Charge and Color Breaking in Supersymmetry and Superstring

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Abstract. Charge and color breaking minima in SUSY theories might make the standard vacuum unstable. In this talk a brief review of this issue is performed. When a complete analysis of all the potentially dangerous directions in the field space of the theory is carried out, imposing that the standard vacuum should be the global minimum, the corresponding constraints turn out to be very strong and, in fact, there are extensive regions in the parameter space of soft SUSY–breaking terms that become forbidden. For instance, in the context of the MSSM with universal soft terms, this produces important bounds, not only on the value of $A$, but also on the values of $B$, $M$ and $m$. In specific SUSY scenarios, as fixed point models, no–scale supergravity, gauge–mediated SUSY breaking and superstrings, the charge and color breaking constraints are also very important. For example, if the dilaton is the source of SUSY breaking in four–dimensional superstrings, the whole parameter space $(m_{3/2}, B)$ is excluded on these grounds. Cosmological analyses are also briefly reviewed.

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1. Introduction and summary

As is well known, the presence of scalar fields with color and electric charge in supersymmetric (SUSY) theories induces the possible existence of dangerous charge and color breaking minima, which would make the standard vacuum unstable \(^1\). This is not necessarily a shortcoming since many SUSY models can be discarded on these grounds, thus improving the predictive power of the theory. A complete analysis of all the potentially dangerous directions in the field space of the minimal supersymmetric standard model (MSSM) was carried out in \(^2\). It was shown there that, imposing that the SUSY standard vacuum should be deeper than the charge and color breaking minima, the corresponding constraints on the soft parameter space are very strong (see also \(^3\)). For instance, in the universal case and assuming radiative symmetry breaking with nothing but the MSSM in between the weak scale and the grand unification scale \(M_{\text{GUT}}\), there are extensive regions of this space that become forbidden producing important bounds, not only on the value of the trilinear scalar parameter \((A)\), but also on the values of the bilinear scalar parameter \((B)\) and the scalar and gaugino masses \((m, M)\) respectively. The above mentioned constraints were used in \(^4\) finding a preferred region of SUSY particle masses after imposing in addition dark matter and naturalness constraints. Very strong bounds can also be obtained applying the above mentioned constraints to particular SUSY scenarios. This is the case of the infrared fixed point model \(^5\) and a SO(10) GUT \(^6\). No–scale supergravity models where the limit \(m = 0\) is obtained would be excluded on these grounds \(^2, 5\). Charge and color breaking constraints were also studied in the context of gauge–mediated SUSY–breaking models \(^7\). In most of them the global vacuum does not preserve QCD. On the other hand, the stability of the corresponding constraints with respect to variations of the initial scale for the running of the soft breaking parameters was analyzed in \(^5\), finding that the larger the scale is, the stronger the bounds become. In particular, by taking the Planck scale rather than \(M_{\text{GUT}}\) for the initial scale, substantially stronger constraints are found. These issues are reviewed in section 2. Let us finally remark that the stability of the standard vacuum also imposes constraints on flavor–mixing trilinear soft terms which are stronger than the laboratory bounds coming from the absence of FCNC \(^8\).

The low–energy limit of four–dimensional superstrings is a SUSY field theory. This allows us to apply the above mentioned general constraints to SUSY/string scenarios. The analysis can be in principle more predictive since in four–dimensional superstrings it is possible to obtain information about the structure of soft SUSY–breaking terms \(^9\). In particular, in the dilaton–dominated SUSY–breaking scenario, the soft terms are universal and depend on only two parameters, the gravitino mass \(m_{3/2}\) and \(B\). It was shown in \(^10\) that charge and color breaking constraints are so important
that the \textit{whole} parameter space is forbidden and, as a consequence, the
dilaton–dominated SUSY breaking is excluded on these grounds. In section
3 this analysis is reviewed. The possibility of assuming that the moduli
fields contribute to SUSY breaking is also discussed.

Finally, section 4 is left for some final comments including cosmological
considerations.

2. Charge and color breaking in supersymmetry

A complete study of this crucial issue is in principle very involved. This is
mainly due to two reasons. First, the enormous complexity of the scalar
potential, $V$, in a SUSY theory. Second, the radiative corrections to $V$
must be included in a proper way. Concerning the first point, the tree–level scalar
potential, using a standard notation, is given by

$$V_0 = V_F + V_D + V_{\text{soft}},$$



where $W$ is the MSSM superpotential

$$W = \sum_{i \equiv \text{generations}} \{ \lambda_u Q_i H_2 u_i + \lambda_d Q_i H_1 d_i + \lambda_e L_i H_1 e_i \} + \mu H_1 H_2,$$

and $\alpha$ runs over all the canonically normalized scalar components of the
chiral superfields. The first observation is that the previous potential is
extremely involved since it has a large number of independent fields. Further-
more, even assuming universality of the soft breaking terms at $M_{\text{GUT}}$,
it contains a large number of independent parameters: $m$, $M$, $A$, $B$, $\mu$.
In addition, there are the gauge ($g$) and Yukawa ($\lambda$) couplings which are
constrained by the experimental data. Notice that $M$ does not appear
explicitly in $V_0$, but it does through the renormalization group equations
(RGEs) of all the remaining parameters.

Concerning the radiative corrections it should be noted that the tree–level scalar potential $V_0$ is strongly dependent on the renormalization
scale $Q$, and the one–loop radiative corrections to it, namely $\Delta V_1 = \sum_{\alpha} \frac{n_\alpha}{64\pi^2} M_\alpha^4 \left[ \log \frac{M_\alpha^2}{Q^2} - \frac{3}{2} \right]$, are crucial to make the potential stable against variations of the $Q$ scale. In the previous expression $M_\alpha^2(Q)$ are the
improved tree–level squared mass eigenstates and $n_\alpha = (-1)^{2s_\alpha}(2s_\alpha + 1)$,
where $s_\alpha$ is the spin of the corresponding particle. Clearly the complete one–loop potential $V_1 = V_0 + \Delta V_1$ has a structure that is even far more involved than $V_0$. Notice that $M_\alpha^2(Q)$ are in general field–dependent quantities since they are the eigenvalues of the $(\partial^2 V_0/\partial \phi_i \partial \phi_j)$ matrix. Hence, the values of $M_\alpha^2(Q)$ depend on the values of the fields and thus on which direction in the field space is being analyzed. This makes in practice the minimization of the complete $V_1$ an impossible task. However, in the region of $Q$ where $\Delta V_1$ is small, the predictions of $V_0$ and $V_1$ essentially coincide. This occurs for a value of $Q$ of the order of the most significant $M_\alpha$ mass appearing in $\Delta V_1$, which in turns depends on what is the direction in the field space that is being analyzed. Therefore one can still work just with $V_0$, but with the approximate choice of $Q$.

Taking into account all the above points one should carry out a complete analysis of all the possible dangerous directions in the field space along which the potential develops a charge and color breaking minimum deeper than the realistic one. The latter, given by $V_{\text{real min}} = -\frac{1}{16}(g'^2 + g^2)(v_2^2 - v_1^2)^2$, where $|H_1^0| = v_1$, $|H_2^0| = v_2$ with $v_1^2 + v_2^2 = 2M_{W^2}/g^2$, corresponds to the standard vacuum. Several comments with respect to this minimum are in order. First, note that result $V_{\text{real min}}$ is obtained by minimizing just the tree-level part of the Higgs potential. As explained above this procedure is correct if the minimization is performed at some sensible scale $Q = M_S$, which should be of the order of the most relevant mass entering $\Delta V_1$. Since we are dealing here with the Higgs–dependent part of the potential, that mass is necessarily of the order of the largest Higgs–dependent mass, namely the largest stop mass. A more precise estimate of $M_S$, using a certain average of typical SUSY masses, can be found in [2]. Second, the requirement of correct electroweak breaking fixes one of the five independent parameters of the MSSM, say $\mu$, in terms of the others $(m,M,A,B)$. Third, we must be sure that the realistic minimum is really a minimum in the whole field space. This simply implies that all the scalar squared mass eigenvalues (charged Higgses, squarks and sleptons) must be positive. This is guaranteed for the charged Higgs fields since in the MSSM the minimum of the Higgs potential always lies at $H^+_1 = H^-_1 = 0$, but not for the rest of the sparticles. Finally, we must go further and demand that all the not yet observed particles have masses compatible with the experimental bounds.

There are two types of charge and color breaking constraints: the ones arising from directions in the field–space along which the (tree–level) potential can become unbounded from below (UFB), and those arising from the existence of charge and color breaking (CCB) minima in the potential deeper than the standard minimum. Since it is not possible to give here an account of the explicit form of the constraints we refer the interested reader to [3]. Here we will mention only their most important characteristics.
Concerning the CCB constraints, let us mention that the “traditional” bound, first studied by Frere et al. and subsequently by others [1], when correctly evaluated (i.e. including the radiative corrections in a proper way) turns out to be extremely weak. However, the “improved” set of analytic constraints obtained in [2], which represent the necessary and sufficient conditions to avoid dangerous CCB minima, is much stronger.

Concerning the UFB directions (and corresponding constraints), there are three of them, labelled as UFB–1, UFB–2, UFB–3 in [2]. It is worth mentioning here that in general the unboundedness is only true at tree level since radiative corrections eventually raise the potential for large enough values of the fields, but still these minima can be deeper than the realistic one and thus dangerous. The UFB–3 direction, which involves the fields \( \{H_2, \nu_{L_i}, e_{L_j}, e_{R_j}\} \) with \( i \neq j \) and thus leads also to electric charge breaking, yields the strongest bound among all the UFB and CCB constraints so it deserves to be exposed in greater detail. The explicit form of this bound is as follows. By simple analytical minimization it is possible to write the value of all the relevant fields along the UFB–3 direction in terms of the \( H_2 \) one. Then, for any value of \(|H_2| < M_{GUT}\) satisfying

\[
|H_2| > \sqrt{\frac{\mu^2}{4\lambda_{e_j}^2} + \frac{4m_{L_i}^2}{g'^2 + g_2^2} - \frac{|\mu|^2}{2\lambda_{e_j}}} \ ,
\]

the value of the potential along the UFB-3 direction is simply given by

\[
V_{UFB-3} = (m_2^2 - \mu^2 + m_{L_i}^2)|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}}(m_{L_j}^2 + m_{e_j}^2 + m_{L_i}^2)|H_2|
\]

\[
- \frac{2m_{L_i}^2}{g'^2 + g_2^2} .
\]

Otherwise

\[
V_{UFB-3} = (m_2^2 - \mu^2)|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}}(m_{L_j}^2 + m_{e_j}^2)|H_2|
\]

\[
+ \frac{1}{8}(g'^2 + g_2^2) \left[|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}}|H_2|\right]^2 .
\]

In [3] and [4] \( \lambda_{e_j} \) is the leptonic Yukawa coupling of the \( j \)-generation and \( m_2^2 \) is the sum of the \( H_2 \) squared soft mass, \( m^2_{H_2} \), plus \( \mu^2 \). Then, the UFB–3 condition reads \( V_{UFB-3}(\hat{Q} = \hat{\tilde{Q}}) > V_{real \_min} \), where \( V_{real \_min} \) was given above and the \( \hat{Q} \) scale is given by

\[
\hat{Q} \sim \text{Max}(g_2|\epsilon|, \lambda_{top}|H_2|, g_2|H_2|, g_2|L_i|, M_S) \text{ with } |\epsilon|=\sqrt{\frac{|\mu|}{\lambda_{e_j}}|H_2|} \text{ and } |L_i|^2=\frac{4m_{L_i}^2}{g'^2 + g_2^2} +(|H_2|^2 + |\epsilon|^2). \]

Notice from [3] and [4] that the negative contribution to \( V_{UFB-3} \) is essentially given by the \( m_2^2 - \mu^2 \) term, which can
be very sizeable in many instances. On the other hand, the positive contribution is dominated by the term $\propto 1/\lambda_e^j$, thus the larger $\lambda_e^j$ the more restrictive the constraint becomes. Consequently, the optimum choice of the $e$–type slepton is the third generation one, i.e. $e_j = \text{stau}$.

Now, we will analyze numerically the above constraints. We will see that they are very important and, in fact, there are extensive regions in the parameter space which are forbidden. Our analysis will be quite general in the sense that we will consider the whole parameter space of the MSSM with the only assumption of universality, i.e. $m$, $M$, $A$, $B$. Let us remark, however, that the constraints reviewed above are general and they could also be applied for the non–universal case. In Fig.1 we have presented in detail, as a guiding example, the (well–known minimal supergravity) case $B = A - m$ with $m=100$ GeV to get an idea of how strong the different constraints are and then we will vary $B$ and $m$ freely in order to obtain the most general results. The excluded regions are plotted in the remaining parameter space ($A/m$, $M/m$). It is worth noticing here that even before imposing CCB and UFB constraints, the parameter space is strongly restricted by the experiment as explains in the Figure caption. The restrictions coming from the UFB constraints are very strong. Most of the parameter space is in fact excluded by the UFB–3 constraint. Notice from Fig.1 that there are areas that are simultaneously constrained by different types of bounds. Besides, the values of $A$ and $M$ are both bounded from below and above in a correlated way. At the end of the day, the allowed region left (white) is quite small.

In order to show now that the CCB and UFB constraints put important bounds not only on the value of $A$ and $M$, but also on the values of $B$ and $m$, we generalize the previous analysis by varying first the value of $B$. For a particular value of $m$, the larger the value of $B$ the smaller the allowed region becomes. In general, for $m \lesssim 500$ GeV (larger values of $m$ would conflict absence–of–fine–tuning requirements for electroweak breaking), $B$ has to satisfy the bound $|B| \lesssim 3.5 m$. The results also indicate that the smaller the value of $m$, the more restrictive the constraints become. In fact, it is possible to find a value of $m$ for which the whole parameter space turns out to be excluded. This interesting lower bound on $m$ is $m \geq 50$ GeV. From this discussion it is evident that the limiting case $m = 0$ is also excluded. Of course, this has obvious implications for no–scale supergravity models since that limit is usually obtained. Figures illustrating these numerical results, as well as a discussion about the physical reasons underlying them, can be found in [2].

Finally, let us remark that the previous analyses were performed assuming universality of the soft terms at $M_{\text{GUT}}$. As mentioned in the introduction, the larger the initial scale for the running of the soft terms is, the stronger the bounds become. This can be understood from our
Figure 1. Excluded regions in the parameter space of the MSSM, with $M_{\text{phys}}^{\text{top}} = 174$ GeV. The central black region is excluded because there is no solution for $\mu$ capable of producing the correct electroweak breaking. The upper and lower black regions are excluded because it is not possible to reproduce the experimental mass of the top due to the infrared fixed point of $\lambda_{\text{top}}$. The filled diamonds indicate regions excluded by the experimental lower bounds on SUSY particle masses. The small filled squares indicate regions excluded by UFB constraints, mainly the UFB-3 one. The circles indicate regions excluded by CCB constraints.

The discussion about the UFB-3 direction above: the larger the initial scale for the running is, the more important the negative contribution $m_{1/2}^2 - \mu^2$ to the potential (see (5) and (6)) becomes. In particular, in the standard supergravity framework, where SUSY is broken in a hidden sector, the natural initial scale to implement the boundary conditions for the soft terms is $M_P \equiv M_{\text{Planck}}/\sqrt{8\pi}$ rather than $M_{\text{GUT}}$. Using the scale $M_P$ the constraints are substantially increased. For instance, regions of large $M$ which were previously allowed for $m > 100$ GeV become now completely excluded, also the above bounds on $B$ and $m$ become $|B| \lesssim 3m$ and $m \geq 55$ GeV respectively. Figures illustrating these results can be found in [5].

The CCB and UFB constraints can be applied to particular SUSY scenarios as mentioned in the introduction. For instance, in the case of the infrared fixed point model, the parameter space turns out to be severely
constrained, including the bound $|M/m| \lesssim 1$. Figures can be found again in [5]. SUSY/string scenarios are reviewed in the next section.

3. Charge and color breaking in superstrings

Let us briefly review the basic ingredients required for this analysis. First we will concentrate on the form of soft SUSY–breaking terms. The general form of the soft SUSY–breaking Lagrangian in the context of the MSSM for instance is given by

$$L_{\text{soft}} = \frac{1}{2} \sum_{a} M_{a} \lambda_{a} \lambda_{a} + \text{h.c.} - V_{\text{soft}},$$

where $\lambda_{a}$ are gaugino canonically normalized fields and $V_{\text{soft}}$ is given in (2). The above soft parameters are free in the context of the pure MSSM but can be obtained dynamically in a supergravity theory through the spontaneous breaking of local SUSY in a hidden sector [9]. In supergravity models obtained from superstring compactifications there is a natural hidden sector built-in: the complex dilaton field $S$ and the complex moduli fields $T_{i}$. Assuming that the auxiliary fields of those multiplets are the seed of SUSY breaking, interesting predictions about soft terms are obtained.

Let us first focus on the very interesting case where the dilaton field is the source of all the SUSY breaking [9]. Since, at string tree–level, the dilaton couples in a universal manner to all particles, this limit is quite model independent. The soft parameters are:

$$m_{3/2}^{2} = \frac{m_{3/2}^{2}}{2}, \quad M_{a} = \pm \sqrt{3} m_{3/2}, \quad A_{a\beta\gamma} = -M_{a},$$

where $A_{a\beta\gamma} = A_{u}, A_{d}, A_{e}$ in a self–explanatory notation. This dilaton–dominated scenario is attractive for its simplicity and for the natural explanation that it offers to the universality of the soft terms. Since the value of $B$ is more model dependent, it is better to take it as a free parameter in order to carry out the most general analysis.

The second basic ingredient of our analysis concerns the constraints associated with the existence of dangerous directions in the field space. These were explained in section 2 for a generic SUSY theory and therefore can be applied for any four–dimensional superstring model. In the particular case of the dilaton–dominated scenario, the restrictions coming from the UFB constraints are very strong and the whole parameter space $(m_{3/2}, B)$ turns out to be excluded. Most of it is in fact excluded by the UFB–3 constraint. Figures illustrating this result can be found in [10].

Given the above dramatic conclusions about the dilaton–dominated scenario, let us briefly discuss a possible way–out. The most straightforward possibility is to assume that also the moduli fields $T_{i}$ contribute to SUSY breaking, which is in fact a more general situation. Then the soft terms are modified, new free parameters beyond $m_{3/2}$ and $B$ appear, and possibly some regions in the parameter space will be allowed. This situation is more model dependent since different compactification schemes have different numbers and types of moduli and different couplings of them to
matter, therefore giving rise to different soft terms. In the simple case of (0,2) symmetric Abelian orbifolds with diagonal moduli and matter metrics the soft terms have been computed. To assume that SUSY breaking is equally shared among $T_i$'s, i.e. the “overall modulus” $T$ scenario is a good starting point in the analysis of charge and color breaking since essentially only one more free parameter must be added [11].

4. Final comments and outlook

We have shown in this review that charge and color breaking constraints on the parameter space of generic SUSY theories are very strong. This is particularly true in the case of SUSY theories deriving from weakly coupled heterotic superstring where information about the structure of soft terms can be obtained. Since the dilaton–dominated SUSY–breaking scenario is excluded on these grounds, it would be very interesting to study possible way–outs to the previous dramatic conclusion. As mentioned in section 3 one possibility is to assume that also the moduli fields contribute to SUSY breaking [11]. Another possibility, is to think that the perturbative and non-perturbative corrections to the “standard” string tree–level dilaton–dominated scenario are important and can modify the previous conclusions. Actually, some one–loop string corrections have been calculated for orbifold models and they are rather small for sensible values of the moduli. However, this is not the case for the string non-perturbative corrections, whose size could be much larger, modifying in a sensible way the formulas for soft terms [12]. Finally, recently some information has been obtained in the sense that all superstring theories seem to correspond to some points in the parameter space of a unique strongly coupled eleven–dimensional underlying theory, M–theory. Once the structure of soft terms is known charge and color breaking constraints should be applied to determine their phenomenological viability.

All the strong constraints on the soft parameter space of SUSY theories that have been reviewed here come from the requirement that the standard vacuum is the global minimum of the theory. In this sense, one possibility to avoid some of the above constraints is to accept that we live in a metastable vacuum, provided that its lifetime is longer than the present age of the universe [13], thus rescuing points in the parameter space. In order to carry out this study one might consider two possibilities: quantum tunneling at zero temperature from the standard vacuum to the charge and color breaking one and thermal effects in the hot early universe. Regarding the latter, although there is a thermal energy to cross the barrier, due to the high temperature of the early universe, the barrier is also higher [14]. In the case of quantum tunneling at zero temperature, for instance the CCB minima associated with the top–quark Yukawa coupling are the only ones to
which the standard vacuum might decay within the lifetime of the universe. The CCB minima associated with other Yukawas are deeper but the height of the barrier ($h \sim 1/\lambda^2$) is too large to allow an efficient tunnelling probability ($\sim e^{-ch}$). In this sense the bounds that we reviewed here are basically the most conservative ones (in the sense of safe ones). Needless to say that in any case, the identification of the dangerous CCB and UFB minima is the first necessary step for the cosmological analysis. In the context of gauge–mediated SUSY–breaking models the standard vacuum seems to be stable cosmologically, but only if certain couplings are sufficiently small.

Let us remark however that the possibility of living in a metastable vacuum poses several problems. First of all, as was first suggested in [10], it is hard to understand how the cosmological constant is vanishing precisely in such local “interim” vacuum. Even if a solution to that problem is found we would still have to face the rather bizarre (but mathematically possible) situation of a future cosmological catastrophe, which does not seem very attractive (at least for our descendants!). Finally, from a more scientific (and less philosophical) point of view one needs to explain (without invoking an anthropic principle) how does the universe manage to reach the standard minimum in the first place in spite of being local and metastable. This requires the analysis of all possible cosmological scenarios. In particular one can consider scenarios where the initial conditions are dictated by thermal effects or inflationary scenarios. In the former the standard vacuum is the closest one to the origin and therefore it is the thermal equilibrium state at large temperatures. The inflationary scenario may be much more dangerous and involved due to large fluctuations of all the scalar fields, that could be driven in this way to the dangerous minima. Whether this is the case or not is a complex issue that is hotly discussed.

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