The perturbative odderon in elastic $pp$ and $p\bar{p}$ scattering

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Different models for the odderon-proton coupling are considered and their effects on the differential cross section in the dip region in elastic $pp$ and $p\bar{p}$ scattering are investigated. An allowed range for the size of a possible diquark cluster in the proton can be obtained from a geometrical model.

1. Introduction

The evidence for the existence of an odderon [1] remains scarce. To date, the best evidence comes from the differential cross section in elastic $pp$ and $p\bar{p}$ scattering. The $pp$ data show a dip structure at $-t \approx 1.3$ GeV$^2$ while the $p\bar{p}$ data only flatten off at that point. The odderon, being odd under charge conjugation, couples with different signs to protons and antiprotons and can hence account for that difference.

We describe the odderon in leading order perturbation theory, as a perturbative three-gluon exchange in a $C = -1$ state. We assume that the scale $1.3$ GeV$^2$ is large enough for perturbation theory to be valid; and we do no log $s$ resummation. We consider three different models for the odderon-proton coupling: one proposed by Fukugita and Kwiecinski, one proposed by Levin and Ryskin and a geometrical model which allows an estimation of the size of a possible diquark cluster in the proton.

2. Method of calculations

2.1. The framework

The data for the differential cross section in $pp$ and $p\bar{p}$ elastic scattering are well described by the Donnachie-Landshoff (DL) fit [2]. The authors use a number of contributions: Pomeron ($P$), Reggeon, triple gluon (odderon), $P P$, $P P P$, $P+$Reggeon, and $P+$double gluon exchange. Their perturbative triple-gluon exchange contribution is charge conjugation odd due to the colour structure of the single Feynman graph taken into account, hence an odderon.

We use the DL fit as a framework for comparing different odderon contributions to experimental data. To that end we replace their triple-gluon exchange amplitude by one of the model odderon contributions. We retain the original parameter values of the fit and make no attempt to improve it.

2.2. A position-space model

Our prime interest was to investigate the influence of the proton structure on the odderon-exchange amplitude and hence on the differential cross section. The scattering amplitude in position space is given by

$$T_\Omega(s, t) = 2 s \int d^2b \ e^{-iqb} \int d^2R_1 \int d^2R_2 \ |\psi(R_1)|^2 \ |\psi(R_2)|^2 \ J(b, R_1, R_2).$$

The two latter integrations are over the size and orientation of the protons in transverse position space. $\psi$ is the proton wave function. $J = S - 1$ is the reduced scattering amplitude or $T$-Matrix element. It is computed with a method developed by Nachtmann [3] based on the functional representation of scattering matrix elements and the WKB approximation. For a complete presentation of this method, please refer to [3].

In our case, it leads to a correlator of six integrals over gluon fields along the paths of the quarks. We then project out the $C = -1$ part to obtain the odderon. See [3] for more details.

We use a Gaussian wave function for the pro-
The parameter $S$ which make up the odderon: factors with the propagators of the three gluons computed by folding two odderon-proton impact space, the odderon exchange amplitude is structure are defined in momentum space. In momentum space impact factors will be able to place a bound on it. As shown in Fig. 2.2. The diquark size is a free parameter. By comparing with experiment, we will be able to place a bound on it.

### 2.3. Momentum space impact factors

Two impact factors we took from the literature are defined in momentum space. In momentum space, the odderon exchange amplitude is computed by folding two odderon-proton impact factors with the propagators of the three gluons which make up the odderon:

$$T_O(s, t) = \frac{s}{32 \pi^2} \frac{1}{6!} \int \frac{d^2 \delta_{1t}^t d^2 \delta_{2t}^t}{(2\pi)^2} \frac{\Phi_p(\delta_{1t}, \delta_{2t}, \Delta_t)}{\delta_{1t}^2 (\Delta_t - \delta_{1t} - \delta_{2t})^2}.$$

The $\delta_{it}$ are the transverse gluon momenta; $\Delta_t = \delta_{1t} + \delta_{2t} + \delta_{3t}$ is the transverse momentum of the odderon.

For reasons of gauge invariance the impact factor has to be of the form

$$\Phi_p(\delta_{1t}, \delta_{2t}, \Delta_t) = 8 (2\pi)^2 g^3 \left[ F(\Delta_t, 0, 0) - \sum_{i=1}^{3} F(\delta_{it}, \Delta_t - \delta_{it}, 0) + 2F(\delta_{1t}, \delta_{2t}, \delta_{3t}) \right],$$

where $F$ is a form factor.

One of the form factors we used was proposed by Fukugita and Kwieciński [5]:

$$F_{FK}(\delta_{1t}, \delta_{2t}, \delta_{3t}) = \frac{A^2}{A^2 + \frac{1}{2} \sum_{i \neq k} (\delta_{it} - \delta_{kt})^2}. \tag{5}$$

The constant $A$ determines the width of the form factor and equals half the rho mass. The other form factor was published by Levin and Ryskin [6]:

$$F_{LR}(\delta_{1t}, \delta_{2t}, \delta_{3t}) = \exp \left( -R^2 \sum_{i=1}^{3} \delta_{it}^2 \right). \tag{6}$$

$R = 0.33$ fm is the proton radius used by the authors. We did not attempt to fit the parameters of either form factor but kept the values supplied by the authors.

### 3. Results

As can be seen from Figs. 2 and 3, all models provide a satisfactory description of the data provided their parameters are adjusted correctly. The data do not favour one model over the others.

In the case of the geometrical model, there are two parameters, the diquark size and the coupling constant. Since the precise value of the coupling constant is unknown in a leading-order calculation, we fitted the diquark size for several values of $\alpha_s$ which are common in the literature at the scale given by $-t \approx 1.3$ GeV$^2$ in the dip region.

Table 1 shows the results. For $\alpha_s \geq 0.3$ the diquark size is $\leq 0.35$ fm. This result is of great importance for nonperturbative calculations where the proton is often described as a colour dipole. For such a small diquark size this is legitimate since soft gluons cannot resolve the diquark.

In the calculations with momentum space impact factors, the only free parameter is the coupling constant. The best fit was $\alpha_s = 0.3$ for the Fukugita-Kwieciński (FK) form factor and $\alpha_s = 0.5$ for the Levin-Ryskin form factor. With a larger value for the coupling constant, eg $\alpha_s = 1$, the curves would overshoot the data by more than an order of magnitude over nearly the whole $t$ range.
This casts some doubt on predictions of diffractive $\eta_c$ production. Three groups have used the FK impact factor with a value of $\alpha_s = 1$ in calculations of the diffractive $\eta_c$ production amplitude. In view of our results, this looks like a significant overestimation.

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Impact factor (LR)
Impact factor (FK)

Our model
DL

\[ -t \text{[GeV}^2] \]
\[ \frac{d\sigma}{dt} \text{[mb/GeV}^2] \]

2.2 2.1 2 1.9 1.8 1.7 1.6 1.5 1.4 1.3 1.2 1.1 1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.01 10^{-3} 10^{-4} 10^{-5}

0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2

Figure 3. Differential elastic cross section for \( p\bar{p} \) scattering for all models for the odderon-proton coupling and the original Donnachie-Landshoff fit compared with experimental data \[8]. The centre-of-mass energy is \( \sqrt{s} = 53 \text{ GeV} \). Again, the data do not favour one model over the others.

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Discussion

M. Boutemeur (Munich): Your model does not describe the \(-t\) distributions at higher energies. Do you have in mind other ingredients to your model to make it fit the data better?

V. Schatz: The DL fit is less good at higher energies, but the odderon alone cannot remedy that. Due to the uncertainties in our calculations—the unknown exact value of the coupling, the possibility of non-perturbative effects playing a role—we were not interested in a precision fit. There is a better fit due to Gauron, Leader and Nicolescu, but it is incompatible with the odderon contributions we investigated.

L. Leśniak (Krakow): The momentum transfer distribution \(d\sigma/dt\) in \(pp\) or \(p\bar{p}\) scattering can depend on five different spin amplitudes. Is the spin dependence of the \(pp\) or \(p\bar{p}\) amplitudes included in your model or do you use only one spin independent amplitude?

V. Schatz: The different spin amplitudes are contained in our amplitudes for the various contributions. However, we did not try to extract or investigate the interplay of different spin amplitudes.