INTERPLAY BETWEEN THE ELECTRIC FIELD AND THE
NONLINEAR INTERACTION
IN ORDERED AND DISORDERED CHAINS

Khaled Senouci and Nouredine Zekri
U.S.T.O., Département de Physique, L.E.P.M.,
B.P.1505 El M’Naouar, Oran, Algeria.

Abstract

A simple Kronig-Penney model is used to study the effect of nonlinear interactions on the electronic properties of ordered and disordered electrified chains. In the case of ordered potentials, we found that the nonlinearity suppresses the Wannier-Stark effect caused by the electric field. In the case of disordered potentials, the nonlinearity gives rise to a transition from superlocalized to weakly localized states.

Keywords: Band spectrum, non-linearity interaction, disorder, Wannier-Stark ladder effect, superlocalization.

\[Corresponding author, e-mail: nzekri@meloo.com\]
1 Introduction

It was well established two decades ago that all electronic states of one dimensional (1D) disordered systems are exponentially localized in the absence of external fields irrespective of the amount of disorder [1]. However, recently some models of disorder introducing the correlation [2, 3] and the nonlinearity [4] have been shown to exhibit extended states at particular energies. The electric field, on the other hand has been shown to delocalize the electronic states in 1D disordered systems where the wave function becomes power-law decaying [5-7] while for sufficiently large field strengths the eigenstates become extended [8, 9]. Furthermore, it can affect the backscattering and the interferences yielding a strong enhancement the localization (Wannier-Stark localization) [9]. In a recent paper, We found that the nonlinearity can either localize or delocalize the electronic states depending on the strength and the sign of the nonlinear potential [10]. Physically, a repulsive nonlinear (NL) potential represents the electron-electron interaction while an attractive one corresponds to the electron-phonon interaction. These interactions are important in various systems such as quantum dots, superlattices etc. [11]. Therefore, the electric field and the nonlinear potential effects can compete and their presence together in the system may lead to the suppression of some effects such as the Wannier-Stark localization. This is the aim of the present letter where we examine the effect of the NL interaction on the electronic properties of a chain of potentials in the presence of a constant electric field. Note that this effect on the resonant transmission has been investigated by Cota et al [12]. These resonances seem to change their structure with the NL strength. However, to the best of our knowledge this effect on the nature of the eigenstates has not been investigated before.
2 Model description

The model studied in this letter is defined by the following nonlinear Schrödinger equation \[\text{(1)}\]

\[
\left\{ -\frac{d^2}{dx^2} + \sum_n (\beta_n + \alpha |\Psi(x)|^2)\delta(x - n) - eFx \right\} \Psi(x) = E\Psi(x)
\]

Here \(\Psi(x)\) is the single particle wavefunction at \(x\), \(\beta_n\) the potential strength at the \(n\)–\(th\) site, \(\alpha\) the nonlinearity strength and \(E\) the single particle energy in units of \(\hbar^2/2m\) with \(m\) the electronic effective mass and \(F\) the electric field. The electronic charge \(e\) and the lattice parameter \(a\) are taken here for simplicity to be unity. The two ends of the system are assumed to be connected ohmically to ideal leads (where the electron moves freely) and maintained at a constant potential difference \(V = FL\). The potential strength \(\beta_n\) is uniformly distributed between 0 and \(W\) in the case of potential barriers and between \(-W\) and 0 in the case of potential wells (\(W\) being the degree of disorder). Equation (1) can be mapped by means the Poincaré map representation in the ladder approximation (i.e, when the field can be approximated as constant between two consecutive sites \[\text{[9]}\]). This approximation is valid for \(eFa \ll E\) to the following recursive equation \[\text{[12]}\]

\[
\Psi_{n+1} = \left[ \cos k_{n+1} + \frac{k_n}{k_{n+1}} \sin k_n \cos k_{n+1} \right] \Psi_n - \frac{k_n}{k_{n+1}} \sin k_n \Psi_{n-1}
\]

where \(\Psi_n\) is the value of the wavefunction at site \(n\) and \(k_n = \sqrt{E + Fn}\) is the electron wave number at the site \(n\). The solution of equation (2) is carried out iteratively by taking the two initial wave functions at sites 1 and 2 of the ideal leads : \(\Psi_1 = \exp(-ik)\) and \(\Psi_2 = \exp(-2ik)\). We consider here an electron with a wave number \(k\) incident at site \(N + 3\) from the right side (by taking the chain length \(L = N\), i.e. \(N + 1\) scatterers \). The transmission coefficient \((T)\) reads

\[
T = \frac{k_0}{k_L} \frac{|1 - \exp(-2ikL)|^2}{|\Psi_{N+2} - \Psi_{N+3}\exp(-ikL)|^2}
\]

\[\text{(3)}\]
where $k_0 = \sqrt{E}$ and $k_L = \sqrt{E + FN}$.

3 Results and discussion

In this section we examine in a first step the effect of nonlinearity on the energy spectrum of a periodic system in the presence of an electric field. We choose in this case $\beta = 1$, $F = 0.01$ and $L = 500$. For linear systems ($\alpha = 0$), the electric field seems to narrow the allowed bands because of the Wannier-Stark localization. Indeed, in this case the transmission coefficient has been shown to decrease abruptly near the band edges while Bloch oscillations appear \[13\]. The nonlinearity, on the other hand was found to delocalize, under certain conditions, the electronic states in periodic systems in the sense that the allowed bands become larger and the gaps get narrowed \[14\].

Figure 1 shows the effect of the NL on a periodic chain of potential barriers in the presence of an electric field. We in particular observe for increasing $\alpha < 0$, an increase of the transmission coefficient in the regions localized by the electric field (i.e. Wannier-Stark localization). This field induced localization tends to disappear for a given NL strength. On the other hand, the amplitude of the Bloch oscillations observed in the linear case (solid line) seems to decrease. This delocalization is however not observed if we consider periodic potential wells with repulsive NL, although we found recently that this type of NL delocalizes the electronic states in the gap for such systems \[10\]. This surprising effect may come from the unstabilities (strong drop of the transmission) observed at certain length scales where any amount of the NL potential strength enhances the localization \[10\]. These unstabilities should appear at larger length scales for the potential barriers.

Let us now examine the effect of NL interactions on disordered chains in the presence of an electric field. It was shown that the wave function becomes power-law decaying in the presence of an electric field \[3, 4, 7\]. On the other hand, the electric field was also found in certain cases to modify the scaling of the transmission in jumps with a behavior as $exp(-L^\gamma)$ (with $\gamma > 1$ and $L$ the length scale) between them \[3\]. This case was shown to correspond to a negative differential resistance \[8\]. Figure 2 shows the
transmission coefficient versus the chain length in the case of disordered potential wells. We choose $E = 5, F = 0.015$ and $W = 2$ with an ensemble averaging over 2000 samples (sufficient for an accuracy about 1%). We observe clearly that the superlocalization before the first jump tends to be suppressed in the presence of a repulsive NL ($\alpha > 0$) and the eigenstates become power-law decaying. The same behavior can be observed in the case of potential barriers (not shown here to avoid a lengthy paper). We note here that for almost cases the unstabilities of $T$ discussed above appear after the first jump of $T$. Therefore, we restricted ourselfe to the first jump.

In figure 2 we observed also a characteristic length $l_c$ separating the superlocalized states for small lengths from the power-law decaying ones for larger length scales. This caracteristic length seems to decrease logarithmically with the NL strength in the case of disordered potential wells while it decreases more rapidly for potential barriers (see Fig.3).
4 Conclusion

We studied in this letter the effect of nonlinearity on electrified periodic and disordered chains using a simple Kronig-Penney model. We found that in periodic potential barriers, the nonlinearity contributes to the delocalization of the Wannier-Stark localized states induced by the electric field. In the case of disordered systems, we found that the superlocalization observed recently in such systems in the presence of an electric field is suppressed progressively by the NL interaction and the wave functions become power-law decaying above a characteristic length $l_c$ (which seems to decrease also at least logarithmically with nonlinearity). However, beyond a certain length scale (corresponding after the first jump), any amount of the NL interaction destroys the transmission in certain samples instead of enhancing it, due to the unstability observed in nonlinear systems. Most probably this unstability predicts very interesting statistical properties of the transmission in such systems and should be carefully examined. This investigation should be the subject of a forthcoming paper.
References

[1] P.W.Anderson, D.J.Thouless, E.Abrahams and D.Fisher, Phys. Rev. B 22, 3519 (1980).

[2] A.Sanchez, E.Macia and F.Domínguez-Adame, Phys. Rev. B 49, 147 (1994).

[3] P.Phillips and H.L.Wu, Science 252, 1805 (1991).

[4] R.Bourbonnais and R.Maynard, Phys.Rev.Lett. 64, 1397 (1990); Y.S.Kivshar, S.A.Gredeskul, A.Sanchez and L.Vasquez, Phys.Rev.Lett. 64, 1693 (1990); Y.Kivshar, Phys.Lett. A 173, 172 (1993).

[5] C.M.Soukoulis, J.V.José, E.N.Economou and P.Sheng, Phys. Rev. Lett. 50, 764 (1983).

[6] F.Delyon, B.Simon and B.Souillard, Phys. Rev. Lett. 52, 2187 (1984).

[7] E.Cota, J.V.Jose and M.Ya.Azbel, Phys. Rev. B 32, 6157 (1985).

[8] K.Senouci, N.Zekri and R.Ouasti, Physica A 234, 23 (1996).

[9] R.Ouasti, N.Zekri, A.Brezini and C.Depollier, J. Phys. Condens. Matter 7, 811 (1995).

[10] K.Senouci, N.Zekri, H.Bahlouli and A.K.Sen, J. Phys. Condens. Matter 11, 1823 (1999).

[11] E.Diez, A.Sanchez and F.Domínguez-Adame, Phys.Lett. A 215, 103 (1996); E.Diez, F.Domínguez-Adame and A.Sanchez, Phys.Lett. A 198, 403 (1995).

[12] E.Cota, J.V.Jose and G. Monsivais, J. Phys. A: Matt. Gen 25, L57 (1992).

[13] N.Zekri, M.Schreiber, R.Ouasti, R.Bouamrane and A.Brezini, Z. Phys. B 99, 381 (1994).

[14] N.Zekri and H.Bahlouli, Phys. Stat. Sol.(b) 205, 511 (1998).
Figure Captions

**Fig.1** $-Ln(T)$ versus the Wavenumber $k$ in units of $\pi/a$ for ordered systems with potential barriers ($\beta = 1$), a length scale $L = 500$ and $E = 1$. Effect of the NL interaction.

**Fig.2** $< -Ln(T) >$ versus $L$ for disordered potentials wells with $E = 5$, $W = 2$, $F = 0.015$. Effect of the NL interaction.

**Fig3** $L_c$ versus $Log(|\alpha|)$ for disordered potential barriers and wells for the same parameters as Fig.2.
FIGURE 1

Wavenumber $k$ (units of $\pi / a$)

$\ln(T)$
FIGURE 2
FIGURE 3

Potential Barriers
Potential Wells