The Frontiers of Observational Cosmology and the Confrontation with Theory

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Abstract. The current state of observational cosmology and the confrontation with theory is presented. The review is divided into the following sections:
- Basic observations on which the models are based.
- Testing the basic assumptions made in the construction of the standard cosmological models.
- Structure formation in the standard models
- Observational tests of the standard models - the confrontation with observation
- Basic problems and approaches to their solution
- Future challenges - the ESA EUCLID mission is given as an example.

1. Introduction
This brief summary of the current status of observational cosmology and the confrontation with theory is based upon the contents of my book *Galaxy Formation* to which reference should be made for many more details of the observations, their interpretation and the underlying astrophysical and cosmological theory (Longair 2008). For illustrative purposes, I use the following values of the cosmological parameters: \( \Omega_0 = 0.3, \) \( \Omega_\Lambda = 0.7, \) \( h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1} = 0.7. \) The dangerous bend sign indicates areas where care is needed.

2. Basic observations
Nowadays astrophysical cosmology begins with observations of the Cosmic Microwave Background Radiation as observed by the Cosmic Background Explorer (COBE) and the Wilkinson Microwave Anisotropy Probe (WMAP). The COBE observations showed that the spectrum of the background radiation is very precisely that of a black-body at temperature \( T = 2.726 \text{ K}. \) A perfect dipole component is detected over the whole sky, corresponding to the motion of the Earth through a frame of reference in which the radiation would be perfectly isotropic. The COBE map of the whole sky had angular resolution of 7\(^\circ\) while the more recent WMAP observations had a higher angular resolution of 0.3\(^\circ\). Both maps display the remarkable isotropy of the background cosmological signal over the whole sky once the Galactic foreground emissions have been subtracted. The large-scale distribution of radiation over the sky is isotropic to better than one part in \( 10^5 \) over the whole sky. Fluctuations in intensity are observed at about the level \( \Delta T/T \sim 10^{-5} \) and these provide key information about cosmological parameters and the development of structure in the Universe. It is wholly convincing that this background
radiation originated at a last scattering surface at a redshift \( z \approx 1000 \), or scale factor \( a \approx 10^{-3} \), at the epoch of recombination of the primordial plasma.

The next issue is the homogeneity of the Universe. This has been established by large surveys of galaxies, starting with the local distribution determined by Geller and Huchra. Their famous picture shows that locally the galaxy distribution is highly inhomogeneous with huge walls and holes, or voids, on physical scales up to about 50–100 Mpc. The structure is ‘sponge-like’, the material of the sponge corresponding to the location of the galaxies. The important feature of this structure is that the material of the sponge is continuously connected as is the distribution of holes – this provides evidence for the gaussianity of the primordial fluctuations from which the structure formed. These structures have now been extended to five times greater distances at the present epoch by the 2dF and SDSS surveys, each containing over 200,000 galaxies. These surveys show the same type of cellular structure seen nearby and statistically, although the distribution of galaxies is highly irregular, the degree of irregularity is found to be the same at the present epoch out to redshifts \( z \sim 0.25 \). The power spectrum of the irregularities is of power-law form, \( n(r) = n_0[1 + \xi(r)] \) with \( \xi(r) \propto r^{-1.8} \) on scales up to about 10 Mpc; on larger scales, the amplitude of the power spectrum decreases more rapidly than this.

For future high precision cosmological studies, it should be noted that one consequence of the presence of this structure is that we observe the distant Universe through a distorting gravitational screen. These corrections are very small at the moment, but many of the future cosmological tests require exquisite control of all the uncertainties in the observations and then these distorting effects should not be ignored.

Finally, we need the redshift-distance relation for galaxies, the *Hubble diagram*. Suffice to say that all classes of galaxy follow the same Hubble’s law \( v = H_0 r \), where \( H_0 \) is Hubble’s constant. Combined with the observed isotropy and homogeneity of the Universe, this means that the Universe as a whole is expanding uniformly.

3. Basic Assumptions
The standard models are based upon two assumptions and a theory of gravity:

- The *Cosmological Principle* states that we are not located at any special location in the Universe. Combined with the observations that the Universe is isotropic, homogeneous and uniformly expanding on a large scale, this leads to the *Robertson-Walker metric*.
- *Weyl’s Postulate* is the statement that the world lines of particles meet at a singular point in the finite or infinite past. This solves the clock synchronisation problem and means that there is a unique world-line passing through every point in space-time.
- *General Relativity* enables us to relate the energy-momentum tensor \( T_{ab} \) of the cosmic fluid to the geometrical properties of space-time.

The Robertson-Walker metric can be written in the following form:

\[
\text{d}s^2 = \text{d}t^2 - \frac{a^2(t)}{c^2} \left[ \text{d}r^2 + \mathcal{R}^2 \sin^2(r/\mathcal{R})(\text{d}\theta^2 + \sin^2\theta \text{d}\phi^2) \right].
\]  

\( t \) is *cosmic time* as measured by a clock carried by a fundamental observer and \( r \) is the *comoving radial distance coordinate* which is fixed to a galaxy for all time. The metric contains one unknown function \( a(t) \), the *scale factor*, and the constant \( \mathcal{R} \), the radius of curvature of the geometry of the Universe at the present epoch. \( a(t) \) is normalised to unity at the present epoch. It follows from this metric that the redshift \( z = (\lambda_{\text{obs}} - \lambda_{\text{em}})/\lambda_{\text{em}} \) is directly related to the scale-factor through the relation \( a(t) = (1+z)^{-1} \), where \( \lambda_{\text{em}} \) is the emitted wavelength and \( \lambda_{\text{obs}} \) the observed wavelength.

A key test of the Robertson-Walker metric is that the same formula which describes the redshift of spectral lines should also apply to time intervals in the emitted and received reference
frames. This test has now been extended to cosmological redshifts $z \approx 0.8$ using Type 1a supernova which have remarkably similar light curves, particularly when account is taken of the observed luminosity-width correlation. A clear time dilation effect has been observed which is exactly proportional to $(1 + z)$, as predicted by the Robertson-Walker metric.

A second test concerns the radiation temperature of the Cosmic Microwave Background Radiation as a function of redshift. In the case of black-body radiation, the form of the spectrum is preserved as the Universe expands, but the radiation temperature $T_r$ changes with redshift as $T_r = T_0 (1 + z)$. The temperature of the background radiation can be inferred from observations of fine structure lines in absorption lines systems in the spectra of distant quasars. Photons of the background radiation excite the fine-structure levels of the ground state of neutral carbon (CI) atoms and the relative strengths of the absorption lines originating from the ground and first excited states are determined by the temperature of the background radiation. This experiment requires very high quality absorption spectra of large redshift quasars. For the quasars Q1331+170 and QSO 0013-004, absorption systems at redshifts $z \sim 1.8$, the inferred radiation temperatures are about $7 - 8$ K, consistent with the expected relation. More recently, in the spectrum of the quasar PSS J1443+2724, there is an absorption system at redshift $z_{\text{abs}} = 4.224$. The expected radiation temperature of the background radiation at this redshift is $14.2$ K is consistent with the observed excitation of the ground and first fine-structure excited states of CI.

How good is general relativity? The present status is splendidly reviewed by Will (2006) in his on-line article. There is no evidence for departures from Einstein’s Equivalence Principle at a level better than about one part in $10^{12}$. Concerning the deviations from general relativity, the four classic tests show consistency with the standard theory in terms of the Parameterised Post-Newtonian (PPN) parameters $\beta$ and $\gamma$. Specifically,

- The gravitational redshift of electromagnetic waves in a gravitational field has been measured using hydrogen masers in rocket payloads. These confirm the prediction of general relativity at the level of about 5 parts in $10^5$.
- The Advance of the Perihelion of Mercury. Continued observations of Mercury by radar ranging have established the advance of the perihelion of its orbit to about 0.1% precision with the result $d\omega/dt = 42.98(1 \pm 0.001)$ arcsec per century. General relativity predicts a value of $d\omega/dt = 42.98$ arcsec per century.
- Gravitational deflection of electromagnetic waves by the Sun has been measured by Very Long Baseline Interferometry and the values found are $(1 + \gamma)/2 = 0.99992 \pm 0.00023$.
- The time delay of electromagnetic waves propagating through a varying gravitational potential, the Shapiro test, has been carried out by the Cassini spacecraft which found a time-delay corresponding to $(\gamma - 1) = (2.12.3) \times 10^{-5}$. Hence the coefficient $(1 + \gamma)/2$ must be within at most 0.0012 percent of unity.

The binary pulsar systems have proved to be very important in testing various aspects of general relativity. In binary pulsars such as PSR 1913+16, the pulsar is one of a pair of neutron stars in binary orbits about their common centre of mass. The radio pulses are assumed to be due to beams of radio emission from the poles of the magnetic field distribution. Many parameters of the binary orbit and the masses of the neutron stars can be measured with very high precision by accurate timing measurements. For this binary pulsar, the kinematics of the orbits are entirely consistent with general relativity and in addition the binary pulsar emits gravitational radiation leading to a speeding up of the stars in the binary orbit. The observed speeding up of the pulsars is entirely consistent with the predicted rate of gravitational radiation of the binary system. Even more impressive is the double pulsar system PSR J0737-3039 in which both neutron stars are pulsars. In addition, the period of the binary is only 2.4 hours and so the system enables general relativity to be tested with even higher precision. After only 2.5
years, the double pulsar enabled general relativity to be tested at a level similar to that of PSR 1913+16. In addition, the eclipse phenomenon observed in this double pulsar enabled the relativistic spin precession of PSR J0737-3039B to be measured with high statistical significance. The spin precession rate was found to be in agreement with general relativity.

Various solar system, astrophysical and cosmological tests have been made to find out if the gravitational constant has varied over cosmological time-scales. For binary pulsar data, the bounds are dependent upon the theory of gravity in the strong-field regime and on the neutron star equation of state. The big-bang nucleosynthesis bounds assume specific forms for time dependence of the gravitational constant. Lunar laser ranging provides the best limit of $\dot{G}/G \leq (4 \pm 9) \times 10^{-13}$ and so there can have been little variation in the value of the gravitational constant over the last $10^{10}$ years.

There has been some debate as to whether or not the fine-structure constant $\alpha$ has changed very slightly with cosmic epoch from observations of fine-structure lines in large redshift absorption line systems in quasars. The Australian observers reported a small decrease in the value of $\alpha$, $\Delta \alpha/\alpha \approx 0.5 \times 10^{-5}$ at redshifts $1-2.5$ compared with the value at the present epoch. The ESO observers found little evidence for such a change.

4. The Growth of Small Density Perturbations

Within the background of the standard cosmological models of general relativity, a key role is played by the development of small perturbations with cosmic epoch. Many of the issues can be understood from the simplest solutions for a non-relativistic fluid on scales less than the cosmological horizon scale. The differential equations which come out of a simple perturbation analysis are:

$$\frac{d\Delta}{dt} = -\nabla \cdot \delta v,$$

$$\frac{d^2\Delta}{dt^2} + \left(\frac{\dot{a}}{a}\right) \frac{d\Delta}{dt} = \Delta \left(4\pi G \rho_0 a^2 - k_c^2 c_s^2 \right),$$

where $\Delta = \delta \rho/\rho_0$ is the density contrast and $c_s^2 = \partial p/\partial \rho$ is the adiabatic sound speed; $k_c$ is the comoving wave number. Equation (2) relates the rate at which the density contrast develops to the peculiar velocity $\delta v$ associated with the collapse of the perturbation. Equation (3) results in a dispersion relation for the growth rate of the small perturbations. The solutions correspond to the Jeans’ instability in an expanding medium. Two limiting solutions are useful. In the critical Einstein-de Sitter model ($\Omega_0 = 1; \Omega_\Lambda = 0$), the perturbation grows linearly with scale factor, $\Delta \propto a = (1+z)^{-1}$. In the empty model ($\Omega_0 = 0; \Omega_\Lambda = 0$), there is no growth of the perturbations $\Delta = constant$. In the more general case, the evolution of the density contrast depends upon both $\Omega_0$ and $\Omega_\Lambda$ and, if the evolution of $\Delta$ with redshift could be measured observationally, this would provide an important way of determining cosmological parameters.

These solutions illuminate the origin of the basic problem of structure formation. Throughout the matter-dominated era, the growth rate of perturbations on large physical scales is $\Delta \propto a = (1+z)^{-1}$. Since galaxies and astronomers certainly exist at the present day $z = 0$, it follows that $\Delta \geq 1$ at $z = 0$ and so, at the last scattering surface, $z \approx 1,000$, fluctuations must have been present with amplitude at least $\Delta = 10^{-3}$. Therefore,

- The slow growth of density perturbations is the source of a fundamental problem in understanding the origin of galaxies – large-scale structures did not condense out of the primordial plasma by exponential growth of infinitesimal statistical perturbations.

1 Examples of the evolution of the density contrast $\Delta$ with cosmic epoch are given in my book *Galaxy Formation* (2008).
Table 1.

| Parameter          | Definition                                           | Standard value | Estimated value |
|--------------------|------------------------------------------------------|----------------|-----------------|
| $h$                | Hubble’s constant                                    |                | 0.72            |
| $\omega_B = \Omega_B h^2$ | baryon density parameter                             | 0.0223         |                 |
| $\omega_D = \Omega_D h^2$ | cold dark matter density parameter                   | 0.127          |                 |
| $\Omega_{\Lambda}$ | dark energy density parameter                        |                | 0.72            |
| $\Omega_k = \Omega_{\Lambda} + \Omega_0$ | space curvature                                      | $\approx 1$   |                 |
| $n_s$              | scalar spectral index                                 | $n_s = 1$      | $\approx 1$     |
| $\tau$             | reionisation optical depth                            | 0.09           |                 |
| $\sigma_8$         | density variance in 8 Mpc spheres                     | 0.74           |                 |
| $w$                | dark energy equation of state                         | $w = -1$       |                 |
| $f_{\nu} = \Omega_{\nu}/\Omega_D$ | massive neutrino fraction                            | $f_{\nu} = 0$ |                 |
| $N_{\nu}$          | number of relativistic neutrino species              | $N_{\nu} = 3$ |                 |
| $\Delta^2_R$       | amplitude of curvature perturbations                 |                |                 |
| $r$                | tensor-scalar ratio                                   |                |                 |
| $A_s$              | amplitude of scalar power-spectrum                   |                |                 |
| $\alpha = d \ln K/d \ln k$ | running of scalar spectral index                    | $\alpha = 0$  |                 |
| $A_{SZ}$           | SZ marginalization factor                             |                |                 |
| $b$                | bias factor                                           |                |                 |
| $z_s$              | weak lensing source redshift                         |                |                 |

- Because of the slow development of the density perturbations, we have the opportunity of studying the formation of structure on the last scattering surface at a redshift $z \approx 1,000$.

The methodology is therefore to follow the evolution of an initial power-spectrum of density perturbations from the early Universe when it was radiation dominated, through the epoch of equality of radiation and matter energy densities to the epoch of recombination at $z \approx 1000$ at which temperature perturbations are imprinted on the Cosmic Microwave Background Radiation. Typically the numerical simulations begin with a power-law spectrum in the early Universe on scales from $10^5$ to $10^{20} M_{\odot}$. The power-law power spectrum of the form $P(k) \propto k$ is known as a *Harrison-Zeldovich power spectrum*.

To cut a long story short, these computations concerning the predicted power-spectrum of the distribution of galaxies and of fluctuations in the Cosmic Microwave Background Radiation have now become one of the most powerful approaches for the determination of cosmological parameters.

5. The Values of the Cosmological Parameters

The remarkable result of the last ten years is that the computations described in the last paragraph above and the observed power spectra are in highly satisfactory agreement with a number of quite independent ways of estimating the cosmological parameters. In Table 1, a list of cosmological parameters is given with estimates of their values which are consistent with observations of the power spectrum of the Cosmic Microwave Background Radiation and its polarisation. The estimates of these parameters are now known to better than 10% and in many cases with much greater precision. In the lower half of the table a list of additional cosmological parameters is given and these can be constrained or determined by future ground- and space-based observations.
Specifically,

- **Supernovae of Type 1a** Type 1a supernovae have dominated methods for extending the redshift-distance relation to large redshifts. They are brightest supernovae and are observed to have remarkably standard properties, particularly when corrections are made for the observed luminosity-width relation. It should be noted that the precise mechanism of the supernova explosion is not yet established. Despite this concern, the empirical evidence is impressive. The ESSENCE project has the objective of measuring the redshifts and distances of about 200 supernovae. The Supernova Legacy Project aims to obtain distances for about 500 supernovae. The observations are consistent with a finite and positive value of the cosmological constant, the best fitting values being \( \Omega_M = 0.73 \); the total density parameter in baryonic and cold dark matter form \( \Omega_0 = \Omega_D + \Omega_B \approx 0.27 \).

- **The power spectrum of galaxies from the Sloan Digital Sky Survey and the AAO 2dF galaxy survey** There is evidence for residual acoustic oscillations in the galaxy power spectrum corresponding to the first maxima observed in the power-spectrum of the Cosmic Microwave Background Radiation – these are referred to as *Baryonic Acoustic Oscillations*.

- **Mass density of the Universe from the infall velocities of galaxies into large scale structures** The two-dimensional correlation function for galaxies selected from the 2dF Galaxy Redshift Survey in the radial and tangential directions on the sky shows a flattening due to the infall of galaxies into large-scale density perturbations. The inferred overall density parameter is \( \Omega_0 = 0.25 \).

- **Formation of the Light Elements by Primordial Nucleosynthesis** The predicted primordial abundances of the light elements is a sensitive function of the present baryon-to-photon ratio \( \eta = 10^{10} n_B / n_\gamma = 274 \Omega_B h^2 \). Steigman finds a best fitting value \( \Omega_B h^2 = 0.022^{+0.003}_{-0.002} \). This can be compared with the independent estimate from the power-spectrum of fluctuations of the Cosmic Microwave Background Radiation \( \Omega_B h^2 = 0.0224 \pm 0.0009 \). The only concern is that recent estimates of the observed abundances of the light elements may differ slightly from the predictions of standard model.

- **Cosmic time scale from the theory of stellar evolution and nucleocosmochronology** Ages of the oldest globular clusters from studies of their Hertzsprung-Russell diagrams for have resulted in ages of typically \( T_0 = 15 \pm 2.4 \) (stat) \(^{+4}_{-1} \) (syst) Gy by Bolte and \( T_0 = (11.5 \pm 1.3) \) Gy by Chaboyer. An interesting case is the star CS 22892-052 which has iron abundance 1000 times less than the solar value. A number of species never previously observed in such metal-poor stars were detected, as well a single line of thorium. A lower limit to the age of the star is \( (15.2 \pm 3.7) \) Gy. Schramm found a lower limit to the age of the Galaxy of 9.6 Gy from nucleocosmochronology. His best estimates of the age of the Galaxy are somewhat model-dependent, but typically ages of about \( (12 - 14) \) Gy.

- **The value of Hubble’s constant** The controversies of the 1970s and 1980s have been resolved thanks to a very large effort by many observers to improve knowledge of the distances to nearby galaxies. The final result of the HST key project was \( H_0 = 72 \pm 7(1\sigma) \) km s\(^{-1}\) Mpc\(^{-1}\). From gravitational time delays a value \( H_0 = 68 \pm 6 \) (stat) \( \pm 8 \) (syst) km s\(^{-1}\) Mpc\(^{-1}\) was found. The Sunyaev-Zeldovich effect in X-ray clusters of galaxies gave the result \( H_0 = 76.9 \pm 4 \) (stat) \( \pm 9 \) (syst) km s\(^{-1}\) Mpc\(^{-1}\).

- **The statistics of gravitational lenses** In the Cosmic Lens All Sky Survey (CLASS), a very large sample of flat spectrum radio sources was imaged by the Very Large Array, Very Long Baseline Array and the MERLIN long-baseline interferometer. 13 sources were multiply imaged out of a total sample of 8958 radio sources. The numbers are consistent with flat world models with density parameter \( \Omega_0 = 0.31 \pm 0.25 \pm 0.12 \) (syst). For a flat universe with dark energy equation of state of the form \( p = w \rho c^2 \), they found an upper limit \( w < -0.55 \pm 0.15 \) consistent with \( w = -1 \).
The power-spectrum and polarisation of the Cosmic Microwave Background Radiation on their own provide excellent estimates of the values of the parameters in the top half of Table 1 but it is striking how well these now agree with the above independent estimates. The caution sign means that there are other parameters may play a role in determining the exact form of the spectrum of the temperature fluctuations of the Cosmic Microwave Background Radiation and these should not be ignored. Here are a few examples of the parameters which might impact the estimates of the values of the cosmological parameters included in Table 1.

- The value of $w$ is found to be close to $-1$, which would correspond precisely to Einstein’s cosmological constant into the field equations of general relativity. If the value differed from unity, this would indicate new physics beyond the standard model of Einsteinian gravity.

- We now know that the known types of standard neutrinos have non-zero rest mass and so at some level their presence must influence the predicted spectrum of the fluctuations in the Cosmic Microwave Background Radiation.

- The presence of primordial gravitational waves would influence the spectrum of perturbations on large angular scales. Already, constraints can be placed upon the tensor to scalar ratio $r$ of these waves. Primordial gravitational waves with spectrum similar to that of scalar perturbations on large angular scales is a prediction of the standard inflationary picture of the early Universe. They could be detected through their characteristic B-mode quadrupole signature at low levels in the Cosmic Microwave Background Radiation, but it is a very demanding observation. In addition, it has to be detected in the presence of a similar B-mode quadrupole signature induced by gravitational lensing by large scale structures.

- A further concern is whether or not it is safe to assume that the initial power-spectrum from which large-scale structures formed was indeed of power-law form. It will be argued that the fact that the simplest inflationary models predict a Harrison-Zeldovich spectrum is compelling evidence that that picture should be taken seriously, but of course, there may well be good reasons why the actual Universe is more complex than that simple picture.

These are all issues which can be addressed by observation, but they are very challenging programmes at and beyond the current limits of technology. I have no doubt, however, that these will be carried out if the appropriate investment is made.

6. The Basic Problems

Granted these successes, there remain a number of fundamental questions.

- Why is the Cosmic Microwave Background Radiation so isotropic? At earlier cosmological epochs, the particle horizon scale $r \sim ct$ encompassed less and less mass and so the scale over which particles could be causally connected became smaller and smaller. On the last scattering layer at redshift $z = 1000$, the distance $r = 3ct$ corresponds to an angle $\theta = 2^\circ$ on the sky. Regions of the sky separated by greater angular distances could not have been in causal communication. Why then is the sky isotropic to one part in $10^5$ on all greater angular scales?

- Why is the Universe geometricaly flat, $\Omega_k = 0$? According to the standard world models, if the Universe were set up with a value of the density parameter differing even slightly from the critical value $\Omega_0 = 1$, it would diverge very rapidly from this value at later epochs.

- Why is the Universe baryon-asymmetric with $N_\gamma/N_B = 1.6 \times 10^9$? If photons were neither created nor destroyed, this ratio is conserved as the Universe expands. In the early Universe, baryon-antibaryon pair production took place with the result that there must have been a very small asymmetry in the baryon-antibaryon ratio in the very early Universe if we are to end up with the correct photon-to-baryon ratio at the present day.
• Why do the cosmological parameters take the values $\Omega_\Lambda = 0.72$, $\Omega_D = 0.23$, $\Omega_B = 0.05$. Furthermore, why is $\Omega_\Lambda$ about $10^{120}$ smaller than the value expected from quantum field theory? It is a surprise that $\Omega_\Lambda$ and $\Omega_0$ are of the same order of magnitude at the present epoch. The matter density evolves with redshift as $a^{-3}$, while the dark energy density is unchanged with cosmic epoch. Why then do we live at an epoch when they have more or less the same values?

• What is the origin of the initial power spectrum of perturbations and why is it of power-law form with power spectrum $P(k) \propto k^n$? The amplitudes of the density perturbations which led to the formation of galaxies had to be of finite amplitude, $\Delta = \delta \rho / \rho \sim 10^{-5}$, on a very wide range of mass scales when they came through the horizon.

I have suggested five possible solutions to these problems.

• That is just how the Universe is – the initial conditions were set up that way.

• There are only certain classes of Universe in which intelligent life could have evolved. This approach involves the *Anthropic Cosmological Principle* according to which the Universe is as it is because we are here to observe it.

• The inflationary scenario for the early Universe.

• Seek clues from particle physics and extrapolate to the earliest phases of the Universe.

• Something else we have not yet thought of, what I sometimes refer to as the *Donald Rumsfeld model* – the things we don’t know we don’t know. This would certainly involve new physical concepts.

There is some merit in each of these approaches. My suspicion is that the answer almost certainly lies in the fifth bullet, but in the mean time the most promising approach seems to be the inflationary picture which surprisingly has the potential to provide answers to at least some of the basic problems.

In the standard inflationary scenario, the Universe went through a period of exponential expansion in its very early phases, in a typical realisation the expansion lasting for about 100 e-folding times from the time the Universe was about $10^{-34}$ s old. This has two effects. First, particles are driven beyond their local particle horizons. If there were sufficient e-folding times, particles which were originally in causal contact in the very early Universe and so could homogenise the material of the Universe, were driven far beyond their causal horizons by the present epoch, solving the horizon problem. Second, the inflationary expansion straightens out the geometry of the Universe, however complex it may have been in the early Universe. The Universe is driven to flat geometry by the end of the inflationary era. The inflationary scenario thus solves the flatness and homogeneity problems.

The considerations of the last paragraph may be thought of as inflation without physics. To put more physics into the model, we appeal to the properties of scalar fields, similar to those which may be responsible for the present exponential expansion of the Universe under the influence of the cosmological constant. Their equation of state can be written

$$\varrho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad ; \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi).$$

(4)

If the first terms in $\dot{\phi}^2$, the kinetic energy terms, are very small compared with the potential energy terms, we obtain a negative pressure equation of state $p_\phi = -\varrho_\phi c^2$. The form of the potential $V(\phi)$ has to be chosen very carefully so that the exponential expansion can last long enough. This is known as slow-roll inflation so that the homogenising effect of the expansion can encompass the whole Universe. Despite a huge amount of theoretical effort, there is not yet a genuinely physical theory of inflation. It looks at first as though we have not made much
progress since we are putting in by hand the sort of potential which results in the Universe we observe today.

The inflationary picture has, however, a remarkable property which was not expected when it was first proposed. We need to consider the quantum fluctuations which are necessarily present in the scalar fields during the inflationary era in the very early Universe. Amazingly, in the simplest model of inflation, the fluctuations in the scalar field result in exactly the Harrison-Zeldovich power-spectrum, \( P(k) \propto k \). In my book *Galaxy Formation*, I demonstrate how this result can be derived using simple concepts from elementary quantum mechanics. As remarked by Liddle and Lyth in their book *Cosmological Inflation and Large Scale Structure*,

‘Although introduced to resolve problems associated with the initial conditions needed for the Big Bang cosmology, inflation’s lasting prominence is owed to a property discovered soon after its introduction. It provides a possible explanation for the initial inhomogeneities in the Universe that are believed to have led to all the structures we see, from the earliest objects formed to the clustering of galaxies to the observed irregularities in the microwave background.’

7. Some Future Challenges

The considerations of this review result in a large number of observational and theoretical challenges. Here is a selection of these.

- The PLANCK mission of ESA, now in its second year in orbit, will result in an order of magnitude improvement in the determination of cosmological parameters, thus providing yet more precise estimates of these and providing yet more stringent tests of what has become the standard cosmological model.
- Is the physics of the standard model accurately understood at the 1% level? If the objective is to improve yet further the accuracy with which we know the cosmological parameters, say to the 1% level, we need to understand the physics of the standard model to better than 1%. This is a serious theoretical challenge.
- Does the evolution of galaxies and large scale structures really agree with the predictions of the standard Λ Cold Dark Matter model?
- Is the dark energy really the cosmological constant? Or, to put it another way, what is the value of \( w \) in the equation of state of the dark energy which drives the present exponential expansion of the Universe?
- Does the dark energy vary with cosmic epoch or is it a constant as expected if it were simply the cosmological constant?
- What clues can be derived from the Large Hadron Collider at CERN – what are the implications of detecting/not detecting Higgs particles? Are dark matter candidates found in the LHC experiments?
- The experiments to make direct detections of dark matter particles have made spectacular progress over the last few years, the present limits already excluding some of the potential supersymmetric candidates for the dark matter.

As an example of just how difficult some of these challenges are, let me give the example of the EUCLID mission of the European Space Agency which is one of the candidates for the next medium-sized mission. High precision dark energy measurements require a combination of two or more cosmological probes. Euclid aims to utilise the most promising dark energy probes: an all-sky survey of weak lensing and huge samples of galaxy redshifts to measure precisely the baryon acoustic oscillations in the distribution of galaxies. Individually, these are the two most powerful measures of dark energy. In combination they will control and measure a wide range
of systematics effects which would affect either of them separately. The combination of these experiments will measure the two Newtonian potentials $\phi(k, z)$ and $\psi(k, z)$, which describe a perturbed $\Lambda$ Cold Dark Matter Universe. In standard general relativity, $\phi(k, z) = \psi(k, z)$. The weak lensing and baryon acoustic oscillations measurements depend in different ways upon $\phi(k, z)$ and $\psi(k, z)$ - weak lensing depends upon both $\phi(k, z)$ and $\psi(k, z)$ whereas the baryon acoustic oscillations depends only on $\psi(k, z)$. Thus, Euclid enables us to test if General Relativity is the best description of gravity on the scale of the largest perturbations we can measure in the Universe.

The way in which the experiment will be carried out is that the weak lensing will reconstruct directly the distribution of the dark matter and the evolution of the growth rate of dark matter perturbations with redshift. The baryon acoustic oscillations act as standard rods, determine $P(k)$ and provide a measure of $H(z)$ and hence $w(z)$. They also map out the evolution of the baryonic component of the Universe. Together, these enable many systematic effects to be controlled, for example, intrinsic alignments in weak lensing and bias factors in baryon acoustic oscillations. The value of $w$ will be measured to 1% and the variation of $w(z)$ with redshift to 10%. This is a very large undertaking, but it is only with efforts on this scale that the future challenges can be addressed.

References
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