Theory of polarized Fermi liquid

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The dispersion law of transverse spin waves known in the Stoner-Hubbard model of itinerant ferromagnetism corresponds to that is well known in more broad and well controlled approach of Fermi-liquid theory. Making use the quantum-field theoretical approach we derive the dispersion law for the transverse spin waves in a weakly polarized Fermi liquid at \( T = 0 \). Along with the dissipationless part inversely proportional to the polarization it contains also the finite zero-temperature damping. It is shown that similar derivation for “ferromagnetic Fermi liquid” taking into consideration the divergency of static transverse susceptibility also leads to the same attenuating spin wave spectrum. Hence, in both cases we deal in fact with spin polarized Fermi liquid but not with isotropic itinerant ferromagnet where the zero temperature attenuation is prohibited by Goldstone theorem. It demonstrates, the troubles of the Fermi liquid formulation of a theory of itinerant ferromagnetic systems.

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I. INTRODUCTION

It is widely accepted that the Hubbard-Stoner model serves as a sort of background for the description of the itinerant ferromagnetism\(^1\). The exchange interaction between the electrons is just ignored here but instead the model deals with the contact or extremely short range repulsion between the fermi particles with the opposite spins. The latter leads to the ground state with finite polarization described by the wave function

\[
Ψ_0 = \prod_{p < p^+} a^\dagger_{p \uparrow} \prod_{p' < p^-} a^\dagger_{p' \downarrow} Ψ_{\text{vac}},
\]  

where \( p^+ \) and \( p^- \) are the Fermi surface radii with spin directions parallel and antiparallel to the magnetization direction. The elementary excitations in the model are the excitations of pairs electron and hole with opposite spins. There is also a collective mode which is the transverse spin waves. Usually it is thought that the Stoner model gives not bad qualitative description of itinerant ferromagnetism but for the creation of more realistic quantitative theory it must be corrected by taking into account of spin-wave -electron-hole interaction. At the same time it is customary accepted to shut our eyes to the qualitative consequences of Stoner model which look strange for a ferromagnet.

First, it is the Stoner criterium of ferromagnetism according to which the ferromagnetic state appears at high enough interactions and density of states of Fermi gas \( UN_o > 1 \). The latter in the case of \( D = 3 \) one band electron system with quadratic dispersion law means the existence of ferromagnetism at high enough densities of electron gas. This property of Hubbard model with short range interaction is just the opposite to the property of electron gas with Coulomb exchange interaction (see for example the book\(^2\)) where it is easy to check that the ferromagnetic state appears in the low density limit as it has been pointed out by F.Bloch\(^3\). The criterium of ferromagnetism looks here as \( e^2m > \hbar^2n^{1/3} \).

Secondly, the reactive part of diffusion constant \( \Re D \) in quadratic spin wave dispersion law

\[
\omega = Dk^2
\]

obtained in frame of Hubbard-Stoner model\(^1\) is proved inversely proportional to the magnetization

\[
\Re D \propto \frac{1}{M} \propto \frac{1}{p^+ - p^-}.
\]
Whereas according to Landau and Lifshits the magnetization in any ferromagnet obeys the equation of motion

$$\frac{\partial \mathbf{M}}{\partial t} = K \nabla^2 \mathbf{M} \times \mathbf{M}. \quad (4)$$

Hence the diffusion constant in the spin wave dispersion law is proportional to magnetization that physically corresponds to the finite rigidity of the ferromagnetic ordering preventing formation of inhomogenous states.

Thirdly, unlike to Landau-Lifshits spin wave dispersion law the Stoner dispersion always contains a dissipation described by the imaginary part of diffusion constant. The presence of dissipation at $T = 0$ in an ordered state contradicts to the Goldstone theorem.

These properties having nothing common with ferromagnetism demonstrate the difficulties of Stoner-Hubbard model which does not take into account the many particle character of ferromagnetic ground state and rather related to the polarized Fermi gas with wave function (1). The same properties are well known for spin polarized neutral Fermi liquid like liquid $^3\text{He}$ or $^3\text{He}-^4\text{He}$ solutions. They can be obtained in frame of more broader and well controlled approach of the theory of Fermi liquid with short range interaction between particles.\(^5–10\)

On the other hand there were known also several publications with pretension to develop a theory of genuine itinerant isotropic ferromagnet based on Landau Fermi liquid theory.\(^11–13\) In particular the derivation of dissipationless (up to the terms of $k^4$ power) spin waves spectrum with diffusion constant proportional to magnetization has been announced.

The goal of the present article is to reconsider in frame of microscopic theory the problem of transverse spin waves in spin-polarized Fermi liquid and in "itinerant ferromagnet" as it has been defined in the papers.\(^11–13\) It is shown that in both cases the microscopic derivation leads to the same spin wave spectrum. Along with the dissipationless part inversely proportional to the polarization it contains also the finite zero-temperature damping. The polarization dependence both dissipative and reactive part of diffusion constant corresponds to dependences found earlier by means of kinetic equation with two-particle collision integral.\(^10\) It is shown that similar derivation for "ferromagnetic Fermi liquid" taking into consideration the divergency of static transverse susceptibility also leads to the same attenuating spin wave spectrum. Hence, in both cases we deal in fact with spin polarized Fermi liquid but not with isotropic itinerant ferromagnet where the zero temperature attenuation is prohibited by Goldstone theorem. It demonstrates, the troubles of the Fermi liquid formulation of a theory of itinerant ferromagnetic systems.

In section II starting from the integral equation for the vortex function for the scattering of two particles with opposite spin direction we derive the Landau-Silin kinetic equation for the transversal spin waves and find its dispersion law. In the third section the latter is done by another way taking into consideration the divergency of static transverse susceptibility. The paper is concluded by Conclusion Section.

II. SPIN WAVES IN POLARIZED FERMI LIQUID

The problem of transverse spin-waves in spin-polarized Fermi liquid has the long story. The calculations of transverse spin-diffusion coefficient in dilute degenerate Fermi gas with arbitrary polarization have been done for the first time in the papers by W.Jeon and W.Mullin where the low temperature saturation of corresponding relaxation time has been established. About the same time A.Meyerovich and K.Musaelyan have generalized the Landau derivation of Fermi liquid kinetic equation from microscopic theory on the case transverse spin kinetics in the polarized Fermi liquid and also come to the same conclusion. A derivation and an exact solution of the kinetic equation in the s-wave scattering approximation for dilute degenerate Fermi gas with arbitrary polarization at $T = 0$ and for a small polarization at $T \neq 0$ have been obtained also in the papers by D.Golosov and A.Ruckenstein. For the treatment of this problem in a Fermi liquid the Matthiessen-type rule arguments and simple relaxation-time approximation for the collision integral have been used.\(^9\) More recently the derivation of transverse spin dynamics in a spin-polarized Fermi liquid from Landau-Silin kinetic equation with general form of two-particle collision integral has been performed.\(^10\) The existence of zero-temperature damping of transverse spin waves has been established.

Experimentally the saturation of the transverse spin wave diffusion constant at temperatures about several millikelvin has been registered by spin-echo technique (see for instance\(^15\)). On the other hand the spin wave experiments demonstrate the behaviour characterized rather by the absence of transverse spin wave damping in the same temperature region.\(^16\) The latter seemed to be a confirmation of the point of view of I.Fomin who has argued for the dissipationless form of transverse spin wave spectrum being obtained from the correction to the systems energy due to the gauge transformation into the coordinate system where the magnetization vector is a constant. The same trick
has been used earlier for the treatment of one-particle and collective excitations dualism in the itinerant ferromagnets by R.Prange and co-workers\textsuperscript{18}, which is in our opinion still unresolved problem (see below). The calculation of the generalized susceptibility coefficient in the expression for the spin current found in\textsuperscript{17} has not been performed, just the reference on such the calculation\textsuperscript{19} in superfluid $^3$He had been given. Indeed, one can calculate susceptibility using the similar procedure. However in the case of polarized Fermi liquid one must use the Green functions with the finite imaginary self energies parts due to collisions between quasiparticles as it was done in\textsuperscript{7,8}, that inevitably leads to the spin waves attenuation.

I.Fomin also used an additional argument in support of absence of attenuation of transversal spin-waves in spin polarized Fermi liquid. This was an analogy with ferromagnetic Fermi liquid where was shown by P.Kondratenko\textsuperscript{12} that attenuation in the spin wave spectrum arises only in the terms of order $\sim k^4$. Indeed, it seems, that the space-time evolution of somehow artificially created magnetization in paramagnetic Fermi liquid in absence of spin-nonconserving interactions will be developed according the same laws as in isotropic itinerant ferromagnet. In reality this is not true. The conservation of magnetization does not accomplish the dissipationless dispersion of magnons. Even in the inhomogeneously rotating coordinate system, where the magnetization vector is a constant, the quasiparticle distribution function of paramagnetic Fermi liquid still is time and coordinate dependent matrix in the spin space.

The inhomogeneously rotating coordinate system, where the magnetization vector is a constant, the quasiparticle distribution function of paramagnetic Fermi liquid still is time and coordinate dependent matrix in the spin space containing odd in momentum off-diagonal part producing the spin current relaxation. In case of a ferromagnet the corresponding redistribution of the particles over the states with different momenta and spin up and down directions is prevented by the rigidity of many electron orbital wave function. The last property is not taken into account in the theory of polarized Fermi liquid and all the attempts to discuss the itinerant ferromagnets as sort of polarized Fermi liquid are incomplete.

The Landau-type derivation of transverse spin dynamics in a weakly spin-polarized Fermi-liquid from microscopic theory has been performed in the paper\textsuperscript{7}. Here we make a similar derivation with the purpose to stress the conditions it needs to be valid, to compare the answer with that had been obtained from kinetic equation at nonzero temperatures\textsuperscript{10}, and to juxtapose this derivation with derivation for ”ferromagnetic” Fermi-liquid\textsuperscript{13} which we also reproduce after.

As in original paper by Landau\textsuperscript{14} we shall consider here a system of fermions at $T = 0$, with arbitrary short range interaction forces. The presence of polarization means that the particle distribution functions for spin-up and spin-down particles have different Fermi momenta $p_{+}$ and $p_{-}$. The Green functions near $p = p_\pm$ and $\varepsilon = \mu_\pm$ have the form

$$ G_\pm(p, \varepsilon) = \frac{a}{\varepsilon - \mu_\pm - v_F(p - p_\pm) + ib(p - p_\pm)|p - p_\pm|}. \quad (5) $$

We shall assume a weak polarization $v_F(p_+ - p_-) \ll \varepsilon_F$ and also that both the Fermi distributions are characterized by the same Landau Fermi liquid parameters. Unlike\textsuperscript{20} we introduce here the general form of imaginary part of self-energy\textsuperscript{21} which is quadratic function of the difference $(p - p_\pm)$ and changes its sign at $p = p_\pm$ correspondingly. The assumption of small polarization in particular means that $G_+$ is given by expression (5) not only near its own Fermi surface $|p| = p_+$ and $\varepsilon = \mu_+$ but in whole intervals $p_- < p < p_+$ and $\mu_- < \varepsilon < \mu_+$ and also near the ”alien” Fermi surface $|p| = p_-$ and $\varepsilon = \mu_-$. The same is true for $G_-$. In general the polarization is nonequilibrium, hence $\mu_+ - \mu_- = \Omega - \omega_L$, where $\omega_L = \gamma H_0$ is the Larmor frequency corresponding to the external field $H_0$ and $\Omega$ is the Larmor frequency corresponding to the effective field which would produce the existing polarization , $v_F(p_+ - p_-) = \Omega/(1 + F_0^a)$.\textsuperscript{22}

Following Landau let us write equation for the vortex function for the scattering of two particles with opposite spin direction and a small transfer of 4-momentum $K = (k, \omega)$

$$ \Gamma(P_1, P_2, K) = \Gamma_1(P_1, P_2) - \frac{i}{(2\pi)^4} \int \Gamma_1(P_1, Q) G_+(Q)G_-(Q + K)\Gamma(Q, P_2, K) d^4Q. \quad (6) $$

If $K$ is small and polarization is also small, the poles of two Green functions are close each other. Let us assume that all other quantities in the integrand are slowly varying with respect to $Q$: their energy and momentum scales of variation are larger than max$\{\Omega, \omega\}$ and max$\{\Omega/v_F, k\}$ correspondingly. Then one can perform the integration in (6) at fixed values of $Q = p_0 = (p_+ + p_-)/2$, $\mu = (\mu_+ + \mu_-)/2$ in the arguments of $\Gamma$ and $\Gamma_1$ functions. Another words, one can substitute in (6)

$$ G_+(Q)G_-(Q + K) = G_+(q, \varepsilon)G_-(q + k, \varepsilon + \omega) $$

$$ = \frac{2\pi i\nu^2}{v_F^2} \delta(\varepsilon - \mu)\delta(|q| - p_0) \frac{\Omega}{\omega - \omega_L + \frac{\Omega^2 F_0 a^4}{1 + F_0^a} + \frac{4\pi^2}{v_F^2(1 + F_0^a)^2}} - k v_F + \frac{2\pi i\nu^2}{v_F^2(1 + F_0^a)^2} + \Phi_{\text{reg}}. \quad (7) $$
For eliminating \( \Gamma_1 \) from (6) we shall rewrite this equation in the operator form

\[
\Gamma = \Gamma_1 - i\Gamma_1(i\Phi + \Phi_{\text{reg}})\Gamma,
\]

where product is interpreted as integral, and \( i\Phi \) denotes the first term from rhs eq. (7). In equation (8), we transpose the term involving \( \Phi_{\text{reg}} \) to the left-hand side, and then apply the operator \( (1 + i\Gamma_1\Phi_{\text{reg}})^{-1} \), obtaining

\[
\Gamma = \Gamma^\omega + \Gamma^\omega \Phi \Gamma,
\]

where

\[
\Gamma^\omega = (1 + i\Gamma_1\Phi_{\text{reg}})^{-1}\Gamma_1.
\]

As it is known, \( \Gamma^\omega(\Omega = 0) \) is directly related to the function determining the Fermi liquid interaction,

\[
\Gamma^\omega(\Omega = 0) = \Gamma((|k|/\omega) \to 0, \Omega = 0) = \frac{F_{nn'}}{a^2N_0}.
\]

At finite \( \Omega \) the \( \Gamma^\omega \) function can be expanded over the polarization as

\[
a^2N_0\Gamma^\omega = F_{nn'} + \frac{i\hat{n}\Omega}{v_F^2(1+F_0^a)}C_{nn'} + O(\Omega^2).
\]

From eqns (9) and (12), we come, according to well known procedure, to kinetic equation

\[
\begin{align*}
\left(\omega - \omega_L + \frac{\Omega F_0^a}{1+F_0^a} + \frac{i\hat{n}\Omega}{v_F^2(1+F_0^a)} - \mathbf{k} \mathbf{n} v_F + \frac{i\hat{n}m_n \mathbf{n} \mathbf{v} \mathbf{F} \mathbf{F}^*}{v_F^2(1+F_0^a)}\right) & \nu(n) \\
& = \left(\frac{\Omega}{1+F_0^a} + \mathbf{k} \mathbf{n} v_F\right) \int \frac{d\nu'}{4\pi} \left(F_{nn'} + \frac{i\hat{n}\Omega}{v_F^2(1+F_0^a)}C_{nn'}\right) \nu(n').
\end{align*}
\]

We limit ourself by the first two harmonics in the Landau interaction function \( F_{nn'} = F_0^a + (\mathbf{n}'\mathbf{n})F_1^a \) and \( C_{nn'} = C_0 + (\mathbf{n}'\mathbf{n})C_1 \). To obtain the spectrum of the spin waves (see below) obeying the Larmor theorem: the system of the spins in a homogeneous magnetic field executes the precessional motion with the Larmor frequency \( \omega_L = \gamma H_0 \), the coefficient \( C_0 \) has to be chosen equal to unity.

Introducing the expansion of the distribution function \( \nu(n) \) over spherical harmonics of direction \( \mathbf{n} = \mathbf{v}_F/v_F \) one can find from (13) that the ratio of amplitudes of the successive harmonics with \( l \geq 1 \) is of the order of \( kv_F/\Omega \). Hence if it is assumed this ratio as a small parameter one can work with distribution function taken in the form

\[
\nu(n) = \nu_0 + (\mathbf{n}\mathbf{k})\nu_1.
\]

The functions \( \nu_0 \) and \( \nu_1 \) obey the following system of linear equations:

\[
\begin{align*}
(\omega - \omega_L)\nu_0 - \frac{kv_F}{3} \left(1 + \frac{F_1^a}{3} - \frac{ib(1-C_1/3)\Omega}{v_F^2(1+F_0^a)}\right) & \nu_1 = 0,
\end{align*}
\]

\[
-kv_F(1+F_0^a)\nu_0 + \left(\omega - \omega_L + \frac{\Omega(F_0^a - F_1^a)}{1+F_0^a} + \frac{ib(1-C_1/3)\Omega^2}{v_F^2(1+F_0^a)^2}\right) \nu_1 = 0.
\]

The equality to zero of the determinant of this system gives the spin waves dispersion law. At long enough wave lengths when the dispersive part of \( \omega(k) \) dependence is much less than \( \omega_L \) we have

\[
\omega = \omega_L + (D'' - iD')k^2,
\]

where

\[
D'' = \frac{v_F^2(1+F_0^a)(1+F_1^a/3)}{3\kappa\gamma H}
\]

is the reactive part of diffusion coefficient,
\( D' = \frac{b(1 - C_1/3)(1 + F_0 \alpha)^2}{3\kappa^2} \) \hspace{1cm} (18)

is dissipative part of diffusion coefficient, \( \kappa = F_0 \alpha - F_1 \alpha / 3 \) and \( H = \Omega / \gamma(1 + F_0 \alpha) \) is effective "internal" field corresponding to effective "external" field \( \Omega / \gamma \) producing the existing polarization. We derived eqns (17) and (18) in the assumption of \( \kappa \neq 0 \).

The expressions for \( D' \) and \( D'' \) have been obtained first by the same method by A.Meyerovich and Musaelyan\(^6\). The former is literally coincides with found in this paper, the latter has the same parametric dependence but depends in different way from Fermi liquid parameters. The reason for this is not clear at the moment. These expressions reproduce the corresponding diffusion constants have been obtained from phenomenological Landau-Silen kinetic equation with two-particle collision integral\(^{10}\) at arbitrary relation between polarization and temperature if we put in the latters \( T = 0 \). In particular \( D' \) proves to be polarization independent whereas \( D'' \) is inversely proportional to polarization.

Thus, the general microscopic derivation confirms the statement about the existence of zero-temperature spin waves attenuation in polarized Fermi liquid. The value of the dissipative part of spin diffusion \( D' \) is determined by the amplitude "b" of the imaginary part of self-energy. Hence it originates of collisions between quasiparticles.

### III. FERMI LIQUID APPROACH TO THE "FERROMAGNETIC" STATE

There are known several attempts to consider an isotropic itinerant ferromagnetic state as some peculiar type of Fermi liquid. This subject has been discussed first phenomenologically by A.A.Abrikosov and I.E.Dzyaloshinskii\(^11\) and after microscopically by P.S.Kondratenko\(^12\). They did not include in the theory a finite scattering rate between quasiparticles and as result they have obtained the dissipationless transverse spin wave dispersion law as it seemed to be in isotropic ferromagnet. The derivation\(^11\) has been critisized by C.Herring\(^24\) who pointed out on the existence of the finite scattering rate and inapplicability of naive Fermi-liquid approach to itinerant ferromagnet (see also discussion in\(^{10}\)). Later I.E.Dzyaloshinskii and P.S.Kondratenko\(^13\) have rederived the spin-wave dispersion law in ferromagnets. Making use as the starting point the Landau equation for the vertex function for the scattering of two particles with opposite spin direction and a small transfer of 4-momentum they have redefined the product of two Green functions \( G_+ G_- \) in such a manner that its resonant part was taken equal to zero at \( \omega = 0 \). This trick gives a possibility to use the \( 1/k^2 \) divergency of transverse static susceptibility which is an inherent property of degenerate systems and occurs both in an isotropic ferromagnet and in spin polarized paramagnetic Fermi-liquid. The latter of course is true in absense of interactions violating of total magnetization conservation. As in previous papers\(^{11,12}\) the authors of\(^13\) did not introduce a scattering rate in the momentum space between the Fermi surfaces for the particles with opposite spins.

Let us see now what kind modifications appeare if we reproduce the derivation proposed in\(^{13}\) with the Green functions (5) taking into account the finite quasiparticle scattering rate in whole interval \( p_- < p < p_+ \). We discuss first an isotropic ferromagnet at equilibrium \( \mu_+ = \mu_- \) in the absence of external field. Following\(^{13}\) we write:

\[
G_+(Q)G_-(Q + K) = G_+(q, \varepsilon)G_-(q + k, \varepsilon + \omega) = \frac{2\pi a^2 v_F \delta(\varepsilon - \mu) \delta(|q| - p_0)}{\omega - v_F \Delta + ib\Delta^2 - kv_F + \frac{ibkv_F \Delta}{v_F}} + \tilde{\Phi}_{\text{reg}}, \hspace{1cm} (19)
\]

where \( \Delta = p_+ - p_- \). Now the eqn (6) is written as

\[
\Gamma = \Gamma_1 - i\Gamma_1(i\tilde{\Phi} + \tilde{\Phi}_{\text{reg}})\Gamma, \hspace{1cm} (20)
\]

where \( i\tilde{\Phi} \) denotes the first term from rhs eq. (19). The equivalent form of this equation is

\[
\Gamma = \Gamma^k + \Gamma^k \tilde{\Phi} \Gamma, \hspace{1cm} (21)
\]

where

\[
\Gamma^k = \Gamma \left( \frac{\omega}{|k|} \to 0 \right) = (1 + i\Gamma_1\tilde{\Phi}_{\text{reg}})^{-1}\Gamma_1. \hspace{1cm} (22)
\]
The isotropic part of $\Gamma^k$ is proportional to the transversal susceptibility. Hence it has the singular form \(^{13}\)

$$
\Gamma^k \propto -\frac{1}{N_0(ck)^2}.
$$

(23)

Here, $c$ is constant with the dimensions of length. It is quite natural to take

$$
c \sim \frac{1}{\Delta}
$$

(24)

such that the divergency (23) disappears in nonpolarized liquid when $\Delta \to 0$. The authors of \(^{13}\) have lost this property by taking $c \sim p_0^{-1}$.

Substitution of eqn (23) to eqn (21) gives the transversal spin waves dispersion law

$$
\omega = v_F \Delta (ck)^2 (1 - \frac{ib\Delta}{v_F}),
$$

(25)

which proves to be attenuating similar to the polarized Fermi-liquid. One can take into consideration a static external field, by working in the rotating with Larmor frequency coordinate frame that is equivalent to the substitution $\omega \to \omega - \omega_L$ (see also \(^{13}\)). As result we obtain the law of dispersion

$$
\omega = \omega_L + v_F \Delta (ck)^2 (1 - \frac{ib\Delta}{v_F}),
$$

(26)

which obviously coincides with (16) after taking into account the relation (24).

The attenuating dispersion of transversal spin waves is not surprising because in both cases we deal in fact with spin polarized Fermi liquid but not with isotropic itinerant ferromagnet where the zero temperature attenuation is prohibited by Goldstone theorem. The Fermi liquid theory leading to the existence of such attenuation is not correct starting point for the construction of a theory of isotropic itinerant ferromagnetism.

IV. CONCLUSION

The inverse proportionality to polarization of reactive part diffusion coefficient is typical for the polarized Fermi liquid. It appears in all the derivations of spin waves dispersion law including Fomin’s \(^{17}\), Prange’s \(^{18}\) and in Stoner-Hubbard model for itinerant ferromagnetism (see for instance the book \(^{1}\), or the paper \(^{25}\) where the same results are obtained by more fashionable now method of functional integration over fermi fields). It has nothing to do with dispersion law for a ferromagnet which must be proportional to the magnetization as it follows from Landau-Lifshits equation (4) taking into account the domain wall rigidity. The latter is the inherent property of ferromagnet and absent in the paramagnetic polarized Fermi liquid. The domain wall rigidity in itinerant ferromagnet is formed because space time variations of momentum dependent off-diagonal, or spin part of quasiparticle distribution function are blocked up by the inevitable alternation of the orbital part of many particle electron wave function being accompanied by huge increase of interaction energy. It is not taken into account in Fermi liquid theory. From this point the famous Stoner-Hubbard model of ferromagnetism overlooks the most important property of a ferromagnet.... So, in our opinion the Fermi liquid theory is applicable to spin-polarized Fermi liquid but not to the itinerant ferromagnets.

In conclusion, we note that making use the quantum-field theoretical approach, one can derive the dispersion law for the transverse spin waves in a weakly polarized Fermi liquid at $T = 0$. Along with the dissipationless part inversely proportional to the polarization it contains also the finite zero-temperature damping. The polarization dependence both dissipative and reactive part of diffusion constants corresponds to dependences found earlier by means of kinetic equation with two-particle collision integral. The same dispersion law is derived by means of another approach where the divergency of the static transverse susceptibility is taken into consideration. These results obtained for the system of fermions with Fermi liquid type ground state are quite natural for spin polarized paramagnetic Fermi liquid. On the other hand in the isotropic itinerant ferromagnet one can expect the dissipationless spin wave spectrum with reactive diffusion constant proportional to magnetization. This demonstrates the troubles of the Fermi liquid formulation of theory of itinerant ferromagnetic systems which has to operate with an ordered type of ground state.
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1 T. Moriya "Spin fluctuations in itinerant electron magnetism", Springer-Verlag, Berlin, 1985.
2 L.D. Landau and E.M. Lifshits, *Statistical Physics Part I*, Butterworth-Heinemann, Oxford (1995).
3 F. Bloch, Z. Physik 57, 545 (1929).
4 L. Landau and E. Lifshits, *Zh. Sowjetunion* 8, 153 (1935).
5 W.J. Jeon and W.J. Mullin, Phys. Rev. Lett. 62, 2691 (1989); W.J. Mullin and W.J. Jeon, Journ. of Low Temp. Phys. 88, 433 (1992).
6 A.E. Meyerovich and K.A. Musaelian, Journ. of Low Temp. Phys. 89, 781 (1992); Phys. Rev. B 47, 2897 (1993); Journ. of Low Temp. Phys. 94, 249 (1994).
7 A.E. Meyerovich and K.A. Musaelian, Journ. of Low Temp. Phys. 89, 781 (1992).
8 D.I. Golosov and A.E. Ruckenstein, Phys. Rev. Lett. 74, 1613 (1995); Journ. of Low Temp. Phys. 112, 265 (1998).
9 A.E. Meyerovich and K.A. Musaelian, Phys. Rev. Lett. 72, 1710 (1994).
10 V.P. Mineev, Phys. Rev. B 69, 144429 (2004).
11 A.A. Abrikosov and I.E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. 35, 771 (1958) [Sov. Phys. JETP 8, 535 (1958)]. This paper is also reproduced as Appendix to the book by A.A. Abrikosov *Fundamentals of the theory of metals*, Elsevier Science Publisher; B.V. (1988).
12 P.S. Kondratenko, Zh. Eksp. Teor. Fiz. 46, 1438 (1964) [Sov. Phys. JETP 19, 972 (1964)].
13 I.E. Dzyaloshinskii, P.S. Kondratenko, Zh. Eksp. Teor. Fiz. 70, 1987 (1976) [Sov. Phys. JETP 43, 1036 (1976)].
14 L.D. Landau, Zh. Eksp. Teor. Fiz. 35, 97 (1958) [Sov. Phys. JETP 8, 70 (1959)].
15 H. Akimoto, D. Candela, J.X. Xia, W. J. Mullin, E.D. Adams and N.S. Sullivan, Phys. Rev. Lett. 90, 105301 (2003).
16 G. Vermeulen and A. Roni, Phys. Rev. Lett. 86, 248 (2001).
17 I.A. Fomin, Pis’ma Zh. Eksp. Teor. Fiz. 65, 717 (1997) [JETP Lett. 65, 749 (1997)].
18 V. Korenman, J.L. Murray, and R.E. Prange, Phys. Rev. B 16, 4032 (1977); ibid. p. 4048 ; ibid. p. 4058; R.E. Prange, V. Korenman, and V. Korenman, ibid. 19, 4691 (1979).
19 K. Maki, Phys. Rev. B 11, 4264 (1976).
20 I.A. Fomin "Spin Dynamics of a Spin-Polarized Fermi Liquid", in "Correlations, coherence and order", edited by D.V. Shopova and D.I. Uzunov (Plenum Press, London - New-York, 1999).
21 E.M. Lifshits and L.P. Pitaevskii, "Statistical Physics", Part 2 (Pergamon Press, Oxford, 1980).
22 A. Rodrigues, G. Vermeulen, Journ. Low Temp. Phys. 108, 103 (1997).
23 A.J. Leggett, J. Phys. C 3, 448 (1970).
24 C. Herring "Exchange Interactions among Itinerant Electrons" Chapter XIV, pp.345-385, in "Magnetism" v.IV, edited by G.T. Rado and H. Suhl, Academic Press, NY and London, 1966.
25 J. Fernandez-Rossier, M. Braun, A.S. Nunez, and A.H. MacDonald, Phys. Rev. B 69, 174412 (2004).