Active Magnetic Bearing Design and Backstepping-Adaptive Control for High-Speed Rotors

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Abstract. Friction and vibrations hindering high-speed are the most pertinent problems facing the rotating machines. Thus, friction and vibration attenuation are essential in improving the overall performance of turbomachines. Conventionally, high-speed flywheel energy storage systems use Active Magnetic Bearings (AMB) to nullify friction losses and to deal with unbalances but, in the present era with increasing demand for high-speed machinery, the applications of AMBs are proliferating. They are not like the traditional bearings; they generate forces through magnetic fields with no contact between bearing and rotor thus decreasing the friction and providing the ability to counteract imbalances actively. This paper investigates an optimized design methodology along with backstepping adaptive control design of an 8-pole radial AMB used for flywheel energy storage systems. Design optimization using GA, along with analytical design constraints are presented. The design details along with finite element simulations are given to verify the optimization results and system requirements. Bearing parameters then obtained are used to develop a linearized mathematical model of the system, an adaptive back-stepping controller is designed for it to regulate the deviation of the rotating shaft from its equilibrium position.

1. Introduction
Active Magnetic Bearing (AMB) is being extensively used in many rotating machine applications, mainly because of their non-contact motion control characteristics. As a consequence, these properties allow novel constructions, high speeds with the possibility of active vibration control, operation with no mechanical wear, less maintenance and, therefore, lower costs. Due to above-mentioned properties Active magnetic bearing (AMB) is a promising solution for many kinds of potentially promising machines such as turbine engines, helicopter, Agile satellites, vacuum pump, motors and flywheel energy storage systems (FESS) [1-7]. Many researchers have presented the capability of producing a controllable electromagnetic force by AMB, among them, Nickolajsen et al. [8] in 1979 introduced its use for vibration damping for the first time. Many authors have investigated their use for vibration suppression [9-12]. They have demonstrated the use of AMB as between bearings to introduce damping and stiffness into the system and compensate for rotor unbalance; their focus has been on suppressing the vibration by changing the pole currents based on position feedback. An active magnetic bearing is purely a Mechatronics product; The hardware is composed of electromechanical components: electromagnets sensors amplifiers and an information processing part, usually a microcontroller [8]. AMB is inherently an unstable system; therefore, a control algorithm is essential.
In the past many researchers have done work on hardware designs and many researchers have done the controller designs. However, an active magnetic bearing system is a mechatronics product; the design involves multidisciplinary challenges; which makes it a thought-provoking task. Hence, this paper presents a design methodology of 8-Pole Radial AMB for an 8kg high-speed composite flywheel presented in [7].

The developed AMB will be exhibited in the following sections. The second section will present a brief overview of the AMB operation. The third section will show the hardware optimization along with Finite element analysis. The fourth section will elaborate the analytical modelling of a designed AMB depicting the relation between current and force generation in electromagnets. In addition, a control technique that combines the merits of both backstepping and adaptive control laws is presented. Finally, concluding remarks are added to summarize the present research.

2. Operation of an Active Magnetic Bearing (AMB)

![Figure 1. (a) 1-DOF Radial AMB model in Differential Driving Mode (b) Radial-AMB Design Sketch](image)

Fig. 1 (a) shows a simple 1-DOF AMB magnetic actuator model. Since the principle for vertical and horizontal plane forces is the same, for simplicity, only 1-DOF AMB is considered here. In Fig.1, i is the coil current, so is a nominal air gap, N is the number of coil rounds on the core, Ap represents the cross-section area of pole face and the angle of each pole relative to the centreline between the poles is \( \theta_p \). The magnetic field generated by the current will create a force in the horizontal axis. According to Ampere’s loop law, we have (1), where \( H \) is the magnetic field, which involves the flux density \( B \).

According to constitutive Law:

\[
B = \mu_0 \mu_r H
\]  

(1)

Ampère’s circuitual law [16]:

\[
\oint H \, ds = l t e H_{fe} + 2s_o H_a = NI
\]

(2)

Combining (1) and (2) will result in a relation (3):

\[
B = \mu_0 \frac{NI}{\mu_r^{-1} 2s_o}
\]

(3)

As \( \mu_r >> 1 \), Hence magnetization of Iron is neglected, resulting in a more simplified form as:

\[
B \approx \frac{\mu_0 NI}{2s_o}
\]

(4)

Thus, the energy stored in the airgap is presented as:

\[
E = \frac{1}{2} B_a H_a (A_p 2s_o)
\]

(5)

The force acting on the ferromagnetic body (\( \mu_r >> 1 \)) is generated by a change of the field energy in the air gap, as a function of the body displacement. For small displacements \( ds \), the magnetic flux \( B_a A_p \) remains constant. When the air gap \( s \) increases by \( ds \), the energy \( E \) in the field increases by an amount \( dE \). This energy has to be provided mechanically by electromagnets. Thus, Force \( F \) is equalled:

\[
F = \frac{\delta E}{\delta s} = B_a H_a A_p = \frac{B_a A_p}{\mu_0} = \frac{1}{4} \mu_0 N^2 A_p \left( \frac{i^2}{s_o^2} \right) = \frac{K i^2}{4 s_o^2}
\]

(6)

In the case of a Radial AMB, the forces of both magnetic poles affect the rotor with an angle \( \theta_p \). So,

\[
F = \frac{K i^2}{4 s_o^2} \cos \theta_p
\]

(7)
3. Active Magnetic Bearing Design for Flywheel

3.1. Design Optimization

In practice, the design of an AMB encounters several restrictions due to peripheral equipment or actuator itself and these restrictions interfere with one another such that tweaking one parameter will also have an effect on other parameters. Therefore, a compromise between different parameters is to be made in order to reach an ideal solution depending upon the application of AMBs. As part of the design process, this section will deal with the design optimisation of a Radial AMB. AMBs are designed to achieve higher load carrying capacity and compact size. Multi-Objective Genetic Algorithm optimisation strategy is used in Matlab®. The preliminary design conditions used in R-AMB design procedure as follows: Two Radial AMBs are to be used for a Flywheel system of total mass (m=10kg) presented in [7]. The nominal air gap is to be kept at (s_o=1mm). Diameter of the shaft is (d=40mm). Number of poles (N_p=8) and Silicon iron is to use with Saturation Flux density of (B_s=1.8Tesla). Fringing-leakage factor is assumed to be (\epsilon=0.8) and permeability (\mu_o=4\pi \times 10^{-7}). Along with the initial conditions, optimization method requires a set of constraints which are part of the design methodology adopted in developing the Radial AMB depicted as follows:

The allowable magnetomotive force:  
\[ MMF = \frac{2s_oB_{sat}}{\mu_o} \]  

Required Coil Diameter:  
\[ \phi = 2 \times \frac{l_{max}}{\sqrt{\pi J_{cu}}} \]  

Minimum Area of Pole:  
\[ A_p \geq \frac{F_{max}\mu_o}{B_{sat}^{2}(\cos\theta_p)} \]  

AMB Pole length and width:  
\[ w_p \leq 0.8(d_r+2s_o)\sin\theta_p - 2.5(s_o\cos\theta_p), \quad L_p \frac{A_p}{w_p} \]  

Required Number of turns:  
\[ N \leq \frac{MMF}{l_{max}} \]  

Winding space limit:  
\[ A_r < A_w \]  

Where \( A_r \) and \( A_w \) are the required winding space and available winding space respectively. Assume the bulking factor as 80% and most effective winding space as the trapezoid region then the winding space available will be presented by equation (14).

\[ A_w = 0.8\left[(d_r+2s_o+l)\sin\theta_p - w_p\cos\theta_p\right]l\cos\theta_p, \quad A_r = 2N \frac{\pi s_o^2}{4} \]  

The diameter of the stator is given by:  
\[ D_{st} = d_r + 2(s_o + w_p + 1.15l) \]  

Objective of the optimization is to minimize the size of the AMB and maximize the force capability within the constraints from eq. (8-17). Considering an AMB as cylinder, the objective of the optimization is to minimize the total volume and to maximize the force produced and get the optimized value of Number of turns ‘N’, current ‘I’, Area ‘A_p’ and Pole width ‘w_p’.

Cost Function 1: \( \text{min}(V) = \frac{\pi d_r^2 A_p}{4w_p} \)  

Cost Function 2: \( \text{max}(F) = \frac{\kappa l^2}{4s^2} \cos\theta_p \)  

Fig.3 shows the optimization results in the form of the Pareto-diagram and Table 1.0 shows design parameters obtained by the preliminary design, GA optimization.

| TABLE 1. Optimization Results |
|-----------------------------|
| PARAMETER                  |
| Current (I_{max})          |
| Number of turns (N)        |
| Area of Pole (A_p)         |
| Width of Pole (W_p)        |
| Length of pole (l)         |
| Stator dia. (D_{st})       |
| Coil dia. (\phi)           |
| Max. Force (F)             |
| UNITS                      |
| A                          |
| N                          |
| mm^2                       |
| mm                         |
| mm                         |
| mm                         |
| mm                         |
| N                          |
| OPTIMUM VALUE              |
| 4                          |
| 360                        |
| 300                        |
| 10                         |
| 33                         |
| 138                        |
| 0.9                        |
| 140                        |
3.2. Finite Element Analysis (FEA)

The AMB model based on the parameters presented in Table 1 is designed and magneto-static analysis is done to simulate the magnetic flux density and magnetic force density. In Fig. 3 the top-plots show the flux density distribution and bottom plots show the force density distribution in the designed AMB and rotor lamination.

![Figure 2. Pareto Front of GA optimization for AMB](image)

**Figure 2.** Pareto Front of GA optimization for AMB

![Figure 3.](image)

**Figure 3.** (a) Flux density distribution, Force density distribution in AMB Rotor-Lamination for $i_x=0A$, $i_y=0A$ (b) Flux density distribution in AMB Force density on Rotor-Lamination for $i_x=0A$, $i_y=2A$. 
To ensure that the rotor is capable of high rotational speeds, the losses should be minimized. To do so, the magnetic circuit is laminated. Silicon Iron laminations are used both in magnetic bearing and on the rotor. Using laminations on Rotor; in the area where AMB’s magnetic field is to be applied results in lower flux loss and helps to avoid excess rotor heating resulting in the long-life span of rotor.

Following the control methodology of Fig 1(a), In the nominal operation when the rotor is at the desired position and reference current $i_1$ and $i_2$ are set to zero, biased current $i_o$ set to $2\text{A}$ then the resulting force-flux distribution is presented in Fig 3(a). It can clearly be seen that the flux is evenly distributed along all axes around the stator and the rotor lamination. Similarly, Fig 3(a) bottom plot presents a detailed view of force density distribution in whole AMB and Rotor lamination. There are attractive magnetic forces between stator poles and rotor lamination, but as amplitudes of these attractive radial forces are the same and the directions are equally distributed, the resultant radial force acting on the rotor is zero. Hence, the rotor will always be stabilized at its equilibrium position unless an external disturbance occurs.

As soon as disturbance occurs and the rotor moves in the negative $y$-axis. The controller then set $i_o$ to $1\text{A}$, the new flux and force distribution plots are given in Fig.3(b). The flux density is increased in the positive $y$-direction side and decreases in the negative $y$-direction side. The magnetic forces in the two air gaps in the $y$-direction are no longer equal and the force in the positive $y$-direction is more significant than that negative $y$-direction as shown in the force density plot in Fig.3(b). Hence, a radial force $F$ equals to 94-Newton is produced in the positive $y$-axis. In both of the plots, the maximum magnetic field produced is not more than 1.4 Tesla which is below the saturation point and force density is strong enough to lift the rotor.

4. Controller Design

4.1. Strict Feedback, AMB Model

For designing an Adaptive Backstepping Controller, a mathematical model of AMB in strict feedback form is developed. Fig.1(a) shows the nominal position of the rotor is $s_o$ and $m$ is the rotor’s mass, but if a disturbance, $F_d$ occurs and displaces the rotor by an amount $s$, then according to Newton’s law:

$$ m\ddot{s} = (F_1 - F_2) + (F_d) \tag{18} $$

Let $s_1$ and $s_2$ are now the air gaps between the rotor and stator from top and bottom, respectively. Replacing $s$ in (7) with $s_1$ and $s_2$, we can derive the two electromagnetic forces $F_1$ and $F_2$:

$$ F_1 = \frac{K}{4} \left( \frac{L_s}{s_1} \right)^2, \quad F_2 = \frac{K}{4} \left( \frac{L_s}{s_2} \right)^2 \tag{19} $$

By Kirchoff’s Voltage Law (KVL), we have:

$$ u_1 = R\dot{i}_1 + L_s \frac{d\dot{i}_1}{dt} + \frac{K}{2} \frac{d}{dt} \left( \frac{i_1}{s_1} \right), \quad u_2 = R\dot{i}_2 + L_s \frac{d\dot{i}_2}{dt} + \frac{K}{2} \frac{d}{dt} \left( \frac{i_2}{s_2} \right) \tag{20} $$

Where: $u_1$ and $u_2$ are input voltages to control the two currents $i_1$ and $i_2$ and $\frac{K}{2} \frac{d}{dt} \left( \frac{i_1}{s_1} \right)$ and $\frac{K}{2} \frac{d}{dt} \left( \frac{i_2}{s_2} \right)$ represents back-electromotive force generated by the air gap flux changes. Nominal states of the systems are represented by $(s_o, i_o, \mu_o)$, but as the system will get disturbed, the new states will be presented by $(s_1, i_1, \mu_1)$ and $(s_2, i_2, \mu_2)$ for top and bottom electromagnets respectively.

$$ s_1 = s_o - s, \quad s_2 = s_o + s \tag{21} $$

$$ i_1 = i_o + i, \quad i_2 = i_o - i \tag{22} $$

$$ u_1 = u_o + u, \quad u_2 = u_o + u \tag{23} $$

Putting (21~23) into (20) and (19) into (18) will result in formulation of a nonlinear AMB model:

$$ \dot{s} = \nu$$

$$ \begin{align*}
    \dot{i}_1 &= \frac{2(s_o-s)}{2L_s(s_o-s)+K} \left[ -R\dot{i}_1 + \frac{K}{2(s_o-s)^2} v\dot{i}_1 + u_1 \right] \\
    \dot{i}_2 &= \frac{2(s_o+s)}{2L_s(s_o+s)+K} \left[ -R\dot{i}_2 + \frac{K}{2(s_o+s)^2} v\dot{i}_2 + u_2 \right]
\end{align*} \tag{24}$$
Where \( v \) is the linear velocity of the rotor. By using Jacobian transformation, (24) can be linearized at its equilibrium states as:

\[
\begin{bmatrix}
\hat{s} \\
\hat{v}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & c \\
0 & d & a
\end{bmatrix} \begin{bmatrix}
\hat{s} \\
\hat{v}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
e
\end{bmatrix} u + \begin{bmatrix}
0 \\
0 \\
f
\end{bmatrix} F_s
\]  
(25)

Where:

\[
a = \frac{2k_i}{m}, \quad b = \frac{2k_i}{m}, \quad c = \frac{-k_i}{L_0 L_s}, \quad d = \frac{-R}{L_0 L_s}, \quad e = \frac{1}{L_0} F_s = F_d,
\]

\[
k_s = \frac{K_{iq}}{2 s_0}, \quad k_i = \frac{K_{iq}}{2 s_0}
\]

and

\[
L = L_o + L_s, \quad L_0 = \frac{K}{2 s_0}, \quad R = \frac{\rho c_{w2}}{A_c}, l_w \text{ is the length of winding and } A_c \text{ the cross-sectional area of conducting wire}
\]

\( A_c \), the cross-sectional area of conducting wire and the bias voltage caused by the coil resistance \( R \) denoted by \( u_o = R l_o \).

Parameters used in (25) are evaluated from Table 1 and the values presented in Table 2.

**Table 2. AMB Parameter Configuration**

| Parameter | Force-Displacement constant \((K_s)\) | Force-Current constant \((K_i)\) | Coil Self Inductance \((L_s)\) | Air Gap Inductance \((L_o)\) | Bias current \((i_o)\) | Disturbances Force \((F_d)\) | Mass per AMB \((m)\) | Resistance \((R)\) |
|-----------|---------------------------------|---------------------------------|-------------------------------|-------------------------------|-----------------|-------------------|----------------|----------------|
| Unit      | N/m                            | N/A                             | mH                            | mH                            | A               | N                 | kg             | ohm            |
| Value     | 97667                          | 49                             | 90                            | 24.4                          | 2               | 5                 | 4.5            | 1              |

For creating a “strict feedback form”, (25) is transformed into system shown in (26), (27) and (28).

Where:

\[
x_1 = \frac{s}{b}, \quad x_2 = \frac{v}{b}, \quad x_3 = l.
\]

\[
x_1' = x_2
\]

\[
x_2' = x_3 + \frac{a}{b} x_1 + \theta = x_3 + \varphi_1(\theta, x_1)
\]

\[
x_3' = u' + cb x_2 + d x_3 = u' + \varphi_2
\]

In (19), (20) and (21), the disturbance force and control effort are defined as \( \theta = \frac{F_s}{b m} = 1.02, \ u' = eu \), respectively. In (20) and (21), \( \varphi_1 \) and \( \varphi_2 \) are defined as \( \varphi_1(\theta, x_1) = \frac{a}{b} x_1 + \theta \) and \( \varphi_2 = \varphi_2(x_2, x_3) = cb x_2 + d x_3 \).

#### 4.2. Back Stepping with Adaptive Disturbance Rejection

From (25) and Table 2, we can calculate the eigenvalues of matrix \( A \), which comes out to be \([112 - 766.99i - 766.99i]\). Since there is a positive eigenvalue for \( A \), the system is inherently unstable. Therefore, primary control objectives are to stabilize the AMB and to drive the position of the rotor to its equilibrium point in the presence of external disturbance and system uncertainties.

The goal is to achieve by implanting two virtual controllers \( \alpha_1 \) and \( \alpha_2 \) in (26) and (27) respectively (i.e. \( \alpha_1 = x_2 \) and \( \alpha_2 = x_3 \)) which will help drive \( x_1 \) and \( x_2 \) to zero.

Starting from \( x_1 \), CLF (control Lyapunov function) for \( x_1 \) will be chosen as \( V = \frac{1}{2} x_1^2 \) and the virtual control action \( \alpha_1 = -k_1 x_1 \) will make \( V' = -k_1 x_1^2 \) which is negative indefinite, making \( x_1 \) asymptotically stable.

However, practically it is not possible and there is always an error between virtual controller \( \alpha \) and real state \( x \).

We construct an error system whose states will compute the difference between the actual states and virtual controls. Such that:

\[
z_1 = x_1, \quad z_2 = x_2 - \alpha_1, \quad z_3 = x_3 - \alpha_2
\]

Now, the control goal is to asymptotically stabilize all the error system states such that they get eventually driven to zero.

CLF for states \( z_1 \) and \( z_2 \) as follow:
\[ V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \]  
(29)

The derivative of \( V_1 \) will be: 
\[ \dot{V}_1 = x_1 x_2 + z_2 \left( x_3 + \varphi_1 - \frac{\partial x_1}{\partial x_1} x_2 \right) \]  
(30)

Virtual function \( x_3 \) that is to make the derivative of negative definite: 
\[ \alpha_2 = x_3 = -k_2 z_2 - \varphi_1 + \frac{\partial x_1}{\partial x_1} x_2 - z_1 \]  
(31)

Putting (31) in (30) results in: 
\[ \dot{V}_1 = -k_1 z_1^2 - k_2 z_2^2 \]

The derivative of \( V_1 \) has become negative semi-definite; hence, \( z_1 \) and \( z_2 \) are eliminated.

- Now for eliminating \( z_i \) new CLF including all the existing errors and displacement is created as:
\[ V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} z_3^2 \]  
(32)

The derivative of \( V_2 \) will be: 
\[ \dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2 + z_3 (z_2 + u' + \varphi_2 - \alpha_2) \]  
(33)

Virtual function \( u' \) that makes the derivative \( \dot{V}_2 \) negative definite:
\[ u' = -k_2 z_3 - z_2 - \varphi_2 + \alpha_2 \]  
(34)

Putting 34 in 33 results in:
\[ \dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 \]  
(35)

Derivative of \( V_2 \) has also become negative semi-definite. So, the control goal is achieved.

The above control law \( u' \) will work efficiently unless there is no external disturbance, but in the case of high-speed flywheels supported by AMB’s, rotor imbalances can cause unknown disturbances. Therefore, we generate an adaptive law to make an estimate disturbance error to zero. The disturbance estimation methodology adopted is as follow:

- Let disturbance be \( \theta \), and estimated disturbance be \( \hat{\theta}_1 = \theta - \hat{\theta}_1 \). We have an estimation error. We add the quadratic form of it to (29) and then form a new CLF (36). Positive real numbers \( \gamma_i \) \( (i=1,2,3) \) are chosen as adaptive coefficients and the derivative is presented in (37).
\[ V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} \gamma_1 \hat{\theta}_1^2 \]  
(36)

Virtual function \( \alpha_2 \) that is to make the derivative of negative definite:
\[ \alpha_2 = -z_1 - k_2 z_2 - \varphi_1 x_1 - \hat{\theta}_1 + \alpha = -(k_1 k_2 + \varphi_1 + 1) \hat{\theta}_1 - (k_1 + k_2) x_2 - \hat{\theta}_1 \]  
(38)

Putting (38) in (37), and selecting adaptive law as \( \hat{\theta}_1 = \gamma_1 z_2 \) will make derivative \( \dot{V}_1 \) negative definite assuming the term \( z_i z_j \) will be cancelled in the future. 
\[ \dot{V}_1 = -k_1 z_1^2 - k_2 z_2^2 + z_3 z_2 \]  
(39)

- If \( \theta_2 \) is the estimate of \( \theta \), and the estimation error is \( \hat{\theta}_1 = \theta_1 - \hat{\theta}_1 \). Then the (32) can be reconstructed as:
\[ V_z = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} \hat{\theta}_1^2 \]  
(40)

From (38) computing \( \alpha_2 \), and putting in control law (34) will result in (41):
\[ u' = -(k_3 + d) x_3 - \left[ k_3 \left( k_1 k_2 + \frac{a}{b} + 1 \right) + k_1 \right] x_1 - \left[ k_3 (k_1 + k_2) + 1 + cb \right] x_2 - k_3 \hat{\theta}_1 + \alpha_2 \]  
(41)

Differentiating the CLF (\( V_z \)) in (40), given the control law (41) and adaptive law as \( \hat{\theta}_2 = -z_3 \gamma_2 \frac{\partial \alpha_2}{\partial x_2} \), the derivative of \( V_z \) becomes:
\[ \dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 \]  
(42)

Now, that the derivative of the final CLF is negative semi-definite, which ensures the system is now stabilized at its equilibrium point. The final back-stepping control law is generated by calculating (41):
\[ u' = - \left[ k_1 \left( \frac{a}{b} - 2 \right) + k_2 \left( \frac{a}{b} - 1 \right) + k_3 (\frac{a}{b} + 1) \right] x_1 - [k_1 k_2 + k_2 k_3 + k_4 k_3 + cb + \varphi_1 + k_1 + k_2 + 3] x_2 - [k_1 + k_2 + k_3 + d] x_3 \]  
(43)
4.3. Controller simulation results

From Table 2, (24) and using the control law $u'$, we construct the Backstepping controlled AMB system in Simulink. The disturbance is added to the system as a step input at 1.5 seconds and all the initial values of state variables are assumed as zeros. For tuning first ACs $\gamma_1$ and $\gamma_1$ are set to 1. Lyapunov coefficients LCs $k_1$, $k_2$ and $k_3$ are then tuned like a P-controllers. The time response of all three states is then computed as shown below:

![Figure 4. Time response of all three states of AMB](image)

Figure 4 shows the time response of AMB with backstepping adaptive control. It has been seen the maximum tracking error was 0.02mm that can guarantee the rotor will not touch stator. The maximum current drawn as disturbance occur is less than $I_{max}$ which guarantees that the controller will not force the AMB to cross its saturation point. To check the robustness of the controller white noise with a sample time of 0.20sec and with noise power of 50 is fed as a disturbance Fig.4 bottom left plot shows the tracking response. The maximum deviation from the desired position was no more than 0.07mm which assures that the system is robust and adaptive part is estimating the disturbances accurately.

5. Conclusion

Due to the outstanding characteristics such as complete contact-free support, no wear, no friction, no lubrication and active vibration control; Active magnetic bearing (AMB) Systems are finding more and more industrial applications. Paper presented a complete design methodology for a Radial AMB; Analytical Modelling, design constraints and the resulting GA Optimization was presented. Force and flux density were evaluated to verify the optimized parameters. Based on the parameters obtained, a mathematical model of an AMB was created and back-stepping adaptive control was introduced to regulate the AMB’s position, this technique combined both the merits of backstepping and adaptive control law and was able to deal with the parametric uncertainties as well as continuously varying external disturbances. Control objectives were to stabilize the closed-loop system and to regulate the displacement of the rotor in the presence of the disturbance and uncertainties. The Lyapunov method proves the stability of the control system despite the presence of disturbance. Magnetostatic and Control Simulation results, verify the effectiveness and robustness of the AMB system.
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