Linear Accelerators

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LINAC APPLICATIONS

 Injectors for synchrotrons

 Medical applications: radiotherapy

 Industrial applications

 Nuclear waste treatment and controlled fission for energy production (ADS)

 Spallation sources for neutron production

 Material testing for fusion nuclear reactors

 National security

 Material treatment

 Ion implantation

 Material/food sterilization

 ~10^4 LINACs operating around the world

 Free Electron Lasers

 LINACs operating around the world
LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.
The overall LINAC has to be designed to obtain the desired beam parameters in terms of:

- Output energy/energy spread
- Beam current (charge)
- Long. and transverse beam dimensions/divergence (emittance)

Having, in general, constraints in terms of:

- Space
- Cost
- Power consumption
- Available power sources
LORENTZ FORCE: ACCELERATION AND FOCUSING

The basic equation that describes the acceleration/bending/focusing processes is the Lorentz Force. Particles are accelerated through electric fields and are bended and focused through magnetic fields.

\[
\frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)
\]

\( \vec{p} = \text{momentum} \)
\( m = \text{mass} \)
\( \vec{v} = \text{velocity} \)
\( q = \text{charge} \)

**ACCELERATION**

To accelerate, we need a force in the direction of motion

**BENDING AND FOCUSING**

2\textsuperscript{nd} term always perpendicular to motion => no energy gain

**Longitudinal Dynamics**

**Transverse Dynamics**
The first historical linear particle accelerator was built by the Nobel prize Wilhelm Conrad Röntgen (1900). It consisted in a vacuum tube containing a cathode connected to the negative pole of a DC voltage generator. Electrons emitted by the heated cathode were accelerated while flowing to another electrode connected to the positive generator pole (anode). Collisions between the energetic electrons and the anode produced X-rays.

The energy gained by the electrons travelling from the cathode to the anode is equal to their charge multiplied the DC voltage between the two electrodes.

\[
\frac{d\hat{p}}{dt} = q \vec{E} \quad \Rightarrow \quad \Delta E = q\Delta V
\]

\[ \hat{p} = \text{momentum} \]
\[ q = \text{charge} \]
\[ E = \text{energy} \]

Particle energies are typically expressed in electron-volt [eV], equal to the energy gained by 1 electron accelerated through an electrostatic potential of 1 volt: 1 eV=1.6x10^{-19} J
PARTICLE VELOCITY VS ENERGY: LIGHT AND HEAVY PARTICLES

Light particles (as electrons) are practically fully relativistic ($\beta \approx 1$, $\gamma >> 1$) at relatively low energy and reach a constant velocity (~c). The acceleration process occurs at constant particle velocity.

Heavy particles (protons and ions) are typically weakly relativistic and reach a constant velocity only at very high energy. The velocity changes a lot during the acceleration process.

This implies important differences in the technical characteristics of the accelerating structures. In particular for protons and ions we need different types of accelerating structures, optimized for different velocities and/or the accelerating structure has to vary its geometry to take into account the velocity variation.
To increase the achievable maximum energy, Van de Graaff invented an electrostatic generator based on a **dielectric belt** transporting positive charges to an isolated electrode hosting an **ion source**. The positive ions generated in a large positive potential were accelerated toward ground by the static electric field.

**LIMITS OF ELECTROSTATIC ACCELERATORS**

DC voltage as large as \(\sim 10\) MV can be obtained (E\(\sim 10\) MeV). The main limit in the achievable voltage is the **breakdown** due to **insulation** problems.

**APPLICATIONS OF DC ACCELERATORS**

DC particle accelerators are in operation worldwide, typically at \(V<15\) MV (\(E_{\text{max}}=15\) MeV), \(I<100\) mA.

They are used for:

- material analysis
- X-ray production,
- ion implantation for semiconductors
- first stage of acceleration (particle sources)

750 kV Cockcroft-Walton Linac2 injector at CERN from 1978 to 1992
Basic idea: the particles are accelerated by the electric field in the gap between electrodes connected alternatively to the poles of an AC generator. This original idea of Ising (1924) was implemented by Wideroe (1927) who applied a sine-wave voltage to a sequence of drift tubes. The particles do not experience any force while travelling inside the tubes (equipotential regions) and are accelerated across the gaps. This kind of structure is called Drift Tube LINAC (DTL).

If the length of the tubes (or, equivalently, the distances between the centers of the accelerating gaps) increases with the particle velocity during the acceleration such that the time of flight between gaps is kept constant and equal to half of the RF period, the particles are subject to a synchronous accelerating voltage and experience an energy gain of $\Delta E = q\Delta V$ at each gap crossing.

In principle a single RF generator can be used to indefinitely accelerate a beam, avoiding the breakdown limitation affecting the electrostatic accelerators.

The Wideroe LINAC is the first RF LINAC.
We consider the acceleration between two electrodes in DC.

Energy-momentum relation

\[ E^2 = E_0^2 + p^2 c^2 \Rightarrow 2EdE = 2pdpc^2 \Rightarrow dE = v \frac{mc^2}{E} dp \Rightarrow dE = vdp \]

Lorentz force

\[ \frac{dp}{dt} = qE_z \Rightarrow \frac{dp}{dz} = qE_z \Rightarrow \frac{dE}{dz} = qE_z \left( \text{and also} \frac{dW}{dz} = qE_z \right) \]

\[ W = E - E_0 \]

\[ \Rightarrow \Delta E = \int_{gap}^{gap} \frac{dE}{dz} dz = \int_{gap} qE_z dz \Rightarrow \Delta E = q\Delta V \]

Rate of energy gain per unit length

Energy gain per electrode pair
We consider now the acceleration between two electrodes fed by an RF generator.

\[ \Delta V = V_{RF} \cos(\omega_{RF} t) \quad \omega_{RF} = 2\pi f_{RF} = \frac{2\pi}{T_{RF}} \]

\[ \Rightarrow \Delta E = q \hat{V}_{acc} \cos(\omega_{RF} t_{inj}) \]

Only these particles are accelerated.

These particles are not accelerated and basically are lost during the acceleration process.

**DC acceleration**

**RF acceleration**

**Bunched beam** (in order to be synchronous with the external AC field, particles have to be gathered in non-uniform temporal structure)
If now we consider a DTL structure, we have that at each gap the maximum energy gain is $\Delta E_n = qV_{\text{acc}}$ and the particle increase its velocity accordingly to the previous relativistic formulae. It is convenient to refer to the center of each gap as follows:

$$\Delta V = V_R F \cos(\omega_{RF} t)$$

$\Rightarrow$ In order to be synchronous with the accelerating field at the center of each gap, the distance between the centers of the gaps ($L_n$) has to be increased as:

$$t_n = \frac{L_n}{\bar{v}_n} = \frac{L_n}{T_R F} \Rightarrow L_n = \frac{1}{2} \bar{v}_n T_R F = \frac{1}{2} \frac{\beta_n c T_R F}{\lambda_{ss}} \Rightarrow L_n = \frac{1}{2} \frac{\beta_n \lambda_{RF}}{\lambda_{ss}}$$

$\bar{v}_n$=average particle velocity between the gap $n$ and $n+1$

$\Rightarrow$The energy gain per unit length (i.e. the average accelerating gradient times $q$) is given by:

$$\frac{\Delta E}{\Delta L} = \frac{qV_{\text{acc}}}{L_n} = \frac{2qV_{\text{acc}}}{\lambda_{RF} \beta_n}$$

[eV/m]
ACCELERATION WITH HIGH RF FREQUENCIES: RF CAVITIES

There are two important consequences of the previous obtained formulae:

\[ L_n = \frac{1}{2} \beta_n \lambda_{RF} \]

The condition \( L_n << \lambda_{RF} \) (necessary to model the tube as an equipotential region) requires \( \beta << 1 \). \( \Rightarrow \) The Wideröe technique can not be applied to relativistic particles.

\[ \frac{\Delta E}{\Delta L} = \frac{qV_{acc}}{L_n} = qE_{acc} = \frac{2qV_{acc}}{\lambda_{RF} \beta_n} \]

Moreover when particles get high velocities the drift spaces get longer and one looses on the efficiency. The average accelerating gradient \( (E_{acc} \ [V/m]) \) increase pushes towards small \( \lambda_{RF} \) (high frequencies).

High frequency, high power sources became available after the 2nd world war pushed by military technology needs (such as radar). Moreover, the concept of equipotential DT can not be applied at small \( \lambda_{RF} \) and the power lost by radiation is proportional to the RF frequency.

\( \Rightarrow \) The solution consists of enclosing the system in a cavity which resonant frequency matches the RF generator frequency.

\( \Rightarrow \) Each cavity can be independently powered from the RF generator

As a consequence we must consider accelerating structures different from drift tubes.
High frequency RF accelerating fields are confined in **cavities**.

The cavities are **metallic closed volumes** where the e.m. fields have a particular spatial configuration (resonant modes) whose components, including the accelerating field $E_z$, oscillate at some specific frequencies $f_{\text{RF}}$ (resonant frequency) characteristic of the mode.

The modes are excited by **RF generators** that are coupled to the cavities through waveguides, coaxial cables, etc...

The resonant modes are called **Standing Wave (SW) modes** (spatial fixed configuration, oscillating in time).

The spatial and temporal field profiles in a cavity have to be computed (analytically or numerically) **by solving the Maxwell equations** with the proper boundary conditions.
Alvarez's structure can be described as a special DTL in which the electrodes are part of a resonant macrostructure.

The DTL operates in 0 mode for protons and ions in the range $\beta = 0.05-0.5$ ($f_{RF} = 50-400$ MHz, $\lambda_{RF} = 6-0.7$ m) 1-100 MeV;

The beam is inside the “drift tubes” when the electric field is decelerating. The electric field is concentrated between gaps;

The drift tubes are suspended by stems;

Quadrupole (for transverse focusing) can fit inside the drift tubes.

In order to be synchronous with the accelerating field at each gap the length of the n-th drift tube $L_n$ has to be:

$$L_n = \beta_n \lambda_{RF}$$
**CERN LINAC 4:** 352 MHz frequency, Tank diameter 500 mm, 3 resonators (tanks), Length 19 m, 120 Drift Tubes, Energy: 3 MeV to 40 MeV, $\beta$=0.08 to 0.31 $\rightarrow$ cell length from 68mm to 264mm.

- Ion species H-
- Output energy 160 MeV
- Bunch frequency 352.2 MHz
- Max. rep.-rate 2 Hz
- Beam pulse length 400 s
- Max. beam duty cycle 0.08%
- Linac current 40mA
- Average current 0.032mA
HIGH β CAVITIES: CYLINDRICAL STRUCTURES
(electrons or protons and ions at high energy)

⇒When the β of the particles increases (>0.5) one has to use **higher RF frequencies** (>400-500 MHz) to increase the accelerating gradient per unit length

⇒the DTL structures became less efficient (effective accelerating voltage per unit length for a given RF power);

Real cylindrical cavity
(TM₀₁₀-like mode because of the shape and presence of beam tubes and couplers)

Cylindrical single (or multiple cavities) working on the TM₀₁₀-like mode are used

For a **pure cylindrical structure** (also called pillbox cavity) the first accelerating mode (i.e. with non zero longitudinal electric field on axis) is the **TM₀₁₀ mode**. It has a well known analytical solution from Maxwell equation.

\[
E_z = AJ_0 \left(2.405 \frac{r}{a}\right) \cos(\omega_{\beta} t)
\]

\[
H_{\theta} = A \frac{1}{Z_0} J_1 \left(2.405 \frac{r}{a}\right) \sin(\omega_{\beta} t)
\]

\[
f_{res} = \frac{2.405 c}{2\pi a}
\]
The shunt impedance is the parameter that qualifies the efficiency of an accelerating mode. The higher is its value, the larger is the obtainable accelerating voltage for a given power. Traditionally, it is the quantity to optimize in order to maximize the accelerating field for a given dissipated power:

\[ R = \frac{V_{acc}^2}{P_{diss}} \]  

**SHUNT IMPEDANCE PER UNIT LENGTH**

\[ r = \frac{(V_{acc}/L)^2}{P_{diss}/L} = \frac{E_{acc}^2}{P_{diss}} \]  

Example:

R ~ 1 MΩ  
P_{diss} = 1 MW  
V_{acc} = 1 MV  

For a cavity working at 1 GHz with a structure length of 10 cm we have an average accelerating field of 10 MV/m

NC cavity Q ~ 10^4  
SC cavity Q ~ 10^{10}
MULTI-CELL SW CAVITIES

*(electrons or protons and ions at high energy)*

- In a multi-cell structure there is **one RF input coupler**. As a consequence the **total number of RF sources is reduced**, with a **simplification of the layout** and reduction of the costs;

- The **shunt impedance is n time** the impedance of a single cavity

- They are **more complicated** to fabricate than single cell cavities;

- The fields of adjacent cells couple through the cell **irises** and/or through properly designed coupling **slots**.
MULTI-CELL SW CAVITIES: $\pi$ MODE STRUCTURES
(electrons or protons and ions at high energy)

- The N-cell structure behaves like a system composed by N coupled oscillators with N coupled multi-cell resonant modes.

- The modes are characterized by a cell-to-cell phase advance given by:

$$\Delta \phi_n = \frac{n\pi}{N-1} \quad n = 0,1,\ldots,N-1$$

- The multi cell mode generally used for acceleration is the $\pi$, $\pi/2$ and 0 mode (DTL as example operate in the 0 mode).

- The cell lengths have to be chosen in order to synchronize the accelerating field with the particle traveling into the structure at a certain velocity.

**EXAMPLE: 4 cell cavity operating on the $\pi$-mode**

$\Rightarrow$ For ions and protons the cell lengths have to be increased along the linac that will be a sequence of different accelerating structures matched to the ion/proton velocity.

$\Rightarrow$ For electron, $\beta=1$, $d=\lambda_{RF}/2$ and the linac will be made of an injector followed by a series of identical accelerating structures, with cells all the same length.
**π MODE STRUCTURES: EXAMPLES**

**LINAC 4 (CERN) PIMS (Pi Mode Structure) for protons:** $f_{RF} = 352$ MHz, $\beta > 0.4$

100 MeV
1.28 m
12 tanks

1.55 m
160 MeV

Each module has 7 identical cells

**European XFEL (Desy): electrons**

800 accelerating cavities
1.3 GHz / 23.6 MV/m

Cryomodule housing: 8 cavities, quadrupole and BPM

All identical $\beta = 1$
Superconducting cavities
MULTI-CELL SW CAVITIES: $\pi/2$ MODE STRUCTURES
(electrons or protons and ions at high energy)

$\Rightarrow$ It is possible to demonstrate that over a certain number of cavities (>10) working on the $\pi$ mode, the overlap between adjacent modes can be a problem (as example the field uniformity due to machining errors is difficult to tune).

$\Rightarrow$ The criticality of a working mode depend on the frequency separation between the working mode and the adjacent mode.

$\Rightarrow$ The $\pi/2$ mode from this point of view is the most stable mode. For this mode it is possible to demonstrate that the accelerating field is zero every two cells. For this reason, the empty cells are put of axis and coupling slots are opened from the accelerating cells to the empty cells.

$\Rightarrow$ this allow to increase the number of cells to >20-30 without problems.
TRAVELLING WAVE (TW) STRUCTURES
(electrons)

⇒ To accelerate charged particles, the electromagnetic field must have an electric field along the direction of propagation of the particle.

⇒ The field has to be synchronous with the particle velocity.

⇒ Up to now we have analyzed the standing standing wave (SW) structures in which the field has basically a given profile and oscillate in time (as example in DTL or resonant cavities operating on the TM_{010}-like).

\[ E_z(z, t) = E_{RF}(z) \cos(\omega_{RF} t) \]

⇒ There is another possibility to accelerate particles: using a travelling wave (TW) structure in which the RF wave is co-propagating with the beam with a phase velocity equal to the beam velocity.

⇒ Typically these structures are used for electrons because in this case the phase velocity can be constant all over the structure and equal to c. On the other hand it is difficult to modulate the phase velocity itself very quickly for a low β particle that changes its velocity during acceleration.
TW Cavities: Circular Waveguide and Dispersion Curve

In TW structures an e.m. wave with $E \neq 0$ travel together with the beam in a special guide in which the phase velocity of the wave matches the particle velocity ($v$). In this case the beam absorbs energy from the wave and it is continuously accelerated.

**Circular Waveguide**

As example if we consider a simple circular waveguide the first propagating mode with $E_z \neq 0$ is the TM$_{01}$ mode. Nevertheless by solving the wave equation it turns out that an e.m. wave propagating in this constant cross section waveguide will never be synchronous with a particle beam since the phase velocity is always larger than the speed of light $c$.

$$E_z|^ {TM_{01}} = E_0(r) \cos(\omega_{RF} t - k^* z) \quad \Rightarrow \quad v_{ph} = \frac{\omega_{RF}}{k^*} > c$$

$$J_0\left( \frac{p_{01}}{a} r \right)$$
In order to slow-down the wave phase velocity, iris-loaded periodic structure have to be used.

The field in this type of structures is that of a special wave travelling within a spatial periodic profile.

\[ E_z|_{TM_{01}} = E_0(r) \cos(\omega_{RF} t - k^* z) \]

\[ E_z|_{TM_{01\text{-like}}} = \hat{E}_{acc}(r, z) \cos(\omega_{RF} t - k^* z) \]

\[ \omega = c \]

\[ \omega_{RF} \]

\[ k = \frac{\pi}{D} \]

⇒ The structure can be designed to have the phase velocity equal to the speed of the particles.

⇒ This allows acceleration over large distances (few meters, hundred of cells) with just an input coupler and a relatively simple geometry.

⇒ They are used especially for electrons (constant particle velocity → constant phase velocity, same distance between irises, easy realization)
TW CAVITIES PARAMETERS: \( r, \alpha, v_g \)

Similarly to the SW cavities it is possible to define some figure of merit for the TW structures.

- **Shunt impedance per unit length** \( [\Omega/m] \): Similarly to SW structures the higher is \( r \), the higher the available accelerating field for a given RF power.

- **Field attenuation constant** \([1/m]\): because of the wall dissipation, the RF power flux and the accelerating field decrease along the structure.

- **Group velocity** \([m/s]\): the velocity of the energy flow in the structure (~1-2% of \( c \)).

- **Working mode** \([\text{rad}]\): defined as the phase advance over a period \( D \). For several reasons the most common mode is the \( 2\pi/3 \)

\[
\dot{V}_{\text{acc}} = \frac{1}{D} \int_{\text{cell volume}} E_{\text{acc}} \, dV \quad \text{single cell accelerating voltage}
\]

\[
\hat{E}_{\text{acc}} = \frac{\dot{V}_{\text{acc}}}{D} \quad \text{average accelerating field in the cell}
\]

\[
P_f = \frac{1}{2} \text{Re} \left( \hat{E} \times \hat{H} \right) \cdot \dot{z} \quad \text{flux power}
\]

\[
P_{\text{diss}} = \frac{1}{2} R_\text{wall} \int_{\text{cell volume}} |H_{\text{in}}|^2 \, dS \quad \text{average dissipated power in the cell}
\]

\[
P_{\text{diss}} = \frac{1}{D} P_{\text{diss}} \quad \text{average dissipated power per unit length}
\]

\[
W = \int_{\text{cell volume}} \left( \frac{1}{4} \rho c \dot{E}^2 + \frac{1}{4} \rho c \dot{H}^2 \right) \, dV \quad \text{stored energy in the cell}
\]

\[
w = \frac{W}{D} \quad \text{average stored energy per unit length}
\]

\[
r = \frac{\hat{E}_{\text{acc}}^2}{P_{\text{diss}}} \quad \text{energy flow in the structure}
\]

\[
\alpha = \frac{P_{\text{diss}}}{2P_F} \quad \text{field attenuation constant}
\]

\[
v_g = \frac{P_F}{w} \quad \text{group velocity}
\]

\[
Q = \omega_{RF} \frac{w}{P_{\text{diss}}} \quad \text{quality factor}
\]

\[
\Delta \phi = kD \quad \text{working mode}
\]
In a TW structure, the RF power enters into the cavity through an input coupler, flows (travels) through the cavity in the same direction as the beam and an output coupler at the end of the structure is connected to a matched power load.

If there is no beam, the input power, reduced by the cavity losses, goes to the power load where it is dissipated.

In the presence of a large beam current, however, a fraction of the TW power is transferred to the beam.

In a purely periodic structure, made by a sequence of identical cells (also called “constant impedance structure”), $\alpha$ does not depend on $z$ and both the RF power flux and the intensity of the accelerating field decay exponentially along the structure:

$$E_{\text{acc}}(z,t) = \frac{E_p(r,z)}{\cos(\omega_{RF}t - k_z^*z)} e^{-\alpha z} \approx E_{\text{IN}} \cos(\omega_{RF}t - k_z^*z) e^{-\alpha z}$$

$$P_F(z) = P_{\text{IN}} e^{-2\alpha z} \quad P_{\text{OUT}} = P_{\text{IN}} e^{-2\alpha L} \quad E_{\text{IN}} = \sqrt{2\alpha r P_{\text{IN}}}$$

The filling time is the time necessary to propagate the RF wave-front from the input to the end of the section of length $L$ is:

$$\tau_F = \frac{L}{v_g}$$

It is possible to demonstrate that, in order to keep the accelerating field constant along the structure, the iris apertures have to decrease along the structure.
LINAC TECHNOLOGY: MATERIALS
The cavities (and the related LINAC technology) can be of different material:

- **copper** for normal conducting (NC, both SW than TW) cavities;
- **Niobium** for superconducting cavities (SC, SW);

We can choose between NC or the SC technology depending on the required performances in term of:

- accelerating gradient (MV/m);
- RF pulse length (how many bunches we can contemporary accelerate);
- Duty cycle (see next slide): pulsed operation (i.e. 10-100 Hz) or continuous wave (CW) operation;
- Average beam current.

\[ P_{\text{diss}} = \int_{\text{cavity wall}} \frac{1}{2} R_s H_{\text{tan}}^2 dS \]

Between copper and Niobium there is a factor $10^5$-$10^6$
The “beam structure” in a LINAC is directly related to the “RF structure”. There are two possible type of operations:

- **CW** (Continuous Wave) operation ⇒ allow, in principle, to operate with a continuous (bunched) beam
- **PULSED** operation ⇒ there are RF pulses at a certain repetition rate (Duty Cycle (DC)=pulsed width/period)

⇒ **SC** structures allow operation at very high Duty Cycle (>1%) up to a CW operation (DC=100%) (because of the extremely low dissipated power) with relatively high gradient (>20 MV/m). This means that a continuous (bunched) beam can be accelerated.

⇒ **NC** structures can operate in pulsed mode at very low DC (10^{-2}-10^{-1}%) (because of the higher dissipated power) with, in principle, larger peak accelerating gradient (>30 MV/m). This means that one or few tens of bunches can be, in general, accelerated. NB: NC structures can also operate in CW but at very low gradient because of the dissipated power.
EXAMPLE: SWISSFEL LINAC (PSI)

2.5 cell copper cavity
2998.8 MHz (S-band)
2 μs pulse length
100 MHz repetition rate
100 μT magnetic field
40°C operating temperature

50 MW system, 3 μs pulse
0.22 GV energy gain per module

Example: SwissFEL Linac (PSI)
EXAMPLES: EUROPEAN XFEL

Nominal Energy: GeV 17.5
Beam pulse length: ms 0.60
Repetition rate: Hz 10
Max. # of bunches per pulse: 2700
Min. bunch spacing: ns 220
Bunch charge: nC 1
Bunch length, $\sigma_z$: $\mu m$ < 20
Emittance (slice) at undulator: $\mu rad$ < 1.4
Energy spread (slice) at undulator: MeV 1

600 $\mu s \rightarrow$ 100 ms

101 cryomodules in total
RF system: 25 RF units. The unit = 4 cryomodules + RF-power source (klystron)

Klystron

Cryomodule housing: 8 cavities, quadrupole and BPM

25 RF stations
5.2 MW each

800 accelerating cavities
1.3 GHz / 23.6 MV/m
LINAC (linear accelerator) is a system that allows to accelerate charged particles through a linear trajectory by electromagnetic fields.
Let us consider a **SW linac structure** made by accelerating gaps (like in DTL) or cavities.

In each gap we have an accelerating field oscillating in time and an integrated accelerating voltage \( V_{\text{acc}} \) still oscillating in time than can be expressed as:

\[
V_{\text{acc}} = \dot{V}_{\text{acc}} \cos(\omega_{RF}t)
\]

Let’s assume that the “perfect” synchronism condition is fulfilled for a phase \( \phi_s \) (called **synchronous phase**). This means that a particle (called **synchronous particle**) entering in a gap with a phase \( \phi_s \) \((\phi_s = \omega_{RF}t_s)\) with respect to the RF voltage receive an energy gain (and a consequent change in velocity) that allow entering in the subsequent gap with the same phase \( \phi_s \) and so on.

For this particle the energy gain in each gap is:

\[
\Delta E = q \frac{\dot{V}_{\text{acc}} \cos(\phi_s)}{V_{\text{acc,s}}} = qV_{\text{acc,s}}
\]

Obviously both \( \phi_s \) and \( \phi_s^* \) are synchronous phases.
Let us consider now the first synchronous phase $\phi_s$ (on the positive slope of the RF voltage). If we consider another particle “near” to the synchronous one that arrives later in the gap ($t_1 > t_s, \phi_1 > \phi_s$), it will see an higher voltage, it will gain an higher energy and an higher velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be shorter, partially compensating its initial delay.

Similarly if we consider another particle “near” to the synchronous one that arrives before in the gap ($t_1 < t_s, \phi_1 < \phi_s$), it will see a smaller voltage, it will gain a smaller energy and a smaller velocity with respect to the synchronous one. As a consequence its time of flight to next gap will be longer, compensating the initial advantage.

On the contrary if we consider now the synchronous particle at phase $\phi_s^*$ and another particle “near” to the synchronous one that arrives later or before in the gap, it will receive an energy gain that will increase further its distance form the synchronous one.

The choice of the synchronous phase in the positive slope of the RF voltage provides longitudinal focusing of the beam: phase stability principle.

The synchronous phase on the negative slope of the RF voltage is, on the contrary, unstable.

Relying on particle velocity variations, longitudinal focusing does not work for fully relativistic beams (electrons). In this case acceleration “on crest” is more convenient.
In order to study the **longitudinal dynamics in a LINAC**, the following variables are used, which describe the generic particle **phase** (time of arrival) and **energy** with respect to the synchronous particle:

- Arrival time (phase) of a **generic particle** at a certain gap (or cavity)
- Arrival time (phase) of the **synchronous particle** at a certain gap (or cavity)
- Energy of a **generic particle** at a certain position along the linac
- Energy of the **synchronous particle** at a certain position along the linac

The **energy gain per cell** (one gap + tube in case of a DTL) of a generic particle and of a synchronous particle are:

\[
\begin{align*}
\Delta E_s &= q\hat{V}_{acc} \cos \phi_s \\
\Delta E &= q\hat{V}_{acc} \cos \phi = q\hat{V}_{acc} \cos(\phi_s + \varphi)
\end{align*}
\]

Dividing by the accelerating cell length \(\Delta L\) and assuming that:

\[
\frac{\Delta W}{\Delta L} = q\hat{E}_{acc} \cos(\phi_s + \varphi) - \cos \phi_s
\]

Approximating

\[
\frac{\Delta w}{\Delta L} \approx \frac{dw}{dz}
\]

**Energy of the synchronous particle at a certain position along the linac**

**Energy of a generic particle at a certain position along the linac**
On the other hand we have that the phase variation per cell of a generic particle and of a synchronous particle are:

\[
\begin{align*}
\Delta \phi_S &= \omega_{RF} \Delta t_s \\
\Delta \phi &= \omega_{RF} \Delta t
\end{align*}
\]

\(\Delta t\) is basically the time of flight between two accelerating cells

\[\Delta \phi = \omega_{RF}(\Delta t - \Delta t_s)\]

\[\Delta \phi / \Delta L = \omega_{RF} \left( \frac{\Delta t}{\Delta L} - \frac{\Delta t_s}{\Delta L} \right) = \omega_{RF} \left( \frac{1}{v} - \frac{1}{v_s} \right) \approx \frac{\omega_{RF}}{cE_0 \beta_s^3 \gamma_s^3} w\]

\(v, v_s\) are the average particles velocities

\(\Delta \phi\) is the phase variation per cell of a generic particle and \(\Delta \phi_s\) is that of a synchronous particle.

This system of coupled (non linear) differential equations describe the motion of a non synchronous particles in the longitudinal plane with respect to the synchronous one.

\[\frac{dw}{dz} = qE_{acc} [\cos (\phi_s + \varphi) - \cos \phi_s] \]

\[\frac{d\varphi}{dz} = -\frac{\omega_{RF}}{cE_0 \beta_s^3 \gamma_s^3} w\]
\[
\frac{dw}{dz} = q \hat{E}_{\text{acc}} [\cos(\phi_s + \varphi) - \cos \phi_s]
\]

Deriving both terms with respect to \( z \)

\[
\frac{d\varphi}{dz} = -\frac{\omega_{RF}}{cE_0 \beta_s^3 \gamma_s^3} w \rightarrow cE_0 \beta_s^3 \gamma_s^3 \frac{d\varphi}{dz} = -\omega_{RF} w
\]

\[
\frac{d^2 \varphi}{dz^2} + q \frac{\omega_{RF} \hat{E}_{\text{acc}} \sin(-\phi_s)}{cE_0 \beta_s^3 \gamma_s^3 \Omega_s^2} \varphi = 0
\]

Assuming small oscillations around the synchronous particle:

\[\cos(\phi_s + \varphi) - \cos \phi_s \equiv \varphi \sin \phi_s\]

\[\Omega_s^2 > 0 \Rightarrow \sin(-\phi_s) > 0 \quad \Rightarrow \quad -\frac{\pi}{2} < \phi_s < 0\]

\[V_{\text{acc}} > 0 \Rightarrow \cos \phi_s > 0 \quad \Rightarrow \quad -\frac{\pi}{2} < \phi_s < 0\]

\[\Omega_s^2 > 0 \Rightarrow \sin(-\phi_s) > 0 \quad \Rightarrow \quad -\frac{\pi}{2} < \phi_s < 0\]

\[V_{\text{acc}} > 0 \Rightarrow \cos \phi_s > 0 \quad \Rightarrow \quad -\frac{\pi}{2} < \phi_s < 0\]

The condition to have stable longitudinal oscillations and acceleration at the same time is:

\[\left\{ \begin{array}{l} \varphi = \hat{\varphi} \cos(\Omega_s z) \\ w = \hat{w} \sin(\Omega_s z) \end{array} \right.\]

The angular frequency is simply: \( \Omega = \Omega_s \beta_s c; \)

The angular frequency scale with \( 1/\gamma^{3/2} \) that means that for ultra relativistic electrons shrinks to 0 (the beam is frozen).

if we accelerate on the rising part of the positive RF wave we have a longitudinal force keeping the beam bunched around the synchronous phase.

\[
cE_0 \beta_s^3 \gamma_s^3 \frac{d^2 \varphi}{dz^2} + cE_0 \left( \frac{d\beta_s^3 \gamma_s^3}{dz} \right) \frac{d\varphi}{dz} = -\omega_{RF} q \hat{E}_{\text{acc}} [\cos(\phi_s + \varphi) - \cos \phi_s]
\]
LARGE OSCILLATIONS AND SEPARATRIX (SMOOTH APPROX)

To study the longitudinal dynamics at large oscillations, we have to consider the non linear system of differential equations without small oscillation approximations (but with adiabatic acceleration approximation). It is possible to easily obtain the following relation between \( w \) and \( \phi \) (that is the Hamiltonian of the system related to the total particle energy):

\[
\frac{1}{2} \frac{\omega_{RF}}{cE_0\beta^3_s\gamma^3_s} w^2 + q\hat{E}_{acc}[\sin \phi - \phi \cos \phi_s] = H
\]

⇒ For each \( H \) we have different trajectories in the longitudinal phase space

⇒ the oscillations are stable within a region bounded by a special curve called separatrix: its equation is:

\[
\frac{1}{2} \frac{\omega_{RF}}{cE_0\beta^3_s\gamma^3_s} w^2 + q\hat{E}_{acc}[\sin \phi + \sin \phi_s - (\phi + \phi_s) \cos \phi_s] = 0
\]

⇒ the region inside the separatrix is called RF bucket. The dimensions of the bucket shrinks to zero if \( \phi_s = 0 \).

⇒ trajectories outside the RF buckets are unstable.

⇒ we can define the RF acceptance as the maximum extension in phase and energy that we can accept in an accelerator:

\[
\Delta \phi|_{\text{MAX}} \simeq 3\phi_s
\]

\[
\Delta w|_{\text{MAX}} = \pm 2\left[\frac{q c E_0 \beta_s^3 \gamma s \hat{E}_{acc} (\phi_s \cos \phi_s - \sin \phi_s)}{\omega_{RF}}\right]^{1/2}
\]
From previous formulae it is clear that there is **no motion** in the longitudinal phase plane for ultrarelativistic particles ($\gamma >> 1$).

It is interesting to analyze what happen if we inject an electron beam produced by a cathode (at low energy) directly in a TW structure (with $v_{ph} = c$) and the conditions that allow to **capture** the beam (this is equivalent to consider instead of a TW structure a SW designed to accelerate ultrarelativistic particles at $v = c$).

Particles enter the structure with velocity $v << c$ and, initially, they are **not synchronous with the accelerating field** and there is a so called slippage.

After a certain distance they can **reach enough energy (and velocity) to become synchronous** with the accelerating wave. This means that they are captured by the accelerator and from this point they are stably accelerated.

If this does not happen (the energy increase is not enough to reach the velocity of the wave) they are lost.

⇒ This is the case of electrons whose **velocity is always close to speed of light** $c$ even at low energies.

⇒ Accelerating structures are designed to provide an accelerating field synchronous with particles moving at $v = c$, like **TW structures** with phase velocity equal to $c$.
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS:

PHASE SPLIPAGE

The accelerating field of a TW structure can be expressed by

\[ E_{acc} = \frac{\dot{E}_{acc} \cos(\omega_{RF} t - k z)}{\phi(z,t)} \]

\[ \sin \phi_{fin} = \sin \phi_{in} + \frac{2\pi E_0}{\lambda_{RF} q \dot{E}_{acc}} \left( \frac{1 - \beta_{fin}}{1 + \beta_{in}} - \frac{1 - \beta_{fin}}{1 + \beta_{fin}} \right) \]

Suppose that the particle reach asymptotically the value \( \beta_{fin} = 1 \) we have:

\[ \sin \phi_{fin} > \sin \phi_{in} \Rightarrow \phi_{fin} > \phi_{in} \]

The equation of motion of a particle with a position z at time t accelerated by the TW is then

\[ \frac{d}{dt}(mv) = q\dot{E}_{acc} \cos \phi(z,t) \Rightarrow m_c \frac{d}{dt}(\gamma \beta) = m_c \gamma^3 \frac{d\beta}{dt} = q\dot{E}_{acc} \cos \phi \]

It is useful to find which is the relation between \( \beta \) and \( \phi \) from an initial condition (in) to a final one (fin)

Suppose that the particle reach asymptotically the value \( \beta_{fin} = 1 \) we have:

\[ \sin \phi_{fin} = \sin \phi_{in} + \frac{2\pi E_0}{\lambda_{RF} q \dot{E}_{acc}} \left( \frac{1 - \beta_{fin}}{1 + \beta_{in}} \right) \]

\[ \sin \beta_{fin} > \sin \beta_{in} \Rightarrow \beta_{fin} > \beta_{in} \]

Should be in the interval [-1,1] to have a solution for \( \phi_{fin} \)

This limits the possible injection phases (i.e. the phase of the electrons that is possible to capture)

This quantity is >0
LONGITUDINAL DYNAMICS OF LOW ENERGY ELECTRONS: CAPTURE EFFICIENCY AND BUNCH COMPRESSION

During the capture process, as the injected beam moves up to the crest, the beam is also bunched, which is caused by **velocity modulation** (velocity bunching). This mechanism can be used to compress the electron bunches (FEL applications).

During the capture process, as the injected beam moves up to the crest, the beam is also bunched, which is caused by velocity modulation (velocity bunching). This mechanism can be used to compress the electron bunches (FEL applications).

\[
\sin \phi_{\text{fin}} = \sin \phi_{\text{in}} + \frac{2\pi E_0}{\lambda_{\text{RF}} q E_{\text{acc}}} \sqrt{1 - \beta_{\text{in}}} \nonumber
\]

\[
\beta_{\text{in}} \approx 0.01
\]

\[
\beta_{\text{fin}} \approx 1
\]

\[
\sin \phi_{\text{fin}} = \sin \phi_{\text{in}} + \frac{2\pi E_0}{\lambda_{\text{RF}} q E_{\text{acc}}} \sqrt{1 - \beta_{\text{in}}} \nonumber
\]

\[
\Delta \phi_{\text{fin}} = \Delta \phi_{\text{in}} \frac{\cos \phi_{\text{in}}}{\cos \phi_{\text{fin}}}
\]

Depending on the injection phase we can have bunch compression or expansion.

\[
\beta_{\text{in}} < 1
\]

\[
\beta_{\text{fin}} = 1
\]

\[
\beta_{\text{in}} = 0.01
\]

\[
\beta_{\text{fin}} \approx 1
\]

These particles are lost during the capture process.

All particles are captured.
In order to increase the capture efficiency of a traveling wave section, pre-bunchers are often used. They are SW cavities aimed at pre-forming particle bunches gathering particles continuously emitted by a source.

Once the capture condition $E_{RF} > E_{RF\_MIN}$ is fulfilled, the fundamental equation of previous slide sets the ranges of the injection phases $\phi_m$ actually accepted. Particles whose injection phases are within this range can be captured, the other are lost.

Bunching is obtained by modulating the energy (and therefore the velocity) of a continuous beam using the longitudinal $E$-field of a SW cavity. After a certain drift space, the velocity modulation is converted in a density charge modulation. The density modulation depletes the regions corresponding to injection phase values incompatible with the capture process.

A TW accelerating structure (capture section) is placed at an optimal distance from the pre-buncher, to capture a large fraction of the charge and accelerate it till relativistic energies. The amount of charge lost is drastically reduced, while the capture section provide also further beam bunching.
LINAC (linear accelerator) is a **system that allows to accelerate charged particles through a linear trajectory** by electromagnetic fields.

**LINAC BEAM DYNAMICS**

- Longitudinal dynamics of accelerated particles
- Transverse dynamics of accelerated particles

**LINAC COMPONENTS AND TECHNOLOGY**

- Particle source
- Accelerating structures
- Accelerated beam
- Focusing elements: quadrupoles and solenoids
The RF fields act on the transverse beam dynamics because of the transverse components of the E and B field.

According to Maxwell equations the divergence of the field is zero and this implies that in traversing one accelerating gap there is a focusing/defocusing term.

\[ \nabla \cdot \vec{E} = 0 \quad \nabla \times \vec{B} = \frac{1}{c^2} \vec{E} \]

\[
E_z(z,t) = E_{RF}(z) \cos(\omega_{RF}t)
\]

\[
F_r = q(E_r - vB_\theta) = -q \left( \frac{\partial E_z}{\partial z} \right) \frac{\beta}{c} \frac{\partial E_z}{\partial t}
\]

\[
F_{r|E} = -q \frac{r}{2} \frac{\partial E_{RF}(z)}{\partial z} \cos\left( \frac{\omega_{RF} z}{\beta c} + \phi_{inj} \right)
\]

\[
F_{r|B} = q \frac{r}{2} \frac{\omega_{RF}}{c} \beta E_{RF}(z) \sin\left( \frac{\omega_{RF} z}{\beta c} + \phi_{inj} \right)
\]

\[ f_{RF} = 350 \text{ MHz} \quad \beta = 0.1 \quad L = 3 \text{ cm} \]
RF DEFOCUSING

From previous formulae it is possible to calculate the **transverse momentum increase** due to the RF transverse forces. Assuming that the velocity and position changes over the gap are small we obtain to the first order:

\[
\Delta p_r = \int_{-L/2}^{+L/2} F_r \frac{dz}{\beta c} = \pi q \hat{E}_{acc} L \sin \phi r 
\]

⇒ transverse **defocusing scales as \( \sim 1/\gamma^2 \)** and disappears at relativistic regime (electrons). In this case we have a compensation between the electric deflection and the magnetic one.

⇒ At relativistic regime (electrons), moreover, we have, in general, \( \phi = 0 \) for maximum acceleration and this completely cancel the defocusing effect

⇒ Also in the **non relativistic regime** for a correct evaluation of the defocusing effect we have to:

⇒ take into account the **velocity change across the accelerating gap**

⇒ the **transverse beam dimensions changes across the gap** (with a general reduction of the transverse beam dimensions due to the focusing in the first part)

Both effects give a **reduction of the defocusing force**
COLLECTIVE EFFECTS: SPACE CHARGE AND WAKEFIELDS

Collective effects are all effects related to the number of particles and they can play a crucial role in the longitudinal and transverse beam dynamics of intense beam LINACs

⇒ Effect of Coulomb repulsion between particles (space charge).

⇒ These effects cannot be neglected especially at low energy and at high current because the space charge forces scales as $1/\gamma^2$ and with the current $I$.

**SPACE CHARGE**

EXAMPLE: Uniform and infinite cylinder of charge moving along $z$

$$\vec{F}_{sc} = q \frac{I}{2\pi \varepsilon_0 R_b \beta \gamma^2} r_q \hat{r}$$

**WAKEFIELDS**

The other effects are due to the wakefield. The passage of bunches through accelerating structures excites electromagnetic field. This field can have longitudinal and transverse components and, interacting with subsequent bunches (long range wakefield), can affect the longitudinal and the transverse beam dynamics. In particular the transverse wakefields, can drive an instability along the train called multibunch beam break up (BBU).

Several approaches are used to absorb these field from the structures like loops couplers, waveguides, Beam pipe absorbers.
MAGNETIC FOCUSING AND CONTROL OF THE TRANSVERSE DYNAMICS

⇒ Defocusing RF forces, space charge or the natural divergence (emittance) of the beam need to be compensated and controlled by focusing forces.

⇒ Quadrupoles are focusing in one plane and defocusing on the other. A global focalization is provided by alternating quadrupoles with opposite signs.

⇒ In a linac one alternates accelerating structures with focusing sections.

⇒ The type of magnetic configuration and magnets type/distance depend on the type of particles/energies/beam parameters we want to achieve.

This is provided by quadrupoles along the beam line. At low energies also solenoids can be used.
Due to the alternating quadrupole focusing system each particle perform transverse oscillations along the LINAC.

The equation of motion in the transverse plane is of the type:

\[
\frac{d^2x}{ds^2} + \left[ k^2(s) - k_{RF}^2(s) \right] x - F_{sc} = 0
\]

Term depending on the magnetic configuration

RF defocusing/focusing term

Space charge term

The single particle trajectory is a pseudo-sinusoid described by the equation:

\[
x(s) = \sqrt{\varepsilon \beta(s)} \cos \left[ \int_s^s ds \frac{1}{\beta(s)} \cos \phi_0 \right]
\]

Characteristic function (Twiss $\beta$-function $[m]$) that depend on the magnetic and RF configuration

Depend on the initial conditions of the particle

The final transverse beam dimensions ($\sigma_{x,y}(s)$) vary along the linac and are contained within an envelope

Focusing period ($L_p$) = length after which the structure is repeated (usually as $N\beta\lambda$).
In case of "smooth approximation" of the LINAC (we consider an average effect of the quadrupoles and RF) we obtain a simple harmonic motion along s of the type ($\beta$ is constant):

$$\sigma = \int_s \frac{ds}{\beta(s)} \approx \frac{L_p}{\langle \beta \rangle}$$

Where $\sigma$ is the phase advance per unit length ($\sigma/L_p$).

Magnetic focusing elements (for a FODO)

$$K_0 = \sqrt{\left( \frac{qGl}{2m_0c\gamma_s\beta_s} \right)^2 - \frac{\pi q\hat{E}_{acc} \sin(-\phi_s)}{m_0c^2\lambda_{RF}} \left( \gamma_s\beta_s \right)^3} - \frac{Z_0ql\lambda_{RF}(1-f)}{8\pi m_0c^2\beta_s^2\gamma_s^3r_x r_y r_z}$$

Space charge term

For ultrarelativistic electrons RF defocusing and space charge disappear and the external focusing is required to control the emittance and to stabilize the beam against instabilities.
Beam dynamics dominated by space charge and RF defocusing forces

- Focusing is usually provided by quadrupoles

- Phase advance per period ($\sigma$) should be, in general, in the range 30-80 deg, this means that, at low energy, we need a strong focusing term (short quadrupole distance and high quadrupole gradient) to compensate for the rf defocusing, but the limited space ($\beta\lambda$) limits the achievable $G$ and beam current

As $\beta$ increases, the distance between focusing elements can increase ($\beta\lambda$ in the DTL goes from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV), and can be increased to 4-10$\beta\lambda$ at higher energy (>40 MeV).

- A linac is made of a sequence of structures, matched to the beam velocity, and where the length of the focusing period increases with energy. As $\beta$ increases, longitudinal phase error between cells of identical length becomes small and we can have short sequences of identical cells (lower construction costs).

- Keep sufficient safety margin between beam radius and aperture
⇒ **Space charge only at low energy and/or high peak current**: below 10-20 MeV (injector) the beam dynamics optimization has to include emittance compensation schemes with, typically solenoids;
⇒ At higher energies no space charge and no RF defocusing effects occur but we have RF focusing due to the ponderomotive force: focusing periods up to several meters
⇒ Optics design has to take into account longitudinal and transverse wakefields (due to the higher frequencies used for acceleration) that can cause energy spread increase, head-tail oscillations, multi-bunch instabilities,...
⇒ Longitudinal bunch compressors schemes based on magnets and chicanes have to take into account, for short bunches, the interaction between the beam and the emitted synchrotron radiation (**Coherent Synchrotron Radiation effects**)
⇒ All these effects are important especially in LINACs for **FEL that requires extremely good beam qualities**
At low proton (or ion) energies ($\beta \approx 0.01$), space charge defocusing is high and quadrupole focusing is not very effective. Moreover cell length becomes small and conventional accelerating structures (DTL) are very inefficient. At this energies it is used a (relatively) new structure, the Radio Frequency Quadrupole (1970).

These structures allow to simultaneously provide:

- Transverse Focusing
- Acceleration
- Bunching of the beam

Electrodes

Courtesy M. Vretenar
RFQ: PROPERTIES

1-Focusing
The resonating mode of the cavity (between the four electrodes) is a focusing mode: Quadrupole mode (TE_{210}). The alternating voltage on the electrodes produces an alternating focusing channel with the period of the RF (electric focusing does not depend on the velocity and is ideal at low $\beta$).

2-Acceleration
The vanes have a longitudinal modulation with period $= \beta \lambda_{RF}$ this creates a longitudinal component of the electric field that accelerate the beam (the modulation corresponds exactly to a series of RF gaps).

3-Bunching
The modulation period (distance between maxima) can be slightly adjusted to change the phase of the beam inside the RFQ cells, and the amplitude of the modulation can be changed to change the accelerating gradient. One can start at -90° phase (linac) with some bunching cells, progressively bunch the beam (adiabatic bunching channel), and only in the last cells switch on the acceleration.

The RFQ is the only linear accelerator that can accept a low energy continuous beam.

Courtesy A. Lombardi

Courtesy M. Vretenar and A. Lombardi
RFQ: EXAMPLES

The 1st 4-vane RFQ, Los Alamos
1980: 100 KeV - 650 KeV, 30 mA, 425 MHz

The CERN Linac4 RFQ
45 keV – 3 MeV, 3 m
80 mA H-, max. 10% duty cycle

TRASCO @ INFN Legnaro
Energy In: 80 keV
Energy Out: 5 MeV
Frequency 352.2 MHz
Proton Current (CW) 30 mA
In general the choice of the accelerating structure depends on:

- **Particle type**: mass, charge, energy
- **Beam current**
- **Duty cycle** (pulsed, CW)
- **Frequency**
- **Cost** of fabrication and of operation

Moreover a given accelerating structure has also a curve of **efficiency** (shunt impedance) with respect to the particle energies and the choice of one structure with respect to another one depends also on this.

As example a very general scheme is given in the Table (absolutely not exhaustive).

| Cavity Type | β Range   | Frequency      | Particles        |
|-------------|-----------|----------------|------------------|
| RFQ         | 0.01–0.1  | 40-500 MHz     | Protons, Ions    |
| DTL         | 0.05–0.5  | 100-400 MHz    | Protons, Ions    |
| SCL         | 0.5–1     | 600 MHz-3 GHz  | Protons, Electrons|
| SC Elliptical| > 0.5-0.7 | 350 MHz-3 GHz  | Protons, Electrons|
| TW          | 1         | 3-12 GHz       | Electrons        |
We can analyze how all parameters \((r, Q)\) scale with frequency and what are the advantages or disadvantages in accelerate with low or high frequencies cavities.

\[
\begin{align*}
R_s & \propto f^{1/2} & \propto f^2 \\
Q & \propto f^{-1/2} & \propto f^{-2} \\
r & \propto f^{1/2} & \propto f^{-1} \\
r/Q & \propto f \\
w_{||} & \propto f^2 \\
w_{\perp} & \propto f^3
\end{align*}
\]

Wakefield intensity: related to BD issues

\(\Rightarrow r/Q \) increases at high frequency

\(\Rightarrow r/Q \) increases at high frequency

\(\Rightarrow r \) increases and this push to adopt higher frequencies

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\(\Rightarrow \) for SC structures the power losses increases with \(f^2\) and, as a consequence, \(r\) scales with \(1/f\) this push to adopt lower frequencies

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\(\Rightarrow\) On the other hand at very high frequencies (>10 GHz) power sources are less available

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\(\Rightarrow\) Beam interaction (wakefield) became more critical at high frequency

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\(\Rightarrow\) Cavity fabrication at very high frequency requires higher precision

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\(\Rightarrow\) but, on the other hand, at low frequencies one needs more material and larger machines

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\(\Rightarrow\) short bunches are easier with higher \(f\)

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**SW SC:** 500 MHz-1500 MHz

**TW NC:** 3 GHz-6 GHz

**SW NC:** 0.5 GHz-3 GHz

**Compromise between several requirements**
Medical applications

Neutron spallation sources

Security: Cargo scans

Injectors for colliders and synchrotron light sources

Industrial applications:
- Ion implantation for semiconductors

THANK YOU FOR YOUR ATTENTION
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