Putting Liouville String Models of (Quantum) Gravity to Test

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Abstract

Critical String Theory is by definition an $S$-matrix theory. In this sense, (quantum) gravity situations where a unitary $S$-matrix may not be a well-defined concept, as a consequence of the existence of macroscopic (global) or microscopic (local) gravitational fluctuations with event horizons, present a challenge to string theory. In this article, we take some modest steps in suggesting alternative treatments of such cases via non-critical (Liouville) strings, which do not have a well-defined $S$ matrix, but for which a superscattering $S$ matrix is mathematically consistent. After a brief review of the underlying mathematical formalism, we consider a specific stringy model of induced non-criticality, with dynamical formation of horizons, associated with the recoil of a $D$-particle defect, embedded in our four-dimensional space time, during its scattering with a (macroscopic) number of closed string states. We study in detail the associated spacetime distortion in the neighbourhood of the defect, which has the form of a finite-radius curved ‘bubble’, matched with a Minkowskian space-time in the exterior. As a consequence of the non-criticality of the underlying $\sigma$-model, the space time is unstable, and has non-trivial stochastic properties: thermal properties due to its “Rindler accelerating nature”, and entropy growth for an asymptotic observer, associated with information being carried away by the ‘recoil’ degrees of freedom. For the validity of our approach it is essential that the string length is a few orders of magnitude larger than the Planck length, which is a typical situation encountered in many $D$-brane-world models. An interesting feature of our model is the emission of high-energy photons from the unstable bubble. As a result, the neighbourhood of the recoiling $D$-particle may operate as a source region of ultra-high-energy particles, which could reach the observation point if the source lies within the respective mean-free paths. This may lead to severe phenomenological constraints on the model, e.g. in connection with apparent “violations” of the GZK cutoff, which we discuss briefly. We also comment on the effects of embedded recoiling $D$-particles on the cosmological evolution of a Friedman-Robertson-Walker type Universe. Specifically, we argue in favour of a relaxing-to-zero vacuum energy a la quintessence, and an eventual removal of global cosmological horizons, and hence stopping of the cosmic acceleration in the far future in such models.

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1 Introduction and Summary

The field of critical string theory is a vast, and well-established, field of modern theoretical physics, where an enormous intellectual effort has been devoted for the past twenty years. Since the anomaly cancellation mechanism of Green and Schwarz [1], which opened up the way for realistic phenomenology of string theory, a considerable amount of very attractive work has been provided, dealing with various aspects of string theories, ranging from low-energy physics to non-perturbative aspects. The recent discovery of string dualities and the associated solitonic structures of ‘string theory’, the so-called (D)irichlet branes [2], has enlarged the spectrum of applicability of string theory to important issues of quantum gravity that could not have been tackled before, such as the counting of microstates of certain species of black holes.

However, despite this considerable effort, there are still important conceptual issues of quantum gravity which are not answered by critical string theory, at least in our opinion. One of them is the compatibility of quantum coherence with the existence of singular space-time configurations, such as black holes, which exist as classical solutions of General Relativity, and are therefore expected to appear (at a microscopic Planckian size) also in the appropriate (and yet-to-be discovered) measure of gravitational path integration. Finding a proper treatment of spacetime singularities, encountered in general relativity, within the context of a unitary quantum theory still remains, in our opinion, a challenge for critical string theory. Such a feeling is also supported by the fact that, at present, a consistent quantization of eternally accelerating Universes is also poorly understood in this context [1, 3, 4].

These problems are associated with the fact that in such situations there are space-time boundaries (microscopic or macroscopic) which prevent a proper definition of a scattering $S$ matrix. Since critical string theory is by construction a theory of an $S$ matrix, this presents an incompatibility. One way out of this impasse, therefore, would be to somehow enlarge the definition of ‘string theory’ so that critical string theory is only a limiting case, and the full theory admits consistent quantization but not necessarily through an $S$ matrix.

A possible resolution of this important issue may be provided by an appropriate extension of a concept already encountered in local field theory of open systems. The idea utilizes the concept of the so-called superscattering or $\$-matrix, which was suggested by Hawking [5] as a way of making sense of theories of quantum gravity. Taking into account that any complete theory of quantum gravity should incorporate microscopic space-time singularities (e.g. black holes) in the appropriate path-integral measure over the gravitational field, Hawking argued that such configurations appear to be incompatible, in general, with the maintenance of quantum coherence of matter theories interacting with them. Indeed, once such configurations are formed from the quantum-gravity ‘vacuum’, they can ‘capture’ part of an initially pure matter quantum state, inside their microscopic event horizons. Since a black hole evaporates in quantum theory, according to the celebrated Hawking radiation effect, the relevant information appears be ‘lost’.

Thus, an observer in the asymptotic future, and far away from the black hole, will necessarily observe a mixed quantum state, in analogy with the case of open quantum mechanical systems [6]. This evolution, from an initially pure quantum matter state to a mixed asymptotic state in the presence of microscopic quantum-gravity vacuum singular configurations, results inevitably in the impossibility of treating the quantum gravity vacuum in a standard unitary field theoretic way. The $\$-matrix relates asymptotic density matrix states, which are not pure states in general. In open systems, where there is ‘loss of information’, an asymptotic observer should trace over such states, $\{\mathcal{M}\}$, to obtain a density matrix $\rho = \text{Tr}_{\mathcal{M}}|\Psi><\Psi|$. The $\$-matrix is then defined:

$$\rho_{\text{out}} = \$\rho_{\text{in}}$$

The reader is invited to make a comparison with the definition of the conventional $S$-matrix of closed systems for which $|\text{out} > = S |\text{in} >$. In such systems one can defined a von-Neumann density matrix $\rho = |\Psi><\Psi|$ for which [4] yields a factorizable $\$-matrix: $\$_{\text{closed}} = SS^\dagger$. But in open systems $\$ is not factorizable, $\$ \neq SS^\dagger$, and unitarity of the theory is lost. This has been argued by Hawking to characterize generic quantum-gravity theories. Such features may lead to effects which can be, in principle, observable [6].
However, the above intuitive description, may be incomplete. The apparent ‘loss of information’ may not be a fundamental problem of quantum gravity, given that the above arguments pertain strictly to the dynamics of only part of the complete system, that of matter living in the ‘exterior’ of the (microscopic) black hole horizon. If complete knowledge of the interior spacetimes (and the associated dynamics) of such microscopic singular configurations were known, then it would be possible, in principle, to trace down the information carried by the relevant internal (‘gravitational’) degrees of freedom of the black hole.

String theory [1], and its modern non-perturbative membrane version (D(irichlet) branes) [2], provides the first, and probably the only up to date, mathematically consistent theory where such issues began to be addressed in a rather rigorous way. Macroscopic Black Holes are viewed as solitonic states in this approach, with a rich internal dynamical structure, that can account for a great deal of the information carried by the relevant degrees of freedom. In general terms, stringy black holes carry an infinity of energy (Planckian) levels, which are thus capable of accounting for the enormous entropy associated with such (macroscopic) objects, and thus for the apparent loss of information. An exact counting of the internal states of black holes has been made possible for specific cases [7], using D(irichlet) branes [2].

The underlying duality symmetries are discrete gauge symmetries in string theory, believed to be exact, which help mapping a strongly coupled non-perturbative problem, such as the counting of microstates of a black-hole soliton, to a dual model where perturbative methods (in some sense) can be applied. In some models the horizon of the black hole itself is treated as a $D$-brane. However, such exact (non-perturbative) results are only possible for very specific string backgrounds, and refer only to macroscopic black holes. At present there is no exact treatment, in our opinion, that allows for generic microscopic black hole configurations, which appear in any complete theory of quantum gravity, to be treated in a satisfactory way. From this point of view, the problem of finding a mathematically and physically consistent theory of stringy quantum gravity is still unsolved. To this problem one should add the above-mentioned problem of quantizing eternally accelerating Universes [3] with global particle horizons within a critical string theory framework.

It is our belief that such problems cannot be treated by restricting oneself to equilibrium field theory methods, such as studies of conformal backgrounds of critical string/membrane theory only, which constitute the dominant part of the contemporary string literature. In fact, there are arguments in the string literature [8], supporting the fact that the process of formation and subsequent evaporation of a black hole, microscopic or macroscopic, cannot be described entirely within the framework of equilibrium field theories, but necessarily involve concepts of quantum decoherence. In other words, pure quantum states cannot form black holes. Such arguments were derived within the above-mentioned modern framework of D(irichlet) branes for counting black-hole microstates. In such a framework, one gives a statistical meaning to the black-hole entropy by matching it with a quantity that counts the internal states. Hence, in the relevant $D$-brane calculations [9], the black hole results for the entropy can be matched only by considering a mixed state, including all the degenerate configurations of the $D$-brane. Moreover, in those works it was found that, in order to calculate the absorption cross section of a scalar mode incident on a $D$-brane configuration, one must include a decoherent ensemble of all such degenerate $D$-brane configurations.

An additional argument in favour of a stochastic treatment of stringy quantum black holes is provided by the fact that $D$-brane calculations refer to weak coupling theories, which are assumed to be connected with strongly-coupled gravitational field theory solutions by means of appropriate duality symmetries. Near the high curvature regions of strong gravitational fields, the concept of space time, at least as we perceive it in the context of low-energy physics, may itself break down. In this sense the apparent black hole singularities may be a property only of the truncation of the dynamics to the low-energy Einstein action, which certainly does not describe the strong gravity region of black hole singularities in string theory. The latter may well resemble the analogous situation in the very Early Universe, where the associated strong gravitational field is treated by many researchers in a stochastic (non-equilibrium) way.

In [10] we have taken some modest steps to formulate mathematically such non-equilibrium processes in string theory, by adopting the point of view that departure from equilibrium, in the
context of string theory, is linked to departure from criticality. Non-critical strings appear to be mathematically consistent in the same way as critical strings are, at least from a (perturbative) world-sheet point of view. The idea of non-critical strings can be summarized as follows: there are some deformations in the $\sigma$-model, which cause deviations of the associated world-sheet theory from conformal invariance. Such deviations imply the coupling of another world-sheet field, the Liouville mode $\phi$, whose presence restores conformal invariance.

There are two cases of non-critical strings, depending on whether the central charge of the deformed $\sigma$-model is less or bigger than a critical (fixed-point) value (which is 25 for bosonic strings and 9 for superstrings). In the former case (subcritical strings), the Liouville mode $\sigma$-model kinetic terms appear with space-like signature, i.e. with the same sign as the $\sigma$-model kinetic terms of the space components of the target space time. In the second case (supercritical string), the Liouville mode has a time-like signature (i.e. it appears as a ghost field in the $\sigma$-model).

In the second case the Liouville mode can be considered as a target time variable. In we have developed an approach, according to which a time-like Liouville field is identified with the observable target time. This approach results in a non-equilibrium treatment, which is manifested in many ways. These include entropy production in such theories, and in general the presence of instabilities. Such features can be seen by many researchers as unwanted features. However, instead of discarding such models, we have adopted the point of view that such instabilities are physical, and in fact constitute an important ingredient of the non-critical string approach to Quantum Gravity. As we have argued, this non-equilibrium approach is similar in spirit with standard non-equilibrium approaches of open-field theoretic systems. In fact, in such models one can give a mathematically consistent definition of a superscattering matrix $S$, which is simply given by Liouville correlation functions on the world sheet among the pertinent vertex operators. Upon the identification of the Liouville mode with the target time, however, such correlators do not yield scattering amplitudes, because they fail to factorize into products of $SS^\dagger$. Nevertheless, as objects in string theory, are well defined.

A natural question arises at this point, concerning the predicting power of such Liouville strings. Leaving aside the possibility of resolving conceptually important formal issues of quantum gravity, the big question is whether non-critical strings lead to low-energy properties which could, in principle, distinguish them experimentally from critical string theories. One would hope, and expect, that the situation is analogous to that of local field theory, where non-equilibrium models of field theory can be distinguished from equilibrium ones due to their stochastic properties. For instance, in many of such models there is entropy production and thermal effects, leading to observable (in principle) violations of various properties and symmetries of equilibrium theories, such as Lorentz symmetry.

It is the point of this article to discuss some physical properties of a specific non-critical string theory model, inspired by the modern $D$-brane approach to string theory. Specifically, we shall consider a toy example of a four-dimensional space time punctured by point-like $D$-particle defects. Such a situation may be viewed as a simple case of intersecting branes in string theory. The defects are scattered by a (macroscopic) number of closed string states, propagating in the space time. The scattering of the string/$D$-particle is treated in the semi-classical impulse approximation, according to which the pertinent $\sigma$-model is deformed by operators expressing a ‘sudden’ movement of the defect, at a given time moment. These deformations obey a logarithmic conformal world-sheet algebra, which lie in the border line between conformal field theories and general two-dimensional field theories, so that they can still be classified by means of conformal data. As such, these $\sigma$-model deformations constitute acceptable string theory backgrounds. From a world-sheet renormalization-group view point, such operators are marginally relevant. Hence they violate (slightly) conformal invariance, thereby necessitating Liouville dressing. The dressing results in induced space time deformations in the neighbourhood of the recoiling defect, whose physical properties we study below in detail.

In the present article we shall consider the $D$-particle defects as being real. In this sense, our analysis below is related to quantum gravity considerations only in that we consider quantum metric fluctuations about a specific background space time. In a $\sigma$-model perturbative context this is achieved by summing up world-sheet topologies (genera). The full non-perturbative problem of
considering virtual space time fluctuations in vacuo is not solved, as it lies beyond our calculational capability at present. Nevertheless, as we shall see, the results obtained from the analysis in this simplified problem, look, at least to the authors, sufficiently interesting and non-trivial to motivate further studies along this direction, and also to imply important thermal properties that might characterize space-time foam situations of quantum gravity [15]. As we shall see, one aspect of our analysis will be an effective violation of Lorentz symmetry, precisely due to the aforementioned thermal properties, which will manifest itself as a non-trivial refractive index. But as we shall argue, such effects will not be detectable in the near future. What could be detectable, though, is the emission of very high energy photons from the recoiling \(D\)-particle defects, which could be linked to some extreme astrophysical phenomena, e.g. those associated with ultra-high-energy cosmic rays. Such a possibility implies severe phenomenological constraints on the model, which will be discussed briefly.

The structure of the article is as follows: in section 2 we study some formal aspects of the Liouville (non-critical) string dynamics, with emphasis on demonstrating their non-equilibrium nature. In section 3 we review in some detail a specific example of non-critical string models, in which the non-criticality is a consequence of the scattering of a \(D\)-particle defect, embedded in our four-dimensional space time, with (a macroscopic number of) closed string states in the semi-classical impulse approximation. In section 4 we study the physical properties of the associated metric distortion, which results in the formation of a bubble spacetime, of finite radius around the defect. We pay particular attention to demonstrating the consistency of the approach with the (lowest order) \(\sigma\)-model conformal invariance conditions. This is a result of the restoration of conformal invariance by means of Liouville dressing, under the identification of the Liouville field with the target time. Specifically, we show that the interior of the bubble is consistent with the effective field theory obtained from strings, with non-trivial tachyon-like and antisymmetric tensor fields. However the tachyonic mode expresses an instability of the distorted spacetime, which is argued to exist also in supersymmetric strings. For the validity of our perturbative approach it is essential that the string length is a few orders of magnitude larger than the Planck length. This is a typical situation encountered in many \(D\)-brane-world models. In section 5 we analyse thermal effects and entropy production, and argue on the emission of high energy photons from such geometries. The entropy production is due to the non-trivial matter content of the interior of the bubble, and is associated with information being carried away by the ‘recoil’ degrees of freedom. In section 6 we analyse the motion of particles in the interior of our unstable space time, and demonstrate capture of particles, associated with loss of information for an asymptotic observer. The latter becomes manifest through entropy production, proportional to the area of the unstable space-time bubble. In section 7 we discuss the appearance of effective refractive indices due to the aforementioned non-trivial thermal effects in the interior of the bubble. In section 8 we speculate on a possible link of these geometries to the physics of extreme astrophysical phenomena, in particular to ultra high energy cosmic rays, and argue how such phenomena can be used to constrain some parameters of our non-critical string theory model. In this respect we argue that observable phenomena from such models may only come if one has a certain population of \(D\)-particles in the Universe, and we use recent astrophysical observations to constrain the maximum number of constituent defects in such populations. In section 9 we make some remarks on the important issue of stability of these \(D\)-particle populations, in the modern context of non-perturbative string theory. In section 10 we comment briefly on the rôle of the embedded recoiling \(D\)-particles on the cosmological evolution of a Friedmann-Robertson-Walker type Universe. We argue in favour of an eventual stopping of the cosmic acceleration, thereby implying the absence of cosmological horizons in such models, and hence the possibility of a proper definition of asymptotic states. This is linked to the fact that our non-critical string model asymptotes to a critical (equilibrium) string for large times. We also demonstrate that such cosmological models may be characterised by a relaxing-to-zero vacuum energy a lá quintessence. Finally, concluding remarks are presented in section 11.
2 Generic Non-Equilibrium Aspects of Liouville Strings

We commence our analysis with a brief review of the Liouville dressing procedure for non-critical strings, with the Liouville mode viewed as a local world-sheet renormalization group scale [10]. Consider a conformal \( \sigma \)-model, described by an action \( S^* \) on the world-sheet \( \Sigma \), which is deformed by (non conformal) deformations \( \int \Sigma g^i V_i d^2 \sigma \), with \( V_i \) appropriate vertex operators.

\[
S_g = S^* + \int \Sigma g^i V_i d^2 \sigma \tag{2}
\]

The non-conformal nature of the couplings \( g^i \) implies that their (flat)world sheet renormalization group \( \beta \)-functions, \( \beta^i \), are non-vanishing. The generic structure of such \( \beta \)-functions, close to a fixed point, \( \{ g^i = 0 \} \) reads:

\[
\beta^i = (h^i - 2)g^i + c^i_{jk} g^j g^k + O(g^3) \tag{3}
\]

where \( h^i \) are the appropriate conformal dimensions. In the context of Liouville strings, world-sheet gravitational dressing is required. The “gravitationally”-dressed couplings, \( \lambda^i(g, \phi) \), which from our point of view correspond to renormalized couplings in a curved space, read to \( O(g^2) \) [11, 16]:

\[
\lambda^i(g, \phi) = g^i e^{\alpha_i \phi} + \frac{\pi}{Q + 2\alpha_i} c^i_{jk} g^j g^k \phi e^{\alpha_i \phi} + O(g^3), \quad Q^2 = \frac{1}{3}(c - c^*) \tag{4}
\]

where \( \phi \) is the (world-sheet zero mode) of the Liouville field, and \( Q^2 \) is the central charge deficit, with \( c = c[g] \) the (‘running’) central charge of the deformed theory [17], and \( c^* \) one of its critical values (conformal point) about which the theory is perturbed by means of the operators \( V^i \). Close to a fixed point \( Q^2 \) may be considered as independent of \( g \), but this is not true in general. Finally, \( \alpha_i \) are the gravitational anomalous dimensions:

\[
\alpha_i(\alpha_i + Q) = 2 - h_i \quad \text{for} \quad c \geq c^* \tag{5}
\]

Below we shall concentrate exclusively to the supercritical string case, \( Q^2 \geq 0 \), which from the point of view of identifying the Liouville mode with target time, corresponds to a Minkowskian signature spacetime manifold.

Due to the renormalization [1], the critical-string conformal invariance conditions, amounting to the vanishing of flat-space \( \beta \)-functions, are now substituted by:

\[
\dot{\lambda}^i + Q \lambda^i = -\beta^i(\lambda) + \ldots \quad \text{for} \quad c \geq c^* \tag{6}
\]

where the overdot denotes derivative with respect to the Liouville mode \( \phi \), and the \( \ldots \) denote higher-order terms, quadratic in \( \dot{\lambda}^i \), \( O\left( (\dot{\lambda}^i)^2 \right) \). As we shall argue later, such terms can either be removed by field redefinitions, or alternatively are negligible if one works in the neighbourhood of a world-sheet renormalization-group fixed point, which is the case we shall consider in this work. The notation \( \beta^i(\lambda) \) denotes flat-world-sheet \( \beta \)-functions but with the formal substitution \( g^i \rightarrow \lambda^i(g, \phi) \). Note the minus sign in front of the flat world sheet \( \beta \)-functions \( \beta^i \) in (3), which is characteristic of the supercriticality of the string [13, 16]. Notice that upon the identification of the Liouville mode \( \phi \) with the target time \( t \) the overdot denotes temporal derivative.

In [1] we have treated the Liouville mode as a local (covariant) world-sheet renormalization-group scale. To justify formally this interpretation, one may write

\[
\phi = -\frac{2}{\alpha} \tau, \quad \tau \equiv -\frac{1}{2} \log A, \quad A = \int \Sigma d^2 \sigma \sqrt{\gamma} = \int \Sigma d^2 \sigma \sqrt{\hat{\gamma}} e^{\alpha \phi}, \quad \alpha = -\frac{Q}{2} + \frac{1}{2} \sqrt{Q^2 + 8}, \tag{7}
\]
where $\gamma$ is a world-sheet metric, and \( \hat{\gamma} \) is a fiducial metric, obtained after the conformal gauge choice in terms of the Liouville mode $\phi$. We thus observe that the Liouville mode is associated with the logarithm of the world-sheet area $A$.

Using (7), we can re-write (4) in a standard “flat-world-sheet” renormalization-group form \[ \frac{d}{d\tau} \lambda^i = (\tilde{h}_i - 2)\lambda^i + \pi \tilde{c}_{jk}^i \lambda^j \lambda^k + \ldots, \]

\[ \tilde{h}_i - 2 = -\frac{2}{\alpha_i}, \quad \tilde{c}_{jk}^i = -\frac{2}{\alpha(Q + 2\alpha_i)} c_{jk}^i. \] (8)

which justifies formally the identification \[10\] of the Liouville mode with a local renormalization-group scale on the world sheet. It also implies that the point $\phi \to \infty$ is an infrared fixed point of the flow, in which case the world-sheet area diverges $|A| \to \infty$.

A highly non-trivial feature of the $\beta^i$ functions is the fact that they are expressed as gradient flows in theory space \[17, 18\], i.e. there exists a ‘flow’ function $F[g]$ such that

\[ \beta^i = G^{ij} \frac{\delta F[g]}{\delta g^j} \] (9)

where $G^{ij}$ is the inverse of the Zamolodchikov metric in theory space \[15\], which is given by appropriate two-point correlation functions between vertex operators $V^i, g_{ij} \sim \langle V_i V_j \rangle$. In the case of stringy $\sigma$-models the flow function $F$ may be identified \[18\] with the running central charge deficit $Q^2[g]$.

An important comment we would like to make concerns the possibility of deriving the set of equations (6) from a target space action. This issue has been discussed in the affirmative in \[10\], where it was shown that the set of equations (6) satisfies the Helmholtz conditions for the existence of an action in the ‘space of couplings’ \{g\} of the non-critical string. The property (9) is crucial to this effect. Upon the identification of target time with the Liouville mode \[10\] this action becomes identical with the target space action describing the off-shell dynamics of the Liouville string. We should stress the fact that the action is off shell, in the sense that the on-shell conditions correspond to the vanishing of the $\beta$-functions $\beta^i$, while in our case $\beta^i \neq 0$.

However, the restoration of conformal invariance by the Liouville mode implies that in an enlarged target space time, with coordinates $(\phi, X^0, X^i)$ the resulting $\sigma$-model will be conformal, for which one would have the normal conformal invariance conditions \[1\]. This means that the set of equations (6) can be cast in a conventional form, amounting to the vanishing of $\beta$ functions of a $\sigma$-model, but in this enlarged space:

\[ \beta^{(D+1)}(\lambda) = 0 \] (10)

where $D$ is the target-space dimensionality of the $\sigma$-model before Liouville dressing, and now there are Liouville components as well in appropriate tensorial coordinates.

For fields of the string multiplet, it can be checked explicitly that (10) and (6) (in D-dimensions) are equivalent \[19\]. For completeness, we shall demonstrate this by considering explicitly the dilaton $\Phi$, graviton $G_{\mu\nu}$ and antisymmetric tensor fields $B_{\mu\nu}$. We shall not consider explicitly the tachyon field, although its inclusion is straightforward and does not modify the results. This is because, as we shall see in the next section, in our non-critical string model the tachyon-like mode expresses only the instability of the spacetime metric, and thus in principle it should also appear in supersymmetric strings, which do not have normal (flat-space-time spectrum) tachyons.

To $O(\alpha')$, the appropriate $\sigma$-model $\beta$-functions for a D-dimensional target spacetime, parametrized
by coordinates $X^\mu$, $\mu = 0, 1, \ldots D - 1$, read \[1\]:

$$\grave{\beta}^{G(D)} = \beta^{G(D)} - \frac{1}{4} G^{\mu\nu} \beta^{G(D)}_{\mu\nu} = \frac{1}{6} \left( C^{(D)} - 26 \right),$$

$$C^{(D)} = D - \frac{3}{2} \alpha' \left( R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - 4 (\nabla \Phi)^2 + 4 \nabla^2 \Phi \right),$$

$$\beta^{G(D)}_{\mu\nu} = \alpha' \left( R_{\mu\nu} + 2 \nabla_\mu \nabla_\nu \Phi - \frac{1}{4} H_{\mu\sigma\rho} H^{\nu\sigma\rho} \right),$$

$$\beta^{B(D)}_{\mu\nu} = \alpha' \left( -\frac{1}{2} \nabla_\rho H^\rho_{\mu\nu} + H^\rho_{\mu\nu} \nabla_\rho \Phi \right).$$

(11)

where $H_{\mu\nu\rho} = 3 \nabla_{[\mu} B_{\nu\rho]}$ is the antisymmetric tensor field-strength, on which the $\beta$-functions depend, as dictated by an appropriate Abelian Gauge symmetry \[1\].

To demonstrate that such $\beta$-functions yield equations of the form \[1\], when they are reduced to a target-space manifold with one lower dimension, we separate from the expressions \[1\] a Liouville component. We first note that there is a special normalization of the $\sigma$-model kinetic term of the Liouville field $\phi$ for which \[1\] is valid, which implies that the enlarged spacetime metric is of “Robertson-Walker” form with respect to $\phi$, i.e.:

$$ds^2 = -d\phi^2 + G_{\mu\nu}(\phi, X^\mu) dX^\mu dX^\nu, \quad \mu, \nu = 0, 1, \ldots D - 1$$

(12)

where the Minkowski signature of the Liouville term is due to the assumed supercriticality of the non-critical string \[1\] \[2\]. This implies that for graviton and antisymmetric tensor $\beta$-functions one has:

$$\grave{\beta}^G_{\phi\phi} = \grave{\beta}^G_{\phi\mu} = 0$$

(13)

which are viewed as additional constraints. However, from the point of view of the enlarged space time such constraints can be easily achieved by an appropriate general coordinate transformation, which from our point of view is a renormalization-scheme choice.

We find it convenient to shift the dilaton \[1\]:

$$\Phi \rightarrow \varphi = 2 \Phi - \log \sqrt{G}$$

(14)

In this case we may write \[1\] as follows (to keep consistency with the previous notation we have denoted the $\beta$-functions in the enlarged spacetime $(\phi, X^\mu)$ by $\grave{\beta}$):

$$0 = C^{(D+1)} - 26 = C^{(D)} - 25 - 3 G^{\phi\phi} (\ddot{\varphi} - (\dot{\varphi})^2),$$

$$0 = \grave{\beta}^G_{\phi\phi} = 2 \ddot{\varphi} - \frac{1}{2} G_{\mu\nu} G^{\nu\sigma} \left( \dot{G}_{\mu\nu} \dot{G}_{\nu\sigma} + \dot{B}_{\mu\nu} \dot{B}_{\nu\sigma} \right),$$

$$0 = \grave{\beta}^G_{\mu\nu} = \beta^{G(D)}_{\mu\nu} - G^{\phi\phi} \left( \dot{G}_{\mu\nu} - \dot{B}_{\mu\nu} \right),$$

$$0 = \grave{\beta}^B_{\mu\nu} = \beta^{B(D)}_{\mu\nu} - G^{\phi\phi} \left( \dot{B}_{\mu\nu} - \dot{\varphi} \dot{B}_{\mu\nu} - 2 G_{\mu\nu} \dot{G}^{\kappa\lambda} \dot{G}_{\kappa\lambda} - \dot{B}_{\mu\nu} \dot{B}_{\nu\lambda} \right).$$

(15)

where the overdot denotes total Liouville scale derivative. In our interpretation of the Liouville field as a (local) renormalization scale \[1\] this is equivalent to a total world-sheet renormalization-group derivative.

In Liouville strings \[1\], the dilaton $\Phi$, as being coupled to the world-sheet curvature, receives contributions from the Liouville mode $\phi$ which are linear. In this sense one may split the dilaton field in $\phi$-dependent parts and $X^\mu$ dependent parts

$$\Phi(\phi, X^\mu) = -\frac{1}{2} Q \dot{\varphi} + \hat{\Phi}(X^\mu)$$

(16)
where $Q^2 = \frac{1}{3} (C^{(D)} - 25)$ is the central charge deficit, and the normalization of the term linear in $\phi$ is dictated by the analysis of [11], in which the Liouville mode has a canonical $\sigma$-model kinetic term. This implies that $\varphi$ is such that:

$$\dot{\varphi} = -Q + O(\sqrt{G} \dot{G})$$

(17)

Therefore, the renormalization-group invariance of the central charge $C^{(D)}$, implies $\dot{Q} = 0$. Note that, in the context of the equations (13), the terms in $\dot{\varphi}$ proportional to $\dot{G}_{\mu \nu}$, will yield terms quadratic in Liouville derivatives of fields.

Upon our interpretation of the Liouville field as a (local) renormalization scale terms quadratic in the Liouville derivatives of fields, i.e. terms of order $O(\ddot{G} \dot{B}, \ddot{G} \dot{G}, \ddot{B} \dot{B})$ become quadratic in appropriate $\beta$-functions. Such quadratic terms may be removed by appropriate field redefinitions, provided the gradient flow property (3) is valid, which can be shown to be true for the Liouville local renormalization-group world-sheet scale $\sigma$. Alternatively, one may ignore such quadratic terms in Liouville derivatives of fields by working in the neighbourhood of a renormalization-group fixed point. Such terms are of higher order in a weak-field/ $\sigma$-model-coupling expansion, and thus can be safely neglected if one stays close to a fixed point. This is the case of the specific example of recoiling $D$-particles, where one has only marginal non-criticality for slow-moving heavy $D$-particles, as we shall see in the next section. Ignoring such higher-order terms, therefore, and taking into account world-sheet renormalizability, one obtains

$$\ddot{\varphi} = \dot{Q} + \cdots = 0$$

(18)

where the $\cdots$ denote the neglected (higher-order) terms.

Taking into account that $G_{\phi \phi} = -1$ for supercritical strings, we observe that, as a result of (17), (18), the first two of the equations (13) are satisfied automatically (up to quadratic terms in Liouville derivatives of fields). The first of these equations is the dilaton equation, which thus becomes equivalent to the definition of $Q^2$, and therefore acquires a trivial content in this context. Notice also that the second of these equations is due to the constraints (13), which should be taken into account together with the set of equations (13). It can be shown that the rest of these constraints do not impose further restrictions, and thus can be ignored, at least close to a fixed point, where the constraints can be solved for arbitrary $G_{\mu \nu}, B_{\mu \nu}$ fields. The rest of the equations (13) then, for graviton and antisymmetric tensor fields, reduce to (13), up to irrelevant terms quadratic in Liouville derivatives of fields. This completes our proof for the case of interest.

What we have shown above is that the Liouville equations (13) can be obtained from a set of conventional $\beta$-function equations (10) if one goes to a $\sigma$-model with one more target-space dimension, the extra dimension being provided by the Liouville field. We now stress that the identification of the Liouville mode with the target time $X^0$, which distinguishes the approach of [11] from the standard Liouville approach described above in which $\phi$ was an independent mode, will be made in expressions of the form (10), pertaining to the enlarged $(D + 1)$-dimensional spacetime $(\phi, X^\mu)$. Then, one should look for consistent solutions of the resulting equations in the $D$-dimensional submanifold $(\phi = X^0, X^i)$. In this sense, the target-space dimensionality remains $D$, but the resulting string will be characterized by the Liouville equations (13), supplemented by the constraint of the identification $\phi = X^0$, and will have a non zero central charge deficit $Q^2$, appearing as target-space vacuum energy (13), in contrast to the case of treating the Liouville mode as an independent coordinate.

To put it in other words, one starts from a critical $\sigma$-model, perturbs it by the recoil deformation, induces non-criticality, but instead of using an extra Liouville $\sigma$-model field, one uses the existing time coordinate as a Liouville mode, i.e. one invokes a readjustment of the time dependence of the various background fields (a sort of back reaction), in order to restore the broken conformal invariance. It is a non-trivial fact that there are consistent solutions to the resulting equations, and this is the topic of the present work, namely we shall consider a specific model of non-critical strings, in which we shall identify the time with the Liouville mode, and we shall present consistent
solutions of \( \mathcal{L} \), under the constraint \( \phi = X^0 \), to \( \mathcal{O}(\alpha') \) in the respective \( \sigma \)-model. This is done in the next two sections.

We would now like to comment briefly on the breakdown of the interpretation of world-sheet correlators in Liouville strings as unitary scattering amplitudes in the target space time. This is one of the most important features which makes Liouville strings non-equilibrium string theories \([10]\). Below we recall briefly our formal arguments \([10]\) supporting the point of view that it is impossible to define a unitary \( S \) matrix in Liouville strings, with the only well-defined object being the \( S \) matrix. This follows from the fact that an \( N \)-point world-sheet correlation function of vertex operators in Liouville strings, \( \mathcal{F}_N \equiv \langle V_{i_1} \cdots V_{i_N} \rangle \), where \( \langle \cdots \rangle \) signifies a world-sheet expectation value in the standard Polyakov treatment, transforms under infinitesimal Weyl shifts of the world-sheet metric \( \gamma \) in the following way:

\[
\delta_{\text{weyl}} \mathcal{F}_N = \left[ \delta_0 + \mathcal{O} \left( \frac{s}{A} \right) \right] \mathcal{F}_N
\]  

(19)

where the standard part \( \delta_0 \) of the variation involves a sum over the conformal dimensions \( h_i \) of the operators \( V_i \) that is independent of the world-sheet area \( A \) (whose logarithm is the world-sheet zero mode of the Liouville field). The quantity \( s \) is the sum of the gravitational anomalous dimensions \([11]\):

\[
s = - \sum_{i=1}^{N} \frac{\alpha_i}{\alpha} - Q \alpha, \quad \alpha_i = - \frac{Q}{2} + \frac{1}{2} \sqrt{Q^2 + 4(2 - h_i)} \quad \alpha = - \frac{Q}{2} + \frac{1}{2} \sqrt{Q^2 + 8}
\]  

(20)

where \( Q^2 \) is the central-charge deficit, that is non-zero for non-critical string.

From \([10]\), and upon the identification of the logarithm of the world-sheet area in the target space, as a result of the explicit \( A \) (i.e. time \( t \)) dependence. Nevertheless, according to the analysis in \([10]\), it is clear that one can construct a well-defined quantity free from such world-sheet area ambiguities. However, this is a \( S \) matrix that is not factorizable into a product of \( S \) and \( S \dagger \). This results from the way one performs the world-sheet path integration over the Liouville mode \( \int D\phi(\ldots) \) in a first-quantized approach to string theory. This integration can be done via a steepest-descent method \([10, 22]\), along the curve in Fig. 1 which is reminiscent of the closed time-like path used in non-equilibrium field theories \([12]\). Close to the ultraviolet fixed point on the world sheet, i.e., in the limit when \( A \to 0 \), there are divergences. These are responsible for the lack of factorization of the \( S \) matrix in this case.

The definition of the Liouville path integral over such a curve is responsible, in this formalism, for a density-matrix rather than a wave-function interpretation of the world-sheet partition function of the Liouville string, in full agreement with its non-equilibrium (open-system) nature. Indeed, the effective action \( \Gamma[g'] \) in the space of couplings \( g' \) of the closed string theory under
Where the identification of the Liouville mode $\phi$ to prevent a consistent definition of a unitary of the accelerating Universe puzzle, where the presence of a cosmological (particle) horizon seems a matrix, provides an argument that non-critical string theory may be the key \cite{4} to a resolution of the matrix $\rho$, models of quantum gravity plagued by microscopic singular fluctuations with event horizons.

Consideration is connected to the Liouville path integral as follows: $$e^{-\Gamma[g]} = \int D\phi DX^i e^{-\int d^2x \mathcal{L}(g, \phi=\delta H, X)}$$ (21)

where $\mathcal{L}$ is the Liouville-dressed world-sheet Lagrangian of the deformed $\sigma$-model (2) after the identification of the Liouville mode $\phi$ with the target time $t$. The extension of the Liouville zero mode integration over the contour of figure \cite{3} implies that the time integration in $\Gamma[g] = \int dt L(g(t), g'(t), t)$ will result in products $\Psi(g', t)\Psi^*(g', t)$ at each time moment, with $\Psi(g', t)$ the wave function at a fixed slice of the Liouville time, being defined along, say, the upper side curve of figure \cite{3} with its complex conjugate $\Psi^*(g', t)$ being defined on the lower side curve. This yields a probability density interpretation of the Liouville $\sigma$-model partition function in the space of couplings $g^i$. In the case of recoil we are considering in the next section, the couplings $g^i$ correspond to collective coordinates of the $D$-particles.

This impossibility of defining a consistent unitary $S$-matrix element in Liouville string, but only a $\mathcal{S}$ matrix, provides an argument that non-critical string theory may be the key \cite{3} to a resolution of the accelerating Universe puzzle, where the presence of a cosmological (particle) horizon seems to prevent a consistent definition of a unitary $S$ matrix \cite{3}, or in general to quantize consistently models of quantum gravity plagued by microscopic singular fluctuations with event horizons.

Thus, in Liouville strings, the state of the observable system may be characterized by a density matrix $\rho$. The full density matrix of string theory might in principle be a pure-state density matrix $\rho = \vert \Psi > < \Psi \vert$. However, in our effective non-critical string picture $\vert \Psi >= \vert \psi, \psi >$, where $\vert \psi >$ denotes a state of the observable system and $\vert \psi >$ the unobserved degrees of freedom that are integrated out in the renormalization-group approach. The reduced observable density matrix $\tilde{\rho} \equiv \int d\psi \rho = \int d\psi |\psi, \psi > < \psi, \psi|$ will in general be mixed as a result of entanglement with the unobserved degrees of freedom $\vert \psi >$, e.g., those that disappear across the macroscopic event horizon in an accelerating Universe, or across a microscopic event horizon in a model of space-time foam.

The renormalization-group equation for the reduced density matrix $\tilde{\rho}$ is easy to derive and can be cast in the form

$$\dot{\tilde{\rho}} = i[\tilde{\rho}, H] + \mathcal{H} \tilde{\rho}$$ (22)

with an explicit form for the non-Hamiltonian operator $\mathcal{H}$ in terms of the non-conformal field couplings $g^i$ \cite{10}:

$$\mathcal{H} \tilde{\rho} = \beta^i G_{ij}[g^j, \tilde{\rho}]$$ (23)

where the $\beta^i$ are the non-trivial world-sheet renormalization-group functions of the couplings/fields $g^i$. We remind the reader that a consistent quantization of the couplings $g^i$, which allows one to view them as quantum-mechanical operators as above, is provided by a summation over world-sheet genera \cite{14}. Notice that an equation of the form (22) is familiar from the theory of quantum-mechanical systems in interaction with an environment \cite{3}, that is provided in our case by the modes $\vert \psi >$ that disappear across the event horizons.

A final comment we would like to make concerns entropy growth, which is another generic non-equilibrium feature of non-critical strings \cite{10}. Indeed, whenever one has entanglement with an environment over which one integrates, as in the non-critical string formalism, one necessarily encounters entropy growth. Within our Liouville renormalization-group formalism, there is a simple formula for the rate of growth of the entropy $S$ \cite{11}:

$$\dot{S} \propto G_{ij} \beta^i \beta^j$$ (24)

which is positive semi-definite. If any of the renormalization functions $\beta^i \neq 0$, entropy will necessarily grow. Notice that in view of the Zamolodchikov’s $c$-theorem \cite{17}, the rate of change of the entropy $S$ is proportional to the rate of change (along a world-sheet renormalization-group trajectory) of the effective (‘running’) central charge deficit. This is in accordance with the fact...
that the deficit counts effective degrees of freedom of the relevant \( \sigma \)-model \cite{17, 23}. Thus, in models with space-time boundaries, there is a natural change in the available degrees of freedom as time evolves, given that states may cross these boundaries and become unobservable from an asymptotic observer’s point of view.

We are ready now to demonstrate the above features by looking at a specific example of induced non-criticality in string theory. As we shall see, this specific example has the virtue of allowing us to approach these non-equilibrium aspects of Liouville strings in a more conventional way i.e. using techniques which lie closer to open-system field theory concepts.

3 A Specific Non-Critical String Model: Recoil-Fluctuating \( D \)-particles Embedded in the Universe

We consider for definiteness the situation depicted in figure 2. Our four-dimensional world is viewed as a three-brane domain wall, punctured by \( D \)-particle defects. From the modern non-perturbative string-theory perspective, this is an example of intersecting brane configurations. On the four-dimensional world we have propagating closed strings. When a (macroscopic) number of them strikes a \( D \)-particle, the latter 'recoils'. The recoil is treated in the impulse approximation, which proves sufficient for the case of heavy \( D \)-particle defects, and weakly coupled string theories, to which we restrict ourselves for the purposes of this work. Throughout our approach we shall ignore bulk fluctuations of the D3 brane worlds, which will be assumed rigid. Such assumptions are compatible with the orbifold constructions of \cite{24}, which have attracted enormous attention recently, in view of their capability of providing a resolution to the mass hierarchy problem on the observable world. We note that fluctuations of the D3 brane world along the bulk directions have been considered, in the present context, in \cite{25}.

From a \( \sigma \)-model viewpoint, the ‘recoil’ of the \( D \)-particle is described by adding the following
deformation to the $\sigma$-model Lagrangian, describing fluctuations of the $D$-brane defect \cite{13, 14}:

$$\mathcal{V}_i = \int_{\partial \Sigma} (\epsilon y_i + u_i X^0) \Theta_{\epsilon}(X^0) \partial_n X^i$$

(25)

where $\partial \Sigma$ denotes the world-sheet boundary, $X^0, X^i, i = 1, 2, 3$ are four-dimensional $\sigma$-model fields, whose zero modes are the space-time coordinates ($X^0$ is the time), $y_i$ describes the initial location of the $D$-particle defect in space, $u_i$ is the recoil velocity in a Galilean approximation due to the heaviness of the defects, and $\partial_n$ denotes the normal derivative on the world-sheet boundary. The deformation \cite{25} describes recoil fluctuations of the $D$-particle in a comoving frame. It can be proven rigorously within the $\sigma$-model framework \cite{14} that energy and momentum conservation is respected by the recoil operators (25), and hence the velocity $u_i = g_s \frac{\Delta p_i}{M_s}$, where $\Delta p_i$ is the momentum transfer during the scattering, $M_s$ is the string scale, $g_s < 1$ is the string coupling, and $M_s/g_s$ is the mass of the $D$-particle. The operators $\Theta_{\epsilon}(X^0)$ describe the impulse approximation, and are regularized Heaviside operators \cite{13}:

$$\Theta_{\epsilon}(X^0) = \frac{1}{i} \int \frac{d\omega}{\omega - i\epsilon} e^{i\omega X^0} \quad \epsilon \to 0^+$$

(26)

The time coordinate $X^0$ obeys Neumann boundary conditions, while the coordinates $X^i, i = 1, \ldots, 3$ obey Dirichlet boundary conditions \cite{2}. In the limiting case $\epsilon \to 0^+$ only the second part of the operator (25), proportional to the $\sigma$-model coupling $u_i$ is dominant.

The impulse approximation to recoil may be thought of as the result of the incidence of a macroscopic number of closed string states on the defect. The impulse approximation is semiclassical, which, however, proves sufficient for a treatment of heavy $D$-particles, in the context of weakly coupled strings with couplings $g_s \ll 1$, we are dealing with here.

By studying the appropriate operator products of the operators (25) with the world-sheet stress energy tensor, it can be shown \cite{13} that the operators are relevant from a world-sheet renormalization-group point of view, with anomalous dimension

$$\Delta = -\frac{\epsilon^2}{2}.$$ 

As such, they cause departure from conformal invariance of the deformed $\sigma$-model, and thus the need for Liouville dressing \cite{11}.

Before doing so, we remark that the deformations (25) obey a logarithmic conformal algebra on the world sheet \cite{15}, provided one has the following relation between the regulating parameter $\epsilon^2$ and the size $L$ of the world-sheet disc, where the analysis is performed \cite{13}:

$$\epsilon^2 \sim \log (L/a)^2$$

(28)

In the above formula, the (omitted) constant proportionality coefficients are of order one, $L$ is the size of the world sheet, and $a$ is an ultraviolet world-sheet cut-off. The logarithmic pair is provided in this case by splitting the operator (25) into position and momentum parts:

$$C(z) = \epsilon y_i \Theta_{\epsilon}(X^0(z)) \partial_n X^i(z), \quad D(z) = \epsilon u_i X^0(z) \Theta_{\epsilon}(X^0(z)) \partial_n X^i(z),$$

(29)

with the following operator product expansion on the world-sheet boundary \cite{13, 14}:

$$< C(z) C(w) > \sim \mathcal{O}(\epsilon^2) \to 0 \quad < C(z) D(w) > \sim \frac{1}{|z - w|^2} \quad < D(z) D(w) > \sim \frac{\log \left( \frac{|z - w|}{L} \right)}{|z - w|^2}$$

(30)

with $z \to w$. These are the relations of logarithmic conformal algebras \cite{25}, which lie in the border line between conformal field theories and general (renormalizable) two-dimensional field
theories. The presence of the logarithmic algebra is intimately connected with the fact that the impulse of the $D$-particle defect implies a sudden change in the background of the $\sigma$-model, and as such it cannot be described by a conformal field theory. Nevertheless, the existence of a logarithmic algebra, which still allows the classification of the appropriate deformations by means of conformal blocks and other conformal field theory data, implies that such deformed $\sigma$-models constitute acceptable backgrounds of string theory.

In the Liouville context, by viewing the Liouville field as a local renormalization-group scale on the world-sheet, as appropriate for curved-space renormalization, the right-hand-side of (28) may be connected to the zero mode of the Liouville field. As such, we observe from (25) that one encounters a situation with running anomalous dimensions for the recoil operators (25). The asymptotic point $\epsilon \to 0^+$ corresponds to a conformal theory. By performing the contour integral of the $\Theta_\epsilon(X^0)$ we obtain the asymptotic behaviour $(X^0 > 0) \Theta_\epsilon(X^0) \sim e^{-\epsilon X^0}$, from which we may conclude that

$$\frac{1}{\epsilon} \sim X^0 \equiv t \gg 0$$

This implies that asymptotically in time our non-critical (non-equilibrium) string theory of recoiling $D$-particles relaxes to a conformal (equilibrium) theory. We shall come back to this point in section 10, when we discuss possible cosmological implications of this property.

Let us now discuss the physical implications of such a Liouville dressing procedure in the context of the above model. For instructive purposes let us first study the simplified case in which the $D$-particle defect is recoiling along one direction, say $z$, in space. From a modern viewpoint the $D$-particle defect is viewed as a zero-space universe (domain ‘wall’), moving in time $X^0$, and the $z$ direction of recoil is viewed as a ‘bulk’ dimension [25]. The situation will be easily generalized to three spatial ‘bulk’ dimensions, as we shall see later on. For reasons of formal convergence of world-sheet path integrals we also assume the time $X^0$ to be Euclidean [13, 25]. As we shall see, the Liouville formalism generates automatically a Minkowskian signature, even in such Euclidean models [10, 13, 25].

To determine the effect of Liouville dressing on this space-time geometry, we write the boundary recoil deformations as bulk world-sheet deformations [1]

$$\int_{\partial \Sigma} u_z X^0 \Theta_\epsilon(X^0) \partial_n z = \int_{\Sigma} \partial_\alpha (u_z X^0 \Theta_\epsilon(X^0) \partial^\alpha z)$$

The bulk world-sheet Liouville-dressed operators read [11]:

$$\mathcal{V}_{z,L} = \int_{\Sigma} e^{\alpha_{0z} z} \phi \partial_\alpha (u_z X^0 \Theta_\epsilon(X^0) \partial^\alpha z)$$

where $\phi$ is the Liouville mode, and $\alpha_{0z}$ is the world-sheet ‘gravitational’ anomalous dimension [11]:

$$\alpha_{0z} = -\frac{Q_b}{2} + \sqrt{\frac{Q_b^2}{4} + \epsilon^2}$$

where $Q_b$ is the central-charge deficit of the bulk world-sheet theory. In the recoil problem at hand, the deviation from conformal invariance is due exclusively to the recoil deformations (25). In this case one can estimate [28]:

$$Q_b^2 \sim \epsilon^2 u_z^2 > 0$$

for weak deformations $u_z \ll 1$, and hence one encounters a supercritical Liouville theory. However, due to the smallness of $\epsilon, u_z$ one stays in the neighbourhood of a renormalization-group fixed point, and hence a perturbative analysis, along the lines discussed in the previous section, is valid. The

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1Equivalently, one can dress the boundary non-critical deformations (25) directly, using methods of boundary non-critical strings. The formal equivalence of the two approaches can be established easily [27].
supercriticality implies a *Minkowskian signature* Liouville-field kinetic term in the respective $\sigma$-model [24], which prompts one to interpret the Liouville field as a time-like target-space field. At present we shall treat the Liouville mode as a second time coordinate [11], which is different from the (Euclideanized) $X^0$. The identification of $\phi = X^0$ will be made eventually. From the expression (33) we conclude (c.f. (34)) that $\alpha_u \sim \epsilon$ to leading order in perturbation theory in $\epsilon$ and $u_z$, to which we restrict ourselves here.

We next remark that, as the analysis of [24] indicates, the $X^0$-dependent field operators $\Theta_\epsilon(X^0)$ scale as follows with $\epsilon$: $\Theta_\epsilon(X^0) \sim e^{\epsilon X^0} \Theta(X^0)$, where $\Theta(X^0)$ is a Heaviside step function without any field content, evaluated in the limit $\epsilon \rightarrow 0^+$. The bulk deformations, therefore, yield the following $\sigma$-model terms:

$$\frac{1}{4\pi \ell_s^2} \epsilon u_z X^0 e^{\epsilon(\phi(0)-X^0(0))} \Theta(X^0(0)) \int_{\Sigma} \partial_0 X^0 \partial^a z \tag{36}$$

where the subscripts (0) denote world-sheet zero modes.

Upon the interpretation of the Liouville zero mode $\phi(0)$ as a (second) time-like coordinate, the deformations (34) yield space-time metric deformations (of the generalized space time with two times). The induced metric components can be interpreted [24] as expressing the distortion of the space-time surrounding the recoiling $D$-brane soliton.

For clarity, we now drop the subscripts (0) for the rest of this paper, and we work in a region of space-time on the $D3$ brane such that $\epsilon(\phi - X^0)$ is finite in the limit $\epsilon \rightarrow 0^+$. The resulting space-time distortion is therefore described by the metric elements

$$G_{\phi \phi} = -1, \quad G_{zz} = 1, \quad G_{00} = 1 \text{(Euclideanized } X^0),$$

$$G_{\phi z} = \epsilon u_z X^0 \Theta(X^0), \tag{37}$$

where the index $\phi$ denotes Liouville ‘time’, not to be confused, at this stage, with the original Euclideanized time $X^0$ (index 0).

The presence of $\Theta(X^0)$ functions (impulse) and the fact that we are working in the region $z > 0$ indicate that the induced space-time is piecewise continuous. The important implications for non-thermal particle production and decoherence for a spectator low-energy field theory in such space-times were discussed in [28, 29].

We now make some important remarks about the metric (37). Upon performing the transformation $\phi \rightarrow \phi - \frac{\epsilon}{2} \epsilon u_z X^0 z$, which from a world-sheet point of view, under the identification of the Liouville mode $\phi$ with a local world-sheet scale [11], may be considered as a renormalization-scheme choice, the line element corresponding to (37) becomes, for $u_z \ll 1, \epsilon \epsilon u_z \ll 1$ [25]:

$$ds^2 = -d\phi^2 + \left(1 - b'^2 \right) (dX^0)^2 + \left(1 + b'^2 (X^0)^2 \right) dz^2 - 2 u_z z dX^0 d\phi,$$

$$b' = \epsilon u_z, \quad u_z = g_s |\Delta p_z|/M_s$$

where $\Delta p_z$ is the momentum transfer along the bulk direction.

For our purposes of eventually applying the results presented here to models of space-time foam, in which virtual $D$-brane configurations are emerging out of the vacuum, we are interested in the case in which there is no world-sheet tree level momentum transfer. This naively corresponds to the case of static intersecting branes. However, the whole philosophy of recoil [13, 14] implies that even in that case there are quantum fluctuations induced by *summing up genera* on the world-sheet. The latter implies the existence of a statistical distribution of logarithmic deformation couplings of Gaussian type about a mean field value $\overline{\sigma}^2 = 0$. Physically, the couplings $u_z$ represent recoil velocities of the intersecting branes, hence the situation of a Gaussian fluctuation about a zero mean value represents the effects of quantum fluctuations about the zero recoil velocity case, which may be considered as a quantum correction to the static intersecting brane case. Such Gaussian

\[2\text{Note that the eventual identification } \phi = X^0 \text{ is thereby consistent with this feature, as yielding a null contribution.}\]
quantum fluctuations arise quite naturally by summing up higher world-sheet topologies \[14\], in particular pinched annuli, which have been argued to be the dominant configurations, and were shown to exponentiate. We therefore consider a statistical average \[\langle \cdots \rangle\] of the line element

\[
\langle \cdots \rangle = \int_{-\infty}^{+\infty} du_z \left( \sqrt{\pi \Gamma} \right)^{-1} \langle \cdots \rangle e^{-u_z^2/\Gamma^2}
\]

where

\[
\langle \cdots \rangle = \left( \frac{1}{4} \right) \left( 1 - \frac{1}{4} \epsilon^2 \langle u_z^2 \rangle \right) (dX^0)^2 + \left( 1 + \frac{1}{4} \epsilon^2 \langle u_z^2 \rangle \right) (X^0)^2 dz^2 - 2 \langle u_z \rangle \langle X^0 \rangle d\phi,
\]

and the width \( \Gamma \) has been calculated in \[14\], after proper summation over world-sheet genera, and is found to be proportional to the string coupling \( g_s \).

From \[40\] the average line element \( ds^2 \) becomes:

\[
\langle ds^2 \rangle = -d\phi^2 + \left( 1 - b' \epsilon \langle u_z \rangle \right) (dX^0)^2 + \left( 1 + b' \epsilon \langle u_z \rangle \right) (X^0)^2 dz^2,
\]

where

\[
b' = \frac{1}{2} \Gamma \]

The definition of \( b' \) comes from evaluating the quantity \( \langle u_z^2 \rangle \) using the statistical distribution \[40\]. Thus, in that case, averaging over quantum fluctuations leads to a metric of the form \[88\], but with a parameter \( b' \) much smaller, being determined by the width (uncertainty) of the pertinent quantum fluctuations \[14\]. The metric \[88\] is exact, in contrast to the metric \[88\] which was derived for \( z \ll 1/b' \).

The width \( b' \) expresses a momentum uncertainty of the fluctuating D-particle. Such uncertainties in general depend on the (kinetic) energy content of the non-relativistic heavy D-particle defect. Even in isotropic situations, in which there is no tree-level average momentum for the D-particle, \( \overline{\overline{p}} = 0 \) such an energy dependence is non-trivial. For details we refer the reader to ref. \[14\]. For our purposes below we simply quote the result for low energies \( E \) compared to string scale \( M_s \). In this case, the parameter \( b' = b'(E) \) is given by:

\[
b'^2(E) = \epsilon^2 g_s^2 \left( 1 - \frac{285}{18} \frac{g_s^2}{M_D} \frac{E}{M_s} \right)
\]

where \( E \) denotes the kinetic energy of the recoiling D-particle, \( g_s \) is the string coupling, assumed weak, and \( M_D = M_s/g_s \) is the D-particle mass, which is formally derived in the logarithmic conformal field theory approach from energy-momentum conservation \[14\].

It should be remarked at this point that, throughout this work, we shall be working with \( g_s \) small but finite. The limit \( g_s \to 0 \) will not be considered, given that when \( g_s = 0 \) the mass of the D-particle \( M_D \to \infty \), and thus the recoil is absent, but on the other hand the curvature of the surrounding spacetime, as a result of the immense mass of the D-particle, should be taken into account. In that limit the scale \( b'(E) \to 0 \), and thus one can no longer consider distances sufficiently far away from the center of the infinite gravitational attraction so that the Schwarzschild curvature effects of the D particle could be ignored.

As one observes from \[12\], the value of \( b' \) decreases with increasing energy, and formally vanishes when the energy is close to \( M_s \). We should note, however, that the above expression \[12\] pertains strictly to slowly moving strings, i.e. \( E \ll M_s \). In general, for arbitrary energies (including intermediate ones, which we shall be interested in below), the precise expression for \( b'(E) \) is not known at present. For our purposes, we shall assume that \( b'(E) \) decreases with increasing \( E \) for all energies. This will be justified later on.
An important feature of the line element (41) is the existence of an horizon at $z = 1/b'$ for Euclidean Neumann coordinates $X^0$. The presence of an horizon raises the issue of how one could analytically continue so as to pass to the space beyond the horizon. The simplest way, compatible, as we shall show later with the low-energy Einstein’s equations, is to take the absolute value of $1 - b'^2 z^2$ in the metric element (41). We therefore consider the following metric defined in all space $z \in \mathbb{R}$:

$$ds^2 = -d\phi^2 + \left| 1 - b'^2 z^2 \right| (dX^0)^2 + \left( 1 + b'^2 (X^0)^2 \right) dz^2,$$

(43)

For small $b'$, which is the case studied here, and for Euclidean Neumann coordinates $X^0$, the scale factor in front of the $dz^2$ term does not introduce any singular behaviour, and hence for all qualitative purposes we may study the following metric element:

$$ds^2 = -d\phi^2 + \left| 1 - b'^2 z^2 \right| (dX^0)^2 + dz^2,$$

(44)

which is expected to share all the qualitative features of the full metric (43) induced by the recoil process in the case of an uncompactified ‘bulk’ Dirichlet dimension $z$ we restrict ourselves here.

The extension of (44) to four dimensions is straightforward, and has been made in [30]. In this case one views the $D$-particle as a zero-(spatial)-dimensional ‘wall’ embedded in a three-dimensional ‘bulk’ space. Here the terminology ‘bulk’ pertains to the longitudinal dimensions of the D3 brane in figure 2, and should not be confused with the real bulk transverse dimensions of the D3 brane domain wall. The four-dimensional analogue of (44) is obtained upon making the following substitutions in the expression for the line element:

$$z^2 \rightarrow r^2 = \sum_{i=1}^{3} x_i^2$$

$$dz^2 \rightarrow \sum_{i=1}^{3} dx_i^2$$

(45)

An important comment concerns the relation of the Liouville mode $\phi$ with the time $t$, under (31). As already mentioned, in Liouville strings [11], for which (5), (12) are valid, there is an implicit normalization of the Liouville mode $\phi$, which yields a canonical kinetic term in the $\sigma$-model action. This normalization implies that the world-sheet zero mode of the Liouville field $\phi$ is connected to the area of the world-sheet $(L/a)^2$ via:

$$\phi \sim Q_b \log(L/a)^2 \sim \epsilon^{-1}$$

(46)

where $Q_b$ is the square root of the central charge deficit of the bulk world-sheet theory, which in our case is of order $\epsilon$ (33), given that the only source of non-criticality is the recoil of the $D$-particle.

From (44) we thus observe that, for large times after the impulse, the Liouville mode and the target time scale similarly. This prompts one to identify these two fields. This is a crucial ingredient of the approach adopted in [10, 30], which is different from the treatment in [25] where the Liouville mode had been fixed to a certain value. Setting $\phi = X^0 = t$ implies, on account of (28), (46), that $\epsilon^{-1}$ will scale as the physical time $t$ [13, 24], for large times after the impulse where our perturbative world-sheet approach is valid. This was expected from (31), and thus it provides a nice consistency check of the approach.

This identification, implies, that inside the initial horizon

$$r < t/b'(E),$$

(47)

where we have redefined $b'$ to be:

$$b'^2(E) = g_s^2 \left( 1 - \frac{285}{18} g_s^2 \frac{E}{M_D} \right)$$

(48)
the induced space-time metric (44), reads (30):

\[ ds^2 = \frac{b^2 r^2}{t^2} dt^2 - \sum_{i=1}^{3} dx_i^2, \quad r^2 = \sum_{i=1}^{3} x_i^2 \]  

(49)

A careful analysis (30) shows that in this case the induced space time acquires a bubble structure, which, as we shall discuss in the next section, matches smoothly a flat Minkowski space time outside the region bounded by the horizon (47), (48). An important point is that this space time is unstable. This can be easily seen by examining the positive energy conditions (30), which are valid only in the interior of the bubble, being violated outside the region (47). For instance the weakest of these conditions

\[ T_{\mu\nu} \zeta^\mu \zeta^\nu \geq 0, \]  

(50)

where \( \zeta^\mu \) is an arbitrary light-like (null) four vector, is easily seen to be valid only inside the region (47) in the context of the space time (44) under the constraint of the identification of time with the Liouville mode \( \phi \). In the next section we shall try to remedy this by demanding that the bubble space time is matched smoothly at the horizon boundary \( r = t/b'(E) \). However, as we shall see the instabilities will remain, in the sense that they will now appear as ‘tachyon-like’ scalar excitations of the string multiplet inside the bubble.

It is the purpose of this work to discuss the physical implication of such instabilities. As we have discussed above, and in (25, 30), the spacetime (44) may be considered as a mean field result of appropriate resummation of quantum corrections for the collective coordinates of the recoiling \( D \)-particle. To lowest order in a weak string \( (g_s < 1) \) \( \sigma \)-model perturbative framework, such quantum fluctuations may be obtained by resumming pinched annuli world-sheets, which can be shown to exponentiate (14), thereby providing a Gaussian probability distribution over which one averages. The metric (19) is the result of such an average, and the eventual identification of the Liouville mode with the target time (10). The resulting instabilities, therefore, may not be considered as pathological, but a characteristic feature of quantum fluctuating space times.

In order for the above procedure to be consistent, the resulting effective field theory must satisfy the appropriate \( \sigma \)-model conformal invariance conditions, which in a target-space framework correspond to appropriate equations of motion derived from a string-effective action. In this work we shall demonstrate that this is indeed the case, at least to lowest order in a Regge slope \( \alpha' \) perturbative expansion where we restrict ourselves. We shall also study some physically important properties of the spacetime (19). We shall argue that an inevitable result of the unstable nature of this spacetime is the emission of high-energy radiation. Then we shall speculate on possible astrophysical applications of this phenomenon. Specifically, we shall argue that, as a consequence of the high-energy-photon emission from the unstable bubble, the neighbourhood of the recoiling \( D \)-particle defect may behave as a source of ultra-high-energy particles, which can reach the observation point, provided that the defect lies at a distance from Earth which is within the respective mean-free paths of the energetic particles. This effect, then, may have a potential connection with the recently observed (31) apparent ‘violations’ of Greisen-Zatsepin-Kuzmin (GZK) cutoff (32).

4 Dynamical Properties of the Recoil (Bubble) Spacetime

We now commence our study of the most important properties of the space time (19). It is our aim in this section to demonstrate that such space times are consistent solutions of the \( \mathcal{O}(\alpha') \) conformal invariance conditions of appropriately deformed closed-string sectors of the \( \sigma \)-model, with target spacetimes dimensionally reduced to four dimensions.

To this end, we remark that, in a four-dimensional spacetime, obtained by appropriate compactification of the higher-dimensional spacetime, where string theory lives, the string massless multiplet consists of a graviton field \( g_{\mu\nu} \), a dilaton \( \Phi \) and an antisymmetric tensor field \( B_{\mu\nu} \), which in four dimensions gives rise, through its field strength \( H_{\mu\nu\rho} = 3 \partial_{[\mu} B_{\nu\rho]} \), to a pseudoscalar axion
field $b$ (not to be confused with the uncertainty parameter $b'$). The latter is defined as follows:

$$H_{\mu\nu\rho} = \frac{1}{\sqrt{-g}} \epsilon_{\mu\nu\rho\sigma} \partial_{\sigma} b$$

(51)

What we shall argue below is that the spacetime (49) is compatible with the equations of motion obtained from a string effective action for the above fields. Equivalently, these equations are the $\sigma$-model conformal invariance conditions to leading order in the Regge slope $\alpha'$. We stress once again the fact that the spacetime metric (49) has been derived upon the non-trivial assumption that the target time is identified with the Liouville field $\alpha$, whose presence is necessitated by the recoil [30, 13, 14]. The fact, to be demonstrated below, that this identification is consistent with the $\sigma$-model conformal invariance conditions to $\mathcal{O}(\alpha')$, is therefore a highly-non-trivial consistency check of the approach.

The components of the Ricci tensor for the metric (49) are:

$$R_{00} = -\frac{2b'^2}{t^2}, \quad R_{ij} = \frac{\delta_{ij}}{r^2} - \frac{x_i x_j}{r^4}, \quad i, j = 1, 2, 3.$$  

(52)

The curvature scalar, on the other hand, reads:

$$R = -\frac{4}{r^2}$$

(53)

which is independent of $b'(E)$ and singular at the origin $r = 0$ (initial position of the $D$-particle). Thus, we observe that the spacetime after the recoil acquires a singularity. However, our analysis is only valid for distances $r$ larger than the Schwarzschild radius of the massive $D$-particle, and hence the locus of points $r = 0$ cannot be studied at present within the perturbative $\sigma$-model approach. It is therefore unclear whether the full stringy spacetime has a true singularity at $r = 0$.

The Einstein tensor $G^E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ has components:

$$G^E_{00} = 0, \quad G^E_{ij} = -\frac{\delta_{ij}}{r^2} - \frac{x_i x_j}{r^4}$$  

(54)

The conformal invariance conditions for the graviton mode of the pertinent $\sigma$-model result in the following Einstein’s equations as usual:

$$G^E_{\mu\nu} = -T_{\mu\nu}$$

(55)

where $T_{\mu\nu}$ contains contributions from string matter, which in our case includes dilaton and antisyymmetric tensor (axion) fields, and probably cosmological constant terms (which will turn out to be zero in our case, as we shall see later on). As we shall also show, there are tachyonic modes necessarily present, which, however, are not the ordinary flat-spacetime Bosonic string tachyons. In fact, despite the fact that so far we have dealt explicitly with bosonic actions, our approach is straightforwardly extendible to the bosonic part of superstring effective actions. In that case, ordinary tachyons are absent from the string spectrum. However, our type of tachyonic modes, will still be present in that case, because as we shall argue later, such modes simply indicate an instability of the spacetime (49).

We find it convenient to use a redefined stress energy tensor $T'_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T_{\alpha}^\alpha$, in terms of which Einstein’s equations become:

$$R_{\mu\nu} = -T'_{\mu\nu}$$

(56)

We also assume that on the D3 brane world the dilaton field is constant. This is a desirable feature for late times, where our approach is valid. In certain cosmological models this assumption may be relaxed, in which case a time-dependent dilaton field plays the rôlè of a quintessence field, as we shall discuss in section 10.
The stress tensor $T'_{\mu\nu}$ for the case of tachyon and axion fields reads:

$$T'_{\mu\nu} = \partial_\mu T \partial_\nu T + \partial_\mu b \partial_\nu b - g_{\mu\nu} V(T)$$  \hspace{1cm} (57)

where $V(T)$ is a potential for the tachyonic mode $T$. The fact that the axion field $b$ does not have a potential is dictated by the abelian gauge symmetry of string effective actions, according to which they depend only on the antisymmetric-field strength $H_{\mu\nu\rho}$ and not on the field $B_{\mu\nu}$. Below we shall show that indeed the field $T$ acquires a tachyonic mass, which however, in contrast to the flat-space time Bosonic string theory tachyons, depends on the parameter $b'(E)$.

In addition to (56), one has the conformal invariance conditions for the tachyon and axion fields, to $O(\alpha')$:

$$\partial^2 T = -V'(T) , \quad \partial^2 b = 0$$  \hspace{1cm} (58)

where the prime denotes differentiation with respect to $T(x_i, t)$.

A solution to (57), (58) is given by:

$$T(x_i, t) = \ln r , \quad b(x_i, t) = b'(E) \ln t$$  \hspace{1cm} (59)

provided that the tachyon potential $V(T)$ is:

$$V(T) = -\exp\left(-2T(r)\right)$$  \hspace{1cm} (60)

Naively, if the solution is extended to all space, we observe that the matter diverges logarithmically. To remedie this fact we restrict the above solution to the range $r \leq t/b'(E)$, and thus we enclose it in a bubble of time-dependent radius $t/b'(E)$. Outside the bubble we demand the spacetime to be the flat Minkowski spacetime, and thus the above upper limit in $r$, $t/b'(E)$ is the locus of points at which the temporal component of the metric (49) becomes unity, and this allows an appropriate matching of the interior and exterior geometries. As we shall see later on, a non-trivial consistency check of this matching will be provided dynamically by an explicit study of the scattering of test particles off the bubble. In this way the phenomenologically unwanted tachyon and probably axion fields (obtained from the antisymmetric tensor field of the string) are confined inside the bubble of radius $t/b'(E)$ (cf. figure 3).
In arriving at the above solution we have restricted ourselves to $O(\alpha')$ because we have ignored terms of higher order in curvature $R$. To justify such an approximation it suffices to note that the ratio of the leading terms to the next to leading ones is:

$$(R/M_s)^2/R = O\left(b'^2 \frac{g_s^2 285}{18 M_D} E\right)$$

provided one does not approach the singularity at $r = 0$, which is in fact consistent with the regime of validity of the logarithmic conformal field theory [13, 14], i.e. distances far away from the defect, and times long enough after its collision with the string. This implies that our analysis is restricted near the boundary of the bubble, which will be sufficient for our purposes in this work. Since the string coupling is assumed weak, it is evident from (61) that the approximation of neglecting the higher-curvature terms is satisfactory near the boundary of the bubble. Then, from the decreasing behaviour of $b'(E)$ with increasing energy, which, as mentioned previously, is assumed here even for intermediate energies, it follows that this approximation becomes even better for higher energy scales appropriate for the early stages of the universe.

A second important remark concerns the fact that in the analysis leading to the metric (49) we have treated the spacetime surrounding the $D$-particle defect as initially (i.e. before the collision with the string) flat. However, even an initially at rest $D$-particle, being a very massive one of mass $M_D = M_s/g_s$, would naturally curve the spacetime around it, producing a Schwarzschild radius $r_S = \ell_P^2/g_s \ell_s$, where $\ell_P$ denotes the four-dimensional Planck length and $\ell_s = 1/M_s$ the string length. From our discussion above, the radius of the bubble of figure 3 is $r_b = \ell_s/g_s$. For consistency of our approach, the approximation of treating the spacetime as initially flat implies that we work at distances considerably larger than the Schwarzschild radius, so as the general relativistic effects due to the mass of the $D$-particle could be safely ignored. This implies that the radius of the bubble must be considerably larger than $r_S$, i.e.

$$1 \gg r_S/r_b = (\ell_P/\ell_s)^2$$

From the modern viewpoint of string/D-brane theory, the string length may not be necessarily of comparable order as the Planck length, but actually could be larger. Thus, the above condition seems consistent.

Notice also that the matching with the flat Minkowski spacetime in the exterior geometry is possible because the matter energy density $T_{00}$ and energy flow $T_{0i}$ in the interior of the bubble of figure 3 are both zero:

$$T_{00} = T_{0i} = 0$$

and thus there is no radiation coming out of or flowing into the bubble.

We next compute the mass squared of the field $T$. To this end, we shall consider the fluctuations $\delta T$ of the tachyonic mode $T$ around the classical solution $T_{cl}$ in the interior of the bubble, but close to its boundary, where the spacetime approaches the Minkowski flat geometry. From (58) we then have:

$$\partial^2 \delta T - 4e^{-2\delta T} \delta T \bigg|_{r \rightarrow t/b(E)} = 0$$

from which we obtain a mass squared term of the form:

$$m^2 = -4 b'^2 (E) t^2$$

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3For instance, in the brane-world models of [24], where our world is the domain wall depicted in figure 2, the bulk space time has a warped factor, characterised by a parameter $\kappa$, in such a way that the four-dimensional Planck Mass is of order $M_P^2 \sim M_s^2/\kappa$. If one assumes that closed strings are propagating along the transverse directions of the brane of figure 3 then string effective action methods in the bulk, including $O(\alpha'^2)$ corrections, lead to $\kappa \sim g_s M_s$. This implies that, in such models, the condition (62) is satisfied upon the assumption of weak string couplings $g_s \ll 1$. 

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The negative value indicates, of course, the fact that the field $T$ is tachyonic, but the interesting issue here is that the induced mass depends on the parameter $b'(E)$, and hence on the initial energy data of the incident string.

A remark we would like to make at this point concerns the time-dependence of the mass (66). As one observes from (63), the tachyon field will eventually disappear from the spectrum (as it becomes massless) asymptotically in time $t$. This fact comes from the specific form of the metric (19). From a non-critical string viewpoint, discussed in section 2, such time-dependent mass terms appearing inside the effective tachyon potential $V(T)$, upon expanding in powers of the field $T$, will be considered as part of the central charge deficit $Q^2$, which thus relaxes to zero asymptotically in time $t$ as

$$Q^2 \sim \frac{1}{t^2}$$

in agreement with (33). Notice that above we have obtained this result in an “apparently conventional” field-theory framework, by matching our unstable bubble, bounded in space by a time dependent horizon (17), with the flat Minkowski space time in the exterior. In this way, the non-equilibrium nature of the underlying non-critical string theory, discussed in section 2, is hidden in this matching procedure involving an unstable spacetime. We think that this is an interesting and non-trivial feature of the model.

However, as argued in (30) quantum effects will eventually stop the expansion of the bubble and may even force it to contract. In this sense, the value of the mass of the tachyonic mode (12) will remain finite and negative, and will never relax to zero. Such effects can be understood in our context by the breathing nature of the evolution of the Liouville field implied by the contour of integration depicted in fig. 1. The infrared world-sheet fixed point (corresponding to an infinite area $A \to \infty$) acts as a ‘bounce’ point for this evolution (10, 22).

An equivalent way of seeing this is to observe that the explicit time-$t$ dependence in the temporal component of the metric (43) may be absorbed by a redefinition of the time: $dt'/l_s = dt/t$. In that case, the metric reads:

$$ds^2 = b^2(E)r^2 dt'^2 - \sum_{i=1}^{3} dx_i^2$$

Under this redefinition, the bubble solution remain, but now the bubble appears to be independent of time, with its radius being $r = 1/b'(E)$, and the mass squared of the tachyon being $m^2 = -4b'^2$. From that we observe that the tachyon mass remains $b'(E)$-dependent and finite. This argument supports the fact that even in the initial coordinate system (19) the time $t$ cannot be such so as to eliminate the $b'(E)$ dependence of the tachyon mass. This system of coordinates corresponds to a frame in which the bubble appears static, hence it corresponds in some sense to a comoving frame. Therefore the mass $m^2 = -4b'^2(E)$ is the rest mass of the tachyonic mode.

Notice that the bounce (breathing bubble) picture is a feature of the microscopic event horizons, being created at a local scale around a recoiling D-particle. In section 10 we shall consider also the recoil of the D-particle in an expanding Friedman-Robertson-Walker Universe, in which case we shall study global effects of this recoil associated with the removal of possible cosmological horizons. In such a context the relaxing-to-zero central charge deficit (16), which here has been associated with instabilities of the bubble space time, will be given physical significance as providing a global, time-dependent vacuum energy, relaxing-to-zero à là quintessence, for large times.

Some clarifying remarks are in order at this point concerning the nature of the tachyonic mode $T$. As we have just seen its mass is dependent on the uncertainty scale $b'(E)$ of the D-particle, and hence is proportional to the string coupling $g_s$ (cf. (43)). For this reason this tachyonic mode

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4Due to the fact that these effects occur in the interior of the bubble, whose radius is a few orders of magnitude larger than the Planck length, and taking into account the rare distribution of D-particles in the Universe, as imposed by the phenomenology of the model, discussed in section 8, we remark that there will be no appreciable contributions to the vacuum energy of the Universe in a global scale from the above phenomena. This could be a feature only of certain cosmological models, to be discussed briefly in section 10.
should be distinguished from the standard tachyon fields in flat-spacetime free Bosonic string theory. In fact, in our approach this tachyonic mode expresses simply the instability of the bubble configuration, and will be present even in superstring effective field theories.

As a result, from such considerations one may obtain an average lifetime $\tau$ for the bubble:

$$\tau = \frac{1}{2b'(E)}$$

in the “comoving frame”, or

$$\tau \sim l_s e^{1/2b'(E)}$$

in the Liouville time frame, where $l_s = M_s^{-1}$ is the string length scale. We remind the reader that for heavy $D$-particle defects (low kinetic recoil energy), we are dealing with here, $b' \sim g_s$ \cite{footnote}. Working with weakly coupled strings, with couplings $g_s \ll 1$, we thus observe that the associated life times \cite{footnote} in the physical Liouville frame, $\tau \sim \ell_s e^{1/2g_s}$, are sufficiently larger than the minimum uncertainty times $\ell_s$, thereby justifying our semiclassical perturbative treatment. We shall give some estimates in section 8, when we discuss possible physical applications of the model.

Some comments are now in order concerning the analysis in the Liouville-time frame. In our approach we prefer to work in this frame, with the initial form of the metric \cite{footnote}, implied directly by the logarithmic conformal field theory approach to recoil \cite{footnote,footnote}. This defines a natural frame for the definition of the observable time. As we shall see this is also important when we discuss cosmological implications in section 10. In this approach, the presence of the recoil degrees of freedom after a time, say, $t = 0$ imply a breaking of the general coordinate invariance by the background, and also an irreversibility of time \cite{footnote}. This comes from the fact that in our problem, time is a world-sheet renormalization group parameter (Liouville mode) \cite{footnote}, which is assumed irreversible, flowing towards a non-trivial infrared fixed point of the world-sheet renormalization group of the Liouville $\sigma$-model \cite{footnote,footnote,footnote}. As we have seen, in this frame the mass of the tachyon appears time dependent, because, as we shall discuss below, the frame is not an inertial one, given that the spacetime of the bubble is a Rindler accelerated spacetime. There is one disadvantage, though, with the explicit use of the Liouville time $t$, in that it would require precise knowledge of the time moment at which the bubble was formed, and started decaying. Obviously, within a quantum string formalism such a process would occur within a (uncertainty) time $\sim l_s$. Such issues do not arise if one uses the “comoving” time $t'$ instead.

5 Thermal Effects and Emission of Radiation from the Bubble Spacetime

In this section we study issues associated with thermal radiation from our unstable bubble spacetime. To this end, we consider the metric \cite{footnote}, pass onto spherical polar coordinates $(r, \theta, \phi)$, and fix the angular part for convenience, as this does not modify the conclusions. We then perform the following coordinate transformations (from now on we work in units $\ell_s = 1$):

$$R = r \cosh(b'(E) \ln t), \quad T = r \sinh(b'(E) \ln t),$$

where $d\Omega$ is the conventional solid angle.

The transformation maps our space time to the right Rindler wedge (R) depicted in figure \ref{fig:4}. The left wedge (L) is described by similar transformations up to a minus sign. In the $(R, T)$ coordinates the line element becomes:

$$ds^2 = dT^2 - dR^2 - (R^2 - T^2) d\Omega^2$$

From the above spacetime, we observe that for distances $R \gg T$ one recovers the flat Minkowski spacetime, whilst for distances $R \sim T$ one obtains the bubble spacetime. Notice that $R^2 - T^2 = r^2$, and the interior of the bubble is defined by $r \leq 1/b'(E)$ in comoving coordinates.
Figure 4: Rindler wedge spacetime arising from the recoil of a $D$-particle, embedded in a four-dimensional spacetime, due to its scattering with a closed string. The set of wedges (LEFT and RIGHT) describe the spacetime for $t > 0$.

An observer comoving with the expanding bubble, placed at position $r$, is accelerated with respect to the $(R, T)$ frame, with proper acceleration $1/r$. According, then, to the standard analysis of accelerated observers [34], such an observer sees the Minkowski vacuum (in the $(R, T)$ coordinates) as having a non-trivial temperature $T_{\text{bubble}}$

$$T_{\text{bubble}} = \frac{1}{2\pi r}, \quad 0 < r < 1/b'(E)$$

(72)

The temperature for the inertial Rindler observer $T_0$ is:

$$T_0 = \sqrt{g_{00}}T_{\text{bubble}} = \frac{b'(E)}{2\pi}$$

(73)

The presence of temperature is expected to imply a non-trivial proper entropy density $s$. For a massless scalar field, in our case the axion $b$, the entropy $S = \int d^3x \sqrt{-g}s$ in four spacetime dimensions is given by:

$$S = \int d^3x \sqrt{-g} \frac{4\pi^2}{30} T_0^3$$

(74)

From the bubble spacetime (49) one then obtains:

$$S = \int d^3x \sqrt{-g} s = 4\pi b'(E) \int_0^{1/b'(E)} dr \, r^3 \frac{4\pi^2}{30} \frac{b'^3(E)}{8\pi^3} = \frac{1}{180}$$

(75)

Thus the bubble carries non-trivial entropy, which turns out to be independent of $b'(E)$. The reader should not be alarmed by the apparent volume independence of the entropy, which at first sight would seem to contradict the fact that the entropy is an extensive quantity. In fact, there is no contradiction in our case, since there is only one scale in the problem, $b'(E)\ell_s$, and the volume of the bubble is itself expressed in terms of this scale.

The presence of entropy production after the recoil implies loss of information which can be understood as follows: one starts from a pure state of a string striking a $D$ particle. There is no entropy in the initial configuration. After the moment of impact, the $D$-particle recoils, and because it is a heavy object it distorts the spacetime around it, producing the bubble phenomenon via its recoil excitation degrees of freedom. Due to the finite lifetime of the bubble, the entropy (75) will be released to the external space, implying information encoded in the recoil degrees of freedom.
Figure 5: Emitted radiation from the unstable bubble. The radiation is not isotropic, but most of it will be emitted in the forward direction, parallel to that of the incident high-energy particle.

which are unmeasurable by an asymptotic observer. This picture is consistent with the loss of conformal invariance of the underlying $\sigma$-model, and the irreversible world-sheet renormalization-group flow of the recoiling system, as discussed in [30, 10], upon the identification of the Liouville mode with the target time.

It should be remarked at this point that the entropy (75) pertains strictly to scalar fields that live inside the bubble. There is no crossing of the surface of the bubble by the interior fields in our construction, at least classically (as we shall discuss below there is a quantum-mechanical escape probability). On the other hand, it should be noticed that for a field which lives in the exterior of the bubble, there appears to be loss of information in the sense that the exterior particle degrees of freedom may enter the bubble and be captured, as we shall discussed in the next section. Thus, an asymptotic observer, far away from the bubble, will necessarily trace out such (unobserved) degrees of freedom in the density matrix formalism, and in this sense the resulting entropy, pertaining to such degrees of freedom, will be proportional to the area of the bubble and not its volume [35].

We shall come back to this important point, in connection with emitted radiation from the bubble later on.

The presence of temperature $T_0$ (73) implies the emission of radiation from the bubble (see figure 5), which can be read off from the Stefan-Boltzman law $\sigma T_0^4$, $\sigma = \pi^2/60$ (in units $\hbar = c = k_B = 1$). Given that the area is $4\pi b^{-2}(E)$, and that the lifetime of the bubble is estimated from (68), one observes that during the life time $\tau$, the following amount of energy is released in the form of radiation:

$$ E_{\text{rad}} \sim 4\pi b^{-2}(E)\tau \sigma T_0^4 = \frac{1}{480\pi} b'(E)M_s = \frac{gs}{480\pi} b'(E)M_D $$

(76)

It is interesting to observe that the same amount of energy represents the thermal energy of the axion field in the interior of the bubble:

$$ E_{\text{th/axion}} = \int d^3x \sqrt{-g}\frac{\pi^2}{30}T_0^4 = \frac{gs}{480\pi} b'(E)M_D $$

(77)

From (78), and taking into account that in the effective field-theory limit we are working here, $E < M_s$, and that $gs \ll 1$ in our weak string framework, one observes that the energy $E_{\text{rad}} = E_{\text{th/axion}} < M_D$.

From energy conservation then, which, notably, is shown to be valid rigorously in the context of our logarithmic conformal field theory (stringy) recoil framework [14], one obtains:

$$ E_{\text{in}} + M_D = M_D + E + E_{\text{th/axion}} $$

(78)
where $E_{\text{in}}$ denotes the total energy of the incident particle/string. From this, one thus sees that there is a threshold for bubble formation:

$$E^{\text{threshold}} = E_{\text{rad}} = E_{\text{th/axion}}$$

From these considerations, one observes that the radiation energy will not cause any mass loss of the $D$-particle, since all the thermal axion energy accounts for that. Hence, despite the instability of the bubble, the stability of the $D$-particle defect is not affected. This will be important for our physical applications, to be discussed later on.

Notice that the energy release cannot make up for the maximum of the thermal energy expected from Wien’s law $\lambda T_{\text{max}} = \text{const}$, where $\lambda$ the wavelength of radiation. Thus the resulting photon spectrum is not thermal, and hence one can only get an estimate for the energies of the emitted radiation. The alert reader might then object to our previous use of the black body (or, in general, equilibrium) laws. In fact their use is indicative, and they can only give qualitative results (e.g. in our case we only obtain a tail of the thermal distribution).

To recapitulate, the physics behind the above properties can be summarized as follows: one needs a highly-energetic incident particle of energy $E_{\text{in}} > E^{\text{threshold}}$, which strikes a $D$-particle, and forms a bubble; the bubble radiates an amount of energy $E_{\text{rad}}$ distributed appropriately among the various photons. The emitted radiation will not be isotropic as a result of (spatial) momentum conservation (see figure 5). This will be useful in physical applications. The fact that a particle, entering and being captured by the bubble, will cause the emission of radiation from the bubble is nicely related to the existence of non-zero entropy measured by an asymptotic observer. This phenomenon is thus not dissimilar to the Hawking process of an evaporating black hole, although in our case the bubble is not a black hole, neither the $D$-particle evaporates, as we discussed above. This picture is in agreement with the non-equilibrium Liouville string framework, on which the approach is based.

6 Motion of Particles in the Bubble Spacetime

We shall now analyze the motion of a particle in the bubble spacetime. For convenience we shall work at the equator of the three sphere (fixed angle $\theta = \pi/2$). The Lagrangian of the particle in the background spacetime in comoving coordinates ($t', r, \phi$) is:

$$L = \frac{1}{2} \left( \frac{ds}{d\lambda} \right)^2 = \frac{b'^2(E) r^2}{2} \left( \frac{dt'}{d\lambda} \right)^2 - \frac{1}{2} \left( \frac{dr}{d\lambda} \right)^2 - r^2 \frac{1}{2} \left( \frac{d\phi}{d\lambda} \right)^2$$

Expressing the Lagrangian in terms of the conserved angular momentum $L = r^2 (d\phi/d\lambda)^2$ and energy $E_{\text{in}} = b'^2(E) r^2 (dt'/d\lambda)$ we obtain:

$$\frac{E_{\text{in}}^2}{2b'^2(E)r^2} - \frac{1}{2} \left( \frac{dr}{d\lambda} \right)^2 - \frac{L^2}{2r^2} = \frac{\mu^2}{2}$$

where $\mu = 0$ for massless particles (e.g. photons), and 1 for massive particles (in which case the quantities $L$ and $E_{\text{in}}$ are the corresponding quantities per unit mass).

Writing the equation of motion as:

$$\frac{1}{2} \left( \frac{dr}{d\lambda} \right)^2 + \frac{\mu^2}{2} = \frac{E_{\text{in}}^2/b'^2(E) - L^2}{2r^2}$$

we can see that the impact parameter $L/E_{\text{in}}$ must be smaller than $1/b'(E)$, for the equation to make sense. This means that if $L/E_{\text{in}} > 1/b'(E)$, then the particle will necessarily travel outside the bubble spacetime, which is thus a dynamical consistency check of our matching assumptions that the spacetime in the region $r \geq 1/b'(E)$ is the flat Minkowskian spacetime. Such outside particles will not be affected by the presence of the bubble, and their trajectory will be undisturbed, that predicted by special relativistic dynamics.
Below we shall study now the case of impact parameters $L/E_{\text{in}} < 1/b'(E)$, for massless and massive particles. In such a case, from (82) the massless particle equation of motion reads:

$$r = r_0 \exp \left( \pm \phi \sqrt{\frac{E_{\text{in}}^2}{b'^2(E)L^2} - 1} \right)$$  \hspace{1cm} (83)$$

where in the case of an incoming photon (from outside the bubble) $r_0 = \frac{1}{b'(E)}$, and we have taken only the ($-$) sign, because this is the only consistent choice. This shows that the massless particle will be captured inside the bubble.

From (82) one observes that, in the case of a massive particle, there exist values of energy and angular momentum such that the particle can escape the bubble spacetime. This happens if the radial velocity on the boundary (as the particle attempts to escape) is non-zero, which implies (we have re-instated the units of $M_s$ for clarity):

$$E_{\text{in}}^2 - L^2b'^2(E)M_s^2 > M_s^2$$  \hspace{1cm} (84)$$

This demonstrates that only highly energetic particles with energies much higher than $M_s$ can escape the bubble spacetime.

Once such a particle is electrically charged, its non-uniform (spiral) motion inside the bubble will cause the emission of radiation. The latter will continue to carry angular momentum and energy of roughly the same order as that of the particle. Because the emission is now taken place at $r_0 < 1/b'(E)$ the positive sign in the exponent of equation (83) is also allowed, implying an escape possibility for the emitted photons (cf. figure 6). In addition, as we shall show in the next section, the bubble has a non-trivial (thermal) refractive index, and thus behaves as a medium.

If there is a beam of charged particles entering the bubble within its short life time, then these particles will experience the (suppressed) phenomenon of transition radiation, i.e. the emission of photons accompanying an electrically-charged particle when crossing the interface separating two media with different refractive indices (in our case, the interior of the bubble and the exterior Minkowski spacetime). A fraction of this radiation will also escape the bubble. Such phenomena, if true, will imply excess of photons accompanying the charged particle. We shall present a more detailed discussion of these issues, and their potential experimental consequences, in a forthcoming publication.

It should be noticed that, if the condition (84) is satisfied, then the particle is deflected by an angle $\Delta \phi$ which can be computed in a standard way to be:

$$\Delta \phi = \pi - \frac{2}{\sqrt{\frac{1}{\rho^2 b'^2(E)} - 1}} \arccosh \left( E_{\text{in}} \sqrt{1 - \rho^2 b'^2(E)} \right), \hspace{0.5cm} \rho = L/E_{\text{in}} = \text{impact parameter}$$  \hspace{1cm} (85)
From this we observe that, for fixed impact parameter $\rho$, highly energetic particles will not be deflected much, as should be expected.

As a final comment we mention that the scattering cross section $\sigma(E)$ for energies and/or angular momenta that violate the condition (84), which are of physical interest, is given by:

$$\sigma(E) = \pi b'^{-2}(E) \tag{86}$$

From the results of ref. [14], which are valid strictly only for low energies, we observe that $b'(E)$ decreases with increasing energy $E$ (48). One would expect intuitively that strings with higher energy would cause larger distortion of the spacetime surrounding the recoiling $D$-particle defect. This point of view is supported by the above results if one extends the behaviour of $b'(E)$ encoded in (48) to intermediate energies as well, and in fact to all energies (up to Planckian), because in that case the distortions of spacetime caused by strings with higher energy will correspond to formation of bubbles with bigger radii $1/b'(E)$.

From (86) we observe that higher energy strings would correspond to larger cross sections. The important point to notice is that the cross section is non zero even for zero energy. This stems from (48), and is associated with the fact that $b'(E)$ is essentially a quantum uncertainty in the momentum of the $D$-particle [14], which is not zero even for vanishing incident energy $E$. Hence, the spatial uncertainty of the position of the $D$-particle, which is associated with the cross section, is not zero, but is bounded from below by the Heisenberg uncertainty principle, which explains naturally the non-zero minimum value of $\sigma(E = 0)$. It also explains the increasing behaviour of the cross section with increasing energy, given that the higher the energy is, the larger the uncertainty is expected to be.

7 Quantum Electrodynamics Effects inside the Bubble and Refractive Index for Photons

So far our considerations have been classical. It is in this sense that we demonstrated capture of photons (and, in general, massless particles) in the interior of the bubble. The presence of finite temperature (73) will create a non-trivial (thermal) vacuum, with broken Lorentz symmetry inside the bubble. In that case, it is known [37] that the effective velocity of light, defined by the quantum propagator of photons, is modified in accordance with the fact that the finite temperature effects provide the notion of a medium.

Specifically, the dispersion relation for a particle of mass $m$ in the non-trivial vacuum at temperature $T$ is: $E^2(p) = p^2 + f(p, T, m)$, where $E$ is the energy and $p$ the momentum, and the function $f$ encodes the quantum effects of the vacuum polarization. The group velocity of the particle is then given by $v = \partial E(p)/\partial p$, and in general depends on $p$.

In the case of photons in a non-trivial quantum electrodynamical vacuum, the function $f$ can be computed, to one loop, from the vacuum-polarization graph of the photon [37]. The latter is of order $f \sim T^2 e^2$, where $e$ is the electron charge [3].

In our case, the induced temperature (73) is much larger than the incident momentum $p$ of the photons. From the results of [3] in this case, the effective velocity of the photons inside the bubble is (in units of speed of light in vacuo $c = 1$):

$$v \sim \frac{p}{eT_0} \sim \frac{p}{eb'(E)} \tag{87}$$

where $p$ is related to the energy $E$ of the photon via the (modified) dispersion relation. The photon in this case becomes effectively massive, with mass $\mu \sim eb'(E) \neq 0$. It is interesting to note that, due to the extreme temperature effects, $p \ll T_0$, the resulting photon is considerably slowed down inside the bubble to non-relativistic velocities.

From our earlier discussion on massive particle trajectories inside the bubble, and the fact that the quantum effects result in an effective photon mass, one is tempted to consider the possibility...
of a photon escaping the bubble. However, from the condition (84) and the induced photon mass \( \mu \sim e b'(E) \), it becomes evident that there is no such possibility.

Nevertheless, there is a non-zero probability of quantum tunnelling through the potential barrier. In this sense, part of the classically captured (incident) photons can escape. Combined with the thermal slowing down (cf. (87)) of all photons inside the bubble, then, this will result in the appearance of delays in the respective arrival times of photon beams from distant astrophysical sources in areas where there are \( D \)-particles. Because these delays will be associated with only part of the photon beam, the final effect will appear as a fluctuation in the arrival time.

The associated delay for a single photon, which passes through a region in space where there is one \( D \)-particle, and is assumed to escape through tunnelling, is estimated to be:

\[
\Delta t \sim \frac{1}{b'(E)v(E)} = \frac{e}{p}
\]

where \( \frac{1}{v(E)} \) is the radius of the bubble, \( e \) is the electron charge, and this formula is applicable for \( p \ll M_s \). For velocities \( p \sim M_s \) there are modifications, which however are not of interest to us here.

The existence of delay effects that depend on the energies of the photons bares some resemblance to quantum spacetime effects, associated with induced refractive index for photons, discussed in [38]. As in those works, so in the present model there appears to be a non-trivial refractive index \( n(E) = 1/v(E) \) inside the bubble. However, there is an important difference, in that here this is due to conventional quantum electrodynamical thermal effects. Because of this reason, the induced refractive index \( n(E) \) is reduced with increasing photon energies, in contrast to the effects of refs. [38], where the quantum spacetime induced refractive index appears to increase with energy. However, the reader should bear in mind that in our model the existence of temperature is due to quantum stringy effects [30, 14] associated with the recoil of the \( D \)-particle. In this sense there is, in our approach, a notion of quantum gravity, in similar spirit to the work of [38].

Unfortunately, for a very dilute gas of \( D \)-particles the maximal delay (88), corresponding to the less energetic observable photons (e.g. infrared background of energies 0.025 eV) is very small, or order \( 10^{-11} \) s. Nevertheless, we should notice that if one considers photon sources from distant astrophysical objects that are at distances corresponding to cosmological redshifts \( z > 1 \), then in those areas of the relatively early universe, the density of \( D \)-particles might have been higher. One then can get an upper bound of such densities by considering the effect (88) as lying inside the present experimental errors.

8 Possible Astrophysical Constraints on the Model:

Clusters of \( D \)-Particles in the Universe and GZK Cutoff

In this section we would like to make some speculative remarks on possible physical applications of the above phenomena. It is by no means a priori obvious that features of Liouville strings can be constrained (or even tested!) experimentally in the immediate future. However, the specific non-critical string/\( D \)-brane model under consideration has the feature that it leads to emission of very high-energy radiation from the recoiling \( D \)-particles. As we shall argue below this feature imposes severe restrictions on the parameters of the model.

Let us assume that there is a rare distribution of \( D \)-particles in the inflated universe (although their density was much higher in the early universe). Due to this rare distribution, the concept of isotropy is not applicable. It is therefore possible that an isolated \( D \)-particle lies between Earth and a distant galaxy. As the galaxy emits particles, some highly energetic and weakly interacting ones, such as neutrinos, strike the \( D \)-particle and induce the formation of a bubble. The latter then emits radiation in the way explained above. The emitted photons, in the direction of the incident particles, will be highly energetic, of typical energy \( b'(E) \). In general, such photons will interact with the background photons of either the microwave background radiation, or the infrared
background, to yield, say, $e^+e^-$ pairs. Because of this, if the $D$-particle lies far away from Earth, outside the average mean free path of such photons, the latter will not arrive on Earth. However, one may imagine a situation in which the isolated $D$-particle lies within the above mean free path distance, which in the case of photons interacting with the infrared background is estimated to be of order of a few Mpc \cite{39}. Then, the weakly interacting incident particles that trigger the phenomenon of bubble formation, e.g. neutrinos, will arrive at the location of the $D$-particle(s) undisturbed. In that case, the emitted high-energy radiation will reach the observation point, and in this sense the recoiling $D$-particle bubble constitutes a novel and relatively nearby source of highly energetic photons. Such scenarios may have applicability to the recently observed highly energetic 30 TeV photons that seem to violate the so-called GZK cut-off \cite{30,31}.

In a similar way, one can also extend the above discussion to incorporate charged particles. Consider, for instance, a beam of protons emitted by a galaxy lying at cosmological distances, whose energies do not exceed the GZK cut-off, and hence they arrive undisturbed until the point where a $D$-particle defect lies. One of the protons will then strike the $D$-particle and create a bubble spacetime. As discussed previously, the proton will be captured inside the bubble, since its energy does not satisfy the escape condition \cite{33}, as being less than $M_s$ \cite{19}.

The bubble will then radiate very high energy photons, which can interact with the remaining protons in the beam, that fly outside the bubble, to create, say, protons and pions etc. The protons (or, in general, the particles) that emerge from such interactions will then be very energetic, and it is conceivable that their energies can be of the order of the observed \cite{31} $3 \times 10^{20}$ eV. In this way, the region around the recoiling $D$-particle defect acts as a source of ultra-high-energy cosmic rays, and if one assumes, as before, that the defect lies within the mean free path of a proton (from Earth), this can easily contribute to observed apparent “violations” of the GZK cut-off \cite{31}.

If the above scenario survives, it may then imply that there is no actual violation of Lorentz symmetry that is responsible for the phenomenon, as claimed by a number of authors \cite{40}, since in our bubble spacetime there is no such violation (except the trivial one due to temperature effects inside the bubble). To put it in simple terms, in our scenario, the source of the ultra-high-energy cosmic rays is in the neighbourhood of the $D$-particle defect, which may lie at a much closer distance from Earth than one naively thought. Of course, it goes without saying that one cannot exclude the possibility that a peculiar combination of phenomena, involving the model discussed here in conjunction with both conventional and unconventional (spacetime foam) physics \cite{38,40}, might actually lie behind such extreme astrophysical phenomena.

We shall now make use of the above considerations to constrain the model by employing current observational data related to ultra-high-energy cosmic rays \cite{31}. Alternatively, from an overly optimistic viewpoint, such a ‘phenomenology’ may be seen as providing a means of possible experimental detection of the $D$-particles, if clusters of them really exist in our galactic neighbourhood! We will show that, under a few assumptions, radiation from a physically acceptable population of $D$-particles in the universe can be observed. Then, an excess of photons somewhere in the observed spectrum of high energy cosmic rays, can be interpreted as coming from this effect and, through that, from the branes. Since high energy photons have a mean free path smaller than 100 Mpc, we shall consider here $D$-particles located inside a volume of radius of order of 10 Mpc, around Earth. However, as we shall show below, compatibility with standard astrophysics implies that only a small part, if at all, of the observed ultra high energy cosmic ray events can be attributed to this kind of exotic physics.

To this end, let us consider a $D$-particle striken by a current of highly-energetic weakly-interacting particles, which can travel undisturbed at great distances. We assume that the energy of the particle is higher than the threshold energy \cite{73} for the formation of the bubble. The highest energy neutrinos from gamma-ray bursters (GRB’s) \cite{11} can provide the high energy flux we need, as in the bursters there is a relatively significant production of particles whose energies can possibly be of order $10^{-3}q_s M_s$, where $q_s$ is assumed smaller than unity, for the validity of the string perturbation expansion. Our interest in neutrinos arises from the fact that these par-

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\footnote{Here we assume that $M_s$ exceeds the GZK cut-off $10^{19}$eV($=10^{10}$ GeV). If this is not the case, and one has a lower $M_s$, then such massive particles may escape. At any rate, our discussion does not depend upon this fact.}
particles can travel undisturbed over very large distances, creating significant uniform and isotropic flux. According to contemporary models \cite{11}, these neutrinos come with appreciable flux only up to energies $10^{18}$eV, so our first assumption will be that the threshold energy must be bounded from above by such a value. This, in turn, implies that the string length scale falls within the experimental sensitivity, if it is at most (c.f. \cite{23})

$$M_s \sim \frac{10^{11}}{g_s} \text{GeV}$$

Then, a few photons will be emitted, with an energy spectrum in a narrow window below $E_{\text{rad}}$, whose highest value has been assumed to be about $10^{18}$eV. Note that, if one uses $g_s \sim 10^{-2}$, which is small, as needed for the validity of our string perturbation expansion, but not unjustifiably small, we observe that the smallest detectable string length, in this approach, is $l_s \sim 10^{-27}$cm, in which case the bubble’s lifetime \cite{69} is \sim $10^{-16}$ sec. This is much larger than the minimum string time scale $\ell_s/c \sim 10^{-38}$ sec, which implies that our perturbative approach is consistent in this case.

We now turn to an estimation of the distribution of $D$-particles in the universe which can produce sufficient rates of photons on Earth. We assume a uniform and isotropic flux $J_{D_{\text{he}}}$ for the highly-energetic particles, and a concentration of $D$-particles $n_D$ in a region of linear dimension $\ell$. Then, it is evident that the flux of the incident particles, which will strike the $D$-particles, is $J_{D_{\text{he}}}(1-e^{-\ell \sigma n_D})$ where $\sigma$ is the effective cross section of the particle/$D$-brane scattering :

$$\sigma \sim l_s^2 = \frac{10^{11}}{g_s} \text{GeV}^{-2} = g_s^2 10^{-50} \text{cm}^2.$$  

(90)

Note that the string length scale $l_s$ determines the natural size of the $D$-brane as seen by an incoming string. For each scattering event there will be a few highly-energetic photons emitted due to the mechanism described above. This implies that, in order of magnitude, the total number of emitted photons is about the same as that of incident particles. Assuming that $\ell \sigma n_D$ is small, it follows that the flux of the emitted photons will be roughly of order $J_{D_{\text{he}}}(1-e^{-\ell \sigma n_D})$. Restricting ourselves from now on to the value $g_s = 10^{-2}$, we obtain $\sigma \sim 10^{-54} \text{cm}^2$.

The emerging photons of the first particle/$D$-brane scattering carry energies less than the threshold energy, hence, when such photons strike another $D$-particle will be captured. If $\ell \sigma n_D$ is small, the final photon flux will still be given by

$$J_{\gamma} = J_{D_{\text{he}}} \ell \sigma n_D$$

(91)

since in this case $e^{-\ell \sigma n_D} \simeq 1$.

As already mentioned, a good candidate for the highly energetic particles with energies above the threshold \cite{11}, are neutrinos emitted from the GRB’s. According to studies of the last few years \cite{11}, neutrinos of such energies can be emitted from the fireball due to the muon decay that follows the pion production, due to the interaction of fireball protons with afterglow photons. The energy of neutrinos of appreciable flux is about two orders of magnitude less than that of protons, which is supposed to be the highest energy of the ultra-high-energy cosmic rays (UHECR). Then, we expect to have neutrinos of flux of order, roughly, $10^{-9}$GeV/cm$^2$sec at energies $10^{18}$ to $10^{19}$eV, depending on the energy and neutrino species \cite{11}.

If one assumes a uniform distribution of the $D$-particle defects inside large regions in space, then a simple calculation shows that, in order to explain the observed events, one needs a phenomenologically unacceptable total mass of $D$-particles. It is, therefore, necessary to assume that the latter form collections of very high density, whose concentration in space, however, is low. We next assume that such collections can produce radiation flux of order of that of the high energy cosmic rays, $J_{\text{CR}} \sim 10^{-20}$/cm$^2$ sec sr, and that the observed UHECR flux is of the same order as the photon flux (\cite{11}). If there are $N$ such ensembles of $D$-particles in a volume around Earth, of average radius in the order of $10$ Mpc, one has

$$J_{\text{CR}} \sim NJ_\gamma \sim NJ_{D_{\text{he}}} \ell \sigma n_D = Ncn_{D_{\text{he}}} \ell \sigma n_D \sim \sim Ncn_{D_{\text{he}}} \sigma N_D/\ell^2$$

(92)
where \( N_D \) denotes the average number of branes in one of these \( D \)-brane-collections, \( c \) is the speed of light in vacuo, and \( n_{he} \) is the concentration of the high-energy particles, assumed to be neutrinos from GRB’s. Their flux, given above, implies a number density of high energy particles \( n_{he} \sim 10^{-28}/\text{cm}^3 \). Then,

\[
N N_D \sim \ell^2 10^{48}/\text{cm}^2.
\]

Using the fact that \( M_D = 10^{-9}\text{gr} \) for the given string scale and coupling, one obtains the total \( D \)-particle mass contained in the 10Mpc-volume as:

\[
M_{\text{total}} \sim 10^6 \ell^2 /\text{cm}^2 \ M_\odot
\]

These formulae show that in the limit of very small \( \ell \) the total number and mass of branes needed can be very small. We have then to determine the lower limit of the linear dimension of the brane collections \( \ell \). First we must take into account a few other requirements. As we have stressed earlier, it is important that the scattering rate of high energy particles passing through the brane ensembles is small, so that

\[
\ell \sigma n_D \sim \frac{\sigma N_D}{\ell^2} \ll 1.
\]

Also, in order to be able to treat the \( D \)-particle collections as distinct spots of scattering, whose internal density is unrelated to their concentration in space, we must have \( n_D \gg \frac{N}{10 \text{ Mpc - volume}} \). Finally, the Schwarzschild radius has to be much smaller than \( \ell \), so that one is far away from the black hole limit. Then, it turns out that the strongest condition is (95), and the last two are trivially satisfied. Since there is no independent lower limit for \( N_D \), we may take for concreteness \( N_D \sim 10 \) assuming that such a configuration forms a stable bound state of \( D \)-particles, an issue that we will discuss later on. Setting \( \ell \) to be ten orders of magnitude larger than the saturated limit, we then get \( \ell \sim 10^{-16}\text{cm} \), which gives a value for the total number of brane ensembles in the 10 Mpc-volume \( N \sim 10^{16} \). These imply a total \( D \)-brane mass in that volume of order \( 10^{-26}M_\odot \sim 10^4 \text{kgr} \). These numbers are quite sensitive in changes of the parameters, but there are large regions of the parameter space where the results are reasonable. The total mass can be very small and hence it does not contribute to the dark matter, although it would be a very good candidate for it, given that branes are “bright” only when very high energetic particles are passing nearby.

The conclusion of the above, rather indicative, results is that a relatively small number of \( D \)-particles is needed to produce an observable effect, if our assumptions are valid. Then, an observed excess of photons in a certain energy channel of the high energy cosmic rays would imply the existence of \( D \)-particles, thereby fixing the value of the product \( g_s M_s \) of the fundamental string parameters. The above estimates provide information only about a portion of the total population of \( D \)-particles in the universe, assuming the latter exist, which contributes to an observable radiation.

It should be remarked that the above considerations were based on the assumption that the maximum energy of the incident particles (neutrinos or other species) is of order \( 10^{19} \text{eV} \). Relaxing this, by allowing the maximum energies to be higher, e.g. of order \( 10^{21} \text{eV} \) [43], would imply that the above scenario could also provide a mechanism for production of UHECR beyond the GZK cut-off [32]. For this to happen, of course, the location of the brane defects must lie within the respective mean free paths. However, we must note that these considerations cannot offer an explanation of the complete set of the observed UHECR events [31]. The so-produced UHECR events, if any, constitute only a contribution to this set, given that our mechanism produces only photons with energies in a rather narrow interval.

9 On the Stability of Clusters of \( D \)-Particles in the Universe

For the validity of the above scenarios it is crucial that the populations of the \( D \)-particle defects are stable. Naively, since the \( D \)-particles are very massive, one expects that at large concentrations,
such as the ones above, they will collapse to form black holes or at least to produce very strong gravitational fields, which would jeopardize our scenario. The problem of preventing such a collapse is equivalent to that of stabilizing $D$-branes, which at present is an important open issue. One expects that, in general, such stability mechanisms would imply a certain distance scale among the constituent $D$-particles in a collection, as well as restrictions on the number of branes in the ensemble and probably on the total number of branes in the universe, something which most likely cannot be explained by the dynamics of their interactions alone.

However, there are recent attempts in the string literature in which—under certain circumstances—one has a no force condition among the $D$-particles. Usually this is the case of sufficient spacetime supersymmetries, where the $D$-particle states are viewed as specific states, saturating the so-called Bogomol’nyi-Prasad-Sommerfield (BPS) bound $^{42}$. However, it has been pointed out $^{43}$ that one may obtain stable non BPS $D$-brane states by considering appropriate combinations of BPS branes, in such a way that spacetime supersymmetry is not preserved. From our point of view we are interested in such stable non-BPS non-supersymmetric $D$-particles.

Such states can be viewed as solutions of certain string theories, which are connected with the phenomenologically relevant string theories, like Heterotic String, assumed to be living on the four-dimensional spacetime (after appropriate compactification), by virtue of certain duality symmetries $^{44, 45}$. One such theory is type IIB string theory. In general, $D$-particle/$D$-particle scattering in type IIB string theory has been studied in the literature $^{44}$. Although the construction of $^{44}$ refers to a specific string model involving a particular orbifold compactification $^{45}$, which may not be necessarily realized in our situation, however we find it generic enough so as to consider it as a prototype for the $D$-particle stability mechanism we need here.

The main idea behind the stability mechanism via the no force condition in this model for $D$-particles lies on the fact that there are additional vector interactions, $U_D(1)$, associated with the specific way of constructing the non-BPS state. We shall not explain the details here, but we refer the interested reader to the relevant literature $^{44, 45, 46}$. Such $U_D(1)$ interactions should not be confused with observable interactions of ordinary matter in our construction. The latter may be assumed neutral under the $U_D(1)$, which thus characterizes only the $D$-particles in the ensemble, which are then assumed ‘electrically’ charged under this $U_D(1)$, and in principle may have both positive and negative charges. The positive charges characterize, say, the $D$-particles, whilst the negative charges the anti-$D$ particles, denoted by $\overline{D}$. The $U_D(1)$ interaction is therefore repulsive among $D$-particles (or $\overline{D}$-particle), and attractive among $D$-particle-$\overline{D}$-particle.

Once we admit both kinds of charges there arises the issue as to how “polarized” $D$-particle collections, characterized by a significant excess of (positive or negative) $U_D(1)$ charge, have been formed in the Galaxies today. This has not been resolved as yet, neither will be the topic of the present work. For our purposes here we merely conjecture that, somehow, after the Big Bang, such polarized regions emerged inside ordinary matter, in such a way that the overall net $U_D(1)$ charge of the Universe is zero. An alternative scenario would be that $\overline{D}$-particles behaved in similar way as antiparticles in ordinary matter, and hence the Universe today consists in its overwhelming majority of $D$-particles only. This, would then trivially solve the above-mentioned problem of “polarization”.

In general, the no force condition may be a property valid at all distances. This is the case of supersymmetric $D$-particles that saturate the BPS bound. However, in the orbifold construction of $^{44}$, the no force condition among the non BPS $D$-particles is found to occur at large scales $r$, as compared with the string length $l_s$, where notably (target space) effective low-energy string action methods are applicable. This feature is exclusive of a critical-radius orbifold compactification.

An important point, however, which should be stressed here is that the construction of $^{44}$ ignored higher-string loop effects. The incorporation of such effects results in general in the destruction of the no force condition $^{44}$, although under some circumstances it may be made valid up to one loop. The effects of string-loop resummation cannot be answered at present, and hence the issue of the no force condition at a non-perturbative string level is still open.
10 Embedded Recoiling $D$-Particles in the Universe, Cosmological Evolution and Particle Horizons

In the previous sections we have demonstrated the dynamical creation of event horizons around a recoiling $D$-particle defect, as a result of the induced distortion of the surrounding spacetime, due to back reaction effects. The original spacetime, in which the defect was embedded had been assumed flat and static (at global scales). The perturbative analysis presented above is valid only for times large after the impulse, and for distances sufficiently away from the defect, i.e. outside its Schwarzschild radius, so that induced space-time curvature effects due to the massive nature of the $D$-particle can be safely ignored.

An interesting question is how the presence of recoiling $D$-particles in a cosmological Universe, of Friedman-Robertson-Walker type, affects the cosmological evolution. This question acquires important physical significance, in view of our previous ‘phenomenological’ analysis. This issue has been examined in detail in [27], and we shall not repeat the details here. For completeness, in what follows we shall review briefly the most important arguments and results.

Let us consider a $D$-particle, located (for convenience) at the origin of the spatial coordinates of a curved four-dimensional space time, which at a time $t_0$ experiences an impulse. In a $\sigma$-model framework, the trajectory of the $D$-particle $y^i(t)$, $i$ a spatial index, is described by inserting the following vertex operator

$$V = \int_{\partial \Sigma} G_{ij} y^j(t) \partial_n X^i$$  \hspace{1cm} (96)

where $G_{ij}$ denotes the spatial components of the metric, $\partial \Sigma$ denotes the world-sheet boundary, $\partial_n$ is a normal world-sheet derivative, $X^i$ are $\sigma$-model fields obeying Dirichlet boundary conditions on the world sheet, and $t$ is a $\sigma$-model field obeying Neumann boundary conditions on the world sheet, whose zero mode is the target time.

This is the basic vertex deformation assumed to describe the motion of a $D$-particle in a curved geometry to leading order at least, where spacetime back reaction and curvature effects are assumed weak. Such vertex deformations may be viewed as a generalization of the flat-target-space case [23]. For times long after the event, where our perturbative approach is valid, the trajectory $y^i(t)$ will be that of free motion in the curved space time under consideration. In the flat space time case, this trajectory was a straight line [13, 14], and in the more general case here it will be simply the associated geodesic.

In [27] we have dealt with space times of Friedman-Robertson-Walker (FRW) form:

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} (dX^i)^2$$  \hspace{1cm} (97)

where $a(t)$ is the FRW scale factor. We shall work with expanding FRW space times with scale factors

$$a(t) = a_0 t^p, \quad p \in \mathbb{R}^+$$  \hspace{1cm} (98)

where the times $t \gg t_0$, i.e. much later after the moment of impulse.

With initial conditions $y^i(t_0) = 0$, and $dy^i/dt(t_0) \equiv v^i$, one easily finds that, for long times $t \gg t_0$ after the event, the path $y_i(t)$, which is a solution of the appropriate geodesic equations for the spacetime [27],[48], acquires the form:

$$y^i(t) = \frac{v^i}{1 - 2p} \left( t^{1-2p} t_0^{2p} - t_0 \right) + \mathcal{O}(t^{1-4p}), \quad t \gg t_0$$  \hspace{1cm} (99)

In that case the deformation [48] becomes [27]:

$$V = \int_{\partial \Sigma} a_0^2 \frac{v^i}{1 - 2p} \Theta_s(t - t_0) \left( t_0^{2p} - t_0 t^{2p} \right) \partial_n X^i$$  \hspace{1cm} (100)
where $\Theta(t-t_0)$ is the regulated Heaviside step function [26], expressing an instantaneous action (impulse) on the $D$-particle at $t = t_0$.

The case $p > 1$ corresponds to an accelerated Universe, $\ddot{a}(t) = p(p - 1)a(t) > 0$, which suffers from the problem of a cosmological horizon, in the sense that the quantity

$$\delta \propto \int_{t_0}^{\infty} \frac{dt}{a(t)} = \int_{t_0}^{\infty} \frac{dt}{a(t)^p} < \infty, \quad \text{for } p > 1$$

is finite. This presents a problem in defining asymptotic states, and hence a proper $S$-matrix [3]. Hence, as mentioned previously, this constitutes a challenge for critical string theory [3].

However, as became clear from our analysis above, non-critical string theory, with the target-time being identified as the Liouville field [10], is not an $S$-matrix theory [4]. In such a case there is a well-defined superscattering matrix $\Sigma$ which connects asymptotic density matrices, rather than pure states.

However, there may be a possibility, which we shall point out in this section, which allows for a relaxation of a non-critical string model to a critical one, thereby allowing for a proper asymptotic (far future) definition of pure states. As we have mentioned previously, this is dictated by the fact that the anomalous dimension of the non-conformal recoil deformations [34] is running with the time/Liouville-field and becomes zero asymptotically in time, thereby implying an approach to a fixed (equilibrium) conformal point.

In the specific cosmological model at hand, this has been demonstrated in [27], and will be reviewed below. Specifically, we shall show that the recoil of the $D$-particle induces, via the associated non-criticality of the string theory involved, a sufficient distortion in the space time global geometry so as to remove the cosmological Horizon and thus stop the cosmic acceleration. This phenomenon presumably will be enhanced by the presence of populations of $D$-particle defects, like the ones assumed in previous sections.

To this end, we first recall [27] that, from a world-sheet viewpoint, the operators (100),(97) obey a higher order (determined by the exponent $p$) logarithmic algebra [26], and are relevant operators in a renormalization-group sense, with anomalous dimension of the form (27). They are, therefore, in need of Liouville dressing. This is similar in spirit to their flat-space counterparts (25), with the important physical difference that here the logarithmic operator pairs do not describe Galilean positions and velocities but rather cosmic velocities and acceleration. The logarithmic nature of the deformations imply that they can still be classified by conformal blocks, and hence are ‘good’ deformations from a conformal-field theory viewpoint. This makes them acceptable backgrounds of string theory. However, as a result of their relevant renormalization-group nature, the resulting $\sigma$-model is non-critical. This is in agreement with the fact that such deformations describe non-equilibrium processes in the Universe.

Dressing the operators with the Liouville mode, to restore conformal invariance, we have

$$V_{L,\text{bulk}} = \int_{\Sigma} e^{\alpha_i \phi} \partial_\alpha \left( y_i(t) \partial^\alpha X^i \right), \quad \alpha_i = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + (2 - h_i)}$$

(102)

where $h_i$ is the conformal dimension of the bulk operator. Above we have followed the method of dressing the non-critical operator, having it first expressed as a world-sheet bulk total derivative operator. One can show [27] that completely equivalent results are obtained if one dresses directly the boundary operator (100).

For this type of non-critical recoil models the central charge deficit $Q$ has the form [27]:

$$Q^2 = Q_0^2 + O(\epsilon^2)$$

(103)

There are two cases, depending on the value of $Q_0$: (i) $Q_0 \neq 0$, and (ii) $Q_0 = 0$, in which case the recoil is the only source of non-criticality in the model. The case (i) is a feature of cosmological models where there may be, in general, additional sources of non-criticality, as compared with the flat-space time case discussed in previous sections, which is characterized by $Q_0 = 0$. This implies that the gravitational anomalous dimensions are of order: $\alpha_i \sim \epsilon^2$ if $Q_0 \neq 0$, and $\alpha_i \sim \epsilon$ if $Q_0 = 0$. 

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Consider first the case $Q_0 \neq 0$. In this case $\alpha_0 \sim \epsilon^2$, $\phi_0 \sim \epsilon^{-2}$ and hence $\alpha_0 \phi_0 \sim \epsilon t = \text{const.}$ We identify now the Liouville direction $\phi$ with that of the target time \cite{10,30}. Given that $t \sim \frac{1}{\epsilon}$ this implies that $\phi \sim t^2$. Under this identification we observe \cite{25} that the terms $e^{\alpha_0 \phi_0}$, and the exponential factors $e^{-\epsilon (t-t_0)}$ appearing in the regulated $\Theta$ functions \cite{13} are all of order one.

From these considerations one obtains an induced non-diagonal metric element $G_{0i}$:

$$G_{0i}d\phi dX^i \simeq -v_i \alpha_i t^2 p d(t^2)dX^i, \quad t \gg t_0$$

Under the fact that one identifies $\epsilon^{-1} = t-t_0 \sim t \gg t_0$, the non-diagonal element of the spacetime metric becomes:

$$G_{0i} \sim vt^2 t^{p-1}$$

We remind the reader that we analyze here the case with horizon, which implies $p > 1$.

It is convenient now to diagonalize the metric, which implies the following line element

$$ds^2 = -\frac{v^2}{a_0^2} t^{2p-2}dt^2 + a_0^2 t^2 (dX^i)^2$$

By redefining the time coordinate to $t' = \frac{\sqrt{\alpha_0}}{\alpha_0} t^p$ one obtains the induced line element:

$$ds^2 = -(dt')^2 + \frac{a_0^4 p^2}{v^2} (t')^2 (dX^i)^2, \quad t \gg t_0$$

From (103), we thus observe that the induced metric has no horizon, and no cosmic acceleration. In other words a recoiling $D$-particle, embedded in a space time which initially appeared to have an horizon, back reacted in such a way so as to remove it! Equivalently, we may say that recoiling $D$-particles are consistent only in spacetimes without cosmological horizons.

Similar conclusions are reached in the case $Q_0 = 0$. In that case, $\phi/Q \sim \epsilon^{-2}$, for reasons associated with the normalization of the Liouville $\sigma$-model kinetic term, as explained previously \cite{10}, and since $Q \sim \epsilon \sim \frac{1}{t}$ in that case (cf. \cite{23}), one has that $\phi \sim t$. Again, the exponential terms $e^{\alpha_0 \phi}$, $\alpha_0 \sim \epsilon$, and those coming from the regulated $\Theta_s(t)$ are of order one. Evidently, the induced non-diagonal metric has the same form (102) as in the case with $Q_0 \neq 0$, and one can thus repeat the previous analysis, implying removal of the cosmological horizon and stopping of cosmic acceleration.

The reader must have noticed that the same conclusion is reached already at the level of the metric (106), before the time transformation, once one interprets the coefficient of the $(dt)^2$ as a time-dependent light velocity. The fact that such situations arise ‘suddenly’, after a time moment $t_0$, might prompt the reader to draw some analogy with scenarios of time-dependent light velocity, involving some sort of phase transitions at a certain moment in the (past) history of our Universe \cite{17}. In our case, as we have seen, one can perform (at late times) a change in the time coordinate in order to arrive at a RW metric \cite{107} \footnote{At this point we would like to draw the reader’s attention to the fact that such transformations depend on the recoil velocity, and thus on the energy content of the matter incident on the $D$-particle. On the other hand, it is not clear how one could extend the results of the present work to space-time ‘foam’ situations \cite{30}, in which several incident particles interact with collections of $D$-particles, which are virtual quantum excitations of the string/brane vacuum. Hence it might be that in such cases one cannot perform simultaneous transformations to diagonalize the metric, thereby obtaining non-trivial refractive indices of the type discussed in \cite{33,30}, implying time delays in the arrival time of massless particles which increase with increasing energy. This should be contrasted to the thermal refractive indices discussed in section 7 above. Such issues fall beyond the scope of the present article.}

From a field-theoretic point of view, the removal of the horizon would seem to imply that one can define asymptotic states and thus a proper $S$-matrix. However, in the context of Liouville strings, with the Liouville mode identified with the time \cite{10}, there is no proper $S$-matrix, independently of the existence of horizons \cite{3}. As we have already discussed, this has to do with the structure of the correlation functions of vertex operators in this construction, which are defined over steepest-descent closed time-like paths in a path-integral formalism, resembling closed-time...
paths of non-equilibrium field theories [10, 4]. In such constructions one can define properly only a (non factorizable) superscattering matrix $S \neq S S^\dagger$. Nevertheless, the removal of the cosmic horizon asymptotically, and the stopping of the acceleration of the Universe, are compatible with the approach of our non-critical string model to a (world-sheet) renormalization-group fixed point. The latter is an equilibrium point, at which the theory becomes a critical string. It is for this reason that one can define proper asymptotic (far future) states.

As a final remark, we would like to point out that in models where the induced non-criticality is an exclusive feature of the impulse distortion (100), the induced four-dimensional vacuum energies are given by the central charge deficit, which asymptotically in time behaves as (c.f. (35),(31)):

$$\Lambda_{4d} = Q^2 \sim \frac{1}{t^2}$$

Such a relaxing-to-zero vacuum energy is compatible with recent observations, and in the above model is due exclusively to gravitational recoil degrees of freedom. The dilaton field has been assumed constant.

On the other hand, in certain non-critical cosmological models with central charge deficits $Q^2 = Q_0^2 + \ldots$, with $Q_0 = \text{constant} \neq 0$, and the $\ldots$ denoting terms that go to zero as $t \to \infty$, there may be non-trivial time-dependent dilaton fields, $\Phi(t) \sim -\log(t)$, for $t \to \infty$. Models with such a behaviour are discussed explicitly in [4, 48], where we refer the reader for details. Here we simply mention that such dilaton fields may operate as quintessence fields, in the sense of yielding four-dimensional relaxing-to-zero vacuum energies of the form:

$$\Lambda_{4d} = e^{2\Phi(t)}Q_0^2 \to \frac{Q_0^2}{t^2}$$

where the dilaton exponential factors appear as a result of the fact that we work in the Einstein frame of string effective actions, in which the Einstein curvature term has the canonical normalization [1].

We therefore see that Liouville strings are compatible with current cosmological observations. Of course we do not claim that there are no conventional explanations of the currently observed acceleration of the Universe, but we think that the above ideas are interesting enough to encourage further studies of non-critical string cosmological models.

11 Instead of Conclusions

In this work we have studied the formation of bubbles as a result of scattering of closed strings off $D$-particles embedded in a four dimensional spacetime. Such configurations may be thought of as a trivial case of intersecting branes, provided one adopts the modern viewpoint that our four-dimensional world is a $D3$-brane. In this sense, the string scale $M_s$, which enters our calculations, is not necessarily the same with the four-dimensional Planck scale, and as we have seen this played an important role in our analysis, as it allowed us to work at distances sufficiently far from the Schwarzschild radius of the $D$-particle, for finite $g_s$ string couplings.

An interesting feature of our approach is the entropy production, which we associated with information carried away by the recoil degrees of freedom. This latter feature may have cosmological implications for mechanisms of entropy production in the early universe, where we expect the density of $D$-particles to be significant, and hence the probability of scattering with closed strings important. The fact that highly energetic charged particles can escape the bubble, with the simultaneous release of radiation, which can also escape and thus is in principle observable, is interesting, and might imply important phenomenological constraints on the order of the density of the $D$-particles in the Universe today, in the way discussed in section 8. An important feature for all such scenarios, which is yet-to-be established is the stability of the relevant population of $D$-particles. This issue is an open issue at present, and is exclusively a non-perturbative feature of string theory, which is beyond our control at present.
A comment we wish to make at this point concerns the impossibility of the extension of the above analysis, with the specific choice of fields, to higher-dimensional spacetimes, of spacetime dimension \( d > 4 \). This comes about because, under the simplest form for the tachyonic mode \( T = \ln r \), consistent with Einstein's equations to order \( \mathcal{O}(\alpha') \), one obtains the following form for the tachyonic-mode potential \( V(T) \sim 1/r^2 \). On the other hand, the equation of motion for the mode itself, consistent with the above form for \( T \), demands that \( dV(T)/dT = -(d-2)V(T) \). Clearly, this is consistent only in \( d=4 \) spacetime dimensions. Hence, despite the fact that in our approach we have restored Lorentz invariance, we still obtain a special role of \( d = 4 \), as in the Lorentz violating scenario of [30], where the sacrifice of Lorentz invariance in the sense of an explicitly space-dependent vacuum energy, lead also to a selection of \( d = 4 \). If this feature survives the inclusion of the complete string matter multiplets, something which is not clear to us at present, then, it might imply that a recoiling Liouville \( D \)-particle cannot be embedded in (intersect with) a target spacetime (viewed itself as a \( D \)-brane), consistently with the \( \sigma \)-model conformal invariance, unless its dimensionality is \( d = 4 \). At present we consider the issue only as a mathematical curiosity of the specific effective field theory at hand, and we do not attribute to it further physical significance. However, surprises cannot be excluded.

Another important result, described above, concerns a potential physical rôle of embedded \( D \)-brane defects on the Cosmological Evolution of a Friedman-Robertson-Walker Universe. It has been argued that the presence of a recoiling \( D \)-particle results in the removal of cosmological horizons and the eventual stopping of the Cosmic acceleration. From a field-theoretic view point this would allow a proper definition of asymptotic states in such models. Moreover one obtains a relaxing to zero vacuum energy, compatible with current astrophysical observations.

The above considerations pertain strictly to the case where the \( D \)-particle defects are considered as real. In quantum space-time foam situations such defects are emerging as virtual excitations of the vacuum. In that case it is not clear how the present results can be extended, given that such situations lie far beyond the perturbative regime assumed above. Nevertheless one might hope that, at least qualitatively, some of the stochastic properties discussed here, e.g. thermal refractive indices and entropy growth, will survive the full quantum treatment, and will constitute characteristic features of non-critical stringy models of space time foam.

What we have described above is an admittedly speculative, but nevertheless interesting, at least in the authors' opinion, scenario on possible physical aspects of a specific Liouville (non-critical) string model of quantum gravity. It is interesting that physical predictions from such models might lie within the sensitivity of immediate-future experimental facilities, both terrestrial and extraterrestrial. The basic feature of non-critical string models is their instability, as a result of their non-equilibrium nature. As such, these theories may be quite relevant for the physics of the Early Universe, and also may be responsible for (part of) observed extreme astrophysical phenomena of cosmological origin, whose evidence has recently started becoming overwhelming. Gamma Ray Bursters, Ultra-High-Energy Cosmic Rays and others, are among the most obvious candidates to search for new physics, either within the (conventional) local field theories, or in the (yet to be established) framework of modern approaches to quantum gravity, like critical strings or even non-critical ones.

Admittedly, the ideas presented in this article are very speculative, and indeed may have nothing to do at the end with real Physics. However, the mathematical and logical consistency of such unconventional models, and their effects, seems, at least to the authors, convincing enough so as not to discard them immediately. Our basic point of view in this article, was that, if one adopts the modern approach of viewing our world as a domain wall (brane), embedded in a higher dimensional (bulk) space time, then the possibility of having intersecting brane configurations cannot be logically excluded. Embedded \( D \)-particles in a four-dimensional domain wall is a special (and the simplest) case of intersecting branes. As we have seen above, the fluctuations and scattering of the \( D \)-particles with other matter in the Universe may lead to physical effects, e.g. in connection with some extreme astrophysical phenomena, which severely constrain their populations.

It is actually very intriguing that stringy defects, which at first sight are not expected to play any rôle in low-energy physics, may actually be partly responsible for some extreme astrophysical
phenomena, whose observation became only recently possible, as a result of the enormous technological advances in both terrestrial and extraterrestrial instrumentation. It is always intellectually challenging, but also expected intuitively in some sense, to think of the vast Universe as being the ‘next-generation’ Laboratory, where ideas on the quantum structure of spacetime may be finally subjected to experimental tests in the not-so-distant future.

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