure 3 shows the initial magnetization curves of the superconducting strip for the different configurations of the magnetic sheath, calculated assuming the two different $j_c$ values ($10^5$ and $10^9 \text{A/m}^2$) as mentioned above. The magnetization $-M$ is the volume density of the magnetic moment of the superconducting strip, which is the magnetic moment generated by the currents flowing in the filled current elements. Note that in the presented normalized graphs the differences between the curves calculated for the same configuration with different $j_c$ value are caused only by the behaviour of the magnetic material.

### 4. Conclusion

We have investigated the behaviour of the long superconducting strip, placed in a perpendicular applied magnetic field and covered by the sheath from a magnetic material, by the means of the numerical simulation. For this purpose we developed a calculation method employing the critical state model and using the idea of the magnetic energy minimization.

The obtained results show that the magnetic sheath significantly changes the magnetic field distribution at the surface of the superconductor, resulting in a considerable shielding effect and straightening of the penetrating flux front. The initial magnetization curves also suggest that the flux penetration into the superconducting strip sheathed by a magnetic material is delayed compared to the bare superconducting strip.

In contrast to our model’s assumption, in the real materials the $j_c$ usually strongly depends on the local value of the magnetic field. This dependence would affect the overall velocity of the flux penetration and the value of the magnetization, but we believe the pronounced straightening of the flux front would be present also in this case.

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