An Assessment Method for Residual Bearing Capacity of RC Bridges After Fatigue Damage

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ABSTRACT

In order to realize the in-situ evaluation of reinforced concrete bridges subjected to fatigue for a long time, an evaluation method for cumulative damage of concrete structures based on unloading elastic modulus is proposed. First, according to the concrete stress-strain curve and the statistical relationship between residual strain and cumulative strain, the calculation method of static equivalent strain and residual strain concrete based on unloading elastic modulus and the method for estimating the strength of concrete after damage are proposed. The detailed steps of field test and analysis and the practical damage indicators of residual strain are given. Then, the evaluation method of existing stress and strain of Reinforced Concrete Bridge under dead load and the concept of “equivalent dead load bending moment” are put forward. On this basis, the paper analyzes the root cause of the decrease of bearing capacity of Reinforced Concrete Bridge after fatigue damage, and points out that the equivalent strain or residual strain of reinforced concrete increases under the fatigue effect, which leads to the decreasing of actual live moment and deformation performance while the ultimate load-carrying capacity remains constant or very little decrease. The evaluation method of structure residual capacity is given, which has reference value for engineering practice.¹

INTRODUCTION

In recent years, with the frequent accidents worldwide caused by overloading of bridges and environmental erosion, the fatigue damage of bridges is becoming more and more serious. A large number of experimental study on concrete beams show that [1-4], the failure characteristics of concrete beam under fatigue load are

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generally behave as one or part of the tensioned main reinforcement fracture due to fatigue, and the bearing capacity of reinforced concrete beam after the failure will be significantly reduced. For the concrete beams without fatigue failure, the average strain of the cross-section is still in accordance with the assumption of plane section. The height of the compression zone in the cross-section is basically the same (relate to the ceiling load). The concrete in the compression area is still in the elastic stage, and the fatigue elastic modulus of concrete decays continuously. The width of the concrete crack in the tension zone increases, resulting in the decrease of the flexural rigidity of the concrete beam, the increase of the residual deformation and the increase of the tensile steel stress. Therefore, the working state and remaining bearing capacity of concrete bridges without fatigue damage has become the research focus of the bridge sector.

In terms of assessing the bearing capacity of concrete bridges, China has formed a more perfect standard system, usually through structural detection combined with the calculation and analysis. The calculation method of bearing capacity adopted in the Technical code for the detection and evaluation of urban bridges (CJJ/T 233-2015) is based on the test results of the bridge material and appearance, but the bearing capacity calculation of structures with more serious damage is still a difficult problem. The method of calculating the bearing capacity of the bridge is based on the test results of the bridge inspection and assessment. The “scoring” method is adopted by the testing and evaluation procedures of Highway and bridge carrying capacity (JTG/T J21-2011), which mainly depends on the bridge damage classification, the refinement and accuracy of scoring standards as well as the experience and technical level of detection personnel, and subject to the influence of subjective factors. Besides, the generally considered reliable method-load test is often used to verify the calculation result. However, the traditional static load test can only evaluate the bridge performance under the “test load”, which is to evaluate the normal bearing capacity according to the change of the structural rigidity or the cross-sectional stiffness, and the bearing capacity is indirectly reflected by the elastic behavior. Thus, a contradiction emerges between conventional static load test and the actual bridge failure test; some bridges which are judged to have a decreasing bearing capacity or fail to meet the requirements by the conventional static load test, are tested to have the same bearing capacity as before by actual bridge failure test. For example, Zhang Jianren [5] carried out an on-site failure test for a 43-year long-term overloaded RC beam subjected to over-limit vehicle loading. The results show that cumulative fatigue damage has no significant effect on the static load-carrying capacity of the bridge. Structural response analysis shows that overload make the mechanical behavior of the structure changes from elastic to plastic. Yu Zhiwu [6] and Sun Xiaoyan [7] also found through the experimental study that, when the fatigue failure doesn’t happen on concrete beams, the corresponding fatigue residual capacity is almost unchanged. Therefore, they believed that fatigue cumulative damage mainly reduces the deformation properties of the structure, but has little
influence on the bearing capacity of the structure, however they didn’t further analysis the reason.

In terms of theoretical analysis, the present research mainly focuses on the analysis of structural damage accumulation and life prediction based on material damage. Wang Chunsheng [8] proposed an evaluation method for fatigue life of concrete bridges based on S-N curves and fracture mechanics. Zhu Jinsong [9] and Wang Qing [10] established the whole process analysis method of concrete bridges fatigue failure. The application process of the above-mentioned analysis method requires to know how the load works, but the functioning course of the actual bridge is often not clear, and the structure or the cross-section stress redistribution after the damage, and the effect of nonlinear coupling of fatigue, creep, temperature and shrinkage during service add more complexity to the evaluation of the cumulative damage degree of the existing structure concrete, and the result obtained by simplified prediction of fatigue cumulative damage analysis is often different from the actual results. Therefore, in view of the urgent need of the engineering community to assess the cumulative damage of existing concrete bridges, an evaluation method that can directly determine the cumulative damage degree and residual bearing capacity is urgently needed.

In this paper, according to the measured elastic modulus of concrete after damage, the calculation method of equivalent strain and residual strain as well as the evaluation method of concrete strength after damage are put forward. Then, the structural analysis method is adopted for the evaluation of the existing stress, strain state and residual bearing capacity of the bridge.

![Stress-strain relations for concrete under fatigue loading.](image-url)

Figure 1. Stress-strain relations for concrete under fatigue loading.
RESIDUAL STRAIN ANALYSIS METHODS

Many concrete fatigue tests [11-13] show that the damage and failure of concrete are internal microcracks extending to macroscopic cracks. When the crack length reaches a certain critical length, it will expand unsteadily until it breaks. And under various time-varying effects, the uniform internal damage of concrete is also caused by the expansion of micro-cracks inside the concrete, which results in deterioration of the mechanical properties of concrete [14]. The elastic modulus and the residual strain can reflect the microscopic damage mechanism of the concrete macroscopically, and have nothing to do with the loading history, which is a good choice for the concrete structure assessment. For various strength concrete, the development of fatigue residual strain is more stable and representative [12,15], but it cannot be tested directly due to the time-varying effect, in consequence, the deformation modulus $E_f$ cannot be measured either, only the elastic modulus of unloading concrete $E_r$ can be tested on-site, however, there’s still some limitations for the application of $E_r$ [16]. The compressive stress-strain curves of concrete under fatigue loading are shown in Fig.1. It can be seen that the residual strain $\varepsilon_p$ is related to the unloaded elastic modulus $E_r$ and the static equivalent strain $\varepsilon_s$. Therefore, how to obtain the residual strain according to the unloaded elastic modulus has become the key to evaluate the cumulative damage of concrete.

Relationship between Concrete Residual Strain and Fatigue Cumulative Strain

Now the residual strain of concrete is basically analyzed by the test statistic method. The Berkeley loading and unloading model, which was a simplified linear model of Karsan and Jirsa [17], illustrates the relationship between the residual strain $\varepsilon_p$ and fatigue cumulative strain $\varepsilon_r$ as shown in formula (1). The statistical formula shown in formula (2) is also derived by Guo Zhenhai [18]. The residual strain calculated by the Berkeley loading and unloading model is close to that calculated by the Guo Zhenhai model which is slightly larger.

Holmen [11] believes that “concrete ultimate strain can be used as a criteria for concrete fatigue failure”, a large number of fatigue test results also verify his point. Therefore, the maximum strain of concrete under fatigue loading is equivalent to the strain corresponding to the maximum stress of the

\[
\begin{align*}
\frac{\varepsilon_t}{\varepsilon_{c,t}} < 2 & \quad \frac{\varepsilon_t}{\varepsilon_{c,t}} = 0.145 \times \left( \frac{\varepsilon_r}{\varepsilon_{c,r}} \right)^2 + 0.13 \times \left( \frac{\varepsilon_t}{\varepsilon_{c,t}} \right) \\
\frac{\varepsilon_t}{\varepsilon_{c,t}} \geq 2 & \quad \frac{\varepsilon_p}{\varepsilon_{c,p}} = 0.707 \times \left( \frac{\varepsilon_r}{\varepsilon_{c,r}} - 2 \right) + 0.834
\end{align*}
\]

(1)
\[
\frac{\varepsilon_p}{\varepsilon_{c,r}} = 0.247 \times \left( \frac{\varepsilon_t}{\varepsilon_{c,t}} \right)^{0.77}
\]  

(2)

monotone loading softening zone [19], such as \( \varepsilon_{un} \) in Figure 1, but this value is related to the maximum stress \( \sigma_{max} \) corresponding to the softening zone of the concrete and is difficult to accurately determine. “The fatigue residual stress \( \varepsilon_p \) is equal to 0.4\( \varepsilon_{c,r} \)” is generally recognized as the practical failure criterion for fatigue failure [2, 19]. The actual structure of the concrete stress is not uniform, the site of damage is often local. And structures which break this limit are usually have serious cracking and can no longer be effectively used. Therefore, this failure criterion is also used in this paper. The relationship between residual strain and cumulative strain as shown in formula (3) is presented. The comparison with Berkeley loading-unloading model and over-sea model is shown in Fig.2. It can be seen that the residual strain curve calculated by the relational expression is between the curve of the Berkeley loading-unloading model and the Guo Zhenhai model, which is close to the curve of Guo Zhenhai model, and also proves the rationality of the relational expression. When the concrete residual strain \( \varepsilon_p = 0.4\varepsilon_{c,r} \), the corresponding cumulative strain is 1.3\( \varepsilon_{c,r} \) according to the formula (3). When the concrete reaches the peak strain \( \varepsilon_{c,r} \), the corresponding \( \varepsilon_p =0.25\varepsilon_{c,r} \).

\[
\frac{\varepsilon_p}{\varepsilon_{c,r}} = 0.186 \times \left( \frac{\varepsilon_t}{\varepsilon_{c,r}} \right)^2 + 0.067 \times \left( \frac{\varepsilon_t}{\varepsilon_{c,r}} \right) \left( \frac{\varepsilon_t}{\varepsilon_{c,r}} \right) \leq 1.3
\]  

(3)

Concrete Static Equivalent Strain and Residual Strain Analysis Methods

Since the residual strain can be calculated from the statistical relationship between the residual strain and the cumulative strain of fatigue on the premise that the cumulative fatigue strain is known. Therefore, this paper presents an analysis method for static equivalent strain and residual strain based on the unloaded elastic modulus of concrete. According to the uniqueness assumption proposed by Sinha [20], the relationship between the load and the deformation will remain the same regardless of the repeated load history of the concrete, so long as the residual deformation is the same and the same repeated load is applied. This hypothesis has now been confirmed by many experiments, so the monotone loading stress-strain relationship can be used to show the fatigue envelope of concrete. On the basis of the constitutive relations of concrete under uniaxial compression provided by Code for concrete structure design (hereinafter referred to as Code), the complexity of the energy dissipation law is deducted, that is to say, the loading and unloading elasticity modulus after concrete damage is assumed to be in the linear mode as shown in Fig.1. Then combing with formula (3), the static equivalent strain analysis model of the cumulative damage of compressive concrete is established.
The residual strain can be calculated by static equivalent strain and the stress and the strength of concrete after damage can be further inferred. Specific test and analysis steps are as follows:

Step 1: Using Non-destructive testing methods such as ultrasonic, blast wave or core sample method to test and analyze the mechanical parameters of non-destructive concrete in representative concrete member and parts, including the concrete compressive strength $f_{c,r}$, the concrete elastic modulus $E_c$ and the peak compressive strain $\varepsilon_{c,r}$.

Step 2: The uniaxial compression stress-strain curve can be made according to the actual compressive strength of concrete $f_{c,r}$ and elastic modulus $E_c$, the formula is shown below:

$$\varepsilon = (1 - d_c)E_c \varepsilon$$  \hspace{1cm} (4)

Where $\sigma$, $\varepsilon$ are the compressive resistance and compressive strain of the concrete; $d_c$ is the evolutionary parameter of the compression damage of the concrete. For details, see C.2.4 of the Code.

Step 3: The unloading elastic modulus $E_r$ of the critical point of the structural concrete (usually the most unfavorable force section of the stressed member) can be measured by using the non-destructive testing method or the static loading and strain testing system; For example, by applying the static load to the structural concrete, the stress increment and strain increment of the concrete are obtained, and the unloading elastic modulus $E_r$ is calculated according to formula (5)

$$E_r = \frac{\Delta \sigma}{\Delta \varepsilon}$$  \hspace{1cm} (5)
In the formula, \( \triangle \sigma, \triangle \varepsilon \) are the stress increment and strain increment of concrete under static loading respectively.

Step 4: The relationship between the unloaded elastic modulus \( E_r \) and the static equivalent strain \( \varepsilon_r \) is established according to Figure 1.

\[
E_i = \frac{(1 - d_r) E_c \varepsilon_i}{\varepsilon_r - \varepsilon_p} \tag{6}
\]

Step 5: According to the relationship between residual strain and fatigue cumulative strain given by formula (3) and formula (6), the static equivalent strain \( \varepsilon_r \) can be calculated.

Step 6: Calculate the static equivalent stress \( \sigma_r \) corresponding to \( \varepsilon_r \) according to equation (7), then the residual strain value \( \varepsilon_p \) of the concrete is obtained by formula (8);

\[
\sigma_r = (1 - d_r) E_c \varepsilon_i \tag{7}
\]

\[
\varepsilon_p = \varepsilon_i - \sigma_r / E_r \tag{8}
\]

Step 7: The residual strength of concrete after damage can be deduced according to the concrete stress-strain curve and static equivalent strain \( \varepsilon_r \): when \( \varepsilon_r \leq \varepsilon_c, r \), the concrete strength is remains to be \( f_{c,r} \); when \( \varepsilon_r > \varepsilon_c, r \), the concrete strength decreases, the static equivalent stress \( \sigma_r \) calculated by formula (7) is the residual strength of the concrete after damage.

Step 8: The practical damage indices \( D_p \) and \( D_\varepsilon \) calculated based on the residual strain and static equivalent strain of concrete are shown in Eqs. (9) and (10),
respectively. The comparison of Dp and Dε applicability is shown in Figure 3. Dε increases linearly with the increasing strain, but Dp can reflect the accelerated development of the damage when the strain is increasing, therefore Dp index is more reasonable.

\[ D_p = \frac{\varepsilon_p}{0.4\varepsilon_{cr}} \]  

\[ D_e = \frac{\varepsilon_e}{1.3\varepsilon_{cr}} \]  

\section*{PRACTICAL EVALUATION OF WORKING STATE AND REMAINING BEARING CAPACITY OF REINFORCED CONCRETE BRIDGES}

\section*{The Working Condition of Rc Bridge Under Constant Load}

According to the above analysis and evaluation of residual strain of concrete, the existing stress and strain state of reinforced concrete beams can be evaluated under the assumption of plane section. In order to consider the effect of residual strain, the concept of “equivalent dead load moment” is put forward to distinguish with the original dead load moment and to evaluate the residual bearing capacity of reinforced concrete beam. Take the common T-section reinforced concrete bending members as an example, as shown in Figure 4, the specific analysis steps are as follows:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{rc beams.png}
\caption{The actual strain distribution and the equivalent bending moment of RC beams.}
\end{figure}
Step 1: According to the practical failure criterion of concrete fatigue failure, the residual strain $\varepsilon_p$ at the edge of compression area are compared with $0.4\varepsilon_{c,r}$, and the damage boundary of reinforced concrete beam is evaluated by the residual strain $\varepsilon_p$ of the concrete:

1. When the concrete residual strain $\varepsilon_p > 0.4\varepsilon_{c,r}$, reinforced concrete beams is damaged seriously and often does not have the value of maintenance and reinforcement, assessment is no longer carried out;

2. When the concrete residual strain $\varepsilon_p \leq 0.4\varepsilon_{c,r}$, the working state of reinforced concrete beams should be evaluated.

Step 2: When the concrete residual strain $\varepsilon_p \leq 0.4\varepsilon_{c,r}$, according to $\sigma_e = \frac{M_g Z_c}{I_{cr}}$, the concrete elastic stress $\sigma_e$ in the stress zone of cross section under constant load can be obtained. Where $M_g$ is the constant load moment of the cross-section; $I_{cr}$ is the converted moment of inertia, $Z_c$ is the measured height of cross-section pressure zone.

Step 3: According to $\varepsilon_c = \sigma_e / E_r$, the concrete elastic strain $\varepsilon_c$ can be obtained, and according to $\varepsilon_c = \varepsilon_e + \varepsilon_p$, the concrete total strain $\varepsilon_c$ can be obtained as shown in Figure 4.

Step 4: The total strain $\varepsilon_s$ of the tensioned longitudinal reinforcement is obtained from $\varepsilon_s = \varepsilon_c \times \frac{(h-c-Z_c)}{Z_c}$, and the elastic modulus of the tensioned steel is basically the same as the elasticity of the tensile steel still works under fatigue loading, and the reinforcement tensile stress can be obtained by $\sigma_s = E_s \times \varepsilon_s$. Considering the influence of non-closed fracture, the actual stress is obtained by multiplying the calculated stress with the increase coefficient of 1.1~1.2 [2, 3]. Where c is the thickness of the concrete protective layer in the tension zone; h is the height of the section.

Step 5: The “equivalent static moment” $M_{eq}$ can be obtained approximately from $M_{eq} = A_s \times \sigma_s \times 0.9h_0$ (rectangular cross-section taken 0.87$h_0$). Where, $M_{eq}$ is the equivalent bending moment of the mid-span section under the influence of the residual strain of the concrete under the constant load (after the longitudinal reinforcement stress increases), which is obviously greater than or equal to the actual dead load moment $M_g$; $A_s$ is the cross-sectional area of tensioned longitudinal reinforcement; $h_0$ is the effective cross-section height, $h_0 = h-c$.

Step 6: For the sake of simplicity, it is safe to approximate the stress-strain curve of the concrete in the dead zone to a triangular shape, as shown in Fig. 4, and the equivalent compressive stress $\sigma_c$ under constant load can be obtained by $\sigma_c = \frac{M_{eq} Z_c}{I_{cr}}$.

Following the above steps, the ultimate bearing capacity and residual bearing capacity of reinforced concrete bridges after fatigue damage can be further analyzed. The results show that the ultimate load-carrying capacity of reinforced concrete bridges is higher than that of concrete beams.
Residual Bearing Capacity of Reinforced Concrete Bridges After Fatigue Damage

Experiments have shown that fatigue damage has little effect on the ultimate load-carrying capacity of the bridge, and the ultimate bearing capacity does not decrease or decrease very little, even increases due to the enhancement of yield strength of longitudinal reinforcement, exceeding the bearing capacity evaluated by the static load test [5-7]. The reason is that the ultimate bearing capacity is controlled by the material properties and the section size of the concrete and the longitudinal reinforcement. If the material properties and the cross-sectional dimension are not changed or the degradation is very small, the ultimate bearing capacity of the reinforced concrete bridge will basically remain the same or show a very little decline. The experimental study and analysis of reinforced concrete members considering low-cycle fatigue have also proved that when the longitudinal reinforcement yielded but did not enter the descending section of the load-deformation curve, the degradation of yield strength and ultimate bearing capacity of the members caused by low-cycle fatigue damage is very small [21,22]. Only when it goes into the descending section of the load-deformation curve, will the bearing capacity decrease gradually, while the low cycle fatigue has a greater influence on the deformation performance. In the use stage, the reinforced concrete bridge can hardly enter the descending section of the load-deformation curve. The serious damage of the compression concrete often accounts for a small area of the whole section, so the ultimate bearing capacity is almost unchanged. And now according to the traditional static load test, the bearing capacity is just the normal carrying capacity in the use stage, which is far less than the ultimate carrying capacity. Based on the above analysis, the bearing capacity after fatigue damage can be assessed based on the degradation of material properties, without considering the size of the cross-section and the impact of corrosion, the T-shaped section shown in Figure 4 can still be used as an example to illustrate the practical evaluation process.

First of all, the ultimate bearing moment $M_{jj}$ without fatigue cumulative damage and the bearing moment $M_{dc}$ in service stage can be calculated based on the measured concrete compressive strength $f_{c,r}$ and the actual cross-section size of reinforced concrete beams. The ultimate bearing moment $M_{jj}$ is calculated as shown in equations (11) and (12). The calculation formula of the maximum loading moment $M_{dc}$ is the same as the formula of the ultimate load moment $M_{jj}$, except that the standard value is substituted by the design value of concrete and longitudinal reinforcement.

When $f_{sk} A_s \leq f_{c,r} h_l$, \[ M_{jj} = f_{c,r} b_l h_f (h_0 - h_f / 2) \]  (11)

When $f_{sk} A_s > f_{c,r} h_l$, \[ M_{jj} = f_{c,r} \left[ b x (h_0 - x / 2) + (b_l - b) h_f (h_0 - h_f / 2) \right] \]  (12)
Where $bf$ is the effective width of the compression flange of the T-section; $hf$ is the effective thickness of the compressed flange of the T-section; $b$ is the width of the T-section web, $f_{sk}$ is the standard value of the tensile strength of the longitudinal reinforcement, $x$ is the height of the compression zone when the section is broken.

Then, according to the practical failure criterion of concrete fatigue failure, whether concrete strength after concrete damage is reduced is judged by static equivalent strain or residual strain of concrete, the actual ultimate bearing moment $M_{zj}$ of reinforced concrete beam after fatigue damage and the bearing moment $M_{zc}$ can be evaluated:

When $\varepsilon_c \leq \varepsilon_{c,r}$ or $\varepsilon_p \leq 0.25 \varepsilon_{c,r}$,
the concrete strength is not changed, \[ M_{zj} = M_{jj}, \quad M_{zc} = M_{dc} \] (13)

When $\varepsilon_{c,r} < \varepsilon_c \leq 1.3 \varepsilon_{c,r}$ or $0.25 \varepsilon_{c,r} < \varepsilon_p \leq 0.4 \varepsilon_{c,r}$,
The concrete strength decreases, \[ M_{zj} = 95\% M_{jj}, \quad M_{zc} = 95\% M_{dc} \] (14)

It should be noted that, according to the existing experimental study [7, 21], the reduction of ultimate bearing capacity caused by fatigue damage is generally less than or equal to 5%, so the bearing capacity reduction coefficient can be taken as 95%. The evaluation of bridge bearing capacity generally requires the live load of the structure as a basis for assessing its capacity to withstand overload (overload does exist) or set limit load. The maximum live load moment $M_{hj}$ can be obtained by $M_{hj} = M_{zj} - M_{eq}$. The corresponding safety factor should be considered in the calculation of the maximum bending moment $M_{hc}$, according to the load factor valuation set by the bridge specification [23], $M_{hc} = \left( M_{zc} - 1.2 M_{eq} \right) / 1.4$.

Based on the above analysis, the ultimate load-carrying capacity of reinforced concrete bridges after fatigue damage has not decreased or decreased very little. And the reason why it is generally believed that the bearing capacity of the structure decreases under the fatigue damage is that the structural concrete strain, residual strain are increasing under fatigue effect, so that the “equivalent dead load moment” of the cross section is greater than the dead load moment, in consequence, the actual live load bending moment is reduced.

The research on the evaluation method of bearing capacity of reinforced concrete bridges after fatigue damage is discussed above, which is applicable for the in-situ evaluation of concrete bridges. The influence of longitudinal reinforcement ratio, concrete strength and pre-stressing force is not analyzed in this paper. In addition, the fatigue effect has a greater influence on the deformation performance of concrete bridge. It can be initially evaluated by the static equivalent strain and residual strain analysis method and residual damage practical damage index proposed in this paper, and the analysis and evaluation of the related structural deformation requires some further study.
CONCLUSIONS

(1) According to the practical relationship among concrete stress-strain curve, residual strain and accumulated strain, the calculation method of concrete equivalent static strain and residual strain based on unloading elastic modulus is put forward, and the specific testing and analysis steps of concrete structure site is detailed described, which is applicable for the acquisition of fatigue cumulative strain and residual strain of structural concrete.

(2) The evaluation method of existing stress and strain and the “equivalent dead load bending moment” concept which taking the reinforced concrete bridge under constant load into account the effect of residual strain are put forward, and it’s applicable for the working state evaluation of existing reinforced concrete bridges.

(3) The main reason why the residual capacity of Reinforced Concrete Bridge is basically invariable or declining after fatigue damage is analyzed, and the practical evaluation method of residual capacity of structure is given. It point out that the main reason is that the equivalent strain or residual strain of structural concrete increase under the influence of fatigue, resulting in the decline of the actual live load level that the bridge can bear.

(4) The concrete residual strain analysis method and the damage index can be used to evaluate the deformation performance of concrete bridge, and the analysis and evaluation of the deformation behavior after damage are worthy of further study.

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