Structural Reliability Analysis Using Group Teaching Optimization Algorithm

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Abstract. Conventional First order reliability method (FORM) usually encounters divergence and non-convergence for high nonlinear performance functions. This paper puts forward an improved FORM coupling with a newly raised metaheuristic optimization algorithm, that is, Group teaching optimization algorithm (GTO). The GTO algorithm is inspired by grouping teaching strategy, featured by no additional control parameters, and has shown excellent global search ability for high nonlinear problems. Thus, the GTO algorithm is utilized to solve the equivalent constrained optimization problem according to the first order reliability theory. The implementation procedure of the proposed method is then presented in this paper. A numerical example is employed to verify the performance of the proposed method. Traditional reliability methods and PSO-based FORM are also introduced to compared with the proposed approach. The simulation results prove that the proposed method provides good performance in terms of computational efficiency and accuracy.

1. Introduction

The inherent uncertainties connected with structural parameters as well as external loading of the structures will inevitably severely affect its safety and serviceability [1], and hence, it is necessary to conduct reliability analysis to ensure the safety of structures [2, 3]. The task of the reliability analysis is to obtain the failure probability \( P_f \). According to reliability theory, the failure probability \( P_f \) can be given as

\[
P_f = \int_{G(X)\leq0} f(X) \, dX
\]  

(1)

where \( X = [x_1, \ldots, x_n]^T \) is a vector with \( n \) random variables, \( G(X) \leq 0 \) represents the failure domain of the limit state function \( G(X) \), \( f(X) \) is the joint probability density function of \( X \). However, the value of \( P_f \) is of many difficulties to be obtained due to the high dimensional integral and irregularly shaped failure domain [4]. Cornell provides the relationship between the failure probability and reliability index [5], described as

\[
P_f = \Phi(-\beta)
\]  

(2)
where $\Phi(\cdot)$ is the cumulative distribution function. Thus, the failure probability can be gained by the reliability index.

Traditional reliability methods can be generally divided into three categories: approximate methods, numerical integration methods, and simulation methods. The approximate methods, such as the FORM [6] and second order reliability method (SORM) [7], expand the limit state function at its design point using the Taylor expansion. Nevertheless, these traditional approximate methods may fail to converge for highly non-linear problems. The point estimation method (PEM) [8] and sparse grid integration method (SGIM) [9] belong to the numerical integration method, is suitable for many reliability problems except for high dimensional problem. The computation cost for computing moments of the limit state function explodes when dealing with the multidimensional nonlinear problems. As for the simulation method, the crude Monte Carlo simulation (MCS) is representative of this category. The MCS is featured by high accuracy and robustness but becomes computational intensive for reliability problems of small probability or complicated performance function.

In the past few decades, to perform efficient and accurate reliability analysis, various methods have been developed for this purpose, especially when the reliability index is regarded as a solution to the optimization problem. Recently, meta-heuristic algorithms have attracted many researchers' interests and are applied successfully for many engineering optimization problems [10–13]. Furthermore, some metaheuristic algorithms have been integrated with the first order reliability method for structural reliability analysis. The literature results prove the metaheuristic algorithm indeed improves the accuracy and applicability of the conventional FORM including chaotic enhanced colliding bodies optimization (CECBO) [4], particle swarm optimization (PSO) [14], improved ray optimization (IRO) [15], etc. However, some of these metaheuristic algorithms such as the PSO and CECBO relies on interior parameter tuning to achieve its best performance. Therefore, this paper used a novel metaheuristic algorithm, that is, Group teaching optimization algorithm (GTO) for structural reliability analysis. The GTO algorithm simulates the group teaching mechanism, characterized by implementation simplicity and no extra parameters, and enables to provide better solutions for most optimization problems [16]. Therefore, this paper first employed the GTO for searching design point and proposed a new method GTO-FORM.

The rest of this paper is organized as follows. Section 2 gives the concepts of the first order reliability theory. In section 3, the details of the Group teaching optimization algorithm are presented. Section 4 provides the procedure of the proposed method GTO-FORM. Validation for the proposed method is presented in section 5, and conclusions are derived in section 6.

2. First Order Reliability Method

In reality, the reliability index $\beta$ is often calculated by the FORM where the reliability index is equivalent to the shortest distance between the origin and the limit state surface in standard normal space (U-space) [6], can be obtained by solving a constrained optimization problem, which can be expressed as

$$
\begin{aligned}
\text{Find} & \quad \mathbf{u}^* \\
\text{min} & \quad \beta = \text{norm} (\mathbf{u}) \\
\text{s.t.} & \quad G (T^{-1}(\mathbf{u})) = 0
\end{aligned}
$$

where $T^{-1}(\cdot)$ is the inverse convert function, $\mathbf{u}^* = [u_1, \ldots, u_n]^T$ is the design point of the limit state function in U-space. The independent normal distribution variables can be converted by

$$
u_i = T(x_i) = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \quad (i = 1, \ldots, n)
$$
where \( x_i \) is the \( i \)th variable of the vector \( \mathbf{X} \) in original space, \( \mu_{x_i} \) and \( \sigma_{x_i} \), respectively, are the mean value and standard deviation of \( x_i \). As for non-normal random variables, the Rosenblatt transformation as equations (5)-(6) is employed to implement the transformation from the original space to standard Gaussian space.

\[
\sigma_{x_i}^* = \frac{\phi\left(\Phi^{-1}\left[F_{x_i}(x_i^*)\right]\right)}{f_{x_i}(x_i^*)} (i = 1, \ldots, n) \tag{5}
\]

\[
\mu_{x_i}^* = x_i^* - \Phi^{-1}\left[F_{x_i}(x_i^*)\right]\sigma_{x_i}^* (i = 1, \ldots, n) \tag{6}
\]

in which, \( \phi(\cdot) \) is the probability density function, \( F_{x_i}(x_i^*) \) and \( f_{x_i}(x_i^*) \) are the probability density function and cumulative distribution function of the original variable \( X_i \) at \( x_i^* \).

3. **Group Teaching Optimization Algorithm**

The group teaching optimization algorithm is established based on a group teaching model. In this model, the students, the subjects offered to students, and the knowledge of students, respectively, are represented by the population, decision variables, and fitness value. Two groups composed the student population, i.e., outstanding group and average group. One teacher is selected to teach both two groups during the teaching phase. Meanwhile, the teacher will consider the discrepancy of the learning ability of two groups thus making different teaching plans and group learning plans for students including self-learning and student interactions.

Six steps form the framework of the GTO algorithm including initialization, teacher allocation phase, teacher phase, student phase, reconstruct population, and termination. The details of these steps are presented in the following subsections.

3.1. **Initialization**

Before the main optimization loop starts, the value of the maximum iteration number \( MaxIt \) should be set, and the current iteration number \( T_{cur} \) must be initialized \( T_{cur} = 0 \). Besides, the population \( \mathbf{X} \) is generated according to the number of population \( N \), the dimensions of the variables \( D \), and the lower bound \( \mathbf{lb} \) and the upper bound \( \mathbf{ub} \), which can be expressed as follows:

\[
X_{i,j} = lb_{j} + \text{rand} \cdot (ub_{j} - lb_{j}) \quad (i = 1, \ldots, N; j = 1, \ldots, D) \tag{7}
\]

where \( \text{rand} \) is a random number between zero and one, \( lb_{j} \) and \( ub_{j} \) are the lower bound and upper bound of the \( j \)th variable, respectively.

3.2. **Teacher Allocation Phase**

\[
T' = \begin{cases} 
X'_{\text{first}} & f(X'_{\text{first}}) \leq f\left(\frac{X'_{\text{first}} + X'_{\text{second}} + X'_{\text{third}}}{3}\right) \\
X'_{\text{first}} + X'_{\text{second}} + X'_{\text{third}} & \frac{3}{3} \end{cases} \tag{8}
\]
where $T^t$ is the selected teacher at the current iteration $t$, $X^t_{\text{first}}$, $X^t_{\text{second}}$, and $X^t_{\text{third}}$, respectively, are the current first, second, third-best student. $f(\cdot)$ is the fitness function.

### 3.3. Teacher Phase

#### 3.3.1. Teacher Phase: Outstanding Group

$$X^t_{\text{teacher}} = X^t_i + a \times \left( T^t - F \times (b \times M^t + c \times X^t_i) \right) \quad \left( i = 1, \ldots, \frac{N}{2} \right)$$

\begin{align}
M^t &= \frac{\sum_{i=1}^{N/2} X^t_i}{\frac{N}{2}} \\
b + c &= 1
\end{align}

$$X^{t+1}_{\text{teacher},i} = \begin{cases} 
X^{t+1}_{\text{teacher},i}, & f(X^{t+1}_{\text{teacher},i}) < f(X^t_i) \\
X^t_i, & f(X^{t+1}_{\text{teacher},i}) \geq f(X^t_i) 
\end{cases} \quad \left( i = 1, \ldots, \frac{N}{2} \right)$$

where $X^{t+1}_{\text{teacher},i}$ represents the knowledge of the $i$th student in the outstanding group learned from the teacher at the current iteration $t$, $X^t_i$ is the knowledge of the $i$th student in the outstanding group. $a$, $b$ and $c$ all are a random number in the range of $(0,1)$. $F$ is a coefficient either equals to 1 or 2. $M^t$ is the mean vector of the outstanding group at the current iteration $t$.

#### 3.3.2. Teacher Phase: Average Group

$$X^{t+1}_{\text{teacher},i} = X^t_i + 2 \times d \times \left( T^t - X^t_i \right) \quad \left( i = \frac{N}{2} + 1, \ldots, N \right)$$

$$X^{t+1}_{\text{teacher},i} = \begin{cases} 
X^{t+1}_{\text{teacher},i}, & f(X^{t+1}_{\text{teacher},i}) < f(X^t_i) \\
X^t_i, & f(X^{t+1}_{\text{teacher},i}) \geq f(X^t_i) 
\end{cases} \quad \left( i = \frac{N}{2} + 1, \ldots, N \right)$$

where $X^{t+1}_{\text{teacher},i}$ represents the knowledge of the $i$th student in the average group learned from the teacher at the current iteration $t$.

### 3.4. Student Phase

$$X^{t+1}_{\text{student},i} = \begin{cases} 
X^{t+1}_{\text{teacher},i} + e \times (X^{t+1}_{\text{teacher},i} - X^{t+1}_{\text{teacher},j}) + g \times (X^{t+1}_{\text{teacher},i} - X^t_i), & f(X^{t+1}_{\text{teacher},i}) < f(X^{t+1}_{\text{teacher},j}) \\
X^{t+1}_{\text{teacher},i} - e \times (X^{t+1}_{\text{teacher},i} - X^{t+1}_{\text{teacher},j}) + g \times (X^{t+1}_{\text{teacher},i} - X^t_i), & f(X^{t+1}_{\text{teacher},i}) \geq f(X^{t+1}_{\text{teacher},j}) 
\end{cases} \quad \left( i = 1, \ldots, N; \ j = 1, \ldots, N \right)$$
\[
X^*_t = \begin{cases} 
X^{t+1}_{\text{teacher},i}, f(X^{t+1}_{\text{teacher},i}) < f(X^{t+1}_{\text{student},i}) \\
X^{t+1}_{\text{student},i}, f(X^{t+1}_{\text{student},i}) \geq f(X^{t+1}_{\text{student},i}) 
\end{cases} 
\quad (i = 1, \ldots, N) 
\quad (16)
\]

where \( e \) and \( g \) both are the random numbers within the range of \((0, 1)\), \(X^{t+1}_{\text{student},i}\) is the knowledge of the \( i \)-th student learning from the student phase at the iteration \( t + 1 \).

### 3.5. Reconstruct Population

\[
X^{t+1} = [X^{t+1}_{\text{out}}, X^{t+1}_{\text{avg}}] 
\quad (17)
\]

in which \(X^{t+1}\) is the updated population, \(X^{t+1}_{\text{out}}\) and \(X^{t+1}_{\text{avg}}\) are the updated outstanding and average group after an iteration, respectively.

### 3.6. Termination Criteria

The optimization process will be terminated if the iteration number \(T_{\text{cur}}\) exceeds the value of \(N \times MaxIt\). If it is not satisfied, the process of optimization is continued from the teacher allocation phase.

### 4. Implementation of the Proposed Method

Herein, the Group teaching optimization algorithm is utilized to solve the equivalent constrained optimization problem equation (3), and the optimization problem can be converted as

\[
\min \left( \mathbf{u}^T \mathbf{u} \right)^{1/2} + \lambda \cdot \max \left( G(T^{-1}(\mathbf{u})), 0 \right) 
\quad \text{s.t.} \quad G(T^{-1}(\mathbf{u})) = 0 
\quad (18)
\]

where \( \lambda \) is the penalty coefficient usually as a positive constant.

The flowchart of the proposed method is shown in figure 1.

**Figure 1.** Flowchart of the proposed method for structural reliability analysis.

As seen from figure 1, the design point \( \mathbf{u}^* \) can be derived by considering equation (18) as the fitness function which is solved by the group teaching optimization, then the reliability index and failure probability are calculated according to equations. (2) and (3), respectively.
5. Validation

This example is excerpted from [17]

\[ G(X_1, X_2) = \frac{1}{80} - \frac{4.3125w}{EI} \]  

(19)

where \( w, \ E \) and \( I \) are random variables following normal distribution, and the statistics of these random variables are displayed in Table 1.

| Random variable | Distribution type | Mean    | Standard deviation |
|-----------------|-------------------|---------|--------------------|
| \( w \)         | Normal            | 10      | 0.4                |
| \( E \)         | Normal            | \( 2 \times 10^{-7} \) | \( 5 \times 10^{6} \) |
| \( I \)         | Normal            | \( 8 \times 10^{-4} \) | \( 1.5 \times 10^{-4} \) |

Four methods are employed to compare the performance of the methods for this example. The boundaries of random variables are \([\mu-5\sigma, \mu+5\sigma]\). The penalty coefficient for this example is 500. Table 2 and figure 2 provide the obtained results and iterative curves of these methods, respectively. The estimator F-evaluations indicates the number of function calls and \( \varepsilon \) is the relative error between the result obtained by other methods and the reliability index obtained by MCS.

| Method      | Design point                  | \( P_f \) | \( \beta \) | F-evaluations | \( \varepsilon \) (%) |
|-------------|--------------------------------|-----------|-------------|---------------|---------------------|
| GTO-FORM    | \((0.1180, -3.1073, -0.6305)^T\) | 0.00100   | 3.0847      | 1500          | 1.87                |
| PSO-FORM    | \((0.0022, -3.0765, -0.3626)^T\) | 0.00097   | 3.0978      | 4000          | 2.30                |
| FORM[4]     | Diverge                       | –         | –           | –             | –                   |
| MCS         | –                              | 0.00123   | 3.0282     | \( 10^6 \)    | –                   |

Figure 2. Comparison of iterative curves by different methods for example 1.
According to table 1, it should be noted that the conventional FORM is unable to deal with this nonlinear reliability problem. In contrast, two metaheuristic algorithms based FORMs both converge to a suitable solution near to $\beta = 3.0282$, but the proposed GTO-FORM yields a result of high accuracy $\varepsilon=1.87\%$. Figure 2 shows that the GTO-FORM quickly converges to the stable result from 4th iteration while the PSO-FORM converging at 8th iteration after a huge fluctuation. Furthermore, the GTO-FORM consumes much less number of function evaluations (F-evaluations=1500) without parameter tuning compared with that of the PSO-FORM (F-evaluations=4000). In this example, the GTO-FORM behaves better and more efficient than the conventional FORM and PSO-FORM.

6. Conclusions
In this paper, an improved FORM based on group teaching optimization algorithm is proposed to perform structural reliability analysis. An equivalent optimization is presented to calculate the reliability index. Then, a novel metaheuristic algorithm without additional control parameters, group teaching optimization algorithm, is utilized to solve this optimization problem and thus obtain the design point of the limit state function to calculate the reliability index.

The efficiency and accuracy of the proposed method are compared with the conventional FORM, PSO-FORM, and MCS through an illustrative nonlinear example. The results show that the conventional FORM encounters non-convergence for the example, while the proposed GTO-FORM providing a better result using lower computational cost compared with other methods. It also indicates that the proposed GTO-FORM is a potentially efficient tool for complex structural reliability analysis in the future.

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