The effect of rotation on resonant frequency of interfacial oscillation of a droplet using electrostatic levitator

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Abstract. Under the microgravity environment, liquid can be treated without container due to weightlessness. Therefore physical properties of a material are expected to be measured with a high accuracy. In surface tension measurement of levitated droplet, it is calculated by using the formula derived under the assumption of linearity of the resonant frequency. However, it is indicated that the resonant frequency of a droplet changes due to oscillation and rotation. And mechanism of frequency shift has not been revealed yet. The final goal of this study is to make clear the mechanism of frequency shift. A droplet is levitated by using electrostatic levitator, and induced to oscillate and rotate. Resonant frequency, amplitude, angular velocity and aspect ratio are measured from droplet motion. It is confirmed oscillation and rotation effect on resonant frequency. And the frequency shift becomes zero under the appropriate combination of amplitude and angular velocity. It is experimentally clarified that the resonant frequency is strongly related with time average of aspect ratio.

1. Introduction

It has become possible to conduct various experiments under the microgravity environment in space. Under the microgravity condition, the buoyant convection and the phase separation of mixing liquid due to density difference does not occur. So, it is expected to establish the new methods of manufacturing new materials and of measuring thermo-physical properties of molten materials utilizing the microgravity condition. Understanding the nature of a levitating liquid drop is indispensable for the containerless processing. Recently, there are a lot of studies with levitating technique under gravity environment on the ground. For example, it has become possible to measure thermo-physical properties of extreme high-temperature molten materials exceeding 3000 K with the electrostatic levitation technique [1-3]. On the measurement of physical properties, the axisymmetric oscillation is applied to a levitating drop, and the surface tension is calculated from the resonant frequency by the oscillating drop method derived by Rayleigh [4]. In this method, the surface tension is calculated by equation (1):

\[
\sigma = \frac{\rho R^3 f_0^2}{8} \tag{1}
\]
where \( \sigma, R_0, f_0 \) are surface tension, droplet radius, initial resonant frequency, respectively. However, the equation was derived under linear approximations; inviscid, very small deformation only in the radius direction, potential flow. Therefore, there is a possibility that a linear theory is not applicable in the case of larger deformation [5]. Figure 1 shows the surface tension measurement of propylene carbonate by the oscillating drop method. Difference between actual measurement value 42.6 mN/m and calculated surface tension is small in small oscillation amplitude. But the difference become large with increase of amplitude. On the measurement of thermophysical properties, the horizontal rotation often has to be applied for stable levitation of sample. However, it is indicated the resonant frequency change due to rotation [6]. Watanabe [7] analyzed the oscillation behavior of rotating droplet numerically. He examined how oscillation amplitude, rotational velocity, and aspect ratio of droplet effect on resonant frequency, respectively (see Figures 2 and 3). It was indicated that resonant frequency is not suffered from the frequency shift if the time-averaged aspect ratio of droplet was maintained unity. However, mechanism of frequency shift has not been revealed yet, and there are only a few experimental data for oscillation of rotating levitated droplet.

The final goal of this study is to make clear the mechanism of frequency shift. In this paper, it is examined the effect of oscillation and rotation on resonant frequency and droplet shape.

**Figure 1.** Effect of oscillation amplitude on surface tension measurement.

**Figure 2.** Dependency of frequency shift on amplitude and rotational velocity (Numerical analysis by Watanabe [7]).

**Figure 3.** Dependency of aspect ratio on amplitude and rotational velocity (Numerical analysis by Watanabe [7]).
2. Experimental approach

Figure 4 shows the conceptual diagram of the electrostatic levitation. Electrostatic levitation apparatus is composed of a pair of cylindrical electrodes, high voltage amplifiers, He-Ne laser and photo detector used to control the droplet position. At first, the sample is placed on the tip of needle located in the bottom electrode. Secondly, positive voltage is applied to the bottom electrode, then the surface of sample charges positively. Thirdly, negative voltage is applied to the upper electrode. So, the electrostatic force lifts up the charged sample in the opposite direction of gravity. In this method, there is no restoring force on the levitating droplet; hence it is necessary to control the sample position. It is performed by the detecting sample position with photo detector, and by adjusting the voltage of the upper electrode through the PID algorithm.

The experimental apparatus in the present study is shown in figures 5 and 6. Experiments were performed in normal pressure at room temperature. The test fluids are shown in table 1. Propylene carbonate and Ethylene glycol are safe liquids, low viscosity and possible to levitate. Droplets of form 1.8 to 2.6 mm in diameter were levitated in the air. The voltage of 2 up to 7 kV was applied to the upper electrode, which was high-speed-feedback-controlled via PID algorithm. It changed due to levitating droplet size and droplet surface charge.

The deformation behaviors of the levitating liquid droplet were observed by using high speed video camera and charge-coupled device camera set on the side of the acrylic chamber. The vertical length of the levitating droplet was measured by a digital measurement system. The resonant frequency was calculated by the spectrum analysis from time series data of the vertical length. However, it is necessary to excite a droplet oscillation for determining the resonance frequency. In the present study, the oscillation in a perpendicular direction was excited by applying sinusoidal voltage to the bottom electrode with a voltage amplifier controlled by a function generator. The amplitude of the droplet oscillation was varied by changing the applied voltage.

The method to induce the rotation of levitating droplet was based on the theoretical prediction by Busse & Wang [8]. In this method, the rotation torque was generated by two orthogonal acoustic standing waves. In the present study, these waves were generated with two speakers set up on the side of the chamber. The phase difference between two waves was set \( \pi/4 \) rad, so acoustic torques was generated in the chamber and rotation in the horizontal direction was excited to a levitating droplet. The angular velocity of the droplet can be changed by adjusting the sound pressure level. The angular velocity of levitating droplet is measured by taking the motion of immiscible fine tracer particle with high speed video camera.

![Diagram of Electrostatic Levitation](image-url)
(a) He-Ne laser (b) Position detector
(c) Line sensor (d) High speed camera
(e) CCD camera (f) Acric chamber
(g) Speaker
(h) electrode

**Figure 5.** Schematic of Experimental apparatus (Top view).

(l) High speed video camera
(b) Upper electrode
(k) Sample fluid
(i) Bottom electrode
(j) Syringe

**Figure 6.** Schematic of Experimental apparatus (Side view).

**Table 1.** Physical properties of test fluids.

| Fluid          | Density $\rho$ [kg/m$^3$] | Surface tension $\sigma$ [mN/m] | Viscosity $\mu$ [mPa·s] | Kinematic viscosity $\nu$ [mm$^2$/s] |
|----------------|---------------------------|---------------------------------|--------------------------|-------------------------------------|
| Propylene carbonate | 1205.7                    | 42.6                            | 2.77                     | 2.30                                |
| Ethylene glycol   | 1113.2                    | 48.4                            | 22.8                     | 20.5                                |
3. Experimental Results

3.1 Oscillation of levitated droplet

Oscillation of levitated droplet is shown in figures 7 and 8. The droplet was propylene carbonate, 1.82 mm in diameter, and record speed was 2250 fps. $T$ and $A$ are oscillation period and amplitude, respectively. $\frac{A}{R_0}$ is dimensionless amplitude divided by droplet radius $R_0$. The droplet shape deformed widely with increase of amplitude. Time series of dimensionless amplitude of droplet are shown in figure 9. The oscillation period of $\frac{A}{R_0} \approx 0.5$ is longer than that of $\frac{A}{R_0} \approx 0.1$, it is confirmed that the resonant frequency decreases due to increase of amplitude. Examination result of resonant frequency shift of both propylene carbonate and ethylene glycol with increase of amplitude is shows in figure 10. The frequency shift is defined as follow:

$$\delta f = \frac{f - f_0}{f_0}$$

where $f$ is the resonant frequency with certain amplitude, $f_0$ is initial resonant frequency when only little oscillation is given. The line in figure 10 is result of numerical analysis by Tsamopoulos and Brown [5]. They analyzed 2nd mode axial oscillation of inviscid and incompressible droplet. The frequency shift decreased with increasing of amplitude, experimental result corresponded to numerical one by Tsamopoulos et al. qualitatively.

![Figure 7](image7.png)

**Figure 7.** Observation result of droplet shape deformation in one period $T (\frac{A}{R_0} \approx 0.1)$.

![Figure 8](image8.png)

**Figure 8.** Observation result of droplet shape deformation in one period $T (\frac{A}{R_0} \approx 0.5)$.

![Figure 9](image9.png)

**Figure 9.** Time series of dimensionless amplitude of droplet.

![Figure 10](image10.png)

**Figure 10.** Effect of oscillation on frequency shift.
3.2 Oscillation of levitated droplet with rotation

Oscillation of rotating levitated droplet is shown in figures 11 and 12. The droplet was propylene carbonate of 2.15 mm diameter, and record speed was 3000 fps. $\Omega$ and $\Omega/\omega_0$ is angular velocity and dimensionless angular velocity divided by initial resonant frequency $\omega_0 (\approx 2\pi f_0)$, respectively. Figures 11 and 12 show rotation of $\Omega/\omega_0 \approx 0.1, 0.4$ with oscillation of $A/R_0 \approx 0.1$, respectively. The droplet shape did not deform so much in vertical direction compared with in horizontal direction because the centrifugal force pulled droplet outward. Time series of dimensionless amplitude of rotating droplet are shown in figure 13. The oscillation period of $\Omega/\omega_0 \approx 0.4$ is shorter than that of $\Omega/\omega_0 \approx 0.1$, it is confirmed that the resonant frequency increases due to increase of angular velocity. Examination result of resonant frequency shift with increase of angular velocity is shows in figure 14. The line is result by Busse [8]. He analyzed 2 mode axial oscillation of inviscid and incompressible rotating droplet. The frequency shift increased due to increase of angular velocity, experimental results agreed with result by Busse qualitatively.

![Figure 11. Observation result of droplet shape deformation in one period $T (\Omega/\omega_0 \approx 0.1)$.

![Figure 12. Observation result of droplet shape deformation in one period $T (\Omega/\omega_0 \approx 0.4)$.

![Figure 13. Time series of dimensionless amplitude of rotational droplet.

![Figure 14. Effect of rotation on frequency shift.](image-url)
3.3 Effect of oscillation amplitude and angular velocity on resonant frequency

Figure 15 shows experimental result of frequency shift of oscillation droplet with rotation. Frequency decreased with increasing amplitude and with decreasing angular velocity. Hence, it is confirmed that the effect of amplitude and rotation on frequency shift is opposite in this experiment in common with numerical analysis (Figure 2).

![Figure 15. Effects of amplitude and angular velocity on frequency shift.](image)

The surface tension which is estimated from frequency is shown in figure 16. It is confirmed that there are measurement errors due to frequency shift. Measurement errors of calculated surface tension, which is in $\Omega/\omega_0 \approx 0.0, 0.1, 0.2, 0.3, 0.4$, is shown in figure 17, and the line is actual measured value. It is shown that the frequency shift directly effects on accuracy of surface tension measurement. Relation between oscillation and rotation makes calculated surface tension be within $\pm 5\%$ error is in figure 18. Taking into account of effect of both amplitude and rotation, the appropriate combination which makes frequency shift zero would exist. This might contribute to improve accuracy of measurement.

![Figure 16. Effects of amplitude and angular velocity on surface tension.](image)
3.4 Relation between droplet shape and frequency shift

Figure 19 shows time series of aspect ratio of oscillating droplet without rotation. Aspect ratio is defined as vertical length b divided by horizontal length a. The line indicates aspect ratio of unity which means spherical shape. Aspect ratio greater than 1.0 is appeared for long time when dimensionless amplitude \( A/R_0 \) is 0.5. Figure 20 shows time series of aspect ratio of rotating droplet with dimensionless amplitude \( A/R_0 \approx 0.1 \). Aspect ratio became smaller than 1.0 due to increase of angular velocity. Figure 21 shows percentage time spent in prolate shape of oscillating and rotating droplet, and the line is numerical analysis result of non-rotating droplet by Tsamopoulos & Brown. The prolate shape indicates aspect ratio is greater than 1.0. Time spent in prolate shape increased when amplitude became larger. In the case of rotating droplet, time spent in prolate shape is smaller than rotating droplet. Thus it is possible time spent in prolate shape shift has something to do with frequency shift. Figure 21 shows time-averaged aspect ratio of oscillating droplet with no rotation and rotation. Averaged aspect ratio \( A_s \) is calculated from average of three oscillation cycles. Average aspect ratio increased with increasing amplitude. In contrast, increase of angular velocity made average aspect ratio decrease. Figure 22 shows aspect ratio of oscillation droplet with no rotation and rotation. From Figures 15 and 22, relation between average aspect ratio and frequency shift is in figure 23. It is suggested that average aspect ratio correlates with frequency shift.
Figure 21. Percentage time spent in prolate shape of oscillating and rotating droplet.

Figure 22. Effects of amplitude and angular velocity on aspect ratio.
4. Conclusion
We observed dynamics of levitated droplet by using electrostatic levitator. Behaviour of droplet shape was analysed when both oscillation amplitude and angular velocity changed. The concluding remarks are obtained as follows:

- The resonant frequency of levitated droplet was measured. It is confirmed that the resonant frequency decreases due to increase of amplitude, increases due to increase of angular velocity.
- It is suggested appropriate combination of oscillation and rotation makes frequency shift 0 in experiment.
- It is suggested that average aspect ratio correlates with frequency shift in experiment.

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