A controlled-NOT logic gate based on conditional spectroscopy has been demonstrated recently for a pair of superconducting flux qubits [Plantenberg et al., Nature 447, 836 (2007)]. Here we study the fidelity of this type of gate applied to a phase qubit coupled to a resonator (or a pair of capacitively coupled phase qubits). Our results show that an intrinsic fidelity of more than 99% is achievable in 45ns.

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I. INTRODUCTION

Approaches to the development of a large-scale quantum computer face numerous practical challenges [1–3]. One such challenge, the construction of a robust controlled-NOT (CNOT) logic gate, has been explored from several perspectives [4–10]. In this work we analyze a CNOT gate based on conditional spectroscopy for a superconducting phase qubit coupled to a resonator. A related construction has already been demonstrated for a pair of superconducting flux qubits [11] and a similar concept has been explored in the context of NMR quantum computing [12]. Although this approach can be applied to a variety of physical systems, the fidelity observed in the experiment of Ref. [11] is not sufficient for practical use. Here we propose a method to improve the intrinsic fidelity and we then calculate the optimal fidelity as a function of total gate time. The phase qubit coupled to resonator system we consider is relevant to the UCSB Rezqu architecture [13].

The idea behind a spectroscopic CNOT gate is simple and has a wide range of applicability: A \( \pi \) pulse is applied to the target qubit with a carefully selected carrier frequency \( \omega_{rf} \). The carrier frequency is close to the qubit transition frequency given that the attached control resonator is in the “on” or \( |1\rangle \) state, which in the \( |qr\rangle \) basis is

\[
\omega_{on} = \frac{E_{11} - E_{01}}{\hbar}. \tag{1}
\]

The Rabi frequency has to be smaller than the detuning to the “off” transition at

\[
\omega_{off} = \frac{E_{10} - E_{00}}{\hbar}. \tag{2}
\]

A direct \( \sigma^z \otimes \sigma^z \) coupling between the devices would of course generate a difference in \( \omega_{on} \) and \( \omega_{off} \), but in the phase qubit plus resonator system—which has no direct \( \sigma^z \otimes \sigma^z \) coupling—such an interaction is generated by level repulsion from the noncomputational \( |2\rangle \) states.

The difference \( |\omega_{on} - \omega_{off}| \) characterizes the sensitivity of the conditioning effect and determines the speed of the resulting gate.

When the qubit and resonator are detuned by an amount larger than the coupling between them, they become weakly coupled. In this limit, the sensitivity for current devices is limited to a few MHz, which is not sufficient for practical application. Therefore to amplify the sensitivity we adiabatically bring the target qubit to a suitable point near resonance with the control resonator, and drive the qubit while it is strongly coupled with the resonator. After performing a \( \pi \) pulse the qubit is adiabatically detuned from the resonator.

Two main sources of intrinsic errors exist in this approach: Although we set the carrier frequency \( \omega_{rf} \) to a value such that we have a \( \pi \) pulse in the qubit when the resonator is in \( |1\rangle \), there is a small probability for the qubit to get rotated even if the resonator is in \( |0\rangle \). The second error comes from the fact that, since we are driving the qubit, it is possible to have leakage to the qubit \( |2\rangle \) state. The fidelity will reach its maximum value when both of these errors are minimized simultaneously. We use the DRAG method [14] to suppress the error due to leakage and adjust all other parameters by optimization.

II. CNOT DESIGN

A. Hamiltonian

In the basis of uncoupled qubits, the Hamiltonian of a qubit capacitively coupled to a resonator (assuming harmonic eigenfunction of 2-states) is given by (suppressing \( \hbar \)),

\[
H = H_0 + H_{ef} + H_{int}, \tag{3}
\]
where,

\[ H_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & 2\epsilon - \Delta \end{pmatrix}_q + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 2\omega \end{pmatrix}_r \]

\[ H_{it} = \Omega(t) \cos(\omega_{rt}(t) + \phi(t)) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}_q \]

\[ H_{int} = g \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -\sqrt{2}_q \\ 0 & \sqrt{2} & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -\sqrt{2}_r \\ 0 & \sqrt{2} & 0 \end{pmatrix} \]

where, \( \epsilon, \omega \) and \( \Delta \) are qubit frequency, resonator frequency, and anharmonicity of the qubit, respectively. \( \Omega, \omega_{rt} \) and \( \phi \) are Rabi frequency, carrier frequency, and phase of the microwave pulse. \( g \) is the (time-independent) interaction strength between qubit and resonator.

The first six energy levels of the system (obtained numerically) are shown in Fig. 1. We use coupling \( g/h = 115 \text{ MHz} \), resonator frequency \( \omega/h = 6.5 \text{ GHz} \) and anharmonicity of the qubit \( \Delta/h = 200 \text{ MHz} \). The simulations discussed below are carried out in a frame rotating with the instantaneous frequency of the qubit.

**B. CNOT Protocol**

Eigenstates of the full Hamiltonian reduce to the eigenstates of the uncoupled Hamiltonian far away from the resonance. We denote the first six eigenstates of the full Hamiltonian by \( E_{00}, E_{01}, E_{10}, E_{02}, E_{11}, E_{20} \) and define a conditional control sensitivity \( S_c \) and leakage sensitivity \( S_l \) as

\[ S_c \equiv |(E_{11} - E_{01}) - (E_{10} - E_{00})|, \]

\[ S_l \equiv |(E_{21} - E_{11}) - (E_{11} - E_{01})|. \] (5)

The conditional control sensitivity is the (magnitude of the) difference between \( \omega_{on} \) and \( \omega_{off} \). Leakage sensitivity is the anharmonicity of the target qubit when control resonator is on. In order to achieve a high fidelity both of these quantities need to be maximized (by varying the detuning \( \epsilon - \omega \)).

A plot of these sensitivities is shown in Fig. 2. Peaks in the control sensitivity (resulting from expected anticrossings) at detuning equal to zero and \( \Delta \) conspire to give the maximum at 100 MHz detuning shown in the blue curve of Fig. 2. Operating near 100 MHz detuning, however, leads to poor performance because of the large leakage error there. Better operation points exist near -100 and 200 MHz detuning; we shall make use of the latter. Fig. 3 shows the behavior of sensitivities vs. coupling at \( (\epsilon - \omega)/h = 215 \text{ MHz} \).

To implement a CNOT gate we begin with a strongly detuned qubit-resonator system. Then the qubit is adiabatically tuned into resonance with the resonator, driven with a \( \pi \) pulse, and finally detuned. In the \( |qr\rangle \) basis this protocol ideally produces

\[ U_{\text{target}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \text{SWAP} \times \text{CNOT} \times \text{SWAP}, \] (6)
where

\[
\text{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

is the swap gate. To obtain this target we also perform \( z \) rotations on the qubit before and after the sequence described above, with angles determined by optimizations.

### C. Local Rotations with DRAG

As far as leakage outside the computational subspace is concerned, we can consider \( E_{01}, E_{11} \) and \( E_{21} \) to be a single 3-level quantum system where we are interested in local rotations between the first two levels and therefore, in order to suppress the errors due to leakage to the third level, local rotations are performed with a DRAG pulse \([14]\), up to \( 5^{th} \) order. In order to do a local rotation of angle \( \theta \) about the \( x \) axis, we set the Rabi pulse of the Hamiltonian to be

\[
\begin{align*}
\Omega(t) \cos(\phi(t)) &= f_\theta + \frac{(\lambda^2 - 4) f^2_{\phi}}{8 \Delta^2} - \frac{(13\lambda^4 - 79\lambda^2 + 112) f^2_{\phi}}{128 \Delta^4}, \\
\Omega(t) \sin(\phi(t)) &= -\frac{f_\phi}{\Delta} + \frac{33(\lambda^2 - 2) f^2_{\phi}}{24 \Delta^2}, \\
\omega_{\text{rf}}(t) &= \omega_c + \frac{(\lambda^2 - 4) f^2_{\phi}}{4 \Delta^2} - \frac{(\lambda^4 - 7\lambda^2 + 12) f^2_{\phi}}{16 \Delta^4},
\end{align*}
\]

where, \( \lambda = \sqrt{2} \) for our case and the first two equations give the amplitudes of \( \cos(\omega_{\text{rf}}(t)) \) and \( \sin(\omega_{\text{rf}}(t)) \) quadratures. Here \( \omega_c \) is found via optimization around \( |E_{01} - E_{11}| \) and \( f_\phi(t) \) is chosen to be a Gaussian function (vertically shifted) such that \( f_\phi(0) = f_\phi(t_g) = 0 \) and \( \int_0^{t_g} f_\phi(t) \, dt = \theta \), \( t_g \) being the time required to perform the local rotation.

### III. GATE OPTIMIZATION

In this section we describe the sources of error and our methodology to compute the intrinsic fidelity curve. As mentioned above, there are two major intrinsic sources of error. In our simulation we use the full nine-level Hamiltonian to compute the fidelity curve and use DRAG to suppress the leakage error while the control error is treated with an adjustment of other controllable parameters via optimization.

The state-dependent process fidelity between two quantum gates \( U \) and \( U_{\text{target}} \) is given by,

\[
F(U_{\text{target}},U,|\psi\rangle) \equiv \left| \langle \psi | U^\dagger_{\text{target}} U | \psi \rangle \right|^2.
\]

The fidelity measure we use in this work is the process fidelity averaged over the four dimensional Hilbert space, that can be shown \([13,16]\) to be exactly equal to a trace formula given by,

\[
F(U_{\text{target}},U) \equiv \frac{\text{Tr}(U U^\dagger)}{20}.
\]

where \( U_{\text{target}} \) is always four dimensional for a two-qubit operation and \( U \) is the time evolution operator projected into the four dimensional computational subspace of the entire Hilbert space. This definition assumes that \( U_{\text{target}} \) is unitary, but \( U \) does not have to be (because the final evolution operator after projection into the four-dimensional computational subspace is not necessarily unitary).

In order to compute the fidelity curve, we perform a 8 dimensional optimization over the control parameters for a given total gate time and obtain the best fidelity. The 8 control parameters optimized here are coupling \( (g) \), qubit frequency in the intermediate state (see Fig.1), carrier frequency of the Rabi pulse \( (\omega_c) \), angle of rotation by the Rabi pulse, duration of each ramp pulse \( (t_{\text{ramp}}) \) and three other parameters related to the areas of the gaussian envelope of DRAG pulse. The result obtained from such an optimization corresponds to a single point in the plot of Fig.4. In order to help to control the phases of the matrix elements developed in the time-evolution operator, we also attach a pre and a post \( z \)-rotation of the qubit. We observe from Fig.2 and Fig.3 that a good range for driving point \( (\epsilon - \omega) \) should be \( 200-250 \) MHz and a good range for coupling strength should be \( 100-125 \) MHz. These numbers are used as guess values of our multidimensional optimization search.

Fig.4 shows the fidelity curve obtained from optimization and TableII shows corresponding coupling strengths, duration of each ramp and driving points. Our
As an example, we show the change of qubit frequency (in GHz) over time (in nanoseconds) in Fig. 5 for the CNOT having total gate time = 45 ns. while the resonator frequency is always fixed at 6.5 GHz and pre and post z-rotation angles are found to be $\vartheta_{\text{pre}} = -1.0915$ and $\vartheta_{\text{post}} = 0.5442$ radian for this case. We use linear pulse for ramps and a gaussian envelope is used for DRAG.

![Qubit frequency vs. time for CNOT at total gate time=45 ns.](image)

FIG. 5: (Color online) A plot of qubit frequency vs. time for CNOT at total gate time=45 ns.

### IV. CONCLUSIONS

We have computed intrinsic fidelity of a CNOT gate based on conditional spectroscopy approach and have shown that it is possible to achieve greater than 99% fidelity within an experimentally practical time scale of 45ns. However, this design requires a large coupling strength, so tunable coupling [17] would probably be required in a multi-qubit system. Although our analysis assumed a phase qubit and resonator, the design also applies to capacitively coupled phase qubits, where a slightly higher fidelity would be expected because of the additional anharmonicity.

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