Improved Jacobi matrix method for the numerical solution of Fredholm integro-differential-difference equations. (English) Zbl 1371.65135 Math. Sci., Springer 10, No. 3, 83-93 (2016).

Summary: This study is aimed to develop a new matrix method, which is used as an alternative numerical method to the other method for the high-order linear Fredholm integro-differential-difference equation with variable coefficients. This matrix method is based on orthogonal Jacobi polynomials and using collocation points. The improved Jacobi polynomial solution is obtained by summing up the basic Jacobi polynomial solution and the error estimation function. By comparing the results, it is shown that the improved Jacobi polynomial solution gives better results than the direct Jacobi polynomial solution, and also, than some other known methods. The advantage of this method is that Jacobi polynomials comprise all of the Legendre, Chebyshev, and Gegenbauer polynomials and, therefore, is the comprehensive polynomial solution technique.

MSC:

65R20 Numerical methods for integral equations
45B05 Fredholm integral equations
45J05 Integro-ordinary differential equations

Keywords:

orthogonal Jacobi polynomials; Fredholm integro-differential-difference equation; residual error technique; matrix method; collocation; error estimation

Software:

Maple

Full Text: DOI

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