Quantum effects in bi-dust plasmas

J Puerta$^{1}$, P Martin$^{2,3}$, F Maass$^{2}$ and F Blanco$^{1}$

$^{1}$ Departamento de Física, Universidad Simon Bolivar, Balle Sartenejas, Caracas, Venezuela
$^{2}$ Departamento de Física, Universidad de Antofagasta, Av. Angamos 601, Antofagasta, Chile

E-mail: jpuerta@usb.ve, pablo.martin.dejulian@uantof.cl, fernando.maass@uantof.cl and fblanco@usb.ve

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Abstract

In this paper a theoretical approach has been carried out using a fluid model on a bi-dust plasma which consist of two dusts negative charged different particles in an electron–ion plasma. In such system, this fluid model describe a dense strong coupled heavy dust grain together with a weak coupled light dusty particle. The bulk is immersed in a system of electrons and ions dominated by a Thomas–Fermi density distribution, where interaction and propagation of solitary dust acoustic solitons will be analyzed. This investigation will be carried out for arbitrary amplitude of solitary dust acoustic waves. These are originated through the presence of both dust grains in the plasma system. Therefore, it is used a set of fluid equation which determine general expressions for the concentrations of different dust particles. The onset of the Sagdeev method is then employed in order to look for influence of the light component on the quantum system. In this way there is an increasing or decreasing in the Sagdeev potential. This is a quantum effect phenomena.

Keywords: bi-dust plasmas, solitons, quantum effects, acoustic waves

(Some figures may appear in colour only in the online journal)

1. Introduction

A dusty plasma, as a general rule, is a plasma system with electrons and ions plasmas with dust particles and neutral ones, whose size ranges in the micron and sub-microns with masses larger than protons [1]. Such plasmas are called complex plasmas, and now its study has increased very rapidly as important field of research. It is remarkable to mention that first experiments carried out in the International Space Station was in industrial plasmas [2]. This gave an idea of the relative importance of complex plasmas. This interesting environment are characterized by contains massive dust grains supporting a large variety of collected effect like solitary waves, voids, plasmas crystals and quasi-crystal and gels. After the excellent work from Rao [3] that predicted theoretically the occurrence of dust acoustic waves, experiments and demonstration of this mode appeared very rapidly. A lot of research has been addressed to study the properties of the dust acoustic solitary wave (DASW) and DASWs in quantum plasmas. These quantum effects appear mostly in dense astrophysical plasmas, laboratory plasmas and laser plasmas [4]. In this work, the properties of propagation of nonlinear DASW has been studied in a strong coupled bi-dust plasmas. It is interesting to point out related to this theme, that some interesting research, mainly theoretically. For example, the work carried out by Misra and Barnau [5] on two positive ions and other one on bi-dust plasmas [6]. The first one analyses the effect of the well known Landau damping in connection with the nonlinear propagation of DASWs with two species of positive ions ‘Landau damping of Gardner solitons in a dusty bi-ion plasma’. On the second one, a ‘Drift instability grow rates in non-ideal inhomogeneous bi-dust plasmas’ has been studied on a bi-dust plasma, where inhomogeneities due to a weak magnetic field generating instabilities in that configuration was used and analyzed. Such studies indicate some interest on this systems. Here, we use this concepts, introducing a light component in our system that is considered and treated as an ideal gas. However, the heavy component, contrary have a strong coupled character due to the high negative charge on the surface. It is important to point out that in virtue of all these effects a large interest in such plasmas with this massive particles has been carried out. The main

3 Author to whom any correspondence should be addressed.

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characteristic of these systems is to acquire significant amount of charge on their surface. Therefore, the grains can suffer high electrostatic forces from their neighbors, as predicted by Ikezi in 1986 [7]. In this way a dusty plasmas enters rapidly in a strong coupled regimen due to the high charge and the low thermal temperature of the dusts. The strong coupling activity is measured by the strong coupled parameter given by $\Gamma_D$. In the case of $\Gamma_D \geq 1$, we speak of a quasi strong regimen. This parameter is defined as

$$\Gamma_D = \frac{Z_D^2 e^2 (n_D)^{2/3}}{\kappa_D}$$

with $\kappa_D = r_D/\lambda_D$ and $r_D = n_D^{1/3}$, and where $\Gamma_D$ is the ratio between Coulomb (subscript $D$ stays for the heavy grains) and thermal energy, $\kappa_D$ is the Boltzmann constant, $T_D$ the thermal temperature, $Z_D$ the charge number, constant in this work and $r_D$ the average particle distances. Now in order to use the fluid approach to investigate DASWs, in bi-dust, the procedure introduced first by Gozadinios will be considered [8]. Numerical models were used to simulate crystalline dusty plasmas under micro-gravity, therefore the state equation following such procedure was of the form

$$P^* = N \Gamma_D \kappa_B T_D n_D (1 + \kappa_D) e^{-\kappa_D} = \kappa_B \tilde{T}_D^* n_D,$$

where $N_D = N_{dust}/3$ is the neighbors number to determine the dusty plasma crystal structure, $\kappa_D = r_D/\lambda_D$, the lattice parameters, $P^*$ is the effective electrostatic pressure, and $\tilde{T}_D^*$ the effective electrostatic temperature that normally is a little bit higher than $T_D$. Here, the electrostatic Debye screening length $\lambda_D$ is

$$\lambda_D^2 = \frac{\epsilon_0 \kappa_B T_{Fe} T_{Fi}}{4 \pi e^2 (n_0 T_{Fe} + n_0 T_{Fi})}.$$ 

This model was developed for the crystalline structure. They demonstrate that this is also still valid for the fluid model in the range

$$1 \ll \Gamma_D \ll \Gamma_{cr},$$

In the quasi-strong coupling regime, the model is that of a ‘quasi-gas’, liquid state.

$$1 \ll \Gamma_D \ll \Gamma_{cr},$$

where $\Gamma_{cr}$ is the critical coupling parameter for the onset of the crystallization. In the case of the second component, we consider that system hold in a quasi-gas, and in formal thinking, it is acceptable to use the standard state equation

$$P_d = \kappa_B T_d n_d,$$

where the subscript $d$ indicates the light component.

In this paper, the main purpose is to study the influence of the light component of the bi-dust system, using the fluid equation model to determine its interactions with the strong coupled components. Here, it is convenient to point out that for electron and ions the quantum coupling parameters $\Gamma_{el,i} \simeq \omega_{pe,i}/T_{Fe,i}$ are much lower than the other one. Therefore, the assumption we are using is that only the heavy components are strong coupled. In order to analyze the influence mentioned before, we treat to make the investigation for arbitrary amplitude for both components in bi-dust system using the fluid model to determine the generalized particle concentration $n_j (M, \varphi)$ and $n_D (M, \varphi)$ where $M$ is the average Mach number, defined elsewhere and $\varphi$ the electrostatic plasma potential. Additionally our motivation is to establish a procedure for arbitrary amplitude. We adopt the pseudo-potential method of Sagdeev in order to determine the general Sagdeev potential $V(M, \varphi, n_j)$ for the whole system in function of $\varphi$ and the other parameters of the system. Here, it is straightforward to visualize (see the corresponding figures) the interaction of the two systems via the Sagdeev potential.

In the case of light particles the inter-particle distance is considered large enough in order that the electrostatic influence can be ignored. In this case this second system is treated as quasi-ideal, and the subscript $d$ is taken ideal for the ‘light grain’. On the other hand, it is well known that quantum dusty plasmas has been observed in astrophysical plasmas. In such systems $\Gamma_d$ can achieves values greater than one, and quantum effects has been observed because there has been observed densities are in the order of $n_{d1} \approx 2.0 \times 10^{21} \text{ m}^{-3}$, $Z_d \approx 10^3$ and $T_e \approx T_i \approx 3000 \text{ K}$ [9, 10]. In this paper the interest is interested on the analysis of nonlinear solitary waves in bi-dust plasmas in a quasi-strong coupling regime in order to observe quantum effect phenomena in presence of a second light particle. In this way the electrons and ions are considered to have the properties of a Thomas–Fermi distribution. For the dust particle a fluid model take in consideration and they characterize the properties of DASW using pseudo potentials with arbitrary amplitudes [9, 11–13]. This study will be of interest in such systems where impurities of particularly light grains could be present.

2. Theoretical mode

In general it is well known that quantum effect related to electrons and ions will become important to determine the dynamic of our system. This is, when and only when, the average of inter-particle distances are comparable with the De Broglie thermal wavelength. In this direction, in order to research about the properties of non-enveloped DASW, we make the assumptions that the propagation happens along an unbounded collision-less and not magnetized negatively charged bi-dust plasma. Due to a lot of quantum effects that could appear in such system, it is acceptable to assume that electrons and ions follow Thomas–Fermi distribution [10]. Therefore, we have

$$n_e = n_{e0} \left(1 + \frac{e \phi}{E_{Fe,i}}\right)^{3/2},$$

$$n_i = n_{i0} \left(1 - \frac{e \phi}{E_{Fi}}\right)^{3/2},$$

where $n_{e0}$ and $n_{i0}$ denotes electrons and ion densities in equilibrium. As Usual, $e$ is the electron charge and $\phi$ the plasma potential. Here, $E_{Fe,i} = 2\kappa_B T_{Fe,i}$ and $E_{Fi} = 2\kappa_B T_{Fi}$ are the Fermi energies with $T_{Fe,i}$ the Fermi temperature defined by [14]

$$T_{Fe,i} = \frac{\hbar^2}{2m_{e,i}} \kappa_B \left(3 \pi^2 n_{e,i}\right)^{2/3}.$$
2.1. Ideal dust plasma system

For the light particles, and the average inter-particle distance are large in comparison with De Broglie length, thus quantum effects can be neglected. A description of the system using the fluid model for arbitrary amplitude, is acceptable. Then, the continuity and momentum equations in dimensionless form are considered, taking into account temperature effects

\[ \frac{\partial n_d}{\partial \tau} + \frac{\partial}{\partial \xi}(n_d v_d) = 0, \] (9)

\[ \frac{\partial v_d}{\partial \tau} + v_d \frac{\partial v_d}{\partial \xi} = \frac{Z_d \varepsilon_F e}{m_d} \varepsilon \frac{\partial \varphi}{\partial \xi} - \frac{1}{n_d m_d} \frac{\partial P_d}{\partial \xi}. \] (10)

Here \( n_d \) is the particle density of the small dust. This is normalized by \( n_{d0} \). The velocity \( v_d \) is also normalized by \( C_D = (Z_{d0} n_{d0} T_{Fi}/m_d)^{1/2} \) and the electrostatic wave potential \( \varphi \) by \( \varepsilon_0 / \varepsilon_F \). Similarly time \( \tau \) will be normalized by the dust plasma frequency \( \omega_{pd}^{-1} \) and space \( x \) by its Debye length. Now, in order to look for this acoustic solitary wave solutions, it is convenient to use the stretched relation \( \xi = x - M \tau \), to make all the dependent variables depend on a single one, and thus we follow the description shown in equation (11) [3, 15, 16]

\[ \frac{\partial}{\partial \tau} = -M \frac{\partial}{\partial \xi}, \] (11)

where \( M \) is the Mach number defined by the ion acoustic solitary wave speed divided by the ion acoustic speed. Equation (11) is introduced into equation (9) obtaining

\[ \frac{\partial n_d}{\partial \xi} (v_d - M) = -n_d \frac{\partial v_d}{\partial \xi}. \] (12)

In order to solve this equation for \( n_d \), we must use appropriate boundary conditions defined by:

\[ \xi \rightarrow \pm \infty \quad n_d \rightarrow 1 \quad V_d \rightarrow 0 \quad \varphi \rightarrow 0 \quad \frac{\partial \varphi}{\partial \xi} \rightarrow 0 \]

Thus, it is obtained

\[ n_d = \frac{M}{v_d - M}. \] (13)

Now by integration of equation (15), and imposing boundary condition as defined before, it yields to

\[ \frac{1}{2} (M - v_d)^2 - \mu_d \varphi + \delta_d \ln n_d = \frac{1}{2} M^2. \] (16)

First pitting equation (13) into equation (16) and solving this equation for \( n_d \) in function of \( \varphi \), and developing \( \ln(n_d) \) in series up to second order [17] (Abramowitz, of \( n_d \gtrsim 0.5 \) the following equation is obtained

\[ n_d = \frac{1}{\sqrt{1 + \frac{2 \varepsilon_0}{\varepsilon_F \varphi}}}. \] (17)

2.2. Heaviest dust plasma system

The heaviest grains with \( \Gamma \geq 1 \) are in presence of a quasi-strong coupling system susceptible to be described with the fluid model as for the light grains, this system is treated as a quasi-ideal system considering solutions in steady state condition. Thus, we can use the continuity equation as well the momentum one, where the thermal effects via \( T_D \) will be ignored because this effect is very low in comparison with the electrostatic one, so only the effective electrostatic influence represented by \( T_D^* \) will be considered. Therefore, equations (7), (18a) and (18b), (together with the generalized Poisson relation (18c) build a closed system of equations:

\[(a) \text{ Continuity } \frac{\partial n_D}{\partial \tau} + \frac{\partial}{\partial \xi}(n_D v_D) = 0 \]

\[(b) \text{ Momentum } \frac{\partial v_D}{\partial \tau} + v_D \frac{\partial v_D}{\partial \xi} = \mu_D \frac{\partial \varphi}{\partial \xi} - \delta_D \frac{1}{n_D} \frac{\partial P_D}{\partial \xi} \]

\[(c) \text{ Poisson } \frac{\partial^2 \varphi}{\partial \xi^2} = \mu_e (1 + \sigma \varphi)^{3/2} - \mu_e (1 - \varphi)^{3/2} + \mu_D n_d + n_D. \] (18)

where

\[ \mu_D = Z_D T_{Fe} / T_D^*; \quad \delta_D = \kappa_B T_D^* / \varepsilon_D; \]

\[ T_D^* = N_0 \Gamma_D T_D (1 + \kappa_D) e^{-\sigma}; \]

\[ \mu_e = \varepsilon_Z / \varepsilon_D; \quad \mu_i = \varepsilon Z_D / \varepsilon_D; \]

\[ \sigma = T_{Fe} / T_{Fe}; \]

\[ \lambda_D^2 = \frac{\kappa_B T_{Fe} T_D}{4 \pi e^2 (n_{d0} T_{Fe} + n_{e0} T_{Fe})}; \]

\[ \omega_{PD}^2 = \frac{n_{d0} Z_D^2 e^2}{4 \pi m_D}; \]

\[ T_0 = \frac{Z_D^2 n_{d0} T_{Fe} T_{Fe}}{n_{d0} T_{Fe} + n_{e0} T_{Fe}}; \]

\[ \varepsilon_{(e,i)} = \kappa_B T_{Fe} (e, i). \] (19)
Now, using equation (11) into the continuity equation (18a), and after several algebraic manipulations the general form for \( n_D \) is obtained. Putting this expression into the momentum equation (18c) together with equation (11) the equation (19) yields to equation (20)

\[
\frac{M^2}{2n_D} = \mu_D \varphi + \delta_D \ln(n_D) = \frac{M^2}{2}.
\]

Integrating and taking into account the boundary conditions we found

\[
\xi \to \pm \infty, \quad \varphi \to 0, \quad n_D \to 1, \quad \nu_D \to 0
\]

\[
\frac{M^2}{2n_D} = \mu_D \varphi + \delta_D \ln(n_D) = \frac{M^2}{2}.
\]

Here, the integration constant could be determined using the usual boundary condition defined before.

### 3. Results

Here it is shown some results for \( n_c \approx 10^{35} \text{ m}^{-3} \), \( Z_d \approx 10^3 \), \( n_d \approx 10^{22} \text{ m}^{-3} \) and also assume steady state condition and arbitrary amplitudes. Now, in several figures will be shown graphically the behavior of the Sagdeev pseudo—potential \( V(\varphi) \) when small grains are present inside of the system characterized by the parameter \( \mu_d \) for different values of the averaged mach number \( M \). Four figures has been considered with the same value of \( \sigma = 0.05 \), but different values of \( \mu_d = 0.07, 0.05, 0.03 \) and 0.00. Different values of \( M = 2.0, 2.1, 2.2, 2.3 \) are also considered. However, in figures 3–5 is shown the behavior of the Sagdeev pseudo potential \( V(\varphi) \) for \( \sigma = 0.05 \) and \( M \) similar as before but for \( \mu_d = 0.07, 0.05, 0.03 \). The effect is a singular reduction of the potential deep and also a significant decreasing of the trapping phenomenon registered in figures 3–5 in comparison with the case presented in figure 6, with \( \sigma = 0.05 \) and \( \mu_d = 0.00 \).
which means, no impurities are present in the system. It is assumed that this effect is due to a significant modification of the equilibrium electrostatic field due to the presence of the small dust grain across the plasma.

4. Discussion

In order to make an adequate analysis about this system, it is necessary to know that the Sagdeev pseudo-potential \( V(\varphi) \) is a very important tool to determine the spectrum and profile of the soliton structure. Due to this property, it is crucial to point out that \( V(\varphi) \) could have solitonic solution 39 only if \( d^2V(\varphi)/d\varphi^2 < 0 \) and therefore \( dV(\varphi)/d\varphi = V(\varphi) \approx 0 \) at \( \varphi = 0 \), also we have a minimum \( \varphi_m = 0 \) and \( V(\varphi_m) \geq 0 \). In this case the pseudo-potential indicate that at this point we have a region where the total energy take the value of zero and the quasi-particle will be reflected. Under this consideration is easy to observe that the quasi-particle have enough potential between \([0, \varphi_m]\) to trap and to participate of an oscillatory process. Figures 3 \( \rightarrow \) 6 shown such a region where \( \varphi_m \approx 0.07 \) considering our sets of initial parameters.

This study of the predicted stationary pulses related with the pseudo-potential theory explained elsewhere, demonstrate the soliton effect in function of the grain densities \( n_D, n_d \), where \( n_D \) stay for the strong coupled character. and the pulse velocity characterized by the the average value of the Mach number \( M \). At this point, it is very important to study the range for the onset where soliton waves could appear. In this order, we use Mathematica program to achieve the minimum and maximum values for \( M \) graphically in function of \( \varphi \), between the quasi-particles bounce back and forth. In the figures 1 and 2, the values for the minimum and maximum of \( M \) are shown: \( M_{\text{lower}} \approx 1.3 \) and \( M_{\text{upper}} \approx 2.4 \). Here we change the values of \( n_d \) while the other ones, remains fixed. Figures 3 \( \rightarrow \) 6 shown the behavior of the observed spectra. Here it is to determine, that the soliton amplitude is a decreasing function of \( n_d \) but contrary increase with the pulse velocity and from the profile structure the depth appear significant lower and better stronger trapping. Here is my be good to point out that such behavior could be very interesting, because in general there exist a lot of spurious particles in several experimental facilities where, the simple fact of their appearing turn on the effect(quantum effect) here described.
5. Conclusion

In this work we introduce a new theoretical analysis of the nonlinear wave particle interaction represented by the DASW in the region of quasi-strong coupling effect generated by a collision less unmagnetized Thomas–Fermi plasma distribution where it would be assumed that the presence of two kind of dust particle of different size, small and big are defined by us as bi-dust plasma. The system dynamic introduced by the small grain could be considered as ideal and treated as a normal plasma system while the other one obey the quantum field dynamic generated by the Thomas–Fermi distribution [6]. The effect on the small grain will be ignored. Here, the main idea is to analyze this plasma when small particles take actions in the system, considered as an impurity affecting the particle trapping usually shown by displaying the phenomenon via the Sagdeev potential $V(\phi)$ for different values of the Mach number $M$ among other parameters that we do not analyze in this work. We think that the Mach number is an important parameter in this discussion for DASWs in bi-dust plasmas to the strong coupling. This new solution here obtained could have a lot of applications and Soliton waves in bi-dust plasmas are studied. One of the dust in plasma is a light one and the other is a heaviest particle. The condition here considered are those characteristic of astrophysics system and some experiments in laboratory with low temperature. In these conditions the ions and electrons have to be treated in quantum way considering the well known Fermi temperature. The Sagdeev potential characteristic of solitons have been determined. In the case where the dust light density is zero, our results are coincident with those found for other authors for this plasma systems [18]. However new Sagdeev potentials have been calculated for these systems considering the presence of different light dust densities and Mach numbers. An analysis of these potentials shows a significant decreasing of the trapping phenomena. This has to be considered as a reduction of the equilibrium condition to the presence of the light dust component in the plasma system, may be altering the dynamic of the electrostatic field present via the assumed distribution.

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ORCID iDs

P Martin @ https://orcid.org/0000-0001-9554-4390

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