Spin Structure Function $g_2(x, Q^2)$ and Twist-3Operators in large-$N_C$ QCD

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Abstract

It is shown in the framework of the operator product expansion and the renormalization group method that the twist-3 part of flavour nonsinglet spin structure function $g_2(x, Q^2)$ obeys a simple Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation in the large $N_C$ limit even in the case of massive quarks ($N_C$ is the number of colours). There are four different types of twist-3 operators which contribute to $g_2$, including quark-mass-dependent operators and the ones proportional to the equation of motion. They are not all independent but are constrained by one relation. A new choice of the independent operator bases leads to a simple form of the evolution equation for $g_2$ at large $N_C$.

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In the experiments of the polarized deep inelastic lepton production we can obtain the information on spin structures of nucleon, which are described by the two functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$. The QCD effects on $g_1$ and $g_2$ have been extensively studied [1] since earlier papers [2]-[4]. Increasingly accurate measurements of $g_1$ have been performed at SLAC, CERN and DESY [5], while the $g_2$ measurements still have limited statistical precision [6].

In the language of the operator-product-expansion (OPE) the twist-2 operators contribute to $g_1$ in the leading order of $1/Q^2$. As for the structure function $g_2$, on the other hand, both twist-2 and twist-3 operators participate in the leading order. Moreover, the number of participating twist-3 operators grows with spin (moment of $g_2$). Due to increase of the number of operators and the mixing among these operators the analysis of the twist-3 part of $g_2$ turns out to be rather complicated [7]-[14]. In other words, the $Q^2$ evolution equation for the moments of the twist-3 part of $g_2$ cannot be written in a simple form but in a sum of terms, the number of which increases with spin.

For the case of the twist-3 flavour nonsinglet $g_2$ it has been observed by Ali, Braun and Hiller (ABH) [15] that in the large $N_C$ limit $g_2$ obeys a simple Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [16]. In their formalism of working directly with the nonlocal operator contributing to the twist-3 part of $g_2$, they showed that local operators involving gluons effectively decouple from evolution equation for large $N_C$. In fact their analysis has been made with massless quarks.

In this paper I reanalyze the $Q^2$ evolution of the flavour nonsinglet twist-3 part of $g_2$ in the framework of the standard OPE and the renormalization group (RG) with massive quarks. Actually the OPE analysis of $g_2$ has been performed already and the anomalous dimensions of the relevant twist-3 operators have been calculated [8][9][11][17][18]. However, to the best of my knowledge, the large $N_C$ limit of $g_2$ has not been thoroughly studied so far in OPE and RG. There are four different types of twist-3 operators which contribute to $g_2$, including quark-mass-dependent operators and the ones proportional to the equation of motion. They are not all independent but are constrained by one relation. It was pointed out recently by Kodaira, Uematsu and Yasui [17] that any choice of the independent operator bases
leads to a unique prediction for the moments. Taking a new basis of the independent operators, I will show that the $Q^2$ evolution of the twist-3 part of $g_2$ obeys a simple DGLAP equation in the $N_C \to \infty$ limit and thus the ABH result on $g_2$ is reproduced even with massive quarks.

The spin structure function $g_2$ receives contributions from both twist-2 and twist-3 operators. However, the twist-2 part of $g_2$ can be extracted once $g_1$ is measured \[19\]:

$$ g^{\text{tw.2}}_2(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} dy. \quad (1) $$

Thus the difference $$ g_2(x, Q^2) = g_2(x, Q^2) - g^{\text{tw.2}}_2(x, Q^2) \quad (2) $$ contains the twist-3 contributions only.

The twist-3 operators which enter the OPE for the flavour nonsinglet $g_2$ are the following (I follow the notation and conventions of Refs.\[17\] \[18\] and omit the flavour matrices $\lambda_i$):

$$ R^{\sigma\mu_1\cdots\mu_{n-1}}_E = \frac{i^{n-1}}{n} \left[ (n-1) \overline{\psi} \gamma_5 \gamma^\sigma D^{(\mu_1 \cdots D^{\mu_{n-1}}} \psi \\
- \sum_{l=1}^{n-1} \overline{\psi} \gamma_5 \gamma^{\mu_1} D^{(\sigma D^{\mu_1} \cdots D^{\mu_{l-1} D^{\mu_{l+1}}} \cdots D^{\mu_{n-1} \psi} \right] \text{ traces}, \quad (3)$$

$$ R^{\sigma\mu_1\cdots\mu_{n-1}}_t = \frac{1}{2n} (V_l - V_{n-1-l} + U_l + U_{n-1-l}), \quad (l = 1, \cdots, n-2) \quad (4)$$

$$ R^{\sigma\mu_1\cdots\mu_{n-1}}_m = i^{n-2} m S' \overline{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2} \gamma^{\mu_{n-1}}} \psi \text{ traces}, \quad (5)$$

$$ R^{\sigma\mu_1\cdots\mu_{n-1}}_E = \frac{i^{n-2} n - 1}{2n} S' \overline{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2} \gamma^{\mu_{n-1}}} (i \not{D} - m) \psi \\
+ \overline{\psi} (i \not{D} - m) \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2} \gamma^{\mu_{n-1}}} \psi \text{ traces}, \quad (6)$$

where $\{ \}$ means complete symmetrization over the Lorentz indices and $m$ represents the quark mass. The symbol $S'$ denotes symmetrization on the indices $\mu_1 \mu_2 \cdots \mu_{n-1}$ and antisymmetrization on $\sigma \mu_i$. The operators in Eq.\[4\] contain the gluon field strength $G_{\mu\nu}$ and its dual tensor $\tilde{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$ and they are given by

$$ V_l = -i^n g S' \overline{\psi} \gamma_5 D^{\mu_1} \cdots G^{\sigma \mu_1} \cdots D^{\mu_{n-2} \gamma^{\mu_{n-1}}} \psi \text{ traces}, \quad (7)$$

$$ U_l = i^{n-1} g S' \overline{\psi} D^{\mu_1} \cdots \tilde{G}^{\sigma \mu_1} \cdots D^{\mu_{n-2} \gamma^{\mu_{n-1}}} \psi \text{ traces}, \quad (8)$$
where $g$ is the QCD coupling constant. The operator $R_F^\mu$ in Eq.(4) is proportional to the equation of motion (EOM operator). The above twist-3 operators are not all independent but they are constrained by the following relation [7][12]:

$$R^{\sigma\mu_1\cdots\mu_{n-1}}_F = \frac{n-1}{n} R^{\sigma\mu_1\cdots\mu_{n-1}}_m + \sum_{l=1}^{n-2} (n-1-l) R^{\sigma\mu_1\cdots\mu_{n-1}}_l + R^{\sigma\mu_1\cdots\mu_{n-1}}_E. \quad (9)$$

Thus in total there are $n$ independent operators contributing to the $(n-1)$-th moment of $\bar{g}_2$. But we will see later that in the $N_C \to \infty$ limit the $(n-1)$-th moment is expressed in terms of one operator $R^{\sigma\mu_1\cdots\mu_{n-1}}_F$.

In all the analyses of $\bar{g}_2$ performed so far in the framework of OPE and RG, operators $R_i, R_m, R_E$ of Eqs.(4)-(6) have been taken as independent bases. In this paper I choose $R_F, R_i, R_E$ as independent operators, replacing $R_m$ with $R_F$ of Eq.(3). The advantage of this choice of operator basis is that the coefficient functions take simple forms at the tree-level. In fact we have [17]

$$E^n_F(\text{tree}) = 1, \quad E^n_l(\text{tree}) = 0 \quad \text{for } l = 1, \ldots, n-2 \quad (10)$$

since the anti-symmetric part of the short distance expansion for the product of two electromagnetic currents can be written at the tree level as

$$i \int d^4x e^{iq \cdot x} T(J_\mu(x)J_\nu(0))|_{\text{anti-symmetric}} = -i\varepsilon_{\mu\nu\lambda\sigma} g^\lambda \sum_{n=1,3,\ldots} \left( \frac{2}{Q^2} \right)^n q_{\mu_1} \cdots q_{\mu_{n-1}} \{ R^{\sigma\mu_1\cdots\mu_{n-1}}_q + R^{\sigma\mu_1\cdots\mu_{n-1}}_F \}$$

$$\cdots, \quad (11)$$

where dots $\cdots$ stands for non-leading terms and

$$R^{\sigma\mu_1\cdots\mu_{n-1}}_q = i^{n-1} \overline{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-1}} \psi - \text{(traces)} \quad (12)$$

are twist-2 operators which contribute to $g_1$ and $g_2^{tw.2}$. It is true that due to the relation, Eq.(9), $R^{\sigma\mu_1\cdots\mu_{n-1}}_F$ can be expressed in terms of other operators. When eliminating $R^\mu_F$, we obtain a different set of coefficient functions. In other words, the (tree-level) coefficient functions are dependent upon the choice of the independent operators [17].
The renormalization constants for this new set of independent operators are written in the matrix form as

\[
\begin{pmatrix}
R^n_F \\
R^n_l \\
R^n_{E, B}
\end{pmatrix}
= \begin{pmatrix}
\tilde{Z}_{FF} & \tilde{Z}_{Fj} & \tilde{Z}_{FE} \\
\tilde{Z}_{lF} & \tilde{Z}_{lj} & \tilde{Z}_{lE} \\
0 & 0 & \tilde{Z}_{EE}
\end{pmatrix}
\begin{pmatrix}
R^n_F \\
R^n_j \\
R^n_{E, R}
\end{pmatrix},
\quad (l, j = 1, \cdots, n - 2),
\]

where the suffix \( R(B) \) denotes renormalized (bare) quantities.

Now we proceed to the moment sum rule for \( \bar{g}_2 \). Define the matrix elements of composite operators between nucleon states with momentum \( p \) and spin \( s \) by

\[
\begin{aligned}
\langle p, s | R^\sigma_{\mu_1 \cdots \mu_{n-1}}F | p, s \rangle &= -\frac{n - 1}{n} d_n (s^\sigma p^{\mu_1} - s^{\mu_1} p^\sigma) p^{\mu_2} \cdots p^{\mu_{n-1}} \\
\langle p, s | R^\sigma_{\mu_1 \cdots \mu_{n-1}}l | p, s \rangle &= -f_n^l (s^\sigma p^{\mu_1} - s^{\mu_1} p^\sigma) p^{\mu_2} \cdots p^{\mu_{n-1}} \\
\langle p, s | R^\sigma_{\mu_1 \cdots \mu_{n-1}}E | p, s \rangle &= 0
\end{aligned}
\]

Normalization is such that for free quark target we have \( d_n = 1 \) and \( f_n^l = \mathcal{O}(g^2) \). It is recalled that physical matrix elements of the EOM operators vanish \([20]\). Using Eqs.(14) - (16), we can write down the moment sum rule for \( \bar{g}_2 \) as,

\[
M_n \equiv \int_0^1 dx x^{n-1} \bar{g}_2(x, Q^2) = \frac{n - 1}{2n} d_n E^n_F(Q^2) + \frac{1}{2} \sum_{l=1}^{n-2} f_n^l E^n_l(Q^2).
\]

The coefficient functions \( E^n_F(Q^2) \) and \( E^n_l(Q^2) \) satisfy the following renormalization group equation,

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_m(g) m \frac{\partial}{\partial m} \right) E_i = \tilde{\gamma}_{ij} E_j \quad \text{for} \quad i, j = F, 1, \cdots, n - 2
\]

where \( \beta(g) \) and \( \gamma_m(g) \) are the QCD \( \beta \) function and the anomalous dimension of mass operator, respectively. The anomalous dimension matrix \( \tilde{\gamma}_{ij} \) of the composite operators \( R_F^n \) and \( R_l^n \) with \( l = 1, \cdots, n - 2 \) is defined as

\[
\tilde{\gamma}_{ij} = \left[ \tilde{Z}^{-1} \mu \frac{\partial \tilde{Z}}{\partial \mu} \right]_{ij} \quad \text{for} \quad i, j = F, 1, \cdots, n - 2
\]

Note that the anomalous dimension matrix which appears in Eq.(18) is a transposed one. This comes from our convention of defining renormalization constants and anomalous dimensions of the operators in Eqs.(13) and (19).
In the leading-logarithmic approximation, the solutions of the RG equations in Eq. (18) are given as follows [21]:

$$E^n_i(Q^2) = \left[ \exp \left\{ \frac{\tilde{\gamma}^{(0)n}_i}{2\beta_0} \ln \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right) \right\} \right]_{F_i} \quad \text{for } i = F, 1, \cdots, n-2. \quad (20)$$

where $\alpha(Q^2)$ is the QCD running coupling constant, $\beta_0$ and $\tilde{\gamma}^{(0)n}_i$ are, respectively, one-loop coefficients of the $\beta$ function and anomalous dimension matrix,

$$\beta(g) = -\beta_0 g^3 + O(g^5), \quad \beta_0 = \frac{1}{(4\pi)^2} \frac{11N_c - 2n_f}{3} \quad (21)$$

$$\tilde{\gamma}^{(0)n}_{ij}(g) = \tilde{\gamma}^{(0)n}_{ij} g^2 + O(g^4), \quad (22)$$

with $n_f$ being the number of flavours, and we have used the fact that $E^n_F(\mu^2) = 1$ and $E^n_i(\mu^2) = 0$ (for $l = 1, \cdots, n-2$) at the lowest-order.

Now we need the information on the anomalous dimensions $(\tilde{\gamma}^{(0)n})_{Fi}$ (for $i = F, 1, \cdots, n-2$). We can get it without embarking on a new calculation of the relevant Feynman diagrams. We utilize the existing results on the anomalous dimension matrix for the operators $R_l, R_m$ and $R_E$. In the case of the conventional choice of $R_k, R_m$ and $R_E$ as independent operators, the renormalization constant matrix takes a triangular form

$$
\begin{pmatrix}
R^n_l \\
R^n_m \\
R^n_E
\end{pmatrix}_B = \begin{pmatrix}
Z_{ij} & Z_{im} & Z_{iE} \\
0 & Z_{mm} & 0 \\
0 & 0 & Z_{EE}
\end{pmatrix} \begin{pmatrix}
R^n_l \\
R^n_m \\
R^n_E
\end{pmatrix}_R, \quad (i, j = 1, \cdots, n-2), \quad (23)
$$

In the MS renormalization scheme $Z_{ij}$ is expressed as

$$Z_{ij} = \delta_{ij} - \frac{g^2}{16\pi^2 \varepsilon} X_{ij} \quad (i, j = 1, \cdots, n-2, m, E), \quad (24)$$

where $\varepsilon = (4 - d)/2$ with $d$ the space-time dimension, and the components $X_{ij}$ have been calculated [8] [9] [11] [18]. The following is the result on $X_{ij}$ taken from Ref. [13]:

$$X_{ij} = C_G \frac{(j + 1)(j + 2)}{(l + 1)(l + 2)(l - j)} + (2C_F - C_G) \left[ (-1)^{l+j} \frac{n-2C_{l-1}}{n-2C_{l-1}} \frac{(n-1+l-j)}{(n-1)(l-j)} + \frac{2(-1)^j}{l(l+1)(l+2)} C_j \right].$$
\[ X_{ll} = C_G \left( \frac{1}{l} - \frac{1}{l+1} - \frac{1}{n-l} - S_l - S_{n-l-1} \right) \]

\[ + (2C_F - C_G) \left[ \frac{1}{n-1} + \frac{2(-1)^l}{l(l+1)(l+2)} - \frac{(-1)^l}{n-l} \right] \]

\[ + C_F (3 - 2S_l - 2S_{n-l-1}) , \]  

\[ (1 \leq j \leq l-1), \]  

\[ (25) \]

\[ X_{lj} = C_G \frac{(n-1-j)(n-j)}{(n-1-l)(n-l)(j-l)} \]

\[ + (2C_F - C_G) \left[ (-1)^{l+j} \frac{n-2C_j}{n-2C_l} \frac{(n-1-l+j)}{(n-1)(j-l)} + (-1)^{n-j} \frac{n-2-lC_{n-2-j}}{n-l} \right] \]

\[ (l+1 \leq j \leq n-2) , \]  

\[ (26) \]

\[ X_{lm} = \frac{4C_F}{nl(l+1)(l+2)} , \quad X_{mm} = -4C_FS_{n-1} . \]  

\[ (27) \]

If we impose that the renormalized and bare operators respectively satisfy the constraint Eq.(9), we find from Eqs.(13) and (23) that \( \tilde{Z} \)'s are related to the conventional \( Z \)'s as follows:

\[ \tilde{Z}_{FF} = Z_{mm} + \frac{n}{n-1} \sum_{l=1}^{n-2} (n-1-l)Z_{lm}, \]  

\[ (29) \]

\[ \tilde{Z}_{Fj} = -(n-1-j)\tilde{Z}_{FF} + \sum_{l=1}^{n-2} (n-1-l)Z_{lj}, \]  

\[ (30) \]

\[ \tilde{Z}_{lF} = \frac{n}{n-1}Z_{lm}, \]  

\[ (31) \]

\[ \tilde{Z}_{lj} = Z_{lj} - \frac{n}{n-1} (n-1-j)Z_{lm}, \]  

\[ (32) \]

where \( l, j = 1, \ldots, n-2 \). Using the MS scheme-rule, \( 1/\varepsilon \to \ln \mu^2 \), we obtain from Eqs.(19) and (22),

\[ -8\pi^2 \gamma_{FF}^{(0)n} = X_{mm} + \frac{n}{n-1} \sum_{l=1}^{n-2} (n-1-l)X_{lm}, \]  

\[ (33) \]

\[ -8\pi^2 \gamma_{Fj}^{(0)n} = -(n-1-j) \left[ X_{mm} + \frac{n}{n-1} \sum_{l=1}^{n-2} (n-1-l)X_{lm} \right] \]

\[ + \sum_{l=1}^{n-2} (n-1-l)X_{lj}, \]  

\[ (34) \]

\[ -8\pi^2 \gamma_{lF}^{(0)n} = \frac{n}{n-1}X_{lm}, \]  

\[ (35) \]
\[-8\pi^2 \tilde{\tau}_{ij}^{(0)n} = X_{ij} \frac{n}{n-1} (n-1-j) X_{lm}, \quad (36)\]

It is straightforward to calculate the above \(\tilde{\tau}_{ij}^{(0)n}\) using the expressions \(X_{ij}\) in Eqs. (25)-(28). Especially we obtain

\[8\pi^2 \tilde{\tau}_{FF}^{(0)n} = 4C_F \left(S_{n-1} - \frac{1}{4} + \frac{1}{2n}\right), \quad (37)\]

\[8\pi^2 \tilde{\tau}_{Fj}^{(0)n} = -(2C_F - C_G) \left[ (n-1-j) \left\{ 2S_{n-1} - S_j - S_{n-j-1} + 1 + \frac{1}{n} \right\} \right.\]

\[\left. + \sum_{l=1}^{j-1} (n-1-l) \left\{ (-1)^{l+j} \frac{n-2C_j}{n-2C_l} \left( \frac{n-1-l+j}{n-1}(j-l) \right) + (-1)^{n-j} \frac{n-2-l}{n-l} \right\} \right]\]

\[+ \left( n-1-j \right) \left\{ \frac{1}{n-1} + \frac{2(-1)^j}{j(j+1)(j+2)} - \frac{(-1)^j}{n-j} \right\} \]

\[+ \sum_{l=j+1}^{n-2} (n-1-l) \left\{ (-1)^{l+j} \frac{n-2C_{j-1}}{n-2C_{l-1}} \left( \frac{n-1+l-j}{n-1}(l-j) \right) + \frac{2(-1)^j}{l(l+1)(l+2)} \right\} \right]\]

\[\text{for } j = 1, \ldots, n-2. \quad (38)\]

Now we see that the mixing anomalous dimension \(\tilde{\tau}_{Fj}^{(0)n}\) turns out to be proportional to \((2C_F - C_G)\). Since

\[C_F = \frac{N_C^2 - 1}{2N_C}, \quad C_G = N_C, \quad (39)\]

we have \(2C_F = C_G\) and thus \(\tilde{\tau}_{Fj}^{(0)n} = 0\) in the \(N_C \to \infty\) limit. Then Eq. (20) gives,

\[E_{ii}^n(Q^2) = \left[ \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right]^{\tilde{\tau}_{FF}^{(0)n}/2\beta_0}, \quad (40)\]

\[E_{ii}^n(Q^2) = 0 \quad \text{for } l = 1, \ldots, n-2. \quad (41)\]

Returning to Eq. (17), we find that, at \(N_C\) going to infinity, the moment sum rule for \(\mathcal{g}_2\) takes a simple form as follows:

\[\int_0^1 dx x^{n-1} \mathcal{g}_2(x, Q^2) = \frac{n-1}{2n} d_n \left[ \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right]^{\tilde{\tau}_{FF}^{(0)n}/2\beta_0}. \quad (42)\]
\[ \frac{\gamma_{FF}^{(0)n}}{2\beta_0} = \frac{2N_C}{\beta_0} \left[ S_{n-1} - \frac{1}{4} + \frac{1}{2n} \right] . \quad (43) \]

In other words, at large \( N_C \), the operators \( R^{\sigma_1\cdots\sigma_{n-1}} \) involving the gluon fields decouple from the evolution equation of \( g_2 \) and the whole contribution is represented by one type of operators \( R^{F\sigma_1\cdots\sigma_{n-1}} \). With the substitution \( C_F = N_C / 2 \) and \( n = j + 1 \), the anomalous dimension \( 8\pi^2 \tilde{\gamma}_{FF}^{(0)n} \) coincides with Eq.(18) of Ref.[15]. This completes the reproduction, in the framework of OPE and RG, of the ABH result on \( g_2 \).

It should be emphasized that we have reproduced the ABH result without assuming massless quarks. A question expected to come up immediately is that the replacement of the mass-dependent operator \( R^{m} \) with \( R^{F} \) may be equivalent to working with massless quarks. The answer is no. Indeed it can be shown that even when we include the mass-dependent operator \( R^{m} \) among the independent operator bases we reach the same conclusion. Let us take, for an example, \( R^{F} \), \( R^{l} \) (with \( l = 2, \cdots, n-2 \)), \( R^{m} \) and \( R^{E} \) as independent operators replacing one quark-gluon operator \( R^{l=1} \) with \( R^{F} \). With this choice of new operator bases, the moment sum rule for \( g_2 \) is written in terms of the coefficient functions \( \hat{E}_p^n(Q^2) \), \( \hat{E}_l^n(Q^2) \) with \( l = 2, \cdots, n-2 \), and \( \hat{E}_m^n(Q^2) \). The renormalization constants for these operators are written as

\[
\begin{pmatrix}
R^{F}_p \\
R^{j}_l \\
R^{m}_n \\
R^{E}_r
\end{pmatrix}_B
= \begin{pmatrix}
\hat{Z}_{FF} & \hat{Z}_{Fj} & \hat{Z}_{Fm} & \hat{Z}_{FE} \\
\hat{Z}_{IF} & \hat{Z}_{Ij} & \hat{Z}_{Im} & \hat{Z}_{IE} \\
0 & 0 & \hat{Z}_{mm} & 0 \\
0 & 0 & 0 & \hat{Z}_{EE}
\end{pmatrix}_R
\begin{pmatrix}
R^{F}_p \\
R^{j}_l \\
R^{m}_n \\
R^{E}_r
\end{pmatrix}_R , \quad (l, j = 2, \cdots, n-2). \quad (44)
\]

Again imposing that the renormalized and bare operators respectively satisfy the constraint Eq.(4), we find that \( \hat{Z} \)’s are related to conventional \( Z \)’s as follows:

\[
\hat{Z}_{FF} = \frac{1}{n-2} \sum_{l=1}^{n-2} (n-1-l) Z_{ll} \quad (45)
\]

\[
\hat{Z}_{Fj} = - (n-1-j) \hat{Z}_{FF} + \sum_{l=1}^{n-2} (n-1-l) Z_{lj}, \quad (j = 2, \cdots, n-2) \quad (46)
\]

\[
\hat{Z}_{Fm} = - \frac{n-1}{n} \hat{Z}_{FF} + \frac{n-1}{n} Z_{mm} + \sum_{l=1}^{n-2} (n-1-l) Z_{lm} \quad (47)
\]

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Then it is easy to obtain the following one-loop coefficients of the relevant anomalous dimensions

\[ 8\pi^2 \hat{\gamma}_{FF}^{(0)n} = 4C_F \left( S_{n-1} - \frac{1}{4} + \frac{1}{2n} \right) \]

+ terms proportional to \((2C_F - C_G)\),

\(8\pi^2 \hat{\gamma}_{Fj}^{(0)n} \propto (2C_F - C_G) \quad \text{for } j = 2, \ldots, n-2 \quad \tag{49}\)

\(8\pi^2 \hat{\gamma}_{Fm}^{(0)n} \propto (2C_F - C_G) \quad \tag{50}\)

Inserting these anomalous dimensions to the solutions of the RG equations for the coefficient functions \(\hat{E}_F^n(Q^2)\), \(\hat{E}_l^n(Q^2)\) \((l = 2, \ldots, n-2)\) and \(\hat{E}_m^n(Q^2)\),

\[ \hat{E}_i^n(Q^2) = \left[ \exp \left\{ \frac{\hat{\gamma}_{FF}^{(0)n}}{2\beta_0} \ln \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right) \right\} \right]_{Fi} \quad \text{for } i = F, 2, \ldots, n-2, m \quad \tag{51}\]

we obtain in the large \(N_C\) limit

\[ \hat{E}_F^n(Q^2) = \left[ \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right]^{\hat{\gamma}_{FF}^{(0)n}/2\beta_0} E_F^n(Q^2) \quad \tag{52}\]

\[ \hat{E}_l^n(Q^2) = 0 \quad \text{for } l = 2, \ldots, n-2 \quad \tag{53}\]

\[ \hat{E}_m^n(Q^2) = 0 \quad \tag{54}\]

Thus we reach the same conclusion Eq.(42) even when we include the mass-dependent operators among the independent operator bases.

A few comments are in order. Firstly, the twist-3 quark-gluon operators \(R_l^n\) decouple from the evolution equation for \(\hat{g}_2\) at large \(N_C\). This might be explained by an argument on quark condensate \([6]\). A hint is that the mixing anomalous dimensions \(\hat{\gamma}_{Fj}^{(0)n}\) turn out to be proportional to \((2C_F - C_G)\). There are two types in the products of colour matrices entering into the calculation of anomalous dimensions for the flavour nonsinglet \(\bar{g}_2\):

\[ T^b T^a T^b = (C_F - \frac{1}{2} C_G) T^a = -\frac{1}{2N_C} T^a \quad \tag{55}\]

\[ T^b T^b T^a = C_F T^a = \frac{1}{2} \left( N_C - \frac{1}{N_C} \right) T^a \quad \tag{56}\]

It is argued in Ref.[7] that the quark condensate contains all colours and at large \(N_C\) the condensate polarization becomes small and that the combination \(T^b T^a T^b\) is connected with condensate polarization effects.
Secondly, we have chosen particular sets of the independent operators and reached a simple form for the moments of $\mathcal{g}_2$ in the large $N_C$ limit. However, arbitrariness in the choice of the operator bases should not enter into physical quantities [17]. A different choice of the operator bases leads to different forms for the anomalous dimension matrix and the coefficient functions. Recall that the constraint, Eq.(1), gives a relation among the tree-level coefficient functions and also a relation among the matrix elements of the operators. After diagonalizing the anomalous dimension matrix and using these relations, we can arrive at the same conclusion for the moments of $\mathcal{g}_2$ in the $N_C \to \infty$ limit. What we did in this paper is that we chose particular sets of bases from the beginning which include an operator that represents the whole contribution to $\mathcal{g}_2$ for large $N_C$.

Finally, the nucleon has other twist-3 distributions, namely, chiral-odd distributions $h_L(x,Q^2)$ and $e(x,Q^2)$ [22]. Just like the $\mathcal{g}_2$ case, the $Q^2$ evolutions of flavour nonsinglet $h_L(x,Q^2)$ and $e(x,Q^2)$ turn out to be quite complicated due to mixing with quark-gluon operators, the number of which increases with spin. However, it has been proved [23] that in the large $N_C$ limit these twist-3 distributions also obey a simple DGLAP equation. The proof holds true only when we work with massless quarks.

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