Time indicators of effective optimization algorithms in group load control modelling

A P Dimitriev¹ and R I Bazhenov²
¹Department of Computer Technologies, Chuvash State University, Cheboksary, Russian Federation
²Department of Information Systems and Mathematics, Sholom-Aleichem Priamursky State University, Birobidzhan, Russian Federation
E-mail: dimitrie1@yandex.ru

Abstract. The purpose of the work is to determine the optimal set of parameters of the genetic algorithm for optimization in the control model of the group load of electricity consumers taking into account time parameters. The analysis based on the approximation of empirical results is used as a research method. The results of two efficient algorithms used for optimization in this model are compared. Computational experiments were carried out and their results were processed, the location of efficiency of the parameters of the combined genetic algorithm was determined.

1. Introduction
In shops on smelting of ferrous and nonferrous metals of large foundries often there are several melting electric furnaces, which can be up to 12 installations. Power consumption in them is sometimes controlled by pulse-width modulation (PWM) power control. It consists in periodic switching on and off of these consumers of the electric power, thus the average power consumption of this or that consumer depends on time of stay in the switched-on state.

It is known that simultaneous activation of all consumers of the electric power causes an overload of an electrical transmission network owing to what penalties from system of power supply are exposed. In order to avoid such situations, it is necessary to rationally distribute in time the work of such consumers, that is, to make and optimize the schedule. Optimization of the schedule of switching on of consumers can serve for increase in the power factor in the network. This allows to reduce the permissible power of supply and reduce general electrical losses in the current-carrying sections of the load group. Thus, optimizing the PWM schedule is an urgent task.

When scheduling PWM should take into account the parameters of each power consumer: first, structurally constant parameter, which is expressed by the height of the pulses and physically means the current, and secondly, the width of the pulses. The period is the same for all consumers and can reach 1800 seconds. This allows for long-term calculations in scheduling. Consumers are turned on and off through controllers that can be connected to a microprocessor or microcontroller, in which is necessary to implement some algorithm for scheduling and optimization. Realization of digital PWM on microprocessors described in [1].

In [2] proposed a method for optimizing the schedules of the PWM. According to the condition, it is necessary to get the maximum power factor, which the range of values from 0 to 1. Introduced designations:
\( \gamma_i \) is the relative pulse duration of the \( i \)-th regulator, \( i = 1, \ldots, n \), where \( n \) is the number of regulators in the group; 

\( I_i \) is the load current of the \( i \)-th regulator.

The power factor is expressed by the equation:

\[
\zeta = \left( \frac{\sum_{i=1}^{n} I_i^2 \gamma_i}{\sum_{i=1}^{n} I_i^2 \gamma_i + 2 \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} I_i I_k \gamma_{ik}} \right)^{-1/2},
\]

where \( \gamma_{ik} \) is the relative duration of the interval of the joint action of the pulses of the \( i \)-th and \( k \)-th regulators. When using the above method, the power factor reaches the value of 0.9723 for the four consumers and the value of 0.9906 for the eight ones. All source values are integers, since in the 1990s, when the problem was set, the processing of integers was faster than real ones. Considered practical problem can be considered from an abstract point of view as a mathematical problem of discrete optimization, if the number \( n \) increases above the really necessary. When \( n = 40 \), the power factor for files [3] is from 0.9829 to 0.99941 using this method.

There are other algorithms of scheduling in this problem, sometimes leading to a greater \( \zeta \), for example, in [3], where the objective function is formulated somewhat differently, which does not change the essence of the problem. Here consumers have parameters: the share of power for the \( i \)-th consumer from the maximum value is set by the duration of the corresponding work, \( p_i \). Each consumer has its own constant importance, denoted by \( t_i \). The time period is divided into \( m \) equal intervals (discrete moments of time). A regulator may be activated at any such time and only once per period. The set of such moments of time \( \{\sigma_{i}, i = 1, \ldots, n\} \) forms a schedule stable for the vector of input data. It is necessary to minimize overloads according to the following objective function:

\[
C = \sum_{h=1}^{m} \sum_{i=1}^{n-1} t_{h,i} \cdot t_{h,k},
\]

where \( m \) is the number of discrete intervals in the period, \( n \) is the number of consumers, \( t_{h,i} \) is the electricity consumption of the \( i \)-th consumer at the \( h \)-th moment of the period, it is either \( t_i \) or zero, which depends on \( p_i \) and the moment of inclusion relative to the beginning of the period.

Equation (2) is included in the denominator of the (1) as an additive component, the other components (1) are constant, so minimization (2) also means minimization (1).

In [3] some other problems of placement for which the (2) is used are given, in particular, a problem of timetabling on high schools, and also a number of the applied algorithms is given. For the timetabling problem, the number \( n \) can often reach the above value of 40, so this value is not so abstract.

2. Used optimization algorithms

At such values of \( n \), it is possible to obtain placements with the values of the objective function \( C \) exceeding the optimal value by less than \( 8 \cdot 10^{-5} \) for 600—900 seconds on a personal computer only with the help of two algorithms from the ones given in [3]. This is a selection sequence optimization (SSO) algorithm based on the simulated annealing method [4], and a combined genetic algorithm (GA).

Consider the SSO algorithm. Algorithm parameters:

- \( Y \) is the initial number of iterations;
- \( T \) is the preset temperature;
- \( t \) is the current temperature;
- \( d \) is the increasing the number of iterations;
- \( R \) is the scatter.

Objects represent consumers. The algorithm is as follows.
1. Sort objects according to some criterion, for example, in ascending order of importance. The sequence of numbers of the sorted list is designated as the starting point \( S \) in the \( n \)-dimensional space.

   Place the objects in the sequence specified by \( S \) in a relatively optimal position (by placing them sequentially in the best positions at the time of location, performing an overview of each of the \( m \) positions for the newly located object).

   Calculate the objective function \( C(S) \) by the (2).

2.0. Number of iterations performed \( j := 0 \).

2.1. In the neighborhood of \( S \), defined by the scatter \( R \), select point, denoted \( S_1 \), by pseudo-random permutation from two to four elements in \( S \).

2.2. Place the objects in the sequence specified by \( S_1 \) in a relatively optimal position (same to the one in step 1).

2.3. Calculate \( C(S_1) \) according to the resulting placement. If \( C(S) > C(S_1) \), then \( S := S_1 \).

2.4. Otherwise, with probability \( t/T \), if \( |C(S) − C(S_1)| \) is less than some constant threshold, then \( S := S_1 \).

2.5. \( j := j + 1 \).

2.6. If \( j > Y \), then \{ \( t := t - 1; Y := Y + d \) \}. Otherwise go to 2.1.

2.7. If \( t > 0 \), go to 2.0.

3. End.

Using this algorithm it is often possible to get the best value for \( C \) than for any other algorithm. However, SSO is little known, unlike various GA [5, 6]. Therefore, the study mainly concerns only GA. The essence of the combined GA is set out below. This algorithm uses some specific parameters. The values of these parameters affect the efficiency of its application. It is necessary to determine the optimal set of GA parameters for optimization in the above model. To do this, requires to:

- compare the results of the GA and SSO;
- determine which parameters should be investigated;
- carrying out computational experiments and process their results.

The analysis based on the approximation of empirical results is used as a research method [7].

First, the approximation of the results of series of three computational experiments using trend lines in Microsoft Excel was performed. The dependence of optimization quality on GA and SSO operation time for f1 [8] and f5 [3] files is shown in the diagram (Figure 1).

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**Figure 1.** Time dependence of the quality of the optimization for different input data and algorithms.
In Figure 1, on the ordinate axis is the deviation of the value $C$ from the found minimum. The trend line for the lower chart is a linear filter, for the rest it is a power one.

A personal computer with a frequency of 3344 MHz was used for plotting the diagram. For each file, the work of the two above-mentioned algorithms was studied.

As follows from the diagram, usually with the help of SSO it is possible to get closer to the optimal results in less time compared to GA. However, when approaching optimal values, sometimes the opposite is true. At the beginning of the optimization process, as a rule, SSO works more efficiently than GA.

The combined GA largely repeats the Genitor [9], but has some features. The parameters of the combined GA include the size of the population and the level of mutations. In addition, the results are affected by the input data, so it is necessary to examine different sets of input data.

When using a combined GA, it should be noted that if "mutation" less than 50 is selected, it is possible to get values of $C$, which closer to the optimal one than if this value is more than 50. Unlike Genitor, this algorithm uses a different number of genes from two parents when creating a new chromosome.

Denote by $V_1$ the proportion of genes from the total number of genes that are copied to a new chromosome from the first ancestor, by $V_2$—respectively from the second ancestor, by $V_3$ the proportion of mutating genes, by $M$ the value of the parameter "mutation", and by $P$ the size of the population (the number of chromosomes).

The algorithm uses a random number generator to distribute genes in $V_1$, $V_2$, $V_3$. According to the algorithm, the average $V_3 \approx M$. If $M < 50$, then $V_1 \approx 1/2$, $V_2 \approx 1 - M$. Otherwise $V_1 \approx 1 - M$, $V_2 = 0$. Thus, $V_1 \approx (1.5 - M - (0.5 - M)) / 2$; $V_2 \approx (0.5 - M + (0.5 - M)) / 2$.

For example, if $M = 30$, then about 50% of genes are used from one individual, about 20% from the second, and about 30% of genes mutate.

Next, a series of five computational experiments for the same set of parameters were carried out to reduce random errors.

3. The analysis parameters of genetic algorithm with 40 consumers

On the computer with a frequency of 1500 MHz was investigated the influence of $P$, to be modified in a wide range, in order to select a more narrow field and the subsequent more precise analysis. This computer is multi-core, so in order to speed up the work in some cases, two equal processes with different parameters were launched simultaneously. The value $M$ was used as a variable within the experimental data acquisition for each size of population. Studied the influence of the following values of $P$ on the optimization result: 3, 10, 30, 100. In Excel, point diagrams of the dependence of the minimum obtained $C$ values from time in the range from 1 to 1000 seconds are plotted in logarithmic scale on both coordinate axes. For each $P$, trend lines of the power type were displayed. Other types of trend lines significantly distorted the result of the approximation.

From the relative position of the trend lines revealed that more promising for further research area closer to the value of $P = 30$. The worst value listed is 100. This is the highest value of the studied ones, therefore, a further increase of $P$ is not effective.

Further experiments were carried out on a computer with a frequency of 3344 MHz. Similarly, the influence of $P$, which is changed in a narrower range, was investigated at a fixed $M$. The following values $P$: 15, 20, 30, 40 were studied. It is determined that $P = 20$ is more optimal, and the worst of these is 40. Thus, neither decreases below 15 nor increases above 40 are effective.

Similarly, the influence of $M$ from the values of 20, 30, 40 at a fixed $P$ was studied. It was found that $M = 20$ is more optimal, and the worst $M$ is 40. However, the number $M = 20$ in some cases leads to far from optimal results. Thus, increasing $M$ to 40 and above is not effective.

As a continuation of the previous studies, the study of the rectangle area in coordinates $(M, P)$ from $(20, 20)$ to $(30, 30)$ with step 5 was carried out. It is revealed that $M = 30$ and $P = 20$ are more optimal, but there is a significant error when using trend lines.
The use of trend line approximation in Excel results in either straight lines or excessively broken lines, which makes it difficult to compare results and can lead to incorrect conclusions. For example, it is possible to conclude that $P = 10$ is better, while in reality 30 is better, because it leads to more optimal values in a reasonable time.

When using the "Data Analysis" item in Excel, an attempt is made to express the contents of some cells through formulas, but these formulas are not suitable in this case. The use of the trend function for interpolation is also not suitable, since in this case a linear trend line is needed [10].

In this regard, developed a Windows Forms application (the name of the program Form3), creating the set values of coordinate points on the basis of the other set. To do this, the program interpolates the data as follows. Creates a new set of points with given abscissa. To obtain the ordinate of a point $(x, y)$, two starting points $(x_1, y_1)$ and $(x_2, y_2)$, abscissa of which are located directly to the left and to the right of the abscissa of the point $(x, y)$, respectively, are used. The required ordinate is determined by the equation:

$$y = y_1 + (y_2 - y_1) \cdot (x - x_1)/(x_2 - x_1).$$

For the leftmost point, a large number is used as $y_1$. For the rightmost point, the $y_2$ is the ordinate of the previous origin. This is possible due to the shape of the graph in the form of a damped exponent, since at the beginning of minimization there are often points with smaller values of $C$ than the previous and closer to the end rarely.

The results of the diagrams based on the data obtained in this way are shown in Figure 2.

![Figure 2](image)

**Figure 2.** Time depending constructed via interpolation.

As can see, it is more convenient to compare charts here than in Figure 1. From Figure 2 it follows that values $P = 20$ and $M = 20$ are more optimal.

4. **The analysis parameters of genetic algorithm with 28 consumers**

It is possible that for the source data files containing information about objects in a different quantity, the found optimal values of $P$ and $M$ may be different. Therefore, a file named in28-100-t10-25-p8-50.txt was created. This file contains information about 28 objects instead of 40 objects, as in previous files, with the number of discrete intervals still being used equal to 100.

The data in this file, together with the values of activation moments denoted by $o_i$ at some near-optimal schedule, are presented in table 1.
Table 1. Data in the file with information about 28 objects.

| i  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| \( t_i \) | 30  | 34  | 27  | 30  | 23  | 34  | 24  | 30  | 22  | 29  | 18  | 11  | 16  | 22  | 31  | 14  | 24  | 12  | 26  | 10  | 15  | 23  | 13  | 23  | 26  | 13  | 16  | 14  |
| \( p_i \) | 58  | 44  | 47  | 41  | 46  | 29  | 40  | 27  | 33  | 21  | 33  | 50  | 32  | 21  | 33  | 22  | 13  | 33  | 12  | 9  | 14  | 8  | 8  | 8  | 14  | 78  | 100  |
| \( o_i \) | 48  | 53  | 97  | 7  | 32  | 83  | 43  | 12  | 99  | 78  | 82  | 28  | 50  | 22  | 39  | 100  | 6  | 78  | 37  | 50  | 78  | 53  | 15  | 66  | 44  | 14  | 29  | 99  |

Similar to the study of the previous files, a series of computational experiments on optimization using GA at constant \( M, P \) with steps of 5 units were carried out. Here, value \( P = 2 \) was used from the left edge of the study area, not \( P = 0 \), since there must be at least two chromosomes in the population for crossing.

For each point, a series of 10 experiments was performed. All the experiments lasted from 600 to 900 seconds on a desktop PC or twice as much on a laptop, because the clock frequency on it is half as low. All of these computers have four CPU, so it was often runs four equivalent process with the same initial data. Since the absolute time under such conditions is unpredictable, the achievement of a given number of crossovers is used as a stop criterion. Since time is proportional to the number \( M \), then at \( M = 20 \) it is equal to 150,000, at \( M = 30 \) it is equal to 100,000, etc. A GA settings file and user interface elements were created to load these settings from a file and save them to a file.

In the study of the file with \( n = 28 \) to reduce the time, the graph was not built, since the construction of one curve of the graph takes 300 seconds of PC user work. Instead, only the \( C \) value reached in the specified time was used. Denote by \( D \) the deviation of the value \( C \) from the found minimum, rounded to 1247,000. The first point of study in connection with the results of studies for files with 40 objects is \( P = 20, M = 20 \). In it, \( D \) averaged is equal to 378.6 units with a standard deviation \( S_0 = 176.4 \) units.

In its neighborhood with a step of \( \pm 5 \) units found point \( P = 15, M = 20 \), where the deviation from 1247011 minimum for four points (excluding the original) and is equal to 362.4 when \( S_0 = 166.6 \), which is 111.6 less than the next ascending value. This point is taken as the current one, and the study continued in its vicinity with a step of \( \pm 5 \) units. The results of the study of the parameter values are shown in Figure 3.

The criterion of transition to the next point in order to study its neighborhood is the condition that \( D \) in it was not less than 400 units. The number 400 is chosen because several points where \( D < 300 \) were found and 400–300 produce the stock of the same order as \( S_0 \) is left.

In the process of constructing points, the number 1247011 found in about 7,000 seconds with the help of SSO was first used as a minimum. Then, for about 360,000 seconds of single-processor time
using GA were found two numbers smaller than this – 1247007 and 1246962. Placement, presented in table 1, obtained using the GA. The power factor for this location is 0.999945, the value $C = 1246962$

It is expected the nature of the diagrams for the investigated file, the same to nature of file f1, i.e., with increasing $P$ or $M$ any improvement effect is not achieved. Thus, the criterion for the termination of the study is the condition: the study area should be closed by the boundary at which all points should have $D$ at least 400 units.

66 points were investigated in accordance with the termination criterion. Here, $S_0$ in the whole population is in the range from 79.5 to 279.2. The average value for $S_0$ is 164.4. The standard deviation for $S_0$ is 40.2. There were no regularities in the distribution of $S_0$ in the study area.

For three points: (2, 35), (10, 20) and (30, 25), in which the minimum values of $D$—259.5, 302.4 and 227, respectively, were found, the series of experiments were repeated. According to the results of 20 experiments at these points $S_0$ is approximately the same as the average value of $S_0$ for the whole set of points and is 150.1, 176 and 130, respectively. The averaged values of $D$ for 20 experiments each time became more, namely 288, 334.3 and 291.3. However, the first and third of these points still have two minimum values of $D$ from the whole population.

Two more points with coordinates (100, 20) and (300, 20) were constructed. For the first point $C = 1247384$ that is 422 greater than the minimum value, and for the second one about 1600. This suggests that the increase in $P$ more than 55 is not effective. This is due to the fact that the larger the population, the slower it is possible to find a chromosome, which leading to more effective values.

Thus, if there are 28 objects, the optimal values are in the region $P = 2\ldots30, \, M = 20\ldots35$.

5. Summary
For the fl file used in a number of scientific publications authors, 20 chromosomes in the population and the value of the parameter "mutation" 20 are more optimal when using a combined GA.

For source data describing other number of consumers or with other qualities, these values may be different, but calculations show that the differences between these values are within 20 units. That is, neither $P$ nor $M$ should be very large. The SSO algorithm achieves near-optimal values faster than the combined GA.

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