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Lepton flavor violation  
in supersymmetric  
low-scale seesaw models

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The minimal supersymmetric standard model with a low scale see-saw mechanism is presented. Within this framework, the lepton flavour violation in the charged lepton sector is thoroughly studied. Special attention is paid to the individual loop contributions due to the heavy neutrinos $N_{1,2,3}$, sneutrinos $\tilde{N}_{1,2,3}$ and soft SUSY-breaking terms. For the first time, the complete set of box diagrams is included, in addition to the photon and $Z$-boson mediated interactions. The complete set of chiral amplitudes and their associate form-factors related to the neutrinoless three-body charged lepton flavor violating decays of the muon and tau, such as $\mu \rightarrow eee$, $\tau \rightarrow \mu\mu\mu$, $\tau \rightarrow e\mu\mu$ and $\tau \rightarrow e\epsilon\mu$, as well as the coherent $\mu \rightarrow e$ conversion in nuclei, were derived.

The obtained analytical results are general and can be applied to most of the New Physics models with charged lepton flavor violation. This systematic analysis has revealed the existence of two new box form factors, which have not been considered before in the existing literature in this area of physics.

In the same model, the systematic study of one-loop contributions to the muon anomalous magnetic dipole moment $a_\mu$ and the electron electric dipole
moment $d_e$ is performed. Special attention is paid to the effect of the sneutrino soft SUSY-breaking parameters, $B_\nu$ and $A_\nu$, and their universal CP phases ($\theta$ and $\phi$) on $a_\mu$ and $d_e$.

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Introduction

In the first part of this thesis the study of charged lepton flavor violation (CLFV) is performed in low-scale seesaw model of minimal supersymmetric standard model ($\nu_R$MSSM) within the framework of minimal supergravity (mSUGRA). There are two dominant sources of CLFV: one originating from the usual soft supersymmetry-breaking sector, and other entirely supersymmetric coming from the supersymmetric neutrino Yukawa sector. Both sources are taken into account within this framework, and number of possible lepton-flavor-violating transitions are calculated. Supersymmetric low-scale seesaw models offer distinct correlated predictions for lepton flavor violating signatures, which might be discovered in current and projected experiments.

In the second part, the same model is used to study the anomalous magnetic and electric dipole moments of charged leptons. The numerical estimates of the muon anomalous magnetic moment and the electron electric dipole moment will be given as a function of key parameters. The electron electric dipole moment is found to be naturally small in this model, and can be probed in the present and future experiments.

The thesis is organized as follows:

- The first chapter gives a brief experimental survey, the current and projected experiments regarding the detection of charged lepton flavor violation and anomalous dipole moments of charged leptons.

- The second chapter presents the theoretical framework which underlines the study of lepton flavor violation and anomalous dipole moments.
given in the thesis.

- The third chapter exposes the analytic and numerical results for various lepton flavor violating transitions, as well as some important physical implications which follow.

- The fourth chapter gives the analysis of the muon anomalous magnetic moment and the electron electric dipole moment in supersymmetric low-scale seesaw models with right-handed neutrino superfields.

- Concluding remarks are given in the fifth chapter.

- The appendices contain technical details regarding the relevant interaction vertices, loop functions and formfactors.

The main results of the thesis are the following:

- The soft SUSY-breaking effects in the $Z$-boson-mediated graphs dominate the CLFV observables for appreciable regions of the $\nu_R$MSSM parameter space in mSUGRA. But for $m_N \lesssim 1$ TeV the box diagrams involving heavy neutrinos in the loop can be comparable to, or even greater than the corresponding $Z$-boson-mediated diagrams in $\mu \to eee$ and $\mu \to e$ conversion in nuclei. Therefore, the usual paradigm with the photon dipole-moment operators dominating the CLFV observables in high-scale seesaw models have to be radically modified.

- Heavy singlet neutrino and sneutrino contributions to anomalous magnetic dipole moment of the muon are small, typically one to two orders of magnitude below the muon anomaly $\Delta a_\mu$. The largest effect on $\Delta a_\mu$ instead comes from left-handed sneutrinos and sleptons, exactly as is the case in the MSSM without right-handed neutrinos. Heavy singlet neutrinos do not contribute to the electric dipole moment (EDM) of the electron either. The main contribution to EDM comes from SUSY-breaking terms, but only if one of the CP phases ($\theta$ and/or $\phi$) introduced to SUSY-breaking sector is nonvanishing.
Neutrino oscillation experiments have provided undisputed evidence of lepton flavor violation (LFV) in the neutrino sector, pointing towards physics beyond the Standard Model (SM). Nevertheless, no evidence of LFV has been found in the charged lepton sector of SM, implying conservation of the individual lepton number associated with the electron $e$, the muon $\mu$ and the tau lepton $\tau$. All past and current experiments were only able to report upper limits on observables of charged lepton flavor violation (CLFV). The experimental detection of CLFV would certainly pave the way to the New Physics.

Measurements of the anomalous magnetic dipole moment of the muon (i.e. its deviation from the SM prediction, $\Delta a_\mu$) can give an important constraint on model-building, since any New Physics contribution must remain within $\Delta a_\mu$ limit. Study of the electric dipole moment of the electron $d_e$ is even more compelling, since the observation of non-zero (i.e. $\gtrsim 10^{-33} \, e \, cm$) value for $d_e$ would signify the existence of CP-violating physics beyond the Standard Model.
§ 1.1 Neutrino oscillations

When a neutrino is produced in some weak interaction process, and it propagates through some finite distance, there is a non-zero probability that it will change its flavor. This well established and observed fact is known as *neutrino oscillation* [1–3], due to the oscillatory dependence of the flavor change probability with respect to the neutrino energy and the distance of the propagation.

There are numerous neutrino experiments which report the lepton flavor violation in the neutrino sector, by observing the disappearances or the appearances of a particular neutrino flavor.

In solar neutrino experiments, first by Homestake [4] and later confirmed by others [5–12], the disappearance of the solar electron neutrino $\nu_e$ is observed. Atmospheric muon neutrinos $\nu_\mu$ and antineutrinos $\bar{\nu}_\mu$ disappeared in Super-Kamiokande experiment [13, 14]. The disappearance of reactor electron antineutrinos $\bar{\nu}_e$ is observed in Kam-LAND reactor [15, 16] and in DOUBLE-CHOOZ experiment [17]. Muon neutrinos $\nu_\mu$ disappeared in the long-baseline accelerator neutrino experiments MINOS [18, 19] and K2K [20]. Short-baseline reactor experiments Daya Bay [21, 22] and RENO [23] report the disappearance of the reactor electron antineutrinos $\bar{\nu}_e$.

The appearance of electron neutrino $\nu_e$ in a beam of muon neutrinos $\nu_\mu$ in long-baseline accelerator is reported by T2K [24] and MINOS [25] experiments.

All these experiments have provided undisputed evidence for neutrino oscillations caused by finite (non-zero) neutrino masses and, consequently, neutrino mixing parameters. Since neutrinos are massive, the transition from the neutrino flavor eigenstate fields $(\nu_e, \nu_\mu, \nu_\tau)$ which makes the lepton charged current in weak interactions to the neutrino mass eigenstate fields $(\nu_1, \nu_2, \nu_3)$ is non-trivial:

$$\nu_l(x) = \sum_{i=1}^{3} U_{li} \nu_i(x), \quad l = e, \mu, \tau.$$  \hspace{1cm} (1.1)
Unitary matrix $U$ is known as Pontecorvo-Maki-Nakagawa-Sakata matrix $^{[1-3]}$ and is usually parametrized as

$$U_{PMNS} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \cdot P,$$

where $P = \text{diag}(1, e^{i\alpha}, e^{i\beta})$, $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$. $\theta_{12}$ denotes solar mixing angle, $\theta_{23}$ atmospheric mixing angle and $\theta_{13}$ reactor mixing angle. Phases $\delta$, $\alpha$ and $\beta$ stand for Dirac CP violating phase and two Majorana CP violating phases, respectively.

Nonzero values of $\theta_{13}$ reported in recent reactor neutrino oscillation experiments $^{[17,21,23]}$ strongly indicate a nontrivial neutrino-flavor structure and possibly CP violation.

## § 1.2 Searching for CLFV

The existence of lepton flavor violation (LFV) in the neutrino sector implies the possibility of LFV in the charged sector as well. However, in spite of intense experimental searches $^{[26-37]}$ no evidence of LFV in the charged lepton sector of the Standard Model (SM) has yet been found.

All past and current experiments searching for the charged lepton flavor violation (CLFV) were only able to report upper limits on the observables associated with CLFV. Recently, the MEG collaboration $^{[26]}$ has announced an improved upper limit on the branching ratio of the CLFV decay $\mu \rightarrow e\gamma$, with $B(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}$ at the 90% confidence level (CL). As also shown in Table $^{[1.1]}$ future experiments searching for the CLFV processes, $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$, coherent $\mu \rightarrow e$ conversion in nuclei, $\tau \rightarrow e\gamma/\mu\gamma$, $\tau \rightarrow 3$ leptons and $\tau \rightarrow \text{lepton + light meson}$, are expected to reach branching-ratio sensitivities to the level of $10^{-13}$ $^{[38,39]} (10^{-14} \ [40])$, $10^{-16}$ $^{[41]} (10^{-17} \ [40])$, $10^{-17}$ $^{[42,45]} (10^{-18} \ [40])$, $10^{-9}$ $^{[48,49]}$, $10^{-10}$ $^{[48]}$ and $10^{-10}$ $^{[48]}$, respectively. The values in parentheses indicate the sensitivities that are ex-
expected to be achieved by the new generation CLFV experiments in the next decade. Most interestingly, the projected sensitivity for $\mu \to eee$ and $\mu \to e$ conversion in nuclei is expected to increase by five and six orders of magnitude, respectively. The history and current status of the experimental search for CLFV is very nicely exposed in Ref [50], which is highly recommended for further reading.

| No. | Observable | Upper Limit | Future Sensitivity |
|-----|------------|-------------|--------------------|
| 1.  | $B(\mu \to e\gamma)$ | $2.4 \times 10^{-12}$ | $1-2 \times 10^{-13}$ |
| 2.  | $B(\mu \to eee)$ | $10^{-12}$ | $10^{-16}$ |
| 3.  | $R^{T}_{\mu e}$ | $4.3 \times 10^{-12}$ | $3-7 \times 10^{-17}$ |
| 4.  | $R^{A}_{\mu e}$ | $7 \times 10^{-13}$ | $3-7 \times 10^{-17}$ |
| 5.  | $B(\tau \to e\gamma)$ | $3.3 \times 10^{-8}$ | $1-2 \times 10^{-9}$ |
| 6.  | $B(\tau \to \mu\gamma)$ | $4.4 \times 10^{-8}$ | $2 \times 10^{-9}$ |
| 7.  | $B(\tau \to eee)$ | $2.7 \times 10^{-8}$ | $2 \times 10^{-10}$ |
| 8.  | $B(\tau \to e\mu\mu)$ | $2.7 \times 10^{-8}$ | $10^{-10}$ |
| 9.  | $B(\tau \to \mu\mu\mu)$ | $2.1 \times 10^{-8}$ | $2 \times 10^{-10}$ |
| 10. | $B(\tau \to \mu e\mu)$ | $1.8 \times 10^{-8}$ | $10^{-10}$ |

Table 1.1: Current upper limits and future sensitivities of CLFV observables under study.

Given that CLFV is forbidden in the SM, its observation would constitute a clear signature for New Physics, which makes this field of investigation ever more exciting.

§ 1.3 Measuring lepton dipole moments

The anomalous magnetic dipole moment (MDM) of the muon, $a_\mu$, is a high precision observable extremely sensitive to physics beyond the Standard Model. Its current experimental value, according to PDG [37], is

$$a_\mu^{\exp} = (116592089 \pm 63) \times 10^{-11}.$$  

The Standard Model prediction of this observable reads

$$a_\mu^{\SM} = (116591802 \pm 49) \times 10^{-11}.$$ (1.4)
The difference between measured and predicted value,

\[
\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (287 \pm 80) \times 10^{-11}
\]

is at the 3.6\(\sigma\) confidence level (CL) and has therefore been called the muon anomaly. This value limits the allowed contributions of New Physics to MDM and consequently can be used as a strong constraint on model-building, or even eliminate some of the proposed New Physics models.

Likewise, the electric dipole moment (EDM) of the electron, \(d_e\), constitutes a very sensitive probe for CP violation induced by new CP phases present in the physics beyond the Standard Model. The present upper limit on \(d_e\) is reported to be \(37, 51, 52\)

\[
d_e < 10.5 \times 10^{-28} \text{ e cm}.
\]

Future projected experiments utilizing paramagnetic systems, such as Cesium, Rubidium and Francium, may extend the current sensitivity to the \(10^{-29} - 10^{-31} \text{ e cm}\) level \(52, 59\). In the Standard Model, the predictions for \(d_e\) range from \(10^{-38} \text{ e cm}\) to \(10^{-33} \text{ e cm}\) depending on whether the Dirac CP phase in light neutrino mixing is zero or not (for details see Ref \(60\)). Therefore, any observation of non-zero value of \(d_e\), i.e. value larger than \(10^{-33} \text{ e cm}\), would signify the existence of CP-violating physics beyond the Standard Model.

For that reason, these observables are of great interest for the investigation of possible scenarios for the New Physics. The announced higher-precision measurement of \(a_\mu\) by a factor of 4 in the future Fermilab experiment E989 \(61, 65\) as well as the expected future sensitivities of the electron EDM down to the level of \(\sim 10^{-31} \text{ e cm}\) \(52\), renders the study of the dipole moments even more actual and interesting.

For further reading, the reader is encouraged to the excellent reviews provided by Refs \(59, 66, 67\).
In this chapter we will expose some basic features of the theoretical framework which underlines the study of lepton flavor violation and anomalous dipole moments given in the thesis.

In the first section, we will give the basic structure of the Minimal Supersymmetric Standard Model (MSSM), as well as some main features regarding the Soft Supersymmetry Breaking in the MSSM. The notation used when discussing the Supersymmetry (SUSY) will correspond to the one used in Drees et al. [68], adapted to Petcov et al. [69]. For further reading regarding SUSY in general and MSSM in particular, the reader is encouraged to consult Refs [68,70–73].

Second section is dedicated to the seesaw mechanisms, with the main focus on the low-scale version of the seesaw mechanism type I.

Finally, the the MSSM extended by low-scale right handed neutrinos (or $\nu_R$MSSM) is introduced.
§ 2.1 Basic features of the MSSM

The basic idea behind all supersymmetric models is that there is a symmetry (conveniently called supersymmetry) which transforms a fermion into the boson and vice versa. The Minimal Supersymmetric Standard Model supersymmetrizes the SM with minimal extension of the SM particle spectrum: every SM particle is accompanied by one superparticle or a superpartner. The superpartners of matter fermions are spin zero particles, called sfermions. They can be further classified into the scalar leptons or sleptons and scalar quarks or squarks. Matter fermions and their superpartners are described by chiral superfields. The superpartners of SM gauge bosons are spin one-half particles called gauginos. They can be further classified into the strongly interacting gluinos and electroweak zino and winos (superpartners of Z and W bosons, respectively). Together with SM gauge bosons, they are described by vector superfields. Superpartners of Higgs bosons are spin one-half particles called higgsinos and, along with the latter, are described by chiral superfields. The electroweak symmetry breaking mixes the electroweak gauginos with higgsinos resulting in physical particles referred to as charginos and neutralinos. Table 2.1 displays full field contents of the MSSM, with the corresponding quantum numbers.

| Superfield | Bosons | Fermions | $SU_c(3)$ | $SU_L(2)$ | $U_Y(1)$ |
|-----------|--------|----------|-----------|-----------|----------|
| gauge     | $G^a$  | gluons   | $g^a$     | $g^a$     | 8 0 0    |
| $\Psi^k$  | electroweak $W^k (W^\pm, Z)$ | wino, zino | $\tilde{\lambda}^k (\tilde{\nu}^\pm, \tilde{\nu})$ | 1 3 0    |
| $\Psi'$   | hypercharge $B$ | bino | $\tilde{\lambda}_0$ | 1 1 0    |
| matter    | $L_1$  | sleptons | $\tilde{L}_1 = (\tilde{\nu}, \tilde{e})_L$ | lepton | $L_e = (e, \nu)_L$ | 1 2 -1 |
| $E_1$     |       | $\tilde{E}_1 = \tilde{e}_R$ | $E_e = e_R$ | 1 1 -2 |
| $Q_1$     | $\tilde{Q}_1 = (\tilde{u}, \tilde{d})_L$ | $Q_l = (u, d)_L$ | 3 2 1/3 |
| $U_1$     | $\tilde{U}_1 = \tilde{u}_R$ | $U_l = u_R$ | 3* 1 -4/3 |
| $D_1$     | $\tilde{D}_1 = \tilde{d}_R$ | $D_l = d_R$ | 3* 1 2/3 |
| Higgs     | $H_1$  | Higgs bosons | $H_1$ | Higgsinos | $\tilde{H}_1$ | 1 2 -1 |
| $H_2$     | $H_2$  | Higgsinos | $\tilde{H}_2$ | 1 2 1    |

Table 2.1: Superfields of the MSSM

As can be seen from Table 2.1 there are two Higgs superfields in the MSSM.
These can be written as

\[ H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix}. \]  

(2.1)

\( H_1 \) field is sometimes referred to as the down type Higgs \((Y = -1)\), superfield containing \( h_1 \) and \( \tilde{h}_{1L} \), while \( H_2 \) is referred to as the up type Higgs superfield containing \( h_2 \) and \( \tilde{h}_{2L} \). The component fields denoted by lower case letters are given by

\[ h_1 \equiv \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}, \quad h_2 \equiv \begin{pmatrix} h_2^0 \\ h_2^+ \end{pmatrix}, \]  

(2.2)

\[ \tilde{h}_{1L} \equiv \begin{pmatrix} \tilde{h}_{1L}^0 \\ \tilde{h}_{1L}^- \end{pmatrix}, \quad \tilde{h}_{2L} \equiv \begin{pmatrix} \tilde{h}_{2L}^0 \\ \tilde{h}_{2L}^+ \end{pmatrix}. \]  

(2.3)

After the spontaneous breakdown of electroweak symmetry, the Higgs vacuum expectation values (VEVs) are given by real, positive quantities \( v_1 \) and \( v_2 \),

\[ \langle h_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle h_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \]  

(2.4)

which arise from the minimization of the Higgs potential. The ratio of these values,

\[ \frac{v_2}{v_1} \equiv \tan \beta \]  

(2.5)

is considered to be a free parameter of the theory, at least regarding the fermion masses.

Let us proceed to the interaction and mass terms in the Lagrangian density \( \mathcal{L}_{\text{MSSM}} \) which partly comes from the exact supersymmetrization of the SM. Full MSSM Lagrangian can be written as the sum of two parts,

\[ \mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{SSB}}. \]  

(2.6)
While $\mathcal{L}_{\text{SUSY}}$ is fully supersymmetric, the $\mathcal{L}_{\text{SSB}}$ contains terms which explicitly break the supersymmetry (acronym SSB stands for SuperSymmetry Breakdown).

Let’s first take a look to the contents of $\mathcal{L}_{\text{SUSY}}$. The supersymmetric part of the MSSM Lagrangian can be further decomposed as

$$
\mathcal{L}_{\text{SUSY}} = \mathcal{L}_g + \mathcal{L}_M + \mathcal{L}_H,
$$

(2.7)

where $\mathcal{L}_g$, $\mathcal{L}_M$ and $\mathcal{L}_H$ are pure gauge, matter and Higgs-Yukawa parts, respectively. Detailed expressions for these terms can be found in the literature [68, pp 171-172]. The part which is most interesting for the purposes of this thesis is the superpotential, which constitutes important part of $\mathcal{L}_H$, and reads

$$
\mathcal{W}_{\text{MSSM}} = \mu H_1 \cdot H_2 + \bar{E}_i h^e_{ij} H_1 \cdot L_j + \bar{D}_i h^d_{ij} H_1 \cdot Q_j + \bar{U}_i h^u_{ij} H_2 \cdot Q_j,
$$

(2.8)

where $h$ matrices are given by

$$
h^e_{ij} = \frac{g_2}{\sqrt{2} M_W \cos \beta} (m_e)_{ij},
$$

(2.9)

$$
h^d_{ij} = \frac{g_2}{\sqrt{2} M_W \cos \beta} (m_d)_{ij},
$$

(2.10)

$$
h^u_{ij} = \frac{g_2}{\sqrt{2} M_W \cos \beta} (m_u)_{ij}.
$$

(2.11)

Here, $m_e$, $m_d$ and $m_u$ represent $3 \times 3$ lepton, down-quark and up-quark mass matrices, respectively. The dot products are defined in two-component notation [72,74] as $A \cdot B \equiv \epsilon_{\alpha \beta} A^\alpha B^\beta$ ($\epsilon_{12} = +1$). Second, third and fourth terms in right-hand side of Eq (2.8) are just supersymmetric generalization of the Yukawa couplings in the Standard Model Lagrangian (for this and other aspects of the SM see Ref [75]). The first term is however new, and can be thought of as a supersymmetric generalization of a higgsino mass term. It can be shown that the consistent incorporation of spontaneous electroweak symmetry breakdown requires $\mu$ to be of the order of the weak scale.

One more thing needs to be addressed at this point, and that is the implicit
assumption of the conservation of $R$-parity defined by a quantum number $R_p$ given by

$$R_p = (-1)^{3(B-L)+2S},$$

where $B$, $L$ and $S$ stand for barion number, lepton number and spin of the particle, respectively. The conservation of $R_p$ in the MSSM may be posited as a natural assumption in a minimal supersymmetric extensions of the SM, due to the barion and lepton number conservations in the SM Lagrangian.

Let’s now turn back to (2.6) and analyse the contents of the $\mathcal{L}_{SSB}$. There are several constraints which need to be put upon the supersymmetry breaking terms. First, they need to be “small” compared to the fully supersymmetric part $\mathcal{L}_{SUSY}$. Second, and most important, they must obey certain mass dimensional constrains in order to preserve the desired convergent behavior of the supersymmetric theory at high energies as well as the nonrenormalization of its superpotential couplings. According to the Symanzik’s rule [76, pp 107-8] this turns out to be possible in all orders in perturbation theory only if the explicit supersymmetry breaking terms are soft [77–80], i.e. that every field operator occurring in $\mathcal{L}_{SSB}$ has mass dimension strictly less then four. The Eq (2.6) is therefore usually written as

$$\mathcal{L}_{MSSM} = \mathcal{L}_{SUSY} + \mathcal{L}_{SOFT}. \quad (2.13)$$

Taking all this into account, one can write down the expression for $\mathcal{L}_{SOFT}$, by collecting all allowed soft SUSY-breaking terms [68 p 185],

$$-\mathcal{L}_{SOFT} = \tilde{q}_{iL}(\mathcal{M}^2_{\tilde{q}})_{ij}\tilde{q}_{jL} + \tilde{u}_{iR}(\mathcal{M}^2_{\tilde{u}})_{ij}\tilde{u}_{jR} + \tilde{d}_{iR}(\mathcal{M}^2_{\tilde{d}})_{ij}\tilde{d}_{jR}$$
$$+ \tilde{\tilde{e}}_{iL}(\mathcal{M}^2_{\tilde{e}})_{ij}\tilde{\tilde{e}}_{jL} + \tilde{\tilde{e}}_{iR}(\mathcal{M}^2_{\tilde{e}})_{ij}\tilde{\tilde{e}}_{jR}$$
$$+ \tilde{q}_{iL}(A^e)_{ij}\tilde{e}_{jR} + \tilde{d}_{iR}(A^d)_{ij}\tilde{d}_{jR}$$
$$+ \tilde{u}_{iR}(A^u)_{ij}\tilde{u}_{jR} + h.c.$$
Practical calculations within the MSSM usually include several simplifying assumptions in order to drastically reduce the number of additional parameters in the model. Different assumptions result in different versions of the Constrained Minimal Supersymmetric Standard Model or CMSSM.

In this thesis, we will adopt the framework of Minimal Super Gravity (mSUGRA) model. Since MSSM fields alone cannot break supersymmetry spontaneously at the weak scale \cite{68}, pp 183-5, spontaneous supersymmetry breakdown needs to be effected in a sector of fields which are singlets with respect to the SM gauge group. This sector is known as the hidden or secluded sector. SUSY breaking is then transmitted to the gauge nonsinglet observable or visible sector by a messenger sector associated by a typical mass scale $M_M$. Unlike the details of the spontaneous SUSY-breaking in the hidden sector, the mechanism of its transmission from hidden sector to the MSSM fields does have an immediate impact on the observable sparticle spectrum and then also on the SUSY phenomenology. The most economical mechanism of this kind uses gravitational strength interactions based on local supersymmetry also known as supergravity \cite{70,81}.

The great benefit in using the mSUGRA model is the fact that it reduces the extra one hundred and five parameters (compared to the nineteen parameters of the SM) to the set \{p\} of just five parameters,

$$\{p\} = \{\text{sign}(\mu), m_0, M_{1/2}, A_0, \tan\beta\},$$

where sign($\mu$) stands for the sign of the $\mu$ parameter in superpotential \cite{2.8}, $m_0$ constitute masses of the scalars ($m_{ij} = m_0\delta_{ij}$), $M_{1/2}$ is common mass of all three MSSM gauginos, $A_0$ is common trilinear coupling constant (higgs-sfermion-sfermion) and $\tan\beta$ is ratio of VEVs defined by Eq \cite{2.5}. These parameters are also referred to as the supersymmetry breaking parameters. Their values are usually imposed on the scale of Grand Unification (GUT),
and then via Renormalization Group Equations (RGE) transmitted down to the weak scale.

There are quite a few reasons to work in the framework of the MSSM with $R$-parity conserved. The MSSM provides a quantum-mechanically stable solution to the gauge hierarchy problem and predicts rather accurate unification of the SM gauge couplings close to the grand unified theory (GUT) scale. The lightest supersymmetric particle (LSP) is stable and, if neutral, such as the neutralino, could represent a good candidate for the dark matter in the Universe. Besides that, the MSSM typically predicts a SM-like Higgs boson lighter than 135 GeV, in agreement with the recent observations for a $\sim 125$ GeV Higgs boson, made by ATLAS [82] and CMS [83, 84] Collaborations.

§ 2.2 Seesaw mechanism

Neutrino oscillation experiments (see Chapter 1) have indisputably shown that neutrinos are not massless, as was once believed to be. This imposes the necessity to extend the Standard Model (as well as the MSSM) in a way that will consistently allow the existence of massive neutrinos. One of the most interesting extensions in that sense is provided by so-called seesaw mechanism. There are three realizations of the seesaw mechanism: the seesaw type one [85–90], the seesaw type two [90–95] and the seesaw type three [96]. These three scenarios differ by the nature of their seesaw messengers needed to explain the small neutrino masses. For the purpose of this thesis, we will explain and adopt a low-scale variant of the seesaw type-I realization, whose messengers are three singlet neutrinos $N_{1,2,3}$. But first let us examine the usual, high-scale variant, seesaw type-I mechanism in order to detect its weaknesses and to demonstrate how low-scale variant can overcome them.

The leptonic Yukawa sector of the SM with massless neutrinos is described
by

\[ \mathcal{L}^{(SM)}_Y = - \left( \overline{\nu}_i \, 
\begin{pmatrix} \bar{\nu}_{i}^t & \bar{\nu}_{i} \end{pmatrix} \right) L_{ij} \left( \phi^+ \phi^0 \right) l_{jR} + \text{h.c.} \] (2.16)

Here, the primes indicate that the fields are not written in the mass basis (so-called physical states), but rather in the interaction basis. \( h^{(l)} \) and \( h^{(\nu)} \) are 3 \( \times \) 3 lepton and neutrino Yukawa matrices, respectively.

The consistent and straightforward extension of this sector by a right-handed neutrinos includes both the extra Yukawa neutrino term and the mass term which is singlet under the SM gauge group,

\[ \mathcal{L}^{(SM+\nu_R)}_Y = - \left( \overline{\nu}_i \, 
\begin{pmatrix} \bar{\nu}_{i}^t & \bar{\nu}_{i} \end{pmatrix} \right) L_{ij} \left( \phi^+ \phi^0 \right) l_{jR} \]

\[ - \left( \overline{\nu}_i \, 
\begin{pmatrix} \bar{\nu}_{i}^t & \bar{\nu}_{i} \end{pmatrix} \right) L_{ij} \left( \phi^{0\dagger} \right) \nu_{jR} \]

\[ - \frac{1}{2} M (\nu^t_R) C \nu_R + \text{h.c.} \] (2.17)

After the spontaneous breakdowns of the electroweak symmetry,

\[ \Phi(x) \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\
\nu \end{pmatrix} , \] (2.18)

one ends with the well-known expression for lepton masses,

\[ (m_l)_{ij} = \frac{v}{\sqrt{2}} h^{(l)}_{ij} \, , \quad (m_D)_{ij} = \frac{v}{\sqrt{2}} h^{(\nu)}_{ij} \, \, , \quad M \] . (2.19)

Here \( m_l \) represents masses of the charged leptons, \( m_D \) stands for the Dirac mass matrix, and \( M \) is the Majorana mass matrix. The former two make
the mass term for neutrinos,

$$\mathcal{L}^{(\text{mass})}_{\nu} = -\frac{1}{2} \begin{pmatrix} \nu'_L \ (\nu'_R)^C \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \begin{pmatrix} (\nu'_L)^C \\ \nu'_R \end{pmatrix}. \quad (2.20)$$

In order to get from the interaction to mass basis, i.e. to write the Lagrangian in terms of physical states, one needs to diagonalize the $\mathcal{M}_{D+M}$ matrix. This is performed with unitary $6 \times 6$ matrix $W$,

$$W^T \mathcal{M}_{D+M} W = \begin{pmatrix} \mathcal{M}_\nu & 0 \\ 0 & \mathcal{M}_N \end{pmatrix}. \quad (2.21)$$

This matrix equation is solved by Taylor expansion, order by order \[97\]. Keeping only the leading term, the solutions of Eq (2.21) read \[98\]

$$\mathcal{M}_\nu \simeq -m_D^T M^{-1} m_D, \quad \mathcal{M}_N \simeq M, \quad (2.22)$$

$$W \simeq \begin{pmatrix} 1_{3 \times 3} & (M^{-1} m_D)^T \\ -M^{-1} m_d & 1_{3 \times 3} \end{pmatrix} \simeq \begin{pmatrix} 1 & \sqrt{m_\nu/m_N} \\ \sqrt{m_\nu/m_N} & 1 \end{pmatrix}. \quad (2.23)$$

Matrix $W$ transforms fields written in the interaction basis to the one written in the mass basis,

$$\begin{pmatrix} (\nu'_L)^C \\ \nu'_R \end{pmatrix} = W \begin{pmatrix} \nu^C_L \\ \nu_R \end{pmatrix} \quad (2.24)$$

Finally one can re-write the Lagrangian \[2.20\] in the mass basis,

$$\mathcal{L}^{(\text{mass})}_{\nu} = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L \ \bar{\nu}_R^C \end{pmatrix} \begin{pmatrix} \mathcal{M}_\nu & 0 \\ 0 & \mathcal{M}_N \end{pmatrix} \begin{pmatrix} \nu^C_L \\ \nu_R \end{pmatrix}. \quad (2.25)$$

If we allow the Yukawa matrices to be of arbitrary form, we have to face two
unpleasant consequences:

1. From Eq (2.22) we see that mass of light neutrinos is roughly given by \( m_\nu \sim m_D^2 / M \). Since the light neutrino masses are of the order \( m_\nu \sim 0.1 \) eV, and if we assume that Yukawa couplings are of order \( \sim 0.1 \), it follows that the heavy singlet neutrinos must assume masses of order \( \sim 10^{12-14} \) GeV. That is inconvenient by itself, since its direct detection is way beyond the reach of experiments in high energy physics.

2. From Eq (2.23) we see that the mixing between light and heavy neutrinos is of the order \( \xi_{\nu N} \sim \sqrt{m_\nu / m_N} \sim 10^{-12} \), for light neutrino masses \( m_\nu \sim 0.1 \) eV. That means that the heavy neutrinos decouple from low-energy processes of CLFV in the SM with right-handed neutrinos, giving rise to extremely suppressed and unobservable rates.

One way to overcome these difficulties is to impose the presence of the approximate lepton flavor symmetries \([99–105]\) in the theory. These symmetries result in a specific structure of Yukawa matrices which, if exact, can provide massless light neutrinos regardless of the masses of heavy neutrinos, so that

\[
M_\nu = -m_D^T M^{-1} m_D + \ldots \equiv 0 .
\]  

Small neutrino masses can then be reproduced by breaking the imposed symmetry by just the right amount. This scenario allows the heavy neutrino mass scale to be as low as 100 GeV. Unlike in the usual seesaw scenario, the light-to-heavy neutrino mixings \( \xi_{\nu N} \) are not correlated to the light neutrino masses \( m_\nu \). Instead, \( \xi_{\nu N} \) are free parameters, constrained by experimental limits on the deviations of the \( W^\pm \) and \( Z \)-boson couplings to leptons with respect to their SM values \([106–109]\).

Approximate lepton flavor symmetries do not restrict the size of the LFV, and so potentially large phenomena of CLFV may be predicted. This feature is quite generic both in the SM \([110]\) and in the MSSM \([111,112]\) extended with low-scale right-handed neutrinos. This new source of LFV, in addition
§ 2.3 MSSM extended with right-handed neutrinos

The SM and the MSSM extended by low-scale right-handed neutrinos in the presence of the approximate lepton-number symmetries will be denoted by $\nu_R\text{SM}$ and $\nu_R\text{MSSM}$, respectively. Although some of the results displayed in this thesis may be applicable to the more general soft SUSY breaking scenarios, this study will be performed within the mSUGRA framework.

The $\nu_R\text{MSSM}$ has some interesting features compared with the MSSM. In particular, the heavy singlet sneutrinos may emerge as a new viable candidates of cold dark matter [120–124]. In addition, the mechanism of low-scale resonant leptogenesis [125–129] could provide a possible explanation for the observed baryon asymmetry in the Universe, as the parameter space for successful electroweak baryogenesis gets squeezed by the current LHC data [130,131].

Given the multitude of quantum states mediating LFV in the $\nu_R\text{MSSM}$, the predicted values for observables of CLFV in this model turn out to be generically larger than the corresponding ones in the $\nu_R\text{SM}$, except possibly for $B(l \to l'\gamma)$ [111,112], where $l, l' = e, \mu, \tau$. The origin of suppression for the latter branching ratios may partially be attributed to the SUSY no-go theorem due to Ferrara and Remiddi [132], which states that the magnetic dipole moment operator necessarily violates SUSY and it must therefore vanish in the supersymmetric limit of the theory.

In this section, we will describe the leptonic sector of the $\nu_R\text{MSSM}$ and introduce the neutrino Yukawa structure of two baseline scenarios based on approximate lepton-number symmetries and universal Majorana masses at the GUT scale. These scenarios will be used to present generic predictions of
the CLFV within the framework of mSUGRA, and to analyze the anomalous magnetic and electric dipole moments within the same framework.

The leptonic superpotential part of the $\nu_R$MSSM reads:

$$W_{\text{lepton}} = \hat{E}^C \hat{h}_L \hat{H} + \hat{N}^C \hat{h}_L \hat{H} + \frac{1}{2} \hat{N}^C \mathbf{m}_M \hat{N}^C,$$

(2.27)

where $\hat{H}_{u,d}$, $\hat{L}$, $\hat{E}$ and $\hat{N}^C$ denote the two Higgs-doublet superfields, the three left- and right-handed charged-lepton superfields and the three right-handed neutrino superfields, respectively. The Yukawa couplings $h_{e,\nu}$ and the Majorana mass parameters $\mathbf{m}_M$ form $3 \times 3$ complex matrices. Here, the Majorana mass matrix $\mathbf{m}_M$ is taken to be SO(3)-symmetric at the $m_N$ scale, i.e. $\mathbf{m}_M = m_N \mathbf{1}_3$.

In the low-scale seesaw models with the presence of approximate lepton symmetries, the neutrino induced LFV transitions from a charged lepton $l = \mu, \tau$ to another charged lepton $l' \neq l$ are functions of the ratios

$$\Omega_{vl} = \frac{v_u^2}{2m_N^2} (h^\dagger_{\nu} h_{\nu})_{vl} = \sum_{i=1}^3 B_{\nu N_i} B_{lN_i},$$

(2.28)

and are not constrained by the usual seesaw factor $m_\nu/m_N$, where $v_u/\sqrt{2} \equiv \langle H_u \rangle$ is the vacuum expectation value (VEV) of the Higgs doublet $H_u$, with $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$. The mixing matrix $B_{lN_i}$ that occurs in the interaction of the $W^\pm$ bosons with the charged leptons $l = e, \mu, \tau$ and the three heavy neutrinos $N_{1,2,3}$ is defined in Appendix A. It is important to note that the LFV parameters $\Omega_{vl}$ do not directly depend on the RGE evolution of the soft SUSY-breaking parameters, except through the VEV $v_u$ defined at the minimum of the Higgs potential.

In the electroweak interaction basis $\{\nu_{e,\mu,\tau L}, \nu_{1,2,3 R}^C\}$, the neutrino mass matrix in the $\nu_R$MSSM takes on the standard seesaw type-I form:

$$\mathbf{M}_\nu = \left(\begin{array}{cc} 0 & \mathbf{m}_D \\ \mathbf{m}_D^T & \mathbf{m}_M^* \end{array}\right),$$

(2.29)
where $m_D = \sqrt{2} M_W \sin \beta g_w^{-1} h^\dagger_\nu$ and $m_M$ are the Dirac- and Majorana-neutrino mass matrices, respectively. Complex conjugation of $m_M$ matrix is a consequence of the Majorana mass term in the superpotential $W_{\text{lepton}}$ (2.27). In this thesis, we consider two baseline scenarios of neutrino Yukawa couplings. The first one realizes a $U(1)$ leptonic symmetry [125–127] and is given by

$$h_\nu = \begin{pmatrix} 0 & 0 & 0 \\ a e^{-i\pi/4} & b e^{-i\pi/4} & c e^{-i\pi/4} \\ a e^{i\pi/4} & b e^{i\pi/4} & c e^{i\pi/4} \end{pmatrix}.$$  \hspace{1cm} (2.30)

In the second scenario, the structure of the neutrino Yukawa matrix $h_\nu$ is motivated by the discrete symmetry group $A_4$ and has the following form [137]:

$$h_\nu = \begin{pmatrix} a & b & c \\ a e^{-2\pi i/3} & b e^{-2\pi i/3} & c e^{-2\pi i/3} \\ a e^{2\pi i/3} & b e^{2\pi i/3} & c e^{2\pi i/3} \end{pmatrix}.$$  \hspace{1cm} (2.31)

In Eqs (2.30) and (2.31), the Yukawa parameters $a$, $b$ and $c$ are assumed to be real. As was explained in the previous section, the small neutrino masses can be obtained by adding small symmetry-breaking terms into these matrices thus making the above mentioned symmetries approximate rather than exact. The predictions for CLFV observables, however, remain independent of the flavor structure of these small terms, needed to fit the low-energy neutrino data. For this reason, the particular symmetry breaking patterns of the above two baseline Yukawa scenarios will not be discussed in this thesis.

Another source of LFV in the models under consideration comes from sneutrino interactions. Specifically, the sneutrino mass Lagrangian in flavor and
mass bases is given by

\[ L(\tilde{\nu}) = (\tilde{\nu}_L^\dagger, \tilde{\nu}_R^C, \tilde{\nu}_L^T, \tilde{\nu}_R^C)^\dagger \begin{pmatrix} \tilde{\nu}_L \\ \tilde{\nu}_R^C \\ \tilde{\nu}_L^T \\ \tilde{\nu}_R^C^* \end{pmatrix} M^2_{\tilde{\nu}} \]

where the block entries are the 3 \( \times \) 3 matrices, namely

\[ M^2_{\tilde{\nu}} = \begin{pmatrix} H_1 & N & 0 & M \\ N^\dagger & H_2^T & M^T & 0 \\ 0 & M^* & H_1^T & N^* \\ M^\dagger & 0 & N^T & H_2 \end{pmatrix} \]

(2.34)

where \( M^2_{\tilde{\nu}} \) is a 12 \( \times \) 12 Hermitian mass matrix in the flavor basis and \( \tilde{M}^2_{\tilde{\nu}} \) is the corresponding diagonal mass matrix in the mass basis. More explicitly, in the flavor basis \( \{ \tilde{\nu}_{e,\mu,\tau L}, \tilde{\nu}_{e,\mu,\tau L}^C, \tilde{\nu}_{e,\mu,\tau L}^C \} \), the sneutrino mass matrix \( M^2_{\tilde{\nu}} \) may be cast into the following form:

\[ \tilde{N}^\dagger U^\dagger M^2_{\tilde{\nu}} U \tilde{N} = \tilde{N}^\dagger \tilde{M}^2_{\tilde{\nu}} \tilde{N}, \]

(2.33)

where \( m_2^2 \tilde{L} \) are the soft SUSY-breaking matrices associated with the left-handed slepton doublets, the right-handed sneutrinos and their trilinear couplings, respectively.

Here, \( m_2^2 \tilde{L} \) and \( A_\nu \) are 3 \( \times \) 3 soft SUSY-breaking matrices associated with the left-handed slepton doublets, the right-handed sneutrinos and their trilinear couplings, respectively.

In the supersymmetric limit, all the soft SUSY-breaking matrices are equal to zero, \( \tan \beta = 1 \) and \( \mu = 0 \). As a consequence, the sneutrino mass matrix \( M^2_{\tilde{\nu}} \) can be expressed in terms of the neutrino mass matrix \( M_\nu \) in (2.29) as
follows:

$$ M^2_{\nu} \xrightarrow{\text{SUSY}} \begin{pmatrix} M_\nu M_\nu^\dagger & 0_{6 \times 6} \\ 0_{6 \times 6} & M_\nu^\dagger M_\nu \end{pmatrix}, $$

resulting with the expected equality between neutrino and sneutrino mixings. Sneutrino LFV mixings do depend on the RGE evolution of the $\nu_R$MSSM parameters, but unlike the LFV mixings induced by soft SUSY-breaking terms, the sneutrino LFV mixings do not vanish at the GUT scale.

The sneutrino LFV mixings are obtained as combinations of unitary matrices which diagonalize the sneutrino, slepton and chargino mass matrices. It is interesting to notice that in the diagonalization of the sneutrino mass matrix $M^2_{\nu}$ in (2.34), the sneutrino fields $\tilde{\nu}_{e,\mu,\tau L}, \tilde{\nu}^C_{1,2,3 R}$ and their complex conjugates $\tilde{\nu}_{e,\mu,\tau L}^*, \tilde{\nu}^C_{1,2,3 R}^*$ are treated independently. As a result, the expressions for $\tilde{\nu}_{e,\mu,\tau L}$ and $\tilde{\nu}^C_{1,2,3 R}$, in terms of the real-valued mass eigenstates $\tilde{N}_{1,2,\ldots,12}$, are not manifestly complex conjugates to $\tilde{\nu}_{e,\mu,\tau L}^*$ and $\tilde{\nu}^C_{1,2,3 R}^*$, thus leading to a two-fold interpretation of the flavor basis fields,

$$ \begin{align*}
\tilde{\nu}_{i}^* & = (\tilde{\nu}_{i})^* = U_{iA}^\dagger \tilde{N}_A, \\
\tilde{\nu}_{i}^* & = U_{i+6A} \tilde{N}_A,
\end{align*} $$

(2.37)

where $\tilde{\nu}_{1,2,3} \equiv \tilde{\nu}_{e,\mu,\tau L}$ and $\tilde{\nu}_{4,5,6} \equiv \tilde{\nu}^C_{1,2,3 R}$, with $i = 1, 2, \ldots, 6$ and $A = 1, 2, \ldots, 12$. For this reason, in Appendix A we include all equivalent forms in which Lagrangians, such as $\mathcal{L}_{\tilde{\tau} \tilde{\chi}^{-}}$ and $\mathcal{L}_{\tilde{\nu} \tilde{N} Z}$, can be written down.

Finally, a third source of LFV in the $\nu_R$MSSM comes from soft SUSY-breaking LFV terms $[13,15]$. These LFV terms are induced by RGE running and, in the mSUGRA framework, vanish at the GUT scale. Their size strongly depends on the interval of the RGE evolution from the GUT scale to the universal heavy neutrino mass scale $m_N$.

All the three different mechanisms of LFV, mediated by heavy neutrinos, heavy sneutrinos and soft SUSY-breaking terms, depend explicitly on the neutrino Yukawa matrix $h_\nu$ and vanish in the limit $h_\nu \to 0$. 

We will end this chapter with a technical remark. The diagonalization of $12 \times 12$ sneutrino mass matrix $M^2_{\tilde{\nu}}$ and the resulting interaction vertices will be evaluated numerically, without approximations. To perform the diagonalization of $M^2_{\tilde{\nu}}$ numerically, the method developed in Ref. [138] for the neutrino mass matrix will be used. This method becomes very efficient if one of the diagonal submatrices has eigenvalues larger than the entries in all other submatrices. It will therefore be assumed that the heavy neutrino mass scale $m_N$ is of the order of, or larger than the scale of the other mass parameters in the $\nu R$MSSM.
In this chapter, the results and key details regarding the calculations for a number of CLFV observables in the $\nu_R$MSSM will be presented.

In the first section, the analytical results for the amplitudes of CLFV decays $l \rightarrow l'\gamma$ and $Z \rightarrow l l'$, as well as their branching ratios will be given. Second section gives analytical expressions for the neutrinoless three-body decays $l \rightarrow l'l_1l_2'$ pertinent to muon and tau decays. Third section will deal with coherent $\mu \rightarrow e$ conversion in nuclei, giving analytical results for transition amplitudes. All analytical results are expressed in terms of one-loop functions and composite form factors defined in the appendices at the end of this thesis.

Finally, last section will present the numerical results for above mentioned processes, accompanied by the brief description of the numerical methods used and corresponding discussion regarding the very results.

These results are presented in Ref [139].
§ 3.1 The Decays \( l \to l'\gamma \) and \( Z \to ll'^C \)

At the one-loop level, the effective \( \gamma l' \) and \( Zl' \) couplings are generated by the Feynman graphs shown in Fig 3.1. The general form of the transition amplitudes associated with these effective couplings is given by

\[
T_{\mu}^{\gamma l'} = \frac{e \alpha_w}{8\pi M_W} \bar{l} \left[ (F^L_{\gamma})_{l'l} (q^2 \gamma_{\mu} - q_{\mu}) P_L + (F^R_{\gamma})_{l'l} (q^2 \gamma_{\mu} - q_{\mu}) P_R \right. \\
\left. + (G^{L}_{\gamma})_{l'l} \gamma_{\mu} P_L + (G^{R}_{\gamma})_{l'l} \gamma_{\mu} P_R \right] l,
\]

\[
T_{\mu}^{Zl'} = \frac{g_w \alpha_w}{8\pi \cos \theta_w} \bar{l} \left[ (F^L_{Z})_{l'l} \gamma_{\mu} P_L + (F^R_{Z})_{l'l} \gamma_{\mu} P_R \right] l,
\]

where \( P_{L(R)} = \frac{1}{2} [1 - (+) \gamma_5] \), \( \alpha_w = g^2_w/(4\pi) \), \( e \) is the electromagnetic coupling constant, \( M_W = g_w \sqrt{v^2_\nu + v^2_\mu}/2 \) is the W-boson mass, \( \theta_w \) is the weak mixing angle and \( q = p_{l'} - p_l \) is the photon momentum. The form factors \( (F^L_{\gamma})_{l'l} \), \( (F^R_{\gamma})_{l'l} \), \( (G^{L}_{\gamma})_{l'l} \), \( (G^{R}_{\gamma})_{l'l} \), \( (F^L_{Z})_{l'l} \) and \( (F^R_{Z})_{l'l} \) receive contributions from heavy neutrinos \( N_{1,2,3} \), heavy sneutrinos \( \tilde{N}_{1,2,3} \) and RGE induced soft SUSY-breaking terms. The analytical expressions for these three individual contributions are given in Appendix C. Note that, according to the normalization used, the composite form factors \( (G^{L}_{\gamma})_{l'l} \) and \( (G^{R}_{\gamma})_{l'l} \) have dimensions of mass, whilst all other form factors are dimensionless.

It is important to remark that the transition amplitudes (3.1) and (3.2) are also constituent parts of the leptonic amplitudes \( l \to l'l_1l_2^C \) and semileptonic amplitudes \( l \to q_1l_2q_2 \), which will be discussed in more detail in Sections 3.2 and 3.3. To calculate the CLFV decay \( l \to l'\gamma \), we only need to consider the dipole moment operators associated with the form factors \( (G^L_{\gamma})_{l'l} \) and \( (G^R_{\gamma})_{l'l} \) in (3.1). Taking this last fact into account, the branching ratios for \( l \to l'\gamma \) and \( Z \to ll'^C + l'^{C}l \) are given by

\[
B(l \to l'\gamma) = \frac{\alpha^3_w s_w^2}{256\pi^2} \frac{m_l^3}{M_W^4 \Gamma_l} \left( |(G^L_{\gamma})_{l'l}|^2 + |(G^R_{\gamma})_{l'l}|^2 \right),
\]

\[
B(Z \to ll'^C + l'^{C}l) = \frac{\alpha^3_w M_W}{768\pi^2 c_w^2 \Gamma_Z} \left( |(F^L_{Z})_{l'l}|^2 + |(F^R_{Z})_{l'l}|^2 \right).
\]

The above expressions are valid up to the leading order in external charged.
lepton masses and external momenta, which constitutes an excellent approximation for our purposes. Thus, in (3.4) we have assumed that the Z-boson mass $M_Z$ is much smaller than the SUSY and heavy neutrino mass scales, $M_{SUSY}$ and $m_N$, and we have kept the leading term in an expansion of small momenta and masses for the external particles. In the decoupling regime of all soft SUSY-breaking and charged Higgs-boson masses, the low-energy sector of the $\nu_R$MSSM becomes the $\nu_R$SM. In this $\nu_R$SM limit of the theory, the analytical expressions for $B(l \rightarrow l'\gamma)$ and $B(Z \rightarrow l'l' + l'l)_{C}$ take on the forms given in Refs [140] and [133–135], respectively.
§ 3.2 Three-Body Leptonic Decays $l \rightarrow l'l_1l_2^C$

We now study the three-body CLFV decays $l \rightarrow l'l_1l_2^C$, where $l$ can be the muon or tau lepton, and $l'$, $l_1$, $l_2$ denote other charged leptons to which $l$ is allowed to decay kinematically.

The transition amplitude for $l \rightarrow l'l_1l_2^C$ receives contributions from $\gamma$- and $Z$-boson-mediated graphs shown in Fig 3.1 and from box graphs displayed in Fig 3.2. The amplitudes for these three contributions are:

$$\mathcal{T}_{\gamma l'l_1l_2} = \frac{\alpha^2}{2M_W^2} \left\{ \delta_{l_1l_2} \vec{p} \left[ (F^L_{\gamma})_{l'l} \gamma_{\mu} P_L + (F^R_{\gamma})_{l'l} \gamma_{\mu} P_R + \frac{(p'-p)}{(p'-p)^2} \right] \right\} ,$$

$$\mathcal{T}_{Z l'l_1l_2} = \frac{\alpha^2}{2M_W^2} \left[ \delta_{l_1l_2} \vec{p} \left[ (F^L_Z)_{l'l} \gamma_{\mu} P_L + (F^R_Z)_{l'l} \gamma_{\mu} P_R \right] l + \right\} ,$$

$$\mathcal{T}_{\text{box} l'l_1l_2} = -\frac{\alpha^2}{4M_W^2} \left\{ B_{l'l_1l_2}^{LL} \vec{p} \gamma_{\mu} P_{l_1} \bar{l}_1 \gamma_{\mu} P_{l_2} + B_{l'l_1l_2}^{RR} \vec{p} \gamma_{\mu} P_{l_1} \bar{l}_1 \gamma_{\mu} P_{l_2} \right\} ,$$

where $g^L_1 = -1/2 + s_w^2$ and $g^R_1 = s_w^2$ are $Z$-boson–lepton couplings and $s_w = \sin \theta_w$. The composite box form factors $B_{l'A}^{XY}$ are given in Appendix C. The labels $V$, $S$ and $T$ denote the form factors of the vector, scalar and tensor combinations of the currents, while $L$ and $R$ distinguish between left and right chiralities of those currents. The box form factors contain both direct and Fierz-transformed contributions (see Appendix D). Equation (3.8) represents a shorthand expression that takes account of all individual contributions to the amplitude $\mathcal{T}_{\text{box} l'l_1l_2}$ induced by box graphs. Explicitly, the matrices $\Gamma^X_{l'A}$
appearing in \textbf{(3.8)} read:

\[
(\Gamma^L_V, \Gamma^R_V, \Gamma^L_S, \Gamma^R_S, \Gamma^L_T, \Gamma^R_T) = (\gamma_\mu P_L, \gamma_\mu P_R, P_L, P_R, \sigma_{\mu\nu} P_L, \sigma_{\mu\nu} P_R).
\]  \tag{3.9}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{feynman_diagram}
\caption{Feynman graphs contributing to the box $l \to l'l_1l_2^C$ amplitudes.}
\end{figure}

As a consequence of the identity $\sigma^{\mu\nu} \gamma_5 = -\frac{i}{2} \varepsilon^{\mu\nu\rho\tau} \sigma_{\rho\tau}$, the tensor form factors $B^{LR}_{TT}$ and $B^{RL}_{TT}$ vanish in the sum \textbf{(3.8)}, i.e. $B^{LR}_{TT} = B^{RL}_{TT} = 0$. A very similar chiral structure is found in the semileptonic box amplitudes defined in the
next section as well. It should also be said that the previous studies of these processes \[114, 141\] do not include in their calculations the chiral structures 

\[ P_L \times P_R \text{ and } P_R \times P_L \]

and their corresponding form factors \( B_{lS}^{LR} \) and \( B_{lS}^{RL} \).

In a three-generation model, the transition amplitude for the decays \( l \to l' l_1 l_2 \) may fall in one of the following three classes or categories \[110\]: (i) \( l' \neq l_1 = l_2 \), (ii) \( l' = l_1 = l_2 \), and (iii) \( l' = l_1 \neq l_2 \). In the first two classes, total lepton number is conserved, whilst in the third class the total lepton number is violated by two units on the current level. Since the predictions for the observables in class (iii) turn out to be unobservably small in the \( \nu_R \text{MSSM} \), these processes will be ignored. Moreover, the universal indices \( l' l \) which appear in the photon and \( Z \)-boson form factors, i.e. \( F_{\gamma}^L, F_{\gamma}^R, F_Z^L \) and \( F_Z^R \), will be dropped out for the sake of readability. Given the above simplification and the notation of the box form factors (3.8), the branching ratios for the class (i) and (ii) of CLFV three-body decays are given by

\[
B(l \to l' l_1 l_2^C) = \frac{m_l^5 a_w^4}{24576 \pi^3 M_W^4 \Gamma_l} \left\{ \left[ 2 s_w^2 (F_{\gamma}^L + F_{Z}^L) - F_{\gamma}^L - F_{Z}^L \right]^2 \right. \\
+ \left. 2 s_w^2 (F_{\gamma}^R + F_{Z}^R) - B_{lS}^{RR} \right]^2 + 2 s_w^2 (F_{\gamma}^L + F_{Z}^L) - B_{lS}^{LR} \right]^2 \\
+ \left. 2 s_w^2 (F_{\gamma}^R + F_{Z}^R) - F_{\gamma} - B_{lS}^{RL} \right]^2 \\
+ \left. 1 \left( |B_{lS}^{LL}|^2 + |B_{lS}^{RR}|^2 + |B_{lS}^{LR}|^2 \right) \right. \\
+ \left. 12 \left( |B_{lT}^{LL}|^2 + |B_{lT}^{RR}|^2 \right) \right. \\
+ \left. \frac{32 s_w^4}{m_l} \left[ \text{Re} \left( (F_{\gamma}^R + F_{Z}^R) G_{\gamma}^{L*} \right) + \text{Re} \left( (F_{\gamma}^L + F_{Z}^L) G_{\gamma}^{R*} \right) \right] \right. \\
- \left. \frac{8 s_w^2}{m_l} \left[ \text{Re} \left( (F_{\gamma}^R + B_{lS}^{RR} + B_{lS}^{RL}) G_{\gamma}^{L*} \right) \right. \right. \\
+ \left. \text{Re} \left( (F_{\gamma}^L + B_{lS}^{LR} + B_{lS}^{LL}) G_{\gamma}^{R*} \right) \right] \right. \\
- \left. \frac{32 s_w^4}{m_l^2} \left( |G_{\gamma}^L|^2 + |G_{\gamma}^R|^2 \right) \left( \ln \frac{m_l^2}{m_\mu^2} - 3 \right) \right\}, \tag{3.10}
\]
§ 3.3. Coherent $\mu \to e$ Conversion in a Nucleus

\[
B(l \to l'l'^{C}) = \frac{m_l^5 \alpha^4_w}{24576 \pi^3 M^4_W \Gamma_l} \left\{ \begin{array}{c} 2 \left[ \frac{1}{2} \left( F^L_\gamma + F^L_Z \right) - \frac{1}{2} B^{LL}_{\ell V} \right]^2 + \left[ 2 s^2_w (F^R_\gamma + F^R_Z) - \frac{1}{2} B^{RR}_{\ell V} \right]^2 \\
+ \left[ 2 s^2_w (F^L_\gamma + F^L_Z) - \frac{1}{2} \left( B^{LL}_{\ell S} + B^{RR}_{\ell S} \right) \right]^2 + 6 \left( |B^{LL}_{\ell T}|^2 + |B^{RR}_{\ell T}|^2 \right) \end{array} \right.
\]

\[
+ \frac{48 s^4_w}{m_l} \left[ \text{Re} \left( (F^R_\gamma + F^R_Z) G^{L*}_\gamma \right) + \text{Re} \left( (F^L_\gamma + F^L_Z) G^{R*}_\gamma \right) \right] - \frac{8 s^2_w}{m_l} \left[ \text{Re} \left( (F^R_\gamma + B^{RR}_{\ell V} + B^{RL}_{\ell V}) G^{L*}_\gamma \right) + \text{Re} \left( (2 F^L_\gamma + B^{LL}_{\ell V} + B^{LR}_{\ell V}) G^{R*}_\gamma \right) \right]
\]

\[
+ \frac{32 s^4_w}{m^2_l} \left( |G^{L}_\gamma|^2 + |G^{R}_\gamma|^2 \right) \left( \ln \frac{m^2_l}{m^2_\ell} - \frac{11}{4} \right) \right\}, \quad (3.11)
\]

where $m_l$ and $m_\ell$, $m_{l_1}$, $m_{l_2}$ are the masses of the initial- and final-state charged leptons and $\Gamma_l$ is the decay width of the charged lepton $l$. It should be emphasized that the transition amplitudes (3.5), (3.6) and (3.7) as well as the branching ratios (3.10) and (3.11) have the most general chiral and form factor structure to the leading order in the external masses and momenta, which makes them applicable to most models of the New Physics containing CLFV. Even more general result can be found in the Appendix D.

These results have been checked in the $\nu_R$SM limit of the theory in which the branching ratios (3.10) and (3.11) go over to the results presented in Ref [110].

§ 3.3 Coherent $\mu \to e$ Conversion in a Nucleus

The coherent $\mu \to e$ conversion in a nucleus corresponds to the process $J_\mu \to e^- J^+$, where $J_\mu$ is an atom of nucleus $J$ with one orbital electron replaced by a muon and $J^+$ is the corresponding ion without the muon. The
transition amplitude for such a CLFV process,

\[ \mathcal{T}^{\mu e; J} = \langle J^+ e^- | T^{d\mu \rightarrow de} | J_\mu \rangle + \langle J^+ e^- | T^{u\mu \rightarrow ue} | J_\mu \rangle, \] (3.12)

depends on two effective box operators,

\[ \mathcal{T}_{\text{box}}^{d\mu \rightarrow de} = -\frac{\alpha_w^2}{4M_W^2} \sum_{X,Y=L,R} \sum_{A=V,S,T} B_{dA}^{XY} \bar{\epsilon} \Gamma_A^X \mu \, \epsilon \Gamma_A^Y d \]
\[ = -\frac{\alpha_w^2}{2M_W^2} (d^\dagger d) \, \bar{\epsilon} (V_d^R P_R + V_d^L P_L) \mu, \] (3.13)

\[ \mathcal{T}_{\text{box}}^{u\mu \rightarrow ue} = -\frac{\alpha_w^2}{4M_W^2} \sum_{X,Y=L,R} \sum_{A=V,S,T} B_{uA}^{XY} \bar{\epsilon} \Gamma_A^X \mu \, \epsilon \Gamma_A^Y u \]
\[ = -\frac{\alpha_w^2}{2M_W^2} (u^\dagger u) \, \bar{\epsilon} (V_u^R P_R + V_u^L P_L) \mu. \] (3.14)

Here \( \mu \) and \( e \) are the muon and electron wave functions and \( d \) and \( u \) are field operators acting on the \( J_\mu \) and \( J^+ \) states, respectively. The form factors \( B_{dA}^{XY} \) and \( B_{uA}^{XY} \) are given in the Appendix C. The composite form factors \( V_d^L, V_d^R, V_u^R, V_u^R \) may be written as

\[ V_d^L = -\frac{1}{3}s_w^2 \left( F_{\gamma}^L - \frac{1}{m_\mu} G_{\gamma}^R \right) + \left( \frac{1}{4} - \frac{1}{3}s_w^2 \right) F_{\gamma}^Z \]
\[ + \frac{1}{4} \left( B_{d\nu}^{LL} + B_{d\nu}^{LR} + B_{d\nu}^{RR} + B_{d\nu}^{RL} \right), \] (3.15)

\[ V_d^R = -\frac{1}{3}s_w^2 \left( F_{\gamma}^R - \frac{1}{m_\mu} G_{\gamma}^L \right) + \left( \frac{1}{4} - \frac{1}{3}s_w^2 \right) F_{\gamma}^Z \]
\[ + \frac{1}{4} \left( B_{d\nu}^{RR} + B_{d\nu}^{RL} + B_{d\nu}^{LL} + B_{d\nu}^{LR} \right), \] (3.16)

\[ V_u^L = \frac{2}{3}s_w^2 \left( F_{\gamma}^L - \frac{1}{m_\mu} G_{\gamma}^R \right) + \left( -\frac{1}{4} + \frac{2}{3}s_w^2 \right) F_{\gamma}^Z \]
\[ + \frac{1}{4} \left( B_{u\nu}^{LL} + B_{u\nu}^{LR} + B_{u\nu}^{RR} + B_{u\nu}^{RL} \right), \] (3.17)

\[ V_u^R = \frac{2}{3}s_w^2 \left( F_{\gamma}^R - \frac{1}{m_\mu} G_{\gamma}^L \right) + \left( -\frac{1}{4} + \frac{2}{3}s_w^2 \right) F_{\gamma}^Z \]
\[ + \frac{1}{4} \left( B_{u\nu}^{RR} + B_{u\nu}^{RL} + B_{u\nu}^{LL} + B_{u\nu}^{LR} \right), \] (3.18)
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where $F^L_\gamma$, $F^R_\gamma$, $F^L_Z$, $F^R_Z$ is the shorthand notation for $(F^L_\gamma)_{e\mu}$, $(F^R_\gamma)_{e\mu}$, $(F^L_Z)_{e\mu}$, $(F^R_Z)_{e\mu}$.

The next step aims to determine the nucleon matrix elements of the operators $u^\dagger u$ and $d^\dagger d$. These are given by

$$
\langle J^+ e^- | u^\dagger u | J_\mu \rangle = (2Z + N) F(-m^2_\mu),
$$

$$
\langle J^+ e^- | d^\dagger d | J_\mu \rangle = (Z + 2N) F(-m^2_\mu),
$$

(3.19)

where the form factor $F(q^2)$ incorporates the recoil of the $J^+$ ion \cite{142}, and the factors $2Z + N$ and $Z + 2N$ count the number of $u$ and $d$ quarks in the nucleus $J$, respectively. Hence, the matrix element for $J_\mu \rightarrow J^+ e^-$ can be written down as

$$
T^{J_\mu \rightarrow J^+ e^-} = -\frac{\alpha^2}{2M^2_W} F(-m^2_\mu) \bar{\epsilon} (Q^L_W P_R + Q^R_W P_L) \mu,
$$

(3.20)

with

$$
Q^L_W = (2Z + N) V^L_u + (Z + 2N) V^L_d,
$$

$$
Q^R_W = (2Z + N) V^R_u + (Z + 2N) V^R_d.
$$

(3.21)

Given the transition amplitude (3.20), the decay rate $J_\mu \rightarrow J^+ e^-$ is found to be

$$
R^{J_\mu}_{\gamma e} = \frac{\alpha^3 \alpha^4 \mu^5_{\mu}}{16\pi^2 M^4_W} \frac{Z^4_{\text{eff}}}{Z} \frac{|F(-m^2_\mu)|^2}{\Gamma_{\text{capture}}} \left( |Q^L_W|^2 + |Q^R_W|^2 \right),
$$

(3.22)

where $\Gamma_{\text{capture}}$ is the capture rate of the muon by the nucleus, and $Z_{\text{eff}}$ is the effective charge which takes into account coherent effects which can occur in the nucleus $J$ due to its finite size. In this analysis, the values of $Z_{\text{eff}}$ quoted in Ref \cite{143} are used. Like before, the branching ratio (3.22) possesses the most general form factor structure to the leading order in external masses and momenta and is relevant to most models of New Physics with CLFV.

The analytical results presented in this section are found to be consistent with the results given in Refs \cite{111,144,145} in the $\nu_R$SM limit of the theory.
§ 3.4 Numerical Results

In this section, the numerical analysis of CLFV observables in the $\nu_R$MSSM will be presented. In order to reduce the number of independent parameters, we adopt the constrained framework of mSUGRA, discussed in Chapter 2. In detail, the model parameters are: (i) the usual SM parameters, such as gauge coupling constants, the quark and charged-lepton Yukawa matrices inputted at the scale $M_Z$, (ii) the heavy neutrino mass $m_N$ and the neutrino Yukawa matrix $h_\nu$ evaluated at $m_N$, (iii) the universal mSUGRA parameters $m_0$, $M_{1/2}$ and $A_0$ inputted at the GUT scale, and (iv) the ratio $\tan \beta$ of the Higgs VEVs and the sign of the superpotential Higgs-mixing parameter $\mu$.

The allowed ranges of the soft SUSY-breaking parameters $m_0$, $M_{1/2}$, $A_0$ and $\tan \beta$ are strongly constrained by a number of accelerator and cosmological data [82–84, 146–148]. For definiteness, we consider the following set of input parameters:

\[ \tan \beta = 10, \quad m_0 = 1000 \text{ GeV}, \quad A_0 = -3000 \text{ GeV}, \quad M_{1/2} = 1000 \text{ GeV}. \]  

(3.23)

Here the $\mu$ parameter is taken to be positive, whilst its absolute value $|\mu|$ is derived from the minimization of the Higgs potential at the scale $M_Z$. Using Refs [149–152], one can verify that the parameter set (3.23) predicts a SM-like Higgs boson with $m_H \approx 125$ GeV, in agreement with the recent discovery at the LHC [82, 84, 147], and is compatible with the current lower limits on gluino and squark masses [84, 146, 147]. The set (3.23) is also in agreement with having the lightest neutralino as the Dark Matter in the Universe [148].

We employ the one-loop RGE equations given in Refs [69, 153] to evolve the gauge coupling constants and the quark and charged lepton Yukawa matrices from $M_Z$ to the GUT scale, while the heavy neutrino mass matrix $m_M$ and the neutrino Yukawa matrix $h_\nu$ are evolved from the heavy neutrino mass threshold $m_N$ to the GUT scale. Furthermore, we assume that the heavy neutrino-sneutrino sector is approximately supersymmetric above $m_N$. For
purposes of RGE evolution, this is a good approximation for $m_N$ larger than
the typical soft SUSY-breaking scale [111]. At the GUT scale, the mSUGRA
universality conditions are used to express the soft SUSY-breaking masses,
in terms of $m_0$, $M_{1/2}$ and $A_0$. Hence, all scalar masses receive a soft SUSY-
breaking mass $m_0$, all gauginos are mass-degenerate to $M_{1/2}$, and all scalar
trilinear couplings are of the form $h_x A_0$, with $x = u, d, l, \nu$, where $h_x$ are
the Yukawa matrices at the GUT scale. The sneutrino mass matrix acquires
additional contributions from the heavy neutrino mass matrix. The sparticle
mass matrices and trilinear couplings are evolved from the GUT scale to
$M_Z$, except for the sneutrino masses which are evolved to the heavy neutrino
threshold $m_N$. Having thus obtained all sparticle and sneutrino mass matri-
ces, one can numerically evaluate all particle masses and interaction vertices
in the $\nu_R$MSSM, without approximations.

To simplify our numerical analysis, two representative scenarios of Yukawa
textures discussed in Chapter 2 are considered. Specifically, the first sce-
nario realizes the U(1)-symmetric Yukawa texture in (2.30), for which we
take either $a = b$ and $c = 0$, or $a = c$ and $b = 0$, or $b = c$ and $a = 0$,
thus giving rise to CLFV processes $\mu \rightarrow eX$, $\tau \rightarrow eX$ and $\tau \rightarrow \mu X$, re-
spectively. Here $X$ stands for the state(s) with zero net lepton number, e.g.
$X = \gamma, e^+e^-, \mu^+\mu^-, q\bar{q}$. The second scenario is motivated by the $A_4$
group and uses the Yukawa texture (2.31), where the parameters $a$, $b$ and $c$
are taken to be all equal, i.e. $a = b = c$.

The heavy neutrino mass scale $m_N$ strongly depends on the size of the
symmetry-breaking terms in the Yukawa matrix $h_\nu$. For instance, for the
model described by Eq (2.30), the typical values of the $U(1)$-lepton-symmetry-
breaking parameters $\epsilon_l \equiv \epsilon_{e,\mu,\tau}$ consistent with low-scale resonant leptogene-
sis is $\epsilon \lesssim 10^{-5}$ [125,127], leading to light-neutrino masses

$$m_\nu \sim \frac{\epsilon_l^2 \nu^2}{m_N} \sim 10^{-2} \text{eV} \left(\frac{\epsilon_l}{10^{-6}}\right)^2 \left(\frac{1 \text{TeV}}{m_N}\right). \quad (3.24)$$

Taking into account the constraint $m_\nu \gtrsim 10^{-1} \text{eV}$ generically derived from
neutrino oscillation data, we may estimate that the heavy neutrino mass scale
$m_N$ is typically restricted to be less than 10 TeV, for $\epsilon_l = 10^{-5}$. If the assumption of successful low-scale leptogenesis is relaxed, the symmetry-breaking parameters $\epsilon_l$ has only to be couple of orders in magnitude smaller than the Yukawa parameters $a$, $b$ and $c$, with $a$, $b$, $c \lesssim 10$. Thus, for $\epsilon_l < 10^{-3} - 10^{-2}$, the heavy neutrino mass scale $m_N$ may be as large as $10^7 - 10^9$ TeV, leading to the decoupling of heavy neutrinos from low-energy observables. As the main interest of this thesis is in the interplay between heavy neutrino, sneutrino and soft SUSY-breaking contributions to CLFV observables, the focus will be only on the parameter space in which $m_N < 10$ TeV.

In the present analysis, we consider that the symmetry-preserving Yukawa parameters $a$, $b$ and $c$ are limited by the perturbativity condition: $\text{Tr} h^\dagger h_p < 4\pi$, which is required to hold true for the entire interval of the RGE evolution: $\ln(M_Z/\text{TeV}) < t < \ln(M_{\text{GUT}}/\text{TeV})$. For the model described by Eq (2.30), this condition translates into the constraint: $a < 0.34$, and for the model described by Eq (2.31), to: $a < 0.23$. For that reason, the numerical values for points in parameter space for which the aforementioned perturbativity condition gets violated will not be displayed.

In Fig 3.3 are displayed numerical predictions for the $\mu$-LFV observables $B(\mu \rightarrow eX)$: $B(\mu \rightarrow e\gamma)$ [blue (solid) line], $B(\mu \rightarrow eee)$ [red (dashed) line], $R^{T\mu e}$ [violet (dotted) line] and $R^{Au}$ [green (dash-dotted) line], as functions of $B(\mu \rightarrow e\gamma)$ (left panels) and the Yukawa parameter $a$ (right panels), for $m_N = 400$ GeV and $\tan \beta = 10$. The upper two panels assume the Yukawa texture in (2.30), with $a = b$ and $c = 0$, whilst the lower two panels correspond to the Yukawa texture in (2.31), with $a = b = c$. In Fig 3.4 we give numerical estimates for the same set of $\mu$-LFV observables, but for $m_N = 1$ TeV. In Figs 3.3 and 3.4 the Yukawa parameter $a$ has been chosen, such that $10^{-20} < B(\mu \rightarrow e\gamma) < 10^{-10}$. Such a range of values includes both the present [26, 29, 33, 36] and future [43, 47, 49, 154, 156] experimental limits. As can be seen from Figs 3.3 and 3.4 the CLFV observables under study depend quadratically on the Yukawa parameter $a$, namely they are proportional to $a^2$. Instead, the quartic Yukawa terms proportional to $a^4$ [111] remain always small, which is a consequence of the imposed perturbativity
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Figure 3.3: Numerical estimates of $B(\mu \to e\gamma)$ [blue (solid)], $B(\mu \to eee)$ [red (dashed)], $R_{\mu e}^{\text{Ti}}$ [violet (dotted)] and $R_{\mu e}^{\text{Au}}$ [green (dash-dotted)], as functions of $B(\mu \to e\gamma)$ (left panels) and the Yukawa parameter $a$ (right panels), for $m_N = 400 \text{ GeV}$ and $\tan \beta = 10$. The upper two panels correspond to the Yukawa texture (2.30), with $a = b$ and $c = 0$, and the lower two panels to the Yukawa texture (2.31), with $a = b = c$.

constraint: $\text{Tr}(h_{\nu}^\dagger h_{\nu}) < 4\pi$, up to the GUT scale.

By analogy, Figs 3.5 and 3.6 present numerical estimates of the $\tau$-LFV observables $B(\tau \to eX)$: $B(\tau \to e\gamma)$ [blue (solid) lines], $B(\tau \to eee)$ [red (dashed) lines] and $B(\tau \to e\mu\mu)$ [violet (dotted) lines], as functions of $B(\tau \to e\gamma)$ (left panels) and the Yukawa parameter $a$ (right panels), for $m_N = 400 \text{ GeV}$ and $m_N = 1 \text{ TeV}$, respectively. The predictions for the fully complementary observables $B(\tau \to \mu X)$: $B(\tau \to \mu\gamma)$, $B(\tau \to \mu\mu\mu)$ and $B(\tau \to \mu\mu\mu)$ are not displayed. The upper panels give our predictions for the Yukawa tex-
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Figure 3.4: The same as in Fig. 3.3 but for $m_N = 1$ TeV.

ture (2.30), with $a = c$ and $b = 0$, and the lower pannels for the Yukawa texture (2.31), with $a = b = c$. In both Figs 3.5 and 3.6, the Yukawa parameter $a$ has been chosen, such that $10^{-16} < B(\tau \to e\gamma) < 10^{-7}$. As can be seen from Figs 3.5 and 3.6, all observables $B(\tau \to eX)$ of $\tau$-LFV (with $X = \gamma, ee, \mu\mu$) exhibit similar quadratic dependence on the small Yukawa parameter $a$. However, close to the largest perturbatively allowed values of $a$, i.e. $a \lesssim 0.34$ for the model (2.30) and $a \lesssim 0.23$ for the model (2.31), some of the observables of $\tau$-LFV exhibit either numerical instability, or the existence of a cancellation region in parameter space, as will be seen below.

Figure 3.7 presents numerical estimates of $B(\mu \to e\gamma)$ [blue (solid) line], $B(\mu \to eee)$ [red (dashed) line], $R^{T_i}_{\mu e}$ [violet (dotted) line] and $R^{Au}_{\mu e}$ [green (dash-dotted) line], as functions of $B(\mu \to e\gamma)$ (left pannels) and the heavy
§ 3.4. Numerical Results

Figure 3.5: Numerical estimates of $B(\tau \to e\gamma)$ [blue (solid)], $B(\tau \to eee)$ [red (dashed)] and $B(\tau \to e\mu\mu)$ [violet (dotted)], as functions of $B(\tau \to e\gamma)$ (left panels) and the Yukawa parameter $a$ (right panels), for $m_N = 400$ GeV and $\tan\beta = 10$. The upper panels present predictions for the Yukawa texture (2.30), with $a = c$ and $b = 0$, and the lower panels for the Yukawa texture (2.31), with $a = b = c$. The heavy neutrino mass is varied within the LHC explorable range: $400$ GeV $< m_N < 10$ TeV. All observables $B(\mu \to eX)$ of $\mu$-LFV (with $X = \gamma$, ee, Ti, Au) exhibit a non-trivial dependence on $m_N$. The branching ratio $B(\mu \to e\gamma)$ shows a dip at $m_N \approx 800$ GeV in both models (2.30) and (2.31), signifying the existence of neutrino mass scale $m_N$ (right panels). In all panels, the Yukawa parameter $a$ is fixed by the condition $B(\mu \to e\gamma) = 10^{-12}$ for $m_N = 400$ GeV, using the benchmark value $\tan\beta = 10$. The upper panels display numerical values for the Yukawa texture (2.30), with $a = b$ and $c = 0$, and the lower panels for the Yukawa texture (2.31), with $a = b = c$. The heavy neutrino mass is varied within the LHC explorable range: $400$ GeV $< m_N < 10$ TeV. All observables $B(\mu \to eX)$ of $\mu$-LFV (with $X = \gamma$, ee, Ti, Au) exhibit a non-trivial dependence on $m_N$. The branching ratio $B(\mu \to e\gamma)$ shows a dip at $m_N \approx 800$ GeV in both models (2.30) and (2.31), signifying the existence of neutrino mass scale $m_N$ (right panels). In all panels, the Yukawa parameter $a$ is fixed by the condition $B(\mu \to e\gamma) = 10^{-12}$ for $m_N = 400$ GeV, using the benchmark value $\tan\beta = 10$. The upper panels display numerical values for the Yukawa texture (2.30), with $a = b$ and $c = 0$, and the lower panels for the Yukawa texture (2.31), with $a = b = c$. The heavy neutrino mass is varied within the LHC explorable range: $400$ GeV $< m_N < 10$ TeV. All observables $B(\mu \to eX)$ of $\mu$-LFV (with $X = \gamma$, ee, Ti, Au) exhibit a non-trivial dependence on $m_N$. The branching ratio $B(\mu \to e\gamma)$ shows a dip at $m_N \approx 800$ GeV in both models (2.30) and (2.31), signifying the existence of neutrino mass scale $m_N$ (right panels). In all panels, the Yukawa parameter $a$ is fixed by the condition $B(\mu \to e\gamma) = 10^{-12}$ for $m_N = 400$ GeV, using the benchmark value $\tan\beta = 10$. The upper panels display numerical values for the Yukawa texture (2.30), with $a = b$ and $c = 0$, and the lower panels for the Yukawa texture (2.31), with $a = b = c$. The heavy neutrino mass is varied within the LHC explorable range: $400$ GeV $< m_N < 10$ TeV. All observables $B(\mu \to eX)$ of $\mu$-LFV (with $X = \gamma$, ee, Ti, Au) exhibit a non-trivial dependence on $m_N$. The branching ratio $B(\mu \to e\gamma)$ shows a dip at $m_N \approx 800$ GeV in both models (2.30) and (2.31), signifying the existence of neutrino mass scale $m_N$ (right panels). In all panels, the Yukawa parameter $a$ is fixed by the condition $B(\mu \to e\gamma) = 10^{-12}$ for $m_N = 400$ GeV, using the benchmark value $\tan\beta = 10$. The upper panels display numerical values for the Yukawa texture (2.30), with $a = b$ and $c = 0$, and the lower panels for the Yukawa texture (2.31), with $a = b = c$. The heavy neutrino mass is varied within the LHC explorable range: $400$ GeV $< m_N < 10$ TeV. All observables $B(\mu \to eX)$ of $\mu$-LFV (with $X = \gamma$, ee, Ti, Au) exhibit a non-trivial dependence on $m_N$. The branching ratio $B(\mu \to e\gamma)$ shows a dip at $m_N \approx 800$ GeV in both models (2.30) and (2.31), signifying the existence of neutrino mass scale $m_N$ (right panels). In all panels, the Yukawa parameter $a$ is fixed by the condition $B(\mu \to e\gamma) = 10^{-12}$ for $m_N = 400$ GeV, using the benchmark value $\tan\beta = 10$. The upper panels display numerical values for the Yukawa texture (2.30), with $a = b$ and $c = 0
of a cancellation region in parameter space, due to the loops involving heavy neutrino, sneutrino and soft SUSY-breaking terms. For $m_N \gtrsim 3$ TeV, all observables tend to a constant value, as a result of the dominance of the soft SUSY-breaking contributions.

In Fig. 3.8 we show contours of the Yukawa parameters $(a, b, c)$ versus the heavy neutrino mass scale $m_N$, for $B(\mu \to e\gamma)$ [blue (solid) line], $B(\mu \to eee)$ [red (dashed) line], $R^T_{\mu e}$ [violet (dotted) line] and $R^A_{\mu e}$ [green (dash-dotted) line]. The Yukawa parameter $a$ and $m_N$ are determined by the condition $B(\mu \to e\gamma) = 10^{-12}$. The labels in the vertical axes indicate the two Yukawa textures in (2.30) and (2.31), which we have adopted in our analysis. The contours for $B(\mu \to e\gamma)$ display a maximum for $m_N \approx 800$ GeV, as a consequence of cancellations between heavy neutrino, sneutrino and soft SUSY-
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Figure 3.7: Numerical estimates of $B(\mu \rightarrow e\gamma)$ [blue (solid)], $B(\mu \rightarrow eee)$ [red (dashed)], $R_{T\mu}^{\text{e}}$ [violet (dotted)] and $R_{\mu\mu}^{\Lambda}$ [green (dash-dotted)], as functions of $B(\mu \rightarrow e\gamma)$ (left pannels) and the heavy neutrino mass scale $m_N$ (right pannels).

In all pannels, the Yukawa parameter $a$ was kept fixed by the condition $B(\mu \rightarrow e\gamma) = 10^{-12}$ for $m_N = 400$ GeV, and $\tan \beta = 10$ was used. The upper pannels display numerical values for the Yukawa texture (2.30), with $a = b$ and $c = 0$, and the lower pannels for the Yukawa texture (2.31), with $a = b = c$.

Breaking contributions (cf Fig 3.7).

Figure 3.9 shows contours of the Yukawa parameters $(a, b, c)$, as functions of $m_N$, for $B(\tau \rightarrow e\gamma)$ [blue (solid) line], where the parameters $a$ and $m_N$ are determined by the condition $B(\tau \rightarrow e\gamma) = 10^{-9}$. The numerical results for $B(\tau \rightarrow \mu\gamma)$ are not given, since these are fully complementary to the ones given for $B(\tau \rightarrow e\gamma)$. Given the above condition on $B(\tau \rightarrow e\gamma)$, no solution exists for the observables $B(\tau \rightarrow eee)$ and $B(\tau \rightarrow e\mu\mu)$.
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Figure 3.8: Contours of the Yukawa parameters \((a, b, c)\) versus \(m_N\), for \(B(\mu \rightarrow e\gamma)\) [blue (solid)], \(B(\mu \rightarrow eee)\) [red (dashed)], \(R_{\mu e}^{Ti}\) [violet (dotted)] and \(R_{\mu e}^{Au}\) [green (dash-dotted)], where \(a\) and \(m_N\) are determined by the condition \(B(\mu \rightarrow e\gamma) = 10^{-12}\). All contours are evaluated with \(\tan \beta = 10\) and for different Yukawa textures, as indicated by the vertical axes labels.

Figure 3.9: Contours of the Yukawa parameters \((a, b, c)\) versus \(m_N\), for \(B(\tau \rightarrow e\gamma)\) [blue (solid)], where \(\tan \beta = 10\) and \(a\) and \(m_N\) are determined by the condition \(B(\tau \rightarrow e\gamma) = 10^{-9}\). No solutions have been found for \(B(\tau \rightarrow eee)\) and \(B(\tau \rightarrow e\mu\mu)\).

In the numerical analysis presented so far, the assumed value of \(\tan \beta\) was fixed to its benchmark value given in (3.23), \(\tan \beta = 10\). In Fig 3.10 this assumption is relaxed, and \(\tan \beta\) is varied in the interval \(5 \lesssim \tan \beta \lesssim 20\), while maintaining agreement with a SM-like Higgs boson mass \(M_H \approx 125\) GeV and taking into account that the combined experimental and theoretical errors are currently of the order of 5–6 GeV. Specifically, in Fig 3.10 we display the dependence of \(B(\mu \rightarrow e\gamma)\) [blue (solid) line], \(B(\mu \rightarrow eee)\) [red (dashed) line], \(R_{\mu e}^{Ti}\) [violet (dotted) line] and \(R_{\mu e}^{Au}\) [green (dash-dotted) line] on \(\tan \beta\). In all pannels, the Yukawa parameter \(a\) is determined by the condition \(B(\mu \rightarrow\)
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Figure 3.10: Numerical estimates of $B(\mu \rightarrow e\gamma)$ [blue (solid)], $B(\mu \rightarrow eee)$ [red (dashed)], $R_{\mu e}^{Ti}$ [violet (dotted)] and $R_{\mu e}^{Au}$ [green (dash-dotted)], as functions of $\tan\beta$. The upper pannels are obtained for $m_N = 400$ GeV and the lower pannels for $m_N = 1$ TeV. The left pannels use the Yukawa texture (2.30), with $a = b$ and $c = 0$, and the right pannels the Yukawa texture (2.31), with $a = b = c$. In all pannels, the Yukawa parameter $a$ is determined by the condition $B(\mu \rightarrow e\gamma) = 10^{-12}$. 
$e\gamma = 10^{-12}$. The upper panels in Fig 3.10 show numerical results for $m_N = 400$ GeV, while the lower panels for $m_N = 1$ TeV. The left panels give the predictions for the Yukawa texture \( (2.30) \), with $a = b$ and $c = 0$, and the right panels for the Yukawa texture \( (2.31) \), with $a = b = c$. In the lower panels, one can observe a suppression of $B(\mu \to e\gamma)$, for values $\tan \beta \approx 7$, due to the cancellation between heavy neutrino, sneutrino and soft SUSY-breaking effects.

It can be instructive to compare the contributions of the magnetic dipole form factors to the CLFV observables, with those originating from the remaining form factors. Specifically, if one assumes that only the magnetic dipole form factors $G_L^\gamma$ and $G_R^\gamma$ contribute in \( (3.10) \), \( (3.11) \) and \( (3.22) \), then the following analytical results are obtained for the ratios:

$$R_1 \equiv \frac{B(l \to l_1l_2l_3)}{B(l \to l\gamma)} = \frac{\alpha}{3\pi} \left( \ln \frac{m^2_l}{m^2_{l'}} - 3 \right)$$

$$R_2 \equiv \frac{B(l \to l'l'l''e)}{B(l \to l\gamma)} = \frac{\alpha}{3\pi} \left( \ln \frac{m^2_l}{m^2_{l'}} - \frac{11}{4} \right)$$

$$R_3 \equiv \frac{R^J_{\mu e}}{B(\mu \to e\gamma)} = 16\alpha^4 \frac{\Gamma_\mu}{\Gamma_{\text{capture}}} ZZ_{eff}^4 |F(-\mu^2)|^2$$

According to the formulae \( (3.25) \)--\( (3.27) \), the predicted $R_1$ values for $\tau \to e\mu\mu$ and $\tau \to \mu ee$ are 1/90 and 1/419 respectively, the predicted $R_2$ values for $\mu \to eee$, $\tau \to eee$ and the $\tau \to \mu\mu\mu$ are 1/159, 1/91 and 1/460 respectively, and the predicted $R_3$ values for Ti and Au are 1/198 and 1/188 respectively.

In Fig 3.11 the numerical estimates are given for the ratios $R_2(\mu \to eee)$, $R_3^{\text{Ti}}$ and $R_3^{\text{Au}}$, as functions of $m_N$. The Yukawa parameter $a$ is fixed by the condition $B(\mu \to e\gamma) = 10^{-12}$, for $m_N = 400$ GeV and $\tan \beta = 10$. In the upper pannel, thick lines show the predicted values obtained by a complete evaluation of $R_2(\mu \to eee)$ [blue (solid) line], $R_3^{\text{Ti}}$ [red (dashed) line] and $R_3^{\text{Au}}$ [violet (dotted) line], while the respective thin lines are obtained by keeping only the magnetic dipole form factors $G_L^\gamma$ and $G_R^\gamma$. Hence, we see that going beyond the magnetic dipole moment approximation may enhance the ratios.
Figure 3.11: Numerical estimates of the ratios $R_2(\mu \to eee)$, $R_{3T}^{Ti}$ and $R_{3A}^{Au}$, as functions of $m_N$. The Yukawa parameter $a$ is fixed by the condition $B(\mu \to e\gamma) = 10^{-12}$, for $m_N = 400$ GeV and $\tan \beta = 10$. In the upper pannel, thick lines give the complete evaluation of $R_2(\mu \to eee)$ [blue (solid)], $R_{3T}^{Ti}$ [red (dashed)] and $R_{3A}^{Au}$ [violet (dotted)], while the respective thin lines are evaluated keeping only the magnetic dipole form factors $G_L^\gamma$ and $G_R^\gamma$. The two middle pannels provide a form factor analysis of $R_2(\mu \to eee)$ and $R_{3A}^{Au}$, in terms of contributions due to $G_\gamma$ and $F_\gamma$ [blue (solid)], $F_Z$ [red (dashed)] and box form factors [violet (dotted)]. The lower two pannels show the separate contributions due to heavy neutrinos $N$ [blue (solid)], sneutrinos $\tilde{N}$ [red (dashed)] and soft SUSY-breaking LFV terms [violet (dotted)]. The green (horizontal) lines in the middle and lower pannels give the predicted values obtained by assuming that only the $G_L^\gamma, R$ form factors contribute to the amplitudes.
$R_{2,3}$ by more than two orders of magnitude.

The two middle pannels of Fig 3.11 provide a form factor analysis of $R_2(\mu \rightarrow eee)$ and $R_3^{Au}$, by considering separately the contributions due to $G_\gamma$ and $F_\gamma$ [blue (solid) line], $F_Z$ [red (dashed) line] and box form factors [violet (dotted) line]. In particular, one observes that heavy neutrino contributions to the box form factors become comparable to and even larger than the $Z$-boson mediated graphs in $\mu \rightarrow e$ conversion in Gold, for heavy neutrino masses $m_N \lesssim 1$ TeV. We have checked that for $m_N \lesssim 1$ TeV, box graphs due to heavy neutrinos also dominate the process of $\mu \rightarrow e$ conversion in Titanium (not explicitly shown in Fig 3.11). Finally, the two lower pannels show the individual contributions due to the heavy neutrinos $N_{1,2,3}$ [blue (solid) line], sneutrinos $\bar{N}_{1,2,...,12}$ [red (dashed) line] and soft SUSY-breaking LFV terms [violet (dotted) line]. From these two lower pannels, it is obvious that for heavy neutrino masses $m_N \gtrsim 1$ TeV, the soft SUSY-breaking effects dominate the CLFV form factors, which are tagged with the superscripts SB in Appendix C. Instead, for $m_N \lesssim 1$ TeV, heavy-neutrino effects start becoming the leading contribution to the CLFV observables associated with the muon. The green (horizontal) lines in the middle and lower pannels serve as reference values obtained by assuming that only the $G_{\gamma}^{L,R}$ form factors contribute to the amplitudes.

An important consistency check for this numerical analysis has been to analytically show that all soft SUSY-breaking effects on the form factors (C.4), (C.8) and (C.14) vanish in the limit of degenerate charged slepton masses. On the other hand, RGE effects from $M_{GUT}$ to $M_Z$ induce sizeable deviations to the charged slepton mass matrix from the unit matrix. As a consequence, unitarity cancellations due to the so-called GIM mechanism become less effective in this case and so render the SB part of the form factors, such as $F_{r\ell Z}^{L,SB}$ and $F_{r\ell Z}^{R,SB}$, rather large.

Another essential check was to show that under the assumptions adopted in part of Ref [157], the form factor $F_{r\ell Z}^{L,N}$ given in Eq (C.7) reduces to $\frac{2\alpha_W}{g} F_L^c$, where $F_L^c$ is one of the form factors defined in [157] Eq (6), which in turn can be shown to vanish. The assumptions in [157] are: (i) the standard
seesaw mechanism with ultra-heavy right neutrinos, (ii) no charged wino or higgsino mixing, and (iii) the dominance of the wino contribution. Under these three assumptions, the interaction vertices occurring in the form factor $F_{\nu lZ}^{L,\tilde{N}}$ simplify as follows:

$$
\hat{B}^{R,1}_{lmA}, \hat{B}^{R,2}_{lmA} \rightarrow -U_{lk}, \quad \tilde{C}^1_{AB}, \tilde{C}^2_{AB}, \tilde{C}^3_{AB}, \tilde{C}^4_{AB} \rightarrow -\frac{1}{2} \delta_{kk'},
$$

$$
V_{mk}^\chi^{-R} \rightarrow c_w^2,
$$

where $A, B$ now assume the restricted range of values $k, k' = 1, 2, 3$ and $U$ is a $3 \times 3$ unitary matrix. Given the simplifications in Eq (3.28), we recover the expression given in Ref [157], resulting in the replacement: $F_{\nu lZ}^{L,\tilde{N}} \rightarrow 2c_w^2 F_L$. The above non-trivial checks provide firm support for the correctness of analytical and numerical results hereby presented. The full-fledged calculation in Ref [157] was performed without the above mentioned assumptions.
Chapter 4

Lepton Dipole Moments

In this chapter we perform the study of anomalous magnetic and electric dipole moments of charged leptons in $\nu_R$MSSM, under the assumption that CP violation originates from complex soft SUSY-breaking bilinear and trilinear couplings associated with the right-handed sneutrino sector.

In the first section, the conventions and notation for the lepton dipole moments will be presented. This will be accompanied by the description of the new sources of CP violation which are considered within the $\nu_R$MSSM.

Second section contains the numerical estimates for the anomalous magnetic moment of the muon ($a_\mu$) and the electric dipole moment of the electron ($d_e$).

Technical details pertinent to the lepton-dipole moment form factors are to be found at the end of this chapter.

These results are presented in Ref [158].
§ 4.1 Magnetic and electric dipole moments

The anomalous MDM and EDM of a charged lepton $l$ can be read off from the Lagrangian \[159\]:

$$\mathcal{L} = \bar{l} \left[ \gamma \mu (i \partial^\mu + e A^\mu) - m_l - \frac{e}{2m_l} \sigma^{\mu\nu} (F_l + iG_l \gamma_5) \partial_\nu A_\mu \right] l.$$  \hspace{1cm} (4.1)

In the on-shell limit of the photon field $A^\mu$, the form factor $F_l$ defines the anomalous magnetic dipole moment (MDM) of the lepton $l$, i.e. $a_l \equiv F_l$, whilst the form factor $G_l$ defines its electric dipole moment (EDM), i.e. $d_l \equiv eG_l/m_l$. Using Eq (3.1), one can write down the general form-factor decomposition of the photonic transition amplitude,

$$i T_{\gamma ll} = i \frac{e\alpha_w}{8\pi M_W^2} \left[ (G_{\gamma}^L)_{ll} i\sigma_{\mu\nu} q^\nu P_L + (G_{\gamma}^R)_{ll} i\sigma_{\mu\nu} q^\nu P_R \right].$$  \hspace{1cm} (4.2)

The anomalous MDM ($a_l$) and the EDM ($d_l$) of a lepton $l$ are then respectively determined by:

$$a_l = \frac{\alpha_w m_l}{8\pi M_W^2} \left[ (G_{\gamma}^L)_{ll} + (G_{\gamma}^R)_{ll} \right].$$  \hspace{1cm} (4.3)

$$d_l = \frac{e\alpha_w}{8\pi M_W^2} i \left[ (G_{\gamma}^L)_{ll} - (G_{\gamma}^R)_{ll} \right].$$  \hspace{1cm} (4.4)

Here and in the following, the notation for the couplings and the form-factors will correspond to the one used in Chapter 3.

As shown in Ref \[160\], the EMD $d_l$ of the lepton vanishes in the MSSM with universal soft SUSY-breaking boundary conditions, if no CP phases are introduced. This result also holds true in the extensions of the MSSM with heavy neutrinos, as long as the sneutrino sector is universal and CP-conserving.

As a minimal departure from the above universal scenario, let it be assumed that only the sneutrino sector is CP-violating due to soft CP phases in the
bilinear and trilinear soft-SUSY breaking parameters:

\[ b_\nu = B_\nu m_M = B_0 e^{i\theta} m_N 1_3, \]  
\[ A_\nu = h_\nu A_0 e^{i\phi}, \]

where \( B_0 \) and \( A_0 \) are real parameters determined at the GUT scale, \( m_N \) is a real parameter inputed at the scale \( m_N \), and \( \theta \) and \( \phi \) are physical, flavor-blind CP-odd phases, and \( h_\nu \) is the \( 3 \times 3 \) neutrino Yukawa matrix given by Eq (2.31). The soft SUSY breaking terms corresponding to the \( b_\nu \) and \( A_\nu \) are obtained from the Lagrangian terms

\[ -(A_\nu)^{ij} \tilde{\nu}_{iR}(h_{uL}^+ \tilde{e}_{jL} - h_{uL}^0 \tilde{\nu}_{jL}) \]
and

\[ (b_\nu m_M)^{ij} \tilde{\nu}_{iRi} \tilde{\nu}_{Rj}, \]

respectively. Correspondingly, \( \tilde{\nu}_{iR}, \tilde{e}_{jL}, h_{uL}^+ \) and \( h_{uL}^0 \) denote the heavy sneutrino, selectron, charged Higgs and neutral Higgs fields. The \( O(3) \) flavor symmetry of the model for the heavy neutrinos assures that the heavy neutrino mass matrix \( m_N \) is proportional to the unit matrix \( 1_3 \) with eigenvalues \( m_N \), up to small renormalization-group effects. To keep things simple, we also assume that the \( 3 \times 3 \) soft bilinear mass matrix \( b_\nu \) is proportional to \( 1_3 \). In the standard SUSY seesaw scenarios with ultra-heavy neutrinos of mass \( m_N \), the CP-violating sneutrino contributions to electron EDM \( d_e \) scale as \( B_0/m_N \) and \( A_0/m_N \) at the one-loop level, and practically decouple for heavy-neutrino masses \( m_N \) close to the GUT scale. Hence, sizeable effects on \( d_e \) should only be expected in low-scale seesaw scenarios, in which \( m_N \) can become comparable to \( B_0 \) and \( A_0 \).

Note that the bilinear soft \( 3 \times 3 \) matrix \( b_\nu \) was neglected in the previous chapter, where it was tacitly assumed that it was small compared to the other soft SUSY-breaking parameters in sneutrino mass matrix given by Eq (2.34). Here, this term will be taken into the account, but with the restricted size of the universal bilinear mass parameter \( B_0 \), such that the sneutrino masses remain always positive and hence physical.
The generation of a non-zero lepton EDM $d_l$ results from the soft sneutrino CP-odd phases $\theta$ and $\phi$, as well as from complex neutrino Yukawa couplings $h_{\nu}$. All these CP-odd phases are present in the photon dipole form factors $G^{\mu N}_{L \gamma}$ and $G^{\mu N}_{R \gamma}$, whose analytical forms may be found in Appendix C. In fact, it can be noticed that $d_l$ may be generated by products of vertices that are not relatively complex conjugate to each other, since they contain the factors

$$\Delta^{LR}_{\text{CP}} = \tilde{B}_{lkA}^{R,1} \tilde{B}_{lkA}^{L,1*} + \tilde{B}_{lkA}^{R,2} \tilde{B}_{lkA}^{L,2*} \quad \Delta^{RL}_{\text{CP}} = \tilde{B}_{lkA}^{R,1} \tilde{B}_{lkA}^{L,1*} + \tilde{B}_{lkA}^{R,2} \tilde{B}_{lkA}^{L,2*} , \quad (4.9)$$

as can be seen from the Eq (C.3).

In the exact supersymmetric limit of softly-broken SUSY theories, the anomalous MDM (as well as EDM) operator is forbidden. This comes as a consequence of the Ferrara and Remiddi no-go theorem [132], which is verified for every particle and its SUSY-counterpart contribution to the anomalous MDM $a_\mu$. Besides the SM contribution, there are three additional contributions in the $\nu_R$MSSM, which originate from: (i) heavy neutrinos, (ii) sneutrinos and (iii) soft SUSY-breaking parameters. In the supersymmetric limit, the latter contribution (iii) vanishes. In the same limit, the heavy neutrino and sneutrino contributions read:

$$(G^{\mu N}_{L \gamma}) \rightarrow \frac{7}{6} B_{lN_a} B_{lN_a}^* ,$$

$$(G^{\mu N}_{R \gamma}) \rightarrow - \frac{7}{6} B_{lN_a} B_{lN_a}^* , \quad (4.10)$$

where $B_{lN_a}$ are the lepton-to-heavy neutrino mixings defined in Refs [104, 105, 110]. Obviously, the sum $(G^{\mu N}_{L \gamma}) + (G^{\mu N}_{R \gamma})$ vanishes, thereby confirming the above mentioned theorem proposed by Ferrara and Remiddi.

In the MSSM, the leading contribution to $a_l$ behaves as [161, 163]

$$a_l^{\text{MSSM}} \propto \frac{m_l^2}{M_{\text{SUSY}}^2} \tan\beta \text{sign}(\mu M_{1,2}) , \quad (4.11)$$

where $M_{\text{SUSY}}$ is a typical soft SUSY-breaking mass scale, $\tan\beta = v_2/v_1$ is
§ 4.2 Numerical results

the ratio of the neutral Higgs vacuum expectation values, and $M_{1,2}$ are the soft gaugino masses associated with the $U(1)_Y$ and $SU(2)$ gauge groups, respectively. As will be seen in the next section, the MSSM contribution (4.11) to $a_\mu$ remains dominant in the $\nu_R$MSSM as well. From Eqs (4.3) and (4.11), one naively expects $d_l$ at the one-loop level to behave

$$d_l^{\text{MSSM}} \propto \sin(\phi_{\text{CP}}) \frac{m_l}{M_{\text{SUSY}}^2} \tan \beta,$$

where $\phi_{\text{CP}}$ is a generic soft SUSY-breaking CP-odd phase. Although there are different dependencies of $d_l$ on $\tan \beta$ possible in the MSSM beyond the one-loop approximation [160,164], it will be shown that within the $\nu_R$MSSM, the $\tan \beta$ dependence is linear at the one-loop level.

§ 4.2 Numerical results

In this numerical analysis, we will adopt the procedure established in Chapter 3. As a benchmark model, we choose a minimally extended scenario of minimal supergravity (mSUGRA), in which we allow for the bilinear and trilinear soft SUSY-breaking terms, $B_\nu$ and $A_\nu$, to acquire at the GUT scale overall CP-violating phases denoted as $\theta$ and $\phi$, respectively. Like before, we choose the sign of the $\mu$-parameter to be positive. As for the neutrino Yukawa coupling matrix $h_\nu$, we consider the $A_4$-symmetric models introduced in previous chapter [see Eq (2.31)].

For definiteness, our numerical analysis in this section is based on the following baseline scenario:

$$m_0 = 1 \text{ TeV}, \quad M_{1/2} = 1 \text{ TeV}, \quad A_0 = -4 \text{ TeV}, \quad \tan \beta = 20,$$

$$m_N = 1 \text{ TeV}, \quad B_0 = 0.1 \text{ TeV}, \quad a = b = c = 0.05,$$

where $m_0$, $M_{1/2}$ and $A_0$ are the standard universal soft SUSY-breaking parameters [cf Eq (2.15)]. All mass parameters except $m_N$ are defined at the
GUT scale and $m_N$ is intaken at $m_N$ scale. It is understood that parameters which are not explicitly quoted in the text assume their default values stated in (4.13).

We will analyze the deviation of $a_\mu$ from the SM value due to the $\nu_R$MSSM, denoted by $\delta a_\mu$, as well as $d_e$ on several key theoretical parameters, by varying them around their baseline value given in (4.13), while keeping the remaining parameters fixed. In doing so, it will be made sure that the displayed parameters can accommodate the LHC data for a SM-like Higgs boson with mass $m_H = 125.5 \pm 2$ GeV \cite{82,84,147} and satisfy the current lower limits on gluino and squark masses \cite{146,147}, i.e. $m_{\tilde{g}} > 1500$ GeV and $m_{\tilde{t}} > 500$ GeV.

\section*{§ 4.2.1 Results for $a_\mu$}

The numerical estimates for $\delta a_\mu$ exhibit a direct quadratic dependence on the muon mass $m_\mu$. In fact, one finds that for the same set of soft SUSY-breaking parameters $m_0$, $M_{1/2}$ and $A_0$, the ratio $\delta a_\mu/\delta a_e$ remains constant to a good approximation, i.e. $\delta a_\mu/\delta a_e \approx m_\mu^2/m_e^2 \approx 42752.0$. In order to understand this parameter dependence, one has to carefully analyze the soft SUSY-breaking contributions to the form-factors:

\begin{align}
G_{LL}^{SB} & = \tilde{V}_{lma}^\dagger \tilde{V}_{lma}^\dagger \left[ m_1 \lambda_{\tilde{e}_a} J_{41}^1 (\lambda_{\tilde{e}_a}, \lambda_{\tilde{\chi}^0_m}) \right] \\
& + \tilde{V}_{lma}^\dagger \tilde{V}_{lma}^\dagger \left[ 2m_{\tilde{\chi}^0_m} \lambda_{\tilde{e}_a} J_{31}^0 (\lambda_{\tilde{e}_a}, \lambda_{\tilde{\chi}^0_m}) \right] , \quad (4.14) \\
G_{RR}^{SB} & = \tilde{V}_{lma}^\dagger \tilde{V}_{lma}^\dagger \left[ m_1 \lambda_{\tilde{e}_a} J_{41}^1 (\lambda_{\tilde{e}_a}, \lambda_{\tilde{\chi}^0_m}) \right] \\
& + \tilde{V}_{lma}^\dagger \tilde{V}_{lma}^\dagger \left[ 2m_{\tilde{\chi}^0_m} \lambda_{\tilde{e}_a} J_{31}^0 (\lambda_{\tilde{e}_a}, \lambda_{\tilde{\chi}^0_m}) \right] , \quad (4.15)
\end{align}

where the different terms that occur in Eq (4.14) and (4.15) are defined in Chapter 3 as well as at the end of this chapter. It is important to note that the neutralino vertices induce a term which is not manifestly proportional to the charged lepton mass, but to the neutralino mass. However, we have numerically confirmed that $\delta a_\mu$ is proportional to $m_1^2$, which means that
the products of the mixing matrices $\tilde{V}_{lR}^{\nu lR}$ and $\tilde{V}_{lR}^{\nu lL}$, as well as $G_L^{\nu l\gamma}$ and $G_R^{\nu l\gamma}$, are themselves proportional to the charged lepton mass $m_l$ (cf Ref [163]). The latter provides a non-trivial powerful check for the correctness of the results presented in this thesis.

In addition, this numerical analysis shows that the contribution to the muon anomalous MDM is almost independent of the neutrino-Yukawa parameters $a$, $b$ and $c$, the heavy neutrino mass $m_N$ and the soft trilinear parameter $A_0$. Hence, our results are almost insensitive to a particular choice for a neutrino Yukawa texture, e.g. as given in (2.30) and (2.31), and also independent of the CP-odd phases $\theta$ and $\phi$.

In Fig 4.1, the numerical estimates for $\delta a_\mu$ are given, as a function of the key theoretical parameters: $\tan \beta$, $M_{1/2}$, $m_0$ and $m_N$. In the frame (a) of this figure, we see that $\delta a_\mu$ depends linearly on $\tan \beta$, as expected from (4.11). Likewise, in Fig 4.1 we have investigated the dependence of $\delta a_\mu$ on the soft SUSY-breaking parameters $m_0$ and $M_{1/2}$, for different kinematic situations, and obtained results consistent with the scaling behaviour of $1/M^2_{\text{SUSY}}$ in (4.11).

In the panel (e) of Fig 4.1, we observe that the effect of the heavy right-handed neutrinos ($N$) and sneutrinos ($\tilde{N}$) on $\delta a_\mu$ is negative, but small, in agreement with our discussion above. The size of their contributions alone to $a_\mu$ ranges from $-10^{-12}$ to $-4.8 \times 10^{-15}$, for $m_N = 0.5 - 10$ TeV. On the other hand, the left-handed sneutrino contributions to $a_\mu$ are approximately independent of the heavy Majorana mass $m_N$, reaching values $\approx 8.5 \times 10^{-11}$. The soft SUSY-breaking contributions are also approximately independent of the heavy Majorana mass $m_N$ and have values $\approx 1.1 \times 10^{-12}$. Note that the light sneutrino contribution to the anomalous magnetic moment is the largest in magnitude, and it is already present in the MSSM contributions to $a_\mu$.

Finally, we have checked the dominance of the MSSM contributions by looking at the dependence of the parameter:

$$\delta' a_\mu = a^{\nu R\text{MSSM}}_\mu - a^{\text{MSSM}}_\mu.$$ (4.16)
Figure 4.1: Numerical estimates for the contribution to the muon anomalous MDM, as functions of $\tan \beta$, $M_{1/2}$, $m_N$, $m_0$ and $m_0 = M_{1/2}$, in the $\nu_R$MSSM. The default parameter set of the baseline model is given in (4.13). The panel (e) shows the heavy neutrino ($N$), sneutrino ($\tilde{N}$), soft SUSY-breaking (SB) and all contributions to $\delta a_\mu$, as a function of $m_N$. The panel (f) displays the absolute value of the relative deviation $\delta' a_\mu / a_\mu$ of the $\nu_R$MSSM and MSSM predictions for $a_\mu$ [cf. (4.16)], as a function of $m_0$. The range of input parameters in all plots satisfy the current LHC constraints on Higgs, gluino and squark masses. The heavy dots on the curves give the predicted values evaluated for the default parameters (4.13).
The difference $\delta a_\mu$ of the predictions for $a_\mu$ within the $\nu_R$MSSM and the MSSM divided by $a_\mu$ is evaluated, and the absolute values of the results are displayed in the pannel (f) of Fig 4.1 as a function of $m_0 = M_{1/2}$. The largest deviation from the MSSM is found for largest allowed parameter value, $m_0 = 3600$ GeV, in which case $\delta a_\mu/a_\mu^{MSSM}$ is as large as $6.2 \times 10^{-2}$.

§ 4.2.2 Results for $d_e$

We will now study the dependence of the electron EDM $d_e$ on several key model parameters, such as $m_0$, $M_{1/2}$, $B_0$, $A_0$, $\tan \beta$, $\theta$ and $\phi$. The predictions for $d_\mu$ may be obtained by using the naive scaling relation: $d_\mu \approx (m_\mu/m_e) d_e \approx 205 d_e$. It is found this scaling behaviour is numerically satisfied very well. The maximal numerical values for $d_e$ obtained are of the order $\sim 10^{-27}$ e cm. The predicted values for $d_\mu$ are therefore always found to be less than $\sim 10^{-25}$ e cm, which is several orders of magnitude below the present experimental upper bound: $d_\mu = 0.1 \pm 0.9 \times 10^{-19}$ e cm [37].

It is noted that heavy singlet neutrinos $N$ do not contribute to $d_e$, even if the soft SUSY-breaking CP-odd phases $\phi$ and $\theta$ are taken to be non-zero. On the other hand, soft SUSY-breaking and right handed neutrino effects induce non-vanishing $d_e$, if either $\theta$ or $\phi$ are non-zero. If both $\phi = 0$ and $\theta = 0$, lepton EDMs $d_l$ numerically vanish. Therefore, the complex products of vertices (4.9) emerging in the $\nu_R$MSSM do not induce the CP violation at one loop level, in accord with the result of Ref [160] obtained in the MSSM with a high-scale seesaw mechanism.

In Fig 4.2 the present numerical estimates of $d_e$ on the $\nu_R$MSSM parameters $\tan \beta$, $m_0$, $M_{1/2}$ and $m_N$, for maximal $A_0$ phase, $\phi = \pi/2$ are presented. The value of $\theta$ is set to zero, since the dependence of $d_e$ on $B_0$ is weaker than the dependence on $A_0$. As shown in pannel (a) of Fig 4.2, $d_e$ exhibits a linear dependence on $\tan \beta$ confirming the $\tan \beta$ naive scaling behaviour in Eq (4.12).

Further, $d_e$ is a decreasing function of $m_0$. As a function of $m_0 = M_{1/2}$, $d_e$ assumes both positive and negative values, and is roughly proportional to
Figure 4.2: Numerical estimates of the electron EDM $d_e$ in the $\nu_{R}$MSSM, as functions of $\tan \beta$, $m_0$, $m_0 = M_{1/2}$ and $m_N$, for $\phi = \pi/2$. The remaining parameters not shown assume the baseline values in (4.13). All input parameters are chosen so as to satisfy the LHC constraints on Higgs, gluino and squark masses. The heavy dots on the curves indicate the predicted values for $d_e$ evaluated for the default parameters (4.13).

$-1 - 2.4 \text{TeV}/m_0 + 6.3 \text{TeV}^2/m_0^2$. There is also a small region of parameter space for $m_0 = M_{1/2} \lesssim 800$ GeV, for which the prediction for $d_e$ is of the order of the experimental upper limit on $d_e$ (1.6). In addition, $d_e$ decreases with increasing $m_N$: for the $m_N$ values from the pannel (d) of Fig 4.2 this behavior can roughly approximated by a function $-0.13 + \text{TeV}^2/2 m_N^{-2}$, in the $m_N$-range $10 \text{TeV} < m_N < 100 \text{TeV}$ $d_e$ roughly scales as $1/m_N$, and above $m_N = 100 \text{TeV}$ it becomes very slowly decreasing function in $m_N$.

In Fig 4.3 we show the predicted numerical values for $d_e$, as functions of the soft SUSY-breaking parameters $A_0$ and $B_0$, and their corresponding CP phases $\phi$ and $\theta$. In all pannels except the pannel (c), where $\phi = 0$ and $\theta$ is a variable, $\phi$ assumes value $\pi/2$ or it is a variable and $\theta$ is taken to be equal zero. In the pannel (a) of Fig 4.3 the soft trilinear parameter $A_0$ is
§ 4.2. Numerical results

Figure 4.3: Predicted numerical values for the electron EDM $d_e$ versus the soft SUSY breaking parameters $A_0$ and $B_0$ and their corresponding soft CP-odd phases $\phi$ and $\theta$ in the $\nu_R$MSSM, for the baseline scenario in (4.13). If not shown $\phi$ assumes value $\pi/2$. The range of input parameters shown in the plots is compatible with the LHC constraints on Higgs, gluino and squark masses. The heavy dots show the predicted values for $d_e$, using the default parameters (4.13).

The electron EDM $d_e$ is a complicated function of $|A_0|$ that slowly rises for $|A_0|$ between 1.8 TeV and 4.5 TeV, slowly decreases for $|A_0|$ between 4.5 TeV and 6 TeV, and steeply rises for $|A_0| > 6$ TeV. This function cannot be pre-
cisely described by a simple Laurent series in $|A_0|$, but in the largest part of the allowed $|A_0|$ interval it can roughly be approximated by a constant. The $\phi$ dependence of $d_e$ is almost sinusoidal with an amplitude few times smaller than the experimental upper bound (1.6). Moreover, $d_e$ is approximately constant function of $B_0$, up to $B_0 \approx 600$ GeV. For larger values, i.e. $B_0 \gtrsim 600$ GeV, $d_e$ steeply rises, suggesting the numerical instability in the diagonalization of the sneutrino mass matrix, which probably makes the results in this regime invalid. For $\phi = \pi/2$, the electron EDM $d_e$ attains values of order the experimental upper limit (1.6), but for $\phi = \theta = 0$, the predictions are numerically consistent with zero. The dependence of $d_e$ on $\theta$ is sinusoidal with an amplitude of order few $\times 10^{-30}$, while its average value strongly depends on the chosen value $\phi$. From Figs 4.2 and 4.3, the following dependence of $d_l$ on $m_l$, $m_0 = M_{1/2}$, $m_N$ and $\tan \beta$ may be deduced:

$$d_l \propto \tan \beta \cdot m_l \cdot \frac{f(m_0)}{m_N^x}, \quad m_N < 10 \text{ TeV}, \quad (4.17)$$

where $x$ assumes values between $2/3$ and $1$, and $f(m_0)$ is roughly proportional to the function $-1 - 2.4 \text{ TeV}/m_0 + 6.3 \text{TeV}^2/m_0^2$. The last factor in Eq (4.17) corresponds to the scaling factor $1/M_{\text{SUSY}}^2$ in the naive approximation (4.12), and in the approximate expressions for lepton EDM derived in Ref [160].

§ 4.3 Technical remarks

Let’s end this chapter with several technical remarks, including the detailed analytical expressions for all the quantities that appear in the form factors $C^{L,SB}_{ll\gamma}$ and $C^{R,SB}_{ll\gamma}$, given in (4.14) and (4.15), respectively. To start with, the variables $\lambda_X$ are defined as $\lambda_X = m_X^2/M_W^2$, for instance $\lambda_\tilde{e} = m_{\tilde{e}}^2/M_W^2$. The integrals $J_{bc}^a$ derived from loop integrations (see Appendix E) are UV finite. These are given by

$$J_{bc}^a = (-1)^{a-n_b-n_c} \int_0^\infty \frac{dx x^{1+a}}{(x+\lambda_b)^{n_b}(x+\lambda_c)^{n_c}}. \quad (4.18)$$
The couplings $\tilde{V}_{lma}^{\ell L}$ and $\tilde{V}_{lma}^{\ell R}$ read:

\[
\tilde{V}_{lma}^{\ell L} = -\sqrt{2}t_w Z^*_{m1}(R^e_R)_{al}^* - \frac{(m_e)_l}{\sqrt{2}c_\beta M_W} Z^*_{m3}(R^e_L)_{al}^* ,
\]

\[
\tilde{V}_{lma}^{\ell R} = \frac{1}{\sqrt{2}c_w}(c_w Z_{m2} + s_w Z_{m1})(R^e_L)_{al}^* - \frac{(m_e)_l}{\sqrt{2}c_\beta M_W} Z_{m3}(R^e_R)_{al}^* ,
\]

(4.19) (4.20)

where $t_w = \tan \theta_w$, $c_w = \cos \theta_w$, $s_w = \sin \theta_w$, $c_\beta = \cos \beta$. The unitary matrices $\mathcal{U}$ and $\mathcal{V}$, which diagonalize the chargino mass matrix, and the unitary matrix $Z$ diagonalizing the neutralino mass matrix are taken from Ref. [68].

Finally, the following lepton-slepton disalignment matrices may be defined:

\[
R^e_{\ell L} = U^{\ell L}_i U^e_{ik}^* ,
\]

\[
R^e_{\ell R} = U^{e R}_{i+3a} U^e_{ik}^* ,
\]

(4.21)

where $U^{e L}$, $U^{e R}$ and $U^{\ell}$ are unitary matrices diagonalizing the lepton and slepton mass matrices, with $a = 1, \ldots, 6$ and $i, k = 1, 2, 3$. 
In Chapter 3 Charged Lepton Flavor Violation was analysed in the MSSM extended by low-scale singlet heavy neutrinos, paying special attention to the individual loop contributions due to the heavy neutrinos $\tilde{N}_{1,2,3}$, sneutrinos $\tilde{N}_{1,2,...,12}$ and soft SUSY-breaking terms. In this analysis, we have, for the first time, included the complete set of box diagrams, in addition to the photon and the $Z$-boson mediated interactions. We have also derived the complete set of chiral amplitudes and their associate form factors related to the neutrinoless three-body CLFV decays of the muon and tau, such as $\mu \rightarrow eee$, $\tau \rightarrow \mu\mu\mu$, $\tau \rightarrow e\mu\mu$ and $\tau \rightarrow ee\mu$, and to the coherent $\mu \rightarrow e$ conversion in nuclei. Our analytical results are general and can be applied to most of the New Physics models with CLFV. In this context, we emphasize that this systematic analysis has revealed the existence of two new box formfactors, which have not been considered before in the existing literature of New Physics theories with CLFV.

This detailed study has shown that the soft SUSY-breaking effects in the $Z$-boson-mediated graphs dominate the CLFV observables, for appreciable regions of the $\nu_R$MSSM parameter space in mSUGRA. Nevertheless, there is a significant portion of parameter space for heavy neutrino masses $m_N \lesssim 1$ TeV, where box diagrams involving heavy neutrinos in the loop can be comparable to, or even larger than the corresponding $Z$-boson-exchange diagrams.
in $\mu \to eee$ and in $\mu \to e$ conversion in nuclei (cf. Fig 3.11). In the same
kinematic regime, due to accidental numerical cancellations, we have also
observed a suppression of the branching ratios for the photonic CLFV de-
cays $\mu \to e\gamma$, as well as for the decays $\tau \to e\gamma$ and $\tau \to \mu\gamma$. As was
already mentioned, such a suppression in low-scale seesaw models is a conse-
quence of a cancellation between particle and sparticle contributions due to
the approximate realization of the SUSY no-go theorem due to Ferrara and
Remiddi [132]. Instead, in high-scale seesaw models such cancellations can
only occur for a particular choice of the neutrino-Yukawa and Majorana-mass
textures [119, 141]. Hence, the results obtained within supersymmetric low-

scale seesaw type-I models, with $m_N \lesssim 10$ TeV, suport the original findings
in Ref [111], where the usual paradigm with the photon dipole-moment opera-
tors dominating the CLFV observables in high-scale seesaw models [114, 118]
gets radically modified, such that $\mu \to eee$ and $\mu \to e$ conversion may also
represent sensitive probes of CLFV.

We have found that, unlike heavy neutrinos, CLFV effects induced by sneu-
trinos remain subdominant for the entire region of the mSUGRA parameter
space. In addition, the perturbativity constraint on the neutrino Yukawa
couplings $h_\nu$, up to the GUT scale renders the quartic coupling contributions
of order $(h_\nu)^4$ small. This study has focused on providing numerical predic-
tions for relatively small and intermediate values of $\tan \beta$, i.e. $\tan \beta \lesssim 20$,
where neutral Higgs-mediated interactions constrained by the recent LHCb
observation of the decay $B_s \to \mu\mu$ are not expected to give sizeable contribu-
tions. A global analysis that includes large $\tan \beta$ effects on CLFV observables
and LHC constraints is one of the goals of the future researches.

Chapter 4 presented the systematical study of the one-loop contributions to
the muon anomalous MDM $a_\mu$ and the electron EDM $d_e$ in the $\nu_R$MSSM.
In particular, special attention was paid to the effect of the sneutrino soft
SUSY-breaking parameters, $B_\nu$ and $A_\nu$, and their universal CP phases, $\theta$
and $\phi$, on $a_\mu$ and $d_e$. As far as one can tell, lepton dipole moments have not
been analyzed in detail before, within SUSY models with low scale singlet
(s)neutrinos.
For the deviation of $a_\mu$ from the SM value due to the $\nu_R$MSSM ($\delta a_\mu$) it is found that the heavy singlet neutrino and sneutrino contributions to $\delta a_\mu$ are small, typically one to two orders of magnitude below the muon anomaly $\Delta a_\mu$. Instead, left-handed sneutrinos and sleptons give the largest effect on $\Delta a_\mu$, exactly as is the case in the MSSM. The dependence of $\delta a_\mu$ on the muon mass $m_\mu$, $\tan \beta$ and the soft SUSY-breaking mass scale $M_{SUSY}$ have been carefully analyzed and their scaling behaviour according to Eq (4.11) has been confirmed. Finally, the dependence of $\delta a_\mu$ on the universal soft trilinear parameter $A_0$, the neutrino Yukawa couplings $h_\nu$ and the heavy neutrino mass $m_N$ are negligible.

Furthermore, the electron EDM $d_e$ in the $\nu_R$MSSM is analysed. The heavy singlet neutrinos do not contribute to $d_e$, and soft SUSY-breaking and sneutrino terms contribute only if the phases $\phi$ and/or $\theta$ have a nonzero value. The contribution from the possible CP violating terms arising from the relatively complex products of the vertices exposed in (4.9) is numerically shown to be equal zero. On the other hand, the contribution due to a non-zero value of $\phi$ is the largest and may give rise to values for the electron EDM $d_e$ comparable to its present experimental upper limit. The effect of the CP-odd phase $\theta$ on $d_e$ is approximately one to two orders of magnitude smaller than that of $\phi$. The size of $d_e$ increases with $\tan \beta$ and mass of the lepton $m_l$, it is approximatively independent of $A_0$ and $B_0$, but it generically decreases, as functions of the soft SUSY-breaking parameters $m_0$, $M_{1/2}$.

Based on this numerical results, the approximate semi-analytical expressions are derived, which differ from those presented in the existing literature for SUSY models realizing a high-scale seesaw mechanism. Specifically, the flavor-blind CP-odd phases lead to a scaling of the lepton EDM $d_l \propto m_l \tan \beta / m_N^y$, where $2/3 < y < 1$. While it is true that $d_l$ generally decreases with $M_{SUSY}$, this dependence cannot be described with a simple scaling law. The dependences on SUSY breaking parameters $A_0$ and $B_0$ are weak in the largest part of the parameter space. The linear dependence on $\tan \beta$ and the dependence on heavy neutrino mass are new results arising from this study.
In comparison, the tan $\beta$ dependence in Ref. [160] is, depending on its magnitude, either cubic or constant. Given the current experimental limits on $d_e$, a significant portion of the $\nu_R$MSSM parameter space is identified with maximal CP phase $\phi = \pi/2$, where the electron EDM $d_e$ can have values comparable to the present and future experimental sensitivities. The effect of sneutrino-sector CP violation on the neutron and Mercury EDMs is expected to be suppressed, which is a distinctive feature for the class of the $\nu_R$MSSM scenarios studied in this thesis.

In his brief review regarding the future of Particle Physics [165], Nobel Prize winner Sheldon Lee Glashow emphasized six points which he personally finds most important for the future of High Energy Physics research. Among others, those include charged lepton flavor violation processes, anomalous magnetic dipole moment of the muon $a_\mu = g_\mu - 2$ and electric dipole moment of the electron $d_e$. The author of this thesis couldn’t agree more.
Appendices
Appendix A

Interaction vertices

In this appendix, the Lagrangians describing the interaction vertices required to calculate the transition amplitudes for the CLFV processes under study are listed. The corresponding interaction vertices for the SM and the MSSM are obtained by adopting the conventions of the public code FeynArts-3.3 [166], FVMSSM.mod [167, 168], adapted to the notation used by Petcov et al. [69]. The Lagrangians of interest to this study include:

1. Vertices from 2HDM sector of the MSSM involving SM particles only,

\[ \mathcal{L}_{\bar{t}uH^-} + \text{h.c.} = \frac{g_w}{\sqrt{2}M_W} V^{*}_{ij} d_j \left( t_\beta m_d^j P_L + t_\beta^{-1} m_u^i P_R \right) u_i H^- + \text{h.c.} \quad (A.1) \]

Here \( H^- \) is the negatively charged Higgs scalar, \( V \) is the Cabibbo–Kobayashi–Maskawa matrix, \( m_d^i \) and \( m_u^i \) are the quark masses and \( c_w = \cos \theta_w \).

2. Vertices of singlet neutrinos in the \( \nu_R \)SM sector of the MSSM,

\[ \mathcal{L}_{\bar{\nu}_a G^-} + \text{h.c.} = \frac{g_w}{\sqrt{2}M_W} B_{ia} \bar{\nu}_i \left( -m_{\nu_i} P_L + m_{\nu_a} P_R \right) n_a G^- + \text{h.c.} \, , \quad (A.2) \]
\[ \mathcal{L}_{\bar{\nu}_a W^-} + \text{h.c.} = -\frac{g_w}{\sqrt{2}} B_{ia} \bar{\nu}_i \gamma^\mu P_L n_a W^-_\mu + \text{h.c.} \, , \quad (A.3) \]
\[ \mathcal{L}_{\bar{\nu}_a Z} = -\frac{g_w}{2c_W} C_{ab} \bar{\nu}_a \gamma^\mu P_L n_b Z_\mu \, . \quad (A.4) \]

Here \( n_a \) and \( m_{\nu_a} \) denote the neutrino mass-eigenstates and their respective
masses and $B$ and $C$ are lepton flavor mixing matrices defined in Refs [104, 105,110]. The matrices $B$ and $C$ satisfy the following set of relations:

$$B_{ia}B^*_{ia} = \delta_{ii'}, \quad C_{ac}C_{bc} = C_{ab}, \quad B_{ib}C_{ba} = B_{ia}, \quad B^*_{ia}B_{ib} = C_{ab}, \quad m_a C_{ac}C_{bc} = 0, \quad m_a B_{ib}C^*_{ba} = 0, \quad m_a B_{ia}B^*_{ia} = 0. \quad (A.5)$$

3. Vertices from the 2HDM sector of the MSSM involving Majorana neutrinos,

$$\mathcal{L}_{\nu H^-} + \text{h.c.} = \frac{g_w}{\sqrt{2}M_W} B_{ia} \bar{e}_i \left( t_\beta m_{e_i} P_L + t^{-1}_\beta m_a P_R \right) m_a H^- + \text{h.c.} \quad (A.6)$$

4. MSSM vertices with sparticlest,

$$\mathcal{L}_{\tilde{\chi}_-\tilde{\mu}} + \text{h.c.} = g_w \tilde{\mu}_j \left( \tilde{V}^{-dL}_{jma} P_L + \tilde{V}^{-dR}_{jma} P_R \right) \tilde{\chi}_m \tilde{\mu}_a + \text{h.c.}, \quad (A.7)$$

$$\mathcal{L}_{\tilde{\chi}^+\tilde{\mu}} + \text{h.c.} = g_w \tilde{\mu}_j \left( \tilde{V}^{+uL}_{jma} P_L + \tilde{V}^{+uR}_{jma} P_R \right) \tilde{\chi}_m \tilde{\mu}_a + \text{h.c.}, \quad (A.8)$$

$$\mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} + \mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} = e \tilde{\chi}_m \gamma_\mu \tilde{\chi}_m A_\mu - e \tilde{\chi}_m \gamma_\mu \tilde{\chi}^+ A_\mu, \quad (A.9)$$

$$\mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} + \mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} = \frac{g_w}{c_W} \tilde{\chi}_m \gamma_\mu \left( \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_L + \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_R \right) \tilde{\chi}_k Z_\mu \quad (A.10)$$

$$\mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} + \mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} = \frac{g_w}{c_W} \tilde{\chi}_m \gamma_\mu \left( \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_L + \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_R \right) \tilde{\chi}_k Z_\mu, \quad (A.11)$$

$$\mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} + \mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} = \frac{g_w}{c_W} \tilde{\chi}_m \gamma_\mu \left( \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_L + \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_R \right) \tilde{\chi}_k Z_\mu, \quad (A.12)$$

$$\mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} + \mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} = \frac{g_w}{c_W} \tilde{\chi}_m \gamma_\mu \left( \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_L + \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_R \right) \tilde{\chi}_k Z_\mu, \quad (A.13)$$

$$\mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} + \mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} = \frac{g_w}{c_W} \tilde{\chi}_m \gamma_\mu \left( \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_L + \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_R \right) \tilde{\chi}_k Z_\mu, \quad (A.14)$$

$$\mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} + \mathcal{L}_{\tilde{\chi}^0\tilde{\mu}} = \frac{g_w}{c_W} \tilde{\chi}_m \gamma_\mu \left( \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_L + \tilde{V}_{\tilde{\chi}_m}^{\gamma} P_R \right) \tilde{\chi}_k Z_\mu, \quad (A.15)$$

where

$$\tilde{V}^{-dL}_{jma} = \frac{m_d}{\sqrt{2}c_\beta M_W} U^{*}_{m2} V^{*}_{ij}(R^L_R)_{ai}, \quad (A.16)$$

$$\tilde{V}^{-dR}_{jma} = -\nu_{1i} V^{*}_{ij}(R^L_R)_{ai} + \frac{m_{ai}}{\sqrt{2}s_\beta M_W} \nu_{2i} V^{*}_{ij}(R^L_R)_{ai}, \quad (A.16)$$
\[ V_{jma}^{+u_L} = \frac{m_{aj}}{\sqrt{2} s_{\beta} M_W} V_{m2}^{*} V_{ji} (R_{L}^{d})_{ai}^{*}, \]
\[ V_{jma}^{+u_R} = -U_{m1} V_{ji} (R_{L}^{d})_{ai}^{*} + \frac{m_{di}}{\sqrt{2} c_{\beta} M_W} U_{m2} V_{ji} (R_{R}^{d})_{ai}^{*}, \]  

(A.17)
\[ V_{mk}^{x_{-L}} = U_{m1} U_{k1}^{*} + \frac{1}{2} U_{m2} U_{k2}^{*} - \delta_{mk} s_{w}^{2}, \]
\[ V_{mk}^{x_{R}} = V_{m1} V_{k1} + \frac{1}{2} V_{m2} V_{k2} - \delta_{mk} s_{w}^{2}, \]  

(A.18)
\[ V_{mk}^{\tilde{X}_{0}} = -\frac{1}{4} (Z_{m3} Z_{k3}^{*} - Z_{m44} Z_{k4}^{*}), \]
\[ V_{mk}^{\tilde{X}_{0} R} = \frac{1}{4} (Z_{m3} Z_{k3}^{*} - Z_{m44} Z_{k4}^{*}), \]  

(A.19)
\[ \tilde{V}_{ab} = \frac{c_{2w}}{c_{w}} (R_{L}^{\tilde{e}})_{ai} (R_{L}^{\tilde{e}})_{bi}^{*} - \frac{s_{w}^{2}}{c_{w}} (R_{R}^{\tilde{e}})_{ai} (R_{R}^{\tilde{e}})_{bi}^{*}, \]
\[ \tilde{V}_{jk}^{\tilde{d}L} = -\sqrt{2} t_{w} Z_{m1}^{*} (R_{L}^{\tilde{d}})_{aj}^{*} - \frac{(m_{a})_{j}}{\sqrt{2} c_{\beta} M_W} Z_{m3}^{*} (R_{L}^{\tilde{d}})_{aj}^{*}; \]  

(A.22)
\[ \tilde{V}_{jk}^{\tilde{d}R} = \frac{1}{\sqrt{2} c_{w}} (c_{W} Z_{m2} + s_{w} Z_{m1}) (R_{R}^{\tilde{d}})_{aj}^{*} - \frac{(m_{a})_{j}}{\sqrt{2} c_{\beta} M_W} Z_{m3}^{*} (R_{R}^{\tilde{d}})_{aj}^{*}, \]  

(A.23)
\[ \tilde{V}_{jk}^{\tilde{u}L} = \frac{2 \sqrt{2}}{3} t_{w} Z_{m1}^{*} (R_{L}^{\tilde{u}})_{aj}^{*} - \frac{(m_{a})_{j}}{\sqrt{2} s_{\beta} M_W} Z_{m4}^{*} (R_{L}^{\tilde{u}})_{aj}^{*}; \]  

(A.24)
\[ \tilde{V}_{jk}^{\tilde{u}R} = -\frac{1}{2} c_{w} (c_{W} Z_{m2} + \frac{1}{3} s_{w} Z_{m1}) (R_{R}^{\tilde{u}})_{aj}^{*} - \frac{(m_{a})_{j}}{\sqrt{2} s_{\beta} M_W} Z_{m4}^{*} (R_{R}^{\tilde{u}})_{aj}^{*}, \]  

(A.25)
\[ \tilde{V}_{jk}^{\tilde{d}L} = -\sqrt{2} s_{w} Z_{m1}^{*} (R_{L}^{\tilde{d}})_{aj}^{*} - \frac{(m_{a})_{j}}{\sqrt{2} c_{\beta} M_W} Z_{m3}^{*} (R_{L}^{\tilde{d}})_{aj}^{*}; \]  

(A.26)
\[ \tilde{V}_{jk}^{\tilde{d}R} = \frac{1}{2} c_{w} (c_{W} Z_{m2} - \frac{1}{3} s_{w} Z_{m1}) (R_{R}^{\tilde{d}})_{aj}^{*} - \frac{(m_{a})_{j}}{\sqrt{2} c_{\beta} M_W} Z_{m3}^{*} (R_{R}^{\tilde{d}})_{aj}^{*}, \]  

(A.27)

and $c_{2w} = \cos 2\theta_{w}$. The unitary matrices diagonalizing the chargino mass matrix $\mathcal{U}$ and $\mathcal{V}$ and the unitary matrix diagonalizing the neutralino mass matrix $Z$ are taken from [68]. The matrices

\[ R_{ak}^{fL} \equiv U_{i a}^{fL} U_{i k}^{L*}, \quad R_{ak}^{fR} \equiv U_{i+3 a}^{fR} U_{i k}^{R*}, \]  

(A.28)

with $f = d, u, e$, $a = 1, 2, \ldots, 6$ and $i, k = 1, 2, 3$, quantify the disalignment between fermions and sfermions. Here $U_{i a}^{fL}$, $U_{i a}^{fR}$ and $U_{i a}^{f}$ are unitary matrices that diagonalize the fermion and sfermion mass matrices, respectively.
5. Sneutrino vertices in the $\nu_R$MSSM,

\[
\mathcal{L}_{\tilde{\chi}^- \tilde{N}} + \text{h.c.} = g_w \tilde{N}_A \tilde{\ell}_i \left( P_L \frac{m_l}{\sqrt{2} c_\beta M_W} \tilde{B}_{lmA}^L + P_R \tilde{B}_{lmA}^R \right) \tilde{\chi}^-_m + \text{h.c.}
\]

\[
\mathcal{L}_{\tilde{N} \tilde{Z}} = \frac{g_w}{c_W} \tilde{C}_{AB} \tilde{N}_A^i \tilde{\ell}_i \tilde{\ell}_j \tilde{\chi}^- \tilde{\ell}_j \tilde{\ell}_i \tilde{N}_B^i \tilde{Z}_\mu \tilde{Z}_\mu
\]

\[
\mathcal{L}_{\tilde{N} \tilde{Z}} = \frac{g_w}{c_W} \tilde{C}_{AB} \tilde{N}_A^i \tilde{\ell}_i \tilde{\ell}_j \tilde{\chi}^- \tilde{\ell}_j \tilde{\ell}_i \tilde{N}_B^i \tilde{Z}_\mu \tilde{Z}_\mu
\]

where

\[
\tilde{B}_{lmA}^L = U_{m2} U_{\ell R}^{t L} U_{\tilde{\nu} A},
\]

\[
\tilde{B}_{lmA}^R = U_{m2} U_{\ell R}^{t L} U_{\tilde{\nu} 6 A},
\]

\[
\tilde{B}_{lmA}^R = -U_{m2} U_{\ell R}^{t L} \frac{m_{n A}}{\sqrt{2} s_\beta M_W} \nu_{m2} U_{\ell R}^{t L} \nu_{6 A} B_{lmA},
\]

\[
\tilde{B}_{lmA}^R = -U_{m2} U_{\ell R}^{t L} \frac{m_{n A}}{\sqrt{2} s_\beta M_W} \nu_{m2} U_{\ell R}^{t L} \nu_{6 A} B_{lmA},
\]

\[
\tilde{C}_{AB}^1 = -\frac{1}{2} U_{A}^{t L} U_{B}^{t L},
\]

\[
\tilde{C}_{AB}^2 = -\frac{1}{2} U_{A}^{t L} U_{B}^{t L},
\]

\[
\tilde{C}_{AB}^3 = -\frac{1}{2} U_{A}^{t L} U_{B}^{t L},
\]

\[
\tilde{C}_{AB}^6 = -\frac{1}{2} U_{A}^{t L} U_{B}^{t L}.
\]

In the above, $U^\nu$ is the unitary matrix diagonalizing the sneutrino mass matrix.

Notice that the weak coupling constant $g_w$ are factored out from all interaction vertices defined above. To better identify chirality-flip mass effects in the CLFV amplitudes, factor $m_l/(\sqrt{2} c_\beta M_W)$ is also pulled out from the interaction vertex $\tilde{B}_{lmA}^L$. 

Appendix B

Loop functions

The CLFV amplitudes are expressed in terms of leading-order one-loop functions. We expand the loop functions with respect to the momenta and masses of the external charged leptons, while keeping only the leading non-zero terms. The leading terms may then be expressed, in terms of the dimensionless loop integrals

$$\bar{J}_{n_1n_2...n_k}^m (\lambda_1, \lambda_2, \ldots, \lambda_k) = \frac{(\mu^2)^{2-D/2}}{(M_W^2)^{D-2-D/2}} \int \frac{d^D \ell}{(2\pi)^D} \frac{(\ell^2)^m}{\prod_{i=1}^k (\ell^2 - m_i^2)^{n_i}}$$

$$= \frac{i(-1)^m \sum n_i}{(4\pi)^D \Gamma(D/2)} \left( \frac{\mu^2}{M_W^2} \right)^{2-D/2} \int_0^\infty \frac{dx x^{D/2-1+m}}{\prod_{i=1}^k (x + \lambda_i)^{n_i}}$$

where $m_i$ are loop particle masses, $n_i$ are the exponents of the propagator denominators, $\lambda_i = m_i^2/M_W^2$ are dimensionless mass parameters and $\mu$ is 't Hooft’s renormalization mass scale. Parameter $\mu$ is chosen to be $M_W$, even though any other scale can be chosen equally well as a reference scale for any of the integrals. For the amplitudes dealt with in this thesis, the integrals are either divergent and satisfy $m + 2 - \sum_i n_i = 0$, or they are convergent with $m + 2 - \sum_i n_i < 0$. For convergent integrals, one may set $D = 4$, whilst for divergent integrals one takes $D = 4 - 2\epsilon$. Factor $i/(4\pi)^2$ is
pulled out from all integrals. Thus, for finite integrals one obtains:

\[
\tilde{J}^m_{n_1n_2...n_k}(\lambda_1, \lambda_2, \ldots, \lambda_k) \equiv \frac{i}{(4\pi)^{\frac{D}{2} - 1 + a}} J^m_{n_1n_2...n_k}(\lambda_1, \lambda_2, \ldots, \lambda_k). \tag{B.2}
\]

Instead, the divergent integrals are written down as a sum of a divergent and constant term and a finite mass-dependent term:

\[
\tilde{J}^m_{n_1n_2...n_k}(\lambda_1, \lambda_2, \ldots, \lambda_k) \equiv \frac{i}{(4\pi)^2} \left[ \frac{1}{\varepsilon} + \text{const} + J^m_{n_1n_2...n_k}(\lambda_1, \lambda_2, \ldots, \lambda_k) \right]. \tag{B.3}
\]

In the CLFV amplitudes, the “divergent+constant” terms vanish in the total sum, or as a result of a GIM-like mechanism. Therefore, all CLFV amplitudes can be expressed in terms of finite mass dependent functions \(J^m_{n_1n_2...}(\lambda_1, \lambda_2, \ldots)\), which we call the \textit{basic integrals}. Those integrals are analytically calculated using Wolfram Mathematica package. We will now describe the procedure used in the calculation and present the exact results thus obtained.

There are three types of \(J\)-functions which are used in this study:

\[
J^{a}_{bc}(x,y) = K \cdot (-1)^{a+b+c} \cdot \int \frac{dt}{(t+x)^b(t+y)^c}, \tag{B.4}
\]

\[
J^{a}_{bcd}(x,y,z) = K \cdot (-1)^{a+b+c+d} \cdot \int \frac{dt}{(t+x)^b(t+y)^c(t+z)^d}, \tag{B.5}
\]

\[
J^{a}_{bcde}(x,y,z,w) = K \cdot (-1)^{a+b+c+d+e} \cdot \int \frac{dt}{(t+x)^b(t+y)^c(t+z)^d(t+w)^e}, \tag{B.6}
\]

where \(K = \frac{i 2^{-D} \pi^{-D/2} \mu^{4-D}}{\Gamma(D/2)} \). \tag{B.7}

The calculation of these integrals is performed in three steps. In the first step, the integral is exactly calculated using \textit{Integrate} function. In the second step, the constant term is isolated from the expression. Because of the specific identities obeyed by the flavor-mixing matrices (see Eqs (2.9) and (2.10) in Ref [110]) the constant terms can effectively be ignored. Finally,
the third step gets rid of these constant terms, and gives simplified result in
the zeroth order over $\epsilon$, with factor $i/16\pi^2$ dropped for better readability of
the result.

For calculation of the $J$-functions type \(\text{(B.4)}\), these steps are performed by
the following Mathematica functions:

**Step one.**

\[
\text{INT3}\[a_-, b_-, c_\] := (-1)^{(a + b + c)}
\]
\[
\text{Assuming}\{\{x > 0, y > 0, D <= 2 (b + c - a - 1)\},
\text{Integrate[}
\]
\[
t^{(D/2 - 1 + a)}/(((t + x)^b) ((t + y)^c) ), \{t, 0, \[Infinity]\}]\]

**Step two.**

\[
\text{CteExtract3}\[expr_\] := Module[{},
\]
\[
cstep1 = \text{Collect[Expand[expr /. Log[x] -> 0], x];}
\]
\[
cstep2 = cstep1 /. x^n___ -> 0 /. x -> 0;
\]
\[
cstep3 = cstep2 /. Log[y] -> 0 /. y^n___ -> 0
\]

**Step three.**

\[
\text{ProcInt3}\[a_-, b_-, c_\] := Module[{},
\]
\[
\text{step1 =}
\]
\[
\text{Series[INT3[a, b, c]*O Fa[D]*(16 \[Pi]^2)/I /.}
\]
\[
D -> 4 - 2 \[Epsilon], \{Epsilon, 0, 0\}] // \text{FullSimplify //}
\]
\[
\text{Normal;}
\]
\[
\text{cte = CteExtract3[step1];}
\]
\[
\text{step2 = step1 - cte // FullSimplify}
\]

In order to evaluate function $J^{\alpha}_{bc}(x, y)$, one only needs to call the function
defined in Step three, e.g. $J241[x_, y_] = \text{ProcInt3}[2, 4, 1]$. The anal-
logous procedure is applied for other types of $J$ loop-functions as well.

For calculation of the $J$-functions type \(\text{(B.5)}\):

**Step one.**

\[
\text{INT4}\[a_-, b_-, c_-, d_\] := (-1)^{(a + b + c + d)}
\]
\[
\text{Assuming}\{\{x > 0, y > 0, z > 0, D <= 2 (b + c + d - a - 1)\},
\text{Integrate[}
\]
\[
t^{(D/2 - 1 + a)}/((t + x)^b (t + y)^c (t + z)^d), \{t, 0, \[Infinity]\}]\]
Appendix B. Loop functions

Step two.
CteExtract4[expr_] := Module[{},
  cstep1 = Collect[Expand[expr /. Log[x] -> 0], x];
  cstep2 = cstep1 /. x^n__ -> 0 /. x -> 0;
  cstep3 = Collect[Expand[cstep2 /. Log[y] -> 0], y];
  cstep4 = cstep3 /. y^n__ -> 0 /. y -> 0;
  cstep5 = cstep4 /. Log[z] -> 0 /. z^n__ -> 0]

Step three.
ProcInt4[a_, b_, c_, d_] := Module[{},
  step1 = Series[INT4[a, b, c, d]*OFa[D]*(16 \[Pi]^2)/I /.
    D -> 4 - 2 \[Epsilon], \{\[Epsilon], 0, 0\}] // FullSimplify //
    Normal;
  cte = CteExtract4[step1];
  step2 = step1 - cte // FullSimplify
  ♦
  For calculation of the $J$-functions type (B.6):

Step one.
INT5[a_, b_, c_, d_, e_] := (-1)^(a + b + c + d + e)*
  Assuming[{x > 0, y > 0, z > 0, w > 0, D < 1},
    Integrate[t^((D/2 - 1 + a)/(t + x)^b (t + y)^c (t + z)^d (t + w)^e),
      \{t, 0, \[Infinity]\}]}

Step two.
CteExtract5[expr_] := Module[{},
  cstep1 = Collect[Expand[expr /. Log[x] -> 0], x];
  cstep2 = cstep1 /. x^n__ -> 0 /. x -> 0;
  cstep3 = Collect[Expand[cstep2 /. Log[y] -> 0], y];
  cstep4 = cstep3 /. y^n__ -> 0 /. y -> 0;
  cstep5 = Collect[Expand[cstep4 /. Log[z] -> 0], z];
  cstep6 = cstep5 /. z^n__ -> 0 /. z -> 0;
  cstep7 = cstep6 /. Log[w] -> 0 /. w^n__ -> 0]
Step three.

ProcInt5[a_, b_, c_, d_, e_] := Module[{},
  step1 =
  Series[INT5[a, b, c, d, e]*OFa[D]*(16 \[Pi]^2)/I /. D -> 4 - 2 \[Epsilon], \[Epsilon], 0, 0] // FullSimplify //
  Normal;
  cte = CteExtract5[step1];
  step2 = step1 - cte // FullSimplify]

In all these expressions, OFa[D] is defined as factor $K$ in the Eq (B.7).

Using this procedure, one comes out with the following results:

\[
\begin{align*}
J_{11}^0(x, y) &= \frac{y \log(y) - x \log(x)}{x - y}, \\
J_{21}^0(x, y) &= \frac{y \log(x) - x + y - y \log(y)}{(x - y)^2}, \\
J_{31}^0(x, y) &= \frac{x^2 - 2xy(\log(x) - \log(y)) - y^2}{2x(x - y)^3}, \\
J_{21}^1(x, y) &= -\frac{x(x - y) + x(x - 2y) \log(x) + y^2 \log(y)}{(x - y)^2}, \\
J_{31}^1(x, y) &= \frac{2y^2(\log(y) - \log(x)) - (x - 3y)(x - y)}{2(x - y)^3}, \\
J_{41}^1(x, y) &= \frac{(x - y)(x^2 - 5xy - 2y^2) + 6xy(\log(x) - \log(y))}{6x(x - y)^4}, \\
J_{41}^2(x, y) &= \frac{6y^3(\log(x) - \log(y)) - (x - y)(2x^2 - 7xy + 11y^2)}{6(x - y)^4},
\end{align*}
\]

\[
\begin{align*}
J_{111}^0(x, y, z) &= \frac{x \log(x)(z - y) + y(x - z) \log(y) + z(y - x) \log(z)}{(x - y)(x - z)(y - z)}, \\
J_{211}^0(x, y, z) &= \frac{1}{(x - y)^2(x - z)^2(y - z)} \left[ \log(x)(y - z)(x^2 - yz) \\
&\quad + z(x - y)^2 \log(z) + (x - z)(y(z - x) \log(y) - (x - y)(y - z)) \right], \\
J_{111}^1(x, y, z) &= \frac{x^2 \log(x)(y - z) + y^2(z - x) \log(y) + z^2(x - y) \log(z)}{(x - y)(x - z)(y - z)}.
\end{align*}
\]
Appendix B. Loop functions

\[ J_{111}^{1}(x, y, z) = \frac{1}{(x - y)^2(x - z)^2(y - z)} \]
\[ \times \left[ (x - z)(y^2 - x\log(y) + x(y - z)(z - y)) + z^2(x - y)^2\log(z) + x\log(x)(y - z)(x(y + z) - 2yz) \right], \]
\[ (B.9) \]

\[ J_{111}^{0}(x, y, z, w) = -\frac{w\log(w)}{(w - x)(w - y)(w - z)} + \frac{x\log(x)}{(w - x)(x - y)(x - z)} \]
\[ + \frac{y\log(y)}{(w - y)(y - x)(y - z)} + \frac{z\log(z)}{(w - z)(x - z)(y - z)}, \]
\[ J_{111}^{1}(x, y, z, w) = -\frac{w^2\log(w)}{(w - x)(w - y)(w - z)} + \frac{x^2\log(x)}{(w - x)(x - y)(x - z)} \]
\[ + \frac{y^2\log(y)}{(w - y)(y - x)(y - z)} + \frac{z^2\log(z)}{(w - z)(x - z)(y - z)}. \]
\[ (B.10) \]

These functions are often evaluated in the limit in which two or more variables are equal to each other or equal to zero. This is easily performed with the Limit function, although it can be rather time consuming. To avoid time consumption, we have evaluated all possible limits only once and then used those results to define the one-loop form factors expressed in Appendix C.
Appendix C

One–loop form factors

Here we present the complete analytical form of the CLFV form factors $F_{\gamma}$, $F_Z$ and $F_{\text{box}}$ defined in Chapter 3 in the Feynman–’t Hooft gauge. In the following, the usual summation convention over repeated indices is implied. The interaction vertices and loop functions used here are given in Appendices A and B respectively.

§ C.1 Photon Form factors

The form factors $F^L_{\gamma}$, $F^R_{\gamma}$, $G^L_{\gamma}$ and $G^R_{\gamma}$ may be explicitly written as follows:

$$
(F^L_{\gamma})_{l'l} = F^N_{l'l} + F^{L,\tilde{N}}_{l'l} + F^{L,\text{SB}}_{l'l},
$$

$$(F^R_{\gamma})_{l'l} = F^N_{l'l} + F^{R,\tilde{N}}_{l'l} + F^{R,\text{SB}}_{l'l},$$

$$(G^L_{\gamma})_{l'l} = m_l(G^N_{l'l} + G^{L,\tilde{N}}_{l'l}) + G^{L,\text{SB}}_{l'l},$$

$$(G^R_{\gamma})_{l'l} = m_l(G^N_{l'l} + G^{R,\tilde{N}}_{l'l}) + G^{R,\text{SB}}_{l'l},$$

(C.1)

where
Appendix C. One–loop form factors

\[ F_{\ell l}^{\gamma} = B_{\ell}^{\gamma} B_{\ell}^{\gamma} \left[ 2 \left( J_{31}^1(1, \lambda_{n_a}) - \frac{1}{6} J_{41}^2(1, \lambda_{n_a}) \right) \right. \\
- \frac{1}{6} \lambda_{n_a} J_{41}^2(1, \lambda_{n_a}) - \frac{1}{6\beta^2} \lambda_{n_a} J_{21}^2(\lambda_{H^+}, \lambda_{n_a}) \left. \right], \\
G_{\ell l}^{\gamma} = B_{\ell}^{\gamma} B_{\ell}^{\gamma} \left[ J_{31}^1(1, \lambda_{n_a}) + J_{41}^2(1, \lambda_{n_a}) + \lambda_{n_a} \left( \frac{1}{2} J_{41}^1(1, \lambda_{n_a}) - J_{31}^0(1, \lambda_{n_a}) \right) \right. \\
+ \lambda_{n_a} \lambda_{H^+} \left( \frac{1}{2\beta} J_{41}^1(\lambda_{H^+}, \lambda_{n_a}) + J_{31}^0(\lambda_{H^+}, \lambda_{n_a}) \right) \right], \\
F_{\ell l}^{L, \tilde{N}} = \frac{1}{2} \left( \tilde{B}_{\ell}^{R,1} \tilde{B}_{\ell}^{R,1} + \tilde{B}_{\ell}^{R,2} \tilde{B}_{\ell}^{R,2} \right) \left[ - \frac{2}{3} J_{41}^2(\lambda_{\tilde{h}_K}, \lambda_{N_A}) + \lambda_{\tilde{h}_K} J_{41}^1(\lambda_{\tilde{h}_K}, \lambda_{N_A}) \right], \\
F_{\ell l}^{R, \tilde{N}} = \frac{m_t m_\nu}{4c_\beta^2 M_W^2} \left( \tilde{B}_{\ell}^{L,1} \tilde{B}_{\ell}^{L,1*} + \tilde{B}_{\ell}^{L,2} \tilde{B}_{\ell}^{L,2*} \right) \\
\cdot \left[ - \frac{2}{3} J_{41}^2(\lambda_{\tilde{h}_K}, \lambda_{N_A}) + \lambda_{\tilde{h}_K} J_{41}^1(\lambda_{\tilde{h}_K}, \lambda_{N_A}) \right], \\
G_{\ell l}^{L, \tilde{N}} = \frac{1}{2} \left( \tilde{B}_{\ell}^{L,1} \tilde{B}_{\ell}^{L,1} + \tilde{B}_{\ell}^{L,2} \tilde{B}_{\ell}^{L,2} \right) \left[ - \frac{m_t^2}{2c_\beta^2 M_W^2} \lambda_{\tilde{h}_K} J_{41}^1(\lambda_{\tilde{h}_K}, \lambda_{N_A}) \right] \\
+ \frac{1}{2} \left( \tilde{B}_{\ell}^{R,1} \tilde{B}_{\ell}^{R,1} + \tilde{B}_{\ell}^{R,2} \tilde{B}_{\ell}^{R,2} \right) \left[ - \lambda_{\tilde{h}_K} J_{41}^1(\lambda_{\tilde{h}_K}, \lambda_{N_A}) \right] \\
+ \sqrt{\frac{2}{c_\beta}} \sqrt{\lambda_{\tilde{h}_K} J_{31}^1(\lambda_{\tilde{h}_K}, \lambda_{N_A})}, \\
G_{\ell l}^{R, \tilde{N}} = \frac{1}{2} \left( \tilde{B}_{\ell}^{L,1} \tilde{B}_{\ell}^{L,1} + \tilde{B}_{\ell}^{L,2} \tilde{B}_{\ell}^{L,2} \right) \left[ - \frac{m_t^2}{2c_\beta^2 M_W^2} \lambda_{\tilde{h}_K} J_{41}^1(\lambda_{\tilde{h}_K}, \lambda_{N_A}) \right] \\
+ \frac{1}{2} \left( \tilde{B}_{\ell}^{R,1} \tilde{B}_{\ell}^{R,1} + \tilde{B}_{\ell}^{R,2} \tilde{B}_{\ell}^{R,2} \right) \left[ - \lambda_{\tilde{h}_K} J_{41}^1(\lambda_{\tilde{h}_K}, \lambda_{N_A}) \right] \\
+ \sqrt{\frac{2}{c_\beta}} \sqrt{\lambda_{\tilde{h}_K} J_{31}^1(\lambda_{\tilde{h}_K}, \lambda_{N_A})}, \\
F_{\ell l}^{L, SB} = \tilde{\gamma}_{\ell}^{0lR} \tilde{\gamma}_{\ell}^{0lR} \left[ - \frac{1}{3} J_{41}^2(\lambda_{e_a}, \lambda_{\chi_m}) \right], \\
F_{\ell l}^{R, SB} = \tilde{\gamma}_{\ell}^{0lL} \tilde{\gamma}_{\ell}^{0lL} \left[ - \frac{1}{3} J_{41}^2(\lambda_{e_a}, \lambda_{\chi_m}) \right], \\
G_{\ell l}^{L, SB} = \tilde{\gamma}_{\ell}^{0lR} \tilde{\gamma}_{\ell}^{0lR} \left[ m_t \lambda_{e_a} J_{41}^1(\lambda_{e_a}, \lambda_{\chi_m}) \right] + \tilde{\gamma}_{\ell}^{0lL} \tilde{\gamma}_{\ell}^{0lL} \left[ m_t \lambda_{e_a} J_{41}^1(\lambda_{e_a}, \lambda_{\chi_m}) \right] \\
+ \tilde{\gamma}_{\ell}^{0lR} \tilde{\gamma}_{\ell}^{0lR} \left[ 2m_{\chi_m} \lambda_{e_a} J_{31}^0(\lambda_{e_a}, \lambda_{\chi_m}) \right],
\[ G^{r,sb}_{\nu \nu} = \tilde{\nu}_{\nu m a}^{\nu} \tilde{\nu}_{\nu m a}^{\nu s} \left[ m_{r} \lambda_{s} J_{11}^{1} (\lambda_{s}, \lambda_{0}^{m}) \right] + \tilde{\nu}_{\nu m a}^{\nu r} \tilde{\nu}_{\nu m a}^{\nu s} \left[ m_{r} \lambda_{s} J_{11}^{1} (\lambda_{s}, \lambda_{0}^{m}) \right] + \frac{2 m_{\chi}_{m}^{0}}{\sqrt{2}} \lambda_{s} J_{11}^{1} (\lambda_{s}, \lambda_{0}^{m}). \] (C.4)

\section{Z-Boson Form factors}

The form factors \( F_{Z}^{r} \) and \( F_{Z}^{R} \) may be decomposed as follows:

\[ (F_{Z}^{r})_{\nu \nu} = F_{\nu iz}^{N} + F_{\nu iz}^{L, N} + F_{\nu iz}^{LSB}, \]
\[ (F_{\gamma}^{R})_{\nu \nu} = F_{\nu iz}^{N} + F_{\nu iz}^{R, N} + F_{\nu iz}^{RSB}, \] (C.5)

where

\[ F_{\nu iz}^{L, N} = B_{\nu \nu} B_{\nu a}^{*} \left[ \frac{5}{2} \lambda_{n_{a}} J_{11}^{1} (1, \lambda_{n_{a}}) \right] \]
\[ + B_{\nu \nu} C_{b a} B_{\nu a}^{*} \left[ - \frac{1}{2} J_{11}^{1} (1, \lambda_{n_{a}}) + \frac{1}{2} \lambda_{n_{a}} \lambda_{n_{b}} J_{11}^{1} (1, \lambda_{n_{b}}, \lambda_{n_{a}}) \right. \]
\[ \left. + \frac{1}{2} \sqrt{2} \lambda_{n_{a}} \lambda_{n_{b}} J_{11}^{1} (\lambda_{n_{a}}, \lambda_{n_{b}}, \lambda_{n_{a}}) \right]. \]

\[ F_{\nu iz}^{R, N} = - \frac{m_{l_{1}} m_{l_{2}}}{4 m_{W}^{2}} B_{\nu \nu} B_{\nu a}^{*} J_{11}^{1} (\lambda_{n_{a}}, \lambda_{n_{p}}, \lambda_{n_{a}}), \] (C.6)

\[ F_{\nu iz}^{L, N} = \frac{1}{2} \left( B_{l_{1} m a}^{R} \tilde{B}_{l_{2} k a}^{R} + \tilde{B}_{l_{1} m a}^{R} \tilde{B}_{l_{2} k a}^{R} \right) J_{11}^{1} (\lambda_{n_{a}}, \lambda_{n_{b}}, \lambda_{n_{a}}) \]
\[ - \left( B_{l_{1} m a}^{R} \tilde{B}_{l_{2} k a}^{R} \tilde{B}_{l_{1} m a}^{R} \tilde{B}_{l_{2} k a}^{R} \right) J_{11}^{1} (\lambda_{n_{a}}, \lambda_{n_{b}}, \lambda_{n_{a}}) \]
\[ + \frac{1}{2} \left( B_{l_{1} m a}^{R} \tilde{B}_{l_{2} k a}^{R} \tilde{B}_{l_{1} m a}^{R} \tilde{B}_{l_{2} k a}^{R} \right) \]
\[ \cdot \left( \frac{1}{2} - s_{w}^{2} \right) J_{11}^{0} (\lambda_{n_{a}}, \lambda_{n_{b}}, \lambda_{n_{a}}) \]
\[ + \frac{1}{4} \left( B_{l_{1} m a}^{R} \tilde{B}_{l_{2} k a}^{R} \tilde{B}_{l_{1} m a}^{R} \tilde{B}_{l_{2} l a}^{R} \right) J_{11}^{1} (\lambda_{n_{a}}, \lambda_{n_{b}}, \lambda_{n_{a}}) \]
\[ + \frac{1}{2} \left( B_{l_{1} m a}^{R} \tilde{B}_{l_{2} k a}^{R} \tilde{B}_{l_{1} m a}^{R} \tilde{B}_{l_{2} l a}^{R} \right) \]
\[ \cdot \left( \frac{1}{2} - s_{w}^{2} \right) J_{11}^{0} (\lambda_{n_{a}}, \lambda_{n_{b}}, \lambda_{n_{a}}) \]
\[ + \frac{1}{2} \left( B_{l_{1} m a}^{R} \tilde{B}_{l_{2} k a}^{R} \tilde{B}_{l_{1} m a}^{R} \tilde{B}_{l_{2} l a}^{R} \right) \]
\[ \cdot \left( \frac{1}{2} - s_{w}^{2} \right) J_{11}^{0} (\lambda_{n_{a}}, \lambda_{n_{b}}, \lambda_{n_{a}}) \]
comes from a chiral amplitude of the form \( \bar{D} \), and
\[ + \frac{1}{2} (\tilde{B}_{VmA} \tilde{B}_{lka} + \tilde{B}_{VmA} \tilde{B}_{lka}^*) (-s_w^2) (J_{21}^i (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{N}_A}) - 2J_{11}^0 (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{N}_A})) \]
\[ + \frac{1}{4} (\tilde{B}_{V'kA} \tilde{C}_B \tilde{B}_{lka} + \tilde{B}_{V'kA} \tilde{C}_B^* \tilde{B}_{lka}^*) + \tilde{B}_{VmA} \tilde{C}_B \tilde{B}_{lka}^* \tilde{B}_{lka}^* \]
\[ + \tilde{B}_{VmA} \tilde{C}_B \tilde{B}_{lka}^* \tilde{B}_{lka}^* ) J_{111} (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{N}_k}, \lambda_{\tilde{N}_A}) \],
\[ (C.7) \]
\[ F_{VIZ}^{L,SB} = -\tilde{V}_{0\ell R} \tilde{V}_{\ell ka}^* J_{111}^1 (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}) \]
\[ + \tilde{V}_{0\ell R} \tilde{V}_{\ell ka}^* (-s_w^2) \left( -\frac{1}{2} J_{21}^1 (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}) + J_{11}^0 (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}) \right) \]
\[ - \frac{1}{2} \tilde{V}_{\ell ka}^* \tilde{V}_{ka} \tilde{V}_{lka}^* J_{111}^1 (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}) , \]
\[ F_{VIZ}^{R,SB} = -\tilde{V}_{0\ell L} \tilde{V}_{\ell ka}^* J_{111}^1 (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}) \]
\[ + \tilde{V}_{0\ell L} \tilde{V}_{\ell ka}^* (-s_w^2) \left( -\frac{1}{2} J_{21}^1 (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}) + J_{11}^0 (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}) \right) \]
\[ - \frac{1}{2} \tilde{V}_{\ell ka}^* \tilde{V}_{ka} \tilde{V}_{lka}^* J_{111}^1 (\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_k}) . \] (C.8)

\[ \textbf{§ c.3 Leptonic Box Form factors} \]

The leptonic box amplitudes are expressed in terms of the chiral structures: \( \bar{V} \Gamma^X \Gamma A \bar{1} \bar{1} \Gamma^Y \Gamma C \) [cf Eq (3.8)]. There are two distinct contributions to the chiral amplitudes. The first one has direct relevance to the original structure given above and that one is denoted with a subscript \( D \). The second contribution comes from a chiral amplitude of the form \( \bar{I}_1 \Gamma^X \Gamma A \bar{1} \bar{1} \Gamma^Y \Gamma C \), which contributes to the original amplitude \( \bar{V} \Gamma^X \Gamma A \bar{1} \bar{1} \Gamma^Y \Gamma C \), after performing a Fierz transformation. This Fierz-transformed contribution is indicated with a subscript \( F \). More explicitly, the leptonic box form factors are given by

\[ B_{\ell V}^{LL} = B_{\ell V,D}^{LL} + B_{\ell V,F}^{LL} , \quad B_{\ell V}^{RR} = B_{\ell V,D}^{RR} + B_{\ell V,F}^{RR} , \]
\[ B_{\ell V}^{LR} = B_{\ell V,D}^{LR} - \frac{1}{2} B_{\ell S,F}^{LR} , \quad B_{\ell V}^{RL} = B_{\ell V,D}^{RL} - \frac{1}{2} B_{\ell S,F}^{RL} , \]
\[ B_{\ell S}^{LL} = B_{\ell S,D}^{LL} + \frac{1}{2} B_{\ell S,F}^{LL} + \frac{3}{2} B_{\ell T,F}^{LL} , \quad B_{\ell S}^{RR} = B_{\ell S,D}^{RR} + \frac{1}{2} B_{\ell S,F}^{RR} + \frac{3}{2} B_{\ell T,F}^{RR} , \]
The form factor contributions from Eq (C.11) read:

\[ B_{\ell S}^{LL} = B_{\ell S,D}^{LL} - 2B_{\ell V,F}^{LL} \]
\[ B_{\ell S}^{RR} = B_{\ell S,D}^{RR} - 2B_{\ell V,F}^{RR} \]
\[ B_{\ell T,D}^{LL} = \frac{1}{2} B_{\ell T,F}^{LL} + \frac{1}{2} B_{\ell S,F}^{LL} \]
\[ B_{\ell T,D}^{RR} = \frac{1}{2} B_{\ell T,F}^{RR} + \frac{1}{2} B_{\ell S,F}^{RR} \].

(C.9)

The \textit{direct} and Fierz-transformed contributions to the form factors are related by the exchange of outgoing leptons

\[ B_{\ell A,F}^{XY} = B_{\ell A,D}^{XY} (l' \leftrightarrow l_1) . \]

(C.10)

The \textit{direct} contributions have \textit{direct} \( N \), \( SB \) and Fierz-transformed \( \tilde{N} \) contributions:

\[ B_{\ell V,D}^{LL} = B_{\ell V,F}^{LL} + B_{\ell V,D}^{LL} + B_{\ell V,D}^{LSB} \]
\[ B_{\ell V,D}^{RL} = B_{\ell V,D}^{RL} + \frac{1}{2} B_{\ell V,F}^{RL,\tilde{N}} + B_{\ell V,D}^{RLSB} \]
\[ B_{\ell S,D}^{LL} = B_{\ell S,D}^{LSB} \]
\[ B_{\ell S,D}^{RL} = \frac{1}{2} B_{\ell S,F}^{RL,\tilde{N}} + B_{\ell S,D}^{RLSB} \]
\[ B_{\ell T,D}^{LL} = B_{\ell T,D}^{LSB} \]
\[ B_{\ell T,D}^{RR} = \frac{1}{2} B_{\ell T,F}^{RR,\tilde{N}} + B_{\ell T,D}^{RRSB} \].

(C.11)

The form factor contributions from Eq (C.11) read:

\[ B_{\ell V,D}^{LL,N} = B_{\ell V,D}^{LL} B_{\ell V,D}^{*} B_{\ell V,D}^{\dagger} \left[ - \left( 1 + \frac{\lambda_{n} \lambda_{m}}{4} \right) J_{211}^{1}(1, \lambda_{n}, \lambda_{m}) + 2\lambda_{n} \lambda_{m} J_{211}^{0}(1, \lambda_{n}, \lambda_{m}) - 2\lambda_{n} \lambda_{m} t_{\beta}^{-2} J_{1111}^{0}(1, \lambda_{H+}, \lambda_{n}, \lambda_{m}) - \frac{1}{2} \lambda_{n} \lambda_{m} t_{\beta}^{-4} J_{211}^{1}(\lambda_{H+}, \lambda_{n}, \lambda_{m}) \right] \]

\[ B_{\ell V,D}^{RL,N} = -B_{\ell V,D}^{RL} B_{\ell V,D}^{*} B_{\ell V,D}^{\dagger} \frac{m_{l} m_{l} t_{\beta}^{2}}{M_{W}} \left( J_{1111}^{1}(1, \lambda_{H+}, \lambda_{n}, \lambda_{m}) + \lambda_{a} \lambda_{b} J_{1111}^{0}(\lambda_{H+}, \lambda_{n}, \lambda_{m}) \right) . \]

(C.12)

\[ B_{\ell V,F}^{LL,\tilde{N}} = (\tilde{B}_{1k}^{R1} \tilde{B}_{2m}^{R1*} + \tilde{B}_{1k}^{R2} \tilde{B}_{2m}^{R2*})(\tilde{B}_{1m}^{R1} \tilde{B}_{2k}^{R1*} + \tilde{B}_{1m}^{R2} \tilde{B}_{2k}^{R2*}) \]

\[ \cdot J_{1111}^{1}(\tilde{\lambda}_{e}, \tilde{\lambda}_{m}, \tilde{\lambda}_{A}, \tilde{\lambda}_{B}) , \]

\[ B_{\ell V,F}^{RL,\tilde{N}} \]
\[ B_{\ell V,F}^{RL,\tilde{N}} = (\mathcal{B}_{1 k B}^{L,1} \mathcal{B}_{2 m B}^{R,1*} + \mathcal{B}_{1 k B}^{L,2} \mathcal{B}_{2 m B}^{R,2*}) (\mathcal{B}_{1 m A}^{R,1} \mathcal{B}_{k A}^{L,1*} + \mathcal{B}_{1 m A}^{R,2} \mathcal{B}_{k A}^{L,2*}) \]
\[ \frac{m_p m_i}{c^2 M_W^2} J_{1111}'(\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_m}, \lambda_{\tilde{N}_A}, \lambda_{\tilde{N}_B}) , \]
\[ B_{\ell S,F}^{RR,\tilde{N}} = (\mathcal{B}_{1 k B}^{R,1} \mathcal{B}_{2 m B}^{L,1*} + \mathcal{B}_{1 k B}^{R,2} \mathcal{B}_{2 m B}^{L,2*}) (\mathcal{B}_{1 m A}^{R,1} \mathcal{B}_{k A}^{L,1*} + \mathcal{B}_{1 m A}^{R,2} \mathcal{B}_{k A}^{L,2*}) \]
\[ \frac{2m_p m_i}{c^2 M_W^2} \sqrt{\lambda_{\tilde{\chi}_k} \lambda_{\tilde{\chi}_m}} J_{1111}'(\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_m}, \lambda_{\tilde{N}_A}, \lambda_{\tilde{N}_B}) , \]
\[ B_{\ell S,F}^{RR,\tilde{N}} = (\mathcal{B}_{1 k B}^{R,1} \mathcal{B}_{2 m B}^{L,1*} + \mathcal{B}_{1 k B}^{R,2} \mathcal{B}_{2 m B}^{L,2*}) (\mathcal{B}_{1 m A}^{R,1} \mathcal{B}_{k A}^{L,1*} + \mathcal{B}_{1 m A}^{R,2} \mathcal{B}_{k A}^{L,2*}) \]
\[ \frac{2m_p m_i}{c^2 M_W^2} \sqrt{\lambda_{\tilde{\chi}_k} \lambda_{\tilde{\chi}_m}} J_{1111}'(\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_m}, \lambda_{\tilde{N}_A}, \lambda_{\tilde{N}_B}) , \] 
\( \text{C.13} \)
Semileptonic form factors have only direct and contributions, with the following \( N, \tilde{N} \) and SB content:

\[
B_{dV}^{LL} = B_{dV}^{LL,N} + B_{dV}^{LL,\tilde{N}} + B_{dV}^{LL,SB}, \quad B_{uV}^{LL} = B_{uV}^{LL,N} + B_{uV}^{LL,\tilde{N}} + B_{uV}^{LL,SB},
\]

and

\[
B_{dA}^{XY} = B_{dA}^{XY,SB}, \quad B_{uA}^{XY} = B_{uA}^{XY,SB},
\]

for \((X, Y, A) \neq (L, L, V)\). The \( N \) and \( \tilde{N} \) contributions are given by

\[
B_{dV}^{LL,N} = B_{dV}^{LL,N} + B_{dV}^{LL,\tilde{N}} + B_{dV}^{LL,SB}, \quad B_{uV}^{LL,N} = B_{uV}^{LL,N} + B_{uV}^{LL,\tilde{N}} + B_{uV}^{LL,SB},
\]

\[
B_{dA}^{XY} = B_{dA}^{XY,SB}, \quad B_{uA}^{XY} = B_{uA}^{XY,SB},
\]

for \((X, Y, A) \neq (L, L, V)\). The \( N \) and \( \tilde{N} \) contributions are given by

\[
B_{dV}^{LL,N} = B_{dV}^{LL,N} + B_{dV}^{LL,\tilde{N}} + B_{dV}^{LL,SB}, \quad B_{uV}^{LL,N} = B_{uV}^{LL,N} + B_{uV}^{LL,\tilde{N}} + B_{uV}^{LL,SB},
\]

and

\[
B_{dA}^{XY} = B_{dA}^{XY,SB}, \quad B_{uA}^{XY} = B_{uA}^{XY,SB},
\]

for \((X, Y, A) \neq (L, L, V)\). The \( N \) and \( \tilde{N} \) contributions are given by

\[
B_{dV}^{LL,N} = B_{dV}^{LL,N} + B_{dV}^{LL,\tilde{N}} + B_{dV}^{LL,SB}, \quad B_{uV}^{LL,N} = B_{uV}^{LL,N} + B_{uV}^{LL,\tilde{N}} + B_{uV}^{LL,SB},
\]

and

\[
B_{dA}^{XY} = B_{dA}^{XY,SB}, \quad B_{uA}^{XY} = B_{uA}^{XY,SB},
\]

for \((X, Y, A) \neq (L, L, V)\). The \( N \) and \( \tilde{N} \) contributions are given by

\[
B_{dV}^{LL,N} = B_{dV}^{LL,N} + B_{dV}^{LL,\tilde{N}} + B_{dV}^{LL,SB}, \quad B_{uV}^{LL,N} = B_{uV}^{LL,N} + B_{uV}^{LL,\tilde{N}} + B_{uV}^{LL,SB},
\]

and

\[
B_{dA}^{XY} = B_{dA}^{XY,SB}, \quad B_{uA}^{XY} = B_{uA}^{XY,SB},
\]
\[ B^{LL,\tilde{N}}_{uV} = -\hat{V}^{-uR}_{lka} \hat{V}^{-uR}_{lma} (\hat{V}^{-\ell R,1*}_{lkA} \hat{V}^{-\ell R,1}_{l'mA} + \hat{V}^{-\ell R,2*}_{lkA} \hat{V}^{-\ell R,2}_{l'mA}) \sqrt{\lambda_{\tilde{N}}, \lambda_{k}} \cdot J_{111}^{0}(\lambda_{\tilde{N}}, \lambda_{k}, \lambda_{\tilde{N}}, \lambda_{d}) . \] (C.18)

The SB form factors \( B^{XY,SB}_{dA} \) and \( B^{XY,SB}_{uA} \), with \( X = L, R \), \( Y = L, R \) and \( A = V, S, T \), are obtained from the direct leptonic form factors \( B^{XY,SB}_{\ell A} \), by making the replacements: \( \ell \to d, \ l_1 \to d, \ l_2 \to d, \ \tilde{e} \to \tilde{d} \) and \( \ell \to u, \ l_1 \to u, \ l_2 \to u, \ \tilde{e} \to \tilde{u} \), in both the interaction vertices and the arguments of the \( J \)-loop functions that carry the index \( b \) in Eq (C.14).
Appendix D

Form factor analysis

In order to calculate three-body decays given by Eqs (3.10) and (3.11), we have developed the *model independent* procedure for calculation of CLFV three-body decay rates. The analytical calculus is for the most part performed using *Wolfram Mathematica*, with the aid of *FeynCalc* package [169] which was found to be very useful in dealing with the Dirac algebra (traces of $\gamma$ matrices, kinematics, etc.).

The starting point of the calculation are the most general photon mediated, $Z$-boson mediated and box-diagram effective operators inducing $l \rightarrow l' l_1 l_2$ LFV transitions,

$$T_\gamma^{l \rightarrow l' l_1 l_2} = \frac{\alpha^2}{M_W^2} \left\{ \bar{l}' \gamma_\alpha P_{LL} \bar{l}_1 \gamma^\alpha P_{LL} \cdot P_1 + \bar{l}' \gamma_\alpha P_{RL} \bar{l}_1 \gamma^\alpha P_{RL} \cdot P_2 \\
+ \bar{l}' \gamma_\alpha P_{LL} \bar{l}_1 \gamma^\alpha P_{RL} \cdot P_3 + \bar{l}' \gamma_\alpha P_{RL} \bar{l}_1 \gamma^\alpha P_{LL} \cdot P_4 \\
+ \bar{l}' i \sigma_{\alpha \beta} q^\beta P_{LL} \bar{l}_1 \gamma^\alpha P_{LL} \cdot \frac{P_{11}}{q^2} + \bar{l}' i \sigma_{\alpha \beta} q^\beta P_{RL} \bar{l}_1 \gamma^\alpha P_{RL} \cdot \frac{P_{12}}{q^2} \\
+ \bar{l}' i \sigma_{\alpha \beta} q^\beta P_{LL} \bar{l}_1 \gamma^\alpha P_{RL} \cdot \frac{P_{13}}{q^2} + \bar{l}' i \sigma_{\alpha \beta} q^\beta P_{RL} \bar{l}_1 \gamma^\alpha P_{LL} \cdot \frac{P_{14}}{q^2} \right\},$$

(D.1)
Appendix D. Form factor analysis

The total effective operator is a sum of the photon-mediated, $Z$ to the terms with different ordering of the lepton fields.

\[
\mathcal{T}_{Z}^{l \to l' l_1 l_2} = \frac{\alpha^2_w}{M_W^2} \left\{ \bar{l}_1 \gamma^\alpha P_L l \cdot Z_1 + \bar{l}_1 \gamma^\alpha P_R l \cdot Z_2 
\right.
\left. + \bar{l}_1 \gamma^\alpha P_R l_2 \cdot Z_3 + \bar{l}_1 \gamma^\alpha P_R l_2 \cdot Z_4 \right\}, \quad (D.2)
\]

\[
\mathcal{T}_{\text{box}}^{l \to l' l_1 l_2} = \frac{\alpha^2_w}{M_W^2} \left\{ \bar{l}_1 \gamma^\alpha P_L l \cdot B_1 + \bar{l}_1 \gamma^\alpha P_R l \cdot B_2 
\right.
\left. + \bar{l}_1 \gamma^\alpha P_R l_2 \cdot B_3 + \bar{l}_1 \gamma^\alpha P_R l \cdot B_4 
\right.
\left. + \bar{l}_1 P_L l_2 \cdot B_5 + \bar{l}_1 P_R l_2 \cdot B_6 
\right.
\left. + \bar{l}_1 P_L l_2 \cdot B_7 + \bar{l}_1 P_R l_2 \cdot B_8 
\right.
\left. + \bar{l}_1 \sigma_{\alpha\beta} P_L l_2 \cdot B_9 + \bar{l}_1 \sigma_{\alpha\beta} P_R l_2 \cdot B_{10} \right\}, \quad (D.3)
\]

expressed in terms of the form factors $P_i$, $Z_i$ and $B_i$ multiplied by the corresponding four-lepton operators. The Higgs mediated contributions were not included since we assured their smallness assuming small $\tan \beta$ ($\tan \beta \lesssim 20$).

Note that the four-lepton operators are all written in the form $\bar{l} \cdot l \cdot \bar{l}_1 \cdot l_2$. This is achieved by applying the Fiertz transformations

\[
(\gamma^\mu P_L \times \gamma_\mu P_L)_{1234} = (\gamma^\mu P_L \times \gamma_\mu P_L)_{1432},
\]

\[
(\gamma^\mu P_R \times \gamma_\mu P_R)_{1234} = (\gamma^\mu P_R \times \gamma_\mu P_R)_{1432},
\]

\[
(\gamma^\mu P_L \times \gamma^\mu P_R)_{1234} = -2(P_R \times P_L)_{1432},
\]

\[
(\gamma^\mu P_R \times \gamma_\mu P_L)_{1234} = -2(P_L \times P_R)_{1432},
\]

\[
(P_R \times P_L)_{1234} = -\frac{1}{2}(\gamma_\mu P_L \times \gamma^\mu P_R)_{1432},
\]

\[
(P_L \times P_R)_{1234} = -\frac{1}{2}(\gamma_\mu P_R \times \gamma^\mu P_L)_{1432},
\]

\[
(P_R \times P_R)_{1234} = \left[ -\frac{1}{2}(P_R \times P_R) + \frac{1}{8} \sigma_{\mu\nu} P_R \times \sigma^{\mu\nu} P_R \right]_{1432},
\]

\[
(P_L \times P_L)_{1234} = \left[ -\frac{1}{2}(P_L \times P_L) + \frac{1}{8} \sigma_{\mu\nu} P_L \times \sigma^{\mu\nu} P_L \right]_{1432}. \quad (D.4)
\]

to the terms with different ordering of the lepton fields.

The total effective operator is a sum of the photon-mediated, $Z$-boson me-
diated and box contributions,

\[
\mathcal{T}_{TOT}^{l_i \rightarrow l'_1 l'_2} = \frac{\alpha_w^2}{M_W^2} \cdot \left\{ \bar{l} \gamma_\alpha P_{L,1} l \bar{l}_1 \gamma^\alpha P_{L,2} \cdot [P_1 + Z_1 + B_1] 
\begin{align*}
&+ \bar{l} \gamma_\alpha P_{R,1} l \bar{l}_1 \gamma^\alpha P_{R,2} \cdot [P_2 + B_2 + Z_2] \\
&+ \bar{l} \gamma_\alpha P_{R,1} l \bar{l}_1 \gamma^\alpha P_{R,2} \cdot [P_3 + Z_3 + B_3] \\
&+ \bar{l} \gamma_\alpha P_{R,1} l \bar{l}_1 \gamma^\alpha P_{R,2} \cdot [P_4 + Z_4 + B_4] \\
&+ \bar{l} P_{L,1} l \bar{l}_1 P_{L,2} \cdot B_5 + \bar{l} P_{R,1} l \bar{l}_1 P_{L,2} \cdot B_6 \\
&+ \bar{l} P_{L,1} l \bar{l}_1 P_{R,2} \cdot B_7 + \bar{l} P_{R,1} l \bar{l}_1 P_{L,2} \cdot B_8 \\
&+ \bar{l} i_\sigma_\alpha q^\beta P_{L,1} l \bar{l}_1 i_\sigma_\alpha q^\beta P_{L,2} \cdot \frac{P_{11}}{q^2} + \bar{l} i_\sigma_\alpha q^\beta P_{R,1} l \bar{l}_1 i_\sigma_\alpha q^\beta P_{R,2} \cdot \frac{P_{12}}{q^2} \\
&+ \bar{l} i_\sigma_\alpha q^\beta P_{L,1} l \bar{l}_1 i_\sigma_\alpha q^\beta P_{R,2} \cdot \frac{P_{13}}{q^2} + \bar{l} i_\sigma_\alpha q^\beta P_{R,1} l \bar{l}_1 i_\sigma_\alpha q^\beta P_{L,2} \cdot \frac{P_{14}}{q^2} \right\}.
\]

\text{(D.5)}

This is the most general form factor CLFV $l \rightarrow l'_1 l'_2$ structure valid in any model. The four-lepton operators with the Lorentz structure $P_L \times P_R$ and $P_R \times P_L$ are novelty, since they have not been considered in the previous publications [114][141].

The total amplitude can be written down in the more compact form,

\[
\mathcal{T}_{TOT}^{l_i \rightarrow l'_1 l'_2} = \frac{\alpha_w^2}{M_W^2} \cdot \sum_{i=1}^{14} \bar{l} \Gamma_{1i} l \bar{l}_1 \Gamma_{2i} l_2 \cdot \mathcal{F}_i,
\text{(D.6)}
\]

where $\Gamma_1$, $\Gamma_2$ and $\mathcal{F}_i$ are given by

\[
\Gamma_{1i} = \left\{ \gamma_\alpha P_L, \gamma_\alpha P_R, \gamma_\alpha P_L, \gamma_\alpha P_R, P_L, P_R, P_L, P_R, \sigma_\alpha P_L, \sigma_\alpha P_R, \right\},
\text{(D.7)}
\]

\[
i_\sigma_\alpha q^\beta P_L, i_\sigma_\alpha q^\beta P_R, i_\sigma_\alpha q^\beta P_L, i_\sigma_\alpha q^\beta P_R \right\},
\text{(D.8)}
\]

\[
\Gamma_{2i} = \left\{ \gamma_\alpha P_L, \gamma_\alpha P_R, \gamma_\alpha P_L, \gamma_\alpha P_R, P_L, P_R, P_L, P_R, \sigma_\alpha P_L, \sigma_\alpha P_R, \right\},
\text{(D.9)}
\]

\[
\gamma^\alpha P_L, \gamma^\alpha P_R, \gamma^\alpha P_L \right\},
\text{(D.10)}
\]
\[ \mathcal{F}_i = \left\{ F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, \frac{F_{11}}{q^2}, \frac{F_{12}}{q^2}, \frac{F_{13}}{q^2}, \frac{F_{14}}{q^2} \right\}. \]  

(D.11)

Evaluating the operator (D.6) between initial and final lepton states of the \( l \to l'l_1l_2 \) transition, one arrives at the amplitude written in the terms of spinors,

\[ \mathcal{F}_{TOT}^{e_1 \to e_2 e_3 e_4} = \frac{\alpha W}{M_W} \left\{ \sum_{i=1}^{14} \bar{u}_2 \Gamma_{1'i}^A u_1 \bar{u}_3 \Gamma_{2'i}^A v_4 \cdot \mathcal{F}_i^A \right. \]
\[ \left. - \sum_{i=1}^{14} \bar{u}_2 \Gamma_{1'i}^B u_1 \bar{u}_2 \Gamma_{2'i}^B v_4 \cdot \mathcal{F}_i^B \right\}. \]  

(D.12)

The contents of the arrays denoted by \( \mathcal{F}_i^A \) and \( \mathcal{F}_i^B \) depends on the leptons in the final state. There are three possible cases:

1) \( \nu' = \nu_1 = \nu_2 \) \( (\tau^- \to \mu^- \mu^- \mu^+; \tau^- \to e^-e^- \mu^+; \mu^- \to e^-e^- e^+) \):

\[ \mathcal{F}_i^A = \left\{ F_1^A, F_2^A, F_3^A, F_4^A, F_5^A, F_6^A, F_7^A, F_8^A, F_9^A, F_{10}^A, \frac{F_{11}^A}{s_{12}}, \frac{F_{12}^A}{s_{12}}, \frac{F_{13}^A}{s_{12}}, \frac{F_{14}^A}{s_{12}} \right\}, \]
\[ \mathcal{F}_i^B = \left\{ F_1^B, F_2^B, F_3^B, F_4^B, F_5^B, F_6^B, F_7^B, F_8^B, F_9^B, F_{10}^B, \frac{F_{11}^B}{s_{13}}, \frac{F_{12}^B}{s_{13}}, \frac{F_{13}^B}{s_{13}}, \frac{F_{14}^B}{s_{13}} \right\}. \]

2) \( \nu' \neq \nu_2, \nu_1 = \nu_2 \) \( (\tau^- \to e^- \mu^- \mu^+; \tau^- \to e^- \mu^- e^+) \):

\[ \mathcal{F}_i^A = \left\{ F_1^A, F_2^A, F_3^A, F_4^A, F_5^A, F_6^A, F_7^A, F_8^A, F_9^A, F_{10}^A, \frac{F_{11}^A}{s_{12}}, \frac{F_{12}^A}{s_{12}}, \frac{F_{13}^A}{s_{12}}, \frac{F_{14}^A}{s_{12}} \right\}, \]
\[ \mathcal{F}_i^B = \left\{ F_1^B, F_2^B, F_3^B, F_4^B, F_5^B, F_6^B, F_7^B, F_8^B, F_9^B, F_{10}^B, 0, 0, 0, 0 \right\}. \]

3) \( \nu' \neq \nu_2, \nu_1 \neq \nu_2 \) \( (\tau^- \to \mu^- e^- e^+; \tau^- \to e^- e^- \mu^+) \):

\[ \mathcal{F}_i^A = \left\{ F_1^A, F_2^A, F_3^A, F_4^A, F_5^A, F_6^A, F_7^A, F_8^A, F_9^A, F_{10}^A, 0, 0, 0, 0 \right\}, \]
\[ \mathcal{F}_i^B = \left\{ F_1^B, F_2^B, F_3^B, F_4^B, F_5^B, F_6^B, F_7^B, F_8^B, F_9^B, F_{10}^B, 0, 0, 0, 0 \right\}. \]

Here, \( s_{12} \) and \( s_{13} \) are Mandelstern variables defined in \( (D.15) \).
Absolute square of the amplitude (D.12) reads

\[ |T_{TOT}|^2 = \frac{\alpha_w^4}{M_W^4} \cdot \left\{ \sum_{i,j} \text{Tr} \left[ (\hat{p}_2 + m_2) \Gamma_{1_i}^A (\hat{p}_1 + m) \Gamma_{1_j}^A \right] \cdot \text{Tr} \left[ (\hat{p}_3 + m_3) \Gamma_{2_i}^B (\hat{p}_4 - m_4) \Gamma_{2_j}^B \right] \cdot \mathcal{F}_i^A \mathcal{F}_j^{A*} \right. \]
\[ + \sum_{i,j} \text{Tr} \left[ (\hat{p}_2 + m_2) \Gamma_{1_i}^B (\hat{p} + m) \Gamma_{1_j}^B \right] \cdot \text{Tr} \left[ (\hat{p}_2 + m_2) \Gamma_{2_i}^B (\hat{p}_4 - m_4) \Gamma_{2_j}^B \right] \cdot \mathcal{F}_i^B \mathcal{F}_j^{B*} \]
\[ - \sum_{i,j} \text{Tr} \left[ (\hat{p}_2 + m_2) \Gamma_{1_i}^A (\hat{p}_1 + m) \Gamma_{1_j}^A (\hat{p}_3 + m_3) \Gamma_{2_i}^A (\hat{p}_4 - m_4) \Gamma_{2_j}^A \right] \cdot \mathcal{F}_i^A \mathcal{F}_j^{A*} \]
\[ - \sum_{i,j} \text{Tr} \left[ (\hat{p}_3 + m_3) \Gamma_{1_i}^B (\hat{p}_1 + m) \Gamma_{1_j}^B (\hat{p}_2 + m_2) \Gamma_{2_i}^B (\hat{p}_4 - m_4) \Gamma_{2_j}^B \right] \cdot \mathcal{F}_i^B \mathcal{F}_j^{B*} \right\} \]  

(D.13)

After evaluating the traces in Eq (D.13), one imposes the kinematics of the three-body process:

\[ p_1 \cdot p_2 = \frac{1}{2} \left( m_2^2 + m_2^2 - s_{12} \right), \]
\[ p_1 \cdot p_3 = \frac{1}{2} \left( m_2^2 + m_3^2 - s_{13} \right), \]
\[ p_3 \cdot p_4 = \frac{1}{2} \left( s_{12} - m_3^2 - m_4^2 \right), \]
\[ p_2 \cdot p_4 = \frac{1}{2} \left( s_{13} - m_2^2 - m_4^2 \right), \]
\[ p_1 \cdot p_4 = \frac{1}{2} \left( m_2^2 + m_3^2 - s_{14} \right) = \frac{1}{2} \left( s_{12} + s_{13} - m_2^2 - m_3^2 \right), \]
\[ p_2 \cdot p_3 = \frac{1}{2} \left( s_{14} - m_2^2 - m_3^2 \right) = \frac{1}{2} \left( m_3^2 + m_4^2 - s_{12} - s_{13} \right), \]  

(D.14)

where \( s_{12}, s_{13} \) and \( s_{14} \) are well-known Mandelstam variables,

\[ s_{12} \equiv (p_1 - p_2)^2 = s_{34}, \]
\[ s_{13} \equiv (p_1 - p_3)^2 = s_{24}, \]
\[ s_{14} \equiv (p_1 - p_4)^2 = s_{23}, \]  

(D.15)

satisfying \( s_{12} + s_{13} + s_{14} = m^2 + m_2^2 + m_3^2 + m_4^2 \).
The last step in the evaluation of the branching ratios is an evaluation of the three-body phase-space integral \[37\]

\[
\Gamma(e_1 \rightarrow e_2 e_3 e_4^c) = \frac{1}{(2\pi)^3} \frac{1}{32m^3} \int_{4\pi^2} ds_{12} \int_{13}^{s_{13}} ds_{13} \left| T_{TOT}^{e_1 \rightarrow e_2 e_3 e_4^c} \right|^2, \quad (D.16)
\]

where

\[
s_{13}^\pm = \varepsilon^2 + \frac{m^2 - s_{12}}{2} \left[ 1 \pm \left( 1 - \frac{4\varepsilon^2}{s_{12}} \right)^{1/2} \right], \quad (D.17)
\]

and \( \varepsilon \) is equal to the external masses (\( \varepsilon = m_2 = m_3 = m_4 \) for \( l' = l_1 = l_2 \) and \( \varepsilon = m_3 = m_4 \) for \( l' \neq l_1, l_1 = l_2 \)). Since the decaying particle is much more massive than the resulting particles, we take \( \varepsilon \to 0 \) whenever possible.

In Eq (D.16), there are seven types of integrals which are divergent in the \( \varepsilon \to 0 \) limit. These integrals were evaluated partly by hand and partly using Mathematica, keeping the leading terms in \( \varepsilon \) expansion,

\[
\begin{align*}
I_1 &= \int_{4\pi^2}^{s_{12}} ds_{12} \int_{13}^{s_{13}} ds_{13} \simeq m^2 \left( \ln \frac{m^2}{\varepsilon^2} - 3 \right), \\
I_2 &= \int_{4\pi^2}^{s_{12}} ds_{12} \int_{13}^{s_{13}} s_{13} ds_{13} \simeq m^4 \left( \frac{1}{2} \ln \frac{m^2}{\varepsilon^2} - \frac{7}{4} \right), \\
I_3 &= \int_{4\pi^2}^{s_{12}} ds_{12} \int_{13}^{s_{13}} s_{13}^2 ds_{13} \simeq m^6 \left( \frac{1}{3} \ln \frac{m^2}{\varepsilon^2} - \frac{4}{3} \right), \\
I_4 &= \int_{4\pi^2}^{s_{12}} ds_{12} \int_{13}^{s_{13}} s_{13} ds_{13} \simeq -\frac{\pi^2}{6} + \frac{1}{2} \ln^2 \frac{m^2}{\varepsilon^2}, \\
I_5 &= \int_{4\pi^2}^{s_{12}} ds_{12} \int_{13}^{s_{13}} s_{13} ds_{13} \simeq \frac{m^6}{6\varepsilon^2} - \ln \frac{m^2}{\varepsilon^2} + 1, \\
I_6 &= \int_{4\pi^2}^{s_{12}} ds_{12} \int_{13}^{s_{13}} s_{13}^2 ds_{13} \simeq m^4 \left( \frac{m^2}{20\varepsilon^2} + \frac{17}{6} - \ln \frac{m^2}{\varepsilon^2} \right), \\
I_7 &= \int_{4\pi^2}^{s_{12}} ds_{12} \int_{13}^{s_{13}} s_{13} ds_{13} \simeq m^2 \left( \frac{m^2}{12\varepsilon^2} + \frac{13}{3} - \ln \frac{m^2}{\varepsilon^2} \right).
\end{align*}
\]
The leading order divergences in the integrals $I_4, I_5, I_6$ and $I_7$ cancel out in the final expression, leaving only $\varepsilon$-divergent terms which comprise $\ln \frac{m^2}{\varepsilon^2}$.

The final result again depends on the lepton content of the final state:

1) $l' = l_1 = l_2$

\[
\left| T_{TOT}^{e_1 \to e_2 e_3 l} \right|^2 = \frac{m_i^6}{12} \cdot \left\{ \begin{array}{l}
( |B_5| + |B_6|^2 ) m_i^2 + 2 ( |B_7|^2 + |B_8|^2 ) m_i^2 + 144 ( |B_9|^2 + |B_{10}|^2 ) m_i^2 \\
- 4 \left[ (B_3 + P_3 + Z_3)^* B_7 + (B_4 + P_4 + Z_4)^* B_8 + c.c \right] m_i^2 \\
+ 8 \left[ (B_3 + P_3 + Z_3) + |B_4 + P_4 + Z_4| \right] m_i^2 \\
- 12 (B_9 B_5 + B_{10} B_6 + c.c ) m_i^2 \\
+ 16 ( |B_1 + P_1 + Z_1| + |B_2 + P_2 + Z_2| ) m_i^2 \\
+ 8 \left( P_{12}^* B_7 + P_{11}^* B_8 + c.c \right) m_i \\
- 16 \left[ (B_4 + P_4 + Z_4)^* P_{11} + (B_3 + P_3 + Z_3)^* P_{12} + c.c \right] m_i \\
- 32 \left[ (B_1 + P_1 + Z_1)^* P_{12} + (B_2 + P_2 + Z_2)^* P_{11} + c.c \right] m_i \\
+ 64 ( |P_{11}|^2 + |P_{12}|^2 ) \left( \ln \frac{m^2}{\varepsilon^2} - \frac{11}{4} \right) \end{array} \right\}. \tag{D.18}
\]

2) $l' \neq l_2, l_1 = l_2$

\[
\left| T_{TOT}^{e_1 \to e_2 e_3 l} \right|^2 = \frac{m_i^6}{12} \cdot \left\{ \begin{array}{l}
( |B_5| + |B_6|^2 ) m_i^2 + 2 ( |B_7|^2 + |B_8|^2 ) m_i^2 + 4 ( |B_3|^2 + |B_4|^2 ) m_i^2 \\
+ 8 (|B_1|^2 + |B_2|^2 ) m_i^2 + 144 ( |B_9|^2 + |B_{10}|^2 ) m_i^2 \\
+ 8 (B_1 + P_1 + Z_1)^* B_1 m_i^2 + 8 (B_2 + P_2 + Z_2)^* B_2 m_i^2 - 4 B_9^* B_3 m_i^2 \\
+ 4 (B_3 + P_3 + Z_3)^* B_3 m_i^2 - 4 B_9^* B_4 m_i^2 + 4 (B_4 + P_4 + Z_4)^* B_4 m_i^2 \\
- 12 B_9^* B_5 m_i^2 - 12 B_{10}^* B_6 m_i^2 - 2 B_3^* B_7 m_i^2 \\
- 2 (B_3 + P_3 + Z_3)^* B_7 m_i^2 - 2 B_9^* B_8 m_i^2 + 2 (B_4 + P_4 + Z_4)^* B_8 m_i^2 \\
- 12 B_9^* B_9 m_i^2 - 12 B_{10}^* B_{10} m_i^2 + 4 B_3^* P_{11} m_i^2 \\
+ 4 (B_1 + P_1 + Z_1)^* P_{11} m_i^2 + 4 B_3^* P_{12} m_i^2 + 4 (B_2 + P_2 + Z_2)^* P_{12} m_i^2 \\
- 2 B_3^* P_{11} m_i^2 + 4 (B_3 + P_3 + Z_3)^* P_{12} m_i^2 - 2 B_3^* P_{12} m_i^2 \\
\end{array} \right\}. \tag{D.18}
\]
Appendix D. Form factor analysis

\[ + 4 \left( B_4 + P_4 + Z_4 \right) P_1 m_1^2 + 4 B_1^* Z_1 m_1^2 + 4 \left( B_1 + P_1 + Z_1 \right) Z_1 m_1^2 \\
+ 4 B_2^* Z_2 m_1^2 + 4 \left( B_2 + P_2 + Z_2 \right) Z_2 m_1^2 - 2 B_2^* Z_3 m_1^2 \\
+ 4 \left( B_3 + P_3 + Z_3 \right) Z_3 m_1^2 - 2 B_3^* Z_4 m_1^2 + 4 \left( B_4 + P_4 + Z_4 \right) Z_4 m_1^2 \\
- 16 P_{12}^* B_4 m_4 - 16 P_{11}^* B_2 m_2 - 8 P_{12}^* B_3 m_3 \\
- 8 P_{11}^* B_3 m_4 + 4 P_{12}^* B_7 m_7 + 4 P_{12}^* B_8 m_8 \\
- 8 P_{11}^* P_1 m_4 - 8 P_{11}^* P_2 m_2 - 8 P_{11}^* P_3 m_3 \\
- 8 P_{11}^* P_4 m_4 - 8 B_2^* P_1 m_1 + 4 B_3^* P_1 m_1 \\
- 8 \left( B_1 + P_1 + Z_1 \right) P_1 m_1 - 8 \left( B_1 + P_1 + Z_1 \right) P_2 m_2 - 8 \left( B_3 + P_3 + Z_3 \right) P_1 m_1 \\
- 8 P_{12}^* Z_1 m_4 - 8 P_{11}^* Z_2 m_2 - 8 P_{12}^* Z_3 m_3 - 8 P_{11}^* Z_4 m_4 \\
+ 32 \left( |P_{11}|^2 |P_{12}|^2 \right) \left( \ln \frac{m^2}{\varepsilon^2} - 3 \right). \tag{D.19} \]

3) \( l' \neq l_2, l_1 \neq l_2 \)

\[
\left| T_{TOT}^{e_1 \rightarrow e_2 e_3 e_4} \right|^2 = \frac{m_8}{12} \cdot \left\{ \right.
16 \left( |B_1|^2 + |B_2|^2 \right) + 8 \left( |B_3|^2 + |B_4|^2 \right) \left( |B_5|^2 + |B_6|^2 \right) \\
+ 2 \left( |B_7|^2 + |B_8|^2 \right) + 144 \left( |B_9|^2 + |B_{10}|^2 \right) \\
+ 4 \left[ B_7^* B_3 + B_8^* B_4 + 3 B_9^* B_5 + 3 B_{10}^* B_6 + \text{c.c.} \right] \right\}. \tag{D.20} \]

The results [D.18], [D.19] and [D.20] were tested in several different manners. One of the main tests was to reproduce the result from Ref. [110], which was performed with success.
Prošireni sažetak

Ova disertacija izlaže minimalni supersimetrični standardni model sa modelom nijihalice na niskoj skali. U okviru tog modela napravljena je detaljna studija narušenja leptonskog okusa u nabijenom leptonskom sektoru. Izveden je cjelovit skup kiralnih amplituda i pridruženih im form-faktora povezanih sa tročestičnim CLFV raspadima miona i tau-leptona bez neutrina, kao što su \( \mu \to eee \), \( \tau \to \mu\mu\mu \), \( \tau \to \mu ee \) i \( \tau \to e\mu e \) prijelazi na atomskim jezgrama. Dobiveni analitički rezultati su općeniti i mogu se primijeniti na većinu modela nove fizike koji uključuju narušenje nabijenog leptonskog broja.

Osim toga, u istom su modelu sustavno izučeni doprinosi na razini jedne petlje anomalnom magnetskom dipolnom momentu miona \( a_\mu \) i električnom dipolnom momentu elektrona \( d_e \).
Prošireni sažetak

Pregled tekućih i budućih eksperimentenata

Kada neutrino nastane u nekom slabo-interakcijskom procesu i propagira se putem neke konačne udaljenosti, postoji konačna vjerojatnost da će promijeniti okus. Ova opažena i dobro utvrđena činjenica poznata je pod nazivom *neutrinske oscilacije* [1–3], porađi oscilatorne ovisnosti vjerojatnosti promjene okusa u odnosu na energiju neutrina i udaljenost propagacije.

Nebrojeni eksperimenti s neutrinima izvještavaju o narušenju leptonskog okusa u neutrinском sektoru, bilo da je riječ o nestanku ili nastanku pojedinog okusa [4–25].

Ti su eksperimenti pružili nedvojbeni dokaz postojanja neutrinskih oscilacija uzrokovane konačnim neutrinskim masama i, posljedično, parametrima miješanja neutrina. Budući su neutrini masivni, prijelaz između neutrinskih polja napisanih u bazi okusa (ν_e, ν_μ, ν_τ) u neutinska polja napisanih u masenoj bazi (ν_1, ν_2, ν_3) postaje netrivijalan,

\[ \nu_l(x) = \sum_{i=1}^{3} U_{li} \nu_i(x) , \quad l = e, \mu, \tau . \]  

Unitarna matrica \( U \) poznata je kao Pontecorvo-Maki-Nakagawa-Sakata matrica [1–3] i obično se parametrizira na sljedeći način:

\[ U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P , \]  

gdje su \( P = \text{diag}(1, e^{i\alpha}, e^{i\beta}) \), \( c_{ij} \equiv \cos \theta_{ij} \) i \( s_{ij} \equiv \sin \theta_{ij} \). \( \theta_{12} \) označava solarni kut miješanja, \( \theta_{23} \) atmosferski kut miješanja, a \( \theta_{13} \) reaktorski kut miješanja. Faze \( \delta, \alpha \) i \( \beta \) označavaju Diracovu, odnosno dvije Majoranine faze koje narušavaju CP simetriju.

Nedavno izvješće reaktorskih neutrinskih eksperimentenata [17, 21, 23] o konačnoj vrijednosti parametra \( \theta_{13} \), snažno upućuje na netrivijalnu okusu
strukturu neutinskog sektora, kao i mogućnost postojanja CP narušenja.

Narušenje leptonskog okusa (LFV) u neutinskome sektoru može implicirati mogućnost postojanja LFV-a i nabijenom sektoru. Međutim, unatoč intensivnoj eksperimentalnoj potrazi [26–37] još uvijek nije pronađen dokaz narušenja leptonskog okusa u nabijenom neutinkom sektoru standardnog modela (SM). Tekući i budući eksperimenti usmjereni na detekciju narušenja leptonskog okusa u nabijenom sektoru (CLFV) uspjeli su odrediti neke gornje granice na pripadajuće opservabile. U donjoj tablici navedene su neke trenutne gornje granice, kao i one koje se očekuju u budućnosti, tokom iduće dvije dekade.

| Br. | Opservabla | Gornja granica | Očekivana buduća osjetljivost |
|-----|------------|----------------|-----------------------------|
| 1.  | $B(\mu \to e\gamma)$ | $2.4 \times 10^{-12}$ | $1 \to 2 \times 10^{-13}$ | $10^{-14}$ | $\mu \to e\gamma$ |
| 2.  | $B(\mu \to eee)$    | $10^{-12}$ | $10^{-16}$ | $10^{-17}$ | $\mu \to eee$ |
| 3.  | $R_{\mu e}^{\tau_{\tau}}$ | $4.3 \times 10^{-12}$ | $3 \to 7 \times 10^{-17}$ | $10^{-18}$ | $\tau \to \mu e\tau$ |
| 4.  | $R_{\mu e}^{\Lambda_{\mu \mu}}$ | $7 \times 10^{-13}$ | $3 \to 7 \times 10^{-17}$ | $10^{-18}$ | $\tau \to \mu e\mu$ |
| 5.  | $B(\tau \to e\gamma)$ | $3.3 \times 10^{-8}$ | $1 \to 2 \times 10^{-9}$ | $10^{-10}$ | $\tau \to e\gamma$ |
| 6.  | $B(\tau \to \mu\gamma)$ | $4.4 \times 10^{-8}$ | $2 \times 10^{-9}$ | $10^{-10}$ | $\tau \to \mu\gamma$ |
| 7.  | $B(\tau \to e\mu\mu)$ | $2.7 \times 10^{-8}$ | $2 \times 10^{-10}$ | $10^{-11}$ | $\tau \to e\mu\mu$ |
| 8.  | $B(\tau \to \mu\mu\mu)$ | $2.1 \times 10^{-8}$ | $2 \times 10^{-10}$ | $10^{-10}$ | $\tau \to \mu\mu\mu$ |
| 9.  | $B(\tau \to e\mu\mu)$ | $1.8 \times 10^{-8}$ | $2 \times 10^{-10}$ | $10^{-10}$ | $\tau \to e\mu\mu$ |

Budući da je CLFV zabranjen u okviru standardnog modela, opažanje takve pojave bio bi jasan signal postojanja nove fizike, što ovo područje istraživanja čini posebno zanimljivim.

Uz opservable koje uključuju CLFV, korisno je i osvrnuti se na leptonske dipolne momente, posebno anomalni magnetski dipolni moment (MDM) miona i električni dipolni moment (EDM) elektrona.

Trenutna eksperimentalna vrijednost MDM-a miona $a_{\mu}$, prema podacima navedenim u *Particle Data Group* [37] iznosi

$$a_{\mu}^{\text{exp}} = (116592089 \pm 63) \times 10^{-11}.$$  (3)
S druge strane, teorijska vrijednost koja proizlazi iz standardnog modela glasi

\[ a_{\mu}^{\text{SM}} = (116591820 \pm 49) \times 10^{-11}. \]  

Razlike između izmjerene i predviđene vrijednosti,

\[ \Delta a_{\mu} \equiv a_{\mu}^{\exp} - a_{\mu}^{\text{SM}} = (287 \pm 80) \times 10^{-11} \]  

nalazi se na nivou pouzdanosti od 3.6\(\sigma\) pa se stoga naziva mionska anomalija. Ova vrijednost ograničava dozvoljene doprinose nove fizike, pa se kao takva često upotrebljava kao ograničavajući faktor u građenju novih modela, a ponekad čak i kao argument za eliminaciju nekih od predloženih modela nove fizike.

U skoroj budućnosti očekujemo znatno preciznija mjerenja ove opservable. Tako Fermilab eksperiment E989 najavljuje povećanje preciznosti mjerenja za faktor 4 [61–65].

Slično tomu, EDM elektrona \(d_e\) može služiti kao iznimno precizan test postojanja narušenja CP simetrije induciranog novim CP fazama koje mogu biti prisutne u fizici izvan standardnog modela. Trenutna gornja granica na \(d_e\) iznosi

\[ d_e < 10.5 \times 10^{-28} \text{ e cm}. \]  

Neki budući eksperimenti mogli bi znatno povećati ovu osjetljivost, čak do reda veličine \(10^{-29} - 10^{-31} \text{ e cm}\) [52,59]. S druge strane, predviđanja standardnog modela za \(d_e\) kreću se između \(10^{-38} \text{ e cm}\) i \(10^{-33} \text{ e cm}\), ovisno o tome jesu li Diracove CP faze u matricama koje opisuju miješanje lakih neutrina različite od nule ili ne (za detalje vidi ref. [60]). Prema tome, svako opažanje EDM-a različitog od nule, tj. opažanje vrijednosti veće od \(10^{-33} \text{ e cm}\), značilo bi postojanje fizike izvan standardnog modela koja u sebi sadrži narušenje CP simetrije.

Iz svega navedenog, vidimo da su ove opservable od velikog interesa za istraživanje mogućih scenarija u okvirima nove fizike. Za više detalja, preporučamo konzultirati neke od izvrsnih preglednih radova navedenih u refer-
Teorijski okvir

Osnovna ideja iza svih supersimetričnih modela jest postojanje simetrije (prikladno nazvane supersimetrija) koja transformira fermion u bozon i obratno. Minimalni supersimetrični standardni model (MSSM) supersimetrizira standardni model (SM) uz minimalno proširenje standardnomodelskog čestičnog spektra: svakoj čestici iz SM-a pridružena je jedna superčestica ili superpartner. Superpartneri fermiona materije su čestice spina nula, nazvani sfermioni. Oni se dalje mogu klasificirati u skalarnje leptone ili sleptone i skalarnje kvarkove ili kvarkove. Fermioni materije i njihovi superpartneri opisani su kiralnim superpoljima. Superparneri baždarnih bozona SM-a čestice su spina 1/2 i zovemo ih gejdžini (engl. gauginos). Oni se dalje mogu klasificirati u jako interagirajući gluino i elektroslabo interagirajući zino i vino (engl. wino) (superpartneri Z odnosno W bozona). Zajedno sa baždarnim bozonima SM-a, oni su opisani vektorskim superpoljima. Superparneri Higsovih bozona čestice su spina 1/2 nazvani higgsini i skupa s njima opisani su kiralnim superpoljima. Lom elektroslabe simetrije miješa elektroslabe gejdžine sa higgsinima, što rezultira fizikalnim česticama koje nazivamo čardžini (engl. charginos) i neutralini. Donja tablica prikazuje sadržaj čestica i polja MSSM-a, zajedno sa pripadajućim kvantnim brojevima.

| Superpolje                  | Bosoni     | Fermioni     | SU$_c$(3) | SU$_L$(2) | U$_Y$(1) |
|-----------------------------|------------|--------------|----------|----------|----------|
| baždarno                    |            |              |          |          |          |
| $G^a$                       | gluon      | $g^a$        | gluino   | $g^b$    | 8        | 0        | 0        |
| $Y^b$                       | slabi      | $W^b$ ($W^\pm, Z$) | vino, zino | $\tilde{a}^b$ ($\tilde{a}^\pm, \tilde{z}$) | 1 | 3       | 0        |
| $V'$                        | hipernalojni | $B$ ($\gamma$) | bino     | $\tilde{b}$ ($\tilde{\gamma}$) | 1 | 1       | 0        |
| materije                   |            |              |          |          |          |
| $L_i$                       | sleptoni   | $\tilde{L}_i$ = $(\tilde{\nu}, \tilde{e})_L$ | leptoni | $L_i = (\nu, e)_L$ | 1 | 2       | -1       |
| $E_i$                       | $\tilde{e}_R$ | $\tilde{e}_R$ | leptoni | $E_i = e_R$ | 1 | 1       | 2        |
| $Q_i$                       | $\tilde{q}_R$ | $\tilde{q}_R$ | kvarkovi | $Q_i = (u, d)_L$ | 3 | 2       | 1/3      |
| $U_i$                       | skvarkovi | $\tilde{U}_i = \tilde{u}_R$ | kvarkovi | $U_i = u_R$ | 3* | 1       | -4/3     |
| $D_i$                       | $\tilde{d}_R$ | $\tilde{d}_R$ | kvarkovi | $D_i = d_R$ | 3* | 1       | 2/3      |
| Higgs                      | $H_1$      | $H_1$        | higgsini | $H_1$    | 1 | 2       | -1       |
| $H_2$                       | Higgsovi   | $H_2$        | higgsini | $\tilde{H}_2$ | 1 | 2       | 1        |
Kao što se vidi iz tablice, u MSSM-u postoje dva Higgsova superpolja, koja se mogu napisati kao

\[ H_1 = \begin{pmatrix} H^1_1 \\ H^2_1 \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^1_2 \\ H^2_2 \end{pmatrix}. \quad (7) \]

Polje \( H_1 \) ponekad se naziva donje Higgsovo superpolje (\( Y = -1 \)), a sastoji se od polja \( h_1 \) and \( \tilde{h}_{1L} \). Polje \( H_2 \) također se naziva gornje Higgsovo superpolje, a sastoji se od polja \( h_2 \) and \( \tilde{h}_{2L} \). Komponentna polja označena malim tiskanim slovima mogu se dalje napisati kao

\[ h_1 \equiv \begin{pmatrix} h^1_1 \\ h^2_1 \end{pmatrix} = \begin{pmatrix} h^0_1 \\ h^-_1 \end{pmatrix}; \quad h_2 \equiv \begin{pmatrix} h^1_2 \\ h^2_2 \end{pmatrix} = \begin{pmatrix} h^0_2 \\ h^+_2 \end{pmatrix}, \quad (8) \]

\[ \tilde{h}_{1L} \equiv \begin{pmatrix} \tilde{h}^1_1 \\ \tilde{h}^2_1 \end{pmatrix} = \begin{pmatrix} \tilde{h}^0_1 \\ \tilde{h}^-_1 \end{pmatrix}_L; \quad \tilde{h}_{2L} \equiv \begin{pmatrix} \tilde{h}^1_2 \\ \tilde{h}^2_2 \end{pmatrix} = \begin{pmatrix} \tilde{h}^0_2 \\ \tilde{h}^+_2 \end{pmatrix}_L. \quad (9) \]

Nakon spontanog loma elektroslabe simetrije, vakuumске očekivane vrijednosti dane su realnim i pozitivnim vrijednostima \( v_1 \) i \( v_2 \),

\[ \langle h_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}; \quad \langle h_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad (10) \]

koji dolaze od minimalizacije Higgsovog potencijala. Omjer ovih vrijednosti, \( \frac{v_2}{v_1} \equiv \tan \beta \), \( (11) \)

smatra se slobodnim parametrom teorije, barem što se tiče fermionskih masa.

Lagrangijan MSSM-a može se napisati kao zbroj dvaju dijelova: prvi koji dolazi od egzaktno supersimetrisuvane standardne teorije, i drugi koji eksplicitno lomi supersimetriju,

\[ \mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{SSB}}. \quad (12) \]
Prvi član možemo dalje pisati po komponentama,

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_g + \mathcal{L}_M + \mathcal{L}_H,$$

gdje su $\mathcal{L}_g$, $\mathcal{L}_M$ i $\mathcal{L}_H$ lagranžijani koji sadrže baždarna polja, polja materije te Higgsova polja. Detaljan prikaz ovih komponenti može se naći u literaturi [68, str. 171-172]. U kontekstu ove disertacije najviše će nas zanimati tzv. superpotencijal, koji čini važan dio lagranžijana $\mathcal{L}_H$ i glasi

$$W_{\text{MSSM}} = \mu H_1 \cdot H_2 + \bar{E}_i h^e_{ij} H_1 \cdot L_j + \bar{D}_i h^d_{ij} H_1 \cdot Q_j + \bar{U}_i h^u_{ij} H_2 \cdot Q_j.$$  \hspace{1cm} (14)

Matrice $h$ dane su sa

$$h^e_{ij} = \frac{g_2}{\sqrt{2} M_W \cos \beta} (m_e)_{ij},$$  \hspace{1cm} (15)

$$h^d_{ij} = \frac{g_2}{\sqrt{2} M_W \cos \beta} (m_d)_{ij},$$  \hspace{1cm} (16)

$$h^u_{ij} = \frac{g_2}{\sqrt{2} M_W \cos \beta} (m_u)_{ij}.$$  \hspace{1cm} (17)

Matrice $m_e$, $m_d$ i $m_u$ su dimenzije $3 \times 3$, a predstavljaju leptonsku masenu matricu, te masene matrice donjih odnosno gornjih kvarkova. Skalarni produkti definirani su u dvokomponentnoj notaciji [72, 74] kao $A \cdot B \equiv \epsilon_{\alpha \beta} A^\alpha B^\beta$ ($\epsilon_{12} \equiv +1$). Drugi, treći i četvrti član na desnoj strani izraza za superpotencijal samo su supersimetrično popočenje Yukawinih vezanja u lagranžijanu standardnog modela [75]. Prvi član međutim predstavlja novost te o njemu možemo razmišljati kao o supersimetričnom poopćenju masenih članova Higgsovog polja. Može se pokazati da konzistentno provođenje loma elektroslabe simetrije zahtijeva da parametar $\mu$ bude reda veličine mase $W$ bozona.

Ovdje je još potrebno reći da je u cijelom ovom razmatranju implicitno pretpostavljeno sačuvanje $R$-pariteta, definiranog kvantnim brojem $R_p$,

$$R_p = (-1)^{3(B-L)+2S},$$  \hspace{1cm} (18)
pri čemu su $B$, $L$ i $S$ barionski i leptonski broj, odnosno spin dane čestice. Sačuvanje ovog kvantnog broja u MSSM-u može se smatrati prirodnim pretpostavkom u minimalnom supersimetričnom proširenju standardnog modela, s obzirom na činjenicu da su barionski i leptonski broj sačuvani u lagranžijanu SM-a.

Pogledajmo sada sadržaj drugog dijela lagranžijana MSSM-a, onog koji eksplicitno lomi supersimetriju ($L_{SSB}$). Koristeći Symanzikovo pravilo \[76, str. 107-8\], može se pokazati da, ukoliko želimo zadržati poželjno konvergentno ponašanje supersimetrične teorije, članovi u $L_{SSB}$ moraju biti *mekani* [77–80], što znači da operatori polja u tom lagranžijanu moraju biti dimenzije strogo manje od četiri. Osim toga, očekujemo da ti članovi budu mali u odnosu na članove u $L_{SUSY}$.

Kada to uzmemo u obzir, možemo pisati [68, str. 185]

$$- L_{SOFT} = \tilde{q}_{lL}(M_{\tilde{q}}^2)_{ij}\tilde{q}_{jL} + \tilde{u}_{lR}(M_{\tilde{u}}^2)_{ij}\tilde{u}_{jR} + \tilde{d}_{lR}(M_{\tilde{d}}^2)_{ij}\tilde{d}_{jR}$$

$$+ \tilde{\bar{t}}_{lL}(M_{\tilde{t}}^2)_{ij}\tilde{\bar{t}}_{jL} + \tilde{\bar{e}}_{lR}(M_{\tilde{e}}^2)_{ij}\tilde{\bar{e}}_{jR}$$

$$+ \left[ h_1 \cdot \tilde{\bar{q}}_{lL}(A_{\tilde{q}})^T\tilde{\bar{e}}_{jR} + h_1 \cdot \tilde{\bar{q}}_{lL}(A_{\tilde{d}})^T\tilde{\bar{d}}_{jR} \right]$$

$$\tilde{\bar{q}}_{lL} \cdot h_2(A_{\tilde{e}})^T\tilde{\bar{q}}_{lL} + h.c.]$$

$$+ \left[ m_1^2|h_1|^2 + m_2^2|h_2|^2 + (B_{ij}h_1 \cdot h_2 + h.c.) \right]$$

$$+ \frac{1}{2}(M_1\tilde{\bar{\lambda}}_0P_L\tilde{\lambda}_0 + M_1^*\tilde{\bar{\lambda}}_0P_R\tilde{\lambda}_0)$$

$$+ \frac{1}{2}(M_2\tilde{\bar{\lambda}}_0P_L\tilde{\lambda}_0 + M_2^*\tilde{\bar{\lambda}}_0P_R\tilde{\lambda}_0)$$

$$+ \frac{1}{2}(M_3\tilde{\bar{\gamma}}^aP_L\tilde{\gamma}^a + M_3^*\tilde{\bar{\gamma}}^aP_R\tilde{\gamma}^a)$$

(19)

Praktični izračuni unutar MSSM-a obično uključuju nekoliko pojednostavljajućih pretpostavki, kako bi se smanjio veliki broj parametara koje smo morali dodati u teoriju. Takve pretpostavke rezultiraju različitim inačicama ograničenog minimalnog supersimetričnog standardnog modela ili CMSSM-a. U ovoj disertaciji usvojiti ćemo jednu takovu inačicu, tзв. minimalni supergravitacijski model, skraćeno mSUGRA. Budući da polja MSSM-a ne mogu sama spontano slomiti supersimetriju na skalama karakteriziranim masom
$W$ bozona, spontani lom supersimetrije mora se odviti u sektoru polja koji su singleti u odnosu na baždarnu grupu standardnog modela. Jedan od najekonomičnijih mehanizama ove vrste koristi gravitacijsku interakciju koja se temelji na lokalnoj supersimetriji poznatoj kao supergravitacija \[\text{[70,81]}\].

Velika korist od ovog modela sastoji se u tome da dodatnih 105 parametara uspjeva svesti na samo pet parametara,

\[
\{p\} = \{\text{sign}(\mu), m_0, M_{1/2}, A_0, \tan \beta\}.
\]  

Ovdje sign($\mu$) označava predznak parametra $\mu$ koji se nalazi u superpotencijalu, $m_0$ označava mase skalara ($m_{ij} = m_0 \delta_{ij}$), $M_{1/2}$ zajedničku masu svih MSSM gejdžina, $A_0$ zajedničku konstantu trilinearnog vezanja (higgs-sfermion-sfermion), a $\tan \beta$ omjer vakuumskih očekivanih vrijednosti kojeg smo definirali ranije. Ove parametre nazivamo parametrima loma supersimetrije. Njihove vrijednosti obično se postavljaju na skali velikog ujedinjenja (GUT), te se putem renormalizacijskih grupnih jednadžbi \[\text{[69]}\] prenose do skale karakterizirane masom $W$ bozona.

Recimo i to da postoji više teorijskih motivacija za rad unutar MSSM-a. MSSM nudi stabilno kvantnomehaničko rješenje problema hijerarhije u baždarnom sektoru i daje prilično preciznu predikciju ujedinjenja baždarnih vezanja SM-a na skali bliskoj GUT skali. Najljakša supersimetrična čestica je stabilna i, ako bi bila neutralna poput neutralina, može predstavljati dobrog kandidata za konstituenta tamne tvari u svemiru. Osim toga, MSSM tipično predviđa da je masa standardnomodelskog Higgsa manja od 135 GeV, što je u skladu s nedavnim opažanjima od strane kolaboracija ATLAS \[\text{[82]}\] i CMS \[\text{[83,84]}\].

U minimalnom supersimetričnom standardnom modelu sa sadržajem polja kakav je dan u gornjoj tablici ne dolazi do narušenja leptonskog okusa u nabijenom leptonskom sektoru (CLFV). To je posljedica odsustva desnih neutrina, što rezultira trivijalnom okusnom strukturuom. Jedan od načina da unutar MSSM-a omogućimo procese koji uključuju CLFV jest da navedeni čestični spektar proširimo s desnim neutrinima čija je masa reda veličine
Proširen sažetak

1 TeV. Takav model označit ćemo sa $\nu_R$MSSM.

Proširenje čestičnog spektra vrši se putem mehanizma njihalice (engl. seesaw mechanism) na niskoj skali. Za razliku od uobičajenog mehanizma njihalice u kojem teški neutrinski singleti poprimaju mase $m_N \sim 10^{12-14}$ GeV, mehanizam njihalice na niskoj skali omogućava da desni neutrini imaju znatno niže mase, već od 100 GeV. Dok je u običajenom mehanizmu njihalice vezanje između teških i lakih neutrina reda veličine $\xi_{\nu N} \sim \sqrt{m_\nu/m_N} \sim 10^{-12}$ (za $m_\nu \sim 10^{-1}$ GeV), u mehanizmu njihalice na niskoj skali $\xi_{\nu N}$ postaje slobodan parametar. Sve te pogodnosti omogućavaju nam da masu desnih neutrina postavimo na skalu eksperimentalne dohvatljivosti, kao i da povećamo učinak CLFV-a u ishodi nezanemarive jakosti vezanja lakih i teških neutrina.

Realizacija mehanizma njihalice na niskoj skali postiže se uvođenjem dodatnih leptonskih simetrija \[99-105\] u teoriju koje razultiraju time da laki neutrini postaju bezmaseni, dok teški neutrini mogu biti na skali $\sim 1$ TeV. Ukoliko bismo željeli reproducirati niskoenergetski maseni spektar lakih neutrina, te simetrije se mogu blago narušiti pa u tom slučaju govorimo o aproksimacijama leptonskim simetrijama.

Ovakav model može biti zanimljiv iz više razloga. Novo uvedeni singletni neutrini mogu biti kandidati za hladnu tamnu tvar \[120-124\]. Uz to, mehanizam rezonantne leptogeneze na niskoj skali \[125-129\] može ponuditi objašnjenje za opaženu barionsku asimetriju u svemiru, što je posebno aktualno u svjetlu činjenice da se parametarski prostor za elektroslabu bariogenezu svakoga dana sve više sužava novim podacima koji pristižu sa LHC-a \[130,131\].

Leptonski dio superpotencijala u $\nu_R$MSSM-u glasi

$$W_{\text{lepton}} = \hat{E}^C h_e \hat{H}_d \hat{L} + \hat{\bar{N}}^C h_\nu \hat{H}_u + \frac{1}{2} \hat{\bar{N}}^C m_M \hat{N}^C,$$  \hspace{1cm} (21)

gdje $\hat{H}_{d,u}, \hat{L}, \hat{E}$ i $\hat{\bar{N}}^C$ označavaju dva superpolja Higgsovih dubleta, tri nabijena leptonska superpolja lijeve i desne kiralnosti te tri neutrinska superpolja desne kiralnosti, redom. Yukawina vezanja $h_{e,\nu}$ i Majoranini maseni parametri $m_M$ čine kompleksnu $3 \times 3$ matricu. Za matricu $m_M$ uzeli smo da na skali $m_N$ bude simetrična s obzirom na okusnu grupu SO(3),...
tj. $m_M = m_N \mathbf{1}_3$.

U ovoj disertaciji posebno ćemo razmatrati dva scenarija neutrinskih Yukawinih vezanja. Prvi realizira $U(1)$ leptonsku simetriju \cite{125, 127} i dan je sa

$$
\begin{pmatrix}
0 & 0 & 0 \\
 ae^{-i\pi} & be^{-i\pi} & ce^{-i\pi} \\
 ae^{i\pi} & be^{i\pi} & ce^{i\pi}
\end{pmatrix}.
$$

U drugom scenariju, struktura neutrinske Yukawine matrice $h_\nu$ motivirana je diskretnom grupom simetrija $A_4$ i poprima sljedeći oblik \cite{137}:

$$
\begin{pmatrix}
 a & b & c \\
 ae^{-2\pi i/3} & be^{-2\pi i/3} & ce^{-2\pi i/3} \\
 ae^{2\pi i/3} & be^{2\pi i/3} & ce^{2\pi i/3}
\end{pmatrix}.
$$

U ovim izrazima pretpostavljamo da su Yukawini parametri $a$, $b$ i $c$ realni. Opservable koje uključuju CLFV ne ovise o niskoenergetskom masenom spektru lakih neutrina, pa ćemo iz praktičnih razloga uzeti da su mase lakih neutrina jednake nuli, tj. da su leptonske simetrije egzaktno realizirane.

Prema tome, u $\nu_R$MSSM-u postoje tri relevantna doprinosa narušenju leptonskog okusa u nabijenom sektoru. Jedan dolazi od teških neutrina ($N$), drugi od sneutrina ($\tilde{N}$), a treći od sektora koji mekano lomi supersimetriju (SB). Svaki od tih doprinosa bit će zasebno analiziran.

**CLFV opservable**

Na razini jedne petljе, efektivna $\gamma l l$ i $Z l l$ vezanja generirana su Feynmanovim dijagramima prikazanim na slici 3.1 \cite{3.1} (str. 27). Opći oblici amplituda prijelaza povezanih s ovim efektivnim vezanjima dani su sa

$$
\mathcal{T}_{\mu}\gamma l l = \frac{e \alpha_w}{8\pi M_W^2} \bar{P} \left[(F_L^L)_{l l} (q^2 \gamma_\mu - \mathbf{\not} q_\mu) P_L + (F_R^L)_{l l} (q^2 \gamma_\mu - \mathbf{\not} q_\mu) P_R \right]
$$
Prošireni sažetak

Iz amplitude možemo izračunati i pripadajuće omjerе grananja,

\[
\mathcal{T}^{l'l} = \frac{g_w \alpha_w}{8\pi \cos \theta_w} \bar{l} \left[ (F^L_{Z})_{l'l} \gamma_\mu P_L + (F^R_{Z})_{l'l} \gamma_\mu P_R \right] l, \tag{23}
\]

gdje je \(P_{L(R)} = \frac{1}{2} \left[ 1 - (+) \gamma_5 \right] \), \(\alpha_w = g_w^2/(4\pi)\), e je elektromagnetska konstanta vezanja, \(M_W = g_w \sqrt{v^2_\text{u} + v^2_\text{d}/2}\) masa W bozona, \(\theta_w\) slabi kut vezanja, a \(q = p_l - p_{l'}\) impuls fotona. Form-faktori \((F^L_{Z})_{l'l}, (F^R_{Z})_{l'l}\) dobijaju doprinose od teških neutrina \(N_{1,2,3}\), teških sneutrina \(\tilde{N}_{1,2,3}\) i članova koji mekano lome supersimetriju induciranih putem renormalizacijskih grupnih jednadžbi (RGE). Analitički izrazi za svaki od ovih doprinosa mogu se naći u Dodatku C.

Iz amplitude možemo izračunati i pripadajuće omjerе grananja,

\[
B(l \rightarrow l') = \frac{\alpha^3 s_w^2}{256\pi^2} \frac{m^3}{M_W^2 \Gamma_l} \left( |(G^L_{\gamma})_{l'l}|^2 + |(G^R_{\gamma})_{l'l}|^2 \right), \tag{24}
\]

\[
B(Z \rightarrow l'l'^C + l'^C l) = \frac{\alpha^3 M_W}{768\pi^2 c^2_W \Gamma_Z} \left( |(F^L_{Z})_{l'l}|^2 + |(F^R_{Z})_{l'l}|^2 \right). \tag{25}
\]

Svi ovi izrazi napisani su u aproksimaciji razvoja do vodećeg reda u vanjskim masama i impulsima, kao i pretpostavci da je masa Z bozona \(M_Z\) znatno ispod supersimetrične skale \(M_{\text{SUSY}}\) i mase teškog neutrina \(m_N\).

U idućem koraku prelazimo na tročestične CLFV raspade \(l \rightarrow l' l_1 l_2^C\), pri čemu \(l\) može biti mion ili tau-lepton, a \(l', l_1, l_2\) označavaju drugi nabijeni lepton u kojeg se lepton \(l\) može raspasti s obzirom na kinematiku.

Prijelazna amplitude za \(l \rightarrow l' l_1 l_2^C\) sadrži doprinose od fotonskih i Z-bozonskih dijagrama prikazanih na slici 3.1 (str. 27), ali i od pravokutnih dijagrama prikazanih na slici 3.2 (str. 29). Amplitude za ova tri doprinosa glase:

\[
\mathcal{T}^{\mu l_1 l_2}_3 = \frac{\alpha^2 s_w^2}{2M_W^2} \left[ \delta_{l_1 l_2} \bar{l}' \left[ (G^L_{\gamma})_{l'l} \gamma_\mu P_L + (G^R_{\gamma})_{l'l} \gamma_\mu P_R \right] + \frac{(p - p')^2}{(p - p')^2} \right] \cdot \left( (G^L_{\gamma})_{l'l} \gamma_\mu P_L + (G^R_{\gamma})_{l'l} \gamma_\mu P_R \right) [l' \leftrightarrow l_1], \tag{26}
\]

\[
\mathcal{T}^{\mu l_1 l_2}_2 = \frac{\alpha^2 s_w^2}{2M_W^2} \delta_{l_1 l_2} \bar{l}' \left[ (F^L_{Z})_{l'l} \gamma_\mu P_L + (F^R_{Z})_{l'l} \gamma_\mu P_R \right] l.
\]
\[ T_{\text{box}}^{ll'l_1l_2} = - \frac{\alpha_w^2}{4M_W^2} \left( B_{\ell_1}^{LL} \tilde{\gamma}_\mu P_L l_1 \gamma^\mu P_L l_2 \tilde{C} + B_{\ell_1}^{RR} \tilde{\gamma}_\mu P_R l_1 \gamma^\mu P_R l_2 \tilde{C} \right) \]

\[ + B_{\ell_1}^{LR} \tilde{\gamma}_\mu P_L l_1 \gamma^\mu P_R l_2 \tilde{C} + B_{\ell_1}^{RL} \tilde{\gamma}_\mu P_R l_1 \gamma^\mu P_L l_2 \tilde{C} \]

\[ + B_{\ell_1}^{IS} \tilde{\gamma}_\mu P_L l_1 \gamma^\mu P_R l_2 \tilde{C} + B_{\ell_1}^{IS} \tilde{\gamma}_\mu P_R l_1 \gamma^\mu P_L l_2 \tilde{C} \]

\[ + B_{\ell_1}^{IT} \tilde{\sigma}_{\mu\nu} P_L l_1 \gamma^\mu P_R l_2 \tilde{C} + B_{\ell_1}^{IT} \tilde{\gamma}_\mu P_R l_1 \gamma^\mu P_L l_2 \tilde{C} \right) . \]

U gornjim izrazima, \( g_L^l = -1/2 + s_w^2 \) i \( g_R^l = s_w^2 \) su Z-bozonska leptonska vezanja, a \( s_w = \sin \theta_w \). Kompozitni form-faktori pravokutnih dijagrama \( B_{\ell_1}^{XY} \) dani su u Dodatku [C] Oznake \( V, S \) i \( T \) označavaju form-faktore vektorskih, skalarnih i tenzorskih kombinacija struja, dok \( L \) i \( R \) razlikuju lijeve i desne kiralnosti tih struja. Form-faktori iz pravokutnih dijagrama sadrže izravne i Fiertz-transformirane doprinose, što se može vidjeti u Dodatku [D] 

S obzirom na tri leptonske generacije, prijelazna amplituda za raspad \( l \to l'l_1l_2 \) može upasti u jednu od tri klase ili kategorije [110]: (i) \( l' \neq l_1 = l_2 \), (ii) \( l' = l_1 = l_2 \), (iii) \( l' \neq l_1 \neq l_2 \). Za prve dvije klase, ukupni leptonski broj je sačuvan, dok je u trećoj klasi ukupni leptonski broj na razini struja narušen za dvije jedinice. Budući da se za predikcije za opservable iz klase (iii) ispostavlja da su neopazivo malene u \( \nu_R \) MSSM-u, ove procese ćemo ignorirati. Omjeri grananja za klase (i) i (ii) glase:

\[ B(l \to l'l_1l_2^c) = \frac{m_l^5 \alpha_w^4}{24576 \pi^3 M_W^2 \Gamma_1} \left\{ \left[ 2s_w^2(F_L^L + F_Z^L) - F_Z^L - B_{\ell_1}^{LL} \right]^2 \right. \]

\[ + \left[ 2s_w^2(F_R^L + F_Z^R) - B_{\ell_1}^{RR} \right]^2 + \left[ 2s_w^2(F_L^R + F_Z^L) - B_{\ell_1}^{RL} \right]^2 \]

\[ + \left[ 2s_w^2(F_R^R + F_Z^R) - B_{\ell_1}^{RL} \right]^2 \]

\[ + \frac{1}{4} \left( |B_{\ell_1}^{IS}|^2 + |B_{\ell_1}^{RR}|^2 + |B_{\ell_1}^{LR}|^2 + |B_{\ell_1}^{RL}|^2 \right) \]

\[ + 12 \left( |B_{\ell_1}^{IS}|^2 + |B_{\ell_1}^{RR}|^2 \right) \]

\[ + \frac{32s_w^4}{m_l} \left[ \text{Re} \left( (F_L^L + F_Z^R) G_{\gamma L}^L \right) + \text{Re} \left( (F_L^R + F_Z^L) G_{\gamma R}^L \right) \right] \]
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\begin{align}
- \frac{8s_w^2}{m_l} \left[ \text{Re}\left( (F_Z^R + B_{\ell V}^{RR} + B_{\ell V}^{RL}) G_{\gamma}^L \right) \right] \\
+ \text{Re}\left( (F_Z^L + B_{\ell V}^{LL} + B_{\ell V}^{LR}) G_{\gamma}^R \right) \\
- 32s_w^4 \left( |G_{\gamma}^L|^2 + |G_{\gamma}^R|^2 \right) \left( \ln \frac{m_{l1}^2}{m_{l2}^2} - 3 \right) \right), \tag{31}
\end{align}

\begin{align}
B(l \to l'l') &= \frac{m_l^5\alpha_w^4}{24576\pi^3 M_W^4 \Gamma_l} \left\{ 2 \left[ 2s_w^2 (F_{\gamma}^L + F_{Z}^L) - F_{Z}^L - \frac{1}{2} B_{\ell V}^{LL} \right]^2 \\
+ \left| 2s_w^2 (F_{\gamma}^R + F_{Z}^R) - \frac{1}{2} B_{\ell V}^{RR} \right|^2 \right\} + \left| 2s_w^2 (F_{\gamma}^L + F_{Z}^R) - B_{\ell V}^{LR} \right|^2 \\
+ \left| 2s_w^2 (F_{\gamma}^R + F_{Z}^L) - (F_{Z}^R + B_{\ell V}^{RL}) \right|^2 + \frac{1}{8} \left( |B_{\ell S}^{LL}|^2 + |B_{\ell S}^{RR}|^2 \right) \\
+ 6 \left( |B_{\ell T}^{LL}|^2 + |B_{\ell T}^{RR}|^2 \right) \\
+ \frac{48s_w^4}{m_l} \left[ \text{Re}\left( (F_{\gamma}^R + F_{Z}^R) G_{\gamma}^L \right) + \text{Re}\left( (F_{\gamma}^L + F_{Z}^L) G_{\gamma}^R \right) \right] \\
- \frac{8s_w^2}{m_l} \left[ \text{Re}\left( (F_{Z}^R + B_{\ell V}^{RR} + B_{\ell V}^{RL}) G_{\gamma}^L \right) \right] \\
+ \text{Re}\left( (2F_{Z}^L + B_{\ell V}^{LL} + B_{\ell V}^{LR}) G_{\gamma}^R \right) \right\} \\
+ \frac{32s_w^4}{m_l} \left( |G_{\gamma}^L|^2 + |G_{\gamma}^R|^2 \right) \left( \ln \frac{m_{l1}^2}{m_{l2}^2} - \frac{11}{4} \right) \right\}, \tag{32}
\end{align}

gdje smo radi jednostavnosti izostavili univerzalne indekse \(l'l'\) koji se pojavljuju u fotonskim i Z-bozonskim form-faktorima. U gornjim izrazima, \(m_{l1}, m_{l2}\) i \(m_{l3}\) predstavljaju mase ulaznih i izlaznih nabijenih leptona, a \(\Gamma_l\) ukupnu širinu raspada nabijenog leptona \(l\).

Prijelaz \(\mu \to e\) na atomskim jezgrama odgovara procesu \(J_\mu \to e^- J^+\), pri čemu je \(J_\mu\) atom jezgre \(J\) u kojem je jedan orbitalni elektron zamijenjen mionom, a \(J^+\) odgovarajući ion bez miona. Prijelazna amplituda za ovaj proces,

\[ \mathcal{T}^{\mu e|J} = \langle J^+e^-|\mathcal{T}^{d\mu\to de}|J_\mu\rangle + \langle J^+e^-|\mathcal{T}^{u\mu\to ue}|J_\mu\rangle, \tag{33} \]
ovisi o dva efektivna operatora pravokutnih dijagrama:

\[ T_{\text{box}}^{d\mu\rightarrow de} = -\frac{\alpha^2_w}{2M_W^2} (d^\dagger d) \bar{e} (V_d^R P_R + V_d^L P_L) \mu, \tag{34} \]

\[ T_{\text{box}}^{u\mu\rightarrow ue} = -\frac{\alpha^2_w}{2M_W^2} (u^\dagger u) \bar{e} (V_u^R P_R + V_u^L P_L) \mu. \tag{35} \]

U gornjem izrazu, \( \mu \) i \( e \) predstavljaju mionsku i elektronsku valnu funkciju, a \( d \) i \( u \) su operatori polja koji djeluju na \( J_\mu \) odnosno \( J^+ \) stanja. Kompozitni form-faktori \( V_d^L \), \( V_u^L \), \( V_d^R \) i \( V_u^R \) mogu se napisati kao:

\[ V_d^L = -\frac{1}{3} s_w^2 \left( F_{\gamma}^L - \frac{1}{m_\mu} G_{\gamma}^R \right) + \left( \frac{1}{4} - \frac{1}{3} s_w^2 \right) F_Z^L \]
\[ + \frac{1}{4} \left( B_{d\gamma}^{LL} + B_{d\gamma}^{LR} + B_{d\gamma}^{RR} + B_{d\gamma}^{RL} \right), \tag{36} \]

\[ V_d^R = -\frac{1}{3} s_w^2 \left( F_{\gamma}^R - \frac{1}{m_\mu} G_{\gamma}^L \right) + \left( \frac{1}{4} - \frac{1}{3} s_w^2 \right) F_Z^R \]
\[ + \frac{1}{4} \left( B_{d\gamma}^{RR} + B_{d\gamma}^{RL} + B_{d\gamma}^{LL} + B_{d\gamma}^{LR} \right), \tag{37} \]

\[ V_u^L = \frac{2}{3} s_w^2 \left( F_{\gamma}^L - \frac{1}{m_\mu} G_{\gamma}^R \right) + \left( -\frac{1}{4} + \frac{2}{3} s_w^2 \right) F_Z^L \]
\[ + \frac{1}{4} \left( B_{u\gamma}^{LL} + B_{u\gamma}^{LR} + B_{u\gamma}^{RR} + B_{u\gamma}^{RL} \right), \tag{38} \]

\[ V_u^R = \frac{2}{3} s_w^2 \left( F_{\gamma}^R - \frac{1}{m_\mu} G_{\gamma}^L \right) + \left( -\frac{1}{4} + \frac{2}{3} s_w^2 \right) F_Z^R \]
\[ + \frac{1}{4} \left( B_{u\gamma}^{RR} + B_{u\gamma}^{RL} + B_{u\gamma}^{LL} + B_{u\gamma}^{LR} \right). \tag{39} \]

Nukeonski matrični elementi operatora \( u^\dagger u \) i \( d^\dagger d \) glase:

\[ \langle J^+ e^- | u^\dagger u | J_\mu \rangle = (2Z + N) F(-m_\mu^2), \]
\[ \langle J^+ e^- | d^\dagger d | J_\mu \rangle = (Z + 2N) F(-m_\mu^2), \tag{40} \]

pri čemu \( F(q^2) \) označava odboj iona \( J^+ \) \[142\], a faktori \( 2Z + N \) i \( Z + 2N \) broj \( u \) odnosno \( d \) kvarkova u jezgru \( J \). Matrični element za \( J_\mu \rightarrow J^+ \mu^- \) se
prošireni sažetak
prema tome može napisati kao:

\[ T_{J \mu \rightarrow J^+ e^-} = -\frac{\alpha^2 w}{2M_W^2} F(-m_\mu^2) \bar{e} (Q^L_W P_R + Q^R_W P_L) \mu, \]  

(41)

uz

\[ Q^L_W = (2Z + N)V^{L}_u + (Z + 2N)V^{L}_d, \]

\[ Q^R_W = (2Z + N)V^{R}_u + (Z + 2N)V^{R}_d. \]

(42)

Koristeći gore navedene izraze, dolazimo do izraza za brzinu prijelaza \( J_\mu \rightarrow J^+ e^- \)

\[ R^{J}_{\mu e} = \frac{\alpha^3 \alpha^4 m^{5}_\mu}{16\pi^2 M^4_W \Gamma_{\text{capture}} Z^4 \left| F(-m^2_\mu) \right|^2 \left( |Q^L_W|^2 + |Q^R_W|^2 \right)}, \]  

(43)

gdje je \( \Gamma_{\text{capture}} \) brzina uhvata miona od jezgre, a \( Z_{\text{eff}} \) efektivni naboj koji uzima u obzir koherentne učinke koji se javljaju u jezgri \( J \) uslijed njezine konačne dimenzije. Vrijednosti za \( Z_{\text{eff}} \) uzete su iz ref. \[143\].

Na osnovi gore izloženih analitičkih rezultata napravljena je numerička analiza. mSUGRA parametri odabrani su tako da zadovoljavaju ograničenja koja je postavio LHC: \( m_H = 125.5 \pm 2 \text{ GeV} \) \[82,84,147\], \( m_{\tilde{g}} > 1500 \text{ GeV} \) i \( m_{\tilde{t}} > 500 \text{ GeV} \) \[146,147\], gdje je \( m_H \) masa standardnomodelskog Higgsa, a \( m_{\tilde{g}} \) i \( m_{\tilde{t}} \) mase gluina odnosno stop kvarka:

\[ \tan \beta = 10, \quad m_0 = 1000 \text{ GeV}, \]

\[ A_0 = -3000 \text{ GeV}, \quad M_{1/2} = 1000 \text{ GeV}. \]  

(44)

Koristenjem renormalizacijske grupne jednadžbe danih u referencama \[69,153\], radimo evoluciju baždarnih vezanja te Yukawinih matrica za kvarkove i nabijene leptone od skale \( M_Z \) do GUT skale, dok neutrinska masena matrica \( (m_M) \) i Yukawina matrica \( (h_\nu) \) matrica evoluiraju od skale određene masom teškog neutrina \( m_N \), do GUT skale.

Analiza je provedena u dva različita scenarija. Prvi realizira U(1) simetriju
Dipolni momenti leptona

S obzirom na trenutno stanje eksperimenta, može se vidjeti da anomalni magnetski moment (MDM) miona i električni dipolni moment (EDM) elektrona zavrijeđuju posebnu pažnju. Zbog toga se učinilo uputnim unutar modela s kojim smo izučavali procese s narušenjem leptonskog okusa u nabiljenom sektoru izvrijedniti i ove opservable.

Anomalni magnetski dipolni moment i električni dipolni moment leptona $l$ može se isčitati iz lagranžijana \[159\]:

$$ L = \bar{l} \left[ \gamma_{\mu} (i \partial^{\mu} + e A^{\mu}) - m_{l} - \frac{e}{2m_{l}} \sigma^{\mu\nu} (F_{l} + iG_{l}\gamma_{5}) \partial_{\nu} A_{\mu} \right] l. $$ \tag{45}

U području u kojem se fotonsko polje $A^{\mu}$ nalazi na ljusci mase, form-faktor $F_{l}$ definira magnetski dipolni moment leptona $l$, t.j. $a_{l} \equiv F_{l}$, dok form-faktor $G_{l}$ definira njegov električni dipolni moment, t.j. $d_{l} \equiv eG_{l}/m_{l}$. Iz prethodnog analitičkog izraza za fotonsku amplitudu možemo napisati opću form-faktorskog dekompoziciju prijelazne amplitude,

$$ iT_{\gamma l}^{\gamma l} = i \frac{e\alpha_{w}}{8\pi M_{W}^{2}} \left[ (G_{\gamma}^{L})_{ll} i\sigma_{\mu\nu} q^{\nu} P_{L} + (G_{\gamma}^{R})_{ll} i\sigma_{\mu\nu} q^{\nu} P_{R} \right]. $$ \tag{46}

Tako dolazimo do izraza za anomalni MDM ($a_{l}$) i EDM ($d_{l}$) leptona $l$,

$$ a_{l} = \frac{\alpha_{w} m_{l}}{8\pi M_{W}^{2}} \left[ (G_{\gamma}^{L})_{ll} + (G_{\gamma}^{R})_{ll} \right], $$ \tag{47}

$$ d_{l} = \frac{e\alpha_{w}}{8\pi M_{W}^{2}} \left[ (G_{\gamma}^{L})_{ll} - (G_{\gamma}^{R})_{ll} \right]. $$ \tag{48}
gdje notacija za vezanja i form-faktore odgovara onoj koju smo koristili i ranije.

Kao što je pokazano u ref. [160], EDM leptona $d_l$ iščežava u u MSSM-u sa univerzalnim rubnim uvjetima mekanog loma supersimetrije bez dodatno uvedenih CP faza. Ovaj rezultat vrijedi i u proširenjima MSSM-a teškim neutrinima, dok god sneutrinski sektor čuva CP simetriju.

Kao minimalno odstupanje od ovog scenarija, pretpostavimo da samo sneutrinski sektor narušava CP simetriju i to putem mehanih CP faza u bilinearnim i trilinearim parametrima u lagranžijanu mekanog loma CP simetrije,

$$b_\nu \equiv B_\nu m_M = B_0 e^{i\theta} m_N 1_3,$$

$$A_\nu = h_\nu A_0 e^{i\phi}.$$  (49)  (50)

pri čemu su $B_0$ i $A_0$ realni parametri određeni na GUT skali, $m_N$ realni parametar unesen na skali $m_N$, $\theta$ i $\phi$ fizikalne CP faze, a $h_\nu$ neutrinska Yukawina $3 \times 3$ matrica dana sa (23). Ovi parametri nalaze se u sljedećim članovima lagranžijana koji mekano lomi supersimetriju:

$$-(A_\nu)^{ij} \tilde{\nu}_R^c (h_{uL}^+ \tilde{e}_{jL} - h_{uL}^0 \tilde{\nu}_{jL}),$$  (51)

$$(b_\nu m_M)^{ii} \tilde{\nu}_{Ri}^c \tilde{\nu}_{Ri}.$$  (52)

Radi jednostavnosti pretpostavljamo da je matrica $b_\nu$ proporcionalna sa jediničnom matricom $1_3$. U uobičajenim SUSY scenarijima sa mehanizmom njihalice uz ultra-teške neutrine mase $m_N$, doprinosi CP narušenja u sneutrinskom sektoru električnom dipolnom momentu $d_l$ ponašaju se kao $B_0/m_N$ i $A_0/m_N$ na razini jedne petlje. A kako su mase $m_N$ u tim scenarijima velike (blizu GUT skale), doprinos EDM-u praktički je zanemariv. Uočljivi doprinosi EDM-u mogu se dakle očekivati u scenarijima sa mehanizmom njihalice na niskoj skali, u kojem masa $m_N$ može biti usporediva sa $B_0$ i $A_0$.

Bilinearnu mekanu matricu $b_\nu$ zanemarili smo kada smo izučavali procese s
narušenjem leptonskog okusa u nabijenom sektoru. Tada smo prešutno pretpostavili da je taj parametar malen u odnosu na druge parametre mekanog loma supersimetrije. Ovdje ćemo ga ipak uzeti u obzir, ali tako da senutrijske mase uvijek ostanu pozitivne, dakle i fizikalne.

U MSSM-u, vodeći doprinos anomalnom magnetskom momentu leptona \( a_l \) ima sljedeće ponašanje [161–163]:

\[
a_l^{\text{MSSM}} \propto \frac{m_l^2}{M_{\text{SUSY}}^2} \tan \beta \ \text{sign}(\mu M_{1,2}) ,
\]

pri čemu je \( M_{\text{SUSY}} \) tipična masena skala mekanog loma supersimetrije, \( \tan \beta = v_2 / v_1 \) omjer očekivanih vakuumskih vrijednosti Higgsovih dubleta, a \( M_{1,2} \) mase gejdžina povezanih sa U(1)\(_Y\) odnosno SU(2) baždarnom grupom. Kao što ćemo vidjeti, doprinos MSSM-a anomalnom magnetskom momentu miona ostaje dominantan i u \( \nu_R^{\text{MSSM}} \).

Uspoređujući izraze za \( d_l \) i \( a_l \), možemo dati naivnu procjenu ponašanja električnog dipolnog momenta leptona \( l \) na razini jedne petlje,

\[
d_l^{\text{MSSM}} \propto \sin(\phi_{\text{CP}}) \frac{m_l}{M_{\text{SUSY}}^2} \tan \beta ,
\]

gdje je \( \phi_{\text{CP}} \) generička CP faza iz sektora koji mekano lomi supersimetriju. Iako su u MSSM-u moguće i drugačije ovisnosti \( d_l \)-a o \( \tan \beta \) [160,164], pokazat će se da je u okviru \( \nu_R^{\text{MSSM}} \)-a ova ovisnost uvijek linearna na razini jedne petlje.

Numerička analiza napravljena je u ovisnosti o mSUGRA parametrima, i to u okolici točke određene parametrima

\[
\begin{align*}
m_0 &= 1 \text{ TeV}, & M_{1/2} &= 1 \text{ TeV}, & A_0 &= -4 \text{ TeV}, & \tan \beta &= 20, \\
m_N &= 1 \text{ TeV}, & B_0 &= 0.1 \text{ TeV}, & a &= b = c = 0.05,
\end{align*}
\]

Kao i ranije, parametri su odabrani tako da zadovoljavaju eksperimentalna ograničenja koja je postavio LHC (\( m_H = 125.5 \pm 2 \text{ GeV} \), \( m_{\tilde{g}} > 1500 \text{ GeV} \), \( m_t > 500 \text{ GeV} \)).
Rezultati numeričke analize prikazani su na slikama 4.1 (str. 56), 4.2 (str. 58) i 4.3 (str. 59).

Zaključak

Narušenje leptonskog okusa (CLFV) izučavano je u okviru minimalnog supersimetričnog standardnog modela (MSSM) proširenog sa singletnim teškim neutrinima na niskoj skali, pri čemu je posebna pažnja posvećena pojedinim doprinosima petlji koji dolaze od teških neutrina $N_{1,2,3}$, sneutrina $\tilde{N}_{1,2,...,12}$ i članova koji mekano lome supersimetriju. U ovoj analizi, po prvi put smo uključili potpuni skup pravokutnih dijagrama, uz dijagrame sa fotonskim i $Z$-bozonskim doprinosima. Također smo izveli potpun skup kiralnih amplituda i pridruženih im form-faktora koji su povezani sa CLFV raspadima miona i tau-leptona bez neutrina, kao što su $\mu \rightarrow eee$, $\tau \rightarrow \mu\mu\mu$, $\tau \rightarrow e\mu\mu$ i $\tau \rightarrow eee\mu$, te $\mu \rightarrow e$ prijelazima na atomskim jezgrama. Dobiveni analitički rezultati su općeniti i mogu se primijeniti na većinu modela nove fizike koji uključuju CLFV. U tom kontekstu valja naglasiti da je sustavna analiza ovih procesa pokazala postojanje dvaju novih form faktora iz pravokutnih dijagrama, koji se ne navode u postojećoj literaturi koja se bavi teorijama sa CLFV-om.

Ova detaljna studija pokazala je da efekti mekanog loma supersimetrije u dijagramima sa $Z$ bozonom dominira u CLFV opservablima, i to u velikom dijelu $\nu_R$MSSM parametarskog prostora u okviru mSUGRA scenarija. Ipak, postoji značajno područje parametarskog prostora (za neutrinske mase $m_N \lesssim 1$ TeV) u kojima pravokutni dijagrami koji uključuju teške neutrine u petlji mogu biti usporedivi, pa čak i veći, od odgovarajućih doprinosa iz dijagrama sa $Z$ bozonom u procesima $\mu \rightarrow eee$ i $\mu \rightarrow e$ prijelazima na atomskim jezgrama (v. sliku 3.11 na str. 45). U istom kinematičkom režimu, uslijed slučajnih numeričkih poništenja, opažamo i potisnuće omjera grananja za fotonske CLFV raspe $\mu \rightarrow e\gamma$, kao i za raspe $\tau \rightarrow e\gamma$ i $\tau \rightarrow \mu\gamma$. Kao što je već rečeno, takva potisnuća u mehanizmu njihalice na niskoj skali dolaze kao posljedica poništenja između čestičnih i s-čestičnih doprinosa uslijed aproksimativne realizacije supersimetričnog no-go teorema kojeg su postavili Ferrara...
i Remiddi [132]. U mehanizmu njihalice na visokoj skali ova se poništenja mogu pojaviti samo za određeni izbor neutrinih Yukawinih matrica i Majoraninih maseni matrica, kao što je pokazano u ref. [119] [141]. Stoga možemo reći da rezultati koje smo dobili unutar supersimetričnog modela njihalice na niskoj skali (uz $m_N \leq 10$ TeV), podržavaju izvorni rezultat iznesen u ref. [111], gdje uobičajena paradigma po kojoj fotonski operatori dipolnog momenta dominiraju CLFV opservablama u modelima njihalice na visokoj skali [114] [118] doživljava radikalnu promjenu. Iz toga možemo zaključiti da raspadi $\mu \rightarrow eee$ i $\mu \rightarrow e$ prijelazi na jezgrama također mogu služiti kao precizan test narušenja leptonskog okusa u nabijenom sektoru.

Otkrili smo da CLFV efekti inducirani od strane sneutrina, za razliku od onih induciranih teškim neutrinima, ostaju potisnuti u cijelom prostoru mSUGRA parametarskog prostora. Uz to, perturbacijsko ograničenje na neutrinaska Yukawina vezanja $h_\nu$ do GUT skale čini kvadrične doprinosе reda $(h_\nu)^4$ malima. Ova studija usmjeren je na davanje numeričkih predikcija za male i umjerene vrijednosti parametra $\tan \beta$ (za $\tan \beta \lesssim 20$), pri čemu se očekuje da neutralne interakcije s Higsovim bozonima ograničene nedavnim opažanjima raspada $B_s \rightarrow \mu \mu$ ne daju značajniji doprinos ovakvima procesima. Globalna analiza koja bi uključivala velike $\tan \beta$ na CLFV opservablama uz LHC ograničenja jedan je od ciljeva budućih istraživanja.

Uz navedeno, disertacija sadrži sustavnu studiju doprinosa jedne petlje mionskom anomalnom magnetskom momentu (MDM) $a_\mu$ te električnom dipolnom momentu (EDM) $d_e$ u okviru $\nu_R$MSSM modela. Posebna pažnja dana je učinku sneutrinskih parametara koji mekano lome supersimetriju, $B_\nu$ i $A_\nu$, kao i njihovim univerzalnim CP fazama, $\theta$ i $\phi$. Koliko znamo, leptonski dipolni momenti u prijašnjoj literaturi nisu detaljno analizirani u okviru supersimetričnih modela sa singletem (s)neutrinima na niskoj skali.

Za anomalni MDM miona $a_\mu$ pokazali smo da su doprinosi teških neutrininskih i sneutrinskih singleta MDM-u maleni, tipično jedan ili dva reda veličine ispod mionske anomalije $\Delta a_\mu$. S druge strane, sneutrin i sleptoni lijeve kiralnosti daju najveći efekt na $\Delta a_\mu$, točno kao i u MSSM-u. Ovisnost MDM-a o masi miona $m_\mu$, $\tan \beta$ i masenoj skali mekanog loma supersimetrije $M_{SUSY}$ pažljivo
su analizirani i potvrđeno je njihovo ponašanje u skladu s jednadžbom \(53\). Konačno smo utvrdili i to da ovisnost \(a_\mu\) o univerzalnom mekom trilinearnom parametru \(A_0\) te neutrinskim Yukawinim vezanjima \(h_\nu\) i masi teškog neutrina \(m_N\) zanemariva.

Nadalje, u okviru istog \(\nu_R\)MSSM modela napravljena je analiza EDM-a elektrona \(d_e\). Teški singletni neutrini ne doprinose EDM-u, a članovi koji mekano lome supersimetriju iz sneutrinskog sektora doprinose samo ako su faze \(\phi\) i/ili \(\theta\) različiti od nule. Numerički je pokazano da je mogući doprinos narušenju iz CP simetrije koji bi dolazili od relativno kompleksnih produkata vrhova (v. jednadžbu \(4.9\) na str. \(52\)) jednak nuli. S druge strane, doprinos koji dolazi od konačnih vrijednosti faze \(\phi\) najveći je i može rezultirati vrijednostima za EDM koji su usporedivi s trenutno postavljenom eksperimentalnom gornjom granicom. Efekt CP faze \(\theta\) na \(d_e\) je od prilike jedan do dva reda veličine manji nego onaj koji dolazi od faze \(\phi\). EDM elektrona \(d_e\) linearno raste sa \(\tan \beta\) i masom leptona \(m_l\), približno je neovisan o parametrima \(A_0\) i \(B_0\), ali općenito pada sa mSUGRA parametrima \(m_0\) i \(M_{1/2}\).

Na temelju ovih numeričkih rezultata, izvedeni su približni poluanalitički izrazi, koji se razlikuju od onih iz postojeće literature o supersimetričnim modelima s mehanizmom njihalice na visokoj skali. Specifično, dodavanje CP faza vodi na ponašanje EDM-a tipa \(d_l \propto m_l \tan \beta/m_N^y\), gdje je \(2/3 < y < 1\). Dok je istina da \(d_e\) općenito pada sa \(M_{SUSY}\), ova ovisnost ne može se opisati jednostavnim potencijskim padom. Ovisnost o mSUGRA parametrima \(A_0\) i \(B_0\) su slabe u najvećem dijelu parametarskog prostora. Linearna ovisnost o \(\tan \beta\) kao i ovisnost o masi teškog neutrina predstavljaju nove rezultate koji proizlaze iz ove studije.

Za usporedbu, ovisnost o \(\tan \beta\) navedena u ref. \([160]\) je, ovisno o veličini, ili kubična ili konstantna. Uzveši u obzir trenutne eksperimentalne granice na \(d_e\) značajan dio \(\nu_R\)MSSM parametarskog prostora identificiran je maksimalnom vrijednošću CP faze \(\phi = \pi/2\), pri čemu EDM elektrona \(d_e\) može poprimiti vrijednosti uspoređive sa trenutnom i budućom eksperimentalnom osjetljivosti. Učinak CP narušenja iz sneutrinskog sektora na električne dipolne momente neutrina i žive trebao bi biti potisnut, pa ovakav tip studije može služiti kao
razlikovni kriterij za $\nu_R$MSSM scenarije koje smo razmatrali u ovoj disertaciji.
U svom kratkom osvrtu o budućnosti fizike elementarnih čestica, dobitnik Nobelove nagrade Sheldon Lee Glashow iznio je šest točaka koje on osobno smatra najvažnijim za budućnost teorijskog istraživanja u fizici visokih energija. Između ostalog, ove točke uključuju procese s narušenjem leptonskog okusa u nabijenom sektoru, anomalni magnetski dipolni moment miona $a_\mu = g_\mu - 2$, kao i električni dipolni moment elektrona $d_e$. Autor ove disertacije rado bi se s time složio.
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- Relativistic quantum physics (4th year course)
- Theory of fields I (4th year course)
- Theory of fields II (5th year course)
- General physics (Faculty of Electrical Engineering, 1st year course)
List of publications

1. Ilakovac A and Popov L 2010 Two-step Lorentz Transformation of Force *Fizika* A 19 3

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Conference proceedings

1. Popov L 2009 Einstein-Podolsky-Rosen (EPR) argument and consequences *Teorija relativnosti i filozofija. Povodom 100. obljetnice Einsteinove Specijalne teorije relativnosti* ed Petković, T (Zagreb: Hrvatsko filozofsko društvo)

Active participation in conferences

1. Ilakovac A, Pilaftsis A and Popov L 2012 Charged Lepton Flavour Violation in Supersymmetric Low-Scale Seesaw Models *LHC Days in Split* Poster