Supplementary material for rejoinder to discussion of the paper "Human life is unlimited – but short"

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An updated version of LATool, the MATLAB toolbox for life length analysis, is available as electronic supplementary material. In the updated version, the estimate of the mean of the exponential distribution of excess lifelengths was 1.42 with 95% confidence interval (1.28, 1.56). Hence, the probability to survive one more year for a supercentenarian was estimated to be 0.50 with 95% confidence interval (0.46, 0.53). The rest of this document contains updated versions of Figure 5, left, and of Tables 2-5 in Rootzén and Zholud (2017).

![Figure 5](image_url)

**Fig. 5** Illustration of effect of size-biased sampling, left plot: Empirical quantiles for IDL supercentenarians who died 1980-1999 in USA plotted against quantiles of exponential distribution with estimated mean (red line, with observations in blue). To illustrate the statistical variation of such plots, we have simulated 100 data sets with the estimated mean for the US 1980-1999 data, and with the same 110 year birthdates and truncations, and inserted the empirical quantile lines (dotted) for these data sets in the plot.

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Table 2  p-values for likelihood ratio tests of the null hypothesis of no difference in mortality between women and men after age 110. Middle column assumes GP distribution, right column exponential distribution. There are only 5 men in the North Europe data set, and N/A means that GP fitting didn’t succeed for them. A "-" means that the test for exponential distribution of data rejects at the 0.001% level for the GRG data set.

| ID             | GP       | exp |
|----------------|----------|-----|
| North Europe   | N/A      | 0.81|
| South Europe   | 0.18     | 0.45|
| Europe         | 0.57     | 0.53|
| North America  | 0.32     | 0.61|
| Japan          | 0.19     | 0.61|
| Europe&America | 0.29     | 0.41|
| World          | 0.26     | 0.33|
| GRG            | 0.06     | -   |

Table 3  p-values for Wald tests of the null hypothesis of no difference in mortality between first and last half of data. "mean" is the estimated parameter of the exponential distribution, not the mean of the observed life lengths; tests assume an exponential distribution.

| ID             | death date | #  | mean  | death date | #  | mean  | p-value |
|----------------|------------|----|-------|------------|----|-------|---------|
| North Europe   | 1968-1997  | 41 | 1.74  | 1998-2006  | 43 | 1.47  | 0.56    |
| South Europe   | 1973-1997  | 50 | 1.62  | 1998-2007  | 64 | 1.19  | 0.25    |
| Europe         | 1968-1997  | 91 | 1.68  | 1998-2007  | 107| 1.30  | 0.18    |
| North America  | 1962-1992  | 141| 1.17  | 1993-2002  | 159| 1.38  | 0.26    |
| Japan          | 1996-2000  | 28 | 1.69  | 2001-2005  | 36 | 2.51  | 0.58    |
| Europe&America | 1962-1994  | 247| 1.32  | 1995-2008  | 255| 1.50  | 0.28    |
| World          | 1962-1996  | 309| 1.30  | 1997-2008  | 257| 1.48  | 0.28    |

Table 4  p-values for likelihood ratio tests of the null hypothesis of no difference in mortality between groups of countries. Middle column assumes GP distribution, right column exponential distribution.

| ID             | GP   | exp |
|----------------|------|-----|
| North Europe vs south Europe | 0.09 | 0.49|
| Europe vs north America       | 0.76 | 0.58|
| Europe&America vs Japan       | 0.09 | 0.46|

Table 5  Estimates of shape parameter $\gamma$ of the generalized Pareto distribution, with 95% asymptotic confidence intervals in parentheses, and p-values for likelihood ratio test of the null hypothesis that data follows an exponential distribution.

| ID             | shape parameter | p-value |
|----------------|-----------------|---------|
| North Europe   | -0.18 (-0.44, 0.08) | 0.25    |
| South Europe   | 0.22 (-0.10, 0.54)  | 0.05    |
| Europe         | 0.09 (-0.10, 0.28)  | 0.27    |
| North America  | 0.10 (-0.08, 0.28)  | 0.23    |
| Europe&America | 0.09 (-0.07, 0.16)  | 0.10    |
| Japan          | -0.38 (-0.79, 0.02) | 0.11    |
| World          | 0.07 (-0.05, 0.19)  | 0.20    |
References

Rootzén, H. and D. Zholud (2017). Human life is unlimited – but short. *Extremes* 20(4), 713–728.