Bubble Wall Velocity in the MSSM \[\text{1}\]

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Abstract

We compute the wall velocity in the MSSM with $W$, tops and stops contributing to the friction. In a wide range of parameters including those which fulfil the requirements of baryogenesis we find a wall velocity of order $v_w \approx 10^{-2}$ much below the SM value.

1 Bubble Wall Equation of Motion in the MSSM

Energy conservation leads to the equations of motion of an electroweak bubble wall interacting with a hot plasma of particles:

\[
\Box h_1 + \frac{\partial V_T(h_1, h_2)}{\partial h_1} + \sum_i \frac{\partial m_i^2}{\partial h_1} \int \frac{d^3 p}{(2\pi)^3 2 E_i} \delta f_i(p, x) = 0,  \\
\Box h_2 + \frac{\partial V_T(h_1, h_2)}{\partial h_2} + \sum_j \frac{\partial m_j^2}{\partial h_2} \int \frac{d^3 p}{(2\pi)^3 2 E_j} \delta f_j(p, x) = 0.
\]

where $f_i = f_{0,i} + \delta f_i$ is the distribution function for a particle species in the heat bath \[\text{2}\]. We have to sum over all particle species $i$. The distribution function is divided up into equilibrium part $f_{0,i}$ and out-of-equilibrium part $\delta f_i$. The equilibrium part has been absorbed into the equilibrium temperature dependent effective potential $V_T(h_1, h_2)$.

In the following we will restrict ourselves to late times leading to a stationarily moving domain wall where the friction stops the bubble wall acceleration. This is a reasonable assumption for the late stage of the bubble expansion where baryogenesis takes place.

\[\text{1} \text{To appear in the Proceedings of SEWM2000, Marseille, June 14-17, 2000}\]
2 Fluid Equations

The deviations $\delta f_i$ from the equilibrium population density are originating by a moving wall. They are derived by Boltzmann equations in the fluid frame:

$$d_t f_i \equiv \partial_t f_i + \dot{x} \frac{\partial}{\partial x} f_i + \dot{p}_x \frac{\partial}{\partial p_x} f_i = -C[f_i],$$

(3)

with the population density $f_i$ and energy $E = \sqrt{p_x^2 + m^2(x)}$. $C[f_i]$ represents the scattering integral. The classical (WKB) approximation is valid for $p \gg 1/L_w$ (“thick wall”). For particles with $E, p \gtrsim gT$ this should be fulfilled. Infrared particles are supposed not to contribute to the friction. This is a crude approximation and there are additional contributions [3] which further lower the wall velocity. In the MSSM the wall thickness $L_w$ is of order $15/T–40/T$, as found in [4, 5], and $L_w \gg 1/T$ is fulfilled. Those particles which couple very weakly to the Higgs are denoted as “light particles”. Particles coupling strongly to the Higgs are heavy in the Higgs phase and therefore called “heavy”. “superheavy” particles as the “left handed” stops do not appear in our calculation besides their remnants in the effective potential. We treat as “heavy” particles top quarks, (right handed) stops, and W bosons. The Higgses are left out. Further contributions produce an even smaller velocity. We assume now that the interaction between wall and particle plasma is the origin of small perturbations from equilibrium. We will treat perturbations in the temperature $\delta T$, velocity $\delta v$ and chemical potential $\delta \mu$ and linearize the resulting fluid equations. Then the full population density $f_i$ of a particle species $i$ in the fluid frame is given by

$$f_i = \frac{1}{\exp\left\{\frac{(E+\delta_i)}{T}\right\} \pm 1}$$

(4)

where we have generally space dependent perturbations $\delta_i$ from equilibrium. In principle one must include perturbations for each particle species. A simplification is to treat all the “light” particle species as one common background fluid. This background fluid obtains common perturbations $\delta v_{bg}$ in the velocity and $\delta T_{bg}$ in the temperature. This leads us to

$$\delta_i = -\left[\delta \mu_i + \frac{E}{T}(\delta T_i + \delta T_{bg}) + p_x(\delta v_i + \delta v_{bg})\right]$$

(5)

for the “heavy” particles. The spatial profiles of all these perturbations depend on the microscopic physics. We treat particles and antiparticles as one species neglecting CP violation which is a minor effect on the friction. It were, of course, important for the calculation of the baryon asymmetry.

Since the perturbations are Lagrangian multipliers for particle number, energy, and momentum, we can expand (3) to a set of three equations, called “fluid equations”, coupled by the collision term $C[\delta \mu, \delta T, gdv]$. Performing the integrals [1, 2] leads to the general pattern

$$\int \frac{d^3p}{(2\pi)^3 T^2} C[f] = \delta \mu \Gamma_{\mu_1} + \delta T \Gamma_{T_1},$$
$$\int \frac{d^3p}{(2\pi)^3 T^3} EC[f] = \delta \mu \Gamma_{\mu_2} + \delta T \Gamma_{T_2},$$
$$\int \frac{d^3p}{(2\pi)^3 T^3} p_x C[f] = \delta v \Gamma_v,$$

(6)
For a stationary wall we can use \( \partial_z \delta_i \to \gamma_w v_w \delta'_i \) and \( \partial_z \delta_i \to \gamma_w \delta'_i \), where the prime denotes the derivative with respect to \( z = \gamma_w (x - v_w t) \). Our equations are similar to those in [1] but there are important additional terms.

For each heavy particle species in the plasma we have three fluid equations. The final form of the fluid equations can be written in a matrix notation:

\[
A \delta' + \Gamma \delta = F, \tag{7}
\]

where \( \Gamma = \Gamma_0 + 1/\bar{c}_4 M \). The matrices \( A, \Gamma, \Gamma_0, \) and \( M \) can be found in [2]. The number \( \bar{c}_4 \) is the heat capacity of the plasma \( \bar{c}_4 = 78c_{4-} + 37c_{4+} \) including light quarks, leptons, and sleptons in the plasma. The perturbations are combined in a vector \( \delta \), the driving terms are combined in the vector \( F \). The driving term containing \( (m^2)' \) can be split up into different contributions

\[
(m^2)' = \frac{\partial m^2}{\partial h_1} h'_1 + \frac{\partial m^2}{\partial h_2} h'_2. \tag{8}
\]

The vectors \( \delta \) and \( F \) for \( k \) particle species read (index \( x \) denotes + or −, for fermions and bosons, respectively, for the \( i \)th particle):

\[
\delta = [\delta \mu_1 \; \delta T_1 \; T \delta v_1 \; \ldots \; \delta \mu_k \; \delta T_k \; T \delta v_k]^T, \tag{9}
\]

\[
F = \frac{\gamma_w v_w}{2T} \begin{bmatrix} c_{1+}(m_1^2)' & c_{2+}(m_2^2)' & 0 & \ldots & c_{1+}(m_k^2)' & c_{2+}(m_k^2)' & 0 \end{bmatrix}^T. \tag{10}
\]

where \( c_{1\pm}, c_{2\pm} \) denote the fermionic(+) or bosonic(-) statistical factors, respectively, defined through \( c_{n\pm} = \int E^{n-2} T^{n+1} (f_0(\pm)) d^3p/(2\pi)^3 \). Eq. (6) has to be solved for \( \delta \). To a first approximation, neglecting \( \delta' \), we obtain \( \delta = \Gamma^{-1} F \). Then, including (right-handed) stop-, top- and \( W \) particles, the equations of motion can be rewritten in the fluid picture as

\[
h''_1 - V'_T = \eta_1 \gamma_w v_w \frac{h_1^2}{T} h'_1, \quad h''_2 - V'_T = \eta_2 \gamma_w v_w \frac{h_2^2}{T} h'_2. \tag{11}
\]

with slightly \( \tan\beta \)-dependent friction constants \( \eta_{1.2} = T/4G_{1.2} \Gamma^{-1} F_{1.2} \), with constant vectors \( F_{1.2} \) and \( G_{1.2} \). Perhaps \( \delta' \) is not negligible, so we have to solve (9) numerically. We compare both resulting velocities later on in Fig. 1.

### 3 Wall Velocity in the MSSM

In order to solve eqs. (11) we derive a virial theorem, based on the necessity that for a stationary wall the pressure to the wall surface is balanced by the friction. The pressure on a free bubble wall can be obtained from l.h.s. of the equations of motion (11) integrated with \( h'_1 \), e.g.

\[
p = \int_0^\infty \left( h''_1 - \frac{\partial V_T}{\partial h_1} \right) h'_1 dx = \Delta V_T = \int \eta_1 \gamma_w v_w \frac{h_1^2}{T} (h'_1)^2. \tag{12}
\]

\( \Delta V_T \) is the difference in the effective potential values at the transition temperature \( T_n \), which is basically the nucleation temperature. Both of the Higgs fields develop friction terms and we have to add the pressure contributions. In the MSSM the approximation [4] to the solution by a kink \( h(x) = h_{crit}/2 (1 + \tanh(x/L_w)) \) is a rather good choice. With \( h_2 = \sin \beta \) and \( h_1 = \cos \beta \) we are left with

\[
\gamma_w v_w = \frac{20L_w}{h_{crit}^4} \frac{\Delta V_T(T_n)T_n}{\sin^4 \beta (\eta_2 + \eta_1 \cot^4 \beta)}. \tag{13}
\]
Figure 1: Wall velocity in dependence on the parameter $\tan \beta$ for $m_U = -60, 0, 60$ GeV; left: $\delta' = 0$, right: the same plus velocities for $\delta' \neq 0$ resulting from the full solution of (7).

Figure 2: $\beta(x)$ for $m_U^2 = -(60\text{GeV})^2$, $m_Q = 2\text{TeV}$, $m_A = 400\text{GeV}$, $\tan \beta = 2$, and no mixing. The back reaction of the friction is negligible small (left). Only artificially setting $\eta_2$ two orders of magnitude larger leads to sizable effects (right).

The missing numbers for $L_w$, $h_{\text{crit}}$, $T_{\text{h}}$, and $\Delta V(T_n)$ can be independently determined with methods described in [4, 5]. We used the 1-loop resummed effective potential. The diagram is calculated for $m_Q = 2\text{TeV}$, and $m_A = 400\text{GeV}$. We find a strong phase transition with $v/T = 0.95$ at 1-loop level for $m_U = -60\text{GeV}$ at $\tan \beta = 2.0$ and $A_t = \mu = 0$ (no stop mixing). At 2-loop level it is even much larger permitting larger $\tan \beta$ for the same strength. Also with mixing in the stop matrix this allows to comply the experimental lower bound on the Higgs mass. In Figure 1 we show $v_w$ vs. $\tan \beta$ for three values of $m_U = -60, 0, +60\text{GeV}$ at zero mixing. Very heavy stops should decouple more and more which would lead to increasing wall velocities again. This behaviour is reproduced with the full numerical solution to (7), see Fig. 1. Nevertheless for increasing $m_U^2$ the used approximations become worse and corrections of order $O(m^2/T^2)$ are important. Our calculations are done with massless outer legs only taking into account changing plasma masses.

Due to the effective potential which couples the equations of motion there may be back-reaction of the different friction contributions to $\partial \beta/\partial z$ leading to a change in $\Delta \beta = \max(\partial \beta/\partial z)$. A larger $\Delta \beta$ were highly welcome to obtain a larger baryon asymmetry. This becomes even more important since we realized [7] that in the MSSM transitional CP violation does not occur. Therefore we must exploit the explicit phases which may nevertheless be strongly restricted by experimental bounds. The determination of $\Delta \beta$ can be done numerically by solving eqs. (11) with extensions of known methods [5]. But only for artificially large friction we obtain sizable effects (see Fig. 2).

Acknowledgements We thank G. Moore for useful discussions. This work was
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