Internal Spatial Oscillations in a Single Trapped Bose–Einstein Condensate

Alexander V. Zhukov

Department of Theoretical Physics, Belgorod State University, 12 Studencheskaya str.,
308007 Belgorod, Russian Federation

(October 28, 2018)

Abstract

I predict the existence of internal spatial currents in a single macroscopic quantum system, namely in trapped dilute-gas at sufficiently low temperatures, when a Bose-Einstein condensation occurs. The spatial profiles of the wavefunctions of low-lying states in such a system are different due to the inhomogeneity, caused by an asymmetry of external trapping potential. This is the reason for appearing of Josephson–like oscillations between atomic subsystems in different states including the ground state as well. Using a simple model for the wavefunctions of three low-lying states we demonstrate how essential this effect can be. The possible applications of the predicted effect are briefly discussed. Particularly, this effect opens the possibility to identify experimentally the low lying excited states of a system.
The recent observations of Bose-Einstein condensation (BEC) in trapped atomic gases [1] have renewed interest in bosonic many-particle systems at low temperatures. The strong inhomogeneity of trapped gases in real experiments makes a system to be even more interesting.

Of course, the most spectacular consequences of Bose-Einstein condensation is superfluidity and high phase coherence [2]. The latter can lead to quite a number of interference phenomena, which were not being observed earlier in macroscopic systems in such a direct way. One of such interference effects was observed by the MIT group [3]. They used a laser beam to separate an atomic cloud into two parts. After switching off the confining potential and the laser, the authors of [3] observed clean interference patterns in the overlapping region.

The authors of [4] showed a possibility of another interesting manifestation of the phase coherence, namely they predicted the existence of Josephson-type effects in a double–well external potential. In this case a difference of chemical potentials, say $\mu_1$ and $\mu_2$, in two condensates leads to appearance of the Josephson current $I \propto \sin[(\mu_1 - \mu_2)/\hbar t]$. Analytic calculations of the spatial current distribution are rather complicated because of necessity to account the boundary conditions of condensates order parameters [2], [4]. Nevertheless, the Josephson oscillations are already observed in experiment [5] and are extensively studied by various groups [6] both theoretically and experimentally.

Here we propose a simple model theory for internal spatial oscillations, which occur in a single condensate. The origin of this effect is a difference of spatial profiles of wavefunctions of the ground state and neighboring excited states. So, the well–known quantum interference between two quantum states can lead to the observable spatial process in the systems under consideration.

It was realized long ago, that the ground state properties of trapped Bose-gas can be well described by the Gross–Pitaevskii equation [7]

$$i\hbar \frac{\partial \Phi(r, t)}{\partial t} = \left\{ -\frac{\hbar^2}{2m} \Delta + U_{\text{trap}}(r) + g|\Phi(r, t)|^2 \right\} \Phi(r, t), \quad (1)$$
where \( \Phi(r,t) = \langle \hat{\psi}(r,t) \rangle \) is the condensate wavefunction, \( U_{\text{trap}}(r) \) is the trapping potential,

\[
g = \frac{4\pi\hbar^2 a}{m}
\]
is the zero-range interaction constant, \( a \) is the \( s \)-wave scattering length. To obtain the ground state properties, one can write the condensate wavefunction as \( \Phi(r,t) = \varphi(r) \exp(-i\mu t/\hbar) \).

In fact, this means we assumed a condensate to be in the stationary state. In this case equation (1) takes the stationary form

\[
\mu \varphi(r) = \left\{ -\frac{\hbar^2}{2m} \Delta + U_{\text{trap}}(r) + g|\varphi(r)|^2 \right\} \varphi(r).
\]

This equation explains the sense of the chemical potential \( \mu \) as an energy of the stationary level. This can be used further for determining of \( \varphi(r) \) and \( \mu \).

In this Letter we want to study the ground state of trapped bosons in an asymmetric harmonic potential. For our aims we must allow for (at least) two single-particle states. Keeping in mind the illustrational purposes we shall consider the simplest case of stationary states, where the single–particle wavefunctions have the oscillator–like form

\[
\Phi_j(r,t) = \varphi_j(r) \exp\left(-\frac{i\mu_j t}{\hbar}\right),
\]

where

\[
\begin{align*}
\varphi_0(r) &= \frac{1}{\pi^{3/4}b^{3/2}} \exp\left(-\frac{r^2}{2b^2}\right), \\
\varphi_1(r) &= \frac{\sqrt{2}}{\pi^{3/4}b^{5/2}} z \exp\left(-\frac{r^2}{2b^2}\right),
\end{align*}
\]

\( b \) is a variational parameter. These have the same shape as the ground state and the first excited state with a zero projection of angular momentum. More generally, we could take \( \varphi_1(r) \) as a linear combination of the states with different angular momentum projections \( m_l = 0, \pm 1 \), but this has no influence on the qualitative picture. Furthermore, below we will operate in fact with the one dimension, say \( z \). This is particularly acquit to view of spatial anisotropy of trapping potential in majority of real experiments. The resulting wavefunction of our simple model system can be presented as a linear combination:
\[ \Phi^{(01)}(r, t) = \varphi_0(r) \exp \left(-i \frac{\mu_0}{\hbar} t \right) + \varphi_1(r) \exp \left(-i \frac{\mu_1}{\hbar} t \right). \]  

(6)

The current density is then given by the well–known relation [10]

\[ I^{(01)} = \frac{i \hbar}{2m} \left\{ \Phi^{(01)}(r, t) \nabla \Phi^{(01)*}(r, t) - \Phi^{(01)*}(r, t) \nabla \Phi^{(01)}(r, t) \right\}. \]  

(7)

Substituting eq. (6) into eq. (7), we come to

\[ I^{(01)} = -\frac{\hbar}{m} \varphi_0(r) \varphi_1(r) \frac{e_z}{z} \sin \left( \frac{\mu_1 - \mu_0}{\hbar} t \right), \]  

(8)

where \( e_z \) is the unit vector along \( z \)–direction. Equation (6) looks very similar to the corresponding expression in [4] for the Josephson current between two condensates, but have rather different sense. This simple example shows that there is an oscillating current which results in oscillations of particle number in each state. For example the temporal variation of the number of ground state particles due to a Josephson-type exchange with the first excited state is given by the formula

\[ \delta n^{(1)}_0(z, t) = \frac{2^{3/2} \hbar^2 z}{\pi^{3/2} mb^6 (\mu_1 - \mu_0)} \exp \left( -\frac{z^2}{b^2} \right) \cos \left( \frac{\mu_1 - \mu_0}{\hbar} t \right). \]  

(9)

Typical dependence of \( \delta n^{(1)}_0(z, t) \) on \( z \) and \( t \) is presented in Fig. 1.

Significance of the discussed effect is especially clearly manifests if we account the second excited state as well. Taking the simple oscillator-type wavefunction [11] for this second excited state, we easily get the following formula

\[ \delta n^{(2)}_0(z, t) = \frac{2^{5/2} \hbar^2 z(1 - z^2/b^2)}{\pi^{3/2} mb^6 (\mu_2 - \mu_0)} \exp \left( -\frac{z^2}{b^2} \right) \cos \left( \frac{\mu_2 - \mu_0}{\hbar} t \right), \]  

(10)

where \( \mu \) stands for the chemical potential of the second excited level. Fig. 2 shows the dependence of a total variation of the particle number in the ground state \( \delta n_0(z, t) = \delta n^{(1)}_0(z, t) + \delta n^{(2)}_0(z, t) \) on time (in units of \( \hbar / (\mu_1 - \mu_0) \)), calculated from expressions (9) and (10). The figure clearly illustrates possible applications of the predicted effect. Namely, if we measure the oscillations of the number of particles in the ground state (at any convenient coordinate), then we get a possibility to recover the low lying excited stationary states (i.e.
to find their chemical potentials, spreading, spatial shape etc.). This becomes even more tempting in view of a difference in spatial dependences of $\delta n_0^{(1)}$ and $\delta n_0^{(2)}$, which is shown on Fig. 3. Thus, all excited states become well distinguishable by measuring the amplitude and period of oscillations. Fig. 4 represents $\delta n_0$ as a function of both $z$ and $t$. So, the character of oscillations is rather different for each mode.

In conclusion, using a simple model for describing the ground and low-lying excited states of the trapped Bose-gas, we showed the principal possibility of the spatial oscillations within a system due to the quantum interference between states with different spatial profiles. These oscillations demonstrate rather different behaviour for different energy levels of the system. This fact provides a tempting possibility to determine the spectrum of stationary states in the system.

In reality the wavefunctions are of course different from those used above due to an interaction. Furthermore, we even do not certainly know whether or not the stationary excited states with well defined chemical potentials exist. The measuring of the considered internal oscillations just can answer this question.
REFERENCES

[1] M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Science 269, 198 (1995); K.B. Davis, M.-O. Mewes, M.R. Andrews, N.J. van Druten, D.S. Durfee, D.M. Kurn, W. Ketterle, Phys. Rev. Lett. 75, 3969 (1995); C.C. Bradley, C.A. Sackett, J.J. Tollet, R.G. Hulet, Phys. Rev. Lett. 75, 1687 (1995); C.C. Bradley, C.A. Sackett, R.G. Hulet, Phys. Rev. Lett. 78, 985 (1997)

[2] F. Dalfovo, S. Giorgini, L.P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 71, 463 (1999); L.P. Pitaevskii, Usp. Fiz. Nauk 168, 641 (1998)

[3] M.R. Andrews, C.G. Townsend, H.-J. Miesner, D.S. Durfee, D.M. Kurn, and W. Ketterle, Science 275, 637 (1997)

[4] F. Dalfovo, L. Pitaevskii, and S. Stringari, Phys. Rev. A 54, 4213 (1996); cond-mat/9604069

[5] J.I. Cirac et al., Phys. Rev. A 57, 1208 (1998); S. Raghavan, A. Smerzi, and V.M. Kenkre, ibid. 60, R1787 (1999); J. Williams et al., ibid. 59, R31 (1999); A. Sinatra and Y. Castin, Eur. Phys. J. D 8, 319 (2000)

[6] A.J. Leggett, Rev. Mod. Phys., to appear; D. Jaksch et al, cond-mat/0101057

[7] E.P. Gross, Nuovo Cimento 20, 454 (1961); L.P. Pitaevskii, Zh. Eksp. Teor. Fiz. 40, 646 (1961) [Sov. Phys. JETP 13, 451 (1961)]

[8] Ø. Elgarøy and C.J. Pethick, Phys. Rev. A 59, 1711 (1999)

[9] E.A. Cornell, J.R. Ensher, and C.E. Wieman, cond-mat/9903109

[10] L.D. Landau and E.M. Lifshitz, Quantum Mechanics (Pergamon Press, Oxford, 1965)
FIGURES

FIG. 1. The dependence of $\delta n_0^{(1)}$ on $z$ and $t$ (here $\tau = (\mu_1 - \mu_0)t/\hbar$). We used the constants $b = 5 \cdot 10^{-3}$ cm, $\mu_0 = 830\hbar$ erg, $\mu_1 = 1.3 \cdot 10^3\hbar$ erg. Note, the real amplitude of oscillations may be, of course, not so large. The parameters of wavefunctions (3) are taken rather approximately to draw the general picture.

FIG. 2. Temporal dependence of $\delta n_0^{(1)} = \delta n_0^{(1)} + \delta n_0^{(2)}$ at $z = b/2$ ($\tau = (\mu_1 - \mu_0)t/\hbar$). The constants are $b = 5 \cdot 10^{-3}$ cm, $\mu_0 = 830\hbar$ erg, $\mu_1 = 1.3 \cdot 10^3\hbar$ erg, $\mu_2 = 3.6 \cdot 10^3\hbar$ erg.

FIG. 3. Spatial dependence of $\delta n_0^{(1)}$ and $\delta n_0^{(2)}$ at $\tau = (\mu_1 - \mu_0)t/\hbar = 6$. All constants are the same as in Figs. 1 and 2.

FIG. 4. The dependence of $\delta n_0 = \delta n_0^{(1)} + \delta n_0^{(2)}$ on $z$ and $t$. All constants are the same as in Figs. 1 and 2.
