Analytic solution for matter density perturbations in a class of viable cosmological $f(R)$ models

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Abstract

For a class of viable cosmological models in $f(R)$ gravity in which deviation from the Einstein gravity decreases as a inverse power law of the Ricci scalar $R$ for large $R$, an analytic solution for density perturbations in the matter component during the matter dominated stage is obtained in terms of hypergeometric functions. An analytical expression for the matter transfer function at scales much less than the present Hubble scale is also obtained.

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I. INTRODUCTION

Recent numerous observational data obtained from angular anisotropy and polarization of the cosmic microwave background radiation, large-scale gravitational clustering of galaxies and observations of distant supernovae explosions prove convincingly that the Universe expands with acceleration at the present time while it was decelerating in the past for redshifts larger than about 0.7. If interpreted in terms of the Einstein general theory of relativity, this acceleration requires the existence of some new component in the right-hand side of the Einstein equations, dubbed dark energy (DE), which remains practically unclustered at all scales at which gravitational clustering of baryonic and dark non-baryonic matter is seen, and which effective pressure $p_{DE}$ is approximately equal to minus its effective energy density $\rho_{DE}$. Thus, its properties are very close to those of a cosmological constant $\Lambda$ (see Refs. [1] - [5] for some reviews). The simplest possibility of DE being exactly $\Lambda$ combined with a non-relativistic non-baryonic dark matter (the standard spatially flat $\Lambda$CDM cosmological model) provides a good fit to all existing observational data [6]. In this case $\Lambda$ acquires the status of a new fundamental physical constant. However, its required value is very small as compared to known atomic and elementary particle scales (not speaking about the Planck ones), so a firm theoretical prediction for this quantity from first principles is lacking currently (although it may arise due to some non-perturbative effects, see e.g. Ref. [7]).

On the other hand, in the second case when a component with qualitatively similar properties is assumed to exist – in the inflationary scenario of the early Universe, we are sure that this “primordial DE” may not be an exact cosmological constant since it should decay long time ago. That is why it is natural to assume by analogy that the present DE is not absolutely stable, too.

An interesting alternative to the standard $\Lambda$CDM model is provided by “geometric” DE models based on $f(R)$ gravity which modify and generalize the Einstein General Relativity by introducing a new function of Ricci scalar, $f(R)$, into the gravitational field action instead of $R - 2\Lambda$ (see Ref. [8] for a recent review). $f(R)$ gravity, which in turn is a particular case of more general scalar-tensor gravity, contains an additional scalar degree of freedom or, in
quantum language, a massive scalar particle. 1 Because of this, viable models of present DE in \( f(R) \) gravity should satisfy a number of conditions which exclude many possible, in principle, forms of \( f(R) \). In particular, in order to have the correct Newtonian limit for \( R \gg R(t_0) \sim H_0^2 \) where \( t_0 \) is the present moment and \( H_0 \) is the Hubble constant, as well as the standard matter-dominated FLRW stage with the scale factor behaviour \( a(t) \propto t^{2/3} \) driven by cold dark matter and baryons, the following conditions should be fulfilled:

\[
|f(R) - R| \ll R, \quad |f'(R) - 1| \ll 1, \quad Rf''(R) \ll 1, \quad R \gg R_0 ,
\]

where the prime denotes the derivative with respect to the argument \( R \). In addition, the stability condition \( f''(R) > 0 \) has to be satisfied that guarantees that the standard matter-dominated FLRW stage remains an attractor with respect to an open set of neighboring isotropic cosmological solutions in \( f(R) \) gravity (in quantum language, this condition means that scalaron is not a tachyon). 2

A number of functional forms has been proposed that can account for accelerated expansion without \( \Lambda \) while passing laboratory and astronomical tests [10, 11].

The present paper considers the evolution of density perturbation in \( f(R) \) model. In Section 2, we review the \( \Lambda \)CDM model. In Section 3, after discussing density perturbation in general \( f(R) \) gravity, we choose a specific inverse power-law form of \( f(R) - R \) which is the limiting form of the models proposed in Refs. [10, 11] for \( R \gg R_0 \) and find an analytic solution for density perturbations.

II. DENSITY PERTURBATIONS IN THE \( \Lambda \)CDM MODEL

Perturbed spatially flat FLRW metric in the longitudinal gauge is

\[
ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)(dr^2 + r^2d\theta + r^2\sin^2\theta d\phi).
\]

1 This particle was dubbed "scalaron" in Ref. [9] where the particular case \( f(R) = R + R^2/6M^2 \) (plus small additional terms) was used to construct the first internally self-consistent inflationary scenario of the early Universe having a graceful exit to the subsequent radiation-dominated Friedmann-Lemaître-Robertson-Walker (FLRW) stage through an intermediate matter-dominated reheating period. This inflationary model still remains viable since it predicts the slope of the primordial spectrum of scalar perturbations \( n_s \) and the tensor-to-scalar ratio \( r \) in agreement with the most recent observational data.

2 The second stability condition \( f'(R) > 0 \) which means that gravity is attractive and graviton is not a ghost is automatically fulfilled in this regime.
During the matter-dominated era, non-zero components of the energy-momentum tensor are given by
\[ T_{0}^{0} = -\rho - \delta \rho, \quad T_{i}^{0} = -\rho \partial_{i} v, \]  
where \( \rho \) and \( \delta \rho \) denote background matter density and its fluctuation, and \( v \) is the velocity potential of scalar perturbation.

From the Einstein equations, we can derive the differential equation for comoving density perturbation defined by
\[ \delta = \frac{\delta \rho}{\rho} + 3HAV. \]  
Fourier transformation of density perturbation is given by
\[ \delta_{k}(t) = \int \frac{d^{3}x}{(2\pi)^{3/2}} \delta(t, \mathbf{x}) e^{i \mathbf{k} \cdot \mathbf{x}}, \]
where \( k \) denotes comoving wavenumber. In the following, we abbreviate \( \delta_{k}(t) \) just \( \delta \) for simplicity. The evolution of \( \delta \) in Fourier space is governed by
\[ \ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho \delta = 0. \]  
In matter dominated era, this equation becomes
\[ \ddot{\delta} + \frac{4}{3t} \dot{\delta} - \frac{2}{3t^{2}} \delta = 0. \]  
While it has two independent solutions proportional to \( t^{2/3} \) and \( t^{-1} \), we are only interested in the growing mode:
\[ \delta_{k}(t) = \delta_{0k} \left( \frac{t}{t_{0}} \right) ^{\frac{2}{3}}. \]
Here, \( \delta_{0k} \) is the initial value \( \delta_{k}(t_{0}) \).

**III. DENSITY PERTURBATIONS IN \( f(R) \) GRAVITY**

We write the action in the following form:
\[ S = \int d^{4}x \sqrt{-g} \left[ \frac{1}{16\pi G} f(R) + \mathcal{L}^{(m)} \right], \]
where \( G \) is the Newton constant and \( \mathcal{L}^{(m)} \) is matter Lagrangian. If we take \( f(R) = R - 2\Lambda \), Eq. (9) reduces to the Einstein-Hilbert action for the \( \Lambda \)CDM model. Below we consider
\( f(R) \) which vanishes for \( R = 0 \), so no cosmological constant is introduced by hand in flat space-time.

In \( f(R) \) gravity, modified Einstein equations have the form:

\[
FR_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_{\mu}\nabla_{\nu})F = 8\pi GT_{\mu\nu},
\]

(10)

where

\[
F(R) \equiv \frac{df}{dR}.
\]

(11)

For dust-like matter, the background equations take the form:

\[
3FH^2 = \frac{1}{2}(FR - f) - 3H\dot{F} + 8\pi G\rho,
\]

(12)

\[
-2FH\dot{H} = \ddot{F} - H\dot{F} + 8\pi G\rho,
\]

(13)

\[
\dot{\rho} + 3H\rho = 0.
\]

(14)

When deviation from the Einstein gravity is small, namely, \( f(R) \approx R \) and \( F(R) \approx 1 \), these equations yield the standard matter-dominated regime \( a(t) = a_0(t/t_0)^{2/3} \).

The differential equation describing a density perturbation in the subhorizon regime is:

\[
\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}}\rho\delta = 0,
\]

(15)

where

\[
G_{\text{eff}} = \frac{G}{F}\left[1 + \frac{4k^2F,R}{a^2F,F,R}\right].
\]

(16)

The above equation was derived by Tsujikawa (see also Ref. [13]) who took the subhorizon limit of the coupled equations of \( \delta \) and metric perturbations before reducing the system to a single equation. Its solutions were numerically studied in Refs. [15] - [18] in the linear regime and in Refs. [19] - [22] in the non-linear one.

On the other hand, de la Cruz-Dombriz et al. [23] derived the evolution equation of \( \delta \) without any approximation (see also Refs. [24, 25]). They then took the subhorizon limit to yield Eq. (33) in Ref. [23]. We find that, at the matter-dominated stage, their result can be

\[ \frac{k^2F,R}{a^2F,F,R} \ll 1 \text{ and } \frac{k^2F,R}{a^2F,F,R} \gg 1, \text{ Eq. (15) reduces to the one derived in Ref. [14] for matter density perturbations in general scalar-tensor cosmology for the values of the Brans-Dicke parameter } \omega_{BD} \gg 1 \text{ and } \omega_{BD} = 0 \text{ correspondingly.} \]
rewritten in the form of Eq. (15) but with

\[ G_{\text{eff}} = \frac{G}{F} \left( \frac{k^2 F}{a^2} \right)^4 \]

(App. 17)

Apparently, Eq. (16) and Eq. (17) have different dependence on \( k \). However, we can see that both of these two equations give practically the same result for observationally relevant \( f(R) \) models as follows.

First, we consider two extreme cases with \( F_r/F \gg a^2/k^2 \) and \( F_r/F \ll a^2/k^2 \). In the former case we see both equations have the same \( G_{\text{eff}} \). In the latter case, Eq. (16) gives \( G_{\text{eff}} = G/F \), while Eq. (17) reduces to \( G_{\text{eff}} = G \). However, since we are assuming the subhorizon regime, we find in this case \( F_r/F \ll a^2/k^2 \ll H^{-2} \), so that \( F \) is practically constant (and in fact equal to unity) at the cosmological time scale. So, both equations give the same result in the two extreme regimes.

Next we consider the intermediate regime with \( F_r/F \sim a^2/k^2 \). In this case, Eq. (16) and Eq. (17) do have different wavenumber dependence, but we still have \( F_r/F \sim a^2/k^2 \ll H^{-2} \) and again \( F \approx 1 \) in cosmological time scale. It has been numerically confirmed in Ref. [23] that in such observationally viable \( f(R) \) models density fluctuations evolve in the same way whichever equations one uses. Hence both equations give the same result in this case, too.

We therefore continue to use Eq. (15) with Eq. (16). From now on, we adopt a specific \( f(R) \) model such that

\[ F(R) \equiv f'(R) = 1 - \left( \frac{R_0}{R} \right)^{n+1}, \quad n > -1 \]

(18)

with \( R_0 \sim H_0^2 \) (but still \( R_0 < R(t_0) \)). This \( f(R) \) corresponds to the models in Refs. [10, 11] in the regime \( R \gg R_0 \), and we shall use it in this regime only. Equation (15) then becomes

\[ \ddot{\delta} + \frac{4}{3t} \dot{\delta} - \frac{2}{3t^2} \frac{1 + 4A(t/t_0)^{2n+8/3}}{1 + 3A(t/t_0)^{2n+8/3}} \delta = 0, \]

(19)

with

\[ A(n, k) = \frac{(n + 1)k^2}{a_0^2 R_0^2}. \]

(20)

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4 The parameter \( n \) used here has the same sense as in Ref. [10], while it is equal to \( 2n \) in the notation of Ref. [11].
A. Asymptotic behavior

We can read asymptotic behavior from the differential equation Eq. (19).

(i) \( t \to 0 \)

In this limit, we can neglect \( A(t/t_0)^{2n+8/3} \) respect to 1. Therefore, Eq. (19) reduces to Eq. (7). This reproduces the result of the \( \Lambda \)CDM model. The solution is Eq. (8),

\[
\delta_k(t) = \delta_{0k} \left( \frac{t}{t_0} \right)^\frac{4}{3}.
\]  

(ii) \( t \to \infty \)

Then Eq. (19) reduces

\[
\ddot{\delta} + \frac{4}{3t} \dot{\delta} - \frac{8}{9t^2} \delta = 0.
\]

In this regime the growing mode is given by

\[
\delta_k(t) = \delta_{0k} C(k) \left( \frac{t}{t_0} \right) ^{-\frac{1+\sqrt{33}}{6}}.
\]

The coefficient \( C(k) \) is the transfer function for matter perturbations which will be derived from the analytic solution found below.

B. Analytic solution

By changing the variable from \( t \) to \( \tau = (t/t_0)^{\alpha} \) with \( \alpha = 2n + 8/3 \), Eq. (19) can be rewritten as

\[
\delta'' + \left( 1 + \frac{1}{3\alpha} \right) \frac{\delta'}{\tau} - \frac{2}{3\alpha^2} \frac{1 + 4A\tau}{1 + 3A\tau} \frac{\delta}{\tau^2} = 0.
\]

Here a prime denotes derivative respect to \( \tau \). This equation can be solved in terms of the hypergeometric function \( _2F_1 \). The two independent solutions are

\[
\delta_k(t) = \delta_{0k} \left( \frac{t}{t_0} \right) ^{-\frac{1+\sqrt{33}}{6}} \binom{\pm 5 - \sqrt{33}}{6\alpha} ; \pm 5 + \sqrt{33}}{6\alpha} ; 1 \pm \frac{5}{3\alpha} ; -3A(n, k) \left( \frac{t}{t_0} \right)^{\alpha}.
\]  

In the following discussion, we consider the upper sign case only, because the other solution corresponds to the decaying mode of perturbations and is singular at \( t \to 0 \).

Let us check the asymptotic behavior of the solution, Eq. (25).
(i) \( t \to 0 \)

\[ \delta_k(t) \to \delta_{0k} \left( \frac{t}{t_0} \right)^{\frac{2}{3}}. \]  

(ii) \( t \to \infty \)

\[ \delta_k(t) \to \delta_{0k} \times \frac{\Gamma \left( 1 + \frac{5}{3\alpha} \right) \Gamma \left( \frac{\sqrt{33}}{3\alpha} \right)}{\Gamma \left( 1 + \frac{5+\sqrt{33}}{6\alpha} \right) \Gamma \left( \frac{5+\sqrt{33}}{6\alpha} \right)} \left[ \frac{3(n+1)k^2}{a_0^2 R_0} \right]^{-\frac{5+\sqrt{33}}{6\alpha}} \left( \frac{t}{t_0} \right)^{\frac{1+\sqrt{33}}{6}}. \]  

In both limits, the asymptotic behavior agrees with that one given by Eq. (21) and Eq. (23), respectively. Furthermore, here we can read off the transfer function, \( C(k) \), which appears in Eq. (23):

\[ C(k) = \frac{\Gamma \left( 1 + \frac{5}{2(3n+4)} \right) \Gamma \left( \frac{\sqrt{33}}{2(3n+4)} \right)}{\Gamma \left( 1 + \frac{5+\sqrt{33}}{4(3n+4)} \right) \Gamma \left( \frac{5+\sqrt{33}}{4(3n+4)} \right)} \left[ \frac{3(n+1)k^2}{a_0^2 R_0} \right]^{-\frac{5+\sqrt{33}}{4(3n+4)}}. \]  

IV. CONCLUSIONS AND DISCUSSION

We have obtained an analytic solution describing the growth of density perturbation at the matter-dominated stage for a specific class of viable cosmological models in \( f(R) \) gravity. Initially, the solution behaves in the same way as in the \( \Lambda \text{CDM} \) model, while it experiences an anomalous growth at late times (redshifts of the order of a few). We also find an analytic expression for the matter transfer function which shows that an initial perturbation power spectrum acquires the additional power-law factor \( \propto k^{\Delta n_s} \) with

\[ \Delta n_s = \frac{-5 + \sqrt{33}}{3n + 4} \]  

at scales much less than the present Hubble scale, as originally shown in Ref. \[11]\[5\]. Clearly, this additional factor is absent in the matter power spectrum at the recombination time. So, by comparing the form of the primordial matter power spectrum derived from CMB data and from galaxy surveys separately, it is possible to obtain an important constraint on the parameter \( n \) characterizing this class of cosmological models in \( f(R) \) gravity, although we do not have much stringent constraints on it at present\[20\]. If we take an upper limit on

\[5\] Note the difference in the notation for \( n \) between Ref. \[11\] and our paper which was mentioned above.
$\Delta n_s$ as $\Delta n_s^{\text{max}} = 0.05$, which is a conservative bound for now\cite{11}, and assume that $R_0$ is not much less than $H_0^2$ (if otherwise $R_0 \ll H_0^2$, deviation of the background FLRW model from the $\Lambda$CDM one is very small), we obtain a constraint

$$n > 4.96 \left( \frac{\Delta n_s^{\text{max}}}{0.05} \right)^{-1} - 1.33.$$  \hspace{1cm} (30)

Future observational data together with a more detailed theoretical analysis may well yield a more stringent bound on $n$.

Of course, the $f(R)$ gravity model \cite{18} is viable for a finite range of $R$ only, in particular, for $R \gg R_0$. For $R \sim R(t_0) \sim H_0^2$, it has to be substituted by a more complicated expression admitting a stable (or, at least metastable) de Sitter solution, e.g. by the models presented in Refs. \cite{10,11}. As a result, the equation for matter density perturbations has to be solved numerically for recent redshifts $z \lesssim 1$. However, evolution in this region may add only a $k$-independent factor to the total matter transfer function. Therefore, the $k$ exponent in Eqs. (28) and (29) does not depend on a concrete form of $f(R)$ for $R \sim R(t_0)$.

Also, the model \cite{18} should not be used for too large values of $R$ for several reasons. First, the effective scalaron mass squared $m_s^2 = 1/3F_{,R}$ (in the regime $F_{,R}H^2 \ll 1$) grows quickly with $R$ to the past and may even exceed the Planck mass making copious production of primordial black holes possible. As was noted in Ref. \cite{11}, this problem may be avoided by adding the $R^2/6M^2$ term to $f(R)$ that bounds the scalaron mass from above just by $M$. Simultaneously, such a change of $f(R)$ at large $R$ removes the “Big Boost” singularity (in terminology of Ref. \cite{27} where such a singularity appeared in a different context), which generic appearance in the model \cite{18} was shown in Ref. \cite{28}. However, the value of $M$ should be sufficiently large in order not to destroy the standard cosmology of the present and early Universe. In particular, its values considered in Refs. \cite{29,30} seem not to be high enough for this purpose. Indeed, the simplest way to solve one more problem of this $f(R)$ cosmological scenario (also noted in Ref. \cite{11}) – overproduction of scalarons in the early Universe – is, as usual, to have an inflationary stage preceding the radiation-dominated one. Then $M$ should not be smaller than $H$ during its last 60 e-folds, and for $M \approx 3 \times 10^{13}$ GeV the scalaron will be the inflaton itself according to the scenario \cite{9}.

Note that for sufficiently large $n$ the corresponding correction to $F(R)$ may become more important than the second term in the right-hand side of Eq. (18) already at the matter-dominated stage. For example, even for $M$ as large as $3 \times 10^{13}$ GeV, the term $R/3M^2$
becomes larger than $(R_0/R)^{n+1}$ for $R = 3 \times 10^{10} R(t_0)$ (corresponding to the matter-radiation equality) if $n \geq 10$. However, this does not affect the exact solution obtained in the previous section since at this moment $F_R/F \ll a^2/k^2$ for all scales of interest, so $G_{\text{eff}} = G$ in Eq. (15) irrespective of an actual structure of $F(R) - 1$. In turn, if $F_R/F \geq a^2/k^2$, the $R^2$ correction is negligible for all scales of interest at the matter-dominated stage. Therefore, this high-$R$ correction to the model (18) needed to obtain a viable cosmological model of the early Universe does not change our results.

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