Electromagnetic draping of merging neutron stars

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Abstract

We first derive a set of equations describing general stationary configurations of relativistic force-free plasma, without assuming any geometric symmetries. We then demonstrate that electromagnetic interaction of merging neutron stars is necessarily dissipative due to the effect of electromagnetic draping - creation of dissipative regions near the star (in the single-magnetized case) or at the magnetospheric boundary (in the double-magnetized case). Our results indicate that even in the single magnetized case we expect that relativistic jets (or “tongues”) are produced, with correspondingly beamed emission pattern.
I. INTRODUCTION

The detection of gravitational waves associated with a short GRB [1] identifies merger of neutron stars as the central engine. It is highly desirable to detect any possible precursor to the main event. [10] [see also 11] argued that magnetospheric interaction during double neutron star (DNS) merger can lead to the production of electromagnetic radiation. The underlying mechanism advocated in those works is a creation of inductive electric field due to the relative motion of neutron stars. Both singly magnetized (1M-DNS) and double magnetized case (2M-DNS) are possible [13]. The 1M-DNS case is similar to the Io-Jupiter interaction [8]. Other relevant works include [5, 16, 17].

Similarly to the DNS merger, in the case of merging black holes, or BH-NS mergers, motion of the black hole through magnetic field (generated ether by the accretion disk or through neutron star magnetosphere) leads to generation of inductively-induced outflows, even by a non-rotating Schwarzschild black hole [2, 13, 14, 18].

The approach taken by [10, 15], heuristically, follows that of [7], in that a quasi-vacuum approximation is used at first. This leads to the generation of dissipative regions, pair production and ensuing nearly-ideal plasma dynamics. Resulting charges and currents modify the magnetospheric structure. In the axisymmetric case this leads to the pulsar equation [8, 19]. The pulsar equation, a variant of the Grad-Shafranov equation [9, 20], is a scalar equation for axially-symmetric relativistic force-free configurations. Axial symmetry allows introduction of an associated Euler potential, which, together with the $\text{div} \mathbf{B} = 0$ and ideal conditions reduce the force-balance to a single scalar equation.

In the case of merger double neutron stars systems, there is no geometrical symmetry that can be used to reduce the force-balance to a single equation. In this paper we first derive equation governing relativistic force-free configurations without assuming axially symmetry, §II. It is a set of two nonlinear elliptic equations for two Euler potential, with initially unknown dependence of the electric potential. It turns out to be prohibitively complicated.

In §III we take an alternative approach: expansion in small electric field (small velocity). We demonstrate that the electromagnetic fields “pile-up” near the surface of the neutron star, creating regions with large electric field. Similar effects occur in 2M-DNS scenario, §IV.
II. RELATIVISTIC FORCE-FREE CONFIGURATIONS

First we derive a set of equations describing general stationary configurations of relativistic force-free plasma, without assuming any geometric symmetries.

Let us represent the magnetic field in terms of Euler potentials $\alpha - \beta$, and stationary electric field in terms of the electrostatic potential $\Phi$ (factors of $4\pi$ are absorbed into definitions of fields)

\[
\begin{align*}
B &= \nabla \alpha \times \nabla \beta \\
E &= -\nabla \Phi
\end{align*}
\]  

(1)

Ideal condition

\[
E \cdot B = \nabla \Phi \cdot (\nabla \alpha \times \nabla \beta)
\]  

(2)

requires $\Phi(\alpha)$ or $\Phi(\beta)$. For definiteness let’s assume $\Phi(\alpha)$. This is an initially unknown function that needs to be found as part of the solution with given boundary conditions.

Also, we impose orthogonality condition

\[
(\nabla \alpha \cdot \nabla \beta) = 0
\]  

(3)

Then vectors $\nabla \alpha$, $\nabla \beta$, and $\nabla \Phi$ form an orthogonal triad. Surfaces of constant $\alpha$, $\beta$, $\Phi$ are mutually orthogonal.

Force balance

\[
\Delta \Phi \nabla \Phi + (\nabla \times B) \times B = 0
\]  

(4)

takes the form

\[
\begin{align*}
\nabla \beta \left( \nabla \alpha \cdot (\nabla \alpha \Delta \beta + L(\alpha, \beta)) \right) + \\
\nabla \alpha \left( (\nabla \alpha \cdot \nabla \alpha) \Phi' \Phi'' + \Delta \alpha (\Phi')^2 - (\nabla \beta \cdot (L(\alpha, \beta) + \Delta \alpha \nabla \beta)) \right) &= 0
\end{align*}
\]  

(5)

where

\[
L(\alpha, \beta) \equiv (\nabla \alpha \cdot \nabla) \nabla \beta - (\nabla \beta \cdot \nabla) \nabla \alpha
\]  

(6)

and primes denote $\Phi' = \partial_\alpha \Phi(\alpha)$.

Both terms in (5) should be zero independently

\[
\nabla \alpha \cdot (\nabla \alpha \Delta \beta + L(\alpha, \beta))
\]  

(7)

\[
(\nabla \alpha)^2 \Phi' \Phi'' + \Delta \alpha (\Phi')^2 = (\nabla \beta \cdot (L(\alpha, \beta) + \Delta \alpha \nabla \beta))
\]  

(8)
Equations (7) - (8), together with constraint (3) represent two equations for two Euler potentials $\alpha$ and $\beta$.

Some further modifications can be done. Eq. (7) can be written as

$$\nabla \alpha \Delta \beta = \mathcal{L}(\alpha, \beta) + g \nabla \beta$$  \hspace{1cm} (9)

where $g$ is an arbitrary function. Scalar product (9) with $\nabla \beta$ gives

$$(\nabla \beta \cdot \mathcal{L}(\alpha, \beta)) = -g (\nabla \beta \cdot \nabla \beta)$$  \hspace{1cm} (10)

Eq. (8) then becomes

$$(\nabla \alpha)^2 \Phi' \Phi'' + \Delta \alpha (\Phi')^2 = (\nabla \beta)^2 (\Delta \alpha - g) = 0$$  \hspace{1cm} (11)

Or

$$\Delta \alpha \left((\Phi')^2 - (\nabla \beta)^2\right) + (\nabla \alpha)^2 \Phi' \Phi'' + g (\nabla \beta)^2 = 0$$  \hspace{1cm} (12)

Equations (9) and (12) can be used instead of (7) - (8). Function $g$ should be chosen to fit the boundary conditions. A simple example is considered in Appendix III B.

The set of equations (7) - (8) - (3) or (9) - (12) - (3) describe general relativistic force-free equilibrium. It’s a nonlinear set of equations for two functions $\alpha$ and $\beta$ with initially unknown $\Phi(\alpha)$.

III. METAL SPHERE MOVING THROUGH FORCE-FREE MAGNETIC FIELD

A. Boundary conditions

Let in the frame of a conducting ball the magnetic field at infinity be along $z$ and electric field is along $y$ axis (so that electromagnetic velocity is along $x$), see Fig. 1. The magnetic field is assumed to be non-penetrating, the ball is unmagnetized.

A set of equations that needs to be solved is

$$\rho_e \mathbf{E} + \mathbf{J} \times \mathbf{B} = 0$$

$$\text{div} \mathbf{J} = 0$$

$$\mathbf{E} \cdot \mathbf{B} = 0$$  \hspace{1cm} (13)
FIG. 1. Geometry of the system. In the frame of the sphere at large $x \to -\infty$ the magnetic field is along $z$ axis, electric field is along $y$ axis, so that plasma is moving in positive $z$ direction with velocity $v_0$.

force balance, stationarity and ideality. Boundary conditions are

$$B_z(x = -\infty) = B_0$$
$$E_y(x = -\infty) = v_0 B_z$$
$$\mathbf{e}_r \cdot \mathbf{B}|_{r=R} = 0$$
$$\mathbf{e}_r \times \mathbf{E}|_{r=R} = 0$$

(14)

The last two imply no normal magnetic field and no tangential electric field on the surface.

In terms of Euler potentials

$$\beta(x = -\infty) = -x$$
$$\alpha(x = -\infty) = B_0 y$$
$$\mathbf{e}_r \cdot (\nabla \alpha \times \nabla \beta)|_{r=R} = 0$$
$$\mathbf{e}_r \times \nabla \alpha|_{r=R} = 0$$

(15)
(Landau gauge for magnetic field is better at \( x = \infty \)). Thus, at \( x = -\infty \) we have \( \Phi = -v_0 B_0 y = -v_0 \alpha \).

The resulting system of nonlinear elliptical equations with unknown \( \Phi(\alpha) \) turns out to be prohibitively complicated, hence we have to resort to approximate methods - expansion in terms of the velocity \( v_0 \).

**B. Metal ball in static magnetic field, \( v_0 = 0 \)**

As a zeroth-order, we start with conducting ball in external magnetic field. In this case magnetic field and vector potential are a sum of constant vertical field \( B_v \) and dipole field \( B_d \)

\[
B_0 = B_v + B_d = \left\{ \left( 1 - R^3/r^3 \right) \cos \theta, -\left( 1 + \frac{R^3}{2r^3} \right) \sin \theta, 0 \right\} B_0
\]

\[
A = \{0, 0, 1 - R^3/r^3\} r \sin \theta B_0/2
\]

\[
B_v = \{\cos \theta, -\sin \theta, 0\} B_0
\]

\[
B_d = \left\{ -R^3/r^3 \cos \theta, -\frac{R^3}{2r^3} \sin \theta, 0 \right\} B_0
\]

Euler potentials are

\[
\alpha_0 = \frac{1}{2} B_0 \sin^2 \theta \left( r^2 - \frac{R^3}{r} \right)
\]

\[
\beta_0 = \phi
\]

Scalar magnetic potential

\[
\Phi_B = \left( 1 + \frac{R^3}{2r^3} \right) r \cos \theta B_0
\]

so that \( B_0 = \nabla \alpha_0 \times \nabla \beta_0 = \nabla \Phi_B \).

Importantly,

\[
(\nabla \alpha_0) \cdot (\nabla \Phi_B) = 0
\]

Thus, Euler potentials \( \alpha_0, \beta_0 \) and \( \Phi_0 \) form a mutually orthogonal triad of surfaces, see Fig. 2

\[
\nabla \alpha_0 \perp \nabla \beta_0 \perp \nabla \Phi_B
\]
FIG. 2. Orthogonal surfaces of constant $\alpha_0$, $\beta_0$ and $\Phi_B$. 
We find
\[ \nabla \alpha_0 = \left\{ \left( r + \frac{R^3}{2r^2} \right) \sin^2 \theta, \frac{(r^3 - R^3) \sin \theta \cos \theta}{r^2}, 0 \right\} B_0 \]
\[ \nabla \beta_0 = \left\{ 0, 0, \frac{1}{r \sin \theta} \right\} \]
\[ (\nabla \alpha_0 \cdot \nabla \beta_0) = 0 \]
\[ \mathcal{L} = \frac{B_0 (-4r^3 + 3R^3 \cos(2\theta) + R^3)}{2 \sin \theta r^4} e_\phi \]
\[ \mathcal{L} \cdot \nabla \alpha_0 = 0 \]
\[ \mathcal{L} \cdot \nabla \beta_0 = \frac{B_0 (-4r^3 + 3R^3 \cos(2\theta) + R^3)}{2 \sin^2 \theta r^5} \]
\[ \Delta \beta_0 = 0 \]
\[ (\nabla \beta_0)^2 = \frac{1}{\sin^2 \theta r^2} \]
\[ (\nabla \alpha_0)^2 = \frac{B_0^2 \sin^2 \theta (3R^3 \cos(2\theta) (R^3 - 4r^3) - 4r^3 R^3 + 8r^6 + 5R^6)}{8r^4} \]
\[ \Delta \alpha_0 = -\frac{B_0 (-4r^3 + 3R^3 \cos(2\theta) + R^3)}{2r^3} \]  \hspace{1cm} (21)

Eq. (9) becomes
\[ \mathcal{L}(\alpha, \beta) + g_0 \nabla \beta_0 = 0 \]
\[ g_0 = \frac{B_0 (4r^3 - 3R^3 \cos(2\theta) - R^3)}{2r^3} \]  \hspace{1cm} (22)

For \( \Phi = 0 \) Eq. (12) becomes
\[ g_0 = \Delta \alpha_0 \]  \hspace{1cm} (23)

And it is indeed satisfied.

C. First order expansion in \( v_0 \)

In Appendix A we demonstrate that in first order expansion in \( v_0 \) the surfaces of constant \( \alpha - \beta - \Phi \) remain unchanged.

Let’s expand the force balance (4) for small velocity \( v_0 \ll 1 \). In the zeroth order
\[ \nabla \times \mathbf{B}_0 = 0 \]  \hspace{1cm} (24)

We expect that electric potential is first order in \( v_0 \)
\[ \Phi \propto v_0 \sim \frac{E}{B_0} \]
\[ \Phi \propto \mathcal{O}(\epsilon) \]  \hspace{1cm} (25)
The key point is that the force balance is second order in \( v_0 \):
\[
\Delta \Phi \nabla \Phi + (\nabla \times \delta B) \times B_0 = 0
\]
\[
\Delta \Phi \nabla \Phi \propto O(\epsilon^2)
\]
\[
(\nabla \times \delta B) \propto O(\epsilon^2),
\]
while the constraint
\[
E \cdot B \propto E \cdot B_0 \propto O(\epsilon)
\]
is first order. Thus, if we are limited to terms linear in \( v_0 \), we need to consider only the constraint: the force balance is violated only in \( v_0^2 \).

For magnetic field \( B_0 \) let’s use the magnetic potential \([18]\). Then we need to find \( \Phi \) such that
\[
(\nabla \Phi) \cdot (\nabla \Phi_B) = 0
\]
Clearly any
\[
\Phi(\alpha_0) f(\phi)
\]
satisfies this condition.

At \( x = -\infty \) the electric potential is
\[
\Phi = -yv_0B_0 = -r \sin \theta \sin \phi v_0B_0
\]
Thus,
\[
\Phi(\alpha_0) = -v_0B_0\sqrt{2\alpha_0}
\]

\[
f(\phi) = \sin \phi
\]
And finally
\[
\Phi = -\sqrt{1 - R^3/r^3} \times r \sin \theta \sin \phi v_0B_0
\]
\[
E = -\nabla \Phi = \left\{ \frac{1 + R^3/2r^3}{\sqrt{1 - R^3/r^3}} \sin \theta \sin \phi, \sqrt{1 - R^3/r^3} \cos \theta \sin(\phi), \sqrt{1 - R^3/r^3} \cos \phi \right\} v_0B_0 \rightarrow
\]
\[
\left\{ \sqrt{3} \frac{\sin \theta \sin \phi}{2\sqrt{\delta_r}}, \sqrt{3} \cos \theta \sqrt{\delta_r} \sin \phi, \sqrt{3} \sqrt{\delta_r} \cos \phi \right\} v_0B_0
\]
\[
\delta_r = r - R
\]
By construction \( E \cdot B_0 = 0 \). The radial component of the electric field diverges - this is the electromagnetic draping. Also, in Appendix \([B]\) we compare electric field \([32]\) with other relevant cases.
Given the electric field (32), the induced charger density is

$$\rho_e = \text{div} \mathbf{E} = -\frac{9 R^6 \sin \theta \sin \phi}{4 r^7 (1 - R^3/r^3)^{3/2}} B_0 v_0 \rightarrow -\frac{\sqrt{3} \sqrt{R} \sin \theta \sin \phi}{4 \delta_r^{3/2}} B_0 v_0$$  \hspace{1cm} (33)

The electromagnetic velocity is

$$\mathbf{v}_{EM} = \frac{\mathbf{E} \times \mathbf{B}_0}{B_0^2}$$

$$v_r = \frac{2 r^3 (2 r^3 + R^3) \sqrt{1 - R^3/r^3}}{4 (r^3 - R^3)^2 - 3 R^3 (R^3 - 4 r^3) \sin^2 \theta} \sin \theta \cos \phi$$

$$v_\theta = \frac{4 r^3 (r^3 - R^3) \sqrt{1 - R^3/r^3}}{4 (r^3 - R^3)^2 - 3 R^3 (R^3 - 4 r^3) \sin^2 \theta} \cos \theta \cos \phi$$

$$v_\phi = -\frac{\sin \phi}{\sqrt{1 - R^3/r^3}} \rightarrow -\frac{v_0 \sin \phi}{\sqrt{3} \sqrt{\delta_r/R}}$$ \hspace{1cm} (34)

see Figs. 3-6.

The condition $\mathbf{v}_{EM} = 1$ is satisfied at approximately

$$\frac{\delta r}{R} = \sin^2 \phi \frac{v_0^2}{3}$$  \hspace{1cm} (35)

This is the estimate of the thickness and location of the draping layer. It is maximal at the plane $x = 0$ ($\phi = \pi/2$). In the $\phi = \pi/2$ plane ($x = 0$) the condition $\beta_{EM} = 1$ is satisfied at

$$\frac{r_{EM}}{R} = \gamma_0^{1/3}$$

$$\gamma_0 = \frac{1}{\sqrt{1 - v_0^2}}$$ \hspace{1cm} (36)

**D. Second order in $v_0$**

As we discussed above, the first order perturbations come not from the dynamics, but from the constraint $\mathbf{E} \cdot \mathbf{B} = 0$. We can then use the $\propto v_0$ terms to construct the second order expansion.

The charge density (33) and the electromagnetic velocity (34) lead to the appearance of charge-separated current

$$\mathbf{J}_{EM} = \rho_e \mathbf{v}_{EM}$$ \hspace{1cm} (37)

(it is of $\mathcal{O}(\epsilon^2)$ order). The current $\mathbf{J}_{EM}$ is not the total current, only its transverse charge-separated part, see below. Naturally,

$$\rho_e \mathbf{E} + \mathbf{J}_{EM} \times \mathbf{B}_0 = 0$$ \hspace{1cm} (38)
FIG. 3. Electric field \((32)\) in the \(x = 0\), \(y = 0\) and \(z = 0\) planes (for \(y = 0\) the electric field is \(\sqrt{(x^2 + z^2)^{3/2} - 1/(x^2 + z^2)^{3/4}} e_y v_0 B_0\)). Red lines indicate regions where \(\beta_{EM} = 1\) (\(v_0 = 0.75\) is assumed for plotting).

The most radially-divergent \(\phi\)-component can be easily found

\[
J^{(2)}_\phi = \frac{9 \sin \theta \sin^2 \phi}{4r (r^3 - R^3)^2} \times R^6 v_0^2 B_0
\]

(39)

Near \(r \to R\)

\[
J_{EM} \approx \left\{-\frac{\sin(2\phi)}{4\delta_r}, 0, \left(\frac{R}{4\delta_r^2} - \frac{3}{4\delta_r}\right) \times \sin \theta \sin^2 \phi\right\} B_0 v_0^2
\]

(40)

The toroidal current increases the most. Since largest gradients are in radial direction, that leads to growth of \(B_\theta\), see Eq. (41).

Using (39), neglecting \(B_r\) component (small near the surface, non-penetrating magnetic field), we find divergent terms

\[
B_\theta = \left(-\frac{3R^2}{4 (r^3 - R^3)} + \frac{\ln \left(\frac{r^2 + R + R^2}{(r-R)^2}\right)}{4r}\right) \sin \theta \sin^2 \phi R B_0 v_0^2
\]

(41)

The most dominant divergent term is

\[
\delta B_\theta = -\frac{R}{4\delta_r} B_0 v_0^2 \sin \theta \sin^2 \phi
\]

(42)

Eq. (42) gives an estimate of the magnetic field perturbation - hence the justification of the first order expansion. The condition \(\delta B_\theta \leq B_0\) implies that the first order expansion is valid for

\[
\frac{\delta_r}{R} \geq v_0^2,
\]

(43)

consistent with (35).
FIG. 4. 3D view of first order electric field [32]. The central sphere is the neutron star. Blue surface is the magnetic field flux surface (magnetic field lines lie on the surface pointing in the $z$ direction. Arrows are electric field sliced at $x = 0, y = 0, z = 0$. In the frame of the neutron star plasma is moving in the $+x$ direction. Bounded ear-like surfaces are regions where $\beta_{EM}$ becomes larger than 1.

Thus, both the electric field and the magnetic field diverge on the surface - this is electromagnetic draping. The electric field diverges in linear terms in $v_0$, magnetic field in $v_0^2$. The ratio of divergent terms in the first order electric field and second order magnetic field
FIG. 5. Flow lines in the $z = 0$ and $y = 0$ plane. A slight disconnection at $x = 0$ is an artifact of the plotting procedure. In the plane $x = 0$ the velocity is $\beta_{EM} = (y^2 + z^2)^{3/4} / \sqrt{(y^2 + z^2)^{3/2} - v_0^2 e_x}$.

Red lines indicate regions where $\beta_{EM} \geq 1$.

Thus, the divergent second order term in magnetic field cannot generally compensate for the divergent first order term in electric field.

Next, the longitudinal current

$$J_\parallel = G(r, \theta, \Phi)B_0$$

follows from stationary condition

$$\text{div} (J_{EM} + J_\parallel) = 0$$

We find

$$\text{div} J_{EM} \approx \left( \frac{1}{\delta_r^2} - 3 \frac{1}{R \delta_r} \right) \sin \phi \cos \phi v_0^2 B_0$$

Function $G$ must be $\propto \sin \phi \cos \phi$, and we find

$$\text{div} (GB_0) \approx (2 \delta_r \cos \theta \partial_r G - \sin \theta \partial_\theta G) \frac{3}{4} \sin(2\phi) \frac{B_0}{R}$$

To match $\theta$-independent $\text{div} J_{EM}$ function $G$ should be necessarily divergent either at $\theta = 0$ (the $\partial_\theta G$ term) or at $\theta = \pi/2$ (the $\partial_r G$ term)
IV. DOUBLE MAGNETIZED (ANTI)ALIGNED CASE

Results of the single magnetized neutron star can be generalized to the double magnetized aligned or anti-aligned case in the case when the reconnection effects are not important and the magnetospheres remain topologically disconnected [see 5, 16 for the case when the
magnetospheres are strongly coupled]. Recall that for a metal ball in external magnetic field, the field is a sum of dipole and external field. For double magnetized case, then the parameter $R$ is the radius where the field of the star matches the external field, Fig. 7. Equivalently, in expression for $B_d$, a change $R^3 B_0 \to \mu$ in Eq. (16), the magnetic moment of the star. In the anti-aligned case, when the magnetic moment opposes the external field, there are no currents; in the opposite aligned case there is a toroidal surface current at $R$.

![Diagram of double magnetized anti-aligned case](image)

FIG. 7. Double magnetized anti-aligned case. Topologically disconnected intrinsic dipolar field matches the external field at $r = R$. The black circle in the center indicates the neutron star.

The location of the boundary between the external magnetic field and that of the neutron star magnetosphere is not fixed now (for single-magnetized case it was the surface of the star). But as we discuss in Appendix A, any distortion of the surfaces is second order in $v_0$. Thus, in the linear regime all the previous derivations for the 1M-DNS case remains valid.

V. DISCUSSION

In this paper we argue that effects of electromagnetic draping - creation of dissipative layer near the merging neutron star may lead to generation of observable precursor emission. The draping effect is well known in space and astrophysical plasmas [4, 6, 12]. In the conventional
MHD limit, when the electric field is not an independent variable, creation of the magnetized layer (for super-Alfvenic motion) does not lead to dissipation, only break-down of the weak-field approximation in the draping layer.

We argue that relativistic plasmas are different. In this case the electric field is an independent dynamic variable; also charge densities are important. As a result, the set of ideal conditions, $\mathbf{B} \cdot \mathbf{E} = 0$ and $B \geq E$, is violated. Since the approach we took - expansion in small velocity - involves step-by-step approximation, it is feasible that higher order effects will smooth-out the divergencies. We think this is unlikely: divergent first-order electric field is not compensated by the second order magnetic field, Eq. (44). Instead, the second order magnetic field is divergent on its own. Divergent electric currents, Eq. (40) will lead to resistive dissipation.

Thus, we expect electromagnetic dissipation near the neutron star (or magnetospheric boundary). Particle will be accelerated and eventually collimated to move that particle along magnetic field lines, Fig. 8.

FIG. 8. Expected jets from a neutron star moving through force-free magnetic field. Yellow regions are dissipative regions, $E \geq B$. Quasi-cylindrical surfaces are magnetic flux surfaces. Dissipation within the $E \geq B$ regions would produce double-tongue-like jet structures.

The effect of collimation may be important for the detection of precursors, since the
expected powers are not very high. The expected powers in the 1M-DNS and 2M-DNS scenarios were discussed by [15]. If a neutron star is moving in the field of a primaries' dipolar magnetic field at orbital separation \( r \), the expected powers is \[ L_1 \sim \frac{GB_{NS}^2 M_{NS} R_{NS}^8}{cr^7} = 3 \times 10^{41} (-t)^{-7/4} \text{ erg s}^{-1} \] (49)
where in the last relations the time to merger \( t \) is measured in seconds. (Index 1 indicates here that the interaction is between single magnetized neutron star and unmagnetized one.) Magnetospheric interaction of two magnetized neutron stars can generate larger luminosity that the case of one star moving in the field of the companion [15]. In this case
\[ L_2 \sim \frac{B_{NS}^2 GM_{NS} R_{NS}^6}{cr^5} = \frac{c^{21/4} B_{NS}^2 R_{NS}^6}{(-t)^{5/4} (GM_{NS})^{11/4}} = 6 \times 10^{42} (-t)^{-5/4} \text{ erg s}^{-1} \] (50)
(Index 2 indicates here that the interaction is between two magnetized neutron star.) The ratio of luminosities of the models 1M-DNS and 2M-DNS is
\[ \frac{L_2}{L_1} = \left( \frac{GM}{c^2 R_{NS}} \right)^{3/2} \sqrt{\frac{(-t)c}{R_{NS}}} \approx 16 \sqrt{-t} \] (51)
Thus \( L_2 \) dominates \( L_1 \) prior to merger. This is due to larger interaction region, of the order of teh magnetospheric radius, instead of the radius of a neutron star.

Qualitatively, for the non-magnetar magnetic field the power (50) is fairly small. Even at the time of a merger, with \( t \sim 10^{-2} \) seconds the corresponding power is only \( L \sim 10^{45} \text{ erg s}^{-1} \) - hardly observable from cosmological distances by all-sky monitors. The best case is if a fraction of the power (50) is put into radio. If a fraction of \( \eta_R \) of the power is put into radio, the expected signal then is
\[ F_R \sim \eta_R \frac{L_2}{4 \pi d^2 \nu} \approx 0.1 \text{ Jy} \eta_R (-t)^{-5/4} \] (52)
This is a fairly strong signal that could be detected by modern radio telescopes.

Our results indicate that even in the single magnetized case we expect relativistic jets (or “tongues”) produced due to electromagnetic interaction of merging neutron stars, with correspondingly beamed emission pattern. Another way to produce higher luminosity is at the moments of topological spin-orbital resonances [5].

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VI. DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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Appendix A: 1st order variation of $\alpha - \beta - \Phi$ are vanishing

Here we demonstrate that in the first order of $v_0$ the variation of $\alpha - \beta - \Phi$ are vanishing. Let us expand

$$\alpha = \alpha_0 + \epsilon \alpha_1$$
$$\beta = \beta_0 + \epsilon \beta_1$$
$$\Phi = \Phi_0 + \epsilon \Phi_1$$ \hspace{1cm} (A1)

The orthogonality constraint

$$(\nabla \alpha) \cdot (\nabla \beta) = 0 \rightarrow (\nabla \alpha_0) \cdot (\nabla \beta_1) + (\nabla \alpha_1) \cdot (\nabla \beta_0) = 0$$ \hspace{1cm} (A2)

implies

$$\nabla \alpha_1 = a_1 \nabla \alpha_0 + a_2 \nabla \Phi_0$$
$$\nabla \beta_1 = b_1 \nabla \beta_0 + b_2 \nabla \Phi_0$$ \hspace{1cm} (A3)

On the other hand,

$$(\nabla \alpha) \cdot (\nabla \Phi) = 0 \rightarrow (\nabla \alpha_0) \cdot (\nabla \Phi_1) + (\nabla \alpha_1) \cdot (\nabla \Phi_0) = 0$$ \hspace{1cm} (A4)

hence

$$\nabla \alpha_1 = a_1 \nabla \alpha_0 + d_2 \nabla \beta_0$$
$$\nabla \Phi_1 = c_1 \nabla \Phi_0 + c_2 \nabla \beta_0$$ \hspace{1cm} (A5)

Thus, to keep all surfaces orthogonal we need

$$\nabla \alpha_1 = a_1 \nabla \alpha_0$$
$$\nabla \beta_1 = b_1 \nabla \beta_0$$
$$\nabla \Phi_1 = c_1 \nabla \Phi_0$$ \hspace{1cm} (A6)

Thus, first order perturbations are “locked in”.
Appendix B: Comparing electric field (32) with other cases

The electric field (32) is not too different from the vacuum case, where for electric field along $y$ direction at infinity

$$\Phi^{(\text{vac})} = E_0 (1 - (R/r)^3) r \sin \theta \sin \Phi$$

$$E_r^{(\text{vac})} = -(1 + 2R^3/r^3) \sin \theta \sin \Phi E_0$$

$$E_\theta^{(\text{vac})} = (1 - (R/r)^3) r \cos \theta \sin \Phi E_0$$

$$E_\phi^{(\text{vac})} = (1 - (R/r)^3) \cos \Phi$$  \hspace{1cm} (B1)

with surface charge density

$$\sigma^{(\text{vac})} = \frac{3}{2\pi} \sin \Phi \sin \theta E_0$$  \hspace{1cm} (B2)

Fields (32) and (B1) have the same angular dependence, but different radial dependence. The electric field (B1) has a non-zero component along $B_0$.

$$\mathbf{E}^{(\text{vac})} \cdot \mathbf{B}_0 = \frac{3 R^3 (r^3 - R^3) \sin(2\theta) \sin \phi}{4r^6} E_0 B_0$$  \hspace{1cm} (B3)

Another possible approximation, that of an incompressible flow around a sphere with kinematically added magnetic field, with velocity

$$\mathbf{v}^{(\text{inc})} = \{ -(1 - R^3/r^3) \sin \theta \cos \phi, -(1 + R^3/(2r^3)) \cos \theta \cos \phi, (1 + R^3/r^3) \sin \phi \} v_0$$  \hspace{1cm} (B4)

would produce electric field with similar angular dependence,

$$\mathbf{E}^{(\text{inc})} = -\mathbf{v}^{(\text{inc})} \times \mathbf{B}_0 =$$

$$E_r = \left(1 + R^3/(2r^3)\right)^2 \sin \theta \sin \phi v_0 B_0$$

$$E_\theta = \left(1 - R^3/(2r^3) - R^6/(2r^6)\right) \cos \theta \sin \phi v_0 B_0$$

$$E_\phi = \left(1 - R^3/(2r^3) - R^6/(2r^6)\right) \cos \phi v_0 B_0$$  \hspace{1cm} (B5)

A drawback of this approach is that the electric field has non-zero curl

$$\nabla \times \mathbf{E}^{(\text{inc})} = \left\{ 0, -\frac{9R^6 \cos \phi}{4r^7}, \frac{9R^6 \cos \theta \sin \phi}{4r^7} \right\} v_0 \mathbf{B}_0$$  \hspace{1cm} (B6)

and hence cannot be stationary.