Dynamic Analysis of Cone Waveguide for Ultrasonic Assisted Technological Processes

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Abstract. Research and development of new materials (ceramics and composites) with advanced mechanical properties also leads to the development and application of new so-called hybrid technological processes, which are based on a combination of conventional technological processes with ultrasonic tool vibrations. The fundamental condition for the optimal realization of technological process is the vibration of ultrasonic waveguide in the resonant state. Therefore, the ultrasonic waveguide must have the required modal properties, which depend on the distribution of its mass and stiffness properties. Design and analysis of dynamic properties of ultrasonic cone waveguide with adaptive modification of modal properties is presented in this paper. Modification of modal properties is performed using an embedded core, which changes the distribution of spatial properties of the waveguide structure. The effect of geometric parameters and material properties of the inserted core on the ultrasonic waveguide modal properties (mode shapes, natural frequency, amplification factor) is presented.

1. Introduction

The high requirements for material properties that are currently used result in the development of qualitatively new materials and material structures. These new advanced materials (ceramics, composites, etc.) are generally characterized by improved physical and mechanical properties. This situation, of course, leads to relatively large problems in their technological processing. One of the ways to perform the required technological process and achieve a quality of required processing of these materials are technological processes using the so-called hybrid technology processes [3-4]. Hybrid technological processes are usually based on a combination of conventional technological processes with ultrasonic vibration. The transmission of vibrations to the place of realization of the technological process (for example in the case of turning to the cutting edge of a turning tool) is performed in an electromechanical excitation system [1]. The mechanical part of electromechanical excitation system (Figure 1) consists of an ultrasonic generator, electromechanical transducer, booster and waveguide (ultrasonic horn or sonotrode). In order to achieve the required parameters of the hybrid technological process, a resonant state must be achieved [2] in the whole mechanical subsystem of the electromechanical excitation system. The ultrasonic waveguide is the most important part of the whole excitation system, which have to be in the resonant state because it performs the transmission of the oscillation amplitude directly into the performed technological process. However, due to the changes that occur during the execution of the technological process, the "detuning" of the resonant
state of the excitation system occurs and the quality of the performed technological process is unsatisfactory. To preform of required technological process, it is necessary to change the parameters of the excitation system so that the mechanical part of the excitation system will be still tuned in the resonant state [2]. Design and analysis of cone ultrasonic waveguide with adaptive change of modal properties performed by embedding of reinforcing core is solved in this paper.

![Model mechanical part of ultrasonic excitation systems](image)

**Figure 1.** Model mechanical part of ultrasonic excitation systems.

2. **Formulation of the problem**

The mathematical formulation of the longitudinal vibration of a general waveguide is derived from the waveguide model shown in Figure 2.

![Ultrasonic waveguide design for general geometrical shape](image)

**Figure 2.** Ultrasonic waveguide design for general geometrical shape.

The partial differential equation (PDE) describing the free longitudinal vibration of general waveguide structure [5-6] is defined in the following form

\[
\rho S(x) dx \frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial}{\partial x} \left( ES(x) \frac{\partial u(x,t)}{\partial x} \right) dx = 0
\]

(1)

where

- \( x \) - coordinate in longitudinal direction,
- \( L_0 \) - waveguide length,
- \( u(x, t) \) - longitudinal displacement of cross section in \( x \) direction,
\( S(x) \) - cross section in \( x \) position,
\( E \) - Young modulus,
\( \rho \) - density of waveguide material,

After the introduction and substitution of dimensionless quantities
- dimensionless coordinate \( \xi = \frac{x}{L_0} \),
- dimensionless displacement \( \tilde{u}(\xi,t) = \frac{u(x,t)}{L_0} \),

the equation (1) has the form

\[
\frac{\partial^2 \tilde{u}(\xi,t)}{\partial t^2} - \frac{c_p^2}{L_0^2} \frac{1}{f_s(\xi)} \frac{\partial}{\partial \xi} \left( f_s(\xi) \frac{\partial \tilde{u}(\xi,t)}{\partial \xi} \right) = 0
\]

(4)

where \( c_p = \sqrt{\frac{E}{\rho}} \) - speed of propagation of longitudinal waves in the \( \xi \)-direction,

\( f_s(\xi) = f_s^2(\xi) \) - function describing the change of waveguide cross section in the direction \( \xi \),
\( f_r(\xi) \) - function describing the change of cross section radius in the direction \( \xi \).

The solution (4) can be assumed in the form

\[
\tilde{u}(\xi,t) = \tilde{U}(\xi)T(t),
\]

(5)

where function \( \tilde{U}(\xi) \) represents the mode shape of the longitudinal vibration of the waveguide and the function \( T(t) \) describes the time dependence of the waveguide longitudinal vibration.

The structural design of ultrasonic cone waveguide with adaptive modal properties is presented in Figure 3. The cone waveguide has input radius \( R_i \), output radius \( R_o \) and length \( L_o \). The ultrasonic cone waveguide body has a drilled hole (radius \( R_c \)) for insertion of core with the length \( L_c \). Position of core in waveguide body from input radius is defined by dimension \( L_s \). The different material properties will be used for body of cone waveguide and embedded core. The longitudinal displacements of interacting points between the cone waveguide body and the core are the same, i.e. perfect adhesion is assumed for the corresponding points in interface between core and waveguide body.

Figure 3. Design of ultrasonic cone waveguide with adaptive modal properties.
In order to generalize the results of the analyzes, the following geometric dimensionless parameters and material properties are introduced

- dimensionless input radius of waveguide cross section
  \[ \delta_R = \frac{R_i}{L_0} \approx R_i \]  
  (6)

- dimensionless output radius of waveguide cross section
  \[ \delta_o \delta_R = \frac{R_o}{L_0} \approx R_o \),  \( \delta_o = \frac{R_o}{R_i} \]  
  (7)

- dimensionless radius of core cross section
  \[ \delta_c \delta_R = \frac{R_c}{L_0} \approx R_c \),  \( \delta_c = \frac{R_c}{R_i} \]  
  (8)

- dimensionless core position
  \[ \xi_s = \frac{L_s}{L_0} \approx L_s \),  
  (9)

- dimensionless core length
  \[ \xi_c = \frac{L_c}{L_0} \approx L_c \),  
  (10)

- dimensionless density of core material
  \[ \kappa_\rho = \frac{\rho_c}{\rho} \],  
  (11)

- dimensionless Young modulus of core material
  \[ \kappa_E = \frac{E_c}{E} \].  
  (12)

The dimensionless geometric parameters for conical shape of the waveguide with the inserted reinforcing core are shown in Figure 4.

**Figure 4.** Dimensionless geometrical parameters of cone waveguide with inserted core.
The finite element method (ANSYS software code environment) is used to create a computational model and perform a modal analysis of a conical waveguide and determine its modal properties (mode shapes, natural frequency, amplification factor). Typical mode shapes (half wave and full wave) of waveguide vibrations in resonant states are shown in Figure 5. The input and output amplitudes of the mode shape are expressed as follows

\[ A_i = \bar{U} (\xi = 0) \quad \text{and} \quad A_o = \bar{U} (\xi = 1). \] (13)

![Figure 5. Typical mode shapes of full cone waveguide vibrations in resonant state.](image)
a) half wave mode shape; b) full wave mode shape

The cone waveguide with adaptive modal properties (Figure 3, Figure 4) has similar mode shapes to those shown for full cone waveguide (Figure 5) without the possibility of modifying modal properties. The performance and efficiency of a waveguide with adaptive modification of modal properties are evaluated using the values of its modal properties, which are modified due to the change of geometric parameters and material properties of the inserted core.

In order to compare the effect of structural modifications of cone waveguide on its modal properties and to generalize the results of the analyzes, the dimensionless modal properties such as dimensionless natural frequency and dimensionless amplification factor are formulated, as follows

- the first dimensionless natural frequency

\[ \psi_1 = \frac{f_{01,m}}{f_{01}}, \] (13)

- the second dimensionless natural frequency

\[ \psi_2 = \frac{f_{02,m}}{f_{02}}, \] (14)

- the first dimensionless amplification factor

\[ \theta_1 = \frac{A_{1,0}}{A_{1,i}}, \] (15)

- the second dimensionless amplification factor

\[ \theta_2 = \frac{A_{2,0}}{A_{2,i}}, \] (16)

where

- \( f_{01,m} \) (\( f_{02,m} \)) - the first (second) natural frequency of modified waveguide structure (with core),
- \( f_{01} \) (\( f_{02} \)) - the first natural frequency of original waveguide structure (without core),
- \( A_{1,i} \) (\( A_{2,i} \)) - input amplitude for the first (second) mode shape of waveguide,
- \( A_{1,0} \) (\( A_{2,0} \)) - output amplitude for the first (second) mode shape of waveguide.
3. Numerical analysis and results

The computational models and half-wave and full-wave mode shapes of cone waveguide vibrations with the possibility of adaptive modification of modal properties, by inserting the core into the waveguide body, for different core positions, are shown in Figure 6.

![Mode shapes of cone waveguide vibrations with adaptive modification modal properties.](image)

The effect of structural changes of the cone waveguide by the inserted core on the first and second natural frequencies are graphically presented in Figures 7-8. Similarly, the effect of the presented structural changes cone waveguide by the inserted nucleus on the first and second amplification factors are presented in Figures 9-10. In these graphs, the effects of changes in the position and length of the core in the cone waveguide body for different dimensionless densities and young modulus of elasticity are presented.

The dependencies of dimensionless natural frequencies and dimensionless amplification factors on the change in structural parameters for the case when the reinforcing core is gradually inserted into the waveguide body (Figure 6) until the entire hole in the waveguide body is filled with the core are shown in Figure 7 and Figure 8. For the structural parameters that have been considered and that can be changed, it is possible to modify the first dimensionless natural frequency in the range from $-16\%$
to +16 % and the second dimensionless natural frequency in the range from −12 % to +16 %. For the first dimensionless amplification factor, the range of the modification is from −26 % to +44 %, and for the second dimensionless amplification factor, the range is from −28 % to 75 %.

**Figure 7.** Dependency of the first (a) and second (b) natural frequencies on the dimensionless core length for parameters $\kappa_E = \{0.5; 1.0; 2.0\}$.

The second case for the modification of the modal properties of a conical waveguide, which concerns the insertion of a reinforcing core of finite length into the waveguide body (Figure 6c), where two dimensionless core lengths $\xi_c$ (0.3 and 0.5) were considered. The modifications of dimensionless natural frequencies and dimensionless amplification factors are shown in Figure 9 and Figure 10.

**Figure 8.** Dependency of the first (a) and second (b) amplification factors on the dimensionless core length for parameters $\kappa_E = \{0.5; 1.0; 2.0\}$.
When the considered structural parameters are modified, the modification of the first dimensionless natural frequency in the range from $-13\%$ to $+7\%$ (for $\xi_c = 0.3$), respectively from $-13\%$ to $+4\%$ (for $\xi_c = 0.5$) and the second dimensionless natural frequency in the range from $-7\%$ to $+3\%$ (for $\xi_c = 0.3$), respectively from $-1.0\%$ to $+1.2\%$ (for $\xi_c = 0.5$). For the first dimensionless amplification factor, the range of the modification is from $-40\%$ to $+1\%$ (for $\xi_c = 0.3$), respectively from $-51\%$ to $0\%$ (for $\xi_c = 0.5$), and for the second dimensionless amplification factor, the range of the modification is from $-59\%$ to $+17\%$ (for $\xi_c = 0.3$), respectively from $-20\%$ to $+16\%$ (for $\xi_c = 0.5$).

**Figure 9.** Dependency of the first (a) and second (b) natural frequencies on the dimensionless core position $\xi_s$ and specific dimensionless core length $\xi_c = \{0.3; 0.5\}$.

**Figure 10.** Dependency of the first (a) and second (b) amplification factors on the dimensionless core position $\xi_s$ and specific dimensionless core length $\xi_c = \{0.3; 0.5\}$. 
Conclusions
Method for modification (tuning) modal properties (natural frequencies and amplification factor) of cone waveguide structure for excitation system for ultrasonic assisted technological processes is presented. The method of modal properties modification of waveguide is based on structural modification of waveguide parameters (i.e. redistribution of mass and stiffness properties), which consists in inserting a reinforcing core into the waveguide body.

By inserting a reinforcing core, it is possible to achieve relatively significant modification of modal properties (natural frequencies and amplification factors) depending on the length of inserted core, on the position of inserted core in the waveguide body and the core material properties.

Based on the results obtained, it can be concluded that this method of modifying the modal properties provides suitable possibilities for "tuning" the conical waveguide for the needs of performing the technological process in the optimal mode.

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