Particle Acceleration by Static Black Holes in a Model of \( f(R) \) Gravity

M. Halilsoy\( ^{\ddagger} \) and A. Ovgun\( ^{\dagger} \)

Physics Department, Eastern Mediterranean University, Famagusta, Northern Cyprus, Mersin 10, Turkey.

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Particle collisions are considered within the context of \( f(R) \) gravity described by \( f(R) = R + 2\alpha\sqrt{R} \), where \( R \) stands for the Ricci scalar and \( \alpha \) is a non-zero constant. The center of mass (CM) energy of head-on colliding particles moving in opposite radial directions near the naked singularity/horizon are considered. Collision of particles in the same direction near the event horizon yields finite energy while the energy of oppositely moving particles grows unbounded. Addition of a cosmological constant does not change the feature. Collision of a massless outgoing photon with an infalling particle and collision of two oppositely moving photons following null-geodesics are also taken into account.

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I. INTRODUCTION

In particle accelerators, physicists routinely accelerate elementary particles and bring them to collision. Banados, Silk and West (BSW) \[1\] have proposed a scenario, where black holes may act as particle accelerators and first showed that the collision of geodesic particles in the vicinity of a black hole horizon yields a total unbounded centre of mass (CM) energy. This amounts to a natural collision process similar to the artificially tested process in the high-energy laboratory at CERN. The difference is that the latter is under strict human control albeit a bit too expensive process whereas the former one is free of charge, occurring in cosmos frequently as an ordinary event. Not only do the black holes create a similar BSW effect, but also naked singularities as well as the throat regions of wormholes \[2\]. Rotating black holes and wormholes act more efficiently in comparison with static ones to yield a high CM energy \[3\]-\[12\]. Another aspect of the BSW effect is that it occurs irrespective of the dimensionality of spacetime or the nature of the underlying theory. That is, even in lower/higher dimensions of 3+1- spacetime we can have an accelerator effect \[13\]-\[37\]. Collision must take place near the horizon of the formed black hole or naked singularity so that the particles get boost from the unlimited attraction/repulsion \[1, 38\]-\[40\].

The main aim of the paper is to study the BSW process which is useful to detect relic cold dark matter particles which are located around the black holes. Massive dark matter particles collide each others and the center of mass energy may reach arbitrarily high energies \[1\]. In our earlier study \[35\] we showed that modification of the Einstein gravity, Horava-Lifshitz gravity, does not always lead to infinite CM energy. Since so far the BSW effect is found, first time in this paper we investigate the possibility of BSW effect in the modified Einstein theory known as the \( f(R) \) gravity with static, diagonal metrics \[41\]. In this theory the Einstein-Hilbert action characterized by the Ricci scalar \( R \) is extended to cover an arbitrary function of \( R \). Theoretically such a concept has an infinite number of extensions which are to be severely restricted by experimental tests. Naturally any higher power of \( R \) hosts higher order derivatives of the metric and expectedly obtaining exact solutions is not an easy task at all. The solution for \( f(R) \) gravity that we shall consider in this study is a static one with \( f(R) = R + 2\alpha\sqrt{R - 4\Lambda} - 2\Lambda \), in which the constant \( \alpha \neq 0 \), so that our model of \( f(R) \) has no vacuum Einstein limit. By comparison with the Schwarzschild - de Sitter line element the second integration constant \( \Lambda \) can be interpreted as a cosmological constant. Since it is expected that most rotating black holes would have faced the BSW effect, in fact modification of the Einstein theory changes the structure of the black hole and the BSW effect in some cases occur and dark matter particles is accelerated. We concentrate here on our main new result, namely computation of the limiting energy for \( f(R) \) black holes. In this paper we also address the issue of CM of high energy collisions in the absence of an event horizon and near the naked singularity.

We consider first \( \alpha < 0 \) case which represents a black hole in which collision of two infalling particles takes place near the event horizon. Oppositely moving particles near the event horizon does yield BSW effect, however, the physical
situation prohibits the existence of outgoing particles from the event horizon \cite{36,37}. For that reason we base our argument on some physical processes that involve decay/ disintegration of particles outside the horizon. Once such a process is assumed valid there will be outgoing as well as infalling particles in the vicinity of a black hole. As a result the existence of outgoing particles/photons will naturally invite the process of collision with different infalling particles/photons. Next, we consider the case $\alpha > 0$, as a naked singularity at $r = 0$ and the collision of two oppositely moving particles near $r \approx 0$. It is found that due to the physical constraints, such as real momenta no unbounded CM energy arises from collisions in the vicinity of naked singularity. We note that the outgoing particle may be attributed due to the repulsive effect of the naked singularity which reverses/rebounds the particles and photons from $r \approx 0$. We cite as an example the case of negative mass Schwarzschild metric which gives rise to repulsive gravitational effect. Particles/photons turning outward can naturally make geodesic collisions with the incoming particles.

The paper is organized as follows: Section II summarizes static spherically symmetric black holes and naked singularities in a model of f(R) gravity. Collision of particles, between outgoing and infalling particles near the naked singularities and event horizons are analysed in Section III. Section IV considers collisions involving photons, both outgoing and infalling. We complete the paper with our conclusion in Section V.

II. SPECIFIC BLACK HOLE/ NAKED SINGULARITY

The action of general, sourceless $f(R)$ gravity theories in four dimension is

$$S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} f(R),$$

in which $f(R)$ is the function of the scalar curvature $R$, and $g$ stands for the determinant of the metric tensor \cite{41}. The spherically symmetric line element is given by

$$ds^2 = -Adt^2 + \frac{1}{A} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

in which the function of $f(R)$

$$f(R) = R + 2\alpha \sqrt{R},$$

with the constant $\alpha \neq 0$, yields the solution

$$A(r) = \frac{1}{2} + \frac{1}{3\alpha r},$$

with the Ricci scalar

$$R = \frac{1}{r^2}.$$  

Inclusion of a cosmological constant $\Lambda$ yields the function $f(R)$ as follows \cite{42}

$$f(R) = R + 2\alpha \sqrt{R - 4\Lambda} - 2\Lambda,$$

and the corresponding metric function of $A(r)$ is

$$A(r) = \frac{1}{2} + \frac{1}{3\alpha r} - \frac{\Lambda}{3} r^2,$$

with the scalar curvature

$$R = \frac{1}{r^2} + 4\Lambda.$$

Note that the fact that $\alpha \neq 0$ is already revealed by the metric function $A(r)$. In the sequel for both cases, $\Lambda = 0$ and $\Lambda \neq 0$ we shall investigate the possibility of BSW effect. Lastly, for the case of $\alpha > 0$, ($\Lambda = 0$) which corresponds to
a naked singular solution at \( r = 0 \) we shall search for the collider effect. For the case of naked singularity, the metric function is calculated (let \( \Lambda = 0 \)) as follows

\[
A = \frac{1}{2} + \frac{1}{3\alpha r}. \tag{9}
\]

Obviously in the two parametric solution employed, \( A \) is a dispensable parameter whereas \( \alpha \) not. That is, our choice of \( f(R) \) gravity lacks the Einstein’s general relativity limit. With deliberation we have made such a choice to see the significance of the BSW effect in a \( f(R) \) model that is not connected with the general relativity. This is precisely the case with \( \alpha \neq 0 \).

### III. PARTICLE COLLISION NEAR \( f(R) \) BLACK HOLE AND NAKED SINGULARITY

We wish to check first the role of event horizon when particles collide in case our metric is static/diagonal in \( f(R) \) gravity. For different cases we investigate the CM energy for the collision, 4-d velocity components of the colliding particles in the background of the 4-d \( f(R) \) black holes by taking the radial motion on equatorial plane \( (\theta = \frac{\pi}{2}) \) (Fig. 1).

![Diagram of particle collision](image)

FIG. 1: The schematic figure of particle collision for which the CM energy can be very large (Particles 1 and 3 are infalling while particle 2 is outgoing)

Our Lagrangian is chosen by

\[
L = \frac{1}{2} \left( -At^2 + \frac{1}{A} \dot{r}^2 + r^2 \dot{\phi}^2 \right), \tag{10}
\]

in which a dot implies derivative with respect to proper time. The velocities follow as

\[
\dot{u}^t = t = \frac{E}{A}, \tag{11}
\]

and

\[
\dot{u}^\phi = \phi = \frac{L}{r^2}, \tag{12}
\]

where \( E \) and \( L \) are the energy and angular momentum constants, respectively. By using the normalization condition \((u,u = -1)\), it is found that the radial velocity is

\[
\dot{u}^r = \dot{r} = \pm \sqrt{E^2 - A \left(1 + \frac{L^2}{r^2} \right)}, \tag{13}
\]
and clearly we are interested in time-like geodesics. We proceed now to present the CM energy of two particles with four-velocities \( u_1^\mu \) and \( u_2^\mu \). We assume that both have rest mass \( m_0 = 1 \). The CM energy is given by,

\[
E_{\text{cm}} = \sqrt{2}(1 - g_{\mu\nu}u_1^\mu u_2^\nu),
\]

so that it can be expressed as

\[
\frac{E_{\text{cm}}^2}{2} = 1 + \frac{E_1E_2}{A} - \kappa \frac{|L_1||L_2|}{r^2} - \kappa \frac{1}{A}\sqrt{E_1^2 - A \left(1 + \frac{L_1^2}{r^2}\right)}\sqrt{E_2^2 - A \left(1 + \frac{L_2^2}{r^2}\right)},
\]

where \( \kappa = \pm 1 \) correspond to particles moving in the same direction (\( \kappa = +1 \)) or opposite direction (\( \kappa = -1 \)), respectively. Furthermore \( E_1 \) and \( E_2/\sqrt{r^2} \) and \( L_1 \) and \( L_2 \) are defined as the energy/ angular momentum constants corresponding to each particle. Upon taking the lowest order terms in the vicinity of the horizon, since \( A \approx 0 \), we can make the expansion

\[
\sqrt{\left(E^2 - A \left(1 + \frac{L_1^2}{r^2}\right)\right)} \approx E \left[1 - \frac{A}{2E^2} \left(1 + \frac{L_1^2}{r^2}\right) + \ldots\right],
\]

so that the CM energy of two particles is obtained as

\[
\frac{E_{\text{cm}}^2}{2} \approx 1 + (1 - \kappa)\frac{E_1E_2}{A} - \kappa \frac{|L_1||L_2|}{r^2} + \kappa \frac{1}{2} \left[\frac{E_2}{E_1} \left(1 + \frac{L_1^2}{r^2}\right) + \frac{E_1}{E_2} \left(1 + \frac{L_2^2}{r^2}\right)\right].
\]

We investigate the BSW effect whether occurs or not for \( A(r) \to 0 \) whenever there is a horizon. In the case of the collision of ingoing/ingoing or outgoing/ingoing particles (i.e. motion in same directions) \( \kappa = +1 \), it reduces to

\[
\frac{E_{\text{cm}}^2}{2} \approx 1 - \left|\frac{L_1||L_2|}{r^2}\right| + \frac{1}{2} \left[\frac{E_2}{E_1} \left(1 + \frac{L_1^2}{r^2}\right) + \frac{E_1}{E_2} \left(1 + \frac{L_2^2}{r^2}\right)\right].
\]

The occurrence of outgoing particles is crucial for a diverging \( E_{\text{cm}}^2 \). Such an outgoing particle may be attributed to a decay/disintegration process in the vicinity of the horizon. While one of the particle falls into the hole its pair moves outward to collide with an infalling particle.

**A. Particle Collision near the \( f(R) \) Black Holes with a Cosmological Constant**

The second case of interest is for the chosen \( f(R) \) black hole model with a cosmological constant in which the metric function \( A \) is

\[
A = \frac{-\Lambda r^2}{3} + \frac{1}{2} + \frac{1}{3\alpha r},
\]

where the event horizon is located at

\[
r_h = \frac{\Xi}{2\alpha \Lambda} + \frac{\alpha}{2},
\]

for

\[
\Xi = \left(4 + 2\alpha^2 \Lambda^2 \sqrt{-2\alpha^2 - 4\Lambda} \Lambda\right)^{1/2}.\]

At this point we must add that we are not interested in the other roots of \( A(r) = 0 \) that specify the inner horizon. It is observed that for real \( r_h \) we must have \( -2\alpha^2 - 4\Lambda > 0 \), which restricts the cosmological constant to the case of \( \Lambda < 0 \).

As in the case of \( \kappa = +1 \) above the CM energy \( E_{\text{cm}}^2 \) is finite. Collision of an infalling and outgoing particle \( \kappa = -1 \) , however, does yield a BSW effect.
B. Particle Collision near the Naked Singularity

There is a naked singularity for our $f(R)$ model at the location of $r = 0$, with $\alpha > 0$, where the metric function is given by (let $\Lambda = 0$)

$$A = \frac{1}{2} + \frac{1}{3\alpha r}. \quad (22)$$

As it is calculated above the collision of two particles generally is

$$\frac{E_{cm}^2}{2} = 1 + \frac{E_1 E_2}{A} - \kappa \frac{|L_1| |L_2|}{r^2} - \kappa \frac{1}{A} \sqrt{E_1^2 - A \left(1 + \frac{L_1^2}{r^2} \right)} \sqrt{E_2^2 - A \left(1 + \frac{L_2^2}{r^2} \right)}. \quad (23)$$

Let us note that each term under the square root must be positive. Such a constraint restricts the range of $r$ to stay away from the naked singularity. Once $r$ is finite the overall CM energy also must be finite and therefore we observe no diverging result from the presence of the naked singularity. Choosing the pure radial motions. (i.e. $L_1 = L_2 = 0$) also does not change the feature of the problem.

IV. COLLISIONAL PROCESSES WITH PHOTONS

A. In Naked Singular Spacetime

The outgoing massless photon presumably reflected from the naked singularity can naturally scatter an infalling particle or vice versa. This phenomenon is analogous to a Compton-like scattering process which was originally introduced for a photon and an electron. The null-geodesics for a photon satisfies

$$\frac{dt}{d\lambda} = t = \frac{E_\gamma}{A}, \quad (24)$$

and

$$\frac{d\varphi}{d\lambda} = \phi = \frac{L_\gamma}{r^2}, \quad (25)$$

$$\dot{r} = \pm \sqrt{\left[\frac{E_\gamma^2}{A^2} - \frac{\kappa L_\gamma^2}{r^2}\right],} \quad (26)$$

where $\lambda$ and $E_\gamma$ are an affine parameter and the photon energy, respectively. Defining $E_\gamma = h \omega_0$, where $\omega_0$ is the frequency (with the choice $\hbar = 1$) we can parametrize the energy of the photon by $\omega_0$ alone. The center-of-mass energy of an outgoing photon and the infalling particle can be taken now as

$$E_{cm}^2 = -(p^\mu + k^\mu)^2, \quad (27)$$

in which $p^\mu$ and $k^\mu$ refer to the particle and photon, 4-momenta, respectively. It is needless to state that for a photon we have $k^2 = 0$. This amounts to (for $\theta = \pi/2$)

$$E_{cm}^2 = m^2 - 2mg_{\mu\nu}p^\mu k^\nu. \quad (28)$$

Since we have for the particle

$$p^\mu = m \left(\frac{E_1}{A}, \sqrt{E_1^2 - A \left(1 + \frac{L_1^2}{r^2} \right)}, 0, \frac{L_1}{r^2} \right), \quad (29)$$

and for the photon

$$k^\mu = \left(\frac{E_\gamma}{A}, \sqrt{E_\gamma^2 - \frac{\kappa L_\gamma^2}{r^2}}, 0, \frac{L_\gamma}{r^2} \right), \quad (30)$$
one obtains
\[ E_{cm}^2 = m^2 + \frac{2mE_1E_2}{A} - \frac{2\kappa m|L_1||L_2|}{r^2} - \frac{2m\kappa}{A} \sqrt{E_2^2 - \frac{AL_1^2}{r^2}} \sqrt{E_1^2 - A \left( 1 + \frac{L_1^2}{r^2} \right)}. \] (31)

which amounts to a collision process that must occur away from the singularity \( r = 0 \), i.e. \( E_{cm}^2 \) remains finite.

**B. Photon - Photon Collision Near the Naked Singularity**

Let us consider the problem of collision between two photons in the naked singular spacetime. The photons follow null geodesics in opposite directions and make head-on collision. In quantum electrodynamics colliding energetic photons can transmute into particles. Since our analysis here is entirely classical we shall refer only to the CM energy of the yield without further specification. The CM energy of the product satisfies
\[ E_{cm}^2 = -(k_1^\mu + k_2^\mu)^2 = -2g_{\mu\nu}k_1^\mu k_2^\nu, \] (32)

where \( k_1 \) and \( k_2 \) correspond to the 4- momenta of respective photons. From the null- geodesic analysis in the \( \theta = \frac{\pi}{2} \) plane, we have
\[ k_1^\nu = \left\{ \frac{E_1}{A}, \sqrt{E_1^2 - \frac{AL_1^2}{r^2}}, 0, \frac{L_1}{r^2} \right\}, \] (33)
\[ k_2^\nu = \left\{ \frac{E_2}{A}, \sqrt{E_2^2 - \frac{AL_2^2}{r^2}}, 0, \frac{L_2}{r^2} \right\}, \] (34)

where \( E_1 \) and \( E_2 \) are the corresponding energies of different photons. Upon substitution into (32) we obtain
\[ \frac{1}{2}E_{cm}^2 = \frac{E_1E_2}{A} - \kappa \frac{1}{A} \sqrt{E_1^2 - \frac{AL_1^2}{r^2}} \sqrt{E_2^2 - \frac{AL_2^2}{r^2}} - \frac{|L_1||L_2|}{r^2}, \] (35)
in which we have to insert \( \kappa = \pm 1 \) to specify the parallel/anti-parallel propagation of the photons.

1) For \( \kappa = +1 \), which implies two parallel photons moving at the speed of light naturally don’t scatter, so we observe no noticeable effect.

2) For \( \kappa = -1 \), however, the photons are moving in opposite directions and inevitably they collide. In classical background each naturally follows a null geodesics. Their corresponding CM energy from our foregoing analysis that \( r_i^2 > \frac{AL_i^2}{E_i^2} \) for each \( i = 1, 2 \) remains also finite.

Let us comment that this is a collision of test photons on a given geometry without backreaction effect. On the other hand, exact collision of electromagnetic shock plane waves in Einstein’s gravity, as a highly non- linear process \cite{44} \cite{45} is entirely different. As a result of mutual focusing, the latter develops null- singularities after the collision process. Our conclusion is that, at the test level, collision of two oppositely moving photons in a naked singular spacetime yields no observable effect.

**C. Photon - Photon Collision Near the Horizon**

From the analysis in part B above the CM energy of two photons is adapted as
\[ \frac{1}{2}E_{cm}^2 = \frac{E_1E_2}{A} - \kappa \frac{1}{A} \sqrt{E_1^2 - \frac{AL_1^2}{r^2}} \sqrt{E_2^2 - \frac{AL_2^2}{r^2}} - \frac{|L_1||L_2|}{r^2}, \] (36)

with the supplement that now we search the case for the limit \( A \to 0 \), instead of \( A \to \infty \), since \( r = r_h = \text{finite} \). We obtain
\[ \frac{1}{2}E_{cm}^2 \approx \frac{E_1E_2}{A} (1 - \kappa) + (\text{finite terms}), \]
which suggests that for $\kappa = -1$, i.e. for oppositely moving photons, it yields an unbounded CM energy. For parallel photons, both ingoing, naturally $\kappa = +1$ and there is no observable effect.

How can an outgoing photon from the near-horizon region can be justified since the Hawking photons remains too weak to cope/scatter with an infalling one? An outgoing photon can be created in an explosion/decay process by a physical particle before it falls into the horizon. Such an assumption yields an outgoing photon and naturally it has the chance to collide with an opposite photon and give rise to an unbounded energy. This is exactly what happens for two colliding oppositely moving electromagnetic plane waves to focus each other and create a null singularity\textsuperscript{[44, 45].}

V. CONCLUSION

Collision of particles near black hole horizons in Einstein’s general relativity, i.e. the BSW effect, has been considered in details during recent years. Oppositely moving particle collisions near static black holes were also considered in \textsuperscript{9, 17}. Besides static, charged and rotating black holes were also investigated. In particular, rotational effects were shown from the original Penrose process long ago \textsuperscript{[46].} This has a significant role in the extraction of energy from the black holes. In this paper, we investigated the idea of BSW to the modified theory known as $f(R)$ gravity. In particular we concentrated on $f(R) = R + 2\alpha\sqrt{R - 4\Lambda} - 2\Lambda$, which arises as an exact, source-free spherically symmetric solution that the external energy-momentum tensor vanishes, but the curvature makes its own source. We can easily set $\Lambda = 0$, however $\alpha \neq 0$ is an essential parameter of the model so that our model does not have the general relativity limit of $f(R) = R$. For $\alpha < 0$ we have the black hole while for $\alpha > 0$ we obtain a naked singularity at $r = 0$.

In case of black hole we show the existence of BSW effect provided that outgoing particles from some physical process is taken for granted. Collision of an ingoing and outgoing particle $\kappa = -1$ near the horizon of the $f(R)$ black holes with/ without a cosmological constant, however, does yield a BSW effect. Near a naked singularity, however, we observe no efficient collision to increase the CM energy unbounded. For oppositely moving particles a similar result can also be obtained for a Compton-like process between a photon and a particle provided that they move in opposite directions. Collision of two oppositely moving photons near the naked singularity also yields no diverging CM energy.

On the other hand, for oppositely moving particles, it yields an unbounded CM energy. Therefore it is clear that CM energy depends on the direction of the particles with the parameter of $\kappa$ and the collision of the oppositely moving particles must be near the horizon of the black hole.

The CM energy distribution of relic cold dark matter particles colliding in lower/higher dimensions will be discussed in a future publication with comparing the observational data that give the possible excess of gamma rays observed in Fermi data at WIMP-scale energies \textsuperscript{[48].} Moreover, it is our belief that seeking an alternative model of gravity, which can lead to BSW effect will be useful in the searching of the dark matter. On this purpose we will investigate the BSW process for black holes/strings or wormholes to look at the CM energy of the colliding neutral/charged particles. This is going to be our next problem in the near future.

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