Stimulated Neutrino Conversion in the CERN Beam

M. C. Gonzalez-Garcia

Theory Division, CERN, CH-1211 Geneva 23, Switzerland.

F. Vannucci

LPNHE, Université Paris VII, F-75251 Paris CEDEX 05, France.

J. Castromonte

Departamento de Física y Matemáticas,
Universidad Peruana Cayetano Heredia, Lima, Perú.

Abstract

We discuss the possibility of searching for anomalous magnetic transitions of neutrinos in the CERN beam induced by the absorption or emission of low-energy photons in a high-quality resonant cavity such as the LEP radio-frequency cavities. With the attainable sensitivities of the present CERN neutrino detectors, this experiment would impose strong limits on this transition and on the radiative decay lifetime of neutrinos with masses in the range of interest to the resolution of the dark matter solar and atmospheric neutrino puzzles.

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The existing indications for non-zero neutrino masses include the deficit of solar electron neutrinos [1], the deficit in the atmospheric muon neutrino flux [2] and the indications favouring a hot component in the dark matter of the Universe [3]. All solar neutrino experiments [1] find fewer \( \nu_e \)'s than predicted theoretically. If the current solar models are correct, the explanation for this deficit relies on the oscillation of \( \nu_e \) to another neutrino species with a mass difference \( \Delta m^2_{\nu_e} \approx 10^{-5} \text{eV}^2 \). Some atmospheric neutrino experiments [2] also observe a deficit in the ratio of experimental-to-expected ratio of muon-like to electron-like events. An explanation for this anomaly relies on \( \nu_\mu \) oscillations into another flavour with a mass difference \( \Delta m^2_{\mu_\nu} \approx 10^{-2} - 10^{-3} \text{eV}^2 \).

Massive neutrinos also play an important role in cosmology as the substance of hot dark matter. Currently, the cosmological best-fit scenario includes a mixture of cold plus hot dark matter [3]. This translates into an upper limit on neutrino masses [4]: \( \sum \nu_i \lesssim 10 \text{eV} \). Putting all this information in a common framework and restricting ourselves to the three known neutrinos, we arrive at the scenario with three almost mass-degenerate neutrinos with the mass differences quoted above [5]. Such spectrum could arise, for instance, from the imposition of a cyclic permutation symmetry among the generations as pointed out by Harrison, Perkins and Scott [6].

Neutrinos with masses in this range could decay radiatively provided that they posses an anomalous transition magnetic moment of the form

\[
L_{\text{trans}} = \frac{1}{2} \bar{\nu'} \sigma_{\alpha\beta} (\mu + d \gamma_5) \nu F^{\alpha\beta} + \text{h.c.},
\]

which would give a lifetime for the \( \nu \to \nu' \gamma \) decay

\[
\tau^{-1} = (|a|^2 + |b|^2) \frac{(\Delta m^2)^3}{8\pi m^3}.
\]

where \(|a| = |\mu| (2 \text{Im}(\mu))\) and \(|b| = |d| (2 \text{Re}(d))\) for Dirac (Majorana) neutrinos.

The supernova SN1987 limits on this decay mode [7] only apply to neutrinos with non-degenerate masses. For mass-degenerate neutrinos there exist limits on this decay mode from a laboratory search for gamma-rays from decaying \( \bar{\nu}_e \)'s produced at a nuclear reactor [8].

In this letter we study the possibility of imposing stringent limits on this transition using the CERN neutrino beam. The CERN neutrino beam produces predominantly \( \nu_\mu \)'s with
fractions of 6% $\bar{\nu}_\mu$’s, 0.7% $\nu_e$’s and 0.2% $\bar{\nu}_e$’s. The mean energy of the beam is 27 GeV. At present two experiments, NOMAD [10] and CHORUS, [11] search for the appearance of $\nu_\tau$ in the CERN beam due to neutrino oscillations. They expect a sample of about 1 million $\nu_\mu$ charged current (CC) events and the experiments can reach sensitivities of few $\times 10^{-4}$ in the $\nu_\mu \rightarrow \nu_\tau$ channel.

The proposal is to use a resonant high-quality cavity intercepting the beam line to stimulate the neutrino conversion by absorption or emission of resonant photons inside the cavity, much as the experiment proposed by Matsuki and Yamamoto in Ref. [12] for solar or reactor neutrinos. The large number of photons in the cavity enhances the conversion and improves substantially the accessible range of lifetimes.

In a realistic vein, we will use as resonant cavity one of the LEP radio-frequency cavities. These are cylindrical cavities with a diameter of 60 cm and a length of 20 cm along the beam direction. The transition amplitude for the conversion is given by

$$T_{fi} = \int d^4 x L_{\text{trans}}(x) .$$

For highly relativistic neutrinos moving along the $z$ direction with helicity $s, s' = \pm 1$ and momentum $k, k'$, the wave function is given by

$$\nu^{(i)}(x) \approx \frac{\exp(-ik^{(i)} \cdot x)}{\sqrt{2V}} \begin{pmatrix} \phi_{s^{(i)}} \\ s^{(i)} \phi_{\bar{s}^{(i)}} \end{pmatrix}$$

with $\phi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\phi_{-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Here, $V$ is the volume in which the neutrino field is quantized.

We are going to consider the first transverse magnetic mode of the resonant cavity. The electric field is then aligned with the incident neutrino beam. This configuration will give the largest conversion rate since it corresponds to photons polarized in the same direction as the neutrino beam. The electromagnetic fields inside the cavity are given by:

$$E_z = E_0 J_0(x_0 r/R) \exp(-i\omega t)$$
$$B_\phi = \mp i E_0 J_1(x_0 r/R) \exp(-i\omega t)$$

where $r$ is the distance to the cavity axis in the transverse plane and $B_\phi$ is the azimuthal component of the magnetic field. All other components are zero. $J_i$ are the Bessel functions.
and $x_{01} = 2.402$ is the first zero of the $J_0$ function. $w$ is the characteristic frequency of the cavity $w = x_{01}/R = 1.4 \times 10^{-6}$ eV. $\alpha = \pm 1$ corresponds to the process of photon emission ($-1$) or absorption ($+1$).

The transition cross section is

$$
\sigma = \int \frac{d^3k'}{(2\pi)^3} \frac{V^2}{T} \sum_\alpha \left| \frac{1}{2\pi} (a + b) (s' - s) \int d^4x \left[ iB_x - sB_y \right] \exp(ik \cdot x) \right|^2
$$

$$
= \frac{4\mathcal{E}_0^2 |a + b|^2 V^2}{\pi^2} \sum_\alpha \int d^3k' \delta(E' - E - \omega) \frac{\sin^2(\beta/2)}{\beta^2} I^2(\xi)
$$

$$
= 8 \mathcal{E}_0^2 |a + b|^2 L \sum_\alpha \int d\xi \left| \frac{E + \alpha \omega}{k_z} \frac{\sin^2(\beta/2)}{\beta^2} \right| I^2(\xi)
$$

where $|a + b| = |\mu - d| (2|\text{Im}(\mu) - \text{Re}(d)|)$ for incident left-handed Dirac (Majorana) neutrinos. $V = \pi R^2 L$, $\beta = L(k'_z - k_z)$, and $\xi = k'_T R = \sqrt{(k'_x^2 + k'_y^2)R}$, and the function $I(\xi)$

$$
I(\xi) = \int_0^1 d\rho J_1(x_{01}\rho)J_1(\xi\rho)
$$

is non-negligible for $\xi \lesssim \mathcal{O}(10)$, as seen if Fig. 1. The last step in Eq.(6) is obtained from integration over the azimuthal angle and over $dk'_z$, this last one using the delta of energy condition.

The interpretation of Eq.(7) is the following. In the first line we see that the transition amplitude vanishes unless the helicity is flipped in the conversion $(ss' = -1)$, as expected from a magnetic transition. The $\delta$-function in the second line represents energy conservation resulting from integration over a large time interval $T \sim L/c \gg \hbar/E$. On the other hand since the space integral is not performed over an infinite volume, the photon momentum is not fixed. If the integral was performed over an infinite volume the functions verify

$$
\lim_{V \to \infty} V \sin(\beta/2)/\beta I(\xi) \to \delta(\Delta k_z)\delta(k'_T^2 - w^2), \text{ i.e. the momentum along the beam direction is conserved since the magnetic field lies in the transverse plane and the transverse momentum transfer must be equal to the photon energy. In consequence the transition would only be possible for energies } E = \frac{|\Delta m^2|}{2w}.
$$

Since we are restricted to the finite volume of the cavity, there is a spread on the photon momenta of the order of the inverse of the spatial dimension of the cavity ($\xi \sim \mathcal{O}(1)$, $\beta \sim \mathcal{O}(1)$). For these values the functions $I(\xi)$ and $\sin(\beta/2)/\beta$ are not negligible and the variables verify the energy conservation condition

$$
\xi^2 = R^2 \left(2\alpha\omega E + \Delta m^2 + w^2 - \frac{\beta^2}{L^2} - 2\frac{\beta}{L}|k| \right)
$$

(8)
independently of the value of the initial energy $E$ provided this is much larger than the photon energy $w$ and that $Ew/\Delta m^2 \gg 1$, i.e. neutrinos with any large enough energy to verify these conditions can absorb or emit a resonant photon in the cavity. For values of $\xi$ not large enough for $I(\xi)$ to be negligible, the condition on $\beta$ does not depend on $\xi$,

$$\beta \simeq \alpha Lw \Rightarrow \Delta k_Z = \alpha w.$$  \hspace{1cm} (9)

Therefore the characteristic energy-momentum transfer in the transition is extremely small compared with the incident neutrino energy. In other words, to a very good approximation the neutrino maintains its energy and momentum during the transition and the interaction with the magnetic field translates only in the change of its flavour and the flip of its helicity. The cross section is consequently independent of the neutrino energy, provided this is large enough.

Finally we can express the field amplitude $\mathcal{E}_0$ in terms of the number of photons in the cavity using the normalization condition

$$N_\gamma w = \frac{1}{2} \int dV (|\mathcal{E}|^2 + |\mathcal{B}|^2) = \frac{1}{2} V \mathcal{E}_0^2 J_1^2(x_{01}) \left[ 1 + \frac{4}{x_{01}^2} \right] = 0.23 V \mathcal{E}_0^2$$  \hspace{1cm} (10)

and we can write the transition rate as

$$R = \frac{\Delta N_\nu}{N_\nu} = \frac{\sigma}{\pi R^2} = \frac{16 N_\gamma |a + b|^2}{0.23 \pi R^2 Lw B}$$

$$\simeq 50 \kappa_B \left( \frac{Q}{10^9} \right) \left( \frac{P}{100 \text{ W}} \right) \left( \frac{m_{\nu}}{\text{eV}} \right) \left( \frac{eV^2}{\Delta m^2} \right) \left( \frac{10^{-6} \text{ eV}}{w} \right) \left( \frac{10 \text{ cm}}{L} \right) \left( \frac{8}{7} \right)$$  \hspace{1cm} (11)

with $B = \sin^2(Lw/2) \int d\xi I_2^2(\xi) \simeq 0.5$. In the last line we have used the relation between the number of photons, the energy, the quality factor $Q$, and the power supplied to the cavity $P$, $N_\gamma = 4 \times 10^{26}(Q/10^9)(P/100 \text{ W})(10^{-6} \text{ eV}/w)^2$ and we have defined

$$\kappa_B = |a + b|^2/(|a|^2 + |b|^2) \sim \mathcal{O}(1).$$  \hspace{1cm} (12)

We now turn to the estimate of the sensitivity attainable at the CERN experiments for this transition rate. Notice that the signature is different for Majorana or Dirac neutrinos as a consequence of the helicity flip in the transition. If neutrinos are Dirac particles the neutrinos in the beam will convert into sterile right-handed neutrinos. The signal would then be the disappearance of $\nu_\mu$’s from the incident beam. If, on the other hand neutrinos
are Majorana particles, the incoming neutrino will transform into a right-handed active neutrino, which will produce positive-charge leptons in its CC interactions with the detector. The signal will then be the appearance of $\tau^+$ or an excess of $e^+$.

For the purpose of illustration we use the NOMAD detector. NOMAD is in essence an electronic bubble chamber, with a continuous target of alternating panels of light material and drift chambers, followed by a transition radiation detector and an electromagnetic calorimeter. All the detectors are located inside a 0.4 T magnetic field. NOMAD measures essentially all the charged tracks and photons in the event, allowing for a good reconstruction of the transverse missing momentum in magnitude and direction. These features permit to study both the $\tau^+$ and the $e^+$ channels. The transverse area of the detector is $S = 3 \times 3 \text{ m}^2$ and the aim is to collect $1.1 \times 10^6 \nu_\mu$ CC interactions in the full detector volume. The expected fraction of $\nu_\mu$ CC interactions from neutrinos intercepted by the cavity is the fraction of solid angle covered by the cavity. This depends on where the cavity is located, since the beam has a Gaussian profile with $\sigma$ growing linearly with the distance to the production point and reaching $\sigma = 1 \text{ m}$ at the detectors. If the cavity is set at the point where the beam emerges from the ground at a distance of about 150 m from the detector $\sigma = 80 \text{ cm}$ and

$$N_{ev}^{cavity} = 1.1 \times 10^6 \frac{2\pi \int_0^{0.3} \exp(-r^2/0.8^2)rdr}{0.8^2 \left( \int_{-1.5}^{1.5} \exp(-x^2)dx \right)^2} = 1.5 \times 10^5. \tag{13}$$

With this number of events one could reach a sensitivity on the measurement of the ratio $R$ in Eq.(11) of the order of 1% for a “disappearance type” experiment. Correspondingly one expects $9 \times 10^3$ CC from $\bar{\nu}_\mu$ interactions which constitute the main source of background for the $\nu_\mu \rightarrow \bar{\nu}_\tau$ appearance. For the $\nu_\mu \rightarrow \bar{\nu}_e$ transition, one should observe an excess of $e^+$ over the $3 \times 10^2$ expected events from the interaction of the $\bar{\nu}_e$’s present in the beam. Comparing the interactions observed with the RF cavity on and off, a sensitivity $R \simeq 10^{-3}$ can be reached for the $e^+$ appearance experiments. It may be possible to push the sensitivity down to $R \simeq 10^{-4}$ for the $\bar{\nu}_\tau$ appearance channel.

Such an experiment would impose a limit on the anomalous transition magnetic moment. To illustrate this, take $d \approx -\mu$. Then

$$|\mu|^2 \lesssim 2 \times (10^{-5} - 10^{-7}) \mu_B^2 \tag{14}$$
for $R \lesssim 10^{-2} - 10^{-4}$. Assuming that there is no cancellation between $d$ and $\mu$, i.e. $\kappa_\gamma \sim \mathcal{O}(1)$ the experiment can also impose a lower limit on the lifetime of the neutrino radiative decay. In Fig. 2 we show the attainable lower limit on the decay lifetime for an applied power $P = 100$ W and a quality factor $Q = 2.5 \times 10^9$, characteristic of the LEP cavities. The limit is shown as a function of the neutrino mass for mass-squared differences in the interesting range for the solar and atmospheric neutrino problem. As can be seen in the figure, the attainable limits are comparable to the age of the Universe, $\tau = 10^{17}$ s, for $\Delta m^2/m < \sim 6 \times 10^{-5}$ eV.

Our calculation is also valid for the case $\Delta m^2 \ll m$, for which we get a limit

$$\left(\frac{\tau}{s}\right) \left(\frac{m}{eV}\right)^3 \gtrsim 2 \times (10^3 - 10^5),$$

much weaker than the existing limits from SN1987A data \cite{7} $(\tau/s)(eV/m) > 1.7 \times 10^{15}$, in the interesting mass range. The supernova limit is based on the non observation of an energetic gamma-ray burst produced by the neutrino decay. For almost-degenerate neutrinos the photon energy is suppressed with respect to the non-degenerate case, by a factor $\Delta m^2/m^2$ making the limit uninteresting for the parameter range indicated by solar, atmospheric and dark-matter data.

Limits on radiative decay are also available from reactor data \cite{8,9}. The limit in Ref. \cite{8} for non-degenerate neutrinos is $(\tau/s)(eV/m) > 3.8 \times 10$ clearly weaker than the supernova limit. For mass-degenerate neutrinos the limit is still valid but now it takes the form $(\tau/s)(eV/m) \gtrsim 3.8 \times 10(\Delta m^2/m^2)^{-a}$, where $(-a)$ is the slope of the neutrino spectrum of the reactor, $a = 3-4$ \cite{13}, becoming also uninteresting. In Ref. \cite{9} a limit is imposed for almost-degenerate neutrinos of the order $(\tau/s)(eV/m) \gtrsim 10^{-1} F(\Delta m^2/m^2)$ where $F(\Delta m^2/m^2) \leq 1$ in the range $10^{-7} \leq \Delta m/m \leq 10^{-1}$ and $F$ vanishes out of this range. This constraint is much weaker than the attainable limit with the experiment proposed here. Notice also that reactor limits only apply to $\nu_e$ radiative decays.

We conclude that an experiment searching for anomalous magnetic neutrino transitions at the CERN beam, stimulated with the use of a LEP radio-frequency cavity would dramatically improve the existing limits on neutrino transition magnetic moments. The characteristic signature would be the appearance of $\tau^+$ or an excess of $e^+$ if the neutrinos are Majorana particles. If neutrinos are Dirac particles one should expect a decrease in the number of detected $\mu^-$. Comparing the data obtained with and without the cavity, it is
possible to impose severe limits on the corresponding radiative decay lifetime for neutrinos with masses in the interesting cosmological range and mass differences compatible with a solution to the solar and atmospheric neutrino puzzles.
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FIG. 1. The form factor $I(\xi)$. 
FIG. 2. Lower limits on the radiative decay lifetime attainable at the CERN experiments by stimulated conversion, assuming a sensitivity $R = 10^{-3}$ as a function of the neutrino mass for almost mass-degenerate neutrinos with mass squared differences in the range $10^{-2} – 10^{-5}$. 