Creation of a Compact Topologically Nontrivial Inflationary Universe

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If inflation can occur only at the energy density $V$ much smaller than the Planck density, which is the case for many inflationary models based on string theory, then the probability of quantum creation of a closed or an infinitely large open inflationary universe is exponentially suppressed for all known choices of the wave function of the universe. Meanwhile under certain conditions there is no exponential suppression for creation of topologically nontrivial compact flat or open inflationary universes. This suggests, contrary to the standard textbook lore, that compact flat or open universes with nontrivial topology should be considered a rule rather than an exception.

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I. INTRODUCTION

The standard textbook description of the cosmological models usually is limited to the description of the three Friedmann models, describing a closed universe, and infinite topologically trivial flat and open universes. However, in his papers Friedmann noted that even flat and open universes may be compact if they have nontrivial topology \(^1\). He described how space could be finite and multi-connected by suitably identifying points, and predicted the possible existence of “ghost” images of astronomical sources. The possibility that we may live in a compact universe with nontrivial topology was mentioned even earlier by Schwarzschild \(^2\). Since that time, there were dozens of papers on the compact flat or open universes, see e.g. \(^3\) \(^4\) \(^5\) \(^6\) \(^7\) \(^8\) \(^9\) \(^10\). Nevertheless, this possibility is not mentioned even as a footnote in most of the textbooks on cosmology and general theory or relativity.

This attitude is quite understandable. First of all, we still do not have a compelling observational evidence of the nontrivial topology of the universe \(^11\), even though the situation may change with the further investigation of the CMB anisotropy. Moreover, inflation typically makes the universe so large that it removes any hope for the observational study of its topological structure, unless one fine-tunes the total number of e-folds of inflation and bends the overall shape of the scalar potential in a rather peculiar way \(^12\). So one may wonder why should we spend our time studying topology of the universe if it is not supported by experimental evidence and theoretical expectations?

As we will argue in this paper, the models of a compact flat or open (hyperbolic) universe with nontrivial topology may play an important role in inflationary cosmology by providing proper initial conditions for low-scale inflation. Most of our arguments will be based on results obtained in the papers on quantum creation of the compact universe with nontrivial topology \(^3\) \(^4\) and in the papers on chaotic mixing \(^5\) \(^6\). However, we will look at these results from a slightly different perspective: We will try to find out how the probability of initial conditions for inflation depends on the effective potential of the inflaton field $V(\phi)$ for various versions of topology of the universe.

II. THE PROBLEM OF INITIAL CONDITIONS FOR THE LOW-SCALE INFLATION

Let us first remember some basic facts related to the problem of initial conditions for inflation \(^13\). Inflation appears in a patch of a closed universe or of an infinite flat or open universe if this patch is sufficiently homogeneous and its size is greater than the size of the horizon, $\Delta l \gtrsim H^{-1} \sim V^{-1/2}(\phi)/M_p$, where $V(\phi)$ is a flat inflationary potential.

The simplest inflationary scenario is based on the theory of a scalar field with the potential $\sim \phi^n$. Formally, inflation in this scenario can start at an arbitrarily large values of $\phi$, i.e. close to the cosmological singularity. In this paper we will use the system of units $M_p^2 = (8\pi G)^{-1} = 1$. If we assume that the standard description of the universe in terms of the classical space-time becomes possible only below the Planck density $M_p^4 = 1$, then inflation may begin, for example, in a closed universe of the smallest possible size $\Delta l = O(1)$ with the total entropy $S = O(1)$. These conditions seem quite natural and easy to satisfy; see e.g. a discussion of this issue in \(^13\).
However, in all inflationary models where the effective potential does not change too much during the last stages of inflation one has a general constraint $V \lesssim 10^{-8}$, which follows from the constraint on the amplitude of gravitational waves generated during inflation. In many inflationary models, such as new inflation, hybrid inflation, etc., inflation occurs at $V(\phi) \ll 10^{-8}$. In these models, the problem of initial conditions becomes more complicated. Consider, for example, a closed universe filled with radiation. At the Planck time, the energy density of radiation $T^4 \sim 1$ was completely dominated the total energy density $\sim T^4 + V(\phi)$. Inflation begins only when the universe cools down and the energy density of radiation becomes much smaller than $V(\phi)$, which happens at $t \sim H^{-1} \sim V^{-1/2}$. The problem is that a radiation dominated closed universe may die before this happens.

Indeed, a closed universe with energy density dominated by radiation collapses within the time $t \sim S^{2/3}$ (in Planckian units), where $S$ is the total entropy of the universe. If we want the universe to survive until its energy density drops down to the total entropy $S_v$ until its energy density drops down to $S_v$, the initial entropy must be $S > S_v$. This means, e.g., that for the models where inflation may occur only at $V < 10^{-12}$, the initial entropy of the universe must be $S > 10^9$. At the Planck time the total mass of such universe would be greater than $10^9$ in Planck units, and it would consist of $10^9$ causally disconnected regions of Planck size. Thus in order to explain why our universe is so large and homogeneous we would need to assume that it was very large and homogeneous from the very beginning.

An estimate of the probability of such event for the universe initially dominated by relativistic matter suggests that it should be suppressed by a factor of $\exp(-S) \sim \exp(-V^{-3/4}) < e^{-10^9}$, and the probability that not only the energy density but also the scalar field in such a universe will be sufficiently homogeneous may be even smaller.

Another way to look at the same problem is to assume that instead of being born in a singularity and passing through the radiation dominated stage, a closed universe was created “from nothing” (i.e. from a state with a scale factor $a = 0$) in an inflationary state dominated by the inflaton potential energy $V(\phi)$. However, the minimal size of a closed inflationary universe is given by $H^{-1} \sim V^{-1/2}$. Therefore the universe with $V(\phi) \ll 1$ cannot classically evolve starting from the point $a = 0$. Since this process is forbidden at the classical level, the universe must tunnel through a potential barrier. An investigation of this issue using Euclidean approach to quantum cosmology suggested that the probability of quantum creation of a closed inflationary universe is exponentially suppressed by a factor of

$$P \sim \exp\left(-\frac{24\pi^2}{V(\phi)}\right). \quad (1)$$

This result has an obvious interpretation. Creation of the universe from nothing implies emergence of a universe with initial volume $H^{-3}$, and with total energy of matter $\Delta E \sim H^{-3}V \sim V^{-1/2}$. For $V \sim 1$ (Planck density), uncertainty relations tell us that such fluctuations can readily occur within the Planck time $\Delta t = O(1)$, but for $V \ll 1$ these fluctuations should be extremely improbable, which is reflected in Eq. (1). In a particular example with $V \sim 10^{-12}$, the probability of quantum creation of inflationary universe is suppressed by the factor of $10^{-10^{14}}$.

Open and flat inflationary universes can classically evolve from $a = 0$ without any tunneling. However, we do not know any way to create an infinite homogeneous flat universe. There are two different ways to create an infinite homogeneous open universe. The first one involves creating a closed inflationary universe of the old inflation type. Then one must ensure that it decays by bubble formation and that the slow-roll inflation occurs inside each bubble. Another possibility is to create an infinite open universe directly, using a different analytic continuation of the instanton of the same type as the one used for the description of creation of a closed universe. Both cases involve tunneling and exponential suppression of the probability of creation of an inflationary universe with $V \ll 1$.

### III. LOW-SCALE INFLATION IN A COMPACT OPEN OR FLAT UNIVERSE

In the previous section we identified the root of the problem of initial conditions for creation of a closed or an infinitely large open inflationary universe: If the universe initially was matter dominated, it should contain a very large nearly homogeneous patch with large initial mass and entropy. If, on the other hand,

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1 The Hartle-Hawking wave function yields a different result, $P \sim \exp(\pm \frac{24\pi^2}{V})$. In this case, the probability of inflation is also exponentially suppressed, as compared with the probability of living in the deepest minimum of $V(\phi)$. In our opinion, this function, as well as its recent modifications, describe the probability of the final conditions in the universe, instead of the initial conditions; see a discussion of this issue in [13, 14]. It is important that in either interpretation, the exponential suppression of probability is due to the tunneling, i.e. due to the classically forbidden evolution.
we consider a possibility of a quantum creation of the universe, the probability of such an event is exponentially suppressed because a part of the trajectory describing creation of the universe is forbidden at the classical level.

However, in a compact flat or open universe none of these problems exist, under certain conditions to be discussed below.

A. Creation of a hot compact universe

Consider for simplicity the flat compact universe having the topology of a torus, $S^3_1$,

$$ds^2 = dt^2 - a^2(t) dx_i^2$$

with identification $x_i + 1 = x_i$ for each of the three dimensions. Consider for simplicity the case $a_1 = a_2 = a_3 = a(t)$. In this case the curvature of the universe and the Einstein equations written in terms of $a(t)$ will be the same as in the infinite flat Friedmann universe with metric $ds^2 = dt^2 - a^2(t) dx^2$. In our notation, the scale factor $a(t)$ is equal to the size of the universe in Planck units $M_p^{-1} = 1$.

Let us assume, as we did in the beginning of the previous section, that at the Planck time $t_p \sim M_p^{-1} = 1$ the universe was radiation dominated, $V \ll T^4 = O(1)$. Let us also assume that at the Planck time the total size of the box was Planckian, $a(t_p) = O(1)$. In such case the whole universe initially contained only $O(1)$ relativistic particles such as photons or gravitons, so that the total entropy of the whole universe was $O(1)$.

In general, in addition to the energy of relativistic particles one should also consider the Casimir energy which appears due to the finiteness of the size of the box. However, Casimir energy is suppressed by supersymmetry. In the absence of supersymmetry, this energy is $O(a^{-4})$, so if $a$ is somewhat greater than 1 at the Planck density, this energy always remains smaller than the thermal energy.

The size of the universe dominated by relativistic particles was growing as $a(t) \sim \sqrt{t}$, whereas the mean free path of the gravitons was growing as $H^{-1} \sim t$. If the initial size of the universe was $O(1)$, then at the time $t \gg 1$ each particle (or a gravitational perturbation of metric) within one cosmological time would run all over the torus many times, appearing in all of its parts with nearly equal probability. This effect, called “chaotic mixing,” should lead to a rapid homogenization of the universe [3, 4]. Note, that to achieve a modest degree of homogeneity required for inflation to start when the density of ordinary matter drops down, we do not even need chaotic mixing. Indeed, density perturbations do not grow in a universe dominated by ultrarelativistic particles if the size of the universe is smaller than $H^{-1}$. As we just mentioned, this is exactly what happens in our model.

Note that this homogenization applies to all light fields, including the inflaton field. Consider, for example, an initial domain of a Planckian size $O(1)$, containing the inhomogeneities of the scalar field with the energy density $(\partial_i \phi)^2 \sim T^4 \leq 1$. Since the mass squared of the inflaton field is much smaller than $V$, and the size of the universe prior to inflation is smaller than $H^{-1}$, perturbations of this field behave as ultrarelativistic particles, the distribution of density of scalar perturbations becomes nearly homogeneous, and the energy density of the gradients of the scalar field will decrease as the energy of radiation (as $a^{-4}$) all the way until the beginning of inflation when $V(\phi)$ begins to dominate.

Chaotic mixing can be very efficient in a compact hyperbolic universe (i.e. in a compactified version of an open universe) [3, 4]. Moreover, the effective energy associated with the curvature of 3D space in the open universe case decreases as $a^{-2}$, i.e. much more slowly than the energy of ordinary matter. Soon after the beginning of expansion, the curvature energy density becomes dominant, and the relative role of normal matter and of its density perturbations become insignificant. This is an additional reason to expect that the density perturbations in a compact hyperbolic universe will rapidly decrease [22]. Therefore the universe should remain relatively homogeneous until the thermal energy density drops below $V$ and inflation begins.

One may wonder whether we need inflation at all if a compact universe can be efficiently homogenized. But the main reason why it becomes homogeneous is because we were able to start with a universe of a very small size. In a non-inflationary universe of the present size, chaotic mixing would not have enough time to occur since the present part of the universe only now becomes causally connected. If one starts with the universe with a typical 3D curvature $O(1)$, then in the absence of inflation our universe would have vanishingly small $\Omega$. Finally, in the absence of inflation one would have no explanation to the large entropy of the observable part of the universe, $S > 10^{87}$, and we would have problems explaining the origin of the large scale structure of the universe, as well as the CMB anisotropy.

Thus without the help of inflation the effects considered above cannot solve the major cosmological problems. However, they can keep the universe sufficiently homogeneous until the onset of inflation.
The authors of Ref. [1] discussed conditions which could lead to damping of initial density perturbations down to $10^{-5}$, with the goal to have only a relatively short stage of inflation and obtain a model of a homogeneous universe with $\Omega \sim 0.3$. Our goals are much more modest: We only need the initial expanding domain of the universe to be sufficiently homogeneous ($\delta \rho / \rho \lesssim 1$) for the onset of the subsequent stage of inflation. It seems rather easy to achieve such a damping of density perturbations at the radiation dominated stage in a domain of the horizon size, or of the size smaller than $H^{-1}$. One could not use this mechanism in the non-inflationary models of a closed or open universe because our universe previously consisted of many causally disconnected domains of horizon size, but one can do it during the pre-inflationary stage in our scenario.

Similarly, it was shown long ago that particle production in the very early universe can make the universe locally isotropic [2a]. However, this process could not make the universe globally isotropic because in the hot big bang scenario the universe consisted of many causally disconnected domains. But such effects may play important role in our scenario by making the initial small patch of the universe isotropic, and then this isotropy will be further enhanced by the subsequent stage of inflation.

It is interesting to estimate the size of a flat universe of initial Planckian size at the moment when inflation begins. This happens when $T^4 \sim a^{-4}$ drops from $O(1)$ to $V(\phi)$, i.e. at $a \sim V^{-1/4}$. Note that in this case the size of the universe at the beginning of inflation is much smaller than the size of dS horizon $H^{-1} \sim V^{-1/2}$. This is a counterexample to the common lore that the size of inflationary universe must be greater than $H^{-1}$.

The probability of creation of a small part of the universe of a size $O(1)$ at the Planck density is not expected to be exponentially suppressed. Most importantly, since at that time the evolution of the universe is dominated by the relativistic matter, this probability practically does not depend on $V(\phi)$. After that, inflation occurs in an expanding and cooling universe automatically, if the inflaton field did not strongly interact with matter and if from the very beginning it was staying at the flat (inflationary) region of the effective potential. Of course, there will be some suppression of the probability of inflationary initial conditions if an inflationary regime occurs only for a narrow range of initial values of the inflaton field. However, this phase space suppression is much milder than the exponential suppression of the type described by Eq. [1]. One can argue that this suppression can be easily compensated by the exponential growth of volume of the inflationary domains.

We conclude that in this class of cosmological models, unlike in the usual case of a closed or an infinite open universe, the probability of inflation is not exponentially suppressed for the theories with $V(\phi) \ll 1$.

B. Quantum creation of a compact inflationary universe

Now we will study a possibility that the universe from the very beginning is dominated by the potential energy density of the scalar field. As we already discussed, in the closed universe case such solutions require tunneling from $a = 0$ to $a = H^{-1} = \sqrt{3} \sqrt{\frac{V}{\Omega}}$. One way to see it is to study the well-known Wheeler-DeWitt equation for a closed de Sitter (dS) universe [12, 17, 24]:

$$\left[ \frac{1}{24\pi^2} \frac{d^2}{da^2} - 6\pi^2 a^2 + 2\pi^2 a^4 V \right] \Psi(a) = 0$$

In this equation we temporarily ignored the evolution of the scalar fields (we will discuss it later, see Eqs. [8], [9]), and assumed that $V(\phi) = \text{const}$, which is a good approximation in inflationary cosmology if the potential $V$ is sufficiently flat. This equation describes tunneling from $a = 0$ to point $a = \sqrt{3} \sqrt{\frac{V}{\Omega}}$ through the potential barrier $V(a) = 6\pi^2 a^2 - 2\pi^2 a^4 V$. Meanwhile for a flat dS universe with a toroidal compactification the cosmological evolution can begin at an arbitrarily small $a$, i.e. there is no need to tunnel through a classically forbidden region of $a$ [3]. As pointed out by Zeldovich and Starobinsky, in this case, unlike in the case of the topologically trivial flat dS universe, the flat compactified dS is geodesically complete [3].

In order to derive the Wheeler-DeWitt equation for the compact flat toroidal universe, one should first consider the gravitational action $S = \int dt d^3x \sqrt{-g} \left( -\frac{1}{2} R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$ and take into account that the volume of the 3D box is equal to $a^3$. (In a closed universe the volume is given by $2\pi^2 a^3$. That is why the coefficients in the Wheeler-DeWitt equation for the toroidal universe will be different from the analogous coefficients for the closed universe.) Let us assume for a moment that $\dot{\phi}$ is constant (see below). In this case one can represent the effective Lagrangian for the scale factor as a function of $a$ and $\dot{a}$,

$$L(a) = -3a^2 \dot{a} - a^3 V.$$  \hspace{1cm} (4)

Finding the corresponding Hamiltonian and using the Hamiltonian constraint $H \Psi(a) = 0$ yields the Wheeler-DeWitt equation

$$\left[ \frac{d^2}{da^2} + 12a^4 V \right] \Psi(a) = 0$$  \hspace{1cm} (5)
Eqs. 6 differs from Eq. 3 not only by numerical coefficients, but mainly by the absence of the term $6a^2a^2$, which was leading to the existence of the barrier in the effective potential $V(a)$ for the closed universe. In the flat toroidal universe case there is no such barrier.

One should note that here we ignored the Casimir effect, which may give an important quantum contribution $O(a^{-4})$ to the vacuum energy density. Under certain conditions, this contribution may be negative, which will again forbid the classical evolution at $a < O(H^{-1})$. However, this effect is suppressed by supersymmetry, it disappears in some anisotropic versions of toroidal compactification, and, as we mentioned in the previous section, it becomes unimportant if one takes into account the usual matter contribution, or the 3D curvature in the open universe case.

For large $a$, the solution of Eq. 6 can be easily obtained in the WKB (semiclassical) approximation, $\Psi \sim a^{-1} \exp[\pm i\frac{2a}{\sqrt{V}}]$; positive sign corresponds to an expanding universe. This approximation breaks down at $a \lesssim V^{-1/6}$. At that time the size of the universe is much greater than the Planck scale, but much smaller than the Hubble scale $H^{-1} \sim V^{-1/2}$. The meaning of this result, to be discussed below in a more detailed way, is that at $a \gg V^{-1/6}$ the effective action corresponding to the expanding universe is very large, and the universe can be described in terms of classical space and time. Meanwhile at $a \lesssim V^{-1/6}$, the effective action becomes small, the classical description breaks down, and quantum uncertainty becomes large. In other words, contrary to the usual expectations, at $a \lesssim V^{-1/6}$ one cannot describe the universe in terms of a classical space-time even though the size of the universe at $a \sim V^{-1/6}$ is much greater than the Planck size, and the density of matter as well as the curvature scalar in this regime remains small, $R = 4V \ll 1$.

In this regime one should go beyond the WKB approximation. The general solution for Eq. 6 can be represented as a sum of two Bessel functions:

$$\Psi(a) = \beta \sqrt{a} \left( J_{\frac{1}{3}} \left( \frac{2\sqrt{V}a^3}{\sqrt{3}} \right) + \gamma J_{\frac{2}{3}} \left( \frac{2\sqrt{V}a^3}{\sqrt{3}} \right) \right),$$

where $\beta$ and $\gamma$ are some complex constants. These functions are shown in Fig. 1.

The expression for the probability to have a universe with a scale factor $a$ is given by $j(a) = -\frac{1}{2} (\Psi^* \partial_t \Psi - \Psi \partial_t \Psi^*)$. An example of the function $\Psi$ describing an expanding universe and satisfying the normalization condition $j(a) = 1$ is given by Eq. 6 with $\beta = \sqrt{2\Gamma(5/6)} \Gamma(7/8)$ and $\gamma = -e^{-i\pi/6}$.

As we see from this figure, quantum mechanical description of the universe near $a = 0$ is quite regular. The universe appears “from nothing” without tunneling and without exponential suppression of the probability of its creation. A similar result is valid for the open universe as well.

One can provide an alternative interpretation of this result, without invoking the Wheeler-DeWitt equation. By substituting the classical solution $a = e^{Ht}$ into the effective Lagrangian, one finds the total action of the universe:

$$S(t) = -\frac{2\sqrt{V}}{\sqrt{3}} a^3(t) = -\frac{2\sqrt{V}}{\sqrt{3}} e^{3Ht}.$$  

As we see, for $a < V^{-1/6}$ the action becomes smaller than 1, which means that one must describe evolution of the universe quantum mechanically. This is what we did above. However, once the universe grows larger

2 During the last 20 years the idea of necessity of tunneling at the moment of the universe creation was so popular that the main effort of the investigation of Casimir effect in supergravity in [4] was devoted to finding versions of supergravity with twisted fields, which break supersymmetry and lead to the negative Casimir energy and to the exponential suppression of the tunneling.

3 After this work was completed we became aware of Ref. [26], where it was also found that the probability of quantum creation of compact flat and open $dS$ universes (in the case $K = 0$, see below) is not exponentially suppressed, which agrees with [4] and with our results.
than $a \sim V^{-1/6}$, its action rapidly becomes exponentially large and its classical description becomes possible. What is most important to us here is that the action at the first (quantum mechanical) part of the evolution of the universe was $O(1)$, i.e., for any $V(\phi)$ there was no exponential suppression of the probability of quantum creation of the universe associated with tunneling.

It is clear, however, that at least one of the properties of our solution is not generic: If one adds any matter contribution to the energy-momentum tensor, then going back in time will result in the curvature singularity. Usually, kinetic energy of the scalar field $\phi$ gives the leading contribution in this limit, since it scales as $a^{-6}$, i.e., much faster than the energy density of relativistic and nonrelativistic matter. Thus, such a solution is not realistic. What is most important to us here is that the kinetic energy can be greater than the Planckian energy only when the size of the universe is smaller than the Planckian size: $a \lesssim 0.5$. In this case the semiclassical (WKB) approximation works only at $a > V^{-1/6}$. Meanwhile, for $48K \gg 1$ the kinetic energy becomes Planckian when the universe is large in Planckian units, at $a = K^{1/6} \gg 1$, and the solution of the WDW equation remains semiclassical and oscillatory for all $a$.

The solutions with $K \gtrsim 1$ may be interesting from the point of view of the probabilistic interpretation of quantum cosmology. One of the main problems there is the absence of the S-matrix approach. Consider, for example, the exponential suppression of quantum creation of a closed de Sitter space, Eq. (10). To study tunneling in quantum mechanics one should consider an incident wave, a reflected wave and a transmitted wave. The total probability current is conserved, but the current corresponding to the transmitted wave is exponentially smaller than the current corresponding to the incident wave, the difference being accounted by the reflected wave.

Meanwhile when one studies creation of a closed universe due to the tunneling from $a = 0$ to $a = \sqrt{\frac{\alpha}{\phi}}$ through the potential barrier $V(a) = 6\pi^2a^2 - 2\pi^2a^4V$, one finds that the potential is positive at small $a$. Therefore the description of tunneling begins under the barrier, where the wave function does not oscillate, so there are no incident and reflected waves. If one adds to $\mathcal{V}(a)$ the kinetic energy term $-2\pi^2K/a^2$, the effective potential $\mathcal{V}(a)$ becomes negative at small $a$, but one can show that for $K < 1/192\pi^4$ the solutions of the WDW equation remain non-oscillatory at small $a$, and the standard description of tunneling
breaks down. A possible interpretation of the exponential suppression \( K = 0 \) for the case \( K = 0 \) can be found in \[27\].

However, for \( K > 1/192\pi^4 \), the WKB approximation is applicable at small \( a \), which leads to the existence of solutions oscillating at \( a \lesssim 1 K^{1/4} \). In terms of the variable \( \alpha = \log a \), one can interpret these solutions as waves coming from \(-\infty\) and almost completely reflected back from the barrier at \( a = (K/3)^{1/4} \). In this case one can use the standard quantum mechanical description of tunneling. Note that for \( K \ll V^{-2} \) an addition of the term \(-2\pi^2 K/a^2\) practically does not affect the shape of the potential barrier at \( a \sim H^{-1} \sim V^{-1/2} \), so the final result for the exponential suppression of tunneling \( \alpha \) remains intact. This allows us to look at the results of the previous sections from a different point of view: If a closed universe is born small, with the total kinetic energy of the scalar field being smaller than \( V^{-2} \) at the Planck time, then, at the classical level, it is going to collapse. It may survive due to tunneling, but the probability of this event is suppressed by \( \exp\left(-\frac{2\pi^2}{V}\right) \).

One can avoid the tunneling if the universe was created with kinetic energy \( K \gtrsim V^{-2} \), but in this case one would need to explain how could it happen that the universe at the Planck density contained matter with a total mass greater than \( V^{-2} \) Planck masses (which was greater than \( 10^{16} \) for \( V < 10^{-38} \)), and why it was homogeneous if it consisted of \( V^{-2} \) causally disconnected domains. As we already discussed in Section \( \dagger \) the probability of such event for \( V \ll 1 \) is expected to be exponentially small, even though under certain conditions it might be greater than the probability given by the square of the tunneling wave function \( \dagger \); see Eqs. (18) and (19) in \[14\].

If we repeat the same investigation for the compact flat or open universes, we will see that for \( K \gg 1/48 \) the incident waves coming from \( \alpha = \log a = -\infty \) are adiabatically (i.e., practically without any reflection) transformed into waves moving towards \( \alpha = \log a = +\infty \). This result, which we have verified numerically, is in agreement with our conclusion that there is no exponential suppression of the probability of creation of such universes.

### IV. DISCUSSION AND SUMMARY

Quantum cosmology is a rather esoteric science. Perhaps for this reason the authors of one of the pioneering papers on creation of the flat universes with nontrivial topology \[3 \] did not want to make any definite statements concerning the relative probability of creation of a closed universe as compared to the probability of creation of a compact flat or open universe. In our opinion, however, the last 20 years of the debates on the probability of quantum creation a closed universe or an infinite open universe \[13, 14, 17, 19, 21\] demonstrated that in all known cases the probability of creation of a low energy density inflationary universe is exponentially suppressed. This is a very important issue since in many inflationary models the energy density of the universe during inflation is many orders of magnitude smaller than the Planck density. Meanwhile, as we have seen, there is no exponential suppression of creation of the low energy density inflationary universe if one considers compact open or flat universes. It would be interesting to investigate the behavior of the wave function of the universe for more general anisotropic cosmological solutions considered in \[3, 23\].

From this perspective, one may argue that when the baby universe was born, it has chosen the path of the least resistance: If inflation may occur only at \( V \ll 1 \), it is exponentially more probable that when the universe was born, it was looking not like a sphere but like a crystal with identified sides. In most versions of inflationary cosmology, initial anisotropy of the universe was completely erased by the subsequent long stage of inflation, so it is rather unlikely that we are going to see any trace of the nontrivial topology of the universe. However, with a specific fine-tuning of inflationary potential \[12\], the inflationary stage can be made sufficiently short, in which case the remnants of the original anisotropy of the universe may become observable \[6\].

The possibility that our universe may be a compact open or flat space may have important implications for string cosmology. String theory is based on the idea that 6 space dimensions are compactified. Therefore it seems natural to assume that initially all space dimensions were compact; later some of them continued expansion, whereas some others become stabilized, e.g. by the KKLT mechanism \[28\]. In all known versions of string cosmology, the process of inflation occurs at a density which is much smaller than the Planck density. It is quite interesting, therefore, that the very idea of compactification, being applied to our universe, may help us to resolve the problem of initial conditions for inflation in string theory \[29\] including the so-called overshooting problem \[30, 51, 32, 53\]. It seems very intriguing that creation of the universe may be facilitated by supersymmetry, which suppresses the Casimir effect. Could it be that supersymmetry is weakly broken in our universe simply because the probability of creation of the universes with strongly broken supersymmetry is exponentially suppressed?
This paper was devoted to the problem of initial conditions for inflation. One should note, however, that in the eternal inflation scenario \[36, 37\] the problem of initial conditions may become irrelevant. One may argue that even if the probability of proper initial conditions for eternal inflation is small, most observers are going to live in the parts of the universe produced by eternal inflation. Moreover, under certain assumptions one can show that the probability distribution to live in the parts with different properties does not depend on the initial conditions at the moment of the universe creation \[36\]. This is an important consideration, because inflation in the string theory landscape scenario is eternal \[37, 38\]. However, evaluation of probabilities in eternal inflation scenario is a rather delicate issue, which requires investigation of measure in quantum cosmology. Moreover, there are many inflationary models that do not lead to eternal inflation. Therefore it is good to have an independent argument that in certain cosmological models there is no exponential suppression of the probability to have inflation with \(V \ll 1\).

Independently of the issues related to eternal inflation, our investigation leads us to a rather unexpected conclusion. If inflation is eternal, the universe should look like an eternally growing fractal \[36, 39\]. If inflation is not eternal, then our investigation suggests that most probably we live in a flat or open compact universe with nontrivial topology. None of these possibilities correspond to the standard textbook models of a closed universe or of an infinite flat or open universe.

We are planning to return to some of the problems discussed above in a separate publication \[40\].

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