Peccei-Quinn Symmetry and Nucleon Decay in Renormalizable SUSY SO(10)

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K.S. Babu, B. Bajc, S. Saad, JHEP 1810 (2018) 135
K.S. Babu, T. Fukuyama, S. Khan, S. Saad, arXiv:1812.11695 [hep-ph]
Outline of the talk

- Shortcomings of the Standard Model
- Why $SO(10)$ GUT?
- Minimal SUSY $SO(10)$ Model: pros and cons
- New Minimal SUSY $SO(10)$ Model
- SUSY $SO(10)$ Model with Peccei-Quinn Symmetry
- Summary
Shortcomings of the Standard Model

- Yukawa couplings are arbitrary parameters, no correlation.

- No understanding of the observed hierarchies in the charged fermion masses and mixings.

- Strong CP problem: why $\theta$-term is so small?

- Neutrinos are massless.

- Why charge is quantization?

- Many scattered fermion multiplets. Unsightly?

- Observed Baryon asymmetry can not be incorporated.

- No dark matter candidate ...
Why $SO(10)$ GUT?

- Gauge coupling unification can be realized.
- Electric charge quantization is understood.
- Unify all fermions into a single irreducible 16 dimensional multiplet.
- Predicts the existence of right-handed neutrinos.
- Seesaw mechanism is a natural candidate to explain neutrino oscillation data.
- Baryon asymmetry can be explained, as for example via Leptogenesis mechanism.
- Dark matter stability is automatic.
Unification of Matter

- 16 members of a family fit into a spinor of $SO(10)$ GUT

Pati, Salam (1974) – Quark-lepton unification
Georgi, Glashow (1974) – $SU(5)$ unification
Georgi (1975); Fritzsch, Minkowski (1975) – $SO(10)$ unification
Georgi, Quinn, Weinberg (1974) – Gauge coupling unification

Cartan-Weyl weights (Table: Stuart Raby)

|       | $SU(1)_Y$ | $SU(3)_C$ | $SU(2)_L$ |
|-------|-----------|-----------|-----------|
| $\nu^c$ | 0         | ++ +      | ++        |
| $e^c$   | 1         | ++ +      | --        |
| $u_{\text{red}}$ | $\frac{1}{6}$ | + -- +    | + --      |
| $d_{\text{red}}$ |           | -- + +    | -- +      |
| $u_{\text{green}}$ | $\frac{2}{3}$ | - + -    | ++ +      |
| $d_{\text{green}}$ |           | + -- +    | -- +      |
| $u_{\text{blue}}$ |           | + + --    | + --      |
| $d_{\text{blue}}$ |           | + + --    | -- +      |
| $\nu^c$ | $\frac{3}{2}$ | - ++     | ++ --     |
| $e^c$   | $\frac{1}{2}$ | + - +    | -- +      |
Unification of Gauge Couplings with SUSY
Minimal SUSY $SO(10)$ model

- $10_H + 126_H + \overline{126}_H + 210_H$.
  
  C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, 04

- 26 real parameters
  
  $= 15$ (Yukawa) + $10$ (symmetry breaking sector) + $1$ (gauge coupling)

- minimal number of parameters among all SUSY GUTs
**Minimal SUSY $SO(10)$ model**

- $10_H + 126_H + \overline{126}_H + 210_H$.
- Neutrino and charged fermion masses and mixings are related.
- Natural generation of neutrino masses and mixings through type I and type II seesaw.
- Good fit to fermion masses and mixings with only two symmetric matrices.
- Automatic and exact low energy R-parity conservation leading to a compelling dark matter candidate.
- Connection of the $b-\tau$ unification and large atmospheric mixing angle in type II seesaw.
Constraints from the Higgs sector ruled out such an attractive model, the minimal SUSY $SO(10)$. 

B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, 05;

S. Bertolini, T. Schwetz and M. Malinsky, 06
Minimal SUSY $SO(10)$: Solutions

A) $10_H + 126_H + \overline{126}_H + 210_H + 120_H$.

$$\mathcal{L}_{yuk} = 16_F^T (Y_{10} 10_H + Y_{126} \overline{126}_H + Y_{120} 120_H) 16_F$$

B. Dutta, Y. Mimura and R. N. Mohapatra, 04;
C. S. Aulakh, I. Garg and C. K. Khosa, 13;
R. N. Mohapatra and M. Severson, 18

$\Rightarrow 6 + 17 + 26$ real parameters

B) $10_H + 126_H + \overline{126}_H + 210_H + 54_H$.

New Minimal SUSY $SO(10)$? (Yukawa sector unaltered)

Babu, Bajc, Saad, JHEP 1810 (2018) 135

$\Rightarrow 0 + 11 + 26$ real parameters
**Yukawa Sector of SUSY $SO(10)$**

\[ 16 \times 16 = 10_s + 120_a + 126_s \]

\[
W_{Yukawa}^{SO(10)} = 16_F^T(Y_{10}^{10H} + Y_{126}^{126H})16_F.
\]

K. S. Babu, R. Mohapatra, 92

\[
M_U = v_u^{10} Y_{10} + v_u^{126} Y_{126} \\
M_D = v_d^{10} Y_{10} + v_d^{126} Y_{126} \\
M_E = v_d^{10} Y_{10} - 3v_d^{126} Y_{126} \\
M_{\nu_D} = v_u^{10} Y_{10} - 3v_u^{126} Y_{126} \\
M_R = Y_{126} V_R
\]

\[
Y_{10} = \begin{pmatrix} a_1 & 0 & 0 \\ a_2 & 0 \\ a_3 \end{pmatrix} \\
Y_{126} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{22} & b_{23} & b_{33} \end{pmatrix}
\]
12 parameters plus 7 phases to fit 18 observed quantities
This setup fits all observables quite well
Large neutrino mixings coexist with small quark mixings
$\theta_{13}$ prediction turned out to be correct

Babu, Mohapatra (1993); Bajc, Senjanovic, Vissani (2001); (2003); Fukuyama, Okada (2002); Goh, Mohapatra, Ng (2003); Bajc, Melfo, Senjanovic, Vissani (2004); Bertolini, Malinsky, Schwetz (2006); Babu, Macesanu (2005); Dutta, Mimura, Mohapatra (2007); Aulakh et al (2004); Bajc, Dorsner, Nemevsek (2009); Joshipura, Patel (2011); Dueck, Rodejohann (2013); Babu, Bajc, Saad (2018)
Strongly Hierarchical Charged Fermion Masses

- **up-type quarks**
  - $m_u \sim 6.5 \times 10^{-6}$
  - $m_c \sim 3.3 \times 10^{-3}$
  - $m_t \sim 1$

- **down-type quarks**
  - $m_d \sim 1.5 \times 10^{-5}$
  - $m_s \sim 3 \times 10^{-4}$
  - $m_b \sim 1.5 \times 10^{-2}$

- **charged leptons**
  - $m_e \sim 3 \times 10^{-6}$
  - $m_\mu \sim 6 \times 10^{-4}$
  - $m_\tau \sim 1 \times 10^{-2}$

(in the unit of $m_t$)
Mixing matrices and Neutrino mass differences

\[ V_{\text{CKM}} \sim \begin{bmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{bmatrix} \]

\[ U_{\text{PMNS}} \sim \begin{bmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{bmatrix} \]

Quark mixing angles are small and Leptonic mixing angles are large

- **neutrinos** (assuming normal hierarchy)

\[ \Delta m^2_{\text{sol}} \sim 7.5 \times 10^{-5} \text{eV}^2 ; \ m_2 \sim 8.5 \times 10^{-12} \text{GeV} \]

\[ \Delta m^2_{\text{atm}} \sim 2.5 \times 10^{-3} \text{eV}^2 ; \ m_3 \sim 5 \times 10^{-11} \text{GeV} \]

Neutrino mass spectrum shows mild hierarchy
# Best Fit Values

| Masses (in GeV) and Mixing parameters | Inputs (at $\mu = M_{\text{GUT}}$) | Fitted values (at $\mu = M_{\text{GUT}}$) | pulls |
|--------------------------------------|-------------------------------------|----------------------------------------|-------|
| $m_u/10^{-3}$                        | $0.450 \pm 0.139$                  | $0.454$                                | 0.028 |
| $m_c$                                | $0.248 \pm 0.007$                  | $0.245$                                | 0.175 |
| $m_t$                                | $84.53 \pm 0.84$                  | $84.49$                                | -0.057|
| $m_d/10^{-3}$                        | $0.951 \pm 0.19$                  | $0.585$                                | -1.92 |
| $m_s/10^{-3}$                        | $18.07 \pm 0.97$                  | $18.46$                                | 0.409 |
| $m_b$                                | $0.961 \pm 0.009$                  | $0.961$                                | 0.048 |
| $m_e/10^{-3}$                        | $0.379457$                        | $0.379468$                            | 0.002 |
| $m_\mu/10^{-3}$                      | $80.1068$                         | $80.0416$                            | -0.081|
| $m_\tau$                            | $1.36781$                         | $1.36799$                            | 0.012 |
| $|V_{us}|/10^{-2}$                    | $22.54 \pm 0.06$                  | $22.54$                                | 0.057 |
| $|V_{cb}|/10^{-2}$                    | $4.14 \pm 0.06$                   | $4.14$                                | 0.013 |
| $|V_{ub}|/10^{-2}$                    | $0.358 \pm 0.012$                 | $0.358$                                | 0.020 |
| $\delta_{\text{CKM}}$               | $1.208 \pm 0.054$                 | $1.222$                                | 0.265 |
| $\Delta m^2_{\text{sol}}/10^{-5}(\text{eV}^2)$ | $8.679 \pm 0.218$             | $8.683$                                | 0.019 |
| $\Delta m^2_{\text{atm}}/10^{-3}(\text{eV}^2)$ | $2.929 \pm 0.046$             | $2.929$                                | -0.011|
| $\sin^2 \theta_{12}^{\text{PMNS}}$ | $0.3219 \pm 0.017$                | $0.3204$                                | -0.029|
| $\sin^2 \theta_{23}^{\text{PMNS}}$ | $0.431 \pm 0.019$                 | $0.4281$                                | -0.0148|
| $\sin^2 \theta_{13}^{\text{PMNS}}$ | $0.0216 \pm 0.00082$           | $0.02148$                              | -0.145|

Total $\chi^2 = 4$  

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# Best Fit Predictions

| Quantity | Predicted Value |
|----------|-----------------|
| $\{m_1, m_2, m_3\}$ (in eV) | $\{3.32 \times 10^{-3}, 9.89 \times 10^{-3}, 5.42 \times 10^{-2}\}$ |
| $\{\delta_{PMNS}, \alpha_{21}^{PMNS}, \alpha_{31}^{PMNS}\}$ | $\{17.0^\circ, 344.13^\circ, 337.45^\circ\}$ |
| $\{m_{\cos}, m_{\beta}, m_{\beta\beta}\}$ (in eV) | $\{6.74 \times 10^{-2}, 6.47 \times 10^{-3}, 6.11 \times 10^{-3}\}$ |
| $\{M_1, M_2, M_3\}$ (in GeV) | $\{1.29 \times 10^{10}, 6.25 \times 10^{11}, 4.13 \times 10^{12}\}$ |

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$d = 5$ Proton Decay

- $\tau(p \rightarrow \bar{\nu}K^+) > 5.9 \times 10^{33} \text{ yrs.}$

(4 such diagrams)

- $m_S \geq 100 \text{ TeV}$

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GUTs solve almost all problems!

Strong CP problem?
**Strong CP problem**

- QCD allows a **CP violating** flavor singlet interaction

\[ \mathcal{L} \supset -\frac{\theta_{QCD}}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + \bar{q}Mq \]

- QCD interactions appear to conserve CP symmetry

\[ \bar{\theta} = \theta_{QCD} + \text{ArgDet}(M) \]

- \(\bar{\theta}\) contributes to **neutron EDM**

\[ d_n \sim 10^{-16} \text{ e-cm} \Rightarrow \bar{\theta} < 10^{-10} \]

- Why is a dimensionless parameters of theory so small?
An elegant solution is the Peccei-Quinn mechanism

\( \bar{\theta} \) is promoted to a dynamical field

Assumes a global \( U(1) \) symmetry that has a QCD anomaly. This \( U(1) \) is broken spontaneously by a scalar field at a scale \( f_a \sim 10^{12} \text{ GeV} \) leading to massless axion

Minimization with respect to \( a \) sets \( \bar{\theta} = 0 \)
**SUSY** $SO(10) \times U(1)_{PQ}$

- $10_H + 126_H + \overline{126}_H + 210_H + 54_H + 10'_H$.

Babu, Bajc, Saad arXiv: 1812.11695 [hep-ph]
## SUSY $SO(10) \times U(1)_{PQ}$

| Fields | $16_{Fi}$ | $210_H$ | $54_H$ | $126_H$ | $126_H$ | $10_H$ | $10'_H$ | $S_1$ | $S_2$ | $S_3$ |
|--------|-----------|---------|--------|---------|---------|--------|---------|------|------|------|
| $U(1)_{PQ}$ | -1 | 0 | 0 | -2 | +2 | +2 | -2 | -8 | +8 | +4 |

- **Singlet sector breaks PQ symmetry in SUSY limit:**

  \[
  W_S = M_S S_1 S_2 + \kappa S_1 S_3^2
  \]

  In SUSY limit, $S_3$ is undetermined, but it gets fixed once SUSY breaking is included.

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Babu, Bajc, Saad arXiv: 1812.11695 [hep-ph]
Symmetry Breaking Chain

\[ SO(10) \times U(1)_{\text{PQ}} \]

\[
\langle 54_H \rangle, \langle 210_H \rangle \quad \rightarrow \quad SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \times U(1)_{\text{PQ}}
\]

\[
\langle 126_H \rangle, \langle 126_H \rangle, \langle S_i \rangle \quad \rightarrow \quad SU(3)_c \times SU(2)_L \times U(1)_Y
\]

\[
\langle 10_H \rangle, \langle 10'_H \rangle \quad \rightarrow \quad SU(3)_c \times U(1)_{\text{em}}
\]

Babu, Bajc, Saad arXiv: 1812.11695 [hep-ph]
Superpotential

Symmetry breaking sector:

\[
W^{PQ}_{SO(10)} = \frac{1}{2} m_{1} 120^{2} H + m_{2} 126_{H} 126_{H} + m_{3}' 10_{H} 10'_{H} + \frac{1}{2} m_{5} 54^{2}_{H} \\
+ \lambda_{1} 120^{3}_{H} + \lambda_{2} 210_{H} 126_{H} 126_{H} + \lambda_{3} 126_{H} 10_{H} 210_{H} \\
+ \lambda_{4}' 126_{H} 10'_{H} 210_{H} + \lambda_{8} 54^{3}_{H} + \lambda_{10} 54_{H} 210^{2}_{H} + \lambda_{13}' 54_{H} 10_{H} 10'_{H} \\
+ \frac{1}{2} \lambda_{5}' 10'_{H}^{2} S_{3}
\]

Minimal Yukawa sector preserved under PQ:

\[
W^{Yuk PQ}_{SO(10)} = 16^{T} (Y_{10} 10_{H} + Y_{126} 126_{H}) 16
\]
$SU(2)_L$ doublet Higgs mass matrix

$$
\begin{pmatrix}
H_d & \bar{\Delta}_d & \Delta_d & \Phi_d & H'_d
\end{pmatrix}
\mathcal{M}_D
\begin{pmatrix}
H_u \\
\Delta_u \\
\bar{\Delta}_u \\
\Phi_u \\
H'_u
\end{pmatrix}
$$

$$
\mathcal{M}_D =
\begin{pmatrix}
0 & \frac{\lambda_3 \Phi_2}{\sqrt{10}} - \frac{\lambda_3 \Phi_3}{2\sqrt{5}} & 0 & 0 & m_3' + \sqrt{3/5} \lambda_1 E \\
0 & \frac{\lambda_2 \Phi_2}{\sqrt{10}} - \frac{\lambda_2 \Phi_3}{2\sqrt{5}} & m_2 + \frac{\lambda_2 \Phi_2}{15\sqrt{2}} + \frac{\lambda_2 \Phi_3}{30} & 0 & \frac{\lambda_2 \bar{\nu}_R}{10} \lambda_1 E \\
\frac{-\lambda_3 \Phi_2}{\sqrt{10} - \frac{\lambda_3 \nu_R}{\sqrt{5}}} & 0 & m_2 + \frac{\lambda_2 \Phi_2}{15\sqrt{2}} + \frac{\lambda_2 \Phi_3}{30} & \frac{\lambda_2 \bar{\nu}_R}{10} \lambda_1 E & 0 \\
m_3' + \sqrt{3/5} \lambda_1 E & 0 & \frac{-\lambda_3 \Phi_2}{\sqrt{10} - \frac{\lambda_3 \nu_R}{\sqrt{5}}} & \frac{\lambda_4 \Phi_2}{\sqrt{10}} - \frac{\lambda_4 \Phi_3}{2\sqrt{5}} & \lambda_5 S_3
\end{pmatrix}
$$

$\langle S_3 \rangle \ll M_{GUT}$:

$$
W_D^{mass} = (H'_d \quad \Delta_d \quad \Phi_d) \mathcal{M}_{D1} (H_u \quad \bar{\Delta}_u \quad \Phi_u)^T + (H_d \quad \bar{\Delta}_d) \mathcal{M}_{DII} (H'_u \quad \Delta_u)^T + \lambda'_5 H'_d H'_u \langle S_3 \rangle.
$$

An extra pair of doublet has mass of order PQ scale $\sim 10^{12}$ GeV

Babu, Bajc, Saad arXiv: 1812.11695 [hep-ph]
**SU(3)_C triplet Higgs mass matrix**

\[
\begin{pmatrix}
H_C & \Delta_C & \Delta_C' & \Phi_C & H'_C
\end{pmatrix}\begin{pmatrix}
M_T
\end{pmatrix}
\]

\[
M_T = \begin{pmatrix}
0 & \frac{\lambda_3 \Phi_2}{\sqrt{30}} - \frac{\lambda_1 \Phi_1}{\sqrt{10}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\frac{-\lambda_1 \Phi_1}{\sqrt{10}} - \frac{\lambda_1 \Phi_2}{\sqrt{30}} & m_2 & 0 & \frac{\lambda_2 \Phi_3}{15 \sqrt{2}} + \frac{\lambda_3 \Phi_2}{30 \sqrt{2}} & m_1 + \frac{-\lambda_2 \Phi_R}{10 \sqrt{3}} + \frac{\lambda_2 \Phi_R}{5 \sqrt{6}} \\
-\sqrt{\frac{2}{15}} \lambda_3 \Phi_3 & \frac{-\lambda_2 \Phi_R}{10 \sqrt{3}} - \frac{\lambda_2 \Phi_R}{5 \sqrt{6}} & m_2 & 0 & \frac{\lambda_1 \Phi_1}{\sqrt{6}} + \frac{\lambda_2 \Phi_2}{3 \sqrt{2}} + \frac{2}{3} \lambda_1 \Phi_3 + \frac{1}{2 \sqrt{15}} \lambda_10 E \\
m' - \frac{2}{\sqrt{15}} \lambda_13 E & \frac{-\lambda_1 \Phi_1}{\sqrt{10}} - \frac{\lambda_4 \Phi_2}{\sqrt{30}} & -\sqrt{\frac{2}{15}} \lambda_4 \Phi_3 & \frac{\lambda_4 \Phi_R}{\sqrt{6}} & \lambda'_5 \langle S_3 \rangle
\end{pmatrix}
\]

- Contains the same parameters as the doublet matrix
- Must be heavy \( \sim M_{GUT} \)

Babu, Bajc, Saad arXiv: 1812.11695 [hep-ph]
Suppressing $d = 5$ Proton Decay

PQ-symmetry: $10^2_H$ direct mass terms are not allowed
Suppression mechanism

\[ W_5 \sim \frac{\lambda'_5 \langle S_3 \rangle}{M_T} \left( \frac{QQQL}{M_T} + \frac{u^c e^c u^c d^c}{M_T} \right). \]
Suppression via PQ-scale, $M_{PQ} \sim 10^{12}$ GeV

- Suppression factor: $(M_{PQ}/M_{GUT})^2 \sim 10^{-8}$

- $SO(10)$ without PQ: $\tau_p \sim 10^{25} - 10^{26}$ yrs. ($m_S \sim$ TeV)

- $SO(10)$ with PQ: $\tau_p \sim 10^{33} - 10^{34}$ yrs. ($m_S \sim$ TeV)

Babu, Bajc, Saad arXiv: 1812.11695 [hep-ph]
Allowed SUSY Scalar Masses

Minimum SUSY scalar mass allowed by proton decay (with Wino mass fixed at 1 TeV)

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MSSM gauge coupling unification does not hold

\[ \phi = (6,1,1/3) + \text{c.c.} \]

\[ M_{\Phi} = 2.76 \times 10^{15} \text{ GeV} \]

\[ M_{\text{GUT}} = 1.65 \times 10^{16} \text{ GeV} \]

\[ \alpha^{-1}(\text{GUT}) = 24.3 \]

\[ \alpha_1^{-1} \]

\[ \alpha_2^{-1} \]

\[ \alpha_3^{-1} \]

\[ \mu \text{ GeV} \]

Babu, Bajc, Saad arXiv: 1812.11695 [hep-ph]
Gauge boson mediated $d = 6$ proton decay

| Multiplet, $\phi$ | Running coefficient $(b_1, b_2, b_3)$ | $M_\phi$ | $M_{\text{GUT}}$ | $\alpha_{\text{GUT}}^{-1}$ | $\tau_p(p \to e^+\pi^0)$ in yrs |
|-------------------|---------------------------------------|----------|-----------------|--------------------------|---------------------------------|
| $(6, 1, \frac{1}{3}) + c.c.$ | $(\frac{4}{5}, 0, 5)$ | $2.76 \times 10^{15}$ GeV | $1.65 \times 10^{16}$ | 24.3 | $7.39 \times 10^{35}$ |
| $(6, 1, -\frac{2}{3}) + c.c.$ | $(\frac{16}{5}, 0, 5)$ | $2.76 \times 10^{15}$ GeV | $9.92 \times 10^{15}$ | 24.46 | $9.78 \times 10^{34}$ |
| $(6, 1, \frac{4}{3}) + c.c.$ | $(\frac{64}{5}, 0, 5)$ | $2.75 \times 10^{15}$ GeV | $5.0 \times 10^{15}$ | 24.68 | $6.42 \times 10^{33}$ |

- **Experimental bound:** $\tau_p > 1.6 \times 10^{34}$ yrs for $p \to e^+\pi^0$

| p decay modes | Branching ratio |
|---------------|-----------------|
| $p \to \bar{\nu}\pi^+$ | 52.5% |
| $p \to e^+\pi^0$ | 40.9% |
| $p \to \mu^+K^0$ | 4.42% |
| $p \to \mu^+\pi^0$ | 1.14% |
| $p \to \bar{\nu}K^+$ | 0.69% |
| $p \to e^+K^0$ | 0.23% |
| $p \to e^+\eta$ | 0.04% |
| $p \to \mu^+\eta$ | 0.001% |
Lepton Flavor Violation

- RGE running from GUT scale to $\nu_R$ scale induces LFV
- Dirac and Majorana masses of neutrinos are all determined
- $\mu \to e\gamma$ is the most prominent process

Limits the sfermion masses

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Conclusions

- **PQ symmetry successfully implemented**
- **PQ symmetry** allows $\sim$ TeV scale SUSY scalar masses
- **Minimal Yukawa** sector of $SO(10)$ is rather **predictive and works quite well**
- Both $d = 5$ and $d = 6$ proton decay rates near current limits
- Proton decay branching ratios may **test** such high scale theories
- $\mu \rightarrow e\gamma$ is close to current experimental limit

THANK YOU!