Dissipative creation of three-dimensional entangled state in optical cavity via spontaneous emission

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We present a dissipative protocol to engineer two 87Rb atoms into a form of three-dimensional entangled state via spontaneous emission. The combination of coupling between ground states via microwave fields and dissipation induced by spontaneous emission make the current scheme deterministic and a stationary entangled state can always be achieved without state initialization. Moreover, this scheme can be straightforwardly generalized to preparation of an N-dimensional entangled state in principle.

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I. INTRODUCTION

For an open quantum system, the dissipation process must be accompanied by entanglement generation, i.e. the populations of quantum states are altered due to entanglement with an external environment. Thus researchers are dedicating themselves to find efficient ways avoiding decoherence during quantum information process. Currently, the feasible methods include an active error-correction approach based on the assumption that the most probable errors occur independently to a few qubits, which can be corrected via subsequent quantum operation [1–5], and alternative passive error-prevention scheme, where the logical qubits are encoded into subspaces which do not decohere because of symmetry [6–10]. Recently, the function of dissipation is reexamined in Ref. [11–19], where the environment along can be used as a resource to preparing entanglement and implementing universal quantum computing. In particular, Kastoryano et al. consider a dissipative scheme for preparing a maximally entangled state of two Λ-atoms in a high finesse optical cavity [14], in which a pure steady singlet state is achieved with no need of state initialization.

Compared with other kinds of entanglement, high-dimensional entangled states have attracted much interest, since it can enhance the violations of local realism and the security of quantum cryptography. In the fields of linear optics, two experiments utilize the spatial modes of the electromagnetic field carrying orbital angular momentum to create high-dimensional entanglement. In the context of cavity quantum electrodynamics (QED), three-dimensional entanglement has also been realized in the unitary evolutionary dynamics based on resonant, and dispersive atom-cavity interactions [20–25]. In this paper, we put forward a dissipative method for preparing a stationary three-dimensional entangled state. The motivation of our proposal is mainly based on the following truth: The typical decoherence factors in cavity QED system consist of atomic spontaneous emission and cavity decay, which have detrimental effects on schemes based on unitary dynamics. However, the loss of cavity can used to stabilize a pure maximally entangled state when a suitable feedback control is applied [26–29]. Thus the spontaneous emission of atom becomes the only one detrimental factor. The result of our work shows that atomic spontaneous emission is able to be a useful resource in respect of entanglement preparation, especially the fidelity of target state can even be better than the unitary evolution based schemes.

The structure of the manuscript is as follows. We derive the Lindblad master equation for preparation of three-dimensional entangled state with effective operator method in section II. We then generalize the scheme to realization of an N-dimensional entangled state via introducing multi-level atoms and multi-mode cavity and discuss the effect of cavity decay on the fidelity in section III. This paper ends up with a conclusion in section IV.
II. EFFECTIVE MASTER EQUATION FOR OPEN QUANTUM SYSTEMS

We take into account a system composed by two $^{87}$Rb atoms trapped in a bi-mode optical field, as shown in Fig. 1. The quantum states $|g_L\rangle$, $|g_0\rangle$, $|g_R\rangle$, and $|g_R\rangle$ correspond to atomic levels $|F = 1, m_f = -1\rangle$, $|F = 1, m_f = 0\rangle$, $|F = 1, m_f = 1\rangle$, and $|F = 2, m_f = 0\rangle$ of $5S_{1/2}$, and $|e_L\rangle$, $|e_0\rangle$, $|e_R\rangle$ correspond to $|F = 1, m_f = -1\rangle$, $|F = 1, m_f = 0\rangle$, and $|F = 2, m_f = 0\rangle$ of $5P_{3/2}$. Without loss of generality, we apply two off-resonance $\pi$-polarized optical lasers, with Rabi frequencies $\Omega_L$ for atoms and cavity modes. The Hamiltonian of the system can be written as

$$\hat{H} = \hat{H}_0 + \hat{H}_g + \hat{V}_+ + \hat{V}_-,$$

where $\hat{H}_0$ characterizes the strong interaction between atoms and quantized cavity fields, and $\hat{H}_g$ and $\hat{V}_\pm$ correspond to the weakly driven fields of microwave and optical lasers, respectively. For simplicity, we set $g_L = g_R = g$, $\Omega_1 = \Omega_2 = \Omega$, and $\omega_1 = -\omega_2 = \omega$ in the following. To gain a better insight into the effect of spontaneous emission on the preparation of entanglement state, we first consider a perfect cavity without decay. According to the effective operator method [30], the excited states of the atoms and the cavity field modes can be adiabatically eliminated, provided that the Rabi frequency $\Omega$ of the optical pumping laser is sufficiently weak enough compared with $g$, $\delta$ and $\Delta$, and the excited states are not initially populated. Then we obtain the effective master equation as

$$\dot{\rho} = i[\hat{H}, \rho] + \sum_j \hat{L}_{eff,j} \rho \hat{L}_{eff,j}^\dagger - \frac{1}{2}(\hat{L}_{eff,j}^\dagger \hat{L}_{eff,j} \rho + \rho \hat{L}_{eff,j}^\dagger \hat{L}_{eff,j}),$$

where

$$\hat{H}_{eff} = \sum_{j} \hat{L}_{eff,j}^\dagger \hat{L}_{eff,j} = \hat{L}_j \hat{H}_{NH}^{-1} \hat{V}_+ + \hat{V}_-(\hat{H}_{NH}^{-1})^\dagger \hat{V}_+ + \hat{H}_g,$$

and

$$\hat{H}_{NH}^{-1} = \frac{\delta/2 - 3\delta\Delta'}{9g^2\Delta' - 3\delta\Delta'^2} |X_1\rangle\langle X_1| + \frac{9}{9} \left[ \frac{\delta}{\Delta' - 3g^2 - \delta\Delta'} |X_2\rangle\langle X_2| - \frac{1}{9} \frac{\delta\Delta'}{\Delta' - 3g^2 - \delta\Delta'} |X_3\rangle\langle X_3| + \Delta' |\rangle\langle \rangle - \langle|\rangle\langle|\rangle \right] - \frac{2\sqrt{2}g^2}{9g^2\Delta' - 3\delta\Delta'^2} |X_2\rangle\langle X_1| + \frac{2\sqrt{2}g^2}{\sqrt{3}(3g^2 - \delta\Delta')} \sum_j |X_3| - \frac{g}{\sqrt{3}(3g^2 - \delta\Delta')} |X_2| - \frac{g}{\sqrt{3}(3g^2 - \delta\Delta')} |X_3| + \text{H.c.},$$

where $\hat{H}_0$ characterizes the strong interaction between atoms and quantized cavity fields, and $\hat{H}_g$ and $\hat{V}_\pm$ correspond to the weakly driven fields of microwave and optical lasers, respectively. For simplicity, we set $g_L = g_R = g$, $\Omega_1 = \Omega_2 = \Omega$, and $\omega_1 = -\omega_2 = \omega$ in the following. To gain a better insight into the effect of spontaneous emission on the preparation of entanglement state, we first consider a perfect cavity without decay. According to the effective operator method [30], the excited states of the atoms and the cavity field modes can be adiabatically eliminated, provided that the Rabi frequency $\Omega$ of the optical pumping laser is sufficiently weak enough compared with $g$, $\delta$ and $\Delta$, and the excited states are not initially populated. Then we obtain the effective master equation as

$$\dot{\rho} = i[\hat{H}, \rho] + \sum_j \hat{L}_{eff,j} \rho \hat{L}_{eff,j}^\dagger - \frac{1}{2}(\hat{L}_{eff,j}^\dagger \hat{L}_{eff,j} \rho + \rho \hat{L}_{eff,j}^\dagger \hat{L}_{eff,j}),$$

where

$$\hat{H}_{eff} = \sum_{j} \hat{L}_{eff,j}^\dagger \hat{L}_{eff,j} = \hat{L}_j \hat{H}_{NH}^{-1} \hat{V}_+ + \hat{V}_-(\hat{H}_{NH}^{-1})^\dagger \hat{V}_+ + \hat{H}_g,$$

and

$$\hat{H}_{NH}^{-1} = \frac{\delta}{2g^2 - \delta\Delta'} (|e_0g_L\rangle\langle e_0g_L| + |e_0g_R\rangle\langle e_0g_R|) + \frac{1}{2} \frac{g^2}{2g^2 - \delta\Delta'^2} (|g_Lg_L\rangle\langle g_Lg_L| + |g_Rg_R\rangle\langle g_Rg_R|) + \frac{g}{2g^2 - \delta\Delta'^2} |g_Lg_L\rangle\langle g_Lg_L| + \frac{g}{2g^2 - \delta\Delta'^2} |g_Rg_R\rangle\langle g_Rg_R| + \text{H.c.}. $$

In the above expression, $\hat{H}_{NH} = \hat{H}_0 - \frac{1}{2} \sum_j \hat{L}_{eff,j}^\dagger \hat{L}_{eff,j}$ is a non-Hermitian Hamiltonian, and its inverted matrix can be written as $\hat{H}_{NH}^{-1} = \hat{H}_{NH1}^{-1} + \hat{H}_{NH2}^{-1} + \hat{H}_{NH3}^{-1}$, explicitly

$$\hat{H}_{NH1}^{-1} = \frac{9g^2\Delta' - 3\delta\Delta'^2}{\delta/2 - 3\delta\Delta'},$$

$$\hat{H}_{NH2}^{-1} = \frac{\delta}{2g^2 - \delta\Delta'} (|e_0g_L\rangle\langle e_0g_L| + |e_0g_R\rangle\langle e_0g_R|) + \frac{1}{2} \frac{g^2}{2g^2 - \delta\Delta'^2} (|g_Lg_L\rangle\langle g_Lg_L| + |g_Rg_R\rangle\langle g_Rg_R|) + \frac{g}{2g^2 - \delta\Delta'^2} |g_Lg_L\rangle\langle g_Lg_L| + \frac{g}{2g^2 - \delta\Delta'^2} |g_Rg_R\rangle\langle g_Rg_R| + \text{H.c.}. $$
\[ \hat{H}_{\text{eff}} = -\frac{\delta}{g^2 - \delta^2} \left( |g_{a_L}e_L\rangle \langle g_{a_L}| + |g_{a_R}e_R\rangle \langle g_{a_R}| + g_{L_e}e_L\rangle \langle g_{L_e}| + g_{R_e}e_R\rangle \langle g_{R_e}| \right) + \frac{\Delta}{g^2 - \delta^2} \left( |g_{a_L}g_{o}\rangle \langle 1_R| + |g_{a_R}g_{o}\rangle \langle 1_R| \right) \]

\[ + g_{a_L} |1_L\rangle \langle g_{a_L}| + |g_{a_R}g_{L}\rangle \langle 1_R| \right) + g_{a_R} |1_L\rangle \langle g_{a_R}| + |g_{R_o}g_{L}\rangle \langle 1_R| \right) + g_{R_o} |1_L\rangle \langle g_{R_o}| + H.c., \]

\[ \text{where } \Delta' = \Delta - \frac{\delta^2}{2} \text{ and the vacuum states of cavity modes are discarded and we have adopted the notation } |X_1\rangle = \frac{1}{\sqrt{3}}(|g_{L_e}e_L\rangle + |g_{R_e}e_L\rangle + |e_0g_{a}\rangle), \]

\[ |X_2\rangle = \frac{1}{\sqrt{3}}(|g_{L_e}e_L\rangle + |g_{R_e}e_L\rangle - 2|e_0g_{a}\rangle, \]

\[ |X_3\rangle = \frac{1}{\sqrt{2}}(|g_{L_e}e_L\rangle - |g_{R_e}e_L\rangle), \]

\[ |+\rangle = \frac{1}{\sqrt{2}}(|g_{L_e}g_{L}\rangle + |g_{R_o}g_{R}\rangle), \]

\[ |-\rangle = \frac{1}{\sqrt{2}}(|g_{L_e}g_{L}\rangle - |g_{R_o}g_{R}\rangle). \]

On the basis of Eq. (7), we have the effective Hamiltonian as

\[ \hat{H}_{\text{eff}} = \Omega^2 \text{Re} \left[ \frac{\delta}{g^2 - \delta^2} \right] (|g_{a_L}g_{a_L}\rangle \langle g_{a_L}| + |g_{a_R}g_{R}\rangle \langle g_{a_R}| + |g_{R_o}g_{R}\rangle \langle g_{R_o}| + |2\rangle \langle T_3| \right) \]

\[ -\Omega^2 \text{Re} \left[ \frac{g^2 - 3\delta}{9g^2\Delta^2 - 3\delta^2} \right] |T_1\rangle \langle T_1| \]

\[ -\Omega^2 \text{Re} \left[ \frac{2g^2}{9g^2\Delta^2 - 3\delta^2} \right] |T_2\rangle \langle T_2| + H.c. \]

\[ -\Omega^2 \text{Re} \left[ \frac{1}{9} \frac{8}{\Delta^2} - \frac{\delta}{3g^2 - \delta^2} \right] |T_2\rangle \langle T_2| + \hat{H}_g(11) \]

where \(|T_1\rangle = \frac{1}{\sqrt{2}}(|g_{L_o}g_{R}\rangle + |g_{R_o}g_{L}\rangle + |g_{a_o}\rangle)|) is the desired three-dimensional entangled state and \(|T_2\rangle = \frac{1}{\sqrt{2}}(|g_{L_o}g_{R}\rangle + |g_{R_o}g_{L}\rangle - 2|g_{a_o}\rangle), \]

\(|T_3\rangle = \frac{1}{\sqrt{2}}(|g_{L_o}g_{R}\rangle - |g_{R_o}g_{L}\rangle). \]

The effective Lindblad operators induced by the spontaneous emission take the form of

\[ \hat{L}_{\text{eff}}^{g_{L,R}(a,R)} = \frac{\Omega}{\sqrt{3}} \left( \frac{1}{g_{L,R}(a,R)} \right) \left[ \left( \frac{g^2 - 3\delta\Delta'}{9g^2\Delta^2 - 3\delta^2} \right) T_1 + \frac{2g^2}{3} \left( \frac{9g^2\Delta^2 - 3\delta^2}{\Delta^2} \right) T_2 \right] \]

\[ - \frac{8}{9} \frac{\delta}{\Delta^2} + \frac{\delta}{3g^2 - \delta^2} \right] \langle T_2 \rangle - \frac{g^2}{\Delta^2} \right] \langle T_2 \rangle \]

\[ \langle T_3 \rangle + \langle T_3 \rangle \right] \right) \left( \frac{1}{2} \left| T_3 \right| + \frac{1}{6} \left| T_1 \right| + \frac{1}{2\sqrt{3}} \left| T_2 \right| \right) \]

\[ \frac{\Omega}{\sqrt{2}} \frac{2g^2}{\Delta^2} \right] \langle T_2 \rangle \right) \]

\[ \langle T_3 \rangle + \langle T_3 \rangle \right] \right) \left( \frac{1}{2} \left| T_3 \right| + \frac{1}{6} \left| T_1 \right| + \frac{1}{2\sqrt{3}} \left| T_2 \right| \right) \]

\[ \left( \frac{1}{2} \left| T_3 \right| + \frac{1}{6} \left| T_1 \right| + \frac{1}{2\sqrt{3}} \left| T_2 \right| \right) \]

\[ \left( \frac{1}{2} \left| T_3 \right| + \frac{1}{6} \left| T_1 \right| + \frac{1}{2\sqrt{3}} \left| T_2 \right| \right) \]

\[ \left( \frac{1}{2} \left| T_3 \right| + \frac{1}{6} \left| T_1 \right| + \frac{1}{2\sqrt{3}} \left| T_2 \right| \right) \]

\[ \left( \frac{1}{2} \left| T_3 \right| + \frac{1}{6} \left| T_1 \right| + \frac{1}{2\sqrt{3}} \left| T_2 \right| \right) \]
where $g_{\text{eff}} = g|\Omega|/\Delta$. The application of microwave fields is crucial to our scheme, because it guarantees $|T_1\rangle$ remains the dark state while other ground states are coupled to each other. Therefore, the three-dimensional entangled state $|T_1\rangle$ is able to be achieved from an arbitrary initial state via the effective dissipation induced by spontaneous emission. In the left panel of Fig. 2 we plot the fidelities $F(|T_1\rangle, \hat{\rho}) = \langle T_1 | \hat{\rho} | T_1 \rangle$ for creation of $|T_1\rangle$ with the full and the effective master equations, from which we see that under the given parameters the full and the effective dynamics of the system are in excellent agreement. In the right panel, we further optimize the parameters to make the entangled state reach stable in a shorter time.

III. GENERALIZATION TO HIGH-DIMENSIONAL ENTANGLED STATE

The successful use of dissipation to deterministic creation of three-dimensional entangled state mainly relies on the effective level structure of atoms, i.e. we require transitions from a common excited (ground) state of first (second) atom to two ground (excited) states coupled by two orthogonal cavity modes, while other transitions are driven by off-resonance optical lasers. Thus it is possible to generalize our model to prepare high-dimensional entangled state if we design the atomic energy-level diagram following the similar rules. In Fig. 3 we suppose two potential multi-level atoms strongly interact with a multi-mode optical cavity, which is a direct extension of Fig. 1. By introducing microwave fields that drive the transitions $|g_0\rangle \leftrightarrow |g_i\rangle$ where $i = 1, \cdots, N - 1$, an $N$-dimensional entangled state $1/\sqrt{N}(|g_0g_0\rangle + |g_1g_1\rangle + |g_2g_2\rangle + \cdots + |g_{N-1}g_{N-1}\rangle)$ will be carried out via spontaneous emission. In confirmation of our assumption, we numerically simulation of the fidelity for generating the four-dimensional entangled state with the full master equation in the left panel of Fig. 4. Compared with the case of three-dimensional entangled state, a longer time is needed to stabilize the target state above the fidelity 90%. Hence it is not difficult to conclude that the increase of dimension is at the cost of convergence time.

Now we briefly discuss the effect of cavity decay on the performance for entanglement preparation. In the right panel of Fig. 4 we plot the fidelity by numerically solving the full master equation of Eq. 1 incorporating $\kappa$, three curves correspond to different parameters of dissipation, i.e. $\kappa = \gamma = 0.05g$, $\kappa = \gamma = 0.1g$ and $\kappa = \gamma/2 = 0.1g$. The decrease of population for $|T_1\rangle$ undoubtedly accompanied by a increase of population for other state. As the system approach to equilibrium, we will obtain a steady-mixcd entanglement state. For certain cavity setup, the coupling strength between atom and cavity $g$, the cavity leakage rate $\kappa$, and the spontaneous emission rate $\gamma$ are fixed, thus we are allowed to modulate other parameters to achieve a three-dimensional entangled state with a relatively high fidelity. Fig. 5 illustrates the evolution of fidelity versus time with cavity parameters extracted from a recent experiment ($g, \kappa, \gamma) \sim 2\pi \times (750, 2.62, 3.5)$MHz [31]. A selection of $\Omega = 0.02\pi$, $\omega = 0.4\Omega$, $\Delta = g$ will lead to a fidelity about 98%, which overwhelms with the value based on the unitary dynamics [23-25].

IV. CONCLUSION

In conclusion, we have achieved a stationary three-dimensional entangled state via using the dissipation caused by spontaneous emission of atoms. The numerical simulation reveals the theory for effective operator agrees well with the full master equation under given parameters. This proposal is then extended to realize the $N$-dimensional entangled state in theory by considering two multi-level atoms interacting with a multi-mode cavity, which is confirmed by the simulation of implementing a four-dimensional entangled state. The cavity decay plays a negative role on the state preparation, thus corresponding to different experimental situations, we need to regulate the Rabi frequencies of both optical and microwave fields accurately so as to obtain a relatively high fidelity. We believe that our work will be useful for the experimental realization of quantum information in the
FIG. 5: (Color online) Fidelity for generation of three-dimensional entangled state using an experimental cavity parameters.

near future.

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