On the $s-\bar{s}$ asymmetry in the nucleon sea

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(Dated: February 2, 2008)

We study the $s-\bar{s}$ asymmetry in the nucleon sea using a model in which the proton wave function includes a Kaon-Hyperon Fock state. Parameters of the model are fixed by fitting the $s-\bar{s}$ asymmetry obtained from global fits to Deep Inelastic Scattering data. We discuss possible effects of the $s-\bar{s}$ asymmetry on the measurement of the Weinberg angle by the NuTeV Collaboration.

I. INTRODUCTION

The first studies about a possible asymmetry in the strange sea of the nucleon dates from 1987, when Signal and Thomas [1] discussed the possibility of a $K^+\Lambda$ pair component in the proton wavefunction. Since then on, several models have been proposed for the nucleon structure, allowing for an asymmetric $s-\bar{s}$ sea [2, 3, 4]. However, no experimental evidence was presented on this subject until the global fit of Deep Inelastic Scattering (DIS) data by Barone et al. [5] in 2000. Most recently, the $s-\bar{s}$ asymmetry in the nucleon sea was called for as a possible explanation [6] for the almost 3 $\sigma$ difference between the NuTeV $\sin^2\theta_W$ result [7] and global fits [8].

From a theoretical point of view, it is interesting to note that although sea quarks in the nucleon originating in gluon splitting necessarily have symmetric momentum distributions, after interacting with the valence quarks and the remaining partons in the sea, their momentum distributions do not have to be equal. This can be interpreted as the formation of a virtual $K^+\Lambda$ pair in the nucleon structure. Being this the case, it is easy to see that, since the $s$ and the $\bar{s}$ quarks are part of the $\Lambda$ and $K^+$ respectively, then their momentum distributions would be different. This difference, which is merely a consequence of the interaction of sea quarks with the remaining partons in the nucleon, has to be understood as part of the non-perturbative dynamics responsible for the formation of the nucleon as a bound state of quarks and gluons. Recall also the $\bar{u}-\bar{d}$ asymmetry and the Gottfried Sum Rule violation, known since the New Muon Collaboration results [9], which can also be explained in terms of a $n\pi^+$ and a $\Delta^{++}\pi^-$ components in the proton wave function [10]. It is interesting to note however, that a small $s-\bar{s}$ asymmetry arises as a NNLO perturbative effect [11]. Nevertheless, the integrated value, $S = \int_{0}^{1} x|s(x) - \bar{s}(x)|dx \simeq -5 \times 10^{-4}$ at $Q^2 = 20$ GeV$^2$, is too small and negative to account for the NuTeV result.

In this work, we shall consider a model for the extrange sea of the proton which can describe the form of the $s-\bar{s}$ asymmetry extracted from global fits to DIS data. After fixing the parameters of the model, in Section III, we shall study the effect of this asymmetry, together with possible effects coming from the non isoscalarity of the target and nuclear medium effects, in the determination of $\sin^2\theta_W$ by the NuTeV Collaboration. Section IV will be devoted to discussion and conclusions.

II. A MODEL FOR THE $s(x) - \bar{s}(x)$ ASYMMETRY

Different models [1, 2, 3, 4] have attempted to predict the $s-\bar{s}$ asymmetry. Among them, the most promising approach seems to be the Meson Cloud Model (MCM). In the MCM fluctuations of the proton to kaon-hyperon virtual states are responsible for the $s-\bar{s}$ asymmetry. Since the $s$ quark belong to the hyperon and the $\bar{s}$ quark to the kaon, the asymmetry arises naturally due to the different momentum carried by the kaon and the hyperon in the fluctuation.

Two different approaches exist within the MCM. The first is based in a description of the form factor of the extended proton-kaon-hyperon vertex [1, 3] and, the second one, in terms of parton degrees of freedom [4]. In the first approach, the knowledge of the form factors is crucial to get a reasonable description of the $s-\bar{s}$ asymmetry (see e.g. Ref. [3]). In the second one, fluctuations are generated through gluon emission from the constituent valence quarks and its subsequent splitting to a $s-\bar{s}$ pair [4]. This $s-\bar{s}$ pair then recombines with constituent quarks to form a kaon-hyperon bound state. In what follows, we will adopt the second approach.

A. The model

We start by considering a simple picture of the nucleon in the infinite momentum frame as being formed by three dressed valence quarks - valons, $\nu(x)$ - which carry all of its momentum [12].

In the framework of the MCM, the nucleon can fluctuate to a meson-baryon bound state carrying zero net
strangeness. As a first step in such a process, we may consider that each valon can emit a gluon which, before interacting, decays perturbatively into a $s\bar{s}$ pair. The probability of having such a perturbative $q\bar{q}$ pair can then be computed in terms of Altarelli-Parisi splitting functions \[13\]

\[
P_{qg}(z) = \frac{4}{3} \frac{1 + (1 - z)^2}{z},
\]

\[
P_{gq}(z) = \frac{1}{2} \left( z^2 + (1 - z)^2 \right).
\]

These functions have a physical interpretation as the probability of gluon emission and $q\bar{q}$ creation with momentum fraction $z$ from a parent quark or gluon respectively. Hence,

$$q(x, Q^2) = \bar{q}(x, Q^2) = N \alpha_s^2(Q^2) (2\pi)^2 \times \int_x^1 \frac{dy}{y} P_{qg} \left( \frac{x}{y} \right) \int_y^1 \frac{dz}{z} P_{gq} \left( \frac{y}{z} \right) v(z)$$

is the joint probability density of obtaining a quark or anti-quark coming from subsequent decays $v \to v + q$ and $g \to q + \bar{q}$ at some fixed low $Q^2$. As the valon distribution does not depend on $Q^2$ \[12\], the scale dependence in eq. (2) only exhibits through the strong coupling constant $\alpha_s$. The range of values of $Q^2$ at which the process of virtual pair creation occurs in this approach is typically below 1 GeV$^2$, as dictated by the valon model of the nucleon. For definiteness, we will use $Q = 0.7$ GeV as in Ref. [12], for which $\alpha_s^2 \sim 0.3$ is still sufficiently small to allow for a perturbative evaluation of the $q\bar{q}$ pair production. Since the scale must be consistent with the valon picture, the value of $Q^2$ is not really free and cannot be used to control the flavor produced at the $q\bar{q}$ vertex. Instead, this role can be ascribed to the normalization constant $N$, which must be such that to a heavier quark corresponds a lower value of $N$.

Once a $s\bar{s}$ pair is produced, it can rearrange itself with the remaining valons so as to form a most energetically favored meson-baryon bound state. When the nucleon fluctuates into a meson-baryon bound state, the meson and baryon probability densities inside the nucleon are not independent. Actually, to ensure the zero net strangeness of the nucleon and momentum conservation, the in-nucleon meson and baryon distributions must fulfill two basic constraints,

\[
\int_0^1 \frac{dz}{z} [P_B(x) - P_M(x)] = 0,
\]

\[
\int_0^1 dx [x P_B(x) + x P_M(x)] = 1,
\]

for all momentum fractions $x$.

The meson, $P_M(x)$, and baryon, $P_B(x)$, probability density functions have to be calculated by means of effective techniques in order to deal with the non-perturbative QCD processes inherent to the dressing of quarks into hadrons. In Ref. [4], these probability densities have been related to the cross section for meson production by recombination, and the model by Das and Hwa [14] was used to obtain them. In this work, since the aim is to compare the model to experimental data by means of a fit, we will follow a different approach. Let us note that, for reasonable valon distributions in the nucleon and sea quark distributions of the form given by eq. (2), the result of using the recombination model gives for the meson probability density a function of the form $x^n(1 - x)^b$.

Then we will assume

$$P_M(x) = \frac{1}{\beta(a_{KN} + 1, b_{KN} + 1)} x^{a_{KN}}(1 - x)^{b_{KN}},$$

which is properly normalized to one. If for the in-nucleon baryon probability we use the same functional form as for the meson,

$$P_B(x) = \frac{1}{\beta(a_{HN} + 1, b_{HN} + 1)} x^{a_{HN}}(1 - x)^{b_{HN}},$$

it is automatically satisfied the requirement of zero net strangeness. In addition, interpreting $a_{KN}$, $b_{KN}$, $a_{HN}$ and $b_{HN}$ as parameters of the model, and recognizing that eq. (4) fix one of them as a function of the remaining three by means of

$$\Gamma(a_{KN} + b_{KN} + 2) \Gamma(a_{KN} + 2) + \Gamma(a_{KN} + b_{KN} + 3) \Gamma(a_{KN} + 1) \Gamma(a_{KN} + b_{KN} + 2) \Gamma(a_{KN} + 2) = 1,$$

then the momentum conservation sum rule is also fulfilled.

The non-perturbative strange and anti-strange sea distributions in the nucleon can be now computed by means of the two-level convolution formulas

\[
s^{NP}(x) = \int_x^1 \frac{dy}{y} P_B(y) s_H(x/y)
\]

\[
\bar{s}^{NP}(x) = \int_x^1 \frac{dy}{y} P_M(y) \bar{s}_K(x/y),
\]

where the sources $s_B(x)$ and $\bar{s}_M(x)$ are primarily the probability densities of the strange valence quark and anti-quark in the baryon and meson respectively, evaluated at the hadronic scale $Q^2$ \[11\]. In principle, to obtain the non-perturbative distributions given by eqs. (9), one should sum over all the strange meson-baryon fluctuations of the nucleon but, since such hadronic Fock states are necessarily off-shell, the most likely configurations are those closest to the nucleon energy-shell, namely $\Lambda^0 K^+$, $\Sigma^+ K^0$ and $\Sigma^0 K^+$, for a proton state.

B. Fit to $xs(x) - \bar{s}(x)$ data

In order to fit to experimental data on the $s - \bar{s}$ asymmetry and to extract the parameters of the model, we
will use
\[
\bar{s}_K(x) = \frac{1}{\beta(a_K + 1, b_K + 1)} x^{a_K}(1 - x)^{b_K}, \quad (10)
\]
\[
s_H(x) = \frac{1}{\beta(a_H + 1, b_H + 1)} x^{a_H}(1 - x)^{b_H}, \quad (11)
\]
which are consistent with the hypothesis that the meson and baryon are formed by valons. Then the \( s - \bar{s} \) asymmetry of the nucleon is given by
\[
x s(x) - x \bar{s}(x) = N^2 \left[ x s^{NP}(x) - x \bar{s}^{NP}(x) \right], \quad (12)
\]

at the valon scale \( Q^2 = 0.49 \text{ GeV}^2 \), and where \( N^2 \) is the probability of the \( |KH \) Fock state in the proton wave function, which is related to the probability of having a \( s - \bar{s} \) quark pair out of eq. (2).

The model has then a total of 8 parameters to be fixed by fits to experimental data.

Results of the fit to experimental data from Ref. [15] are shown in Fig. 1 and Table I. For the experimental data, we extracted 27 segments from the allowed band in Figure 2 of Ref. [12] and assumed that the midpoint of each segment is the most probable value which we interpreted as the value for the asymmetry, while the half length of the segment was interpreted as the error bar. Notice however that this procedure has to be taken only as a way to fit our model inside the allowed region for the \( s - \bar{s} \) asymmetry.

The fit was performed by minimizing the \( \chi^2 \) using MINUIT. In the fitting procedure, since the allowed bars for the \( s - \bar{s} \) asymmetry are given at \( Q^2 = 20 \text{ GeV}^2 \), the parameters where chosen, then the asymmetry was evolved from \( Q^2 = 0.49 \text{ GeV}^2 \) to \( Q^2 = 20 \text{ GeV}^2 \), the \( \chi^2 \) was evaluated and the procedure was repeated until a minimum was reached.

The evolution has been done at NLO and a value of \( S^- = 0.87 \times 10^{-4} \) at \( Q^2 = 20 \text{ GeV}^2 \) has been obtained, which is also the average \( Q^2 \) reported by NuTeV [5, 10]. Although this value of \( S^- \) is small to account for the anomalous result for \( \sin^2 \theta_W \) reported by NuTeV, it is positive. It is also conceivable that, by performing a NNLO evolution, the positive contribution of the perturbative asymmetry in \( s - \bar{s} \) be compensated by a bigger positive non-perturbative asymmetry.

In Fig. 2 the \( x s \) and \( x \bar{s} \) distributions are displayed at the \( Q^2 \) scale where the evolution starts, namely, \( Q^2 = 0.49 \text{ GeV}^2 \).

### III. THE EFFECT OF \( s(x) - \bar{s}(x) \) ON THE DETERMINATION OF \( \sin^2 \theta_W \)

The weak mixing angle is one of the basic parameters of the standard model of electroweak interactions. An experimental value for \( \sin^2 \theta_W \) has been obtained, using the on-shell renormalization scheme, from a global fit to the precise electroweak measurements performed by the LEP experiments at CERN and the SLD experiment at SLAC, together with data from several other experiments at Fermilab [8]. This global analysis lends a value of

\[
\sin^2 \theta_W = 0.2227 \pm 0.0004,
\]

excluding the data from CCFR and NuTeV experiments.

The NuTeV collaboration reported a value of \( \sin^2 \theta_W \) extracted from the analysis of neutrino and antineutrino
charged current (CC) and neutral current (NC) scattering data. The on shell value obtained by this experiment is
\[
\sin^2 \theta_W = 0.22773 \pm 0.00135 \text{ (stat)} \pm 0.00093 \text{ (syst)},
\]
which is \(\sim 3\sigma\) away from the the electroweak global fit value.

Due to an inevitable contamination of the \(\bar{\nu}_\mu(\nu_\mu)\) beams with \(\nu_e(\bar{\nu}_e)\), and the impossibility of separating CC \(\nu_e(\bar{\nu}_e)\) induced interactions from the NC \(\nu_\mu(\bar{\nu}_\mu)\) induced ones on an event by event bases, the NuTeV result was obtained by performing a full simulation of the whole experiment, in which the value of the weak mixing angle was adjusted so that the Monte Carlo yield the best description of the experimental data. The NuTeV Monte Carlo included, among may other things, a detailed simulation of the neutrino(antineutrino) beam, a detailed model of the \(\nu(\bar{\nu})N\) cross section, QED radiative corrections, charm-production-threshold effects, strange and charm sea scattering, quasi-elastic scattering, neutrino-electron scattering, non-isoscalar-target effects, higher twist effects, etc. The experimental data and the Monte Carlo results compare very well.

The NuTeV Monte Carlo simulation did not assume any asymmetry in the strange-antistrange sea of the nucleons. The effect of this asymmetry and possible effects due to nuclear medium modifications to the parton distributions have been explored as possible explanations for the discrepancy between the results for the \(\sin^2 \theta_W\) obtained by NuTeV and those obtained by the global fit to the DIS data.

The effect of the \(s-\bar{s}\) asymmetry on the determination of \(\sin^2 \theta_W\) can be accounted for through the use of the Paschos-Wolfenstein (PW) relation
\[
R^-= \frac{\sigma^{\nu N}_{NC} - \sigma^{\nu N}_{CC}}{\sigma^{\bar{\nu} N}_{CC} - \sigma^{\bar{\nu} N}_{CC}} = \frac{1}{2} - \sin^2 \theta_W, \quad (13)
\]
where \(\sigma^{\nu (\bar{\nu}) N}_{NC}\) refer to the neutrino(antineutrino)-nucleon neutral(charged) current cross sections.

Equation (13) does not assume any asymmetry in the strange sea of the nucleon, and has to be corrected when the target is a nucleus, due to effects of the nuclear medium. In most experimental cases, as for example the NuTeV experiment, the target is not isoscalar either and this effect has to be taken into account. A generalized PW relation that includes all these modifications can easily be obtained. The extraction of the \(\sin^2 \theta_W\) by NuTeV did not resort to any PW relation because of the impossibility to separate effectively the charged current form the neutral current signals.

Through the use of a generalized PW relation one can estimate the effect of the presence of a nucleon-strange-sea asymmetry over the extraction of the \(\sin^2 \theta_W\) done by NuTeV. The same procedure can also allow to estimate the effect of different nuclear medium modifications.

The generalized PW relation can be written as:
\[
R^- = \frac{Z(\sigma^{\nu p}_{NC} - \sigma^{\nu p}_{CC}) + N(\sigma^{\nu n}_{NC} - \sigma^{\nu n}_{NC})}{Z(\sigma^{\bar{\nu} p}_{CC} - \sigma^{\bar{\nu} p}_{CC}) + N(\sigma^{\bar{\nu} n}_{CC} - \sigma^{\bar{\nu} n}_{CC})} \times \\
\left( \frac{1}{4} - \frac{2}{3} \sin^2 \theta_W \right) \left( ZU^- + ND^- \right) + \left( \frac{1}{4} - \frac{1}{3} \sin^2 \theta_W \right) \left( ZD^- + NU^- \right) + \left( \frac{1}{4} - \frac{1}{3} \sin^2 \theta_W \right) \left( N + Z \right) S^- \right], \quad (14)
\]
where,
\[
U^- = \int_0^1 x[u(x) - \bar{u}(x)] dx, \quad \text{(15)}
\]
\[
D^- = \int_0^1 x[d(x) - \bar{d}(x)] dx, \quad \text{(16)}
\]
\[
S^- = \int_0^1 x[s(x) - \bar{s}(x)] dx, \quad \text{(17)}
\]
and \(Z\) and \(N\) are the proton and neutron numbers of the target nucleus.

The structure functions measured in DIS experiments with nuclear targets differ from those of the nucleon. This is due to modifications caused by the nuclear medium over the parton distributions. The modified distributions can be parametrized as
\[
q_i^A(x, Q^2) = R_i^A(x, Q^2) q_i(x, Q^2). \quad (18)
\]
Parametrizations of the $R^4(x, Q^2)$ for different nuclei can be found in the literature, for example those presented in reference [19] (available in CERNLIB).

In order to include nuclear-medium modifications in the generalized PW relation it is enough to modify the asymmetry integrals as

$$
U^{-}(A) = \int_{0}^{1} xR_u^4(x, Q^2)[u(x) - \bar{u}(x)]dx,
$$

$$
D^{-}(A) = \int_{0}^{1} xR_d^4(x, Q^2)[d(x) - \bar{d}(x)]dx,
$$

$$
S^{-}(A) = \int_{0}^{1} xR_s^4(x, Q^2)[s(x) - \bar{s}(x)]dx.
$$

The NuTeV Collaboration extracted $\sin^2 \theta_W$ from a full simulation of the experiment in which a symmetric strange sea was assumed ($S^- = 0$). From this, and the use of the generalized PW relation, one could determine the value of $\bar{R}^-$ consistent with the results of NuTeV, by evaluating

$$
\overline{R}_{\text{NuTeV}} = \overline{R}^{-}[S^- = 0, Z, N, \sin^2 \theta_W^{\text{NuTeV}}] = 0.2613,
$$

where $N$ and $Z$ correspond to the iron target used, and $\sin^2 \theta_W^{\text{NuTeV}}$ to the value reported by the experiment.

Assuming $\overline{R}_{\text{NuTeV}}$, the value of $\sin^2 \theta_W$ from the global analysis of the DIS data ($\sin^2 \theta_W^{\text{Global}}$), and the parametrizations of Gluck, Reya and Vogt for the parton distributions of $u(x)$ and $d(x)$ [20], one can evaluate the level of asymmetry in the strange sea of the nucleon that could explain the NuTeV result,

$$
S^- = \frac{1}{\left[\frac{2}{3} \overline{R}_{\text{NuTeV}} - \left(\frac{1}{3} - \frac{2}{3} \sin^2 \theta_W^{\text{Global}}\right)\right]} (N + Z)
\times \left[\left(\frac{1}{4} - \frac{2}{3} \sin^2 \theta_W^{\text{Global}}\right) (ZU^- + ND^-)\right.
\left.\left.\left(\frac{1}{4} - \frac{1}{3} \sin^2 \theta_W^{\text{Global}}\right) (ZD^- + NU^-)\right]
\left.\frac{R_{\text{NuTeV}}}{2} [(3N - Z)U^- + (3Z - N)D^-]\right],
$$

$$
= 0.004013.
$$

By including nuclear medium effects according to the parametrizations of Eskola et al. [19], the value for the strange sea asymmetry that could explain the NuTeV result is

$$
S^{-(A)} = 0.003868.
$$

The strange asymmetries predicted by our model, with and without nuclear medium effects, are

$$
S^- = 0.000087, \\
S^{-(A)} = 0.000047,
$$

which are two orders of magnitude smaller. Since our parametrizations for $x(s(x) - \bar{s}(x))$ are in agreement with the experimental data from Ref. [12], one can conclude that the anomalous value for $\sin^2 \theta_W$ reported by NuTeV cannot be explained in terms of a possible asymmetry in the strange sea of the nucleon.

IV. CONCLUSIONS

We have presented a model, based in fluctuations of the proton wavefunction to a generic Hyperon-Kaon Fock state, that closely reproduces experimental data on the strange sea asymmetry of the nucleon. The model has a total of 8 parameters which have been fixed by fits to experimental data. No NNLO effects in the evolution of the $xs$ and $xs$ have been considered, however the negative asymmetry introduced by NNLO evolution effects should be compensated by a large and positive asymmetry coming from the non-perturbative dynamics associated to the confining phase of QCD.

We investigated also the effect of such an asymmetry on the result presented by the NuTeV experiment on the measurement of $\sin^2 \theta_W$. In the study we considered, in addition, effects coming from the non isoscalarity of the NuTeV target and effects associated to the nuclear medium. Considering all together, we found that the effect of the $s - \bar{s}$ asymmetry in the nucleon sea is too small to account for the almost $3\sigma$ difference among the $\sin^2 \theta_W$ result by NuTeV and the world average.

Acknowledgments

J. Magnin would like to thanks the warm hospitality at the Physics Department, Universidad de los Andes, where part of this work was done. J.C. Sanabria would like to thanks also the warm hospitality during his visit to CBPF. This works was supported by the Brazilian Council for Science and Technology and FAPERJ (Brazil) under contract Project No.: E-26/170.158/2005.

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