Decoherence Driven Quantum Transport

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We propose a new mechanism to generate a dc current of particles at zero bias based on a noble interplay between coherence and decoherence. We show that a dc current arises if the transport process in one direction is maintained coherent while the process in the opposite direction is incoherent. We provide possible implementations of the idea using an atomic Michelson interferometer and a ring interferometer.

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The rates of emission and absorption between two quantum states are equal as governed by the principle of detailed balance. Under special conditions, however, the detailed balance can be broken, and one of the processes can even be completely suppressed. One relevant example is the so-called lasing without inversion (LWI) in quantum optics [1]. LWI is achieved in an ensemble of atoms that have a pair of nearly degenerate ground-state levels, say, a and b (Fig. 1). The atoms are prepared in a coherent superposition of a and b. This coherent superposition can be realized by applying microwave resonant with the ground-state splitting, $E_b - E_a$. The excitation probability to a upper level c, $T_{ab\rightarrow c}$, undergoes two-path interference, and hence can be modulated by changing the relative phase of the levels a and b in the coherent superposition. On the contrary the decaying probability from the state c to both a and b, $T_{c\rightarrow ab}$, is just the summation of the probabilities of two spontaneous decays since it is incoherent processes. When the phase of the microwave field is adjusted to make $T_{ab\rightarrow c}$ smaller than $T_{c\rightarrow ab}$, the lasing operation is possible without population inversion.

A dc current of particles can be achieved by applying an external dc bias; e.g., an electrical potential difference for charges, density difference for masses, temperature difference for heat, and so on. In these examples the external bias breaks directly the detailed balance between the currents in opposite directions. It is also possible to obtain a dc current by breaking the detailed balance indirectly. In rectification, a dc current is generated by an external ac voltage. In “ratchets”, the directed motion of particles can be caused from even random fluctuations [2]. Quantum mechanics provides still another ways to generate dc current by applying ac fields, e.g., (mesoscopic) photo-voltaic effect [3], quantum pumps [4], and quantum version of classical ratchet [5].

In this Letter we propose a new mechanism to generate dc currents at zero-bias by generalizing the concept of LWI to the transport problem. The basic idea is simple: A dc current will arise if the transport process in one direction is coherent while the process in the opposite direction is incoherent. One can easily check the idea by noting that the transmission probability of the coherent transport varies with the relative phases of multiple paths, which does not affect the incoherent one. The important question is then how to realize such spatially anisotropic, coherent/incoherent transport processes. It is clearly distinguished from the original idea of LWI since the asymmetry in coherence/decoherence of our concern addresses spatial directions rather than the excitation and relaxation of energy.

The scheme of our implementation is quite general, but for definiteness, here we take two specific examples; one based on Michelson interferometer and the other on a ring interferometer.

Let us first consider the scheme based upon the Michelson interferometer; see Fig. 2. We have an atomic Michelson interferometer and two reservoirs, 1 and 2, of two-level atoms at the ends of the two input/output channels of the interferometer. The atoms from a reservoir enter the interferometer, experience scattering and/or interference, and is either reflected back to the original reservoir or transmitted to the other. In addition we have an important component, the microcavity (C) between the reservoir 2 and the atomic beam splitter (BS). The cavity is set resonant to the level splitting $\Delta$ of the two-level atoms, so that the atoms entering the cavity in the ground state come out of the cavity in the excited state.
Therefore, when entering the interferometer, the atoms from the reservoir 2 are in the excited state while those from the reservoir 1 remain in the ground state. This difference in the energy state between the atoms entering the interferometer can cause a significant difference in the coherence of their center-of-mass (CM) motions in the interferometer.

To see this, let \( L_\tau = v \tau \), where \( v \) is the velocity of the atoms and \( \tau \) is the lifetime of the excited energy level. Provided that \( L_\tau < 2L \), with \( L \) being the lengths of the arms (i.e., the paths from the atomic BS to the mirrors) of the interferometer (the lengths of the arms are assumed to be equal), an excited atom in the interferometer will relax back to its ground state emitting a photon; see Fig. 2(b). In ideal case, the photon enables us to locate the atom definitely on one of the two arms of the interferometer. The excited atoms thus never experience an interference through the Michelson interferometer. In this sense, the CM motion of the atoms from reservoir 2 is incoherent. Furthermore, starting from the just located arm (whichever it is), the atom is transmitted to reservoir 1 with probability 0.5 and reflected back to reservoir 2 with 0.5 (we consider a 50:50 BS); see Fig. 2(b).

On the other hand, the atoms from reservoir 1 (ground-state atoms) do not allow such relaxation and will experience coherent interference as long as \( L_\phi \gg 2L \), where \( L_\phi \) is the coherence length of the CM motion of ground-state atoms; see Fig. 2(a). Due to the constructive interference, an atom from reservoir 1 is perfectly transmitted to the reservoir 2; see Fig. 2(a). Comparing these two transport processes, one can see that 50% of the incoming atoms contribute to the net dc current. Namely, when the currents from the reservoirs 1 and 2 are equal, \( I_1 = I_2 = I \), the net current from 1 to 2 is given by

\[
I_{12} = I_1 - 0.5I_2 = 0.5I. \tag{1}
\]

The current expressed in Eq. (1) is the maximum current attainable in our scheme assuming idealistic situations. Certain imperfections in reality will diminish the current. Firstly, only a fraction \( P_{ex} \) of the atoms from the reservoir 2 may be excited by the cavity. Secondly, an excited atom entering the interferometer may not necessarily relax to the ground state inside the interferometer. The probability \( P_\tau \) for such an event to occur is given by

\[
P_\tau \approx \int_0^{2L/v} dt \ e^{-t/\tau} \]

ignoring the distance between the BS and the cavity. Thirdly, even if the excited atom relaxes inside the interferometer and a photon is emitted, the atom cannot contribute to the net current unless the photon gives enough information about which path of the interferometer the atom takes. For example, if the wavelength \( \lambda \) of the photon is comparable to or larger than the size of the interferometer \( L (\lambda \gtrsim L) \), then one cannot get enough which-path information and the CM motion still remains coherent. Fourthly, while the decoherence due to photon emission is of particular type and of our primary concern, in general, the CM motion is subject to additional decoherence of usual type due to the “environment” even when the atom keeps its internal state (either ground or excited). Unlike the former, however, the latter is spatially isotropic and tends to reduce the net current.

The simple inspections in Eq. (1) and the effects of the imperfections mentioned above can be treated more rigorously based on the scattering theory. Important ingredients to be included in the formalism are the time dependence and the wave-packet description. It is because the decoherence process due to photon emission should locate the atom within the interferometer and the subsequent scattering process of the located atom is separated in time from that of the incoming atom. Another important element is an effective description of decoherence. To provide an unified description of both types of decoherence (see above), we adopt the framework of the

FIG. 2: (color online) The scheme based on the Michelson interferometer. 1 and 2 represent reservoirs. The (blue) dashed and the (red) solid lines represent the trajectories of the the ground-state and excited atoms, respectively. The horizontal and the vertical thick lines are mirrors, and the tilted thick lines in the middle are an atomic beam splitter. The box with “C” is the microcavity. (a) The coherent process: An atom from the reservoir 1 undergoes an constructive interference and reaches the reservoir 2 with unit probability. The cavity does not affect the transmission of this atom. (b) The incoherent process: An atom from the reservoir 2 is excited at the cavity, and spontaneously emits a photon within the vertical path. The atom is then transmitted to either reservoir with equal probability 0.5.
unitary representation. A 50:50 BS is described by the unitary scattering matrix (both for the wave entering and leaving the interferometer):

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}, \quad (2)$$

ignoring the weak energy-dependence in the range of interest. The state vector of the atom that has come from the reservoir 2 is given inside the interferometer by:

$$|\Psi(t)\rangle = \int \frac{dk}{\sqrt{2}\pi} \phi(k)e^{-i\omega(k)t} \times \left[ (\alpha |g\rangle + \beta e^{-i\Delta\tau/h} |e\rangle) \otimes |0\rangle \otimes (|e_1\rangle + |e_2\rangle) + \gamma |g\rangle \otimes (|\nu_1\rangle \otimes |e_1\rangle + e^{i\Delta\tau/h} |e_2\rangle) \right], \quad (3)$$

where $\phi(k)$ is the envelope function of the wave packet, $\omega(k)$ is the dispersion of the CM motion, $x_1$ ($x_2$) is the position from the BS along the vertical (horizontal) arm of the interferometer. $|g\rangle$ ($|e\rangle$) is the internal ground (excited) state of the atom. The photon emitted from the vertical (horizontal) arm is represented by $|\nu_1\rangle$ ($|\nu_2\rangle$) whereas $|0\rangle$ is the vacuum state. $\langle \nu_1 | \nu_2 \rangle = 0$ implies that the photon provides a sufficient which-path information and one can locate perfectly the atom on one of the two arms. The environment that couples to the CM motion and causes the decoherence of usual type has the state $|e_1\rangle \otimes (|e_2\rangle)$ when the atom takes the vertical (horizontal) arm, whereas $|0\rangle \otimes (|e_1\rangle + |e_2\rangle)$ when the atom takes the horizontal (vertical) arm. The atom momentum $\hbar k$ of the atom is sufficiently larger than that of the photon $\hbar \nu$. The atom scatters again off the BS to get out of the interferometer and then has the state vector

$$|\Psi(t)\rangle = i \int \frac{dk}{\sqrt{2}\pi} \phi(k)e^{i\nu L - i\omega(k)t} \times \left[ e^{-i\frac{\nu_1}{2}x_1} (\alpha |g\rangle + \beta e^{-i\Delta\tau/h} |e\rangle) \otimes |0\rangle \otimes (|e_1\rangle + |e_2\rangle) + ie^{-i\frac{\nu_2}{2}x_2} (\alpha |g\rangle + \beta e^{-i\Delta\tau/h} |e\rangle) \otimes |0\rangle \otimes (|e_1\rangle - |e_2\rangle) + \frac{\gamma}{2} e^{-i\frac{\nu_1}{2}x_1} |g\rangle \otimes (|\nu_1\rangle \otimes |e_1\rangle + |\nu_2\rangle \otimes |e_2\rangle) + \frac{i\gamma}{2} e^{-i\frac{\nu_2}{2}x_2} |g\rangle \otimes (|\nu_1\rangle \otimes |e_2\rangle - |\nu_2\rangle \otimes |e_1\rangle) \right]. \quad (4)$$

Therefore the probability that an atom from the reservoir 2 reach the reservoir 1 is given by $P(1 \leftrightarrow 2) = (|\alpha|^2 + |\beta|^2)(1 + |\langle e_1 | \nu_2 \rangle|^2)/2 + |\gamma|^2(1 + Re(\langle e_1 | \nu_2 \rangle \langle \nu_1 | \nu_2 \rangle))/2$, which leads to the net current

$$I_{12} = 0.5P_{exP_\tau Re \{ (1 - \langle \nu_1 | \nu_2 \rangle) \langle e_1 | \nu_2 \rangle \} I}. \quad (5)$$

Equation (5) shows a sharp contrast between the roles of the two types of decoherence. The decoherence that is due to photon emission and described in effect by $|\nu_2\rangle$ enhances the current while the usual decoherence process (described by $|\nu_1\rangle$) due to the coupling to the environment suppresses the current.

At this point, it will be interesting to address the question: Does this spontaneous dc current violate the second law of thermodynamics? Consider four atoms, two from each reservoir. One will end up with (on average) three atoms in the reservoir 1 but one in the reservoir 2, which corresponds to the decrease of the entropy by $\log(3/2)$. However, the increase in entropy induced by the decoherence is enough to compensate this decrease and give a net increase in total entropy. To see this, note that the complete decoherence makes the off-diagonal components of the density matrix zeros, which gives rise to the increase of the entropy by $\log 2$. Therefore the net increase in total entropy is $(1/2) \log 2 - (1/4) \log(3/2) \approx \log 1.3$ per atom.

Now we turn to the second example, i.e., the scheme based on the ring geometry; see Fig. 6. This scheme is interesting in the light that the coherent electron transport through the ring has been extensively investigated in condensed matter physics. The basic principle is exactly the same as in the first scheme: The coherent propagation [blue dashed line in Fig. 6(a)] of the atoms from reservoir 1 enables them to reach the reservoir 2 with unit probability. The atoms from the reservoir 2, on the other hand, are excited before entering the interferometer, and experiences decoherence within the interferometer; Fig. 6(b). This incoherent propagation reduce the probability for the atoms to reach the reservoir 1. Overall we have a net dc current from reservoir 1 to 2. To estimate the amount of the current, we calculate the transmission probability for incoherent process using the same formalism as in the Michelson interferometer except that we replace the scattering matrix in Eq. (1) for the BS with that for the three-terminal junction [11]:

$$J = \begin{pmatrix} -(a + b) & \sqrt{\epsilon} & \sqrt{\epsilon} \\ \sqrt{\epsilon} & a & b \\ \sqrt{\epsilon} & b & a \end{pmatrix} \quad (6)$$

with $a = (\sqrt{1 - 2\epsilon} - 1)/2$ and $b = (\sqrt{1 - 2\epsilon} + 1)/2$. $\epsilon$ is the coupling parameter: $\epsilon = 1/2$ gives no reflection when the waves enter the ring from the reservoirs. For $\epsilon = 0$, the ring and the lead are completely decoupled. For simplicity, let us assume that $\epsilon = 1/2$. 


Now suppose the atom emits a photon, say, in the lower arm. The atom (now in the ground-state) starts coherent propagation. Simple inspection shows that the final transmission probability is 0.5, and there is no reflection. Because of the conservation of the number of the particles, the remaining one half should be trapped in the ring!

Let us consider this striking result more carefully. In Fig. 3(b) the wave starting from the upper arm scatters on the right junction. \[\sqrt{1/2}\] of the wave amplitude (probability 1/2) escapes from the ring, \(-1/2\) is reflected back to the upper arm, and 1/2 is transmitted to the lower. The phase accumulation during the passage of either arm is ignored for the moment. The two transmitted and the reflected waves are scattered again in the left junction. Since these two waves are out of phase with the same magnitude, they interfere destructively in the lead attached to the reservoir 2. Repeating similar analysis shows that 50% of the wave is trapped in the ring when the spontaneous decay takes place in the ring. In reality, due to various sources of decoherence the atom should finally escape from the ring to the right or left reservoir with the same probability. Roughly, 75% of the incoming wave is transmitted and 25% reflected. Compared with the coherent case, 25% of the transmission probability decreases, which results in the net dc current. The trapping probability depends on the geometry of the ring such as \(\varepsilon\) and the length of the arm, \(L\) [12].

In conclusion, we have proposed a new mechanism to generate dc current using a noble interplay between coherence and decoherence. Two specific schemes of implementation have been presented based on the Michelson and the ring interferometers. A coherent superposition of states has more information (or equivalently less entropy) than incoherent ones. In some sense, this extra information has been exploited to generate a dc current. Thus it will be interesting to compare our work with another striking proposal by Scully et al. [13], a quantum heat engine operating from a single heat bath prepared in a certain coherent superposition and with a greater efficiency than a classical Carnot engine. The idea presented here may hopefully shed light on deeper understanding of the nature of decoherence and the subtle boundary between classical and quantum physics.

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\footnotesize
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