Research on Signal Processing in φ-OTDR System Based on Single Frequency IIR Trap

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Abstract: In the traditional phase sensitive optical time domain reflectometer (φ-OTDR) system, after locating of the vibration source, the spectrum diagram of the vibration signal can be obtained through the non-uniform Discrete Fourier Transform (NDFT) by extracting the data at this position. However, spectral aliasing noise included in the spectrum of the obtained vibration signal at this time. Because spectrum aliasing noise is generated in the process of NDFT transformation, traditional noise reduction methods have little effect on it. Therefore, this paper proposes a kind of IIR notch device based on single frequency for use in φ-OTDR. The change of standard deviation in discrete frequency domain is used to locate the vibration source, the large frequency components are filtered out from the time domain to solve the small signal components. The performance of IIR notch filter is verified by simulation. The results show that the notch filter can be used to calculate the small signal components of non-uniformly sampled signals successfully. It is convenient for φ-OTDR to use time-frequency images of different vibration types to identify vibration signals, so as to improve the recognition efficiency and monitoring ability of the system.

1. Introduction
Phase sensitive optical time-domain reflectometer (φ-OTDR) is a rising star in the distributed optical fiber sensor (DOFS), which can detect weak vibration events. It has the characteristics of high sensitivity, fast response speed and distributed sensing. Therefore, it has great application potential in security system intrusion monitoring, power line monitoring and optical fiber communication line monitoring. φ-OTDR uses high coherence light source to send out detection light pulse and inject it into optical fiber, then, the coherent fading phenomenon of backward Rayleigh scattering is used for sensing. When the vibration event ACTS on the optical fiber, the scattering rate and position of the scattered point will change, which will cause the backscattering phase change in the light pulse width. The light intensity at this location fluctuates with the vibration events, and by monitoring such fluctuations, the vibration events on the optical fiber can finally be located and analyzed.

Infinite Impulse Response (Infinite Impulse moisturiser Response and IIR) notch filter design has been developed for decades, and its purpose is to suppress or filter out some specific frequency component in digital signal, and the impact on other frequency component, because of its hardware to smaller footprint can provide very satisfying trap bandwidth is widely applied to eliminate sinusoidal interference signal, is a kind of basic processing unit, is the design of a digital communication system often choose module. It has been widely used in communication systems, military electronic countermeasures and communication line monitoring [1]. When the interference frequency is known, a fixed IIR notch can be used. In the study of fixed IIR notch, the obtained notch has some defects by using pole assignment. At the beginning of designing the notch, its indexes have been given. At this
time, it needs to be improved by combining with other strategies to make its performance closer to that of the ideal notch. The φ-OTDR system in literature [2] has a distance of 175km without relay sensing, but the corresponding highest response frequency is only 285Hz. To solve this problem, Zhang proposed in 2017 that the non-uniform sampling method could be used to detect the time interval of the pulses through random modulation, to break through the limitations of Nyquist sampling theorem and increase the maximum response frequency of φ-OTDR system. In a system with a maximum sampling frequency of 10kHz, 1.153mhz single-frequency vibration can be detected [3]. In literature [4], Peramuna S A further analyzed the optimal value of each parameter in this method. However, he only analyzed how the non-uniform sampling method detects the vibration frequency, and did not study how this method is used for positioning and signal recognition.

In this paper, a single frequency IIR notch is used in the φ-OTDR system based on non-uniform sampling to improve the accuracy of vibration signal identification. Since the NDFT is irreversible, small frequency components cannot be calculated by removing large frequency components directly from the frequency domain. In order to facilitate the measurement of small frequency components, a single frequency IIR notch is introduced to filter out the large frequency components of the positioning data from the time domain. As a result, better small signal components are obtained, improving the performance of vibration signal recognition.

2. Detection Principle of φ-OTDR System

2.1. System composition
As shown in Figure 1, the φ-OTDR System Chart, the light source (LS) emits a stable ultra-narrow line-width laser. The laser is modulated as a periodic light pulse by the acoustic optical modulator (AOM), then it passes the erbium-doped optical fiber amplifier, and finally is injected into the optical fibre link by a circulator after filtering noise. In the process of light pulse (LP) transmission, the backward rayleigh scattering light (RSL) will be constantly produced along the optical fiber, and the scattered light produced will be interfered by multiple beams because the rayleigh scattering belongs to elastic scattering. The interference light returned will be coupled with the reference light of the other circuit, and then it will become an electrical signal after entering the photoelectric detector (APD). Upon passing the data acquisition (DAQ), it is converted to a digital signal, and then a rayleigh scattering curve distributed along the timeline is achieved at the display terminal (DT), with the time points corresponding to the optical fiber distance. If a certain point on the optical fiber vibrates, the rayleigh scattering optical phase at the corresponding position will change, which further results in the change of rayleigh scattering curve at the receiving terminal. Based on the change detection, the vibration source (VS) can be positioned and the vibrational frequency will be worked out.

![Fig. 1. φ-OTDR system block diagram](image)

2.2. Frequency Solution of Small Signal Component
After the φ-OTDR system locates the vibration source by using the non-uniform sampling method, the data at this position will be extracted to work out the frequency spectrum diagram of the vibration signals through NDFT. The expression formula of the vibration signal s(t) is set as:
\[ s(t) = \sum_{i=1}^{K} A_i \cos(2\pi f_i t) \quad (1) \]

Where, \( s(t) \) is the superposition of \( K \) vibration signals with different frequencies, and \( A_i \) is the amplitude of each vibration component. Here, the relationship between the variance \( \sigma^2 \) of frequency spectrum aliasing noise and the amplitude of the vibration component can be expressed as:

\[ \sigma^2 \propto \sum_{i=1}^{K} A_i^2 \quad (2) \]

It can be seen, from the above formula, that if large amplitude components exist in the vibration signals, the aliasing noise in the frequency domain will cover other smaller frequency components, and the vibration signal frequency obtained won't be accurate any more. Therefore, for the purpose of detecting other smaller frequency components, another way is required to be found to filter out the large frequency components from signals. The small frequency components can't be worked out by directly deleting large frequency components from the frequency domain due to the irreversible NDFT process. Thus, it's necessary to find a way to filter out the large frequency components from the time domain. Based on the comparison and analysis, the single frequency IIR notch filter will be selected to process in the frequency spectrum analysis module of the \( \phi \)-OTDR system.

3. Analysis and Design of Single Frequency IIR Notch Filter in the \( \phi \)-OTDR System

3.1. Principle of Single Frequency IIR Notch Filter

The single frequency IIR notch filter belongs to the digital filter, and its transfer function is zero only at a certain frequency point, while the transfer function is 1 at other frequency points. Therefore, it's also known as point-stop filter. The transfer function of single frequency IIR notch filter can be expressed as[1]:

\[ H(z^{-1}) = b_0 \frac{1 + az^{-1} + z^{-2}}{1 + a \rho z^{-1} + \rho^2 z^{-2}} \quad (3) \]

Where, \( b_0 \) represents the gain coefficient, \( \rho \) represents the polar radius, and \( \omega_0 \) parameters are related to the notch frequency.

It can be seen from the definition of the notch filter that its transfer function has and only has one null point on the unit circle of the convergence domain of Z transformation, and the molecule part is extracted from Formula (3), with the following formula:

\[ Y(z^{-1}) = 1 + az^{-1} + z^{-2} \quad (4) \]

The stop-band angular frequency is set as \( \omega_0 \), namely, when \( z = \exp(j\omega_0 T_s) \) is zero, the relation \( a \) with the notch frequency \( \omega_0 \) can be obtained as:

\[ a = -2\cos(\omega_0 T_s) \quad (5) \]

where, \( T_s \) is the sampling period.

The frequency at other positions of the notch filter transfer function approximately equals to 1 except for the frequency points, and it shall ensure that other frequencies will not be affected by the notch filter in addition to the notch frequency. To meet this characteristic, the polar radius \( \rho \) must be a
constant which is close to 1 but a little bit less than 1. The closer $\rho$ is to 1, the closer the pole of the transfer function is to the unit circle, and the steeper the filter transfer function obtained is at the position of notch frequency, as shown in Figure 2:

$$\rho = 0.98$$

$$\rho = 0.92$$

Fig.2. Relationship between the size of the value and the notch width

In the figure above, the notch frequency is 200Hz, when $\rho$ are 0.98 and 0.92 respectively, $b_\phi = \rho$ it can be seen that the closer $\rho$ is to 1, the narrower the notch is. When the filter transfer function is 0 at 200Hz, while the other frequencies outside of the notch frequency is 1, it will only filter out the target frequency when filtering, and it hardly has any influence on other frequencies.

3.2. Design of Notch Filter

The notch function is of the periodicity in the frequency domain. To ensure that there is only one notch stop-band in the response frequency band of the $\varphi$-OTDR system, the sampling frequency of the notch filter shall be greater than or equal to 2 times the system response bandwidth $B$, and the parameter $a$ value at this time can be expressed as:

$$a = -2 \cos(2\pi \frac{f_0}{2B}) = -2 \cos(\pi \frac{f_0}{B})$$

(6)

When $\rho = 0.98$, and the bandwidth $B=50$KHz, $f_0=30$KHz, $a = 0.618$, after putting them into the Formula (3), the transfer function graph of the notch filter will be obtained as shown in Figure 3.
Generally, the stopband of the filter is required to be narrower at low frequencies of the signal and wider at high frequencies in the practical application. Therefore, the frequency window of the filter stopband has to be adaptively adjusted as the magnitude of the center frequency. It is known that the width of the notch stopband is related to the value of $\rho$, and this width is very sensitive to the change of the $\rho$ value at $\rho \to 1$. For convenience, the independent variable $x$ is added to express the value of $\rho$ in the exponential form:

$$\rho = 1 - 10^{-x} \quad (7)$$

By changing the value of $x$, you can adjust the size of $\rho$ and then change the stop-band width of the notch filter $f_k$. The relation between $x$ and the stop-band width of the notch filter $f_k$ is as shown in Figure 4:

![Fig.3 Notch filter transfer function image at B=50KHz](image1)

Fig.3 Notch filter transfer function image at B=50KHz

![Fig.4. Relationship between x and the stopband width of the trap](image2)

Fig.4. Relationship between $x$ and the stopband width of the trap
The expression formula $x = g(f_k)$ of $x$ and $f_k$ can be got by making a fitted curve. Suppose the specific value $\eta = f_k/f_0$ between the stop-band width $f_k$ and the center frequency $f_0$, and in this way, the formula for $\rho$ can be written as:

$$\rho = 1 - 10^{g(\eta f_0)}$$  \hspace{1cm} (8)

As long as the value of $\eta$ is given, $\rho$ will be subject to self-adapting adjustment with the center frequency $f_0$, and maintains the specific value between $f_k$ and $f_0$ constant forever.

As can be seen from the above analysis, when $f_0$ and $B$ are known, parameters $\alpha$ and $\rho$ can be got through formulas (6) and (8). Based on this, the transfer function of z domain of Formula (5) can be converted to power series expansion form:

$$H(z) = \sum_{n=0}^{\infty} c_n z^{-n}$$ \hspace{1cm} (9)

The number of terms of power series in the above formula is infinite. In practice, only by computing impulse sequence of limited items, it can reach the filtering effect. (a) and (b) in Figure 5 are respectively the frequency domain images of the notch filter at Item 50 and Item 100.

![Fig.5. Image of notch filter transfer function represented by power series polynomial](image)

(a) The polynomial term is 50  \hspace{1cm} (b) The polynomial term is 100

It can be seen, within the allowable error, the operational requirements are basically met as long as expanded to Item 100. Figure 6 is the enlarged image at the notch tip of filter in Figure 5 (b), and as can be seen, the notch filter at this time equivalently zooms out some frequency component of the signal to 0.1 times of the original.
Fig. 6. The notch filter transfer function tip magnified image with a polynomial term of 100

Formula (9) can be converted to the expression formula of the impulse sequence of the time domain:

\[ h(n) = \sum_{n=0}^{\infty} c_n \delta(t - nT) \]  

Where, \( \delta(t) \) is the impulse function, taking the first 100 items, and by fitting the optimum curve of \( h(n) \), you can get a continuous function \( h(t) \), as shown in Figure 7. When the non-uniform sampling signal is known, input the sampling time of the signal to \( h(t) \), you can get the pulse polynomial function of the corresponding non-uniform sampling:

\[ h(k) = \sum_{k=0}^{60} c_k \delta(t_k) \]  

Making convolution computation by using the above formula and the non-uniform sampling signal, we can filter out the target frequency from the time domain.

Fig. 7 Time domain continuous function image of notch filter

In the spectrum analysis module, find out the big frequency component in the input data and filter out it by IIR notch filter, then find out new large frequency component and filter out again, repeat this process, until there is no large frequency component in the frequency spectrum. All the large frequency components found are input into the vibration identification module and output as the
vibration spectrum information. By recording the vibration spectrum information input at several times, we can draw the time-frequency image and compare it to the database, then judge the vibration type and output judgment results.

4. Simulation Verification
Simulate and verify the effectiveness of single-frequency IIR notching method, suppose the vibration signal $s(T)$, and

$$s(t) = \cos(2\pi f_0 t) + 0.1\cos(2\pi f_1 t) + 0.01\cos(2\pi f_2 t) + \sigma(t)$$

(12)

Where, $f_0 = 5\text{kHz}$, $f_1 = 20\text{kHz}$, $f_2 = 40\text{kHz}$ and $\sigma(t)$ are white gaussian noises, and the signal-to-noise ratio between such noise $f_0$ and signal component is 10dB, execute non-uniform sampling (sampling interval time $[2\text{ms}, 4\text{ms}]$ and meeting uniform distribution) to $s(t)$, and get 600 sampling points, the image of frequency domain drawn by NDFT is as shown in Figure 8:

![Fig.8 Original signal spectrum without notch filtering](image)

It is known that the original vibration signal is composed of three frequency components, but only the frequency spectrum $f_0$ is displayed in the figure. At this time, $f_1$ and $f_2$ are covered up by spectrum aliasing noise, and we can filter out the frequency $f_0$ by the notch filter. It is known that the system bandwidth $B = 50\text{KHz}$, putting it into the formula (4.4.6), we can get $a = -1.902$. Suppose the specific value $\eta = 0.1$ and put it into Formula (8), we can obtain $\rho = 0.998$. To ensure the passband of filter is equivalent to 1, let $b_0 = \rho$. Make the signal be subject to notch processing by the designed transfer functions, we can get the frequency spectrum as the processed signal is subject to NDFT, as shown in Figure 9.
We can see \( f_1 \), there appears maximum value at the frequency position in the new frequency spectrum, showing that through the notch filter, we can realize frequency solution of small signal component in vibration signal. However, we still can see spectral line in the frequency spectrogram \( f_0 \), this is because the number of items of the power series polynomial of the notch filter can only be took to 100, so we cannot filter out the signal completely but only zoom out the signal frequency spectrum to 0.1 times of the original, which is just the amplitude range of the signal component with a frequency of \( f_1 \), therefore, two spectral lines can be seen in the frequency spectrogram. At this time, the frequency component \( f_2 \) still cannot be seen. Design the notch filter to filter out \( f_0 \) and \( f_1 \) frequencies, and calculate the frequency spectrum of the derived signal, as shown in Figure 10:

![Signal spectrum after two notch filtering](image)

Fig.10 Signal spectrum after two notch filtering

We can see from the above figure, the spectral line of \( f_2 \) does not show up, as the signal is added with noise, the signal-to-noise ratio of such noise and \( f_2 \) frequency component is -10dB, and the frequency component \( f_2 \) is drowned out by the noise. Therefore, for especially small signal component, it cannot be detected temporarily by the designed IIR notch filter.

5. Conclusion
The paper designs a single-frequency IIR notch filter applying to non-uniform sampling \( \varphi \)-OTDR system, simulates and verifies the performance of the single-frequency IIR notch filter. The results
show that, the application of the single-frequency IIR notch filter can successfully obtain the small signal component of the non-uniform sampling signal. However, it is difficult to detect the signal component for especially small signal component. Later, the notch filter will be further optimized with the combination of corresponding intelligent algorithms, to improve the monitoring capability of the submarine cable communication system by mutual combination with φ-OTDR.

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