Imbibition of liquids in fibrous porous media

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Abstract. The present paper proposes a theoretical and experimental approach to predict the imbibition process into a fibrous porous media where the displacement of air by the liquid is seen to be characterized by the presence of a clearly-defined liquid front. The imbibition process is defined as the absorption of liquid in a porous media, mainly due to surface tension. In the most imbibition applications, the Washburn equation is used to predict the rise of the liquid front as a function of time when the inertial and gravitational energies can be neglected. The objectives of this article are oriented to the modeling process of the imbibition through highly compressible porous layers, and model validation through experimental simulations which were done on different unwoven porous materials. It was shown that the imbibition predictions by the theoretical model are in agreement with the experimental results.

1. Introduction

The load capacity generated under compression of porous layers imbibed with liquids is based on an innovative mechanism of lubrication, called ex-poro-hydrodynamic (XPHD) lubrication [1], [2]. An essential problem in the modeling process of the squeeze with applications to successive external loads is the reimbibition capacity of porous material during decompressing.

Imbibition, which is the displacement of one fluid by a liquid in a porous matrix, is observable in many everyday processes, ranging from paper and textile treatment to oil recovery and groundwater hydrology [3]. For nearly a century [4], [5], imbibition in homogeneous porous media has been a topic of interest for scientists and engineers due to its inherent complexity and practical importance in many technological areas. Previous research on imbibition process can be divided into two groups: study of the relation between the imbibition rate and imbibition time and a study of the relation between the imbibition rate and the porous medium characteristics.

The relation between imbibition rate and time was studied by Lucas [6] and Washburn [7]. They concluded that the relation between the imbibition rate and the square root of time is linear. In the most imbibition applications, the Lucas-Washburn equation is used when the inertial and gravitational energies can be neglected. Szekely et al. [8] took inertial and gravity forces into account and developed a differential equation for the imbibition rate. Unfortunately, the resulting equation is a non-linear ordinary differential equation that cannot be solved analytically.

Study of the imbibition rate relationship with the porous medium characteristics, especially the fiber orientation, was also a major direction of previous investigations. Chwastiak [9], Scher [10] and Hodgson and Berg [11] studied the imbibition rate along fibers. The imbibition rate across the fibers was studied by Fowkes [12] and Williams et al. [13]. In recent years, You-Lo-Hsieh [14] studied liquid wetting and analyzed its relation to fiber morphology and chemistry.
Imbibition tests are used to evaluate the absorption efficiency and liquid transport in the porous materials. Miller [15] experimentally investigated the effect of gravity on upward imbibition. Russel and Mao [16] proposed an apparatus and a method to investigate the in-plane anisotropic liquid absorption in nonwoven fabrics. Jeong [17] investigated the slip boundary condition on a porous wall using Stokes approximation. Mao and Russel [18] proposed a theoretical approach to predict the liquid absorption in homogeneous 3-D nonwoven structures. They also [19] presented a theoretical analysis of fluid flow in a 2-D nonwoven structure by using Darcy’s law. Lockington and Parlange [20] presented a new approach to predict water absorption in porous materials based on sorptivity. Gane et al. [21] numerically and experimentally studied the absorption rate and volume dependency on the complexity of porous networks.

Fiber swelling is an important phenomenon that affects imbibition into a porous media, as it is one of the swelling mechanisms that occur during water-fiber interaction in all bio-fiber materials such as paper. Schuchardt and Berg [22] studied the swelling phenomenon experimentally and modified the Washburn equation for application in some swelling materials. This model, built on the assumption that pore radii in a swelling porous medium decrease linearly with time, compares better with the experimental data than the conventional Washburn model.

In this paper, the study of liquid absorption in highly compressible porous layer (HCPL) was analysed theoretically (analytical and numerical approach) and validated through convincing experimental imbibition tests. The experimental studies allowed measuring the liquid absorption rate into unwoven textile materials using an original device, which was designed and manufactured in the laboratory activities. These experimental results have shown a good correlation comparing with the theoretical model proposed by Lucas-Washburn equation.

2. Theoretical model (Lucas-Washburn equation)

The imbibition of liquid into porous media is often modeled as a fully saturated flow behind a clearly-defined fluid front.

In the first few attempts to model the imbibition of a liquid into a porous medium, the porous structure was treated as a bundle of capillary tubes, and the inertial and gravitational forces were neglected in the liquid motion modeling. The most used law which would allow the calculation of the absorption rate, known under the name of Lucas–Washburn law [6,7] considers the liquid flow behind the front through porous media like a one-dimensional imbibitional flow along the direction of capillary tubes.

The use of the Washburn equation implies that the complicated flow paths of liquids in a real porous medium are replaced by straight and hypothetical flow paths through cylindrical tubes as can be seen in figure 1.

To demonstrate Lucas-Washburn equation in steady regime were taken into account only the surface tension forces and the viscous friction forces, and the mass and the pressure forces were neglected.

![Figure 1. Capillary with concave meniscus.](image-url)
In the case of capillary, the surface tension forces, \( F_t \) acting tangent in each point of the meniscus have a component directed along the tube. The result of these components is directed outwards of the liquid for concave meniscus.

The relation that gives expression of surface tension force is:

\[
F_t = 2\pi R_p \gamma \cos \theta
\]

(1)

where \( R_p \) – capillary radius, \( \gamma \) – the surface tension of the liquid and \( \theta \) – the angle of contact.

However, the side (lateral) surface of the tube is carried by the internal friction forces which makes that a part of the fluid energy to be consumed through mechanical work done by friction. Viscous friction assessed at walls, \( F_f \), is directly proportional to the dynamic viscosity, \( \eta \), and to the side surface of the cylindrical capillary, \( S \):

\[
F_f = -\eta \frac{du}{dr} S
\]

(2)

The minus sign indicates that the friction force is opposite to the fluid flow.

From equation (2) it is noted that the dynamic viscosity, \( \eta \) can be considered as the friction force of a layer exerted on the other layer per unit area when velocity gradient modulus in the perpendicular direction to the surface is equal to unity.

As shown in figure 2 for \( \gamma = R_p \), equation (2) becomes:

\[
F_f |_{y=R_p} = -2\pi \eta R_p l \frac{2u_{max}}{R_p}
\]

(3)

*Figure 2. Parabolic velocity distribution along the capillary tube.*

On balance, when the fluid moves at a certain speed, we can write:

\[
\sum F_i = 0
\]

(4)

Substituting equations (1) and (3) in equation (4) gives:

\[
\eta l \frac{2u_{max}}{R_p} = \gamma \cos \theta
\]

(5)

It is noted that in a laminar flow through a horizontal tube of constant cross section, velocity is distributed as a paraboloid of revolution (figure 2). In this case, the mean velocity of the fluid, \( u_m \), can be expressed as:

\[
u_m = \frac{u_{max}}{2}
\]

(6)
Taking into account equation (6), and expressing mean velocity in function of the length of penetration, \( u_m = l/t \), we get the final form of Lucas-Washburn equation:

\[
 l = \sqrt{\frac{R_p \gamma \cos \theta}{4\eta}} t
\]

(7)

It can be observed that the length filled by the liquid is directly proportional to the square root of saturation time, \( t \). The quantity \( \gamma \cos \theta / 4\eta \) measures the penetrating power of a liquid and is called the coefficient of penetration or the penetrability of the liquid. Its dimensions are obviously those of velocity.

In the case of porous materials, using the "effective-section radius" concept (see Appendix) can be considered that \( R_p = R \). The effective-section radius, \( R \) may be expressed as:

\[
 R = \gamma \left( \frac{2\pi}{3\sqrt{3\sigma}} - 1 \right)
\]

(8)

where \( \gamma \) is fiber radius and \( \sigma \) is the compactness of the porous material.

3. Numerical approach

A numerical model based on finite differences was developed to model the imbibition of liquids into porous materials, where a sharp flow-front is clearly visible during the imbibition process, using the flow physics of single-phase flow in porous media for flow behind the front. A computer program called COMSOL Multiphysics based on the finite element/control volume (FE/CV) algorithm for the Darcy’s law based single-phase flow was used to model the 2-D imbibition flows in porous media of different porosities.

Since the liquid front is moving, so the FE/CV method is employed to track the flow surface. This method uses an Eulerian fixed mesh to track the flow front, and is highly efficient computationally as it avoids frequent remeshing of the flow domain as opposed to a Lagrangian algorithm.

The velocity normal to the boundary is zero except at the inlet, where a non-zero value is automatically satisfied by the finite element method (often referred to as the natural boundary condition).

The pressure boundary conditions implemented in our model are:

\[
 P = p_{atm} \quad \text{at} \quad x = 0
\]

(9)

and

\[
 P = (p_{atm} - p_s) + \rho gl \quad \text{at} \quad x = l
\]

(10)

where \( p_{atm} \) is atmospheric pressure and \( p_s = 2\gamma \cos \theta / R_p \) is the capillary suction-pressure.

In the proposed algorithm, the transient fluid flow in the porous material involving a moving-boundary is divided into multiple time steps. After assuming a quasi-steady condition during each time step, Laplace equation \( \nabla^2 P = 0 \) is first solved for pressure in the porous region that has been saturated by the moving liquid-front.

We then use the computed pressure field to estimate the velocity field, and then use the field to find the new location of liquid front at each time step:

\[
 u = -\frac{\phi}{\eta} \frac{dP}{dx}
\]

(11)

where \( \phi \) is permeability of the porous material.

We have defined a filling factor, \( f_l \) which is used to track saturation around any element, \( i \) and can be expressed as:

a) \( f_l = 0 \) for volumes beyond the liquid front in the dry region;

b) \( 0 < f_l < 1 \) for volumes near the liquid front that filled partially with the liquid;
c) \( f_i = 1 \) for volumes behind the liquid front that are filled with the liquid.

The time increment of each time step for advancing the flow front is determined by the shortest time to completely fill one control volume expressed as:

\[
\Delta t = \min \left[ \frac{eV_i(1-f_i)}{Q_i} \right]
\]  

(12)

where \( Q_i \) is the flow rate related to control volume \( V_i \).

After each time-step, \( t_n \), we have to update the filling factor before going into next time-step, \( t_{n+1} \); the following expression is used to modify the filling factor for each time increment:

\[
f_i^{t_{n+1}} = f_i^{t_n} + \frac{Q_i}{eV_i} \Delta t
\]  

(13)

For a concise overview about numerical modeling of the imbibition process, the following hypotheses are accepted:

a. The stress exerted by the non-wetting gas phase on the gas-liquid interface is negligible.

b. The imbibition flow is horizontal enough that the gravity effects can be neglected.

c. The liquid is an incompressible and Newtonian fluid (water) and the two-dimensional flow is sufficient in the modeling of imbibition flow in the present paper.

Numerical simulations for imbibition modeling were made considering three different initial porosities: \( \varepsilon_1 = 0.90 \) for the first case; \( \varepsilon_2 = 0.93 \) for the second case, respectively \( \varepsilon_3 = 0.78 \) for the last.

Figure 3 shows the liquid-front positions as a function of time for the considered cases as predicted by Comsol.

![Surface: Liquid front location (cm) versus imbibition time (s)](image)

Figure 3. Liquid front location vs. imbibition time.

Initially, it can be seen that the imbibition velocities in each case are almost identical. During imbibition, interface advancement beyond the average front position is slowed down while parts of the interface lagging behind are drawn forward.

Note that at \( t = 200 \) s from the beginning of the process, meniscus position is more advanced in the first material. According to figure 3, the highest imbibition rate is in the first case \( (t_1 \approx 255 \) s), while the lowest is in the last \( (t_3 \approx 370 \) s).
Figure 4 plots the evolution of the liquid-front length till the full saturation. Note that the slope of these plots is equal to the speed of the liquid front. Since the imbibition rate (or absorption speed) is inversely proportional to the measured times, so the plot is also indicative of the relative absorption rates in porous media considered.

It is clear that the liquid-front has travelled the farthest in the first case. For the other two cases, the speeds of liquid fronts are almost identical beyond $t = 50 \text{s}$; hence the curves are almost parallel beyond this point.

4. Experimental studies

The experiments carried out in this paper were done on porous unwoven materials characterized by different porosities, $\varepsilon$, with open pores (communicating) – V1, V2, V3. The material V1 ($\varepsilon = 0.9$) contains cellulose fibers and cotton, and V2 ($\varepsilon = 0.93$) and V3 ($\varepsilon = 0.78$) are composed of synthetic microfibers.

Another characteristic of the porous materials is the compactness, denoted by $\sigma$, defined as the complement of porosity:

$$\sigma = 1 - \varepsilon$$

The characteristics measured for porous materials are presented in table 1.

| Material | Mean fiber radius $r_f$ | Initial porosity $\varepsilon$ | Initial compactness $\sigma$ |
|----------|------------------------|-------------------------------|-----------------------------|
| V1       | 11 $\mu$m              | 0.9                           | 0.1                         |
| V2       | 6.5 $\mu$m             | 0.93                          | 0.07                        |
| V3       | 6 $\mu$m               | 0.78                          | 0.22                        |

The experimental studies allowed measuring the liquid imbibition rate into textile materials using an original device [23], which was designed and manufactured in the laboratory activities. The tests consisted in measuring of the time at which a liquid (water) from a tray penetrates the porous material fixed on a horizontal plexiglass plate, thus the effect of gravity is negligible. During experimental activities were measured the values of two parameters: the imbibition time and the meniscus position (or liquid front), the end of the test being when the porous material is totally saturated.
Figure 6 plots the evolution of the dimensionless liquid-front length for porous materials V1, V2 and V3. The experimental results are compared with the predictions from the Washburn equation using the "effective-section radius" concept.

![Figure 5](image1.png)

**Figure 5.** Length of the liquid-front in function of the imbibition time in the materials V1, V2, respectively V3.

As can be seen in figure 5, the theoretical model approximated very well the results obtained. A comparison with Washburn model for material V1 indicates a very good prediction. Note that from the beginning of the imbibition, liquid-front velocity for material V1 is higher compared to V2 and V3, and according to these plots the time till of full saturation will be lower ($t \approx 216$ s).

In figure 6 one can observe that the porous materials V2 and V3 have a similar behavior in terms of imbibition process, even at $t = 20$ s the length of penetration into the material V3 is about 42% higher than in V2.

During experimental activities, it was observed a little bulging of the front due to an enlargement in the cross-sectional area. It implies that the pores in the middle regions face higher flow-rates as compared to the pores in the border regions, and hence the liquid front is a little higher in the middle. Also, in the case of material V1 which is formed from a network of cellulose and superabsorbent fibers, experiments put in evidence the swelling effect that changes the structure and molecular arrangement of material.

The experimental study was done using new materials, and also under the conditions of one cycle (dry-imbibed-drained-dry-reimbibed), respectively two cycles.
In this regard, figure 6 proves clearly that material V1 behaves differently compared to the other two.

![Graph](image)

**Figure 6.** The comparison imbibition-reimbibition till the time of full saturation in the materials V1, V2, respectively V3.

After drying process of the porous material V1 that contains cellulose appears its reinforcing tendency. This observation indicates that a domain of "capillary bridges" existence can be defined, structures formed by a small liquid volume adhering to the surface of the pores or absorbed in the fibres. For V2 and V3, the reinforcing tendency is much lower than in V1. So that, the rigidity of a dry porous material has a major influence on the liquid reimbusition process.

5. **Concluding remarks**

The imbibition of a liquid by a porous medium due to the capillary suction pressure, is the focus of the present investigation. In most porous materials, pore space is highly interconnected, resulting in a continuous liquid-gas interface, whose advancement is correlated due to an effective surface tension.

Analytical solution given by the Lucas-Washburn equation for the imbibition of liquid into a porous media was presented in this paper. Also, a 2-D numerical model based on finite differences was developed with the software COMSOL and which approximated very well the results given by the conventional Washburn model.

The experimental results obtained and presented in this paper are repeatable and validate the theoretical model developed previously, adapted using the "effective-section radius" concept.
It was observed experimentally a swelling effect of the interaction between water and a porous media such as cellulose fibres which can lead to errors. Also, it was remarked a reinforcing tendency in the case of material formed from a network of cellulose.

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6. Appendix A

In the Washburn equation presented in Section 2, $R_p$ represents "effective-section radius" for constitutive "equilateral triangle" cells (figure A1), that describe better the structure of the porous unwoven material [24].

![Figure A1. The sketch of constitutive "equilateral triangle" cell.](image)

In the case of the "equilateral triangle" arrangement of fibres, the dimensionless "effective-section radius" (in relation to the fibre radius, $r_f$) and the compactness of the porous structure as a function of dimensionless distance between fibres, $\bar{a}$ are calculated as follows:

- **effective-section radius:**
  \[
  \bar{R} = \frac{(2+\bar{a})}{\sqrt{3}} - 1
  \]  
  \[\text{(A.1)}\]

- **compactness:**
  \[
  \sigma = \frac{2\pi}{(2+\bar{a})^2 \sqrt{3}}
  \]  
  \[\text{(A.2)}\]

where $\bar{R} = \frac{R}{r_f}$ and $\bar{a} = \frac{a}{r_f}$.

For the case of the constitutive "square" cells, analytical expressions become:

- **effective-section radius:**
  \[
  \bar{R} = (1 + 0,5\bar{a}) \sqrt{2} - 1
  \]  
  \[\text{(A.3)}\]

- **compactness:**
  \[
  \sigma = \frac{\pi}{(2+\bar{a})^2}
  \]  
  \[\text{(A.4)}\]
Nomenclature

**Latin letters**

| Symbol | Description |
|--------|-------------|
| $a$    | distance between fibers [m]; |
| $\bar{a}$ | dimensionless distance between fibers, $\bar{a} = \frac{a}{r_f}$ [-]; |
| $f$    | filling factor [-]; |
| $F_f$  | friction force [N]; |
| $F_t$  | acceleration tension force [N]; |
| $g$    | length of imbibition [m]; |
| $l$    | pressure [Pa]; |
| $p$    | capillary suction-pressure $p_s = 2\gamma \cos \theta / R_p$ [Pa]; |
| $P$    | modified pressure, $P = (p_{atm} - p_s) + \rho gl$ [Pa]; |
| $Q$    | flow rate [m$^3$/s]; |
| $r$    | radial coordinate [m]; |
| $R$    | "effective-section" radius [m]; |
| $\bar{R}$ | dimensionless "effective-section" radius, $\bar{R} = \frac{R}{r_f}$ [-]; |
| $R_p$  | capillary radius [m]; |
| $S$    | side surface of the cylindrical capillary [m$^2$]; |
| $t$    | time [s]; |
| $u$    | velocity of the fluid [m/s]; |
| $V$    | control volume [m$^3$]; |
| $x, y$ | axial coordinate [m]; |

**Greek letters**

| Symbol | Description |
|--------|-------------|
| $\gamma$ | surface tension of the liquid [N/m]; |
| $\varepsilon$ | porosity [-]; |
| $\eta$ | viscosity of the liquid [Pa·s]; |
| $\theta$ | angle of contact [degree]; |
| $\rho$ | density of the liquid [kg/m$^3$]; |
| $\sigma$ | compactness [-]; |
| $\phi$ | permeability [m$^2$]; |

**Subscripts**

| Symbol | Description |
|--------|-------------|
| atm    | atmospheric; |
| $f$    | regarding fibers; |
| $i$    | corresponding to the solid particle (element $i$); |
| $m$    | mean value; |
| $min$  | minimum value; |
| $max$  | maximum value; |
| $s$    | suction; |
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