Towards Extending Forward Kinematic Models on Hyper-Redundant Manipulator to Cooperative Bionic Arms

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Abstract. Forward Kinematics is a stepping stone towards finding an inverse solution and subsequently a dynamic model of a robot. Hence a study and comparison of various Forward Kinematic Models (FKMs) is necessary for robot design. This paper deals with comparison of three FKM on the same hyper-redundant Compact Bionic Handling Assistant (CBHA) manipulator under same conditions. The aim of this study is to project on modeling cooperative bionic manipulators. Two of these methods are quantitative methods, Arc Geometry HTM (Homogeneous Transformation Matrix) Method and Dual Quaternion Method, while the other one is Hybrid Method which uses both quantitative as well as qualitative approach. The methods are compared theoretically and experimental results are discussed to add further insight to the comparison. HTM is the widely used and accepted technique, is taken as reference and trajectory deviation in other techniques are compared with respect to HTM. Which method allows obtaining an accurate kinematic behavior of the CBHA, controlled in the real-time.

1. Introduction

Hyper-redundant manipulators represent a rapidly growing research field. There are many robots inspired from natural behaviours like elephant trunk [1], tentacles [2] and octopus [3] etc. They are made from flexible materials to make them compatible of narrow and congested areas. Most of the soft robots are actuated by pneumatic actuators to bend etc., due to smooth bending these robots have many degree of freedoms. Therefore it is very complex to model such type of robots.

Forward Kinematic Modelling can be done using qualitative as well as quantitative methods. The Arc Geometry Method under the assumption of constant or circular curvature is used for most of the continuum robots [4]. This method is common and it is better to apply when we know all of the parameters of circular arc and this method is validated by [5]. In [6], the issues regarding singularity are realized particularly for a trunk robot and also the constraints for limit of length and workspace that can be reached. The application of this method is also in modelling concentrating tube based robot [7], which are called “Active Cannulas” too. The actuation is done by using three telescopic tubes for rotation as well as translation. A micro-robot which is placed at the tip of a catheter [8] is also modelled using this method. Forward Kinematic of Compact Bionic Handling Assistant (CBHA) is also done by Arc Geometry Method [9] and is validated. Kinematic model of Bionic Handling Assistant (BHA) is also done by using geometrical approach in [10]. Bionic soft robots are also modelled with modified Denavit-Hartemberg convention [11], the assumption of constant curvature is also integrated [5]. This
model is validated on the robot called "elephant trunk". A Dual Quaternion representation based kinematics is applied on a pneumatic muscle [12] which is a continuum arm. The Model Shape Functions are also used to model hyper-redundant robots [13] but with this the robots remain specified for applications because in this the shapes are restricted means less shapes. Modelling of multisection continuum manipulators is also purposed by Curve Parametric (CP) kinematics [14]. This computes structurally accurate results, are computational efficient and are better to interpret physically. But Curve Parametric can not model straight position of arm because it has singularity inherently. Neural Network Qualitative method is used by [15] to control Compact Bionic Handling Assistant. Qualitative approach goal babbling learning approach is used to calculate the kinematic model of BHA [17]. A Hybrid approach is used by [16] to model kinematics of CBHA. This hybrid approach consists of quantitative as well as qualitative approaches. Geometric method is used to derive equations but due to non linear nature of equations, Neural Network is used to approximate the solution of these equations.

There are many type of models which models kinematics of continuum robots. Modelling of continuum robots is very complex because of there hyper-redundancy and non-linearities, even due to soft material there are hysteresis and memory errors. Therefore modelling of co-operation between more than one continuum robots is more complex than that of one robot. In this paper, the three Kinematic model are applied on a hyper-redundant CBHA, which are applicable on this robot. Then results are compared theoretically as well as experimentally with respect to arc geometry HTM Method to find out one method for appropriate condition. In case of continuum robots, the most widely used method for modeling is arc geometry having the assumption of constant curvature. Arc geometry method is presented as a general method by [18] and it is also used for the modeling of several continuum robots [4] [6]. This work is done So that when the work space calculation of collaborative system is to be calculated, then we can apply more appropriate method for each manipulator to make overall model of co-operative system more efficient. In fig. 1 the collaboration between two manipulators is shown but it can be more than two also.

The remaining paper is organized as follows: Section [2] describes the structure of CBHA. Section [3] briefly explains the three forward kinematic methods and also differentiate them theoretically. Experimental setup is explained in section [4]. Experimental results and discussions are presented in section [5]. Co-operation model using three CBHAs is discussed in section [6]. Finally the conclusions of this paper are described in section [7].
2. Compact Bionic Handling Assistant
The CBHA manipulator is assembled on the top of an omni-drive mobile platform. This mobile robot with bionic manipulator is known as RobotinoXT, shown in Fig. 2.

![Figure 2. RobotinoXT](image)

The manipulator is made up of 2 sections, which are of elastic material, a rotational wrist and a compliant gripper as shown in Fig. 3. Flexibility of 2 sections is the main advantage of the CBHA, which are formed by 3 polyamide electro-pneumatic tubes. These tubes extend when a pressure is applied through them, moreover, since each section is formed by 3 tubes, by applying differential pressures to the tubes, the bending of a section can be achieved.

![Figure 3. The CBHA of the RobotinoXT](image)

If equal pressures are applied to each tube, the manipulator extends in a straight line. The linear extension is limited by a non-extensible cable routed through the middle of the manipulator. The length of the tube can be measured by string potentiometers that deliver a voltage proportional to the channel length $l_{ij}$, where $i = 1, \ldots, 3$ refers to the potentiometer number and $j = 1, 2$ corresponds to the section number. The CBHA is made only by longitudinal pneumatic-actuated bellows, therefore the torsion of the backbone is not possible.

3. Forward Kinematic Methods: Description and differences
The description as well as differences between the FKMs are given as:

3.1. Arc Geometry HTM
This is the geometric quantitative method [9]. Assumptions used in this method are: 1. CBHA's shape is modeled as two cylinders. 2. CBHA is considered as a non-extensible arm. 3. Curvature of the tubes
is assimilated to a perfect arc of a circle. No torsion because tubes are interconnected rigidly at each vertebra. The equations 1 are deduced from the geometrical model of CBHA as in fig. 4:

\[
\begin{align*}
L_j &= \frac{l_i + l_{i+1} + l_{i+2}}{3} \\
\phi_j &= \tan^{-1}\left(\frac{\sqrt{3}(l_{i+2} - l_{i+1})}{2l_i - l_{i+1} - l_{i+2}}\right) \\
r_j &= \frac{(l_i + l_{i+1} + l_{i+2})}{D_j} d_j \\
\theta_j &= \frac{D_j}{r_j} \frac{d_j}{d_j} \\
D_j &= 2\sqrt{l_i^2 + l_{i+1}^2 + l_{i+2}^2 - l_il_{i+1} - l_il_{i+2} - l_{i+1}l_{i+2}}
\end{align*}
\]

Where, \(i\) refer to the number of the tube, \(j\) refer to the number of the section, \(r_j\) refer to the curvature radius of the \(j\)th section, \(\phi_j\) refer to the angle between x-axis and the projection of the \(j\)th section, \(\theta_j\) refer to the curvature angle of the \(j\)th section and \(L_j\) refer to the length of the \(j\)th section. Final position of manipulator is calculated by using above equations and transformation matrix. Transformation matrix for one section of CBHA is eq. 2.

\[
\begin{bmatrix}
R & P \\
0 & 1
\end{bmatrix}
\]

Here,

\[P = \begin{bmatrix} r_j c\phi_j (1 - c\theta_j) \\ r_j s\phi_j (1 - c\theta_j) \\ r_j s\theta_j \end{bmatrix}\]

and,

\[R = \begin{bmatrix}
c^2\phi_j c\theta_j + s^2\phi_j & c\phi_j s\phi_j (c\theta_j - 1) & c\phi_j s\theta_j \\
c\phi_j s\phi_j (c\theta_j - 1) & s^2\phi_j c\theta_j + c^2\phi_j & c\phi_j s\theta_j \\
c\phi_j s\theta_j & -s\phi_j s\theta_j & c\theta_j
\end{bmatrix}\]

Where \(c\) and \(s\) are cos and sin respectively also \(j\) is the section number, and \(i\) is the tube number.
3.2. Dual Quaternion Method

Dual Quaternion representation [12] made up of ordered pair of quaternions composed by applying Clifford dual number algebra on quaternions. One for position of the joint and the other for the orientation. Same assumptions are taken in this method as Arc Geometry HTM because it is just a different representation of matrix. Dual Quaternion is represented as in equ. 3:

\[ Q_i = s_i + \varepsilon t_i \]  \hspace{1cm} (3)

Here \( Q_i \) is the Quaternion containing rotational part \( s_i \) and translational part as \( t_i \). \( \varepsilon \) separates the rotational part and translational part. Let \( T \) is the sample transformation matrix, so the conversion of transformation matrix \( T \) into dual quaternions \( s_i \) and \( t_i \) is as follows:

\[
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} & a_{14} \\
    a_{21} & a_{22} & a_{23} & a_{24} \\
    a_{31} & a_{32} & a_{33} & a_{34} \\
    a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\]  \hspace{1cm} (4)

\[ s_i = [q_0 \quad q_1 \quad q_3 \quad q_4] \]  \hspace{1cm} (5)

\[ t_i = 0.5 \ast [a_{14} \quad a_{24} \quad a_{34} \quad 0] oS_i \]  \hspace{1cm} (6)

In equ. 6 operator 'o' represents dual quaternion multiplication.

\[
\begin{align*}
q_0 &= \sqrt{1 + a_{11} + a_{22} + a_{33}}/2 \\
q_1 &= (a_{32} - a_{23})/4q_0 \\
q_2 &= (a_{13} - a_{31})/4q_0 \\
q_3 &= (a_{21} - a_{12})/4q_0
\end{align*}
\]  \hspace{1cm} (7)

By using these equations, the transformation matrix of each section of CBHA is converted into dual quaternion form. Let \( Q_i \) be the dual quaternion for first section and \( Q_j \) be the dual quaternion for the second section then the dual quaternion of both combined sections is given by:

\[ Q_i o Q_j = s_i o s_j + \varepsilon(s_i o t_j + t_i o s_j) \]  \hspace{1cm} (8)

3.3. Geometric-Neural Network Hybrid Method

Hybrid approach is used to compute FKM [16]. In this approach the assumption of constant curvature is not there but the extra assumption in this is that the CBHA is made up of 16 vertebrae. This hybrid approach composes of both qualitative as well as quantitative methods: Geometric modelling as well as neural network technique. The CBHA is considered to be a concatenation of inter-vertebra along the backbone of the CBHA. Each inter-vertebra is represented by a parallel robot of type 3UPS-1UP (Universal -Prismatic spherical) , as shown in the fig. 5. The controls of the position and the orientation of the upper vertebra relative to the lower vertebra are done by three kinematic serial UPS joints, but only the prismatic joints are active. In the case of the CBHA, the translation of the upper vertebrae relative to the lower vertebrae is perpendicular. Moreover, the torsion is not possible. For these constrains, the serial UP is used. The inverse kinematics equation (IKE) of parallel robots is easy to establish with some elementary geometric relationships [16]. The model of an inter-vertebra, represented by eqs 9, 10 and 11, are obtained by calculating the joint variable \( q_{m,k} \) as shown in the fig. 5 , where \( m=1..3 \) number of prismatic active and \( k=1..17 \) number of vertebra, corresponding to the position \( z_k \) and orientation \( k \) (pitch angle) and \( k \) (roll angle) of the upper vertebra relative to the lower vertebra.
\[ q_{1k}^2 = Z_k^2 + 2r_kZ_k \sin(\Theta_k) - 2r_k r_{k-1} \cos(\Theta_k) + r_k^2 + r_{k-1}^2 \]  

(9)

\[ q_{2k}^2 = Z_k^2 + r_kZ_k \left( \sqrt{3} \cos(\Theta_k) \sin(\Psi_k) - \sin(\Omega_k) \right) + r_k^2 + r_{k-1}^2 \]

\[- r_k^2 r_{k-1}^2 \left( \frac{\sqrt{3}}{2} \sin(\Theta_k) \sin(\Psi_k) + \frac{3}{2} \cos(\Psi_k) + \frac{1}{2} \cos(\Theta_k) \right) \]  

(10)

\[ q_{3k}^2 = Z_k^2 - r_kZ_k \left( \sqrt{3} \cos(\Theta_k) \sin(\Psi_k) + \sin(\Omega_k) \right) + r_k^2 + r_{k-1}^2 \]

\[ + r_k^2 r_{k-1}^2 \left( \frac{\sqrt{3}}{2} \sin(\Theta_k) \sin(\Psi_k) - \frac{3}{2} \cos(\Psi_k) - \frac{1}{2} \cos(\Theta_k) \right) \]  

(11)

Unlike the IKE, the solution of the forward kinematic equation is relatively complex, because the eqs 9, 10 and 11 are highly nonlinear. Thus, Two neural network are generated to provide approximated solution of the IKE, allowing to obtain the position and orientation to from the joint variable \( q_{m,k} \), as shown in fig 6. Then, the transformation matrix \( \frac{k}{k+1} T \) of the upper vertebra frame relative to the lower vertebra, represented by eq 12, allows to estimate Cartesian coordinates of the tip arm, from to homogeneous transformation matrices. However, The CBHA dispose only of six potentiometer, a block is added to estimate the joint variable from to tube length.

\[ \frac{k}{k+1} T = \begin{bmatrix}
  c\Theta_k & s\Theta_k s\Psi_k & s\Theta_k c\Psi_k & 0 \\
  0 & c\Psi_k & -s\Psi_k & 0 \\
 -s\Theta_k & s\Psi_k c\Theta_k & c\Theta_k c\Psi_k & Z_k \\
  0 & 0 & 0 & 1
\end{bmatrix} \]  

(12)

Where c and s are cos and sin respectively.

Figure 5. Modelling of Inter-Vertebra

Theoretical differences between Arc Geometry HTM, Dual Quaternion Method and Hybrid Approach.

Arc Geometry HTM

- This is Quantitative Approach.
- Two sections of CBHA are modelled as two cylinders.
- Need to remove singularity in this approach.
Transformations are in the form of 4x4 Matrix, so rotation as well as translation is represented by 16 elements.

- For two or more than two sections of CBHA, there is multiplication of two matrices for addition of each section.

### Dual Quaternion Method
- This is Quantitative Approach.
- Two sections of CBHA are modelled as two cylinders.
- No singularity.
- Transformations are in the form of dual quaternion, so rotation as well as translation are represented by 8 elements only. Therefore, it needs less memory.
- For two or more than two sections of CBHA, there is multiplication of two dual quaternions for addition of each section. Therefore, this is computationally efficient because dual quaternion multiplication need less calculations than matrices multiplication.

### Hybrid Approach
- This is Quantitative as well as Qualitative approach.
- Each inter-vertebra is modelled, therefore taking into account the conical shape of CBHA.
- No singularity.
- It uses Neural Network approach, so more memory.
- There are a lot of calculations in the beginning for neural network so it is also not efficient computationally.

### 4. Experiment and Apparatus
The experiment is done on the CBHA to compare these three FKMs. The experiment is done on same set-up on the same time to do comparison between different methods. In case of CBHA we can control pressure, so five different values to the pressure sets are given to generate a trajectory. For example, \([p1 p2 p3 p4 p5 p6]\) is a one pressure set. Therefore the data is collected regarding position and orientation of all three methods. The data contains 4200 points during all of the trajectory. Therefore all plots are plotted against 4200 points. Experimental set-up is as shown in fig. 7. This experiment is done as free load condition means without gripper.
5. Results and Discussion

5.1. Trajectory Tracking
From the results, trajectories of both of representations HTM as well as DQ are exactly same. Because both of them are having the same assumptions just the difference is in representation of transformation matrix. So there is no difference in the output position value. The trajectory of Hybrid approach is some deviated but having same trend as shown in fig. 8.

5.2. Cartesian Error
Cartesian errors are calculated for both of methods DQ as well as Hybrid from Arc Geometry HTM. Errors of DQ are approximately zero because it has overlapped trajectory as shown but errors of Hybrid Method are observable and are shown in fig. 9. The x-error is varying between -8.5mm to 21mm, y-error is varying from -14mm to 2mm, z-error is varying from -8.5mm to 6.5mm approximately.

5.3. Orientation Comparison
From fig. 10, the orientation $\psi$, $\phi$ and $\theta$ is compared for all three methods. But from the graphs, it is clear that in all three methods have approximately same trend. So it means all of them converge approximately same in case of orientation. The orientations are deduced by using eqs 13.

Figure 7. Experimental Set-up

Figure 8. Trajectory Comparison of HTM, DQ and Hybrid Method
\[ \phi = \arctan 2(r_{21}, r_{11}) \]
\[ \psi = \arctan 2(r_{13} \sin \phi - r_{23} \cos \phi, -r_{12} \sin \phi + r_{22} \cos \phi) \]
\[ \theta = \arctan 2(-r_{31}, r_{11} \cos \phi + r_{21} \sin \phi) \]  
(13)

Where \( r_{nm} \) are the elements of rotational matrix having \( n \) rows and \( m \) columns.

6. Forward Kinematic Model of Co-operation of three CBHAs

In fig. 11 the co-operation between three CBHAs is shown. All CBHAs are attached on the fixed platform from the base and the tip points of them are attached to a movable frame. By using forward kinematic model the center point C can be deduced but for that the FKMs of all three CBHAs are needed. Therefore the efficient forward kinematic model is to be used to apply on all CBHAs to reduce complexity.

In fig. 11 \( Y_{nm} \) and \( Z_{nm} \) are used as \( Y \) and \( Z \) axis of a frame where \( n \) = number of the CBHA and \( m \) = number of the frame.
7. Conclusions
In this paper, FKMs are compared using theoretical and experimental data. From the comparison it can be concluded that as the number of sections in CBHA increases, it is advantageous to use Dual Quaternion Method as it demands less computational cost as compared to HTM and avoid the use of Hybrid Method because with increase in sections computations are increased. Hybrid Method should be avoided due to its oscillatory nature of the errors and should be used only in cases when utilisation of other methods is not possible. In future work, the work space of the cooperation between two or more CBHA is to be calculated which needs the more efficient FKM because there are more calculations if there is a big system. Therefore, this work is done to choose an efficient model.

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