SMBH growth parameters in the early Universe of Millennium and Millennium-II simulations

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ABSTRACT

We make BH merger trees from Millennium and Millenium-II Simulations to find under what conditions $10^6 M_\odot$ SMBH can form by redshift $z = 7$. In order to exploit both: large box size in the Millennium Simulation; and large mass resolution in the Millennium-II Simulation, we develop a method to combine these two simulations together, and use the Millennium-II merger trees to predict the BH seeds to be used in the Millennium merger trees. We run multiple semi-analytical simulations where SMBHs grow through mergers and episodes of gas accretion triggered by major mergers. As a constraint, we use observed BH mass function at redshift $z=6$. We find that in the light of the recent observations of moderate super-Eddington accretion, low mass seeds ($100 M_\odot$) could be the progenitors of high redshift SMBHs ($z \sim 7$), as long as the accretion during the accretion episodes is moderately super-Eddington, where ($f_{\text{Edd}} = 3.7$) is the effective Eddington ratio averaged over 50 Myr.

Key words: quasars: supermassive black holes - galaxies: dark matter haloes

1 INTRODUCTION

Super-massive black holes (SMBHs) with masses $10^6 \text{M}_\odot$ to $10^{10} \text{M}_\odot$ populate centres of spiral galaxies with central bulge and centres of massive elliptical galaxies (Kormendy & Richstone 1995). These SMBHs grow through mergers and episodes of gas accretion. During accretion phase, black holes (BHs) are indirectly observed as AGNs or quasars, and episodes of gas accretion. During accretion phase, black holes (BHs) are indirectly observed as AGNs or quasars, and episodes of gas accretion. During accretion phase, black holes (BHs) are indirectly observed as AGNs or quasars, and episodes of gas accretion. Obviously question is: How massive were the BH seeds from which SMBHs formed so early and what are the key SMBH growth parameters?

Several mechanisms for BH seeds formation have been proposed. We will briefly discuss the three most popular mechanisms.

Pop III stars, the first metal-free stars that started to form at $z \sim 20$, might have left BH seeds that grew to SMBHs observed in the first billion years after the Big Bang (Madau & Rees 2001; Heger et al. 2003; Wise & Abel 2005). Masses of those BH seeds depend on the masses of Pop III stars, which could be in range $60 \sim 300 M_\odot$ (Bromm et al. 2009), or even $1000 M_\odot$ (Hirano et al. 2013). Some simulations showed that Pop III stars were not so massive and that they formed in binary systems or in clusters (Turk, Abel & O’Shea 2009; Clark et al. 2011; Stacy et al. 2012), but other authors showed that Pop III stars were formed isolated (O’Shea & Norman 2008; Hirano et al. 2013). It is typically assumed in the literature that Pop III stars form BH seeds with masses close to $100 M_\odot$.

Previous attempts to form $\sim 10^9 M_\odot$ SMBHs at $z \sim 6$ from $100 M_\odot$ seeds required continuous accretion close to or exceeding the Eddington limit and radiative efficiencies of $\epsilon \lesssim 0.1$ (Haiman & Loeb 2001; Tyler, Janus & Santos-Noble 2003; Volonteri & Begelman 2010; Whalen & Fryer 2012; Johnson et al. 2012, 2013). Even with combination of BH mergers and accretion, low mass BH seeds must grow at the Eddington limit for a significant fraction of time or they need to have early stages of super-Eddington accretion in order to explain the observed high redshift quasars (Yoo & Miralda-Escudé 2004; Volonteri & Rees 2005, 2006; Li et al. 2007; Pelupessy, Di Matteo & Ciardullo 2007; Sijacki, Springel & Haehnelt 2009; Tanaka & Haiman 2009; Tanaka, Perna & Haiman 2012; Madau, Haardt & Dotti 2014). There are theoretical uncertainties whether BH seeds can sustain such high accretion rates for a long time (Milosavljević, Couch & Bromm 2009; Alvarez, Wise & Abel 2009; Jeon et al. 2011). It is even more difficult to form $\sim 10^9 M_\odot$ SMBHs

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at $z \sim 7$ considering that the time available for accretion is even smaller than at $z \sim 6$.

Another possible mechanism for BH seed formation is direct collapse of gas into BH. This process produces BH seeds with masses in range $10^4 - 10^6 M_\odot$ (Loeb & Rasio 1994; Eisenstein & Loeb 1995; Oh & Haiman 2002; Bromm & Loeb 2003; Kouwenhoven, Bullock & Dekel 2004; Begelman, Volonteri & Rees 2006; Lodato & Natarajan 2006; Begelman, Rossi & Armitage 2008). It has been shown that massive BH seeds formed by direct collapse can reach $10^7 M_\odot$ (Begelman, Rossi & Armitage 2008). It has been shown that those models are not viable because molecular cooling has no mechanism for efficient cooling, that is if there are not gas inflows and if formation of H$_2$ molecules is prevented via strong UV background. Only extremely rare haloes satisfy this condition. Direct collapse is expected to occur in haloes with virial temperature $T_{\text{vir}} \gtrsim 10^4 K$ and masses $10^7 - 10^8 M_\odot$ at $10 < z < 20$. Recently, models have been proposed where direct collapse might be possible even without a strong UV radiation background (Inayoshi & Omukai 2012; Tanaka & Li 2013), but Vishal, Haiman & Greg (2014) have shown that those models are not viable because molecular cooling will still occur, as the gas density increases, which leads to fragmentation. Alternatively, major mergers may lead to a rapid gas inflow. In such case, turbulence may be the inhibitor of fragmentation, and the requirement of metal-free gas may be relaxed (Mayer et al. 2010).

The third possible mechanism for BH seed formation is collapsing star clusters. Devecchi & Volonteri (2009) suggested model where mergers of Pop II stars lead to the formation of a very massive star which collapses into a BH with mass $\sim 10^3 M_\odot$, at $z \sim 10 - 20$. Davies, Miller & Bellovary (2011) proposed a model of a star cluster which contains only BHs and main sequence low mass stars. More massive BHs sink to the centre and form BH cluster which further collapses and forms massive BH with mass $10^5 M_\odot$, at $z > 10$. These seeds can grow via mergers and accretion to form SMBHs. Some authors showed that such seeds could explain observed quasars at $z < 5$ but have trouble explaining quasars at high redshifts (Chisumaki et al. 2001; Volonteri, Haardt & Madau 2003; Islam, Taylor & Silk 2003).

BH mergers could contribute to the SMBH growth. Johnson et al. (2013) argued that growth of the most massive BH at high redshift is solely due to the accretion and that mergers can be neglected. Their conclusion is based on: the fact that only a small number of high redshift haloes is capable of hosting Pop III stars; Pop III star formation rate is reduced due to Lyman-Werner (LW) radiation field; and the results of the recent large scale cosmological simulations tracking the buildup of SMBH, where it is claimed that mergers do not affect the growth of SMBHs. Simulation by DeGraf et al. (2012) shows that negligible fraction of high redshift SMBH mass is acquired in mergers but this is a consequence of the poor mass resolution. Their simulation does not form haloes smaller than $5 \times 10^{10} h^{-1} M_\odot$. While true that Pop III star formation cannot take place in smallest haloes, it does occur in haloes with $T_{\text{vir}} \sim 1000 - 2000 K$ which corresponds to $M_{\text{halo}} \gtrsim 10^9 M_\odot$ at redshift $z > 6$. Also, haloes with $M_{\text{halo}} \gtrsim 10^8 M_\odot$ are most likely self-shielded from LW radiation due to the large H$_2$ column density (Wise & Abel 2008). Hence, it is clear that haloes with mass $10^9 M_\odot < M_{\text{halo}} \lesssim 10^{10} M_\odot$ host Pop III stars and if these stars produce massive BHs, then their mergers should be relevant to the growth of high redshift SMBHs.

We investigate if light BH seeds (100 $M_\odot$) planted into haloes of Millennium simulation ($M_{\text{halo}} > 10^{10} M_\odot$) and Millennium-II simulation ($M_{\text{halo}} > 10^9 M_\odot$) can grow into SMBHs that have been observed at $z \sim 7$.

We also investigate if gravitational wave recoil could prevent the formation of SMBH. During a BH merger gravitational wave radiation is produced. Asymmetric emission of gravitational radiation can lead to BH kick. Gravitational waves carry a non-zero net linear momentum, which establishes a preferential direction for the propagation of the waves and the centre of mass of the binary recoils in the opposite direction (Redmount & Rees 1989). The magnitude of the gravitational wave recoil depends on the mass ratio of BHs, the spin magnitude and orientation with respect to the binary orbital plane and the eccentricity of the orbit (Campanelli, Lousto & Zlochower 2007; Schnittman & Buonanno 2007; Baker et al. 2002). Gravitational wave recoil can eject a newly formed BH from the host halo if the BH speed is larger than the escape velocity from the halo centre. BHs can be kicked with a speed as large as $\sim 4000$ km s$^{-1}$ in special orbital configurations (Herrmann et al. 2007; Gonzales et al. 2007a, 2007b; Campanelli et al. 2007; Schnittman & Buonanno 2007; Koppitz et al. 2007). At high redshift, dark matter haloes generally have small masses and thus small escape velocities, so BH with speed $\sim 150$ km s$^{-1}$ can be ejected even from the most massive haloes at redshift $z > 11$ (Merritt et al. 2004; Micic, Abel & Sigurdsson 2006; Volonteri 2007; Schnittman 2007; Sesana 2007; Volonteri, Gulletek & Dotti 2010; Micic, Rolley-Bockelmann & Sigurdsson 2011). This effect may play a major role in suppressing the growth of SMBH through mergers.

1.1 Growth Parameters and Their Values

Large scale structure formation and galaxy dynamics lead to BH mergers. If those mergers are ignored, BH growth depends on three gas accretion parameters. Those parameters are radiative efficiency, Eddington ratio and the time that a BH spends accreting.

Radiative luminosity of a BH, $L$, which is accreting at a rate $\dot{M}_{\text{BH}}$ is given by $L = \epsilon \dot{M}_{\text{BH}} c^2 / (1 - \epsilon)$, where $c$ is the speed of light and $\epsilon$ is the radiative efficiency. Eddington ratio is $E_{\text{edd}} = \frac{c L}{4 \pi G M_{\text{BH}}}$, where $L_{\text{edd}} = 1.26 \times 10^{38} (\dot{M}_{\text{BH}} / M_\odot) [\text{erg s}^{-1}]$ is Eddington luminosity. The accretion rate at which a BH will radiate at a given Eddington ratio is given by:

$$\dot{M}_{\text{BH}} = \frac{(1 - \epsilon) L_{\text{edd}}}{\epsilon c^2}.$$

After integration the final BH mass $M_{\text{BH}, f}$, as a function of its initial mass $M_{\text{BH}, 0}$ is:

\[ M_{\text{BH}, f} = \frac{1}{\epsilon c^2} \int_{M_{\text{BH}, 0}}^{M_{\text{BH}, f}} \frac{(1 - \epsilon) L_{\text{edd}}}{\dot{M}_{\text{BH}}} \, \text{d}M_{\text{BH}}. \]
$M_{\text{BH}} = M_{\text{BH,0}} \times \exp \left[ \frac{f_{\text{Edd}} (1 - \epsilon)}{\epsilon} \right], \quad (2)$

where $t_{\text{Edd}} = 450 \ \text{Myr}$, $t_1$ and $t_i$ are the ages of the universe when the BH attains its final mass and at the time of seed formation, respectively [Johnson et al. 2013].

Radiative efficiency is the efficiency of conversion of rest-mass into energy during accretion and it depends on the BH spin. Radiative efficiency can take values from 0.057 for accretion onto Schwarzschild BHs to 0.42 for fast rotating Kerr BHs [Shapiro, 2005]. Mean value of radiative efficiency for quasars can be estimated by comparing the local SMBH mass density with the total AGN luminosity per unit volume in the Universe integrated over time [Soltan, 1982]. This is Soltan’s argument. Previous works based on Soltan’s argument showed that the mean value of radiative efficiency is $\epsilon \geq 0.1$ [Elvis, Risaliti & Zamorani, 2002; Yu & Tremaine, 2002; Davis & Laor, 2011]. Some authors have found that radiative efficiency changes with the redshift (Wang et al., 2009; Li, Wang & Ho, 2012) and increases with the mass of the accreting BH [Davis & Laor, 2011; Shankar, Weinberg & Miralda-Escudé, 2011; Li, Wang & Ho, 2012].

It is usually assumed that BH luminosity during accretion cannot be greater than the Eddington luminosity. However, evidence for super-Eddington accretion has been growing recently. Kelly & Shen (2013) used a sample of ~58,000 quasars at $z \sim 0.3 - 5$ from SDSS DR7 catalog [Schneider et al., 2010] to estimate their Eddington ratios. They found that the highest observed Eddington ratio for quasars is $f_{\text{Edd}} \sim 3$, but that those quasars which radiate above the Eddington limit are rare. Du et al. (2014) observed three quasars, Mrk 335, Mrk 142 and IRAS F12397+3333, using the 2.4-m Shangri-La telescope at the Yunnan Observatory in China. One of their goals was to measure BH masses and Eddington ratios. They found that the lower limits on the Eddington ratios for these objects are 0.6, 2.3, and 4.6. Page et al. (2013) have shown that soft X-ray spectrum of the highest redshift quasar yet found, ULAS J112001.48+064124.3 at $z = 7.085$ (Mortlock et al., 2011), obtained with Chandra and XMM-Newton, suggests that the quasar is accreting above the Eddington limit, $f_{\text{Edd}} = 5^{+15}_{-1}$. Their findings of moderate super-Eddington accretion are consistent with the Eddington ratio estimated from the UV luminosity of that object, $f_{\text{Edd}} = 1.2^{+0.6}_{-0.5}$ (Mortlock et al., 2011). In order to fit the observed statistics of far-infrared and X-ray spectra of AGNs at $z \geq 2$ [Lapi et al., 2006, 2014] assumed model where Eddington ratio depends on the redshift. During the exponential growth of the BH, the maximum Eddington ratio is $f_{\text{Edd}} \sim 4$ for $z = 6$, and $f_{\text{Edd}} \sim 1$ for $z = 2$, with constant radiative efficiency $\epsilon = 0.15$. Similar results have been found by Li (2013), who has also explored the possibility of having a short super-Eddington accretion followed by a sub-Eddington accretion in order to explain the presence of BHs with masses $\sim 10^8 M_\odot$ SMBHs at $z \sim 6$.

A recent analysis of BH scaling relations showed that the normalization of the BH mass-bulge relation should be increased by a factor of five, from the previously accepted value of $M_{\text{BH}} = 0.1% M_{\text{bulge}}$ to $M_{\text{BH}} = 0.5% M_{\text{bulge}}$. This increases the local mass density in BHs by the same factor and decreases the required mean radiative efficiency to values that cannot be reasonably explained in terms of luminous thin-disc accretion. This may be an evidence for the radiatively-inefficient super-Eddington accretion [Novak, 2013] [Kormendy & Ho, 2013]. These works have shown that quasars, at least at some point in their evolution, can accrete at the super-Eddington luminosities. Recent numerical simulations [McKinney et al., 2013; Sadowski et al., 2013] of super-Eddington accretion suggest that those sources are most likely to be characterized by strongly collimated outflows or jets. Accretion in these simulations is mildly super-Eddington, $f_{\text{Edd}} = 1 - 10$.

Theoretical works by Volonteri & Rees (2005, 2006) assumed early stages of super-Eddington quasi-spherical accretion estimated using the Bondi-Hoyle formula [Bondi & Hoyle, 1944]. In this case, when the inflow rate is super-critical, the radiative efficiency drops. Hence, the Eddington luminosity is not greatly exceeded ($L_{\text{Edd}} = \epsilon \dot{M}_{\text{Edd}} c^2$). The accretion rate is initially super-critical by a factor of 10 and then grows up to a factor of about $10^4$. They showed that BH seeds with initial mass of 1000 M_\odot could explain SMBH at $z \sim 6$ if they go through early phases of super-Eddington accretion and BH mergers. Similar approach has been recently argued by Volonteri & Silk (2014). They showed that short-lived intermittent episodes of super-Eddington accretion ($f_{\text{Edd}} > 10$) may increase the BH mass by several orders of magnitude in $\sim 10^7$ years. The authors assumed slim disc solution where the luminosity during accretion depends logarithmically on the accretion rate. Since ($f_{\text{Edd}} = \epsilon \dot{M}_{\text{Edd}}$, if the effective radiative efficiency is low, accretion rate can be highly super-Eddington while the emergent luminosity is only mildly super-Eddington and feedback is limited. Madau, Haardt & Dotti (2014) extended these works and showed that light BH seeds (100 M_\odot) could explain quasars with $10^9 M_\odot$ SMBH observed at $z > 6$ if they have a few episodes of super-Eddington accretion via slim accretion disc. Eddington ratio in these works is $\sim 1 - 10$.

Dehnen & King (2013) proposed different mechanism for super-Eddington accretion. Momentum driven feedback from an accreting BH gives significant orbital energy but little angular momentum to the surrounding gas. Once central accretion drops, the feedback weakens and gas falls back towards the BH, forming a small scale accretion disc. The feeding rates into the disc typically exceed Eddington by factor of a few.

Based on the mentioned works it can be concluded that super-Eddington accretion is inevitable for the growth of SMBHs.

Another important question is for how long accretion can be sustained? Quasars lifetimes can be estimated using Soltan’s argument. Amount of matter accreted onto quasars during their lifetime, represented by the luminosity density due to accretion, should be less than or equal to the space density of remnant BHs in the local Universe [Soltan, 1982]. This method depends on the value of radiative efficiency. Another way to estimate this parameter is to ask what value of quasars lifetime is required if all bright galaxies go through a quasar phase. This approach is not sensitive to the value of radiative efficiency, but it is affected by the assumed value of Eddington ratio and galaxy mergers [Martini, 2004]. Computed values for the duration of accretion using this approach are from 10^9 to 10^8 yr.
Typical accretion time for majority of quasars is Salpeter’s time \( t_a = M/M = 4.5 \times 10^7 \left(\frac{\epsilon}{0.1}\right) \left(\frac{L}{L_{\text{Edd}}}\right)^{-1} \). (3)

A single massive BH seed \((10^5 - 10^6 M_\odot)\) needs to continuously accrete at Eddington luminosity for 380-500 Myr (which corresponds to 9 - 11 e-folding times) to be able to grow to \( > 2 \times 10^8 M_\odot \) at redshift \( z > 6 \). Light BH seed \((100 M_\odot)\) requires 19 e-folding times or 840 Myr of accretion. Alternatively, it would take 280 Myr at Eddington ratio 3 (also 19 e-folding times). This means that it would be very hard to grow SMBH from light seeds at the Eddington limit by \( z = 6 \) and impossible by \( z = 7 \). Some authors assumed such long quasar lifetimes in order to match observed quasars number density at high redshifts (Haiman & Loeb 2001; Tyler et al. 2003; Sijacki et al. 2009; Tanaka & Haiman 2009; Tanaka et al. 2012; Johnson et al. 2012, 2013).

Continuous accretion at or above the Eddington limit might be problematic due to the feedback. Radiation and kinetic power in matter outflows could be so strong that once the accreting BH gets too big, it blows out all of the gas in the centre of its host galaxy, shutting down the accretion onto the BH (Coppi 2003). If continuous growth at the Eddington limit cannot be maintained, the main alternative to these models are short periods of super-Eddington accretion (Volonteri & Rees 2005, 2006; Dehnen & King 2013; Volonteri & Silk 2014; Madau, Haardt & Dotti 2014). However, observations show that, in most cases, quasar lifetimes are comparable with Salpeter’s time and Eddington ratios are between 0.1 and 1. Time that SMBH spends accreting depends on radiative efficiency, Eddington ratio and BH mass (Martini 2001). Yu & Tremaine (2002) showed that quasar lifetime is a function of BH mass. They found that the mean lifetime is \( 3 - 13 \times 10^7 \) yr for \( \epsilon = 0.1 - 0.3 \) and \( 10^8 < M_{\text{BH}} < 10^9 M_\odot \). It is obvious that if one assumes these typical values of the growth parameters, it would be impossible to grow SMBH at high redshift even if we start with seeds as massive as \( 10^6 M_\odot \). Accretion would have to be either super-Eddington or prolonged for more than Salpeter’s time (many e-folding times instead of one).

Our goal is to examine if BH mergers combined with accretion episodes in merger trees of Millennium Simulation (Springel et al. 2005) and Millennium-II Simulation (Boylan-Kolchin et al. 2009) contribute to the growth of \( 10^9 M_\odot \) SMBH at \( z \sim 7 \). If they do, then values of Eddington ratios and numbers of e-folding times during the merger induced accretion episodes in merger tree branches can be lowered to a more reasonable values. Growth of a SMBH in halo that undergoes several major mergers does not require continuous accretion for many e-folding times. Every major merger will reset previous accretion episode and trigger a new one which effectively reduces number of e-folding times. That in turn could increase the likelihood of low mass BHs as SMBH seeds.

In Section 2 we describe the method. In Section 3 we present our results. We summarize and discuss our results in Section 4.

2 METHOD

Our goal is to produce SMBH with mass \( > 10^9 M_\odot \) at \( z \sim 7 \) from light BH seeds using publicly available data from both Millennium Simulation (Springel et al. 2005) and Millennium-II Simulation (Boylan-Kolchin et al. 2009). As a constraint we use comparison between the observed and calculated BH mass function from our model at \( z \sim 6 \).

2.1 Millennium Simulation and Millennium-II Simulation

Millennium Simulation (Springel et al. 2005) is a large N-body simulation which follows \( 2163^3 \) particles within a periodic simulation cube of side length \( L = 500 h^{-1} \text{ Mpc} \). Each simulation particle has mass \( 8.61 \times 10^8 M_\odot \). The \( \Lambda \)CDM cosmology used for the Millennium Simulation is:

\[
\Omega_{\text{tot}} = 1.0, \quad \Omega_m = 0.25, \quad \Omega_b = 0.045, \quad \Omega_\Lambda = 0.75, \quad h = 0.73, \quad \sigma_8 = 0.9, \quad n_s = 1,
\]

where \( h \) is the Hubble constant at \( z = 0 \) in units of \( 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \sigma_8 \) is the rms amplitude of linear mass fluctuations in \( 8 h^{-1} \text{ Mpc} \) spheres at \( z = 0 \), and \( n_s \) is the spectral index of the primordial power spectrum.

Millennium-II Simulation (Boylan-Kolchin et al. 2009) uses the same cosmology and the same number of particles as the Millennium Simulation, but in a 5 times smaller box \((L = 100 h^{-1} \text{ Mpc})\) and thus with 125 times better mass resolution. Each simulation particle in Millennium-II Simulation has mass \( 6.885 \times 10^8 M_\odot \). Millennium-II simulation uses GADGET-3 code, updated version of GADGET code (Springel, Yoshida & White 2001; Springel 2005).

Current dominant cosmological paradigm is CDM, which predicts bottom-up mode of structure formation, that is, small dark matter haloes forming first and then merging into larger haloes later in the life of the Universe.

Since Millennium-II Simulation has 125 times better mass resolution than Millennium Simulation, it allows us to track BH growth by mergers of highest redshift low mass haloes which are too small to be resolved due to the lower mass resolution of Millennium Simulation. On the other hand, it is questionable if it is possible to produce a \( 10^9 M_\odot \) SMBH at \( z \sim 7 \) in a simulation box with sides of length that Millennium-II Simulation has due to low population of highest mass haloes. In order to have both small haloes and large box, we develop a method to use BHs produced in merger trees of Millennium-II Simulation, as BH seeds in merger trees of Millennium Simulation, which enable us to cover both haloes that are forming very early and abundant highest mass haloes later.

2.2 Combining Millennium-II and Millennium merger trees

With 100 M\(_\odot\) BH seeds placed in Millennium-II haloes we produce BH growth history. From it, because of the higher mass resolution of Millennium-II Simulation, we have information about earliest halo formation history.

First, we make merger trees which track dark matter halo merger history in Millennium-II Simulation from redshift \( z = 23.79 \) to \( z = 6.2 \). Halo is defined as a self-bound
structures with at least 20 particles. Merger history of BHs corresponds to the merger history of haloes assuming that each halo hosts one BH and that two BHs merge right after their host halo merge. We have distinguished between minor and major mergers. Merger is major if \( \frac{M_{\text{halo},1}}{M_{\text{halo},2}} > 0.3 \) for \( M_{\text{halo},1} < M_{\text{halo},2} \). In the case of a minor merger, mass of the newly formed BH is a simple sum of the previous BH masses. In the case of a major merger, newly formed BH is also accreting gas according to the accretion recipe described in following subsections. In each halo of Millennium-II Simulation we place one BH seed with 100\( M_\odot \) and assume fixed values for radiative efficiency and accretion timescale in every accretion episode. Hence, the only variable in our model is the Eddington ratio.

Next, we use Millennium Simulation which has simulation box 5 times larger \((L = 500 h^{-1} \text{Mpc})\) in order to compare final BH masses in merger trees of this simulation to the observed BH mass function. We apply the same BH growth recipe to the haloes of Millennium Simulation. The difference is, this time the seed BH mass is the most common BH mass estimated in the merger trees of Millennium-II Simulation.

We use Millennium-II Simulation to determine a typical BH mass for a specific host halo mass. We do it by binning masses of resulting BHs at each snapshot and halo mass interval and then choosing central value of the most occupied bin. Hence, at every redshift we know the range of BH masses hosted by haloes with a specific mass. We select the typical (most common mass) BH and use it as a BH seed in each newly formed halo of Millennium Simulation at the matching redshift and halo mass obtained from Millennium-II, thus extending our sample to include highest mass haloes. Note that this is a conservative way to populate haloes since we are losing instances when BH mass is in reality more massive than the common one.

### 2.3 Merger tree

First, we use Millennium-II database to select haloes with masses \( > 10^{10} M_\odot \) at redshift \( z = 6.2 \). For all selected haloes we find their progenitor haloes, i.e. haloes that have merged at the previous snapshot to form selected haloes. Halos have merged if they have the same descendant halo. We repeat this procedure up to redshift \( z = 23.79 \) where first mergers are recorded. Once we find all of the haloes that take part in formation of the selected haloes, we form merger trees for each of them.

Final haloes at redshift \( z = 6.2 \) are called main haloes. Mergers of the main halo form the main branch of the merger tree. Other haloes that have had merged with the main halo are side haloes and their previous mergers form side branches. Every merger tree contains main branch and side branches. We follow not only mergers of the main halo, but we take into account mergers in all side branches.

In a single merger event the most massive progenitor halo is called primary halo and other progenitor haloes are satellite haloes. In the Millennium-II simulation output is printed with time resolution \( \Delta z \sim 1 \), so the common case is that large number of haloes have the same descendant halo (case with largest number of progenitors being 186). The question is, which halo is the primary halo and which haloes are satellite haloes? To solve this we first find the most massive progenitor halo (primary halo) and then using the comoving coordinates of the centre of mass, we calculate distances to other progenitor haloes (satellite haloes), sort them in the order from closest to furthest away and assume that they have merged in that order. We calculate halo mass ratio of the merging haloes and distinguish minor and major mergers.

We repeat the same procedure for haloes in Millennium Simulation that have masses \( > 10^{11} M_\odot \) at redshift \( z = 6.2 \).
ratio. In each run we assign the same values for the effective Eddington ratio to each accretion episode. We make sure not to overproduce SMBH at $z = 6.2$ (Millennium snapshot closest to redshift $z = 6$) and once this condition is satisfied, we check if we have $10^9 M_\odot$ SMBH at $z = 7$ for the specific choice of the effective Eddington ratio.

3 RESULTS

We find that if the effective Eddington ratio is $f_{\text{Edd}} = 3.7$ both conditions of our model are satisfied: BH mass function is consistent with the observed BH mass function at $z \sim 6$ (Willott et al. 2010b) and our merger tree produces $10^9 M_\odot$ SMBH at $z = 7$.

Figure 1 shows mass function of BHs that populate $> 10^{11} M_\odot$ haloes of Millennium Simulation at $z = 6.2$. BH masses are the result of one semi-analytic simulation in which we chose initial BH masses to be $100 M_\odot$, effective radiative efficiency $\epsilon = 0.1$, effective Eddington ratio $f_{\text{Edd}} = 3.7$ and each accretion episode is limited to 50 Myr. In order to calculate BH mass function in our model we bin masses of all BHs in haloes of Millennium Simulation at redshift $z = 6.2$. Points depicted by plus symbols in Figure 1. represent numbers of BHs in each bin per $\text{Mpc}^3$. Bins have width of 0.1 dex (in logarithmic scale). We compare our BH mass function to the BH mass function at $z = 6$ given by Willott et al. (2010b) (dashed and solid lines). Willott et al. (2010b) modeled BH mass functions to produce luminosity functions which are then fitted to the observed luminosity function of Canada-France High-z Quasar Survey (CFHQS) and Sloan Digital Sky Survey (SDSS) quasars at $5.74 < z < 6.42$ (Willott et al. 2010a). Dashed lines in Figure 1. represent their upper and lower limits, while the solid line is their best-fitting to the data.

Kelly & Shen (2013) showed that accretion with similar Eddington ratios exists in high redshift quasars. They found that the maximum observed value of Eddington ratio for the observed quasars is $f_{\text{Edd}} \sim 3$. However, other authors suggest that even larger values of Eddington ratio are possible ($f_{\text{Edd}} = 4.6$ (Du et al. 2014), or even $f_{\text{Edd}} = 10$ (Collin & Kawaguchi 2004)).

Assuming that a moderate super-Eddington accretion is possible for a prolonged period of time (more than one e-folding time), in the further analysis we focus on the main halo with the most massive SMBH at redshift $z = 7$ and its progenitors.

Figure 2. shows the merger tree of a $10^9 M_\odot$ SMBH at redshift $z = 7$ for $f_{\text{Edd}} = 3.7$. It follows the entire growth of a SMBH through all mergers and accretion episodes, as a function of the age of the Universe. Black circles represent BH masses in side haloes at the snapshot-times, while red asterisks represent BH masses in the main halo. Dotted blue lines follow BH growth by minor mergers and solid red lines show major mergers which are always followed by the gas accretion.

Accretion episodes at $t < 300 \text{Myr}$ occur in haloes of Millennium-II Simulation, while accretion episodes at $t > 600 \text{Myr}$ represent major mergers of haloes in Millennium Simulation. The absence of accretion episodes between $t = 300 \text{Myr}$ and $t = 600 \text{Myr}$ is the consequence of the fact that we do not know the exact merger history of low mass haloes of Millennium Simulation. We approximate BH masses in those haloes with typical BHs that populate haloes of Millennium-II Simulation at the matching redshift.

Each accretion episode lasts 50 Myr. Salpeter’s time for a BH accreting at the effective Eddington ratio $f_{\text{Edd}} = 3.7$ is $\sim 12 \text{Myr}$ (equation 3), which means that e-folding time in each accretion episode is $\sim 4$. 

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Super-Eddington accretion for such long period of time might seem unrealistic, but it could be more possible than the accretion at the Eddington limit for almost a billion years. To explain formation of \( > 10^9 \, M_\odot \) SMBH with a former classical values of the accretion parameters (\( M_{BH,0} = 10^5 \, M_\odot \), \( \epsilon = 0.1 \) and \( f_{Edd} = 1 \)) it can be calculated that continuous accretion for 500 Myr is needed (equation 2) and e-folding time is \( \sim 11 \). If we start with those parameters and smallest BH seed of 100 \( M_\odot \), we calculate that e-folding time is \( \sim 19 \) and the accretion lasts for 840 Myr in case where \( f_{Edd} = 1 \) and 280 Myr in case where \( f_{Edd} = 3 \).

In our model, growing SMBH at \( z = 7 \) does not require e-folding time larger than 4 for 100 \( M_\odot \) BH seeds. This is a consequence of accretion being restarted in major mergers. Instead of one BH constantly accreting we have several BHs in shorter accretion episodes, where each of them is triggered in a major merger. SMBH in the main halo grows through two accretion episodes while the rest of the accretion occurs in side haloes (Figure 2). Above 10\(^4\) \( M_\odot \) two accretion episodes occur in a side halo. BH in that side halo grows parallel with BH in the main halo. When the side and the main haloes merge side halo hosts more massive BH than the main halo because it has greater number of major mergers in its history. This approach increases the impact of mergers, and reduces importance of accretion, which in turn alleviates need for super-Eddington accretion with the large number of e-folding times.

Figure 3. represents accumulative contribution to the SMBH growth from minor mergers (black solid line), major mergers (blue dashed line) and direct accretion (red dash dot line). Below 10\(^7\) \( M_\odot \) major mergers and accretion are equally important while above that mass SMBH growth is due to direct accretion.

![Figure 3](image1.png)

**Figure 3.** Growth of the SMBH in the main halo by minor mergers (black solid line), major mergers (blue dashed line) and direct accretion (red dash dot line). Below 10\(^7\) \( M_\odot \) major mergers and accretion are equally important while above that mass SMBH growth is due to direct accretion.

![Figure 4](image2.png)

**Figure 4.** Main and side haloes masses as a function of BHs mass ratio in both major and minor halo mergers. Red diamonds represent haloes where the sum of two merging BHs is \( > 10^7 \, M_\odot \). Those mergers are the most important in BH growth and they are not affected by gravitational wave radiation. Mergers of BHs with mass ratio \( \sim 1 \) might suppress the growth of SMBH by gravitational wave recoil since they reside in low mass haloes. BHs that form in larger haloes are protected from ejection by the large gravitational potentials.

To further investigate the influence of the gravitational wave recoil, we calculate and compare kick value with the value of the halo escape velocity.

Kick velocities are taken from Micic et al. (2011). The authors used parametrized fit of [Campanelli, Lousto & Zlochower (2007)] to calculate the kick velocity as a function of the merging BHs mass ratio, BHs spin and the alignment to the orbital angular momentum. Kick velocity is (their equations 7-10):

\[
v_{kick} = \sqrt{(v_m + v_{\perp} \cos \xi)^2 + (v_{\perp} \sin \xi)^2 + (v_{\parallel})^2},
\]

where

\[
v_m = A \frac{q^2 (1 - q)}{(1 + q)^2} \left[ 1 + B \frac{q}{(1 + q)^2} \right],
\]

and

\[
v_{\perp} = H \frac{q^2}{(1 + q)^2} (a_{\perp} - qa_{\parallel}),
\]

where

\( q = M_{BH,2}/M_{BH,1} \), \( a_{\perp} \) and \( a_{\parallel} \) are the BH spin in the meridional and parallel directions.

The SMBH growth parameters

\( SMBH growth parameters \)
The fitting constants are $A = 1.2 \times 10^4 \text{km s}^{-1}$, $B = -0.93$, $H = (7.3 \pm 0.3) \times 10^4 \text{km s}^{-1}$ and $K \cos (\Theta - \Theta_0) = (6.0 \pm 0.1) \times 10^4$, $q$ is mass ratio of the merging BHs, $\alpha_i$ is the reduced spin parameter and the orientation of the merger is specified by angles $\Theta$ and $\xi$. The authors show distribution of gravitational recoil for two models: when the spin parameters are chosen from a uniform distribution and when it is assumed that the BH spins are aligned with the orbital angular momentum. We accepted their model when BH spins are aligned with the orbital angular momentum (their Figure 2., red line) to assign values of kick velocities to the merging BHs in our model.

Escape velocity is calculated as described in O’Leary & Loeb (2009). Circular velocity of a halo at the virial radius is (Barkana & Loeb 2001):

$$v_c = 24 \frac{M_{\text{halo}}}{10^9 h^{-1} M_\odot} \left( \frac{\Omega_m}{\Omega_m^{\text{crit}}} \right)^{1/3} \frac{\Delta_c}{18\pi^2}^{1/6} \left( \frac{1 + z_{\text{merge}}}{10} \right)^{1/2} \text{km s}^{-1},$$

(8)

where $\Delta_c = 18\pi^2 + 8d - 39d^2$, $d = \Omega_m^\star - 1$ and $\Omega_m^\star = \Omega_m (1 + z)^3 / (\Omega_m (1 + z)^3 + \Omega_\Lambda)$ at the merger redshift. The authors assumed that the dark matter haloes are described with NFW profile (Navarro, Frenk & White 1996) with a concentration parameter $c = 4$ out to the virial radius, as expected for a newly formed dark-matter halo (Wechsler et al. 2002). Under those assumptions the escape velocity from the halo’s centre is $v_{\text{esc}} \approx 2.8c$.

Figure 5. shows the number of BH mergers as a function of BH mass. Grey filled histogram shows BHs whose kick velocities are larger than the escape velocities from their host haloes, so they are sensitive to the gravitational wave recoil. Blue histogram with horizontal lines shows BHs that are not affected by gravitational wave recoil.

$$v_{\parallel} = K \cos (\Theta - \Theta_0) - \frac{q^2}{(1 + q)^2} (\alpha_1 - q \alpha_1^\perp).$$

(7)

The fitting constants are $A = 1.2 \times 10^4 \text{km s}^{-1}$, $B = -0.93$, $H = (7.3 \pm 0.3) \times 10^4 \text{km s}^{-1}$ and $K \cos (\Theta - \Theta_0) = (6.0 \pm 0.1) \times 10^4$, $q$ is mass ratio of the merging BHs, $\alpha_i$ is the reduced spin parameter and the orientation of the merger is specified by angles $\Theta$ and $\xi$. The authors show distribution of gravitational recoil for two models: when the spin parameters are chosen from a uniform distribution and when it is assumed that the BH spins are aligned with the orbital angular momentum. We accepted their model when BH spins are aligned with the orbital angular momentum (their Figure 2., red line) to assign values of kick velocities to the merging BHs in our model.

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where $\Delta_c = 18\pi^2 + 8d - 39d^2$, $d = \Omega_m^\star - 1$ and $\Omega_m^\star = \Omega_m (1 + z)^3 / (\Omega_m (1 + z)^3 + \Omega_\Lambda)$ at the merger redshift. The authors assumed that the dark matter haloes are described with NFW profile (Navarro, Frenk & White 1996) with a concentration parameter $c = 4$ out to the virial radius, as expected for a newly formed dark-matter halo (Wechsler et al. 2002). Under those assumptions the escape velocity from the halo’s centre is $v_{\text{esc}} \approx 2.8c$.

Figure 5. shows the number of BH mergers as a function of the mass of the newly formed BH. If BH kick velocity is larger than the escape velocity from the host halo, newly formed BH could be ejected from the halo (gray filled histogram). Blue histograms with horizontal lines represents BHs whose kick velocities are smaller than the escape velocities. The figure shows that low mass BHs reside in low mass haloes and have their kick velocities larger than the escape velocities from their host haloes, so they are sensitive to the gravitational wave recoil. On the other hand, we use simple model in which we assume that each newly formed halo in Millennium-II Simulation hosts a BH with the initial mass of 100 $M_\odot$. Possibility that two merging BHs have exactly the same mass is very small. First BHs have initial mass function (IMF), with masses in wide range. IMF for Pop III BH remnants is not fully understood due to uncertainty in primordial gas fragmentation during Pop III star formation. Pop III stars with masses in range $\sim 25 - 140 M_\odot$ are believed to leave BH remnants with masses $M_{\text{BH}} \sim 10 - 50 M_\odot$ while more massive stars, $\gtrsim 260 M_\odot$, leave BHs with masses $M_{\text{BH}} \sim 100 - 600 M_\odot$ (e.g., Heger & Woosley 2002). Such initial mass function would significantly reduce the mass ratio of merging BHs. In turn this would lower the value of gravitational wave recoil.

4 DISCUSSION AND CONCLUSIONS

In this paper we test under which conditions light BH seeds (100 $M_\odot$) placed in haloes of Millennium-II Simulation (Boylan-Kolchin et al. 2009) and Millennium Simulation (Springel et al. 2005) merger trees can grow to SMBHs with masses $10^7 M_\odot$ that have been observed at $z \sim 7$.

We make merger trees which track dark matter halo merger history from redshift $z = 23.79$ to $z = 6.2$. We assume that each halo hosts one BH and that two BHs merge right after their host haloes merge. BH can grow in BH mergers and by gas accretion. We have distinguished between minor and major mergers. In the case of a minor merger, mass of the newly formed BH is a simple sum of the previous BH masses. Major merger leads to the formation of a new BH and it triggers the gas accretion onto that BH. Final BH mass depends on the initial BH mass, Eddington ratio, radiative efficiency and quasars lifetime (equation 2). Effective radiative efficiency is fixed at $\epsilon = 0.1$ (Elvis, Risaliti & Zamorani 2002 [Yu & Tremaine 2002] Davis & Loeb 2011) and each accretion episode is limited to 50 Myr, which is $\sim$ Salpeter’s time for accretion at the Eddington limit. Eddington ratio is a free parameter in our model, but it has fixed value for each accretion episode in one simulation run. If a new major merger occurs before the maximum allowed time for accretion has passed, accretion is reseted and the new accretion episode begins. Also, accretion can be stopped if BH mass exceeds $8 \times 10^{-4}$ of the host halo mass, so we make sure that the gas reservoir never gets depleted.

We combine Millennium-II and Millennium merger trees in order to have both: early halo formation history with low mass haloes (to track BH growth history); and a large box giving us abundant highest mass haloes later (in which $10^9 M_\odot$ SMBH at $z \sim 7$ can be produced). First we place 100 $M_\odot$ BH seeds in haloes of Millennium-II Simulation which has 125 times better mass resolution to make BH growth history. Then we take most common BHs from Millennium-II Simulation as seeds for Millennium Simulation to produce $10^9 M_\odot$ SMBH at $z \sim 7$.

We run the set of semi-analytical simulations where we assign same initial masses of 100 $M_\odot$ to all BH seeds and the same value for the effective Eddington ratio during the
episodes of accretion. We investigate what value of the effective Eddington ratio match two conditions: BH mass function in our model cannot exceed BH mass function given by Willott et al. (2010b) and our merger trees need to produce $10^9 M_\odot$ SMBH at $z = 7$.

We find that remnants of Pop III stars can produce $10^9 M_\odot$ SMBH at redshifts $z = 7$ if at each accretion episode they are able to accrete at the effective Eddington ratio of $f_{\text{Edd}} = 3.7$. Recent observations have suggested that moderate super-Eddington accretion ($1 < f_{\text{Edd}} < 10$) might be possible (Kelly & Shen 2013; Du et al. 2014; Page et al. 2013; Novak 2013).

The question is then how long can accretion maintain rates above the Eddington limit? Some authors argued that quasars lifetimes are much shorter then the Salpeter’s timescale (e.g., Richstone et al. 1998; Wyithe & Padmanabhan 2006). In that case, even massive BH seeds do not have enough time to grow to $10^9 M_\odot$ SMBH at $z \gtrsim 7$. A single BH seed with mass $10^9 M_\odot$ requires 9 e-folding times to produce $> 10^9 M_\odot$ SMBH at $z = 7$, while 100 $M_\odot$ seed requires 19 e-folding times for the same SMBH. Because of this requirement some previous works rejected Pop III star remnants as a possible candidates for SMBH seeds (e.g., Johnson et al. 2012, 2013).

In our model, growing SMBH at $z = 7$ does not require e-folding time larger than $\sim 4$ for $100 M_\odot$ BH seeds and effective $f_{\text{Edd}} = 3.7$. Thus, we have managed to reduce the time that BH needs to spend accreting. In our model, every major merger restarts the accretion, hence, instead of one BH constantly accreting for a long time (large number of e-folding times) we have several BHs in shorter (small number of e-folding times) accretion episodes. This approach increases the importance of mergers on SMBH growth.

Our model requires prolonged super-Eddington accretion which could produce strong feedback that stops the inflow of gas toward SMBH and disrupts the SMBH accretion long before 50 Myr. This could occur if we would assume the classical ”thin disc” accretion. However, we do not assume any accretion model in advance. The chosen values for the radiative efficiency and the Eddington ratio in our model should be regarded as the effective values, averaged over 50 Myr. BH could also grow through a sequence of short-lived intermittent phases of super-Eddington accretion ($f_{\text{Edd}} > 3.7$) without the feedback. Eddington ratio averaged over 50 Myr would then be $f_{\text{Edd}} = 3.7$ and effectively this model would produce the same SMBH as in the ”thin disc” model. The absence of feedback in the ”slim disc” model comes from $f_{\text{Edd}} = \epsilon \frac{M_{\text{BH}}}{M_{\text{Edd}}}$. BH mass growth rate could stay the same as radiative efficiency decreases and $f_{\text{Edd}}$ increases. In principle, given a sufficiently low efficiency, a super-Eddington BH may be emitting at sub-Eddington luminosity, thus eliminating BH feedback completely (Volonteri & Silk 2014).

In light of the recent observations of super-Eddington accretion mentioned above, we show that with a moderate super-Eddington accretion ($f_{\text{Edd}} = 3.7$) averaged over 50 Myr, even low mass seeds ($100 M_\odot$) could be progenitors of high redshift SMBHs ($z \sim 7$).

Note that our model is conservative for three reasons:

1. All BH seeds in our model have the same mass of $100 M_\odot$. It is possible that Pop III stars left BH seeds with masses up to $300 M_\odot$ (Bromm et al. 2009), or even $1000 M_\odot$ (Hirano et al. 2013). Larger mass of BH seeds would significantly reduce need for accretion which would, in turn, effectively lower the value of the effective Eddington ratio.

2. We combine two simulations which is also a conservative approximation. We do not know the exact merger history of low mass haloes in Millennium Simulation, but instead we take it from Millennium-II Simulation. We always chose the most common BHs from Millennium-II Simulation to be the seeds for newly formed haloes of Millennium Simulation. If we would have chosen just a few BHs with mass greater than the most common mass, that would strongly affect BH growth history. This in turn would reduce the Eddington ratio necessary for SMBH growth.

3. In our model BH growth is limited by the amount of gas that BH can accrete ($8 \times 10^{-4}$ of the host halo mass). This constraint comes from the BH mass scaling relations in the local Universe. However, the same scaling relation may not hold in the early Universe. Recently, Barnett et al. (2014) measured $\dot{M}_{\text{bh}}/\dot{M}_{\text{bulge}} \gtrsim 0.2$ for quasar ULAS J1120+0641 at $z = 7.1$. They found that the BH was growing much faster then the bulge relative to the mass measured in the local Universe. If BHs at high redshift had more gas available for the accretion, BH growth would be much easier which would also reduce the value of the effective Eddington ratio.

Some authors have speculated that BH mergers have a limited role in SMBH growth due to gravitational wave recoil (e.g., Merritt et al. 2004; Volonteri 2007). We calculate BH kick velocities and compare it to the escape velocities of their host haloes. We find that BHs with low masses reside in low mass haloes and have their kick velocities larger than the escape velocities from their host haloes, so they are sensitive to the gravitational wave recoil. This is a consequence of a simple model where all BH seeds have the same mass which is highly unlikely. Using a proper initial BH mass function with a wide range of possible masses would lower the value of gravitational wave recoil.

We note that, in order to get more accurate results, our model should be applied to the merger trees of a simulation that would have both resolution of Millennium-II Simulation, and the box size as in Millennium Simulation.

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