Lyα absorbers in motion: consequences of gravitational lensing for the cosmological redshift drift experiment

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ABSTRACT
The evolution of the expansion rate of the Universe results in a drift in the redshift of distant sources over time. A measurement of this drift would provide us with a direct probe of expansion history. The Lyman α (Lyα) forest has been recognized as the best candidate for this experiment, but the signal would be weak and it will take next generation large telescopes coupled with ultrastable high-resolution spectrographs to reach the cm s$^{-1}$ resolution required. One source of noise that has not yet been assessed is the transverse motion of Lyα absorbers, which varies the gravitational potential in the line of sight and subsequently shifts the positions of background absorption lines. We examine the relationship between the pure cosmic signal and the observed redshift drift in the presence of moving Lyα clouds, particularly the collapsed structures associated with Lyman limit systems and damped Lyα systems. Surprisingly, the peculiar velocities and peculiar accelerations both enter the expression, although the acceleration term stands alone as an absolute error, whilst the velocity term appears as a fractional noise component. An estimate of the magnitude of the noise reassures us that the motion of the Lyα absorbers will not pose a threat to the detection of the signal.

Key words: gravitational lensing – methods: analytical – intergalactic medium – quasars: absorption lines – cosmology: theory.

1 INTRODUCTION
The large-scale homogeneity and isotropy of the Universe is encompassed in Friedmann–Lemaître–Robertson–Walker geometry (see Hobson, Efstathiou & Lasenby 2006 for further details). One of the key aspects of this cosmology is the evolution of the scale-factor, $R(t)$; modern cosmology aims to characterize this evolution, distinguish between competing cosmological models and predict the fate of the Universe. Observationally, this has been a rather difficult science, with so-called standard candles and standard rulers playing leading roles in indirect measurements. Sandage (1962) recognized that a direct detection of the evolution of the expansion rate was, in theory, possible; just as the cosmological redshifts of extragalactic objects in all directions are a consequence of isotropic expansion, a positive or negative drift in the redshift of a single object over time would indicate acceleration or deceleration with respect to the observer. The redshift drift is very small, but as we head towards an era of cm s$^{-1}$ resolution spectroscopy, the prospect of detection becomes very real. In addition to identifying the appropriate instrumentation and spectroscopic techniques required, it is crucial to account for any systematic biases and consider if any source of noise may hinder the detection of the cosmic signal. Previous studies have covered some of this ground but the potential for the transverse motion of Lyman α (Lyα) clouds to hide this signal remains yet unexplored. In the present paper, we aim to address this issue and estimate the magnitude of the bias or noise that is introduced.

In Section 2, we review the current cosmological paradigm along with relevant observational tests. We rederive the expression for the cosmic signal being sought in Section 3. In Section 4, we introduce the transverse moving-lens effect. We derive the expression for the observed redshift drift in the presence of Lyα clouds in Section 5 and discuss the results in Section 6.

2 BACKGROUND
The linear correlation between the recession velocities and luminosity distances of galaxies within a few Mpc was first discovered by Hubble (1929).¹ This result, the first Hubble diagram, set the observational precedent for the notion of an expanding Universe.
at a time when a static Universe was philosophically favoured. More recently, two independent measurements of the luminosities of 'standard candle' Type Ia supernovae (SNIa) by Riess et al. (1998) and Perlmutter et al. (1999) have revealed that, assuming a metric theory of gravity, the Universe is currently undergoing a period of acceleration (Shapiro & Turner 2006), and there is evidence for past deceleration beyond a transition redshift of $z \sim 0.5$ from Hubble Space Telescope (HST) observations of high-redshift $(0.2 \lesssim z \lesssim 1.6)$ SNIa (Riess et al. 2004).

Whilst deceleration is expected in a matter-only universe, recent acceleration requires a 'dark-energy' component. To uncover the underlying physical cause of the acceleration, it is necessary to map out the evolution of the scalefactor; the functional form is characterized by the relative densities (in units of the critical density $\Omega_0$) of the various matter–energy components of the cosmological fluid $\Omega_X$, where $X$ may be one of $r$ (radiation), $M$ (matter), dark energy or $k$ (curvature) and the subscript 0 denotes the current values. The growing evidence points towards a flat universe that is currently dominated by a cosmological constant, $\Lambda$ (a 'dark-energy' component with an equation of state $w = -1$), a subdominant pressureless dark matter component and a small amount of baryonic matter.

The expansion rate is normalized by its value today, i.e. the Hubble constant:

$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}.$$ 

The HST Key Project has combined several independent distance methods, establishing the most accurate measurement as $H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Freedman et al. 2001). This is in good agreement with the current best estimates for the values of the cosmological parameters:

$$\Omega_{M,0} = 0.27, \quad \Omega_{\Lambda,0} = 0.73 \quad \text{and} \quad h = 0.71,$$

based on observations of the cosmic microwave background (CMB) radiation by the Wilkinson Microwave Anisotropy Probe (WMAP; Komatsu et al. 2009), SNIa, and baryonic acoustic oscillations (BAOs; Percival et al. 2007) in the galaxy distribution. Each of the measurements individually provides constraints on various degenerate combinations of the parameters in question; the Hubble parameter constraints are model dependent.

Whilst precision cosmology converges upon a concordance in the parameters with complementary methods, the expansion remains an underlying assumption upon which observed phenomena, such as cosmological redshifts, are interpreted. Few direct tests are available to support this hypothesis. One observable consequence of expansion is time dilation by a factor of $(1 + z)$. Wilson (1939) suggested the use of SNIa light curves as 'cosmic clocks', and the results of high-redshift SNIa observations have confirmed the expansion hypothesis and excluded other models (Leibundgut 1996; Goldhaber et al. 2001; Blondin et al. 2008). Another prediction is the decrease in the surface brightness (SB) of galaxies with a $(1 + z)^{-4}$ redshift dependence, independent of all other cosmological parameters (Tolman 1930, 1934; Hubble & Tolman 1935). The Tolman SB test has been performed in several studies, each confirming the expanding geometry. Pahre, Djorgovski & de Carvalho (1996) and Lubin & Sandage (2001) rule out the $(1 + z)^{-4}$ dependence expected by the alternative non-expanding tired-light model at the $5\sigma$ and $10\sigma$ confidence level, respectively. Furthermore, the temperature of the CMB is predicted to increase linearly with redshift:

$$T_{\text{CMB}}(z) = T_0(1 + z),$$

the temperature at the present epoch has been accurately determined to be $T_0 = 2.725 \pm 0.002$ K (Mather et al. 1999). Atomic fine structure transitions identified in quasar absorption spectra probe the CMB temperature at the absorption redshift (Bahcall & Wolf 1968). This method generally provides an upper limit to the temperature only, but the observations, which cover redshifts up to $z \lesssim 3$ are consistent with the standard cosmological model (e.g. Songaila et al. 1994; Srianand, Petitjean & Ledoux 2000; Molaro et al. 2002, and references therein). Alternatively, the CMB temperature can be estimated at the redshift of galaxy clusters by observing the thermal Sunyaev-Zel’dovich effect (Fabbri, Melchiorri & Natale 1978; Rephaeli 1980); measurements using this method have been carried out for clusters at a range of redshifts $0.02 < z < 0.6$, again finding the redshift dependence to be consistent with an expansion hypothesis (Battistelli 2002; De Petris et al. 2002; Lazzi et al. 2009).

Redshift drift is a direct probe of the dynamics of expansion, as the Hubble parameter $H(z)$ is measured at particular redshifts. It allows a unique insight into the validity of our cosmological model, sometimes with complementary observations. For example, by characterizing the redshift dependence of the drift, the experiment can constrain evolving dark-energy models (Corasaniti, Hutner & Melchiorri 2007) and map the equation of state (Lake 2007), although many-parameter dark–energy models are harder to constrain (Balbi & Quercellini 2007). If we live in a $\Lambda$CDM universe, the Chaplygin gas model (Kamenshchik, Moschella & Pasquier 2001) and interacting dark-energy model would be rejected at high confidence (Balbi & Quercellini 2007), but the Lemaitre–Tolman–Bondi (LTB) void models would be ruled out even earlier (Quartin & Amendola 2009). Furthermore, when combined with luminosity distance data, the redshift drift experiment provides a test of the Copernican principle (Uzan, Clarkson & Ellis 2008b) and an independent measurement of spatial curvature (Nakamura & Chiba 1999).

Many of these tests rely on accurate measurements of the drift, and detection alone is only possible with next generation large telescopes. All systematic biases and sources of noise must, therefore, be identified and the threat they pose to the detection must be quantified. With this in mind, we examine one such threat: the changes to observed redshifts of objects caused by the transverse motion of Lyα clouds.

### 3 THE COSMIC DECELERATION SIGNAL

The general form for the redshift drift dependence on redshift and the expansion rate was originally derived by McVittie (1962, see equation 4A). We briefly rederive it here.

The Hubble parameter is defined in terms of the scalefactor and expansion rate:

$$H(t) \equiv \frac{R(t)}{R(t)}.$$ 

The cosmological redshift of an emitter is related to the ratio of the scalefactors at emitter and observer:

$$1 + z = \frac{R(t_0)}{R(t)},$$

where $t_0$ is the time of observation and $t$ the time of emission.

As an aside, the cosmological fluid components appear in the redshift dependence of the Hubble parameter as derived from the cosmological field equations. The expression is given by:

$$H(z) = H_0 \sqrt{\Omega_{M,0}(1 + z)^3 + \Omega_{\Lambda,0} + \Omega_k(1 + z)^2},$$

where we neglect radiation and assume dark energy to be in the form of a cosmological constant.
Consider now the redshift after some time interval, when the scalefactors have evolved at both emitter and observer:

\[ 1 + z + \Delta z = \frac{R(t_0 + \Delta t_0)}{R(t + \Delta t)} \]

\[ \approx \frac{R(t_0) + \dot{R}(t_0)\Delta t_0}{R(t) + \dot{R}(t)\Delta t} \]

\[ = (1 + z)(1 + \frac{H(t_0)\Delta t_0}{H(t)\Delta t}) \]

\[ = (1 + z)(1 + \frac{H_0\Delta t_0 - H(z)\Delta t}{H(z)\Delta t}) \]

The scalefactors have been expanded to the first order in \( \Delta t \). The Hubble parameter at the time of observation is relabelled: \( H_0 \equiv H(t_0) \), whilst the Hubble parameter at emission is expressed in terms of the redshift of the emitter, rather than the time of emission, i.e. \( H(z) \) rather than \( H(t) \). Subtracting to obtain the change in redshift:

\[ \Delta z = (1 + z)[H_0\Delta t_0 - H(z)\Delta t] \]

If both emitter and observer are at rest in comoving coordinates, then time intervals in their respective frames are related by

\[ \Delta t = \Delta t(1 + z) \]

and thus we obtain the expression for the redshift drift:

\[ \Delta z = [1 + z - E(z)]H_0\Delta t_0 \]

\[ \text{where} \]

\[ E(z) \equiv \frac{H(z)}{H_0} \]

The exact functional form is dependent on the equation of state and density of the cosmological fluid components, but an order of magnitude estimate

\[ \Delta z \approx 1 \times 10^{-10}\frac{H[1 + z - E(z)]}{\text{yr}} \left( \frac{\Delta t_0}{\text{yr}} \right) \]

using the density parameters determined from WMAP+BAO+SNIa and evaluated, for example at \( z = 3 \), gives an estimated cosmic signal of \( |\Delta z| \sim 4 \times 10^{-10} \), or \( \Delta v \sim 3 \times 10^{-2} \) m/s, per decade between epochs of observation.

Sandage (1962) acknowledged that the required spectral resolution (~1 cm/s per decade cadence) was beyond the capabilities of the instrumentation of that generation. The redshift drift and its potential to constrain cosmological parameters have been studied within the contexts of various cosmological models (Ebert & Trümper 1975; Rüdiger 1980; Lake 1981), but without hope of detection.

3.1 The new challenge

Loeb (1998) identified the Ly\( \alpha \) forest as the best candidate for a redshift drift experiment. The Ly\( \alpha \) forest is a phenomenon observed in the spectra of quasars as a series of absorption lines bluewards of the quasar Ly\( \alpha \) emission line, each at a rest wavelength of \( \lambda_{Ly\alpha} = 1216 \) Å. The absorption lines are fingerprints of intervening neutral hydrogen clouds. Although absorbers may lie anywhere between the observer and the quasar, they are not detected or distinguishable below a certain redshift cut-off; the reasons are two-fold. Quasar spectra are usually observed by ground-based optical telescopes, but low-redshift absorption lines lie in the ultraviolet, so only absorbers at \( z \gtrsim 1.7 \) are detected.\(^2\) Contamination by the Ly\( \beta \) emission line (\( \lambda_{Ly\beta} \equiv 1025 \) Å), and the associated Ly\( \beta \) forest, causes confusion below \( 1 + z = (1 + z_0) \lambda_{Ly\beta}/\lambda_{Ly\alpha} \). The redshift range thus probed with this experiment does not include the current acceleration phase. The Ly\( \alpha \) forest supplies a large sample of spectral lines at high redshift, so while the linewidths are relatively large (~20 km/s) 1\(^-\) 1, and the cosmic signal weak, the sheer number density of lines per redshift bin should yield the necessary statistical accuracy. Loeb found that existing spectroscopic techniques, such as those employed in extrasolar planet searches, could produce a marginal detection.

This experiment has become one of the science drivers for next generation 30–60 m extremely large telescopes (ELTs). The instrumental challenges for the COSmic Dynamics Experiment (CODEX) spectrograph proposed for the European-ELT have been discussed by Pasquini et al. (2005), Murphy et al. (2007) and Corasaniti et al. (2007); the latter finds CODEX capable of detecting the redshift drift in the Ly\( \alpha \) forest over ~10 yr. Liske et al. (2008, hereafter L08) conducted an extensive study into the feasibility of detection with a 42-m ELT, concluding that ~4000 h of observing time over a 20 yr interval would reveal the redshift drift at a ~3\( \sigma \) significance.

But before the search for a weak signal such as this is undertaken, all possible systematic biases and sources of noise must be accounted for. Peculiar accelerations of the Ly\( \alpha \) clouds or their associated galaxies present perhaps the most obvious obstacle. The magnitude of the noise has been studied by Phillipps (1982), Lake (1982), L08 and Uzan, Bernardeau & Mellier (2008a); after some initial controversy, the accelerations were determined to have little impact on the signal.

The noise from galactic feedback and optical depth variations was evaluated by L08 and found to have a negligible effect. L08 also noted that the evolution of gravitational potential wells in the line of sight was a source of error not yet accounted for. We identify Ly\( \alpha \) absorbers to be culprits; the positions of absorption lines may be shifted by foreground Ly\( \alpha \) clouds, which are effectively weak lenses with non-zero transverse velocities.

4 TRANSVERSE MOVING LENSES

As light passes near a massive object, it is gravitationally lensed, but if the lens gravitational potential field varies during the time of passage, the light experiences a net redshift or blue shift. This leads to the well-known Rees–Sciama effect (Rees & Sciama 1968) in the context of a collapsing galaxy cluster lens; a similar phenomenon occurs when a (non-evolving) lens moves across the line of sight. Birkinshaw & Gull (1983) were the first to describe how lenses with transverse relativistic peculiar velocities asymmetrically distort background emission, and the general relativistic treatment was later developed by Pyne & Birkinshaw (1993).

This effect (hereafter the BG effect) has garnered attention largely in the context of galaxy clusters moving across the sky, inducing secondary anisotropies in the CMB brightness temperature (Birkinshaw & Gull 1983; Birkinshaw 1989; Aghanim et al. 1999).\(^3\) The influence of cluster lens transverse motion on weakly and strongly lensed background galaxies has also been investigated (Molnar & Birkinshaw 2003).

The induced change in wavelength by the BG effect is proportional to the lens velocity and deflection angle. For a given lens speed, the magnitude of the change in wavelength is maximized if

\(^2\) Although space-based ultraviolet spectrographs may detect the low-redshift Ly\( \alpha \) forest, they would not have the stability required for such an experiment; the ultrastable high-resolution instrumentation is necessarily ground based.

\(^3\) The induced CMB anisotropy derived by Birkinshaw & Gull (1983) was incorrect by a factor of two. This was corrected by Birkinshaw (1989).
the lens is moving in the plane of the sky and towards (or away from) the image of the background source. For the purposes of this study, we need only consider this special case. We thus have

$$\frac{\delta \lambda}{\lambda} = \gamma \frac{v}{c} \hat{\alpha},$$

(2)

where \( v \) is the lens speed, \( \gamma \) is the Lorentz factor associated with the total lens velocity and \( \hat{\alpha} \) is the deflection angle. This manifests as a change in the observed redshift

$$\Delta z = (1 + z_s) \frac{\delta \lambda}{\lambda},$$

(3)

$$= (1 + z_s) \gamma \frac{v}{c} \hat{\alpha},$$

where \( z_s \) is the source (not lens) redshift. It is useful to consider the order of magnitude of the redshift:

$$\Delta z \approx 1.6 \times 10^{-8} (1 + z_s) \gamma \frac{v}{(1000 \text{ km s}^{-1})} 1 \text{ arcsec}.$$  

A typical cluster lens (\( v \sim 1000 \text{ km s}^{-1}, \hat{\alpha} \sim 30 \text{ arcsec} \)) can change the redshift of a source at \( z \sim 3 \) by \( \Delta z \sim 2 \times 10^{-6} \) or \( \Delta v \sim 150 \text{ m s}^{-1} \), while a galaxy lens may produce an effect an order of magnitude smaller.

In this paper, we interpret Ly\( \alpha \) absorbers as a series of numerous moving lenses and consider the impact of all foreground absorbers on the observed wavelengths of absorption lines from more distant Ly\( \alpha \) clouds. In the context of the redshift drift, however, we must determine the differential BG effect, i.e. how the shift in the observed wavelengths at the second epoch of observation compare to the shift at the first epoch. Thus we will consider the transverse accelerations, \( a_x \), as well.

5 THE EFFECT OF LYMAN-\( \alpha \) CLOUDS ON THE OBSERVED REDSHIFT DRIFT

We consider here the dynamic nature of Ly\( \alpha \) clouds and how, as lenses, they can affect the redshift drift observed over some time interval. The expression we will derive below is parametrized by the cadence between observations, the number of Ly\( \alpha \) absorbers, their peculiar velocities and accelerations as well as the deflection angle they induce.

5.1 Ly\( \alpha \) clouds

Ly\( \alpha \) clouds are typically distinguished by their neutral hydrogen column densities, \( N_{\text{HI}} \), in units of cm\(^{-2}\). Beyond \( \log(N_{\text{HI}}) \gtrsim 17 \), the optically thick nature of the absorbers results in a discontinuity at (bluewards of) 912 Å (rest frame); these absorbers are classified as Lyman limit systems (LLSs). Clouds with \( \log(N_{\text{HI}}) \gtrsim 20 \) exhibit damping wings about the absorption line profile leading to large Ly\( \alpha \) equivalent widths; these are labelled damped Ly\( \alpha \) systems (DLAs).

The exact nature of these absorbers is somewhat a mystery [see Rauch (1998) for a review]. Low column density absorption occurs in filamentary structures, although overdense knots will move along these, as seen in cosmological N-body simulations. In this context, equation (2), which strictly assumes a compact lens, would require an additional term to compensate for the extended shapes of the overdensities, rendering the BG effect less pronounced. The BG effect here may also arise from galaxy clusters that lie at the ends of the associated filaments. A moderate-sized cluster (\( 10^{13} \text{ M}_\odot \) galactic halo (\( v \sim 300 \text{ km s}^{-1}, \hat{\alpha} \sim 4 \text{ arcsec} \)) could alter positions of background absorption lines at redshift \( z \) by as much as \( \Delta z \sim 2 \times 10^{-8} (1 + z) \). This is indicative of the BG effect as can be expected from the high column density absorbers, such as LLSs and some DLAs.

The lower column density absorbers are associated with galaxy haloes and discs responsible for LLSs and DLAs being detected over only 10–100 km s\(^{-1}\). Beyond \( \log(N_{\text{HI}}) \sim 20 \), a galaxy lens may produce an effect an order of magnitude smaller.

5.2 Velocities of lenses

Previous studies have provided insight into the peculiar motions of Ly\( \alpha \) clouds. Rauch (2005) measured the velocity shear between pairs of absorbers common to multiple images of gravitationally lensed quasars; their lines of sight are close together. Their results indicate that while the large-scale motions correspond to the Hubble flow, there is tentative evidence for the clouds undergoing gravitational collapse. L08 used hydrodynamic simulations of the intergalactic medium (IGM) to determine the peculiar motions of Ly\( \alpha \) absorbers. They find that the distribution of peculiar velocities peak at \( v_{\text{pec}} \sim 190–200 \text{ km s}^{-1} \) for gas at \( 2 \lesssim z \lesssim 4 \) with an approximate dispersion of \( \sigma_{v_{\text{pec}}} \sim 80 \text{ km s}^{-1} \).

A single high-velocity cloud that is present in the line of sight, with absorption taking place at an impact parameter of 1 kpc from a \( 10^{11} \text{ M}_\odot \) galactic halo (\( v \sim 300 \text{ km s}^{-1}, \hat{\alpha} \sim 4 \text{ arcsec} \)), could alter the positions of background absorption lines at redshift \( z \) by as much as \( \Delta z \sim 2 \times 10^{-8} (1 + z) \). This is indicative of the BG effect as can be expected from the high column density absorbers, such as LLSs and some DLAs.

5.3 Time interval between observation epochs

The duration of the interval between observations of quasar spectra has by far the largest impact on the chance to detect the cosmic signal. The chosen cadence must clearly be large enough to allow for some appreciable evolution in the scalefactor (over all redshifts \( 2 \lesssim z \lesssim 5 \)). We will, however, still have \( H_0 \Delta t \sim 1 \text{ yr} \) and \( \Delta z \ll 1 \). By combining the signal from \( \sim 100 \) uncorrelated quasars, the signal may be detected over only \( \Delta t \sim 10–20 \text{ yr} \) (Loeb 1998; Corasaniti et al. 2007; Liske et al. 2008). These intervals are proposed in the context of spectrographs mounted on 30–60 m ELTs proposed for the near future.

5.4 The observed redshift drift

Here, we derive the observed redshift drift over some time interval and compare this to the drift resulting purely due to cosmic deceleration. Fig. 1 shows how the wavelength of a photon varies as it
travels from a distant source to an observer being lensed by moving \( \text{Ly} \alpha \) clouds along the way. We have \( 0 < z_i < z_j < z_q \) for \( i < j \), although we are not particularly concerned with the redshift of the source quasar nor the wavelength of the photon upon emission, \( \lambda_* \).

We are, instead, interested in the redshift of an absorption line with redshift, \( z \), and that which is observed, \( z_{\text{obs}} \). The true redshift is given by

\[
1 + z^* = \frac{R(t_0)}{R(t)}
\]

while the observed redshift is defined as:

\[
1 + z_{\text{obs}} = \frac{\lambda_{\text{obs}}}{\lambda}.
\]

As the photon travels from cloud \( j \) (or the source quasar) to the next cloud \( j - 1 \) (or observer), it is redshifted as a result of the difference in scalefactors at the two clouds. Thus, the (true) redshifts of the two clouds are related by

\[
\frac{1 + z^*_{j - 1}}{1 + z^*_{j}} = \frac{\lambda_{j - 1}}{\lambda_j}.
\]

We recognize that the true redshift of the \( N \)th cloud is the product of the ratios of scalefactors at consecutive pairs of foreground clouds, and so

\[
1 + z^*_N = \prod_{i=1}^{N} \frac{\lambda_{i - 1}}{\lambda_i} = \frac{\lambda_{\text{obs}}}{\lambda_N} \prod_{i=1}^{N} \frac{\lambda_i}{\lambda_q} = \left( 1 + z_{\text{obs}}^* \right) \left[ 1 - \sum_{i=1}^{N} \frac{\delta \lambda_i}{\lambda_i} \right],
\]

since \( \delta \lambda \ll \lambda \). We have defined \( \lambda_0 \equiv \lambda_{\text{obs}} \). After some time interval \( \Delta t_0 \) (in the observer rest frame), the true redshift has evolved to be:

\[
1 + z^*_N + \Delta z^*_N \approx \frac{R_0 + R_0 \Delta t_0}{R_N + R_N \Delta t_N} = (1 + z^*_N) \left[ 1 + \frac{H_0 \Delta t_0}{1 + H(z^*_N) \Delta t_N} \right] \approx 1 + z^*_N + H_0 \Delta t_0 [1 + z^*_N - E(z^*_N)].
\]

where \( \Delta t_N \) denotes the time passed in the rest frame of the \( N \)th cloud:

\[
\Delta t_0 = \Delta t_N (1 + z^*_N).
\]

We observe a drifted redshift, analogous to equation (5):

\[
1 + z^*_N + \Delta z^*_N = (1 + z_{\text{obs}}^* + \Delta z_{\text{obs}}^*) (1 - S) - (1 + z_{\text{obs}}^*) S_{\Delta t},
\]

where the summations are denoted thus:

\[
S = \sum_{i=1}^{N} \frac{\delta \lambda_i}{\lambda_i},
\]

\[
S_{\Delta t} = \Delta t_0 \sum_{i=1}^{N} \frac{\alpha_i}{c} \dot{\alpha}_i
\]

with \( \alpha_i \) the tangential peculiar acceleration of the \( i \)th \( \text{Ly} \alpha \) cloud.

Any change in the deflection angle is dependent on the mass profile of the cloud and vanishes in the case of a singular isothermal sphere. In any case, the impact parameter changes by only a fraction of a parsec so we neglect this term.

We subtract the observations at the two epochs to find the observed drift rate, i.e. equate equations (6) and (7) and subtract equation (5):

\[
H_0 \Delta t \left[ 1 + z^*_N - E(z^*_N) \right] = \Delta z_{\text{obs}}^* (1 - S) - (1 + z_{\text{obs}}^*) S_{\Delta t}.
\]

As a sanity check, if we neglect the effect of all clouds, we recover the McVittie expression (equation 1).

### 5.5 The magnitude of the noise

Equation (8) describes how the observed redshift drift after a time interval \( \Delta t_0 \) differs from the drift expected purely due to the evolution of the expansion rate. The peculiar velocities of each absorber lens are contained within the summation \( S \); even in the absence of peculiar acceleration, the velocities will affect the observed magnitude of the redshift drift by entering the expression as a fractional offset. The accelerations are contained within the summation \( S_{\Delta t} \), which stands alone as an absolute error. Below, we estimate the order of magnitude of these effects.

Each term in \( S \) and \( S_{\Delta t} \) may be positive or negative depending on the direction of motion and acceleration of the \( \text{Ly} \alpha \) cloud. Thus the uncertainties are in the form of noise, rather than a systematic bias; they are the result of a random walk. We would expect the noise to increase with absorption line redshift as there would be more lenses in the foreground.

The denser absorbers, such as LLSs, may produce deflection angles of at most a few arcseconds, and at speeds of \( v \sim 300 \text{ km s}^{-1} \) would dominate \( S \) with \( \Delta \lambda \ll \lambda \sim 10^{-8} \), although we are unlikely to find more than one or two in a single quasar spectrum. Even if we coupled the high column density absorbers with the expected large number of low column density absorbers (\(~1000\) per line range).
of sight), and coincidentally aligned the directions of motions, we would still find $|S| \ll 1$. If $S$ was non-negligible, the net result would depend on how the lines are binned for the experiment. If the binned absorption lines come from one spectrum only, then $S$ for each line will be highly correlated, and a strong bias will be present. However, if multiple spectra are observed for the same experiment, which is likely to be the case, the value of $S$ for each spectrum would be independent, even for a particular redshift bin, and thus the effect would still average out.

L08 also plotted the narrow distribution of peculiar acceleration of the IGM within hydrodynamic simulations, establishing a tight peak at $d\sigma_{\text{pec}} \sim 10^{-11} \text{ cm s}^{-2}$. Again, coupling this acceleration with a large deflection angle of a few arcseconds, a large number density ($\sim 1000$) and a time interval of 20 yr between epochs of observation, we find $S_{\text{pec}} \sim 10^{-15}$. Though this appears as an absolute error in the measurement, the redshift drift expected is $\Delta z \sim 10^{-10}$, many orders of magnitude larger. Again, binning within a single spectrum can amplify this offset, but the use of multiple spectra renders the offset small again, leaving the signal unaffected.

6 CONCLUDING REMARKS

The redshift drift experiment has been recognized as a direct test of the cosmic expansion, and carries with it the potential to independently constrain cosmological parameters and distinguish between evolving dark-energy models. The proposed next generation of ELTs and ultrastable high-resolution spectrographs make the detection of the cosmic signal possible in the near future. However, as the signal is weak, it is necessary to account for all possible systematic biases and sources of noise. One source of noise that has, until now, remained unexplored is the variation in the gravitational field due to the transverse motion of Lyα absorbers. In this paper, we have examined this effect and characterized and quantified the noise introduced to the signal. To summarize, we find that the peculiar velocities and accelerations of the Lyα clouds both enter the expression relating the observed redshift to the cosmic signal as noise terms; the former introduces a fractional offset, the latter absolute. The evolution of the gravitational potential wells in the line of sight due to transverse motion of the Lyα absorbers will not, in fact, introduce significant noise or bias into the detection of the cosmic signal.

We consider now whether this is the final word on evolving gravitational fields. Lyα absorbers were the most obvious source of foreground interference; they, by definition, intersect the line of sight. Galactic haloes at impact parameters of 1–20 kpc are thought to produce the rarer absorption effects, such as LLs and DLAs; these have been the extreme cases studied in this work. Estimates of the frequency and redshift distribution of these objects are hindered by reddening, which can dim quasars and exclude them from flux-limited samples, although the magnification bias can counteract this effect. Both effects can be expected, particularly if the absorbers are massive galaxy haloes at low impact parameters. The resulting evolution of the gravitational field within the filaments and sheets associated with the low-density Lyα forest is difficult to study analytically. The filaments feed into more massive structures, such as galaxy clusters, with absorption taking place at impact parameters of $\sim 1$ Mpc. The transverse peculiar motion of such objects will have a similar ‘moving-lens’ effect, the rarity of which has not been assessed. Their presence may not be noticed in the quasar spectra, though perhaps in direct imaging (at least at low redshift). Further studies using dark matter simulations will probe the expected motions of all mass scales in the vicinity of the line of sight to distant quasars, allowing the determination of the full influence of the BG effect in future redshift drift experiments; this will form the basis of future contributions.

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REFERENCES

Aghanim N., Prunet S., Forni O., Bouchet F. R., 1998, A&A, 334, 409
Bahcall J., Wolf R. A., 1968, ApJ, 152, 701
Balbi A., Quercellini C., 2007, MNRAS, 382, 1623
Bartelmann M., Loeb A., 1996, ApJ, 457, 529
Battistelli E. S. et al., 2002, ApJ, 580, L101
Birkhainsh M., 1989, in Moran J., Hewitt J., Lo K. Y., eds, Moving
  Gravitational Lenses. Springer-Verlag, Berlin, p. 59
Birkhainsh M., Gull S. F., 1983, Nat, 302, 315
Blondin S. et al., 2008, ApJ, 682, 724
Corasaniti P.-S., Huterer D., Melchiorri A., 2007, Phys. Rev. D., 75, 062001
De Petris M. et al., 2002, ApJ, 574, L119
Ebert R., Trümper M., 1975, Ann. New York Acad. Sci., 262, 470
Fabbri R., Melchiorri F., Natale V., 1978, Ap&SS, 59, 223
Freedman W. L. et al., 2001, ApJ, 553, 47
Goldhaber G. et al., 2001, ApJ, 558, 359
Hobson M. E., Efstathiou G., Lasenby A. N., 2006, General Relativity: An
  Introduction for Physicists. Cambridge Univ. Press, Cambridge
Hubble E., 1929, Proc. Natl. Acad. Sci., 15, 168
Hubble E., Tolman R. C., 1935, ApJ, 82, 302
Kamenschik A., Moschella U., Pasquier V., 2001, Phys. Lett. B, 511, 265
Komatsu E. et al., 2009, ApJS, 180, 330
Kowalski M. et al., 2008, ApJ, 686, 749
Lake K., 1981, ApJ, 247, 17
Lake K., 1982, Astrophys. Lett., 23, 23
Lake K., 2007, Phys. Rev. D, 76, 063508
Leibundgut B. et al., 1996, ApJ, 466, L21
Liske J. et al., 2008, MNRAS, 386, 1192
Loeb A., 1998, ApJ, 499, 111
Lubin L. M., Sandage A., 2001, AJ, 122, 1084
Luzzi G., Shimon M., Lamagna L., Rephaeli Y., De Petris M., Conte A., De
  Gregori S., Battistelli E. S., 2009, ApJ, in press (arXiv:0909.2815v1)
Mather J. C., Fixsen D. J., Shafer R. A., Mosier C., Wilkinson D. T., 1999,
  ApJ, 512, 511
McVittie G. C., 1962, ApJ, 136, 334
Ménard B., 2005, ApJ, 630, 28
Molaro P., Levshakov S. A., Dessauges-Zavadsky M., D’Odorico S., 2002,
  A&A, 381, L64
Molnar S. M., Birkinshaw M., 2003, ApJ, 586, 731
Murphy M. T. et al., 2007, MNRAS, 380, 839
Nakamura T., Chiba T., 1999, MNRAS, 306, 696
Pahre M. A., Djorgovski S. G., de Carvalho R. R., 1996, ApJ, 456, L79
Pasquini L. et al., 2005, The Messenger, 122, 10
Percival W. J., Cole S., Eisenstein D. J., Nichol R. C., Peacock J. A., Pope
  A. C., Szalay A. S., 2007, MNRAS, 381, 1053
Pierlmann S. et al., 1999, ApJ, 517, 565
Phillipp S., 1982, Astrophys. Lett., 22, 123
Pyne T., Birkhainsh M., 1993, ApJ, 415, 459
Quartin M., Amendola L., 2009, preprint (arXiv:0909.4954v1)
Rauch M., 1998, ARA&A, 36, 267
Rauch M., 2005, ApJ, 632, 58
Rees M. J., Sciama D. W., 1968, Nat, 217, 511
Rephaeli Y., 1980, ApJ, 241, 858
Riess A. G. et al., 1998, ApJ, 116, 1009
Riess A. G. et al., 2004, ApJ, 607, 665
Rüdiger R., 1980, ApJ, 240, 384
Sandage A., 1962, ApJ, 136, 319
Shapiro C., Turner M. S., 2006, ApJ, 649, 563
Songaila A., Cowie L. L., Hogan C. J., Rugers M., 1994, Nat, 368, 599
Srianand R., Petitean P., Ledoux C., 2000, Nat, 408, 931
Tolman R. C., 1930, Proc. Natl. Acad. Sci., 16, 511
Tolman R. C., 1934, Relativity, Thermodynamics and Cosmology. Oxford Univ. Press, Oxford
Uzan J.-P., Bernardeau F., Mellier Y., 2008a, Phys. Rev. D, 77, 021301
Uzan J.-P., Clarkson C., Ellis G. F. R., 2008b, Phys. Rev. Lett., 100, 191303
Wilson O. C., 1939, ApJ, 90, 634

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