ABSTRACT

We give a global analysis of mass transfer variations in low-mass X-ray binaries and cataclysmic variables whose evolution is driven by the nuclear expansion of the secondary star. We show that limit cycles caused by irradiation of the secondary by the accreting primary are possible in a large class of these binaries. In the high state, the companion transfers a large fraction of its envelope mass on a thermal timescale. In most cases this implies super-Eddington transfer rates and would thus probably lead to common-envelope evolution and the formation of an ultrashort-period binary. Observed systems with (sub)giant secondaries stabilize themselves against this possibility either by being transient or by shielding the secondary from irradiation in some way.

Subject headings: accretion, accretion disks — binaries: close — instabilities — novae, cataclysmic variables

1. INTRODUCTION

Semidetached binaries in which a compact object (white dwarf, neutron star, or black hole) accretes material via Roche lobe overflow from a companion on or near the main sequence are of great interest in current astrophysics. The evolution of such systems is driven by orbital angular momentum losses via gravitational radiation and magnetic braking (see, e.g., King 1988 for a review). Thus most properties of the binary, particularly the mean mass transfer rate, depend essentially only upon the secondary mass braking (see, e.g., for a review). Thus most King 1988 momentum losses via gravitational radiation and magnetic evolution of such systems is driven by orbital angular sequence are of great interest in current astrophysics. The Roche lobe overflow from a companion on or near the main

Of course semidetached evolution is not restricted to systems with main-sequence companions. In systems with orbital periods \( P \gtrsim 1 \) day, the orbital evolution and hence the rate of mass transfer are either determined or strongly influenced by nuclear evolution of the companion. In this paper, we discuss the stability of mass transfer in systems containing a giant or a subgiant companion and consider the possibility of irradiation-driven mass transfer cycles similar to those thought to exist in CVs. For this purpose, we generalize the analysis of Paper I to include the effects of nuclear evolution on the radius variations by using a simple core-envelope model for the companion. This is a good representation of systems with low-mass giant secondaries, which constitute the great majority of long-period compact binaries. However, this description does not apply to the recently discovered black hole transient GRO J1655-40, in which the companion star appears to be crossing the Hertzsprung gap (Orosz & Bailyn 1997).

2. GLOBAL ANALYSIS OF MASS TRANSFER VARIATIONS

In this section we follow closely the formalism developed in Paper I, casting the equations governing time-dependent mass transfer in a semidetached binary in a form suitable for global analysis including the effects of nuclear evolution. We restrict our analysis to the effect of variations in the radius \( R_2 \) of the lobe-filling star on the transfer rate, as this is the simplest way of causing mass transfer variations (other ways can easily be accommodated; Paper I gives an example). Note that this part of the analysis is quite general, in that there is no presumption at this point that the radius variations result from irradiation.

Radius changes can result from dynamical, thermal, or secular processes. For example, local adjustments in the structure of the atmospheric layers can take place on the shortest (dynamical) timescale, while adjustments of the stellar radius in response to secular mass loss and nuclear evolution occur on the longest (secular) driving timescale, \( t_{dr} \), defined more precisely below. We represent the radius variation as

\[
\frac{\dot{R}_2}{R_2} = \zeta \frac{\dot{M}_2}{M_2} + K_{\text{ad}}(R_2, \dot{M}_2) + K_{\text{nuc}}.
\]

Here \( \zeta \) is the adiabatic mass-radius exponent (about \( -\frac{1}{3} \) for a fully convective star), \( K_{\text{ad}}(R_2, \dot{M}_2) \) represents radius variations due to thermal relaxation and irradiation of the star by the primary, and \( K_{\text{nuc}} = (\partial \ln R_2 / \partial t)_{\text{nuc}} \) represents the secular changes due to nuclear evolution. We considered examples of specific forms of \( K_{\text{ad}}(R_2, \dot{M}_2) \) in Paper I and will discuss these again later. The change of the mass transfer rate is given by

\[
\dot{M}_2 = \frac{M_2}{H} \left( \frac{\dot{R}_2}{R_2} - \frac{R_2}{R_1} \right) \approx \dot{M}_2 \frac{R_2}{H} \left( \frac{\dot{R}_2}{R_2} - \frac{R_1}{R_2} \right),
\]

where \( R_1 \) is the critical Roche radius and \( H \) is the pressure scale height in the secondary star’s atmosphere. The approximation given by equation (2) is justified since even
for giant companions $|R_2 - R_L| \ll R_L, R_2$. The response of the Roche radius to mass loss is described by

$$\frac{\dot{R}_L}{R_L} = \xi_R \frac{M_2}{M_2} + 2 \frac{J}{J},$$

where $\xi_R$ is a function of the mass ratio $M_2/M_2$, approximately given by $\xi_R \approx 2M_2/M_2 - 5/3$ for conservative mass transfer, and $J$ is the rate of loss of orbital angular momentum. Inserting equations (1) and (3) into equation (2), we obtain

$$\dot{M}_2 = \frac{M_2 R_L}{H} \left[ (\xi_e - \xi_R) \frac{M_2}{M_2} + K_{nuc} + K_{nuc} - 2 \frac{J}{J} \right].$$

For the more general case considered here, the evolution of the binary is driven by the combined effects of nuclear evolution and angular momentum losses. We introduce the effective driving timescale $t_{dr}$, as follows:

$$\frac{1}{t_{dr}} = \frac{1}{t_{nuc}} + \frac{2}{t_J},$$

where $t_{nuc} = K_{nuc}^{-1}$ is the nuclear timescale and $t_J = -J/J$ is the timescale for angular momentum losses. The nuclear timescale is a strong function of the core mass and is $\sim 10^7$-$10^9$ yr for giants and much longer for low-mass main-sequence dwarfs. The angular momentum loss timescale $t_J$ is typically $\sim 10^8$-$10^9$ yr for main-sequence companions but could be much longer for evolved companions. Thus orbital evolution is driven mainly by angular momentum losses in systems with main-sequence companions (e.g., CVs) and nuclear processes in systems with evolved companions. The introduction of $t_{dr}$ allows us to treat both cases simultaneously. We define a dimensionless mass transfer rate

$$x = \frac{-\dot{M}_2}{(-\dot{M}_2)_{ad}} = \frac{\dot{M}_2}{\dot{M}_2} (\xi_e - \xi_R) t_{dr},$$

where $(-\dot{M}_2)_{ad}$ is the adiabatic mass transfer rate, i.e., the steady rate implied by equation (4) with $K_{nuc} = 0$. A system undergoing stable mass transfer will typically do so at $x \sim 1$ (see § 3). We also introduce the dimensionless stellar radius

$$r = R_2/R_e,$$

where $R_e = R_4(M_2, M_4)$ is the radius of the secondary in thermal equilibrium for a given total mass $M_4$ and a core mass $M_2$. The radii and luminosities of lower giant branch stars are virtually independent of the mass of the envelope but depend strongly upon the mass of the degenerate helium core (Refsdal & Weigert 1970). We allow $R_e$ formally to depend upon the total mass because then the equations obtained are identical in form to those derived in Paper I and have wider applicability.

As we discuss in § 3, the secular equilibrium radius of the companion under stable mass transfer differs slightly from $R_e$. Although the equilibrium radius $R_e$ is only attained in the absence of mass transfer, we write formally

$$\frac{\dot{R}_e}{R_e} = \xi_e \frac{M_2}{M_2} + K_{nuc},$$

where $\xi_e$ is defined by the above equation. Taking the time derivative of equation (6), and neglecting secular variations of $H$, we obtain

$$-\dot{M}_2 = \frac{M_2}{(\xi_e - \xi_R) t_{dr}} \dot{x} + \frac{\dot{M}_2 x}{(\xi_e - \xi_R) t_{dr}} + \frac{M_2}{t_{dr}} \frac{dln[(\xi_e - \xi_R) t_{dr}]}{dt}.$$ 

Finally, we introduce $e = H/R_e$, which is typically $\sim 10^{-3}$ for lower giant branch stars (see below). We define the dimensionless thermal relaxation function $p(r, x) = K_{nuc} t_{dr}$ and the dimensionless time $\tau = t_{dr} e t_{dr}$. With these assumptions and definitions, equation (9) becomes

$$\frac{dx}{d\tau} = rx[1 + p(r, x) - x] + e x \left\{ \frac{x}{(\xi_e - \xi_R)} + t_{dr} \frac{dln[(\xi_e - \xi_R) t_{dr}]}{dt} \right\}.$$ 

Note that since the terms inside the braces are of order unity and are multiplied by $e$, they can be safely neglected. Thus equation (10) reduces to

$$\frac{dx}{d\tau} = rx[1 + p(r, x) - x].$$

With the variations of the equilibrium radius given by equation (8), and using the above conventions, the radius equation (eq. [1]) becomes

$$\frac{dr}{d\tau} = er \left[ p(r, x) - \frac{x}{\lambda} \right],$$

where $\lambda = (\xi_e - \xi_R)/(\xi_e - \xi_e)$. The equations describing radius and mass transfer variations (eqs. [11], [12]) are identical in form to those derived in Paper I. Thus we can take over from Paper I all the results for the phase-plane motion of the system, the critical curves $\tau = 0$ and $\dot{x} = 0$, the fixed point(s) $(r_0, x_0)$ at the intersection(s) of these critical curves, the stability analysis for the fixed point(s), and the conditions for limit cycles, with the modification that now $t_{dr}$ is a more general driving time that includes both systemic angular momentum losses and nuclear evolution. In the limit $t_{nuc} \rightarrow \infty$, we recover the case studied in Paper I.

From equations (11) and (12) it is easy to see that the stationary (secular mean) mass transfer rate at the fixed point $x_0 = (\xi_e - \xi_R)/(\xi_e - \xi_R)$ depends upon the properties of the companion and the driving rate. In particular, it is independent of $p(r, x)$ (as it must be), and in most realistic cases $x_0 \sim 1$. The radius $r_0$ at the fixed point, on the other hand, is given implicitly in terms of $p(r, x)$; again we typically have $r_0 \sim 1$ (see Paper I for further details).

### 3. Binaries with Subgiant and Giant Companions

In their study of the evolution of compact binaries containing a lower giant branch companion, Webbink, Rappaport, & Savonije (1983) introduced simple analytic expressions for the luminosity and radius in terms of the core mass. For our purposes it suffices to take only the first two terms of their approximate formulae for the radius,

$$R_e(M_2) = 12.55 R_\odot \left( \frac{M_2}{0.25 M_\odot} \right)^{5.1},$$

where
and the luminosity,

\[ L_a(M_s) = 33.1 \, L_\odot \left( \frac{M_s}{0.25 \, M_\odot} \right)^{8.1}, \]  

(14)

in thermal equilibrium. Using these equations we can readily estimate \( \epsilon \) for these stars:

\[ \epsilon = \frac{H}{R_e} = 2.1 \times 10^{-3} \mu^{-1} \left( \frac{M_s}{0.25 \, M_\odot} \right)^{4.6} \left( \frac{M_2}{1 \, M_\odot} \right)^{-1}, \]  

(15)

where \( \mu \) is the mean molecular weight in the stellar atmosphere. From equations (13) and (14), the nuclear timescale for radial variations is\( t_{\text{nuc}} = M_s/(5.1 \, M_\odot) \), where \( M_s \) is calculated from the equilibrium luminosity assuming a hydrogen mass fraction of 0.7 and an energy yield of \( 6 \times 10^{18} \) ergs g\(^{-1}\). Thus

\[ t_{\text{nuc}} = 1.0 \times 10^8 \, \text{yr} \left( \frac{M_s}{0.25 \, M_\odot} \right)^{-7.1}. \]  

(16)

The Kelvin-Helmholtz timescale is defined for the whole star as \( t_{\text{KH}} = GM_2^2/(R_s L_s) \), so

\[ t_{\text{KH}} = 7.6 \times 10^4 \, \text{yr} \left( \frac{M_2}{1 \, M_\odot} \right)^2 \left( \frac{M_s}{0.25 \, M_\odot} \right)^{-13.2}. \]  

(17)

The ratio of these timescales plays a major role in the discussion of thermal relaxation and stability. We define

\[ \rho_{\text{nuc}} = \frac{t_{\text{nuc}}}{t_{\text{KH}}} = 1.4 \times 10^5 \left( \frac{M_s}{0.25 \, M_\odot} \right)^{6.1}. \]  

(18)

Since no explicit dependence of \( R_s \) upon the total mass of the companion \( M_2 \) is allowed in equations (13) and (14), it appears that \( \zeta_e = 0 \). However, both computed models of lower giant branch stars and analytic treatment of nearly fully convective stars following the approximations used by Kippenhahn & Weigert (1990, p. 226) for the Hayashi line show a weak dependence upon the total stellar mass \( M_2 \). Taking a photospheric opacity of the form \( \kappa = \kappa_0 P^\alpha T^\beta \), and adopting a polytropic index \( n = 3/2 \) for the envelope, one can show that

\[ \zeta_e = -\frac{a + 3}{5.5a + b + 1.5}. \]  

(19)

With \( a = 1 \) and \( b = 3 \), the above expression yields \( \zeta_e = -0.4 \), while direct estimates from numerical stellar models indicate \( \zeta_e \approx -0.2 \) to \(-0.3 \). We adopt the latter range in describing the reaction of the star to mass loss, although equation (13) is adequate for computing the nuclear and thermal timescales.

We now describe the effects of thermal relaxation and irradiation on the radius of the evolved companion. We do this here by using a simple homologous model and making the same approximations as in King et al. (1995) and in Paper I, with slight changes appropriate for giants:

1. Homology relations can be used to describe the stellar structure, i.e., the effective temperature \( T_e \) on the unirradiated portion of the stellar surface remains essentially constant (Hayashi line) and any stellar radius change changes the surface luminosity as \( L \propto R_e^2 \). In contrast, \( L_{\text{nuc}} \) remains unchanged, as given by equation (14).

2. If a fraction \( s \) of the stellar surface is exposed to a uniform irradiating flux \( F \), the luminosity of the star, i.e., the loss of energy per unit time from the interior, is reduced by the blocking luminosity

\[ L_b = sL \tanh \left( k \frac{F}{F_*} \right), \]  

(20)

where \( F_* = \sigma T_4^4 \) is the unperturbed stellar flux and \( k \) is a parameter that is adjusted to approximate the results of more realistic model calculations (e.g., those by Ritter, Zhang, & Hameury 1996a; Ritter, Zhang, & Kolb 1996b). The above Ansatz for \( L_b \) is motivated by the facts that \( L_b = 0 \) if \( F = 0 \), (b) \( L_b \rightarrow sL \) if \( F \gg F_* \), and (c) the transition between \( F = 0 \) and \( F \gg F_* \) is smooth and monotonic.

With these assumptions, we obtain the thermal relaxation function including irradiation effects:

\[ p(r, x) = -fp \left( 1 - s \tanh \left( \frac{x}{x_c} \right) \right) r^{3/2} - r \]  

(21)

(cf. eq. [8] of King et al. 1995 and eq. [38] of Paper I). Note that the quantity \( \rho = t_{\text{nuc}}/t_{\text{KH}} \) introduced above differs from the \( \rho = t_{\text{nuc}}/t_{\text{KH}} \) used in Paper I (see eq. [5]). The ratio \( f = t_{\text{KH}}/t_{\text{e}}, \) with \( t_{\text{e}} \) being the thermal timescale for the convective envelope, is a constant that depends upon the internal structure (i.e., mainly on the ratio of envelope mass to the total mass of the secondary), and

\[ x_c = \frac{2(\zeta - \zeta_e) \rho M_2 R_1}{k \eta M_1 R_e} \left( \frac{a}{a} \right)^2. \]  

(22)

This is the dimensionless critical mass transfer rate, which yields an irradiating flux \( F = F_*/k \). Cycles typically occur provided that \( x_c \) is less than or roughly equal to the secular mean transfer rate \( x_0 \). In equation (22), \( a \) is the orbital separation and \( \eta \) is a dimensionless efficiency factor relating the irradiating flux \( F \) to the mass transfer rate via

\[ F = \frac{\eta \, GM_1 (\Delta M)}{8\pi \, R_1 \, a^2}. \]  

(23)

The critical mass transfer rate for CVs is \( x_c \approx 0.1 \rho/(k \eta) \ll 1 \), where the value of \( k \) depends upon the type of companion star assumed, while \( \eta \) depends mainly upon the radiation spectrum (typical values are \( 0.1 \leq \eta \leq 5 \) and \( 0.2 \leq k \leq 0.9 \), with \( \eta \approx 0.1 \) a few percent). As this critical rate is of the order of the secular mean mass transfer rate \( x_0 \ll 1 \), irradiation can potentially drive cycles in CVs, as we found in Paper I. For low-mass X-ray binaries (LMXBs) with similar mass ratios, the much smaller neutron star or effective black hole radius implies \( x_c \approx 0.0001 \rho/(k \eta) \ll 0.1 \). Thus we can potentially expect cycles in these systems also. This contrasts with the case of LMXBs with unevolved companions (cf. Paper I), in which the companion is too strongly irradiated to yield cycles (critical transfer rates \( x_c \ll x_0 \approx 1 \)). For LMXBs with evolved companions, the large ratio \( \rho \) of driving to thermal timescales almost compensates the effect of the smaller primary radius, to yield \( x_c \approx 0.1 \).

4. STABILITY OF MASS TRANSFER FROM AN EVOLVED COMPANION

The simple thermal relaxation function given in the previous section allows us to discuss the stability of mass transfer in close binaries with evolved companions. More detailed calculations using the bipolytrope approximation (Ritter et al. 1996b) or integrating the full equations of
stellar structure with appropriate changes in the boundary conditions (Hameury & Ritter 1996) produce results that, to a first approximation, can be represented by this analytic form with suitable \( k, \lambda \sim 1 \). In Paper I we noted that a necessary condition for limit cycles is that the fixed point is unstable, which in turn requires \( p_\rho < 0 \) and \( p_x > 1 \) simultaneously. These derivatives are easily calculable for the simple form of equation (21), namely,

\[
p(r_0, x_0) = -fp \left[ 1 - s \tanh \left( \frac{x_0}{x_c} \right) \right] r_0^3 \left( \frac{x_0}{x_c} \right)^3 \left( \frac{x_0}{x_c} \right)^3 - \left( \frac{x_0}{x_c} \right)^3 - \left( \frac{x_0}{x_c} \right)^3 - \left( \frac{x_0}{x_c} \right)^3,
\]

(24)

\[
p_x(r_0, x_0) = \frac{fps}{x_c} r_0^3 \sech^2 \left( \frac{x_0}{x_c} \right).
\]

(25)

For physically reasonable situations, \( s < 0.5 \) and \( r_0 > 1 \), and thus clearly \( p_\rho < 0 \) as required. The second condition, \( p_x > 1 \), for instability can be rewritten as in Paper I:

\[
s \frac{x_0}{x_c} \sech^2 \left( \frac{x_0}{x_c} \right) > \frac{x_0}{fpr_0^3},
\]

(26)

where the left-hand side is a function that has a maximum value of \( \approx 0.448s \) at \( x_0/x_c \approx 0.78 \) and vanishes at both small and large \( x_0/x_c \). As emphasized in Paper I, the right-hand side of equation (26) depends mainly upon the type of companion and the driving mechanism. Clearly, the large values of \( \rho \) for giant or subgiant companions make these systems extremely vulnerable to the irradiation instability. Note that if \( s = 0 \) in equation (26), i.e., irradiation is not included, or the flux is somehow blocked (e.g., by a thick disk), the inequality is violated and mass transfer driven by nuclear evolution is stable, as assumed in conventional treatments of these systems.

Thus the simple model described above suggests that most binaries with irradiated lower giant branch companions cannot transfer mass at a stable rate. More detailed models, discussed later, confirm this result. While the linearized equations are adequate to discuss the stability of the fixed point, as in Paper I, we need the full nonlinear evolution equations to understand the ultimate fate of the system. It turns out that the phase point representing the evolution of the system away from an unstable fixed point is trapped in its vicinity and will settle into a limit cycle, which is described in more detail below.

5. PROPERTIES OF THE LIMIT CYCLE

The simple thermal relaxation functions \( p(r, x) \) given by equation (21) for evolved companions and equation (38) of Paper I for main-sequence companions yield approximate analytic expressions for a number of properties of the limit cycle. The two cases can be treated simultaneously by taking

\[
p(r, x) = -fp \left[ 1 - s \tanh \left( \frac{x}{x_c} \right) \right] r^3 - r^{-v(2)} \]

(27)

with \( \rho \) as defined in this paper and \( v = 5-6 \) and \( v = -3 \) for main-sequence and giant companions, respectively. Some of the results given below depend upon having \( \rho \gg 1 \), which is satisfied very well by giants (for which \( \rho \approx \rho_{\nu c}; \) see eq. [18]) but not always by main-sequence companions. Nevertheless, since most of the analytic results described below can also be obtained for main-sequence companions, we shall quote them here too.

From equation (11) the critical curve \( \dot{x} = 0 \) is given by \( 1 + p(r, x) - x = 0 \). The chosen form of \( p(r, x) \) yields two branches of this curve with different slopes: a low branch with \( x \sim x_c \) and a high branch where \( x \gg x_c \). For the low branch one can easily show, using the approximation \( \tanh (x/x_c) \approx x/x_c \), that

\[
x_L \approx \frac{fpsr^3 - r^{-v-2} - 1}{fpsr^3/x_c - 1}.
\]

(28)

The denominator in the above equation can be rewritten by using equation (25) as \( p_\rho \cosh^2 (x/x_c) - 1 \). The condition \( p_\rho > 1 \) for instability ensures that this denominator is positive near \( x_c \); as \( x_c \) must be finite, the denominator must remain positive everywhere on the lower branch, implying a positive slope there. Since in general the radial variations are small departures from equilibrium, we can linearize around \( r = 1 \) to obtain \( x_L = 0 \) at \( r = 1 + 1/[fps(5 + v)] \). In general, the slope of this critical curve is given by

\[
\frac{dx}{dr} \bigg|_{x=0} = -\frac{p_\rho}{p_x - 1},
\]

(29)

where \( p_\rho < 0 \) for physically relevant cases. If \( fps \gg 1 \), as in the case of evolved companions, then the slope of the lower branch at \( x = 0 \) is approximately \( (5 + v)x_c/s \).

The upper branch is obtained by setting the tanh factor to unity, and thus

\[
x_U \approx 1 + fpsr^3 - fps(3 - r^{-v-2}).
\]

(30)

For small departures from the equilibrium radius, this reduces to \( x = 1 + fps - fps(5 + v - 3s)(r - 1) \), showing that the upper branch has a large negative slope and is therefore stable (as \( s < 0.5 \), we have \( v = 5 + v - 3s > 3.5 + v \), which is greater than 0.5 even for giant companions, with \( s = -3 \)). At some intermediate value of the transfer rate \( x_t \), there is a turning point at which \( (dr/dx)|_{x=0} = 0 \) or, equivalently, where \( p_x = 1 \). This turning point therefore satisfies the equation

\[
\cosh^2 \left( \frac{x_t}{x_c} \right) = \frac{fpsr^3}{x_c},
\]

(31)

where \( r_t \) is the dimensionless radius at the turning point. In general, \( r_t \) and \( x_t \) must be obtained by solving equation (31) together with \( 1 + p(r, x) - x = 0 \), which can easily be done iteratively. Note also that as in Paper I, the axis \( x = 0 \) is formally part of the \( x = 0 \) curve, although it takes infinite time to reduce \( x \) to zero (see Fig. 1). From the considerations above we can see that the fixed point is unstable when \( x_c > x_0 > 0 \).

The maximum expansion stage of the secondary is reached close to \( r_t \). We could estimate \( r_t \) by asking for the radius at which \( x_L \approx x_L \), but we prefer to use a physical argument based upon the assumptions made in the model: the maximum expansion of the companion is reached when all the luminosity generated is emitted by the unilluminated side at the unperturbed effective temperature. This yields \( p(r_t, x_t) \approx 0 \) and

\[
r_t \approx (1 - s_{\text{eff}})^{-1}v + 5 \approx 1 + \frac{s_{\text{eff}}}{v + 5},
\]

(32)

where \( s_{\text{eff}} = s \tanh (x/x_c) \sim s \). Formally, if \( p(r, x) = 0 \), for nonvanishing illumination we again obtain the adiabatic mass transfer rate \( x = 1 \), and thus \( x \approx 0 \). This simple argu-
one sees that the critical curve \( \dot{r} = 0 \) must go through the point \((r = 1, x = 0)\). The slope of this curve at that point is also positive, but smaller than the slope of the \( \dot{x} = 0 \) curve at slightly larger \( r \) if a fixed point with \( x_0 > 0 \) exists. This is obvious from geometric arguments but can also be shown explicitly.

A typical limit cycle is shown in Figure 1, where we have stretched the \( r \)-variable and exaggerated the separation between the two critical curves for the sake of clarity. Given that the \( \dot{r} = 0 \) curve intersects the \( r \)-axis at \( r = 1 \), the entire cycle satisfies \( r > 1 \), i.e., the star is always somewhat oversized.

The critical curves cross at the unstable fixed point \((r_0, x_0)\). The four points labeled ABCD along the cycle identify the locations at which the phase point crosses critical curves. These naturally divide the cycle into four phases, which we discuss in turn. At point A, the mass transfer reaches its peak value \( x_A = M_2 \frac{M_2}{(\frac{r}{x} - \frac{3}{2})^2} \), and we have maximum contact: \( R_2 - R_1 \approx H \ln(x_A/x_0) \). We can estimate \( x_A \) from the fact that point A lies almost vertically above the point where \( x_L = 0 \). From equations (28) and (30), we have \( x_A \approx f_{\rho} \).

In physical units this rate is

\[
(-M_2)_{\text{max}} \approx x_A \frac{M_2}{(\frac{r}{x} - \frac{3}{2})^2} = \frac{sM_2}{(\frac{r}{x} - \frac{3}{2})^2} ,
\]

where \( t_{ce} \) is the timescale for thermal relaxation of the convective envelope, which can be significantly shorter than \( t_{KII} \). At point B the companion is very close to its maximum size, while the binary orbit has expanded so that \( R_1 \approx R_2 \approx rR \). The degree of overfilling of the Roche lobe has been reduced to \( R_2 - R_1 \approx H \ln(x_A/x_0) \). The stellar expansion rate \( \dot{R}_2/\dot{R}_1 = \dot{t}_{\text{nucl}} \) is now too slow compared with that of the lobe to sustain the high mass transfer rate that in turn drives the radius expansion. (This is inevitable since the transfer rate cannot indefinitely remain above the secular mean driven by nuclear expansion and angular momentum losses.) Thus the companion rapidly loses contact and the mass transfer drops very sharply, while the star shrinks back toward its equilibrium radius. At point C minimum mass transfer is reached because the system is maximally detached, with \( R_2 - R_1 \approx -(r - 1) \), so that \( x \) is essentially zero in the low state. At this point \( R_3 \) is again very close to zero, and nothing will happen until the combined effects of nuclear evolution and angular momentum losses bring the system close to contact again. At point D the secondary is expanding slightly \( \dot{R}_3/\dot{R}_2 = \dot{t}_{\text{nucl}} \), so that \( \dot{r} = 0 \).

The companion radius \( R_3 \) is now within a few scale heights of \( R_1 \). This raises the transfer rate, making the companion expand more rapidly under irradiation, which in turn increases the transfer rate, so that the cycle restarts.

We can also estimate the timescales for the different phases of the cycle and evaluate the total mass transferred in a cycle. We estimate the rise time by assuming that the mass transfer initially increases because the thermal imbalance due to irradiation forces the secondary into deeper contact. The characteristic rate of radial expansion when a fraction \( s \) is fully blocked is \( \dot{R}_2/\dot{R}_1 \approx s/\dot{t}_{ce} \). Therefore the rise time is approximately the time required to expand by \( \ln(x_A/x_0) \) scale heights,

\[
t_{DA} \approx \frac{\ln(x_A/x_0)}{R_2} \frac{H t_{ce}}{s} \text{ or } \tau_{DA} \approx \frac{\ln(x_A/x_0)}{x_A} ,
\]

where we have used our estimate of \( x_A = s\rho \) to obtain the final expression. During the time \( t_{AB} \) spent on the high

![Image of figure 1 showing a phase plane for the ODE system (eqs. [11], [12]). The two critical curves where \( \dot{x} = 0 \) (dot-dashed curve) and \( \dot{r} = 0 \) (dashed curve) are shown for a typical \( p(r, x) \). These curves intersect at the fixed point \((r_0, x_0)\) and divide the \((r, x)\)-plane into four regions in which the motion of the system point is indicated by the arrows. The limit cycle intersects the critical curves at the points ABCD, yielding the four phases of the cycle discussed in the text: a high state AB during which the companion expands, a moderately rapid contraction phase BC, a long low state CD, and an extremely fast rise DA to peak mass transfer.](Image)
branch, the rate of expansion of the secondary decreases monotonically from the maximum rate estimated above, which yields the peak mass transfer rate \( x_d \), until \( d \ln R_2/dt = 1/t_{\text{nuc}} \).

At that point (B) we have \( \dot{r} = 0 \), and the expansion rate falls below the driving rate so that the system detaches, going rapidly into a low state. We adopt the following Ansatz for the radius as a function of time:

\[
R_2(t) = R_A + (R_B - R_A)(1 - e^{-t/T_+}) .
\]

Equating \( d \ln R_2(0)/dt = s/t_{ce} \) and \( d \ln R_2(t_{AB})/dt = 1/t_{\text{nuc}} \), we obtain both the characteristic radial expansion timescale \( T_+ \) and the time \( t_{AB} \) spent on the high branch,

\[
T_+ \approx \frac{r_r - 1}{s} t_{ce} \sim \frac{t_{ce}}{v + 5} \quad \text{or} \quad \tau_+ \approx \frac{r_r - 1}{e x_A} ,
\]

\[
t_{AB} \approx \frac{r_r - 1}{s} t_{ce} \ln \left( \frac{x_A}{r_r} \right) \sim \frac{t_{ce}}{v + 5} \ln \left( \frac{x_A}{r_r} \right)
\]

or

\[
\tau_{AB} \approx \frac{r_r - 1}{e x_A} \ln \left( \frac{x_A}{r_r} \right) ,
\]

where we have taken \( R_A \approx R_c \) and \( R_B \approx r_r R_c \). As soon as the system detaches, irradiation ceases and the companion contracts rapidly while the orbit and Roche lobe remain at the size attained at the end of the high state. The contraction is even more rapid than the expansion, because the star is more luminous. One can show that the characteristic thermal contraction timescale is

\[
\tau_- \approx (1 - s)^{(v + 5)/r_+} ,
\]

leading to

\[
t_{BC} \approx \frac{r_r - 1}{s} t_{ce} \ln \left( \frac{r^2 s_{\text{eff}} d_{t_r}}{t_{ce}} \right)
\]

or

\[
\tau_{BC} \approx \frac{r_r - 1}{r_r^3 s_{\text{eff}} d_{t_r}} \ln \left( \frac{r^2 s_{\text{eff}} d_{t_r}}{t_{ce}} \right) .
\]

Clearly, this effect is more pronounced in giant companions than in main-sequence secondaries, for reasons that have already been mentioned. In a few times \( \tau_- \) (i.e., typically a time \( \lesssim 0.5 t_{ce} \)), the mass transfer formally reaches a minimum value \( x_c \approx 0 \), and a long detached (low) state now follows while the driving tries to bring the system back to contact. The time spent in the low state is thus dominated by the time \( t_{CD} \),

\[
t_{CD} \approx (r_r - 1)d_{t_r} \sim \frac{s}{v + 5} t_{dr} \quad \text{or} \quad \tau_{CD} \approx \frac{r_r - 1}{\epsilon} .
\]

| Parameter | Definition |
|-----------|------------|
| \( t_{dr} = (1/t_{\text{nuc}} + 2/t_J)^{-1} \) | Driving timescale |
| \( t_{ce} = (t_{J}/t_{\text{Ku}})^{-1} \) | Thermal timescale of the convective envelope |
| \( \zeta_c = (\partial R_2/\partial \ln M_2)_{c} \) | Adiabatic mass-radius exponent |
| \( \zeta_0 = (\partial R_2/\partial \ln M_2)_{0} \) | Thermal equilibrium mass-radius exponent |
| \( e = H(R_2/R_\text{eff}) \) | Photospheric scale height in units of equilibrium radius |

| \( v \) | \( \frac{5}{6} \), for MS donors \( \frac{3}{2} \), for giant donors \( (-v - 3) \) is the radius exponent of the nuclear luminosity |

\[ s_{\text{eff}} = s \tan(x/x_0) \approx s \approx 0.3 \] | Effective blocked surface fraction |

**TABLE 1**

**Characteristic Properties of the Limit Cycle**

| Parameter | Definition |
|-----------|------------|
| \(-M_2\)_0 | \( \frac{M_2}{\zeta_0 - \zeta_R} t_{dr} \) |
| \(-M_2\)_\text{eff} | \( \frac{M_2}{\zeta_0 - \zeta_R} s_{\text{eff}} t_{ce} \) |
| \( R_2 \text{, eff} \) | \( R_2 \approx (1 - s_{\text{eff}})^{-1/(v + 5)} R_2 \) |
| \( \Delta M_2 \) | \( r_r - 1 \) |
| \( t_{AB} \approx t_{cr} \ln \left( \frac{s_{\text{eff}} d_{t_r}}{r_r t_{ce}} \right) \) | \( x_0 \approx \zeta_0 - \zeta_R \) |
| \( t_{BC} \approx t_{ce} \ln \left( \frac{x_A}{r_r} \right) \) | \( x_4 \approx \zeta_0 - \zeta_R \) |
| \( t_{CD} \approx \frac{r_r - 1}{\epsilon} \) | \( \tau_{AB} \approx \frac{r_r - 1}{x_4} \ln \left( \frac{x_A}{r_r} \right) \) |

In Physical Units

| Parameter | Definition |
|-----------|------------|
| \(-M_2\)_0 | \( \frac{M_2}{\zeta_0 - \zeta_R} t_{dr} \) |
| \(-M_2\)_\text{eff} | \( \frac{M_2}{\zeta_0 - \zeta_R} s_{\text{eff}} t_{ce} \) |
| \( R_2 \text{, eff} \) | \( R_2 \approx (1 - s_{\text{eff}})^{-1/(v + 5)} R_2 \) |
| \( \Delta M_2 \) | \( r_r - 1 \) |
| \( t_{AB} \approx t_{cr} \ln \left( \frac{s_{\text{eff}} d_{t_r}}{r_r t_{ce}} \right) \) | \( \tau_{AB} \approx \frac{r_r - 1}{x_4} \ln \left( \frac{x_A}{r_r} \right) \) |
| \( t_{BC} \approx t_{ce} \ln \left( \frac{x_A}{r_r} \right) \) | \( \tau_{BC} \approx \frac{r_r - 1}{x_4} \ln \left( \frac{x_A}{r_r} \right) \) |
| \( t_{CD} \approx \frac{r_r - 1}{\epsilon} \) | \( \tau_{CD} \approx \frac{r_r - 1}{x_4} \ln \left( \frac{x_A}{r_r} \right) \) |

In Dimensionless Form
The total mass transferred during a cycle can now be estimated as
\[ \Delta M_2 \approx x_A (-M_2)_a d T_+ \]
which yields the simple—withehindsight perhaps obvious—result
\[ \Delta M_2 \approx (-M_2)_a d(r_t - 1)t_{dr} = \frac{r_t - 1}{\zeta_s - \zeta_R} M_2 \]
\[ \approx \frac{\zeta_{eff}}{(\zeta_s - \zeta_R)(v + 5)} M_2. \]  

Thus, irradiated main-sequence stars \((v = 5-6)\) transfer at most a few percent of their mass per cycle. In contrast, giants \((v = -3)\) transfer a significant fraction, \(~s\), of their total mass \(M_2\) in the high state, which may amount to most of the convective envelope. Since \(t_{se}\) is also much shorter for giants, we expect that if the irradiation instability is allowed to grow unchecked in such systems it will produce accretion rates that are super-Eddington in LMXBs. The resulting common-envelope evolution would probably lead to the formation of an ultrashort-period binary. Clearly, the irradiation instability must be quenched in observed LMXBs with evolved companions. In §7, we discuss ways in which this can happen. For reference we collect together the analytic expressions for the various properties of the limit cycle in Table 1.

6. NUMERICAL RESULTS

We have integrated the evolution equations (eqs. [11], [12]) from arbitrary initial states for a variety of parameter choices. In this section we present a few examples and compare them with the analytic estimates given in the previous section. The values quoted in the text are those obtained by numerical integration, with the corresponding analytic estimate in parentheses. The analytic estimates for peak rates, rise times, and maximum expansion radius are in all cases calculated in very good agreement with numerical results. In general, we integrated the equations for several cycles (three to six) and noted that the first outburst is usually somewhat atypical (slightly higher for the same radii and a higher peak rate) because of initial conditions, whereas later outbursts are virtually identical. We therefore quote values taken from later outbursts.

Figure 2 shows a case with core mass \(M_\odot = 0.15 M_\odot\), the lowest reasonable value for which the approximations used in §3 are still valid. We have also taken \(s = 0.5\), which is perhaps unrealistically large unless there is significant scattering from a disk corona or similar structure. The luminosity and radius of the companion are at the lower end of the subgiant range, and therefore the instability is weakest.

Nevertheless, a very large amplitude outburst results, with a peak transfer rate \(x_A = 152.8 (154.9)\) that is attained rapidly,
in $\tau_{DA} = 0.032 (0.033)$. The companion continues to expand in the high state until the turning point is reached at $r_t = 1.41421 (1.41421)$. The duration of the high state can be read off the graph directly and is $\tau_{AB} = 8.1 (12.5)$. While the behavior of $r$ as shown in Figure 2 appears to obey equation (35), closer examination shows that the expansion phase is actually faster than exponential while the contraction phase is slower than the assumed exponential. Nevertheless, we have estimated the characteristic radial expansion and contraction timescales graphically, obtaining $\tau_+ = 2.4 (2.67)$ and $\tau_-$ = 1.0 (0.95). Despite the fact that equation (35) holds only approximately, the expressions based upon it are better than order-of-magnitude estimates. For the case shown, the total duration of the cycle is 415.9, and the duration of the low state is $\tau_{CD} \approx 407.8 (414.2)$.

Figure 3 shows another case with a higher core mass, $M_x = 0.25 M_\odot$, and $s = 0.3$, appropriate for illumination by a point source. The instability in this case is more violent, with shorter rise times $\tau_{DA} = 0.0023 (0.0024)$ and higher peak rates $x_A = 3326 (3609)$ but a smaller turning radius, $r_t = 1.19529 (1.19523)$. The total duration $\tau_{AB} = 0.45 (0.43)$ of the high state is relatively short. The characteristic $e$-folding times for radial expansion, $\tau_+ = 0.051 (0.054)$, and contraction, $\tau_- = 0.04 (0.031)$, are also relatively shorter because of the higher luminosity. The analytic estimates for the properties of the radial variations during the limit cycle are even more accurate than in the case in Figure 2, because the neglected terms $\sim (fps)^{-1}$ are still smaller here. The total duration of the cycle is 188.6, and the low state lasts for $\tau_{CD} \approx 188.1 (195.2)$.

7. IRRADIATION INSTABILITY IN LOW-MASS BINARIES

We can now discuss the application of the theory developed here to various types of close binaries encountered in nature. The only restriction is that the companions must have a significant convective envelope and thus a low mass, $M_x \leq 1.5 M_\odot$. The instability criterion implying cycles given in equation (26) can be rewritten as

$$fps > \frac{x_0}{r_0} \frac{x_T}{x_0} \cosh^2 \left( \frac{x_0}{x} \right).$$

(43)

Here the factor $x_0/r_0 \sim 1$ does not vary much, whereas $fps$ is mainly sensitive to the type of companion and the mechanism driving the binary evolution, while $\xi = x_0/x$, depends upon both the accretor and donor type. In Figure 4, we plot $x_0/x$ along the abscissa (the compact object axis) and $fps$ along the ordinate (the companion axis). Changing the type of companion causes displacements along both axes, whereas changing the primary causes only horizontal displacements. The stability/instability boundary plotted is simply $fps = \xi^{-1} \cosh^2 \xi$. The locations of various possible evolutionary sequences for CVs and LMXBs are also shown in Figure 4. Changing $s$ by screening or scattering causes purely vertical displacements since $\xi$ is not affected: clearly, for any binary there is a value of $s$ below which mass transfer is stable. Within the limitations of the simple theory developed here [i.e., the form chosen for $p(r, x)$], the results displayed in Figure 4 show the following:

1. CVs above the period gap and above a certain period (or companion mass) can be unstable, depending upon the value of $\eta$ (see Paper I). CVs below the gap are stable.
2. Long-period CVs with companions having small core masses are stable, whereas companions with larger core masses, likely to have even longer orbital periods, are unstable. A detailed analysis shows that GK Per and V1017 Sgr are unstable if $\eta \geq 0.08$ and 0.04, respectively.
3. Short-period LMXBs with main-sequence or partially evolved companions are stable because they are so strongly irradiated that they have reached saturation ($x_0 \sim x$). Thus variations in $x$ do not cause radius variations, eliminating the feedback necessary for instability.
4. Long-period LMXBs with (sub)giant companions are unstable: the larger the core mass and the smaller the total companion mass, the more violent the instability.

The irradiation instability may well cause the formation of ultrashort-period systems (e.g., the AM CVn's) from long-period CVs and LMXBs. However, since the rise to the high state is so rapid, it is extremely improbable that we currently observe any long-period LMXB undergoing cycles. We must therefore consider ways of quenching the instability in these systems. The most obvious possibility is screening of the companion from the irradiation, which is the basic cause of the instability. Screening may well result from the
extensive disk coronae inferred in LMXBs (see, e.g., White & Mason 1985). Formally we can consider this possibility by decreasing $s$ in our simulations. As can be seen from equation (31), the effect of a small $s$ is to lower the high branch and thus reduce the amplitude of the cycle in $x$. Since the lower branch remains unaffected, the turning radius and hence the radial amplitude will also be reduced. This suggests that screening could reduce the amplitude of the cycle and increase the frequency of outbursts, because it will take less time to drive the system into contact again once it has detached following an outburst. However, this argument applies only to a fixed $s$. If $s(x)$ is itself a function of the instantaneous mass transfer rate, as a result of a varying geometric thickness of the accretion disk or varying optical depth through the outflow, a more gradual transition occurs. For example, if screening results in a rapid reduction of $s$ beyond some critical $x_{scr}$, the amplitude of the cycle in $x$ is reduced, saturating somewhere close to $x_{scr}$, whereas the radial amplitude remains large; thus the outburst does not recur any sooner than in the unscreened case. We have performed some numerical experiments to simulate these effects and verify the above statements. These simulations and the discussion above imply that if $x_{scr} > x_t$, the radial amplitude of the cycle and recurrence time are not significantly different from the unscreened case. However, if $x_{scr} < x_t$ then both the radial amplitude and the duration of the low state are reduced. Also, the duration of the high state and the amount of mass transferred per cycle are decreased. Clearly, if $x_{scr}$ is reduced to $\sim x_t$, we will find that the cycles disappear altogether. Thus efficient screening can either eliminate the cycles entirely or render them relatively harmless as far as the binary evolution is concerned.

While screening must play a role in stabilizing some systems against the irradiation instability, a second way of quenching the instability appears to be more common. This mechanism uses the fact that the companion swells only when its own intrinsic luminosity is blocked by the irradiating flux, and that the blocking effect saturates once the latter is comparable to the intrinsic flux. A variable accretion rate, as seen in, e.g., soft X-ray transients, severely reduces the efficiency of irradiation in expanding the companion: the very high irradiating flux during outbursts has no more effect than a much weaker value, while the star can cool between outbursts. We might thus expect a highly modulated accretion rate with a duty cycle $d \ll 1$ to mimic irradiation by a steady accretion rate a factor of $d$ smaller. This expectation is largely fulfilled, as the following simple calculation shows.

We assume the dimensionless accretion rate to vary periodically in time over $t_{rec}$ as

$$x_{acc} = \begin{cases} x_h, & 0 < t \leq t_h, \\ x_l, & t_h < t \leq t_{rec}. \end{cases}$$

(44)

Mass conservation requires that the dimensionless transfer rate obey

$$x = dx_h + (1 - d)x_l = x_h(d + \frac{1 - d}{A}),$$

(45)

where $d = t_h/t_{rec}$ is the duty cycle and $A = x_h/x_l$ is the amplitude of the accretion rate variation. The reaction of the companion to this intermittent irradiation is governed by the thermal relaxation function (eq. [27]), which now becomes

$$p = -fp\left[\left(1 - s \tanh \frac{x}{x_c} \frac{A}{d(A - 1) + 1}\right)r^3 - r^{-(v + 2)}\right]^{(0 < t \leq t_h)},$$

(46)

$$p = -fp\left[\left(1 - s \tanh \frac{x}{x_c} \frac{1}{d(A - 1) + 1}\right)r^3 - r^{-(v + 2)}\right]^{(t_h < t \leq t_{rec}).}$$

(47)

Now assuming that the cycle time $t_{rec}$ is $\ll t_{ee}$, we can define a mean value

$$\langle p \rangle = \frac{1}{t_{rec}} \int_0^{t_{rec}} p(x, r)dt. \quad (48)$$

Performing the integration, we can compute the derivative $\langle p \rangle_x$, which governs the stability of the fixed point $(x_0, r_0)$. If $x_h > x_t$, i.e., $A \rightarrow \infty$, as is characteristic of soft X-ray transients and dwarf novae, we have

$$\langle p(x_0, r_0) \rangle_x = \frac{f_{ps}}{x_c} \frac{x_0}{r_0} \sech^2 \left(\frac{x_0}{dx_c}\right),$$

(49)

and the criterion for instability $\langle p(x_0, r_0) \rangle_x > 1$ can be written as

$$f_{ps} > \frac{x_0}{x_c} \frac{x_c}{r_0} \cosh^2 \left(\frac{x_0}{dx_c}\right). \quad (50)$$

This criterion is very similar to the earlier one (eq. [43]) assuming steady accretion, to which it of course reduces as $d \rightarrow 1$. For $x_0/dx_c \leq 1$, the two criteria are identical. Thus in Figure 4 the stability/instability boundary for $d < 1$ is given simply by sliding the $d = 1$ curve parallel to itself along the asymptote for small $x_0/x_c$, by a displacement of $-\log d$ in $x_0/x_c$. We may thus draw a further conclusion from Figure 4:

5. Typical soft X-ray transient duty cycles $d \lesssim 10^{-2}$ are probably enough to stabilize most LMXBs with evolved companions against the irradiation instability.

By contrast, extremely short duty cycles, $d \lesssim 10^{-4}$, would be required for dwarf nova outbursts to stabilize CVs with evolved secondaries. We note that most LMXBs with periods $\gtrsim 1$ day are transient (King, Kolb, & Burderi 1996b), and indeed this holds for a very large fraction of long-period systems also (King et al. 1996b, 1997). We shall consider the application of the stability criteria to individual systems in a future paper.

8. CONCLUSIONS

We have shown that irradiation of an evolved low-mass companion in LMXBs and CVs can drive mass transfer cycles. These cycles do not need the intervention of any further effect, unlike the case of main-sequence companions (Paper I), in which a modest increase in the driving angular momentum loss rate is required in the high state if cycles are to occur in many cases. The cycles with evolved companions are also considerably more violent than in the main-sequence case. This is a direct consequence of two facts. First, the nuclear luminosity of an evolved star is insensitive to the stellar radius, so that blocking of the intrinsic stellar flux by irradiation requires the star to expand so as to maintain the same unblocked area. This leads to much larger expansions than in the main-sequence case, in which the nuclear luminosity drops sharply as the star expands.
Second, the ratio $t_{dr}/t_{ce} = fp$ of driving to thermal timescales is much larger in evolved stars than on the main sequence, making the expansion very rapid. In the high state an evolved star would lose a significant fraction of its total envelope mass on a thermal timescale.

In a CV with a low core mass, the implied accretion rate would probably turn the system into a supersoft X-ray source. In CVs with higher core masses, the nuclear burning causes the white dwarf to develop an extensive envelope while, in long-period LMXBs, the high-state accretion rates greatly exceed the Eddington limit. If the instability is not quenched, all these systems would undergo a common-envelope phase. They may merge entirely, or reappear as ultrashort-period systems like AM CVn’s (for white dwarf primaries), or helium star LMXBs, or detached systems with low-mass white dwarf companions. However, there are at least two ways in which the instability can be quenched and the systems (particularly LMXBs) stabilized: shielding of the companion by, e.g., an extensive accretion disk corona, and intermittent accretion. The typical duty cycles $d \lesssim 10^{-2}$ observed in soft X-ray transients are short enough to stabilize them, while observed dwarf nova duty cycles are unable to stabilize CVs with evolved companions.

Although both means of stabilization seem to occur in nature, it is clear that the irradiation instability is so violent that it must play a major role in any discussion of the evolution of CVs and LMXBs with evolved companions: the systems must somehow stave it off or evolve catastrophically. We shall consider some of the observational consequences in a future paper.

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