One-side Forward-backward Asymmetry in Top Quark Pair Production at CERN Large Hadron Collider

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Both D0 and CDF at Tevatron reported the measurements of forward-backward asymmetry in top pair production, which showed possible deviation from the standard model QCD prediction. In this paper, we explore how to examine the same higher-order QCD effects at the more powerful Large Hadron Collider.

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The top quark is the heaviest ever known fermion and is thought to be related to the mechanism of electroweak symmetry breaking and physics beyond the standard model (SM). Since it was discovered more than one decade ago, measuring its properties is one of the most active fields. Most of the measured properties such as mass, width, production rate and so on are consistent with SM predictions, however the CDF and D0 Collaborations have observed a possible deviation in the SM is calculated to be $7.8\%$ in Ref. [6–8], which is larger than $5\%$. Therefore they differ by a factor of $k \sim 1.3$.

Such FB asymmetry is equivalent to the charge asymmetry provided that $CP$ is conserved. It is strange at first glance that vector like theory QCD can induce FB asymmetry. The fact is that such asymmetry arising from higher-order effects, namely, the interference between tree-level and virtual box diagrams of $t\bar{t}$ production, as well as among diagrams of real processes of $q\bar{q} \rightarrow t\bar{t}g$ (cf. Figs. 1&3). Similar asymmetry of QED was noticed even 37 years ago.

Obviously only less than $3\sigma$ deviation is not the evidence that the SM is failed. Though the pursuit of possible new physics beyond the SM (BSM) implied by the deviation is exciting, the investigation of the same interference effect at more powerful Large Hadron Collider (LHC) is more necessary. Once the deviation is confirmed at the LHC, the measurements may be the first BSM signature. Unfortunately, the FB asymmetry defined at the proton-antiproton collider Tevatron is not applicable at the proton-proton collider LHC, as LHC does not have the preferred direction in the laboratory frame. In order to solve this issue, the central charge asymmetry has been proposed,

$$ A_C = \frac{\sigma_t(|Y| \leq Y_C) - \sigma_t(|Y| \leq Y_C)}{\sigma_t(|Y| \leq Y_C) + \sigma_t(|Y| \leq Y_C)}. \quad (3) $$

Here $A_C$ is defined as the ratio between the difference and the sum of the events of the top and the antitop quark in a central region $|Y| < Y_C$ in the laboratory frame. The disadvantage of this definition is that at the LHC, such asymmetry is quite small. The reason is that the central region cut $|Y| < Y_C$ can not remove the symmetric $t\bar{t}$ events.

\[ A_{FB} = \frac{\sigma(|Y| > 0) - \sigma(|Y| < 0)}{\sigma(|Y| > 0) + \sigma(|Y| < 0)), (1) \]

where $\Delta Y \equiv Y_t - Y_{\bar{t}}$ is the difference between rapidity of the top and antitop quark, which is invariant under $t\bar{t}$ or $p\bar{p}$ rest frame.

The measurements of CDF and D0 are $0.158 \pm 0.072 \pm 0.017$, with $5.3fb^{-1}; A_{FB}^{C} = 0.08 \pm 0.04 \pm 0.01$, with $4.3fb^{-1}$. (2)

The measurements are consistent with previous ones $[3, 4]$. The corresponding SM predictions from Monte Carlo(MC) simulations are $5.8 \pm 0.9\%$ by MCFM and $1\%$ by MC@NLO. Here D0's measurement and the corresponding MC@NLO prediction can not be compared directly with SM ones because they are not normalized by selecting efficiency. The $A_{FB}$ in the SM is calculated to be $7.8\%$ in Ref. [6–8], which is larger than $5.8 \pm 0.9\%$ by MCFM. The reason is that the denominator of $A_{FB}$ in the MCFM is the cross section at the next leading order QCD, while the leading order cross section in Ref. [6–8]. Therefore they differ by a factor of $k \sim 1.3$.

\[ A_{FB} = \frac{\sigma(|Y| > 0) - \sigma(|Y| < 0)}{\sigma(|Y| > 0) + \sigma(|Y| < 0)). (1) \]

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via $gg$ fusion efficiently. In this paper, we propose a new definition of FB asymmetry, namely the one-side FB asymmetry $A_{\text{OFB}}$, to conquer this difficulty. $A_{\text{OFB}}$ can be large and arises from the same $O(\alpha_s^2)$ contributions which induce the observed FB asymmetry at Tevatron. This quantity can be examined at the LHC and cross-checked to the corresponding measurements at the Tevatron.

At the LHC, up to next-to-leading order (NLO) QCD, top pair events can be generated through the channels $q\bar{q} \to t\bar{t}$, $q\bar{q} \to t\bar{t}g$, $gg \to t\bar{t}q$ or $gq \to t\bar{t}q$ and $gg \to t\bar{t}$ at the partonic level. Being a proton-proton collider, LHC does not have the preferred directions in the laboratory rest frame. However except the symmetric gluons, the incoming partons do have preferred direction. Usually the valence quark momentum is larger than that of the sea quark. For example, for the process $u\bar{u} \to t\bar{t}$ (taking the momentum of the $u$ quark as the positive $z$ direction), momentum of $u$ is most probably larger than that of $\bar{u}$. On average, this will induce the $z$ direction $t\bar{t}$ total momentum in lab frame $P_{t\bar{t}}^z > 0$. So even for the $pp$ collider, $u\bar{u} \to t\bar{t}$ can contribute an asymmetric $t\bar{t}$ distribution. However, this asymmetry is completely canceled with the opposite direction $\bar{u}u \to t\bar{t}$ events. If we observe only one-side $t\bar{t}$ events, i.e. $P_{t\bar{t}}^z > 0$, such asymmetry will be kept. To maintain the partonic asymmetry and suppress the symmetric events, we require a cut $|P_{t\bar{t}}^z| > P_{\text{cut}}^z$ on the $z$ direction top pair momentum of the final $t\bar{t}$ pair in the $pp$ rest frame. One may argue that determination of the momentum in beam line direction may be problematic, especially when one neutrino is missing when using the associated charged lepton to trigger the top/antitop event. This issue can be solved by requiring invariant mass of the neutrino and charged lepton just equal to that of the $W$ boson, which is assumed to be the decay product of the top quark. Thus $P_{t\bar{t}}^z$ is still a measurable quantity.

The new one-side forward-backward asymmetry $A_{\text{OFB}}$ can be defined in the $pp$ rest frame as

$$A_{\text{OFB}} = \frac{\sigma(\Delta Y > 0) - \sigma(\Delta Y < 0)}{\sigma(\Delta Y > 0) + \sigma(\Delta Y < 0)} |P_{t\bar{t}}^z > P_{\text{cut}}^z, M_{t\bar{t}} > M_{\text{cut}}$$

or

$$A_{\text{OFB}} = \frac{F_- + B_-}{F_+ + B_+} = \frac{\sigma^A}{\sigma}, \quad (5)$$

with

$$F_\pm = (\sigma(\Delta Y > 0) \pm \sigma(\Delta Y < 0)) |P_{t\bar{t}}^z > P_{\text{cut}}^z, M_{t\bar{t}} > M_{\text{cut}}$$

$$B_\pm = (\sigma(\Delta Y < 0) \pm \sigma(\Delta Y > 0)) |P_{t\bar{t}}^z < -P_{\text{cut}}^z, M_{t\bar{t}} > M_{\text{cut}}$$

The asymmetry defined in Eq.(5) is the same as that in Eq.(4) except the statistics are doubled. We will adopt the asymmetry definition in Eq.(5) in the following evaluation. The goal to apply constraint on $P_{t\bar{t}}^z$ and $M_{t\bar{t}}$ is to exclude the symmetric $gg \to t\bar{t}$ events. In Eq.(5), the asymmetric cross section in the numerator arises from $O(\alpha_s^2)$ in QCD, and the denominator is the total cross section. Although some high order effects in $t\bar{t}$ production have been considered, such as soft gluon resummation \cite{14, 15} and the exclusive next-to-leading order cross section of $t\bar{t} + jet$ production \cite{16, 17}, the exact inclusive next-to-leading order asymmetric cross section which involves the two-loop contributions is still unknown. For consistency, we choose the lowest order result of total cross section at $O(\alpha_s^2)$ as a rough estimation.

The typical Feynman diagrams of $O(\alpha_s^2)$ for the denominator in Eq.(4) are drawn in Fig.1.

![Fig. 1: Typical Feynman diagrams for $t\bar{t}$ pair production at LHC at $O(\alpha_s^2)$.](image)

In the SM, the leading asymmetric cross section arises at $O(\alpha_s^3)$. The related Feynman diagrams at partonic level can be classified into three categories: (1) the interference among virtual box in Fig.2 and the leading diagrams for the process $q\bar{q} \to t\bar{t}$ in Fig.1 (2) the interference among initial and final gluon radiation diagrams of $q\bar{q} \to t\bar{t}g$ in Fig.3 and (3) contributions from diagrams of $gq \to t\bar{t}q$ and $gq \to t\bar{t}q$ in Fig.4.

The asymmetric cross section at the parton level was given analytically in Ref.\cite{17}. However, we carry out independent calculations \cite{19} with the help of FeynCalc \cite{20}, FormCalc \cite{21} and QCD-Loop \cite{22}.

The asymmetric cross section $\sigma^A$ contributed from each of the above three categories is UV and
collinear divergences free. The real gluon radiation of category (2) can be divided into a soft part and a hard part by introducing a soft cut \( \delta^s \) \[23\]. Soft divergences are canceled when adding the virtual (1) and soft part to form a virtual-soft part. \( \delta^s \) independence is checked by adding the virtual-soft and hard part \[19\].

In our numerical calculations, we choose cteq6l for the leading order calculation and cteq6m for higher-order estimations. The scales are chosen as \( \mu_r = \mu_f = m_t \) and \( \alpha_S(m_Z) = 0.118 \).

To check our Fortran code, forward-backward asymmetry at Tevatron is recalculated independently using our code. \( A_{FB} \) is calculated to be \( 7.1\% \), which is in good agreement with the existing results \[8, 24\].

**FIG. 2:** Typical NLO virtual Feynman diagrams which contribute to asymmetric cross section.

**FIG. 3:** Typical real gluon emission Feynman diagrams which contribute to asymmetric cross section.

Figures 5 and 6 show our predictions on \( A_{OFB} \) and significance to discover the asymmetric events which is defined as \( \text{sig} = \sqrt{L} \sigma^A/\sqrt{\sigma} \) (with \( L = 10\text{fb}^{-1} \)) at LHC with \( \sqrt{s} = 7 \text{ TeV} \) and 14 TeV, respectively. For no cuts at all \( (M_{t\bar{t}} > 2m_t, P_{t\bar{t}}^z = 0) \), the \( A_{OFB} \) is not zero but very small, 1.2% and 0.58% for \( \sqrt{s} = 7 \text{ TeV} \) and 14 TeV, respectively. The reason is simply due to the large denominator \( \sigma \) which arises mainly from \( gg \rightarrow t\bar{t} \). To increase \( A_{OFB} \), we apply cuts on \( P_{t\bar{t}}^z \) and \( M_{t\bar{t}} \). The key point is to suppress \( gg \) fusion contributions to the denominator while decreasing the numerator \( \sigma^A \) as small as possible. From the figures, we can see clearly that \( P_{t\bar{t}}^z \) cut can increase \( A_{FB} \) greatly while \( M_{t\bar{t}} \) cut is not so efficient. The \( P_{t\bar{t}}^z \) cut has two impacts on \( A_{OFB} \). First, as symmetric events \( gg \rightarrow t\bar{t} \) lie mainly in the small \( P_{t\bar{t}}^z \) region, the \( P_{t\bar{t}}^z \) cut can remove them effectively. Second, as mentioned above, it is most probably that the valence quark’s momentum is larger than that of the sea quark, but it does has some small probability that the valence quark’s momentum is smaller than that of the sea quark. This will cause an opposite con-
tributions to the asymmetric cross section in our definition of $A_{OFB}$. The $P_{cut}$ cut can reduce such kind of pollution. In figures we also show the significance to discover the asymmetric events from the background symmetric events. Such measure can be utilized to optimize the cut conditions, namely $M_{tt} > 2m_t \; P_{cut}^z \sim 500$ GeV is the optimal one at $\sqrt{s} = 7$ TeV and $M_{tt} > 2m_t \; P_{cut}^z \sim 1.2$ TeV is the optimal one at $\sqrt{s} = 14$ TeV though $A_{OFB}$ is not the largest here. Note that $\text{sig} \propto \sqrt{L}$, here we choose $L = \infty/\text{fb}^{-∞}$. The value of $\text{sig}$ varies with the integrated luminosity but optimal cut criterion should be stable.

It’s interesting to compare $A_{OFB}$ proposed in this paper with the central charge asymmetry $A_C$ proposed before [cf. Eq. (3)]. First, $A_C$ only accounts for the single top or antitop in $|Y| < Y_C$ regions, while the new asymmetry $A_{OFB}$ needs to include the top pair kinematical information for every events. Second, $A_C$ is based on a central region $|Y| < Y_C$, while $A_{OFB}$ is defined in a region $|P_{cut}^z| > P_{cut}^z$. Because of the different momentum distribution of the quarks and the corresponding sea quarks in the proton, the top pair events produced via $q\bar{q}$ annihilation are very likely be boosted in the $z$ direction. For the central charge asymmetry, a small $Y_C$ cut will include nearly equal number of $t$ and $\bar{t}$ events, which makes $A_C$ approach zero [11]. $A_C$ vanishes if the whole rapidity spectrum is integrated. Optimal $Y_C$ for $A_C$ is about 1. Our $A_{OFB}$ can cover the top pair events that reach to the edge of the detector, so it can have more statistics. Third, the denominator $\sigma$ in both cases is mainly composed of the symmetric $gg \to t\bar{t}$ events. This makes both asymmetries small, especially for $A_C$ [11]. A kinematic feature of $gg \to t\bar{t}$ events is that they are mostly located in a small $P_{cut}^z$ region. So by requiring a higher $P_{cut}^z$, top pair events via $gg$ fusion can be removed efficiently. The central charge asymmetry does not take this advantage so $A_C$ is smaller than $A_{OFB}$.

To summarize, both CDF and D0 at Tevatron reported the measurements of forward-backward asymmetry in top pair production. Theoretically such asymmetry is due to the higher-order QCD processes. The measurements showed a possible deviation from the theoretical prediction. In this paper, we explore how to examine the same higher-order QCD effects at the more powerful LHC. Unlike Tevatron, the proton-proton LHC has no preferred direction in the laboratory frame. Thus we define a new one-side forward-backward asymmetry $A_{OFB}$ [cf. Eqs. (4) and (5)] in terms of the top pair kinematical information. Our studies show that the cut on top pair momentum in the $z$ direction can increase asymmetry greatly. Provided that huge $t\bar{t}$ events will be produced at the LHC, $A_{OFB}$ can be precisely measured and compared with the corresponding measurements at the Tevatron.

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