Alfvén waves have been invoked as an important mechanism of particle acceleration in stellar winds of cool stars. After their identification in the solar wind they started to be studied in winds of stars located in different regions of the HR diagram. We discuss here some characteristics of these waves and we present a direct application in the acceleration of late-type stellar winds.

1. Introduction

In 1942 combining fluid hydrodynamics and electromagnetism Hannes Alfvén discovered a new mode of waves, thus opening a new field of research. He studied the mutual interaction between conducting fluid motion and electromagnetic fields and discovered these waves (Alfvén, 1942) that later on were named Alfvén waves. However, to be verified the waves needed to be detected in laboratory experiments. This was done in the late 40s and beginning of the 50s, in experiments with mercury and liquid sodium. It was discovered that in liquid metals the waves are strongly damped. In the 60s, experiments in plasma demonstrated weakly damped Alfvén waves with the properties predicted by the theory. Since then this field has expanded mainly due to space studies and thermonuclear research, making possible, among other things, to create high temperature plasma in laboratories.

An Alfvén wave propagating in a plasma is a traveling oscillation of the ions and the magnetic field. The ion mass density provides the inertia and the magnetic field line tension provides the restoring force. The wave vector can either propagate in the parallel direction of the magnetic field or at oblique incidence. The waves efficiently carry energy and momentum along the magnetic field lines.

Identification of Alfvén waves in the solar wind by means of spacecraft measurements was first achieved in late 60s. In 2007, Tomczyk et al. (2007) reported the detection of Alfvén waves in images of the solar corona with the Coronal Multi-Channel Polarimeter instrument at the National Solar Observatory, New Mexico. Also in 2007, several research groups reconfirmed that Alfvén waves have sufficient energy to heat the solar corona and momentum to accelerate the solar wind (De Pontieu et al., 2007; Okamoto et al., 2007). The Sun is a benchmark for stellar research providing an example of a cool main sequence star losing mass.

Since the discovery of the high velocity winds from O and B supergiant stars by Morton (1967) and his co-workers (Morton, Jenkins & Brooks, 1969), the theories of stellar winds
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had their interest amplified. On another part of the HR diagram, the winds from evolved K and M stars, characterized by low velocities, demanded a different theoretical approach. Although presenting very different terminal velocities, in the range of 600 - 3500 km s\(^{-1}\) for early-type and 10 - 100 km s\(^{-1}\) for late-type K and M stars, these stars present a similarity: both luminous early and late-type stars are losing mass at a rate as high as \(10^{-5} \, M_\odot \, \text{yr}^{-1}\) (Lamers & Cassinelli, 1999).

The idea that a flux of Alfvén waves could accelerate winds of stars in many regions of the HR diagram remount from a few decades ago (Hollweg, 1973, 1987; Hartmann & MacGregor, 1980; Holzer, Fla & Leer, 1983; Underhill 1983; Hartmann & Avrett, 1984; Jatenco-Pereira and Opher, 1989a,b,c; dos Santos, Jatenco-Pereira and Opher, 1993; Jatenco-Pereira, Opher and Yamamoto, 1994; among others). The belief that Alfvén waves are present in the magnetized winds of cool giant stars is supported by the fact that these waves are observed in the solar wind (Smith et al., 1995; Balogh et al., 1995). If there are oscillations in the magnetic field at the base of the wind, Alfvén waves will be generated. As they propagate outward, the dissipation of their energy and the transfer of their momentum to the plasma can heat and accelerate the wind. As an example of how these waves can be used as an acceleration mechanism of a stellar wind, we show later on this paper a simple model to explain the mass-loss in a typical supergiant K5 star (Section 3). In Section 2, we present the issue of the damping of the Alfvén waves. In Section 4, we apply this model to a typical K5 supergiant star and in Section 5, we draw our conclusions.

2. The damping mechanism for Alfvén waves

For cool red giant stars the flux of Alfvén waves can provide an efficient driving mechanism for the wind. In this mechanism it is the wave magnetic pressure that accelerates the wind. Hartmann & MacGregor (1980) showed that if the waves are not damped, the resultant terminal velocities are much higher than observed. Assuming that the waves are damped with a constant damping length, they obtained terminal velocities and mass-loss rate consistent with the observations.

In order to improve the model presented by Hartmann & MacGregor (1980), Jatenco-Pereira & Opher (1989a) discarded the use of an artificial constant damping length for the waves and considered three different physical damping mechanisms: (i) the nonlinear damping, (ii) the resonant surface damping, and (iii) the turbulent damping. They showed that, for an isothermal atmosphere with \(T \sim 10^4 \, \text{K}\), their model can reproduce the observed large mass-loss rates and the small ratios between the terminal velocity and the escape velocity at the base (\(u_\infty/v_e\)) of these stellar winds.

To illustrate this mechanism, we present a simplified version of this model (see Vidotto & Jatenco-Pereira, 2006; Falceta-Gonçalves, Vidotto & Jatenco-Pereira, 2006 for a more complete analysis). We focus our study on the resonant surface damping for the Alfvén waves. A surface wave exists at the interface separating two environments, e.g., open magnetic flux tubes. Magnetohydrodynamic surface waves may decay through a process called “resonance absorption” (Hollweg, 1987), that leads to a concentration of the surface wave energy into the thin resonant layer. In this layer, the energy may be dissipated locally as heat or concentrated in the form of turbulent Alfvén waves which may propagate further
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and be dissipated elsewhere, or concentrated in other forms, such as high energy particles, electric currents, etc. In the present paper we suggest that resonant absorption dissipates the energy locally as heat with the damping length $L$ given by (Jatenco-Pereira & Opher, 1989a)

$$L = L_0 \left( \frac{v_A}{v_{A0}} \right)^2 \sqrt{\frac{A(r_0)}{A(r)}(1 + M)},$$

where $L_0$ is the damping length of the wave at the base of the wind, $M = u/v_A$ is the Alfvén-Mach number, the Alfvén speed is given by $v_A = (B/\sqrt{4\pi\rho})$ and $A(r)$ is the cross-section of a flux tube at a radial distance $r$. The index “0” indicates that the parameter is being evaluated at the stellar surface, i.e., at $r = r_0$. If a large damping length is considered (i.e., there is a smooth damping of the wave throughout the wind), there will be a non negligible outward force acting beyond the critical point of the wind, creating thus a high-speed wind. Hence, the damping length cannot be too large. This will also assure that the WKB approximation is valid in the wind we study.

3. Model

The first attempts in accelerating winds by means of Alfvén waves considered that the waves were propagating in an isothermal ambient. This assumption simplifies the equations, since the energy equation simply reads $T = \text{constant}$. However, if we are interested in modeling the temperature profile of the wind, there is a need to solve a consistent energy equation simultaneously with the mass and momentum equations of the wind. In this way, we are able to obtain both the temperature and the velocity profiles of the wind and compare them with observations.

3.1. Geometry of the Magnetic Field Lines

For a long time, it is known that the coronal holes are the source of the high-speed solar wind streams at the Earth’s orbit (e.g., Peter & Judge, 1999). Solar coronal holes observations show a superradial expansion of the magnetic flux tubes: the area expansion of a solar coronal hole is $5 - 13$ times greater than for a radial expansion (Lie-Svendsen, Hansteen & Leer, 2002; Esser et al., 2005; Tu et al., 2005). Jatenco-Pereira & Opher (1989a) implemented the Alfvén wave driven wind model by taking into account our knowledge on coronal holes in the Sun. We apply the same assumption for cool giant stars. For simplicity, we assume that in the wind of a typical K5 supergiant star, the non-radial expansion factor is

$$F = \frac{\Omega}{\Omega_0} = 10,$$

where $\Omega_0$ and $\Omega$ are the solid angles measured at the stellar surface and at the edge of the coronal hole, respectively. Hence, in order to model the diverging geometry for the magnetic field, we follow the idea given by Kuin and Hearn (1982). Thus, the cross-section of a flux tube at a radial distance $r$ is given by

$$A(r) = \begin{cases} A(r_0)(r/r_0)^S & \text{if } r \leq r_t \\ A(r_0)(r_t/r_0)^S(r/r_t)^2 & \text{if } r > r_t, \end{cases}$$
where $S$, the superradial index, is a parameter that determines the divergence of the geometry from the stellar radius $r_0$ up to the transition radius $r_t$, as shown in Figure 1. For a given $S > 2$, $r_t$ is obtained by

$$F = \frac{\Omega}{\Omega_0} = \frac{A(r_t)/r_t^2}{A(r_0)/r_0^2} = \left(\frac{r_t}{r_0}\right)^{S-2}. \tag{3}$$

Hence, we have

$$r_t = F^{1/(S-2)} r_0. \tag{4}$$

Considering this geometry, the conservation of magnetic flux yields a magnetic field intensity of

$$B(r) = B_0 \frac{A(r_0)}{A(r)}. \tag{5}$$

A detailed study of the diverging geometry of the magnetic field in the winds of late-type stars can be found in Jatenco-Pereira & Opher (1989a).

3.2. Wind Equations

The equation of mass continuity expresses the conservation of mass. If the flow velocity is denoted by $u$ and the gas density is $\rho$, we have in steady-state

$$\rho u A(r) = C. \tag{6}$$

The constant $C$ in equation (6) is computed at the wind base, i.e., $C = \rho_0 u_0 A(r_0)$. 
Assuming a steady flow, the equation of motion is written as
\[
\rho u \frac{du}{dr} = -\rho \frac{GM_*}{r^2} - \frac{dP}{dr} - \frac{d}{dr} \langle (\delta B)^2 \rangle / 8\pi,
\]
where \(-\rho GM_*/r^2\) is the gravitational attraction force, \(-dP/dr\) is the pressure gradient, and \(d[\langle (\delta B)^2 \rangle/(8\pi)]/dr\) is the Alfvén wave magnetic pressure gradient. The parameter \(\delta B\) is the magnetic field amplitude of the wave. It is related to the energy density \(\epsilon\) of the wave by
\[
\epsilon = \frac{\langle (\delta B)^2 \rangle}{4\pi}.
\]

The energy equation is determined through the balance between wave heating, adiabatic expansion, and radiative cooling (Hartmann, Edwards, & Avrett, 1982). Assuming an ideal gas, we can write the gas pressure as \(P = \rho k_B T/m\), where \(k_B\) is the Boltzmann constant, \(T\) is the gas temperature, and \(m\) is the mean mass per particle. Thus, neglecting conduction, we write the energy equation as
\[
\rho u \frac{d}{dr} \left( \frac{u^2}{2} + \frac{5 k_B T}{2 m} - \frac{GM_*}{r} \right) + \frac{u d\epsilon}{2 dr} = (Q - P_R).
\]

The term \((u/2)d\epsilon/dr\) is the rate at which the waves do work on the gas. \(Q\) is the wave heating rate, i.e., the rate at which the gas is being heated due to dissipation of wave energy, and \(P_R\) is the radiative cooling rate, both in erg cm\(^{-3}\) s\(^{-1}\). The wave heating can be written as (Hollweg, 1973)
\[
Q = \frac{\epsilon}{L}(u + v_A)
\]
and the radiative cooling is given by
\[
P_R = \Lambda n_e n_H,
\]
where \(n_e\) is the electron density, \(n_H\) is the hydrogen density and \(\Lambda\) is the radiative loss function. Here, we adopt \(\Lambda\) given by Schmutzler & Tscharnuter (1993) and calculate \(n_e\) as Hartmann & MacGregor (1980).

If the waves are damped, the wave energy is dissipated and we write
\[
\epsilon = \epsilon_0 \frac{M_0}{M} \left( \frac{1 + M_0}{1 + M} \right)^2 \exp \left[ - \int_{r_0}^r \frac{1}{L} dr' \right].
\]

The initial energy density \(\epsilon_0\) and the initial wave flux \(\phi_{A_0}\) are related to each other by (Jatenco-Pereira & Opher, 1989a)
\[
\phi_{A_0} = \epsilon_0 v_{A_0} \left( 1 + \frac{3}{2} M_0 \right).
\]

From the equations above, we write the temperature variation as
\[
\frac{dT}{dr} = \frac{2 T}{3 r} \left[ \frac{r(Q - P_R)}{\rho u (k_B T/m)} - \left( Z + \frac{r du}{u dr} \right) \right],
\]
where, we defined $Z$ as

$$Z = \begin{cases} S & \text{if } r \leq r_t \\ 2 & \text{if } r > r_t. \end{cases}$$

As $\epsilon = \rho \langle (\delta v)^2 \rangle$, where $\delta v$ is the velocity fluctuation of the wave, the velocity gradient is given by

$$\frac{1}{u} \frac{du}{dr} \left[ u^2 - \frac{5 k_B T}{3 m} - \frac{\langle (\delta v)^2 \rangle}{4} \left( \frac{1 + 3 M}{1 + M} \right) \right] = \frac{Z}{r} \left[ \frac{5 k_B T}{3 m} - \frac{2 r(Q - P_B)}{3 Z \rho_u} - \frac{GM_*}{r Z} + \frac{\langle (\delta v)^2 \rangle}{2 L Z} r + \frac{\langle (\delta v)^2 \rangle}{4} \left( \frac{1 + 3 M}{1 + M} \right) \right].$$

Thus, in order to obtain self-consistently the velocity and the temperature profiles of the wind, it is necessary to solve equations (6), (14), and (15) simultaneously.

4. Results and discussions

The model is applied here to the wind of a typical K5 supergiant star with a mass of 16 $M_\odot$ and a radius of 400 $R_\odot$. We adopt a temperature of 3500 K at the base of the wind, and assume that the magnetic field intensity is $\sim 10$ G at this location. In our model, we adopt an initial wave flux of $\phi_{A_0} \sim 10^6$ erg cm$^{-2}$ s$^{-1}$, which is of the same order of magnitude as the one estimated for the Sun (Banerjee et al., 1998), and take $L_0 = 0.2 r_0$ and $S = 5.2$. The initial density is $\rho_0 = 1.07 \times 10^{-13}$ g cm$^{-3}$.

We perform the calculations until $r = 300 r_0$, where the terminal velocity has been already reached.

4.1. Typical K5 supergiant star

Similarly to the wind first studied by Parker (1960), the momentum equation (eq. [15]) has a critical point when the term in brackets on the left-hand side becomes zero, and hence the bracketed term on the right-hand side must necessarily be zero too. The requirement that the wind velocity should increase through the critical point determines the initial velocity at the base of the wind, i.e., $du(r = r_0)/dr \geq 0$. Hence, to find the critical solution for a given set of initial conditions, we iterate the initial velocity until the solution passes through the critical point. Together with the initial density, the initial velocity determines the mass-loss rate.

Our results are shown in Figure 2, where the velocity and the temperature profiles obtained were plotted for $r \leq 30 r_0$. We obtained a terminal velocity of $u_\infty \sim 63$ km s$^{-1}$ $\sim 0.5v_e$ and a mass-loss rate of $\dot{M} \simeq 1.2 \times 10^{-7}$ $M_\odot$ yr$^{-1}$. These results compares favorably with observations: observationally, the terminal velocity should be lower than the surface escape velocity in a ratio around 1:2, and the mass-loss rate should range from $10^{-8}$ to $10^{-5}$ $M_\odot$ yr$^{-1}$ (Linsky 1998).

According to Roddier & Roddier (1985), the CII emission detected in cool winds of giant stars is associated to regions with temperatures of 7000 – 9000 K. Carpenter, Robinson, & Judge (1995) obtained, for the wind of $\gamma$ Cru (M3.4III), a maximum temperature of $\sim 9000$ K. Carpenter et al. (1999) found a maximum temperature of $\sim 10000$ K for the wind of $\lambda$ Vel (K5Ib). For $\zeta$ Aur (K4Ib), Eaton (1993) found that the wind temperature
reaches 12000 K at \( \sim 2r_0 \). We can see in Figure 2 an abrupt rise in the temperature of the wind: the temperature rises from \( 3.5 \times 10^3 \) K to \( \sim 10^4 \) K in the range from \( 1r_0 \) to \( \sim 2r_0 \). This heating is mainly due to the dissipation of the Alfvén waves, i.e., caused by the wave damping. At large distances, when almost all the flux of Alfvén waves has been dissipated, the flow expands adiabatically. Hence, in the absence of strong heating, the temperature tends to fall monotonically with the adiabatic exponent \( 4/3 \).

As to exemplify the effects of the choice of \( L_0 \) in our model, we have run a second model taking into account a different value of initial damping length: \( L_0 = 0.4r_0 \). Comparing this second model with the previous one, we obtained a value of \( \sim 100 \) km/s while for the first model a flow with a terminal velocity of \( \sim 63 \) km/s was achieved. This situation is explained by the position where a large fraction of the energy is deposited in the wind. If energy is mainly deposited in the super-critical part of the flow, as is the case for the second model, the initial structure of the wind is not modified, i.e., it maintains the same mass-loss rate, but the super-critical part is accelerated. Comparing the mass-loss rate, we obtained for the first model \( \dot{M} \approx 1.2 \times 10^{-7} \) M\(_\odot\) yr\(^{-1}\), while for the second \( \dot{M} \approx 1.3 \times 10^{-7} \) M\(_\odot\) yr\(^{-1}\).

5. Conclusions

We presented here some characteristics of Alfvén waves and showed how these waves can be used as an acceleration mechanism of stellar winds. As a particular case, we evaluated the wind temperature and velocity profiles of a typical K5 supergiant star using an outward-directed flux of Alfvén waves as the main acceleration mechanism of the wind. For a typical K5 supergiant star, we obtained a high mass-loss rate and a low terminal velocity consistent with observations.
It is important to notice that after the recent report on the unambiguously detection of Alfvén waves in the solar corona, these waves will increase their applicability in the Sun and in several astrophysical environments.

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References

Alfvén, H. 1942, Nature, 150, 405.
Balogh, A., Smith, E. J., Tsurutani, B. T., Southwood, D. J., Forsyth, R. J., & Horbury, T. S. 1995, Science, 268, 1007.
Banerjee, D., Teriaca, L., Doyle, J. G., & Wilhelm, K. 1998, A&A, 339, 208.
Carpenter, K. G., Robinson, R. D., & Judge, R. G. 1995, ApJ, 444, 424.
Carpenter, K. G., Robinson, R. D., Harper, G. M., Bennett, P. D., Brown, A., & Mullan, D. J. 1999, ApJ, 521,382.
De Pontieu, B., McIntosh, S. W., Carlsson, M., Hansteen, V. H., Tarbell, T. D., Schrijver, C. J., Title, A. M., Shine, R. A., Tsuruta, S., Katsukawa, Y., Ichimoto, K., Suematsu, Y., T. Shimizu, T., & S. Nagata, S. 2007, Science, 318, 1574.
dos Santos, L. C., Jatenco-Pereira, V., & Opher, R. 1993, ApJ, 410, 732.
Eaton, J. A. 1993, ApJ, 404, 305.
Esser, R., Lie-Svendsen, Ø., Janse, Å. M., & Killie, M. A. 2005, ApJ, 629, 61.
Falceta-Gonçalves, D., Vidotto, A., & Jatenco-Pereira, V. 2006, MNRAS, 368, 1145.
Hartmann, L., & MacGregor, K. B. 1980, ApJ, 242, 260.
Hartmann, L., Edwards, S., & Avrett, E. 1982, ApJ, 261, 279.
Hartmann, L., & Avrett, E. H. 1984, ApJ, 284, 238.
Hollweg, J. V. 1973, ApJ, 181, 547.
Hollweg, J. V. 1987, ApJ, 312, 880.
Holzer, T. E., Fla, T., & Leer, E. 1983, ApJ, 275, 808.
Jatenco-Pereira, V., & Opher, R. 1989a, A&A, 209, 327.
Jatenco-Pereira, V., & Opher, R. 1989b, MNRAS, 236, 1.
Jatenco-Pereira, V., & Opher, R. 1989c, ApJ, 344, 513.
Jatenco-Pereira, V., Opher, R., & Yamamoto, L. C. 1994, ApJ, 432, 409.
Kuin, N. P. M., & Hearn, A. G. 1982, A&A, 114, 303.
Lamers, H. J. G. L. M., & Cassinelli, J. P. 1999, Introduction to stellar winds (New York: Cambridge University Press.
Lie-Svendsen, Ø., Hansteen, V. H., & Leer, E. 2002, ApJ, 566, 562.
Linsky, J. L. 1998 in ESA Special Publication, Vol. 413, Ultraviolet Astrophysics Beyond the IUE Final Archive, ed. W. Wamsteker, R. Gonzalez Riestra & B. Harris, 83
Morton, D. C. 1967, ApJ, 147, 1017.
Morton, D. A., Jenkins, E. B. & Brooks, N. 1969, ApJ, 155, 875.
Okamoto, T. J., Tsuruta, S., Berger, T. E., Ichimoto, K., Katsukawa, Y., Lites, B. W., Nagata, S., K. Shibata, K., Shimizu, T., Shine, R. A., Suematsu, Y., T. D. Tarbell, T. D. & Title, A. M. 2007, Science, 318, 1577.
Parker, E. N. 1960, ApJ, 132, 821.
Peter, H., & Judge, P. G. 1999, ApJ, 522, 1148.
Roddier, F., & Roddier, C. 1985, ApJ, 295, 21.
Schmutzler, T., & Tscharnuter, W. M. 1993, A&A, 273, 318.
Smith, E. J., Balogh, A., Neugebauer, M., McComas, D. 1995, Geophys. Res. Lett., 22, 3381.
Tomczyk, S., McIntosh, S. W., Keil, S. L., Judge, P. G., Schad, T., D. H. Seeley, D. H., & Edmondson, J. 2007, Science, 317, 1192.
Tu. C.-Y., Shou, C., Marsch, E., Xia, L.-D., Zhao, L., Wang, J.-X., & Wilhelm, K. 2005, Science, 308, 519.
Underhill, A. B. 1983, ApJ, 268, L127.
Vidotto, A., & Jatenco-Pereira, V. 2006, ApJ, 639, 416.