Hadronization cross–sections
at the chiral phase transition of a quark plasma

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ABSTRACT: Hadronization at finite temperature $T$ is discussed in the framework of the Nambu–Jona-Lasinio model. The differential cross-section for the conversion of a quark–antiquark pair into two pions to first order in a $1/N_c$ expansion is calculated as a function of the c.m. energy $s$ and temperature $T$. In particular, approaching the temperature $T_c$ of the chiral phase transition, the hadronization cross-section diverges like $\ln|1 - \frac{T}{T_c}|$.

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Our current understanding of the hadronic world via quantum chromodynamics (QCD) appears to be characterized by (a) confinement, in that no free quarks and gluons are observed, and (b) broken chiral symmetry, manifesting itself in a non-vanishing value of the quark condensate $< \bar{\psi} \psi >$ and in a small pion mass. In particular, chiral symmetry is essential in determining the static properties of the light hadrons. In ultra-relativistic heavy ion collisions, nuclear matter is heated up to a degree where one might penetrate into the region of both a chirally symmetric and a deconfined phase of matter, at a transition temperature in the region $T_c \sim 100 - 200$ MeV [1,2]. In the cooling process the deconfined partons will recombine into the observed hadrons, which are mainly pions. It is thus important to understand how hadronization rates for processes such as the conversion of a quark–antiquark pair into two pions, $q\bar{q} \rightarrow 2\pi$, behave as a function of the c.m. energy $\sqrt{s}$ and the temperature $T$, and especially what the influence is of a phase transition on these rates.

In this paper, we study the hadronization rates for $q\bar{q} \rightarrow 2\pi$ in a model system that displays chiral symmetry, the flavor $SU(2)$ Nambu–Jona-Lasinio (NJL) model [3,4]. We are aware of the deficiencies of the model— the lack of confinement and non-renormalizability of the effective 4-fermion interaction, but also of its strengths—the transparent description of the chiral phase transition at critical temperature $T_c$, with the attendant binding of quarks into mesons. For temperatures $T \geq T_c$, no hadrons exist in the model, and one has a system of interacting quarks with mass $m = 0$ in the chiral limit. For $T \leq T_c$, in the domain of broken symmetry, a quark-meson plasma is formed with $m \neq 0$, pion mass $m_\pi = 0$ and sigma mass $m_\sigma = 2m$. While the situation for $T > T_c$ is fairly realistic (except for the absence of gluons), the coexistence of quarks and mesons below $T_c$ is an artifact of the NJL-model. Yet with due caution, one may still extract meaningful, physical information from the NJL model about hadronization in the vicinity of the phase transition itself.

The NJL Lagrange density determining the chiral dynamics of quarks and mesons in
flavor SU(2) is
\[
\mathcal{L}(x) = \bar{\psi}(x) (i \gamma \not{\partial} - m_0) \psi(x) + G [(\bar{\psi}(x) \psi(x))^2 + (\bar{\psi}(x) i \gamma_5 \vec{\tau} \psi(x))^2],
\]
(1)
where \(G\) is a coupling constant of dimension [Mass]^{-2}, and \(m_0\) is the current quark mass. In this paper, we work for the most part in the chiral limit \(m_0 = 0\), where the system displays a phase transition at a critical temperature \(T_c\). For simplicity we restrict ourselves to \(\mu = 0\), i.e. zero baryon density, which corresponds to the central rapidity region of heavy ion collisions. A three dimensional regulator \(\Lambda\) will be imposed on all divergent quantities. Since the NJL model is a strong–coupling theory, perturbation theory is inapplicable, and we require a selection procedure for the relevant Feynman graphs. We choose the \(1/N_c\) expansion [5-7], and work to lowest order. To \(O(1/N_c)\), the relevant T-matrix amplitudes for the reaction \(q\bar{q} \rightarrow 2\pi\) are sketched in Fig.1. Figure 1(a) depicts the s-channel amplitude \(T^{(s)}\) in which an intermediate \(\sigma\)-meson is produced, while Figure 1(b) displays quark exchange in the t-channel, the amplitude being denoted by \(T^{(t)}\). It is understood that the crossed diagrams are included for a process in which the final state particles are identical, e.g. for \(u\bar{u} \rightarrow 2\pi_0\).

Using an obvious notation for the T-matrix amplitudes, one has
\[
T^{(s)}(k, p_3; T) = \bar{v}_2 u_1 \frac{2G}{1 - 2G \Pi_\sigma(k; T)} g_\pi^2 A_{\sigma\pi\pi}(k, p_3; T) \delta_{c_1, c_2},
\]
(2)
where the symbol \(\delta_{c_1, c_2}\) refers to the color degree of freedom and \(A_{\sigma\pi\pi}\) represents the amplitude of the triangle vertex \(\sigma \rightarrow \pi\pi\). In terms of the intermediate \(\sigma\)-meson four–momentum \(k\) and the four–momentum \(p_3\) for one external pion, \(A_{\sigma\pi\pi}\) is obtained by analytic continuation of
\[
A_{\sigma\pi\pi}(i\nu_l, \vec{k}; i\mu_m, \vec{p}_3; T) = \sum_n e^{i\omega_n \eta} \int \frac{d^3 q}{(2\pi)^3} S(i\omega_n, \vec{q}) i \gamma_5 \vec{\tau} S(i\omega_n + i\mu_m, \vec{q} + \vec{p}_3) i \gamma_5 \vec{\tau} S(i\omega_n + i\nu_l, \vec{q} + \vec{k}).
\]
(3)
Here \(\omega_n = (2n+1)\pi/\beta, n = 0, \pm 1, \pm 2, ...\) and \(\mu_m = 2m\pi/\beta, \nu_l = 2l\pi/\beta, m, l = 0, \pm 1, \pm 2, ...\) are fermionic and bosonic Matsubara frequencies respectively. The quark propagator is
denoted by \( S(i\omega_n, \vec{q}) = [\gamma_0(i\omega_n) - \vec{\gamma}\vec{q} + m]/[(i\omega_n)^2 - E_q^2] \), with \( E_q^2 = \vec{q}^2 + m^2 \), where \( m = m(T) \) is the dynamically generated quark mass, to be calculated using the usual gap equation in the Hartree approximation \([4]\). In Eq.(3), \( Tr \) refers to the trace over color, flavor and spinor indices. Finally, \( \Pi_\sigma(k; T) \) in Eq.(2) is the standard mesonic polarization function for the \( \sigma \)-meson \([4]\), and \( g_\pi \) is the pion-quark coupling strength, determined in the model via \([4]\)

\[
g_\pi^{-2}(T) = \frac{\partial \Pi_\pi(k_0, \vec{0}; T)}{\partial k_0^2} \bigg|_{k_0^2 = m_\pi^2}. \tag{4}
\]

The indices 1 and 2 of the quark and antiquark spinors \( u \) and \( \bar{v} \) in Eq.(2) refer to both momenta and helicity.

After evaluation of the Matsubara sum in Eq.(3), it is useful, in the analytic continuation to real variables, to move to the center of mass frame of the incident \( q\bar{q} \) pair. While the differential cross-section of any two-body reaction \( d\sigma(s, t)/dt \) is invariant under a Lorentz-boost, this is no longer so in a heat bath. Here the cross section not only depends on the temperature \( T \), but also on the c.m. velocity of the initial pair with respect to the heat bath. By assuming the \( q\bar{q} \) system to be at rest in the heat bath, we avoid this complication, but also restrict ourselves to only part of the solution. Then one has \( iv_l \rightarrow k_0 = \sqrt{s}, i\mu_m \rightarrow p_3^0 = \sqrt{s}/2, |\vec{p}_3| = \sqrt{s}/4 - m_\pi^2 \) and \( p_3k = s/2 \), leading to the form

\[
A_{\sigma\pi\pi}(s; T) = 2mN_cN_f\int \frac{d^3q}{(2\pi)^3} \frac{f(E_q) - f(-E_q)}{E_q^{2}} \frac{2sE_q^2 - s^2/2 - 8(\vec{q}\vec{p}_3)^2 + (4m_\pi^2 + 2s)\vec{q}\vec{p}_3}{(s - 4E_q^2)((m_\pi^2 - 2\vec{q}\vec{p}_3^2) - sE_q^2)} \tag{5}
\]

where the Fermi function \( f(x) = (1 + e^{\beta x})^{-1} \). One sees that the amplitude \( A_{\sigma\pi\pi} \) and the T-matrix element \( T(s) \) are simply functions of the center of mass energy \( s = (p_1 + p_2)^2 \). Note that the amplitude \( A \) in Eq.(5) is proportional to the dynamically generated quark mass \( m \), a fact that is general to any 3-meson vertex process. Therefore the s-channel diagrams become unimportant at the chiral phase transition in the chiral limit. Since \( m = 0 \) for \( T \geq T_c \), one has \( A_{\sigma\pi\pi} = 0 \), while for \( T < T_c, m_\pi = 0 \).

For the real and imaginary parts of \( A_{\sigma\pi\pi} \), semi-analytic expressions can be found.
Explicitly, one has
\[ \text{Re} A_{\sigma\pi\pi}(s; T) = -\frac{mN_cN_f}{2\pi^2} \int dq \left( 4q^2 + \frac{sq}{2E_q} \ln \left| \frac{E_q - q}{E_q + q} \right| \frac{f(E_q) - f(-E_q)}{E_q(s - 4E_q^2)} \right) \] (6)
and
\[ \text{Im} A_{\sigma\pi\pi}(s, T) = \frac{mN_cN_f}{8\pi\sqrt{s}} \left( 2\sqrt{s - 4m^2} + \sqrt{s} \ln \left| \frac{\sqrt{s - 4m^2} - \sqrt{s}}{\sqrt{s - 4m^2} + \sqrt{s}} \right| \right) \left( f\left( \frac{\sqrt{s}}{2} \right) - f\left( -\frac{\sqrt{s}}{2} \right) \right). \] (7)

Next we turn to the transition amplitude \( T(t) \) in the t-channel shown in Fig.1(b), which reads
\[ T(t)(t; T) = g^2\bar{\psi}_2\gamma_5 \frac{\not{p}_1 - \not{p}_3 + m}{t - m^2} \gamma_5 u_1, \] (8)
in which the \( m \) and \( t = (p_1 - p_3)^2 \) dependence is made explicit. This amplitude \( T(t) \) forms the dominant contribution for \( T \to T_c \), since, unlike in the s-channel, it is not directly proportional to the dynamically generated quark mass. When approaching the phase transition and \( m \to 0 \), one sees that the range of the "force" due to the exchange of a constituent quark tends to infinity, and we thus expect that the cross-section becomes infinite at this point.

As an example, we consider in detail the hadronization process \( u\bar{u} \to 2\pi_0 \). The s-channel crossed graph equals the direct term, while the t-channel crossed graph gives rise to a distinct exchange term. The differential cross-section can be written as
\[ \frac{d\sigma_{u\bar{u}\to 2\pi_0}}{dt}(s, t; T) = \frac{1}{16\pi s(s - 4m^2)} \sum_{c,s} l\left| 2T(s)(s; T) + T^{(t,\text{dir})(t; T)} + T^{(t,\text{exc})(t; T)} \right|^2. \] (9)
Here the prime on the summation indicates the average over spin and color degrees of freedom of the quarks in the initial states. The prefactor multiplying the differential cross-section is due to the initial flux. The direct contribution to the T-matrix, \( T^{(t,\text{dir})} \), is as given in (9) while the exchange contribution \( T^{(t,\text{exc})} \) can be derived from the u-channel to be
\[ T^{(t,\text{exc})}(t; T) = g^2\bar{\psi}_2\gamma_5 \frac{\not{p}_3 - \not{p}_4 + m}{u - m^2} \gamma_5 u_1. \] (10)
The individual contributions to the cross-section are listed in Table 1.

The integrated cross-section multiplied by a factor which takes into account the Bose-Einstein statistics in form of the distribution function $f_B(x) = (e^{\beta x} - 1)^{-1}$ for the final state pions is

$$\sigma_{u\bar{u}\rightarrow 2\pi_0}(s; T) = \int_{t_{\text{min}}}^{t_{\text{max}}} \frac{d\sigma_{u\bar{u}\rightarrow 2\pi_0}}{dt}(s, t; T)(1 + f_B(\frac{\sqrt{s}}{2}))^2.$$ (11)

The limits $t_{\text{max}}$ and $t_{\text{min}}$ of the integral in Eq.(11) are given as

$$t_{\text{max}} = -\frac{s}{2} + m^2 + \frac{1}{2}s\sqrt{(s - 4m^2)}$$ (12)

and

$$t_{\text{min}} = -\frac{s}{2} + m^2.$$ (13)

Before we discuss our numerical results in Figs.2 and 3, we introduce the hadronization cross-section into charged pions. The hadronization cross-sections $u\bar{u} \rightarrow \pi^+\pi^-$ and $u\bar{u} \rightarrow \pi^+\pi^0$ are obtained by simple modifications of the $\pi^0\pi^0$ results. Since the final state does not involve identical particles, $t_{\text{min}}$ is replaced by

$$t_{\text{min}} = -\frac{s}{2} + m^2 - \frac{1}{2}s\sqrt{(s - 4m^2)}$$ (14)

and no crossed diagrams occur. The isospin trace factor leads however to additional factors of 2 or $\sqrt{2}$ in the t-channel of the T-matrix element. One obtains the scattering cross-section for the charged final states on replacing the primed sum in Eq.(9) by

$$\sum_{c,s}'|T(s)|^2 + 2|T(t)|^2$$ (15)

for $u\bar{u} \rightarrow \pi^+\pi^-$ and

$$\sum_{c,s}'|\sqrt{2}T(t)|^2$$ (16)

for $u\bar{u} \rightarrow \pi^+\pi^0$. We define the total hadronization cross-section of a single quark as

$$\sigma_{u}^{\text{had}} = \sigma_{u\bar{u}\rightarrow \pi^0\pi^0} + \sigma_{u\bar{u}\rightarrow \pi^+\pi^-} + \sigma_{u\bar{d}\rightarrow \pi^+\pi^0}.$$ (17)
This is a measure of how quickly a $u$-quark hadronizes from a charge symmetric plasma. The corresponding cross-section for elastic scattering [9] is defined as

$$
\sigma_{\text{ela}}^{u} = \sigma_{u\bar{u}\rightarrow u\bar{u}} + \sigma_{u\bar{u}\rightarrow \bar{d}d} + \sigma_{u\bar{d}\rightarrow \bar{u}d} + \sigma_{u\bar{u}\rightarrow uu} + \sigma_{u\bar{d}\rightarrow ud}.
$$

Figures 2 and 3 show our numerical results. In the calculation, the parameters of the NJL model have been chosen in the standard way [4], to fit $f_\pi = 93$ MeV and $<\bar{\psi}\psi> = -(0.25\text{GeV})^3$ at $T = 0$, leading to a value [8] $G = 5.02\text{GeV}^{-2}$ and $\Lambda = 0.65 \text{ GeV}$. The critical temperature at which a second order phase transition occurs is $T_c = 1.95 \text{ GeV}$. Because of the use of a cutoff, the energy range is restricted to $4m^2 \leq s \leq 4(m^2 + \Lambda^2)$.

In Fig.2, we show the integrated hadronization cross-section $\sigma_{q}^{\text{had}} = \sigma_{u}^{\text{had}} = \sigma_{d}^{\text{had}}$ as a function of the c.m. energy $s$ for various values of the temperature. The hadronization cross-section is rather flat, except for a singularity that occurs at the threshold value $s_{th} = 4m^2$. Note that the threshold is a function of temperature since $m = m(T)$. While the shape of the $s$-dependence of $\sigma_{q}^{\text{had}}$ remains similar with increasing temperature, its magnitude increases dramatically when approaching the phase transition temperature (about a factor 4 between $T = 0.15 \text{ GeV}$ and $0.19 \text{ GeV}$). At $T = 0.19 \text{ GeV}$, i.e. 5 MeV below $T_c$, the hadronization cross-section and the elastic cross-section are of comparable magnitude (about 5 mb) with a singularity for both at the threshold. There is however a difference between $\sigma_{q}^{\text{ela}}$ and $\sigma_{q}^{\text{had}}$. While $\sigma_{q}^{\text{ela}}(s; T)$ remains relatively constant in shape and amplitude, when going through the phase transition, $\sigma_{q}^{\text{had}}(s; T)$ displays a singularity at $T = T_c$ for all values of $s$. This is shown in Fig.3, where $\sigma_{q}^{\text{had}}(s; T)$ is displayed as a function of temperature for several values of $s$. Note that there is no hadronization cross-section for $T > T_c$, since for $T > T_c$, we have strict deconfinement even in the NJL model.

The singularity structure of $\sigma_{q}^{\text{had}}(s; T)$ which is evident from the numerical calculations can be summarized by the two statements (considering always $T \leq T_c$):

(i) $\sigma_{q}^{\text{had}}(s; T) \rightarrow \infty$ for $s \rightarrow 4m^2$ and all $T$,
(ii) $\sigma_{q}^{\text{had}}(s; T) \rightarrow \infty$ for $T \rightarrow T_c$ and all $s$. 

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Case (i) can be understood to be of kinematical and dynamical origin. One finds for $s \to 4m^2$,
\begin{align}
t_{\text{max}} &\sim -m^2, \\
t_{\text{max}} - t_{\text{min}} &\sim \sqrt{s(s-4m^2)} \to 0,
\end{align}
so that (for $\sqrt{s-4m^2} << \sqrt{s}$, i.e. for $T < T_c$)
\begin{align}
\sigma^{\text{had}}(s) &\simeq \sqrt{s(s-4m^2)} \frac{d\sigma}{dt}_{t=t_{\text{max}}} \\
&= \frac{1}{\sqrt{s(s-4m^2)}} (C^{(t)} + C^{(s)} \frac{m^2}{s-4m^2}),
\end{align}
where $C^{(t)}$ and $C^{(s)}$ are smooth functions of $m$ and $s$. The singularity proportional to $(s-4m^2)^{-1/2}$ is expected, since the process $q\bar{q} \to 2\pi$ is exothermic: the entrance channel has a threshold energy $\sqrt{s} = 2m > 0$, while the exit channel has $\sqrt{s} = 0$. This singularity thus appears for any dynamics. The contribution from $\sigma$-exchange in the $s$-channel, the term proportional to $C^{(s)}$, increases the power of the singularity to $1/(s-4m^2)^{3/2}$. It arises from the $s$-channel resonance due to the excitation of the $\sigma$-meson. However, because of the factor of $m^2$, the effect of the $\sigma$-meson exchange diminishes when approaching $T_c$.

Case (ii) mentioned above, the singularity for $T \to T_c$, has its origin in the chiral dynamics of the model and arises from the diagram in Fig. 1 b. The leading order behavior from the $t$-channel exchange is given by
\begin{align}
\frac{d\sigma}{dt} &\sim \frac{1}{(t - m^2)},
\end{align}
which, in combination with the limiting value $t_{\text{max}} \simeq -m^4/(16s) \to 0$ and $t_{\text{min}} = -s$, and the relation [4] $m = -2G < \bar{\psi}\psi > \sim (T_c - T)^{1/2}$, leads to the behavior
\begin{align}
\sigma^{\text{had}}(s;T) &\sim \ln \left| \frac{t_{\text{max}} - m^2}{t_{\text{min}} - m^2} \right| \\
&\sim \ln | < \bar{\psi}\psi > (T) | \sim \ln |1 - \frac{T}{T_c}| 
\end{align}
as $T \to T_c$. Since we work in the chiral limit where the phase transition is of second order, the hadronization cross-section diverges at $T = T_c$. Note that this logarithmic singularity is present for all values of $s$. 
In this paper we have investigated the influence of the chiral phase transition on the hadronization phenomenon in a quark plasma. We have used a particular model, the NJL model, which has a deconfined phase above the chiral phase transition temperature \( T_c \) but a meson-quark plasma for \( T < T_c \). Although we have calculated the hadronization cross-section \( \sigma_{\text{had}}^q(s; T) \) for all temperatures, the results in the neighborhood of \( T_c \) are the most interesting and may be most useful. We find that the hadronization cross-section is always strongest for small values of the c.m. energy of the quark–antiquark pair, and it diverges for all values of \( s \) when \( T \to T_c \).

We have also considered the case of a non-vanishing current quark mass \( m_0 \) (\( m_0 = 5 \) MeV). Then the phase transition is washed out, and so are the singularities in the cross-sections at the transition temperature. The behavior at the threshold in \( s \) (shown in Fig.2) remains, while the logarithmic singularity seen in Fig.3 at \( T = T_c \) vanishes completely.

Although our results are derived in a particular model, we expect them to be of more general validity. The main conclusions are: (i) The hadronization cross section in the neighborhood of the phase transition is of the order of several mb, and strongly depends on the relative momentum of the quark–antiquark pair. (ii) The dynamics of the phase transition (here via temperature dependent masses) plays an essential role and leads to several singularities in the cross-sections.

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Table 1

Products of the T-matrix amplitudes (in the chiral limit), which contribute to the hadronization cross sections, averaged over initial and summed over final spin and color degrees of freedom.

\[
\sum_{c,s} |T(s)|^2 = \frac{2G^2 g_\pi^4 (s - 4m^2)}{N_c} \left| \frac{A_{\sigma\pi\pi}(s;T)}{1 - 2G\Pi_\sigma(s;T)} \right|^2
\]

\[
\sum_{c,s} |T(t,\text{dir})|^2 = \frac{-g_\pi^4}{2N_c} \left[ 1 + \frac{s}{t - m^2} \right]
\]

\[
\sum_{c,s} |T(t,\text{exc})|^2 = \frac{g_\pi^4}{2N_c} \frac{t - m^2}{m^2 - s - t}
\]

\[
\sum_{c,s} T(s)T^*(t,\text{dir}) = \frac{mg_\pi^4}{N_c(t - m^2)}(s + 2t - 2m^2)Re \frac{A_{\sigma\pi\pi}(s;T)}{1 - 2G\Pi_\sigma(s;T)}
\]

\[
\sum_{c,s} T(s)T^*(t,\text{exc}) = \frac{-mg_\pi^4}{N_c(m^2 - s - t)}(s + 2t - 2m^2)Re \frac{A_{\sigma\pi\pi}(s;T)}{1 - 2G\Pi_\sigma(s;T)}
\]

\[
\sum_{c,s} T(t,\text{dir})T^*(t,\text{exc}) = -\frac{g_\pi^4}{2N_c}
\]

Figure Captions.

Fig.1. The diagrams considered in this paper for the hadronization of a quark–antiquark pair (single lines) into two pions (double lines). The convention is chosen that they should be read from left to right. Fig.1 a shows the annihilation of a \( q\bar{q} \) via the intermediate formation of a \( \sigma \)-meson in the s-channel, while in Fig.1 b, the hadronization proceeds via quark exchange in the t-channel.

Fig.2. The integrated hadronization cross-section \( \sigma_{\text{had}}^q \) as a function of the c.m. energy \( s \) of the \( q\bar{q} \) pair for three values of the temperature. For comparison, also the integrated elastic cross-section is shown as a function of \( s \) at a temperature slightly below the critical value.
Fig. 3. The integrated hadronization cross-section as a function of $T$ for three values of the center of mass energy $s$. Note the (logarithmic) singularity at the critical temperature $T_c$. 
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