Verifiable Remote Voting with Paper Assurance

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Abstract

We propose a protocol for verifiable remote voting with paper assurance. It is intended to augment existing postal voting procedures, allowing a ballot to be electronically constructed, printed on paper, then returned in the post. It allows each voter to verify that their vote has been correctly cast, recorded and tallied by the Electoral Commission. The system is not end-to-end verifiable, but does allow voters to detect manipulation by an adversary who controls either the voting device, or (the postal service and electoral commission) but not both. The protocol is not receipt-free, but if the client honestly follows the protocol (including possibly remembering everything), they cannot subsequently prove how they voted. Our proposal is the first to combine plain paper assurance with cryptographic verification in a (passively) receipt-free manner.

1 Introduction

The biggest form of remote voting is postal voting—nearly half the US presidential votes in 2020 were cast by mail [12]. Postal voting suffers from the same coercion problems that all remote voting approaches do, but it provides easy cast-as-intended verification via the intuitive checking of the plaintext ballot. This simplicity greatly favours postal voting over online remote voting. Most online voting schemes require a level of trust in the device for integrity or involve complex and difficult cast-as-intended verification procedures. Failure to conduct the audits can have a catastrophic impact on integrity. However, postal voting produces no evidence that the vote was accurately included and tallied—this is what we add.

In an ideal world remote voting in any form would be deployed only where absolutely necessary, for example to enfranchise a house-bound voter. Unfortunately, there is a global trend towards increases in remote voting, either by mail [51, 62, 63] or worse, online voting [24, 32, 60]. The global coronavirus pandemic has accelerated the trend.

Postal voting has seen demonstrated instances of fraud in recent years [21, 50, 61]; those that have been detected may be only a small indication of a larger problem. There have been repeated problems with online voting systems [17, 23, 30, 57–59], including flaws in cryptographic verification mechanisms that have been demonstrated only after they had been relied upon in an election [29]. Resilience against cryptographic failure is another great advantage of a plain paper backup—the continued push towards paperless remote voting runs counter to both academic opinion and the established experiences of deployed systems. However, Electoral Commissions (ECs) are under pressure to deliver some form of remote voting.

We propose a new remote voting system that combines electronic construction of the ballot with a familiar and easy-to-verify paper record. Using their own device, the voter generates a ballot, prints it, and returns it by post. This halves the use of the postal channel, thus increasing the time available for voters to construct and cast their ballots. The approach offers immediate plaintext cast-as-intended verification to the voter, with an option for verifying the vote was recorded and tallied properly if the voter conducts a simple electronic check that her vote is properly included on the bulletin board.

The system is not end-to-end verifiable, but it provides verifiability against an adversary who controls either the voter’s device or (the postal system and the EC) but not both. The verification protocols detect manipulation by an adversary who corrupts the EC and the post, as long as the client’s randomness remains secret. Although it is not Receipt-Free, clients who execute the protocol honestly (including remembering their randomness) cannot prove how they voted.

Our proposal offers a new choice of tradeoffs: easy cast-as-intended verification, defence against a cheating post and EC (unlike traditional postal voting) and some defence against coercion (though it does introduce coercion opportunities not present in traditional postal voting). Whilst our recommendation is not to conduct remote voting unless absolutely necessary, this may be the best choice in some scenarios.
1.1 Our contribution / protocol properties

The scheme resembles existing end-to-end verifiable voting schemes, except that “the system” whose behaviour the voter needs to verify includes her device, the postal service, and the EC’s vote-receiving process. We use a web bulletin board (WBB) (such as [18]), which is an authenticated broadcast channel with memory. The voter verifies that her vote has been cast as she intended by reading it on a plain paper printout, which she puts in a post box along with other verification artifacts. At the end of the election, the voter checks that her ID appears among the confirmed votes on the WBB. The proof of proper tallying is universally verifiable.

A great advantage of our scheme is that it falls back to postal voting integrity guarantees even if all the electronic devices are compromised. If voters check their printouts, and the postal service can be trusted, and the processes for opening the envelopes and counting the ballots are properly observed, then the election outcome is correct. We refer in the text to the places where scrutineers may watch the paper processing to gain the evidence they need to have trust in the paper-only tally, independent of any cryptography. The cryptographic protocol adds the option for these processes to be verified electronically by people who are not present at the counting location or do not trust the post. Scrutineers are welcome, but not required, to do anything to support the security claims of the cryptographic protocol.

Of course, the scheme also adds the possibility to fabricate problems, for example by submitting inconsistent values to make it appear that there was cheating when there was not—a correct result may look suspicious. Accountability (and other defences against this) is a topic for future work.

Receipt Freeness [8] means that the system does not allow voters to prove how they voted. Our system offers a weaker version we call honest-but-remembering Receipt Freeness—if a voter’s device executes the protocol honestly, she cannot subsequently prove how she voted (assuming her mail isn’t read and her channel to the EC is not tapped), even if the device remembers the randomness used to generate the ciphertexts. Thus it is strictly better than Helios (which does not claim to be Receipt-Free). She can, however, produce a receipt by actively deviating from the protocol. For example, a voter who posts commitments given to her by the coercer can later prove to that coercer how she voted. In this sense our receipt freeness property is weaker than that proposed by Benaloh [8] and proven for some attendance polling-place systems [47]—see Section 4.2. Note this is also true for traditional postal voting e.g. if the voter films her voting process. We do not claim this is sufficient for government elections. It is, however, better than the coercion-resistance properties of any remote end-to-end verifiable e-voting system.

Our proposal is the first remote voting system to combine plain paper assurance with cryptographic verification in an honest-but-remembering receipt-free manner.

Our protocol has the following security properties:

- privacy from an adversary that does not collude with the voting client, post, or EC, given threshold trust for vote decryption and proper opening of paper ballots,
- honest-but-remembering receipt-freeness against an adversary who sees the WBB but does not tap the voter-EC communication channels or collude with the EC,
- easy cast-as-intended verifiability based on plain paper,
- recorded-as-intended verifiability secure against an attacker who controls (the post and the EC) or the voter’s device, but not both.

This is the first proposal to include all four of these advantages. Correct tallying is universally verifiable. If verification is properly performed, the proof of integrity is significantly better than postal voting.

A complete prototype implementation, including voting, tallying, and verifying, is available at: https://github.com/eleanor-em/papervote

1.2 Protocol main idea

The key innovation is in the recorded-as-cast step, which allows the voter to check that the vote she put in the post was correctly recorded on the WBB. We use a Carter and Wegman [11] universal hash on the WBB to bind the vote without allowing people to prove how they voted. Before voting, the voter’s device randomly generates two secrets $a$ and $b$ in $\{1, \ldots, q-1\}$, where $q$ is a large prime known to all voters. The device posts a Pedersen commitment [48] to these secrets on the WBB. When a voter wishes to cast a ballot, her device computes a MAC as $MAC = a \cdot Vote + b \mod q$ and sends the MAC and vote, in encrypted form, to the EC. The EC re-randomises the encryptions (for receipt-freeness) and posts them on the WBB.

The voter sends two pieces of paper to the EC by post.

Paper 1 contains her plaintext vote and encrypted commitment openings (with a proof of knowledge).

Paper 2 contains her plaintext VoterID in human- and machine-readable form.

These do not have to be made by one device, and splitting the task could improve privacy—this is explored in Section 6.

As well as printing her VoterID and sending it with her paper vote, the voter does whatever is usual for postal voting in her country, such as writing her name and address on an outer envelope or signing it. This is used at the EC for checking eligibility and identity against the electoral roll.

The Carter-Wegman hash provides two useful properties:
• Without knowing $a$ and $b$, an attacker cannot generate a valid alternative $(\text{Vote}, \text{MAC})$ pair except with very small probability.

• Even after $a$ and $b$ are exposed, a voter can plausibly claim to have cast any vote (if the system does not reveal $\text{Vote}$ or $\text{MAC}$ to the coercer)—she simply claims the correct $\text{MAC}$ for whichever $\text{Vote}$ the coercer demands.

The main idea of our verifiability proof is that anyone who intercepts the envelope, including a corrupt EC, cannot (except with small probability) change the MAC and the plaintext vote consistently, unless they can guess the values of $a$ and $b$ before the EC posts the vote and MAC on the WBB. Using perfectly hiding commitments for $a$ and $b$ means that they are hidden unless the client exposes them—obviously this assumes the client keeps them secret. This protects the vote from manipulation by a corrupt postal service or EC, even if all the decryption authorities collude. So EC-re-randomization provides honest-but-remembering receipt freeness, while the Carter-Wegman hash prevents dishonest EC re-randomization. The homomorphic property of the encryption scheme is then used to recreate the right MAC from the paper vote received in the mail—if they match, the vote is accepted.

Thus verification depends on a secrecy assumption, but one in which the voter generates their own secret. The protocol is not end-to-end verifiable, but it contrasts favourably with protocols such as code voting (described below) in which integrity depends on the secrecy of values that are generated centrally and sent to the voter.

1.3 Related work on remote recorded-as-intended verification and receipt freeness

Neither the scientific literature nor the remote electronic systems used in practice have good solutions for cast-as-intended verification. They are either too hard for ordinary voters to use easily, or they are dependent on a secrecy assumption that is unverifiable and outside the voter’s control. Ordinary postal voting has clear and simple cast-as-intended verification (assuming we take a vote to be “cast” when it is put in the mailbox) but no recorded-as-cast verification at all.

The Helios voting system [1] offers end-to-end verifiability in a remote all-electronic setting. A diligent voter gets very good evidence that her vote is cast as she intended and properly included, followed by a universally verifiable count. However, the verification is difficult enough that ordinary voters may be tricked into not performing it properly [38], and the recommended challenge strategies do not form Nash equilibria in a remote setting [19]. Even more importantly, Helios is not (and has never claimed to be) receipt free: a voter can prove how she voted if her client remembers the randomness used to encrypt her vote. Thus Helios assumes low-coercion environments, and diligent voters who perform the cast-as-intended verification step on an independent device. The Estonian Internet voting system [31, 59] is similar.

The Civitas Internet voting system [14], based on a protocol by Juels et al. [37], provides a very strong form of coercion resistance but no cast-as-intended verification. The Selene Internet voting system [54] can be used to enhance it with cast-as-intended verification [35], or as an adjunct to Helios-style systems. In Selene, election trustees generate a unique tracker for each user, who uses it later to verify that their (plaintext, electronic) vote was correctly included.

In Code Voting systems, the voter verifies cast-as-intended and recorded-as-cast in a single step, using a code sheet sent (usually) by paper mail. Remotegrity [65] is a remote version of the Scantegrity II voting system, with extra codes for confirming a properly-verified vote. In Pretty Good Democracy [55] (PGD), there are codes for sending the vote and only one return code, to acknowledge receipt.

In Code-return voting systems, such as the Norwegian [24] and Swiss Internet voting systems, voters cast an encrypted vote and then receive a confirmation code, which they check against a code sheet received in the mail. All these systems allow some subsequent verification of the tally.

Although code voting and code-return voting are convenient, they suffer from a major drawback: the integrity of the outcome is dependent on the secrecy of the codes. In principle, secrecy is impossible to verify. Printing the code sheets securely and sending them privately is the major practical problem in these schemes. Thus these systems are not really end-to-end verifiable. (Ours is also not end-to-end verifiable, because verification is also dependent on a secrecy assumption, but one in which the voter generates their own secret.)

Although Remotegrity and PGD have been adapted for instant runoff voting (also called ranked-choice voting), code voting becomes unwieldy as the number of preferences grows.

A plain paper mail step could also be added to the Internet voting solutions described above. With Helios, this would result in properties incomparable to our scheme: genuine end-to-end verifiability, a simple cast-as-intended step for the paper backup, but no receipt freeness.

It seems less useful to combine code-voting-style solutions with a plain paper return. Firstly, it requires paper mail in both directions, thus removing much of the benefit of an electronic solution. Second, it doesn’t solve the problem that a malicious authority (or sufficient collusion among trustees) can fabricate a successful-looking verification. This seems strictly worse than our solution, in which even a completely corrupted EC cannot cheat undetectably unless it also controls the client.

Prêt à voter [53] uses preprinted auditable ciphertexts which the voter selects or arranges to express their vote. Although designed for pollsite voting, the idea could be extended to a remote setting, but auditing the printouts would be cumbersome. Belenios VS [15] extends on this idea in a remote setting, allowing voters to receive their preprinted ciphertexts by mail and verify them with a device that is assumed to be in-
dependent of their voting client. It also incorporates eligibility verifiability and receipt freeness. Its main disadvantage is that all its security properties depend on non-collusion between the registration server and the voting server. However, Beleinos VS preserves privacy even against a corrupted voting client. (Our system can be expanded to do so—see Section 6.)

Several existing designs combine plain paper ballots with cryptographic verification for pollsite voting [5, 6, 13, 52]. These are not designed for a remote setting—the aim of this paper is to take that design philosophy to remote voting.

Also note that the usability of even the simplest cast-as-intended verification mechanisms is questionable, with practical failure rates shown for both code voting [43] and the checking of plaintext printouts [10, 20, 22, 41].

In summary, no existing solution provides both receipt-freeness and easily-usable recorded-as-intended verification in a remote setting, while protecting integrity against a fully corrupt authority. Our contribution is to fill this gap. Compared with code-return systems, our scheme has two important differences: the code secrecy assumption is on the client, not the electoral authorities, and the system easily accommodates arbitrary ballots. Table 1.3 compares the properties of our proposal to other schemes used or proposed.

Verifiable postal voting [7] is closest to our setting. This proposal improves on that work by achieving honest-but-remembering receipt freeness, and having a much higher probability of detecting manipulation. Complex voting schemes raise extra challenges for election privacy and verification. Aditya et al. [3, 4] first examined cryptographic election verification for instant runoff elections. We reuse their idea of getting an authority to re-randomize votes before publication in order to achieve Receipt Freeness, but in our scheme the authority does not need to be trusted to re-randomize honestly, as long as the client keeps its secrets.

1.4 Structure of this Paper

Cryptographic tools are described in the next section, followed by the protocol in Section 3, including algorithms for casting, receiving, and tallying votes. Sketched security arguments are given in Section 4, with formal proofs in the Appendix. A prototype implementation and small trial are discussed in Section 5. In Section 6 we discuss some simple extensions, while limitations and further work are described in Section 6.

2 Cryptographic Background

Let $T$ be a set of election trustees. We use the following:

ElGamal encryption scheme The encryption scheme has parameters $\mathbb{G} = (G, g, q)$ where $G$ is a cyclic group of prime order $q$ generated by $g$.\footnote{The particular group chosen does not matter as long as the decisional Diffie-Hellman (DDH) problem is hard in $G$.} A secret key $sk$ is jointly generated amongst the trustees $T$ using $k$-out-of-$n$ Pedersen key generation [49] as an extension of Shamir secret sharing [56], with corresponding public key $pk$. The scheme encrypts a message $m$ by setting $e = (g^r, m \cdot (pk)^r)$ for a blinding factor $r$. We will sometimes use $m = g^r$ to encrypt some value $v$—this allows for homomorphic addition of encrypted data by elementwise multiplication of ciphertexts.

We write $\{m\}_{pk}$ for an ElGamal encryption of message $m$ with public key, optionally producing (as defined in [9]):

$$\text{PrfEnc}_{G, pk}(m, c): \text{ an adaptively secure noninteractive zero knowledge proof (ZKP) that } c \text{ is an encryption of } m;$$

$$\text{PrfKnow}_{G, pk}(e = \{m\}_{pk}): \text{ an adaptively secure noninteractive ZKP of knowledge of the message } m.$$ We write $\{m_1, m_2\}_{pk}$ to mean multiple encryptions of different plaintexts $m_1$ and $m_2$. We also sometimes abuse notation and write $\text{PrfKnow}(S)$ for a vector of ciphertexts $S$ to mean a vector of proofs of knowledge, one for each element in $S$.

Threshold decryption of a ciphertext $\{m\}_{pk}$ (as defined in [16]) produces the plaintext $m$ and a universally-verifiable proof of proper decryption $\text{PrfDec}_{G, pk}(e, m)$, which is a vector of at least $t$ adaptively secure NIZKs of equality of discrete logarithms. We write $\text{Decrypt}_{G, pk}(\{m\})$ to mean a pair of a plaintext and its decryption proof.

ElGamal ciphertexts may be re-randomised, written $\text{Rand}_{G, pk}(e_1, e_2)$ which means generating a random $r \in \mathbb{Z}_q$ and setting $\text{Rand}_{G, pk}(e_1, e_2) = (e_1g^r, e_2pk^r)$. This produces a new encryption of the same ciphertext.

Finally, we use plaintext equivalence proofs (PEPs), a universally-verifiable ZKP that two ciphertexts encrypt the same message, as defined in [46] based on [36].

Pedersen commitment The commitment scheme [48] has parameters $\mathbb{P} = (G, h_1, h_2)$ where $G$ is a cyclic group in which discrete logarithms are hard, and $h_1, h_2$ are generators chosen such that nobody knows $\log_{h_1} h_2$ (as described in [39]). We write $\text{Com}_{\mathbb{P}}(a; r_a)$ to mean a Pedersen commitment to the value $a$ using randomness $r_a$ (i.e. the value $h_1^a \cdot h_2^{r_a}$).

Mixing We write $\text{Mix}_{G, pk}(S)$ to mean a universally-verifiable distributed mix of the vector $S$ (with the associated ZKPs), as in [28, 64]. Each trustee performs one stage of the mix, so that as long as at least one trustee is honest, the resulting mix is private (the link between inputs and outputs is unknown). Note: If $S$ contains plaintexts, they will be encrypted before the mix process.

Web bulletin board We model the WBB as a public broadcast channel with memory. That is, items cannot be removed
from the bulletin board once they are published, and every participant’s final view of the WBB is identical [18, 33]. In practice this means that we need the voter to have access to the WBB via a channel independent from the Client Device. We assume that it is available both during and after the election period, implying that a malicious authority cannot block uploads (though a malicious client might fail to upload).

3 The Protocol

Assume a list of VoterIDs constructed so that

- each voter can recognise their own VoterID, and
- no two voters have the same VoterID.

The latter assumption is important for preventing clash attacks [44], in which two voters are convinced that the same VoterID corresponds to an eligible voter. VoterIDs could be a simple function of the voter’s name and address. If a voter uses someone else’s VoterID we assume it can be detected (by the owner of that ID). For eligibility verifiability, we need to assume that the public has some way of assessing whether a VoterID corresponds to an eligible voter.

Recall the election secret key sk is shared amongst the set of electoral trustees T. This is important for voter privacy: we will assume that n − k + 1 of the trustees are honest so that no k of them collude to decrypt data they are not supposed to.

Setup is shown in Algorithm 1. The electoral commission prepares a set of encrypted VoterIDs, and places them inside a smaller envelope. The smaller envelope is placed in a larger envelope marked with the corresponding plaintext. The double-envelope is called D_{VoterID}. It can be generated in advance or on demand, and will be used during vote receiving to link voters to votes in a private manner. Note that the mechanism that generates these double-envelopes is the only part of the EC that is trusted for privacy.

Algorithm 1 Setup: System setup protocol

1: \[ \text{The following are posted to the WBB:} \]
2: \( (\text{VoterID}_1) \): a list of IDs of eligible voters. We assume these are assigned one-on-one to each voter.
3: \( G \leftarrow (G, g, q) \): the public parameters of an ElGamal encryption scheme as discussed in Section 2.
4: \( \text{pk} \): an ElGamal public key generated jointly among the trustees with the ElGamal parameters G and security parameter \( \lambda \). The corresponding secret key sk is shared among the trustees. This may be done e.g. following [49].
5: \( P \leftarrow (P, h_1, h_2) \): the public parameters of a Pedersen commitment scheme as discussed in Section 2.
6: \( \text{The EC creates the following double envelopes:} \)
7: Outer envelope of \( D_{\text{VoterID}} \leftarrow \text{VoterID} \)
8: Inner envelope of \( D_{\text{VoterID}} \leftarrow e_{\text{VoterID}} = \{ \text{VoterID}_1 \} \text{pk} \)

3.1 Voting

The voter’s experience is extremely straightforward and is detailed in Algorithm 2 (Cast). To generate a ballot, the voter’s device chooses two random secrets a, b, ∈ G, and randomness \( r_a, r_b \), then publishes commitments \( c_a = \text{Com}_G(a, r_a) \) and \( c_b = \text{Com}_G(b, r_b) \) on the WBB (Steps 1–3). When the voter chooses a vote, the device computes a message authentication code \( MAC = a \cdot \text{Vote} + b \), and sends encryptions of the vote and MAC to the EC, which re-randomises the values and posts them on the WBB indexed by the VoterID (Steps 4–9). After the EC has posted these values, the device prints two pieces of paper (Steps 10–12):

\[ \text{To encode votes as an integer, the EC could e.g. publish an ordered list of choices indexed from 0 to represent candidates or more complex preferences.} \]
Paper 1 -- Vote: Alice: 2  Bob: 3  Eve: 1

Encryptions:

Proofs:

Paper 1 -- Vote: Alice: 2  Bob: 3  Eve: 1

Encryptions:

Proofs:

Figure 2: Paper 1: The voter only needs to check the plaintext vote at the top. This example is a ranking: Eve first, Alice next, Bob last. Each encryption QR code contains two ciphertexts, overall including \( \{ a, b, r_a, r_b \} \). The proof QR code has the corresponding proofs of plaintext knowledge.

Paper 1 contains the human-readable plaintext vote, encryp-
tions of \( a, b, r_a, r_b \) (i.e. the commitment openings) and proofs of plaintext knowledge of those ciphertexts.

Paper 2 contains the human-readable plaintext VoterID.

Example printouts from our prototype are in Figures 2 and 3. They use QR codes for the non-human-readable values.

The voter must mail both to the EC (using the standard postal voting procedures in her country, which may include signing the envelope) by placing Paper 1 in a smaller envelope, and both the smaller envelope and Paper 2 in the larger envelope. This mirrors standard postal voting practices.

The voter must check that Paper 1 contains a correct human-readable printout of her vote, and Paper 2 contains a correct human-readable printout of her VoterID. If she wants to check that her vote has not been dropped, she needs to visit the WBB after the election to check that her VoterID is in the list of included IDs. Note that she does not have to do anything to verify that the QR codes on her printouts are not maliciously generated—this will be detected by subsequent verification (assuming the attacker model given in the Introduction).

3.2 Receiving votes

When the EC receives votes, they must be handled carefully to maintain voter privacy, so that the voter’s identity is checked...
and forgotten before their vote is revealed. This seems complicated but is not much different from the common double-envelope system for protecting postal vote privacy, except for the matching of encrypted and plaintext VoterIDs.

The protocol (Process vote) is shown in Algorithm 3. Scrutineers may be present to observe the plaintext-VoterID and plain-paper vote acceptance. The (outer) envelope is opened, and only Paper2 is removed. The received VoterID is checked against the identification on the (outer) envelope (e.g. the voter’s name, address or signature); if this step fails, the entire ballot is placed in a reject pile (which scrutineers may see) and RecVoterID is added to the WBB list Brejected.

Algorithm 3 Process vote: Vote receiving protocol

1: \(\triangleright\) Run for each ballot \((\text{Paper}_1, \text{Paper}_2)\) received by mail
2: \(\triangleright\) Primed variables \(\text{VoterID}'\) are used to indicate the EC may receive different values to those the voter sent
3: \(\text{Paper}_2 \rightarrow \text{EC}: \text{VoterID}'\)
4: EC: Checks \(\text{VoterID}'\) matches electoral roll
5: EC: Retrieves \(D_{\text{VoterID}'}\) using \(\text{VoterID}'\)
6: EC: Joins \(e_{\text{VoterID}}\) (inner envelope of \(D_{\text{VoterID}'}\)) to \(\text{Paper}_1\)
7: EC: Destroy \(\text{Paper}_2\)
8: EC: Shuffle batches of \(\text{Paper}_1\) attached to \(e_{\text{VoterID}}\)
9: \(\text{Paper}_1 \rightarrow \text{EC}: e_{\text{VoterID}}, e_{\text{comm}}', e_{\text{Params}}', \text{PrfKnow}_{G, pk}(e_{\text{Params}}')\)
10: EC: Verifies \(\text{PrfKnow}_{G, pk}(e_{\text{Params}}')\). On failure, post \(\text{VoterID}'\) to \(B_{\text{rejected}}\) and skip ballot.
11: \(\text{EC} \rightarrow \text{WBB}\)
12: Add \((\text{Vote}', e_{\text{VoterID}}, \text{Rerand}(e_{\text{Params}}'))\) to \(B_{\text{received}}\)

Next, the EC retrieves, or generates, the corresponding prepared envelope \(D_{\text{VoterID}}\). The inner envelope containing the corresponding ciphertext \(e_{\text{VoterID}}\) is attached to \(\text{Paper}_1\) (e.g. by stapling, or by inserting into the unmarked inner envelope) without looking at \(\text{Paper}_1\). Finally, \(\text{Paper}_2\) is destroyed, leaving no plaintext link from VoterID to vote.

The resulting \((\text{Paper}_1, e_{\text{VoterID}})\) pairs are shuffled physically to remove any link to the order in which envelopes were opened. Next they are opened. For each \(\text{Paper}_1\), the proofs of knowledge are verified, again putting the ballot in a reject pile and adding \(e_{\text{VoterID}}\) to \(B_{\text{rejected}}\) if it fails. (These are shuffled and decrypted after opening all ballots.) Finally, \((\text{Paper}_1, e_{\text{VoterID}})\) pairs with verified proofs have their contents re-randomised and posted to the WBB list \(B_{\text{received}}\).

### 3.3 Tallying

Tallying is shown in Algorithm 4, performed jointly by all trustees. First we decrypt the secrets and check they are a correct opening of the commitments. We then construct a second MAC from the committed vote on the WBB, and check that it matches the committed MAC. If so, then with high probability the vote was cast as the voter intended, assuming that her secrets \(a, b\) were not exposed. If any verifications fail, the trustees should mark the VoterID and go to the next vote.

Algorithm 4 Tally votes: Vote tallying protocol for trustees \(T\)

1: \(\triangleright\) Mix \(B_{\text{received}}\) to produce re-randomised encryptions of each item, permuted consistently, indicated by \(\overline{\text{Vote}}\), etc.
2: \(T \rightarrow \text{WBB}: B_{\text{received}}' = \text{Mix}_{G, pk}(B_{\text{received}})\)
3: \(\triangleright\) Decrypt
4: for \(B_{\text{received}}'\) do
5: \(T: (a, b, r_a, r_b), \text{PrfDec}_{c_1} \leftarrow \text{Decrypt}_{G, sk}(\overline{\text{Params}})\)
6: \(T: (\text{RecVoterID}, \text{PrfDec}_{c_2}) \leftarrow \text{Decrypt}_{G, sk}(\overline{\text{VoterID}})\)
7: \(T \rightarrow \text{WBB}\)
8: \(B_{\text{mixed}} = (\overline{\text{Vote}}, (a, b, r_a, r_b), \text{RecVoterID}, \text{PrfDec}_{c_1}, \text{PrfDec}_{c_2})\)
9: \(\triangleright\) Join by matching \(\text{VoterID}\) to \(\text{RecVoterID}\)
10: for \(B_{\text{mixed}}\) do
11: if \(B_{\text{RecVoterID}}\) is empty then
12: skip to the next iteration.
13: \(\text{WBB} \rightarrow T: (c_a, c_b) = B_{\text{RecVoterID}}\)
14: if \(c_a = \text{Comp}_P(a, r_a)\) and \(c_b = \text{Comp}_P(a, r_b)\) then
15: \(B_{\text{mixed}}\) is a correct opening for \(B_{\text{RecVoterID}}\)
16: \(\triangleright\) For each \(\text{VoterID}\) with one correct opening, recreate the MAC to check it matches the EC’s committed one.
17: for all \(\text{VoterID}\) do
18: if \(B_{\text{RecVoterID}}\) or \(B_{\text{comm}}\) is empty then
19: skip to the next iteration
20: if \(B_{\text{RecVoterID}}\) has a unique correct opening \(B_{\text{mixed}}\) then
21: \(\text{WBB} \rightarrow T: (\overline{\text{Vote}}, \overline{\text{MAC}}) = \text{Cons}^{\text{commit}}\)
22: \(T \rightarrow \text{WBB}: \text{PlaintextEquivalent}_G, pk(\overline{\text{Vote}}, \overline{\text{MAC}})\)
23: \(\overline{\text{MAC}} \leftarrow (\overline{\text{Vote}})^a + \{g_b^{k_c}\}_{pk}\)
24: \(T \rightarrow \text{WBB}: \text{PlaintextEquivalent}_G, pk(\overline{\text{MAC}}, \overline{\text{MAC}})\)
25: if plaintext equivalence proofs pass then
26: \(T \rightarrow \text{WBB}: \overline{\text{Vote}}\) on WBB do
27: \(\triangleright\) Mix and decrypt to produce final tally. \(\overline{\text{Vote}}\) represents the mix re-randomisation; we ignore \(\overline{\text{Vote}}\).
28: \(T \rightarrow \text{WBB}: B_{\text{tally}} = \text{Decrypt}_{G, pk}(\overline{\text{Vote}})\)
29: for \(B_{\text{tally}}\) do
30: \(T \rightarrow \text{WBB}: B_{\text{tally}} = \text{Decrypt}_{G, pk}(\overline{\text{Vote}})\)
3.4 Verification protocols

We define the vote to be *cast* when the voter puts the envelope in the mail (or in a box at the EC), *recorded* when the EC posts it (encrypted) on the bulletin board, and *counted* when the list of decrypted votes is published on the WBB.\(^3\)

Each voter must check the printed paper vote, then use a device to verify that her ID appears in the final mix. Scrutineers verify the plain-paper aspects of the election. The WBB transcript is publicly verifiable. The procedure for the WBB transcript verification is in Algorithm 6 (GlobalVerify).

3.4.1 By election scrutineers

Scrutineers may observe the process of receiving paper ballots. This is not relevant to the security properties proven in this paper, but is relevant to the claim that the system fails back to traditional postal vote assumptions if the cryptography is broken, or if clients and EC are both dishonest.

Scrutineers present when the envelopes are opened must:

1. verify the received VoterID is on the electoral roll and has not already had a vote included;
2. verify the vote posted to the WBB (in step 9) matches the vote on Paper\(_1\).

3.4.2 By the voter

The system is easy for a voter to verify. Before sending her vote by mail, the voter checks that the printed ballot paper matches the vote she intends to cast, and that the plaintext VoterID on Paper 1 is correct.

Once the receiving process is complete, she should check that her voter ID appears on the accepted list. The precise voter-verification protocol is shown in Algorithm 5 (VoterVerify). The voter must check the paper vote herself while casting it, but the WBB check can be outsourced to anyone. Indeed, the voter only needs to check so that she can detect the non-arrival of her paper (or interference by an adversary).

### Algorithm 5 VoterVerify: Voter’s recorded-as-intended verification protocol

1. The voter checks their printouts to verify
2. that the vote on Paper\(_1\) matches their intended vote, and
3. that the VoterID on Paper\(_1\) is correct.
4. At the end of the election, the voter checks that their VoterID appears in \(B^{\text{accepted}}\) on the WBB.

3.4.3 Public Tally verification

This consists of verifying the proofs that the tally protocol has been properly conducted. It is described in Algorithm 6.

### Algorithm 6 GlobalVerify: Verification protocol for facts asserted on the WBB

1. Verify the mix proof for \(B^{\text{received}}\) in Step 2 of Tally
2. Verify the decryption proofs in Steps 5 and 6 of Tally
3. Verify all PET proofs in Steps 22 and 24 of Tally
4. Verify the mix proof for \(B^{\text{accepted}}\) in Step 28 of Tally
5. Verify the decryption proofs in Step 30 of Tally
6. for each row of \(B^{\text{registered}}\) do
7. Verify that VoterID is unique in \(B^{\text{registered}}\)
8. for each row of \(B^{\text{commit}}\) do
9. Verify that VoterID is unique in \(B^{\text{commit}}\)
10. for each row of \(B^{\text{accepted}}\) do
11. Verify that VoterID is unique in \(B^{\text{mixed}}\) and does not appear in \(B^{\text{rejected}}\)
12. Verify that exactly one opening in \(B^{\text{registered}}\) is a correct opening for \(c_r, c_h\)
13. Verify that the PETs in Steps 22 and 24 of Tally pass

3.5 Interpretation of the outcome

We need to be precise about the election outcome, since it depends on a combination of paper and electronic votes. At the end of the election, the WBB transcript contains four sets:\(^4\)

1. registered voter IDs \(L^{\text{registered}}\) drawn from \(B^{\text{registered}}\), i.e. those who have uploaded a VoterID and commitments,
2. received voter IDs \(L^{\text{received}}\) drawn from the plaintext ballots \(B^{\text{received}}\) posted by the EC in Step 12 of Process Vote, and decrypted in Step 8 of Tally votes.
3. rejected voter IDs \(L^{\text{rejected}}\) drawn from the ballots \(B^{\text{rejected}}\) that arrived with invalid proofs, and
4. accepted voter IDs \(L^{\text{tally}}\) of those ballots \(B^{\text{tally}}\) that uniquely matched a registered voter’s commitments, posted to the WBB in Step 26 of Tally votes.

If \(L^{\text{tally}}\) is not a subset of \(L^{\text{registered}} \cup L^{\text{received}}\), then something has gone badly wrong (and verification should fail). But in the normal course of an election we expect some deviation: some voters will register but never vote, some votes will go astray in the mail, or some votes will be misrecorded on arrival. We want to devise a reasonable definition of an acceptable election outcome that can detect fraud but not cause the election to fail if small deviations are observed.

The paper record consists of all ballots that passed traditional paper acceptance. It includes the votes in \(B^{\text{rejected}}\).

The votes corresponding to the IDs in \(L^{\text{tally}}\) are those for which everything worked out perfectly—they should be accepted. Their deviation from the plaintext paper ballots is an indication of one type of problem: possible substitution

---

\(^3\)We omit questions of proper counting of complex ballots—when the list of accepted votes is public, we assume someone counts them properly.

\(^4\)For each of these lists one voter ID may have many corresponding WBB posts, but the list is a set so we don’t count the same voter ID multiple times.
of paper ballots in the mail or by the EC. Another type of problem is voters who registered but did not have a unique commitment match at Step 20 of Tally, or did not pass PEPs in Step 24 or Step 22 of Tally. Depending on the exact nature of the problem, this could be evidence of attempted fraud or a legitimate decision to register but not vote. In summary, \( L_{tally} \) provides an arguable election outcome, while \( L_{registered} \), \( L_{received} \), and \( L_{rejected} \) provide some indication as to the extent of errors or manipulation attempts. Call the amount of detected error \( \epsilon = |L_{registered} \cup L_{received} - L_{tally}| \).

Each democracy would have to decide how to deal with inconsistent results or evidence of problems. Let \( O \) be the outcome of the election according to the paper record (e.g. a tally of votes made for each candidate) with margin \( M \) (e.g. half the difference in vote counts between the top two candidates). For a given WBB transcript \( \tau \), define the acceptable number of caught errors to be \( d \). One obvious formula would be: accept \( O \) if the demonstrated error in received votes was below the margin, (i.e. \( d = M \)). Another could be: accept \( O \) if the demonstrated error in received votes was below the margin, ignoring voters who registered but for whom a vote was not received (i.e. \( d = M + |L_{registered} - L_{received}| \)).

We abstract these choices out by defining the result to be:

\[
\text{Result}(\tau, O) = \begin{cases} 
O, & \text{if } \epsilon < d \text{ and } \text{GlobalVerify}(\tau) \text{ passes} \\
\perp, & \text{otherwise,}
\end{cases}
\]

where \( \epsilon = |L_{registered} \cup L_{received} - L_{tally}| \) and \( d \) is determined by policy.

To be confident that there were at most \( d \) errors\(^5\), at least \( \theta = |\mathcal{V'}| - (M - d) \) voters must correctly verify their votes (where \( \mathcal{V'} \) is the set of voters). Thus there is an inverse relationship between the allowed deviation and the number of voters that are allowed to not perform verification. We assume

- the voter’s receipt (later referred to as \( \alpha_i \) for voter \( i \)) consists of their VoterID,
- there is some (out of scope) way for voters to check that their VoterID is unique, for example, it could be their name and address (which is bad for privacy, but ensures that it doesn’t clash with someone else’s),
- there is some (out of scope) way for observers to check that all registered voters are eligible.

We do not attempt to defend against denial-of-service attacks: votes can be scratched from the tally, e.g. if an adversary knows the target’s VoterID. However, this will be detected. Preventions of this are a topic for future work.

\(^5\)In [40], \( d \) represents undetected errors since the verification procedure is probabilistic. In our protocol we should be able to detect every error, so the interpretation is slightly different.

3.6 Sketch of security arguments

We provide below an English outline of the arguments we will use to prove security and privacy properties.

A client colluding with the EC or post can cheat, because then the adversary knows \( a, b \), so it can change the paper ballot and generate a fake (MAC, Vote) pair. In this threat model our system is no better than plain-paper postal voting.

Clearly a cheating EC can write a bad MAC, Vote or VoterID onto the WBB, rather than re-randomisations of what it received. This will force the MAC match to fail and the electronic vote to be excluded. A cheating client can do similarly. Each of these cheating individually will be detected.

We prove that if either the EC or the client is honest, then the vote cannot be substituted undetectably. Informally, suppose the client is honest, then consider what value the corrupt EC posts as that voter’s re-randomised MAC and vote in Step 3 of Cast. If it is a valid MAC for the encrypted vote, then the EC must know \( a, b \) and has been very lucky. (The proof that it knows \( a, b \) is that, if it could decrypt the encrypted Vote and MAC it received, it would know two different points on the line defined by \( m = a \cdot v + b \).) If it is not a valid MAC for that Vote, but somehow passes the PEP in the final step, then the cheating EC must have broken either the mix or the PEP.

This is formalised and proved in Section 4.3. Section 4.4 shows why the paper vote defends against a cheating client.

Informally, honest-but-remembering Receipt Freeness is achieved because for any vote she wishes to pretend to have cast, a voter can always generate a MAC consistent with her commitments to her \( a, b \) (which she can open honestly to a cooperator). Server-side re-randomisation achieves Receipt Freeness (as in [4]), because the voter’s client does not know the randomness used to generate the ciphertexts posted on the WBB. However, this applies to a cooperator who sees only the bulletin board, and does not defend against a cooperator who corrupts the EC or taps the channel between the EC and the voting client. This is made more precise in Section 4.2 after a privacy proof (with no client collusion) in Section 4.1.

4 Security proofs

4.1 Privacy

We follow Kiayias et al. [40] in defining voter privacy of an election via a Voter Privacy game denoted by \( G^{\mathcal{P}}_{\text{priv}} \) that is played between an adversary \( \mathcal{A} \) and a challenger \( \mathcal{C} \); we do not use a simulator in this version because the adversary is simply trying to guess which voter selection made it into the results. For clarity, we removed some qualifiers from the definitions which do not apply to the scheme we wish to prove. This simplification serves only to strengthen the definitions and improve readability. The game is parameterised by the security parameter \( \lambda \), the number of voters \( n \), and the number of candidates \( m \). We consider a set of candidates \( \mathcal{P} \), a set of
voters \( \mathcal{V} \), and a set of allowed candidate selections \( \mathcal{U} \), and introduce an election evaluation function \( f((\mathcal{U}_1, \ldots, \mathcal{U}_n)) \) that outputs a vector whose \( i \)-th index is the number of times candidate \( i \) was voted for (\( \mathcal{P} \) may be a complex set of choices).

**Definition 1 (Privacy Game).** Denoted by \( G_{\text{priv}}^A(1^k, n, m) \).

1. \( A \) on input \( 1^k, n, m \) chooses a list of candidates \( \mathcal{P} = \{P_1, \ldots, P_m\} \), a set of voters \( \mathcal{V} = \{V_1, \ldots, V_n\} \), and the set of allowed candidate selections \( \mathcal{U} \), providing \( C \) with the sets \( \mathcal{P}, \mathcal{V}, \) and \( \mathcal{U} \).

2. \( A \) flips a coin \( b \in \{0, 1\} \) and performs the Setup protocol on input \( (1^k, \mathcal{P}, \mathcal{V}, \mathcal{U}) \) to obtain \( sk, (G, g, q, pk) \), providing \( A \) with \((G, g, q, pk)\).

3. The adversary \( A \) and the challenger \( C \) engage in an interaction where \( A \) schedules Cast protocols of all voters which may run concurrently. For each voter \( V_i \in \mathcal{V} \), the adversary chooses whether \( V_i \) is corrupted:
   - If \( V_i \) is corrupted, \( A \) plays the role of \( V_i \) and \( C \) plays the role of the EC in the Cast protocol.
   - If \( V_i \) is not corrupted, \( A \) provides two candidate selections \( \{\mathcal{U}_0^i, \mathcal{U}_1^i\} \) to the challenger \( C \). They must do so such that \( f((\mathcal{U}_0^i)_{V_i \in \mathcal{V}}) = f((\mathcal{U}_1^i)_{V_i \in \mathcal{V}}) \), where \( \mathcal{V} \) is the set of honest voters (that is, the election result w.r.t. the honest voters does not leak \( b \)).

4. \( C \) performs the Tally protocol playing the role of the election trustees. \( A \) is allowed to observe the WBB.

5. Finally, \( A \) using all the information collected above (including the contents of the WBB) outputs a bit \( b^* \).

Denote the set of corrupted voters as \( \mathcal{V}_c \), and the set of honest voters as \( \mathcal{V} = \mathcal{V} \setminus \mathcal{V}_c \). The game returns a bit which is \( 1 \) if and only if \( b = b^* \). We say that a voting scheme achieves voter privacy if for any PPT adversary \( A \):

\[
\Pr[G_{\text{priv}}^A(1^k, n, m) = 1] - 1/2 = \text{negl}(\lambda).
\]

**Assumptions** For privacy, assume the following are honest:

- the Electoral Commission (EC)
- the postal channel
- all threshold sets of the Election Tellers (ET)
- the voter’s device

**Theorem 1.** For any constant \( m \in \mathbb{N} \) and \( n = \text{poly}(\lambda) \), the e-voting system described in section 2 is private with respect to the privacy game \( G_{\text{priv}}^A(1^k, n, m) \).

A proof is in Appendix A. It considers the information visible to the adversary. During **Cast** the adversary sees VoteID, \( c_a, c_b \) and VoterID, \( \text{Rerand}\{g^{\text{MAC}}\}_{pk}, \{g^{\text{Vote}}\}_{pk} \).

During **Tally** the adversary sees Vote, \{VoterID\}_{pk}, \{a, b, r_a, r_b\}_{pk}, \text{PrfKnow}_{G, pk}(\{a, b, r_a, r_b\}_{pk})
\{(ReceivedVote, \text{Rerand}\{RecVoterID\}_{pk}, \text{Rerand}\{a, b, r_a, r_b\}_{pk})
\{(ReceivedVote\}_{pk}, \{a, b, r_a, r_b\}_{pk}, \text{RecVoterID}, \text{decryption proof}\}
\text{Decrypt}_{G, pk}(\{g^{\text{Vote}}\}_{pk})

Crucially, the adversary cannot use \{VoterID\}_{pk} in the above to relate Vote with VoterID, since this relationship is forgotten when attaching \{VoterID\}_{pk} to Paper1. Plaintext commitment openings are likewise not linkable to the plaintext vote.

### 4.2 Honest-but-remembering RF

We prove honest-but-remembering receipt-freeness (which is a stronger notion than privacy) against a weaker adversary.

Consider a coercer who does not collude with the EC, but does make demands of the voting client. We can prove only a passive form of receipt freeness, in which the colluding client follows the protocol honestly except for recording all its secrets. We also have to assume that the channel to the EC is not tapped by the adversary, which models an attacker who does not have the capacity to intercept communications (such as TLS) over the Internet\(^{6}\). (At least one untappable channel in one direction is necessary and sufficient [34], though we have two, and have not here considered an untrustworthy EC.)

**Assumptions** The receipt-freeness game \( G_{\text{RecFree}}^{A,5}(1^k, n, m) \) is in Appendix B. It is very similar to the privacy game, except that the adversary does not collude with the EC and cannot tap the channel between the voter and the EC. The adversary may view only the WBB. It may, however, demand to see a (possibly faked) view from any honest voter.

**Setup** The coercer will demand that the voter cast some vote \( v \), and then provide the coercer with a transcript describing the setup, ballot generation and ballot casting for \( v \).

**Proof main idea** The Voter’s coercion-resistance strategy is to truthfully reveal \( a, b, r_a, r_b \) but claim to have sent \( \text{MAC}_{cr} = a \cdot v + b \mod q \) as their MAC. We rely on the re-randomised encrypted MAC that the EC posts on the WBB being indistinguishable from a re-randomised encryption of \( \text{MAC}_{cr} \).

---

\(^{6}\)TLS is not an untappable channel—people can prove what they sent by exposing the AES key.
Theorem 2. For any constant \( m \in \mathbb{N} \) and \( n = \text{poly}(\lambda) \), the e-voting system described in section 2 has receipt freeness with respect to the game \( \Gamma_{\text{RecFree}}^{A,S}(1^\lambda, n, m) \)

A proof is given in Appendix B.

What this means in practice The assumption that a coercer cannot tap the electronic channel from client to EC excludes adversaries associated with any network-based attacker, including those who see only encrypted TLS traffic. The assumption that the coercer cannot tap communications through the paper channel excludes an attacker who is physically present to watch the voter generate and post their vote. It also assumes that a voter filmaking themselves creating and posting the envelope would not be convincing. We do not know how hard it is to fake such a video in practice, but note that our protocol does not add anything to such a video (such as specific ciphertexts) that would make it any more convincing than any other, except through collusion with the EC.

Honest-but-remembering receipt-freeness is better than no receipt freeness in the following practical scenario: suppose that the coercer has compromised the voter’s computer, and seemingly has read access to all of their communications but doesn’t know whether this access is genuine or simulated (e.g. if the voter is running the client in a virtual machine and controlling what the attacker sees). With Helios, the attacker would be able to distinguish these two cases by verifying that the encrypted vote constructed by the device was posted to the bulletin board. With our system, assuming the attacker can’t verify what was sent to the EC, the system does not provide a way for the attacker to distinguish whether the voter sent the vote it seems to have sent, or intercepted it outside the coercer’s view and sent something else. Hence read-only access does not allow coercion. However, if the coercer can instruct the voter to deviate from the protocol then coercion does succeed.

4.3 Verifiability against a cheating EC

Here we formalise the argument that, if the corrupt EC successfully posts a valid MAC for the claimed ReceivedVote then it knows \( a, b \), so this happens only for a negligible number of votes without client collusion. We use a modified version of the end-to-end verifiability game from [40]; our version does not allow the adversary to control the client and the EC simultaneously. The definition uses a vote extractor algorithm \( E \), which given an election transcript \( \tau \) and a set of honest voter receipts \( \alpha \), outputs the set of dishonest votes \( \{ \mathcal{U}_i \}_{\forall i \in \mathcal{V} \setminus \tilde{\mathcal{V}}} \). (We will use the metric \( d_1 \), meaning the absolute difference in number of votes for each candidate.)

Definition 2 (EC Verifiability Game (after [40])). We denote the game by \( \Gamma_{\text{EC-Ver}}^{A,E,d,\theta}(1^\lambda, m, n) \).

1. \( A \) on input \( 1^\lambda, n, m \), choose a list of candidates \( \mathcal{P} = \{ P_1, ..., P_m \} \), a set of voters \( \mathcal{V} = \{ V_1, ..., V_n \} \), and the set of allowed candidate selections \( \mathcal{U} \). It provides \( C \) the sets \( \mathcal{P}, \mathcal{V}, \) and \( \mathcal{U} \).

2. \( A \) performs the Setup protocol on input \( (1^\lambda, \mathcal{P}, \mathcal{V}, \mathcal{U}) \) to obtain \( \mathcal{S}, (G, g, q, pk) \), providing \( C \) with \( (G, g, q, pk) \).

3. The adversary \( A \) and the challenger \( C \) engage in an interaction where \( A \) schedules Cast protocols of all voters which may run concurrently. For each voter \( V_i \in \mathcal{V} \), \( A \) can either completely control the voter or allow \( C \) to operate on their behalf, in which case \( A \) provides a candidate selection \( \mathcal{U}_i \) to \( C \). Then, \( C \) engages with the adversary \( A \) in the Cast protocol so that \( A \) plays the role of the EC and the postal service. If the protocol terminates successfully, \( C \) obtains the receipt \( \alpha \) = VoterID on behalf of \( V_i \) Let \( \tilde{\mathcal{V}} \) be the set of honest voters (i.e. those controlled by \( C \)) that terminated successfully.

4. \( A \) posts the election transcript \( \tau \) to the WBB.

The game returns a bit which is 1 iff the following conditions are true:

1. \( |\{ i \in [n] \mid \text{VoterVerify}(\alpha_i) \text{ passes} \} | \geq \theta \) (i.e. at least \( \theta \) honest voters verified successfully);

2. \( \text{Result}(\tau, O) \neq \bot \); and

3. for the metric \( d_1 \) and election outcome function \( f \):

\[
\text{Result}(\tau, O) \neq \bot \text{ and } d_1(\text{Result}(\tau, O), f((\mathcal{U}_1, ..., \mathcal{U}_n))) > d
\]

where \( \{ \mathcal{U}_i \}_{V_i \in \mathcal{V} \setminus \tilde{\mathcal{V}}} \leftarrow \mathcal{E}(\tau, (\alpha_i)_{V_i \in \mathcal{V}}) \) (That is, the deviation from the true result is larger than the accepted error \( d \).)

We say that a voting scheme achieves EC verifiability if for any PPT adversary \( A \):

\[
\Pr[\Gamma_{\text{EC-Ver}}^{A,E,d,\theta}(1^\lambda, m, n) = 1] = \text{negl}(\lambda).
\]

4.3.1 A simplified protocol for proving verifiability

Consider a simplified version of the protocol in which there is only one decryption authority (this is not the privacy game after all). Remember that the attacker can modify the plaintext ballots as well as the EC’s computations, though it does not control the voting client of the honest voters.

Theorem 3. For any constant \( m \in \mathbb{N} \) and \( n = \text{poly}(\lambda) \), a specified result function \( \text{Result}(\tau, O) \) defining a threshold \( 0 < d < M \) for an election with margin \( M \), and \( \theta = |\mathcal{V}|- (M - d) \), the simplified single-decryptor version of the protocol satisfies EC verifiability.
Proof. We begin by defining the vote extractor \( E \). For each corrupt voter ID, it considers the commitment pair posted by the voter’s device in Step 3 of Cast, and the encrypted vote-MAC pair posted by the EC in Step 9 of Cast. It inspects the WBB transcript \( \tau \) and outputs:

1. zero, if the VoterID has no matches in Step 9 of Tally votes or no correct opening in Step 20.
2. zero, if the VoterID has more than one such match or correct opening
3. zero, if there is a unique match and correct opening but either of the PETs in Steps 22 and 24 are not successful
4. ReceivedVote otherwise.

The first three cases correspond to a vote that was not submitted, or a verification failure. Case 4 represents successful verification of a vote that makes it into the tally. We will argue that the adversary has a negligible probability of successfully (and undetectably) substituting a vote with a different one voter’s device. (Remember that the BB is not under the adversary’s control and hence the cheating EC cannot prevent the client from uploading its initial commitments.) Also note, that since GlobalVerify passes only one commitment is present for each Voter and the voter’s device checked that the commitment was the one it uploaded. At Step 12 of Process Vote (Algorithm 3), the EC must choose a particular vote and encrypted commitment openings \( a,b \) to post alongside the vote and encrypted RecVoterID. There are three possibilities for such a commitment opening, compared to the commitment posted alongside VoterID in Step 3 of Cast.

1. The opening may match the commitment.
2. The opening may match a different voter’s commitment.
3. The opening may match no voter’s commitment.

Case 1 is the successful case where the correct commitment is opened; the security properties of Pedersen commitments guarantee the opening is legitimate except with only negligible probability \( \eta_2 = \text{negl}(\lambda) \). Note that the EC cannot submit many possible openings and hope that one is a successful forgery — the uniqueness condition in Step 10 of GlobalVerify prevents multiple attempted openings from being accepted. Case 2 will not pass verification, since only openings where \( \text{RecVoterID} = \text{VoterID} \) should be accepted in Step 10 of GlobalVerify. Similarly, Case 3 will not pass verification at the same step. We therefore discount the possibility of forged commitments for the remainder of the discussion.

We now arrive at the key argument of the voting scheme. We will demonstrate that even a computationally-unbounded adversary cannot cheat in these circumstances with non-negligible probability. This adversary receives the genuine voter ID and ciphertexts \( \{g_{\text{vote}}^a\}_pk, \{g_{\text{MAC}}_a\}_pk \) during Cast, which they can brute-force to produce plaintexts Vote, MAC. They will post encryptions of different values Vote\_cheat, MAC\_cheat to the WBB in Step 9 of Cast. The PETs verified in Steps 3 and 10 of GlobalVerify (which we assume are honest) ensure that

\[
a \cdot \text{Vote}_{\text{cheat}} + b = \text{MAC}_{\text{cheat}} \text{ with Vote}_{\text{cheat}} \neq \text{Vote}
\]

But the adversary also knows that \( a \cdot \text{Vote} + b = \text{MAC} \), and thus knows two points on the line defined by \( a \) and \( b \). The adversary has therefore extracted \( a \) and \( b \) from the information it had received by Step 9 of Cast, which included only one point on the line and two perfectly-hiding commitments to \( a \) and \( b \). However, given a fixed pair \( a,b \in \{1,\ldots,q-1\} \), a vote, and a MAC there are \( q-2 \) other pairs

\[
a' = a + k, \quad b' = b - k \cdot \text{Vote} \text{ for } k \in \{1,\ldots,q-1\}
\]

such that \( a' \cdot \text{Vote} + b' = \text{MAC} \).

Perfectly-hiding commitments leak no information; the adversary must therefore have guessed \( a \) and \( b \). Since \( a \) and \( b \) were chosen uniformly at random, the adversary can do so with probability \( \frac{1}{q-1} \).

We are left with three ways the adversary can succeed: by forging ZKPs (with probability \( \eta_1 \)), by forging commitments (with probability \( \eta_2 \)), or by forging MAC/vote pairs (with probability \( \frac{1}{q-1} \)). If the adversary does not forge a ZKP, it
must forge commitments or MAC/vote pairs for at least $d$ votes — but the probability for forging these for even one vote is negligible. All told, any PPT adversary must therefore have advantage at most $\eta_1 + \eta_2 + \frac{1}{q^{\tau}} = \text{negl}(\lambda)$. □

**4.4 Recorded-as-cast Verifiability against a cheating client**

We claimed that the protocol allowed voters to detect cheating against an adversary who controls either the client or (the postal service and the EC) but not both. We therefore assume that if the client is corrupted, the paper ballot is properly received and processed at the electoral commission. We assume that $\theta = |V| - (M - d)$ voters check their plain paper printout with their vote and their VoterID. Note that the Voter is honest, but an honest voter’s client may be malicious.

The client verifiability game is defined in Appendix C and is very similar to that in [40]. The cheating clients win if the election tally is accepted, but is substantially different from the true result. The proof relies on the assumption that the paper ballot is properly posted and processed.

**Theorem 4.** For any constant $m \in \mathbb{N}$ and $n = \text{poly}(\lambda)$, a specified result function $\text{Result}(\tau, O)$ defining a threshold $0 \leq d < M$ for an election with margin $M$, and $\theta = |V| - (M - d)$, the simplified single-decryptor version of the protocol satisfies client verifiability.

The definition of the game and proof of the Theorem are given in Appendix C.

**5 Implementation and pilot**

We implemented a prototype in Rust. The ElGamal cryptosystem was implemented over the prime-order Ristretto subgroup of Curve25519 using curve25519-dalek [45].

For the shuffle, we re-implemented a variant of the Verification shuffle [64] where each row contains multiple ciphertexts, based on the presentation in [27]. The implementation was tested for efficiency, as a real-world system may need to handle millions of votes. Using an Intel i7-10750H mobile CPU to run a shuffle on 100000 rows, each with 6 ciphertexts, the code was able to generate a shuffle and corresponding proof in 38.34 seconds, and was able to verify this proof in 26.43 seconds. Practicality is a key benefit of our protocol—for $n$ votes, we require only $O(n)$ PEPs, and the shuffle proof requires only $O(n)$ elliptic curve additions and multiplications.

A real-world pilot of the protocol was run with three trustees A small number of volunteers were asked to rank candidates Alice, Bob, and Eve—the example votes from Section 3.1 were taken from this pilot. Seven ranked-choice votes were submitted and physically mailed to one of the authors acting in the role of EC. Five of them were scanned and tallied. The other two were (unintentionally) lost due to human error in the testing process, which gave us a nice opportunity to test our basic verification steps. The verification protocol successfully verified four of the votes and revealed that one was missing. The other two voters did not use the verification protocol to check their votes. Although this is nothing close to a full usability study or realistic test, it shows that our system is complete and that simple failures can be detected.

**6 Limitations and possible enhancements**

**Trust in the Electoral Commission for privacy** Our protocol trusts the EC to forget the link between VoterID and the encrypted VoterID it generates. In traditional postal voting there is analogous trust in the Electoral Commission to not misbehave, for example, by opening both the inner and outer envelope at the same time. However, this can be observed by scrutineers without infringing on the secrecy of the ballot, and is therefore a relatively low, observable, risk. Trusting an electronic component not to leak is much more problematic.

**Non-collusion between EC and client** Another important limitation is the assumption that the client and the EC are not both compromised. In an ideal world, people would download and compile an open-source voting app from an independent entity they trusted. In practice voters generally get their voting instructions and software from the same EC that will be receiving their votes. (Two of the authors worked on an end-to-end verifiable e-voting project in which the electoral authority refused to issue any cast-as-intended verification instructions at all.) This is an important practical question for the true security of our scheme. However, most verifiable e-voting systems suffer from some version of the same problem, hence the huge motivation to fall back to plain paper mail.

**Revealing which voters’ MAC matched** The current version of this protocol reveals VoterIDs during the MAC matching process. This produces public information about who cast a valid vote and who didn’t. The protocol could be altered to hide this information, thus making it secret which MACs matched, though the total numbers would be public. This would change the verifiability property from an individual to a group one: voters would not be able to tell whether their own vote had been dropped, though everyone would be able to see the total numbers of dropped and invalid votes.

**Trusting the client for privacy** The client is trusted for privacy: although a client controlled by the voter can lie to a coercer, a client controlled by the coercer knows which MAC was submitted and hence which vote was sent.

However, the MAC generation, vote encryption, and data upload (to the EC) steps of Cast do not all need to be performed by the same device. One device could generate ciphertexts (without knowing which were uploaded), and a different
device could upload them (without knowing their contents). This has significant advantages for privacy against the client, effectively splitting the information about how the person voted between two devices.

Also, since the voter’s ability to defend against a cheating EC relies on the client to keep the random $a$ and $b$ values secret, it would be beneficial to expand the protocol so that $a$ and $b$ were generated in a distributed way by multiple devices.

**Accountability** The current protocol emphasises verifiability but does not attempt to provide accountability or defence against voters who maliciously claim that there was a problem when there was none. The most obvious attack is to post fake votes with someone else’s VoterID. This will not forge a vote, but it will appear as an indication of a problem. Although this isn’t a verifiability failure, it would cause votes to be cancelled (detectably) and to give an impression of fraud. A standard authentication mechanism would address this. For example, voters could be given a secret nonce without which an upload for their VoterID would not be accepted to the WBB.

**WBB** We also make strong assumptions on the existence and security properties of a web bulletin board. In practice, this would be implemented in some specific way involving either threshold trust on a set of peers, or a final verification step requiring some extra work from each voter.

### 7 Conclusion

We provide a significant step forward in the design of verifiable remote voting protocols. Our protocol combines very simple cast-as-intended verification of a plaintext printout with strong guarantees of verifiability for those observers who choose to check. We also provide a (passive, honest-but-remembering) version of receipt-freeness against an attacker who cannot tap either of the voter’s communication channels. This is the first work to provide all these advantages.

For politically binding elections, trusting the Electoral Commission for privacy, and trusting it not to collude with the client for verifiability, may be unacceptable. However, for non-political elections, where independent scrutineers may not be present and possibly even double envelopes are not used, the trust assumptions may be equivalent to what they are in typical postal voting. In such scenarios, the additional benefit of verifiability remains valuable.

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We prove Theorem 1 (vote privacy) assuming a threshold of with respect to the honest voters is going to be as follows: Cast adversary, with encryptions of nonsense without the adversary noticing. Specifically in game 3, the adversary’s view during Cast with respect to the honest voters is going to be as follows plus some random ciphertexts and simulated proofs:

\[(\text{VoterID}, c_a, c_b)\] (2a)

\[\text{VoterID}\] (2b)

\[\text{VoterID}\] (2c)

During Tally the adversary will see:

\[\text{Vote}\] (3a)

\[\text{ReceivedVote}\] (3b)

\[((a, b, r_a, r_b), \text{RecVoterID})\] (3c)

\[s^{\text{Vote}}\] (3d)

The mixing that EC performs between (3c) and (4a), between (4b) and (4c), and also between (4c) and (4d) hides the order in which the votes were submitted, preventing the vote from being trivially matched with the voter ID.

**Proof.** Define the advantage between game \(G_i\) and \(G_j\) to be

\[\text{Adv}_{G_i, G_j}(\mathcal{A}) := \frac{1}{2} |\Pr[\mathcal{A} = 1 | G_i] - \Pr[\mathcal{A} = 1 | G_j]|\]

Consider the following sequence of games.

**Game \(G_0\):** The actual game \(G^q_{\text{priv}}(1^\lambda, n, m)\). By definition \(\text{Adv}_{G_0, G^q_{\text{priv}}(1^\lambda, n, m)}(\mathcal{A}) = 0\).

**Game \(G_1\):** Let \(G_1\) be the same as Game \(G_0\) except that all PETs and decryptions are simulated using knowledge of the plaintext rather than decryption keys. This is allowable since all ciphertexts being decrypted or tested for plaintext equivalence are either produced by the challenger, or they are produced by the adversary but are accompanied by zero-knowledge proofs of knowledge (and therefore the challenger can extract them with the zero-knowledge extractor). By the soundness properties of the zero-knowledge proof of knowledge proofs \(\text{Adv}_{G_1, G_0}(\mathcal{A}) = \text{negl}(\lambda)\).

**Game \(G_2\):** Let \(G_2\) be the same as Game \(G_1\) except all the ZKPs used to demonstrate correct mixing, correct decryption, and correct PETs that the challenger performs are simulated via the zero-knowledge simulator. At this point, the challenger no longer uses the secret key corresponding to \(pk\) for any purpose. The mixing must be simulated to avoid leaking information as to the permutation or randomness used. Since the proofs of knowledge are non-malleable and the EC filters for duplicates, none of the adversary’s proofs depend on the simulated proofs; we can thus continue to use the extractor on these proofs. By their zero-knowledge properties, \(\text{Adv}_{G_2, G_1}(\mathcal{A}) = 0\).

**Game \(G_3\):** Let \(G_3\) be the same as Game \(G_2\) except that all the values to be encrypted are replaced by random values from an oracle and all values to be re-encrypted are replaced with fresh encryptions of random values. Decryption and plaintext equivalence is always simulated, so we never provide a decryption oracle; this allows us to rely on the IND-CPA property of ElGamal, guaranteeing that \(\text{Adv}_{G_3, G_2}(\mathcal{A}) = \text{negl}(\lambda)\).

In Game \(G_3\), the ciphertexts and proofs contain random values with the exception of the (decoupled) VoterIDs and Votes. The VoterIDs and Vote are decoupled as a result of the mixing which applies a random permutation to these lists—this is secret assuming the honesty of at least one mixing trustee and the honesty of \(n - k + 1\) decrypting trustees (for \(k\)-out-of-\(n\) secret sharing). Recall that the second criterion of the game says that the adversary loses if the set of honest votes

\[a, b, r_a, r_b\]
leaks $b$. Therefore, the adversary cannot have any advantage in winning $G_3$. Following the chain of games yields

$$Adv_{G_3,G^A_{prv}}(1^λ,n,m) = \negl(λ)$$

so $A$’s advantage in $G^A_{prv}(1^λ,n,m)$ is negligible.

We note that it is possible to trust the voter’s device less and the EC more by changing the adversary’s view and the privacy proof follows in much the same manner. This change only affects the definition in point 3; the adversary is allowed to observe $a$ and $b$ but not Paper$_2$.

### B Definition and proof of honest-but-remembering Receipt Freeness

Informally, the adversary is attempting to coerce a voter $V_l$ into submitting a vote for candidate selection $U_l^0$. The game is defined as follows.

**Definition 3 (Honest-but-remembering Receipt-freeness Game).** We denote the game by $G^{A,S}_{RecFree}(1^λ,n,m)$.

1. $A$ on input $1^λ,n,m$, chooses a list of candidates $P = \{P_1,...,P_m\}$, a set of voters $V' = \{V_1,...,V_n\}$, and the set of allowed candidate selections $U$. It provides $C$ the sets $\mathcal{P}$, $V'$, and $U$.

2. $C$ flips a coin $b ∈ \{0,1\}$ and performs the Setup protocol on input $(1^λ,\mathcal{P},V',\mathcal{U})$ to obtain $sk,(G,g,q,pk)$, providing $A$ with $(G,g,q,pk)$.

3. The adversary $A$ and the challenger $C$ engage in an interaction where $A$ schedules Cast protocols of all voters which may run concurrently. For each voter $V_l ∈ V'$, the adversary chooses whether $V_l$ is corrupted:

   - If $V_l$ is corrupted, they engage in a Cast protocol where $A$ plays the role of $V_l$ and $C$ plays the role of EC.
   - If $V_l$ is not corrupted, $A$ provides two candidate selections $(\mathcal{U}_0^l,\mathcal{U}_1^l)$ to the challenger $C$. They must do so such that $f((\mathcal{U}_0^l)_{V_l∈\mathcal{V}}) = f((\mathcal{U}_1^l)_{V_l∈\mathcal{V}})$ where $\mathcal{V}$ is the set of honest voters (that is, the election result w.r.t. the honest voters does not leak $b$).

4. $C$ operates on $V_l$’s behalf, using $\mathcal{U}_0^l$ as the voter $V_l$’s input. The adversary is allowed to observe WBB only, where $C$ plays the role of $V_l$ and the EC. When the Cast protocol terminates, the challenger $C$ provides to $A$:

   - (a) the receipt consisting of the VoterID for voter $V_l$, and
   - (b) if $b = 0$, the current view, $view_l = (a,b,r_a,r_b,v,MAC,(g_{Vote}^l)_{pk},(g_{MAC}^l)_{pk},\{a\}_{pk},\{b\}_{pk},\{r_a\}_{pk},\{r_b\}_{pk},\{VoterID\}_{pk})$, of the voter $V_l$ that the challenger obtains from the Cast execution. If $b = 1$, the challenger instead provides a simulated view of the internal state of $V_l$ produced by $S(view_l)$.7

5. $C$ performs the Tally protocol playing the role of the election trustees ET. $A$ is allowed to observe the WBB.

Denote the set of corrupted voters as $\mathcal{V}_{corr}$ and the set of honest voters as $\mathcal{V} = \mathcal{V} \setminus \mathcal{V}_{corr}$. The game returns a bit which is 1 if and only if $b = b^∗$.

We say that a voting scheme achieves receipt-freeness if there is a PPT voter simulator $S$ such that for any PPT adversary $A$:

$$\Pr[G^{A,S}_{RecFree}(1^λ,n,m) = 1] - 1/2 = \negl(λ).$$

The proof of receipt-freeness relies on defining a coercion-resistance strategy in which the voter tells the truth about the secret values $(a,b)$ it has committed to, but lies about the vote and then claims a MAC corresponding to the claimed vote and the truthful $(a,b)$. We show that for a receipt-freeness adversary this is indistinguishable from obedience.

Since we do not rely on secrecy of the $a,b$ values for receipt freeness, we can use an intermediate game in which $a$ and $b$ are known to the EC.

We now state and prove the main theorem.

**Theorem 5.** For any constant $m ∈ \mathbb{N}$ and $n = poly(λ)$, the e-voting system described in section 2 has honest-but-remembering receipt freeness with respect to the game $G^{A,S}_{RecFree}(1^λ,n,m)$

**Proof.** We briefly recap the information visible to the adversary. During Cast the adversary sees

$$\{(VoterID,c_a,c_b) \mid (VoterID, Rerand\{g_{MAC}\}_{pk}, Rerand\{g_{Vote}\}_{pk}) \}$$

The main difference from the privacy game is that the adversary may demand the voter’s view, including secret information. After Cast, $A$ sees the possibly-simulated view

$$\{(VoterID,a,b,r_a,r_b,v,MAC,(g_{Vote}^l)_{pk},(g_{MAC}^l)_{pk},\{a\}_{pk},\{b\}_{pk},\{r_a\}_{pk},\{r_b\}_{pk},\{VoterID\}_{pk}) \}$$

7Intuitively, if $b = 0$ the voter honestly gives its view to the adversary. If instead $b = 1$ the voter simulates a fake view and gives that to the adversary, voting however they please.
During Tally the adversary sees

\[
(\text{ReceivedVote}, \text{Rand}\{\text{RecVoterID}\}_pk, \\
\text{Rand}\{\{a\}_pk, \{b\}_pk, \{r_a\}_pk, \{r_b\}_pk\})
\]

(\{\text{ReceivedVote}\}_pk, (a, b, r_a, r_b), \text{RecVoterID}, \text{decryption proof})

\[
\text{Decrypt}_{G, pk}(\{g^{\text{Vote}}\}_pk)
\]

**Defining the simulator** The simulator \(S\) for each honest voter \(V_i\) receives the voter’s view (including candidate selections (\(\mathcal{U}_i^0, \mathcal{U}_i^1\)) and randomness for all the encryptions)

\[
(\text{VoterID}, a, b, r_a, r_b, \text{Vote} = \mathcal{U}_i^1, \text{MAC}, \{g^{\text{Vote}}\}_pk, \\
\{g^{\text{MAC}}\}_pk, \{a\}_pk, \{b\}_pk, \{r_a\}_pk, \{r_b\}_pk, \{\text{VoterID}\}_pk)
\]

Then \(S\) outputs the fake view

\[
(\text{VoterID}, a, b, r_a, r_b, \text{Vote}' = \mathcal{U}_i^0, \text{MAC}' = a \cdot \text{Vote} + b, \{g^{\text{Vote}}\}_pk \\
\{g^{\text{MAC}}\}_pk, \{a\}_pk, \{b\}_pk, \{r_a\}_pk, \{r_b\}_pk, \{\text{VoterID}\}_pk)
\]

Define the advantage between game \(G_i\) and \(G_j\) to be

\[
\text{Adv}_{G_i, G_j}(\mathcal{A}) := \frac{1}{2}|Pr[\mathcal{A} = 1|G_i] - Pr[\mathcal{A} = 1|G_j]|
\]

Consider the following sequences of games.

**Game \(G_0\)**: The actual game \(G_{\text{RecFree}, (\lambda, n, m)}^S\), where the challenger uses \(\mathcal{U}_i^0\) in the Cast protocol and the above simulator is invoked when \(b = 1\). (That is, voters vote as they wish and run the coercion-resistance strategy.)

By definition \(\text{Adv}_{G_0, G_0}(\mathcal{A}) = \text{negl}(\lambda)\).

**Game \(G_1\)**: The same as Game \(G_0\), except the decryptions and plaintext equivalence tests are simulated with knowledge of the plaintext as in Theorem 1; \(\text{Adv}_{G_1, G_0}(\mathcal{A}) = \text{negl}(\lambda)\).

**Game \(G_2\)**: The same as Game \(G_1\), except the proofs used to demonstrate correct decryption, plaintext equivalence, and correct mixing are simulated with their zero-knowledge simulators as in Theorem 1. The challenger replaces the re-randomised ciphertexts from the mixing with fresh encryptions to ensure the link is destroyed. We have \(\text{Adv}_{G_2, G_1}(\mathcal{A}) = \text{negl}(\lambda)\).

**Game \(G_3\)**: The same as Game \(G_2\), except when \(b = 1\):

1. In Step 9 of Cast, the challenger posts an encryption of the claimed MAC, \(\{g^{\text{MAC}}\}_pk\), instead of a re-randomised encryption of the actual MAC \(\{g^{\text{MAC}}\}_pk\).
2. In Step 12 of Process Vote, the challenger changes the posted (re-randomised) encryptions of RecVoterID and \(a, b, r_a, r_b\) so that they appear together with the votes they claimed to have cast.

\(Tally\) can then proceed as usual; we have changed the votes and MACs consistently so that they are still plaintext-equivalent. Since all we have done is change encryptions for which the adversary does not know the randomness and the mixing breaks the link between successive encrypted votes, the IND-CPA property of ElGamal yields \(\text{Adv}_{G_3, G_2}(\mathcal{A}) = \text{negl}(\lambda)\).

**Game \(G_4\)**: The same as Game \(G_3\), except the challenger (acting as the honest voters) ignores the value of \(b\) and always obeys the adversary. Since the adversary does not see anything different to what it saw in Game \(G_3\), \(\text{Adv}_{G_4, G_3}(\mathcal{A}) = 0\).

The adversary has no advantage in Game \(G_4\) because the value of \(b\) is ignored. Following the chain of games then yields

\[
\text{Adv}_{G_4, G_{\text{RecFree}, (\lambda, n, m)}^S} = \text{negl}(\lambda)
\]

so \(\mathcal{A}\)’s advantage in \(G_{\text{RecFree}, (\lambda, n, m)}^S)\) is negligible.

**C Client Verifiability definition and proof**

**Definition 4** (Client Verifiability Game (after [40])). We denote the game by \(G^S_{\text{C-Ver}}(\lambda, m, n)\) for a vote extractor algorithm \(E\) (which may be super-polynomial).

1. \(A\) on input \(\lambda, m, n\), chooses a list of candidates \(P = \{P_1, ..., P_m\}\), a set of voters \(V = \{V_1, ..., V_n\}\), and the set of allowed candidate selections \(\mathcal{U}\). It provides \(C\) the sets \(P, V, \) and \(\mathcal{U}\).
2. \(A\) performs the Setup protocol on input \((\lambda, P, V, \mathcal{U})\) to obtain \(sk(G, g, q, pk)\), providing \(C\) with \((G, g, q, pk)\).
3. The adversary \(\mathcal{A}\) and the challenger \(C\) engage in an interaction where \(\mathcal{A}\) schedules Cast protocols of all voters which may run concurrently. For each voter \(V_i \in V\), \(\mathcal{A}\) can either completely control the voter or allow \(C\) to operate on their behalf, in which case \(\mathcal{A}\) provides a candidate selection \(\mathcal{U}_i\) to \(C\). Then, \(C\) engages with the adversary \(\mathcal{A}\) in the Cast protocol so that \(C\) plays the role of the voting client. The postal system and the EC execute honestly. If the protocol terminates successfully, \(C\) obtains the receipt VoterID on behalf of \(V_i\).

Let \(\hat{V}\) be the set of honest voters (i.e. those controlled by \(C\)) that terminated successfully.

4. Finally, the (honest) EC posts the election transcript \(\tau\) to the WBB.

The game returns a bit which is 1 iff the following conditions are true:

1. \[
\left|\left\{l \in [n] \mid \text{VoterVerify}(a_l) \text{ passes} \right\}\right| \geq \theta \text{ (i.e. at least } \theta \text{ honest voters verified successfully)};
\]
2. Result(τ, O) ≠ ⊥; and

3. for the metric $d_1$ and election outcome function $f$:
   \[d_1(\text{Result}(\tau, O), f(\langle \mathcal{U}_1, \ldots, \mathcal{U}_t \rangle)) > d\]
   where $\mathcal{U}_i \in \forall \forall; \forall \leftarrow \mathcal{E}(\tau, \{\omega_i\}_{i \in \forall})$ (That is, the deviation from the true result is larger than the accepted error $d$.)

We say that a voting scheme achieves client verifiability if for any PPT adversary $A$:
\[\Pr[c_{\text{Client-\text{ver}}}(1^\lambda, n, m) = 1] = \text{negl}(\lambda).\]

**Theorem 6.** For any constant $m \in \mathbb{N}$ and $n = \text{poly}(\lambda)$, a specified result function $\text{Result}(\tau, O)$ defining a threshold $0 \leq d < M$ for an election with margin $M$, and $\theta = |\mathcal{V}| - (M - d)$, the simplified ZKP-based version of the protocol satisfies client verifiability.

**Proof.** We assume that $\theta = |\mathcal{V}| - (M - d)$ voters run Algorithm 5 (Voter verification) correctly, including checking whether their vote is included in $\mathcal{B}_{\text{accepted}}$. These are the honest voters, though note that their client may be controlled by the adversary. The vote extractor $\mathcal{E}$ is the same as in the proof of Theorem 6.

Consider one honest voter with a possibly-malicious client. Consider the following cases, of which the first three are the only ways of making a malformed ballot. Note, of course, that none of them prevent the cheating client from sending other ballots either electronically or by colluding with another voter.

**Case 1** Suppose the client printed on Paper$_1$, or sent to the EC in Step 7 of Algorithm 2, at least one proof $\text{PrfKnow}_{G, pk}(e_{\text{Params}})$, $\text{PrfKnow}_{G, pk}(e_{\text{MAC}})$ or $\text{PrfKnow}_{G, pk}(e_{\text{Vote}})$, that contains a false statement but passes verification. By the adaptive soundness of the proofs, this is done with negligible probability $\eta_1$.

**Case 2** Suppose client printed on Paper$_1$, or sent to the EC in Step 7 of Algorithm 2, at least one proof $\text{PrfKnow}_{G, pk}(e_{\text{Params}})$, $\text{PrfKnow}_{G, pk}(e_{\text{MAC}})$ or $\text{PrfKnow}_{G, pk}(e_{\text{Vote}})$ that does not pass verification.

These are checked in Step 10 of Process vote (Algorithm 3). Since we assume an honest EC, the VoterID will be included in $\mathcal{B}_{\text{accepted}}$, and the vote will not pass voter verification (Algorithm 5).

**Case 3** Suppose the client printed on Paper$_1$ values $\{a\}_{pk}, \{b\}_{pk}, \{c_a\}_{pk}, \{c_b\}_{pk}$ that are not valid commitment openings of the $c_a, c_b$ commitments posted in Step 3 of Algorithm 2 (where Paper$_1$ here means the one that the honest voter verified, though the dishonest client may have printed other values onto other instances of Paper$_1$ and fraudulently inserted them into the post).

In this case, either there will be no commitment opening in Step 15, or there will be multiple matching VoterIDs in Step 20, of Tally votes / Algorithm 4. The honest EC will therefore not add VoterID to $\mathcal{B}_{\text{accepted}}$, so the vote will not pass verification.

**Case 4** Everything else.

To count Case 4, observe that cases 1–3 are the only ways a ballot (Paper$_1$) can be malformed. We can hence suppose for this case that the paper ballots are well-formed. By assumption of postal/EC honesty, both the plaintext ReceivedVote and the encrypted RecVoterID must be posted properly on the WBB at line 12 of Algorithm 3, because that is what it specifies when the ZKPs are valid. So there is at least one valid commitment opening received and posted.

We now count the number of RecVoterID’s in each possible category for Step 9 of Tally votes.

If there are multiple valid commitment openings for VoterID at Step 20, VoterID will not appear in $\mathcal{B}_{\text{accepted}}$. If there are other matching RecVoterIDs, GlobalVerify fails (Step 11).

So now assume there is a unique valid commitment opening at Step 20 and exactly one matching RecVoterID. By EC honesty, it must match what the voter checked on the paper ballot. If the cheating client did not send true encryptions of the vote and MAC at Step of Cast, the plaintext equivalence tests checked at Steps 24 and 22 will fail and VoterID will not appear in $\mathcal{B}_{\text{accepted}}$.

Therefore, if the voter runs the verification protocol correctly, either verification fails, or their vote has been correctly included in the tally, except with negligible probability $\eta_1$.

We have computed the probability that one client cheated without detection, which implies that the probability that the client of any honest voter cheated without detection is also negligible. Since all but $\theta = |\mathcal{V}| - (M - d)$ voters verified their vote successfully this is also an upper bound on the adversary’s success probability.

\[\square\]