Outage Analysis of $2 \times 2$ MIMO-MRC in Correlated Rician Fading

Prathapasinghe Dharmawansa*, Kumara Kahatapitiya*, Saman Atapattu‡, and Chintha Tellambura§
*Department of Electronic and Telecommunication Engineering, University of Moratuwa, Moratuwa, Sri Lanka
‡Department of Electrical and Electronic Engineering, University of Melbourne, Victoria, Australia
§Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Canada

Abstract—This paper addresses one of the classical problems in random matrix theory—finding the distribution of the maximum eigenvalue of the correlated Wishart matrix. In particular, we derive a new exact expression for the cumulative distribution function (c.d.f.) of the maximum eigenvalue of a $2 \times 2$ correlated non-central Wishart matrix with rank-1 mean. By using this new result, we derive an exact analytical expression for the outage probability of $2 \times 2$ multiple-input multiple-output maximum-ratio-combining (MIMO-MRC) in Rician fading with transmit correlation and a strong line-of-sight (LoS) component (rank-1 channel mean). We also show that the outage performance is affected by the relative alignment of the eigen-spaces of the mean and correlation matrices. In general, when the LoS path aligns with the least eigenvector of the correlation matrix, the performance improves with the increasing correlation. Moreover, we show that as $K$ (Rician factor) grows large, the outage event can be approximately characterized by the c.d.f. of a certain Gaussian random variable.

Index Terms—Maximum eigenvalue, MIMO-MRC, Non-central Wishart matrix, outage probability, Rician fading

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems are critical for 3G (third generation) and current 4G (fourth generation) wireless networks [1]. They are also a key enabler for 5G (fifth generation) and beyond wireless networks to accommodate the ever-increasing data demand [2]. For example, in cell-free massive MIMO, where a large number of access points are distributed over a wide area, users and access point may have only one or two antennas [3]. Therefore, the simplest $2 \times 2$ MIMO channel becomes the most common channel and hence is of paramount importance for small-cell wireless networks [4]. Moreover, for such small-size MIMO channels, accurate channel state information (CSI) estimation is fully viable, which enables the use of MIMO maximum-ratio-combining (MRC) receivers.

The performance of MIMO-MRC has been extensively analyzed for an arbitrary number of transmit and receive antennas for different channel models (5–9 and references therein). However, exact analytical characterization of the MIMO-MRC over the correlated Rician channel remains elusive. This challenge arises because the $2 \times 2$ correlated non-central Wishart matrix does not admit analytically tractable joint eigenvalue density [10]. While general MIMO systems are considered in the literature, only few studies have focused on small-scale practical MIMO architectures [11]–[14]. For instance, [11] derives the channel capacity of $2 \times 2$ or $2 \times 3$ MIMO channel for Nakagami-$m$ fading. In [12], the statistical properties of the Gram matrix $W = HH^H$, where $H$ is a $2 \times 2$ complex central Gaussian matrix whose elements have arbitrary variances, have been investigated, resulting in exact distributions of $W$ and its eigenvalues. In [13], the exact and asymptotic largest eigenvalue distributions of $W$ are derived when $H$ is a complex Gaussian matrix with unequal variances in the real and imaginary parts of its entries, or equivalently $H$ belongs to the non-circularly-symmetric Gaussian subclass. These results have been then leveraged to analyze the outage performance of multi-antenna systems with MRC over Nakagami-$q$ (Hoyt) fading.

To the best of our knowledge, no exact performance analysis is available for the important case of $2 \times 2$ MIMO correlated Rician fading channel with rank-1 channel mean, i.e., a strong line-of-sight (LoS) path exists between transmitter and receiver [15], [16]. To develop exact analytical results, we must first statistically characterize the maximum eigenvalue of the correlated non-central Wishart matrix. However, the distribution of this maximum eigenvalue remains an open problem in both wireless and even wider statistics literature. The main technical challenge is that the joint eigenvalue density of the correlated non-central Wishart matrix has no tractable representation [10]. Since the invariant polynomial representation for the joint eigenvalue density given in [10] is not amenable to further manipulations, the direct technique of obtaining marginal densities by integrating the joint density of the eigenvalues over a suitable multi-dimensional region cannot be applied in the problem at hand. To circumvent this difficulty, here we follow an alternative approach [17], where the matrix variate density is directly integrated instead of the joint eigenvalue density, in order to find the maximum-eigenvalue distribution.

Specifically, we first derive an exact c.d.f. of the maximum eigenvalue of a $2 \times 2$ complex correlated none-central Wishart matrix with rank-1 mean, which is the key contribution of this paper. Some recent results on the maximum eigenvalue are symbolic, and not amenable to further processing, e.g., [9], [17]. We thus believe this to be the first tractable exact result on the maximum eigenvalue of such matrices. Subsequently, we also provide an exact expression for the outage probability. Additionally, we also characterize the effect of the Rician factor $K$ on the outage probability for different signal-to-noise...
ratio (SNR) regimes and establish stochastic convergence limit of outage probability for large $K$ values. Moreover, the effect of the eigenspaces of the mean and the correlation matrices on the outage probability is discussed in detail.

The following notation is used throughout this paper. The superscript $(\cdot)^\dagger$ indicates the Hermitian-transpose, and $(\cdot)^T$ stands for the matrix transpose. We use $\det(\cdot)$ to represent the determinant of a square matrix, $\text{tr}(\cdot)$ to represent trace, and $\etr(\cdot)$ stands for $\exp(\text{tr}(\cdot))$. Positive definiteness of a square matrix $A$ is represented by $A \succ 0$, and $A \succ B \succ 0$. The square root of a positive definite matrix $G$ is denoted by $G^{\frac{1}{2}}$ and $\text{diag}\{s_1,s_2\}$ denotes a $2 \times 2$ diagonal matrix with the real diagonal entries $s_1$ and $s_2$. We use $\lambda_{\text{max}}(A)$ and $\lambda_{\text{min}}(A)$ to denote, respectively, the maximum and the minimum eigenvalues of a square matrix $A$. The real and imaginary parts, modulus and conjugate of a complex number $z$ are denoted, respectively, by $\Re(z)$, $\Im(z)$, $|z|$ and $z^*$. The Euclidean norm of a vector $w$ is denoted by $||w||$. $|x|$ denotes the floor function, defined as $|x| = \max\{m \in \mathbb{Z} | m \leq x\}$. Finally, the union of two measurable sets $R_1$ and $R_2$ is denoted by $R_1 \cup R_2$.

II. CDF OF THE MAXIMUM EIGENVALUE

This section derives the new expression for the distribution of the maximum eigenvalue of a $2 \times 2$ correlated complex non-central Wishart matrix with rank-1 mean matrix. Before proceeding with the derivations, we present some fundamental statistical characteristics of the complex correlated non-central Wishart matrix.

**Definition 1.** Let $X$ be an $n \times m$ ($n \geq m$) complex Gaussian random matrix distributed as $CN_{n,m}(\Psi, \mathbf{I}_n \otimes \Psi)$, where $\Psi \in \mathbb{C}^{m \times m} \succ 0$ and $\mathbf{I}_n \in \mathbb{C}^{n \times n}$. Then $W = X^\dagger X$ has a complex non-central Wishart distribution $W_{m}(n, \Psi, \Theta)$ with the density function [18]

$$f(W) = \frac{\etr(-\Theta) \det^{n-m}(W)}{\Gamma_m(n) \det^n(\Psi)} \etr(-\Psi^{-1}W) \Gamma_1(n ; \Theta \Psi^{-1}W)$$

(1)

where $\Theta = \Psi^{-1}Y^\dagger Y$ is the non-centrality parameter, $\Gamma_1(n \mid \cdot)$ denotes Bessel type complex hypergeometric function of matrix argument and the complex multivariate gamma function is defined as $\Gamma_m(n) = \pi^{\frac{m(m-1)}{2}} \prod_{j=1}^{m} \Gamma(n-j+1)$ with $\Gamma(\cdot)$ being the gamma function.

Next we define the joint eigenvalue distribution of $W$ which consists of invariant polynomials due to Davis [19, 20].

**Corollary 1.** The joint density of ordered eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_m > 0$, of the complex non-central Wishart matrix $W$ is given by [10, Eq. 5.4]

$$g(A) = \frac{\pi^{m(m-1)} \etr(-\Theta)}{\Gamma_m(n) \Gamma_m(n) \det^n(\Psi)} \prod_{k=1}^{m} \sum_{\lambda_k \succ \lambda_l} \prod_{k<l} (\lambda_k - \lambda_l)^2$$

$$\times \sum_{k,l=0,\kappa,\tau, \rho} \sum_{k \neq l} C_{\rho}^{\kappa} C_{\rho}^{\kappa} (\Lambda, \Lambda)$$

(2)

where $\Lambda$ is the diagonal matrix having eigenvalues of $W$ along the main diagonal, $C_{\rho}^{\kappa}(\cdot, \cdot)$ denotes an invariant polynomial with two matrix arguments [19, 20] and the complex hypergeometric coefficient $[n]_{\tau}$ is defined as $[n]_{\tau} = \prod_{j=1}^{m} (n - j + 1)_{\tau}$ in which $\tau = (\tau_1, \tau_2, \ldots, \tau_m)$ is a partition of the integer $t$ into $m$ parts such that $\sum_{j=1}^{m} \tau_j = t$ and $\tau_1 \geq \tau_2 \geq \ldots \tau_m \geq 0$. Also, $(a)_k$ is the Pochhammer symbol given by $(a)_k = \frac{x^k}{\Gamma(a+k)}$ with $(a)_0 = 1$.

A. Cumulative Distribution Function of the Maximum Eigenvalue

Here, to derive the c.d.f. of the maximum eigenvalue, the most straightforward method is to find the probability that the interval $[x, \infty)$ is free from the eigenvalues [21–24]. As such, we may write

$$F_{\lambda_{\text{max}}}(x) = \Pr(\lambda_m < \lambda_{m-1} < \ldots < \lambda_1 < x)$$

$$= \int_D g(A) d\lambda_1 \ldots d\lambda_m$$

(3)

where $D = \{0 < \lambda_m < \ldots < \lambda_1 < x\}$. This direct method, however, is cumbersome because of the invariant polynomials in eq (2). Despite their theoretical significance and frequent appearance in multivariate distribution theory, the invariant polynomials do not seem to admit simple forms in terms of the eigenvalues of its argument matrices even for the $2 \times 2$ case [19, 20]. To circumvent this difficulty, we will adopt an alternative approach based on integrating directly over matrix variate distribution, instead of the joint eigenvalue distribution [10, 17, 19, 25–27]. More precisely, we write the c.d.f. of the maximum eigenvalue as

$$F_{\lambda_{\text{max}}}(x) = \Pr(W < xI_m)$$

(4)

which facilitates the use of the probability density function of $W$ instead of its eigenvalue distribution. Now we apply the change of variable $W = xZ$, where $Z$ is Hermitian positive definite with $dW = x^{m^2} dZ$, in (4) to yield

$$F_{\lambda_{\text{max}}}(x) = \frac{x^{mn} \etr(-\Theta)}{\Gamma_m(n) \det^n(\Psi)} \int_0^{xI_m} \etr(-\Psi^{-1}Z) \Gamma_1(n ; x\Theta \Psi^{-1}Z) dZ$$

(5)

As shown in [10] and [17], this integral does not admit simple form for arbitrary values of $m$ and $n$ even for rank one mean matrix. However, as we now show, it can be solved in terms of simple functions for the important configuration of $m = n = 2$.

In the case of $m = n = 2$, (5) simplifies to

$$F_{\lambda_{\text{max}}}(x) = \frac{x^{4} \etr(-\pi^2)}{\pi \det^2(\Psi)} \int_0^{xI_2} \etr(-x\Psi^{-1}Z) \Gamma_1(2 ; x\Theta \Psi^{-1}Z) dZ$$

(6)

where $\eta = \text{tr}(\Theta)$. Observing the fact that the matrix $\Psi$ is Hermitian positive definite having the eigen-decomposition $\Psi = \mathbf{U} \Omega \mathbf{U}^\dagger$, where $\mathbf{U} \in \mathbb{C}^{2 \times 2}$ is unitary and $\Omega =$$

1Although our general approach is valid even for $n > 2$, here we focus on $n = 2$ in view of obtaining a relatively not so complicated answer.
we can simplify (11) to obtain
\[ I_k(x) = \sum_{p=0}^{\frac{1}{2}k} \sum_{j=0}^{k-2p} a_1(k, p, j) (x\sigma_2)^j (p + 1)! \left[ \frac{1}{(c_k, p, j) + 3 + 3x\sigma_1} - \sum_{i=0}^{j+p+1} \frac{(c_k, p, j) + 1)!}{(c_k, p, j) + i + 2)!} x(i + 1; c_k, p, j + i + 3; x(\sigma_1 - \sigma_2)) \right]. \]

\[ J_k(x) = \sum_{p=0}^{\frac{1}{2}k} \sum_{j=0}^{k-2p} a_2(k, p, j) \exp(-x\sigma_2) \sum_{l=0}^{p+1} \frac{a_4(k, p, j, l)}{(x\sigma_2)^j (p + j + 3; c_k, p, j + 3; x\sigma_2)} \]

where

\[ Q_k(x) = \int_{0 < \eta < 1} \text{etr}(-\eta \Sigma Y) \left( \alpha^\dagger \alpha \right)^k dY. \]
Outage probability characterizes the quality of service (QoS) provided by the system, and is a more generic performance measure of user experience. It is formally defined as the probability of the maximum received SNR being below a certain threshold value, \( \gamma_{\text{th}} \), which determines the minimum SNR level for satisfactory reception. Following Theorem 1, the outage probability can be written as

\[
P_{\text{out}} \left( \frac{\gamma_{\text{th}}}{\bar{\gamma}} \right) = \Pr \left\{ \gamma < \gamma_{\text{th}} \right\} = \Pr \left\{ \lambda_{\max} < \frac{\gamma_{\text{th}}}{\bar{\gamma}} \right\}
\]

(19)

where \( \lambda_{\max} \) is the leading eigenvalue of \( \bar{H} \bar{H}^\dagger \). This clearly demonstrates that the performance of MIMO-MRC is tightly coupled with the statistics of \( \lambda_{\max} \). We next focus on evaluating the outage probability of this system.

A. Outage Probability

The outage probability characterizes the quality of service (QoS) provided by the system, and is a more generic performance measure of user experience. It is formally defined as the probability of the maximum received SNR being below a certain threshold value, \( \gamma_{\text{th}} \), which determines the minimum SNR level for satisfactory reception. Following Theorem 1, the outage probability can be written as

\[
P_{\text{out}} \left( \frac{\gamma_{\text{th}}}{\bar{\gamma}} \right) = \Pr \left\{ \gamma < \gamma_{\text{th}} \right\} = \Pr \left\{ \lambda_{\max} < \frac{\gamma_{\text{th}}}{\bar{\gamma}} \right\}
\]

(19)

where \( \lambda_{\max} \) is the leading eigenvalue of \( \bar{H} \bar{H}^\dagger \). This clearly demonstrates that the performance of MIMO-MRC is tightly coupled with the statistics of \( \lambda_{\max} \). We next focus on evaluating the outage probability of this system.

A. Outage Probability

The outage probability characterizes the quality of service (QoS) provided by the system, and is a more generic performance measure of user experience. It is formally defined as the probability of the maximum received SNR being below a certain threshold value, \( \gamma_{\text{th}} \), which determines the minimum SNR level for satisfactory reception. Following Theorem 1, the outage probability can be written as

\[
P_{\text{out}} \left( \frac{\gamma_{\text{th}}}{\bar{\gamma}} \right) = \Pr \left\{ \gamma < \gamma_{\text{th}} \right\} = \Pr \left\{ \lambda_{\max} < \frac{\gamma_{\text{th}}}{\bar{\gamma}} \right\}
\]

(19)

where \( \lambda_{\max} \) is the leading eigenvalue of \( \bar{H} \bar{H}^\dagger \). This clearly demonstrates that the performance of MIMO-MRC is tightly coupled with the statistics of \( \lambda_{\max} \). We next focus on evaluating the outage probability of this system.

A. Outage Probability

The outage probability characterizes the quality of service (QoS) provided by the system, and is a more generic performance measure of user experience. It is formally defined as the probability of the maximum received SNR being below a certain threshold value, \( \gamma_{\text{th}} \), which determines the minimum SNR level for satisfactory reception. Following Theorem 1, the outage probability can be written as

\[
P_{\text{out}} \left( \frac{\gamma_{\text{th}}}{\bar{\gamma}} \right) = \Pr \left\{ \gamma < \gamma_{\text{th}} \right\} = \Pr \left\{ \lambda_{\max} < \frac{\gamma_{\text{th}}}{\bar{\gamma}} \right\}
\]

(19)

where \( \lambda_{\max} \) is the leading eigenvalue of \( \bar{H} \bar{H}^\dagger \). This clearly demonstrates that the performance of MIMO-MRC is tightly coupled with the statistics of \( \lambda_{\max} \). We next focus on evaluating the outage probability of this system.
Fig. 2: Outage probability vs normalized SNR threshold $\gamma_{th}/\bar{\gamma}$ for different $K$ values.

simulation purposes, we have used $\mathbf{T} = \Psi$ and $\tilde{\mathbf{H}} = \Upsilon$ matrices given in (15). Note that for these analytical curves, the infinite sum (14) has been truncated to a maximum of 15 terms, thereby demonstrating a fast convergence rate for each case. The figure displays a close match between the simulations and analytical results, which verifies the accuracy of the analytical derivations. A counterintuitive trend is also visible - namely the outage does not uniformly decreases with the strength of the LoS component for all $\bar{\gamma}$ values. In particular, in the large $\bar{\gamma}$ regime (i.e., low $\gamma_{th}/\bar{\gamma}$ regime), outage performance improves with the increasing strength of the LoS component. In contrast, the opposite trend can be observed in the low $\bar{\gamma}$ regime. Therefore, when the transmit SNR is high, the outage is benefited by having a strong LoS component, whereas in the low transmit SNR regime, a rich scattering environment certainly helps improve the outage. Thus, from an outage point of view and counterintuitively, it is not always beneficial to have stronger LoS links.

B. The Effect of $K$

We now investigate the effect of Rician factor $K$ on the outage probability. In particular, we focus on the large $K$ regime. The following proposition characterizes the outage probability as $K \to \infty$:

**Proposition 1.** Let $\tilde{\mathbf{H}}^\dagger \tilde{\mathbf{H}} = 4\mathbf{vv}^\dagger$, where $||\mathbf{v}|| = 1$. Then we have, as $K \to \infty$

$$\sqrt{\frac{K}{8\bar{\gamma}^2}} \frac{(\gamma - 4\bar{\gamma})}{\sqrt{\mathbf{v}^\dagger \mathbf{T} \mathbf{v}}} \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1)$$

(20)

where $\mathcal{D}$, $\mathcal{N}(0, 1)$ denotes the convergence in distribution to a standard normal random variable.

**Proof.** Omitted due to space limitation.

An immediate consequence of Proposition 1 is

$$\lim_{K \to \infty} \Pr \left\{ \sqrt{\frac{K}{8}} (\gamma - 4\bar{\gamma}) \leq \gamma_{th} \right\} = \Phi \left( \frac{\gamma_{th} - \bar{\gamma}}{\sqrt{\mathbf{v}^\dagger \mathbf{T} \mathbf{v}}} \right)$$

(21)

where $\Phi(z)$ is the c.d.f of the standard normal random variable. Clearly, once properly centered and scaled, the asymptotic outage depends on the channel mean and correlation through the positive definite quadratic form $\mathbf{v}^\dagger \mathbf{T} \mathbf{v}$. It is noteworthy that this quadratic form depends on $\mathbf{T}$ but not its inverse. Therefore, the asymptotic outage expression remains valid even when $\mathbf{T}$ is rank deficient. Since we are interested in outage probability, we may use (21) to approximately characterize it, for sufficiently large $K$ values, as

$$p_{out}^{\text{large } K} \left( \frac{\gamma_{th}}{\bar{\gamma}} \right) \approx \Phi \left( \frac{\gamma_{th} - 4}{\sqrt{8\mathbf{v}^\dagger \mathbf{T} \mathbf{v}/K}} \right).$$

(22)

It is interesting to observe that the parameter $K$ is completely decoupled from the other parameters in (22).

The effect of large $K$ on the outage probability is depicted in Fig. 3a. As we already know, for $K \to \infty$, the maximum eigenvalue tends to concentrate around 4 (i.e., $\gamma$ around $4\bar{\gamma}$). Therefore, as the figure shows, the outage curves bend more sharply at the critical point and ultimately converges to the vertical barrier at normalized SNR of 4.

Let us now investigate the joint effect of $\mathbf{v}$ and $\mathbf{T}$ on the outage probability. To this end, we first note that $\mathbf{v}^\dagger \mathbf{T} \mathbf{v}$ is maximized when $\mathbf{v}$ aligns with the leading eigenvector of $\mathbf{T}$, whereas $\mathbf{v}^\dagger \mathbf{T} \mathbf{v}$ is minimized when $\mathbf{v}$ aligns with the least eigenvector of $\mathbf{T}$. Therefore, when $\lambda_{\text{max}}(\mathbf{T})$ and $\lambda_{\text{min}}(\mathbf{T})$ represent, respectively, the maximum and the minimum eigenvalue of $\mathbf{T}$, we have the inequality $\lambda_{\text{min}}(\mathbf{T}) \leq \mathbf{v}^\dagger \mathbf{T} \mathbf{v} \leq \lambda_{\text{max}}(\mathbf{T})$. Thus, keeping in mind that $\Phi(z)$ is a monotonically increasing function, we have from (22), for $\frac{\gamma_{th}}{\bar{\gamma}} < 4$, $\Phi \left( \frac{\gamma_{th} - 4}{\sqrt{8\lambda_{\text{min}}(\mathbf{T})/K}} \right)$, and for $\frac{\gamma_{th}}{\bar{\gamma}} < 4$, $\Phi \left( \frac{\gamma_{th} - 4}{\sqrt{8\lambda_{\text{max}}(\mathbf{T})/K}} \right)$.

Figure 3b numerically illustrates the above insights. As shown in the figure, below the critical threshold (i.e., high transmit SNR regime), alignment of the mean vector with the least eigenvalue of the transmit correlation matrix outperforms the other mode of alignments. Also the inverse behavior is depicted above the critical threshold (i.e., low transmit SNR regime).
The joint effect of particular integral regions. To this end we first parameterize the
in the literature. Therefore, we now take a closer look at this
This more complicated integral region in turn makes the above
matrix integral intractable with the available tabulated results
Clearly, this integral is performed over the space of Hermitian
positive definite matrices
We first derive a new expression for the c.d.f. of the maximum
eigenvalue of a $2 \times 2$ correlated non-central Wishart matrix
in turn facilitates the derivation of the outage, contains a fast
converging infinite series of functions. Our analysis demonstrates that as the Rician factor ($K$) grows large, the outage
can be approximately characterized by the c.d.f. of a certain
Gaussian random variable. Interestingly, our analysis shows that a strong LoS path is not always beneficial from the outage perspective.

Appendix A
Proof of Theorem [1]
Here we provide an outline of the main proof due to space limitations.
For clarity let us recall (13)
$$Q_k(x) = \int_{0 < \textbf{Y} < I_2} \text{etr} \left( -x\textbf{Y} \right) \left( \alpha^+\textbf{Y}\alpha \right)^k \text{d}\textbf{Y}. \quad (23)$$
Clearly, this integral is performed over the space of Hermitian positive definite matrices $\textbf{Y} \in \mathbb{C}^{2 \times 2}$ such that $I_2 - \textbf{Y} > 0$. This more complicated integral region in turn makes the above matrix integral intractable with the available tabulated results in the literature. Therefore, we now take a closer look at this particular integral region. To this end we first parameterize the Hermitian positive definite matrix $\textbf{Y}$ as
$$\textbf{Y} = \begin{pmatrix} y_{11} & y_{12} \\ y_{12}^* & y_{22} \end{pmatrix}$$
(24)
where $y_{11}, y_{22} > 0$, $y_{12} \in \mathbb{C}$ and $y_{11}y_{22} - |y_{12}|^2 > 0$. Since we require $I_2 - \textbf{Y} > 0$ to be satisfied, the above parameters should also fulfill
$$\left| y_{11} \right| > \left( 1 - y_{11} \right) \left( 1 - y_{22} \right) - |y_{12}|^2 > 0, \quad y_{11}, y_{22} \in (0, 1). \quad (25)$$
Now keeping in mind the representation $y_{12} = \Re y_{12} + i\Im y_{12}$, we can combine the above results to write the integration region corresponding to $0 < \textbf{Y} < I_2$ as
$$\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2 \quad (26)$$
where
$$\mathcal{R}_1 = \left\{ \left( y_{11}, y_{22}, \Re y_{12}, \Im y_{12} \right) : y_{11} + y_{22} < 1, \right.$$\left.$$ \left| y_{12} \right|^2 < y_{11}y_{22}, \quad y_{11}, y_{22} \in (0, 1) \} \right.$$\left.$$\mathcal{R}_2 = \left\{ \left( y_{11}, y_{22}, \Re y_{12}, \Im y_{12} \right) : y_{11} + y_{22} > 1, \right.$$\left.$$ \left| y_{12} \right|^2 < \left( 1 - y_{11} \right) \left( 1 - y_{22} \right), \quad y_{11}, y_{22} \in (0, 1) \} \right.$$\left.$$.$$
Capitalizing on the above facts we can simplify the matrix integral in (23) to yield the scalar form
$$Q_k(x) = P_1(k, x) + P_2(k, x) \quad (27)$$
where
$$P_1(k, x) = \int_{\mathcal{R}_1} \exp \left( -x\sigma_1 y_{11} - x\sigma_2 y_{22} \right) \times \left( |\alpha_{11}|^2 y_{11} + |\alpha_{12}|^2 y_{22} + 2\Re[\alpha_{11}^* \alpha_{12} y_{12}] \right)^k \text{d}\textbf{Y}$$
$$P_2(k, x) = \int_{\mathcal{R}_2} \exp \left( -x\sigma_1 y_{11} - x\sigma_2 y_{22} \right) \times \left( |\alpha_{11}|^2 y_{11} + |\alpha_{12}|^2 y_{22} + 2\Re[\alpha_{11}^* \alpha_{12} y_{12}] \right)^k \text{d}\textbf{Y}$$
and we have used the differential relation $\text{d}\textbf{Y} = dy_{11} dy_{22} d\Re y_{12} d\Im y_{12}$. The final answer follows by evaluating the above four-dimensional integrals.

Fig. 3: Asymptotically Gaussian behavior of outage as $K \to \infty$. 

IV. CONCLUSION
This paper has focused on characterizing $2 \times 2$ MIMO-MRC system over correlated Rician fading and a strong LoS component (rank-1 mean) with respect to outage probability. We first derive a new expression for the c.d.f. of the maximum eigenvalue of a $2 \times 2$ correlated non-central Wishart matrix with rank-1 non-centrality parameter. This expression, which in turn facilitates the derivation of the outage, contains a fast
converging infinite series of functions. Our analysis demonstrates that as the Rician factor ($K$) grows large, the outage
can be approximately characterized by the c.d.f. of a certain
Gaussian random variable. Interestingly, our analysis shows that a strong LoS path is not always beneficial from the outage perspective.
