Generalized Covariant Derivative on Extra Dimension and Weinberg-Salam Model

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Abstract

The generalized covariant derivative on 5-dimensional space including 1-dimensional extra compact space is defined, and, by use of it, the Weinberg-Salam model is reconstructed. The spontaneous breakdown of symmetry takes place owing to the extra dimension under the settings that the Higgs field exists in the extra dimensional space depending on the argument $y$ of this extra space, whereas the gauge and fermion fields do not depend on $y$. Both Yang-Mills-Higgs and fermion Lagrangians in Weinberg-Salam model are correctly reproduced.

1 Introduction

The supersymmetric string theory [1] is consistently formulated in 10 dimensional space. The extra 6-dimensional space over $M_4$ has to be compactified in order not to be observed by the present experimental facilities. However, this extra 6-dimensional space after compactified has the profound possibilities to explicate many unresolved problems such as particle generations, internal flavor symmetry and its spontaneous breakdown, CKM matrix, particle masses and so on. For these purposes, the extra 6-dimensional space has recently attracted much attentions.

Manton [2] initiated efforts to derive the Weinberg-Salam model from the Yang-Mills theory in 6-dimensional space containing extra compact two dimensional space. He elucidated in his work that the Higgs field is a part of gauge fields. Meanwhile, Connes [3] proposed non-commutative geometry and applied it to construct the spontaneous broken gauge theory on two-sheeted discrete space followed by $M_4$, which provided a geometrical understanding of the Higgs mechanism without extra physical degrees of freedom as in the Kaluza-Klein theory. There are several versions of this approach [4]. However, in any case gauge and Higgs fields are written together and yield the Yang-Mills-Higgs Lagrangian.

In this paper, we try to extend the generalized covariant derivative method proposed by Sogami [5] into that on the product space of $M_4$ and extra continuous compact space. This compact space does not need to be specified, but we call it $S_1$ with argument $y$. We first define the generalized covariant derivatives on $M_4 \times S_1$ and then, we obtain the generalized field strength following the usual procedure of gauge theory, from which the Yang-Mills-Higgs Lagrangian is derived. This Lagrangian contains the term which provokes the correct spontaneous symmetry breakdown. In order to yield the Lagrangian in Weinberg-Salam model, we have to assume that the Higgs field intrudes into the extra dimensional space $S_1$ so as to depend on the argument $y$ whereas the gauge field as well as leptonic fields do not contain $y$, and exist uniformly in $S_1$. We do not consider the Kaluza-Klein mode in this paper. The leptonic Lagrangian is also obtained by use of the generalized covariant derivatives, and we can successfully reconstruct the Weinberg-Salam model.

This paper consists of four sections. The next section presents the basic formulation of the generalized covariant derivatives in order to yield the Yang-Mills Higgs Lagrangian as well as fermionic Lagrangian. In the third section, the Weinberg-Salam model is reconstructed. The last section is devoted to concluding remarks.

2 Generalized covariant derivative

Sogami [5] reconstructed the spontaneous broken gauge theories such as standard model and grand unified theory by use of the generalized covariant derivative smartly defined by him. Let us explain his method in the version of Weinberg-Salam model. He divided the space of fermion fields into two sectors which consist of the left-handed and right-handed fermions, respectively.

$$\psi = \psi_L |L> + \psi_R |R>, \quad (2.1)$$

where $|L>$ and $|R>$ are the base of left- and right-handed fermion spaces, respectively and $\psi_L$ and $\psi_R$ are the left- and right-handed fermion fields denoted...
We write this 5-dimensional space to be

\[
\psi_L = \left( \begin{array}{c} \nu_L \\ e_L \end{array} \right), \quad \psi_R = e_R.
\] (2.2)

Then, he defined the generalized covariant derivative

\[
\mathcal{D}_\mu \equiv \partial_\mu - ig A_L \mu |L \rangle \langle L| - ig A_R \mu |R \rangle \langle R|
- \frac{1}{4} g' \mu (\Phi |L \rangle \langle R| + \Phi^\dagger |R \rangle \langle L|) + c + c_5 \gamma_5,
\] (2.3)

from which the generalized field strength is yielded by the equation

\[
[D_\mu, D_\nu] = -ig F_{\mu\nu} - ig' F_{R\mu\nu} - \frac{i}{4} \mathcal{F}^{(0)}_{\mu\nu}.
\] (2.4)

He succeeded in reconstructing the spontaneous symmetry broken gauge theory by use of these items.

In this paper, we apply his idea to reconstruct the Weinberg-Salam model on the 5-dimensional space including one extra compact space. Though the extra space is not necessary to be \(S_1\), we write this 5-dimensional space to be \(M_4 \times S_1\) with the argument \(x_\mu\) and \(y\). We define the generalized covariant derivative on 5-dimensional space as

\[
D_\mu = \partial_\mu + A_L \mu |L \rangle \langle L| + A_R \mu |R \rangle \langle R|,
\]

\[
D_y = \partial_y + \Phi |L \rangle \langle R|,
\]

\[
D_y = \partial_y + \Phi^\dagger |R \rangle \langle L|.
\] (2.5)

According to (24) and (25), we can describe the leptonic Lagrangian

\[
\mathcal{L}_D(x, y) = \bar{\psi} \left( i \gamma^\mu D_\mu + g_Y D_y + g_Y D_y \right) \psi
= \bar{\psi}_L i \gamma^\mu (\partial_\mu + A_L \mu) \psi_L + \bar{\psi}_R i \gamma^\mu (\partial_\mu + A_R \mu) \psi_R
+ g_Y \bar{\psi}_L \Phi \psi_R + g_Y \bar{\psi}_R \Phi^\dagger \psi_L.
\] (2.6)

Then, field strengths are derived in usual way

\[
\mathcal{F}_{\mu\nu} = [D_\mu, D_\nu]
= (\partial_\mu A_L \nu - \partial_\nu A_L \mu + [A_L \mu, A_L \nu]) |L \rangle \langle L| + (\partial_\mu A_R \nu - \partial_\nu A_R \mu + [A_R \mu, A_R \nu]) |R \rangle \langle R|,
\]

\[
\mathcal{F}^\mu_y = [D_\mu, D_y]
= (\partial_\mu \Phi + A_L \mu \Phi - \Phi A_R \mu) |L \rangle \langle R| - \partial_y A_L \mu |L \rangle \langle L| - \partial_y A_R \mu |R \rangle \langle R|,
\]

\[
\mathcal{F}^\nu_y = [D_\nu, D_y]
= (\partial_\nu \Phi^\dagger + A_R \nu \Phi^\dagger - \Phi^\dagger A_L \nu) |R \rangle \langle L| - \partial_y A_R \nu |R \rangle \langle R| - \partial_y A_L \nu |L \rangle \langle L|,
\]

\[
\mathcal{F}^\mu y = [D_\mu, D_y]
= (\partial_\mu \Phi^\dagger + A_R \mu \Phi^\dagger - \Phi^\dagger A_L \mu) |R \rangle \langle L| - \partial_y A_R \mu |R \rangle \langle R| - \partial_y A_L \mu |L \rangle \langle L|,
\]

\[
\mathcal{F}^\nu y = [D_\nu, D_y]
= (\partial_\nu \Phi + A_L \nu \Phi - \Phi A_R \nu) |L \rangle \langle R| - \partial_y A_L \nu |L \rangle \langle L| - \partial_y A_R \nu |R \rangle \langle R|,
\]

\[
= \bar{\psi}_L \bigg( \psi_L \partial_\mu + \frac{g_Y}{2} |L \rangle \langle L| - \frac{g_Y}{2} |R \rangle \langle R| + \frac{1}{2} \mathcal{F}^{(0)}_{\mu\nu} \bigg)
+ \frac{1}{2} \text{Tr} \left( \mathcal{F}^{(0)}_{\mu\nu} \right) \bigg( \psi_L \partial_\nu + \frac{g_Y}{2} |L \rangle \langle L| - \frac{g_Y}{2} |R \rangle \langle R| + \frac{1}{2} \mathcal{F}^{(0)}_{\mu\nu} \bigg) \psi_L + \frac{1}{2} \text{Tr} \left( \mathcal{F}^{(0)}_{\mu\nu} \right) \bigg( \psi_R \partial_\nu + \frac{g_Y}{2} |R \rangle \langle R| - \frac{g_Y}{2} |L \rangle \langle L| + \frac{1}{2} \mathcal{F}^{(0)}_{\mu\nu} \bigg) \psi_R
\]

In order to obtain the Yang-Mills-Higgs Lagrangian with correct signs, we define the counter covariant derivatives to (2.5).

\[
\mathcal{D}^\mu = \partial^\mu + A_L^\mu |L \rangle \langle L| + A_R^\mu |R \rangle \langle R|,
\]

\[
D^\mu = \partial^\mu - \Phi |L \rangle \langle R|,
\]

\[
\bar{D}^\mu = \partial^\mu - \Phi^\dagger |R \rangle \langle L|,
\]

from which the counter field strengths to (2.7) are derived.

\[
\mathcal{F}^{\mu\nu} = \bigg( \partial^\mu A_L^\nu - \partial^\nu A_L^\mu + [A_L^\mu, A_L^\nu] \bigg) |L \rangle \langle L| + \bigg( \partial^\mu A_R^\nu - \partial^\nu A_R^\mu + [A_R^\mu, A_R^\nu] \bigg) |R \rangle \langle R|,
\]

\[
\mathcal{F}^{\mu y} = \bigg( \partial^\mu \Phi + A_L^\mu \Phi - \Phi A_R^\mu \bigg) |L \rangle \langle R| - \partial_y A_L^\mu |L \rangle \langle L| - \partial_y A_R^\mu |R \rangle \langle R|,
\]

\[
\mathcal{F}^{\nu y} = \bigg( \partial^\nu \Phi^\dagger + A_R^\nu \Phi^\dagger - \Phi^\dagger A_L^\nu \bigg) |R \rangle \langle L| - \partial_y A_R^\nu |R \rangle \langle R| - \partial_y A_L^\nu |L \rangle \langle L|.
\] (2.7)

Then, we define the Lagrangian by use of field strengths in (24) and (25).

\[
\mathcal{L}(x, y) = \frac{1}{2g_Y^2} \text{Tr} \chi < \mathcal{F}^{\mu\nu}(x, y), \mathcal{F}_{\mu\nu}(x, y) >=
- \frac{1}{2g_Y^2} \text{Tr} \chi < \mathcal{F}^{\mu y}(x, y), \mathcal{F}_{\mu y}(x, y) >
- \frac{1}{2g_Y^2} \text{Tr} \chi < \mathcal{F}^{\nu y}(x, y), \mathcal{F}_{\nu y}(x, y) >
\]

\[
= - \frac{1}{2g_Y^2} \text{Tr} \chi F^{\mu\nu} F_{\mu\nu} - \frac{1}{2g_Y^2} \text{Tr} \chi F^{\mu y} F_{\mu y}
+ \left( \frac{1}{g_Y^2} + \frac{1}{g_Y^2} \right) \text{Tr} \chi \left( (D^\nu \Phi)^\dagger \right) (D^\mu \Phi)
- \left( \partial^\mu A_L^\nu \right)^\dagger (\partial_y A_L^\mu) - \left( \partial^\mu A_R^\nu \right)^\dagger (\partial_y A_R^\mu) \bigg)
+ \frac{2}{g_Y^2} \text{Tr} \chi \left( (D^\nu \Phi)^\dagger (D^\mu \Phi) - (D^\nu \Phi) (D^\mu \Phi) \right).
\] (2.10)

where

\[
F^{\mu\nu}_L = \partial^\mu A_L^\nu - \partial^\nu A_L^\mu + [A_L^\mu, A_L^\nu],
F^{\mu\nu}_R = \partial^\mu A_R^\nu - \partial^\nu A_R^\mu + [A_R^\mu, A_R^\nu],
\]

\[
D^\mu \Phi = \partial^\mu \Phi + A_L^\mu \Phi - \Phi A_R^\mu.
\] (2.12)

Here, we address the gauge transformation of the present formulation. The transformation function is denoted by

\[
g(x) = g_L(x) |L \rangle \langle L| + g_R(x) |R \rangle \langle R|,
\]

in which we should note that \(g(x)\) does not depend on the argument \(y\) of \(S_1\). It is evident that the
covariant derivatives are gauge covariant as they should be.

\[ g(x)\mathcal{D}_\mu g^{-1}(x) = \partial_\mu + A^2_{\mu\nu}|L \ll | \]
\[ + A^B_{\mu\nu}|R \ll | = \mathcal{D}_\mu, \]
\[ g(x)\mathcal{D}_y g^{-1}(x) = \partial_y + \Phi^{\mu}|L \ll | \]
\[ = \mathcal{D}_y, \]
\[ g(x)\mathcal{D}_y g^{-1}(x) = \partial_y + \Phi^{\mu}|R \ll | = \mathcal{D}_y, \]

Similarly, we can prove the covariant derivatives given in (2.13) are also covariant for the gauge transformation. According to (2.14), field strengths expressed in (2.17) and (2.18) are gauge covariant which yields that the Lagrangian in (2.10) is gauge invariant. It is also evident that the leptonic Lagrangian in (2.16) is gauge invariant.

3 Reconstruction of Weinberg model

Let us specify gauge fields in W-S model as

\[ A_{L\mu} = -i \frac{1}{2} \left\{ \sum_{k=1}^{3} \sigma^k g A^k_{L\mu} + \alpha \sigma^0 y' B_{\mu} \right\}, \]
\[ A_{R\mu} = -i \frac{1}{2} b y' B_{\mu}, \]
\[ \Phi = \left( \phi^+, \phi^0 \right), \]

where \( A^k_{\mu\nu} \) and \( B_{\mu} \) are \( SU(2) \) and \( U(1) \) gauge fields with coupling constants \( g \) and \( g' \), respectively and, \( a \) and \( b \) are the \( U(1) \) hypercharges of left- and right-handed leptons, respectively. From (2.17), we can form the fields strengths

\[ F^\mu_{L\mu\nu} = -i \frac{1}{2} g \sum_{k=1}^{3} \sigma^k \left( \partial^\mu A^k_{L\mu} - \partial^\nu A^k_{L\mu} + g f_{ijk} A^j_{L\mu} A^k_{L\nu} \right), \]
\[ -i \frac{1}{2} g \sigma^0 \left( \partial^\mu B^\nu - \partial^\nu B^\mu \right), \]
\[ F^\mu_{R\mu\nu} = -i \frac{1}{2} b y' \sigma^0 \left( \partial^\mu B^\nu - \partial^\nu B^\mu \right), \]
\[ \mathcal{D}_\mu \Phi = \left\{ \partial_\mu - i \frac{1}{2} \left( \sum_{k=1}^{3} \sigma^k g A^k_{L\mu} - \sigma^0 y' B_{\mu} \right) \right\} \Phi. \]

After insertion of (3.2) into (2.11) and rescaling of fields, the Lagrangian takes the form, with constant parameters \( \alpha, \beta_L, \beta_R \) and \( \lambda' \) resulting from proper calculation

\[ \mathcal{L}(x, y) = -\frac{1}{4} \sum_{i=1}^{3} F^\mu_{L\mu\nu} F^\nu_{L\mu\nu} - \frac{1}{4} B^\mu_{\nu\mu} B_{\mu\nu} \]
\[ + (D^\mu \Phi)^\dagger (D^\mu \Phi) - \lambda' (\Phi^\dagger \Phi)^2 + \alpha^2 (D^\mu \Phi)^\dagger (D^\mu \Phi) \]
\[ - \beta_L^2 \sum_{i=1}^{3} (\partial^\mu A^i_{L\mu}) (\partial^\nu A^i_{L\nu}) - \beta_R^2 (\partial^\mu B^\nu) (\partial^\nu B_{\mu}), \]

where

\[ F^\mu_{L\mu\nu} = \partial_\mu A^i_{L\mu} - \partial^i A^i_{L\mu} + g f_{ijk} A^j_{L\mu} A^k_{L\nu}, \]
\[ B_{\mu\nu} = \partial_\mu B_{\nu} - \partial_\nu B_{\mu}, \]
\[ \mathcal{D}_\mu \Phi = \left\{ \partial_\mu - i \frac{1}{2} \left( \sum_{k=1}^{3} \sigma^k g A^k_{L\mu} + \sigma^0 y' B_{\mu} \right) \right\} \Phi, \]

with \( a = -1 \) and \( b = -2 \) for left- and right-handed leptons in (2.22), respectively.

In this paper, we consider the case that only Higgs field is infiltrated into the extra dimensional space such as

\[ \Phi(x, y) = \phi(x)f(y), \]

where \( f(y) \neq 0 \) and we normalize the function \( f(y) \) such as

\[ \frac{1}{2\pi R} \int_0^{2\pi R} f^2(y) dy = 1, \]

whereas gauge fields penetrate the extra dimension uniformly and therefore don’t depend on the argument \( y \) of extra dimensional space.

\[ A^i_{L}(x, y) = A^i_{L}(x), \quad B_{\mu}(x, y) = B_{\mu}(x). \]

From these settings, we find the 4-dimensional Yang-Mills-Higgs Lagrangian

\[ L_{YMH} = \frac{1}{2\pi R} \int_0^{2\pi R} \mathcal{L}(x, y) dy \]
\[ = -\frac{1}{4} \sum_{i=1}^{3} F^i_{\mu\nu\sigma}(x) F^i_{\mu\nu\sigma}(x) - \frac{1}{4} B_{\mu\nu\sigma}(x) B_{\mu\nu\sigma}(x) \]
\[ + (D_\mu \phi(x))^\dagger (D_\mu \phi(x)) \]
\[ + \alpha^2 \int_0^{2\pi R} f^2(y) dy \left( \phi^\dagger(x)\phi(x) \right) \]
\[ - \lambda' \int_0^{2\pi R} f^4(y) dy \left( \phi^\dagger(x)\phi(x) \right)^2, \]

where we should notice that the metric structure is

\[ \partial_y = \partial^y. \]

The effective potential in tree level is known to be

\[ V(\phi) = \lambda (\phi^\dagger(x)\phi(x))^2 - \mu^2 (\phi^\dagger(x)\phi(x)) \]

(3.10)
with the parameters \( \lambda = \frac{\alpha}{2 \pi R} \int_0^{2 \pi R} f^2(y)dy \) and \( \mu^2 = \frac{\alpha^2}{2 \pi R} \int_0^{2 \pi R} f^4(y)dy \). Here, we adopt the unitary gauge and then Higgs field is expressed as

\[
\phi = \left( \begin{array}{c} 0 \\ \varphi + v \end{array} \right),
\]

(3.11)

where \( \phi^0 = \left( 0, v/\sqrt{2} \right) \) gives the minimal point to the effective potential \( V(\phi) \), and so \( v = \mu/\sqrt{\lambda} \). The field \( \varphi \) is the neutral Higgs boson. The effective potential as a function of \( \varphi \) is

\[
V(\varphi) = \frac{\lambda}{4} \varphi^4 + \lambda v \varphi^3 + \mu^2 \varphi^2
\]

(3.12)

except for the constant term. The covariant derivative of \( \phi \) is written as

\[
D_{\mu} \phi = \left( \begin{array}{c} 0 \\ \partial_{\mu} \varphi \end{array} \right) - i \frac{\sqrt{2} g W^+_{\mu}}{\sqrt{g^2 + g'^2}} \frac{\varphi + v}{\sqrt{2}},
\]

(3.13)

where \( W^+_{\mu} \) and \( Z_{\mu} \) are the charged and neutral weak boson fields, respectively. Finally, we obtain the Yang-Mills-Higgs Lagrangian

\[
L_{YMH} = -\frac{1}{4} \sum_{i=1}^{2} \left[ F_{L,\mu \nu}^{\mu} (x) F_{L}^{\mu \nu} (x) - \frac{1}{4} F_{Z}^{\mu \nu} (x) F_{Z}^{\mu \nu} (x) \right] + \frac{1}{2} \left( \frac{g^2}{g^2 + g'^2} W^+_{\mu} W^-_{\mu} + \frac{g^2 + g'^2}{2} Z_{\mu} \right)
\]

(3.14)

where \( F_{L,\mu \nu} \), \( F_{Z,\mu \nu} \) and \( F_{\mu \nu} (x) \) are the field strengths of charged, neutral weak gauge fields and photon field, respectively. From \( B_{i1} \), the famous mass relation \( m_W = m_Z \cos \theta_W \) follows.

Under the assumption that leptons also stay at \( S_1 \) uniformly, the 4-dimensional Dirac Lagrangian obtained by integrating \( \Phi \) takes the form

\[
L_D = \frac{1}{2 \pi R} \int_0^{2 \pi R} L_D dy
\]

\[
= \bar{\psi} L \psi \left( \partial_{\mu} - i \frac{\lambda}{2} \sum_{k=1}^{3} \frac{\sigma^k g A_{\mu}^k}{4} - \sigma^0 g' B_{\mu} \right) + \bar{\psi} R \psi \left( \partial_{\mu} + ig B_{\mu} \right) + g \bar{\psi} \psi \varphi + m_{\nu} \bar{\nu} \nu,
\]

(3.15)

where \( g_{\varphi} = \frac{\sqrt{\lambda}}{2 \sqrt{2} \pi R} \int_0^{2 \pi R} f(y)dy \) and the electron mass \( m_{\nu} = g_{\varphi} v \). The equation \( \frac{\partial \varphi}{\partial y} = 0 \) is equal to the lepton part of Lagrangian in the Weinberg-Salam model.

4 Conclusions

We have reconstructed the Weinberg-Salam model based on the generalized covariant derivative method on the product space \( M_4 \times S_1 \), where the gauge symmetry is spontaneously broken owing to the penetration of the Higgs field into the extra compact space \( S_1 \). This breakdown of symmetry is realized without considering the quantum effects. This is favorable point of our model.

It is assumed that the gauge and fermion fields do not depend on the argument \( y \) in \( S_1 \), and therefore the Kaluza-Klein modes of those fields do not appear on the stage. This assures the renormalizability and stability of our model formulated in this paper since the \( y \) derivative terms of gauge fields in \( (3.11) \) give the imaginary masses to the Kaluza-Klein modes. The existence of the Kaluza-Klein modes of Higgs field is also undesirable since it increases the gauge boson mass. This non-existence of Kaluza-Klein modes is a clear difference from other model \( \Phi \). LHC or more powerful machine will decide whether Kaluza-Klein modes exists or not.

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