FERMION MASSES IN SO(10)

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ABSTRACT

Yukawa coupling constant unification together with the known fermion masses is used to constrain SO(10) models. We consider the case of one (heavy) generation, with the tree-level relation $m_b = m_\tau$, calculating the limits on the intermediate symmetry breaking scales. This analysis extends previous analyses which addressed only the simplest symmetry breaking schemes. In the case where the low energy model is the standard model with one Higgs doublet, there are very strong constraints due to the known limits on the top mass and the tau neutrino mass. The two Higgs doublet case is less constrained. Finally we address the role of a speculative constraint on the tau neutrino mass, arising from the cosmological implications of anomalous $B + L$ violation in the early universe.

1. Motivation

Non-zero neutrino masses arguably provide one of the more well motivated extensions of the standard model. Theoretically, there is a prejudice for a see-saw neutrino mass mechanism since the relative smallness of the neutrino masses is thereby naturally explained. Such a mechanism is readily available in grand-unified theories which display spontaneous violation of lepton number at a high scale. The simplest such model is the $SO(10)$ GUT. Of the two possible maximal subgroups which can appear in the symmetry breaking chain, only $SU(2)_L \times SU(2)_R \times SU(4)$ is viable phenomenologically. This is the Pati-Salam intermediate unification, and displays the left-right symmetry directly. In many ways, $SO(10)$ is the canonical implementation of grand unification with spontaneous violation of lepton number. Thus it seems worthwhile to study neutrino masses in this model, in particular their relation to the other fermion masses so that we can understand the constraints on the model from measured (or constrained) masses.

Two major features of the current experimental situation have led to a revival in GUT calculations. First, there is the increasing precision of coupling constant measurements at the $Z$ resonance. Second, there is the increasing lower bound on the top quark mass. Precision measurements of gauge couplings provide constraints on intermediate symmetry breaking scales, and a large $m_\tau$-$m_b$ splitting may be difficult to reconcile with a unified fermion multiplet without some gymnastics. Furthermore, such a large splitting has important consequences for neutrino masses, since the lepton sector must mirror the quark sector.

The first step in gaining a quantitative understanding of fermion masses in $SO(10)$ is to calculate radiative corrections to the tree-level mass relations within

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1Presented at DPF 92 Meeting, Fermilab, November, 1992.
the fermion multiplet. The results of these calculations and their implications are discussed in Ref. [4].

Considering the nature of this note, references are sparse. For a more complete list of references, we refer to Ref. [4].

2. The Model

$SO(10)$ has nothing to say about the repetition of generations exhibited by nature, and as a first approximation we deal only with the heavy generation. One generation of fermions can Yukawa couple only to the scalar representations $\phi(10)$ or $\phi(126)$, and it must have Dirac Yukawa couplings predominantly [5] to the low-energy Higgs doublets which derive from $\phi(10)$. This gives the tree-level relations $m_b = m_\tau$ and $m_t = m_{\nu_D}$. Note that a small admixture of $\phi(126)$ mass relations is required in order to produce the correct pattern of mixing in the full multi-generation model, but this can arise in ways which do not require a finely tuned upset to these tree-level relations [3].

Of course, we must realize that there is a great deal of freedom for engineering the fermion spectrum of the model by introducing arbitrary complications in the Higgs sector. Therefore, before making predictions, we must choose a philosophy for a minimal model. This is less necessary when dealing with only one generation than it is when dealing with a full multi-generation model, considering the above remarks about couplings. We choose the minimal model to have only the Yukawa coupling of $\phi(10)$ necessary to generate the tree-level Dirac masses and the Yukawa coupling of $\phi(126)$ necessary to generate the tree-level Majorana mass for the right-handed neutrino. Our philosophy is to constrain the minimal model as much as possible.

The last piece of information necessary to specify the model is the number of Higgs doublets that we wish to have in the low-energy theory. Derived from a single $\phi(10)$, there can be one or two Higgs doublets. This choice is important for the radiative corrections, embodied by the Yukawa coupling renormalization group equations. In the one doublet model, the large $m_t$-$m_b$ splitting requires a large splitting in the top and bottom Yukawa couplings. In the two doublet model, the mass splitting is accounted for by the large ratio of the two Higgs doublet vevs, $\tan \beta = v_u/v_d \simeq 30$.

The Yukawa coupling beta functions are detailed in Ref. [4].

3. Results

The constraints we place on the model are as follows. First, the gauge coupling unification must be consistent with the measured values of gauge couplings; $\alpha_S(M_Z) = 0.11 \pm 0.01$, $\sin^2 \theta_W = 0.233 \pm 0.003$, and $\alpha(M_Z) = 0.00781$ with negligible error for our analysis. Second, the fermion masses must be consistent with $m_t = 1.78$ GeV, $m_t > 91$ GeV, and $m_b = 4.3 \pm 0.3$ GeV. The latter of these is clearly an important parameter, and an attempt is made in Ref. [4] not to hide the dependence on $m_b$. Remember this value of $m_b$ is the current mass and not the constituent mass which is fit in potential models to be $m_b^{\text{Cons}} \simeq 4.8$ GeV. Also note that we use a value of $\alpha_S(M_Z)$ which is somewhat smaller than the direct determinations. Such a value can arise
from certain analyses \[7\]. This is a conservative assumption for our purposes, as will be explained below. Finally, we apply the weakest of the cosmological constraints on the \(\nu_\tau\) mass \[8\] together with the direct limit, \(m_{\nu_\tau} < 70\,\text{eV} \) or \(1\,\text{MeV} < m_{\nu_\tau} < 35\,\text{MeV}\).

Consider first the case of one low-energy Higgs doublet. Previously, it had been shown that it was difficult to reconcile the large \(m_t-m_b\) splitting with unification and the measured \(m_b/m_\tau\) ratio, in a restricted symmetry breaking scheme \[9\]. By relaxing the constraint on the symmetry breaking scheme, we allow the top quark a bit more range, but the basic conclusion remains the same. For \(\alpha_s(M_Z) > 0.105\) the one Higgs doublet case is ruled out at the one sigma level, unless the tau neutrino mass lies in its upper window. Larger values of \(\alpha_s(M_Z)\) produce an unacceptably large \(m_b/m_\tau\), and thus we see why our choice of “smaller” values for \(\alpha_s(M_Z)\) is a conservative one.

The case of two low-energy Higgs doublets is almost unconstrained by the \(m_b/m_\tau\) ratio. This is because the beta function for the bottom quark Yukawa coupling is slightly more positive in this case, and \(m_b\) evolves into the middle of its allowed range.

Finally, we come to what may be the strictest of the constraints on these models. This is the constraint due to the observed baryon asymmetry of the universe. If lepton-number violating processes are in equilibrium with anomalous \(B+L\) violating processes in the early universe, the baryon asymmetry will be washed away \[10\][11]. At tree-level this constrains \(m_{\nu_\tau}\) sufficiently that the model cannot support the known lower bound on the top mass, with no regard to details such as the number of low-energy Higgs doublets. However, we find that the radiative corrections are large enough that this conclusion is tempered to a bound (roughly) \(m_t < 120\,\text{GeV}\). The real test of this sort of argument remains to be completed. The question must be addressed in the light of a full leptogenesis calculation, incorporating CP violation in the lepton sector and realistic multi-generation mass matrices \[12\].

4. References

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