Spin Alignment of Heavy Meson Revisited

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Abstract

Using heavy quark effective theory a factorized form for inclusive production rate of a heavy meson can be obtained, in which the nonperturbative effect related to the heavy meson can be characterized by matrix elements defined in the heavy quark effective theory. Using this factorization, predictions for the full spin density matrix of a spin-1 and spin-2 meson can be obtained and they are characterized only by one coefficient representing the nonperturbative effect. Predictions for spin-1 heavy meson are compared with experiment performed at $e^+e^-$ colliders in the energy range from $\sqrt{s} = 10.5$GeV to $\sqrt{s} = 91$GeV, a complete agreement is found for $D^*$- and $B^*$-meson. There are distinct differences from the existing approach and they are discussed.

Heavy quark effective theory (HQET) is a powerful tool to study properties of heavy hadrons which contain one heavy quark $Q$ [1,2]. With HQET it allows such a study starting directly from QCD. HQET is widely used in studied of decays of heavy hadrons. In comparison, only in a few works it is used to study productions of heavy hadrons. In 1994 Falk and Peskin used HQET to predict spin alignment of a heavy hadron in its inclusive production [3]. In this talk we will reexamine the subject and restrict ourself to the case of spin-1 meson.

A spin-1 heavy meson $H^*$ is a bound state of a heavy quark $Q$ and a system of light degrees of freedom in QCD, like gluons and light quarks. In the work [3] the total angular momentum $j$ of the light system is taken as $1/2$. In the heavy quark limit, the orbital angular momentum of $Q$ can be neglected and only the spin of $Q$ contributes to the total spin of $H^*$. Once the heavy quark $Q$ is produced, it will combine the light system into $H^*$. Because parity is conserved in QCD, the probabilities for the light system with positive- and negative helicity is the same. Therefore, one can predict the probabilities for production of $H^*$ with a left-handed heavy quark $Q$ as:

$$P(\bar{B}^*(\lambda = -1)) : P(\bar{B}^*(\lambda = 0)) : P(\bar{B}^*(\lambda = 1)) = \frac{1}{2} : \frac{1}{4} : 0,$$

where $\lambda$ is the helicity of $H^*$. These results are easily derived, however, three questions or comments can be made to them:
(a). In general the spin information is contained in a spin density matrix, probabilities are the
diagonal part of the matrix. The question is how about the non-diagonal part. It should be noted
that this part is also measured in experiment.

(b). It is possible that the light system can have the total angular momentum \( j = 3/2 \) to form
\( H^\ast \). One may argue that the production of such a system is suppressed. Is it possible to derive
full spin density matrix without the assumption of \( j = 1/2 \)?

(c). How can we systematically add corrections to the approximation which lead to the results
in Eq.(1)?

To make responses to these questions let us look at a inclusive production of \( H^\ast \) in detail. In its
inclusive production a heavy meson is formed with a heavy quark \( Q \) and with other light degrees
of freedom, the light degrees can be a system of light quarks and gluons. Because its large mass
\( m_Q \) the heavy quark is produced by interactions at short distance. Therefore the production can
be studied with perturbative QCD. The heavy quark, once produced, will combine light degrees of
freedom to form a hadron, the formation is a long-distance process, in which momentum transfers
are small, hence the formed hadron will carry the most momentum of the heavy quark. The
above discussion implies the production rate can be factorized, in which the perturbative part is
for the production of a heavy quark, while the nonperturbative part is for the formation. For the
nonperturbative part an expansion in the inverse of \( m_Q \) can systematically be performed in the
framework of HQET. This type of the factorization was firstly used in parton fragmentation into
a heavy hadron [4].

In this talk we will not discuss the factorization in detail, the details can be found in the work
[4]. We directly give our results and make a comparison with experiment. It should be noted
that the factorization can be performed for any inclusive production of \( H^\ast \). Because the most
experiments to measure spin alignment are performed at \( e^+e^-\)-colliders, we present the results for
the inclusive production at \( e^+e^-\)-colliders. We consider the process
\[
e^+(p) + e^-(p) \rightarrow H^\ast(k) + X,
\]
where the three momenta are given in the brackets. In the process we assume that the initial
beams are unpolarized. We denote the helicity of \( H^\ast \) as \( \lambda \) and \( \lambda = -1, 0, 1 \). All information
about the polarization of \( H^\ast \) is contained in a spin density matrix, which may be unnormalized or
normalized, we will call them unnormalized or normalized spin density matrix, respectively. The
unnormalized spin density matrix can be defined as
\[
R(\lambda, \lambda', p, k) = \sum_X \langle H^\ast(\lambda)X | T | e^+ e^- \rangle \cdot \langle H^\ast(\lambda')X | T | e^+ e^- \rangle^*.
\]
where the conservation of the total energy-momentum and the spin average of the initial state is
implied. \( T \) is the transition operator. The cross-section with a given helicity \( \lambda \) is given by:
\[
\sigma(\lambda) = \frac{1}{2s} \int \frac{d^3k}{(2\pi)^3} R(\lambda, \lambda, p, k).
\]
From Eq.(3) the normalized spin density matrix is defined by
\[
\rho_{\lambda\lambda'}(p, k) = \frac{R(\lambda, \lambda', p, k)}{\sum_\lambda R(\lambda, \lambda, p, k)}.
\]
It should be noted that the normalized spin density matrix is measured in experiment. It is straightforward to perform the mentioned factorization for the unnormalized spin density matrix in the rest frame of $H^*$, which is related to the moving frame only by a Lorentz boost. In the rest frame we can define a creation operator for $H^*$:

$$|H^*(\lambda)\rangle = a^\dagger(\lambda)|0\rangle = \epsilon(\lambda) \cdot a^\dagger|0\rangle.$$  \hspace{1cm} (6)

where $\epsilon(\lambda)$ is the polarization vector. In the rest frame the field $h_\nu$ of the heavy quark $Q$ in HQET has two non-zero components. We denote them as:

$$h_\nu(x) = \begin{pmatrix} \psi(x) \\ 0 \end{pmatrix}.$$  \hspace{1cm} (7)

With these notations we define two operators:

$$O(H^*) = \frac{1}{6} \text{Tr} \psi a_i^\dagger a_i \psi, \quad O_s(H^*) = \frac{i}{12} \text{Tr} \sigma_i a_j^\dagger a_k \psi^\dagger \epsilon_{ijk},$$  \hspace{1cm} (8)

where $\epsilon_{ijk}$ is the totally antisymmetric tensor and $\sigma_i (i = 1, 2, 3)$ is the Pauli matrix. The results for the unnormalized spin density matrix read:

$$R(\lambda, \lambda', p, k) = \frac{1}{3} a(p, k) \langle 0 | O(H^*) | 0 \rangle \epsilon^*(\lambda) \cdot \epsilon(\lambda') + \frac{i}{3} b(p, k) \cdot [\epsilon^*(\lambda) \times \epsilon(\lambda')] \cdot \langle 0 | O_s(H^*) | 0 \rangle + O(m_Q^{-2}).$$  \hspace{1cm} (9)

The quantities $a(p, k)$ and $b(p, k)$ characterize the spin density matrix of the heavy quark $Q$ produced in the inclusive process:

$$e^+(p) + e^-(p) \rightarrow Q(k, s) + X$$  \hspace{1cm} (10)

where $s$ is the spin vector of $Q$ in its rest frame and the rest frame is related to the moving frame only by a Lorentz boost. The unnormalized spin density matrix $R_Q(s, p, k)$ of $Q$ can be defined by replacing $H^*(\lambda)$ with $Q(k, s)$ in Eq.(3). This matrix can be calculated with perturbative theory because of the heavy mass. The result in general takes the form

$$R_Q(s, p, k) = a(p, k) + b(p, k) \cdot s$$  \hspace{1cm} (11)

where $a(p, k)$ and $b(p, k)$ are the same in Eq.(9). The physical interpretation for Eq.(9) is the following: The coefficients $a(p, k)$ and $b(p, k)$ characterize the production of $Q$ and they can be calculated with perturbative QCD, while the two matrix elements defined in HQET characterize the nonperturbative effects of the formation of $H^*$ with the heavy quark $Q$. With Eq.(9) we obtain:

$$\rho(p, k) = \frac{1}{3} \begin{pmatrix} 1 + P_3, & -P_+, & 0 \\ -P_-, & 1, & -P_+ \\ 0, & -P_-, & 1 - P_3 \end{pmatrix},$$  \hspace{1cm} (12)
with

\[ P_3 = \frac{b_3(p, k)}{a(p, k)} \cdot \langle 0 | O_s(H^*) | 0 \rangle \]

\[ P_\pm = \frac{b_1(p, k) \pm ib_2(p, k)}{\sqrt{2}a(p, k)} \cdot \langle 0 | O_s(H^*) | 0 \rangle \]

The indices of the matrix in Eq.(12) run from -1 to 1. Without knowing the coefficients and the matrix elements we can already predict that \( \rho_{00} = 1/3 \) and \( \rho_{1-1} = \rho_{-11} = 0 \).

With these results we are in position to compare with experiment. The experiments to measure the polarization of \( B^* \) are performed at LEP with \( \sqrt{s} = M_Z \) by different experiments groups. To measure the polarization the dominant decay \( B^* \to \gamma B \) is used, where the polarization of the photon is not observed. Because the parity is conserved and the distribution of the angle between the moving directions of \( \gamma \) and of \( B^* \) is measured, one can only determine the matrix element \( \rho_{00} \). If we denote \( \theta \) the angle between the moving directions of \( B^* \) and of \( \gamma \) in the \( B^* \) rest frame and \( \phi \) is the azimuthal angle of \( \gamma \), then the angular distribution is given by

\[ W_{B^* \to \gamma B}(\theta, \phi) \propto \sum_{\lambda \lambda'} \rho_{\lambda \lambda'} (\delta_{\lambda \lambda'} - Y_{1 \lambda}(\theta, \phi) Y_{1 \lambda'}^*(\theta, \phi)) \]

Integrating over \( \phi \) and using our result \( \rho_{00} = 1/3 \), the distribution of \( \theta \) is isotropic. In experiment one indeed finds that the distribution is isotropic in \( \theta \). The experimental results at \( \sqrt{s} = M_Z \) are [3] [8]:

\[ \rho_{00} = 0.32 \pm 0.04 \pm 0.03, \quad \text{DELPHI} \]
\[ \rho_{00} = 0.33 \pm 0.06 \pm 0.05, \quad \text{ALEPH}, \]
\[ \rho_{00} = 0.36 \pm 0.06 \pm 0.07, \quad \text{OPAL}. \]

These results agree well with our prediction \( \rho_{00} = 1/3 \).

The polarization measurement for \( D^* \)-meson has been done with different \( \sqrt{s} \), in some experiments the non-diagonal part of the spin density matrix has also been measured by measuring azimuthal angular distribution in \( D^* \) decay, where the decay mode into two pseudo-scalars, i.e., \( D^* \to D \pi \), is used. Denoting \( \theta \) as the angle between the moving directions of \( D^* \) and of \( \pi \) in the \( D^* \)-rest frame and \( \phi \) as the azimuthal angle of \( \pi \), then the angular distribution of \( \pi \) is given by

\[ W_{D^* \to D \pi}(\theta, \phi) \propto \sum_{\lambda \lambda'} \rho_{\lambda \lambda'} Y_{1 \lambda}(\theta, \phi) Y_{1 \lambda'}^*(\theta, \phi) \]

Integrating over \( \phi \) and using our result \( \rho_{00} = 1/3 \), the distribution of \( \theta \) is again isotropic. The experimental results are summarized in Table 1 and also partly summarized in [3].

From Table 1, we can see that the \( \rho_{00} \) measured by all experimental groups is close to the prediction \( \rho_{00} = 1/3 \); the most precise result is obtained by CLEO, its deviation from the prediction is 2%, the largest deviation of the prediction is from the result made by OPAL at \( \sqrt{s} = 90 \text{GeV} \), it is 20%. In general, \( \rho_{00} \) depends on the energy of \( H^* \). Our results in Eq.(17) give that \( \rho_{00} \) is a constant in the heavy quark limit, or the energy dependence is suppressed by \( m_Q^{-2} \). In experiment only a very weak energy dependence is observed, e.g., in CLEO results [3]. From our results \( \rho_{1-1} \) is exactly zero in the heavy quark limit, the results from TPC and from HRS are in consistent with our result, a non zero value is obtained by OPAL, which has a 3\( \sigma \) deviation from zero. These deviations may be explained with effects of higher orders in \( m_c^{-1} \); these effects are expected to be substantial, because \( m_c \) is not so large. It is interesting to note only results from OPAL at \( \sqrt{s} = 91 \text{GeV} \) have the largest deviations from our predictions, while results from other groups agree well with our predictions. At \( \sqrt{s} = 10.5 \text{GeV} \) or 29GeV, the effect of the Z-boson exchange can be neglected, hence the parity is conserved. We obtain \( \rho_{10} = 0 \). This prediction is also in agreement with the experimental result made by TPC and by HRS.
Table 1. Experimental Results for $D^*$

| Collaboration | $\sqrt{s}$ in GeV | Results          |
|--------------|-------------------|-----------------|
| CLEO [10]   | 10.5              | $\rho_{00} = 0.327 \pm 0.006$ |
| HRS [11]    | 29                | $\rho_{00} = 0.371 \pm 0.016$  
                     |                    | $\rho_{1-1} = 0.04 \pm 0.03$  
                     |                    | $\rho_{10} = 0.00 \pm 0.01$   |
| TPC [12]    | 29                | $\rho_{00} = 0.301 \pm 0.042 \pm 0.007$  
                     |                    | $\rho_{1-1} = 0.01 \pm 0.03 \pm 0.00$  
                     |                    | $\rho_{10} = 0.03 \pm 0.03 \pm 0.00$   |
| SLD [13]    | 91                | $\rho_{00} = 0.34 \pm 0.08 \pm 0.13$  
                     |                    | $\rho_{00} = 0.40 \pm 0.02 \pm 0.01$  
                     |                    | $\rho_{1-1} = -0.039 \pm 0.014$   |

Since our results are derived without knowing the total angular momentum $j$ of the light degrees of freedom in the heavy meson, the agreement of our results with experiment can not be used to extract the information about $j$ from the experimental data in Eq.(19) and in Table 1., although $\rho_{00} = 1/3$ can also be obtained by taking $j = 1/2$. One way to extract $j$ may be to measure the difference between $\rho_{11} - \rho_{-1-1}$, but it seems not possible, because the polarization of $H^*$ is measured through its parity-conserved decay and the polarization of decay products is not observed in experiment. In the heavy quark limit, the nondiagonal element $\rho_{1-1}$ and $\rho_{-11}$ are zero, while the other nondiagonal matrix elements are nonzero if the parity is not conserved and the initial state is unpolarized. At higher orders in $m_Q^{-1}$ this can be changed, e.g., $H^*$ can have tensor polarization.

The factorization can also be done for inclusive productions of a spin-2 meson. The results can be found in the work [5]. Experimentally only the spin alignment of $D_2^*(2460)$ is measured with large errors. But the experimental results seem to be not in agreement with the predictions. The reason can be the large effect from corrections at higher orders of $m_c^{-1}$.

To summarize: Using the approach of QCD factorization and employing HQET, we obtain predictions of full spin density matrices of spin-1- and spin-2 heavy meson. The leading order predictions for a spin-1 meson agree well with experiment. Within the approach the three questions asked before are answered. Although we have given in this talk detailed predictions for inclusive production of a spin-1 heavy meson at an $e^+e^-$ collider, our approach can be easily generalized to other inclusive productions, testable predictions can be made without a detailed calculation, for example, in inclusive production of $B^*$ at an electron-hadron- or a hadron-hadron collider we always have the prediction $\rho_{00} = 1/3$ and $\rho_{-11} = \rho_{1-1} = 0$ in the heavy quark limit.
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