Reinforcement learning in market games

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Abstract

Financial markets investors are involved in many games – they must interact with other agents to achieve their goals. Among them are those directly connected with their activity on markets but one cannot neglect other aspects that influence human decisions and their performance as investors. Distinguishing all subgames is usually beyond hope and resource consuming. In this paper we study how investors facing many different games, gather information and form their decision despite being unaware of the complete structure of the game. To this end we apply reinforcement learning methods to the Information Theory Model of Markets (ITMM). Following Mengel, we can try to distinguish a class $\Gamma$ of games and possible actions (strategies) $a^i_m$ for $i$–th agent. Any agent divides the whole class of games into analogy subclasses she/he thinks are analogous and therefore adopts the same strategy for a given subclass. The criteria for partitioning are based on profit and costs analysis. The analogy classes and strategies are updated at various stages through the process of learning. We will study the asymptotic behaviour of the process and attempt to identify its crucial stages, eg existence of possible fixed points or optimal strategies. Although we focus more on the instrumental aspects of agents behaviours, various algorithm can be put forward and used for automatic investment. This line of research can be continued in various directions.

Key words: econophysics, market games, learning

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Motto:

"The central problem for gamblers is to find positive expectation bets. But the gambler also needs to know how to manage his money, i.e. how much to bet. In the stock market (more inclusively, the securities markets) the problem is similar but more complex. The gambler, who is now an investor, looks for excess risk adjusted return."

Edward O. Thorp

1 Introduction

Noise or structure? We face this question almost always while analyzing large data sets. Pattern discovery is one of the primary concerns in various fields in research, commerce and industry. Models of optimal behaviour often belong to that class of problems. The goal of an agent in a dynamic environment is to make optimal decision over time. One usually have to discard a vast amount of data (information) to obtain a concise model or algorithm. Therefore prediction of individual agent behaviours is often burdened with large errors. The prediction game algorithm can be described as follows.

\[
\text{FOR } n = 1, 2, \ldots \\
\text{Reality announces } x_n \in X \\
\text{Predictor announces } \gamma_n \in \Gamma \\
\text{Reality announces } y_n \in Y \\
\text{END FOR,}
\]

where \( x_n \in X \) is the data upon which the prediction \( \gamma_n \in \Gamma \) is made at each round \( n \). The prediction quality is measured by some utility function \( v : \Gamma \times Y \to \mathbb{R} \). One can view such a process as a communication channel that transmit information from the past to the future \([1]\). The gathering of information, often indirect and incomplete, is referred to as measurements. Learning theory deals with the abilities and limitations of algorithms that learn or estimate functions from data. Learning helps with optimal behaviour decisions by adjusting agent’s strategies to information gathered over time. Agents can base their action choices on prediction of the state of the environment or on reward received during the process. For example, Markov decision process can be formulated as a problem of finding a strategy \( \pi \) that maximizes

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the expected sum of discounted rewards:

\[ v(s, \pi) = r(s, a_\pi) + \beta \sum_{s'} p(s'|s, a_\pi) v(s', \pi), \]

where \( s \) is the initial state, \( a_\pi \) is the action induced by the strategy \( \pi \), \( r \) is the reward at stage \( t \) and \( \beta \) is the discount factor; \( v \) is called the value function. 

\( p(s'|s, a_\pi) \) denote the (conditional) probability\(^1\) of reaching the state \( s' \) from the state \( s \) as result of action \( a_\pi \). It can be shown that, in the case of infinite horizon, an optimal strategy \( \pi^* \) such that (Bellman optimality equation)

\[ v(s, \pi^*) = \max_a \{ r(s, a) + \beta \sum_{s'} p(s'|s, a) v(s', \pi^*) \} \]

exists. In reinforcement learning, the agent receives rewards from the environment and uses them as feedback for its action. Reinforcement learning has its roots in statistics cybernetics, psychology, neuroscience, computer science ... . In its standard formulation, the agent must improve his/her performance in a game through trial-and-error interaction with a dynamical environment. There are two ways of finding the optimal strategy:

- **strategy iteration** – one directly manipulates the strategy;
- **value iteration** – one approximates the optimal value function.

Therefore two classes of algorithms are considered: strategy (policy) iteration algorithms and value iteration algorithms. In the following section we discuss the adequacy of reinforced learning in market games.

## 2 Reinforcement learning in market games

Can reinforcement learning help with market games analysis? Could it be used for finding optimal strategies? It not easy to answer this question because it involves the problem of real-time decision making one often have to (re-)act as quickly as possible. Consider model-free reinforcement learning, Q-learning\(^2\).

In this approach one defines the value of an action \( Q(s, a) \) as a discounted return if action \( a \) following from the strategy \( \pi \) is applied:

\[ Q^*(s, a) = r(s, a) + \beta \sum_{s'} p(s'|s, a) v(s', \pi^*) \]

\(^1\) In a more formal setting it would be a transition kernel of for the process of consecutive actions and observations.

\(^2\) This is obviously a value iteration, but in market games there is a natural value function – the profit.
then
\[ \nu(s, \pi^*) = \max_a Q^*(s, a). \]

In Q-learning, the agent starts with arbitrary \( Q(s, a) \) and at each stage \( t \) observes the reward \( r_t \) and updates the value of \( Q \) according to the rule:
\[ Q_{t+1}(s, a) = (1 - \alpha_t) Q_t(s, a) + \alpha_t (r_t + \beta \max_b Q_t(s, b)) , \]

where \( \alpha_t \in [0, 1) \) is the learning rate that needs to decay over time for the learning algorithm to converge. This approach is frequently used in stochastic games setting. Watkins and Dayan proved that this sequence converges provided all states and actions have been visited/performed infinitely often [5]. Therefore we anticipate weak convergence ratios. Indeed, various theoretical and experimental analyses [6,7,8] showed that even in very simple games might require \( \sim 10^8 \) steps! If a well-shaped stock trend is formed, one can expect that there are sorts of adversarial equilibria (no agent is hurt by any change of others’ strategies)
\[ R_i(\pi_1, \ldots, \pi_n) \leq R_i(\pi_1', \ldots, \pi_{i-1}', \pi_i', \pi_{i+1}', \ldots, \pi_n') \]

or coordination equilibria (all agents achieve their highest possible return)
\[ R_i(\pi_1, \ldots, \pi_n) = \max_{a_1, \ldots, a_n} R_i(a_1, \ldots, a_n). \]

Here \( Rs \) denote the pay-off functions and \( \pi s \) the one-stage strategies. The problem is they can be easily identified with technical analysis tools and there is no need to recall to learning algorithms. In the most interesting classes of games neither adversarial equilibria nor coordination equilibria exist. This type of learning is much more subtle and, up to now, there is no satisfactory analysis in the field of reinforcement learning. Therefore a compromise is needed, for example we must be willing to accept returns that might not be optimal. The models discussed in the following subsections belong to that class and seem to be tractable by leaning algorithms.

2.1 Kelly’s criterion

Kelly’s criterion [9] can be successfully applied in horse betting or blackjack when one can discern biases [10] even though its optimality and convergence can be proven only in the asymptotic cases. The simplest form of Kelly’s formula is:
\[ \Theta = W - (1 - W)/R \]

where:
\footnote{We understand the term technical analysis as simplified hypothesis testing methods that can be applied in real time.}
\( \Theta \) = percentage of capital to be put into a single trade.
\( W \) = historical winning percentage of a trading system.
\( R \) = historical average win/loss ratio.

Originally, Kelly’s formula involves finding the ”bias ratio” in a biased game. If the game is infinitely often repeated then one should put at stake the percentage of one’s capital equal to the bias ratio. Therefore one can easily construct various learning algorithms that perform the task of finding an environment so that Kelly’s approach can be effectively applied (bias search + horizon of the investment) \[11,12\].

2.2 MMM model

Piotrowski and Śladkowski have analysed the model where the trader fixes a maximal price he is willing to pay for the asset \( \Theta \) and then, if the asset is bought, after some time sells it at random \[13\]. One can easily reverse the buying and selling strategies. The expected value of the of the profit after the whole cycle is

\[
\rho_\eta(a) = \frac{-\int_{-\infty}^{a} p \eta(p) \, dp}{1 + \int_{-\infty}^{a} \eta(p) \, dp}
\]

where \( a \) is the withdrawal price. The maximal value of the function \( \rho \), \( a_{\text{max}} \), lies at a fixed point of \( \rho \), that is fulfills the condition \( \rho(a_{\text{max}}) = a_{\text{max}} \). The simplest version of of the strategy is as follows: there an optimal strategy that fixes the withdrawal price at the level historical average profit.\[4\] Task: find an implementation of reinforced learning algorithm that can be used effectively on markets. We should control both, the probability distribution \( \eta \) and the profit ”quality”.

2.3 Learning across games

An interesting approach was put forward by Mengel \[14\]. One can easily give examples of situations where agents cannot be sure in what game they are taking part (e.g. the games may have the same set of actions). Distinguishing all possible games and learning separately for all of them requires a large amount of alertness, time and resources (costs). Therefore the agent should try to identify some classes of situations she/he perceives as analogous and therefore takes the same actions. The learning algorithm should update both the partition of the set of games and actions to be taken:

\[4\] Or else: do not try to outperform yourself.
• Agents are playing repeatedly a game (randomly) drawn from a set \( \Gamma \)
• Agents partition the set of all games into subsets (classes) of games they learn not to discriminate (see them as analogous)
• Agents update both propensities to use partitions \( \{G\} \) and attractions towards using their possible strategies/actions

Asymptotic behaviour and computation complexity of such process is discussed in Ref. [14]. Stochastic approximation is working in this case (approximation through a system of deterministic differential equations is possible). It would be interesting to analyse the following problems. Problem 1.: Identify possible “classes of market games” Problem 2.: Identify “universal” set of strategies. For example, on the stock exchange one can try the brute force approach. Admit as strategies buying/selling at all possible price levels and identify classes of games with trends. Unfortunately, the number of approximations generates huge transaction costs. This can be reduced on the derivative markets as due to the leverage the ratio of transaction cost to price movements is much lower. We envisage that an agent may try to optimize among various classes of technical analysis tools.

3 Conclusion

As conclusions we would like to stress the following points.

Algorithms are simple but computation is complex, time and resource consuming.
Learning across games could be used to “fit” technical analysis models.
Dynamic proportional investing (Kelly) should be the easiest to implement.
But here we envisage problems analogous to related to heat (entropy) in thermodynamics, and exploration of knowledge might involve in cases of high effectiveness paradoxes [11] analogous to those of arising when speed approaches the speed of light [12].
One can envisage learning (information) models of markets/portfolio theory. Implementation should be carefully tested – transaction costs can ”kill” even crafty algorithms [15].
Quantum algorithms/computers, if ever come true might change the situation in a dramatic way: we would have powerful algorithms at our disposal and the learning limits would certainly broaden [16][17][18].

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