Relation between discrete and continuous teleportation using linear elements

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We discuss the relation between discrete and continuous linear teleportation, i.e., teleportation schemes that use only linear optical elements and photodetectors. For this the existing qubit protocols are generalized to qudits with a discrete and finite spectrum but with an arbitrary number of states or alternatively to continuous variables. Correspondingly a generalization of linear optical operations and detection is made. It is shown that linear teleportation is only possible in a probabilistic sense. A general expression for the success probability is derived which is shown to depend only on the dimensions of the input and ancilla Hilbert spaces. From this the known results \( P = 1/2 \) and \( P = 1 \) for the discrete and continuous cases can be recovered. We also discuss the probabilistic teleportation scheme of Knill, Laflame and Milburn and argue that it does not make optimum use of resources.

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I. INTRODUCTION

Recently Knill, Laflamme and Milburn (KLM) \(^1\) proposed a scheme for linear optical quantum computing using photons as qubits. This scheme works efficient if one can implement deterministic discrete quantum teleportation. The original proposal for discrete teleportation by Bennett et.al. \(^2\) involves the projection on two-particle Bell states, which, as shown in \(^3\) \(^4\), cannot be implemented with linear elements in a deterministic way, unless entanglement in additional degrees of freedom is used \(^5\) \(^6\). Discrete teleportation with linear elements is only possible using post selection with a success probability of 50 % e.g., using the scheme suggested by Weinfurter \(^7\). On the other hand in the case of continuous variables, perfect teleportation with linear elements can be achieved as shown by Braunstein and Kimble \(^8\). This raises the question about the origin of this manifestly different behavior. Vaidman and Yoran suggested that the beam-splitter used in the Braunstein-Kimble experiment would lead to an effective “quantum-quantum” interaction \(^9\) \(^10\). The proposal of KLM hints in a different direction: The success probability of the KLM teleportation protocol tends toward 100 % when an increasing number of additional ancilla photons is used \(^1\), which indicates that the difference could be connected to the resources used.

We here discuss a general relation between discrete and continuous teleportation using linear elements. We show that the success probability of a generalized linear protocol is only limited by the dimensions of the input and ancilla spaces. It is argued that the KLM scheme is inefficient from this point of view as it does not reach this limit. Recently Franson et.al. \(^11\) suggested a modification of the KLM scheme with a better scaling of the fidelity with the number of ancilla photons at the expense that the input state is never reconstructed exactly. This scheme also does not reach the limit obtained in the present paper.

Guided by continuous variable teleportation protocols \(^8\) \(^10\) we generalize teleportation from qubits to qudits with a bounded discrete spectrum of states. Also the notion of linear elements is generalized reducing however to photodetection and / or homodyne detection in the two limits. We will restrict the discussion first to schemes that faithfully reproduce the input state, i.e., in which Alice and Bob can decide whether the teleportation was successful. At the end we will also discuss the connection to teleportation schemes of the Franson-type with non-unity fidelity but higher success probability.

II. A GENERALIZED LINEAR TELEPORTATION PROTOCOL

Let us briefly recall the general properties of a teleportation protocol: Alice possesses an unknown quantum state \( | \Psi \rangle \) in a Hilbert space of dimension \( d \) (qubits for \( d = 2 \) or qudits in the general case), which she wants to transfer to Bob by only applying local operations including measurements and exchanging classical information. In the following we will restrict ourselves to discrete generalizations of a qubit, note however that also continuous generalizations can be discussed in a similar way. For the teleportation Alice and Bob make use of an ancilla system with state \( | \Phi \rangle \), which is an entangled pair of qudits which they share. Alice then performs an appropriate measurement on her qudit of the ancilla and the unknown quantum state. This measurement projects Bob’s ancilla qudit to a certain state which is up to local operations identical to the unknown one. Alice’s measurement must be such that the knowledge about its outcome, transmitted via classical channels, is sufficient for Bob to reconstruct the initial state \( | \Psi \rangle \) by local operations. If Alice is able to perform all possible measurements in the joint Hilbert space of her qudit of the ancilla state and the
unknown state, unconditional teleportation is always possible. Unfortunately this is in general a very difficult task. For example in order to measure the four Bell states in the Hilbert space of two qubits, as required in the teleportation protocol of Bennett et al., quantum gate operations must be performed. As shown in [23] for the case of photonic qubits, only two out of the four Bell states can be distinguished without gate operations by linear optical elements and photodetection. On the other hand in the case of continuous teleportation linear elements suffice. Until now the limitations of teleportation with linear elements are not completely understood. Since linear elements are easy to implement such an understanding is very important for the practical realization of quantum information processing. We thus restrict the discussion in the following to teleportation protocols that use only certain generalizations of linear optical operations and photodetection.

Let us consider an unknown input state decomposed into a set of orthonormal basis states $|q\rangle$, which are (non-degenerate) eigenvectors of an observable $\hat{q}$

$$\hat{q} |q\rangle = q |q\rangle.$$

The real numbers $q$ are the eigenvalues which are assumed to lie in the symmetric interval $\{-a, a\}$ with integer steps. Their total number is $2a + 1$. Thus Alice’s state is given by

$$|\Psi\rangle = \sum_{q_1 = -a}^{a} \alpha_{q_1} |q_1\rangle_A.$$

The ancilla state shared by Alice and Bob has the general form:

$$|\Phi\rangle = \sum_{q_2, q_3 = -b}^{b} \beta_{q_2 q_3} |q_2\rangle_A |q_3\rangle_B,$$

where we have assumed for the sake of simplicity that $q_2$ and $q_3$ have the same symmetric interval of allowed values $\{-b, b\}$, again with integer steps. In order to teleport an unknown state from a Hilbert space of dimension $2a + 1$, there must be at least the same number of pairs of bi-orthogonal ancilla states, i.e. the Schmidt number of the ancilla state must be larger or equal to $2a + 1$, implying $b \geq a$. Without loss of generality we here require that the coefficients $\beta$ are such that only states that fulfill the relation $q_3 = -q_2$ have a non-vanishing amplitude, and that the Schmidt number is $b$. This choice is quite general since any state in which there is a unique relation between the eigenvalues $q_2$ and $q_3$ can be brought into this form by reordering. One further needs that

$$|\beta_{q_2}| = \text{const.} \quad \forall q_2.$$

This condition can be deduced in a general way for tight teleportation schemes [12], but will also become clear in the course of the present discussion. For simplicity we choose all $\beta_{q_2}$ to be equal, i.e.

$$|\Phi\rangle = \frac{1}{\sqrt{2b + 1}} \sum_{q_2 = -b}^{b} |q_2\rangle_A |q_2\rangle_B.$$

The above scheme includes in particular the input and ancilla states of the Bennett protocol as well as the KLM protocol. In the latter case the state $|q_i\rangle$ is a quantum state of $n$ modes occupied by $q_i$ photons.

For the teleportation Alice needs to measure the total state $|\Gamma_0\rangle = |\Phi\rangle \otimes |\Psi\rangle$ in a way that the input state is restored in Bob’s qudit up to a local operation which is uniquely defined by the outcome of the measurement. As discussed above, we will not allow for all possible measurements but restrict ourselves to generalized \textit{linear} measurements, which will be defined in the following.

In an optical realization the $\hat{q}_i$ correspond to either photon numbers or quadrature amplitudes of an electromagnetic field which can be measured by direct photon counting or homodyne detection. A measurement scheme that uses only linear optical elements like beamsplitters etc. can only project onto eigenstates of linear combinations of photon number or quadrature amplitude operators. We thus call a generalized linear measurement a projection onto eigenstates of any linear combination of operators $\hat{x}_1$ and $\hat{x}_2$, i.e. of operators $\hat{X}$ in $\mathcal{H}_1 \otimes \mathcal{H}_2$.

Without loss of generality we consider here a projection onto eigenstates of

$$\hat{Q}_+ = \hat{q}_1 + \hat{q}_2,$$

which corresponds to the setup used in continuous teleportation. Measuring $\hat{Q}_+$ can lead to only $2a + 2b + 1$ different outcomes. The total Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$ is however of dimension $(2a + 1) \times (2b + 1)$ and thus the spectrum of
eigenstates of $\hat{Q}_+$ must be degenerate. In order to project onto a non-degenerate state a second measurement is required. In the continuous teleportation protocol of Braunstein and Kimble, where $\hat{q}_{1,2}$ are equivalent to position operators, the difference between the two momenta $\hat{P}_- = \hat{p}_1 - \hat{p}_2$ is measured in addition to $\hat{Q}_+$. We now define an analogue measurement in the discrete case. To this end we construct another orthonormal basis $|p\rangle$ with the property

$$|\langle p|q\rangle| = \text{const.} \quad \forall p, q.$$  \hfill (6)

We define the states

$$|p\rangle := \frac{1}{\sqrt{2b+1}} \sum_{q=-b}^{b} \exp \left\{ \frac{2\pi i pq}{2b+1} \right\} |q\rangle \quad \hfill (7)$$

with $p$ running in integer steps from $-b$ to $b$. Furthermore we assume that the quantum number $p$ can be detected by measuring the observable

$$\hat{p} = \sum_{p=-b}^{b} p |p\rangle\langle p|.$$  \hfill (8)

Again we allow only measurements of linear combinations of the operators $\hat{p}_1$ and $\hat{p}_2$, i.e. linear measurements. In particular Alice could measure in addition to $\hat{Q}_+$ the quantity

$$\hat{P}_- = \hat{p}_1 - \hat{p}_2,$$  \hfill (9)

in close analogy to continuous teleportation. The measurement operators $\hat{Q}_+$ and $\hat{P}_-$ do not commute, except in a limiting case, but this does not matter as we will see. For simplicity of the discussion we consider a measurement outcome with $Q \geq 0$ and $P \geq 0$. Then the measurement outcome is described by the projection operators:

$$\hat{\Pi}_Q = \sum_{q=Q-b}^{b} |Q-q,q\rangle\langle Q-q,q| \quad \hfill (10)$$

$$\hat{\Pi}_P = \sum_{p=-b}^{b-P} |P+p,p\rangle\langle P+p,p|.$$  \hfill (11)

The subsequent measurement of $\hat{Q}_+$ and $\hat{P}_-$ thus projects the initial state $|\Gamma_0\rangle$ onto

$$|\Gamma_1\rangle = \frac{\hat{\Pi}_P \hat{\Pi}_Q |\Gamma_0\rangle}{||\hat{\Pi}_P \hat{\Pi}_Q |\Gamma_0\rangle||}.$$  \hfill (12)

To calculate this expression it is convenient to express the projection operators in one common basis, so we express $\hat{\Pi}_P$ in terms of the states $|q\rangle$ via eqn. (7):

$$\hat{\Pi}_P = \frac{1}{2b+1} \sum_{p=-b}^{b-P} \sum_{q',q''=-b}^{b} \sum_{q'''=-b}^{b} |q',q''\rangle\langle q',q''|$$

$$\times \exp \left\{ \frac{2\pi i}{2b+1} (q' P + q'' P + q''' q' - q'' P + q''' q') \right\}.$$  \hfill (13)

The total projection operator is thus given by

$$\hat{\Pi}_P \hat{\Pi}_Q = \frac{1}{2b+1} \sum_{p=-b}^{b-P} \sum_{q',q''=-b}^{b} \sum_{q=Q-b}^{b} |q',q''\rangle\langle q',q''|$$

$$\times \exp \left\{ \frac{2\pi i}{2b+1} (q' P + q'' P + q''' q' - q'' P + q''' q') \right\}$$

$$= \frac{1}{2b+1} \sum_{p=-b}^{b-P} \sum_{q',q''=-b}^{b} \sum_{q=Q-b}^{b} |q',q''\rangle\langle q-q| \exp \left\{ \frac{2\pi i}{2b+1} (q' P + q'' P + Q + q P - Q P) \right\}. \hfill (14)$$
Applying this operator on the initial state $|\Gamma_0\rangle$ finally yields

$$
\hat{\Pi}_P \hat{\Pi}_Q |\Gamma_0\rangle = \frac{1}{(2b+1)^{3/2}} \sum_{p=-b}^{b-P} \sum_{q',q''=-b}^{b} \sum_{q=Q-b}^{b} |q',q''\rangle_A \otimes |q\rangle_B \\
\times \alpha_{Q-q} \exp \left\{ \frac{2\pi i}{2b+1} (q'P + q'p + q''p - QP + qP - Qp) \right\}
$$

$$
= \frac{1}{(2b+1)^{3/2}} \sum_{p=-b}^{b-P} \sum_{q',q''=-b}^{b} \exp \left\{ \frac{2\pi i}{2b+1} (q'P + (q' + q'')p - Q(P - p)) \right\} |q',q''\rangle_A \otimes |\Gamma_2\rangle_B
$$

(15)

where $|\Gamma_2\rangle_B$ is Bob's (unnormalized) state after the measurement of $\hat{Q}_+$ and $\hat{P}_-$

$$
|\Gamma_2\rangle_B = \sum_{q=-Q-b}^{b} \alpha_{Q-q} \exp \left\{ \frac{2\pi i qP}{2b+1} \right\} |q\rangle_B.
$$

(16)

This represents the initial unknown state with shifted amplitudes $\alpha_{Q-q}$ and some phase factors $e^{\frac{2\pi iqP}{2b+1}}$. Both, the index shift $Q$ and the phase factor proportional to $P$ are known to Alice after the joint measurement of $\hat{Q}_+$ and $\hat{P}_-$ and the corresponding information can be transmitted to Bob by classical channels. Bob can then apply appropriate shift and phase-rotation operations to his quantum state to recover a replica of the input state. We do see however that due to the index shift and the finite dimension of the Hilbert space of Bob's state, some of the original state amplitudes $\alpha_q$ may be lost, depending on the value of $Q$. Consequently the teleportation has only a finite success probability.

### III. PROBABILITY OF SUCCESS

We now discuss the probability of success of the described teleportation protocol. For this we generalize the above discussion and drop the requirements $Q \geq 0$ and $P \geq 0$. Then eqn. (17) can be rewritten as

$$
|\Gamma_2\rangle \sim \sum_{q=\max[-b,-b+Q]}^{\min[b,b+Q]} \alpha_q \exp \left\{ \frac{2\pi i }{2b+1} (q + Q)P \right\} |q + Q\rangle.
$$

(18)

We see that only state amplitudes $\alpha_q$ survive for which $q \in \{\max[-b,-b+Q],\min[b,b+Q]\}$. Thus a sufficient condition for a successful teleportation of an arbitrary qudit state is that the measured eigenvalue $Q$ fulfills

$$
|Q| \leq b - a.
$$

(19)

If this inequality holds, the knowledge of $Q$ and $P$ is sufficient for Bob to reproduce the initial state by local operations.

We now want to calculate the success rate for the given teleportation protocol, i.e. the probability that condition (19) is fulfilled. The probability to measure a specific eigenvalue $Q$ is

$$
p(Q) = |\langle Q|\Gamma\rangle|^2
$$

$$
= \frac{1}{2b+1} \sum_{q_1=-a}^{a} \sum_{q_2=-b}^{b} |\alpha_{q_1}|^2 \delta_{q_1+q_2,Q}
$$

(20)

The probability of success of the teleportation protocol is therefore

$$
P = \sum_{Q=-(b-a)}^{b-a} p(Q) = \frac{1}{2b+1} \sum_{Q=-(b-a)}^{b-a} \sum_{q_1,q_2} |\alpha_{q_1}|^2 \delta_{q_1+q_2,Q}.
$$

(21)

Independent of the value of $\alpha_{q_1}$ the summation over $q_2$ and $Q$ yields

$$
\sum_{q_2=-b}^{b} \sum_{Q=-(b-a)}^{b-a} \delta_{q_2,Q-q_1} = 2(b-a) + 1.
$$
This is because \( a - b \leq Q \leq b - a \) and \(-a \leq q_1 \leq a\) always imply \(-b \leq Q - q_1 \leq b\) and so one has \(2(b - a) + 1\) non-vanishing contributions regardless of \(q_1\). With this we eventually arrive at the success probability

\[
P = \frac{2(b - a) + 1}{2b + 1} \sum_{q_1} |\alpha_{q_1}|^2
\]

\[
= \frac{2(b - a) + 1}{2b + 1} = 1 - \frac{2a}{2b + 1}
\]  \(\text{(22)}\)

which does not depend on the input state. We can rewrite this result noting that the dimension of the Hilbert space of the input state is \(\dim\{\mathcal{H}_i\} = 2a + 1\) and the dimension of the ancilla Hilbert space is \(\dim\{\mathcal{H}_a\} = 2b + 1\):

\[
P = 1 - \frac{\dim\{\mathcal{H}_i\} - 1}{\dim\{\mathcal{H}_a\}}.
\]  \(\text{(23)}\)

Eq.(23) is the main result of our paper. It shows that a linear teleportation protocol is in general always probabilistic and that its success probability only depends on the relative Hilbert dimensions of the input and ancilla spaces. In the case of qubits, i.e. for \(\dim\{\mathcal{H}_i\} = \dim\{\mathcal{H}_a\} = 2\) we obtain \(P = 1/2\) as realized e.g. in the scheme of Weinfurter \([7]\). Likewise in the case of large Hilbert spaces with \(\dim\{\mathcal{H}_a\} \gg \dim\{\mathcal{H}_i\}\) as in the Braunstein-Kimble proposal \([8]\) in the limit of perfect squeezing, a unit success probability can be achieved.

It should be noted that an analogous calculation can be performed in the case of continuous variables with a constant density of states but bounded spectrum. In this case the input and the ancilla states are given by

\[
|\Psi\rangle = \int_{-a}^{a} dq_1 \alpha(q_1)|q_1\rangle
\]  \(\text{(24)}\)

\[
|\Phi\rangle = \frac{1}{\sqrt{2b}} \int_{-b}^{b} dq_2 \int_{-b}^{b} |q_2\rangle| - q_2\rangle.
\]  \(\text{(25)}\)

Then \(\hat{q}\) and \(\hat{p}\) are position and momentum operators or quadrature amplitudes. One obtains the similar result

\[
P_{\text{cont}} = 1 - \frac{a}{b}.
\]  \(\text{(26)}\)

The only difference in this formula is the missing of the term \(+1\) which arises from the different treatment of the end points.

### IV. EFFICIENCY OF THE KLM TELEPORTATION SCHEME

At first sight the KLM \([1]\) teleportation scheme seems to implement the generalized discrete teleportation scheme discussed in this paper where the quantum number \(q\) is the total number of photons in Alice’s modes since its probability of success scales as

\[
P = 1 - \frac{1}{n + 1}.
\]  \(\text{(27)}\)

The entangled ancilla state consist of \(n + 1\) terms containing \(n\) photons each. This is exactly the result one would obtain from eqn. \(24\) if we insert \(\dim\{\mathcal{H}_i\} = 2\) (qubits) and \(\dim\{\mathcal{H}_a\} = n + 1\). But this interpretation is not correct because the dimension of the Hilbert space \(\mathcal{H}_a\) of the \(n\) ancilla photons is much larger. Although the true Hilbert-space dimension of \(n\) photons distributed over \(2n\) modes is much larger than \(n + 1\), one could argue that in fact only a small subspace of this large Hilbert space is actually used. This leads us to the question what is the relevant dimension of the Hilbert space of the ancilla photons.

One definition that gives a lower bound for the dimension of the used Hilbert space is the number of distinguishable measurement outcomes. In the KLM case one applies a discrete \(n + 1\) point Fourier transformation on the first \(n + 1\) modes followed by photodetection of these modes. One has to measure not only the total photon number but also their distribution over the modes. In fact every possible distribution of \(k = 0, \ldots, n + 1\) photons over \(n + 1\) modes does occur. As photons in the same mode are indistinguishable the number of different measurements is smaller than the physical dimension of the Hilbert space and one has a number of

\[
N_{\text{KLM}} = \sum_{k=0}^{n+1} \binom{n + k}{k}
\]

\[
= \frac{(2n + 2)!}{((n + 1)!)^2}
\]  \(\text{(28)}\)
different measurement outcomes. In contrast, the measurement of \( \hat{Q}_+ \) and \( \hat{P}_- \) discussed in this paper can only result into \( 2(a + b) + 1 \) resp. \( 4b + 1 \) different measurement outcomes. I.e. one can distinguish

\[
N = (2(a + b) + 1)(4b + 1)
\]  

(29)
different measurement results. We see that for the discussed teleportation scheme the number of distinguishable measurement outcomes scales quadratically in \( b \) whereas for the KLM teleportation scheme this number scales much less favorable with \( (2n + 2)!/(n+1)!^2 \). Since the KLM scheme does not scale optimal in the sense of the probability of success calculated above, it seems feasible that other linear teleportation schemes may be developed whose success probability scales better. This is of particular interest for the practical implementation of linear optical quantum computing, for example in the protocol described by Yoran and Reznik \[12\], where the resources depend critical on the scaling of the probability of success of the single quantum gates.

V. SUCCESS VS. FIDELITY

Recently Franson, \textit{et al.} \[11\] suggested a modification of the KLM scheme which always succeeds but whose fidelity, i.e. the overlap of the teleported to the input state is not unity. According to reference \[11\] we will rather consider the square of the fidelity

\[
F = ||\langle \Psi_{\text{in}}|\Psi_{\text{out}} \rangle||^2.  
\]  

(30)

In the proposal of Franson, \textit{et al.} the square of the fidelity scales as \( F = 1 - 1/n^2 \). This approach is different from the present one and the one of Knill, Laflamme, and Milburn, where the teleportation reproduces exactly the input state if it is successful. Whether or not the teleportation is successful is hereby uniquely determined by the measurement result. This has the advantage that by post-measurement selection of a teleported ensemble a sub-ensemble of exact replica of the input state can be generated. The obvious advantage of the Franson scheme is that in any case a state is teleported which is similar to the input one.

To make a comparison to the Franson scheme we could ask the question what is the average fidelity of our scheme if we do not discard those events where \( Q \) falls outside of the interval given by eq.(19). Then the mean squared fidelity of the discussed teleportation scheme is:

\[
\bar{F} = \sum_{Q=-b-a}^{b+a} p(Q) |\langle \Gamma_2^Q |\Psi \rangle|^2  
\]  

(31)

where \( |\Gamma_2^Q \rangle \) is Bob’s state after the measurement with result \( Q \). \( p(Q) \) denotes the probability to obtain this outcome and \( P \) denotes the probability of success derived in previous sections, corresponding to an exact reproduction of the input state. To estimate the fidelity we assume in the following that the basis states \( |q_i \rangle \) have an equal a priori probability to appear in the input state of the teleportation such that we have the mean value of the coefficients \( \alpha_{q_i} \):

\[
\langle |\alpha_{q_i} | \rangle = \text{const.} = \frac{1}{\sqrt{2a+1}}  
\]  

(32)

Then given a measurement outcome with \( Q > b - a \), the overlap between the input and the output state is

\[
|\langle \Gamma_2^Q |\Psi \rangle| = \frac{1}{2a+1} \sum_{q=Q-b}^{b} \sum_{q'=-a}^{a} \delta_{q,q'}  
\]  

(33)

= \frac{a + 1 + b - Q}{2a + 1}.

Considering as an example qubits that can only assume two values (i.e. \( a = 1/2 \)) the teleportation does not reproduce the exact input state iff \( Q = b + 1/2 \) or \( Q = -b - 1/2 \). In this case it is \( |\langle \Gamma_2^Q |\Psi \rangle|^2 = \frac{1}{4} \) and \( p(b+1/2) = p(-b-1/2) = \frac{1}{2(2b+1)} \) and thus we find for the mean squared fidelity of our teleportation protocol:

\[
\bar{F} = 1 - \frac{1}{2b+1} + \frac{1}{4(2b+1)}.  
\]  

(34)
The first two terms represent the non unity probability to exactly reproduce the input state, whereas the last term describes the finite but nonvanishing overlap of the teleported state with the initial state in the previously unsuccessful cases. We see that allowing for a nonperfect reproduction of the state the fidelity can be enhanced as compared to the result of the last section. Still $1 - F$ scales linear in $1/b$ as compared to the quadratic scaling in $1/n$ of the Franson scheme. One has to keep in mind however that the number $n$ in the KLM or Franson scheme is not the dimension of the ancilla Hilbert space, as we pointed out in the preceding section.

VI. CONCLUSION

In the present paper we discussed the question why continuous teleportation can be performed with linear elements in a deterministic way while the discrete counterpart requires nonlinear elements in form of quantum gates in order to be successful in all cases. For this we introduced a teleportation protocol similar to that used in the continuous case applied to qudits with a discrete and finite set of basis states. We also generalized the notion of linear elements and detection to the measurements of operators $\hat{Q}_+ = \hat{q}_1 + \hat{q}_2$ and $\hat{P}_- = \hat{p}_1 - \hat{p}_2$ which are linear combinations of the basic observables $\hat{q}$ and $\hat{p}$ in the input and ancilla spaces. We have shown that this protocol which uses only linear elements allows only for a probabilistic teleportation, a result recently shown to be true for any linear protocol \cite{14}, and has a success probability $P$ which is determined only by the Hilbert space dimensions of the input (dim{H$_i$}) and ancilla states (dim{H$_a$}). In the case of qubits we recover the value $P = 1/2$, which is the known limit for discrete teleportation with linear elements. On the other hand for the continuous teleportation protocol of Braunstein and Kimble the requirement for a local oscillator field with infinite squeezing, and thus infinite photon number, implies that the effective dimension of the ancilla Hilbert space is much larger than that of the input space. In this case the success probability of linear teleportation approaches unity. Thus the difference between discrete and continuous teleportation appears to result from the difference in the ancilla resources used rather than hidden nonlinearities as suggested in \cite{9}.

We have also compared the Knill Laflame Milburn proposal for linear teleportation \cite{1} as well as the one by Franson and coworkers \cite{11} with our abstract scheme and found that both do not scale optimal. This suggests that it may be possible to construct specific linear teleportation schemes for photons which use much less resources than the KLM and Franson schemes.

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\cite{1} E. Knill, R. Laflamme, G.J. Milburn, Nature 409, 46 (2001).
\cite{2} C.H. Bennett et al., Phys. Rev. Lett. 70, 1895 (1993).
\cite{3} N. Lütkenhaus, J. Calsamiglia, and K.-A. Suominen, Phys. Rev. A 59, 3295 (1999).
\cite{4} J. Calsamiglia, Phys. Rev. A 65, 030301 (2002)
\cite{5} N.J. Cerf, C. Adami, and P.G. Kwiat, Phys. Rev. A 57 R1477 (1998)
\cite{6} P.G. Kwiat, H. Weinfurter, Phys. Rev. A 58, R2623 (1998).
\cite{7} H. Weinfurter, Europhys. Lett. 25, 559 (1994).
\cite{8} S.L. Braunstein and H.J. Kimble, Phys. Rev. Lett. 80, 869 (1998).
\cite{9} L. Vaidman and N. Yordan, Phys. Rev. A 59, 116 (1999).
\cite{10} L. Vaidman, Phys. Rev. A 49, 1473 (1994).
\cite{11} J.D. Franson et al., Phys. Rev. Lett. 89, 137901 (2002).
\cite{12} R.F. Werner, J. Phys. A 34, 7081 (2001).
\cite{13} N. Yordan and B. Reznik, quant-ph/0303008 (2003).
\cite{14} Peter van Loock and Norbert Lütkenhaus, quant-ph/0304057 (2003).