New Strategies to Extract $\beta$ and $\gamma$ from

$$B_d \to \pi^+\pi^- \text{ and } B_s \to K^+K^-$$

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Abstract

The decays $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ are related to each other by interchanging all down and strange quarks, i.e. through the $U$-spin flavour symmetry of strong interactions. A completely general parametrization of the CP-violating observables of these modes is presented within the framework of the Standard Model, allowing the determination of the angles $\beta$ and $\gamma$ of the unitarity triangle. This strategy is affected neither by penguin contributions nor by any final-state-interaction effects, and its theoretical accuracy is only limited by $U$-spin-breaking corrections. If the $B_d^0$–$\bar{B}_d^0$ mixing phase $2\beta$ is determined separately, for example with the help of $B_d \to J/\psi K_S$, $\gamma$ can be extracted with a reduced $U$-spin flavour symmetry input. A variant of this strategy to determine $\gamma$, which uses $B_d \to \pi^\pm K^\mp$ instead of $B_s \to K^+K^-$ and relies – in addition to the $SU(3)$ flavour symmetry – on a certain dynamical assumption, is also briefly discussed.
1 Introduction

The exploration of CP violation in the $B$-meson system and the determination of the three angles $\alpha$, $\beta$ and $\gamma$ of the usual non-squashed unitarity triangle \cite{1} of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) \cite{2} is one of the central goals of future $B$-physics experiments. However, only the determination of $\beta$ with the help of the “gold-plated” mode $B_d \to J/\psi K_S$ is “straightforward” \cite{3}, whereas the determination of $\alpha$ and $\gamma$ is considerably more involved \cite{4}.

In the literature, the decay $B_d \to \pi^+\pi^-$ usually appears as a tool to determine $\alpha = 180^\circ - \beta - \gamma$. Unfortunately, penguin contributions are expected to affect this determination severely \cite{5}. Although there are several strategies on the market to take care of these penguin uncertainties \cite{4, 5}, they are usually very challenging from an experimental point of view. In this paper, a new way of making use of the CP-violating observables of the decay $B_d \to \pi^+\pi^-$ is proposed. Combining them with those of the mode $B_s \to K^+K^-$, a simultaneous determination of $\beta$ and $\gamma$ becomes possible. The decays $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ are related to each other by interchanging all down and strange quarks, i.e. through the so-called “$U$-spin” subgroup of the $SU(3)$ flavour symmetry of strong interactions. The utility of $B_s \to K^+K^-$ to probe $\gamma$ was already pointed out in several previous publications \cite{6}. The new strategy proposed here is not affected by penguin topologies – it rather makes use of them – and does not rely on certain “plausible” dynamical or model-dependent assumptions. Moreover, final-state interaction effects \cite{7}, which led to considerable attention in the recent literature in the context of the determination of $\gamma$ from $B \to \pi K$ decays \cite{8}, do not lead to any problems, and the theoretical accuracy is only limited by $U$-spin-breaking effects. A conceptually quite similar strategy to extract $\gamma$ using $B_{s(d)} \to J/\psi K_S$ and $B_{d(s)} \to D^+_{d(s)}D^-_{d(s)}$ decays was recently proposed in \cite{9}.

The determination of $\gamma$ is a crucial element in the test of the Standard Model description of CP violation. In particular, this angle should be measured in a variety of ways to check whether one finds the same result consistently. During the recent years, several methods to accomplish this task were developed \cite{4}. Since the $e^+e^-$ $B$-factories operating at the $\Upsilon(4S)$ resonance will not be in a position to explore $B_s$ decays, a strong emphasis was given to decays of non-strange $B$ mesons. However, also the $B_s$ system provides interesting strategies to determine $\gamma$ (see, for example, \cite{10}). In order to make use of these methods, dedicated $B$-physics experiments at hadron machines, such as LHCb, are the natural place. The $B_s$ system exhibits several peculiar features. Within the Standard Model, the weak $B^0_s\rightarrow\bar{B}^0_s$ mixing phase is very small, and studies of $B_s$ decays involve very rapid $B^0_s\rightarrow\bar{B}^0_s$ oscillations, which are due to the large mass difference $\Delta M_s \equiv M^{(s)}_H - M^{(s)}_L$ between the mass eigenstates $B^H_s$ (“heavy”) and $B^L_s$ (“light”). Future $B$-physics experiments at hadron machines should be in a position to resolve these oscillations. In contrast to the $B_d$ case, there may be a sizeable width difference $\Delta \Gamma_s \equiv \Gamma^{(s)}_H - \Gamma^{(s)}_L$ between the $B_s$ mass eigenstates \cite{11}, which may allow studies of CP violation with “untagged” $B_s$ data samples, where one does not distinguish between initially, i.e. at time $t = 0$, present $B^0_s$ or $\bar{B}^0_s$ mesons \cite{12}. In such untagged rates, the rapid $B^0_s\rightarrow\bar{B}^0_s$ oscillations cancel.
There are a few theoretically clean strategies to determine $\gamma$, making use of pure “tree” decays, for example of $B^\pm \to DK^\pm$ or $B_s \to D_s^\pm K^\mp$ modes. Since no flavour-changing neutral-current (FCNC) processes contribute to the corresponding decay amplitudes, it is quite unlikely that they – and the extracted value of $\gamma$ – are affected significantly by new physics. In contrast, the strategies discussed in this paper rely on interference effects between “tree” and “penguin”, i.e. FCNC, processes. Therefore, new physics may well show up in the corresponding decay amplitudes, thereby affecting the CP-violating observables and the extracted value of $\gamma$.

The outline of this paper is as follows: in Section 2, a completely general parametrization of the $B_d \to \pi^+ \pi^-$ and $B_s \to K^+ K^-$ decay amplitudes, as well as of the corresponding CP-violating observables, is given within the Standard Model. The strategies to determine $\beta$ and $\gamma$ with the help of these observables are discussed in Section 3, where also an approach, which uses $B_d \to \pi^\mp K^\pm$ instead of $B_s \to K^+ K^-$ and relies – in addition to the $SU(3)$ flavour symmetry – on a certain dynamical assumption, is briefly discussed. In Section 4, the $U$-spin-breaking corrections affecting these strategies are investigated in more detail, and the conclusions are summarized in Section 5.

2 Decay Amplitudes and CP-violating Observables

The decay $B^0_d \to \pi^+ \pi^-$ originates from $\bar{b} \to \bar{u}u d\bar{d}$ quark-level processes, as can be seen in Fig. 1. Its transition amplitude can be written as

$$A(B^0_d \to \pi^+ \pi^-) = \lambda^{(d)}_u (A^{u+c}_{cc} + A^u_{pen}) + \lambda^{(d)}_c A^c_{pen} + \lambda^{(d)}_t A^t_{pen},$$

(1)

where $A^{u+c}_{cc}$ is due to current–current contributions, and the amplitudes $A^{j}_{pen}$ describe penguin topologies with internal $j$ quarks ($j \in \{u, c, t\}$). These penguin amplitudes take into account both QCD and electroweak penguin contributions. The quantities

$$\lambda^{(d)}_j \equiv V_{jd}V^*_{jb}$$

(2)
are the usual CKM factors. If we make use of the unitarity of the CKM matrix and apply the Wolfenstein parametrization [13], generalized to include non-leading terms in $\lambda$ [14], we obtain
\[ A(B_d^0 \to \pi^+\pi^-) = e^{i\gamma} \left(1 - \frac{\lambda^2}{2}\right) C \left[1 - d e^{i\theta} e^{-i\gamma}\right], \] (3)
where
\[ C \equiv \lambda^3 A R_b \left(A_{cc}^u + A_{pen}^u\right), \] (4)
with $A_{pen}^u \equiv A_{pen}^u - A_{pen}^t$, and
\[ d e^{i\theta} \equiv \frac{1}{(1 - \lambda^2/2) R_b} \left(\frac{A_{pen}^c}{A_{cc}^u + A_{pen}^u}\right). \] (5)

The quantity $A_{pen}^c$ is defined in analogy to $A_{pen}^u$, and the CKM factors are given by
\[ \lambda \equiv |V_{ud}| = 0.22, \quad A \equiv \frac{1}{\lambda^2} |V_{cb}| = 0.81 \pm 0.06, \quad R_b \equiv \frac{1}{\lambda} \frac{|V_{ub}|}{|V_{cb}|} = 0.41 \pm 0.07. \] (6)

For the following considerations, time-dependent CP asymmetries play a key role. In the case of a general $B_d$ decay into a final CP eigenstate $|f\rangle$, satisfying
\[ (CP)|f\rangle = \eta |f\rangle, \] (7)
we have [1]
\[ \alpha_{CP}(B_d(t) \to f) = \frac{\Gamma(B_d^0(t) \to f) - \Gamma(B_d^0(t) \to f)}{\Gamma(B_d^0(t) \to f) + \Gamma(B_d^0(t) \to f)} = A_{CP}^{dir}(B_d \to f) \cos(\Delta M_d t) + A_{CP}^{mix}(B_d \to f) \sin(\Delta M_d t). \] (8)

If the $B_d^0 \to f$ decay amplitude takes the same form as (3), we obtain the following expressions for the “direct” and “mixing-induced” CP-violating observables:
\[ A_{CP}^{dir}(B_d \to f) = -\left[\frac{2 d \sin \theta \sin \gamma}{1 - 2 d \cos \theta \cos \gamma + d^2}\right], \] (9)
\[ A_{CP}^{mix}(B_d \to f) = \eta \left[\frac{\sin(\phi_d + 2\gamma) - 2 d \cos \theta \sin(\phi_d + \gamma) + d^2 \sin \phi_d}{1 - 2 d \cos \theta \cos \gamma + d^2}\right], \] (10)
where $\phi_d = 2\theta$ denotes the $B_d^0 - \bar{B}_d^0$ mixing phase, which can be determined, for instance, with the help of the “gold-plated” mode $B_d \to J/\psi K_S$. Strictly speaking, mixing-induced CP violation in $B_d \to J/\psi K_S$ probes $2\theta + \phi_K$, where $\phi_K$ is related to the weak $K^0 - \bar{K}^0$ mixing phase and is negligibly small in the Standard Model. Due to the small value of the CP-violating parameter $\varepsilon_K$ of the neutral kaon system, $\phi_K$ can only be affected by very contrived models of new physics [15].
In the case of $B_d \to \pi^+\pi^-$, $\eta$ is equal to +1, and for negligible values of the “penguin parameter” $d$, we have $\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-) = + \sin(\phi_d + 2\gamma) = \sin(2\beta + 2\gamma) = -\sin(2\alpha)$. However, the penguin contributions are expected to play an important role. This feature is already indicated experimentally by recent CLEO results on $B \to \pi K$ modes.

Let us now turn to the decay $B_s^0 \to K^+K^-$. It originates from $b \to u\bar{u}s$ quark-level processes, as can be seen in Fig. 1. Using a notation similar to that in (3), we obtain

$$A(B_s^0 \to K^+K^-) = e^{i\gamma}\lambda C' \left[ 1 + \left( \frac{1 - \lambda^2}{\lambda^2} \right) d' e^{i\theta'} e^{-i\gamma} \right],$$

(11)

where

$$C' \equiv \lambda^3 A R_b \left( A_{cc}^{u'} + A_{\text{pen}}^{u'} \right)$$

(12)

and

$$d' e^{i\theta'} \equiv \frac{1}{(1 - \lambda^2/2)R_b} \left( \frac{A_{\text{pen}}^{t'} A^{t'}_{\text{pen}}}{A^{u'}_{\text{pen}} + A^{u'}_{\text{pen}}} \right)$$

(13)

correspond to (4) and (5), respectively. The primes remind us that we are dealing with a $\bar{b} \to \bar{s}$ transition. It should be emphasized that (3) and (11) are completely general parametrizations of the $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ decay amplitudes within the Standard Model, relying only on the unitarity of the CKM matrix. In particular, these expressions take into account also final-state interaction effects, which can be considered as long-distance penguin topologies with internal up- and charm-quark exchanges.

As we have already noted, there may be a sizeable width difference $\Delta\Gamma_s \equiv \Gamma_H^{(s)} - \Gamma_L^{(s)}$ between the $B_s$ mass eigenstates, which may allow studies of CP violation with “untagged” $B_s$ data samples. Such untagged rates take the following form:

$$\Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f) \propto R_He^{-\Gamma_H^{(s)}t} + R_Le^{-\Gamma_L^{(s)}t},$$

(14)

whereas the time-dependent CP asymmetry is given by

$$a_{\text{CP}}(B_s(t) \to f) \equiv \frac{\Gamma(B_s^0(t) \to f) - \Gamma(\bar{B}_s^0(t) \to f)}{\Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f)}$$

$$= 2 e^{-\Gamma_s t} \left[ \frac{\mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \to f) \cos(\Delta M_s t) + \mathcal{A}_{\text{CP}}^{\text{mix}}(B_s \to f) \sin(\Delta M_s t)}{e^{-\Gamma_H^{(s)} t} + e^{-\Gamma_L^{(s)} t}} + \mathcal{A}_{\Delta\Gamma}(B_s \to f) \right]$$

(15)

with $\mathcal{A}_{\Delta\Gamma}(B_s \to f) = (R_H - R_L)/(R_H + R_L)$. If the $B_s^0 \to f$ decay amplitude takes the same form as (11), we have

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \to f) = \frac{2 \tilde{d} \sin \theta' \sin \gamma}{1 + 2 \tilde{d}' \cos \theta' \cos \gamma + \tilde{d}^2}$$

(16)

$$\mathcal{A}_{\text{CP}}^{\text{mix}}(B_s \to f) = \eta \frac{\sin(\phi_s + 2\gamma) + 2 \tilde{d}' \cos \theta' \sin(\phi_s + \gamma) + \tilde{d}^2 \sin \phi_s}{1 + 2 \tilde{d}' \cos \theta' \cos \gamma + \tilde{d}^2}$$

(17)

$$\mathcal{A}_{\Delta\Gamma}(B_s \to f) = -\eta \frac{\cos(\phi_s + 2\gamma) + 2 \tilde{d}' \cos \theta' \cos(\phi_s + \gamma) + \tilde{d}^2 \cos \phi_s}{1 + 2 \tilde{d}' \cos \theta' \cos \gamma + \tilde{d}^2}.$$
These observables are not independent quantities, and satisfy the relation
\[
\left[ A_{\text{CP}}^{\text{dir}}(B_s \to f) \right]^2 + \left[ A_{\text{CP}}^{\text{mix}}(B_s \to f) \right]^2 + \left[ A_{\Delta \Gamma}(B_s \to f) \right]^2 = 1. \tag{19}
\]
In the general expressions (13)–(18), we have introduced the abbreviation
\[
d' \equiv \left( \frac{1 - \lambda^2}{\lambda^2} \right) d',
\]
and \( \phi_s \equiv -2\delta\gamma = 2 \arg(V_{ts}^* V_{tb}) \) denotes the \( B_0^s-B_0^s \) mixing phase. Within the Standard Model, we have \( 2\delta\gamma \approx 0.03 \) due to a Cabibbo suppression of \( \mathcal{O}(\lambda^2) \), implying that \( \phi_s \) is very small. This phase can be probed – and even determined – with the help of the decay \( B_s \to J/\psi \phi \) (see, for example, [17]). Large CP-violating effects in this decay would signal that \( 2\delta\gamma \) is not tiny, and would be a strong indication for new-physics contributions to \( B_0^s-B_0^s \) mixing.

3 Extracting \( \beta \) and \( \gamma \)

Since the decays \( B_d \to \pi^+\pi^- \) and \( B_s \to K^+K^- \) are related to each other by interchanging all strange and down quarks, the \( U \)-spin flavour symmetry of strong interactions implies
\[
d' = d, \tag{21}
\]
\[
\theta' = \theta. \tag{22}
\]
In contrast to certain isospin relations, electroweak penguins do not lead to any problems in the \( U \)-spin relations (21) and (22). Consequently, if we assume that the \( B_0^s-B_0^s \) mixing phase \( \phi_s \) is negligibly small, or that it is determined with the help of the decay \( B_s \to J/\psi \phi \), the four observables provided by the time-dependent CP asymmetries of the modes \( B_d \to \pi^+\pi^- \) and \( B_s \to K^+K^- \) depend on the four “unknowns” \( d, \theta, \phi_d = 2\beta \) and \( \gamma \) in the strict \( U \)-spin limit. These quantities can therefore be determined simultaneously. In order to extract \( \gamma \), it suffices to consider \( A_{\text{CP}}^{\text{mix}}(B_s \to K^+K^-) \) and the direct CP asymmetries \( A_{\text{CP}}^{\text{dir}}(B_s \to K^+K^-) \) and \( A_{\text{CP}}^{\text{dir}}(B_d \to \pi^+\pi^-) \). If we make use, in addition, of \( A_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-) \), \( \beta \) can be determined as well.

Let us now give the formulae to implement this approach in a mathematical way. Using the general parametrization of the CP-violating \( B_s \to K^+K^- \) observables given in the previous section, we obtain
\[
\tilde{d}' = \sqrt{\frac{1}{k^2}} \left[ l' \pm \sqrt{l'^2 - h'k'} \right], \tag{23}
\]
\[
2 \tilde{d}' \cos \theta' = - \left( u' + v' \tilde{d}'^2 \right), \tag{24}
\]
\[
2 \tilde{d}' \sin \theta' = \left[ 1 - u' \cos \gamma + (1 - v' \cos \gamma) \tilde{d}'^2 \right] \frac{A_{\text{CP}}^{\text{dir}}(B_s \to K^+K^-)}{\sin \gamma}, \tag{25}
\]
where
\[ h' = u'^2 + D'(1 - u' \cos \gamma)^2 \]  
(26)
\[ k' = v'^2 + D'(1 - v' \cos \gamma)^2 \]  
(27)
\[ l' = 2 - u'v' - D'(1 - u' \cos \gamma)(1 - v' \cos \gamma) \]  
(28)
with
\[ u' = \frac{A_{\text{mix}}^{\text{CP}}(B_s \to K^+K^-) - \sin(\phi_s + 2\gamma)}{A_{\text{mix}}^{\text{CP}}(B_s \to K^+K^-) \cos \gamma - \sin(\phi_s + \gamma)}, \]  
(29)
\[ v' = \frac{A_{\text{mix}}^{\text{CP}}(B_s \to K^+K^-) - \sin \phi_s}{A_{\text{mix}}^{\text{CP}}(B_s \to K^+K^-) \cos \gamma - \sin(\phi_s + \gamma)}, \]  
(30)
and
\[ D' = \left[ \frac{A_{\text{dir}}^{\text{CP}}(B_s \to K^+K^-)}{\sin \gamma} \right]^2. \]  
(31)

Taking into account (20) and the U-spin relations (21) and (22), these expressions allow us to determine the direct CP asymmetry \( A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) \) (see (9)) as a function of \( \gamma \). The measured value of \( A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) \) then fixes \( \gamma \) and \( d e^{i\theta} \). Inserting \( \gamma \) and \( d e^{i\theta} \) thus determined into (10), the measured value of the mixing-induced CP asymmetry \( A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-) \) allows us to determine \( \beta \). The U-spin-breaking corrections to (21) and (22) will be discussed in Section 4. Needless to note that also \( d e^{i\theta} \) is an interesting quantity, allowing valuable insights into the hadronization dynamics of \( B_d \to \pi^+\pi^- \).

Since the strategy discussed in the previous paragraph appears quite abstract, let us illustrate it by considering a simple example. If we neglect the \( B_s^0 - \bar{B}_s^0 \) mixing phase \( \phi_s \) and assume \( 2\beta = 53^\circ \) and \( \gamma = 76^\circ \), lying within the ranges allowed at present for these angles, implied by the usual indirect fits of the unitarity triangle, as well as \( d = d' = 0.3 \) and \( \theta = \theta' = 210^\circ \), we obtain

\[ A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) = +24\%, \quad A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-) = +4.4\%, \]  
\[ A_{\text{dir}}^{\text{CP}}(B_s \to K^+K^-) = -17\%, \quad A_{\text{mix}}^{\text{CP}}(B_s \to K^+K^-) = -28\%. \]  
(32)

In Fig. 3 we show the dependence of \( A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) \) on \( \gamma \), obtained as described above. The dotted line in this figure represents the “measured” value of +24%. In this example, we get two solutions for \( \gamma \), the “true” value of 76°, and a second one of 126°. Inserting these values for \( \gamma \) and the corresponding ones for \( d \) and \( \theta \) into the expression (10) for \( A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-) \), we obtain the curves shown in Fig. 3, where the solid line corresponds to \( \gamma = 76^\circ \), and the dot-dashed line corresponds to the second solution of \( \gamma = 126^\circ \). The dotted line represents the “measured” value \( A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-) = +4.4\% \). If we look at Fig. 3, we observe that \( \beta \) can be determined in this example up to a fourfold ambiguity. Here we have assumed that \( \beta \in [0^\circ, 180^\circ] \), as implied by the measured value of \( \varepsilon_K \). A similar comment applies to the range for \( \gamma \). If the twofold ambiguity in the extraction of \( \gamma \) could be resolved, we were left with a twofold ambiguity for \( \beta \).
Figure 2: The dependence of $A_{\text{CP}}^\text{dir}(B_d \rightarrow \pi^+\pi^-)$ on $\gamma$ fixed through the CP-violating $B_s \rightarrow K^+K^-$ observables for a specific example discussed in the text.

Figure 3: The dependence of $A_{\text{CP}}^\text{mix}(B_d \rightarrow \pi^+\pi^-)$ on $\beta$ fixed through the CP-violating $B_s \rightarrow K^+K^-$ observables for a specific example discussed in the text.
The $B^0_d\rightarrow\bar{B}^0_d$ mixing phase $\phi_d = 2\beta$ can be determined in a reliable way with the help of the “gold-plated” mode $B_d \rightarrow J/\psi K_S$ \[3\]. Strictly speaking, a measurement of mixing-induced CP violation in $B_d \rightarrow J/\psi K_S$ allows us to determine only $\sin(2\beta)$, i.e. to fix $\phi_d = 2\beta$ up to a twofold ambiguity for $\beta \in [0^\circ, 180^\circ]$. However, several strategies to resolve this ambiguity were proposed in the literature \[18\], which should be feasible for “second-generation” $B$-physics experiments. Consequently, $\phi_d = 2\beta$ should be known reliably and unambiguously in the era of these experiments, thereby providing a different strategy to combine the observables \(3\) and \(10\) with \(16\) and \(17\). The point is that these quantities allow us to fix contours in the $\gamma$–$d$ planes as functions of the $B^0_d\rightarrow\bar{B}^0_d$ and $B^0_s\rightarrow\bar{B}^0_s$ mixing phases in a theoretically clean way. In the $B_s \rightarrow K^+K^-$ case, these contours are described by \(23\) with \(20\). On the other hand, in the case of $B_d \rightarrow \pi^+\pi^-$, we have

\[
d = \sqrt{\frac{1}{k} [l \pm \sqrt{l^2 - h k}]},
\]

\[
d d \cos \theta = u + v d^2
\]

\[
2 d \sin \theta = - \left[1 - u \cos \gamma + (1 - v \cos \gamma) d^2 \right] \frac{A_{\text{dir}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-)}{\sin \gamma},
\]

where $h, k$ and $l$ take the same form as \(26\)–\(28\) with $u', v'$ and $D'$ replaced by

\[
u = \frac{A_{\text{mix}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-) - \sin \phi_d}{A_{\text{mix}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-) \cos \gamma - \sin(\phi_d + \gamma)}
\]

\[
v = \frac{A_{\text{mix}}^{\text{mix}}(B_d \rightarrow \pi^+\pi^-) - \sin \phi_d}{A_{\text{mix}}^{\text{mix}}(B_d \rightarrow \pi^+\pi^-) \cos \gamma - \sin(\phi_d + \gamma)}
\]

and

\[
D = \left[\frac{A_{\text{dir}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-)}{\sin \gamma}\right]^2.
\]

The contours in the $\gamma$–$d$ plane are fixed through \(33\). Using the $U$-spin relation \(21\), the intersection of the theoretically clean $B_s \rightarrow K^+K^-$ and $B_d \rightarrow \pi^+\pi^-$ contours described by \(23\) (with \(20\)) and \(33\) allows us to determine $\gamma$. Analogously, it is possible to fix theoretically clean contours in the $\gamma$–$\theta^{(c)}$ planes through \(23\)–\(25\) and \(33\)–\(35\), and to determine $\gamma$ with the help of the $U$-spin relation \(22\).

In the case of the illustrative example discussed above, we obtain the contours in the $\gamma$–$d^{(c)}$ planes shown in Fig. 4. Here the dot-dashed and solid lines correspond to \(23\) and \(33\), respectively. The intersection of these lines yields a twofold solution for $\gamma$, given by $51^\circ$ and by the “true” value of $76^\circ$. The dotted line in Fig. 4 is related to

\[
K \equiv -\frac{1}{\epsilon} \frac{A_{\text{dir}}^{\text{dir}}(B_d \rightarrow \pi^+\pi^-)}{A_{\text{dir}}^{\text{dir}}(B_s \rightarrow K^+K^-)} = \left(\frac{d \sin \theta}{d' \sin \theta'}\right) \left(\frac{1 + 2 \tilde{d} \cos \theta' \cos \gamma + \tilde{d}^2}{1 - 2 \tilde{d} \cos \theta \cos \gamma + \tilde{d}^2}\right),
\]

where

\[
\epsilon \equiv \frac{\lambda^2}{1 - \lambda^2}
\]

(40)
Figure 4: The contours in the $\gamma$–$d^{(')}$ planes fixed through the CP-violating $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ observables for a specific example discussed in the text.

Figure 5: The contours in the $\gamma$–$d$ plane fixed through the observables $A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-)$, $A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-)$ and $K$ for a specific example discussed in the text.
If we use the $U$-spin relations (21) and (22), as well as the mixing-induced CP asymmetry $\mathcal{A}_{\text{mix}}^{\text{CP}}(B_s \to K^+K^-)$, we obtain

$$d = \epsilon \frac{(K - 1 + (1 + \epsilon K) u' \cos \gamma)}{1 - \epsilon^2 K - (1 + \epsilon K) v' \cos \gamma},$$

which fixes the dotted line in Fig. 4. Combining all contours in this figure, we obtain a single solution for $\gamma$ in this example, which is given by the “true” value of 76°.

It is interesting to note that an alternative way to fix certain contours through the CP-violating observables $\mathcal{A}_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-)$ and $\mathcal{A}_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-)$ was proposed in [19]. In this paper, a different parametrization of the $B_d^0 \to \pi^+\pi^-$ decay amplitude was chosen, and in the corresponding contours, $\alpha$ was set in correlation with a hadronic parameter $|P/T|$, which represents – sloppily speaking – the ratio of the “penguin” to the “tree” contributions to $B_d^0 \to \pi^+\pi^-$. Unfortunately, it is very difficult to fix this parameter in a theoretically reliable way, in particular as it is also affected by final-state-interaction effects. In the strategies proposed above, there are no problems of this kind, and the theoretical accuracy is only limited by $U$-spin-breaking corrections, which will be discussed in more detail in Section 1.

Before turning to these corrections, let us discuss some other interesting implications of the $U$-spin flavour symmetry. Employing once more (21) and (22), we obtain

$$K = \frac{1}{\epsilon} \left| \frac{c}{\mathcal{C}} \right|^2 \left[ \frac{M_{B_s}}{M_{B_d}} \frac{\Phi(M_{\pi}/M_{B_d}, M_{\pi}/M_{B_s}) \phi_{B_d}}{\Phi(M_{K}/M_{B_s}, M_{K}/M_{B_d}) \phi_{B_s}} \right] \frac{\text{BR}(B_s \to K^+K^-)}{\text{BR}(B_d \to \pi^+\pi^-)},$$

where

$$\Phi(x, y) \equiv \sqrt{[1 - (x + y)^2] [1 - (x - y)^2]}$$

is the usual two-body phase-space function, and $\text{BR}(B_s \to K^+K^-)$ and $\text{BR}(B_d \to \pi^+\pi^-)$ denote the “CP-averaged” branching ratios, which can be extracted from the “untagged” rates (41). Moreover, taking into account (39), we arrive at the $U$-spin relation

$$\frac{\mathcal{A}_{\text{dir}}^{\text{CP}}(B_s \to K^+K^-)}{\mathcal{A}_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-)} = - \frac{\left| \frac{c'}{\mathcal{C}} \right|^2 \left[ \frac{M_{B_s}}{M_{B_d}} \frac{\Phi(M_{K}/M_{B_s}, M_{K}/M_{B_d}) \phi_{B_s}}{\Phi(M_{\pi}/M_{B_d}, M_{\pi}/M_{B_s}) \phi_{B_d}} \right] \frac{\text{BR}(B_s \to K^+K^-)}{\text{BR}(B_d \to \pi^+\pi^-)}}{\text{BR}(B_s \to K^+K^-)}.$$

An analogous relation holds between the CP-violating asymmetries of the decays $B^\pm \to \pi^\pm K$ and $B^\pm \to K^\pm K$ [20]. In the strict $U$-spin limit, we have $|c'| = |c|$. Corrections to this relation can be calculated within the “factorization” approximation, yielding

$$\left| \frac{c'}{\mathcal{C}} \right|^2 \approx \frac{f_K}{f_\pi} \frac{F_{B_s,K}^2(M_K^2; 0^+)}{F_{B_d,\pi}^2(M_{\pi}^2; 0^+)} \left( \frac{M_{B_s}^2 - M_{K}^2}{M_{B_d}^2 - M_{\pi}^2} \right)^2,$$

where $f_K$ and $f_\pi$ denote the kaon and pion decay constants, and the form factors $F_{B_s,K}(M_K^2; 0^+)$ and $F_{B_d,\pi}(M_{\pi}^2; 0^+)$ parametrize the hadronic quark-current matrix elements $\langle K^-|(\bar{b}u)_{V-A}|B_s^0 \rangle$ and $\langle \pi^-|(\bar{b}u)_{V-A}|B_d^0 \rangle$, respectively [21]. Consequently, we have

$$\frac{\mathcal{A}_{\text{dir}}^{\text{CP}}(B_s \to K^+K^-)}{\mathcal{A}_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-)} \approx -1.56 \times \left[ \frac{F_{B_s,K}(M_K^2; 0^+)}{F_{B_d,\pi}(M_{\pi}^2; 0^+)} \right]^2 \frac{\tau_{B_s}}{\tau_{B_d}} \frac{\text{BR}(B_d \to \pi^+\pi^-)}{\text{BR}(B_s \to K^+K^-)}.$$

10
where \( F_{B_s K}(M_{K}^2; 0^+) \approx F_{B_d \pi}(M_{\pi}^2; 0^+) \) and \( \tau_{B_s} \approx \tau_{B_d} \). As we will see in the following section, the U-spin relations (21) and (22) do not receive U-spin-breaking corrections within the “factorization” approximation. However, it should be emphasized that also non-factorizable contributions, which are not included in (13) and in the analysis of Section 4, may play an important role.

Interestingly, the ratio \(|C^+ / C|\) can be determined with the help of (14), allowing valuable experimental insights into U-spin breaking. Alternatively, (12) and (15) allow us to determine \( \gamma \) by using only the CP-averaged \( B_d \to \pi^+ \pi^- \) and \( B_s \to K^+ K^- \) branching ratios. Employing once again the U-spin relations (21) and (22), as well as the general expression (10) for the mixing-induced CP asymmetry \( \mathcal{A}_{CP}^{mix}(B_d \to \pi^+ \pi^-) \), we obtain

\[
d = \sqrt{\frac{\epsilon^2 (K - 1) - \epsilon (1 + \epsilon K) u \cos \gamma}{1 - \epsilon^2 K + \epsilon (1 + \epsilon K) v \cos \gamma}},
\]

which corresponds to (11). Consequently, if \( \phi_d = 2\beta \) is determined through \( B_d \to J/\psi K_S \), the CP-violating observables \( \mathcal{A}_{CP}^{dir}(B_d \to \pi^+ \pi^-) \), \( \mathcal{A}_{CP}^{mix}(B_d \to \pi^+ \pi^-) \) and the CP-averaged \( B_d \to \pi^+ \pi^- \), \( B_s \to K^+ K^- \) branching ratios allow us to determine \( \gamma \) with the help of the contours in the \( \gamma - d \) plane described by (33) and (17). This approach is illustrated in Fig. 3, where the solid and dotted lines correspond to (33) and (17), respectively. Although this approach suffers from theoretical uncertainties due to (15) larger than those of the strategies described above, it does not require a time-dependent measurement of \( B_s \to K^+ K^- \), i.e. to resolve the \( \Delta M_s t \) oscillations in this decay.

Since the decays \( B_s \to K^+ K^- \) and \( B_d \to \pi^\pm K^\pm \) differ only in their spectator quarks, we have

\[
\mathcal{A}_{CP}^{dir}(B_s \to K^+ K^-) \approx \mathcal{A}_{CP}^{dir}(B_d \to \pi^\pm K^\pm)
\]

\[
\text{BR}(B_s \to K^+ K^-) \approx \text{BR}(B_d \to \pi^\pm K^\pm) \frac{\tau_{B_s}}{\tau_{B_d}},
\]

allowing us to fix \( K \) through the \( B_d \to \pi^\pm K^\pm \) and \( B_d \to \pi^\pm \pi^- \) observables with the help of the formulae given above. In contrast to \( B_d \to \pi^\pm K^\pm \), \( B_s \to K^+ K^- \) receives also contributions from “exchange” and “penguin annihilation” topologies, which are usually expected to play a minor role (22), but may be enhanced through certain rescattering effects (4). Although these topologies do not lead to any problems for the strategies using \( B_d \to \pi^\pm \pi^- \) and \( B_s \to K^+ K^- \) discussed in this section – even if they should turn out to be sizeable – they may affect (18) and (19). Consequently, these relations rely – in addition to the \( SU(3) \) flavour symmetry – on a certain dynamical assumption. If we perform the appropriate replacements in (14) and (15), we obtain

\[
\frac{A_{CP}^{dir}(B_d \to \pi^\pm K^\pm)}{A_{CP}^{dir}(B_d \to \pi^\pm \pi^-)} \approx \left( \frac{f_K}{f_\pi} \right)^2 \frac{\Phi(M_\pi/M_{B_d}, M_K/M_{B_d})}{\Phi(M_\pi/M_{B_d}, M_\pi/M_{B_d})} \frac{\text{BR}(B_d \to \pi^\pm \pi^-)}{\text{BR}(B_d \to \pi^\pm K^\pm)},
\]

which provides an interesting cross check. The importance of the “exchange” and “penguin annihilation” topologies contributing to \( B_s \to K^+ K^- \) can be probed with the help of the decay \( B_s \to \pi^\pm \pi^- \). The naive expectation for the corresponding branching ratio
is $\mathcal{O}(10^{-8})$; a significant enhancement would signal that the “exchange” and “penguin annihilation” topologies cannot be neglected. Another interesting decay in this respect is $B_d \to K^+ K^-$, for which already stronger experimental constraints exist [23].

If the $B_d \to \pi^+ \pi^-$ and $B_d \to \pi^\pm K^\pm$ observables are measured and $\phi_d$ is fixed through $B_d \to J/\psi K_S$, $\gamma$ can be determined with the help of the contours described by (53) and (57), as illustrated in Fig. 5. All time-dependent measurements that are required for this strategy can in principle be performed at the asymmetric $e^+ e^-$-factories operating at the $\Upsilon(4S)$ resonance, i.e. at BaBar or BELLE. Whereas $B_d \to \pi^\pm K^\pm$ has already been observed by the CLEO collaboration with a CP-averaged branching ratio of $(1.4 \pm 0.3 \pm 0.2) \times 10^{-5}$, at present only the upper limit $\text{BR}(B_d \to \pi^+ \pi^-) < 0.84 \times 10^{-5}$ (90% C.L.) is available from CLEO [24]. If we use (50) and $\text{BR}(B_s \to K^+ K^-) \approx \text{BR}(B_d \to \pi^\pm K^\pm) \approx 1.4 \times 10^{-5}$, we obtain, for the example discussed above, $\text{BR}(B_d \to \pi^+ \pi^-) \approx 0.66 \times 10^{-5}$, satisfying the present upper limit from CLEO. After these remarks, let us come back to the $B_d \to \pi^+ \pi^-$, $B_s \to K^+ K^-$ strategies illustrated in Figs. 3 4, 5, and let us investigate the impact of the $U$-spin-breaking corrections in the following section.

4 A Closer Look at the $U$-spin-breaking Corrections

In order to analyse non-leptonic $B$-meson decays theoretically, one uses low-energy effective Hamiltonians, which are calculated in renormalization-group-improved perturbation theory, and take the following form [25]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \lambda_u^{(q)} \sum_{k=1}^{2} C_k(\mu) Q_k^{uq} + \lambda_c^{(q)} \sum_{k=1}^{2} C_k(\mu) Q_k^{cq} - \lambda_t^{(q)} \sum_{k=3}^{10} C_k(\mu) Q_k^t \right].$$

(51)

Here $Q_k^{uq}$ and $Q_k^{cq}$ ($j \in \{u, c\}$, $q \in \{d, s\}$) are the usual current–current operators, $Q_k^{d, d}, Q_k^{s, s}$ and $Q_k^{d, s}, Q_k^{s, d}$ denote the QCD and electroweak penguin operators, respectively, and $\mu = \mathcal{O}(m_b)$ is a renormalization scale. Applying the Bander–Silverman–Soni mechanism [21], and following the formalism developed in [27], we obtain

$$d e^{i\theta} \approx \frac{1}{(1 - \lambda^2/2) R_b} \left[ \mathcal{A}_t + \mathcal{A}_c \right] \mathcal{A}_T \mathcal{A}_t + \mathcal{A}_u,$$

(52)

where

$$\mathcal{A}_T = \frac{1}{3} \overline{C}_1 + \overline{C}_2$$

(53)

$$\mathcal{A}_t = \frac{1}{3} \left[ \overline{C}_3 + \overline{C}_9 + \chi \left( \overline{C}_5 + \overline{C}_7 \right) \right] + \overline{C}_4 + \overline{C}_{10} + \chi \left( \overline{C}_6 + \overline{C}_8 \right)$$

(54)

$$\mathcal{A}_j = \frac{\alpha_s}{3 \pi} \left[ \frac{10}{9} - G(m_j, k, m_b) \right] \left[ \frac{1}{3} \overline{C}_2 + \frac{1}{3 \alpha_s} \left( 3 \overline{C}_1 + \overline{C}_2 \right) \right] (1 + \chi),$$

(55)

with $j \in \{u, c\}$. The coefficients $\overline{C}_k$ refer to $\mu = m_b$ and denote the next-to-leading order scheme-independent Wilson coefficient functions introduced in [28]. The quantity

$$\chi = \frac{2 M_W^2}{(m_u + m_d)(m_b - m_u)}$$

(56)
is due to the use of the equations of motion for the quark fields, whereas the function
\( G(m_j, k, m_b) \) is related to the one-loop penguin matrix elements of the current–current
operators \( Q_{i,j}^{\mu} \) with internal \( j \) quarks. It is given by

\[
G(m_j, k, m_b) = -4 \int_0^1 dx \, x (1 - x) \ln \left( \frac{m_j^2 - k^2 x (1 - x)}{m_b^2} \right),
\]

(57)

where \( m_j \) is the \( j \)-quark mass and \( k \) denotes some average four-momentum of the virtual
 gluons and photons appearing in corresponding penguin diagrams \( \text{[27]} \). Kinematical
considerations at the quark level imply the following “physical” range for this parameter:

\[
\frac{1}{4} \lesssim \frac{k^2}{m_b^2} \lesssim \frac{1}{2}.
\]

(58)

Since the quantity \( de^{i\theta} \) is defined in \( \text{[3]} \) as a ratio of certain amplitudes, the decay
 constants and form factors arising typically in the “factorization” approximation, as can be seen, for example, in \( \text{[47]} \), cancel in \( \text{[52]} \). The expression for the \( B_s \to K^+K^- \)
parameter \( d'e^{i\theta'} \) takes the same form as \( \text{[52]} \), where \( \chi \) is replaced in \( \text{[54]} \) and \( \text{[55]} \) by

\[
\chi' = \frac{2M_K^2}{(m_u + m_s)(m_b - m_u)}.
\]

(59)

Consequently, in our approach to evaluate \( de^{i\theta} \) and \( d'e^{i\theta'} \), the \( U \)-spin-breaking corrections are only due to the parameters \( \chi \) and \( \chi' \). However, up to small electromagnetic
corrections, the chiral structure of strong interactions implies

\[
\frac{M_Z^2}{m_u + m_d} = \frac{M_K^2}{m_u + m_s},
\]

(60)

leading – among other things – to the Gell-Mann–Okubo relation (see, for example, \( \text{[29]} \)).
In our case, this expression has the interesting implication

\[
\chi = \chi',
\]

(61)

so that the \( U \)-spin relation

\[
d e^{i\theta} = d'e^{i\theta'}
\]

(62)

is not affected by \( U \)-spin-breaking corrections within our formalism. Moreover, \( \chi \) and \( \chi' \)
are suppressed by the bottom-quark mass. Although \( \text{[52]} \) is a simplified expression, which
may be affected by non-factorizable contributions, it strengthens our confidence into \( \text{[62]} \).
Unless such non-factorizable effects have a dramatic impact, the \( U \)-spin-breaking corrections to this relation are probably moderate.

Before giving the conclusions in the following section, let us emphasize again that the
strategies illustrated in Figs. 2 and 3 rely on \( \text{[52]} \), i.e. both on \( \text{[21]} \) and \( \text{[23]} \), whereas the extraction of \( \gamma \) illustrated in Fig. 4 makes only use of \( \text{[24]} \). In particular, the solid and
dot-dashed contours in this figure are \textit{theoretically clean}. Using \( \text{[52]} \), we obtain values
for \( d \) and \( \theta \) of the same order of magnitude as those employed in the example given in
the previous section.
5 Conclusions

The time evolutions of the decays \( B_d \to \pi^+\pi^- \) and \( B_s \to K^+K^- \) provide interesting strategies to extract the angles \( \beta \) and \( \gamma \) of the unitarity triangle of the CKM matrix. These methods, which make use of “penguin” topologies and are therefore not affected by their presence, take into account final-state interaction effects “automatically”. Moreover, they do not rely on any model-dependent or “plausible” dynamical assumptions, and their theoretical accuracy is only limited by \( U \)-spin-breaking corrections. Within a certain model-dependent approach making use – among other things – of “factorization” to estimate the relevant hadronic matrix elements, these \( U \)-spin-breaking corrections vanish. Although this approach is very simplified and may be affected by non-factorizable effects, it strengthens our confidence into the \( U \)-spin relations used for the extraction of \( \beta \) and \( \gamma \) from the decays \( B_d \to \pi^+\pi^- \) and \( B_s \to K^+K^- \). Moreover, an interesting relation between the CP-averaged branching ratios of these modes and the corresponding direct CP asymmetries may provide valuable experimental insights into \( U \)-spin breaking.

If the \( B_0^d - \overline{B}_0^d \) mixing phase \( \phi_d = 2\beta \) is fixed through \( B_d \to J/\psi K_S \), certain theoretically clean contours can be determined with the help of the CP-violating \( B_d \to \pi^+\pi^- \) and \( B_s \to K^+K^- \) observables, allowing the extraction of \( \gamma \) with a reduced \( U \)-spin flavour symmetry input. Since both \( \bar{b} \to \bar{d} \) and \( \bar{b} \to \bar{s} \) “penguin”, i.e. FCNC, processes contribute to the \( B_d \to \pi^+\pi^- \) and \( B_s \to K^+K^- \) decay amplitudes, the extracted value of \( \gamma \) may well be affected by new physics. In such a case, discrepancies would show up with the values of \( \gamma \) determined from pure “tree” decays, for example from \( B_s \to D_{\pm}^s K_{\mp} \) modes. Possible new-physics contributions to \( B_0^s - \overline{B}_0^s \) mixing can be probed through \( B_s \to J/\psi \phi \), and can also be taken into account with the help of this decay. A similar comment applies to the \( B_0^d - \overline{B}_0^d \) mixing phase \( \phi_d \). If new physics should shift \( \phi_d \) from its Standard Model value, it could still be fixed through the “gold-plated” mode \( B_d \to J/\psi K_S \).

The strategies proposed in this paper are very interesting for “second-generation” \( B \)-physics experiments performed at hadron machines, for example LHCb, where the very interesting physics potential of the \( B_s \) system can be fully exploited. At the asymmetric \( e^+e^- \) \( B \)-factories operating at the \( \Upsilon(4S) \) resonance, which will start taking data very soon, this is unfortunately not possible. However, there is also a variant of the strategy to determine \( \gamma \), where \( B_d \to \pi^\pm K^\mp \) is used instead of \( B_s \to K^+K^- \). This approach has the advantage that all required time-dependent measurements can in principle be performed at the asymmetric \( e^+e^- \) machines. On the other hand, it relies – in addition to the \( SU(3) \) flavour symmetry – on the smallness of certain “exchange” and “penguin annihilation” topologies, which may be enhanced by final-state-interaction effects. Consequently, its theoretical accuracy cannot compete with the “second-generation” \( B_d \to \pi^+\pi^-, B_s \to K^+K^- \) approach, which is not affected by such problems.

Hopefully, studies of the kind discussed in this paper will eventually guide us to the physics lying beyond the Standard Model.

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