Dexterous holographic trapping of dark-seeking particles with Zernike holograms

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The intensity distribution of a holographically-projected optical trap can be tailored to the physical properties of the particles it is intended to trap. Dynamic optimization is especially desirable for manipulating dark-seeking particles that are repelled by conventional optical tweezers, and even more so when dark-seeking particles coexist in the same system as light-seeking particles. We address the need for dexterous manipulation of dark-seeking particles by introducing a class of “dark” traps created from the superposition of two out-of-phase Gaussian modes with different waist diameters. Interference in the difference-of-Gaussians (DoG) trap creates a dark central core that is completely surrounded by light and therefore can trap dark-seeking particles rigidly in three dimensions. DoG traps can be combined with conventional optical tweezers and other types of traps for use in heterogeneous samples. The ideal hologram for a DoG trap being purely real-valued, we introduce a general method based on the Zernike phase-contrast principle to project real-valued holograms with the phase-only diffractive optical elements used in standard holographic optical trapping systems. We demonstrate the capabilities of DoG traps (and Zernike holograms) through experimental studies on high-index, low-index and absorbing colloidal particles dispersed in fluid media.

I. INTRODUCTION

Holographic optical trapping uses the forces and torques exerted by computer-generated holograms to manipulate microscopic objects. Most of the literature of holographic trapping focuses on micromanipulation of dielectric particles with refractive indexes higher than the refractive index of the medium, $n_p > n_m$. Such high-index particles tend to be drawn toward regions of high light intensity, such as the focal point of strongly focused optical tweezers. Low-index particles, reflecting particles and particles that absorb light all tend to be repelled by bright light, and therefore are difficult to manipulate with standard optical traps. Successful two-dimensional manipulation of dark-seeking particles has been achieved in the dark regions of interference patterns [1] and the dark core of optical vortices [2, 3]. Full three-dimensional control has been demonstrated with cages of light created with rapidly scanned optical tweezers [4], and with optical bottles created from superpositions of Bessel beams [5, 7]. The bright cages that define these traps all have dark gaps through which trapped particles can escape. Proposals to close the gaps have focused on the properties of vector beams of light with non-trivial polarization structure [8, 9]. These vector traps, however, cannot be projected with standard holographic trapping systems.

Here, we report a class of optical traps for dark-seeking particles that is based on scalar diffraction theory and so is compatible with standard holographic trapping techniques. The ideal hologram encoding these traps is entirely real-valued, which poses a challenge for standard implementations that rely on phase-only spatial light modulators. We therefore introduce a method to transform amplitude-only holograms into phase-only holograms for convenient projection at high diffraction efficiency. The resulting dark tweezers can be combined with conventional bright tweezers in a standard holographic trapping system to enable manipulation of heterogeneous colloidal dispersions. We demonstrate the dark traps’ capabilities through experimental studies on model colloidal dispersions containing mixtures of low-index, high-index and absorbing spheres.

II. DARK OPTICAL TWEEZERS

Conventional optical tweezers are created by bringing a Gaussian laser beam to a diffraction-limited focus with a high-numerical-aperture (NA) lens. Analogous dark optical tweezers can be created by superposing two confocal Gaussian beams with equal amplitudes, different waist diameters and a relative phase of $\pi$ rad. The amplitude profile of such a superposition in the focal plane of the lens is

$$u(r) = u_0 \left[ \exp\left(-\frac{r^2}{4\alpha^2}\right) - \exp\left(-\frac{r^2}{4\beta^2}\right) \right],$$

where $\beta$ is the radius of the dark core and $\alpha$ is the radius of the enclosing region of light. As in the case of conventional optical tweezers, $\beta$ is constrained by the Abbe diffraction limit to $\beta \geq \lambda/2$ for light of wavelength $\lambda$ in the medium. The dark trap furthermore requires $\alpha > \beta$, with the difference ideally exceeding $\lambda/2$. Similar modes have been described previously [10] but do not appear to have been used to create optical traps.

Although the superposition described by Eq. (1) could be implemented with conventional optical elements, it is more conveniently projected with a holographic optical trapping system [11, 12] such as the example shown schematically in Fig. (1a). The standard implementation imprints a hologram on the wavefronts of a conventional laser using a phase-only spatial light modulator (SLM) and then relays the modified beam to an objective lens that focuses it into a sample. The ideal hologram for the difference-of-Gaussians trap described by Eq. (1) therefore may be computed as the Fourier transform of the...
FIG. 1: (a) Schematic representation of an integrated instrument for holographic optical trapping at 1064 nm and holographic video microscopy at 447 nm. The sample consists of a heterogeneous dispersion of low-index colloidal spheres, high-index spheres and particles that strongly absorb the trapping light. (b) Zernike phase hologram, $\phi_Z(r)$, encoding a difference-of-Gaussians dark optical trap. (c) Measured in-plane intensity, $I(r)$, at the axial position indicated by the dashed line in (d). The extent of the dark trap in the $(x,y)$ plane can be tuned by selecting the widths of the component Gaussian fields. (d) Reconstructed axial intensity profile, $I(r)$, projected by the hologram in (b). This slice in the $xz$ plane features a dark central focal volume completely surrounded by light.

field in the focal plane [13],

$$h(r) = \frac{1}{u_0} \int u(x) \exp \left( -i \frac{k}{f} r \cdot x \right) d^2x$$  (2a)

$$= \alpha^2 \exp \left( -\frac{k^2}{f^2} \alpha^2 r^2 \right) - \beta^2 \exp \left( -\frac{k^2}{f^2} \beta^2 r^2 \right),$$  (2b)

where $f$ is the focal length of the lens and $k = 2\pi n_m/\lambda$ is the wave number of light in a medium of refractive index $n_m$. Unfortunately, the hologram in Eq. (2b) is purely real-valued and so cannot be projected with a standard phase-only SLM. We therefore introduce an approach inspired by the Zernike phase-contrast technique to project real-valued holograms with phase-only diffractive optical elements.

### III. ZERNIKE HOLOGRAMS

A complex-valued hologram may be factored into real-valued amplitude and phase profiles,

$$h(r) = u(r) \exp(i\phi(r)),$$  (3)

both of which can be imprinted onto the wavefronts of a laser beam using suitable projection techniques [14] [15]. Most holographic trapping systems, however, rely on phase-only diffractive optical elements that only modify the phase profile. A common expedient is to ignore the amplitude profile by setting $u(r) = 1$, and to imprint only the phase profile, $\phi(r)$, onto the laser beam’s wavefronts [16] [18]. The resulting phase-only hologram,

$$H(r) = \exp(i\phi(r)),$$  (4)

is a superposition of the ideal hologram, $h(r)$, with an error field,

$$\Delta h(r) = \left[ 1 - u(r) \right] \exp(i\phi(r)).$$  (5)

The effect of $\Delta h(r)$ on the projected optical trapping pattern depends on the complexity and symmetries of the ideal pattern [18].

More complicated trapping patterns tend to be more amenable to simple phase-only projection. A hologram encoding multiple optical traps can be composed by superposing single-trap holograms,

$$h(r) = \sum_{n=1}^{N} \alpha_n h_n(r),$$  (6)

where $h_n(r)$ is the ideal hologram for the $n$-th trap and $\alpha_n$ is a complex coefficient setting the relative amplitude and phase of that trap. An individual trap within such a pattern can be translated by $\Delta r$ in three dimensions by adding a suitable parabolic profile to the phase of its hologram [17] [19],

$$\phi_{\Delta r}(r) = \left( -\frac{k}{f} r + \frac{k r^2}{2 f^2} \right) \cdot \Delta r.$$  (7)
As the complexity of the trapping pattern increases, the overall amplitude profile, $u(r)$, develops increasingly rapid spatial variations. The error term, $\Delta h(r)$, therefore tends to redirect light away from the intended trapping pattern and outward toward the edges of the instrument’s field of view. This means that the phase-only hologram, $H(r)$, can project a near-ideal rendition of the intended trapping pattern within a limited volume. In such cases, the error term principally reduces the hologram’s diffraction efficiency into the desired mode by redirecting light elsewhere.

While the simple phase-only conversion described by Eq. (4) is fast and effective, more sophisticated algorithms [20] can refine a phase-only hologram to improve diffraction efficiency and to mitigate artifacts. These refinements typically are too slow for real-time operation, however, and are reserved for the most exacting applications.

Purely real-valued holograms pose a particular challenge for phase-only projection. Sign changes in $h(r)$ can be absorbed into the phase profile with Euler’s theorem, thereby ensuring that the amplitude, $u(r)$, is non-negative [20]. The remaining amplitude variations can be approximated by multiplexing the desired hologram with another grating to deflect light [20]. These methods either discard most of the light in the field of view or discard most information about the amplitude profile. They typically create holograms with undesirably low diffraction efficiency.

We improve both the fidelity and the diffraction efficiency of both real- and complex-valued holograms by transferring more information about the amplitude profile into the phase profile using a variant of Zernike’s phase-contrast approximation,

$$u(r) \approx i \left[ 1 - \exp(iu(r)) \right], \quad (8)$$

under the assumption that $u(r) < 1$. Using the phase profile to encode amplitude information complements Zernike’s original purpose, which was to explain how phase variations contribute to observable contrast in microscope images. The phase profile implied by Eq. (8),

$$\phi_Z(r) = \frac{1}{2} u(r), \quad (9a)$$

serves as a phase-only approximation to the real-valued amplitude profile, yielding the phase-only Zernike approximation to the ideal complex-valued hologram,

$$H_Z(r) = \exp\left( i \left[ \phi_Z(r) + \phi(r) \right] \right). \quad (9b)$$

The associated error field,

$$\Delta h_Z(r) \approx \left[ 1 - \left(1 - \frac{i}{2}\right) u(r) \right] e^{i\phi(r)}, \quad (9c)$$

improves upon the standard result because its complex amplitude tends to cancel ghost traps and other projection artifacts.

Figure 1(b) shows the phase hologram encoding a dark trap that is obtained by applying Eq. (9) to the purely real-valued hologram for a DoG trap, Eq. (2b). Although it superficially resembles a standard Fresnel lens, the pattern of concentric phase rings has very different behavior. Figure 1(c) and 1(d) show volumetric reconstructions [21] of the dark trap projected by the hologram in Fig. 1(b). The trap is created by imprinting the hologram on the wavefronts of a TEM$_{50}$ laser beam at a vacuum wavelength of 1064 nm (fiber laser, IPG Photonics, YLR-LP-SF) using a liquid crystal spatial light modulator (Hololens PLUTO). The modified beam is relayed to the input pupil of an objective lens (Nikon Plan Apo, 100×, numerical aperture 1.4, oil immersion) that focuses the light into the intended optical trap with a focal length of $f = 180 \mu m$. The trapping beam is diverted into the objective lens with a dichroic beamsplitter (Semrock) that has a reflectivity of 99.5% at the trapping wavelength.

Images of the projected intensity pattern are obtained by mounting a front-surface mirror in the focal plane of the objective lens [21]. The reflected light is collected by the objective lens, and a small proportion passes through the dichroic mirror. This transmitted light is collected with a 200 mm tube lens and is recorded with a video camera (Flir Flea3, monochrome) with an effective system magnification of 0.048 µm/pixel. Transverse intensity slices, such as the example in Fig. 1(c), are obtained by translating the trap in steps of $\Delta z = 48 \mu m$ along the axial direction using Eq. (7). A stack of slices then is combined to obtain the axial section that is presented in Fig. 1(d). These images confirm that the Zernike hologram for a DoG trap successfully projects a beam of light that focuses to a dark volume surrounded by light on all sides. Planned and measured intensity profiles are compared in Fig. 2(a) for a trap with $\alpha = 0.96 \mu m$ and $\beta = \alpha/2$. This trap is used for experimental validation measurements in Section V.

IV. THEORETICAL PERFORMANCE
FIG. 2: (a) Measured intensity (discrete points) of a DoG trap with $\alpha = 0.96$ and $\beta = \alpha/2$ measured in the plane indicated by the horizontal line in Fig. 1(d). The dashed blue curve is the ideal intensity distribution, $|u_0(x)|^2$, for this trap. (b) Dashed curves show the potential energy computed with Eq. (10) for three different sizes of silica spheres dispersed in DMSO solution and localized in the trap from (a). Solid curves show the associated Hookean wells computed from the experimentally measured stiffness of the trap for each of the particles.

A dark-seeking particle at distance $r$ from the focus of a DoG trap has a potential energy that depends on its overlap with the light. We model this overlap as

$$U(r) = A \int_0^{2\pi} d\theta \int_0^{a_p} dx \, x \, \zeta(x) \, u^2(x-r)$$

$$= 4\pi a_p^3 u_0^2 A \left[ g(r,a_p,\alpha^2) + g(r,a_p,\beta^2) - 2g(r,a_p,\frac{2\alpha^2\beta^2}{\alpha^2 + \beta^2}) \right],$$

where $u(r)$ is the DoG trap field from Eq. [1] and $\zeta(x) = 2\sqrt{a_p^2 - x^2}$ is the chord length through a sphere of radius $a_p$ at a distance $x$ from its center. This solution depends on the overlap integral

$$g(r,a_p,\alpha^2) = e^{-\frac{r^2}{2\alpha^2}} \int_0^1 y \sqrt{1-y^2} I_0 \left( \frac{a_p r}{\alpha^2} y \right) e^{-\frac{a_p^2 y^2}{2\alpha^2}} dy,$$

whose integrand depends on the modified Bessel function, $I_0(\cdot)$. We treat the overall scale of the trapping potential, $A$, as an adjustable parameter. This constant accounts phenomenologically for the contrast in refractive index between the sphere and the medium as well as the effect of refraction within the sphere, details of which are beyond the scope of this model. Numerical solutions for the dark trap’s potential energy landscape are plotted in Fig. 2(b) for three different particle sizes in a trap with $\alpha = 0.96\mu m$ and $\beta = \alpha/2$.

As expected, DoG traps have a potential-energy minimum at $r = 0$. The transverse trapping stiffness therefore can be obtained from Eq. (10) as

$$\kappa_\perp(a_p,\alpha,\beta) \equiv \lim_{r \to 0} \frac{\partial^2 U}{\partial r^2}$$

$$= 2\pi a_p u_0^2 A \left[ f \left( \frac{a_p}{\alpha} \right) + f \left( \frac{a_p}{\beta} \right) - 2f \left( \frac{a_p}{\alpha \beta} \sqrt{\frac{\alpha^2 + \beta^2}{2}} \right) \right],$$

which depends on the Dawson integral, $F(\cdot)$, through

$$f(x) = 1 - \sqrt{2} \left( x + \frac{1}{x} \right) F \left( \frac{x}{\sqrt{2}} \right).$$

The associated trapping efficiency is

$$Q_\perp(a_p,\alpha,\beta) = \frac{1}{P} \kappa_\perp(a_p,\alpha,\beta),$$
where $P \propto n_0^2$ is the laser power delivered to the trap. For the particular choice of $\beta = \alpha/2$, the optimal value of the trap width is $\sigma^* = 0.952 \sigma_p$. The corresponding optimal efficiency, $Q(a_p, \alpha, \alpha/2) = 1.44 a_p A$, depends on the relative refractive index of the particle through the phenomenological scale factor, $A$. Predictions from Eq. (11) can be compared with the measured performance of DoG traps for low-index dielectric particles.

V. MEASURED PERFORMANCE

The performance of an optical trap can be assessed by tracking thermally-driven fluctuations in a trapped particle’s position [23]. We measure these fluctuations using the instrument’s holographic microscopy subsystem, as illustrated schematically in Fig. 1. In-line holograms of the particles are recorded by illuminating the sample with the collimated beam from a diode laser (Coherent Cube) operating at a vacuum wavelength of $\lambda = 447$ nm. Light scattered by a particle interferes with the rest of the beam in the focal plane of the microscope. The intensity of the magnified interference pattern is recorded by the video camera at 30 frames/s. The camera’s 10 μs exposure time is short enough to avoid motion blurring [23, 24]. A holographic snapshot can be analyzed with predictions of Lorenz-Mie theory to measure each particle’s radius, $r_p = (x_p, y_p, z_p)$, relative to the center of the microscope’s focal plane [25]. The typical precision for such holographic characterization and tracking measurements is $\sigma_{x_p} = \pm 2 \text{ nm}$, $\sigma_{y_p} = \pm 1 \times 10^{-3}$, $\sigma_{x_p} = \sigma_{y_p} = \pm 2 \text{ nm}$ and $\sigma_{z_p} = \pm 5 \text{ nm}$ [26]. We use these capabilities to differentiate high-index, low-index and absorbing particles and to track their movements in situ through holographic particle characterization [25].

Treating a sphere’s thermal fluctuations within the trap as an Ornstein-Uhlenbeck process, an estimate for the trap stiffness along the other three coordinates for each of the three particle sizes. Values for the transverse stiffness plotted in Fig. 3(a) reveal no anisotropy, and thus no dependence on the trap’s polarization, which is directed along $\hat{x}$. This is consistent with the performance of conventional optical tweezers for light-seeking particles that are larger than the wavelength of light [27, 25]. Also like conventional optical traps, the axial stiffness of DoG traps, plotted in Fig. 3(b), differs significantly from the transverse stiffness.

Figures 3(c) and 3(d) show how the transverse and axial trapping efficiencies, $Q_{\perp}(a_p, \alpha) = (\kappa_x(a_p, \alpha) + \kappa_y(a_p, \alpha))/2P$ and $Q_z(a_p, \alpha) = \kappa_z(a_p, \alpha)/P$, depend on particle radius, $a_p$. Interestingly, the measured transverse trapping efficiency, $Q_{\perp}(a_p, \alpha)$, depends non-monotonically on particle size. This observation is consistent with the prediction of Eq. (11), which is plotted as a blue dashed curve in Fig. 3(c), using only $A$ as an adjustable parameter. The model predicts that the DoG trap is optimally stiff when trapping low-index dielectric particles that are slightly larger than trap’s dark core, in agreement with experimental observations.

The complementary results in Figs. 3(e) and 3(f) show how the trapping efficiency depends on $\alpha$ for fixed particle size, $a_p = 1.15 \text{ μm}$. This result illustrates in more

measurement error, $\sigma_a \leq 5 \text{ nm}$, over the range of laser powers considered [19]. A 2000-frame holographic video recorded over one minute provides enough information to measure the trap stiffness, $\kappa_x$, to within 1% in each of the three dimensions.

Figure 3(a) shows how the measured stiffness of the DoG trap from Fig. 2(a) depends on laser power and particle radius for colloidal silica spheres dispersed in an aqueous solution of dimethylsulfoxide (DMSO, Acros Organics, CAS Number 67-68-5). Silica has a refractive index of $n_p = 1.42$. The DMSO solution has a refractive index of $n_m = 1.53$, as measured by an Abbe refractometer (Edmund Scientific, Model 52-975). The silica spheres therefore have a lower refractive index than their medium and cannot be trapped with conventional optical tweezers. The data in Fig. 3(a) and (b) were acquired for three particle sizes, $a_p = 0.505 \text{ μm}$ (Polysciences Inc., 24326), $1.15 \text{ μm}$ (Bangs Laboratories, SS04001) and $1.75 \text{ μm}$ (Bangs Laboratories, SS05001). The diameters and refractive indexes of these particles were confirmed in situ through holographic particle characterization [25].

The DoG trap used for these measurements has $\alpha = 0.96 \text{ μm}$ and $\beta = \alpha/2 = 0.48 \text{ μm}$, which most closely matches the optimal width for the particles with radius $a_p = 1.15 \text{ μm}$. The power, $P$, projected into the trap is measured with a slide-mounted thermal power sensor (Thorlabs model S175C). As might reasonably be expected, the trap stiffness increases linearly with $P$ in all three coordinates for each of the three particle sizes. Values for the transverse stiffness plotted in Fig. 3(a) reveal no anisotropy, and thus no dependence on the trap’s polarization, which is directed along $\hat{x}$. This is consistent with the performance of conventional optical tweezers for light-seeking particles that are larger than the wavelength of light [27, 25]. Also like conventional optical traps, the axial stiffness of DoG traps, plotted in Fig. 3(b), differs significantly from the transverse stiffness.

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The complementary results in Figs. 3(e) and 3(f) show how the trapping efficiency depends on $\alpha$ for fixed particle size, $a_p = 1.15 \text{ μm}$. This result illustrates in more
FIG. 3: (a) Transverse stiffness $\kappa_\perp(\alpha, a_p)$, of a DoG trap with $\alpha = 0.96 \, \mu m$ and $\beta = \alpha/2$ as a function of laser power. Results obtained from thermal fluctuations in the $(x, y)$ plane are plotted for silica spheres of radii $a_p = 0.5 \, \mu m$, $1.15 \, \mu m$ and $1.75 \, \mu m$. (b) Power dependence of the axial stiffness, $\kappa_z(\alpha, a_p)$. (c) Size dependence of the transverse trapping efficiency, $Q_\perp(\alpha, a_p)$, for fixed $\alpha$. (d) Size dependence of the axial trapping efficiency, $Q_z(\alpha, a_p)$. (e) Transverse trapping efficiency as a function of trap shape parameter, $\alpha$, for a silica sphere with $a_p = 1.15 \, \mu m$. The blue curves in (c) and (e) are a comparison with predictions of Eq. (11). (f) Axial trapping efficiency as a function of $\alpha$.

detail how the trapping efficiency of DoG traps is optimized when the trap is designed to fit the particle. This kind of optimization is facilitated by the combining holographic trapping with real-time holographic particle characterization. The peak trapping efficiency around 30 pN $\mu m^{-1} W^{-1}$ for a particle with size parameter $k a_p = 10$ and relative refractive index $n_p/n_m = 0.93$ is comparable to the computed efficiency of a conventional optical tweezer for a complementary high-index particle with relative refractive index $n_p/n_m = (0.93)^{-1} = 1.07$ [28].

VI. SIMULTANEOUS MANIPULATION OF DARK- AND LIGHT-SEEKING PARTICLES

Figure 4 presents holographically measured trajectories of high- and low-index particles being translated simultaneously in a conventional optical tweezer and a DoG trap, respectively. The figure also includes one frame from the holographic video (Supplementary Video 1) that was used to measure the particles' trajectories. The two types of particles are co-dispersed in an aqueous DMSO solution with a measured refractive index of $n_m = 1.53$. The low-index particle is a silica sphere with a radius of $a_p = 1.15 \, \mu m$, and a holographically measured refractive index of $n_p = 1.42$. The high-index particle is composed of polystyrene with a radius of $a_p = 0.5 \, \mu m$ (ThermoFisher Scientific, 5100B) and a refractive index, $n_p = 1.60$, that exceeds the index of the medium. The dispersion is contained in the volume created by sealing the edges of a glass coverslip to the face of a glass microscope slide. This chamber has an optical path length of $L = 25 \, \mu m$.

The discrete plot symbols in Fig. 4 represent the measured three-dimensional trajectories of a (low-index) silica sphere and a (high-index) polystyrene sphere as they are transported in opposite directions around nominally circular trajectories in the vertical $yz$ plane. The particles follow the programmed trap trajectories at 5.0 $\mu m s^{-1}$ given a laser power of 100 mW for the conventional optical trap and 50 mW for the DoG trap as estimated by imaging photometry [21]. Comparatively high power is needed to manipulate the polystyrene particle because it is nearly index matched to the medium. The traps’ relative intensities are adjusted by appropriately weighting each trap’s contribution to the superposition...
Although the two traps were programmed to move in circles of equal radius, the recorded trajectories are elliptical. The two particles move over equal horizontal ranges, but the polystyrene particle moves further than expected in the axial direction and the silica particle moves less far. Similar axial deviations have been reported for particles in conventional optical tweezers and can be ascribed to differences in the particles’ buoyant densities and to changes in the traps’ Rayleigh ranges as they are displaced along the optical axis. These deviations can be corrected through adjustments to the axial displacements in Eq. (7) using feedback from real-time holographic tracking.

VII. MANIPULATING ABSORBING PARTICLES

DoG traps also are useful for manipulating particles that absorb light strongly. Such particles are repelled by conventional optical tweezers through transfer of momentum from absorbed light. Absorption also mediates heating, which can propel the particles through self-thermophoresis, and in extreme cases can destroy the particles or boil the fluid medium. We have demonstrated the ability of DoG traps to stably trap and transport absorbing particles through experiments on composite particles made of hematite cubes embedded in dielectric spheres and superparamagnetic particles composed of hematite nanoparticles dispersed within dielectric spheres. The dielectric spheres used for these demonstrations are created by emulsion polymerization of 3-methacryloxypropyl trimethoxysilane (TPM) an organosilicate with a refractive index of $n_p = 1.495$. Hematite absorbs infrared light strongly, and both types of hematite-loaded particles tend to be ejected from conventional optical tweezers. Both types of particles are readily trapped and transported in three dimensions with DoG traps. Supplementary Video 2 shows the simultaneous three-dimensional control over a 1 µm-diameter polystyrene particle and a 1.8 µm TPM particle enclosing a 0.8 µm hematite cube.

VIII. DISCUSSION

This study introduces a class of “dark” optical traps that are created by superposing two Gaussian modes of different waist diameters and opposite phases. The hologram encoding these Difference-of-Gaussians (DoG) traps is purely real-valued. We therefore introduce a technique based on the Zernike phase-contrast approximation to project real-valued holograms with phase-only diffractive optical elements, such as the spatial light modulators commonly used in holographic optical trapping systems. We have demonstrated DoG traps’ ability to trap dark-seeking particles and to move them in three dimensions. We further have characterized the efficiency of DoG traps for localizing low-index particles and have explained the observed nonmonotonic dependence of the transverse trapping efficiency on particle size. When a DoG trap is optimally matched to the trapped particle, its trapping efficiency is comparable to that of conventional optical tweezers for light-seeking particles.

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DATA AVAILABILITY.

Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

The open-source software used to project holographic optical traps, record in-line holographic microscopy data and analyze those data is available online at [https://github.com/davidgrier/](https://github.com/davidgrier/).
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