On Soft Breaking and CP Phases in the Supersymmetric Standard Model

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ABSTRACT

We consider a class of N=1 supersymmetric extensions of the Standard Model in which the soft breaking sector is CP conserving at the GUT scale. We study the question of whether the presence of explicit CP violation in the Yukawa sector of the theory induces through renormalization effects CP violating phases in the soft terms, which could lead to observable effects. A clear pattern appears in the structure of phases in the soft sector. In particular, the inclusion of intergenerational mixing induces large phases in the flavour mixing entries of the trilinear soft breaking terms, whereas the diagonal entries remain real. A mechanism is proposed for generating through chargino exchange a contribution to the neutron electric dipole moment which can be a few orders of magnitude larger that that of the Standard Model, although still out of reach of experimental tests. We comment on the possible relevance of these phases for baryogenesis at the weak scale in minimal supersymmetric scenarios, recently considered in the literature.
Introduction

In this letter we reconsider the problem of CP violation in supersymmetric extensions of the standard Glashow-Weinberg-Salam model (SM) of the electroweak interactions. In particular, we want to address the question of whether the presence of a CP violating phase ($\delta_{KM}$) in the Yukawa sector of the supersymmetric theory may induce, through the running of the relevant parameters from the GUT scale to the Fermi scale, CP violation in the soft breaking sector.

In general, it is well known that additional phases may appear in the minimal supersymmetric version of the SM (MSSM) due to the complexity of the soft SUSY breaking couplings. On the other hand, it is also well known that the presence of these extra phases (collectively $\delta_{soft}$) would induce a 1-loop contribution to the electric dipole moment (EDM) of the neutron, which for squarks and gluino masses of $O(100 \text{ GeV})$ is too large and requires $\delta_{soft} < 0.01$. From this point of view it may be natural demanding the absence of these phases altogether by assuming that the soft breaking sector, as derived from the flat limit ($M_{Planck} \rightarrow \infty$) of an underlying supergravity theory, is CP conserving. However, the presence of an explicit CP violation in the Yukawa sector of the “effective” theory at the GUT scale may reintroduce some CP violation in the soft breaking sector at the Fermi scale, thus posing again the question of its relevance for present day CP odd observables.

As a matter of fact, the presence of a small explicit CP violation $\delta_{soft} \simeq 10^{-5} - 10^{-6}$ in the trilinear component of the soft breaking potential has been advocated by the authors of ref. [2] in order to trigger the baryon–antibaryon asymmetry in the early universe in a finite temperature MSSM scenario. It is worth stressing that the correct sign of the baryon–antibaryon asymmetry depends crucially on the sign of the explicit soft breaking phase. It is therefore interesting to study whether the $\delta_{KM}$ induced phases previously mentioned have the correct size and sign to support such a scenario.

Our main conclusion is that within the simplified approach of ref. [2], where flavour mixing in the effective potential is neglected, no appreciable phases are induced in the relevant soft breaking parameters. It turns out, however, that due to the presence of flavour mixing large CP violating phases may be induced in the off–diagonal components of the trilinear soft breaking couplings. We study the implications of such a large phases for the EDM of the neutron and find that they induce a contribution consistent with the present upper bound, although it can be as much as four orders of magnitude larger than the SM contribution to the elementary EDM of the quarks. The relevance of these off-diagonal components for the analysis of ref. [2] deserves a separate discussion.
The Minimal Supersymmetric Extension of the Standard Model

We are here interested in the class of N=1 supergravity based minimal supersymmetric extensions of the SM, in which the spontaneous breaking of $SU(2) \times U(1)$ gauge symmetry is achieved via radiative corrections (RMSSM) \cite{3}. These models, implemented in a Grand Unified (GUT) scenario, exhibit the least number of free parameters and are consequently the most predictive ones.

The SUSY-GUT theory is obtained from the $M_{\text{Planck}} \to \infty$ limit of spontaneously broken N=1 supergravity \cite{4}; the flat limit leaves a globally supersymmetric lagrangian, corresponding to the supersymmetrization of the chosen GUT model (minimally an $SU(5)$ GUT) which is explicitly broken by soft terms \cite{5}. After integration of the heavy GUT fields the soft terms can be casted in the following form:

\[
- \sum_{ij} m_{ij}^2 z_i^* z_j - (f_{\Gamma^A}(z) + B\mu h_1 h_2 - \sum_{\alpha} \frac{M_\alpha}{2} \lambda_\alpha \lambda_\alpha + h.c.)
\]  

(1)

where $z_i$ denote all scalar fields present in the theory. The first term is a mass term common to all the scalars, whereas $f_{\Gamma}$ is the part of the superpotential that extends the standard Yukawa potential

\[
f_{\Gamma} = H_1^0 D_L^t \Gamma_D D_L^c + H_1^0 E_L^t \Gamma_E E_L^c + H_2^0 U_L^t \Gamma_U U_L^c
\]

\[
- (H_1^- U_L^t \Gamma_D D_L^c + H_1^- E_L^t \Gamma_E E_L^c + H_2^+ D_L^t \Gamma_U U_L^c)
\]  

(2)

where the three $\Gamma_x$ ($x = U, D, E$) are $3 \times 3$ matrices in flavour space, and the upper index $t$ indicates trasposition in flavour space. Notice that in eq. (1) the dimensionless couplings $\Gamma$ have been replaced with the massive parameters $\Gamma^A$, and the superfields with their scalar counterparts. The third term of eq. (2) arises as the scalar counterpart of the term

\[
\mu H_1 H_2 = \mu (H_1^0 H_2^0 - H_1^+ H_2^-)
\]  

(3)

present in the superpotential, while the last one is a mass term for the gauginos.

If one assumes having a flat Kähler metric, the form of the soft supersymmetry breaking terms turns out to be quite simple at the scale of local supersymmetry breaking, that we assume to be the SUSY-GUT scale required by gauge couplings unification, namely $M_X \simeq 3 \cdot 10^{16}$ GeV. In fact at that scale we have:

\[
\Gamma^A_x = A_G \cdot \Gamma_x \quad x = D, \ E, \ U
\]  

(4)

where $A_G$ is a massive universal coefficient (henceforth the subindex $G$ denotes GUT scale quantities). In addition, each scalar in the theory gets the same mass term

\[
m_G^2 z_i^* z_i
\]  

(5)
This form ensures the absence of large flavour violating effects in neutral currents at the low energy scale. We will also assume that the three gaugino masses are equal at this unification scale:

\[ M_\alpha = M_G \quad \alpha = 1, 2, 3 \]  

Following a widely used notation we will distinguish the superpartner fields using a tilde (for instance \( \tilde{u}_L \) is the scalar partner of the left up quark \( u_L \), both belonging to the superfield \( U_L \)).

**Complex Parameters in the RMSSM**

We have already mentioned that the 4 parameters \( A_G, B_G, M_G, \mu_G \) can be a-priori complex parameters. In fact, by a \( R \)–rotation, with \( R \)–charges \( Q_R = 1 \) for lepton and quark superfields and \( Q_R = 0 \) for the vector and the Higgs superfields, the gaugino mass parameter \( M_G \) can be made real; moreover, multiplying by a common phase the 2 Higgs superfields (with opposite hypercharge) \( B_G \cdot \mu_G \) becomes real as well. We conclude that, in addition to the usual Kobayashi-Maskawa (KM) phase \[ 6 \], there are at most two tipically supersymmetric phases that are physically relevant, say

\[
\arg(A_G) \quad \text{and} \quad \arg(B_G) = -\arg(\mu_G).
\]

These two parameters have an important impact in the phenomenology of CP violation. They can induce an electric dipole moment of the quarks and leptons at the 1-loop level through diagrams of the type shown in fig. 1 (let us recall that in the SM the elementary edm of quarks arises at the third loop \[ 7 \]). If the masses of the particles running in the loop are \( O(100) \) GeV the EDM of the neutron is predicted to be

\[
d_{n}^{SUSY} = O(10^{-23}) \sin \delta_{soft} \ e \ cm
\]

where \( \delta_{soft} \) is a typical SUSY phase (say \( \arg(A) \)). This prediction has to be compared with that obtained in the Standard Model from the elementary EDM of the up and down quarks (for a review see ref. \[ 7 \]) :

\[
d_{n}^{SM}(\text{quarks}) = O(10^{-34}) \ e \ cm.
\]

As a matter of fact, it is likely that the neutron EDM is dominated by long-distance (LD) effects \[ 8 \], which lead to:

\[
d_{n}^{SM}(LD) = O(10^{-32}) \ e \ cm.
\]

Present experiments give the upper bound \[ 9 \]:

\[
d_{n}^{EXP} < 12 \cdot 10^{-26} \ e \ cm
\]
which, as already mentioned, excludes values of the supersymmetric phases larger than $10^{-2}$ (or else requires squark masses to be at the $TeV$ scale \cite{11}); A measure of a non-zero value of the EDM of the neutron in the next generation of experiments would certainly be a signal of new physics, among which a supersymmetric scenario with superpartners at the reach of the future hadron colliders.

Running of the Soft Breaking Parameters

If one believes that squark and gluinos are just around the corner, then a most conservative and natural assumption for CP violation in the soft sector is that the two phases in eq. (7) vanish identically at the GUT scale, due to CP conservation in the sector responsible for SUSY breaking. However in this case, attention must be paid to the KM phase, explicitly present in the model, which may induce analogous phases in the soft breaking sector through the renormalization of the soft breaking parameters.

The complete set of 1-loop renormalization group equation (RGE) for the SUSY model here considered can be found for instance in ref. \cite{11} and in explicit matrix form in ref. \cite{12}. From inspection of the relevant RGE one easily realizes that the gaugino masses do not change their phase during the evolution, so that, within our hypothesis, they remain real; an analogous conclusion holds for the parameter $\mu$, which depends on the matrices $\Gamma$ only through the trace of the hermitian and non-negative combinations

$$\alpha_x \equiv \frac{1}{4\pi} \Gamma_x \cdot \Gamma_x^\dagger \quad x = D, E \text{ or } U. \quad (12)$$

In order to discuss the evolution of the parameters $B$ and $\Gamma_x^A$ we find convenient to define the matrices $A_x$ in the following way:

$$A_x \equiv \Gamma_x^A \cdot \Gamma_x^{-1} \quad x = D, E, \text{ or } U \quad (13)$$

Defining $\dot{A} \equiv dA/dt$, with

$$t \equiv \frac{1}{4\pi} \log(\frac{Q}{Q_0}) \quad (14)$$
we obtain the following RGE:

\[
\begin{align*}
\dot{A}_E &= 2 \left( 3\alpha_2 M_2 + 3\alpha_1 M_1 \right) I \\
&+ 2 \text{Tr}(A_E A_E + 3A_D \alpha_D) I \\
&+ 5\alpha_E A_E + A_E \alpha_E \\
\dot{A}_U &= 2 \left( \frac{16}{3} \alpha_3 M_3 + 3\alpha_2 M_2 + \frac{13}{9} \alpha_1 M_1 \right) I \\
&+ 2 \text{Tr}(3A_U \alpha_U) I \\
&+ 5\alpha_U A_U + A_U \alpha_U + \alpha_D A_D - A_U \alpha_D + 2A_D \alpha_D \\
\dot{A}_D &= 2 \left( \frac{16}{3} \alpha_3 M_3 + 3\alpha_2 M_2 + \frac{7}{9} \alpha_1 M_1 \right) I \\
&+ 2 \text{Tr}(A_E \alpha_E + 3A_D \alpha_D) I \\
&+ 5\alpha_D A_D + A_D \alpha_D + \alpha_U A_U - A_D \alpha_U + 2A_U \alpha_U \\
\dot{B} &= 2 \left( 3\alpha_2 M_2 + \alpha_1 M_1 \right) \\
&+ 2 \text{Tr}(3A_U \alpha_U) I \\
&+ 2 \text{Tr}(A_E \alpha_E + 3A_D \alpha_D + 3A_U \alpha_U)
\end{align*}
\] (15)

The \(A_x\) are a-priori generic \(3 \times 3\) matrices, and the form of the initial conditions is given in eqs. (4–6).

Notice that one can study independently the two cases a) \(M_G = 0, A_G \neq 0\) and b) \(M_G \neq 0, A_G = 0\), and that the GUT value for \(B\), namely \(B_G\), is always additive.

In order to analyze the evolution of the parameters we resorted to a numerical study of this system of RGE, coupled to the RGE for a) the gauge coupling constants \(\alpha_i\), \(i = 1, 2\) or 3, b) the gaugino masses \(M_\alpha\) and c) the “Yukawa couplings” \(\alpha_x, x = U, D, E\).

The initial values of the matrices \(\alpha_x\) at the \(M_Z\) scale are calculable once the value of \(\tan \beta\), the value of masses of the quarks and of the lepton and the Kobayashi-Maskawa matrix are assigned. In fact, by writing the matrices \(\Gamma_x\) in biunitary form,

\[
\Gamma_x = L_x^t \gamma_x R_x
\] (16)

where \(\gamma_x\) are diagonal non-negative matrices, and making unitary redefinitions of the quark and lepton superfields, one finds that the parameters in eq. (2) can be chosen to be:

\[
\begin{align*}
\Gamma_E &= \gamma_E \\
\Gamma_D &= \gamma_D \\
\Gamma_U &= K^t \gamma_U
\end{align*}
\] (17)

where \(K\) is the \(3 \times 3\) Kobayashi-Maskawa matrix (down-diagonal basis).

Denoting by \(v_i\) the two vacuum expectation values of the Higgs fields, \(i.e. \langle h_i^0 \rangle = v_i\) \((i = 1, 2)\), the matrices \(\gamma_x\) are related to the (diagonal) mass matrices for leptons and
quarks as following

\[
\begin{align*}
\gamma_E &= \frac{M_E}{v_1} \\
\gamma_D &= \frac{M_D}{v_1} \\
\gamma_U &= \frac{M_U}{v_2}
\end{align*}
\]

This basis is very useful when performing the RGE analysis since the initial conditions are given in terms of masses and KM mixings and it leaves the $SU(2)_L$ symmetry explicit, having applied the same unitary rotation to the $U$ and $D$ superfields.

To switch to the superfield basis in which the terms of the superpotential with the neutral Higgs superfields are flavour-diagonal (i.e. the quark mass eigenstate basis), we just have to redefine the $U_L$ flavour multiplet of superfields as

\[
U_L \rightarrow K^\dagger U_L .
\]

In this basis the mixing matrix $K$ appears

i) in the interactions involving the charged vector superfields

ii) in the interactions of the charged Higgs superfields in eq. (2),

iii) and in the analogous terms for the charged Higgs fields in eq. (1).

Our numerical results will be shown in this basis.

In order to obtain the numerical solution of the system of RGE we used the following set of electroweak input parameters:

1) $\tan \beta = v_2/v_1$ in the range $1 \div 40$,

2) the $\overline{MS}$ running quark masses at $m_Z$ (for a recent review see ref. [13]); in particular, we tested for $m_{top}$ in the range $100 \div 200 \, GeV$.

3) the values of the three KM angles and phase consistent with the results of the analysis reported in ref. [14].

At the GUT scale we spanned values for the SUSY soft breaking parameters which lead to a consistent SUSY mass spectrum at the electroweak scale.

We wrote the set of the relevant RGE in matrix form and left to a program of symbolic manipulation the task to expand them in vector form, suited for using the variable-step NAG algorithm for the solution of ordinary differential equations. The results of our
numerical analysis revealed that, for any choice of the parameters, the amount of the imaginary part of $B$ and of the diagonal elements in $A_x$, induced at the $M_Z$ mass scale is completely negligible ($\text{Im}\{A_{ii}\}/\text{Re}\{A_{ii}\} < 10^{-20}$), whereas the off-diagonal elements of $A_U$ and $A_D$ can have large phases (although the matrices $A_U$ and $A_D$ remain, up to 1 part per $10^{-3} - 10^{-4}$, hermitian).

The following numerical example illustrates the typical patterns we found. Consider for instance $\tan \beta = 10$ and $m_t = 130$ GeV. Then the values of the Yukawa matrices $\gamma_x$ at the $M_Z$ scale read

$$
\gamma_U = \text{diag}(0.14 \times 10^{-4}, 0.40 \times 10^{-2}, 0.75) \\
\gamma_D = \text{diag}(0.29 \times 10^{-2}, 0.55 \times 10^{-2}, 0.17) \\
\gamma_E = \text{diag}(0.29 \times 10^{-4}, 0.75 \times 10^{-2}, 0.98 \times 10^{-1})
$$

while for the KM angles and phase let us take

$$(\theta_{12}, \theta_{23}, \theta_{13}, \delta_{KM}) = (0.221, 0.043, 0.005, 0.86)$$

where we tried to maximize the CP violation effect from the KM matrix through large values of the mixing between the first and the third family and $\delta_{KM}$ of order 1, according to the present experimental constraints (see for instance ref. [14]).

Finally, for $A_G = 1$ GeV, $M_G = 0$ (notice that in this case the RGE solutions for $A_x$ scale as $A_G$), and $B_G$ arbitrary we obtain

$$
A_U = \begin{pmatrix}
0.78, & (-1.3 - i0.7)10^{-6}, & (-1.0 - i1.7)10^{-5} \\
(-1.3 + i0.7)10^{-6}, & 0.78, & -1.7 \times 10^{-4} \\
(-1.0 + i1.7)10^{-5}, & -1.7 \times 10^{-4}, & 0.56
\end{pmatrix}
$$

$$
A_D = \begin{pmatrix}
0.99, & (2.2 - i1.3)10^{-5}, & (-5.1 + i3.1)10^{-4} \\
(2.2 + i1.3)10^{-5}, & 0.99, & (3.1 + i0.1)10^{-3} \\
(-5.1 - i3.1)10^{-4}, & (3.1 - i0.1)10^{-3}, & 0.90
\end{pmatrix}
$$

$$
A_E = \text{diag}(0.99, 0.99, 0.98) \\
B = -0.12 + B_G
$$

Increasing the top mass up to 200 GeV affects little the elements of $A_U$, while it modifies by factors 2 – 3 the entries of $A_D$. In addition, due to the $1/\cos \beta$ ($1/\sin \beta$) dependence of the down-quark (up-quark) Yukawa couplings, the off-diagonal elements of $A_U$ increase roughly as $(\tan \beta)^2$, while those of $A_D$ remain roughly constant. The two features above are in fact related and may be qualitatively understood by analyzing the interplay between the down and up Yukawa couplings in the RGE for $A_U$ and $A_D$, (eqs. (22)).

As mentioned before, the matrices $A_U$ and $A_D$ turn out to be quasi-hermitian (non-hermiticity appears at the 1 part per $10^{-3} - 10^{-4}$ level in the off diagonal elements and we
have neglected it). Notice also that the matrix $A_E$ remains in any case diagonal, because of the absence of any source of lepton flavour violation.

A consequence of the “hermiticity” of the matrices $A_x$ is the reality of $B$ (up to a part in $10^{-20}$). In fact, the evolution of $B$ depends on the trace of the product of two hermitian matrices ($\alpha_y \cdot A_x$), which is real.

The fact that the diagonal entries of $A_x$ are to an extremely good approximation real bears important implications for the phenomenology of SUSY induced CP violation, which we will shortly discuss. This feature can be understood as follows. Suppose decomposing the matrix $A_x$ in its hermitian and antihermitian parts $A_x = A'_x + A''_x$ respectively. It is then evident that the boundary conditions for $A_x$ at the GUT scale, eq. (3), imply $A''_{x,G} = 0$, so that the antihermitian part can be generated only through $[A'_x, \alpha_i]$; but, due to the diagonality of $A'_{x,G}$ also the commutator is zero at the GUT scale. Moreover, to get a non-zero value of $(A''_x)_{ii}$ from this commutator one readily verifies that a certain amount of non-diagonality in $(A'_x)_{ij}$, $i \neq j$, is also needed, which is only induced radiatively.

The Neutron Electric Dipole Moment

We shall now consider some consequences of the previous results. Since both $\mu$ and $B$ remain real, also the mass parameter $m_3^2 = -B\mu$ responsible for the mixing of the two scalar higgses remains real at the end of the running. Analogously the gaugino mass matrices, i.e. the chargino and the neutralino mass matrices, are real; this fact implies that only real orthogonal matrices are needed to reach the basis of mass eigenstates. Also the sleptons mass matrices are real (and diagonal), so that the CP violating effects in the leptonic sector, such as the electron or muon EDMs are zero at 1-loop level in the RMSSM as in the SM.

Let us now consider the role the complex off-diagonal parameters in the matrices $A_x$ for the squark mass matrices, and begin our discussion with the up-squark matrix. The $6 \times 6$ scalar quark mass matrix can be written in terms of $3 \times 3$ submatrices as follows:

$$M_{\tilde{u}}^2 = \begin{pmatrix} M_{\tilde{u},LL}^2 & M_{\tilde{u},LR}^2 \\ M_{\tilde{u},RL}^2 & M_{\tilde{u},RR}^2 \end{pmatrix}$$

The two $LL$ and $RR$ blocks are determined evolving the scalar mass parameters in eq. (5) with the appropriate RGE; the parameters $A_x$ enter only the $LR$ (and $RL$) blocks

$$M_{\tilde{u},LR}^2 = (\Gamma_A^U v_1 + \mu \cdot v_2 \cdot I) \cdot \gamma_U$$

$$= (A_U + \frac{\mu}{\tan \beta} \cdot I) \cdot M_U$$

(24)
The basis of squark mass eigenstates is reached using the unitary rotation \( S_\tilde{u} \), defined by:

\[
S_\tilde{u} M^2_\tilde{u} S^\dagger_\tilde{u} = \text{diag}(m^2_{\tilde{u}_k})
\] (25)

It is useful to split \( S_\tilde{u} \) into two \( 6 \times 3 \) submatrices

\[
S_\tilde{u} \equiv (S_{\tilde{u},L}, S_{\tilde{u},R})
\] (26)

which relate the the scalar partners of the left and right-handed quarks to the scalar mass eigenstates.

Notice that in the case of the down squarks, since \( M^2_{d,LR} \) is proportional to \( M_d \), the unitary matrix \( S_{\tilde{d}} \) is block diagonal up to \( O(M_D/m_{\text{SUSY}}) \); as we will see this fact suppresses the up-quark contribution to the electric dipole moment of the neutron.

Let us finally come to the neutron EDM. Using a non-relativistic quark model, the elementary EDM of the quarks \( d_u \) and \( d_d \) are related to the dipole of the neutron according to:

\[
d_n = \frac{4}{3} d_d - \frac{1}{3} d_u.
\] (27)

Let us consider first the d quark. The dipole can be computed from the imaginary part of the amplitude \( A^{dd\gamma}_{LR} \) for the chirality-flipping \( d_R \rightarrow d_L + \gamma \) process according to

\[
d_d = -\text{Im}(A^{dd\gamma}_{LR})
\] (28)

In the SUSY model there are no 1-loop contributions to the neutron EDM induced by \( W^+ \) or \( H^+ \) exchange. The only possibility left is to examine typically SUSY diagrams with squarks running in the loop (together with gluinos, charginos or neutralinos), and resort to that component of the amplitude in which the needed helicity flip is realized in the loop. From a qualitative mass insertion analysis of the relevant diagrams it is easy to convince oneself that a non vanishing (and possibly complex) LR mass insertion in the squark line is needed. In fact, when gluinos (or neutralinos) are considered, the down-quark EDM turns out to be proportional at the leading order to \( \text{Im}\{(A_D)_{11}\} \), which is zero in the scenario here considered. One might circumvent this problem by invoking flavour changing (FC) effects at the gluino-quark-squark vertex so to involve complex off-diagonal entries in the soft insertion. However these FC effects are of radiative origin, and for squark and gluino masses heavier than 100 GeV turn out to give quite a small contribution to the dipole amplitude (see ref. [12]).

We are therefore lead to consider chargino exchange, with up-squarks running in the loop. Analogously to the gluino case if one considers the “diagonal” exchange of \( \tilde{u}_{u,c,t} \)
no effect arises due to the reality of the diagonal entries of the trilinear soft breaking parameters. On the other hand, there exist now also “non-diagonal” components of the amplitude at the leading order (see fig. 1) which are proportional to off-diagonal elements of the \( A_U \) matrix and to complex combinations of the relevant KM mixings. In particular we may expect that the dominant contribution arises through the exchange of the “higgsino” component of the chargino field, via the quark-squark “flavour-chain”

\[
d_R \rightarrow (\tilde{u}_L, \tilde{c}_L) \rightarrow \tilde{t}_R \rightarrow d_L
\]

since we can take advantage of the presence of the large top Yukawa coupling. For large \( \tan \beta \), then, also the presence of the down Yukawa coupling becomes important as an enhancement factor.

More concretely, using the interaction lagrangian (see for instance the appendix of ref. [12])

\[
\mathcal{L}_{\tilde{\chi} \tilde{u}} = \overline{\tilde{\chi}^-} \tilde{u}^\dagger \left[ P_L(-V_{a1}\tilde{u}_a,Lg + V_{a2}\tilde{u}_a,R\gamma_u)K + P_R(U_{a2}\tilde{u}_a,LK\gamma_d) \right] d + h.c.
\]

where \( U \) and \( V \) are the \( 2 \times 2 \) orthogonal matrices responsible for the diagonalization of the chargino (\( \tilde{\chi}^- \)) mass matrix (see for instance ref. [15]), we find that the contribute to the d-quark EDM is can be written as

\[
d_d = \frac{1}{(4\pi)^2} \sum_{a=1}^{2} \frac{1}{m_{\tilde{\chi}_a}^2} V_{a2} U_{a2} \sum_{k=1}^{6} \mathrm{Im} \left[ K^{\dagger} \gamma_u \left( S_{\tilde{u}_a,R} F \left( \frac{m_{\tilde{u}_a}^2}{m_{\tilde{\chi}_a}^2} \right) S_{\tilde{\chi}_a,L} \right) K \gamma_d \right]_{11} e \text{ cm}
\]

The function \( F(x) \) is given by

\[
F(x) = \frac{1}{6(1-x)^3} (5 - 12x + 7x^2 + 2x(2 - 3x) \ln(x))
\]

A formula analogous to eq. (31) holds for \( d_u \). However, the appearance of the down quark mass matrix in \( M^2_{d,L,R} \) (compare with eq. (24)) suppresses the EDM of the up-quark compared to that of the d-quark.

To numerically estimate this effect we considered the lightest chargino to be close to the present experimental limit, say \( m_{\tilde{\chi}_1} \approx 50 \text{ GeV} \), lightest squarks masses of \( O(100 \text{ GeV}) \) and maximize the amplitude in the remnant parameters. We therefore find that the SUSY contribution to the neutron EDM in this class of models generated by the elementary EDM of the quarks can be as large as

\[
d_n^{\text{SUSY}} = O\left(10^{-30}\right) \left(\frac{\tan \beta}{10}\right) e \text{ cm}
\]
where the linear dependence on $\tan \beta$ for large $\beta$ comes from the presence of the down quark Yukawa coupling (the dependence on the top mass in the studied range is weaker).

This result has to be compared with the analogous SM contribution of eq. (3). In spite of the absence of phases in the diagonal entries of the trilinear soft breaking terms, the smaller off-diagonal terms may induce an elementary quark EDM four orders of magnitude larger than that of the SM, and give a neutron EDM still two orders of magnitude larger than that induced by long distance effects, eq. (10). It is obvious, however, that this mechanism cannot explain a measurement of the neutron EDM close to the present experimental bound.

For what concerns the possible implications of these results for the analysis of ref. [2], on the generation of baryon-antibaryon asymmetry at the weak scale in a finite temperature MSSM scenario, no phases of $O(10^{-5} \sim 10^{-6})$ are induced in the diagonal (top) entries of the trilinear terms as suggested by the authors. However, the aforementioned analysis just neglects intergenerational mixing in the calculation of the effective potential. The possibility that the large phases induced in the off-diagonal entries of $A_x$ may actually become relevant and trigger the transition to a baryon dominated universe remains open. Answering this question requires a detailed analysis which is beyond the scope of the present letter.

**Figure Captions**

Figure 1. The leading SUSY contribution to the elementary EDM of the quarks in the class of models described in the paper is shown in the interaction eigenstate basis. The photon is attached in all possible ways.

**References**

[1] W. Buchmüller and D. Wyler, *Phys. Lett. B* 121 (1983) 321; F. de l’Aguila, M.B. Gavela, J.A. Grifols and A. Mendez, *Phys. Lett. B* 126 (1983) 71; J. Polchinski and M.B. Wise, *Phys. Lett. B* 125 (1983) 393; E. Franco and M. Mangano, *Phys. Lett. B* 135 (1984) 445.

[2] D. Comelli and M. Pietroni, Phys. Lett. B 306 (1993) 67; D. Comelli, M.Pietroni and A. Riotto, preprint SISSA-93-50-A.
[3] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita,  *Prog. Theor. Phys.* **68** (1982) 927;  *ibid.* **71** (1984) 413; L.E. Ibanez and G.G. Ross,  *Phys. Lett.* B **110** (1982) 214; H.P. Nilles,  *Phys. Lett.* B **118** (1982) 193; L. Alvarez-Gaumé, M. Claudson and M.B. Wise, *Nucl. Phys.* B **207** (1982) 96; L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, *Nucl. Phys.* B **221** (1983) 495.

[4] H. P. Nilles,  *Phys. Rep.* **110** (1984) 1; A.B. Lahanas and D.V. Nanopoulos,  *Phys. Rep.* **145** (1987) 1.

[5] L. Girardello, M. T. Grisaru,  *Nucl. Phys.* B **194** (1982) 65.

[6] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* 49, (1973) 652.

[7] E.P. Shabalin, Sov. Phys. Usp. 26 (1983) 297.

[8] D.V. Nanopoulos, A. Yildiz, and P.H. Cox, Phys. Lett. 87B (1979) 61; I.B. Kriplovich and A.R. Zhitnisky, Phys. Lett. 109B (1981) 490;

[9] I.S. Altarev et al., JETP Lett. 44 (1986) 460; K.F. Smith et al., Phys. Lett. B234 (1990) 2347.

[10] Y. Kizukuri and N. Oshimo, Phys. Rev. D46 (1992) 3025.

[11] N.K. Falck,  *Zeit. für Physik* C **30** (1986) 247.

[12] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, *Nucl. Phys.* B **353** (1991) 591.

[13] H. Arason et al.,  *Phys. Rev.* D **46** (1992) 3945.

[14] A.J. Buras and M.K. Harlander, in *Heavy Flavours*, Eds. A.J. Buras and M. Lindner, World Scientific, Singapore, 1992.

[15] H.E. Haber and G.L. Kane, Phys. Rep. 117 (1985) 75; J.F. Gunion and H.E. Haber, *Nucl. Phys.* B272 (1986) 1.
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