Accelerating Bianchi Type-V Cosmology with Perfect Fluid and Heat Flow in Sáez-Ballester Theory

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Abstract

In this paper we discuss the law of variation of scale factor $a = (t^{k}e^{t})^{\frac{1}{n}}$ which yields a time-dependent deceleration parameter (DP) representing a new class of models that generate a transition of universe from the early decelerated phase to the recent accelerating phase. Exact solutions of Einstein’s modified field equations with perfect fluid and heat conduction are obtained within the framework of Sáez-Ballester scalar-tensor theory of gravitation and the model is found to be in good agreement with recent observations. We find, for $n = 3$, $k = 1$, the present value of DP in derived model as $q_{0} = -0.67$ which is very near to the observed value of DP at present epoch. We find that the time-dependent DP is sensible for the present day Universe and give an earmark description of evolution of universe. Some physical and geometric properties of the models are also discussed.

Key words: Bianchi type-V universe, Exact solution, Alternative gravitation theory, Accelerating universe

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1 Introduction

In the last few decades several new theories of gravitation, carefully weighed to be alternative to Einstein’s theory of gravitation, have been developed according to an orderly plan. In alternative theories of gravitation, scalar tensor theories proposed by Brans andDicke [1], Nordvedt [2], Wagoner [3], Rose [4], Dun [5], Sáez and Ballester [6], Barber [7], Lau and Prokhovnik [8] are most important among them. There are two categories of gravitational theories involving a classical scalar field $\phi$. In first category the scalar field $\phi$ has the dimension of the inverse of the gravitational constant $G$ among which the Brans-Decke theory [1] is of considerable importance and the role of the scalar field is confined to its effect on gravitational field equations. Brans and Decke formulated a scalar-tensor theory of gravitation which introduces an additional scalar field $\phi$ besides the metric tensor $g_{ij}$ and a dimensionless coupling constant $\omega$. This theory goes to general relativity for large values of the coupling constant $\omega > 500$. In the second category of theories involve a dimensionless scalar field. Sáez and Ballester [6] developed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field, an anti-gravity regime appears. This theory suggests a possible way to solve the missing-matter problem in non-flat FRW cosmologies. The Scalar-Tensor theories of gravitation play an important role to remove the graceful exit problem in the inflation era [9]. In earlier literature, cosmological models within the framework of Sáez-Ballester scalar-tensor theory of gravitation, have been studied by Singh and Agrawal [10, 11], Ram and Tiwari [12], Singh and Ram [13]. Mohanty and Sahu [14, 15] have studied Bianchi type-VI\textsubscript{0} and Bianchi type-I models in Sáez-Ballester theory. In recent years, Tripathi et al. [16], Reddy et al. [17, 18], Reddy and Naidu [19], Rao et al. [20-22], Adhav et al. [23], Katore et al. [24], Sahu [25], Singh [26], Pradhan and Singh [27], Socorro and Sabido [28] and Jamil et al. [29] have obtained the solutions in Sáez-Ballester scalar-tensor theory of gravitation in different context. Recently, Naidu et al. [30, 31] and Reddy et al. [32] have studied LRS Bianchi type-II models in Sáez and Ballester scalar tensor theory of gravitation in different context.
Recently, Ram et al. [33] obtained Bianchi type-V cosmological models with perfect fluid and heat flow in Sáez and Ballester theory by considering a variation law for Hubble’s parameter with average scale factor which yields constant value of the deceleration parameter. In literature it is common to use a constant deceleration parameter [34–40] as it duly gives a power law for metric function or corresponding quantity. But it is worth mentioned here that the universe is accelerated expansion at present as observed in recent observations of Type Ia supernova [41–45] and CMB anisotropies [46–48] and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a Universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping [49–51]. So, in general, the DP is not a constant but time variable. Recently, Pradhan et al. [52, 53] investigated some new exact Bianchi type-I cosmological models in scalar-tensor theory of gravitation with time dependent deceleration parameter.

Motivated by the above discussions and observational facts, in this paper, we propose to study Bianchi type-V universe with perfect fluid and heat flow in Sáez-Ballester scalar-tensor theory of gravitation by considering a law of variation of scale factor as increasing function of time which yields a time dependent DP. The out line of the paper is as follows: In Sect. 2, the metric and basic equations are described. Section 3 deals with the field equations and their quadrature solutions. The law of variation of scale factor is given in Sect. 4. Subsection 4.1 deals with the physical and geometric properties of the universe. Finally, conclusions are summarized in the last Sect. 5.

2 The Metric and Basic Equations

We consider anisotropic Bianchi type-V line element, given by

\[ ds^2 = dt^2 - A^2(t)dx^2 - e^{2mz}[B^2(t)dy^2 + C^2(t)dz^2], \]  

where \( A, B \) and \( C \) are metric functions and \( m \) is a constant.

We define the following parameters to be used in solving Einstein’s field equations for the metric (1).

The average scale factor \( a \) of Bianchi type-V model (1) is defined as

\[ a = (ABC)^\frac{1}{3}. \]  

A volume scale factor \( V \) is given by

\[ V = a^3 = ABC. \]

In analogy with FRW universe, we also define the generalized Hubble parameter \( H \) as

\[ H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3), \]

where \( H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B} \) and \( H_3 = \frac{\dot{C}}{C} \) are directional Hubble factors in the directions of \( x \)-, \( y \)- and \( z \)-axes respectively. Here, and also in what follows, a dot indicates ordinary differentiation with respect to \( t \).

Further, the deceleration parameter \( q \) is given by

\[ q = \frac{\ddot{a}}{a^2}. \]

We introduce the kinematical quantities such as expansion scalar \( (\dot{\theta}) \), shear scalar \( (\sigma^2) \) and anisotropy parameter \( (A_m) \), defined as follows:

\[ \theta = u^i_{,i}, \]

\[ \sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij}, \]

\[ A_m = \frac{1}{3} \sum_{i=1}^{3} \left( \frac{H_i - H}{H} \right)^2, \]
where \( u^i = (0, 0, 0, 1) \) is the matter 4-velocity vector and

\[
\sigma_{ij} = \frac{1}{2} \left( u_{i;\alpha} P_j^\alpha + u_{j;\alpha} P_i^\alpha \right) - \frac{1}{3} \theta P_{ij},
\]  

(9)

Here the projection tensor \( P_{ij} \) has the form

\[
P_{ij} = g_{ij} - u_i u_j.
\]  

(10)

These dynamical scalars, in Bianchi type-V, have the forms

\[
\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C},
\]  

(11)

\[
2\sigma^2 = \left[ \left( \frac{\dot{A}}{A} \right)^2 + \left( \frac{\dot{B}}{B} \right)^2 + \left( \frac{\dot{C}}{C} \right)^2 \right] - \frac{\theta^2}{3},
\]  

(12)

### 3 Field Equations and their Quadrature Solutions

The scalar tensor theories are the generalization of Einstein’s theory of gravitation in which the metric is generated by a scalar gravitational field together with non-gravitational field (matter). We, consider the simple case of a homogeneous but anisotropic Bianchi type-I model with matter term with a scalar field \( \phi \). Our model is based on a non-standard scalar-tensor theory, defined in Sáez and Ballester [6] with a dimensionless scalar field \( \phi \) and tensor field \( g_{ij} \). This alternative theory of gravitation is combined scalar and tensor fields in which the metric is coupled with a dimensionless scalar field. We assume the Lagrangian

\[
L = R - \omega \phi^k \phi_i \phi^i,
\]  

(13)

\( R \) being the scalar curvature, \( \phi \) a dimensionless scalar field, \( \omega \) and \( k \) arbitrary dimensionless constants and \( \phi^i \) the contraction \( \phi_{\alpha} g^{\alpha i} \). Here a comma (,) and a semicolon (;) stand for partial and covariant derivative with respect to cosmic time \( t \) respectively.

From the above Lagrangian we can establish the action

\[
I = \int\sum (L + 8\pi L_m) (-g)^{1/2} dx^1 dx^2 dx^3 dx^4,
\]  

(14)

where \( L_m \) is the matter Lagrangian, \( g \) is the determinant of the matrix \( g_{ij} \), \( x^i \) are the coordinates, \( \sum \) is an arbitrary region of integration. When \( k = 0 \), our model is just the Einstein gravity with a massless minimally coupled scalar field coupled to gravity. By considering arbitrary independent variations of the metric and the scalar field vanishing at the boundary of \( \sum \), the variation principle

\[
\delta I = 0,
\]  

(15)

leads to a generalized Einstein equation

\[
G_{ij} - \omega \phi^k \left( \phi_i \phi_j - \frac{1}{2} g_{ij} \phi^l \phi^l \right) = -8\pi T_{ij},
\]  

(16)

\[
2\phi^k \phi^i_j + k \phi^{k-1} \phi_i \phi^l = 0,
\]  

where \( G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} \) is the Einstein tensor; \( T_{ij} \) is the stress-energy tensor of the matter Lagrangian \( L_m \).

Since the action \( I \) is a scalar, it can be easily proved that the equation of motion

\[
T_{ij}^{i} = 0,
\]  

(17)

are consequences of the field equations.
The energy-momentum tensor is the source of gravitational field through which the effect of the perfect fluid with heat flow in the evolution of the universe is performed. The energy-momentum tensor of a perfect fluid with heat flow has the form

\[ T_{ij} = (\rho + p)u_i u_j - p g_{ij} + h_i u_j + h_j u_i, \]  

where \( \rho \) is the energy density, \( p \) the thermodynamic pressure, \( u_i \) the four-velocity of the fluid and \( h_i \) is the heat flow vector satisfying

\[ g_{ij} u_i u_j = 1, \]  

and

\[ h^i u_i = 0. \]

We assume that the heat flow is in \( x \)-direction only so that \( h_i = (h_1, 0, 0, 0) \), \( h_1 \) being a function of time. For the energy-momentum tensor [13] and Bianchi type-V space-time [11], the Einstein’s modified field equations [10], yield the following six independent equations as

\[ \ddot{B} + \frac{\dot{C}}{C} + \frac{\ddot{B}}{B} C + \frac{m^2}{A^2} = -p + \frac{1}{2} \omega \phi^r \dot{\phi}^2, \]  
\[
\ddot{A} + \frac{\dot{C}}{C} + \frac{\ddot{A}}{A} C + \frac{m^2}{A^2} = -p + \frac{1}{2} \omega \phi^r \dot{\phi}^2, \]  
\[
\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} A + \frac{\dot{A} \dot{B}}{A B} - \frac{m^2}{A^2} = -p + \frac{1}{2} \omega \phi^r \dot{\phi}^2, \]  
\[
\frac{\ddot{A} \ddot{B}}{A B} + \frac{\dot{A} \dot{C}}{A C} + \frac{\dot{B} \dot{C}}{B C} - \frac{3 m^2}{A^2} = p - \frac{1}{2} \omega \phi^r \dot{\phi}^2, \]  
\[
\dot{h} + \phi \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \frac{r}{2} \dot{\phi}^2 = 0. \]

The law of energy-conservation equation \( T^{ij}_{;j} = 0 \) gives

\[ \dot{\rho} + (p + \rho) \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{2m}{A^2} h_1. \]  

Equations (21)-(24) can be written in terms of \( H, q, \sigma^2 \) and \( \phi \) as

\[ p = H^2 (2q - 1) - \sigma^2 + \frac{m^2}{A^2} + \frac{1}{2} \omega \phi^r \dot{\phi}^2, \]  
\[
\rho = 3H^2 - \sigma^2 - \frac{3 m^2}{A^2} + \frac{1}{2} \omega \phi^r \dot{\phi}^2. \]

To solve the field equations (21)-(24), we follow the well established method of quadrature. For this, subtracting Eq. (21) from (22), Eq. (21) from (23) and Eq. (22) from (23), we get the following relations respectively:

\[ \frac{A}{B} = d_1 \exp \left( k_1 \int \frac{dt}{a^3} \right), \]  
\[ \frac{A}{C} = d_2 \exp \left( k_2 \int \frac{dt}{a^3} \right), \]  
\[ \frac{B}{C} = d_3 \exp \left( k_3 \int \frac{dt}{a^3} \right), \]

where \( d_1, d_2, d_3 \) and \( k_1, k_2, k_3 \) are constants of integration. From Eqs. (30)-(32), the metric functions can be obtained explicitly as

\[ A(t) = l_1 a \exp \left( \frac{X_1}{3} \int \frac{dt}{a^3} \right), \]
\[ B(t) = l_2 a \exp \left( \frac{X_2}{3} \int \frac{dt}{a^3} \right), \quad (34) \]

\[ C(t) = l_3 a \exp \left( \frac{X_3}{3} \int \frac{dt}{a^3} \right), \quad (35) \]

where

\[ l_1 = \sqrt[3]{d_1 d_2}, \quad l_2 = \sqrt[3]{d_1^{-1} d_3}, \quad l_3 = \sqrt[3]{(d_2 d_3)^{-1}}, \]

\[ X_1 = k_1 + k_2, \quad X_2 = k_3 - k_1, \quad X_3 = -(k_2 + k_3), \]

and the constants \( X_1, X_2, X_3 \) and \( l_1, l_2, l_3 \) satisfy the relations

\[ X_1 + X_2 + X_3 = 0, \quad l_1 l_2 l_3 = 1. \quad (36) \]

The quadrature expression for the dimensionless scalar field function \( \phi \), from Eq. (26), is found as

\[ \phi = \left[ \frac{\phi_0 (r + 2)}{2} \int \frac{dt}{a^3} \right]^{2/(r+2)}, \quad (37) \]

where \( \phi_0 \) is a constant.

It is clear from Eqs. (33)-(37) that once we get the value of the average scale factor \( a \), we can easily calculate the metric functions \( A, B, C \) and the scalar function \( \phi \). In the next section, we are going to assume the value of average scale factor \( a \).

### 4 Law of Variation of Scale Factor \( a = (t^k e^{t})^{\frac{1}{n}} \)

Following Yadav [53] and Pradhan and Amirhashchi [55], we assume that

\[ a = (t^k e^{t})^{\frac{1}{n}}, \quad (38) \]
where $k$ and $n$ are positive constants.

From (38), we obtain the time varying deceleration parameter as

$$q = \frac{nk}{(t + k)^2} - 1.$$  \hfill (39)

From Eq. (39), we observe that $q > 0$ for $t < \sqrt{nk} - k$ and $q < 0$ for $t > \sqrt{nk} - k$. It is observed that for $n \geq 3$ and $k = 1$, our model is evolving from decelerating phase to accelerating phase. Also, recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range $-1 < q < 0$. It follows that in our derived model, one can choose the value of DP consistent with the observation. Figure 1 depicts the deceleration parameter $(q)$ versus time which gives the behaviour of $q$ from decelerating to accelerating phase for different values of $(n, k)$ which is consistent with recent observations of Type Ia supernovae (Perlmutter et al. [11]; Riess et al. [42, 43]; Tonry et al. [43]; Clocchiatti et al. [44]).

Using (38) in Eqs. (33)-(35), we obtain the following expressions for scale factors:

$$A(t) = l_1(t^k e^t)^{1/n} \exp \left( \frac{X_1}{3} \int \frac{dt}{(t^k e^t)^{3/n}} \right),$$  \hfill (40)

$$B(t) = l_2(t^k e^t)^{1/n} \exp \left( \frac{X_2}{3} \int \frac{dt}{(t^k e^t)^{3/n}} \right),$$  \hfill (41)

$$C(t) = l_3(t^k e^t)^{1/n} \exp \left( \frac{X_3}{3} \int \frac{dt}{(t^k e^t)^{3/n}} \right).$$  \hfill (42)

Hence the geometry of the universe (11) is reduced to

$$ds^2 = dt^2 - l_1^2(t^k e^t)^{2/n} \exp \left( \frac{2X_1}{3} \int \frac{dt}{(t^k e^t)^{3/n}} \right) dx^2.$$
Figure 3: The plot of energy density $\rho$ versus $t$. Here $\omega = \alpha = \beta_2 = X_1 = m = \phi_0 = 1$.

$$e^{2mx} \left[ l_2^2 (t^k e^t)^{2/n} \exp \left( \frac{2X_2}{3} \int \frac{dt}{(t^k e^t)^{3/n}} \right) dy^2 + l_3^2 (t^k e^t)^{2/n} \exp \left( \frac{2X_3}{3} \int \frac{dt}{(t^k e^t)^{3/n}} \right) dz^2 \right].$$

(43)

4.1 Some Physical and Geometric Properties of Model

In derived model (43), the present value of DP is estimated as

$$q_0 = -1 + \frac{n}{mH_0^2 t_0},$$

(44)

where $H_0$ is the present value of Hubble’s parameter and $t_0$ is the age of the universe at present epoch. If we set $n = 3$ and $k = 1$ in Eq. (44), we obtain $q_0 = -0.67$. This value is very near to the observed value of DP (i.e., $q_0 \approx -0.77$) at present epoch (see Cunha et al. [56]). Hence, we restraint $n = 3$ and $k = 1$ in the left over discussions of the model and graphical display of physical parameters.

The solution for scalar function $\phi$, from (37), is obtained as

$$\phi = \frac{\phi_0 (r+2)}{2} \int \frac{dt}{(t^k e^t)^{3/n}} \right]^{2/(r+2)}.$$  

(45)

By using the values of the metric functions from eqs. (40)-(42) into Eq. (46), the expression for the heat flow function $h_1$ is given by

$$h_1 = \frac{m\beta_1}{3(t^k e^t)^{3/n}},$$

(46)

where $\beta_1 = 2X_1 - X_2 - X_3$. 
From Eqs. (28) and (29) the isotropic pressure \( p \) and the energy density \( \rho \), for model (43), are obtained as

\[
p = \frac{2k}{nt^2} - \frac{3}{n^2} \left(1 + \frac{k}{t}\right)^2 + \left[\frac{1}{2}\omega_0^2 - \frac{\beta_2}{18}\right] \frac{1}{(tk^e t)^6/n}
\]

\[
+ \left[\frac{m^2}{t^2 (tk^e t)^2/n} \exp\left(\frac{-2X_1}{3}\int \frac{dt}{(tk^e t)^3/n}\right)\right],
\]

\[
\rho = \frac{3}{n^2} \left(1 + \frac{k}{t}\right)^2 - \left[\frac{1}{2}\omega_0^2 - \frac{\beta_2}{18}\right] \frac{1}{(tk^e t)^6/n}
\]

\[
- \left[\frac{3m^2}{t^2 (tk^e t)^2/n} \exp\left(\frac{-2X_1}{3}\int \frac{dt}{(tk^e t)^3/n}\right)\right],
\]

In view of (36), it is observed that the above set of solutions satisfy the energy conservation equation (27) identically and hence represent exact solutions of the Einstein’s modified field equations (21)-(26). From Eqs. (47) and (48), we observe that isotropic pressure \( p \) and the energy density \( \rho \) are always positive and decreasing function of time and both approach to zero as \( t \to \infty \). Figures 2 and 3 depict \( p \) and \( \rho \), respectively, versus time \( t \) showing the positive decreasing function of \( t \) and approaching to zero at \( t \to \infty \).

The expressions for physical parameters such as spatial volume \( V \), directional Hubble parameters \( H_i, i = 1, 2, 3 \), Hubble parameter \( H \), scalar of expansion \( \theta \), shear scalar \( \sigma \) and the anisotropy parameter \( A_m \) for model (43) are, respectively, given by

\[
V = (tk^e t)^\frac{2}{3},
\]

\[
H_i = \frac{1}{n} \left(1 + \frac{k}{t}\right) + \frac{X_i}{3(tk^e t)^{3/n}},
\]

\[
\theta = 3H = \frac{3}{n} \left(1 + \frac{k}{t}\right),
\]

Figure 4: The plot of anisotropic parameter \( A_m \) versus \( t \). Here \( \alpha = \beta_2 = 1 \).
The dynamics of the mean anisotropic parameter depends on the constant \( \beta_2 = X_1^2 + X_2^2 + X_3^2 \). From Eq. (53), we observe that at late time when \( t \to \infty \), \( A_m \to 0 \). Thus, our model has transition from initial anisotropy to isotropy at present epoch which is in good harmony with current observations. Figure 4 depicts the variation of anisotropic parameter \( (A_m) \) versus cosmic time \( t \). From the figure, we observe that \( A_m \) decreases with time and tends to zero as \( t \to \infty \). Thus, the observed isotropy of the universe can be achieved in our model at present epoch.

It is important to note here that \( \lim_{t \to 0} \left( \frac{\rho}{T^2} \right) \) spread out to be constant. Therefore the model of the universe goes up homogeneity and matter is dynamically negligible near the origin. This is in good agreement with the result already given by Collins [58].

The flow of heat along the x-direction was maximum in early universe, and it diminishes as \( t \to \infty \). Figure 5 describe the variation of heat flow versus cosmic time \( t \) which shows the nature of \( h_1 \). From Eqs. (46) and (52), we also observe that \( \frac{\sigma^2}{n^3} = \text{constant} \) which shows that shear scalar is proportional to heat conduction.
5 Concluding Remarks

In this paper we have studied a spatially homogeneous and anisotropic Bianchi type-V space-time within the framework of the scalar-tensor theory of gravitation proposed by Sáez and Ballester [6]. The field equations have been solved exactly with suitable physical assumptions. The solutions satisfy the energy conservation Eq. (27) identically. Therefore, new, exact and physically viable Bianchi type-V model has been obtained. To find the deterministic solution, we have considered scale factor which yields time dependent deceleration parameters. As we have already discussed in Introduction that for a Universe which was deceleration in past and accelerating at present time, the DP must show signature flipping [49-51] and so there is no scope for a constant DP. The main features of the model are as follows:

- The model is based on exact and new solutions of Einstein’s modified field equations for the anisotropic Bianchi type-V space-time filled with perfect fluid and heat flow.
- Our special choice of scale factor yields a time dependent deceleration parameter which represents a model of the Universe which evolves from decelerating phase to an accelerating phase. This scenario is consistent with recent observations (Perlmutter et al. [41]; Riess et al. [42]; Tonry et al. [43]; Clocchiatti et al. [44]).
- Our whole discussions have been concentrated by restraining \( n = 3, k = 1 \). By this choice, we find the present value of deceleration parameter in derived model as \( q_0 = -0.67 \). This value is very near to the observed value of DP (i.e., \( q_0 \approx -0.77 \)) at present epoch (see Cunha et al. [56]).
- For different choice of \( n \) and \( k \), we can generate a class of viable cosmological models of the universe in Bianchi type-V space-time. For example, if we set \( n = 2 \) in Eq. (38), we find \( a = \sqrt{t e^t} \) which is used by Pradhan and Amirhashchi [55] in studying the accelerating dark energy models in Bianchi type-V space-time and Pradhan et al. [53] in studying Bianchi type-I in scalar-tensor theory of gravitation. If we set \( k = 1, n = 2 \) in Eq. (38), we find \( a = \sqrt{t e^t} \) which is utilized by Amirhashchi et al. [59] in studying interacting two-fluid scenario for dark energy in FRW universe. If we set \( k = 1, n = 1 \) in Eq. (38), we find \( a = t e^t \) which is exercised by Pradhan et al. [60] to study the dark energy model in Bianchi type-VI\(_0\) universe. It is observed that such models are also in good harmony with current observations.
- It has been observed that \( \lim_{t \to 0} \left( \frac{\rho}{\theta} \right)^2 \) turn out to be constant. Thus the model approaches homogeneity and matter is dynamically negligible near the origin.
- We also observe that \( \frac{\sigma^2}{h_1} = \text{constant} \) which shows that shear scalar is proportional to heat conduction (i.e. \( \sigma \propto h_1 \)).

Thus, the solutions demonstrated in this paper may be useful for better understanding of the evolution of the universe in Bianchi type-V space-time within the framework of Sáez-Ballester scalar-tensor theory of gravitation. The solutions presented here can be one of the potential candidates to describe the observed universe.

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References

[1] C.H. Brans, R.H. Dicke, Phys. Rev. A, 124, 925 (1961).
[2] K. Nordverdt, The Astrophys. J. 161, 1059 (1970).
[3] R.V. Wagoner, Phys. Rev. D 1, 3209 (1970).
[4] D.K. Ross, Phys. Rev. D 5, 284 (1972).
[5] K.A. Dunn, J. Math. Phys. 15, 2229 (1974).
[6] D. Sáez and V.J. Ballester, Phys. Lett. A 113, 467 (1985).
[7] G.A. Barber, Gen. Rel. Gravit. 14, 117 (1985).
[8] D. La, P.J. Steinhardt, Phys. Rev. Lett. 62, 376 (1989).
[9] L.O. Piemental, Mod. Phys. Lett. A 12, 1865 (1997).
[10] T. Singh, A.K. Agrawal, Astrophys. Space Sci. 182, No. 2, (1991), 289-312.
[11] T. Singh, A.K. Agrawal, Astrophys. Space Sci. 191, 61 (1992).
[12] S. Ram, S.K. Tiwari, Astrophys.Space Sci. 259, 91 (1998).
[13] C.P. Singh and S. Ram, Astrophys. Space Sci. 284, 1999 (2003).
[14] G. Mohanty, S.K. Sahu, Astrophys. Space Sci. 288, 509 (2003).
[15] G. Mohanty, S. K.Sahu, Astrophys. Space Sci. 291, 75 (2004), 75-83.
[16] S.K. Tripathi, S.K. Nayak, S.K. Sahu, T.R. Routray, Int. J. Theor. Phys. 48, 213 (2009).
[17] D.R.K. Reddy, R.L. Naidu, V.U.M. Rao, Astrophys. Space Sci. 306, 185 (2006).
[18] D.R.K. Reddy, P. Govinda, R.L. Naidu, Int. J. Theor. Phys. 47, 2966 (2008).
[19] D.R.K. Reddy, R.L. Naidu, Astrophys. Space Sci. 312, No. 99 (2007).
[20] V.U.M. Rao, T. Vinutha, M.V. Shanthi, G.S.D. Kumari, Astrophys. Space Sci. 317, No. 89 (2008).
[21] V.U.M. Rao, M.V. Shanthi, T. Vinutha, Astrophys. Space Sci. 317, 27 (2008).
[22] V.M.U. Rao, G.S.D. Kumari, K.V.S. Sireesha, Astrophys. Space Sci. 335, 635 (2011).
[23] K.S. Adhav, M.R. Ugale, C.B. Kale, M.P. Bhende, Int. J. Theor. Phys. 46, 3122 (2007).
[24] S.D. Katore, K.S. Adhav, A.Y. Shaikh, N.K. Sarkate, Int. J. Theor. Phys. 49, 2558 (2010).
[25] S. K. Sahu, Jour. Mod. Phys. 1, 67 (2010).
[26] C. P. Singh, Braz. Jour. Phys. 39, 669 (2009).
[27] A. Pradhan and S.K. Singh, Elec. Jour. Theor. Phys. 7, 407 (2010).
[28] J. Socorro, M. Sabido, Revista Mexicana De Fisica, 56, 166 (2010).
[29] M. Jamil, S. Ali, D. Momeni, R. Myrzakulov, arXiv:1201.0895[physics.gen-ph] (2012).
[30] R.L. Naidu, B. Satyanarayana, D.R.K. Reddy, Astrophys. Space Sci. 338, 333 (2012).
[31] R.L. Naidu, B. Satyanarayana, D.R.K. Reddy, Astrophys. Space Sci. 338, 351 (2012).
[32] D.R.K. Reddy, B. Satyanarayana, R.L. Naidu, Astrophys. Space Sci. DOI 10.1007/s10509-012-1007-8 (2012).
[33] S. Ram, M. Zeyauddin, C.P. Singh, Pramana - Journal of Physics, 72, 415 (2009).
[34] A.K. Yadav, Astrophys. Space Sci. 335, 565 (2011).
[35] S. Kumar, A.K. Yadav, Mod. Phys. Lett. A 26, 647 (2011).
[36] Ö. Akarsu, C.B. Kilinc, Gen. Relat. Gravit. 42, 119 (2010).
[37] A. Pradhan, H. Amirhashchi, Astrophys. Space Sci. 332, 441 (2011).
[38] H. Amirhashchi, A. Pradhan, H. Zainuddin, Int. J. Theor. Phys. 50, 3529 (2011).
[39] M.K. Verma, H. Zeyauddin, S. Ram, Rom. Jour. Phys. 56, 616 (2011).
[40] A.K. Yadav, Rom. Jour. Phys. **56**, 609 (2011).
[41] S. Perlmutter, et al., Astrophys. J. **517**, 565 (1999).
[42] A.G. Riess, et al., Astron. J. **116**, 1009 (1998).
[43] J.L. Tonry, et al., Astrophys. J. **594**, 1 (2003).
[44] A. Clocchiatti, et al., Astrophys. J. **642**, 1 (2006).
[45] A.G. Riess, et al., Astrophys. J. **607**, 665 (2004).
[46] C.L. Bennett, et al., Astrophys. J. Suppl. **148**, 1 (2003).
[47] P. de Bernardis, et al., Nature, **404**, 955 (2000).
[48] S. Hanany, et al., Astrophys. J. **545**, L5 (2000).
[49] T. Padmanabhan and T. Roychowdhury, Mon. Not. R. Astron. Soc. **344**, 823 (2003).
[50] L. Amendola, Mon. Not. R. Astron. Soc. **342**, 221 (2003).
[51] A.G. Riess, et al., Astrophys. J. **560**, 49 (2001).
[52] A. Pradhan, A.S. Dubey, R.K. Khare, Rom. Jour. Phys. **57**, No. 3-4, (2012) (to appear).
[53] A. Pradhan, A.K. Singh, H. Amirhashchi, arXiv:1204.5173[physics.gen-ph] (2012).
[54] A.K. Yadav, arXiv:1204.3620[physics.gen-ph] (2012).
[55] A. Pradhan, H. Amirhashchi, Mod. Phys. Lett. A **26**, 2261 (2011).
[56] C.E. Cunha, M. Lima, H. Ogaizu, J. Frieman, H. Lin, Mon. Not. Roy. Astron. Soc. **396**, 2379 (2009).
[57] M.A.H. MacCallum, Commun. Math. Phys. **20**, 57 (1971).
[58] C.B. Collins, J. Math. Phys. **18**, 2116 (1977).
[59] H. Amirhashchi, A. Pradhan, B. Saha, Chin. Phys. Lett. **28**, 039801 (2011).
[60] A. Pradhan, R. Jaiswal, K. Jotania, R. K. Khare, Astrophys. Space Sci. **337**, 401 (2012).