Simulating single-photon sources based on backward-wave spontaneous parametric down-conversion in a periodically poled KTP waveguide

A A Shukhin¹, D O Akatiev², I Z Latypov¹, A V Shkalikov¹ and A A Kalachev¹,²
¹Zavoisky Physical-Technical Institute, 10/7, Sibirsky tract str., 420029, Kazan, Russia
²Kazan Federal University, 18, Kremlevskaya str., 420008, Kazan, Russia

E-mail: spaun73@mail.ru

Abstract. The properties of the backward-wave spontaneous parametric down-conversion (SPDC) in a periodically poled potassium titanyl phosphate (KTP) waveguide are studied in the context of creating narrowband heralded sources of single-photon states. The effective index of refraction and spatial profile of different waveguide modes, efficiency of different SPDC processes and purity of heralded photons are calculated numerically for a given waveguide. Compared to the usual co-propagating SPDC, spectral narrowing of the backward-wave SPDC was observed as should be expected.

1. Introduction
Developing narrowband sources of single photons and correlated photon pairs is an important part of quantum optics and quantum information [1]. In particular, single-photon wave packets of a sub-GHz or GHz bandwidth can effectively interact with atomic systems and therefore are demanded for implementation of quantum computing and quantum communication protocols involving optical quantum memories [2]. A promising approach to the problem is based on the backward-wave spontaneous parametric down-conversion (SPDC) in a nonlinear waveguide [3-5]. First, as with all SPDC-based sources, the photon number correlation between the down-conversion fields can be exploited to herald the existence of one photon by detection of its partner. In doing so, the vacuum state contribution is automatically removed from the output state while the multiphoton state contribution can be removed by using photon number resolving detectors. Second, emitting correlated photons in opposite directions allows one to reduce both SPDC bandwidth and spectral correlations within each photon-pair [3, 4]. As a result, it is possible to prepare single-photon states of high purity and GHz bandwidth at room temperature. Third, enclosing a nonlinear waveguide in a cavity allows one to realize SPDC in a single cavity mode of MHz bandwidth without additional filtering [5] thereby achieving high efficiency of the source. Finally, such a cavity-enhanced SPDC makes it possible to prepare single-photons of various wave forms by modulating the pump field [6].

In the present work, we calculate the properties of such a promising regime of SPDC in a periodically poled potassium titanyl phosphate (KTP) waveguide and report experimental results demonstrating spectral narrowing of the SPDC field.
2. Waveguide model

Our waveguide chip (AdvR Inc.) is fabricated from an X-cut KTP crystal by ion exchange. The area with Rb-diffusion (RTP) has a bigger refractive index than the KTP substrate, forming a channel waveguide under the surface. Along the \(-z\)-direction, the index profile is usually described as

\[ n_j(z) = n_{j,KTP} + \Delta n_j \text{erfc}\left(-z / z_0\right), \]

where \( j = Y, Z \), \( \Delta n_j = n_{j,RTP} - n_{j,KTP} \), and the crystal surface is assumed to be located at \( z = 0 \). Figure 1 presents the microscope images of the waveguide cross section with coupled white light for the channels with different width. The extracted average value of the depth parameter \( z_0 \) proves to be equal to \( \approx 8.9 \, \mu m \). The length of the waveguide chip is 24 mm.

![Microscope images of the waveguide cross section with coupled white light and corresponding concentration profiles of the Rb dopants (in boxes). The width of the waveguides is 2 \( \mu m \) (left), 3 \( \mu m \) (center) and 4 \( \mu m \) (right).](image)

The effective index of refraction \( n_{\text{eff}}(\omega) \) and spatial profile \( u_{\text{eff}}(\rho) \), \( \rho = (y, z) \) of different waveguide modes were calculated using commercial software package Comsol Multiphysics with RF module. The Sellmeier equations for \( n_{j,KTP}(\lambda) \) and \( \Delta n_j(\lambda) \) were taken from [7] and [8], respectively. Figure 2a shows the simulation region consisting of two rectangles filled with KTP, a rectangular strip of RTP between them and an air region above. The simulation region was chosen to be large enough to make boundary effects negligible. The fundamental modes are shown in figure 2b.

![The simulation domain (a) and intensity profile of the fundamental waveguide modes (b) for the pump (532 nm), signal (810 nm) and idler (1550 nm) field polarized along the Z axis. The width of the waveguide is equal to 4 \( \mu m \).](image)

The number of modes supported by the waveguide decreases with increasing wavelength, as should be expected. In particular, there are 16 modes at the wavelength around 532 nm, 6 modes at around 800 nm and only 1 mode at the wavelength 1500 nm. The calculated effective refractive indexes of some low order modes are shown in figure 3.
Figure 3. The effective index of refraction for some low order modes \((m,n)\) as a function of the free space wavelength for polarization along the crystal \(Y\) and \(Z\) axes.

3. Numerical analysis

The process of SPDC involves sending a pump laser beam into the nonlinear waveguide, where the nonlinear interaction occasionally leads to the annihilation of a high-frequency pump photon and the simultaneous creation of two lower frequency photons, which are usually referred to as signal and idler photons. The photons have the same vertical (horizontal) polarization in type 0 (type I) SPDC and have orthogonal polarization in type II SPDC. Possible variants of SPDC processes are shown in figure 3. The two-photon state of the generated light can be calculated by applying the first-order perturbation theory of quantum mechanics, and the state vector of the generated light can be written as

\[
|\psi\rangle = |0\rangle + A \eta_{m,n} \int d\omega_1 d\omega_2 \eta_{m,n,m,n,m,n} F(\omega_1,\omega_2) a_{m,n}^+ (\omega_1) a_{m,n}^+ (\omega_2) |0\rangle,
\]

where \(|0\rangle\) denotes the vacuum state, \(a_{m,n}^+ (\omega)\) is a creation operator corresponding to the waveguide spatial mode \(u_{m,n} (\rho)\), \(d^2\rho = dydz\), and frequency \(\omega\),

\[
\eta_{m,n,m,n,m,n} = \int d^2\rho u_{m,n}^* (\rho) u_{m,n}^* (\rho) u_{m,n} (\rho) u_{m,n} (\rho),
\]

is the overlap integral between the three interacting fields,

\[
F(\omega_1,\omega_2) = E_p (\omega_1 + \omega_2) \text{sinc} \left[ \Delta k (\omega_1,\omega_2) L/2 \right] \exp \left[ i \Delta k (\omega_1,\omega_2) L/2 \right]
\]

is the joint spectral amplitude (JSA), \(E_p (\omega)\) is the pump spectral amplitude, and

\[
\Delta k (\omega_1,\omega_2) = \frac{\omega_p n_{eff} (\omega_1)}{c} - \frac{\omega_p n_{eff} (\omega_2)}{c} + \frac{\omega_i n_{eff} (\omega_1)}{c} - \frac{\omega_i n_{eff} (\omega_2)}{c} - \frac{2\pi m}{\Lambda}
\]

is the momentum mismatch taking into account quasiphasematching vector \(2\pi / \Lambda\) in the order \(m\). The sign “+” before the third term on the right hand side of equation (5) corresponds to the backward propagating idler wave. The factor \(\eta_{m,n} = \int d^2\rho u_{m,n} (\rho) E_m (\rho)\) takes into account the overlap.
between the incident (Gaussian) pump mode \( E_\text{p}(z) \) and the corresponding pump mode in the waveguide, and \( A \) includes all other constants.

**Figure 3.** Possible types of SPDC in the considered waveguide and relevant nonlinear coefficients. The vertical (V) and horizontal (H) polarizations correspond to the Z and Y axes, respectively.

**Figure 4.** Coupling constants \( |\eta_{m_1m_2m_3m_4}|^2 \) for different SPDC processes in the considered waveguide of 4 \( \mu \)m width. In each case the maximum values correspond to the interaction between the fundamental modes.

The waveguides have an effective poling period of \( \Lambda = 2.03 \mu \)m, which supports SPDC via fifth-order quasi-phase matching (\( m = 5 \)) for 1.5-\( \mu \)m photons emitted in the backward direction. The wavelength of the conjugated photons depends strongly on that of the pump field, which is necessary for maintaining the momentum conservation condition \( \Delta k = 0 \). In possession of the solutions for the transverse wave equations, we evaluate the interaction efficiency for different mode triplets and different SPDC types, which can be observed under the excitation by 532.2 nm. The most significant variants are shown in figure 4. In a general case, the down-converted fields will be emitted in a superposition of different mode triplets each having a distinct coupling efficiency. However, for a Gaussian pump beam properly aligned with the center of the waveguide most of the energy can be deposited into the (00)-mode and very little is coupled into higher-order waveguide modes. As a result, the process \((0,0) \rightarrow (0,0) + (0,0)\) becomes much more efficient than the other ones.
Regarding the single photon generation, it is important to characterize the spectral or temporal properties of photon pairs through their interference properties. Two photons interfere in a Hong-Ou-Mandel (HOM) interferometer with perfect visibility if they are indistinguishable. In the case where the two interfering photons are heralded from independent sources, indistinguishability leads to the requirement that the JSA is factorable. To be more precise, consider the experimental situation depicted in figure 5a, where two heralded photons corresponding to the same mode of two identical sources interfere at a beam splitter. The four-photon coincidence rate \( P_c(\tau) \) as a function of the delay between the two interfering photons \( \tau \) is calculated as \[ P_c(\tau) = \frac{1}{2} \left[ 1 - \int \int d\omega_1 d\omega_2 d\omega_3 d\omega_4 F(\omega_1, \omega_2) F(\omega_3, \omega_4) F^*(\omega_1, \omega_4) F^*(\omega_3, \omega_2) e^{i(\omega_2 - \omega_1)} \right], \] where JSA is assumed to be normalized. Then the HOM visibility \( V = 1 - 2P_c(0) \) corresponds to the purity of the heralded photons. In particular, consider type II SPDC process in the fundamental modes (figure 5b) and the pump field of the wavelength 532.2 nm. Figures 5c and 5d present calculated HOM dips which should be observed for the optimal duration of the pump pulse (about 6 ps). The latter is assumed to be Gaussian one. The corresponding purity of the photons is about 0.96. The duration of the single-photon pulse (FWHM) emitted in the backward direction is about 300 ps, which corresponds to the spectral width 2.9 GHz. Similar results are obtained for the other SPDC processes.

**Figure 5.** Scheme of double-source HOM interferometer (a), geometry of considered SPDC process (b), and calculated HOM dips for interfering heralded photons emitted in forward (c) and backward (d) directions. The duration of the Gaussian pump pulse is 6 ps.

4. Experimental results
In our experiment we pumped the nonlinear waveguide with the second harmonic of a CW neodymium laser (ALPHALAS MONOPOWER-532-100-SM) and measured the spectrum of the
radiation emitted in the forward direction at around 810 nm using a single-photon detector (Perkin Elmer SPCM-AQR-14FC). The pump beam was focused on the waveguide with a 10x microscope objective and the pump beam spot at the entrance of the waveguide was aligned to excite mostly the fundamental pump mode. The emitted radiation, which was collected by a 40x objective, went through an interference filter of 10-nm bandwidth, polarization filter and an air spaced Fabry-Perot resonator (IT 51-30). In doing so we managed to identify a narrow peak corresponding to type-0 SPDC on a strong fluorescence background. The expected bandwidth of the signal field (0.006 nm) proves to be much smaller than that of our Fabry-Perot resonator (0.03 nm), and we were not able to measure the spectrum of the SPDC field directly. However, the observed peak corresponds to the signal field bandwidth not larger than the resonator one. The latter in turn is five times smaller than the bandwidth of usual co-propagating SPDC (~ 0.15 nm) which would be observed under the same conditions. Therefore, we can conclude that significant reduction of SPDC bandwidth was observed.

5. Conclusion
We have studied the spectral properties of two-photon states and purity of heralded one-photon states produced by backward-wave spontaneous parametric down-conversion in a periodically poled KTP waveguide, which provides fifth order quasiphasematching for such a nonlinear interaction. Compared to the usual co-propagating SPDC regime in the same material, significant reduction of SPDC bandwidth is observed. According to the numerical simulations, a single-photon source with the purity about 0.95 and bandwidth about 3GHz may be realized on the basis of the investigated material.

Acknowledgments
This research was supported by the Russian Science Foundation (Grant No. 14-12-00806).

References
[1] Eisaman M D, Fan J, Migdall A and Polyakov S V 2011 *Rev. Sci. Instrum.* **82** 071101
[2] Sangouard N, Simon C, de Riedmatten H and Gisin N 2011 *Rev. Mod. Phys.* **83** 33
[3] Booth M C, Atatüre M, Di Giuseppe G, Saleh B E A, Sergienko A V and Teich M C 2002 *Phys. Rev. A* **66** 023815
[4] Christ A, Eckstein A, Mosley P J and Silberhorn C 2009 *Optics Express* **17** 3441
[5] Chuu C S and Harris S E 2011 *Phys. Rev. A* **83** 061803(R)
[6] Kalachev A 2010 *Phys. Rev. A* **81** 043809
[7] König F and Wong F N C 2004 *Appl. Phys. Lett.* **84** 1644
[8] Cheng L K, Cheng L T, Galperin J, Hotsenpiller P A M and Bierlein J D 1994 *J. Cryst. Growth* **137** 107
[9] Fiorentino M, Spillane S M, Beausoleil R G, Roberts T D, Battle P and Munro M W 2007 *Optics Express* **15** 7479
[10] U’Ren A B, Banaszek K and Walmsley I A 2003 *Quantum Inf. Comp.* **3** 480