Conformal transformations to achieve unidirectional behavior of light

Xu Jiang\textsuperscript{1,2}, Kan Yao\textsuperscript{1}, Qiannan Wu\textsuperscript{1}, Yadong Xu\textsuperscript{1} and Huanyang Chen\textsuperscript{1,3}

\textsuperscript{1} School of Physical Science and Technology, Soochow University, Suzhou, Jiangsu 215006, People’s Republic of China
\textsuperscript{2} Department of Physics, University of Maryland, College Park, MA 20742, USA
E-mail: chy@suda.edu.cn

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Abstract. The method of optical conformal mapping is used to design two isotropic devices through which light travels in a unidirectional manner. The first device is a directional emitter. By setting a line current source in a properly tuned refractive index profile, fields can be radiated in only one direction or its opposite without using any reflector or metallic structure. The second proposal is a dual-functional device. It works not only as a directional emitter for an embedded source but also as a quasi-diode for beams, thus having potential on-chip applications. Functionalities of the two designs are verified by finite-element-based simulations. We further investigate the spatial dependence of the refractive index near singularities, and corresponding optimization is proposed in the interests of experimental consideration. Numerical results show that the one-way property is well preserved after the parameter reduction.

\textsuperscript{3} Author to whom any correspondence should be addressed.

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1. Introduction

Transformation optics (TO) [1, 2] has been a hot topic since the exciting proposal of invisibility cloaks was first demonstrated by experiment [3]. The form invariance of Maxwell’s equations allows one, in theory, to manipulate the behavior of fields or light via spatial coordinate transformation [1, 2, 4]. This powerful tool is used to design advanced devices [5–16] and explore physics outside electromagnetics/optics [6, 17–20]. On the one hand, the development of metamaterials has largely enabled the realization of TO devices, which are normally anisotropic in material and require strong magnetic response. On the other hand, however, due to the big challenge of tackling absorption and anisotropy in optical material fabrication, the early experimental results are mainly limited to the microwave regime [3, 21, 22]. To date, the most effective approach to defeating this difficulty from the theoretical side might be through the use of conformal mapping [1, 10, 12, 16] or quasi-conformal maps [5, 13]. By satisfying special conditions in transformations, the resulting materials are of less and constant anisotropy or even of strict isotropy, which leads to a series of remarkable advances in experiment at optical frequencies [23–26].

Among the applications of TO, in addition to some fancy omnidirectional devices such as cloaks, superlenses, field concentrators and rotators [1, 2, 4–7], devices that behave unidirectionally also constitute an important aspect. For instance, the design of an antenna is an actively studied issue. A fact worth noting is that metallic structures, e.g. reflectors, horns or grids, are always necessary when high-directional emission is desired [9, 10, 27–29], regardless of the dielectric components. Without metals, the instruments have only been designed to work as a multi-beam emitter [11, 12]. Can we achieve unidirectional/one-way emission, or further, behavior of light without using any reflector but only by TO? We should emphasize that such unidirectional effects here would come from the spatial asymmetry due to the coordinate transformations. In contrast, conventionally, we need to break the time-reversal symmetry of photonic crystals by applying external magnetic fields or nonlinear systems, which is referred to as nonreciprocal effects. For example, such effects can be achieved via unidirectional waveguides [30–33] and optical diodes [34–36]. In this paper, we design two isotropic devices from the method of optical conformal mapping. Both designs are aimed at achieving unidirectional behavior of light with dielectrics only. The first device is a directional emitter. Fields could only be radiated to a special direction or its opposite when a line current source is embedded at proper locations. The second element possesses two functionalities. For an embedded line current source, it acts as a directional emitter like the first one, but...
the positions of the source and the emitting direction are unique. Meanwhile, for incoming waves along the direction of emission, the device becomes a quasi-diode, which guides parallel incident beams around a region like a splitter and blocks the transmission of antiparallel incident beams. Such effects differ from nonreciprocal effects but could be more or less categorized as unidirectional effects with spatial asymmetry; see, for example, [37–41]. The performances of both instruments are verified by finite-element-based simulations. Since the refractive index profiles tend to 0 and infinity at some locations, we further investigate their spatial dependence around singularities and propose a corresponding recipe for reduction. Numerical results show that the one-way property is well preserved with optimized parameters.

2. Directional emitter

Unlike general transformations applied in the design of cloaks and embedded TO devices [2, 8], in many cases, e.g. Maxwell’s fisheye [42] and invisibility devices [1], conformal maps lead to refractive index profiles occupying the whole space. When the device is constructed to achieve exotic control of light, material singularities (points where the index profile tends to 0 or infinity) need to be included as well. Further operations usually aim at the removal of singularities [43–46], but sometimes they could also be utilized for different purposes. For instance, since the inversion of a refractive index profile does not change the number of singular points but only swaps their positions, it is worth discussing whether the new profile performs another exotic functionality.

Here we begin by considering an index profile \( n^\prime = |(z^2 - a^2)/z^2| \) derived from the Joukowsky transformation, where \( z = x + iy \) denotes the spatial coordinates of a complex plane. This profile, containing three singular points at \( z = 0 \) and \( \pm a \), was once used in the invention of invisibility devices [1]. Without loss of generality, we choose \( a = 1 \) hereafter and write the inversion as

\[
n = \frac{1}{n^\prime} = \left| \frac{z^2}{(z^2 - 1)} \right|.
\]  

(1)

The medium covers the entire plane and \( n \) tends to unity when \( z \) is far from the origin. Three singular points are maintained at \( z = \pm 1 \) and 0. To explore the behavior of light in the medium, one can try to work out the corresponding transformation based on the refractive index profile. Although this attempt might not be a general way of studying arbitrary index profiles, here we obtain such a solution for (1) by which the light ray trajectories can be rigorously predicted. According to the theory of TO, a conformal map \( w = w(z) \) yields an index profile on the \( z \)-plane as \( n = n_0 |\partial w/\partial z| \), in which \( n_0 \) is the refractive index on the \( w \)-plane. Conversely, one can work out the map \( w(z) \) by integral as long as the index distribution only depends on \( z \). Noting that equation (1) could be decomposed into \( n = |1 + 1/[2(z - 1)] - 1/[2(z + 1)]| \), the corresponding map follows:

\[
w = u + iv = z + \frac{1}{2}[\ln(z - 1) - \ln(z + 1)].
\]  

(2)

The contours of \( u \) and \( v \) on the \( z \)-plane and the \( w \)-plane are depicted in figures 1(a) and (b), respectively, where the corresponding trajectories are highlighted with the same color.

In the virtual space, namely the \( w \) plane, the dielectric medium is uniform and therefore light propagates along straight lines, such as the coordinate lines, namely the contours of \( u \) and \( v \). However, equation (2) defines a ‘unidirectional’ branch cut so that light rays propagating
Figure 1. Two complex planes, the \(z\)-plane (a) and the \(w\)-plane (b), are connected by a conformal map \(w = w(z)\) in equation (2). The dielectric medium with uniform refractive index in the \(w\)-plane (or the virtual space) is mapped to the \(z\)-plane (or the real space) as \(n = |\partial w/\partial z|\). The complicated behavior of light in the real space (a) could be ascertained from the corresponding trajectories in the virtual space (b), where light travels along straight lines, such as the coordinate lines. In \(w\) space (b), equation (2) creates a 'directional' branch cut involving three Riemann sheets. For light rays to propagate from left to right (in blue and orange) on the upper sheet and cross the branch cut (in green), they pass onto the middle sheet and keep traveling along straight lines. However, in the opposite direction, the branch cut (in red) is open only for a bundle of light rays propagating from right to left on the lower sheet (not shown here as it lies beneath), through which light (in purple) enters the middle sheet. For other cases, i.e. rays beyond the branch cut (in brown) and rays parallel to the branch cut (in cyan), light misses the entrance and is not allowed to travel between different sheets. Seen from the real space (a), the upper and lower sheets (\(w_3\) and \(w_2\)) are bounded by two touching droplet-shaped curves (the branch cuts). The rest area of the \(z\)-plane, \(w_1\), is transformed from the middle sheet. Sources at the poles in \(w_2\) and \(w_3\) emit light to the left and right, respectively. Inversely, incoming light from left or right also converges at the poles and would be absorbed there due to the singularities in refractive index, which correspond to infinities on the upper and lower Riemann sheets.

along different directions have different behavior, as shown in figure 1(b). The virtual space contains three Riemann sheets, and they connect through the branch cuts in such a way that the left upper sheet is joined to the right middle sheet, whereas the left middle sheet is joined to the right lower sheet. Therefore, in the direction perpendicular to the branch cut, light shows unidirectional behavior once crossing the cut. The conformal map in equation (2) preserves this phenomenon in the real space (the \(z\)-plane); see figure 1(a). The branch cuts in virtual space are transformed into two touching droplet-shaped curves (in green and red), enclosing the regions projected from the left upper sheet (\(w_3\)) and the right lower sheet (\(w_2\)). When a source is embedded at the right pole in \(w_3\), which corresponds to infinity on the left upper sheet, light can be only radiated to the right side. Symmetrically, the source at the left pole in \(w_2\) produces a leftward emission. We note that the device does not use any metallic component such as a reflector but only consists of dielectric media.
Figure 2. Normalized electric field distribution of the directional emitter by feeding (a) one line current source at (1, 0) and (b) two line current sources at (−1, 0) and (1, 0) with a phase difference of $\pi/2$.

To check the performance of the above directional emitter, we use the finite-element-based commercial software COMSOL Multiphysics to calculate the field distribution. In simulations, the two poles are fixed at (1, 0) and (−1, 0) according to equation (1) or equation (2) and the wavelength is chosen as $\lambda = 1$ (arbitrary units). Figure 2(a) shows the normalized electric field distribution due to a line current source embedded at the right pole (1, 0). It is seen that a strong beam is oriented to the right side despite some divergence caused by diffraction. Although the fields emitted from the source are cylindrical waves uniform in all directions, the wave vectors as well as the Poynting vectors are soon bent to the right half of the space. In figure 2(b), we add two sources with a $\pi/2$ phase difference at the two poles. The sources work independently and emit high-directional beams to the left and right in the half-spaces they locate. Furthermore, based on the principle of reversibility of light, the device also has a unidirectional blocking effect on incoming beams from left and right.

3. Dual-functional device

The transformation in equation (2) contains two logarithm functions. These logarithm terms create an exotic branch cut and further lead to the rightward and leftward directional emitting effects each. As verified and depicted in figure 2, the directional emitter optically separates the left and right half-spaces so that two sources work independently, which means one can achieve different unidirectional behavior of light along different directions by modifying the combining method of the logarithm terms. This property could provide interesting designs of
Figure 3. Light propagation in a conformal dual-functional device based on equation (3). When a line current source is embedded at the pole (surrounded by the branch cut in green), light can only travel to the right side after leaving the source (in blue) and becomes a collimated beam. In turn, an opposite beam incoming from the right side will converge at the pole and is blocked due to absorption. These functionalities illustrate half of the field pattern of the conformal directional emitter from equation (2). Moreover, for paraxial light (in red) incident from the left side, rays are split into two bundles and pass around the pole. Since light can propagate from left to right but not vice versa, the device performs as a quasi-diode in addition to as a directional emitter.

multi-functional elements. For example, we consider a simpler map,

\[ w = z + \frac{\ln z}{2} \]  \hspace{2cm} (3)

in which only one logarithm function is included. Figure 3 shows the contours of the real part and the imaginary part of equation (3) on the \( z \)-plane. Light rays propagating along coordinate lines on the \( w \)-plane will be transformed to follow these contours in the real space. Similar to figure 1(a), the logarithm function induces a pole that emits light only in one direction when embedding a line current source or inversely attracts paraxial light incident from that direction. However, on the opposite side, the device does not repeat the same performance but shows a splitting effect, which allows light to pass around the pole and approach infinity on the right side. This unidirectional passage of light makes the device look like a quasi-diode for beams besides its inherent function as a directional emitter. Although in spirit they are somewhat different, we can also comprehend the coexistence of the dual functions with the inspiration of Babinet’s principle. Based on the splitting effect, the region surrounding the pole acts as if it is an opaque body for a rightward plane wave. Therefore, to fill the shadow with a rightward beam, another incidence should be introduced in the shadow region and that is the waves emitted from the pole, which corresponds to a beam incoming from another Riemann sheet in the virtual space. The branch cut is illustrated by the green curve in figure 3.

The refractive index profile determined by equation (3) is given as

\[ n = |\partial w/\partial z| = |1 + 1/(2z)|. \]  \hspace{2cm} (4)
Figure 4. The normalized electric field distribution of the dual-functional device when (a) a line current source is embedded at (0, 0) to perform the unidirectional emitting and blocking effects and (b) a Gaussian beam is incident rightward to perform the splitting effect. The wavelengths are chosen as $\lambda = 1$ and $\lambda = 0.1$, respectively.

Using COMSOL, we calculate the normalized electric field distribution with two different schemes. In figure 4(a), a line current source is embedded at the origin, which is the pole with infinite refractive index. As shown, the cylindrical waves emitted from the origin are sharply bent to a collimated beam propagating to the right side, and correspondingly, with an opposite wave vector, a leftward beam should be guided into the pole and absorbed there \[1\]. This result confirms the contours in figure 3, and will be even better when frequency increases to satisfy the short-wavelength limit of ray optics. Figure 4(b) verifies the device’s response to a beam incoming from the opposite direction at higher frequency. As expected, a rightward Gaussian beam shows very different behavior from a leftward one. The waves are divided at the midpoint and each half is shifted sideways but keeps propagating. Therefore, this device performs dual functions. It works actively as a directional emitter and passively as a quasi-diode for beams.

4. Singularity removal

Although the above devices are strictly isotropic as a result of the conformal transformations applied, there are still some limitations in putting them into practice. The first problem is about the size, since both refractive index profiles occupy the whole space. However, noting that the indices tend to unity rapidly when getting far from the poles, one can choose a cut-off radius, which does not need to be large, to make the devices compact in size. In our cases, a cut-off around 5 is good enough (simulations not shown). The second problem is the singularities in the refractive indices at the poles for both devices, which are theoretically equal to infinity. Besides, there are also points where the indices are zero. In practice, they must be removed since real dielectric parameters are always finite. From figures 2 and 4, we can conclude that the unidirectional emitting and blocking effects seem to locally depend on the media around poles with infinite index of refraction, while the splitting effect relies on a large region containing the zero-index point (ZIP). Therefore, we first look into the spatial dependence of the profile around the infinite-index pole. Without loss of generality, we only consider the dual-functional device described by equation (3), in which the pole is located at the origin (0, 0).
Figure 5. Schematic diagrams for singularity reduction. (a) Blowup of the space around the pole at (0, 0). Contours of the real part of equation (5), which represent the geometry of wavefronts, are close to concentric circles centered at the pole, while light (in color) is emitted almost radially. The brightness of the purple background indicates the refractive index profile of the original design. (b) The singular index profile is refined by replacing the media within the pole-centered circle with radius $r_1 = 0.055$ (a.u.) by a dielectric with constant index $n_1 = 10$, which is chosen to approximately match the impedance around.

Because conformal transformations preserve angles, on the $z$-plane, as depicted in figure 3, the contours of the real part and the imaginary part of $w(z)$ in equation (3) are still orthogonal like on the $w$-plane, where they lie along the coordinate lines. If we consider one of the two sets, e.g. the contours of the imaginary part of $w(z)$, to represent the light ray trajectories, the other set, i.e. the contours of the real part, can be considered as the geometry of wavefronts. Expanding equation (3) in the polar coordinates $(r, \theta)$, we have

$$w = r \cos \theta + (\ln r)/2 + i(r \sin \theta + \theta/2).$$

When approaching the pole, the logarithm function is large and becomes the dominant term in the real part. Thus the nearby wavefronts are quasi-$r$-dependent. Figure 5(a) shows the blowup of contours around the singularity. The closed curves correspond to the real part of $w(z)$ or the wavefronts, which are almost concentric circles centered at the pole, and light rays approximately coincide with the radii in all directions. Therefore, for a source located at (0, 0), a substitution of surrounding media with a uniform inclusion will not destroy the configuration above and thereby the field distribution outside, but the singularity can be removed. By this principle, a reduced index profile using a uniform core is proposed; see figure 5(b) for details.

For the ZIP at $(-0.5, 0)$, a similar recipe is adopted. In simulations, we substitute the index profile inside the ZIP-centered circle with radius $r_2 = 0.1$ (a.u.) by a constant but near impedance-matched index $n_2 = 0.1$. The circle is larger than the pole removal case since the local reliance of the splitting effect on ZIP is not so strong. In fact, the radius $r_2$ could be chosen even larger for more achievable substitutes.

Simulations are carried out for the dual-functional device after singularity removal, as shown in figure 6. Compared with figure 4(a), although the fields distribute within a wide angle...
Figure 6. Performance of the dual-functional device with reduced index profile. The wavelengths are the same as in figure 4 for the convenience of comparison. (a) The normalized electric field distribution due to a line current source embedded at (0, 0). Although waves seem to radiate in a wide angle in the near field, in the far field, however, most energy is confined to a collimated beam in the right direction. (b) The normalized power flow pattern reflects that the directivity of emission is well preserved. The color map is saturated for the sake of contrast enhancement. (c) The rightward Gaussian beam is clearly split although the ZIP has been removed.

near the source in figure 6(a), when coming to the far field, a collimated beam could still be observed on the right side. In figure 6(b), the power flow pattern gives a clearer view of this phenomenon. On the other hand, the directional blocking effect will be more or less influenced. The inserted finite refractive index material cannot absorb leftward incident waves but scatters them back; hence most energy is still prevented from entering the left half-space. Figure 6(c) displays the splitting effect. It is seen that the field distribution is almost the same as in figure 4(b), implying that the reduced index profile does not severely affect the functionality. In short, both the unidirectional emitting and one-way properties of the device are preserved with the reduced parameters.

5. Conclusions

Based on the method of optical conformal mapping, we design two isotropic devices through which light travels in a unidirectional manner. Both elements rely on only the refractive index profiles rather than external metallic components, structures or fields. The first device is a directional emitter, through which fields radiated from two line current sources at proper locations can be confined in two opposite directions. Then, by modifying the method of transformation, we obtain the second design—a dual-functional device. It not only draws half of the radiation pattern of the first design that emits light in a unique direction with an embedded source, but also works as a quasi-diode for incoming waves. Specifically, taking the direction of emission as the reference, parallel incident beams are guided around a region via a
splitting effect, while the antiparallel incident beams are stopped from passing through. Since the refractive index profiles still contain singular values that tend to 0 and infinity, a scheme for singularity reduction is discussed and verified at last.

In this paper, the devices we propose are obtained from basic conformal maps and are therefore just prototypes to show the potential of this methodology. However, we believe that practical instruments with more feasible parameters and better performance could be achieved by utilizing advanced mathematical tools and engineering techniques.

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References

[1] Leonhardt U 2006 Optical conformal mapping Science 312 1777–80
[2] Pendry J B, Schurig D and Smith D R 2006 Controlling electromagnetic fields Science 312 1780–82
[3] Schurig D, Mock J J, Justice B J, Cummer S A, Pendry J B, Starr A F and Smith D R 2006 Metamaterial electromagnetic cloak at microwave frequencies Science 314 977–80
[4] Chen H, Chan C T and Sheng P 2010 Transformation optics and metamaterials Nature Mater. 9 387–96
[5] Li J and Pendry J B 2008 Hiding under the carpet: a new strategy of cloaking Phys. Rev. Lett. 101 203901
[6] Leonhardt U and Philbin T G 2006 General relativity in electrical engineering New J. Phys. 8 247
[7] Yan M, Yan W and Qiu M 2008 Cylindrical superlens by a coordinate transformation Phys. Rev. B 78 125113
[8] Rahm M, Cummer S A, Schurig D, Pendry J B and Smith D R 2008 Optical design of reflectionless complex media by finite embedded coordinate transformations Phys. Rev. Lett. 100 063903
[9] Tichit P-H, Burokur S N and de Lustrac A 2009 Ultradirective antenna via transformation optics J. Appl. Phys. 105 104912
[10] Yao K and Jiang X 2011 Designing feasible optical devices via conformal mapping J. Opt. Soc. Am. B 28 1137–42
[11] Jiang W X, Cui T J, Ma H F, Zhou X Y and Cheng Q 2008 Cylindrical-to-plane-wave conversion via embedded optical transformation Appl. Phys. Lett. 92 261903
[12] Schmiele M, Varma V S, Rockstuhl C and Lederer F 2010 Designing optical elements from isotropic materials Phys. Rev. A 81 033837
[13] Landy N I, Kundtz N and Smith D R 2010 Designing three-dimensional transformation optical media using quasiconformal coordinate transformations Phys. Rev. Lett. 105 193902
[14] Kwon D-H and Werner D H 2008 Polarization splitter and polarization rotator designs based on transformation optics Opt. Express 16 18731–8
[15] Xu H Y, Zhang B, Barbarastathis G and Sun H D 2011 Compact optical waveguide coupler using homogeneous uniaxial medium J. Opt. Soc. Am. B 28 2633–6
[16] Aubry A, Lei D Y, F-Domínguez A I, Sonnefraud Y, Maier S A and Pendry J B 2010 Plasmonic light-harvesting devices over the whole visible spectrum Nano Lett. 10 2574–9
[17] Genov D A, Zhang S and Zhang X 2009 Mimicking celestial mechanics in metamaterials Nature Phys. 5 687–92
[18] Chen H, Miao R-X and Li M 2010 Transformation optics that mimics the system outside a Schwarzschild black hole Opt. Express 18 15183–8
[19] Anderson T H, Mackay T G and Lakhtakia A 2010 Ray trajectories for a spinning cosmic string and a manifestation of self-cloaking Phys. Lett. A 374 4637–41
McCall M W, Favaro A, Kinsler P and Boardman A 2011 A spacetime cloak, or a history editor J. Opt. 13 024003

Chen H, Hou B, Chen S, Ao X, Wen W and Chan C T 2009 Design and experimental realization of a broadband transformation media field rotator at microwave frequencies Phys. Rev. Lett. 102 183903

Liu X, Li C, Yao K, Meng X, Feng W, Wu B and Li F 2009 Experimental verification of broadband invisibility using a cloak based on inductor–capacitor networks Appl. Phys. Lett. 95 191107

Valentine J, Li J, Zentgraf T, Bartal G and Zhang X 2009 An optical cloak made of dielectrics Nature Mater. 8 568–71

Gabrielli L H, Cardenas J, Poitrus C B and Lipson M 2009 Silicon nanostructure cloak operating at optical frequencies Nature Photonics 3 461–3

Ergin T, Stenger N, Brenner P, Pendry J B and Wegener M 2010 Three-dimensional invisibility cloak at optical wavelengths Science 328 337–9

Zentgraf T, Liu Y, Mikkelsen M H, Valentine J and Zhang X 2011 Plasmonic Luneburg and Eaton lenses Nature Nanotechnol. 6 151–5

Enoch S, Tayeb G, Sabouroux P, Guérin N and Vincent P 2002 A metamaterial for directive emission Phys. Rev. Lett. 89 213902

Bonod N, Devilez A, Rolly B, Bidault S and Stout B 2010 Ultracompact and unidirectional metallic antennas Phys. Rev. B 82 115429

Shegai T, Miljković V D, Bao K, Xu H, Nordlander P, Johansson P and Käll M 2011 Unidirectional broadband light emission from supported plasmonic nanowires Nano Lett. 11 706–11

Haldane F D M and Raghu S 2008 Possible realization of directional optical waveguides in photonic crystals with broken time-reversal symmetry Phys. Rev. Lett. 100 013904

Wang Z, Chong Y, Joannopoulos J D and Soljačić M 2009 Observation of unidirectional backscattering-immune topological electromagnetic states Nature 461 772–5

Yu Z, Veronis G, Wang Z and Fan S 2008 One-way electromagnetic waveguide formed at the interface between a plasmonic metal under a static magnetic field and a photonic crystal Phys. Rev. Lett. 100 023902

Liu S, Lu W, Lin Z and Chui S T 2010 Magnetically controllable unidirectional electromagnetic waveguiding devices designed with metamaterials Appl. Phys. Lett. 97 201113

Ramezani H, Kottos T, El-Ganainy R and Christodoulides D N 2010 Unidirectional nonlinear PT-symmetric optical structures Phys. Rev A 82 043803

Biancalana F 2008 All-optical diode action with quasiperiodic photonic crystals J. Appl. Phys. 104 093113

Lepri S and Casati G 2011 Asymmetric wave propagation in nonlinear systems Phys. Rev. Lett. 106 164101

Wang C, Zhou C-Z and Li Z-Y 2011 On-chip optical diode based on silicon photonic crystal heterojunctions Opt. Express 19 26948–55

Feng L, Ayache M, Huang J, Xu Y-L, Lu M-H, Chen Y-F, Fainman Y and Scherer A 2011 Nonreciprocal light propagation in a silicon photonic circuit Science 333 729–33

Fan S et al 2012 Comment on ‘Nonreciprocal light propagation in a silicon photonic circuit’ Science 335 38b

Feng L, Ayache M, Huang J, Xu Y-L, Lu M-H, Chen Y-F, Fainman Y and Scherer A 2012 Response to comment on ‘Nonreciprocal light propagation in a silicon photonic circuit’ Science 335 38c

Aouani H, Mahboub O, Bonod N, Devaux E, Popov E, Rigneault H, Ebbesen T W and Wenger J 2011 Bright unidirectional fluorescence emission of molecules in a nanoaperture with plasmonic corrugations Nano Lett. 11 637–44

Luneburg R K 1964 Mathematical Theory of Optics (Berkeley, CA: University of California Press) pp 172–82
[43] Tyc T and Leonhardt U 2008 Transmutation of singularities in optical instruments New J. Phys. 10 115038
[44] Ma Y G, Ong C K, Tyc T and Leonhardt U 2009 An omnidirectional retroreflector based on the transmutation of dielectric singularities Nature Mater. 8 639
[45] Danner A J 2010 Singularity removal in optical instruments without reflections or induced birefringence New J. Phys. 12 113008
[46] Perczel J, Garcia-Meca C and Leonhardt U 2011 Partial transmutation of singularities in optical instruments, J. Opt. 13 075103