Topography Change and Unitarity in Quantum Black Hole Dynamics

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Abstract

We discuss to what extent semiclassical topology change is capable of restoring unitarity in the relaxation of perturbations of eternal black holes in thermal equilibrium. The Poincaré recurrences required by unitarity are not correctly reproduced in detail, but their effect on infinite time-averages can be mimicked by these semiclassical topological fluctuations. We also discuss the possible implications of these facts to the question of unitarity of the black hole S-matrix.¹

¹Talk delivered by JLFB at ERES2004 “Beyond General Relativity”. Miraflores de la Sierra, Madrid, September 2004.
1 Introduction

The AdS/CFT correspondence [1] provides a nonperturbative model of quantum gravity in which black holes seem to form and evaporate as standard unitary processes in quantum mechanics. In this construction, quantum gravity in a $d$-dimensional asymptotically Anti-de Sitter spacetime (AdS) of curvature radius $R$ is defined in terms of a conformal field theory (CFT) on a spatial sphere $S^{d-2}$ of radius $R$. The effective expansion parameter on the gravity side $1/N^2 \sim G_N/R^{d-2}$, maps to an appropriate large $N$ limit of the CFT. For example, for two-dimensional CFT’s $N^2$ is proportional to the central charge. When the CFT is a gauge theory and the AdS side is a string theory, $N$ is the rank of the gauge group, and the string perturbative expansion in powers of $g_s \sim 1/N$ is identified with ’t Hooft’s $1/N$ expansion in the gauge theory side.

According to this definition, the formation and evaporation of small black holes with Schwarzschild radius $R_S \ll R$, should be described by a unitary process in terms of the CFT Hamiltonian. Thus, there is no room for violations of coherence, independently of the manner in which the process is encoded in the CFT. Unfortunately, the CFT states corresponding to small black holes are hard to describe, and it remains a challenge to put the finger on the precise error in the standard semiclassical analysis [2] in that case.

Figure 1: The energy spectrum of a CFT representing AdS$_d$ quantum gravity. The spectrum is discrete on a sphere of radius $R$, with gap of order $1/R$. The asymptotic energy band of very dense “black hole” states sets in beyond energies of order $N^2/R$. The corresponding density of states is that of a conformal fixed point in $d-1$ spacetime dimensions.

For large eternal AdS black holes with Schwarzschild radius $R_S \gg R$ one may attempt to rise to the challenge, since they are thermodynamically stable and can exist in equilibrium with thermal radiation at fixed (high) temperatures $1/\beta \gg 1/R$. Indeed, the corresponding Bekenstein–Hawking entropy scales like that of $N^2$ conformal degrees of freedom at high energy,

$$ S \sim N^{\frac{2}{d-2}} (E R)^{\frac{d-4}{d-2}} \sim N^2 (R/\beta)^{d-2}. $$

Therefore, large AdS black holes with large Hawking temperature $\beta^{-1} \gg R$ describe the
leading approximation to the thermodynamical functions of the canonical CFT state

\[ \rho_\beta = \frac{e^{-\beta H}}{Z(\beta)}, \quad Z(\beta) = \text{Tr} \exp(-\beta H). \] (2)

This suggests that we can test the semiclassical unitarity argument by careful analysis of slight departures from thermal equilibrium, rather than studying a complete evaporation instability in the vacuum. Ref. [3] proposes to look at the very long time structure of correlators of the form

\[ G(t) = \text{Tr} \left[ \rho A(t) A(0) \right], \] (3)

for appropriate Hermitian operators \( A \). In this expression, \( \rho \) denotes the density matrix of the initial state. In the semiclassical approximation, one expects these correlators to decay as \( \exp(-\Gamma t) \) with \( \Gamma \sim \beta^{-1} \). However, because the CFT lives in finite volume, the spectrum is actually discrete (c.f. Fig 1), and the correlator must show nontrivial long time structure in the form of Poincaré recurrences, in particular it does not vanish (see [4, 5]). This result, which is straightforward from the boundary theory point of view, has far reaching consequences as far as the bulk physics is concerned.

Figure 2: A detailed analysis of dissipation of fluctuations in a finite thermal system can reveal the effect of large quantum fluctuations in which a black hole turns into thermal radiation and vice versa. In the semiclassical approximation to quantum gravity, these processes are represented by a coherent sum over saddle points of different topology. In the case at hand we can use AdS space as an effective finite-volume box.

Hence, the non-vanishing of \( G(t) \) as \( t \to \infty \) can be used as a criterion for unitarity preservation beyond the semiclassical approximation. This argument can be made more explicit by checking the effect of coherence loss on the long-time behaviour of \( G(t) \). Using the results of [6] one can simulate the required decoherence by coupling an ordinary quantum mechanical system to a random classical noise. It is then shown in [7] that
this random noise forces \( G(t) \) to decay exponentially for large \( t \), despite having a discrete energy spectrum. This indicates that the long-time behaviour of correlators probes the strict quantum coherence of the bounded system.

At the same time, one would like to identify what kind of systematic corrections to the leading semiclassical approximation are capable of restoring unitarity. A proposal was made in [3] in terms of topology-changing fluctuations of the AdS background. Our purpose here is to investigate these questions and offer an explicit estimate of the instanton effects suggested in [3]. We conclude (c.f. [7]) that large topological fluctuations are unlikely to restore unitarity in full detail, although they represent a step forward. In particular, certain coarse-grained properties, such as the time averages of the correlators (3), are reproduced in order of magnitude. Related work, especially in the \( d = 3 \) case, can be found in refs. [8, 9, 10].

These considerations should also shed light on the recent proposal in [11], where topological diversity is credited with the restoration of S-matrix unitarity in black hole formation and evaporation.

2 Long-time details of thermal quasi-equilibrium

Poincaré recurrences occur in general bounded systems. Classically they follow from the compactness of available phase space, plus the preservation of the phase-space volume in time (Liouville’s theorem). Quantum mechanically, they follow from discreteness of the energy spectrum (characteristic of spatially bounded systems) and unitarity. The correlator (3)

\[
G(t) = \sum_{j,k} C_{jk} e^{i(E_j - E_k)t}, \quad \text{with} \quad C_{jk} = \sum_i \rho_{ij} A_{ki} A_{jk},
\]

defines a quasiperiodic function of time, provided the matrix elements \( C_{ij} \) are sufficiently bounded so that \( G(t) \) is well defined. After initial dissipation on a non-universal time scale \( \Gamma^{-1} \), where \( \Gamma \) measures the approximate width of the matrix elements \( C_{ij} \) in the energy basis, the correlator will show large “resurgences” when most of the relevant phases complete a whole period (c.f. Fig 3). The associated time scale is \( t_H \equiv 1/\langle \omega \rangle \), with \( \langle \omega \rangle = \langle E_i - E_j \rangle \) an average frequency in (4). We can estimate \( \langle \omega \rangle \) as \( \Gamma/\Delta n_{\Gamma} \), where \( \Delta n_{\Gamma} \) is the number of energy levels in the relevant band of width \( \Gamma \). This must be proportional to the density of levels, or the exponential of the entropy, i.e. we have

\[
t_H \sim \Gamma^{-1} e^{S(\beta)}.
\]

Following [12] we call this the Heisenberg time scale. Poincaré times can be defined in terms of quasiperiodic returns of \( G(t) \) with a given \textit{a priori} accuracy. In a sense, the Heisenberg time is the smallest possible Poincaré time.

A more quantitative, albeit more inclusive criterion can be used by defining a normalized positive correlator, \( L(t) \), satisfying \( L(0) = 1 \), and its infinite time average,

\[
L(t) \equiv \frac{|G(t)|^2}{G(0)^2}, \quad \mathcal{L} \equiv \lim_{T \to \infty} \frac{1}{T} \int_0^T dt \, L(t).
\]
The profile of $L(t)$ is sketched in Fig 3. The time average can be estimated by noticing that the graph of $L(t)$ features positive “bumps” of height $\Delta L$ and width $\Gamma$, separated a time $t_H$, so that

$$L \sim \frac{\Delta L}{\Gamma t_H}.$$  

(7)

For the case at hand $\Delta L \sim 1$, $t_H \sim \Gamma^{-1} e^S$, and we obtain (c.f. [3, 7])

$$L \sim \exp \left( -S(\beta) \right).$$  

(8)

Hence, the “recurrence index” scales as $\overline{L} \sim \exp(-N^2)$ in the high-temperature phase. Since $N^2 \sim G_N^{-1}$ in the AdS/CFT dictionary, the index scales as a nonperturbative effect in the semiclassical approximation.

### 3 Absence of recurrences in semiclassical black holes

The previous considerations suggest that recurrences should be invisible in gravity perturbation theory, i.e. in an expansion in powers of $1/N^2$ around a black hole solution, and this is indeed what is found. The reason is that the relevant eigenfrequencies $\omega$ (the so-called normal modes of the black hole) form a continuous spectrum to all orders in the $1/N$ expansion. For a static metric of the form

$$ds^2 = -g(r) \, dt^2 + \frac{dr^2}{g(r)} + r^2 \, d\Omega_{d-2}^2,$$

(9)

the normal frequency spectrum follows from the diagonalization of a radial Schrödinger operator

$$\omega^2 = -\frac{d^2}{dr_*^2} + V_{\text{eff}}(r_*),$$

(10)
Figure 4: The effective potential determining the semiclassical normal frequency modes in a large AdS black hole background (left). In Regge–Wheeler coordinates the horizon is at \( r_\ast = -\infty \), whereas the boundary of AdS is at \( r_\ast = \pi R/2 \) (only the region exterior to the horizon appears). There is a universal exponential behaviour in the near-horizon (Rindler) region. The effective one-dimensional Schrödinger problem represents a semi-infinite barrier with a continuous energy spectrum. This contrasts with the analogous effective potential in vacuum AdS with global coordinates (right). The domain of \( r_\ast \) is compact and the spectrum of normal modes is discrete with gap of order \( 1/R \).

with

\[
V_{\text{eff}} = \frac{d - 2}{2} g(r) \left( \frac{g'(r)}{r} + \frac{d - 4}{2r^2} g(r) \right) + g(r) \left( -\frac{\nabla^2_\Omega}{r^2} + m^2 \right)
\]

(11)

for a scalar field of mass \( m \) (analogous potentials can be deduced for higher spin fields). Here we have defined the Regge–Wheeler or “tortoise” coordinate \( dr_\ast = dr/g(r) \).

We have shown in Fig. 4 the form of the resulting effective potentials for large AdS black holes, compared with the case of the vacuum AdS manifold. The vacuum AdS manifold, corresponding to the choice \( g(r) = 1 + r^2/R^2 \) in (9), behaves like a finite cavity, as expected. The distinguishing feature of the black-hole horizon is a non-degenerate zero, \( g(r_0) = 0 \), which induces the universal scaling

\[
V_{\text{eff}}(r_\ast) \propto \exp(4\pi r_\ast/\beta) \quad \text{as} \quad r_\ast \to -\infty,
\]

(12)

with \( 1/\beta = g'(r_0)/4\pi \) the Hawking temperature and the horizon \( r = r_0 \) appearing at \( r_\ast = -\infty \). Notice that the near-horizon behaviour (12) only depends on the Hawking temperature, i.e. the curvature at the horizon, and is independent of other long-distance features of the gravitational background. The spectrum is discrete in pure AdS, and continuous in the AdS black hole. Physically, this just reflects the fact that the horizon is an infinite redshift surface, so that we can store an arbitrary number of modes with finite total energy, provided they are sufficiently red-shifted by approaching the horizon \( [13] \). Since the thermal entropy of perturbative gravity excitations in the vacuum AdS spacetime scales as \( S(\beta)_{\text{AdS}} \sim N^0 \), we see that the perturbative Heisenberg time of the AdS spacetime is of \( O(1) \) in the large-\( N \) limit, leading to \( T_{\text{AdS}} = O(1) \). On the other hand, we have \( T_{\text{bh}} = 0 \) in this approximation.

Although these results are based on the leading perturbative approximation in the classical black hole background, it is unlikely that higher-order perturbative effects will
Figure 5: The Euclidean black hole manifold \( X \) is simply connected, unlike standard thermal manifolds in quantum field theory.

render the frequency spectrum discrete, because this feature appears as an infrared property of the potentials in Fig. 4 (c.f. [14]).

Another argument for the robustness of \( I_{\text{bh}} \) in perturbation theory comes from the Euclidean formalism, obtained by \( t = -i\tau \) in (9), followed by an identification \( \tau \equiv \tau + \beta \). The resulting metric for the vacuum AdS spacetime has a non-contractible \( S^1 \) given by the \( \tau \) compact direction. We call \( Y \) this Euclidean manifold. On the other hand, the black hole spacetime with \( g(r_0) = 0 \) is simply connected, since the thermal \( S^1 \) shrinks to zero size at \( r = r_0 \). The choice \( 1/\beta = g'(r_0)/4\pi \) ensures smoothness at \( r = r_0 \). We call this Euclidean black hole manifold \( X \).

The real-time correlation functions in the black hole background, \( G(t)_X \), follow by analytic continuation from their Euclidean counterparts. Since \( X \) is a completely smooth manifold in the \( 1/N \) expansion, so is the Euclidean correlator \( G(it)_X \) for \( t > 0 \). The continuous spectrum arising in the spectral decomposition of \( G(t)_X \) is a consequence of the contractible topology of \( X \), since the foliation by \( \tau = \text{constant} \) hypersurfaces is singular at the horizon (c.f. [1]).

Since the continuous spectrum finds its origin in the topological properties of \( X \), this particular fact will not be affected by perturbative corrections. Incidentally, the same peculiar behaviour with respect to the Hamiltonian conjugate to \( \tau \) is responsible for the existence of a formally classical entropy. Namely, the Euclidean action

\[
I(X) = \frac{1}{16\pi G_N} \int_X R - \frac{1}{8\pi G_N} \oint_{\partial X} (K + \text{C.T.})
\]  

(13)

with appropriately defined counterterms, \text{C.T.}, is not just given by \( \beta M_{\text{ADM}} \), despite the fact that \( \partial_{\tau} \) is a Killing vector on \( X \). Rather, one finds [15]

\[
I(X) = \beta M(X) - S_{\text{bh}}(X)
\]  

(14)

with \( S_{\text{bh}} = A_H/4G_N \) the Bekenstein–Hawking entropy. The microscopic interpretation of this entropy must be referred back to the dual CFT. This point of view suggests that the information encoded in the geometry of \( X \) is fundamentally coarse-grained, so that the continuous spectrum of frequencies would also be a reflection of this coarse-graining.
In the vicinity of $r = r_0$, the manifold $X$ is well-approximated by the product of a flat disk and the $S^{d-2}$ at the horizon. Equal-time hypersurfaces of Hamiltonian foliations on $X$ have a fixed point at $r = r_0$. This fact is responsible for both the classical contribution to the entropy, and the continuous spectrum of normal frequencies.

### 4 Topological diversity and unitarity

Our discussion in the previous section suggests that improving on the semiclassical prediction $\mathcal{L}_{\text{bh}} = 0$ requires some sort of topology-change process. The proposal of [3] is precisely that: instead of evaluating the semiclassical correlators on $X$, one should sum coherently the contribution of $X$ and $Y$. Normally one neglects the contribution of $Y$ on a quantitative basis (at high temperatures $R \gg \beta$). However, here the contribution of $X$ to $\mathcal{L}$ vanishes and one is forced to consider the first correction. Since $Y$ has a discrete spectrum in perturbation theory, the net result for $\mathcal{L}$ should be non-vanishing in this approximation. Physically, this superposition of Euclidean saddle points (or master fields, in the language of the CFT) corresponds to large-scale fluctuations in which the AdS black hole is converted into a graviton gas at the same temperature and vice versa.

The resulting time profile in the instanton approximation takes the form (c.f. [4])

$$L(t)_{\text{inst}} = L(t)_X + C e^{-2\Delta I} L(t)_Y$$

(15)

where $C = O(N^0)$, $\Delta I = I_Y - I_X$ and $I = -\log Z(\beta)$, calculated in the classical gravity approximation. Since $I_Y \sim -N^0$ and $I_X \sim -N^2$, the exponential suppression factor is of order $\exp(-2|I_X|) \sim \exp(-N^2)$. The resulting structure is shown in Fig 8. The instanton approximation to the normalized correlator features the normal dissipation with lifetime $\Gamma^{-1} \sim \beta$ coming from the contribution of $X$. However, the resurgences are controlled by $L(t)_Y$, damped by a factor $\exp(-2\Delta I) \sim \exp(-N^2)$, and separated a time $t_H(Y) \sim N^0$.

We can also find the time scale $t_c$ for which the large-scale instantons considered here are quantitatively important on the graph of $L(t)$. This is shown in Fig. 8 and yields $t_c \sim \Delta I / \Gamma \sim N^2$.

We see that the instanton approximation does not reproduce the expected pattern of recurrences, particularly at high temperatures. It is interesting to find out how much this depends on the temperature above the phase transition.
Figure 7: Summing over large-scale fluctuations of the thermal ensemble in which a black hole spontaneously turns into radiation (and vice versa) is represented in the Euclidean formalism as the coherent sum of thermal saddle points of different topology. The “cigar-like” geometry $X$ represents the black-hole master field (in the CFT language) and the cylindrical topology $Y$ represents the thermal gas of particles.

For large AdS black holes, positive specific heat sets in for $r_0 > R \left( \frac{d-3}{d-1} \right)^{1/2}$, but these black holes do not dominate the leading large-$N$ thermodynamics until $r_0 = R$, the location of the Hawking–Page phase transition \[16\]. In the immediate vicinity of the transition the statistical weight of the two backgrounds is approximately the same, since $\Delta I \approx 0$. However, the entropy increases by a factor of order $N^2$ across the transition, and we would expect a sharp change of behaviour of $t_H$ as a function of $\beta$, as well as the long-time structure of $L(t)$.

On the other hand, in the instanton approximation the resurgence are controlled by $t_H(Y)$, which is of $O(1)$ in the large-$N$ limit on both sides of the phase transition. The only difference is that $\Delta I$ starts increasing away from zero as the temperature increases. The bumps, spaced $t_H(Y)$ apart and initially of height $O(1)$, decrease accordingly in size. When $(r_0 - R)/R \sim 1/N^2$, we reach $\Delta I \sim N^2$ and the pattern in Fig. 8.

In the limit of very high temperatures, there are some limitations to be expected. The free energy of the $Y$ manifold scales as that of a $D$-dimensional thermal gas, with $D = 10$ or $D = 11$ depending on the particular model of AdS/CFT duality considered, i.e. $I(Y) \sim -(R/\beta)^{D-1}$. Hence, at temperatures of order

$$R/\beta \sim N^{\frac{d-2}{d-3}}$$

the $Y$ manifold would dominate again over $X$ (c.f. \[17\]). However, perturbative instabilities of $Y$ appear before this threshold. For example, in the standard case of AdS$_5 \times$ S$^5$ duality, the $Y$ manifold reaches the Hagedorn instability at temperatures $R/\beta \sim (g_sN)^{1/4}$, and the Jeans instability at temperatures $R/\beta \sim N^{1/5}$.

Despite all these caveats, the instanton approximation yields an interesting value for the infinite time average, at all temperatures.

$$\mathcal{T}_{\text{inst}} \approx C e^{-2\Delta I}.$$
Figure 8: The instanton approximation to the correlator $L(t)_{\text{inst}}$ features the expected exponential decay $e^{-\Gamma t}$ induced by the contribution of the $X$-manifold, whereas the resurgences are entirely due to the interference with the $Y$-manifold, leading to small bumps of order $e^{-2\Delta I} \sim e^{-N^2}$, separated a time $t_H(Y) \sim N^0$. These bumps are noticeable against the background of the $X$-manifold after a time $t_c \sim \Delta I/\Gamma$. In the dashed line we plot the very different expected behaviour in the exact CFT: large $O(1)$ bumps separated by time intervals of order $e^{N^2}$. Despite the gross differences between both profiles, their time averages coincide in order of magnitude.

Namely, we have

$$L_{\text{inst}} \sim \frac{\Delta L}{\Gamma t_H} \sim e^{-N^2} \sim \frac{1}{\Gamma \cdot \Gamma^{-1}} e^{N^2} \sim L_{\text{CFT}}. \quad (18)$$

The first estimate obtains $L_{\text{inst}} \sim e^{-N^2}$ from the Boltzman suppression of the $Y$ manifold, despite the fact that $t_H(Y) \sim O(1)$, whereas the second estimate is based on $O(1)$ recurrences with very large Heisenberg time. It is important to stress that (18) holds up to factors of order $e^{-cN^2}$ with $c = O(1)$, because in general $S_X \neq -2|I_X|$, even at high temperatures [7]. For large AdS black holes, the instanton calculation (18) gives a larger value of the index than the estimate based on the quantum mechanical density of states [8].

Thus, we find that a coarse-grained question, such as the infinite-time averages of correlators, is better accounted for by the semiclassical instanton approximation than a “detailed” question, such as the concrete time structure of the correlators. This is another indication of the fundamentally thermodynamical features of relativistic horizons.

A more complicated set of Euclidean saddle points can be analyzed for the three-dimensional case of BTZ black holes. The authors of [9, 10] conclude that resummation of an infinite family of $SL(2, \mathbb{Z})$ saddle points is unlikely to alter the conclusions presented here on the basis of the leading instanton approximation. The authors of Ref. [9] also point out that only a finite set of black-hole saddle points remains under the control of the semiclassical approximation after a time of order $t_c \sim c$, where $c$ is the central charge of the CFT.
Figure 9: A pictorial representation of the compact phase space at very large energy. Poincaré recurrences consist on the time development $U_t(W)$ of a region $W$ intersecting itself after a period larger than $t_H$. We have separated the dominant black-hole like states from the relatively scarce thermal gas states in the phase space. The instanton approximation is only sensitive to the recurrences in the small patch of thermal gas states, because the spectrum of black-hole states is effectively treated as continuous.

5 Conclusions

The study of very long time features of correlators in black hole backgrounds is a potentially important approach towards unraveling the mysteries of black hole evaporation and the associated physics at the spacelike singularity. We have seen that large scale topology-changing fluctuations proposed in [3] begin to restore some of the fine structure required by unitarity, but fall short at the quantitative level. Presumably the appropriate instantons occur on microscopic scales and involve stringy dynamics.

While semiclassical black holes do faithfully reproduce “coarse grained” inclusive properties of the system such as the entropy (c.f. [15]), additional dynamical features of the horizon may be necessary to resolve finer details of the information loss problem. Roughly, one needs a systematic set of corrections that could generate a “stretched horizon” of Planckian thickness [18, 19]. The crudest model of such stretched horizon is the brick-wall model of ’t Hooft [13]. In this phenomenological description one replaces the horizon by a reflecting boundary condition at Planck distance $\epsilon \sim \ell_P$ from the horizon. This defines a “mutilated” $X_\epsilon$ manifold, of cylindrical topology, leading to a discrete spectrum of the right spacing in order of magnitude. Notice that, in line with our previous discussion, the discrete spectrum on the effective manifold $X_\epsilon$ is tied with the absence of classical contribution to the entropy, whose leading contribution is obtained at one loop order: $S(X_\epsilon) \sim A_H/\epsilon^{d-2}$.

We have also seen that the characteristic time for large topological fluctuations to be important is $t_c \sim O(N^2)$ in the semiclassical approximation. In [20] it was argued that semiclassical two-point functions probe the black hole singularity on much shorter characteristic times, thereby justifying the analysis on the single standard black hole manifold. However, detailed unitarity is only restored on time scales of order $t_H \sim$
Figure 10: Different topological contributions to the path integral of the scattering matrix. We have drawn classical black hole and white hole spacetimes (for CPT invariance), as well as a spacetime of trivial topology. According to [11], trivial topology contributions would be enough to restore unitarity of the S-matrix.

\[ \exp(N^2). \] Thus \( t_c \ll t_H \) and we conclude that such semiclassical analysis of the singularity is bound to be incomplete, as it misses whatever microphysics is responsible for the detailed unitarity restoration in the quantum mechanical time evolution.

It is natural to ask at this point what possible lessons can be drawn regarding the related problem of the black hole S-matrix. In particular, Ref. [11] uses the main idea of [3], extrapolating it to the S-matrix problem, and claiming that trivial-topology spacetimes contributing to the path integral are enough to restore unitarity in the complete quantum amplitudes (c.f. Fig. 10).

The standard black hole spacetimes in Fig. 10 violate quantum coherence because either the In or Out asymptotic surfaces do not support the complete In or Out Hilbert space, whereas the topologically trivial spacetimes do have complete In and Out asymptotic surfaces. Thus, the claim would be that asymptotic completeness for off-shell histories is enough to cure the lack of asymptotic completeness of many approximately on-shell histories. Since asymptotic completeness is the key property guaranteeing S-matrix unitarity (c.f. [21, 6]), it is hard to imagine that a path integral featuring both topologically trivial and topologically nontrivial spacetimes could be exactly unitary. A more natural expectation would be that such topological diversity could at best achieve some approximate restoration of unitarity.

One obstacle in assessing the proposal of [11] is the technical difficulty in estimating the quantitative effect of trivial topologies in the complete path integral. For initial conditions that would classically produce a black hole, the spacetimes of trivial topology are far from any semiclassical saddle point, and therefore their contribution is difficult to evaluate with the required precision. In fact, the existence of a classical saddle point with trivial topology; the manifold \( Y \), was the main reward for choosing thermal equilibrium states in Ref. [3], as opposed to S-matrix boundary conditions.

The situation with the topologically trivial spacetimes in the S-matrix is perhaps analogous to that of the \( Y \) manifold at temperatures above the Jeans instability, when
it becomes unstable and hence ceases to be a good saddle point of the path integral. At sufficiently high temperatures, the Jeans length $\ell_J \sim \sqrt{\beta/G_N}$ falls below the curvature radius of AdS, and some thermal fluctuations of wavelength $\lambda > \ell_J$ develop an imaginary effective mass. In real time, this corresponds to exponential behaviour of linearized perturbations, proportional to $\exp(\pm t/\ell_J)$, thus entering the non-linear regime beyond the applicability of perturbation theory. On physical grounds, we expect that the endpoint of this “tachyonic instability” is the large AdS black hole at the corresponding temperature, thus reverting back to the $X$ manifold as the unique stable saddle point.

Therefore, if we are to draw inspiration from the study of thermal boundary conditions, we would suggest that topological diversity *per se* is not enough to restore unitarity of the S-matrix, unless some coarse-graining is imposed on the S-matrix itself (the analog of calculating of $\mathcal{L}$ as opposed to $\mathcal{L}(t)$)

On general grounds, the AdS/CFT correspondence suggests that spacetime topology can be unambiguously defined only in the context of the semiclassical approximation. Perhaps the best we can do in representing geometrically the black-hole S-matrix is the effective spacetime suggested by the principle of black hole complementarity (c.f. [19]), which involves a topologically trivial spacetime with a stretched horizon that behaves as a boundary equipped with an effective Hilbert space and effective Hamiltonian yet to be found (c.f. Fig. 11). In this effective description of the stretched horizon, all the quantum fluctuations would be included, and thus no further summation over topological sectors would be needed.

Perhaps the following metaphor will turn out to be a useful guide line. In massless QCD an infrared scale is required to control the large perturbative infrared fluctuations. The scale is indeed dynamically generated in asymptotically free systems. Although gravity is *a priori* equipped with an intrinsic Planck scale, in the near-horizon region this scale is red-shifted away and the system seems to possess no scale (c.f. the universal
potential of (12)). A dynamically produced scale would presumably allow the formation of a stretched horizon and the restoration unitarity. Some universal features of such stretched horizon were recently pointed out in [22].

Acknowledgements

E. R. would like to thank the KITP at Santa Barbara for hospitality during the completion of this work, under grant of the National Science Foundation No. PHY99-07949. The work of J.L.F.B. was partially supported by MCyT and FEDER under grant BFM2002-03881 and the European RTN network HPRN-CT-2002-00325. The work of E.R. is supported in part by the BSF-American Israeli Bi-National Science Foundation, The Israel Science Foundation and the European RTN network MRTN-CT-2004-512194.

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