Structural system identification of plane frames based on frequency domain decomposition-natural excitation technique (FDD-NExT)

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Abstract. Nowadays, system identification of structures has become an important subject in the civil engineering community. The reason is that with a good system identification method, modal parameters such as natural frequencies, mode shapes, and damping ratios can be obtained accurately, leading to an excellent structural health monitoring system. The two systems identification techniques are categorized as frequency-based and time-based domain identifications. In addition, the identifications based on the output-only measurements are gaining more attention from the researchers recently. One of the advantages of this type of identification is that the modal parameters can be obtained without knowing the excitation forces. In other words, it is not necessary to know the input load of the structures. This paper considers output-only identification based on Frequency Domain Decomposition (FDD) and Natural Excitation Technique (NExT) as FDD-NExT. FDD is one of the methods that relies on the output-only measurements such as displacement, velocity, or acceleration, in order to obtain the natural frequencies and mode shapes of the structures. On the other hand, NExT has good ability for determining the damping ratios of structures. This paper aims to apply the FDD-NExT to obtain the modal parameters and the damping ratio of plane frame structures. Two plane frame structures are considered in this paper to show the ability of the proposed method to predict modal parameters and the damping ratio of the structures. The first structure is a one-story frame with 6 degrees of freedom, while the second structure is a four-story frame structure with 48 degrees of freedom. For comparison, Modal Assurance Criterion (MAC) has been used to determine the difference between the modal parameters obtained by FDD and finite element analysis. The results show a good correlation between those two methods.

1. Introduction

A critical step in structural health monitoring is structural system identification. Structural health monitoring system aims to detect structural damage. It can only be achieved with good structural system identification. One technique of structural system identification, which is quite prominent for predicting the modal parameters of structures, is the Frequency Domain Decomposition (FDD) method. FDD technique is based on the output system only, where the response due to an ambient environment is used as the input, and no external excitation is needed. That is why it becomes more popular compared to other methods, where usually the input excitation is used as the data. Generally, the modal parameters needed for structure health monitoring systems are natural frequency (ω), mode shape (φ), and damping ratio (ζ). In this paper, FDD was used for predicting the natural frequency and mode shape. To obtain the damping ratio, Natural Excitation Technique (NExT) was used. NExT is...
also output system only, similar to FDD, where it needs the response of the structure due to the ambient condition. Both FDD and NExT have been applied to a variety of the structures to determine the modal parameters. Both techniques have shown the robustness, especially in predicting modal parameters of bridges and wind turbines structures [1, 2]. In the previous research, FDD-NExT has been applied to simple shear buildings, beams, and plane truss structures [3-6]. In this paper, FDD-NExT was applied in the plane frame structures with different structural configuration to investigate the capability of FDD-NExT to predict the modal parameters of those structures.

2. Frequency domain decomposition (FDD)
Frequency domain decomposition (FDD) is one of the popular techniques for structural identifications, especially in structural health monitoring issues. The advantage of this method is that it is based on the output system only. Output responses of structure can be displacement, velocity, or acceleration response. The obtained output response from the ambient vibration can be used to determine the modal parameters of structures without knowing the external input vibration. FDD was used in this paper to obtain modal parameters such as natural frequencies and mode shapes of the structure. Generally, there are three steps to use FDD to obtain the modal parameters of the structures, as follows [7]:

1. Estimate the power spectral density based on the output response for each discrete frequency.
2. Do the singular value decomposition (SVD) on power spectral density matrix using
   \[ G_{yy}(j\omega) = U_i S_i U_i^H \]  
   where \( U_i = [u_{i1}, u_{i2}, u_{i3}, \ldots, u_{in}] \) is a unitary matrix that consist of singular matrix, \( u_{ij} \), \( S_i \) is a diagonal matrix consist the scalar value of \( S_{ij} \).
3. For every n degree of freedom, n modes that dominate on power spectral density was taken. It should be noted that the first singular value of PSD generally is taken to predict the modal parameters of the structure.

3. Natural excitation technique (NExT)
Natural excitation technique (NExT) is a technique to determine modal parameters of structures due to the ambient environment. NExT was first applied for wind turbine testing subjected to wind load [2]. After that, NExT becomes popular and widely used for other projects in order to find the modal parameters of structures. Generally, NExT is used to determine the modal parameters such as natural frequencies, mode shapes, and damping ratios of structure. There are four steps to apply NExT, but since the damping ratio of the structure is the only needed parameter, only the second step was used, i.e., measuring the cross-correlation function from the structure. First, before measuring the cross-correlation function, the fast inverse Fourier transform from the power spectral density function had to be determined. The correlation functions are expressed as the summations of decaying sinusoids in which each decaying sinusoid has a damped natural frequency and damping ratio that is identical to that of corresponding structural modes.

4. Modal assurance criterion (MAC)
Modal assurance criterion (MAC) is one of the techniques with the ability to determine the difference between two mode shapes [8]. In the beginning, MAC was used to determine the quality of two modal vectors, which are obtained from the experimental results and frequency response functions [9]. MAC is a statistical measurement between the estimated modal vector and real modal vector. The value of 0 to 1 is the interval of the MAC value. If the MAC value is 1, the two vectors show consistency, and the consistency will decrease until the MAC value is 0, which mean the two vectors are not consistent. In this paper, the MAC value was modified as follows:

\[ MAC_i = \frac{\left| \phi_{FE}^T | \phi_{FDD}^T \right|^2}{\left| \phi_{FE}^T \right| \left| \phi_{FDD}^T \right|^2} \]  

(2)
where $\{\phi_{FE}\}_i$ is the $i$-th mode shape obtained from finite element method and $\{\phi_{FDD}\}_i$ is the $i$-th mode shape obtained from the frequency domain decomposition. If the MAC value is 1, the mode shape between finite element and frequency domain decomposition is identical, otherwise if the MAC value is 0, there is no correlation between mode shapes obtained by finite element and frequency domain decomposition.

5. Applications

5.1. Application 1: one story plane frame with 6-DOF

Figure 1 shows a one-story plane frame to be considered as an example, similar to the one in [3]. The total number of nodes is 4, with six degrees of freedom. Table 1 shows the material and section properties of this plane frame structure. Due to the modest number of degrees of freedom in this plane (only 6 degrees of freedom), all degree of freedom were considered for the analysis, as shown in figure 2.

![Figure 1. One story plane frame.](image)

![Table 1. Material and section property of plane frame.](table)

![Figure 2. Six degrees of freedom.](image)
Table 2 shows the natural frequencies and mass participating ratios obtained by the finite element method, while table 3 shows six mode shape of the structure. A white noise signal was used to obtain the responses of the structure. The frequency sample was taken as 333 Hz, where the acceleration response was taken as the response of the structure. Figure 3 shows the first singular value of the spectral power density matrix. Based on figure 3 and table 4, the first three natural frequencies are 4.2252 Hz, 17.8757 Hz, and 44.2668 Hz, respectively.

**Table 2.** Natural frequency and mass-participating factor of the first application obtained using finite element method.

| Mode | \( f \) (Hz) | \( \omega \) (rad/s) | Mass Participating Factor (%) |
|------|--------------|---------------------|------------------------------|
| 1    | 4.2013       | 26.3973             | 20.7566                      |
| 2    | 14.3613      | 90.2349             | 0.0000                       |
| 3    | 18.0283      | 113.2753            | 45.8684                      |
| 4    | 42.6251      | 267.8214            | 0.0000                       |
| 5    | 43.1300      | 270.9935            | 33.3333                      |
| 6    | 43.3577      | 272.4243            | 0.0416                       |

**Table 3.** Mode shape of the first application obtained using finite element method.

| DOF | Mode Shape (\( \phi \)) |
|-----|--------------------------|
| 1   | \(-1.0000\) \(0.0361\) \(-0.1961\) \(-1.0000\) \(0.0000\) \(0.0011\) |
| 2   | \(-0.0056\) \(0.0000\) \(0.0345\) \(0.0000\) \(-1.0000\) \(1.0000\) |
| 3   | \(0.1959\) \(-1.0000\) \(-1.0000\) \(-0.0361\) \(0.0000\) \(0.0343\) |
| 4   | \(-1.0000\) \(-0.0361\) \(-0.1961\) \(1.0000\) \(0.0000\) \(0.0011\) |
| 5   | \(0.0056\) \(0.0000\) \(-0.0345\) \(0.0000\) \(-1.0000\) \(-1.0000\) |
| 6   | \(0.1959\) \(1.0000\) \(-1.0000\) \(0.0361\) \(0.0000\) \(0.0343\) |

**Figure 3.** First singular value of the spectral power density matrix for the first application.
Table 4. The natural frequency of the first application obtained using FDD.

| Mode | f (Hz)  | ω (rad/s) |
|------|---------|-----------|
| 1    | 4.2252  | 26.5477   |
| 3    | 17.8757 | 112.3163  |
| 5    | 44.2668 | 278.1365  |

Note that each natural frequency corresponds to a particular mode shape in table 5. From the results of table 2, the second, fourth, and sixth mode were not considered, because all mass participating ratios are equal to zero. Therefore, only the first, third, and fifth modes were obtained. Table 5 shows the mode shapes of the first, third, and fifth modes obtained by using FDD.

Table 5. Mode shape of first application obtained using frequency domain decomposition.

| DOF | Mode Shape (φ) |
|-----|----------------|
|     | 1  | 3  | 5  |
| 1   | -1.0000 | -0.1970 | -0.03506 |
| 2   | 0.0006  | -0.0016 | -1.0000   |
| 3   | 0.2041  | -1.0000 | -0.14933  |
| 4   | -1.0000 | -0.1970 | -0.03506  |
| 5   | 0.0120  | -0.0681 | -0.94818  |
| 6   | 0.2041  | -1.0000 | -0.14933  |

MAC was applied to see the correlation between the results of predicted mode shape (FDD) and finite element. Table 6 shows the MAC value of each mode. All of the MAC values are almost equal to 1, which means there is a strong correlation between mode shapes obtained by finite element and FDD. The mode shapes obtained by finite element and FDD are identical.

Table 6. MAC value between the finite element and frequency domain decomposition of the first application.

| Mode | MAC |
|------|-----|
| 1    | 0.9999 |
| 3    | 0.9988 |
| 5    | 0.9751 |

The next step is to use NExT to predict the assumed damping ratio. Table 7 shows the damping ratio obtained by FDD and the assumed damping ratio. All of the assumed damping ratios match with the predicted damping ratio using NExT.
Table 7. Assumption and the predicted damping ratio of the first application using NExT.

| Mode | $\zeta_s$ | $\zeta_{NExT}$ |
|------|----------|----------------|
| 1    | 0.05     | 0.0474         |
| 3    | 0.05     | 0.0419         |
| 5    | 0.05     | 0.0450         |

5.2. Application 2: four-story plane frame with 48-DOF

The second case is a four-story plane frame with 48-DOF. Figure 4 shows the structure’s geometry. The elastic modulus of the material is similar to the first case that is $23.5 \times 10^6$ kN/m$^2$ but with different member sections. For columns, the section area is 0.16 m$^2$. For beams, the section area is 0.18 m$^2$. Because only vertical and horizontal acceleration responses were assumed to be recorded, only the vertical and horizontal degrees of freedom were considered. In this case, the static condensation technique was used to condense out the degree of freedom.

Table 8 shows the participating mass ratio and the comparison between FE and FDD in frequency and natural frequency of the second application. There are 32 modes recorded, which are the same as the number of DOFs. However, only the first three modes that have the highest modal participating ratio were considered.

![Figure 4. Four story plane frame.](image)
Figure 5. First singular value of power spectral density matrix for the second application.

Due to the space limitations, only the first and second mode shapes are shown. Figure 6 shows the comparison between FE and FDD for the first and second mode shapes. The MAC values were presented in table 9.

![Figure 6](image)

(a) First mode shape of the first case, (b) Second mode shape of the second case.

According to table 9, the MAC values of the first and second mode are 0.9997 and 0.9944, respectively, but The MAC value of the third mode is 0.2422. It means that FDD can accurately predict the mode shapes with high participating mass ratios and incorrectly predicts modes with small participating mass ratios. For the damping ratios, table 10 shows the comparison between the assumed damping ratio and the obtained damping ratio that uses NExT. The result shows that there is no significant difference between the assumed damping ratio and the obtained damping ratio that uses NExT.
6. Results and discussion

Based on the result obtained on the first application and the second application, the FDD-NExT was successfully applied for determining the modal parameters of both frames. The FDD-NExT is a combination of two techniques, which are frequency domain decomposition (FDD) and natural excitation technique (NExT) that were sequentially applied. On the first stage, FDD is used to determine the natural frequency and mode shapes of plane frames. The second stage after that is applying NExT to obtain the damping ratio of the plane frames. On the first application, due to the less degree of freedom on the first application (6-DoF), all acceleration response of structure during the white noise signal was considered as the output response of the structure, this output was used to determine the modal parameters of the plane frame. It means that the vertical translation motion, horizontal translation motion, and rotational motion were also considered. The result shows that FDD can only predict modes with high participating mass ratio. On the second application, due to the high number of degrees of freedom (48-DoF), only the translational degrees of freedom (vertical and horizontal degrees of freedom) were considered. Hence, the rotational motions of plane frames were not recorded. Due to this assumption, the static condensation technique was used to condense the rotational degrees of freedom so that only the translation degrees of freedom remained. Moreover, this assumption, which was used to memorize the response of rotational degrees of freedom is more difficult to obtain than the response of translational degrees of freedom in a real structure. According to first and second application results, the FDD-NExT can be applied on structures to predict the modal parameters with high participating mass ratio. This is similar to the result from [3] in which the FDD was applied in the plane truss structure and can only accurately predict modal parameters with high participating mass ratio.

All degree of freedom had been considered (translational and rotational) in both applications. In the first application, it showed that the result of modal parameters between FDD and finite element demonstrated no significant difference. In the second application, the modal parameters, which was obtained via FDD-NExT showed a proper alignment with the obtained modal parameters using the finite element method, although the rotational degrees of freedom were condensed. It means that the static condensation technique was successfully applied in the second application. Furthermore, this technique can save computational time for obtaining the result.

7. Conclusions

This paper presents the application of FDD-NExT to predict the modal parameter of plane frames such as natural frequencies, mode shapes, and damping ratios of plane frame structures by investigating two structures with different configurations. Based on the result, the accuracy of FDD-NExT to predict the modal parameters depends on the participating mass ratio of structures. FDD-NExT can accurately

| Mode | MAC |
|------|-----|
| 1    | 0.9997 |
| 2    | 0.9944 |
| 3    | 0.2422 |

| Mode | $\zeta_s$ | $\zeta_{\text{NExT}}$ |
|------|---------|--------------------|
| 1    | 0.05    | 0.0542             |
| 2    | 0.05    | 0.049              |
predict the modal parameters of mode with high mass participating ratio. However, its ability to predict will decrease with the decreasing participating mass ratio.

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