Suggested New Modes in Supersymmetric Proton Decay

K.S. Babu\textsuperscript{a,1}, Jogesh C. Pati\textsuperscript{a,b,2} and Frank Wilczek\textsuperscript{a,3}

\textsuperscript{a}School of Natural Sciences, Institute for Advanced Study, Princeton, NJ, USA 08540

\textsuperscript{b}Department of Physics, University of Maryland, College Park, MD, USA 20742

Abstract

We show that in supersymmetric unified theories such as $SO(10)$, implementation of the see–saw mechanism for neutrino masses introduces a new set of color triplet fields and thereby a new source of $d = 5$ proton decay operators. For neutrino masses in a plausible range, these operators are found to have the right strength to yield observable, but not yet excluded, proton decay rates. The flavor structure of the new operators is distinctive. Proton decay modes into a charged lepton, such as $\ell^+\pi^0$, $\ell^+K^0$ and $\ell^+\eta$ where $\ell = e$ or $\mu$, can become prominent, even for low or moderate values of $\tan\beta \lesssim 10$, along with the $\pi K^+$ and $\pi\pi^+$ modes. A distinctive feature is the charged lepton modes involving an $e^+$ and/or a $\mu^+$ with the ratio $\Gamma(\ell^+K^0) : \Gamma(\ell^+\pi^0) \simeq 2 : 1$.

\textsuperscript{1}Email: babu@ias.edu, \textsuperscript{2}pati@umdhep.umd.edu, \textsuperscript{3}wilczek@ias.edu
Proton decay, if discovered, will constitute impressive evidence for the placement of quarks and leptons in common multiplets and for the unification of the separate gauge interactions of the Standard Model \[1\]. Already, the fact that the three gauge couplings meet at a common scale \(M_X \approx 2 \times 10^{16} \text{ GeV} \[2\], provided they are extrapolated from their measured values in the context of supersymmetry, supports the idea of supersymmetric unification.

Supersymmetric unified theories (GUTs), however, bring two new features to proton decay: (i) First, by raising \(M_X\) to a higher value as above, they strongly suppress the gauge–boson–mediated \(d=6\) proton decay operators, for which \(e^+\pi^0\) would have been the dominant mode. (In the most straightforward interpretation one obtains \(\tau(p \to e^+\pi^0)|_{d=6} \approx 10^{36 \pm 1.5} \text{ yr.}, \) where the uncertainty reflects those from the hadronic matrix element and from the masses of the relevant gauge bosons.) (ii) Second, they generate \(d=5\) proton decay operators \[3\] of the form \(Q_iQ_jQ_kL_l/M\) in the superpotential, through the exchange of color triplet Higgsinos, which are the GUT partners of the standard Higgs(ino) doublets. Assuming that the color triplets acquire heavy GUT–scale masses, while the doublets remain light, these “standard” \(d=5\) operators, suppressed by just one power of the heavy mass and the small Yukawa couplings, provide the dominant mechanism for proton decay in supersymmetric GUT, with a lifetime \(\tau_p \sim (10^{30} - 10^{35}) \text{ yr.} \[4, 5, 6, 7\]. This range is consistent with present limits, and might be within reach of SuperKamiokande.

The flavor structure of the standard \(d=5\) operators are constrained by three factors: (a) Bose symmetry of the superfields in \(QQQL/M\), (b) color antisymmetry, and especially (c) the hierarchical Yukawa couplings of the standard Higgs doublets. Because of these, it turns out that these operators lead to a strong preference for the decay of the proton into channels involving a \(\nu\) rather than \(e^+\) or (even) \(\mu^+\) and those involving an \(s\) rather than an \(d\) \[4\]. Thus they lead to dominant \(\nu\bar{K}^0\) and comparable \(\nu\pi^+\) modes and in some circumstances (i.e., for large \(\tan\beta > 40\)) to prominent \(\mu^+K^0\) mode; but in all cases to highly suppressed \(e^+\pi^0\) and \(e^+K^0\) decay modes. For example, in minimal \(SU(5)\), with contributions only from the standard \(d=5\) operators, one finds that for \(\tan\beta \lesssim 10\) (see eg., Ref. \[5\] [4]):

\[
\begin{align*}
\frac{\Gamma(e^+K^0)}{\Gamma(\nu\bar{K}^0)_{\text{std}}} &\approx \left( \frac{m_u m_d}{m_c m_s \sin \theta_C} \right)^2 \frac{R_{eK}}{|(1 + Y_{tK})|^2} \approx 1.2 \times 10^{-7}, \\
\frac{\Gamma(e^+\pi^0)}{\Gamma(\nu\bar{K}^0)_{\text{std}}} &\approx \left( \frac{m_u m_d}{m_c m_s} \right)^2 \frac{R_{e\pi}}{|(1 + Y_{tK})|^2} \approx 6 \times 10^{-8}, \\
\frac{\Gamma(\mu^+K^0)}{\Gamma(\nu\bar{K}^0)_{\text{std}}} &\approx \left( \frac{m_u}{m_c \sin^2 \theta_C} \right)^2 \frac{R_{\mu K}}{|(1 + Y_{tK})|^2} \approx 7 \times 10^{-4}, \\
\end{align*}
\]
Here $R_{e\pi} \simeq R_{\mu\pi} \simeq 1.2$ and $R_{eK} \simeq R_{\mu K} \simeq 0.12$ are the products of the matrix element and the phase space factors for $e^+\pi^0$ mode etc., relative to the $\nu_\mu K^+$ mode. The factor $Y_{tK}$ refers to the third family contribution relative to the second, $|Y_{tK}| \simeq |(m_t V_{td} V_{ts})/(m_c V_{cs} V_{cd})|$, and we have neglected any flavor dependence in the squark/slepton masses in writing Eq. (1).

The purpose of this note is to point out that there exists a new set of color triplets and thereby plausibly a new source of $d = 5$ operators, in supersymmetric unified models like $SO(10)$ [8], which assign heavy Majorana masses to the right-handed neutrinos to generate light neutrino masses via the see-saw mechanism [9]. With a desirable pattern of the neutrino masses, in accord with the MSW solution for the solar neutrino puzzle and $\nu_\tau$ serving as the hot component of dark matter, these new $d = 5$ operators are found to compete favorably with the standard ones described above. At the same time, the flavor structure of the new operators, related to the neutrino masses, appear to be rather universal, and different from the standard ones. These new operators allow in general, prominent or even dominant charged lepton decay modes of the proton—i.e., $p \to \ell^+\pi^0$, $\ell^+K^0$, and $\ell^+\eta$, where $\ell^+ = e^+$ or $\mu^+$, even for low values of $\tan\beta \lesssim 10$, along with the neutrino modes $p \to \nu K^+$ and $\nu\pi^+$. A distinguishing test of the new mechanism is provided by the prominence of the charged lepton modes involving an $e^+$ and/or a $\mu^+$, together with the prediction $\Gamma(\ell^+K^0) : \Gamma(\ell^+\pi^0) \simeq 2 : 1$. This, as we will discuss, can distinguish the new contributions not only from the standard $d = 5$ operators, but also from certain gauge boson mediated effects.

2. Using standard notations for quark and lepton doublets and also singlets, the Yukawa couplings of a color triplet $(H_C)$ and antitriplet $(H'_C)$ are given by the superpotential

$$W_{\text{Yukawa}}(H_C, H'_C) = F_{ij} \left[\frac{1}{2} Q_i Q_j + u^c_i \ell^+_j\right] H_C + G_{ij} \left[Q_i L_j + u^c_i d^c_j\right] H'_C ,$$

where $i, j$ are family indices. In the minimal $SU(5)$ model, $F$ and $G$ are the usual Yukawa coupling matrices of the standard Higgs doublets with the up and the down quarks respectively. To allow for a different flavor structure in the couplings of new color triplets, we shall keep $F$ and $G$ general.

After integrating out $H_C$ and $H'_C$ superfields, the effective $\Delta B \neq 0$ superpotential is

$$W_{\Delta B \neq 0} = \frac{1}{M_C} F_{ij} G_{kl} \left[\frac{1}{2} (Q_i Q_j)(Q_k L_l) + (u^c_i \ell^+_j)(u^c_k d^c_l)\right]$$

(3)
where $M_C$ is the mass of the superheavy color triplet Higgsino ($W \supset M_C H_C H_C'$).

The $SU(3)$ and $SU(2)$ contractions in Eq. (3) are as follows:

$$
(Q_i Q_j)(Q_i L_l) = \epsilon_{\alpha\beta\gamma}(u_\alpha^i d_\beta^j - d_\alpha^i u_\beta^j)(u_\gamma^l \ell_l - d_\gamma^l \nu_l)
$$

$$(u_\ell^i \ell_j^l)(u_\nu^i \nu_j^l) = \epsilon_{\alpha\beta\gamma}(u_\alpha^i \ell_j^l)(u_\gamma^l \nu_j^l) .$$

(4)

In terms of component fields, Eq. (3) corresponds to a vertex with 2 fermions and 2 scalars. For proton decay, the two scalars (which are heavier than the proton) should be converted to ordinary fermions by dressing the vertex with a wino or a gluino. The contributions of the gluino, which conserves flavor, turn out to be suppressed, compared to those of the wino except for the case of large tan$\beta \sim 40$ (see e.g., Ref. [4]). For the case of dominant wino contributions, which is what we will mostly consider, only the first term $(QQ)(QL)$ in Eq. (3) is relevant.

3. Neutrino masses and new dimension–5 proton decay operators: Now let us identify new candidates for $H_C, H_C'$ related to neutrino masses. Majorana masses for the right–handed neutrinos, which are needed to implement the see–saw mechanism, can arise in $SO(10)$ by utilizing the vacuum expectation value (VEV) of either a $\mathbf{126}_H$ or a $\mathbf{16}_H$. In the case of $\mathbf{126}_H$, one can use the renormalizable coupling to matter multiplets ($\mathbf{16}_i$) of the form $f_{ij}(\mathbf{16}_i \mathbf{16}_j)\mathbf{126}_H$; while for the case of $\mathbf{16}_H$, one needs to use the effective higher dimensional operator $\tilde{f}_{ij}(\mathbf{16}_i \mathbf{16}_j)(\mathbf{16}_H \mathbf{16}_H)/M$. Either will generate $d = 5$ proton decay operators. We will now discuss each, in turn.

The case of $\mathbf{126}_H$: In this case, the relevant standard model singlet in the $\mathbf{126}_H$ that acquires a VEV has the quantum numbers of a di–neutrino “$\nu_R \nu_R$”. This breaks $SO(10)$ to $SU(5)$, and as is well known, it has the advantage that it changes $(B - L)$ by two units and thereby automatically conserves $R$–parity [10]. Such a symmetry neatly forbids potentially dangerous $d = 4$ proton decay operators.

With the $\mathbf{126}_H$ acquiring a VEV, there must exist a conjugate $\mathbf{126}_H$, also acquiring VEV, to cancel the $D$ term. The $\mathbf{126}_H$ however has no coupling to the $\mathbf{16}_i$ because of $SO(10)$. The only relevant coupling is therefore

$$
W_{\mathbf{126}} = f_{ij}(\mathbf{16}_i \mathbf{16}_j) \mathbf{126}_H .
$$

(5)

Here $i, j = 1, 2, 3$ refer to generation indices in the gauge–basis.

The Yukawa couplings $f_{ij}$ may be determined (approximately) as follows. The light neutrino masses are given by the see–saw formula: $m(\nu^L_i) \simeq m(\nu^D_i)^2/M_{iR}$, where $m(\nu^D_i)$ denotes the Dirac mass of the $i$th neutrino, and $M_{iR}$ are related to (but are not equal to) the physical Majorana masses of $\nu_R$. $M_{iR}$ are given in terms of the matrix elements $M_{ij} \equiv f_{ij}(\mathbf{126}_H)$ as\footnote{This pattern emerges if one assumes a hierarchical structure for the Dirac masses without any significant hierarchy in the Majorana elements $M_{ij}$.} $M_{iR} \simeq \{M_{11}, (M_{11}M_{22} -}$
The successful $SO(10)$ mass relation $m_b(M_X) = m_\tau(M_X)$ suggests that at least the third family fermions receive their masses primarily through the Yukawa couplings $16, 16, 10_H$. This in turn implies that $m_{\nu_\tau} \approx m_\tau(M_X) \approx (100 - 120)\text{ GeV}$. The empirical relation $m_\mu(M_X) \approx 3m_\tau(M_X)$ [1], however, suggests that dominant contributions to the masses of the second family come from the Higgs component transforming as $(2, 2, 15)$ of $SU(2)_L \times SU(2)_R \times SU(4)_C \equiv G_{224}$, which contributes in the proportion $(1, 1, 1, -3)$ to the four colors. Such a Higgs component, with a VEV of the electroweak scale, may arise effectively either from the same $126_H$ which gives Majorana masses to the $\nu_R$ (see Eq. (5)), or alternatively, and in fact preferably, through an effective operator $16, 16, 10_H \langle 45_H \rangle / M$. Now $10_H \times 45_H$ contains the desired submultiplet $(2, 2, 15) \subset 120$, which contributes only to the off–diagonal mixing (with $i, j = 2, 3$), as well as a $(2, 2, 1)$ component. Taking both these contributions including the see–saw off–diagonal mixing into account, it can be verified that reasonable fits to the second family masses and $V_{ub}$ can lead to $m_{\nu_\mu} \approx (3 - 12) \times m_e(M_X) \approx (1 - 4)\text{ GeV}$. Although not essential for our arguments, guided by the masses of $u, d,$ and $e$, it seems reasonable to take $m_{\nu_\mu} \approx (1 - 10)\text{ MeV}$.

Thus, with the values of $m_{\nu_\mu} \sim \{(1 - 10)\text{ MeV}, (1 - 4)\text{ GeV} \text{ and } (100 - 120)\text{ GeV}\}$ for $i = e, \mu, \tau$, which are motivated by the observed pattern of masses of the quarks and the leptons, one gets, via the see–saw formula $^{2}$ $m(\nu^c_i) \sim \{(\frac{1}{4} - 30) \times 10^{-9}, (\frac{1}{4} - 5) \times 10^{-3} \text{ and } (2 - 3)\}$ eV, provided $M_{iR}$ are nearly flavor universal, within a factor of 2 to 3, with $M_{1R} \sim M_{2R} \sim M_{3R} \sim (1 - 3) \times 10^{12}\text{ GeV}$. It is interesting that this pattern of masses for the light neutrinos is precisely the one that goes well with the MSW solution for the solar neutrino puzzle, involving $(\nu_e - \nu_\mu)$ oscillations (which requires $m_{\nu_\mu} \approx (2 - 4) \times 10^{-3} \text{ eV}$), and with $\nu_\tau$ serving as the hot component of dark matter.

Thus we see that considerations based on quark–lepton masses as well as neutrino masses suggest — although they do not mandate — a non–hierarchical pattern for the Yukawa couplings of the $126_H$, with a rather universal Majorana mass $M_{iR} \sim (1 - 3) \times 10^{12}\text{ GeV}$. This contrasts with the large hierarchy exhibited in the Yukawa couplings of the $10_H$ to the three families.

In the absence of other information, it is reasonable to take the VEVs of all relevant Higgs fields (e.g., $126_H, 54_H$ and $45_H$) which break $SO(10)$ to the standard model symmetry to be nearly equal to the GUT scale, $M_X \approx 2 \times 10^{16}\text{ GeV}$. This also ensures that the simple meeting of the gauge couplings within the MSSM framework is preserved. With $\langle 126_H \rangle \sim M_X$ and $M_{iR} \approx f_{ij} \langle 126_H \rangle \approx (1 - 3) \times 10^{12}\text{ GeV}$, we

$^{2}$These values of light neutrino masses include a reduction of about 50% owing to their running from the GUT to the electroweak scale.
get $f_{ij} \approx (\frac{1}{2} - 1) \times 10^{-4}$. This leads to a strength for the new $d = 5$ operators (see below) of order $f^2/M_X \sim (10^{-8} \text{ to } 10^{-9})/M_X$, which is of just the right order to yield proton decay rate in an observable range.

To see the origin of the new dimension–5 proton decay operators, let us now examine the decomposition of $126$ under the subgroup $G_{224} \equiv SU(2)_L \times SU(2)_R \times SU(4)_C$: $126 = (1, 3, \overline{10}) + (3, 1, 10) + (2, 2, 15) + (1, 1, 6)$. The real $(1, 1, 6)$ component contains the color triplet which we call $\hat{H}_C$ and an anti–triplet $(\hat{H}_C')$. Note that Eq. (5) contains the di–quark and lepto–quark couplings of the $\hat{H}_C$ and $\hat{H}_C' \subset 126_H$ respectively (compare with Eq. (2)). Observe that for this case $F_{ij} = G_{ij} \equiv f_{ij} = f_{ji}$.

Thus if a $(1, 1, 6).(1, 1, 6)$ mass term is present, there will be dimension–5 proton decay arising from the diagram shown in Fig. 1. In order to break the $SU(5)$ symmetry that is preserved by the VEV of $126_H$, there must exist other Higgs representations. A $45_H + 54_H$ is a simple choice. The invariant couplings

$$W \supset \lambda (126_H 126_H 54_H) + \overline{\lambda} (126_H 126_H 54_H)$$

are then allowed. The $54_H$ acquires a VEV along the $(1, 1, 1)$ direction under $G_{224}$. This supplies the required $(1, 1, 6).(1, 1, 6)$ mass term to induce the new effective dimension–5 operator of Fig. 1. Since there is also a $126_H 126_H$ mass term, the two $(1, 1, 6)$ multiplets coming from $126_H$ and $\overline{126}_H$ will now mix with an angle parameter $\theta$. This means that the effective $M_C$ in Eq. (3) is $[\cos^2 \theta/M_1 + \sin^2 \theta/M_2]$ where $M_{1,2}$ are the two mass eigenvalues of the color triplet system arising from $126_H$ and $\overline{126}_H$.

It will be seen later (Section 5) that the interaction of the $54_H$ in Eq. (6) is desirable in connection with an attractive mechanism for doublet–triplet splitting, to obtain masses for potential Nambu–Goldstone multiplets. Thus there is an intricate link between the neutrino masses, doublet–triplet splitting and the proton decay operators in this case.

Only the $(1, 1, 6)$ component of the $126_H$ contributes to dimension–5 proton decay operator. It is easy to verify that although the $(1, 3, \overline{10})$ and $(3, 1, 10)$–components of
The case of $\mathbf{126}_H$: In this case, the relevant standard model singlet that acquires a VEV has the quantum numbers of a neutrino $\nu_R$. This too breaks $SO(10)$ to $SU(5)$, but it changes $(B - L)$ by one unit. One can define a $Z_2$ discrete symmetry under which $\mathbf{16}_i$’s are odd, but all other multiplets (including $\mathbf{16}_H$ and $\mathbf{10}_H$) are even. This symmetry can serve as $R$–parity.

Now, with $\mathbf{16}_H$ acquiring a VEV, one needs a $\mathbf{16}_H$, acquiring the same VEV, to cancel the $D$–term. $SO(10)$–invariance, together with the $Z_2$ symmetry (under which $\mathbf{16}_i$ is even), now allows the superpotential terms

$$W_{\mathbf{16}} = \frac{1}{M} \tilde{f}_{ij}^{a} (\mathbf{16}, \mathbf{16}) \left( \mathbf{16}_H \mathbf{16}_H \right)_a + \frac{1}{M} \tilde{g}_{ij} (\mathbf{16}, \mathbf{16}) (\mathbf{16}_H \mathbf{16}_H) + M_{ij} \mathbf{16}_H \mathbf{16}_H . \quad (7)$$

Here $a$ in the first term refers to the two possible $SO(10)$–contractions.

While the first two non–renormalizable terms in Eq. (7) might be taken as quasi–fundamental, to be cut off by gravity or string effects at short distances, it is interesting to examine their possible origins through renormalizable operators. A simple way to generate the first term in Eq. (7) that induces the Majorana masses of the $\nu_R$, is via the couplings $\mathbf{16}, \mathbf{45}, \mathbf{16}_H + M_{ij} \mathbf{45}, \mathbf{45}$. With this coupling alone, which appears to be almost inevitable to induce neutrino masses, there are new contributions to $d = 5$ proton decay.

The relevant diagrams are shown in Figures 2 and 3. In Fig. 2, one of the vertices arises from the effective neutrino mass operator, Eq. (7), while the other vertex is the standard operator $\mathbf{16}, \mathbf{16}_j \mathbf{10}_H$ proportional to the down quark mass matrix. The coupling $\mathbf{16}_H \mathbf{16}_H \mathbf{10}_H$ is allowed by all the symmetries, and is also compatible with the doublet–triplet splitting mechanism. This term may be desirable since it modifies the relation $\tan \beta = m_t/m_b$ that often occurs in $SO(10)$ models. ($\tan \beta \sim m_t/m_b$ is generally problematic for the standard $d = 5$ proton decay.) This modification arises...
because the $Y = -1/2$ light Higgs doublet, will now be partly from the $10_H$ and partly from the $16_H$ (see eg., Ref. [12]). The up-down symmetry preserved by the usual $16,16,10_H$ Yukawa couplings will now be broken, resulting in $\tan \beta \neq m_t/m_b$. Fig. 3 makes use of the fact that the light fermions are not entirely in the $16$, but are also contained in $45$, due to the mixing from $16,45,16_H$.

Analogous to the case of $\langle 126_H \rangle$, neutrino masses would determine $\tilde{f}_{ij}$ (approximately) by noting that $M_{iiR} \approx \tilde{f}_{ij} \langle 16_H \rangle^2/M \approx (1-3) \times 10^{12}$ GeV. Thus we find $\tilde{f}_{ij} \langle 16_H \rangle/M \sim (1/2 - 1) \times 10^{-4}$, with no strong hierarchy in its elements. Consequently the new contributions from these terms allow charged lepton decay modes of the proton to become prominent (see below).

The $16,16,16_H 16_H$ term in Eq. (7) is not directly related to neutrino masses. However, it is similar in form to the first term of Eq. (7). It has been used in the past to induce realistic fermion masses and mixings in $SO(10)$ [12]. Indeed, note that with a single $10_H$ coupling to $16,16_j$, the up and down matrices are proportional and the CKM matrix reduces to the identity. To correct these bad $SO(10)$ relations, one needs some additional contributions to the mass matrices. A simple solution is to induce the effective operator $16,16,16_H 16_H$ through the (renormalizable) couplings $16,10_j 16_H + M_{ij} 10_i 10_j$ involving superheavy $10_i$. Then if the mixing term $\langle 16_H \rangle 10_H$ is also present, so that the $SU(2)_L$ doublet from the $16_H$ acquires a VEV, the proportionality relations will be corrected and non-zero CKM mixings will be induced. From a fit to the masses and the CKM mixing angles, one finds that the matrix elements $\tilde{g}_{ij}$ are all of the same order, of the order of the strange quark Yukawa coupling, within a factor of 10, $\tilde{g}_{ij} \sim (10^{-3} - 10^{-4})$. These terms would still leave the bad relations of minimal $SU(5)$ $m_s(M_X) = m_\mu(M_X)$ and $m_d(M_X) = m_e(M_X)$. One possible way to correct them, while simultaneously inducing the CKM mixing angles, is to utilize the couplings $16,16 45_H + 16,16 10_H + 16_H 16_H 10_H$ involving vector-like pairs of matter multiplets in $16 + \overline{16}$ (see Fig. 4). This would induce an
Figure 4: Possible contribution to $16_i 16_j 16_H 16_H$ operator relevant to proton decay and fermion masses.

effective operator $(16_i 16_j)(16_H 16_H)(45_H 45_H)$, which can also serve the purpose of the $\tilde{g}_{ij}$ terms in Eq. (7). In this case, again, one sees from a fit to the quark and lepton masses and mixings, that the effective couplings $\tilde{g}_{ij}$ are nearly universal and of order the strange quark Yukawa coupling.

From all this, it seems natural to assume $\tilde{g}_{ij} \sim \tilde{f}_{ij}$. The $d = 5$ proton decay amplitude (see below) then turns out to have the right strength, as in the case of $\overline{126}_H$, to yield observable rates.

Let us focus on the proton decay operators arising from utilizing all three terms of Eq. (7). The $SU(5) \times U(1)_X$ decomposition relevant to Eq. (7) is: $16 = 1^{-5} + 5^3 + 10^{-1}$ where the superscript indicates the $U(1)_X$ charges. So the first term in Eq. (7) contains the terms

$$\left(1^{-5}_i 1^{-5}_j\right)\left(1^5_H 1^5_H\right) + \left(10^{-1}_i 10^{-1}_j\right)\left(5^{-3}_H 1^5_H\right) \quad (8)$$

while the second term contains

$$\left(\overline{5}_i \overline{5}_j\right)\left(10^{-1} 1^{-5}_H\right) + \left(\overline{5}_i 10^{-1}_j\right)\left(\overline{5}^3_H 1^{-5}_H\right). \quad (9)$$

Once the $1^{-5}_H$ from the $\overline{16}_H$ and the $1^{-5}_H$ from the $16_H$ acquire (equal) GUT–scale VEVs, the first term in Eq. (8) will induce superheavy Majorana masses for the $\nu_R$. The second term in Eq. (8) along with the second term in Eq. (9) will lead to dimension–5 proton decay. (Recall that the $\overline{5}^3_H + \overline{5}^3_H$ do not belong to the Nambu–Goldstone supermultiplet in the $SO(10)/SU(5)$ coset space.) The effective operator is given by Eq. (3), with the identification of $F$ and $G$ matrices with the $\tilde{f} \langle 1^5_H \rangle / M$ and $\tilde{g} \langle 1^{-5}_H \rangle / M$ matrices of Eq. (7) respectively. (The first term in Eq. (9) does not lead to proton decay.) As in the case of $\overline{126}_H$, the coupling matrices $F$ and $G$ are now non–hierarchical. This turns out to be very significant for proton decay.

4. **Proton decay Modes:** We can write down the effective $\Delta B \neq 0$ four–fermion interactions that arise after $W^\pm$ dressing of the operator in Eq. (3). We do this in
Here the color indices \((\alpha, \beta, \gamma)\) have been to emphasize that this need not be so. It has been argued that the third term dominates; one of our main points here has been that all the matter fields in Eq. (10) belong to weak isodoublets. Traditionally \(F\) is not the same as \(d\) and for the neutrinos \(\nu_i' = V_{ij}^{\nu} \nu_j\) with \(V\) and \(V^\ell\) being the CKM matrix in the quark sector and the lepton sector. Then the effective \(\Delta B \neq 0\) four–fermion interaction is

\[
\mathcal{L}_{\Delta B \neq 0} = \frac{1}{M_C 4\pi} \alpha_2 \hat{F}_{ij} \hat{G}_{kl} \epsilon_{\alpha \beta \gamma} \\
\times \left[ (u_i^\alpha d_j^\beta) (d_k^\gamma \nu_l') \left( f(d_i', u_j) + f(u_k, \ell_i^-) \right) \\
+ (d_i^\alpha u_j^\beta) (u_k^\gamma \ell_l^-) \left( f(u_i, d_j') + f(d_k', \nu_l') \right) \\
+ (d_i^\alpha u_k^\beta) (d_j^\gamma \nu_l') \left( f(u_i, d_j') + f(u_j, \ell_l^-) \right) \\
+ (u_i^\alpha d_k^\beta) (u_j^\gamma \ell_l^-) \left( f(d_i', u_k) + f(d_j', \nu_l') \right) \right]. \quad (10)
\]

Here the color indices \((\alpha, \beta, \gamma)\) and the flavor indices \((i, j, k, l)\) are understood to be summed over, and the fermion fields paired together in parentheses are spin–contracted to singlets. \(f\) is a loop integral, with magnitude of \(M_{\text{SUSY}}^{-1}\), defined as

\[
f(a, b) = \frac{M_W}{m_a^2 - m_b^2} \left( \frac{m_a^2}{m_a^2 - m_W^2} \ln \frac{m_a^2}{m_W^2} - [a \to b] \right). \quad (11)
\]

Note that all the matter fields in Eq. (10) belong to weak isodoublets. Traditionally it has been argued that the third term dominates; one of our main points here has been to emphasize that this need not be so.

Since Eq. (10) is in the mass eigenbasis, the Yukawa couplings \(\hat{F}_{ij}\) and \(\hat{G}_{ij}\) are not the same as \(F_{ij}\) and \(G_{ij}\) of Eq. (2). \(\hat{F}\) remains symmetric, it is related to \(F\) by \(\hat{F} = V_u^T F V_u\) while \(\hat{G} = (V_u^T G V_u) V'\), where \(V_u\) is the unitary matrix that rotates the left–handed up quarks in going to the mass eigenbasis. \(V'\) is another unitary matrix that parameterizes the mismatch between the up quark and the charged lepton mass matrices \([13]\), \(V' = V_u^T V_{\ell}\). Note that \(\hat{G}\) is not symmetric.

The proton decay rate and branching ratios can be obtained from Eq. (10). As we have emphasized, several considerations suggest the matrices \(\hat{F}\) and \(\hat{G}\) may not be hierarchical, so proton decay into alternative flavor modes could a priori have similar rates. Consider first the decay into charged leptons. The relevant interactions are the second and the fourth terms in Eq. (10). In the second term, we must put \(j = k = 1\) since the operator has to have only \(u\) quarks. Flavor antisymmetry then requires that \(i = 2\) or 3. Similarly in the fourth term, \(i = j = 1, k = 2, 3\). Noting that \(d_2' \simeq V_{cd} d + V_{cs}s\) and \(d_3' \simeq V_{cd} d + V_{ts}s\) for proton decay, the amplitude for charged lepton decay can be written as

\[
A(p \to \ell^+) \propto \left\{(u^\alpha d^\beta)(u^\gamma \ell^-) \left[ V_{cd} \left( \hat{G}_{11} \hat{F}_{21} - \hat{G}_{21} \hat{F}_{11} \right) + V_{td} \left( \hat{G}_{11} \hat{F}_{31} - \hat{G}_{31} \hat{F}_{11} \right) \right] \\
+ (u^\alpha s^\beta)(u^\gamma \ell^-) \left[ V_{cs} \left( \hat{G}_{11} \hat{F}_{21} - \hat{G}_{21} \hat{F}_{11} \right) + V_{ts} \left( \hat{G}_{11} \hat{F}_{31} - \hat{G}_{31} \hat{F}_{11} \right) \right] \right\}. \quad (12)
\]
Now if the matrices $\hat{F}$ and $\hat{G}$ have no strong hierarchy in their flavor–dependence, then the terms proportional to $V_{td}$ and $V_{ts}$ can be ignored in Eq. (12). The error introduced is only of order $\lambda_C^2 \simeq 1/20$, ($\lambda_C \equiv \sin \theta_C \simeq 0.22$). This observation leads to predictions for the branching ratios of certain (in general) prominent modes, which are independent of the flavor structure in $\hat{F}$, $\hat{G}$:

$$\frac{\Gamma(p \to \ell^+\pi^0)}{\Gamma(p \to \ell^+K^0)} \simeq \sin^2 \theta_C \left(1 - \frac{m_K^2}{m_p^2}\right)^{-2} R . \quad (13)$$

Here $\ell^+ = e^+$ or $\mu^+$ and $R$ is the ratio of the two relevant hadronic matrix element–squared. The chiral Lagrangian estimate for $R$ given in Ref. [5] is

$$R = \frac{|(1 + D + F)|^2}{2 \left|1 - \frac{m_p}{m_H}(D - F)\right|^2} . \quad (14)$$

Using $D = 0.81, F = 0.44$ for the chiral Lagrangian factors and with $m_H = m_A = 1150 \text{ MeV}$ (as in [2]), we obtain $R \simeq 5$. Thus

$$\frac{\Gamma(p \to \ell^+\pi^0)}{\Gamma(p \to \ell^+K^0)} \simeq 0.5 . \quad (15)$$

Another interesting mode is $p \to \ell^+\eta$ for which one has

$$\frac{\Gamma(p \to \ell^+\eta)}{\Gamma(p \to \ell^+\pi^0)} = \left(1 - \frac{m_\eta^2}{m_p^2}\right)^2 \frac{\left|1 - \frac{1}{3}(D - 3F)\right|^2}{|(1 + D + F)|^2} \simeq 0.35 . \quad (16)$$

While all charged lepton modes are expected to have similar rates, the ratios such as $\Gamma(p \to e^+\pi^0)/\Gamma(p \to \mu^+\pi^0)$ cannot be predicted quantitatively since they are sensitive to the flavor structure of $\hat{F}$ and $\hat{G}$:

$$\frac{\Gamma(p \to e^+\pi^0)}{\Gamma(p \to \mu^+\pi^0)} \simeq \frac{\left|\hat{G}_{11}\hat{F}_{21} - \hat{G}_{21}\hat{F}_{11}\right|^2}{\left|\hat{G}_{12}\hat{F}_{21} - \hat{G}_{22}\hat{F}_{11}\right|^2} . \quad (17)$$

If one uses $\mathbf{16}_H$ to generate the $\nu_R$ Majorana masses, the matrices $\hat{G}$ and $\hat{F}$ are independent, so the ratio in Eq. (17) is in general expected to be of order unity. However, if a single $\mathbf{126}_H$ is used for this purpose, one has the asymptotic relation $\hat{G} = \hat{F}V'$. The flavor–dependent renormalization of this relation is small. If in addition the off–diagonal entries in the Jarlskog matrix $V'$ are small, then one has a cancellation in the amplitude for $e^+\pi^0$. In this case, one would expect the $\mu^+\pi^0$ mode to dominate over the $e^+\pi^0$ mode. Note that in the $\mathbf{16}_H$ option, however, since $\hat{G}$ and $\hat{F}$ are independent, the two modes are expected to be comparable even if $V'$ has small off–diagonal entries.
In the case where \( \overline{126}_H \) is used to generate the see–saw neutrino masses, as noted earlier, new \( d = 5 \) operators arise simply from the first term in Eq. (7). Then the factors \( \tilde{F}_{ij} \), being related to the \( \nu_{iR} \) masses, are non–hierarchical, while the factors \( \hat{G}_{ij} \simeq V^*_{ij}m^d_j \) exhibit a hierarchy. The charged lepton modes become prominent even in this case. For the amplitudes, one obtains: 
\[
A(\mu^+K^0) \propto (\hat{F}_{11} - \sin\theta_C\hat{F}_{21})m_s, \quad \text{while} \quad A(\mu^+\pi^0) \propto \sin\theta_C(\hat{F}_{11} - \sin\theta_C\hat{F}_{21})m_s,
\]
where terms of order \( \lambda_C^2 \simeq 1/20 \) have been dropped. Again one finds that \( \Gamma(\mu^+K^0) : \Gamma(\mu^+\pi^0) \simeq 2 : 1 \), as in Eq. (15). A similar remark applies to \( \Gamma(e^+K^0) : \Gamma(e^+\pi^0) \). Now, however, \( A(e^+K^0) \propto (\hat{F}_{21} + \sin\theta_C\hat{F}_{11})m_d \), so that \( \Gamma(\mu^+K^0) \) is expected to be considerably larger than \( \Gamma(e^+K^0) \) (barring fortuitous cancellations).

It is worth noting that, independent of the relative importance of positron modes, the \( \mu^+\pi^0 \) and \( \mu^+K^0 \) modes arising through the new \( d = 5 \) operators can still compete favorably with or supersede the \( \nu K^+ \) modes, for even small or moderate values of \( \tan\beta \lesssim 10 \). By contrast, for the standard \( d = 5 \) operator, the \( \mu^+\pi^0 \) and \( \mu^+K^0 \) modes can be prominent only for very large \( \tan\beta \gtrsim 40 \) through gluino dressing. Thus a study of these decay modes and determination of \( \tan\beta \) could distinguish between the standard and the new \( d = 5 \) operators, even for the case of \( \overline{126}_H \).

The decays \( p \to \nu K^+ \) and \( p \to \nu\pi^+ \) from the new dimension–5 operators have rates comparable with the charged lepton modes. The expectation for \( \Gamma(p \to \nu K^+) / \Gamma(p \to \nu\pi^+) \) is similar to but not exactly the same as the case of minimal supersymmetric \( SU(5) \). The flavor structure relevant for the \( \nu \) decay is given by (in the limit of neglecting terms of order \( \sin\theta_C \simeq 1/20 \)):

\[
A(p \to \nu\ell) \propto \{ (u^a d^\beta)(s^\gamma\nu\ell)[V_{ud}V_{cs}(\hat{F}_{11}\hat{G}_{2\ell} - \hat{F}_{12}\hat{G}_{1\ell}) - V_{cd}V_{cs}(\hat{F}_{22}\hat{G}_{1\ell} - \hat{F}_{12}\hat{G}_{2\ell})] \\
- (u^a s^\beta)(d^\gamma\nu\ell)[V_{us}V_{cd}(\hat{F}_{11}\hat{G}_{2\ell} - \hat{F}_{12}\hat{G}_{1\ell}) - V_{us}V_{cd}(\hat{F}_{11}\hat{G}_{1\ell} - \hat{F}_{12}\hat{G}_{2\ell})] \\
+ (u^a d^\beta)(d^\gamma\nu\ell)[V_{us}V_{cd}(\hat{F}_{11}\hat{G}_{1\ell} - \hat{F}_{12}\hat{G}_{1\ell}) - V_{cd}^2(\hat{F}_{22}\hat{G}_{1\ell} - \hat{F}_{12}\hat{G}_{2\ell})]\}.
\]

Note that in Eq. (18), the \( V_{ud}V_{cs} \) term (the first term) has no mixing angle suppression, but all the remaining terms are suppressed by factors of at least \( \lambda_C \). It turns out that the matrix element for the \( (u^a s^\beta)(d^\gamma\nu\ell) \) is suppressed by a factor \( \simeq \lambda_C \), compared to the \( (u^a d^\beta)(s^\gamma\nu\ell) \) term, leading to an overall suppression factor of \( \sim \lambda_C^2 \) in the terms in the second line. Neglecting these subleading terms one finds

\[
\frac{\Gamma(\nu\pi^+)}{\Gamma(\nu K^+)} \simeq \sin^2\theta_C \left( 1 - \frac{m^2_{\pi^0}}{m^2_K} \right)^{-2} \left| 1 + D + F \right|^2 \left| 1 + \frac{m_{\mu^+}}{3m_H}(D + 3F) \right|^2 \simeq \frac{1}{5}.
\]

This prediction holds if the contributions from the new \( d = 5 \) operators dominate. The analogous number for the standard \( d = 5 \) operators in minimal \( SU(5) \) is near \( \frac{1}{3} \), but there is considerable uncertainty in this case owing to possible cancellation between
the second and the third generation contributions. It is difficult to make quantitative estimate of ratios such as $\Gamma(\ell^+K^0)/\Gamma(\tau K^+)$ etc., arising from the new $d = 5$ operators, since there is some flavor dependence. Comparing the flavor structure in Eq. (12) and Eq. (18), one infers that the rates for both these modes are similar, with the $\tau K^+$ mode slightly preferred over the $\ell^+K^0$ mode, owing to a matrix element enhancement and the availability of final states with all three neutrino flavors. However, there are terms that differ in the two amplitudes which are of order $\lambda_C \approx 1/4.5$, and so it is difficult to make a more precise quantitative estimate.

The decay rate of the neutron can be obtained from a general operator analysis \cite{14}. For example, $\Gamma(p \to \ell^+\pi^0)/\Gamma(n \to \ell^+\pi^-) \approx \frac{1}{2}$ and $\Gamma(n \to \tau K^0)/\Gamma(p \to \tau K^+) \approx 1.8$.

5. Doublet–triplet splitting and the standard $d = 5$ proton decay: An important issue that any realistic GUT model faces is the question of doublet–triplet splitting. While the $SU(2)_L$ doublets in the $10_H$ of $SO(10)$ have to be light in order to trigger electroweak symmetry breaking, their color triplet GUT partners have to remain heavy at the GUT scale, since they mediate proton decay. One attractive feature of supersymmetric $SO(10)$ is the existence of a natural doublet–triplet splitting mechanism \cite{15,16}. This mechanism can bring in a numerical suppression in proton decay arising from the usual $(16,16_j)10_H$ coupling that gives rise to the quark and lepton masses. These operators, which lead to $p \to \tau K^+$ as the dominant mode, are somewhat problematic in supersymmetric $SU(5)$ since the predicted rate is near the experimental limit \cite{3,7}.

The doublet–triplet splitting mechanism in $SO(10)$ utilizes the superpotential couplings

$$W_{DT} = \Lambda 10_H 45_H 10'_H + M' 10'_H 10'_H ,$$

(20)

where the $10_H$ is the Higgs superfield which contains the two Higgs doublets of MSSM and where $10'_H$ is another field with a GUT scale mass $M'$. Once the $45_H$ acquires a VEV along the $(B - L)$ direction, $\langle 45_H \rangle = \text{diag}(a, a, a, 0, 0) \otimes \tau_2$, the color triplet mass matrix $\mathcal{M}$ and the $SU(2)$–doublet mass matrix $\mathcal{M}'$ become

$$\mathcal{M} = \begin{pmatrix} 0 & \Lambda a \\ \Lambda a & M' \end{pmatrix} ; \quad \mathcal{M}' = \begin{pmatrix} 0 & 0 \\ 0 & M' \end{pmatrix} .$$

(21)

This gives GUT scale masses to all the triplets from the $10_H$ and $10'_H$ while one pair of Higgs doublets from $10_H$ remains light. Proton decay amplitude mediated by the color triplets in $10_H$ is now proportional to $(\mathcal{M}^{-1})_{11} = M'/\Lambda^2a^2$, so by choosing $M'$ somewhat smaller than $\Lambda a$, one obtains a numerical suppression of proton decay. $M'$ cannot be too small compared to the GUT scale, however, since that would result in
a large positive contribution to the predicted value of $\alpha_3(M_Z)$. The shift in $\alpha_3(M_Z)$ from the doublet–triplet sector alone is given by

$$\Delta \alpha_3(M_Z) = -\frac{\alpha_3(M_Z)^2}{2\pi} \frac{9}{7} \ln \left( (M^{-1})_{11} M_X \right).$$

(22)

If $(M^{-1})_{11} = [10^{17} \text{ GeV}]^{-1}$, $\Delta \alpha_3 = +0.005$, which might be acceptable. If $(M^{-1})_{11} = [10^{18} \text{ GeV}]^{-1}$ so that proton decay from this operator is unobservable, then $\Delta \alpha_3(M_Z) \simeq 0.011$ which seems excessive and would require a cancellation from some other threshold effects. This implies that $p \to \pi K^+$ cannot be suppressed to an unobservable level, at least in this simple doublet–triplet splitting scheme.

In order for the VEV of $45_H$ to be along the $(B - L)$ direction to a great accuracy, the $45_H$ should not couple or should couple only weakly to the $16_H + \overline{16}_H$ sector. There is a danger of having pseudo–Goldstone multiplets, which could upset the unification of the gauge couplings, if such cross–couplings are prevented. The potential pseudo–Goldstones are the $(3, 2, \frac{1}{6}) + (3^*, 1, -\frac{2}{3}) + H.c.$ components, under $SU(3)_C \times SU(2)_L \times U(1)_Y$, from the $16_H$ and the $45_H$. This issue has been addressed in the case of using $16_H + \overline{16}_H$ to break $SO(10)$ to $SU(5)$ [19]. Here we point out that the same mechanism will work if the $16_H + \overline{16}_H$ is replaced by a $126_H + \overline{126}_H$. The couplings used in Eq. (6) that were relevant for the proton decay operators are precisely the ones that can give masses to all these would–be–Goldstone multiplets. So in this case the proton decay amplitude is closely related to the doublet–triplet splitting mechanism.

6. **Gauge boson–mediated $d = 6$ versus the new $d = 5$ proton decay operators:** So far we have focussed on the $d = 5$ proton decay operators. In the simplest supersymmetric GUT models, the gauge boson mediated $d = 6$ operators are suppressed relative to the $d = 5$ operators. However, enhancement of the $d = 6$ operators could occur for a variety of reasons. Possibility arises in flipped $SU(5) \times U(1)$ [17], in non–supersymmetric two–step breaking of $SO(10)$ models [18], or possibly even in supersymmetric $SU(5)$ [19] and $SO(10)$ models with large threshold corrections, wherein the relevant $(X, Y)$ gauge boson masses are of order $10^{15} \text{ GeV}$. Exchange of these particles would lead to a dominant $e^+\pi^0$ mode, with proton lifetime $\sim (10^{32} - 10^{35})$ yrs., compatible with current limits. We wish to point out that should the $e^+\pi^0$ decay mode of the proton be observed, one can empirically decide whether it has its origin in the gauge–boson–mediated $d = 6$ or in the new $d = 5$ operators discussed here. For the gauge–mediated case, $e^+K^0$ will be strongly suppressed compared to the $e^+\pi^0$ mode, by the Cabibbo angle ($\sin^2\theta_C \simeq 1/20$), phase space ($\simeq 1/2$) and relevant matrix element-squared ($\simeq 1/2$), so that $[\Gamma(e^+K^0)/\Gamma(e^+\pi^0)]_{d=6} \approx 1/80$. By contrast, for the new $d = 5$ operators, we have shown that $e^+K^0$ exceeds $e^+\pi^0$.
rate by about a factor of two.

7. In conclusion the following remarks are in order.

(i) While the new $d = 5$ proton decay operators seem to be best motivated by their link to neutrino masses, we wish to note that the results presented here are more general. Indeed, the prominence of charged lepton modes and the predictions for certain branching ratios depend only on the assumed non–hierarchical nature of $\hat{F}$ and $\hat{G}$ in Eq. (10). This condition might be satisfied in other contexts as well. For example, if the CKM angles are induced in $SO(10)$ by coupling the fermions to two $10_H$ of Higgses, a reasonable fit may be obtained when the $SU(2)_L$ doublets and the color triplets from one of the $10_H$ has non–hierarchical couplings, with strength of order the strange quark Yukawa coupling. The exchange of these color triplets would lead to results similar to the ones presented here.

(ii) As has been discussed in the literature, $d = 5$ proton decay operators such as $QQQL/M$, belonging to $16,16,16,16/16$, could be induced not only by the exchange of GUT–related color triplets, but also by other effects including exchange of the heavy tower of color triplet string states. These are allowed by $SO(10)$ symmetry as well as as $R$–parity (or the $Z_2$ symmetry mentioned in Sec. 3). In any supersymmetric theory, these non–renormalizable operators must somehow be suppressed at least by a factor of $10^{-7}$ (if $M \sim M_{Pl}$) in order not to conflict with observed limits on proton lifetime. For a discussion of this issue and its possible resolutions through the use of flavor symmetries, in the context of string–derived solutions, see Ref. [20], and in a non–string context, see for example, Ref. [21].

(iii) It is worth noting that there may be circumstances where the potentially dangerous GUT–related color triplets are projected out, e.g., below the compactification scale of a string theory that leads to a non-GUT symmetry like $G_{224} \subset SO(10)$ [22], but the components of $16_H$ and $\overline{16}_H$ providing Majorana masses of the right–handed neutrinos may still exist below the string scale. In this case, the standard $d = 5$ operators will be absent, but the new $d = 5$ operators discussed here could still be effective.

In summary, we have shown that in a class of grand unified theories including supersymmetric $SO(10)$, there can be a significant link between the neutrino masses and proton decay. In the process of generating neutrino masses one typically induces a new source of $d = 5$ proton decay interactions with an interesting strength. The flavor structure of these new $d = 5$ operators is distinctive. In contrast to the standard $d = 5$ operators, the new ones can lead to prominent (or even dominant) charged lepton decay modes, such as $\ell^+ \pi^0, \ell^+ K^0$ and $\ell^+ \eta$, where $\ell = e$ or $\mu$, even for low or moderate values of $\tan \beta \lesssim 10$, along with $\pi K^+$ and $\pi \pi^+$ modes. A distinguishing
feature of the new mechanism, relative to $d = 6$ vector exchange, is the predicted ratio $\Gamma(\ell^+ K^0) : \Gamma(\ell^+ \pi^0) \simeq 2 : 1$.

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