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Parasitic Capacitance in Copper-Foiled Medium-Voltage Filter Inductors

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Abstract—This paper characterizes three parasitic capacitances in copper-foiled medium-voltage inductors. It is found that the conventional modeling method overlooks the effect of the fringe field, which leads to inaccurate modeling of parasitic capacitances in copper-foiled inductors. To address this problem, the parasitic capacitances contributed by the fringe field is identified first, and a physics-based analytical modeling method for the parasitic capacitances contributed by the fringe field is proposed, which avoids using any empirical equations. The total parasitic capacitances are then derived for three different cases with three different core potentials, from which a three-terminal equivalent circuit is derived, and thus, the parasitic capacitances in copper-foiled inductors are explicitly identified. The calculated results show a close agreement with the measured capacitance by using an impedance analyzer.

Index terms—Parasitic capacitance, copper-foiled, medium-voltage, filter inductor, fringe field, physics-based modeling.

I. INTRODUCTION

Thanks to the advances in the wide-band-gap (WBG) power semiconductor devices, Silicon Carbide (SiC) Metal-Oxide-Semiconductor Field-Effect Transistors (MOSFETs) are widely used in modern power conversion systems [1]. With the use of SiC MOSFETs, power converters can be designed with higher switching frequency and less switching losses [2], [3]. Yet, the high dv/dt problem becomes more significant during the switching transitions of SiC MOSFETs [4]. Under the high dv/dt conditions, the parasitic capacitances of passive and active power components, such as the common-mode capacitance of gate drivers [5], [6], the ground capacitance of heatsink [7], the parasitic capacitance in power modules [8], [9], and the parasitic capacitance in transformers [10-12] and inductors [13-15], can bring large common-/differential-mode current into the converter circuit [13], causing electromagnetic interferences [4] and accelerating the aging of power components [16].

The problems become even worse in the applications of medium-voltage (MV) SiC MOSFETs. It has been reported that the current spikes on filter inductors are more significant than the rated current, due to the larger parasitic capacitance caused by the extra insulation and higher inductance with a larger number of turns [13]. Further, the harmonic spectra of MV converter switching waveforms tend to have higher magnitudes due to the larger dv/dt introduced by the WBG devices, which also increase the capacitive current [15]. The accurate modeling of parasitic capacitance of MV filter inductors becomes critical to reduce the capacitive current caused by the high dv/dt of MV SiC MOSFETs.

The windings of inductors are commonly constructed with round cables for low cost. Yet, in high-frequency applications, both the copper foil and the litz wire are commonly used for a lower ac-resistance [17], and the copper-foil is more popular in the high-voltage and high-power applications [17-20], due to its higher power density and more flexibility than round cables and Litz wires in the manufacturing process. However, the intra-winding capacitance of the copper-foiled inductors and transformers is large, due to the extensive interleaving of the windings [17], which is not desirable in applications with a high dv/dt operations. Therefore, it is important to model the parasitic capacitances in copper-foiled inductors when they are used with MV SiC MOSFETs.

A few works have been reported on modeling the parasitic capacitance of copper-foiled magnetic components [17], [20], [21]. Basically, the parasitic capacitance in magnetic devices is divided into static capacitance and dynamical capacitance.

1) Static capacitance [21], [22]. It represents the capacitance between two planes when disconnected, where there is no ohmic voltage drop on each plane. Therefore, the static capacitance is merely dependent on the geometrical structure and material information of the two planes.

2) Dynamical capacitance [21], [22]. This capacitance represents the total electrical field energy stored between two planes, which is related to the voltage potential distribution. In practice, the voltage potential on each plane is not a constant value, due to the current flows through the winding, which results in ohmic voltage drops in the windings. The dynamical capacitance can be calculated by the static capacitance and dynamical voltage potential distribution.

In [17], the parasitic capacitance of a copper-foil-based transformer between the primary side and secondary side is calculated by using the formula of parallel plate capacitance, which is, however, merely a static capacitance, and thus fails to capture the dynamical capacitance in practice [14], [21], [22]. An air-coiled inductor wounded by copper foils is modeled in [20], where the first resonant frequency caused by the intra-winding capacitance is identified, yet the modeling of parasitic capacitance is not addressed. In [21], both the static and dynamical capacitances are calculated, considering the eddy current effects. However, the core is assumed to be always floating in this work, where only one equivalent capacitance can be calculated. An improved modeling method is reported in [22] to consider the grounding effects of the magnetic core, yet it is only focused on the round-cable based...
inductors. As found later in this paper, the method reported in [22] fails to characterize the capacitive couplings between the core and terminals of the copper-foiled inductor, where the fringe field between the winding and core is significant for modeling the parasitic capacitance in typical copper-foiled inductors.

In [23], a modeling method is proposed to calculate the parasitic capacitance of a high-power transformer, where the fringe field between different sections are considered by using the empirical equations oriented from very-large-system-integration (VLSI) applications [24]. The parasitic capacitance contributed by the fringe field is usually neglected in most previous modeling methods [10, 12, 14, 20, 25]. The parasitic capacitance introduced by the fringe field has also been discussed in semiconductor devices [26], antenna applications [27], and transmission lines [28]. However, these empirical equations are actually restricted by the geometrical structure [24], which is not applicable to model the fringe-field capacitance between the winding and core in copper-foiled inductors, due to a more complex geometrical structure. Detailed research in copper-foiled inductors are emerging but still empty. Yet, it is important to systematically and physically model the parasitic capacitance in copper-foiled inductors, where the unique challenges need to be addressed and solved.

Therefore, this article is aimed to physically model the parasitic capacitance in copper-foiled MV filter inductors, where the correct capacitive couplings are addressed with considering the fringe field effects. The fringe field in typical copper-foiled inductors is identified first, where the static and dynamical capacitance contributed by the identified fringe field is derived in sequence. Then, the total capacitance of the typical inductors is obtained by summing the dynamical capacitance contributed by the fringe field, the electrical field between the inner layer and core, the electrical field between windings, and the intra electrical field between two adjacent layers. The theoretical calculations show good agreement with the measurements of a practical copper-foiled inductor by using an impedance analyzer.

II. MV COPPER-FOILED INDUCTORS

An MV copper-foiled inductor (30 mH) is taken as an example in this study. The MV inductor is designed for a 5kHz 2-level voltage source converter based on SiC MOSFETs with 4.16 kV line-to-line ac voltage and 6 kV dc-link voltage. The insulation level of this inductor is 10 kV, and the current rating is 8 A (rms). The windings and insulation of the inductor are constructed by copper- and mylar-foils, which are arranged in multiple layers. Two U-type amorphous cores are used for the magnetic loop with an air gap in between.

The schematics of the studied MV copper-foiled inductor are given in Fig. 1. Fig. 1 (a) shows the front view and Fig. 1(b) gives the cross-section view. Two windings are connected in parallel for sharing current. Spacers are used to reduce the capacitive couplings between the inner layer and core, and they will be modeled later.

The parameters of the studied MV foil-based inductor are summarized in Table I, along with the definitions of symbols used in Fig. 1.

![Fig. 1. Schematics of the studied MV copper-foiled inductor. a) Front view, b) Cross-sectional view schematic of an MV copper-foiled inductor.](image)

| Description                              | Symbol | Value  |
|------------------------------------------|--------|--------|
| Thickness of the copper foil             | d_c    | 0.05 mm|
| Width of the copper foil                 | W_C    | 30 mm  |
| Thickness of the mylar foil              | d_m    | 0.05 mm|
| Width of the mylar foil (Height of the winding) | w_m   | 60 mm  |
| Distance between the two windings        | p_w    | 19 mm  |
| Width of the winding                     | w      | 55.5 mm|
| Width of the spacer                      | w_b    | 10 mm  |
| Thickness of the core                    | d_0    | 40 mm  |
| Distance of the air gap between the bobbin and core | p_0    | 1.5 mm |
| Distance between the inner layer and bobbin | p_1   | 12 mm  |
| Thickness of the bobbin                  | d_l    | 2 mm   |
| Number of layers                         | m      | 190    |
| Total inductance (designated value)      | L      | 30 mH  |
The insulation material is selected as Dupont Mylar A [29]. The material of bobbin and spacer is Durethan BKV 30 H3 [30], which is based on Polyamide 6 with 30% glass-reinforced. The values for the relative permittivity of materials are listed in Table II, which are identified from the datasheet.

The conductivity of the amorphous core is 130 µΩ/cm, according to the datasheet [32].

In Fig. 1, the terminal labeled as “0 mH” is also defined as the hot terminal $P_{\text{hot}}$, which locates at the inner layer. The terminal labeled as “30 mH” is defined as the cold terminal $P_{\text{cold}}$, which locates at the outer layer. The core/frame is also labeled as G.

The focus of this paper is to analytically model the equivalent capacitance between $P_{\text{hot}}$ and $P_{\text{cold}}$, between $P_{\text{hot}}$ and G $C_{\text{c1}}$, and between $P_{\text{cold}}$ and G $C_{\text{c2}}$, based on the geometrical and material information of the copper-foiled inductors.

### III. Modeling of the Fringe Field Capacitance

In this section, the electrical field in the copper-foiled inductors are identified. Based on [22], most capacitive couplings in copper-foiled inductors are identified, which includes the couplings between the inner layer and core, between two different layers, and between two windings. Due to the special structure of copper-foiled inductor, there is only one turn in a single layer. Therefore, the couplings between turns are the same as between those layers.

However, the fringe field between winding and core is not considered in the modeling in [22], which will be proved that is not neglectable in copper-foiled inductors by the later sections of this paper.

#### A. Identify the fringe field

A finite element method (FEM) simulation is given to identify the fringe field in the copper-foiled inductors by using Ansys, which is presented in Fig. 2. In the simulation, the voltage potential along the layers of the copper-foils is configured equally, therefore, the electrical field strength between two adjacent layers is zero in this simulation. The core is configured as a reference ground. 19-layer copper-foil and 20-layer mylar-foil are used to construct the winding. In this simulation, the thickness of the copper- and mylar-foil is configured as 0.5mm, where the distance between the inner-layer and core is 4 mm.

It is worth mentioning that the geometrical parameters of the copper-foiled inductor in FEM simulations are not exactly the same as the designed value since the main target here is to identify the all possible electrical field existed around the copper-foiled inductor.

According to Fig. 2, although the electrical field is strongest between the inner-layer and core, the fringe field between the winding and core are still obvious on both edges. Both the fringe field between the sidewall of the inductor and core, and the fringe field between the top-surface and core can be identified from Fig. 2.

The energy stored in the fringe field will contribute to an equivalent capacitance. This capacitance cannot be neglected in the copper-foiled inductors since the extensive area of the sidewall and top surface of the copper-foiled inductors. Usually, the fringe field is neglected for modeling the parasitic capacitance of inductors in previous research since its impacts are limited. However, in the copper-foiled inductor, due to a large number of layers, the sidewall area of the copper-foils is significant. Therefore, the impacts of the fringe field may not be neglected for the purpose of modeling the parasitic capacitances.

### Table II. Relative permittivity of the material

| Description                      | Symbol | Value      |
|----------------------------------|--------|------------|
| Relative permittivity of the mylar foil [29] | $\varepsilon_m$ | 3.25       |
| The permittivity of the bobbins and spacers [30] | $\varepsilon_b$ | 4.0        |
| The permittivity of vacuum [31] | $\varepsilon_0$ | 8.82×10⁻¹² F/m |

**Table II. Relative permittivity of the material.**

The permittivity of vacuum $\varepsilon_0 = 8.82\times10^{-12}$ F/m

---

**Fig. 2** Fringe field between the winding and core of the copper-foil inductors

**Fig. 3** Identifies the capacitive couplings in copper-foiled MV inductors.
B. Empirical equation in VLSI applications

In VLSI applications [24], a schematic to illustrate the parasitic capacitance with considering the fringe field effect is presented in Fig. 4.

![Fig. 4 Schematic of a chip and ground in VLSI applications](image)

In Fig. 4, $C_1$, $C_2$, $C_3$, and $C_4$ are the parasitic capacitance contributed by the fringe field. $C_{pp}$ is the parallel capacitance between the bottom surface and reference ground. An empirical equation [25], which is widely used in VLSI applications, is given to calculate the total parasitic capacitance.

$$C_{total} = C_1 + C_2 + C_3 + C_4 + C_{pp}$$

$$\approx \epsilon_r \epsilon_0 \times \left[ \frac{w}{h} + 0.77 + 1.06 \times \left( \frac{w}{h} \right)^{0.25} + 1.06 \times \left( \frac{t}{h} \right)^{0.5} \right]$$

(1)

However, there are some restrictions for applying the empirical equation in calculating the parasitic capacitance of copper-foiled inductors:

1) Only applicable for the cases with simple structures. In VLSI applications, the structure of the conductor is with only one layer. The conductor is assumed to be surrounded by the same material, therefore, only one relative permittivity is used in (1).

2) Only applicable for the cases with specific geometrical structures. The empirical equation requires $w/h > 0.3$ and $t/h < 10$. Otherwise, it can introduce significant errors.

3) The calculated capacitance by the empirical equation is the static capacitance, where the voltage potential on the inductor is assumed to be the same.

For the copper-foiled inductor illustrated in Fig. 1, it has a more complex geometrical structure than the structure in Fig. 4. Besides, the voltage potential is distributed linear on the windings in practice, where the calculated static capacitance from the empirical equation cannot be correctly revealed the energy stored in the electrical field, which is typically represented by the dynamical capacitances.

C. Physics-based modeling of the static capacitance

The parasitic capacitance contributed by the fringe field is modeled as two independent capacitances, which are, the fringe field capacitance between sidewall and core of inductor, and the fringe field capacitance between the top-surface and core. The static capacitances are derived firstly. The dynamical capacitance will be modeled based on the derived static capacitance.

Some assumptions are made before modeling the fringe field capacitance:

1) The electrical lines between the sidewall of winding and core is assumed to be an arc of a 90-degree sector, which is illustrated in Fig. 5(a), where the electrical field is orthogonal to the conductor surfaces.

2) The electrical lines between the top-surface and core of the inductor are the arc of a half-circle plus a straight line, which is also illustrated in Fig. 5(b). The electrical field is orthogonal to the conductor surfaces. This is an approximation for describing the field line between the top-surface and core of the inductor shown in Fig. 5.

3) The voltage potential on the sidewall of winding is continuous. The sidewall of the mylar-foil is assumed to have a virtual voltage potential, which is contributed by the fringe field from the top and bottom surface of each copper-foil approximately. However, it is only applicable for the cases that $w_i \gg d_s$, which is not strict for normal copper-foils. A schematic to illustrate the assumption is presented in Fig. 6.

4) The core is a perfect conductor. The voltage potential on the core is always the same.

5) The voltage potential on the same layer is assumed to be the same. The error introduced by this assumption can become less when the number of layers is larger.

![Fig. 5 Elementary capacitances contributed by the fringe field. a) elementary capacitance between the sidewall and core; 2) elementary capacitance between the top-surface and core.](image)

![Fig. 6 Approximation made for distributing the voltage potential continuously on the sidewall](image)

C.1. Static capacitance

In order to calculate the static capacitance contributed by the fringe field, all copper-foils are assigned to have the same voltage potential. The capacitance contributed by the fringe field is classified into two parts, which are the capacitance...
between the sidewall of winding and core, and between the top-surface and core, respectively.

**C.1.1 Static capacitance** between the sidewall of winding and core

The elementary static capacitance contributed by the fringe field between the sidewall of winding and core is derived in (2) according to Fig. 6.

\[ C_{\text{ele_sc}} = \frac{2\varepsilon_0\varepsilon_{\text{dl}} dl}{\pi (l_1 + p + d_1)} \]  
(2)

\( l_1 \) is the direct distance between the elementary capacitance and the start point. \( \varepsilon_{\text{dl}} \) is the dynamical relative permittivity, which is contributed by both air and mylar-foils. At different layers, the contributed ratio on the equivalent permittivity of the mylar foil and air are different, due to the different geometrical structure. Therefore, \( \varepsilon_{\text{dl}} \) should be related to the position of the elementary capacitance, which can be approximately presented as (3).

\[ \varepsilon_{\text{dl}} \approx \frac{p_1 + d_1 + \frac{1 + \varepsilon}{2} l_1}{l_1 + p + d_1} \]  
(3)

The equivalent static capacitance between the two sidewalls of winding and core for single winding in 2-dimension is presented in (4).

\[ C_{2d,sc} = 2\int_0^{l_{\text{winding}}} C_{\text{ele_sc}} \]  
(4)

\[ = 2\int_0^{l_{\text{winding}}} \frac{2\varepsilon_0\varepsilon_{\text{dl}}}{\pi (l_1 + p + d_1)} dl_1 \]

A coefficient two is used in (4) since there are two sidewalls in each winding.

**C.1.2 Static capacitance** between the top-surface of winding and core

Similarly, the elementary static capacitance contributed by the fringe field between the top-surface of winding and core is derived in (5) according to Fig. 5.

\[ C_{\text{ele_tc}} = \frac{\varepsilon_0\varepsilon_{\text{dl}} dl_2}{\pi l_2 + p + d_1 + t_{\text{winding}}} \]  
(5)

\( l_2 \) is the direct distance between the elementary capacitance and the start point. \( \varepsilon_{\text{dl}} \) is the dynamical relative permittivity, which can be presented as (6).

\[ \begin{align*} 
\varepsilon_{\text{dl}} &= \frac{p_1 + d_1 + \frac{1 + \varepsilon}{2} t_{\text{winding}} + \pi l_2}{p_1 + d_1 + t_{\text{winding}} + \pi l_2} \quad (l_2 \leq \frac{w_{\text{in}} - w_{\text{w}}} {2}) \\
\varepsilon_{\text{dl}} &= 1 \quad (l_2 \geq \frac{w_{\text{in}} - w_{\text{w}}} {2}) 
\end{align*} \]  
(6)

The equivalent static capacitance between the top surface of winding and core in 2-dimension is presented in (7).

\[ C_{2d,tc} = 2\int_0^{l_{\text{winding}}} C_{\text{ele_tc}} \]  
(7)

\[ = 2\int_0^{l_{\text{winding}}} \frac{\varepsilon_0\varepsilon_{\text{dl}}}{\pi l_2 + p + d_1 + t_{\text{winding}}} dl_2 \]

Due to the same voltage potential on all copper-foils, there is no parasitic capacitance between adjacent layers. The equivalent parasitic capacitance between the inner layer and core is derived as (8).

\[ \begin{align*} 
C_{2d,lc} &= \frac{\varepsilon_0\varepsilon_{\text{lc}} w_{\text{foil}}}{d_m + p_1 + d_1} \\
\varepsilon_{\text{lc}} &= \frac{d_m\varepsilon_0 + p_1 + d_m\varepsilon_{\text{w}}} {d_m + p_1 + d_1} 
\end{align*} \]  
(8)

Then, the total 2D equivalent capacitance illustrated in Fig. 4 is presented as (9).

\[ C_{2d,\text{total}} = C_{2d,sc} + C_{2d,tc} + C_{2d,lc} \]  
(9)

By substituting the same geometrical and material parameters used in Fig. 2, a comparison between the FEM simulation and calculated 2D capacitance versus the number of copper-foil layers is given in Fig. 7, where the proposed model shows a better agreement with FEM simulations than using the empirical equation (1).

**C.2. Dynamical capacitance**

The dynamical capacitance is dependent on both static capacitance and practical voltage potential disturbance. Three cases with different voltage potential distributions are considered in this paper. The dynamical capacitance between the sidewall of winding and core, between the top-surface and core are calculated, respectively.

In Fig. 8, the voltage potential on the inner layer is assumed to be 0, where the voltage potential on the outer layer is assumed to be \( V_1 \).

Case 1: Core is floating. The schematic of Case 1 is illustrated in Fig. 8(a). [14] indicates that the core potential is floated around \((0+V_1)/4\) in inductors with multiple layer structures. 0 and \( V_1 \) are the voltage potential at the terminals of the inductor.

Case 2: Core is connected to the hot layer, where the core potential is equal to 0. The schematic of Case 2 is illustrated in Fig. 8(b).

Case 3: Core is connected to the cold layer, where the core potential is equal to \( V_1 \). The schematic of Case 3 is illustrated in Fig. 8(c).
W_{sc\_case1} = \frac{1}{2} C_{eq\_sc\_case1} (V_2 - V_1)^2 \tag{12}

Similarly, the equivalent sidewall-to-core capacitance in Case 2 and Case 3 is presented as (13) and (14), respectively.

\begin{align}
W_{sc\_case2} &= 2 \int_{A_1} \frac{1}{2} \frac{2\varepsilon_{dl} \varepsilon_0}{\pi (l_i + p_i + d_i)} \left[ (V_1 - 0) - \frac{l_i}{l_{winding}} \right]^2 dl_i \\
C_{eq\_sc\_case2} &= 2 W_{sc\_case2} \left( \frac{V_1}{V_1 - V_i} \right)^2 \tag{13}
\end{align}

\begin{align}
W_{sc\_case3} &= 2 \int_{A_1} \frac{1}{2} \frac{2\varepsilon_{dl} \varepsilon_0}{\pi (l_i + p_i + d_i)} \left[ (0 - V_i) + (V_i - 0) - \frac{l_i}{l_{winding}} \right]^2 dl_i \\
C_{eq\_sc\_case3} &= 2 W_{sc\_case3} \left( \frac{V_1}{V_1 - V_i} \right)^2 \tag{14}
\end{align}

C.2.2 Dynamical capacitance between the top-surface of winding and core

In Case 1, the elementary energy stored between the top surface and core is presented as (14).

\begin{align}
dW_{sc\_case1} &= \frac{1}{2} C_{ele\_sc} \left[ \left( V_1 - \frac{0 + V_i}{4} \right) + \left( V_i - V_1 \right) - \frac{l_i}{l_{winding}} \right]^2 \\
&= \frac{1}{2} \frac{2\varepsilon_{dl} \varepsilon_0}{\pi (l_i + p_i + d_i)} \left[ \left( 0 - \frac{0 + V_i}{4} \right) + \left( V_i - 0 \right) - \frac{l_i}{l_{winding}} \right]^2 dl_i \\
&= \frac{1}{2} \frac{2\varepsilon_{dl} \varepsilon_0}{\pi (l_i + p_i + d_i)} \left[ \left( 0 - \frac{0 + V_i}{4} \right) + \left( V_i - 0 \right) - \frac{l_i}{l_{winding}} \right]^2 \tag{15}
\end{align}

If the boundary surface of the top layer in the winding is defined as A2 in 3-dimensional, the equivalent top-surface to core capacitance in Case 1 is obtained as

\begin{align}
W_{sc\_case1} &= 2 \int_{A_2} \frac{1}{2} \frac{2\varepsilon_{dl} \varepsilon_0}{\pi (l_i + p_i + d_i + l_{winding})} \left[ \left( 0 - \frac{0 + V_i}{4} \right) + \left( V_i - 0 \right) - \frac{l_i}{l_{winding}} \right]^2 dl_i \\
C_{eq\_sc\_case1} &= 2 W_{sc\_case1} \left( \frac{V_1}{V_1 - V_i} \right)^2 \tag{16}
\end{align}

Similarly, the equivalent top-surface to core capacitance in Case 2 and Case 3 are calculated as (17) and (18), respectively.
\[ W_{l,c,cased} = 2 \frac{1}{2 \pi l_1^2} \left( \frac{d_2 + d_3}{p_1 + d_1 + t_{winding}} \right) \left( V_1 - 0 \right) \left( V_1 - V_1 \right) \frac{l_2}{l_{winding}} \frac{1}{2} dl_2 \]

\[ C_{eq,l,c,cased} = \frac{2W_{l,c,cased}}{(V_1 - 0)^2} \]

\[ W_{l,c,cased} = 2 \frac{1}{2 \pi l_1^2} \left( \frac{d_2 + d_3}{p_1 + d_1 + t_{winding}} \right) \left( V_1 - V_1 \right) \frac{l_2}{l_{winding}} \frac{1}{2} dl_2 \]

\[ C_{eq,l,c,cased} = \frac{2W_{l,c,cased}}{(V_1 - 0)^2} \]

\[ \frac{v}{w} = \frac{w + d_2}{w_2} \]

\[ \frac{m}{m} = \frac{-p_1 + d_1 + p_0}{2} \]

C.2.1 Dynamical capacitance in total

To sum \( C_{eq,1} \) and \( C_{eq,2} \) in the three cases, the dynamical parasitic capacitance \( C_{eq,\text{fringe}} \) contributed by the fringe field in the single winding is presented in (19).

\[ C_{eqfringe,case1} = C_{eq,sc,case1} + C_{eq,tc,case1} \]

\[ C_{eqfringe,case2} = C_{eq,sc,case2} + C_{eq,tc,case2} \]

\[ C_{eqfringe,case3} = C_{eq,sc,case3} + C_{eq,tc,case3} \]

IV. TOTAL CAPACITANCE

Besides the fringe field capacitance, the inner layer to core capacitance, layer to layer capacitance, and winding-to-winding capacitance need to be considered in order to obtain the total capacitance of the copper-foiled inductor, where the basic principle has been introduced in [22]. However, the equations of the copper-foiled inductor are not exactly the same due to the different geometrical structures.

The equivalent inner layer to core capacitance \( C_{eq,1} \) is calculated for Case 1, Case 2, and Case 3, respectively.

\[ C_{eq,1,case1} = \frac{1}{16} \left( 1 - \frac{1}{4m} + \frac{1}{3m^2} \right) \int_{A1} \frac{C_{d_c,1}}{w} \]

\[ C_{eq,1,case2} = \frac{1}{3m} \int_{A1} C_{d_c,1} \]

\[ C_{eq,1,case3} = \frac{3m^2 - 3m + 1}{3m^2} \int_{A1} C_{d_c,1} \]

where \( C_{d_c,1} \) is the elementary capacitance between the inner layer and core, \( A1 \) is the boundary surface of the inner layer of the copper-foil inductor.

Based on the geometrical structure of the researched copper-foil inductor illustrated in Fig. 1 and the parameters are given in Table I, the equivalent permittivity \( \varepsilon_{\text{eq}} \) between the inner layer and core of the researched copper-foil inductor, which is dependent on the geometrical structure and material, is given as

\[ \varepsilon_{eq} = \frac{w_2 + (d_2 - w_2)}{w_2 + d_2} \]

\[ p_1 + d_0 + p_0 \]

Then, the equivalent permittivity used for calculating the fringe field capacitance between the sidewall of winding and core, which is fully dependent on the geometrical structures and material information of designed inductors, is given in (22), approximately.
are the equivalent permittivities in Region I and II, respectively. $C_{eq,ww}$ is the sum of the equivalent capacitance in Region I and II.

It is worth to mention that the elementary capacitances $C_{ele,lc}$, $C_{ele,ll}$, $C_{ele,ww}$ can be calculated according to [22].

The total capacitances for three cases are obtained by summing the equivalent capacitances in (19), (20), (24), and (25).

\[
\begin{align*}
C_{eq}_{\text{total}1} &= C_{eq}_{\text{fringe,case 1}} + C_{eq}_{\text{shell,case 1}} + C_{eq}_{\text{ll}} + C_{eq,\text{ww}} \\
C_{eq}_{\text{total}2} &= C_{eq}_{\text{fringe,case 2}} + C_{eq}_{\text{shell,case 2}} + C_{eq}_{\text{ll}} + C_{eq,\text{ww}} \\
C_{eq}_{\text{total}3} &= C_{eq}_{\text{fringe,case 3}} + C_{eq}_{\text{shell,case 3}} + C_{eq}_{\text{ll}} + C_{eq,\text{ww}}
\end{align*}
\]

(26)

A three-terminal equivalent circuit is illustrated in Fig. 10 for representing the copper-foil inductor.

\[P_{hot} \quad C_{u} \quad C_{lc1} \quad P_{cold}\]

Fig. 10 Three-terminal equivalent circuit to represent the copper-foil MV filter inductor

$P_{hot}$ is the terminal at the inner layer (hot layer), $P_{cold}$ is the terminal at the outer layer (cold layer). The equivalent capacitance between the two terminals is $C_u$, where the equivalent capacitance between the terminals and core are $C_{lc1}$ and $C_{lc2}$, respectively.

Based on [22], the three equivalent capacitances in Fig. 10 are calculated by

\[
\begin{align*}
C_{eq}_{\text{total}1} &= C_u + \frac{C_{lc1} C_{lc2}}{C_{lc1} + C_{lc2}} \\
C_{eq}_{\text{total}2} &= C_u + C_{lc2} \\
C_{eq}_{\text{total}3} &= C_u + C_{lc1}
\end{align*}
\]

(27)

V. MODEL VALIDATIONS

The parasitic capacitances of the copper-foil inductor introduced in Section II are analytically calculated by using the equations derived in Section IV.

A 10 kV/8 A copper-foil inductor is manufactured based on the schematic given in Fig. 1, where the pictures are presented in Fig. 11.

A. Theoretical calculations

By using the derived equations (2)-(27) and physical parameters of the copper-foil inductor, the calculated equivalent fringe field, inner layer to core, layer-to-layer, and winding-to-winding capacitances for the three different cases are listed in Table III. It is worth to mention that the calculated capacitances are only valid before the first resonant frequency of the copper-foil inductor due to the assumptions used for calculating the elementary capacitance.

Based on (27), the three-terminal equivalent circuit of the researched inductor is presented in Fig. 12. The inductance value is used as the rated value of the researched inductor.

\[
\begin{align*}
0 \text{mH} (P_{hot}) &\quad \quad 52.1 \text{pF} \\
27.8 \text{pF} \quad 30 \text{mH} (P_{cold}) &\quad \quad 13.2 \text{pF}
\end{align*}
\]

Fig. 12 Calculated three-terminal equivalent circuit of the researched copper-foil MV inductor

B. Experimental verifications

In this section, the parasitic capacitances of the copper-foil inductor are measured to verify the theoretical analysis using a Keysight E4990A impedance analyzer and its adapter 16047. Both the conventional measurement method and the guarding measurement method are used to measure the parasitic capacitance for the three different cases, where the core potential is different, as discussed in Section V. The principles of the two measurement methods are well described in [33].

Fig. 13 shows the comparisons between the calculations and measurements. The value of inductance in the The impedance obtained using the calculations is derived with the known value of the designed (rated) inductance. Fig. 13(a)-(c) is the comparison between the theoretical calculations and measured impedance using the conventional measurement method. Fig. 13(d)-(f) is the comparison between the theoretical calculations and measured impedance when using guarding technology. Since there are no damping resistors in the calculated equivalent circuit, the magnitude at the resonance point in Fig. 13(a)-(d) is infinitely high. Therefore, the frequency of the first resonant points matches well, which means the calculations are close to the measured results. In Fig.
13(e) and (f), the calculated impedance before the first resonant point (is smaller than 1MHz in this paper) is close to the measured impedance. The numerical comparisons are also given in Table IV. Overall, the theoretical calculations show good agreement with the measurements.

C. Comparisons

The modeling method introduced in [22] is used to calculate the parasitic capacitance of the same copper-foiled inductor, with the same geometrical and material parameters. The comparisons among the calculated three-terminal equivalent circuit with using the method in [22], proposed method, and two different modeling method are presented in Table IV.

For the three cases, compared to the measurements, the calculated parasitic capacitance without considering the fringe field has 15.0%, 20.2%, and 13.8% error in three cases, respectively. The errors of calculations using the proposed method in the three cases are 3.3%, 3.0%, and 8.2%, respectively. The proposed modeling method has better accuracies with considering the fringe field effect. For the calculated three-terminal equivalent circuits, it can be found that the measured capacitance between \( P_{\text{cold}} \) and \( G \) is 11.7 pF by using the guarding technology, instead of the theoretical calculations 0 pF by using the modeling method in [22], which means the modeling method [22] fails to characterize the capacitance between the terminals and core. However, this capacitance is successfully characterized by using the proposed modeling method, where the fringe field effects is important in modeling the parasitic capacitances in copper-foiled inductors.

D. Error analysis

The values of the geometrical and material parameters in the manufactured inductor cannot be the same as the designed value since the complex structure is utilized in the copper-foiled inductors. Several assumptions are made to simplify the electrical field distribution for analytically modeling the parasitic capacitance, which can introduce errors to the calculations. Besides, the measurements can also introduce errors. Especially the values can be easily changed by temperature, humidity, and so on. As can be seen from Table IV, the maximum difference between using two different measurement methods is close to 2%. However, in this article, the maximum error is observed as less than 10%, which is acceptable compared to relevant research [14], [22], [33]-[35].

VI. CONCLUSIONS

The parasitic capacitances in copper-foiled MV filter inductor paper have been modeled in this article. Besides the conventional elementary capacitances, the elementary capacitances contributed by the fringe field have been identified. A physics-based modeling method has proposed to analytically calculate the parasitic capacitance contributed by the fringe field, which is computationally efficient. A three-terminal equivalent circuit was further developed to characterize the couplings between the terminals and the core of the inductors. The measurements on the a copper-foiled inductor have been presented. The results verified the correctness of the proposed modeling method, where the parasitic capacitance between the cold terminal and ground was failed to be revealed in the conventional modeling method without considering the fringe effects.

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Fig. 13 Comparisons between the calculations and measured impedance. (a)-(c) are Case 1, Case 2 and Case 3, respectively, by using the conventional measurement method; (d)-(f) are Case 1, Case 2 and Case 3, respectively, by using the measurement method with guarding technology.

Table IV Numerical comparisons of parasitic capacitances between the measurements and calculations

| Case       | Calculations (w/o considering the fringe field [22]) | Calculations (Proposed method) | Measured capacitance (Normal measurements) | Converted capacitance based on the measurements with guarding method |
|------------|-------------------------------------------------------|--------------------------------|-------------------------------------------|-----------------------------------------------------------|
| Case 1     | 53.7 pF                                               | 61.1 pF                        | 63.2 pF                                   | 63.0 pF                                                   |
| Case 2     | 53.7 pF                                               | 65.3 pF                        | 67.3 pF                                   | 66.0 pF                                                   |
| Case 3     | 75.1 pF                                               | 80.0 pF                        | 87.1 pF                                   | 88.8 pF                                                   |
| Two-terminal equivalent circuit | P_{hot} = 53.7 pF/30 mH P_{cold} = 0 pF | P_{hot} = 61.1 pF/30 mH P_{cold} = 0 pF | P_{hot} = 63.2 pF/30 mH P_{cold} = 0 pF | P_{hot} = 63.0 pF/30 mH P_{cold} = 0 pF |
| Three-terminal equivalent circuit | P_{hot} = 54.6 pF/21.5 pF P_{cold} = 0 pF | P_{hot} = 52.1 pF/27.8 pF P_{cold} = 13.2 pF | P_{hot} = 53.3 pF/33.8 pF P_{cold} = 14.0 pF | P_{hot} = 54.3 pF/34.5 pF P_{cold} = 11.7 pF |

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