Composite shell of toroidal balloons of breathing apparatus with meridional winding of threads

M A Komkov¹, A L Galinovskiy¹, Kyaw Myo Htet¹

¹ Department of rocket and space mechanical engineering, Bauman Moscow State Technical University, 105005, Russian Federation, Moscow.

Email: m_komkov@list.ru

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Abstract: The prospects of using toroidal composite high-pressure cylinders for breathing apparatus, which are more ergonomic in comparison with similar cylindrical cylinders, are described. Through the analytical solution of the equilibrium equations, the cross-sectional shape of the composite shells of toroidal balloons, which are intersecting loop-like curves, is determined. A constructive diagram of a toroidal composite balloon, formed by the intersection of the upper and lower branches of the shell reinforced along the meridians, and the profiled annular layer of threads, installed at the point of their intersection, is proposed.

1. Introduction

Currently, the global market demand for cylinders for breathing apparatus of all types is 350,000 pieces per year. It includes steel or metal composite 2...9 liter cylindrical cylinders weighing 2.5...12.0 kg for a working pressure of 20...24 or 30 MPa. At the same time, ergonomic tests of breathing apparatus with toroidal cylinders [1] on the back of a person (climbing stairs, crawling, passing hatches, water drains, clearing debris, etc.) showed significant advantages of using toroidal cylinders in comparison with their cylindrical counterparts.

The widespread use of toroidal high-pressure cylinders made by spiral winding of tape prepreg at angles of ±β to the torus meridian [2] was hindered by the absence of CNC toron-winding machines. Composite toroidal cylinders with a volume of up to 9 liters, a working pressure of up to 30 MPa and a weight of 5…6 kg are promising for use in breathing apparatus [3-4] for firefighters, EMERCOM soldiers, industrial workers. Composite toroidal cylinders for breathing apparatus are among the promising new high-pressure cylinders. They were successfully assembled in breathing apparatus for various purposes. Moreover, they have good mass characteristics and great opportunities for further improvement and organization of mass production.

At the Bauman Moscow State Technical University designed and manufactured in a single copy a small-sized toron-winding machine SNT-2A [5] with a control system for the trajectory of winding (laying) threads in a spiral from a worm-cam mechanism. The machine (Fig. 1, a) is intended for the manufacture of toroidal composite structures up to 400 mm in diameter and up to 120 mm in diameter, having a circular (Fig. 1, b), elliptical or rectangular cross-section by the orbital winding method. Spiral winding of toroidal shells of cylinders is a rather laborious process; it is much easier and more efficient to manufacture toroidal structures by the method of transverse winding of threads or tapes along the meridians.
Figure 1. General view of a toro winding machine SNT-2A (a) and a fiberglass toroidal balloon (Dbal = 402 mm; dsection = 80 mm), made by spiral winding of threads with an angle $\beta_0 = 48^\circ 8'$ to the meridian on a large torus diameter (b).

At the same time, a number of companies [6] produce toron-winding machines with a programmed control system for the manufacture of large-size electrical coils with transverse winding ($\beta_0 = 0$) of the wire. Such machines with a slight modification of the drive of the mandrel movement can be used for the manufacture of toroidal composite or combined shells of cylinders [7] and annular box frames [5], transverse winding of prepreg tape [8] along the torus meridians.

The aim of the work is to design and determine the structural and geometric parameters of the uniformly stressed composite shell of toroidal balloons, made by orbital winding of filaments along the meridians.

2. Formulation of the problem

An analytical study of the optimal shape of the shells of pressure vessels formed by the meridional winding of one family of threads was been previously considered in [9-10]. In [11], it was proposed to manufacture the bottoms of cylindrical cylinders using two systems of threads. The first system of threads had laid along the meridians, when approaching the pole hole it touches its contour and returns to the area of large radii. The second system of threads fits around the circumference of the pole hole, it winds the first system of threads and receives the linear forces from it. In our opinion, a similar problem can be solved for an equally stressed toroidal shell formed by meridional and circumferential winding of filaments.

3. Determining the shape of the meridian

To determine the shape of the generatrix of the toroidal shell made by winding threads along the meridians (Fig. 2), we use the dependence for the angular coordinate on the radius of rotation of the toroidal shell (3.64, [5]). Substituting into it the angle of winding of the threads $\beta = \beta_0 = 0$, we get:

$$
\cos \alpha = \frac{(r^2 - c^2) \cos \beta_0}{(\bar{r}_0^2 - c^2) \cos \beta} = \frac{(r^2 - c^2)}{(\bar{r}_0^2 - c^2)},
$$

(1)

Where $\alpha$ - angular coordinate, $0 \leq \alpha \leq \pi$; $r$ - radius of rotation, $\bar{r}_0 \geq r \leq r_\pi$; $c$ - distance from the axis of rotation to the top of the torus; $\bar{r}_0$ - radius of rotation of the shell at the maximum diameter (equator) of the torus. Since the generatrix of the shell intersects the small equator of the torus $r = \bar{r}_\mu$ at a right angle, the angle $\alpha = \pi$, $\cos \alpha = -1$ and the geometric parameters of the toroidal shell (1) will be related by the equalities:

$$
c^2 = \left(\bar{r}_0^2 + r_\mu^2\right)/2; \quad \bar{c}^2 = (1 + \mu^2)/2,
$$

(2)

Where $\bar{c} = c/\bar{r}_0$ and $\mu = \bar{r}_\mu/\bar{r}_0$ - respectively, the relative distance from the axis of rotation to the top and small equator of the torus.
The relationship between the current radius of rotation and the angular coordinate of the shell $\alpha$ found from solution (1) with allowance for (2):

$$r^2 = r_0^2 \frac{1}{2}\{1 + \mu^2 + (1 - \mu^2)\cos\alpha\}. \quad (3)$$

Replacing in equation (1) with its differential ratio, according to the designations of Figure 2, b, we write:

$$y' = \frac{dy}{dr} = - \text{ctg}\alpha; \quad \cos \alpha = - \frac{y'}{\sqrt{1 + y'^2}}. \quad (4)$$

performing algebraic transformations, we obtain:

$$y' = \frac{dy}{dr} = - \frac{(r^2 - c^2)}{\sqrt{(r_0^2 - r^2)(r^2 - c^2)}}. \quad (5)$$

Figure 2. Sectional shape of the toroidal shell of the pressure vessel, reinforced with threads (a): 1 - meridian; 2 - threads and to the definition of linear force and differential ratios in a toroidal shell (b).

$$y = - \int_{r_0}^{r} \frac{(r^2 - c^2) dr}{\sqrt{(r_0^2 - r^2)(r^2 - c^2)}}. \quad (6)$$

Changing the limits of integration in equation (6) and the sign of the integral, we represent it as the sum of two integrals, which can be expressed in terms of algebraic functions and elliptic integrals as follows:

$$y(r) = \pm r_0 [E(\theta, k) - E(\theta, k)], \quad (7)$$

where $F(\theta, k)$ and $E(\theta, k)$ - elliptic integrals of the I and II kind, $k = \sqrt{1 - \mu^2}$ and $\theta = \arcsin \sqrt{(1 - r^2/r_0^2)/(1 - \mu^2)}$ - module and argument of elliptic integrals; $\bar{r} = r/r_0$ – the relative radius of shell rotation.

Because $\sin \theta = \sqrt{(1 - \bar{r}^2)/(1 - \mu^2)}$, after substituting $\bar{r}^2$ from (3) into it, we obtain: $\sin \theta = \sqrt{(1 - \cos \alpha)^2/2} = \sin \alpha / \sqrt{2}$. It gives us $\theta = \alpha / \sqrt{2}$. If $\bar{r} = \mu$, then the argument $\theta = \pi / 2$ and the elliptic integrals $F(\theta, k)$ and $E(\theta, k)$ transform into complete elliptic integrals of the first $K(\pi / 2, k)$ and second $E(\pi / 2, k)$ kind.
From equations (7) it follows that when winding threads with an angle $\beta_0 = 0$, the generatrix of a toroidal shell of uniformly stressed fibers is a loop-shaped curve that defines an infinitely long corrugated pipe (Fig. 2, a) for each torus parameter $\mu$ except $\mu = 0$.

4. The discussion of the results

Analysis of equation (7) shows that the upper and lower branches of the shell meridian (Fig. 3) intersect at a radius $r_1 > r_0$, where the ordinate of the generator $y (r_i) = y (r_0) = 0$. If, at the point of their intersection at an angle $\alpha_i$, a wound annular layer of threads 2 has installed, then it will perceive linear tensile forces from the meridional layer 1. In this case, the first system of threads has wound along the meridian from the major equator to the minor equator, bends around the annular layer of threads 2, and returns to the region of large radii of rotation of the shell.

To determine the parameters of the ring system of threads, we project the linear forces $N_{1r1}$ of the upper surface of the meridional layer onto the plane of the large equator of the torus. The distributed load per unit length of the annular layer (frame) in the circumferential direction $q_1$, will be:

$$q_1 r_1 = N_{1r1} r_1 \sin \alpha_1 = \sigma h_{cm1} r_1 \sin \alpha_1 = F r_1,$$  

(8)

where $h_{cm1}$ in the area of intersection of the generatrices of the torus at $r = r_1$; $\sigma$ - tensile strength of unidirectional composite material (PCM); $F$ - half of the cross-sectional area of the frame.

![Figure 3](image)

**Figure 3.** Geometry and shape of the section of an equally stressed toroidal shell formed by the intersection of the upper and lower branches of the loop-like curve of the torus: 1 - meridian; 2 - annular layer of threads (frame)

Writing down the equilibrium condition for the part of the shell cut off by conical and cylindrical sections in the direction of the OY axis (Fig. 2, b), we obtain $2\pi r N_1 \cos \alpha = \pi (r^2 - c^2) P$, whence the meridional linear force

$$N_1 = \sigma h_{cm} = \frac{p (r^2 - c^2)}{2 r \cos \alpha},$$  

(9)

where $p$ is the pressure in the shell. Linear forces $N_{1r1}$ with radius $r = r_1$, we find from the condition of equilibrium of the shell (9):

$$N_{1r1} = \sigma h_{cm1} = \frac{p (r_1^2 - c_1^2)}{2 r_1 \cos \alpha_1},$$  

(10)

After determining $h_{cm1}$ from (10) and $\cos \alpha_1$ and $\sin \alpha_1$ from (1) and substituting them into (8) at $p = P_{lines}$, we find the calculated value of half the cross-sectional area of the circular threads of the frame:
The mass of the entire ring or circumferential winding that holds the threads of the meridional layer of the shell from the action of internal pressure, taking into account (11):

\[ M_{\text{circ}} = 2F \pi r_1 \rho_{\text{cm}} = 2\pi r_1^3 \rho_{\text{times}} \tilde{r}_1 \sqrt{(1 - \tilde{r}_1^2)(\tilde{r}_1^2 - \mu^2) / \sigma / \rho_{\text{cm}}} \]  

(12)

The mass of the entire meridional layer of the wound toroidal shell, taking into account expressions (9) and (6):

\[ M_{\text{mer}} = 2\pi r_0 h_0 L_{\text{mer}} \rho_{\text{cm}} = 2\pi r_0^3 \rho_{\text{times}} (1 - \tilde{c}^2)^2 F_1(\theta_1, k) / \sigma / \rho_{\text{cm}}. \]  

(13)

Where the length of the thread in the direction of the meridian is found from the equation:

\[ L_{\text{mer}} = \int_{r_0}^r \sqrt{1 + y'^2} \, dr = r_0 (1 - \tilde{c}^2)^2 F_1(\theta_1, k), \]

(14)

Wherein:

\[ y' = -\frac{1 - \tilde{c}^2}{\sqrt{(1 - \tilde{c}^2)(\tilde{c}^2 - \mu^2)}}, \quad \theta = \arcsin \frac{1 - \tilde{r}_1^2}{\sqrt{1 - \mu^2}}, \quad k = \sqrt{1 - \mu^2}, \]

and the thickness of the shell at the large equator of the torus is determined from (9) at \( r = r_0, \alpha = \alpha_0; \) \( p = \rho_{\text{times}} \) and will be equal: \( h_0 = \rho_{\text{times}} r_0 (1 - \tilde{c}^2) / 2\sigma. \)

Summing up the masses of layers (12) and (13) of the toroidal composite shell, we find:

\[ M_{\text{cm,sum}} = M_{\text{mer}} + M_{\text{circ}} = 2\pi r_0^3 P_{\text{times}} \rho_{\text{cm}} \frac{(1 - \tilde{c}^2)^2 F_1(\theta_1, k) + \bar{r}_1 \sqrt{(1 - \tilde{r}_1^2)(\tilde{r}_1^2 - \mu^2)}}{\sigma / \rho_{\text{cm}}}. \]  

(15)

The volume of the toroidal shell in the range of radii from \( r_0 \) to \( r_1 > r_\sigma \) and \( y' \) from (13) determined by the sum of integrals that can be expressed in terms of algebraic functions and an elliptic integral of the first kind (p. 260, [12]):

\[ V_{\text{tor}} = 2\pi \int_{r_1}^{r_0} r^2 y' \, dr = 2\pi \left[ \int_{r_1}^{r_0} \frac{r^4}{\sqrt{(r_0^2 - r^2)(r_0^2 - r_1^2)}} - c^2 \int_{r_1}^{r_0} \frac{r^2}{\sqrt{(r_0^2 - r^2)(r_0^2 - r_1^2)}} \right] = \frac{2}{3} \pi r_0^3 \left[ (1 - \tilde{c}^2)^2 F_1(\theta_1, k) + \bar{r}_1 \sqrt{(1 - \tilde{r}_1^2)(\tilde{r}_1^2 - \mu^2)} \right]. \]  

(16)

Expressing the value for \( \pi r_0^3 \) from (15) and substituting it into (14), we find the mass of the equally stressed toroidal shell of the pressure vessel and determined by the formula: \( M_{\text{c,min}} = M_{\text{circ}} + M_{\text{mer}} = 3P_{\text{times}} V_{\text{tor}} \rho_{\text{c}} / \sigma. \) Which is valid for all equally stressed composite shells of pressure vessels formed by winding threads along the geodesic lines of the surface of revolution.
5. Conclusion

Analytical dependences of the meridional curves of uniformly stressed toroidal shells of pressure vessels reinforced by winding threads along the torus generatrix were obtained.

The possibility of creating and design parameters of uniformly stressed toroidal shells of vessels made by the method of meridional and circumferential winding of threads is shown.

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