General Rule Predicting the Hidden Induced Order Parameters and the Formation of Quartets and Patterns of Condensates

Georgios Varelogiannis
Department of Physics, National Technical University of Athens, GR-15780 Athens, Greece

We reveal the rule with which we predicted, and verified by detailed numerical calculations on dozens of specific cases over the last decade, the emergence of hidden induced states and the formation of quartets of order parameters. The rule stipulates that if \( N \) order parameters are such that their cyclic products equal \( \pm 1 \) then the presence of any \( N-1 \) of them will necessarily induce the missing order parameter as a hidden order. We demonstrate that the rule results from general microscopic mean field theory and is thus universally valid. Of exceptional physical interest are the cases \( N=4 \) called quartets, and all \( N > 4 \) cases decompose into overlapping quartets which are the building blocks of observable patterns of condensates. Quartets may be found in two regimes, the hierarchy regime and the equity regime, and transitions between the two regimes exhibit universal characteristics. Following the rule, we predict many characteristic examples of quartets involving all types of particle-hole and superconducting condensates.

PACS numbers: 74.70.Xa, 74.20.-z

Mean field theory is a universal tool in the study of phase transitions. The archetype of a microscopic mean field theory for a quantum phase transition is BCS theory of superconductivity \[\text{[1]}\] involving one order parameter. With the discovery of unconventional superconductivity emerged the need to consider more order parameters enhancing the dimensions of the symmetry space \[\text{[2, 3]}\]. Moreover, in many unconventional superconductors various types of magnetic and/or charge ordered phases coexist or compete with superconductivity and this requires a further enhancement of the symmetry space \[\text{[4, 5]}\]. However, with the exception of some early attempts \[\text{[6, 7]}\], the use of microscopic mean field theories for multiple order parameters situations is avoided because of the rising complexity, and more practical phenomenological Ginzburg Landau schemes are adopted \[\text{[7]}\].

Here we reveal that in multiple order parameter spinor microscopic mean field theories, there is an universal coupling that emerges between the parameters that satisfy a simple rule, their cyclic products equal either \( +1 \) or \( -1 \). This interaction between order parameters is not anticipated from group theory arguments and is unrelated with details of the hamiltonians. Order parameters that obey this (anti)unitary cyclic rule aggregate into quartets that represent the building blocks of quantum complexity. A quartet is a set of four order parameters that may include the kinetic terms of the Hamiltonian, that are such that if any three of them are present the fourth is necessarily present as well. This is the generic mechanism through which hidden induced phases emerge.

To demonstrate the relevance of the rule, we have studied in detail dozens of specific quartets over the last decade. We prove that, it is beyond any doubt, that the above (anti)unitary cyclic product rule should be used a priori in any quantum spinor mean field theory to make real physical predictions of great importance that may affect the symmetry and even the topological characteristics of the system under examination. Moreover, qualitatively new phenomena reported recently, like the sudden emergence of a quartet of coexisting condensates at high fields and low temperatures in CeCoIn\(_5\) \[\text{[8]}\] reflect in fact global characteristics of quartets which may be tuned to switch from the usual hierarchy regime, where members of the quartet are dominating, to the equity regime where all members manifest fully. We provide characteristic quartets that result from the SU(8) Solomon and Birman theory \[\text{[9]}\] for particle-hole condensates and superconductivity.

We consider as an example an eight dimensional spinor, however our arguments are straightforwardly generalized to higher dimensional spinors as well. A mean field Hamiltonian is written as \( H = \Psi_k^\dagger \tilde{E}_k \Psi_k \) where \( \Psi_k \) are spinors and \( \tilde{E}_k \) is the \( 8 \times 8 \) energy matrix and the resulting Green’s function is:

\[
\hat{G}_\alpha(k, i\omega_n) = \frac{1}{i\omega_n - \tilde{E}_k} = f_0(k, i\omega_n) + f_1(k, i\omega_n)\tilde{E}_k + \ldots + f_7(k, i\omega_n)\tilde{E}_k^7
\]

Suppose now the Hamiltonian includes the order parameters \( \alpha \) and \( \beta \) with corresponding matrices \( \hat{A}, \hat{B} \), and we examine if the order parameter \( \gamma \) with matrix \( \Gamma \) may be induced as a hidden order. All matrices are projected into a basis of tensor products of Pauli matrices, \( \hat{\tau}_i \otimes \hat{\rho}_j \otimes \hat{\sigma}_l \)

In the self-consistency equation that provides \( \gamma \)

\[
\gamma_k = T \sum_{k'} \sum_n V_{kk'}^n \frac{1}{8} Tr\{\hat{G}_\alpha(k', i\omega_n)\}
\] (2)

a non-zero trace exists only if a term proportional to the matrix \( \hat{\Gamma} \) is generated and this can happen only if \( \hat{A}\hat{B} = c\hat{\Gamma} \). However, this contribution results from some power of \( \tilde{E}_k \), and is always accompanied by an additional contribution analogous to \( \hat{B}\hat{A} \). The condition for having \( \gamma \) induced as a hidden order from the order parameters \( \alpha \) and \( \beta \) becomes \( \{\hat{A}, \hat{B}\} \neq 0 \). It is straightforward that we can only have \( \hat{A}\hat{B} = \hat{B}\hat{A} \), or \( \hat{A}\hat{B} = -\hat{B}\hat{A} \) but the
second case is irrelevant. Therefore, $\gamma$ is induced as a hidden order if $\hat{A}\hat{B} = \hat{B}\hat{A} = c\hat{\Delta}$. From that we get $\hat{B} = c\hat{A}$ and and $\hat{B} = c\hat{A}$ and multiplying the last two equations we obtain $c = \pm 1$. While $c = 1$ is anticipated, $c = -1$ is sometimes ignored even in this simple case. With this, we now repeat the procedure in the other two order parameter channels, and show that there is a term proportional to $\alpha\gamma$ in the $\hat{B}$ channel and one proportional to $\beta\gamma$ in the $\hat{A}$ channel of $\hat{G}_o(k, \omega_n)$. This implies that any one of $\alpha, \beta, \gamma$ will emerge necessarily as a hidden order parameter if the other two are present. Such a trio of coupled order parameters in $\hat{A}$ of which the coexistence of two implies the third satisfies $\hat{A}\hat{B} = \hat{B}\hat{A} = \pm\hat{\Delta}$, $\hat{A}\hat{B} = \hat{B}\hat{A} = \pm\hat{\Delta}$ which is equivalent to our cyclic rule:

$$\hat{\Delta}\hat{A}\hat{B}\hat{\Delta} = \hat{\Delta}\hat{A}\hat{B}\hat{\Delta} = \pm 1$$

The physically relevant quartets emerge from a similar procedure, supposing that we have three order parameters, $\alpha$, $\beta$, $\gamma$, and we examine whether a hidden fourth order parameter $\delta$ with corresponding matrix $\hat{\Delta}$ can be induced: $\hat{A}\hat{B}\hat{\Delta} = c\hat{\Delta}$. As before, because this product may emerge by some power of $\hat{E}_k$ the necessary condition becomes $\hat{A}\hat{B}\hat{\Delta} + \hat{\Delta}\hat{B}\hat{\Delta} + \hat{\Delta} = \pm 0$ which is equivalent to

$$\{\hat{A}, \hat{B}\}\hat{\Delta} + \{\hat{A}, \hat{B}\}\hat{\Delta} + \{\hat{B}, \hat{\Delta}\}\hat{A} \neq 0$$

In fact, one can prove that $\delta$ will be induced if either one, or all three pairs of $\alpha$, $\beta$, and $\gamma$ commute. By repeating the procedure as before to the other three order parameter channels, we conclude that a quartet is made of four order parameters satisfying the cyclic product rule

$$\hat{\Delta}\hat{A}\hat{B}\hat{\Delta} = \hat{\Delta}\hat{A}\hat{B}\hat{\Delta} = \hat{\Delta}\hat{A}\hat{B}\hat{\Delta} = \pm 1$$

Similarly, we can generalize to the case of four order parameters inducing a hidden fifth: $\hat{A}\hat{B}\hat{\Delta} = c\hat{E}$, and the necessary and sufficient conditions are now more complicated:

$$\{\hat{A}, \hat{B}\}\hat{\Delta} + \{\hat{A}, \hat{\Delta}\}\hat{B}\hat{\Delta} + \{\hat{B}, \hat{\Delta}\}\hat{A}\hat{\Delta} + \{\hat{\Delta}, \hat{\Delta}\}\hat{A} \neq 0$$

leading to a similar cyclic product rule. However, this is useless since one can show that the many order parameter states decompose into a number of eventually overlapping quartets. In fact, proceeding recursively, we can construct patterns of coexisting condensates involving more than one quartet as we will illustrate in a specific example. This becomes more transparent, and computationally more tractable for many order parameters, by noting that the gap equation can be written as

$$\gamma_k = T \sum_{k',\eta} V_{kk'} \frac{1}{\lambda_2\lambda_3...\lambda_8 + \lambda_1\lambda_3...\lambda_8 + ... + \lambda_1...\lambda_7}{\lambda_1\lambda_2...\lambda_8}$$

| Q | $\tilde{\rho}_3$ | particle hole asymmetry, chemical potential |
|---|---|---|
| Y | $\tau_3\rho_3$ | nesting, particle hole symmetry, Pomeranchuk |
| $S^\ast_1\rho_2\sigma_1$ | conventional ferromagnet along y |
| $S^\ast_2\sigma_2$ | conventional ferromagnet along y |
| $S^\ast_3\rho_3\sigma_3$ | conventional ferromagnet along z |
| $A^\ast_1\tau_3\rho_3\sigma_1$ | d-wave ferromagnet, spin Pomeranchuk along x |
| $A^\ast_2\sigma_2\rho_3\sigma_3$ | d-wave ferromagnet, spin Pomeranchuk along y |
| $A^\ast_1\tau_3\rho_3\sigma_1$ | d-wave ferromagnet, spin Pomeranchuk along z |
| $C_{Q_1}\tau_1\rho_1$ | conventional CDW |
| $J_{C_1}\tau_2$ | d-wave CDW, orbital antiferromagnet |
| $S_{Q_1}\tau_1\rho_1\sigma_1$ | conventional SDW polarized along x |
| $S_{Q_2}\tau_1\sigma_2$ | conventional SDW polarized along y |
| $S_{Q_3}\tau_1\rho_3\sigma_3$ | conventional SDW polarized along z |
| $J_{Q_1}\tau_2\sigma_1$ | d-wave SDW polarized along x |
| $J_{Q_2}\tau_2\rho_2\sigma_2$ | d-wave SDW polarized along y |
| $J_{Q_3}\tau_2\sigma_3$ | d-wave SDW polarized along z |

where $\lambda_i$ are the eigenvalues of $\hat{G}_o^{-1}(k', \omega_n)\hat{\Gamma}$ which because $\text{Det}[\hat{\Gamma}] = \pm 1$ are in fact given by

$$\text{Det} [\hat{G}_o^{-1}(k', \omega_n) - \lambda\hat{\Gamma}] = 0$$

Finally, we briefly note that the order parameters may be similarly mixed by self-energy terms $\tilde{\Sigma}$. For example, if we renormalize the propagator by $\hat{G} \approx \hat{G}_o + \tilde{\Sigma}\hat{G}_o$ and take $\tilde{\Sigma} \approx nU\tilde{\rho}_3$, then in the channel of $\alpha$ the product $\beta\gamma$ emerges if $\hat{\Gamma}\tilde{\rho}_3\hat{B} = \hat{B}\hat{\rho}_3\hat{\Gamma}$ or equivalently $\hat{A}\hat{B}\hat{\Delta} = \pm\hat{\Delta}$.

We have tested the real physical implications of our rule on a spinor space involving unconventional superconducting and particle-hole condensates, many of which have been identified experimentally in real material systems and are related to some of the most exciting phenomena in many body physics and nanophysics. The 63 order parameters involved are in fact generators of the SU(8) spectrum generating algebra of Solomon and Birman [9]. Our discussion is based on the spinor $\Psi_k$. The invariance symmetry order parameters of Table I, with the exception of $Y$, $Q$, $C_Q$ and $J_{S}^{\pm,y,-}$ all the others break time reversal invariance. By conventional we mean any condensate that is even in translation and this includes $d_{xy}$ states, whereas d-wave is only indicative referring to the well known $d_{x^2-y^2}$ states from high-$T_c$.
cuprates, however it is as well valid for any other representation that is odd in translation. The coexistence of a $d_{xy} C_Q$ with $J_C$ imply the break of both parity in two dimensions and time reversal, forming thus a chiral state that accounts for the anomalous Nernst phenomena in the pseudogap of the cuprates [12]. If on the other hand the $d_{xy} C_Q$ coexists with $J_S^{x,y,z}$ they form an helical state [13] which is also a topological state with higher angular momentum differing essentially on the fact that edge currents of different spin orientations flow in opposite directions. Clearly, the aggregation of condensates into quartets may affect the topologic characteristics of the system. Multiplying with $\tilde{\rho}_3$ all states of Table I except Q provides the corresponding odd in inversion particle-hole condensates that we note with a tilde. For example the odd in inversion spin Pomeranchuk state along $z$ is written $\tilde{A}^z = \tilde{\tau}_3 \tilde{\sigma}_3$. Breaking of the inversion obliges the order parameters that were breaking time reversal to restore it, and the others to break it. Indeed, now the time reversal symmetry is only broken by $Y$, $C_Q$ and and $J_S^{x,y,z}$. A general discussion of unconventional density waves can be found in [14] and Pomeranchuk states were discussed in [15, 16].

In Table II are given the SC condensates that are even under inversion. Unless specified by an index 3, the SC condensates are real. Only the imaginary partners that result replacing $\tilde{\rho}_3$ by $\tilde{\rho}_1$ break time reversal invariance. The 10 real SC condensates that break inversion result by multiplying the inversion symmetric SC condensates with $\tilde{\rho}_3$ and are given in Table III. The imaginary partners are now produced by changing $\tilde{\rho}_1$ in $\tilde{\rho}_2$ and break time reversal invariance. The non-centrosymmetric superconductors [17] are the natural playground of odd SC condensates. However, the coexistence of $p_x$ with $p_y^3$ is a chiral state proposed for SrRuO$_3$ [18] and LiFeAs [20, 21], and there are also proposals for odd topological superconductivity in Cu$_4$Bi$_2$Se$_3$ as well [22], all these materials being centrosymmetric.

Having the full palette of order parameters, it is easy to exploit the present rule finding a very large number of quartets. Note that members of quartets may be the kinetic terms of the hamiltonian $Y$ and $Q$, as well as the ferromagnetic order parameters $S^{x,y,z}$ that are equivalent to an applied Zeeman field. To produce the quartets we are interested on we proceed as follows: we start with a pair of condensates relevant to the considered problem, then according to our rule, they will interact with a third condensate if and only if, the three pairs that these three condensates can form either all commute or only one of the three pairs commutes. Then it is sufficient to make the product of the there condensates to identify the fourth with which they form a quartet. The correct mean field approach to the problem should include all four members of the quartet. In Table IV are reported examples of quartets involving only the even in inversion order parameters that are generators of an SO(8) algebra and in Table V some quartets that involve a pair of even and a pair of odd under inversion order parameters.

Remarkably, almost all interesting quartets that we report in Table IV involve orders that break translational invariance. Unless is specified 3 in a SC order of a quartet, it is understood that the quartet is valid for both the real and the imaginary SC orders. Note that a pair of order parameters belongs to more than one quartets. Indeed the quartet $(Q, C_Q, s-SC, \eta-SC)$ with the quartets $(Q, J_C, d-SC, \eta-SC)$, $(Y, Q, s-SC, d-SC)$ and $(Y, C_Q, d-SC, \eta-SC)$, have in common two order parameters. We can form a closed pattern of condensates out of these quartets $(Y, Q, C_Q, J_C, s-SC, d-SC, \eta-SC)$. Considering the real and imaginary parts of the SC order parameters

| TABLE II: Inversion symmetric real SC condensates. For the imaginary partners replace $\tilde{\rho}_2$ by $\tilde{\rho}_1$ and note with index 3. |
|---|---|
| $s$-SC | $\tilde{\rho}_2 \tilde{\sigma}_3$ |
| $d$-SC | $\tau_1 \tilde{\rho}_2 \tilde{\sigma}_3$ |
| $\eta$-SC | $\tilde{\tau}_2 \tilde{\rho}_2 \tilde{\sigma}_3$ |
| $\pi_x$-SC | $\tilde{\tau}_1 \tilde{\rho}_2 \tilde{\sigma}_3$ |
| $\pi_y$-SC | $\tilde{\tau}_2 \tilde{\rho}_2 \tilde{\sigma}_3$ |
| $\pi_z$-SC | $\tilde{\tau}_2 \tilde{\rho}_2 \tilde{\sigma}_3$ |

| TABLE III: Real, odd in inversion SC condensates For the imaginary partners replace $\tilde{\rho}_1$ by $\tilde{\rho}_2$ and note with index 3. |
|---|---|
| $s$-SC | $\tilde{\rho}_1 \tilde{\sigma}_3$ |
| $s$-SC | $\tilde{\rho}_1 \tilde{\sigma}_3$ |
| $s$-SC | $\tilde{\rho}_1 \tilde{\sigma}_3$ |
| $p_x$-SC | $\tilde{\tau}_1 \tilde{\rho}_2 \tilde{\sigma}_3$ |
| $p_y$-SC | $\tilde{\tau}_1 \tilde{\rho}_2 \tilde{\sigma}_3$ |
| $p_z$-SC | $\tilde{\tau}_1 \tilde{\rho}_2 \tilde{\sigma}_3$ |
| $\eta$-SC | $\tilde{\tau}_1 \tilde{\rho}_2 \tilde{\sigma}_3$ |
| $\eta$-SC | $\tilde{\tau}_1 \tilde{\rho}_2 \tilde{\sigma}_3$ |
| $\pi$-SC | $\tilde{\tau}_1 \tilde{\rho}_2 \tilde{\sigma}_3$ |

| TABLE IV: Examples of quartets with inversion symmetric order parameters |
|---|---|
| Quartets | Quartets |
| $Y, Q, A^{x,y,z}, S^{x,y,z}$ | $Y, Q, s-SC, d-SC$ |
| $Q, S^{x,y,z}_Q, A^{x,y,z}, J_S^{x,y,z}$ | $Q, C_Q, S^{x,y,z}_Q, S^{x,y,z}_Q$ |
| $Q, J_C, J_S^{x,y,z}, S^{x,y,z}_Q$ | $Q, C_Q, s-SC, \eta-SC$ |
| $Q, \pi_x$-SC, $\pi_x$-SC, $S^{x,y,z}_Q$ | $Q, s-SC, \pi_x$-SC, $J_S^{x,y,z}$ |
| $Y, d-SC, \pi_x$-SC, $J_S^{x,y,z}$ | $Y, s-SC, \pi_x$-SC, $S^{x,y,z}_Q$ |
| $Q, d-SC$, $\pi_x$-SC, $S^{x,y,z}_Q$ | $Q, \eta-SC$, $\pi_x$-SC, $A^{x,y,z}$ |
| $Y, \pi_x$-SC, $\pi_x$-SC, $S^{x,y,z}_Q$ | $Y, C_Q, \eta-SC, d-SC$ |
| $Y, C_Q, A^{x,y,z}_Q, S^{x,y,z}_Q$ | $Y, C_Q, J_S^{x,y,z}, S^{x,y,z}_Q$ |
| $Y, S^{x,y,z}_Q$, $J_S^{x,y,z}, S^{x,y,z}_Q$ | $Y, J_S^{x,y,z}, S^{x,y,z}_Q, S^{x,y,z}_Q$ |
| $S^{x,y,z}, d^-SC, \pi_x$-SC, $S^{x,y,z}_Q$ | $S^{x,y,z}, s-SC, \eta-SC, S^{x,y,z}_Q$ |
| $S^{x,y,z}, s-SC, \pi_x$-SC, $J_C$ | $S^{x,y,z}, s-SC, d-SC, A^{x,y,z}$ |
| $S^{x,y,z}, S^{x,y,z}$, $\pi_x$-SC, $\pi_x$-SC, $J_C$ | $S^{x,y,z}, s-SC, d-SC, A^{x,y,z}$ |
| $S^{x,y,z}, C_Q, \pi_x$-SC, $\pi_x$-SC | $S^{x,y,z}, C_Q, \pi_x$-SC, $\pi_x$-SC |

| TABLE V: Some quartets that involve a pair of even and a pair of odd under inversion order parameters |
|---|---|
| Quartets | Quartets |
| $Y, Q, A^{x,y,z}, S^{x,y,z}$ | $Y, Q, s-SC, d-SC$ |
| $Q, S^{x,y,z}_Q, A^{x,y,z}, J_S^{x,y,z}$ | $Q, C_Q, S^{x,y,z}_Q, S^{x,y,z}_Q$ |
| $Q, J_C, J_S^{x,y,z}, S^{x,y,z}_Q$ | $Q, C_Q, s-SC, \eta-SC$ |
| $Q, \pi_x$-SC, $\pi_x$-SC, $S^{x,y,z}_Q$ | $Q, s-SC, \pi_x$-SC, $J_S^{x,y,z}$ |
| $Y, d-SC, \pi_x$-SC, $J_S^{x,y,z}$ | $Y, s-SC, \pi_x$-SC, $S^{x,y,z}_Q$ |
| $Q, d-SC$, $\pi_x$-SC, $S^{x,y,z}_Q$ | $Q, \eta-SC$, $\pi_x$-SC, $A^{x,y,z}$ |
| $Y, \pi_x$-SC, $\pi_x$-SC, $S^{x,y,z}_Q$ | $Y, C_Q, \eta-SC, d-SC$ |
| $Y, C_Q, A^{x,y,z}_Q, S^{x,y,z}_Q$ | $Y, C_Q, J_S^{x,y,z}, S^{x,y,z}_Q$ |
| $Y, S^{x,y,z}_Q$, $J_S^{x,y,z}, S^{x,y,z}_Q$ | $Y, J_S^{x,y,z}, S^{x,y,z}_Q, S^{x,y,z}_Q$ |
| $S^{x,y,z}, d^-SC, \pi_x$-SC, $S^{x,y,z}_Q$ | $S^{x,y,z}, s-SC, \eta-SC, S^{x,y,z}_Q$ |
| $S^{x,y,z}, s-SC, \pi_x$-SC, $J_C$ | $S^{x,y,z}, s-SC, d-SC, A^{x,y,z}$ |
| $S^{x,y,z}, C_Q, \pi_x$-SC, $\pi_x$-SC | $S^{x,y,z}, C_Q, \pi_x$-SC, $\pi_x$-SC |
we obtain 10 order parameters that can be generators of a singlet SO(5) model potentially relevant for high-T_c cuprates. We will present in a following publication with collaborators, how the particle-hole condensates in a pattern of condensates could help the emergence of pairs of SC condensates enhancing the critical temperature.

The examples of quartets reported in Tables IV and V involve order parameters associated with virtually all relevant phenomena in many body physics and their potential implications will be discussed elsewhere. We focus only to what we call the fundamental quartet: chemical potential Q, conventional CDW and SDW, and FM order parameter from the hierarchy to equity regime. In the red case there is only one dominating order in the hierarchy regime. In the blue case a tiny hidden order is induced in the hierarchy regime as well. Full lines correspond to sharp first order transitions. We have induced the transition from one regime to the other with doping or with applying a magnetic field. In a following publication with collaborators we discuss the correct set of parameters for any spinor mean field theory, predicts examples of quartets that we associate with experimentally observed domes around expected quantum critical points in many different material systems are the places where specific quartets fully develop. In a following publication with collaborators we discuss in detail the hierarchy to equity transition on specific examples of quartets that we associate with experimentally observed domes.

In conclusion, we have verified that a simple rule resulting from general microscopic mean field theory, predicts hidden order parameters and the formation of quartets of order parameters and should be used apriori to predict the correct set of parameters for any spinor mean field theory. The quartet interactions between order parameters that we report are independent of the details of the hamiltonians involved and exhibit universal characteristics. The quartets are the fundamental building blocks from which more complicated patterns of condensates may emerge. We believe our results should be considered in any field of physics where there is need for a

| Quartets | Quartets |
|----------|----------|
| Y, $S^{x,y,z}$, Y, $S^{x,y,z}$ | Y, $S^{x,y,z}$, Y, $S^{x,y,z}$ |
| Y, $A^{x,y,z}$, Y, $A^{x,y,z}$ | Y, $C^{x,y,z}$, Y, $C^{x,y,z}$ |
| Y, $J_{C}$, Y, $J_{C}$ | Y, $J_{C}^{x,y,z}$, Y, $J_{C}^{x,y,z}$ |
| Q, $S^{x,y,z}$, Y, $A^{x,y,z}$ | Q, $A^{x,y,z}$, Y, $S^{x,y,z}$ |
| $S^{x,y,z}$, $S^{x,y,z}$, Y, $J_{C}^{x,y,z}$ | $S^{x,y,z}$, $p_{-}$-SC, $p_{+}$-SC |
| Y, $S^{x}$, $p_{0}$-SC, $n_{y}$-SC | Y, $A^{x}$, $p_{+}$-SC, $p_{-}$-SC |
| Y, $A^{x}$, $p_{0}$-SC, $n_{y}$-SC | Y, $d$-SC, $A^{x,y,z}$, $p_{x,y,z}$-SC |
| Y, $C^{x}$, $p_{x,y,z}$-SC, $p_{x,y,z}$-SC | Q, $S^{x}$, $p_{-}$-SC, $p_{0}$-SC |
| Q, $A^{x}$, $s_{z}$-SC, $p_{0}$-SC | Q, $s$-SC, $A^{x,y,z}$, $p_{x,y,z}$-SC |
| Q, Y, $p_{x,y,z}$-SC, $s_{x,z}$-SC | Q, $d$-SC, $S^{x,y,z}$, $p_{x,y,z}$-SC |
| $S^{x,y,z}$, $s$-SC, $C^{x}$, $p_{x,y,z}$-SC | $S^{x,y,z}$, $d$-SC, $C^{x}$, $p_{x,y,z}$-SC |
| $C^{x}$, $s$-SC, $J_{C}^{x,y,z}$, $p_{x,y,z}$-SC | $C^{x}$, $d$-SC, $A^{x,y,z}$, $p_{x,y,z}$-SC |
spinor formalism. 
I am indebted to Peter Littlewood, Gil Lonzarich and Ben Simons for numerous enlightening and stimulating comments and discussions, collaboration on various parts of the project and continuous encouragement. I am also grateful to my PhD and diploma students for their precious help during the period of the project: A. Aperis, M. Georgiou, G. Giannopoulos, P. Kotetes, S. Kourtis, G. Livanas, G. Roumpos and S. Tsonis. Work has been supported by the PEBE program of NTUA.

[1] J. Bardeen, L.N. Cooper, J.R. Schrieffer, Phys. Rev. 108, 1175 (1957).
[2] M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239 (1991).
[3] G. Volovik, The Universe in a Helium Droplet, Clarendon Press, Oxford (2003).
[4] S.C. Zhang, Science 275, 1089 (1997); E. Demler, W. Hanke, S.C. Zhang, Rev. Mod. Phys. 76, 909 (2004).
[5] G. C. Psaltakis and E.W. Fenton, J. Phys. C 16, 3913 (1983).
[6] M. J. Nass, K. Levin, and G. Grest, Phys. Rev. Lett. 46, 614 (1981); Phys. Rev. B 25, 4541 (1982).
[7] G.C. Milward, M.J. Calderon, and P.B. Littlewood, Nature (London) 433, 607 (2005).
[8] A. Aperis, G. Varelogiannis and P.B. Littlewood, Phys. Rev. Lett. 104, 216403 (2010).
[9] A.I. Solomon and J. Birman, J. Math. Phys. 28, 1526 (1987).
[10] S. Tsonis, G. Varelogiannis, F. Marchetti, B.D. Simons, and P.B. Littlewood, Physica B 378 - 380, 428 (2006).
[11] Wei Min Zhang, Phys. Rev. B 65, 104513 (2002).
[12] P. Kotetes and G. Varelogiannis, Phys. Rev. Lett. 104, 106404 (2010).
[13] C.H. Hsu, S. Raghu and S. Chakravarty, Phys. Rev. B 82, 155111 (2011).
[14] P. Thalmeier, Z. Phys. B 100, 387 (1996); C. Nayak, Phys. Rev. B 62, 4880 (2000).
[15] S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature (London) 393, 550 (1998); H. Yamase and H. Kohno, J. Phys. Soc. Jpn. 69, 332 (2000); 69, 2151 (2000); C. J. Halboth and W. Metzner, Phys. Rev. Lett. 85, 5162 (2000).
[16] C. Wu and S.-C. Zhang, Phys. Rev. Lett. 93, 036403 (2004); C. Wu, K. Sun, E. Fradkin and S.-C. Zhang, Phys. Rev. B 75, 115103 (2007).
[17] E. Bauer et al., Phys. Rev. Lett. 92, 027003 (2004); P. A. Frigeri, D. F. Agterberg, I. Milat, and M. Sigrist, Eur. Phys. J. B 54, 435 (2006); Y. Yanase and M. Sigrist, J. Phys. Soc. Jpn 77, 124711 (2008).
[18] T. Takimoto and P. Thalmeier, J. Phys. Soc. Jpn 78, 103703 (2009).
[19] T.M. Rice and M. Sigrist, J. Phys. Condensed Matter, 7, L643 (1995).
[20] T. Hanke et al., Phys. Rev. Lett. 108, 127001 (2012).
[21] A. Aperis and G. Varelogiannis, arXiv:1303.2231.
[22] L. Fu and E. Berg, Phys. Rev. Lett. 105, 097001 (2010).
[23] G. Varelogiannis, Phys. Rev. Lett. 85, 4172 (2000).
[24] B.A. Volkov and Yu.V. Kopaev, JETP Lett. 19, 104 (1973); B.A. Volkov, Yu.V. Kopaev, and A.I. Rusinov, Sov. Phys. JETP 41, 952 (1975); ibid 43, 589 (1976).
[25] M.E. Zhitomirsky, T.M. Rice and V.I. Anisimov, Nature (London) 402, 251 (1999).
[26] V. Barzykin and L. Gorkov, Phys. Rev. Lett. 84, 2207 (2000).
[27] S. Tsonis, PhD thesis, National Technical University of Athens (2007).
[28] A. Aperis, PhD thesis, National Technical University of Athens, (2012).
[29] S. Tsonis, P. Kotetes, G. Varelogiannis, and P.B. Littlewood, Journal of Physics: Condensed Matter 20, 434234 (2008).
[30] A. Aperis, G. Varelogiannis, P.B. Littlewood and B.D. Simons, Journal of Physics Condensed Matter 20, 434235 (2008).
[31] A. Aperis, G. Varelogiannis and P.B. Littlewood, Journal of Phys. Conf. Series 150, 042007 (2009).