Axionic Domain Wall and Warped Geometry

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Abstract

We discuss how a three-brane with an associated non-factorizable (warped) geometry can emerge from a five dimensional theory of gravity coupled to a complex scalar field. The system possesses a discrete $Z_2$ symmetry, whose spontaneous breaking yields an 'axionic' three-brane and a warped metric. Analytic solutions for the wall profile and warp factor are presented. The Kaluza-Klein decomposition and some related issues are also discussed.
1 Introduction

Theories with extra spacelike dimensions have recently attracted a great deal of attention. It was observed [1, 2] that for suitably large extra dimensions, it is possible to lower the fundamental mass scale of gravity $M_f$ down to a few TeV. This suggests a new way for a solution of the gauge hierarchy problem without invoking supersymmetry (SUSY). In this approach all the standard model particles are localized on a 3-brane, and only gravity propagates in the bulk. Assuming that the $n$ compact dimensions have a typical size $R$, the four dimensional Planck scale is expressed as:

$$M_{Pl}^2 \sim M_f^{2+n} R^n. \quad (1)$$

In order to reproduce the correct behavior of gravity one should take $R \lesssim \text{mm}$ (the behavior of gravity at this distance is now being studied). Interestingly, already for $n = 2$, $R \sim 1 \text{ mm}$ and $M_f \sim \text{few-TeV}$, $M_{Pl}$ has the required magnitude $\sim 10^{19} \text{ GeV}$. Detailed studies of phenomenological and astrophysical implications of these models, were presented in [2]. We note that our Universe as a membrane embedded in higher dimensional spacetime was also considered in earlier works [3].

An alternative solution of the gauge hierarchy problem invoking an extra dimension was presented in [4]. The desired mass hierarchy is generated through a non-factorizable metric obtained from higher dimensional gravity (see also [5]). In the minimal setting [4] there are two three branes - hidden and visible, separated by an appropriate distance. The non-factorizable metric is given by:

$$ds^2 = e^{-2k|y|} ds^2_{3+1} - dy^2, \quad (2)$$

where $y$ denotes the fifth spacelike dimension, $ds^2_{3+1}$ is the ordinary 4D interval, and $k$ is a mass parameter close to the fundamental scale $M_f$. On the visible brane all mass parameters are rescaled due to the warp factor in (2), such that $m_{\text{vis}} = M_f e^{-k|y_0|}$ ($y_0$ is the distance between branes). For $M_f \sim 10^{19} \text{ GeV}$ and $k|y_0| \simeq 37$ one finds that $m_{\text{vis}} \sim \text{few} \cdot \text{TeV}$, the desired magnitude. It was also shown that Newton’s law still holds on the visible brane. It is worth noting that in this approach the extra dimension can be infinite [8], provided it’s volume remains finite. Generalization of this non-factorizable model to scenarios with open codimensions and with intersecting multiple branes was presented in [7].

It is clearly important to inquire about the origin of the 3-branes in the above scheme with the warped metric. In this paper we present one such scenario with a complex scalar field coupled to 5D gravity. The theory possesses 5D Poincare invariance and $Z_2$ discrete symmetry. The 3-brane and warped geometry emerge dynamically from spontaneous breaking of the $Z_2$ symmetry. The 3-brane describes a topologically stable domain wall,
an axion-type solution of the sine-Gordon equation in curved space-time. Analytical solutions for the domain wall profile and warp factor are presented. As expected, the 5D space turns out to be Anti-de-Sitter (AdS). Questions of compactification, Kaluza-Klein (KK) decomposition, graviphoton mass and other related issues are also discussed.

2 The Model

In this section we will consider higher dimensional \((D = 5)\) gravity plus a complex scalar field which turns out to possess a non-factorizable solution of equation (2). The motivation for the choice of complex scalar is that with the help of \(Z_2\) symmetry we naturally obtain a potential with a cosine profile familiar from axion models. This yields a non trivial analytical solution for the \(\theta\)-domain wall whose core can be identified as a 3-brane.

2.1 Complex Scalar Coupled to 5D Gravity

Consider 5D gravity coupled to a complex scalar field \(\Phi\) through the action

\[
S = \int d^5x \sqrt{G} \left( -\frac{1}{2} M^3 R - \Lambda + \mathcal{L}(\Phi) \right),
\]

with

\[
\mathcal{L}(\Phi) = \frac{1}{2} G^{AB} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) - V(\Phi).
\]

Here \(G_{AB}\) is the 5D metric tensor and \(G = \text{Det}G_{AB} \ (A, B = 1, \cdots, 5)\). The Einstein equation derived from (3) is given by

\[
R_{AB} - \frac{1}{2} G_{AB} R - \frac{\Lambda}{M^3} G_{AB} = \frac{V}{M^3} G_{AB} + \frac{1}{M^3} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) - \frac{1}{2 M^3} G_{AB} G^{CD} (\partial_C \Phi^* \partial_D \Phi + \partial_D \Phi^* \partial_C \Phi),
\]

while the equation of motion for \(\Phi\) follows from

\[
\frac{\delta \mathcal{L}}{\delta \Phi} = \frac{1}{\sqrt{G}} \partial_A \left( \sqrt{G} \frac{\delta \mathcal{L}}{\delta (\partial_A \Phi)} \right).
\]

Terms on the right hand side of (5) effectively play the role of energy-momentum tensor \(T_{AB}\), which will be the source for the dynamical generation of the 3-brane and yield a non-factorizable geometry.

\(^3\)For higher dimensional non-factorizable scenarios, extended with real scalar fields, see [8]-[10].
Before proceeding to the specific model, which fixes \( V(\Phi) \), let us derive the appropriate equations of motion [from (5), (6)]. We are looking for a metric of the form:

\[
G_{AB} = \text{Diag} (A(y), -A(y), -A(y), -A(y), -1) ,
\]

which conserves 4D Poincare invariance:

\[
ds^2 = A(y)\bar{g}_{\mu\nu}dx^\mu dx^\nu - dy^2 , \quad \bar{g}_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} ,
\]

where

\[
\eta_{\mu\nu} = \text{Diag}(1, -1, -1, -1)
\]

and \( \bar{h}_{\mu\nu} \) denotes the 4D graviton \((\mu, \nu = 1, \ldots, 4)\). The \((1, 1)\) and \((5, 5)\) components of (3) respectively give

\[
\frac{A''}{A} = -\frac{2 \Lambda + V}{3 M^3} - \frac{2}{3 M^3} (\Phi^*)' \Phi' ,
\]

\[
\left( \frac{A'}{A} \right)^2 = -\frac{2 \Lambda + V}{3 M^3} + \frac{2}{3 M^3} (\Phi^*)' \Phi' ,
\]

where primes denote derivatives with respect to the fifth coordinate \( y \). Subtracting (11) and (10), we get:

\[
- \frac{A''}{A} + \left( \frac{A'}{A} \right)^2 = \frac{4}{3 M^3} (\Phi^*)' \Phi' .
\]

Using the substitutions:

\[
\Phi = v \cdot e^{i \theta} ,
\]

\[
A = A_0 \cdot e^{-\sigma} ,
\]

and assuming that \( v \) in (13) does not depend on \( y \) [see discussion in sec. (1.2)], from (12) and (11) we derive:

\[
\sigma'' = \frac{4 v^2}{3 M^3} \theta'^2 ,
\]

\[
\sigma' = -\frac{2 \Lambda + V}{3 M^3} + \frac{2 v^2}{3 M^3} \theta'^2 .
\]

With our assumption \( v = \text{const.} \), from (3) we obtain the equation of motion for \( \theta \):
The three equations (15)–(17) are not independent. Namely, differentiating (16) and using (15), we obtain (17). However, in order for these three equations to have a solution, one fine tuning between the parameters is unavoidable. This can be seen from the following discussion: Solving equations (16) and (17) we have three parameters of integration \( \theta(y_0) \), \( \theta'(y_0) \) and \( \sigma(y_0) \), where \( y_0 \) is some arbitrary point (In principle there is also a fourth parameter which expresses translation invariance. But it is irrelevant, since the equations are invariant under translations). \( \sigma(y_0) \) is also irrelevant, since the equations contain only derivatives of \( \sigma \). From (15), \( \theta(y_0) \) is also irrelevant. Since for the brane solution we have to impose the condition \( \theta'(\infty) = 0 \), the third parameter \( \theta'(y_0) \) is fixed from this condition. Therefore, there remains no free parameters, and for satisfying (15), one fine tuning must be done (for detailed discussions about this issue see [10]). This will be explicitly seen for the model discussed below.

### 2.2 Axionic Brane and Warped Geometry

We introduce a \( Z_2 \) symmetry under which \( \Phi \to -\Phi \). The relevant potential is given by

\[
V = \frac{\lambda_1}{4} (\Phi^* \Phi - v^2)^2 - \frac{\lambda}{2} (\Phi^2 + \Phi^*^2) .
\]  

(18)

The first term in (18) is \( (U(1)) \) invariant under \( \Phi \to e^{i\theta} \Phi \), while the last term explicitly breaks it to \( Z_2 \). This avoids the appearance of a Goldstone mode because of non-zero \( \Phi \) VEV\(^4\). We restrict our attention in (18) to terms needed to implement the scenario. The couplings \( \Phi^4 + \Phi^*^4 \) can be included if so desired, but this makes analytic calculations more difficult. As noted in [12], such terms are absent in some models. Higher powers in \( \Phi \) and \( \Phi^* \) would complicate the discussion even further. We assume that the \( U(1) \) violating term is such that

\[
\lambda_1 v^2 \gg \lambda .
\]  

(19)

Therefore, the VEV \( \langle |\Phi| \rangle \) is mainly determined by the first term in (18),

\[
\langle \Phi^* \Phi \rangle \simeq v^2 ,
\]  

(20)

from which

\[
\Phi \simeq v e^{i\theta} .
\]  

(21)

\(^4\)For models with \( Z_2 \) replacing the PQ symmetry and avoiding an undesirable axion, see papers [11, 12], where various phenomenological and cosmological implications are also studied.
Substituting (21) in (18), the \( \theta \) dependent part of the potential is given by

\[
V_{\theta} = -\lambda v^2 \cos 2\theta .
\] (22)

This type of potential was also used for brane formation in [8]. In our case we have obtained it through a \( Z_2 \) symmetry acting on a complex scalar field \( \Phi \). Assuming \( \lambda > 0 \), (22) acquires its minima for \( \theta = 0, \pi \). The \( \langle \theta \rangle \) VEV breaks the symmetry \( \theta \to -\theta \). This causes the creation of topologically stable domain wall. The wall is stretched between two energetically degenerate minima, \( \theta = 0 \) and \( \theta = \pi \). With assumption (19) it is consistent to consider \( v \) to be (essentially) \( y \)-independent.

Introducing the dimensionless coordinate \( \xi \)

\[
\xi = \sqrt{2\lambda} y ,
\] (23)

(15) and (17) respectively become:

\[
2 \frac{\partial^2 \theta}{\partial \xi^2} - 4 \frac{\partial \sigma}{\partial \xi} \frac{\partial \theta}{\partial \xi} - \sin 2\theta = 0 ,
\] (24)

\[
\frac{\partial^2 \sigma}{\partial \xi^2} = \alpha \left( \frac{\partial \theta}{\partial \xi} \right)^2 ,
\] (25)

where

\[
\alpha = \frac{4v^2}{3M^3} .
\] (26)

Nontrivial solutions of (24) and (25), with boundary conditions

\[
\theta(-\infty) = 0 , \quad \theta(+\infty) = \pi , \quad \sigma(\pm\infty) \propto \pm y ,
\] (27)

[Note that due to the breaking of the U(1) symmetry to \( Z_2 \) in [18], the wall here is not 'bounded by strings', a phenomenon encountered in \( SO(10) \) and axion models [13].] will indicate the existence of 'warped' geometry and the axion(or \( \theta \))-brane (since \( \langle \theta \rangle \) breaks 5D invariance). The point \( \theta = \frac{\pi}{2} \) will be identified as the location of the 3-brane describing 4D theory.

Using the substitution

\[
\theta = 2 \arctan f(\xi) ,
\] (28)

(24), (23) can be rewritten as

\[
-(f^2 - 1)f'' + 2f(f''f - f'^2) - 2\sigma'(f^2 + 1)f' + f(f^2 - 1) = 0 ,
\] (29)
\[(f^2 + 1)^2 \sigma'' = 4\alpha f'^2 , \]

where primes denote derivatives with \(\xi\). The form for \(f\)

\[ f = ae^{m\xi} , \]

is a reasonable choice, where the parameters \(a, m > 0\) are undetermined for the time being. Substituting (31) in (30), the latter can be integrated:

\[ \sigma' = s_0 - 2\alpha m \frac{1}{1 + f^2} , \]

where \(s_0\) is some constant. Substituting (32) into (28) and taking into account that \(f'' = mf' = m^2 f\), we find:

\[ - (f^2 - 1)m^2 - 2m(s_0(f^2 + 1) - 2\alpha m) + f^2 - 1 = 0 . \]

Comparing appropriate powers of \(f\) in (33), it is easy to verify that (33) is satisfied if

\[ m = \frac{1}{\sqrt{1 + 2\alpha}}, \quad s_0 = \frac{\alpha}{\sqrt{1 + 2\alpha}} . \]

Integration of (32) gives

\[ \sigma = \alpha \ln[\cosh(m\xi + \delta)] + \ln C , \quad \delta = \ln a \]

\((C = \text{constant and we have taken into account (34)}).\)

Finally, for \(\theta\) and the warp factor \(A(= A_0 e^{-\sigma})\) we will have:

\[ \theta = 2 \arctan(ae^{m\xi}) , \]

\[ A = A_0[\cosh(m\xi + \delta)]^{-\alpha} . \]

where the constant \(C\) is now absorbed in \(A_0\), and \(a\) still remains undetermined, which reflects translational invariance in the fifth direction \(\xi (y)\).

Let us note here that these solutions are obtained for \(\lambda > 0\) and a negative sign in front of the last term in (18). In case of a positive sign, the potential is minimized for \(\theta = \pm \frac{\pi}{2}\), and instead of the solution (30), we would have \(\tilde{\theta} = \theta - \frac{\pi}{2}\). Indeed, under these modifications, equations (24), (25) are satisfied [for this case the sign in front of \(\sin \tilde{\theta}\) in (24) will be positive, which reflects a change of sign of the last term in (18)].

From (37), taking into account (34), we will get the desirable asymptotic forms for \(A\):

\[ A \sim e^{s_0\xi} , \quad \xi \to -\infty \]
\[ A \sim e^{-s_0 \xi}, \quad \xi \rightarrow +\infty. \quad (38) \]

The solutions (35), (36) should also satisfy (16), which in terms of \( \xi \) has the form

\[ \sigma^{'2} = -\frac{\Lambda + V}{3\lambda M^2} + \frac{\alpha}{2} \theta^{'2}. \quad (39) \]

From (32), (36) and (34) we have

\[ \sigma' = \alpha m f^2 - 1 \quad \frac{f^2 + 1}{f^2 + 1}, \quad \theta' = \frac{2mf}{f^2 + 1}, \quad \cos 2\theta = 1 - \frac{8f^2}{(f^2 + 1)^2}. \quad (40) \]

Substituting all of this in (39), we can see that the latter is satisfied if

\[ \Lambda = \lambda v^2 (1 - 4\alpha m^2) = \lambda v^2 \frac{1 - 2\alpha}{1 + 2\alpha}. \quad (41) \]

Therefore, as we previously mentioned, one fine tuning between the parameters of the theory is necessary. The effective 5D cosmological constant is determined to be

\[ \Lambda_{\text{eff}} = \Lambda + \langle V \rangle = \Lambda + V_0 (\theta = 0, \pi) = -4\lambda v^2 \frac{\alpha}{1 + 2\alpha}. \quad (42) \]

As expected, the initial 5D space-time is AdS.

The warp factor (37) reaches its maximum at \( \xi_0 = -\ln a/m \) and decays exponentially far from \( \xi_0 \). For a realistic model which solves the gauge hierarchy problem, we may regard the axion wall as a hidden brane, located at \( \xi_0 \). By placing the visible brane (which can describe our 4D Universe) at a distance \( \Delta \xi \approx 74/(\alpha m) \) from \( \xi_0 \), all masses on the visible brane will be rescaled as \( m_{\text{vis}} = M \cdot A(\xi_0 + \Delta \xi)^{1/2} \approx M \cdot e^{-\alpha m \Delta \xi / 2} \sim 10^{-16} \cdot M \). For \( M \sim 10^{19} \text{ GeV} \), the desired scale \( m_{\text{vis}} \sim \text{few} \cdot \text{TeV} \) will be naturally generated.

### 3 Kaluza-Klein Decomposition

In this section we will present the KK reduction of the 5D model to 4D. We will calculate the graviphoton [(\( \mu, 5 \)) component of metric tensor] mass, as well as the effective 4D Planck scale and the radius of the extra dimension. For KK reduction within models with non-factorizable geometry, also see papers in [14, 15], while for works addressing the effects of spontaneous breaking of higher Poincare invariance, Goldstone phenomenon and other relevant issues within models with non-warped geometry, see [16].

For performing a KK decomposition, it is convenient to rewrite the metric in (8) in a conformally 'flat' form:

\[ ds^2 = \Omega^2(z)g_{MN}dx^M dx^N, \quad (43) \]
where:
\[
dz = A^{-1/2}(y)dy, \quad \Omega^2(z) = A(y(z)),
\]
\[
G_{MN} = \Omega^2 g_{MN}.
\]
With the standard KK decomposition
\[
g_{MN} = \left( \bar{g}_{\mu\nu} - k^2 A_\mu A_\nu, \frac{k A_\mu}{k} \right), \quad g^{MN} = \left( \frac{\bar{g}^{\mu\nu}}{k A^\nu} \frac{k A^\mu}{k^2 A^a A_a - 1} \right),
\]
where \( A_\mu \) is the graviphoton, equation (43) reads
\[
ds^2 = \Omega^2 \left( \bar{g}_{\mu\nu} dx^\mu dx^\nu - (dz + k A^\mu dx_\mu)^2 \right).
\]
We omit the graviscalar field in (46) since it is not relevant for our discussion. See [14] for a discussion involving this field. Eq. (47) acquires the ‘usual’ form for \( A_\mu = 0 \). For \( A_\mu \neq 0 \), it is invariant under the following transformations:
\[
x'\mu = x_\mu, \quad z' = z + \epsilon(x_\mu),
\]
\[
A'_\mu = A_\mu - \frac{1}{k} \partial_\mu \epsilon.
\]
Note that this is a \( U(1) \) transformation for \( A_\mu \), where \( z \) plays the role of Goldstone field. The \( \mathbb{Z}_2 \) symmetry breaking creates the brane and translational invariance in the fifth direction is spontaneously broken. The breaking of the corresponding generator gives rise to a massive \( A_\mu \) field. By considering \( z \) as \( x_\mu \) dependent (which corresponds to brane vibrations), the term \((\partial_\mu z)^2\) (see below) appear in the 4D action. This tell us that from the point of view of 4D observer, \( z(x_\mu) \) is a Goldstone field which becomes the longitudinal component of \( A_\mu \). From this discussion it is clear that the fields \( z(x_\mu) \) and \( A_\mu \) reside on the 4D brane.

We now calculate the graviphoton mass. Taking into account (49), for the Einstein Tensor
\[
G_{MN} = R_{MN} - \frac{1}{2} G_{MN} R,
\]
we have
\[
G^G_{MN} = G^g_{MN} + (D - 2) \left( \nabla_M \ln \Omega \nabla_N \ln \Omega - \nabla_M \nabla_N \ln \Omega \right) +
(D - 2) g_{MN} \left( \nabla_P \nabla^P \ln \Omega + \frac{1}{2} (D - 3) \nabla_P \ln \Omega \nabla^P \ln \Omega \right),
\]
where $G^G$ and $G^g$ are calculated using $G$ and $g$ respectively. The covariant derivatives $\nabla_M$ are built from $g$, such that for a scalar function $S$

$$\nabla_M S = \partial_M S \ ,$$

while for a vector $\mathcal{V}$

$$\nabla_M \mathcal{V}^N = \partial_M \mathcal{V}^N + \Gamma^N_{MP} \mathcal{V}^P \ , \quad \nabla_M \mathcal{V}_N = \partial_M \mathcal{V}_N - \Gamma^P_{MN} \mathcal{V}_P \ .$$

From (49) we have

$$R = -\frac{2}{D-2}G^{MN}G_{MN} \ ,$$

and taking into account (50), we get:

$$R(G) = \Omega^{-2} \left( R(g) - 2(D-1)\nabla_M \nabla^M \ln \Omega - (D^2 - 3D + 2)\nabla_M \ln \Omega \nabla^M \ln \Omega \right) + \ ,$$

$$\Omega^{-2} \left( R(g) - 2(D-1)\frac{\nabla_M \nabla^M \Omega}{\Omega} - (D^2 - 5D + 4)\frac{\nabla_M \Omega \nabla^M \Omega}{\Omega^2} \right) \ .$$

Calculating $R(g)$ through (53) and keeping only relevant terms, we have

$$\sqrt{G} = \sqrt{-\bar{g}} \Omega^5 \ ,$$

$$R(g) = \bar{R}(\bar{g}) + \frac{k^2}{4}F_{\mu\nu}F^{\mu\nu} + \ldots \ ,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ ,$$

and $\bar{R}(\bar{g})$ is the 4D curvature, built from the physical 4D metric $\bar{g}_{\mu\nu}$.

Taking into account (51), (52) and (46), we have:

$$\nabla_M \nabla^M \Omega = \left( \partial^\mu \partial_\mu \Omega + 2kA^\mu \partial_\mu \Omega' + (k^2A^\mu A_\mu - 1)\Omega'' + \ldots \right) \ ,$$

$$\nabla_M \Omega \nabla^M \Omega = \left( \partial^\mu \partial_\mu \Omega + 2k\Omega' A^\mu \partial_\mu \Omega + (k^2A^\mu A_\mu - 1)(\Omega')^2 \right) \ ,$$

where primes here denote derivatives with respect to $z$. Using

$$\partial_\mu = \frac{\partial z}{\partial x^\mu} \frac{\partial}{\partial z} = \partial_\mu \bar{z} \cdot \frac{\partial}{\partial z} \ , \quad \bar{z} \equiv z(x_\mu) \ ,$$

from (51), (57) and (58) it finally follows that
\[ R(G) = \Omega^{-2} R(g) - \Omega^{-2} \left( 2(D - 1) \frac{\Omega''}{\Omega} + (D^2 - 5D + 4) \frac{\Omega'^2}{\Omega^2} \right) \times \left( \partial^\mu \bar{z} \partial_{\mu} \bar{z} + 2k A^\mu \partial_{\mu} \bar{z} + k^2 A^\mu A_\mu - 1 \right). \] \tag{60}

From (60) we see that the field \( \bar{z} \) can be absorbed by \( A_\mu \) by a suitable \( U(1) \) transformation.

From the Einstein equation (5) we have:

\[-(\Lambda + V) = \frac{M^3}{2} \frac{D - 2}{D} R + \frac{1}{2} \frac{D - 2}{D} G^{AB} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) \] \tag{61}

and substituting this in (3), we get:

\[ S = \int d^5 x \sqrt{G} \left[ - \frac{M^3}{2} \frac{2(D - 1)}{D} R + \frac{2(D - 1)}{D} \frac{1}{2} G^{AB} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) \right]. \] \tag{62}

With

\[ \frac{1}{2} G^{AB} (\partial_A \Phi^* \partial_B \Phi + \partial_B \Phi^* \partial_A \Phi) = \Omega^{-2} v^2 \theta'^2 \left( \partial^\mu \bar{z} \partial_{\mu} \bar{z} + 2k A^\mu \partial_{\mu} \bar{z} + k^2 A^\mu A_\mu - 1 \right), \] \tag{63}

After integrating over the fifth dimension in (62), we obtain the reduced 4D action:

\[ S^{(4)} = \int d^4 x \sqrt{-g} \left( - \frac{M^3_{\text{Pl}}}{2} \bar{R}(\bar{g}) - T - \frac{k^2}{4} B_{\mu\nu} B^{\mu\nu} + M^2_V (B_\mu + \frac{1}{k} \partial_\mu Z)(B^\mu + \frac{1}{k} \partial^\mu Z) \right), \] \tag{64}

where the 4D Planck mass is

\[ M^2_{\text{Pl}} = \frac{2(D - 1)}{D} M^3 \int \Omega^3 dz, \] \tag{65}

the 4D brane tension is

\[ T = M^3 \frac{D - 1}{D} \int \Omega^3 \left[ 2(D - 1) \frac{\Omega''}{\Omega} + (D^2 - 5D + 4) \frac{\Omega'^2}{\Omega^2} + \frac{2v^2}{M^3} \theta'^2 \right] dz, \] \tag{66}

and the mass of the graviphoton is

\[ M^2_V = \frac{M^3}{M^2_{\text{Pl}}} \frac{D - 1}{D} k^2 \int \Omega^3 \left[ 2(D - 1) \frac{\Omega''}{\Omega} + (D^2 - 5D + 4) \frac{\Omega'^2}{\Omega^2} + \frac{2v^2}{M^3} \theta'^2 \right] dz. \] \tag{67}
In obtaining (64) we have used
\[ B_\mu = M_{Pl} A_\mu , \quad B_{\mu\nu} = M_{Pl} F_{\mu\nu} , \quad Z = M_{Pl} \bar{z} . \] (68)

Comparing (66) and (67),
\[ M_{V}^2 = \frac{T}{M_{Pl}^2} k^2 = \frac{T}{g^2_{V} M_{Pl}^2} . \] (69)

Simplifying (66) yields:
\[ T = \frac{4}{5} M^3 \int dy A^2 \left( 4 \frac{A_y''}{A} + \frac{A_y'^2}{A^2} + \frac{2v^2}{M^3} \theta_y'^2 \right) = \frac{16}{5} \sqrt{2\lambda} M^3 A_0^2 m \left( 2\alpha^2 I(2\alpha) + (\alpha/2 - 2\alpha^2) I(2\alpha + 2) \right) , \] (70)

where we have put \( D = 5 \), the subscript \( y \) denotes derivatives with respect to \( y \), and

\[ I(\alpha) = \int_0^1 \left( 1 - \rho^2 \right)^\alpha \frac{d\rho}{1 - \rho^2} = \int_0^{\frac{\pi}{2}} (\cos 2t)^{\alpha-1} dt . \] (71)

is some finite number whose value depends on the positive parameter \( \alpha \). For \( \alpha = 1 \), \( I = \pi/4 \), and for \( \alpha = 2 \), \( I = 1/2 \).

Note that the relation (63) between the graviphoton mass and brane tension, has same form as for models with non warped geometry [16].

Simplifying (63) one finds:
\[ M_{Pl}^2 = \frac{8}{5} M^3 \int_{-\infty}^{+\infty} A(y) dy = M^3 R_{eff} , \] (72)

where

\[ R_{eff} = \frac{8}{5\sqrt{2\lambda}} \int_{-\infty}^{+\infty} A(\xi) d\xi = \frac{8A_0}{5\sqrt{2\lambda}} \int_{-\infty}^{+\infty} \left[ \cosh(m\xi + \delta) \right]^{-\alpha} d\xi = \frac{32A_0}{5m\sqrt{2\lambda}} I(\alpha) . \] (73)

Thus, even though the extra dimension \( y \) is non-compact, its ‘effective’ size \( R_{eff} \) is finite. In this sense the extra space is effectively compact. Expression (72) resembles the well known relation \( M_{Pl}^2 \sim M^{2+n}L^n \) (for \( n = 1 \)), which relates the effective 4D Planck scale to the fundamental scale \( M \) and the volume \( (\sim L^n) \) of the \( n \) extra dimensions [1, 2]. The crucial difference from models [1, 2] is that even for values \( M \sim M_{Pl} \), \( R_{eff} \sim 1/M_{Pl} \) in (72), the desired hierarchy is obtained, thanks to the warped geometry.

In conclusion, it would be interesting to investigate the possibility of introducing a second domain wall, located at a suitable distance from the first and characterized by the
TeV scale. 'Double wall' solutions that are dynamically stabilized in axion type models with Minkowski background have been studied in ref. [12]. Some extension of the model considered here may well be required to implement such a scenario.

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