Direct CP Violation of $b \to s\gamma$ and CP Asymmetries of Non-Leptonic $B$ Decays in Squark Flavor Mixing

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We study the contribution of the squark flavor mixing from the $LR$ ($RL$) component of the squark mass matrices to the direct CP violation of the $b \to s\gamma$ decay and the CP-violating asymmetry in the non-leptonic decays of $B$ mesons. The magnitude of the $LR$ ($RL$) component is constrained by the branching ratio and the direct CP violation of $b \to s\gamma$. We predict the correlation of the CP asymmetries among $A_{CP}^{B_d}$, $S_{CP}^{B_dK_S}$ and $S_{CP}^{B_sK^0}$ of the $B$ decays. The precise data of these CP violations will give us the crucial test for our framework of the squark flavor mixing.

Subject Index: 113, 151, 152, 156, 158

§1. Introduction

New physics is expected to be observed at the LHC experiments. Although new particle has not been discovered yet, LHCb has reported new data of the CP violation of $B$ mesons and the branching ratios of rare $B$ decays. New physics is also expected to be found in the $B$ meson decays.

The CP violation in the $K$ and $B_d$ mesons has been successfully understood within the framework of the standard model (SM), so-called Kobayashi-Maskawa (KM) model.1) The source of the CP violation is the KM phase in the quark sector with three families. However, there could be new sources of the CP violation if the SM is extended to the supersymmetric (SUSY) models. The CP-violating phases appear in soft scalar mass matrices. These phases contribute to flavor changing neutral currents with the CP violation. Therefore, we expect the SUSY contribution in the CP-violating phenomena in the $B$ meson decays.

The typical contribution of SUSY is the gluino-squark mediated flavor changing process.2)−11) In our previous paper,12) we have already discussed the effect of the squark flavor mixing on the CP violation in the non-leptonic decays of $B^0_d$ and $B^0_s$ taking account of the recent LHCb experimental data. We have found the deviation from the SM predictions in the asymmetries of the penguin dominated decays $B^0_d \to \phi K_S$ and $B^0_d \to \eta' K^0$. In that framework of the SUSY contribution, we assume that $LR$ and $RL$ components of the squark mass matrices are neglected. The $LL$ and $RR$ components of squark mass matrices contribute considerably to the...

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penguin processes for the case of large $\mu \tan \beta$, $\mathcal{O}(10 \text{ TeV})$. However, if $LR$ and $RL$ components of squark mass matrices dominate the penguin decays, the asymmetries of $B_d^0 \to \phi K_S$ and $B_d^0 \to \eta' K^0$ are deviated from the SM predictions even for the case of the smaller $\mu \tan \beta$, $\mathcal{O}(1 \text{ TeV})$. Then, these contributions of the new physics are correlated with the direct $CP$ violation of the $b \to s \gamma$ decay. In this paper, we present the numerical analyses in the case that $LR$ and $RL$ components of squark mass matrices dominate the penguin decays. In this case, the $LR(RL)$ components do not contribute to the dispersive part $M_{12}^q$ of $B_q \to \bar{B}_q$ ($q = d, s$) mixing.

In §2, we summarize the effect of new physics in the $CP$ violations of the neutral $B$ mesons including the recent experimental data. In §3, we discuss our framework of the squark flavor mixing in the $CP$ violation of $B$ mesons. We also discuss the constraints from the direct $CP$ violation in the $b \to s \gamma$ process. In §4, we show the numerical result of the $CP$ violation in the $B$ mesons. Section 5 is devoted to a summary.

§2. New physics of $CP$ violation in $B$ mesons

Let us discuss the effect of new physics in the non-leptonic decays of $B$ mesons. The contribution of new physics to the dispersive part $M_{12}^q(q = d, s)$ is parameterized as

$$M_{12}^q = M_{12}^{q, \text{SM}} + M_{12}^{q, \text{SUSY}} = M_{12}^{q, \text{SM}}(1 + h_q e^{2i\sigma_q}), \quad (q = d, s)$$

(2.1)

where $M_{12}^{q, \text{SUSY}}$ is the SUSY contribution, and the SM contribution $M_{12}^{q, \text{SM}}$ is given as

$$M_{12}^{q, \text{SM}} = \frac{G_F M_B^2}{12\pi^2} M_{WV_b V_{i_q}^*}^2 \hat{\eta}_B S_0(x_t) f_B^2 B_q.$$

(2.2)

The time dependent $CP$ asymmetry decaying into the final state $f$ is defined as

$$S_f = \frac{2 \text{Im}\lambda_f}{|\lambda_f|^2 + 1},$$

(2.3)

where

$$\lambda_f = \frac{q}{p} \bar{\rho}, \quad q = \sqrt{\frac{M_{12}^{q^*} - \frac{i}{2} \Gamma_{12}^{q^*}}{M_{12}^{q} - \frac{i}{2} \Gamma_{12}^{q}}}, \quad \bar{\rho} \equiv \frac{A(B_0^q \to f)}{A(B_0^0 \to f)}.$$

(2.4)

In the $B_d^0 \to J/\psi K_S$ decay, $\lambda_{J/\psi K_S}$ is given in terms of the new physics parameters $h_d$ and $\sigma_d$ as

$$\lambda_{J/\psi K_S} = -e^{-i\phi_d}, \quad \phi_d = 2\beta_d + \text{arg}(1 + h_d e^{2i\sigma_d}),$$

(2.5)

by putting $|\bar{\rho}| = 1$ and $q/p \simeq \sqrt{M_{12}^{q^*}/M_{12}^q}$, where the phase $\beta_d$ is given in the SM. The CKMfitter provided the allowed region of $h_d$ and $\sigma_d$, where the central values are $h_d \simeq 0.3$, $\sigma_d \simeq 1.8$ rad.

(2.6)

Since penguin processes are dominant in the case of $f = \phi K_S, \eta' K^0$, the loop induced new physics could contribute considerably on the $CP$ violation of those decays. Then,
$S_f$ is not any more the same as $S_{J/\psi K_S}$ if new physics gives new phases. Those predictions provide us good tests for new physics.

In the $B_s^0 \to J/\psi \phi$ decay, we parametrize as
\begin{equation}
\lambda_{J/\psi \phi} = e^{-i\phi_s}, \quad \phi_s = -2\beta_s + \arg(1 + h_s e^{2i\sigma_s}), \quad (2.7)
\end{equation}
where $\beta_s$ is given in the SM. Recently the LHCb presented the observed $CP$-violating phase $\phi_s$ in $B_s^0 \to J/\psi \pi^+ \pi^-$ decay using about 1 fb$^{-1}$ of data.\(^{17}\) This result leads to
\begin{equation}
\phi_s = -0.019^{+0.173+0.04}_{-0.174-0.03} \text{ rad}, \quad (2.8)
\end{equation}
which is consistent with the SM prediction\(^{15}\)
\begin{equation}
\phi_{s,J/\psi \phi,SM} = -2\beta_s = -0.0363 \pm 0.0017 \text{ rad.} \quad (2.9)
\end{equation}
Taking account of these data, the CKMfitter has presented the allowed values of $h_s$ and $\sigma_s$.\(^{15,16}\) The allowed region is rather large including zero values. In order to investigate possible contribution of new physics, we take the central values
\begin{equation}
h_s = 0.1, \quad \sigma_s = 0.9 - 2.2 \text{ rad,} \quad (2.10)
\end{equation}
as a typical parameter set in our work.

We remark on numerical inputs of phases $\phi_d$ and $\phi_s$ in our calculation. The phase $\phi_d$ is derived from the observed value $S_f = 0.671 \pm 0.023$ in $B_d^0 \to J/\psi K_S$\(^{18}\) as seen in Eqs. (2.3) and (2.5). On the other hand, we use the SM value of $\beta_s$ and the values of the new physics parameters, $h_s$ and $\sigma_s$ in Eq. (2.10) to estimate $\phi_s = -2\beta_s + \arg(1 + h_s e^{2i\sigma_s})$. We do not use the observed value of $\phi_s$ in $B_s^0 \to J/\psi \phi$ because of the large experimental error in Eq. (2.8).

Since the $B_d^0 \to J/\psi K_S$ process occurs at the tree level in SM, the $CP$-violating asymmetry originates from $M_{12}^d$. Although the $B_d^0 \to \phi K_S$ and $B_d^0 \to \eta' K^0$ decays are penguin dominant ones, their asymmetries also come from $M_{12}^d$. Then, asymmetries of $B_d^0 \to J/\psi K_S$, $B_d^0 \to \phi K_S$ and $B_d^0 \to \eta' K^0$ are expected to be same magnitude in SM.

On the other hand, if the squark flavor mixing contributes to the decay at the one-loop level, its magnitude could be comparable to the SM penguin one in $B_d^0 \to \phi K_S$ and $B_d^0 \to \eta' K^0$, but it is tiny in $B_d^0 \to J/\psi K_S$. Endo, Mishima and Yamaguchi proposed the possibility to find the SUSY contribution in these asymmetries.\(^{19}\) The present data suggest the deviation from SM in these time dependent asymmetries of $B_d^0$ decays such as
\begin{equation}
S_{J/\psi K_S} = 0.671 \pm 0.023, \quad S_{\phi K_S} = 0.39 \pm 0.17, \quad S_{\eta' K^0} = 0.60 \pm 0.07, \quad (2.11)
\end{equation}
however, precise data are required to justify the new physics contribution.

New physics contributes to the $b \to s\gamma$ process. The observed $b \to s\gamma$ branching ratio (BR) is $(3.60 \pm 0.23) \times 10^{-4}$\(^{18}\) on the other hand the SM prediction is given as $(3.15 \pm 0.23) \times 10^{-4}$ at $\mathcal{O}(\alpha_s^2)$.\(^{20,21}\) Therefore, the contribution of new physics should be suppressed. New physics is also constrained by the direct $CP$ violation
\begin{equation}
A_{CP}^{b \to s\gamma} \equiv \frac{\Gamma(\bar{B} \to X_s\gamma) - \Gamma(B \to X_s\gamma)}{\Gamma(\bar{B} \to X_s\gamma) + \Gamma(B \to X_s\gamma)}. \quad (2.12)
\end{equation}
Since SM prediction $A_{CP}^{b\to s\gamma} \simeq 0.005$ is tiny, new physics may appear in this $CP$ asymmetry. The present data $A_{CP}^{b\to s\gamma} = -0.008 \pm 0.029$ has rather large error bar, and so the constraint of new physics is not so severe. However, improved data will provide the crucial test for new physics.

§3. Squark flavor mixing and $CP$ violations of $B$ mesons

Let us consider the flavor structure of squarks in order to estimate the $CP$-violating asymmetries of $B$ meson decays. We take the most popular anzatz, which is to postulate a degenerate SUSY breaking mass spectrum for down-type squarks. Then, in the super-CKM basis, we can parametrize the soft scalar masses squared $M_{d_{LL}}^2$, $M_{d_{RR}}^2$, $M_{d_{LR}}^2$, and $M_{d_{RL}}^2$ for the down-type squarks as follows:

$$M_{d_{LL}}^2 = m_q^2 \left( 1 + (\delta_{d_{LL}})_{11} \right),$$

$$M_{d_{RR}}^2 = m_q^2 \left( 1 + (\delta_{d_{RR}})_{11} \right),$$

$$M_{d_{LR}}^2 = (M_{d_{RL}}^2)^\dagger = m_q^2 \left( (\delta_{d_{LR}})_{11} \right),$$

where $m_q$ is the average squark mass, and $(\delta_{d_{LL}})_{ij}$, $(\delta_{d_{RR}})_{ij}$, $(\delta_{d_{LR}})_{ij}$, and $(\delta_{d_{RL}})_{ij}$ are called the mass insertion (MI) parameters. The MI parameters are supposed to be much smaller than 1.

The contribution of the gluino-squark box diagram to the dispersive part of the effective Hamiltonian for the $B_q\to B_q$ mixing is written as

$$M_{12}^{q,SUSY} = A_1 q_2 \left[ A_2 \left( (\delta_{d_{LL}})_{ij} \right)^2 + (\delta_{d_{RR}})_{ij} \right] + A_3 q_2 \left( (\delta_{d_{LR}})_{ij} \right)^2 + A_4 q_2 \left( (\delta_{d_{RL}})_{ij} \right)^2,$$

where

$$A_1 = -\frac{\alpha_3^2}{216m_q^2} \frac{2}{3} M_{B_q} f_{B_q}^2, \quad A_2 = 24x f_6(x) + 66 \tilde{f}_6(x),$$

$$A_3 = \left\{ 384 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 + 72 \right\} x f_6(x) + \left\{ -24 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 + 36 \right\} \tilde{f}_6(x),$$

$$A_4 = \left\{ 132 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 \right\} x f_6(x), \quad A_5 = \left\{ -144 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 - 84 \right\} \tilde{f}_6(x).$$

Here, we use $x = m_{g_i}^2/m_q^2$, where $m_{g_i}$ is the gluino mass. For the cases of $q = d$
and $q = s$, we take $(i, j) = (1, 3)$ and $(i, j) = (2, 3)$, respectively, where $m_1 = m_d$, $m_2 = m_s$ and $m_3 = m_b$. The loop functions $f_6(x)$ and $f_8(x)$ are shown in 12).

For the case of $x \approx 1$, we get $A_2 \approx -1$, $A_3 \approx 30$, $A_4 \approx -10$ and $A_5 \approx 10$. Therefore, each term on the r.h.s. of Eq. (3-2) may contribute to $M_{12}^{SU SY}$ comparably. However, magnitudes of $(\delta_d^{LR})_{ij}$ and $(\delta_d^{RL})_{ij}$ are constrained severely by the $b \rightarrow s\gamma$ decay as discussed later.

The squark flavor mixing can be tested in the $CP$-violating asymmetries in the neutral $B$ meson decays. Let us present the framework of these calculations. The effective Hamiltonian for $\Delta B = 1$ process is defined as

$$H_{eff} = \frac{4G_F}{\sqrt{2}} \left[ \sum_{q'=u,c} V_{q'b}V_{q's}^* \sum_{i=1,2} C_iO_i^{(q')}-V_{tb}V_{ts}^* \sum_{i=3-6,7,8G} \left( C_iO_i + \tilde{C}_i\tilde{O}_i \right) \right], \quad (3.4)$$

where the local operators are given as

$$O_1^{(q')} = (\bar{s}\gamma_\mu P_Lq_\beta)(\bar{q}_\beta\gamma_\mu P_Lb_\alpha), \quad O_2^{(q')} = (\bar{s}\gamma_\mu P_Lq_\alpha)(\bar{q}_\beta\gamma_\mu P_Lb_\beta),$$

$$O_3 = (\bar{s}\gamma_\mu P_Lb_\alpha) \sum_q (\bar{q}\gamma_\mu P_Lq_\beta), \quad O_4 = (\bar{s}\gamma_\mu P_Lb_\beta) \sum_q (\bar{q}\gamma_\mu P_Lq_\alpha),$$

$$O_5 = (\bar{s}\gamma_\mu P_Lb_\alpha) \sum_q (\bar{q}\gamma_\mu P_Rq_\beta), \quad O_6 = (\bar{s}\gamma_\mu P_Lb_\beta) \sum_q (\bar{q}\gamma_\mu P_Rq_\alpha),$$

$$O_7 = \frac{e}{16\pi^2} m_b\bar{s}_\alpha\sigma^{\mu\nu}P_Rb_\alpha F_{\mu\nu}, \quad O_8G = \frac{g_s}{16\pi^2} m_b\bar{s}_\alpha\sigma^{\mu\nu}P_R T^a_{\alpha\beta} b_\beta G^{a\mu\nu}. \quad (3.5)$$

where $P_R = (1 + \gamma_5)/2$, $P_L = (1 - \gamma_5)/2$, and $\alpha$ and $\beta$ are color indices, and $q$ is taken to be $u, d, s, c$. Here, $C_i$'s and $\tilde{C}_i$'s are the Wilson coefficients, and $O_i$'s are the operators by replacing $L(R)$ with $R(L)$ in $O_i$. In this paper, $C_i$ includes both SM contribution and gluino one, such as $C_i = C_i^{SM} + C_i^{\tilde{g}}$, where $C_i^{SM}$ is given in Ref. 25) and $C_i^{\tilde{g}}$ is presented as follows:\textsuperscript{26)

\begin{align*}
C_3^{\tilde{g}} & \simeq \frac{\sqrt{2}a_s^2}{4G_F V_{tb}V_{ts}^* m_q^2} (\delta_d^{LL})_{23} \left[ -\frac{1}{9} B_1(x) - \frac{5}{2} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right], \\
C_4^{\tilde{g}} & \simeq \frac{\sqrt{2}a_s^2}{4G_F V_{tb}V_{ts}^* m_q^2} (\delta_d^{LL})_{23} \left[ -\frac{7}{3} B_1(x) + \frac{1}{3} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right], \\
C_5^{\tilde{g}} & \simeq \frac{\sqrt{2}a_s^2}{4G_F V_{tb}V_{ts}^* m_q^2} (\delta_d^{LL})_{23} \left[ \frac{10}{9} B_1(x) + \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right], \\
C_6^{\tilde{g}} & \simeq \frac{\sqrt{2}a_s^2}{4G_F V_{tb}V_{ts}^* m_q^2} (\delta_d^{LL})_{23} \left[ -\frac{2}{3} B_1(x) + \frac{7}{6} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right], \\
C_7^{\tilde{g}} & \simeq -\frac{\sqrt{2}a_s\pi}{6G_F V_{tb}V_{ts}^* m_q^2} \left( \delta_d^{LL} \right)_{23} \left( \frac{8}{3} M_3(x) - \mu \tan \beta \frac{m_\tilde{g}^2}{m_q^2} \frac{8}{3} M_1(x) \right) + (\delta_d^{LR})_{23} \frac{m_\tilde{g}^2}{m_b^2} M_1(x) \right], \\
\end{align*}
\[ C_{8G}^{\tilde{q}} \simeq -\frac{\sqrt{2}\alpha_s\pi}{2G_FV_{tb}V_{ts}^*m_{\tilde{q}}^2} \left[ (\delta_{d}^{LL})_{23} \left\{ \left( \frac{1}{3}M_3(x) + 3M_4(x) \right) \right\} \right. \]
\[ \left. - \mu \tan \beta \frac{m_{\tilde{q}}}{m_{\tilde{g}}} \left( \frac{1}{3}M_a(x) + 3M_b(x) \right) \right\} + (\delta_{d}^{LR})_{23} \frac{m_{\tilde{g}}}{m_b} \left( \frac{1}{3}M_1(x) + 3M_2(x) \right) \right]. \]

Here the double mass insertion is included in \( C_{7G}^{\tilde{q}} \) and \( C_{8G}^{\tilde{q}} \). The Wilson coefficients \( \tilde{C}_i^{\tilde{q}} \)'s are obtained by replacing \( L(R) \) with \( R(L) \) in \( C_i^{\tilde{q}} \)'s. The loop functions, which we use in our calculations, are presented in our previous paper.\(^{12}\)

The \( CP \)-violating asymmetries \( S_f \) in Eq. (2.3) are calculated by using \( \lambda_f \), which is given for \( B_d^0 \to \phi K_S \) and \( B_d^0 \to \eta' K^0 \) as follows:

\[ \lambda_{\phi K_S, \eta' K^0} = -e^{-i\phi_d} \frac{\sum_{i=3-6,7\gamma,8G} \left( C_i^{SM} \langle O_i \rangle + C_i^{\tilde{q}} \langle O_i \rangle + \tilde{C}_i^{\tilde{q}} \langle \tilde{O}_i \rangle \right)}{\sum_{i=3-6,7\gamma,8G} \left( C_i^{SM*} \langle O_i \rangle + C_i^{\tilde{q}^*} \langle O_i \rangle + \tilde{C}_i^{\tilde{q}^*} \langle \tilde{O}_i \rangle \right)}, \]  

where \( \langle O_i \rangle \) is the abbreviation of \( \langle f | O_i | B_d^0 \rangle \). It is noticed that \( \langle \phi K_S | O_i | B_d^0 \rangle = \langle \phi K_S | \tilde{O}_i | B_d^0 \rangle \) and \( \langle \eta' K^0 | O_i | B_d^0 \rangle = -\langle \eta' K^0 | \tilde{O}_i | B_d^0 \rangle \) because of the parity of the final state. We have also \( \lambda_f \) for \( B_d^0 \to \phi \phi \) as follows:

\[ \lambda_{\phi \phi} = e^{-i\phi_s} \frac{\sum_{i=3-6,7\gamma,8G} C_i^{SM} \langle O_i \rangle + C_i^{\tilde{q}} \langle O_i \rangle + \tilde{C}_i^{\tilde{q}} \langle \tilde{O}_i \rangle}{\sum_{i=3-6,7\gamma,8G} C_i^{SM*} \langle O_i \rangle + C_i^{\tilde{q}^*} \langle O_i \rangle + \tilde{C}_i^{\tilde{q}^*} \langle \tilde{O}_i \rangle}, \]  

with \( \langle \phi \phi | O_i | B_d^0 \rangle = -\langle \phi \phi | \tilde{O}_i | B_d^0 \rangle \).

In these non-leptonic decays, the \( C_{8G}^{\tilde{q}} \langle O_{8G} \rangle \) dominates these amplitude, but small contributions from other Wilson coefficients are also taken into account in our calculations. Therefore, we estimate each hadronic matrix elements by using the factorization relations in Ref. 27).

Let us discuss the each contribution of the mass insertion parameters to \( C_{8G}^{\tilde{q}} \) in Eq. (3.6). Taking into account that the loop functions \( M_i(x) \) are of same order and \( m_{\tilde{q}} \simeq m_{\tilde{g}} \), the ratio of LL component and LR one is \( (\delta_{d}^{LL})_{23} \times \mu \tan \beta / m_{\tilde{q}} \) to \( (\delta_{d}^{LR})_{23} \times m_{\tilde{q}} / m_b \). If \( O(\mu \tan \beta) \sim O(m_{\tilde{q}}) \) and \( m_{\tilde{q}} \gtrsim 1 \text{ TeV} \), the LR component may contribute significantly to \( C_{8G}^{\tilde{q}} \) due to the enhancement factor \( m_{\tilde{q}} / m_b = O(10^2) \). For example, in the case of \( (\delta_{d}^{LL})_{23} = 10^{-2} \) and \( (\delta_{d}^{LR})_{23} = 10^{-3} \), the LR component dominate \( C_{8G}^{\tilde{q}} \), while it is minor in \( M_{12}^{q,SUSY} \) as seen in Eq. (3.2). This situation is also kept in the \( b \to s \gamma \) decay.

The \( b \to s \gamma \) decay is a typical one to investigate new physics. The branching ratio is given as

\[ \frac{BR(B \to X_s \gamma)}{BR(B \to X_c e \bar{\nu}_e)} = \frac{|V^*_{ts} V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(z)} |C_{7G}^{\tilde{q}}|^2, \]
where
\[ f(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z, \quad z = \frac{m_c}{m_b}. \] (3.10)

Here, \( C^\text{eff}_{\gamma} \) includes both contributions from the SM and the gluino-squark flavor mixing \( C_G^{\tilde{g}} \). As seen in Eq. (3.2), both \( C_G^{\tilde{g}} \) and \( C^{\tilde{g}}_{8G} \) have the similar dependence of \( (\delta_{\tilde{d}}^{LR})_{23} \). Therefore, we should discuss carefully the contribution from \( (\delta_{\tilde{d}}^{LR})_{23} \) in our numerical calculations.

We can discuss the direct \( CP \) violation \( A_{CP}^{b \rightarrow s\gamma} \) in the \( b \rightarrow s\gamma \) decay, which is given as \(^2\)
\[
A_{CP}^{b \rightarrow s\gamma} = \frac{\Gamma(B \rightarrow X_s\gamma) - \Gamma(B \rightarrow X_z\gamma)}{\Gamma(B \rightarrow X_s\gamma) + \Gamma(B \rightarrow X_z\gamma)} |_{E_\gamma > (1-\delta)E_{\gamma}^{\max}} = \frac{\alpha_s(m_b)}{|C_{\gamma}|^2} \left[ -\frac{40}{81} \text{Im}[C_2 C_{\gamma}^*] - \frac{8z}{9} [v(z) + b(z, \delta)] \text{Im}\left( 1 + \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}} \right) C_2 C_{\gamma}^* \right] - 4 \frac{\text{Im}[C_{\gamma} C_{\gamma}^*]}{9} \frac{b(z, \delta)}{27} \text{Im}\left( 1 + \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}} \right) C_2 C_{\gamma}^*,
\]
where \( v(z) \) and \( b(z, \delta) \) are explicitly given in \(^2\), and \( C^\text{eff} \) includes both the SM and SUSY contributions. Although the experimental data has still large error bar, we can discuss the SUSY contribution to \( A_{CP}^{b \rightarrow s\gamma} \).

Let us set up the framework of our calculations. Suppose that \( \mu \tan \beta \) is at most \( O(1) \) TeV. Then, magnitudes of \( (\delta_{\tilde{d}}^{LL})_{23} \) and \( (\delta_{\tilde{d}}^{RR})_{23} \) are constrained by \( M_{12}^2 \) as seen in Eq. (3.2). Taking account of \( h_s = 0.1 \) in Eq. (2.10), we obtain \( |(\delta_{\tilde{d}}^{LL})_{23}| \simeq |(\delta_{\tilde{d}}^{RR})_{23}| \simeq 0.02 \) in our previous work.\(^12\) Then, these contributions to \( C_G^{\tilde{g}} \) and \( C^{\tilde{g}}_{8G} \) are minor.

On the other hand, \( (\delta_{\tilde{d}}^{LR})_{23} \) and \( (\delta_{\tilde{d}}^{RL})_{23} \) are severely constrained by \( C^\text{eff}_{\gamma} \) and \( C^\text{eff}_{8G} \). We show the constraint for \( (\delta_{\tilde{d}}^{LR})_{23} \) and \( (\delta_{\tilde{d}}^{RL})_{23} \) in our following calculations. In our convenience, we suppose \( |(\delta_{\tilde{d}}^{LR})_{23}| = |(\delta_{\tilde{d}}^{RL})_{23}| \). Then, we can parametrize these parameters as follows:
\[
(\delta_{\tilde{d}}^{LR})_{23} = |(\delta_{\tilde{d}}^{LR})_{23}| e^{2i\theta_{\tilde{d}}^{LR}}, \quad (\delta_{\tilde{d}}^{RL})_{23} = |(\delta_{\tilde{d}}^{RL})_{23}| e^{2i\theta_{\tilde{d}}^{RL}}, \quad (3.11)
\]
where \( \theta_{\tilde{d}}^{LR} \) and \( \theta_{\tilde{d}}^{RL} \) are taken in the region \([0 - \pi]\). By using this set up, we show numerical analyses in the next section.

§4. Numerical analyses

In this section, we show the numerical analyses of the \( CP \) violation in the \( B \) mesons. In our following numerical calculations, we fix the squark mass and the gluino mass as
\[
m_{\tilde{q}} = 1000 \text{ GeV}, \quad m_{\tilde{g}} = 1500 \text{ GeV}, \quad (4.1)
\]
which are consistent with recent lower bound of these masses at LHC.\(^28\) We use relevant parameters as given in Table 1 of \(^12\) to estimate the SM contribution.
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Fig. 1. The direct CP violation $A_{CP}^{b\rightarrow s\gamma}$ versus $|\langle \delta^{LR}_{d} \rangle_{23}|$, where the green line denotes the SM prediction and red lines denote the upper and lower bounds of the experimental data with 90% C.L.

Fig. 2. The predicted branching ratio of $b\rightarrow s\gamma$ versus $|\langle \delta^{LR}_{d} \rangle_{23}|$, where the experimental constraint of $A_{CP}^{b\rightarrow s\gamma}$ is taken into account. Predicted value at $|\langle \delta^{LR}_{d} \rangle_{23}| = 0$ is the SM one.

At first, we discuss the $b\rightarrow s\gamma$ decay. The observed $b\rightarrow s\gamma$ branching ratio is $(3.60 \pm 0.23) \times 10^{-4}$, on the other hand the SM prediction is given as $(3.15 \pm 0.23) \times 10^{-4}$ at $O(\alpha^2)$. Since $\mu \tan \beta$ is supposed to be lower than $O(1)$ TeV, the contribution of $|\langle \delta^{LL}_{d} \rangle_{23}| \approx |\langle \delta^{RR}_{d} \rangle_{23}|$ is negligibly small. The contribution of $|\langle \delta^{LR}_{d} \rangle_{23}|$ becomes important through the interference with the SM component in the decay amplitude. On the other hand, since $|\langle \delta^{RL}_{d} \rangle_{23}|$ does not interfere with the SM component, its contribution is minor. The branching ratio gives the constraint for the magnitude of $|\langle \delta^{LR}_{d} \rangle_{23}|$. The direct CP violation of the $b\rightarrow s\gamma$ is also useful to constraint $|\langle \delta^{LR}_{d} \rangle_{23}|$. We show the $A_{CP}^{b\rightarrow s\gamma}$ versus $|\langle \delta^{LR}_{d} \rangle_{23}|$ in Fig. 1, where the upper and lower bounds of the experimental data with 90% C.L. are denoted red lines, and the predicted value of the SM is shown by the green line as the eye guide. As far as $|\langle \delta^{LR}_{d} \rangle_{23}| \leq 10^{-3}$, the predicted value is within the experimental allowed region.

In Fig. 2, we show the $|\langle \delta^{LR}_{d} \rangle_{23}|$ dependence of the branching ratio taking into account the constraint of $A_{CP}^{b\rightarrow s\gamma}$ as seen in Fig. 1. Here, the allowed region at $|\langle \delta^{LR}_{d} \rangle_{23}| = 0$ is the SM prediction. As the magnitude of $|\langle \delta^{LR}_{d} \rangle_{23}|$ increases, the predicted region of the branching ratio splits into the larger region and smaller one. The excluded region between two regions is due to the constraint of $A_{CP}^{b\rightarrow s\gamma}$. Then, the predicted branching ratio becomes inconsistent with the experimental data at $|\langle \delta^{LR}_{d} \rangle_{23}| \geq 5.5 \times 10^{-3}$.∗

To see the role of the phase $\theta^{LR}_{23}$, We show $A_{CP}^{b\rightarrow s\gamma}$ versus $\theta^{LR}_{23}$ for $|\langle \delta^{LR}_{d} \rangle_{23}| = 10^{-3}$ (blue) and $|\langle \delta^{LR}_{d} \rangle_{23}| = 10^{-4}$ (orange) in Fig. 3. The pink horizontal lines denote the experimental upper and lower bounds at $1\sigma$ level. As seen in this figure, we find that the reduction of the experimental error-bar will constrain the SUSY phase $\theta^{LR}_{23}$ severely.

In Fig. 4, we plot the allowed region on the $|\langle \delta^{LR}_{d} \rangle_{23}| - \theta^{LR}_{23}$ plane by putting the experimental data at 90% C.L. of the branching ratio and the direct CP violation

∗) We take into account only the contribution of gluino-squark couplings. However the charged Higgs and chargino also contribute to the $b\rightarrow s\gamma$ process in SUSY. In our analysis, this contribution is neglected. Comprehensive analysis will be presented elsewhere.
The allowed region of $|\langle \delta^{LR}_{d} \rangle_{23}|$ is cut at $5.5 \times 10^{-3}$, where $\theta^{LR}_{23}$ is tuned around $\pi/2$. Around $\pi/4$ and $3\pi/4$, $A^{b \to s\gamma}$ give the severe constraint as seen in Fig. 3. This CP violation phase also contributes on the CP-violating asymmetry of the non-leptonic decays of $B^0_\ell$ and $B^0_s$ mesons.

Let us discuss $S_f$, which is the measure of the CP-violating asymmetry, for $B^0_\ell \to J/\psi K_S$, $\phi K_S$, $\eta' K^0$. As discussed in §2, these $S_f$’s are predicted to be same ones in the SM. On the other hand, if the squark flavor mixing contributes to the decay process at the one-loop level, these asymmetries are different from among as seen in Eq. (3.7).

Since the phase $\theta^{LR}_{23}$ contributes to $A^{b \to s\gamma}$, $S_{\phi K_S}$ and $S_{\eta' K^0}$ of $B^0_\ell$ decays, we expect the correlations among them. We fix $|\langle \delta^{LR}_d \rangle_{23}| = 10^{-4}$ (orange) and $10^{-3}$ (blue) for typical values in the following calculations. We show the predicted regions on the $A^{b \to s\gamma}_{CP}-S_{\phi K_S}$ and $A^{b \to s\gamma}_{CP}-S_{\eta' K^0}$ planes in Figs. 5 and 6, respectively. The experimental data is denoted by red lines at 90% C.L. We also present the predicted region on the $S_{\phi K_S}-S_{\eta' K^0}$ plane in Fig. 7, the slant dashed line denotes the SM prediction $S_{J/\psi K_S} = S_{\phi K_S} = S_{\eta' K}$, where the observed value $S_{J/\psi K_S} = 0.671 \pm 0.023$ is put. The reduction of the experimental error of $A^{b \to s\gamma}_{CP}$ will give us severe predictions for $A^{b \to s\gamma}_{CP}$.
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Fig. 7. Predicted region on the $S_{\phi K_S}-S_{\eta'K^0}$ plane, where the slant dashed line denotes the SM prediction.

Fig. 8. Predicted asymmetry $S_{\phi\phi}$ in the $B_S^0$ decay versus $A_{CP}^{b\to s\gamma}$.

$S_{\phi K_S}$ and $S_{\eta'K^0}$. It is noticed that this predicted region is different from the one in the previous work,\cite{previous_work} where $(\delta_{d}^{LR})_{23}$ is neglected and $\mu \tan \beta = \mathcal{O}(10)\,\text{TeV}$.

As seen in Figs. 5, 6 and 7, the reduction of the experimental errors will provide powerful tool to find the contribution of the squark flavor mixing. At last, we show the correlation between $A_{CP}^{b\to s\gamma}$ and $S_{\phi\phi}$ of the $B_S^0$ decay in Fig. 8. We expect the observation of the $CP$ violation in $B_S^0 \to \phi\phi$ at LHCb.

§5. Summary

The $CP$ violation of the neutral $B$ meson is the important phenomenon to search for new physics. We have discussed the contribution of the squark flavor mixing from $(\delta_{d}^{LR})_{23}$ and $(\delta_{d}^{RL})_{23}$ on the direct $CP$ violation of the $b \to s\gamma$ decay and the $CP$-violating asymmetry in the non-leptonic decays of $B_d^0$ and $B_s^0$ mesons.

The magnitude of $|A_{CP}^{b\to s\gamma}|$ is bounded by the branching ratio of $b \to s\gamma$ with the constraint of $A_{CP}^{b\to s\gamma}$. The predicted branching ratio becomes inconsistent with the experimental data at $|A_{CP}^{b\to s\gamma}| \geq 5.5 \times 10^{-3}$. We have obtained the allowed region on the $|A_{CP}^{b\to s\gamma}|$ vs $\theta_{23}^{LR}$ plane. While the $|A_{CP}^{b\to s\gamma}|$ is cut at $5.5 \times 10^{-3}$, $CP$-violating phase $\theta_{23}^{LR}$ is severely constrained at $|A_{CP}^{b\to s\gamma}| \geq 2 \times 10^{-3}$. This $CP$-violating phase also contribute to the $CP$-violating asymmetry in the non-leptonic decays of $B_d^0$ and $B_s^0$ mesons.

We have predicted the correlation among $A_{CP}^{b\to s\gamma}$ and $S_{\phi\phi}$ of the $B_d^0$ and $B_s^0$ decays. These $CP$-violating asymmetries could deviate from the SM predictions.

Since we suppose rather small $\mu \tan \beta$, $\mathcal{O}(1)\,\text{TeV}$, the contributions from $LL(RR)$ components are minor in these $CP$-violating asymmetries. In this case, the new physics contribution is minor in $M_{12}^q$ of $B_q - \bar{B}_q\ (q = d, s)$ mixing since $|A_{CP}^{b\to s\gamma}|$ is at most $\mathcal{O}(10^{-3})$. This result is consistent with the recent result of the $CP$ violations at LHCb as discussed in §2.

In the near future, the precise data of the direct $CP$ violation of $b \to s\gamma$ and $CP$-violating asymmetries in the non-leptonic decays of $B_d^0$ and $B_s^0$ mesons give us the crucial test for our framework of the squark flavor mixing.
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