Didactical design of mathematical reasoning on three dimensional in high school

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Abstract. Mathematical reasoning ability can be obtained through the process of understanding students in learning mathematics. However, not a few students experience learning barriers, one of which is in studying three-dimensional material. The purpose of this study is to identify the epistemological obstacle focused to Reasoning and Proof indicator in the three dimensional and overcome them by developing mathematical learning designs in high school. This research was held at SMAN 9 Tangerang. The research method is Didactical Design Research. This method is consist of three stages, namely analysis before learning (prospective analysis), during learning (metapeda didactic), and after learning (retrospective). Based on the results of the preresearch, 80,48% of 38 students who took the identification of student obstacle test had epistemological obstacles in the concept three dimensional. To overcome the epistemological obstacle in the concept of three dimensional, are needed learning design that are developed based on student’s learning obstacle, repersonalization, and recontextualization, so that it results the hypothetical learning trajectory which composed the various student activities and predictions of student responses follows with the anticipation, and result the generates student worksheet. The results of the study show that the design can be used to overcome student difficulties, it can be seen from the effectiveness of anticipation given to overcome student difficulties during learning.

1. Introduction

The term mathematics is formed from various language absorptions, some of which come from the Latin term Mathematica which originally took the Greek term Mathematike which means related to science. The word Mathematike is also related to other related words, namely Mathenein or in French les mathématiques which means learning [1]. Based on this, the term mathematics has the meaning of knowledge obtained by the learning process. Mathematics has been so closely related to our lives since we were in Kindergarten to Middle School and even in universities we are still studying mathematics. Mathematics is the basis for the development of other scientific fields such as biology, chemistry, physics and others. If in the past mathematics was only used to calculate simple things, now mathematics is used in calculators or software on computers [1]. Thus, mathematics always develops and plays a role over time so it is very important for us to learn and master it.

The National Council of Teachers of Mathematics stipulates that there are five process skills that students must master in learning mathematics, namely: problem-solving (problem-solving), reasoning (reasoning and proof), communication (communication), connections (connections), and representation (representation) [2]. In addition, Permendiknas no. 22 of 2006 explained that the subject of mathematics aims to make students have the following abilities: First, understand mathematical
concepts, explain the interrelationships between concepts and apply concepts or algorithms flexibly, accurately, efficiently, and precisely in problem-solving. Second, using reasoning on patterns and traits, performing mathematical manipulations in generalizing, compiling proofs, or explaining mathematical ideas and statements. Third, solving problems, including understanding issues, mathematical design models, completing models and interpreting the solutions obtained. Fourth, communicating ideas with symbols, tables, diagrams, or other media to clarify the situation or problem. Fifth, having an attitude of appreciating the usefulness of mathematics in life, namely having curiosity, attention, and interest in learning mathematics, as well as a tenacious and confident attitude in problem-solving [3].

The ability to reason is one of the important skills to be mastered by students in learning mathematics at school. Reasoning is a thought process that starts from sensory observation (empirical observation), which produces some concepts and understandings [1]. People who often reason and think analytically will get impacts such as paying attention to patterns or structures, questioning the origin of the appearance of the design or structure, then making mathematical guesses, then proving the conjecture using properties and definitions that no longer need to be proven true, so that these people will derive conclusions from the results of their analysis [2]. An example of a branch of mathematics that deals with reasoning abilities are Geometry. Geometry is a place for students, especially high school students, to prove the theorems they have previously obtained by using visualization skills, spatial reasoning, and geometric modeling to solve problems [2].

According to 2013 revised 2018 curriculum, one of the geometry materials studied by high school students is three-dimensional and is only taught in class XII in odd semesters. The three-dimensional material taught is the concept of distance at points, lines, and planes. In three-dimensional material, students are expected to have a strong spatial analysis power to improve the students' mathematical reasoning and communication abilities. Sumaryanta, in his journal entitled Mapping of National Examination Results, said that in the matter of geometry and trigonometry, the average value of 2015/2016 National Examination was 48.78; in 2016/2017, the average was 37.45. In 2017/2018, the average was 39.30 [4]. This indicates that in the last three years, the national exam results on geometry material have decreased. Although there was an increase in the 2017/2018 academic year, the results were still below the results in 2015/2016.

Based on a preliminary study conducted at SMAN 9 Tangerang, the mathematical reasoning ability of class XII science students is low. This is shown from the results of observations in written tests regarding mathematical reasoning abilities on three-dimensional material and interviews conducted with mathematics teachers. The mathematics teacher revealed that during the process of learning mathematics, especially on geometry material, students often had difficulty explaining how they found answers. Sometimes students just write down the answers directly, likewise withdrawing a conclusion. In addition, students tend to be silent and have difficulty compiling their arguments. Student geometry learning activities in class include listening to teacher explanations, reading sourcebooks, and practicing questions. The obstacles experienced by students in learning are called learning obstacles. Brousseau suggests that the emergence of teaching obstacles is caused by three factors, namely ontogeny (mental readiness to learn), didactic (teaching teachers or teaching materials), and epistemological (student knowledge that has a limited application context) [5].

Previously, research on the concept of distance from a point to a line and from a point to a plane was conducted by Redi Hermanto. It was found that the learning obstacles experienced by 32 students were: First. determine the location of the projection of a point on the line. Second, determine the location of the projection of a point on the plane. Third, creating and recognizing the shape of a plane containing points and line segments (which includes the results of point projections) [6]. However, in this study, Redi was only limited to identifying learning obstacles without conducting further research to overcome these learning obstacles.

Based on the observations, the researchers concluded that students had difficulties in learning related to mathematical reasoning abilities. Therefore, researchers are interested in overcoming learning obstacles in three-dimensional material based on students' mathematical reasoning abilities by compiling a didactic design that makes students truly understand the concept of distance at points, lines, and planes in three-dimensional space. In addition, the didactic method is expected to reduce the
learning barriers experienced by students by determining the anticipation of student responses that will appear.

2. Method

2.1 Research Subject
This research was carried out at SMA Negeri 9 Tangerang for class XII IPA students in the odd semester of the 2018/2019 academic year. In the initial study, an initial learning obstacle test was conducted with the subject of several high school students majoring in science who had studied the concept of distance at points, lines, and planes in three-dimensional space. This initial research was conducted to identify any learning obstacles in the concept of distance at points, lines, and planes. After that, further research was carried out to develop a didactic design based on the learning obstacles that emerged in the initial research. The didactic design that has been prepared will be implemented for students of class XII science in the same school as the initial research.

2.2 Research Method
The method used in this research is the qualitative method. Using this qualitative method is because the problems encountered are complex and dynamic, so quantitative research methods can't capture data on social situations. In addition, researchers also intend to understand social situations in-depth, find patterns, hypotheses and theories [7].

The design used in this research is a Didactical Design Research. Didactic design compiles mathematics teaching materials based on didactic situations by paying attention to student responses. Didactic Design Research consists of three stages, namely: First, prospective analysis. Second, Metapedaddactic analysis. Third, retrospective analysis, namely an analysis that links the results of the hypothesis didactic situation analysis with the results of the metapedaddactic analysis [8].

2.3 Data Collection Techniques
The main data collection techniques in qualitative research are participant observation, in-depth interviews, documentation studies, and triangulation [7]. Observations were carried out during the learning obstacle test, the implementation of the didactic design took place, and the post-test. While the documentation is carried out during the research.

2.4 Data Analysis Techniques
Data analysis in this study is divided into three stages, namely: First, data analysis before entering the field, namely compiling an initial didactic design which includes predicting student responses and anticipating student responses when in the field later. Second, data analysis while in the area, namely analyzing the situation of various student responses when the initial didactic design was applied. Third, data analysis after completion in the area consists of relate response predictions and anticipations made previously with the situation of various responses that occur during implementation, analyzing the test results of the instruments used to identify learning obstacles to students who have obtained learning with a didactic design, analyzing the effectiveness of the didactic design that has been prepared qualitatively compares the learning barriers of students who have received learning using the didactic design and those who have not, make a revised didactic design that has been refined if there are still deficiencies in the previous didactic design.

3. Results and Discussion

3.1 Didactic Situation Analysis (Prospective)
The prospective analysis is an analytical process that is carried out before learning takes place. This analysis consists of 3 stages, namely repersonalization, recontextualization, and development of teaching materials. The prospective study results are Hypothetical Learning Trajectory (HLT) and worksheets.

3.1.1 Repersonalization
Teaching materials prepared by a teacher are generally designed based on a curriculum with core competencies and basic competencies as established standards. Therefore, in the repersonalization stage, the researcher analyzed the mathematics textbooks used by the teacher to teach students in the
classroom. The books used are two math textbooks for SMA/MA/SMK/MAK class XII, which are by the curriculum applied in schools, namely the 2013 revised 2018 curriculum.

The researcher found that there is a concept map in publisher I book, while in publisher II there is no concept map, but the order of the material presented is not much different. It was found that there were deficiencies found in these two books, namely that there was no material for the sub-chapter of the distance between the line to line, and line to field, while questions related to these sub-chapters were often found in exam questions. The presentation of distance material in three dimensions in the publisher's book is quite detailed and clear. However, the researcher found a deficiency in the publisher's text I problem presentations in the point-to-point distance subsection. The problem is the distance of cities A, B, C, and D, which is likened to a point, and the distance is a line. Students are asked to write down possible travel routes from city A to city C and write down the length of the path. The command for students is only to determine the length of the path so that there will be many possible routes and the length of the path. Students are not instilled with the concept of the ideal distance, which is the distance with the shortest path. This will make students confused to determine the actual distance.

3.1.2 Recontextualization
At this stage, the researcher analyzed the students' reasoning abilities regarding the three-dimensional material, which was adjusted to the level of difficulty of the students through the learning obstacle test to students who had previously studied three-dimensional material. Thus, a learning obstacle test was conducted on 38 students of class XII MIPA 3 SMA Negeri 9 Tangerang to identify learning barriers related to three-dimensional material. The results of the learning obstacle test of students' reasoning abilities related to three-dimensional material are: First, show examples and non-examples of the distance between points and lines in daily life. Second, calculating the distance between points in the shape of the space by using the definition of distance and the properties of the shape. Third, summing up what types of lines can and cannot be calculated the distance between the lines. Fourth, estimating the distance between a point and a plane in three dimensions and the solution process to determine the fifth, determine the pattern formed to find the distance between the line and the plane in three dimensions. Based on the results of the researcher's analysis by considering the effectiveness and efficiency of learning activities in the classroom and using indicators of students' mathematical reasoning abilities, the learning flow is obtained as follows:

![Learning Flowchart](image)

Figure 1. Learning flowchart.
3.1.3 Didactic Design Development

The results of the development didactic design are in the form of Hypothetical Learning Trajectory (HLT) and Student Worksheets. The two results are formed based on the results of the analysis of learning obstacles that appear in the three-dimensional material. This didactic design consists of five encounters with different materials arranged to strengthen the concept of distance between points, lines, and planes in three dimensions. The materials contained in this didactic design are: the distance between points in space, the distance between a point and a line in space, the distance between a point and a plane in space, the distance between lines in space, and the distance between a line and a plane in space.

These five designs are based on several theories from experts—first, the Van Hiele theory. Van Hiele divides five stages in learning geometry. First, in the visualization stage, students observe real illustrations about the distance between points, lines, and planes before students are faced with problems in building space directly. At the abstraction stage, students can conclude or what we know as deductive thinking [9]. Second, Vygotsky's theory. In this theory, it is explained that communication is important [10]. Through the communication process, students can express their ideas to each other, and with the teacher's guidance, it can stimulate students' thinking processes. In addition to communicating with the teacher, the role of peers also helps in the discussion process so that students can exchange ideas with each other in solving problems given by the teacher.

Therefore, the author made a design so that it could be done in groups. Third, Ausubel's theory Ausubel says that meaningful learning will help students' learning process [11]. For example, by linking the knowledge, he already had before. At this stage, the elements and properties of geometric shapes that students have studied previously are used to determine the distance between points, lines, and planes in three dimensions. Fourth, Bruner's theory. In Bruner's theory, it is said that the active role of students is very important because the experience will be gained to develop their thinking processes [12]. In the enactive stage, the teacher will give the cube nets to students to observe the elements to understand better where the points, lines, and planes are in the third dimension. After that, students will be given examples of other situations to make them easier to understand at the iconic stage. The last stage is symbolic, at this stage, students are used to visualizing the given problem and converting it to a mathematical model.

Learning mathematics requires the active involvement of students to find knowledge from their thinking processes and learning experiences, so that students can connect previous concepts with the concepts to be studied. It is the teacher's job to prepare in detail what will be done in the learning process so as to produce the right design. According to Callahan & Clark teaching will not be effective if it is not prepared in writing, because teachers need to think in detail about what will be done during the learning process [13]. According Isnawan, teachers in teaching mathematics must prepare all things related to learning so as to produce a good learning process design [14]. The learning design includes learning objectives, learning strategies, teaching materials and learning assessments.

3.2 Metapedadidactic Analysis

This didactic design was implemented on August 6, 2018, during the seventh and eighth lesson hours. This didactic design is carried out to know the definition of the distance between a point and a line in three dimensions based on a statement, and students can estimate the answer and the solution process to determine the distance between a point a line in space. The teacher asks students to form 8 groups containing four members to discuss and work on the worksheets given by the teacher.

One of the didactic situations given is an illustration of a paper boat floating on the surface of a pond. If the wind blows, the paper boat will move towards the edge of the pond. On the pond's surface, an illustration of the distance between a paper boat is given, which is like a point and the edge of the pool, which is like a line. In addition, three dotted lines are given in black, green, and blue to illustrate the direction the paper boat moves towards the edge of the pool. This didactic situation is given to students who know the definition of the distance between a point and a line in space.

In the first assignment, students are asked to determine which line is the distance between the paper boat to the pool's edge and which line is not. In this assignment, the researcher predicts two student responses. First, the students answered that the green line was the distance from the paper boat to the pool's edge but did not know why. This response was found in several groups, so the teacher felt the
need to anticipate the response by directing students to observe the difference in the shape of the trajectory between the three lines, there is one line that is perpendicular to the edge of the pool while the other is not so that students can estimate which line is the distance between points and lines. The second response predicted by the researcher was that students answered other than the green line, but this response did not occur in class.

The second assignment asked students to define the distance between a point and a line in space based on the previous illustration. The researcher gave two responses to this assignment. Students first understand the concept of distance between points and lines in space, but cannot conclude it into a sentence. The anticipation given by the teacher is quite effective, namely by directing students to observe the difference in the shape of the trajectory between the three lines, there is one line that is perpendicular to the edge of the pool while the other is not so that students can estimate which line is the distance between the point and the line. The second response is that students cannot conclude anything, but this response does not occur.

After completing the first and second assignments in the first didactic situation, students are given a didactic situation again to apply the definition of the distance between points and lines that students already know in the previous assignment by using it to a three-dimensional space. The didactic situation given is a bedroom wall measuring 4m x 3m x 2m. The wall will be decorated with Tumblr lights from the bottom corner to the top corner of the bedroom wall diagonally. In the lower corner of the other wall will be placed a large flower vase. The flower vase is placed on the same side of the wall as the Tumblr lamp.

In the first assignment, students were asked to sketch the given situation and indicate the distance between the flower vase and the series of lights. The researcher predicts two responses to this assignment. First, students have difficulty substituting information about known elements into their sketches. The teacher's anticipation is asking students to discuss again with their group members. The teacher directs students by illustrating the problem through other objects around the classroom and the teacher's standing position. Next, the teacher sketches on the blackboard. Then do a question and answer with the students so that the sketch is formed. Another response was that students had difficulty determining the distance between a flower vase and a series of lights. This response also occurs in the classroom. The teacher anticipates by reminding the definition of the distance between a point and a line and then asking students to draw a perpendicular line between the two objects. Again, anticipation is effective so that students know the distance between the vase and the lamp.

The second assignment asked students to name and explain what concepts were used to determine the distance between a flower vase and a series of Tumblr lights. This assignment will show whether, by understanding the given problem, students can estimate how the steps to complete the task are. The researcher predicts the first response; namely, students only mention the concept without explaining its use. This response was not found in students, but the second response occurred in almost every group. Namely, students could not give any answers. The teacher provides anticipation by giving other similar examples so that students understand the steps to determine the distance from a point to a line in space. The teacher provides an example by using a similar but simpler question to answer this assignment correctly.

In the third assignment, students were asked to determine the distance between the flower vase and a series of Tumblr lights based on their previously estimated steps. The researcher predicts the response that students will find it difficult to determine which line should be counted to determine the length of the band. In the learning process, this response does not occur. However, new difficulties arise, namely, students have difficulty finding the distance between lamps and flower vases with the formula for the ratio of the area of a triangle. The teacher immediately took new anticipation by asking students to redraw the triangle formed from the sketch he had made to make it easier. The teacher makes sure students know that there are two triangles in the picture with different bases and heights so that students can enter that information in the formula for the ratio of the area of a triangle. Following are the students' answers after being anticipated:
In the fourth assignment, students were asked to determine the distance between the flower vase and the Tumblr light arrangement if the flower vase was moved. This task aims to re-assure whether the concept of distance between points that students have obtained can be applied again if the situation at hand is different from what students have worked on before. The response predicts that students cannot determine the distance between a flower vase and a series of Tumblr lights. Next, the teacher takes anticipation by reminding students again on the definition of the distance between a point and a line in space. After that, the teacher asks students to determine the distance sought by directing students to draw a perpendicular line connecting the point.

Based on the metapedadidactic observation analysis of three-dimensional learning, 32 response predictions occur from 49 response predictions compiled by the author, so the percentage of response predictions that occur is 65.31%. Meanwhile, for response predictions that do not occur in the learning process, as many as 17 of 49 response predictions, so the percentage is 34.69%. Next, there are two anticipations out of 31 anticipations that have been prepared by the author which turned out to be ineffective, namely from the first meeting in the first assignment in the second didactic situation and the third meeting in the second assignment in the first didactic situation, so that the percentage effectiveness of the anticipation was 93.55%. While 3 of the 49 assignments were made, new, unexpected difficulties emerged, namely at the first meeting in the fourth assignment in the second didactic situation, the second meeting in the third assignment in the second didactic situation and the third meeting in the first assignment in the second didactic situation. Based on this percentage, the three dimensions’ initial distance learning design between points, lines, and planes is considered effective. However, new difficulties arise, so it is necessary to improve the initial didactic design by the anticipation given to the new challenges that arise.

After the didactic design was implemented, a post-test was conducted with the same instrument as the Learning Obstacle test. This test is performed to find out whether students’ learning barriers are reduced or not. The following table shows the final Learning Obstacle test results.

**Table 1. Table of final learning obstacle test results.**

| No | Obstacle                                                                 | First   | Final Result |
|----|--------------------------------------------------------------------------|---------|--------------|
| 1  | Students have difficulty explaining the position of points and lines in three dimensions. | 52.63%  | 26.32%       |
| 2  | Students have difficulty making models or pictures based on the problems contained in the questions. Students have difficulty illustrating the distance between points they want to know. Students find it difficult to apply the properties of geometric shapes to help determine distances in three dimensions. | 89.47%  | 36.84%       |
Based on the results of the final learning obstacle test table above, it can be seen that the learning obstacles experienced by students are reduced. The decrease in learning difficulties, in general, is the difficulty of students to understand the concept of distance between points, lines, and planes in three dimensions. In addition, the difficulty of students experiencing a decline is quite a lot in concluding a statement, estimating and determining the distance between points, lines, and fields. Students already understand all the concepts contained in the three-dimensional material so that many students can answer properly and correctly. However, the percentage decrease in learning difficulties experienced by students is not very significant. Therefore, revisions or new didactic designs are needed to improve student's abilities in the material on the distance between points, lines and planes in three dimensions. Nevertheless, the decrease in the learning obstacle material distance between points, lines, and planes in the three dimensions contained in the three-dimensional chapter of high school students shows that the initial didactic design provided was effective.

3.3 Retrospective Analysis
According to Surya, student-oriented learning designs (student center) need to pay attention to student learning flows (learning trajectory) [15]. According Confrey learning trajectory is a description of the ideas that arise during the learning process and a series of tasks so that students understand the material and support their cognitive development [16]. Atsman states that learning trajectory is a series of activities that children go through in understanding concepts or solving problems [17]. The use of learning trajectory in the mathematics learning process is expected to help develop students' mathematical thinking skills. In addition, by paying attention to the learning trajectory in learning mathematics, it is expected to help teachers understand students in learning and thinking [18].

The retrospective analysis relates the results of the hypothetical didactic situation analysis with the results of the metapedadidactic study. Through this retrospective analysis, conclusions will be drawn whether the didactic design has been effective enough in overcoming student learning barriers or needs to be revised. The researcher changed the didactic design based on student responses. The results of the revision carried out by this researcher are in the form of a revised didactic design. After conducting a metapedadidactic analysis during the learning process, the researcher found several new difficulties that arose outside the response predictions prepared in the initial didactic design. Furthermore, the revised didactic design will combine the initial didactic design with new difficulties that arose in the previous metapedadidactic stage and their anticipation. With the revision that has been given, there are additional commands and instructions on the assignment, but the assignment itself is not reduced or added.

Table 2. Changes in hypothetical learning trajectory.

| Didactic Design | Hypothetical Learning Trajectory | Revised Learning Trajectory |
|-----------------|---------------------------------|----------------------------|
| First Learning  | Students are asked to draw a sketch of the given situation. | 1) Students are asked to name the known and unknown elements. Students are asked to explain the relationship between the elements. |
|                 | Students are asked to name the known and unknown elements. Students are asked to explain the relationship between the elements. | 2) Students are asked to name the known and unknown elements. Students will be asked to draw a sketch. |

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|                 | Students are asked to name the known and unknown elements. Students are asked to explain the relationship between the elements. | 2) Students are asked to name the known and unknown elements. Students will be asked to draw a sketch. |
Didactic Design | Hypothetical Learning Trajectory | Revised Learning Trajectory
--- | --- | ---
Second Learning | Students are asked to determine the distance between a flower vase and a series of Tumblr lights. | 1) Students are asked to redraw a more specific sketch describing the distance between the lam and the flower vase in assignment 2a. 2) Students are asked to determine the distance between a flower vase and a series of Tumblr lights.

Third Learning | Students are asked to explain the position of the pole EA, rope EB, EC, ED on the field by drawing their respective illustrations. | 1) Students are asked to explain the position of the pole EA, rope EB, EC, ED on the field by drawing their respective illustrations. 2) Students are asked to show the direction of the right angles in the illustrations depicted so that the differences between them are more visible.

Fourth Learning | Students are asked to draw a sketch of the given situation. | 1) Students are asked to draw a sketch of the given situation. 2) Students are asked to name the known and unknown elements.

Fifth Learning | Students are asked to draw a sketch of the given situation. | Students will be asked to draw a sketch. 1) Students are asked to draw a sketch of the given situation. 2) Students are asked to name the known and unknown elements.

| 4. Conclusion |
In research on the didactic design of three-dimensional mathematical reasoning abilities in high school, the initial didactic design was prepared based on epistemological barriers that had been previously tested on students. The design is then developed into a hypothetical learning trajectory which contains didactic situations, assignments, response predictions, and anticipation of these responses as well as student worksheets.

From the results of the metapedadidactic observation analysis of three-dimensional learning, there are 32 response predictions that occur from 49 response predictions compiled by the author. As for the response predictions that did not occur in the learning process as many as 18 of 49 response predictions. Next, there are 2 anticipations out of 31 anticipations that have been prepared by the author which turned out to be ineffective, namely from the first meeting in the first assignment in the second didactic situation and from the third meeting in the second assignment in the first didactic situation. While 3 of the 49 assignments made, new unexpected difficulties emerged, namely at the first meeting in the fourth assignment in the second didactic situation, the second meeting in the third assignment in the second didactic situation and the third meeting in the first assignment in the second didactic situation. There are new difficulties that arise so that it is necessary to improve the initial didactic design in accordance with the anticipation given to the new difficulties that arise.

The percentage of students' learning barriers who did not receive learning using a didactic design was 80.48%, while students who received learning using a didactic design were 39.47%. This indicates that the percentage of learning barriers in three-dimensional material has decreased so that the initial didactic design is quite effective in overcoming student difficulties. The revised didactic design on three-dimensional material is in the form of additions and subtractions to assignments, response predictions, and anticipations. This is due to the emergence of several new difficulties so that new
anticipations emerge. In addition, there are some response predictions that do not occur so that they must be removed or replaced.

The recommendation from the results of this study is that it is necessary to conduct didactic design research using other abilities besides mathematical reasoning such as mathematical problem solving, critical thinking, creative thinking, mathematical literacy and so on by involving other mathematical materials.

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References
[1] Didi Haryono 2015 Filsafat Matematika: Suatu Tinjauan Epistemologi dan Filosofis (Bandung: Alfabeta) p. 6
[2] National Council of Teacher of Mathematic (NCTM) 2018 Executive Summary Principles and Standards for School Mathematics, p. 4 (https://www.nctm.org)
[3] Peraturan Menteri Pendidikan Nasional Republik Indonesia No. 22 tahun 2006 tentang Standar Isi untuk Satuan Pendidikan Dasar dan Menengah
[4] Sumaryanta 2019 Pemetaan Hasil Ujian Nasional Matematika Indonesian Digital Journal of Mathematics and Education 6 no 1 pp. 547
[5] G Brousseau 1997 Theory of Didactical Situation in Mathematic Drodrechf : Kluwer Academic Publisher p. 86
[6] Ebih A R Arhasy 2015 Prosiding Seminar Nasional Pendidikan Matematika Tasikmalaya: FKIP Universitas Siliwangi p. 73
[7] Sugiyono 2015 Metode Penelitian Pendidikan: Pendekatan Kuantitatif, Kualitatif dan R&D, (Bandung: Alfabeta), p. 399.
[8] Didi Suryadi 2010 Didactical Design Research (DDR) dalam Pengembangan Pembelajaran Matematika Seminar Nasional pembelajaran MIPA di UM Malang p. 1
[9] Abdussakir 2010 Pembelajaran Geometri sesuai teori van hiele. Jurnal Kependidikan dan Keagamaan 7(2) UIN Maliki Malang
[10] Vygotsky L S 1978 Mind In Society. Cambridge: Harvard University Press
[11] Ausubel, David P 1960 The Use of Advanced Organizermin The Learning And Retention of Meaningful Verbal Material Journal of Education Pshycology 51 pp. 267-272
[12] Bruner J 1977 The Process of Education. Cambridge: Harvard University Press
[13] Callahan J F & Clark L H 1988 Planning for competence. Macmillan Publishing Co
[14] Isnavan M G, Wicakseno A B & Kunc K 2018 Model Desain Pembelajaran Matematika. Indonesian Journal of Mathematics Education 1(1), pp 47–52
[15] Surya A 2018 Learning Trajectory Pada Pembelajaran Matematika Sekolah Dasar (SD) Jurnal Pendidikan Ilmiah 4(2), 22–2
[16] Confrey J, Gianopulos G, Megowan W, Shah M 2017 Scaffolding learner-centered curricular coherence using learning maps and diagnostic assessments designed around mathematics learning trajectories ZDM Mathematics Education https://doi.org/10.1007/s11858-017-0869-1
[17] Atsnan M F 2016 Keterlaksanaan Learning Trajectory Pada Pembelajaran Matematika Lentera Jurnal Ilmiah Kependidikan 11(1) 57–63
[18] Zaman W I, Hunafii A A 2017 Learning Trajectory Dalam Mengembangkan Kompetensi Berfikir Matematik Jurnal Pendidikan Surya Edukasi (JPSE) 3(2) 34–4