Corrigendum: Dynamical nuclear spin polarization induced by electronic current through double quantum dots (2012 New J. Phys. 13 053010)

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We have recently discovered that there is a hidden conservation law in our rate equations (equations (15) and (17)) used to describe the electronic transport through a double quantum dot coupled to a nuclear spin environment. This implies that the obtained dynamical nuclear polarization (DNP) depends on the initial occupations of the double quantum dot as some of the authors recently explained [1] (see section V.A). Therefore, our rate equation description in the paper breaks down.

In fact, if hyperfine interaction is the only spin relaxation mechanism causing the leakage current in a spin-blockade configuration, then a finite DNP cannot build up. The reason is as follows. Hyperfine interaction can lift a spin-blockade by producing transitions from the blocking triplet states \( T_+ = \uparrow \uparrow \) and \( T_- = \downarrow \downarrow \). These transitions will polarize the nuclei in opposite directions and therefore, on average, the nuclear polarization does not change, since the rates of tunneling into the \( T_\pm \) states are equal. This is true even if the escape rates from \( T_\pm \) are very different. However, if the hyperfine interaction is competing with one or more alternative ways to escape from the triplet states, then a finite DNP can build up, as pointed out in [1–4]. In other words, a finite DNP can occur if more than one spin-relaxation mechanism contributes to the leakage current.

Now, in order to avoid the hidden conservation law and the following breakdown of our rate equation description, we should add an additional escape rate from the triplets in our rate equations.

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equations. Therefore, equation (15) in our paper should be replaced by

\[ \dot{\rho}_\nu = -\rho_\nu \sum_{\nu'} \left( W_{\nu',\nu} + \Gamma_{\nu',\nu} + \Gamma^{\text{ine}}_{\nu',\nu} \right) + \sum_{\nu'} \left( W_{\nu,\nu'} + \Gamma_{\nu,\nu'} + \Gamma^{\text{ine}}_{\nu,\nu'} \right) \rho_{\nu'} \]  

(1)

where \( \dot{\rho}_\nu \) denotes the time derivative of the occupation \( \rho_\nu \) of the double quantum dot states \( \nu = \{ \uparrow \downarrow, \downarrow \uparrow, \uparrow \uparrow, \downarrow \downarrow \} \). Here the additional inelastic rates \( \Gamma^{\text{ine}}_{\nu,\nu'} \) could be provided by cotunneling or spin–orbit mediated spin relaxation. Moreover, \( W_{\nu,\nu'} \) describes the hyperfine induced transition rates and \( \Gamma_{\nu,\nu'} \) are the tunneling rates. Further details are provided in [1] (see equations (9) and (A1)). Furthermore, the polarization rate equation (17) is correct apart from an overall spin-flip normalization factor (see equation (10) in [1]). The details of the double quantum dot modeling (section 2.1), the hyperfine rates and the tunneling rates (section 2.3) in the paper are not affected by the rate equation description breakdown and thereby remain correct.

The results of studying the corrected rate equations with a constant extra inelastic escape rate are provided in [1], where the same hyperfine and tunneling rates as in our paper are used. Furthermore, an alternative simplified model including the inelastic rates can be found in section III.A of [1], which replaces the considerations in section 2.1.1 of this paper.

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References

[1] Lunde A M, López-Monís C, Vasiliadou I A, Bonilla L L and Platero G 2013 Phys Rev. B 88 035317
[2] Rudner M S and Levitov L S 2007 Phys. Rev. Lett. 99 036602
[3] Pfund A, Shorubalko I, Ensslin K and Leturcq R 2007 Phys. Rev. Lett. 99 036801
[4] Rudner M S and Levitov L S 2010 Nanotechnology 21 274016
Dynamical nuclear spin polarization induced by electronic current through double quantum dots

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Abstract. We analyse electron-spin relaxation in electronic transport through coherently coupled double quantum dots (DQDs) in the spin blockade regime. In particular, we focus on hyperfine (HF) interaction as the spin-relaxation mechanism. We pay special attention to the effect of the dynamical nuclear spin polarization induced by the electronic current on the nuclear environment. We discuss the behaviour of the electronic current and the induced nuclear spin polarization versus an external magnetic field for different HF coupling intensities and interdot tunnelling strengths. We take into account, for each magnetic field, all HF-mediated spin-relaxation processes coming from different opposite spin level approaches. We find that the current as a function of the external magnetic field shows a peak or a dip and that the transition from a current dip to a current peak behaviour is obtained by decreasing the HF coupling or by increasing the interdot tunnelling strength. We give a physical picture in terms of the interplay between the electrons tunnelling out of the DQD and the spin-flip processes due to the nuclear environment.

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1. Introduction

In the last decade, solid-state spintronics and quantum computing have experienced great development. In particular, quantum dots (solid-state fabricated zero-dimensional devices) have been widely investigated both experimentally and theoretically. For quantum computing and quantum information they have become major candidates not only for implementing quantum bit units \([1]\) (qubits), but also from a fundamental point of view; since quantum dots resemble artificial atoms, they are highly interesting systems for studying basic atomic physics. In this context, spin decoherence and relaxation are among the most desired to be understood mechanisms since they represent the main sources of quantum computing errors.

We investigate spin relaxation in double quantum dots (DQDs). Spin blockade (SB) \([2]\) is a very suitable regime for attempting a single electron-spin manipulation, because two electrons are trapped in a DQD, since the Pauli exclusion principle avoids electron transport through the dots. Moreover, SB is attainable through transport experiments in DQDs in several materials. However, spin-relaxation processes can partially destroy the SB releasing the trapped electrons, leading to a small, but still measurable leakage current (\(\sim pA–fA\)) \([2]–[6]\). Spin–orbit coupling \([7]\), cotunnelling \([8, 9]\) and hyperfine (HF) interaction between the DQD electrons and the surrounding nuclei spins of the host material represent the main mechanisms of spin relaxation. Depending on the material, one or more mechanisms may be involved collaborating or competing \([3, 4, 6]\).

In this paper, we present a theoretical study of spin relaxation in a DQD in the SB regime due to HF interaction with lattice nuclei spins. Over the last few years this has been a very active field both experimentally \([3, 4, 6], [10]–[17]\) and theoretically \([18]–[23]\). We pay special attention to the dynamical nuclear spin polarization induced by the electronic leakage current \([13, 18, 22, 24, 25]\) emerging from the spin–relaxation transitions (figure 1). This effect is often not taken into account when studying the current through the DQD in the SB regime.

In the present work, we calculate both the electronic leakage current and the nuclei spin polarization induced by electrons tunnelling through the DQD. In the SB regime, two electrons in the DQD can be in either a triplet or a singlet state. Nevertheless, current is allowed to pass through the DQD only when they are in a singlet; otherwise they remain trapped in a triplet state. HF interaction mixes triplet and singlet subspaces and thereby lifts SB. The mixing is due...
(a) A coherently coupled DQD is attached through tunnelling barriers to leads and to the surrounding nuclei of the host material. (b) Transport scheme. Electrons tunnel from the left lead into the DQD states (solid blue arrows online). When electrons fall into the singlet states ($S_\pm$), tunnelling out of the DQD to the right lead is allowed (solid blue arrows online). However, when electrons fall into a triplet state ($T_\pm$) the Pauli exclusion principle prevents them from tunnelling out of the DQD into the right lead; thus, SB occurs. Finally, if electrons tunnel into the $T_x$ state (a mixture of a singlet state and the antiparallel spins triplet $T_0$), they can tunnel out of the DQD to the right lead only if there is a net nuclear spin polarization (dashed blue arrow online); otherwise $T_x$ becomes $T_0$, which is also a blocked state. Electrons trapped in $T_\pm$ states interact with the nuclei and relax to a singlet state or to the $T_x$ state (solid red arrows online) through spin-flip processes. However, electrons in $T_x$ state can also spin-flip back to a SB state (red dashed arrows online). Therefore, when electrons are in the $T_x$ state there is a competition between the tunnelling rate from the $T_x$ to the right lead, and the spin-flip rate from $T_x$ to the $T_\pm$ triplets (a detailed discussion is given in section 2). We will show that this competition will give rise to different physical features in the tunnelling current as a function of an external magnetic field (see section 3).

In previous works [22, 24, 25], interdot tunnel coupling between the quantum dots was considered incoherent, namely, the system was assumed to be in the sequential interdot tunnelling regime. However, in the present work we focus on the resonant tunnel regime and we consider coherently coupled quantum dots (figure 1(a)). We have found in this case different behaviour for both the current and the induced nuclear spin polarization with respect to previous works. In the coherent coupling regime, in addition to the two spin parallel triplets, there is a spin antiparallel triplet. Thus, in addition to the spin-flip transitions between the singlet and triplet states, spin-flip processes between triplet states are also present and contribute to the nuclear spin polarization and the electronic leakage current. Furthermore, in the present model to different HF interaction strengths, i.e. two different effective magnetic fields (the Overhauser fields induced on electrons by the nuclei), within each dot. In addition, the scattering processes between electron and nuclei spins that lead to spin relaxation, also induce a non-negligible nuclear spin polarization as the current flows through the DQD. Moreover, this induced nuclear spin polarization itself, as we shall see below, modifies the mixing between the triplet and singlet subspaces, acting back on the electronic current through the DQD. Furthermore, we find that the Overhauser field, proportional to the electron current-induced nuclear spin polarization, is in general larger than the one obtained when just considering HF interaction as a random stationary magnetic field acting on the electron spins.
we find that the transition rates between the DQD and the leads depend on the nuclear spin polarization (figure 1(b)). This effect will lead to new features in the current through the DQD.

This paper is organized as follows. The Hamiltonian of the DQD coupled to the leads and the surrounding nuclei spins, and the rate equations for the occupation of DQD levels and for the nuclei spin polarization, are considered in section 2. The results and discussions are considered in section 3. Finally, the conclusions are presented in section 4.

2. Model

2.1. Hamiltonian and double quantum dot eigenstates

The system we investigate is a DQD coupled to two uncorrelated electron reservoirs (leads) and to the nuclei spins of the surrounding host material (figure 1(a)). We consider a spin-up and a spin-down level in each dot. The Hamiltonian is the following:

\[ \hat{H} = \hat{H}_{\text{DQD}} + \hat{H}_{\text{leads}} + \hat{V}_{LR} + \hat{V}_T + \hat{V}_\text{HF}, \]  

(1)

where \( \hat{H}_{\text{DQD}} \) and \( \hat{H}_{\text{leads}} \) are the Hamiltonians of the isolated DQD and the electron reservoirs, respectively, \( \hat{V}_{LR} \) is the inter-dot tunnelling Hamiltonian and \( \hat{V}_T \) and \( \hat{V}_\text{HF} \) correspond to the DQD coupling with the leads and the HF interaction with the nuclei spins, respectively. We neglect cotunnelling processes because we consider that the tunnelling coupling through the contact barriers is much smaller than the thermal energy and the bias voltage [26]. Moreover, the energy of the DQD levels is strongly detuned with respect to the chemical potential of the contacts [8, 9].

We consider contact HF interaction, as we assume electronic wave functions with \( s \)-like symmetry [27]. \( \hat{H}_{\text{DQD}} \) and \( \hat{V}_{LR} \) are

\[ \hat{H}_{\text{DQD}} = \sum_{l=1}^{N} \sum_{\sigma} \epsilon_{l\sigma} \hat{n}_{l\sigma} + \sum_{l} U_{l} \hat{n}_{l\uparrow} \hat{n}_{l\downarrow} + U_{LR} \sum_{\sigma} \hat{n}_{L\sigma} \hat{n}_{R\sigma} + \sum_{l} g \mu_B B_{\text{ext}} \hat{S}_l \cdot \hat{S}_l, \]  
\[ \hat{V}_{LR} = \sum_{\sigma} \left( \hat{\gamma}_{L,\sigma} \hat{\gamma}_{R,\sigma} + \text{h.c.} \right), \]

where \( l = L \) (left dot), \( R \) (right dot) and \( \sigma = \uparrow, \downarrow \). \( \hat{\gamma}_{l,\sigma} \) (\( \hat{\gamma}_{l,\sigma}^\dagger \)) creates (annihilates) an electron with spin \( \sigma \) and energy \( \epsilon_l \) in the \( l \)-th dot. \( \hat{n}_{l\sigma} = \hat{\gamma}_{l,\sigma}^\dagger \hat{\gamma}_{l,\sigma} \) is the occupation number operator and \( \hat{S}_l \) the electron-spin operator. \( U_l \) (\( U_{LR} \)) is the intradot (interdot) Coulomb interaction, \( B_{\text{ext}} \) the external magnetic field and \( \gamma_{LR} \) the tunnelling matrix element between the dots. We do not focus on any particular material; thus we take \( g = 2 \). \( \hat{H}_{\text{leads}} \) and \( \hat{V}_T \) are

\[ \hat{H}_{\text{leads}} = \sum_{l=1}^{N} \sum_{\sigma} \epsilon_{l\sigma} \hat{c}_{l,\sigma}^\dagger \hat{c}_{l,\sigma}, \]
\[ \hat{V}_T = \sum_{l=1}^{N} \left( \hat{\gamma}_{l,\sigma} \hat{\gamma}_{l,\sigma}^\dagger + \text{h.c.} \right), \]

(2)

where \( \hat{c}_{l,\sigma} \) (\( \hat{c}_{l,\sigma}^\dagger \)) creates (annihilates) an electron in the \( l \)-th lead with momentum \( k \), spin \( \sigma \) and energy \( \epsilon_{l\sigma} \). \( \gamma_{l} \) are the tunnelling matrix elements between the dots and the contacts. Finally, the HF interaction term \( \hat{V}_\text{HF} \) is

\[ \hat{V}_\text{HF} = \sum_{l=L,R} \sum_{i=1}^{N} A_{l,i} \hat{S}_l \cdot \hat{S}_i, \]

(3)
where \( A_i = v A |\Psi_0^i(r_i)|^2 \) is the HF coupling [28] between the \( i \)th nuclei spin \( \hat{l}_i \) at site \( r_i \) and the electron spin \( \hat{S}_i \), where \( v \) is the volume of a unit cell containing one nuclear spin, \( A \) characterizes the HF coupling strength and \( \Psi_0^i(r_i) \) is the single-particle electronic wave function for dot \( i \), evaluated at site \( r_i \). For simplicity, we only consider spin-\( \frac{1}{2} \) nuclear species. The HF couplings for the left and right dots depend on the square modulus of the electronic wave functions at the position of the nuclei; therefore, for realistic dots they are in principle different for each dot [29, 30]. We consider the case of homogeneous HF couplings [31] within each dot; hence, \( A_i = A_i/N \), where \( N \) is the total number of nuclear spins, and we split \( \hat{V}_{HF} \) (see (4)) into the following two terms:

\[
\hat{V}_{HF} = \sum_{l=L,R} A_i \sum_{i=1}^N \left( \frac{1}{2} (\hat{S}_{i+} \hat{l}_{i-} + \hat{S}_{i-} \hat{l}_{i+}) + \hat{S}_{iz} \hat{l}_{iz} \right) = \hat{V}_{sf} + \hat{V}_{z},
\]

where \( \hat{S}_{iz} = \hat{S}_{i} \pm \hat{l} \hat{S}_y \) and \( \hat{l}_{iz} = \hat{l}_z \pm \hat{i} \hat{l}_y \) are the raising and lowering operators for the electron and the nuclei spins, respectively. \( \hat{V}_{sf} \) corresponds to the \( x \)-\( y \)-components, which are perpendicular to the external magnetic field and thus is responsible for the flip-flop transitions between electron and nuclei spins; and \( \hat{V}_{z} \) corresponds to the \( z \)-component, which is parallel to the external field and, hence, contributes to the Zeeman splitting, as we will see below. We treat these two terms separately. For \( \hat{V}_{z} \) we perform a mean-field approximation that gives [22]

\[
\hat{V}_{z} \rightarrow \hat{V}_{z}^{MF} = \frac{1}{2} \sum_{l} A_i \hat{S}_{iz} P,
\]

where \( P = (\langle N^z_i \rangle - \langle N^z_i \rangle) / N \) is the net nuclear spin polarization and \( \langle N^z_i \rangle \) is the average number of spin up (down) nuclei. \( \hat{V}_{z}^{MF} \) is now equivalent to an effective magnetic field within each dot induced by the nuclei on the electrons (the Overhauser field), and proportional to the nuclear polarization given by

\[
P_{\text{nuc}}^{1(R)} = \frac{A_{L(R)} P}{2g \mu_B}.
\]

This effective field is in general different for each dot [19] and gives rise to an effective Zeeman splitting within each dot that adds to the one produced by the external field. Below we shall see that this inhomogeneity is essential for lifting SB [3]. We consider \( \hat{V}_{T} \) and \( \hat{V}_{sf} \) as perturbations. Thus, the unperturbed Hamiltonian of the isolated DQD is the following:

\[
\hat{H}_0 = \hat{H}_{DQD} + \hat{V}_{LR} + \hat{V}_{z}^{MF}.
\]

SB occurs when the source–drain voltage is tuned so that the number of electrons in the DQD varies between one and two. The right quantum dot is always occupied with one electron, whereas another electron can tunnel from the source to the drain through the DQD. The transport scheme is the following:

\[
(0, \sigma) \rightarrow (\sigma', \sigma) \rightarrow \begin{cases} (0, \uparrow \downarrow) \quad \text{if} \quad \sigma \neq \sigma', \\ (\sigma, \sigma) \quad \text{(SB)}, \quad \text{if} \quad \sigma = \sigma', \end{cases}
\]

where, in \((n, m)\), the variable \( n \) \((m)\) accounts for the population of the left (right) dot level, and \( \sigma, \sigma', \sigma'' = \uparrow, \downarrow \). Inter-dot tunnelling (\( \hat{V}_{LR} \)) is allowed only between states with the same total spin. Since the state \((0, \uparrow \downarrow)\) has total spin zero and the states with \( \sigma = \sigma' \) have total spin one, the transitions \((\sigma, \sigma) \rightarrow (0, \uparrow \downarrow)\) are forbidden. The Hilbert space that we have considered consists of the \textit{atomic basis} \{\(0, \uparrow\), \(0, \downarrow\), \(|\uparrow, \downarrow\), \(|\downarrow, \uparrow\), \(|\uparrow, \uparrow\), \(|\downarrow, \downarrow\), \(|0, \uparrow \downarrow\}\}. We investigate the

\[\text{http://www.njp.org/}\]
coherent resonant transport regime. In this regime, the energies of the DQD two-electron states \( |\sigma, \sigma'\rangle \) and \( |0, \uparrow \downarrow\rangle \) are degenerate in the absence of a magnetic field, i.e. the so-called zero detuning regime. In this case \( \hat{H}_0 \) (see (7)) is exactly diagonalizable. Its eigenenergies and eigenstates are the following:

\[
E_{T_\pm} = \epsilon_L + \epsilon_R + U_{LR} \\
E_{S_\pm} = \epsilon_L + \epsilon_R + U_{LR} \pm \sqrt{2} N t_{LR} \\
E_{T_x} = \epsilon_L + \epsilon_R + U_{LR} \pm \left( g \mu_B B_{\text{ext}} + \frac{A_+}{2} P \right)
\]

and

\[
|T_\pm\rangle = |\uparrow, \uparrow\rangle, \\
|T_-\rangle = |\downarrow, \downarrow\rangle, \\
|T_x\rangle = \frac{1}{\sqrt{N}} (|T_0\rangle - x |S_{02}\rangle), \\
|S_\pm\rangle = \frac{1}{\sqrt{2}} \left( |S_{11}\rangle \pm \frac{1}{\sqrt{N}} (|S_{02}\rangle + x |T_0\rangle) \right),
\]

where

\[
|T_0\rangle = \frac{1}{\sqrt{2}} (|\uparrow, \downarrow\rangle + |\downarrow, \uparrow\rangle), \\
|S_{11}\rangle = \frac{1}{\sqrt{2}} (|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle), \\
|S_{02}\rangle = |0, \uparrow \downarrow\rangle,
\]

being

\[
N = \sqrt{1 + x^2}, \\
x = \frac{1}{\sqrt{2}} \frac{A_+ P}{2 t_{LR}}, \\
A_\pm = \frac{1}{2} (A_L \pm A_R).
\]

Thus, the two-electron molecular basis is \( \{ |\pm\rangle, |S_\pm\rangle, |T_x\rangle, |T_\pm\rangle \} \), where \(|\pm\rangle = |0, \uparrow (\downarrow)\rangle\) are the single-electron states. Equation (10) shows that \( \hat{V}_{LR} \) mixes \( S_{11} \) and \( S_{02} \) singlets, and \( \hat{V}_{z}^{\text{MF}} \) mixes the \( T_0 \) triplet with \( S_{11} \) and \( S_{02} \) singlets when \( A_L \neq A_R \). The singlet–triplet (ST) mixing is given by the weight \( x/N \). This quantity is the ratio between the Zeeman splitting difference within each dot (\( A_- P/2 \)) and the exchange energy defined as \( |E_{S_\pm} - E_{T_x}| = \sqrt{2} N t_{LR} \). Therefore, the ST mixing depends on the competition between these two energy scales. A large (small) difference between the HF-coupling intensities and small (large) interdot tunnelling strength increases (decreases) the ST mixing (see (10) and (12)). Furthermore, \( x \) is zero when either the HF couplings have the same value for both dots (\( A_L = A_R \)) or the nuclei spins are completely depolarized (\( P = 0 \)). In both cases the usual ST basis is recovered. Due to the mixing with the \( T_0 \) triplet state, \( S_\pm \) are not pure singlet states anymore; however, for simplicity we shall continue calling them singlet states. Finally, note that now the eigenenergies of singlet states also depend on the nuclear spin polarization through \( N \) (see (9) and (12)).
The transport scheme in the molecular basis is the following:

\[
\begin{align*}
(0, \sigma) & \rightarrow \begin{cases} 
S_\pm & \rightarrow (0, \sigma'), \\
T_x & \rightarrow \begin{cases} 
(0, \sigma'), & \text{if } x \neq 0, \\
T_0 (SB), & \text{if } x = 0,
\end{cases} \\
T_\pm (SB).
\end{cases}
\end{align*}
\] (13)

This transport scheme shows that for the coherent interdot tunnelling regime, when having different Overhauser fields in the dots \((x \neq 0)\), an additional current channel \((0, \sigma) \rightarrow T_x \rightarrow (0, \sigma')\) is opened. Therefore, there are two possible situations: (i) \(x = 0\). In this case the incoming electron will fall either in a singlet state \((S_\pm)\) or in a triplet state \((T_0 \text{ or } T_\pm)\). Thus, there are two transport channels and three SB states; (ii) \(x \neq 0\). The incoming electron will fall either in a singlet state \((S_\pm)\), the \(T_x\) state, or in the \(T_\pm\) triplets. Therefore, now there are three transport channels and two SB states. However, once the electrons fall into a SB state, the current drops to zero.

2.2. Rate equations

The Hamiltonian (1) can be written now as follows:

\[
\hat{H} = \hat{H}_0 + \hat{V}_T + \hat{V}_{sf} + \hat{H}_{\text{leads}},
\] (14)

where the eigenstates of \(\hat{H}_0\) (see (10)) are the unperturbed states. \(\hat{V}_T\) induces transitions between the leads and the DQD, namely between one-electron and two-electron states. \(\hat{V}_{sf}\) is responsible for the spin flip-flop transitions between the DQD electron spins and the surrounding nuclei spins. The time evolution of the DQD molecular states is obtained with the following rate equations:

\[
\begin{align*}
\dot{\rho}_{T_\pm} &= W_{T_\pm,s_i} \rho_{s_i} + W_{T_\pm,t_i} \rho_{t_i} + \Gamma_{T_\pm} \rho_{T_\pm} - (W_{s_i,T_\pm} + W_{s_i,T_\pm} + W_{T_\pm,T_\pm}) \rho_{T_\pm}, \\
\dot{\rho}_{T_x} &= W_{T_x,s_i} \rho_{s_i} + W_{T_x,t_i} \rho_{t_i} + \Gamma_{T_x} \rho_{T_x} - (W_{s_i,T_x} + W_{s_i,T_x} + W_{T_x,T_x} + \Gamma_{T_x} + \Gamma_{T_x}) \rho_{T_x}, \\
\dot{\rho}_{S_\pm} &= W_{S_\pm,s_i} \rho_{s_i} + W_{S_\pm,t_i} \rho_{t_i} + \Gamma_{S_\pm} \rho_{S_\pm} - (W_{s_i,S_\pm} + W_{S_\pm,S_\pm} + \Gamma_{S_\pm} + \Gamma_{S_\pm} + \Gamma_{S_\pm}) \rho_{S_\pm}, \\
\dot{\rho}_\pm &= \Gamma_{\pm,s_i} \rho_{s_i} + \Gamma_{\pm,t_i} \rho_{t_i} - (\Gamma_{s_i,\pm} + \Gamma_{s_i,\pm} + \Gamma_{t_i,\pm} + \Gamma_{t_i,\pm}) \rho_{\pm},
\end{align*}
\] (15)

where \(\rho_i\) is the occupation of the \(i\)th state. \(\Gamma_{i,f} (W_{i,f})\) are the tunnelling (spin-flip) rates between an initial DQD state \(|i\rangle\) and a final DQD molecular state \(|f\rangle\). Both the tunnelling and the spin-flip rates are computed in section 2.3.

The electron–nuclei spin flip-flop processes induce a non-negligible nuclear spin polarization, which will be positive or negative depending on the specific spin-flip process. In the following scheme, we show the spin-flip processes that contribute to each nuclear spin polarization direction:

\[
\begin{align*}
T_x \rightarrow \{S_+, S_-, T_x\} & \Rightarrow \dot{P} > 0, \\
T_\pm \rightarrow \{S_+, S_-, T_x\} & \Rightarrow \dot{P} < 0, \\
\{S_+, S_-, T_\pm\} \rightarrow T_\pm & \Rightarrow \dot{P} < 0, \\
\{S_+, S_-, T_x\} \rightarrow T_\pm & \Rightarrow \dot{P} > 0.
\end{align*}
\] (16)

For example, the process \(T_x \rightarrow \{S_+, S_-, T_x\}\) flips down an electron spin and up a nuclear spin and thus polarizes positively the nuclei spins. Furthermore, the nuclear spin polarization becomes dynamical due to the electrons tunnelling through the DQD. Therefore, we describe
the time evolution of the induced nuclear spin polarization using the following rate equation:

$$
\dot{P} = (W_{T..S_+} - W_{T..S_-}) \rho_{S+} + (W_{T..S_-} - W_{T..S_+}) \rho_{S-} + (W_{T..T_+} - W_{T..T_-}) \rho_{T_+} + (W_{T..T_-} + W_{S..T_+} + W_{T..T_+} - (W_{S..T_-} + W_{S..T_-} + W_{T..T_-}) \rho_{T_-} - W_{\text{rel}} P,
$$

(17)

where $W_{\text{rel}}$ is a phenomenological rate that accounts for the nuclear dipole–dipole spin interaction that is responsible for nuclear spin depolarization. In this equation, we have assumed that in the absence of spin-flip processes ($W_{f,i}$ all zero), the nuclei spins completely depolarize, namely that the temperature is much larger than nuclei spin level splittings. As will be shown in section 2.3, both the tunnelling and the spin-flip rates depend on nuclear spin polarization. Therefore, (15) and (17) form a set of eight nonlinear equations that we must solve numerically (section 3).

2.2.1. Reduced model. In order to get physical insights into the results that will be discussed in section 3, we have developed a simplified model for the rate equations (15) and (17) through the following assumptions: (i) we consider that spin-flip processes are effective when the electrons are in an SB state. Thus, once the electrons are in a singlet state it is much more probable for them to tunnel out of the DQD than to flip their spins to a $T_\pm$ triplet state. Therefore, neglect the singlet-to-triplet spin-flip transitions ($W_{T..S_\pm}$ and $W_{T..T_\pm}$). (ii) The singlet states and the one-electron states empty much faster than the $T_\pm$ triplets (SB) and the $T_\pm$ state. Thus, in the time scale at which spin-flip transitions are relevant, we assume that the occupation of the one-electron state and that of the singlet states have reached their stationary value, namely $\dot{\rho}_\pm \approx \dot{\rho}_{S\pm} \approx 0$. (iii) The nuclei relaxation time due to the spin dipole–dipole interaction is large enough to be neglected ($W_{\text{rel}} \to 0$). Under these conditions, the rate equation for the nuclei spin polarization (17) is related to the rate equations for the triplet states (15) through the relation $\dot{\rho}_{T_-} - \dot{\rho}_{T_+} = P$; thus, $\rho_{T_-} - \rho_{T_+} = P$ (where we consider $\rho_{T_-}(0) - \rho_{T_+}(0) - P(0) = 0$ and $\rho_{T_+}(0) = \rho_{S_+}(0) = 0$ as the initial conditions). Taking these considerations into account, the rate equations (15) become

$$
\dot{\rho}_T = (2\omega_T^I + \beta) \rho_{T_+} - (\omega_T^I + \omega_T^I) \rho_T + (\omega_T^I + \omega_T^I) P
$$

(18a)

$$
\dot{\rho}_{T_+} = (\omega_T^I + \omega_T^I) \rho_T - (\omega_T^I + \omega_T^I) P - (2\omega_T^I + \beta) \rho_{T_+},
$$

(18b)

$$
\dot{P} = 2\omega_T^I \rho_{T_+} + (\omega_T^I + \omega_T^I) \rho_T - (\omega_T^I + \omega_T^I) P,
$$

(18c)

where $\rho_T = \rho_{T_+} + \rho_{T_-}$ and

$$
\omega_T^I = \frac{1}{2}(W_{T..T_+} \pm W_{T..T_-}),
$$

$$
\omega_T^I = \frac{1}{2} \left( W_{S..T_+} + W_{S..T_-} \right) \pm \left( W_{S..T_+} + W_{S..T_-} \right),
$$

(19)

$$
\beta = \frac{2\Gamma_{T_+}}{1 + \Gamma_{T_+}/\Gamma_{S_+}}.
$$

We have considered the same value of the tunnelling coupling for both contact barriers, $\Gamma_L = \Gamma_R = \Gamma$ (see section 2.3.2). Finally, by summing (18a) and (18b), we find that $\rho_T + \rho_{T_+} = 1,$
where we consider the following as the initial condition: $\rho_T(0) + \rho_{T_1}(0) = 1$; thus, the rate equations become
\[
\dot{\rho}_T = (\omega_{\uparrow\uparrow} + \omega_{\downarrow\downarrow}) - (3\omega_{\uparrow\downarrow} + \beta + \omega_{\downarrow\uparrow})\rho_T - (\omega_{\uparrow\downarrow} + \omega_{\downarrow\uparrow})P,
\]
\[
\dot{P} = (\omega_{\uparrow\uparrow} + \omega_{\downarrow\downarrow}) + (\omega_{\uparrow\downarrow} - \omega_{\downarrow\uparrow})\rho_T - (\omega_{\uparrow\downarrow} + \omega_{\downarrow\uparrow})P.
\]
(20)

The rates $\omega_{\uparrow\downarrow}$ account for the SB lifting due to ST spin relaxation: $T_{\pm} \xrightarrow{\omega_{\uparrow\downarrow}} \{S_+, S_-, \}$. $\omega_{\downarrow\uparrow}$ ($\beta$) account for the spin-flip (tunnelling) rates from $T_+$ to $T_{\pm}$ ($T_+$ to $|\pm\rangle$). In section 2.3.2, we will see that the rate $\Gamma_{+, S_+} \simeq \Gamma/2$; hence, it can be regarded as constant. In section 3, it will be shown that the current through the DQD depends on the ratio between $\omega_{\uparrow\downarrow}$ and $\beta$. The following scheme offers a qualitative description of this dependence for the case of $A_+ \neq 0$:

\[
T_+ \xrightarrow{\omega_{\uparrow\downarrow} < \beta} (0, \sigma) \xrightarrow{\omega_{\uparrow\downarrow} > \beta} T_{\pm} \xrightarrow{\beta} \text{High current regime},
\]
\[
T_+ \xrightarrow{\omega_{\downarrow\uparrow} > \beta} \text{Low current regime}.
\]
(21)

When $\omega_{\uparrow\downarrow} \gg \beta$, electrons in the $T_+$ state have a higher probability to spin-flip to the $T_{\pm}$ triplets than to tunnel out of the DQD, and only the leakage current coming from the $T_{\pm} \rightarrow \{S_+, S_-\}$ transitions provides a leakage current, which we define as the low-current regime. In contrast, when $\omega_{\uparrow\downarrow} \ll \beta$, electrons in the $T_+$ state have a higher probability to tunnel out of the DQD than to spin-flip to the blocked $T_{\pm}$ triplets, and the current is enhanced. We define this case as the high-current regime. In section 3, we will show that the competition between spin-flip and tunnelling transition rates determines the behaviour of the leakage current. Moreover, we will show that the transition between the low-current regime and the high-current regime is obtained by varying either the interdot tunnelling strength or the HF coupling intensity.

2.3. Transition rates

2.3.1. Spin-flip rates. The transition rates between $\hat{H}_0$ eigenstates (see (10)) due to spin-flip processes are calculated in perturbation theory by means of Fermi’s Golden Rule [20, 22, 26]. The left (right) dot electron-spin $z$-projection is given by $m_{L(R)} = \pm 1/2$; hence, the total spin projection in the $z$-direction for a DQD two-electron state is $M = m_L + m_R$ ($M = -1, 0, 1$ for triplets, and $M = 0$ for singlets). The spin-flip interaction term $\hat{V}_{sf}$ (see (4)) increases an electron spin by one and also decreases a nuclei spin by one (and vice versa); thus, $M : -1 \leftrightarrow 0 \leftrightarrow 1$. We consider the initial state $|i_N\rangle|\alpha_M\rangle$ that consists of the initial nuclei state $|i_N\rangle$ and the DQD electron state $|\alpha_M\rangle$. $|i_N\rangle$ is given by $|i_N\rangle = |m_1, m_2, \ldots, m_j, \ldots, m_N\rangle$, where $m_j = \pm 1/2$ is the spin of the $j$th nuclei, and $|\alpha_M\rangle \in \{|S_\pm\rangle, |T_+\rangle, |T_\pm\rangle\}$. The final state $|f_N\rangle|\beta_{M'}\rangle$ is connected to the initial one by having the $j$th nuclear spin flipped and a different electronic state $|\beta_{M'}\rangle$ with $|M - M'| = 1$. Therefore, the spin-flip rate of the transition that flips up an electron spin and down a nuclei spin is

\[
W_{\beta_{M+1}\alpha_M} = 2\pi \sum_{j=1}^{N} \sum_{i_N} |\langle \beta_{M+1} | (f_N | \hat{V}_{sf} | i_N) | \alpha_M \rangle|^2 W_{i_N} \delta(E_{\beta_{M+1}} - \delta_j - E_{\alpha_M})
\]
\[
= 2\pi \frac{1}{N^2 2^3} \left| \sum_{l=L,R} A_l |\beta_{M+1} | \hat{S}^+_l | \alpha_M \rangle \right|^2 \sum_{j=1}^{N} \sum_{i_N} |\langle i_N | \hat{I}_j^+ \hat{I}_j^- | i_N \rangle| W_{i_N} \delta(E_{\beta_{M+1}} - \delta_j - E_{\alpha_M}),
\]
(22)
where $|f_N\rangle = \hat{T}_j^+|i_N\rangle$ and we have taken $\hbar = 1$. The sum over initial states runs over all configurations of the internal degrees of freedom, $i_N$, that give the state $|i_N\rangle$. Each state is weighted by the probability of having that configuration, which is given by the distribution function $\mathcal{W}_{i_N}$. $E_{\alpha M}$ ($E_{\beta M+1}$) is the energy of the initial (final) electronic state $|\alpha M\rangle$ ($|\beta M+1\rangle$), and $\delta_j$ is the energy splitting between the up–down levels of the $j$th nuclei spin. We assume independent nuclei spins; hence, $\mathcal{W}_{i_N} = \mathcal{W}_{m_1} \times \mathcal{W}_{m_2} \times \ldots \times \mathcal{W}_{m_N}$, where $\mathcal{W}_{m_j}$ is the probability that nuclear spin $j$ has the value $m_j$. Thus, the sum over initial nuclear states becomes

$$\sum_{i_N} (i_N|\hat{T}_j^+\hat{T}_j^-|i_N\rangle \mathcal{W}_{i_N} = \sum_{m_j} (m_j|\hat{T}_j^+\hat{T}_j^-|m_j\rangle \mathcal{W}_{m_j}, \tag{23}$$

where we have used the normalization condition $\mathcal{W}_{m_j=1/2} + \mathcal{W}_{m_j=-1/2} = 1$ to eliminate all other sums over $m_j$ except $m_j$. The probability of having nuclear spin $j$ in a certain state is related to the overall nuclear spin polarization by

$$\mathcal{W}_{m_j=1/2} = \frac{\langle N_j \rangle}{N} = \frac{1 + P}{2}, \quad \mathcal{W}_{m_j=-1/2} = \frac{\langle N_1 \rangle}{N} = \frac{1 - P}{2}. \tag{24}$$

In general, the $g$-factor is much smaller for nuclei spins than for electrons [32]. Therefore, under experimental conditions [2], [4]–[6], [12], the nuclear splitting is usually negligible compared to the temperature and the energy difference between electronic levels; hence, $\delta_j \forall j = 1, \ldots, N$ can be safely neglected. Thus, the spin-flip rate becomes

$$W_{\beta M+1 \alpha M} = \frac{\pi}{2N} \sum_{j=L,R} A_j \langle \beta_{M+1} | \hat{S}_j^+ | \alpha_M \rangle^2 \frac{1 + P}{2} \delta(E_{\beta M+1} - E_{\alpha M}). \tag{25}$$

Repeating the same procedure, we have that the spin-flip rate of the transition that flips down an electron spin and up a nuclei spin is the following:

$$W_{\beta M-1 \alpha M} = \frac{\pi}{2N} \sum_{j=L,R} A_j \langle \beta_{M-1} | \hat{S}_j^- | \alpha_M \rangle^2 \frac{1 - P}{2} \delta(E_{\beta M-1} - E_{\alpha M}). \tag{26}$$

Note that these spin-flip rates depend on the extent to which the nuclei are polarized. When the nuclei spins are fully polarized in the positive (negative) direction, $W_{\beta M-1 \alpha M}$ ($W_{\beta M+1 \alpha M}$) vanishes.

We see that the derived spin-flip rate requires energy conservation and hence, strictly speaking; it leads to zero spin-flip when $E_{\alpha M} \neq E_{\beta M \pm 1}$. However, in reality it is possible to exchange energy [33] with the environment, e.g. as phonons. We model this by replacing the Dirac delta by the following expression:

$$\delta(E_{\beta M \pm 1} - E_{\alpha M}) \rightarrow \frac{1}{\pi} \frac{\gamma}{(E_{\beta M \pm 1} - E_{\alpha M})^2 + \gamma^2} \times C_{\beta M \pm 1 \alpha M} \tag{27},$$

where

$$C_{\beta M \pm 1 \alpha M} = \begin{cases} 1, & \text{if } E_{\beta M \pm 1} > E_{\alpha M} \rightarrow \text{Energy emission,} \\ \exp\left(\frac{E_{\beta M \pm 1} - E_{\alpha M}}{k_B T}\right), & \text{if } E_{\beta M \pm 1} < E_{\alpha M} \rightarrow \text{Energy absorption,} \end{cases} \tag{28}$$

where $T$ is the temperature and $k_B$ the Boltzmann constant. The Lorentzian is maximal for the elastic case and falls off with increasing energy exchange on the characteristic scale $\gamma$. 

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This parameter is assumed to be of the order of the typical phonon energy, $\gamma \sim \mu$eV \cite{33, 34}. The function $C_{\beta M \pm |aM}$ accounts for the low-temperature energy emission/absorption asymmetry \cite{33, 34}, i.e. it is much easier to emit than to absorb energy from e.g. a phonon bath. Formally, one can include the electron–phonon coupling as a perturbation together with $\hat{V}_{\text{e}},$ and thereafter use a $T$-matrix approach to find the phonon-mediated HF spin-flip rate \cite{35}–\cite{37}. However, here we do not pursue an exact modelling of the way the energy is exchanged with the environment, but simply include the fact that the spin-flip rate decreases as the energy involved in the flip-flop processes increases \cite{20, 22}. Therefore, the spin-flip rates we obtain are the following:

$$W_{\beta M \pm |aM} = \frac{1}{2N} \left| \sum_{l=L,R} A_l \langle \beta_{M \pm} | \hat{S}^\pm_{l} | \alpha_M \rangle \right|^2 \frac{1 \pm P}{2} \frac{\gamma}{(E_{\beta M \pm} - E_{\alpha M})^2 + \gamma^2} C_{\beta M \pm |aM}. \quad (29)$$

In section 3, we will show that the amount of nuclear spin polarization induced depends on the competition between emission and absorption processes. The energies that appear in the function $C_{\beta M \pm |aM}$ (see \cite{28}) are the eigenenergies shown in \cite{9}. Thus, the energy differences in $C_{\beta M \pm |aM}$ depend on the interdot tunnel strength, the HF coupling and the external magnetic field. Therefore, for a given temperature the emission/absorption asymmetry can be controlled through these parameters. Finally, for the only matrix elements between the DQD states that are different from zero in \cite{29} are

$$(T_\uparrow | (A_L \hat{S}^+_{L} + A_R \hat{S}^+_{R}) | T_\uparrow) = \frac{\sqrt{2} A_+}{N}, \quad (30a)$$

$$(S_\pm | (A_L \hat{S}^-_{L} + A_R \hat{S}^-_{R}) | T_\uparrow) = -\left(A_+ \mp \frac{x A_+}{N}\right), \quad (30b)$$

$$(S_\pm | (A_L \hat{S}^+_{L} + A_R \hat{S}^+_{R}) | T_-) = A_+ \pm \frac{x A_+}{N}. \quad (30c)$$

From these expressions, we distinguish between two different HF-mediated spin-relaxation processes: (i) the triplet–triplet relaxation (\ref{30a}) and (ii) the ST relaxation (\ref{30b} and \ref{30c}). Note that if the HF coupling intensities have the same value for each dot ($A_- = 0$ and then $x = 0$) only the triplet–triplet spin relaxation survives ($T_\uparrow \rightarrow T_\uparrow = T_0$), whereas ST relaxation probabilities become zero. Thus, SB lifting implies $A_- \neq 0,$ as discussed in section 2.1 (see \cite{13}). Finally, the matrix elements also depend on the interdot tunnel through the ST mixing parameter $x$ (see \cite{12}).

### 2.3.2. Tunnelling rates

The tunnelling rates between the leads and the DQD are calculated by using Fermi’s Golden Rule \cite{26, 34}. Given an initial DQD state with $n$ electrons $|\alpha_n\rangle,$ the tunnelling rate for an incoming electron to the final DQD state $|\beta_{n+1}\rangle$ with $n + 1$ electrons is

$$\Gamma^{\uparrow}_{\beta_{n+1}, \alpha_n} = \Gamma f(\mu_D - \mu_i) \sum_\sigma |\langle \beta_{n+1} | \hat{d}^\dagger_\sigma | \alpha_n \rangle|^2. \quad (31)$$

The tunnelling rate of an outgoing electron to the final DQD state $|\beta_{n-1}\rangle$ with $n - 1$ electrons is

$$\Gamma^{\uparrow}_{\beta_{n-1}, \alpha_n} = \Gamma (1 - f(\mu_D - \mu_i)) \sum_\sigma |\langle \beta_{n-1} | \hat{d}_\sigma | \alpha_n \rangle|^2, \quad (32)$$
where \( l = L, R \). Here \( \Gamma_i = 2\pi |\gamma_{ik}|^2 D_i \), where it is assumed that the density of states in both leads \( D_i \) and the tunnelling couplings \( \gamma_{ik} \) (see (2)) are energy independent. \( f(\mu_L - \mu_{L(R)}) \) is the Fermi distribution function for the left (right) lead, \( \mu_{L(R)} \) is the chemical potential of the left (right) lead and \( \mu_D \) is the DQD chemical potential. The DQD states that appear in the matrix elements in the tunnelling rates are the eigenstates of \( \hat{H}_0 \) (see (10)). The chemical potentials \( \mu_{L(R)} \) are tuned such that the system is in the SB regime, and we define \( \Gamma_{\beta n,\pm 1,\alpha} = \sum_{l=L,R} \Gamma_{\beta n,\pm 1,\alpha}^l \). Therefore, the tunnelling rates different from zero are

\[
\Gamma_{T_{\pm, \pm}} = \Gamma_L,
\Gamma_{S_{\pm, \pm}} = \frac{\Gamma_L}{4} \left( 1 + \frac{x^2}{N^2} \mp \frac{2x}{N} \right),
\Gamma_{S_{\pm, \mp}} = \frac{\Gamma_L}{4} \left( 1 + \frac{x^2}{N^2} \pm \frac{2x}{N} \right),
\Gamma_{T_{\pm, \pm}} = \frac{\Gamma_L}{2} \frac{1}{N^2}
\]

for electrons tunnelling from the leads into the DQD and

\[
\Gamma_{\pm, S_{\pm}} = \Gamma_{\pm, S_{-}} = \frac{\Gamma_R}{2} \frac{1}{N^2},
\Gamma_{\pm, T_{\pm}} = \Gamma_R \frac{x^2}{N^2}
\]

for electrons tunnelling out of the DQD to the leads. As pointed out in section 2.2, the tunnelling rates also depend on the nuclear spin polarization. Additionally, \( \Gamma_{\pm, T_{\pm}} \) becomes zero if \( x = 0 \) (see (12)), i.e. if the HF couplings within each dot are the same \( (A_+ = 0) \), or if the nuclei spins are completely depolarized \( (P = 0) \).

Finally, the tunnelling current through the DQD is given by [26]

\[
I_l = (-e) \sum_n \sum_{\alpha_n, \beta_n, \pm 1} \left( \Gamma_{\beta n+1, \alpha_n}^l - \Gamma_{\beta n-1, \alpha_n}^l \right) \rho_{\alpha_n},
\]

where \( l = L, R \).

### 3. Results and discussion

In this section, we present the results obtained by solving numerically the full system of rate equations derived in section 2 ((15) and (17)). The system has three main relaxation time scales: (i) the tunnelling through the contact barriers, (ii) the electron–nuclei spin-flip and (iii) the nuclear spin relaxation \( (\propto W_{\text{rel}}) \). The nuclear relaxation time is the largest one (of the order of approximately minutes \( [3, 6, 38] \)). We are mainly interested in investigating the system on a time scale at which the tunnelling and spin-flip are the relevant relaxation times, whereas the nuclear spin-relaxation time is much larger.

We will discuss the behaviour of the electronic current and induced nuclei spin polarization versus the external magnetic field for different HF couplings and interdot tunnelling strengths. The HF interaction constant of all materials is not known. Thus, we have considered values of the HF coupling intensity in the range \( A_L = 70–90 \mu\text{eV} \) \( [4, 6, 28] \). The difference between the HF couplings in each dot is held constant for all cases \( (A_R = 0.8A_L) \). The plots described below
are given in the following scheme:

| T− | S+ | S+ | T+ |
|-----|-----|-----|-----|
| S+ | T− | T+ | S+ |
| Tx | Tx | Tx | Tx |
| S− | T+ | T− | S− |

(a) (b) (c) (d)

Figure 2. Schematic energy level arrangement of the DQD eigenstates in different external magnetic field regions that are investigated. (a) $B_{\text{ext}} < B_{S±,T±}$, (b) $B_{S±,T±} < B_{\text{ext}} < B_{TT}$, (c) $B_{TT} < B_{\text{ext}} < B_{S±,T±}$ and (d) $B_{S±,T±} < B_{\text{ext}}$.

have been obtained by solving numerically the rate equations derived in section 2, equations (15) and (17), sweeping the external magnetic field from negative to positive values. We have performed calculations for three cases: (i) In order to observe only the effect of the dynamical nuclear spin polarization, we consider that the nuclei spins are initially fully depolarized for each value of the external magnetic field. Thus, the sweeping rate is much lower than the nuclear spin-relaxation rate $W_{\text{rel}}$ (see (17)). Nevertheless, for each value of the external field, the rate equations are solved in a time scale much smaller than that of the nuclear spin-relaxation rate in order to capture only the tunnelling and spin-flip events. (ii) Initially the nuclei spins are completely depolarized; however, while the magnetic field is swept, nuclear spin polarization is built up. Thus, the sweeping rate is much higher than the nuclear spin-relaxation rate, and a feedback process between nuclear spins and electronic current occurs. (iii) We proceed as in (ii) sweeping the external field forwards and backwards. In this case, we observe hysteresis, as has been widely observed experimentally in different DQD devices [3, 4, 6, 10].

As stated in section 2, we consider the zero detuning case (i.e. $|\sigma, \sigma'|$ and $|S_0\rangle$ are degenerate). In previous works [24, 25], finite detuning was considered; therefore only transitions close to one ST level crossing were important. Now, different energy level approaches participate and we take into account all HF-mediated electron-spin relaxation processes, at a fixed external magnetic field. Figure 2 shows the energy levels for the different regimes considered. $B_{S±,T±}$ corresponds to the value of the external field at which the $S±T±$ crossings occur, and $B_{TT}$ the value at which the $T±T±$ crossing occurs. Note that different $S±T±$ crossings occur for the same value of the external field (see (9)) and the same happens for different $S±T±$ crossings. The energy emission spin-flip rates that contribute for each range of the external field in figure 2 are given in the following scheme:

1. $B_{\text{ext}} < B_{S±,T±} \Rightarrow W_{S±,T±}, W_{T±,T±}, W_{T±,T±} \Rightarrow P < 0$, (36a)

2. $B_{S±,T±} < B_{\text{ext}} < B_{TT} \Rightarrow \left\{ \begin{array}{l} W_{S±,T±}, W_{T±,T±}, W_{T±,T±} \Rightarrow P < 0, \\ W_{S±,T±} \Rightarrow P > 0, \end{array} \right.$ (36b)

3. $B_{TT} < B_{\text{ext}} < B_{S±,T±} \Rightarrow \left\{ \begin{array}{l} W_{S±,T±} \Rightarrow P < 0, \\ W_{S±,T±}, W_{T±,T±}, W_{T±,T±} \Rightarrow P > 0, \end{array} \right.$ (36c)

4. $B_{S±,T±} < B_{\text{ext}} \Rightarrow W_{S±,T±}, W_{T±,T±}, W_{T±,T±} \Rightarrow P > 0$, (36d)

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where the sign of the nuclear spin polarization in each case is described in (16). Only emission spin-flip rates are shown because at low temperatures they dominate over the absorption rates; hence, they will determine the sign of nuclear spin polarization. Thus, depending on the intensity of the external magnetic field, there will be different spin-flip processes dominating.

3.1. Dependence on the hyperfine interaction intensity

3.1.1. Small magnetic fields. In this section, we consider small external magnetic field intensities. We will focus on the $B_{S±, T∓} < B_{ext} < B_{S±, T±}$ range of magnetic fields. The main spin-flip transition rates corresponding to this range of magnetic fields are shown in (36b) and (36c), and the energy level scheme is plotted in figures 2(b) and (c) (see also figure 4). In this case, the transitions around the $T±T∓$ crossing give the main contribution to the current, although other possible opposite spin level transitions participate (see (36b) and (36c)). The $S± T∓$ and $S± T±$ crossings occur for larger magnetic field intensities and will be studied in section 3.1.3. Figure 3 shows the induced nuclear spin polarization and the leakage current through DQD versus the external magnetic field for different values of HF coupling ($A_L$), in this range of external field.
A plot of the induced nuclear spin polarization versus the external field is shown in figure 3(a) (without feedback), figure 3(b) (with feedback) and figure 3(c) (sweeping forwards and backwards with feedback). Figure 4 shows the energy levels. When $B_{\text{ext}} < B_{TT}$ ($B_{\text{ext}} > B_{TT}$), the net nuclear spin polarization induced is negative (positive): the emission processes are stronger than the absorption processes and, hence, mainly determine the sign of nuclear spin polarization. In (36b) it is shown that when $B_{\text{ext}} < B_{TT}$ ($B_{\text{ext}} > B_{TT}$) the dominating spin-flip rates are $W_{S_+T_-}$, $W_{T_+T_-}$ and $W_{T_+T_+}$ ($W_{S_+T_+}$, $W_{T_+T_+}$ and $W_{T_-T_+}$), which polarize the nuclei negatively (positively). Nevertheless, in this regime absorption processes have the effect of partially compensating for emission processes, resulting in a finite but not complete spin polarization of the nuclei. In figures 3(a)–(c) it is shown that increasing the HF coupling increases the nuclei spin polarization. In this region, since $|E_{S_+} - E_{T_+}| \gg |E_{S_-} - E_{T_-}|$, the $T_\pm \leftrightarrow T_\pm$ transitions are the most important (see figures 2(b) and (c) and figure 4). The absolute energy difference between these levels is given by the total effective Zeeman splitting (see (9)):

$$
\Delta_{\text{tot}} = \left( g\mu_BB_{\text{ext}} + \frac{A_+}{2}P \right).
$$

(37)

As the HF coupling intensity increases, so does the energy difference between the initial and the final states. Therefore, absorption processes become weaker with respect to the emission processes as $A_L$ increases (see (28)), allowing the nuclei spins to become more polarized (figure 3).

Figure 4 shows the energy levels of the DQD versus the external magnetic field. Electron–nuclei spin feedback is taken into account, and the external field is swept forwards and backwards. Figure 4(a) shows the energy levels for the smallest HF coupling considered ($A_L = 70 \mu eV$). In this case: (i) the induced Overhauser field is always parallel to the external field (see (6)) and (ii) the $T_\pm T_\pm$ crossing occurs at $B_{\text{ext}} = B_{TT} = 0$ T. Figure 4(b) shows that for the largest value of the HF coupling ($A_L = 90 \mu eV$) considered here, the effect of including the feedback is to renormalize the electronic energy levels in such a way that the $T_\pm T_\pm$ crossing occurs at a larger absolute magnetic field than in the previous case. When sweeping forwards (backwards) from $B_{\text{ext}} < 0$ ($B_{\text{ext}} > 0$) through $B_{\text{ext}} = 0$, there is a negative (positive) nuclei spin

Figure 4. Energy levels versus external magnetic field taking the spin electron-nuclei feedback into account and sweeping forwards and backwards $B_{\text{ext}}$, for $A_L = 70 \mu eV$ (a) and $A_L = 90 \mu eV$ (b). $T_+$ (solid, red online), $T_-$ (dotted, blue online), $T_x$ (dashed, green online). The same parameters and initial conditions as those in figure 3 were used. The magnetic field range considered just includes the $T_+T_-$ crossing. Singlet states are far away in energy. As in figure 3, hysteresis is only observed for the largest value of HF coupling ($A_L = 90 \mu eV$).
polarization built up (figure 3(c)); hence, it is still necessary to increase (decrease) the external field in order to compensate for the accumulated Overhauser field and reach the $T_{±}T_{x}$ crossing. This crossing occurs now at $B_{TT} > 0$ ($B_{TT} < 0$) when sweeping forwards (backwards). Thus, figure 3(b) shows a small region of positive values of the external field where the Overhauser field and the external field are antiparallel. Finally, figures 3(c) and 4(b) show that precisely in this region where the external and the induced fields are antiparallel, hysteresis is observed as the external field is swept backwards. Note that hysteresis is observed only for the largest value of the HF coupling intensity considered. Moreover, the size of the hysteresis loop increases with the HF-coupling intensity (not shown in the figures). Summarizing, (i) the amount of polarization induced in the nuclei spins depends on the competition between the absorption and emission spin-flip processes; (ii) nuclear spin polarization increases with HF coupling; and (iii) for large values of HF coupling, the nuclear spin polarization versus the external magnetic field presents a bistable region where hysteresis is observed. In this region, the external field and the Overhauser field are antiparallel.

We will now analyse the leakage current behaviour versus the external magnetic field. Figure 3(d) (without feedback), figure 3(e) (with feedback) and figure 3(f) (swinging forwards and backwards with feedback) show different behaviour of the current depending on the intensity of the HF coupling. The smallest HF coupling intensity considered ($A_L = 70 \mu$eV) presents a current dip around the $T_{±}T_{x}$ crossing (figure 4), whereas the largest one ($A_L = 90 \mu$eV) presents a current peak. This behaviour can be understood recalling the current behaviour studied in the scheme given in (21). Briefly, this scheme defines a low and a high current regime, comparing the $T_x \rightarrow T_x$ spin-flip rates ($W_{T_{±},T_{x}}$) with the $T_x \rightarrow |± \rangle$ tunnelling rates ($\Gamma_{±,x}$). These rates are shown in figure 5. Figure 5(a) ($A_L = 70 \mu$eV) shows that around the $T_{±}T_{x}$ crossing $W_{T_{±},T_{x}} > \Gamma_{±,T_{x}}$ ($\omega_{±}^{dt} > \beta$ in the scheme (21)). Therefore, it is more probable for electrons to tunnel from $T_x \rightarrow T_x$ than to tunnel from $T_x \rightarrow |± \rangle$ through the contact barrier, so a current dip is observed (low-current regime). By contrast, figure 5(c) ($A_L = 90 \mu$eV) shows that around the $T_{±}T_{x}$ crossing, $W_{T_{±},T_{x}} < \Gamma_{±,T_{x}}$ ($\omega_{±}^{dt} < \beta$ in the scheme (21)). Therefore, it is more probable for electrons to tunnel from $T_x \rightarrow |± \rangle$ than to spin-flip from $T_x \rightarrow T_x$, so current is enhanced and a peak is observed (high-current regime). Furthermore, figure 3(a) shows that for the smallest intensity considered for the HF coupling ($A_L = 70 \mu$eV), the nuclear spin polarization goes to zero around the $T_{±}T_{x}$ crossing and so does $\Gamma_{±,T_{x}}$ (see (34) and figure 5(a)); thus, a current dip is observed (figure 3(d)). However, for the largest intensity considered ($A_L = 90 \mu$eV), the nuclear spin polarization is finite around the $T_{±}T_{x}$ crossing and $\Gamma_{±,T_{x}} > W_{T_{±},T_{x}}$ (figure 5(c)); thus, a current peak is observed (figure 3(d)).

Figures 3(d)–(f) show how the leakage current increases as the HF coupling intensity increases when the external field is close to zero. With increasing HF coupling, the tunnelling rate $\Gamma_{±,T_{x}}$ (see (34)) increases (figure 5) and so does the current. At $A_L = 70 \mu$eV and $A_L = 80 \mu$eV, the current drops to zero at $B_{ext} = B_{TT} = 0$, whereas at $A_L = 90 \mu$eV a current peak (figure 3(d)) is observed. This peak occurs at $B_{ext} = B_{TT} ≠ 0$ when feedback is considered (figure 3(e)). Moreover, figure 3(e) shows that the effect of the electron nuclei spin feedback is appreciable only for the largest value of HF coupling ($A_L = 90 \mu$eV). Therefore, only in this case is current hysteresis observed when sweeping backwards the external field (figure 3(f)).

3.1.2. Simplified model around the triplet–triplet crossing. In the regime, described in section 3.1.1, the relations $|E_{T_{±}} - E_{S_{±}}| \gg \gamma$ and $|E_{T_{±}} - E_{S_{±}}| \gg \gamma$ are well satisfied, so the rates $\omega_{±}^{dt}$ that appear in the reduced rate equations (20) can be safely neglected (see (19) and (29)).
Figure 5. Spin-flip rates ($\omega_{\perp}^T$, see (19); solid, red online) and tunnelling rates through contact barriers ($\beta$, see (19); dashed, blue online) versus external magnetic field. The same parameters and initial conditions as in figure 3 are used. In figures 5(a) and (b) there are two regions: (i) $\beta > \omega_{\perp}^T$, namely, when electron tunnelling from $T_x$ to the right lead is more effective than electron spin-flip from $T_x$ to $T_\pm$ triplets (high-current regime); and (ii) $\beta < \omega_{\perp}^T$, namely, when electron tunnelling from $T_x$ to the right lead is less effective than electron spin-flip from $T_x$ to $T_\pm$ triplets (low-current regime). In this case, the current shows a dip around zero external magnetic field (figure 3). In figure 5(c), $\beta > \omega_{\perp}^T$ for all values of the external field. In this case, the current shows a peak (figure 3).

and (20) becomes

$$\dot{\rho}_{T_x} = \omega_{\perp}^T - (3\omega_{\perp}^T + \beta)\rho_{T_x} - \omega_{\parallel}^T P,$$

$$\dot{P} = \omega_{\perp}^T + \omega_{\parallel}^T \rho_{T_x} - \omega_{\parallel}^T P.$$  

(38)

Considering the stationary limit, we obtain the following equations for the stationary solutions:

$$\rho_{T_x} = \frac{1 - p_t^2}{3 + \frac{\beta}{\omega_{\perp}^T} + p_t^2},$$  

(39a)

$$P = \frac{4 + \frac{\beta}{\omega_{\perp}^T} + p_t^2}{3 + \frac{\beta}{\omega_{\perp}^T} + p_t^2} p_t,$$  

(39b)

where $p_t = \omega_{\parallel}^T/\omega_{\perp}^T$. The rates $\omega_{\perp}^T$ and $\beta$ are complicated functions of $P$ (see (19), (29) and (30a)); thus, (39b) cannot be solved analytically. We have obtained numerical solutions for the $B_{ext} = 0$ case (figure 6), although (39a) and (39b) hold also for finite external fields. Figure 6 shows that increasing the HF coupling intensity produces a bifurcation in the induced nuclei spin polarization. For $A_L \leq 80.43 \mu eV$, the nuclei spins have one stable solution at $P = 0$; namely, the nuclei spins are fully depolarized (solid line). However, for $A_L > 80.43 \mu eV$, the $P = 0$ solution becomes unstable (dashed line), and two stable solutions (solid lines), with the same absolute value but with opposite signs, appear. This behaviour is the same as that found in the full numerical solution (figure 3(a)). We have found that the system undergoes a bifurcation as the slope of the linear term of the expansion around $P = 0$ of the right-hand side of (39b) is varied [39]. The slope is given by

$$s = \frac{1}{3} \left( \frac{A_x}{k_B T} - 4 \right).$$  

(40)
Figure 6. Solutions of (39b) for $B_{\text{ext}} = 0$. The induced nuclei spin polarization presents a bifurcation. When $A_L \leq 80.43 \, \mu \text{eV}$, there is only one stable solution that corresponds to having the nuclei spins fully depolarized (solid line). However, when $A_L > 80.43 \, \mu \text{eV}$, $P = 0$ becomes an unstable solution (dashed line) and two stable solutions show up (solid lines). The same parameters as in figure 3 were used.

When $s < 1$, $P = 0$ is the only stable solution, whereas when $s > 1$ the system presents the two stable solutions mentioned above. The bifurcation occurs at $s = 1$. Putting the parameters for which figure 6 is obtained, into (40) we find that the bifurcation takes place when $A_L = 80.43 \, \mu \text{eV}$. Therefore, (40) provides an expression that relates the HF-coupling intensity to the temperature and the bifurcation, which, in addition, can be observed through the hysteresis plots measured for the current through DQD versus the external field [3, 4, 6, 10]. Finally, we find that there is very good agreement between the induced nuclei spin polarization obtained with full calculation for $B_{\text{ext}} = 0$ and that obtained with this simplified model.

From the current through the right contact barrier $I_R = (\Gamma_+, \Gamma_-) \rho_T$ (see (35)) and from (39a), we obtain the following expression for the current through the DQD:

$$I = \left(1 + \frac{1}{2N^2}\right) \frac{1 - p_i^2}{3 + \frac{L_T}{\omega_T} + p_i^2} \beta.$$

(41)

Note that $\beta = 0$ when $P = 0$ (see (19)); thus, the current will be zero when the nuclei spins are fully depolarized. In the previous discussion, we have shown that the induced nuclei spin polarization shows a bifurcation with increasing HF-coupling intensity. Therefore, in the range of $A_L$ where the nuclei spins are fully depolarized (figure 6), no current flows through the DQD at $B_{\text{ext}} = 0$. However, in the range of $A_L$ where the nuclei have a nonzero spin polarization (figure 6), a finite current flows through the DQD at $B_{\text{ext}} = 0$, since $I(B_{\text{ext}} = 0) \propto P^2$ to lowest order in $P$. Furthermore, (41) shows that the current depends on the ratio $\beta/\omega_T$. These rates are shown in figure 5, and motivate the physical picture we have used to understand the transition from a current dip to a current peak. Therefore, this simplified model shows that the bifurcation obtained for the induced nuclei spin polarization, together with the ratio $\beta/\omega_T$, describes the transition from a current dip to a current peak (figure 3(d)).

3.1.3. Large magnetic fields. In this section, we consider larger magnetic fields than in the previous case in order to account for the ST crossings. This means sweeping the external...
Figure 7. (a, b) Energy levels versus external magnetic field. $T_+$ (solid, red online), $T_-$ (dotted, blue online) and $T_x$, $S_+$, $S_-$ (dashed, green online); (c, d) induced nuclear spin polarization, and (e, f) leakage current versus external magnetic field sweeping forwards (solid, red online) and backwards (dashed, green online). The same parameters and initial conditions as in figure 3 were used. The range of magnetic fields considered includes both the ST and the $T_\pm T_x$ crossings. Hysteresis is observed for both the values of HF-coupling intensity at the ST crossings. Recall that in figure 3, where only the $T_\pm$ crossing was shown, hysteresis was found only for the largest value of HF intensity ($A_L = 90 \mu eV$). The current shows now three peaks corresponding mainly to each of the level crossings.

field from $B_{ext} < B_{S_\pm T_\mp}$ to $B_{ext} > B_{S_\pm T_\pm}$ (see the schematic figures 2(a) and (d)). Therefore, all three level crossings ($S_\pm T_\mp$, $S_\pm T_x$ and $T_\pm T_x$) will be considered (see figures 7(a) and (b)). Figure 7 shows the DQD energy levels, the induced nuclear spin polarization and the leakage current through the DQD versus the external magnetic field for the case of $A_L = 70 \mu eV$ and $A_L = 90 \mu eV$. In this case, all figures include feedback and the external field is swept forwards and backwards.
A plot of nuclear spin polarization versus the external field is shown in figure 7(c) \( (A_L = 70 \, \mu eV) \) and figure 7 \( (A_L = 90 \, \mu eV) \). The external field sweeping starts when \( B_{ext} < B_{S_z,T_x} \). Initially, the nuclei spins are completely depolarized. The DQD energy level distribution shows that the \( T_+ \) triplet is the ground state (see figures 7(a) and (b)). In this case, all spin-flip emission rates polarize negatively the nuclei spins (see (36a)); hence, they become almost completely negatively polarized \( (P \simeq -1) \). Furthermore, since the probability of finding a nuclei with spin \( +1/2 \) is nearly zero when \( P \sim -1 \) (see (24)), all emission rates become approximately zero (see (29)), and the nuclei spin polarization remains roughly constant until \( B_{ext} \) reaches the \( S_{\pm}T_x \) crossings. Only close enough to the \( S_{\pm}T_x \) level crossings spin-flip absorption processes become significant and the nuclei spins dramatically depolarize (figures 7(c) and (d)). In the \( B_{S_x,T_x} < B_{ext} < B_{S_x,T_x} \) range of magnetic fields (schematic figures 2(b) and (c)), several spin-flip rates compete in order to polarize the nuclei spins in opposite directions (see (36b) and (36c)). When \( B_{S_x,T_x} < B_{ext} < B_{TT} \) (scheme in figure 2(d)), most of the spin-flip emission rates polarize negatively the nuclei spins (see (36b)), and the resulting nuclear spin polarization is negative in this region. However, as the levels approach the \( T_{\pm}T_x \) crossings, spin-flip absorption processes become more relevant, and the nuclei spins slowly depolarize. When \( B_{TT} < B_{ext} < B_{S_x,T_x} \) (scheme in figure 2(c)), most of the spin-flip emission rates polarize positively the nuclei spins (see (36c)), and the nuclear spin polarization is positive in this region. Finally, when \( B_{ext} > B_{S_x,T_x} \), the \( T_+ \) triplet is the ground state (scheme in figure 2(d)), and (36d) shows that all spin-flip emission rates polarize positively the nuclei spins; thus, they become almost completely positively polarized \( (P \simeq 1) \). The probability of finding a nucleus with spin \( -1/2 \) is nearly zero as \( P \sim 1 \) (see (24)); thus, all emission rates become approximately zero (see (29)), and the nuclei spin polarization remains roughly constant.

Figures 7(c) and (d) show that the feedback between electron and nuclei spins produces hysteresis in the nuclear polarization around the ST crossings. Recall that for small magnetic fields (section 3.1.1), where only \( T_{\pm}T_x \) crossings participate, hysteresis was observed not for small HF intensities, but only the largest HF-coupling intensity considered here \( (A_L = 90 \, \mu eV) \). For larger external magnetic fields, however, hysteresis shows up at ST crossings even in the case of the smallest HF intensity considered \( (A_L = 70 \, \mu eV) \). Finally, as is the case for small magnetic fields, hysteresis becomes larger as HF coupling increases.

Plots of the leakage current through DQD versus the external field is shown in figure 7(e) \( (A_L = 70 \, \mu eV) \) and figure 7(f) \( (A_L = 90 \, \mu eV) \). The current presents three peaks, each of them corresponding to one of the three possible level crossings. When \( B_{ext} < B_{S_x,T_x} \) \( (B_{ext} > B_{S_x,T_x}) \), the current is zero because electrons are trapped in the \( T_x \) \( (T_-) \) triplet state (figures 7(a) and (b)); thus, in these ranges of the external field spin-flip emission rates are nearly zero. In between these crossings (when \( B_{S_x,T_x} < B_{ext} < B_{S_x,T_x} \) the current is strongly quenched but nevertheless finite. This case has been discussed in section 3.1.1. Finally, the feedback between the nuclei and the electron spins also produces hysteresis in the current around the ST crossings when sweeping the external field forwards and backwards.

3.2. Dependence on the interdot tunnelling strength

In this section, we show the leakage current and the induced nuclear spin polarization dependence on the interdot tunnelling intensity. The interdot tunnel varies considerably from one experiment to another and can be externally tuned. For instance, it is estimated to be about 30 \( \mu eV \) in [2] and about 0.2 \( \mu eV \) in [3]. This justifies the large difference between the two values.
that we have chosen. We consider small external magnetic fields and two different interdot tunnel values (figures 8(a) and (b)). For the largest value of the interdot tunnel \( t_{LR} = 50 \mu eV \), only the \( T_\pm T_x \) crossing participates in the current. However, at the smallest value of the interdot tunnel \( t_{LR} = 0.01 \mu eV \), all the crossings, \( S_\pm, T_\mp, T_\pm T_s \) and \( S_\pm T_\mp \), participate in the current for the same range of external field. Figure 8 shows also the induced nuclear spin polarization (figures 8(c) and (d)) and the leakage current through the DQD (figures 8(e) and (f)) versus the external magnetic field for the two interdot tunnelling intensities chosen, \( A_L = 50 \mu eV \) and \( t_{LR} = 0.01 \mu eV \), and for \( A_L = 80 \mu eV \).

**Figure 8.** (a, b) Energy levels versus external magnetic field. \( T_+ \) (solid, red online), \( T_- \) (dotted, blue online) and \( T_\pm, S_\pm, S_\mp \) (dashed, green online); (c, d) induced nuclear spin polarization and (e, f) leakage current versus external magnetic field. \( A_L = 80 \mu eV \). The remaining parameters and the initial conditions are the same as those in figure 3. The current shows a dip (peak) for the largest (smallest) value of the interdot tunnel intensity. Although feedback between the electron and the nuclear spin polarization is taken into account in these figures, hysteresis is not observed for this set of parameters.
Figures 8(c) and (d) show that for the smallest interdot tunnelling ($t_{LR} = 0.01 \mu eV$), the behaviour of the polarization versus external magnetic field is smoother than that for the largest interdot tunnelling considered ($t_{LR} = 50 \mu eV$), due to the stronger competition between the energy absorption and emission processes in the former case. For small interdot tunnelling, absorption is more efficient than that for large interdot tunnelling.

Figures 8(e) and (f) show that with decreasing interdot tunnelling, the current versus magnetic field again presents a transition from a dip to a peak. As in the previous case where we discussed the current behaviour as a function of the intensity of HF interaction, a dip or a peak feature observed in the current can be understood by comparing the spin-flip rate between $T_{\pm}$ and $T_x$ and the tunnelling rate through the barrier contact between $|\pm\rangle$ states (figure 5(b)). However, unlike in the previous case where the HF coupling had to be increased to observe the transition, in this case, in order to go from a dip to a peak the interdot tunnelling must decrease.

4. Conclusions

We have studied the leakage current through a coherently coupled DQD in the SB regime. Spin relaxation due to HF interaction between the spins of the electrons in the DQD and the nuclei spins lifts SB producing leakage current. Moreover, the spin interaction between electrons tunnelling through the DQD and nuclei induces dynamical nuclear spin polarization that is in general non-negligible. We have investigated the behaviour of both the induced nuclear spin polarization and the leakage current as a function of an external magnetic field. Our three main results are as follows. (i) The leakage current shows a dip or a peak depending on the intensities of both the HF interaction and the interdot tunnel strength. We have shown that for large (small) HF coupling (interdot tunnelling) intensities the current shows a peak. In contrast, for small (large) HF coupling (interdot tunnelling) intensities the current shows a dip. Large (small) HF couplings (interdot tunnelling) indicate strong mixing between the ST subspaces; namely, the effective Zeeman splitting difference between the dots is non-negligible with respect to the exchange energy. In this case, the leakage current shows a peak. In contrast, small (large) HF couplings (interdot tunnelling) indicate a weak mixing between the ST subspaces, the Zeeman splitting difference between dots becomes negligible with respect to the exchange energy, and the system is mostly blocked in the triplet subspace. In this case, the leakage current shows a dip. The crossover from a dip to a peak is, thus, obtained by increasing (decreasing) the HF interaction (interdot tunnelling strength). (ii) For a wide external magnetic field sweeping range, we have shown that the leakage current shows three main peaks: two satellite peaks corresponding to the ST crossings and a central peak corresponding to the triplets crossing. (iii) We have observed hysteresis in both the leakage current and the induced nuclear spin polarization as a function of an external magnetic field. This hysteretic behaviour is a consequence of the dynamical nuclear spin polarization interacting with the electron spin that tunnels through the DQD structure. Finally, we have shown that the size of the hysteresis region strongly depends on the HF interaction intensity and the interdot tunnel strength.

Our results are a contribution to ongoing efforts to understand and control spin relaxation in qubits, which represents a limitation for quantum information and quantum computation purposes.
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References

[1] Loss D and DiVincenzo D P 1998 Phys. Rev. A 57 120
[2] Ono K, Austing D G, Tokura Y and Tarucha S 2002 Science 297 1313
[3] Koppens F H L et al 2005 Science 309 1346
[4] Pfund A, Shorubalko I, Ensslin K and Leturcq R 2007 Phys. Rev. Lett. 99 036801
[5] Shaji N et al 2008 Nat. Phys. 4 540
[6] Churchill H O H et al 2009 Nat. Phys. 5 321
[7] Khaetskii A V and Nazarov Y V 2000 Phys. Rev. B 61 12639
[8] Johnson A C, Petta J R, Marcus C M, Hanson M P and Gossard A C 2005 Phys. Rev. B 72 165308
[9] Vorontsov A B and Vavilov M G 2008 Phys. Rev. Lett. 101 226805
[10] Ono K and Tarucha S 2004 Phys. Rev. Lett. 92 256803
[11] Baugh J, Kitamura Y, Ono K and Tarucha S Phys. Rev. Lett. 99 096804
[12] Reilly D J et al 2008 Science 321 817
[13] Danon J et al 2009 Phys. Rev. Lett. 103 046601
[14] Gullans M et al 2010 Phys. Rev. Lett. 104 226807
[15] Rudner M S and Levitov L S 2007 Phys. Rev. Lett. 99 036602
[16] Qassemi F, Coish W A and Wilhelm F K 2009 Phys. Rev. Lett. 102 176806
[17] Khaetskii A, Loss D and Glazman L 2003 Phys. Rev. B 67 195329
[18] Slichter C P 1963 Principles of Magnetic Resonance (New York: Harper and Row)
[19] Blum K 1996 Density Matrix Theory and Applications (New York: Plenum)
[20] Kim J H, Vagner I D and Xing L 1994 Phys. Rev. B 49 16777
[21] Erlingsson S I, Nazarov Y V and Fal’ko V I 2001 Phys. Rev. B 64 195306
[22] Prada M, Blick R H and Joynt R 2008 Phys. Rev. B 77 115438
[23] Hüttel A K et al 2004 Phys. Rev. B 69 073302
[24] Strogatz S H 2000 Nonlinear Dynamics and Chaos (Cambridge: Westview Press)