The main structural element of a reinforced concrete supports: mathematical modeling of mechanical vibrations

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Abstract. The problem of mathematical modeling of a reinforced concrete supports mechanical vibrations is considered. The basic vibrations of the main structural element (a road) are taken into account. The eigenfrequencies of the first three vibration modes of the rod are determined without and with prestressing of the rod. Participation factor of the basic, second and third modes is investigated.

1. Introduction
At present, the electrical contact network of railways is located on prestressed reinforced concrete supports installed along the roads. As a result of the influence of various factors, such as weather conditions, train traffic, presence of leakage currents, etc. the process of destruction of supports takes place. The presence of emerging defects and various deformations of the supports is rather difficult to determine, since some of them may be located in the underground part of the support. To find them, relatively "simple" methods, involving the extraction of the support from the ground or foundation, are very laborious and time-consuming. Thus, there is a great need for more effective, in comparison with "simple" methods of diagnostics of the support state. One of these methods is the acoustic emission method.

It is based, in essence, on the creation of mechanical vibrations in the support and the subsequent registration of its acoustic emission [1–5]. Without going deep into the analysis of known technical solutions, we can say that their differences lie in different ways of excitation of vibrations in the support and algorithms for processing the results of measuring acoustic emission.

2. Mathematical model
In this work, the task is to determine the spectrum of natural mechanical vibrations of the support based on mathematical modeling. For this purpose, the COMSOL Multiphysics software package was used. This package is based on the finite element method and allows to solve problems from various branches of physics, and also allows to take into account the influence of various chemical processes.

A single structural steel bar was chosen as the initial simplest support member model, the characteristics of which are contained in the built-in COMSOL Multiphysics library. The rod was 1 meter long and 6 mm in diameter (Figure 1). Damping was set through hysteresis losses with a factor of 0.01.
Figure 1. Model of a structural steel bar support element.

The left end of the rod was rigidly fixed, and at the opposite end, all movements were limited, except for movement in the direction perpendicular to the end of the rod. When constructing the computational grid, the bar was divided into 400 cells (Figure 2).

Figure 2. Mesh geometry on the bar.

First, the eigenfrequencies of the first three vibration modes of the rod were determined (Figures 3–5).

Figure 3. Basic mode.

Figure 4. Second mode.

Figure 5. Third mode.

Figures 3–5 show the shapes of the first three modes with the oscillation frequency 18.696+ i 0.093477 Hz, 60.847+ i 0.30423 Hz, 127.75+ i 0.63875 Hz, respectively. The presence of imaginary parts at the frequencies indicates that there is damping in the rod. The next step was to study the vibrations of the bar based on calculations in the frequency domain. For this purpose, a force equal to 1 MPa was applied to the rod at its central point and directed perpendicular to the rod axis. The calculation results are shown in Figure 6.
Figure 6. Mean square deviation of the rod’s center point on the frequency.

It is seen that the expected resonance phenomena appear at frequencies corresponding to the resonances of the first two modes. For significant differences in mean square deviation, explanations will be given below.

The next step was to add a prestressed state. For this, a stationary force of 10 MPa was applied to the right end of the rod, directed perpendicular to the end, so that the rod was stretched.

Subsequently, the same studies were carried out over the rod as before. Since the types of modes have similar images, their frequencies, together with the results of previous calculations, are presented in Table 1.

Table 1. Eigenfrequencies of the rod’s vibrations (without and with prestressing).

| № mode    | Without prestressing | With prestressing |
|-----------|----------------------|-------------------|
| Basic mode| 18.696+i0.093477 Hz  | 26.826+i0.13412 Hz|
| Second mode| 60.847+i0.30423 Hz | 71.412+i0.35705 Hz|
| Third mode| 127.75+i0.63875 Hz  | 139.25+i0.69624 Hz|

As can be seen from the table, as a result of the application of force, there is a "shift" of frequencies. This is due to an increase in the rod tension caused by the force applied to the right end of the rod. The study of prestressed rod vibrations in the frequency domain was also performed (Figure 7).
The difference in the amplitudes of the responses to the force action, observed in Fig. 6-7 is explained using the so-called “participation factor”. This factor can be calculated using the COMSOL Multiphysics software when performing analysis on eigenfrequencies (Table 2).

| № mode      | Frequency, Hz | Participation factor | № mode      | Frequency, Hz | Participation factor |
|-------------|---------------|----------------------|-------------|---------------|----------------------|
| Basic mode  | 18.696+i0.093477 | 0.48721              | Basic mode  | 26.826+i0.13412 | 0.48992              |
| Second mode | 60.847+i0.30423 | 0.046724             | Second mode | 71.412+i0.35705 | 0.03427              |
| Third mode  | 127.75+i0.63875 | 0.18923              | Third mode  | 139.25+i0.69624 | 0.18606              |

It can be seen from the table 2 that the main mode has a participation factor that is an order of magnitude higher than the second mode. Thus, already at the stage of analysis for natural frequencies, it is possible to determine which of the vibration modes and how will affect the resonance responses.

3. Following research

The next step in the study was a cylindrical concrete rod with the diameter of 60 mm and the length of 1 meter (Figure 8). Thanks to the capabilities of COMSOL, it is enough to use the physical interface "TRUSS" (truss structures) to integrate the reinforcement into the concrete rod. As a result, it is possible to introduce metal rods without additional operations, such as removing "extra" parts from concrete, which facilitates the process of graphical construction of the model, as well as the possibility of "painless" removal of rods by disconnecting them from the model. Thus, this introduction also allows you to reduce the calculation time, if the rod was made in the interface "SOLID MECHANICS" (solid body) like a concrete rod.

Figure 8. Concrete rod with a diameter of 60 mm and a length of 1 meter.

At this stage, a single rod with a diameter of 6 mm and a length of 1 meter is used as a reinforcing link (Figure 9).

Figure 9. Reinforcement of concrete rod.
To link two parts of the model (concrete and metal rods), the COMSOL operator "General Extrusion" was used, as a result of which all variables associated with the rod movements are equated to the variables of the concrete rod, thereby forming one body combined of the two.

The research plan at this stage is to compare the results obtained from models of concrete stubble without reinforcement, reinforced concrete rod without prestressing, with prestressing, as well as with the inclusion of additional modules for the behavior of concrete and metal rod. The main task of this stage of research is seen as COMSOL's capabilities in the study of prestressed reinforced concrete, as well as ways to model it in this package.

4. Conclusion

The performed mathematical modeling allows us to conclude that the complication of the support model will lead to a noticeable increase in the calculation time, which also significantly depends on the power of the computer used. To speed up the calculation, you can reduce the amount of calculated data. To select not very significant data, one can use the results of the analysis of natural frequencies, and also determine the factor of participation of each of the modes. Thus, knowing these data, it is possible to limit the studied values to a certain interval in which, based on the information obtained, the phenomena of interest will occur.

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