Quantum gravitational decoherence from Generalized Uncertainty Principle with stochastic deformation parameter

Eissa Al-Nasrallah
Department of Physics, Kuwait University, P.O. Box 5969, Safat 13060, Kuwait and
Energy and Building Research Center, KISR, P.O. Box 24885, Safat 13109, Kuwait

Saurya Das
Theoretical Physics Group and Quantum Alberta,
Department of Physics and Astronomy, University of Lethbridge,
4401 University Drive, Lethbridge, Alberta, T1K 3M4, Canada

Fabrizio Illuminati and Luciano Petruzziello
Dipartimento di Ingegneria Industriale, Universita degli Studi di Salerno,
Via Giovanni Paolo II, 132 - 84084 Fisciano, Salerno, Italy
INFN, Sezione di Napoli, Gruppo collegato di Salerno, Italy

Elias C. Vagenas
Theoretical Physics Group, Department of Physics,
Kuwait University, P.O. Box 5969, Safat 13060, Kuwait

Several theories of quantum gravity propose the existence of a minimal measurable length and maximum measurable momentum near the Planck scale. When integrated into the framework of quantum mechanics, such restrictions lead to the generalization of the Heisenberg uncertainty principle, which is commonly referred to as the generalized uncertainty principle (GUP). The GUP has been applied to a plethora of physical models in order to provide further insights on our understanding of the Universe, both in the macroscopic and the microscopic regime. Nevertheless, the corrections related to GUP are expected to be extremely small, which thus renders experimental verification a difficult task. Therefore, phenomenological approaches on a qualitative level are typically addressed with the aim of enhancing our possibility of detecting these tiny effects. Motivated by a recent work on a universal decoherence mechanism stemming from the standard formulation of GUP, we extend the ideas contained therein by considering a more general form of GUP which includes linear and quadratic momentum terms to examine its implications. To achieve such a picture, we equip the deformation parameter appearing in front of all momentum terms with a stochastic nature, so that the effective decoherence mechanism emerges by taking the average over fluctuations. We find that, despite the apparently small differences among the two generalizations of the Heisenberg uncertainty principle, the consequences at the level of the universal decoherence mechanism they entail is significant, as they predict decoherence times that are completely uncorrelated and distinct.

I. INTRODUCTION

Quantum decoherence has been an active area of research for more than 50 years. It was originally proposed by Zeh as an explanation for the quantum-to-classical transition [1]. The conundrum of quantum systems behaving classically at the macroscopic scale has been a source of various interpretations and intense discussions. The phenomenon of decoherence attempts to solve this issue by taking into account the unavoidable entanglement shared between any quantum system and its environment. A similar entanglement affects the set of possible observable states of the system in a measurement and in turn gives rise to a net loss of quantum information. In other words, the system tends to lose all the characteristic quantum features (such as for instance the superposition of states which are predicted quantum mechanically and yet unobserved on a macroscopic scale, and in some circumstances even prohibited [2–6]).

In the quest for seeking a faithful and rigorous representation of natural phenomena, it has been observed that our most successful theories, namely quantum field theory (QFT) and general relativity, encounter non-trivial adversities that prevent them from providing an accurate description of reality when approaching the Planck scale. This stimulated the search for a unifying theory of everything which has introduced many valuable candidates; among these, it
is worth mentioning String Theory, Loop Quantum Gravity, Causal Dynamical Triangulations, Modified Dispersion Relations (MDR), and Doubly Special Relativity (DSR). All these models predict a minimum measurable length which is typically proportional to the Planck length \( \mathcal{O}(\ell_p) \), where \( \ell_p \sim 10^{-35} \) m. Such a constraint on the minimum measurable length necessarily requires a modification of the Heisenberg Uncertainty Principle (HUP), which goes by the name of Generalized Uncertainty Principle (GUP) (see Refs. [7, 21] and therein for a comprehensive overview of the subject).

Understanding decoherence in the framework of quantum gravity is thought to bring an insight on the nature of quantum gravitational interactions and provide possibilities for their experimental tests. Decoherence stemming from general relativity and different models of quantum gravity has been thoroughly treated in the recent literature (see for instance Refs. [22–31] and therein). Among these proposals, two of the present authors have investigated the consequences of deformed canonical commutation relation (DCCR) inspired by quantum gravitational theories to understand decoherence in the framework of quantum gravity. Following the analysis of Ref. [33], we derive the Lindblad-type master equation in the context of LQGUP. First, we derive a Lindblad-type master equation valid for a generic quantum system and obtain the ensuing decoherence time associated with the DCCR [7, 32, 34]

\[
[x_i, p_j] = i\hbar \left[ \delta_{ij} + \beta \delta_{ij} p^2 + 2\beta p_i p_j \right], \quad \beta = \frac{\ell_0^2}{\hbar^2},
\]

which leads to an uncertainty relation of the form

\[
\Delta x \Delta p \geq \frac{\hbar}{2} (1 + \beta \Delta p^2).
\] (2)

In this paper, we extend the above considerations by addressing a more general form of GUP that includes both linear and quadratic momentum terms known as LQGUP. Such a generalization has been analyzed in different contexts (see Refs. [35–39]) and it is compatible with DSR. Specifically, it takes the form

\[
\Delta x \Delta p \geq \frac{\hbar}{2} \left[ 1 - 2\alpha \langle p \rangle + 4\alpha^2 \langle p^2 \rangle \right]
\geq \frac{\hbar}{2} \left[ 1 + \left( \frac{\alpha}{\sqrt{\langle p^2 \rangle}} + 4\alpha^2 \right) \Delta p^2 + 4\alpha^2 \langle p \rangle^2 - 2\alpha \sqrt{\langle p^2 \rangle} \right].
\] (3)

In this case, the DCCR becomes

\[
[x_i, p_j] = i\hbar \left[ \delta_{ij} - \alpha \left( p_i \delta_{ij} + \frac{p_i p_j}{p} \right) + \alpha^2 (p^2 \delta_{ij} + 3p_i p_j) \right],
\] (4)

where \( x_i \) and \( p_i \) are defined as

\[
x_i = x_{0i}, \quad p_i = p_{0i}(1 - \alpha p_0 + 2\alpha^2 p_0^2),
\] (5)

with

\[
[x_{0i}, p_{0j}] = i\hbar \delta_{ij},
\] (6)

and \( [x_i, x_j] = 0, [p_i, p_j] = 0, \) \( p^2 = \sum_{j=1}^{3} p_j p_j, \) \( p_0^2 = \sum_{j=1}^{3} p_{0j} p_{0j}. \) The parameter \( \alpha \) is defined as \( \alpha = \frac{\alpha_0}{Mpc} = \frac{\alpha_0 \ell_0}{\hbar} \) where \( \alpha_0 \) is often assumed to be of order unity [35].

The paper is structured as follows: in Section II we derive the Lindblad-type master equation in the context of LQGUP, whereas in Section III we obtain the entropy variation and the decoherence time due to the LQGUP-modified Lindblad master equation. Finally, in Section IV we summarize our results and provide final remarks.

II. LQGUP-INDUCED MASTER EQUATION

Following the analysis of Ref. [33], we derive the Lindblad-type master equation in the context of LQGUP. First, we note that we can start without loss of generality from the canonical Schrödinger equation

\[
\text{i} \hbar \partial_t |\psi\rangle = H |\psi\rangle = \left( \frac{p^2}{2m} + V \right) |\psi\rangle
\] (7)

and use the definition of the GUP-modified momentum as given by Eq. (5) so as to get

\[
\text{i} \hbar \partial_t |\psi\rangle = \left( \frac{p_0^2 \left( 1 - \alpha p_0 + 2\alpha^2 p_0^2 \right)}{2m} + V \right) |\psi\rangle = \left( \frac{p_0^2 \left( 1 - 2\alpha p_0 + 5\alpha^2 p_0^2 - 4\alpha^3 p_0^3 + 4\alpha^4 p_0^4 \right)}{2m} + V \right) |\psi\rangle
\] (8)
and thus
\[ i\hbar \partial_t |\psi\rangle = \left( \frac{p_0^2}{2m} + V - \frac{\alpha}{m} + 5\alpha^2 \frac{p_0^4}{2m} - 2\alpha^3 \frac{p_0^6}{m} + 2\alpha^4 \frac{p_0^8}{m} \right) |\psi\rangle . \] (9)

We observe that several orders of the deformation parameter appear in the perturbation of the standard Hamiltonian. For our purposes, we will consider a first-order treatment of the factor \(\alpha\). However, for the sake of completeness, we show in Appendix A that considerations involving second-order corrections produce the same outcome. Hence, if we account for a first-order treatment only of the GUP parameter \(\alpha\) and ignore factors that go like \(O(\alpha^2)\), then the Schrödinger equation is given by
\[ i\hbar \partial_t |\psi\rangle = H |\psi\rangle = \left( H_0 + H_1 + O(\alpha^2) \right) |\psi\rangle , \] (10)
where
\[ H_0 = \frac{p_0^2}{2m} + V, \quad H_1 = -2\alpha \frac{p_0^3}{2m} . \] (11)

Now, by introducing the density matrix
\[ \varrho = |\psi\rangle \langle \psi| , \] (12)
we can cast the Liouville-von Neumann equation as follows
\[ \partial_t \varrho = -\frac{i}{\hbar} [H_0 + H_1, \varrho] . \] (13)

Next, to solve the equation for the density matrix we shift our analysis to the interaction picture, where the state takes the form
\[ |\psi\rangle = e^{-iH_0 t/\hbar} |\tilde{\psi}\rangle . \] (14)
According to this representation, we can rewrite Schrödinger equation as
\[ i\hbar \partial_t |\tilde{\psi}\rangle = \tilde{H}_1 |\tilde{\psi}\rangle \] where \(\tilde{H}_1 = e^{iH_0 t/\hbar} H_1 e^{-iH_0 t/\hbar} |\tilde{\psi}\rangle , \] (15)
and substituting back in the Liouville-von Neumann equation, i.e., Eq. (13), we get
\[ \partial_t \tilde{\varrho}(t) = -\frac{i}{\hbar} [\tilde{H}_1(t), \tilde{\varrho}(t)] . \] (16)
A formal solution of the above equation can be achieved by integrating both sides over the interval \([0, t]\), which thus yields
\[ \int_0^t \partial_t \tilde{\varrho}(t')dt' = \int_0^t -\frac{i}{\hbar} [\tilde{H}_1(t'), \tilde{\varrho}(t')]dt' \] (17)
and results in
\[ \tilde{\varrho}(t) = \tilde{\varrho}(0) - \frac{i}{\hbar} \int_0^t [\tilde{H}_1(t'), \tilde{\varrho}(t')]dt' . \] (18)
Subsequently, Eq. (18) is inserted into the r.h.s. of Eq. (16), thereby giving
\[ \partial_t \tilde{\varrho}(t) = -\frac{i}{\hbar} [\tilde{H}_1(t), \tilde{\varrho}(0)] - \frac{1}{\hbar^2} \int_0^t [\tilde{H}_1(t), [\tilde{H}_1(t'), \tilde{\varrho}(t')]]dt' . \] (19)

The next step consists in relying on the Born-Markov approximation to let the density matrix of the r.h.s. of the above equation depend on \(t\) rather than \(t'\), i.e.,
\[ \partial_t \tilde{\varrho}(t) = -\frac{i}{\hbar} [\tilde{H}_1(t), \tilde{\varrho}(0)] - \frac{1}{\hbar^2} \int_0^t [\tilde{H}_1(t), [\tilde{H}_1(t'), \tilde{\varrho}(t')]]dt' . \] (20)
This is a common procedure in the derivation of Lindblad-type master equations for weak interactions between the system and reservoir [33 40 41]. However, we note that it has been argued that this approximation may not be valid for certain regimes, such as composite systems in strong coupling regime [22] and low temperature systems [43].

At this point, we observe that the GUP based on theories such as Loop Quantum Gravity and DSR should contain the information of space-time fluctuations near the Planck scale, which is a typical by-product of the aforementioned quantum gravity models. To comply with the existence of a fluctuating minimum length scale, one can think of encoding a similar feature in the present analysis by demanding a fluctuating deformation parameter, which requires a stochastic treatment of the problem. By averaging over the fluctuations of the GUP parameter $\alpha$ we can allow for the development of the master equation from the mean stochastic Liouville-von Neumann equation. Therefore, we have to regard $\alpha$ as a random variable; specifically, for the sake of simplicity we can interpret it as being a Gaussian white noise with fixed mean and sharp auto-correlation. Inspired by the considerations contained in Ref. [33], we can then impose that the dimensionless GUP parameter $\alpha_0$ is given by

$$\alpha_0 = \sqrt{t_p} \chi(t), \quad \langle \chi(t) \rangle = \bar{\alpha}_0, \quad \langle \chi(t) \chi(t') \rangle = \delta(t - t'),$$

with $t_p$ being the Planck time. In order to be in accordance with the standard literature, we can assume $\alpha_0$ to be of order unity on average. For this purpose, if we average over its fluctuations we can take $\langle \alpha_0 \rangle = \sqrt{t_p} \langle \chi(t) \rangle = 1$ and consequently define the fixed mean as $\bar{\alpha}_0 = 1/\sqrt{t_p}$. Alternatively, we can follow a more conservative approach and impose the mean value of $\alpha_0$ to not exceed the experimental bound available for the deformation parameter, that is $\langle \alpha_0 \rangle \simeq 10^{15}$. In any case, we will show that the arbitrary choice made to fix $\bar{\alpha}_0$ will not play a relevant role in the upcoming investigation, as the important quantity is represented by the auto-correlation and not the mean value. Now, by averaging over the fluctuations and introducing the shorthand notation $\langle \varrho \rangle = \rho$, the derivative will take the form

$$\partial_t \langle \varrho \rangle = \partial_t \rho(t),$$

and Eq. (20) can be rewritten as

$$\partial_t \rho(t) = \left\langle -\frac{i}{\hbar} \left[ \hat{H}_1(t), \varrho(0) \right] \right\rangle - \frac{1}{\hbar^2} \int_0^t \left\langle \left[ \hat{H}_1(t), \left[ \hat{H}_1(t'), \varrho(t') \right] \right] \right\rangle dt'.$$

The first term on the r.h.s. of Eq. (23) vanishes since $\varrho(0)$ is constant. For the second term, in order to further manipulate the factor inside the integral, we recall the definition of $H_1$ in Eq. (11) to substitute it and obtain

$$\partial_t \rho(t) = - \frac{4}{\hbar^2} \int_0^t \langle \alpha(t) \alpha(t') \rangle \left[ \frac{\hat{p}_0^2(t)}{2m}, \frac{\hat{p}_0^2(t')}{2m}, \rho(t) \right] \right\rangle dt'.$$

In terms of $\alpha = \alpha_0 \ell_p / \hbar$, we rewrite the above equation as

$$\partial_t \rho(t) = - \frac{4 \ell_p^2}{\hbar^4} \int_0^t \langle \alpha_0(t) \alpha_0(t') \rangle \left[ \frac{\hat{p}_0^2(t)}{2m}, \frac{\hat{p}_0^2(t')}{2m}, \rho(t) \right] \right\rangle dt'.$$

Now since $\alpha_0$ is a Gaussian white noise as explained in Eq. (21), the integral can be easily evaluated (See Appendix A), thereby yielding

$$\partial_t \rho(t) = - \sigma \left[ \frac{\hat{p}_0^2(t)}{2m}, \frac{\hat{p}_0^2(t)}{2m}, \rho(t) \right],$$

where $\sigma = 4t_p \ell_p^2 / \hbar^4$. At this point, we can recover the density matrix in the Schrödinger representation by means of the following transformation:

$$\rho(t) = e^{-i \mu_0 t} \hat{\rho}(t) e^{i \mu_0 t},$$

so that we can go back to the Liouville-Von Neumann equation given by Eq. (13) with the dissipator term that depends on the characteristic features of the stochastic LQGUP model treated so far. In particular, by requiring the system to not be subject to any external potential, i.e., $V = 0$, we are left with

$$\partial_t \rho(t) = - \frac{i}{\hbar} \left[ \frac{\hat{p}_0^2}{2m}, \rho(t) \right] - \sigma \left[ \frac{\hat{p}_0^2}{2m}, \frac{\hat{p}_0^2}{2m}, \rho(t) \right].$$
By employing the definitions in Eq. (11), the above equation can be rewritten in terms of Hamiltonians as

$$\partial_t \rho(t) = -\frac{i}{\hbar} [H_0, \rho(t)] - 2m\sigma \left[ H_0^{3/2}, \left[ H_0^{3/2}, \rho(t) \right] \right].$$

(29)

Equations (28) and (29) represent the LQGUP-modified Lindblad-type master equation, where the dissipator is identified with the second term of the r.h.s. Clearly, in the limit $\sigma \approx 0$ we recover the standard unitary dynamics predicted by the unmodified Liouville-von Neumann equation. Furthermore, it is worth remarking that the power of $H_0$ appearing in Eq. (29) is different from the usual one that appears in other gravitational decoherence mechanisms, according to which the dependence on the free Hamiltonian is typically linear $[29, 30]$. This unconventional behavior is shared also with the other work dealing with a GUP-induced decoherence process $[33]$, but interestingly the power of $H_0$ is different in the two scenarios, thus suggesting that the intrinsic characteristics of the emergent decoherence phenomenon strictly depend on the GUP model used for computations.

### III. APPLICATION AND PHYSICAL RESULTS

In this Section, we look at the temporal evolution of an entropy quantifier, i.e., linear entropy (for which calculations can be carried out analytically), to analyze the irreversibility of the decoherence mechanism and provide a solution for the differential equation Eq. (29). By means of this study, we are able to derive the decoherence time associated with the LQGUP, which foresees energy localization in momentum space as it occurs also in Ref. $[33]$.

#### A. Entropy Variation

Linear entropy corresponds to the mixedness of the quantum state and is therefore associated with the quantum state purity. It is defined as

$$S(t) = 1 - tr \left( \rho^2(t) \right),$$

(30)

and we recall that a given state is pure if and only if the correlated density matrix is idempotent, namely $\rho^2 = \rho$. Now, the time evolution of the linear entropy is obtained by differentiating Eq. (30), that is

$$\partial_t S(t) = \partial_t (1 - tr \left( \rho^2(t) \right)) = -2tr \left( \rho(t) \partial_t \rho(t) \right).$$

(31)

At this point, by resorting to the LQGUP-modified Lindblad-type master equation from the previous Section, namely Eq. (28), we can substitute the factor $\partial_t \rho(t)$ so as to give

$$\partial_t S = \frac{2i}{\hbar} tr \left( \rho(t) \left[ \frac{p_0^6}{2m}, \rho(t) \right] \right) + 2 \sigma tr \left( \rho(t) \left[ \frac{p_0^3}{2m}, \left[ \frac{p_0^3}{2m}, \rho(t) \right] \right] \right).$$

(32)

By manipulating the r.h.s of the above equation and using the cyclic property of the trace, it is possible to obtain that

$$\partial_t S = 2 \sigma \left[ 2 tr \left( \rho \frac{p_0^6}{4m^2} \right) - 2 tr \left( \rho \frac{p_0^3}{2m} \right)^2 \right].$$

(33)

If we now introduce operator $O$ defined as

$$O = \left[ \frac{p_0^3}{2m}, \rho \right],$$

(34)

then it can be shown that

$$\left[ 2 tr \left( \rho \frac{p_0^6}{4m^2} \right) - 2 tr \left( \rho \frac{p_0^3}{2m} \right)^2 \right] = tr \left( O^\dagger O \right).$$

(35)

Next, by going back to Eq. (33), we are left with

$$\partial_t S = 2 \sigma \ tr \left( O^\dagger O \right) \geq 0,$$

(36)

where $\sigma$ and $tr \left( O^\dagger O \right)$ are positive quantities, which entails that the linear entropy is monotonically increasing with time and the initial quantum state tends to increase mixedness. Furthermore, this achievement clearly conveys the intrinsic irreversible nature of the studied process, as the entropy can never decrease with time.
B. Decoherence Time

We are now ready to calculate the decoherence time associated with the LQGUP-modified Lindblad-type master equation, i.e., Eq. (29). To this aim, we point out that the computation is easier if performed in the momentum representation, where the elements of the density matrix are

\[
\rho_{p,p'}(t) = \langle p | \rho(t) | p' \rangle ,
\]

and consequently

\[
\partial_t \rho_{p,p'}(t) = \partial_t \langle p | \rho(t) | p' \rangle = \langle p | \partial_t \rho(t) | p' \rangle .
\]

Therefore, by using the master equation, i.e., Eq. (29), we get

\[
\partial_t \rho_{p,p'}(t) = -\frac{i}{\hbar} \left( \langle p | [H_0, \rho] | p' \rangle - \langle p | \rho H_0 | p' \rangle \right) - 2m\sigma \left( \left| E^3/2(p) \right| \rho | p' \rangle \langle p | \rho | p' \rangle - 2 E^3/2(p) E^3/2(p') \langle p | \rho | p' \rangle \langle p' | \rho | p' \rangle + E^3(p') \langle p | \rho | p' \rangle \langle p | \rho | p' \rangle \right) .
\]

By opening the commutators, one can easily check that

\[
\partial_t \rho_{p,p'}(t) = -\frac{i}{\hbar} \left( \langle p | [H_0, \rho] | p' \rangle - \langle p | \rho H_0 | p' \rangle \right) - 2m\sigma \left( \left| E^3/2(p) \right| \rho | p' \rangle \langle p | \rho | p' \rangle - 2 E^3/2(p) E^3/2(p') \langle p | \rho | p' \rangle \langle p' | \rho | p' \rangle + E^3(p') \langle p | \rho | p' \rangle \langle p | \rho | p' \rangle \right) .
\]

In the momentum representation, we know that \( H(p) | p \rangle = E(p) | p \rangle \). Moreover, by using the fact that the Hamiltonian is a Hermitian operator, we obtain

\[
\partial_t \rho_{p,p'}(t) = -\frac{i}{\hbar} \left( E(p) \langle p | \rho | p' \rangle - E(p') \langle p | \rho | p' \rangle \right) - 2m\sigma \left( E^3(p) \langle p | \rho | p' \rangle - 2 E^3/2(p) E^3/2(p') \langle p | \rho | p' \rangle + E^3(p') \langle p | \rho | p' \rangle \right) .
\]

By writing the density matrix elements as in Eq. (37), the previous equation reads

\[
\partial_t \rho_{p,p'}(t) = -\frac{i}{\hbar} \left( E(p) - E(p') \right) \rho_{p,p'} - 2m\sigma \left( E^3(p) - 2 E^3/2(p) E^3/2(p') + E^3(p') \right) \rho_{p,p'}
\]

and introducing the notation \( \Delta E^3/2 \equiv E^3/2(p) - E^3/2(p') \), we get

\[
\partial_t \rho_{p,p'} = \left[ -\frac{i}{\hbar} \left( E(p) - E(p') \right) - 2m\sigma (\Delta E^3/2)^2 \right] \rho_{p,p'} .
\]

It is easily seen that Eq. (43) is a separable differential equation which can be solved to yield

\[
\rho_{p,p'}(t) = \exp \left[ -\frac{i}{\hbar} \left( E(p) - E(p') \right) t - 2m\sigma (\Delta E^3/2)^2 t \right] \rho_{p,p'}(0)
\]

from which it is possible to deduce the decoherence time as being equal to

\[
\tau_D = \frac{1}{2m\sigma (\Delta E^3/2)^2} = \frac{\hbar^4}{8m^2 \ell_p^2 (\Delta E^3/2)^2} .
\]

As expected, as long as the diagonal part, i.e., \( \Delta E^3/2 = 0 \), is concerned, there is no time evolution. However, when considering the off-diagonal terms (for which \( \Delta E^3/2 \neq 0 \)), we can recognize an exponential damping that is a typical signature of the decoherence mechanism with a characteristic time which precisely equals the decoherence time. In order to better emphasize the behavior of \( \tau_D \), in Fig. 1, we plot the decoherence time as a function of the energy and the mass of a given quantum system to show that it gets smaller and smaller as the macroscopic scale is approached.

It is noteworthy to mention that in Ref. [33], the decoherence time obtained from considerations involving a GUP of the form shown in Eq. (2) (which is only quadratic in the momentum) has the following form:

\[
\tau_D = \frac{\hbar^6}{16m^2 \ell_p^4 \Delta E^2}, \quad \Delta E^2 = E^2(p) - E^2(p') .
\]

In Fig. 2, we exhibit the comparison between the two forms of the decoherence times. As the picture clearly conveys, the decoherence time obtained from LQGUP arguments is shorter than the one obtained in Ref. [33] in the mesoscopic regime below the Planck size but it is longer beyond the same scale.
IV. CONCLUSIONS

In this paper, we have shown that decoherence phenomena arising from a GUP with linear and quadratic terms in momentum can be deemed as a viable candidate for the quantum-to-classical transition. This can be achieved by assuming a stochastic nature for the GUP parameter $\alpha$; indeed, we have regarded it as a Gaussian white noise with constant mean and sharp auto-correlation. The justification of the above requirement can be found in the assumption of a fluctuating minimal length, which in turn is legitimated by space-time fluctuations that lead to the stochastic treatment of $\alpha$. This very consideration has allowed us to derive a Lindblad-type master equation, from which we have been able to deduce several physical implications of the GUP-induced gravitational decoherence mechanism, such as the entropy variation and decoherence time. Specifically, we have seen how the linear entropy monotonically increases with time, thus leading to a continuous increase of the state mixedness and to an irreversible evolution process. On the other hand, the decoherence time was shown to decrease for increasing energy and mass of the system. This occurs at a rate faster than the decoherence due to GUP without linear momentum term (as derived in Ref. [33]) for mesoscopic regimes beyond the Planck size, whilst the same trend is inverted below the Planck scale. Such an awareness entails that there is a strong bond between the form of GUP and the ensuing decoherence time.

As a matter of fact, the above deep connection can be employed to model phenomenologically meaningful uncertainty relations, capable of explaining the quantum-to-classical transition and quantum gravitational effects. Since the decoherence mechanism stemming from the GUP strictly depends on the deviation from the HUP, the next future high-precision laboratory experiments may be able to progress and provide valuable pieces of evidence that are essential for the construction of a consistent and detectable quantum gravity model. Even if such experiments are not attained, we could potentially set strong bounds on the plethora of generalized uncertainty principles available at present, so as to eventually reach the purported correct description of natural manifestations occurring at scales which are still inaccessible via direct experimental probes. Clearly, more work is inevitably required along this line and is currently being undertaken.
FIG. 2. Comparison between decoherence times as a function of energy derived from LQGUP with both linear and quadratic momentum terms (present paper, solid line) and from GUP with only quadratic momentum terms (Ref. [33], dashed line).

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Appendix A: Second-order perturbation of $\alpha$ in the Hamiltonian

In this Appendix, we show how the main outcome of the current work is left untouched even if we push our analysis beyond the first-order approximation. Indeed, if we consider second-order approximations in $\alpha$ for $H_1$ in the Hamiltonian of Eq. (9), we get

$$i\hbar \partial_t |\psi\rangle = H_1|\psi\rangle = \left(H_0 + H_1 + \mathcal{O}(\alpha^3)\right)|\psi\rangle,$$

(A1)

with

$$H_0 = \frac{p_0^2}{2m} + V, \quad H_1 = -\frac{\alpha}{m} p_0^3 + \frac{5\alpha^2}{2m} p_0^4,$$

(A2)

where we ignore terms that go like $\mathcal{O}(\alpha^3)$.

We note that the addition of the second order $\alpha$ term will begin to have a significant effect only in correspondence of Eq. (23), which can be written as

$$\partial_t \tilde{\rho}(t) = -\frac{1}{\hbar^2} \int_0^t \left\langle \tilde{H}_1(t)\tilde{H}_1(t')\tilde{\varrho}(t) - \tilde{H}_1(t)\tilde{\varrho}(t')\tilde{H}_1(t') - \tilde{H}_1(t')\tilde{\varrho}(t)\tilde{H}_1(t) + \tilde{\varrho}(t)\tilde{H}_1(t')\tilde{H}_1(t) \right\rangle dt'.$$

(A3)
In this case, if we substitute the new value for $H_1$ we obtain
\[ \partial_t \tilde{\rho}(t) = -\frac{4}{\hbar^2} \int_0^t \left\langle \alpha(t) \alpha(t') \left( \frac{\tilde{p}_0^3(t')}{2m} \frac{\tilde{p}_0(t')}{2m} \tilde{\rho}(t') - \frac{\tilde{p}_0^3(t)}{2m} \frac{\tilde{p}_0(t)}{2m} \tilde{\rho}(t) - \frac{\tilde{p}_0^3(t')}{2m} \frac{\tilde{p}_0(t)}{2m} \tilde{\rho}(t) + \frac{\tilde{p}_0^3(t)}{2m} \frac{\tilde{p}_0(t')}{2m} \tilde{\rho}(t) \right) \right\rangle dt'. \] (A4)

If we keep the terms up to $O(\alpha^2)$, we are left with
\[ \partial_t \tilde{\rho}(t) = -\frac{4}{\hbar^2} \int_0^t \left\langle \alpha(t) \alpha(t') \left( \frac{\tilde{p}_0^3(t')}{2m} \frac{\tilde{p}_0(t')}{2m} \tilde{\rho}(t') - \frac{\tilde{p}_0^3(t)}{2m} \frac{\tilde{p}_0(t)}{2m} \tilde{\rho}(t) + \frac{\tilde{p}_0^3(t)}{2m} \frac{\tilde{p}_0(t')}{2m} \tilde{\rho}(t) \right) \right\rangle dt'. \] (A5)

\[ \partial_t \tilde{\rho}(t) = -\frac{4}{\hbar^2} \int_0^t \left\langle \alpha(t) \alpha(t') \left[ \frac{\tilde{p}_0^3(t')}{2m}, \tilde{\rho}(t) \right] \right\rangle dt'. \] (A6)

that is the same integral obtained in Eq. (24) in which we only considered terms up to $O(\alpha)$. Therefore, by following the same steps and recalling that $\alpha(t) = \alpha_0 \frac{\ell_p}{\hbar} = \ell_p \sqrt{\lambda(t)} \hbar$, we obtain
\[ \partial_t \tilde{\rho}(t) = -\sigma \int_0^t \left\langle \chi_0(t) \chi_0(t') \left[ \frac{\tilde{p}_0^3(t')}{2m}, \tilde{\rho}(t) \right] \right\rangle dt'. \] (A7)

which becomes
\[ \partial_t \tilde{\rho}(t) = -\sigma \int_0^t \delta(t - t') \left[ \frac{\tilde{p}_0^3(t')}{2m}, \tilde{\rho}(t) \right] dt'. \] (A8)

Finally, we derive the expression
\[ \partial_t \tilde{\rho}(t) = -\sigma \left[ \frac{\tilde{p}_0^3(t)}{2m}, \tilde{\rho}(t) \right] \] (A9)

which is precisely Eq. (26).

By recovering the Schrödinger representation and going back to the Liouville-Von Neumann equation, i.e., Eq. (28), by vanishing the external potential
\[ \partial_t \rho(t) = -\frac{i}{\hbar} \left[ \frac{\tilde{p}_0^3}{2m}, \rho(t) \right] - \sigma \left[ \frac{\tilde{p}_0^3}{2m}, \rho(t) \right]. \] (A10)

Because of the appearance of a double Hamiltonian in Eq. (A3), it is clear that we can safely neglect all higher-order terms above $O(\alpha)$, which by the way are extremely suppressed due to the smallness of the corrections compared to the unmodified scenario.

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