Entropic corrected Newton’s law of gravitation and the Loop Quantum Black Hole gravitational atom

R.G.L. Aragão, C.A.S.Silva

*Instituto Federal de Educação Ciência e Tecnologia da Paraíba (IFPB),
Campus Campina Grande - Rua Tranquilino Coelho Lemos, 671, Jardim Dinâmica I.

Abstract

One proposal by Verlinde [1] is that gravity is not a fundamental, but an entropic force. In this way, Verlinde has provide us with a way to derive the Newton’s law of gravitation from the Bekenstein-Hawking entropy-area formula. On the other hand, since it has been demonstrated that this formula is susceptible to quantum gravity corrections, one may hope that these corrections could be inherited by the Newton’s law. In this way, the entropic interpretation of Newton’s law could be a prolific way in order to get verifiable or falsifiable quantum corrections to ordinary gravity in an observationally accessible regimes. Loop quantum gravity is a theory that provide a way to approach the quantum properties of spacetime. From this theory, emerges a quantum corrected semiclassical black hole solution called loop quantum black holes or self-dual black holes. Among the interesting features of loop quantum black holes is the fact that they give rise to a modified entropy-area relation where quantum gravity corrections are present. In this work, we obtain the quantum corrected Newton’s law from the entropy-area relation given by loop quantum black holes. In order to relate our results with the recent experimental activity, we consider the quantum mechanical properties of a huge gravitational atom consisting in a light neutral elementary particle in the presence of a loop quantum black hole.
1. Introduction

Since the rising of black hole thermodynamics, in the seventies, through the Hawking demonstration that all black holes emit blackbody radiation \[2\], investigations about these objects break up the limits of astrophysics. In fact, black holes have been put in the heart of the debate of the most fascinating issues in theoretical physics. Among these issues, the search for a better understanding of the quantum nature of gravity, since the quantum behavior of spacetime must be revealed within the presence of a black hole strong gravitational field.

Among the most important lessons from black hole thermodynamics, arises the Bekenstein-Hawking formula which establishes that, in a different way from other usual thermodynamical systems, the entropy of a black hole is not given as proportional to its volume, but to its horizon area: \[ S = k_B c^3 A / 4 h G \]. A deep intersection between gravity, quantum mechanics, and thermodynamics could be contained in Bekenstein-Hawking formula, since it gives us one of the few situations in physics where the Newton’s gravitational constant \(G\) and the speed of light \(c\) meet the Planck constant \(\hbar\) and the Boltzmann constant \(k_B\). In fact, it has been shown by String theory and Loop Quantum Gravity that the black-hole thermodynamics must have its origin in the atomic structure of the spacetime \[3, 4, 5\]. Moreover, in \[6, 7, 8\] it has been argued that a topology change process due to the dynamics of the quantum spacetime could be the origin of black hole entropy and the Generalized Second Law of black hole thermodynamics.

In 1995, a surprising result by Jacobson has deepened the significance of the Bekenstein-Hawking formula. Assuming the proportionality between entropy and horizon area, Jacobson derived the Einstein’s field equations by using the fundamental Clausius relation \[3\]. The procedure behind this result is to require that the Clausius relation, \(\delta Q = T dS\), associating heat, temperature and entropy, holds for all the local Rindler causal horizon through each spacetime point, with \(\delta Q\) and \(T\) interpreted, respectively, as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. In this way, the spacetime could be viewed as a kind of gas whose entropy is given by the Bekenstein-Hawking formula, and the Einstein’s field equation as an equation of state describing this gas.

Following Jacobson’s results, several authors have addressed the issue of the relation of Einstein’s equations and thermodynamics (For a review and a voluminous list of references see \[10\]). More recently, Verlinde \[11\] conjectured that gravity is a non fundamental interaction but would be explained as an entropic force. In this way, the second law of Newton is obtained when one tie up the entropic force with the Unruh temperature. On the other hand, Newton’s law of gravitation is obtained when associating these arguments with the holographic principle and using the equipartition law of energy. Verlinde’s formalism, have been used in several contexts including cosmological ones \[12\].

On the other hand, by using the measurement result of quantum states of ultra-cold neutron under the Earth’s gravity, Kobakhidze presented an argument in opposition to Verlinde’s proposal \[12\]. The problem pointed by Kobakhidze comes from the fact that the entropy formula defined by Verlinde formalism, in principle, leads to a quantum neutron mixed state. However, it disagrees with the results from the ultra-cold neutron experiment. Kobakhidze’s criticism have been questioned in \[13\] and one resolution was suggested by Abreu et al \[14\]. This resolution can be found out by abandoning the implicit assumption in \[12\] that the entropy on the holographic screen is additive.

On the other hand, it is also known that, in other contexts than Einstein’s gravity, the area formula of black hole entropy may not be held. For example, when higher order curvature term appears in some gravity theory, the area formula has to be modified \[15\]. Modifications to Bekenstein-Hawking formula also appear when quantum gravity effects are included. For example, when a Generalized Uncertainty Principle (GUP) is taken into account \[16, 17\]. In this way, it was investigated modifications of the entropic force due to corrections imposed on the area law by quantum effects and extra dimensions \[18\]. Quantum gravity corrections to Bekenstein-Hawking formula also appear in the context of Loop Quantum Gravity. The most popular form to these corrections appear as logarithmic corrections which arises due to thermal equilibrium fluctuations and quantum fluctuations \[19, 20, 21, 22\].

Another way to get quantum corrections to Bekenstein-Hawking formula, which we shall follow in this work, arises in the context of loop quantum black holes \[23, 24, 25, 26, 27, 28\]. A loop black hole, also called self-dual black hole, consists in a quantum gravity corrected Schwarzschild black hole that appears from a simplified model of Loop Quantum Gravity. One of the most interesting results of the loop black hole scenario is the resolution of the black hole singularity by the self-duality property. This property guarantees that an asymptotic flat region corresponding to a Planck-sized wormhole arises in the place of the black hole singularity. The wormhole throat is described by the Kantowski-Sachs solution. The thermodynamical properties of loop black holes has been addressed in the references \[24, 25, 26, 27, 28\]. Moreover, in the reference \[29\], the thermodynamical properties of loop quantum black holes were obtained by the use of a tunneling method with the introduction of back-reaction effects. On the other hand, in the reference \[30\], the tunneling formalism has been applied in order to include corrections due to a Generalized Uncertainty Principle to loop quantum black hole’s thermodynamics. Among the results related with the thermodynamics of loop black holes, we have a quantum corrected Bekenstein-Hawking...
formula for the entropy of a black hole in which quantum
gravity ingredients have been included.

Experimental issues related with loop quantum black
holes have also been addressed in the literature. In this
way, gravitational lenses effects due to this kind of black
holes have been investigated in [31]. On the other hand,
loop quantum black hole’s quasinormal modes have been
calculated in [32], [33] and [34]. In the last, axial gravi-
tational perturbations have been taken into account.

In the present work, we shall address how the New-
ton’s law of gravitation would be modified in the presence
of a loop quantum black hole. In particular, we
consisting in a light neutral elementary particle in the
presence of a loop quantum black hole. In particular, we
apply the Bohr Somerfeld formalism to this system, by
the use of the modified Newtons potential, in order to
obtain its energy levels.

2. Loop quantum black holes

Loop quantum black holes (LQBHs) appeared at the
first time from a simplified model of Loop Quantum Grav-
ity(LQG) [23]. The LQBH’s scenario is described by a
quantum gravitationally corrected Schwarzschild metric,
and can be written in the form

$$ds^2 = -G(r)dt^2 + F^{-1}(r)dv^2 + H(r)d\Omega^2 \quad (1)$$

with

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \quad , \quad (2)$$

where, in the equation [1], the metric functions are given by

$$G(r) = \frac{(r-r_+)(r-r_-)(r+r_*)^2}{r^4+a_0^2} \quad , \quad (3)$$

$$F(r) = \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4+a_0^2)} \quad , \quad (4)$$

and

$$H(r) = r^2 + \frac{a_0^2}{r^2} \quad , \quad (5)$$

where

$$r_+ = 2m \quad ; \quad r_- = 2mP^2 .$$

In this way, two horizons appears in the LQBH’s scenario
- an event horizon at $r_+$ and a Cauchy horizon at $r_-$. Furthermore, we have that

$$r_+ = \sqrt[r_+]{r_-} = 2mP . \quad (6)$$

where $P$ is the polymeric function given by

$$P = \sqrt{1 + \epsilon^2 - 1 \over \sqrt{1 + \epsilon^2 + 1 , \quad (7)$$

and

$$a_0 = {A_{\text{min}} \over 8\pi} \quad , \quad (8)$$

where $A_{\text{min}}$ is the minimal value of area in LQG.

In the metric [1], since $g_{\theta\theta}$ is not just $r^2$, $r$ is only
asymptotically the usual radial coordinate. From the
form of the function $H(r)$, one obtains a more physical
radial coordinate given by

$$R = \sqrt{r^2 + {a_0^2 \over r^2}} . \quad (9)$$

In this way, the proper circumferential distance is mea-
sured by $R$.

Moreover, the parameter $m$ in the solution is related to
the ADM mass $M$ by

$$M = m(1 + P)^2 . \quad (10)$$

The equation [9] reveals important aspects of the
LQBH’s internal structure. From this expression, we have
that, as $r$ decreases from $\infty$ to $0$, $R$ first decreases
from $\infty$ to $\sqrt{2a_0}$ at $r = \sqrt{a_0}$ and then increases again to $\infty$. The value of $R$ associated with the event horizon is
given by

$$R_{\text{EH}} = \sqrt{H(r_+)} = \sqrt{(2m)^2 + \left( {a_0 \over 2m} \right)^2} \quad . \quad (11)$$

A peculiar feature in LQBH’s scenario is the prop-
erty of self-duality. This property says that if one in-
trduces the new coordinates $\tilde{r} = a_0/r$ and $\tilde{t} = \tilde{t} = \sqrt{a_0}$, with $\tilde{r}_\pm = a_0/r_\pm$ the metric preserves its form. The dual
radius is given by $r_{\text{dual}} = \tilde{r} = \sqrt{a_0}$ and corresponds to the
minimal possible surface element. Moreover, since the equation [9] can be written as $R = \sqrt{r^2 + \tilde{r}^2}$, it is
clear that, in the LQBH’s scenario, we have another
asymptotically flat Schwarzschild region in the place of
the singularity in the limit $r \to 0$. This new region cor-
responds to a Planck-sized wormhole. Figure [1] shows
the Carter-Penrose diagram for the LQBH.
The derivation of the black hole’s thermodynamical properties from the metric (1) proceeds in the usual way. The Bekenstein-Hawking temperature $T_{BH}$ can be obtained by the calculation of the surface gravity $\kappa$ by

$$T_{BH} = \frac{\kappa}{2\pi},$$

with

$$\kappa^2 = -g^{\mu\nu}g_{\rho\sigma}\nabla_\mu \chi^\rho \nabla_\nu \chi^\sigma = -\frac{1}{2}g^{\mu\nu}g_{\rho\sigma}\nabla^\rho \Gamma_{0\mu}^\sigma \nabla^\nu \Gamma_{0\nu}^\sigma,$$ (12)

where $\chi^\mu = (1, 0, 0, 0)$ is a timelike Killing vector and $\Gamma_{\rho\mu}^\sigma$ are the connections coefficients.

By connecting with the metric, one obtains that the LQBH temperature is given by

$$T_H = \frac{(2m)^3(1 - P^2)}{4\pi[(2m)^4 + a_0^4]}.$$ (13)

It is easy to see that one can recover the usual Hawking temperature in the limit of large masses. However, differently from the Hawking case, the temperature (13) goes to zero for $m \to 0$, as have been shown in the figure 2. In this point, we remind that the black holes ADM mass $M = m(1 + P)^2 \approx m$, since $P \ll 1$.

The black hole’s entropy can be found out by making use of the thermodynamical relation $S_{BH} = \int dm/T(m)$.

$$S = \frac{4\pi(1 + P)^2}{(1 - P^2)} \left[ \frac{16m^4 - a_0^2}{16m^2} \right].$$ (14)

Moreover, the black hole entropy can be expressed in terms of its area [28]

$$S = \pm \sqrt{A^2 - A_{min}^2} \frac{(1 + P)}{4} \frac{(1 + P)}{(1 - P)},$$ (15)

where we have set the possible additional constant to zero. $S$ is positive for $m > \sqrt{a_0}/2$ and negative otherwise.

The double possibility in the signal of the loop black hole entropy is related with the two possible physical situations that arise from LQBH structure [27]. In the first of these possibilities, the event horizon stays outside the wormhole throat. In order to have this situation, the condition $r_+ > \sqrt{a_0}$ is necessary, which implies that $m > \sqrt{a_0}/2$. In this case, the bounce takes place after the black hole forms for a super-Planckian LQBH and the exterior, is similar, in a qualitative way, to that would be produced by a Schwarzschild black hole with the same mass. In this way, outside the event horizon, the LQBH scenario is different from the Schwarzschild’s one only by Planck-scale corrections. On the other hand, in the sub-Planckian regime, we have a more instigating situation. In this case, the event horizon becomes the other side of the wormhole throat. Moreover, the deviations from the Schwarzschild metric are very expressive and the bounce takes place before the event horizon forms. Consequently, even large event horizons (which it will be for $m \ll m_P$) it will be invisible to observers at $r > \sqrt{a_0}$.

The thermodynamics properties of LQBHs has been also obtained through the Hamilton-Jacobi version of the tunneling formalism [29]. By the use of this formalism, back-reaction effects could be included. Moreover, extensions of the LQBH solution to scenarios where charge and angular momentum are preset can be found in [35].

The issue of information loss has been also addressed in the context of loop black holes. In this case, it has been pointed that the problem of information loss by black holes could be relieved in this framework [26, 29, 36]. This result may be related with the absence of a singularity in the loop black hole interior, and consists in a forceful result in benefit of this approach.

Another interesting result in the realm of LQBH is the fact that, as have been demonstrated in [37], it can been sees as the building blocks of Loop Quantum Cosmology (LQC), in the sense that, starting from the LQBH
entropy expression, LQC equations can be obtained through the use of Jacobson formalism to obtain the Einstein’s gravitational equations.

In the next sections, following the formalism developed by Verlinde [1], we will derive the quantum corrected Newton’s from the modified entropy-area relation given by the equation (15).

3. Quantum corrected Newton’s law from loop quantum black holes

Recently, Verlinde conjectured that gravity is not fundamental but can be explained as an entropic force. In this section, following the Verlind’s entropic force approach to gravity, we shall derive quantum corrected Newton’s law of gravitation from LQBHs entropy-area relation (15).

We have that in thermodynamics, if the number of states depends on position $\Delta x$, entropic force $F$ arises as thermodynamical conjugate of $\Delta x$. In this case, the first law of thermodynamics can be written as

$$ F\Delta x = T\Delta S \quad (16) $$

Based on the Bekenstein’s entropy bound, Verlinde postulated that when a test particle moves approaching a holographic screen, the change of entropy on this screen is proportional to the mass $m$ of the particle, and the distance $\Delta x$ between the test particle and the screen

$$ \Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x \quad (17) $$

to derive the entropic force hypothesis, should hold at least when $\Delta x$ is smaller than or comparable with the Compton wave-length of the particle.

The temperature that appears in (16) can be understood in two ways: one can relate temperature and acceleration using Unruh’s rule

$$ k_B T = \frac{\hbar a}{2\pi c} \quad (18) $$

or relate temperature, energy and the number of used degrees of freedom using equipartition rule

$$ E = \frac{1}{2} N k_B T. \quad (19) $$

It is necessary to point that the temperature $T$ in equations (18) and (19) have different meaning. In the first equation, the temperature is defined in the bulk. However, in the second, the temperature is defined on the holographic screen. To admit these two temperatures to be equal is an further supposition in Verlind’s paper.

From the equations (16), (17) and (18) one obtains the second Newton’s law $F = ma$. In order to obtain the Newton’s law of gravitation, one must have a way to relate the number of bits on the holographic screen with the black hole horizon area. Assuming that the number of bits on the screen is proportional to the horizon entropy, from the equation (15), we shall assume that this relation is given by

$$ N = \frac{(1 + P) \sqrt{A^2 - A_{\text{min}}^2}}{L_p^2} \quad (20) $$

In this way, from the equation above together with (16), (19) and $E = Mc^2$, we shall have

$$ F = -\frac{GMm (1 + P)}{R^2} \times \frac{1}{(1 - \frac{A_{\text{min}}^2}{16\pi^2R^4})} \quad (21) $$

Moreover, for the gravitational potential $V(r) = -\int F(R)dR$, we shall have

$$ V(r) = -\frac{GMm (1 + P)}{R} \left( 1 - \frac{A_{\text{min}}^2}{160\pi^2R^4} - \frac{A_{\text{min}}^4}{18432\pi^4R^9} + ... \right) $$

In this way, corrections to Newton’s gravitational law can be obtained from LQBH entropy-area relation. As we can see, the deviations on the Newton’s law depend on the value of minimal area $A_{\text{min}}$ in LQG, as well as on the polymeric parameter $P$. In this way, the corrections found out are important in the case of submillimeter distances, even though it could be realized in the context of large distances through the dependence on the parameter $P$.

4. The loop quantum black hole atom

In the seventies, Hawking introduced the possibility that a free charged particle could be capture by a primordial charged black hole forming neutral and non-relativistic ultra-heavy black hole atoms [38]. After, the term gravitational atom was coined by V. V. Flambaum and J. C. Berengut in 2001 [39] for gravitationally bound neutral black hole and a charged particle.

An interesting fact about gravitational atoms is that they have been pointed as an important constituent of dark matter. In fact, primordial black hole remnants left after the Hawking evaporation have been considered as a source of dark matter by several authors for more than two decades [40] [41] [42] [43] [44] [45] [46] [47] [48] [49] (for a review see [50] [51] [52]). However, a central question is whether some remnants could leave after the Hawking evaporation have been considered as a source of dark matter by several authors for more than two decades [40] [41] [42] [43] [44] [45] [46] [47] [48] [49] (for a review see [50] [51] [52]). However, a central question is whether some remnants could leave after the Hawking evaporation have been considered as a source of dark matter by several authors for more than two decades [40] [41] [42] [43] [44] [45] [46] [47] [48] [49] (for a review see [50] [51] [52]). However, a central question is whether some remnants could leave after the Hawking evaporation have been considered as a source of dark matter by several authors for more than two decades [40] [41] [42] [43] [44] [45] [46] [47] [48] [49] (for a review see [50] [51] [52]).
for a Schwarzschild black hole the temperature always increases as its mass decreases and vice versa (see the dashed line in the fig. (2)).

On the other hand, new phenomenon emerges in the LQBH scenario. From equation (13), all light enough LQBHs would radiate until their temperature cools until the point it would be in thermal equilibrium with the CMB. In fact a stable thermal equilibrium occurs for a point it would be in thermal equilibrium with the CMB. In fact a stable thermal equilibrium occurs for a point it would be in thermal equilibrium with the CMB. In fact a stable thermal equilibrium occurs for a black hole mass given by $m_{\text{stable}} \approx 10^{-19} \text{kg}$. Based on this feature of LQBH, Modesto et al have yet pointed to the possibility that these objects could be an important component of dark matter [24]. In this way, one could think about the possibility of gravitational atoms where a LQBH could appear as the atomic nucleus.

In order to give a first glance on these kind of system, let us use the expression for the gravitational force between a LQBH and a neutral particle orbiting it given by the equation (21):

$$F = \frac{G m (1 + P)}{R^2 (1 - P)} \times \frac{1}{\sqrt{1 - A_{\text{min}}^2 / 16\pi^2 R^4}} = \frac{m v^2}{R},$$

where $v$ is the particle velocity in the orbit.

In this way, we shall have:

$$v = \left[ \frac{(1 + P) G M}{(1 - P) R} \right]^{1/2} \times \left( 1 - A_{\text{min}}^2 / 16\pi^2 R^4 \right)^{-1/4}$$

Using the Bohr-Sommerfeld quantization method, $m v R = j \hbar$, we shall get the following equation

$$R^6 - \frac{(1 - P)}{(1 + P)} \left( \frac{G M m}{2} \right)^2 R^4 + \frac{(1 - P)}{(1 + P)} \left( \frac{G M m}{2} \right)^2 = 0,$$

whose only real solution is

$$R_j = \frac{\hbar^4 j^4 (P - 1)}{3 m^4 G^2 M^2 (P + 1)} + \left( \frac{\hbar^4 j^4 A_{\text{min}} (P - 1) \times \sqrt{27 m^8 A_{\text{min}}^2 G^4 M^4 (P + 1)^2 - 4 \hbar^8 j^8 (P - 1)^2}}{2 (3^{3/2}) m^8 G^4 M^4 (P + 1)^2} + \frac{\hbar^4 j^4 (1 - P) [2 \hbar^8 j^8 (P - 1)^2 - 27 m^8 A_{\text{min}}^2 G^4 M^4 (P + 1)^2]}{54 m^12 G^6 M^6 (P + 1)^3} \right)^{1/4} + \frac{\hbar^12 j^{12} (P - 1)^3}{9 m^8 G^4 M^4 (P + 1)^3} \times \left( \frac{\sqrt{27 m^8 A_{\text{min}}^2 G^4 M^4 (P + 1)^2 - 4 \hbar^8 j^8 (P - 1)^2}}{2 (3^{3/2}) m^8 G^4 M^4 (P + 1)^2} - \frac{\hbar^8 j^8 (P - 1)^2 - 27 m^8 A_{\text{min}}^2 G^4 M^4 (P + 1)^2}{54 m^{12} G^6 M^6 (P + 1)^3} \right)^{1/3} \right)^{1/2}$$

and can be expanded as

$$R_j = \frac{\sqrt{(P + 1) (P - 1) \hbar^2 j^2}}{m^2 G M (P + 1)} - \frac{\sqrt{(P + 1) (P - 1) m^6 G^4 M^4 (P + 1)}}{2 \hbar^8 j^8 (P - 1)^2} A_{\text{min}}^2 + \cdots$$

where the first term corresponds to the usual gravitational atom radius, unless the $P$ parameter factors.

The energy levels $E_j$ of the LQBH gravitational atom are obtained from the expressions (22), (23) and (25).

$$E_j = \frac{1}{2} m v^2 + V = - m^3 G^2 M^2 (P + 1)^{3/2} + m^{11} G^6 M^4 (P + 1)^{7/2} A_{\text{min}}^2 + m^{11} G^6 M^6 (P + 1)^{7/2} \times$$

$$\left[ 15 m^4 G^4 M^4 (P + 1)^2 - (512 \pi^2 - 128) \hbar^8 j^8 \pi^2 (P - 1)^2 \right] + \cdots$$

where the first term corresponds to the usual expression to the gravitational atom energy levels (unless the dependence on the polymeric parameter), which can be obtained in the limit where the quantum gravity corrections goes to zero.

5. Conclusions and Remarks

We have derived quantum corrected Newton’s graviation law from the LQBH’s entropy-area relation using the Verlinde entropic force interpretation to gravity. Our results points to some quantum deviation from classical Newton’s law that must have a important rule in submillimeter distances where Newton’s gravitation theory has not been tested yet.

Due to its self-duality property, LQBHs can have a mass lower than Planck mass. Particularly, for $m_{\text{stable}} \approx 10^{-19} \text{kg}$, a LQBH would assume a stable thermal equilibrium with the CMB, which makes possible that this kind of black holes can be seen as a good candidate for dark matter. In this way, impelled by the current experimental activity, we investigate the energy spectrum of a huge gravitational atom composed by a neutral particle orbiting a LQBH. As have been demonstrated, this frequency depends on the quantum gravitational corrections inherited from the LQBH metric.

6. Acknowledgements

The authors would like to thank to Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPQ/Brazil for the financial support.
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