Ballistic Target Translation Compensation Based on Corner Detection Algorithm

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Abstract. During the mid-flight of the ballistic target, there are two movement modes: translational and micro-movement. The existence of translation will cause tilt and translation of the time-frequency curve of the scatter point echo, which will affect the extraction of ballistic missile micro-motion information. In response to this problem, this paper proposes a Shi-Tomasi corner detection algorithm from the perspective of image processing, which can effectively extract the corner information in the image. Afterwards, regression analysis is performed according to the obtained corner point information to realize the estimation of translational parameters. Through simulation experiments, it is concluded that the algorithm can better realize the compensation of translational components.

1. Introduction
At present, with the increasing maturity of split-guided multiple warheads and missile decoy technologies, traditional feature recognition techniques based on coating characteristics, structural characteristics and conventional motion characteristics are difficult to identify true warheads from complex target groups [2]. Victor C. Chen of the U.S. Naval Laboratory first proposed the concept of micro-motion and micro-Doppler [3] and introduced it into the field of target recognition. The micro-motion information of the target is unique, so this feature can be used to accurately identify ballistic missiles [4].

During the mid-flight of a ballistic missile, its motion form consists of two motion modes: translational and micro-movement. The superposition of the translational component and the micro-movement component will cause the time-frequency curve of the scattering point echo to appear tilt and translation, and affect the extraction of the micro-motion information of the ballistic missile. Therefore, in order to extract the feature information of the micro-motion of the ballistic missile, the translational component and the micro-motion component should be separated first. At present, a lot of research has been done on translational compensation and some results have been obtained. Literature [5] proposed a parameter estimation method for the two-dimensional spectrum vector, which refines the echo signal spectrum, and uses the energy distribution and the position of the center of gravity to estimate the relevant parameters. Literature [6] uses a high-order ambiguity function to search for image peaks to estimate translational parameters to achieve the translational compensation of the target, but the solution requires relatively long observation time. Literature [7] proposed the Polar format algorithm (PFA) to decouple rotation and translation two-dimensionally, and use the autofocus algorithm to extract the phase, thereby achieving translational compensation. Literature [8] realizes the translational compensation of the target through multi-stage delay conjugate multiplication processing.

Based on the above research status, this paper proposes the Shi-Tomasi corner detection algorithm, which estimates the translational parameters and realizes the translational compensation of the target.
2. Precession Model of Cone Ballistic Target

During the flight, the ballistic missile itself will produce a lot of debris to form a target group. The target group in the middle of the trajectory differs from each other in the form of fretting and fretting parameters. The warhead spins around its symmetry axis to maintain stability. In the process of releasing the bait, the warhead will be disturbed to produce a conical motion around a certain axis in space. The superposition of the two motions will eventually form the precession motion posture of the projectile [9].

![Figure 1. Scattering model of precession cone ballistic target](image)

Figure 1 shows the precession scattering point model of the cone ballistic target. The coordinate origin $O$ is the center of mass of the cone ballistic target, and the $Z$ axis is the precession axis of the cone movement. At the initial moment, the azimuth angle of the radar line of sight $\text{LOS}$ in the coordinate system $O–XYZ$ is $\alpha$, the elevation angle is $\beta$, the angle with the spin axis is $\varphi$, and the angle between the cone rotation angle and the spin angle, that is, the precession angle is $\theta$. The initial distance between the center of mass of the target and the radar is $R$ (not shown here). The cone spin angular velocity of the target is $c\omega$, the spin angular velocity is $s\omega$, and the translational velocity is $v$. The distance from the center of mass $A$ to the scattering point is $l_1$, and the distance from the center of the bottom circle is $l_2$.

According to the literature [10], the formula for the change of the distance of the scatter point is as follows:

$$
\begin{align*}
    r_A &= l_1 \cos \beta(t) \\
    r_B &= -l_2 \cos \beta(t) - r \sin \beta(t) \\
    r_C &= -l_2 \cos \beta(t) + r \sin \beta(t)
\end{align*}
$$

Among them:

$$
\cos \beta(t) = \cos \theta \cos \alpha + \sin \theta \sin \alpha \sin(\omega t - \varphi_0)
$$

$\varphi_0$ is the angle between the projection of the target symmetry axis on the plane $XOY$ and the $OX$ axis at the initial time $t = 0$. Since the included angle $\varphi_0$ only affects the initial phase of the micro-Doppler, it has no effect on the period and amplitude, so we can take any value, without loss of generality, we can make $\varphi_0 = 0$ [10].

Regardless of the occlusion effect, all three scattering points can generate echoes, which are superimposed to form a total echo. Take the scattering point $A$ as an example below to analyze the relationship between the frequency and movement of the scattering point, as shown in Figure 2.
Figure 2. Schematic diagram of the distance between the radar and the scatter point of the cone target

The radar is located at the point position. When the radar recognizes and detects a target, its size is much smaller than the distance from the radar. Therefore, \( |O'A| \approx |O'H| \) and \( |O'A| = |O'O| + |OA| \) can be approximated, namely \( |O'H| = R + r_a \).

When there is a relative speed between the target and the radar, the carrier frequency of the received echo signal has a frequency shift relative to the carrier frequency of the transmitted signal, and its value is shown in the following formula:

\[
f_d = \frac{2v_r}{\lambda}
\]  

where \( f_d \) is the Doppler frequency shift, the unit is Hz; \( v_r \) is the radial velocity of the scattering point along the line of sight of the radar, the unit is \( m/s \); \( \lambda \) is the carrier wavelength, the unit is m.

According to Figure 2, \( v_r = \frac{dR_i}{dt} \) can be obtained, so the Doppler frequency of the scattering point is:

\[
f_{dt} = \frac{2v_r}{\lambda} = -2 \frac{dR_i}{\lambda} + f_i = f_i + f_{ii}
\]  

where \( f_i \) and \( f_{ii} \) are the Doppler frequency caused by target translation and micro-movement respectively. The above formula can show the simple superposition of translation and micro-movement. Therefore, as long as the translational component is compensated, the micro-motion information can be obtained and the relevant characteristic parameters can be extracted.

If the observation time is short, the translational component can be expressed approximately by a second-order mathematical polynomial, namely:

\[
R = R_0 + a_1 t + a_2 t^2 / 2
\]

where \( R_0 \) represents the initial distance of the cone target from the radar, \( a_1 \) represents the radial velocity of the cone target along the line of sight of the radar, and \( a_2 \) represents the radial acceleration of the cone target along the line of sight of the radar.

Therefore, the frequency generated by the cone target movement is:

\[
f_0 = f_i + f_{ii} = -2 \frac{dR}{\lambda} dt + f_i = -\frac{2}{\lambda} (a_1 + a_2 t) + f_i
\]

Suppose the radar transmits a single-frequency signal with carrier frequency \( f_0 \), namely:

\[
s_i(t) = \exp(j2\pi f_0 t)
\]

Then the echo of each scattering point received by the radar is:

\[
s_i(t) = \sum_j A_j \exp(j2\pi f_i t - j\frac{4\pi}{\lambda} R_j(t))
\]
where $A_i$ is the coefficient of the scattering point echo, $R_i(t)$ is the distance between the scattering point and the radar at time $t$.

The received echo signal includes carrier frequency signal $\exp(j2\pi f_c t)$, which can be removed by quadrature demodulation before sampling [11]. The demodulated baseband signal of the scattering point can be expressed as:

$$s(t) = \sum A_i \exp(-j\frac{4\pi}{A} R_i(t))$$  \hspace{1cm} (8)

According to the above formula, the time-frequency graph of the scattering point can be obtained. The time-frequency graph is formed by superimposing the time-frequency curves of multiple scattering points. Observing the time-frequency graph, it is found that there are multiple intersection points on the time-frequency graph. According to mathematical analysis, at these intersections, the micro-Doppler frequencies of the cone scattering points $A$, $B$, and $C$ are all 0, that is, $f_c=0$, which is only related to the Doppler frequency generated by the translational component. Therefore, the translational component can be obtained by extracting the intersection point information, and then the translational compensation for the scattering point can be realized.

3. Extract Image Features of Corner Points
According to the second section, in order to realize translation compensation, it is necessary to determine the position of the intersection in the time-frequency curve. For this reason, the Shi-Tomasi corner detection algorithm is introduced.

![Figure 3. Schematic diagram of window movement](image)

As shown in Figure 3, the window moves on the plane. If it is on a smooth plane, as shown in Figure 3(a), the window does not change in any direction; if it is on the edge area, as shown in Figure 3(b), the window does not change in the direction corresponding to the edge; if it is on the corner area, as shown in Figure 3(c). The window changes in all directions. According to Figure 3, the Shi-Tomasi algorithm uses window movement to determine whether a change occurs in any direction, and detects corners in the image.

At image $(x, y)$, the window is translated, the amount of translation is $[u,v]$, then the gray value of the image before and after the movement changes to

$$E(u,v) = \sum_{i,j} w(x,y) [I(x+u,y+v) - I(x,y)]^2$$  \hspace{1cm} (9)

where $w(x,y)$ is the weighting function, the window located at $(x,y)$, $I(x,y)$ is the gray value of the pixel at $(x,y)$, and $I(x+u,y+v)$ is the gray value of the pixel at $(x+u,y+v)$.

According to the total differential formula:

$$I(x+u,y+v) = I(x,y) + I_u u + I_v v + o(u^2, v^2)$$  \hspace{1cm} (10)

Bring into equation (9) to get
\[
E(u,v) = \sum_{x,y} w(x,y)(u^2I_x^2 + 2uvI_xI_y + v^2I_y^2) \quad (11)
\]

Change the above formula to matrix form as shown below

\[
E(u,v) = [u,v]^T \left[ \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \right] \begin{bmatrix} u \\ v \end{bmatrix} \quad (12)
\]

Make:

\[
M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \quad (13)
\]

\(M\) is a \(2 \times 2\)-dimensional matrix, then formula (12) can be simplified to

\[
E(u,v) = [u,v]^T M [u,v] \quad (14)
\]

Let the eigenvalues of matrix \(M\) be \(\lambda_1\) and \(\lambda_2\), define

\[
R = \min(\lambda_1, \lambda_2) \quad (15)
\]

Perform threshold processing on function \(R\), namely

\[
R > \text{threshold} \quad (16)
\]

Then the local maximum of \(R\) can be extracted. According to the local maximum value \(R\), the non-maximum value is suppressed to realize the corner detection. The specific algorithm steps are as follows:

Step 1: Set the gray value \(I(x,y)\) at image \((x,y)\), filter each pixel through the difference equation to obtain \(I_x\), \(I_y\), and then calculate \(I_x^2\), \(I_y^2\), and \(I_xI_y\).

Step 2: Gaussian filtering is performed on \(I_x^2\), \(I_y^2\) and \(I_xI_y\) to eliminate isolated points in the image, and the result obtained is used as an element in the matrix \(M\).

Step 3: Determine candidate corner points. Compare the gray values of the edge pixels in the window with the center pixels, determine whether they are within the given threshold, get the number of similar pixels, and then determine whether they are corner points to be selected.

Step 4: Calculate the pixel response function \(R\) using formula (16).

Step 5: Perform non-maximum value restriction to determine whether there are corner points in the pixels.

4. Simulation Experiment

Suppose the radar transmits a signal with a carrier frequency of \(f = 6\text{GHz}\) and a pulse repetition frequency of \(\text{PRF} = 600\text{Hz}\). The distance \(l_1 = 1.8m\) from the center of mass of the cone target to the vertex, the distance \(l_2 = 0.6m\) from the center of the bottom surface, and the radius \(r = 0.6m\) from the bottom surface. The angle \(\alpha = \frac{2\pi}{3}\) between the line of sight of the radar and the cone rotation axis, the precession angle \(\theta = \frac{\pi}{18}\), and the cone rotation frequency \(\omega_c = 4\pi\). Assume that the cone target is the initial distance \(R_0 = 10km\) from the radar, the radial velocity \(a_t = 2m/s\), and the radial acceleration \(a_z = 2m/s^2\). Assume that the signal-to-noise ratio is \(\text{SNR} = 10dB\). The simulation is carried out according to the formula and related parameters deduced above. Compare the simulation results with the hypothetical values and calculate the relative error. If the relative error is within the
acceptable range, the algorithm is effective and can realize the compensation of translational component.

The time-frequency curve of the echo generated by the scatter point of the cone target is shown in Figure 4. From the figure, we can see that the time-frequency graph is not horizontal, indicating that there is a translational component. At the same time, it can be observed that the three curves have multiple intersection points. According to the above, these intersection points only contain translational component information, so these intersection points can be used to compensate for translational components.

![Figure 4. Time-frequency curve of scattering points](image1)

![Figure 5. Binarized image](image2)

![Figure 6. Skeleton extraction image](image3)

Figure 4 shows the one-dimensional range image of the target. Since the one-dimensional distance profile has side lobes, this will affect the search for the characteristic points of the curve. Therefore, before corner detection, it is necessary to suppress the side lobes of the one-dimensional range image. Therefore, the time-frequency curve should be smoothed first. In this paper, Gaussian filtering is used to smooth the curve to eliminate the influence of noise around the curve, and then further set an appropriate threshold to convert it into a binary image, as shown in Figure 5. Afterwards, the skeleton extraction is performed, as shown in Figure 6.

Perform corner detection on the time-frequency graph of FIG. 6 and extract relevant corner points, as shown in Fig. 7. Figure 8 is a distribution diagram of the eigenvalues of the autocorrelation matrix corresponding to each corner point. Looking at Figure 7, we find that most corners are not what we need, so we need to eliminate the unnecessary corners. Restrict the generation conditions of the corner points to obtain the distribution diagram of the corner points, as shown in Figure 9. Through the algorithm, the corner points can be accurately detected, and the position information of the relevant
Intersection points can be determined, and the translational component information can be obtained. According to the corresponding relationship between the number of image rows and columns and the time-frequency diagram coordinates, the determined position is converted into time-frequency information. Figures 4 to 9 are simulation diagrams obtained by performing one simulation. Performing multiple Monte Carlo simulations, the radial velocity \( a_1 = 2.00155 \, \text{m/s} \) and the radial acceleration \( a_2 = 1.9861 \, \text{m/s}^2 \) can be estimated using the least square method. Comparing the hypothetical data, calculate the relative errors as \( \varepsilon_{a_1} = 0.155\% \) and \( \varepsilon_{a_2} = 1.345\% \) respectively.

![Corner diagram](image1)

**Figure 7.** Corner diagram

![Eigenvalue distribution](image2)

**Figure 8.** Eigenvalue distribution

![Feature points displayed in the image](image3)

**Figure 9.** Feature points displayed in the image

According to formula (6), by compensating the translational component, the fretting component of the scattering point can be obtained. Figure 10 is a time-frequency graph of the target scattering point after compensating for the translational component. According to the image, the translational component has been removed, and a better time-frequency graph of the micro-motion component is obtained. Verify the effectiveness of the algorithm.
Below, under different signal-to-noise ratios, the detection effect of the algorithm proposed in this paper is further analyzed. Next, perform multiple Monte Carlo simulations under different signal-to-noise ratio conditions. Other simulation parameters are consistent with the above settings. The simulation results obtained are shown in Table 2. At the same time, the overall error analysis of the translational parameters, can obtain the combined relative error of the parameters, as shown in Table 1.

Table 1. Relative error under different signal to noise ratio.

| Signal to noise ratio (dB) | 4    | 6    | 8    | 10   | 12   | 14   |
|---------------------------|------|------|------|------|------|------|
| $\varepsilon_{a_1}$      | 0.5025% | 0.595% | 0.167% | 0.155% | 0.41% | 0.04% |
| $\varepsilon_{a_2}$      | 0.925% | 1.155% | 1.653% | 1.345% | 1.39% | 1.2875% |

According to the data obtained in the above table, it can be seen that under different signal-to-noise ratios, the relative error produced by the translational component remains basically stable, so the algorithm has good robustness. There are very few data in the table that fluctuate greatly, which may be due to the large deviation of the extracted corner points.

5. Conclusion

The time-frequency curve of a ballistic target includes translational components. Therefore, in order to realize the identification of the ballistic target and analyze the relevant characteristics, translational compensation is required. This paper proposes the Shi-Tomasi corner detection algorithm, which compensates the translational component by analyzing the characteristics of the corners. Simulation experiments verify the effectiveness of the algorithm. The algorithm realizes the detection of the anisotropy of the image, which is relatively simple. At the same time, the algorithm is optimized, which shortens the running time and improves the efficiency in practical applications.

It can be seen from Table 1 that due to the inaccuracy of the extracted corner points, a large error occurs when estimating the relevant parameters of translation compensation. Therefore, we can continue to improve the algorithm to make the extracted corners more accurate.

6. References

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