Between theory and practice: guidelines for an optimization scheme with genetic algorithms - Part I:
single-objective continuous global optimization

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Abstract

The rapid advances in the field of optimization methods in many pure and applied science pose the difficulty of keeping track of the developments as well as selecting an appropriate technique that best suits the problem in-hand. From a practitioner point of view is rightful to wander “which optimization method is the best for my problem?”. Looking at the optimization process as a “system” of interconnected parts, in this paper are collected some ideas about how to tackle an optimization problem using a class of tools from evolutionary computations called Genetic Algorithms. Despite the number of optimization techniques available nowadays the author of this paper thinks that Genetic Algorithms still play a central role for their versatility, robustness, theoretical framework and simplicity of use. The paper can be considered a “collection of tips” (from literature and personal experience) for the non-computer-scientist that has to deal with optimization problems both in the science and engineering practice. No original methods or algorithms are proposed.

1 Introduction: optimization-as-a-process

Generally in literature, with the term "optimization" is related to (the output of) a mathematical technique or algorithm used to identify the extreme value of an arbitrary objective

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function through the manipulation of a known set of variables and subject to a list of con-
straining relationships. These mathematical and algorithmic techniques are the focus of the
majority of optimization literature. In this paper we want to look at the this issue from a
more abstract stance. In this context “optimization” also has a more general usage referring
to a system of physical objects (computers, machines) abstract ideas and people combined for
discovering and verifying the best implementable solution to high-dimensional design problems
in across a broad range of engineering and sciences application. In this more general usage, op-
timization can be conceptualized as a (complex) system composed with different sub-systems
is a multiphase process, in which applying a particular maximization algorithm or technique
is but one step. The success or the failure of the optimization process can depend on a single
step (right implementation of a algorithm e. g. what we refer to as the “maximization step”)
or for a not correct interaction of the different parts. So in what follow we will refer as the
technical-mathematical part of the optimization process as the:.

Maximization Problem\(^{1}\) A maximization problem with an explicit objective can in general
be expressed in the following generic form:

$$\max_{x \in \mathcal{H}} f(x),$$

where \(x\) is a give vector in a generic multidimensional space \(\mathcal{H}\) and \(f : \mathcal{H} \to \mathbb{R}\) is a scalar
function of the vector \(x\) and \(\mathcal{H} \subset \mathbb{R}^n\)is a (discrete or continuous) subset of the multidimensional
real Euclidean space. From now on we will refer to \(\mathcal{H}\) as the search space\(^2\).

According to systems theorists a system is comprised of a large number of (possibly non-
linearly) interacting elements \(^2\). The task to decompose the optimization system into smaller,
simpler sub-elements is partly arbitrary and can produce different conceptualizations.

The starting point to the systemic conceptualization of the optimization process is the
basic fact that optimization algorithms don’t exist in isolation but they represent one side

\(^{1}\) Of course it is always possible to transform a minimization problem into a maximization one trough the
transformation \(g(x) = -f(x)\).

\(^{2}\) Even if there is no explicit mention of constrains here the formulation is nonetheless enough general since
they can be incorporated through an appropriate definition of the search space \(\mathcal{H}\).
of a coin where the other side is represented by a problem (a function, fitness or cost) to be optimized. In turn a fitness can be the mathematical approximation of a physical process or system. There is no space here to fully develop the implication of a system-theoretic approach to the optimization process. Nonetheless this framework will stay at the background in the consideration that will follow. Two premises are important to be stressed at the outset. First the view of the optimization practice as a self-adjusting process. In a given application context can be useful to start with class of optimization algorithms and testing them with the problem at hand. The information collected about the problem and the response obtained from algorithms can allow updating both the parameters configuration of the single algorithm, the algorithm used and the overall optimization strategy itself. The “optimization-as-adaptive-process” framework means basically that maximization operations and knowledge about a give problem must precede hand-in-hand, like two sides of a coin, and that the information from one side can help improvements in the other. Regarding the specif algorithm to apply today the spectrum of methods for solving optimization problems are actually vast. Just to name a few: genetic algorithms, simulated annealing, tabu search, particle swarm, ant colony optimization, cross-entropy, etc. From a practitioner point of view is rightful to wander “which optimization method is the best for our problem?”. According to the most common understanding of the so called No Free Lunch Theorems [26] there is no optimization method superior to others for all possible optimization problems. Moreover, an algorithm that performs well on one class of problems must perform worse than random search on all remaining problems. Running an algorithm on a small number of problems with a small range of parameter settings may not be a good predictor of that algorithm’s performance on other problems, especially problems of a different type. It follows also that running after the last algorithm published in the literature claiming to be able to outperform in this or that class of test problems is not necessarily a clever approach. As a starting overall “optimization philosophy” the following steps can be followed [28]:

- The first approach essentially consists of using calculus tools on the target problem.

If the function is simple, use the stationary conditions (first derivatives must be zero)
and extreme points (boundaries) to find the optimal solution(s). If this is not possible then some well established conventional methods such as linear/nonlinear programming, convex optimization, and algorithms based on differential calculus such as the steepest descent method should be tried.

• If this again fails, more established evolutionary algorithms such as genetic algorithms and simulated annealing can be tried to tackle the problem.

• If these two options do not provide any satisfactory solutions, then try more exotic, nature-inspired metaheuristic algorithms such as particle swarm optimization, ant-bee algorithms, or rely algorithms or other class algorithms like Distribution Estimation, Cross-Entropy, MCMC etc.

Remember. It is important to understand why a given algorithm fails before trying a new one.

For this reason it important to get as more knowledge as possible about the problem. Only in this way it will be possible to adapt the best strategy to the actual problem. In this paper we will focus on optimization strategies based on the use of Genetic Algorithms (GA).

2 What you need to know about genetic algorithms

Before introducing a practical discussion about optimization strategy schemes, a few introductory comments on Genetic Algorithms are in order. GAs are a subclass of Evolutionary Algorithms and also an example of metaheuristic.

Remember. A heuristic is a technique (consisting of a rule or a set of rules) which seeks good solutions at a reasonable computational cost but it does not guarantee optimality. Metaheuristic is a high-level problem-independent algorithmic framework that provides a set of guidelines or strategies to develop heuristic optimization algorithms. They deal with a dynamic balance between diversification (global exploration of different areas of the search space) and intensification (to focus on the local fine-tune refinement of the so-far best candidate solution).
GAs are global, parallel, search and optimization methods that mimic the process of natural selection developed by Charles Darwin. Evolution is performed using a set of stochastic genetic operators, which select individuals for reproduction, produce new individuals based on those selected, and determine the composition of the population at the subsequent generation. Crossover and mutation are two well known operators [11]. Terminology of GA is borrowed from natural genetic and evolution theory. The basic idea of the evolution theory states that individuals with a greater “fitness to the environment” have a greater probability of surviving and a greater probability of winning the fights for mating. In such a way the genetic content of the best individuals will be more and more present in the following generations, since it will be transmitted by the offspring [16].

An Individual in genetic algorithm is identified with a chromosome. Information encoded in chromosome is called genotype. Phenotype is values of source task variables corresponding to genotype. In other words phenotype is decoded genotype. In the simplest codification of a genetic algorithm chromosomes are binary string of finite length. A gene is a bit of this string. Allele is value of gene, 0 or 1. One could, in principle, use any representation conceivable for encoding the variables and indeed it strictly depends on the problem: one needs to decide on a structure which is able to represent every possible solution to the desired problem. When the variables are naturally quantized, the binary GA fits nicely. However, when the variables are continuous, it is more logical to represent them by floating-point numbers and this is very common nowadays (for real encoding see the chapter 3 of [12]).

Remember. The way in which candidate solutions are encoded is a central, if not the central, factor in the success of a genetic algorithm. In their earlier work, Holland and his students concentrated on binary encoding (i.e., bit strings) and GA practice has tended to follow this lead. Much of that theory can be extended to apply to non binary encoding, but such extensions are not as well developed as the original theory [19].

Population is a finite set of chromosome. The fitness function (i.e. the corresponding values of the objective function \( f(x) \) in the maximization problem or some transformations of it) compares elements of the population, assigning each chromosome a fitness value. A chro-
mosome $x$ has a better fitness with respect to $y$ if $f(x) > f(y)$. This provides a means of ranking chromosomes from best to worst. Fitness of individual is value of fitness function on phenotype corresponding individual. In simple genetic algorithm size of population $N$ and binary string length $m$ is fixed and don’t changes in process of evolution [22]. Starting with a randomly generated population of chromosomes, a GA carries out a process of fitness-based selection and recombination to produce a successor population, the next generation. During recombination, parent chromosomes are selected and their genetic material is recombined to produce child chromosomes. After this step, in practical implementation a mutation operator is applied. Mutation perturbs the recombined solutions slightly to explore their immediate neighborhood. These then pass into the successor population. As this process is iterated, a sequence of successive generations evolves and the average fitness of the chromosomes tends to increase until some stopping criterion is reached. In this way, a GA “evolves” a best solution to a given problem. In process of evolution one population is replaced by another and so on, thus we select individuals with best fitness. So in the mean each next generation (population) is fitter than it predecessors. Genetic algorithm produces maximal fitness population, so it solves the maximization problem (fig. 2.1).

**Remember.** Initialization is typically just creating some $N$ individuals at random. However, if something about the likely initial “good” regions of the search space is known, it is a good strategy approach is to incorporate this knowledge into the initialization. In some cases can be convenient to seed the initial population with individuals selected from some sub-regions of the search domain according to the problem-specific information ([17], pp.
Summarizing, a GA works through the following steps: (1) creation of a random initial population of \( N \) potential solutions to the problem and evaluation of these individuals in terms of their fitness values; (2) selection of a pair of individuals as parents; (3) recombination through crossover of the parents, with generation of two children; (4) replacement in the population, so as to maintain the population number \( N \) constant; (5) genetic mutation. Genetic operators have been subject to intensive discussion, over both the composition and purpose of the various operators. Essentially, crossover tends to direct the search to superior areas of the search space, whilst mutation acts to explore new areas of the search space and to ensure that genetic material cannot be irretrievably lost. Choice of genetic operators must be made together with choice of representation. Choices of values for the parameters of the operators, such as mutation rate and mutation size, are critical to the success of the algorithm (see below).

Although GAs have enjoyed significant practical success, attempts to establish a theoretical account of their precise operation have proved more difficult. There are two goals for a satisfactory theory of GAs. The first is to explain which classes of problem GAs are particularly suitable for and why. The second is to provide techniques and approaches for optimal design and implementation of GAs, as there are many choices of structure and parameters to be made. The Schema Theorem [13] has stood for a long time as a central result of GA theory for the binary-encoded case. It attempts to explain how the evolutionary processes in a GA can locate optimal or near-optimal solutions, even though they sample only a tiny fraction of the set of all possible solutions (for more details see [18] and [1]). In essence, selection, recombination, and mutation work together to combine small pieces of salient information (called building blocks) from different chromosomes to form “good” solutions. When GA work well, chromosomes containing good building blocks will, on average, outperform chromosomes containing inferior building blocks. Thus, each successive generation is populated with chromosomes containing more and more good building blocks (i.e., better quality solutions). The goal of the GA is to ultimately find a chromosome that is comprised of the best building blocks (i.e., an optimal solution to the problem). When a GA converges to a non-optimal solution, it is either
because crossover does not exchange the correct material or mutation does not explore the proper material. It has been demonstrated that premature convergence due to mining failure is primarily due to a poor match between encoding and genetic operators [21].

GA have demonstrated their potential in numberless projects since they are easy to use. They are problem independent, i.e. they neither need a special representation for candidate solutions nor put any other restricting requirements on the problems to be tackled. Thus they can be viewed as universally applicable. However, practice shows that any successful application depends on on careful tuning of operators, parameters, and problem-dependent aspects. This observation is supported by the No Free Lunch theorems which have shown that, on average, any deterministic or stochastic optimization algorithm is as good as any other algorithm on sets of problems that are closed under the permutation of fitness values. This implies that, on the one hand, each algorithm has a niche within the space of all problems and, on the other hand, for each optimization problem the algorithm must be tailored to the problem ([24]).

Practitioners must be aware that a GA is nothing else than a random search, together with a mechanism that, according to the evolutionary logic, tries to guess the next potential solution to evaluate. This can leave skeptical about the effectiveness of GAs, because there is no guarantee that such an algorithm (based on random choices) will always find the global optimum. According to [6], as an empirical rule in this case we can say that:

- Given a particular maximization problem class, if the researcher already knows the best solution for a number of representative in that class and

- if, over a significant number of runs, the proposed GA finds a solution that in average is 99% as good as the known optimum values then,

- one can reasonably expect that on a new instance of the problem for which the solution is not known, the solution found by the GA will be 99% as good as the unknown optimum over a significant number of runs.
Despite the number of optimization techniques available nowadays the author of this paper thinks that Genetic Algorithms still play a central role in coping with a wide variety of optimization problems without any a priori assumptions about their continuity and differentiability. Even if they still lack a strong theoretical foundation able to understand or predict the dynamics other than a superficial level, the biological analogy is conceptually easy to understand and it is relatively simple to develop program code for any kind of application. This is common to most of nature-inspired computational techniques and can explain the speed of their diffusion virtually in every area of applied sciences far outside the field of computer sciences. Several tricks exist for improving the performance of GAs and affect the final result. Some of them will be examined in the rest of the paper.

2.1 Implementation issues

Several GAs have been developed; beyond the common basic idea (mimicking the evolution of a species), they can have relevant differences. A genetic algorithm for a particular problem must have the following five components ([5], pp. 94).

1. A representation for potential solutions to the problem.

2. A way to create an initial population of potential solutions.

3. An evaluation function that plays the role of the environment rating solutions in terms of their fitness.

4. Genetic operators that alter the composition of children.

5. Values for various parameters that the genetic algorithm uses (population size probabilities of applying genetic operators etc.)

As already stated, representation of candidate solutions is a critical component of a GA. One may either use the original representation of candidate solutions or one may map the original representation into binary strings, real-valued vectors [27] or another representation. A good starting point is to use the original representation for the particular problem with no significant
modifications or transformations. In the initial stage, modifications should be done only if the chosen implementation necessitates them. If the results obtained are not satisfactory, alternative representations should be examined. As far as operators are concerned, different possibilities are described in the literature. In the case of real encoding, very common nowadays for global mono-objective optimization, an intermediate recombination operator can be the best choice. This is usually based on some kind of average/mixture among multiple parents: if \( x = (x_1, \ldots, x_n) \) and \( y = (y_1, \ldots, y_n) \) are two parents, then we can define the component of the children:

\[
x_i' = \alpha x_i + (1 - \alpha) y_i \quad \alpha \in [0, 1]
\]

The above formula can be extended to more than two parents \( x_i' = \alpha_1 x_i + \alpha_2 y_i + \alpha_3 z_i + \cdots \) with \( \Sigma_i \alpha_i = 1 \).

The mutation operator than will operate of on (a fraction of) genes of the children. The mutation operator in the real encoded case can be defined as component wise addition of normal distributed random numbers. A mutation of the individual’s parameters vector is calculated as

\[
x_i' = x_i + \sigma N(0, 1)
\]

where \( N(0, 1) \) is the Gaussian distribution of mean 0 and standard deviation 1 and \( \sigma \) is a real parameter that define the actual standard deviation wanted\(^3\).

There are a number of resources freely available on the Internet for those interested in applying GAs in their own area. Tool-kits are available in many programming languages and vary widely in the level of programming skill required to utilize them. Those implementation most likely includes numerous operators and predefined set of parameters to consider; sticking with the default choices should be a good starting point. If the results are not satisfactory, different operators may be examined and specialized operators may be designed. There are many details that are outside of the scope of this paper, but they can be found in specialized publications like [12] and [19]. Once a GA implementation is up and running, it is important

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\(^3\) In some variants the value of \( \sigma \) can be also adapted during the execution of the algorithm trough a mechanism that change (usually reduce) its value generation after generation. For more details see: [9].
to determine how well it is working, and to adjust things if it is not working well. One of the simplest analysis involves observing the top fitness vs. generation time plot. One easy way to begin experimenting with GAs is the Genetic Algorithm Toolbox in MATLAB [4].

### 3 Typical problems in optimization

In practice, the complexity of a given problem can be ascribed to the following basic causes:

- **High number of independent variables.** The large number of candidate solutions to an optimization problem makes it computationally very hard to be attacked by evolutionary algorithms because the number of candidate solutions grows exponentially with increasing dimensionality. This fact, which is frequently named *the curse of dimensionality*, is well known by practitioners that have to handle problems with hundreds of variables. This phenomenon can be easily understood by first considering an $n$-dimensional binary search space. Here, adding another dimension to the problem means a doubling of the number of candidate solutions. So in order to obtain reliable optimization result with GA, the amount of data required to be sampled from the search space will grow exponentially with the dimensionality. The way to overcome this limitation is one of the most intriguing theoretical and practical area of research at the intersection of mathematics, statistics and computer science that has already produced a vast literature.\footnote{For the non-expert in the field can be a good starting point to consult the Wikipedia entry \url{http://en.wikipedia.org/wiki/Curse_of_dimensionality}}

- **Very complex or irregular response surface.** In landscape surface with weak (low) causality, small changes in the candidate solutions often lead to large changes in the objective values, i.e. ruggedness. Stated informally, a landscape is rugged if there are many local optima of highly varying fitness concentrated in any constrained region of the space. It then becomes harder to decide which region of the problem space to explore and the optimizer cannot and reliable gradient information to follow. A small modification of a very bad candidate solution may then lead to a new local optimum and the best candidate solution currently known may be surrounded by points that are inferior to all other
• **Fitness evaluation.** Evaluating a solution in the objective space can be by far the most computationally expensive step of any optimization process for difficult or large-size optimization problems. Finding the optimal solution to complex high dimensional, multi-modal problems often requires very expensive fitness function evaluations. For most evolutionary algorithms, a large number of fitness evaluations (performance calculations) are needed before a well acceptable solution can be found [14]. In this case, it may be necessary to forgo an exact evaluation and use an approximated fitness (also called *metamodel* or *surrogate fitness*) that is computationally more efficient. Functional surrogate models are in practice algebraic representations of the true problem functions. The most popular ones are polynomials (often known as *response surface methodology* see section 4.1), Interpolation and regression polynomial techniques can be classified in this category. Other several models are now commonly used for fitness approximation, like the Kriging model, neural networks, including multi-layer perceptrons, radial-basis-function networks and the support vector machines. One can say that functional models are typically based on the following components: a class of basis functions, a procedure for sampling the true functions, a regression or fitting criterion, and some deterministic or stochastic mathematical technique to combine them all. For a comprehensive review see [23] and [7].

These are very difficult cases to be attacked by evolutionary algorithms, no special tricks exist which can directly mitigate for example the effects of rugged fitness landscapes. In GA, using large population sizes and applying methods to increase the diversity can decrease the influence of ruggedness, but only up to a certain degree. The lower the causality of an optimization problem, the more rugged its fitness landscape is, which leads to a degradation of the performance of the optimizer. This does not necessarily mean that it is impossible to find good solutions, but it may take very long to do so [25].
4 Tips for an optimization scheme

One important objective of a preliminary optimization process is how to get a better understanding of the objective space. A useful concept commonly adopted in the field of meta-heuristics is the notion of a fitness landscape, i.e. the (hyper)surface obtained by applying the fitness function to every point in the search space over which search is being executed. Given a specific landscape structure – defined by a search space, objective function, and search operators (crossover, mutation), a GA metaheuristic can be viewed as a strategy for navigating this structure using the information provided by the guiding fitness function. This is a commonly used metaphor; it allows interpreting the search in terms of well-known topographical objects such as peaks, valleys, mesas, etc., of great utility to visualize the behavior of the search \[8\]. Knowledge of the fitness landscape structure is key to developing effective algorithms, and consequently it has been a primary focus in the theoretical analysis of metaheuristic methods. Here we define fitness landscape analysis as the step of exploring information about the structure of the problem through an analysis of the fitness function to be maximized. For a continuous global optimization problem, there are several traditional ways to categorize the objective function \(f(x)\) according to some properties like continuity, geometry, symmetries, multi-modality, ruggedness, etc. The knowledge of these properties can give insights about the best algorithm to use or appropriately refining the region of interest to be sampled in the search space. For example, can be important to understand if the function is decomposable. Decomposability is sometimes also referred to as separability, i.e. functions expressed as the sum of element functions on which small subsets of variables have disjoint effects. The optimization process of a decomposable objective function can be performed in a sequence of \(N\) independent optimization processes, where each parameter is optimized independently. In the case where the function manifest an additive decomposability property it will be probable the search along the coordinate axes. Since optimization methods in real-life industrial design problems strictly depend on the “geometry” of the problem to be solved, the more about the response surface is known the better is possible to tune the optimization strategy. To this end a series of tests in which changes in the input variables are set in order to recognize the
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reasons for changes in the output response.

4.1 Fitness landscape analysis

Response Surface Methodology (RSA) stems from the area of experimental design where the aim is to extract the maximum amount of information from a given system (represented by a mathematical function) with few and selective experiments. More technically, RSM is a collection of mathematical and statistical techniques for empirical model building. Through a series of computer experiments carefully designed it is possible to identify the behavior of changes in the fitness output. Due to the lack of data and the "curse" of the high dimensionality of the search space, can be very difficult to obtain a perfect global functional approximation of the original fitness function. A desirable design of experiments should provide a distribution of points throughout the region of interest, which means to provide as much information as possible on the problem. The "space-filling" methods like Latin hypercube sampling are now easily accessible in dedicated scientific software like MATLAB. If there is no space for a systematic exercise of RSA, GAs can be applied preliminary as a tool to scan the fitness landscape. A great advantage of GAs (and other population-based strategies) over the classical techniques is that at the end of the elaboration the user is given not just an "optimal" solution, but also a population of extremely good solutions, usually having very similar responses.

When the optimization problem includes less than three variables, graphical methods can be fruitfully used to gain understanding on the nature of the search space. Graphical methods are often used even when the number of variables exceeds two. In that case, a practical stratagem can be to allow two variable to vary and to freeze the others. The 3D plot of the outcome can give some insights about the fitness landscape but we must be aware this approach is limited and can conduct to misleading conclusions sometimes.

4.2 Managing the parameters

Evolutionary algorithms are characterized by many parameters which may be used for tuning an algorithm for a specific problem. Having selected an encoding, there are many other
choices to make. These include: the form of the fitness function; population size; crossover and mutation operators and their respective rates; the evolutionary scheme to be applied; and appropriate stopping/restart conditions. The usual design approach is a combination of experience, problem-specific modeling and experimentation with different evolution schemes and other parameters. Those parameters cannot be understood in isolation but rather as an interweave network. Changing one parameter has significant impact on the effect of other parameters. Good parameter settings differ from problem to problem and cannot be transferred to algorithms using operators with different characteristics. Using GA, good suggestions for the setting of the parameters are as follows.

Population. An important feature of a population is its genetic diversity: if the population is too small, the scarcity of genetic diversity may result in a population dominated by almost equal chromosomes and then, after decoding the genes and evaluating the objective function, in the quick convergence of the latter towards an optimum which may well be a local one. At the other extreme, in too large populations, the overabundance of genetic diversity can lead to clustering of individuals around different local optima. The population size should increase with problem size \( n \) (the number of bits or variables), and the larger the population size, the better the obtained solution should be [3]. Computational efficiency issues must be considered. The total CPU time used in an optimization run is proportional to: \((\text{population number}) \times (\text{number of generation}) \times (\text{time required for each fitness function evaluation})\). This leads to a trade-off between having large, diverse populations that explore parameter space widely, and having smaller populations that explore longer. In practice, the choice is problem dependent. As a starting point for initial runs, the population size \( N_{\text{pop}} \) may be set to the problem size, \( N_{\text{pop}} = n \). If using \( N_{\text{pop}} = n \) is infeasible, one may start with smaller values of \( N_{\text{pop}} \), such as \( N_{\text{pop}} = 50 \). Changing the population size and rerunning the GA with larger and smaller populations (e.g. \( N_{\text{pop}}/2 \) and \( 2N_{\text{pop}} \)) provides an indication of whether the current population size is adequate. If increasing the population size leads to better solutions, the population size should be increased. If increasing the population size leads to solutions of about the
same quality, the current population size is likely to suffice. Of course, if the evaluation of the objective function is computationally intensive, large populations may become intractable and, to run a sufficient number of generations, one is often forced to use a relatively small population size.

Reproduction. It is important to adopt “elitism”: the $k$ best individuals of each generation go directly to the next generation. The use of elitism guarantees that the maximum fitness of the population never decreases from one generation to the next and it normally produces a faster convergence of the population. Furthermore, the time required for a generation is smaller. With high elitism the risk is that all the chromosomes are quite similar, around a good maximum, and that it will be impossible to get out of that region: the only possibility would be landing by chance on a higher peak, with a higher response. Summarizing, a reproduction without elitism has a higher exploration, while the higher the elitism the higher the exploitation. Here too, the problem is finding a good balance.

Crossover. The probability of crossover in GAs is typically quite large. Some authors suggest that $pc = 0.6$ regardless of the number of bits $n$. Many researchers use an even larger probability of crossover, for example, $pc = 0.9$ or $pc = 1$.

Mutation. The probability of mutation is typically set so that the expected number of modified bits is fixed regardless the number of parameters considered and that only a few bits are expected to change on average. For example, one may use $pm = 1/n$. Some researchers use larger values of $pm$, but in many cases the reasons for increasing the probability of mutation are that the remaining parameter settings are inadequate [20].

4.3 Hybrid strategies

Experimental results show that for most GAs (initialized with random values), evolution makes extremely rapid progress at first, as the diverse elements in the initial population are combined and tested. Over time, the population begins to converge, with the separate individuals resembling each other more and more. Effectively this results in the GA narrowing
its search in the objective space and reducing the size of any changes made by evolution until eventually the population converges to a single solution. In contrast, “hill-climbing” or “gradient ascent” algorithms, which initialize at some point and then move greedily toward the nearest local optima, are maximally exploitative, since they make no attempt to explore alternative solutions away from its current trajectory. In hybrid algorithms, a GA with good exploration capacities are often used to locate some promising zones within the wide search space, while the local optimization methods exploit the located promising zones to achieve the best solution quickly and accurately. So the hybrid algorithms not only have the exploration capabilities, parallelism and combination of GAs, but also obtain the exploitation power of local search methods \[10\]. Also Holland himself, the “father” of genetic algorithms, suggested that the genetic algorithm should be used as a pre-processor for performing the initial search, before invoking a local search method to optimize the final population. Simply stated, the theory suggests that the key role of the global searcher is to find one or a number of good regions and the key role of the local searcher is to and local optima within these regions. Typically a GA followed by quasi deterministic gradient search type algorithm is the easiest choice when the fitness is known to be differentiable. If not, other possibilities for a local search step can be explored from the family of random search like the Simulated Annealing algorithm\(^5\) (see appendix 1). So summarizing we have:

\[
\text{GeneticAlgorithm + LocalSearchAlgorithm = HybridAlgorithm.}
\]

One of the key issues in the design of efficient hybrid procedures is the appropriate coordination of the global and local search. In this case it is important how to manage the improved solution that is produced by the local search. The two main possibilities are captured in the literature by the terms Lamarckian and Baldwinian hybrid \[15\]. In a Lamarckian hybrid, the local search procedure again uses the global procedure’s value as a starting point, searches until reaching an optimum, and then passes back the function value found with back-substitution of the solution value found. That is, suppose that individual \(x\) belongs to the population \(P_t\) in generation \(t\) and that the fitness of \(x\) is \(f(x)\). Furthermore, suppose that the local search pro-

\(^5\) In this case, parameters of SA must be tuned carefully to avoid expensive global search overlap.
duces a new individual $x'$ with a better fitness ($f(x') > f(x)$ for the maximization problem). The designer of the algorithm will replace $x$ with $x'$, in which case $P_{t+1} = P_t \setminus \{x\} \cup \{x'\}$ and the genetic information in $x$ is lost and replaced with that of $x'$. In a Baldwinian hybrid, the local search procedure uses the GA value as a starting point, searches until reaching an optimum, and then passes back the function value found without back-substituting the solution value found. In this case the genetic information of $x$ is kept but its fitness altered: $f(x) = f(x')$. At the first sight, Lamarckian hybrids may seem more intuitive, but in some cases back-substitution of solution values found by local search has been found to reduce solution variance to a point such that subsequent exploration is diminished. Various rules of thumb have suggested Baldwinian moves with a small percentage of Lamarckian moves as desirable in practice (for a comprehensive analysis on this see [10]). Finding an adequate way to split resources between the global and local search requires a systems-level understanding of the roles of the two procedures.

5 Appendix 1

Simulated Annealing

The simulated annealing algorithm for optimization imitates the annealing process in metallurgy, a technique involving heating and controlled cooling of a material to increase the size of its crystals and reduce defect. By analogy with this physical process, each step of the simulated annealing algorithm replaces the current solution by a new solution with a probability that depends on the difference of objective function values at the two solution points and a parameter called the temperature. A simulated annealing algorithm always accepts moves that decrease the value of the objective function (to be maximized). Moves that increase the value of the objective function are accepted with probability $p = e^{\Delta/T}$ where $\Delta$ is the change in the value of the objective function and $T$ is the temperature. A random number generator that generates numbers distributed uniformly on the interval $(0, 1)$ is sampled, and if the sample is less than $p$, the move is accepted. The temperature $T$ is initially high. Therefore,

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See chapter 12 of [28]
the probability of accepting a move that increases the objective function is initially high. The temperature is gradually decreased as the search progresses. I.e., the system is cooled slowly. In the end, the probability of accepting a move that increases the objective function becomes vanishingly small. In general, the temperature is lowered in accordance with an annealing schedule procedure like for example $T_{k+1} = \beta T_k$, $0 < \beta < 1$.

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