Supervisor Localization of Discrete-Event Systems based on State Tree Structures

Kai Cai and W.M. Wonham

Abstract

Recently we developed supervisor localization, a top-down approach to distributed control of discrete-event systems in the Ramadge-Wonham supervisory control framework. Its essence is the decomposition of monolithic (global) control action into local control strategies for the individual agents. In this paper, we establish a counterpart supervisor localization theory in the framework of State Tree Structures, known to be efficient for control design of very large systems. In the new framework, we introduce the new concepts of local state tracker, local control function, and state-based local-global control equivalence. As before, we prove that the collective localized control behavior is identical to the monolithic optimal (i.e. maximally permissive) and nonblocking controlled behavior. In addition, we propose a new and more efficient localization algorithm which exploits BDD computation. Finally we demonstrate our localization approach on a model for a complex semiconductor manufacturing system.

I. INTRODUCTION

Recently we developed a top-down approach, called supervisor localization [1], [2], to the distributed control of discrete-event systems (DES) in the language-based Ramadge-Wonham (RW) supervisory control framework [3], [4]. We view a plant to be controlled as comprised of independent asynchronous agents which are coupled implicitly through control specifications. To make the agents ‘smart’ and semi-autonomous, our localization algorithm allocates external supervisory control action to individual agents as their internal control strategies, while preserving the optimality (maximal permissiveness) and nonblocking properties of the overall monolithic (global) controlled behavior. Under the localization scheme, each agent controls only its own events, although it may very well need to observe events originating in other (typically neighboring) agents. We call such a scheme distributed control architecture;

The authors are with the Systems Control Group, Department of Electrical and Computer Engineering, University of Toronto, 10 King’s College Road, Toronto, Ontario M5S 3G4 Canada. Emails: kai.cai@scg.utoronto.ca, wonham@control.utoronto.ca.
in a general sense it is common in the design and implementation of applications like multi-robot teams and mobile sensor networks (e.g. [5]).

Distinct, though related, control architectures are decentralized, hierarchical, and heterarchical (for recent developments see e.g. [6]–[9]). Both the distributed and the latter modular approaches aim to achieve efficient computation and transparent control logic, while realizing monolithic optimality and nonblocking. With modular supervision, global control action is typically allocated among specialized supervisors enforcing individual specifications. By contrast, with our distributed supervision it is allocated among the individual active agents ( [1], [2] provide further discussion of this distinction).

In this paper we continue our investigation of supervisor localization, but in the (dual) state-based framework of DES. We adopt the recently developed formalism of State Tree Structures (STS) [10], [11], adapted from Statecharts [12], which has been demonstrated to be computationally efficient for monolithic (i.e. fully centralized) supervisor synthesis in the case of large systems. Our aim is to exploit the computational power of STS to solve distributed control problems in that case as well.

STS efficiently model hierarchical and concurrent organization of the system state set. The latter is structured as a hierarchical state tree, equipped with modules (holons) describing system dynamics. For symbolic computation, STS are encoded into predicates. A second feature contributing to computational efficiency is the use of binary decision diagrams (BDD) [13], a data structure which enables a compact representation of predicates that admits their logical manipulation. With BDD representation of encoded STS models, the computational complexity of supervisor synthesis becomes polynomial in the number of BDD nodes (|nodes|), rather than in the ‘flat’ system state size (|states|). In many cases |nodes| ≪ |states|, thereby achieving computational efficiency. In localization, we exploit both these features of STS.

The contributions of this paper are the following. First, we establish supervisor localization theory in the STS framework: formulate the distributed control problem, define the notion of control cover for localization, and prove control equivalence between local controllers and the monolithic one. Compared to [1], this state-based localization theory has several new features: (1) Localization is implemented not by automata but by a state tracker and control functions (see Section II); the corresponding notions of local state tracker and local control function appear here for the first time; (2) the new concept of state-based control equivalence between local and monolithic supervision, which differs from the language-based notion in [1]; (3) an explicit definition of the event sets of local controllers, which determine inter-agent communication structure. Our second contribution is a symbolic localization algorithm which computes local controllers via predicates represented by BDDs; this algorithm is shown to be more efficient than that in [1].
Third, the state size reduction brought about by our localization algorithm can increase the transparency of control logic for large systems, as compared to the monolithic STS synthesis of [10], [11]; the latter can produce complex supervisors with very many BDD nodes. We illustrate this empirical result by a case study of the industrial example Cluster Tool taken from [8], [9]. Fourth, we extend localization to the case where component agents may share events, thus achieving greater formal completeness. As seen in Section IV-C a local controller is computed for each controllable event; when the latter is shared by several agents its (case-dependent) implementation is spelled out.

We note that there is a different approach, based on “polynomial dynamic systems”, to implement the monolithic supervisor by a set of distributed supervisors with communication [14]. The approach fixes a priori subsets of observable events for individual agents, which may practically rule out the existence and/or global optimality of the monolithic supervisor. By contrast, our localization approach always guarantees existence and global optimality, and the observation scopes of individual agents will result automatically as part of the solution. We also note that in [15], [16], the authors proposed a multi-agent coordination scheme in the RW framework similar in general terms to the distributed control architecture of our supervisor localization. Their synthesis procedure is essentially, however, a combination of the existing standard RW supervisor synthesis with partial observation [4] and supervisor reduction [17]; and no approach is presented to handle large systems. In this paper we establish our original supervisor localization in the STS framework, intended for large complex systems such as Cluster Tool.

The rest of the paper is organized as follows. In Section II we provide preliminaries on STS. In Section III we formulate the distributed control problem. Section IV develops the supervisor localization theory and presents a symbolic localization algorithm for computing local controllers. In Section VI we provide the Cluster Tool case study. Finally in Section VII we state conclusions.

II. PRELIMINARIES ON STATE TREE STRUCTURES

This section provides relevant preliminaries on the STS-based supervisory control theory, summarized from [10], [11].

A state tree structure (STS) $G$ for modeling DES is a 6-tuple:

$$G = (ST, \mathcal{H}, \Sigma, \Delta, ST_0, ST_m).$$

Here $ST$ is the state tree organizing the system’s state set into a hierarchy; $\mathcal{H}$ is the set of holons (finite automata) matched to $ST$ that describes the ‘local’ behavior of $G$; $\Sigma$ is the finite event set, partitioned into the controllable subset $\Sigma_c$ and the uncontrollable subset $\Sigma_u$. Let $ST(ST)$ denote the
set of all sub-state-trees of \( ST \). Then \( \Delta : ST(ST) \times \Sigma \rightarrow ST(ST) \) is the ‘global’ transition function; \( ST_0 \in ST(ST) \) is the initial state tree; and \( ST_m \subseteq ST(ST) \) is the set of marker state trees. A special type of sub-state-tree of \( ST \) is the basic (state) tree, each of which corresponds to one ‘flat’ system state in the RW framework. Let \( B(ST) \subseteq ST(ST) \) be the set of all basic trees of \( ST \). A predicate \( P \) defined on \( B(ST) \) is a function \( P : B(ST) \rightarrow \{0,1\} \) where 0 (resp. 1) stands for logical ‘false’ (resp. ‘true’). The predicate \( \text{false} \) (true) is identically 0 (1). Thus, \( P \) can be identified by the subset \( B_P \) of basic trees \( B_P := \{b \in B(ST) \mid P(b) = 1\} \). We shall often write \( b \models P \) for \( P(b) = 1 \). Also for a sub-state-tree \( T \in ST(ST) \), we define \( T \models P \) if and only if \( (\forall b \in B(T)) b \models P \). Given the initial predicate \( P_0 \) with \( B_{P_0} := \{b \in B(ST) \mid b \models P_0\} = B(ST_0) \), and the marker predicate \( P_m \) with \( B_{P_m} := \{b \in B(ST) \mid b \models P_m\} = \bigcup_{T \in ST_m} B(T) \), the STS \( G \) in (1) can be rewritten as

\[
G = (ST, \mathcal{H}, \Sigma, \Delta, P_0, P_m).
\]

Next write \( \text{Pred}(ST) \) for the set of all predicates on \( B(ST) \), and define propositional logic connectives for its elements as follows: for every \( P, P' \in \text{Pred}(ST) \) and \( b \in B(ST) \), (i) \( b \models (\neg P) \) iff \( \neg(b \models P) \); (ii) \( b \models (P \land P') \) iff \( (b \models P) \land (b \models P') \); (iii) \( b \models (P \lor P') \) iff \( (b \models P) \lor (b \models P') \). Introduce for \( \text{Pred}(ST) \) the partial order \( \leq \) defined by \( P \leq P' \) iff \( (P \land P') \models P \); namely \( P \leq P' \) holds exactly when \( b \models P \Rightarrow b \models P' \) for every \( b \in B(ST) \). Under the identification of \( \text{Pred}(ST) \) with the power set \( \text{Pwr}(B(ST)) \) and \( \leq \) with subset containment \( \subseteq \), it is clear that \( (\text{Pred}(ST), \leq) \) is a complete lattice. The top element is \( \text{true} \), the bottom element \( \text{false} \).

Important elements in \( \text{Pred}(ST) \) are the reachability and coreachability predicates. Let \( P \in \text{Pred}(ST) \). The reachability predicate \( R(G, P) \) holds on just those basic trees that can be reached in \( G \), from some \( b_0 \models P \land P_0 \), via a sequence of state trees all satisfying \( P \). Dually, the coreachability predicate \( CR(G, P) \) is defined to hold on those basic trees that can reach some \( b_m \models P \land P_m \) in \( G \) by a path of state trees all satisfying \( P \). It holds that \( R(G, P) \leq P \) and \( CR(G, P) \leq P \). A predicate \( P \) is nonblocking (with respect to \( G \)) if \( R(G, P) \leq CR(G, P) \), i.e. every basic tree reachable from some initial state tree can also reach some marker state tree in \( G \).

Another key property of a predicate is controllability (cf. controllability of a language [4]). For \( \sigma \in \Sigma \) define a map \( M_{\sigma} : \text{Pred}(ST) \rightarrow \text{Pred}(ST) \) by \( b \models M_{\sigma}(P) \) iff \( \Delta(b, \sigma) \models P \). Thus \( M_{\sigma}(P) \) identifies the largest subset of basic trees from which there is a one-step transition \( \sigma \) into \( B_P \), or at which \( \sigma \) is not defined (i.e. \( \Delta(b, \sigma) = \emptyset \)). A predicate \( P \) is called weakly controllable if \( (\forall \sigma \in \Sigma_u) P \leq M_{\sigma}(P) \). Thus \( P \) is weakly controllable if it is invariant under the dynamic flow induced by uncontrollable events. For an arbitrary predicate \( P \in \text{Pred}(ST) \) bring in the family \( NC(P) \) of nonblocking and weakly controllable
subpredicates of $P$, $\mathcal{NC}(P) := \{K \preceq P \mid K \text{ is nonblocking and weakly controllable}\}$. Then $\mathcal{NC}(P)$ is nonempty (since $K = \text{false}$ belongs) and is closed under arbitrary disjunctions $\lor$; in particular the supremal element $\sup \mathcal{NC}(P) := \bigvee \{K \mid K \in \mathcal{NC}(P)\}$ exists in $\mathcal{NC}(P)$.

Now define a state feedback control (SFBC) $f$ to be a function $f : \mathcal{B}(\mathcal{S}T) \to \Pi$, where $\Pi := \{\Sigma' \subseteq \Sigma \mid \Sigma_u \subseteq \Sigma'\}$. Thus $f$ assigns to each basic tree $b$ a subset of events that always contains the uncontrollable events. For $\sigma \in \Sigma$ define a control function (a predicate) $f_\sigma : \mathcal{B}(\mathcal{S}T) \to \{0, 1\}$ according to $f_\sigma(b) = 1$ iff $\sigma \in f(b)$. Thus the control action of $f$ is fully represented by the set $\{f_\sigma | \sigma \in \Sigma\}$. By definition $f_\sigma(\cdot) = \text{true}$ for every uncontrollable event $\sigma$. The closed-loop STS formed by $G$ and $f$ is then written as

$$G^f = (\mathcal{S}T, \mathcal{H}, \Sigma, \Delta^f, P^f_0, P^f_m),$$

where $P^f_0 = R(G^f, \text{true}) \land P_0$, $P^f_m = R(G^f, \text{true}) \land P_m$, and the transition function (under $f$) $\Delta^f(b, \sigma) = \Delta(b, \sigma)$ if $f_\sigma(b) = 1$ and $\Delta^f(b, \sigma) = \emptyset$ otherwise. A SFBC $f$ is nonblocking if $R(G^f, \text{true}) \preceq CR(G^f, \text{true})$.

**Theorem 1.** [10, Theorem 3.2] Let $P \in \text{Pred}(\mathcal{S}T)$ and $P_0 \land \sup \mathcal{NC}(P) \neq \text{false}$. Then there exists a nonblocking SFBC $f$ such that $R(G^f, \text{true}) = R(G, \sup \mathcal{NC}(P))$.

Theorem 1 is the main result for STS on synthesizing an optimal (in the sense of supremal, or maximally permissive) and nonblocking supervisor. The SFBC $f$ in Theorem 1 is represented by the control functions $f_\sigma$, $\sigma \in \Sigma$, defined by

$$f_\sigma := M_\sigma(\sup \mathcal{NC}(P)).$$

Thus for every $b \in \mathcal{B}(\mathcal{S}T)$, $f_\sigma(b) = 1$ if and only if $\Delta(b, \sigma) \models \sup \mathcal{NC}(P)$.

We close this section by describing how to set up a control problem in STS, as will be needed in Section III. Recall [4] that a finite-state automaton $P$ is defined by

$$P := (Q, \Sigma, \delta, q_0, Q_m),$$

where $Q$ is the state set, $q_0 \in Q$ is the initial state, $Q_m \subseteq Q$ is the subset of marker states, $\Sigma$ is the finite event set, and $\delta : Q \times \Sigma \to Q$ is the (partial) state transition function. In the RW (language-based) framework, a control problem is typically given in terms of a plant automaton $P$ and a specification automaton $S$ that imposes control requirements on $P$. We can convert the pair $(P, S)$ into an STS $G$ with a predicate $P$ specifying the illegal basic trees that $G$ is prohibited from visiting. Conversion is illustrated by the example displayed in Fig. 1. Here the plant $P$ consists of two ‘machines’ $M1, M2,$
and the specification automaton is the buffer BUF of capacity one. First assign to each of the three automata a state variable which takes values in the corresponding state set; then bring in a root state \( x_0 \) which links the assigned state variables \( x_1, x_2, y \) by cartesian product. Thereby we obtain the STS \( \mathcal{G} \). Finally we determine the predicate \( P \) for illegal basic trees according to the control requirements imposed by the specification \( \mathcal{S} \). In the example \( \mathbf{BUF} \) conveys two requirements: (i) disabling event \( \alpha_2 \) at state \( y_0 \) (so the buffer is protected from underflow) and (ii) disabling \( \beta_1 \) at \( y_1 \) (to prevent overflow). While the disablement of the controllable event \( \alpha_2 \) is legal, that of the uncontrollable \( \beta_1 \) is illegal. Hence \( P = (x_1 = x_{11}) \land (y = y_1) \), where \( \beta_1 \) is defined at \( x_{11} \) and \( y_1 \).

### III. Problem Formulation

Consider a plant automaton \( \mathbf{P} \) (as defined in [5]) consisting of \( n \) component automata \( \mathbf{P}_k, k = 1, \ldots, n \), called ‘agents’.

**Assumption 1.** The agents \( \mathbf{P}_k, k = 1, \ldots, n \), are defined over pairwise disjoint alphabets, i.e. \( \Sigma_k \cap \Sigma_j = \emptyset \) for all \( k \neq j \in [1, n] \). For every \( k \in [1, n] \) let \( \Sigma_k = \Sigma_{c,k} \cup \Sigma_{u,k} \), the disjoint union of the controllable event subset \( \Sigma_{c,k} \) and uncontrollable event subset \( \Sigma_{u,k} \). Then the plant \( \mathbf{P} \) is defined over \( \Sigma := \Sigma_c \cup \Sigma_u \), where \( \Sigma_c := \bigcup_{k=1}^n \Sigma_{c,k} \) and \( \Sigma_u := \bigcup_{k=1}^n \Sigma_{u,k} \).
Assumption 1 is made in order to simplify the main development and presentation of results. In Section IV-C below, we will remove this assumption, and study the case where agents may share events.

Assumption 2. A specification automaton $S$ is defined over $\Sigma$, imposing a behavioral constraint on $P$.

As stated at the end of Section II, we convert the pair $(P, S)$ of plant and specification into an STS $G = (ST, H, \Sigma, \Delta, P_0, P_m)$ with a predicate $P$ specifying the illegal basic trees. The supremal nonblocking and weakly controllable subpredicate of $\neg P$ is $\operatorname{supNC}(\neg P)$, and we suppose $\operatorname{supNC}(\neg P) \wedge P_0 \neq \text{false}$ to exclude the trivial solution. Let

$$S := R(G, \operatorname{supNC}(\neg P)), \quad BS := \{b \in B(ST) \mid b \models S\}.$$  \hspace{1cm} (6)

Then by Theorem 1 there exists a nonblocking SFBC $f$ (defined in (4)) such that $R(G^f, \text{true}) = S$, with

$$P_0^f = R(G^f, \text{true}) \wedge P_0 \quad \text{and} \quad P_m^f = R(G^f, \text{true}) \wedge P_m.$$  \hspace{1cm} (7)

The SFBC $f$ represented by the control functions $f_\sigma$, $\sigma \in \Sigma$, can be written explicitly as follows:

$$(\forall b \in B(ST)) \quad f_\sigma(b) = \begin{cases} 1, & \text{if either } \Delta(b, \sigma) = \emptyset \text{ or } \Delta(b, \sigma) \neq \emptyset \text{ and } \Delta(b, \sigma) \models S; \\ 0, & \text{if } \Delta(b, \sigma) \neq \emptyset \text{ and } \Delta(b, \sigma) \models \neg S. \end{cases}$$  \hspace{1cm} (8)

The pair $(G^f, f)$ is the monolithic optimal and nonblocking supervisor for the control problem $(G, P)$, where $G^f$ is the state tracker with state set $B_S$ which supports dynamic evolution of the controlled system, and $f$ is the SFBC which issues disablement commands based on the state where $G^f$ currently resides. Since $f$ can be represented by the set of control functions $\{f_\sigma \mid \sigma \in \Sigma_c\}$, the supervisor $(G^f, f)$ may be implemented as displayed on the left of Fig. 2 (cf. [11]). Here the controllable events are grouped with respect to individual agents $P_k$.

In this implementation, the state tracker $G^f$ is a global entity, inasmuch as it reports each and every basic tree in $B_S$ that the system visits to all $f_\sigma$ for their decision making. For a purely distributed implementation, we propose to localize $G^f$ to the individual agents so that each of them is equipped with its own local state tracker, denoted by $G^f_k$, $k = 1, \ldots, n$. As will be seen in Section IV each $G^f_k$ will be constructed by finding a suitable cover $C_k = \{B_{k,i} \subseteq B_S \mid i \in I_k\}$ on $B_S$; here $B_{k,i} (\neq \emptyset)$ is called a cell of $C_k$, $I_k$ is an index set, and $\bigcup_{i \in I_k} B_{k,i} = B_S$. There will also be a set of marked cells $I_{m,k} \subseteq I_k$. Thus a local state tracker $G^f_k$ reports system state evolution only in terms of cells (subsets) of basic trees, rather than singleton basic trees. This requires that the associated local control functions $g_\sigma$, $\sigma \in \Sigma_{c,k}$, take subsets of basic trees as arguments, i.e. $g_\sigma : \text{Pwr}(B(ST)) \rightarrow \{0, 1\}$. It is then required that $G^f_k$ track exactly the information sufficient for its associated $g_\sigma$ to issue correct local control. This
distributed implementation is displayed on the right of Fig. 2. Finally, we emphasize that in the absence of monolithic tracking, the local state trackers \( G^f_k \) must communicate\footnote{Formally, we consider that communication is by way of event synchronization; and for simplicity assume that events are communicated instantaneously, i.e. with no delay.} in order to give correct reports on system state evolution. The communication network topology, namely who communicates with whom, is not given \emph{a priori} but will be generated systematically as part of our localization result.

As usual we require this distributed implementation to preserve the optimality and nonblocking properties of the monolithic supervisory control. Fix an arbitrary \( k \in [1, n]\) and \( \sigma \in \Sigma_{c,k} \). Suppose that the controlled system is currently visiting a basic tree \( b \in B_S \); then there must exist a cell \( B_{k,i}, i \in I_k \), of the cover \( C_k \) to which \( b \) belongs. As displayed in Fig. 3 the monolithic state tracker reports \( b \) to \( f_\sigma \) which then makes the control decision \( f_\sigma (b) \); on the other hand, a local state tracker reports the whole cell \( B_{k,i} \) to \( g_\sigma \) which then makes the control decision \( g_\sigma (B_{k,i}) \). We say that the two pairs \((G^f, f_\sigma)\) and \((G^g_k, g_\sigma)\) are \emph{control equivalent} if for every \( b \in B_S \), there exists \( i \in I_k \) such that \( b \in B_{k,i} \) (a cell of \( C_k \))
and

\[ \Delta(b, \sigma) \neq \emptyset \Rightarrow \left[ f_\sigma(b) = 1 \text{ if and only if } g_\sigma(B_{k,i}) = 1 \right]; \quad (9) \]

\[ b \models P^f_m \text{ if and only if } b \models P_m \land i \in I_{m,k}. \quad (10) \]

Thus (9) requires equivalent enabling/disabling action, and (10) requires equivalent marking action. This form of control equivalence is distinct from the language-based equivalence in [1].

We can now formulate the **Distributed Control Problem**. Given a plant automaton \( P \) (as defined in (5)) of component agents \( P_1, \ldots, P_n \) and a specification automaton \( S \) satisfying Assumptions 1 and 2, let \( \text{SUP} := (G^f, \{f_\sigma | \sigma \in \Sigma_c\}) \) be the corresponding STS monolithic supervisor, where \( G \) is the STS converted from \( (P, S) \). Construct a set of local state trackers \( \text{LOC}_{st} := \{G^f_k | k \in [1, n]\} \), one for each agent, with a corresponding set of local control functions \( \text{LOC}_{cf} := \{g_\sigma, \sigma \in \Sigma_{c,k} \} \) such that \( \text{LOC} := (\text{LOC}_{st}, \text{LOC}_{cf}) \) is control equivalent to \( \text{SUP} \): that is, for every \( k \in [1, n] \) and every \( \sigma \in \Sigma_{c,k} \), the pairs \( (G^f, f_\sigma) \) and \( (G^f_k, g_\sigma) \) are control equivalent in the sense defined in (9) and (10).

For the sake of easy implementation and comprehensibility, it would be desired in practice that the number of cells of local state trackers be much less than the number of basic trees of their ‘parent’ monolithic tracker, i.e. \( \forall k \in [1, n] \) \( |G^f_k| << |G^f| = |B_S| \), where \( |\cdot| \) denotes the size of the argument. Inasmuch as this property is neither precise to state nor always achievable, it will be omitted from the formal problem statement; in applications, nevertheless, it should be kept in mind.

### IV. Supervisor Localization

We solve the Distributed Control Problem by developing a supervisor localization procedure in the STS framework. Although the procedure is analogous to the development in the RW framework of [1], we will formally present the new notions of local state tracker and local control function, explicitly define the event sets of local controllers, and provide a new proof which establishes the state-based control equivalence between local and monolithic supervision.

**A. Construction Procedure**

We need some notation from [10]. Let \( \sigma \in \Sigma \) and \( P \in \text{Pred(ST)} \). Then \( \Gamma(P, \sigma) \) is the predicate which holds on the largest set of basic trees, each of which can reach a basic tree in \( B_P \) by a one-step transition \( \sigma \). Also \( \text{Next}_G(\sigma) \) is the predicate which holds on the largest set of basic trees of \( G \) that is reachable by a one-step transition \( \sigma \). Define the legal subpredicate \( N_{\text{good}}(\sigma) \) of \( \text{Next}_G(\sigma) \) by

...
\[ N_{\text{good}}(\sigma) := \text{Next}_G(\sigma) \land S, \] and the illegal subpredicate \[ N_{\text{bad}}(\sigma) := \text{Next}_G(\sigma) \land \neg S, \] where \( S \) is the supervisor predicate in (6).

Now fix an arbitrary \( k \in [1, n] \). We develop a localization procedure which decomposes the monolithic state tracker \( G^f \) into a local state tracker \( G^f_k \) for agent \( P_k \) defined over \( \Sigma_k \). First, we establish a control cover on \( B_S \) (in (5)), the state set of \( G^f \), based solely on the control and marking information pertaining to \( \Sigma_{c,k} \), as captured by the following four functions. Let \( \sigma \in \Sigma_{c,k} \). Define \( E_\sigma : B_S \rightarrow \{0, 1\} \) by

\[ E_\sigma := \Gamma(N_{\text{good}}(\sigma), \sigma) \land S. \] (11)

Thus \( E_\sigma \) is the characteristic function of the set of basic trees in \( B_S \) where \( \sigma \) is enabled. By this definition, for every \( b \in B_S \), \( b \models E_\sigma \) if and only if \( \Delta(b, \sigma) \neq \emptyset \) and \( f_\sigma(b) = 1 \) (\( f_\sigma \) defined in (8)). Next define \( D_\sigma : B_S \rightarrow \{0, 1\} \) by

\[ D_\sigma := \Gamma(N_{\text{bad}}(\sigma), \sigma) \land S. \] (12)

Namely, \( D_\sigma \) is the characteristic function of the set of basic trees in \( B_S \) where \( \sigma \) must be disabled by the supervisory control action of \( S \). Thus for every \( b \in B_S \), \( b \models D_\sigma \) if and only if \( f_\sigma(b) = 0 \). Also define \( M : B_S \rightarrow \{0, 1\} \) according to

\[ M(b) = 1 \text{ if and only if } b \models P^f_m, \text{ } P^f_m \text{ in (7)}. \] (13)

Thus \( M \) holds on the set of basic trees which are marked in \( B_S \) (i.e. in \( G^f \)). Finally define \( T : B_S \rightarrow \{0, 1\} \) according to

\[ T(b) = 1 \text{ if and only if } b \models P_m, \text{ } P_m \text{ in (2)}. \] (14)

So \( T \) holds on the set of basic trees originally marked in \( G \). Note that for each \( b \in B_S \), we have by \( P^f_m = R(G^f \cup \text{true}) \land P_m \) (in (7)) that \( T(b) = 0 \Rightarrow M(b) = 0 \) and \( M(b) = 1 \Rightarrow T(b) = 1 \). Based on the above four functions of the control and marking information of \( \Sigma_{c,k} \), we define the following key binary relation \( R_k \) on \( B_S \).

**Definition 1.** Let \( R_k \subseteq B_S \times B_S \). We say that \( R_k \) is a control consistency relation (with respect to \( \Sigma_{c,k} \)) if for every \( b, b' \in B_S \), \( (b, b') \in R_k \) if and only if

\[ (i) \ (\forall \sigma \in \Sigma_{c,k}) \ E_\sigma(b) \land D_\sigma(b') = \text{false} = E_\sigma(b') \land D_\sigma(b); \]

\[ (ii) \ T(b) = T(b') \Rightarrow M(b) = M(b'). \]

Informally, a pair of basic trees \( (b, b') \) is in \( R_k \) if there is no event in \( \Sigma_{c,k} \) that is enabled at \( b \) but is disabled at \( b' \), or vice versa (consistent disablement information); and (ii) \( b \) and \( b' \) are both marked or
unmarked in $B_S$ provided that they are both marked or unmarked in $G$ (consistent marking information). It is easily verified that $R_k$ is reflexive and symmetric, but need not be transitive, and consequently not an equivalence relation (analogous to [1]); see Fig. 4. This fact leads to the following definition of control cover. Recall that a cover on a set $B_S$ is a family of nonempty subsets (or cells) of $B_S$ whose union is $B_S$.

Definition 2. Let $I_k$ be some index set, and $C_k = \{B_{k,i} \subseteq B_S | i \in I_k\}$ be a cover on $B_S$. We say that $C_k$ is a control cover (with respect to $\Sigma_{c,k}$) if

(i) $\forall i \in I_k, \forall b, b' \in B_{k,i}$ $(b, b') \in R_k$;

(ii) $\forall i \in I_k, \forall \sigma \in \Sigma \left[ (\exists b \in B_{k,i}) \Delta^f(b, \sigma) \neq 0 \Rightarrow (\exists j \in I_k)(\forall b' \in B_{k,i}) \Delta^f(b', \sigma) \subseteq B_{k,j} \right]$.  

A control cover $C_k$ groups basic trees in $B_S$ into (possibly overlapping) cells $B_{k,i}, i \in I_k$. According to (i), all basic trees that reside in a cell $B_{k,i}$ have to be pairwise control consistent; and (ii), for each event $\sigma \in \Sigma$, all basic trees that can be reached from any basic trees in $B_{k,i}$ by a one-step transition $\sigma$ have to be covered by a certain cell $B_{k,j}$ (not necessarily unique). Hence, recursively, two basic trees $b, b'$ belong to a common cell in $C_k$ if and only if (1) $b$ and $b'$ are control consistent, and (2) two future states that can be reached from $b$ and $b'$, respectively, by the same string are again control consistent. In the special case where $C_k$ is a partition on $B_S$, we call $C_k$ a control congruence.
Having defined a control cover $C_k$ on $B_S$, we construct a local state tracker
\[
G^f_k = (I_k, \Sigma_{l,k}, \delta_k, i_{0,k}, I_{m,k})
\] (15)
by the following procedure.

(P1) Each state $i \in I_k$ of $G^f_k$ is a cell $B_{k,i}$ of $C_k$. In particular, the initial state $i_0 \in I_k$ is a cell $B_{k,i_0}$ where the basic tree $b_0$ belongs, i.e., $b_0 \in B_{k,i_0}$, and the marker state set $I_{m,k} := \{i \in I_k | B_{k,i} \cap \{b \in B_S | \vdash P \}_{m} \neq \emptyset \}$.

(P2) Choose the local event set $\Sigma_{l,k}$. For this, define the transition function $\delta'_k : I_k \times \Sigma \rightarrow I_k$ over the entire event set $\Sigma$ by
\[
\delta'_k(i, \sigma) = j \text{ if } (\exists b \in B_{k,i}) \Delta^f(b, \sigma) \neq \emptyset \text{ and } (\forall b' \in B_{k,i}) \Delta^f(b', \sigma) \subseteq B_{k,j}.
\] (16)
Choose $\Sigma_{l,k}$ to be the union of $\Sigma_k$ of agent $P_k$ with events in $\Sigma \setminus \Sigma_k$ which are not selfloop transitions of $\delta'_k$. Thus $\Sigma_{l,k} := \Sigma_k \cup \Sigma_{com,k}$, where
\[
\Sigma_{com,k} := \{\sigma \in \Sigma \setminus \Sigma_k | (\exists i, j \in I_k) i \neq j \text{ and } \delta'_k(i, \sigma) = j\}.
\] (17)
The set $\Sigma_{com,k}$ determines the subset of agents $P_j$ ($j \neq k$) that $P_k$ communicates with.

(P3) Define the transition function $\delta_k$ to be the restriction of $\delta$ to $\Sigma_{l,k}$, namely $\delta_k := \delta|_{\Sigma_{l,k}} : I_k \times \Sigma_{l,k} \rightarrow I_k$.
Thus the above constructed local state tracker $G^f_k$ is an automaton, which reports system state evolution in terms of cells (subsets) of basic trees which are crucial for, and only for, the local control and marking with respect to $\Sigma_{c,k}$ of agent $P_k$. Owing to the possible overlapping of cells in $C_k$, the choices of $i_0$ and $\delta_k$ may not be unique, and consequently $G^f_k$ may not be unique. In that case we take an arbitrary instance of $G^f_k$. Clearly if $C_k$ happens to be a control congruence, then $G^f_k$ is unique.

Finally, we define local control functions $g_{\sigma}, \sigma \in \Sigma_{c,k}$, to be compatible with $G^f_k$. Let $\sigma \in \Sigma_{c,k}$. Define $g_{\sigma} : I_k \rightarrow \{0, 1\}$ by
\[
g_{\sigma}(i) = 1 \text{ if and only if } (\exists b \in B_{k,i}) b \models \Gamma(N_{good}(\sigma), \sigma).
\] (18)
So $g_{\sigma}$ will enable $\sigma$ at a state $i$ of the tracker $G^f_k$ whenever there is a basic tree in the cell $B_{k,i}$ at which $\sigma$ is enabled.

We have now completed the localization procedure for an arbitrarily chosen agent $P_k$, $k \in [1, n]$. The procedure is summarized and illustrated in Fig. 5. Applying the same procedure for every agent,

2The issue of minimal communication in a distributed system is studied in [18]. Although outside the scope of this paper, minimal communication is an interesting future topic for localization.
we obtain a set of local state trackers $\text{LOC}_{st} := \{G^l_k | k \in [1,n]\}$ with a corresponding set of local control functions $\text{LOC}_{cf} := \{g_\sigma, \sigma \in \Sigma_{c,k} | k \in [1,n]\}$. Our main result, below, states that this pair $\text{LOC} := (\text{LOC}_{st}, \text{LOC}_{cf})$ is a solution to the Distributed Control Problem.

**Theorem 2.** The pair $\text{LOC} := (\text{LOC}_{st}, \text{LOC}_{cf})$ of local state trackers and local control functions is control equivalent to the optimal and nonblocking supervisor $\text{SUP} := (G^l, \{f_\sigma | \sigma \in \Sigma_c\})$; namely, for every $k \in [1,n]$, $\sigma \in \Sigma_{c,k}$, and $b \in B_S$, there exists $i \in I_k$ such that $b \in B_{k,i}$ and

((i)) $\Delta(b, \sigma) \neq \emptyset \Rightarrow \left[ f_\sigma(b) = 1 \text{ if and only if } g_\sigma(i) = 1 \right]$;

((ii)) $b \models P^f_m$ if and only if $b \models P_m$ & $i \in I_{m,k}$.

The proof, below, establishes the state-based control equivalence between local and monolithic supervision. It is distinct from, and more concise than, the language-based proof in [11].

**Proof.** Let $k \in [1,n]$, $\sigma \in \Sigma_{c,k}$, and $b \in B_S$. Then there must exist a state $i \in I_k$ of the tracker $G^l_k$, corresponding to a cell $B_{k,i}$ of the control cover $C_k$, such that $b \in B_{k,i}$. For (i), suppose that $\Delta(b, \sigma) \neq \emptyset$; it will be shown that $f_\sigma(b) = 1$ if and only if $g_\sigma(i) = 1$. (If) Let $g_\sigma(i) = 1$, i.e. there is $b' \in B_{k,i}$ such that $b' \models \Gamma(N_{\text{good}}(\sigma), \sigma)$. Since $b'$ is also in $B_S$, we have $b' \models \Gamma(N_{\text{good}}(\sigma), \sigma) \land S = E_\sigma$. It follows from $b \in B_{k,i}$ that $(b, b') \in R_k$ and $E_\sigma(b') \land D_\sigma(b') \equiv \text{false}$. Hence $D_\sigma(b) \equiv \text{false}$, which is equivalent to $f_\sigma(b) = 1$ by the definition of $D_\sigma$ in [12]. (Only if) Let $f_\sigma(b) = 1$. Since $\Delta(b, \sigma) \neq \emptyset$ and $b$ is in
$B_S$, we have by the definition of $E_\sigma$ in (11) that $b \models E_\sigma = \Gamma(N_{good}(\sigma), \sigma) \wedge S$. We then conclude from $b \in B_{k,i}$ and the definition of $g_\sigma$ in (18) that $g_\sigma(i) = 1$.

Now we show (ii). (If) Let $b \models P_m$ (i.e. $T(b) = 1$) and $i \in I_{m,k}$. Then there is $b' \in B_{k,i}$ such that $b' \models P_m'$; so $M(b') = 1$, and also $T(b') = 1$. Since $(b, b') \in R_k$ and $T(b) = T(b')$, we have $M(b) = M(b') = 1$. (Only if) Let $b \models P_m$ (i.e. $M(b) = 1$). Then $T(b) = 1$, i.e. $b \models P_m$, and also $i \in I_{m,k}$ by the construction of the tracker $G_f^l$.

In essence Theorem 2 asserts that every set of control covers generates a solution to the Distributed Control Problem. This raises the converse question: is every solution to the Distributed Control Problem, then the covers $C_k$, $k \in [1, n]$, are control covers. We answer this question in the next subsection.

B. Necessary Structure

Let $\text{SUP} = (G^f, \{f_\sigma | \sigma \in \Sigma_c\})$, with $G^f$ in (3) and $f_\sigma$ in (8), be the monolithic optimal and nonblocking supervisor for a given control problem. Also let $C_k = \{B_{k,i}|i \in I_k\}, k \in [1, n]$ and $I_k$ some index set, be an arbitrary cover on the state set $B_S$ (as in (6)) of $G^f$; namely $\emptyset \neq B_{k,i} \subseteq B_S$, and $\bigcup_{i \in I_k} B_{k,i} = B_S$. For the cover $C_k$ on $B_S$, apply the procedure (P1)-(P3), above, to obtain an automaton $G^f_k = (I_k, \Sigma_{l,k}, \delta_k, i_{0,k}, I_{m,k})$ as in (15). We impose a normality requirement on $G^f_k$ with respect to $G^f$ (cf. (17)).

Definition 3. We say that $G^f_k = (I_k, \Sigma_{l,k}, \delta_k, i_{0,k}, I_{m,k})$ with $\delta'_k : I_k \times \Sigma \rightarrow I_k$ in (16) is normal with respect to $G^f = (ST, H, \Sigma, \Delta^f, P_0^f, P_m^f)$ if

$$(\forall i \in I_k, \forall \sigma \in \Sigma)(\exists b \in B_{k,i}) \Delta^f(b, \sigma) \neq \emptyset \Rightarrow \exists j \in I_k \delta'_k(i, \sigma) = j.$$

Thus normality of $G^f_k$ requires that if an event $\sigma$ is defined at $b \in B_{k,i}$ in $G^f$, then $\sigma$ must be defined by $\delta'_k$ at $i \in I_k$ of $G^f_k$. This requirement in turn imposes a condition on the cover $C_k$ from which $G^f_k$ is constructed, as illustrated in Fig.6 We will see below that the condition imposed on $C_k$ is indeed one requirement of a control cover.

Now let $\text{LOC}_{st} = \{G^f_k | k \in [1, n]\}$, and $\text{LOC}_{cf} = \{g_\sigma, \sigma \in \Sigma_{c,k} | k \in [1, n]\}$ with $g_\sigma$ defined in (18). We say that the pair $\text{LOC} = (\text{LOC}_{st}, \text{LOC}_{cf})$ is normal if every $G^f_k$, $k \in [1, n]$, is normal with respect to $G^f$. The following result asserts that if the normal pair $\text{LOC} = (\text{LOC}_{st}, \text{LOC}_{cf})$ is a solution to the Distributed Control Problem, then the covers $C_k$, $k \in [1, n]$, must all be control covers.

Theorem 3. If the normal pair $\text{LOC} = (\text{LOC}_{st}, \text{LOC}_{cf})$ is control equivalent to $\text{SUP}$, then the covers $C_k$, $k \in [1, n]$, are control covers.
This means, by (11), is not defined at the same argument. In (9) that E is normal with respect to G. In (b) the transitions α are not defined in G because the condition in (10) is violated for σ = α.

**Proof.** Fix an arbitrary k ∈ [1, n]. According to Definition[2] we must prove the following two conditions for cover C:

(i) (∀i ∈ I_k, ∀b, b' ∈ B_k) (b, b') ∈ R_k;

(ii) (∀i ∈ I_k, ∀σ ∈ Σ) |∃b ∈ B_k) Δ_f(b, σ) ≠ ∅ ⇒ (∃j ∈ I_k)(∀b' ∈ B_k) Δ_f(b', σ) ⊆ B_k_j.

For (ii), let i ∈ I_k, σ ∈ Σ, and suppose there exists b ∈ B_k such that Δ_f(b, σ) ≠ ∅. Since G is normal with respect to G_f, by Definition[3] there exists j ∈ I_k such that δ_k(i, σ) = j. It then follows from (10) that (∀b' ∈ B_k) Δ_f(b', σ) ⊆ B_k_j.

Next for (i), let i ∈ I_k and b, b' ∈ B_k; it will be shown that (b, b') ∈ R_k (Definition[1]). First, let σ ∈ Σ_ck; if Δ(b, σ) = ∅ then E(b) = D(b) = false by (11) and (12) (resp. E(b') = D(b') = false). Hence there holds E(b) ∧ D(b') = false = E(b') ∧ D(b) = false if σ is not defined at b or b' or at both of them. Now suppose that Δ(b, σ) = Δ(b', σ) ≠ ∅ and E(b) = true. This means, by (11), f(b) = 1. Using the assumption that LOC = (LOC_st, LOC_σ) is control equivalent to SUP, in particular (9), we derive gσ(i) = 1. Since b' ∈ B_k and Δ(b', σ) ≠ ∅, it follows again from (9) that f(b') = 1. This implies D(b') = false by (12), and therefore E(b) ∧ D(b') = false. The same argument shows E(b') ∧ D(b) = false.

Second, if T(b) = T(b') = 0, then M(b) = M(b') = 0, and there holds T(b) = T(b') ⇒ M(b) = M(b'). Now suppose that T(b) = T(b') = 1; by (14) b, b' |= P_m. Assume on the contrary that M(b) = 1 and M(b') = 0, i.e. b |= P_m and b' ∈ P_m (the other case where M(b) = 0 and M(b') = 1 is similar).
\(\beta\) 

\(\gamma\) 

\(\alpha\) 

\(G_f\) with \(C_k = \{\{0, 1\}, \{1, 2\}\}\) 

\(G_f^k\) normal (not unique) 

\(\alpha \in \Sigma_c, k\) 

\(\alpha\) is enabled at state 0, i.e. \(E_\alpha(0) = 1, D_\alpha(0) = 0\) 

\(\alpha\) is not defined at state 1, i.e. \(E_\alpha(1) = 0, D_\alpha(1) = 0\) 

\(\alpha\) is disabled at state 2, i.e. \(E_\alpha(2) = 0, D_\alpha(2) = 1\) 

Fig. 7. Example: \(G_f^k\) constructed from control cover \(C_k = \{\{0, 1\}, \{1, 2\}\}\) is normal with respect to \(G_f\), control equivalent to \(\text{SUP}\) with respect to event \(\alpha\), and \(|G_f^k| < |G_f^f|\). Consider the partitions \(C_1^k = \{\{0, 2\}, \{1\}\}, C_2^k = \{\{0, 1\}, \{2\}\}\), and \(C_3^k = \{\{0\}, \{1, 2\}\}\). One verifies that they are not control congruences: for \(C_1^k\) condition (i) of Definition 2 fails, and for \(C_2^k, C_3^k\) condition (ii) of Definition 2 fails. Thus no control congruence can realize \(|G_f^k| < |G_f^f|\).

By the definition of \(I_{m,k}\) in (P1) of the procedure, above, and \(b \in B_{k,i} \cap \{b \in B_S | b | = P_f^f\}\), we obtain \(i \in I_{m,k}\). On the other hand, since \(\text{LOC} = (\text{LOC}_{st}, \text{LOC}_{cf})\) is control equivalent to \(\text{SUP}\), it follows from (10) and \(b' \not\equiv P_f^f\) that \(i \notin I_{m,k}\). We have thus derived a contradiction, so \(M(b) = M(b')\) after all. 

That the \(C_k\) are covers in Theorem 3 is important if the state size of \(G_f^f\) is required to be smaller than that of \(G_f^f\) (as is usually the case in practice). In particular, if control cover is replaced by control congruence, then there may not exist a normal pair \(\text{LOC} = (\text{LOC}_{st}, \text{LOC}_{cf})\) that is control equivalent to \(\text{SUP}\) and with \(|G_f^k| < |G_f^f|\); see Fig. 7.

C. Event Sharing

So far our STS localization theory has been developed under the assumption that component agents have pairwise disjoint alphabets (i.e. Assumption 1). Now we remove this assumption and discuss the case where agents may share events. This also provides an extension of [1]. Our localization scheme in the event sharing case is first to synthesize a local state tracker and a local control function for each controllable event, rather than for each agent, and then allocate the synthesized local state trackers and local control functions among the set of agents.

Fix a controllable event \(\sigma \in \Sigma_c\). We decompose the monolithic state tracker \(G_f^f\) into a local state tracker \(G_f^f_\sigma\) for the event \(\sigma\). The decomposition procedure is the same as before, but with some definitions
revised as follows. Define \( \mathcal{R}_\sigma \subseteq B_S \times B_S \) to be a control consistency relation with respect to \( \sigma \) by 
\( \forall (b, b' \in B_S) \ (b, b') \in \mathcal{R}_\sigma \) if and only if 
\[
(i) \ E_\sigma(b) \land D_\sigma(b') = false = E_\sigma(b') \land D_\sigma(b);
(ii) \ T(b) = T(b') \Rightarrow M(b) = M(b').
\]

Similar to the relation \( \mathcal{R}_k \) in Definition 1, \( \mathcal{R}_\sigma \) is reflexive and symmetric, but need not be transitive, and consequently leads to a cover on the set \( B_S \). Let \( I_\sigma \) be some index set, and \( \mathcal{C}_\sigma = \{ B_{\sigma,i} \subseteq B_S \mid i \in I_\sigma \} \) be a cover on \( B_S \). Define \( \mathcal{C}_\sigma \) to be a control cover with respect to \( \sigma \) by 
\[
(i) \ (\forall i \in I_\sigma, \forall b, b' \in B_{\sigma,i}) \ (b, b') \in \mathcal{R}_\sigma;
(ii) \ (\forall i \in I_\sigma, \forall \sigma' \in \Sigma) \left[ (\exists b \in B_{\sigma,i}) \ \Delta^f(b, \sigma') \neq \emptyset \Rightarrow \right. \nonumber \\
\left. (\exists j \in I_\sigma) (\forall b' \in B_{\sigma,j}) \ \Delta^f(b', \sigma') \subseteq B_{\sigma,j} \right].
\]

Based on a control cover \( \mathcal{C}_\sigma \) on \( B_S \), we construct using the procedure (P1)-(P3) in Section IV-A a local state tracker \( \mathbf{G}^f_\sigma = (I_\sigma, \Sigma_{i,\sigma}, \delta_\sigma, i_0, I_m, \sigma) \) for the event \( \sigma \). Finally, we define a corresponding local control function \( g_\sigma : I_\sigma \rightarrow \{0, 1\} \) for \( \sigma \) by \( g_\sigma(i) = 1 \) if and only if \( (\exists b \in B_{\sigma,i}) \ b \models \Gamma(N_{\text{good}}(\sigma), \sigma) \).

Now for each controllable event \( \sigma \in \Sigma_c \) we derive a local state tracker \( \mathbf{G}^f_\sigma \) and a local control function \( g_\sigma \). Let \( \mathbf{LOC} = \{ (\mathbf{G}^f_\sigma, g_\sigma) \mid \sigma \in \Sigma_c \} \) be the set of local controllers, and \( \mathbf{SUP} := (\mathbf{G}^f, \{ f_\sigma \mid \sigma \in \Sigma_c \}) \) be the optimal and nonblocking monolithic supervisor. Then we have the following result.

**Proposition 1.** \( \mathbf{LOC} \) is control equivalent to \( \mathbf{SUP} \); namely, for every \( \sigma \in \Sigma_c \) and \( b \in B_S \), there exists \( i \in I_k \) such that \( b \in B_{\sigma,i} \) and 
\[
(i) \ \Delta(b, \sigma) \neq \emptyset \Rightarrow [f_\sigma(b) = 1 \text{ if and only if } g_\sigma(i) = 1];
(ii) \ b \models P^f_m \text{ if and only if } b \models P_m \land i \in I_{m,k}.
\]

**Proof.** Let \( \sigma \in \Sigma_c \) and \( b \in B_S \). Then by the definition of control cover \( \mathcal{C}_\sigma \) on \( B_S \), there must exist a state \( i \in I_\sigma \) of the tracker \( \mathbf{G}^f_\sigma \) corresponding to a cell \( B_{\sigma,i} \) of the cover \( \mathcal{C}_\sigma \) such that \( b \in B_{\sigma,i} \). The rest of the proof follows similarly to that of Theorem 2. \( \square \)

Finally, we allocate the derived local state trackers and local control functions (with respect to individual controllable events) among the set of component agents \( \mathbf{G}_k, \ k \in [1, n] \). There may be different ways of allocation, allowing case-dependent choices. For example, if \( \mathbf{G}_k \) and \( \mathbf{G}_j \) share a controllable event \( \sigma \), i.e. \( \sigma \in \Sigma_{c,k} \cap \Sigma_{c,j} \), then the local state tracker \( \mathbf{G}^f_\sigma \) and local control function \( g_\sigma \) can be allocated to either agent or to both. Allocating to both agents may increase robustness against faults because even if one
fails, the other can continue operating; on the other hand, allocating to either agent would be cheaper for implementation. So in practice there is often a tradeoff between robustness and cost.

Among other others, the following is a convenient allocation, in the sense that every $G_{f_{\sigma}}$ and $g_{\sigma}$ is implemented by exactly one agent.

\[ G_1 : \ \forall \sigma \in \Sigma_{c,1} \ G_{f_{\sigma}}, g_{\sigma} \]

\[ G_2 : \ \forall \sigma \in \Sigma_{c,2 \setminus \Sigma_{c,1}} \ G_{f_{\sigma}}, g_{\sigma} \]

\[ \vdots \]

\[ G_n : \ \forall \sigma \in \Sigma_{c,n \setminus (\Sigma_{c,n-1} \cup \cdots \cup \Sigma_{c,1})} \ G_{f_{\sigma}}, g_{\sigma} \]

Choosing this or (obvious) alternative ways of allocation would be case-dependent.

V. Symbolic Localization Algorithm

In this section we design an STS localization algorithm for computing local controllers, which is more efficient than the counterpart algorithm in [1].

We have seen in the preceding section that Theorems 2 and 3 together establish the same conclusion as in the RW framework [1]: namely every set of control covers generates a solution to the Distributed Control Problem, and every normal solution to the Distributed Control Problem can be constructed from some set of control covers. In particular, a set of state-minimal local state trackers (possibly non-unique) can in principle be defined from a set of suitable control covers. It would thus be desirable to have an efficient algorithm that computes such a set of covers; however, the minimal state problem is known to be NP-hard [17]. Nevertheless, a polynomial-time localization algorithm was proposed in [1] which generates a control congruence (instead of a control cover), and empirical evidence [2] shows that significant state size reduction can often be achieved. In the following, we propose a new localization algorithm which
is based on STS. The advantage of using STS is that the efficiency of the new algorithm is improved compared to the one in [1], as will be shown below.

We sketch the idea of the algorithm as follows. Let $B_S$ in (6) be labeled as $B_S = \{b_0, \ldots, b_{N-1}\}$, and $\Sigma_{c,k} \subseteq \Sigma_c$ be the controllable events of agent $P_k$, $k \in [1, n]$. Our algorithm will generate a control congruence $C_k$ on $B_S$ (with respect to $\Sigma_{c,k}$). This is done symbolically. First introduce the set $\tilde{B}_S = \{\tilde{b}_0, \ldots, \tilde{b}_{N-1}\}$, $\Sigma_{c,k} \subseteq \Sigma_c$, $\Sigma_{c,k} \subseteq \Sigma_c$ be the controllable events of agent $P_k$, $k \in [1, n]$. Our algorithm will generate a control congruence $C_k$ on $B_S$ (with respect to $\Sigma_{c,k}$). This is done symbolically. First introduce the set $\tilde{B}_S = \{\tilde{b}_0, \ldots, \tilde{b}_{N-1}\}$, where $\tilde{b}_i : B(ST) \rightarrow \{0, 1\}$ are predicates defined by $\tilde{b}_i(b) = 1$ if and only if $b = b_i$. Two elements of $\tilde{B}_S$ may be merged (by “$\lor$”) if (i) their corresponding basic trees are control consistent (line 10 in the pseudocode below, where $\tilde{R}_k : Pwr(B_S) \rightarrow \{0, 1\}$ is defined by $B_1 \models \tilde{R}_k$ if and only if $(\forall b, b' \in B_1)(b, b') \in R_k$); and (ii) all corresponding downstream basic trees reachable from $b, b'$ by identical strings are also control consistent (line 12, where $\tilde{\Delta} : P_{red}(ST) \times \Sigma \rightarrow P_{red}(ST)$ is the predicate counterpart of $\Delta$ in [1]). We note that since $\tilde{\Delta}$ can handle one-step transitions of a predicate corresponding to a subset of basic trees, in each call of the CHECK$\_MERGE$ function we may also check control consistency by applying $\tilde{R}_k$ to this subset; this is more efficient than the algorithm in [1] which in each call of the CHECK$\_MERGE$ function checks control consistency only for a pair of flat states (corresponding to basic trees). Finally, after checking all the elements in $\tilde{B}_S$, the algorithm at line 8 generates a control congruence $C_k$ each cell of which consists of the basic trees $b_i$ whose corresponding predicates $\tilde{b}_i$ are merged together in $\tilde{B}_S$.

Theorem 4. The STS localization algorithm terminates, has (worst-case) time complexity $O(N^3)$, and the generated $C_k$ is a control congruence on $B_S$.

Before proving Theorem 4 we remark that the STS localization algorithm realizes the same functionality as the one in [1], and moreover improves the time complexity from $O(N^4)$ in [1] to $O(N^3)$. This is achieved by the fact that the (global) transition function of STS can handle subsets of basic trees simultaneously, which makes checking the control consistency relation in each call of the CHECK$\_MERGE$ function more efficient.

The following is the pseudocode of the algorithm. Notation: “\" denotes set subtraction; $x \prec y$ means $x \leq y$ and $x \neq y$.

1: procedure MAIN()
2: for $i := 0$ to $N - 2$ do
3: for $j := i + 1$ to $N - 1$ do
4: $B = \tilde{b}_i \lor \tilde{b}_j$;
5: $W = \emptyset;$

Theorem 4. The STS localization algorithm terminates, has (worst-case) time complexity $O(N^3)$, and the generated $C_k$ is a control congruence on $B_S$.
\begin{verbatim}
6: \textbf{if} Check\_Merge\(B, W, i, \tilde{B}_S\) = \textit{true} \textbf{then}
7: \quad \tilde{B}_S = (\tilde{B}_S \cup W) \setminus \{b \in \tilde{B}_S \mid (\exists w \in W) b < w\};
8: \textbf{return} \(C_k = \{\cup_i b_i \mid \forall_i \tilde{b}_i \in \tilde{B}_S\}\);
9: \textbf{function} Check\_Merge\(B, W, i, \tilde{B}_S\)
10: \quad \textbf{if} \{\exists b \in B(ST) \mid b \models B\} \not\models \mathcal{R}_k \textbf{then return} \textit{false};
11: \quad W = (W \cup B) \setminus \{w \in W \mid w \prec B\};
12: \quad \textbf{for} each \(\sigma \in \Sigma\) with \(\tilde{\Delta}(B, \sigma) \land S \neq \textit{false}\) \textbf{do}
13: \quad \quad \textbf{if} \(\tilde{\Delta}(B, \sigma) \land S \leq w\) for some \(w \in W \cup \tilde{B}_S\) \textbf{then continue};
14: \quad \quad \textbf{if} \(\tilde{\Delta}(B, \sigma) \land S \nRightarrow \tilde{b}_r\) for some \(r < i\) \textbf{then return} \textit{false};
15: \quad \quad B = (\tilde{\Delta}(B, \sigma) \land S) \lor (\bigvee \{w \mid w \in W \land w \land (\tilde{\Delta}(B, \sigma) \land S) \neq \textit{false}\});
16: \quad \textbf{if} Check\_Merge\(B, W, i, \tilde{B}_S\) = \textit{false} \textbf{then return} \textit{false};
17: \quad \textbf{return} \textit{true};
\end{verbatim}

\textbf{Proof of Theorem}\[4\] Since both \(B\) at line 4 and \(\tilde{\Delta}(B, \sigma) \land S\) at line 15 are the join “\(\lor\)” of the predicates in \(\tilde{B}_S\), so is each element of \(W\) which is updated only at line 11. Thus, the size of \(\tilde{B}_S\), which is updated only at line 7, is non-increasing. Because the initial size \(N\) is finite, the algorithm must terminate. In the worst case, there can be \(N(N - 1)/2\) calls (by lines 2, 3) made to the function \text{CHECK\_MERGE}, which can then make \(N\) calls (by lines 12, 13) to itself. So the worst-case time complexity is \(N^2(N-1)/2 = O(N^3)\).

It is left to show that \(C_k\) generated at line 8 is a control congruence. First, the control consistency of every pair of basic trees in the same cell of \(C_k\) is guaranteed by the check at line 10; so \(C_k\) is a control cover. Second, the set subtraction “\(\setminus\)” when updating \(W\) at line 11 and \(\tilde{B}_S\) at line 7 ensures that the cells of \(C_k\) are pairwise disjoint; thus \(C_k\) is a partition on \(B_S\). Therefore, we conclude that \(C_k\) is a control congruence. \(\square\)

\textbf{Example} 1. We provide an example, displayed in Fig. [8] to illustrate the STS localization algorithm. Initially, \(\tilde{B}_S = \{b_0, b_1, \tilde{b}_2, \tilde{b}_3\}\). The ranges of indices \(i\) and \(j\) at lines 2 and 3 are \(i \in [0, 2]\) and \(j \in [i+1, 3]\), respectively.

(1) \((\tilde{b}_0, \tilde{b}_1)\) cannot be merged. First, \(B = \tilde{b}_0 \lor \tilde{b}_1\) and the test at line 10 is passed since \(\{b_0, b_1\} \models \mathcal{R}_k\); so \(W = \tilde{b}_0 \lor \tilde{b}_1\). Second, \(B\) is updated at line 15 to \(B = \tilde{b}_0 \lor \tilde{b}_1 \lor \tilde{b}_2\) and the test at line 10 is still passed since \(\{b_0, b_1, b_2\} \models \mathcal{R}_k\); so \(W = \tilde{b}_0 \lor \tilde{b}_1 \lor \tilde{b}_2\). Third, \(B\) is updated at line 15 to \(B = \tilde{b}_0 \lor \tilde{b}_1 \lor \tilde{b}_2 \lor \tilde{b}_3\) but now the test at line 10 fails since \(\{b_0, b_1, b_2, b_3\} \not\models \mathcal{R}_k\) (indeed, \((b_i, b_3) \not\in \mathcal{R}_k\), \(i = 0, 1, 2\)). Note that when \(B = \tilde{b}_0 \lor \tilde{b}_1 \lor \tilde{b}_2\) the global transition function \(\tilde{\Delta}(B, \sigma)\) at lines 12-15 handles the local transitions
at basic trees $b_1, b_2, b_3$ simultaneously:

$$\Delta(\bar{b}_0 \lor \bar{b}_1 \lor \bar{b}_2, \alpha) = \bar{b}_0 \lor \bar{b}_1 \lor \bar{b}_2 \quad \text{&} \quad \Delta(\bar{b}_0 \lor \bar{b}_1 \lor \bar{b}_2, \beta) = \bar{b}_2 \lor \bar{b}_3.$$ 

This operation is more efficient than the localization algorithm in [1]; there, only a pair of basic trees of \{b_1, b_2, b_3\} and the associated transitions can be processed at a single step.

(2) $(\bar{b}_0, \bar{b}_2)$ can be merged. First, $B = \bar{b}_0 \lor \bar{b}_2$ and the test at line 10 is passed since $\{b_0, b_2\} \models \tilde{R}_k$; so $W = \bar{b}_0 \lor \bar{b}_2$. Second, $B$ is updated at line 15 to $B = \bar{b}_1$ and the test at line 10 is trivially passed; so $W = \{\bar{b}_0 \lor \bar{b}_2, \bar{b}_1\}$. Now one verifies that the condition at line 13 is satisfied for both transitions $\alpha$ and $\beta$ defined at $b_1$, so the “for”-loop from line 12 to line 16 is finished without calling the CHECK_MERGE function. Hence true is returned at line 6 and $\tilde{B}_S$ is updated at line 7 to $\tilde{B}_S = \{\bar{b}_0 \lor \bar{b}_2, \bar{b}_1, \bar{b}_3\}$.

(3) $(\bar{b}_0, \bar{b}_3)$ cannot be merged because $(b_0, b_3) \notin R_k$ and the test at line 10 fails.

(4) $(\bar{b}_1, \bar{b}_2)$ cannot be merged. First, $B = \bar{b}_1 \lor \bar{b}_2$ and the test at line 10 is passed since $\{b_1, b_2\} \models \tilde{R}_k$; so $W = \bar{b}_1 \lor \bar{b}_2$. Second, $B$ is updated at line 15 to $B = \bar{b}_1 \lor \bar{b}_2 \lor \bar{b}_3$ but the test at line 10 fails since $\{b_1, b_2, b_3\} \notin R_k$.

(5) $(\bar{b}_1, \bar{b}_3)$ cannot be merged because $(b_1, b_3) \notin R_k$ and the test at line 10 fails.

(6) $(\bar{b}_2, \bar{b}_3)$ cannot be merged because $(b_2, b_3) \notin R_k$ and the test at line 10 fails.

Finally, $\tilde{B}_S = \{\bar{b}_0 \lor \bar{b}_2, \bar{b}_1, \bar{b}_3\}$ and line 8 generates a control congruence $C_k = \{\{b_0, b_2\}, \{b_1\}, \{b_3\}\}$.

The normal local state tracker (unique in this case) constructed from $C_k$ is displayed in Fig. 8.

VI. CASE STUDY CLUSTER TOOL

In this section, we demonstrate STS supervisor localization on Cluster Tool, an integrated semiconductor manufacturing system used for wafer processing (e.g. [19]). Starting with a decentralized approach (see [20] for a recent development in STS), we apply localization to establish a purely distributed control
architecture for Cluster Tool. A second purpose of this case study is to compare our results with those reported in [8], [9] for the same (structured) system; by imposing additional specifications we derive straightforward coordination logic, which provides global nonblocking control.

As displayed in Fig. 9, Cluster Tool consists of (i) two loading docks \((L_{\text{in}}, L_{\text{out}})\) for wafers entering and leaving the system, (ii) eleven vacuum chambers \((C_{11}, C_{12}, \ldots, C_{52})\) where wafers are processed, (iii) four buffers \((B_1, \ldots, B_4)\) where wafers are temporarily stored, and (iv) five robots \((R_1, \ldots, R_5)\) which transport wafers in the system along the following production sequence:

\[
L_{\text{in}} \rightarrow C_{51} \rightarrow B_4 \rightarrow \cdots \rightarrow B_2 \rightarrow C_{21} \rightarrow B_1 \rightarrow C_{11} \downarrow
\]

\[
L_{\text{out}} \leftarrow C_{52} \leftarrow B_4 \leftarrow \cdots \leftarrow B_2 \leftarrow C_{21} \leftarrow B_1 \leftarrow C_{12} \downarrow.
\]

The five robots are the plant components; their automaton dynamics and state trees are displayed in Fig. 10. Each robot \(R_i\) has 8 events, all assumed controllable. Note that the robots have pairwise disjoint alphabets; thus Assumption 1 at the beginning of Section III is satisfied.

Next, we describe control specifications for Cluster Tool. (1) Fig. 11 (a): at each chamber \(C_{ij}\) a wafer is first dropped in by robot \(R_i\), then processed, and finally picked up by \(R_i\). Thus a chamber behaves essentially like a one-slot buffer; our first control specification is to protect each \(C_{ij}\) against overflow and underflow. Note also that the event \(\text{Process}_{ij}\) (designated uncontrollable) can be viewed as an internal transition of chamber \(C_{ij}\); so for its corresponding two states “10” and “11”, we introduce a hierarchy in \(C_{ij}\)’s state tree model. (2) Fig. 11 (b): each buffer \(B_i\) has capacity one, and may be incremented by \(R_i\) from the right (resp. \(R_{i+1}\) from the left) and then decremented by \(R_{i+1}\) from the left (resp. \(R_i\) from the right). Our second control specification is to protect all buffers against overflow and underflow. Thus far we have described specifications related to physical units – chambers and buffers; the
final requirement, denoted by $D_i$ ($i \in [1, 3]$), is purely logical, and coordinates the operations between neighboring robots. (3) Fig. 11 (c): once robot $R_i, i \in [1, 3]$, picks up a wafer from chamber $C_{i,2}$, it may not do so again until robot $R_{i+1}$ empties chamber $C_{i+1,2}$. The rationale for imposing this specification is as follows (refer to Fig. 9): once a wafer is picked up by $R_i, i \in [1, 3]$, it needs to be transported through $B_i \rightarrow R_{i+1} \rightarrow C_{i+1,2} \rightarrow R_{i+1} \rightarrow B_{i+1}$; here buffers $B_i, B_{i+1}$ and robot $R_{i+1}$ can be viewed as shared resources, and if chamber $C_{i+1,2}$ is full, then the above wafer transportation may cause blocking. Hence a reasonable (but possibly restrictive) requirement to avoid system deadlock is to guarantee an empty slot in $C_{i+1,2}$ before $R_i$ initiates the wafer transportation. Note that we do not impose the same specification between $R_4$ and $R_5$, because $R_5$ can drop wafers out of the system without capacity constraint.

Putting together plant components (Fig. 10) and control specifications (Fig. 11), we obtain the state
tree model $ST_{CT}$ of Cluster Tool, displayed in Fig. 12. The system is large-scale – the (uncontrolled) total state size is approximately $3.6 \times 10^{11}$. Moreover, apart from satisfying all imposed control specifications, the system will require nontrivial coordination to prevent deadlocks caused by conflicts in using multiple shared resources (robots and buffers). Consequently, the overall optimal and nonblocking control of Cluster Tool is a challenging design exercise.

Directly applying the monolithic supervisor synthesis of STS [10], [11] results in an optimal and nonblocking supervisor $S$ (predicate as in (6)); the corresponding global state tracker has 3227412 basic trees, and the associated control functions of certain events have a large number of BDD nodes:

- $f_{\text{pick-}$C_{12}$} : 205$ nodes, $f_{\text{pick-}$C_{21}$} : 284$ nodes,
- $f_{\text{pick-}$C_{22}$} : 319$ nodes, $f_{\text{pick-}$C_{31}$} : 686$ nodes,
- $f_{\text{pick-}$C_{32}$} : 571$ nodes, $f_{\text{pick-}$C_{41}$} : 1561$ nodes,
- $f_{\text{pick-}$C_{34}$} : 777$ nodes, $f_{\text{pick-}$C_{53}$} : 5668$ nodes.

Because of the large sizes, it is difficult to grasp the control logic, and to implement the state tracker and control functions in practice.

Our synthesis goal is to derive, by applying STS supervisor localization, a set of local state trackers and local control functions for each of the five robots such that (1) the corresponding control logic is transparent, and moreover (2) the collective local control action is identical to the monolithic optimal and nonblocking control action of $S$. Specifically, we will derive the distributed control architecture displayed in Fig. 13 as will be shown, the interconnection/communication among robots involves only nearest neighbors.

Remark 1. The setup of our Cluster Tool is borrowed from [8], [9], except for the following. (1) Our system has one more robot (and the corresponding buffer and chambers), so the total state size is of order $10^3$ larger than the system size in [8], [9]. (2) The control specifications $D_i$, $i \in [1, 3]$ (Fig. 11(c)), were not imposed in [8], [9]. The specifications $D_i$ make the system’s behavior more restrictive; as we shall see, however, with $D_i$ imposed we derive straightforward control and coordination logics which achieve global optimal and nonblocking supervision, despite the larger state size of our case. In addition, as explained above, these specifications $D_i$ are themselves reasonable requirements to prevent system deadlock.

Decentralized control and supervisor localization. To facilitate applying STS supervisor localization on the large system at hand, we use a decentralized approach. Since each control specification (Fig. 11(c))
Fig. 13. Distributed control architecture for Cluster Tool: each robot is supervised by its own set of local state trackers and local control functions, as well as interacting (e.g. through event communication) with its immediate left and right neighbors.

Fig. 14. Local state trackers and local control functions obtained by localizing decentralized supervisors $S_{C_{ij}}^j$, $i \in [1, 5], j \in [1, 2]$. Here each state tracker (e.g. $B_{\text{pick-}C_{11}}$) has 2 states, which are encoded by one BDD node in the corresponding control function (e.g. $g_{\text{pick-}C_{11}}$) taking binary values either 0 (displayed by dashed line) or 1 (solid line).

relates (in the sense of sharing events) to no more than two plant components (robots), a corresponding optimal and nonblocking decentralized supervisor may be synthesized as in (6). Then the developed localization algorithm is applied to decompose each decentralized supervisor. In this way both the supervisor synthesis and the localization may be based merely on the relevant robot(s), thereby making the computations more efficient. Concretely, we proceed as follows.

(1) Chamber specifications $C_{ij}$ (Fig. 11(a)). Each $C_{ij}$, $i \in [1, 5]$, shares events only with robot $R_i$. Thus we treat $R_i$ as plant, $C_{ij}$ as specification, and compute an optimal and nonblocking decentralized
supervisor $S_{C_{ij}}$ (predicate as in (6)); the corresponding state tracker has 8 states. Then we apply our algorithm to localize each $S_{C_{ij}}$, and obtain a set of local state trackers and local control functions for the relevant controllable events, as displayed in Fig. 14. For each $S_{C_{ij}}$ there are two events requiring control action. We explain control logic for the case where $i \in [1,5]$ and $j = 1$; the other cases are similar. One such event is pick-$C_{ij}$, which must be disabled ($R_i$ may not pick up a wafer from chamber $C_{ij}$) if $C_{ij}$ is empty; this is to protect chamber $C_{ij}$ against underflow. The other event requiring control action is $R_i$-pick-l, which must be disabled ($R_i$ may not pick up a wafer from left) if chamber $C_{ij}$ is full. This rule prevents a deadlock situation: if $R_i$ took a wafer and $C_{ij}$ were full, then $R_i$ could neither drop the wafer to $C_{ij}$ nor pick up a wafer from $C_{ij}$. The rule at the same time prevents chamber $C_{ij}$ from overflow. Note that controlling just event drop-$C_{ij}$ suffices to prevent overflow, but cannot prevent deadlock.

(2) Buffer specifications $B_i$ (Fig. 11(b)). Each $B_i$, $i \in [1,4]$, shares events with two robots, $R_i$ and $R_{i+1}$. Treating $R_i$ and $R_{i+1}$ as plant, $B_i$ as specification, we compute a decentralized supervisor $S_{B_i}$ (predicate as in (6)); the corresponding state tracker has 55 states. Localizing each $S_{B_i}$, we obtain a set of local state trackers and associated control functions for the relevant controllable events, as displayed in Fig. 15. For each $S_{B_i}$ there are six events requiring control action. Events $R_i$-drop-l and $R_{i+1}$-drop-r must be disabled ($R_i$ or $R_{i+1}$ may not drop a wafer into buffer $B_i$) when $B_i$ is full – this is to prevent buffer overflow. On the other hand, events $R_i$-pick-l and $R_{i+1}$-pick-r must be disabled ($R_i$ or $R_{i+1}$ may not pick up a wafer from buffer $B_i$) when $B_i$ is empty – this is to prevent buffer underflow. In addition to preventing buffer overflow and underflow, event pick-$C_{i2}$ must be disabled ($R_i$ may not pick up a wafer from chamber $C_{i2}$) unless there is no wafer on the path $R_i$-$B_i$-$R_{i+1}$. This logic is to prevent the deadlock situation where both $R_i$ and $R_{i+1}$ pick up a wafer to transport through $B_i$, but neither can do so because the buffer has capacity of only one. For the same reason, event pick-$C_{i+1,1}$ must be disabled ($R_{i+1}$ may not pick up a wafer from chamber $C_{i+1,1}$) unless there is no wafer on the path $R_i$-$B_i$-$R_{i+1}$.

(3) Specifications $D_i$ (Fig. 11(c)). Like buffer specifications, each $D_i$, $i \in [1,3]$, shares events with robots $R_i$ and $R_{i+1}$. Treating $R_i$ and $R_{i+1}$ as plant, $D_i$ as specification, we first synthesize a decentralized supervisor $S_{D_i}$ (the corresponding state tracker has 50 states), and then apply localization to compute a set of local state trackers and associated control functions for the relevant controllable events, as displayed in Fig. 16. For each $S_{D_i}$ only the event pick-$C_{i2}$ requires control action: it must be disabled ($R_i$ may not pick up a wafer from chamber $C_{i2}$) if the neighboring chamber $C_{i+1,2}$ is full. This logic is to prevent blocking while wafers are transported from right to left in the system, as we explained above when the specifications were imposed.
Localize decentralized supervisor \( S_{D_i}, i \in [1, 4] \)

![Diagram showing state trackers and control functions](image)

Fig. 15. Local state trackers and local control functions obtained by localizing decentralized supervisors \( S_{D_i} \). Here the state tracker \( B_{\text{pick-} C_{i2}} \) (resp. \( B_{\text{pick-} C_{i+1,1}} \)) has 3 states, which are encoded by two BDD nodes in the corresponding control function \( g_{\text{pick-} C_{i2}} \) (resp. \( g_{\text{pick-} C_{i+1,1}} \)). For example, state 2 of \( B_{\text{pick-} C_{i2}} \) is encoded as \((B_{2 \text{pick-} C_{i2}}, B_{1 \text{pick-} C_{i2}}) = (1, 0)\).

Localize decentralized supervisor \( S_{D_i}, i \in [1, 3] \)

![Diagram showing state trackers and control functions](image)

Fig. 16. Local state trackers and local control functions obtained by localizing decentralized supervisors \( S_{D_i} \).

**Coordination logic and STS verification.** We have obtained a set of decentralized supervisors \( S_{C_{i,j}} \) (\( i \in [1, 5], j \in [1, 3] \) if \( i = 1 \) and \( j \in [1, 2] \) otherwise), \( S_{B_k} (k \in [1, 4]) \), and \( S_{D_m} (m \in [1, 3]) \). Viewing these decentralized supervisors as predicates defined on the set \( B(ST_{CT}) \) of basic trees of \( ST_{CT} \), we define their joint behavior \( S_{\text{joint}} \) by

\[
S_{\text{joint}} := \left( \bigwedge_{i,j} S_{C_{i,j}} \right) \land \left( \bigwedge_{k} S_{B_k} \right) \land \left( \bigwedge_{m} S_{D_m} \right).
\]

Unfortunately \( S_{\text{joint}} \) is not the same as the monolithic supervisor \( S \). Indeed, there exists conflict among the...
Fig. 17. Coordinators $C_{\text{pick-C}_{11}}, i \in [2, 5]$, each having $2i$ states.

| Events | $S_{C_{ij}}$ in Fig. 14 | $S_{B_{i}}$ in Fig. 15 | $S_{D_{i}}$ in Fig. 16 | $C_{\text{pick-C}_{11}}$ in Fig. 17 | BDD node numbers of control functions of monolithic supervisor $S$ |
|--------|--------------------------|------------------------|------------------------|---------------------------------|--------------------------------------------------|
| pick-$C_{12}$ | 2 | 3 | 2 | $\_\_\_$ | 205 |
| pick-$C_{21}$ | 2 | 3 | $\_\_\_$ | 4 | 284 |
| pick-$C_{22}$ | 2 | 3 | 2 | $\_\_\_$ | 319 |
| pick-$C_{31}$ | 2 | 3 | $\_\_\_$ | 6 | 686 |
| pick-$C_{32}$ | 2 | 3 | 2 | $\_\_\_$ | 571 |
| pick-$C_{41}$ | 2 | 3 | $\_\_\_$ | 8 | 1561 |
| pick-$C_{43}$ | 2 | 3 | $\_\_\_$ | $\_\_\_$ | 777 |
| pick-$C_{51}$ | 2 | 3 | $\_\_\_$ | 10 | 5668 |

TABLE I

STATE/BDD NODE SIZE COMPARISON BETWEEN DISTRIBUTED AND MONOLITHIC APPROACHES.

decentralized supervisors, so that there are non-coreachable basic trees satisfying the joint behavior $S_{\text{joint}}$. This is verified as follows: let $S_{\text{joint}}$ be the plant (so the decentralized supervisors’ state trackers are the plant components), let the predicate $\text{true}$ be the specification (i.e. no additional control requirement is imposed), and compute the corresponding optimal and nonblocking supervisor. This supervisor turns out to be the same as the monolithic $S$, but there are four events – pick-$C_{i1}, i \in [2, 5]$ – requiring disablement action; their control functions have BDD node sizes 14, 78, 471, and 2851, respectively. Since no new control constraint was imposed, the above required disablement action controllably removes the blocking basic trees of $S_{\text{joint}}$ and thereby reproduces the nonblocking monolithic $S$.

One could use as coordinators the above computed control functions of events pick-$C_{i1}, i \in [2, 5]$.

$^3$Algorithm 3 in [20] can also be used for the verification.
Because of the large BDD node sizes of certain events (especially pick-$C_{41}$ and pick-$C_{51}$), however, the control logic is still difficult to grasp. Instead we propose, based on analyzing the structure of Cluster Tool and the wafer transportation route (Fig. 9), the coordinators $\text{CO}_{\text{pick-}C_{i1}}$, $i \in [2,5]$, as displayed in Fig. 17. We explain the coordination logic. Observe in Fig. 9 that once a wafer is picked up from chamber $C_{i1}$ (i.e. pick-$C_{i1}$ occurs), it will be transported by robot $R_i$ to the right, and so all the way to $R_1$ and then back to the left to $R_i$ – a looping route. For example, when $R_3$ takes a wafer from $C_{31}$, the loop is as follows:

$$C_{31} \xrightarrow{R_3} B_2 \xrightarrow{R_2} C_{21} \xrightarrow{R_2} B_1 \xrightarrow{R_1} C_{11} \Downarrow R_1$$

$$C_{32} \xleftarrow{R_3} B_2 \xleftarrow{R_2} C_{21} \xleftarrow{R_2} B_1 \xleftarrow{R_1} C_{12} \Downarrow R_1.$$ 

Since the loop has limited capacity to hold wafers, control is needed at the entrance and exit of the loop to prevent ‘choking’ the loop with too many wafers. The logic of the coordinators in Fig. 17 specifies that event pick-$C_{i1}$ must be disabled if the number of wafers input exceeds wafers output by $2i - 1$. Note that the loop capacity $2i - 1$ is exactly the number of chambers in the loop; this is because robots and buffers are shared resources, and if all the chambers are full, inputting one more wafer to the loop will clearly cause deadlock. We remark that the proposed coordination rule requires global knowledge; for example, to disable event pick-$C_{51}$, the coordinator $\text{CO}_{\text{pick-}C_{51}}$ needs to know a priori the capacity of the whole loop on the right. Upon knowing the loop capacity, however, each coordinator may be implemented locally because it suffices just to count the numbers of wafers input and output to the corresponding loop.

We now verify that the four proposed coordinators $\text{CO}_{\text{pick-}C_{i1}}$, $i \in [2,5]$ (Fig. 17), indeed resolve all the conflicts among the decentralized supervisors. Let the four coordinators and the decentralized supervisors’ state trackers be the plant components, the predicate $\text{true}$ be the specification, and compute the corresponding optimal and nonblocking supervisor. This supervisor turns out to be the same as the monolithic supervisor $S$, and now no event requires further control action. This computation shows that the proposed coordinators and the decentralized supervisors together provide the same global optimal and nonblocking control action as the monolithic supervisor $S$ did.

On the other hand, by Theorem 2 each pair comprising a local state tracker and a local control function in Figs. 14,16 is control equivalent to the corresponding decentralized supervisor. Therefore, the set of controllers and coordinators in Figs. 14,17 is control equivalent to the monolithic supervisor $S$. Finally
grouping them with respect to the individual robots $R_i$, $i \in [1, 5]$, we derive the distributed control architecture displayed earlier in Fig. 13 where each robot interacts only with its nearest neighbor(s).

Remark 2. In the work of [8], [9] on Cluster Tool, the primary focus was on reducing computational complexity in achieving global optimal and nonblocking control. There the authors proposed an efficient “distributed supervisor” synthesis, based on abstraction and coordination techniques, which solves the Cluster Tool problem by involving state sizes of order only $10^2$ in the computations. It is not clear, however, what the resulting control and coordination rules are. Engineers, on the other hand, expect comprehensible rules for easy implementation and safe management, especially when the plant itself has an intelligible structure; in this case, the system components are connected in a loop.

In our results, by contrast, every control/coordination rule is transparent, as displayed in Figs. 14-17. The control rules are derived by applying our developed STS supervisor localization; the coordination logic is designed by analyzing the loop structure of the system. As a comparison to the monolithic result computed above, we see from Table I that substantial size reduction is achieved by supervisor localization and coordination design. Finally, for verification that the derived control and coordination action is globally optimal and nonblocking, we rely on the computational power of STS and BDD.

VII. CONCLUSIONS

To solve a distributed control problem of discrete-event systems, we have developed the top-down supervisor localization approach in the STS framework. The approach establishes a purely distributed control architecture, in which every active agent is endowed with its own local state trackers and local control functions, while being coordinated with its fellows through event communication in such a way that the collective local control action is identical to the global optimal and nonblocking action. Such a control scheme facilitates distributed and embedded implementation of control strategies into individual agents. Compared to the language-based RW counterpart [1], we have designed a more efficient symbolic localization algorithm by exploiting BDD computation. Furthermore, we have demonstrated our localization approach in detail on a complex semiconductor manufacturing system, Cluster Tool.

REFERENCES

[1] K. Cai and W. M. Wonham, “Supervisor localization: a top-down approach to distributed control of discrete-event systems,” IEEE Trans. Autom. Control, vol. 55, no. 3, pp. 605–618, 2010.

[2] ———, “Supervisor localization for large discrete-event systems – case study production cell,” Int. J. of Advanced Manufacturing Technology, vol. 50, no. 9-12, pp. 1189–1202, 2010.
[3] P. J. Ramadge and W. M. Wonham, “Supervisory control of a class of discrete event processes,” *SIAM J. of Control and Optimization*, vol. 25, no. 1, pp. 206–230, 1987.

[4] W. M. Wonham, “Supervisory control of discrete-event systems,” Systems Control Group, ECE Dept, University of Toronto, updated July 1, 2012. Available online at http://www.control.toronto.edu/DES.

[5] R. Olfati-Saber, J. A. Fax, and R. M. Murray, “Consensus and cooperation in networked multi-agent systems,” *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[6] L. Feng and W. M. Wonham, “Supervisory control architecture for discrete-event systems,” *IEEE Trans. Autom. Control*, vol. 53, no. 6, pp. 1449–1461, 2008.

[7] K. Schmidt, T. Moor, and S. Perk, “Nonblocking hierarchical control of decentralized discrete event systems,” *IEEE Trans. Autom. Control*, vol. 53, no. 10, pp. 2252–2265, 2008.

[8] R. Su, J. H. van Schuppen, and J. E. Roorda, “Aggregative synthesis of distributed supervisors based on automaton abstraction,” *IEEE Trans. Autom. Control*, vol. 55, no. 7, pp. 1627–1640, 2010.

[9] ——, “Maximum permissive coordinated distributed supervisory control of nondeterministic discrete-event systems,” *Automatica*, to appear, 2012.

[10] C. Ma and W. M. Wonham, *Nonblocking Supervisory Control of State Tree Structures*. Springer-Verlag, 2005.

[11] ——, “Nonblocking supervisory control of state tree structures,” *IEEE Trans. Autom. Control*, vol. 51, no. 5, pp. 782–793, 2006.

[12] D. Harel, “Statecharts: a visual formalism for complex systems,” *Science of Computer Programming*, vol. 8, no. 3, pp. 231–274, 1987.

[13] R. Bryant, “Graph-based algorithms for boolean function manipulation,” *IEEE Trans. on Computers*, vol. C-35, no. 8, pp. 677–691, 1986.

[14] A. Mannani and P. Gohari, “Formal modeling and synthesis of state-transferring (event-transferring) communication among decentralized supervisors for discrete-event systems,” in *Proc. IEEE Int. Conf. on Systems, Man and Cybernetics*, San Antonio, TX, 2009, pp. 3231–3242.

[15] K. T. Seow, M. T. Pham, C. Ma, and M. Yokoo, “Coordination planning: applying control synthesis methods for a class of distributed agents,” *IEEE Trans. Control Syst. Technol.*, vol. 17, no. 2, pp. 405–415, 2009.

[16] M. T. Pham and K. T. Seow, “Discrete-event coordination design for distributed agents,” *IEEE Trans. Automation Science and Engineering*, vol. 9, no. 1, pp. 70–82, 2012.

[17] R. Su and W. M. Wonham, “Supervisor reduction for discrete-event systems,” *Discrete Event Dynamic Systems*, vol. 14, no. 1, pp. 31–53, 2004.

[18] K. Rudie, S. Lafortune, and F. Lin, “Minimal communication in a distributed discrete-event system,” *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 957–975, 2003.

[19] J. Yi, S. Ding, M. T. Zhang, and P. van der Meulen, “Throughput analysis of linear cluster tools,” in *Proc. 3rd IEEE Int. Conf. on Automation Science and Engineering*, Scottsdale, AZ, 2007, pp. 1063–1068.

[20] W. Chao, Y. Gan, Z. Wang, and W. Wonham, “Modular supervisory control and coordination of state tree structures,” *Int. J. Control*, to appear, 2012. Available online at http://www.tandfonline.com/doi/abs/10.1080/00207179.2012.715754.