Trusted center verification model and classical channel remote state preparation

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Abstract

The classical channel remote state preparation (ccRSP) is an important two-party primitive in quantum cryptography. Alice (classical polynomial-time) and Bob (quantum polynomial-time) exchange polynomial rounds of classical messages, and Bob finally gets random single-qubit states while Alice finally gets classical descriptions of the states. In [T. Morimae, arXiv:2003.10712], an information-theoretically-sound non-interactive protocol for the verification of quantum computing was proposed. The verifier of the protocol is classical, but the trusted center is assumed that sends random single-qubit states to the prover and their classical descriptions to the verifier. If the trusted center can be replaced with a ccRSP protocol while keeping the information-theoretical soundness, an information-theoretically-sound classical verification of quantum computing is possible, which solves the long-standing open problem. In this paper, we show that it is not the case unless BQP is contained in MA. We also consider a general verification protocol where the verifier or the trusted center first sends quantum states to the prover, and then the prover and the verifier exchange a constant round of classical messages. We show that the first quantum message transmission cannot be replaced with a ccRSP protocol while keeping the information-theoretical soundness unless BQP is contained in AM. Furthermore, we also study the verification with the computational soundness. We show that if a ccRSP protocol satisfies a certain condition even against any quantum polynomial-time malicious prover, the replacement of the trusted center with the ccRSP protocol realizes a computationally-sound classical verification of quantum computing. The condition is weaker than the verifiability of the ccRSP. At this moment, however, there is no known ccRSP protocol that satisfies the condition. If a simple construction of such a ccRSP protocol is found, the combination of it with the trusted center verification model provides another simpler and modular proof of the Mahadev’s result. We finally show that the trusted center model and its variant with the ccRSP have extractors for low-energy states.

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I. INTRODUCTION

Whether quantum computing is classically verifiable or not is one of the most important open problems in quantum information science [1–3]. If the soundness is the computational one, the Mahadev’s breakthrough [4] solves the open problem affirmatively. Or, if more than two provers, who are entangled but not allowed to communicate with each other, are allowed, the information-theoretical soundness is possible for a classical verifier [5–9]. In this paper, we focus on the single prover setup and the information-theoretical soundness (except for Secs. V and VII). Furthermore, we require that the honest prover is quantum polynomial-time, and therefore the well-known fact $\text{BQP} \subseteq \text{IP}$ does not solve the open problem.

In Ref. [10], an information-theoretically-sound non-interactive protocol for the verification of quantum computing was proposed. In this protocol, the verifier is classical, but the trusted center is assumed. The trusted center first sends random BB84 states (i.e., $|0\rangle$, $|1\rangle$, $|+\rangle \equiv \frac{|0\rangle + |1\rangle}{\sqrt{2}}$, and $|-\rangle \equiv \frac{|0\rangle - |1\rangle}{\sqrt{2}}$) to the prover, and their classical descriptions to the verifier. The prover then sends a classical message to the verifier. The verifier finally does classical polynomial-time computing to make the decision. (For details, see Ref. [10]. In Sec. III of this paper, we explain the protocol for the convenience of readers.)

The classical channel remote state preparation (ccRSP) is an important primitive in quantum cryptography. It is a two-party protocol between Alice and Bob where Alice is classical polynomial-time, and Bob is quantum polynomial-time. Alice and Bob exchange polynomial rounds of classical messages, and Bob finally gets random single-qubit states while Alice finally gets their classical descriptions. The concept of the remote state preparation was first introduced in Ref. [11] in the context of blind quantum computing. Ref. [12] studies the remote state preparation in an abstract framework for blind quantum computing. Computationally-secure ccRSP protocols have been constructed under the standard assumption in cryptography that the LWE is hard for quantum computing [13–16].

If the trusted center of the protocol of Ref. [10] can be replaced with a ccRSP protocol while keeping the information-theoretical soundness, the information-theoretically-sound classical verification of quantum computing is possible, which solves the open problem affirmatively. In this paper, we show that it is not the case unless $\text{BQP} \subseteq \text{MA}$. Because $\text{BQP} \subseteq \text{MA}$ is not believed to happen, our result suggests that the trusted center cannot
be replaced with the ccRSP while keeping the information-theoretical soundness. (Actually, what we obtain is a slightly stronger result, $\text{BQP} \subseteq \text{MA}_{\text{BQP}}$, where $\text{MA}_{\text{BQP}}$ is MA with honest quantum polynomial-time Merlin. Because $\text{MA}_{\text{BQP}} \subseteq \text{MA}$, we obtain $\text{BQP} \subseteq \text{MA}$.)

The no-go result can be shown even for approximate ccRSP protocols where the prover and the verifier succeed with some probability $p_{\text{succ}}$ even if the prover is honest, and what the prover gets is close to the ideal state.

Replacing the trusted center of Ref. [10] with the ccRSP is a natural approach to solve the open problem, but our result shows that it does not work. It does not mean the impossibility of the (information-theoretically sound) classical verification of quantum computing, because there might be another approach, but at this moment we do not know any promising approach. (For example, the combination of the Fitzsimons-Kashefi (FK) protocol [17] with the ccRSP will not work, because the malicious unbounded prover can learn all trap information. See Appendix [B]) On the other hand, showing the impossibility of the (information-theoretically sound) classical verification of quantum computing is also difficult, because it means the separation between BQP and BPP. (If we define $\text{IP}_{\text{BQP}}$ as the set of decision problems that are verified by an IP protocol with an honest quantum polynomial-time prover, we have $\text{BPP} \subseteq \text{IP}_{\text{BQP}} \subseteq \text{BQP}$. Therefore, $\text{IP}_{\text{BQP}} \neq \text{BQP}$ means $\text{BPP} \neq \text{BQP}$.)

We also consider a general verification protocol where the verifier or the trusted center first sends quantum states to the prover, and then the prover and the verifier exchange a constant round of classical messages. We show that the first quantum message transmission cannot be replaced with a ccRSP protocol unless BQP is contained in AM. (More precisely, what we actually obtain is $\text{BQP} \subseteq \text{IP}_{\text{BQP}}[\text{const}]$, where $[\text{const}]$ means a constant round, but it leads to $\text{BQP} \subseteq \text{AM}$ because $\text{IP}_{\text{BQP}}[\text{const}] \subseteq \text{IP}[\text{const}] \subseteq \text{AM}$.)

The second proof technique can also be applied to show that replacing the trusted center in the protocol of Ref. [10] with the ccRSP is impossible unless BQP $\subseteq$ AM, but we can show a stronger result, namely, BQP $\subseteq$ MA, by using the specific structure of the protocol of Ref. [10].

We also study the verification with the computational soundness. We show that if a ccRSP protocol satisfies a certain condition even against any quantum polynomial-time malicious prover, the replacement of the trusted center of the protocol of Ref. [10] with the ccRSP protocol realizes a computationally-sound classical verification of quantum computing. The
The condition is weaker than the verifiability of the ccRSP. It was believed that the verifiability of a ccRSP is necessary if it is used as a subroutine of a protocol of the verification of quantum computing, but this result suggests that it is not necessarily the case. At this moment, however, no ccRSP protocol is known that satisfies the condition. If a ccRSP protocol that satisfies the condition is constructed in a simple way, the combination of it with the protocol of Ref. [10] provides another simpler and modular proof of the Mahadev’s result.

The condition is satisfied in the protocol where the prover sends quantum states to the verifier and the verifier does measurements. It means that we can construct an off-line-quantum verification protocol where the quantum message is sent from the prover to the verifier.

We also show that the trusted center model and its variant with the ccRSP have extractors for low-energy states. A quantum proof of quantum knowledge was first introduced in Refs. [18, 19], and a classical proof of quantum knowledge was introduced in Ref. [20].

Finally, let us mention a recent related work. The paper [21] showed three results on the ccRSP in the context of blind quantum computing. First, they showed that the ccRSP cannot be composable secure under the no-cloning theorem. There is, however, a possibility that the BFK protocol [22] combined with a ccRSP protocol is still composable secure. Their second result is that it is not the case unless the no-signaling principle is violated. Finally, they showed that the BFK protocol combined with the Qfactory protocol [15] satisfies the game-based security.

This paper is organized as follows. In Sec. II we review the verification protocol of Ref. [10]. In Sec. III we show our first result, and then in Sec. IV we show the second result on the general setup. We study the verification with the computational soundness in Sec. V. We introduce the off-line-quantum verification protocol with quantum communication from the prover to the verifier in Sec. VI. We finally show the existence of extractors in Sec. VII. The computational soundness is considered only in Sec. V and Sec. VII. In other sections, we implicitly assume that the malicious prover is unbounded.

II. THE VERIFICATION PROTOCOL OF REF. [10]

In this section, we review the verification protocol of Ref. [10]. The protocol is given in Fig. II. It was shown in Ref. [10] that the protocol can verify any BQP problem:
**Theorem 1 (Ref. [10])** For any promise problem $A = (A_{yes}, A_{no})$ in BQP, Protocol [II] satisfies both of the following with some $c$ and $s$ such that $c - s \geq \frac{1}{\text{poly}(|x|)}$:

- If $x \in A_{yes}$, the honest quantum polynomial-time prover’s behavior makes the verifier accept with probability at least $c$.

- If $x \in A_{no}$, the verifier’s acceptance probability is at most $s$ for any (even unbounded) prover’s deviation.

In Ref. [10], the completeness and the soundness are shown by introducing virtual protocols where the prover teleports quantum states to the verifier. In Appendix A, we give a direct proof of the completeness and the soundness for the convenience of the readers.

**III. REPLACEMENT OF THE TRUSTED CENTER**

Let us consider Protocol [2], which is the same as Protocol [I] except that the trusted center is replaced with a ccRSP protocol. As a ccRSP, we consider an approximate one: if the prover behaves honestly, the verifier and the prover succeed with probability $p_{\text{succ}}$. If they are successful, the verifier gets $(h, m) \in \{0, 1\}^{N+1}$ and the prover gets an $N$-qubit state $\sigma_{h,m}$ with probability $P(h, m)$, where

$$\frac{1}{2} \left\| \sum_{h,m} P(h, m) \sigma_{h,m} - \frac{1}{2^{N+1}} \sum_{h,m} \bigotimes_{j=1}^{N} H^h |m_j\rangle \langle m_j| H^h \right\|_1 \leq \epsilon$$

is satisfied for a certain small $\epsilon$. Even if the prover behaves honestly, they fail with probability $1 - p_{\text{succ}}$. Furthermore, we assume that $p_{\text{succ}}$ is samplable in classical polynomial-time, which is a reasonable assumption because the description of the ccRSP protocol is known to the verifier.

We show that such a modified protocol is not an information-theoretically-sound verification protocol unless $\text{BQP} \subseteq \text{MA}_{\text{BQP}}$.

Before stating the result, let us define the class $\text{MA}_{\text{BQP}}$.

**Definition 1** A promise problem $A = (A_{yes}, A_{no})$ is in $\text{MA}_{\text{BQP}}$ if and only if there exists a classical probabilistic polynomial-time verifier such that

- If $x \in A_{yes}$, there exists a quantum polynomial-time prover that sends a classical polynomial-length bit string to the verifier such that the verifier accepts with probability at least $\frac{2}{3}$.
0. The input is an instance \( x \in A \) of a promise problem \( A = (A_{\text{yes}}, A_{\text{no}}) \) in BQP, and a corresponding \( N \)-qubit local Hamiltonian

\[
H = \sum_{i<j} \frac{p_{i,j}}{2} \left( \frac{I^{\otimes N} + s_{i,j} X_i \otimes X_j}{2} + \frac{I^{\otimes N} + s_{i,j} Z_i \otimes Z_j}{2} \right)
\]

with \( N = \text{poly}(|x|) \) such that if \( x \in A_{\text{yes}} \) then the ground energy is less than \( \alpha \), and if \( x \in A_{\text{no}} \) then the ground energy is larger than \( \beta \) with \( \beta - \alpha \geq \frac{1}{\text{poly}(|x|)} \). Here, \( I \equiv |0\rangle\langle 0| + |1\rangle\langle 1| \) is the two-dimensional identity operator, \( X_i \) is the Pauli \( X \) operator acting on the \( i \)th qubit, \( Z_i \) is the Pauli \( Z \) operator acting on the \( i \)th qubit, \( p_{i,j} > 0 \), \( \sum_{i<j} p_{i,j} = 1 \), and \( s_{i,j} \in \{+1, -1\} \).

1. The trusted center uniformly randomly chooses \((h, m_1, ..., m_N) \in \{0, 1\}^{N+1} \). The trusted center sends \( \bigotimes_{j=1}^{N} (H^h |m_j\rangle) \) to the prover. The trusted center sends \((h, m)\) to the verifier, where \( m \equiv (m_1, ..., m_N) \in \{0, 1\}^N \).

2. The prover does a POVM measurement \( \{\Pi_{x,z}\}_{x,z} \) on the received state. When the prover is honest, the POVM corresponds to the teleportation of a low-energy state \(|E_0\rangle\) of the local Hamiltonian \( H \) as if the states sent from the trusted center are halves of Bell pairs. The prover sends the measurement result, \((x, z)\), to the verifier, where \( x \equiv (x_1, ..., x_N) \in \{0, 1\}^N \) and \( z \equiv (z_1, ..., z_N) \in \{0, 1\}^N \).

3. The verifier samples \((i, j)\) with probability \( p_{i,j} \), and accepts if and only if \( \langle -1 \rangle^{m_i'} \langle -1 \rangle^{m_j'} = -s_{i,j} \), where \( m_i' \equiv m_i \oplus (h z_i + (1-h) x_i) \).

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**FIG. 1.** The verification protocol of Ref. [10].

- If \( x \in A_{\text{no}} \), for any polynomial-length classical bit string from the prover (who can be unbounded), the verifier’s acceptance probability is at most \( \frac{1}{3} \).

It is easy to show that \( \text{MA}_{\text{BQP}} \subseteq \text{MA} \). Now let us show our first result.

**Theorem 2** Assume that Protocol 2 can verify any BQP problem. It means that for any promise problem \( A = (A_{\text{yes}}, A_{\text{no}}) \) in BQP, Protocol 2 satisfies both of the following with some \( c \) and \( s \) such that \( c - s \geq \frac{1}{\text{poly}(|x|)} \):

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• If $x \in A_{yes}$, the honest quantum polynomial-time prover’s behavior makes the verifier accept with probability at least $c$.

• If $x \in A_{no}$, the verifier’s acceptance probability is at most $s$ for any (even unbounded) prover’s deviation.

Then, $\text{BQP} \subseteq \text{MA}_{\text{BQP}}$.

0. The same as the step 0 of Protocol [1].

1. The verifier and the prover run a ccRSP protocol. If the prover behaves honestly, they succeed with probability $p_{\text{succ}}$. If they are successful, the verifier gets $(h, m_1, ..., m_N) \in \{0, 1\}^{N+1}$ and the prover gets an $N$-qubit state $\sigma_{h,m}$ with probability $P(h, m)$. If they fail, the verifier rejects.

2. The same as the step 2 of Protocol [1].

3. The same as the step 3 of Protocol [1].

FIG. 2. The modified protocol.

Before showing a proof, there is a remark. It is clear from the proof that what we require for the ccRSP is only the (approximate) correctness. Neither the blindness nor the verifiability is required: The correctness means that Alice and Bob get correct outputs when they are honest. In the present case, the correct outputs are $\bigotimes_{j=1}^{N} H^h |m_j\rangle$ for Bob and uniformly random $(h, m) \in \{0, 1\}^{N+1}$ for Alice. Usually when we use a ccRSP, we require the blindness or the verifiability. The blindness means that $h$ or $m$ are “hidden” to even malicious Bob, and the verifiability means that even if Bob is malicious Alice can guarantee that Bob gets the correct state (up to Bob’s operation). Our theorem requires the ccRSP to satisfy only the minimum requirement, namely, the correctness. (Furthermore, not the exact correctness, but the approximate correctness is enough.)
Proof. Let $A = (A_{yes}, A_{no})$ be any BQP promise problem. For any yes instance $x \in A_{yes}$, the verifier’s acceptance probability $p_{acc}^{honest}(x)$ of Protocol 2 is

$$
p_{acc}^{honest}(x) = p_{succ} \sum_{h,m} P(h, m) \sum_{x,z} \text{Tr}(\Pi_{x,z} \sigma_{h,m}) \sum_{i<j} p_{i,j} \frac{1 - s_{i,j}(-1)^{m'_i + m'_j}}{2}$$

$$= p_{succ} \sum_{x,z} \text{Tr}[\Pi_{x,z} \sum_{h,m} P(h, m) \sigma_{h,m}] \sum_{i<j} p_{i,j} \frac{1 - s_{i,j}(-1)^{m'_i + m'_j}}{2}$$

$$\leq p_{succ} \sum_{x,z} \text{Tr}[\Pi_{x,z} \frac{1}{2^{N+1}} \sum_{h,m} (H^{\otimes N})^h |m\rangle \langle m| (H^{\otimes N})^h] \sum_{i<j} p_{i,j} \frac{1 - s_{i,j}(-1)^{m'_i + m'_j}}{2} + \epsilon$$

$$= p_{succ}[1 - \text{Tr}(H|E_0\rangle \langle E_0|)] + \epsilon,$$

(1)

where $|m\rangle \equiv \bigotimes_{j=1}^{N} |m_j\rangle$. For the last equality, see Appendix A.

Let $x \in A_{no}$ be any no instance. Let us consider the following malicious unbounded prover’s attack:

1. When the prover and the verifier run the ccRSP protocol, the prover classically simulates prover’s honest quantum behavior. (The verifier cannot distinguish whether the prover is really doing the honest quantum procedure or simulating it classically. See Appendix B) If they are successful, the verifier gets $(h, m) \in \{0,1\}^{N+1}$ with probability $P(h, m)$. The prover can learn $(h, m)$ because the prover has the classical description of $\sigma_{h,m}$. (See Appendix B)

2. If $h = 0$, the prover chooses $(x, z) \in \{0,1\}^N \times \{0,1\}^N$, where $x$ is sampled from a certain distribution $D$, and $z$ is uniformly randomly chosen. The prover sends $(x \oplus m, z)$ to the verifier. Here, $x \oplus m \equiv (x_1 \oplus m_1, ..., x_N \oplus m_N)$. If $h = 1$, the prover chooses $(x, z) \in \{0,1\}^N \times \{0,1\}^N$, where $z$ is sampled from the distribution $D$, and $x$ is uniformly randomly chosen. The prover sends $(x, z \oplus m)$ to the verifier. Here, $z \oplus m \equiv (z_1 \oplus m_1, ..., z_N \oplus m_N)$.
The verifier’s acceptance probability $q_{acc}^{malicious}(x)$ under this prover’s attack is

$$q_{acc}^{malicious}(x) = p_{succ} \sum_{m} P(0, m) \sum_{x,z} \frac{1}{2^N} D(x) \sum_{i<j} p_{i,j} \frac{1}{2} \left(1 - (-1)^{m_i + (x_i + m_i) + m_j + (x_j + m_j)} s_{i,j} \right) + p_{succ} \sum_{m} P(1, m) \sum_{x,z} \frac{1}{2^N} D(z) \sum_{i<j} p_{i,j} \frac{1}{2} \left(1 - (-1)^{m_i + (x_i + m_i) + m_j + (x_j + m_j)} s_{i,j} \right)$$

$$= p_{succ} \sum_{m} P(0, m) \sum_{x,z} \frac{1}{2^N} D(x) \sum_{i<j} p_{i,j} \langle x \mid \frac{I^{\otimes N} - s_{i,j} Z_i \otimes Z_j}{2} \mid x \rangle + p_{succ} \sum_{m} P(1, m) \sum_{x,z} \frac{1}{2^N} D(z) \sum_{i<j} p_{i,j} \langle z \mid \frac{I^{\otimes N} - s_{i,j} Z_i \otimes Z_j}{2} \mid z \rangle$$

$$= p_{succ} \text{Tr} \left[ (I^{\otimes N} - \mathcal{H}_Z) \sum_{k \in \{0,1\}^N} D(k) \ket{k} \bra{k} \right], \quad (2)$$

where $|x\rangle \equiv \otimes_{j=1}^N |x_j\rangle$, $|z\rangle \equiv \otimes_{j=1}^N |z_j\rangle$, $|k\rangle \equiv \otimes_{j=1}^N |k_j\rangle$, and

$$\mathcal{H}_Z \equiv \sum_{i<j} p_{i,j} \frac{I^{\otimes N} + s_{i,j} Z_i \otimes Z_j}{2}.$$

On the other hand, let us consider Protocol 3. For any $x \in A_{yes}$, the verifier’s acceptance probability $q_{acc}^{honest}(x)$ of Protocol 3 is

$$q_{acc}^{honest}(x) = p_{succ} \frac{1}{2} \sum_{h \in \{0,1\}} \sum_{m \in \{0,1\}^N} |m\rangle \langle (H^{\otimes N})^h | E_0 \rangle|^2 \sum_{i<j} p_{i,j} \frac{1}{2} \left(1 - (-1)^{m_i + m_j} s_{i,j} \right)$$

$$= p_{succ} \frac{1}{2} \sum_{h \in \{0,1\}} \sum_{m \in \{0,1\}^N} \langle m \mid (H^{\otimes N})^h | E_0 \rangle \langle E_0 | (H^{\otimes N})^h \sum_{i<j} p_{i,j} \frac{I^{\otimes N} - s_{i,j} Z_i \otimes Z_j}{2} | m \rangle$$

$$= p_{succ} \frac{1}{2} \sum_{h \in \{0,1\}} \text{Tr} \left[ | E_0 \rangle \langle E_0 | (H^{\otimes N})^h \sum_{i<j} p_{i,j} \frac{I^{\otimes N} - s_{i,j} Z_i \otimes Z_j}{2} (H^{\otimes N})^h \right]$$

$$= p_{succ} \text{Tr} \left[ | E_0 \rangle \langle E_0 | (I^{\otimes N} - \mathcal{H}) \right]$$

$$\geq q_{acc}^{honest}(x) - \epsilon,$$

where the last inequality is from Eq. (1).

For any $x \in A_{no}$, the malicious prover samples $m$ from any probability distribution $D$. The verifier’s acceptance probability $q_{acc}^{malicious}(x)$ is

$$q_{acc}^{malicious}(x) = p_{succ} \sum_{m \in \{0,1\}^N} D(m) \sum_{i<j} p_{i,j} \frac{1}{2} \left(1 - (-1)^{m_i + m_j} s_{i,j} \right)$$

$$= p_{succ} \sum_{m \in \{0,1\}^N} D(m) \langle m \mid \sum_{i<j} p_{i,j} \frac{I^{\otimes N} - s_{i,j} Z_i \otimes Z_j}{2} | m \rangle$$

$$= p_{succ} \text{Tr} \left[ (I^{\otimes N} - \mathcal{H}_Z) \sum_{m} D(m) \ket{m} \bra{m} \right]$$

$$= q_{acc}^{malicious}(x).$$
where $|m⟩ \equiv \bigotimes_{j=1}^{N} |m_j⟩$ and the last equality is from Eq. (2). Therefore, if $p_{\text{honest}}^{\text{acc}}$ and $p_{\text{malicious}}^{\text{acc}}$ have $\frac{1}{\text{poly}(|x|)}$ gap, and $\epsilon$ is sufficiently small, then $q_{\text{honest}}^{\text{acc}}$ and $q_{\text{malicious}}^{\text{acc}}$ also have $\frac{1}{\text{poly}(|x|)}$ gap, which means $A$ is in $\text{MA}_{\text{BQP}}$. Hence we have shown that $\text{BQP} \subseteq \text{MA}_{\text{BQP}}$. □

1. If the prover is honest, it uniformly randomly chooses $h \in \{0,1\}$, generates a low-energy state $|E_0⟩$ of the local Hamiltonian $H$, and measures each qubit of $|E_0⟩$ in the computational (Hadamard) basis if $h = 0$ ($h = 1$). The prover sends $m \equiv (m_1, ..., m_N) \in \{0,1\}^N$ to the verifier, where $m_i$ is the measurement result on the $i$th qubit. If the prover is malicious, the prover sends any $m$ to the verifier.

2. The verifier rejects with probability $1 - p_{\text{succ}}$. With probability $p_{\text{succ}}$, the verifier samples $(i,j)$ with probability $p_{i,j}$, and accepts if and only if $(-1)^{m_i + m_j} = -s_{i,j}$.

FIG. 3. The $\text{MA}_{\text{BQP}}$ protocol.

IV. MORE GENERAL SETUP

In this section, we study a more general setup and show a similar no-go result. Let us consider the verification protocol, Protocol 4. In the first step, the verifier (or the trusted center) generates quantum states $\{\rho_i\}_i$. We assume that this quantum process is a simple one (for example, $\rho_i$ is an $N$-tensor product of random BB84 states), because the verifier’s (or the trusted center’s) quantum burden should be minimum. (If the verifier can do complicated quantum computing, there is no point in delegating quantum computing to the prover: the verifier can do the quantum computation by itself. Furthermore, if a trusted center that can do complicated quantum computing is available, the verifier has only to use it instead of interacting with the untrusted prover.)

We show that the first quantum message transmission (step 1) of Protocol 4 cannot be replaced with a ccRSP protocol unless $\text{BQP} \subseteq \text{IP}_{\text{BQP}}[\text{const}]$, where $\text{IP}_{\text{BQP}}[\text{const}]$ is the IP with a constant round and a honest quantum polynomial-time prover. Because $\text{IP}_{\text{BQP}}[\text{const}] \subseteq \text{IP}[\text{const}] \subseteq \text{AM}$, it means $\text{BQP} \subseteq \text{AM}$.
Let us consider Protocol 5 that is equivalent to Protocol 4 except that the first quantum step of Protocol 4 is replaced with a ccRSP protocol. We consider a general setup where the ccRSP protocol is an approximate one: even if the prover is honest, they succeed with probability $p_{\text{succ}}$, and what the prover gets is a state $\rho'_i$ with probability $p'_i$, where $\rho'_i$ is close to $\rho_i$ and $\{p'_i\}_i$ is close to $\{p_i\}_i$. Furthermore, we assume that $p_{\text{succ}}$ is known, $\{p'_i\}_i$ is samplable in classical polynomial-time, and $\rho'_i$ can be generated in quantum polynomial-time. These assumptions are reasonable, because the description of the ccRSP protocol is known to the verifier, and $\{\rho'_i\}_i$ and $\{p'_i\}_i$ are close to $\{\rho_i\}_i$ and $\{p_i\}_i$, respectively.

**Theorem 3** Assume that Protocol 5 can verify any BQP problem. It means that for any promise problem $A = (A_{\text{yes}}, A_{\text{no}})$ in BQP, Protocol 5 satisfies both of the following with some $c$ and $s$ such that $c - s \geq \frac{1}{\text{poly}(|x|)}$:

- If $x \in A_{\text{yes}}$, the honest quantum polynomial-time prover’s behavior makes the verifier accept with probability at least $c$.
- If $x \in A_{\text{no}}$, the verifier’s acceptance probability is at most $s$ for any (even unbounded) prover’s deviation.

Then, $\text{BQP} \subseteq \text{IP}_{\text{BQP}[\text{const}]}$.

**Remark.** Again, the theorem requires only the correctness for the ccRSP. Neither the blindness nor the verifiability is required.

**Proof.** Let $A = (A_{\text{yes}}, A_{\text{no}})$ be any BQP promise problem. For any yes instance $x \in A_{\text{yes}}$, let $p_{\text{acc}}^{\text{honest}}(x)$ be the verifier’s acceptance probability when the prover is honest in Protocol 5.

For any no instance $x \in A_{\text{no}}$, let us consider the following malicious unbounded prover’s attack in Protocol 5:

1. When the prover and the verifier run the ccRSP protocol, the prover classically simulates prover’s honest quantum behavior. (See Appendix B) If they succeed, the verifier gets $[\rho'_i]$ with probability $p'_i$. The prover can learn $[\rho'_i]$, because the prover has the classical description of $\rho'_i$. (See Appendix B)

2. When the prover and the verifier exchange classical messages, the prover does any malicious behavior.
1. The verifier generates a state $\rho_i$ with probability $p_i$, and sends it to the prover. Or, the trusted center generates a state $\rho_i$ with probability $p_i$, sends it to the prover, and sends its classical description $[\rho_i]$ to the verifier.

2. The prover and the verifier exchange a constant round of classical messages. The honest prover is quantum polynomial-time, but the malicious prover is unbounded. The verifier is classical probabilistic polynomial-time.

3. The verifier finally makes the decision.

FIG. 4. The general protocol with quantum channel.

1. The prover and the verifier run a ccRSP protocol. If the prover is honest, with probability $p_{\text{succ}}$, the prover gets a state $\rho'_i$ with probability $p'_i$, and the verifier gets the classical description $[\rho'_i]$ of $\rho'_i$. With probability $1 - p_{\text{succ}}$, they fail, and the prover and the verifier get an error message. If they fail, the verifier rejects.

2. The same as the step 2 of Protocol 4.

3. The same as the step 3 of Protocol 4.

FIG. 5. The general protocol with ccRSP.

Let us consider Protocol 6. For any yes instance $x \in A_{\text{yes}}$, let $q_{\text{acc}}^{\text{honest}}(x)$ be the verifier’s acceptance probability with the honest prover in Protocol 6. Obviously,

$$p_{\text{acc}}^{\text{honest}}(x) = q_{\text{acc}}^{\text{honest}}(x).$$

For any no instance $x \in A_{\text{no}}$, let $q_{\text{acc}}^{\text{malicious}}(x)$ be the verifier’s acceptance probability in Protocol 6 with the malicious prover. It is also easy to see that

$$p_{\text{acc}}^{\text{malicious}}(x) = q_{\text{acc}}^{\text{malicious}}(x).$$
Therefore, if Protocol 5 can verify the promise problem $A$, Protocol 6 can also verify it, which means that $A$ is in $\text{IP}_{\text{BQP}}[\text{const}]$.  

1. With probability $p_{\text{succ}}$, the verifier chooses $i$ with probability $p_i'$ and sends $i$ to the prover.
   
   If the prover is honest, it generates $\rho_i'$. With probability $1 - p_{\text{succ}}$, the verifier rejects.

2. The same as the step 2 of Protocol 5.

3. The same as the step 3 of Protocol 5.

FIG. 6. The $\text{IP}_{\text{BQP}}[\text{const}]$ protocol.

V. COMPUTATIONAL SOUNDNESS

We have seen that the replacement of the trusted center in the protocol of Ref. [10] with the ccRSP does not realize the information-theoretically-sound classical verification of quantum computing. What happens if we consider the computational soundness? In this section, we show that if a ccRSP protocol satisfies a certain condition, the protocol of Ref. [10] with the ccRSP is the classical verification of quantum computing (with the computational soundness).

**Theorem 4** Assume that a ccRSP protocol satisfies the following: For any quantum polynomial-time malicious prover’s deviation, the verier gets $(h, m) \in \{0, 1\}^{N+1}$ with probability

$$P(h, m) \equiv \frac{1}{2} \text{Tr} \left[ \left( I_{B_1}^{\otimes M} \otimes |\phi_{h,m}\rangle\langle\phi_{h,m}|_{B_2} \right) \rho_{B_1,B_2} \left( I_{B_1}^{\otimes M} \otimes |\phi_{h,m}\rangle\langle\phi_{h,m}|_{B_2} \right) \right],$$

and the prover gets a state

$$\sigma_{h,m} \equiv \frac{1}{2P(h, m)} \text{Tr}_{B_2} \left[ \left( I_{B_1}^{\otimes M} \otimes |\phi_{h,m}\rangle\langle\phi_{h,m}|_{B_2} \right) \rho_{B_1,B_2} \left( I_{B_1}^{\otimes M} \otimes |\phi_{h,m}\rangle\langle\phi_{h,m}|_{B_2} \right) \right]$$

(up to a CPTP map on it), where $B_1$ is a subsystem of $M$ qubits, $B_2$ is a subsystem of $N$ qubits, $|\phi_{h,m}\rangle \equiv \bigotimes_{j=1}^{N} H^h|m_j\rangle$, $\rho_{B_1,B_2}$ is any $(M + N)$-qubit state (that could be chosen by the prover), and $\text{Tr}_{B_2}$ is the partial trace over the subsystem $B_2$. Then, if we replace
the trusted center of the protocol of Ref. [10] with the ccRSP protocol, it is the classical verification of quantum computing (with the computational soundness).

Before showing the theorem, we have three remarks. First, note that when

$$\rho_{B_1,B_2} = \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}} \frac{|00\rangle}{\sqrt{2}} + \frac{|11\rangle}{\sqrt{2}}\right)^\otimes N,$$

$$P(h,m) = \frac{1}{2^{N+1}}$$ for any $\langle h,m \rangle$ and $\sigma_{h,m} = \bigotimes_{j=1}^{N} H^h |m_j\rangle \langle m_j| H^h$, which corresponds to the honest prover case.

Second, the above condition is not satisfied against the unbounded malicious prover, because, as is shown in Appendix B, the unbounded malicious prover can get the classical description of $\sigma_{h,m}$ and therefore what the prover gets is not $\sigma_{h,m}$ but, for example, $\sigma_{h,m} \otimes |h,m\rangle \langle h,m|$. Third, it was believed that the verifiability is necessary for a ccRSP protocol when it is used as a subroutine of the verification of quantum computing: even if malicious Bob deviates during the ccRSP protocol, it should be guaranteed that the correct state is generated in Bob’s place (up to his operation on it). Theorem 4 suggests that it is not necessarily the case: as long as it is guaranteed that Bob does the correct measurement (i.e., the projection $|\phi_{h,m}\rangle \langle \phi_{h,m}|$) on any state, the soundness of the verification protocol holds. It is easy to see that the verifiability is a special case of our condition: In our condition, $\rho_{B_1,B_2}$ is any, but the verifiability requires that $\rho_{B_1,B_2}$ is the $N$-tensor product of the Bell pair. Our condition is therefore weaker than the verifiability.

**Proof.** Let $A = (A_{yes}, A_{no})$ be any promise problem in BQP. The completeness is obvious. For any yes instance $x \in A_{yes}$, it is clear that the verifier’s acceptance probability with the honest prover is $p_{acc} = 1 - \text{Tr}(|E_0\rangle \langle E_0| \mathcal{H}) \geq 1 - \alpha$. (See Appendix A)

Let us next consider the soundness. The verifier’s acceptance probability $p_{acc}$ with the
malicious prover is
\[
\rho_{\text{acc}} = \sum_{h,m} P(h,m) \sum_{x,z} \text{Tr}(\Pi_{x,z} \sigma_{h,m}) \sum_{i<j} p_{i,j} \frac{1 - s_{i,j}(-1)^{m'_i+m'_j}}{2}
\]
\[
= \sum_{h,m} P(h,m) \sum_{x,z} \frac{1}{2P(h,m)} \text{Tr} \left[ (\Pi_{x,z} \otimes |\phi_{h,m}\rangle \langle \phi_{h,m}|) \rho_{B_1,B_2} \right] \sum_{i<j} p_{i,j} \frac{1 - s_{i,j}(-1)^{m'_i+m'_j}}{2}
\]
\[
= \frac{1}{2} \sum_{h,m} \sum_{x,z} \text{Tr} \left[ \rho_{B_1,B_2} \right]
\]
\[
\left\{ \Pi_{x,z} \otimes (H \otimes N)^h |m\rangle \langle m| X^{hz+(1-h)x} \sum_{i<j} p_{i,j} \frac{I \otimes N - s_{i,j} Z_i \otimes Z_j}{2} X^{hz+(1-h)x} (H \otimes N)^h \right\}
\]
\[
= \frac{1}{2} \sum_{h} \sum_{x,z} \text{Tr} \left[ \rho_{B_1,B_2} \right]
\]
\[
\left\{ \Pi_{x,z} \otimes (H \otimes N)^h X^{hz+(1-h)x} \sum_{i<j} p_{i,j} \frac{I \otimes N - s_{i,j} Z_i \otimes Z_j}{2} X^{hz+(1-h)x} (H \otimes N)^h \right\}
\]
\[
= \sum_{x,z} \text{Tr} \left[ \rho_{B_1,B_2} \left\{ \Pi_{x,z} \otimes Z^z X^x (I \otimes N - \mathcal{H}) X^z Z^z \right\} \right]
\]
\[
= 1 - \text{Tr}(\mathcal{H}\eta)
\]
\[
\leq 1 - \beta,
\]
where \(X^x \equiv \bigotimes_{j=1}^{N} X^{x^j}, \ Z^z \equiv \bigotimes_{j=1}^{N} Z^{z^j}, \ X^{hz+(1-h)x} \equiv \bigotimes_{j=1}^{N} X^{hz_j+(1-h)x_j}, \ |m\rangle \equiv \bigotimes_{j=1}^{N} |m^j\rangle\),
and
\[
\eta \equiv \text{Tr}_{B_1} \left[ \sum_{x,z} (\sqrt{\Pi_{x,z} \otimes X^z Z^z}) \rho_{B_1,B_2} (\sqrt{\Pi_{x,z} \otimes Z^z X^z}) \right]
\]
is an \(N\)-qubit state. \(\square\)

VI. OFF-LINE-QUANTUM COMMUNICATION FROM PROVER TO VERIFIER

The trusted center model [10] (Protocol I) does not need any quantum communication between the prover and the verifier. The FK protocol requires quantum communication from the verifier to the prover. The posthoc protocol [23] requires quantum communication from the prover to the verifier. A difference between the FK protocol and the posthoc protocol is that the FK protocol is off-line-quantum but the posthoc protocol is on-line-quantum. It means that in the FK protocol, the first quantum message from the verifier to the prover is independent of the instance that the verifier wants to verify, but in the posthoc protocol, the quantum message from the prover to the verifier depends on the instance.
Is it possible to construct a verification protocol with off-line-quantum communication from the prover to the verifier? Theorem 4 answers to the question. The condition of Theorem 4 is satisfied when the prover generates a quantum state $\rho_{B_1,B_2}$, and sends $B_2$ register to the verifier. Let us consider Protocol 7. From Theorem 4 it is easy to see that the protocol is a verification protocol with off-line-quantum communication from the prover to the verifier.

0. The same as the step 0 of Protocol 7.

1. The prover generates a state $\rho_{B_1,B_2}$ and sends the register $B_2$ to the verifier. If the prover is honest, $\rho_{B_1,B_2}$ is the $N$-tensor-product of Bell pairs.

2. The verifier uniformly randomly chooses $h \in \{0,1\}$. If $h = 0$ ($h = 1$) the verifier measures each qubit sent from the prover in the computational (Hadamard) basis. Let $m_j \in \{0,1\}$ be the measurement result on the $j$th qubit ($j = 1, 2, ..., N$).

3. The same as the steps 2 and 3 of Protocol 7.

FIG. 7. The off-line-quantum prover-to-verifier protocol.

VII. EXTRACTORS

In this section, we show that the trusted center verification protocol of Ref. 10 and its variant with the ccRSP studied in Sec. IV have extractors for low-energy states.

Theorem 5 The protocol of Ref. 10 has a quantum polynomial-time extractor that satisfies the following. When a prover $P^*$ makes the verifier accept an instance $x \in A$ with probability at least $1 - \epsilon$, the extractor that oracle accesses to $P^*$ outputs a state $\eta$ whose expectation energy $\text{Tr}(\eta \mathcal{H})$ on the local Hamiltonian $\mathcal{H}$ corresponding to $x$ is less than $\epsilon$.

Proof. The verifier’s acceptance probability $p_{\text{acc}}$ against the prover $P^*$ whose POVM
measurement is \{\Pi_{x,z}\}_{x,z} is
\[
p_{\text{acc}} = \frac{1}{2^{N+1}} \sum_{h,m} \sum_{x,z} \text{Tr} \left[ \Pi_{x,z} (H^o_N)^h |m\rangle \langle m| (H^o_N)^h \right] \sum_{i<j} p_{i,j} \frac{1 - s_{i,j} (-1)^{m_i+m_j}}{2}
\]
\[
= \frac{1}{2^{N+1}} \sum_{m} \sum_{x,z} \langle m| \Pi_{x,z} Z^z X^x \sum_{i<j} p_{i,j} I^o_N - s_{i,j} Z_i \otimes Z_j X^z Z^z |m\rangle
\]
\[
+ \frac{1}{2^{N+1}} \sum_{m} \sum_{x,z} \langle m| H^o_N \Pi_{x,z} Z^z X^x \sum_{i<j} p_{i,j} I^o_N - s_{i,j} X_i \otimes X_j X^z Z^z H^o_N |m\rangle
\]
\[
= 1 - \text{Tr} [\mathcal{H} \eta],
\]
where \( |m\rangle \equiv \bigotimes_{j=1}^{N} |m_j\rangle \), \( X^x \equiv \bigotimes_{j=1}^{N} X^{x_j} \), \( Z^z \equiv \bigotimes_{j=1}^{N} Z^{z_j} \), and
\[
\eta \equiv \frac{1}{2^N} \sum_{x,z} Z^z X^x \Pi_{x,z} X^x Z^z
\]
is an \( N \)-qubit state.

Assume that \( p_{\text{acc}} \geq 1 - \epsilon \). Then, \( \text{Tr} (\mathcal{H} \eta) \leq \epsilon \). The extractor that outputs \( \eta \) can be constructed in the following way. The extractor first generates \( \frac{\mathcal{I}^o_N}{2^N} \). It then does the POVM measurement \( \{\Pi_{x,z}\}_{x,z} \) to obtain the post-measurement state
\[
\sum_{x,z} \sqrt{\Pi_{x,z} \frac{\mathcal{I}^o_N}{2^N}} \sqrt{\Pi_{x,z}} \otimes |x, z\rangle \langle x, z|.
\]
After the application of the controlled-\( XZ \) operation and the tracing out of the second register, the extractor obtains \( \eta \).

**Theorem 6** Assume that a ccRSP protocol satisfies the conditions of Theorem 4 and \( \rho_{B_1, B_2} \) can be generated in quantum polynomial-time. Then, the protocol of Ref. 10 with the ccRSP has a quantum polynomial-time extractor that satisfies the following. When a prover \( P^* \) makes the verifier accept an instance \( x \in A \) with probability at least \( 1 - \epsilon \), the extractor that oracle accesses to \( P^* \) outputs a state \( \eta \) whose expectation energy \( \text{Tr}(\eta \mathcal{H}) \) on the local Hamiltonian \( \mathcal{H} \) corresponding to \( x \) is less than \( \epsilon \).

**Proof.** The verifier’s acceptance probability is
\[
p_{\text{acc}} = \sum_{h,m} P(h, m) \sum_{x,z} \text{Tr} (\Pi_{x,z} \sigma_{h,m}) \sum_{i<j} p_{i,j} \frac{1 - s_{i,j} (-1)^{m_i+m_j}}{2}
\]
\[
= 1 - \text{Tr} (\mathcal{H} \eta),
\]
where \( \eta \) is the \( N \)-qubit state defined by
\[
\eta \equiv \text{Tr}_{B_1} \left[ \sum_{x,z} (\sqrt{\Pi_{x,z}} \otimes X^x Z^z) \rho_{B_1, B_2} (\sqrt{\Pi_{x,z}} \otimes Z^z X^x) \right].
\]
The extractor that outputs $\eta$ can be constructed in the following way. The extractor does the POVM measurement on the $B_1$ register of $\rho_{B_1,B_2}$ to generate

$$\sum_{x,z} (\sqrt{\Pi_{x,z}} \otimes I^\otimes N) \rho_{B_1,B_2} (\sqrt{\Pi_{x,z}} \otimes I^\otimes N) \otimes |x,z\rangle\langle x,z|.$$
prover is

\[
p_{\text{acc}} = \frac{1}{2} \sum_{h \in \{0,1\}} \frac{1}{2^N} \sum_{m \in \{0,1\}^N} \sum_{x \in \{0,1\}^N} \sum_{z \in \{0,1\}^N} \sum_{j=1}^N \left( \prod_{j=1}^N \langle \phi_{x_j,z_j} | \right) \left[ |E_0 \rangle \langle E_0| \otimes (H \otimes \mathcal{N})^h |m\rangle \langle m| (H \otimes \mathcal{N})^h \right] \left( \prod_{j=1}^N |\phi_{x_j,z_j} \rangle \right)
\]

\[
\times \sum_{i<j} p_{i,j} \frac{1 - s_{i,j}(-1)^{m_i'+m_j'}}{2}
\]

\[
= \frac{1}{2} \sum_{h} \frac{1}{2^N} \sum_{m} \sum_{x,z} \left( \frac{1}{2^N} \langle m| (H \otimes \mathcal{N})^h Z^x X^x |E_0\rangle \langle E_0| X^x Z^z (H \otimes \mathcal{N})^h |m\rangle \right)
\]

\[
\times \sum_{i<j} p_{i,j} \frac{1 - s_{i,j}(-1)^{m_i'+m_j'}}{2}
\]

\[
= \frac{1}{2} \sum_{h} \frac{1}{2^N} \sum_{m} \sum_{x,z} \left( \frac{1}{2^N} \langle m| (H \otimes \mathcal{N})^h Z^x X^x |E_0\rangle \langle E_0| X^x Z^z (H \otimes \mathcal{N})^h \right)
\]

\[
\times X^{hz+(1-h)x} \sum_{i<j} p_{i,j} \frac{I \otimes \mathcal{N} - s_{i,j} Z_i \otimes Z_j}{2} X^{hz+(1-h)x} |m\rangle
\]

\[
= \frac{1}{2} \sum_{h} \frac{1}{2^N} \sum_{x,z} \left( \frac{1}{2^N} \langle E_0| \langle E_0| \frac{1}{2} \sum_{h} X^x Z^z (H \otimes \mathcal{N})^h \right)
\]

\[
\times X^{hz+(1-h)x} \sum_{i<j} p_{i,j} \frac{I \otimes \mathcal{N} - s_{i,j} Z_i \otimes Z_j}{2} X^{hz+(1-h)x} \left( (H \otimes \mathcal{N})^h Z^z X^x \right)
\]

\[
= \text{Tr} \left[ |E_0\rangle \langle E_0| (I \otimes \mathcal{N} - \mathcal{H}) \right]
\geq 1 - \alpha.
\]

Here, in the second equality, we have used the following result: for any \(\alpha, \beta, h, m \in \{0,1\}\) and any single-qubit state \(\rho\),

\[
\langle \phi_{\alpha,\beta} | (\rho \otimes H^h) |m\rangle \langle m| H^h |\phi_{\alpha,\beta} \rangle = \frac{1}{2} \langle m| H^h Z^\beta X^\alpha \rho X^\alpha Z^\beta H^h |m\rangle.
\]

Next we show the soundness. Let \(\{\Pi_{x,z}\}_{x,z}\) be the POVM that the malicious prover
applies. The verifier’s acceptance probability is
\[
p_{\text{acc}} = \frac{1}{2} \sum_{h} \frac{1}{2N} \sum_{m} \sum_{x,z} \langle m | (H^\otimes N)^h \Pi_{x,z} (H^\otimes N)^h | m \rangle \sum_{i<j} p_{i,j} \frac{1 - s_{i,j} (-1)^{m'_i + m'_j}}{2}
\]
\[
= \frac{1}{2} \sum_{h} \frac{1}{2N} \sum_{m} \sum_{x,z} \langle m | (H^\otimes N)^h \Pi_{x,z} (H^\otimes N)^h X^{hz+(1-h)x} \times \sum_{i<j} p_{i,j} \frac{I^\otimes N - s_{i,j} Z_i \otimes Z_j}{2} X^{hz+(1-h)x} (H^\otimes N)^h (H^\otimes N)^h | m \rangle
\]
\[
= \frac{1}{2} \sum_{h} \frac{1}{2N} \sum_{x,z} \text{Tr} \left[ (H^\otimes N)^h \Pi_{x,z} (H^\otimes N)^h X^{hz+(1-h)x} \times \sum_{i<j} p_{i,j} \frac{I^\otimes N - s_{i,j} Z_i \otimes Z_j}{2} X^{hz+(1-h)x} (H^\otimes N)^h (H^\otimes N)^h \right]
\leq \text{Tr} \left[ (I^\otimes N - H) \sigma \right]
\leq 1 - \beta,
\]
where \(\sigma \equiv \frac{1}{2N} \sum_{x,z} X^x Z^z \Pi_{x,z} Z^z X^x\), and the last inequality is from the fact that \(\sigma\) is a state because \(\text{Tr}(\sigma) = 1\) and \(\sigma \geq 0\). \(\Box\)

Appendix B: Unbounded prover can learn \((h, m)\)

In this Appendix, we show that the unbounded malicious prover can learn \((h, m)\). Without loss of generality, a ccRSP protocol when the prover is honest is described as follows:

1. The verifier sends a classical message \(a_1\) to the prover.
2. The prover generates a state \(\rho_1(a_1)\).
3. The prover measures some qubits of \(\rho_1(a_1)\) in the computational basis to obtain a result \(b_1\). The prover sends \(b_1\) to the verifier. Let \(\rho'_1(a_1, b_1)\) be the post-measurement state.
4. The verifier sends a classical message \(a_2\) to the prover.
5. The prover applies a unitary on \(\rho'_1(a_1, b_1)\) to generate a state \(\rho_2(a_1, b_1, a_2)\). The prover measures some qubits of \(\rho_2(a_1, b_1, a_2)\) in the computational basis to obtain a result \(b_2\). The prover sends \(b_2\) to the verifier. Let \(\rho'_2(a_1, b_1, a_2, b_2)\) be the post-measurement state.
6. The verifier sends a classical message $a_3$ to the prover.

... 

k. The verifier outputs $(h, m) \in \{0, 1\}^{N+1}$. The prover has a state $\sigma_{h,m} \otimes \rho_{\text{junk}}$.

The unbounded prover can simulate the above process classically as follows:

1. The verifier sends a classical message $a_1$ to the prover.

2. The prover classically computes the classical description of $\rho_1(a_1)$.

3. The prover classically samples $b_1$ with probability $\text{Tr}[(|b_1\rangle\langle b_1| \otimes I)\rho_1(a_1)]$. The prover sends $b_1$ to the verifier. Let $\rho'_1(a_1, b_1)$ be the post-measurement state. The prover classically computes the classical description of $\rho'_1(a_1, b_1)$.

4. The verifier sends a classical message $a_2$ to the prover.

5. The prover classically computes the classical description of $\rho_2(a_1, b_1, a_2)$. The prover classically samples $b_2$ with probability $\text{Tr}[(|b_2\rangle\langle b_2| \otimes I)\rho_2(a_1, b_1, a_2)]$. The prover sends $b_2$ to the verifier. Let $\rho'_2(a_1, b_1, a_2, b_2)$ be the post-measurement state. The prover classically computes the classical description of $\rho'_2(a_1, b_1, a_2, b_2)$.

6. The verifier sends a classical message $a_3$ to the prover.

... 

k. The verifier outputs $(h, m) \in \{0, 1\}^{N+1}$. The prover has a classical description of $\sigma_{h,m} \otimes \rho_{\text{junk}}$.

The verifier cannot distinguish whether the prover is doing the honest quantum procedure or simulating it classically. Because the prover has the classical description of $\sigma_{h,m}$, the prover can learn $(h, m)$.

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