REAL TIME NONEQUILIBRIUM DYNAMICS OF QUANTUM PLASMAS. QUANTUM KINETICS AND THE DYNAMICAL RENORMALIZATION GROUP.

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We implement the dynamical renormalization group (DRG) using the hard thermal loop (HTL) approximation for the real-time nonequilibrium dynamics in hot plasmas. The focus is on the study of the relaxation of gauge and fermionic mean fields and on the quantum kinetics of the photon and fermion distribution functions. As a concrete physical prediction, we find that for a QGP of temperature $T \sim 200$ MeV and lifetime $10 \leq t \leq 50$ fm/c there is a new contribution to the hard ($k \sim T$) photon production from off-shell bremsstrahlung ($q \rightarrow q\gamma$ and $\bar{q} \rightarrow \bar{q}\gamma$) at order $O(\alpha)$ that grows logarithmically in time and is comparable to the known on-shell Compton scattering and pair annihilation at $O(\alpha^2\alpha_s)$.

The study of nonequilibrium phenomena under extreme conditions play a fundamental rôle in the understanding of ultrarelativistic heavy ion collisions and early universe cosmology.

Thermal field theory provides the tools to study the properties of plasmas in equilibrium, but the consistent study of nonequilibrium phenomena in real time requires the methods of nonequilibrium field theory (see also Refs. for further references). The study of the equilibrium and nonequilibrium properties of abelian and non-abelian plasmas as applied to the QGP has as ultimate goal a deeper understanding of the potential experimental signatures of the formation and evolution of the QGP in ultrarelativistic heavy ion collisions. Amongst these, photons and dileptons (electron and/or muon pairs) produced during the early stages of the QGP are considered as some of the most promising signals. Since photons and lepton pairs interact electromagnetically their mean free paths are longer than the estimated size of the QGP fireball $\sim 10 \sim 50$ fm and unlike hadronic signals they do not undergo final state interactions. Therefore photons and dileptons produced during the early stages of QGP carry clean information from this phase.

The goals our programme. aim to provide a comprehensive study of several relevant aspects of the nonequilibrium dynamics of abelian QED plasma as well as nonabelian QCD plasma in real time.

This programme about the real time evolution of field theory started in 1995 studying scalar theories and was later generalized to hot scalar QED. Our starting point is the Schwinger-Dyson equation for the expectation value of the field. For example, for the spatial Fourier transform of the transverse gauge field in
QED $a_T(\vec{x},t)$,
\[
\left( \frac{\partial^2}{\partial t^2} + k^2 \right) a_T(\vec{k},t) + \int_{-\infty}^{t} dt' \Pi_T(\vec{k},t-t') a_T(\vec{k},t') = 0,
\]
and we neglect non-linear contributions of the order $O(a_T^2)$. Here $\Pi_T(\vec{k},t-t')$ is the transverse part of the retarded photon self-energy.

Eq. (1) can be readily solve by Laplace transform with solution
\[
\tilde{a}_T(s,\vec{k}) = \frac{1}{s} \left[ 1 - \frac{k^2}{s^2 + k^2 + \Pi_T(s,\vec{k})} \right] a_T(\vec{k},0),
\]
where
\[
\tilde{a}_T(s,\vec{k}) = \int_0^{\infty} dt e^{-st} a_T(\vec{k},t).
\]

We focus on the following:

(i) The real time evolution of gauge mean fields in linear response in the HTL approximation. The goal here is to study directly in real time the relaxation of (coherent) gauge field configurations in the linearized approximation to leading order in the HTL program. The analogous study for scalar QED is given in ref.

(ii) The quantum kinetic equation that describes the evolution of the distribution function of photons in the medium, again to leading order in the HTL approximation. This aspect is relevant to study photon production via off-shell effects directly in real time. This quantum kinetic equation, obtained from a microscopic field theoretical approach based on the dynamical renormalization group (DRG) displays novel off-shell effects that cannot be captured via the usual kinetic description that assumes completed collisions.

(iii) The evolution in real time of fermionic mean fields features anomalous relaxation arising from the emission and absorption of magnetic photons (gluons) which are only dynamically screened by Landau damping. The Bloch-Nordsieck approximation for the fermion propagator provides a resummation of the infrared divergences associated with soft photon (or gluon) bremsstrahlung in the medium. In ref. we implement the DRG to study the evolution of fermionic mean fields providing an alternative to the Bloch-Nordsieck treatment.

(iv) We obtain in ref. the quantum kinetic equation for the fermionic distribution function for hard fermions via the DRG. The DRG leads to a quantum kinetic equation directly in real time bypassing the assumption of completed collisions and leads to a time-dependent collision kernel free of infrared divergences.
We studied the relaxation of a gauge mean field in linear response to leading order in the HTL approximation both for soft momentum $k \leq eT$ and for semihard momentum $eT \ll k \ll T$ under the assumption of weak electromagnetic coupling.

**Soft momentum** ($k \sim eT$): in this case the relaxation of the gauge mean field is dominated by the end-point contribution of the Landau damping cut. As a consequence, the soft gauge mean field relaxes with a power law long time tail of the form

$$a_T(\vec{k}, t) \propto a_T(\vec{k}, 0) \left[ \frac{k^2 Z_T(k)}{\omega_T^2(k)} \cos[\omega_T(k)t] - \frac{12}{e^2 T^2} \frac{\cos kt}{t^2} \right],$$

where $\omega_T(k)$ is the transverse photon pole and $Z_T(k)$ is the corresponding residue. We note that in spite of the power law tail the gauge mean field relaxes towards the oscillatory mode determined by the transverse photon pole. This reveals that the soft collective excitation in a plasma is stable in the HTL approximation.

**Ultrasoft momentum** ($k \ll eT$): In the region of ultrasoft momentum the spectral density divided by the frequency features a sharp Breit-Wigner peak near zero frequency in the region of Landau damping, with width $\Gamma_k = 12 k^3 / \pi e^2 T^2$. We find that the amplitude of a mean field of transverse photons is given by

$$a_T(\vec{k}, t) \propto a_T(\vec{k}, 0) \left[ \frac{k^2 Z_T(k)}{\omega_T^2(k)} \cos[\omega_T(k)t] + e^{-\gamma_k t} \right].$$

**Semi-hard momentum** ($eT \ll k \ll T$): In this region both the HTL approximation and the perturbative expansion are formally valid. However the spectral density in the Landau damping region is sharply peaked near $\omega = k$ and the transverse photon pole approaches the edge of the Landau damping region from above. Although the perturbative expansion is in principle valid, the sharp spectral density near the edge of the continuum results in a breakdown of the perturbative expansion. The DRG provides a consistent resummation of the lowest order HTL perturbative contributions in real time, leading for the relaxation at intermediate asymptotic times:

$$a_T(\vec{k}, t) \propto a_T(\vec{k}, 0) A_T(\vec{k}, t) \left( \frac{t}{\tau_0} \right) \frac{2 e^2}{1 + e^{2 e^2 T^2 / \pi}},$$

where $\tau_0 \sim 1/k$ and $A_T(\vec{k}, t)$ is an oscillating function. The anomalous exponent is a consequence of an infrared enhancement arising from the sharp spectral density near the threshold of the Landau damping region for semihard momentum. The crossover to exponential relaxation due to collisional processes at higher orders is discussed in [8].

Using the techniques of nonequilibrium field theory and the DRG, we obtain the quantum kinetic equation for the distribution function of semihard photons $eT \ll k \ll T$ to lowest order in the HTL approximation assuming that the fermions are thermalized. An important result is that the collision kernel is time-dependent and the DRG reveals that detailed balance emerges during microscopic time scales, i.e,
much shorter than the relaxation scales. In the linearized approximation we find that the departure from equilibrium of the photon distribution function relaxes as:

\[ \delta n_k^\gamma(t) = \delta n_k^\gamma(t_0) \left( \frac{t - t_0}{\tau_0} \right) - e^{\frac{-4\pi^2}{18} \left( \frac{k}{\tau_0} \right)^2} \]  

for \( k(t - t_0) \gg 1 \),

where \( \tau_0 \sim 1/k \), and \( t_0 \) is the initial time. Furthermore, this quantum kinetic equation allows us to study photon production by off-shell effects, which to leading order in the HTL approximation is of order \( \alpha \). Extrapolating the result from QED to thermalized QGP with two flavors of light quarks, we find that the total number of hard photons at time \( t \) per invariant phase space volume to lowest order is

\[ N(t) = \frac{5\alpha T^3}{18\pi^2 k^2} \left\{ \ln \left[ 2k(t - t_0) \right] + \gamma_E - 1 \right\} \]  

for \( k(t - t_0) > 1 \).  

We find that for a quark-gluon plasma at temperature \( T \sim 200 \text{ MeV} \) and of lifetime \( 10 \leq (t - t_0) \leq 50 \text{ fm/c} \), this new hard (\( k \sim T \)) photon production by off-shell bremsstrahlung is comparable to the known contribution from Compton scattering and pair annihilation of order \( \alpha \alpha_s \). This is a noteworthy result and our main point: we find photon production to lowest (one-loop) order arising solely from off-shell effects: \( (q \to q\gamma \text{ and } \bar{q} \to \bar{q}\gamma) \) and annihilation of quarks \( (q\bar{q} \to \gamma) \).

We implement the DRG resummation to study the real-time relaxation of a fermion mean field for hard momentum. The emission and absorption of magnetic photons which are only dynamically screened by Landau damping introduce a logarithmic divergence in the spectral density near the fermion mass shell. The DRG resums these divergences in real time and leads to a relaxation of the fermion mean field for hard momentum given by:

\[ \psi(\vec{k}, t) \mid_{kt \gg 1} = e^{-\alpha T \left[ \ln(\omega_P t) + 0.12652 \ldots \right]} \times \text{oscillating phases} \]

with \( \omega_P \) being the plasma frequency.

We obtain a quantum kinetic equation for the distribution function of hard fermions using non-equilibrium field theory and the DRG resummation. In the linearized approximation the distribution function relaxes as:

\[ \delta n_k^f(t) \mid_{kt \gg 1} = \delta n_k^f(t_0) e^{-2\alpha T (t - t_0) \left[ \ln(\omega_P t) + 0.12652 \ldots \right]} \]

where the anomalus relaxation exponent is twice that of the mean field.

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Fig. 1. The ratio of the number of hard photons produced by off-shell processes according to eq. (3) ($q \rightarrow q\gamma$ and $\bar{q} \rightarrow \bar{q}\gamma$) to that produced by on-shell processes ($qg \rightarrow q\gamma$ and $qg \rightarrow gg\gamma$) as a function of QGP lifetime $t$ for $\alpha = 1/137$, $\alpha_s = 0.4$ and $T = 200$ MeV.

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