Unobservable Higgs Boson and Spontaneous Violation of Lorentz Invariance

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The standard theory of elementary particle physics is modified in such a way that the Higgs boson becomes unobservable and Lorentz invariance is slightly violated at the level of the S-matrix. The basic technique of realizing these properties without violating the unitarity of the physical S-matrix is the use of the complex-ghost quantum field theory.

§1. Introduction

The standard theory of elementary particle physics (electroweak theory plus quantum chromodynamics) is a very successful theory. It is formulated as a manifestly covariant quantum field theory quite satisfactorily and its predictions have no clear contradictions with any high-energy experimental results, provided that the right-handed neutrinos are taken into account. The only possible troublesome experimental facts against the standard theory are the non-observation of the Higgs boson and the observation of ultra-high energy cosmic-ray particles indicating “slight” violation of Lorentz invariance. Of course, it is quite likely that they may be resolved by further investigation. But there is also the possibility that these troubles are experimentally confirmed. The purpose of the present paper is to propose a possible minimum modification of the standard theory which is consistent with the absence of the Higgs boson and with slight violation of Lorentz invariance.

The only role of the Higgs boson in the standard theory is to give non-zero masses to weak bosons, leptons and quarks. Its particle contents are not only unnecessary but also unwelcome. Indeed, its self-energy Feynman integrals are quadratically divergent; hence its radiative mass is very sensitive to the value of a cutoff parameter; this trouble is known as the hierarchy problem. This problem is usually supposed to be resolved by the supersymmetric theory (SUSY), but since no superparticles are observed yet, it is quite unlikely that SUSY is realized in the nature. Then it is quite difficult to resolve the hierarchy problem by means of a renormalizable theory.

Soon after the existence of cosmic background radiation had been discovered, it was pointed out that there exists an upper bound for the energies of cosmic-ray particles which can reach the earth because of their collisions with background photons, provided that Lorentz invariance is assumed to hold strictly. Though not yet confirmed, it seems that some cosmic-ray particles whose energies exceed...
the above bound have been observed. If this is true, it may imply that Lorentz invariance is slightly violated in high-energy reactions.\textsuperscript{4)}

The purpose of the present paper is to propose a theory of modifying the standard theory in such a way that there is no observable Higgs boson and that Lorentz invariance is slightly violated. In this modification, we require that the following properties should be kept: 1) manifest covariance of the fundamental Lagrangian density, 2) renormalizability of the theory, 3) $SU(3) \times SU(2)_L \times U(1)$ gauge symmetry, and, most importantly, 4) unitarity of the physical $S$-matrix. The essential idea of realizing the above program is to make the Higgs boson a complex ghost.

In the indefinite-metric theory,\textsuperscript{5)} the eigenvalues of a hermitian operator are not necessarily real; for example, those of the Hamiltonian can be complex in general. The states with complex energy are called complex ghosts. They are unphysical; they should not appear in the final state of the physical $S$-matrix. Remarkably enough, it is known that the energy conservation law forbids not only the appearance of a single complex ghost but also that of a pair of a complex ghost and its complex-conjugate ghost owing to relativistic kinematics (more precisely, the probability of a pair production is of measure zero). Thus the complex-ghost quantum field theory is consistent with the unitarity of the physical $S$-matrix.\textsuperscript{6)}–\textsuperscript{8)} Instead, it was shown that Lorentz invariance is violated spontaneously in the Feynman diagram involving a complex-ghost-pair intermediate state.\textsuperscript{9)} Because of this result, the complex-ghost quantum field theory has been regarded as unrealistic. However, if Lorentz invariance is actually broken in high-energy phenomena, the use of complex ghosts becomes quite welcome because the violation of Lorentz invariance is realized without violating the manifest covariance of the fundamental Lagrangian density.\textsuperscript{10)}

Our strategy is as follows. We introduce a new gauge symmetry, which is the local dilatation invariance, that is, the Weyl gauge symmetry, and indefinite-metric Higgs-like fields. The form of the interaction term between the Higgs field and the indefinite-metric Higgs-like fields is uniquely determined by the requirements of $SU(2)_L$ and local dilatation invariances and of renormalizability. Owing to the presence of this term, it is possible that the Higgs field and the indefinite-metric Higgs-like fields reduce to a pair of complex-ghost fields together with Nambu-Goldstone (NG) fields. Then these fields become unphysical without violating the unitarity of the physical $S$-matrix.

The present paper is organized as follows. In §2, we review the manifestly covariant formalism of the complex-ghost quantum field theory, where we make an extension to the case in which the masses of the fundamental fields are not equal. In §3, we propose a new Higgs-sector Lagrangian density and analyze the Higgs mechanism about it; we show that the Higgs field and the indefinite-metric Higgs-like fields can reduce to a pair of complex-ghost fields together with NG fields. The final section is devoted to discussion.
§2. Complex-ghost quantum field theory

We consider a positive-norm scalar field \( \phi_1(x) \) and a negative-norm scalar field \( \phi_2(x) \). Their free Lagrangian density with a mass-mixing term is given by

\[
\mathcal{L} = \frac{1}{2} \sum_{j=1}^{2} (-1)^{j-1} (\partial^\mu \phi_j \cdot \partial_\mu \phi_j - m_j^2 \phi_j^2) - \gamma \phi_1 \phi_2, \tag{2.1}
\]

where the masses \( m_1 \) and \( m_2 \) are, in general, unequal, and a mixing parameter \( \gamma \) is positive.

Field equations are

\[
(\Box + \alpha + \beta) \phi_1 + \gamma \phi_2 = 0, \quad -(\Box + \alpha - \beta) \phi_2 + \gamma \phi_1 = 0, \tag{2.2}
\]

where we set \( \alpha = (m_1^2 + m_2^2)/2 \) and \( \beta = (m_1^2 - m_2^2)/2 \). The non-vanishing equal-time commutation relations are

\[
[\partial_0 \phi_j(x), \phi_k(y)]_{x^0 = y^0} = -(-1)^{j-1} i \delta(x - y). \tag{2.3}
\]

We set

\[
[\phi_j(x), \phi_k(y)] \equiv i \Delta_{jk}(x - y) \tag{2.4}
\]

and

\[
\Delta \equiv \text{matrix}(\Delta_{jk}). \tag{2.5}
\]

We then have the following Cauchy problem:

\[
[(\Box^x + \alpha) \sigma_3 + \beta + \gamma \sigma_1] \Delta(x - y) = 0 \tag{2.6}
\]

together with

\[
\Delta(x - y)|_{x^0 = y^0} = 0, \\
\partial_0 \Delta(x - y)|_{x^0 = y^0} = -\sigma_3 \delta(x - y), \tag{2.7}
\]

where \( \sigma_i \) denotes the Pauli matrix.

We solve the above Cauchy problem by diagonalizing (2.6). Extending the analysis made previously\(^{11} \) to the unequal-mass case, we obtain the following solution:

\[
\Delta(x-y) = \frac{1}{2 \sqrt{\gamma^2 - \beta^2}} \left[ \left( \sqrt{\gamma^2 - \beta^2} \sigma_3 - i \beta + i \gamma \sigma_1 \right) \Delta(x-y; \alpha + i \sqrt{\gamma^2 - \beta^2}) + \text{c.c.} \right], \tag{2.8}
\]

where c.c. denotes complex conjugate. In (2.8), we have assumed

\[
\gamma^2 > \beta^2; \tag{2.9}
\]

this is the important condition for the existence of complex ghosts. The complex-mass \( \Delta \)-function \( \Delta(\xi; M^2) \) with \( M^2 \) complex is simply defined by analytic continuation with respect to \( M^2 \).

\(^{(*)} \) If negative, change the sign of \( \phi_2 \).
Introducing the complex-mass “positive-energy” \( \Delta \)-function \( \Delta^{(+)}(\xi; M^2) \), we can give the explicit expressions for Wightman functions \( \langle 0|\phi_j(x)\phi_k(y)|0 \rangle \). Then the Feynman propagators \( \langle 0|\phi_j(x)\phi_k(y)|0 \rangle \) are calculated. In matrix form, they are given by

\[
\Delta_F(x-y) = \frac{1}{2\sqrt{\gamma^2 - \beta^2}} \left[ \left( \sqrt{\gamma^2 - \beta^2} \sigma_3 - i\beta + i\gamma \sigma_1 \right) \Delta_F(x-y; \alpha + i\sqrt{\gamma^2 - \beta^2}) + \left( \sqrt{\gamma^2 - \beta^2} \sigma_3 + i\beta - i\gamma \sigma_1 \right) \Delta_F(x-y; \alpha - i\sqrt{\gamma^2 - \beta^2}) \right].
\]

(2.10)

Here the complex-mass Feynman function \( \Delta_F(\xi; M^2) \) with \( \Re M^2 > 0 \) is defined by

\[
\Delta_F(\xi; M^2) = \frac{i}{(2\pi)^4} \int dp \int dp_0 \frac{e^{-ip\xi}}{p^2 - M^2},
\]

(2.11)

where the contour \( C \) runs from \(-\infty\) to \(+\infty\) below the pole located at \( p_0 = -\sqrt{M^2 + p^2} \) and above the pole located at \( p_0 = \sqrt{M^2 + p^2} \). Hence \( C \) is strictly a complex contour if \( \Im M^2 > 0 \), while it can be taken to be the real axis if \( \Im M^2 < 0 \).

We note that \( i\Delta_F(x-y) \) is a fundamental solution to (2.6), that is,

\[
[(\Box + \alpha)\sigma_3 + \beta + \gamma \sigma_1]i\Delta_F(x-y) = \delta^4(x-y).
\]

(2.12)

Now, we introduce an interaction Lagrangian density and work in the interaction picture. The Dyson S-matrix can be defined if one employs a Gaussian adiabatic factor \( e^{-\varepsilon^2(x^0)^2} \). Everything goes in the same way as in the ordinary case except for carrying out the integrations over time variables. As mentioned above, the energy variable is inevitably complex-valued in \( \Delta_F(\xi; M^2) \), and therefore we cannot naively take the \( \varepsilon \to 0 \) limit so as to yield a \( \delta \)-function. We must extend the concept of the \( \delta \)-function to the “complex \( \delta \)-function”, which is defined in the following way.

Let \( \varphi(k_0) \) be a test function, which is an arbitrary function holomorphic in an appropriate strip domain including the real axis; then the complex \( \delta \)-function, \( \delta_c(k_0 - E) \), for \( E \) complex is defined by

\[
\int_{-\infty}^{\infty} dk_0 \varphi(k_0) \delta_c(k_0 - E) = \frac{1}{2\pi i} \int dq \int dk_0 \varphi(k_0) \frac{\delta(k_0)}{k_0 - E},
\]

(2.13)

where the contour goes around \( k_0 = E \) in the anticlockwise direction. Of course, if \( E \) is real, the complex \( \delta \)-function reduces to the ordinary \( \delta \)-function.

By using the complex \( \delta \)-function, we can calculate the \( \varepsilon \to 0 \) limit of the Dyson S-matrix, that is, we obtain the momentum-space expression for it, to which the conventional Feynman rules are applicable except for the modification of the Feynman \(-i\varepsilon \) prescription for Feynman propagators. For example, the Feynman integral involving a complex-ghost-pair intermediate state is given by

\[
\int dq \int_C dq_0 \frac{1}{(q^2 - M^2)[(p - q)^2 - M^2]^2},
\]

(2.14)

\(^{10}\) “Positive energy” means that the function considered is a boundary value of an analytic function of \( \xi^0 \) from the lower-half plane.
where the contour $C$ runs from $-\infty$ to $+\infty$, passing below the two poles located in the left and above the two poles located in the right. Carrying out the integration over $q_0$, we find that there is no unitarity cut on the real axis in the $p_0$ plane. This is a consequence of the simple kinematical fact that the total energy,

$$p_0 = \sqrt{M^2 + q^2} + \sqrt{M'^2 + (p - q)^2},$$

is not real except for the special values of $q$ satisfying $q^2 = (p - q)^2$. This fact guarantees that the unitarity of the physical S-matrix is not broken.

As is easily proved and also confirmed by explicit calculation, however, (2.14) is not Lorentz invariant. That is, Lorentz invariance is spontaneously violated. One may suspect that this result could be avoided if the concept of the complex $\delta$-function were not employed. This is not the case, however. The old-fashioned (non-covariant) perturbation theory also yields the same result. The complex $\delta$-function must be introduced only for the manifestly covariant perturbation theory.

§3. Modification of the standard theory

In this section, we propose a modified version of the standard theory in which the Higgs boson becomes unobservable.

We introduce a new gauge symmetry, which is the local dilatation invariance, that is, the Weyl gauge symmetry\(^*\). The Lagrangian density (including a gauge-fixing term) proper to the Weyl gauge field $\tilde{A}_\mu$ is the well-known one. Our main concern is the modification of the Higgs sector.

We introduce a pair of hermitian scalar fields, denoted by $\tilde{\Phi}(x)$ and $\tilde{\Phi}^*(x)$; under the Weyl gauge transformation they transform as $\tilde{\Phi}(x) \rightarrow \tilde{\Phi}(x)e^{\Lambda(x)}$ and $\tilde{\Phi}^*(x) \rightarrow \tilde{\Phi}^*(x)e^{-\Lambda(x)}$. The Higgs sector consists of the Higgs field $\Phi(x)$, which is, of course, an SU(2)\(_L\)-doublet non-hermitian scalar field, and the newly introduced indefinite-metric Higgs-like fields $\tilde{\Phi}(x)$ and $\tilde{\Phi}^*(x)$. The most general expression for the Higgs-sector Lagrangian density which is consistent with the gauge invariances and with renormalizability is given by

$$L_{\text{Higgs}} = (D^\mu_L \tilde{\Phi})^\dagger (D_{L\mu} \Phi) + \mu^2 \tilde{\Phi}^\dagger \tilde{\Phi} - \frac{1}{2} \lambda(\tilde{\Phi}^\dagger \tilde{\Phi})^2$$

$$- \left( \partial^\mu + \tilde{g} \tilde{A}^\mu \right) \tilde{\Phi}^\dagger \cdot (\partial_\mu - \tilde{g} \tilde{A}_\mu) \tilde{\Phi} - \tilde{\mu}^2 \tilde{\Phi}^\dagger \tilde{\Phi} + \frac{1}{2} \tilde{\lambda}(\tilde{\Phi}^\dagger \tilde{\Phi})^2 - \xi \tilde{\Phi}^\dagger \tilde{\Phi} \tilde{\Phi}^\dagger \tilde{\Phi},$$

where $D_{L\mu}$ denotes the SU(2)\(_L\) covariant differentiation. The parameters $\mu^2$, $\lambda$, $\tilde{\mu}^2$ and $\tilde{\lambda}$ are taken to be positive so as to realize the usual Higgs mechanism; $\tilde{g}$ and $\xi$ must be nonzero. (If $\xi = 0$ then the Higgs-like fields decouple from the main part.)

As in the standard theory, $\Phi$ has a nonvanishing vacuum expectation value. We denote the second component of $\langle 0 | \Phi | 0 \rangle$ by $v/\sqrt{2}$; without loss of generality, we may assume for it to be positive. Likewise, both $\tilde{\Phi}(x)$ and $\tilde{\Phi}^*(x)$ have a nonvanishing

\(^*\) If, instead, we introduce such a symmetry as $U(1)$ or SU(2), then the corresponding gauge field becomes a tachyon.
vacuum expectation value. Without loss of generality, we may assume that
\[
\langle 0|\tilde{\Phi}(x)|0 \rangle = \langle 0|\tilde{\Phi}^*(x)|0 \rangle = \frac{\tilde{v}}{\sqrt{2}} > 0. \tag{3.2}
\]
The doublet non-hermitian field \( \Phi \) is decomposed into a singlet hermitian field \( \varphi \) and a triplet hermitian field \( \chi^a \) \((a = 1, 2, 3)\); the latter is an NG field. Likewise, we set
\[
\tilde{\Phi} = \frac{1}{\sqrt{2}}(\tilde{v} + \tilde{\varphi} + \tilde{\chi}), \quad \tilde{\Phi}^* = \frac{1}{\sqrt{2}}(\tilde{v} + \tilde{\varphi} - \tilde{\chi}), \tag{3.3}
\]
where \( \tilde{\chi} \) is an NG field.

The NG fields become unphysical owing to the subsidiary conditions in the BRS-invariant operator formalism of the gauge theory.¹⁴ There are no linear terms involving the NG fields; the quadratic terms involving the NG fields are absorbed into the mass terms of the gauge fields by transforming the gauge fields. It should be noted that the Weyl gauge field \( \tilde{A}_\mu \) becomes a massive vector field but not a tachyon field. Thus, hereafter, we may concentrate our attention to the discussion on \( \varphi \) and \( \tilde{\varphi} \).

The Higgs potential part of (3.1) is written
\[
\frac{1}{2} \mu^2 (v + \varphi)^2 - \frac{1}{8} \lambda (v + \varphi)^4 - \frac{1}{2} \tilde{\mu}^2 (\tilde{v} + \tilde{\varphi})^2 + \frac{1}{8} \tilde{\lambda} (\tilde{v} + \tilde{\varphi})^4 - \frac{1}{4} \xi (v + \varphi)^2 (\tilde{v} + \tilde{\varphi})^2. \tag{3.4}
\]
The linear terms of (3.4) must vanish, that is, we have
\[
\mu^2 v - \frac{1}{2} \lambda v^3 - \frac{1}{2} \xi v \tilde{v}^2 = 0, \\
-\tilde{\mu}^2 \tilde{v} + \frac{1}{2} \tilde{\lambda} \tilde{v}^3 - \frac{1}{2} \xi v^2 \tilde{v} = 0. \tag{3.5}
\]
Solving (3.5), we obtain
\[
v^2 = \frac{2(\mu^2 \lambda - \tilde{\mu}^2 \xi)}{\lambda \lambda + \xi^2}, \\
\tilde{v}^2 = \frac{2(\tilde{\mu}^2 \lambda + \mu^2 \xi)}{\lambda \lambda + \xi^2}. \tag{3.6}
\]
Therefore, \( \xi \) must satisfy the inequalities
\[
\frac{\mu^2 \lambda}{\tilde{\mu}^2} > \xi > -\frac{\tilde{\mu}^2 \lambda}{\mu^2}. \tag{3.7}
\]
The quadratic part of (3.4) is
\[
\left( \frac{\mu^2}{2} - \frac{3 \lambda v^2}{4} - \frac{\xi \tilde{v}^2}{4} \right) \varphi^2 + \left( -\frac{\tilde{\mu}^2}{2} + \frac{3 \tilde{\lambda} \tilde{v}^2}{4} - \frac{\xi v^2}{4} \right) \tilde{\varphi}^2 - \xi v \tilde{v} \varphi \tilde{\varphi}. \tag{3.8}
\]
In this way, we find that the free Lagrangian density for \( \varphi \) and \( \tilde{\varphi} \) becomes
\[
\mathcal{L}_{\text{Higgs}}^{(0)} = \frac{1}{2} \left( \partial^\mu \varphi \cdot \partial_\mu \varphi - \lambda v^2 \varphi^2 \right) - \frac{1}{2} \left( \partial^\mu \tilde{\varphi} \cdot \partial_\mu \tilde{\varphi} - \tilde{\lambda} \tilde{v}^2 \tilde{\varphi}^2 \right) - \xi v \tilde{v} \varphi \tilde{\varphi}, \tag{3.9}
\]
where use has been made of (3.6).

The condition (2.9) becomes

$$\xi v \tilde{v} > |\lambda v^2 - \tilde{\lambda} v^2|.$$  \hspace{1cm} (3.10)

It is certainly possible to satisfy this inequality if the values of the parameters are chosen appropriately. Thus, \( \varphi \) and \( \tilde{\varphi} \) become complex-ghost fields.

\section{Discussion}

In the present paper, we have successfully proposed a modified version of the standard theory in which the Higgs boson becomes unobservable without violating the unitarity of the physical S-matrix. The essential technique is the use of the complex-ghost field theory.

It seems that some people dislike to use the indefinite-metric theory in the physical context; according to them, indefinite metric is no more than such auxiliary means as a regulator. Such assertion is, of course, inadequate; indeed, the local quantum field theories of gauge fields and gravity can be formulated only in the framework of the indefinite-metric theory.

It is true that it is extremely difficult to formulate the indefinite-metric quantum field theory in the mathematically rigorous manner, but this fact does not mean that the correct physical theory must be the positive-metric quantum field theory. The most fundamental principle of quantum theory is the superposition principle, which is a linear property, while the norm positivity is a nonlinear property. In the axiomatic quantum field theory, the norm positivity is merely postulated as an axiom; it is not the fact which is proved in some general context. Although the constructive field theory showed the existence of some nontrivial examples of the positive-metric quantum field theory, the magnitude of the coupling constant must be restricted in general. It is quite likely that if the coupling constant becomes larger beyond the restriction, the norm positivity no longer remains valid. Indeed, such a phenomenon is seen to exist in exactly solvable 2-dimensional models; furthermore, in the Bethe-Salpeter formalism, the appearance of ghost bound states is known to be inevitable for large values of the coupling constant.\textsuperscript{15} The present author believes that the use of indefinite metric is quite natural in the framework of the Lagrangian quantum field theory.

It is expected that, in near future, the Large Hadron Collider (LHC) experiment will clarify whether or not the Higgs boson is really an observable particle. If it is not observed, our model should be examined more closely. The physical predictions of our model are, in addition to the physical absence of the Higgs boson, slight violation of Lorentz invariance and the existence of a new massive gauge boson.

If the Higgs boson is observed, our model must be abandoned, but the complex-ghost theory can still be used for explaining spontaneous violation of Lorentz invariance. Furthermore, complex ghosts can be utilized as finite-mass regulators; for example, the quadratic divergence of the Higgs-boson self-energy Feynman integrals caused by the Yukawa interactions can be removed by introducing two pairs of \textit{bosonic} Weyl-spinor fields.
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