Formation lengths of hadrons in lepto-production

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Abstract

The average formation lengths of the hadrons produced during the deep inelastic scattering (DIS) of leptons on protons are studied in the framework of the symmetric Lund model. It is shown that these formation lengths essentially depend on the electric charges of the hadron. For electro-production and charged current (CC) neutrino-production, the average formation lengths of positively charged particles are larger than those of negatively charged antiparticles. This situation is reversed for CC antineutrino-production. In all the mentioned cases, the main mechanism is the direct production of hadrons. The additional mechanism of hadron production, through the decay of resonances, is essential only for pions and leads to a decrease in the average formation lengths.
1 Introduction

The study of the space-time evolution of hadronization during lepto-production on nuclear targets is relatively straightforward because in this case, only one string (two jets of hadrons) is produced. The possible influence of the two string mechanism was studied in Ref. [1], in which its contribution was determined to be small. Another important issue is the selection of conditions under which cascading processes in a nucleus do not occur, i.e., the hadronization occurs beyond the nucleus. Consequently, for the investigation of the hadronization process, it is very important to know the formation lengths of hadrons in lepto-production on the elementary nuclear target (nucleon).

Such investigations were performed in Refs. [2, 3]. In [3], it was claimed that for hadrons as composite systems, the notion of formation length is ambiguous as the different constituents of the hadrons originate at different distances. Thus, the question of which of the following two length scales plays the more important role in the hadronization process is an open and model-dependent question: (i) the constituent formation length \( l_c \), which is the distance between the DIS and the first constituent production points, or (ii) the yo-yo formation length \( l_y \), which is the distance between the DIS and yo-yo formation points, where yo-yo is the object with the quantum numbers of the final hadron but without its "sea".

In Refs. [4, 5, 6], the formation lengths of pions were presented in the form \( l = (1-\omega)l_c + \omega l_y \), where \( \omega \) is the probability of the formation length being \( l_y \). When \( l_c \) and \( l_y \) were obtained in the framework of the Lund model, comparison with experimental data gave \( \omega = 0.068-0.088 \). This result has confirmed the conclusion of Ref. [3] on the importance of the constituent formation length. We will further consider \( l_c \) as a formation length. The parameter \( l_c \) is a function of the variables \( \nu \) and \( z \) (the energy of a virtual photon and the fraction of this energy carried away by the final hadron with energy \( E_h (z = E_h/\nu) \), respectively).

The aim of this work to obtain the average formation lengths of hadrons in CC neutrino- and CC antineutrino-production on a proton target and to compare these values with those for electro-production, which were obtained earlier.

The paper is organized as follows. In Section 2, we briefly present a description of the model. The results and discussion, as well as necessary information for the calculations, are presented in Section 3. The conclusions are presented in Section 4.

2 Description of model

2.1 Distribution functions

We begin by considering the distribution of the constituent formation lengths \( l \) of hadrons carrying away fractional energy \( z \):

\[
C_{p1}^h(\nu/\kappa)\theta(l)(L - zL) + C_{p2}^h \sum_{i=2}^{n} D_{ci}^h(L, z, l)\theta(l)\theta(L - zL - l) ,
\]

where \( L = \nu/\kappa \) is the full hadronization length and \( \kappa \) is the string tension.

\( f(z) \) is the scaling function. It is defined by the condition that \( f(z)dz \) is the probability that the first hierarchy (rank 1) primary hadron carries away the fraction of energy \( z \) of the initial string in the small interval \( dz \). We use the symmetric Lund scaling function [7, 8]

\[
f(z) = N z^{-1} (1 - z)a \exp(-b m^2_\perp / z),
\]

where \( a \) and \( b \) are parameters of the model, \( m_\perp = \sqrt{m^2_h + p^2_\perp} \) is the transverse mass of the emitted hadron and \( N \) is a normalization factor.

Information about the functions \( C_{p1}^h \) and \( C_{p2}^h \) is presented in the next subsection.
The functions $D^h_{c_i}(L, z, l)$ are distributions of the constituent formation lengths $l$ of the rank $i$ hadrons carrying fractional energy $z$. To calculate the distribution functions, we use the recursion equation of Ref. [3].

### 2.2 Functions $C^h_{p_i}$

The functions $C^h_{p_1}$ and $C^h_{p_2}$ are the probabilities of obtaining in some process the compositions of valence quarks for leading (rank 1) and subleading (rank 2) hadrons on the proton target. For hadrons of rank greater than $i = 2$, the condition $C^h_{p_i} = C^h_{p_2}$ is fulfilled. We consider the three types of processes: electro-production, CC neutrino- and CC antineutrino-production. $C^h_{p_2}$ does not depend on the process type. The baryons in our scheme are constructed of the valence quark and diquark. The function $C^h_{p_1}$ is a composition of three factors: (i) the probability that as a result of DIS, the first constituent (quark, antiquark) of the final meson is knocked out; (ii) the probability that as a result of the first break of the string, the second constituent of the final meson arises; and (iii) the probability that these partons transform into the desired meson. The function $C^h_{p_2}$ is composed of two factors: (i) the probability that as a result of two consequent breaks of the string, the first and second constituents of the final meson arise and (ii) the probability that these partons transform into the desired meson. For final baryons, the antiquark should be replaced by a diquark. These functions for electro-production were presented in Refs. [10], [11], and for (anti)neutrino-production, they can be obtained using Refs. [10] [11] [12] [13].

### 2.3 Average formation length

We present here a realistic approach for the calculation of the average formation lengths considering the type of process in which the hadron was produced, the types of hadron and the target. Unfortunately, for the symmetric Lund scaling function, the analytic summation of the produced hadrons sequence over all ranks is impossible. Therefore, we restrict ourselves to $n = 10$ in eq.(1).

In the general case, the function $L^h_c$, $L^h_c = < l_c >$ can be written in the form:

$$L^h_c = \int_0^{\infty} dl \left( \alpha_h D^h_c(L, z, l) \right) + \alpha_R \sum_{R} D^{R/h}_c(L, z, l) \int_0^{\infty} dl \left( \alpha_h D^h_c(L, z, l) \right) + \alpha_R \sum_{R} D^{R/h}_c(L, z, l) . (3)$$

When combined with direct production, a few resonances may also contribute. Here, $\alpha_h$ ($\alpha_R$) is the probability that the composite parton system transforms into a hadron (resonance). We use the condition $\alpha_h = \alpha_R = 1/2$.

The distribution functions $D^h_c(L, z, l)$ and $D^{R/h}_c(L, z, l)$ were described in detail in Refs. [10], [11].

### 3 Results and Discussion

All the calculations were performed at fixed values of $E$, $\nu$ and $Q^2$ equal to 27.5GeV, 10GeV and 2.5GeV$^2$, respectively.

The scaling function $F(z)$ in eq.(2) has two parameters [8] $a = 0.3$, $b = 0.58GeV^{-2}$ and depends on the type of produced hadron. In the calculations, the types of observed hadrons and their parent resonances were considered, while for other hadrons, a summation over their types was performed. The string tension was fixed at a value of $\kappa = 1GeV/fm$.

The leading order parton distribution functions from [9] were used for the quarks (antiquarks) in the proton.

We assume that new $q\bar{q}$ pairs are $u\bar{u}$ with probability $\gamma_u$, $d\bar{d}$ with probability $\gamma_d$ and $s\bar{s}$ with probability $\gamma_s$. It follows from isospin symmetry that $\gamma_u = \gamma_d = \gamma_q$. For
For these calculations, we use the set of values for $\gamma$: $\gamma_u : \gamma_d : \gamma_s = 1 : 1 : 0.3$.

For baryon production, we use the set of probabilities for the production of diquark-antidiquark pairs. For simplicity, we express them using the probabilities of light quark-antiquark pairs production. We use:

$$\gamma_{ud} = 0, \quad \gamma_{q}^{ud} = 0.1\gamma_q, \quad \gamma_{dd} = 0, \quad \gamma_{uu} = 0.015\gamma_q, \quad \gamma_{us} = 0, \quad \gamma_{ds} = 0.012\gamma_q$$

The normalized average formation lengths of the pseudoscalar mesons on the proton target, calculated using the symmetric Lund model as functions of $z$, are presented in Fig. 1 for electro- (panels a, b), CC neutrino- (panels c, d), and CC antineutrino- (panels e, f) production. The formation lengths are represented for pions in panels a, c, and e, and for kaons in panels b, d, and f. The values of the parameters of the symmetric Lund model are also presented. The following parameters are indicated: the formation lengths for the direct pions (dashed-dotted lines); the sum of the direct pions and the pions from the decay of $\rho$ mesons (dotted lines); the sum of the direct pions and the pions from the decays of the $\rho$ and $\omega$ mesons (dashed lines); and the sum of the direct pions and the pions from the decays of the $\rho$, $\omega$, and $K^*$ mesons (solid lines). The other indicated parameters are as follows: the formation lengths for the direct kaons (dotted lines); the sum of the direct kaons and the kaons from the decay of $K^*$ mesons (dashed lines); and the sum of the direct kaons and the kaons from the decays of the $K^*$ and $\phi$ mesons (solid lines). For electro- and CC neutrino-production, the upper and lower curves of each type represent the formation lengths of positively and negatively charged hadrons, respectively. For CC antineutrino-production, the upper and lower curves represent the formation lengths of negatively and positively charged hadrons, respectively.

In Fig. 2, the normalized average formation lengths of baryons and antibaryons on the proton target as functions of $z$.

Figure 2: Normalized average formation lengths of baryons and antibaryons on proton target as functions of $z$. 

Figure 1: Normalized average formation lengths of pseudoscalar mesons on proton target as functions of $z$. 
are presented in panels a and b, c and d, and e and f for electro-, CC neutrino- and CC antineutrino-production, respectively. The results for protons and antiprotons are presented in panels a, c and e, and the results for Λ and ¯Λ are presented in panels b, d and f. The contribution of Δ (Δ) resonance and Σ (Σ) resonance is considered for protons (antiprotons) and Λ ( ¯Λ), respectively.

From Figs. 1 and 2, certain general features of the average formation lengths are apparent: (i) For electro- and CC neutrino-production, the positively charged hadrons (π+, K+ and proton) have larger formation lengths than the negatively charged hadrons (π−, K− and antiproton); for CC antineutrino-production, the situation is reversed. (ii) All the curves have a characteristic form with values equal to zero on two boundary points along z and one maximum in the vicinity of z = 0.2 for pions, z = 0.25 for kaons, z = 0.35 for protons and z = 0.4 for Λ. The magnitudes of the maxima of the distribution do not exceed 0.5L. (iii) The contribution from the decay of resonances is maximal for pions. This contribution reaches ∼ 20% for π+ (π−) meson for the electro- and CC neutrino- (CC antineutrino-) production processes. As expected, the maximal contribution originates from the ρ meson. The contribution of resonances can be neglected for kaons and baryons. (iv) A large difference between the formation lengths of oppositely charged hadrons is observed for kaons and protons. The maximal difference occurs for CC neutrino-production. (v) For electro-production, the difference between the average formation lengths of particles and antiparticles vanishes at z > 0.85; for CC neutrino- and CC antineutrino-production, the difference is large enough for z to equal unity. (vi) Unlike charged hadrons, the formation length of a neutral baryon (Λ) is larger than that of the corresponding antibaryon ( ¯Λ) for all three types of the production processes; however, the difference between the average formation lengths of Λ and ¯Λ is very small.

Let us briefly discuss why the average formation lengths of positively (negatively) charged hadrons are larger than those of negatively (positively) charged ones for electro- and CC neutrino- (CC antineutrino-) production. In electro-production, this phenomenon occurs because of the large probability of knocking out a u quark as a result of DIS. The knocked out quark enters a composition of the leading hadron, which has a maximal formation length. The K+ meson has an average formation length that is larger than the π+ meson because, in the first case, the influence of resonances is smaller. The K− meson has a smaller average formation length than the π− meson because it is constructed from the ”sea” quarks of the proton and cannot be a leading hadron, whereas the π− meson can be leading because of the d quark entering its composition. In CC neutrino- (CC antineutrino-) production, the knocked out parton obtains a positive (negative) electric charge, which implies that the leading hadron can be preferably positively (negatively) charged. It should be noted that in the string model, the formation length of the leading (rank 1) hadron \( l_{c1} = (1 - z)\nu/\kappa \), does not depend on the type of process or on the hadron or target type.

4 Conclusions

In this work, for the first time, the average formation lengths of pseudoscalar mesons, baryons and antibaryons for CC neutrino- and CC antineutrino-production on a proton target in the framework of the symmetric Lund model have been determined.

The results of the calculations were compared with the previously determined results of electro-production (see Refs. [10, 11]). It was shown that despite certain differences in shape and magnitude, the average formation lengths for electro- and CC
neutrino-production have similar behaviors. CC antineutrino-production differs from the others by the replacement of the particles and antiparticles on antiparticles and particles, respectively.

Finally, we would like to discuss the possible application of the obtained results. Recently, we have used the simplified version of the average formation lengths calculation for pions obtained in the framework of the symmetric Lund model to fit SIDIS data, in which the possibility of daughter pions production was neglected in eq.(3) and the condition $C_{h1}^{p} = C_{h2}^{p} = 1$ was applied. These simplifications were necessary to reduce the computation time. The two-parametric fit yielded satisfactory agreement with the data [6].

References

[1] N.Akopov, L.Grigoryan, Z.Akopov, Eur.Phys.J. C52 (2007) 893
[2] T.Chmaj, Acta Phys.Polon. B18 (1987) 1131
[3] A.Bialas, M.Gyulassy, Nucl.Phys. B291 (1987) 793
[4] N.Akopov, L.Grigoryan, Z.Akopov, Eur.Phys.J. C44 (2005) 219
[5] N.Akopov, L.Grigoryan, Z.Akopov, Eur.Phys.J. C70 (2010) 5
[6] L.Grigoryan, arXiv:1208.2339 [hep-ph]
[7] B.Andersson et al., Phys.Rep.97 (1983) 31
[8] T.Sjöstrand, L.Lonnblad, S.Mrenna, hep-ph/0108264 (2001);LU TP 01-21
[9] M.Glück, E.Reya, A.Vogt Z.Phys.C67 (1995) 433
[10] L.Grigoryan, Phys.Rev. C81 (2010) 045207
[11] L.Grigoryan, Phys.Rev. C83 (2011) 014904
[12] V.Barone, C.Pascaud and F.Zommer, Eur.Phys.J.C12 (2000) 243
[13] W.-M.Yao et al., Journal of Physics G 33 (2006) 1