Cascade seesaw for tiny neutrino mass

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Abstract

The accessibility to physics responsible for tiny neutrino mass suggests that the mass should better originate from certain higher dimensional operators. The conventional three types of seesaw operate at dimension five with the help of either a new fermion or scalar multiplet. Here we propose a seesaw that generates neutrino mass through a dimension-$(5+4n)$ operator. The seesaw is functioned by a fermion of isospin $n+1$ and zero hypercharge and a sequence of scalar multiplets that share unity hypercharge but have isospin from $\frac{3}{2}$ to $n+\frac{1}{2}$ at a step of unity. Only the scalar of the highest isospin can couple to the relevant fermions while only the scalar of the lowest isospin can directly develop a naturally small vacuum expectation value (VEV). The VEV is then transmitted to scalars of higher isospin through a cascading process. No global symmetry is required to forbid lower dimensional operators. A neutrino mass of desired order can thus be induced with a relatively low seesaw scale without demanding too small couplings.

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1 Introduction

The tiny neutrino mass can be accommodated in the effective theory of standard model (SM) by higher dimensional operators. The first such operator appears at dimension five and is unique, \( \mathcal{O}_5 \equiv (F_L^C H^*) (F_L H^*) \), where \( H \) and \( F_L \) are the Higgs and left-handed lepton doublets in SM. When \( H \) develops a vacuum expectation value (VEV), \( \langle H \rangle \), the operator yields a neutrino mass of order \( m_\nu \sim \lambda \langle H \rangle^2 / \Lambda \). Here \( \lambda \) is a product of couplings and \( \Lambda \) a typical heavy mass scale of the underlying high energy theory that produces the effective operator at low energy.

It has been known for some time that there are exactly three ways to realize the above operator via tree level interactions of heavy particles with the Higgs and lepton particles in SM [2]. They correspond to the conventional three types of seesaw mechanisms or underlying theories [3]-[9]. It would be highly desirable to discriminate amongst the three theories by looking for other detectable effects. But this is hard to manage since a sub-eV neutrino mass generally implies that the new particles are either extremely heavy or interact with known particles too feebly.

The way out to the above phenomenological problem is clear from the point of view of effective theories. The demand for a heavy scale or small couplings may be alleviated by pushing the neutrino mass operators to even higher dimensions. Although with a single SM Higgs field the operators are again unique at each higher dimension [10,11], one anticipates more possible underlying theories that can realize the operators. But with a lowered scale or enhanced couplings one expects to be able to distinguish them by invoking effects that would otherwise be unobservable. Roughly speaking, there are two approaches to do so. In the first approach, one attributes the tiny neutrino mass to a purely quantum nature arising from radiative effects of heavy particles. For this purpose one generally employs new particles in small irreducible representations of the SM gauge group. However, to forbid the operators to appear at a lower dimension or loop level, one has to design certain global symmetries that are exact or softly broken [12]-[18] or quantum numbers like color [19, 20] that the SM leptons and Higgs do not have. In the second approach, one composes new fields in higher irreducible representations of the gauge group in such a manner that one has to go through several steps of interactions between heavy and SM particles to form a neutrino mass operator at tree level [21]-[30]. This effectively pushes up the latter’s dimension. In this approach one does not appeal to global symmetries, but instead chooses representations judiciously so that low dimensional operators indeed do not occur at tree level. When they appear at a loop level, they are more suppressed than those that are available at tree level if the new particles are not very heavy. It is also possible to combine the two approaches to new variants [31]. For attempts to solve the related flavor hierarchy problem using higher dimensional operators involving scalar fields, see Refs. [32]-[34].

In this paper we work in the spirit of the second approach. We attempt to push the neutrino mass operators to higher dimensions with a minimal set of new heavy particles, though it may be hard to specify what is minimal. For instance, with a heavy fermion multiplet of isospin 3 and a heavy scalar multiplet each of isospin \( \frac{3}{2} \) and \( \frac{5}{2} \), the operator first appears at dimension thirteen. At such a high dimension the tension mentioned above between the tiny neutrino
and viable phenomenology would not be a problem.

The paper is organized as follows. In the next section we motivate our approach by analyzing the quantum numbers of the new fields. Then we show in sec 3 how the scalar fields of increasing isospin develop naturally smaller and smaller VEV’s through cascading interactions with the SM Higgs. The exact form of neutrino mass is spelt out in the last section together with a brief mention of possible phenomenology.

2 Quantum numbers

Our goal is to build with the help of a given set of heavy particles the lowest dimensional operator that gives neutrino mass upon spontaneous symmetry breaking in SM. Such an operator must only involve the SM fields $F_L$ and $H$ and has the unique form, $O_{5+2m} = O_5 (H^*H)^m$. While it requires fields in a higher representation to push up the operator’s dimension, we should compose a minimal set of fields that can do the job. Since at tree level gauge fields cannot enter the relevant operators, we restrict ourselves to new fermions and scalars.

We start with the familiar cases but present them in a manner that motivates our general analysis. Suppose there is a new scalar alone. The fermion factor in a potential mass operator must be $F_L^c F_L$, which has the quantum numbers $I = 1$, $Y/2 = -1$ under the SM gauge group. (We suppress the lepton generation index here but will recover it in the last section.) It would therefore Yukawa couple to a heavy scalar $\xi$ with $I = 1$, $Y/2 = 1$ that would in turn interact with the SM Higgs $H$ via a trilinear coupling. The latter induces a VEV of $\xi$ out of that of $H$. Since the coexistence of the two couplings necessarily breaks lepton number, their product can be naturally small. This is the type II seesaw.

In the opposite case with a new fermion $\chi$ alone, it must couple $F_L$ to $H$ and therefore may have $I = 0, 1$ and $Y/2 = 0, -1$. (The case with $Y/2 = +1$ can be covered using an appropriately conjugated field.) The choice $Y = 0$ corresponds to the type I and III seesaw respectively. A $\chi$ with $Y/2 = -1$ should be vector-like to avoid chiral anomaly. When it is a singlet, it has the same quantum numbers as the SM lepton singlet $f_R$ and thus only offers mixing with $f_R$ but not a neutrino mass. When $\chi$ is a triplet, it couples to $F_L$ and $H$ in the form, $\tilde{F}_L \tau^a H \chi^a$. In addition to the mixing between the singly charged fermions, a linear combination of the neutral components $\chi_L^0$ and $\nu_L$ pairs with $\chi_R^0$ to become a Dirac fermion, while the orthogonal one remains massless. Namely, it does not change the numbers of massless and massive modes. From the viewpoint of seesaw, the lepton number has to be broken to yield a light Majorana neutrino mass, which however is not the case here.

The conclusion from the above analysis is that with new scalars or fermions but not both one cannot get anything else but the conventional seesaws. A question then naturally arises: what do we do to go beyond those seesaws, or what is the general case when new fermion and scalar fields are both present? As this turns out to be too broad a question with many answers, we focus in what follows on the more specific case when there is one new fermion $\Sigma$ with quantum numbers $(I_\Sigma, Y_\Sigma)$ and one new scalar $\Phi$ with $(I_\Phi, Y_\Phi)$. Let us first consider what restrictions should be imposed on them to arrive at a high dimensional mass operator. For definiteness,
we assume without losing generality $\Sigma \geq 0$ and $\Phi \geq 0$, which can always be arranged with the help of conjugate fields. The first restriction, called (R1) below, is that we exclude the choices $(I, \Sigma) = (0,0), (1,0)$ and $(I, \Phi, \Sigma) = (1,2)$ that cover the conventional seesaws, and $(I, \Sigma) = (0,2)$ which only causes trivial mixing of charged fermions. Second, $|I - \Phi| = 1/2$, so that isospin allows to couple $\Phi$ and $\Sigma$ to $F_L$ (R2). With a single chain of fermion lines in a seesaw diagram that starts and ends with $F_L$, the scalar $\Phi$ must develop a VEV if it is relevant to the mass generation at all. It is a separate issue how $\Phi$ can develop a VEV, which will be taken up in the next section. This means that both $I$ and $\Phi$ are half-integer (integral) with $0 \leq \Phi/2 \leq I$, called (R3). Similarly, for $\Sigma$ to be relevant and considering the restriction (R2), both $I_\Sigma$ and $\Sigma_\Phi/2$ must be accordingly integral (half-integral) with $0 \leq \Sigma_\Phi/2 \leq I_\Sigma$. There are thus no fractionally charged particles. The fourth restriction, (R4), comes from the neutrality in $Y$ that allows (R4a) $\Sigma_\Phi + \Phi = 1$ for the Yukawa coupling $(F_L \Sigma \Phi)$, or (R4b) $\Sigma_\Phi - \Phi = 1$ for $(F_L \Sigma \Phi)$, or (R4c) $\Sigma_\Phi - \Phi = -1$ for $(F_L \Sigma \Phi)$. Finally, the lepton number $L$ must be explicitly broken (R5).

Consider first the case when both $H$ and $\Phi$ can be connected to $F_L$, $\Sigma$ to induce a neutrino mass, as shown in Fig. 1(a). The above restrictions also apply to the SM Higgs field $H$. (R2) gives $I_\Sigma = 0, 1$, and (R4) requires $Y_\Sigma = 0$ for $(F_L \Sigma H)$ or $(F_L \Sigma \Phi)$, or $Y_\Sigma = 2$ for $(F_L \Sigma \Phi)$. All choices but $(I, \Sigma) = (1,2)$ are excluded by (R1). Then (R2) implies $I = 1/2$ or $3/2$. If $I = 1/2$, we must have $\Phi = 1$, since with $\Phi = 0$, neither Yukawa coupling of (R4a, R4b, R4c) is possible. This however amounts to a second copy of the SM Higgs with Yukawa coupling $(F_L \Sigma \Phi)$, so that $L$ is conserved. The choice $I = 1/2$ thus has to be discarded. For the remaining choice $I = 3/2$, one may have $\Phi = 1$ for $(F_L \Sigma \Phi)$, or $\Phi = 3$ for $(F_L \Sigma \Phi)$. Only for the option $\Phi = 3$ can one break $L$ together with the scalar potential. This is exactly the model suggested in [29] that corresponds to a dimension seven seesaw. Our above analysis shows that it is a unique option for the seesaw shown in Fig. 1(a).

![Fig. 1 Seesaw via Yukawa couplings](image)

Now we examine the next simplest or more symmetric case in Fig. 1(b) that will give the new mechanism discussed in this work. Here only the new scalar $\Phi$ can couple to $(\Sigma, F_L)$ while the SM Higgs $H$ cannot. First of all, $(I, \Sigma) = (1,2)$ is excluded to avoid coupling $H$ to $(F_L, \Sigma)$ as in Fig. 1(a), together with those cases covered in (R1). Next, we choose from (R4) two forms of Yukawa couplings involving $(\Phi, \Sigma, F_L)$ that together with the scalar potential will violate $L$. The combination (R4a,R4b) preserves lepton number and is thus dropped, while the combination (R4b,R4c) is simply not possible. This leaves us with the single choice (R4a,R4c) that yields $Y_\Sigma = 0$ and $\Phi = 1$. Then (R3) implies that $I_\Sigma$ is integral and $I_\Phi$ half-integer. To avoid the type I and III seesaws, we require $I_\Sigma \geq 2$ and then $I_\Phi \geq 3/2$ from (R2). The minimal choice is
\( (I_L, Y_L) = (2, 0) \) and \( (I_\Phi, Y_\Phi) = (3/2, 1) \), which will give a dimension nine mass operator. This is simpler than the model suggested in \([30]\), which has to employ a pair of new scalars with isospin 3/2 due to their different choices of hypercharges for the new fields.

The above analysis for Fig. 1(b) generalizes to arbitrarily high isospin. The single fermion field \( \Sigma \) has quantum numbers \( (I, Y) = (n + 1, 0) \) with \( n \geq 1 \), and the scalar of the highest isospin, \( \Phi^{(n+\frac{3}{2})} \), has \( (I, Y) = (n + 1/2, 1) \). To induce a neutrino mass, \( \Phi^{(n+\frac{3}{2})} \) must develop a naturally small VEV. Since a scalar of isospin \( \frac{3}{2} \) can get a VEV from the \( L \)-violating term \( \sim \kappa \Phi^{(\frac{3}{2})} \tilde{H}H \tilde{H} \) while \( \Phi^{(n+\frac{3}{2})} \) with \( n > 1 \) cannot, the shortest path is to introduce a sequence of scalar multiplets \( \Phi^{(n+\frac{3}{2})} \) that share the same hypercharge but have an isospin decreasing at a step of unity, i.e., \( 1 \leq m \leq n \). These scalars with intermediate isospin cannot couple to \( (F_L, \Sigma) \) due to too small isospin, but will assist \( \Phi^{(n+\frac{3}{2})} \) to develop a VEV via a cascading process to be described in the next section.

A few remarks are in order. It would be tempting to ask in this context why we do not employ two different new scalars in Fig. 1(b). If their isospins are equal but hypercharges different as in the model of \([30]\), this is a matter of simplicity: why should we introduce one more field when one is sufficient to do the job? If their isospins are different, the one with a lower isospin when assigned a correct hypercharge will get a VEV that is less suppressed as we will show in the next section. Thus the seesaw employing one scalar with a lower isospin operates at a lower dimension than the case using two scalars of different isospin, and thus dominates. Second, a fermion with isospin \( n + 1 \) may couple to \( F_L \) through a scalar of either isospin \( n + 1/2 \) or \( n + 3/2 \). But for the same reason as described above, the seesaw mass induced from \( \Phi^{(n+\frac{3}{2})} \) dominates since its VEV is less suppressed compared to that of \( \Phi^{(n+\frac{3}{2})} \). Finally, the neutrino mass operator induced from the symmetric seesaw in Fig. 1(b) jumps in dimension at a step of four with increasing isospin. To increase dimension at a step of two it would require several fermion multiplets and in particular scalar fields of integral isospin that would develop a VEV through trilinear couplings with the SM Higgs field. We will not pursue this possibility in the remainder of this work.

## 3 Vacuum expectation values

The issue now becomes how a scalar \( \Phi^{(n+\frac{3}{2})} \) with quantum numbers \( I = n + \frac{1}{2}, Y = 1 \) develops a naturally small VEV. To make our discussion transparent, we start with the lowest-isospin case, \( n = 1 \), whose scalar potential contains the terms:

\[
V^{(\frac{3}{2})} \supset -\mu_H^2 H^\dagger H + \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_H (H^\dagger H)^2 - \left[ \kappa (\Phi \tilde{H}H \tilde{H})_0 + \text{h.c.} \right],
\]

where \( \Phi \equiv \Phi^{(\frac{3}{2})} \) for brevity and the subscript 0 denotes the isospin-zero combination of the inside product. The \( \kappa \) term breaks lepton number (together with Yukawa couplings) and can thus be considered naturally small. It is important that other parameters are such that \( \Phi \) would not develop a VEV without the \( \kappa \) term, in particular \( \mu_\Phi^2 > 0 \). Otherwise it would result in an unwanted massless Goldstone boson when \( \kappa = 0 \), or a too light scalar when \( \kappa \) is small. The \( \kappa \) term and a small \( \langle \Phi \rangle \) have a negligible effect on \( \langle H \rangle \), and cause small mixing between the \( \Phi \) and
$H$. For $\mu^2_H \gg \mu^2_\Phi$ and perturbative couplings, we have to good precision, $\langle H \rangle \approx \sqrt{\nu^2_H/(2\lambda_H)}$ which is assumed real positive without losing generality, and $\langle \Phi \rangle \approx \kappa^* \langle H \rangle^3/\mu^2_\Phi$. Note that once a small $\langle \Phi \rangle$ develops from the $\kappa$ term all other terms only make a subleading correction to it. These include both $L$-conserving terms like $(\Phi^* \Phi H \tilde{H})_0$, $(\Phi^* \Phi \Phi)_0$, and $L$-breaking terms like $(\Phi^* \Phi \Phi^* \Phi h.c.)_0$ and $(\Phi^* \Phi \tilde{H} \tilde{H})_0$. In other words, the dominant contribution to $\langle \Phi \rangle$ comes from the $L$-breaking quartic term that is linear in $\Phi$ and contains as many factors of $H$ as possible.

When yet another scalar of a higher isospin, $\Phi^{(\frac{3}{2})}$, is introduced, there will be more quartic terms in the potential. But most of them are not of our concern here since they only provide quartic interactions amongst scalars, and their mass splitting, mixing and trilinear couplings that are VEV-suppressed. The point here is that we should consider the largest possible contribution to VEV’s for a given set of new fields. This in turn corresponds to the lowest dimension operator responsible for neutrino mass that is available in the model. Since $L$ is necessarily violated by the $\kappa$ term to induce a small $\langle \Phi^{(\frac{3}{2})} \rangle$, we only need to consider $L$-conserving ones for all other terms that would induce a $\langle \Phi^{(\frac{3}{2})} \rangle$. This will give the least suppressed term in $\langle \Phi^{(\frac{3}{2})} \rangle$. But to guarantee that it is naturally small, it is again necessary that the parameters are such that it would vanish if $L$ were not broken. Since it is not possible to form a $\kappa$-like term for $\Phi^{(\frac{3}{2})}$, the only way to connect $\langle \Phi^{(\frac{3}{2})} \rangle$ to $L$ breaking is through an $L$-conserving quartic term that transfers VEV from $\Phi^{(\frac{1}{2})}$ to $\Phi^{(\frac{3}{2})}$. As in the case of $\Phi^{(\frac{1}{2})}$, the term that dominates $\langle \Phi^{(\frac{3}{2})} \rangle$ should contain as many factors of $H$ as possible. It is thus linear in both $\Phi^{(\frac{3}{2})}$ and $\Phi^{(\frac{1}{2})}$, and is unique upon specifying an equal lepton number, $-\lambda_{(2)}(\Phi^{(\frac{3}{2})} \Phi^{(\frac{1}{2})} \tilde{H} \tilde{H})_0 + h.c.$ All other terms involve more factors of the fields $\Phi^{(\frac{3}{2})}$ and $\Phi^{(\frac{1}{2})}$, and make a correction to the VEV’s that is suppressed by the small VEV’s themselves.

The above analysis generalizes obviously to a sequence of scalars, $\Phi^{(k+\frac{1}{2})}$ with $1 \leq k \leq n$, whose isospin differs by unity. The terms relevant for consideration of VEV’s are

$$V^{(n+\frac{1}{2})} \supset -\mu^2_H H^\dag H + \sum_{k=1}^n \mu^2_{(k)} \Phi^{(k+\frac{1}{2})} \Phi^{(k-\frac{1}{2})}$$

$$+ \lambda_H (H^\dag H)^2 - \sum_{k=1}^n \left[ \lambda_{(k)} (\Phi^{(k+\frac{1}{2})} \Phi^{(k-\frac{1}{2})} \tilde{H} \tilde{H})_0 + h.c. \right],$$

(2)

where identifications $\Phi^{(\frac{1}{2})} = H$ and $\lambda_{(1)} = \kappa$ are understood. For a field $\phi$ of isospin $j$, its conjugate field $\tilde{\phi}$ that transforms under isospin exactly as $\phi$ is formed as $\tilde{\phi} = \tau \phi^*$, where $\tau$ is a matrix with entry $\tau_{m,n} = (-1)^{j-m} \delta_{m,-n}$ ($-j \leq m, n \leq j$) in the eigenstate basis of the third isospin component. It is evident that the isospin invariant of the quartic term shown is unique, and can be worked out in terms of the Clebsch-Gordan coefficients:

$$\langle \Phi^{(k+\frac{1}{2})} \Phi^{(k-\frac{1}{2})} \tilde{H} \tilde{H} \rangle_0 = \frac{1}{2\sqrt{2k+1}} \Phi^{(k+\frac{1}{2})} (\Phi^{(k-\frac{1}{2})})^* |H_0|^2 + \cdots,$$

(3)

where the subscript 0 to a field denotes its neutral component, and the dots stand for the terms
not relevant to VEV’s. The vanishing first derivatives of $V^{(n+\frac{1}{2})}$ at VEV’s give

$$0 = \mu_{(n)}^2 \langle \Phi_0^{(n+\frac{1}{2})} \rangle - \frac{|\langle H_0 \rangle|^2}{2\sqrt{2n+1}} \lambda^{*}_{(n)} \langle \Phi_0^{(n-\frac{1}{2})} \rangle,$$

$$0 = \mu_{(k)}^2 \langle \Phi_0^{(k+\frac{1}{2})} \rangle - \frac{|\langle H_0 \rangle|^2}{2\sqrt{2k+1}} \lambda^{*}_{(k)} \langle \Phi_0^{(k-\frac{1}{2})} \rangle - \frac{|\langle H_0 \rangle|^2}{2\sqrt{2k+3}} \lambda^{*}_{(k+1)} \langle \Phi_0^{(k+\frac{1}{2})} \rangle,$$

for $n-1 \geq k \geq 1$. The first equation yields

$$\langle \Phi_0^{(n+\frac{1}{2})} \rangle = \frac{1}{2\sqrt{2n+1}} \frac{|\langle H_0 \rangle|^2}{\mu_{(n)}^2} \lambda^{*}_{(n)} \langle \Phi_0^{(n-\frac{1}{2})} \rangle,$$

which means that $|\langle \Phi_0^{(n+\frac{1}{2})} \rangle| \ll |\langle \Phi_0^{(n-\frac{1}{2})} \rangle|$ for $\mu_{(n)}^2 \gg |\langle H_0 \rangle|^2$ and perturbative couplings. This implies that the last term in eq (5) at $k = n - 1$ is doubly suppressed compared to the first one for $\mu_{(n-1)}^2 \gg |\langle H_0 \rangle|^2$, and can be ignored. The analysis applies to all $k$, so that

$$\langle \Phi_0^{(k+\frac{1}{2})} \rangle = \frac{1}{2\sqrt{2k+1}} \frac{|\langle H_0 \rangle|^2}{\mu_{(k)}^2} \lambda^{*}_{(k)} \langle \Phi_0^{(k-\frac{1}{2})} \rangle, \quad n \geq k \geq 1,$$

and thus,

$$\langle \Phi_0^{(n+\frac{1}{2})} \rangle = \langle H_0 \rangle |\langle H_0 \rangle|^2 \prod_{k=1}^{n} \frac{1}{2\sqrt{2k+1}} \frac{\lambda^{*}_{(k)}}{\mu_{(k)}^2}.$$

This mechanism of inducing a smaller VEV for a field of a higher isospin from that of a lower isospin is depicted in Fig. 2 as a cascading process. Note that the first cascade is suppressed by an $L$-violating coupling while the sequential cascades are suppressed by the heavy scalar masses.

![Fig. 2 VEV’s induced via a cascading process. Here $\Phi^{(\frac{1}{2})} = H$, $\lambda_{(1)} = \kappa$.](image)

### 4 Neutrino mass and discussions

We are now ready to work out the neutrino mass in a seesaw model that contains a heavy fermion with $I = n + 1$, $Y = 0$ and a sequence of heavy scalar multiplets with $Y = 1$ and $I = n + \frac{1}{2}, n - \frac{1}{2}, \ldots, \frac{3}{2}$. The bare mass and Yukawa terms are

$$-L_{\text{Yuk+mass}} = m_\Sigma \Sigma + \left[ v_{ij} \tilde{F}_{ij} H f_R + x_{ij} \tilde{F}_{ij} \Phi^{(n+\frac{1}{2})} \Sigma \right)_0 + z_{ij} (\tilde{\Sigma} \Phi^{(n+\frac{1}{2})} \tilde{F}_{ij})_0 + \text{h.c.}.$$
where \(i, j\) denote the lepton generation. \(\bar{\Sigma}\) should be correctly understood as the Dirac-barred conjugate field that transforms under \(SU(2)\) just as \(\Sigma\) itself; in the order of decreasing charge, its components are,

\[
\Sigma_{-n-1}, \ -\Sigma_{-n}, \ -\Sigma_{-n+1}, \ldots, \ -\Sigma_{n}, \ -\Sigma_{n+1}.
\]

(10)

The invariant form for a product of three fields \(\psi, \phi, \chi\) with isospin \(n + 1, n + \frac{1}{2}, \frac{1}{2}\) respectively is unique:

\[
(\psi\phi\chi)_0 = \frac{1}{\sqrt{(2n + 3)(2n + 2)}} \sum_{m=-n-1}^{n+1} (-1)^{n+1+m} \frac{1}{\sqrt{m}} (\psi L_j \phi^{(n+\frac{1}{2})} \Sigma_0 + \cdots)
\]

\[
(\bar{\Sigma}\Phi^{(n+\frac{1}{2})} F_{Lj})_0 = \frac{1}{\sqrt{2(2n + 3)}} \sum_0 \Phi^{(n+\frac{1}{2})} v_{Lj} + \cdots.
\]

(11)

(12)

The seesaw neutrino mass can be calculated from Fig. 1(b) with the above vertices or by solving the equation of motion for the heavy \(\Sigma\) field. We find

\[
m_{jk}^\nu = (x_j z_k + x_k z_j) \frac{1}{m_\Sigma} \left(\frac{1}{m_\Sigma} \right) \left(\frac{1}{m_\Sigma} \right)^2
\]

\[
= (x_j z_k + x_k z_j) \frac{1}{m_\Sigma} \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2
\]

\[
\left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2
\]

\[
= (1 - x_j z_k + x_k z_j) \frac{1}{m_\Sigma} \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2
\]

\[
= \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2
\]

\[
= \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2
\]

\[
= \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2 \left(\frac{1}{m_\Sigma} \right)^2
\]

which is suppressed by \(4n + 1\) powers of heavy scales and corresponds to the neutrino mass operator of dimension \(5 + 4n, \mathcal{O}_{5+4n}\). Since \(m^\nu\) as a matrix in flavor space is a product of two vectors, there is always one massless neutrino. By judicious gauge transformations it can be shown \[35, 36\] that there are only two real and one complex physical parameters in the general complex 3-vectors \(x\) and \(z\). For instance, in the normal hierarchy case, one can parameterize without losing generality, \(x^T = (0, 0, x)\) and \(z^T = (0, z, c_z)\), where \(x\) and \(z\) are real positive and \(c_z\) is complex. This provides a convenient relation amongst the neutrino masses, mixing, and the Yukawa couplings. For an order of magnitude estimate, we ignore the Clebsch-Gordan coefficients and make the simple-minded approximations: \(x \sim z, m_\Sigma \sim \mu_{(k)} \sim M\), and \(\lambda_{(k)} \sim \lambda\) (for \(k > 1\)). Then the two massive neutrinos have a mass of order, \(m \sim \lambda^2 \lambda^{2(n-1)} \mu_k^2 \langle H_0 \rangle^{2+4n} M^{-1-4n}\).

For example, with \(x \sim 10^{-2}, \lambda \sim 10^{-1}, \kappa \sim 10^{-3}\), and \(\langle H_0 \rangle \sim 174\) GeV, the heavy mass is about 490 GeV at \(n = 1\) and 190 GeV at \(n = 2\), to give a desired neutrino mass around 0.1 eV.

The above numerical example also illustrates the point that in physical applications we do not really need a long chain of cascading VEV’s. With a fermion of isospin 2 plus a scalar of isospin \(\frac{3}{2}\) (i.e., \(n = 1\)), or at most with a fermion of isospin 3 plus a scalar of isospin \(\frac{5}{2}\) and a
scalar of isospin $\frac{3}{2}$ ($n = 2$), a neutrino mass can be readily induced at the desired level in the range of parameters that would be phenomenologically interesting. Introduction of fields of even higher isospin would over-suppress the neutrino mass. On the other hand, as we briefly mentioned in the Introduction, a neutrino mass operator of a lower dimension can be induced at the loop level that formally amounts to connecting a pair of the $(H, \bar{H})$ fields in Feynman graphs. A detailed study shows [37] that its contribution is subdominant for a not-too-heavy mass $M$: for instance, $M < 1$ TeV at $n = 1$, and $M < 1.4$ TeV at $n = 2$, which lies indeed in the mass range that motivates the current approach.

A salient feature of the seesaw model proposed here is that there are multiply and equally charged heavy fermions and scalars. They participate in electroweak gauge interactions and Yukawa couple to the ordinary leptons. As shown in the above example, these new particles are not necessarily very heavy, and thus could potentially be produced at high energy colliders via gauge boson fusion processes for instance. With enhanced Yukawa couplings with the ordinary leptons the lightest of them would decay into ordinary leptons with feasible signatures. On the other hand, the existing precision data on the rare flavor-changing transitions of the charged leptons could put stringent constraints on those Yukawa couplings. Our experience shows (see e.g., [38] and [36]) that it is feasible with the above illustrated numbers to accommodate the strong constraints in the $\mu e$ sector; instead, the challenge always rests on whether it is possible to enhance the rare decays in the $\tau$ sector to a level that would not be too low compared to the experimental sensitivity available in the near future. Furthermore, the new scalars mix with the SM Higgs particle via interactions in the potential, and thus will modify the production and decays properties of the latter at high energy colliders. This could offer additional information on the origin of neutrino mass. We leave this more comprehensive phenomenological analysis for the future study [39].

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