Loop Effects of Exotic Leptons on Vector-Boson Pair Production at $e^+e^-$ Colliders

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Abstract

We study quantum effects of exotic heavy leptons on the production of two vector bosons in $e^+e^-$ annihilation. We present closed analytic expressions for vector-boson self-couplings and differential cross sections, within the framework of two favourable new-physics scenarios: (i) the fourth-generation Majorana-neutrino model and (ii) an $E_6$-inspired model with sequential mirror isodoublets. We constrain these models by requiring that their contributions to the oblique electroweak parameters be compatible with a recent global fit to high-precision data. We then systematically analyze the loop-induced deviations in the LEP2 cross section of $e^+e^- \rightarrow W^+W^-$ predicted by the models thus confined.

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1 Introduction

Neutrinos are the most weakly coupled particles observed in nature, a fact that makes their detailed experimental study rather difficult. Understanding the underlying properties of these massless or almost massless neutral fermions will definitely shed light on a number of fundamental questions in particle and astro-particle physics. In the minimal Standard Model (SM), the neutrinos are strictly massless by construction, due to the absence of right-handed neutrino states. However, all other fermionic matter, i.e., the charged leptons and quarks, require the existence of right-handed fields. How should one understand theoretically this asymmetry of the fermionic degrees of freedom in the SM? Another long-standing puzzle is related to the energy deficit of the solar and atmospheric neutrinos. A possible solution to all these problems may be achieved by means of the see-saw mechanism [1], which requires the existence of right-handed neutrino fields in addition to the left-handed ones of the minimal SM. Then, very large Majorana masses can be present in the extended SM Lagrangian, and, together with Dirac terms of order of charged-lepton or quark masses, form the entries of the see-saw mass matrix. Diagonalization of the see-saw mass matrix naturally gives rise to non-zero, but very small neutrino masses, in agreement with experiment, as well as to ultra-heavy neutrinos not yet discovered. Furthermore, a possible explanation of the atmospheric and solar neutrino problems may be based on the Mikheyev-Smirnov-Wolfenstein mechanism [2], which also requires that, contrary to the SM, the known neutrinos be massive. Finally, reconciliation of the atmospheric, solar, and Large Scale Neutrino Detector (LSND) anomalies is only possible if four low-mass neutrinos exist [3]. The latter may be regarded as a strong indication in favour of fourth-generation extensions of the SM.

If all neutrino problems can indeed be resolved by the presence of new heavy neutral leptons, one has then to investigate possible consequences emanating from these exotic particles. There are strict experimental bounds from direct searches at LEP1 and, to a lesser extend, at LEP2 for new heavy fermions with appreciable couplings to the Z boson. However, even if heavy particles are not directly accessible at present energies, they can still leave their imprints by inducing noticeable deviations at the quantum level through the oblique electroweak parameters, which have now been tightly constrained by LEP1 measurements. Such heavy particles may also have an observable impact on the triple-gauge-boson couplings, which are directly accessible at LEP2.

In this paper, we study loop effects of exotic heavy leptons on the cross sections of the following four processes: (i) $e^+e^- \rightarrow \gamma \gamma$; (ii) $e^+e^- \rightarrow \gamma Z$; (iii) $e^+e^- \rightarrow ZZ$; and (iv) $e^+e^- \rightarrow W^+W^-$. To that end, we consider two favourable models, which are renormalizable. In the first model, which was first discussed by Hill and Paschos (HP) [4], the fermionic sector of the SM is extended by adding one sequential weak isodoublet, one right-handed neutrino, and one right-handed charged lepton. After spontaneous symmetry breaking, the left-handed neutrinos couple to the right-handed neutrino with Dirac masses proportional to the charged lepton masses, whereas the right-handed neutrino develops a Majorana mass at the electroweak scale. The HP model can naturally predict a mass hierarchy between the different neutrino species. Since the charged lepton
of the fourth generation must be heavy for phenomenological reasons, the fourth doublet neutrino then turns out to be heavy as well. Furthermore, inter-family mixing between the three generations and the fourth one is suppressed by the usual see-saw relations and can thus be safely neglected. To render the HP model anomaly-free, however, one should include one additional quark isodoublet and one pair of up-type and down-type right-handed quarks.

The second favourable model that could potentially allow for large loop effects of exotic leptons is due to Ma and Roy (MR) \[5\]. As was argued in Ref. \[3\], the MR model may be a viable low-energy limit of \(E_6\) unified theories. The MR model is slightly more economical than the HP model, since it only contains two colourless doublets with opposite weak isospins and one isosinglet neutrino in addition to the SM field content. Because of this mirror assignment of the two weak isodoublets, the MR model is anomaly free by itself; the quark sector does not need to be extended. Apart from Dirac masses, the isosinglet neutrino can introduce a lepton-number-violating Majorana mass into the Lagrangian. As we shall see, the two mirror isodoublets can form a singlet mass term for the heavy charged lepton. Among other things, the MR model leads to both left- and right-handed couplings of the \(W\) boson to the heavy charged lepton and the heavy neutrinos.

Recently, an analysis of heavy-lepton effects on the reaction \(e^+e^- \rightarrow W^+W^-\) in the context of the HP model has been reported \[6\]. We have checked that our results agree with those of Ref. \[6\]. The salient difference between our analysis and that of Ref. \[6\] resides in the fact that our numerical predictions are obtained after imposing the constraints on the parameters of the HP model that come from experimental bounds on the oblique electroweak parameters and from direct searches for new heavy leptons. To our knowledge, similar studies have not yet been performed for the production of neutral vector bosons or for the MR model.

This paper is organized as follows. In Section 2, we describe the basic low-energy structure of the HP and MR scenarios. Section 3 contains, in analytic form, the resulting one-loop corrections to the cross sections of processes (i)–(iv). Technical details of the calculation are relegated to the Appendices. In Section 4, we present numerical predictions, relevant for LEP2, for the loop effects due to exotic leptons within the context of the HP and MR models, after the parameter spaces of these models have been constrained by imposing limits derived from oblique electroweak parameters and direct searches. Our conclusions are summarized in Section 5.

### 2 Outline of the models

In the following, we present a brief outline of the HP and MR models. Both models are based on the SM gauge group \(SU(2)\times U(1)\), but contain new isodoublet and isosinglet fields. The resulting gauge interactions are different for the two models, which in turn may lead to different phenomenological consequences.
2.1 Hill-Paschos model

In the HP model \cite{4}, the lepton and quark isodoublets \((\nu', E)_L\) and \((T, B)_L\) as well as the four right-handed fields \(E_R, N'_R, T_R,\) and \(B_R\) are added to the SM. To avoid tight limits coming from the observed sector, we assume the absence of any mixing between the ordinary matter and the new fields introduced in the model. The heavy quarks \(T\) and \(B\) are needed to cancel the triangle anomalies. They are usually considered to be mass-degenerate, \(i.e., m_T = m_B,\) in order to avoid large contributions to the electroweak \(\rho\) parameter.

In the lepton sector, the HP model predicts a heavy charged lepton with mass \(m_E,\) which is to be constrained by electroweak radiative corrections. Moreover, the isosinglet parameter. \(\rho\) mass-degenerate, \(i.e., m_B = m_D,\) so that the mass Lagrangian of the neutrinos has the non-standard form

\[
\mathcal{L}^\nu_M = -\frac{1}{2} \begin{pmatrix} N'_L, N'^C_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \begin{pmatrix} N'^C_L \\ N'_R \end{pmatrix} + \text{H.c.} \tag{2.1}
\]

The \(2 \times 2\) neutrino mass matrix in Eq. (2.1), which we call \(M^\nu,\) can always be diagonalized through a unitary transformation as \(U^T M^\nu U = \tilde{M}^\nu.\) After the diagonalization of \(\mathcal{L}^\nu_M,\) we obtain two heavy Majorana neutrinos, \(\nu\) and \(N.\) In the limit \(m_D \ll m_M,\) \(\nu\) is predominantly isodoublet, whereas \(N\) is isosinglet. In the HP model, \(m_D \sim m_E\) and \(m_M\) is of the order of the electroweak scale \(G^{1/2}_F,\) \(i.e., m_M \lesssim 1\) TeV.

The interactions of the two Majorana neutrinos \(n_1 \equiv \nu\) and \(n_2 \equiv N\) with the \(W\) and \(Z\) bosons and the charged lepton \(E\) is described by the Lagrangian

\[
\mathcal{L}_{\text{int}} = \frac{e}{\sqrt{2} s_w} W^-_\mu \sum_{i=1,2} B_{Ei} \bar{E} \gamma^\mu P_- n_i + \text{H.c.} \\
+ \frac{e}{4c_w s_w} Z_\mu \sum_{i,j=1,2} \bar{n}_i \gamma^\mu (i\bar{3} m C_{ij} - \gamma_5 \text{Re} C_{ij}) n_j, \tag{2.2}
\]

where \(P_\pm = (1 \pm \gamma_5)/2\) are the right/left-handed helicity projectors, \(e\) is the electron charge magnitude, \(c_w^2 = 1 - s_w^2 = M^2_W/M^2_Z,\) and, according to the conventions of Ref. \cite{4}, \(B_{Ei} = U^*_i\) and \(C_{ij} = U_{1'i} U^*_i.\) The mixing \(B_{Ei}\) and \(C_{ij}\) may be expressed in terms of the physical Majorana masses \(m_\nu\) and \(m_N\) as \cite{5}

\[
B_{E\nu} = \sqrt{\frac{m_N}{m_\nu + m_N}}, \quad B_{EN} = i \sqrt{\frac{m_\nu}{m_\nu + m_N}}, \\
C_{\nu\nu} = \frac{m_N}{m_\nu} C_{NN} = \frac{m_N}{m_\nu + m_N}, \quad C_{\nu N} = -C_{N\nu} = i \frac{m_\nu m_N}{m_\nu + m_N}. \tag{2.3}
\]

For later convenience, we rewrite the interaction Lagrangian (2.2) as

\[
\mathcal{L}_{\text{int}} = e W^-_\mu \sum_{i=1,2} \bar{E} \gamma^\mu \left( g^+_{E\nu,WW} P_+ + g^-_{E\nu,WW} P_- \right) n_i + \text{H.c.} \\
+ e Z_\mu \sum_{i,j=1,2} \bar{n}_i \gamma^\mu \left( g^+_{n_i n_j Z} P_+ + g^-_{n_i n_j Z} P_- \right) n_j, \tag{2.4}
\]
where
\[ g_{EnW}^+ = 0, \quad g_{EnW}^- = \frac{B_{Ei}}{\sqrt{2}s_w}, \]
\[ g_{n,n,Z}^+ = -\frac{C_{ij}^*}{4c ws_w}, \quad g_{n,n,Z}^- = \frac{C_{ij}}{4c ws_w}. \quad \tag{2.5} \]

The remaining interactions of the gauge bosons with the quarks and charged leptons are of the SM type, with couplings
\[ g_{BTW}^+ = 0, \quad g_{BTW}^- = \frac{1}{\sqrt{2}s_w}, \]
\[ g_{FFZ}^+ = -\frac{s_wQ_F}{c_w}, \quad g_{FFZ}^- = \frac{T_F}{c_w s_w} - \frac{s_wQ_F}{c_w}, \]
\[ g_{FF\gamma}^\pm = -Q_F, \quad \tag{2.6} \]

where \( Q_F \) and \( T_F \) are the electric charge and the third component of the weak isospin of \( F = E, T, B \), respectively.

### 2.2 Ma-Roy model

In the MR model \[5\], only the lepton sector of the SM is extended. Specifically, two colorless doublets \( (N_1, E)_L \) and \( (E^C, N_2)_L \) and a colorless isosinglet \( N_{3L} \) are added. Similarly to the HP model, we assume that the new fields do not mix with the observed leptons. Since the two new isodoublets have opposite hypercharges, the triangle anomalies cancel, so that there is no need to extend the quark sector as well. As is argued in Ref. \[5\], such an anomaly-free representation may be the low-energy limit of unified E\(_6\) models.

In addition to the Majorana mass term \( m_M N_{3L}^C N_{3L} \), the two lepton isodoublets can form the gauge-invariant isosinglet mass term of the type
\[
m_E \left[ E_L^C \right]^T C^{-1} E_L - N_{2L}^C C^{-1} N_{1L} \right] + m_E \left[ E_L^T C^{-1} E_L^C - N_{1L}^T C^{-1} N_{2L} \right].
\]

In this way, the charged lepton \( E \) of the MR model acquires a SU(2)×U(1)-invariant mass term. Furthermore, after spontaneous symmetry breaking, the neutrino mass Lagrangian takes the form
\[
L^\nu_M = -\frac{1}{2} \left( \bar{N}^C_{1L}, \bar{N}^C_{2L}, \bar{N}^C_{3L} \right) \begin{pmatrix}
0 & -m_E & m_1 \\
-m_E & 0 & m_2 \\
m_1 & m_2 & m_M
\end{pmatrix} \begin{pmatrix}
N_{1L} \\
N_{2L} \\
N_{3L}
\end{pmatrix} + \text{H.c.} \quad \tag{2.7}
\]

For simplicity, we assume that the 3 × 3 neutrino mass matrix, \( M^\nu \), is real. Obviously, the MR model predicts three heavy Majorana neutrinos, which are denoted by \( n_1, n_2, \) and \( n_3 \). The diagonalization of \( M^\nu \) as well as the explicit analytic form of the three mass eigenvalues and eigenvectors are given in Appendix \[A\].
The interactions of the $Z$ and $W$ bosons with the heavy Majorana neutrinos $n_i$ ($i = 1, 2, 3$) and the charged lepton $E$ are determined by the Lagrangian

\[
\mathcal{L}_{\text{int}} = \frac{e}{2\sqrt{2}s_w} W_\mu \sum_{i=1}^{3} \bar{E} \gamma^\mu [(U_{1i} - U_{2i}^*) - \gamma_5 (U_{1i} + U_{2i}^*)] n_i + \text{H.c.} \\
+ \frac{e}{4c_w s_w} Z_\mu \left[ \sum_{i,j=1}^{3} \bar{n}_i \gamma^\mu (i \Im m C_{ij} - \gamma_5 \Re e C_{ij}) n_j + 2(2s_w^2 - 1) \bar{E} \gamma^\mu E \right], \tag{2.8}
\]

where $C_{ij} = U_{1i}^* U_{1j} - U_{2i}^* U_{2j}$. Notice that the coupling of the charged lepton $E$ to the $Z$ boson is purely vectorial, whereas the $W$ boson couples with both chiralities to the $E$ and $n_i$ fields. This makes the MR scenario very distinctive from the HP model. Using a parameterization analogous to Eq. (2.4), complemented by the $Z\bar{E}E$ interaction Lagrangian

\[
\mathcal{L}^Z_{\text{int}} = eZ_\mu \bar{E} \gamma^\mu (g^{+}_{EEZ} P_+ + g^{-}_{EEZ} P_-) E, \tag{2.9}
\]

the couplings of the model read

\[
\begin{align*}
g^{+}_{En,W^-} &= -\frac{U_{2i}^*}{\sqrt{2}s_w}, & g^{-}_{En,W^-} &= \frac{U_{1i}}{\sqrt{2}s_w}, \\
g^+_{n_j Z} &= -\frac{C_{ij}^*}{4c_w s_w}, & g^-_{n_j Z} &= \frac{C_{ij}}{4c_w s_w}, & g^\pm_{EEZ} &= \frac{2s_w^2 - 1}{2c_w s_w}. \tag{2.10}
\end{align*}
\]

3 Analytic results

In this section, we calculate the quantum corrections to the cross sections of processes (i)–(iv) specified in the Introduction which are induced by the exotic heavy fermions of the HP and MR models. As a reference, we also list the tree-level results. For the reader’s convenience, technical details such as the definitions of the relevant matrix elements, one-loop functions, and renormalization constants are relegated to Appendices B, C, and D, respectively.

We denote the four-momenta of $e^+$, $e^-$, and the two produced vector bosons, $V_1$ and $V_2$, by $p_+, p_-$, $k_1$, and $k_2$, and define the Mandelstam variables as $s = (p_+ + p_-)^2$, $t = (p_+ - k_1)^2$, and $u = (p_+ - k_2)^2$. Neglecting the electron mass, we have $s + t + u = M_1^2 + M_2^2$, where $M_1$ and $M_2$ are the masses of $V_1$ and $V_2$, respectively. In this limit, also the $s$-channel contributions due to Higgs-boson exchanges vanish. Because each of the processes (i)–(iv) has more than one tree-level diagram, it proves convenient to present the analytic results in terms of helicity amplitudes. In the centre-of-mass (c.m.) system, the electron and positron have opposite helicities, so that one helicity label $\kappa = \pm$ for left/right-handed electron helicity suffices. Calling the c.m. helicities of $V_1$ and $V_2$, $\lambda_1$ and $\lambda_2$, the differential cross section may be cast into the generic form

\[
\frac{d\sigma}{d\Omega} = \frac{\lambda^{1/2}(s, M_1^2, M_2^2)}{64\pi^2 s^2} \frac{1}{4} \sum_{\kappa,\lambda_1,\lambda_2} |M^\kappa(\lambda_1, \lambda_2, s, t)|^2, \tag{3.1}
\]
where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$ is the Källén function and the factor $1/4$ stems from the average over the initial spins. In the following, we suppress the arguments of the helicity amplitudes $M^\kappa$ if confusion is impossible. It is convenient to express the amplitudes $M^\kappa$ as linear combinations of the standard matrix elements $M^\kappa_i$ ($i = 0, \ldots , 9$) listed in Eq. (B.1), which have well-known helicity representations \cite{9}. In addition, it is useful to introduce the following combinations pertinent to the amplitudes $M^\kappa$ of the helicity amplitudes $M^\kappa$ stems from the average over the initial spins. In the following, we suppress the arguments of the polarization, $\Pi_{V,T}^\kappa$.

\begin{align}
M_s^\kappa &= \frac{2e^2}{s} (M_1^\kappa - M_2^\kappa - M_3^\kappa), \\
M_t^\kappa &= -\frac{e^2}{t} M_6^\kappa, \\
M_u^\kappa &= -\frac{e^2}{u} (M_6^\kappa + 2M_1^\kappa - 2M_2^\kappa - 2M_3^\kappa). \\
\end{align}

(3.2)

In the one-loop approximation, each helicity amplitude $M^\kappa$ is expanded in the fine-structure constant $\alpha = e^2/(4\pi)$ as $M^\kappa = M_{\text{Born}}^\kappa + \delta M^\kappa$, where $M_{\text{Born}}^\kappa$ and $\delta M^\kappa$ are the tree-level and one-loop contributions, respectively. In turn, $\delta M^\kappa$ receives contributions from diagrams containing self-energy corrections, vertex corrections, and counterterm insertions, i.e., $\delta M^\kappa = \delta M_{S}^\kappa + \delta M_{T}^\kappa + \delta M_{U}^\kappa$. Consequently, through order $\alpha^3$, the differential cross section $\sigma(\gamma\gamma)$ may be written as

\begin{align}
\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Born}} + \frac{\lambda^{1/2}}{128\pi^2 s^2} \sum_{\kappa, \lambda_1, \lambda_2} \text{Re}[(M^\kappa_{\text{Born}})^* \delta M^\kappa]. \\
\end{align}

(3.3)

First, we present $M_{\text{Born}}^\kappa$ for processes (i)–(iv). As is well known \cite{5}, we have

\begin{align}
M_{\text{Born}}^\kappa(\gamma\gamma) &= M_{1\kappa} + M_{1\kappa}^T, \\
M_{\text{Born}}^\kappa(\gamma Z) &= \tilde{g}_{eeZ}^\kappa (M_{1\kappa} + M_{1\kappa}^T), \\
M_{\text{Born}}^\kappa(ZZ) &= (\tilde{g}_{eeZ}^\kappa)^2 (M_{1\kappa}^T + M_{1\kappa}^T), \\
M_{\text{Born}}^\kappa(WW) &= M_{1T}^\kappa \left(1 - \tilde{g}_{eeZ}^\kappa \frac{c_w}{s_w} \frac{s}{s - M_Z^2}\right), \\
M_{\text{Born}}^\kappa(WW) &= M_{1S}^\kappa \left(1 - \tilde{g}_{eeZ}^\kappa \frac{c_w}{s_w} \frac{s}{s - M_Z^2}\right) + \frac{M_{1T}^\kappa}{2s_w^2}, \\
\end{align}

(3.4)

where $\tilde{g}_{eeZ}^\kappa = s_w/c_w$ and $\tilde{g}_{eeZ}^\kappa = (2s_w^2 - 1)/(2s_w c_w)$ are the SM couplings of the electron to the $Z$ boson in the notation of Eq. (2.6).

Next, we present the contributions to the $\gamma\gamma$, $ZZ$, and $WW$ self-energies and to the $\gamma Z$ mixing amplitude induced by the exotic fermions of the HP and MR models. These will enter the expressions for $\delta M_{S}^\kappa$ and $\delta M_{T}^\kappa$. Instead of listing all individual contributions separately, we present a generic expression, which depends on the coupling parameters defined in Eqs. (2.3), (2.6), and (2.10). Since all the gauge bosons involved in processes (i)–(iv) couple to conserved currents, only the transverse parts of the above-named vacuum polarizations, $\Pi_T^{V_1 V_2}$ with $V_1 V_2 = \gamma\gamma, \gamma Z, ZZ, WW$, need be calculated. In dimensional
regularization with \( D \) space-time dimensions, we have

\[
\Pi_{T}^{V_2}(p^2) = \frac{1}{D-1} \left( g^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{p^2} \right) \sum_{(j_i,j_f)} \frac{\alpha}{4\pi} N_{c}^{j_i} N_{S}^{j_f} \left( \frac{2\pi \mu}{i\pi^2} \right)^{4-D} \int d^D q \text{ Tr} \left[ \gamma_{\nu} \left( g^{\mu}_{j_i j_f} P_+ + g^{\mu}_{j_f j_i} P_- \right) \frac{1}{q + \not{p} - m_j + i\varepsilon} \right] \\
+ \left( g^{\mu}_{j_i j_f} P_+ + g^{\mu}_{j_f j_i} P_- \right) \frac{1}{q - m_i + i\varepsilon} \right] \\
\sum_{(j_i,j_f)} \pi\alpha N_{c}^{j_i} N_{S}^{j_f} \left\{ \left( g_{j_i j_f}^{+} g_{j_f j_i}^{+} + g_{j_i j_f}^{-} g_{j_f j_i}^{-} \right) \left[ \Pi_V(p^2, m_i, m_j) + \Pi_V(p^2, m_i, -m_j) \right] \\
+ \left( g_{j_i j_f}^{+} g_{j_f j_i}^{-} + g_{j_i j_f}^{-} g_{j_f j_i}^{+} \right) \left[ \Pi_V(p^2, m_i, m_j) - \Pi_V(p^2, m_i, -m_j) \right] \right\} , \tag{3.5}
\]

where \( p \) is the external four-momentum, the sum runs over all possible pairings \((f_i, f_j)\) of the exotic fermions, \( N_{c}^{j_i} = 1 \) (3) for leptons (quarks), we have introduced the short-hand notations \( m_i = m_{f_i} \) and \( g^{\mu}_{j_i j_f} = g^{\mu}_{j_f j_i} \), and the \( \Pi_V \) function is defined in Eq. (C.1). The combinatorial factor \( N_{S}^{j_f} \) accounts for the Majorana properties of the heavy neutrinos; it takes the value \( N_{S}^{j_i} = 2 \) if \( f_i \) and \( f_j \) are both Majorana neutrinos, while \( N_{S}^{j_i} = 1 \) otherwise. The unphysical \( 't \) Hooft mass scale, \( \mu \), is introduced to keep the coupling constants dimensionless; it cancels along with the ultraviolet singularities upon renormalization. Equation (3.5) generalizes the results for the HP model found in Refs. [8,10].

With the help of Eq. (3.5), we may present \( \delta M_{S}^{\gamma\gamma} \) for processes (i)–(iv) as

\[
\delta M_{S}^{\gamma\gamma} = \delta M_{S}^{\gamma\gamma} + \delta M_{S}^{\gamma Z} = \delta M_{S}^{\gamma \gamma} ,
\]

\[
\delta M_{S}^{WW} = M_{S}^{WW} \left[ -\frac{\Pi_{T}^{Z}(s)}{s} + \left( \frac{c_{w}}{s_{w}} - g_{eeZ}^{\kappa} \right) \frac{\Pi_{T}^{Z}(s)}{s - M_{Z}^{2}} + g_{eeZ}^{\kappa} \frac{s_{\Pi_{T}^{Z}(s)}}{(s - M_{Z}^{2})^{2}} \right] . \tag{3.6}
\]

The first line of Eq. (3.6) reflects the fact that processes (i)–(iii) do not involve virtual vector bosons at tree level.

The mass and coupling counterterms as well as the wave-function renormalization constants that are relevant for our analysis can all be expressed in terms of the \( \Pi_{T}^{V_2} \) functions of Eq. (3.3) and their derivatives with respect to \( p^2 \). The corresponding relations are summarized for the on-shell renormalization scheme in Eq. (D.2). The counterterm amplitudes \( \delta M_{S}^{\gamma\gamma} \) for the processes (i)–(iv) emerge from the respective Born amplitudes \( \delta M_{S}^{\gamma\gamma} \) by scaling the couplings and masses and by including the appropriate wave-function renormalizations. The appropriate relations between the bare and renormalized parameters and fields are collected in Eq. (D.1). In this way, we obtain

\[
\delta M_{C}^{\gamma\gamma} = M_{S}^{\gamma\gamma} \left( 2 \frac{\delta c}{c} + \frac{\delta Z_{\gamma\gamma} + g_{eeZ}^{\kappa}}{2} \right) ,
\]

\[
\delta M_{C}^{\gamma Z} = M_{S}^{\gamma Z} \left( 2 \frac{\delta c}{c} + \frac{\delta g_{eeZ}^{\kappa}}{2} + \frac{\delta Z_{\gamma\gamma}}{2} + \frac{g_{eeZ}^{\kappa}}{2} \delta Z_{\gamma Z} + \frac{\delta Z_{\gamma Z}}{2} \right) ,
\]

\[
\delta M_{C}^{ZZ} = M_{S}^{ZZ} \left( 2 \frac{\delta c}{c} + \frac{\delta g_{eeZ}^{\kappa}}{2} + \frac{\delta Z_{\gamma\gamma}}{2} + \frac{\delta Z_{\gamma Z}}{2} \right) ,
\]

\[
\delta M_{C}^{\gamma\gamma} = M_{S}^{\gamma\gamma} \left( 2 \frac{\delta c}{c} + \frac{\delta g_{eeZ}^{\kappa}}{2} + \frac{\delta Z_{\gamma\gamma}}{2} + \frac{\delta Z_{\gamma Z}}{2} \right) ,
\]

\[
\delta M_{C}^{\gamma Z} = M_{S}^{\gamma Z} \left( 2 \frac{\delta c}{c} + \frac{\delta g_{eeZ}^{\kappa}}{2} + \frac{\delta Z_{\gamma\gamma}}{2} + \frac{\delta Z_{\gamma Z}}{2} \right) .
\]
\[
\delta M^\kappa_C(WW) = M^\kappa_{\text{Born}}(WW) \left( \frac{2\delta e}{e} + \delta Z_W + \frac{\delta M_Z^2}{M_Z^2} \frac{s}{s - M_Z^2} \right), \\
\delta M_C(WW) = \frac{M_s}{s - M_Z^2} \left[ \left( \frac{2\delta e}{e} + \delta Z_W \right) \left( \frac{s}{2s_w^2} - M_Z^2 \right) + \left( \frac{1}{2s_w^2} - 1 \right) \delta M_Z^2 \frac{s}{s - M_Z^2} \right] \\
- \frac{\delta s_w}{s_w^3} \right] + \frac{M_s}{2s_w^2} \left( \frac{2\delta e}{e} + \delta Z_W - 2 \delta s_w \right) \left( \frac{s}{s - M_Z^2} \right).
\]

(3.7)

Notice that, since processes (i)–(iii) do not involve virtual gauge bosons at the tree level, their counterterm amplitudes only receive contributions from coupling and wave-function renormalization.

Finally, we present the helicity amplitudes \(\delta M^\kappa_V\) containing the fermionic corrections to the triple-gauge-boson vertices in the generic form:

\[
\delta M^\kappa_V = \sum_{\kappa_1, \kappa_2, \kappa_3 = \pm} \sum_{B=\gamma, Z} \sum_{(f_1, f_2, f_3)} \alpha^2 N_{f_1}^{f_2} N^{f_3}_{\kappa} \epsilon_{\kappa_1}(k_1, \lambda_1) \epsilon_{\kappa_2}(k_2, \lambda_2) \epsilon_{\kappa_3}(k_3, \lambda_3)
\times \mathcal{V}_{\kappa_1\kappa_2\kappa_3}(\lambda_1, \lambda_2, s, t),
\]

(3.8)

where the inner sum runs over all possible triplets \((f_1, f_2, f_3)\) of exotic loop fermions and

\[
\mathcal{V}_{\kappa_1\kappa_2\kappa_3}^\kappa = \bar{v}(p_+) \gamma^\mu P^\kappa u(p_-) \epsilon_{\kappa_1}(k_1, \lambda_1) \epsilon_{\kappa_2}(k_2, \lambda_2) \epsilon_{\kappa_3}(k_3, \lambda_3) \times \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^Dq \text{Tr} \left( \gamma_\mu P_{\kappa_1} \frac{1}{q - k_2 - m_k} \gamma_\rho P_{\kappa_2} \frac{1}{q - m_i} \gamma_\nu P_{\kappa_3} \frac{1}{q + k_1 - m_j} \right).
\]

(3.9)

The combinatorial factor \(N_{f_1}^{f_2}\) accounts for the Majorana properties of the heavy neutrinos; it takes the values \(N_{f_1}^{f_2} = 2^n\), where \(n\) is the number of \(n_i n_j Z\) couplings in the respective term of Eq. (B.8). The values of \(g^\kappa_{f_1 f_2 f_3, V}\) for the exotic leptons of the HP and MR models are listed in Eqs. (2.5) and (2.10), respectively; the remaining SM-type couplings are given in Eq. (2.4). Analytic expressions for \(\mathcal{V}_{\kappa_1\kappa_2\kappa_3}^\kappa\) in terms of standard one-loop tensor integrals may be found in Eq. (C.3). The first (second) term contained within the parentheses of Eq. (3.8) stems from the direct (crossed) triangle diagram. For \(V_1 V_2 = W^+ W^\pm\), the direct (crossed) triangle corresponds to the case when \(f_i\) is down-type (up-type) and \(f_j\) and \(f_k\) are up-type (down-type). If \(V_1 V_2 = \gamma \gamma, \gamma Z, ZZ\), then, in the SM and the HP and MR models, only those parts of \(\mathcal{V}_{\kappa_1\kappa_2\kappa_3}^\kappa\) in Eq. (C.3) which carry an extra factor of \(\kappa\) contribute to Eq. (3.8). The SM version of Eq. (3.8) agrees with Eq. (5.23) of Ref. [9] if we multiply \(C^2_3\) in that equation by \(k_2^2/k_1^2\).

It is interesting to note that, in contrast to the HP model, the MR model can admit CP violation, if the mass parameters \(m_1, m_2\), and \(m_M\) are all complex with different phases. Here, we focus our attention on CP-conserving contributions to the triple-gauge-boson couplings. For a detailed discussion of CP-violating form-factors in Majorana-neutrino models, we refer to Ref. [10].
4 Numerical results and discussion

We are now in a position to explore the phenomenological implications of our results. We focus our attention on LEP2 with c.m. energy $\sqrt{s} = 192$ GeV. We adopt the SM parameters from Ref. [12]. We assume the fourth-generation quarks $T$ and $B$ of the HP model to be degenerate with mass $m_T = m_B = 250$ GeV. The other input parameters are varied. We present the cross sections and their radiative corrections in the $G_F$ formulation of the on-shell renormalization scheme; i.e., we eliminate $\alpha$ via the relation

$$G_F = \frac{\pi \alpha}{\sqrt{2} \varepsilon^2 M_W^2} \frac{1}{1 - \Delta r},$$

where $\Delta r$ [13] contains those radiative corrections to the muon lifetime which the SM or its extensions introduce on top of the purely photonic corrections from within the Fermi model. Specifically, we fix $\alpha = \sqrt{2} G_F s^2 M_W^2 / \pi$ and, in turn, substitute $\delta e/e \to \delta e/e - \Delta r/2$ in Eq. (3.7). In Ref. [10], $\Delta r$ has been calculated together with the $S$, $T$, and $U$ parameters [14] in the HP model. Reference [10] also provides general expressions for these quantities in terms of the vacuum polarizations $\Pi_{1V1V}$ defined in Eq. (3.5), which allow us to obtain the corresponding results for the MR model.

We start by restating the tree-level cross sections of processes (i)–(iv). Figure 1(a) shows the differential cross sections $d\sigma/d\cos\theta$ as functions of the scattering angle $\theta$ in the c.m. frame. The results for $\gamma\gamma$ and $\gamma Z$ production exhibit $t$- and $u$-channel poles in the forward and backward directions, at $\theta = 0^\circ$ and $180^\circ$, which are artifacts of our approximation of neglecting the electron mass. However, these poles are avoided if we impose the experimental acceptance cut $|\cos\theta| < 0.966$, which excludes the regions around the beam pipe not covered by the detectors. The integrated cross sections of processes (i)–(iv), evaluated with this angular cut, are displayed in Fig. 1(b) as functions of $\sqrt{s}$. In the LEP2 energy range and above, $e^+e^- \to W^+W^-$ has the largest cross section of all the vector-boson pair-production processes. This process is being extensively studied at LEP2, since it offers a unique opportunity to experimentally establish the non-Abelian nature of the SM, through the $\gamma WW$ and $ZW W$ vertices. In the remainder of this section, we thus focus our attention on $e^+e^- \to W^+W^-$. The parameter spaces of the HP and MR models are significantly constrained by a wealth of electroweak high-precision data. It is convenient to extract these constraints by means of the $S$, $T$, and $U$ parameters [14]. According to a recent global data analysis [15], at the 68% confidence level, the new-physics contributions to $S$, $T$, and $U$ are confined within the ranges

$$-0.43 < S_{\text{new}} < 0.14, \quad -0.38 < T_{\text{new}} < 0.28, \quad -0.36 < U_{\text{new}} < 0.48.$$  

(4.11)

These limits include the uncertainty in the Higgs-boson mass, which is varied in the range $60 < M_H < 1000$ GeV. In addition, there exist limits from direct searches for new heavy fermions at LEP1 and LEP2. Specifically, we require the masses of all fourth-generation neutrinos to be larger than $M_Z/2$, so as to avoid an observable contribution to the invisible
width of the Z boson. Furthermore, we take the masses of the fourth-generation charged leptons to be larger than $\sqrt{s}/2$, for otherwise they would have been produced and directly detected at LEP2. Finally, we must exclude the production of two heavy neutrinos, $n_i$ and $n_j$, by $e^+e^-$ annihilation via a virtual Z boson followed by the decay of $n_i$ and/or $n_j$ into a heavy charged lepton $E$ and an off-shell $W$ boson. In summary,

(i) $m_{n_i} > M_Z/2$ for all $n_i$;

(ii) $m_E > \sqrt{s}/2$;

(iii) if $m_{n_i} + m_{n_j} < \sqrt{s}$, then $m_{n_i}, m_{n_j} < m_E$.

The shaded areas in Figs. 2 and 3 indicate the allowed regions of parameter space for the HP and MR models, respectively. In the four parts of Fig. 2, the $(m_E, m_D)$ plane is scanned for $m_M = 0, 0.25, 1,$ and $5$ TeV, respectively. The case $m_M = 0$ corresponds to a fourth-generation Dirac neutrino. The effects of the various constraints mentioned above are easily recognized. The $S$ parameter determines the left edges of the allowed areas, while the right edges are controlled by the $T$ parameter. Finally, constraints from direct searches confine the shaded regions from below by horizontal lines. Note that the $U$ parameter does not yield any further constraint on the HP model.

Next, we present a similar analysis for the MR model, varying $m_1$ and $m_2$ continuously and $m_M$ and $m_E$ discretely. In Figs. 3(a)–(d), the $(m_1, m_2)$ plane is scanned for $m_M = 0, 0.25, 1,$ and $5$ TeV, respectively. The four parts of each figure refer to $m_E = 0.1, 0.25, 0.5,$ and $1$ TeV, respectively. The reflection symmetry with respect to the diagonal $m_1 = m_2$, which is common to all figures, is a property of the neutrino mass matrix of the MR model. Similarly to the HP model, the $U$ parameter does not lead to any actual constraint. Obviously, the allowed parameter space of the MR model is larger than that of the HP model. This is mainly due to the facts that, in the MR model, fourth-generation quarks are absent and that, besides the $T$ parameter, also the $S$ parameter may take negative values.

In the remainder of this section, we study the radiative corrections to the cross section of $e^+e^-$ → $W^+W^-$ induced by the heavy fermions of the HP and MR models taking into account the above constraints. Figures 4 and 5 display the angular dependence of the radiative corrections to the differential cross section for the HP and MR models, respectively, with typical parameter sets in compliance with the above bounds. We see that the angular dependence is asymmetric. In general, the loop effects are more significant in the backward direction. In the HP model, they may be as large as 1%, for $m_E = 500$ GeV, $m_D = 250$ GeV, and $m_M = 5$ TeV, while, in the MR model, they may not exceed the 0.1% level, for $m_E = m_1 = m_2 = 250$ GeV and $m_M = 0$.

In Figs. 6(a) and (b), contours of constant correction to the integrated cross section are drawn in the $(m_E, m_D)$ plane of the HP model with $m_M = 0$ and $1$ TeV, respectively. As in Fig. 2, the shaded areas indicate the allowed parameter space. Figures 7(a)–(c) describe a similar analysis for the $(m_1, m_2)$ plane of the MR model with (a) $m_E = 250$ GeV and $m_M = 0$, (b) $m_E = 250$ GeV and $m_M = 1$ TeV, and (c) $m_E = 1$ TeV and $m_M = 10$ TeV, respectively. From this analysis, we conclude that the shifts in the integrated cross section
of $e^+e^- \rightarrow W^+W^-$ due to the fourth fermion generations of the constrained HP and MR models cannot exceed 0.1%.

5 Conclusions

We have studied quantum effects due to heavy exotic leptons in the reactions $e^+e^- \rightarrow \gamma\gamma$, $\gamma Z$, $ZZ$, and $W^+W^-$ at LEP2 energies within two viable scenarios of new physics, the HP and MR models. Both models extend the lepton sector of the SM by additional sequential isodoublets. These models also admit the presence of Majorana mass terms of the order of the electroweak scale, which may lead to lepton-number violating signals through the production of heavy Majorana neutrinos at TeV energies. If the direct production of heavy neutrinos predicted by the HP and MR models is impossible at present energies, such neutrinos may still give rise to significant shifts in the oblique electroweak parameters $S$, $T$, and $U$. Exploiting recent experimental information on the $S$, $T$, and $U$ parameters, we have systematically constrained the parameter spaces of the HP and MR models. We have then quantitatively analyzed the loop-induced shifts in the cross section of $e^+e^- \rightarrow W^+W^-$ which arise in the HP and MR models thus constrained. We have found that these shifts cannot exceed the benchmark of 0.1%. In conclusion, cross-section measurements at LEP2 are unlikely to improve the bounds on the parameters of the HP and MR models already established by electroweak high-precision data. In other words, observable deviations from the SM predictions would require alternative explanations. This could not be anticipated without explicit calculation because, in contrast to $Z$-resonance and low-energy physics, vector-boson pair production at LEP2 is already sensitive to the triple-gauge-boson couplings at the tree level.

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A Mass eigenvalues and eigenvectors in the MR model

The couplings $g_{n_i n_j}^\pm$ and $g_{E_{n_i} E_{n_j}}^\pm$ of the MR model given in Eq. (2.10) implicitly depend on the masses of the heavy neutrinos $n_i$ and the charged lepton $E$, through the unitary matrix $U$. Therefore, it is important to express all the parameters, including the mixing angles, in terms of a minimal set of independent variables. We choose this set to be $m_M$, $m_1$, $m_2$, and $m_E$, which we assume to be real so as to avoid CP violation.

Defining the auxiliary variables

\begin{align}
    p &= \frac{1}{3} \left( \frac{m_M^2}{3} + m_1^2 + m_2^2 + m_E^2 \right), \\
    q &= \frac{m_M}{3} \left( \frac{m_M^2}{9} + \frac{m_1^2 + m_2^2}{2} - m_E^2 \right) - m_1 m_2 m_E, \\
    \phi &= \arccos \frac{q}{p^{3/2}},
\end{align}

we may write the three eigenvalues $m_{n_k}$ ($k = 1, 2, 3$) of the mass matrix in Eq. (2.7) in the compact form

\begin{equation}
    m_{n_k} = \frac{m_M}{3} + 2 \sqrt{p} \cos \phi + \frac{2\pi(k-1)}{3}.
\end{equation}

Solving the eigenvalue problem for the MR neutrino mass matrix, we obtain the following set of real, orthogonal vectors:

\begin{equation}
    O_k = N_k \left( m_1 m_{n_k} - m_2 m_E, m_2 m_{n_k} - m_1 m_E, m_{n_k}^2 - m_E^2 \right),
\end{equation}

where $N_k$ are normalization constants defined such that $O_k O_k^T = 1$. In fact, $O_k$ are eigenvectors of the neutrino mass matrix, with real eigenvalues, $\pm m_{n_k}$. If we assume that $m_M, m_1, m_2,$ and $m_E$ are all positive, then we find that the physical mass eigenvectors are $O_1, iO_2,$ and $O_3.$ In this parameter range, the $3 \times 3$ unitary matrix $U$ is given by

\begin{equation}
    U = (O_1^T, iO_2^T, O_3^T).
\end{equation}

B Standard matrix elements

On general grounds, the radiatively corrected helicity amplitudes $\mathcal{M}^\nu(\lambda_1, \lambda_2, s, t)$ of processes (i)–(iv) can all be written as linear combinations of ten independent standard matrix elements. Following Ref. [2], we define

\begin{align*}
    \mathcal{M}_0^\nu &= \bar{v}(p_+) \gamma_1 (\not{k_1} - \not{p_+}) \gamma_2 P_\lambda u(p_-), \\
    \mathcal{M}_1^\nu &= \bar{v}(p_+) \gamma_1 P_\lambda u(p_-) \varepsilon_1 \cdot \varepsilon_2, \\
    \mathcal{M}_2^\nu &= \bar{v}(p_+) \gamma_1 P_\lambda u(p_-) \varepsilon_2 \cdot k_1, \\
    \mathcal{M}_3^\nu &= -\bar{v}(p_+) \gamma_2 P_\lambda u(p_-) \varepsilon_1 \cdot k_2, \\
    \mathcal{M}_4^\nu &= \bar{v}(p_+) \gamma_2 P_\lambda u(p_-) \varepsilon_2 \cdot p_-,
\end{align*}
\[ \mathcal{M}_5'^* = -\bar{v}(p_+) \gamma_2 P_{\kappa} u(p_) \varepsilon_1 \cdot p_+, \]
\[ \mathcal{M}_6'^* = \bar{v}(p_+) \gamma_1 P_{\kappa} u(p_) \varepsilon_1 \cdot p_+ \varepsilon_2 \cdot p_-, \]
\[ \mathcal{M}_7'^* = \bar{v}(p_+) \gamma_1 P_{\kappa} u(p_-) \varepsilon_1 \cdot p_+ \varepsilon_2 \cdot k_1, \]
\[ \mathcal{M}_8'^* = \bar{v}(p_+) \gamma_1 P_{\kappa} u(p_-) \varepsilon_1 \cdot k_2 \varepsilon_2 \cdot p_-, \]
\[ \mathcal{M}_9'^* = \bar{v}(p_+) \gamma_1 P_{\kappa} u(p_-) \varepsilon_1 \cdot k_2 \varepsilon_2 \cdot k_1, \quad (B.1) \]

where we have suppressed the arguments of \( \varepsilon_1(k_1, \lambda_1) \) and \( \varepsilon_2(k_2, \lambda_2) \). In our application, \( \mathcal{M}_6'^* \), \( \mathcal{M}_7'^* \), and \( \mathcal{M}_8'^* \) do not occur. This would be subject to change if we also allowed for inter-family mixing between standard and exotic fermions.

**C One-loop functions**

In this paper, we evaluate the loop amplitudes using dimensional regularization in \( D \) space-time dimensions along with the reduction algorithm of Ref. [16]. In contrast to Ref. [16], we use the Minkowskian metric, \( g^{\mu \nu} = \text{diag}(1, -1, \ldots, -1) \). As usual, we introduce an unphysical 't Hooft mass scale, \( \mu \), to keep the coupling constants dimensionless.

The vector two-point function occurring in Eq. (3.5) is defined as [8]

\[
\Pi_V(q^2, m_1, m_2) = \frac{1}{12 \pi^2} \left\{ \left[ q^2 - \frac{m_1^2 + m_2^2}{2} \right] + 3m_1 m_2 - \frac{(m_1^2 - m_2^2)^2}{2q^2} \right\} B_0(q, m_1, m_2) \\
+ m_1^2 \left( -1 + \frac{m_1^2 - m_2^2}{2q^2} \right) B_0(0, m_1, m_1) + m_2^2 \left( -1 + \frac{m_2^2 - m_1^2}{2q^2} \right) \\
\times B_0(0, m_2, m_2) - \frac{q^2}{3} + \frac{(m_1^2 - m_2^2)^2}{2q^2} \right\}, \quad (C.1)
\]

where

\[
B_0(p, m_1, m_2) = \frac{(2\pi \mu)^{4-D}}{i\pi^2} \int d^D q \frac{1}{(q^2 - m_1^2)(q^2 - m_2^2)} \quad (C.2)
\]

is the standard two-point scalar integral [16].

Next, we list analytic expressions for the triple-gauge-boson vertex functions \( \mathcal{V}_{\kappa_1 \kappa_2 \kappa_3}^\kappa \) of Eq. (3.3) in terms of standard three-point integrals [16]. There is a total of 16 possible helicity combinations. Keeping \( \kappa = \pm \) generic, we have

\[
\mathcal{V}_{\pm \pm \pm}^\kappa = 2 \left[ \frac{2}{3} - 2 (2C_2^0 + C_3^{01} + C_3^{02}) - M_1^2 \left( C_1^1 + 2C_2^1 + C_2^3 + C_3^3 \right) \right. \\
\left. - M_2^2 \left( C_1^2 + 2C_2^2 + C_3^2 + C_3^{21} \right) + (s - M_1^2 - M_2^2) \left( 2C_2^{12} + C_3^{12} + C_3^{21} \right) \right] \mathcal{M}_1'^* \\
+ 2 \left[ -\frac{2}{3} + 2 (C_2^0 - C_3^{01}) - M_1^2 \left( C_2^1 + C_3^{11} \right) + M_2^2 \left( C_1^2 + C_2^2 - C_3^{21} \right) \right. \\
\left. + (s - M_1^2 - M_2^2) C_3^{12} \right] \mathcal{M}_2'^*
\]
of the standard three-point tensor integrals functions appearing in Eq. (C.3) are the scalar coefficients in the Lorentz decompositions

\[ V = \alpha \]

We work in the on-shell renormalization scheme, which uses the fine-structure constant \( \alpha \). Analytic expressions for the Lorentz coefficients may be found in Ref. [9].

\[ \nu_{++} = \frac{2m_{m_j}}{(C_0 + C_1) M_1^2 + C_1 M_2^2} \]

\[ \nu_{+++} = \frac{2m_{m_k}}{(C_0 + C_1 + C_2) M_1^2 + C_1 M_2^2 + C_2 M_3^2} \]

\[ \nu_{++-} = \frac{2m_{m_i}}{(C_0 + C_1 + C_2 + C_3) M_1^2 + C_1 M_2^2 + C_2 M_3^2 + C_3 M_4^2} \]

(C.3)

Here, \( \nu_{+++} \) stands for \( \nu_{++-} \) or \( \nu_{++-} \) and similarly for the other expressions. The \( C \) functions appearing in Eq. (C.3) are the scalar coefficients in the Lorentz decompositions [16] of the standard three-point tensor integrals

\[ \{C_0, C_{\mu}, C_{\mu\nu}, C_{\mu\nu\rho}\}(k_1, -k_2, m_i, m_j, m_k) \]

\[ = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{1, g_{\mu}, g_{\mu\nu}, g_{\mu\nu\rho}}{(q^2 - m_i^2)((q + k_1)^2 - m_j^2)(-q - k_2)^2 - m_k^2}. \]

(C.4)

In the notation of Ref. [1], we have

\[ C_{\mu} = k_{1\mu} C_1^1 - k_{2\mu} C_1^2, \]

\[ C_{\mu\nu} = g_{\mu\nu} C_0^0 + k_{1\mu} k_{1\nu} C_1^1 + k_{2\mu} k_{2\nu} C_2^2 - (k_{1\mu} k_{2\nu} + k_{2\mu} k_{1\nu}) C_1^{12}, \]

\[ C_{\mu\nu\rho} = (g_{\mu\rho} k_{1\nu} + g_{\nu\rho} k_{1\mu} + g_{\mu\nu} k_{1\rho}) C_0^{01} - (g_{\mu\rho} k_{2\nu} + g_{\nu\rho} k_{2\mu} + g_{\mu\nu} k_{2\rho}) C_0^{02} + k_{1\mu} k_{1\nu} k_{1\rho} C_1^{12} - k_{2\mu} k_{2\nu} k_{2\rho} C_2^{12} - (k_{1\mu} k_{2\nu} + k_{1\mu} k_{2\rho} + k_{2\mu} k_{1\nu} + k_{2\mu} k_{1\rho}) C_3^{12} + (k_{2\mu} k_{2\nu} k_{1\rho} + k_{2\mu} k_{1\nu} k_{2\rho} + k_{1\mu} k_{2\nu} k_{2\rho}) C_3^{21}. \]

(C.5)

Analytic expressions for the Lorentz coefficients may be found in Ref. [1].

**D Renormalization constants**

We work in the on-shell renormalization scheme, which uses the fine-structure constant \( \alpha = e^2/(4\pi) \) and the physical particle masses as basic parameters. As usual, we express
the bare parameters and wave functions, which carry the subscript 0, in terms of the respective renormalized quantities and counterterms as

\[
e_0 = e + \delta e, \quad s_{w,0} = s_w + \delta s_w, \quad g_{eeZ,0}^c = g_{eeZ}^c + \delta g_{eeZ}^c,
\]

\[
M_{W,0}^2 = M_W^2 + \delta M_W^2, \quad M_{Z,0}^2 = M_Z^2 + \delta M_Z^2, \quad W_0^\pm = W^\pm \left(1 + \frac{\delta Z_W}{2}\right),
\]

\[
\begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} 1 + \frac{\delta Z_{ZZ}}{2} & \frac{\delta Z_{Z\gamma}}{2} \\ \frac{\delta Z_{\gamma Z}}{2} & 1 + \frac{\delta Z_{\gamma\gamma}}{2} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}.
\]

(D.1)

In the absence of mixing between the standard and exotic fermion generations, the electron mass and wave function are not affected by the new physics at one loop. The counterterms in Eq. (D.1) can all be expressed in terms of the transverse vacuum-polarization functions defined in Eq. (3.5), as

\[
\delta M_W^2 = \text{Re} \Pi_T^{WW} (M_W^2), \quad \delta M_Z^2 = \text{Re} \Pi_T^{ZZ} (M_Z^2),
\]

\[
\delta Z_W = -\text{Re} \frac{\partial \Pi_T^{WW} (p^2)}{\partial p^2} \bigg|_{p^2=M_W^2}, \quad \delta Z_{ZZ} = -\text{Re} \frac{\partial \Pi_T^{ZZ} (p^2)}{\partial p^2} \bigg|_{p^2=M_Z^2},
\]

\[
\delta Z_{\gamma\gamma} = -\frac{\partial \Pi_T^{Z\gamma} (p^2)}{\partial p^2} \bigg|_{p^2=0}, \quad \delta Z_{Z\gamma} = 2 \frac{\Pi_T^{Z\gamma} (0)}{M_Z^2}, \quad \delta Z_{Z\gamma} = -2 \text{Re} \frac{\partial \Pi_T^{Z\gamma} (M_Z^2)}{M_Z^2},
\]

\[
\frac{\delta e}{e} = -\frac{1}{2} \left( \delta Z_{\gamma\gamma} + \frac{s_w}{c_w} \delta Z_{Z\gamma} \right), \quad \frac{\delta s_w}{s_w} = \frac{c_w^2}{2 s_w^2} \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right),
\]

\[
\frac{\delta g_{eeZ}^c}{g_{eeZ}^c} = \frac{1}{c_w^2} \frac{\delta s_w}{s_w}, \quad \frac{\delta g_{eeZ}^c}{g_{eeZ}^c} = \frac{1}{c_w^2 (s_w^2 - c_w^2)} \frac{\delta s_w}{s_w}.
\]

(D.2)

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Figure Captions

Fig. 1: (a) Differential and (b) integrated cross sections of $e^+e^- \rightarrow \gamma\gamma$, $\gamma Z$, $ZZ$, and $W^+W^-$ in the Born approximation.

Fig. 2: Allowed parameter space of the HP model with $m_M = 0, 0.25, 1, \text{ and } 5 \text{ TeV}.$

Fig. 3: Allowed parameter space of the MR model with $m_E = 0.1, 0.25, 0.5, \text{ and } 1 \text{ TeV}$ and (a) $m_M = 0$, (b) $m_M = 0.25 \text{ TeV}$, (c) $m_M = 1 \text{ TeV}$, and (d) $m_M = 5 \text{ TeV}$.

Fig. 4: Radiative correction relative to the differential cross section of $e^+e^-$ $\rightarrow$ $W^+W^-$ in the HP model as a function of the scattering angle.

Fig. 5: Radiative correction relative to the differential cross section of $e^+e^-$ $\rightarrow$ $W^+W^-$ in the MR model as a function of the scattering angle.

Fig. 6: Contour levels of the radiative correction relative to the integrated cross section of $e^+e^-$ $\rightarrow$ $W^+W^-$ in the HP model with (a) $m_M = 0$ and (b) $m_M = 1 \text{ TeV}$. The allowed regions of parameter space are shaded.

Fig. 7: Contour levels of the radiative correction relative to the integrated cross section of $e^+e^- \rightarrow W^+W^-$ in the RM model with (a) $m_E = 0.25 \text{ TeV}$ and $m_M = 0$, (b) $m_E = 0.25 \text{ TeV}$ and $m_M = 1 \text{ TeV}$, and (c) $m_E = 1 \text{ TeV}$ and $m_M = 10 \text{ TeV}$, respectively. The allowed regions of parameter space are shaded.
Fig. 1a

Fig. 1b
Fig. 2
Fig. 3a
Fig. 3b
Fig. 3c
Fig. 3d
Fig. 4

Fig. 5
Fig. 6a

Fig. 6b
