Component based modelling of piezoelectric ultrasonic actuators for machining applications

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Abstract. Ultrasonically Assisted Machining (UAM) is an emerging technology that has been utilized to improve the surface finishing in machining processes such as turning, milling, and drilling. In this context, piezoelectric ultrasonic transducers are being used to vibrate the cutting tip while machining at predetermined amplitude and frequency. However, modelling and simulation of these transducers is a tedious and difficult task. This is due to the inherent nonlinearities associated with smart materials. Therefore, this paper presents a component-based model of ultrasonic transducers that mimics the nonlinear behaviour of such a system. The system is decomposed into components, a mathematical model of each component is created, and the whole system model is accomplished by aggregating the basic components’ model. System parameters are identified using Finite Element technique which then has been used to simulate the system in Matlab/SIMULINK. Various operation conditions are tested and performed to demonstrate the system performance.

1. Introduction
Ultrasonically Assisted Turning (UAT) enhances and improves machining of hard-to-cut alloys. This technique depends on producing a vibration on the cutting tool with a frequency of 20 kHz and amplitude of 30µm peak-to-peak. Vibrating the cutting tool in this manner reduces the cutting forces up to 60% [1] and a considerable improvement in the surface finish can be obtained [2]. However, this technique has not yet been widely introduced in the industry due to instability problems in the cutting process. Instability in tool vibration causes a poor surface finish and, hence, prevents the full implementation of this process in the industry. Despite all research conducted in this area to make this machining process stable [2-5], an applicable solution is still not found. To tackle the stability issues of such machining processes, comprehensive investigation has to be conducted to find out practical solutions and all that starts from having an accurate and a comprehensive mathematical model for ultrasonic transducers.

Formulating mathematical models for ultrasonic transducers is essential to analyze and investigate the system performance in order to facilitate the controllers’ development. The main challenge in modelling ultrasonic transducers is modelling the driving component in the vibrating system (i.e., the piezoelectric material). Piezoelectric materials exhibit obvious nonlinear characteristics that make it more challenging when modelling the vibrating system. To overcome the nonlinearity inherent in the
model’s governing equations, it has become a common practice to replace it with a linearized version which provides a reasonable description of the system dynamics around an operating point. However, this approach cannot predict the global behaviour of the positioning system which is considered as important as a particular solution around the operating point. Significant work has been conducted to develop a proper nonlinear model for the piezoelectric actuators in order to describe the inherent hysteresis phenomenon. The most popular models are the Preisach model [6, 7, and 8], Prandtl–Ishlinskii (PI) model [9, 10], and Krasnosel’skii Pokrovskii (KP) model [11]. Bashash and Jalili [12] introduced a nonlinear model for piezoelectric actuators combining the Prandtl–Ishlinskii hysteresis operator with a second-order linear dynamics whereas Yeh et al. [13], used a non-linear spring element in the hysteresis model and utilized a Maxwell slip structure. Recently, Ru et al. [14] proposed a new mathematical model for the hysteresis by using polynomial fitting. Salah et al. [15] has utilized Coleman-hysteresis model for modelling the dynamic behaviour of the piezoelectric actuators. They proposed a nonlinear robust control strategy to actively control the displacement of the effective tip. Lyapunov based analysis tools were employed to prove that the tip tracking error is practically regulated to zero.

In this paper, a component-based modelling technique is adopted to create a nonlinear model of ultrasonic transducers. The transducer is decomposed into its basic components and each component is modelled separately. The model of the whole system is created by simply aggregating all components’ models. Simulation results show that the model could predict the behaviour of the system satisfactorily under different conditions.

2. Component-based modelling for ultrasonic transducers

Component-based techniques, in general, have been used extensively in systems development due to their great potential to reduce time and cost, improve system maintainability and flexibility, and enhance system quality [16]. Therefore, adopting this technique to model and simulate ultrasonic transducers provides the following potentials: (1) component-based model is valid over the whole working range of the transducer. This is due to including the nonlinear phenomena of piezoelectric material without the need for linearization, (2) this technique provides flexibility of configuration choice and direct interpretation, (3) changes in the real system components can be reflected easily in the model, (4) the capability of modelling a wide variety of ultrasonic types and handling different materials and shapes, (5) facilitate the modelling of different complexity in order to allow for variation in the future.

Ultrasonic transducers can be decomposed into three main components as shown in Figure 1. These components are: one piezoelectric ring (PZT), and two mass-spring-dampers (MSD 1 and MSD 2). The Load component mimics the external force that acts on the transducer as a result of machining. In this model, the back section of the transducer is neglected because of the existence of a nodal point between the piezoelectric rings and the transducer's back section that prevents vibration waves to propagate back. The two mass-spring-damper components represent the concentrator part. The Kelvin–Voigt component models the effect of the resultant axial load from the machining process.

2.1. PZT component model

The piezoelectric material is represented by a component that has three inputs and two outputs as depicted in Figure 1. The inputs are the voltage, speed and displacement of the driven load (i.e. the first mass-spring-damper which is attached directly to the component). The speed and displacement of the driven load are required to calculate the load force on the piezoelectric ring while the input voltage excites the piezoelectric material to produce the required displacement and speed. The outputs of this component are the displacement and speed of the piezoelectric component. The dynamic behaviour of the piezoelectric component can be expressed as [15].

\[ m_p \ddot{x}_o = \left( \frac{T_{em}}{C_C} \right) q - \left( \frac{T_{em}}{C_C} \right) x_o - \bar{F} \]  

(1)
Figure 1. (a) Ultrasonic transducer; (b) proposed component-based model of ultrasonic transducers

and

$$\vec{F} = k_p x_o + b_p \dot{x}_o + k_1 (x_o - x_1) + b_1 (\dot{x}_o - \dot{x}_1)$$

(2)

where $x_o(t), \dot{x}_o(t), \ddot{x}_o(t)$ are the displacement, speed, and acceleration of the effective tip (where the turning tool is attached), respectively, $x_1(t), \dot{x}_1(t)$ are the displacement and speed of the MSD 1, respectively, $x_2(t), \dot{x}_2(t)$ are the displacement and speed of the MSD 2, respectively, $q(t)$ is the charge induced within the piezoelectric, $m_p$ is the mass of the piezoelectric material, $T_{em}$ is the elongation constant, $C_c$ is the internal capacitance, $k_p, b_p, k_1, b_1$ are the spring constants and damping coefficients of the PZT and MSD 1, respectively, and $\vec{F}$ is the perpendicular force acting on the piezoelectric actuator.

A nonlinear hysteresis model (i.e., $q = H(V_h)$) can be defined to describe the relationship between the input voltage, $V_h(t)$, and the induced charge, $q(t)$ as [17]

$$H = f + d$$

(3)

where the input voltage, $V_h(t)$, is defined as $V_h = V - V_c$. The signal $V_c(t)$ is the voltage across the internal capacitor of the piezoelectric and $V(t)$ is the voltage applied on the piezoelectric component and represents the control input. The function $f(V_h)$ is a subsequently defined signal and the function $d(V_h)$ is defined as
\[
d(\cdot) = \begin{cases} 
q_0 - f(V_{ho})e^{-\delta[V_h-V_{ho}]} + e^{-\delta V_h} \int_{V_{ho}}^{V_h} (g(\tau) - f'(\tau))e^{\delta \tau} d\tau, & \text{when } V_h(t) > 0 \\
q_0 - f(V_{ho})e^{\delta[V_h-V_{ho}]} - e^{\delta V_h} \int_{V_{ho}}^{V_h} (g(\tau) - f'(\tau))e^{-\delta \tau} d\tau, & \text{when } V_h(t) < 0 
\end{cases}
\]

where \(q_0\) is the induced charge at \(t = t_o\), \(V_{ho}\) is the input voltage at \(t = t_o\), \(\delta\) is a positive constant, \(f'(\tau) = \frac{\partial f(V_{h})}{V_{h}}\), and \(g(\cdot)\) is a subsequently designed signal. To validate the expressions in equation (4), the signals \(d(V_{h})\), \(f(V_{h})\) and \(g(V_{h})\) have to satisfy certain properties that can be found in [17, 15]

### 2.2. Mass-Spring-Damper (MSD) component model

The MSD component represents a mass that is connected with two springs and two dampers. As shown in Figure 1, the component has four inputs and two outputs. The inputs are speed and displacement of the previous and subsequent masses that are connected to this component whereas the outputs are the resultant speed and displacement of the current mass. Parameters required for this component are stiffness and damping coefficients of the current and adjacent systems. If this component is not connected to other systems, then all corresponding parameters and inputs are set to zero.

In order to be consistent in modelling different components in the system, the transducer can be modelled with two separate MSDs, as shown in Figure 1. The equation of motion for each MSD component block can be written as follows:

\[
m_n \ddot{x}_n = -k_n(x_n - x_{n-1}) - b_n(\dot{x}_n - \dot{x}_{n-1}) + k_{n+1}(x_{n+1} - x_n) + b_{n+1}(\dot{x}_{n+1} - \dot{x}_n)
\]

where \(m_n\), \(k_n\), and \(b_n\) denote the mass, stiffness coefficient, and damping coefficient, respectively, of the \(n\)th component and \(k_{n+1}\) and \(b_{n+1}\) are the stiffness and damping coefficients of the subsequent mass.

### 2.3. Load component model

The load component is considered as the load force from the machining process (i.e., milling, turning, or drilling). This component is important to study the influence of the machining process on the ultrasonic transducer behaviour. The component has two inputs and two outputs as shown in Figure 1. The inputs are the ultrasonic resultant displacement and speed whereas the outputs are the resultant displacement and speed during the machining process.

Application of the load (i.e., machining process) causes nonlinearity and instability in the ultrasonic transducer behaviour [18]. Therefore, it is important to include the load model in the proposed transducer system to simulate and investigate its influence on the system behaviour. This investigation assists in the development of control systems.

The mathematical model of this component can be created based on Kelvin–Voigt model. This model describes one-dimensional contact interaction between the ultrasonic transducer and a workpiece. Equation (6) represents the load model:

\[
F_L = \frac{1}{2}(1 + \text{sgn}(x))(kx + b\dot{x})
\]

where \(x = x_2 - \Delta\), \(k\) is the contact stiffness, \(b\) is the contact damping, \(\Delta\) is the initial interference/gap, \(x_2\) is the resultant displacement of the ultrasonic transducer, and \(\text{sgn}(\cdot)\) is the standard signum function.

In order to validate the proposed system model, system parameters should be identified for each component as well as the resonant frequencies of the system in order to get maximum amplitude from the system. In this work, parameters identification is done theoretically using the simple beam model as mentioned in the previous section. The mass, damping and stiffness parameters are extracted and used to compute the natural frequencies of the reduced system. The computed natural frequencies are
compared with those computed from the full finite element model. Also, the frequency response of the reduced model is compared with the frequency response of the full finite element model.

3. Numerical simulation
The component-based model of the ultrasonic transducer is created using Matlab/Simulink. All systems parameters (namely the mass, stiffness, and damping coefficient of MSD1 and MSD2) are set based on the identification results obtained from the FEM. To investigate the properties of the proposed ultrasonic transducer model, the amplitude-frequency curve has been created as shown in Figure 2 by exciting the system with a sinusoidal signal at different frequencies and measuring the MSD2 displacement. It can be noticed that the system has three resonant frequencies at 6500Hz, 18000Hz, and 20500Hz which are correspond to $m_p$, $m_2$, and $m_3$ respectively. Although the maximum amplitude was attained at 6500Hz, high frequencies are utilized in the subsequent analyses since it was proved that the machining at 20kHz gives the best results [1].

![Figure 2. Frequency response of the ultrasonic transducer model](image)

Based on the frequency response of the system, a sinusoidal input voltage with a 20.5kHz is applied to the system. The displacement of the ultrasonic transducer tip (cutting tool) is shown in Figure 3. It is clear that the amplitude of the vibration is 30µm peak-peak at a frequency of 20kHz as required for the machining. However, it can be observed that the peaks of the displacement amplitude are not consistent. This is due to the hysteresis effect between the system input (applied voltage) and output (tip displacement) which are shown in Figure (4). Piezoelectric materials are hysteretic in nature, and therefore they may exhibit oscillatory closed-loop behavior, as well as poor tracking and potential instability [19].

![Figure 3. Displacement of the ultrasonic transducer tip](image)
In Figure 4, the response of system components (i.e., PZT, MSD 1, and MSD 2) are presented. It is clear how the hysteresis effect propagates throughout the system components and influences their response.

![Fig4](image1)

**Figure 4.** Hysteresis effect on the ultrasonic transducer's components: (A) MSD 2's displacement, (B) MSD 1's displacement, and (C) PZT's displacement

To investigate the influence of the nonlinearity and hysteresis of piezoelectric actuator, load contact stiffness was increased gradually on the transducer's tip (cutting tool) as depicted in Figure 5. One of the most important observations in the system's response is the attenuation of the peak-to-peak displacement (i.e., amplitude) of the vibration as shown in Figure 6. Moreover, the hysteresis effect on the overall system (i.e., ultrasonic transducer) is increased. This is clear from the high deficiency between the peaks of the output signal (i.e., MSD 2 displacement).

![Fig5](image2)

**Figure 5.** Load contact stiffness profile applied on the ultrasonic transducer
4. Concluding remarks
In this paper, a component-based model for piezoelectric ultrasonic transducers is introduced. The proposed technique is based on decomposing the entire system into individual components that can be used in different configuration and represented by mathematical expressions. The components are configured to be combined in series to formulate the ultrasonic transducer. To investigate the performance of such component-based modelling, the combined system was simulated in Matlab/Simulink. In the simulation, the system parameter values were identified using Finite Element technique. Various operation conditions were applied to test the performance of the system. From the performed numerical simulations, it was clear that the nonlinearity due to the inherent piezoelectric actuator's hysteresis has a big impact on the performance of the machining process and that it should be considered when control strategies are introduced.

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References
[1] Ahmed N, Mitrofanov A V, Babitsky V I and Silberschmidt V V 2007 Analysis of forces in
ultrasonically assisted turning \textit{J. Sound Vib.} \textbf{308}(3-5) 845-854

[2] Babitsky V I, Kalashnikov A N, and Molodtsoy F V 2004 Autoresonant control of ultrasonically assisted cutting. \textit{Mechatronics} \textbf{14} 91-114

[3] Babitsky V I., Kalashnikov A and Astashev V K 2004 Autoresonant control of nonlinear mode in ultrasonic transducer for machining applications. \textit{Ultrasonics} \textbf{42}(1-9) 29-35

[4] Babitsky V I, Mitrofanov A V and Silberschmidt VV 2004 Ultrasonically assisted turning of aviation materials: simulations and experimental study \textit{Ultrasonics} \textbf{42} 81-86

[5] Maurotto A, Muhammad R, Roy A and Silberschmidt V 2013 Enhanced ultrasonically assisted turning of a β-titanium alloy \textit{Ultrasonics} \textbf{53}(7) 1242-1250

[6] Croft D, Shad G, and Devasia S. 2001 Creep, hysteresis, and vibration compensation for piezoelectric actuators: Atomic force microscopy application \textit{Trans. ASME J. Dyn. Syst. Meas. Contr.} \textbf{123}(1) 35–43

[7] Mayergoyz I D 1991 \textit{Mathematical Models of Hysteresis} (New York: Springer)

[8] Natale C, Velardi F and Visone C 2001 Identification and compensation of Preisach hysteresis models for magnetostrictive actuators \textit{Physica B} \textbf{306}(1) 161–165

[9] Brokate M and Sprekels J 1996 \textit{Hysteresis and Phase Transitions} (New York: Springer)

[10] Visintin A 1994 \textit{Differential models of hysteresis} (New York: Springer)

[11] Krasnoselskii M A and Pokrovskii A V 1989 \textit{Systems with hysteresis}. (New York: Springer)

[12] Bashash S and Jalili N 2007 Robust multiple frequency trajectory tracking control of piezoelectrically driven micro/nanopositioning systems \textit{IEEE Trans. Contr. Syst. Technol.} \textbf{15} (5) 867–878

[13] Yeh T, Lu S and Wu T 2006 Modeling and identification of hysteresis in piezoelectric actuators \textit{ASME J. Dyn. Syst. Meas. Contr.} \textbf{128} 189–196

[14] Ru C, Chen L, Shao B, Rong W and Sun L 2009 A hysteresis compensation method of piezoelectric actuator: model, identification and control \textit{Contr Eng. Pract.} \textbf{17} 1107–1114

[15] Salah M H, McIntyre M L, Dawson D M, Wagner J R and Tatlicioglu E 2012 Charge feedback-based robust position tracking control for piezoelectric actuators \textit{IET Contr. Theo. Appl.}, \textbf{6}(5) 1–14

[16] Pour G 1998 Moving toward component-based software development approach \textit{Proc. Tech. Obj.-Orient. Lang.} 296-300

[17] Coleman B and Hodgdon M 1987 On a class of constitutive relations for ferromagnetic hysteresis \textit{Arch. Rat. Mech. and Analys.} \textbf{99}(4) 375-396

[18] Voronina S and Babitsky V 2008 Autoresonant control strategies of loaded ultrasonic transducer for machining applications \textit{J. Sound Vib.} \textbf{313}(3-5) 395-417

[19] Al-Janaideh M, Sumer D, Yan J, Amato A M D’, Drincic B, Aljanaideh K, and Bernstein D S 2012 Adaptive control of uncertain linear systems with uncertain hysteretic input nonlinearities \textit{ASME 2012 5th Annual Dynamic Systems and Control Conference joint with the JSME 2012 11th Motion and Vibration Conference October 17-19, 2012, Fort Lauderdale, Florida, USA.}