On the Twisted $N = 2$ Superconformal Structure in 2d Gravity Coupled to Matter

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Abstract

It is shown that the two dimensional gravity, described either in the conformal gauge (the Liouville theory) or in the light cone gauge, when coupled to matter possesses an infinite number of twisted $N = 2$ superconformal symmetries. The central charges of the $N = 2$ algebra for the two gauge choices are in general different. Further, it is argued that the physical states in the light cone gauge theory can be obtained from the Liouville theory by a field redefinition.
In the last few years, two dimensional conformal field theories coupled to two dimensional gravity has been studied in great detail [1] in two distinct formulations. In one formulation, known as the matrix models, the string worldsheet is discretized into many triangles in a careful way and the summation over all possible triangulations is thus equivalent to the integral of the metric over all possible geometries [2]. In the other formulation, known as the continuum approach, the two dimensional metric is fixed by a suitable gauge and the quantization is performed subsequently. This allows a choice in fixing the gauge and generally one chooses either the conformal gauge [3] or the light cone gauge [4]. Though both the gauge choices gave equivalent results but it was realized that the conformal gauge is more suitable not only for computational reasons but also for further developments of the theory. In fact, some of the matrix model results are obtained directly in the continuum approach following the conformal gauge choice [5], where the conformal degree of freedom of the metric is taken as the Liouville field and thus the gravity sector is realized by the Liouville action. In a different development, it has been shown that the 2d topological gravity coupled to topological matter gives a field theoretic description of the matrix model formulation of 2d quantum gravity [6]. In fact, it is proven in ref.[7] that the matrix model formulation of 2d gravity, 2d topological field theories and the intersection theory of the moduli space of Riemann surfaces are all equivalent [8]. Although, one still needs to have a complete understanding of the matrix model results in terms of the continuum approach, yet it might be natural to expect a topological structure in the continuum approach. Such a structure is already revealed in the continuum approach [9], again with the conformal gauge choice of the metric. More recently, it has been shown that almost all string theories, including the bosonic string, the superstring and W-string theories, possess a topologically twisted $N = 2$ superconformal symmetry [10] which is a signal that there might be a connection between topological theories and the field theoretical approach of gravity coupled to matter system.

As mentioned above, in the continuum approach, 2d gravity can be treated in the light cone gauge where the the metric degrees of freedom are fixed by $h_{++} = h_{--} = 1/2$ and $h_{+-} = 0$ [4]. It was argued there that the renormalizability of the theory can be best understood in this gauge. A remarkable feature of this theory is the presence of an
unexpected $SL(2, R)$ current algebra which is responsible for the complete solvability of
the system. As a technical interest, it is desirable to examine if this formulation of 2d
gravity coupled to conformal matter also possesses the above topologically twisted $N = 2$
superconformal symmetry. In this letter, we show that this is indeed true. Furthermore,
we indicate briefly how the physical states in this theory are same as those in the conformal
gauge (including the discrete states) up to a redefinition of the fields.

Let us recapitulate briefly how the twisted $N = 2$ superconformal symmetry arises in
the conformal gauge gravity coupled to matter. The $(p, q)$ minimal models ($gcd(p, q) = 1$)
coupled to the Liouville field can be described in terms of Coulomb gas representation
where the energy momentum tensors for the matter and Liouville sector are given as

\[ T_M(z) = -\frac{1}{2} : \partial X \partial X : + iQ_M \partial^2 X \]
\[ T_L(z) = -\frac{1}{2} : \partial \phi \partial \phi : + iQ_L \partial^2 \phi \]

where $X$ and $\phi$ represent the matter and Liouville fields respectively, whereas $2Q_M$ and
$2Q_L$ denote the background charges. Since, the total central charge of the combined matter
and Liouville system should add up to 26 and the matter sector is characterized by the
Virasoro central charge $1 - \frac{6(p-q)^2}{pq}$, it, therefore, follows that

\[ 2Q_M = \sqrt{\frac{2p}{q}} - \sqrt{\frac{2q}{p}} = (\alpha_+ + \alpha_-) \]
\[ 2Q_L = \pm i(\sqrt{\frac{2p}{q}} + \sqrt{\frac{2q}{p}}) = (\beta_+ + \beta_-) \]

In the BRST quantization scheme, the BRST current for this system is :

\[ J_B(z) = : c(z) [T_M(z) + T_L(z) + \frac{1}{2} T^{bc}(z)] : \]

where $T^{bc}$ is the energy momentum tensor for the reparametrization ghost system, con-
sisting of ghost field $c(z)$ and the anti-ghost field $b(z)$ with conformal weight $-1$ and $2$
respectively and is given by

\[ T^{bc}(z) = -2 : b(z) \partial c(z) : - : \partial b(z) c(z) : \]
It has been noted before that the generators
\[ T(z) = T_M(z) + T_L(z) + T^{bc}(z) \]
\[ G^+(z) = J_B(z) \]
\[ G^-(z) = b(z) \]
\[ J(z) = :c(z)b(z): \]
satisfy an almost topological $N = 2$ superconformal algebra; however, with the above choice of the generators, the algebra does not close but produce two new fields $c(z)$ and $c\partial c(z)$ \[11\] which can be seen from the following OPEs
\[ G^+(z) G^+(w) \sim -10 \frac{c\partial c(w)}{(z-w)^3} - 5 \frac{\partial(c\partial c)(w)}{(z-w)^2} - \frac{3}{2} \frac{\partial^2(c\partial c)(w)}{(z-w)} \] \[ J(z) G^+(w) \sim \frac{G^+(w)}{(z-w)} - \frac{\partial c(w)}{(z-w)^2} + \frac{c(w)}{(z-w)^3} \]

It has been found in refs \[9,10\] that it is possible to modify the BRST current in (3) and the ghost number current $J(z)$ in (5) by adding total derivative terms (it does not affect the BRST charge) in such a way that the modified generators would form a closed $N = 2$ algebra. To be precise, taking the modified generators as
\[ G^+(z) = J_B(z) + a_1 \partial(c\partial\phi)(z) + a_2 \partial(c\partial X)(z) + a_3 \partial^2 c(z) \]
\[ J(z) = :c(z)b(z): + a_4 \partial\phi(z) + a_5 \partial X(z) \]
where $a_i$ ($i = 1, 2, 3, 4, 5$) are arbitrary parameters, we find the following twisted $N = 2$ superconformal algebra:
\[ T(z)T(w) \sim 2 \frac{T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)} \]
\[ T(z)G^\pm(w) \sim \frac{1}{2} \frac{3 \mp 1}{(z-w)^2} G^\pm(w) + \frac{\partial G^\pm(w)}{(z-w)} \]
\[ T(z)J(w) \sim -\frac{1}{3} c^{N=2} \frac{c^{N=2}}{(z-w)^3} + \frac{J(w)}{(z-w)^2} + \frac{\partial J(w)}{(z-w)} \]
\[ J(z)J(w) \sim \frac{1}{3} c^{N=2} \frac{c^{N=2}}{(z-w)^2} \]
\[ J(z)G^\pm(w) \sim \pm \frac{G^\pm(w)}{(z-w)} \]
\[ G^+(z)G^-(w) \sim \frac{1}{3} c^{N=2} + \frac{J(w)}{(z-w)^2} + \frac{T(w)}{(z-w)} \]
\[ G^\pm(z)G^\pm(w) \sim 0 \]
provided the $a_i$’s satisfy the following conditions:

$$
\begin{align*}
    a_1 + a_4 &= 0 \\
    a_2 + a_5 &= 0 \\
    a_1^2 + a_2^2 + 2a_3 - 1 &= 0 \\
    2iQ_M a_2 + 2iQ_L a_1 - 2a_3 + 3 &= 0
\end{align*}
$$

(10)

and where $c^{N=2} = 6a_3$ is the central charge of the corresponding untwisted $N = 2$ superconformal algebra. Note that here we have three unknown parameters but only two independent conditions governing them. Thus eqn (10) can be satisfied in many ways and consequently we have infinite number of $N = 2$ algebras (i.e. with different central charges) as the underlying symmetry of this theory. In ref.[10], a particular solution to eqn (10) i.e. $a_2 = 0$ is chosen so that (taking the ‘$-$’ branch for $Q_L$ in eqn (2)) we have $a_1 = -\sqrt{\frac{2q}{p}}$ and $c^{N=2} = 6a_3 = 3(1 - \frac{2q}{p})$. However, it was pointed out in ref.[12] that there is an ambiguity in choosing the current $\partial \phi$ for deforming the generators $G^+$ and $J$ because of the fact that when the cosmological constant in the Liouville action is taken to be non-zero, the Liouville equation of motion implies that $\partial \phi$ can not be considered as holomorphic current any more. This situation, of course, will correspond to the case of putting $a_1 = 0$ and we will be left with only two $N = 2$ superconformal algebras (by the interchange of $p$ and $q$ every where in the above discussion). However, we will not face such a problem if we choose the light cone gauge instead of conformal gauge as we will see below.

As shown in [4], in the light cone gauge, the non-zero component of the metric $h_{++}$ admits a decomposition in terms of the three generators of the non-compact group $SL(2, R)$ which satisfy the following current algebra:

$$
\begin{align*}
    j^a(z) j^b(w) &\sim \frac{f^{ab}_{c(d)} j^c(w)}{(z-w)} + \frac{k \eta^{ab}}{(z-w)^2}
\end{align*}
$$

(11)

where $a, b = 0, \pm$ are $SL(2, R)$ indices, $k$ is the level of the current algebra, $\eta$ is the Killing metric with non-zero components $\eta^{+-} = -2\eta^{00} = 2$ and the non-zero structure constants are given as $f^{0+}_+ = -f^{0-}_- = -\frac{1}{2} f^{0+}_0 = -1$. The residual gauge invariance is generated by current $j^+$ and the energy momentum tensor $T_G$. The latter is given by
the modified Sugawara form

\[ T_G(z) = \frac{1}{k-2} : \eta_{ab} j^a(z)j^b(z) : - \partial j^0(z) \]  

(12)

and the associated Virasoro central charge is \( \frac{3k}{k-2} + 6k \). With respect to this energy-momentum tensor, the currents \( j^+ \), \( j^0 \) and \( j^- \) have conformal dimension 0, 1 and 2 respectively which can be seen from the following operator product expansions:

\[ T_G(z) j^+(w) \sim \frac{\partial j^+(w)}{(z-w)} \]
\[ T_G(z) j^0(w) \sim \frac{-k}{(z-w)^3} + \frac{j^0(w)}{(z-w)^2} + \frac{\partial j^0(w)}{(z-w)} \]  
\[ T_G(z) j^-(w) \sim \frac{2j^-(w)}{(z-w)^2} + \frac{\partial j^-(w)}{(z-w)} \]  

(13)

Including the matter coupling, the total energy-momentum tensor in this case is

\[ T(z) = T_G(z) + T_M(z) + T^{bc}(z) + \partial \zeta \epsilon(z) : \]  

(14)

where the extra ghost system \( (\zeta, \epsilon) \) having conformal weights (0,1) and ghost-number \((-1,1)\) is the consequence of the symmetry associated with the generator \( j^+ \). This extra ghost system has Virasoro central charge \(-2\). Thus, taking the matter system again as the \((p,q)\) minimal conformal matter, the central charge balance equation, now reads as

\[ \frac{3k}{k-2} + 6k + 1 - \frac{6(p-q)^2}{pq} - 26 - 2 = 0 \]  

(15)

which admits two solutions for \( k \) i.e. either \( k = \frac{p}{q} + 2 \) or \( k = \frac{q}{p} + 2 \). We expect that at these values of the level of the current algebra, the combined matter and gravity theory to possess twisted \( N = 2 \) superconformal algebra and we will see below that this is indeed true.

The BRST current for this system is given by [13]

\[ J_B(z) = : c(z) [T_G(z) + T_M(z) + \frac{1}{2} T^{bc}(z) + T^{\zeta \epsilon}(z)] : + : \epsilon(z) j^+(z) : \]  

(16)

with \( T^{\zeta \epsilon}(z) =: (\partial \zeta)\epsilon(z) : \) and \( T_G(z), T_M(z) \) as given in (12) and (1) respectively. However, as in the case of the conformal gauge, this algebra also does not close. Not only the fields
but also a new field \((c\epsilon j^+)\) is generated in the algebra which can be seen from the following OPE:

\[
J_B(z) J_B(w) \sim -10c\partial c(w) (z-w)^3 - 5\partial (c\partial c)(w) (z-w)^2 - \frac{3}{2} \partial^2 (c\partial c)(w) (z-w) + \frac{\partial (c\epsilon j^+)(w)}{(z-w)}
\] (17)

Nevertheless, as in the previous case, we can define the \(N=2\) generators by modifying the BRST current as well as the ghost number current to obtain a closed algebra which is again a topologically twisted \(N=2\) superconformal algebra. To be definite, we take \(T(z)\) as in (14), \(G^-(z) = b(z)\) and

\[
G^+(z) = J_B(z) + A_1 \partial (c\zeta \epsilon)(z) + A_2 \partial^2 c(z) + A_3 \partial(cj^0)(z) + A_4 \partial(c\partial X)(z)
\]

\[
J(z) = :c(z)b(z): + A_5 :\epsilon(z)\zeta(z): + A_6 j^0(z) + A_7 \partial X(z)
\] (18)

where \(J_B\) is as given in (16). We find that these generators satisfy the topologically twisted \(N=2\) superconformal algebra exactly as given in (9) with \(c^{N=2} = 6A_2\) provided the \(A_i\)'s \((i = 1, 2, \ldots, 7)\) obey the following relations:

\[
\begin{align*}
A_1 - A_5 &= 0 \\
A_3 + A_6 &= 0 \\
A_4 + A_7 &= 0 \\
A_1 + A_3 - 1 &= 0 \\
A_1 + 2A_2 + kA_3 - 2iQ_MA_4 - 3 &= 0 \\
2A_1^2 + 4A_1 + 4A_2 + kA_3(4 - A_3) - 2A_4(A_4 + 4iQ_M) - 10 &= 0
\end{align*}
\] (19)

Again we notice that there are three independent unknown parameters but two relations governing them. We can fix two of them in terms of the third one as follows:

\[
\begin{align*}
A_1 &= \frac{1}{k-2} \left[ 1 \pm \sqrt{(k-3)^2 - 2A_4(k-2)(A_4 + 2iQ_M)} \right] \\
A_2 &= 1 + iQ_MA_4 - \frac{k-1}{2(k-2)} \left[ (k-3) \pm \sqrt{(k-3)^2 - 2A_4(k-2)(A_4 + 2iQ_M)} \right]
\end{align*}
\] (20)

Thus for different values of \(A_4\) we have a topologically twisted \(N=2\) superconformal algebra with different central charges given by \(6A_2\). In particular, restricting to \(A_4 = 0\)
and substituting for $k$ in terms of $p$ and $q$, as found earlier, we obtain that the central charge of the corresponding $N = 2$ theory is given by

$$c^{N=2} = 6\left(\frac{p}{q} - \frac{q}{p} + 1\right)$$

or 6, which is a particular case $p = q$ of the above and corresponds to the case of $c_M = 1$ coupled to gravity, described in the light cone gauge. Comparing to the corresponding expression for $c^{N=2}$ in the conformal gauge, which is $c^{N=2} = 3\left(1 - \frac{2q}{p}\right)$, we observe that the underlying $N = 2$ theory have different central charge for the two different gauge choices of the metric. In fact, this is true for the generic case also. This in turn implies that unless we can establish an automorphism under which the generators of the $N = 2$ algebra, in these two gauges, have one to one correspondence and the central charge is same for both the cases, it may be ambiguous to determine the physical state spectrum of the matter coupled to gravity theory by relying on the $N = 2$ symmetry alone. We defer further discussion on this issue for a later occasion and proceed to show how the physical states, including the discrete states, of this theory corresponding to the above two gauge choices are related by a field redefinition.

We recall that the physical states are obtained by studying the cohomology classes of the BRST charge, which we denote for the two cases as $Q_{B}^{Conf,LC}$, defined as

$$Q_{B}^{Conf,LC} = \int dz J_{B}^{Conf,LC}(z)$$

where $J_{B}^{Conf}, J_{B}^{LC}$ are the BRST currents and defined in (3) and (16) respectively. The physical states of the theory are the states which are in the kernel of the BRST charge modulo its image. The analysis of the physical states for standard ghost number states have been done long ago both for conformal gauge [3] and for the light cone gauge [13]. However, in recent years, the discovery of discrete states associated with different ghost numbers [14] has drawn much attention. By a simple argument, we will see that there is a one to one correspondence among the elements of the physical state spaces of the above two cases, as it should be. For this purpose, we consider the free field realization of the $SL(2, R)$ current algebra for the level $k$, known as Wakimoto construction [15] and write
the $SL(2,R)$ currents as

\[ j^+(z) = \beta(z) \]
\[ j^0(z) = :\beta(z)\gamma(z) : + \sqrt{\frac{k-2}{2}} \partial \phi(z) \]
\[ j^-(z) = :\beta(z)\gamma^2(z) : + 2\sqrt{\frac{k-2}{2}} \gamma(z) \partial \phi(z) + k \partial \gamma(z) \]

where $\phi, \beta, \gamma$ are free bosonic fields with OPE $\beta(z)\gamma(w) \sim (z-w)^{-1}$, $\phi(z)\phi(w) \sim -\log(z-w)$. In terms of these free fields, the gravitational energy momentum tensor as in (12) takes the form

\[ T_G(z) = - :\partial \beta(z)\gamma(z) : - \frac{1}{2} :\partial \phi(z)\partial \phi(z) : + iQ_L \partial^2 \phi(z) \]

where $Q_L$ is as given in (2). Note that $\beta$ and $\gamma$ have conformal dimensions 0 and 1 respectively with respect to this $T_G$. It is now clear that if we identify the Wakimoto field $\phi$ as the free Liouville field, then we have

\[ T_G(z) = T^{\beta\gamma}(z) + T_L(z) \]

where $T^{\beta\gamma}(z) = - : (\partial \beta(z))\gamma(z) :$. The BRST current then reads as

\[ J_B^{LC}(z) = J_B^{Conf} + :c(z)[T^{\beta\gamma}(z) + T^{\varepsilon\zeta}(z)] : + :\varepsilon(z)\beta(z) : \]

Thus we have

\[ Q_B^{LC} = Q_B^{Conf} + Q_B^{(1)} + : [Q_B^{(1)} , \int dz \varepsilon(z)\partial \zeta(z)\gamma(z)] : \]

where $Q_B^{(1)} = \int dz \varepsilon(z)\beta(z)$. Note that both $Q_B^{Conf}$ and $Q_B^{(1)}$ are independently nilpotent and anticommute with each other. Now from the knowledge of the physical state space which are in the cohomology class of $Q_B^{Conf}$, we can derive the states in the cohomology class of $Q_B^{LC}$ as follows. Let $|\psi >^{Conf}$ be a physical state in the conformal gauge i.e.

\[ Q_B^{Conf} |\psi >^{Conf} = 0 \]

The solution to this equation can be written symbolically as

\[ |\psi >^{Conf} = \mathcal{P} (\partial X, \partial \phi, b, c) V(X) V(\phi) |0 > \]
where $P$ is a differential polynomial, $V$’s are vertex operators and $|0\rangle$ is the $SL(2,C)$ vacuum. Introducing an unitary operator $U$ as

$$U = e^{-\int dz c(z) \gamma(z) \partial \zeta(z)}$$  

(28)

we find a relation between the BRST charges in the conformal and the light cone gauges as follows,

$$Q_{LC}^{B} = U \left[ Q_{B}^{Conf} + \int dz \epsilon(z) \beta(z) \right] U^{-1}$$  

(29)

The physical states in the light cone gauge will be rotated accordingly,

$$|\psi>_{LC} = U |\psi>_{Conf}$$  

(30)

The effect of $U$ on $|\psi>_{Conf}$ is just to shift the field $b$ by $b + \gamma \partial \zeta$. To be explicit, let us recall [16] that the physical state spectrum of the 2d gravity in the conformal gauge coupled to $(p,q)$ minimal matter are generated by three types of operators $x$, $y$ and $w(w^{-1})$. Here $x$, $y$ are the spin zero, ghost number zero operators and are called the ground ring generators, whereas $w(w^{-1})$ has ghost number $-1(+1)$. Any physical operator at ghost number $-n$ can be written as

$$O_{n,i,j} = w^n x^i y^j$$  

(31)

where $i$, $j$ are integers with the restriction $0 \leq i \leq p - 2$, $0 \leq j \leq q - 2$. The ground ring generators in the light cone gauge take the form

$$x = [bc + \gamma \partial \zeta c + \frac{3}{4} \sqrt{\frac{2q}{p}} (i \partial X + \partial \phi)] e^{i \alpha_{1,2} X + i \beta_{1,2} \phi}$$  

$$y = [bc + \gamma \partial \zeta c - \frac{3}{4} \sqrt{\frac{2p}{q}} (i \partial X - \partial \phi)] e^{i \alpha_{2,1} X + i \beta_{2,1} \phi}$$  

(32)

where $\alpha_{m,m'} = \frac{1}{2} [(1-m)\alpha_+ + (1-m')\alpha_-]$ and $\beta_{n,n'} = \frac{1}{2} [(1-n)\beta_+ + (1-n')\beta_-]$. The other generators $w$, $w^{-1}$ with ghost number $-1$, $+1$ in general would have the form

$$w = P(\partial X, \partial \phi, b + \gamma \partial \zeta, c) e^{i \alpha_{q-1,1} X + i \beta_{1,p+1} \phi}$$  

$$w^{-1} = ce^{i \alpha_{q-1,1} X + i \beta_{-q+1,1} \phi}$$  

(33a)

or

$$w = P(\partial X, \partial \phi, b + \gamma \partial \zeta, c) e^{i \alpha_{1,p-1} X + i \beta_{q+1,1} \phi}$$  

$$w^{-1} = ce^{i \alpha_{1,p-1} X + i \beta_{1,-p+1} \phi}$$  

(33b)
where $P$ is a differential polynomial of conformal weight $(p + q - 1)$ and ghost number $-1$. We have two sets of $w(w^{-1})$ because we note that since $w \cdot w^{-1} \sim I$ their multiplication is well defined if we take $w$ from (33a) and $w^{-1}$ from (33b) or vice versa. Since, for general $(p, q)$ model the form of $w$ is quite complicated we give its form for $(3, 2)$ model which is pure Liouville gravity

$$
\begin{align*}
  w &= \left( \frac{1}{2} \partial^2 b + \frac{1}{2} \partial^2 \gamma \partial \zeta + \partial \gamma \partial^2 \zeta + \frac{1}{2} \gamma \partial^3 \zeta - 3 \partial bb \gamma + 3 \partial bc \gamma \partial \zeta \right. \\
  &\quad - 3 b c \partial \gamma \partial \zeta - 3 b c \gamma \partial^2 \zeta + 3 c \gamma^2 \partial \zeta \partial^2 \zeta - \frac{\sqrt{3}}{2} \partial b \partial \phi - \frac{\sqrt{3}}{2} \partial \gamma \partial \zeta \partial \phi \\
  &\left. - \frac{\sqrt{3}}{2} \gamma \partial^2 \zeta \partial \phi + \sqrt{3} b \partial^2 \phi + \sqrt{3} \gamma \partial \zeta \partial^2 \phi \right) e^{\sqrt{3} \phi} 
\end{align*}
$$

(34)

It is now a simple exercise to check that the operators in (32) and (34) belong to the relative cohomology of the full BRST charge given in (25). The Physical states for $c_M = 1$ matter coupled to 2d light cone gauge gravity have been discussed in a recent paper [17]. Following closely the analysis of ref.[18], it has been found there, that the oscillator part of the physical operators gets a shift $b + \gamma \partial \zeta$ in place of $b$. This is precisely the field redefinition we obtain for $c_M < 1$ comparing the BRST charges in two different gauges.

To conclude, we have shown here that the two dimensional gravity, described by either the Liouville theory or in the light cone gauge, when coupled to conformal matter, possesses an infinite number of topologically twisted $N = 2$ superconformal symmetries. The topological central charge for the two gauge choices are found not to be the same. This indicates that the analysis of physical states by using the underlying $N = 2$ symmetry may not be unique. By performing a rotation we also argued that the physical states of the 2d light cone gauge gravity are same as the physical states found in the conformal gauge after a shift of the anti ghost field. Doing this rotation in the opposite way, we notice that the physical states and the BRST operators for the two cases are same if we ignore the $\beta, \gamma, \epsilon, \zeta$ degrees of freedom. Though there is no a priori reason, but if we use this information in the analysis of $N = 2$ algebra (with Wakimoto realization), we observe that the generators and the central charge for both the gauge choices are in one to one correspondence with the Wakimoto field $\phi$ being identified as the Liouville field.
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