The modulation of the eigen oscillation in the harbours by tide

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Abstract. In the port harbors there are reoccurring a reciprocating movements of water, which are caused by strong waves at sea. Threatening the safety of ships and berthing facilities, these movements, called harbor oscillations, are caused by the infragravity waves. They occur when their periods are close to the periods of eigen resonant oscillations of the water area. As a result of the observations, it was found that the seiches modulated with the frequency of tidal waves. This phenomenon cannot be explained by analogy with the modulation of infragravity waves. Observations show that the primary reason is an enhancement of energy loss over the low-tide surfzone bottom profile, while in the port harbour significant currents are observed only in the harbor entrance zone. It is shown that the reason of modulation is a change in the water level in the harbor, which occurs nonlinearly. Using the Duffing equation, the dynamics of a nonlinear dynamic system a liquid oscillating in a port under the external excitation by tidal waves and swell, is analyzed. It is shown that under the influence of swell at the harbor entrance can lead to chaotic vibrations in the harbor, which can be dangerous for ships.

1. Introduction
It is known that even in “well-protected” ports there are strong reciprocating movements of water, which lead to the impact of ships on the pier or against each other, the breakage of moorings, and violation of loading and unloading operations [1, 2, 3, 4]. This phenomenon has been called harbor oscillations. They are caused, infragravity (IG) waves, when their characteristic periods coincide or are close to the periods of eigen oscillations of the port water area – seiches [1]. Therefore, the study of the eigen oscillations of the harbors is interest for practical purposes – to ensure the safety of navigation.

In the study of wave processes in port harbors or partially enclosed water areas, an interesting effect was found-periodic change in the periods of eigen oscillations of water areas – modulation. This phenomenon has a clearly visible connection with the tidal waves in all spectra of sea level fluctuations in partially enclosed water areas, for example, in the water areas of Uglegorsk and Boshnyakovo ports, Sakhalin region (figure 1).

Before, we studied the effect of modulation of IG waves by the tide, which showed that the primary reason for the modulation is an enhancement of energy loss over the low-tide surfzone bottom profile [5]. Tidal energy is transferred to higher-frequency motions in the surfzone through near-resonant nonlinear interactions between triads of wave components. These nonlinear transfers are sensitive to the surfzone bottom profile thus tidal sea level variations over the nonuniform beach produce tidal changes in the IG energy observed offshore. This study is supported by the conclusion of the paper [6].
At the same time, our data on sea level fluctuations in the port harbor showed a pronounced modulation of seiches with a tidal period. In this case, the change in the period can reach 20%, while the modulation of the IG waves value does not exceed 10%. Moreover, tidal currents are manifested only at the entrance of harbor and therefore are unlikely to have modulating effects on seiches. As a result, it was decided to study the mechanism of modulation seiches in the port harbors of Uglegorsk and Boshnyakovo, located on the West coast of Sakhalin Island.

2. The obtained data
During field experiments in 2010, digital self contained wave recorders were installed in the port harbors of the Western coast of Sakhalin Island. One of them was installed in the Northern part of the port of Uglegorsk at the quay wall. As a result of observations, a record of sea level fluctuations with a one second discreteness from December 2009 to June 2010 was obtained, a fragment of which is shown in figure 1. In addition, in 2008 the observations of sea-level fluctuations were performed in the Boshnakova harbor. The time series were similar in nature for both water areas and clearly indicated the presence of modulation.

The water area of the port of Uglegorsk has a rectangular shape and is divided into unequal parts by the pier (figure 2). In such closed water areas with one open entrance sea level fluctuations are formed. The period of seiches depends on the geometrical shape and size of the water area and in harbors with a high q-factor, seiches can be very dangerous.

To simplify the problem of calculating the eigen oscillations, we will focus only on some aspects that are directly related to the fluctuations in the harbors, bays, i.e., with the waters areas with an open border (entrance), through which their connection with the external water area is carried out.

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**Figure 1.** Spectra of sea level fluctuations in the water area of Uglegorsk and Boshnyakovo ports.
Figure 2. The water areas of the port of Bosnyakovo with length 190 and width 120 meters, a depth of 2.2 m and the port of Uglegorsk: $L = 264$ m, $L_1 = 80$ m, $L_2 = 184$ m, width 72 m, depth at the berth from 1.9 to 2.2 m.

Determination of eigen resonance frequencies of partially enclosed rectangular basins of length $L$ and homogeneous depth $H$ can be performed using the Merian’s formula [1, 3]:

$$T_n = \frac{2L}{n\sqrt{gH}}$$, where $n = 1, 2, 3, \ldots$ (1)

In this case, since there is a possibility of excitation of seiches in each segment of the port, the calculation of seiches were made for each part and for the total port. The results are given in table. 1. There’s also, for comparison, shows the periods of seiches defined from the spectra of the experimental data. It should be noted that the calculation was also carried out by the equation for calculating the period of eigen oscillations of the liquid in a rectangular basin given in paper [7], which showed results close to the calculations by the formula (1) is shown at table 1, where $\Delta H(m)$ – average depth, $T_{ch}(s)$ – computed period for high tide, $T_{cl}(s)$ – computed period for low tide, $T_{oh}(s)$ – observed period for high tide, $T_{ol}(s)$ – observed period for low tide.

The analysis of the table showed that the computed and observed periods of the seiches are close, except for the periods of the port segment adjacent to the entrance, and this fact apparently affects the calculation. The others differ by no more than 10%, that is a good result for the calculated periods. It can be considered that the peaks detected with using the measurements correspond to the first mode of the seiches. The calculated short-period transverse seiches for the port of Uglegorsk have a better agreement with the observed periods of the seiches and the peak in spectra correspond to these transverse seiches apparently.
### Table 1. Computed eigen oscillations periods of the first mode and periods of the observed seiches.

| Port         | L(m) | H(m) | ΔH(m) | $T_{ch}(s)$ | $T_{cl}(s)$ | $T_{oh}(s)$ | $T_{ol}(s)$ | Note       |
|--------------|------|------|-------|-------------|-------------|-------------|-------------|------------|
| Uglegorsk   | 80   | 2.05 | 0.45  | 32.3        | 35.7        | —           | —           | longitudinal |
| Uglegorsk   | 184  | 2.05 | 0.9   | 68.4        | 82.1        | 51.1        | 55.9        | longitudinal |
| Uglegorsk   | 264  | 2.05 | 0.41  | 107.5       | 117.8       | 116.4       | 130.9       | longitudinal |
| Uglegorsk   | 72   | 2.05 | 0.55  | 28.5        | 32.1        | 29.1        | 31.7        | lateral     |
| Bosnyakovo  | 190  | 2.2  | 0.65  | 71.9        | 81.8        | 70          | 79          | longitudinal |
| Bosnyakovo  | 120  | 2.2  | 0.4   | 47.4        | 51.7        | 41.2        | 43.9        | lateral     |

### 3. The model of dynamic system

The dynamic system considered here – the oscillating sea water in the water area is nonlinear, since the change in the water level during the day, occurs according to the nonlinear and at the presence of an external excitation by tidal wave. Above, we have determined the periods of seiches for the extreme positions of sea levels in the port at high and low tide only. It is of interest to consider the behavior of the system as a whole with using computer simulation.

In this case, we will take into account that in addition to the influence of tidal waves on the seiches, the system has a second external excitation – swell (or wind waves), which will also affect the oscillations in the system. It is usually assumed that if an external force acts on the oscillator, the frequency of which is much less than the natural frequency of the oscillator, then this effect can be considered as quasi-stationary. However, studies carried out in the 1990s proved that under certain conditions the interaction of low and high - frequency oscillations can be significant and leads to the occurrence of chaotic vibrations [8, 9]. It should be noted that in [10] the dynamic system with the influence of two close frequencies is considered. These results were interesting in the analysis of the system considered here.

Oscillations in a nonlinear system can be described by one ordinary differential Duffing equation, which describes a system of the 2nd order with irregular oscillations and external periodic excitation [11, 12]. In this paper, we will use the Duffing equation for computer modeling, which describes oscillations in a system with two external influences – swell and tidal waves. The equation for this case is [13]:

$$\ddot{x} + kx + \omega_0^2 x - \beta x^3 + \alpha x^3 = F_l \cos(\Omega t) + F \cos(\omega t),$$  

(2)

where $F_l$, $\Omega$ and $F$, $\omega$ – amplitudes and frequencies of the external long-period tidal excitation (period $T_l$) and short-period excitation (period $T$), in this case, the waves swell; $\omega_0$ – frequency oscillator ($T_0$); $\alpha$ and $\beta$ – coefficients of non-linearity.

Using the equation (2) and developed by us program PUAN [14], phase portraits and the form of oscillations in the system for the port of Uglegorsk were calculated with $T = 12$ s, $T_l = 24$ hours, $T_0 = 130.9$ s, $k = 0.01$, $\alpha = 2$, $\beta = 2$, at changing the amplitude of external excitations - tide and swell. As can be seen from the figure, in the absence of swell $F = 0$ phase portrait of the system and the shape of oscillations (figure 3, a, b) corresponds to the weak dissipation of the phase space element at the spiral trajectory to a stable fixed point.

An increase in the amplitude of the swell leads to a change in the shape of the oscillations and their transition to a quasi-periodic regime. At the equality of the amplitudes of tidal wave and swell the regime is observed when during one period the movement is carried out in a small orbit, and during the another, a large orbit (figure 3, c). The shape of the oscillations in this case corresponds to the quasi-periodic (figure 3, d). And in this case we do not see the damping, which indicates a transfer of swell energy to seiches.
Figure 3. The phase portraits, shape of the oscillations in the system and the Poincare section for $T = 12$ s, $T_l = 24$ hours, $T_0 = 130.9$ s, $k = 0.01$, $\alpha = 2$, $\beta = 2$, at changing the amplitudes of the external excitations – tide and swell: (a, b) $F_l = 0.1$, $F = 0$; (c, d) $F_l = 0.1$, $F = 0.1$; (e) $F_l = 0.1$, $F = 3.5$; (f) the Poincare section for $F_l = 0.1$ and $F = 3.5$.

A further increase in the amplitude of the swell leads to a phase portrait of the system, which is called a strange chaotic attractor (figure 3, e), corresponding to the forced oscillations of the Duffing oscillator under external excitations of both tidal wave and swell with a period of 12 s and for the period of system oscillations 130.9 s (figure 3, e, f).

Also, bifurcation diagrams were calculated for the case of influence on the dynamic system of waves of swell and tidal. They are shown in figure 4, for the damping parameter of 0.01, and nonlinearity factor of 2. At the diagrams calculating we taken amplitudes $F_l = 0.5$ and $F_l = 0$, i.e. in the second case the system was excited by only one wave with a swelling period. It is seen that in the presence of the second excitation with a swell period significantly changes the moments of occurrence of chaotic vibration.
depending on the amplitude of the swell. This indicates the need when studying such dynamic systems to take into account both excitations, even with very different frequencies.

Detailed analysis of the computed bifurcation diagram for two external excitations (figure 4, a) showed at what values of \( F_l \) and \( F \) the system transition to chaos occurs and for this case, the phase portrait of the system and the Poincare section shown in (figure 3, d, e), which confirmed the presence of chaotic vibrations in the system. It was also found that the change in \( F \) observed bifurcation doubling, followed by chaotic vibrations. At the same time, the occurrence of chaotic vibrations is often observed without prior bifurcation of doubling.

Figure 4. Bifurcation diagrams of \( x \) versus the external excitation amplitude \( F \) for damping parameter 0.01, nonlinearity factor 2.0 and for the cases: (a) \( F_l = 0.5 \); (b) \( F_l = 0 \). Bifurcation diagrams of \( x \) versus \( F \) for \( T_l = 0.5 \) and nonlinearity factor (c) \( \alpha = 7 \); (d) damping parameter \( k = 0.1 \)

The analysis with computer simulations showed that an increase in the nonlinearity factor leads to an increase in the regions with chaotic oscillations (figure 4, c), and an increase in the damping parameter leads to reduce the complexity of the system (figure 4, d). A further increase in the damping parameter to \( k = 0.3 \) leads to the transition of the dynamic system to periodic oscillations.

4. Conclusion

The analysis of the waves mode in the port of Boshnyakovo and the port of Uglegorsk was performed. It is shown that the seiches are observed in both harbors. In the port of Uglegorsk seiches excited on three different periods in the range of dangerous sea phenomena, which are manifested in the form of reciprocating movements of the water and can pose a danger to ships and piers.

It is established that seiches are modulated by the frequency of the tidal wave. The reason for this modulation is a nonlinear change in sea level in the harbors with a tidal frequency. Calculation of periods of seiches in high and low tides performed using the formula Meriana, demonstrates a good agreement with field data.

Computer simulation of the dynamics of oscillations in the dynamic system - water mass in the water area of the port of Uglegorsk was performed using the Duffing equation. It was taken into account that
the dynamic system is affected by two external excitations - tidal waves and swell waves, significantly different in values of periods of these waves.

The obtained phase portraits of the dynamic system showed that the oscillations in the system under the influence of tides corresponded to the weak dissipation of the phase space element at the spiral trajectory to a stable fixed point. With increasing damping the parameter decreases the number of turns of the spiral phase portrait. The transition to shorter periods of oscillations of the system corresponding to the periods of the seiches of other parts of the water area leads to a decrease in the number of turns of the spiral, without changing the nature of the phase portrait.

Numerical simulation for the influence of short-period swell waves showed the complexity of the dynamics of the considered nonlinear system, including periodicity, quasi-periodicity and chaos. The intermittency of chaotic and quasi-periodic oscillations is observed in a wide range of periods of external short-period excitation and with increasing its amplitude the duration of the chaos and quasi-periodicity sites is reduced. An increase in the dissipation of short-period oscillations in the system, i.e. an increase in the damping parameter in the system, leads to periodic oscillations.

The presence of the second excitation with a tidal period in the system significantly changes the moments of occurrence of chaotic vibrations depending on the amplitude of the swell. This indicates the need to take into account both excitations for the study of such dynamic systems, even with very different frequencies.

The results of studies of chaotic movements help to understand the transition from ordered to turbulent motion [12]. This is also of interest at studying of hydrophysical processes, especially for scales comparable to the size of wind waves and swell, since such turbulence can have a significant impact on the seaworthiness of ships.

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