Strange hadron production in a quark combination model in Au+Au collisions at RHIC energies

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We apply a quark combination model to study yield densities and transverse momentum (p_T) spectra of strange (anti-)hadrons at mid-rapidity in central Au+Au collisions at \( \sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27, 39 \) and 200 GeV. We show that experimental data for p_T spectra of (anti-)hadrons in these collisions can be systematically described by the equal velocity combination of constituent quarks and antiquarks at hadronization. We obtain p_T spectra of quarks and antiquarks at hadronization and study their collision energy dependence. We also reproduce the yield densities of hadrons and anti-hadrons. In particular, we demonstrate that yield ratios of anti-hadrons to hadrons \( K^-/K^+ \), \( \bar{p}/p \), \( \Lambda/\bar{\Lambda}, \Xi^+/\Xi^- \) and \( \Omega^+/\Omega^- \) simply correlate with each other and their experimental data except \( \Omega^+/\Omega^- \) at \( \sqrt{s_{NN}} = 7.7 \) GeV are systematically described by the model. These results suggest that the equal velocity combination mechanism for quarks and antiquarks at hadronization plays an important role in Au+Au collisions at low RHIC energies (\( \sqrt{s_{NN}} \geq 11.5 \) GeV).

I. INTRODUCTION

Hadron production from final state partons in high energy collisions is a complex Quantum Chromodynamics (QCD) process. Due to the difficulty of non-perturbative QCD, phenomenological mechanisms and models have to be applied to describe the production of hadrons at hadronization [1–8]. In relativistic heavy-ion collisions at high RHIC and LHC energies, quark-gluon plasma (QGP) is created in early stage of collisions and the hadronization of QGP can be microscopically described by the quark (re-)combination/coalescence mechanism [9–14]. The enhanced ratio of baryon to meson and number of constituent quark scaling for elliptic flows of hadrons at the intermediate transverse momentum (p_T) are typical experimental signals for quark combination mechanism at hadronization and have been widely observed in relativistic heavy-ion collisions [15–22].

In our recent studies in high-multiplicity events in pp and p-Pb collisions at LHC energies where the mini-QGP is possibly created and re-scattering of hadrons is weak, we found an interesting quark number scaling property for p_T spectra of hadrons at mid-rapidity [23, 24]. This scaling property is a direct consequence of equal velocity combination (EVC) of constituent quarks and antiquarks at hadronization. Our studies showed that the EVC of up/down, strange and charm quarks can provide a good and systematic description on p_T spectra of light, strange and charm hadrons in ground-state in pp and p-Pb collisions at LHC energies [23–27]. In latest work [28], we further found that experimental data for p_T spectra of \( \Omega \) and \( \phi \) at mid-rapidity in heavy-ion collisions in a broad collision energy range (\( \sqrt{s_{NN}} = 11.5 - 2760 \) GeV) also satisfy the quark number scaling property. This is a clear signal of EVC at hadronization even in heavy-ion collisions. Therefore, it is interesting to systematically test this mechanism by production of more hadron species in relativistic heavy-ion collisions.

Recently, STAR collaboration reported their precise experimental data for the production of strange hadrons in Au+Au collisions at \( \sqrt{s_{NN}} = 7.7-39 \) GeV [29]. This provides us a good opportunity to systematically study the EVC mechanism of hadron production in these collisions. In this paper, we apply a quark combination model with EVC to carry out a systematic study on yield densities and p_T spectra of strange hadrons in Au+Au collisions at \( \sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27, 39 \) and 200 GeV. We put particular emphasis on the self-consistency of the model in explaining experimental data for different kinds of hadrons and on the regularity in multi-hadron production correlations which is sensitive to hadronization mechanism. Taking advantage of precise data for strange hadrons, we also discuss the strangeness neutralization in the midrapidity region in these collisions. We extract quark p_T distributions at hadronization from data of hadrons and study properties of the relative abundance for strange quarks as the function of collision energy. Furthermore, we discuss the key physics in current quark combination model which are responsible for explaining successfully experimental data of hadronic p_T spectra and yields, and discuss the creation of QGP or the deconfinement at low RHIC energies.

The paper is organized as follows. In Sec. II, we introduce a quark combination model with equal velocity combination approximation. In Sec. III, we study the strangeness neutralization in mid-rapidity region. In Sec. IV, we show results of p_T spectra for hadrons and compare them with experimental data. In Sec. V, we study the split in yield between hadrons and their antiparticles. In Sec. VI, we study properties for the obtained numbers and p_T spectra of quarks at hadronization at different collision energies. The summary and discussion are given at last in Sec. VII.
II. A QUARK COMBINATION MODEL

The quark combination is one of phenomenological mechanisms for hadron production at hadronization. The basic idea of quark combination was firstly proposed in 1970s [3-5] and has many successful applications in high energy reactions [6, 30-32]. In relativistic heavy-ion collisions, quark combination mechanism is often used to describe the hadron production at QGP hadronization [9-11, 13, 14, 33, 34]. Recently, we found that quark combination can also well explain experimental data of hadron production in high-multiplicity pp and p-Pb collisions at LHC energies [23-25, 35]. In this paper, inspired by the quark number scaling property of hadronic $p_T$ spectra [23, 24, 28], we adopt a specific version of quark combination model [23, 25]. This model assumes the combination of constituent quarks and antiquarks with equal velocity to form baryons and mesons at hadronization. The unknown non-perturbative dynamics at hadronization are parameterized and their values are assumed to be stable in high energy reactions and are fixed by experimental data. This model is essentially a statistical model based on the constituent quark degrees of freedom at hadronization and the constituent quark structure of hadrons. It is different from the popular re-combination/coalescence models which adopt the Wigner wave function method under instantaneous hadronization approximation [9, 10]. The brief description of the model is as follows.

We start from the general formula for the production of the baryon $B_j$ composed of $q_1q_2q_3$ and the production of the meson $M_j$ composed of $q_1\bar{q}_2$ in quark combination mechanism

$$f_{B_j}(p_B) = \int dp_1 dp_2 dp_3 \mathcal{R}_{B_j}(p_1, p_2, p_3; p_B)$$

$$f_{M_j}(p_M) = \int dp_1 dp_2 \mathcal{R}_{M_j}(p_1, p_2; p_M) f_{q_1\bar{q}_2}(p_1, p_2).$$

Here $f_{q_1\bar{q}_2}(p_1, p_2, p_3)$ is the joint momentum distribution for $q_1$, $q_2$ and $q_3$. The combination kernel function $\mathcal{R}_{B_j}(p_1, p_2, p_3; p_B)$ denotes the probability for a given $q_1q_2q_3$ with momenta $p_1$, $p_2$ and $p_3$ to combine into a baryon $B_j$ with momentum $p_B$. It is similar for mesons.

We emphasize that Eqs. (1) and (2) are generally suitable for hadron production in momentum space of any dimension. In this paper, we study the one-dimensional transverse momentum ($p_T$) distribution of hadrons at midrapidity $y = 0$. In this case, $p_1$ simply denotes $p_T$, and the distribution function $f_h(p)$ denotes the $dN_h/dp_T$ at midrapidity.

For the joint momentum distribution of quarks and/or antiquarks, we adopt the factorization form

$$f_{q_1q_2q_3}(p_1, p_2, p_3) = f_{q_1}(p_1)f_{q_2}(p_2)f_{q_3}(p_3),$$

$$f_{q_1\bar{q}_2}(p_1, p_2) = f_{q_1}(p_1)f_{\bar{q}_2}(p_2).$$

The combination functions $\mathcal{R}_{B_j}(p_1, p_2, p_3; p_B)$ and $\mathcal{R}_{M_j}(p_1, p_2; p_M)$ contain the key information of combination dynamics which is not clear at present due to the non-perturbative difficulty of hadronization. In our recent works [23, 24, 28], we observed an interesting quark number scaling property for the $p_T$ spectra of hadrons in high-multiplicity pp and p-Pb collisions as well as in heavy-ion collisions. This scaling property supports the combination of constituent quarks and antiquarks with equal velocity. This suggests an effective form for the combination kernel functions, i.e.,

$$\mathcal{R}_{B_j}(p_1, p_2, p_3; p_B) = \kappa_{B_j} \prod_{i=1}^{3} \delta(p_i - x_ip_B),$$

$$\mathcal{R}_{M_j}(p_1, p_2; p_M) = \kappa_{M_j} \prod_{i=1}^{2} \delta(p_i - x_ip_M).$$

Here momentum fraction $x_i$ is determined by the masses of constituent quarks because $p_i = m_i \gamma \beta \propto m_i$. Specifically, we have $x_i = m_i/(m_1 + m_2 + m_3)$ for baryon $B_j$ with $x_1 + x_2 + x_3 = 1$ and $x_i = m_i/(m_1 + m_2)$ for meson $M_j$ with $x_1 + x_2 = 1$. $m_i$ is the constituent mass for quark of flavor $i$, and is taken as $m_u = m_d = 0.3$ GeV and $m_s = 0.5$ GeV. $\kappa_{B_j}$ and $\kappa_{M_j}$ are coefficients independent of momentum.

Substituting Eqs. (3)-(6) into Eqs. (1) and (2), we obtain

$$f_{B_j}(p_B) = \kappa_{B_j} f_{q_1}(x_1p_B)f_{q_2}(x_2p_B)f_{q_3}(x_3p_B),$$

$$f_{M_j}(p_M) = \kappa_{M_j} f_{q_1}(x_1p_M)f_{\bar{q}_2}(x_2p_M).$$

$\kappa_{B_j}$ and $\kappa_{M_j}$ carry the information of $p_T$-independent combination dynamics. In order to determine their forms, we express momentum distributions of hadrons in another form

$$f_{B_j}(p_B) = N_{B_j} f_{B_j}^{(n)}(p_B),$$

$$f_{M_j}(p_M) = N_{M_j} f_{M_j}^{(n)}(p_M).$$

$N_{B_j}$ and $N_{M_j}$ are numbers of $B_j$ and $M_j$, respectively. $f_{B_j}^{(n)}(p_B)$ and $f_{M_j}^{(n)}(p_M)$ are distribution functions normalized to one when integrating over momentum, which can be obtained by those of quarks and antiquarks,

$$f_{B_j}^{(n)}(p_B) = A_{B_j} f_{q_1}^{(n)}(x_1p_B)f_{q_2}^{(n)}(x_2p_B)f_{q_3}^{(n)}(x_3p_B),$$

$$f_{M_j}^{(n)}(p_M) = A_{M_j} f_{q_1}^{(n)}(x_1p_M)f_{\bar{q}_2}^{(n)}(x_2p_M).$$

Here, $f_{q_i}^{(n)}(p) = f_{q_i}(p)/N_{q_i}$ is the normalized distribution function of quark $q_i$. Normalization coefficients $A_{B_j}$
and $A_{M_j}$ are defined as
\[
A_{B_j}^{-1} = \int f_{q_1}^{(n)}(x_1p_B) f_{q_2}^{(n)}(x_2p_B) f_{q_3}^{(n)}(x_3p_B) \, dp_B, \quad (13)
\]
\[
A_{M_j}^{-1} = \int f_{q_1}^{(n)}(x_1p_M) f_{q_2}^{(n)}(x_2p_M) \, dp_M. \quad (14)
\]

Substituting Eqs. (11) and (12) into Eqs. (9) and (10) and then comparing them with Eqs. (7) and (8), we obtain
\[
N_{B_j} = N_{q_1}N_{q_2}N_{q_3} \frac{\kappa_{B_j}}{A_{B_j}}, \quad (15)
\]
\[
N_{M_j} = N_{q_1}N_{q_2} \frac{\kappa_{M_j}}{A_{M_j}}. \quad (16)
\]

From above two equations, we can read out the physical meaning of $\kappa_{B_j}$ and $\kappa_{M_j}$. It is obvious that $\kappa_{B_j}/A_{B_j}$ denotes the $p_T$-integrated probability of $q_1q_2q_3$ forming a baryon $B_j$ and $\kappa_{M_j}/A_{M_j}$ denotes that of $q_1q_2$ forming a meson $M_j$. Two probabilities can be further decomposed as
\[
\frac{\kappa_{B_j}}{A_{B_j}} = P_{q_1q_2q_3 \to B_j} = C_{B_j} N_{iter} \frac{N_B}{N_q^3}, \quad (17)
\]
\[
\frac{\kappa_{M_j}}{A_{M_j}} = P_{q_1q_2 \to M_j} = C_{M_j} \frac{N_M}{N_{q_2}}. \quad (18)
\]

Taking meson for example, $\frac{N_M}{N_q N_{q_2}}$ is used to denote the averaged probability of a $q\bar{q}$ forming a meson. Here, $N_q N_{\bar{q}}$ is the all possible number of $q\bar{q}$ pair, where $N_q = N_u + N_d + N_s$ is the number of all quarks and $N_{\bar{q}} = N_{\bar{u}} + N_{\bar{d}} + N_{\bar{s}}$ is that of all antiquarks. $N_M$ is the average number of mesons produced by the hadronization of quark system with given quark number $N_q$ and antiquark number $N_{\bar{q}}$. The factor $C_M$ denotes further sophisticated tune for the production probability of $M_j$ on the basis of the averaged probability $N_M/N_q N_{\bar{q}}$. The baryon formula is similar to meson except a factor $N_{iter}$. This factor is to assure $\sum_{q_1q_2q_3} N_{iter} N_{q_1} N_{q_2} N_{q_3} = N_q^3$ and equals to 1, 3, 6 for $q_1q_2q_3$ with three identical flavor, two identical flavor, and three different flavors, respectively. Finally, we obtain yield formulas of hadrons
\[
N_{B_j} = C_{B_j} N_{iter} N_{q_1} N_{q_2} N_{q_3} \frac{N_B}{N_q^3}, \quad (19)
\]
\[
N_{M_j} = C_{M_j} N_{q_1} N_{q_2} \frac{N_M}{N_{q_2}}. \quad (20)
\]

$N_B$ and $N_M$ are functions of $N_q$ and $N_{\bar{q}}$ [36],
\[
N_B = \frac{x}{2} \left[ 1 - z \left( \frac{1 + z^a}{1 + z} - \frac{1}{2} \right) \right], \quad (21)
\]
\[
N_B = \frac{xz}{3} \left( \frac{1 + z^a}{1 + z} - \frac{1}{2} \right), \quad (22)
\]
\[
N_B = \frac{xz}{3} \left( \frac{1 + z^a}{1 + z} - \frac{1}{2} \right), \quad (23)
\]

where $x = N_q + N_{\bar{q}}$ and $z = (N_q - N_{\bar{q}})/x$. Parameter $a = 1 + (N_M/N_B)_{iter}/3/3$ denotes the production competition of baryon to meson and is tuned to be $a \approx 4.86 \pm 0.01$ according to our recent work [35].

In this paper, we only consider the production of the ground state $J^P = (0^+), (1^-)$ mesons and $J^P = (1/2)^+, (3/2)^+$ baryons in flavor SU(3) group. In meson formation, we introduce a parameter $R_{V/P}$ to describe the relative weight of a quark-antiquark pair forming the state of spin 1 to that of spin 0. Here, we take $R_{V/P} = 0.55 \pm 0.05$ in order to reproduce the measured $K^*/K$ and $\phi/K$ data in high energy reactions [37, 38]. Factor $C_{M_j}$ is then parameterized as
\[
C_{M_j} = \begin{cases} 
\frac{1}{1 + R_{V/P}} & \text{for } J^P = 0^+ \text{ mesons} \\
\frac{1}{1 + R_{V/P}} & \text{for } J^P = 1^- \text{ mesons.} 
\end{cases} \quad (24)
\]

In baryon formation, we introduce a parameter $R_{D/O}$ to describe the relative weight of spin 3/2 state to 1/2 state for three quark combination. We take $R_{D/O} = 0.5 \pm 0.04$ by fitting the experimental data of $\Xi^*/\Xi$ and $\Sigma^*/\Lambda$ in high energy collisions [39]. Then we have for $C_{B_j}$
\[
C_{B_j} = \begin{cases} 
\frac{1 + R_{D/O}}{1 + R_{D/O}} & \text{for } J^P = (1/2)^+ \text{ baryons} \\
\frac{1 + R_{D/O}}{1 + R_{D/O}} & \text{for } J^P = (3/2)^+ \text{ baryons,} 
\end{cases} \quad (25)
\]

except that $C_{\Lambda} = C_{\Sigma^0} = 1/(2 + R_{D/O})$, $C_{\Sigma^*0} = R_{D/O}/(2 + R_{D/O})$, $C_{\Delta^{++}} = C_{\Delta^{+}} = C_{\Omega^+} = 1$.

Taking $f_{\Lambda}(p)$ as model inputs, we can calculate $f_H(p)$ and $N_h$ of hadrons directly produced at hadronization. Finally, we take the decay contribution of short-life resonances into account according to experimental measurements, and obtain results of final-state hadrons
\[
f_{i}^{(final)}(p) = f_{i}(p) + \sum_{i \neq j} \int dp' f_h(p') D_{ij}(p', p), \quad (26)
\]

where the decay function $D_{ij}(p', p)$ is determined by the decay kinematics and decay branch ratios [40].

### III. STRANGENESS NEUTRALIZATION IN HEAVY-ION COLLISIONS

In relativistic heavy-ion collisions, strange quark and antiquark are always created in pair in collisions and therefore strangeness is globally conserved. However, for a finite kinetic region such as the mid-rapidity region, the strangeness neutralization is not so explicit, in particular, at low collision energies. In this section, using the precise data for yield densities and $p_T$ spectra of identified hadrons [29, 41–46], we study the local strangeness in the midrapidity region in Au+Au collisions at STAR BES energies.
A. strangeness at mid-rapidity

In this subsection, we estimate strangeness density \( dN_s/dy - dN_s/dy \) in the mid-rapidity region in relativistic heavy-ion collisions. We write \( N_s - N_s \) for short in the following. Since experimental measurements are mainly for hadrons in ground state in flavor SU(3), we estimate the strangeness by yield densities of the following hadrons

\[
N_s - N_s = (K^+ + K^0 + K^{*+} + K^{*0}) - (K^- + \bar{K}^0 + K^{-*} + \bar{K}^{*0}) \nonumber \\
- (\Lambda + \Sigma^{0,\pm} + \Sigma^{0,\ast \pm} + 2\Xi^{-,0} + 2\Xi^{*-,0} + 3\Omega^{-}) \nonumber \\
+ (\bar{\Lambda} + \bar{\Sigma}^{0,\pm} + \bar{\Sigma}^{0,\ast \pm} + 2\bar{\Xi}^{+,0} + 2\bar{\Xi}^{*+,0} + 3\bar{\Omega}^{+}) .
\]

Here we use \( h \) to denote \( dN_h/dy \) for convenience. We note that the contribution of higher mass resonances can be also included effectively if we identify above ground state hadrons as the measured ones.

The strangeness in meson sector can be calculated as

\[
K^+ + K^0 + K^{*+} + K^{*0} - K^- - \bar{K}^0 - K^{-*} - \bar{K}^{*0} = (K^+ - K^-)_{\text{final}} + (K^0 - \bar{K}^0)_{\text{final}},
\]

where we use the subscript \( \text{final} \) to denote that \( K^+ - K^- \) and \( K^0 - \bar{K}^0 \) have received the decay contribution of \( K^* \). Since neutral kaons are not measured, we use the approximation

\[
(K^+ - K^-)_{\text{final}} \approx (K^0 - \bar{K}^0)_{\text{final}}.
\]

For strangeness contained in hyperons with one strange quark, we decompose them into two parts: experimentally measured net-\( \Lambda \) and experimentally un-measured net-\( \Sigma^{\pm} \). Using the property of S&EM decays for hyperons \([40]\), we have

\[
(\Lambda + \Sigma^{0,\pm} + \Sigma^{0,\ast \pm}) - (\bar{\Lambda} + \bar{\Sigma}^{0,\pm} + \bar{\Sigma}^{0,\ast \pm}) = (\Lambda - \bar{\Lambda})_{\text{final}} + (\Sigma^{\pm} - \bar{\Sigma}^{\pm})_{\text{final}}
\]

with

\[
(\Lambda - \bar{\Lambda})_{\text{final}} = (\Lambda - \bar{\Lambda}) + (\Sigma^0 - \bar{\Sigma}^0) + 0.94 (\Sigma^{\ast \pm} - \bar{\Sigma}^{\ast \pm}) + 0.88 (\Sigma^{0,\pm} - \bar{\Sigma}^{0,\pm})
\]

and

\[
(\Sigma^{\pm} - \bar{\Sigma}^{\pm})_{\text{final}} = (\Sigma^{\pm} - \bar{\Sigma}^{\pm}) + 0.06 (\Sigma^{\ast \pm} - \bar{\Sigma}^{\ast \pm}) + 0.12 (\Sigma^{0,\pm} - \bar{\Sigma}^{0,\pm}) .
\]

Applying our formula of hadronic yields in Sec. II, we obtain

\[
\frac{(\Sigma^{\pm} - \bar{\Sigma}^{\pm})_{\text{final}}}{} = 1.1 + 0.68R_{D/O} + 0.099R^2_{D/O} \approx 0.55 .
\]

\[
\frac{(\Lambda - \bar{\Lambda})_{\text{final}}}{} = 1.096 + 2.62R_{D/O} + R^2_{D/O} \approx 0.55 .
\]

For strangeness contained in hyperons with two strange quarks, after considering S&EM decays, we have

\[
(\Xi^{-,0} - \bar{\Xi}^{+,0}) + (\Xi^{*-,0} - \bar{\Xi}^{*+,0}) = (\Xi^{-,0} - \bar{\Xi}^{+,0})_{\text{final}}
\]

and we use the approximation

\[
(\Xi^{-,0} - \bar{\Xi}^{+})_{\text{final}} \approx (\Xi^{-,0} - \bar{\Xi}^{+})_{\text{final}} .
\]

By the sum over the strangeness in meson and baryon sectors, we obtain the net-strangeness of the system

\[
N_s - N_s \approx 2 (K^+ - K^-)_{\text{final}} - 1.57 (\Lambda - \bar{\Lambda})_{\text{final}}
\]

\[
- 4 (\Xi^{-,0} - \bar{\Xi}^{+})_{\text{final}} - 3 (\Omega^- - \bar{\Omega}^+) ,
\]

where subscript \( \text{final} \) denotes the yield including S&EM decay contributions.

The total number of strange quarks and strange antiquarks is

\[
N_s + N_s \approx 2 (K^+ + K^-)_{\text{final}} + 1.57 (\Lambda + \bar{\Lambda})_{\text{final}}
\]

\[
+ 4 (\Xi^{-} - \bar{\Xi}^{+})_{\text{final}} + 3 (\Omega^- + \bar{\Omega}^+)
\]

\[
+ 2\phi + 2 \left( \frac{2}{3} \eta + \frac{1}{3} \eta' \right) .
\]

Because the strangeness \( N_s - N_s \) is explicitly dependent on collision energies and collision centralities, we define the relative asymmetry factor

\[
z_S = \frac{N_s - N_s}{N_s + N_s}
\]

which is convenient to compare results in different situations.

In Table I, we show results for \( z_S \) in most central Au+Au collisions at different collisions energies \([47]\). Experimental data for yield densities of hadrons that are used in calculation are also presented \([41-43] [43-45] [29, 48] \). Because some data for \( \Omega^- \) and \( \phi \) are results in 0-10% centrality, we re-scale them by multiplying a factor \( N_{\text{part}}(0-5%) / N_{\text{part}}(0-10%) \) according to the participant nucleon number \( N_{\text{part}} \). We do the similar re-scaling for data of \( \Omega^- \) in 0-60% centrality at \( \sqrt{s_{NN}} = 7.7 \) GeV. Because data of \( \eta \) and \( \eta' \) mesons are usually absent, we neglect them and therefore the calculated \( z_S \) is over-estimated. We see that \( z_S \) at seven collision energies are quite small. Therefore, strangeness neutralization \( N_s = N_s \) is well satisfied in mid-rapidity region in heavy-ion collisions at RHIC energies.
Table I. Strangeness asymmetry factor $z_S$ calculated by yield data of strange hadrons and anti-hadrons in central Au+Au collisions [41–43][43–45][29, 48].

| $\sqrt{s_{NN}}$ (GeV) | $K^{+-}$ | $\Lambda$ (A) | $\Xi^-$ ($\Xi^+$) | $\Omega^-$ ($\Omega^+$) | $\phi$ | $z_S$ |
|-----------------------|----------|---------------|-------------------|-----------------------|-------|-------|
| 200                   | 48.9 ± 6.3 | 16.7 ± 1.1   | 2.17 ± 0.2        | 0.53 ± 0.06           | 7.95 ± 0.11  | -0.004 ± 0.06 |
| 62.4                   | 37.6 ± 2.7 | 15.7 ± 2.3   | 1.63 ± 0.2        | 0.212 ± 0.028         | 3.52 ± 0.08  | -0.019 ± 0.04 |
| 39                    | 32.0 ± 2.9 | 11.02 ± 0.03 | 1.54 ± 0.01       | 0.191 ± 0.006         | 3.38 ± 0.03  | -0.002 ± 0.05 |
| 27                    | 31.1 ± 2.8 | 11.67 ± 0.04 | 1.57 ± 0.01       | 0.154 ± 0.008         | 3.01 ± 0.04  | -0.006 ± 0.05 |
| 19.6                   | 29.6 ± 2.9 | 12.58 ± 0.04 | 1.62 ± 0.02       | 0.155 ± 0.01          | 2.57 ± 0.04  | -0.0002 ± 0.05 |
| 11.5                   | 25.0 ± 2.5 | 14.17 ± 0.08 | 1.35 ± 0.02       | 0.082 ± 0.012         | 1.72 ± 0.04  | -0.004 ± 0.05 |
| 7.7                    | 20.8 ± 1.7 | 15.3 ± 0.11  | 1.19 ± 0.03       | 0.0271 ± 0.0048       | 1.21 ± 0.06  | -0.021 ± 0.04 |

B. $p_T$ spectrum symmetry

In this subsection, we study the symmetry property between $p_T$ spectrum of strange quark $f_\Omega(p_T)$ and that of strange antiquark $f_\Omega(p_T)$. For this purpose, we choose $\Omega^- (\Omega^+)$ which consists of only strange quarks (antiquarks). Using Eq. (7), we have

$$f_\Omega(3p_T) = \kappa_\Omega f_\bar{\Omega}(3p_T)$$ \hspace{1cm} (39)

$$f_\Omega(3p_T) = \kappa_\Omega f_\bar{\Omega}(3p_T)$$ \hspace{1cm} (40)

from which we have

$$\frac{f_\bar{\Omega}(p_T)}{f_\Omega(p_T)} = \kappa_\bar{\Omega}\Omega \left[ \frac{f_\bar{\Omega}(3p_T)}{f_\Omega(3p_T)} \right]^{1/3} \propto \left[ f_\bar{\Omega} (3p_T) \right]^{1/3}$$ \hspace{1cm} (41)

where $\kappa_\bar{\Omega}\Omega = (\kappa_\bar{\Omega}/\kappa_\Omega)^{1/3}$ is independent of $p_T$ but is dependent on quark numbers. We emphasize that $\kappa_\bar{\Omega}\Omega$ is not equal to one at nonzero net quark number.

Using data of $p_T$ spectra for $\Omega^-$ and $\Omega^+$ in central Au+Au collisions [29, 48], we calculate the ratio $f_\bar{\Omega}(p_T)/f_\Omega(p_T)$ at different collision energies and present results in Fig. 1. Since we have shown $N_s = N_{\bar{s}}$ in the previous subsection, we multiply a proper constant before data of $f_\bar{\Omega}(3p_T)/f_\Omega(3p_T)$ to satisfy $N_s = N_{\bar{s}}$ and therefore we can directly compare $f_\bar{\Omega}(p_T)/f_\Omega(p_T)$ in/at different collision centralities/energies. Because of finite statistics of $\Omega^-$ and $\Omega^+$, data points of $f_\bar{\Omega}(p_T)/f_\Omega(p_T)$ show a certain fluctuations around one. On the whole, we can see that $f_\bar{\Omega}(p_T) = f_\Omega(p_T)$ is a good approximation at the studied collision energies.

IV. $p_T$ SPECTRA OF HADRONS

In this section, we use the quark combination model (QCM) in Sec. II to study $p_T$ spectra of identified hadrons in Au+Au collisions at RHIC energies. The inputs of model are $p_T$ spectra of quarks and antiquarks at hadronization. Here, we take $f_\bar{s}(p_T) = f_s(p_T)$ based on the study of strangeness neutralization in Sec. III. We also assume the iso-spin symmetry $f_u(p_T) = f_d(p_T)$ and $f_s(p_T) = f_s(p_T)$ in the mid-rapidity region. Therefore, we have altogether three inputs $f_u(p_T), f_d(p_T)$ and $f_s(p_T)$, which can be fixed by using our model to fit experimental data of $p_T$ spectra for proton, anti-proton and $\phi$ [44, 46, 48], respectively. The extracted results for quark $p_T$ spectra in central Au+Au collisions at six RHIC energies are shown in Fig. 2.

In Fig. 3, we show the calculated results for $p_T$ spectra of hadrons in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and compare them with experimental data [21,
but for central Au+Au collisions at $\sqrt{s_{NN}} = 7.7$–27 GeV are scaled for clarity as shown in the figure.

Figure 2. $p_T$ spectra of quarks at hadronization at mid-rapidity in central Au+Au collisions. Spectra at $\sqrt{s_{NN}} = 7.7$–27 GeV are scaled for clarity as shown in the figure.

Spectra of some hadrons are scaled for clarity as shown in the figure. In Figs. 2–5, the available data at $\sqrt{s_{NN}} = 7.7$ GeV cover smaller $p_T$ range and $\Omega$ data in the most central collisions are absent. Therefore the comparison at $\sqrt{s_{NN}} = 7.7$ GeV is not as conclusive as $\sqrt{s_{NN}} = 27$ GeV. Experimental data are from [29, 46, 48].

Figure 3. $p_T$ spectra of hadrons at mid-rapidity in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Symbols are experimental data [21, 42, 43] and lines are results of our model. Spectra of some hadrons are scaled for clarity as shown in the figure.

We see that the agreement between our model results and experimental data is satisfactory. Although there exists many successful explanations on these experimental data in the framework of quark combination mechanism in previous works [9, 10, 13, 14, 34, 49], here we would like to emphasize that the current EVC model can systematically explain these data in a simple way. Furthermore, the good agreement at top RHIC energy provides important basis for the application of our model to lower RHIC energies.

In Figs. 4, 5, 6, 7 and 8, we show results for $p_T$ spectra of hadrons in central Au+Au collisions at $\sqrt{s_{NN}} = 39, 27, 19.6, 11.5$ and 7.7 GeV and compare with experimental data [29, 46, 48]. At $\sqrt{s_{NN}} = 39, 27, 19.6, 11.5$ GeV, we see a good agreement between our model results and experimental data [29, 46, 48]. In particular, we see that experimental data for baryons ($p, \Lambda, \Xi, \Omega$) and meson $\phi$ can be explained by the model very well. At $\sqrt{s_{NN}} = 7.7$ GeV, we also see that model results are in good agreement with available experimental data. However, compared with data in Figs. 3–7, the available data at $\sqrt{s_{NN}} = 7.7$ GeV cover smaller $p_T$ range and $\Omega$ data in the most central collisions are absent.
those at other five higher energies. We should study this energy point further in the future when more precise data are available.

Using these systematic calculations and comparisons, we would like to emphasize the equal-velocity combination of quarks and antiquarks as the effective description at hadronization. This is manifested by the following two points. First, from panel (d) in Figs. 3-7, we see that experimental data of $\Omega$ and $\phi$ can be perfectly explained by the same $f_s (p_T)$ in a very simple way,

$$f_\Omega (p_T) = \kappa_{\Omega} f_s^3 (p_T/3),$$

$$f_\phi (p_T) = \kappa_{\phi} f_s^2 (p_T/2).$$

We call this property as the quark number scaling for hadronic $p_T$ spectra. We have shown that this property not only exists in relativistic heavy-ion collisions but also exists in high-multiplicity $pp$ and $p$-Pb collisions at LHC energies [23, 24, 28]. Second, we see that data of $\Lambda$ and $\Xi^-$ can be simultaneously explained by $f_s (p_T)$ from $\phi$ and $f_u (p_T)$ from proton,

$$f_\Lambda (p_T) = \kappa_{\Lambda} f_u^2 \left( \frac{1}{2 + r} p_T \right) f_s \left( \frac{r}{2 + r} p_T \right),$$

$$f_{\Xi^-} (p_T) = \kappa_{\Xi^-} f_u \left( \frac{1}{1 + 2r} p_T \right) f_s^2 \left( \frac{r}{1 + 2r} p_T \right),$$

after further including the decay contribution of heavier baryons. Here, $r = m_\Xi/m_u$. This is a clear support for the equal-velocity combination for quarks with different flavors.

Because masses of kaons and pions are small due to their Goldstone nature, the direct combination of constituent quarks and antiquarks can not form the on-shell pion and/or kaon. Here, we have adopted a naive treatment [25] by letting $u + \bar{u} \rightarrow K^+ + X$ in kaon formation and $u + \bar{d} \rightarrow \pi^+ + X$ in pion formation. $X$ denotes some soft degrees of freedom at hadronization and we identify them as soft pions. Because of this treatment, kaon and pion are not direct probes to test our model and extract quark distributions at hadronization. We show results of kaons and pions in Figs. 3-8 panel (a). We see that data of kaons and pions in the low $p_T$ range can be described by our model.
a more direct test for the quark combination mechanism at hadronization in relativistic heavy-ion collisions. Experimentally measured protons and anti-protons contain complex decay contributions of heavier baryons and anti-baryons,

\[ p^{(\text{final})} = p + \Delta^{++} + \frac{2}{3}\Delta^+ + \frac{1}{3}\Delta^0 + 0.64\Lambda + 0.517\Sigma^+ + 0.64\Sigma^0 + 0.633\Sigma^{*+} + 0.594\Sigma^{*0} + 0.602\Sigma^{*-} + 0.64(\Xi^0 + \Xi^- + \Xi^{*0} + \Xi^{*+}) + 0.64\Omega^- \]  

(46)

where we use the particle name with superscript (final) to denote the yield density of final-state hadron receiving the decay contributions and the particle name without superscript in the right hand side of the equation to denote yields of directly-produced hadrons by hadronization. The ratio \( \bar{p}/p \) finally behaves as

\[ \frac{\bar{p}^{(\text{final})}}{p^{(\text{final})}} \approx \left( \frac{1 - z}{1 + z} \right)^{0.99a} \]  

(47)

where \( z \) is net-quark fraction and the factor \( a \approx 4.86 \) is related to the baryon-meson production competition [35, 36], see Eq. (23) and texts below. For yields of kaons, we take the decay contributions of \( K^* \) (892) and \( \phi \) into account and have

\[ \frac{K^-}{K^+} = \frac{K^- + \frac{4}{3}K^* - \frac{4}{3}K^0 + 0.489\phi}{K^+ + \frac{4}{3}K^* + \frac{4}{3}K^0 + 0.489\phi} = \left( \frac{1 - z}{1 + z} \right)^{a - 1} \left( 1 + \lambda_s \frac{z}{1 + z} \right)^{-2} \]  

(48)

where \( \lambda_s = N_s/N_B \).

For yields of \( \Lambda, \Xi^- \) and their anti-particles, we consider the S&EM decay contributions,

\[ \frac{\bar{\Lambda}}{\Lambda} = \frac{\bar{\Lambda} + \Sigma^0 + 0.94(\Sigma^{*+} + \Sigma^{*0}) + 0.88\Sigma^{*0}}{\Lambda + \Sigma^0 + 0.94(\Sigma^{*+} + \Sigma^{*-}) + 0.88\Sigma^{*0}} = \left( \frac{1 - z}{1 + z} \right)^{a - 1} \left( 1 + \lambda_s \frac{z}{1 + z} \right)^{-2} \]  

(49)

and

\[ \frac{\Xi^+(\text{final})}{\Xi^-} = \frac{\Xi^+ + \frac{1}{3}\Xi^{*+} + \frac{1}{3}\Xi^{*0}}{\Xi^- + \frac{1}{3}\Xi^{*+} + \frac{1}{3}\Xi^{*0}} = \left( \frac{1 - z}{1 + z} \right)^{a - 2} \left( 1 + \lambda_s \frac{z}{1 + z} \right)^{-1} \]  

(50)

For \( \Omega^- \), we directly have

\[ \frac{\Omega^+}{\Omega^-} = \left( \frac{1 - z}{1 + z} \right)^{a - 3} \]  

(51)

In Eqs. (48-50), the power term \( (1 - z)/(1 + z) \) dominates the behavior of three ratios. Therefore, ratios

V. HADRONDON YIELDS AND MULTI-PARTICLE CORRELATIONS

In this section, we study the \( p_T \)-integrated yields of identified hadrons. In heavy-ion collisions at RHIC energies, the net baryon numbers deposited in the mid-rapidity region will influence the production of hadrons and anti-hadrons to a certain extent. One of the consequences for non-zero baryon number density is the asymmetry in yield between hadrons and their anti-particles. We study this yield asymmetry with our model by focusing on multi-particle yield correlations.

In Fig. 9, we show yield densities of hadrons and anti-hadrons divided by participant nucleon number at mid-rapidity in central Au+Au collisions at different collision energies. Open symbols are experimental data [29, 42, 44–46, 50, 51] and solid symbols are model results. Experimental data show that the yield split between \( K^+ \) and \( K^- \) in (a) is much smaller than that between \( p \) and \( \bar{p} \) in (b). Yield split between \( \Lambda \) and \( \bar{\Lambda} \) in (c) and that between \( \Xi^- \) and \( \bar{\Xi}^+ \) in (d) are larger than kaon split in (a) but are smaller than proton split in (b). Comparing with experimental data, we see that the model provides a globally good description for yield densities of hadrons and anti-hadrons.

In order to further study the split in yield between hadrons and anti-hadrons, we calculate the ratio of yield for anti-hadron to that for hadron. Yield ratio is not sensitive to the numbers of quarks and antiquarks at hadronization and is also not sensitive to some hadronization details such as parameters \( R_{\ell/F} \) and \( R_{D/O} \) introduced in our model. Therefore, it can be used to provide

![Figure 9. Hadronic yield densities divided by participant nucleon number at mid-rapidity in Au+Au collisions at different collision energies. Open symbols are experimental data [29, 42, 44–46, 50, 51] and solid symbols are model results.](image)
K−/K+, p/p, Λ/Λ, Ξ+/Ξ− and Ω+/Ω− in our model are simply correlated with each other by the net-quark fraction z.

Based on Eqs. (47-51), we can build several multi-hadron correlations as more sensitive tests of quark combination mechanism. In Fig. 10 (a), we firstly show the correlation between K−/K+ and p/p. This correlation shows how the production of baryon and that of meson in heavy-ion collisions are simultaneously influenced by the baryon quantum number density characterized by net-quark fraction z in our model. Symbols are experimental data at mid-rapidity in central and semi-central collisions [29, 42, 44-46, 50-52]. Error bars are the quadratic sum of statistical and systematic uncertainties. The dashed line is the result of QCM by Eqs. (47) and (48). Different from our previous work [36] and previous experimental measurements [52], here we show the correlation in double-logarithmic coordinates in order to provide a full and clear presentation because ratio p/p changes much faster than K−/K+. We see that experimental data in double-logarithmic coordinates behave as a straight line and our model can well reproduce this correlation.

In Fig. 10 (b) we show the correlation between Λ/Λ and p/p. We see that experimental data, symbols in the figure, exhibit a linear behavior in double-logarithmic coordinates. This behavior can be perfectly reproduced in our model by Eqs. (47) and (49). In addition, we note that ratio Λ/Λ decreases slower than p/p by a factor (1 − z)/(1 + z) = N_q/N_s. This is because that, compared with p/p, ratio Λ/Λ involves a strangeness neutralization N_s/N_s = 1.

In Fig. 10 (c) we show the correlation between Ξ+/Ξ− and p/p. Experimental data also exhibit a linear behavior in double-logarithmic coordinates. Since Ξ+/Ξ− involves double effect of strangeness neutralization (N_s/N_s)2 = 1, Ξ+/Ξ− decreases slower than Λ/Λ as the function of p/p. Our model result Eqs. (47) and (50), the dashed line in the figure, can well describe data at √s_nn ≥ 11.5 GeV and slightly under-estimates Ξ+/Ξ− at √s_nn = 7.7 GeV.

In Fig. 10 (d), we further show the correlation between Ω+/Ω− and Ξ+/Ξ−. Experimental data in double-logarithmic coordinates also exhibit a linear behavior. Because Ω−(Ω+) completely consists of strange (anti)quarks, ratio Ω+/Ω− involves triple effect of strangeness neutralization and therefore it decreases slower than Ξ+/Ξ−. The model result by Eqs. (50) and (51), the dashed line in the figure, can roughly describe data at √s_nn ≥ 11.5 GeV and under-estimates Ω+/Ω− at √s_nn = 7.7 GeV to a certain extent.

Some discussions on above results are necessary.

First, we emphasize the key physics in our model relating to multi-hadron correlations shown in Fig. 10. As shown by Eqs. (47)-(51), correlations among yield ratios of anti-hadron to hadron are not sensitive to absolute numbers of quarks and antiquarks at hadronization and non-perturbative parameters R_{V/P} and R_{D/O} introduced in the model. Therefore, these yield correlations are only related to two basic features of quark combination in our model. (1) free combination. Newborn quarks and antiquarks, net-quarks are all treated as individual (anti-)quarks and freely take part in combination. (2) flavor independent combination probability. We take N_{B}/N_{q} as the averaged probability of q1q2q3 forming a baryon and N_{M}/(N_q N_q) as the averaged probability of q1q2 forming a meson. No sophisticated flavor correction is made at the moment. From Fig. 10, we see that such a global quark combination model provides a systematic description on multi-hadron yield correlations in Au+Au collisions, at least at √s_nn ≥ 11.5 GeV. Therefore, this is a clear signal of quark combination mechanism at hadronization in these collisions.

Second, the comparison between our model calculation and experimental data in Au+Au collisions at √s_nn = 7.7 GeV may indicate some physics beyond the key features of quark combination discussed above. As p/p and Λ/Λ ratios are reproduced, we see that model results for K−/K+, Ξ+/Ξ− and Ω+/Ω− are smaller than experimental data to a certain extent. This may be related to the point (1) discussed above. In Au+Au collisions at low energy, the colliding nucleons may not break completely. A part of nucleon fragments may do not behave as the individual up/down quarks and freely take part in the combination with newborn quarks and antiquarks; Instead, they behave as diquarks and can form proton by combining with a up/down quark or form Λ by combining with a strange quark. Because these net-quarks only contribute to proton and Λ production, net-quark fraction z used in Eqs. (48), (50) and (51) for K−/K+, Ξ+/Ξ− and Ω+/Ω− should be smaller than that in p/p and Λ/Λ. This consid-
eration can increase ratios $\bar{K}^-/K^+$, $\Xi^+/\Xi^-$ and $\Omega^+/\Omega^-$ at the given $p_T$ and $\Lambda/\Lambda$ ratios and therefore can qualitatively improve the description at $\sqrt{s_{NN}} = 7.7$ GeV in the current model. Such a sophisticated effect of net-quarks is worthwhile to be studied in detail in the future works.

VI. PROPERTIES OF QUARK DISTRIBUTIONS

By studying experimental data of hadronic $p_T$ spectra, we have obtained $p_T$ spectra of quarks at hadronization, which are shown in Fig. 2. In this section, we study properties of these obtained quark distributions at hadronization.

We firstly calculate the ratio $f_s(p_T)/f_{\bar{u}}(p_T)$ and study its $p_T$ dependence. Fig. 11 shows results in Au+Au collisions at $\sqrt{s_{NN}} = 200, 39, 27, 19.6$ and 11.5 GeV. We see that ratios at these collision energies in the low $p_T$ region ($p_T \lesssim 1.3$ GeV/c) all increase with $p_T$ and then turn to decrease at larger $p_T$. This behavior is the consequence of the complex non-perturbative evolution in partonic phase stage. We note that the increase of the ratio at low $p_T$ can be qualitatively understood by thermal statistics. In the case of thermal equilibrium such as the Boltzmann distribution $dN/(p_T dp_T) \propto \exp\left(-\frac{p_T^2}{2T} + \frac{m^2}{T}\right)$ in two-dimensional transverse momentum space, heavier mass will lead to flatter shape of the $p_T$ distribution and thus lead to the increase of the ratio.

![Figure 11. Spectrum ratio $f_s(p_T)/f_{\bar{u}}(p_T)$ in central Au+Au collisions at $\sqrt{s_{NN}} = 200, 39, 27, 19.6$ and 11.5 GeV.](image)

The Boltzmann distribution at hadronization temperature in the rest frame can not directly describe the obtained quark distributions in Fig. 2. We should further take into account the contribution of the collective radial flow in the prior partonic phase evolution in heavy-ion collisions. As an illustration, we consider a simple situation, that is, Boltzmann distribution in the two-dimensional transverse momentum space boosted with a radial flow velocity $v_\perp$. In this case the distribution is

$$\frac{dN}{p_T dp_T} \propto \frac{1}{E^*} E^* (v_\perp) \exp[-E^* (v_\perp)/T]$$

with $E^* (v_\perp) = \gamma (E - v_\perp p_T)$ and $E = \sqrt{p_T^2 + m^2}$. If we assume that quarks of different flavors at hadronization are thermalized and boosted with the same radial velocity, we can use the above formula to simultaneously describe the extracted $f_s(p_T)$ and $f_{\bar{u}}(p_T)$ in the low $p_T$ range ($p_T \leq 1.3$ GeV/c) in Fig. 2. According to our previous work [28], the hadronization temperature is taken as $T = (0.164, 0.163, 0.162, 0.161, 0.156)$ GeV in central Au+Au collisions at $\sqrt{s_{NN}} = (200, 39, 27, 19.6, 11.5)$ GeV, respectively. We obtain radial flow velocity $v_\perp/c \approx (0.39, 0.28, 0.27, 0.25, 0.23)$ at these collision energies.

We find that these results for radial flow velocity are consistent with our previous extraction by a hydrodynamics-motivated blast-wave model [53] fit of $f_s(p_T)$ with the same hadronization temperature [28]. We also find that $f_\bar{u}(p_T)$ and $f_u(p_T)$ extracted in this work can also be consistently described in blast-wave mode. Here, taking central Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ GeV as an example, we show in Fig. 12 (a) the fit results of quark $p_T$ spectra by Eq. (52) and those by blast-wave model at the same hadronization temperature 0.161 GeV and radial flow velocity 0.25 c. We see that two fit methods give the consistent description on quark $p_T$ spectra. In addition, we see from Fig. 12 (b) that the increase part of the ratio $f_s(p_T)/f_{\bar{u}}(p_T)$ in the low $p_T$ range can be reasonably described.

![Figure 12. (a) Fit results for quark $p_T$ spectra at hadronization in central Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ GeV by Boltzmann formula Eq. (52) and by blast-wave model at the same hadronization temperature 0.161 GeV and radial flow velocity 0.25 c. (b) ratio $f_s(p_T)/f_{\bar{u}}(p_T)$ by two fit methods. Symbols are quark $p_T$ spectra and lines are fit results.](image)

Fig. 11 also shows the ratio $f_s(p_T)/f_{\bar{u}}(p_T)$ globally increases with the decrease of collision energies. To study this energy dependence of strangeness, we calculate the strangeness factor

$$\lambda_s = \frac{N_s}{N_{\bar{u}}}$$

and present results in Fig. 13. Here, results of $\lambda_s$ in central Au+Au collisions at $\sqrt{s_{NN}} = 62.4$ and 130 GeV
are also shown. The uncertainty of \( \lambda_s \) is caused by that of experimental data for yield ratios such as \( K/\pi \), \( \Lambda/p \), \( \bar{\Lambda}/\bar{p} \). We note that these new results of \( \lambda_s \) are consistent with our previous works \[49, 54, 55\]. Compared with \( \lambda_s \sim 0.3 \) in elementary collisions such as \( e^+e^- \) and \( pp \) reactions, we see that \( \lambda_s \gtrsim 0.42 \) in heavy-ion collisions is obviously enhanced. We also see that \( \lambda_s \) increases as the decrease of collision energy.

The dependence of \( \lambda_s \) on collision energy is related to the varied baryon quantum number density. In this paper, we study this energy dependence in the framework of thermodynamics for quark system. We consider a thermal system consisting of constituent quarks and antiquarks. In our quark combination model, constituent quarks and antiquarks are regarded as the effective degrees of freedom at hadronization and they freely combine into baryons and/or mesons at hadronization. Therefore, we can treat such a constituent quark system as the classical gas.

Because we study the production of hadrons in mid-rapidity range, we firstly discuss the case of grand-canonical ensemble. The number of quark under Fermi-Dirac statistics in the grand-canonical ensemble is

\[
N_f = g \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\exp \left( \frac{(E - \mu_f)}{T} \right) + 1} = \frac{V m^2 T}{2\pi^2} \sum_{n=1} \frac{(-1)^{n+1}}{n} K_2 \left( \frac{m}{T} \right) \exp \left( \frac{\mu_f}{T} \right),
\]

where \( V \) is system volume and \( T \) is temperature. \( g = 6 \) is degeneracy factor of quark, \( m \) is quark mass and \( \mu_f \) is chemical potential of quark \( f \). \( K_2 \) is the modified Bessel function of the second kind. As discussions of local strangeness conservation in Sec. III, we set \( \mu_s = \mu_\bar{s} = 0 \) to get \( N_s = N_{\bar{s}} \). We assume the iso-spin symmetry between up and down quarks, then we have \( \mu_u = \mu_d = -\mu_\bar{u} = -\mu_\bar{d} = \mu_B/3 \). Then, the strangeness factor in grand-canonical ensemble (GCE) of free quark gas is

\[
\lambda_s^{(GCE)} = \frac{m^2 n_s}{m \sum_{n=1} \frac{(-1)^{n+1}}{n} K_2 \left( \frac{m}{T} \right) \exp \left( \frac{\mu_B}{3T} \right)} \approx \lambda_s^{(GCE)} = 0 \exp \left( \frac{\mu_B}{3T} \right).
\]

In the second line, we consider only the leading term \( n = 1 \), which is corresponding to the Boltzmann statistics. As we know, baryon chemical potential is increased as the decrease of the collision energy. Therefore, the collision energy dependence of \( \lambda_s \) is qualitatively understood.

For a demonstrative calculation for the collision energy dependence of \( \lambda_s^{(GCE)} \), we first apply Eq. (55) with the re-tuned \( m_u = 0.3 \) GeV and \( m_s = 0.54 \) GeV to give a proper strangeness at vanishing baryon chemical potential \( \lambda_{s,\mu_B=0} \approx 0.42 \). Then, we apply Eq. (54) to fit the total quark number \( x = N_q + N_{\bar{q}} \) and net-quark number \( xz = N_q - N_{\bar{q}} \) integrated from Fig. 2 and obtain \( V \) and \( \mu_B \) of quark system at hadronization. Here, the hadronization temperature is taken as before. Then we substitute \( \mu_B \) and \( T \) into Eq. (55). The calculated results for \( \lambda_s^{(GCE)} \) at RHIC energies are shown as triangles with the dashed line in Fig. 13. We see that the extracted \( \lambda_s \) in quark combination model can be explained by grand-canonical ensemble of quark gas system at \( \sqrt{s_{NN}} > 20 \) GeV. At lower two collision energies, GCE results are higher than our extraction.

Considering the longitudinal rapidity space of heavy-ion collisions is finite, in particular, \( y_{beam} < 2.5 \) at \( \sqrt{s_{NN}} \leq 11.5 \) GeV, the studied mid-rapidity range \( |y| < 0.5 \) is not a tiny fraction of the whole system, the hadron production in the midrapidity range may be influenced by effects of global charge conservation not only strangeness conservation but also baryon quantum number conservation. Therefore, grand-canonical treatment may be not perfectly suitable. We now consider the result of canonical ensemble for free quark system. We apply the method of canonical statistics \[56\] to obtain the property of quark number distribution under the constraint of finite baryon quantum number and strangeness. We put the detailed derivation into Appendix A. The inputs of canonical ensemble are volume \( V \), temperature \( T \), and charges \( (B,Q,S) \). The temperature is set to the hadronization temperature whose values at different collision energies are taken as before. As discussions in Sec. III, we take \( S = 0 \), \( V \), \( B \) and \( Q \) can be determined by fitting the quark and antiquark numbers integrated from Fig. 2. Results of \( \lambda_s^{(CE)} \) for canonical ensemble of quark system are shown as solid circles with the dotted line in Fig. 13. We see that \( \lambda_s^{(CE)} \) is also increased with the decrease of collision energy and is smaller than \( \lambda_s^{(GCE)} \), in particular, at low collision energies. Our extracted \( \lambda_s \) is roughly located in the middle of two ensembles.
VII. SUMMARY AND DISCUSSIONS

In this paper, we have applied a quark combination model with equal-velocity combination approximation to systematically study the production of hadrons in Au+Au collisions at RHIC energies. The model applied in this work is motivated by our recent findings for the constituent quark number scaling property of hadronic $p_T$ spectra in high energy $pp$, $pA$ and AA collisions. After systematic study of $p_T$ spectra and yields for hadrons, we found that our quark combination model provides a good explanation on the experimental data in Au+Au collisions at $\sqrt{s_{NN}} \geq 11.5$ GeV. This suggests that the constituent quark degrees of freedom still play important role even at low RHIC energy, which is closely related to the deconfinement at these collision energies.

By study of hadronic $p_T$ spectra and yields in these collisions, we obtained $p_T$ spectra and numbers of constituent quarks and antiquarks at hadronization. We calculated the net strangeness $N_s - N_{\bar{s}}$ at mid-rapidity and showed the strangeness neutralization is well satisfied at mid-rapidity in heavy-ion collisions at RHIC energies. We studied the spectrum ratio of strange quarks to newborn up/down quarks $f_s (p_T)/f_{\bar{s}} (p_T)$ and the $p_T$ integrated number ratio $\lambda_s = N_s/N_{\bar{s}}$ at different collision energies. We applied the basic thermal statistics for free constituent quark system to understand these properties of strange quarks relative to up/down quarks.

We emphasize the key physics in our model which are responsible for successfully explaining experimental data of hadronic $p_T$ spectra and yields. First, the model takes the constituent quarks and antiquarks as the effective interface connecting the strongly-interacting system before hadronization and that after hadronization. We assume the constituent quarks and antiquarks as effective degrees of freedom for the strongly-interacting quark-gluon system just before hadronization. On the other hand, we take the constituent quark model to describe the static structure of hadrons in the ground state. In this scenario, the equal-velocity combination of these constituent quarks and antiquarks is a reasonable approximation and is indeed supported by the quark number scaling property for $p_T$ spectra of hadrons observed in experiments [23, 24, 28]. The study in this paper further showed that the equal-velocity combination can systematically describe the production of different kinds of hadrons in heavy-ion collisions at RHIC energies. Second, the model includes reasonable considerations for the unity of hadronization and the linear response property, see detailed discussions in [36]. This is very important to reproduce the multi-hadron yield correlations shown in Fig. 10. For example, in a naive inclusive view of combination, we have $N_0 \propto N_s^3$ and $N_0 \propto N_s^3$ and therefore $\Omega^+ / \Omega^- \propto (N_s/N_0)^3 \approx 1$ which is independent of collision energy. However, yield of $\Omega^-$ in our model is not only dependent on $N_s^3$ but also dependent on the global system information shown as in Eq. (17); thus we have $\Omega^+/\Omega^- = [(1-z)/(1+z)]^{a-3}$ in Eq. (51) which decreases with the decrease of collision energy.

VIII. ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China under Grant No. 11975011, Shandong Province Natural Science Foundation under Grant Nos. ZR2019YQ06 and ZR2019MA053, and Higher Educational Youth Innovation Science and Technology Program of Shandong Province (2019KJ010, 2020KJ004).

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If we consider the small asymmetry in number between down quarks and up quarks coming from colliding nucleons, we should modify Eq. (29) by multiplying a factor $N_u/N_d$ in right hand of equations and the corresponding coefficients in Eqs. (36) and (37). According to the number of newborn quark extracted in this paper and net-quarks from colliding nucleons, we estimate, for example, $N_u/N_d \approx 0.99$ at $\sqrt{s_{NN}} = 200$ GeV and $N_u/N_d \approx 0.93$ at 7.7 GeV. The resulting $z_B$ is $(-0.004, -0.018, -0.0014, 0.0002, -0.003, -0.009)$ at $\sqrt{s_{NN}} = (200, 62.4, 39, 27, 19.6, 11.5, 7.7)$ GeV, respectively.

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where $\beta = 1/T$ is the inverse temperature, $E_{\text{state}}$ is the energy of the state and $Q_{\text{state}}$ are abelian charges of the state

$$Q_{\text{state}} = \sum_{j=1}^{K} q_j N_j. \quad (A2)$$

Here $N_j$ is the number of particle $j$ in the current state and $q_j = (B_j, Q_j, S_j)$ is the quantum number vector for the $j$-th particle. $K$ is the number of particle species.

The multiplicity distribution of particles can be obtained from the generating function associated to the canonical partition function $Z(Q)$ [56] and is expressed as

$$P(\{N_j\}) = \frac{1}{Z(Q)} \prod_{j=1}^{K} \left\{ \sum_{j=1}^{N_j} \prod_{j=1}^{N_i} \left[ \frac{[z_j(n_j)]^{h_{n_j}}}{n_j!} \right] \right\} \delta_{Q,\sum_j q_j N_j}, \quad (A3)$$

where the summation takes different configurations for $\{h_{n_j}\}$ into account under the condition $\sum_{n_j=1}^{\infty} n_j h_{n_j} = N_j$.

$$z_j(n_j) = (\mp)^{n_j+1} \frac{gV}{(2\pi)^{3}} \int d^3p e^{-n \beta E}$$

$$= (\mp)^{n_j+1} \frac{gV}{2\pi^2 n_j \beta} K_2(n_j \beta m). \quad (A4)$$

Now, we consider the thermal system consisting of constituent quarks and antiquarks. In our quark combination model, constituent quarks and antiquarks are regarded as the effective degrees of freedom at hadronization and they freely combine to form baryons and/or mesons at hadronization. Therefore, we can simply apply above formula to obtain the number distribution of these “free” constituent quarks and antiquarks under canonical statistics. Here, we consider up, down, strange quarks and their antiparticles. Index $j$ denotes $u, d, s, \bar{u}, \bar{d}, \bar{s}$ and $K = 6$.

Eq. (A3) can be further denoted as

$$P(\{N_j\}) = \frac{1}{Z(Q)} \left[ \prod_{j=1}^{K} Z(S_N) \right] \delta_{Q,\sum_j N_j} q_j, \quad (A5)$$

where $Z(S_N)$ is cycle-index polynomial of symmetric group. $Z(S_N)$ can be numerically evaluated using the recurrence relation

$$Z(S_N) = \frac{1}{N} \sum_{l=1}^{N} z(l) Z(S_{N-l}) \quad (A6)$$

with $Z(S_0) = 1$ and $Z(S_1) = z(1)$.

As discussed in Sec. III, we take strangeness neutralization $S = 0$ and therefore $N_u = N_d$. For constraints of baryon charge $B$ and electric charge $Q$, we denote them as $N_u - N_d = B + Q$ and $N_d - N_d = 2B - Q$. Finally the joint distribution function of quark numbers and antiquark numbers is

$$P(N_d, N_u, N_s, N_d, N_u, N_s)$$

$$= \frac{1}{Z(Q)} Z(S_{N_d}) Z(S_{N_u}) \delta_{N_u, N_d+2B-Q}$$

$$\times Z(S_{N_s}) Z(S_{N_u}) \delta_{N_u, N_d+B+Q}$$

$$\times Z(S_{N_s}) Z(S_{N_u}) \delta_{N_u, N_s}. \quad (A7)$$

We obtain the averaged number of quarks by

$$\overline{N}_s = \sum_{\{N_i\}} N_s P(\{N_d, N_u, N_s, N_d, N_u, N_s\}), \quad (A8)$$

$$\overline{N}_{\bar{u}} = \sum_{\{N_i\}} N_{\bar{u}} P(\{N_d, N_u, N_s, N_d, N_u, N_s\}), \quad (A9)$$

and calculate strangeness factor $\lambda_s^{(CE)} = \overline{N}_s/\overline{N}_{\bar{u}}$ in canonical ensemble of free quark system.