Resolved photon and multi-component model for $\gamma^*p$ and $\gamma^*\gamma^*$ total cross section *

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We generalize our previous model for $\gamma^*p$ scattering to $\gamma\gamma$ scattering. Performing a new simultaneous fit to $\gamma^*p$ and $\gamma\gamma$ total cross section we find an optimal set of parameters to describe both processes. We propose new measures of factorization breaking for $\gamma^*\gamma^*$ collisions and present results for our new model.

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1. Introduction

In the last decade the photon-proton and photon-photon reactions became a testing ground for different QCD-inspired models. The dipole model was one of the most popular and successful. In the simplest version of the model only quark-antiquark Fock components of the photon are included in order to describe the total cross sections. In contrast, the more exclusive processes, like diffraction [1], jet [2] or heavy quark [3] production, require inclusion of higher Fock components of the photon.

In Ref. [4] we have constructed a simple hybrid model which includes the resolved photon component in addition to the quark-antiquark component. With a very small number of parameters we were able to describe the HERA $\gamma^*p$ total cross section data with an accuracy similar to that of very popular dipole models. In Ref. [5] we have generalized our hybrid model also to photon-photon collisions. Application to $\gamma\gamma$ processes requires some modifications of the model.

2. Formulation of the model

In our model the total cross section for $\gamma^*p$ is a sum of three components illustrated graphically in Fig[1]

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For the dipole component

\[ \sigma_{\text{dip}}^{\text{tot}}(W, Q^2) = \sum_q \int dz \int d^2 \rho \sum_{T,L} \left| \Psi_{T,L}^{\gamma^* \to q\bar{q}}(Q, z, \rho) \right|^2 \cdot \sigma_{(q\bar{q})N}(x, \rho) \]  

(1)

and for the vector meson component

\[ \sigma_{\text{VDM}}^{\text{tot}}(W, Q^2) = \sum_V \frac{4\pi M_V^2 \sigma_{N}^{V}(W)}{\gamma_V(Q^2 + M_V^2)^2} \cdot (1 - x). \]  

(2)

The last component in Fig. 1 becomes important only at large \( x \), i.e. small \( W \).

We take the simplest diagonal version of VDM with \( \rho, \omega \) and \( \phi \) mesons included. The vector-meson-nucleon cross section is approximated by pion(kaon)-proton cross section. A simple Regge parametrization by Donnachie and Landshoff [7] is used to parametrize the pion(kaon)-proton total cross section. We take \( \gamma \)'s calculated from the leptonic decays of vector mesons, including finite width corrections.

In the same spirit, the total cross section for \( \gamma^* \gamma^* \) scattering can be written as a sum of five terms shown in Fig. 2.

The formulae for the direct term can be found in Ref. [8].

If both photons fluctuate into perturbative quark-antiquark pairs, the interaction is due to gluonic exchanges between quarks and antiquarks represented in Fig. 2 by the blob. Formally this component can be written in terms of the photon perturbative "wave functions" and the cross section for
the interaction of both dipoles

\[
\sigma_{dip-dip}^{tot}(W, Q_1^2, Q_2^2) = \sum_{a,b=1}^{N_f} \int_0^1 dz_1 \int d^2 \rho_1 |\Psi_T^a(z_1, \rho_1)|^2 \\
\cdot \int_0^1 dz_2 \int d^2 \rho_2 |\Psi_T^b(z_2, \rho_2)|^2 \sigma_{dd}^{a,b}(x_{ab}, \rho_1, \rho_2). \tag{3}
\]

In paper [9] a phenomenological parametrization for the azimuthal-angle averaged dipole-dipole cross section has been proposed:

\[
\sigma_{dd}^{a,b}(x_{ab}, \rho_1, \rho_2) = \sigma_0^{a,b} \left[1 - \exp \left(-\frac{\rho_{eff}^2}{4R_0^2(x_{ab})}\right)\right] \cdot S_{thresh}(x_{ab}). \tag{4}
\]

Our formula for \(x_{ab}\) is different from the one used in Ref. [9]. As discussed in Ref. [3] our formula provides correct behaviour at threshold energies. Different prescriptions for \(\rho_{eff}\) have been considered in Ref. [9], with \(\rho_{eff}^2 = \frac{\rho_1^2 \rho_2^2}{\rho_1^2 + \rho_2^2}\) being probably the best choice [9].

Following our philosophy of explicitly including the nonperturbative resolved photon, in photon-photon collisions completely new terms must be included (the last two diagrams in Fig. 2). If one of the photons fluctuates into a quark-antiquark dipole and the second photon fluctuates into a vector meson, or vice versa

\[
\sigma_{SR1}^{tot}(W, Q_1^2, Q_2^2) = \int d^2 \rho_2 \int dz_2 \sum_{V_1} \frac{4\pi}{f_{V_1}^2} \left(\frac{m_{V_1}^2}{m_{V_1}^2 + Q_1^2}\right)^2.
\]
\[ \cdot |\Psi(\rho_2, z_2, Q_2^2)|^2 \sigma_{V_1d}^{\text{tot}}(W, Q_2^2), \quad (5) \]

\[ \sigma_{SR2}^{\text{tot}}(W, Q_1^2, Q_2^2) = \int d^2\rho_1 \int dz_1 \sum_{V_2} \frac{4\pi}{f_{V_2}^2} \left( \frac{m_{V_2}^2}{m_{V_2}^2 + Q_2^2} \right)^2 \cdot |\Psi(\rho_1, z_1, Q_1^2)|^2 \sigma_{V_2d}^{\text{tot}}(W, Q_1^2). \quad (6) \]

In the formulae above:

\[ \sigma_{V_1d}^{\text{tot}}(W, Q^2) = \sigma_0 \left( 1 - \exp \left( -\frac{\rho_1^2}{4R_0^2(x_g)} \right) \right) \cdot S_{\text{thresh}}. \quad (7) \]

In the present calculation we take \( m_f = m_0 \) for \( u/\bar{u} \) and \( d/\bar{d} \) (anti)quarks and \( m_f = m_0 + 0.15 \text{ GeV} \) for \( s/\bar{s} \) (anti)quarks.

If each of the photons fluctuates into a vector meson the corresponding component is called double resolved. The corresponding cross section reads:

\[ \sigma_{DR}^{\text{tot}}(W, Q_1^2, Q_2^2) = \sum_{V_1V_2} \frac{4\pi}{f_{V_1}^2} \left( \frac{m_{V_1}^2}{m_{V_1}^2 + Q_1^2} \right)^2 \cdot \frac{4\pi}{f_{V_2}^2} \left( \frac{m_{V_2}^2}{m_{V_2}^2 + Q_2^2} \right)^2 \sigma_{V_1V_2}^{\text{tot}}(W). \quad (8) \]

The total cross section for \( V_1-V_2 \) scattering must be modeled. In the following we assume Regge factorization and use a simple parametrization which fits the world experimental data for hadron-hadron total cross sections [7]. More details can be found in Ref.[5].

3. Results

In Ref.[4] we have adjusted the parameters of our model to \( \gamma^*p \) collisions. Let us try to use these parameters to describe \( \gamma\gamma \) total cross section. In Fig.3 we present the total cross section as a function of center-of-mass energy. The sum of all components of Fig.2 (thick-solid line) exceeds the experimental data by a factor of two or even more. The individual components are shown explicitly as well. The direct component (dash-dotted line) dominates at low energies only. At high energies the dipole-dipole (thin-solid line), single-resolved (dashed line) and double-resolved (dotted line) components are of comparable size. The overestimation of the experimental data suggests a double-counting.

In Ref.[4] it was assumed that the coupling constants responsible for the transition of photons into vector mesons are the same as those obtained
The total $\gamma \gamma$ cross section as a function of photon-photon energy with parameters from Ref. [4] (panel a) and with the new set of parameters. The experimental data are from [11, 12].

Fig. 3.

from the leptonic decays of vector mesons, i.e. the on-shell approximation was used. In our case we need the corresponding coupling constants rather at $Q^2 = 0$ and not on the meson mass shell ($Q^2 = m_V^2$). We replace $\frac{4\pi f_{V_i}}{f_{V_i}^2} F_{\text{off}}(Q^2, m_V^2)$ and extrapolate from meson mass shell to $Q^2 = 0$ by means of

$$F_{\text{off}}(Q^2, m_V^2) = \exp \left( -\frac{(Q^2 + m_V^2)}{2\Lambda_E^2} \right).$$  \hspace{1cm} (9)

The parameter $\Lambda_E$ is a new nonperturbative parameter of our new model. Secondly, the “photon-wave functions” commonly used in the literature allow for large quark-antiquark dipoles. This is a nonperturbative region where the pQCD is not expected to work. Furthermore this is a region which is taken into account in the resolved photon components as explicit vector mesons. Therefore we propose the following modification of the “perturbative” photon wave function:

$$|\Psi(\rho, z, Q^2)|^2 \rightarrow |\Psi(\rho, z, Q^2)|^2 \exp \left( -\frac{\rho}{\rho_0} \right).$$  \hspace{1cm} (10)

The parameters $\Lambda_E$ and $\rho_0$ were obtained by fitting our modified model formula to the experimental data. The $\gamma \gamma$ data is not sufficient for this
purpose as different combinations of the two parameters lead to equally
good description. Therefore we were forced to perform a new fit of the
model parameters to both $\gamma p$ and $\gamma\gamma$ scattering.

In Fig. 3 we show the resulting $\sigma_{\gamma\gamma}^{\gamma\gamma}$ together with the experimental data
of the PLUTO (solid triangles) and OPAL (open circles) collaborations. We also show the individual contributions of different processes from Fig. 2. The relative size of the contributions has changed when compared to the old set of parameters. Now the sum of the single resolved components, included here for the first time, dominates in the broad range of center-of-mass energies. The double resolved component is now much weaker and constitutes 10-15% of the total cross section only. In Fig. 4 we show the analogous description of the $\gamma p$ data. The agreement with the HERA data is similar as in our previous paper [4].

![Graph showing $\sigma_{\gamma\gamma}^{\gamma\gamma}$ vs $W$]({#graph})

Fig. 4. The total $\gamma^*p$ cross section as a function of photon-proton energy. The experimental HERA data are from [10].

In our fit we have included $\gamma^*p$ and $\gamma\gamma$ experimental data. In Ref. [5] we have compared the predictions of our model for total cross sections for one virtual – one real photon with existing experimental data.

In data processing, in particular in extrapolations to small photon virtualities one often assumes the following relation

$$\sigma_{\gamma^*\gamma^*}^{\gamma^*\gamma^*}(W, Q_1^2, Q_2^2) = \Omega(Q_1^2) \cdot \Omega(Q_2^2) \cdot \sigma(W)$$  (11)
known as factorization. We have considered two quantities which measure factorization breaking. They read:

\[
\begin{align*}
    f_{fb}^{(1)}(W, Q_1^2, Q_2^2) &\equiv \frac{\sigma_{\gamma^*\gamma^*}(W, Q_1^2, Q_2^2) \sigma_{\gamma^*\gamma^*}(W, 0, 0)}{\sigma_{\gamma^*\gamma^*}(W, Q_1^2, Q_2^2) \sigma_{\gamma^*\gamma^*}(W, 0, 0)} , \\
    f_{fb}^{(2)}(W, Q_1^2, Q_2^2) &\equiv \frac{\sigma_{\gamma^*\gamma^*}(W, Q_1^2, Q_2^2) \sigma_{\gamma^*\gamma^*}(W, Q_2^2, Q_2^2)}{\sigma_{\gamma^*\gamma^*}(W, Q_1^2, Q_2^2) \sigma_{\gamma^*\gamma^*}(W, Q_2^2, Q_1^2)} .
\end{align*}
\]  

(12)

For the factorized Ansatz (11) \( f_{fb}^{(1,2)}(W, Q_1^2, Q_2^2) = 1 \) for any \( Q_1^2 \) and \( Q_2^2 \).

4. Conclusions

We have generalized our previous model for \( \gamma^*p \) total cross section to the case of \( \gamma\gamma \) scattering. In the last case a few new components appear.

The naive generalization of our former model for \( \gamma^*p \) total cross section leads to a serious overestimation of the \( \gamma\gamma \) total cross sections. A priori, this fact can be due either to a nonoptimal set of model parameters found in our
previous study, double counting, or due to some model simplifications like
off-shell effects. We have suggested to include such an effect by introduc-
ing new form factors. When including the quark-antiquark continuum one
usually takes into account the perturbative quark-antiquark "photon wave
function". This is justified and reasonable for small size dipoles only. In
order to avoid double counting the large-size dipoles have been eliminated
using a simple exponential function in transverse dipole size. We have per-
formed a new fit of our generalized-model parameters to the $\gamma^*p$ and $\gamma\gamma$
total cross sections.

When trying to extrapolate the experimental cross sections for the $\gamma^*\gamma^*$
scattering to real photons one often assumes factorization. We have quanti-
tified the effects of factorization breaking in our model with parameters fixed
to describe the $\gamma^*p$ and $\gamma\gamma$ data. We have proposed two new functions
which can be used as a measure of the factorization breaking.

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