3D Textured Model Encryption via 3D Lu Chaotic Mapping

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Abstract. In the coming Virtual/Augmented Reality (VR/AR) era, 3D contents will be popularized just as images and videos today. The security and privacy of these 3D contents should be taken into consideration. 3D contents contain surface models and solid models. The surface models include point clouds, meshes and textured models. Previous work mainly focus on encryption of solid models, point clouds and meshes. This work focuses on the most complicated 3D textured model. We propose a 3D Lu chaotic mapping based encryption method of 3D textured model. We encrypt the vertexes, the polygons and the textures of 3D models separately using the 3D Lu chaotic mapping. Then the encrypted vertexes, edges and texture maps are composited together to form the final encrypted 3D textured model. The experimental results reveal that our method can encrypt and decrypt 3D textured models correctly. In addition, our method can resistant several attacks such as brute-force attack and statistic attack.

Keywords: 3D model, Surface model, Textured model, 3D model encryption, 3D Lu chaotic mapping

1 Introduction

Today, images and videos are everywhere in our daily life. Besides images and videos, the 3D models are increasingly growing with the image based 3D modeling and 3D print technologies. Some Apps on the smartphone such as Autodesk 123D Catch allow users to shot photos of one subject from various views and upload all the photos to Autodesk cloud server. Then the 123D service on the cloud server will return a 3D model of subject to the users. Desktop software such as Google Sketchup also makes one editing 3D models easily. The 3D models are going into our daily life step by step. In the industry, the virtual reality and augmented reality technologies are now hot topics, which need plenty of 3D models to build the virtual world. It is anticipated that the market for virtual
reality and augmented reality will reach $1.06 billion by 2018 at a Compound Annual Growth Rate (CAGR) of 15.18% from 2013 to 2018 [10]. The governments are scanning the whole city into 3D virtual city models by laser scanners and multi-view cameras. The 3D contents encryption technologies are required in order to accomplish a high level of security, integrity, confidentiality and to prevent unauthorized access of sensitive information during 3D contents storage or transmission over an insecure channels.

The 3D digitalized objects are defined by means of two types of 3D contents: 3D solid models and 3D surface (shell/boundary) models. A solid model defines the volume of the physical object that represents, whereas a surface model represents the surface, not the volume. In [3], Rey has addressed the encryption of 3D solid models. The surface models include point clouds, meshes and textured models.

![Fig. 1. A full 3D surface model contains vertices, polygons and textures.](image)

Current methods in this direction only consider the solid models [3], the point clouds models [9,8], the meshes [4] and the textures [10]. This paper focuses on encryption of the most complicated 3D surface models. A full 3D textured surface model often contains vertices, polygons and textures, as shown in Fig. 1. We propose a chaotic mapping based encryption method of 3D textured models. We encrypt the vertexes, the polygons and the textures of 3D models separately using the 3D Lu chaotic mapping. Then the encrypted vertexes, edges and texture maps are composed together to form the final encrypted 3D textured model. The experimental results reveal that our method can encrypt and decrypt 3D textured models correctly. In addition, our method can resistant several attacks such as brute-force attack and statistic attack.

2 Previous Work

The particular properties of chaos [16], such as sensitivity to initial conditions and system parameters, pseudo-randomness, ergodicity and so on, have granted chaotic dynamics as a promising alternative for the conventional cryptographic algorithms [19,21,2,1,20]. The inherent properties connect it directly with cryptographic characteristics of confusion and diffusion, which is presented in Shannon’s works [15]. High-dimensional chaotic system is more reliable to design secure image encryption scheme because of its high complexity [6,11,12,13].
Some cryptosystems, which are based on a low-dimensional chaotic map, have obvious drawbacks, such as short period and small key space [23].

Meanwhile, due to massive parallelism, huge storage, ultra-low power consumption and the massive research on DNA computing [17], DNA cryptography emerged as a new cryptographic field [17]. Recently, DNA-based image encryption has become more and more popular [14] [18] [22]. DNA-based image encryption is generally categorized into two phases: firstly, using DNA theory to encode plain image pixels into DNA sequence. Then a gray pixel value is decomposed into four DNA elements, which can increase the efficiency of image confusion and diffusion. Secondly, the encoded plain image pixels generate a key image based on DNA operation rules and form the cipher image [23].

Current methods in this direction only consider the solid models [3], the point clouds models [9,8], the meshes [28] and the textures [10]. This paper focus on encryption of the most complicated 3D surface models.

3 Preliminaries

3.1 Chaotic Systems

We adopt a high-order chaotic mapping. 3D Lu mapping is a 3D chaotic map, which is described by Eq. 1.

\[
\begin{align*}
\dot{x} &= a(y - x) \\
\dot{y} &= -xz + cy, \\
\dot{z} &= xy - bz
\end{align*}
\]  

(1)

where \((x, y, z)\) are the system trace. \((a, b, c)\) are the system parameters. When \(a = 36, b = 3, c = 20\), the system contain a strange attractor and being in chaotic state.

3.2 DNA Encoding

A DNA sequence contains four kinds of nucleic acids. \(A - T\) is a couple, and \(G - C\) is a couple. As we all know, in the binary, \(1 - 0\) is a couple. Thus \(00 - 11\) is a pair, \(10 - 01\) is a pair. In this paper, we use \(A, G, C, T\) to replace \(00, 01, 10\) and \(11\). For each 8-bit image pixel, 4 nucleic acids can be used to represent it. For example, the pixel value 123 in decimal can be represented as a binary vector 01111011 and can be further encoded as a DNA sequence \(AGTG\) [23].

Based on the above encoding rule, we use three kinds of operations for DNA encoding, as shown in Fig. 1.

4 3D Textured Model Encryption

In this section, we describe the proposed encryption method of 3D textured model. Firstly, we decompose the 3D textured model into vertices, polygons and
Table 1. The DNA addition, DNA subtraction and DNA complement. The Cmp(X) is the complement of X. X ∈ \{A, T, G, C\}

| DNA+ | T | A | C | G |
|------|---|---|---|---|
| DNA- | T | C | G | T |
| DNA- | A | G | C | T |
| DNA- | C | T | A | G |
| DNA- | G | A | T | C |

| DNA+ | T | A | C | G |
|------|---|---|---|---|
| DNA- | T | C | G | T |
| DNA- | A | G | C | T |
| DNA- | C | T | A | G |
| DNA- | G | A | T | C |

texture. Then these three parts are encrypted using 3D Lu chaotic mapping, which is described in Section 3.1. At last, the encrypted vertices, polygons and texture are composited into the encrypted 3D textured model.

Fig. 2. Our proposed method for 3D textured model encryption. The decryption method is the inverse version of the encryption method.

4.1 Vertices Encryption

The vertices in a 3D textured model are in the form of a list of triplets:

\[ V = \{(X_1, Y_1, Z_1), \ldots, (X_N, Y_N, Z_N)\}, \]  

where \((X_i, Y_i, Z_i)\) is the 3D coordinate of a vertex. \(N\) is the number of the vertices. We use the 3D Lu mapping defined in Eq. 1 to produce a random vector with dimensions of \(3N\):

\[ LV = \{(LV_1, LV_2, LV_3), \ldots, (LV_{3N-2}, LV_{3N-1}, LV_{3N})\}. \]
Then we make element by element product of $V$ and $LV$:

$$V LV = \{(X_1 LV_1, Y_1 LV_2, Z_1 LV_3), ..., (X_N LV_{3N-2}, Y_N LV_{3N-1}, Z_N LV_{3N})\}. \quad (4)$$

The new vector $VLV$ contains novel coordinates of the original 3D vertices:

$$(X_i, Y_i, Z_i) \rightarrow (X_i LV_3(i-1), Y_i LV_3(i-1)+1, Z_i LV_3(i-1)+2), 1 \leq i \leq N \quad (5)$$

### 4.2 Polygons Encryption

The polygons (taking the triangle as an example) in a 3D textured model are in the form of a list of triplets:

$$P = \{(A_1, B_1, C_1), ..., (A_i, B_i, C_i), ..., (A_M, B_M, C_M)\}, \quad (6)$$

where $(A_i, B_i, C_i)$ represents the 3 vertices of a triangle in the form of the indices of vertices. $1 \leq i \leq M, 1 \leq A_i, B_i, C_i \leq N$. $N$ is the number of the vertices. We use the 3D Lu mapping defined in Eq. [1] to produce a random vector with dimensions of $3M$:

$$LP = \{(LP_1, LP_2, L_3), ..., (LP_{3M-2}, LP_{3M-1}, LP_{3M})\}. \quad (7)$$

We make the element to element correspondences between $P$ and $LP$:

$$\begin{align*}
A_i & \leftrightarrow LP_{3(i-1)+1} \\
B_i & \leftrightarrow LP_{3(i-1)+2} \\
C_i & \leftrightarrow LP_{3(i-1)+3}
\end{align*} \quad (8)$$

Then we make ascending sort of $LP$. The sorted $LP$ is denoted as $LP^{\text{sort}}$. According to the new order in $LP^{\text{sort}}$, we reorder the element in $P$ using the correspondences described in Eq. [8]. The vector with new order of $P$ is denoted as $P'$.

$$P' = \{(A'_1, B'_1, C'_1), ..., (A'_i, B'_i, C'_i), ..., (A'_{M}, B'_{M}, C'_{M})\}, \quad (9)$$

where $(A'_i, B'_i, C'_i)$ is the new triangle of the encrypted 3D model.
Table 2. The secret keys of the 3D Lu maps in Eq. In all the 3 encryption phases, $a = 36, b = 3, c = 20.$

| Encryption Phases | Keys |
|-------------------|------|
| vertices encryption | $x_v^0 = -6.045, y_v^0 = 2.668, z_v^0 = 16.363$ |
| polygons encryption | $x_p^0 = -5.045, y_p^0 = 2.668, z_p^0 = 16.363$ |
| texture encryption (1) | $x_t^{10} = -6.045, y_t^{10} = 2.668, z_t^{10} = 20.363$ |
| texture encryption (2) | $x_t^{20} = -5.045, y_t^{20} = 3.668, z_t^{20} = 16.363$ |

4.3 Textures Encryption

The textures in textured 3D model is represented as 2D images with corresponding texture coordinates. We use image encryption method based on 3D Lu mapping and DNA encoding [7] to encrypt the texture images. We first separate the texture image into RGB channels. Then each channel of the texture image is encoded by DNA encoding. Then, we use 3D Lu mapping to generate a random matrix with the same size of the texture image and use DNA addition to add it to the encoded result. After that, another random matrix with the same size of the texture image is generated by 3D Lu mapping and convert it to a binary matrix with the threshold 0.5. The DNA addition result is then converted to the DNA complement result when the corresponding value in the second random matrix is 1. The last step is DNA decoding to obtain the 8-bit encryption result.

5 Simulation Results

We use plenty of 3D textured models to test our method, as shown in Fig. 3, with the secret keys shown in Table 2.

We use 4 Lu maps in our method. The texture encryption contains 2 Lu maps. The 3D textured model with various contents are tested. All the encryption results can be correctly decrypted to the original plain 3D models with the correct secret keys. We can see that the simulation results are quite satisfactory.

6 Security and Performance Analysis

A well designed 3D model encryption scheme should be robust against different kinds of attacks, such as brute-force attack and statistical attack. In this section, we analyse the security of the proposed encryption method.

6.1 Resistance to the brute-force Attack

Key Space The key space of the image encryption scheme should be large enough to resist the brute-force attack, otherwise it will be broken by exhaustive search to get the secret key in a limited amount of time. In our encryption method, we have the key space of 12 key values shown in Table 2.
Fig. 3. The simulation results. We test our method on 3D models with various contents.

Fig. 4. Decrypted with wrong key. We slightly change the key and get the wrong decrypted result.
Fig. 5. The Viewpoint Feature Histogram (VFH) of 3D textured models before and after encryption.
Fig. 6. The distribution of occupied positions per z-column of the 3D textured models before and after encryption.
Table 3. Slightly change the key values.

| Secret Keys | Original Values | Novel Values |
|-------------|-----------------|--------------|
| $x^0_0$     | -6.045          | -6.0450000001|
| $y^0_0$     | 2.668           | 2.668000000001|
| $z^1_0$     | 20.363          | 20.36300000001|

The precision of 64-bit double data is $10^{-15}$, thus the key space is about $(10^{15})^{12} = 10^{180} \approx 2^{599}$, which is much larger than the max key space ($2^{256}$) of practical symmetric encryption of the AES. Our key space is large enough to resist brute-force attack.

**Sensitivity of Secret Key** The chaotic systems are extremely sensitive to the system parameter and initial value. A light difference can lead to the decryption failure. To test the secret key sensitivity of the image encryption scheme, we change the secret key as shown in Table 3.

![Fig. 7.](image)

Fig. 7. The encryption and decryption time costs of our proposed method for 3D model encryption against the number of vertices.

We use the changed key to decrypt the encrypted 3D textured model in Fig. 4, while the other secret keys remain the same. The decryption results are shown in Fig. 4. We can see that the decrypted 3D model is completely different from the original plain 3D model. The test results of the other secret key are similar. The experiments show that the 3D model encryption scheme is quite sensitive to the secret key, which also indicates the strong ability to resist exhaustive attack.
6.2 Resistance to the Statistic Attack

The Histogram Analysis For vertices, the Viewpoint Feature Histogram (VFH) is a representation for point clusters for the problem of cluster recognition and 6 DOF pose estimation. We use the VFH for the evaluation of our 3D vertices encryption. As shown in Fig. 5, the VFHs of the encrypted results by both our method are completely different from the VFH of the original 3D model, which makes statistical attacks impossible.

Distribution of Occupied Positions We further analyse the the occupied positions of the 3D vertices. As defined in [3], we compute the occupied position per \( x \)-column, \( y \)-column and \( z \)-column of a 3D lattice \( Z = (z_{ijk}) \).

The matrices obtained for the plain 3D vertices and the encrypted 3D vertices are far different. Moreover, the distributions of the number of occupied positions per \( z \)-column in the plain 3D vertices and the corresponding encrypted 3D vertices are shown in Fig. 6. The distributions of occupied positions are far apart. In the case of the plain 3D vertices some clusters appears, whereas in the case of encrypted 3D vertices the distribution seems to be homogeneously.

6.3 The Speed of the Encryption and Decryption

The 3D textured model encryption scheme is implemented by Matlab on personal computer with AMD A10 PRO-7800B, 12 Computer Cores 4C+8G 3.4GHz and 4.00G RAM. The encryption and decryption consumption time is recorded for the 3D models of different number of vertices. The larger size of the 3D model, the more time it needs for encryption and decryption, as show in Fig. 7. When our implementation in Matlab 2015a transplanted to other implement environment, like C/C++, the speed can be much faster, which can satisfy practical demand.

7 Conclusion and Discussion

Previous work mainly focus on encryption of solid models, point clouds and meshes. This work focuses on the most complicated 3D textured model. We propose a chaotic mapping based encryption method of 3D textured model. We encrypt the vertexes, the polygons and the textures of 3D models separately using the 3D Lu chaotic mapping. Then the encrypted vertexes, edges and texture maps are composed together to form the final encrypted 3D textured model. The experimental results reveal that our method can encrypt and decrypt 3D textured models correctly. In addition, our method can resistant several attacks such as brute-force attack and statistic attack.

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