By calculating a Casimir energy for the acoustic phonons of Graphene, we find some temperature-dependent corrections for the pretension of a Graphene sheet suspended on a trench. We obtain values of the order of few mN/m for these corrections in fully as well as doubly clamped Graphene on a narrow trench with one nanometer width, at room temperature. These values are considerable compared to the experimental values, and can increase the fundamental resonance frequency of the Graphene. The values of these corrections increase by increasing the temperature, and so they can be utilized for tuning the Graphene pretension.

Keywords
Graphene sheet, Acoustic Phonons, pretension, Resonance frequency, Casimir energy, Temperature correction, Nanoelectromechanical systems.

1 Introduction

Nowadays Casimir forces, as important macroscopic manifestations of the quantum zero-point energies, have been investigated for various systems, see e.g. [1–3] as reviews. Primarily the Casimir forces, as attractive interactions between some chargeless conductors being microscopically apart, are arisen from the quantum zero-point energies of electromagnetic modes discretized in the presence of the metallic boundary walls. However one can generally calculate a Casimir energy by finding a finite value for the total zero-point energy of any system with a mode spectrum discretized by some appropriate physical boundaries. As we know, the Casimir forces are significant in microscopic distances, so they can imply magnificent results for tiny scale systems, see e.g. [4–6] as reviews.

Graphene, a one-atom-thick layer of Carbon atoms covalently bound in a honeycomb hexagonal lattice, as the thinnest known material, has represented some remarkable mechanical and electrical properties [7, 8] and is expected to provide a vast area of promising applications in nano scale devices known as the nanoelectromechanical systems.
(NEMS), see e.g. [9][11]. As we know, a NEMS is prototypically a nano-resonator, i.e. a nanoscale beam of an appropriate material being actuated by an applied external force, to vibrate in a particular resonance mode, see e.g. [12][14].

It has been shown that the fundamental resonance frequency of a monolayer Graphene sheet suspended over a trench, is highly influenced by a significant pre-tension induced specifically by the strong van der Waals forces which clamp the Graphene to the sidewalls of the trench, see Refs. [16][22]. In a previous work [27] we have shown that the zero-point energy of the Graphene acoustic phonons can have a considerable influence on this pre-tension. In fact, in that work, by regarding the acoustic modes of a membrane as the oscillation modes of a massless bosonic field having two polarizations (corresponding to transverse and longitudinal vibrations), and being confined in a rectangle cavity (corresponding to a rectangular trench), we have obtained a “phononic” Casimir energy for a suspended Graphene sheet being fully clamped to the sidewalls of a rectangular trench. Then by assigning a Casimir force to this energy, and interpreting it as a correction to the initial tension of the membrane (see also Ref. [26]), we have obtained some temperature-dependent correctional terms for the Graphene pretension. We have shown that for a monolayer Graphene sheet suspended on a narrow rectangular trench with a width $\sim 10^{-3}$ nm and length $\sim 10^{-4}$ µm, at room temperature, this correction can be noticeable ($\sim 10^{-1}$ mN/m) compared to the experimental values of the Graphene pretension ($\sim 1–10$ mN/m) given in Refs. [20][22]. Note that this “phononic” Casimir forces are different from the conventional “photonic” Casimir interactions e.g. between two Graphene sheets or between a Graphene sheet and a substrate, which are arisen actually from the zero-point oscillations of the electromagnetic field. As shown in Refs. [28][31] these electromagnetic Casimir interactions also can be significantly strong for the Graphene sheets at room temperature.

In this work we find a similar correction for a Graphene sheet being doubly clamped to the sidewalls of a trench. We also explore the influence of flexural acoustic modes on this correction for fully as well as doubly clamped Graphene. As we know, the flexural modes, unlike the longitudinal/transverse modes, are described by a non-linear dispersion relation, so here we may need to partly change some calculations. In the next section, having the flexural modes been included, we find the phononic Casimir correction to the Graphene pretension, by using similar approach as in the previous work [27]. In the section 3, using a rather similar approach we calculate these corrections for the case of a doubly clamped Graphene. We see that for doubly as well as fully clamped Graphene sheet over a trench of few nanometer width, at room temperature, these corrections can be significant $\sim \text{mN/m}$, one order of magnitude larger than that of our previous work.

2 Fully clamped Graphene

The dispersion relations for longitudinal (L), transverse (T), and flexural (F) acoustic phonons in Graphene, near the center of the Brillouin zone, are well known, see e.g. [32],

$$\omega_{L,T} \approx v_{L,T}k; \quad v_L \approx 21.3 \text{ km/s}, \quad v_T \approx 13.6 \text{ km/s},$$

$$\omega_F \approx \alpha k^2; \quad \alpha \approx 6.2 \times 10^{-7} \text{ m}^2/\text{s},$$

(1)
in which $\omega$ and $k$ are mode frequency and mode wavenumber, respectively, of the acoustic modes, and $v$’s are the sound velocities in the Graphene sheet. For a fully clamped sufficiently tensioned few-layer Graphene sheet suspended on a rectangular trench, see Fig. 1, the resonance frequencies are given with a good accuracy by the familiar modenumbers 

$$k_{n,m} = \pi \sqrt{\frac{n^2}{a^2} + \frac{m^2}{b^2}}, \quad a \geq b, \quad n, m = 1, 2, ...$$

(2)

in which, “$a$” and “$b$” are the side-lengths of the trench. Then regarding the mentioned Graphene sheet, as a 2-dimensional gas of phonons in a rectangle cavity, one can write the total zero-point energy (see e.g. [1]) of the Graphene acoustic phonons as

$$E_0(a, b, T) = \frac{k_B T}{2} \sum_{n,m=1}^{\infty} \sum_{l=-\infty}^{\infty} \left( \sum_{I=L,R} \ln \left[ t^2 + \lambda_I^2 (x^2 n^2 + m^2) \right] \right)$$

$$+ \ln \left[ t^2 + \lambda_F^2 (n^2 x^2 + m^2)^2 \right],$$

(3)

in which $x \equiv b/a$, “$T$” is the Graphene temperature, $\hbar$ and $k_B$ are the (reduced) Planck and the Boltzmann constants, respectively, and we have introduced some dimensionless parameters $\lambda_I = \theta_I/T$ and $\lambda_F = \theta_F/T$ with the effective temperatures

$$\theta_I = \frac{\hbar v_I^2}{2bk_B}, \quad \theta_F = \frac{\pi \hbar \alpha}{2b^2 k_B}.$$  

(4)

Note that in Eq. (3) we have discarded an irrelevant constant term to make the logarithm arguments dimensionless. Then as is conventional, one can rewrite Eq. (3) as a parametric integral,

$$E_0(a, b, T) = -\frac{k_B T}{2} \lim_{s \to 0} \frac{\partial}{\partial s} \left( \sum_{I=L,R} \int_0^{\infty} \frac{dt}{t} \frac{t^s}{\Gamma(s)} \sum_{n,m=1}^{\infty} \exp \left[ -t\lambda_I^2 (x^2 n^2 + m^2) \right] \right)$$

$$+ 2 \sum_{I=L,R} \int_0^{\infty} \frac{dt}{t} \frac{t^s}{\Gamma(s)} \sum_{l,m,n=1}^{\infty} \exp \left[ -t(l^2 + \lambda_I^2 [x^2 n^2 + m^2]) \right]$$

$$+ \int_0^{\infty} \frac{dt}{t} \frac{t^s}{\Gamma(s)} \sum_{n,m=1}^{\infty} \exp \left[ -t\lambda_F^2 (x^2 n^2 + m^2)^2 \right].$$

3
\[ +2 \int_0^\infty \frac{dt}{t} \frac{t^s}{\Gamma(s)} \sum_{l,m,n=1} \exp \left[ -t(l^2 + \lambda_\nu^2 [x^2 n^2 + m^2]^2) \right] \] \quad (5)

The first two lines of the above equation, i.e. the contributions of the longitudinal and the transverse acoustic modes, has been calculated exactly in Ref. [27]. Aside from contribution of the unbounded space which must be subtracted, the second line of the above equation contains some exponential and/or Bessel function terms, which for sufficiently small \( \lambda \)'s can be neglected with a good degree of accuracy. By a similar discussion, one can neglect the fourth line of the above equation. Therefore for sufficiently small \( \lambda \)'s, that is, for sufficiently large temperatures compared to the effective temperatures \( \theta \), the zero-point energy \( E_0 \) can be approximated by its first and third lines;

\[ E_0(a, b, T) \approx -\frac{k_B T}{2} \lim_{s \to 0} \frac{\partial}{\partial s} \left( \sum_{l=L,R} \int_0^\infty \frac{dt}{t} \frac{t^s}{\Gamma(s)} \sum_{n,m=1} \exp \left[ -t\lambda_\nu^2 (x^2 n^2 + m^2) \right] \right) \quad (6) \]

Note that at room temperature (\( \approx 300K \)), the above approximation is acceptable for \( b \gtrsim 0.5\)nm, see Eq. (4) (having \( \hbar \approx 1.05 \times 10^{-34} m^2 kg/s \) and \( k_B \approx 1.38 \times 10^{-23} m^2 kg/s^2 K \)). Now the first line of the above equation can be simplified by using the relation;

\[ \sum_{n=1}^\infty \exp \left[ -\alpha n^2 \right] = -\frac{1}{2} + \frac{1}{2} \sqrt{\frac{\pi}{\alpha}} + \frac{\sqrt{\pi}}{\alpha} \sum_{n=1}^\infty \exp \left[ -\frac{\pi^2 n^2}{t\alpha} \right] ; \quad \alpha > 0 \quad (7) \]

which can be obtained directly by using the Poisson summation formula (see e.g. [34]),

\[ \sum_{n=1}^\infty f(n) = -\frac{f(0)}{2} + \int_0^\infty f(x)dx + 2 \sum_{n=1}^\infty \int_0^\infty f(x) \cos(2\pi nx)dx. \quad (8) \]

Then by utilizing the Riemann zeta function \( \zeta(s) = \sum_{n=1}^\infty n^{-s} \), and the integral relation

\[ \int_0^\infty t^r \exp \left[ -x^2 t - y^2/t \right] dt = 2(x/y)^{-r-1} K_{r-1}(2xy) \quad (9) \]

with the Modified Bessel function

\[ K_\nu(z) = \frac{(z/2)\nu \Gamma(1/2)}{\Gamma(\nu + 1/2)} \int_1^\infty e^{-zt} (t^2 - 1)^{(2\nu-1)/2} dt \quad (10) \]

one can find

\[ \int_0^\infty \frac{dt}{t} \frac{t^s}{\Gamma(s)} \sum_{n,m=1} \exp \left[ -t\lambda_\nu^2 (x^2 n^2 + m^2) \right] = \]

\[ -\frac{\zeta(2s)}{2(x\lambda_f)^{2s}} + \frac{\sqrt{\pi}}{2\lambda_f} \frac{\Gamma(s - 1/2) \zeta(2s - 1)}{\Gamma(s)} (x\lambda_f)^{2s-1} \]

Note that at room temperature (\( \approx 300K \)), the above approximation is acceptable for \( b \gtrsim 0.5\)nm, see Eq. (4) (having \( \hbar \approx 1.05 \times 10^{-34} m^2 kg/s \) and \( k_B \approx 1.38 \times 10^{-23} m^2 kg/s^2 K \)).
\[
+2 \frac{\sqrt{\pi}}{\lambda I \Gamma(s)} \sum_{n,m=1}^{\infty} \left( \frac{n \lambda I}{m \pi / \lambda I} \right)^{-s+1/2} K_{-s+1/2}(2\pi nm x) \quad (11)
\]

But the second line of Eq. (6), which comes from the contribution of the flexural modes, can not be simplified, in its present form, by directly using a relation of the form (7). However we can re-parametrize the mentioned expression as

\[
\int_0^{\infty} \frac{dt}{t} \frac{t^s}{\Gamma(s)} \exp \left[ -t \lambda_F^2 (x^2 n^2 + m^2)^2 \right] = \left[ \lambda_F^2 (x^2 n^2 + m^2)^2 \right]^{-s} = \lambda_F^{-2s} \int_0^{\infty} \frac{dt}{t} \frac{t^{2s}}{\Gamma(2s)} \exp \left[ -t (x^2 n^2 + m^2) \right] \quad (12)
\]

Now one can apply the summation (7) to the \(m\)-sum in the second line of the above equation, and after some calculations similarly as carried out for the first line of Eq. (6), we obtain

\[
\int_0^{\infty} \frac{dt}{t} \frac{t^s}{\Gamma(s)} \sum_{n,m=1}^{\infty} \exp \left[ -t \lambda_F^2 (x^2 n^2 + m^2)^2 \right] = \lambda_F^{-2s} \left[ - \frac{\zeta(4s)}{2x^{4s}} + \frac{\sqrt{\pi} \Gamma(2s-1/2) \zeta(4s-1)}{\Gamma(2s)} x^{4s-1} \right. \\
+ \left. 2 \frac{\sqrt{\pi}}{\Gamma(2s)} \sum_{n,m=1}^{\infty} \left( \frac{nx}{m \pi} \right)^{-2s+1/2} K_{-2s+1/2}(2\pi nm x) \right] \quad (13)
\]

Eventually after a rather long calculation by using

\[
\lim_{s \to 0} \frac{\partial}{\partial s} \frac{g(s)}{\Gamma(s)} = g(0) ; \\
\lim_{s \to 0} \frac{\partial}{\partial s} \frac{K_{-s-r}(x)}{\Gamma(s)} = 0 ; \quad r \geq 0 \quad (14)
\]

in Eq. (6) we obtain an expression for the phononic Casimir energy of the Graphene sheet;

\[
E_C(a, b, T) \approx -k_B T \left( \frac{\pi}{6} x - \ln x - \frac{\ln(\lambda_L \lambda_T \lambda_F)}{4} \right) \\
+ 2 \sum_{n,m=1}^{\infty} \frac{\exp(-2mn \pi x)}{m} ; \quad x \equiv b/a , \quad \lambda_{L,T,F} \ll 1 \quad (15)
\]

Then the phononic Casimir forces can be introduced as

\[
F_{C,a}(a, b, T) \equiv -\frac{\partial E_C}{\partial a} \approx \frac{k_B T}{a} \left( 1 - \frac{\pi b}{6a} + \frac{b}{a} S \left( \frac{b}{a} \right) \right) \\
F_{C,b}(a, b, T) \equiv -\frac{\partial E_C}{\partial b} \approx \frac{k_B T}{a} \left( \frac{\pi}{6} - S \left( \frac{b}{a} \right) \right) \quad (16)
\]
in which
\[ S(x) \equiv \sum_{n,m=1}^{\infty} 4n\pi \exp(-2mn\pi x); \quad x \leq 1. \] (17)

One can see that the forces (16) are just twice the corresponding forces in Ref. [27], where the contribution of the flexural modes had not been taken into account. Hence the contribution of the flexural modes, to the phononic Casimir forces of the Graphene sheet, equals just twice the contribution of transverse/longitudinal modes.

Now, as shown in Ref. [27], the above phononic Casimir forces can be interpreted as corrections to the pretension of Graphene sheet;
\[ \tau_a = \tau_0 - \frac{1}{b} F_{C,a} \]
\[ \tau_b = \tau_0 - \frac{1}{a} F_{C,b} \] (18)
in which, \( \tau_0 \) is the Graphene pretension. As a result one can write
\[ \tau_{a,b}(a, b, T) \approx \tau_0 + \Delta_{a,b}(a, b, T) \] (19)
with
\[ \Delta_a(a, b, T) \equiv -\frac{k_B T}{b^2} \left[ \frac{1}{2} - \frac{\pi}{12} \left( \frac{b}{a} \right)^2 + \left( \frac{b}{a} \right)^2 S \left( \frac{b}{a} \right) \right] \]
\[ \Delta_b(a, b, T) \equiv \frac{k_B T}{b^2} \left[ -\frac{\pi}{12} \left( \frac{b}{a} \right)^2 + \left( \frac{b}{a} \right)^2 S \left( \frac{b}{a} \right) \right] \] (20)

By numerical computations, one can see that \( S(x) \geq S(1) \approx 0.02 \), so (the absolute value of) \( \Delta_a \) as well as \( \Delta_b \) increases by decreasing the ratio \( b/a \). However, \( \Delta_a \) is always negative i.e. subtractive to the pretension, while \( \Delta_b \) is negative for \( b/a \gtrsim 0.5 \), and positive, i.e. additive to the pretension, for \( b/a \lesssim 0.5 \).

For a narrow trench \((b/a \ll 1)\) we can write
\[ \tau_{a,b}(a, b, T) \approx \tau_0 \mp \frac{k_B T}{a^2} S \left( \frac{b}{a} \right); \quad b/a \ll 1 \] (21)
in which “+” (“−”) corresponds to the index “a” (“b”). For instance, for a suspended fully clamped Graphene sheet on a narrow trench with \( a = 1 \mu m, b = 1 \text{nm}, \) at room temperature (\( \approx 300\text{K} \)), using \( S(0.001) \approx 5.2 \times 10^5 \) one obtains
\[ \tau_{a,b}(1 \mu m, 1 \text{nm}, 300\text{K}) - \tau_0 \approx \mp 2.1 \text{ mN/m}. \] (22)
This is a considerable value compared to the experimental values for the Graphene pretension \( \sim 1–10 \text{ mN/m}, \) see Refs. [20][22]. Note that the above correction is one order of magnitude larger than that of Ref. [27], which is a consequence of taking the contribution of flexural modes into account, as we mentioned before. However for
rectangular trenches having widths \((b)\) larger than few nanometer, one can see that the correction \((21)\) would be negligible.

The change in the Graphene tensions \((\Delta \tau_{a,b})\) due to the change \(\Delta T\) in the Graphene temperature, can be written as

\[
\Delta \tau_{a,b} \approx \pm \frac{k_B \Delta T}{a^2} S \left( \frac{b}{a} \right)
\]

Note that these corrections generally break the (tensinal) isotropy of the Graphene \((\tau_a \neq \tau_b)\). As a result, the fundamental resonance frequency \((f_{11})\) of the Graphene sheet \([15,24]\), changes by temperature as

\[
\Delta f_{11}^2 = \frac{\sigma}{4a \rho_0} \left( \frac{\Delta \tau_a}{a^2} + \frac{\Delta \tau_b}{b^2} \right) \\
\approx \frac{\sigma k_B S (b/a)}{4a^2 \rho_0} \Delta T.
\]

in which, \(\Delta f_{11}^2\) is the change in the square-value of \(f_{11}\), \(\sigma = 340 \text{N/m}\) and \(\rho_0 = 7.4 \times 10^{-7} \text{kg/m}^2\) are the in-plane stiffness and the density of the mono-layer Graphene, respectively, and \(\alpha = \rho_{\text{total}}/\rho_0 \sim 1–10\) is the adsorbed mass coefficient, see also \([14,20,21]\).

### 3 Doubly clamped Graphene

For a doubly clamped Graphene sheet suspended on a narrow trench as in Fig. 2, the phononic zero-point energy \([3]\) takes the form

\[
E_0(a, b, T) = \frac{k_B T}{2} \int_0^\infty \frac{d\lambda I}{2} \sum_{i=-\infty}^{\infty} \left( \sum_{l=-L,T} \ln \left[ l^2 + \lambda I^2 \left( y^2 k^2 + n^2 \right) \right] \\
+ \ln \left[ l^2 + \lambda F^2 \left( y^2 k^2 + n^2 \right)^2 \right] \right).
\]

in which \(y \equiv a/L\), \(\lambda_I = \theta_I/T\) and \(\lambda_F = \theta_F/T\) with the effective temperatures

\[
\theta_I = \frac{\hbar \nu_I}{2ak_B}, \quad \theta_F = \frac{\pi \hbar \alpha}{2a^2k_B}.
\]
As a result similar to Eq. (6) one can write

\[
E_0(a, b, T) \approx -\frac{k_B T}{2} \lim_{s \to 0} \frac{\partial}{\partial s} \left( \sum_{l=L,R} \int_0^\infty dt \frac{t^s}{\Gamma(s)} \sum_{n=1}^\infty \exp \left[-t\lambda_l^2(y^2 k^2 + n^2)\right] \right)
+ \int_0^\infty dt \frac{t^s}{\Gamma(s)} \sum_{n=1}^\infty \exp \left[-t\lambda_F^2(y^2 k^2 + n^2)^2\right]; \quad T \gg \theta_{L,T,F}
\] (27)

with \( k \)'s as the wavenumbers of continuous acoustic modes propagating parallel to the trench. Note that at room temperature, the above approximation is valid for \( a \gtrsim 0.5 \text{nm} \).

The contribution of T- and L-modes (in the first line of the above equation) can be directly simplified by using a Gaussian integral for “\( k \)” and a zeta regularization for the \( n \)-sum;

\[
\int_0^\infty \frac{dt}{t} \frac{t^s}{\Gamma(s)} \sum_{n=1}^\infty \exp \left[-t\lambda_l^2(y^2 k^2 + n^2)\right] = \frac{\sqrt{\pi}}{2y\lambda_l^{2s}} \int_0^\infty \frac{dt}{t} \frac{t^{s-1/2}}{\Gamma(s)} \sum_{n=1}^\infty \exp \left[-tn^2\right]
= \frac{\sqrt{\pi}}{2y\lambda_l^{2s}} \frac{\Gamma(s - \frac{1}{2})}{\Gamma(s)} \zeta(2s - 1)
\] (28)

Similar calculation can be done for the contribution of F-modes (in the second line of Eq. (27)), after a reparametrization;

\[
\int_0^\infty \frac{dt}{t} \frac{t^s}{\Gamma(s)} \sum_{n=1}^\infty \exp \left[-t\lambda_F^2(y^2 k^2 + n^2)^2\right] = \lambda_F^{2s} \int_0^\infty \frac{dt}{t} \frac{t^{2s}}{\Gamma(2s)} \sum_{n=1}^\infty \exp \left[-t(y^2 k^2 + n^2)\right]
= \frac{\sqrt{\pi}}{2y\lambda_F^{2s}} \frac{\Gamma(2s - \frac{1}{2})}{\Gamma(2s)} \zeta(4s - 1)
\] (29)

Finally substituting Eq. (28) and (29) into the zero-point energy (27) and using the relations (14), one can obtain the phononic Casimir energy of the doubly clamped Graphene;

\[
E_C(a, T) \approx -\frac{\pi k_B T L}{6a}; \quad T \gg \theta_{L,T,F}, \quad a \ll L
\] (30)

As a result, similarly as the previous section, the corrected pretension of the Graphene can be given as

\[
\tau(a, T) = \tau_0 - \frac{1}{L} F_C
\approx \tau_0 + \frac{\pi k_B T}{6a^2}
\] (31)

As is seen, the correction term is always additive. For a narrow trench having the width \( a = 1 \text{nm} \), at room temperature, one obtain

\[
\tau(1\text{nm}, 300K) - \tau_0 \approx 2.2 \text{ mN/m}
\] (32)

which is again considerable compared to the experimental values \( \tau_0 \sim 1–10 \text{mN/m} \) \cite{20,22}. Note that the above correction for the doubly clamped Graphene is approximately
equal to the corresponding correction for the Fully clamped Graphene, see Eq. (22). The tension change in terms of temperature change can be written as

$$\Delta \tau \approx \frac{\pi k_B \Delta T}{6a^2}. \hspace{1cm} (33)$$

which changes the first resonance frequency of the doubly clamped Graphene as

$$\Delta f_{11}^2 = \frac{\sigma}{4\alpha \rho_0} \Delta \tau \approx \frac{\pi \sigma}{24\alpha \rho_0} \frac{k_B \Delta T}{a^2}. \hspace{1cm} (34)$$

4 Concluding remarks

We have obtained the Casimir energy for acoustic phonons in fully as well as doubly clamped Graphene sheet suspended on a trench, at finite temperature. We have introduced Phononic Casimir forces as changes in the mentioned Casimir energy due to infinitesimally changing the trench sidelengths, and interpreted these Casimir forces as corrections to the Graphene pretension. Then by numerical computations, for narrow trenches of 1nm width at room temperature, we have obtained values in the order of few mN/m for these corrections, which are considerable compared to the experimental values of Refs. [20,22]. It is also interesting to note that the values of these corrections increase by increasing the temperature, see Eqs. (21) and (33), while the Graphene pretension decreases by increasing the temperature [23,25]. Hence these corrections can be even more considerable by increasing the temperature, and so they can be utilized for tuning the Graphene pretension.

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References

[1] M. Bordag, U. Mohideen and V. M. Mostepanenko, Phys. Rep. 353, 1205 (2001).
[2] K. A. Milton, The Casimir Effect (World Scientific, Singapore, 2001).
[3] S. K. Lamoreaux, Reports on Progress in Physics, 68,1 (2004)
[4] M. Kardar and R. Golestanian, Rev. Mod. Phys. 71, 1233 (1999).
[5] G. L. Klimchitskaya and V. M. Mostepanenko Contemporary Physics, 47,3 (2006)
[6] G. L. Klimchitskaya, U. Mohideen and V. M. Mostepanenko, Rev. Mod. Phys., 81, 4, (2009)
[7] Neto, A.C., Guinea, F., Peres, N.M., Novoselov, K.S. and Geim, A.K., 2009. The electronic properties of graphene. Reviews of modern physics, 81(1), p.109.

[8] D.S.L. Abergela, V. Apalkovb, J. Berashevicha, K. Zieglerc and T. Chakraborty, Advances in Physics 59, 261–482, 2010

[9] A. K. Geim, Science, 2009, 324, 5934, 1530–1534

[10] J. S. Bunch, A. M. Van Der Zande, S. S. Verbridge, I. W. Frank, D. M. Tanenbaum, J. M. Parpia, H. G. Craighead, P. L. McEuen, Science 2007, 315 (5811), 490–493.

[11] J. T. Robinson, M. Zalalutdinov, J. W. Baldwin, E. S. Snow, Z. Wei, P. Sheehan, B. H. Houston, Nano Lett. 2008, 8 (10), 3441–3445.

[12] Craighead, H. G. (2000), Science 290(5496): 1532–1535.

[13] Ekinci, K. L. and M. L. Roukes (2005), Review of Scientific Instruments 76(6): 061101

[14] Bunch J. S., Mechanical And Electrical Properties Of Graphene Sheets, PhD Thesis, Cornell University (2008)

[15] Van Der Zande A., The structure and mechanics of atomically-thin graphene membranes, PhD Thesis, Cornell University (2011).

[16] S. P. Koenig, N. G. Boddeti, M. L. Dunn, J. S. Bunch, Nature Nanotechnology 6, 543–546 (2011)

[17] C. Lee, X. Wei, J.W. Kysar, J. Hone, Science 321 (2008) 385–388.

[18] R.A. Barton, B. Ilic, A.M. van der Zande, W.S. Whitney, P.L. McEuen, J.M. Parpia, H.G. Craighead, Nano Lett. 11 (2011) 1232–1236.

[19] T. Yoon, W. C. Shin, T. Y. Kim, J. H. Mun, T. S. Kim, and B. J. Cho, Nano Lett. 2012, 12, 1448–1452

[20] J.S. Bunch, S.S. Verbridge, J.S. Alden, A.M. van der Zande, J.M. Parpia, H.G. Craighead, P.L. McEuen, Nano Lett. 8 (2008) 2458-2462.

[21] C. Gómez-Navarro, M. Burghard, and K. Kern, Nano Lett. 8, 7, 2008

[22] J. W. Suk, R. D. Piner, J. An, and R. S. Ruoff, ACS Nano, 4, 11, 6557–6564 (2010)

[23] C. Chen, S. Rosenblatt, K. I. Bolotin, W. Kalb, P. Kim, I. Kymissis, H. L. Stormer, T. F. Heinz, and J. Hone, Nature Nanotechnology 4, 861 - 867 (2009)

[24] A. M. van der Zande, R. A. Barton, J. S. Alden, C. S. Ruiz-Vargas, W. S. Whitney, P. H. Q. Pham, J. Park, J. M. Parpia, H. G. Craighead, and P. L. McEuen, Nano Lett. 2010, 10, 4869–4873

[25] V. Singh, S. Sengupta, H. S. Solanki, R. Dhall, A. Allain, S. Dhara, M. M. Deshmukh, Nanotechnology 2010, 21 (16), 165204.
[26] Koohsarian, Y. and Shirzad, A., Progress of Theoretical and Experimental Physics, 2014 (11).

[27] Koohsarian, Y., Javidan, K. and Shirzad, A., 2017 , EPL (Europhysics Letters), 119(4), p.48002.

[28] M. Bordag, I. V. Fialkovsky, D. M. Gitman, and D. V. Vassilevich, Phys. Rev. B 80, 245406 (2009).

[29] I. V. Fialkovsky, V. N. Marachevsky, and D. V. Vassilevich, Phys. Rev. B 84, 035446 (2011)

[30] M. Bordag, B. Geyer, G. L. Klimchitskaya, and V. M. Mostepanenko, Phys. Rev. B 74, 205431, 2006

[31] D. Drosdoff and L. M. Woods, Phys. Rev. B 82, 155459 (2010)

[32] E. Pop, V. Varshney, and A. K. Roy, MRS Bull. 37, 1273 (2012).

[33] M. Bordag, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, Advances in the Casimir Effect, Oxford Science Publications, 2009.

[34] E. C. Titchmarsh, Introduction to the Theory of Fourier Integrals, Oxford University Press, Oxford (1948).