Quantum oscillations in non-Fermi liquids: Implications for high-temperature superconductors

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We address quantum oscillation experiments in high $T_c$ superconductors and the evidence from these experiments for a pseudogap versus a Fermi liquid phase at high magnetic fields. As a concrete alternative to a Fermi liquid phase, the pseudogap state we consider derives from earlier work within a Gor’kov-based Landau level approach. Here the normal state pairing gap in the presence of high fields is spatially non-uniform, incorporating small gap values. These, in addition to $d$-wave gap nodes, are responsible for the persistence of quantum oscillations. Important here are methodologies for distinguishing different scenarios. To this end we examine the temperature dependence of the oscillations. Detailed quantitative analysis of this temperature dependence demonstrates that a high field pseudogap state in the cuprates may well “masquerade” as a Fermi liquid.

The surprising discovery of quantum oscillations in the underdoped cuprate high-temperature superconductor$^{12,13}$ can potentially elucidate the normal, non-superconducting state of these materials. However, a number of experiments seem to indicate the presence of phenomena that may be associated with the application of high magnetic fields, rather than with the intrinsic normal phase. The oscillatory frequency is very small, which suggests a small and possibly reconstructed Fermi surface.$^{14,15}$ More recently, static or quasi-static charge density wave order in the high-field regime has been observed, which also suggests substantial differences between the zero-field normal state and the high-field state.$^{16,17}$ Changes in magnetization, as well as transport coefficients, have also been found at high fields.$^{18,19}$

It is notable that detailed studies of the temperature dependence of the oscillatory amplitude$^{14}$ report excellent agreement with a Fermi-Dirac dependence, providing no sign of the pseudogap state that has been observed at magnetic field $H = 0$. Adding to the complexity, specific heat measurements suggest an overall temperature dependence of $\sqrt{H}$ that is consistent with the presence of a $d$-wave pairing gap.$^{20,21}$ These observations have led to theoretical proposals in which there are Fermi liquid-like features, perhaps co-existing with a pseudogap. Two notable scenarios both introduce a low frequency, third peak into the spectral function, while maintaining the two other peaks at finite $\omega$ which reflect a gap structure.$^{22,23}$ However, the question of whether the quantum oscillation measurements require this peak (or more broadly, Fermi liquid-like behavior near the Fermi surface) is an open and very important one. The answer bears on the proper microscopic description of the cuprates.

In this paper we address this issue more phenomenologically. We quantitatively compare a high field pseudogap scenario and an alternative “co-existing pseudogap and Fermi liquid” approach with a strict Fermi liquid. We do so via the measured$^{24}$ temperature dependences of the quantum oscillations which have been interpreted to strongly support a Fermi liquid phase at high $H$. Our high field pseudogap scenario was derived earlier using a Gor’kov-based theory$^{22,23}$ and incorporating Landau level physics. Although some model systems do display substantial deviations, discrepancies for the parameter sets appropriate to the cuprates are not large enough to be detected in existing experiments. Thus we conclude that a non-Fermi liquid phase supports quantum oscillations with $T$ dependence presently indistinguishable from that of a Fermi liquid.

**Theory of the Pseudogap in High Magnetic Fields** At $H = 0$ the commonly used pseudogap self-energy $\Sigma(k, i\omega_n)$, has been derived from a theory of pairing fluctuations$^{22,23}$ and has also been obtained phenomenologically by fitting angle resolved photoemission experiments.$^{22,23}$ It is given by

$$\Sigma(k, i\omega_n) = -i\gamma + \frac{\Delta^2(k)}{i\omega_n + \xi_{-k} + i\gamma}. \quad (1)$$

where $i\omega_n$ is the fermionic Matsubara frequency, $\xi_k$ the
single-particle dispersion, and $\gamma$ and $\gamma'$ damping coefficients associated with the pairing gap ($\Delta(k)$) and single particles, respectively.

Using Gor'kov theory, we and others have shown earlier that in the presence of large magnetic fields, with intra-Landau level pairing, the general BCS-like structure of the Green’s functions (and hence self energy as in Eq. [1]) is maintained. For this quasi-two dimensional pseudogap state, we incorporate Landau levels via $\xi(k) \rightarrow \xi(n, q) = (n + \frac{1}{2})\hbar\omega_c$, where $n$ is the Landau level and $q$ the degenerate quantum index. The latter is based on a magnetic translation group approach which is associated with the superconducting and pseudogap phases.\textsuperscript{23}

Importantly, a gap squared contribution (as in Eq. [1]) persists\textsuperscript{23} into the normal phase. This reflects pairing (as distinct from phase) fluctuations which arise from short-lived, preformed pairs; they are to be associated with stronger-than-BCS attractive interactions (consistent with high transition temperatures) and they lead to a pseudogap. Furthermore, Gor'kov theory at high fields requires the introduction of inhomogeneity in the gap function $\Delta$.\textsuperscript{[18]}

Similarly, in the normal high-field state the pairing gap $\Delta(k)$ must be dependent on the $(n, q)$ parameters defined above, and this will lead to real-space inhomogeneity. Physically, these inhomogeneities reflect excited pair states which were shown\textsuperscript{23} to correspond to small distortions or excitations of the optimal (condensate) vortex configuration. As such, they represent blurred lattice patterns. This “precursor vortex” state is illustrated in Fig. [1].

For the purposes of this paper, the exact dependence of $\Delta(n, q)$ (where $\Delta$ is the (real) magnitude of the gap) on $q$ need not be determined. Instead only the distribution of $\Delta$ values over $q$ is fixed. Throughout the paper we use the normalization that $\langle |\Delta(n, q)|^2 \rangle = \Delta$, the specified gap magnitude. One can reasonably approximate this distribution by taking $\Delta(n, q)$ independent of $n$, as the distribution should change only slightly for moderate to large Landau levels. While the high field normal state gap inhomogeneity (“pseudovortex”) is present for any pairing symmetry, it should be noted that nodal effects from $d$-wave pairing in a Landau level basis also lead to real-space inhomogeneity.\textsuperscript{23} For simplicity, we calculate the distribution for $n = 8$ from previous work on $d$-wave pairing at high magnetic fields within the magnetic translation group\textsuperscript{23} and use this as a model distribution throughout the paper.

Fig. [1] shows the density plot of $|\Delta(q)|^2$ while the Fig. [2] inset shows the histogram used. We stress that deviations in vortex locations, pseudogap inhomogeneities, and inter-Landau level pairings should only change this distribution slightly, and leave the conclusions unaffected. Thus, with this method we have extended the zero-field self energy, Eq. [1], to a form appropriate for the high-field pseudogap state.

It is useful to compare this high field normal state picture with others in the literature, which focus on quantum oscillations, as we have shown in similar work near zero field.\textsuperscript{23} This is consistent with the quantum oscillations seen previously within the superconducting phase of extreme type-II superconductors.\textsuperscript{23,13,14}

A critical component of these oscillations is the presence of nodes or near-nodal $q$ states in $\Delta(q)$, as Figs. [1] and [2], inset show are present in this system. If on the other hand one presumed a constant $\Delta(q)$, Fig. [2b] (which plots the density of states) shows that the amplitude of the oscillations is reduced to zero. For the $d$-wave case the near-nodal states, in contrast, preserve the single-particle Landau level dispersion and thus the quantum oscillations.

Temperature Dependence of Oscillations

To distinguish a non-Fermi liquid from a Fermi liquid on the basis of quantum oscillations, we next focus on the temperature dependence of the oscillation amplitude. Importantly, this temperature dependence has previously been measured in YBa$_2$Cu$_3$O$_{6+\delta}$ and shown to have excellent agreement with Fermi liquid theory.\textsuperscript{13} We compare the expected temperature dependence of the high field pseudogap state developed here with that of an admixed Fermi liquid/pseudogap scenario for the normal state.\textsuperscript{16} Throughout we presume $\Delta(T)$ is roughly

$$I(T) = I(0) e^{-\frac{T}{T_c}}$$
T-independent, since it depends on much larger temperature scales than those accessed by quantum oscillations.

The temperature dependence is computed by investigating the total energy of quasiparticles for a grand canonical system with fixed $\mu$:[35]

$$E(\mu, T) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_{k} (\omega + \xi_{k} + 2\mu) A(k, \omega)f(\omega)$$  \hspace{1cm} (2)$$

where $f(\omega) = (1 + \exp(\omega/T))^{-1}$ is the Fermi function. Because the oscillations are created by quasiparticles near the Fermi surface, $\omega$ and $\xi_{k}$ are both much less than $\mu$, and we can approximate $\omega + \xi_{k} + 2\mu \to 2\mu$.

Following earlier work,[34,37] we take the large Landau level limit and use the Poisson resummation formula to extract the fundamental frequency of oscillation. After

$$E(\mu, T) = -2\mu \int_{-\infty}^{\infty} dy \frac{df(y)}{dy} \int_{-\infty}^{y} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\xi^{0}}{\hbar\omega_{c}} \times \sum_{q} e^{2\pi i q_{0}/(\hbar\omega_{c})} A(\xi^{0} - \mu, q, \omega),$$  \hspace{1cm} (3)$$

where $\xi^{0} = \xi(n) + \mu$. In a Fermi liquid system, in which $A$ only depends on $\xi - \omega$, this becomes a convolution and gives a total amplitude equal to the Fourier transform of $|df(y)/dy|$ as in previous work.[14,15] (Note that throughout we take $A$ to be temperature independent.) For a general non-Fermi liquid system, the above amplitude will not be a convolution, and the temperature-dependent amplitude can instead be calculated using an analytic form of $A$:[35]

Results The calculated temperature dependence of this non-Fermi liquid with a large pseudogap is shown as the dashed line in Fig. 3a and gives excellent agreement with Fermi liquid theory (solid green line). In contrast,
the dotted curve with a much smaller $\Delta = h\omega_c$ displays large deviations from the Fermi liquid. In the cuprates $\Delta$ is much larger than $h\omega_c$. (For example, in Ref. 14 $h\omega_{c,\text{max}} = \hbar B_{\text{max}}/m^* = 3.8$ meV, whereas $\Delta$ can be tens of meV.\textsuperscript{5} This indicates that it can be possible for a pseudogapped system to “masquerade” as a Fermi liquid for quantum oscillation measurements in the cuprates.

The discrepancy in the energy scales of $\Delta$ (Ref. 35) and $h\omega_c$ is the primary cause of this similarity to Fermi liquid behavior. As shown by the dashed line in Fig. 3, because these scales are so different, the underlying density of states has very little curvature. This, as well as the damping and inhomogeneity, creates a flat density of states near the Fermi surface which in turn leads to Fermi liquid-like behavior in the oscillations. Specifically, Fig. 3, plots the non-oscillating integrand $N(\omega) = \int_{\infty}^{\omega} d\xi A(\xi - \mu, q, \omega)$, essentially a zero-field density of states. As quantum oscillations are primarily sensitive to the innermost Landau level to the Fermi surface, Eq. 3 is mostly sensitive to effects within $h\omega_c$ of the Fermi surface. Although systems with $\Delta = h\omega_c$ do display variations on this scale, in more physical situations as studied here they are negligible.

In order to more quantitatively determine the quality of fit, we calculate the moments of the Fourier transform of the amplitude, as in Ref. 14 which are displayed in Table I. In the table, Case (1) uses the large $\Delta = 10h\omega_c$ and $\gamma = 0.5\Delta$ considered throughout the paper, and shows excellent agreement with the theoretical Fermi liquid values. Case (2) provides an example of a non-Fermi liquid that does display large deviations from a Fermi liquid system, with a small $\Delta = h\omega_c$, as can clearly be seen in Fig. 3, as well. This provides evidence that this technique can be useful for the discrimination of non-Fermi liquids in some systems. Finally, in Case (3) we consider

| Moment $\mu_K$ | $\mu_6$ | $\mu_8$ | $\mu_{10}$ |
|----------------|---------|---------|------------|
| FL Theory     | 874.8   | $2.697 \times 10^4$ | $9.937 \times 10^3$ |
| Ref. 14       | 873     | $2.65 \times 10^4$  | $9.49 \times 10^3$  |
| (1) $\Delta = 10h\omega_c$ | 872.6  | $2.682 \times 10^4$ | $9.859 \times 10^3$ |
| (2) $\Delta = h\omega_c$ | 797.1  | $2.163 \times 10^4$ | $6.997 \times 10^3$ |
| (3) Ref. 16   | 875.0   | $2.698 \times 10^4$ | $9.948 \times 10^3$ |

TABLE I. A table of $z$-limited moments $\mu_K = \int_{-\gamma}^{\gamma} z^k |f(z)| dz$ for different cases. (1) and (2) use the parameters from the dashed black line and dotted red line of Fig. 2, respectively. (3) uses the spectral function in Ref. 16 which incorporates Landau-level based pairing, real-space inhomogeneity, and $d$-wave pairing symmetry. Our non-Fermi liquid scenario is to be contrasted with hybrid Fermi liquid-pseudogap approaches which view the “normal” high field phase as a vortex liquid,\textsuperscript{10,17} a concept about which there is not yet unanimity.\textsuperscript{22,29} Within our model, two major components are responsible for the robust oscillations. First, the nodal or near-nodal effects leading to small values of $\Delta(q)$ are critical in retaining the visibility of quantum oscillations in these systems. Second, the large discrepancy in energy scales between the gap\textsuperscript{22} and the cyclotron frequency makes the system appear relatively Fermi liquid-like on the scale of the oscillations.

The fact that existing experimental work has not distinguished between these scenarios may allow for a simple resolution between oscillatory measurements and the specific heat measurements which suggest the continued presence of a $d$-wave gap at high fields.\textsuperscript{35} We note that the temperature dependent formalism outlined in this paper may serve as a vehicle for testing future theories of these oscillatory phenomena. Additionally, our work has shown that an extremely high level of precision will be required in future experiments to distinguish among different theoretical scenarios of the oscillations.

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We take $\gamma/\Delta$, which would be zero in the superconducting state, to be as large as 0.5 based on estimates which use the Fermi arc size in zero-field. The choice of $\gamma'$ has little effect on the conclusions, but $\gamma'=0.2$ strikes a balance between computability (with no extremely sharp features, as opposed to large $\gamma'$) and the visibility of distinct oscillations (as opposed to large $\gamma'$). Parameters used for the model from Ref. 15 preserve that paper’s choice: $\Gamma/\Delta$ and $\gamma/\Delta$, while $\Delta$ itself is scaled to match the other.

For our calculation, we make one more transformation, by splitting up the $\omega$ integral in Eq. (3) (with $I$ the integrand): $\int_{-\infty}^{\infty} d\omega \, I(\omega) \to \int_{-\infty}^{0} d\omega \, I(\omega) + \int_{0}^{\infty} d\omega \, I(\omega)$. We know that as $T \to \infty$, the result of Eq. (3) must go to zero, and with $\int_{-\infty}^{\infty} dy \, (dy)/(dy) = 1$, we find that $\int_{-\infty}^{\infty} d\omega \, I(\omega) = -\lim_{T \to \infty} \int_{-\infty}^{\infty} dy \, (dy)/(dy) \int_{0}^{\infty} d\omega \, I(\omega)$. This latter formula is used to compute the constant part $\eta$ based on estimates which means we will not distinguish between their descriptions here.

Note that in $s$-wave gaps, nodal states are created solely by real-space inhomogeneity. However, in $d$-wave contributions to nodal states from pairing symmetry and Landau level-based real-space inhomogeneity are both present and become essentially inseparable, which means we will not distinguish between their descriptions here.

For calculation, we make one more transformation, by splitting up the $\omega$ integral in Eq. (3) (with $I$ the integrand): $\int_{-\infty}^{\infty} d\omega \, I(\omega) \to \int_{-\infty}^{0} d\omega \, I(\omega) + \int_{0}^{\infty} d\omega \, I(\omega)$. We know that as $T \to \infty$, the result of Eq. (3) must go to zero, and with $\int_{-\infty}^{\infty} dy \, (dy)/(dy) = 1$, we find that $\int_{-\infty}^{\infty} d\omega \, I(\omega) = -\lim_{T \to \infty} \int_{-\infty}^{\infty} dy \, (dy)/(dy) \int_{0}^{\infty} d\omega \, I(\omega)$. This latter formula is used to compute the constant part of the original integral, so that each individual calculation only requires computation of the $\int_{0}^{\infty} d\omega \, I(\omega)$ term.

Note that unlike Eq. (1) in Ref. [1] no $\eta_{\text{lim}}$ is used here.