GENERALIZED PARTON DISTRIBUTIONS: A NEW AVENUE TO COLOR TRANSPARENCY PHENOMENA

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Color Transparency studies have been since long suggested as a means to study the occurrence and relevance of small size hadronic configurations, predicted within QCD to dominate exclusive scattering processes, by monitoring the passage of hadrons through the nuclear medium at large four momentum transfer, $Q^2$. The validation through experiments of this picture – the dominance of short separation components – is however not straightforward. This situation motivated us to explore a description of Color Transparency in terms of Generalized Parton Distributions (GPDs) using a recent interpretation in impact parameter representation.

1 Introduction

A most intensively studied question in Quantum Chromo Dynamics (QCD) is the space-time structure of high energy exclusive reactions. In the hard scattering approach these are expected to be dominated by the Fock space components of their wave function with the minimum number of quarks (anti-quarks). Such configurations, in turn, are only possible if the constituents are located within a small relative transverse distance, $\approx 1/\sqrt{Q^2}$, $Q^2$ being the (high) four-momentum transfer squared in the reaction. It was suggested in Refs. 2, 3 that performing (quasi)-elastic reactions off nuclear targets can provide an experimental test of the dominance of short separation components. Nuclei can in fact function as both “passive” or “active” probes for the small partonic separation components. At very large $Q^2$, small size configurations were in fact predicted to be less subject to rescatterings inside a nucleus with $A$ nucleons. This is in turn a consequence of the fact that their cross section is expected to be proportional to their transverse size within the one gluon exchange QCD dipole model of high energy hadron-hadron scattering. Small distances can also be filtered at finite (moderate) $Q^2$, and varying $A$, by observing that large separations will gradually be blocked by the strong interactions occurring in the nucleus, as $A$ increases.
From a practical point of view, however, current searches for CT might appear to be in a stall as all experiments performed so far seem not to show either any systematics or any marked trend for the onset of this phenomenon. Additional observables and new experiments have been recently proposed in order to better interpret the present situation, at the light, also, of emerging critical observations that the transverse size of exclusive hard processes might not be small due to the persistence of large endpoint contributions of the hadron’s wave function.\(^5\)

In summary, whether or not a pQCD description of hadrons holds at the \(Q^2\) values presently available, or at reach at future experimental programs, it has now become imperative to investigate the basic question of the existence and observability of small size hadronic configurations. Generalized Parton Distributions (GPDs) have been recently shown to provide a theoretical tool for studying the (deeply inelastic) spatial structure of hadrons.\(^6\) Color Transparency and Nuclear Filtering are aimed at providing measurements of the spatial extension of different hadronic components. The usage of these two tools in combination represent a promising whole new dimension in studies of hadronic structure.

2 Quantitative study of the transverse structure of the proton

We present here the results of a comprehensive study of the transverse size of the proton, using GPDs. GPDs were introduced a few years ago with the main aim of providing a framework to describe in a partonic language the concept of orbital angular momentum carried by the nucleon’s constituents.\(^8\)\(^9\)\(^10\). In the deeply virtual Compton scattering reaction \(e p \rightarrow e' p \gamma\), where the final photon is emitted from the proton’s blob, one can describe the soft part of the reaction by introducing two GPDs, \(H, E\), corresponding to the two possibilities or the final particle’s helicity. The relevant kinematical variables in the process are: \(P\) and \(P'\), the initial and final nucleon’s momenta in the exclusive process, \(\bar{P} = (P + P')/2\), the average nucleon momentum, \(k\) the active quark momentum, \(q\) the virtual photon momentum, \(\Delta = P - P'\) from which one obtains the relativistic invariants for the process: \(x = k^+/P^+, \xi = -\Delta^+/2\bar{P}^+, Q^2 = -q^2\), and \(t = \Delta^2\) (for a review see \(e.g.\) Ref.\(^{11}\)).

More recently,\(^6\) a relationship was found between GPDs and the Impact Parameter dependent Parton Distributions (IPPDs) defined as the joint distribution: \(dn/dxd\bar{b} = q(x, \bar{b})\) — the number of partons of type \(q\) with momentum fraction \(x = k^+/P^+\), located at a transverse distance \(b\) (\(b\) is the impact parameter) from the center of \(P^+\) of the system.\(^{12}\) The connection is obtained by observing that for a purely transverse four momentum transfer, namely for \(\Delta \equiv (\Delta_0 = 0; \Delta, \Delta_3 = 0)\) and \(\xi = 0\), \(H_q(x, 0, -\Delta^2)\), and \(q(x, b)\) can be related as follows:

\[
q(x, b) = \int \frac{d^2\Delta}{(2\pi)^2} e^{-ib\cdot\Delta} H_q(x, 0, -\Delta^2) 
\]

\[
H_q(x, 0, -\Delta^2) = \int d^2b e^{ib\cdot\Delta} q(x, b). 
\]

Since \(q(x, b)\) satisfies positivity constraints and it can be interpreted as a probability distribution, \(H_q(x, 0, -\Delta^2)\) is also interpreted as a probability distribution, namely the Fourier transformed joint probability distribution of finding a parton \(i\) in the proton with longitudinal momentum fraction \(x\), at the transverse position \(b\), with respect to the center of momentum of the nucleon.

As shown in\(^{13}\) the radius of the system of partons, which is needed for quantitative CT studies, is:

\[
\langle r^2(x) \rangle^{1/2} = MAX \left\{ \langle b^2(x) \rangle^{1/2}, \frac{x}{1-x} \right\} 
\]

In what follows, we describe the behavior of: \(i)\) the hadronic configuration’s radius, \(\langle r^2(x) \rangle^{1/2}\); \(ii)\) the intrinsic transverse momentum, \(k\); \(iii)\) the average value of \(x\) in elastic scattering from
This paper

Radyushkin (98)
Stoler (02)
Burkardt (04)

\[ \langle b^2(x) \rangle = N_b \int d^2b \, q(x, b) \, b^2, \]
\[ \langle k^2(x) \rangle = N_k \int d^2k \, f(x, k) \, k^2, \]
\[ \langle x(\Delta) \rangle = N_x \int_0^\Delta dx \, H(x, \Delta), \]

where \( N_b, N_k \) and \( N_x \) are normalization factors, and \( \Delta = \sqrt{-t} \). Furthermore, \( \langle r^2(x) \rangle^{1/2} \) is obtained inserting Eq. (4) in Eq. (3); \( f(x, k) \), the Unintegrated Parton Distribution (UPD), is defined as:

\[ f(x, k) = \int d^2b \int d^2b' \, e^{i k \cdot (b-b')} \, q(x, b, b'), \]

where \( q(x, b, b') \) is the non-diagonal IPDF. Notice that \( \langle x \rangle \) is calculated directly in terms of the GPD, \( H \).

We compare three different models (see also [13]), respectively characterized by: (a) a soft gaussian type distribution for \( q(x, b) \) [14], (b) the “semi-hard” large \( x \) model of [15], (c) a quark-diquark model, characterized by large intrinsic transverse momentum components \( \propto 1/k^4 \).

Results are presented in Fig. 1. One can see that while the “soft” model of Ref. [14] predicts an unphysically large value for the proton’s radius at large \( x \) (large dots in left panel), it is however not dominated by large \( x \) components at large momentum transfer (dotted curve in right panel). On the other side, the model of Ref. [15] gives a vanishing value of the radius at large \( x \) (not shown in this figure), it is dominated by large \( x \) components at large \( \Delta \) (dot-dashed curve in right panel), at the expense, however, of introducing very large intrinsic momentum components (dot-dashed curve in central panel). Finally, the quark-diquark model stands in between the previous two models in that it predicts physically acceptable although non vanishing behaviors for both the radius and the intrinsic \( k \) at large \( x \); at the same time it is not dominated by large \( x \) components at large momentum transfer.

In summary, while it can be challenging to unambiguously disentangle the amount and nature of hard components responsible for the large \( t \) behavior of the hadronic form factors, one might
gain a better insight by requiring models to simultaneously describe the hadrons transverse spatial distribution, and in particular the possible onset of small transverse configurations. The diquark model presented here seems to provide a satisfactory starting point for such studies.

If configurations with small radii indeed exist, they can be isolated in principle by performing CT and/or nuclear filtering type experiments \[13,16\]. By considering \((e, e'p)\) processes with an "unmodified" proton in the nuclear medium one has \[13\]:

\[
H_A(t) = \int_0^{b_{\text{max}}(A)} db \, b \, q(x, b) J_0(b\Delta), 
\]

(8)

where we introduced a nuclear filter for the large transverse size components by multiplying the IPDF, \(q(x, b)\), by a square function:

\[
\Pi(b) = \begin{cases} 
1 & b < b_{\text{max}}(A) \\
0 & b \geq b_{\text{max}}(A) 
\end{cases},
\]

\(b_{\text{max}}(A)\) being the size of the filter. The transparency ratio is then defined as:

\[
T_A(Q^2) = \left[ \frac{\int_0^1 dx H_A(x, \Delta)}{\int_0^1 dx H(x, \Delta)} \right]^2, 
\]

(9)

where scattering in free space is calculated setting \(b_{\text{max}} = \infty\) in the denominator. Based on this result, one can fit the available data, using different distributions \(q(x, b)\), and varying the parameter \(b_{\text{max}}\). In Fig.2 we compare: (a) a soft distribution \[14\] characterized by a parameter \(\alpha \approx (1 - x)\), with (b) the harder ones proposed e.g. in \[13,15\], characterized by \(\alpha \approx (1 - x)^2\) (see
The effect of the filter is to produce both damping and oscillations in $H_A$. In Fig. 2, we show for illustration, the ratio $R = H_A(x, \Delta)/H(x, \Delta)$ plotted vs. $\Delta$ for two different values of $x$, in case (a) and (b), and for different values of the filter size, $b_{\text{max}}$. The results shown in the figure allow us to understand for varying $x$, the different effects due to variations in the size of $\alpha$, which in turn is a feature of different models of GPDs. For instance, in the upper panel, $x = 0.9$, for large $b \equiv b_{\text{max}}$, CT is attained, independently from the model. For small $b \equiv b_{\text{max}}$, the soft model (case (a)), is highly attenuated with respect to the hard one (case (b)).

In conclusion, our study of CT using the new concept of GPDs, will both improve our knowledge of nuclear filtering phenomena and allow for a more detailed understanding of the transverse components involved at large momentum transfer. In particular, we hope to have provided a connection between $b$- and $k$-space that will help to systematically address both the role of Sudakov effects in the endpoints of the hadronic wave function and the role of power corrections in the large longitudinal momentum region.

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