ABSTRACT Data Envelopment Analysis (DEA) cross-efficiency has been used to replace the self-evaluation system, which requires decision-makers to rank a series of decision-making units (DMUs) according to cross-efficiency scores, and finally determine the order of each DMU, so as to provide decision-making basis for decision-makers. However, this method has certain deficiencies: one is that the cross-efficiency value is not unique; the other one is that the cross-efficiency evaluation uses the same weight to aggregate cross-efficiency. In this paper, Technique for order performance by similarity to ideal solution (TOPSIS) and The Best Worst Method (BWM) methods are used to aggregate the aggressive cross-efficiency values and benevolent cross-efficiency values. Besides, the high consistency of the BWM method improves the shortcomings of the TOPSIS method. The result of TOPSIS method is not necessarily close to the ideal solution, and far away from the negative ideal solution. At the same time, the distance measured by the TOPSIS method replaces the preference of the BWM method, which makes BWM method more objective. Finally, a numerical example is given to verify the feasibility and effectiveness of the method.

INDEX TERMS Cross-efficiency aggregation, data envelopment analysis, best worst method, TOPSIS.
used secondary goals to get the weights which effectively avoid the problem of zero weight. For cross-efficiency aggregation problems, previous studies obtained the weight of each cross-efficiency to summarize the cross-efficiency by using nuclear and Shapley values [16], [17], Shannon entropy [18]–[20], ordered weighted average operators [21], evidence reasoning methods [22], Gini Guidelines [23] and the ordered weighted averaging (OWA) [24], [25]. In addition, Liang et al. [26] proposed the game cross-efficiency model in which DMUs were regarded as players in the non-cooperative game, Wu et al. [27] applied to the performance evaluation of participating countries in the Olympic Games. Some scholars extended cross-efficiency into the uncertain environment, introduced hesitant fuzzy sets [28], interval values [29]–[32].

First, the aggressive and benevolent cross-efficiency models are the most classical methods to improve the non-unique cross-efficiency. Second, TOPSIS and BWM are used to improve the average cross-efficiency result. So, in order to improve the original cross-efficiency method, the paper applies the TOPSIS and BWM methods to integrate the cross-efficiency values based on the aggressive and benevolent cross-efficiency model. Lai et al. [33] proposed the TOPSIS method which is multi-criteria decision-making (MCDM) method by detecting the distance between alternatives and solutions of ideal and non-ideal. Because its intuitionist, reliable and real, it has been applied to many practical problems. BWM proposed by Rezaei [34] is a method in which experts and decision-makers firstly determine the best criterion and the worst criterion, then compare the best criterion and the worst criterion with other criteria, finally draw a conclusion by using linear programming method and obtained a comprehensive score. It has been applied in many fields. Rezaei et al. [35] used BWM method to evaluate suppliers; Li et al. [36] proposed intuitionist fuzzy multiplication optimization-optimal method under the fuzzy environment, Mou et al. [37] introduced multi-criterion decision-making method of probabilistic hesitation fuzzy and BWM, Li et al. [38] proposed linear BWM method. Rezaei [38] used BWM method to measure the social sustainability of manufacturing enterprise supply chain; Ahmadi et al. [39] proposed an integrated model of failure mode and effects analysis (FMEA) based on fuzzy BWM, relative entropy and VIKOR method, and so on.

The innovation of this paper lies in combining BWM, TOPSIS, and cross-efficiency for the first time. In order to increase the objectivity of BWM, the efficiency value will be selected to the best and worst solution, and the distance in the TOPSIS method is used instead of the expert determine the comparative preference in BWM; on the contrary, the high consistency of the BWM method also addresses the shortcomings of the final result of the TOPSIS method, which is not necessarily closest from the ideal solution and farthest from the negative ideal solution. Combining TOPSIS and BWM with cross-efficiency will solve the shortcomings of a simple weighting of cross-efficiency.

The rest of the article is organized as follows: Section 2 introduces the cross-efficiency model, TOPSIS method and BWM method; Section 3 proposes Cross-efficiency method based on BWM-TOPSIS; Section 4 uses a numerical example to prove the feasibility and practicability of the method proposed in this paper; Section 5 makes a summary and prospect.

II. PRELIMINARIES

A. THE CROSS-EFFICIENCY MODEL

Consider $n$ DMUs that are to be evaluated with $m$ inputs and $s$ outputs. For the $j$th DMU, $j = 1, 2, ..., n$, note $i$th input and $r$th output are $x_{ij} (i = 1, 2, ..., m)$ and $y_{rj} (r = 1, 2, ..., s)$. Respectively, for any evaluated $d^{th}$ DMU, the efficiency value $E_{dd}$ under the CCR model can be obtained by solving the program (1):

$$
\begin{align*}
\max E_{dd} &= \sum_{r=1}^{s} u_{rd} y_{rj}, \\
\text{s.t.} & \sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} \vartheta_{id} x_{ij} \leq 1, \\
& u_{rd}, \vartheta_{id} \geq 0, \\
& i = 1, 2, ..., m, \\
& r = 1, 2, ..., s, \\
& j = 1, 2, ..., n.
\end{align*}
$$

Among them, variables $\vartheta_{id}$ and $u_{rd}$ represent the weights given to the $i$th input and the $r$th output for DMU $d$ for the above fractional program (1), the following equivalent forms of linear programming problems can be obtained through the classical Charnes et al. [2] transformation:

$$
\begin{align*}
\max E_{dd} &= \sum_{r=1}^{s} \mu_{rd} y_{rj}, \\
\text{s.t.} & \sum_{i=1}^{m} \vartheta_{id} x_{ij} = 1, \\
& \sum_{r=1}^{s} \mu_{rd} y_{rj} - \sum_{i=1}^{m} \vartheta_{id} x_{ij} \leq 1, \\
& \vartheta_{id}, \mu_{rd} \geq 0, \\
& i = 1, 2, ..., m, \\
& r = 1, 2, ..., s, \\
& j = 1, 2, ..., n.
\end{align*}
$$

where $j = 1, 2, ..., n$ indicate that there are $n$ subjects to be evaluated, that is DMU, $i = 1, 2, ..., m$ indicate that there are $m$ inputs, $r = 1, 2, ..., s$ indicate that there are $s$ outputs; $E_0$ represents the efficiency value of the evaluated DMU. $E_0$ is between 0 and 1, the closer $E_0$ to 1, the higher the efficiency of the evaluated DMU. Note that the optimal solution of the model (2) is $(\vartheta_{id}^*, \mu_{rd}^*)$ and for the other DMU, $(\vartheta_{id}^*, \mu_{rd}^*)$ can be obtained.

The cross-efficiency of $DMU_j (j = 1, 2, ..., n)$ can be obtained by solving program (3):

$$
E_{dj} = \sum_{r=1}^{s} \frac{\mu_{rd}^* y_{rj}}{\vartheta_{id}^* x_{ij}}, \quad j = 1, 2, ..., n
$$
The DEA cross-efficiency matrix can be obtained by equation (3):

The element $E_{ij}$ in the matrix is the cross-efficiency value $DMU_j$ obtained by the weight of $DMU_i$, and the element on the diagonal represents the efficiency value of the self-evaluation of $DMU_i$. For $DMU_j (j = 1, 2, \ldots, n)$, all the average $E_{ij} (d = 1, 2, \ldots, n)$, $E_{ij} = \frac{1}{n} \sum_{d=1}^{n} E_{ijd}$ represents the final cross-efficiency value of $DMU_j$.

The traditional cross-efficiency may be non-unique because the optimal weight may not be unique. Therefore, some secondary gold cross-efficiency models were proposed. The most classic models are aggressive and benevolent models. The aggressive model minimizes other $DMU_j$'s average efficiencies while ensuring that its self-evaluation efficiency is constant. Conversely, the benevolent model maximizes other $DMU_j$'s as much as possible while ensuring that the self-evaluation efficiency is constant.

The aggressive model:

$$\min \sum_{r=1}^{s} \mu_{rd} \left( \sum_{d=1, d \neq d}^{n} y_{ij} \right),$$

$$\text{s.t.} \sum_{i=1}^{m} \theta_{id} \left( \sum_{d=1, d \neq d}^{n} x_{ij} \right) = 1,$$

$$\sum_{r=1}^{s} \mu_{rd} y_{ij} - \sum_{i=1}^{m} \theta_{id} x_{ij} \leq 0,$$

$$\sum_{r=1}^{s} \mu_{rd} y_{id} - E_{dd} \sum_{i=1}^{m} \theta_{id} x_{id} = 0,$$

$$\mu_{rd}, \theta_{id} \geq 0,$$

$$i = 1, 2, \ldots, m,$$

$$r = 1, 2, \ldots, s,$$

$$j = 1, 2, \ldots, n.$$  (4)

The benevolent model:

$$\max \sum_{r=1}^{s} \mu_{rd} \left( \sum_{d=1, d \neq d}^{n} y_{ij} \right),$$

$$\text{s.t.} \sum_{i=1}^{m} \theta_{id} \left( \sum_{d=1, d \neq d}^{n} x_{ij} \right) = 1,$$

$$\sum_{r=1}^{s} \mu_{rd} y_{ij} - \sum_{i=1}^{m} \theta_{id} x_{ij} \leq 0,$$

$$\sum_{r=1}^{s} \mu_{rd} y_{id} - E_{dd} \sum_{i=1}^{m} \theta_{id} x_{id} = 0,$$

$$\mu_{rd}, \theta_{id} \geq 0,$$

$$i = 1, 2, \ldots, m,$$

$$r = 1, 2, \ldots, s,$$

$$j = 1, 2, \ldots, n.$$  (5)

### B. TOPSIS

TOPSIS was proposed by Lai et al. [33], the steps for TOPSIS are as follows:

**Step 1:** Use the vector specification method to obtain the normative decision matrix.

Let the decision matrix of the multi-attribute decision problem be $Y = \{y_{ij}\}$, and the normalized decision matrix is $F = \{f_{ij}\}$, then:

$$f_{ij} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^{n} y_{ij}^2}}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n$$  (6)

**Step 2:** Construct a weighted normalized decision matrix $X = \{x_{ij}\}$.

Let the weight vector of each attribute given by the decision maker be $W = (w_1, w_2, \ldots, w_n)^T$, where the weight $w_j$ is the weight of the $j$ attribute or criterion, and $\sum_{j=1}^{n} w_j = 1$. Then:

$$x_{ij} = w_j f_{ij}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n$$  (7)

**Step 3:** Determine the ideal solution and the negative ideal solution:

The ideal solution:

$$x^+_j = \begin{cases} \max x_{ij}, & j \in J_1 \\ \min x_{ij}, & j \in J_2 \end{cases}$$  (8)

The negative ideal solution:

$$x^-_j = \begin{cases} \min x_{ij}, & j \in J_1 \\ \max x_{ij}, & j \in J_2 \end{cases}$$  (9)

Among them, $J_1$ and $J_2$ respectively represent the benefit indicator set and the cost indicator set.

**Step 4:** Calculate the distance from each alternative to the ideal solution and the negative ideal solution:

The distance from the alternative to the ideal solution is:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} \left(x_{ij} - x^+_j \right)^2}, \quad i = 1, \ldots, m$$  (10)

The distance from the alternative to the negative ideal solution is:

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left(x_{ij} - x^-_j \right)^2}, \quad i = 1, \ldots, m$$  (11)

**Step 5:** Calculate the comprehensive evaluation index of each alternative:

$$C_i = \frac{d_i^-}{d_i^- + d_i^+}, \quad i = 1, \ldots, m$$  (12)

Larger $C_i$ indicates that the alternative $i$ is relatively better, whereas smaller $C_i$ indicates this alternative $i$ is relatively poorer. Then the alternatives are ranked according to their $C_i$. 

---

**TABLE 1.** The cross-efficiency matrix.

| Department (DMU) | Target DMU 1 | Target DMU 2 | Target DMU 3 | Target DMU n |
|------------------|-------------|--------------|--------------|--------------|
| 1                | $E_{i1}$    | $E_{i2}$    | $E_{i3}$    | $E_{in}$    |
| 2                | $E_{i2}$    | $E_{i2}$    | $E_{i3}$    | $E_{in}$    |
| 3                | $E_{i3}$    | $E_{i2}$    | $E_{i3}$    | $E_{in}$    |
| ...              | ...         | ...         | ...         | ...         |
| n                | $E_{n1}$    | $E_{n2}$    | $E_{n3}$    | $E_{nm}$    |

Average cross-efficiency $E_1 = \sum_{i=1}^{n} E_{i}$.
C. BWM

BWM proposed by Rezaei [34], the specific steps of BWM are as follows:

Step 1: Determine a set of decision criteria.

Step 2: Determine the best (most desirable, most important) and worst (least desirable, least important) decision criteria based on the opinions of the decision-maker or expert.

Step 3: Determine the preference of the best criterion over all the other criteria using a number between 1 and 9. The resulting Best-to-Others vector would be:

\[ B = (a_{B1}, a_{B2}, \ldots, a_{Bn}) \]

where \( a_{Bj} \) represents the preference of the best criterion \( B \) over criterion \( j \), it is clear that \( a_{BB} = 1 \).

Step 4: Determine the preference of all the criteria over the worst criterion using a number between 1 and 9. The resulting Others-to-Worst vector would be:

\[ W = (a_{1W}, a_{2W}, \ldots, a_{nW})^T \]

where \( a_{jW} \) represents the preference of the criterion \( j \) over the worst criterion \( W \), it is clear that \( a_{WW} = 1 \).

Step 5: Find the optimal weights \( (w^*_1, w^*_2, \ldots, w^*_n) \):

The optimal weight for the criteria is the one where, for each pair of \( w_B/w_j \) and \( w_j/w_W \), we have \( w_B/w_j = a_{Bj} \) and \( w_j/w_W = a_{jW} \). Therefore, to get as close as possible to the ideal situation, we should minimize the maximum among the set of \( \{ |w_B - a_{Bj}w_j|, |w_j - a_{jW}w_W| \} \), and the problem can be formulated as follows:

\[
\begin{align*}
\min \quad & \max \left\{ |w_B - a_{Bj}w_j|, |w_j - a_{jW}w_W| \right\}, \\
\text{s.t.} \quad & \sum_{j=1}^n w_j = 1, \\
& w_j \geq 0, \quad \forall j.
\end{align*}
\]

(13)

Problem (13) can be transferred to the following problem:

\[
\begin{align*}
\min \xi^L, \\
\text{s.t.} \quad & |w_B - a_{Bj}w_j| \leq \xi^L, \quad \forall j, \\
& |w_j - a_{jW}w_W| \leq \xi^L, \quad \forall j, \\
& \sum_{j=1}^n w_j = 1, \\
& w_j \geq 0, \quad \forall j.
\end{align*}
\]

(14)

Solving equation (14), we can get the optimal weights \( (w^*_1, w^*_2, \ldots, w^*_n) \) and consistency index \( \xi^{L*} \). The larger weight \( w^*_j \) is, the more important the index \( j \) is. \( \xi^{L*} \) is the consistency index. The closer \( \xi^{L*} \) is to 0, the higher the consistency.

Step 6: Calculate the final score.

Let the decision matrix of the multi-attribute decision problem is \( Y = (y_{ij}) \), and the normalized decision matrix is \( F = (f_{ij}) \), where:

\[
f_{ij} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^m y_{ij}^2}}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n
\]

(15)

### TABLE 2. Aggressive (or benevolent) cross-efficiency matrix.

| Department (DMU) | Target DMU |
|------------------|------------|
|                  | 1 2 3 ... n |
| 1                | \( E_{11} \) \( E_{12} \) \( E_{13} \) ... \( E_{1n} \) |
| 2                | \( E_{21} \) \( E_{22} \) \( E_{23} \) ... \( E_{2n} \) |
| 3                | \( E_{31} \) \( E_{32} \) \( E_{33} \) ... \( E_{3n} \) |
| ...              | ... ... ... ... ... |
| n                | \( E_{n1} \) \( E_{n2} \) \( E_{n3} \) ... \( E_{nn} \) |

### TABLE 3. Cross-efficiency matrices containing ideal and negative ideal solutions.

| Department (DMU) | Target DMU |
|------------------|------------|
|                  | 1 2 3 ... n |
| 1                | \( E_{11} \) \( E_{12} \) \( E_{13} \) ... \( E_{1n} \) |
| 2                | \( E_{21} \) \( E_{22} \) \( E_{23} \) ... \( E_{2n} \) |
| 3                | \( E_{31} \) \( E_{32} \) \( E_{33} \) ... \( E_{3n} \) |
| ...              | ... ... ... ... ... |
| n + 1(\( Z_j^* \)) | \( Z_j^* \) \( Z_j^* \) \( Z_j^* \) ... \( Z_j^* \) |
| n + 2(\( Z_j^* \)) | \( Z_j^* \) \( Z_j^* \) \( Z_j^* \) ... \( Z_j^* \) |

Obtain the multi-attribute, and the final score \( V_i \) is:

\[
V_i = \sum_{j=1}^n w_j^* f_{ij}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n
\]

(16)

Larger \( V_i \) indicates that the alternative \( i \) is better. Then the alternatives are ranked according to their \( V_i \).

### III. THE BWM-TOPSIS CROSS-EFFICIENCY METHOD

This paper combines aggressive and benevolent cross-efficiency methods with BWM and TOPSIS methods to introduce the BWM-TOPSIS aggressive and benevolent cross-efficiency methods. The specific steps are as follows:

Step 1: Derive an aggressive (or benevolent) cross-efficiency matrix from the above description:

Step 2: Based on the aggressive (or benevolent) cross-efficiency matrix, find the ideal solution \( Z_j^+ \) and the negative ideal solution \( Z_j^- \), where:

- The ideal solution:

\[
Z_j^+ = \max_{d} E_{dj}, \quad d = 1, 2, \ldots, n
\]

(17)

- The negative ideal solution

\[
Z_j^- = \min_{d} E_{dj}, \quad d = 1, 2, \ldots, n
\]

(18)

Then, we can get aggressive (or benevolent) cross-efficiency matrixes containing ideal and negative ideal solutions:
TABLE 4. Data for seven academic departments in a university.

| Department (DMU) | Inputs | Outputs |
|------------------|--------|---------|
|                  | x₁     | x₂     | x₃    | y₁    | y₂    | y₃    |
| 1                | 12     | 400    | 20    | 60    | 35    | 17    |
| 2                | 19     | 750    | 70    | 139   | 41    | 40    |
| 3                | 42     | 1500   | 70    | 225   | 68    | 75    |
| 4                | 15     | 600    | 100   | 90    | 12    | 17    |
| 5                | 45     | 2000   | 250   | 253   | 145   | 130   |
| 6                | 19     | 730    | 50    | 132   | 45    | 45    |
| 7                | 41     | 2350   | 600   | 305   | 159   | 97    |

Note: x₁- Number of academic staff; x₂- Academic staff salaries in thousands of pounds; x₃- Support staff salaries in thousands of pounds; y₁- Number of undergraduate students; y₂- Number of postgraduate students; y₃- Number of research papers.

Step 3: Calculate the distance \( d^+_d \) from the ideal solution and the distance \( d^-_d \) from the negative ideal solution:

\[
\begin{align*}
    d^+_d &= \sqrt{\sum_{j=1}^{n} (E_{dj} - Z_j^+)^2}, \quad d = 1, 2, \ldots, n, n+1, n+2 \\
    d^-_d &= \sqrt{\sum_{j=1}^{n} (E_{dj} - Z_j^-)^2}, \quad d = 1, 2, \ldots, n, n+1, n+2 
\end{align*}
\]  

(19)

Step 4: Derive the best preference vector, the worst preference vector:

The optimal weight for the criteria is the one where, for each pair of \( w_B/w_d \) and \( w_d/w_W \), we have \( w_B/w_d = a_{Bd} \) and \( w_d/w_W = a_{dW} \).

The best preference vector:

\[
A_B = (a_{B1}, a_{B2}, \ldots, a_{B(n+2)})
\]

where \( a_{Bd} = d^+_d + 1 \), it represents the preference of the best criterion \( B \) over criterion \( d \), it is clear that \( a_{BB} = 1 \).

The worst preference vector:

\[
A_W = (a_{W1}, a_{W2}, \ldots, a_{W(n+2)})^T
\]

where \( a_{dW} = d^-_d + 1 \), it represents the preference of the criterion \( d \) over the worst criterion \( W \), it is clear that \( a_{dW} = 1 \).

Step 5: Get the final score:

To get as close as possible to the ideal situation, we should minimize the maximum among the set of \(|w_d - a_{Bd}w_d|, |w_d - a_{dW}w_W|\), and the problem can be formulated as follows:

\[
\begin{align*}
    \min \max_{d} & \left| w_d - a_{Bd}w_d \right|, \left| w_d - a_{dW}w_W \right|, \\
    \text{s.t.} & \sum_{d=1}^{n+2} w_d = n + 2, \\
    & w_d \geq 0, \forall d. 
\end{align*}
\]  

(20)

IV. NUMERICAL EXAMPLE

In this section, we provide a numerical example to illustrate the feasibility and performance of the BWM-TOPSIS Cross-efficiency method.

The three inputs and three outputs are given to evaluate the seven academic departments in a university recorded.
TABLE 7. Comparison.

| Department (DMU) | Target 1 | Target 2 | Target 3 | Target 4 | Target 5 | Target 6 | Target 7 |
|------------------|----------|----------|----------|----------|----------|----------|----------|
|                  | Value 1  | Value 2  | Value 3  | Ranking  | Value 1  | Value 2  | Value 3  | Ranking  |
| 1                | 1.0000   | 0.8452   | 0.9333   | 0.6878   | 1.0000   | 0.9333   | 0.7521   | 0.8788   | 1        | 0.7934  | 1.3374  | 1        |
| 2                | 0.3347   | 1.0000   | 0.6178   | 1.0000   | 0.7017   | 0.8426   | 0.5564   | 0.7219   | 4        | 0.5348  | 0.9871  | 4        |
| 3                | 0.5551   | 0.8481   | 1.0000   | 0.7351   | 0.5551   | 1.0000   | 0.4175   | 0.7301   | 3        | 0.5868  | 1.0067  | 3        |
| 4                | 0.0686   | 0.7551   | 0.2800   | 0.8197   | 0.2417   | 0.4413   | 0.2063   | 0.4018   | 7        | 0.1416  | 0.7042  | 7        |
| 5                | 0.3314   | 0.662    | 0.3148   | 0.7646   | 1.0000   | 0.4778   | 0.8309   | 0.6259   | 5        | 0.4719  | 0.8830  | 5        |
| 6                | 0.5143   | 1.0000   | 0.8213   | 0.9507   | 0.7915   | 1.0000   | 0.6107   | 0.8126   | 2        | 0.6660  | 1.1432  | 2        |
| 7                | 0.1514   | 0.6044   | 0.1581   | 0.9985   | 0.9854   | 0.2783   | 1.0000   | 0.5966   | 6        | 0.4387  | 0.7845  | 6        |

(A. THE AGGRESSIVE CROSS-EFFICIENCY METHOD BASED ON BWM-TOPSIS)

Step 1: The aggressive cross-efficiency matrix of seven DMUs is obtained based on the raw data:

The closer the efficiency value is to 1, the DMU is better, and the closer the efficiency value is to 0, the DMU is worse.

Step 2: Find the ideal solution and the negative ideal solution based on the cross-efficiency matrix:

The ideal solution:

$$Z^+_j = (1.000, 1.000, 1.000, 1.000, 1.000, 1.000, 1.000)$$

The negative ideal solution:

$$Z^-_j = (0.0686, 0.6044, 0.1581, 0.6878, 0.2417, 0.2783, 0.2063)$$

Obtain an aggressive cross-efficiency matrix containing the ideal solution and the negative ideal solutions:

Step 3: Calculate the distance from the ideal solution and the distance from the negative ideal solution:

$$d^+_j = (0.437932, 0.948282, 0.910178, 1.730755, 1.177909, 0.682126, 1.451373, 0, 1.885762)$$

$$d^-_j = (1.681775, 1.090278, 1.292587, 0.285546, 1.052511, 1.36043, 1.134213, 1.885762, 0)$$

Step 4: Derive the best preference vector and the worst preference vector:

The best preference vector:

$$A_B = (1.437932, 1.948282, 1.910178, 2.730755, 2.177909, 1.682126, 2.451373, 1, 2.885762)$$

The worst preference vector:

$$A_W = (2.681775, 2.090278, 2.292587, 1.285546, 2.052511, 2.36043, 2.134213, 2.885762, 1)^T$$

Step 5: Get the final score:

TABLE 8. Seven DMUs benevolent cross-efficiency matrix.

| Department (DMU) | Target DMU | Value 1 | Value 2 | Value 3 |
|------------------|------------|---------|---------|---------|
|                  |            | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
| 1                | 1.0000     | 0.9219  | 1.0000  | 0.6875  | 1.0000  | 1.0000  | 1.0000  |
| 2                | 0.9812     | 1.0000  | 0.8510  | 1.0000  | 0.8461  | 0.9812  | 0.9812  |
| 3                | 0.7690     | 0.7719  | 1.0000  | 0.7349  | 0.6651  | 0.7690  | 0.7690  |
| 4                | 0.6411     | 0.7013  | 0.4542  | 0.8197  | 0.4135  | 0.6411  | 0.6411  |
| 5                | 0.9382     | 0.8990  | 0.4950  | 0.7650  | 1.0000  | 0.9382  | 0.9382  |
| 6                | 1.0000     | 1.0000  | 1.0000  | 0.9506  | 0.9104  | 1.0000  | 1.0000  |
| 7                | 1.0000     | 0.2941  | 1.0000  | 1.0000  | 1.0000  | 1.0000  | 1.0000  |

TABLE 9. Benevolent cross-efficiency matrix containing ideal and negative ideal solutions.

| Department (DMU) | Target DMU | Value 1 | Value 2 | Value 3 |
|------------------|------------|---------|---------|---------|
|                  |            | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
| 1                | 1.0000     | 0.9219  | 1.0000  | 0.6875  | 1.0000  | 1.0000  | 1.0000  |
| 2                | 0.9812     | 1.0000  | 0.8510  | 1.0000  | 0.8461  | 0.9812  | 0.9812  |
| 3                | 0.7690     | 0.7719  | 1.0000  | 0.7349  | 0.6651  | 0.7690  | 0.7690  |
| 4                | 0.6411     | 0.7013  | 0.4542  | 0.8197  | 0.4135  | 0.6411  | 0.6411  |
| 5                | 0.9382     | 0.8990  | 0.4950  | 0.7650  | 1.0000  | 0.9382  | 0.9382  |
| 6                | 1.0000     | 1.0000  | 1.0000  | 0.9506  | 0.9104  | 1.0000  | 1.0000  |
| 7                | 1.0000     | 0.2941  | 1.0000  | 1.0000  | 1.0000  | 1.0000  | 1.0000  |

To solve the linear programming problem of the program (14), the conclusions are: $W_1 = 1.337, W_2 = 0.9871, W_3 = 1.0067, W_4 = 0.7042, W_5 = 0.88230, W_6 = 1.14323, W_7 = 0.7845$.

The above table is the comparison among BWM-TOPSIS cross-efficiency method, the aggressive cross-efficiency model and the cross-efficiency improved by TOPSIS. The value 1 represents the efficiency calculated by the aggressive...
cross-efficiency model; the value 2 represents the efficiency calculated by the cross-efficiency improved by TOPSIS, and the value 3 represents the efficiency value calculated by BWM-TOPSIS cross-efficiency method proposed in this paper. The results calculated by the three methods are consistent, which shows that the BWM-TOPSIS cross-efficiency proposed in this paper is feasible.

B. THE BENEVOLENT CROSS-EFFICIENCY METHOD BASED ON BWM-TOPSIS

Step 1: Based on the raw data, the benevolent cross-efficiency matrix of seven DMUs is obtained:

The closer the efficiency value is to 1, the DMU is better, and the closer the efficiency value is to 0, the DMU is worse.

Step 2: Find the ideal solution and the negative ideal solution based on the cross-efficiency matrix:

The ideal solution:

\[ Z_j^+ = (1.000, 1.000, 1.000, 1.000, 1.000, 1.000, 1.000) \]

The negative ideal solution:

\[ Z_j^- = (0.641, 0.701, 0.294, 0.688, 0.414, 0.641, 0.641) \]

Obtain a benevolent cross-efficiency matrix containing positive ideal solution and negative ideal solution:

Step 3: Calculate the distance from the ideal solution and the distance from the negative ideal solution:

\[ d_d^+ = (0.322112, 0.216671, 0.628131, 1.072398, 0.576115, 0.102316, 0.7059, 0, 1.189783) \]

\[ d_d^- = (1.130208, 1.015462, 0.786069, 0.207627, 0.833211, 1.135815, 0.957751, 1.189783, 0) \]

Step 4: Derive the best preference vector and the worst preference vector:

The best preference vector:

\[ A_B = (1.322112, 1.216671, 1.628131, 2.072398, 1.576115, 1.102316, 1.7059, 1, 2.189783) \]

The worst preference vector:

\[ A_W = (2.130208, 2.015462, 1.786069, 1.207627, 1.833211, 2.135815, 1.957751, 2.189783, 1)^T \]

Step 5: Get the final score:

To solve the linear programming problem of the program (14), the conclusions are: \[ W_1=1.132381, \ W_2=1.230517, \ W_3=0.919542, \ W_4=0.722417, \ W_5=0.949889, \ W_6=1.303083, \ W_7=0.877622 \].

The above table is the comparison among BWM-TOPSIS cross-efficiency method, the benevolent cross-efficiency model and the cross-efficiency improved by TOPSIS. The value 1 represents the efficiency calculated by the benevolent cross-efficiency model; the value 2 represents the efficiency calculated by the cross-efficiency improved by TOPSIS, and the value 3 represents the efficiency value calculated by BWM-TOPSIS cross-efficiency method, the benevolent cross-efficiency model and the cross-efficiency improved by TOPSIS. The value 1 represents the efficiency calculated by the benevolent cross-efficiency model; the value 2 represents the efficiency calculated by the cross-efficiency improved by TOPSIS, and the value 3 represents the efficiency value calculated by BWM-TOPSIS cross-efficiency method proposed in this paper. Compared with the results of benevolent cross-efficiency model, the order of the department 3, department 5, and department 7 have changed. Where department 7 is the DMU farthest from the negative ideal solution, but it is also closest from the ideal solution; department 5 is closest to the ideal solution, and it is not closest to the negative ideal solution; department 3 is closest to the negative ideal solution, but it is not farthest from the ideal solution. Compared with the results of the cross-efficiency improved by TOPSIS, there is a discrepancy between the department 3 and the department 7, and the gap between the two results is small. Compared with department 3, although department 7 is the furthest from the negative ideal solution, it is not as close to the ideal solution as department 3. In summary, the method proposed in this paper doesn’t only rely on the absolute value of the average efficiency, but it also considers the relative distance.

V. CONCLUSION

In view of the drawbacks when the ultimate average cross-efficiency scores are used to evaluate and rank DMUs, we improve the assumption of average and utilize the
BWM-TOPSIS technique to rank cross-efficiency. There are some advantages to this method: (1) The method is clear, and the calculation is simple; (2) Combining TOPSIS, BWM and cross-efficiency methods for the first time. The biggest feature of the BWM method is that it greatly improves the consistency, but the method is more subjective. Adding efficiency values will more objectively select the best and worst solution. At the same time, using the distance in TOPSIS instead of the preference vector in BWM will also increase the objectivity of the method; on the contrary, the high consistency of the BWM method also solves the shortcomings of the final result of the TOPSIS method, that is, the final decision made by the TOPSIS method is not necessarily closest to the ideal solution and farthest from the negative ideal solution; combining these two methods with cross-efficiency will solve the aggregation of cross-efficiency. The results of the rank order make full use of the original data information in cross-efficiency matrix and can quantitatively reflect the virtues or defect degree of different evaluation programs. In order to prove the effectiveness of the proposed approach, a numerical example is illustrated finally. The disadvantage of this method is that there are many cross-efficiency improvement models. The aggressive and benevolent cross-efficiency models can only solve certain deficiencies. Then, Improvements can be made based on more complex cross-efficiency. Secondly, the weight of each cross-efficiency is not determined in this method, which needs to be explored in the future research.

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