Modeling viscous-plastic extrusion of oil-bearing materials pertaining to Bingham rheology

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Abstract. This paper defines the boundaries of changing the screw turn pitch to ensure maximum productivity in transporting a viscous-plastic fluid. A technique is proposed to determine the capacity of screw conduits of oil presses using Bingham viscous-plastic model of extrusion. The numerical modeling has shown a possibility for a significant increase in productivity of the FP oil press by means of optimizing the geometry of the press screw by changing its pitch to the optimal value.

1. Introduction
Increasing the efficiency of vegetable oil production resulting from studying physical and chemical peculiarities of the processes is an important task for the food industry [1]. At that, studies of theoretical and practical foundations of mass transfer in the production processes of oil extraction and refining [2] provide significant additional possibilities for further improvement of the processes and equipment in the food industry.

Using the balance equations of material and concentration flows allows significantly simplifying the formulation of the hydrodynamic models [3] when transitioning from the microlevel to the macrolevel. The systems of ordinary differential equations derived from the mass, thermal and pressure balances during the process [4] allows neglecting the concentration and thermal arrays, using only their average values. At that, the equations of simultaneous transfer of heat and mass are usually based on models from the similarity theory [5] and indirectly account for the hydrodynamics of the unfolding processes. The extrusion devices generate mixing or moving forces for relatively short periods of time, making them highly energy efficient in vegetable oil production.

Analysis of a one-dimensional model of flow rheology in the FP screw press has shown that the principal factors influencing its operation are the rotational velocity of the screw shaft and the width of the output slot of the screw escapement device [3]. This fact indirectly supports the necessity to use a more detailed rheological model for flow inside screw presses. Analysis of the pressing process in the screw press with a one-dimensional model revealed a necessity to apply a sigmoid function for dependence of the head conductivity on pressure [13]. Analysis of operation of the FP screw press escapement device has shown that its throughput capacity depends not only on geometry, but on shaft rotational velocity as well [5], thus supporting the necessity to use the Bingham rheological model to determine its throughput capacity. Studies of oil-bearing suspension flow through the screw conduit [6] have shown that the suspension flows as a rigid core [7], lubricated with a thin film at the wall of the conduit, while inside the core the particles are held together with mutual attraction [8]. Thus, the extruded vegetable oily material shows a significant non-linearity in the flow equations and pertains to
elastically yielding disperse materials. In this case, the movement of the disperse medium in the interturn conduit is determined by the external friction forces and may be represented as a turn of the screw on the basis of the elastic-plastic flow model of the Bingham structure. The main parameters influencing the distribution of the friction forces are geometric characteristics of the screw blade bordering on the body (pressing cage): turn diameter $D_b$, turn pitch $S_b$, shaft diameter $d_v$ [9]. Let us consider the friction forces arising on the surfaces of an equivalent rectangular conduit during transportation with a screw (Figure 1).

![Figure 1. Geometry of material movements in the conduit formed by the screw and the body of the pressing cage of the FP oil extraction machine.](image)

The angle of slope $\beta$ of the cross-section of this rectangular channel is determined with the equation:

$$\beta = \arctan \left( \frac{S_b}{\pi \cdot D_b} \right);$$  

(1)

The width $L_n$ of the channel is determined from the angle of slope $\beta$:

$$L_n = \sin(\beta) = \sin \left[ \arctan \left( \frac{S_b}{\pi \cdot D_b} \right) \right];$$  

(2)

The height of the cross-section is determined by a difference between the diameters of the screw turn and the shaft $W_n = \frac{D_b - d_v}{2}$. The angle of induced displacement of the material inside the rectangular conduit of the turn $\alpha$ is determined from the ratio of friction coefficients of the Bingham elastic-plastic core against the body ($\mu_{\beta}$) and against the turn conduit ($\mu_{\alpha}$)

$$\alpha = \arccos \left( \frac{\mu_{\beta} \cdot dS_{bc}}{\mu_{\alpha}} \right) - \beta$$  

(3)

As a result of rotation of the turn's blade, the material bordering on the top part of the pressing cage moves from point A to point B in a time $dt$. Then, the displacement velocity vector $V_{ab} = V_{ac} + V_{cb}$. From the ABC triangle (Figure 1), let us determine the displacement $V_{ac}$ that is proportional to the angular velocity of the shaft rotation ($\omega$):

$$V_{ac} = \frac{D_b}{2} \cdot \omega \cdot \cos(\beta);$$  

(4)

where $\beta$ is the inclination angle of the turn conduit. Taking (4) into account, the displacement of material along the turn conduit ($V_{cb}$) is determined by the ratio:

$$V_{cb} = \frac{V_{ac} \cdot \sin(\alpha)}{\cos(\beta) \cdot \sin(\alpha) + \cos(\alpha) \cdot \sin(\beta)};$$  

(5)

where $\alpha$ is the angle of the displacement vector $V_{ab}$. During the displacement from point A to point Be, the material experiences friction forces against the turn conduit ($F_b$) and against the body of the screw ($F_k$). Under stabilized mode of operation, projections of these forces onto the axis of the turn conduit
are equal to zero, thus \( \frac{F_L}{F_K} = \cos(\alpha + \beta) \). As the forces are proportional to the area of contact between the material and the screw conduit and the body [10], we may write down the following, taking into account the ratio between these areas \( dS_{bk} \):

\[
dS_{bk} = \frac{d_L}{D_b} + \frac{2}{\pi \cdot D_b \cdot S_B} \cdot \frac{\partial}{\partial r} \left[ \sqrt{(2 \cdot \pi \cdot r)^2 + S_B^2} \right] dr ;
\]

(6)

knowing the friction coefficients of the material against the body \( (\mu_3) \) and the screw conduit \( (\mu_a) \), we may determine the angle \( \alpha \) of the displacement vector \( V_{AB} \) taking into account the ratio between the friction coefficients against the screw and the body, and accounting for (5) – the displacement velocity of the material along the screw turn conduit \( (V_{CB}) \):

\[
V_{CB} = \frac{V_{AC} \cdot \sin(\alpha)}{\sin(dS_{bk})}.
\]

(7)

From (7), let us determine the throughput of the turn conduit \( (q_\theta) \):

\[
q_\theta = V_{AC} \cdot L_n \cdot W_n.
\]

(8)

Thus, we finally have the turn conduit throughput (8) with accounts for (9) and (4) as a function of the screw geometry, friction coefficients and shaft rotational frequency. Differentiating the turn conduit throughput \( (q_\theta) \) with respect to the turn pitch and equating it to zero \( \frac{\partial}{\partial S_B} q_\theta (S_B) = 0 \), we determine the maximum throughput for a given depth ratio of the turn conduit. As the body forms one of two surfaces bounding the material inside the screw conduit, while the screw forms another one, then displacement of material along the body would require that friction of material against the interior surface of the body is less than that against the screw. Otherwise, the material is going to rotate with the screw without moving in the axial direction [11].

Determining the friction coefficient against the turn conduit \( (\mu_a = 0.3) \) and the friction coefficient against the body \( (\mu_3 = 0.7) \) we may evaluate the throughput capacity of the turn (8) in the absence of resistance at the press escapement [12,13].

Thus, the throughput capacity of the rectangular equivalent turn conduit depends on the ratio between the diameter of the turn and that of the shaft, turn pitch and the friction coefficients against the conduit and the body. Transition to actual geometry of extruder will require taking into account the volume of the turn blade that depends on the length of the blade \( L_B \), width of the top part of the turn blade \( W_{eff} \), adjacent to the pressing cage and the width of the bottom part of the blade \( W_{eff} \). Taking into account a criterion of hydraulic radius, the cross-section area \( (A_T) \) and perimeter \( (P_T) \) of the conduit are its important characteristics:

\[
A_T = \frac{(D_a - d_a) \left[ \pi \cdot D_b \cdot S_B + \pi \cdot d_a \cdot S_B - w_{eff} - w_{eff} \right]}{4} \left[ \sqrt{(\pi \cdot D_b)^2 + S_B^2} + \sqrt{(\pi \cdot d_a)^2 + S_B^2} \right]
\]

(9)

\[
P_T = \frac{\pi \cdot D_b \cdot S_B}{\sqrt{(\pi \cdot D_b)^2 + S_B^2}} + \frac{\pi \cdot d_a \cdot S_B}{\sqrt{(\pi \cdot d_a)^2 + S_B^2}} - w_{eff} - w_{eff} + \left( D_a - d_a \right)^2 + \left( w_{eff} - w_{eff} \right)^2
\]

(10)

Knowing the ratio between the area (9) and perimeter (10), we may determine a rectangular conduit equivalent to the trapezoid conduit from the following ratio of its width \( (w_s) \) to height \( (h_s) \):

\[
w_s = \frac{P_T + \sqrt{P_T^2 - 16 \cdot A_T}}{4} ;
\]

(11)
\[ h_z = P_z - \frac{P_z^2 - 16 \cdot A_v}{4} \] (12)

Having determined the dimensions of the equivalent rectangular conduit from equations (11), (12) we now have a possibility to model the throughput capacity of an actual extruder from solutions of the following system of equations:

\[
\begin{align*}
W_z \left[ P_z \left( D_B, S_B, d_a, w_{all}, w_{all} \right), A_T \left( D_B, S_B, d_a, w_{all}, w_{all} \right) \right] &= L \left( D_a, S_a \right) \\
H_z \left[ P_z \left( D_B, S_B, d_a, w_{all}, w_{all} \right), A_T \left( D_B, S_B, d_a, w_{all}, w_{all} \right) \right] &= W_n \left( D_a, S_a \right) \\
q_a \left( \omega, D_B, S_B, d_a, \mu_a, \mu_3 \right) &= q_n \left( \omega, D_z, S_z, d_z, \mu_a, \mu_3 \right)
\end{align*}
\] (13)

where \( D_z \) is the equivalent turn diameter; \( d_z \) is the equivalent shaft diameter; \( S_z \) is the equivalent turn pitch. Determining the geometry of a turn conduit equivalent by its hydraulic radius from the system of equations (13) \((D_z; d_z; S_z)\), we have got a possibility to optimize the turn pitch of this conduit with respect to its throughput employing a gradient method \( \frac{\partial}{\partial S_n} q_a \left( S_n \right) = 0 \). Let us consider application of this algorithm to geometry of the first turn of the FP oil press machine (Figure 1) at a rotation velocity of 15 rpm. From equations (9), (10) we obtain the cross-section area of the actual screw \( A_T = 14.028 \text{ mm}^2 \), as well as its perimeter \( P_T = 578 \text{ mm} \). We then use the data to calculate the dimensions of an equivalent rectangular conduit of the same hydraulic radius employing the formulas (11), (12) and obtain the values \( w_z = 227 \text{ mm}; h_z = 62 \text{ mm} \). Determining the throughput capacity of the conduit from the ratio (8) – \( q_a \left( \omega, D_B, S_B, d_a, \mu_a, \mu_3 \right) = 10939 \text{ liter/hour} \), we substitute these calculated values into the system of equations (13) and determine the equivalent geometry of the conduit \((D_z = 323 \text{ mm}; d_z = 200 \text{ mm}; S_z = 233 \text{ mm})\). We then use these values to perform the gradient optimization of the turn pitch \((S_z)\) under otherwise equal conditions (Figure 2).

![Figure 2. Gradient optimization of the turn pitch (S).](image)

Having determined the optimal turn pitch \( S_{\text{opt}} = 651 \text{ mm} \), we use the formula (1) to calculate the slope angle of the section \( \beta = 33^\circ \) of this rectangular conduit. From the angle of slope we may determine the optimal value of pitch for the actual turn \( S_{\beta, \text{opt}} = \pi \cdot D_B \cdot \tan(\beta) = 497 \text{ mm} \). Using this value, we...
calculated the optimal throughput capacity of the actual turn conduit as being equal to \( q_{\text{opt}} = 14.257 \text{ l/h} \).

Let us consider the throughput capacity of the screw conduits of the FP oil pressing machine (Figure 3).

![Figure 3. Screw shaft geometry of the FP oil pressing machine.](image)

Let us calculate the throughput capacity of the FP oil pressing machine's conduits for the rotational velocity of 15 rpm (Table 1).

| Turn | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    |
|------|------|------|------|------|------|------|------|------|
| \( D_B \) | 247  | 247  | 247  | 197  | 197  | 217  | 217  | 237  |
| \( S_B \) | 290  | 235  | 155  | 130  | 115  | 110  | 100  | 84   |
| \( d_a \) | 122  | 122  | 122  | 134  | 174  | 184  | 204  |      |
| \( w_{W1} \) | 32   | 30   | 24   | 23   | 21   | 20   | 20   | 20   |
| \( w_{W2} \) | 22   | 20   | 15   | 13   | 14   | 14   | 14   |      |
| \( S_2 \) | 233  | 191  | 129  | 108  | 96   | 92   | 82   | 67   |
| \( D_2 \) | 323  | 334  | 344  | 256  | 253  | 269  | 271  | 314  |
| \( d_2 \) | 200  | 209  | 219  | 181  | 190  | 226  | 238  | 281  |
| \( q_{W1} \) | 10939 | 8999 | 5547 | 2231 | 1633 | 1200 | 852  | 765  |
| \( S_1^{\text{opt}} \) | 651  | 669  | 687  | 496  | 480  | 488  | 484  | 556  |
| \( S_2^{\text{opt}} \) | 497  | 496  | 493  | 382  | 374  | 395  | 387  | 419  |
| \( q_{W1}^{\text{opt}} \) | 14257 | 14252 | 14246 | 4840 | 3867 | 2936 | 2189 | 2592 |

As the data show (Table 1), the throughput capacity reduces monotonously, thus ensuring the extraction of the liquid phase from the porous oily material. An important indicator of oil operation is its productivity, which largely depends on the throughput capacity of the first, feeding turn of the screw. The most realistic variant for upgrading this turn is changing the turn pitch \( S_2 \), whose influence on the throughput capacity has been modeled (Figure 2). Thus, using the data on the contact between the material and the body, the shaft and the side surface of the blade, we obtained average optimal values for the screw turns.
A technique is proposed to determine the capacity of screw conduits of oil presses using Bingham viscous-plastic model of extrusion. The numerical modeling has shown a possibility for a significant increase in productivity of the FP oil press by means of optimizing the geometry of the press screw by changing its pitch to the optimal value.

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