Evidence of Solar-cycle-related Structural Changes in the Solar Convection Zone

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Abstract

While it has been relatively easy to determine solar-cycle related changes in solar dynamics, determining changes in structure in the deeper layers of the Sun has proved to be difficult. By using helioseismic data obtained over two solar cycles, and sacrificing resolution in favor of lower uncertainties, we show that there are significant changes in the solar convection zone, and perhaps even below it. Using Michelson Doppler Imager (MDI) data, we find a relative squared sound-speed difference of $(2.56 \pm 0.71) \times 10^{-5}$ at the convection-zone base between the maximum of solar Cycle 23 and the minimum between Cycles 23 and 24. The squared sound-speed difference for the maximum of Cycle 24 obtained with Helioseismic and Magnetic Imager (HMI) data is $(1.95 \pm 0.69) \times 10^{-5}$. Global Oscillation Network Group (GONG) data support these results. We also find that the sound speed in the solar convection zone decreases compared to the sound speed in the layers below it as the Sun becomes more active. We find evidence of changes in the radial derivative of the sound-speed difference between the solar minimum and other epochs at the base of the convection zone, implying possible small changes in the position of the convection-zone base; however, the results are too noisy to make any definitive estimates of the change.

Unified Astronomy Thesaurus concepts: Solar cycle (1487); Helioseismology (709); Solar interior (1500); Solar activity (1475)

1. Introduction

Solar frequencies, and frequency splittings, change with solar activity. This has been known for decades (Woodard & Noyes 1985; Palle et al. 1989; Elsworth et al. 1990; Libbrecht & Woodard 1990) and is one of the earliest, and most robust, findings of helioseismology. The frequency changes were shown to be correlated with the average line-of-sight magnetic field over the solar surface (Woodard et al. 1991); this was confirmed later using much longer data sets (e.g., Howe et al. 2002). The correlation between frequency shifts and activity is also seen for other solar activity proxies (Jain et al. 2009; Howe et al. 2018a, etc.).

The solar-cycle dependent changes in solar oscillation frequencies have been successfully used to determine activity-related changes in solar dynamics (see review by Howe 2009 for Cycle 23 results, subsequent results can be found in Antia & Basu 2013; Howe et al. 2013, 2018b; Basu & Antia 2019; Kosovichev & Pipin 2019, etc.). While changes in solar dynamics have been quite easy to detect, detecting changes in solar structure has been very difficult.

Solar dynamics, and changes thereof, are determined from frequency splittings, while the structure is determined from the frequencies themselves. Solar-cycle dependent frequency changes are predominantly a function of frequency, with the changes increasing with frequency. It is reminiscent of the "surface term," (i.e., changes or differences in frequencies caused by near-surface changes or differences). The apparent lack of a degree-dependence in the frequency shifts has led to the view that solar-cycle related changes in the Sun are confined to a thin layer close to, or even above, the surface (Libbrecht & Woodard 1990; Goldreich et al. 1991; Evans & Roberts 1992; Nishizawa & Shibahashi 1995, etc.). Other studies seemed to confirm this (Howe et al. 1999; Basu & Antia 2000, etc.). Despite this, there have been attempts to look for changes deeper inside the Sun.

The solar dynamo is believed to be situated in the tachocline, which is more or less coincident with the base of the solar convection zone (Kosovichev 1996; Basu 1997; Charbonneau et al. 1999; Basu & Antia 2019). There has been an expectation that the large magnetic fields generated there will be visible as changes in solar structure close to the convection-zone base. Consequently, there have been attempts, with various degrees of success, in detecting such changes (Dziembowski et al. 2000; Basu 2002; Eff-Darwich et al. 2002; Chou & Serebryanskii 2005, etc.). Baldner & Basu (2008) did a principal component analysis of the helioseismic data available until 2007 May to find a change in sound speed at the base of the convection zone; the results at other radii were not statistically significant. None of the other efforts have shown a clear, unambiguous pattern of changes that are correlated with solar activity. Attempts to determine changes in relatively shallow layers were, however, successful. Baldner & Basu (2008) showed that there was evidence of changes in the outer 10%–15% of the Sun, but at low latitudes, while Basu et al. (2007) found evidence of changes even at high latitudes, but their work was only sensitive to the outer 2% of the Sun. Basu & Mandel (2004) examining signatures caused by the acoustic glitch in the HeII ionization zones detected a solar-cycle-dependent change; this was subsequently confirmed by Watson & Basu (2020) using solar frequencies for Cycles 23 and 24, but while the result was obtained with data sensitive to changes at all latitudes, the change was localized to a shallow layer.

Watson & Basu (2020) showed that the use of two solar cycles worth of data helps in demonstrating whether or not there is a solar-cycle dependent change when the changes are small. In this paper, we use helioseismic data for two solar cycles to examine whether or not there are changes in the deeper layers of the Sun. We do a differential analysis, (i.e., we analyze frequency differences between different epochs of the two solar cycles and the solar minimum between Cycles 23 and 24). We perform solar structure inversions; however, unlike usual inversions, we sacrifice radial resolution in order to reduce uncertainties in the results.

The rest of the paper is organized as follows: We describe the data in Section 2, the frequency differences are examined in...
Section 3, the inversion method and possible systematic errors are described in Section 4, and we present our results in Section 5. We discuss the implications of our results in Section 6.

2. Data Used

For this work, we use solar oscillation frequencies obtained by the ground-based Global Oscillation Network Group (GONG; Hill et al. 1996) and the space-based Michelson Doppler Imager (MDI) on board the Solar and Heliospheric Observatory spacecraft (Scherrer et al. 1995) and the Helioseismic and Magnetic Imager (HMI; Scherrer et al. 2012) on board the Solar Dynamics Observatory.

The GONG data we use cover a period from 1995 May 5 to 2020 May 18. The data are designated by GONG “months,” with each “month” being 36 days long. Solar oscillation frequencies and splittings of sets starting GONG Month 2 are obtained using 108 day (i.e., 3 GONG months) time series. There is an overlap of 72 days between different data sets, (i.e., GONG Month 2 frequencies were obtained from data of GONG Months 1, 2, and 3, those for GONG Month 3 from GONG Months 2, 3, and 4, etc.). We use data for GONG Months 2–254. While the overlapping sets are useful in looking at time variations, they can bias statistics, consequently all statistical inferences are drawn from non-overlapping data sets. These data sets are publicly available from GONG archives.

Data from MDI cover the period from 1996 May 1 to 2011 April 24. Solar oscillation frequencies and splittings for these data are obtained from 72 day time series and the sets have no overlap in time. HMI started obtaining data on 2010 April 30 and these are also obtained from 72 day time series. We use all MDI data, and all HMI sets up to set 10,000, which has an end date of 2020 July 30. These sets are available to all researchers through the Joint Science Operations Center of the Solar Dynamics Observatory.

We use the 10.7 cm flux as a measure of solar activity (Tapping 2013; Tapping & Morton 2013). These data are also public.¹

For the purpose of looking at time variations and solar-activity-related changes, we use GONG data as one set and combine the MDI and HMI data to form another set—if both sets show the same trends, we can be confident that we are not seeing an instrumental effect. Since we are doing a differential study, we choose one reference frequency set for each of the ground-based and space-based data. In each case, we choose sets from the minimum between Cycle 23 and Cycle 24. For the GONG sets, we use the averaged frequencies of GM 133, 136, and 139 as the reference set. These are non-overlapping sets and have an average 10.7 cm flux of 68.1 SPU.² For the space-based set, we use a specially created 720 day data set spanning 2007 July 27 to 2009 July 16 as the reference set. This set has an average 10.7 cm flux of 69.3 SPU.

3. A Clue to Cycle-dependent Structure Changes

If the solar-cycle related changes of frequencies were a result only of changes localized to the very shallow, near-surface layers of the Sun, the changes would be a function of frequency alone, at least at low and intermediate degrees; any degree differences can be accounted for by the differences in the inertia of the modes. As in the case of all surface terms, it is usual to fit an ad hoc, slowly varying function of frequency to model the solar-cycle-related change.

However, the form of the surface term need not be completely arbitrary. Libbrecht & Woodard (1990) showed that the shifts that they found could be modeled as the cube of the frequency divided by the mode inertia. This was given some theoretical justification by Goldreich et al. (1991) who showed that a photospheric perturbation could produce such a term. Gough (1990) proposed an additional term, one that depends on the inverse of the frequency, again scaled by the mode inertia. The cubic term could arise from the presence of some factor that affects sound speed but not density, such as magnetic flux tubes. The inverse term would reflect factors such as a change in the pressure scale height near the surface. Ball & Gizon (2014) first used this to describe the frequency dependence of the surface term in low-degree solar and asteroseismic data.

The Ball & Gizon (2014) parameterization is given by:

\[ \delta \nu = \frac{1}{E_{\nu i}^2} \left[ a_1 \left( \frac{\nu}{\nu_{ac}} \right)^{-1} + a_3 \left( \frac{\nu}{\nu_{ac}} \right)^3 \right], \]

where \( E_{\nu i} \) is the inertia of a model of degree \( l \) and order \( \nu \), and \( \nu_{ac} \) is the acoustic cutoff frequency. Howe et al. (2017) showed that this can be applied to both low-degree and intermediate-degree solar modes. What had not been examined directly in that work is how the fits behave as a function of time, (i.e., how the reduced \( \chi^2 \) behaves as a function of time). The reduced \( \chi^2 \) for the frequency differences between the individual GONG sets and the GONG reference set, as well as the frequency differences of the MDI and HMI sets with respect to the MDI reference set, are shown in Figure 1. What is clear immediately is that there is a solar-cycle-dependent change in the reduced \( \chi^2 \) and that the \( \chi^2 \) is largest at the solar maxima.

The variation of the reduced \( \chi^2 \) is an indication that there is more to the solar cycle-dependence of the frequency differences than just a contribution from the near-surface layers. In Figure 2 we show how the average residuals for modes with lower-turning points between 0.7125 ± 0.0125 \( R_\odot \) vary as a function of time. Since modes that have their lower turning point within any radius range can have different frequencies—recall that the classical lower turning point of a mode is \( r_t = c(r_t)\sqrt{\ell(\ell + 1)}/\omega \), where \( c(r_t) \) is the sound speed at \( r_t \) and \( \omega = 2\pi \nu \)—a frequency-dependent solar-cycle-related change alone cannot explain a solar-cycle-related change in the frequency differences of modes with a given lower turning point. Figure 2 points to changes in the structure around 0.7125 ± 0.0125 \( R_\odot \), which means we cannot rule out changes at other radii either. We look for changes in the internal sound speed by inverting the frequency difference between the different data sets and the reference sets.

4. Inversions: Methods and Systematic Errors

Inversions for solar structure start with the linearized form of the oscillation equations obtained using the variation principle:

\[ \frac{\delta \nu_i}{\nu_i} = \int K_{\nu_i,\rho_c} \frac{\delta \rho_c}{\rho_c} d\rho + \int K_{\rho,\nu_c} \frac{\delta \rho}{\rho_c} d\rho + \frac{F(\nu_i)}{E_i}, \]

where \( \delta \nu_i/\nu_i \) is the relative frequency difference of the \( i \)th mode, \( \rho \) is the density, \( K_{\nu_i,\rho_c} \) and \( K_{\rho,\nu_c} \) are known functions of a

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¹ Data are available from http://www.spaceweather.gsfc.nasa.gov/solar/sx-en.php, and https://omniweb.gsfc.nasa.gov.
² 1 solar flux unit or SPU is 10^4 Jy, (i.e., 10^22 W m^-2 Hz^-1).
reference solar model, $F(\nu)$ is a slowly varying function of frequency to account for the surface term, and $E_i$ is the inertia of the $i$th mode. The aim of inversions is to determine $\delta c^2/c^2$ or $\delta \rho/\rho$ given a set of $\delta \nu/\nu$.

There are two primary ways of inverting Equation (2) that have different aims, these are the Regularized Least Squares (RLSs) method, and the method of Optimally Localized Averages (OLA). RLS aims to find the $\delta c^2/c^2$ and $\delta \rho/\rho$ profiles that give the best fit to the data (i.e., give the smallest residuals) while keeping uncertainties small; the aim of OLA is not to fit the data at all, but to find linear combinations of the frequency differences in such a way that the corresponding combination of kernels, the averaging or resolution kernel, provides a localized average of the unknown function, again while keeping uncertainties small. Details of how these techniques are implemented can be found in Basu (2016). OLA and RLS inversions are complementary in nature (Sekii 1997) and inversions can be trusted if both inversion techniques return the same results.

There are two variants of OLA, the original version, which in helioseismic literature is referred to as multiplicative OLA (MOLA) because a weighting function used in the construction of the averaging kernels are multiplied with the mode kernels, and subtractive OLA (SOLA; Pijpers & Thompson 1992), where the difference between the averaging kernels and a target kernel. SOLA has the advantage that the target kernels can be selected for specific inversions, such as the derivatives of the function rather than the function itself (Pijpers & Thompson 1994). Details of how SOLA and MOLA implementations differ can be found in Basu (2016).

We use SOLA of the results obtained in this paper since it is faster and allows us to directly invert for derivatives. We use mode kernels from Model S (Christensen-Dalsgaard et al. 1996); our tests have shown that the kernels from other standard solar models give identical results. In Figure 3 we show the inversion results for MDI set 3232 and the 2 yr MDI set obtained using three different inversion methods. Note that the results for the different methods agree at radii below about 0.88 $R_\odot$. Although not shown in the figure, the match between the results deteriorates below about 0.45 $R_\odot$.

Since there is a trade-off between the resolution of the inversions and the uncertainty in the results, we sacrificed resolution in order to increase the precision of the results. The averaging kernels at a few different positions obtained for the SOLA inversions are shown in Figure 4. As can be seen, the results at most radii will be correlated.

A possible source of systematic error is the structure of the averaging kernel far from the target radius. This is caused mainly because of the lack of high degree modes in the data set. In particular, the worry is that since the averaging kernels are...
not strictly zero close to the surface, if there is a large solar-cycle related change in the surface, this would leak into the results at other radii giving a false impression about the deeper layers. Recall that the SOLA inversion results are in effect (Basu 2016)

\[
\left( \frac{\delta c^2}{c^2} \right)_{\text{inv}} = \sum_i c_i \frac{\delta \nu_i}{\nu_i} = \sum_i c_i \int K_{i,p} \frac{\delta \rho}{\rho} \, dr,
\]

assuming that \( \sum_i c_i \int K_{i,p} \delta \rho \, /\rho \, dr \) and \( \sum_i c_i F(\nu_i / \nu) \), are small; \( c_i \) are the inversion coefficients. The term \( \sum_i c_i K_{i,p} \) is nothing but \( K \), the averaging kernel, and \( \sum_i c_i K_{i,2} \) is usually called the “cross term” kernel. Since the inverted value of the sound speed at a radius \( r_0 \) is in effect \( \int K(r_0, r) \frac{\delta c^2}{c^2} \, dr \), any structure in the averaging kernel away from \( r_0 \) will contribute to the solution. As can be seen from Figure 4, the averaging kernels are not zero close to the surface.

To try and quantify the effect of the surface structure of the averaging and cross-term kernels on the results, we assume a large change in structure (10%) at the surface (see the top panel of Figure 5), and we also assume that there are changes as deep as the second helium ionization zone since there are indirect detections of change in structure in those layers (Watson & Basu 2020). We then determine the effect of this change on the solutions at different radii. We show the results in the lower panel of Figure 5. We show the results assuming that there is a difference only in sound speed, a change in both sound speed and density with both changing the same way, and in one case where the changes in sound speed and density have the same magnitude but opposite sign. As can be seen in the figure, the systematic error because of unresolved surface changes of 10% is less than a part in 10^5 for radii below 0.8 \( R_\odot \) and rises thereafter. This will need to be kept in mind while interpreting the results at different radii. Clearly, for us to claim that we see changes in \( c^2 \), they need to be greater than one part in 10^5.

5. Results

5.1. Changes in Sound Speed

In Figures 6 and 7 we show the inversion results obtained with space based data (MDI+HMI) and GONG, respectively, at three different radii. The results are plotted both as a function of time and of the 10.7 cm flux. In the case of GONG data, the results of all sets are plotted against time, but only nonoverlapping sets are plotted against the radio flux. It can be seen very clearly that there is a time variation in the result. Note that the results are with respect to the minimum between Cycle 23 and 24, and the results show that the sound speed is lower at high activity than at low activity, particularly in the convection zone. The changes are large enough that they...
cannot be explained as systematic error due to changes in the near-surface layers. Visually, the GONG results look more significant, but that is probably because the sets overlap in time. The figures appear to show that there is a variation in sound speed even below the base of the convection zone, but as can be seen from Figure 4, the averaging kernel for 0.65 $R_\odot$ samples the convection zone too.

As expected from the time variation, the changes are well correlated with the 10.7 cm flux at all radii. The correlation coefficients and their statistical significance are listed in Table 1. Note that the lower the $p$ value, the more the significance of the slope; a $p$-value of 0.05 corresponds to a 95% confidence level. The slopes of the relation between the changes and the 10.7 cm flux obtained for the space-based and ground-based data do not completely agree—GONG results are steeper. This is most likely due to the fact that the mode sets used are different for GONG and MDI+HMI, and also that for the GONG sets we have used a reference set of data that spans slightly less than a year, while for MDI+HMI we subtract out a 2 yr set. What is important though, is that both sets show that there is an activity-related change in structure. We also note that the change seems to be larger within the convection zone than below it, but as mentioned earlier the results below the convection zone have a substantial contribution from those in the convection zone.

Figure 6. Inversion results for MDI and HMI data sets at three radii. Left: Results plotted as a function of time, with the curve in the background showing the 10.7 cm flux plotted on a scale from 0 to 250 (not shown). The horizontal line in each panel is the weighted average of the results. Note that while the results at 0.65 $R_\odot$ and 0.85 $R_\odot$ are independent (see Figure 4), the result at 0.75 $R_\odot$ has an overlap with both 0.65 $R_\odot$ and 0.85 $R_\odot$. Right: The same results plotted as a function of the 10.7 cm flux. The horizontal line is again the weighted average. The red line shows a weighted linear least-squares fit to the points with the 95% confidence level shown as the dotted lines.

| Table 1 |
|---------|
| **Correlations and Statistical Significance** |
| Quantity | Radius ($R_\odot$) | Pearson Coeff. | Spearman Coeff. | $t$ Value | $p$ Value |
|----------|-------------------|-----------------|-----------------|-----------|-----------|
| MDI+HMI  |                   |                 |                 |           |           |
| $\frac{\delta c^2}{c^2}$ | 0.650 | -0.54 | -0.46 | -5.83 | <0.00001 |
| $\frac{\delta c^2}{c^2}$ | 0.750 | -0.71 | -0.69 | -10.63 | <0.00001 |
| $\frac{\delta c^2}{c^2}$ | 0.850 | -0.80 | -0.74 | -12.14 | <0.00001 |
| $\Delta(\frac{\delta c^2}{c^2})$ | 0.713 | 0.37 | 0.33 | 3.91 | 0.00015 |
| $\frac{\delta (\delta c^2)}{\delta c^2}$ | 0.613 | -0.02 | -0.04 | -0.48 | 0.64 |
| $\frac{\delta (\delta c^2)}{\delta c^2}$ | 0.713 | -0.37 | -0.35 | -4.21 | 0.00006 |
| $\frac{\delta (\delta c^2)}{\delta c^2}$ | 0.813 | -0.28 | -0.18 | -2.06 | 0.04 |
| $\frac{\delta (\delta c^2)}{\delta c^2}$ | 0.750 | -0.61 | -0.60 | -8.17 | <0.00001 |
| GONG    |                   |                 |                 |           |           |
| $\frac{\delta c^2}{c^2}$ | 0.650 | -0.58 | -0.72 | -6.43 | <0.00001 |
| $\frac{\delta c^2}{c^2}$ | 0.750 | -0.88 | -0.88 | -17.29 | <0.00001 |
| $\frac{\delta c^2}{c^2}$ | 0.850 | -0.85 | -0.83 | -13.74 | <0.00001 |
| $\Delta(\frac{\delta c^2}{c^2})$ | 0.713 | 0.40 | 0.37 | 3.60 | 0.00027 |
| $\frac{\delta (\delta c^2)}{\delta c^2}$ | 0.613 | -0.24 | -0.23 | -2.81 | 0.33 |
| $\frac{\delta (\delta c^2)}{\delta c^2}$ | 0.713 | -0.45 | -0.50 | -4.71 | <0.00001 |
| $\frac{\delta (\delta c^2)}{\delta c^2}$ | 0.813 | -0.14 | 0.05 | -0.45 | 0.016 |
| $\frac{\delta (\delta c^2)}{\delta c^2}$ | 0.750 | -0.88 | -0.89 | -18.01 | <0.00001 |
5.2. Differences Below and Above the Convection Zone

The results in Figures 6 and 7 point to differences below and above the convection zone; however, as mentioned earlier, the differences below the convection zone are affected by differences above, and hence merely subtracting the results will not give us the true difference. We make use of the fact that for SOLA inversions we can define a suitable target averaging kernel that will give us the quantity we seek, in this case, the difference in the results at two locations. The averaging kernel obtained for directly inverting for the difference in sound speed below and above the base of the convection zone is shown in Figure 8. Note that we get a clean averaging kernel with no overlap below and above the base of the convection zone. The results of the inversion are shown in Figure 9. There is indeed a small, time-varying difference between the layer below and above the convection zone, with the sound speed in the convection zone being lower than that below it when the Sun is active. The differences are small; however, the systematic error in the results for an artificial profile like that in Figure 5 is at most three parts in $10^6$, (i.e., smaller than the changes seen).

5.3. Is There a Change in the Position of the Convection-zone Base?

The base of the convection zone is marked by a change in the temperature gradient, consequently the sound-speed difference for two models with different $r_\text{p}$, the position of the convection-zone base shows a sharp rise and a steep gradient near $r_\text{p}$, as is shown in Figure 10.

We use the SOLA technique to determine the derivative of $\delta c^2/c^2$ directly; these are shown for three different radii in Figure 11. Since the averaging kernels are quite wide, we cannot expect to see the steep gradients that are seen in Figure 10(b), instead, the derived results will be smaller and smoother, as shown in Figure 10(c). However, Figure 10(c)
shows that even though the inversions will smooth out the underlying gradient, the obtained difference will be largest near the position of the convection-zone base.

The inversion results at 0.713 $R_\odot$ (the base of the solar convection zone as per Christensen-Dalsgaard et al. 1991; Basu 1998) can be seen in Figure 12. The results appear to show a variation with time and activity; however, the results are noisy enough that to actually determine the amount of change in the position of the convection-zone base is extremely difficult. We do not show the results at other radii since they are not very statistically significant. Additionally, as seen from Figure 10, if there is a change in the position of the convection-zone base, it will be the largest in the region around it. If we try to interpret Figure 12 (top and bottom left panels) at face value with the aid of Figure 10, it would appear that the convection zone becomes deeper at activity maximum.

5.4. Is the Surface Term Fooling Us?

Unlike earlier attempts in determining solar-cycle-related structural changes, we have used a very specific form of the surface term, that is given in Equation (1). Thus one could
wonder whether the results are simply an artifact of using that form of the surface term, and perhaps that form does not really account for all near-surface effects. To examine whether this is indeed the case, we repeated the inversions with a conventional surface term. The results at $0.75 R_e$ are shown in Figure 13, and as can be seen, we see the same type of time variation as in Figures 6 and 7, and the slope of the change against the 10.7 cm radio flux is almost identical. Thus the form of the surface term is not causing the time-variation that we find.

6. Discussion

We have used two solar cycle’s worth of helioseismic data to show that there is indeed a solar-cycle related change in structure, particularly in the convection zone. While Watson & Basu (2020) had to use indirect means, we observe the changes more directly by using conventional inversions; however, using current mode sets, we can only probe changes between about $0.6 R_e$ and $0.88 R_e$. Although we started the study using the Ball & Gizon (2014) formulation of the surface term, we find that the time dependence remains when we use the convection forms of the surface term. The changes, although small, are statistically significant.

Baldner & Basu (2008) had found a change $\delta c^2/c^2$ of $(7.23 \pm 2.08) \times 10^{-5}$ near the base of the convection zone between the solar minimum and maximum. The changes they find are localized at $(0.71 \pm 0.09)^{+0.0097}_{-0.0028} R_e$. In contrast, we get a change of $(2.56 \pm 0.71) \times 10^{-5}$ using MDI data and $(3.99 \pm 0.65) \times 10^{-5}$ using GONG data. The discrepancy in the results is most likely due to differences in the radial resolution of the two sets of inversions; in this work, we sacrificed resolution to lower the uncertainties in the solution. Our uncertainties are smaller by nearly a factor of 3 compared with that of Baldner & Basu (2008). Unlike the highly oscillatory nature of the Baldner & Basu (2008) solution, our differences are smooth at all radii (see Figure 3), which again points to the effects of the radial resolution of the inversions.

The changes in solar dynamics between Cycles 23 and 24 were quite different (Howe et al. 2017; Basu & Antia 2019). In particular, changes in the tachocline did not show the same activity-related change in the two cycles—the change in the rotation rate across the tachocline was different for the same value of the 10.7 cm flux. Given the uncertainty in our results, we are not able to examine cycle-to-cycle changes between the response of internal sound speed to solar activity. A principal component analysis of the two cycles, done separately, might be able to detect cycle-to-cycle changes since there is evidence that the frequency response to solar activity was different in the two cycles (Broomhall & Nakariakov 2015).

Our results show that as solar activity increases, the sound speed in the region above the base of the convection zone, (i.e., the tachocline region), decreases compared with that below the convection-zone base. This result is consistent with the assumption that the sound-speed changes are a result of magnetic fields.

By directly inverting for the gradient of the sound-speed differences, we have shown that there is a change in the
gradient with the solar cycle. The change is most significant at the base of the convection zone. Interpreted most simply, this implies a change in the position of the convection-zone base. However, with currently available frequencies, finding the change at any given epoch is difficult. There had been earlier attempts to determine if the position of the convection-zone base showed any change without much success (Basu & Antia 1999); however, those used a very limited data set (for instance, Basu & Antia 1999 only had access to data from 1995–1998), and did not use a differential method.

The results are intriguing enough to examine the results further. Given that the current data sets are not ideal, the analyses need to be repeated with frequencies obtained from time series that are longer; while longer time series will smooth out some of the solar-cycle-related changes, overlapping sets will still reveal the changes. Two year data sets at the solar maxima, two years at the minima, and a few two year sets in between will help.

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Facilities: GONG, MDI, HMI, Penticton Radio Observatory.

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