Gluon Propagator and Heavy Quark Potential in an Anisotropic QCD Plasma

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Abstract
The hard-loop resummed propagator in an anisotropic QCD plasma in general linear gauges are computed. We get the explicit expressions of the gluon propagator in covariant gauge, Coulomb gauge and temporal axial gauge. Considering one gluon exchange, the potential between heavy quarks is defined through the Fourier transform of the static propagator. We find that the potential exhibits angular dependence and that there is stronger attraction on distance scales on the order of the inverse Debye mass for quark pairs aligned along the direction of anisotropy than for transverse alignment.

Key words: gluon propagator, anisotropic plasma, heavy quark potential
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1. Introduction

Information on quarkonium spectral functions at high temperature has started to emerge from lattice-QCD simulations; we refer to ref. [1] for recent work and for links to earlier studies. This has motivated a number of attempts to understand the lattice measurements within non-relativistic potential models including finite temperature effects such as screening [2]. In this paper, we consider the effects due to a local anisotropy of the plasma in momentum space on the heavy-quark potential. Such deviations from perfect isotropy are expected for a real plasma created in high-energy heavy-ion collisions, which undergoes expansion. We derive the HTL propagator of an anisotropic plasma for general linear gauges, which allows us to define a non-relativistic potential via the Fourier transform of the propagator in the static limit.
2. Hard-Loop resummed gluon propagator in an anisotropic plasma

The retarded gauge-field self-energy in the hard-loop approximation is given by 3

\[ \Pi^{\mu\nu}(p) = g^2 \int \frac{d^3k}{(2\pi)^3} \nu^\mu \frac{\partial f(k)}{\partial k^\nu} \left( g^{\nu\beta} - \frac{v^\nu p^\beta}{p \cdot v + i\epsilon} \right). \]  

(1)

Here, \( v^\mu \equiv (1, k/|k|) \) is a light-like vector describing the propagation of a plasma particle in space-time. The self-energy is symmetric, \( \Pi^{\mu\nu}(p) = \Pi^{\nu\mu}(p) \), and transverse, \( p_\mu \Pi^{\mu\nu}(p) = 0 \).

In a suitable tensor basis the components of \( \Pi^{\mu\nu} \) can be determined explicitly. For anisotropic systems there are more independent projectors than for the standard equilibrium case [4,5,6]. We use a four-tensor basis developed in ref. [7] and the self-energy can now be written as \( \Pi^{\mu\nu} = \alpha A^{\mu\nu} + \beta B^{\mu\nu} + \gamma C^{\mu\nu} + \delta D^{\mu\nu} \) with

\[ A^{\mu\nu} = -g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} + \frac{\tilde{m}^\mu \tilde{m}^\nu}{\tilde{m}^2}, \quad B^{\mu\nu} = -\frac{p^2}{(m \cdot p)^2} \tilde{m}^\mu \tilde{m}^\nu, \]

\[ C^{\mu\nu} = \frac{\tilde{m}^2 p^2}{\tilde{m}^2 p^2 + (m \cdot p)^2} \left[ \tilde{m}^\mu \tilde{n}^\nu - \frac{\tilde{m} \cdot \tilde{n}}{\tilde{m}^2} (\tilde{m}^\mu \tilde{n}^\nu + \tilde{n}^\mu \tilde{m}^\nu) + \frac{\tilde{m} \cdot \tilde{n}}{\tilde{m}^4} (\tilde{m}^\mu \tilde{m}^\nu + \tilde{m}^\nu \tilde{m}^\mu) \right], \]

\[ D^{\mu\nu} = \frac{p^2 m}{m \cdot p} \left[ 2 \frac{\tilde{m} \cdot \tilde{n}}{\tilde{m}^2} \tilde{m}^\mu \tilde{m}^\nu - (\tilde{n}^\mu \tilde{n}^\nu + \tilde{m}^\mu \tilde{m}^\nu) \right]. \]  

(2)

Here, \( m^\mu \) is the heat-bath vector, which in the local rest frame is given by \( m^\mu = (1, 0, 0, 0) \), and \( \tilde{m}^\mu = m^\mu - m \cdot p \tilde{p}^\mu \) is the part that is orthogonal to \( p^\mu \). The direction of anisotropy in momentum space is determined by the vector \( n^\mu = (0, n) \), where \( n \) is a three-dimensional unit vector. We choose \( n = (0, 0, 1) \) in this paper. As before, \( \tilde{n}^\mu \) is the part of \( n^\mu \) orthogonal to \( p^\mu \).

In order to determine the four structure functions explicitly we employ the following ansatz: \( f(p) = f_{\text{iso}}(\sqrt{p^2 + \xi (p \cdot n)^2}) \). The parameter \( \xi \) is used to determine the degree of anisotropy. Thus, \( f(p) \) is obtained from an isotropic distribution \( f_{\text{iso}}(|p|) \) by removing particles with a large momentum component along \( n \). We do not list the rather cumbersome explicit expressions for the four structure functions \( \alpha, \beta, \gamma, \) and \( \delta \) here since they have already been determined in ref. [3].

The retarded propagator \( i\Delta_{ab}^{\mu\nu} \) is diagonal in color and so color indices will be suppressed. Using Dyson-Schwinger equation, its inverse is given by

\[ (\Delta^{-1})^{\mu\nu}(p, \xi) = -p^2 g^{\mu\nu} + p^\mu p^\nu - \Pi^{\mu\nu}(p, \xi) + G, \]  

(3)

where \( G \) is a gauge fixing term. In covariant gauge, Coulomb gauge and temporal axial gauge, it has the following form

\[ G_{\text{cova}} = -\frac{1}{\eta} p^\mu p^\nu, \quad G_{\text{coul}} = -\frac{1}{\eta} (p^\mu - \omega m^\mu)(p^\nu - \omega m^\nu), \quad G_{\text{temp}} = -\frac{1}{\eta} m^\mu m^\nu. \]  

(4)

Here, \( \omega \equiv m \cdot p \) and \( \eta \) is the gauge parameter. Upon inversion, the propagator in covariant gauge is written as

\[ \Delta_{\text{cova}}^{\mu\nu} = \frac{A^{\mu\nu} - C^{\mu\nu}}{p^2 - \alpha} + \Delta G \left[ (p^2 - \alpha - \gamma) \frac{\omega^4}{p^4} B^{\mu\nu} + (\omega^2 - \beta) C^{\mu\nu} + \delta \omega^2 D^{\mu\nu} \right], \]  

(5)

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where $\Delta_G^{-1} = (p^2 - \alpha - \gamma)(\omega^2 - \beta) - \delta^2 [p^2 - (n \cdot p)^2]$.

In Coulomb gauge, we have

$$\Delta_{\text{coul}}^{\mu\nu} = \frac{A^{\mu\nu} - C^{\mu\nu}}{p^2 - \alpha} + \Delta_G \left[ \left( p^2 - \alpha - \gamma \right) \frac{\omega^2}{p^2} B^{\mu\nu} + (\omega^2 - \beta) C^{\mu\nu} + \delta \frac{\omega^2}{p^2} D^{\mu\nu} \right],$$  \hspace{1cm} (6)

where the two new projectors $B^{\mu\nu}$ and $D^{\mu\nu}$ are defined as

$$B^{\mu\nu} = m^\mu m^\nu, \quad D^{\mu\nu} = \frac{p^2}{m \cdot p} \left[ 2 \frac{\tilde{m} \cdot \tilde{n}}{m^2} m^\mu m^\nu - (\tilde{n}^\mu m^\nu + m^\mu \tilde{n}^\nu) \right],$$  \hspace{1cm} (7)

with $\tilde{n}^\mu = n^\mu + \frac{n \cdot p}{p^2} p^\mu$.

In temporal axial gauge, we have

$$\Delta_{\text{temp}}^{\mu\nu} = \frac{A^{\mu\nu} - C^{\mu\nu}}{p^2 - \alpha} + \Delta_G \left[ \left( p^2 - \alpha - \gamma \right) B^{\mu\nu} + (\omega^2 - \beta) C^{\mu\nu} + \delta D^{\mu\nu} \right],$$  \hspace{1cm} (8)

where the two new projectors $\tilde{B}^{\mu\nu}$ and $\tilde{D}^{\mu\nu}$ are defined as

$$\tilde{B}^{\mu\nu} = \frac{(\omega m^\mu - p^\mu)(\omega m^\nu - p^\nu)}{p^2}, \quad \tilde{D}^{\mu\nu} = (p^\mu - \omega m^\mu) \tilde{n}^\nu + \tilde{n}^\mu (p^\nu - \omega m^\nu),$$  \hspace{1cm} (9)

with $\tilde{n}^\mu = n^\mu - \frac{n \cdot p}{p^2} (\omega m^\mu - p^\mu)$.

In the expressions of the propagators, we drop the gauge fixing term. Actually, the gauge fixing term in covariant gauge, Coulomb gauge and temporal axial gauge are $-\frac{2}{p^2} p^\mu p^\nu$, $-\frac{2}{p^2} p^\mu p^\nu$ and $-\frac{2}{p^2} p^\mu p^\nu$, respectively.

It is easy to show that we recover the isotropic propagator by setting $\xi = 0$. In addition, if the gauge parameter $\eta = 0$, we can check in covariant gauge, $p_\mu \Delta^{\mu\nu}(p) = 0$ because of the gauge condition $\partial^\mu A_\mu = 0$. In Coulomb gauge, due to the fact that $\partial^\mu A_\mu = 0$, we have $p_\mu \Delta^{\mu\nu}(p) = 0$ and in isotropic case, we have $\Delta^{0i}(p) = 0$. In temporal axial gauge, we can check $\Delta^{0i} = \Delta^{00} = 0$ as a result of the gauge condition $A_0 = 0$.

3. Heavy Quark Potential in an anisotropic plasma

We determine the real part of the heavy-quark potential in the non-relativistic limit, at leading order, from the Fourier transform of the static gluon propagator,

$$V(r, \xi) = -g^2 C_F \int \frac{d^3p}{(2\pi)^3} e^{ip \cdot r} \Delta^{00}(\omega = 0, p, \xi) \int \frac{d^3p}{(2\pi)^3} e^{ip \cdot r} \frac{p^2 + m_0^2 + m_3^2}{(p^2 + m_0^2 + m_3^2)(p^2 + m_3^2) - m_0^2}. \hspace{1cm} (10)$$

Here, $C_F$ is the color factor and the $\xi$-dependent masses $m_0^2$, $m_3^2$, $m_3^2$ and $m_3^2$ are given in ref. [7]. This definition of potential should be gauge independent. One can check that no matter the plasma is isotropic or anisotropic, the definition is equivalent in covariant gauge and Coulomb gauge. In temporal axial gauge, due to the fact that $A_0 = 0$, it fails to define the potential. However, there is a simple relation between these gauges: in static limit, the quantity $\frac{p^2}{p^2} \Delta^{01}$ in temporal axial gauge is identical to the quantity $\Delta^{00}$ in covariant gauge and Coulomb gauge if the gauge fixing term vanishes. In general, we
find the quantity $|\frac{e^2}{p^2} \Delta^{ii} - \Delta^{00}|$ in static limit is gauge independent which can be used as a more general definition of the potential.

Generally, the integral in (11) has to be performed numerically. The poles of the function are integrable. They are simple first-order poles which can be evaluated using a principal part prescription. The numerical results have been show in ref. [7]. In general, screening is reduced, i.e. that the potential at $\xi > 0$ is deeper and closer to the vacuum potential than for an isotropic medium. This is partly caused by the lower density of the anisotropic plasma. However, the effect is not uniform in the polar angle. Overall, one may therefore expect that quarkonium states whose wave-functions are sensitive to the regime $\hat{r} \sim 1$ are bound more strongly in an anisotropic medium. Here, $\hat{r} \equiv r m_D$ and $m_D$ is the Debye mass.

4. Conclusions

We have determined the HTL gluon propagator in an anisotropic plasma in general linear gauges. Its Fourier transform at vanishing frequency defines a non-relativistic potential induced by one gluon exchange for static sources. We find that, generically, screening is weaker than in isotropic media and so the potential is closer to that in vacuum. Also, there is stronger binding of the quark pairs in the anisotropic system. Our results are applicable when the momentum of the exchanged gluon is on the order of the Debye mass $m_D$ or higher, i.e. for distances on the order of $\lambda_D = 1/m_D$ or less. The binding energy for quarkonium can be estimated analytically from this potential [8] if the quark mass is very large and the temperature is very high. In this case we can neglect the non-perturbative string contribution. For those states whose length scale is larger, to determine the binding energy, we should solve the Schrödinger equation with a potential which contains the medium-dependent contributions due to one-gluon exchange and due to the string [9] (for a derivation of the non-relativistic potential model see [10]). This is work in progress.

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