Note on holographic entanglement entropy and complexity in

Stückelberg superconductor

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Abstract

The holographic superconductors, as one of the most important application of gauge/gravity duality, promote the study of strongly coupled superconductors via classical general relativity living in one higher dimension. One of the interesting properties in holographic superconductor is the appearance of first and second order phase transitions. Recently, another active studies in holographic framework is the holographic entanglement entropy and complexity evaluated from gravity side. In this note, we study the properties of the holographic entanglement entropy and complexity crossing both first and second order phase transitions in Stückelberg superconductor. We find that they behave differently in two types of phase transitions. We argue that holographic entanglement entropy and complexity conjectured with the volume can also be a possible probe to the type of superconducting phase transition.

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I. INTRODUCTION

In the last decades, the AdS/CFT correspondence [1–3], has made significant progress and development. In many applications of gauge/gravity duality, AdS and condensed matter physics duality (AdS/CMT) has attracted plenty of interest, which provides a powerful approach to study the strongly coupled field theory in terms of weakly coupled gravitational systems. In particular, physicist has investigated kinds of holographic superconductors which was first proposed in [4, 5] including s-wave model [6], p-wave model [7] and d-wave model [8, 9]. Considerable efforts in holographic superconductors have been made in the related research and discussion, and for reviews, readers can see for example, [10, 11] and references therein.

In the study of holographic superconductors and other condensed matter physical models like holographic superfluidity, many study on the types of phase transition analysis have been done. The first proposal on holographic s-wave superconductor in probe limit was found to undergo a second order phase transition from normal state[5]. Later, It was addressed in [12] that the holographic superconductor via the Stückelberg mechanism allows the first order phase transition to occur when the model parameter surpasses a critical value, which was extended in [13–15]. First order phase transition in holographic s-wave superconductor has also been observed in superfluidity model[16], $N = 8$ gauged supergravity model[17] and so on. Besides, in p-wave holographic superconductor models, some works for instance [18–21] showed the zero-order, first-order and second-order phase transition in their models, and discovered the retrograde condensation in certain parameter space.

On the other hand, holographic entanglement entropy[22] as a measure of the degrees of freedom in a strongly coupled system, has been firstly evaluated in holographic metal/superconductor phase transition in [17]. It was addressed that the entanglement entropy in superconducting case is always less than that in the normal phase. Beside the entanglement entropy is continuous but its slope in terms of temperature is discontinuous at the transition temperature $T_c$ for the second order phase transition, while for the first order phase transition, the entanglement entropy presents a discontinuous drop at the critical temperature. Thus, the authors of [17] argued that the holographic entanglement entropy can be used to determine the orders of superconducting phase transition. Later, the authors of [23] found that the entanglement entropy has a different behavior near the contact interface of the superconducting to normal phase due to the proximity effect. Further effort in using holographic entanglement entropy as a probe of phase transition has been made in [24–31] where the behaviors of holographic entanglement entropy in different orders phase transition are different. They also argued that the holographic entanglement entropy may not be universal in holographic superconduction models, which explains the existence of the difference between in first order phase transition and second order phase transition.

In this note, we will investigate the properties of holographic complexity measured by
the volume\[32\] in St"uckelberg holographic superconductor model. The complexity essentially measures the difficulty of turning a quantum state into another and so it could reflect a phase transition on the boundary field theory. We note that holographic complexity has been studied in one dimensional s-wave superconductor in \[33, 34\] and p-wave superconductor in \[35\]. It was found that holographic complexity behave in the different way with holographic entanglement entropy and both of them can reflect the one dimensional phase transitions.

Here we are aiming to study the properties of holographic complexity for a strip subregion when the system crosses the first order and second order St"uckelberg superconducting phase transition. We will consider only the $\psi^4$ coupling term in St"uckelberg model, which brings in first order phase transition when the coupling parameter is bigger than a critical value in probe limit case \[12\] and in backreaction case\[31\]. We will see that the holographic complexity increase monotonously as the temperature becomes lower for the second order phase transition, while it present a multivalue area in high temperature region in first order case which appears in the same temperature region in free energy diagram. Besides, similar to the holographic entanglement entropy, at the second order phase transition temperature, the complexity is continuous and its slope in terms of the temperature have a jump, while in the first order phase transition case, the complexity presents a jump at the critical temperature. Thus, we are expecting that similar to the holographic entanglement entropy, the holographic complexity can also be a good probe to the order of superconducting phase transition.

Our paper is organized as follows. We briefly show the St"uckelberg superconducting phase transition, the condensation and computing the free energy of the dual system in section II. In section III, we first review the holographic setup of entanglement entropy and complexity for a strip subregion in AdS/CFT framework, and then numerically evaluate them in the superconducting model. The last section contributes to our conclusion and discussion.

II. ST"UCKELBERG SUPERCONDUCTING PHASE WITH BACKREACTION

The generalized four dimensional action containing a $U(1)$ gauge field and the scalar field coupled via a generalized St"uckelberg Lagrangian is \[13\]

$$S = \int d^4x \sqrt{-g} \left[ \left( R + \frac{6}{l^2} \right) + L_M \right],$$

where $l$ is the AdS radius which will be set to be unity in the following discussion. $L_M$ is the generalized St"uckelberg Lagrangian

$$L_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (\partial \psi)^2 - m^2 |\psi|^2 - |\mathcal{F}(\psi)| (\partial p - qA)^2,$$
where $\mathcal{F}(\psi)$ is a function of $\psi$. In this paper, we will consider $\mathcal{F}(\psi) = \psi^2 + c_4 \psi^4$ where $c_4$ is a model parameter\(^1\). Considering the gauge symmetry $\bar{A}_\mu \rightarrow \bar{A}_\mu + \partial \Lambda$ and $p \rightarrow p + \Lambda$, we fix the gauge $p = 0$ by using the gauge freedom.

To include the backreaction, we consider the metric ansatz

$$ds^2 = \frac{-f(z)}{z^2}e^{-\chi(z)}dt^2 + \frac{dz^2}{z^2 f(z)} + \frac{dx^2 + dy^2}{z^2}.$$  \hspace{1cm} (3)

The Hawking temperature is expressed as

$$T = \frac{-f'(z_H)e^{-\chi(z_H)/2}}{4\pi},$$  \hspace{1cm} (4)

and the event horizon $z_H$ satisfies $f(z_H) = 0$. Then, considering the ansatz of the matter fields as $\psi = \psi(z), A = \phi(z)dt$, we obtain the equations of motion from the action (1) under the metric (3)

$$\psi'' - \left(\frac{2}{z} + \frac{\chi'}{2} - \frac{f'}{f}\right)\psi' + \frac{q^2 \phi^2 e^\chi}{f^2} \left(\psi + 2q^2 c_4 \psi^3\right) - \frac{m^2}{z^2 f}\psi = 0,$$  \hspace{1cm} (5)

$$\phi'' + \frac{\chi'}{2} \phi' - \frac{q^2 \phi}{z^2 f} \left(\psi^2 + q^2 c_4 \psi^4\right) = 0,$$  \hspace{1cm} (6)

$$\chi' - \frac{z}{2} \left(\psi'^2 + \frac{q^2 \phi^2 e^\chi}{f^2} \left(\psi^2 + q^2 c_4 \psi^4\right)\right) = 0,$$  \hspace{1cm} (7)

$$f' - \frac{zf}{4} \left(\psi'^2 + \frac{12}{z^2} + \frac{z^2 e^\chi}{f} \phi'^2\right) - \frac{\psi^2}{4z} \left(m^2 + \frac{z^2 q^2 \phi^2 e^\chi}{f} \left(\psi^2 + q^2 c_4 \psi^4\right)\right) + \frac{3}{z} = 0.$$  \hspace{1cm} (8)

In our following study, we will take $q = 1$ and $m^2 = -2$ without loss of generality, though our analysis can be extended into other proper parameters. Thus, the behavior of various field in the asymptotical AdS boundary is

$$\psi = \psi_1 z + \psi_2 z^2 \ldots, \phi = \mu - \rho z + \ldots, f = 1 + f_3 z^3 \ldots, \chi = \chi_0 + \chi_1 z \ldots.$$  \hspace{1cm} (9)

According to AdS/CFT dictionary, $\mu$ and $\rho$ are considered as the chemical potential and charge of the boundary field theory, respectively. $\psi_1$ and $\psi_2$ can be dual to the source while the other is the vacuum expectation value due to the choice of standard quantization or alternative one. As is pointed out in [6] that $\psi_1$ as expectation value is always divergent at zero temperature limit, i.e., it may be not physical. So we will choose $\psi_1$ as the source and $\psi_2$ is related to the expectation value $(O_2)$. Moreover, near the horizon, the regular condition requires that the Maxwell field satisfies $\phi(r_H) = 0$ and the scalar field is not vanished.

We numerically solve the equations of motion (5)-(8) by setting the source $\psi_1 = 0$. Our numerical results of the condensation are explicitly shown in figure 1 which indicates that

\[1\] We note that the holographic superconductor with $\mathcal{F}(\psi) = \psi^2 + c_6 \psi^4$ coupling with backreaction has been studied in [36].
the type of phase transition is affected by the values of coupling parameter \( c_4 \). There exists a critical value of \( c_4 \sim 0.5 \) below which the system undergoes a second superconducting phase transition while above which a first order phase transition occurs in the system. Similar phenomena was also found in [12].

![FIG. 1: The condensates of the scalar operator \( \langle O_2 \rangle \) for \( c_4 = 0 \) and \( c_4 = 0.6 \). The left curve describes the second-order phase transition and the right multi-value curve describes the first-order phase transition.](image)

For the second order phase transition, the critical temperature \( T_c \) is straightforward to be read off, at which the condensation appears. We find that the critical temperature is around 0.0358912 which is not explicitly dependent on coupling parameter. This is an obvious result indicating that the physical process is stable when the system has the second-order phase transition. Similar phenomena has also been found in the studies[12, 31]. However, for the first order phase transition, it is not that direct to fix \( T_c \) because of the multivalue of the condensation. Then, a usual way to fix it is to calculate the free energy of the solutions.

In order to further determine \( T_c \) for the phase transitions and its order, we will compute the free energy of the system. We will work in the canonical ensemble, in which the charge is fixed. According to [37, 38], the free energy of the boundary field theory is connected with on-shell Euclidean bulk action as \( F = -TS_E \) and in our model

\[
S_E = -(S_{EH} + S_\Psi + S_A) + \int_{z=0} dx^3 \sqrt{-g_\infty} (-2K + 4/L^2),
\]

where \( g_\infty \) is the induced metric on the boundary, and \( K \) is the trace of extrinsic curvature. Then considering the equations of motion (5)-(8) and the boundary behaviors (9), the free energy density in our model is evaluated by

\[
\frac{F}{V_2} = f_3
\]

where \( V_2 = \int dx dy \) which will be set to be unit and \( f_3 \) is a coefficient in (9). We notice that for the normal state, the free energy is \( f_3 = \mu/4 - 1 \).

The results of \( \Delta F = F - F_{RN} \), which is the difference between the free energy of the condensed state and normal state, are shown in figure 2. The blue solid lines are for the
superconducting state while the black lines represent the normal state. For $c_4 = 0$ in the left plot, it is explicit that when $T > T_c$, the RN solution is physically favorable; when $T = T_c$ which is marked by the red dashed vertical line, second order phase transition occurs and the superconducting phase become physically favorable as the temperature further decreases. For $c_4 = 0.6$ in the right plot, the multivalue region of the $\Delta F$ appears which implies that first order phase transition occurs. Though the temperature $T = 0.03559$ represents the appearance of condensation function, but physical phase transition happens at the interaction point $T \approx 0.038143$, below which the superconducting state is thermodynamically favorable.

Thus, from the free energy, we can read off $T_c$ for the first order transition for different $c_4$ which are listed in table I. $T_c$ for the first order phase transition grows as the coupling parameter, which behaves differently with that in the second order phase transition. Comparing the two types of the superconducting phase transition, the second-order phase transitions has lower but stable critical temperatures, so more drastic condensation and phase transition process is hinted in the first order case.

![Graph showing free energy for $c_4 = 0$ and $c_4 = 0.6$.]

**FIG. 2:** The free energy of the system for $c_4 = 0$ and $c_4 = 0.6$.

| $c_4$  | 0.52  | 0.55  | 0.60  | 0.70  | 0.80  |
|------|-------|-------|-------|-------|-------|
| $T_c$| 0.0365796 | 0.0372203 | 0.0381431 | 0.0403314 | 0.0423508 |

**TABLE I:** The critical temperature for the first order phase transition which grows with the coupling parameter.

Next, we shall study holographic entanglement entropy and holographic complexity in the two types of phase transitions. We expect that the holographic entropy and complexity would perform different properties in the two types of phase transition, and suggest more deeply physics about the superconducting phase transitions in the dual systems.
III. HOLOGRAPHIC ENTANGLEMENT ENTROPY AND HOLOGRAPHIC COMPLEXITY

A. The setup

For a system $A$ in the boundary CFT which has a gravity dual, Ryu and Takayanagi proposed that the information included in a subsystem $B$ is evaluated by the entanglement entropy $S_A$ as

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N^{(d+2)}},$$  \hspace{1cm} (12)

and Alishahiha proposed that the holographic complexity for this system $A$ is measured by the volume enclosed by the aforementioned minimal surface as

$$C = \frac{\text{Volume}(\gamma_A)}{8G_N^{(d+2)}},$$  \hspace{1cm} (13)

where $\gamma_A$ is a $d$-dimensional minimal surface whose boundary is given by the $(d-1)$-dimensional manifold $\partial \gamma_A = \partial A$. $G_N^{(d+2)}$ is the Newton constant of the general gravity in AdS$_{d+2}$ theory.

Following [17], we will consider the subsystem $A$ with a straight strip geometry which is described by $-\frac{l}{2} \leq x \leq \frac{l}{2}, 0 \leq y \leq L$. Here $l$ is the size of region $A$ and $L$ is a regulator which can be set to be infinity. Therefore, the induced metric of the hypersurface $\gamma_A$ who has the same boundary as the stripe does is

$$ds_{\text{induced}}^2 = \frac{1}{z^2} \left[ \left( \frac{1}{f} + x'(z) \right) dz^2 + dy^2 \right].$$  \hspace{1cm} (14)

Then, the holographic entanglement entropy connecting with the area of the surface can be evaluated as

$$4G_4 S = \text{Area}(\gamma_A) = \int dy \int_{-\ell/2}^{\ell/2} dx \frac{1}{z^2} \sqrt{1 + \frac{1}{f(z)} \left( \frac{dz}{dx} \right)^2},$$ \hspace{1cm} (15)

The condition $\left( \frac{dz}{dx} \right)^2 = f(z) \left( \frac{d}{dz} - 1 \right)$ gives the minimal area in (15) where $z_*$ is the location in $z$ with the smooth extremal surface[39]. Integrating the condition gives us

$$x(z) = \int_z^{z_*} \frac{z^2 dz}{\sqrt{(z_*^4 - z^4)f(z)}}.$$ \hspace{1cm} (16)

Subsequently, the entanglement entropy (12) is geometrized in terms of AdS/CFT dictionary as

$$S = \frac{L}{2G_4} \int_{\epsilon}^{z_*} \frac{z_*^2}{z^2} \frac{dz}{\sqrt{(z_*^4 - z^4)f(z)}} = \frac{L}{2G_4} \left( \frac{1}{\epsilon} + s \right),$$ \hspace{1cm} (17)
where $L = \int dy$ and $\epsilon \to 0$ is the UV cutoff. And according to [32, 33], the holographic complexity (13) is holographically related to the volume in the bulk enclosed by $\gamma_A$ as

$$C = \frac{1}{4\pi G_4} \int_\epsilon^{z_*} dz \frac{x(z)}{z^3 \sqrt{f(z)}},$$

(18)

where $x(z)$ is expressed in (16).

B. The results

We shall analyze the properties of the holographic entanglement entropy and complexity of the dual system in different types of superconducting phase transitions.

The numerical results of holographic entanglement entropy are shown in figure 3 where the red dashed lines are for the normal state and the blue lines are for the condensed state. For the second order phase transition with $c_4 = 0$ (left plot), $s$ decreases monotonously as the temperature becomes lower. When the temperature reaches the critical temperature $T_c = 0.0358912$, the holographic entropy separates into two branches which implies a phase transition. And the values for the superconducting state is always lower than that for the normal state. This is reasonable because the cooper pairs form in the superconducting state which supress the degree of freedom of the system. Contrasting to the monotonous behavior of second order phase transition, the entanglement entropy are always non-monotonous for the first order phase transition with $c_4 = 0.6$(right plot), and has a discontinuous drop to the minimal entropy at the critical temperature $T_c = 0.038143$. Similar phenomena has also been found in the first order of insulator/superconducting phase transition[31, 36]. We note that we always expect new degrees of freedom to emerge in new phases, so that the discontinuity or drop of the entropy at transition point may imply non-trivial reorganization of the degrees of freedom in the system.

We turn to study the properties of holographic complexity shown in figure 4. Different from the entanglement entropy, the complexity grows up as the temperature decreases which is similar as that found in [34]. For $c_4 = 0$, the holographic complexity in the the second order
phase transition case increases monotonously as $T$ becomes lower, until $T$ reaches the critical value, then $c$ in superconducting state is always larger than that in normal state. Similarly, in the case of first order phase transition with $c_4 = 0.6$, When the temperature decreases, the numerical holographic complexity smoothly increases firstly and presents multi-values until the transformation point $T = 0.03559$. But the real process of complexity has a jump to the maximal complexity at $T_c = 0.038143$ in the superconducting state.

Thus, we argue that the drop of the entanglement entropy and the jump of the complexity at the critical point may be quite general features for the first order superconducting phase transition. Both of holographic entanglement entropy and complexity can be a possible probe of phase transition in our superconductor model.

IV. CONCLUSION AND DISCUSSION

In this paper, we firstly reviewed the holographic superconductor phase transition in the St"uckelberg model with $\mathcal{F}(\psi) = \psi^2 + c_4\psi^4$. For the value of coupling parameter is less than the critical value, there is the second order phase transition; or else, when the coupling parameter is larger than the critical one, the first order phase transition occurs. According to the analysis of free energy for the system, it is obvious that the system is experiencing more drastic condensation during the first order phase transition. Comparing to the critical temperature of phase transition, for the second order phase transition, the system reflects lower but has more stable critical temperature which is not affected by the coupling strength. For the first order case, the relation between critical temperature and coupling strength is almost linearly increasing. Our results are similar to that found in the previous literatures[12, 31].

Then we studied the holographic entanglement entropy and holographic complexity for the two types of phase transition. For the second order phase transition, the holographic entanglement entropy monotonously decreases while the complexity increases as the temperature becomes lower. On the other hand, they have a multivalue area in high temperature region in first order case, which appears in the same temperature region in free energy di-
agram. One of the multivalue lines is not physical and does not exist in the real system. According to (17) and (18), the behavior of the metric function \( f(z) \) takes charge of the multivalue region of holographic entanglement entropy and complexity. We show the profiles of \( f(z) \) for samples of temperatures in both cases in figure 5. It is obvious that in the first order case, the metric function \( f(z) \) has unusual behaviors in low temperature like the case in [17].

![Graph 1](image1.png)

**FIG. 5:** Left: The profile of \( f(z) \) with \( c_4 = 0 \). The different colors blue, yellow, green and red correspond to \( T_c, 0.8T_c, 0.6T_c \) and \( 0.4T_c \), respectively; Right: The profile of \( f(z) \) with \( c_4 = 0.6 \). The different colors blue, yellow, green and red correspond to \( 1.01T_c, T_c, 0.80T_c \) and \( 0.70T_c \), respectively.

Both holographic entanglement entropy and complexity behaves differently crossing the second order and first order superconducting phase transition point. At the second order phase transition point, they are continuous and the slopes in terms of the temperature have a jump. While in the first order superconducting phase transition, there exists the drop for the entanglement entropy and the jump for the complexity at the critical temperature. Thus, we argued that both of them can be used as a probe to the order of phase transition in our holographic superconducting model. We note that the complexity is deeply connected with fidelity susceptibility, which is known to be able to probe phase transition[40–42]. It would be very interesting to pursue the deep physics of difference in the probes by studying the fidelity susceptibility in holographic superconducting models.

In this note, we mainly focused on the difference of the qualitative behavior of holographic complexity for different orders of superconducting phase transitions. Very recently, in the paper [43], the authors studied the scaling of complexity on the temperature effected by the superconductor model parameters, as well as the time dependent complexity when the system undergoes second order phase transition. Study on this phenomena in superconducting models with different types of phase transition so as to further understand the deep physics in holographic superconductor should be another interesting direction.
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