Quarks-excited states or gluon-excited states, are the $J^{PC} = 1^{-+}$ hybrid mesons confirmed?

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Abstract

The existence of hadrons containing gluons as constituent particles (like hybrid mesons) will involve the necessity to admit an hadronic spectroscopy beyond the naïve quark model. In the QGC model, the lowest $J^{PC} = 1^{-+}$ hybrid mesons may be built in two different modes: the GE-mode corresponding to an angular momentum between the gluon and $qq$ system, while the QE-mode corresponding to an angular momentum between $q$ and $\overline{q}$. In this paper, we give an analysis of the possible $1^{-+}$ hybrid mesons in the 1.32-2.04 GeV mass range, and predictions on their decay. We discuss on the inconsistency between some theoretical results and experimental data. We find that the $1^{-+} (1400 \text{ MeV})$, dominated by the QE-mode will decay in the favourite $\rho \pi$ channel, while the GE-$1^{-+} (1600 \text{ MeV})$, and the $1^{-+} (2.0 \text{ GeV})$, pure GE-hybrids, will decay preferably into $b_1 \pi$ channel (never observed); we indicate problems concerning the observation of decay channels which are forbidden by theoretical present models. The total decay widths of the $1^{-+} (2.0 \text{ GeV})$ and the GE-$1^{-+} (1600 \text{ MeV})$ being very large, this leads to a problem of their observability as resonances.

We may conclude that at this time, there is no absolute confirmation of their existence, but some asked questions remain not solved.

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1 Introduction

The naïve quark model interprets the meson as a bound state of a quark and an antiquark, and predicts that the charge conjugation ($C$) and the parity ($P$) of a meson with spin $S$ and angular momentum $L$ are respectively:

$$C = (-)^{L+S}$$

$$P = (-)^{L+1}.$$  

Besides these conventional mesons, Quantum Chromodynamics, actually considered as the theory of strong interactions, predicts that states containing gluons (namely glueballs and hybrids) may exist. Several models (Lattice-QCD, QCD Sum-rules, Flux Tube Model, Constituent Glue Model, ...) suggest that lightest glueballs occur in the 1-2 GeV region and $q\bar{q}g$ hybrids occur around 2 GeV in mass, but without any clear evidence.

The collective behaviour of gluons in the strong interaction regime of QCD is not yet established, and the confirmation of presence of gluons in hadrons as constituent particles will change our understanding about the rôle of the gluons in QCD. From the theoretical side such state may be in color-singlet; however this does not imply the existence of resonance: one must demonstrate a state with consider quantum numbers and which has the total decay width smaller compared to the level spacing (if the total decay width is larger than the level spacing, the considered is simply a part of the non-resonant background) and a long life time before decaying.

The most clear-out signal for hybrid meson is to reach for $J^{PC}$ quantum numbers not allowed in the naïve quark model, such as $J^{PC} = 0^{-+}, 0^{--}, 1^{--}, 2^{++}, ...$ There has been considerable recent interest in such hybrid mesons, both from the perspective of models and from the experimental data.

From experimental efforts at IHEP, KEK, CERN, and BNL, hybrid $1^{++}$ candidates have been observed by AGS at BNL[1, 2, 3, 4, 5], by Cristall Barrel at LEAR[6], by GAMS at CERN[7], by VES at IHEP[8, 9], by FNAL[10], ... Furthermore, several experimental programs are proposed for further investigation of hybrids. COMPASS Program at CERN[11] intends to study the possibility to Primakov production of hybrids near 1.4 GeV; hybrids with $J^{PC} = 0^{--}, 2^{+-}$ and $1^{--}$ would be produced by photoproduction at CEBAF[12] and $2^{++}$ at JLAB[13]. It is proposed to investigate mainly the $J^{PC} = 1^{-+}$: programs are proposed to search for resonances decaying in $b_1\pi$ (at SLAC[14]) and $\rho\pi$ (at BEPC/VES [15], and by photoproduction [16]). Then, this intense experimental activity should contribute significantly to the understanding of the hybrids, and would give answer to the question of their existence.
In this paper, we give an analysis of the different hybrid modes possible to construct in the 1.32-2.04 GeV mass range and we give predictions on their decay. We discuss on the inconsistency between some theoretical results and experimental data, concerning the favourite channels (non observed) and observed channels (non allowed); we suggest to attribute a particular importance to the problem of the existence of hybrid mesons, since earlier theoretical results lead to this interrogation. In a preceding work [17], we have studied the states of the $J^{PC} = 1^{--} c\bar{c}g$ charmonium hybrid with a mass around 4 GeV; due to the very large total decay width, we concluded that the $c\bar{c}g$ at 4 GeV cannot be a resonance and then not a physical observable object.

Furthermore, the study of the $J^{PC} = 1^{-+}$ hybrid candidates with masses 1.4 and 1.6 GeV [18] give $b_1\pi$ as preferred decay channel (which is not observed) and $\eta\pi$ forbidden (which is observed). We give also in this work our interpretation of the exotic 2.0 GeV resonance claimed to be observed by the AGS Collaboration at BNL [4], decaying in $f_1\pi$.

2 The two-modes hybrid mesons and their masses

In the quark-gluon constituent model [19, 20], the gluon is considered as a massive constituent particle ($m_g \simeq 800$ MeV) moving in the framework of $q\bar{q}$ pair center of mass. Restricting ourselves to the lightest states, the hybrid $1^{-+}$ states may be built in two ways: with $l_{q\bar{q}} = 1$, $S_{q\bar{q}} = 0$, $l_g = 0$ (which is the quarks-excited mode: QE) and with $l_{q\bar{q}} = 0$, $S_{q\bar{q}} = 1$, $l_g = 1$ (which is the gluon-excited mode: GE); we examine also the possibility to have a mixed state (QE+GE). The calculations of the hybrid masses, using two different confining potential models to describe the hybrid states, show a difference between the masses of the two modes, the GE-hybrid being much heavier:

- a non-relativistic quark model with a confining chromo-harmonic potential [21]
- a more realistic potential in good agreement with QCD characteristics (with coulombien and linear terms), and taking in account some relativistic corrections[22].

N.B: We note the quantum numbers of the hybrid meson:

- $l_g$: is the relative orbital momentum of the gluon in the $q\bar{q}$ center of mass;
$l_{q\bar{q}}$ : is the relative orbital momentum between $q$ and $\bar{q}$;
$S_{q\bar{q}}$ : is the total quark spin;
$j_g$ : is the total gluon angular momentum;
$L : l_{q\bar{q}} + j_g$.
The parity and charge conjugation of the hybrid are given by:

$$
P = (-)^{l_{q\bar{q}} + j_g};
C = (-)^{l_{q\bar{q}} + S_{q\bar{q}} + 1}.
$$

Wave functions and masses have been calculated through the equation:

$$
\left\{ \sum_{i=1}^{N} \left( \frac{\vec{p}_i^2}{2M_i} + \frac{M_i}{2} + \frac{m_i^2}{2M_i} \right) + V_{eff} \right\} \Psi(\vec{r}_i) = E \Psi(\vec{r}_i); (2)
$$

which take into account relativistic corrections in the Hamiltonian; $V_{eff}$ is
the average over the color space of chromo-spatial potential[22]:

$$
V_{eff} = \langle V \rangle_{color} = \left\langle - \sum_{i<j=1}^{N} \vec{F}_i \cdot \vec{F}_j \ v(r_{ij}) \right\rangle_{color}
= \sum_{i<j=1}^{N} \alpha_{ij} v(r_{ij}); (3)
$$

where $v(r_{ij})$ is the phenomenological potential term, having a (Coulomb+linear)
or harmonic oscillator form.

Defining the Jacobi coordinates:

$$
\vec{\rho} = \vec{r}_q - \vec{r}_{\bar{q}};
\vec{\lambda} = \vec{r}_{\bar{q}} - \frac{M_q \vec{r}_q + M_{\bar{q}} \vec{r}_{\bar{q}}}{M_q + M_{\bar{q}}}
$$

We develop the spatial wave functions:

$$
\psi^{j_{q\bar{q}}l_g}(\vec{\rho}, \vec{\lambda}) = \sum_{n=1}^{N} a_n \varphi_n^{j_{q\bar{q}}l_g}(\vec{\rho}, \vec{\lambda}); (4)
$$

where $\varphi_n^{j_{q\bar{q}}l_g}(\vec{\rho}, \vec{\lambda})$ are the Gaussian-type functions.
2.1 Masses of the GE- and QE-hybrids

We present in Table 1 estimates of hybrid mesons masses, using the harmonic oscillator confining potential and the (Coulomb + Linear) potential, for both light and heavy flavors; we take 800MeV for the mass of the gluon.

| potentiels model | u,d | s | c | b |
|------------------|-----|---|---|---|
| Coul.+Lin. QE    | 1.31| 1.57| 4.09| 10.34 |
| GE               | 1.70| 2.00| 4.45| 10.81 |
| Harm.Oscill. QE  | 1.47| 1.68| 4.83| 11.40 |
| GE               | 1.61| 1.84| 4.98| 11.58 |

*Table 1: estimates of hybrid mesons masses (in GeV) for different flavors (without spin effects)*

In Table 2, we give results of light hybrid mesons masses, calculated using the (Coulomb + Linear) potential, and taking in account spin-spin effects.

| S = 1 | uūg | 1.32 | 1.45 | 1.58 | QE Mode (l_{q\bar{q}} = 1; l_g = 0 and S_{q\bar{q}} = 0) |
|-------|-----|------|------|------|--------------------------------------------------|
| S = 0 | uūg | 1.56 | 1.72 | 1.87 | GE Mode (l_{q\bar{q}} = 0; l_g = 1 and S_{q\bar{q}} = 1) |
| S = 1 | sūg | 1.69 | 1.84 | 1.99 | GE Mode (l_{q\bar{q}} = 0; l_g = 1 and S_{q\bar{q}} = 1) |
| S = 2 | sūg | 1.75 | 1.89 | 2.04 | GE Mode (l_{q\bar{q}} = 0; l_g = 1 and S_{q\bar{q}} = 1) |

*Table 2: masses of 1−+ light hybrid mesons (in GeV), calculated (using Cb+Lin. pot.) within spin-spin corrections* [23]

2.2 Masses of the mixed (GE+QE) states

Representing the hybrid meson wave function in the cluster approximation [22]:

\[
\Psi_{JM}(\vec{\rho}, \vec{\lambda}) = \sum_{n, l_{q\bar{q}}, l_g} a_n^{l_{q\bar{q}}l_g} \sum_{j_g, L, \langle m \rangle, \langle \mu \rangle} \phi_n^{l_{q\bar{q}}l_g}(\vec{\rho}, \vec{\lambda}) e^{\mu_g} \chi_{S_{q\bar{q}}}^{\mu q} \langle l_g m_g l_{\mu_g} | J_g M_g \rangle \times \langle l_{q\bar{q}} m_{q\bar{q}} J_g M_g | Lm \rangle \langle Lm S_{q\bar{q}} \mu_{q\bar{q}} | JM \rangle .
\]

the expansion of the wave function for a mixed (GE+QE) state may be written:
\[
\Psi_{1-}(\vec{\rho}, \vec{\lambda}) \simeq \sum_{n=1}^{N} a_n^{QE} \varphi_n^{QE}(\vec{\rho}, \vec{\lambda}) + \sum_{n=1}^{N} a_n^{GE} \varphi_n^{GE}(\vec{\rho}, \vec{\lambda}).
\] (6)

For the spin states we choused \(\{|S_{q\bar{q}}, s_g; S\}\) (\(s_g = 1\) and \(S = S_{q\bar{q}} + s_g\)).

Using (Cb+Lin.) confining potential, we give the results in Table 3:

| hybrid state | 2-modes mixing | mass (in GeV) |
|--------------|----------------|--------------|
| \(1^-+(u\bar{u}g)\) | \(-0.999|QE\rangle + 0.040|GE\rangle\) | 1.34 |
| \(1^-+(u\bar{u}g)\) | \(-|GE\rangle\) | 1.72 |
| \(1^-+(s\bar{s}g)\) | \(-0.999|QE\rangle + 0.050|GE\rangle\) | 1.60 |
| \(1^-+(s\bar{s}g)\) | \(-|GE\rangle\) | 2.02 |
| \(1^-+(c\bar{c}g)\) | \(-0.999|QE\rangle - 0.040|GE\rangle\) | 4.10 |
| \(1^-+(c\bar{c}g)\) | \(-0.031|QE\rangle - 0.999|GE\rangle\) | 4.45 |

Table 3: predictions of mixed (QE+GE) states

The results show that the QE-hybrid and the GE-hybrid mix very weakly.

3 The decay of the two-modes hybrid mesons

We use the quark-gluon constituent model, in which the gluon moves in the framework of the \(q\bar{q}\) pair center of mass. It is important to note that the decay occurs through two diagrams, the gluon annihilating into a \(q\bar{q}\) pair (namely \(q = u, d\) and \(s\)) and the decay amplitude will then be the sum of two corresponding amplitudes to each contribution[17, 18].

Then the decay of an hybrid state \(A\) into two mesons \(B\) and \(C\) is represented by the matrix element of the Hamiltonian annihilating a gluon and creating a quark pair:

\[
\langle BC | H | A \rangle = gf(A, B, C) (2\pi)^3 \delta_3 (p_A - p_B - p_C);
\] (7)

where \(f(A, B, C)\) representing the decay amplitude by:

\[
f(A, B, C) = \sum_{(m),(\mu)} \Phi \Omega X \left( \mu_{q\bar{q}}, \mu_g; \mu_B, \mu_C \right) \left( m_{q\bar{q}}, m_g; m_B, m_C, m \right) R_{m_{q\bar{q}} m_g, m_B, m_C, m} \times \langle L | J_g M_g | L' \rangle \langle J g' | J g M g | L m' \rangle \langle J g' M g' | S_{q\bar{q}} S_{q\bar{q}} \mu_{q\bar{q}} | J M \rangle \times \langle l_B m_B S_B | J_B M_B \rangle \langle l_C m_C S_C \mu_C | J_C M_C \rangle.
\] (8)
where $\Phi, \Omega, X$ and $I$ are the flavor, color, spin and spatial overlaps. $\Omega$ is given by:

$$\Omega = \frac{1}{24} \sum_a \text{tr} (\lambda^a)^2 = \frac{2}{3}. \quad (9)$$

From:

$$\chi_{\mu_1}^+ \sigma^\lambda \chi_{\mu_2} = \sqrt{3} \langle \frac{1}{2} \mu_2 \lambda \left| \frac{1}{2} \mu_1 \right\rangle, \quad (10)$$

we obtain the spin overlap:

$$X(\mu_{q\bar{q}}, \mu_g; \mu_B, \mu_C) = \sum_S \sqrt{2} \left[ \begin{array}{ccc}
\frac{1}{2} & \frac{1}{2} & S_B \\
\frac{1}{2} & \frac{1}{2} & S_C \\
S_{q\bar{q}} & 1 & S
\end{array} \right] \times \langle S_{q\bar{q}} \mu_{q\bar{q}} \mu_g | S (\mu_{q\bar{q}} + \mu_g) \rangle \langle S_B \mu_B S_C \mu_C | S (\mu_B + \mu_C) \rangle; \quad (11)$$

where

$$\left[ \begin{array}{ccc}
\frac{1}{2} & \frac{1}{2} & S_B \\
\frac{1}{2} & \frac{1}{2} & S_C \\
S_{q\bar{q}} & 1 & S
\end{array} \right] = \sqrt{3} (2S_B + 1) (2S_C + 1) (2S_{q\bar{q}} + 1) \left\{ \begin{array}{ccc}
\frac{1}{2} & \frac{1}{2} & S_B \\
\frac{1}{2} & \frac{1}{2} & S_C \\
S_{q\bar{q}} & 1 & S
\end{array} \right\}.$$ 

The spatial overlap is given by:

$$I(m_{q\bar{q}}, m_g; m_B, m_C, m) = \int \int \frac{d\vec{p}_1 d\vec{k}}{(2\pi)^n \sqrt{2\omega}} \Psi_{q\bar{q}}^{l_{q\bar{q}} \gamma m_{q\bar{q}} m_g} \left( \vec{P}_B - \vec{p} - \frac{\vec{k}}{2} \right) \times \Psi_{q\bar{q}}^{l_{q\bar{q}} \gamma m_{q\bar{q}}} \left( \vec{P}_B - \vec{p} - \frac{\vec{k}}{2} \right) \cdot \langle \Omega_B \rangle d\Omega_B, \quad (12)$$

where:

$$\vec{p}_1 = \frac{m_{q\bar{q}}}{m_q + m_{q\bar{q}}} \vec{P}_B - \vec{p} - \frac{\vec{k}}{2} \quad (13)$$

$$\vec{p}_2 = -\frac{m_{q}}{m_{q\bar{q}} + m_q} \vec{P}_B + \vec{p} - \frac{\vec{k}}{2}. \quad (14)$$

$l, m$ label the orbital momentum between the two final mesons. 
Finally,
\[ \Phi = \begin{bmatrix} i_1 & i_3 & I_B \\ i_2 & i_4 & I_C \\ \eta_{\overline{q}\eta} & 1 & I_A \end{bmatrix} \eta \epsilon, \] (15)

where I's (i's) label the hadron (quark) isospins, \( \eta = 1 \) if the gluon goes into strange quarks and \( \eta = \sqrt{2} \) if it goes into non-strange ones. \( \epsilon \) is the number of diagrams contributing to the decay. Indeed one can check that since P and C are conserved, two diagrams contribute with the same sign and magnitude for allowed decays while they cancel for forbidden ones. In the case of two identical final particles, \( \epsilon = \sqrt{2} \).

The partial width is then given by:

\[ \Gamma (A \rightarrow BC) = 4\alpha_s |f (A, B, C)|^2 \frac{P_B E_B E_C}{M_A}; \] (16)

with

\[ P_B^2 = \frac{\left[M_A^2 - (m_B + m_C)^2\right]\left[M_A^2 - (m_B - m_C)^2\right]}{4M_A^2}; \] (17)

\[ E_B = \sqrt{P_B^2 + m_B^2}; \]

\[ E_B = \sqrt{P_B^2 + m_C^2}. \] (18)

3.1 The Quarks-Excited mode, decaying in \( \rho \pi \)

Using the conservation of angular momentum, parity, isospin and G-parity for the state with:

- \( l_{\overline{q}q} = 1, \ l_g = 0, \ S_{\overline{q}q} = 0, \ L = 1 \) and \( J = 1 \)

the \( J^{PC} = 1^{-+} \)-QE hybrid is allowed to decay into (the decay channels, with the widths for the \( J^{PC} = 1^{-+} \) hybrid meson at 1.4 and 1.6\( GeV \) are given in [17, 18]):

- \( \rho \pi, \ \rho \omega, \ \rho(1450)\pi, \ K^*(1410)K \) and \( K^*K \).

All this channels verify the selection rule for the \( 1^{-+} \) QE-hybrid mode which is allowed to decay only into two fundamental mesons \( (L = 0 + L = 0) \) [17, 18].
The preferred decay channel of the $1^{-+}$ QE-hybrid meson is $\rho(770)\pi$, and the calculations give a width $\Gamma = 296\text{MeV}$ (for $M = 2.0\text{GeV}$). The decay channel $K^*(892)K$ is also important ($\Gamma = 103\text{MeV}$); the other channels are about $10\text{MeV}$. The total decay width is around $600\text{MeV}$.

Table 4 give the results:

|                  | $\Gamma_{\rho(770)\pi}$ | $\Gamma_{\rho(770)\omega}$ | $\Gamma_{\rho(1450)\pi}$ | $\Gamma_{K^*(892)K^-}$ | $\Gamma_{K^*(1410)K^-}$ | $\Gamma_{\text{tot}(2.0)}$ |
|------------------|--------------------------|----------------------------|--------------------------|-------------------------|--------------------------|-----------------------------|
| $\text{MeV}$    | 296                      | 100                        | 75                       | 103                     | 7                        | 581                         |

Table 4: Predicted widths in $\alpha_s\text{MeV}$ for the decay of a QE $1^{-+}$-hybrid meson of mass 2.0GeV.

### 3.2 The Gluon-Excited mode, decaying in $b_1\pi$

Concerning the GE mode, we find three hybrid states with an orbital excitation between the gluon and the $q\bar{q}$ system, with:

$l_g = 1$, $l_{q\bar{q}} = 0$, $S_{q\bar{q}} = 1$,

and respectively for the three states:

$J_g = L = 0, 1, 2$.

For an hybrid of mass 2.0GeV, only

$f_1(1520)\pi^-$, $f_1(1420)\pi^-$, $f_1(1285)\pi^-$, $b_1(1235)\pi^-$, $a_1(1260)\eta$,

$K^+_1(1400)K^-$, $K^-_1(1400)K^0$, $K^0_1(1270)K^-$ and $K^-_1(1270)K^0$.

Final states are allowed on general grounds: conservation of angular momentum, parity, isospin and G-parity. All this channels are allowed by the selection rule for $1^{-+}$ GE-hybrid meson which decays only into one fundamental meson and one orbitally excited meson ($L = 0 + L = 1$)\cite{20}.

The quark-gluon constituent model predictions for the decay of a 2.0GeV $1^{-+}$ GE-hybrid meson are given in Table 5:
Table 5: Predicted widths in $\alpha_s \text{MeV}$ of a GE $1^-_+^+$-hybrid meson of mass 2.0 GeV.

4 The experimental data

i) Hybrid $1^-_+^+$ candidates (with masses 1400 MeV, 1600 MeV, 1740 MeV, 2.0 GeV) have been claimed to be observed[1, 10], decaying in $\eta \pi, \eta' \pi, f_1 \pi$ and $\rho \pi$:

with masses around 1400 MeV:

- by BNL (in $\pi N$ interactions at 18 GeV) at $(1370 \pm 16)$ MeV, with decay width $\Gamma = (385 \pm 40)$ MeV, decaying in $\eta \pi, \eta' \pi, f_1 \pi$[1, 3].
- by Cristall Barrel Coll. at LEAR (in $\bar{p}n$ interactions), with a mass $(1400 \pm 20)$ MeV, and $\Gamma = (310 \pm 50)$ MeV, decaying in $\eta \pi$[6].
- by GAMS Coll. at CERN, (in $\rho \pi$ interactions at 100 GeV), with a mass 1400 MeV, and $\Gamma = 180$ MeV, decaying in $\eta \pi$[7].
- by VES Coll. with mass 1405 MeV, decaying in $\eta \pi$ and $\eta' \pi$[8, 9].

with mass 1600 MeV:

- by E852 at BNL with mass $(1593 \pm 8)$ MeV and $\Gamma = (168 \pm 20)$ MeV, decaying in $\rho \pi$[2, 3].
- by E852 at BNL with mass $(1597 \pm 10)$ MeV and $\Gamma = (340 \pm 40)$ MeV, decaying in $\eta' \pi$[5].
with mass 1740 MeV:

- at FNAL, with mass $(1740 \pm 12)\text{ MeV}$ and $\Gamma = (136 \pm 30)\text{ MeV}$, decaying in $f_1\pi$.

with mass 2.0 GeV:

- by AGS Coll. at BNL, decaying in $f_1\pi$ and $\eta(1295)\pi$.

ii) The analysis of $e^+e^-$ data\cite{24} shows the possible existence of $J^{PC} = 1^{--}$ states beyond the naïve quark model, like a system of mixing of a $q\bar{q}g$ hybrid meson with $q\bar{q}$ state (for the 1.1GeV state).

iii) A suggestion to explain the $\Psi$ anomaly observed at CDF is by the mixing of an hybrid meson with a mass around 4.1GeV and a conventional $\Psi(3S)$ charmonium; the hadronic decays of the states $\Psi(4040)$ and $\Psi(4160)$ are predicted dominated by the $\Psi(3S)$ component\cite{25}. We have studied the states of the $1^{--}c\bar{c}g$ charmonium hybrid with mass around 4GeV, and the possibility to mixing with the conventional $c\bar{c}$ charmonium meson\cite{17}.

5 Theoretical results and comparison

We recapitulate the results of the predicted $1^{-+}$ hybrid mesons in Table 6; we pay our attention on the 1.32-2.04 GeV mass range (corresponding to the mass range of the experimental candidates). We give the predicted masses with the corresponding modes (QE or GE) and the favourite decay channels.
Then, around the mass 1.4 GeV, the hybrid 1−+ mesons are predicted in the QE-mode, or in a mixed (QE+GE)-mode, dominated by QE ($\theta_{\text{max}} \sim 4.6^\circ$, $\lambda^2 \sim 0.006$); and the decay should occur into $\rho \pi, K^* K$.

Around 1.6 GeV, we may obtain hybrid mesons in the two modes, decaying preferably into $b_1 \pi$ for the GE-hybrid, and $\rho \pi$ for the QE-hybrid.

Around 1.7 GeV, the predicted hybrid 1−+ mesons will be built in pure GE-mode, or in a dominated by GE mode; the decay should occur preferably in $b_1 \pi$.

Around 2.0 GeV, we obtain pure GE-mode hybrid 1−+ mesons, or dominated by GE; the preferred decay channels are into $b_1 \pi$, or $K_1 K$.

In our model, considering the different possibilities to construct a $J^{PC} = 1^{−+}$ hybrid meson, relative to the two modes (GE and QE), we obtain the following predictions for candidates with masses 1.4, 1.6, 1.7 and 2.0 GeV, compared with the experimental data (Table 7):

| mass (in GeV) | mode | favourite decay channels |
|---------------|------|-------------------------|
| 1.32          | QE   | $\rho \pi, K^* K$       |
| 1.34          | mixing dominated by QE | $b_1 \pi, f_1 \pi$ |
| 1.45          | QE   | $\rho \pi, K^* K, \rho \omega$ |
| 1.56          | GE   | $b_1 \pi, f_1 \pi$ |
| 1.58          | QE   | $\rho \pi, K^* K, \rho \omega$ |
| 1.6           | mixing dominated by QE | $b_1 \pi, f_1 \pi$ |
| 1.69          | GE   | $b_1 \pi, f_1 \pi$ |
| 1.72          | GE   | $b_1 \pi, a_1 \eta, f_1 \pi, K_1 K$ |
| 1.72          | mixing dominated by GE | $b_1 \pi, f_1 \pi$ |
| 1.75          | GE   | $b_1 \pi, a_1 \eta, f_1 \pi, K_1 K$ |
| 1.84          | GE   | $b_1 \pi, a_1 \eta, f_1 \pi, K_1 K$ |
| 1.87          | GE   | $b_1 \pi, a_1 \eta, f_1 \pi, K_1 K$ |
| 1.89          | GE   | $b_1 \pi, a_1 \eta, f_1 \pi, K_1 K$ |
| 1.99          | GE   | $b_1 \pi, a_1 \eta, f_1 \pi, K_1 K$ |
| 2.02          | mixing dominated by GE | $b_1 \pi, a_1 \eta, f_1 \pi, K_1 K$ |
| 2.04          | GE   | $b_1 \pi, a_1 \eta, f_1 \pi, K_1 K$ |

Table 6: Predicted hybrid 1−+ mesons with preferred decay channels.
Table 7: Predicted decay channels and rates for hybrid candidates with masses 1.4, 1.6, 1.7 and 2.0 GeV.

Note that for the GE-mode, we have reported the results only for L=0.

PS: the results for the $J^{PC} = 1^{-+}$ hybrid mesons at 1.4 and 1.6 GeV are given from [18].

Table 8 presents our total decay widths predictions for 1.4, 1.6, and 2.0 GeV hybrid candidates.

| Mass (in GeV) | Mode | Channels | Decay Widths (in MeV) | Exp. Data |
|---------------|------|----------|----------------------|-----------|
| 1.4           | QE   | $\rho\pi$ | 72                   | -         |
| 1.6           | QE   | $\rho\pi$ | 142                  | 168       |
|               | GE   | $b_1\pi$  | 230                  | -         |
|               | GE   | $f_1(1285)\pi$ | 186            | seen       |
|               | GE   | $f_1(1420)\pi$ | 62              | -         |
| 1.7           | GE   | $b_1\pi$  | 231                  | -         |
|               | GE   | $f_1(1285)\pi$ | 188            | 136       |
|               | GE   | $f_1(1420)\pi$ | 62              | -         |
| 2.0           | GE   | $K_1K$    | 175                  | -         |
|               | GE   | $b_1\pi$  | 166                  | -         |
|               | GE   | $f_1\pi$  | 160                  | observed  |
|               | GE   | $a_1\eta$ | 156                  | -         |

Table 8: The total decay widths of the $1^{-+}$ hybrid mesons at 1.4, 1.6, and 2.0 GeV.

In our model, the gluon-excited $1^{-+}$ hybrid meson do decay preferably in $b_1\pi$; this result was confirmed by the flux-tube model [26]. The quarks-excited $1^{-+}$ hybrid meson do decay preferably into $\rho\pi$.

The decay channels into $\eta\pi$, $\eta'\pi$, $\eta(1295)\pi$ are forbidden, both in the two modes: in the GE-mode, due to the selection rule which suppresses this channel (and allows a channel with one fundamental meson and the other one orbitally excited: (L=0)+(L=1)); in the QE-mode, due to the total spin conservation in the final state. This is in contradiction with experiment.
The total decay widths of the 1.6 (in the GE-mode) and 2.0 GeV 1−+ hybrid meson are found to be very large, which means that such states do not really emerge from the continuum of the two meson spectrum. In other words, the GE-hybrids in the 1.6-2.0 GeV mass range do decay before they had time to really exist as hadrons: they do not exist as resonances.

On the contrary, the QE-1.6 candidate has a total decay width around 165 MeV, with a ρπ preferred decay channel, in agreement with BNL. Our conclusion is that this resonance may be considered as a hybrid meson in the quarks-excited mode.

6 Discussion and Conclusion

We have presented here some theoretical results and experimental data relative to exotic mesons, which are possibly interpreted as hybrid q̅qg candidates. We used the quark-gluon constituent model, implemented with a confining potential between the hybrid constituents, which is inspired from gluon-exchange in QCD. However, the flux-tube model assumes that hybrids are predominately quark-antiquark states moving on an adiabatic surface generated by an excited color flux-tube; there is no constituent gluon in FTM model, the gluon degrees of freedom are treated as collective excitations of the color flux; in practice, the color is treated as a string, and the excited states are represented by the excited modes of the string.

Restricting ourselves to the lightest states, we have three hybrids (L=0, 1, 2) with an orbital excitation between the gluon and the q̅q system (GE-mode), and one with an orbital excitation between the q and the q̅ (QE-mode). Mixed (QE+GE) states may exist, but the results show that the QE-hybrid and the GE-hybrid mix very weakly. The QE-mode is lighter than the GE-one (see Table 1, Table 2), due to the confining interaction inside each mode.

In our model, the GE-hybrid and the QE-hybrid are allowed to decay respectively into two mesons S+P states and into two ground state mesons.

The (b1π) and (f1π) are the preferred decay channels of the hybrid mesons in the GE-mode; the first channel was confirmed by the flux-tube model [20], but was never observed, and it will be necessary to check this channel in order to verify our selection rule concerning the GE-mode. The latter channel was observed by AGS Coll. at BNL[4], in the 1.6-2.2 GeV mass range, but without precision on the corresponding branching ratio.
The \( (\rho \pi) \) is the favourite decay channel for the QE-mode and was observed only for the 1.6 GeV resonance by E852 Coll. at BNL [2], with a width \( \Gamma = (168 \pm 20) \text{ MeV} \), which is in good agreement with our prediction \( \sim 142 \) MeV.

The \( \eta \pi, \eta' \pi, \eta(1295)\pi \) channels are forbidden for every resonances and both in the two-modes (QE and GE). They have been observed. Unhappily, it becomes difficult to understand the \( \sim 400 \) MeV width of the E852 Coll. [1] as an evidence for an exotic resonance at 1.4 GeV, which is totally forbidden in our model.

Theoretical studies by QCD-Sum Rules give \( (\rho \pi) \) as dominant decay channel compared to \( \eta \pi \) and \( \eta' \pi \) [27, 28].

The total decay width of the 1.4GeV \( 1^{-+} \) hybrid, dominated by the QE-mode, is around 72 GeV, and the decay channel is into \( \rho \pi \) preferably.

The total decay widths of the GE-1.6 and 2.0 GeV hybrids are respectively around 700 MeV and 1270 MeV for \( L = 0 \) and even larger for \( L \geq 1 \). The increase of the widths with the resonance mass is obviously due to the increasing phase space. Then the GE-1.6 and the 2.0 GeV hybrids do not exist as resonances.

Ref[26] rejects the hypothesis that the observed 1.4 and 1.6 GeV states are both hybrid mesons, because of the experimental width which is larger for the 1.4 state than the 1.6 state, which should be opposite. They interpret the experimental observed peak in the \( \eta \pi \) channel, which is suppressed by symmetrization selection rules, as a sizable final state interaction and they suggest that the E852 \( \eta \pi \) peak is due to the interference of a Deck-type background with an hybrid resonance of higher mass, for which the \( \tilde{\rho} \) at 1.6 GeV is an obvious candidate.

Then from our analysis, we conclude the following:

1. The \( 1^{-+} \) resonance at 1.4GeV would be a good hybrid candidate if it has been observed in \( \rho \pi \), which is predicted around 70 MeV. However, it is unplausible to interpret the \( \sim 400 \) MeV width in \( \eta \pi \) reported by E852 experiment.

2. The \( 1^{-+} \) resonance at 1.6GeV is a good hybrid candidate in the QE-mode, with a total decay width around 165 MeV dominated by \( \rho \pi \), in good agreement with BNL.

3. For the \( 1^{-+} \) resonance at 2.0GeV, dominated by the GE-mode, it has a total decay width around 1.3 GeV for \( L=0 \), and even larger for \( L \geq 1 \),
making the hybrid interpretation rather unbelievable.

Finally, to check the existence of the hybrid mesons, namely the $1^{-+}$ ones, and have more informations about the physics of these objects, it will be important to have (from the experiments) the observed branching ratio in $f_1\pi$ in the $1.6-2.2\text{GeV}$ mass range, and to check the $b_1\pi$ channel. Concerning the $1.4\text{GeV}$, it will be necessary to check the $\rho\pi$ channel. We believe that these experimental tests are crucial to remove any doubt concerning the interpretation of these objects and to lead to the understanding of the hybrid phenomenology.

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