QUINTESSENCE, COSMOLOGY, AND FANAROFF-RILEY TYPE IIB RADIO GALAXIES

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ABSTRACT

Fanaroff-Riley type Iib (FR Iib) radio galaxies provide a modified standard yardstick that allows constraints to be placed on global cosmological parameters. This modified standard yardstick is analogous to the modified standard candle provided by Type Ia supernovae. The radio galaxy and supernova methods provide a measure of the coordinate distance to high-redshift sources, and the coordinate distance is a function of global cosmological parameters. A sample of 20 FR Iib radio galaxies with redshifts between 0 and 2 are compared with the parent population of 70 radio galaxies to determine the coordinate distance to each source. The coordinate-distance determinations are used to constrain the current mean mass-energy density of quintessence \( \Omega_Q \), the equation of state of the quintessence \( w \), and the current mean mass-energy density of nonrelativistic matter \( \Omega_m \). Zero space curvature is assumed. Radio galaxies alone indicate that the universe is currently accelerating in its expansion (with 84% confidence); most of the allowed parameter space falls within the accelerating universe region on the \( \Omega_m w \) plane. This provides verification of the acceleration of the universe indicated by high-redshift supernovae and suggests that neither method is plagued by systematic errors. It is found that \( \Omega_m \) must be less than about 0.5 and the equation of state \( w \) of the quintessence must lie between \(-0.25\) and \(-2.5\) at about 90% confidence. Fits of the radio galaxy data constrain the model parameter \( \beta \), which describes a relation between the beam power of the active galactic nucleus (AGN) and the total energy expelled through large-scale jets. It is shown that the empirically determined model parameter is consistent with models in which the outflow results from the electromagnetic extraction of rotational energy from the central compact object. A specific relation between the strength of the magnetic field near the AGN and the spin angular momentum per unit mass of the central compact object is predicted.

Key words: cosmological parameters — cosmology: observations — cosmology: theory — dark matter — galaxies: active

1. INTRODUCTION

There are several independent ways to determine the global cosmological parameters that describe the current state of the universe. It is important to have several complementary and independent methods that yield consistent results since any given method could be plagued by unknown systematic errors. A particularly useful way to determine global cosmological parameters is through measurements of the coordinate distance to high-redshift sources. This method is particularly useful because the coordinate distance depends only on global cosmological parameters and is independent of the clustering properties of the mass-energy components that control the expansion rate of the universe, as long as each component is homogeneous and isotropic on very large scales. The coordinate distance is also independent of whether different components cluster differently, known as biasing, and of whether the dark matter is cold, warm, or hot (although it does depend on the equation of state of each component).

Two cosmological tools that are particularly sensitive to the coordinate distance as a function of redshift are Fanaroff-Riley type Iib (FR Iib) radio galaxies (a subset of FR II sources defined by Fanaroff & Riley 1974), which provide a modified standard yardstick (Daly 1994; Daly & Guerra 2002a, Guerra 1997; Guerra & Daly 1998; Guerra, Daly, & Wan 2000, hereafter GDW00), and Type Ia supernovae, which provide a modified standard candle (see, e.g., Riess et al. 1998; Perlmutter et al. 1999). Some aspects of the methods are compared in § 4.

The coordinate distance to a source at a given redshift depends on the present value of the mean mass-energy density of each component and on the redshift evolution of the mass-energy density of each component. Nonrelativistic matter, including baryonic matter and clustered dark matter, has a mean mass-energy density that evolves as \((1 + z)^3\). There is a contribution from radiation and neutrinos left over from the big bang, currently negligible, which has an energy density that evolves as \((1 + z)^4\). There may also be a cosmological constant, with constant energy density, and so it evolves as \((1 + z)^0\).

The redshift evolution of the mean mass-energy density of a component depends on the equation of state \( w = P/\rho \) of the component. There could exist a dynamical vacuum energy density called quintessence (Caldwell, Dave, & Steinhardt 1998). Quintessence would have an equation of state \( w \) such that the density evolves as \((1 + z)^n\), where \( n = 3(w + 1) \) (see Turner & White 1997; Bludman & Roos 2001; Wang et al. 2000; or the Appendix). For example, pressureless dust (i.e., nonrelativistic matter) is described by \( P = w = 0 \) and \( n = 3 \); a relativistic fluid is described by \( w = \frac{1}{3} \) and \( n = 4 \), and a cosmological constant is described by \( w = -1 \) and \( n = 0 \).
Two components of the universe are known to exist with certainty: there is a nonrelativistic component (including baryons and clustered dark matter), with zero-redshift, normalized mean mass-energy density $\Omega_m$, and a primordial relativistic component that is negligible for the epochs of interest here (including microwave background photons and neutrinos).

There must be at least one more component, which remains to be determined. This third component could be a cosmological constant, quintessence, space curvature, or something else. In the simplest case, there is only one additional component that is significant and has a measurable impact over the redshift interval where the coordinate distance is used to constrain cosmological parameters and the properties of the unknown component.

Here the determination of the coordinate distances to 20 radio galaxies with redshifts between 0 and 2 are used to constrain the equation of state and current mean mass-energy density of quintessence assuming a spatially flat universe as indicated by measurements of the microwave background radiation (de Bernardis et al. 2000; Balbi et al. 2000).

The radio galaxy method is briefly reviewed in § 2. Constraints on cosmological parameters and the equation of state of quintessence obtained using FR IIb radio galaxies as a modified standard yardstick are presented in § 3. The supernova and radio galaxy methods and results are compared in detail in § 4. The relation between beam power and total energy for FR IIb radio jets is discussed in § 5. Here it is shown that the underlying hypothesis of the radio galaxy method is consistent with current models of large-scale jet production in active galactic nuclei (AGNs). The results are summarized in § 6.

2. COSMOLOGY WITH FR IIb RADIO GALAXIES

The use of FR IIb radio galaxies as a modified standard yardstick to determine global cosmological parameters has been presented in detail elsewhere (see Daly 1994; Daly & Guerra 2002a; Guerra & Daly 1998; GDW00). Radio images of the 20 sources used here can be found in Leahy, Muxlow, & Stephens (1989), Liu, Pooley, & Riley (1992), and GDW00. The radio galaxy method is based on the following premises: First, it is assumed that the forward region of the radio bridge of an FR IIb source represents a strong shock front, and the equations of strong-shock physics are applied at this boundary. Second, it is assumed that there is a power-law relation between the average beam power of the source $L_j$ and the total lifetime of the outflow $t*$: $t* \propto L_j^{3/5}$ (see § 5). Third, it is assumed that at a given redshift, the sources studied will have an average source size similar to that of the parent population of FR IIb sources at the same redshift, where the source size is defined as the separation between the radio hot spots. All of these points are discussed in detail in previous work (e.g., Daly 2002; Daly & Guerra 2002a; GDW00).

In addition, in its simplest interpretation the model works in a straightforward way if the radio power of a given FR IIb radio galaxy does not decrease monotonically and measurably with time. There is substantial evidence that this is the case for radio galaxies that fall into the FR IIb category, as do all of the radio galaxies considered here (Daly 2002a). For example, Neeser et al. (1995) find that there is no correlation between radio size and radio power. The low-frequency radio surface brightness of these sources can be explained using adiabatic expansion alone if the radio powers of the sources are roughly time-independent (Wellman, Daly, & Wan 1997). Gopal-Krishna, Kulkarni, & Wiita (1996) find that a roughly constant radio power followed by an exponential drop in radio power would explain the relative sizes of radio galaxies and radio-loud quasars in the context of the orientation unified model, and it is shown in § 5 that the beam power is independent of core–hot-spot separation, which would not be expected if the radio power of a given source decreases with time. Thus, there is no empirical evidence that the radio power of a given FR IIb radio galaxy decreases with time and substantial evidence that it is roughly constant over the lifetime of a given source, followed by a precipitous drop. The Mach numbers of FR IIb radio galaxies are about 2.5–10 (Wellman et al. 1997), implying overpressures of 10–200, so the lobe will rapidly expand and drop in radio power when the beam power shuts off, as suggested by Neeser et al. (1995). Some authors (e.g., Blundell, Rawlings, & Wilott 1999) have studied large heterogeneous samples that include sources with all types of radio structure and power and both radio-loud quasars and radio galaxies. To explain the properties of this heterogeneous sample, a model is proposed that allows for the decrease in the radio power of a given source as a function of time. There is, however, no evidence that this applies to the select subset of sources (the most powerful FR II radio galaxies) studied here and substantial evidence that the radio powers of FR IIb radio galaxies do not monotonically decline over the lifetime of a given source.

Since FR IIb radio galaxies form a remarkably homogeneous population and all the sources at a given redshift are quite similar, the average size of a given source $D_{\ast}$ will equal the average size of the full population $\langle D \rangle$ of FR IIb radio galaxies with similar redshifts. The dependence of $D_{\ast}$ and $\langle D \rangle$ on the coordinate distance, and hence on cosmological parameters, is quite different, and

$$\frac{\langle D \rangle}{D_{\ast}} \propto (\alpha_0 r)^{2/3+3/7},$$

(1)

since $\langle D \rangle \propto (\alpha_0 r)$ (see Guerra & Daly 1998, eq. [6]). By construction, the ratio $\langle D \rangle / D_{\ast}$ should be a constant, independent of redshift. Because the model is based on a comparison of individual source characteristics with the characteristics of the parent population at the same redshift, selection effects, such as radio power selection effects, are unlikely to affect the constraints placed on cosmological parameters. All of the sources for which values of $D_{\ast}$ are determined belong to the parent population. The sources for which $D_{\ast}$ is determined are subject to selection effects that are identical to those of the parent population, used to determine $\langle D \rangle$. Thus, this method is one of the few methods that is unlikely to be affected by observational selection effects. Equation (1) allows constraints to be placed on the model parameter $\beta$ and the cosmological parameters that enter into the determination of the coordinate distance.

3. CONSTRAINTS ON QUINTESSENCE FROM RADIO GALAXIES

Recent measurements of the cosmic microwave background anisotropy suggest that the universe has zero space curvature (de Bernardis et al. 2000; Balbi et al. 2000). Here a universe with zero space curvature ($k = 0$) and quintessence is considered. The coordinate distance to a source at redshift
z in a spatially flat universe is

\[
\langle a r \rangle = H_0^{-1} \int_0^z \frac{dz}{E(z)}, \tag{2}
\]

where \( E(z) = \left[ \Omega_m (1 + z)^3 + (1 - \Omega_m)(1 + z)^n \right]^{1/2} \). The coordinate distance is obtained by substituting this into equation (2) and solving for \( \langle a r \rangle \) (see, e.g., Turner & White 1997; Bludman & Roos 2001; Appendix).

The equation of state \( w \) of quintessence is assumed to be time-independent over the redshift interval from 0 to 2 (Wang et al. 2000); radio galaxy constraints on an evolving scalar field modes are discussed by Podariu et al. (2002). Thus, the mean mass-energy density of quintessence will evolve as \((1 + z)^w\), where \( w = \frac{1}{3} - 1 \). For a present value of the normalized mass-energy density of quintessence \( \Omega_Q \) and that of nonrelativistic matter \( \Omega_m \), we have \( \Omega_Q + \Omega_m = 1 \) at \( z = 0 \) since space curvature is taken to be zero. This implicitly assumes that there is only one unidentified component, \( \Omega_Q \), that contributes significantly over the redshift interval where the coordinate distance will be determined.

The minimization of the \( \chi^2 \) of \( (D)/D_* = \text{const} \) leads to constraints on \( \Omega_m, w, \) and the model parameter \( \beta \). The one- and two-dimensional confidence intervals have been determined in the usual way (see, e.g., Press et al. 1992).

Figure 1 shows the 68%, 90%, and 95% one-dimensional confidence contours in the \( \Omega_m, w \) plane. At 90% confidence, \( \Omega_m \) is less than about 0.5 for any value of \( w \). A cosmological constant, represented by \( w = -1 \), is clearly consistent with the radio data. This figure differs slightly from the two-dimensional confidence intervals presented in Figure 2 of Daly & Guerra (2002b). At 68% confidence, \( \Omega_m < 0.25 \) and \(-1.3 < w < -0.45 \). At 90% confidence, \( \Omega_m < 0.5 \) and \(-2.6 < w < -0.25 \). Note that some models for the dark energy invoke mechanisms that behave like quintessence with \( w \) less than \(-1 \) (see, e.g., Frampton 2002).

The 54 supernovae identified by Perlmutter et al. (1999) and used to obtain their “primary fit” have been similarly fitted and imply strong constraints on \( \Omega_m \), although the supernovae do not place a lower bound on \( w \), as shown in the second panel of Figure 2.

As described by Turner & White (1997) and presented in some detail in the Appendix, the expansion rate of the universe is accelerating at the present epoch if

\[
1 + 3w(1 - \Omega_m) < 0. \tag{3}
\]

This line is drawn on the \( \Omega_m, w \) plane (see Figs. 1 and 2); points below the line represent solutions for which the universe is currently accelerating, while those above the line represent solutions for which the universe is currently decelerating. Radio galaxies alone (Fig. 1) indicate that the expansion of the universe is accelerating (\( q_0 < 0 \)); this is an 84% confidence result.

It is very important to consider whether significant covariance exists between the different parameters deter-
mined by the fit. Figure 3 shows the 68%, 95%, and 99% one-dimensional confidence contours in the $w$-$\beta$ plane. Clearly, there is no covariance between the equation of state $w$ of quintessence and the model parameter $\beta$: $w$ and $\beta$ are independent.

Figure 4 shows the 68%, 95%, and 99% one-dimensional confidence contours in the $\Omega_m$-$\beta$ plane. Clearly, the two parameters are independent. The data suggest that $\beta \approx 1.75 \pm 0.25$ independent of the cosmological parameters $\Omega_m$ and $\Omega_Q$ or the equation of state $w$ of quintessence.

4. COMPARISON OF SUPERNOVA AND RADIO GALAXY METHODS

The observation of the acceleration of the universe through measurements of the coordinate distance to high-redshift sources is so important that it needs to be independently verified using different methods. In $\S$ 3, it was shown that the radio galaxy method alone indicates at about 84% confidence that the universe is accelerating in its expansion at the present epoch if the universe has zero space curvature. The supernova teams report similar results at stronger confidence levels (Riess et al. 1998; Perlmutter et al. 1999), as illustrated in Figure 2. It is important to consider the similarities and differences between the methods to ensure that the results of each are independent and to compare the quantities that are actually determined in each method. The two methods are compared in Table 1.

The two methods are similar in their dependence on the coordinate distance and in the sense that both provide a modified standard rather than an absolute standard. They are somewhat similar in their redshift interval of coverage and in the number of sources studied. The radio galaxy and supernova methods differ in the way the "modification" relation is derived; one is empirical, and one is based on physical arguments. They also differ in that the radio galaxy method is not normalized at zero redshift and is independent of the properties of local sources and of the local distance scale; in fact, nearly identical results are obtained.

| Parameter                        | Supernovae                                      | Radio Galaxies                      |
|----------------------------------|------------------------------------------------|-------------------------------------|
| Source population                | Type Ia                                        | Type Ib                             |
| Dependence on coordinate distance| $\propto (\theta_J)^2.0$                       | $\propto (\theta_J)^1.6$            |
| Redshift coverage                | $0 < z < 1$                                    | $0 < z < 2$                         |
| Sources                          | ~100                                           | 20 (70 in parent population)        |
| Type of standard                 | Modified standard candle                       | Modified standard yardstick         |
| Approach                         | Light curve implies peak luminosity            | Radio bridge implies average length  |
| Type of relation                 | Empirical                                      | Physical                            |
| Written in terms of              | Observables                                    | Physical variables                  |
| Normalization                    | Model parameter determined at $z = 0$         | Model parameter not determined at $z = 0$ |
| Dependencies                     | May depend on local distance scale             | Independent of local distance scale |
| Result                           | Universe is accelerating                       | $\Omega_m$ is low; universe is accelerating if $k = 0$ |
| Theoretical understanding        | Some                                           | Good                                |
| Empirical state                  | Well tested                                    | Needs more testing                  |
4. Comparison Allowing for Space Curvature

It is interesting to compare the constraints placed on cosmological parameters assuming that three main terms control the expansion of the universe at the present epoch: nonrelativistic matter with contribution $\Omega_m$, a cosmological constant with contribution $\Omega_{\Lambda}$, and space curvature.

Figures 5 and 6 show constraints placed by the microwave background radiation (Bond et al. 2002) and the high-redshift supernovae project (Riess et al. 1998); the results obtained by the Supernova Cosmology Project (Perlmutter et al. 1999) are similar and are presented in Daly & Guerra (2002b). Each method carves out a different part of parameter space because each method covers a different redshift range, with the supernovae being at the lowest redshift, the radio galaxies at higher redshift, and the microwave background radiation at still higher redshift. The three methods are thus complementary in their redshift coverage.

The radio galaxy and supernova methods are independent of the initial power spectrum of density fluctuations and, hence, are independent of the index of the primordial fluctuation spectrum. These methods are also independent of the Hubble constant and of the baryon fraction. The radio galaxy and supernova methods depend only on the global cosmological parameters $\Omega_m$ and $\Omega_{\Lambda}$, allowing for space curvature, or $\Omega_m$ and $w$ allowing for quintessence, where $\Omega_m = 1 - \Omega_{\Lambda}$ and assuming zero space curvature. They are therefore particularly important to study since most other methods depend on several other parameters, such as the initial spectrum of fluctuations, the Hubble constant, and/or the baryon fraction.

5. Lifetime, Beam Power, and Energy in FR IIb Radio Galaxies

The application of the radio galaxy cosmology method indicates that $\beta \simeq 1.75 \pm 0.25$ irrespective of cosmological model, as discussed here for quintessence, by GDW00 in cosmologies that include space curvature, and summarized in Daly & Guerra (2002a). The total energy released over the lifetime of the FR IIb in the form of a highly collimated outflow is $E_* = L_{j,*}$ assuming that $L_j$ is roughly constant over the lifetime of a given source, which is supported by lack of correlation between $L_j$ and the hot-spot–to–core distances in these sources (see Wan, Daly, & Guerra 2000; Guerra & Daly 2001; Fig. 7). Values of $L_j$ and $t_*$ are obtained independently, using different aspects of the radio data for a given source. Two values are determined for each source, one on each side of each source. An indicator of possible problems with the model assumptions or selection biases is whether the beam powers, total source lifetimes, or total energies depend strongly on the distance of the radio hot spot from the location of the AGN; no such problems are indicated. Each of these quantities is plotted as a function of hot-spot distance from the AGN in Figures 7, 8, and 9. There are 40 data points on each figure, two for each of the 20 radio galaxies studied here.

It is interesting to note that the total energy processed through large-scale jets has a rather small range, from roughly a few times $10^7 M_\odot$ to roughly a few times $10^6 M_\odot$, although it does increase systematically with redshift, as shown in Figure 10.

Because $t_* \propto L_{j,-3/2}$ and $E_* = L_{j,t_*}$, the relation between beam power and energy released by an FR IIb is $L_j \propto E_*^{2/3-\beta} \propto E_*^{2/3}$, where $q = 2 - 3$ for $\beta = 1.5 - 2.0$. In terms of the total lifetime of the source, $t_*$ is proportional to $L_{j,-1/2}$, $L_{j,-7/12}$, and $L_{j,-2/3}$ for values of $\beta$ of 1.5, 1.75, and 2.0, respectively. Clearly, $\beta$ remains far from the limiting cases of $\beta = 0$ or $\beta = 3$ (see Figs. 3 and 4).
5.1. Electromagnetic Energy Extraction from Rotating Holes

Blandford (1990) gives a summary of the energy and beam power of outflows associated with the electromagnetic extraction of the spin energy of a rotating black hole. The beam power and total energy available are

\[ L_j = L_{EM} \sim 10^{45} \left( \frac{a}{m} \right)^2 B_4^2 M_8^2 \text{ ergs s}^{-1} \propto \left( \frac{a}{m} \right)^2 B^2 M^2, \]

(4)

\[ E_* = E \sim 5 \times 10^{61} \left( \frac{a}{m} \right)^2 M_8 \text{ ergs s}^{-1} \propto \left( \frac{a}{m} \right)^2 M \]

(5)

for \( a/m \ll 1 \), where \( M \) is the mass of the black hole, \( M_8 \) is the mass in units of \( 10^8 M_\odot \), \( a \) is the spin angular momentum \( S \) per unit mass \( M [a = S/(Mc)] \), with \( c \) the speed of light, \( m \) is the gravitational radius \( m = GM/c^2 \), \( B \) is the magnetic field strength, and \( B_4 \) is the magnetic field strength in units of \( 10^4 \text{ G} \) (Blandford 1990).

Empirically, \( E_* \propto L_j^{1-\beta/3} \) with \( \beta = 1.75 \pm 0.25 \). This is consistent with equations (4) and (5) when the magnetic field strength satisfies

\[ B \propto M^{(2/3-1/2)(3-\beta)} \left( \frac{a}{m} \right)^{\beta/(3-\beta)}. \]

This is particularly simple when \( \beta = 1.5 \); in this case, \( B \propto a/m \). For the cases \( \beta = 1.75 \) and 2, the empirical constraint is consistent with equations (4) and (5) when \( B \propto (a/m)^{1.4} M^{0.2} \) and \( B \propto (a/m)^{3/2} M^{1/2} \), respectively.

Typical values for the beam power, total energy processed by a jet, and lifetime are \( L_j \sim 10^{45} \text{ ergs s}^{-1}, E_* \sim 5 \times 10^{61} c^2 M_8 \), and \( t_* \sim 10^7 \text{ yr} \) (see Figs. 7, 8, and 9). Equations (4)}
and (5) imply a total source lifetime of $t_s \sim 10^9/(m_b B_4^2)$, indicating a lifetime of about $10^7$ yr for $m_b B_4^2 \sim 10^2$. This is satisfied for $m_b \sim 10$ and $B_4 \sim 3$. In this case, the beam power is $\sim 10^{45}$ ergs s$^{-1}$ for $a/m \sim 1/30$, and the total energy is $\sim 5 \times 10^5 c^2 M_8$. These values for $m_b$, $B_4$, and $a/m$ seem quite reasonable. The scaling between variables required by the empirically determine relation between total energy and beam power are discussed above.

6. CONCLUSION

Type IIb radio galaxies and Type Ia supernovae are particularly important methods to develop to constrain the global cosmological parameters $\Omega_m$, $\Omega_\Lambda$, and space curvature, or $\Omega_m$, $\Omega_Q$, and the equation of state of quintessence $w$. These methods depend only on the properties of global cosmological parameters and are independent of other factors such as the index of the primordial power spectrum, the Hubble constant, the baryon fraction, the properties of the dark matter that clusters around galaxies and clusters of galaxies, or any biasing of dark relative to luminous matter. The radio galaxy and supernova methods are completely independent and have completely different potential systematic errors, as discussed in § 4.

Radio galaxies may be used to constrain the mass-energy density and equation of state of quintessence. These results are presented here assuming a spatially flat universe, which is supported by recent measurements of fluctuations of the cosmic microwave background. Radio galaxies alone suggest that the universe is accelerating in its expansion at present (see § 3), consistent with results obtained by the supernova teams, indicating that both methods are working well and probably are not plagued by unknown systematic errors.

The implications for models of energy extraction for the cases of an outflow related to electromagnetic energy extraction of rotational energy are considered. If the outflow is produced by the electromagnetic extraction of energy from a rotating black hole, then the magnetic field strength must be related to the spin angular momentum of the rotating black hole $S$, the mass of the black hole $M$, and the gravitational radius of the black hole $m$, as described in § 5. The relation is particularly simple if $\beta = 1.5$ and implies that the magnetic field strength satisfies $B \propto a/m$, where $a = S/(Mc)$.

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APPENDIX

The deceleration parameter is defined to be $q_0 = -(\ddot{a}/a^2)$ evaluated at $z = 0$, where $a$ is the cosmic scale factor. This may also be written $q_0 = -\ddot{a}/(a_0 H_0^2)$, where quantities evaluated at $z = 0$ have a subscript zero and $H_0 = a_0/a_0$. It is straightforward to show that

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum (\rho_i + 3 p_i) = -\frac{4\pi G}{3} \sum \rho_i (1 + 3 w_i)$$

(e.g., Peebles 1993, eq. [5.15]), where $p_i$ is the pressure, $\rho_i$ is the mean mass-energy density, $w_i$ is the equation of state of the $i$th component, $w_i = p_i/\rho_i$, and the quintessence or cosmological-constant term is included in the summation.

For each component, mass-energy conservation implies

$$\dot{\rho}_i = -3(\rho_i + p_i) \left(\frac{\dot{a}}{a}\right)$$

(e.g., Peebles 1993, eq. [5.16]). By definition, the equation of state is $w_i = p_i/\rho_i$, so equation (A2) implies $\dot{\rho}_i/\rho_i = -3(1 + w_i)(\dot{a}/a)$. When the equation of state $w_i$ is time-independent, the solution to this equation is $\rho_i = \rho_i(1 + z)^{3(1 + w_i)}$, where $(1 + z) = a_0/a$. Thus, a com-
ponent with equation of state \( w_i \) and present mean mass-energy density \( \rho_0 \) will have a mean mass-energy density at redshift \( z \) of \( \rho = \rho_0 (1 + z)^{n_i} \), where \( n_i = 3(1 + w_i) \).

When \( k = 0 \),
\[
\left( \frac{a'}{a} \right)^2 = 8 \pi G \rho_0 \frac{3}{3} \sum \rho_i ,
\]
(A3)
where \( \rho_1 = \rho_{c,0} (1 + z)^{n_i} \). It follows that \( H_0^2 = (a_0^2/a^2) = (8 \pi G / 3) \sum \rho_i,0 \). For \( k = 0 \), \( \sum \rho_{i,0} = \rho_{c,0} \) where \( \rho_{c,0} \) is the critical density at redshift zero. Thus,
\[
H_0^2 = 8 \pi G \rho_{c,0} \quad \text{(A4)}
\]
for \( k = 0 \). The deceleration parameter \( q_0 = -a_0^2 (a_0 H_0^2) \) then becomes
\[
q_0 = -\sum \Omega_i (1 + 3w_i) , \quad \text{(A5)}
\]
where \( \Omega_i = \rho_{i,0} / \rho_{c,0} \) and equations (A1) and (A4) have been used.

The universe is accelerating rather than decelerating when \( q_0 < 0 \). This can only occur if \( 1 + 3w_i < 0 \), or \( w_i < -\frac{1}{3} \), which is a necessary but not sufficient condition to have an accelerating universe at the present epoch. If there are only two significant types of mass-energy controlling the expansion rate of the universe at the present epoch, quintessence and nonrelativistic matter, then the deceleration parameter is \( q_0 = \Omega_m + \Omega_{\text{w}} (1 + 3w) \). The universe will be accelerating in its expansion when
\[
1 + 3w (1 - \Omega_m) < 0 , \quad \text{(A6)}
\]
which follows since \( \Omega_Q = 1 - \Omega_m \). Thus, a curve can be drawn on the \( \Omega_m\text{-}w \) plane; points below the curve represent solutions for which the universe is currently decelerating. For example, this curve is shown in Figure 1, which illustrates the constraints obtained using radio galaxies alone. Note that radio galaxies alone place interesting constraints on \( \Omega_m \) and \( w \), and these results are consistent with those obtained using other methods (see, e.g., Wang et al. 2000).

The coordinate distance to a source at redshift \( z \) follows from the equation
\[
\int \frac{dr}{\sqrt{1 - kr^2}} = \int \frac{dt}{a(t)} = \frac{1}{a_0} \int \frac{1}{a} \left( \frac{a}{a_0} \right)^{-1} dz \quad \text{(A7)}
\]
(Weinberg 1972). For a spatially flat universe, the left-hand side of the equation reduces to the coordinate distance \( r \). Equations (A3) and (A4) imply that
\[
\left( \frac{a}{a_0} \right)^2 = H_0^2 \sum \Omega_i (1 + z)^{n_i} . \quad \text{(A8)}
\]

Following the notation of Peebles (1993), this reads
\[
a/a = H_0 E(z) , \quad \text{(A9)}
\]
Thus, the coordinate distance to a source at redshift \( z \) in a spatially flat universe is
\[
(a_0 r) = H_0^{-1} \int_0^z \frac{dz}{E(z)} . \quad \text{(A10)}
\]
(see eq. [A7]).

A spatially flat universe has \( \sum \Omega_i = 1 \). The components that must be included in equation (A9) are those that contribute from redshift zero out to the redshift that the coordinate distance is being determined. Two components are considered here: nonrelativistic mass with normalized mean mass-energy density at \( z = 0 \) of \( \Omega_m \) and quintessence with normalized mean mass-energy density at \( z = 0 \) of \( \Omega_Q \). Thus, \( \Omega_m + \Omega_Q = 1 \), and equation (A9) becomes
\[
E(z) = \frac{\Omega_m (1 + z)^3 + (1 - \Omega_m) (1 + z)^{n_i} / 2}{2} . \quad \text{(A11)}
\]

The coordinate distance is obtained by substituting this into equation (2) and solving for \( (a_0 r) \).

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