Is it possible to test directly General Relativity in the gravitational field of the Moon?

Lorenzo Iorio

1) Dipartimento Interateneo di Fisica,
Via Amendola 173, 70126, Italy

Abstract

In this paper the possibility of measuring some general relativistic effects in the gravitational field of the Moon via selenodetic missions, with particular emphasis to the future Japanese SELENE mission, is investigated. For a typical selenodetic orbital configuration the post-Newtonian Lense-Thirring gravitomagnetic and the Einstein’s gravitoelectric effects on the satellites orbits are calculated and compared to the present-day orbit accuracy of lunar missions. It turns out that for SELENE’s Main Orbiter, at present, the gravitoelectric periselenium shift, which is the largest general relativistic effect, is 1 or 2 orders of magnitude smaller than the experimental sensitivity. The systematic error induced by the mismodelled classical periselenium precession due to the first even zonal harmonic $J_2$ of the Moon’s non-spherical gravitational potential is 3 orders of magnitude larger than the general relativistic gravitoelectric precession.

1. Introduction

Some of the most interesting effects that general relativity predicts for the orbit of a test body in the field of a central mass are the secular Lense-Thirring gravitomagnetic precessions of the node $\Omega$ and the pericenter $\omega$, generated by the off-diagonal terms of the stationary part of the spacetime metric (Lense and Thirring, 1918; Ciufolini and Wheeler, 1995), and the secular Einstein’s gravitoelectric precession of the pericenter, induced by the Schwarzschild’s static part of the spacetime metric (Ciufolini and Wheeler, 1995). Their expressions, in the weak-field and slow-motion approximation, referred to an asymptotically inertial frame of reference with its origin in the center of mass of the central body, are

\[
\dot{\Omega}_{GM} = \frac{2GJ}{c^2a^3(1-e^2)^{\frac{3}{2}}}, \quad \dot{\omega}_{GM} = -\frac{6GJ\cos i}{c^2a^3(1-e^2)^{\frac{3}{2}}}, \quad \dot{\omega}_{GE} = \frac{3nGM}{c^2a(1-e^2)},
\]

Fax: +390805443144, E-mail: Lorenzo.Iorio@ba.infn.it
in which $G$ is the Newtonian gravitational constant, $c$ is the speed of light in vacuum, $J$ and $M$ are the proper angular momentum and the mass, respectively, of the central body, $a$, $e$ and $i$ are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit of the test mass and $n = \sqrt{\frac{GM}{a^3}}$ is its mean motion.

In the weak field of solar system such features are very tiny and difficult to measure because of lots of other competing classical effects which in many cases are quite larger than them. To-day, the Lense-Thirring effect has been measured, with 20% accuracy, in the gravitational field of the Earth by means of the laser-ranged data to LAGEOS and LAGEOS II satellites (Ciufolini, 2000). Moreover, the ambitious GP-B mission (Everitt et al., 2001), which is scheduled to be launched in spring 2002, will measure the effect of the gravitomagnetic field of the Earth on orbiting gyroscopes at a claimed precision of 1%. The Einstein’s precession of the pericenter has been detected in the well-known radar ranging measurements of the Mercury’s perihelion advance in the field of the Sun with 1% or better accuracy (Shapiro, 1990).

In general, it should be pointed out that the experimental basis of General Relativity is not yet particularly wide because, from one hand, it is difficult to work out new predictions of the theory which could be effectively detected, and, from the other hand, it is not easy to perform experiments, even for those effects which are already well known theoretically, or which have been already tested in some particular contexts. The gravitomagnetic Lense–Thirring drag and the gravitoelectric Einstein pericenter shift fall in this category, so that, in the author’s opinion, every effort aimed to the investigation of the possibility of enlarging the experimental foundations of General Relativity in various contexts and by using different techniques deserves attention.

Recent years have seen increasing efforts in selenodetic activity conducted by several lunar missions like Clementine and Lunar Prospector. One of the main goals of such missions is the mapping of the lunar gravity field (Lemoine et al., 1997; Konopoliv et al., 1998; 2001). Such kinds of selenodetic missions seem to be just the natural candidates in order to detect direct general relativistic effects of the lunar gravitational field on the orbit of test bodies. Indeed, about the Moon dynamics, we can speak, in a global sense, of the motions of the Moon’s center of mass and, in a local sense, of the motions around its center of mass. Recently, many general relativistic features of the global motion of the Moon in the Sun’s gravitational field, among which the most famous is the geodetic or de Sitter precession, have been investigated both theoretically (Mashhoon and Theiss, 1991; 2001) and experimentally (Ciufolini, 2001; Nordvedt, 2001). In this paper we investigate quantitatively the possibility of measuring the impact of General Relativity in the local dynamics of a satellite–Moon system with particular emphasis to the SELENE mission. Knowing the present–day level of experimental sensitivity to two of the most important general relativistic direct effects in the Moon’s gravitational field may help to focus further attentions to the lunar arena or to redirect them to other, more feasible and promising scenarios.
The plan of the work is the following. In section 2 the relativistic effects on the SELENE subsatellites are analyzed. In section 3 their values are compared to the obtainable experimental accuracy. Section 4 is devoted to the conclusions.

2. Relativistic effects on the orbits of the SELENE satellites

The forthcoming Japanese SELENE mission (Ohta et al., 2000) is of relevant scientific interest. It is currently under development by National Space Development Agency of Japan (NASDA) and Institute of Space and Astronautical Science (ISAS), and will be launched in 2004 to elucidate lunar origin and evolution. SELENE is composed of Main Orbiter, and two micro sub-satellites; the Relay Satellite (Rstar) and the VLBI Radio Satellite (Vstar) which will be used for selenodesy experiments. The range-rate accuracy for the three subsatellites should amount to $10^{-1}$ mm s$^{-1}$ (Heki et al., 1999) (sampling interval 20-60 seconds, Doppler). The designed nominal orbital parameters for Main Orbiter and Rstar are in Tab.1. Their lifetimes should amount to 1-3 years. By using in eqs.(1)-(3) the values of Tab. 1 and those for the Moon quoted in Tab. 2 it is possible to obtain for the relativistic effects on the SELENE sub-satellites’ orbits the values quoted in Tab. 3. They hold in a selenocentric inertial frame.

Table 1. Orbital parameters of SELENE sub-satellites.

| Orbital Parameter | Main Orbiter | Rstar | Units |
|-------------------|--------------|-------|-------|
| $a$               | 1838         | 3000  | km    |
| $e$               | 0.01         | 0.38  | -     |
| $i$               | 95           | 95    | deg   |
| $n$               | $8.83 \times 10^{-4}$ | $4.26 \times 10^{-4}$ | s$^{-1}$ |

Table 2. Moon parameters.

| Parameter                   | Value              | Units               |
|-----------------------------|--------------------|---------------------|
| $M$ mass                    | $7.349 \times 10^{25}$ | g                   |
| $J$ proper angular momentum | $2.32 \times 10^{36}$ | g cm$^2$s$^{-1}$    |
| $GM$                        | $4.9 \times 10^{18}$ | cm$^3$s$^{-2}$      |
| $R$ radius                  | $1.738 \times 10^8$ | cm                  |
| $\omega$ proper angular velocity | $2.66 \times 10^{-6}$ | rad s$^{-1}$       |
| $I$ moment of inertia       | $8.74 \times 10^{41}$ | g cm$^2$            |
| $\frac{GM}{I}$              | $5.452 \times 10^{-3}$ | cm                 |
| $\frac{\delta J_2}{J_2}$   | $1.71 \times 10^8$ | cm$^3$s$^{-1}$    |
| $J_2$ mass quadrupole moment| $2.03428 \times 10^{-4}$ | -                 |
| $\delta J_2$               | $9 \times 10^{-8}$ | -                   |
Table 3. Relativistic effects on the nodes and periselenia of SELENE sub-satellites.

| Relativistic precession | Main Orbiter | Rstar | Units   |
|------------------------|--------------|-------|---------|
| \(\dot{\Omega}_{GM}\)  | \(5.4 \times 10^{-17}\) | \(1.2 \times 10^{-17}\) | rad s\(^{-1}\) |
| \(\dot{\omega}_{GM}\)   | \(1.4 \times 10^{-17}\)  | \(3.3 \times 10^{-18}\) | rad s\(^{-1}\) |
| \(\dot{\omega}_{GE}\)   | \(7.8 \times 10^{-14}\)   | \(3.1 \times 10^{-14}\) | rad s\(^{-1}\) |

3. The confrontation with the present-day orbit accuracy

We will focus our attention to the gravitoelectric post-Newtonian periselenium advance which is more than 3 orders of magnitude larger than the gravitomagnetic effects, as can be inferred from Tab. 3. Indeed, it should be noticed that the present-day quality of force models applicable to low lunar satellites does not allow highly precise determination of their orbits. The largely unknown gravitational potential of the Moon remains the most important perturbation force acting upon low lunar satellites (Floberghagen et al., 1999).

The relativistic gravitoelectric perturbation affects only the pericenter of the orbit of a test particle, as can be inferred from a straightforward calculation based on the radial, transverse and cross-track components of the related disturbing acceleration (Grafarend and Joos, 1992). This implies that the relativistic effect of interest can be mapped onto an along-track shift. Indeed, in general (Christodoulidis et al., 1988) the along-track perturbation can be written as

\[
\Delta s = a \sqrt{1 + \frac{e^2}{2}} \left[ \Delta \mathcal{M} + \Delta \omega + \Delta \Omega \cos i + \sqrt{(\Delta e)^2 + (e \Delta \mathcal{M})^2} \right].
\]  

(4)

The element \(\mathcal{M}\) is the mean anomaly. In the case of the gravitoelectric pericenter shift it reduces to

\[
\Delta s_{GE} = a \sqrt{1 + \frac{e^2}{2}} \Delta \omega_{GE}.
\]  

(5)

For Main Orbiter and Rstar it yields the effects quoted in Tab. 4 for different arc lengths which are of common use, e.g., in typical lunar gravity field mapping missions. In order

Table 4. Along-track relativistic gravitoelectric effects on the SELENE sub-satellites.

| Arc length | Main Orbiter | Rstar     |
|------------|--------------|-----------|
| 1 week     | \(8.6 \times 10^{-2}\) m | \(5.6 \times 10^{-2}\) m |
| 2 weeks    | \(1.7 \times 10^{-1}\) m  | \(1.1 \times 10^{-1}\) m |
| 4 weeks    | \(3.7 \times 10^{-1}\) m  | \(2.4 \times 10^{-1}\) m |

to investigate if such effects are within the present-day or near forthcoming orbit accuracy
of lunar missions we rely on the results presented in (Floberghagen et al., 1999). In it the RMS of orbit fit for a test case where the satellite polar orbits are determined through low-low satellite-to-satellite tracking (SST) in combination with Earth-based ground tracking is presented. As input models for the gravity field and the albedo the GLGM-2 lunar gravity model (Lemoine et al., 1997) and DLAM-1 lunar albedo model (Floberghagen et al., 1999), respectively, are adopted. It should be noticed that, due to the non-conservative nature of the albedo force, the longer the arc, the larger the orbit error. For the integrated SST Doppler measurements a sampling rate of 20 s is assumed with a precision of $10^{-1}$ mm s$^{-1}$. For arc lengths of one week of a polar 100 km altitude satellite the along-track RMS is of the order of some meters or even of $10^1$ m, as can be inferred from Tab. 2 of (Floberghagen et al., 1999). Unfortunately, it is almost 1-2 orders of magnitude larger than the general relativistic gravitoelectric effect on SELENE Main Orbiter quoted in Tab. 4.

These results seem to be confirmed also by the estimates of the contribution of differential VLBI ($\Delta$VLBI) to the standard 2-way Doppler which allows for three dimensional orbit determination of the SELENE Main Orbiter summarized in Fig. 2 of (Heki et al., 1999). Over an arc length of 12 hours the along-track accuracy amounts almost to 1 m, while the along-track gravitoelectric relativistic shift, over the same time span is $6.2 \times 10^{-3}$ m.

In order to get an order of magnitude of the main systematic error which could affect such kind of measurement we will calculate the mismodelled classical precession of the periselenium of Main Orbiter induced by the first even zonal harmonic $J_2$ of the Moon’s gravity field according to the LP75G lunar gravity model (Konopoliv et al., 1998). The rate of the classical pericenter advance due to the first even zonal harmonic is

$$\dot{\omega}_{\text{class}} = -\frac{3nJ_2}{4(1-e^2)^2} \left(\frac{R}{a}\right)^2 \left(1 - 5\cos^2 i\right).$$

(6)

According to Tab. 1 and Tab. 2 we have $\delta \dot{\omega}_{\text{class}} = 5.1 \times 10^{-11}$ s$^{-1}$ which is 3 orders of magnitude larger than the relativistic gravitoelectric periselenium shift, as can be inferred from Tab. 3. Anyway, this specific kind of problems could be overcome by adopting suitable orbital residuals combinations, as done in the terrestrial field for the Lense-Thirring LAGEOS experiment (Ciufolini, 2000). However, it should be also considered that, in the case of an orbital motion around the Moon the long-term perturbations due to the odd zonal harmonics of the lunar gravitational field play a role much more important that in the Earth case (Knežević and Milani, 1998), especially for the eccentricity $e$ and the pericenter $\omega$.

4. Conclusions

The present-day level of accuracy of satellite selenodesy does not allow the detection of the general relativistic gravitomagnetic and gravitoelectric precessions of the satellites’ orbits in the field of the Moon. For example, such effects on the SELENE subsatellites are,
at most, 1 or 2 orders of magnitudes smaller than the obtainable orbit accuracy. This holds for the gravitoelectric periselenium advance of Main Orbiter. The systematic errors which would be induced by the poor knowledge of the lunar gravity field are almost 3 orders of magnitude larger than the gravitolectric periselenium advance of Main Orbiter.

Acknowledgment

I wish to thank J. Ping for the useful information kindly given me and A. Sengoku for the attention and encouragement to this work.

References

Christodoulidis, D., D. E. Smith, R. G. Williamson and S. M. Klosko (1988): Observed tidal braking in the Earth/Moon/Sun system, J. Geophys. Res., (B6), 6216-6236.

Ciufolini, I. and J. A. Wheeler (1995): Gravitation and Inertia, Princeton University Press, 498p.

Ciufolini, I. (2000): The 1995-99 measurements of the Lense-Thirring effect using laser-ranged satellites, Class. Quantum Grav., 17(12), 2369-2380.

Ciufolini, I. (2001): Lense–Thirring effect and de Sitter precession, in Pascual-Sánchez, J.F., L. Flóra, A. San Miguel and F. Vicente (eds), Proceedings of the XXIII Spanish Relativity Meeting on Reference Frames and Gravitomagnetism, World Scientific, Singapore, 367p.

Everitt, C.W.F., and other members of the Gravity Probe B team, Gravity Probe B (2001): Countdown to Launch, in Lämmerzahl, C., C.W.F. Everitt and F.W. Hehl (eds), Gyros, Clocks, Interferometers...:Testing Relativistic Gravity in Space, Lecture Note in Physics 562, Springer Verlag, Berlin, 507p.

Floberghagen R., P. Visser, and F. Weischede (1999): Lunar albedo force modeling and its effect on low lunar orbit and gravity field determination, Adv. Sp. Res. 23(4), 733-738.

Grafarend, E. W., and G. Joos (1992): Relativistic computation of geodetic satellite orbits, in Linkwitz, K., and U. Hangleiter (eds), Proceedings of the 21nd International Workshop on High Precision Navigation, 19-29, Dümmler Verlag, Bonn.

Heki, K., K. Matsumoto, and R. Floberghagen (1999): Three-dimensional tracking of a lunar satellite with differential very-long-baseline-interferometry, Adv. Sp. Res. 23(11), 1821-1824.

Knežević, Z., and A. Milani (1998): Orbit maintenance of a lunar polar orbiter, Planet. Space Sci., 46(11/12), 1605–1611.

Konopoliv, A.S., A.B. Binder, L.L. Hood, A.B. Kucinskas, W.L. Sjögren, and J.G. Williams (1998): Improved Gravity Field of the Moon from Lunar Prospector, Science, 281, 1476-1480.

Konopoliv, A.S., S.W. Asmar, E. Carranza, W.L. Sjörgen and D.N. Yuan (2001): Recent Gravity Models as a Result of the Lunar Prospector Mission, Icarus, 150, 1–18.

Lemoine, F.G., M. Zuber, G. Neumann, and D. Rowlands (1997): A 70th degree lunar gravity model (GLGM-2) from Clementine and other tracking data, J. Geophys. Res. (Planets) 102(E7), 16339-16359.

Lense, J. and H. Thirring (1918): Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie, Phys. Z., 19, 156-163, translated by Mashhoon, B., F. W. Hehl and D. S. Theiss (1984): On the Gravitational Effects of Rotating Masses: The Thirring-Lense Papers, Gen. Rel. Grav., 16, 711-750.
Mashhoon, B., and D.S. Theiss (1991): Relativistic Lunar Theory, Il Nuovo Cimento 106B, 545-571.

Mashhoon, B., and D.S. Theiss (2001): Relativistic Effects in the Motion of the Moon, in Lämmerzahl, C., C.W.F. Everitt and F.W. Hehl (eds), Gyros, Clocks, Interferometers...:Testing Relativistic Gravity in Space, Lecture Note in Physics 562, Springer Verlag, Berlin, 507p.

Nordvedt, K (2001): Lunar Laser Ranging, in Lämmerzahl, C., C.W.F. Everitt and F.W. Hehl (eds), Gyros, Clocks, Interferometers...:Testing Relativistic Gravity in Space, Lecture Note in Physics 562, Springer Verlag, Berlin, 507p.

Ohta, K., K. Yonekura, Y. Takizawa, R. Nagashima, Y. Iijima and S. Sasaki (2000): System Concept and Status of the Selenological and Engineering Explorer (SELENE), Proc. 51st Int. Astronautical Congress, IAF-00-Q.4.04.

Shapiro, I. (1990): Solar system tests of general relativity: recent results and present plans, in Proceedings of the 12th International Conference on General Relativity and Gravitation, University of Colorado at Boulder, 1989, edited by N. Ashby, D. Bartlett, and W. Wyss, Cambridge University Press, Cambridge, 313-330.