Acousto-optic figure of merit search

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Abstract

We present a method for searching for Bragg matched acousto-optic geometries that provide the largest acousto-optic figure of merit $M_2$ for both isotropic and anisotropic diffraction. We apply our method to LiNbO$_3$ and discuss the calculated results.

Keywords: acousto-optics; $M_2$; LiNbO$_3$

1. Introduction

We present a method for searching for acousto-optic (AO) interaction geometries that will maximize the diffraction efficiency of an AO device. AO devices are a versatile technology used to electronically, precisely, and rapidly control the intensity, frequency, and propagation direction of a laser beam. Applications include acousto-optic scanners, filters, mode lockers, and modulators [Xu and Stroud (1992)]. In order to maximize the diffraction efficiency of an AO device, we have developed a search method that calculates the acousto-optic figure of merit $M_2$ for every possible Bragg match AO geometry for a fixed acoustic frequency in order to plot and then identify interaction geometries that maximize the diffraction efficiency. In this paper we apply our method to lithium niobate LiNbO$_3$ and discuss our results.

2. $M_2$ calculation

We search for an AO geometry with a large diffraction efficiency by maximizing the AO figure of merit $M_2 = \frac{n_0 n_d^2 \rho}{\nu_a^3}$, where $n_0(\lambda)$ and $n_d(\lambda)$ are the index of refraction of the incident and diffracted light, $\rho$ is the medium density, $\nu_a$ is the acoustic wave velocity, and $p$ is the effective photoelastic coefficient. For LiNbO$_3$ we need to include the piezoelectric effect when calculating the acoustic velocity $\nu_a$ with the Christoffel equation. The piezoelectric effect increases the velocity of the acoustic waves. $M_2$ is proportional to the inverse of the acoustic velocity cubed $\nu_a^3$, so a faster acoustic wave greatly decreases $M_2$. We also need to include the piezoelectric effect when calculating the effective photoelastic coefficient $p$. The strain from an acoustic wave can induce a change in the electric field through the piezoelectric effect in the material. The electro-optic effect will then change the index of refraction. LiNbO$_3$ is also birefringent ($\delta n = 0.083$ at 632nm) that we must include the Nelson and Lax local rotary correction when calculating $p$. For our

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M2 calculations for LiNbO3, we use the same physical parameters as [?], who apply their own acousto-optic figure of merit search to LiNbO3.

3. Bragg-matching

In an AO devices the acoustic wave diffracts the incident light by exchanging momentum \( \vec{k}_d = \vec{k}_i \pm \vec{K}_A \), where \( \vec{K}_A \) is the acoustic wavevector, \( \vec{k}_d \) and \( \vec{k}_i \) are the diffracted and incident light wavevectors respectively, and the + and – signs correspond to Doppler up-shifting and down-shifting interactions. Momentum mismatch \( \Delta k \) of the AO interaction is often considered to arise when the wavevectors of the incident light and the acoustic wave do not sum to a wavevector that is a solution of the optical wave equation. In a finite medium, an alternative viewpoint is to ascribe a momentum uncertainty \( \Delta \hat{k} = \frac{\Delta \vec{k}}{L} \), due to the finite length of the interaction, that corresponds to the width of the distribution of Fourier components of the acoustic wave in momentum space. The diffraction efficiency scales as \( \text{Sinc}^2 \left( \frac{\Delta \hat{k} L}{\pi} \right) \) where \( L \) is the AO interaction length. The strongest diffraction occurs when the wavevectors are Bragg matched (\( \angle \Delta \hat{k} = 0 \)); thus, we want to design an AO device with an interaction geometry that ideally sets the momentum mismatch \( \Delta \hat{k} = 0 \).

We only consider Bragg matched AO geometries for our M2 calculations, and we use \( \vec{k} \)-surfaces as a geometrical tool to solve for the Bragg matched AO geometries. Conservation of energy gives a Doppler shift \( \omega_d = \omega_i \pm \Omega_A \), where \( \omega_d \), \( \omega_i \), and \( \Omega_A \) are the angular frequencies of the diffracted, incident, and acoustic waves, but negligibly effects the Bragg matching geometry.

Fig. 1a is a XZ cross-section of an uniaxial \( \vec{k} \)-surface and shows the acoustic wave \( \vec{K}_A \) deflecting the incident beam \( \vec{k}_i \) from the optical extraordinary momentum surface to the diffracted beam \( \vec{k}_d \) on the optical ordinary momentum surface. The AO interaction illustrated in Fig. 1a is perfectly Bragg matched because the acoustic wavevector \( \vec{K}_A \) and the incident wavevector \( \vec{k}_i \) sum to a diffracted wavevector \( \vec{k}_d \) that lies exactly on the diffracted \( \vec{k} \)-surface.

To solve for a Bragg matched AO interaction geometry, we first solve for the incident and diffracted \( \vec{k} \)-surfaces. Next we displace the incident \( \vec{k} \)-surface by the acoustic wavevector \( \vec{K}_A \). The intersections of the displaced incident \( \vec{k} \)-surface with the diffracted \( \vec{k} \)-surface are possible solutions for the perfectly Bragg-matched diffracted wavevector \( \vec{k}_d \). To find the incident wavevector \( \vec{k}_i \), simply subtract the acoustic wavevector \( \vec{K}_A \) from the diffracted wavevector \( \vec{k}_d \). Fig. 1b illustrates a 2D cross-section of the incident \( \vec{k} \)-surface being displaced by the acoustic wavevector \( \vec{K}_A \) and the solution for the exact Bragg-matched diffraction occurring where the displaced \( \vec{k} \)-surface intersects with the incident \( \vec{k} \)-surface.

In order to solve for the Bragg-matched geometry in three dimensions for isotropic or the ordinary mode of uniaxial crystals, we need to solve for the intersections of a sphere with a sphere. For the extraordinary optical polarization of uniaxial crystals, we need to solve for the intersection of an ellipsoid with an ellipsoid. For the very important
anisotropic polarization-switching AO interaction, we need to solve for the intersection of a sphere with an ellipsoid. Such quartic intersection problems have been solved numerically in a very general context for computer graphics [Lazard et al. (2006); Wang et al. (2002); Levin (1976)], but we have solved for the special cases of optical momentum surfaces of uniaxial crystals such as LiNbO₃ where the sphere and ellipsoid have one radius in common. Fig. 1c illustrates our solution for the intersection of a sphere with an ellipsoid displaced from the origin.

To find a Bragg-matched AO interaction, we first choose a propagation direction $\hat{l}$ of the acoustic wave. We then use the Christoffel equation to solve for the acoustic wave velocity $V_\alpha$, from which we can calculate the acoustic wavevector $\vec{K}_\alpha = \frac{2\pi}{V_\alpha} \hat{l}$, where $f$ is the chosen acoustic frequency. With the acoustic wavevector $\vec{K}_\alpha$ we can calculate all possible optical Bragg-matched geometries to solve for the incident and diffracted optical wavevectors. Once we know all of the acoustic and optical wavevectors, we can calculate the eigen-polarizations and velocities for each optical and acoustic mode as needed for an $M_2$ calculation.

In order to do an $M_2$ calculation, we first decide the propagation direction of the acoustic wave. For each direction, there will be three different acoustic velocities with their respective polarizations. Each acoustic wave can cause four different types of optical diffraction: (1) ordinary-to-ordinary, (2) extraordinary-to-extraordinary, (3) extraordinary-to-ordinary, and (4) ordinary-to-extraordinary polarization diffraction. The last two diffractions yield identical geometries, so we only consider the first three cases. Therefore, there are a total of 9 different ways to diffract the optical light for a given acoustic propagation direction, and thus 9 different $M_2$ surfaces to calculate, each a function of the polar angle, $\theta$, and azimuth angle, $\phi$, of the acoustic propagation direction unit vector $\hat{l}(\phi, \theta)$.

To find the maximum $M_2$ value, we sweep through all of the possible directions the acoustic wave can propagate in three-dimensional space using spherical coordinates. For each of the three acoustic wavevectors for a given direction, we calculate the optical wavevectors that are Bragg matched, usually in the form of the Bragg degenerate ellipse from intersecting the two surfaces, and parameterized by the optical rotation angle, $\beta$, around the acoustic $\vec{K}$-vector. We then calculate the $M_2$ values for each Bragg matched geometry and only store the maximum value obtained for any $\beta$: $M_2^{\text{max}}(\theta, \phi) = \max_\beta M_2(\theta, \phi, \beta)$. This procedure is then repeated for each of the 9 possible types of acousto-optic diffraction.

Table 1: Maximum $M_2$ values and corresponding acousto-optic geometries for a give type of diffraction in LiNbO₃ with a 500MHz acoustic wave.

| Acous. Mode | Diffraction Type | $\vec{K}_d$ | $\vec{K}_a$ | $M_2 (10^{-15} \text{s}^3/\text{kg})$ |
|-------------|------------------|-------------|-------------|---------------------------------|
| Quasi-Long. | $\hat{\alpha} \rightarrow \hat{\alpha}$ | 210.00 | 30 | 1.10 |
| Quasi-Long. | $\hat{\epsilon} \rightarrow \hat{\epsilon}$ | 149.20 | 60 | 4.47 |
| Quasi-Long. | $\hat{\epsilon} \rightarrow \hat{\alpha}$ | 323.45 | 81 | 1.20 |
| Quasi-Shear Fast | $\hat{\alpha} \rightarrow \hat{\alpha}$ | 134.44 | 87 | 6.75 |
| Quasi-Shear Fast | $\hat{\epsilon} \rightarrow \hat{\epsilon}$ | 305.21 | 67 | 7.43 |
| Quasi-Shear Fast | $\hat{\epsilon} \rightarrow \hat{\alpha}$ | 91.73 | 67 | 9.95 |
| Quasi-Shear Slow | $\hat{\alpha} \rightarrow \hat{\alpha}$ | 225.661 | 81 | 6.63 |
| Quasi-Shear Slow | $\hat{\epsilon} \rightarrow \hat{\epsilon}$ | 258.24 | 21 | 10.89 |
| Quasi-Shear Slow | $\hat{\epsilon} \rightarrow \hat{\alpha}$ | 30.00 | 30 | 16.17 |

Fig. 2 illustrates the results of our figure or merit $M_2$ search in LiNbO₃. Each surface illustrates the maximum $M_2$ value as a function of propagation direction of certain acoustic wave in polar coordinates for a certain type of diffraction. Table 1 lists the maximum $M_2$ values for each surface with the largest $M_2$ value of $10.88 \ (10^{-15} \text{s}^3/\text{kg})$ coming from the slow shear acoustic wave for extraordinary-to-extraordinary optical polarization diffraction. Notice how the maximum $M_2$ values are at complicated, off-axes angles that requiring a search method to find. With our search method, we can hopefully find AO geometries previous undiscovered to improve the performance of AO devices.

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Fig. 2: $M_2$ surfaces for different types of acousto-optic diffraction in LiNbO$_3$ for an acoustic frequency of 80MHz.

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