Correlation Functions of Operators and Wilson Surfaces in the $d = 6$, $(0, 2)$ Theory in the Large $N$ Limit

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Abstract

We compute the two and three-point correlation functions of chiral primary operators in the large $N$ limit of the $(0, 2)$, $d = 6$ superconformal theory. We also consider the operator product expansion of Wilson surfaces in the $(0, 2)$ theory and compute the OPE coefficients of the chiral primary operators at large $N$ from the correlation functions of surfaces.

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1. Introduction

The existence of the (0, 2) tensor multiplet in $d = 6$ has been known for a long time, but, until the last few years, there was no evidence suggesting the existence of nontrivial field theories for $d > 4$. This situation has changed with the discovery that interacting (0, 2) superconformal theories exist in compactifications of the IIB string on a K3 surface [1], as well as on the worldvolume of the M-theory 5-brane [2, 3]. These theories are of great interest in themselves, but they are also important because they have applications [4–6] to matrix theory [7], as well to the study of 4-dimensional field theories, including the large $N$ limit of QCD [8].

Although many features of these nontrivial superconformal field theories were evident within their string and M-theory realizations, it is only recently that methods were developed to permit detailed computations within them. Important steps toward a deeper understanding of these theories were made after realizing their importance in the compactification of matrix theory. Soon afterwards, it was realized that the (0, 2) theory had its own description as a matrix model [9–11]. This idea was carefully developed in [12], where the construction of states and the computation of correlation functions in a discrete light-cone quantization was discussed.

Another route towards the study of interacting superconformal field theories appears in the correspondence between anti–de Sitter (AdS) spaces and conformal field theories (CFTs) [13–15]. One aspect of this connection is that the large $N$ limit of the (0, 2) theory is described by supergravity on AdS$_7 \times S^4$. The correspondence between SCFT operators and supergravity Kaluza-Klein modes in this case was carried out in a series of papers [16–19].

In this paper, motivated by the success in computing correlation functions of the $\mathcal{N} = 4$, $D = 4$ SCFT from supergravity, we compute correlation functions in the large $N$ limit of the (0, 2) theory.

In section 2, we review several features of the (0, 2) theory, in particular the chiral primary operators that we will be interested in for our computations. In section 3, we perform the supergravity computations that allow us to extract the 3-point function of the chiral primary operators of the (0, 2) theory. We use the quadratic corrections to the equations of motion to find the necessary cubic terms in the action to compute the 3-point function of the chiral primary operators, as well as the 3-point function of two chiral primaries with a certain higher dimensional operator. In an appendix, we point out some subtleties in the computation of the quadratic action for the scalars corresponding to the chiral primary operators.

In section 4, we consider the operator product expansion (OPE) of Wilson surfaces of the (0, 2) theory. We identify the operators which are allowed to appear in the OPE and explicitly
compute the OPE coefficients of the chiral primary operators by computing correlation functions of Wilson surfaces.

In the conclusion, we comment on our results and point to avenues of further research. Several appendices collect results which were useful in obtaining the correlation functions.

2. Review of the Properties of the (0, 2) Theory

In the following, we will briefly review some of the features of the (0, 2) field theory. For an additional discussion, see [20] and the references therein.

In six dimensions, the vector of $SO(5, 1)$ appears in the antisymmetric product of two $4$s of the $Spin(4) \sim SU(2) \times SU(2)$ little group, $4 \times 4 = 6_a \oplus 10_a$, so that spinors in six dimensions may be taken to be symplectic-Majorana-Weyl. The supercharges of (0, 2) SUSY, $Q^a_\alpha$, carry a $Spin(4)$ spinor index, $\alpha$, and an $Sp(2)_R \sim SO(5)$ index, $a$, labeling the $4$. The supersymmetry algebra is

$$\{Q^a_\alpha, Q^b_\beta\} = 2\omega^{ab}_\gamma \gamma_\mu^\gamma P^\mu + \gamma_\mu^\gamma Z^{ab}_\mu,$$

where $\omega^{ab}$ is the $Sp(2)$-invariant tensor. A central charge $Z^{ab}_\mu$ is allowed. This central charge must transform as a Lorentz vector (as such, it couples to a string) and under the $5$ of $Sp(2)_R$. The only massless representation of (2.1) that does not involve a graviton is the tensor multiplet. Labeling fields by their $SU(2) \times SU(2) \times Sp(2)_R$ representations, the tensor multiplet contains a 2-form, $B_{\mu\nu}$, in the $(1, 3; 1)$ whose field strength $H = dB$ is (anti) self-dual, four fermions, $\lambda^a_\alpha$, in the $(2(1, 2); 4)$, and five scalars, $\phi^{ab}$, in the $(1, 1; 5)$.

The tensor multiplet appears on the M5-brane in the following fashion. The M5-brane is a BPS state in $D = 11$ supergravity which preserves 16 of the 32 bulk supercharges. By considering the spectrum of fluctuations around the 5-brane supergravity solution, one finds 16 fermionic and eight bosonic zero modes [21]. Five of the bosonic zero modes are translational, corresponding to fluctuations in the directions transverse to the brane, so they form a $5$ of the $Spin(5)$ component of the broken 11-dimensional Lorentz group. The other 3 bosonic modes come from fluctuations of the bulk 3-form and are arranged in an antisymmetric 2-form on the 5-brane worldvolume. The fermionic degrees of freedom form a $4$ of $Spin(5)$. This is exactly the content of the (0, 2) tensor multiplet. An interacting field theory is obtained when one considers a configuration of $N$ parallel 5-branes. In this case, M2-branes are allowed to stretch between the 5-branes [2], leading to $N + 1$ massless tensor multiplets on the worldvolumes of the M5-branes. Since the worldvolume theory is chiral, there are no additional massless states arising when the 5-branes are coincident. The scalars in one linear combination of these tensor multiplets corresponds to the center-of-mass motion.
of the collection of 5-branes, so that tensor multiplet is free and decouples from the rest of the theory. The moduli space of scalars in the remaining part of the theory is $\mathbb{R}^{5N}/S_{N+1}$. The scalars, $\phi^i_A$, are in the irreducible $N$ of $S_{N+1}$, labeled by $A$, while $i$ labels the 5 of the $SO(5)$ R-symmetry. The expectation values of the scalars determine the tensions of the BPS strings which appear at the $(2 \perp 5|1)$ brane intersections. Away from the orbifold fixed points, the theory on the 5-branes is free. At the fixed points of the $S_{N+1}$ group, some number of 5-branes coincide. There the theory is an interacting superconformal field theory.

2.1. Chiral Primary Operators of the $(0,2)$ Theory

We are interested in the chiral primary operators of the $(0,2)$, $d = 6$, $U(N)$ field theory. In [22,23,12], unitarity bounds on the dimensions of operators are obtained. For states which are Lorentz singlets and have $Sp(2)$ Dynkin labels $(l,k)$, there are superconformal primaries at dimension $2(l+k)$ with a null vector at level $\frac{1}{2}$, at dimension $2(l+k) + 2$ with a null vector at level 1, at dimension $2(l+k) + 4$ with a null vector at level $\frac{3}{2}$, and at dimension $2(l+k) + 6$ with a null vector at level 2. All of these states belong to short multiplets, so we expect that the corresponding operators have conformal dimensions which are protected. We will refer to the primary operators of the shortest of these multiplets as chiral primary operators.

From these results, it is evident that a rank $k$ symmetric, traceless tensor representation of $SO(5)$, with $Sp(2)$ Dynkin label $(0,k)$, is a chiral primary field of dimension $\Delta = 2k$. That such states actually exist in the $(0,2)$ theory was found in the discrete light-cone quantization (DLCQ) approach of [12]. Away from the $S_{N+1}$ fixed point, operators of these dimensions and quantum numbers have a realization in terms of $S_{N+1}$ invariant, symmetric traceless (in the $SO(5)$ indices) products of $k$ free scalars. At the fixed point, it is not clear that this realization makes any sense, so we will not pursue it further. We will simply denote these chiral primaries as $O^I$.

Correlation functions of these operators are restricted by conformal invariance. The two-point function is determined up to a free constant, which can be taken to be unity

$$\langle O^{I_1}(x_1)O^{I_2}(x_2) \rangle = \frac{\delta^{I_1I_2}}{|x_1 - x_2|^{4k}}. \quad (2.2)$$

Three-point functions are similarly constrained

$$\langle O^{I_1}(x_1)O^{I_2}(x_2)O^{I_3}(x_3) \rangle = \frac{c_3}{|x_1 - x_2|^{4\alpha_3}|x_2 - x_3|^{4\alpha_1}|x_3 - x_1|^{4\alpha_2}} \langle C^{I_1}C^{I_2}C^{I_3} \rangle \quad (2.3)$$

where we have denoted the symmetric, traceless $SO(5)$ tensors by $C^{I}_{i_1\cdots i_k}$, defined such that the spherical harmonics on $S^4$ are $Y^I = C^{I}_{i_1\cdots i_k} x^{i_1} \cdots x^{i_k}$, with a normalization such that
The quantity
\[
\langle C^{I_1} C^{I_2} C^{I_3} \rangle = C^{I_1}_{i_1 \ldots i_k} C^{I_2}_{i_{a_2} \ldots i_{a_3}} C^{I_3}_{i_{a_3+1} \ldots i_{a_3+a_2} j_{j_1} \ldots j_{j_3}}
\]
\[\alpha_i = \frac{1}{2} \sum_{j=1}^{3} k_j - k_i, \]
\[\Sigma = k_1 + k_2 + k_3, \]
is the \(SO(5)\) invariant contraction of three \(C^{I_1}_{i_1 \ldots i_k}\). Our goal in section 3 will be to obtain the coefficient of this 3-point function in the large \(N\) limit.

2.2. The \((0,2), d = 6\) SCFT and Supergravity on \(AdS_7 \times S^4\)

Information about the \((0,2)\) theory at the fixed point can be obtained from its realization on the M5-brane worldvolume. One begins by considering the M-Theory solution for a large number, \(N\), of coincident M5-branes. Next, the limit \(M_P \to 0\) is taken, while keeping the tensions of the strings on the 5-branes (due to membranes which stretch between the 5-branes) fixed. The result is M-Theory on \(AdS_7 \times S^4\). For large enough \(N\), the curvature is very small in Planck units, so that classical supergravity is a good approximation to the physics. On the other hand, this is precisely the same limit that leads to the decoupling of bulk modes from the \((0,2)\) theory on the 5-brane worldvolume. One is lead to the conclusion that the large \(N\) limit of the \((0,2)\) field theory is described by 11D supergravity on \(AdS_7 \times S^4\) [13].

A prescription for a generating functional for the correlation functions of operators was given within the AdS/CFT correspondence in [14, 15]. Each operator in the CFT which is chiral, so that its conformal dimension is protected against renormalization, corresponds to a mode of the KK compactification of the supergravity on the compact manifold. The mass of the mode corresponding to an operator of dimension \(\Delta\) is \(m^2 = \Delta(\Delta - d)\), where \(d\) is the number of CFT spacetime dimensions. Then correlation functions may be computed from the formula
\[
\langle e^{\int \phi_0^I O^I} \rangle_{CFT} \sim e^{-S_{\text{sugra}}[\phi_0^I]}, \tag{2.5}
\]
where \(\phi_0\) is the boundary data for the supergravity mode \(\phi^I\) corresponding to the operator \(O^I\) and \(S_{\text{sugra}}[\phi_0^I]\) is the effective action computed from all tree graphs with external \(\phi\) legs.

In the case of the \((0,2)\) theory, this correspondence was discussed by [16–19]. The chiral primary operators have conformal weight \(\Delta = 2k\), so they correspond to KK-scalars, \(s^I\) with masses \(M_s^2 = 4k(k - 3)\). These scalars arise as linear combinations of the harmonic modes of the trace of the metric and the 3-form with indices on the sphere.
In order to compute 2 and 3-point functions in the large \( N \) limit of the \((0, 2)\) theory via the prescription (2.5), the supergravity action must be computed to cubic order in the modes \( s^I \). In the next section, we discuss in detail the identification of the mode \( s^I \) and the other supergravity modes which are related to it via constraint equations. We then follow the methodology of [24] to compute the cubic action from quadratic corrections to the supergravity equations of motion. The cubic action is then used to obtain 2 and 3-point correlation functions of the \((0, 2)\) theory at large \( N \).

3. Correlation Functions from Supergravity

The spectrum of states of supergravity on \( \text{AdS}_7 \times S^4 \) have been worked out via linearization around a Freund-Rubin background in [25,26], as well as via the oscillator formalism in [27]. Here we present the equations of motion and background appropriate to the correspondence with the \((0, 2)\) theory and identify the modes \( s^I \) corresponding to the CPOs. We then consider quadratic corrections to the equation of motion [24] and extract the normalization of the quadratic action for \( s^I \) and the cubic \( s-s-s \) vertex. With this action, we derive the large \( N \) result for the 3-point function (2.3) of normalized operators.

3.1. 11D Supergravity on \( \text{AdS}_7 \times S^4 \): Background and Scalar Fluctuations

We start with the action for 11D supergravity

\[
S_{\text{sugra}} = \frac{1}{2\kappa^2} \int d^{11}x \left[ \sqrt{-G} \left( R - \frac{1}{24} (F_{m_1 \cdots m_4})^2 \right) + \frac{2\sqrt{2}}{(144)^2} \epsilon^{m_1 \cdots m_{11}} F_{m_1 \cdots m_4} F_{m_5 \cdots m_8} A_{m_9 \cdots m_{11}} \right].
\]

We choose units in which \( R_{\text{AdS}} = 1 \), then \( R_{S^4} = 1/2 \) and

\[
\frac{1}{2\kappa^2} = \frac{2N^3}{\pi^5}.
\]

Varying (3.1) with respect to the metric results in the equation of motion (after eliminating \( R \))

\[
R_{mn} = \frac{1}{6} \left( F_{mm_1 \cdots m_3} F_{n}^{m_1 \cdots m_3} - \frac{1}{12} g_{mn} F^2 \right).
\]

Varying with respect to the 3-form \( A_{mn} \) yields the Maxwell equations

\[
\nabla^m F_{mm_1 \cdots m_3} = -\frac{\sqrt{2}}{1152} \epsilon^{m_1 \cdots m_3 m_4 \cdots m_{11}} F_{m_4 \cdots m_7} F_{m_8 \cdots m_{11}}.
\]
The AdS$_7 \times S^4$ background with $R_{\text{AdS}} = 1$ is given by

$$ds^2 = \frac{1}{z^2} (dz^2 + \eta_{ij} dx^i dx^j) + \frac{1}{4} d\Omega_4^2, \quad \eta_{ij} = \text{diag}(-1, 1, \ldots, 1),$$

$$R_{\mu_1 \ldots \mu_4} = -(g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} - g_{\mu_1 \mu_4} g_{\mu_2 \mu_3}), \quad R_{\mu \nu} = -6 g_{\mu \nu}, \quad R^{(7)} = -42,$$

$$R_{\alpha_1 \ldots \alpha_4} = 4(g_{\alpha_1 \alpha_3} g_{\alpha_2 \alpha_4} - g_{\alpha_1 \alpha_4} g_{\alpha_2 \alpha_3}), \quad R_{\alpha \beta} = 12 g_{\alpha \beta}, \quad R^{(4)} = 48,$$

$$\bar{F}_{\alpha_1 \ldots \alpha_4} = 3\sqrt{2} \varepsilon_{\alpha_1 \ldots \alpha_4}.$$

(3.5)

where $\mu, \nu, \ldots$ correspond to AdS indices, while $\alpha, \beta, \ldots$ correspond to the sphere. Now we would like to expand around the background (3.5). This is simplified if we Weyl-rescale the metric on AdS. Then the fluctuations are defined as:

$$G_{mn} = g_{mn} + h_{mn},$$

$$h_{\mu \nu} = h'_{(\mu \nu)} + \left( \frac{h'}{7} - \frac{h_2}{5} \right) g_{\mu \nu},$$

$$h_{\alpha \beta} = h_{(\alpha \beta)} + \frac{h_2}{4} g_{\alpha \beta},$$

$$F_{m_1 \ldots m_4} = \bar{F}_{m_1 \ldots m_4} + f_{m_1 \ldots m_4}, \quad f_{m_1 \ldots m_4} = 4\nabla_{[m_1} a_{m_2 m_3 m_4]},$$

(3.6)

where $(mn)$ denotes symmetrization and removal of the trace. The dependence on the coordinates of the sphere is best determined by expanding the fluctuations in a basis of spherical harmonics. With the gauge choices $\nabla^\alpha h_{(\alpha \beta)} = \nabla^\alpha h_{\mu \alpha} = \nabla^\alpha a_{\alpha mn} = 0$, the fluctuations which we are interested in have the expansions

$$h'_{(\mu \nu)} = \sum_I h'_{(\mu \nu)} Y^I, \quad h_{(\alpha \beta)} = \sum_I \phi^I Y^I_{(\alpha \beta)},$$

$$h' = \sum_I h'^I Y^I, \quad h_2 = \sum_I h_2^I Y^I,$$

$$f_{\alpha_1 \ldots \alpha_4} = -6 \sum_I b'^I \varepsilon_{\alpha_1 \ldots \alpha_4} \nabla^\beta \nabla_\beta Y^I,$$

$$f_{\mu \alpha_1 \ldots \alpha_3} = 6 \sum_I \nabla_\mu b'^I \varepsilon_{\alpha_1 \ldots \alpha_3} \nabla^\alpha_4 Y^I.$$

(3.7)

By expanding the $\alpha \beta$ components of (3.3) to linear order and projecting onto linearly independent spherical harmonics, one finds the field equations and constraint

$$-\frac{1}{8} \left( \left( \nabla^\mu \nabla_\mu + \frac{1}{10} \nabla^\gamma \nabla_\gamma - 72 \right) h'^I + \nabla^\gamma \nabla_\gamma h'^I - 192 \sqrt{2} \nabla^\gamma \nabla_\gamma b'^I \right) Y^I = 0,$$

$$-\frac{1}{2} \left[ \nabla^\mu \nabla_\mu + \nabla^\gamma \nabla_\gamma - 8 \right] \phi^I Y^I_{(\alpha \beta)} = 0,$$

(3.8)

(3.9)
\[ -\frac{1}{2} \left[ h'^\mu - \frac{9}{10} h^0_2 \right] \nabla_{(\alpha} \nabla_{\beta)} Y^I = 0. \quad (3.10) \]

Among the \(\mu \alpha\) components, we find the constraint
\[ \frac{1}{2} \left[ \nabla^\nu h'^{\mu \nu} + 3 \nabla_\mu \left( \frac{3}{40} h^0_2 - \frac{1}{7} h'^\mu + 6\sqrt{2} b^I \right) \right] \nabla_\alpha Y^I = 0, \quad (3.11) \]

where we neglect terms involving \(h_{\mu \alpha}\). From the \(\nabla^m F_{m \alpha_1 \ldots \alpha_3}\) component of (3.4), we find another field equation,
\[ -6 b^I \nabla_\alpha \nabla_\gamma \nabla_\gamma Y^I - 6 \left[ \nabla^\mu \nabla_\mu b^I - \frac{1}{2\sqrt{2}} h'^\mu + \frac{3\sqrt{2}}{5} h^0_2 \right] \nabla_\alpha Y^I = 0. \quad (3.12) \]

When \(\nabla_{(\alpha} \nabla_{\beta)} Y^I \neq 0\), (3.10) can be solved for \(h'\) in terms of \(h^0_2\). Then from (3.8) and (3.12), after applying the eigenvalue equation (D.3) for the spherical harmonics, we find the coupled equations of motion
\[ \left[ \nabla^\mu \nabla_\mu + 4 \begin{pmatrix} -k(k+3) & 9 \\ -4k(k+3) & -k(k+3) - 18 \end{pmatrix} \right] \left( \frac{48\sqrt{2} b^I}{h^0_2} \right) Y^I = 0. \quad (3.13) \]

The mass eigenvalues and eigenvectors of this pair of equations are
\[ s^I = \frac{k}{2k+3} \left( h^0_2 + 32\sqrt{2}(k+3)b^I \right), \quad m_{s^I}^2 = 4k(k-3), \quad k \geq 2, \]
\[ t^I = \frac{k+3}{2k+3} \left( h^0_2 - 32\sqrt{2}kb^I \right), \quad m_{t^I}^2 = 4(k+3)(k+6), \quad k \geq 0. \quad (3.14) \]

The mode \(s^I\) has the right mass to correspond to a \(\Delta = 2k\) operator. Since it has an expansion on \(S^4\) in terms of scalar spherical harmonics of degree \(k \geq 2\), each mode transforms in the rank \(k\) symmetric traceless tensor representation of \(SO(5)\). These are the same representations as the chiral primaries \(O^I\), so we conclude that the \(s^I\) are the corresponding supergravity modes [16–19].

We will also be interested in the modes \(\phi^I\) appearing in (3.9). Applying the eigenvalue formula for the tensor spherical harmonics (D.13), we find from (3.9) that these have masses
\[ m_{\phi^I}^2 = 4k(k+3), \quad k \geq 2. \quad (3.15) \]

The \(SO(5)\) representation of these modes is that of the rank 2 symmetric, traceless spherical harmonics, namely the \((2, k)\), where we again use the \(Sp(2)\) Dynkin labels to denote the representation. From the bounds quoted in section 2.1, an operator in this representation of dimension \(\Delta = 2(2 + k) + 2 = 2k + 6\) will be protected. This is precisely the conformal dimension one predicts from the mass formula (3.15). We will denote these operators as \(\Phi^I\).
3.2. The Action to Cubic Order for the Modes $s^I$ and $\phi^I$

Now we would like to derive the action to cubic order for these modes. We first identify the supergravity fields that depend on $s^I$. For our purposes, we can ignore the mass eigenstate $t^I$ in (3.14). This mode corresponds to a scalar operator with dimension $\Delta = 2k + 12$ and is not of immediate relevance to our discussion. We can therefore solve (3.14) for $h^I_2$ and $b^I$ in terms of $s^I$. From the constraint (3.10), the modes $h^I$ are obtained in terms of $s^I$ through their dependence on $h^I_2$. Additionally, we have to solve the constraint (3.11) to find the dependence of $h^I_{(\mu\nu)}$ on $s^I$. To do this, we note that the only way to get a symmetric traceless tensor from a scalar is to act with covariant derivatives, so we choose the ansatz

$$h^I_{(\mu\nu)} = H^I_{(\mu\nu)} + \alpha \nabla_{(\mu} \nabla_{\nu)} s^I,$$

(3.16)

where $H^I_{(\mu\nu)}$ is independent of $s^I$ (so we can ignore it for our purposes) and is divergenceless, $\nabla^\mu H^I_{(\mu\nu)} = 0$. We then compute the normalization $\alpha$ by putting $s^I$ on shell and solving the constraint (3.10). We find that

\[
\begin{align*}
  h^I_{(\alpha\beta)} &= h^I_{(\alpha\beta)} + g_{\alpha\beta} V(k) s^I, \\
  h^I_{\mu\nu} &= W(k) \nabla_{(\mu} \nabla_{\nu)} s^I + g_{\mu\nu} U(k) s^I, \\
  b^I &= X(k) s^I,
\end{align*}
\]

(3.17)

where

\[
\begin{align*}
  V(k) &= \frac{1}{4}, & U(k) &= -\frac{1}{14}, \\
  W(k) &= \frac{3}{16k(2k+1)}, & X(k) &= \frac{1}{32\sqrt{2k}}.
\end{align*}
\]

(3.18)

For the $\phi^I$ modes, from the supergravity action (3.1) we find the quadratic Lagrangian

$$L[\phi^I] = -\frac{B_I}{2} \left[ (\nabla_\mu \phi^I)^2 + 4k(k+3)(\phi^I)^2 \right],$$

(3.19)

$$B_I = \frac{z(k)}{2} \frac{2N^3}{\pi^5}.$$

We note that this action is canonical and that the mode dependence of the normalization is due solely to the properties of the spherical harmonics. In particular, no subtleties arising from the boundary of AdS$_7$ are encountered.

For the modes, $s^I$, the computation of the quadratic action is more complicated. Imposing the constraint (3.11) by simple substitution of the solution (3.16) leads to terms in the action with higher derivatives, of the form $(\nabla_{(\mu} \nabla_{\nu)} s^I)^2$ and $(\nabla^\rho \nabla_{(\mu} \nabla_{\nu)} s^I)^2$. In [24], where the analogous modes on AdS$_5 \times S^5$ were studied, it was proposed to deal with these terms by
computing the action on shell as a function of boundary values. We find that this procedure fails, both for $\text{AdS}_5 \times S^5$ and $\text{AdS}_7 \times S^4$, to produce a normalization consistent with the quadratic equations of motion. For the completeness of our discussion we present the details of these computations in Appendix B and comment on why they fail, but we will not need the (misleading) results to compute the correlation functions.

It turns out that the simplest way of obtaining both the quadratic\(^1\) and cubic terms in the action for the $s^I$ is by computing quadratic corrections to the equations of motion. Once we have these corrections, we promote them to an action, as explained in [24]. To this end, the quadratic corrections to equations (3.8), (3.10), (3.12), and (3.9) are computed in appendix C.

With $Q^I_\phi$ from (C.13), we can compute the $s-s-\phi$ vertex. Note that the $ss$ corrections to (3.9) can be written in the form

\[
(\nabla^\mu \nabla_\mu - f_1) \phi^{I_1} = 2Q^I_\phi = D^\phi_{I_1I_2I_3} s^{I_2} s^{I_3} + E^\phi_{I_1I_2I_3} \nabla_\mu s^{I_2} \nabla_\mu s^{I_3} + F^\phi_{I_1I_2I_3} \nabla_\nu s^{I_2} \nabla_\mu \nabla_\nu s^{I_3}. \tag{3.20}
\]

The change of variables

\[
\phi^{I_1} = \tilde{\phi}^{I_1} + J^\phi_{I_1I_2I_3} \tilde{s}^{I_2} \tilde{s}^{I_3} + L^\phi_{I_1I_2I_3} \nabla_\mu \tilde{s}^{I_2} \nabla_\mu \tilde{s}^{I_3},
\]

\[
J^\phi_{I_1I_2I_3} = \frac{1}{2} F^\phi_{I_1I_2I_3} - \frac{1}{4} (m_2^2 + m_3^2 - f_1 - 12) F^\phi_{I_1I_2I_3}, \tag{3.21}
\]

\[
L^\phi_{I_1I_2I_3} = \frac{1}{2} L^\phi_{I_1I_2I_3},
\]

leaves the normalization of the quadratic action for $s^I$ and $\phi^I$ invariant, but removes the derivatives from the right-hand side of (3.20)

\[
(\nabla^\mu \nabla_\mu - f_1) \tilde{\phi}^{I_1} = \lambda^\phi_{I_1I_2I_3} \tilde{s}^{I_2} \tilde{s}^{I_3}, \tag{3.22}
\]

where

\[
\lambda^\phi_{I_1I_2I_3} = D^\phi_{I_1I_2I_3} - (m_2^2 + m_3^2 - f_1) J^\phi_{I_1I_2I_3} - \frac{2}{3} m_2^2 m_3^2 L^\phi_{I_1I_2I_3} = -\frac{9a(k_1, k_2, k_3)}{64z(k_1)} \frac{\alpha_1(\alpha_1 - 1)(2\alpha_1 - 3)\Sigma(\Sigma + 1)(\Sigma + 3)}{k_2(2k_2 + 1)k_3(2k_3 + 1)} \langle T^{I_1} C^{I_2} C^{I_3} \rangle. \tag{3.23}
\]

The cubic $s-s-\phi$ vertex will be obtained from this by multiplication by the normalization $B_I$ of the quadratic action (3.19) for $\phi^I$,

\[
G^{\phi ss}_{I_1I_2I_3} = -\frac{9N^3}{64\pi^5} \frac{\alpha_1(\alpha_1 - 1)(2\alpha_1 - 3)\Sigma(\Sigma + 1)(\Sigma + 3)}{k_2(2k_2 + 1)k_3(2k_3 + 1)} a(k_1, k_2, k_3) \langle T^{I_1} C^{I_2} C^{I_3} \rangle. \tag{3.24}
\]

\(^1\)We would like to thank Sangmin Lee for pointing out to us that the computation of the $s-s-\phi$ vertex allows the computation of the proper quadratic normalization for $s^I$. 
This vertex may also be computed by studying corrections of the form \( s\phi \) to the equation of motion (C.4) for the \( s^I \) modes, which take the form

\[
(\nabla^\mu \nabla_\mu - 4k_1(k_1 - 3)) s^I = D_{I1}^{(s\phi)} \phi^I s^I + E_{I1}^{(s\phi)} J_{I2}^{(s\phi)} \nabla_\mu J_{I2}^{s\phi} \nabla_\nu s^I + F_{I1}^{(s\phi)} \nabla_{(\mu} J_{I2}^{s\phi} \nabla_{(\nu} s^I. \tag{3.25}
\]

Again, by a change of variables, this time with

\[
s'^I = s^I + J_{I1}^{(s\phi)} \phi^I s'^I + L_{I1}^{(s\phi)} \phi^I s'^I,
\]

we can remove the higher derivative terms, obtaining

\[
(\nabla^\mu \nabla_\mu - 4k_1(k_1 - 3)) s'^I = \lambda_{I1}^{(s\phi)} \phi^I s'^I,
\]

\[
\lambda_{I1}^{I1} = \frac{-a(k_1, k_2, k_3)}{4z(k_2)} \frac{a_1(k_1 - 1)(2a_1 - 3)\Sigma(\Sigma + 1)(\Sigma + 3)}{(k_2 - 1)(2k_2 + 3)k_3(2k_3 + 1)} \langle T_1 C_{12} C_{13} \rangle. \tag{3.27}
\]

The \( \phi-s-s \) vertex in (3.24) should be obtained by multiplying \( \lambda_{I1}^{I1} \) by the quadratic normalization \( A_I \) for the mode \( I_2 \). By comparison of these expressions, taking into account a factor of two because the vertex is quadratic in the \( s^I \), we find that the linearized equation of motion for \( s^I \) can be obtained from a canonical quadratic action

\[
L[s^I] = -\sum_I \frac{A_I}{2} [(\nabla s^I)^2 + 4k(k_1 - 3)(s^I)^2],
\]

\[
A_I = \frac{9(k_1 - 1)(2k_2 + 3)z(k)}{16k(2k_1 + 1)} \frac{2N^3}{\pi^5}. \tag{3.28}
\]

The \( s-s-s \) vertex is computed by considering the \( ss \) terms in (C.4)

\[
(\nabla^\mu \nabla_\mu - 4k_1(k_1 - 3)) s^I = D_{I1} s^I s^I s^I + E_{I1} s^I \nabla_\mu s^I \nabla_\nu s^I + F_{I1} \nabla_{(\mu} s^I \nabla_{(\nu} s^I, \tag{3.29}
\]

where the higher derivative terms are removed by the change of variables

\[
s'^I = s^I + J_{I1} s^I s'^I + L_{I1} s^I \nabla_\mu s^I \nabla_\nu s^I,
\]

\[
J_{I1} s^I = \frac{1}{2} F_{I1} - \frac{1}{4}(m_2^2 + m_3^2 - m_1^2 - 12) F_{I1}, \tag{3.30}
\]

\[
L_{I1} = \frac{1}{2} F_{I1},
\]

\[
\]
so that
\[(\nabla^\mu \nabla_\mu - 4k_1(k_1 - 3)) \tilde{z}^I = \lambda_{I_1I_2I_3} \tilde{z}^{I_2} \tilde{z}^{I_3},\]
\[\lambda_{I_1I_2I_3} = -\frac{3a(k_1, k_2, k_3)}{32z(k_1)} \frac{\alpha_1 \alpha_2 \alpha_3 (\Sigma - 2)(\Sigma^2 - 1)(\Sigma^2 - 9)}{(k_1 - 1)(2k_1 + 3)k_2(2k_2 + 1)k_3(2k_3 + 1)} \langle C^{I_1} C^{I_2} C^{I_3} \rangle.\] (3.31)

Multiplying this by \(A_{I_1}\), as given in (3.28), we obtain a totally symmetric function
\[G_{I_1I_2I_3} = -\frac{27N^3}{256\pi^5} \frac{\alpha_1 \alpha_2 \alpha_3 (\Sigma - 2)(\Sigma^2 - 1)(\Sigma^2 - 9)}{k_1(2k_1 + 1)k_2(2k_2 + 1)k_3(2k_3 + 1)} a(k_1, k_2, k_3) \langle C^{I_1} C^{I_2} C^{I_3} \rangle.\] (3.32)

Putting these results together, we have computed the following terms in the action for the modes \(s^I\) and \(\phi^I\)
\[L[s^I, \phi^I] = -\sum_I \left[ \frac{A_I}{2} [\nabla s^I]^2 + 4k(k - 3)(s^I)^2 \right] + \frac{B_I}{2} [\nabla \phi^I]^2 + 4k(k + 3)(\phi^I)^2 \]
\[+ \sum_{I_1, I_2, I_3} \left[ G_{I_1I_2I_3}^{\phi s s} \phi^{I_1} s^{I_2} s^{I_3} + \frac{1}{3} G_{I_1I_2I_3} s^{I_1} s^{I_2} s^{I_3} \right] + \cdots.\] (3.33)

### 3.3. Two and Three-Point Correlation Functions at Large \(N\)

With an action for the modes \(s^I\) computed to cubic order, it is now simple to compute correlation functions of the corresponding chiral primary operators. We are assuming that the operators are normalized (2.5), so the coupling appearing in the generating functional could involve a proportionality constant, i.e.,
\[\left\langle e^{\mathcal{N}^I s^I} \mathcal{O}^I \right\rangle_{\text{CFT}} \sim e^{-S_{\text{sugra}}[s^I]}.\] (3.34)

The \(\mathcal{N}^I\) will be chosen such that the large \(N\) two-point function is normalized. From the formula (A.13) that generates the two-point function, we find
\[\langle \mathcal{O}^{I_1}(x_1) \mathcal{O}^{I_2}(x_2) \rangle = \frac{1}{(\mathcal{N}^I)^2} \frac{1}{A_I} \frac{4(k - 1)(2k - 1)(2k - 3)}{\pi^3} \frac{\delta^{I_1I_2}}{|x_1 - x_2|^{4k}},\] (3.35)
so that we set
\[\mathcal{N}^I = -\frac{2^{3(k+1)/2}(2k - 3)}{3N^{3/2}} \sqrt{\frac{(2k - 1)(2k + 1)\Gamma(k + \frac{3}{2})}{\Gamma(k)}}.\] (3.36)

We have chosen a minus sign in this expression so that the coefficient of the 3-point function below is positive.
From this normalization and the formula for the three-point function (A.15), we find the three-point function for chiral primary operators, valid in the large \( N \) limit,

\[
\langle \mathcal{O}^{I_1}(x_1) \mathcal{O}^{I_2}(x_2) \mathcal{O}^{I_3}(x_3) \rangle
= \frac{1}{\sqrt{\pi} N^{3/2}} \left[ \prod_{i=1}^{3} \Gamma(\alpha_i + \frac{1}{2}) \right] \frac{4k_i^2 - 1}{\Gamma(k_i) \Gamma(k_i + \frac{3}{2})} \frac{2^{(\Sigma/2-15)/2} \Gamma(k_i)}{|\bar{x}_1 - \bar{x}_2|^{4\alpha_1} |\bar{x}_2 - \bar{x}_3|^{4\alpha_1} |\bar{x}_3 - \bar{x}_1|^{4\alpha_2}} \left( \frac{\alpha_3}{\alpha_1 \alpha_2} \right) \tag{3.37}
\]

In terms of the conformal weights, we can express this as

\[
\langle \mathcal{O}^{I_1}(x_1) \mathcal{O}^{I_2}(x_2) \mathcal{O}^{I_3}(x_3) \rangle
= \frac{2^{\Delta_1+\Delta_2+\Delta_3-30}}{\sqrt{\pi} N^{3/2}} \Gamma \left( \frac{\Delta_1+\Delta_2+\Delta_3}{2} \right) \frac{(\Delta^2 - 1)(\Delta_2^2 - 1)(\Delta_3^2 - 1)}{\Gamma \left( \frac{\Delta_1}{2} \right) \Gamma \left( \frac{\Delta_2+3}{2} \right) \Gamma \left( \frac{\Delta_3+3}{2} \right) \Gamma \left( \frac{\Delta_1+3}{2} \right)} \left( \frac{\alpha_3}{\alpha_1 \alpha_2} \right) |\bar{x}_1 - \bar{x}_2|^{\Delta_1+\Delta_2-\Delta_3} |\bar{x}_2 - \bar{x}_3|^{\Delta_2+\Delta_3-\Delta_1} |\bar{x}_3 - \bar{x}_1|^{\Delta_3+\Delta_1-\Delta_2} \langle C^{I_1} C^{I_2} C^{I_3} \rangle. \tag{3.38}
\]

The expressions (3.37) and (3.38) are certainly more formidable than the expression found in [24] for the analogous correlation functions of the CPOs of the \( \mathcal{N} = 4, D = 4 \) SCFT. One\(^2\) can ask if these expressions simplify, given an explicit expression for the \( \langle C^{I_1} C^{I_2} C^{I_3} \rangle \), which are related to the Clebsch-Gordan coefficients for \( SO(5) \). Since we know of no closed-form expression for arbitrary values of \( k \), let us consider the very degenerate case that we have only the highest and lowest weight states

\[
Y^{(k_1)} = \frac{1}{2^{3k_1/2} \pi^{1/4}} (x_1 + \frac{i}{2} x_2)^{k_1}, \tag{3.39}
\]

\[
Y^{(k_2)} = \frac{1}{2^{3k_2/2} \pi^{1/4}} (x_1 + \frac{i}{2} x_2)^{k_2},
\]

\[
Y^{(k_3)} = \frac{1}{2^{3k_3/2} \pi^{1/4}} (x_1 - \frac{i}{2} x_2)^{k_3},
\]

where the normalization has been chosen to agree with (D.5). In this case, we find

\[
\langle C^{(k_1)} C^{(k_2)} C^{(k_3)} \rangle = \frac{\delta_{k_1+k_2+k_3}}{\pi^{1/4} \Gamma(k_1 + k_2 + \frac{3}{2})}. \tag{3.40}
\]

For \( k_3 = k_1 + k_2 \), the 3-point function simplifies quite a bit, but the use of (3.40) doesn’t

\(^2\) We thank J. Distler for actually asking the question.
result in any further simplification,

\[
\langle O^{I_1}(x_1)O^{I_2}(x_2)O^{I_3}(x_3) \rangle
\]

\[
= \frac{2^{(k_1+k_2-15)/2}}{N^{3/2}} \frac{\Gamma(k_1+\frac{1}{2}) \Gamma(k_2+\frac{1}{2}) \Gamma(k_1+k_2)}{|x_2-x_3|^{4k_1} |x_3-x_1|^{4k_2}} \frac{C^{(k_1)} C^{(k_2)} C^{(k_3)}}{|x_2-x_3|^{4k_1} |x_3-x_1|^{4k_2}}
\]

(3.41)

So, unfortunately, knowing the Clebsch-Gordan coefficients doesn’t appear to lead to more illuminating expressions for the correlation functions.

One further observation we can make about the 3-point functions is that there don’t appear to be any additional zeros beyond those arising from the group-theoretic factor \( \langle C^{I_1} C^{I_2} C^{I_3} \rangle \). This is, at least, consistent with the fact that no additional zeros were found in an analysis, based solely on the properties of the six-dimensional superconformal algebra, of the 4-point function [22].

From the \( G^{\phi_{ss}}_{I_1 I_2 I_3} \) vertex, we can also compute the correlation function \( \langle \Phi^{I_1} O^{I_2} O^{I_3} \rangle \). We must first identify the normalization constant appearing in the coupling of \( \phi_0^I \) to \( \Phi^I \) in the generating function. From the 2-point function \( \langle \Phi^{I_1} \Phi^{I_2} \rangle \), we find

\[
N_\phi^I = -\frac{2^{(3k/2+1)}(2k+3)}{N^{3/2}} \sqrt{\frac{(k+2)\Gamma(k+\frac{7}{2})}{\Gamma(k+1)}}.
\]

(3.42)

We then compute, in the large \( N \) limit,

\[
\langle \Phi^{I_1}(x_1)O^{I_2}(x_2)O^{I_3}(x_3) \rangle
\]

\[
= \frac{2^{11\Sigma/2-9}}{N^{3/2}} \frac{\Sigma(\Sigma+1)(\Sigma+3)\Gamma(\Sigma+6)}{\Gamma(\Sigma+5/2)} \prod_{i=1}^{3} \frac{\Gamma(2\alpha_i + 3)\Gamma(k_i + \frac{5}{2})}{\Gamma(\alpha_i + 1)\Gamma(2k_i + 4)} \sqrt{\frac{\Gamma(k_i + \frac{7}{2})}{\Gamma(k_i)}} \cdot \alpha_1(\alpha_1 - 1)(2\alpha_1 - 3) \frac{2k_1 + 5(2k_2 - 3)(2k_3 - 3)}{k_1} \frac{1}{(k_2 - 1)(k_3 - 1)} \sqrt{(4k_2^2 - 1)(4k_3^2 - 1)}
\]

\[
\cdot \sqrt{\frac{\alpha_2 + 6}{\alpha_2 + 6}} \cdot \prod_{i=1}^{3} |x_i-x_{i+1}|^{4\alpha_i + 6} |x_i-x_{i+2}|^{4\alpha_i - 6} |x_i-x_{i+3}|^{4\alpha_i + 6}.
\]

(3.43)
Once again, we can express this in terms of the conformal weights,

\[
\langle \Phi^I_1(x_1) \mathcal{O}^I_2(x_2) \mathcal{O}^I_3(x_3) \rangle = \frac{2^{(\Delta_1-\Delta_2-\Delta_3-23)/2}\pi}{N^{3/2}} \frac{\Gamma\left(\frac{\Delta_1+\Delta_2+\Delta_3+\Delta_4+6}{2}\right)}{(\Delta_2-3)(\Delta_3-3)(\Delta_2-2)(\Delta_3-2)\Gamma\left(\frac{\Delta_1+\Delta_2+\Delta_3-4}{4}\right)}
\]

\[
\cdot \frac{\Gamma\left(\frac{\Delta_1}{2}-2\right)\Gamma\left(\frac{\Delta_1+\Delta_2-\Delta_3}{2}\right)^2\Gamma\left(\frac{\Delta_1+\Delta_2-\Delta_4}{2}\right)}{\Gamma(\Delta_1)\Gamma\left(\frac{\Delta_1+\Delta_2+\Delta_3}{2}\right)\Gamma\left(\frac{\Delta_1+\Delta_2+\Delta_4}{2}\right)\Gamma\left(\frac{\Delta_1+\Delta_2-\Delta_4}{2}\right)\Gamma\left(\frac{\Delta_1+\Delta_2+\Delta_3-10}{4}\right)}
\]

\[
\cdot \sqrt{\frac{\Gamma\left(\Delta_1-2\right)\Gamma\left(\frac{\Delta_1+1}{2}\right)\Gamma\left(\frac{\Delta_1}{2}\right)}{\Gamma\left(\frac{\Delta_1}{2}-2\right)\Gamma\left(\frac{\Delta_1+1}{2}\right)}} \frac{\Gamma\left(\frac{\Delta_1}{2}\right)\Gamma\left(\frac{\Delta_1}{2}\right)}{\Gamma\left(\frac{\Delta_1}{2}-2\right)\Gamma\left(\frac{\Delta_1+1}{2}\right)}
\]

\[
\cdot \sqrt{\frac{(\Delta_1-2)(\Delta_2)(\Delta_3-2)}{(\Delta_2-1)(\Delta_3-1)(\Delta_2+3\Delta_3-\Delta_1)(\Delta_2+3\Delta_3-\Delta_1)(\Delta_2+3\Delta_3-\Delta_1)}}
\]

\[
\cdot \langle T^{I_1I_2}C^{I_3} \rangle
\]

\[
\cdot |\vec{x}_1 - \vec{x}_2|^\Delta_1+\Delta_2-\Delta_3 |\vec{x}_2 - \vec{x}_3|^\Delta_2+\Delta_3-\Delta_1 |\vec{x}_3 - \vec{x}_1|^\Delta_3+\Delta_1-\Delta_2.
\]

As we saw for the case of \( \langle \mathcal{O}^I_1 \mathcal{O}^I_2 \mathcal{O}^I_3 \rangle \), we don’t expect that an explicit expression for \( \langle T^{I_1I_2}C^{I_3} \rangle \) will cause these expressions to simplify.

## 4. The Operator Product Expansion of Wilson Surfaces

In [28], it was shown that one could use the AdS description of the large \( N \) limit of the (0, 2) superconformal field theory in six dimensions to compute Wilson surface observables [29]. The chiral primaries for these theories are known [16–18], so one can use the AdS formalism to write an operator product expansion for such a surface. Analogously to the case of the Wilson loop [30], we expect that there exists an operator product expansion for the Wilson surface when it is probed from distances large compared to its characteristic size \( a \),

\[
W(S) = \langle W(S) \rangle \left[ 1 + \sum_{i,n} c_i^{(n)} a^{\Delta_i^{(n)}} \mathcal{O}_i^{(n)} \right],
\]

where the \( \mathcal{O}_i^{(n)} \) are a set of operators with conformal weights \( \Delta_i^{(n)} \). Here \( \mathcal{O}_i^{(0)} \) denotes the \( i^{th} \) primary field, while the \( \mathcal{O}_i^{(n)} \) for \( n > 0 \) are its conformal descendants. For a spherical Wilson surface, the expectation value of all operators other than the identity vanishes, so that the coefficient of the identity is the expectation value of the loop.

In [30] the spherical Wilson surface solution was studied. The scalar charge of the surface was taken to be constant (a point on \( S^4 \)) and the Wilson surface was the boundary (a 2-sphere) of a minimal area membrane worldvolume in \( \text{AdS}_7 \times S^4 \). A convenient parameter-
ization of the solution was given in terms of the Poincare coordinates as

\[
x_1 = \sqrt{a^2 - z^2} \cos \theta \\
x_2 = \sqrt{a^2 - z^2} \sin \theta \cos \psi \\
x_3 = \sqrt{a^2 - z^2} \sin \theta \sin \psi,
\]

where \(0 \leq z \leq a\), \(0 \leq \theta \leq \pi\), and \(0 \leq \psi \leq 2\pi\).

The volume of the membrane was found to be divergent

\[
S = T(2) \int dV = T(2) 4\pi \int_0^a \frac{dz \sqrt{a^2 - z^2}}{z^3} = \pi T(2) \left[ + \frac{2a^2}{\epsilon^2} - 2 \ln \frac{2a}{\epsilon} - 1 + O(\epsilon) \right],
\]

where the tension of the membrane is \(T(2) = 2N/\pi\) in our units. The quadratic divergence is proportional to the area of the surface and was present in the case of a rectangular Wilson surface [28]. The logarithmic divergence was found to be proportional to the “rigid string” action [31], where for a general 2-surface \(\Sigma_2\),

\[
S_{\text{rigid}} = \int_{\Sigma_2} d^2 \sigma \sqrt{\gamma} (\nabla^2 X^i)^2,
\]

where \(\gamma\) is the induced metric on the Wilson surface and the \(X^i\) are the coordinates on \(\mathbb{R}^6\) describing the surface. It was conjectured in [30] that tensionless strings in the (0, 2) six-dimensional field theory might be governed by some supersymmetric form of the action (4.4). In fact, this instance of a conformal anomaly is part of a much more general structure, as discussed in [32].

One implication of the logarithmic divergence in (4.3) is that the expectation value of the Wilson surface is not well defined, since we can add any constant to the logarithmic subtraction. Furthermore, it seems to indicate that the expectation value of a Wilson surface is scale dependent. Despite this, the connected correlation functions of Wilson surfaces do not receive extra divergent contributions. Therefore their correlators can be calculated in a completely analogous fashion to the Wilson loops in [30], allowing the extraction of OPE coefficients of (4.1). In this section, we investigate the OPE for the spherical Wilson surface. We consider a Wilson surface whose characteristic size is much smaller than its distance from any probe in the theory. Then we identify the operators of low conformal dimension that are allowed to appear in the OPE and we compute the necessary correlation functions to extract the OPE coefficients.
4.1. The Operator Product Expansion of the Spherical Wilson Surface

The possible operators which can appear in the Wilson surface must have the same symmetry properties as the Wilson surface itself, so the operators $O_i^{(n)}$ should be bosonic and $S_{N+1}$ invariant. The Wilson loop and surface solutions of [28] require a massive “quark” in the theory, and hence a non-zero Higgs VEV. We consider the case where $\theta_I(s) = \theta_I$ is a constant, which breaks the R-symmetry group from $SO(5) \rightarrow SO(4)$. So we should look for operators in $SO(5)$ representations which have $SO(4)$ singlets in their decomposition. We also consider only operators whose dimensions are protected. The appearance in the OPE of operators with large anomalous dimensions will be suppressed.

The first level at which operators can appear is at dimension $\Delta = 4$, where we have the first CPO. This operator is in the $14 \rightarrow 1 \oplus 4 \oplus 9$ under $SO(4)$, so it is allowed. In general, the CPOs, being in the symmetric traceless representations of $SO(5)$, always contain one $SO(4)$ singlet, and hence all of them may appear in the OPE.

At dimension $\Delta = 5$, there are two operators to consider. There is a $Spin(5,1)$ vector operator in the $10 \rightarrow 4 \oplus 6$ which does not contribute a singlet. There is also a Lorentz 3-form operator in the $5 \rightarrow 1 \oplus 4$ which is allowed to contribute, depending on the orientation, $\hat{\sigma}^{\mu\nu\rho}$, of the membrane. This is the first operator in a series of Lorentz 3-form, $SO(5)$ rank $k$ symmetric traceless tensor operators of dimension $\Delta = 2k + 3$, and all of these can appear. Below we denote these operators as $O_{\mu\nu\rho}^I$.

At dimension $\Delta = 6$, we have the CPO at $k = 3$ which, as discussed above, is permitted in the OPE. Additionally there is the first ($k = 0$) member of a series in the $20'$ of $Spin(5,1)$ and the rank $k$ symmetric traceless tensor of $SO(5)$, with dimensions $\Delta = 2k + 6$.

Higher dimension operators can be analyzed in the same fashion. We arrive at the following expression for the OPE

$$\frac{W(S)}{\langle W(S) \rangle} = 1 + c_2^{(0)} a^4 Y^I(\theta) O^I + c_3^{(0)} a^5 Y^I(\theta) \hat{\sigma}^{\mu\nu\rho} O_{\mu\nu\rho}^I + \cdots ,$$

(4.5)

where the $Y^I(\theta)$ denote spherical harmonics on $S^4$ and the coefficients $c_i^{(n)}$ are what we wish to compute.

The coefficients of the operator product expansion (4.5) can be determined in a few ways. One can compute the correlator of the Wilson surface with each operator that is expected to appear in the OPE. This correlator gets contributions only from the given conformal primary and its descendents. For a Wilson surface of size $a$, separated from an operator by
a distance $L$,

$$\frac{\langle W(S)O_i^{(0)} \rangle}{\langle W(S) \rangle} = c_i^{(0)} \frac{a^{\Delta_i^{(0)}}}{L^{2\Delta_i^{(0)}}} + \sum_{m>0} c_i^{(m)} a^{\Delta_i^{(m)}} \langle O_i^{(m)} O_i^{(0)} \rangle,$$  

(4.6)

where we have assumed that the operators have been normalized. Here we have isolated the contribution from the descendents and their mixings with the primaries in the second term. One can also compute the correlator of a pair of Wilson surfaces that are separated by a distance $L$ which is large compared to their size. This correlator can be calculated from the operator product expansion for the two Wilson surfaces

$$\frac{\langle W(S,L)W(S,0) \rangle}{\langle W(S,L) \rangle \langle W(S,0) \rangle} = \sum_{i,j;m,n} c_i^{(m)} c_j^{(n)} a^{\Delta_i^{(m)} + \Delta_j^{(n)}} \langle O_i^{(m)}(L)O_j^{(n)}(0) \rangle$$

$$= \sum_i (c_i^{(0)})^2 a^{2\Delta_i^{(0)}}$$

$$+ \sum_{i,\{m,n\} \neq \{0,0\}} c_i^{(m)} c_i^{(n)} a^{\Delta_i^{(m)} + \Delta_i^{(n)}} \langle O_i^{(m)}(L)O_i^{(n)}(0) \rangle.$$

(4.7)

In the last line, the first term is due solely to the primary fields, while the second contains the contributions from descendents.

4.2. Wilson Surface Correlators and OPE Coefficients from Supergravity

We would now like to investigate the process of extracting the large $N$ values of the OPE coefficients by using the AdS/CFT correspondence to compute correlation functions like (4.6) or (4.7). If we consider two closely separated Wilson surfaces, then, analogously to the case of two Wilson loops [33], for small enough separation, instead of having two separate membranes forming the surfaces, a single membrane worldsheet with both surfaces as its boundary will be the configuration of minimal surface area. As the surfaces are separated beyond a critical distance, this worldsheet will degenerate into a long, thin cylinder connecting two minimal surfaces. In a worldsheet picture, this long, thin cylinder would represent the bulk picture of propagation of light supergravity states between the two membranes.

This picture tells us that the correlation functions (4.6) and (4.7) can be calculated by treating the membrane as a source for the fields propagating in anti-de Sitter spacetime and then computing the effective action for the propagation of supergravity states from the surface to the point where the other operator, or the other surface, is located. This is, of course, the analog of the method used for the Wilson loop in [30].

We would like to compute the coupling of the membrane to the bulk supergravity fields. We will be content with considering only the lightest scalar modes, which are the most rel-
evant when the surfaces are separated by a distance which is large compared to their size. The lightest states include the modes \( s^I \) from section 3, corresponding to the appearance of the CPOs in the OPE. We saw that these modes were related to several components of the supergravity fluctuations in (3.6). Couplings to the modes \( b^I \) will be through world-volume fermions, and should be suppressed in the small spacetime curvature limit that we are considering. The couplings to modes of the graviton can be obtained by linearizing the membrane effective action

\[
S_{\text{eff.}} = T^{(2)} \int d^3 \sigma \sqrt{\det G_{mn}} \partial_\alpha x^m \partial_\beta x^n \\
= T^{(2)} \int d^3 \sigma \sqrt{\det g_{mn}} \partial_\alpha x^m \partial_\beta x^n \left[ 1 + \frac{1}{2} (g_{mn} \partial_\alpha x^m \partial_\beta x^n)^{-1} h_{mn} \partial_\alpha x^m \partial_\beta x^n + \cdots \right].
\] (4.8)

Since the membrane is living at a point on \( S^4 \), we are only interested in the components \( h^I_{\mu\nu} \), which, from (3.17), may be written in terms of \( s^I \) as

\[
h^I_{\mu\nu} = -\frac{1}{14} g_{\mu\nu} s^I + \frac{3}{16k(2k+1)} \nabla_\mu \nabla_\nu s^I. \] (4.9)

To aid in computing the derivatives, we can use an argument from [30], which was also used to derive (B.5). Namely, we are looking for the leading terms in the expansions in \( a/L \) of (4.6) and (4.7). These leading contributions arise from derivatives with respect to \( z \) which act on the numerator of the expansion into a Green’s function, e. g., (A.9). So we find

\[
h^I_{\mu\nu} \sim -\frac{1}{8} \left( g_{\mu\nu} + 3 \delta^z_{\mu} \delta^z_{\nu} g_{zz} \right) s^I. \] (4.10)

We find that the coupling to the membrane is given by

\[
S_{s^I} = -\frac{3T^{(2)}}{8} \int d\mathcal{V} \frac{z^2}{a^2} s^I. \] (4.11)

The contributions to the correlator of two Wilson surfaces due to exchange of \( s^I \) modes are then expressed as

\[
\frac{\langle W(S, L)W(S, 0) \rangle^{(s)}}{\langle W(S, L) \rangle \langle W(S, 0) \rangle} = \exp \left[ \sum_k \left[ Y^I(\theta) \right]^2 \left( \frac{3T^{(2)}}{8} \right)^2 \int d\mathcal{V} d\mathcal{V'} \frac{(zz')^2}{a^4} G_k(W) \right]. \] (4.12)

If the distance between the surfaces is much larger than their size, then we may approximate the Green’s function for propagation between the two loops by the expression (A.8). We find that

\[
\frac{\langle W(S, L)W(S, 0) \rangle^{(s)}}{\langle W(S, L) \rangle \langle W(S, 0) \rangle} \sim \sum_k 2^{3k-1} \pi \frac{\Gamma(k)}{\Gamma(k - \frac{1}{2})} [Y^I(\theta)]^2 \left( \frac{a}{L} \right)^{4k}, \] (4.13)
where the \( \sim \) refers to the fact that subleading terms at each order of the \( a/L \) expansion have been dropped but the numerical coefficient corresponding to the coefficient of the contribution of each primary field is precise. We can similarly obtain the correlation function between the surface and one of the chiral primary operators \( \mathcal{O}^I \),

\[
\frac{\langle W(S, L)\mathcal{O}^I(0) \rangle^{(s)}}{\langle W(S, L) \rangle} \sim \frac{1}{N^I} \frac{\delta}{\delta s_0^I(x)} Y^I(\theta) \int dV d^6x' \left( \frac{3T(2)}{8} \frac{z^2}{a^2} \right) K_\Delta(x, z; x') s_0^I(x') \\
\sim -2^{(3k-1)/2} \sqrt{\frac{\pi}{N}} \frac{\Gamma(k)}{\Gamma(k - \frac{1}{2})} Y^I(\theta) a^{2k} L_{4k}^{-1},
\]

(4.14)

where we have set the normalization of the operator by the 2-point function (3.35). We note that the coefficient here is simply the square root of that appearing in (4.13). From these correlation functions, we find the operator product coefficients of the chiral primary operators

\[
c_{\Delta}^{\text{CPO}} = -2^{(3\Delta-2)/4} \sqrt{\frac{\pi}{N}} \frac{\Gamma(\frac{\Delta}{2})}{\Gamma(\frac{\Delta-1}{2})},
\]

(4.15)

where the minus sign is a result of choosing the 3-point correlation function of the \( \mathcal{O}^I \) to be positive.

5. Conclusions

In this paper we have succeeded in computing 3-point correlation functions of chiral primary operators in the large \( N \) limit of the \((0, 2)\) superconformal field theory. We also studied the operator product expansion for a spherical Wilson surface, and found that we could compute correlation functions of two surfaces as well as that for a surface with a local operator. We used these correlation functions to extract the explicit large \( N \) values of operator product coefficients for the chiral primary operators. Extracting more information about the \((0, 2)\) theory from our results is an exciting prospect to be left for future work.

It would also be interesting to study the 3-point function from the perspective of the DLCQ formulation of the \((0, 2)\) theory using the prescription of [12]. A discussion of the comparison of DLCQ theories of M5-branes and the AdS/CFT correspondence appears in [34]. In particular, in the large \( N \) limit of the DLCQ, there should be exact agreement with our results.
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A. Green’s Functions and Correlation Functions

In this appendix, we list some properties of Green’s functions on AdS which will be useful to us in the main text. We also discuss the method we use to ensure that the limit $z \to 0$ is taken properly when computing correlation functions. Scalar Green’s functions on anti-de Sitter space have been discussed in a large number of papers, including [14, 15, 18, 35–37].

In the text, we consider a real scalar field $\phi$ on anti-de Sitter spacetime of radius one. With source $J$, the action and equation of motion are

$$S = -\int_{\text{AdS}} d^{d+1}x \sqrt{g} \left[ \frac{A}{2} (\nabla \phi)^2 + m^2 \phi^2 \right] \phi J,$$

$$A(-\nabla_x^2 + m^2)\phi = J,$$  

(A.1)

where $A$ is some constant. Imposing the boundary condition $\phi|_{\partial M} = 0$ yields a unique solution for $\phi$, as long as the operator $-\nabla_x^2 + m^2$, is positive definite. This is the case for all $m^2 \geq -d^2/4$ [38].

Solutions for $\phi$ which minimize the action (A.1) are given by the integral equation

$$\phi(x) = \int_M d^{d+1}x' \sqrt{g(x')} G(x, x').J(x'),$$  

(A.2)

where the kernel $G(x, x')$ is the covariant Green’s function for the equation of motion (A.1), satisfying

$$A(-\nabla_x^2 + m^2)G(x, x') = \frac{1}{\sqrt{g(x')}} \delta^{(d+1)}(x - x').$$  

(A.3)

In the upper-half space representation of anti-de Sitter spacetime, with metric

$$ds^2 = \frac{1}{z^2} (dz^2 + \eta_{ij} dx^i dx^j),$$  

(A.4)

the scalar Green’s function can only depend on the distance between the sources,

$$W = \frac{zz'}{(z - z')^2 + \sum_{i=1}^d |x_i - x'_i|^2},$$  

(A.5)
The solution for (A.3) which goes to zero at the boundary is given in terms of the hypergeometric function

\[ G(W) = \frac{\alpha_0}{A} W^\Delta \, _2F_1(\Delta, \Delta + \frac{1-d}{2}, 2\Delta - d + 1; -4W). \]  

(A.6)

where \( \Delta = d/2 + \sqrt{m^2 + d^2/4} \) will be the conformal weight of the associated operator and \( \alpha_0 \) is

\[ \alpha_0 = \frac{\Gamma(\Delta)}{2\pi^{\frac{d}{2}}\Gamma(\Delta - \frac{d}{2} + 1)} = \frac{(\Delta - 1)(\Delta - 2)}{2\pi^3} \quad \text{for} \quad d = 6. \]  

(A.7)

We will be particularly interested in fields in the bulk which are produced by a source on the boundary. We take a source with support very close to the boundary, \( \text{supp}(J(x')) = \{ z'|0 \leq z' < \epsilon \} \). In this region \( W \sim 0 \), so we can approximate

\[ G(W) \sim \frac{\alpha_0}{A} W^\Delta. \]  

(A.8)

Then the field produced in the bulk is given by

\[ \phi(x) = \int_M d^{d+1}x' \sqrt{g(x')} \, G(x, x') J(x') \sim \frac{\alpha_0}{A} \int_{\partial M} d^d x' \left( \frac{z}{z^2 + |\vec{x} - \vec{x}'|^2} \right)^\Delta \int_0^\epsilon dz' z^{\Delta - d - 1} J(x') \]  

(A.9)

Here we have defined the boundary-to-bulk propagator

\[ K_\Delta(\vec{x}, z; \vec{x}') = \frac{\Gamma(\Delta)}{A\pi^{\frac{d}{2}}\Gamma(\Delta - \frac{d}{2})} \left( \frac{z}{z^2 + |\vec{x} - \vec{x}'|^2} \right)^\Delta \]  

(A.10)

and the boundary sources

\[ \phi_0(\vec{x}') = \frac{1}{2\Delta - d} \int_0^\epsilon dz' z^{\Delta - d - 1} J(x'). \]  

(A.11)

The normalization of the boundary-to-bulk propagator (A.10) has been chosen specifically to agree with that of [36]. By evaluating the action (A.1) in terms of the boundary sources,

\[ S^{(2)} = \frac{1}{2} \int_M \int_{M'} d^{d+1}x \int d^{d+1}x' \sqrt{g(x)} J(x) G(x, x') \sqrt{g(x')} J(x') \]  

\[ = \frac{1}{2A} \frac{2\Delta - d}{\Delta} \frac{\Gamma(\Delta + 1)}{\pi^{\frac{d}{2}}\Gamma(\Delta - \frac{d}{2})} \int d^d x d^d x' \frac{\phi_0(\vec{x})\phi_0(\vec{x}')}{|\vec{x}_1 - \vec{x}_2|^2\Delta}, \]  

(A.12)
we can compute the two-point function

\[ \langle O(x_1)O(x_2) \rangle = \frac{\delta^2 S^{(2)}}{\delta \phi_0(x_1)\delta \phi_0(x_2)} = \frac{1}{A} \frac{2\Delta - d}{\Delta} \frac{\Gamma(\Delta + 1)}{\pi^{\frac{d}{2}} \Gamma(\Delta - \frac{d}{2}) |\vec{x}_1 - \vec{x}_2|^{2\Delta}}, \]  

in agreement with the “corrected” result found in [36]. We note in particular that the infamous factor of \((2\Delta - d)/\Delta\) which was very carefully obtained in [36] appears here very naturally. Of course, we are free to adjust the normalization coefficient in (A.11) so that the corresponding operators are normalized.

In the text, we are also interested in the three-point function computed from a cubic term in the supergravity action of the form

\[ L^{(3)} = \sum_{I_1I_2I_3} G_{I_1I_2I_3} \phi^{I_1} \phi^{I_2} \phi^{I_3}. \]  

The computation of the three-point function from this vertex can be found in [36]. Including the appropriate combinatorial factor, we find

\[ \langle O^{I_1}(x_1)O^{I_2}(x_2)O^{I_3}(x_3) \rangle = \frac{3\Gamma(\Delta_1 + \Delta_2 - \Delta_3)\Gamma(\Delta_2 + \Delta_3 - \Delta_1)\Gamma(\Delta_3 + \Delta_1 - \Delta_2)}{A_{I_1}A_{I_2}A_{I_3} \pi^d \Gamma(\Delta_1 - \frac{d}{2}) \Gamma(\Delta_2 - \frac{d}{2}) \Gamma(\Delta_3 - \frac{d}{2})} \cdot \frac{G_{I_1I_2I_3}}{|\vec{x}_1 - \vec{x}_2|^{\Delta_1 + \Delta_2 - \Delta_3} |\vec{x}_2 - \vec{x}_3|^{\Delta_2 + \Delta_3 - \Delta_1} |\vec{x}_3 - \vec{x}_1|^{\Delta_3 + \Delta_1 - \Delta_2}}. \]

**B. The Quadratic Action for the \( s^I \) Modes**

In the text we need a Lagrangian for the modes \( s^I \) up to cubic order. In order to obtain the cubic \( s-s-s \) vertex from the quadratic equation of motion (3.31), we needed to know the normalization of the quadratic \( s^I \) action. In section 3, we were able to obtain this normalization by comparison of the equations of motion (3.20) and (3.27), used to extract the \( s-s-\phi \) vertex. In [24, 39] the computation of the quadratic action for the \( s^I \) modes on AdS\(_5 \times S^5\) was performed ([39] actually considers the quadratic action for every mode) using two different methods.

We can first attempt to use the procedure described in [24]. Namely, we expand the Lagrangian (3.1) to quadratic order in all fields which depend on \( s^I \), as determined by (3.18). We keep all terms in the expansion of \( \sqrt{-G} R \), including total derivatives, and apply the equations of motion whenever possible. To enforce the constraint (3.10), we directly substitute the solution for \( h^I_{(\mu\nu)} \) given by (3.16). Via this method, we also succeeded in recovering
equation (3.22) of [24] when expanding around the AdS$_5 \times S^5$ background. In the present case, we find the Lagrangian quadratic in the modes $s^I$

\[ L[s^I] = -\frac{2N^3z(k)}{\pi^5} \left( \frac{9(11k^2 + 21k - 63)}{896k^2} (\nabla_\mu s^I)^2 + \frac{9}{224}(k - 3)(11k + 3)(s^I)^2 \right. 
+ \left. \frac{9(2k + 6k - 1)}{512k^2(2k + 1)^2} (\nabla_\mu \nabla_\nu s^I)^2 \right. 
+ \left. \frac{9}{1024k^2(2k + 1)^2} (\nabla_\mu \nabla_\nu s^I)^2 \right), \tag{B.1} \]

where $z(k)$ is the value of the integral over spherical harmonics given in (D.6).

We wish to use the action (B.1) to compute the 2-point function of the CPOs. According to the prescription of [14, 15], we should compute the action as a function of boundary values. In order to do this, we must carefully separate the boundary terms from the bulk in the higher-derivative terms. For this, we compute

\[ (\nabla_\mu \nabla_\nu s^I)^2 = \nabla_\mu (\nabla^\nu s^I \nabla_\mu \nabla_\nu s^I) - 2(2k(k - 3) - 7)(\nabla_\mu s^I)^2 \]

\[ = \nabla_\mu (\nabla^\nu s^I \nabla_\mu \nabla_\nu s^I) - 2(2k(k - 3) - 7)(\nabla_\mu s^I)^2 \]

\[ + \frac{4}{7}(4k(k - 3)(6k(k - 3) - 35) + 147)(\nabla_\mu s^I)^2 \]

\[ - 32k^2(k - 3)^2(s^I)^2, \]

on shell. We use these to rewrite (B.1) as

\[ L[s^I] = -\frac{2N^3z(k)}{\pi^5} \left( \frac{9(k + 1)(6k^2 + 2k + 3)}{128k(2k + 1)^2} [(\nabla s^I)^2 + 4k(k - 3)(s^I)^2] \right. 
+ \left. \frac{27}{256k^2(2k + 1)} \nabla^\mu (\nabla^\nu s^I \nabla_\mu \nabla_\nu s^I) \right. 
+ \left. \frac{9}{1024k^2(2k + 1)^2} \nabla^\rho (\nabla^\mu \nabla^\nu s^I \nabla_\rho \nabla_\mu \nabla_\nu s^I) \right), \tag{B.3} \]

which demonstrates that the quadratic action vanishes on shell in the bulk, as expected.

In order to finish the computation, we need to collect the surface terms in (B.3) and compute them as a function of boundary values [14, 15, 36] (see (A.9))

\[ s^I(z, \vec{x}) = \frac{\Gamma(2k)}{A\pi^3 \Gamma(2k - 3)} \int d\vec{x}' \left( \frac{z}{z^2 + |\vec{x} - \vec{x}'|} \right)^{2k} s_0^I(\vec{x}'). \tag{B.4} \]

For this it is convenient to use a prescription that was found useful in [30]. We note that the leading singularities in $1/|\vec{x} - \vec{x}'|$ arise from terms which do not have derivatives with
respect to the boundary acting on \( s^I \). Furthermore, terms with \( z \)-derivatives acting on the denominator of the boundary-to-bulk Green’s function are also subleading. As only a conformal structure is defined on the boundary of AdS, the induced metric on the boundary is defined by rescaling by a power of \( z \to 0 \). After this rescaling, these subleading terms will vanish as the boundary is approached. Therefore, we only need to compute the surface terms which involve

\[
\partial_z s^I \sim 2k s^I / z \\
\partial_z^2 s^I \sim 2k(2k - 1) s^I / z^2.
\]

(B.5)

We use \( \sim \) to indicate that terms which vanish in the limit that \( z \to 0 \) have been dropped, but the numerical coefficients are precise.

We find that

\[
\nabla^\mu (\nabla^\nu s^I \nabla_\mu \nabla_\nu s^I) \sim 4k^2 \nabla^\mu (s^I \nabla_\mu s^I)
\]

\[
\nabla^\rho (\nabla^\mu s^I \nabla_\rho \nabla_\mu \nabla_\nu s^I) \sim 8k^2 (2k^2 + 3) \nabla^\mu (s^I \nabla_\mu s^I),
\]

(B.6)

so that

\[
L[s^I] = -\frac{9(2k + 3) z(k)}{128 k} \frac{2N^3}{\pi^5} \nabla^\mu (s^I \nabla_\mu s^I).
\]

(B.7)

This is the same result that we would have obtained from a canonical lagrangian (with no higher derivatives)

\[
L[s^I] = -\frac{A_I}{2} [(\nabla s^I)^2 + 4k(k - 3)(s^I)^2],
\]

(B.8)

\[
A_I = \frac{9(2k + 3) z(k)}{128 k} \frac{2N^3}{\pi^5}.
\]

Unfortunately, this normalization is inconsistent with the value (3.28) obtained from the equations of motion of the theory at second order. In fact, it is obvious that the \( k \)-dependence of this result is completely incompatible with the totally symmetric form of the \( s-s-s \) vertex we obtain from (3.31).

This procedure is also inconsistent in the case of AdS\(_5 \times S^5\). While we reproduce the result (3.22) (the higher derivative Lagrangian) of [24], when we evaluate it as a function of boundary values using the appropriate versions of the formulæ (B.2) and (B.6), we recover

\[
A_I^{(AdS_5)} = 32k(k + 2) z(k),
\]

(B.9)

which differs by a factor of \((k - 1)/(k + 1)\) from the result quoted as eq. (3.23) of [24]. Nevertheless, the \( k \)-dependence of the result reported there is certainly correct, as it led to the cyclic symmetry of the 3-point vertex there.
We conclude that this method of evaluating the quadratic action is flawed.

The authors of [39] note that it is conventional to supplement the Einstein-Hilbert action with boundary terms involving the second fundamental form of the metric $G$ as well as the metric induced on the boundary [40, 41, 15, 42, 43]. In our case, the contribution of the standard extrinsic curvature term can be easily computed and its addition to the action above does not result in a correct normalization.

The motivation of [41] (see an extended discussion in [44]) to add the extrinsic curvature term was to remove terms in the Einstein-Hilbert action that were linear in second derivatives. As mentioned in [42, 43], this occurs for the physical graviton because certain terms in the Einstein-Hilbert action vanish if an appropriate gauge choice is made. Here we are dealing with the action for scalars that arises from the supergravity action and the terms in question cannot be made to vanish. Therefore adding the extrinsic curvature term does not cancel all of the boundary terms in the action for the scalar modes.

Another motivation for adding boundary terms will arise from supersymmetry. The supergravity action is supersymmetric up to total derivative terms, so we should expect that additional boundary terms must be added via the Noether procedure to restore supersymmetry on a space with boundary$^3$. The process of finding all such terms seems to be a much more difficult procedure than the method we use in the text, so we will not attempt to do that here.

Despite these remarks, the authors of [39] seem to obtain the correct normalization for the $s^I$ action in the case of $\text{AdS}_5 \times S^5$. Applying what we understand their methods to be did not result in the correct normalization for $\text{AdS}_7 \times S^4$.

C. Quadratic Corrections to the Equations of Motion

We must compute quadratic corrections $Q^I_1$ to (3.8), (3.10), and (3.12),

$$\frac{-1}{8} \left[ \left( \nabla^\mu \nabla_\mu + \frac{1}{10} \nabla^\gamma \nabla_\gamma - 72 \right) h_2^I + \nabla^\gamma \nabla_\gamma h^I - 192 \sqrt{2} \nabla^\gamma \nabla_\gamma b^I \right] Y^I + Q^I_1 Y^I = 0, \quad (C.1)$$

$$\left[ -\frac{1}{2} \left( h^I - \frac{9}{10} h_2^I \right) + Q_2^I \right] \nabla_{(\alpha} \nabla_{\beta)} Y^I = 0. \quad (C.2)$$

$$-6 b^I \nabla_\alpha \nabla^\gamma \nabla_\gamma Y^I + \left[ -6 \left( \nabla^\mu \nabla_\mu b^I - \frac{1}{2\sqrt{2}} h^I - \frac{3\sqrt{2}}{5} h_2^I \right) + Q_3^I \right] \nabla_\alpha Y^I = 0. \quad (C.3)$$

$^3$We thank Jacques Distler for reminding us of this.
We then find that the equation of motion for the \( s^I \) takes the form

\[
(\nabla^\mu \nabla_\mu - 4k(k-3)) s^I = \frac{8k}{2k+3} \left( Q_1^I + (k+3)(k+4)Q_2^I + \frac{2\sqrt{2}(k+3)}{3}Q_3^I \right),
\]

(C.4)

We also compute quadratic equations to the field equation (3.9) for the \( \phi^I \) modes

\[
\left[ -\frac{1}{2} (\nabla^\mu \nabla_\mu + \nabla^\gamma \nabla_\gamma - 8) \phi^I + Q_\phi^I \right] Y_{(\alpha \beta)}^I = 0,
\]

(C.5)

The terms \( Q_1^I \) and \( Q_2^I \) are determined by computing the quadratic corrections to the \( \alpha \beta \) components of (3.3). For the \( ss \) terms, using (3.17), we find that

\[
\delta^2 R_{\alpha \beta|ss} = Z_{\alpha \beta} + g_{\alpha \beta} Y,
\]

\[
\delta^2 \left[ \frac{1}{6} \left( F_{\alpha m_1 \ldots m_3} F_{\beta m_1 \ldots m_3} - \frac{1}{12} g_{\alpha \beta} F^2 \right) \right]_{ss} = \zeta_{\alpha \beta} + g_{\alpha \beta} \psi,
\]

where

\[
Z_{\alpha \beta} = \frac{1}{4} \left[ (7U_3 + 2V_3) (\nabla_\alpha s_2 \nabla_\beta s_3 + 2s_2 \nabla_\alpha \nabla_\beta s_3) + 2V_2 (7U_3 + 2V_3) \nabla_\alpha s_2 \nabla_\beta s_3
+ W_2 W_3 (\nabla^\mu \nabla^\nu) \nabla_\alpha s_2 \nabla_{(\mu \nabla_\nu) \nabla_\beta s_3} + 2\nabla^\mu \nabla^\nu s_2 \nabla_{(\mu \nabla_\nu) \nabla_\alpha \nabla_\beta s_3} \right],
\]

(C.7)

\[
Y = \frac{1}{2} \left[ U_2 V_3 \nabla^\mu (s_2 \nabla_\mu s_3) + W_2 V_3 \nabla^\mu (\nabla_{(\mu \nabla_\nu) s_2} \nabla_\nu s_3) + V_2 V_3 \nabla^\gamma (s_2 \nabla_\gamma s_3)
- \frac{V_2}{2} (7U_3 + 2V_3) (\nabla^\mu s_2 \nabla_\mu s_3 + \nabla^\gamma s_2 \nabla_\gamma s_3) \right],
\]

and

\[
\zeta_{\alpha \beta} = -36X_2 X_3 \nabla^\mu \nabla_\alpha s_2 \nabla_\mu \nabla_\beta s_3,
\]

\[
\psi = 24X_2 X_3 (\nabla^\gamma \nabla_\gamma s_2 \nabla^\delta \nabla_\delta s_3 + \nabla^\mu \nabla^\gamma s_2 \nabla_\mu \nabla_\gamma s_3)
+ 72\sqrt{2} V_2 X_2 s_2 \nabla^\gamma \nabla_\gamma s_3 + 72V_2 V_3 s_3 s_2.
\]

(C.8)

Here \( U_2 = U(k_2) \), etc., are the coefficients defined in (3.18), while \( s_2 = s^I_{ss} \), etc.

The term \( Q_3^I \) is obtained from the quadratic terms in the Maxwell equation. For the \( ss \) terms we compute

\[
\delta^2 (\nabla^m F_{\alpha m_1 \alpha_2 \alpha_3})_{ss} = \epsilon^\delta_{\alpha_1 \alpha_2 \alpha_3} \left[ W_2 \left( 6X_3 - \frac{3\sqrt{2}}{2} W_3 \right) \nabla^\mu \nabla^\nu s_2 \nabla_{(\mu \nabla_\nu) \nabla_\delta s_3}
+ 3 \left( -5U_2 + 2V_3 + 12W_2 \left( \frac{m_2^4}{7} - 1 \right) \right) X_3 \nabla^\mu s_2 \nabla_\mu \nabla_\delta s_3
+ \left( 6(U_2 m_2^2 - V_2 f_3) X_3 - 3\sqrt{2}(7U_2 U_3 - 10V_2 V_3)
- 18X_2 f_2 V_3 \right) s_2 \nabla_\delta s_3 \right].
\]

(C.9)
Computation of the projections onto spherical harmonics is facilitated by the integral identities in appendix D. Projection of \( \frac{1}{4}(Z - \zeta)_{\gamma} + Y - \psi \) onto \( Y^I \) yields the correction to (3.8),

\[
Q_1^I = \frac{1}{4z_1} \left[ \frac{1}{4} \left( W_2 W_3 b_{123} + \frac{8(W_2 V_3 + V_2 W_3 - W_3 V_2)(f_2 + f_3)}{2} a_{123} \right) \nabla(\mu \nabla^\nu) s_2 \nabla(\mu \nabla^\nu) s_3 \\
+ \left( -60X_2 X_3 b_{123} + \frac{7(U_2 V_3 + V_2 U_3) + 6(W_2(m_2^2 - 7)V_3 + V_2 W_3)(m_2^2 - 7)}{7} a_{123} \right) \nabla^\mu s_2 \nabla_\mu s_3 \\
+ \left( -96(3V_2 V_3 + X_2 X_3 f_2 f_3) + (U_2 V_3 m_3^2 + V_2 m_2^2 U_3) \\
+ 144\sqrt{2}(V_2 X_3 f_3 + X_2 f_2 V_3) - \frac{(7U_2 U_3 + 6V_2 V_3)(f_2 + f_3)}{4} \right) a_{123} \\
+ \left( 7U_2 U_3 + 10V_2 V_3 \right) b_{123} \right] s_2 s_3 \langle C^{I_1} C^{I_2} C^{I_3} \rangle. 
\]

(C.10)

Projecting \( (Z - \zeta)_{(\alpha\beta)} \) onto \( \nabla^{(\alpha \nabla^\beta)} Y^I \) yields the correction to (3.10),

\[
Q_2^I = \frac{1}{4q(k_1)f(k_1)z(k_1)} \left[ W_2 W_3 (c_{123} + d_{213} + d_{321}) \nabla(\mu \nabla^\nu) s_2 \nabla(\mu \nabla^\nu) s_3 \\
+ 144X_2 X_3 c_{123} \nabla^\mu s_2 \nabla_\mu s_3 \\
+ (7U_2 U_3 + 2V_2 V_3)(c_{123} + d_{213} + d_{321}) s_2 s_3 \right] \langle C^{I_1} C^{I_2} C^{I_3} \rangle. 
\]

(C.11)

Projecting the terms in (C.9) onto \( \epsilon^\delta_{\alpha_1\alpha_2\alpha_3} \nabla_\delta Y^I \), we find

\[
Q_3^I = \frac{b_{213}}{f_1 z_1} \left[ \left( 3(W_2 X_3 + X_2 W_3) - \frac{3\sqrt{2}}{2} W_2 W_3 \right) \nabla(\mu \nabla^\nu) s_2 \nabla(\mu \nabla^\nu) s_3 \\
+ \left( -15(U_2 X_3 + X_2 U_3) + 6(V_2 X_3 + X_2 V_3) \right) \frac{36(W_2(m_2^2 - 7)X_3 + X_2 W_3(m_2^2 - 7))}{7} \nabla^\mu s_2 \nabla_\mu s_3 \\
+ \left( -\frac{3\sqrt{2}}{2}(7U_2 U_3 - 10V_2 V_3) + 3(U_2 X_3 + X_2 U_3) \\
- 12(X_2 f_2 V_3 + V_2 X_3 f_3) \right) s_2 s_3 \right] \langle C^{I_1} C^{I_2} C^{I_3} \rangle. 
\]

(C.12)

Projecting \( (Z - \zeta)_{(\alpha\beta)} \) onto the symmetric tensor spherical harmonics \( Y^I_{(\alpha\beta)} \) yields the correction to (3.9),

\[
Q_4^I = \frac{b_{123}}{4z_1} \left[ \left( -W_2 W_3 \nabla(\mu \nabla^\nu) s_2 \nabla(\mu \nabla^\nu) s_3 + 144X_2 X_3 \nabla^\mu s_2 \nabla_\mu s_3 \\
- (7U_2 U_3 + 2V_2 V_3) s_2 s_3 \right) \langle T^{I_1} C^{I_2} C^{I_3} \rangle. 
\]

(C.13)

For the \( s_\phi \) terms in the \( \alpha\beta \) components of (3.3), we obtain

\[
\delta^2 R_{\alpha\beta}|_{s_\phi} = Z^0_{\alpha\beta} + g_{\alpha\beta} Y^\phi; \\
\delta^2 \left[ \frac{1}{6} \left( F_{m_1 \ldots m_3} F_{\alpha \beta}^{m_1 \ldots m_3} - \frac{1}{12} g_{\alpha\beta} F^2 \right) \right]_{s_\phi} = \zeta^0_{\alpha\beta} + g_{\alpha\beta} \psi^\phi, 
\]

(C.14)
where

\[ Z^\phi_{\alpha\beta} = -\frac{5U}{4}\nabla^\mu h_{(\alpha\beta)} \nabla_\mu s + \frac{U}{2}\nabla^\mu \nabla_\mu h_{(\alpha\beta)} s + \frac{V}{2} (s \nabla^\gamma \nabla_\gamma h_{(\alpha\beta)} + \nabla^\gamma s \nabla_\gamma h_{(\alpha\beta)}) \]

\[ + \frac{W}{2} \nabla_\mu (\nabla_\mu h_{(\alpha\beta)} \nabla^\mu s) + \frac{7U}{4} (\nabla_\alpha h_{(\beta\gamma)} + \nabla_\beta h_{(\alpha\gamma)} - \nabla_\gamma h_{(\alpha\beta)}) \nabla^\gamma s \]

\[ - \frac{V}{2} (h_{(\alpha\gamma)} \nabla^\gamma \nabla_\beta s + h_{(\beta\gamma)} \nabla^\gamma \nabla_\alpha s) - 16Vh_{(\alpha\beta)} s, \]

\( Y^\phi = \frac{V}{2} h_{(\gamma\delta)} \nabla^\gamma \nabla^\delta s, \)

and

\[ \zeta^\phi_{\alpha\beta} = -24(\sqrt{2}X \nabla^\gamma \nabla_\gamma s + 2V s) h_{(\alpha\beta)}, \]

\[ \psi^\phi = 0. \]

The \( s\phi \) corrections to the Maxwell equations are

\[ \delta^2 (\nabla^m F_{m\alpha_1 \alpha_2 \alpha_3}) = \epsilon^\delta_{\alpha_1 \alpha_2 \alpha_3} \left[ (12\sqrt{2}V - 6X f) h_{(\gamma\delta)} \nabla^\gamma s \right] \]

\[ + \epsilon^\nu_{\alpha_1 \alpha_2 \alpha_3} \left[ 18X \nabla^\mu h_{(\alpha_1 \delta)} \nabla_\mu \nabla^\nu s + (18\sqrt{2}V - 18X f) s \nabla_\nu h_{(\alpha_1 \delta)} \right] \]

\[ + 9\sqrt{2}V h_{(\alpha_1 \delta)} \nabla^\nu s \].

Projecting these expressions onto the appropriate spherical harmonics, we find

\[ Q_{1}^{\phi I_1} = -\frac{V_3 h_{213}}{4z_1} \phi_2 s_3 \left( C^{I_1} T^{I_2} C^{I_3} \right), \]

\[ Q_{2}^{\phi I_1} = \frac{h_{213}}{q_{1 f_1} z_1} \left[ -\frac{W_3}{2} \nabla^\mu \nabla_\mu s_3 + \left( \frac{5U_3}{4} - 3W_3 \left( \frac{m_3^2}{7} - 1 \right) \right) \nabla^\mu \phi_2 \nabla_\mu s_3 \right] \]

\[ + \left( 24\sqrt{2}X_3 f_3 + \left( 7 + \frac{7}{8} f_1 + \frac{17}{8} f_2 - \frac{7}{8} f_3 \right) U_3 \right) \phi_2 s_3 \]

\[ + \left( -34 + \frac{3}{4} f_2 - \frac{3}{4} f_3 \right) V_3 \phi_2 s_3 \left( C^{I_1} T^{I_2} C^{I_3} \right), \]

\[ Q_{3}^{\phi I_1} = \frac{3h_{213}}{f_1 z_1} \left[ -2X_3 \nabla^\mu \phi_2 \nabla_\mu s_3 + (3\sqrt{2}V_3 - 2X_3 f_3) \phi_2 s_3 \right] \left( C^{I_1} T^{I_2} C^{I_3} \right). \]

D. Spherical Harmonics on \( S^4 \)

Here we collect some results about the spherical harmonics on \( S^4 \), following the appendix of [24]. We take the radius of the sphere to be \( 1/2 \), as in the text. We embed the \( S^4 \subset \mathbb{R}^5 \) and represent the spherical harmonics by

\[ Y^I = C^I_{i_1 \cdots i_k} x^{i_1} \cdots x^{i_k}, \]
where we normalize $C^{I_1\cdots I_k}_{i_1\cdots i_k}C^{I_2}_{i_1\cdots i_k} = \delta^{I_1I_2}$. For each $k$, there are

$$n(k) = (2k + 3)(k + 2)(k + 1)/6$$

(D.2)

harmonics and the eigenvalues of the spherical harmonics on a sphere of radius $1/2$ are given by

$$\nabla^\alpha \nabla_\alpha Y^I = -4k(k + 3)Y^I.$$  

(D.3)

Integrals over products of spherical harmonics are facilitated by the use of the formula

$$\int_{S^4} x^{i_1} \cdots x^{i_{2m}} = \frac{3\omega_4}{2^{3m+2}\Gamma(m + \frac{5}{2})}(\delta^{i_1i_2} \cdots \delta^{i_{2m-i}i_{2m}} + \text{perms.}),$$

(D.4)

where $\omega_4 = 8\pi^2/3$ and the permutations are computed in the following manner: fix the first index, then perform $(2m - 1)$ cyclic permutations, then, for each of the terms thus generated, fix the first three indices and perform $(2m - 3)$ cyclic permutations, etc. We find the integral formulæ

$$\int_{S^4} Y^{I_1} Y^{I_2} = z(k)\delta^{I_1I_2},$$

$$\int_{S^4} \nabla^\alpha Y^{I_1} \nabla_\alpha Y^{I_2} = f(k)z(k)\delta^{I_1I_2},$$

$$\int_{S^4} \nabla^{(\alpha} \nabla^{\beta)} Y^{I_1} \nabla_{(\alpha} \nabla_{\beta)} Y^{I_2} = q(k)f(k)z(k)\delta^{I_1I_2},$$

where

$$z(k) = \frac{3\omega_4\Gamma(k + 1)}{2^{3k+2}\Gamma(k + \frac{5}{2})},$$

$$f(k) = 4k(k + 3),$$

$$q(k) = 3(k - 1)(k + 4).$$

(D.6)

For any given $k$, the sum over the $n(k)$ degenerate eigenfunctions, labeled by $A$, is a constant

$$\sum_A [Y^I_A(\theta)]^2 = \frac{(2k + 3)\Gamma(k + 3)}{2^{3(k+1)}\Gamma(k + \frac{5}{2})}.$$  

(D.7)

Further, we find

$$\int_{S^4} Y^{I_1} Y^{I_2} Y^{I_3} = a(k_1, k_2, k_3)\langle C^{I_1} C^{I_2} C^{I_3} \rangle,$$

$$a(k_1, k_2, k_3) = \frac{3\omega_4}{2^{3\Sigma/2+2}\Gamma(\Sigma + 5/2)}\frac{\Gamma(k_1 + 1)\Gamma(k_2 + 1)\Gamma(k_3 + 1)}{\Gamma(\alpha_1 + 1)\Gamma(\alpha_2 + 1)\Gamma(\alpha_3 + 1)},$$

(D.8)
where \( \langle C^I C^I C^I \rangle, \alpha_i, \) and \( \Sigma \) are defined as in (2.4).

Rank two, symmetric traceless tensor spherical harmonics are defined via

\[
Y^I_{(\alpha\beta)} = e_{\alpha a} e_{\beta b} T^I_{a_1 \cdots a_k} x^{i_1} \cdots x^{i_k},
\]

(D.12)

and satisfy

\[
\nabla^\gamma \nabla_\gamma Y^I_{(\alpha\beta)} = -4(k(k + 3) - 2) Y^I_{(\alpha\beta)}.
\]

(D.13)

We have the integral formulæ

\[
\int_{S^4} Y^I_{(\alpha\beta)} Y^J_{(\gamma\delta)} g^{\alpha\gamma} g^{\beta\delta} = z(k) \langle T^I T^J \rangle,
\]

(D.14)

\[
\langle T^I T^J \rangle = T^I_{a_1 \cdots a_k} T^J_{a_1 \cdots a_k},
\]

\[
\int_{S^4} Y^I_{(\alpha\beta)} Y^J_{(\gamma\delta)} g^{\alpha\gamma} g^{\beta\delta} Y^I = a(k_1, k_2, k_3) \langle T^I_1 T^I_2 C^I_3 \rangle,
\]

(D.15)

\[
\langle T^I_1 T^I_2 C^I_3 \rangle = T^I_{a_1 \cdots a_2} T^I_{a_3 \cdots a_4} C^I_{a_1 \cdots a_2 + a_3 \cdots a_4 - 1} C^I_{a_1 \cdots a_2 + a_3 \cdots a_4 - 1},
\]

\[
\int_{S^4} Y^I_{(\alpha\beta)} \nabla^\alpha Y^J_{(\gamma\delta)} \nabla^\beta Y^I = h(k_1, k_2, k_3) \langle T^I C^I C^I \rangle,
\]

(D.16)

\[
\langle T^I C^I C^I \rangle = T^I_{a_1 \cdots a_2} C^I_{a_3 \cdots a_4} C^I_{a_5 \cdots a_6} j_{a_1 \cdots a_2} j_{a_3 \cdots a_4} j_{a_5 \cdots a_6},
\]

\[
h(k_1, k_2, k_3) = (\Sigma + 3) a(k_1, k_2, k_3),
\]

30
\[ \int_{S^4} Y_{(\alpha \beta)} I^I \nabla^\alpha Y^{I_2} \nabla^\gamma Y^{I_3} = \frac{1}{2} (f(k_2) + f(k_3) - f(k_1) - 16) \cdot h(k_1, k_2, k_3) \langle T^I C^{I_2} C^{I_3} \rangle, \]

\[ \int_{S^4} Y_{(\alpha \beta)} I^I \nabla^\alpha Y^{I_2} \nabla^\gamma Y^{I_3} = \frac{1}{2} \left( \frac{1}{2} f(k_2) + f(k_3) - f(k_1) - 16 \right) \cdot h(k_1, k_2, k_3) \langle T^I C^{I_2} C^{I_3} \rangle, \]  

\[ \int_{S^4} \nabla^\gamma Y_{(\alpha \beta)} I^I \nabla^\alpha Y^{I_2} \nabla^\gamma Y^{I_3} = \frac{1}{2} (f(k_2) - f(k_1) - f(k_3) - 24) \cdot h(k_1, k_2, k_3) \langle T^I C^{I_2} C^{I_3} \rangle, \]

\[ \int_{S^4} \nabla^\gamma Y_{(\alpha \beta)} I^I \nabla^\gamma Y^{I_2} \nabla^\beta Y^{I_3} = \frac{1}{2} (f(k_1) + f(k_2) - f(k_3) - 8) \cdot h(k_1, k_2, k_3) \langle T^I C^{I_2} C^{I_3} \rangle. \]  

\[ \text{(D.17)} \]

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