Gravitational waves from rotating neutron stars

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Abstract.
In this review we examine the dynamics and gravitational wave detectability of rotating strained neutron stars. The discussion is divided into two halves: triaxial stars, and precessing stars. We summarise recent work on how crustal strains and magnetic fields can sustain triaxiality, and suggest that Magnus forces connected with pinned superfluid vortices might contribute to deformation also. The conclusions that could be drawn following the successful gravitational wave detection of a triaxial star are discussed, and areas requiring further study identified. The latest ideas regarding free precession are then outlined, and the recent suggestion of Middleditch et al (2000a,b) that the remnant of SN1987A contains a freely precessing star, spinning-down by gravitational wave energy loss, is examined critically. We describe what we would learn about neutron stars should the gravitational wave detectors prove this hypothesis to be correct.
1. Introduction

Broadly speaking, gravitational wave emission from rotating neutron stars can be divided into two classes: emission due to the normal modes of oscillation of the fluid core, and emission due to some non-fluid agent deforming the star, such as the crust or an internal magnetic field. As a perusal of journal archives readily reveals, most research into gravitational wave emission from rotating stars falls into the first class. The main reason for this is that fluid oscillations can undergo the Chandrasekhar-Friedman-Schutz (CFS) instability (Friedman & Schutz 1978a,b), where gravitational radiation reaction amplifies a mode that propagates forward with respect to the inertial frame, but backward with respect to the rotating star. Whether or not such a mode grows to large amplitude is then determined by the strength of dissipative processes that tend to sap its energy, such as viscosity of the stellar fluid (see Lindblom’s article in this volume for a review of one such gravitationally unstable mode - the r-mode).

In this review will concentrate entirely on the relatively neglected second class of emission, i.e. that due to the neutron star not being a simple fluid, but instead being able to support some sort of strain. Such a star could then be triaxial (i.e. support some sort of ‘mountain’), and radiate gravitationally at twice its rotation rate. There is no gravitational wave instability for such a star, but equally there will be no viscous processes within it, allowing a perfectly efficient conversion of kinetic into gravitational wave energy. Traditionally, the two candidates for producing triaxiality have been strains within the crystal lattice of the crust, and a deformation due to a strong internal magnetic field. In section 2, we will describe some recent developments in modelling these effects, and also describe some new possibilities for producing deformations, which make use of the pinned component of the neutron star fluid predicted by glitch modellers.

In addition to triaxiality, a deformed neutron star can radiate gravitationally by undergoing free precession (sometimes referred to simply as ‘wobble’). Such a motion is in the rather unfortunate position of not being susceptible to the gravitational wave instability of the fluid modes, and yet still suffering from viscous dissipation within the star. Perhaps because of this, very little work has been done to investigate the detectability of freely precessing stars. In section 3, we will summarise some recent work on this problem, and comment on the recent hypothesis of Middleditch et al (2000a,b) that a gravitationally spinning down freely precessing pulsar exists in the remnant of supernova 1987A.

2. Triaxial neutron stars

2.1. Crustal strain

The most obvious way of deforming the shape of a neutron star from that of the equivalent rotating fluid is to suppose there is a strain in its solid crust. Indeed, very soon after glitches were first observed, a crust-quake model was proposed (Baym & Pines 1971, Pines & Shaham 1972). The key idea was that the crust first solidified when the star was spinning very rapidly, and was therefore very oblate. As the spin rate gradually decreased, so did the oblateness, straining the crust until some critical breaking strain was reached, whereupon the crust cracked. The consequent decrease in moment of inertia of the star resulted in the small fractional increase in spin rate observed as a glitch.
This model soon went out of favour when it was found that not all glitching pulsars could be described by this model, as the amount of elastic energy that needed to be stored far exceeded that likely to be available (Lyne & Graham-Smith 1998). Nevertheless, the machinery developed to describe glitches can be used to estimate how large a non-axisymmetry the crust can support. For convenience, we will consider a non-rotating star, with an axisymmetric deformation whose size we will characterise by the dimensionless small number $\epsilon$ (this might be the change in radius along some direction divided by the average radius). Suppose the crust would be entirely unstrained if $\epsilon$ took some particular value $\epsilon_0$. The energy of such a star can be written as:

$$E(\epsilon) = E_{\text{spherical}} + A\epsilon^2 + B(\epsilon - \epsilon_0)^2,$$

where $E_{\text{spherical}}$ is the energy the star would have if it were spherical, $A\epsilon^2$ is the increase in gravitational potential energy due to the non-sphericity, and $B(\epsilon - \epsilon_0)^2$ is the strain energy stored in the crust. Note that the constant $A$ is of order the total gravitational binding energy of the star, while the constant $B$ is of the order of the total electrostatic binding energy of the crust.

The actual shape of the star will minimise this energy, so that:

$$\epsilon = \frac{B}{A + B} \epsilon_0.$$  

(2)

This is the estimate of crustal deformation we required. (Strictly, we are interested in the case where a small strain of the above size is superimposed upon the shape of a rapidly rotating star, but including the effects of rotation in our analysis would not change the key result, i.e. equation [3]). Fortunately, estimates of $B$ and $A$ are readily available (see Jones & Andersson 2001). The coefficient $B$ is a property of the crust, and so depends mainly on the rather well known low-density equation of state. For a canonical $M = 1.4M_\odot$ $R = 10^6$ cm neutron star, a value of around $10^{47}$ ergs is found. The value will be higher for lower mass stars, as they have thicker crusts. The coefficient $A$, being of order of the gravitational binding energy of the whole star, is found to be rather insensitive to the total mass, taking a value of around $10^{52}$ ergs. We therefore find:

$$\epsilon \sim 10^{-5} \epsilon_0.$$  

(3)

for a canonical neutron star. In words, the electromagnetic forces are very much weaker than the gravitational ones, so that the deformation induced by the crust is five orders of magnitude smaller than the ‘zero strain’ shape of the crust.

This mismatch between the actual shape of the star and its zero strain shape results in a strain in the crust, of dimensionless size $\sim |\epsilon - \epsilon_0| \approx |\epsilon_0|$. This strain cannot exceed the crustal breaking strain, $u_{\text{break}}$, and so we obtain for the upper bound on the triaxiality:

$$\epsilon < \frac{B}{A + B} u_{\text{break}}.$$  

(4)

Unfortunately, the breaking strain is rather uncertain. By extrapolation of breaking strains of terrestrial materials Ruderman (1992) estimates this to lie in range $10^{-2} \rightarrow 10^{-4}$. Combining with the estimates of $A$ and $B$ we finally obtain:

$$\Rightarrow \epsilon_{\text{max}} \sim 10^{-7} \rightarrow 10^{-9}.$$  

(5)

In words, we can expect the non-axisymmetric deformations to correspond to mountains of height in the range $10^{-1} \rightarrow 10^{-3}$ cm.
So much for the bound placed on triaxiality by neutron star structure. The question remains: Why would a non-axisymmetry be formed in the first place? Very little work has been done on this problem. One possibility would be that when very young, a hot, entirely fluid neutron star undergoes a CFS-type instability, resulting in large violent motions of the fluid near the surface. As the star cools, the crust begins to solidify. If this solidification occurs while the mode is still active, an ‘imprint’ of the mode might remain in the zero-stress shape of the crust, leaving a mountain whose structure reflects that of the mode that created it.

Another rather subtle way of producing a non-axisymmetry was proposed recently by Bildsten (1998), motivated by the possibility that the low-mass X-ray binary stars (LMXBs, see Van der Klis 2000) might have their spins limited by gravitational wave emission. Bildsten proposed that, providing a slight non-axisymmetry in the temperature was present, the compression-induced burning of accreted nuclei will occur in a non-axisymmetric way, thereby building up an asymmetry in the mass quadrupole. This idea was investigated in detail by Ushomirsky, Cutler & Bildsten (2000), who solved the combined equations of elasticity and heat flux for an accreting star. They found that it was indeed possible to build up a sufficiently large mass quadrupole for the gravitational spin-down torque to balance the spin-up accretion torque, provided that temperature asymmetries at the 5% level were present, and that the crustal breaking strain is as large as $10^{-2}$, i.e. at the upper end of the range estimated by Ruderman.

This is a very exciting result for gravitational wave physicists, as one LMXB in particular, Sco X-1, might be visible to first generation interferometer detectors, providing sufficiently accurate search templates are available. We will discuss gravitational wave detectability of triaxial stars in section 2.4.

### 2.2. Magnetic strain

The second of the two standard mechanisms for producing triaxiality concerns neutron star magnetic fields. Indeed, most of our observational data on neutron stars comes from the pulsars, whose defining characteristic is a non-axisymmetry in their magnetic field structure. The standard formula for estimating this deformation comes from balancing the total magnetostatic energy stored by a uniform field $B_{\text{mag}}$ within the star to the gravitational binding energy:

$$\epsilon \sim \frac{B_{\text{mag}}^2 R^3}{GM^2/R} \sim 10^{-12} \left( \frac{B_{\text{mag}}}{10^{12} \text{G}} \right)^2$$

for a canonical pulsar. For typical magnetic fields of order $10^{12}$ G this is several orders of magnitude smaller than the sorts of deformation that crustal strain can produce. However, the size and geometry of the magnetic field within a neutron star is highly uncertain. As discussed by Bonazzola & Gourgoulhon (1996), if the interior fluid is a type I superconductor, the magnetic field will be expelled from the core fluid, and confined to the crustal region. Such a confinement will result in internal magnetic fields much larger than the external polar magnetic field, and can produce deformations several orders larger than formula (6) would suggest.

### 2.3. Magnus strain

The two forms of deformation considered above have both made use of electrostatic forces. However, in the course of attempting to explain glitches, theorists have realised
that other sorts of force may play an important role in neutron stars. If the core is a superfluid, it rotates by forming an array of vortices, with the number per unit area being proportional to the rotation rate. About 1% of the core fluid penetrates the solid crust. The vortices in this region are predicted to ‘pin’ to nuclei in the solid lattice, so that the rotation rate of this fluid is fixed, even when the crust is spun-up or spun-down. This creates an angular velocity difference between the fluid and the vortices, which in turn results in a Magnus force on the vortices. This force is transferred, via the pinning sites, to the crust itself.

The effect of this Magnus force on the crust has been considered by Ruderman (1976, 1991, see also Ruderman et al 1998), who has shown that, provided that the vortex pinning does not break first, the Magnus force can crack the crust. Indeed, Ruderman has used this force to construct a theory of pulsar magnetic dipole evolution, where the dipole, frozen into the highly conducting crust, migrates with the fracturing crustal plates toward the rotational equator of a spinning down star. A simple estimate of the size of deformation the Magnus force can produce is straightforward. Following Ruderman (1991), the force per unit volume on the crystal lattice is:

\[ F = 2\omega \times (\Omega \times r) \rho_n f_{\text{pin}} \]  

where \( \omega \) is the angular velocity difference between the superfluid and the lattice, \( \Omega \) the angular velocity of the star, \( \rho_n \) the superfluid neutron density and \( f_{\text{pin}} \) a geometric factor likely to be of order unity. The total force on the crust is then of order the crustal volume \( \Delta V \) time this. The deformation is of the order of this force divided by the gravitational force at the surface \( GM^2/R^2 \), giving

\[ \epsilon \sim \frac{2\omega \Omega R^3 \Delta M}{GM^2} \]  

where the crust mass \( \Delta M \sim \rho_n \Delta V \). Parameterising:

\[ \epsilon \sim 5 \times 10^{-7} \left( \frac{\omega}{1 \text{ Hz}} \right) \left( \frac{f}{\text{kHz}} \right) \left( \frac{R}{10^6 \text{ cm}} \right)^3 \left( \frac{\delta M}{0.01 M_\odot} \right) \left( \frac{1.4 M_\odot}{M} \right)^2 \left( \frac{f_{\text{pin}}}{1} \right). \]  

This is larger than the deformations due to conventional crustal strain, and so is clearly of interest.

For a spinning-down star (such as a pulsar) \( \omega \) is parallel to \( \Omega \), and equation (7) shows that the Magnus force is directed away from the rotation axis, making the crust more oblate than its rotational equilibrium shape. Conversely, for a spinning-up star (such as a LMXB before the compact has has reached spin equilibrium) the Magnus force tends to make the crust less oblate. (There must be an equal and opposite force on the fluid, so the actual change in shape of the star is not obvious).

However, we are not interested in axi-symmetric deformations. We would like to know how the Magnus force might induce triaxiality. This question has not, as far as we are aware, been addressed previously, and is clearly of great interest for gravitational wave emission. One (speculative) mechanism would apply if the Bildsten type asymmetries occur in an accreting system: Any asymmetry in the atomic number or temperature of nuclei would result in different pinning strengths. Certain areas of crust might then pin nuclei perfectly, thereby feeling the full Magnus force. Other area might pin nuclei relatively weakly. The slippage of the vortices through these areas would then lead to a somewhat weaker Magnus force being transmitted to the lattice. Whether or not this could lead to a significant triaxiality is clearly a problem of interest.
2.4. Gravitational waves

Having reviewed the physics behind triaxiality, we now reach our intended destination: a discussion of gravitational wave detection. Figure 1 is a modern version of the classic ‘upper bound on the gravitational wave amplitude’ diagram. In the figure we plot gravitational wave amplitudes and detector noises versus frequency. Each black dot represents a pulsar of known period and period derivative. These two pieces of data, together with an estimate of the distance from Earth and the star’s moment of inertia (assumed to be $10^{45} \text{g cm}^2$ for all) allows an estimate of the gravitational wave amplitude, assuming all of the spin-down energy is converted to gravitational wave energy. A similar diagram could have been drawn at any point over the last twenty years.

![Figure 1. Upper bound on $h$ from spin-down rates for pulsars and accretion luminosity for LMXBs. Pulsar data from Princeton catalogue and Lorimer (2001), LMXB data from Bildsten (1998).](image)
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The spin rate of an accreting star allows a calculation of a gravitational wave amplitude. The key assumption here is that the accreting plasma exerts a torque at the stellar surface appropriate to it being in a Keplerian orbit just prior to accretion. (Note that there is some uncertainty in the exact spin rates of these objects, connected with the interpretation of quasi-periodic oscillations in their electromagnetic fluxes—see van der Klis 2000).

Clearly, the wave amplitudes in the figure represent upper bounds, probably rather optimistic ones in the case of the pulsars, whose spin-down energy budget probably involves electromagnetic energy losses in addition to gravitational ones. Nevertheless, we should be ready, in the event of a successful gravitational wave detection, to try and draw conclusions regarding neutron star structure.

The first thing to note is that there are two separate groups of stars in the figure that lie above or close to the first-generation noise curves. At the low frequency end we have the young pulsars (e.g. the Crab and Vela), while at the high frequency end we have the old millisecond pulsars and the LMXBs. A successful detection of a triaxial star would lead to very different conclusions being drawn, depending upon whether it is a low or high frequency source that is observed.

Working backwards, the degree of triaxiality required to explain the gravitational wave spin-down of a typical young pulsar in the diagram is of order $10^{-3}$, approximately four orders of magnitude larger than the largest triaxialities discussed previously. If such a gravitational wave signal were to be observed, we would be forced to conclude that our picture of neutron star interiors is seriously lacking. If magnetic fields were responsible, an internal field in excess of $10^{16}$ G would be required (equation 6), around four orders of magnitude larger than the external polar field. Magnus forces are certainly too weak to produce such a triaxiality. The remaining possibility might be that our knowledge of the high density equation of state is poor, and that a solid, highly rigid core exists within the star, capable of supporting a much larger triaxiality than the relatively low density crust. Such a result would be very exciting, and fulfill the promise that gravitational wave astronomy can probe deeper into neutron star interiors than electromagnetic observations.

Observation of a gravitational wave signal at or close to the strengths indicated from one of the high frequency sources would lead to very different conclusions. The degree of triaxiality required to explain this strength of emission is around $10^{-8}$, comfortably within the range of $\epsilon$ values discussed previously. In terms of our theoretical expectations, these high frequency sources seem much better bets than the low frequency ones, an expectation that may guide gravitational wave data analysts in their searches. However, this creates a new problem: If such a source were to be observed, how could we decide which of the three deformation mechanisms described above (if any) is producing the deformation?

One partial solution suggests itself, based on the fact that if it is a strong magnetic field that is producing the triaxiality, the quadrupole moment (and therefore the gravitational wave phase and amplitude) should not be significantly affected by a glitch, whereas if either crustal strain or Magnus forces are responsible, the gravitational wave amplitude and possibly the phase should both change abruptly. However, in over 30 years of observation, a millisecond pulsar has not yet been observed to glitch (Lyne & Graham-Smith 1998), while the spin rates of the LMXBs are not known accurately or monitored frequently (van der Klis 2000), so further distinguishing features need be found. Clearly, this is a tricky problem, and one which seems not to have been investigated previously. Nevertheless, its resolution is required...
Figure 2. The geometry of free precession. The symmetry axis of the deformation lies along \( \mathbf{n} \), which moves in a cone of half-angle \( \theta \) around the angular momentum vector \( \mathbf{J} \). The angular velocity vector, denoted by \( \mathbf{\Omega} \), lies in this plane also.

if we are to ever to fulfil our promise of using gravitational wave data to learn about the interiors of these stars.

3. Free precession

Free precession is the most general motion of a rigid body (Landau & Lifshitz 1976). For an axisymmetric rigid body, the motion consists of the symmetry axis moving in a cone of half-angle \( \theta \) (known sometimes as the ‘wobble angle’) about the fixed angular momentum vector \( \mathbf{J} \). The angular velocity vector \( \mathbf{\Omega} \) lies in this plane also, as illustrated in figure 2. The whole plane rotates around \( \mathbf{J} \) at a rate that we will call \( \dot{\phi} \). In addition to this rotation, there is a superimposed slow rotation of the star about \( \mathbf{n} \), at a rate \( \dot{\psi} \) known as the body-frame precession frequency. The two rotation rates are related by:

\[
\dot{\psi} = -\frac{\Delta I}{I_0} \dot{\phi}
\]

where \( \Delta I \) is the non-spherical piece of the moment of inertia tensor symmetric about \( \mathbf{n} \), and \( I_0 \) is the total moment of inertia along this axis. Free precession was first discussed as a source of gravitational radiation by Zimmermann (1978) and Zimmerman & Szenes (1979), who showed that mass quadrupole gravitational radiation was produced at the frequencies \( \dot{\phi} \) and \( 2\dot{\phi} \). If the star radiates electromagnetically as a pulsar, the pulsation rate is given by a rather complicated time-varying combination of \( \dot{\phi} \) and \( \dot{\psi} \) (Jones & Andersson 2000).

Of course real neutron stars are not rigid—they consist of a thin elastic shell containing a superfluid core, possibly with part of the fluid pinned to the crust. With the obvious exception of the superfluidity and pinning, the Earth itself has a similar structure, so the machinery that geophysicists have evolved to describe the Earth’s free
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precession (known as the ‘Chandler wobble’) can be used to begin to describe neutron stars (Pines & Shaham 1972). One important point is that, because the crust is elastic, during the course of one free precession period its shape changes, corresponding to the movement of the ‘centrifugal bulge’, symmetric about $\Omega$, in a cone around $n$. This places a limit on the wobble angle given by:

$$\theta_{\text{max}} \approx \frac{u_{\text{break}}}{\epsilon_{\text{rotation}}} \approx 0.45 \left(\frac{100 \text{ Hz}}{f}\right)^2 \frac{u_{\text{break}}}{10^{-3}}. \quad (11)$$

where $\epsilon_{\text{rotation}} \approx \Omega^2 R^3/GM$, is the (dimensionless) size of the centrifugal deformation. The effects of pinning can be added relatively straightforwardly (Shaham 1977). The final picture that emerges is that the basic geometry described above applies even to realistic neutron stars, although the free precession timescale depends upon the internal structure in a complicated way (Jones & Andersson 2000).

Gravitational wave emission from realistic precessing stars was examined by Alpar & Pines (1985), who pointed out that for a long-lived freely precessing star to exist, some mechanism must be found to continually excite the wobble motion, as dissipative processes within the star tend to damp the motion. The magnitude of the internal dissipation was estimated by Alpar & Sauls (1988), who found that, in the absence of any excitation mechanism, free precession is damped in between $400$ and $10^4$ free precession periods.

There have been three very recent developments in this field. First of all, Stairs, Lyne & Shemar (2000) have reported evidence of a freely precessing pulsar, identified by careful analysis of the electromagnetic pulse timing data. This pulsar spins very slowly, and so is not of gravitational wave interest, except inasmuch as it allows us to test our understanding of the physics of precession. As noted by Jones & Andersson (2000), the timing data were consistent with theoretical expectations, provided that none (or virtually none) of the superfluid is pinned to the crust, seemingly in conflict with the standard glitch model of pulsar behaviour. A resolution to this seeming paradox was supplied by Link & Cutler (2001), who showed that the Magnus forces induced by the precessional motion might be sufficient to break the vortex pinning, provided that the pinning is sufficiently weak. In this way, the standard pulsar model could continue to explain glitches in non-precessing pulsars, while still accounting for the observations of Stairs et al.

Secondly, and of most interest to us, Jones & Andersson (2001) have recently undertaken a systematic study of the likely free precession gravitational wave amplitudes likely to exist in Nature (see also Jones 2001). The problem is one of finding sufficiently strong wobble-pumping mechanisms to counteract the dissipation of precessional energy within the star. In this study, many different pumping mechanisms were considered, including accretion torques, electromagnetic torques on pulsars, wobble induced by the violent birth of a neutron star in a supernova, and near collisions between a neutron star and another star. In all cases it was found that the dissipation of precessional energy within the neutron star limited the gravitational wave amplitude to a level undetectable even by a second generation laser interferometer. These conclusions were reached assumed that no superfluid was pinned to the crust, is agreement with the Stairs et al (2000) observation and the Link & Cutler (2001) pinning model. If a pinned component is included, the internal dissipation rate is even higher, reinforcing this pessimistic conclusion: freely precessing neutron stars don’t seem to be good sources of gravitational radiation.
However, the third recent development in this field is a possible observational of a gravitationally spinning-down pulsar, reported by Middleditch et al (2000a,b). The pulsar in question was found in the remnant of supernova 1987A, and was observed only intermittently between 1992 and 1997, with no reported sightings since. The free precession period $2\pi/\dot{\psi}$ seemed to vary, spanning a range of 935s to 1430s.

The main piece of evidence Middleditch et al. cite to support the gravitational wave spin-down hypothesis is that the frequency derivative $\dot{f}$ correlates rather well with the inverse square of the long modulation period $P_{fp}$, with $\dot{\Omega}$ varying by a factor of $\sim 4$, while $P_{fp}$ varies by a factor of 2. The spin-down rate of a precessing pulsar is given by (Cutler & Jones 2000):

$$\dot{\Omega} = \frac{32G}{5c^5} \Omega^5 \left(\frac{\Delta I}{I_{\text{star}}}\right)^2 \sin^2 \theta (\cos^2 \theta + 16 \sin^2 \theta)$$ (12)

Combining equations (10) and (12) we see that just such a correlation is to be expected, provided that the wobble angle remains roughly constant. However, this is a somewhat curious state of affairs: it is difficult to imagine how the deformation in the moment of inertia tensor $\Delta I$ varies by a factor of two, while maintaining a roughly constant orientation with respect to the angular momentum vector.

The hypothesis of gravitational wave spin-down via free precession can be tested further. Note that $\Omega$, $\dot{\Omega}$, the ratio $\Delta I/I_0$ can all be extracted from the observational data, while $I_0$ estimated as $10^{43}$ g cm$^2$ if just the crust participates in the free precession, and $I_{\text{star}} \sim 10^{45}$ g cm$^2$. Then equation (12) can be inverted to give $\theta = 25^\circ$. Using equation (11) we see that this requires a crustal breaking strain of approximately $2 \times 10^{-2}$, higher than even the most optimistic estimate of Ruderman (1992). This difficulty is relieved somewhat if the whole star is assumed to participate in the free precession; then equation (12) gives $\theta = 9^\circ$, and equation (11) gives a crustal breaking strain of $8 \times 10^{-3}$, a somewhat more plausible value. In summary, if this pulsar really is spinning-down due to gravitational wave energy loss, the breaking strain of the crust must be very high, particularly if only the crust is participating in the free precession. By coincidence(?) the breaking strain required, approximately $10^{-2}$, is close to the value required by Ushomirsky, Cutler & Bildsten (2000) to support the triaxiality of the LMXBs.

To sum up, if gravitational wave observations were to confirm the free precession hypothesis for this pulsar, we will have learnt three important facts about neutron stars. Firstly, their deformations can vary significantly over rather short timescales (of order a few years); their crusts are rather strong, with breaking strains at the upper end of theoretical estimates; and finally that they can be set into free precession during, or soon after, birth. Clearly, when the laser interferometers reach the necessary sensitivity, a search for this precession candidate will be of great interest.

4. Conclusions

In this review we have examined our current understanding of the dynamics and gravitational wave detectability of rotating strained neutron stars. We divided our discussion into two halves: triaxial stars, and precessing stars. Both fields, having been rather quiet for some years, have enjoyed an injection of interest recently: The observations of spin frequencies in the LMXBs exciting interest into the triaxial problem, and observations from Stairs et al (2000) and Middleditch et al (2000a,b) exciting interest in the free precession case.
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For triaxial stars we noted that, for first generation gravitational wave detectors at least, an observation of gravitational wave spin-down from one of the young pulsars would require us to profoundly rethink our picture of neutron star interiors. A successful detection of gravitational waves from a millisecond pulsar or LMXB would be less spectacular, as this level of emission is consistent with current theories of stellar deformation. However, the problem of deciding just what was the mechanism responsible for the deformation has not yet been addressed, and stands in the way of our fulfilling our promise of using gravitational wave observations to probe neutron star interiors. To complicate the issue further, we have speculated that, in addition to the traditional crustal strain and magnetic field-induced deformation, triaxiality might be produced by the Magnus force on superfluid vortices pinned to the crust. A study of how such a force might set up a non-axisymmetry in a pulsar is clearly of great interest.

For freely precessing stars, the recent study of Jones & Andersson (2001) suggests that precessing stars do not seem to be a promising source of gravitational radiation. The reason for this is that dissipative forces within the stellar interior rapidly damp the precessional motion. However, Middleditch et al (2000a,b) have recently presented evidence of a pulsar within the remnant of supernova 1987A, and have suggested that it is spinning-down due to gravitational waves generated by free precession. We have investigated this hypothesis, and found that if correct we would learn a number of surprising things about neutron stars, not least that their deformations can vary in size significantly on a timescale of a few years.

To sum up, after having been rather quiet for sometime, the field of gravitational wave emission from strained neutron stars has heated up. Although still a poor cousin of the unstable fluid modes and binary inspiral, the types of source considered here will be high on the target lists of gravitational wave data analysts. As we hope this review makes clear, much could be learnt from a successful detection, but more work still need to be done to turn observations into constrains on the physics, and thereby turn gravitational wave astronomy into gravitational wave astrophysics.

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