Approximation characteristic between LEMP on earth surface and channel-base current

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Abstract. Based on the dipole technique, the exact expressions and approximate expressions for LEMP on earth surface are derived from the TL model and specified channel-base current described by a pulse function. According to the comparisons between the exact results and approximate results, the waveforms of LEMP and its derivative are similar to the lightning channel-base current and its derivative, respectively. In near-field, the electric field and its derivative waveforms of the approximate results are essentially coincident with the corresponding waveforms of the exact results within 100 m. The magnetic field waveforms of approximate results are essentially coincident with the waveforms of the exact results within 500 m, and the magnetic field derivative waveforms of the approximate results are essentially coincident with the waveforms of the exact results within 200 m. In far-field, the waveforms of the approximate results are essentially coincident with the waveforms of the exact results in the rising edge before the initial peak value. At the distance beyond 100 km, the waveforms of the radiation field component and the scaled channel-base current appear identical. The difference between the derivative waveforms of the LEMP field and the scaled channel-base current in the first three microseconds is slight and can be ignored.

1. Introduction

The lightning electromagnetic pulse (LEMP) generated by the lightning return stroke current has caused wide concern in recent years due to the proliferation of sensitive loads [1-5]. A series of experiments were performed to measure the electric field, magnetic field, and time derivative of the electromagnetic field [6-16]. And lots of researches on the theoretical modeling and calculation methods of LEMP were also carried out by different researchers [17, 18]. Rakov et al. defined four classes of lightning return stroke models [17], including the gas dynamic models, the electromagnetic models, the distributed-circuit models, and the engineering models. Among them, the engineering models, which can be defined as equations relating the longitudinal current along the lightning channel at any height and any time to the current at the channel origin (e.g. at ground level), is widely used. The engineering models such as the Bruce-Golde model (BG) [19], the transmission line model (TL) [20], the traveling current source model (TCS) [21], the modified transmission-line model with linear current decay with height (MTLL) [22], the modified transmission line model with exponential current decay with height (MTLE) [23], and the Diendorfer-Uman model (DU) [24] were proposed one after the other to describe the lightning return stroke process. In addition to the propagation direction and production mode of the channel current, different engineering models are mainly distinguished by the

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propagation characteristic (e.g. velocity, and attenuation law) of the return stroke current along the channel. Based on these engineering models, several analytic methods, such as variable separation method, conformal mapping method, the monopole and the dipole techniques, were adopted to calculate the electromagnetic field. Rubinstein presented a comparison between the two different techniques (monopole and dipole) for calculating the electric and magnetic fields from lightning, and showed that the monopole approach is applicable only to upward-traveling current waves and hence is not particularly useful in the realistic modeling of lightning return strokes [25, 26]. Cooray, who used the dipole technique, derived analytical expressions for LEMP fields from Maxwell’s equations [27]. Uman, comparing the calculation results with experimental results, showed that the dipole technique has a high computing accuracy [28]. Henceforth, the dipole technique is widely used in the antenna theory and the calculation of electromagnetic fields.

With the increasing demand by customers for good quality in the electronic system, security testing is a necessary procedure for the sensitive electronic equipments. In order to meet the test requirements of LEMP environment effects for the sensitive loads, LEMP simulation technology is suggested to resolve this key issue. The above theoretical calculation methods provide the possibility for LEMP simulation. Moreover, Uman et al. demonstrated the approximation characteristics between the far-field of LEMP and channel-base current, and showed that the far-field within a certain distance can be approximated as a function of channel-base current [29]. Similar fine structure was also observed by Jerauld et al. in $dE/dt$ and $di/dt$ waveforms produced by an unusual rocket-triggered lightning stroke, which involved a downward dart-stepped leader and a pronounced upward connecting leader [30]. These above researches connect LEMP simulation with the channel-base current of lightning return stroke closely. In order to establish the further relationship between LEMP fields and the channel-base current in theory, the analytical expressions of electric and magnetic fields based on the TL model are derived according to the dipole method. The approximation characteristics between LEMP fields and the channel-base current, and the approximation characteristics between the derivatives of LEMP fields and the derivative of channel-base current in near field and far field are analyzed, respectively. And the applicable ranges of the approximate expressions are also discussed.

2. Exact expressions of LEMP on earth surface
The return-stroke channel can be regarded as a vertical perfectly antenna, as shown in figure 1. The transient current propagates along the vertical perfectly conductor (the lightning channel) above a perfectly conducting plane (the ground).

According to the dipole method, the LEMP field due to an upward-moving return stroke for the case of a field point (in a cylindrical coordinate system) can be described as follows [28]:

\[
E_z = \frac{1}{4\pi \varepsilon_0} \int_{-h}^{h} \left[ \frac{3r(z - z')}{R^3} \int_{-\infty}^{t'} i(z', t - R/c) \, dt + \frac{3r(z - z')}{cR^4} i(z', t - R/c) + \frac{r(z - z')}{c^2 R^3} \frac{\partial i(z', t - R/c)}{\partial t} \right] \, dz' \quad (1)
\]

\[
E_z = \frac{1}{4\pi \varepsilon_0} \int_{-h}^{h} \left[ \frac{2(z - z')}{c R^4} i(z', t - R/c) + \frac{r(z - z')}{c^2 R^3} \frac{\partial i(z', t - R/c)}{\partial t} \right] \, dz' \quad (2)
\]

\[
H_\theta = \frac{1}{4\pi} \int_{-h}^{h} \left[ \frac{r}{R^2} i(z', t - R/c) + \frac{r}{c R^2} \frac{\partial i(z', t - R/c)}{\partial t} \right] \, dz' \quad (3)
\]

Where $R = \sqrt{(z - z')^2 + r^2}$ is the distance between the return-stroke current waveform and the field point, $L$ is the length of the return-stroke channel, $r$ is the horizontal distance between the return-stroke channel and the observation point, $z$ is the height of the field point, $z'$ is the position of the waveform, $h$ is the height of $z'$, $c$ is the speed of light, $i$ is the return-stroke current, $E_z$ is the radial electric field, $E_z$ is the vertical electric field, $H_\theta$ is the poloidal magnetic field. In equations (1) and (2), the first
term represents the electrostatic field, the second term is the induction electric field (i.e. intermediate-field), and the third term is the radiation electric field (i.e. far-field). In equation (3), the first term represents the induction magnetic field and the second term is the radiation magnetic field.

**Figure 1.** The antenna model of lightning return stroke.

In this paper, the emphasis is placed on the electromagnetic fields on earth surface (i.e. the height of the field point $z=0$). Thus, $E_z=0$, $R = \sqrt{h^2 + r^2}$.

In the engineering models, the relationship between the channel current $i(z',t)$ and channel-base current $i(0,t)$ can be expressed by the following equation except DU model [17]:

$$i(z',t) = u(t - z'/v_f)P(z')i(0, t - z'/v)$$  \hspace{1cm} (4)

Where $u(\xi)$ is the Heaviside function equal to unity for $\xi > 0$ and zero otherwise, $P(z')$ is the height-dependent current attenuation factor, $v_f$ is the upward-propagating front speed, and $v$ is the current-wave propagation speed. Different $P(z')$ and $v$ corresponds to different engineering model.

Here, the TL model is adopted to calculate the lightning electromagnetic field. Thus, $P(z') = 1$, $v = v_f$. And the return stroke current can be written as:

$$i(z', t) = u(t - z'/v_f) - i(0, t - z'/v)$$  \hspace{1cm} (5)

Moreover, the pulse function is chosen to describe the channel-base current, expressed as follows:

$$i(0, t) = \frac{I_0}{\eta} \left[1 - \exp(-t/\tau_1)\right] \exp(-t/\tau_2)$$  \hspace{1cm} (6)

Where $\eta = [n\tau_z/(\tau_1 + n\tau_z)]^n [\tau_1/(\tau_1 + n\tau_z)]^{1/\eta}$. In this paper, we set $n=2$.

Thus, each component of equations (2) and (3), and the total electric and magnetic field can be described as follows:

**Electrostatic field component:**

$$E_z(\text{electrostatic}) = \frac{i(z', t)}{2\pi\varepsilon_0} \left\{ \frac{-th + \frac{2h^2}{v} + \frac{r^2}{v}}{(h^2 + r^2)^{3/2}} - \frac{1}{rv} - \frac{1}{2cr} \left[\tan^{-1} \left( \frac{h}{r} \right) - \frac{3hr}{h^2 + r^2} \right] \right\}$$  \hspace{1cm} (7)

**Induction electric field component:**

$$E_z(\text{induction}) = -\frac{i(z', t)}{4\pi\varepsilon_0 cr} \left[\tan^{-1} \left( \frac{h}{r} \right) - \frac{3hr}{h^2 + r^2} \right]$$  \hspace{1cm} (8)

**Radiation electric field component:**
\( E_z(\text{radiation}) = -\frac{i(z',t)}{2\pi\varepsilon_0} \frac{r^2}{\left( h^2 + r^2 \right)^{3/2}} \) 

Induction magnetic field component:

\( H_\phi(\text{induction}) = \frac{i(z',t)}{2\pi} \frac{h}{r \left( h^2 + r^2 \right)^{1/2}} \) 

Radiation magnetic field component:

\( H_\phi(\text{radiation}) = \frac{i(z',t)}{2\pi} \frac{r}{\left( h^2 + r^2 \right)^{1/2}} \) 

Total electric field on earth surface:

\( \vec{E}(r,0,t) = \frac{i(z',t)}{2\pi\varepsilon_0} \left[ -\frac{2h^2 + r^2}{v \left( h^2 + r^2 \right)^{3/2}} - \frac{1}{rv} \frac{r^2}{c^2 \left( h^2 + r^2 \right)^{3/2}} \left( \frac{1}{v} + \frac{h}{c\sqrt{h^2 + r^2}} \right) \right] \vec{\alpha}_z \) 

Total magnetic field on earth surface:

\( \vec{H}(r,0,t) = \frac{i(z',t)}{2\pi} \left[ \frac{h}{r \left( h^2 + r^2 \right)^{1/2}} + \frac{r}{v \left( h^2 + r^2 \right)^{1/2}} \right] \vec{\alpha}_\phi \) 

Where \( h = \beta - \sqrt{(\beta ct)^2 + r^2 (1 - \beta^2)} \) \( \frac{1}{1 - \beta^2} \), \( \beta = v/c \). 

3. Approximation characteristic of LEMP on earth surface and its derivative in near-field

3.1. Derivation of approximate expressions for LEMP in near-field

Let \( \partial f(z',t)/\partial t = i(z',t) \), according to the TL model:

\[ \int_{-\infty}^{t} i(z',\tau) d\tau = \int_{-\infty}^{t} i(0,\tau - z'v) d\tau = \int_{-\infty}^{t} i(0,\tau - z'v) d\tau = \int_{0}^{t-z'v} i(0,s) ds \]

\[ = f(0,t) \big|_{0}^{t-z'v} = f(0,t-z'v) - f(0,0) = F(t-z'v) \] 

Thus, the current integration in the electrostatic field component of equation (2) is:

\[ \int_{-\infty}^{t} i(z',\tau - R/c - z'v) d\tau = \int_{-\infty}^{t} i(0,\tau - R/c - z'v) d\tau = F(t-R/c - z'v) \]

For the near-field, we view \( r \ll H. 1/R \) attenuates to zero very rapidly with the increasing \( z' \). Thus, using the Taylor series expansion, \( F(t-R/c - z'v) \) can be written as:

\[ F(t-R/c - z'v) = F(t-r/c) - F'(t-r/c)[(t-r/c) - (t-R/c - z'v)] + o(c^{-2}) \]

\[ \approx F(t-r/c) - i(0,t-r/c)[(R-r)/c + z'v] \] 

where \( \partial F(t-r/c)/\partial t = i(0,t-r/c) \).

Thus, the electrostatic field component can be expressed approximately as:
The radiation electric field component is extremely small, of the order of \( c^{-2} \), and can be ignored. Thus, we obtain the first-level approximation of the electric field in near field as:

\[
E_z' (\text{electricstatic}) = \frac{1}{2\pi\varepsilon_0} \int_0^h \frac{2R^2 - 3r^2}{R^2} \, dz' \int_0^t i(z', \tau - R/c) \, d\tau
\]

\[
= \frac{1}{2\pi\varepsilon_0} \int_0^h \frac{2R^2 - 3r^2}{R^2} \, dz' i(0, t - r/c) \delta(z')
\]

\[
= \frac{F(t - r/c)}{2\pi\varepsilon_0} \left[ \frac{h}{R(h)^3} \right] - \frac{i(0, t - r/c)}{2\pi\varepsilon_0} \delta(z')
\]

\[
\times \left\{ \frac{1}{c} \left( \frac{rR}{R(h)^2} - 2 \tan^{-1} \left( \frac{r}{2R(h)^2} \right) \right) + \frac{1}{v} \left( \frac{r^2}{R(h)^2} - \frac{2}{R(h)} + \frac{1}{r} \right) \right\}
\]

(17)

The radiation electric field component is extremely small, of the order of \( c^{-2} \), and can be ignored. Thus, we obtain the first-level approximation of the electric field in near field as:

\[
E_z' \approx -\frac{F(t - r/c)}{2\pi\varepsilon_0} \left( \frac{h}{R(h)^3} - \frac{i(0, t - r/c)}{2\pi\varepsilon_0} \left[ \frac{rR}{R(h)^2} - \frac{2}{R(h)} + \frac{1}{r} \right] \right)
\]

(19)

Likewise, we can obtain the first-level approximation of the magnetic field in near field as:

\[
H' \approx H'^{(\text{induction})} = \frac{1}{2\pi} \int_0^h \frac{2R^2 - 3r^2}{R^4} \, dz' \int_0^t i(0, t - r/c) \delta(z')
\]

\[
= \frac{i(0, t - r/c)}{2\pi\varepsilon_0 c} \left[ \tan^{-1} \left( \frac{h}{2r} \right) - \frac{3h}{2r} \right]
\]

(18)

As \( r \ll H \) in near-field, \( R(h) \) approaches \( h \) gradually along with the current propagating upward in the channel. Ignore the \( R^{-1} \) term and \( R^{-3} \) term in equation (19), we can obtain the second-level approximation of the electromagnetic field in near field as:

\[
E_z'' \approx -\frac{1}{2\pi\varepsilon_0 vr} i(0, t - r/c)
\]

(21)

\[
H'' \approx \frac{1}{2\pi r} i(0, t - r/c)
\]

(22)

Performing the derivation on both sides of equations (21) and (22), we obtain:

\[
\frac{dE_z(r,t)}{dt} \approx \frac{dE''(r,t)}{dt} \approx \frac{dE_z''(r,t)}{dt} \approx -\frac{1}{2\pi\varepsilon_0 vr} \frac{di(0, t - r/c)}{dt}
\]

(23)

\[
\frac{dH_\phi(r,t)}{dt} \approx \frac{dH''(r,t)}{dt} \approx \frac{dH_z''(r,t)}{dt} \approx \frac{1}{2\pi r} \frac{di(0, t - r/c)}{dt}
\]

(24)

3.2. Comparisons of LEMP in near-field between the exact results and approximate results

By comparing the waveforms of the first-level approximation, the second-level approximation in near-field with the waveforms of the exact solution, the LEMP field waveforms are similar to the channel-base current waveform of lightning return stroke, only with a coefficient difference. In order to verify the reliability of the approximate expressions in near-field, the lightning channel-base current with a typical waveform 8/20μs is adopted to calculate the lightning electromagnetic fields. The height of the lightning channel is set to \( H=7.5 \) km, and the return stroke speed is \( v=1.3\times10^8 \) m s\(^{-1}\). The parameters of the pulse function are set as follows: \( I_0 = 30\) kA, \( \tau_1 = 4.0\times10^{-6} \) s, \( \tau_2 = 6.25\times10^{-6} \) s. Comparisons of
the electric and magnetic field waveforms from the approximate results, including the first-level approximation and the second-level approximation, with the exact results at different distances are given in figures 2 to 9. And comparisons of the electric and magnetic field derivative waveforms from the approximate results, including the first-level approximation and the second-level approximation, with the exact results at different distances are given in figures 10 to 16.

**Figure 2.** Comparison of the electric and magnetic field waveforms from the approximate results with the exact results at a distance of 5 m.

**Figure 3.** Comparison of the electric and magnetic field waveforms from the approximate results with the exact results at a distance of 8 m.

**Figure 4.** Comparison of the electric and magnetic field waveforms from the approximate results with the exact results at a distance of 10 m.
Figure 5. Comparison of the electric and magnetic field waveforms from the approximate results with the exact results at a distance of 20 m.

Figure 6. Comparison of the electric and magnetic field waveforms from the approximate results with the exact results at a distance of 50 m.

Figure 7. Comparison of the electric and magnetic field waveforms from the approximate results with the exact results at a distance of 100 m.

Figure 8. Comparison of the electric and magnetic field waveforms from the approximate results with the exact results at a distance of 200 m.
Figure 9. Comparison of the electric and magnetic field waveforms from the approximate results with the exact results at a distance of 500 m.

Figure 10. Comparison of the electric and magnetic field derivative waveforms from the approximate results with the exact results at a distance of 5 m.

Figure 11. Comparison of the electric and magnetic field derivative waveforms from the approximate results with the exact results at a distance of 10 m.

Figure 12. Comparison of the electric and magnetic field derivative waveforms from the approximate results with the exact results at a distance of 20 m.
Figure 13. Comparison of the electric and magnetic field derivative waveforms from the approximate results with the exact results at a distance of 50 m.

Figure 14. Comparison of the electric and magnetic field derivative waveforms from the approximate results with the exact results at a distance of 100 m.

Figure 15. Comparison of the electric and magnetic field derivative waveforms from the approximate results with the exact results at a distance of 200 m.

Figure 16. Comparison of the electric and magnetic field derivative waveforms from the approximate results with the exact results at a distance of 500 m.
As follows from figures 2 to 9, the LEMP waveforms of the approximate results are similar to the waveforms of the exact results. The deviations of the approximate results from the exact results increase with the distance \( r \) increases. The closer the distance is, the less the deviations are. The electric field waveforms of the approximate results are essentially coincident with the waveforms of the exact results within 100 m. When the distance exceeds 500 m, there is a great deal of difference between the electric field waveforms of the two approximate results and the exact results. By comparing the waveforms of electric and magnetic fields in figures 2 to 9, it can be found that the difference between the magnetic field and its two approximate results is smaller than the difference between the electric field and its two approximate results within near range. The magnetic field waveform of the two approximate results can not been distinguished from the exact results until the distance exceeds 500 m.

As follows from figures 10 to 16, the deviations of the approximate derivative waveforms from the exact derivative waveforms increase with the distance \( r \) increases. The closer the distance is, the less the deviations are. The electric field derivative waveforms of the approximate results are essentially coincident with the derivative waveforms of the exact results within 100 m. When the distance exceeds 500 m, there is a great deal of difference between the electric field derivative waveforms of the two approximate results and the exact results. By comparing the waveforms of electric and magnetic field derivatives in figures 2 to 9, it can be found that the difference between the magnetic field derivative and its two approximate results is smaller than the difference between the electric field derivative and its two approximate results within near range. The magnetic field derivative waveforms of the two approximate results can not been distinguished from the exact results until the distance exceeds 200 m.

### 4. Approximation characteristic of LEMP on earth surface and its derivative in far-field

#### 4.1. Derivation of approximate expressions for LEMP in far-field

For the far-field, namely \( R \approx r \) and \( r \gg L \), the radiation field is the dominant component of the electromagnetic field produced by lightning return strokes, and the electrostatic, and induction terms tend to 0. For the transmission-line-type models, we can simplify the expressions as following because of the continuity of the upward-moving front [28]:

\[
E_z \approx E_z(\text{radiation}) \approx -\frac{1}{2\pi\varepsilon_0 c^2 r} \int_0^b \frac{\partial i(0,t-r/c-z'/v)}{\partial t} \, dz'
\]  

(25)

\[
H_\phi \approx H_\phi(\text{radiation}) \approx -\frac{1}{2\pi c r} \int_0^b \frac{\partial i(0,t-r/c-z'/v)}{\partial t} \, dz'
\]  

(26)

Since \( v \) is constant, we obtain:

\[
\frac{\partial i(0,t-r/c-z'/v)}{\partial t} = -v \frac{\partial i(0,t-r/c-z'/v)}{\partial z'}
\]  

(27)

Thus, the above equations (25) and (26) can be written as:

\[
E_z(\text{radiation}) \approx -\frac{v}{2\pi\varepsilon_0 c^2 r} \int_0^b \frac{\partial i(0,t-r/c-z'/v)}{\partial z'} \, dz'
\]  

(28)

\[
H_\phi(\text{radiation}) \approx \frac{v}{2\pi c r} \int_0^b \frac{\partial i(0,t-r/c-z'/v)}{\partial z'} \, dz'
\]  

(29)

Performing the integration, we get:

\[
E_z(\text{radiation}) \approx -\frac{v}{2\pi\varepsilon_0 c^2 r} \left[ i(0,t-r/c) - i(0,t-r/c-h/v) \right]
\]  

(30)

\[
H_\phi(\text{radiation}) \approx \frac{v}{2\pi c r} \left[ i(0,t-r/c) - i(0,t-r/c-h/v) \right]
\]  

(31)

Since \( i(0,\tau) = 0 \), when \( \tau \leq 0 \). If \( t \leq h/v + r/c \), the equations (30) and (31) can be expressed as:
From equations (32) and (33), we can obtain that the electromagnetic field waveforms are essentially coincident with the waveform of the lightning return stroke channel-base current as long as the return stroke current has not reached the top of the channel. The negative sign in equation (32) represents that the direction of the electric field is opposite to the direction of the current propagation. Performing the derivation on both sides of equations (32) and (33), we obtain:

\[
\frac{dE_x(r,t)}{dt} \approx \frac{\partial E_x(r,t)}{\partial t} = \frac{v}{2\pi\varepsilon_0 c^2} i(0, t-r/c)
\]

(34)

\[
\frac{dH_y(r,t)}{dt} \approx \frac{\partial H_y(r,t)}{\partial t} = \frac{v}{2\pi c} i(0, t-r/c)
\]

(35)

4.2. Comparisons of LEMP in far-field between the exact results and approximate results

The rising height of the return stroke current on the channel is generally several hundreds of meters while the electromagnetic field reaches the initial peak value. Thus, at the distance beyond 10km, the far-field approximation condition of the electromagnetic field derivative can be basically satisfied, and the approximate expressions of the far-field derivative have a wider applicable range. In order to verify the reliability of the approximate expressions in far-field calculation, the lightning channel-base current with a typical waveform 1.2/50 \( \mu \)s is adopted to calculate the lightning electromagnetic fields. Accordingly, the parameters of the pulse function are set as follows: \( I_0 = 30kA \), \( \tau_1 = 4.05 \times 10^{-7}s \), \( \tau_2 = 6.80 \times 10^{-5}s \).

And the height of the lightning channel is set to \( H=7.5km \), and the return stroke speed is \( v=10^8 \) m s\(^{-1} \). Comparisons of the waveforms from the radiation fields, and the scaled channel-base current with the total electromagnetic fields at different distances beyond 5 km are given in figures 17 to 20. And comparisons of derivative waveforms from the radiation fields, and the scaled channel-base current with the total electromagnetic fields at different distances are given in figures 21 to 23. Here, two different coefficients are introduced in the scaled channel-base current: While comparing with the electric field and its derivative, the multiplying factor of the current and its derivative is \( q_1 \). While comparing with the magnetic field and its derivative, the multiplying factor of the current and its derivative is \( q_2 \). The values of \( q_1 \) and \( q_2 \) are set as follows:

\[
q_1 = \frac{v}{2\pi\varepsilon_0 c^2}, \quad q_2 = \frac{v}{2\pi c}
\]

**Figure 17.** Comparison of the waveforms from the approximate results (2) radiation field component, and (3) scaled channel-base current with (1) the exact results at a distance of 5 km.
Figure 18. Comparison of the waveforms from the approximate results (2) radiation field component and (3) scaled channel-base current with (1) the exact results at a distance of 10 km.

Figure 19. Comparison of the waveforms from the approximate results (2) radiation field component, and (3) scaled channel-base current with (1) the exact results at a distance of 50 km.

Figure 20. Comparison of the waveforms from the approximate results (2) radiation field component and (3) scaled channel-base current with (1) the exact results at a distance of 100 km.

Figure 21. Comparison of the derivative waveforms from the approximate results (2) radiation field component and (3) scaled channel-base current with (1) the exact results at a distance of 5 km.
As follows from figures 17 to 20, the deviations of the radiation fields, and the scaled channel-base current from the total electromagnetic fields decrease with the distance $r$ increases. The closer the distance is, the greater the deviations are. However, the rising edge of the radiation field waveforms, and the scaled channel-base current waveforms are essentially coincident with that of the total electromagnetic field waveforms. As shown in figures 17 and 18, at a distance of 5 to 10 km, the deviations of the radiation fields from the total electromagnetic fields increase gradually after the peak value, because the distance of 5 to 10 km located in the range of intermediate field, and the electrostatic, and induction components can not yet be ignored. Moreover, the deviations between the radiation fields and the scaled channel-base current are relatively small. These deviations mainly result from the approximate substitution of $r$ for $R$ in equations (32) and (33). As shown in figure 19, at a distance of 50 km, the electric and magnetic fields increase to a peak value then decay. Thus, the approximation characteristic between the total electromagnetic fields and the approximate results in the first several microseconds before reaching the initial peak value can basically represents the approximation characteristic of the full waveform. Furthermore, there is still some difference between the total electromagnetic fields, the radiation fields and the scaled channel-base current because the distance of 50km is only about 7 times larger than the height of the channel, which can not be viewed as $r \gg L$ (namely, the far-field approximation condition can not be met perfectly). When the distance exceeds 100 km, as shown in figure 20, the radiation field waveforms are essentially coincident with the scaled channel-base current waveforms. There is a little difference, which emerges after dozens of microseconds, between the total electromagnetic field and the radiation field component.

As follows from figures 21 to 23, the deviations of the radiation field derivatives, and the scaled channel-base current derivatives from the total electromagnetic field derivatives decrease with the distance $r$ increases. The closer the distance is, the greater the deviations are. However, the radiation field waveforms, and the scaled channel-base current waveforms are essentially coincident with the total electromagnetic field derivatives.
electromagnetic field waveforms in the first several microseconds. The difference between the scaled channel-base current derivatives and the total electromagnetic field derivatives in the first three microseconds is almost negligible at the distance beyond 5 km, because the rising height of the return stroke current on the channel is generally several hundreds of meters in the first several microseconds. At the distance beyond 5 km, the far-field approximation condition of the electromagnetic field derivative can be basically satisfied. With time going by and the further rising of the return stroke current on the channel, the distance requirement of the far-field approximation becomes farther gradually. Note that the electromagnetic field changes sharply only in the first several microseconds, then becomes slow finally. The far-field approximation characteristic in the first several microseconds between the total electromagnetic field derivatives and the approximate derivatives can basically represents the far-field approximation characteristic of the full waveform. It can be stated that the approximation condition of the far-field derivative is looser than the approximation condition of the far-field. The approximate expressions of the far-field derivative can be applied in the field derivative calculation at the distance beyond several kilometers, and the approximate expressions of the far-field can be applied only in the field calculation at the distance beyond 50 km.

Moreover, by comparing figures 17 to 20 with figures 21 to 23, it can be found that the difference among the total electromagnetic field derivatives, the radiation field derivatives, and the scaled channel-base current derivatives is smaller than the difference among the total electromagnetic fields, the radiation fields, and the scaled channel-base currents at the same distance.

5. Conclusions

According to the approximate derivation of the general expressions for LEMP on earth surface, the approximation characteristic between the near-field and far-field of LEMP and the channel-base current is obtained. The dipole method, based on the TL model, is used to calculate the near-field and far-field of LEMP on earth surface. The approximate results obtained from the approximate expressions are in a good agreement with the exact calculation results in the near-field and far-field, which indicates that the approximate derivation of the near-field and far-field expressions on earth surface is feasible, and the approximation characteristic between the near-field and far-field waveforms of LEMP and the channel-base current waveform is credible. This conclusion provides a theoretical basis for the LEMP wave simulation of near-field and far-field on earth surface.

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