Infrared supercontinuum generation in multiple quantum well nanostructures

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Abstract
The paper presents a theoretical study of broadband mid-infrared supercontinuum generation at low power in semiconductor multiple quantum wells (MQWs) facilitated by electromagnetically induced transparency. Pulses of 200 W peak power and 700 fs duration at 9.963 μm have been used to study the supercontinuum generation dynamics in a 1.374 μm long MQW system. The supercontinuum spectrum is 13.0 μm broad and asymmetric about the pump wavelength. Although the spectral broadening is dominated by self-phase modulation, four-wave mixing, modulation instability and soliton generation also contribute to the broadening.

Keywords: electromagnetically induced transparency, multiple quantum wells, self-phase modulation, supercontinuum generation

(Some figures may appear in colour only in the online journal)

1. Introduction
Recently optical supercontinuum (SC) generation has drawn considerable attention [1–10], both theoretically and experimentally, due to its important applications in optical coherence tomography [5], optical metrology [6], spectroscopy, optical frequency comb generation [7, 8] and wavelength division multiplexing [9]. Since its discovery [1], SC generation has been studied in different nonlinear media including optical fibers [2–8], silicon photonic nanowires [10], chalcogenide waveguides [11, 12] and silica waveguides [13, 14]. Although SC generation has been experimentally achieved in different media, photonic crystal fibers (PCFs) have emerged as the most popular nonlinear media for SC generation due to the feasibility of low dispersion and large nonlinearity [2, 3]. The discovery of non-silica PCFs, characterized by large optical nonlinearity, has further enhanced their popularity as nonlinear media for successful SC generation [15, 16].

SC generation is characterized by dramatic spectral broadening of an optical field that occurs when an intense narrowband light pulse propagates through a nonlinear medium [2–4]. SC generation is the result of the interplay between linear and nonlinear processes. The nonlinear optical processes that contribute to the SC generation are self-phase modulation, cross-phase modulation, modulation instability, soliton fission, Raman scattering, dispersive wave generation, four-wave mixing, self-steepening [2–4] etc. The group velocity dispersion profile and specifically the location of the pumping wavelength against the zero dispersion wavelength determines the set of nonlinear processes responsible for the SC generation. The dispersion profile has the strongest influence on the shape of the SC and the gain bandwidth is influenced by the nonlinear response. The coherence of the generated SC is governed mainly and firstly by the pump pulse duration, the input (pump) soliton order and then later by the dispersion profile. High power input pulses are generally launched near the zero dispersion wavelength in a highly nonlinear fiber to generate broadband SC. Very low dispersion ensures phase matching for four-wave mixing and generation of dispersive waves. Thus, for broadband flat SC generation a large effective nonlinearity and low uniform dispersion profile are essential. Therefore, identification of appropriate highly nonlinear material is a central issue to the SC generation at low power. Since in comparison to PCFs, giant optical nonlinearity could be experienced in semiconductor quantum wells (QWs) under appropriate experimental configuration, it may be possible to achieve SC generation in QWs at a much lower power level. Moreover,
they can be easily integrated with other semiconductor devices.

In recent years, quantum coherence and interference effects in semiconductor nanostructures, particularly in QWs, have received tremendous attention due to the widespread use of semiconductor components in optical computing, optical communication and quantum information processing. Quantum coherent phenomena such as gain without inversion (GW), coherent population oscillations, electromagnetically induced transparency (EIT) and slow light propagation have been explored both theoretically and experimentally in QWs [17–22]. The implementation of EIT in semiconductor-based devices is very attractive from the viewpoint of applications. Devices based on intersubband transitions in semiconductor QW structures have many inherent advantages over other systems. For example, large electric dipole moments due to the small effective electron mass in a semiconductor QW gives rise to giant optical nonlinearity. Moreover, a semiconductor QW provides a great flexibility in device design since transition energies, dipole moments and symmetries in it can be engineered as desired by choosing appropriate structure dimensions and materials. Based on quantum coherence and interference effects, Kerr nonlinearity can be enhanced enormously, while linear absorption and even two-photon absorption can be suppressed. Particularly, facilitated by EIT larger optical nonlinearity can be engineered in QWs in comparison to PCFs and other nonlinear media, thus, it is prudent to examine the SC generation in semiconductor QWs. Hence, the main thrust of this communication is to theoretically examine the possibility of generation of SC in quantum well nanostructures utilizing giant optical nonlinearity created under EIT. Therefore, in the present communication, we first identify large Kerr nonlinearity in a ladder type three level semiconductor MQW system driven by a pump laser pulse and controlled by an additional coupling field. In the second step, we utilize this EIT enhanced large nonlinearity to study the SC generation of the pump pulse at low peak power level.

The organization of the paper is as follows: section 2 presents the theoretical model of a three-level MQW interacting with a pump and a control field. The derivation of susceptibilities is also covered in this section. In section 3, the properties of linear and nonlinear susceptibilities of the MQW system are examined. The dynamics of supercontinuum generation under EIT are discussed in section 4 and finally, a summary is added in section 5.

2. Theoretical model and equations

We consider a MQW structure with three energy levels as shown schematically in figure 1(a). This system was first investigated both experimentally as well as theoretically by Dynes et al in 2005 [23]. The MQW is grown by molecular beam epitaxy method, with the lattice matched to an undoped InP substrate. It consists of 30 coupled well periods each with a 4.8 nm In0.47Ga0.53As/0.2 nm Al0.48In0.52As/4.8 nm In0.47Ga0.53As coupled quantum well separated by modulation doped 36 nm Al0.48In0.52As barriers. In this MQW, a weak pump pulse with angular frequency ωp and amplitude $E_p(z, t)$ couples the transition between the states $|1\rangle$ and $|2\rangle$, simultaneously a strong continuous wave (CW) control laser beam with angular frequency $\omega_c$ and amplitude $E_c$ couples the transition between the states $|2\rangle$ and $|3\rangle$, that forms a ladder-type excitation scheme, as shown schematically in figure 1(b).

The total electric field of the pump pulse and control beam can be written as

$$\hat{E} = \hat{e}_p E_p(z, t) \exp\{i(k_p z - \omega_p t)\} + \hat{e}_c E_c \exp\{i(k_c z - \omega_c t)\} + c.c.,$$

where $\hat{e}_p$ and $\hat{e}_c$ are the unit vectors along the polarization directions of the pump and control field, respectively; $k_p$ and $k_c$ respectively are the wave number of the pump and control fields.

The Hamiltonian of the system can be written as

$$\hat{H} = \hat{H}_0 + \hat{H}',$$

where $\hat{H}_0$ describes the free Hamiltonian in the absence of any external field and $\hat{H}'$ describes the perturbed Hamiltonian due to the interaction between MQW and the fields. In the Schrödinger picture, with rotating wave approximation, these two parts of the Hamiltonian can be written as,

$$\hat{H}_0 = \sum_{i=1}^{3} E_i |i\rangle \langle i|,$n

$$\hat{H}' = -\hbar \{\Omega_p e^{i(k_p - \omega_p t)} |2\rangle \langle 1| + \Omega_c e^{i(k_c - \omega_c t)} \times |3\rangle \langle 2| + h.c., \}$$

where $\Omega_p$ and $\Omega_c$ are the half Rabi frequencies of the pump and control field, respectively, which are defined as $\Omega_p = \frac{\langle \hat{p}_z \rangle \langle \hat{e}_p^* \rangle}{\hbar}$ and $\Omega_c = \frac{\langle \hat{p}_z \rangle \langle \hat{e}_c^* \rangle}{\hbar}$, while $\hat{p}_z = \epsilon \langle j | z | k \rangle$ is the dipole matrix element for the transition $|j\rangle \rightarrow |k\rangle$. To analyze the light–matter interaction process in the MQW, we adopt the density matrix formalism in which the evolution of density operator $\rho$ of the system is governed by Liouville’s equation which with the decay mechanism can be written as [20–24],

$$\frac{\partial}{\partial t} \rho_{jk} = \frac{1}{\hbar} \sum_m (\hat{H}_{jm} \rho_{mk} - \rho_{jm} \hat{H}_{mk}) - \frac{1}{2} \sum_m (\Gamma_{jm} \rho_{mk} - \rho_{jm} \Gamma_{mk}),$$

where $\rho_{jk}$ is the $j$, $k$th matrix element. The first term of the right hand side of equation (4) is due to the coherent evolution and second term is due to the decay process within the system. The relaxation matrix $\Gamma$ is related to the decay rates of the energy levels, which are defined by $\Gamma_{jk} = \langle j \Gamma | k \rangle = \gamma_{jk}$. The Hamiltonian ($\hat{H}$) in above density matrix equation contains the Rabi frequency $\Omega_p$ of the pump pulse that has been expressed in terms of the pump field $E_p(z, t)$, whose explicit form will contain a phase term that will carry all phase information about the pump pulse including self-phase modulation. The explicit form of the pump pulse in the normalized form shall be expressed subsequently in section 4, where we will discuss self-phase modulation and supercontinuum generation. By virtue of standard procedure [20, 21, 23], we get the following
\[ \begin{align*}
\dot{\rho}_{11} &= i\Omega_{\gamma} \rho_{21} - i\Omega_{\gamma} \rho_{12}, \\
\dot{\rho}_{22} &= -\gamma_2 \rho_{22} + i\Omega_{\gamma} \rho_{12} + i\Omega_{\gamma}^* \rho_{32} - i\Omega_{\gamma}^* \rho_{21} - i\Omega_{\gamma} \rho_{23}, \\
\dot{\rho}_{33} &= -\gamma_3 \rho_{33} - i\Omega_{\gamma} \rho_{23} + i\Omega_{\gamma}^* \rho_{32}, \\
\dot{\rho}_{21} &= i(\Delta_p + \frac{\gamma_2}{2}) \rho_{21} + i\Omega_{\gamma} \rho_{11} - \rho_{22} + i\Omega_{\gamma}^* \rho_{31}, \\
\dot{\rho}_{32} &= i(\Delta_e + \frac{\gamma_3}{2}) \rho_{32} + i\Omega_{\gamma} \rho_{22} - \rho_{33} - i\Omega_{\gamma}^* \rho_{31}, \\
\dot{\rho}_{31} &= i(\Delta_p + \Delta_e + \frac{\gamma_3}{2}) \rho_{31} + i\Omega_{\gamma} \rho_{21} - \rho_{32}.
\end{align*} \]

where \( \gamma_j \) (\( j = 2, 3 \)) are the population decay rates that are dominated by the inelastic emission of longitudinal optical (LO) phonons. The \( \gamma_{jk} \) represent the total coherence relaxation rates given by \( \gamma_{21} = \gamma_2 + \gamma_{21}^{\text{ph}}, \gamma_{31} = \gamma_3 + \gamma_{31}^{\text{ph}} \) and \( \gamma_{32} = \gamma_2 + \gamma_{32}^{\text{ph}} + \gamma_{32}^{\text{el}} \), where \( \gamma_{jk}^{\text{ph}} \) comprises the sum of quasi-elastic acoustic phonon scattering and the elastic interface Raman scattering. The detunings \( \Delta_p \) and \( \Delta_e \) are defined as \( \Delta_p = \omega_p - \omega_{21} \) and \( \Delta_e = \omega_e - \omega_{32} \), where \( \omega_{kj} \) are the angular frequencies of the resonant transition between states \( |j\rangle = |k\rangle \). Density matrix equations (5.1) to (6.6) are supplemented by the population conservation condition, \( \rho_{jk} \sum_{j=1}^3 = 1 \). We now assume that all the electrons are initially in the ground state \( |1\rangle \), i.e., at \( t = 0 \), \( \rho_{11} \approx 1 \) while \( \rho_{22} \approx \rho_{33} \approx 0 \). Notice that the pump field is much weaker than the control field (i.e., \( \Omega_p \ll \Omega_{\gamma} \)), therefore, due to quantum coherence and interference effects the ground state is not depleted too much. We introduce the perturbation expansion \( \rho_{jk} = \sum_{m=0}^k \rho_{jk}^{(m)} \), where \( \rho_{jk}^{(m)} \) is the \( m \)-th order of \( \rho_{jk} \). Within the above perturbation, it can be shown that \( \rho_{jk}^{(0)} = 0 \) (\( j = k \)), while \( \rho_{jk}^{(1)} = \rho_{jk}^{(2)} = 0 \). Taking the Fourier transform of equations (5.4) and (6.5), we get

\[ \begin{align*}
(\omega + \Delta_p + \frac{i\gamma_2}{2}) \beta_{21}^{(1)} + \Lambda_p + \Omega_{\gamma} \beta_{31}^{(1)} &= 0, \\
(\omega + \Delta_p + \Delta_e + \frac{i\gamma_3}{2}) \beta_{31}^{(1)} + \Omega_e \beta_{21}^{(1)} &= 0.
\end{align*} \]

where \( \beta_{jk}^{(1)} \) and \( \Lambda_p \) are the Fourier transform of \( \rho_{jk}^{(1)} \) and \( \Omega_p \), respectively, and \( \omega \) is the Fourier transform variable. The evolution of the pump field is governed by Maxwell’s wave equation

\[ \nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2 (P)}{\partial t^2}, \]

with polarization \( P = N (d) \), where \( N \) is the electron density in the quantum well, \( d \) is the dipole moment operator, \( c \) is the velocity of light in vacuum and \( \varepsilon_0 \) is the vacuum permittivity. Under slowly varying envelope approximation, the pump laser field obeys

\[ \frac{\partial \alpha_{21}}{\partial t} + \frac{1}{\varepsilon_0} \frac{\partial \alpha_{31}}{\partial t} = \frac{i\alpha_{11}}{2\varepsilon_0} \beta_{21}^{(1)} \]

Equation (8), after some simple algebra, reduces to

\[ \frac{\partial \alpha_{21}}{\partial t} + \frac{1}{\varepsilon_0} \frac{\partial \alpha_{31}}{\partial t} = \kappa \beta_{21}^{(1)} - \frac{i\omega}{2\varepsilon_0} \chi_p, \]

where \( \kappa = N \frac{\Omega_p}{2\varepsilon_0 \gamma_{21}^{\text{ph}}} \) and \( \chi_p = N \frac{\Omega_p}{2\varepsilon_0 \gamma_{21}^{\text{ph}}} \beta_{21}^{(1)} \). In view of the adiabatic condition described earlier, we use the following relations \( \beta_{21} \approx \beta_{21}^{(1)} \approx 1 \), \( \beta_{21} \approx \beta_{21}^{(1)} \approx 0 \), \( \beta_{31} \approx \beta_{31}^{(1)} \approx 0 \) and the normalisation condition \( \sum_{j=1}^3 |\beta_{1j}|^2 = 1 \) to reduce equation (9) in the following form

\[ \frac{\partial \alpha_{21}}{\partial t} + \frac{1}{\varepsilon_0} \frac{\partial \alpha_{31}}{\partial t} = \kappa \beta_{21}^{(1)} - \frac{i\omega}{2\varepsilon_0} (|\beta_{21}^{(1)}|^2 + |\beta_{31}^{(1)}|^2) \beta_{21}^{(1)}. \]

To this end we solve the equations (6.1) and (6.2) to get

\[ \beta_{21}^{(1)} = \frac{\Lambda_p}{\Delta_p - \Delta_e + i\frac{\gamma_3}{2}} \text{ and } \beta_{31}^{(1)} = \frac{\Omega_p}{\Delta_p - \Delta_e + i\frac{\gamma_3}{2}} \]

where \( \Lambda_p = \frac{\partial D_p}{\partial \omega_p} \) and \( \Omega_p = \frac{\partial D_p}{\partial \omega_p} \). The induced polarization \( P \) at the pump frequency \( \omega_p \) can be written as \( P(\omega_p) = \varepsilon_0 \chi_p (\omega_p, r) \). The susceptibility \( \chi_p \) can be expressed as a sum of linear and nonlinear terms, i.e., \( \chi_p = \chi(1) + \chi^{(3)} |E_p|^2 \), where we have retained only up to the third-order term and neglected higher-order terms since they are usually small. Thus, we obtain the following

**Figure 1.** (a) Schematic of the band structure of a single period of the multiple quantum well. Each InGaAs/AlInAs wells has thickness of 9.8 nm that is covered by 36 nm AlInAs barriers. (b) Energy level diagram of the quantum well. Arrows represent the ladder type excitation scheme. 

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expressions for the first- and third-order susceptibilities:

\[ \chi^{(1)} = -\frac{N(\mu_{12}^2 E_2(t_0))}{\hbar \epsilon_0}, \]  
\[ \chi^{(3)} = \frac{N(\mu_{12}^4 E_2(t_0) \partial E_2(t_0))}{\hbar^3 \epsilon_0}. \]

Applying the Fourier transform on the linear version of equation (10), we get

\[ \frac{\partial \Lambda_p}{\partial \xi} - i \frac{\omega}{c} \Lambda_p - \kappa \frac{D_0(\omega)}{D(\omega)} \Lambda_p = \frac{\partial \Lambda_p}{\partial \xi} - i \beta \Lambda_p = 0. \]

To solve equation (12), we use the following ansatz for \( \Lambda_p \):

\[ \Lambda_p(\omega, \omega) = \Lambda_p(0, \omega) \exp[i \beta(\omega) z], \]

where \( \beta(\omega) = \frac{\omega}{c} - \kappa \frac{D_0(\omega)}{D(\omega)} \). The propagation constant \( \beta(\omega) \) can be expanded in the Taylor series around the central frequency of the pump field (\( \omega = 0 \)) as

\[ \beta(\omega) = \beta(0) + \beta_1(0) \omega + \frac{1}{2} \beta_2(0) \omega^2 + \frac{1}{6} \beta_3(0) \omega^3 + \ldots, \]

where \( \beta_n(0) = \frac{\partial^{n+1} \beta(\omega)}{\partial \omega} \bigg|_{\omega=0} \). The group velocity is given by,

\[ v_g = \text{Re} \left[ \frac{1}{\omega \epsilon(0) \partial \omega} \right] \]  
and \( \beta_2(0) = \frac{\partial^2 \beta(\omega)}{\partial \omega^2} \bigg|_{\omega=0} \) represents the group velocity dispersion (GVD) of the pump pulse leading to the change of shape of the propagating pulse, while \( \beta_3(0) \) represents the third-order dispersion (TOD) which causes additional spreading of the pulse. In order to investigate the nonlinear dynamics of the pump pulse inside the MQW, equation (10) is now recast keeping terms up to order \( \omega^3 \) in \( \beta(\omega) \) in the left hand side to yield the following:

\[ \left( \frac{\partial}{\partial \xi} + i \beta_1(0) \omega - \frac{1}{2} \beta_2(0) \omega^2 - \frac{1}{6} \beta_3(0) \omega^3 \right) \Lambda_p e^{i \beta(\omega) z} = -i \kappa \left( |p_{21}^{(1)}|^2 + |p_{31}^{(1)}|^2 \right) \beta_1^{(1)}, \]

Performing an inverse Fourier transform of the above equation and substituting \( \beta_1^{(1)} \) and \( \beta_3^{(1)} \) we get the following equation,

\[ i \left( \frac{\partial \Omega_p}{\partial \xi} + i \beta_1(0) \frac{\partial \Omega_p}{\partial \xi} - \frac{1}{2} \beta_2(0) \frac{\partial \Omega_p}{\partial \xi} - i \beta_3(0) \frac{\partial \Omega_p}{\partial \xi} \right) + \frac{W \Omega_p^2}{\hbar} \Omega_p = 0, \]

where \( W = \kappa \left( \frac{\Omega_p^2}{D(\omega)^2} \right) \frac{D_0(\omega)}{D(\omega)} \) corresponds to the Kerr nonlinear coefficient. Introducing the retarded frame \( z = \xi - Tc \) and \( T = t - zc(0) \), and after suitable rescaling, the above equation can be recast as:

\[ i \left( \frac{\partial A}{\partial \xi} - \frac{1}{2} \beta_2(0) \frac{\partial A}{\partial \xi} - i \beta_3(0) \frac{\partial A}{\partial \xi} + \gamma |A|^2 A \right) = 0, \]

where \( A = \frac{\hbar^2 \epsilon_0^3}{\mu_{12}} \Omega_p, \gamma = W \left( \mu_{12}/\hbar \sqrt{\frac{\epsilon_0 \hbar c}{2}} \right)^2 \); \( n \) and \( S \) are the linear refractive index of the medium and cross-sectional area of the pump laser pulse, respectively. The above equation is the modified nonlinear Schrödinger equation that has been derived incorporating second- and third-order dispersions and Kerr nonlinearity. Equation (17) can be used to study the spectral broadening due to combined effects of group velocity dispersion, third-order dispersion and nonlinearity induced self-phase modulation (SPM).

3. Linear and nonlinear susceptibilities

In this section we numerically examine the linear and nonlinear susceptibilities of the MQW considered in the present communication. However, we first focus on the linear (first-order) susceptibility of the system with the objective of achieving low absorption, which may provide a useful starting point in the subsequent section. The system parameters taken in the present study are: \( N = 10^{22} m^{-3} \), \( \mu_{12} = 23.35 eA, \) \( \omega_p = 18.84 \times 10^{13} s^{-1} \), thus, \( \kappa = 4.69 \times 10^{17} m^{-1} s^{-1} \); decay rates \( \gamma_1 = 0.5 \times 10^{12} s^{-1} \) and \( \gamma_2 = 1.0 \times 10^{12} s^{-1} \). We demonstrate the variations of \( \text{Im}(\chi(1)) \) and \( \text{Re}(\chi(1)) \) with normalized pump detuning (\( \Delta_p/\gamma_1 \)) for different values of the control field (\( \Omega_c/\gamma_1 \)) in figures 2(a) and (b), respectively. From figure 2(a), it is amply clear that, in the absence of the control field (\( \Omega_c/\gamma_1 = 0 \)), the pump field experiences large absorption (i.e., large \( \text{Im}(\chi(1)) \)) when it is at resonance (\( \Delta_p/\gamma_1 = 0 \)). When a suitable control field is applied (\( \Omega_c/\gamma_1 = 2 \)), the absorption profile splits into two separate peaks, which is the signature of formation of the EIT window. The transparency window (TW) widens with the increase in the value of the control field, which is evident from the curves in the figure for control fields (\( \Omega_c/\gamma_1 = 2 \) and 4). Meanwhile, from figure 2(b), it can be seen that, initially for \( \Omega_c/\gamma_1 = 0 \), the profile of \( \text{Re}(\chi(1)) \) possesses a negative slope as \( \Delta_p/\gamma_1 \) changes from negative to positive values within the TW. This signifies a change of group velocity from normal to anomalous dispersion region. When the control field is turned on and tuned to suitable values, say at \( \Omega_c/\gamma_1 = 2 \) (dot - dash) and \( \Omega_c/\gamma_1 = 4 \) (solid). (a) Imaginary part of \( \chi(1) \) and (b) real part of \( \chi(1) \).
We now proceed to study the nonlinear (third-order) susceptibility. To begin with, in figure 3(a) we have demonstrated the variation of the real part of third-order nonlinear susceptibility $\chi^{(3)}$ as a function of $\Delta_p/\gamma_{31}$ for three different values of the control field Rabi frequency. Control field detuning $\Delta_c/\gamma_{31} = 0$. In both cases, (i) dashed line $\Omega_c/\gamma_{31} = 0$, (ii) dashed-dot line $\Omega_c/\gamma_{31} = 2$ and (iii) solid line $\Omega_c/\gamma_{31} = 4$. Im($\chi^{(1)}$) has been depicted to show that nonlinearity is large even within the EIT window. (b) Variation of $\text{Re}(\chi^{(3)})$ (top-3 panels) and imaginary part of $\chi^{(1)}$ (bottom panel) as a function of $\Delta_p/\gamma_{31}$ for three different values of control detuning. Rabi frequency of the control field $\Omega_c/\gamma_{31} = 4$. In both cases, (i) dashed line $\Delta_c/\gamma_{31} = 3$, (ii) dashed-dot line $\Delta_c/\gamma_{31} = 0$ and (iii) solid line $\Delta_c/\gamma_{31} = -3$. Im($\chi^{(1)}$) has been depicted to confirm the existence of large nonlinearity within TW.

Figure 3. (a) Variations of $\text{Re}\chi^{(3)}$ (top-3 panels) and imaginary part of $\chi^{(1)}$ (bottom panel) as a function of $\Delta_p/\gamma_{31}$ for three different values of the control field Rabi frequency. Control field detuning $\Delta_c/\gamma_{31} = 0$. In both cases, (i) dashed line $\Omega_c/\gamma_{31} = 0$, (ii) dashed-dot line $\Omega_c/\gamma_{31} = 2$ and (iii) solid line $\Omega_c/\gamma_{31} = 4$. Im($\chi^{(1)}$) has been depicted to show that nonlinearity is large even within the EIT window. (b) Variation of $\text{Re}(\chi^{(3)})$ (top-3 panels) and imaginary part of $\chi^{(1)}$ (bottom panel) as a function of $\Delta_p/\gamma_{31}$ for three different values of control detuning. Rabi frequency of the control field $\Omega_c/\gamma_{31} = 4$. In both cases, (i) dashed line $\Delta_c/\gamma_{31} = 3$, (ii) dashed-dot line $\Delta_c/\gamma_{31} = 0$ and (iii) solid line $\Delta_c/\gamma_{31} = -3$. Im($\chi^{(1)}$) has been depicted to confirm the existence of large nonlinearity within TW.

We now proceed to study the nonlinear (third-order) susceptibility. To begin with, in figure 3(a) we have demonstrated the variation of the real part of third-order nonlinear susceptibility $\text{Re}(\chi^{(3)})$ as a function of $\Delta_p/\gamma_{31}$ for different strengths of the control field ($\Omega_c/\gamma_{31}$) and zero detuning (i.e., $\Delta_c/\gamma_{31} = 0$). The third-order susceptibility is quite large and possesses a single peak when no control field is applied. With the application of finite control field, an additional peak in the susceptibility appears and the separation between these two peaks increases with the increase in the value of the control field. For clarity in understanding, the imaginary part of the linear susceptibility has been depicted at the bottom of the figure 3(a). Note that the third-order nonlinearity possesses a large value within the EIT window, which will be subsequently exploited in the next section to generate supercontinuum.

In order to examine the influence of the control field detuning ($\Delta_c/\gamma_{31}$) on the third-order nonlinearity, we have demonstrated in the figure 3(b) the variation of $\text{Re}(\chi^{(3)})$ with $\Delta_p/\gamma_{31}$ for different values of $\Delta_c/\gamma_{31}$. It is evident from the figure that, in the absence of any control field detuning ($\Delta_c/\gamma_{31} = 0$), the variation of $\text{Re}(\chi^{(3)})$ shows antisymmetric behaviour, while for finite detuning ($\Delta_c/\gamma_{31} = 0$) it loses antisymmetric property. Therefore, the peak value of $\text{Re}(\chi^{(3)})$ can be shifted to any desired pump frequency by varying the Rabi frequency and detuning of the control field. At this
stage, a comparison of values of $\chi^{(3)}$ exhibited by different materials reported in the literature would be worthwhile, which are summarized in Table 1. From the values listed in the table, it is amply clear that the value of $\chi^{(3)}$ as identified in the present investigation is extremely large in comparison to that exhibited by other materials, particularly photonic crystal fibers, which have been widely used for the generation of optical supercontinuum.

### Table 1. Comparison of values of $\chi^{(3)}$ for different materials, QW nanostructures and the MQW system considered in the present investigation.

| Materials                          | $\chi^{(3)} (m^2 V^{-2})$ | Wavelength ($\mu$m) | References |
|-----------------------------------|---------------------------|---------------------|------------|
| Fused Silica                      | $1.9 \times 10^{-22}$     | 0.800               | [31]       |
| PBG-08 PCF                        | $2.7 \times 10^{-21}$     | 0.900               | [31]       |
| Silicate N-F2 PCF                 | $1.4 \times 10^{-21}$     | 1.000               | [31]       |
| Semiconductor Doped Glass fiber   | $4.5 \times 10^{-19}$     | 0.740               | [32]       |
| P-toluene sulphonate (PTS)        | $3.7 \times 10^{-18}$     | 1.060               | [33]       |
| **Quantum Wells**                 |                           |                     |            |
| InGaAs/AlAs/AlAsSb                | $5.8 \times 10^{-17}$     | 1.550               | [34]       |
| GaN/AlN                           | $2.2 \times 10^{-16}$     | 1.550               | [35]       |
| Si doped GaN–AlN                  | $2.2 \times 10^{-15}$     | 1.500               | [36]       |
| GaAs/AlInAs                       | $4.4 \times 10^{-14}$     | 9.963               | Present paper |

#### 4. Supercontinuum generation under EIT

In this section, we adopt a numerical simulation and investigate optical supercontinuum generation, exploiting the large nonlinearity in the MQW. For the generation of supercontinuum at low power levels, we need to select the wavelength of the pump pulse such that it experiences negligible absorption, low dispersion and large nonlinearity. Therefore, we choose pump detuning $\Delta_{\nu}/\gamma_{31}$ = 0.8, which corresponds to the pump wavelength $\lambda_{\nu} = 9.963 \mu$m. At this pump detuning, without the application of control field $\text{Im}(\chi^{(3)}) = 0.5323$, while it reduces to 0.0438 when the control field $\Omega_c/\gamma_{31} = 4.0$ and detuning $\Delta_{\nu}/\gamma_{31} = -3.3$. Thus within the EIT window absorption is considerably reduced. In the present investigation the length of the MQW is 1.374 $\mu$m.

Since the propagation distance is short and the absorption is also reduced, we have neglected the attenuation while investigating the SC generation. The values of different relevant parameters are: $\text{Re}(\chi^{(3)}) = 4.43 \times 10^{-14} \text{m}^2 \text{V}^{-2}$, $\beta_2 = -1.05 \times 10^{-20} \text{s}^2 \text{m}^{-1}$, $\beta_3 = 1.10 \times 10^{-33} \text{s}^3 \text{m}^{-1}$, $W = 4.42 \times 10^{-21} \text{s}^2 \text{m}^{-1}$ and $\gamma = 1.22 \times 10^5 \text{W}^{-1} \text{m}^{-1}$.

The dispersion parameter $D = -\frac{2\pi}{\lambda_{\nu}} \beta_2$, and the value of this parameter at the pump wavelength $\lambda_{\nu}$ turns out to be $0.11 \times 10^3 \text{ps} \cdot \text{nm}^{-1} \text{m}^{-1}$. In figures 4(a) and (b), we have demonstrated the variations of $\beta_2$ with $\Delta_{\nu}/\gamma_{31}$ and $D$ with wavelength, respectively. At the pump wavelength, the dispersion is anomalous, which is marked by A in figure 4(b).

Before proceeding to study the SC generation, it is worth investigating exclusively the nonlinearity induced SPM phenomenon. In order to do this, we use equation (17) neglecting all terms involving dispersion, the modified equation reads

$$i \frac{d\phi}{dx} + \gamma |A|^2 A = 0.$$  \hspace{1cm} (18)

We introduce the transformation $A = \sqrt{P_0} Q$, where $P_0$ corresponds to the peak power and $Q$ represents the form of the pulse. With this transformation, equation (18) reduces to

$$i \frac{d\phi}{dx} + \gamma P_0 |Q|^2 Q = 0.$$  \hspace{1cm} (19)

The solution of the above equation is $Q(\xi, T) = Q(0, T) \exp[i\Phi_{nl}(\xi, T)]$, where $\Phi_{nl}(\xi, T) = |Q(0, T)|^2 \delta$ is the nonlinear phase shift due to intensity dependent change in

![Figure 4.](image-url)
nonlinear refractive index, $\delta = \gamma P_0 \zeta$. The explicit form of the electric field of the pump pulse now turns out to be

$$E_p = \hat{E}_p \left\{ \int_{-\infty}^{0} Q(0, T) e^{i \phi_p(\omega_0 T - \omega_0 T)} dT \right\}.$$ 

The pulse is phase modulated due to its own intensity profile and is referred to as self-phase modulation. The time dependent phase shift induces a time dependent frequency shift that is given by

$$\delta \omega(T) = -\frac{\partial \delta \omega}{\partial T} = -\frac{\partial^2}{\partial T^2} |Q(0, T)|^2.$$ \hspace{1cm} (20)

The temporal variation of frequency chirp is caused by the SPM and increases in magnitude with the propagation distance. The $\delta \omega(T)$ leads to negative frequency chirp on the leading edge of the pulse and positive frequency chirp on the trailing edge. The chirp is almost linear and positive over a large central region of the pulse. The nonlinear phase shift $\phi_p$, translates into optical spectral broadening due to nonlinear temporal dependence of chirp $\delta \omega(T)$, thus generating new frequencies as the pump pulse travels inside MQW. Note that the temporal shape of the pulse remains unaffected. The normalized intensity spectrum $I(\omega)$ of the pulse can be obtained by taking the Fourier transform of $Q(\xi, T)$, which is given by

$$I(\omega) = \left[ \int_{-\infty}^{\infty} Q(\xi, T) \exp[i(\omega - \omega_0) T] dT \right]^2.$$ \hspace{1cm} (21)

In order to numerically examine the temporal variation of frequency chirp due to SPM, we launch unchirped hyperbolic secant pulses at the entry of the MQW at $\xi = 0$. These pulses can be expressed as $A(\xi = 0, T) = P_0 \text{sech}(T/T_0)$, where the intensity full width at half maximum pulse duration is given by $T_{\text{FWHM}} = 1.763 T_0$. The temporal variation of frequency chirp due to the nonlinearity induced SPM for a 700 fs (FWHM) pulse has been demonstrated in figure 5. From figure 5, it is evident that the leading edge of the pulse undergoes red shift, whereas the trailing edge undergoes blue shift crafting a down-chirp around the central portion of the pulse. The spectral broadening due to the sole effect of SPM for different values of $\delta$ has been depicted in figure 6, which demonstrates that the spectrum is associated with several internal oscillations whose number increases with the increase in the value of $\delta$. The width of the broadened spectrum also increases with the increase in the value of $\delta$. The typical spectral broadening is associated with a large number of oscillations, which arise solely due to the nonlinearity induced SPM phenomenon, and is now well understood [27, 28].

At this stage it would be appropriate to discuss super continuum generation due to Kerr nonlinearity induced SPM alone, hence, we ignore all dispersive terms in the nonlinear Schrödinger equation. In order to investigate the SC generation adopting numerical simulation, we have launched the pulses at 9.963 $\mu$m wavelength in the MQW. Peak power and FWHM pulse duration of these pulses are 200 W and 700 fs, respectively. Quantum cascade lasers in the wavelength range 2.75–160 $\mu$m are available on the market, which can be readily employed to obtain such pulses [29]. The spectral and temporal evolutions of injected pulses at different distances inside the MQW have been demonstrated in figure 7. From figure 7(a), it is evident that significant spectral broadening is achieved at the end of the MQW. The spectrum consists of multiple oscillations throughout its breadth, which is the signature of SPM induced broadening. At 200 W peak power and for a propagation distance 1.374 $\mu$m, $\delta (= \gamma P_0 \zeta)$ turns out to be $\sim 33.5$. At this value of $\delta$, the two ends of the SPM broadened spectrum extend approximately up to 6.8 $\mu$m and

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**Figure 5.** Variation of nonlinear frequency shift $\delta \omega(T)$ of the sech pulse due to SPM for three different values of $\delta$.

**Figure 6.** The SPM broadened spectra of the pulse for different phase shift $\delta (= \gamma P_0 \zeta)$. (a) $\delta = 0$, (b) $\delta = 16.7$, (c) $\delta = 25.1$, and (d) $\delta = 33.5$. 
18.7 \mu m, respectively. From figure 7(b), which depicts the temporal profile of these pulses at different distances, it is evident that these pulses do not broaden in the temporal domain throughout their propagation in the MQW, which is the characteristic of SPM induced spectral broadening.

To this end we proceed to investigate numerically the dynamics of SC generation considering Kerr nonlinearity and up to third-order dispersion terms, hence, the relevant nonlinear Schrödinger equation becomes

\[ i \frac{\partial A}{\partial z} - \frac{1}{2} \beta_2 (0) \frac{\partial^2 A}{\partial z^2} - \frac{1}{6} \beta_3 (0) \frac{\partial^3 A}{\partial z^3} + \gamma |A|^2 A = 0. \] (22)

The second and third terms in the nonlinear Schrödinger equation are, respectively, due to second- and third-order dispersions, while the last term is due to intensity dependent Kerr nonlinearity. Since these dispersive terms are linear, their independent action cannot lead to the generation of new frequencies. On the other hand, nonlinearity induced SPM phenomenon alone leads to the generation of new frequencies characterized by multiple oscillations in the spectrum and without any temporal broadening in the pulse. However, second- and third-order dispersion in conjunction with the nonlinearity induced SPM can lead to the generation of new frequencies via four-wave mixing (FWM) and modulation instability (MI).

Higher-order dispersive terms. This equation has limited applicability and it is inadequate for short pulses of pulse duration less than 500 fs, since self-steepening and higher-order dispersive terms are important for such pulses. Hence, we continue our investigation with pulses of pulse duration greater than 500 fs where self-steepening and higher-order dispersive terms may be ignored without loss of generality. Therefore, we have chosen pulses of 700 fs FWHM pulse duration and 200 W peak power. The pumping is at 9.963 \mu m, the zero dispersion wavelength is located at 9.90 \mu m and the MQW at this wavelength exhibits anomalous dispersion. The dispersion length \( L_D = \frac{\beta_2}{\beta_1} \) and nonlinear length \( L_{NL} = \frac{1}{\gamma P_0} \) of the MQW for these pulses turn out to be 15 and 0.04 \mu m, respectively. Thus, the injected pulses correspond to a soliton number \( N = \frac{L_D}{L_{NL}} \approx 19 \). Since the pumping is in the anomalous dispersion regime, important nonlinear phenomena in the anomalous dispersion regime are expected to occur. The numerically simulated spectral profiles of the SC as well as the temporal profiles of these pulses at different lengths of the MQW have been depicted in figure 8. At the initial stage of the SC generation dynamics, the nonlinearity induced SPM leads the continuum. Due to SPM phenomenon, the continuum spreads almost symmetrically on both sides of the pump wavelength. The SPM dominated spectral broadening continues up to a distance of 0.3 \mu m, which is evident from multiple oscillations in the central region of the spectrum. As the pulse propagates further inside the MQW, second- and third-order dispersion in conjunction with the nonlinearity induced SPM can lead to the generation of new frequencies via four-wave mixing (FWM) and modulation instability (MI).
Since in the present case FWM is an important nonlinear process that is responsible for spectral broadening, we investigate FWM phase matching condition. Since we have neglected $\beta_4$, therefore, the value of phase-mismatch $k_m$ of the FWM is given as:

$$k_m = 2\gamma P_0 + \beta_2 \Omega^2,$$

where $\Omega$ is the angular frequency shift from the pump in the FWM process. The parametric gain of the sidebands can be written as:

$$g = \sqrt{-\left(\frac{1}{2} \beta_2 \Omega^2 + 2\gamma P_0\right)^2 + \beta_2 \Omega^2}.$$

We have depicted the phase-mismatch and the parametric gain for the MQW in figure 9 for three different pulse peak powers, particularly 100 W, 150 W and 200 W. From figure, it is evident that at a given power ($P_0$), the gain is maximum at phase-mismatch $\kappa_m = 0$. However, significant sideband gain is also possible even though $\kappa_m \neq 0$. At 200 W peak power, the shift in the angular frequency at which maximum gain is possible turns out to be $\Omega \approx \frac{2\pi P_0}{|\chi_3|} \approx 68.14$ THz. Thus in the SC spectrum, two sidebands, one at $m_7$ and other at $m_{15.57}$ are expected. These two sidebands are present in the SC spectrum that is depicted in the top panel of figure 8(a).

As the pulse propagates beyond 0.3 $\mu$m, the modulation instability, which is a four-wave mixing process, is start developing and creating internal structure in the temporal...
domain of the pulse. The characteristic length scale ($L_{MI}$) for modulation instability can be easily determined which turns out to be $L_{MI} \approx 1.6 \times 10^{-4} \mu m$. In addition to causing the spectral broadening, MI also leads to the splitting of the pulse in the temporal domain, which is evident from figure 9(b). This figure also indicates that the modulation instability lead pulse splitting is accompanied with soliton generation. The broadening of the pulse in the temporal domain is attributed to second- and third-order dispersions. The spectral broadening is strongly asymmetric. The long-wavelength side of the spectrum broadens more in comparison to the broadening in the short-wavelength side. This is due to efficient
dispersive wave generation in the long-wavelength side of the spectrum. A dominant portion of the spectral broadening occurs within 1.0 μm of the MQW. Further propagation leads to minor smoothening in the long-wavelength side of the spectrum keeping the short-wavelength side unchanged. The spectrum consists of several oscillations throughout the length whose number increases with the increase in the distance of propagation, which is a signature of dominance of SPM. The final spectral broadening of the pulse is attributed to SPM, FWM, modulation instability and also soliton generation.

In order to examine the influence of pulse peak power on the generated supercontinuum, we have injected 700 fs pulses of different peak powers, particularly 100 W, 150 W and 200 W into the MQW and examined SC dynamics. The spectral, as well as the temporal, profiles have been depicted in figure 10(a) from which it is amply clear that for all the three cases the mechanism of initial spectral broadening is nonlinearity induced SPM phenomenon, which is characterized by no broadening in the temporal domain. Kindly note from the temporal profile (panel (ii) of figure 10(a)) that up to a length of 0.5 μm inside MQW, the injected pulse does not broaden in the temporal domain. With further propagation of the pulse, FWM, modulation instability and soliton generation take place (as evident from the temporal domain), which contribute considerable broadening of the spectrum. Note that the long-wavelength side of the spectrum broadens more in comparison to the short-wavelength side. In the temporal domain too the pulse broadens considerably. Figure 10(b) demonstrates the spectral and temporal profiles at the end of the MQW system. The bandwidth of the broadened spectrum increases with the increasing peak power. With the increase in power, the amplitude of oscillations in the SC spectrum decreases while the number of oscillations increases.

5. Conclusion

In conclusion, we have explored the possibility of broadband mid-infrared supercontinuum generation at low pump power in a 30 period 1.374 μm long semiconductor multiple quantum well system facilitated by electromagnetically induced transparency. We have derived an expression for Kerr nonlinearity exhibited in the quantum well system due to quantum interference of a weak pump pulse and a controlling laser beam. It is found that within the EIT window, the exhibited Kerr nonlinearity is very large, which has been exploited to generate infrared supercontinuum. The SC generation has been examined adopting numerical simulation and launching 700 fs pulses of 200 W peak power at 9.963 μm. Significant spectral broadening is achievable at the end of the MQW. The broadened spectrum is asymmetric about the pump wavelength and consists of multiple oscillations, which is a signature of the domination of SPM phenomenon in the spectral broadening. The generated SC is attributed to self-phase modulation, four-wave mixing, modulation instability and also soliton generation. A key advantage of the proposed scheme is that the supercontinuum source could be easily integrated with other semiconductor devices.

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