A NEW METHOD FOR RANKING DECISION MAKING UNITS USING COMMON SET OF WEIGHTS:
A DEVELOPED CRITERION

Gholam Hassan Shirdel* and Somayeh Ramezani-Tarkhorani
Department of Mathematics, Faculty of Sciences, University of Qom
Alghadir Bld., Postal code:3716146611, Qom, Iran
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Abstract. In this paper we have developed a new model by altering Liu and Peng’s approach [20] toward ranking method using CSW. In fact, we have adopted a new criterion which is stronger in terms of maximizing efficiencies. After showing advantages of our model theoretically and illustrating it geometrically, two examples demonstrated how the proposed method is practically more capable.

1. Introduction. Data Envelopment Analysis (DEA) is a mathematical Programming based approach for measuring the relative efficiency of a group of homogeneous Decision Making Units (DMUs). The basic DEA models, CCR model initially proposed by Charnes et al. [5], and BCC model initially proposed by Banker et al. [4] determine the maximum relative efficiency for each DMU. Although, this is considered as an advantage for the reorganization of inefficient DMUs, there are also some disadvantages as all efficient DMUs gain the same ranking score of 1, and there is no distinction among efficient units. Hence, numerous approaches with different points of view are presented by researches for ranking performance of efficient units. A number of these methods have been reviewed by Adler et al. [1], Hossein zadeh et al. [12] and Aldamak et al. [3]. Classical DEA models calculate the best relative efficiency of units under different conditions which make their performance incomparable. Common weights methodology is one of the ranking approaches which eliminates this disadvantage and allow Decision Maker (DM) to evaluate and to compare performance of the units (the set of efficient units or all units) fairly under common base. This methodology was first introduced by Cook et al. [8] and Roll et al. [26]. The purpose of these methods is generally to determine a common set of weights (CSW) to maximize the efficiency of units in which the common benchmark level for the efficiencies is equal to 1. The criterion utilized to reach this aim has a significant role in linearity or non linearity of the model to generate CSW. In some of the models the maximum relative efficiencies of units are considered as the ideal efficiencies and a CSW based upon which the efficiencies are as close as possible to these efficiencies is searched. Some of such models deal with this issue directly and aim to minimize the distance that leads to a non linear model. Furthermore, it

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* Corresponding author: Gholam Hassan Shirdel.
imposes a considerable computational burden because it’s necessary to solve a DEA model per unit before solving the model. In the following, we review some of these methods. Despotis [9] proposed an approach to obtain a CSW the idea behind which was to minimize the derivation between DEA efficiency scores and the efficiencies based upon the CSW. To this aim, he searched a CSW that the convex combination of the mean derivation and the maximal derivation based upon which is minimum. The convex combinations, with allocating different amounts of parameters from the interval of [0, 1], lead to generate different CSWs. Kao and Hung [16] presented a method for finding a CSW based on the compromise solution approach. They first solved the standard DEA models and set the efficiencies as the ideal solution to be achieved. Then, they searched a CSW in which the vector of efficiency scores is the closest to the ideal solution. To this aim, they used the generalized measure of distance, and so their model to generate a CSW was non-linear. Wang et al. [32] have developed two non-linear models to generate alternative common weights based on regression analysis. They calculated DEA efficiencies and set them as the target efficiencies. Their aim was to obtain a CSW whose extracted efficiency vector was as close to target efficiency vector as possible. In some other methods, to generate a CSW maximizing the efficiencies of units, the focus is directly on the efficiency fractions. In such methods, the aim is to maximize the efficiency fractions and to close them to 1 as much as possible. Dealing with this issue directly may lead to a non-linear model. For example, Ganley and Cubbin [10] presented a DEA model to determine common weights maximizing DMUs’ efficiencies. To this aim, they searched a CSW that sum of the efficiencies based upon which is maximum. Chiang and Tzeng [7] also proposed a multi objective programming to find a set of weights with the purpose of maximizing all units’ efficiencies. In order to solve the multi objective problem, they used max-min approach. In this way, they presented a non-linear programming to search a CSW that the minimum efficiency based upon which is the maximum among the set of feasible solutions of the model. But in this context, Liu and Peng [20] describing CWA-methodology introduced a linear criterion to measure the magnitude of efficiency fractions and their closeness to benchmark level of 1. Using this criterion, they suggest a linear programming to find a CSW to maximize efficiencies. They also propose another linear model to choose only one optimal solution (if there is any alternative) as the CSW. They presented CWA-ranking rules to rank DMUs. Their method was used to efficient units, but finally, it was extended to the set of all units. Ramezani et al. [25] made some modifications on their work and also presented the same criteria to be used to rank all CWA-efficient and CWA-inefficient DMUs. Jahanshahloo et al. [15] proposed two ranking methods. In the first one, they defined an ideal line as a straight line that passes through the origin and positive ideal DMU with slope of 1. Where, the virtual positive ideal DMU is considered to be a DMU, in which inputs and outputs are fixed to be the best among all DMUs. Then, they determined a CSW for efficient DMUs and obtained a new efficiency score for their ranking method. Their suggested model was a linear programming similar to Liu and Peng’s [20] with an additional constraint related to the positive ideal DMU. In the second method, a special line was defined, and all efficient DMUs were compared with it and they were ranked accordingly. Chiang et al. [6] presented a model to generate CSW to maximize DMU’s efficiencies. They use an auxiliary vector whose components stand between the numerator and denominator of efficiency fractions (the virtual outputs
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and inputs) of DMUs. They used the same criterion as Liu and Peng’s [20] for maximization of the efficiency fractions. Sun et al. [30] proposed two linear models to generate CSW with the purpose of maximizing the efficiencies. In the first method, they introduced a virtual positive ideal DMU whose input and output components are set to be the best among corresponding components of DMUs. They suggested a linear programming with regard to the virtual ideal unit as the reference object whose efficiency score is equal to 1, and all of the units were pursuing to maximize their efficiency scores. Their point of view for increasing the efficiencies of DMUs was minimizing the distance between each unit and the ideal DMU. They used the same criterion as Liu and Peng’s [20] for the purpose of maximizing the efficiency fractions. Also, they proposed another approach based upon this point of view that if a DMU has the farthest distance from the worst solution, it is the most efficient one. They introduced an anti-ideal DMU which its components were set in the opposite way rather than the ideal DMU. In this regard, they considered the feasible region composed of the set of weights in which the efficiency scores of DMUs are greater than or equal to 1 where the efficiency value of virtual anti-ideal unit equals to 1. Then, they searched a CSW based upon which the obtained efficiencies have maximum distance from the anti-ideal DMU. They proposed a non-linear criterion to choose a unique optimal solution as the CSW per model. Their method requires that all data be positive. Now, we review some other works in the field of common weights methodology. Saati et al. [27] presented a two-phase algorithm. In the first step, they defined an ideal DMU (IDMU) which was a hypothetical DMU consuming the least inputs to secure the most outputs. In the second step, they used the IDMU in a LP with a small number of constraints to determine a CSW. Hatami et al. [11] proposed an alternative DEA model for centrally imposed resource or output reduction across the reference set. They determined the amount of input and output reduction needed for each DMU to increase the efficiency scores of all the DMUs. They used a CSW method based on the Goal Programming (GP) to control the total weight flexibility in the conventional DEA models. Shirdel et al. [29] presented a method to rank DMUs with interval data using common weights. Their purpose was to find a CSW maximizing efficiencies of all DMUs. For a given CSW, the absolute efficiency of a DMU in IDEA lies in an interval. They searched a CSW based upon which the minimum of the set of possible efficiencies per unit was maximum. Their proposed model was linear and the utilized criterion to maximize the efficiencies was the same as Liu and Peng’s [20]. They ranked DMUs based upon the middle points of their efficiency interval. Aghayi et al. [2] presented a robust DEA model with a common set of weights under varying degrees of conservatism and data uncertainty. Salahi et al. [28] presented a linear model to find a CSW under interval uncertainties on input and output data. They computed the optimistic counterpart of multiplier form of CCR model under interval uncertainties. Their model considers the uncertainties of data under the worst-case. Hu et al. [13] used methodology of common weights to measure the relative efficiency of DMUs with fuzzy data on a common base. Pourmahmoud and Zenali [23] used common weights for network structures in DEA. Their proposed model to this aim was nonlinear. Li et al. [18] suggested a new general framework to allocate multiple resources and set multiple targets across DMUs based on DEA-based models across the DMUs by taking into account common weights and the efficiency invariance principle, simultaneously. Kritikos [17] produces common weights for full ranking of DMUs based on five dummy DMUs. We also mention some other works as follows. Rahman
et al. [24] present an accurate multi-criteria decision making methodology (AMD) which empirically evaluated and ranked classifiers and allow end users or experts to choose the top ranked classifier for their applications to learn and build classification models for them. Nan et al. [22] propose an effective linear programming technique for solving matrix games in which the payoffs are expressed with intervals and the choice of strategies for players is constrained. Lin et al. [19] presented a new proportional sharing model to determine a unique fixed cost allocation under two assumptions: efficiency invariance and zero slack. In this paper, we present a ranking method using common weights. A linear programming is suggested to generate CSW to maximize all DMU’s efficiencies. Our approach for maximizing the efficiency fractions is focused on them directly to approximate them to the benchmark level of 1. To this aim, we propose a new linear criterion to obtain a CSW maximizing the efficiencies. In this method, there is no need to pre-computation for solving the model to generate CSW. We investigate our criterion theoretically and geometrically. Furthermore, since this criterion can be treated as a promotion to the well-known criterion proposed by Liu and Peng [20], we discuss and compare them in these aspects, too. Our ranking procedure is simple with proposing two different approaches depending to DM’s tendency. Finally, we consider two numerical examples. In the first one, we implement our ranking method, in details, on a set of real data and discuss the results. Regarding that both of our method and Liu and Peng’s [20] uses linear criteria to maximize the efficiencies and approximate them to 1, we compare them practically in the second numerical example. To this aim, we consider various data samples. Then, the models to generate CSWs are compared just in terms of their capability in maximizing the efficiencies.

Rest of this paper is organized as follows. We review some elementary notions in Section 2, we discuss theoretical and geometrical aspects of our methodology in Section 3. Our proposed model to determine a CSW is presented in Section 4, and the ranking procedure is explained in Section 5. We implement both our method and Liu and Peng’s [20] on various data samples and discuss the results in Section 6.

2. Preliminaries. In this section, we have a brief mention of some essential subjects.

2.1. CCR model. Consider a group of $n$ homogeneous DMUs, $DMU_j, j = 1, \ldots, n$. $X_j \in \mathbb{R}^m$ denotes the vector of consumed inputs and $Y_j \in \mathbb{R}^s$ denotes the vector of produced outputs for $DMU_j$. These vectors are assumed to meet the following conditions:

\[
0 \neq X_j \geq 0, \ 0 \neq Y_j \geq 0 \quad j = 1, \ldots, n \\
\forall i \in \{1, \ldots, m\}, \exists j \in \{1, \ldots, n\} : (x_{ij} > 0) \\
\forall r \in \{1, \ldots, s\}, \exists j \in \{1, \ldots, n\} : (y_{rj} > 0)
\]

Let $U$ and $V$ be the vectors of output and input weights respectively ($U^t \in \mathbb{R}^s, V^t \in \mathbb{R}^m$).

The absolute efficiency of $DMU_o, o \in \{1, \ldots, n\}$, based upon $(U, V)$ is equal to:

\[
E_o = \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}}
\]

And its relative efficiency is equal to:

\[
RE_o = \frac{\sum_{r=1}^s u_r y_{ro}}{\max_{j=1,\ldots,n} \sum_{r=1}^s u_r y_{rj}}
\]
The input-oriented CCR model in the multiplier form to evaluate \( DMU_o \) is as follows:

\[
CCR_o : \quad \theta_o^* = \max \quad UY_o \\
\text{s.t.} \quad VX_o = 1 \\
\quad UY_j - VX_j \leq 0 \quad j = 1, \ldots, n \\
\quad U \geq 0 \ , \ V \geq 0
\] (4)

The optimal value of \( CCR_o \) is the maximum relative efficiency for \( DMU_o \). Regarding constraints of (4), \( \theta_o^* \) is always less than or equal to 1. The maximum relative efficiency of each DMU is obtained from individual set of weights.

Dual of multiplier form of CCR model is called envelopment form of CCR model.

**Definition 2.1.** \( DMU_o \) is called CCR-efficient if \( \theta_o^* = 1 \) and there is at least one optimal solution with strictly positive components to (4); otherwise, it is called CCR-inefficient.

2.2. A brief review of Liu and Peng’s method [20]. Liu and Peng [20] proposed a method for ranking all efficient DMUs using common set of weights, and then they extended it to the set of all DMUs. They aimed to determine a CSW to maximize the efficiencies. The extended version of their model to generate CSW considering all DMUs is as follows:

\[
\min \quad \sum_{j=1}^{n} \Delta_j^O + \Delta_j^I \\
\text{s.t.} \quad \frac{UY_j + \Delta_j^O}{VX_j - \Delta_j^I} = 1 \quad j = 1, \ldots, n \\
\quad \Delta_j^O, \Delta_j^I \geq 0 \quad j = 1, \ldots, n \\
\quad U \geq \varepsilon 1_s^t , \ V \geq \varepsilon 1_m^t
\] (5)

(5) can be formulated as follows:

\[
\min \quad \sum_{j=1}^{n} \Delta_j \\
\text{s.t.} \quad UY_j - VX_j + \Delta_j = 0 \quad j = 1, \ldots, n \\
\quad \Delta_j \geq 0 \quad j = 1, \ldots, n \\
\quad U \geq \varepsilon 1_s^t , \ V \geq \varepsilon 1_m^t
\] (6)

Where \( U^t \in R^s \), \( V^t \in R^m \) and \( \varepsilon \) is a non-Archimedean infinitesimal constant, \( 1_s \in R^s \) and \( 1_m \in R^m \) with vector components of 1. They also proposed a linear programming to choose one of the optimal solutions of (6) as CSW. They introduced CWA-ranking rules to rank DMUs. As mensioned in previous Section, some later works take advantage of the CWA-methodology presented by them. (More details in Liu and Peng [20]).

3. Theoretical and geometrical viewpoints of our methodology. In this section, the idea behind our proposed criterion is explained from theoretical and geometrical points of view:

3.1. Theoretical point of view. For a given \((U,V)\), where \( U \in R^{s+}, V \in R^{m+} \), the absolute efficiency for \( DMU_j \) is equal to \( \frac{UY_j}{VX_j} \). In our models to generate CSW (Section 4), the absolute efficiency fractions of DMUs are positive, and the common benchmark level is equal to 1. Thus, each absolute efficiency fraction belongs to the set of \( B = \{ \frac{y}{x} \mid (x,y) \in A \} \) where, \( A = \{ (x,y) \mid 0 < y \leq x \} \). For a given fraction of \( \frac{y}{x} \) the fraction \( \frac{\zeta}{x} \) is greater than \( \frac{y}{x} \) if and only if \( a(d-c) < c(b-a) \). Let \( \Delta_{ba} = b - a \) and \( \Delta_{dc} = d - c \). Whatever the value of \( c \) is more and the value of \( \Delta_{dc} \) is less, it
is more probable to have greater magnitude of $\frac{c}{b}$ than that of $\frac{a}{b}$. This is noticeable that there is no linear criterion to recognize the bigger fraction between $\frac{a}{b}$ and $\frac{c}{b}$. Hence, a linear criterion cannot definitely choose the maximum member of a subset of $B$. Therefore, we intend to consider a more stronger linear criterion in order to use for our proposed model to generate CSW maximizing efficiencies. Given a fraction $\frac{a}{b} \in B$, whatever the value of $y$ is more and the value of $\Delta_{xy} = x - y$ is less, the fraction of $\frac{y}{x}$ has a more favorable condition regarding magnitude. Hence, we utilize the linear criterion $\Delta_{xy} = x - y$ to measure the magnitude of the fraction $\frac{x}{y} \in B$. Less amount of the criterion is more favorable to attain the aim. Therefore, the factors involving to measure the magnitude of the efficiency fractions in our method is more than that of Liu and Peng method, which only consider $\Delta_{xy}$ to measure the magnitude of the fraction $\frac{x}{y} \in B$. In the next section, we investigate our proposed criterion geometrically and also in comparison with the Liu and Peng [20]

3.2. Geometrical point of view. Here, we discuss and compare each linear criterion used by Liu and Peng [20] and us geometrically. Let $P$ be an arbitrary point with the Cartesian coordinate $(x_P, y_P) \in A$. We called the line bisecting the first and third quarters the ideal-line. Then, $P$ is located on a region on the first quarter of the coordinate system along and below the ideal-line except x-axis. We indicate the line crossing the origin and the point $P$ with $l$ and its slope with $m_P$. We investigate the mentioned linear criteria and compare them in regard to choosing the points which are unfavorably preferred to an assumed point in terms of magnitude of the line slope. Suppose that $\Delta_{l_P}$ and $\Delta_{O_P}$ indicate respectively the horizontal and vertical distances between $P$ and its image on the ideal-line, $P_{image}$.

**Theorem 3.1.** The horizontal and vertical distances between the point $P \in A$ and the ideal-line is equal to $\Delta_{l_P} + \Delta_{O_P}$.

**Proof.** Let $P_{image} = (x_{P_{image}}, y_{P_{image}})$. Then, $x_{P_{image}} = x_P - \Delta_{l_P}$, $y_{P_{image}} = y_P + \Delta_{O_P}$. Let $P_{l_1} = (x_{P_{l_1}}, y_{P_{l_1}})$ be the image of $P$ on the ideal-line along with the line $y = 0$. Thus, $x_{P_{l_1}} = y_{P_{l_1}} = y_P$. The horizontal distance between $P$ and ideal-line is equal to the distance between $P$ and $P_{l_1}$. It is equal to $x_P - x_{P_{l_1}} = x_P - y_P$. $x_P - x_{P_{l_1}} = (x_P - x_{P_{image}}) + (x_{P_{image}} - x_{P_{l_1}}) = (x_P - x_{P_{image}}) + (y_{P_{image}} - y_P) = \Delta_{l_P} + \Delta_{O_P}$. There is a similar discussion about the vertical distance between $P$ and the ideal-line, and we are done about that.

We indicate $x_P - y_P$ with $\Delta_P$. The line $x - y = \Delta_P$, we called it $l_{\Delta_P}$, contains the points in which the difference between the $x$-coordinate and $y$-coordinate is the same as $P$. It’s obvious that by traversing $l_{\Delta_P}$ in the direction of increasing $y$ (increasing $x$), slope of the lines crossing the origin and the points located on that would also increase.

**Theorem 3.2.** If $0 < \Delta' < \Delta$ then all of the points in which the difference between their length and width is equal to $\Delta'$ locate on a line, say $l_{\Delta'}$, between $l_{\Delta}$ and ideal-line, and vice versa.

**Proof.** It is easily proven by elementary geometry.
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we indicate it with $R_1$ (Figure 2). $R_1$ contains all the points belonging to the set $A$ which are preferred to $P$ in terms of Liu and Pengs criterion. Hence, these points are preferred to $P$ while the slopes of the lines crossing the origin and these points are less than that of $P$, that is not favored. The slope of the lines crossing the origin and these unfavorable points lie in the interval of $(0, m_P)$.

Now, we investigate the new criterion proposed in our model. Let $M$ be an arbitrary constant such that $x_P < M$. We define $A_M = \{(x, y) \mid (x, y) \in A, x \leq M\}$. Obviously, $P \in A_M$. There are 3 possible cases:

i. $x_P - 2y_P < 0$
ii. $x_P - 2y_P = 0$
iii. $x_P - 2y_P > 0$
We consider each case in the following:

i. $x_P - 2y_P < 0$

In this case, the points surrounded in the region $PQ_3Q_4$, we call it $R_2$, (Figure 3) are preferred to the point $P$ by our proposed criterion. It means that the value of $\Delta_{xy} - y$ which is equal to $x - 2y$ is less than $x_P - 2y_P$. However, slope of the lines crossing the origin and these points are less than $m_P$, and so they are not favorable. The line slopes lie in the interval $(m_{Q_3}, m_P)$, where $m_{Q_3}$ is the slope of the line crossing the origin and the point $Q_3$. We have:

$$0 < m_{Q_3} = \frac{M - \Delta_P + 2y_P}{2M}$$

\[\text{Figure 3. Slopes of crossing lines from the origin and the points located in the areas } R_1 \text{ and } R_2 \text{ are less than } m_P\]

ii. $x_P - 2y_P = 0$

In this case, all of the points preferred to $P$ by our criterion meet our purpose. Therefore, there is no unfavorable point regarding our criterion.

iii. $x_P - 2y_P > 0$

In this case, the points surrounded in the region $OQ_5P$ are preferred to the point $P$ by our criterion, while slopes of the lines crossing the origin and the points are less than $m_P$, and so they are unfavorable. The slope lines lie in the interval $(0, m_P)$ (Figure 4).

Regarding to the above discussion, we compare the line slopes of the unfavorable points preferred to $P$ by either of the linear criterions.

For the Liu and Peng’s criterion, the slope lines always lie in the interval $(0, m_P)$. However, for our proposed criterion, it depends on the coordinate of $P$.

If $x_P - 2y_P < 0$, the line slopes lie in an interval $(t, m_P)$, where $t \in R^+(t = m_{Q_3})$. Hence, range of the line slopes has a better statement than that for Liu and Peng’s.

If $x_P - 2y_P = 0$, there is no unfavorable point preferred to $P$. Hence, in this case our criterion absolutely has a better statement than Liu and Peng’s.

If $x_P - 2y_P > 0$, the line slopes lie in the interval $(0, m_P)$ the same as those of Liu and Peng’s. However, the region of unfavorable points preferred to $P$ with regard to our criterion is smaller than Liu and Peng’s.

Therefore, the set of unfavorable points preferred to $P$ with regard to our criterion has a better statement than that of Liu and Peng’s in terms of line slopes or the size.
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Figure 4. Range of slopes of the lines crossing the origin and the points located in $R_5$ is the same as that for the points located in the area enclosed by the segments $OP, OQ_6$ and $Q_6P$. 

Now, assume that the horizontal and vertical axes of the two-dimensional coordinate system in Figure 5 correspond to the virtual inputs (weighted sum of inputs) and the virtual outputs (weighted sum of outputs), respectively. Let $V^t \in R^{m^+}$ and $U^t \in R^{s^+}$. The point $P^o(U, V) = (VX_o, UY_o)$, $o \in \{1, \ldots, n\}$, is corresponding to $DMU_o$ in respect to $(U, V)$ in the two-dimensional coordinate system. Slope of the line crossing the origin and $P^o(U, V)$ is equal to the absolute efficiency of $DMU_o$ based upon $(U, V)$, $E^o(U, V) = \frac{UY_o}{VX_o}$. The difference between length and the width of $P^o(U, V)$ is equal to $\Delta^o(U, V) = VX_o - UY_o$. The greater slope of the line crossing the origin and $P^o(U, V)$; the greater $E^o(U, V)$. Let $E^o(U, V) \leq 1$, according to the former discussion, the value of our linear criterion to measure how much close it is to 1, is equal to $\Delta^o(U, V) - UY_o = VX_o - 2UY_o$.

Let $P_i(U_i, V_i)$, $i = 1, \ldots, 5$, show the corresponding points to $DMU_o$ based upon $(U_i, V_i)$. As Figure 5 illustrates, $(U_i, V_i)$, $i = 1, \ldots, 5$, are preferred to $(U, V)$ by our proposed criterion. Among them, only $(U_5, V_5)$ is not favorable because $E^o(U_5, V_5)$ is less than $E^o(U, V)$. ($\text{slop of the line crossing the origin and } P^o(U_5, V_5)$ is less than that of $P^o(U, V)$). Regarding to Liu and Peng’s criterion, $(U_i, V_i)$, $i = 1, \ldots, 4$, are preferred to $(U, V)$. Among them, only $(U_4, V_4)$ is not favorable. With regard to the previous discussion, $(U_1, V_1)$ is the best of the mentioned sets of weights in terms of maximizing the absolute efficiency of $DMU_o$.

In the next section, we are going to show how this affects the model and make it different from Liu and Pengs [20].

4. The proposed model. In this section, we suggest a new linear programming to generate a common set of weights. The purpose of this model is to find a CSW to maximize absolute efficiencies of all DMUs. To this aim, according to the previous discussions, we regard for minimizing the differences among virtual inputs...
and outputs and maximizing the virtual outputs. The original model to generate a CSW is as follows:

\[
\begin{align*}
\text{min} & \quad \{ VX_1 - UY_1, \ldots, VX_n - UY_n \} \\
\text{max} & \quad \{ UY_1, \ldots, UY_n \} \\
\text{s.t.} & \quad \frac{UY_j}{X_j} \leq 1 \quad j = 1, \ldots, n \\
& \quad U1_s + V1_m = 1 \\
& \quad U \geq \varepsilon 1_s \quad V \geq \varepsilon 1_m
\end{align*}
\]

(7) is transformed to Multi Objective Linear Programming (MOLP) (8):

\[
\begin{align*}
\text{min} & \quad \{ VX_1 - UY_1, \ldots, VX_n - UY_n, -UY_1, \ldots, -UY_n \} \\
\text{s.t.} & \quad UY_j - VX_j \leq 0 \quad j = 1, \ldots, n \\
& \quad U1_s + V1_m = 1 \\
& \quad U \geq \varepsilon 1_s \quad V \geq \varepsilon 1_m
\end{align*}
\]

(8)

(7) is transformed to Multi Objective Linear Programming (MOLP) (8):

We can solve the linear programming (9) in order to find an efficient solution of MOLP (8) (More details can be found in [14]):

\[
\begin{align*}
\text{min} & \quad z = \sum_{j=1}^{n} (VX_j - UY_j) - \sum_{j=1}^{n} UY_j \\
\text{s.t.} & \quad UY_j - VX_j \leq 0 \quad j = 1, \ldots, n \\
& \quad U1_s + V1_m = 1 \\
& \quad U \geq \varepsilon 1_s \quad V \geq \varepsilon 1_m
\end{align*}
\]

(9)

Finally, (9) is rewritten as (10):

\[
\begin{align*}
\text{min} & \quad z = V \sum_{j=1}^{n} X_j - 2U \sum_{j=1}^{n} Y_j \\
\text{s.t.} & \quad UY_j - VX_j \leq 0 \quad j = 1, \ldots, n \\
& \quad U1_s + V1_m = 1 \\
& \quad U \geq \varepsilon 1_s \quad V \geq \varepsilon 1_m
\end{align*}
\]

(10)
Where, the constraints $UY_j - VX_j \leq 0 \ (j = 1, \ldots, n)$ guarantee that the efficiencies of all DMUs are less than or equal to 1. $U1_s + V1_m = 1$ is a normalizing constraint that has a considerable role on preventing (10) to be unbounded. $\varepsilon$ is a non-Archimedean infinitesimal constant value which is used to avoid the appearance of zero weights in the CSW. So, all of the indicators will be involved in the evaluation.

We accept the optimal solution of (10) as the CSW. It is noteworthy that in the methods of this context, ranking methods are usually dependent on the generated common set of weights. Hence, having alternative optimal solutions may lead to different ranking results. Some researches such as [20] and [30] deal with this issue and provide an extra model to choose only one optimal solution as the CSW. Thus, we can use the criterion presented in [30] to ensure choosing just one of the optimal solutions to (10) as the CSW.

As it will be proven later (Theorem 4.1), the points corresponding to the DMUs in respect to each feasible solution of (10) are located on a bounded region similar to what discussed in Section 3. Accordingly, we utilize the new linear criterion introduced in that Section to measure magnitude of efficiencies of the DMUs.

**Theorem 4.1.** The set of all points corresponding to $DMU_1, \ldots, DMU_n$ in respect to each feasible solution of (10) are located on a bounded region of $R^2$.

**Proof.** Let $(U, V)$ be a feasible solution of 10, then

$$U1_s + V1_m = 1 , \ U \geq \varepsilon 1_s, V \geq \varepsilon 1_m$$

$$\Rightarrow u_r < 1 \ (r = 1, \ldots, s), v_i < 1 \ (i = 1, \ldots, m)$$

Hence,

$$\forall j \in \{1, \ldots, n\} \ (UY_j \leq 1_sY_j \leq \max 1_sY_j | j \in \{1, \ldots, n\})$$

We set $M_x = \max\{1_mX_j | j = 1, \ldots, n\}$ and $M_y = \max\{1_sY_j | j = 1, \ldots, n\}$. Then,

$$\forall j \in \{1, \ldots, n\} \ (UY_j \leq M_y , VX_j \leq M_x)$$

With regard to (1), $0 < VX_j, UY_j \ (j = 1, \ldots, n)$. Also, with regard to the constraints in (10), $UY_j \leq VX_j \ (j = 1, \ldots, n)$. Let $M = \max\{M_x, M_y\}$. Then,

$$\forall j \in \{1, \ldots, n\} \ (VX_j, UY_j) \in \{(x, y) | 0 < y \leq x \leq M\} = A_M$$

Thus, the set of all points corresponding to the DMUs in respect to each feasible solution to (10) is a subset of $A_M \subseteq R^2$, and we are done. \hfill \Box

$\varepsilon$ is a non-Archimedean infinitesimal constant, for which in practice we set a fixed value to solve the model using software. In this regard the following Theorem is stated:

**Theorem 4.2.** (10) is feasible for a $\varepsilon$ which is small enough.

**Proof.** Assume $S = \{(U, V) \ | \ UY_j - VX_j \leq 0 , j = 1, \ldots, n , \ U \geq \varepsilon 1_s, V \geq \varepsilon 1_m\}$. Regarding [21] we can find $\varepsilon$ such that $S$ is feasible. We indicate the feasible region of (10) as $S_k$:

$$S_k = \{(U, V) \ | \ UY_j - VX_j \leq 0, j = 1, \ldots, n, U1_s + V1_m = 1, U \geq \varepsilon 1_s, V \geq \varepsilon 1_m\}$$

Let $(\bar{U}, \bar{V}) \in S$, then

$$\forall k > 0 \ k \bar{U}Y_j - k\bar{V}X_j \leq 0 \ j = 1, \ldots, n$$

Suppose $\bar{U}1_s + \bar{V}1_m = \bar{\alpha}$, it’s evident that $\bar{\alpha} > 0$. It’s easily noticed that if $\bar{\alpha} \leq 1$ then $\frac{1}{\bar{\alpha}} (\bar{U}, \bar{V}) \in S_k$, and so $S_k$ is feasible. Let $1 < \bar{\alpha}$, in this case the $\varepsilon$-constraints
should be checked. If \( \varepsilon \mathbf{1}_s \leq \frac{1}{\alpha} \bar{U} , \varepsilon \mathbf{1}_m \leq \frac{1}{\alpha} \bar{V} \), then \( \frac{1}{\alpha}(\bar{U}, \bar{V}) \in S_8 \), and so (10) is feasible. otherwise, we set \( \varepsilon = \frac{1}{\alpha} \varepsilon \). It’s clear that \( \frac{1}{\alpha}(U, V) \in S_8 \) in this case, and so \( S_8 \) is feasible.

**Theorem 4.3.** (10) has finite optimal value.

**Proof.** Let \( S_8 \) be the feasible region of (10) for a small enough \( \varepsilon \) and let \((U, V)\) be an arbitrary member of \( S_8 \). Then,

\[
U \mathbf{1}_s + V \mathbf{1}_m = 1 , U \geq \varepsilon \mathbf{1}_s , V \geq \varepsilon \mathbf{1}_m \Rightarrow ||U||_\infty, ||V||_\infty < 1
\]

Let \( X_{sum} \) denotes \( \sum_{j=1}^{n} X_j \) and \( Y_{sum} \) denotes \( \sum_{j=1}^{n} Y_j \), and \( M = \sum_{j=1}^{n} X_{sum} \leq M = \sum_{j=1}^{n} Y_{sum} \leq 2 \). \( M \) is a positive constant in this problem, so the set of all objective values of (10) is bounded. Hence (10) has finite optimal value, and we are done.

Let \((U^*, V^*)\) be the common set of weights. So the absolute efficiency of \( DMU_j \), \( j = 1, \ldots, n \), based upon it will be equal to \( E_j^* = \frac{\sum_{j=1}^{n} u_j^* v_{ji} x_i}{\sum_{i=1}^{n} v_{ji} x_i} \).

**Definition 4.4.** \( DMU_o, o \in \{1, \ldots, n\} \), is called efficient in our method if \( E_o^* \) is equal to one.

**Theorem 4.5.** Each efficient \( DMU \) in our method is CCR-efficient.

**Proof.** Suppose \( DMU_o \) be efficient in our method, and \((U^*, V^*)\) be the CSW. It is evident that \( \frac{1}{\sum_{j=1}^{n} u_j^* v_{ji} x_i} (U^*, V^*) \) is a feasible solution for (4) which has the objective value of 1, and so it is an optimal solution with positive components. Therefore, \( DMU_o \) is considered CCR-efficient.

5. **Ranking approach.** The purpose of our method is to evaluate the performance of DMUs under the same conditions. Hence, the absolute efficiencies of DMUs based upon generated CSW are considered as the ranking criterion. The DMUs are ranked based upon the magnitude of the efficiencies. In the case of having two DMUs with the same efficiencies, another criterion can be utilized for more distinction. Before stating our approach, it should be mentioned that Liu and Peng used a criterion to distinguish the performance of CWA-inefficient units with the same CWA-efficiencies which can’t be used for CWA-efficient ones, more details in [20]. This criterion was considered in [25] from another point of view and it was extended to be used for both of the categories. In this case, our approach is based on [25], and we use a developed approach. Suppose that the absolute efficiencies of \( DMU_i \) and \( DMU_j \) are the same, \( E_i^* = E_j^* \). Then,

\[
\frac{U^* Y_j}{V^* Y_j} = \frac{V^* X_j}{V^* X_i} = \frac{\Delta_j^*}{\Delta_i^*} \tag{11}
\]

where, \( \Delta_j^* = V^* X_j - U^* Y_j \) and \( \Delta_i^* = V^* X_i - U^* Y_i \). It is evident that the unit with better (worse) performance in terms of inputs consumption, under the given conditions, has worse (better) performance in terms of outputs production.
Our ranking rules are stated as follows:
Let \( i, j \in \{1, \ldots, n\} \)
If \( E_i^* < E_j^* \) then the ranking score of \( DMU_j \) is better than that of \( DMU_i \).
If \( E_i^* = E_j^* \) then:
In input-oriented approach:
If \( V^* X_i > V^* X_j \) then the ranking score of \( DMU_j \) is better than that of \( DMU_i \).
In output-oriented approach:
If \( U^* Y_i < U^* Y_j \) then the ranking score of \( DMU_j \) is better than that of \( DMU_i \).

We adopt either input-oriented or output-oriented approaches depending on the DM’s tendency. If less inputs consumption (greater outputs production) is more important for DM, input-oriented (output-oriented) approach is utilized for the ranking procedure.

In our method, the ranking criteria are simple and identical for both efficient and inefficient DMUs. However, it’s noticeable that the performance of inefficient DMUs with the same efficiencies may be distinguished by increasing the decimal precision.

![Graph showing two pairs of DMUs with the same efficiency scores.](image)

Figure 6. Two pairs of DMUs with the same efficiency scores.

As Figure 6 shows, \( E_1^* = E_2^* = 1, E_3^* = E_4^* < 1 \). Both of \( DMU_1 \) and \( DMU_2 \) are efficient, and both of \( DMU_3 \) and \( DMU_4 \) are inefficient in our method. Suppose that \( R_j \) \((j = 1, \ldots, 4)\) shows the ranking score of \( DMU_j \). Then, according to the input-oriented approach, \( R_4 < R_3 < R_2 < R_1 \). Also, According to the output-oriented approach, \( R_3 < R_4 < R_1 < R_2 \).

6. **Numerical Examples.** In this section, we consider two numerical examples with different points of view. In the first one, our purpose is to implement our ranking method in details on a real data set. Besides, we use CWA- ranking rules [20]. Finally, we discuss and compare the ranking results.

6.1. **Numerical example 1:** In this example, we implement our method on a real data set from 16 hospitals in Japan [31]. We use both of input-oriented and output-oriented approaches. Table 1 shows the input and output data of these hospitals.

The input indicators are as follows:
\( I_1 \): Total hours worked by doctors in the survey period.
$I_2$: Total hours worked by nurses.
$I_3$: Total hours worked by technical workers.
$I_4$: Total hours worked by office staff

The output indicators are as follows:
$O_1$: Total medical insurance points for outpatients.
$O_2$: Total medical insurance points for inpatients.

Table 1. Input and Output data of DMUs

| DMU  | $x_{1j}$ | $x_{2j}$ | $x_{3j}$ | $x_{4j}$ | $y_{1j}$ | $y_{2j}$ |
|------|----------|----------|----------|----------|----------|----------|
| $DMU_1$ | 995      | 6205     | 1375     | 2629     | 4127     | 1678     |
| $DMU_2$ | 917      | 5898     | 1379     | 2047     | 3721     | 1277     |
| $DMU_3$ | 3178     | 10049    | 3615     | 3511     | 2706     | 2051     |
| $DMU_4$ | 813      | 5833     | 1124     | 1730     | 2176     | 1538     |
| $DMU_5$ | 1236     | 8639     | 2486     | 4990     | 5220     | 2042     |
| $DMU_6$ | 1146     | 7610     | 1600     | 3589     | 3517     | 1856     |
| $DMU_7$ | 705      | 5600     | 1557     | 3623     | 2352     | 2060     |
| $DMU_8$ | 2871     | 11524    | 2880     | 2452     | 1755     | 1664     |
| $DMU_9$ | 1098     | 8998     | 1730     | 2823     | 4412     | 2334     |
| $DMU_{10}$ | 2032     | 9383     | 2421     | 4454     | 5386     | 2080     |
| $DMU_{11}$ | 1414     | 10468    | 2140     | 3649     | 5735     | 2691     |
| $DMU_{12}$ | 1967     | 11260    | 2759     | 3178     | 6079     | 2804     |
| $DMU_{13}$ | 1851     | 9880     | 2335     | 4570     | 5893     | 2495     |
| $DMU_{14}$ | 3100     | 15649    | 5487     | 2940     | 5248     | 3692     |
| $DMU_{15}$ | 5016     | 18010    | 4008     | 3567     | 7800     | 4852     |
| $DMU_{16}$ | 1924     | 12682    | 2490     | 2975     | 6040     | 3396     |

Also, we use CWA-ranking method [20] for comparing the results. We utilize a common criterion used in [30] in order to determine just one CSW from each set of optimal solutions to (10) and to (6). Table 2 shows the CSWs obtained from both of the methods. $\varepsilon$ is set to $10^{-6}$ for both the models.

Table 2. The generated common set of weights

|              | $v_1^*$   | $v_2^*$  | $v_3^*$   | $v_4^*$   | $u_1^*$  | $u_2^*$  |
|--------------|-----------|----------|-----------|-----------|----------|----------|
| Our method   | 0.000001  | 0.181081 | 0.000001  | 0.124392  | 0.116466 | 0.578059 |
| Liu and Peng’s method [20] | 0.000001  | 0.000001 | 0.000001  | 7.50E-07  | 8.70E-07 | 0.000004 |

The obtained efficiencies based on the CSWs are shown in Table 3. Moreover, CCR-efficiencies are calculated for more information. In each method, there are 3 units with the efficiency scores of 1. Although the set of such DMUs are the same in both of the methods, it is not always this way. In both methods, existing the efficiency score of 1 in the results caused the obtained absolute efficiencies to be interpreted as relative efficiencies. Now, we compare the results in terms of magnitude of the obtained efficiencies. $56.2\%$ of the units obtained greater efficiency scores in our method rather than Liu and Peng’s. Moreover, $18.7\%$ of DMUs in our method obtained as much efficiency scores as Liu and Peng’s (with the efficiency
scores of 1). 25% of DMUs obtained less efficiency scores in our method rather than Liu and Peng’s, but the differences are low such that the maximum of them is less than 0.03 depending to $DMU_9$.

Table 3. The efficiencies

| $DMU_j$ | $E_j^*$ | CWA-Efficiency | CCR-Efficiency |
|---------|---------|----------------|----------------|
| $DMU_1$ | 1       | 1              | 1              |
| $DMU_2$ | 0.885761| 0.877007       | 1              |
| $DMU_3$ | 0.665101| 0.55716        | 0.690363       |
| $DMU_4$ | 0.89857 | 0.911488       | 1              |
| $DMU_5$ | 0.818436| 0.807723       | 1              |
| $DMU_6$ | 0.812557| 0.823997       | 0.881091       |
| $DMU_7$ | 1       | 1              | 1              |
| $DMU_8$ | 0.48762 | 0.440469       | 0.555791       |
| $DMU_9$ | 0.940676| 0.968986       | 1              |
| $DMU_{10}$ | 0.812047 | 0.774854      | 0.863042      |
| $DMU_{11}$ | 0.94638  | 0.963253      | 0.996068      |
| $DMU_{12}$ | 0.956693 | 0.920591      | 1             |
| $DMU_{13}$ | 0.90288  | 0.884299      | 0.915511      |
| $DMU_{14}$ | 0.858084 | 0.75116       | 1             |
| $DMU_{15}$ | 0.960084 | 0.867478      | 1             |
| $DMU_{16}$ | 1       | 1              | 1              |

We consider the average of obtained efficiencies as an evaluation index of the methods in terms of magnitude of the obtained efficiencies. Table 4 shows the average of the obtained efficiencies in both methods. The ratio of the averages is reported, too. As the content shows, the average of obtained efficiencies in our method is greater than that of Liu and Peng [20].

Table 4. The average of the efficiencies in each method and their ratio

| Our method | Liu and Peng’s method [20] | The ratio |
|------------|----------------------------|----------|
| 0.871556   | 0.846779                   | 1.02926  |

Table 5 shows the ranking scores of DMUs based upon the input-oriented and output-oriented approaches in our method.

The last two columns show the CWA-ranking scores [20] and CCR-ranking scores. The units are ranked based on magnitude of efficiency scores. As seen in Section 5 either input-oriented or output-oriented approaches can be utilized for more distinction. Here, we run both approaches. However, only one of them is used in practice, depending on DM’s decision. If he/she has no predetermined idea, one of the approaches can be chosen randomly. According to our definition, in each approach the range of the ranking scores of the DMUs with equal efficiency scores are the same but in reverse, whereas ranking scores of the rest of DMUs are independent from these approaches. Also, we use CWA-ranking rules to distinguish the DMUs with the same efficiency scores of 1 in Liu and Pengs method [20]. As the results
show, some of the CCR-inefficient DMUs have gained greater ranking scores than some of the CCR-efficient DMUs (e.g. see $DMU_2$ and $DMU_{13}$). In fact, there may be conditions, under which a CCR-inefficient DMU may have better performance than some CCR-efficient DMUs which don’t dominate that. For more clarification, assume that $DMU_p$ be a DMU which its status in one input or output indicator is uniquely the best, with a little difference from the others. Regarding the rest of indicators, it has a significantly bad status compared to other DMUs. This DMU is Pareto-optimal, and so CCR-efficient. The maximum relative efficiency of $DMU_p$ may occur under a special condition when the weight of the mentioned indicator is greater than the weight of the rest of indicators. If the generated CSW doesn’t meet this condition regarding $DMU_p$, its efficiency may be less than some other units. On the other hand, assume that $DMU_q$ be a CCR-inefficient unit having the best status in all input and output indicators except some, for each of which there is at least one CCR-efficient unit with a insignificantly better statement. Under some special conditions (some specific set of weights), this CCR-inefficient DMU may gain greater efficiency than the efficiencies of some CCR-efficient DMUs which don’t dominate it. Similarly, this CCR-inefficient DMU may gain greater efficiency than the efficiencies of some CCR-efficient DMUs which don’t dominate that under some special conditions (some specific set of weights). Therefore, it wouldn’t be meaningless if the efficiency score of a CCR-inefficient DMU based upon the CSW were better than that of a CCR-efficient DMU which doesn’t dominate it.

In the second numerical example, we have just a comparative point of view. Our method and Liu and Peng’s [20] have similar strategies to find a CSW to maximize efficiencies of units. In fact, both of them consider to make the obtained efficiencies close to the common benchmark level of 1 and use linear criterion to measure the closeness. Earlier, we investigate our criterion theoretically and geometrically in

| $DMU_j$ | Our method: Input-oriented approach | Our method: Output-oriented approach | CWA-ranking | CCR |
|---------|------------------------------------|------------------------------------|-------------|-----|
| $DMU_1$ | 1                                  | 3                                  | 2           | 1   |
| $DMU_2$ | 10                                 | 10                                 | 9           | 1   |
| $DMU_3$ | 15                                 | 15                                 | 15          | 6   |
| $DMU_4$ | 9                                  | 9                                  | 7           | 1   |
| $DMU_5$ | 12                                 | 12                                 | 12          | 1   |
| $DMU_6$ | 13                                 | 13                                 | 11          | 4   |
| $DMU_7$ | 2                                  | 2                                  | 3           | 1   |
| $DMU_8$ | 16                                 | 16                                 | 16          | 7   |
| $DMU_9$ | 7                                  | 7                                  | 4           | 1   |
| $DMU_{10}$ | 14                                 | 14                                 | 13          | 5   |
| $DMU_{11}$ | 6                                  | 6                                  | 5           | 2   |
| $DMU_{12}$ | 5                                  | 5                                  | 6           | 1   |
| $DMU_{13}$ | 8                                  | 8                                  | 8           | 3   |
| $DMU_{14}$ | 11                                 | 11                                 | 14          | 1   |
| $DMU_{15}$ | 4                                  | 4                                  | 10          | 1   |
| $DMU_{16}$ | 3                                  | 1                                  | 1           | 1   |
comparison with Liu and Peng's. Now, we compare them practically in terms of magnitude of the obtained efficiencies.

6.2. **Numerical example 2:** Here, we concentrate on comparison of the capability of the models (10) of our method and (6) of Liu and Peng's in terms of magnitude of obtained efficiencies in practice. To this aim, we consider some sample data in various sizes of 6, 12, 25, 50, 100 and 200 with different number of indicators. In each case, we solve the models and calculate the absolute efficiencies of DMUs. The evaluation index is set to the average of obtained efficiencies.

With regard to the large amount of data, we avoid statement of the details. In each case, only the amount of evaluation index, the average of the obtained efficiencies, is presented for comparison (see Table 6). Besides, the ratio of the averages are calculated.

| Size of the sample | Our method | CWA-Efficiencies | The ratio |
|--------------------|------------|------------------|----------|
| 6                  | 0.916128   | 0.906961         | 1.010107 |
| 12                 | 0.539086   | 0.539082         | 1.000006 |
| 25                 | 0.689937   | 0.148176         | 4.656189 |
| 50                 | 0.550724   | 0.553320         | 0.995309 |
| 100                | 0.639243   | 0.471678         | 1.355254 |
| 200                | 0.582310   | 0.580711         | 1.002753 |

With regard to our evaluation index, among all samples except the one of the size 50, our model is superior in terms of maximizing the efficiencies. However, in the sample of size 50, the average of efficiency scores obtained from (10) in our method is about 0.99 times compared to that extracted from solving (6) in the Liu and Peng's method [20]. In the sample of size 25, the average of efficiency scores in ours is significantly greater than that of Liu and Peng's (about 4.6 times). In this numerical example, in about 83.3% of the cases, our model is superior to Liu an Peng's [20] in terms of magnitude of obtained efficiency scores. In these samples, the sets of efficient DMUs obtained from each method don’t have special dependence regarding common members or size.

7. **Conclusion.** In this paper, we present a fully ranking method using common weights. We suggest a linear programming to generate a CSW to maximize efficiency scores of all DMUs. Our ranking procedure is simple but meaningful. the ranking criteria are applied to all efficient and inefficient DMUs in our method disregarding to their categories. In this methodology, not only performance of efficient DMUs are compared under the same conditions but also the inefficient DMUs find opportunity to be evaluated in comparison with the efficient units. We implement our ranking method on a real data set and discuss the results. As the ranking results show, under a given conditions, an inefficient DMU may even have a better performance than an efficient one does. However, it can never outrank the efficient units which dominate it. In the ranking methods with common weights, the aim is generally to search a CSW to maximize the efficiencies. As the literature shows, the criterion applied to this aim has a considerable role in the linearity or non-linearity and the computational burden of the obtained model to generate a CSW.
Liu and Peng [20] suggested a model to generate a CSW in which the criterion that was used to measure the magnitude of efficiencies is linear and does not need pre-computation. Our proposed method not only enjoys the advantages of being linear and not requiring pre-computation but also is stronger in terms of maximizing magnitude of the efficiencies. Although using linear criteria is more desirable, they may make some limitations in determining the best choice in order to maximize efficiencies as the CSW. We investigated and compared our criterion and Liu and Peng’s in some aspects. Theoretically, our criterion is more equipped than the Liu and Peng’s. Geometrically, it generally has a better statement in terms of preferring undesirable points. Practically, by considering some different data samples, our criterion is more capable in terms of maximizing efficiency scores.

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E-mail address: g.h.shirdel@qom.ac.i
E-mail address: sramezani_j@yahoo.com