String Universality and Non-Simply-Connected Gauge Groups in 8d

Mirjam Cvetič,1, 2 Markus Dierigl,1 Ling Lin,3 and Hao Y. Zhang1

1Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104-6396, USA
2Center for Applied Mathematics and Theoretical Physics, University of Maribor, Maribor, Slovenia
3Department of Theoretical Physics, CERN, 1211 Geneva 23, Switzerland

I. INTRODUCTION

One of the important lessons from string theory is that consistency conditions of quantum gravity are highly restrictive. In the low energy limit, they result in a small and (up to deformations) possibly finite subset of effective descriptions, leaving behind a vast “Swampland” of seemingly consistent quantum field theories coupled to gravity [1]. Recent attempts to specify the Swampland’s boundary (cf. [2] for reviews) have since reinforced the idea of String Universality: every consistent quantum-gravitational theory is in the string landscape.

In this work, we address an important characterizing feature of supergravity theories, namely the global structure of their gauge group. By deriving a field theoretic consistency condition for a gauge group to take the form $G/Z$, we rule out many seemingly acceptable eight-dimensional (8d) $\mathcal{N} = 1$ supergravity theories without known string constructions as consistent quantum gravity theories, thus providing strong evidence for String Universality in 8d.

The prototypical examples of string universality are in eleven and ten dimensions, where the low energy limits of M- and string theory give rise to the only consistent supergravity theories in the respective dimensions. In ten dimensions, this requires more subtle field theoretic arguments [3], or incorporating extended dynamical objects of the theory [4], to “drain” the 10d supergravity Swampland.

In lower dimensions, one observes a broader spectrum of string-derived supergravity theories, but nevertheless with some intricate structures not naïvely expected from field theory considerations. For example, the rank $r_G$ of the gauge group in known string compactifications is bounded by $r_G \leq 26 - d$ in $d$ dimensions, and satisfies $r_G \equiv 1 \mod 8$ and $r_G \equiv 2 \mod 8$, respectively, in $d = 9$ and $d = 8$, respectively. Likewise, not all gauge algebras have string realizations. In particular, being relevant to our discussion, there are no string compactifications to 8d with $so(2n+1)$ ($n \geq 3$), $f_4$ and $g_2$. Again, novel Swampland constraints [5, 6] and refined anomaly arguments [7] reproduce these restriction, and therefore downsizes the 9d and 8d Swampland considerably.\(^1\)

The goal of this work is to provide similar constraints for the global structure of the gauge group of 8d $\mathcal{N} = 1$ supergravity theories. Taking inspiration from F-theory [8], where the global gauge group structure is encoded in the Mordell–Weil group of the elliptically fibered compactification space [9–11], it appears that the allowed gauge groups $G/Z$, with $Z \subset Z(G)$ a discrete subgroup of the center of $G$, are heavily restricted. For example, there are no 8d string compactifications, including non-F-theoretic constructions, that have gauge group $SU(n)/\mathbb{Z}_n$, whereas models with $SU(n)$ gauge group are straightforwardly constructed in F-theory.

These restrictions are mathematically well-known from the classification of elliptic K3 surfaces [12, 13] (see also [14]). Focusing on $G$ a simply-connected non-Abelian Lie group\(^2\), the geometry bounds the size of $Z$, e.g., when $Z \cong \mathbb{Z}_4$, then $\ell \leq 8$. Moreover, for each of the cases $\ell = 7, 8$, there is exactly one elliptic K3 on which F-theory compactifies to $SU(7)^3/\mathbb{Z}_7$ and $[SU(8)^2 \times SU(4)/SU(2)]/\mathbb{Z}_8$, respectively.

A natural question is then, whether these geometric restrictions reflect limitations of string theory, or previously unknown consistency conditions of quantum gravity in 8d.

In this work, we show that the latter is the case. The key is to realize a non-simply connected group $G/Z$ by gauging a Z 1-form symmetry [15]. Then, the above restrictions can be understood as requiring the absence of certain anomalies that would obstruct this gauging. Turning tables around, charting the Swampland of gauge groups $G/Z$ (in any dimension) can be equivalently tackled by studying consistency conditions for gauged Z 1-form symmetries in gravitational theories. As we will discuss below, in 8d $\mathcal{N} = 1$ theories, one such condition

\(^1\) As $g_2$ does not suffer similar anomalies, it remains an open question if it truly belongs to the 8d Swampland.

\(^2\) More precisely, the most general gauge group is $G/Z \times \mathbb{Z}_{2}$, with $Z \subset Z(G)$, i.e., $Z \cap U(1)^r = \{1\}$. In this work we consider constraints for $Z$ exclusively, leaving a more detailed study including $Z_{f} \subset Z(G \times U(1)^r) \cong Z(G) \times U(1)^r$, based on [11], for the future.
is the absence of a mixed anomaly involving center 1-form symmetries.

The anomaly originates from a generalization of the familiar $\theta$-term, $\theta \text{Tr}(F^2)$, in 4d. There, the fractional shift of the instanton density $\text{Tr}(F^2)$, due to the presence of a background field for the 2d 1-form symmetry, breaks the $2\pi$-periodicity of $\theta$ [15–18]. In higher dimensions, $\text{Tr}(F^2)$ can now couple to higher-form fields (e.g., to vector fields in 5d and tensors in 6d), which themselves enjoy gauge symmetries. These can now lead to mixed anomalies between these (continuous) symmetries and the 2d 1-form symmetry [19, 20].

The analogous coupling in 8d would be $B_4 \wedge \text{Tr}(F^2)$, where $B_4$ is a 4-form field that can be viewed as the gauge field for a 3-form $U(1)$ symmetry. Crucially, while this term is absent in a pure 8d supersymmetric gauge theory (as there are no appropriate fields $B_4$ in the $\mathcal{N} = 1$ vector multiplet), this coupling necessarily exists if one includes a gravity multiplet, which contains a 2-tensor $B_2$ that is dual to $B_4$ [23]. Since the $U(1)$ 3-form symmetry of $B_4$ must be gauged for a consistent supergravity theory, a mixed anomaly involving a 2d 1-form symmetry obstructs the gauging of the latter. The vanishing of this anomaly is therefore a necessary condition to gauge (part of) the center symmetry to obtain a non-simply connected gauge group $G/Z$. Remarkably, this condition turns out to be structurally the same as geometric properties of elliptic K3 manifolds! In particular, it heavily constrains possible combinations of simply-connected $G$ and $Z \subset Z(G)$, consistent with known string constructions in 8d. Using this condition, we can consequently “drain” large portions of the 8d Swampland.

In the following, we will outline in section II key geometrical facts about the global gauge group structure in F-theory, and in 8d it is constrained through properties of elliptic K3s. The more field-theoretically inclined reader can safely move directly to section III, where we will elaborate on the anomaly that provides the physical counterpart to these constraints. Crucially, these constraints are also compatible with non-F-theoretic constructions. In section IV, we will discuss outliers that hint at further, more subtle constraints, whose quantification poses interesting open problems.

II. MORDELL–WEIL TORSION AND THE GAUGE GROUP IN F-THEORY

A broad class of 8d $\mathcal{N} = 1$ supergravity theories arise from F-theory compactified on elliptic K3 surfaces. We refer to reviews [24–26] for broader introduction to the relevant background material, and focus in the following on two aspects key to the present discussion. First, the non-Abelian gauge algebras $\mathfrak{g}_i$ (associated to simply-connected groups $G_i$) are captured by reducible Kodaira-fibers of ADE-type $\mathfrak{g}_i$ [8]. Second, the global structure of the gauge group, $\prod G_i/Z$, is determined by the torsional part $Z$ of the Mordell–Weil group of sections [9] (see especially [26] for a pedagogical introduction for this).\(^4\)

In general, the notation $G/Z$ requires a specification of the subgroup $Z \subset Z(G) = \prod_i Z(G_i)$ of the full center $Z(G)$. In F-theory, this is determined by the intersection pattern between the generating sections of the Mordell–Weil group and the components of the $\mathfrak{g}_i$-Kodaira fiber that form the affine Dynkin diagram of $\mathfrak{g}_i$ [10, 11].

For definiteness, we restrict ourselves to compactifications with $\mathfrak{g}_i = \mathfrak{su}_{n_i}$, $i = 1, \ldots, s$, realized by K3 surfaces with only $I_{n_i}$ fibers.\(^5\) Each reducible $I_{n_i}$ fiber consists of $n_i$ irreducible components that form a loop, reflecting the structure of the affine $\mathfrak{su}_{n_i}$ Dynkin diagram. Starting with the affine node (determined by the intersection with the zero-section) we label the components by $0, \ldots, n_i - 1$ as we go around the loop of the $i$-th fiber. Then, an $\ell$-torsional section $\tau$ is uniquely characterized by the tuple $(k_1, \ldots, k_s)$ which labels the $k_i$-th component in the $i$-th fiber met by $\tau$ [12]. Moreover, one has $k_i \ell \equiv 0 \mod n_i$.

As explained in [10, 11], $\tau = (k_1, \ldots, k_s)$ corresponds precisely to the element $(k_1, \ldots, k_s) \in \prod_i \mathbb{Z}_{n_i} = Z(\prod_i SU(n_i))$, which is of order $\ell$. This element acts trivially on all matter states of the F-theory compactification, hence giving rise to the gauge group $G/\langle \tau \rangle \cong G/\mathbb{Z}_\ell$.

The allowed combinations of $G$ and $Z_G$ is heavily constrained geometrically by the following fact pertaining to intersection patterns between torsional sections and fiber components. For a K3 $X$ with only $I_{n_i}$ fibers, the non-affine components of each fiber span a sublattice $R \subset H_2(X, \mathbb{Z})$ with $\text{rank}(R) \leq 18$. Then, one can associate to $R$ a so-called discriminant-form group [12]

$$G_R \cong \mathbb{Z}_{n_1} \times \cdots \times \mathbb{Z}_{n_s}. \quad (1)$$

This group inherits from the lattice structure of $H_2(X, \mathbb{Z})$ a quadratic form

$$q : G_R \rightarrow \mathbb{Q}/\mathbb{Z}, \quad (x_1, \ldots, x_s) \mapsto \sum_{i=1}^s \frac{1 - n_i}{2n_i} x_i^2 \mod \mathbb{Z}. \quad (2)$$

Notice that $G_R = \prod_{i=1}^s Z(SU(n_i)) = Z(G)$. Then, by regarding a torsional section $\tau = (k_1, \ldots, k_s)$ as an element of $G_R$, it can be shown [12] that

$$q(k_1, \ldots, k_s) = \sum_{i=1}^s \frac{1 - n_i}{2n_i} k_i^2 \equiv 0 \mod \mathbb{Z}. \quad (3)$$

---

\(4\) To be precise, in the notation of footnote 2, the torsional part of the Mordell–Weil group is isomorphic to $Z[9]$, which is the factor of the center that we will focus on in this work.

\(5\) If $n_i = 1$, the fiber is singular but irreducible, and contributes no gauge factor. They will play no role for our discussion.

---

3 See also [21] for recent treatments of higher-form symmetries in higher-dimensional setups and [22] for an analysis of the global gauge group in 6d SCFTs.
This constraint is particularly powerful when $\ell$ is the power of a prime number. For such $\ell \geq 9$, one can quickly show that there are no possible sets $\{(n_i, k_i)\}$ satisfying (3) and $\sum_i (n_i - 1) \leq 18$ (corresponding to $\text{rank}(R) = \text{rank}(G) \leq 18$). For $\ell = 7$, there is exactly one configuration of fibers, namely three $I_7$'s, and $\tau = (1, 2, 3)$, in accordance with the classification of K3s. Likewise, this condition only allows, in the case $\ell = 8 = 2^4$, for the fiber configuration $\{I_8, J_8, I_4, I_2\}$ and $\tau = (1, 3, 1, 1)$.\footnote{This identification, as well as for $\ell = 7$, is up to reordering the fibers and exchanging $k_i \leftrightarrow -k_i \equiv n_i \rightarrow k_i$, which geometrically corresponds to the two different ways of going around the loop of the $I_n$ fiber.} Furthermore, if we also take into account another geometric property of K3s with only $I_n$ fibers — namely that there is always one $I_n$ fiber with $\ell$ a divisor of $n_j$, $\ell | n_j$ — we can show that there are no possible fiber configurations for all $\ell \geq 10$.

In the following, we will see that (3), and its strong limitations on gauge groups $G/Z$ in F-theory, can also be derived from purely field-theoretical arguments.

### III. Mixed Anomaly for Center Symmetries in 8d Supergravity

Let $G = \prod_i G_i$ be a non-Abelian group, where $G_i$ are simple simply-connected Lie groups with algebra $\mathfrak{g}_i$. In 8d $\mathcal{N} = 1$, the gauge potential $A_i$, with field strength $F_i$, of the $(0\text{-form})$ $\mathfrak{g}_i$ gauge symmetry comes in a vector multiplet with adjoint fermions. There are no other massless charged matter states, so at low energies one expects a discrete $Z(G) = \prod_i Z(G_i)$ 1-form symmetry [15]. Moreover, since the only massless fermions transform in a real representation, there are no pure gauge anomalies.\footnote{See also [24] for a discussion of continuous anomalies including gravity.}

Besides the vector multiplets, 8d $\mathcal{N} = 1$ supergravity contains the gravity multiplet with a 2-form gauge field $B_2$ as one of its component fields [27]. In the dual formulation where $B_2$ is replaced by a 4-form $B_3$, the most general Lagrangian contains the coupling [23]

$$\int_{M_8} \sum_i B_i \wedge \frac{1}{2} \text{Tr}(F_i \wedge F_i),$$

where the trace is normalized such that the instanton density $I_4(G_i) = \int_{M_8} \frac{1}{2} \text{Tr}(F_i \wedge F_i) = 1$ for a one-instanton configuration of a $G_i$-bundle over a 4-manifold $M_4$. Due to this coupling, the partition function generally vanishes for topologically non-trivial instanton configurations when integrating over flat $B_4 \in H^4(M_8, \mathbb{R}/\mathbb{Z})$, analogous to configurations in 6d discussed in [19, 28]. This can be cured by including the electrically charged objects for $B_3$, similar to a tadpole cancellation condition, as long as the required charges for the objects are integers.

The center 1-form symmetry of $G_i$ can be coupled to a 2-form background gauge field $C_2^{(i)}$ which takes values in $Z(G_i)$. When $C_2^{(i)}$ is non-trivial, it twists the $G_i$-bundle into a $G_i/Z(G_i)$-bundle with second Stiefel–Whitney class $w_2(G_i/Z(G_i)) = C_2^{(i)}$ [15, 16]. This twist leads to a contribution to (4),

$$I_4(G_i/Z(G_i)) \equiv \alpha_{G_i} \mathfrak{P}(C_2^{(i)}) \mod \mathbb{Z},$$

with $\mathfrak{P}$ the Pontryagin square. The contribution is in general fractional due to the coefficients $\alpha_{G_i}$ derived in [17], which we reproduce here:

| $G_i$     | $Z(G_i)$ | $\alpha_{G_i}$ |
|-----------|---------|---------------|
| $SU(n)$   | $\mathbb{Z}_n$ | $\frac{n-1}{2n}$ |
| $Sp(n)$   | $\mathbb{Z}_2$ | $\frac{1}{2}$ |
| $Spin(2n+1)$ | $\mathbb{Z}_2$ | $\frac{1}{2}$ |
| $Spin(4n+2)$ | $\mathbb{Z}_4$ | $\frac{2n+1}{8}$ |
| $Spin(4n)$ | $\mathbb{Z}_2^{(L)} \times \mathbb{Z}_2^{(R)} (\frac{1}{3}, \frac{1}{2})$ |
| $E_6$     | $\mathbb{Z}_3$ | $\frac{2}{3}$ |
| $E_7$     | $\mathbb{Z}_2$ | $\frac{1}{4}$ |

Analogous to the situation in 6d [19], the coupling (4) combines this fractional instanton configuration with a large $U(1)$ transformation $B_4 \to B_4 + b_4$, with $b_4$ a closed 4-form with integer periods, into a phase $2\pi i A(b_4, C_2^{(i)})$ for the partition function, with

$$A(b_4, C_2^{(i)}) = \sum_i \alpha_{G_i} \int_{M_8} b_4 \cup \mathfrak{P}(C_2^{(i)}).$$

For arbitrary $b_4$, the integral $\int_{M_8} b_4 \cup \mathfrak{P}(C_2^{(i)}) \in \mathbb{Z}$ cannot in general absorb the denominator of $\alpha_{G_i}$, thus leading to a non-trivial phase.

Regarding the 8d theory as living on the boundary $\partial M_9 = M_8$ of a 9d bulk $M_9$, this shift is the boundary contribution of a 9d action of the form

$$S_{9d} = 2\pi \sum_i \alpha_{G_i} \int_{M_9} b_5 \cup \mathfrak{P}(C_2^{(i)}),$$

where $C_2^{(i)}$ are extensions to the 9d bulk and $h_5$ is a 5-cocycle which shifts by $b_5$ under transformations, where $b_5|_{\partial M_9}$ restricts to $b_4$. On closed $M_9$ with $h_5$ a 5-cocycle, this generally integrates to non-trivial values, which by arguments in [18] shows that there cannot be an 8d topological sector canceling the shift. Furthermore, by generalizing the arguments presented in [19], the electrically charged objects for $B_3$ would acquire a fractional charge in the non-trivial background of $C_2^{(i)}$ if $\sum_i \alpha_{G_i} \mathfrak{P}(C_2^{(i)})$ is not an integral cocycle. Since this violates charge quantization, the fractional shift (6) cannot be compensated and can be understood as an anomaly between the large
$U(1)$ transformations of $B_4$ and the center 1-form symmetries. As the former symmetry is gauged, one cannot allow for background fields $C_2^{(i)}$ where (6) is non-trivial.

Note, however, that several gauge factors can conspire in a way that a combination of the individual center symmetries becomes free of the anomaly (6). Assuming that there are no other obstructions to switch on a background for this subgroup $Z$ of the center, this anomaly-free combination should be gauged, in line with common lore that, in quantum theories of gravity, no global symmetries (including discrete and higher-form symmetries) are allowed [29]. This would then lead to the gauge group $G/Z$.

### A. Condition for anomaly-free center symmetries

In the following, we will discuss how to determine subgroups $Z_\ell \cong Z \subset Z(G)$, for which a 1-form symmetry background has no fractional contribution (6) — a necessary condition to gauge $Z$.

Let $Z(G) = \prod_{i=1}^s \mathbb{Z}_{n_i}$, and $(k_1, \ldots, k_s) \in \prod_{i=1}^s \mathbb{Z}_{n_i}$ be the generator for $Z \cong \mathbb{Z}_\ell$. This necessarily means that $\ell$ is the smallest integer such that $k_i\ell \equiv 0 \mod n_i$ for all $i$.

The generic background field for the $Z(G)$ 1-form symmetry consists of backgrounds $C_2^{(i)}$ for each $Z_{n_i}$ factor of $Z(G)$. Specifying a background for a subgroup then amounts to correlate the a priori independent $C_2^{(i)}$'s [17]. In particular, the background $C_2$ for $Z_{\ell} = \langle (k_1, \ldots, k_s) \rangle$ corresponds to setting $C_2^{(i)} = k_iC_2$.

For concreteness, let us further assume $G = \prod_{i=1}^s SU(n_i)$. Then, the total contribution to the anomalous phase (6) in the presence of the background field $C_2$ of the $Z_\ell$ subgroup is

$$A(h_4, C_2^{(i)}) = \left( \sum_{i=1}^s \frac{n_i - 1}{2n_i} k_i^2 \right) \int_{M_4} h_4 \cup \mathfrak{p}(C_2), \quad (8)$$

where it is crucial that $\mathfrak{p}(kC) = k^2\mathfrak{p}(C)$. Thus, the anomaly vanishes if the coefficient is integral.

Note that the anomaly contribution of non-$SU$ groups can be written as a sum of contributions from $SU(n_i)$-subgroups [17]. Therefore, by further restricting ourselves to rank($G$) $\leq 18$ (which is the bound in 8d from other quantum gravity arguments [6]), we can exhaustively scan for all possible groups $G$ that have an anomaly-free $Z_\ell \subset Z(G)$ with given $\ell$, by finding s pairs of integers $(n_i, k_i)$ such that

$$\sum_{i=1}^s \frac{n_i - 1}{2n_i} k_i^2 \in \mathbb{Z}, \quad \text{with } k_i \cdot \ell \equiv 0 \mod n_i. \quad (9)$$

This precisely reproduces the geometrical constraint for elliptically fibered K3 manifolds with only $I_n$ singularities in (3), thus leading to the same restrictions, e.g., $G = SU(7)^3$ for $\ell = 7$ and no solutions for prime powers $\ell \geq 9$.

### B. Examples of anomaly-free centers

To further showcase the constraining power of the field-theoretic anomaly argument (6), we remark that it immediately implies that an 8d $\mathcal{N} = 1$ supergravity with $G/Z(G)$ gauge group, where $G$ is a simple Lie group, is not consistent unless $G = Sp(n)$ with $n$ a multiple of 4.

In total, with rank($G$) $\leq 18$, the only $G/Z$ theories, with a single simple factor $G$, that are free of the anomaly (6), are

$$\begin{align*}
SU(16) \frac{\mathbb{Z}_2}{\mathbb{Z}_2}, & \quad SU(18) \frac{\mathbb{Z}_3}{\mathbb{Z}_2}, \quad Spin(32) \frac{\mathbb{Z}_2}{\mathbb{Z}_2}, \\
Sp(4) \frac{\mathbb{Z}_2}{\mathbb{Z}_2}, & \quad Sp(8) \frac{\mathbb{Z}_2}{\mathbb{Z}_2}, \quad SU(8) \frac{\mathbb{Z}_3}{\mathbb{Z}_2}, \quad SU(9) \frac{\mathbb{Z}_2}{\mathbb{Z}_2}, \quad Spin(16) \frac{\mathbb{Z}_2}{\mathbb{Z}_2}.
\end{align*} \quad (10)$$

The first line of theories correspond to the only cases with simple $G$ realizable via F-theory on elliptic K3s. The second line of theories fits into holonomy reductions of the 9d CHL string [30]. Thus, we see that the anomaly arguments are compatible with other possible string constructions of 8d $\mathcal{N} = 1$ theories.

On the other hand, the two examples in the last line do not have known string constructions. They point towards other, more subtle field theoretic constraints that go beyond the mixed anomaly (6), which we will discuss momentarily.

### IV. DISCUSSION AND OUTLOOK

In this work we have provided strong evidence for String Universality for non-simply-connected non-Abelian gauge groups $G/Z$ in eight dimensions. The key idea is to interpret a $G/Z$ gauge theory as having gauged a subgroup $Z$ of the center 1-form symmetry. Utilizing a subtle interplay between these discrete 1-form symmetries and large gauge transformations of higher-form fields of the gravity multiplet, we have found a non-removable anomalous phase of the partition function in the presence of a 1-form symmetry background. A subgroup $Z$ can only be gauged if the phase vanishes in the presence of a $Z$ background, thus allowing for a group $G/Z$.

This restriction is heavily constraining, and rules out almost all 8d theories without known string constructions. Moreover, we find a beautiful agreement between the cancellation condition for this anomaly and geometric properties of elliptic K3 surfaces, which are known to control the global structure of the gauge group in 8d F-theory compactifications. Therefore, the anomaly provides a purely physical explanation for the intricate patterns of realizable non-trivial gauge groups in F-theory.

However, there are outlier theories free of this anomaly, which nevertheless have no known string construction.
As already mentioned above, the gauge groups $Sp(16)/Z_8$ and $Sp(16)/Z_2^2$ fall into this category. One plausible explanation could be that $Sp$ groups in 8d are somehow special [7]8, and therefore come with additional consistency constraints.

More generally, any product $Z_\ell \times Z_m$ of anomaly-free factors would again be anomaly-free. For example, this would lead to an acceptable gauge group $[SU(5)^2/Z_5] \times [SU(2)^4/Z_2] = [SU(5)^2 \times SU(2)^4]/Z_{10}$, which has no known string realization. Observe, in particular, that this would violate the bound $\ell < 9$ for possible $Z_\ell$. In F-theory, the geometry of K3 guarantees that one always has an $SU(n)$ factor with $\ell | n$.

Currently, we do not know an adequate physical argument that can exclude such products, and also enforce the existence of an appropriate $SU(n)$ factor. However, it is clear that there are other geometric constraints for K3s that have not yet been translated into purely field theoretic arguments. In the spirit of the Swampland program, the structure of these geometric constraints could point towards further subtle interplay between quantum field theory and gravity.9

Specifically, there could be other discrete symmetries of the theory that interact non-trivially with 1-form center symmetries. For example, it has been pointed out [3] that the gauge symmetry of the $E_8 \times E_8$ heterotic string should be augmented by an outer automorphism $Z_2$ which exchanges the two factors, so that the full gauge group is $(E_8 \times E_8) \times Z_2$. In fact, the 9d CHL string can then be understood as the $S^1$-reduction with holonomies in this $Z_2$. Such an identification would also be possible for, e.g., $[SU(2)^4/Z_2] \times [SU(2)^4/Z_2]$, which in 8d is free of the anomaly (6), but not realized in terms of a string compactification. If one could establish other field theory / Swampland arguments for why the $Z_2^2$ outer automorphism must be gauged in this case, then there could be other mixed anomalies involving the 1-form symmetries such that only a diagonal $Z_2$ center survives, leading to an $[SU(2)^8]/Z_2$ theory which is realized in F-theory.

Moreover, taking inspiration from the Standard Model gauge group, one could envision that $U(1)$s play a central role here. Namely, for $Sp(8)/Z_m$ with $\ell$ and $m$ coprime, it may be that other field theory mechanisms enforce the presence of a $U(1)$, into which the product $Z_\ell \times Z_m = Z_{\ell m}$ embeds, so that the full gauge group is actually $[G_1 \times G_2 \times U(1)]/Z_{\ell m}$. Such a theory would not be in contradiction to F-theory models, as center symmetries embedded in $U(1)$s are generally encoded in the free part of the Mordell–Weil group [11] (see [32] for direct implications for 4d particle physics models). Moreover, in 8d F-theory, there are additional $U(1)$ factors coming from harmonic (1,1)-forms on K3s that are not divisors, whose center-mixing with non-Abelian gauge factors needs further investigation. To complete the geometric picture from the field theoretic side, one must also extend the discussion of the anomalies to include $U(1)$ gauge sectors, which we defer to future studies.

Finally, it would be exciting to compare and combine these ideas with other quantum gravity criteria. This could be a milestone towards completely draining the 8d Swampland.

**ACKNOWLEDGMENTS**

We thank Miguel Montero and Cumrun Vafa for helpful comments. M.D. and L.L. further thank Fabio Apruzzi for valuable discussions and collaboration on related work [19]. M.D. is grateful to Miguel Montero for valuable discussions. The work of M.C. is supported in part by the DOE (HEP) Award DE-SC0013528, the Fay R. and Eugene L. Langberg Endowed Chair, the Slovenian Research Agency (ARRS No. P1-0306) and by the Simons Foundation Collaboration on “Special Holonomy in Geometry, Analysis, and Physics,” ID: 724069. The work of M.D. is supported by the individual DFG grant DI 2527/1-1.

---

8 Note that the anomalies studied in this reference appear for sp gauge algebras and are not directly related to the global form of the gauge group.

9 We anticipate that additional constraints would be contained in gravitational couplings such as $B_4 \wedge \text{Tr}(R^2)$, or quartic contributions in the field strengths and curvature 2-forms involving the scalar field in the gravity multiplet.
[10] C. Mayrhofer, D. R. Morrison, O. Till, and T. Weigand, JHEP 10, 016 (2014), arXiv:1405.3656 [hep-th].

[11] M. Cvetiæ and L. Lin, JHEP 01, 157 (2018), arXiv:1706.0521 [hep-th].

[12] R. Miranda and U. Persson, Mathematische Zeitschrift 201, 339 (1989); Problems in the theory of surfaces and their classification (Cortona, 1988), 167–192, Sympos. Math., XXXII, Academic Press, London (1991).

[13] I. Shimada, arXiv Mathematics e-prints , math/0505140 (2005), arXiv:math/0505140 [math.AG].

[14] N. Hajouji and P.-K. Oehlmann, JHEP 04, 103 (2020), arXiv:1910.04905 [hep-th].

[15] R. Miranda and U. Persson, Mathematische Zeitschrift 201, 339 (1989); Problems in the theory of surfaces and their classification (Cortona, 1988), 167–192, Sympos. Math., XXXII, Academic Press, London (1991).

[16] I. Shimada, arXiv Mathematics e-prints , math/0505140 (2005), arXiv:math/0505140 [math.AG].

[17] N. Hajouji and P.-K. Oehlmann, JHEP 04, 103 (2020), arXiv:1910.04905 [hep-th].

[18] C. Closset, S. Schafer-Nameki, and Y.-N. Wang, (2020), arXiv:2007.15600 [hep-th]; M. Del Zotto, I. García-Étxebarria, and S. S. Hosseini, (2020), arXiv:2007.15603 [hep-th]; L. Bhardwaj and S. Schafer-Nameki, (2020), arXiv:2008.09600 [hep-th].

[19] M. Awada and P. Townsend, Phys. Lett. B 156, 51 (1985).

[20] W. Taylor, (2011), arXiv:1104.2051 [hep-th].

[21] T. Weigand, Proceedings TASI 2017, PoS TASI2017, 016 (2018), arXiv:1806.01854 [hep-th].

[22] M. Cvetiæ and L. Lin, Proceedings TASI 2017, PoS TASI2017, 020 (2018), arXiv:1809.00012 [hep-th].

[23] A. Salam and E. Sezgin, Phys. Lett. B 154, 37 (1985).

[24] C. Crdova, Y. Tachikawa, and K. Yonekura, (2020), arXiv:2003.11550 [hep-th].

[25] J. Polchinski, Int. J. Mod. Phys. A 19S1, 145 (2004), arXiv:hep-th/0304042; T. Banks and N. Seiberg, Phys. Rev. D 83, 084019 (2011), arXiv:1011.5120 [hep-th]; D. Harlow and H. Ooguri, (2018), arXiv:1810.05338 [hep-th].

[26] S. Chaudhuri, G. Hockney, and J. D. Lykken, Phys. Rev. Lett. 75, 2264 (1995), arXiv:hep-th/9505054.

[27] M. Cvetiæ, L. Lin, M. Liu, and P.-K. Oehlmann, JHEP 09, 089 (2018), arXiv:1807.01320 [hep-th]; M. Cvetiæ, J. Halverson, L. Lin, M. Liu, and J. Tian, Phys. Rev. Lett. 123, 101601 (2019), arXiv:1903.00009 [hep-th]; M. Cvetiæ, J. Halverson, L. Lin, and C. Long, Phys. Rev. D 102, 026012 (2020), arXiv:2004.00630 [hep-th].