Toward coupling flow driven and magnetically driven dynamos

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Abstract. Most large-scale dynamo research for astrophysical rotators focuses on interior flow driven helical dynamos (FDHDs), but larger scale coronal fields most directly influence observations. It is thus important to understand the relationship between coronal and interior fields. Coronal field relaxation is actually a type of magnetically dominated helical dynamo (MDHD). MDHDs also occur in fusion plasma devices where they drive a system toward its relaxed state in response to magnetic helicity injection that otherwise drives the system away from this state. Global scale fields of astrophysical rotators and jets are thus plausibly produced by a direct coupling between an interior FDHD and a coronal MDHD, interfaced by magnetic helicity transport through their mutual boundary. Tracking the magnetic helicity also elucidates how both FDHD and MDHDs evolve and saturate. The utility of magnetic helicity is unhampered by its non-gauge invariance since physical fields can always be recovered.
1. Introduction

As emphasized in [1], from which the present paper has evolved, the term ‘dynamo’ can induce confusion because it has different meanings not only within the astrophysical context but also when compared to laboratory plasma dynamos of magnetically dominated environments. Many astrophysicists think of dynamos as the flow driven amplification of magnetic energy. Others think specifically of the ‘large-scale’ flow driven helical dynamo (FDHD), where field is amplified on scales larger than the input driving flow. Researchers immersed in the problem of flow driven amplification of an initially weak magnetic field sometimes wonder what role a dynamo could possibly play in a magnetically dominated environment, and yet these are the environments for which laboratory plasma dynamos operate.

In this focus issue article I pursue several goals: (i) I streamline, update and re-organize the conceptual relation between helical dynamos types developed in [1]; (ii) I strongly emphasize the need to couple FDHDs and magnetic driven helical dynamos (MDHDs) in astrophysics culminating with a simple steady-state relation between the coronal and interior dynamos to stimulate specific further work. In section 2, I describe the differences and similarities between FDHDs and MDHDs, the environments in which they occur, and the energy budgets. In section 3 all of the equations needed to follow their evolution are given. In sections 4 and 5, I summarize how closed volume and open volume FDHD and MDHD and their respective saturations can be described by different cases of a unified framework that tracks the spatial and spectral flow of magnetic helicity. Section 6 outlines the most minimalistic paradigm of how coupled FDHDs and MDHDs operate symbiotically in astrophysics. Section 7 is the conclusion.

2. Conceptually distinguishing dynamo types and their environments

Three types of dynamos need to be distinguished: (i) non-helical flow driven (ii) FDHD and (iii) MDHD. Non-helical dynamos [2]–[7] describe the flow-driven amplification of magnetic
energy via random walk line stretching, folding, and shear. No helicity of any kind is involved. The magnetic field is amplified at and below the driving scale, with negligible amplification on larger scales. I will not discuss non-helical dynamos further herein.

FDHD describe how an initially weak large-scale field is subsequently amplified via strong, large amplitude smaller scale helical velocity fluctuations often in combination with large-scale velocity shear \([8]–[11]\). In astrophysics, the source of energy for the velocity flows is ultimately gravity in the case of disks, and gravity + fusion in the case of stars. This type of dynamo is required to explain the large-scale field of the Sun and the solar cycle. An important characteristic of FDHDs is that the large-scale magnetic flux is amplified and sustained on time and/or spatial scales significantly larger than that of the driving turbulence. The key quantity which makes FDHDs work is magnetic field aligned mean electromotive force \(\bar{\mathbf{E}}_\parallel = (\mathbf{v} \times \mathbf{b})_\parallel \neq 0\), where \(\parallel\) indicates along the mean field \(\mathbf{B}\), and the overbar indicates a spatial, temporal or ensemble average depending on the specific application. This quantity is the commonality between FDHD and MDHDs.

An important source of \(\alpha\) in the relation \(\bar{\mathbf{E}}_\parallel \propto \alpha \mathbf{B}\) for the FDHD is the kinetic helicity \(\mathbf{v} \cdot \nabla \times \mathbf{v}\), a pseudoscalar correlation arising, for example, from the interplay between stratified turbulence and rotation. Inside of a rotator with an outward decreasing density, kinetic helicity can in principle be sustained as rising eddies expand and rotate oppositely to the underlying mean rotation to conserve angular momentum. Falling eddies rotate in the same direction as the mean rotation. Rising and falling eddies thus statistically contribute the same sign of \(\mathbf{v} \cdot \nabla \times \mathbf{v}\) in each hemisphere, with net opposite signs in each hemisphere.

As the large-scale field in an FDHD grows, a turbulent small-scale dynamo can generate magnetic fluctuations with energy density comparable to that in the velocity fluctuations before the large scale field saturates. The effect of the small-scale field growth and boundary terms on the FDHD are subjects of considerable research.

Unlike FDHDs, MDHDs occur in laboratory plasma fusion confinement configurations (not to be confused with the laboratory liquid metal experiments designed to test FDHDs \([12]–[17]\)). These configurations include spheromaks and the reversed field pinch (RFP) \([18, 19]\), where the MDHD describes the dynamic evolution toward the relaxed state by a magnetic field aligned electric field (or equivalently, external injection of one sign of magnetic helicity \([19]–[28]\)) in the presence of external driving away from this state. The source of energy in the laboratory case is either an electric potential placed across a gap between electrodes bounding the plasma, which then drives a direct current, or an externally time dependent current from which an electric potential and current generate helical fields within the plasma.

Although the magnetic helicity injection drives the system away from the relaxed state, the driving electric potential injects current along an initially toroidal magnetic field, for example, in the case of the RFP. This generates a poloidal field which becomes unstable to small amplitude fluctuations via kink mode instabilities from large currents, or tearing modes from current gradients. The fluctuations produce a finite \(\bar{\mathbf{E}}_\parallel\) which reduces the field aligned current to restore stability by driving a spatial flow of magnetic helicity. This in turn, enables the magnetic field structure to evolve toward the largest helical scale available (subject to boundary conditions), as this would be the lowest energy state \([29]\) if it could be reached. The continuous injection of magnetic helicity leads to a quasi-steady dynamical equilibrium, with possible sawtooth oscillations, as the system is driven away from, and evolves back toward the relaxed state. The competing effects are the helicity injection and the MDHD. If the injection is turned off, the fully relaxed state can be reached, but the field then eventually resistively decays. Via the
MDHD, the helicity injection therefore also sustains the large-scale helical field, highlighting that this energy is also injected. Note, however, that the injected energy does not go only into the large-scale field but also into small and large-scale velocity flows and heat as well.

Note that classical Taylor relaxation and fossil field relaxation are often presented as if relaxation is distinct from a dynamo. But these processes are very much linked to the MDHD which provides the dynamical evolution to the relaxed state. The fully relaxed Taylor state is typically found by minimizing the magnetic energy subject to boundary conditions and is achieved only when the driving is turned off and the dissipation, and velocity flows are neglected. In general, the relaxed state can include finite non-magnetic energies as well and generalized MDHD equations can be used to model the evolution of hydrodynamic and magnetic quantities (e.g. [34]).

The coronae above astrophysical rotators are likely magnetically dominated [30]–[33]. Coronal loops are sites for MDHDs driven by helical field injection from the astrophysical rotator below [34, 35]. If the helical field injected from below is produced by a FDHD inside the rotator, the full description of the origin of the large-scale fields involves a coupling between the FDHD and MDHD. The MDHD component provides a direct analogy to the laboratory case just discussed and is also implicitly revealed itself in specially designed spheromak experiments to study the formation of astrophysical jets [36]. In the lab direct electric potential drop across the footpoints injects the helicity. In coronae, this is supplied directly as a buoyant twisted field rises or by a velocity shear between or within a loop’s footpoints that twists the field (and induces the analogous electric potential to that of the laboratory case). The ultimate source of energy for the coronal magnetic fields are the sources of energy for the FDHD (gravity and fusion as discussed above).

For all extra-terrestrial astrophysical rotators except galaxies, we observe at most coronal fields, not interior fields. Coronal holes of the Sun are sites of large-scale ‘open’ along which the solar wind propagates [31]. Jets from accretion engines in young stellar objects, active galactic nuclei and γ-ray bursts or magnetic towers [37]–[39] are the analog to coronal holes. These large-scale coronal fields can be produced by the MDHD relaxation of smaller scale loops emerging from a FDHD inside the rotator below [40].

3. Unifying helical dynamos

Progress toward understanding helical dynamos and their saturation has evolved from a combination of numerical and analytic work that dynamically incorporates magnetic helicity evolution [20], [41]–[56]. Following [1], I review the derivation of the evolution equations for mean, fluctuating and total magnetic helicity respectively, and show that both MDHD and FDHD emerge naturally from different limits.

The electric field is

\[
E = -\nabla \Phi - \partial_t A,
\]

where \(\Phi\) and \(A\) are the scalar and vector potentials. Taking the average (spatial, temporal or ensemble), and denoting averaged values by the overbar, we have

\[
\overline{E} = -\nabla \overline{\Phi} - \partial_t \overline{A}.
\]
Subtracting (2) from (1) gives the equation for the fluctuating electric field
\[ e = -\nabla \phi - \partial_t a, \]
where \( \phi \) and \( a \) are the fluctuating scalar and vector potentials.

Using \( \mathbf{B} \cdot \partial_t \mathbf{A} = \partial_t (\mathbf{A} \cdot \mathbf{B}) + \mathbf{E} \cdot \mathbf{B} - \nabla \cdot (\mathbf{A} \times \mathbf{E}) \), where the latter two terms result from Maxwell’s equation \( \partial_t \mathbf{B} = -\nabla \times \mathbf{E} \), and the identity \( \mathbf{A} \cdot \nabla \times \mathbf{E} = \mathbf{E} \cdot \mathbf{B} - \nabla \cdot (\mathbf{A} \times \mathbf{E}) \), we take the dot product of (1) with \( \mathbf{B} \) to obtain the evolution of the magnetic helicity density
\[ \partial_t (\mathbf{A} \cdot \mathbf{B}) = -2(\mathbf{E} \cdot \mathbf{B}) - \nabla \cdot (\Phi \mathbf{B} + \mathbf{E} \times \mathbf{A}) = -2(\mathbf{E} \cdot \mathbf{B}) - \nabla \cdot (2\Phi \mathbf{B} + \mathbf{A} \times \partial_t \mathbf{A}). \]
Similarly, dotting (2) and (3) with \( \mathbf{B} \) and \( \mathbf{b} \), respectively, the time evolution for mean large-scale and mean fluctuating magnetic helicity densities are
\[ \partial_t (\overline{\mathbf{A}} \cdot \mathbf{B}) = -2\overline{\mathbf{E}} \cdot \mathbf{B} - \nabla \cdot (\overline{\Phi} \mathbf{B} + \mathbf{E} \times \overline{\mathbf{A}}) = -2\overline{\mathbf{E}} \cdot \mathbf{B} - \nabla \cdot (2\overline{\Phi} \mathbf{B} + \overline{\mathbf{A}} \times \partial_t \overline{\mathbf{A}}), \]
and
\[ \partial_t \mathbf{a} \cdot \mathbf{b} = -2\mathbf{e} \cdot \mathbf{b} - \nabla \cdot (\mathbf{\phi} \mathbf{b} + \mathbf{e} \times \mathbf{a}) = -2\mathbf{e} \cdot \mathbf{b} - \nabla \cdot (2\mathbf{\phi} \mathbf{b} + \mathbf{a} \times \partial_t \mathbf{a}). \]

To eliminate the electric fields from (4) to (6) we use Ohm’s law with only a resistive term:
\[ \mathbf{E} = -\nabla \times \mathbf{B} + \eta \mathbf{J}, \]
where \( \mathbf{J} \) is the current density and \( \eta \) is the resistivity in appropriate units. Taking the average gives
\[ \overline{\mathbf{E}} = -\overline{\nabla} \times \overline{\mathbf{B}} + \eta \overline{\mathbf{J}}, \]
where \( \overline{\mathbf{E}} \equiv \mathbf{v} \times \mathbf{b} \) is the turbulent electromotive force. Subtracting (8) from (7) gives
\[ \mathbf{e} = \overline{\mathbf{E}} - \mathbf{v} \times \mathbf{b} - \mathbf{v} \times \overline{\mathbf{B}} - \nabla \times \mathbf{b} + \eta \mathbf{j}. \]
Plugging (7) into (4), gives
\[ \partial_t (\mathbf{A} \cdot \mathbf{B}) = -2\eta (\mathbf{J} \cdot \mathbf{B}) - \nabla \cdot (\Phi \mathbf{B} + \mathbf{E} \times \mathbf{A}) = -2\eta (\mathbf{J} \cdot \mathbf{B}) - \nabla \cdot (2\Phi \mathbf{B} + \mathbf{A} \times \partial_t \mathbf{A}). \]
Plugging (8) into (5) and (9) into (6) give, respectively,
\[ \partial_t (\overline{\mathbf{A}} \cdot \mathbf{B}) = 2\overline{\mathbf{E}} \cdot \mathbf{B} - 2\eta \overline{\mathbf{J}} \cdot \mathbf{B} - \nabla \cdot (\overline{\Phi} \mathbf{B} + \mathbf{E} \times \overline{\mathbf{A}}) = 2\overline{\mathbf{E}} \cdot \mathbf{B} - 2\eta \overline{\mathbf{J}} \cdot \mathbf{B} - \nabla \cdot (2\overline{\Phi} \mathbf{B} + \overline{\mathbf{A}} \times \partial_t \overline{\mathbf{A}}) \]
and mean fluctuating magnetic helicity
\[ \partial_t \mathbf{a} \cdot \mathbf{b} = -2\mathbf{e} \cdot \mathbf{b} - 2\eta \overline{\mathbf{J}} \cdot \mathbf{b} - \nabla \cdot (\mathbf{\phi} \mathbf{b} + \mathbf{e} \times \mathbf{a}) = -2\mathbf{e} \cdot \mathbf{b} - 2\eta \overline{\mathbf{J}} \cdot \mathbf{b} - \nabla \cdot (2\mathbf{\phi} \mathbf{b} + \mathbf{a} \times \partial_t \mathbf{a}). \]
Note that in the absence of flux and resistive terms, the growth of the mean large and mean small-scale magnetic helicities are equal and opposite. This will be important for the kinematic regime of dynamo growth discussed later.

Dotting (9) with \( \mathbf{b} \) and averaging reveals the important relation
\[ \overline{\mathbf{E}} \cdot \mathbf{B} = \mathbf{e} \cdot \mathbf{b} - \eta \overline{\mathbf{J}} \cdot \mathbf{b}. \]
Both FDHDs and MDHDs can be derived from equations (11)–(13) and require \( \overline{\mathbf{E}} \mathbf{j} \equiv \overline{\mathbf{E}} \cdot \mathbf{B}/\mathbf{B}^2 \neq 0. \)
4. Closed volume FDHD versus closed volume MDHD

Here, we employ spatial averages and distinguish global closed volume averages, indicated by brackets, from local averages (associated with wavenumber \( k = 1 \) in periodic boxes) indicated by an overbar. Global and local averages can be time dependent. This formalism applies to e.g. the numerical simulations of [47], the analytic study of [48] for the FDHD and in analytic studies of the MDHD in [34, 35]. For large enough scale separation between fluctuating and overlined scales, the distinction between overline and bracket for mean small-scale scalars and pseudoscalars can often be ignored.

Globally averaging (11) and (12) gives

\[
\partial_t \langle \mathbf{a} \cdot \mathbf{b} \rangle = -2\langle \mathbf{\nabla} \cdot \mathbf{B} \rangle - 2\eta \langle \mathbf{j} \cdot \mathbf{B} \rangle
\]

and

\[
\partial_t \langle \mathbf{A} \cdot \mathbf{B} \rangle = 2\langle \mathbf{\nabla} \cdot \mathbf{B} \rangle - 2\eta \langle \mathbf{J} \cdot \mathbf{B} \rangle.
\]

The time evolution equation for \( \mathbf{E} \) is given by

\[
\partial_t \mathbf{E} = \partial_t \mathbf{v} \times \mathbf{b} + \mathbf{v} \times \partial_t \mathbf{b}
\]

and the small-scale momentum density and induction equations for \( \nabla \cdot \mathbf{v} = 0 \) are

\[
\partial_t \mathbf{b} = \mathbf{B} \cdot \nabla \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{B} + \mathbf{v} \times (\mathbf{v} \times \mathbf{b}) - \nabla \times \mathbf{v} \times \mathbf{b} + \lambda \nabla^2 \mathbf{b},
\]

and

\[
\partial_t v_q = P_{qi}(\mathbf{B} \cdot \nabla b_i + \mathbf{b} \cdot \nabla B_i - \mathbf{v} \cdot \nabla v_i + \nabla \cdot \mathbf{v} v_i + \mathbf{b} \cdot \nabla b_i - \mathbf{b} \cdot \nabla b_i) + \nu \nabla^2 v_q + f_q,
\]

where \( f \) is a divergence-free forcing function, \( \lambda \) is the magnetic diffusivity, \( \nu \) is the viscosity and \( P_{qi} \equiv (\delta_{qi} - \nabla^2 \nabla_q \nabla_i) \) is the projection operator that arises after taking the divergence of the momentum density equation to eliminate the fluctuating pressure (magnetic + thermal). For the ‘minimal \( \tau \)’ closure (MTC) [48] (discussed below) \( f \) can be uncorrelated with \( \mathbf{b} \) [57]. Reynolds rules [58] allow the interchange of averages and time or spatial derivatives, so the 5th term of (17) and the 4th and 6th terms in the parentheses of (18) do not contribute when put into correlation averages with a fluctuating quantity.

The manipulations required to obtain a practical evolution equation for \( \mathbf{E} \) by combining (16)–(18) without using the first-order smoothing approximation (FOSA) and retaining triple correlations using the MTC, have been discussed at length elsewhere (e.g. [48, 56]). The result is

\[
\partial_t \mathbf{E} = \tilde{a} \mathbf{B}^2 / |\mathbf{B}| - \tilde{b} \mathbf{B} \cdot \nabla \times \mathbf{B} / |\mathbf{B}| - \tilde{\zeta} \mathbf{E},
\]

where \( \tilde{a} = (1/3)(\mathbf{b} \cdot \nabla \times \mathbf{b} - \mathbf{v} \cdot \nabla \times \mathbf{v}) \), \( \tilde{b} = (1/3)\mathbf{v}^2 \) and \( \tilde{\zeta} \sim k_i \nu \) accounts for microphysical dissipation terms, and triple correlations. It is often reasonable to assume that the left side (19) vanishes as the \( \mathbf{E} \) can saturate long before the dynamo does [48, 51]. Then (19) can be rearranged to give an explicit expression for \( \mathbf{E} \) similar to what appears in standard textbooks [8], with the distinction that the time constant \( 1/\tilde{\zeta} \) here is an eddy turnover time, not an approximation to a correlation [48, 59]. Although \( \tilde{\zeta} \) can be a function of wavenumber [60], even the simple MTC is a significant improvement over the traditional FOSA [8, 9, 49] in which triple correlations are ignored. The MTC is simpler than the eddy-damped quasi-normal Markovian closure [41] and has also been successful for scalar diffusion [59, 61].

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The equations to be solved for both the closed FDHD or MDHD are (14), (15) and (19). For the FDHD, kinetic helicity is injected \( \nu \cdot \nabla \times \nu \) while for the MDHD magnetic helicity or current helicity \( j \cdot b \) is injected. In either case, for a periodic box, the large-scale field which grows is fully helical at \( k = 1 \). The results and differences between the closed volume FDHD and MDHD are summarized in the next subsections.

4.1. FDHD case

The closed box isotropically forced FDHD [48] is a nonlinear version of the \( \alpha^2 \) dynamo (e.g. [8]) that incorporates the dynamical backreaction of the magnetic field on the kinetic helicity driving the flow and the evolution of magnetic helicity. Keeping in mind that (14), (15), and (19) are the coupled equations to be solved, the essence of the dynamo growth and saturation is as follows: the initial system is driven with a finite \( \nu \cdot \nabla \times \nu \simeq \langle \nu \cdot \nabla \times \nu \rangle \). This grows a finite \( \mathcal{E}_\parallel \) which then grows mean large-scale magnetic helicity of the opposite sign to the driving kinetic helicity via (15). At early times, the growth is kinematic because \( \tilde{\alpha} \) is dominated by the kinetic helicity, \( \mathcal{E}_\parallel \) is substantial, and the resistive terms in (14) and (15) are negligible. Thus \( \langle a \cdot b \rangle \) (and thus \( \langle j \cdot b \rangle \)) grows with opposite sign to that associated with \( \langle A \cdot B \rangle \). This in turn quenches \( \tilde{\alpha} \) and \( \mathcal{E}_\parallel \). The helical mean field reached at the end of the kinematic phase is first estimated by setting the mean small-scale helical field at quenching from \( |\langle b \cdot \nabla \times b \rangle| \simeq |\langle \nu \cdot \nabla \times \nu \rangle| \) and then using magnetic helicity conservation to get the large-scale helical magnetic field. That is \( \mathcal{B}_L^2 \sim k_1 f_h (v^2)/k_1 \), where \( f_h \equiv \langle \nu \cdot \nabla \times \nu \rangle/\langle v^2 \rangle \), and \( k_1 \) is the wavenumber associated with \( \mathcal{B} \).

Because the resistive term in (14) is larger than that of (15) in the kinematic phase, \( |\langle A \cdot B \rangle| \) can grow to exceed \( |\langle a \cdot b \rangle| \) beyond the kinematic regime, but at a microphysical resistively limited rate. Eventually, however, the system reaches a true steady state in which growth balances decay. This occurs when the value of the mean current helicity grows large enough such that the dissipation terms in (14) and (15) are equal.

For \( \alpha^2 \) dynamos of standard texts [8], the equation for the mean magnetic field is solved with an imposed \( \mathcal{E}_\parallel \) and linear growth results. Blackman and Brandenburg [53] emphasize that magnetic helicity is not conserved even in the kinematic regime (neither in the equations nor in the diagrams) for dynamos in the standard texts. In the modern version just discussed, the additional time dependent equations for mean small-scale helicity \( \mathcal{E} \) evolution are coupled into the theory dynamically and magnetic helicity evolution is properly evolved. The approach can be generalized to an \( \alpha - \Omega \) dynamo [51] where the mean large-scale magnetic helicity evolution equation has been replaced by the vector equation for \( \mathcal{B} \) [51].

The two-scale analytic approach has been generalized to a four-scale approach [54] to assess whether the mean small-scale magnetic helicity is more dominated by the dissipation scale or forcing scale (with the mean large-scale magnetic helicity migrating toward the even larger box scale). The analysis shows that the mean small-scale magnetic helicity is first dominated near the resistive scale but migrates toward the forcing scale before the end of the kinematic regime. Numerical simulations of helical dynamos in a periodic box [47, 50] also show that the magnetic helicity in saturation peaks with opposite signs at the forcing scale and box scale, respectively. In short, the system achieves bihelical equilibrium in which the mean small and mean large-scale magnetic helicities are dominated by the largest scales available to them respectively. The driving kinetic helicity ensures that these two scales are distinct and prevents the mean small-scale magnetic helicity from migrating to the box scale.
4.2. MDHD case

For the MDHD case, mean small-scale current helicity \( \langle \mathbf{j} \cdot \mathbf{b} \rangle = k_l^2 \langle \mathbf{a} \cdot \mathbf{b} \rangle \) is injected into the \( \tilde{\alpha} \) of (19). Again, because (14), (15) and (19) are the equations to be solved, like the FDHD, the MDHD growth of mean large-scale and mean small magnetic helicity is also mediated by the difference between the mean small-scale kinetic and current helicities.

Like the FDHD, for the MDHD the mean large-scale and mean small-scale magnetic helicities have growth rates of the opposite sign. But for the MDHD, the mean large scale magnetic helicity grows with the same sign as that of the injected helicity, not the opposite sign as in the FDHD case. That the MDHD drives magnetic helicity to large scales exemplifies that the MDHD is essentially a generalized dynamical Taylor relaxation. It is thus appropriate to refer to the MDHD as ‘dynamical magnetic relaxation’. The lowest energy state of a unihelical configuration is one in which the magnetic helicity resides at the largest scale, subject to available boundary conditions or imposed forcing away from the relaxed state. In this context, both the steadily forced case and the case for which forcing is turned off have been studied [34, 35].

In the forced case, the large-scale magnetic helicity grows to about 1/2 of the forced value of small-scale magnetic helicity kinematically (independent of dissipation) after which the true steady state evolves via a viscously limited phase. This viscously limited phase is analogous to the resistively limited phase of the FDHD and arises because the kinetic helicity is the backreactor in the MDHD case and so its dissipation facilitates the slow growth phases after the kinematic regime. Eventually a steady MDHD results, analogous to the FDHD case. In the MDHD case like the FDHD case most of the magnetic energy resides on the largest \( k = 1 \) scale in the saturated steady state. Unlike the FDHD case, the forcing scale and the \( k = 1 \) scale magnetic helicities have the same sign. In addition, the total magnetic energy at the forcing scale dominates the kinetic energy there. The saturation to the steady state in the forced case arises because the injected magnetic helicity also induces a growth of kinetic helicity at the injection scale and this which plays the role of the backreacting agent. This depletes the \( \tilde{\alpha} \) in (19) and saturates the growth of the large-scale magnetic helicity. The roles of the kinetic and current helicities are therefore reversed for the MDHD compared to the FDHD.

For the case in which the small-scale helicity is injected only initially, the kinematic phase proceeds similar to the forced phase, but then all quantities eventually decay, with the large-scale helicity decaying the slowest.

The MDHD regime, unlike the FDHD regime, has not been fully tested with 3D MHD numerical experiments, but similar simulations in a periodic box for which magnetic helicity is injected at some wavenumber, say \( k_l \sim 5 \), with an initially negligible velocity would be appropriate. The overall evolution of the magnetic helicity and kinetic helicity spectra could then be measured as a function of time.

5. Open volume FDHD versus open volume MDHD

Here, we assume overlined averages are taken over large scales with respect to fluctuating quantities but over scales equal to or smaller than the scale of the bracketed averages. For example, in an accretion disk, the bracketed averages could be taken over an entire hemisphere within the disk, whereas the overlined averages could be taken over full azimuth and half scale height, but remain local in radius. Note that neither average would include the corona, and so global vertical fluxes can remain.

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In the limit that time evolution and resistive terms are ignored but the divergence terms are kept, equations (11) and (12) give
\[ 0 = 2 ( \mathbf{E} \cdot \mathbf{B} ) - \nabla \cdot ( \Phi \mathbf{B} + \mathbf{E} \times \mathbf{A} ) \] (20)
and
\[ 0 = -2 ( \mathbf{E} \cdot \mathbf{B} ) - \nabla \cdot ( \phi \mathbf{b} + e \times \mathbf{a} ) . \] (21)
Combining these two equations reveals that the fluxes of mean large and mean small-scale helicity through the system boundary are equal and opposite. This is important for an FDHD inside of an astrophysical rotator, an MDHD outside of an astrophysical rotator and an MDHD in laboratory plasmas: the $\mathbf{E}_\parallel$ for these cases is supplied by a helicity flux.

5.1. Open FDHD case

If helical motions sustain kinetic helicity inside of the rotator, then $\mathbf{E}$ is sustained by $\mathbf{a}$ from (19), and the averaging in (20) and (21) is taken over scales less than or equal to the interior hemisphere of the rotator. A steady-state with open boundaries would imply that the rotator supplies each hemisphere of its corona with one sign of magnetic helicity on large scales and the other on smaller scales [53, 62]. The coronal energy deposition rate associated with these helicity fluxes [62] is consistent time-averaged steady coronae of the Sun [31] and AGN accretion disks [32, 33]. The bihelical nature of the field and the sign dependence of the injected helicity for any single structure can also be influenced by whether additional localized surface shear operates on a scale larger or smaller than that of a given loop’s footpoint separation [63]. A statistical approach is therefore needed to infer the dominant scale dependent sign of magnetic helicity.

5.2. Open MDHD case

In the context of an astrophysical rotator, the MDHD case applies when the volume averages of (20) and (21) are that of a corona. The FDHD supplies magnetic helicity via the flux terms which then drive the coronal MDHD. The evolution of coronal magnetic structures is analogous to the evolution of magnetically dominated laboratory plasma configurations [28, 36] subject to injection of magnetic helicity. The experiment of [36] reveals a direct analogy to helical loops of flux rising into an astrophysical corona from its rotator below. The loops coalesce at the symmetry axis and form a magnetic tower. For large enough helicity injection, the tower can break off a spheromak blob from the kink instability. The helicity flux from the loops seeds subsequent dynamical magnetic relaxation via an MDHD. The relaxation opens field lines that form coronal holes or jets.

The astrophysical corona, taken as a single entity, can also be modeled as a statistical aggregate of loops and the corona can be thought of as a single dynamical entity [34] which statistically receives injected helicity of both signs in each hemisphere such that bihelical relaxation [35] applies. However, for a single structure within that corona, the net sign may be positive or negative. It is in this sense that the guiding principles of the MDHD in a corona for each structure can then be understood by analogy to laboratory plasma configurations. The most direct analogy comes from comparing a toroidal laboratory configuration such as an RFP with a single coronal loop of magnetic flux, subject to footpoint twisting that injects helicity.
into the loop. We can think of each loop as a cut torus with a net injection of helicity of one sign.

It is useful to discuss the MDHD in the context of a torus, streamlining the treatment of [1]. Overlined mean quantities are time averages and spatial averages over periodic directions φ (locally ẑ) and θ, but not over radius r (where r = 0 corresponds to an azimuthal ring at the center of the torus’ cross-section). The steady-state limit of (12) is

$$\mathcal{E} = \frac{\mathbf{B}}{B^2} (\nabla \cdot \mathbf{h} - \eta \mathbf{j} \cdot \mathbf{b}),$$  \tag{22}

where \(\mathbf{h} \equiv -(\mathbf{\phi B} + \frac{1}{2} \mathbf{a} \times \partial \mathbf{a})\). Dotting (22) with \(\mathbf{B}\), and using (8) gives

$$\mathcal{E} \cdot \mathbf{B} = \nabla \cdot \mathbf{h} - \eta \mathbf{j} \cdot \mathbf{b} = \eta \mathbf{J} \cdot \mathbf{B} - \mathbf{E} \cdot \mathbf{B},$$  \tag{23}

where, from equation (5), we also have

$$\mathbf{E} \cdot \mathbf{B} = -\nabla \cdot (\mathbf{\Phi B} + \frac{1}{2} \mathbf{A} \times \partial \mathbf{A}) = -\nabla \cdot (\mathbf{\Phi B}),$$  \tag{24}

where the latter equality follows for the assumed steady-state.

The RFP dynamo emerges when an \(\mathbf{E}\) of sufficient strength is externally applied along the initial toroidal magnetic field. A finite \(\mathcal{E}_\parallel\) then results from fluctuations induced by tearing or kink mode instabilities. If there were no \(\mathcal{E}\), the two terms on the right of (23) would balance. For sufficiently large applied \(\mathbf{E}_\parallel\), RFP experiments [18, 19, 64, 65] reveal that \(\mathbf{E} \cdot \mathbf{B} = \eta \mathbf{J} \cdot \mathbf{B} > 0\) only at a single radius \(0 < r < r_c < a\), where \(a\) is the minor radius of the torus and \(r_c\) is measured from the toroidal axis. For \(r < r_c\), \(\mathbf{E} \cdot \mathbf{B} > \eta \mathbf{J} \cdot \mathbf{B} > 0\) and for \(r > r_c\), \(\eta \mathbf{J} \cdot \mathbf{B} > 0 > \mathbf{E} \cdot \mathbf{B}\). Excluding pressure gradient and inertial terms in Ohm’s law, such measurements imply that \(\mathcal{E}_\parallel \neq 0\). Since \(\eta \mathbf{J} \cdot \mathbf{B} - \mathbf{E} \cdot \mathbf{B}\) changes sign from negative to positive moving outward through \(r_c\) (while \(\mathbf{J} \cdot \mathbf{B}\) keeps the same sign), \(\mathcal{E}_\parallel\) must also change from negative to positive across \(r = r_c\).

If the third term in (22) is negligible, (22) shows that \(\nabla \cdot \mathbf{h}\) must change sign through \(r_c\).

Volume integrating (23) eliminates the \(r\) dependence. Using (24), this gives

$$\int \mathcal{E} \cdot \mathbf{B} \, dV = \int \mathbf{h} \cdot dS = \int (\eta \mathbf{J} \cdot \mathbf{B} - \mathbf{E} \cdot \mathbf{B}) \, dV = \int \eta \mathbf{J} \cdot \mathbf{B} \, dV + \int (\mathbf{\Phi B}) \cdot dS,$$  \tag{25}

dropping the third term of (22) (justified by experiment [18, 19, 64, 65]), and using Gauss’ theorem to obtain surface integrals, keeping in mind that for doubly connected topologies this requires \(\mathbf{h}\) to be analytic everywhere. The latter is ensured because our averaged quantities depend only on radius. Taking the surface integral in (25) over the cut faces of the torus we have

$$\int \mathbf{\Phi B} \cdot dS = \Delta \mathbf{\Phi} \int \mathbf{B} \cdot dS = V_s \Psi_s,$$  \tag{26}

where we have assumed that \(\mathbf{\Phi}\) has a constant value on each of the surface faces, \(V_s = \Delta \mathbf{\Phi} = \int \mathbf{E} \cdot dz\) is the externally applied potential drop between toroidal faces, and \(\Psi_s\) is the toroidal magnetic flux through both surfaces. Equation (26) represents the helicity injection; a similar term drives the instabilities of spheromak experiments of [36].

The parallel component of the electromotive force can be written \(\mathcal{E}_\parallel = \alpha \mathbf{B}\). The pseudoscalar \(\alpha\) for the laboratory case is given from (13) and (23) by

$$\alpha = \frac{\mathbf{E} \cdot \mathbf{B}}{B^2} \sim \frac{1}{B^2} \nabla \cdot \mathbf{h} = \frac{\mathbf{e} \cdot \mathbf{b}}{B^2} \sim \frac{\mathbf{e}_\perp \cdot \mathbf{b}_\perp}{B^2},$$  \tag{27}
where the last similarity follows because the fluctuations are primarily perpendicular to the strong mean fields. The measured right side of (27) is consistent with the \( \mathcal{E} \) needed for MDHD models \([65]\).

5.3. Time-dependent, open, FDHD

In general, both the time dependent and the flux terms in equations (11) and (12) can be included dynamically rather than taking the simpler steady-state of the previous sections. Two recent calculations of FDHDs, applied to the interiors of astrophysical disks, incorporate the time dependence using different sets of approximations.

In the context of the Galaxy, \([57, 66]\) solved the mean field induction equation for \( \mathbf{B} \) with \( \mathcal{E}_\parallel \) determined from setting \( \partial_t \mathcal{E}_\parallel = 0 \) in (2). The \( \mathcal{E}_\parallel \) involves the difference between the kinetic and current helicities which can be related to small-scale magnetic helicity in the Coulomb gauge. References \([57, 66]\) formally use a gauge invariant helicity density \([67]\) \( \chi \) to replace the magnetic helicity density but the role of the boundary terms is conceptually independent of this. Effectively, the induction equation is solved for \( \mathbf{B} \) (which depends on \( \mathcal{E}_\parallel \) and thus \( \langle \mathbf{a} \cdot \mathbf{b} \rangle \)) and equation (12) for \( \langle \mathbf{a} \cdot \mathbf{b} \rangle \). The divergence term in (12) can be rigorously \([67]\) replaced with one of the form \( \propto \nabla \cdot (\chi \mathbf{V}) \), where \( \mathbf{V} = (0, 0, V_z) \) is the mean velocity advecting the small-scale helicity out of the volume. This mean velocity also appears in the induction equation for \( \mathbf{B} \), highlighting that the loss terms in the small-scale helicity equation also implies advective loss of mean field. For the estimates in \([57, 66]\) the distinction between \( \chi \) and \( \langle \mathbf{a} \cdot \mathbf{b} \rangle \) in the Coulomb gauge is not essential.

This approach supports the concept \([45]\) that a flow of small-scale helicity toward the boundary may help to alleviate the backreaction of the small-scale magnetic helicity on the kinetic helicity which drives the dynamo in \( \mathcal{E}_\parallel \). However, if \( V_z \) is too large, it may carry away too much of the mean field which the dynamo is trying to grow in the first place. In general, more work is needed to calculate \( V_z \) from first principles, and its effect on large and small-scale fields. The helicity flows in the time-dependent case can also seed a time dependent coronal MDHD.

A more restrictive time dependent FDHD dynamo that includes boundary terms, maintains the time dependence in (11), but implicitly assumes that equation (12) reaches a steady-state, has also been studied \([49]\). Although the approach involves assumptions that have now been avoided in more general calculations of helicity fluxes \([55]\) (one being the FOSA which can be avoided by the MTC discussed earlier), \([49]\) does identify how a time dependent flow-driven dynamo in a Keplerian shear flow might be sustained by a magnetic helicity flux. Here the relevant forms of (11) and (12) are

\[
\partial_t (\mathbf{A} \cdot \mathbf{B}) = 2\mathcal{E} \cdot \mathbf{B} - \nabla \cdot (\mathbf{\Phi B} + \mathbf{E} \times \mathbf{A}) \tag{28}
\]

and

\[
0 = -2\mathcal{E} \cdot \mathbf{B} - \nabla \cdot (\mathbf{\Phi B} + \mathbf{e} \times \mathbf{a}). \tag{29}
\]

Using (29), \( \mathcal{E}_\parallel \) can be directly written in terms of the small-scale helicity flux as

\[
\mathcal{E}_\parallel = -\frac{\mathbf{B}}{B^2} \nabla \cdot (\mathbf{\Phi B} + \mathbf{e} \times \mathbf{a}) = -\frac{\mathbf{B}}{B^2} \nabla \cdot (-\nabla \Phi + \mathbf{e}) \times \mathbf{a}, \tag{30}
\]

then use (30) in the equation for the mean magnetic field applied to an accretion disk whose mean quantities are axisymmetric. The mean field equation is

\[
\partial_t \mathbf{B} = \nabla \times \mathbf{E} + \nabla \times (\nabla \times \mathbf{B}) + \lambda \nabla^2 \mathbf{B}. \tag{31}
\]
Solving (31) requires the use of (30). Vishniac and Cho [49] invoke a correlation time $\tau_c$ such that $\mathbf{a} \simeq - (\mathbf{e} + \nabla \phi) \tau_c$, (note: [49] defines $e_{mf} \equiv - \mathbf{e}$ and work with $e_{mf}$). This reduces the last term of (30) to

$$2 \frac{\mathbf{B}}{B^2} \tau_c \nabla \cdot \mathbf{e} \times \nabla \phi \equiv - \frac{\mathbf{B}}{B^2} \nabla \cdot \mathbf{J}_H,$$

which defines the helicity flux $J_H$. (Vishniac and Cho [49] is missing a factor of 2). Assuming incompressible flow for the fluctuations,

$$J_{H,i} \sim 2 l^2 \tau_c \mathbf{B} \cdot \nabla \mathbf{v},$$

where $l$ is the outer turbulent scale. This current is then used in (31) to allow growth of $\mathbf{B}$.

Authors of [55, 56] show that (33) is one of a number of current terms that emerge in a more general calculation which avoids the FOSA (see section 3.2) with additional fluxes arising when $\mathbf{V}$ is included in $\mathbf{e}$. Nevertheless, the importance of equation (33) is that it shows how dynamo growth of the large-scale field can be driven entirely by the small-scale helicity flux without any kinetic helicity. Determining whether this works in practice is an active area of research. The authors of [68–70] collectively show that the Vishniac-Cho flux can sustain large-scale field growth via boundary flow in the absence of kinetic helicity, but requires the field to already exceed a double-digit percentage of the equipartition value of the turbulent velocity before this effect kicks in.

The role of the boundary flux to alleviate catastrophic dynamo quenching [45] may actually be more important for the Sun than the Galaxy since the former requires a rapid, unquenched, cycle period [53]. However, as also emphasized in the previous subsections, large-scale helicity flux likely accompanies any small-scale helicity flux. Significant loss of the large-scale field would imply removal of the large-scale field that the dynamo is invoked to generate inside the rotator thereby lowering its maximum value inside the rotator. Care in identifying the relative amount of large and small-scale helicity flux is warranted.

6. Coupling flow driven and magnetically driven dynamos

As emphasized earlier, global scale field growth in astrophysical rotators plausibly involves a coupling of the interior FDHD to the exterior dynamo MDHD. The fields which are large scale with respect to the FDHD in the interior are small scale with respect to the corona and the large scale fields of coronae are on a larger ‘global’ scale. An example of an FDHD produced large-scale field would be a flux tube with each footpoint on the scale of order of the thickness of the turbulent zone of the underlying rotator, i.e. a disk scale height, or convection zone thickness.

In a steady state, both signs of magnetic helicity would be injected into the corona on separate scales, as per (20) and (21). Consider here an ensemble of individual structures whose mean ‘large-scale’ helicity injected from the interior to the corona is coherent in order to further exploit the analogy to the steady-state laboratory MDHD case discussed in section 5. We take the divergence term in (20) to indicate the flux of mean ‘small-scale’ helicity to the corona. That is, we posit a steady state with the correspondence

$$\nabla \cdot \langle \Phi \mathbf{B} + \mathbf{E} \times \mathbf{A} \rangle_{int} = - \nabla \cdot \langle \phi \mathbf{b} + \mathbf{e} \times \mathbf{a} \rangle_{cor}.$$

Then

$$0 = 2 \langle \mathbf{E} \cdot \mathbf{B} \rangle_{cor} - \nabla \cdot \langle \Phi \mathbf{B} + \mathbf{E} \times \mathbf{A} \rangle_{cor}$$

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and

\[ 0 = -2(\mathbf{E} \cdot \mathbf{B})_{\text{cor}} - \nabla \cdot (\mathbf{\phi} \mathbf{b} + e \times \mathbf{a})_{\text{cor}}. \]  

(36)

The subscripts ‘int’ and ‘cor’ indicate averaging scales chosen to be taken within a hemisphere interior to the rotator and in the corona, respectively. The mean scales for the corona structure and the disk structure can be quite different. When both the disk and the coronal region are assumed to be in a mutually steady state, equations (34)–(36) and (28) and (29) imply

\[ (\mathbf{E} \cdot \mathbf{B})_{\text{cor}} = (\mathbf{E} \cdot \mathbf{B})_{\text{rot}}. \]  

(37)

The coronal electromotive force is driven by the flux into the corona which itself can be ultimately driven by the helical turbulence within the rotator. Since it is \((\mathbf{E} \cdot \mathbf{B})_{\text{rot}}\) which sources the large-scale field of the corona, the assumed steady state implies a constant generation and loss of large-scale magnetic helicity in the corona. The quasi-steady state can be established for a fixed coronal volume via loss in e.g. a Poynting flux jet.

The amount of helicity injected into the corona also provides a lower limit on the energy injected into astrophysical coronae and is consistent with that needed for the Sun and active galactic nuclei [62]. The role of CMEs for the Sun deserves special attention as a mechanism by which magnetic helicity is ejected into the corona. Blackman and Brandenburg [53] suggest that both signs of magnetic helicity are ejected by CMEs, in both hemispheres. This serves to keep the solar cycle fast, but also lowers the net saturation value of the mean field in the solar interior that would otherwise arise without ejection. The MDHD is the process by which CMEs relax in the corona.

7. Conclusions

By following the dynamical evolution of magnetic helicity, a unifying framework for FDHDs and MDHDs and their nonlinear saturation emerges. Both FDHDs and MDHDs fit into this framework. Standard \(\alpha^2\) and \(\alpha - \Omega\) type dynamos are FDHDs while coronal dynamical magnetic relaxation (or dynamical Taylor relaxation), and laboratory plasma dynamos are MDHDs.

A unifying principle for FDHD and MDHD is that both require a turbulent electromotive force aligned with the mean magnetic field. For closed FDHD fed by kinetic helicity, the kinetic helicity drives large-scale magnetic helicity growth which in turn corresponds to the large-scale magnetic field growth. But since no net magnetic helicity is injected into the system, the opposite sign of magnetic helicity must grow on small scales to compensate. For the closed MDHD, the system is injected with magnetic helicity of one sign on small scales and the dynamo proceeds by relaxing this field to large scales whilst depleting the helicity from small scales.

In astrophysics, open FDHDs and open MDHDs are coupled via the boundary between an astrophysical rotator and its corona. The FDHD can feed the MDHD via magnetic helicity injection at coronal loop footpoints. Although isolated FDHDs have received most of the attention in astrophysics, it is in fact coronal relaxation via the MDHD that produces the fields most directly observed. Energy released as field configurations relax via MDHDs can heat coronae. More work is definitely needed to understand how these principles apply in specific systems.

Finally, one might raise the concern that because magnetic helicity is a gauge non-invariant quantity, its prominent conceptual role in recent dynamo theory work is puzzling. In fact,
magnetic helicity evolution can be employed entirely as a calculation tool with a convenient gauge chosen. At the end of the day, one can always convert back to the magnetic and electric fields. It is only when one specifically demands a physically measurable variant of magnetic helicity \cite{71,72} for its own sake, that the issue of defining a gauge invariant quantity arises.

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