Investigations on Linear Manoeuvres of Underwater Towed Cable in Current

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Abstract. In this paper, a numerical method for dynamic characteristics of cable in marine linear towage is proposed. The governing equation of the towing system is established based on the lumped mass method. The continuous dynamic characteristic partial differential equation is obtained by the discretization of cascade nodes. The equation is solved by the finite difference time domain method. Through numerical simulation, the tension variation curves and cable response configuration under three different operating conditions-when there is no current, along with and against sea current directions-are given respectively, which can be used for dynamic load estimation and motion trajectory prediction in towed-array systems. The comparison between the numerical example results and the experimental demonstrates the validity of the method and is of practical significance for improving maritime safety and operational efficiency.

1. Introduction

The deep-sea towing system is usually composed of three parts: towing ship, towing armor cable, the depth controller and towing body. It is widely used in various fields such as seabed resource exploration, marine military defense, and marine geography research. The towing system can be connected to a towed sonar, multi-body array, etc [1, 2]. The typical towing system is shown in Figure 1. During the process of releasing and recovering of the cable, i.e. the process of submarine entering and retrieving from the water, various problems will be encountered. The drastic changes in the depth and tension of the cable affect the normal operation of various sensors in the submarine, threaten to the safety of the tow body and the streamer, and even lead to breakage and loss of the submarine. Therefore, it is necessary to predict and simulate the dynamic response of the towing system [2, 3].

Due to the strong nonlinearity of the flexible towline motion, the time-varying fluid field, the uncertainties of the surrounding flow field and the towing direction, a numerical method is introduced to approximate the solution. Ablow and Schechter [4], Howell [5] obtained the cable control equation by finite difference approximation. The system equations are implicit, second-order approximation, hyperbolic partial differential equations, which takes a lot of time to solve. Yuan [6] analyzed the nonlinear dynamics of the tow system to maintain the depth by coupling the dynamic relationship
between the streamer and the towed body. Wang [7] used a floating cable to control the towing speed of the underwater vehicle. Hover, F.S [8] applied the mooring system theory into the underwater towed system response. Pang [9] analyzed the effect of variable length towed armor cables on the motion of the secondary tractor. In order to simplify the towage model, this paper assumes that the streamer is a flexible cable, ignoring the bending and torsion effects of the cable, and discretizing the cable at the beginning of the modeling, converting it into cascading nodes of finite length. Based on Newton's second law or the D'Alembert principle, the governing equation of the towed system is established by the lumped mass method, and the 3N differential equation of motion is solved by the finite difference time domain method. The solving process is intuitive, clear in physical meaning and extensible. It can be implemented in complex conditions such as unsteady, non-uniform and oscillating cables. Compared with the experimental results, the numerical results have higher reliability, and the dynamic properties of cable configuration and tension distribution can be obtained.

Figure 1. Components of a typical towing system.

2. Mathematical model
Figure 1 is a schematic diagram of system towing in two-dimensional coordinates. In the underwater towing system, four coordinate systems are established respectively, i.e. $O - XYZ$ inertial coordinate system, $O' - XYZ'$ local coordinate system at the streamer node, $o - xyz$ coordinate system of the parent ship and $o_b - x_b y_b z_b$ coordinate system of the towed body. The Euler angle ($\theta$) is the angle between the tangent line of the streamer and the y-axis, and the attitude angle ($\phi$) is the lifting angle of the cable, which can be defined as,

$$
\theta = \begin{cases} 
\arcsin(dx / dl \cdot \cos \phi) & dy / dl \geq 0 \\
\pi - \arcsin(dx / dl \cdot \cos \phi) & dy / dl < 0 
\end{cases} \\
\phi = \arcsin(dz / dl)
$$

(1)
Where, $dl$ is the unit length of node, $dx, dy, dz$ are the microelements in three directions respectively, \( \theta \in \left[ -\pi/2, 3\pi/2 \right] \), \( \phi \in \left[ -\pi/2, \pi/2 \right] \). The pitching motion of the towed object is ignored here. The local coordinate system \((b, t, n)\) and the inertial coordinate system \((x, y, z)\) can be converted to each other by two rotations.

The force relationship between the node mass units is shown in Figure 2. According to Newton’s second law, the kinematics equation of the $j$th node is obtained as follows:

$$m_j\ddot{x}_j + \frac{1}{2}e_{j+1/2}x_{N|j+1/2} + \frac{1}{2}e_{j-1/2}x_{N|j-1/2} = F_j$$

(2)

Where $m_j$ is the mass of the cable node; Respectively, $e_{j+1/2}, e_{j-1/2}$ is the additional mass between the $j$th, $j+1$ nodes and the $j-1, j$ nodes. $x_j$ is the node displacement matrix, \( \ddot{x}_{N|j+1/2}, \ddot{x}_{N|j-1/2} \) is the acceleration normal component on the adjacent node element, and $N$ is the $N$th discrete unit. $F_j$ is the node force matrix, including the end driving force, fluid resistance, mooring tension, gravity and buoyancy [10].

Expanding equation (2) in a fixed coordinate system yields:

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} \ddot{x}_j \\ \ddot{y}_j \\ \ddot{z}_j \end{bmatrix} = \begin{bmatrix} F_{xj} \\ F_{yj} \\ F_{zj} \end{bmatrix}$$

(3)

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \frac{1}{2}(\rho_{j-1/2}l_{j-1/2} + \rho_{j+1/2}l_{j+1/2})I + \frac{1}{2}(e_{j-1/2} + e_{j+1/2})A$$

(4)

$$A = \begin{bmatrix} 1 - \sin^2\theta \cos^2\phi & -\sin\theta \cos\theta \cos^2\phi & -\sin\theta \sin\phi \cos\phi \\ -\sin\theta \cos\theta \cos^2\phi & 1 - \cos^2\theta \cos^2\phi & -\cos\theta \sin\phi \cos\phi \\ -\sin\theta \sin\phi \cos\phi & -\cos\theta \sin\phi \cos\phi & \cos^2\phi \end{bmatrix}$$

(5)
\[ e_{j+1/2} = \rho k_{am, j+1/2} l_{j+1/2} \sigma_{j+1/2}, \quad e_{j-1/2} = \rho k_{am, j-1/2} l_{j-1/2} \sigma_{j-1/2} \]  

(6)

Where, \( I \) is the unit matrix of 3X3, \( \rho_{ij-1/2}, \rho_{ij+1/2} \) represent the mass per unit length of the cable; \( l_{j-1/2}, l_{j+1/2} \) represent the length of the node unit, and \( k_{am}, \sigma \) denote the additional mass coefficient, cross-sectional area, respectively.

\[ F_j = \Delta F^T_j + F^B_j + G_j + F^D_j \]  

(7)

\( \Delta F^T_j, F^B_j, G_j, F^D_j \) of Eq.(7) denote the elastic tension, buoyancy, gravity and drag resistance of the node \( j \). According to Hooke’s law, available:

\[ \Delta F^T_j = F^T_{j+1/2} - F^T_{j-1/2}, \quad F^T_{j+1/2} = E \sigma (\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} / l_{j+1/2} - 1) \zeta \]  

(8)

Similarly,

\[ F^T_{j-1/2} = E \sigma (\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} / l_{j-1/2} - 1) \zeta \]  

(9)

Where, \( \zeta \) is the unit tangent vector, as shown in figure 2. \( E \) is the Young’s modulus, \( \Delta x \) is the difference between nodes.

\[ F^B_j + G_j = -\frac{1}{2} \rho (l_{j+1/2} \sigma_{j+1/2} + l_{j-1/2} \sigma_{j-1/2}) g + m_j g \]  

(10)

\( g \) is the acceleration of gravity.

The drag resistance referencing Ablow and Schechter[4] is divided into tangential and normal directions respectively.

\[ F^D_j = (F^D_{j+1/2} + F^D_{j-1/2}) / 2 \]  

(11)

\[ F^D_{j+1/2} = \frac{1}{2} \rho C_t^{j+1/2} l_{j+1/2} d_{j+1/2} (v - J)_t (v - J)_t - \frac{\pi}{2} \rho C_n^{j+1/2} l_{j+1/2} d_{j+1/2} (v - J)_n (v - J)_n \]  

(12)

Considering the deformation \( \varepsilon = |F^T|/(E\sigma) \), Eq.(12) can be approximated as,

\[ F^D_{j+1/2} = -\frac{1}{2} \rho \sqrt{1 + \varepsilon} C_t^{j+1/2} l_{j+1/2} d_{j+1/2} (v - J)_t (v - J)_t - \frac{\pi}{2} \rho \sqrt{1 + \varepsilon} C_n^{j+1/2} l_{j+1/2} d_{j+1/2} (v - J)_n (v - J)_n \]  

(13)

Where,

\[ v = \{ v_x, v_y, v_z \} = \{ (\dot{x}_{j+1} + \dot{x}_j) / 2, (\dot{y}_{j+1} + \dot{y}_j) / 2, (\dot{z}_{j+1} + \dot{z}_j) / 2 \}, \quad J = \{ J_x, J_y, J_z \} \]  

(14)

Combining Eq.(4) and Eq.(13), then,

\[ (v - J)_t = A [v_x - J_x, v_y - J_y, v_z - J_z]^T, (v - J)_n = (I - A) [v_x - J_x, v_y - J_y, v_z - J_z]^T \]  

(15)

Where \( v, J \) are the speed of towing ship and current respectively given in inertial frame \( (x, y, z) \), the direction and magnitude of current can change in the spatial and temporal domains. \( C_t, C_n \) are tangential and normal drag coefficients, the subscripts \( t, n \) represent the tangential and normal components, respectively.

Combining Eq.(3) and Eq.(7), then,
\[
F_{yj} = F_{yj}^{T} \sin \theta_{j-1/2} \cos \phi_{j-1/2} - F_{yj}^{T} \sin \theta_{j-1/2} \cos \phi_{j-1/2} + \frac{1}{2} (F_{yj}^{Dx} + F_{yj}^{Dx})/2
\]
\[
F_{yj} = F_{yj}^{T} \cos \theta_{j-1/2} \cos \phi_{j-1/2} - F_{yj}^{T} \cos \theta_{j-1/2} \cos \phi_{j-1/2} + \frac{1}{2} (F_{yj}^{Dy} + F_{yj}^{Dy})/2
\]
\[
F_{yj} = F_{yj}^{T} \sin \phi_{j-1/2} - F_{yj}^{T} \sin \phi_{j-1/2} + \frac{1}{2} (F_{yj}^{Dz} + F_{yj}^{Dz}) + \frac{1}{2} \rho (l_{j-1/2} \sigma_{j-1/2} + l_{j+1/2} \sigma_{j+1/2}) g - m_{yj} g
\]

3. Boundary and Initial Conditions

The governing equation (3) is not sufficient for a fully specified solution. Boundary conditions and initial conditions must be specified.

3.1. Forms of boundary conditions

There are many different types of boundary conditions, depending on the physical form of the upper and lower endpoints [10, 11].

1) A form of the cable is fixed at both ends:

\[x(0) = y(0) = z(0) = 0, \; x(L) = x_L, y(L) = y_L, z(L) = z_L\]  

2) A form in which one end of a cable is fixed and one end bears known forces:

\[x(0) = y(0) = z(0) = 0, \; T \frac{dx}{dR} |_{x=L} = T_x, \; T \frac{dy}{dR} |_{x=L} = T_y, \; T \frac{dz}{dR} |_{x=L} = T_z \]

Where \(R\) is the strain arc length, \(x_L, y_L, z_L, T_x, T_y, T_z\) are all given conditions.

3.2. Boundary conditions of a typical towing system

In this paper, the hull movement and the towing system are decoupled, and the influence of wave force is not included. One end of the rope is tied to the towed ship at sea and the other end is connected to the underwater towed body.

1) Upper end boundary condition (hull movement)

\[x_N = x_U(t), y_N = y_U(t), z_N = z_U(t)\] (19)

\(x_U(t), y_U(t), z_U(t)\) represent the displacement of the tow-ship as a function of the given \(f(t)\).

2) Lower end boundary condition

According to formula (2):

\[m_{11} = M_0 + \frac{1}{2} \rho \mu_1 \frac{l_{1/2}}{2} + \frac{1}{2} \mu_2 (1 - \sin^2 \theta_{1/2} \cos^2 \phi_{1/2})\]
\[m_{22} = M_0 + \frac{1}{2} \rho \mu_1 \frac{l_{1/2}}{2} + \frac{1}{2} \mu_2 (1 - \cos^2 \theta_{1/2} \cos^2 \phi_{1/2})\]
\[m_{33} = M_0 + \frac{1}{2} \rho \mu_1 \frac{l_{1/2}}{2} + \frac{1}{2} \mu_2 \cos^2 \phi_{1/2}\] (20)

Combining Equations (4), (16) and (20), the following equation is obtained.

\[
\begin{bmatrix}
m_{11} & m_{12} & m_{13} \\
m_{21} & m_{22} & m_{23} \\
m_{31} & m_{32} & m_{33}
\end{bmatrix} = \left( \frac{1}{2} \rho_1 l_{1/2} + M_0 \right) I + \frac{1}{2} \mu_2 A_{1/2}
\] (21)
\[ F_{x0} = F_{v2} \sin \theta_{v2} \cos \phi_{v2} + \frac{1}{2} F_{v2}^D + F_{ux} \]
\[ F_{y0} = F_{v2} \cos \theta_{v2} \cos \phi_{v2} + \frac{1}{2} F_{v2}^D + F_{uy} \]
\[ F_{z0} = F_{v2} \sin \phi_{v2} + \frac{1}{2} F_{v2}^D + \rho \left( \frac{1}{2} I_{j+1/2} \sigma_{j+1/2} + V_0 \right) \mathbf{g} - (M_0 + m_0) \mathbf{g} + F_{uz} \]  

\( M_0, V_0 \) denote the mass and volume of the towed body, respectively; \( A_{1/2} \) is the Euler angle matrix between nodes 0-1; \( F_{ux}, F_{uy}, F_{uz} \) include active or passive power, drag force and other forces in the x, y, z direction, respectively.

### 3.3. Initial conditions

The initial value condition is the position and speed state value of the cable when \( t = 0 \).

\[ x_j(0) = x_j^0, \quad y_j(0) = y_j^0, \quad z_j(0) = z_j^0 \]  

\[ \dot{x}_j(0) = v_{xj}^0, \quad \dot{y}_j(0) = v_{yj}^0, \quad \dot{z}_j(0) = v_{zj}^0 \]  

\( x_j^0 \) represent the displacement of the j node at time \( t = i \).

### 4. Numerical approach

The governing equations of the system are simplified and the time domain finite difference method is introduced. The time state quantities are respectively subjected to first-order backward difference and second-order centre difference [12].

\[ \dot{x}_j' = (x_j' - x_j^{i-1}) / \Delta t, \quad \dot{y}_j' = (y_j' - y_j^{i-1}) / \Delta t, \quad \dot{z}_j' = (z_j' - z_j^{i-1}) / \Delta t \]  

\[ \ddot{x}_j' = (x_j'^{i+1} - 2x_j' + x_j'^{i-1}) / \Delta t^2, \quad \ddot{y}_j' = (y_j'^{i+1} - 2y_j' + y_j'^{i-1}) / \Delta t^2, \quad \ddot{z}_j' = (z_j'^{i+1} - 2z_j' + z_j'^{i-1}) / \Delta t^2 \]  

Combining Equations (8), (25) and (26), then,

\[ (F_{f_{j+1/2}} / E\sigma_{j+1/2} + 1)^2_{j+1/2} (F_{f_{j+1/2}} / E\sigma_{j+1/2} + 1) = \left( [\mathbf{x}]^{k_{j+1}} [\mathbf{x}]^{k_{j+1}} + [\mathbf{y}]^{k_{j+1}} [\mathbf{y}]^{k_{j+1}} + [\mathbf{z}]^{k_{j+1}} [\mathbf{z}]^{k_{j+1}} \right) \boldsymbol{\xi} \]  

\( F_{f_{j+1/2}} \) denote the elastic tension of nodes \((j, j+1)\) at time \( i+1 \).

Combining Equations (25), (26) and (27), then,

\[ \begin{bmatrix} P_{j_{-1/2}}^{i+1} \\ P_{j_{+1/2}}^{i+1} \end{bmatrix} = \mathbf{Q}^T \begin{bmatrix} P_{j_{-1/2}}^{i} \\ P_{j_{+1/2}}^{i} \end{bmatrix} - \begin{bmatrix} E\sigma_{j-1/2} \\ E\sigma_{j+1/2} \end{bmatrix} \]  

\[ \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} -M_{3x1} & O_{3x1} \\ M_{3x1} & -N_{3x1} \\ O_{3x1} & N_{3x1} \end{bmatrix}_{3x2}, \quad \mathbf{M} = \begin{bmatrix} E\sigma_{j-1/2} & x/q_0 \\ E\sigma_{j-1/2} & y/q_0 \\ E\sigma_{j-1/2} & z/q_0 \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} E\sigma_{j+1/2} & x/q_1 \\ E\sigma_{j+1/2} & y/q_1 \\ E\sigma_{j+1/2} & z/q_1 \end{bmatrix} \]
\[ q_0 = I_{\mu/2}^2 (F_{j+1/2}^{T_y} / E \sigma_{j+1/2} + 1), \quad q_1 = I_{\mu/2}^2 (F_{j+1/2}^{T_y} / E \sigma_{j+1/2} + 1) \]  

(30)

Where, \( O_{3x1} \) is 3x1 zero matrix; \( P_j \) represent the displacement vector of the \( j \)-node relative to the inertial coordinate system. For simplicity, the parameter that is not marked with the superscript is represented as the state at time \( t=i \).

Combining the governing equation(2) of the towing cable, and the elastic tension equation (28) of nodes \((j, j+1)\), then,

\[
AP_{i+1} - t^2 A \begin{bmatrix} F_{j+1/2}^{T_y} \\ F_{j+1/2}^{T_y} \end{bmatrix}_{3xN} = t^2 \begin{bmatrix} F_{w1} \\ F_{w1} \end{bmatrix} + 2AP^i - AP^{i+1} 
\]

(31)

\[
A = \begin{bmatrix}
-\sin \theta_{j-1/2} \cos \phi_{j-1/2} & \sin \theta_{j+1/2} \cos \phi_{j+1/2} \\
-\cos \theta_{j-1/2} \cos \phi_{j-1/2} & \cos \theta_{j+1/2} \cos \phi_{j+1/2} \\
-\sin \phi_{j-1/2} & \sin \phi_{j+1/2}
\end{bmatrix}_{3x2}
\]

(32)

The governing equation obtains 3N algebraic equations through differential transformation, and iteratively solves 3N unknowns \( P_i^{i+1} = \{x_j^{i+1}, y_j^{i+1}, z_j^{i+1}\}_{j=1,2,\ldots,N} \), and the remaining state variables can be solved in turn [12, 13].

Figure 3. Numerical scheme and Cable configuration.

5. Numerical Simulations and Discussions

5.1. Case 1: Steady-state Motion of Linear Towing

In this section, the effect of different initial value attitude angles on a steady-state motion is studied, as shown in Figure 3. When the towing system stabilizes direct drag in the water, the entire system is in a relatively static state, which is called steady motion. It is difficult to obtain an analytical solution for the steady-state towing solution, which is often obtained through numerical simulation. The entire streamer consists of two components, the upper end is a negative buoyancy-guided streamer and the lower is a near-zero buoyancy tail rope, which is mainly used for stabilization. Their physical parameters are shown in Table 1 below.

Table 1. Properties of the towed cable.

|            | \( \rho(\text{kg/m}^3) \) | \( L(\text{m}) \) | \( d(\text{mm}) \) | \( u(\text{kg/m}) \) | \( C_i \) | \( C_n \) | \( E(\text{N/m}^2) \) |
|------------|-----------------|-------------|-------------|----------------|-----|-----|----------------|
| Trail-rope | 1024.0          | 52.5        | 35          | 1.00           | 0.049 | 1.58 | 1.6x10^{10}   |
| Streamer   | 200             | 17.5        | 0.95        | 0.02           | 1.21 | 5.5x10^{10} |
Setting the added mass coefficient to 1, and now suppose that cable is towed along the Y axis at the speed of 4 m/s. The steady-state cable and load distribution are shown in Figure 4, respectively. When considering only the longitudinal section (ie \( \theta=0 \)), the final end steady state value is \( z = -32.716 \) m, and the tension at the towing-ship is \( T = 4181.76 \) N. The upper boundary condition is set to \( x_N, y_N, z_N = 0 \) m, lower boundary condition is set to \( x_0 = 0, y_0 = -R \cos \phi, \ z_0 = -R \sin \phi, \ R = 186.3 \) m.

![](image)

**Figure 4.** Steady state solution of towed cable.

**Table 2.** Initial values at different angles \( \Phi \).

| \( \Phi \)    | 90°     | 60°     | 30°     | 10°     | 0°     |
|--------------|---------|---------|---------|---------|--------|
| Tension Variation(N) | 2757.85 | 2977.64 | 3440.52 | 3979.22 | 4085.18 |
| Tension steady state moment (s) | 144     | 132     | 108     | 48      | 84     |
| Cable Variation(m)    | 221.02  | 185.55  | 93.72   | 2.81    | 28.72  |
| Tension steady state moment (s) | 268     | 260     | 244     | 96      | 160    |
Figure 5. Cable profiles at different times of different initial value attitude angles.
Figure 5 shows cable profiles at different times of different initial value attitude angles, the tow-point tension curve and the lower end depth curve. The comparison shows that the towing system can reach steady state at different initial attitude angles. However, the tension, the cable profile and the time requirement for reaching the steady state are different during quasi-steady state motion entering the steady state, as shown in Table 2. When the system is running at a speed of 4 m/s, the rising angle at the tow point is 7.3°. Table 2 shows that the closer the initial value angle Φ is to the rising angle, the faster it can reach the steady state. Therefore, the initial state of the towed system dynamic motion simulation is set to steady state, which can greatly reduce the system calculation time and improve the stability of the algorithm. In this paper, the step length Δt is 4 ms, which is much smaller than the response period T, the convergence of the calculation is guaranteed.

5.2. Case 2: Motion Simulation of System Maneuver

In this section, in order to verify the reliability of the simulation algorithm, the towing simulation is carried out according to the Rispin [14] test: First, the towing ship moves at a uniform speed of 18.5 knots(9.5m/s), and its direction is in the positive direction of Y axis. Then the radius is 640m. For the rotational motion, after 440 seconds (about 374.8° rotation); Finally, run for 300 seconds in the tangential direction of the circumference. The total running length is 7052m. Figure 6 shows a schematic diagram of the streamer. The specific physical parameters are shown in Table 3 below, from Milinazzo etc. For monitoring point A, the results of this paper and the test results of Rispin are listed in Table 4, and the results of calculations by Ablow & Schechter [4], Milinazzo et al. [15], Grindheim et al.[16]etc. Take $E=1.00\times 10^9$N/m² the depth changes of points A and B, and the towing system trajectory are shown in Fig. 6.

The "initial depth" is the steady state initial value in Table 4. The difference in the last row in Table 4 (final - minimum) is more important than the third line result (initial - minimum) because the initial valve depth is a steady-state solution dedicated to analog initialization. The last row of data in Table 4 (final-minimum) is more relevant to experimental data than other codes, which proves the effectiveness of the proposed method.

### Table 3. Physical Properties of Towed Cable.

| Element | L(m)  | D(mm) | u(kg/m) | C₁   | Cₙ   |
|---------|-------|-------|---------|------|------|
| 1       | 723.0 | 40.60 | 1.567   | 0.01500 | 2.0   |
| 2       | 8.23  | 79.375| 5.067   | 0.00898 | 1.8   |
| 3       | 71.02 | 79.375| 5.067   | 0.00898 | 1.8   |
| 4       | 156.36| 79.375| 5.067   | 0.00898 | 1.8   |
| 5       | 38.71 | 79.375| 5.067   | 0.00898 | 1.8   |
| 6       | 30.48 | 25.40 | 0.057   | 0.02168 | 1.8   |

#### Figure 6. Diagram of towed array system cable.
Table 4. Comparison of Verification Simulation Results for Node A.

| Node A     | Result | Data from Rispin | Ablow | Milinazzo | Grindheim |
|------------|--------|-----------------|-------|-----------|-----------|
| Initial depth (m) | 12.21  | 10.04           | 10.95 | 11.87     | 11.91     | 12.02     | 12.20     |
| Minimum depth (m)  | 3.42   | 2.51            | 3.54  | 3.46      | 3.40      | 3.40      | 3.42      |
| Difference(initial-min)(m) | 8.79   | 7.52            | 7.41  | 8.41      | 8.51      | 8.62      | 8.78      |
| Final depth (m)     | 10.95  | 10.16           | 10.82 | 12.47     | 12.45     | 12.63     | 10.73     |
| Difference(final-min)(m) | 7.53   | 7.65            | 7.28  | 9.01      | 9.05      | 9.23      | 7.30      |

Figure 7. Graph between the velocity and cable depth, angle.
5.2.1. Acceleration/deceleration movement. In this section, the transient characteristics of the underwater towing system under the influence of ocean currents are studied under the assumption that the towing ship travels linearly along the Y-axis in the longitudinal section. The steady-state solution is taken as the initial state of dynamic motion simulation. Linear towed variable speed motion is:

$$v = (v_f - v_0)(t / T) + v_0$$ (33)

Where $v_0$, $v_f$ are the initial and final velocity of tow-ship; $T$ is the response time to the end speed [17].

a) Towing ship accelerates from 4-12 m/s. The steady-state configuration and tension value during the acceleration without current is basically consistent with the corresponding value of VK Srivastava [17]. In addition, the cable depth curve, the tension extreme curve and the angle variation during the acceleration, when there is no current, along with and against sea current directions respectively are also given, as shown in Fig.7(a). Cable depth varies from 45.9 to 19.1m and the towed point tension varies from 12.83kn to 90.12kn under three different operating conditions. It can be seen that the cable depth reaches the maximum and the tension of the tow-point minimum during the acceleration along the current direction, and vice versa anti.

b) Towing ship decelerates from 12-4 m/s. The cable depth curve, the tension extreme curve, and the angle variation during the deceleration, when there is no current, along with and against sea current directions respectively are given, as shown in Fig.7(b). Cable depth varies from 10.7 to 17.3m and the towed point tension varies from 46.39kn to 162.85kn under three different operating conditions. It can be seen that the cable depth reaches the maximum and the tension of the tow-point minimum during the deceleration along the current direction, and vice versa anti.

c) Towing ship accelerates from 4-12 m/s & decelerates from 12-4 m/s subsequently. The cable depth curve, the tension extreme curve, and the angle variation during the acceleration & deceleration, when there is no current, along with and against sea current directions respectively are given, as shown in Fig.7(c). Cable depth varies from 53.5m to 18.9m and the towed point tension varies from 12.83kn to 93.21kn under three different operating conditions. It can be seen that the cable depth reaches the maximum and the tension of the tow-point minimum during the acceleration & deceleration along the current direction, and vice versa anti.

| Table 5. Cable Depth and Towed Point Tension Variation in Linear Towed Profile. |
|-----------------------------------------------|
| Linear towed profile                        |
| 4-12m/s                  | 12-4m/s                  | 4-12m/s&12-4m/s          |
| Depth(m) | Tension(kn) | Depth(m) | Tension(kn) | Depth(m) | Tension(kn) |
|   J=0   | 39.9-20.9  | 16.73-82.18 | 10.8-16.4  | 53.95-150.08 | 39.6-19.7  | 16.73-88.17 |
|   J=0.5 | 45.9-23.8  | 12.83-77.44 | 11.8-17.3  | 46.39-137.84 | 53.5-32.4  | 12.83-78.16 |
|   J=-0.5| 37.8-19.1  | 21.14-90.12 | 10.7-15.8  | 63.66-162.85 | 34.6-18.9  | 21.14-93.21 |

6. Conclusion
In this paper, the governing equation of towing system is established based on the lumped mass method. The cable is discretized at the beginning of modeling, and converted into cascading nodes of finite length. According to Newton’s second law or D’Alembert’s principle, the partial differential equations of continuous dynamic characteristics are obtained. The 3N differential equation of motion is solved by the finite difference time domain method. The numerical simulation shows that the initial of the towing system is set to steady-state in dynamic characteristics simulation, which can greatly reduce the system calculation time and improve the stability of the algorithm. The dynamic response of the streamer in the linear towing profile is further analyzed, and the tension disturbance curve and the cable response configuration under three different operating conditions-when there is no current, along and against sea current directions-are given respectively, which can be used for dynamic load estimation and motion
trajectory prediction in towed-array systems. This is of practical significance to improve the safety and efficiency of offshore operations.

Acknowledgments
Projects supported by the National Key R&D Program of China (2016YFE0205700), National Natural Science Foundation of China (Grant No. 51705288), Shandong Provincial Natural Science Foundation(ZR2017QEE001), Shandong Provincial Natural Science Foundation (2018GHY115006), Open Foundation of the State Key Laboratory of Fluid Power and Mechatronic Systems (GZKF-201805).

References
[1] Qi Z F, Jia L J, Qin Y F, et al. Dynamic Modeling and Simulating Analysis of Submersible Buoy System [J]. Applied Mechanics and Materials, 2013, 475-476: 6.
[2] Li S, Wei J, Guo K, et al. Nonlinear Robust Prediction Control of Hybrid Active– Passive Heave Compensator with Extended Disturbance Observer [J]. IEEE Transactions on Industrial Electronics, 2017, PP (99): 1-1.
[3] Dewey R K. Mooring design and dynamics - A Matlab® package for designing and analyzing oceanographic moorings [J]. Marine Models, 1999, 1 (1-4): 103-157.
[4] Ablow C M, Scheckter S. Numerical simulation of undersea cable dynamics [J]. Ocean Engineering, 2012, 10 (6): 443-457.
[5] Howell C T. Numerical Analysis of 2-D Nonlinear Cable Equations with Applications to Low-Tension Problems [J]. International Journal of Offshore & Polar Engineering, 1991, 2 (2).
[6] Z. Yuan and L. Jin, A dynamic model to maintain the depth of underwater towed system, Proc. International Conference on System Science & Engineering, 2012.
[7] Y.B. Wang, L. Li, T. Fu and S. Zhou, "[IEEE 2014 33rd Chinese Control Conference (CCC) - Nanjing, China (2014.7.28-2014.7.30)] Proceedings of the 33rd Chinese Control Conference - Research on underwater vehicle floating cable towing speed control technology," 2014.
[8] F.S. HoverM.A. Grosenbaugh and M.S. Triantafyllou, "Calculation of Dynamic Motions and Tensions in Towed Underwater Cables," Oceanic Engineering IEEE Journal of, vol. 19, no. 3, pp. 449-457, 1994.
[9] S.K. Pang, J.Y. Liu, C. Hong, W. Jian and Y. Hong, Analysis of motion state of the tow-part underwater towed vehicle system during cable deployment, Proc. Oceans, 2017.
[10] Wang F, Huang G L, Deng D H, et al. A study on dynamic response of cable-seabed interaction [J]. Journal of Shanghai Jiaotong University (Science), 2009, 14 (4): 443-449.
[11] Wang F, Huang G L, Deng D H. Dynamic response analysis of towed cable during deployment/retrieval [J]. Journal of Shanghai Jiaotong University (Science), 2008, 13 (2): 245.
[12] Walton T S, Polachek H. Calculation of transient motion of submerged cables [J]. Mathematics of Computation, 1960, 14 (69): 27-46.
[13] Delmer T N, Stephens T C, Tremills J A. Numerical simulation of cable-towed acoustic arrays [J]. Ocean Engineering, 1988, 15 (6): 511-548.
[14] Rispin P. Data Package No.1 for Cable and Array Maneuvering. David W. Taylor Naval Ship Research and Development Center, Bethesda, Maryland (1980).
[15] Milinazzo F, Wilkie M, Latchman S A. An efficient algorithm for simulating the dynamics of towed cable systems [J]. Ocean Engineering, 1987, 14 (6): 513-526.
[16] Grindheim J V, Revhaug I, Pedersen E. Utilizing the EnKF (Ensemble Kalman Filter) and EnKS (Ensemble Kalman Smoother) for Combined State and Parameter Estimation of a 3D Towed Underwater Cable Model [J]. Journal of Offshore Mechanics & Arctic Engineering, 2017, 139 (6).
[17] Array U T C, Dynamics C, Systems T, et al. Vineet Kumar Srivastava, YVSS Sanyasiraju, Mohammad Tamsir, Dynamic Behavior of Underwater Towed-cable in Linear Profile [J]. 201.