QCD predictions for spin dependent photonic structure function $g_1^\gamma(x, Q^2)$ in the low $x$ region of future linear colliders

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Abstract

Spin dependent structure function $g_1^\gamma(x, Q^2)$ of the polarised photon is analysed within the formalism based upon the unintegrated spin dependent parton distributions incorporating the LO Altarelli-Parisi evolution and the double $ln^2(1/x)$ resummation at low values of Bjorken parameter $x$. We analyse the effects of the double $ln^2(1/x)$ resummation on the behaviour of $g_1^\gamma(x, Q^2)$ in the low $x$ region which may be accessible in future linear $e^+e^-$ and $e\gamma$ colliders. Sensitivity of the predictions on the possible nonperturbative gluon content of the polarised photons is analysed. Predictions for spin dependent gluon distribution $\Delta g^\gamma(x, Q^2)$ are also given.

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1 Introduction

The small $x$ behaviour of the spin dependent structure functions, where $x$ is the Bjorken parameter, may be influenced in QCD by the novel effects coming from the double $\ln^2(1/x)$ resummation \cite{1, 2, 3, 4, 5, 6}. These effects generate more singular behaviour in the limit $x \to 0$ than that given by LO (or NLO) Altarelli-Parisi evolution equations with non-singular input. The double $\ln^2(1/x)$ resummation effects have been quantitatively analysed for spin dependent structure function $g_1(x, Q^2)$ of the nucleon \cite{6}, and it has been found that they can in principle significantly affect the structure function in the small $x$ region which can be probed at possible polarised HERA measurements \cite{7, 8}. The formalism developed in \cite{6} contains important subleading $\ln^2(1/x)$ effects which follow from including the complete splitting functions and the running QCD coupling. It may however be still possible that other subleading terms can appreciably reduce the magnitude of the leading double $\ln^2(1/x)$ resummation \cite{4}.

In this paper we would like to investigate spin structure function $g_1^\gamma$ of the polarised photon in the low $x$ region which could become accessible in future linear $e^+e^-$ or $e\gamma$ linear colliders \cite{3, 4, 1, 12, 13}. The $\gamma$ mode, i.e. deep inelastic electron (positron) scattering on a photon beams obtained through the Compton back-scattering of the laser photon beams \cite{14}, would be particularly suitable for probing the photon structure in the region of low values of Bjorken parameter $x$ \cite{13}. Possible potential of probing the spin dependent structure functions of the photon at both $e^+e^-$ and $e\gamma$ colliders is discussed in ref. \cite{11}. The spin dependent parton distributions of polarised photons could also be studied through the dijet photoproduction in polarised $ep$ collisions at HERA \cite{12}.

The content of our paper is as follows: in the next section we discuss the double $\ln^2(1/x)$ resummation for the case of the spin dependent parton distributions and for spin dependent structure function $g_1^\gamma(x, Q^2)$ of the photon. The novel feature of the corresponding integral equations for the unintegrated parton distribution functions is the presence of the additional contribution to the inhomogeneous terms which is generated by the point-like coupling of the photon to quarks and antiquarks. We present analytic solution of these integral equations using the approximation of the double $\ln^2(1/x)$ resummation by that contribution which corresponds to the ladder diagrams. In Sec. 3 we extend the formalism by including complete leading order (LO) Altarelli-Parisi evolution at large values of $x$. We also include the complete double
\( \ln^2(1/x) \) resummation by adding the corresponding higher order terms to the kernels of the integral equations. This extension leads to the system of integral equations for the unintegrated spin dependent parton distributions in the photon which embodies the complete LO Altarelli-Parisi evolution at large values of \( x \) and the double \( \ln^2(1/x) \) resummation at small \( x \). In Sec. 4 we discuss the solutions of these equations and show predictions for spin dependent structure function \( g_1^\gamma(x, Q^2) \) and for the spin dependent gluon distributions in the photon. We study sensitivity of the predictions upon the assumptions concerning the non-perturbative spin dependent gluon distributions in the photon. We analyse two possible scenarios: (a) the case in which the non-perturbative part of the quark and gluon distributions are neglected and the partonic content of the polarised photon is entirely driven by the point-like coupling of the photon to quarks and antiquarks and (b) the case in which one introduces in a model dependent way the non-perturbative gluon distributions in the photon. Finally in Sec. 5 we give summary of our results.

2 Double \( \ln^2(1/x) \) resummation effects for the spin dependent parton distributions in the polarised photon

Low \( x \) behaviour of photonic spin structure function is influenced by double logarithmic \( \ln^2(1/x) \) contributions i.e. by those terms of the perturbative expansion which correspond to the powers of \( \ln^2(1/x) \) at each order of the expansion, similarly as for DIS spin dependent structure function \( g_1 \) of the nucleon \([4, 5]\). In the following we will apply the double \( \ln^2(1/x) \) resummation scheme based on unintegrated parton distributions \([3, 5, 15]\). Conventional integrated spin dependent parton distributions \( \Delta p_l(x, Q^2) \) \((p = q, g)\) are related to unintegrated parton distributions \( f_l(x', k^2) \) in the following way:

\[
\Delta p_l(x, Q^2) = \Delta p_l^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f_l(x' = x(1 + \frac{k^2}{Q^2}), k^2),
\]

where \( \Delta p_l^{(0)}(x) \) is the nonperturbative part of the distribution, \( k^2 \) denotes the transverse momentum squared of the probed parton, \( W^2 \) is the total energy in the center of mass \( W^2 = Q^2 \left( \frac{1}{x} - 1 \right) \), and index \( l \) specifies the parton flavour. The parameter \( k_0^2 \) is the infrared cut-off which will be set equal to 1 GeV\(^2\). The nonperturbative part \( \Delta p_l^{(0)}(x) \) can be viewed upon as originating from the integration over non-perturbative region.
Figure 1: Ladder diagram generating double logarithmic terms in photonic spin structure function $g_1^\gamma$.

$k^2 < k_0^2$, i.e.

$$\Delta p_i^{(0)}(x) = \int_0^{k_0^2} \frac{d k^2}{k^2} f_i(x, k^2).$$

(2)

Photonic structure function $g_1^\gamma(x, Q^2)$ is related in a standard way to the (integrated) parton distributions describing the parton content of polarized photon:

$$g_1^\gamma(x, Q^2) = \frac{\langle e^2 \rangle}{2} [\Delta q_{NS}(x, Q^2) + \Delta q_S(x, Q^2)],$$

(3)

where $N_f$ denotes the number of active flavours ($N_f = 3$). For convenience we have introduced in (3) the non-singlet and singlet combinations of the spin dependent quark and antiquark distributions defined as:

$$\Delta q_{NS}(x, Q^2) = \sum_{l=1}^{N_f} \left( e_l^2 \langle e^2 \rangle - 1 \right) (\Delta q_l^\gamma(x, Q^2) + \Delta \bar{q}_l^\gamma(x, Q^2)),$$

(4)

$$\Delta q_S(x, Q^2) = \sum_{l=1}^{N_f} (\Delta q_l^\gamma(x, Q^2) + \Delta \bar{q}_l^\gamma(x, Q^2)),$$

(5)

where $\langle e^m \rangle = \frac{1}{N_f} \sum_{l=1}^{N_f} (e_l)^m$.

The full contribution to the double $ln^2(1/x)$ resummation comes from the ladder diagrams with quark and gluon exchanges along the ladder (see e.g. Fig. 1) and the non-ladder bremsstrahlung diagrams [10]. The latter ones are obtained from the ladder..
diagrams by adding to them soft bremsstrahlung gluons or soft quarks \[1, 2, 16\], and they generate the infrared corrections to the ladder contribution.

The relevant region of phase space generating the double \(\ln^2(1/x)\) resummation from ladder diagrams corresponds to ordered \(k_n^2/x_n\), where \(k_n^2\) and \(x_n\) denote respectively the transverse momenta squared and longitudinal momentum fractions of the proton carried by partons exchanged along the ladder \[17\]. It is in contrast to the leading order Altarelli - Parisi evolution alone which corresponds to ordered transverse momenta.

The structure of the corresponding integral equations describing unintegrated distributions \(f_{NS}(x', k^2)\), \(f_S(x', k^2)\) and \(f_g(x', k^2)\) in a photon is the same as for the case of the partonic structure of a hadron \[6\] which read:

\[
f_{NS}(x', k^2) = f_{NS}^{(0)}(x', k^2) + \frac{\alpha_S}{2\pi} \Delta P_{qq}(0) \int_{x'}^1 dz \int_{k_0^2}^{k^2/z} \frac{dk^2}{k^2} f_{NS} \left(\frac{x'}{z}, \frac{k^2}{z^2}\right),
\]

\[
f_S(x', k^2) = f_S^{(0)}(x', k^2) + \frac{\alpha_S}{2\pi} \int_{x'}^1 dz \int_{k_0^2}^{k^2/z} \frac{dk^2}{k^2} \left[ \Delta P_{qg}(0) f_S \left(\frac{x'}{z}, \frac{k^2}{z^2}\right) + \Delta P_{gq}(0) f_S \left(\frac{x'}{z}, \frac{k^2}{z^2}\right) \right],
\]

\[
f_g(x', k^2) = f_g^{(0)}(x', k^2) + \frac{\alpha_S}{2\pi} \int_{x'}^1 dz \int_{k_0^2}^{k^2/z} \frac{dk^2}{k^2} \left[ \Delta P_{qg}(0) f_g \left(\frac{x'}{z}, \frac{k^2}{z^2}\right) + \Delta P_{gg}(0) f_g \left(\frac{x'}{z}, \frac{k^2}{z^2}\right) \right]
\]

(7)

with splitting functions \(\Delta P_{ij}(0) \equiv \Delta P_{ij}(z = 0)\) equal to :

\[
\Delta P(0) \equiv \left( \begin{array}{cc} \Delta P_{qq}(0) & \Delta P_{qg}(0) \\ \Delta P_{gq}(0) & \Delta P_{gg}(0) \end{array} \right) = \left( \begin{array}{cc} \frac{N_F^2-1}{2N_C} & -N_F \\ \frac{N_F^2-1}{2N_C} & 4N_C \end{array} \right),
\]

(8)

where \(\alpha_S\) denotes the QCD coupling which at the moment is treated as a fixed parameter. The variables \(k^2(k'^2)\) denote the transverse momenta squared of the quarks (gluons) exchanged along the ladder. In the case of the parton distributions of a hadron the inhomogeneous driving terms \(f_{l}^{(0)}(x', k^2)\) are entirely determined by non-perturbative parts \(\Delta p_{l}^{(0)}(x')\) of the (spin dependent) parton distributions. The novel feature of the photon case is the appearance of the additional contributions to these inhomogeneous terms describing point-like interaction of polarized photon with quarks and antiquarks. If the non-perturbative parts of the parton distributions were neglected then the inhomogeneous terms would be given by:

\[
f_{l}^{(0)}(x', k^2) = k_l^2(x'),
\]

(9)
where functions \( k_i^\gamma(x') \) are defined as \[9, 10\]:

\[
\begin{align*}
  k_{NS}^\gamma(x) &= \frac{N_C N_f}{2\pi} \left( \frac{\langle e^4 \rangle}{\langle e^2 \rangle} - \langle e^2 \rangle \right) \kappa(x), \\
  k_S^\gamma(x) &= \frac{N_C N_f}{2\pi} \langle e^2 \rangle \kappa(x), \\
  k_g^\gamma(x) &= 0
\end{align*}
\]  

(10)  

(11)  

(12)

with function \( \kappa(x) \) given by

\[
\kappa(x) = 2\alpha_{em}(x^2 - (1 - x)^2),
\]

(13)

where \( N_C \) denotes number of colours (\( N_C = 3 \)), and \( \alpha_{em} \) is the electromagnetic coupling constant. To be precise, in the genuine double \( \ln^2(1/x) \) approximation we should set \( f^{(0)}_i(x) = k_i^\gamma(0) \).

For the inhomogeneous terms set equal to \( k_i^\gamma(0) \) the solution(s) of equations (6,7) is found to be:

\[
\begin{align*}
  f_{NS}(x, k^2) &= d_{NS} I \left( 2\sqrt{D_{NS}} \frac{y}{\rho}, \frac{y}{\rho} \right), \\
  f_S(x, k^2) &= \frac{x^- d_s}{x^- - x^+} I \left( 2\sqrt{D_+} \frac{y}{\rho}, \frac{y}{\rho} \right) - \frac{x^+ d_s}{x^- - x^+} I \left( 2\sqrt{D_-} \frac{y}{\rho}, \frac{y}{\rho} \right), \\
  f_g(x, k^2) &= \frac{x^- x^+ d_s}{x^- - x^+} I \left( 2\sqrt{D_+} \frac{y}{\rho}, \frac{y}{\rho} \right) - \frac{x^+ x^- d_s}{x^- - x^+} I \left( 2\sqrt{D_-} \frac{y}{\rho}, \frac{y}{\rho} \right)
\end{align*}
\]

(14)  

(15)  

(16)

where \( y = \ln(1/x) \), \( \rho = \ln[k^2/(k_0^2 x)] \), \( \bar{\alpha}_S = \alpha_S/(2\pi) \) and

\[
\begin{align*}
  d_{NS} &= -2 N_F N_C \left( \frac{\langle e^4 \rangle}{\langle e^2 \rangle} - \langle e^2 \rangle \right), \\
  d_s &= -2 N_f N_C \langle e^2 \rangle, \\
  D_{NS} &= \bar{\alpha}_S \lambda_{NS}, \\
  D_+ &= \bar{\alpha}_S \lambda_+, \\
  D_- &= \bar{\alpha}_S \lambda_-
\end{align*}
\]

(17)

The function \( I(2\sqrt{u}, v) \) is defined by

\[
I(2\sqrt{u}, v) = \alpha_{em} [(1 - v) I_0(2\sqrt{u}) + \frac{v}{\sqrt{u}} I_1(2\sqrt{u})],
\]

(18)

where \( I_i(z) \) are the modified Bessel functions. The coefficients \( x^-, x^+ \) are defined as :

\[
x^\pm = \frac{\lambda^\pm - \Delta P_{qq}}{\Delta P_{qg}},
\]

(19)
while \( \lambda_{NS} = \Delta P_{qq}(0) \), and \( \lambda_+, \lambda_- \) correspond to the eigenvalues of matrix \( \Delta P(0) \) (cf. [3]).

In the limit of \( x \to 0 \) the dominant term in singlet distribution \( f_S(x, Q^2) \) is given by the first term in eq. (15) which is proportional to function \( I \left( 2\sqrt{D_+ y \rho}, \frac{2}{\rho} \right) \), since \( D_+ > D_- \). It can be checked, however, that coefficient \( x^- \) which multiplies this function is smaller by about a factor equal to three than coefficient \( x^+ \) multiplying function \( I \left( 2\sqrt{D_- y \rho}, \frac{2}{\rho} \right) \). This implies that the asymptotic behaviour controlled by the leading term is delayed to the very small values of \( x \) and that for moderately small values of \( x \) the singlet quark distributions are dominated by the second term in eq. (15). This effect is closely related to the approximation in which possible non-perturbative (i.e. hadronic) parts of the (spin dependent) distributions are neglected. It will also be present in the more elaborate and realistic treatment of the distributions which will be discussed in the subsequent Sections. The contribution of the leading eigenvalue can be enhanced by the non-perturbative spin dependent (input) gluon distributions in the photon.

Since \( y = \ln(1/x) \) and \( \rho = \ln[k^2/(k_0^2 x)] \) we find that \( y \rho \sim \ln^2(1/x) \) and \( y/\rho \sim 1 \) in the small \( x \) limit. For large values of \( u \) and for fixed \( v \) function \( I(2\sqrt{u}, v) \) behaves like \( \exp(2\sqrt{u}) \) modulo power corrections. This implies the power-law behaviour \( \sim x^{-2\sqrt{D_+}} \) (modulo logarithmic corrections) of the (singlet) parton distributions and of structure function \( g_1^\gamma(x, Q^2) \).

Besides the ladder diagrams contributions the double logarithmic resummation does also acquire corrections from non-ladder bremsstrahlung contributions. It has been shown in ref. [3] that these contributions can be included by adding the higher order terms to the kernels of integral equations (14). These terms can be obtained from the matrix:

\[
\tilde{F}_8^{\omega^2}(z) G_0,
\tag{20}
\]

where \( \tilde{F}_8^{\omega^2}(z) \) denote the inverse Mellin transform of the octet partial wave matrix (divided by \( \omega^2 \)), and the matrix \( G_0 \) reads:

\[
G_0 = \begin{pmatrix}
\frac{N_c^2-1}{2N_c} & 0 \\
0 & N_c
\end{pmatrix}.
\tag{21}
\]
Following ref. [6], we shall use Born approximation for the octet matrix which gives:

\[
\tilde{F}_{8}^{\text{Born}}(z) = 4\pi^2 \bar{\alpha}_S M_8 \ln^2(z),
\]

where \(M_8\) is the splitting functions matrix in colour octet t-channel, and it takes the form:

\[
M_8 = \begin{pmatrix}
-\frac{1}{2N_c} & -\frac{N_F}{2N_c} \\
\frac{N_F}{2N_c} & 2N_c
\end{pmatrix}.
\]

### 3 Unified treatment of the LO Altarelli Parisi evolution and of the double \(\ln^2(1/x)\) resummation.

In the region of large values of \(x\) the integral equations (6), (7) describing pure double logarithmic resummation \(\ln^2(1/x)\), even completed by including non-ladder contributions, are inaccurate. In this region one should use the conventional Altarelli - Parisi equations [3, 8, 13] with complete splitting functions \(\Delta P_{ij}(z)\) and not restrict oneself to the effect generated only by their \(z \to 0\) part. Following refs. [5, 6], we do therefore extend equations (6,7) and add to their right hand side(s) the contributions coming from the remaining parts of splitting functions \(\Delta P_{ij}(z)\). We also allow coupling \(\alpha_S\) to run, setting \(k^2\) as the relevant scale. In this way we obtain unified system of equations which contain both the complete leading order Altarelli - Parisi evolution and the double logarithmic \(\ln^2(1/x)\) effects at low \(x\). The corresponding system of equations reads:

\[
f_{NS}(x', k^2) = f_k^{(0)}(x', k^2) + \frac{\alpha_S(k^2)}{2\pi} \frac{4}{3} \int_{x'}^1 \frac{dz}{z} \int_{k_0^2}^{k^2/z} \frac{dk'^2}{k'^2} f_{NS}\left(\frac{x'}{z}, k'^2\right)
\]

(Ladder)

\[
+ \frac{\alpha_S(k^2)}{2\pi} \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2} \frac{4}{3} \int_{x'}^1 \frac{dz}{z} \frac{(z + z^2)f_{NS}(z', k'^2) - 2zf_{NS}(x', k'^2)}{1 - z}
\]

(Altarelli – Parisi)

\[
- \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^1 \frac{dz}{z} \left[ \frac{\tilde{F}_8}{\omega^2}(z) \frac{G_0}{2\pi^2} \right]_{qq} \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2} f_{NS}\left(\frac{x'}{z}, k'^2\right)
\]

(Non – ladder)
\[ f_S(x', k^2) = f_S^{(0)}(x', k^2) + \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \int k_0^2 \frac{dk^2}{k'^2} \frac{4}{3} f_S \left( \frac{x'}{z}, k^2 \right) \]

\[ - \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \int k_0^2 \frac{dk^2}{k'^2} N_F \frac{dk^2}{k'^2} f_g \left( \frac{x'}{z}, k^2 \right) \frac{\left( \frac{x'}{z}, k^2 \right)}{1 - z} \]

\[ + \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \int k_0^2 \frac{dk^2}{k'^2} \frac{4}{3} \int_{x'}^{1} \frac{dz}{z} \frac{1 + \frac{8}{3} \ln(1 - x')} {2 - 2z} f_S(x', k^2) \]

\[ + \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \int k_0^2 \frac{dk^2}{k'^2} \int_{x'}^{1} \frac{dz}{z} \frac{2zN_F f_g \left( \frac{x'}{z}, k^2 \right)} {1 - z} \]

\[ (\text{Ladder}) \]

\[ - \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \left( \frac{\hat{F}_8}{2\pi} \right) \frac{\left( \frac{x'}{z}, k^2 \right)} {2 - 2z} f_S \left( \frac{x'}{z}, k^2 \right) \]

\[ - \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \int k_0^2 \frac{dk^2}{k'^2} \frac{4}{3} f_S \left( \frac{x'}{z}, k^2 \right) \frac{\left( \frac{x'}{z}, k^2 \right)} {2 - 2z} f_g \left( \frac{x'}{z}, k^2 \right) \]

\[ (\text{Altarelli – Parisi}) \]

\[ f_g(x', k^2) = f_g^{(0)}(x', k^2) + \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \int k_0^2 \frac{dk^2}{k'^2} \frac{8}{3} f_S \left( \frac{x'}{z}, k^2 \right) \]

\[ + \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \int k_0^2 \frac{dk^2}{k'^2} \frac{12}{5} f_g \left( \frac{x'}{z}, k^2 \right) \]

\[ \left( \frac{x'}{z}, k^2 \right) \]

\[ + \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \int k_0^2 \frac{dk^2}{k'^2} \int_{x'}^{1} \frac{dz}{z} \frac{11}{2} \frac{f_g \left( \frac{x'}{z}, k^2 \right)} {1 - z} \]

\[ - \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \left( \frac{\hat{F}_8}{2\pi} \right) \frac{\left( \frac{x'}{z}, k^2 \right)} {2 - 2z} f_S \left( \frac{x'}{z}, k^2 \right) \]

\[ \left( \frac{x'}{z}, k^2 \right) \]

\[ - \frac{\alpha_S(k^2)}{2\pi} \int_{x'}^{1} \frac{dz}{z} \int k_0^2 \frac{dk^2}{k'^2} \frac{4}{3} f_S \left( \frac{x'}{z}, k^2 \right) \frac{\left( \frac{x'}{z}, k^2 \right)} {2 - 2z} f_g \left( \frac{x'}{z}, k^2 \right) \]

\[ (\text{Non – ladder}) \]

In equations (24), (25), (26) we group separately terms corresponding to ladder diagram contributions to the double \( \ln^2(1/x) \) resummation, contributions from the non-
singular parts of the Altarelli - Parisi splitting functions and finally contributions from
the non-ladder bremsstrahlung diagrams. We label those three contributions as "ladder", "Altarelli - Parisi " and "non-ladder" respectively.

Inhomogeneous terms \( f_i^{(0)}(x', k^2) \) \((i = NS, S, g)\), as stated above, may be expressed as:

\[
\begin{align*}
  f_{NS}^{(0)}(x', k^2) &= k^2_{NS}(x) + \frac{\alpha_s(k^2)}{2\pi} \left[ \frac{2}{3} \int_{x'}^1 \frac{dz}{z} \left( 1 + z^2 \right) \Delta q_{NS}^{(0)}(\frac{x'}{z}) - 2z \Delta q_{NS}^{(0)}(x') \right] \\
  f_S^{(0)}(x', k^2) &= k^2_S(x) + \frac{\alpha_s(k^2)}{2\pi} \left[ \frac{2}{3} \int_{x'}^1 \frac{dz}{z} \left( 1 + z^2 \right) \Delta q_S^{(0)}(\frac{x'}{z}) - 2z \Delta q_S^{(0)}(x') \right] \\
  f_g^{(0)}(x', k^2) &= k^2_g(x) + \frac{\alpha_s(k^2)}{2\pi} \left[ \frac{2}{3} \int_{x'}^1 \frac{dz}{z} \left( 1 + z^2 \right) \Delta q_S^{(0)}(\frac{x'}{z}) - 2z \Delta q_S^{(0)}(x') \right] \\
  &\quad + \frac{\alpha_s(k^2)}{2\pi} N_F \int_{x'}^1 \frac{dz}{z} \left( 1 - 2z \right) \Delta g^{(0)}(\frac{x'}{z}), \\
  &\quad + \frac{\alpha_s(k^2)}{2\pi} \left[ \frac{11}{2} - \frac{N_F}{3} + 6 \ln(1 - x') \right] \Delta g^{(0)}(x') \\
  &\quad + \frac{\alpha_s(k^2)}{2\pi} 6 \int_{x'}^1 \frac{dz}{z} \left[ \Delta g^{(0)}(\frac{x'}{z}) - z \Delta g^{(0)}(x') \right] + (1 - 2z) \Delta g^{(0)}(\frac{x'}{z}).
\end{align*}
\]

Equations (24), (25), (26) together with (27), (28) and (1) reduce to the leading order Altarelli - Parisi evolution equations for photonic structure function with starting (integrated) distributions \( \Delta q_i^{(0)}(x) \) \((i = NS, S)\) and \( \Delta g^{(0)}(x) \) after we set the upper integration limit over \( dk'^2 \) equal to \( k^2 \) in all terms in equations (24), (25), (26), neglect the higher order terms in the kernels, and set \( Q^2 \) in place of \( W^2 \) as the upper integration limit of the integral in eq. (1). In this approximation the cut-off parameter \( k^2_0 \) is equal to the magnitude of the scale \( Q^2 \) for which the parton distributions in the photon are entirely specified by the hadronic component of the photon and are equal to \( \Delta q_i^{(0)}(x) \) \((i = NS, S)\) and \( \Delta g^{(0)}(x) \).

\section{Numerical results.}

We have solved equations (24), (25), (26), assuming the input parametrizations which satisfy the constraint for the first moment of photonic structure function \( g_1(x, Q^2) \) [20]:

\[
\int_0^1 g_1(x, Q^2) = 0
\]

(29)
Figure 2: Structure function $g_1^\gamma(x, Q^2)/\alpha_{em}$ for $Q^2 = 10 GeV^2$ derived after solving eqs. (24), (25), (26) with input parametrization (31) plotted as function of $x$. Solid line corresponds to the calculations which contain full $ln^2(1/x)$ resummation with both bremsstrahlung corrections and LO Altarelli - Parisi kernel included, dashed line shows pure LO Altarelli - Parisi evolution, dotted line represents NLO Altarelli - Parisi evolution.

which, in turn, implies the non-perturbative input to fulfill the sum rule ($l = NS, S$) :

$$\int_0^1 \Delta q_l^{(0)}(x) = 0,$$

$$\int_0^1 \Delta g_{(0)}(x) = 0. \quad (30)$$

We have considered two input parametrizations and tested their influence on the behaviour of $g_1^\gamma(x, Q^2)$. The first parametrization assumed no non-perturbative contribution at all and followed the lower limit parametrization by Stratmann et al. [9, 10]:

$$\Delta q_l^{(0)}(x) = 0,$$

$$\Delta g^{(0)}(x) = 0. \quad (31)$$

In the second parametrization we allowed for the non-zero input gluon distribution which was based on the vector meson dominance (VMD) model. Assuming the dominance of the $\rho$ and $\omega$ meson contribution, one obtains :

$$\Delta g^{(0)}(x) = \alpha_{em} \left( g_\rho^2 \Delta g_\rho^{(0)}(x) + g_\omega^2 \Delta g_\omega^{(0)}(x) \right), \quad (32)$$

where $g_\rho$ and $g_\omega$ are constants characterizing the coupling of the $\rho$ and $\omega$ mesons to the photon, and:

$$g_\rho^2 + g_\omega^2 \sim 0.5. \quad (33)$$
Coefficient $\Delta g^V(0)(x)$ ($V = \rho, \omega$) denotes the nonperturbative part of the spin dependent gluon distribution in a photon which was parametrized in the following form:

$$\Delta g^V(0)(x) = C^V_g (1 - x)^3 (1 + a_g x),$$

so that we have $\Delta g^V(0)(x) \rightarrow C^V_g$ in the limit $x \rightarrow 0$. Similar behaviour was assumed in ref. [6] for the nonperturbative part of the spin dependent gluon distribution in the nucleon, i.e. $\Delta g^N(0)(x) \rightarrow C^N_g$. Constant $C^V_g$ was related to constant $C^N_g$ as below

$$C^V_g = \frac{2}{3} C^N_g,$$

and it reads $C^V_g = 4.5$. Finally, coefficient $a_l$ is determined by imposing condition (30) on the distributions (32) so that $a_l = -5$.

In Figs. 2, 3 we present results obtained for photonic structure function $g_1^\gamma(x, Q^2)/\alpha_{em}$ after numerical solving of eqs. (24), (25), (26) with parametrizations (31), (32) respectively.

In Fig. 2 predictions for $g_1^\gamma(x, Q^2)$ based on eqs. (24), (25), (26) with input parametrization (31) are plotted and confronted with the results obtained from the solution of LO and NLO Altarelli - Parisi evolution equations with the input distribution given by eq. (31). One may see that double logarithmic contributions manifest at low $x$, however, not as strong as for the nucleon structure function case (cf. [1]). There is a
characteristic decrease of photonic structure function till \( x \sim 10^{-4} \), then the function starts to increase rapidly, and it is expected to grow steeply in the ultra-asymptotic region \( x < 10^{-5} \). We note that in the small \( x \) region which may be probed in linear \( e\gamma \) colliders (i.e. \( x > 10^{-4} \)) the leading asymptotic behaviour has not been established yet, and that it is delayed to the ultra-asymptotic region. This is manifestation of the effect discussed in Sec. 2.

In Fig. 3 we show results for \( g_1^\gamma(x, Q^2) \) which were obtained assuming the presence of the non-perturbative part of the spin dependent gluon distribution in the photon (see eq. (32)). One may see from this plot that the absolute magnitude of the structure function is significantly enhanced in comparison to the previous case. This is related to the enhancement of the leading asymptotic term by the non-perturbative gluon distribution. The structure function would stay negative in the limit \( x \to 0 \).

In both cases we notice that the NLO approximation of the Altarelli-Parisi equation contains significant part of the double \( \ln^2(1/x) \) resummation in the structure function \( g_1(x, Q^2) \) at low values of \( x \). It should be emphasised that this resummation embodies the leading singular part of the NLO approximation which contains the lowest order term of the double \( \ln^2(1/x) \) resummation. We also notice that the difference between the structure function \( g_1(x, Q^2) \) calculated from equations (1), (3), (24), (25), (26) and that obtained within the LO Altarelli-Parisi formalism is also present at large values of
Figure 5: Sensitivity of predictions for $g_1^\gamma(x, Q^2)$ ($Q^2 = 10 GeV^2$) with input parametrization \cite{12} to the magnitude of the infrared cut-off parameter $k_0^2$. Solid line corresponds to the calculations with $k_0^2 = 1 GeV^2$, dashed line shows $g_1^\gamma(x, Q^2)$ for $k_0^2 = 2.0 GeV^2$ and dotted line represents $g_1^\gamma(x, Q^2)$ calculations for $k_0^2 = 0.5 GeV^2$.

This fact is caused by different upper integration limit in eq. (1) which for the LO approximation is equal to $Q^2$ instead of $W^2 = Q^2(1/x - 1)$. Although this difference is formally subleading it is logarithmically enhanced at both small and large values of $x$. At large values of $x$ the unintegrated parton distributions $f_i(x, k^2)$ are dominated by the inhomogeneous parts $k_i^\gamma(x)$ (see eq. (1)), and we get $g_1^\gamma(x, Q^2) \sim \kappa(x)ln(W^2/k_0^2)$ (modulo less singular terms) while in LO approximation we have $g_1^\gamma(x, Q^2) \sim \kappa(x)ln(Q^2/k_0^2)$, where the function $\kappa(x)$ is defined by eq. (13). We do therefore notice that structure function $g_1^\gamma(x, Q^2)$ calculated within our scheme contains at large values of $x$ an additional term proportional to $\kappa(x)ln(1/x - 1)$ in comparison to the structure function calculated within the LO approximation, since $ln(W^2/k_0^2) = ln(Q^2/k_0^2) + ln(1/x - 1)$. This additional term is in fact equal to the singular part of the photonic coefficient $\Delta C_\gamma(x)$ in the NLO approximation in the \overline{MS} scheme \cite{9}. These effects are of course absent in the hadronic case and so the spin dependent nucleon structure function calculated within the formalism generating the double $ln^2(1/x)$ resummation approaches at large values of $x$ that calculated within the LO approximation \cite{4}.

In Figs. 4, 5 we show sensitivity of our predictions for $g_1^\gamma(x, Q^2)$ at low values of $x$ to the magnitude of the infrared cut-off parameter $k_0^2$. We see that the magnitude of the structure function can change by about a factor equal to 1.8 for $k_0^2$ varying between 0.5 and 2 $GeV^2$. The cut-off parameter $k_0^2$ does also slightly affect the shape of $g_1^\gamma(x, Q^2)$ as the function of $x$. 
In Fig. 6 we plot the ratio \( g_1^\gamma(x, Q^2)/F_1^\gamma(x, Q^2) \) which measures the asymmetry, using the structure function \( F_1^\gamma(x, Q^2) \) obtained from the LO analysis presented in ref. [21]. We see that the magnitude of this ratio is very small for \( x < 10^{-3} \) (i.e. smaller than \( 10^{-2} \)), and it may be very difficult to measure such a low value of the asymmetry.

We have also plotted spin dependent photonic gluon distribution \( \Delta g_1^\gamma \) derived for both parametrizations [31], [32] (see Figs. 7, 8). We observe that the gluon distributions are very different. We note in particular that \( \Delta g_1^\gamma(x, Q^2) \) is negative at small \( x \) for the first case, i.e. when this distribution is generated purely radiatively. It is possible to obtain a positive gluon distribution assuming the (positive) non-perturbative part of this distribution (see Fig. 8). These two possible scenarios for the gluon distributions could be discriminated by the measurements which would access more directly the spin dependent gluon distributions in the photon like, for instance, the measurement of the dijet production in (polarised) \( \gamma\gamma \) scattering etc.

5 Summary and conclusions

In this paper we have studied the possible effects of the double \( \ln^2(1/x) \) resummation upon behaviour of spin dependent structure function \( g_1^\gamma(x, Q^2) \) of the photon. This
Figure 7: Spin dependent gluon distribution in the photon $\Delta g^\gamma(x, Q^2)/\alpha_{em}$ for $Q^2 = 10 GeV^2$ derived after solving eqs. (24), (25), (26) with input parametrization (31) plotted as function of $x$. Solid line corresponds to the calculations which contain full $ln^2(1/x)$ resummation with both bremsstrahlung corrections and LO Altarelli - Parisi kernel included, dashed line shows pure LO Altarelli - Parisi evolution.

quantity could in principle be measured in the future linear colliders, in particular in the $e\gamma$ mode. We have extended the formalism developed by us for the case of the spin dependent structure function of the nucleon to the case of the photon structure functions by including the suitable inhomogeneous terms in the corresponding integral equations. These inhomogeneous terms describe the point-like coupling of the (real) photons to quarks and antiquarks. We have studied sensitivity of our predictions upon the possible hadronic component of the spin dependent distributions, and found that the presence of the (input) gluon distribution in the hadronic component can significantly enhance the absolute magnitude of the structure function in the low $x$ region.

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Figure 8: Spin dependent gluon distribution $\Delta g(x, Q^2)/\alpha_{em}$ for $Q^2 = 10 GeV^2$ derived after solving eqs. (24), (25), (26) with input parametrization (32) plotted as function of $x$. Solid line corresponds to the calculations which contain full $\ln^2(1/x)$ resummation with both bremsstrahlung corrections and LO Altarelli - Parisi kernel included, dashed line shows pure LO Altarelli - Parisi evolution.

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