Expansion of the Weak Mixing Matrix

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Abstract

We perform a $1/m_b$ and $1/m_t$ expansion of the Cabibbo-Kobayashi- Maskawa mixing matrix. Data suggest that the dominant parts of the Yukawa couplings are factorizable into sets of numbers $|r>$, $|s>$, and $|s'>$, associated, respectively, with the left-handed doublets, the right-handed up singlets, and the right- handed down singlets. The first order expansion is consistent with Wolfenstein parameterization, which is an expansion in $\sin \theta_c$ to third order. The mixing matrix elements in the present approach are partitioned into factors determined by the relative orientations of $|r>$, $|s>$, and $|s'>$ and the dynamics provided by the subdominant mass matrices. A short discussion is given of some experimental support and a generalized Fritzsch model is used to contrast our approach.

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It is not an overstatement that a very perplexing and yet most challenging problem in particle physics lies in explaining the disparateness of fermion masses and in some of the off-diagonal elements of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. There has been a great amount of activities in this area. Generally speaking, it has become somewhat of an art form to postulate certain textures [1] in the quark mass matrices to explain these peculiarities. While this may well be a first step towards formulating some dynamical principle and to fathom a Nature’s deep secret, we would like to take a different approach here. Our emphasis is on the empirical fact of the strong decoupling of the third family from the first two in the CKM matrix, which will not be perceived as numerical accidents. We shall provide a short discussion of the relevant experimental evidence and analyse a generalized Fritzsch model for contrast in Appendices.

Our starting point is an attempt to separate out the large from the small at the very beginning. By this, we mean to work in a framework in which the mass of the top quark $m_t$ is $ab initio$ taken as the largest scale in a theory with three families. The mass of the bottom quark $m_b$ is also initially built into our analysis as the heaviest member of the down-type quarks [2]. For the CKM matrix, we shall regard the mixing of the first two families predominantly due to approximate mass degeneracy relative to $m_t$ and $m_b$. The magnitude of these elements is taken to be of order unity $O((1/m_t)^0, (1/m_b)^0)$. The lifting of degeneracy in $m_u \neq m_c$ and $m_d \neq m_s$ provides the dynamics for their eventual assumed values. In the following, we shall organize in a way which we perceive as natural for an expansion in the small parameters. We shall see that upon a conjecture to be made shortly, an expansion to first order in $m_b^{-1}$ and $m_t^{-1}$ will give the same accuracy for the CKM matrix as in Wolfenstein parameterization [3].

We can diagonalize, for example, the up-type quark mass matrix $M$ by a biunitary transformation [4]. Let $|\hat{r} >$ and $< \hat{s}|$ be, respectively, the normalized right and left eigenvector for the top quark. Then we write

$$M = \bar{m}_t |\hat{r} >< \hat{s}| + \epsilon M,$$

where $\epsilon M$ can be written as a sum of two terms pertaining to the up and the charm quark, which have the corresponding structure as the first term. However, we shall refrain from doing that, because unless we know what the mass matrix is, we would not know what these other eigenvectors are like in relation to $|\hat{r} >$. Fortunately, for our immediate purpose, we need to note only that $\epsilon M$ is relatively small. Similarly, for the down-type quark mass matrix $M'$, we pick out the right $|\hat{s}' >$ and left $< \hat{r}'|$ eigenvectors for the bottom quark and write

$$M' = m_b |\hat{r}' >< \hat{s}'| + \epsilon' M'.$$

We now make the following conjecture: $|\hat{r} >$ and $|\hat{r}' >$ are almost aligned, i.e. if we replace $|\hat{r}' >$ by $|\hat{r} >$ in $M'$, the difference, which will be put into $\epsilon' M'$, is small compared to $m_b$. Thus,

$$M' = \bar{m}_b |\hat{r} >< \hat{s}'| + \epsilon' M'.$$  (2)

In the above, $\epsilon$ and $\epsilon'$ are counting parameters, which will be set to unity after the count. As we shall see, this momentarily gives small off-diagonal matrix elements $V_{ts}$, $V_{td}$, $V_{ub}$, and $V_{cb}$.
One may forgo the presentation we just made, which is motivated from simple mathematical consideration. One takes note that the left-handed up and down type quarks belong to the same doublets. Our observation is equivalent to the proposal that the dominant piece of the Yukawa coupling matrices are factorizable into a factor $\sim |r>$ which is connected with the left-handed doublets $Q_L = (U_L, D_L)$, a factor $\sim |s>$ with the right-handed up singlets $U_R$, and another factor $\sim |s'>$ with the right-handed down singlets $D_R$. This may hint at some exchange-type of mass generating mechanism (Fig. 1). To accommodate this probable interpretation, and also to facilitate writing expressions for the up-type and the down-type symmetrically, we shall assume that

\[ |\bar{t} \rangle, |\bar{s} \rangle \text{ and } |\bar{s}' \rangle \text{ in Eqs.}(1) \text{ and } (2) \text{ are not the exact eigenvectors for the top and the bottom fields. There are } O(\epsilon, \epsilon') \text{ corrections. By the same token, } m_t \text{ differs from the true mass } m_t \text{ of the top quark by } O(\epsilon), \text{ and } \bar{m}_b \text{ from } m_b \text{ by } O(\epsilon'). \]

It proves convenient at this point to introduce a set of basis vectors for the up and the down flavor spaces. Because the CKM matrix connects left-handed quarks, we need to deal with only $MM^\dagger$ and $M'M'^\dagger$. For the up-sector, the basis vectors we choose are $|\hat{r} \rangle$, $|\hat{s} \rangle$ and $|\hat{s}' \rangle$ can be equated with $\bar{t}$, $\bar{s}$ and $\bar{s}'$ in (1) and (2) are not the exact eigenvectors for the top and the bottom fields. There are $O(\epsilon, \epsilon')$ corrections. By the same token, $m_t$ differs from the true mass $m_t$ of the top quark by $O(\epsilon)$, and $\bar{m}_b$ from $m_b$ by $O(\epsilon')$.

\[ |\hat{t} \rangle \equiv (|\bar{s} \rangle - \langle \bar{s} | \bar{r} \rangle)/[(1 - |\bar{r} \rangle |\bar{s} \rangle)^2]^{1/2}, \]

and

\[ |\hat{n} \rangle \equiv |\hat{r} \rangle \times |\hat{s} \rangle /[(1 - |\bar{r} \rangle |\bar{s} \rangle)^2]^{1/2}. \]

Similar expressions with $s \rightarrow s'$ will be used for $|\hat{t}' \rangle$ and $|\hat{n}' \rangle$ for the down-sector. It may seem odd that the intrinsically right-handed vectors $|\bar{s} \rangle$ and $|\bar{s}' \rangle$ are also used in the left spaces, but they are certainly the preferred vectors and may in fact tell us something about the much discussed left-right symmetry, or the lack thereof.

We take $MM^\dagger$ and $M'M'^\dagger$ and determine the masses and eigenvectors to an accuracy such that the CKM matrix will be obtained to $O(\epsilon)$ or $O(\epsilon')$. For the up-sector, it is easy to arrive at the normalized mass eigenstates

\[ |y_{u,c} \rangle \equiv |y_{u,c}^0 \rangle - \langle \epsilon/m_t | \bar{s} \rangle |\hat{M}^\dagger| y_{u,c}^0 \rangle |\hat{r} \rangle, \]

\[ |y_t \rangle = |\hat{r} \rangle + \langle \epsilon/m_t | \bar{s} \rangle |y_u^0 \rangle + \langle \epsilon/m_t | \bar{s} \rangle |y_c^0 \rangle, \]

where $m_t$ can be equated with $\bar{m}_t$ to this order, and $|y_{u,c}^0 \rangle$ are the two 0-th order eigenvectors for the up and the charm quarks

\[ |y_i^0 \rangle \equiv |\hat{i} \rangle |y_i^0 \rangle + |\hat{n} \rangle |y_i^0 \rangle, \]

determined by the equations

\[ < \hat{i}|\hat{H}|\hat{i} > < \hat{i}|y_i^0 \rangle + < \hat{i}|\hat{H}|\hat{n} > < \hat{n}|y_i^0 \rangle = \lambda_i < \hat{i}|y_i^0 \rangle, \]

and

\[ < \hat{n}|\hat{H}|\hat{i} > < \hat{i}|y_i^0 \rangle + < \hat{n}|\hat{H}|\hat{n} > < \hat{n}|y_i^0 \rangle = \lambda_i < \hat{n}|y_i^0 \rangle. \]
Here the dynamics is given by the subtracted 'Hamiltonian'

\[ \mathcal{H} = \mathcal{M} \mathcal{M}^\dagger - \mathcal{M} |\hat{s} > < \hat{s} | \mathcal{M}^\dagger \]
\[ = \mathcal{M} |\hat{v} > < \hat{v} | \mathcal{M}^\dagger + \mathcal{M} |\hat{n} > < \hat{n} | \mathcal{M}^\dagger , \]

with

\[ |\hat{v} > = < \hat{s} | \hat{r} > |\hat{t} > - (1 - | \hat{r} | \hat{s} > | \hat{s} > )^{1/2} |\hat{r} > . \] (8)

Please note that the subspace which has been subtracted out is |\hat{s} >. There can be leakage of dynamics from the approximate top state |\hat{r} > into the two low-lying members. We have taken the liberty to factor out a common factor \( \varepsilon \) in Eq.(6), i.e. \( m_i^2 = \varepsilon^2 \lambda_i , i = u, c. \)

We can write down similar expressions for the down-type quarks with the replacements \( \varepsilon \rightarrow \varepsilon' , \mathcal{M} \rightarrow \mathcal{M}' , \) etc.

For the CKM matrix, we just form the scalar products \( V_{ij} = < y_{u,c,t} | y_{d,s,b}^0 > . \) We shall adjust the phases so that

\[ V_{ud} = < y_u^0 | y_d^0 > + O(\varepsilon^2, \varepsilon'^2, \varepsilon e') = \cos \theta_c , \]
\[ V_{us} = < y_u^0 | y_s^0 > + O(\varepsilon^2, \varepsilon'^2, \varepsilon e') = \sin \theta_c , \]
\[ V_{cd} = < y_c^0 | y_d^0 > + O(\varepsilon^2, \varepsilon'^2, \varepsilon e') = -\sin \theta_c , \]

and

\[ V_{cs} = < y_c^0 | y_s^0 > + O(\varepsilon^2, \varepsilon'^2, \varepsilon e') = \cos \theta_c , \] (9)

in which \( \theta_c \) stands for the Cabibbo angle. These four elements can be calculated from Eqs.(7)-(8) and the corresponding set for the d-s system, once we postulate some dynamics for \( \mathcal{H} \) and \( \mathcal{H}' \). The other elements are

\[ V_{ub} = (\varepsilon'/m_b) < y_d^0 | \mathcal{M}' | \hat{s}' > \cos \theta_c + (\varepsilon'/m_b) < y_s^0 | \mathcal{M}' | \hat{s}' > \sin \theta_c - (\varepsilon/m_t) < y_u | \mathcal{M} | \hat{s} > , \]
\[ V_{td} = (\varepsilon/m_t) < \hat{s} | \mathcal{M}^\dagger | y_u > - \cos \theta_c - (\varepsilon/m_t) < \hat{s} | \mathcal{M}^\dagger | y_c > - (\varepsilon'/m_b) < \hat{s}' | \mathcal{M}'^\dagger | y_d^0 > , \]
\[ V_{cb} = -(\varepsilon'/m_b) < y_d^0 | \mathcal{M}' | \hat{s}' > \sin \theta_c + (\varepsilon'/m_b) < y_s^0 | \mathcal{M}' | \hat{s}' > \cos \theta_c - (\varepsilon/m_t) < y_c | \mathcal{M} | \hat{s} > , \]
\[ V_{ts} = (\varepsilon/m_t) < \hat{s} | \mathcal{M}^\dagger | y_u > \sin \theta_c + (\varepsilon/m_t) < \hat{s} | \mathcal{M}^\dagger | y_c > \cos \theta_c - (\varepsilon'/m_b) < \hat{s}' | \mathcal{M}'^\dagger | y_s^0 > , \]

and

\[ V_{tb} = 1 + O(\varepsilon^2, \varepsilon'^2, \varepsilon e') , \]

from which it follows

\[ V_{ub} \cong - V_{td}^* \cos \theta_c - V_{ts}^* \sin \theta_c , \]

and

\[ V_{cb} \cong V_{td}^* \sin \theta_c - V_{ts}^* \cos \theta_c . \] (10)

Let us be reminded that the expansion parameters in the present analysis are \( 1/m_t \) and \( 1/m_b \). One can easily check that unitarity holds to the first order in the CKM matrix we just constructed. Interestingly enough, the Wolfenstein representation [3], which is
obtained under the assumption that \( \sin \theta_c \) is the only parameter of expansion and which is accurate to \( O((\sin \theta_c)^3) \), yields relations in Eq.(10) trivially. Besides, the neglected terms in our approach are of order \( (m_s/m_b)^2 \), which is \( \approx O((\sin \theta_c)^4) \). This may not be a numerical accident.

By design, all the matrix elements of \( \mathcal{M} \) and \( \mathcal{M}' \) should be at most of order \( m_c \) or \( m_s \), respectively. Thus, making use of the fact that \( m_s/m_b \gg m_c/m_t \), we have

\[
V_{td,ts} \cong -\frac{\epsilon'}{m_b} < \hat{s}'|\mathcal{M}'|y_{d,s}' > ,
\]

which provide the absolute normalization for the mixing of the third to the first and the second families, \( \sim m_d/m_b \) and \( \sim m_s/m_b \), respectively.

We now discuss the mixing of the first two families, where

\[
< y_{i=u,c}^0 |y_{j=d,s}^0 > = < y_i^0 |\hat{t} > < t'|y_j^0 > < t|\hat{t}' > + < y_i^0 |\hat{n} > < n'|y_j^0 > < n|\hat{n}' >
+ < y_i^0 |\hat{t} > < n'|y_j^0 > < t|\hat{n}' > - < y_i^0 |\hat{n} > < t'|y_j^0 > < n|\hat{n}' > .
\]  

This is composed of intra-space dynamics, determined by Eqs.(7)-(8) and a similar set for the down-space, and interspace 'geometry'

\[
< \hat{t}|\hat{t}' > = < \hat{n}|\hat{n} > ; < \hat{t}|\hat{n}' > = - < \hat{t}'|\hat{n} > .
\]  

At one extreme, one model may be that \( |\hat{s} > \) and \( |s' > \) are aligned much better than \( O((m_d/m_s)^{1/2}) \). Then, \( < \hat{t}|\hat{t}' > = 1 \) and \( < \hat{t}|\hat{n}' > = 0 \). A Fritzsch-type [5] model

\[
\mathcal{M}_{ia=i,\hat{n} b=\hat{v},\hat{n}} = \left( \begin{array}{cc} -(m_c - m_u) & (m_c m_u)^{1/2} \\ (m_c m_u)^{1/2} & 0 \end{array} \right),
\]

\[
\mathcal{M}'_{ia=i',\hat{n} b=\hat{v}',\hat{n}'} = \left( \begin{array}{cc} -(m_s - m_d) & (m_s m_d)^{1/2} \\ (m_s m_d)^{1/2} & 0 \end{array} \right)
\]

will dictate the intraspace dynamics and lead to \( \sin \theta_c \approx - (m_d/m_s)^{1/2} \).

At the other extreme, we may have a model in which the 'Hamiltonian' of Eq.(8) is already in the diagonal form, with e. g.

\[
< y_u^0 |\hat{t} > = 1, \quad < y_u^0 |\hat{n} > = 0; \quad < y_c^0 |\hat{t} > = 0, \quad < y_c^0 |\hat{n} > = 1;
\]

\[
< t'|y_d^0 > = 1, \quad < n'|y_d^0 > = 0; \quad < t'|y_s^0 > = 0, \quad < n'|y_s^0 > = 1.
\]  

This will give

\[
\cos \theta_c = < \hat{t}|\hat{t}' >, \quad \sin \theta_c = < \hat{t}|\hat{n}' >,
\]

which are completely due to the geometric orientations of \( |\hat{r} >, |\hat{s} > \) and \( |\hat{s}' > \). Clearly, some clarifying dynamical principle is awaited to give some credence. Work is in progress in this as well as to calculate \( < \hat{s}'|\mathcal{M}'|y_{d,s}' > \) of Eq.(11).

In conclusion, we find data in suggestion that the left eigenvector of the top quark is very much aligned with the left eigenvector of the bottom quark. This may be elevated
to a conjecture that the dominant Yukawa couplings factorize into a set of numbers $|r>$ assigned to the left-handed quark doublets $Q_L = (U_L, D_L)$, a set $|s>$ to the right-handed up singlets $U_R$, and another set $|s'>$ to the right-handed down singlets $D_R$. The first order expansion in $1/m_t$ and $1/m_b$ is in agreement with Wolfenstein parameterization, as indicated by Eq.(10). This approach also sharpens the separation between the large and the small, particularly in their dynamics and geometry.

Added Note: Professor A. Kagan has kindly pointed out to me some earlier related work.$^5$

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Appendices:

Experimental data

The experimental values of the CKM matrix elements have been summarized by the Particle Data Group. Let us look at them, particularly at those obtained under the assumption of three families and constrained by unitarity:

\[
(|V_{ij}|) = \begin{pmatrix}
0.9747 \text{ to } 0.9759 & 0.218 \text{ to } 0.224 & 0.002 \text{ to } 0.005 \\
0.218 \text{ to } 0.224 & 0.9738 \text{ to } 0.9752 & 0.032 \text{ to } 0.048 \\
0.004 \text{ to } 0.015 & 0.030 \text{ to } 0.048 & 0.9988 \text{ to } 0.9995
\end{pmatrix}
\]

From this table, we have:

(i) \[1 - \{|V_{ud}| |V_{cs}| + |V_{us}| |V_{cd}|\} = 1 - \{0.9747 \times 0.9738 + (0.224)^2\} \text{ or} \]
\[= 1 - \{0.9757 \times 0.9752 + (0.218)^2\} \approx 0.0008,\]

(ii) \[1 - |V_{tb}| \approx 0.0012,\]

(iii) \[|V_{ud}|^2 - |V_{cs}|^2 \ll (0.9759)^2 - (0.9738)^2 = 0.0041.\]

These are various measures of how the third family decouples from the first two. While somewhat indirect, we interpret these to indicate that the differences are all of order \((\frac{m_s}{m_b})^2\). The situation will be much improved when we have a better determination of \(|V_{cd}| = 0.204 \pm 0.017\) and \(|V_{cs}| = 1.01 \pm 0.18.\)

One can also relate these to the Wolfenstein parametrization

\[
V = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & \lambda^3 A (\rho - i \eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & \lambda^2 A \\
\lambda^3 A (1 - \rho - i \eta) & -\lambda^2 A & 1
\end{pmatrix},
\]

which is an expansion in the (assumed single) parameter \(\lambda = \sin \theta_c\) to the third order. The neglected terms are of order \(\lambda^4 \approx (\frac{m_s}{m_b})^2\). Taken together, the data correspond to:

(i) \[V_{tb} = 1 + O((\frac{m_s}{m_b})^2),\]

(ii) \[V_{ud} = V_{cs} + O((\frac{m_s}{m_b})^2),\]

(iii) \[|V_{us}| = |V_{cd}| + O((\frac{m_s}{m_b})^2),\]
\(|V_{td}| \approx |V_{ub}| \sim \frac{m_d}{m_b}, \quad |V_{ts}| \approx |V_{cb}| \sim \frac{m_s}{m_b}.

We have used:\(^7\)

\[
\frac{m_s}{m_b}(1\text{Gev}) = \frac{0.199}{7.005} = 0.0284 = \frac{m_s}{m_b}(m_w) = \frac{0.087}{3.063},
\]
\[
\frac{m_d}{m_b}(1\text{Gev}) = \frac{0.0999}{7.005} = 0.0014 = \frac{m_d}{m_b}(m_w) = \frac{0.00433}{3.063}.
\]

\(\text{Fritzsch-type models}\)

Clearly, there are far too many parameters in the mass matrices. In attempts to decrease the number, Fritzsch and others proposed to put some of the elements to zero, which could lead to relations between masses and mixing matrix elements. This has been coined as ‘texture studies’ or more poetically ‘stitching the Yukawa quilt’.

Let us take a typical example to point out what needs to be done and perhaps the potential troubles in all such models. Here\(^9\)

\[
M = \begin{pmatrix}
0 & x & 0 \\
x^* & \alpha & b \\
0 & b^* & a
\end{pmatrix}, \quad M' = \begin{pmatrix}
0 & y & 0 \\
y & \beta & f \\
0 & f & d
\end{pmatrix}
\]

where by phase choice, all the entries other than \(x = |x|e^{i\delta_x}\) and \(b = |b|e^{i\delta_b}\) are real. The original Fritzsch model corresponds to setting \(\alpha = \beta = 0\), which turns out to be inconsistent with \(m_t \approx 174\text{ Gev}\). In order to enforce the sequencing of eigenvalues \(m_u < m_c < m_t\) and \(m_d < m_s < m_b\), one has \(m_u - m_c < \alpha < m_t - m_c\) and \(m_d - m_s < \beta < m_b - m_s\). Furthermore, to retain some vestige of a radiative mass hierarchy interpretation, which was the motivation for the model, one should confine the ranges to \(|\alpha| < m_c\) and \(|\beta| < m_s\).

For the decoupling indicators, one can easily obtain

(i)

\[
1 - \{|V_{ud}| |V_{cs}| + |V_{us}| |V_{cd}|\} \approx \frac{1}{2} \Delta,
\]

(ii)

\[
1 - |V_{tb}| \approx \frac{1}{2} \Delta,
\]

(iii)

\[
|V_{ud}|^2 - |V_{cs}|^2 \approx \Delta,
\]
where
\[ \Delta = \Delta_t + \Delta_b - 2\Delta_t^{1/2}\Delta_b^{1/2}\cos\delta_b, \]
\[ \Delta_b = \frac{\beta + m_s - m_d}{m_b}, \quad \Delta_t = \frac{\alpha + m_c - m_u}{m_t}. \]

In order to agree with data, we need
\[ \frac{1}{2}\Delta \approx 2\left(\frac{m_s}{m_b}\right)^2, \]
which requires a tuning of
\[ \alpha \approx -\frac{1}{2}m_c \quad \beta \approx -m_s. \]

We do not know of any symmetry which demands this. We may remark that once we decide to tune \( \Delta \) to decrease to \( \approx O\left(\left(\frac{m_s}{m_b}\right)^2\right) \), then there are other extra terms to the same order for the right hand sides of the decoupling indicators above. However, the simple predictions
\[ |V_{(ub, cb, ts, td)}| \approx N_{(u, c, s, d)}\Delta^{1/2}, \]
where
\[ N_{(u, c)} = \sqrt{\frac{m_{(u, c)}}{m_u + m_c}}, \quad N_{(d, s)} = \sqrt{\frac{m_{(d, s)}}{m_d + m_s}}, \]
and
\[ |V_{(ud, cs)}| \approx \cos\theta_c, \quad |V_{(us, cd)}| \approx \sin\theta_c, \]
where
\[ \cos\theta_c = (N_u^2N_d^2 + N_c^2N_s^2 + 2N_uN_cN_dN_s\cos\delta_x)^{1/2}, \]
\[ \sin\theta_c = (N_u^2N_s^2 + N_c^2N_d^2 - 2N_uN_cN_dN_s\cos\delta_x)^{1/2}. \]

still hold. Corrections are of order \( (m_s/m_b)^2 \) after the tuning.

From this brief exposition, it is evident that to save the radiative mass hierarchy picture, it is necessary to tune some parameters, \( \alpha \) and \( \beta \). The sole purpose is to accelerate the decoupling of the third family from the first two. Our proposed formulation incorporates this at the very start.
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Figure Caption:

Figure 1: A possible dominant mass generating mechanism which causes factorization of
Yukawa couplings related to mass matrices. The internal fermion and boson should be
flavor neutral.
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