On the origin of current quark mass within nonperturbative QCD

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Abstract

We develop the theory of fermion induced phononlike excitations of the instanton liquid. It suggests a mechanism of current quark mass generation which is easily understandable by calculating the corresponding functional integral in the tadpole approximation. We systematically study the quark condensate excitations influenced by the phononlike modes and rederive the relation Gell-Mann–Oakes–Renner with realistic pion mass. The picture of $\sigma$-meson as being mixed with the soft scalar glueball-like excitation is discussed.

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I. Introduction

Nowadays, there are no doubts the model of QCD vacuum as the instanton liquid (IL) is the most practical instrument on the chiral scale of QCD. It provides, as the lattice calculations recently confirmed, not only the theoretical background for describing spontaneous chiral symmetry breaking (SCSB) but is mostly powerful in the phenomenology of the QCD vacuum and in the physics of light quarks while considered to propagate by zero modes arising from instantons. The origin of gluon and chiral condensates turns out in this picture easily understandable and both are quantitatively calculated getting very realistic values defined by $\Lambda_{QCD}$ and parameters of instanton and anti-instanton ensemble, for example, $-i \langle \psi^\dagger \psi \rangle \sim -(250 \text{ MeV})^3$. Moreover, the scale for dynamical quark masses, $M \sim 350 \text{ MeV}$, naturally appears and pion decay constant, $f_\pi \sim 100 \text{ MeV}$, is then transparently calculated.

Another significant advantage of this approach is that the initial formulation starts basically from the first principles and subsequent approximations being well grounded and reliably controlled are plugged in [1], [2]. It becomes clear especially in recent years when the impressive progress has been reached in understanding the instanton physics on the lattice [3]. Further we are summarizing several things we have learned thinking of the IL theory and trying to answer the challenging questions.

Let us start on that stage of the IL approach when its generating functional has already been taken as factorized one into two factors

$$Z = Z_g \cdot Z_\psi,$$

where eventually $Z_g$ provides nontrivial gluon condensate and the fermion part $Z_\psi$ is responsible to describe the chiral condensate in instanton medium and its excitations. It is usually supposed the
functional integral of $Z_g$ is saturated by the superposition of the pseudo-particle (PP) fields which are the Euclidean solutions of the Yang-Mills equations called the (anti-)instantons

$$A_\mu(x) = \sum_{i=1}^{N} A_\mu(x; \gamma_{i}).$$

Here $A_\mu(x; \gamma_{i})$ denotes the field of a single (anti-)instanton in singular gauge with $4N_c$ (for the $SU(N_c)$ group) coordinates $\gamma = (\rho, z, U)$ of size $\rho$ centred at the coordinate $z$ and colour orientation defined by the matrix $U$. The nontrivial bloc of corresponding $N_c \times N_c$ matrices of PP is a part of potential

$$A_\mu(x; \gamma) = \frac{\bar{n}_{\mu\nu} y_{\nu}}{g} \frac{\rho^2}{y^2 + \rho^2} U^\dagger \tau_a \ U, \quad y = x - z, \quad a = 1, 2, 3,$$

where $\tau_a$ are the Pauli matrices, $\eta$ is the 't Hooft symbol $[4]$, $g$ is the coupling constant and for anti-instanton $\bar{\eta} \rightarrow \eta$. For the sake of simplicity we do not introduce the distinct symbols for instanton ($N_+)$ and anti-instanton ($N_-)$ and consider topologically neutral IL with $N_+ = N_- = N/2$. Utilizing the variational principle the following estimate of $Z_g$ was found $[2]$

$$Z_g \simeq e^{-\langle S \rangle}$$

with the action of IL defined by the following additive functional $[1]$

$$\langle S \rangle = \int dz \int d\rho \ n(\rho) \ s(\rho).$$

The integration should be performed over the IL volume $V$ along with averaging the action per one instanton

$$s(\rho) = \beta(\rho) + 5 \ln(\Lambda \rho) - \ln(\bar{\beta}^{2N_c} + \beta \xi^2 \bar{\rho}^2) \int d\rho_1 \ n(\rho_1) \rho_1^2,$$

weighted with instanton size distribution function

$$n(\rho) = C \ e^{-s(\rho)} = C \ \rho^{-5} \bar{\beta}^{2N_c} e^{-\beta(\rho) - \nu \bar{\rho}^2/\nu},$$

$$\nu = \frac{b - 4}{2}, \quad b = \frac{11 N_c - 2 N_f}{3}, \quad (\bar{\rho}^2)^2 = \frac{\nu}{\beta \xi^2 n},$$

where $\bar{\rho}^2 = \int d\rho \ \rho^2 \ n(\rho)/n$, $n = \int d\rho \ n(\rho) = N/V$ and $N_f$ is the number of flavours. The constant $C$ is defined by the variational maximum principle in the selfconsistent way and $\beta(\rho) = \frac{8 \pi^2}{g^2} = -\ln C_{N_c} - b \ln(\Lambda \rho) \ (\Lambda = \Lambda_{M S} = 0.92 \Lambda_{P V})$ with constant $C_{N_c}$ depending on the renormalization scheme, in particular, here $C_{N_c} \approx \frac{4.66 \ \exp(-1.68 N_c)}{\pi^4 (N_c - 1)! (N_c - 2)!}$. The parameters $\beta = \beta(\bar{\rho})$ and $\bar{\beta} = \beta + \ln C_{N_c}$ are fixed at the characteristic scale $\bar{\rho}$ (an average instanton size). The constant $\xi^2 = \frac{27}{4} \frac{N_c}{N_c - 1} \pi^2$ characterizes, in a sense, the PP interaction and Eqs. $[3]$, $[4]$ and $[5]$ describe the equilibrium state of IL. The minor modification of variational maximum principle (see Appendix) leads to the explicit form of the mean instanton size $\bar{\rho} A = \exp\{-\frac{2 N_c}{2 \nu - 1}\}$ and, therefore, to the direct definition of the IL parameters unlike the conventional variational principle $[2]$ which allows one to extract those parameters solving numerically the transcendental equation only.

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1. In fact, the additive property results from the supposed homogeneity of vacuum wave function in metric space. Eq. $[3]$ looks like a formula of classical physics although it describes the ground state of quantum instanton ensemble. Intuitively clear, this definition will be still valid even when the wave function is nonhomogeneous with the nonuniformity scale essentially exceeding average instanton size or, precisely speaking, being larger (or of the order) than average size of characteristic saturating field configuration. Then each instanton liquid element of such a distinctive size will provide a partial contribution depending on the current state of IL, see next Section.
The quark fields are considered to be influenced by the certain stochastic ensemble of PPs, Eq. (1), while calculating the quark determinant

\[ Z_\psi \simeq \int D\psi^\dagger D\psi \langle\langle e^{S(\psi,\psi^\dagger,A)} \rangle\rangle_A. \]

Besides, dealing with dilute IL (small characteristic packing fraction parameter \( n \bar{\rho}^4 \)) one neglects the correlations between PPs and utilizes the approximation of \( N_c \to \infty \) where the planar diagrams only survive. In addition the fermion field action is approached by the zero modes what means the quark Green function is considered as the superposition of the free Green function \( S_0 = (-\hat{i}\partial)^{-1} \) and fermion zero modes \( \Phi_\pm(x-z) \) which are the solutions of the Dirac equation \( i(\hat{D}(A_\pm) + m) \Phi_\pm = 0 \) in the field of (anti-)instanton \( A_\pm \) centred at \( z \), i.e.

\[ S_\pm = S_0 - \frac{\Phi_\pm \Phi_\pm^\dagger}{i m}, \]

here \( m \) is the current quark mass and the quark zero mode possesses the following analytic form

\[ [\Phi_\pm(x)]_{ic} = \frac{\rho}{\sqrt{2\pi|x|(x^2 + \rho^2)^{3/2}}} \left[ \hat{x} \pm \frac{\gamma_5}{2} \right] \varepsilon_{jd} U_{dc} \]

with the colour \( c,d \) and the Lorentz \( i,j \) indices and the antisymmetric tensor \( \varepsilon \). In fact, there exists the singular term in the limit \( m \to 0 \) but it is selconsistently fixed by the saddle point calculation of quark determinant \( Z_\psi \). In spite of the fact the exact Green function (at \( m \neq 0 \)) including the terms of the whole series is well known [5] and, moreover, the Green function of instanton molecule has been also established [6], the simple zero mode approximation (6) is still the most practical in the concrete evaluations. In particular, at \( N_f = 1 \) the quark determinant reads [2]

\[ Z_\psi \simeq \int D\psi^\dagger D\psi \exp \left\{ \int dx \, \psi(x) i\hat{\partial}\bar{\psi}(x) \right\} \left( \frac{Y^+}{VM} \right)^{N_+} \left( \frac{Y^-}{VM} \right)^{N_-}, \]

\[ Y^\pm = i \int dz \, dU \, d\rho \, n(\rho)/n \int dx dy \, \psi(x) i\hat{\partial}_x \Phi_\pm(x-z) \Phi_\pm^\dagger(y-z) i\hat{\partial}_y \psi(y), \]

where the factor \( M \) makes the result dimensionless and is also fixed by the saddle point calculation. Amazingly this relatively crude approximation turns out so fruitful to develop (even quantitatively!) the low energy phenomenology of light quarks. The generating functional beyond the chiral limit was obtained in Ref. [7].

Thus, the IL approach at the present stage of its development looks very indicative, well theoretically grounded and reasonably adjusted phenomenologically. The proper form of generating functional obtained and its verisimilar parameter dependence indicated provide enough predictive power and justify, hence, the approximations made. It dictates the improvements to be done within the approach but, on the other hand, calculating some corrections has clearly no serious prospect. Taking this message as a guiding one we are going to demonstrate that amplifying the approach with an inverse influence of quarks upon the instanton ensemble which is intuitively small effect leads, however, to rather unobvious important conclusions.

We describe this influence (not getting beyond IL and SCSB approach) as a small variation of instanton liquid parameters \( \delta n \) and \( \delta \rho \) around their equilibrium values of \( n \) and \( \bar{\rho} \) being in full analogy with the description of chiral condensate excitations. Indeed, the result of nontrivial calculation of the functional integral (treating substantially from physical viewpoint the zero quark modes in the fermion determinant) comes to 'encoding' the IL state with just those two parameters. Moreover, the IL density appears in the approach via the packing fraction parameter \( n\bar{\rho}^4 \) only (clear from dimensional analysis) what means one independent parameter existing in practice. It is just the instanton size. The analysis of the quark and IL interaction is addressed in this paper developing...
our idea of phononlike excitations of IL resulting from the adiabatic changes of the instanton size. Thus, we will suppose the inverse quark influence should be described by these deformable (anti-)instanton configurations which are the field configurations Eq. (2) characterized by the size $\rho$ depending on $x$ and $z$, i.e. $\rho \rightarrow \rho(x, z)$.

The paper is organized as follows: in Section II we discuss the modification of quark determinant when the functional integral is saturated by the deformable modes at the minimal number of flavours. Then in Section III we develop the approximate calculation (tadpole approximation) which is based on the saddle point method and the corresponding iteration procedure. Section IV is devoted to the generalization for the multiflavour picture. The meson excitations of quark condensate and calculation of the Gell-Mann–Oakes–Renner relation when it is provided by the mechanism of quark current mass generation related to the phononlike excitations of IL are analyzed in Section V. The paper includes also Appendix where the fault finding reader gets a chance to control the explicit formulae of the IL parameters and to improve our calculations if is able.

II. Supplementing phononlike excitations

Apparently, the gist of what we discuss here could be illuminated in the following way. Saddle point method calculation of the functional integral implies the treatment of the action extremals which are the solutions of classical field equations. For the case in hands the action $S[A, \psi, \psi^\dagger]$ is constructed including the gluon fields $A_\mu$, (anti-)quarks $\psi^\dagger, \psi$ and extremals, which are given by the solutions of the consistent system of the Yang-Mills and Dirac equations. As a trial configuration in the IL theory the superposition of (anti-)instantons which is the approximate solution of the Yang-Mills equations (with no reverse influence of the quark fields) and an external field for the Dirac equation simultaneously is considered. We believe it is reasonable to utilize the deformable (crampled) (anti-)instantons $A_\pm(x; \gamma(x))$ as the saturating configurations. They just admit of varying the parameters $\gamma(x)$ of the Yang-Mills sector of the initial consistent system in order to describe the influence of quark fields in the appropriate variables for the quark determinant.

Let us remind first of all that deriving Eq. (3) we should average over the instanton positions in a metric space. Clearly, the characteristic size of the domain $L$ which has to be taken into account should exceed the mean instanton size $\bar{\rho}$. But at the same time it should not be too large because the far ranged elements of IL are not ‘causally’ dependent. The ensemble wave function is expected to be homogeneous (every PP contributes to the functional integral being weighted with a factor proportional to $\sim 1/V, V = L^4$) on this scale. The characteristic configuration which saturates the functional integral is taken as the superposition Eq. (1) with $N$ of PP in the volume $V$. It is easy to understand that because of an additivity of the functional Eq. (3) describes properly even non-equilibrium states of IL when the distribution function $n(\rho)$ does not coincide with the vacuum one and, moreover, allows us to generalize it for the non-homogeneous liquid when the size of the homogeneity obeys the obvious requirement $\lambda \geq L > \bar{\rho}$. Besides, we should deal with the weak

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2Then we are allowed to take the slowly changing deformation field beyond the integral while calculating the action of deformed instanton.
(comparing to the instanton $G_{\mu\nu}$ fields $g_{\mu\nu}$ ($g_{\mu\nu} \ll G_{\mu\nu}$) and long-length wave (on the scale of the instanton size $\bar{\rho}$) perturbations $\left| \frac{\partial \rho(x, z)}{\partial x} \right| \ll O(1)$ when the ensemble saturating the functional integral is very close to the vacuum (instanton) one. Then this smoothness or the adiabatic change of instanton size practically dictates another essential simplification defining everywhere the field in the center of instanton $\left| \frac{\partial \rho(x, z)}{\partial x} \right| \sim \left. \frac{\partial \rho(x, z)}{\partial x} \right|_{x=z}$ as a characteristic deformation field and the exact instanton definition in the singular gauge, $A^a_{\mu}(x, z) = -\frac{\bar{\eta}_{\mu\nu}}{g} \frac{\partial}{\partial x_{\nu}} \ln \left( 1 + \frac{\rho^2}{y^2} \right)$, leads to the following correction to the potential

$$a^a_{\mu}(x) = \Phi^a_{\mu\nu}(x, z) \frac{\partial \rho}{\partial x_{\nu}},$$

where $\Phi^a_{\mu\nu}(x, z) = -\frac{\bar{\eta}_{\mu\nu}}{g} \frac{2\rho(x, z)}{y^2 + \rho^2(x, z)}$. Keeping ourselves within the precision accepted here we could take $\rho(x, z) \simeq \bar{\rho}$, i.e. $\Phi^a_{\mu\nu}(x, z) = \Phi^a_{\mu\nu}(x - z)$. On the other hand the most general form of the correction to the instanton field influenced by the quarks might be calculated if the gluon Green function in the instanton medium is known

$$a^a_{\mu}(x, z) = \int d\xi \ D^{ab}_{\mu\nu}(x - z, \xi - z) \ J^b_{\nu}(\xi - z_\psi),$$

where $J^b_{\nu}$ is the current of external quark source, $z_\psi$ belongs to the region of long-length wave disturbance and, at last, $D^{ab}_{\mu\nu}$ is the Green function of PP in the instanton medium. In fact, this function is not well defined [5] but, seems, for the case in hands we could develop the selfconsistent way to calculate the regularized Green function. The nonsingular propagator behaviour in the soft momentum region is defined by the mass gap of phononlike excitations. Fortunately, the exact form of the Green function occurs unessential here (we are planning to return to the problem of regularized Green function calculation in the forthcoming publication). In the coordinate space it is peaked around the averaged PP size being in nonperturbative regime and, hence, integral Eq. (9) is appraised to be

$$a^a_{\mu}(x, z) \simeq F^{ab}_{\mu\nu}(x - z) \ J^b_{\nu}(z - z_\psi).$$

Appealing now to Eq. (8) we are capable to get immediately for $\rho_{\nu}$ the following equation

$$\Phi^a_{\mu\nu}(x - z) \frac{\partial \rho(x, z)}{\partial x_{\nu}} = F^{ab}_{\mu\nu}(x - z) \ J^b_{\nu}(z - z_\psi).$$

In fact, the current $J$ might be taken constant in the long-length wave approximation. Then neglecting the gradients the following change appears to be justified everywhere $\left| \frac{\partial \rho(x, z)}{\partial x} \right| \sim \left| \frac{\partial \rho(z)}{\partial z} \right|$ since there is no other fields in the problem at all (in the adiabatic approximation). The deformable mode contribution to the functional integral (when the corrections coming from the deformation fields of PP are absorbed) may be estimated as [8]

$$\langle S \rangle \simeq \int dz \int d\rho \ n(\rho) \ \left\{ \frac{\kappa}{2} \left( \frac{\partial \rho}{\partial z} \right)^2 + s(\rho) \right\},$$

where $\kappa$ is the kinetic coefficient being derived within the quasiclassical approach. Our estimate of it gives the value of a few instanton actions $\kappa \sim c \beta$ with the coefficient $c \sim 1.5 \div 6$ depending quantitatively on the ansatz supposed for the saturating configurations. Although this estimate is not much meaningful because there is no the vital $\kappa$ dependence eventually (becomes shortly clear). Thus, this coefficient should be fixed on a characteristic scale, for example $\kappa \sim \kappa(\bar{\rho})$ if we are planning not to be beyond the precision peculiar to the approach. Actually, it means adding the small contribution of kinetic energy type to the action per one instanton only. Such a term
results from the scalar field of deformations and affects negligibly the pre-exponential factors of the functional integral. In one’s turn pre-exponential factors do the negligible influence on the kinetic term as well [3]. If we strive for to be within the approximation we should retain the small terms of the second order in deviation from the point of action minimum only \( \frac{ds(\rho)}{d\rho}\bigg|_{\rho=\rho_c} = 0 \) supposing approximately

\[
s(\rho) \simeq s(\bar{\rho}) + \frac{s^{(2)}(\bar{\rho})}{2} \varphi^2 ,
\]

where \( s^{(2)}(\bar{\rho}) \simeq \frac{d^2 s(\rho)}{d\rho^2}\bigg|_{\rho_c} = \frac{4\nu}{\rho^2} \) and the scalar field \( \varphi = \delta \rho = \rho - \rho_c \simeq \bar{\rho} - \rho \) is the field of deviations from the equilibrium value of \( \rho_c = \bar{\rho} \left(1 - \frac{1}{2\nu}\right)^{1/2} \approx \bar{\rho} \). Consequently, the deformation field is described by the following Lagrangian density

\[
\mathcal{L} = \frac{n\kappa}{2} \left\{ \left( \frac{\partial \varphi}{\partial z} \right)^2 + M^2 \varphi^2 \right\}
\]

with the mass gap of the phononlike excitations

\[
M^2 = \frac{s^{(2)}(\bar{\rho})}{\kappa} = \frac{4\nu}{\kappa\rho^2}
\]

which is estimated for IL with \( N_c = 3 \), for example, in the quenched approximation to be

\[
M \approx 1.21 \Lambda
\]

if \( c = 4, \bar{\rho} \Lambda \approx 0.37, \beta \approx 17.5, n \Lambda^{-4} \approx 0.44 \) (for the details see the tables of Appendix). The deformation fields \( \varphi \) (with corresponding Jacobian) contribute to the generating functional on the same footing as the quark fields and it looks like

\[
Z'_g \sim \int D\varphi \frac{\delta A}{\delta \varphi} \exp \left\{ -\frac{n\kappa}{2} \int dz \left[ \left( \frac{\partial \varphi}{\partial z} \right)^2 + M^2 \varphi^2 \right] \right\}
\]

in full analogy with the fields \( \psi^\dagger, \psi \) entering with the functional measure \( D\psi^\dagger, D\psi \). Actually the Jacobian contribution [3] should be omitted in what follows as discussed above.

Analyzing the modifications arising now in the quark determinant \( Z_\psi \) we take into account the variation of fermion zero modes resulting from the instanton size perturbed

\[
\Phi_{\pm}(x - z, \rho) \simeq \Phi_{\pm}(x - z, \rho_c) + \Phi_{\pm}^{(1)}(x - z, \rho_c) \delta \rho(x, z) ,
\]

where \( \Phi_{\pm}^{(1)}(u, \rho_c) = \frac{\partial \Phi_{\pm}(u, \rho)}{\partial \rho}\bigg|_{\rho=\rho_c} \) and because of the adiabaticity it is valid \( \delta \rho(x, z) \simeq \delta \rho(z, z) = \varphi(z) \). The additional contributions of scalar fields generate the corresponding corrections in the factors of the kernels \( Y_{\pm}^{\dagger} \) of Eq. (11) which are treated in the linear approximation in \( \varphi \), i.e.

\[
i\hat{\partial}_x \Phi_{\pm}(x - z, \rho) \frac{\Phi_{\pm}^{\dagger}(y - z, \rho)}{i\hat{\partial}_y} \simeq \Gamma_{\pm}(x, y, z, \rho_c) + \Gamma_{\pm}^{(1)}(x, y, z, \rho_c) \varphi(z) ,
\]

here we introduced the notations

\[
\Gamma_{\pm}(x, y, z, \rho_c) = i\hat{\partial}_x \Phi_{\pm}(x - z, \rho_c) \Phi_{\pm}^{\dagger}(y - z, \rho_c) i\hat{\partial}_y ,
\]

[3]Generally, the deformation field \( \rho_c \) and integration variable \( a^\mu_0 \) are related via the rotation matrix: \( \Omega_{a\mu} \Phi_{\mu}(x - z)\rho_c = a^\mu_0 \) and in the long-wavelength approximation \( \Phi \) might be constant \( \Phi_{\mu}(0) \simeq \) \( \Phi_{\mu}(0) \simeq \) \( (x \sim z) \). With the rotation matrix spanning the colour field \( a_\mu = \Omega^{-1}a_\mu \) on the fixed axis (on the \( z \) axis for \( SU(2) \) group, for instance) we can conclude that the vectors \( a_\mu \) and \( \rho_c \) are, in fact, in one to one correspondence (of course, being within one loop approximation and up to this unessential colour rotation). Thus, the Jacobian occurs an unessential constant.
\[ \Gamma^{(1)}_{\pm}(x, y, z, \rho_c) = i \partial_x \Phi^{(1)}_{\pm}(x - z, \rho_c) \Phi^{\dagger}_{\pm}(y - z, \rho_c) i \partial_y + i \partial_x \Phi^{(1)}_{\pm}(x - z, \rho_c) \Phi^{(1)}_{\pm}(y - z, \rho_c) i \partial_y \]

The physical meaning of the basic phenomenon behind this Lagrangian seems pretty transparent. The propagation of the fields in turn

\[ (2\pi)^4 \delta(k - l) \gamma_0(k, k) + \gamma_1(k, l) \varphi(k - l) \]

with \( \gamma_0(k, k) = G^2(k) \), \( G(k) = 2\pi \rho_c F(k \rho_c/2) \), \( \gamma_1(k, l) = G(k)G'(l) + G'(k)G(l) \),

\[ G'(k) = \frac{dG(k)}{d\rho} \bigg|_{\rho = \rho_c} , \quad F(x) = 2x \left[ I_0(x)K_1(x) - I_1(x)K_0(x) \right] - 2I_1(x)K_1(x) , \]

where \( I_i, K_i (i = 0, 1) \) are the modified Bessel functions.

In fact, the functional integral of Eq. (7) including the phononlike component may be exponentiated in the momentum space with the auxiliary integration over the \( \lambda \)-parameter (see, for example [4])

\[ Z \simeq \int \frac{d\lambda}{2\pi} \exp \left\{ N \ln \left( \frac{N}{i\lambda V M} \right) - N \right\} \times \]

\[ \int D\psi \dagger D\psi \exp \left\{ \int \frac{dk dl}{(2\pi)^8} \psi \dagger(k) \left[ (2\pi)^4 \delta(k - l) \left( -\hat{k} + i\lambda \gamma_0(k, k) \right) + i\lambda \gamma_1(k, l) \varphi(k - l) \right] \psi(l) \right\} \]

we dropped out the factor normalizing to the free Lagrangian everywhere. It is pertinent to mention here the Diakonov-Petrov result comes to the play precisely if the scalar field is switched off.

In order to avoid a lot of the needless coefficients in the further formulae we introduce the dimensionless variables (momenta, masses and vertices)

\[ \frac{k \rho_c}{2} \rightarrow k , \quad \frac{M \rho_c}{2} \rightarrow M , \quad \gamma_0 \rightarrow \rho_c^2 \gamma_0 , \quad \gamma_1 \rightarrow \rho_c \gamma_1 , \quad (14) \]

the fields in turn

\[ \varphi(k) \rightarrow (n\kappa)^{-1/2} \rho_c^3 \varphi(k) , \quad \psi(k) \rightarrow \rho_c^{5/2} \psi(k) , \quad (15) \]

and eventually for \( \lambda \) we are using \( \mu = \frac{\lambda \rho_c^3}{2N_c} \). Then the generating functional takes the following form

\[ Z \simeq \int d\mu Z'' \int \int D\psi \dagger D\psi \ d\varphi \exp \left\{ -N \ln \mu - \int \frac{dk}{\pi^4} \frac{1}{2} \varphi(-k) 2 \left[ k^2 + M^2 \right] \varphi(k) \right\} \times \]

\[ \times \exp \left\{ \int \frac{dk dl}{\pi^8} \psi \dagger(k) \left[ 2 \pi^4 \delta(k - l) \left( -\hat{k} + i\mu \gamma_0(k, k) \right) + \frac{i\mu}{(n\rho_c^2 \kappa)^{1/2}} \gamma_1(k, l) \varphi(k - l) \right] \psi(l) \right\} \]

where \( Z'' \) is a part of gluon component of the generating functional which survives after expanding the action per one instanton Eq. (11). The functional obtained describes the IL state influenced by the quarks when all the terms containing the scalar field are collected (see also Appendix).

\footnote{In the metric space we have the nonlocal Lagrangian of the phononlike deformations \( \varphi(z) \) interacting with the quark fields \( \psi \), \( \psi \), i.e.

\[ L = \int dx \psi \dagger(x)(x) \partial_x \psi(x) - \int dz \frac{n\kappa}{2} \left\{ \left( \frac{\partial \varphi}{\partial z} \right)^2 + M^2 \varphi^2(z) \right\} + \]

\[ + \frac{i\lambda}{N_c} \int dxdy dz dU \psi \dagger(x) \left( \Gamma_{\pm}(x, y, z, \rho_c) + \Gamma_{\pm}^{(1)}(x, y, z, \rho_c) \varphi(z) \right) \psi(y) \].

The physical meaning of the basic phenomenon behind this Lagrangian seems pretty transparent. The propagation of quark fields through the instanton medium is accompanied by the IL disturbance (the analogy with well known polaron problem embarrasses us strongly in this point).}
mentioned above we believe this influence analogous to the reversal impact of phononlike deformations on the quark determinant does not considerably change the numerical results of the IL and SCSB theory. But it is invisible from Eq. (16) directly how this smallness is reasoned. The scalar field enters the generating functional formally at the same order of the $\mu$-expansion as the term providing SCSB (compare the second and third terms of the second exponential function in Eq. (16)). The suppression arises because the dominant contribution of the Yukawa interaction comes from the quark field condensate which maintains the additional $\mu$ smallness. This result prompts, in fact, rather natural scheme of the approximate calculation of the generating functional

$$\psi^\dagger\psi \varphi = \langle \psi^\dagger\psi \rangle \varphi + \{ \psi^\dagger\psi - \langle \psi^\dagger\psi \rangle \} \varphi .$$

If we believe, for example, the first term ascribes some gluon component of the generating functional then being linearly dependent on the scalar field it produces the small shift from the equilibrium value of the instanton size ($\rho_c \sim \bar{\rho}) \varphi = \varphi' + \delta \varphi$. And the shift $\delta \varphi$ generates the mass term in the quark sector what means the scheme should be pushed forward to the following expression

$$\psi^\dagger\psi \varphi = \langle \psi^\dagger\psi \rangle (\varphi' + \delta \varphi) + \{ \psi^\dagger\psi - \langle \psi^\dagger\psi \rangle \} (\varphi' + \delta \varphi) .$$

Thus, we face the conventional mechanism of the mass generation when it is related to the insignificant variation of the equilibrium instanton size $\rho_c$ (or, in usual terms, to falling the scalar field condensate down) produced by the quark condensate. Moreover, it is clear if the variations happen to be of the opposite sign then the field $\gamma_5 \psi$ (the chiral partner of the field $\psi$) should develop the true mass. Before turning to the mechanism of the mass generation below let us calculate the quark determinant integrating formally over the scalar field $D\varphi$, then find the quark Green function in the tadpole approximation and formulate the equation for the saddle point. It is supposed the phonon component contribution does not affect substantially the results of the SCBS theory. Here we are interested in projecting upon the scale inherent for the scalar field.

### III. Tadpole approximation

The integration leads us to the four fermion interaction and the functional integral can not be calculated exactly. However, due to smallness of scalar field corrections we may find the effective Lagrangian substituting the condensate value in lieu of one of the pairs of quark lines (see Fig. 1.)

![Figure 1: The tadpole contribution.](image)

$$\psi^\dagger(k)\psi(l) \rightarrow \langle \psi^\dagger(k)\psi(l) \rangle = -\pi^4 \delta(k - l) Tr S(k) .$$

In such an approach the diagram with four fermion lines in the lowest order of the perturbation theory in $\mu$ is reduced to the two-legs diagram with one tadpole contribution (there are two such

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5Let us emphasize the first term is responsible, in a sense, for the quark condensate fluctuations what allows the $\pi$ meson to gain its mass. In the second term such fluctuations are suppressed.
Then the quark determinant after integrating over the scalar field \( s \) reads

\[
\frac{4 (i\mu)^2}{n\bar{\rho}^4\kappa} \int \frac{dkdl}{\pi^{16}} \gamma_1(k, l) \gamma_1(k', l') \psi^\dagger(k)\psi(l) \psi^\dagger(k')\psi(l') \langle \varphi(k-l) \varphi(k'-l') \rangle \approx \\
\simeq \frac{4 \mu^2}{n\bar{\rho}^4\kappa} \int \frac{dk}{\pi^2} \gamma_1(k, k) \psi^\dagger(k)\psi(k) \int \frac{dl}{\pi^2} \gamma_1(l, l) Tr S(l) D(0) ,
\]

where the natural pairing definition was introduced

\[
\langle \varphi(k) \varphi(l) \rangle = \pi^4 \delta(k-l) D(k), \quad D(k) = \frac{1}{4 \left(k^2 + M^2\right)} .
\]

It is obvious the factors surrounding \( \psi^\dagger(k)\psi(k) \) has a meaning of quark mass

\[
m_f(k) = \frac{\mu}{(n\bar{\rho}^4\kappa)^{1/2}} \gamma_1(k, k) \frac{(-2i\mu)}{(n\bar{\rho}^4\kappa)^{1/2}} \int \frac{dl}{\pi^4} \gamma_1(l, l) Tr S(l) D(0) ,
\]

(17)

(account at the same order of the \( \mu \) expansion while calculating the saddle point equation)

\[
- \frac{2 \mu^2}{n\bar{\rho}^4\kappa} \left[ \int \frac{dk}{\pi^4} \gamma_1(k, k) Tr S(k) \right]^2 \pi^4 \delta(0) D(0) = -\frac{1}{2} \frac{\mu^2}{n\bar{\rho}^4\nu} \frac{V}{\bar{\rho}^4} \left[ \int \frac{dk}{\pi^4} \gamma_1(k, k) Tr S(k) \right]^2 .
\]

Here we used the natural regularization of the \( \delta \)-function \( \delta(0) = \frac{1}{\pi} \frac{V}{\bar{\rho}^4} \) in the dimensionless units. Then the quark determinant after integrating over the scalar fields reads

\[
Z \sim \int d\mu \int D\psi^\dagger D\psi \exp \left\{ -N \ln \mu + \frac{2N^2_c}{n\bar{\rho}^4\nu} \frac{V}{\bar{\rho}^4} \mu^4 c^2(\mu) + \int \frac{dk}{\pi^4} \psi^\dagger(k) 2 \left[ -\hat{k} + i\Gamma(k) \right] \psi(k) \right\} = \\
\int d\mu \exp \left\{ -N \ln \mu + \frac{2N^2_c}{n\bar{\rho}^4\nu} \frac{V}{\bar{\rho}^4} \mu^4 c^2(\mu) + \frac{V}{\bar{\rho}^4} \int \frac{dk}{\pi^4} Tr \ln[-\hat{k} + i\Gamma(k)] \right\} ,
\]

(18)

where the vertex function is defined as

\[
\Gamma(k) = \mu \gamma_0(k, k) + m_f(k) ,
\]

and we introduced the function \( c(\mu) \) convenient for the practical calculations

\[
c(\mu) = -\frac{i}{2\mu} \frac{N_c}{\bar{\rho}^4} \int \frac{dk}{\pi^4} \gamma_1(k, k) Tr S(k) .
\]
As it is clear from Eq. (18) the Green function of the quark field is self-consistently defined by the following equation

$$2 \left[ -\hat{k} + i\Gamma(k) \right] S(k) = -1.$$ 

Searching the solution in the form

$$S(k) = A(k) \hat{k} + i B(k),$$

we get

$$A(k) = \frac{1}{2} \frac{1}{k^2 + \Gamma^2(k)}, \quad B(k) = \frac{1}{2} \frac{\Gamma(k)}{k^2 + \Gamma^2(k)}.$$

Using Eq. (17) and the definitions of $\Gamma(k)$ and $B(k)$ we have the complete integral equation

$$\Gamma(k) = \mu \gamma_0(k,k) + \frac{N_c}{n\rho^4\nu} \mu^2 \gamma_1(k,k) \int \frac{dl}{\pi^4} \gamma_1(l,l) \frac{\Gamma(l)}{l^2 + \Gamma^2(l)},$$

which drives to have the convenient representation of the solution

$$\Gamma(k) = \mu \gamma_0(k,k) + \frac{N_c}{n\rho^4\nu} \mu^3 c(\mu) \gamma_1(k,k).$$

What concerns the function $c(\mu)$ it is not a great deal to obtain

$$c(\mu) = \frac{1}{\mu} \int \frac{dk}{\pi^4} \gamma_1(k,k) \frac{\Gamma(k)}{k^2 + \Gamma^2(k)},$$

and, therefore, the complete integral equation for the function $c(\mu)$ which is shown in Fig. 3 for $N_f = 1$. Let us underline the $N_f$-dependence of the $c(\mu)$ function in the interval of $\mu$ determined by
saddle point value is unessential. Then we easily obtain for the current quark mass

$$m_f(k) = \frac{N_c}{n\rho^4} \frac{\mu^3}{\nu} c(\mu) \gamma_1(k, k) ,$$

and see the cancellation of the kinetic coefficient $\kappa$ in $m_f$. Thus, it means the precise value of the coefficient is unessential as declared.

We have the following equation for the saddle point of the functional of Eq. (18)

$$2N_c \int \frac{dk}{\pi^4} \frac{\mu [\Gamma^2(k)]'}{k^2 + \Gamma^2(k)} + 2N_c^2 \frac{\mu}{n\rho^4\nu} \frac{[\mu^4 c^2(\mu)]'}{n\rho^4} = 1 ,$$

where the prime is attributed to the differentiation in $\mu$.

It results from assuming the stationary IL parameters, what is not accurate. We should include another effect produced by the shift of equilibrium instanton size thanks to the quark condensate presence. The modification of the IL parameters $(n(\mu), \cdots)$ caused by the Yukawa interaction comes from simple tadpole graph of Fig. 5 in the leading order

$$\Delta = -\frac{2i\mu}{(n\rho^4\kappa)^{1/2}} \int \frac{dk}{\pi^4} \gamma_1(k, k) Tr S(k) \varphi(k - l) = \frac{4N_c}{(n\rho^4\kappa)^{1/2}} \mu^2 c(\mu) ,$$

The solution is unique within the $\mu$ interval of our interest. It is curious to notice when $\mu$ is larger than $\sim 4 \cdot 10^{-2}$ several solution branches for $c(\mu)$ emerge (see Fig. 4). Surely, it could be interesting to clarify if there is any correspondence between these branches and the saddle point equation resulting from Eq. (18). However, it is clear wittingly those objects should be heavier than the scale of several hundred $MeV$ characteristic for SCSB. It is quite possible such solutions might be associated with some `heavy' particles if they do exist.
Figure 5: The tadpole.

and this term generates the small shift of the equilibrium instanton size $\rho_c \sim \bar{\rho}$ (remind here $\varphi = \rho - \rho_c$, and $\varphi(0) = \int dz \, \varphi(z)$ is the scalar field in momentum representation)\textsuperscript{7}. The tadpole contribution could be absorbed in making more sophisticated variational procedure of saddle point calculation. Actually, it includes also the variation of the IL parameters as a function of $\mu$. But in practice it comes about highly effective to use the simple iterating procedure. At the first step this variation of the IL parameters is not taken into account and the saddle point $\mu(\rho_c)$ is calculated from Eq. (20). Then getting new IL parameters (see, Appendix) we have to resolve Eq. (21) again and etc. Five or six iterations are quite enough if we are satisfied with the same precision as in calculating the integrals. In the Table 1 the numerical results (M.S.Z.) are shown for $N_f = 0, 1$ ($N_f = 0$ corresponds to the quenched approximation for the IL parameters) comparing to those of Diakonov and Petrov (D.P.) where the disturbance of instanton medium was not considered.

Table 1.

| $N_f$ | $\mu$ | $M(0)$ | $-i \langle \psi^\dagger \psi \rangle$ | $N_f$ | $\mu$ | $M(0)$ | $-i \langle \psi^\dagger \psi \rangle$ | $m_f$ |
|-------|-------|--------|---------------------------------|-------|-------|--------|---------------------------------|------|
| 0     | 5.68 \times 10^{-3} | 341 | $-(301)^3$ | 0     | 5.51 \times 10^{-3} | 366 | $-(297)^3$ | 4.29 |
| 1     | 5.14 \times 10^{-3} | 376 | $-(356)^3$ | 1     | 5.07 \times 10^{-3} | 385 | $-(328)^3$ | 5.13 |

The parameters indicated in the Table 1 are the dynamical quark mass

$$M(0) = \Gamma(0) \left( \frac{2}{\rho_c} \right) \ [MeV],$$

the quark condensate

$$-i \langle \psi^\dagger \psi \rangle = -i \langle \psi^\dagger \psi \rangle \bigg|_{x=0} = \frac{N_c}{4} \int dk \, \frac{\Gamma(k)}{\pi^2 k^2 + \Gamma^2(k)} \left( \frac{2}{\rho_c} \right)^3 \ [MeV]^3,$$

and, at last, the current quark mass $m_f \ [MeV]$, which is defined now being mixed with the quark condensate as

$$m_f = \frac{\int dk \, m_f(k) \, \Gamma(0) \, S(k)}{\int dk \, \Gamma(0) \, S(k)} . \tag{22}$$

It is just what is dictated by the Gell-Mann–Oakes–Renner relation which we calculate below. Through this paper the value of renormalization constant is fixed by $\Lambda = 280 \, MeV$. Then the IL parameters are slightly different from their conventional values $\rho \sim (600 \, MeV)^{-1}$, $\tilde{R} \sim (200 \, MeV)^{-1}$ (see the corresponding tables in Appendix). However, with the minor $\Lambda$ variation the parameters could be optimally fitted. As expected the change of quark condensate is insignificant, the order of

\textsuperscript{7}This shift in phononlike component of Lagrangian $-4 \frac{M^2}{2} \varphi^2 + \Delta \cdot \varphi = -4 \frac{M^2}{2} \varphi^2 + \cdots$ with the definitions $\varphi' = \varphi - \delta \varphi$ and $\delta \varphi = \frac{\Delta}{4 M^2}$ generates the mass term in quark sector $m'_f(k) = \frac{\mu}{(n \rho^2 \kappa)^{1/2}} \gamma_1(k, k) \delta \varphi$, which has obviously the same form as that of Eq. (19).
several $MeV$, what hints the existence of new soft energy scale established by the disturbance which accompanies the quark propagation through the instanton medium.

**IV. Multiflavour approach**

In order to match the approach developed to phenomenological estimates we need the generalization for $N_f > 1$. Then the quark determinant becomes

$$Z_{\psi} \simeq \int D\psi^\dagger D\psi \exp \left\{ \int dx \sum_{f=1}^{N_f} \psi^\dagger_f(x) i\tilde{\partial}\psi_f(x) \right\} \left\{ \left( \frac{Y^+}{VM^{N_f}} \right)^{N_+} \left( \frac{Y^-}{VM^{N_f}} \right)^{N_-} \right\} ,$$

$$Y^\pm = i^{N_f} \int dz \ dU \ d\rho(n)/n \prod_{f=1}^{N_f} \int dx_f dy_f \ \psi^\dagger_f(x_f) i\tilde{\partial}_{x_f}\Phi_\pm(x_f - z) \ \Phi^\dagger_\pm(y_f - z) i\tilde{\partial}_{y_f} \psi_f(y_f) .$$

With phononlike component included every pair of the zero modes $\sim \Phi \Phi^\dagger$ acquires the additional term similar to Eq. (12). The appropriate transformation driving the factors $Y^\pm$ to their determinant forms is still valid here since the correction term differs from the basic one with the scalar field $\varphi$. The complete integration over $dz$ leads (in the adiabatic approximation $\varphi(x, z) \to \varphi(z)$) to the transparent Lagrangian form with the momentum conservation of all interacting particles. Besides, we keep the main terms of $Y^\pm$ in $\varphi$ expansion. The quark zero modes generate the factor similar to Eq. (13) with $1/N_c$ being changed by the factor $\left( \frac{1}{N_c} \right)^{N_f}$ and then in the leading $N_c$ order we have

$$Y^\pm = \left( \frac{1}{N_c} \right)^{N_f} \int dz \ \det_{N_f}\left( i\ J^\pm(z) \right) ,$$

$$J^\pm_f(z) = \int \frac{dk dl}{(2\pi)^6} \left[ e^{i(k-l)dz}\gamma_0(k, l) + \int \frac{dp}{(2\pi)^4} e^{i(k-l+p)dz}\gamma_1(k, l) \varphi(p) \right] \psi^\dagger_f(k) \frac{1 \pm \gamma_5}{2} \psi_g(l) .$$

While providing the Gaussian form for the functional we perform the integration over the auxiliary parameter $\lambda$ together with the bosonization resulting in the integration over the auxiliary matrix $N_f \times N_f$ meson fields

$$\exp \left\{ \lambda \ \det\left( \frac{i\ J}{N_c} \right) \right\} \simeq \int d\mathcal{M} \ \exp \left\{ i \ \text{Tr}[\mathcal{M}J] - (N_f - 1) \left( \frac{\det[\mathcal{M}N_c]}{\lambda} \right)^{\frac{N_f - 1}{2}} \right\} .$$

As a result the generating functional may be written as

$$Z = \int \frac{d\lambda}{2\pi} \ Z_g'' \exp(-N\ln\lambda) \int D\varphi \ \exp \left\{ - \int \frac{dk}{(2\pi)^4} \ \frac{n\kappa}{2} \ \varphi(-k) \ \left[ k^2 + M^2 \right] \varphi(k) \right\} \cdot \int D\mathcal{M}_{L,R} \ \exp \int dz \left\{ -(N_f - 1) \left[ \left( \frac{\det[\mathcal{M}_L N_c]}{\lambda} \right)^{\frac{1}{N_f - 1}} + \left( \frac{\det[\mathcal{M}_R N_c]}{\lambda} \right)^{\frac{1}{N_f - 1}} \right] \right\} \cdot$$

$$\cdot \int D\psi^\dagger D\psi \ \exp \left\{ \int \frac{dk}{(2\pi)^4} \ \sum_f \psi^\dagger_f(k)(-\hat{k})\psi_f(k) + i \ \int dz \left( \text{Tr}[\mathcal{M}_L J^+] + \text{Tr}[\mathcal{M}_R J^-] \right) \right\} .$$

(23)

Now scalar field interacts with the quarks of the different flavours, nevertheless, the dominant contribution is expected from the tadpole graphs where any pair of the quark fields is taken in the condensate approximation as it happens at $N_f = 1$

$$\psi^\dagger_f(k)\psi_g(l) \to \langle \psi^\dagger_f(k)\psi_g(l) \rangle = -\pi^4\delta_{fg} \ \delta(k - l) \ \text{Tr} \ S(k) .$$
As for the condensate itself we obtain it as the nontrivial solution of saddle point equation. For example, it is

$$(\mathcal{M}_{L,R})_{PQ} = \mathcal{M} \delta_{PQ}$$

for the diagonal meson fields. The dimensionless convenient variables (in addition to Eqs. (4), (15)) are the following

$$\frac{\mathcal{M}}{2} \frac{\bar{\rho}^3}{\rho} \to \mu, \quad \left( \frac{\lambda \bar{\rho}^4}{(2N_c \rho)^N_f} \right)^{\frac{1}{N_f-1}} \to g.$$  

Then the effective action \( (Z = \int d\bar{g} d\mu \exp\{-V_{eff}\}) \) in new designations has the form

$$V_{eff} = N(N_f - 1) \ln g - \frac{V}{\bar{\rho}^4} (N_f - 1) \frac{2\mu^N_f}{g} - \frac{V}{\bar{\rho}^4} \frac{2N_f^2N_c^2}{g^4} \mu^4 c^2(\mu) - 2N_fN_c \frac{V}{\bar{\rho}^3} \int \frac{d\mu}{\bar{\rho}^4} \ln\{k^2 + \Gamma^2(k)\},$$

and the saddle point equation reads

$$\frac{2N_c}{n\bar{\rho}^4} \int \frac{k^2}{\bar{\rho}^4} \frac{\mu}{k^2 + \Gamma^2(k)} \left[ \Gamma^2(k) \right]_\mu \frac{2}{n\bar{\rho}^4} \frac{N_f}{\mu^4} \left[ c^2(\mu) \right]_\mu = 1.$$  

The quark current mass in Eq. (19) gains the additional factor \( N_f \) because the scalar nature of the phononlike field requires to match the tadpole quark field condensates of all \( N_f \) flavours to every vertex

$$m_f(k) = \frac{N_fN_c}{n\bar{\rho}^4} \frac{\mu^3}{\nu} c(\mu) \Gamma_1(k, \mu).$$

Table 2 complements the Table 1 with the calculations at \( N_f = 2 \)

| D.P. | M.S.Z. |
|------|-------|
| \( \mu \) | \( \mu \) |
| \( M(0) \) | \( M(0) \) |
| \( -i\langle \psi^\dagger \psi \rangle \) | \( -i\langle \psi^\dagger \psi \rangle \) |
| \( f_\pi \) | \( f_\pi \) |
| \( f'_\pi \) | \( f'_\pi \) |
| 4.66 \cdot 10^{-3} | 4.61 \cdot 10^{-3} |
| 422 | 416 |
| \( -(427)^3 \) | \( -(374)^3 \) |
| 135 | 118 |
| 111 | 97.2 |
| 12.7 | |

where \( f_\pi \) \([MeV]\) is the pion decay constant and

$$f^2_\pi = \frac{N_cN_f}{8} \int \frac{d\mu}{\bar{\rho}^4} \frac{\Gamma^2(k)}{(k^2 + \Gamma^2(k))^2} \left( \frac{2}{\rho_c} \right)^2,$$

\( f'_\pi \) \([MeV]\) is its approximated form

$$f'^2_\pi = \frac{N_cN_f}{8} \int \frac{d\mu}{\bar{\rho}^4} \frac{\Gamma^2(k)}{(k^2 + \Gamma^2(k))^2} \left( \frac{2}{\rho_c} \right)^2,$$

here \( \Gamma'(k) = \frac{d\Gamma(k)}{dk} \), and the condensate \(-i\langle \psi^\dagger \psi \rangle\) is implied for the quarks of every flavour.

**V. Meson excitations of the chiral condensate**

Here we discuss the meson excitations of the chiral condensate adapting the effective Lagrangian to the suitable variables. The meson degrees of freedom are introduced into the effective Lagrangian \( (23) \) with \( N_f = 2 \) by the following substitutions

$$\mathcal{M}^L = \mathcal{M} \tilde{\mathcal{M}}^L, \quad \mathcal{M}^R = \mathcal{M} \tilde{\mathcal{M}}^R,$$

\( (23) \) with \( N_f = 2 \) by the following substitutions

$$\mathcal{M}^L = \mathcal{M} \tilde{\mathcal{M}}^L, \quad \mathcal{M}^R = \mathcal{M} \tilde{\mathcal{M}}^R.$$
where $\mathcal{M}$ denotes the condensate and $\sigma$, $\eta$ are the scalar and pseudoscalar meson fields, $\pi^a$ is the isorotiplate of the $\pi$-mesons, $\sigma^a$ is the vector meson isorotiplate. The effective action for the meson field excitations looks then as

$$ V_{\text{eff}} = - \int \frac{dp}{\pi^4} \left\{ \pi^a(p) R_{\pi^a}(p) \pi^a(-p) + \sigma^a(p) R_{\sigma^a}(p) \sigma^a(-p) + \sigma(p) R_\sigma(p) \sigma(-p) + \eta(p) R_\eta(p) \eta(-p) \right\}, $$

here $R_{\pi^a}$, $R_{\sigma^a}$, $R_\sigma$, $R_\eta$ are the inverse propagators of the corresponding particles. Their exact forms are listed in Ref. [2] but for the $\pi$- and $\sigma$-mesons with the phononlike contributions included they are calculated below.

The contributions generated by the quark determinant if the phononlike fields ignored are well investigated in the leading order of the $N_c$-expansion. In particular, one of the contributions comes from the diagram A). It is of the first order in the $\mu$-expansion with one quark loop and one external meson leg in which the meson fields are maintained up to the quadratic terms

$$ A) \quad 2i \int dz \int \frac{dk}{\pi^4} (-Tr S(k)) \Gamma(k) \left\{ \tilde{\mathcal{M}}^L_{ff}(z) \frac{1 + \gamma_5}{2} + \tilde{\mathcal{M}}^R_{ff}(z) \frac{1 - \gamma_5}{2} \right\}. $$

Besides, another contribution comes from the diagram B) being of the second order in $\mu$ with one quark loop and two external meson legs

$$ B) \quad 2 \int \frac{dkdl}{\pi^8} Tr[S(k)S(l)] \Gamma(k,l) \Gamma(l,k) \left\{ \tilde{\mathcal{M}}^L_{ff}(l-k) \frac{1 + \gamma_5}{2} \tilde{\mathcal{M}}^L_{gf}(k-l) \frac{1 + \gamma_5}{2} + (L,\gamma_5) \rightarrow (R,-\gamma_5) \right\}, $$

where we introduced $\Gamma(k,l) = \mu \gamma_0(k,l) + \frac{\mu}{(n^0)^2} \gamma_1(k,l) \delta \varphi$, $\gamma_0(k,l) = G(k)G(l)$. The determinant terms of the generating functional at $N_f = 2$ develop the form

$$ det \tilde{\mathcal{M}}^L + det \tilde{\mathcal{M}}^R = 1 + \sigma^2 + \eta^2. $$

In addition to this diagrams we have the tadpole diagram C) in which the meson fields are maintained up to the quadratic terms and which describes the effect of equilibrium instanton size fluctuations when the meson fields are present. These fluctuations influence the IL parameters [4]
In particular, we have for the inverse propagator of the \( \pi \)-meson field

\[
R_{\pi^a}(p) = \int \frac{dk}{\pi^4} \{ -2 N_c N_f \frac{(k, k + p) + \Gamma(k)\Gamma(k + p)}{[k^2 + \Gamma^2(k)][(k + p)^2 + \Gamma^2(k + p)]} \Gamma(k, k + p) \Gamma(k + p, k) + \\
+ 2N_c N_f \frac{\Gamma^2(k)}{k^2 + \Gamma^2(k)} + i N_f m_f(k) Tr (-S(k)) \}.
\]

When the pion momentum goes to zero \( p \to 0 \) the first two terms do not generate the massive term because of the quark determinant symmetry. As a result, these terms at small values of \( p \) originate only term proportional to \( p^2 \) and we limit ourselves with the approximate result for \( \Gamma = \mu \gamma_0 \) while calculating it. Using the expansion obtained in Ref. [2] we have

\[
R_{\pi^a}(p) = i N_f \int \frac{dk}{\pi^4} m_f(k) Tr (-S(k)) + \beta n p^2
\]

with \( \beta n = \frac{N_c N_f}{16} \int \frac{dk}{\pi^4} \frac{\Gamma^2(k) - \frac{k^2}{2} \Gamma'(k) \Gamma(k) + \frac{k^2}{4} (\Gamma'(k))^2}{(k^2 + \Gamma^2(k))^2} \left( \frac{2}{p_c} \right)^2 \). Eq. (24) combined with the definition (22) can be given as

\[
R_{\pi^a}(p) = \beta n \left\{ \frac{m_f i\langle \bar{\psi}\psi \rangle N_f}{\beta n} + p^2 \right\}.
\]

From here we have for the \( \pi \)-meson mass \( m^2_\pi = \frac{m_f i\langle \bar{\psi}\psi \rangle N_f}{\beta n} \) or if the relation between \( \beta \) and the pion constant \( f_\pi \) (\( f^2_\pi = 2\beta n \)) plugged in we obtain

\[
m^2_\pi = 2 \frac{m_f i\langle \bar{\psi}\psi \rangle N_f}{f^2_\pi},
\]

what displays Gell-Mann–Oakes–Renner relation

\[
m^2_\pi = \frac{(m_u + m_d)}{f^2_\pi} \frac{i\langle \bar{u}u \rangle + \langle \bar{d}d \rangle}{f^2_\pi}.
\]

The factor 2 in the numerator of Eq. (25) corresponds just the sum of the \( u \) and \( d \) quark masses and \( N_f \) means the summation of condensates.

The light particle which we introduced and which imitates the scalar glueball properties does not affect significantly the SCSB parameters and correctly describes the soft pion excitations of quark condensate. Meanwhile, the experimental status of this light scalar glueball is very vague. We believe the phononlike excitations could manifest themselves being mixed with the excitations of the quark

\[\text{Footnote 8: The problems related to the effective chiral Lagrangian (when the phononlike fields included) and its symmetries will be discussed in the separate publication.}\]
condensate in the scalar channel. To illuminate the point let us consider the inverse propagator of the \( \sigma \)-meson which is given by the contributions of the diagram \( B \) and determinant

\[
R_\sigma(p) = -2 N_c N_f \int \frac{dk}{\pi^2} \frac{(k, k + p) - \Gamma(k)\Gamma(k + p)}{(k^2 + \Gamma^2(k))[\Gamma(k, p)\Gamma(k + p)]} \Gamma(k, k + p) \Gamma(k + p, k) + n \bar{\rho}^4.
\]

Holding the highest terms of the \( \mu \)-expansion only, when \( \Gamma = \mu \gamma_0 \), we receive the following result by means of the identity (see, [2])

\[
R_\sigma(p) = N_c N_f \int \frac{dk}{\pi^2} \frac{[\Gamma(k)(k + p)_\mu + \Gamma(k + p) k_\mu]^2}{(k^2 + \Gamma^2(k))[\Gamma(k, p)\Gamma(k + p)]}.
\]

In the course of this exercise we have to include the contribution of the 'shifting' diagram of \( C \) type where the field \( \varphi \) (more exactly \( \varphi' \)) should be treated as the dynamical one, i.e.

\[
C' = 2i\mu \int \frac{dk}{\pi^2} Tr (-S(k)) \frac{\gamma_1(k, l)}{(n \bar{\rho}^4)^{1/2}} \int dz \varphi(z) \left\{ f_{\tau}^L(z) \frac{1 + \gamma_5}{2} + f_{\tau}^R(z) \frac{1 - \gamma_5}{2} \right\}.
\]

Then the term of interacting scalar fields which we are interested in looks like \( V_{\varphi\sigma} = \int dz \Delta \varphi(z) \sigma(z) \), where \( \Delta \) is defined by Eq. (21). At low momenta the diagrams contribute to the effective action (in the dimensionless variables) as

\[
-2 (p^2 + M^2) \varphi^2 - 2 n \bar{\beta}_\sigma (p^2 + M_\sigma^2) \sigma^2 + \Delta \varphi \sigma
\]

with \( \bar{\beta}_\sigma \) standing for the kinetic coefficient of the \( \sigma \)-meson and \( M_\sigma \) is its mass. Both quantities are derived from the expansion of the inverse propagator in the low momentum region as \( R_\sigma(p) \simeq 2 n \bar{\beta}_\sigma (p^2 + M_\sigma^2) \). Diagonalizing the quadratic form

\[
-2 (p^2 + M^2) \varphi^2 - 2 (p^2 + M_\sigma^2) \bar{\sigma}^2 + \bar{\Delta} \varphi \bar{\sigma} \, ,
\]

(presented in more adequate variables \( \bar{\sigma} = (n \bar{\beta}_\sigma)^{1/2} \sigma \), \( \bar{\Delta} = \Delta (n \bar{\beta}_\sigma)^{1/2} \)) leads to the following definition of the composite particle masses

\[
M_{1,2}^2 = \frac{M^2 + M_\sigma^2}{2} \pm \frac{\sqrt{[M_\sigma^2 - M^2]^2 + \bar{\Delta}^2}}{2}.
\]

It is well known that SCSB is inapplicable when we are going to deal with heavy meson masses. Specifically, it leads to the wrong predictions for the \( \sigma \)-meson because the corresponding mass obtained is of an order \( 1/\bar{\rho} \sim 1 \text{ GeV} \) (moreover, the adiabaticity approximation is certainly broken then). The result Eq. (20) aggravates, in a sense, the situation since the hard component leaves for the region of harder masses whereas the light component persists to become lighter. On the other hand, the present observations signal rather the existence of the scalar meson of mass about 0.5 GeV what apparently does not coincide with the light component \( M_2 \). Nevertheless, our consideration having no claims of the quantitative agreement shows the scalar meson could be the mixed particle of pretty large width (similar to the superposition of oscillators with the various fundamental frequencies) describing the excitations of chiral and gluon condensates.

### VI. Conclusion

In this paper we have developed the consistent approach to describe the interaction of quarks with IL. Theoretically, it is based (and justified) on the particular choice of the configurations saturating the functional integral what occurs to be not merely a technical exercise. They are the deformable
(crampled) (anti-)instantons with the variable parameters \( \gamma(x) \) and in the concrete treatment of this paper we play with the variation of the PP size \( \rho(x,z) \). In a sense, such an ansatz is strongly motivated by the form of quark determinant which is solely dependent on the average instanton size in the SCSB theory. We have demonstrated that in the long-length wave approximation the variational problem of the deformation field optimization turns into the construction of effective Lagrangian for the scalar phononlike \( \varphi \) and quark fields with the Yukawa interaction. Physically, it allows us to analyze the inverse influence of quarks on the instanton vacuum. We have pointed out this influence on the IL parameters as negligible. The modification of the SCSB parameters occurs pretty poor as well. In particular, the scale of quark condensate change amounts to a few \( MeV \) only. Nevertheless, switching on the phononlike excitations of IL leads to several qualitatively new and interesting effects. The propagation of the quark condensate disturbances over IL happens in this approach to be in close analogy with well-known polaron problem. We imply a necessity to take into account the medium feedback while elementary excitations propagating. The intriguing conclusion comes with realizing it leads to generation of current quark masses \( m_f \) within QCD itself with their values entirely corresponding to the conventional phenomenological results for \( u \) and \( d \) quarks. Moreover, the \( \pi \)-meson being massless pseudo-Goldstone particle in the standard SCSB theory acquires its mass obeying the Gell-Mann–Oakes–Renner relation when the IL deformations enter the game. Besides, it hints that fitting the parameters \( \bar{\rho} \), \( n \) and renormalization constant \( \Lambda \) all together with the alteration of \( s(\rho) \) profile function we might achieve suitable agreement not only in the order of magnitude. The difficulties which confronted us here illuminate the fundamental problem of gluon field penetration into the vacuum (the instanton vacuum in this particular case) as the most principle one. Indeed, it is a real challenge to answer the question about the strong interaction carrier in the soft momentum region. Perhaps, the light particle of scalar glueball properties which appears inherently in our approach and should manifest itself in the mixture with the excitation of quark condensate in scalar channel (\( \sigma \)-meson) is not bad candidate for that role. By the way, it could be experimentally observed as a wide resonance.

Summarizing, we understand our calculation can not pretend to the precise quantitative agreement with experimental data and see many things to be done. We are planning shortly to consider the problem of instanton profile \([12]\), to make more realistic description of the PP interaction, to push our ansatz beyond the long-length wave approximation analyzing more precisely ‘instanton Jacobian’ \( \left| \frac{\delta A}{\delta \varphi} \right| \).

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Appendix

The contribution of the quark determinant to the IL action is given by the tadpole diagram Eq. (21) which takes the following form when returned to the dimensional variables (see, Eq. (23))

\[
\Delta \varphi = \Delta \left( \frac{n\kappa}{\bar{\rho}^3} \right)^{1/2} \varphi(0) = \Delta \left( \frac{4}{\bar{\rho}^3} \right)^{1/2} \int d\rho \frac{n(\rho)}{n} \int \frac{dz}{\bar{\rho}^3} \frac{\rho(z) - \rho_c}{\bar{\rho}} .
\]

Then the IL action, Eq. (3), acquires the additional term

\[
\langle S \rangle = \int dz \ n \left\{ \langle s \rangle - \langle \Delta' \frac{\rho - \rho_c}{\bar{\rho}} \rangle \right\} ,
\]
where $\Delta' = \frac{4N}{n^2} \mu^2 c(\mu)$ and the mean action per one instanton is given by the following functional

$$\langle s_1 \rangle = \int d\rho \, s_1(\rho) n(\rho)/n$$

with

$$s_1(\rho) = \beta(\rho) + 5 \ln(\Lambda\rho) - \ln \beta^{2N_c} + \beta \xi^2 \rho^2 n(\rho)^2 - \Delta' (\rho - \rho_c)/\bar{\rho}.$$ 

In order to evaluate the equilibrium parameters of IL we treat the maximum principle

$$\langle e^{-S} \rangle \geq \langle e^{-S_0} \rangle e^{-\langle S - S_0 \rangle}$$

adapting it to the simplest version (when the approximating functional is trivial, $S_0 = 0$). In a sense, this choice of the approximating functional should be a little worse than in Ref. [2]. Its only advantage comes from the possibility to get the explicit formulae for the IL parameters in lieu of solving the complicated transcendental equation. In equilibrium the instanton size distribution function $n(\rho)$ should be dependent on the IL action only, i.e. $n(\rho) = C e^{-s(\rho)}$ where $C$ is a certain constant. This argument corresponds to the maximum principle of Ref. [2]. Indeed, if one is going to approach the functional [2] as a local form $\langle s \rangle = \int d\rho \, s_1(\rho) n(\rho)/n$ where $s_1(\rho) = \beta(\rho) + 5 \ln(\Lambda\rho) - \ln \beta^{2N_c} + \beta \xi^2 \rho^2 n(\rho)^2$, it makes the approach self-consistent. The functional difference $\langle s \rangle - \langle s_1 \rangle = \int d\rho \{s(\rho) - s_1(\rho)\} e^{-s(\rho)}/n$ being varied over $s(\rho)$ leads then to the result $s(\rho) = s_1(\rho) + \text{const}$ keeping into the mind an arbitrary normalization. The maximum principle results in getting the mean action per one instanton as the IL parameters function, for instance, average instanton size $\bar{\rho}$. The corrections generated by the 'shifting' terms turn out to be small and we consider them in the linear approximation in the deviation $\Delta$. The following schematic expansion exhibits how the major contribution appears

$$\langle s_1 \rangle = \frac{\langle (s + \delta) e^{-s-\delta} \rangle}{\langle e^{-s-\delta} \rangle} \simeq \frac{\langle s e^{-s} \rangle + \langle \delta e^{-s} \rangle}{\langle e^{-s} \rangle} + \frac{\langle s e^{-s} \rangle \langle \delta e^{-s} \rangle - \langle s \delta e^{-s} \rangle \langle e^{-s} \rangle}{\langle e^{-s} \rangle^2},$$

(27)

here $\delta(\Delta)$ stands for a certain small 'shifting' contribution and $s$ is the action generated by the gluon condensate contribution only. The last term in Eq. (27) is small comparing to the first one and we neglect it. Then it is clear that evaluating the mean action per one instanton is permissible to hold the gluon condensate contribution $s$ only (without the 'shifting' term $\delta$) in the exponential. Hence we have for the mean action per one instanton $\langle s_1 \rangle = \int d\rho \, s_1(\rho) n_0(\rho)/n_0$, and $n_0(\rho)$ is the distribution function which does not include the 'shifting' term $\delta$.

It is possible to obtain for the average squared instanton size and the IL density that

$$r^2 \bar{\rho}^2 = \nu \left\{ 1 + \frac{\Delta'}{r \bar{\rho}} \frac{\Gamma(\nu + 1/2)}{2\nu \Gamma(\nu)} \right\} \simeq \nu \left\{ 1 + \Delta' \frac{\Gamma(\nu + 1/2)}{2\nu^{3/2} \Gamma(\nu)} \right\},$$

(28)

$$n = C \, C_{N c} \beta^{2N_c} \frac{\Gamma(\nu)}{2 \nu^{2/\nu}},$$

(29)

The 'shifting' term changes the mass of phononlike excitation insignificantly. The equilibrium instanton size as dictated by the condition $\left. \frac{ds}{d\rho} \right|_{\rho = \rho_c} = 0$ is equal then to $\rho_c = (\alpha + \Delta' / \beta) \bar{\rho}$, $\alpha = \left(1 - \frac{1}{2\nu}\right)^{1/2}$, $\beta = \frac{1}{4\nu} \left\{ 1 - \frac{\Gamma(\nu + 1/2)}{\nu^{3/2} \Gamma(\nu)} \right\}$ and the second derivative of action in the equilibrium point equals to $s''(\rho_c) = \frac{4\nu}{\rho^2} + \frac{2\nu}{\rho^2} \Delta' \left\{ \frac{\Gamma(\nu + 1/2)}{\nu^{3/2} \Gamma(\nu)} - \frac{1}{2\nu\alpha} \right\}$. Another source of corrections to the kinetic coefficient appears while one considers

the instanton profile change $A \to A + a$ where the field of correction is $a \sim \frac{\partial \rho(x, z)}{\partial x} \bigg|_{x=z}$. This mode could appear within the superposition ansatz Eq. (20) and leads to the modifications of quark zero mode $(D(A + a)\psi = 0)$. Fortunately, both corrections to the kinetic term occurs to be numerically small.

In order not to overload the formulae with the factors making the results dimensionless, which are proportional to the powers of $\Lambda$, we drop them out hoping it does not lead to the misunderstandings.
where the parameter \( r^2 \) equals to

\[
\tau^2 = \beta^2 n \beta^2 .
\]  

(30)

Expanding \( \ln \rho = \ln \hat{\rho} + \frac{\rho - \hat{\rho}}{\rho} + \frac{1}{2} \left( \frac{\rho - \hat{\rho}}{\rho} \right)^2 + \cdots \) and using Eq. (28) we can show that

\[
\int \frac{d\rho}{\rho_0(\rho)} \ln \rho = \ln \bar{\rho} + \Phi_1(\nu) , \quad \int \frac{d\rho}{\rho_0(\rho)} \rho = \bar{\rho} + \Phi_2(\nu) ,
\]

where \( \Phi_1, \Phi_2 \) are the certain function of \( \nu \) independent of \( \bar{\rho} \). Besides, the average squared instanton size within the precision accepted obeys the equality \( r^2 \bar{\rho}^2 = \Phi(\nu) \), and \( \Phi(\nu) \) is the function of \( \nu \) only. Then the mean action per one instanton looks like

\[
\langle s_1 \rangle = -2N_c \ln \beta + (2\nu - 1) \ln \rho + F(\nu)
\]

\((F(\nu)\) is again the function of \( \nu \) only and its explicit form is unessential for us here). Calculating its maximum in \( \rho \) we receive

\[
\bar{\rho} = \exp \left\{ \frac{2N_c}{2\nu - 1} \right\} , \quad \beta = \frac{2bN_c}{2\nu - 1} - \ln C_{N_c} .
\]

From Eqs. (28), (30) we find the IL density to be as

\[
n = \nu e^{\frac{2N_c}{\beta \xi^2}} \left( 1 + \Delta' \Gamma(\nu + 1/2) \frac{\Gamma(\nu)}{2\nu^3/\Gamma(\nu)} \right) ,
\]

and handling Eq. (29) we determine the constant \( C \).

The IL parameters occur to be close to the parameter values of the Diakonov-Petrov approach [2] and are shown in the following

Table 3.

| \( N_f \) | D.P. | M.S.Z. |
| --- | --- | --- |
| \( \bar{\rho} \lambda \) | \( n/A^4 \) | \( \beta \) | \( \bar{\rho} \lambda \) | \( n/A^4 \) | \( \beta \) |
| 0 | 0.37 | 0.44 | 17.48 | 0 | 0.37 | 0.44 (0.49) | 17.48 |
| 1 | 0.30 | 0.81 | 18.86 | 1 | 0.33 | 0.63 (0.71) | 18.11 |
| 2 | 0.24 | 1.59 | 20.12 | 2 | 0.28 | 1.03 (1.17) | 18.91 |

Here \( N_f \) is the number of flavours, \( N_c = 3 \), and the IL density at \( \Delta \neq 0 \) when the iteration process completed is shown in the parenthesis. It is curious to notice the quark influence on the IL equilibrium state provokes the increase of the IL density.

In Table 4 we demonstrate the mass gap magnitude \( M \) and the wave length in the ‘temporal’ direction \( \lambda_4 = M^{-1} \). To make it more indicative we show also the average distance between PPs from which is clear, in fact, that the adiabatic approximation \( \lambda \geq L \sim \bar{R} > \bar{\rho} \) is valid for the long-length wave excitations of the \( \pi \)-meson type. All the parameters are taken at \( \kappa = 4\beta \) but the primed ones correspond to the kinetic term value of \( \kappa = 6\beta \).

Table 4.

| \( N_f \) | \( M \lambda^{-1} \) | \( \lambda \lambda \) | \( M' \lambda^{-1} \) | \( \lambda' \lambda \) | \( \bar{R} \lambda \) |
| --- | --- | --- | --- | --- | --- |
| 0 | 1.21 | 0.83 | 0.99 | 1.01 | 1.23 (1.2) |
| 1 | 1.34 | 0.75 | 1.09 | 0.91 | 1.12 (1.09) |
| 2 | 1.45 | 0.69 | 1.18 | 0.84 | 0.99 (0.96) |

Here the parameters in the parenthesis designate the distance between PP \( \bar{R} = n^{-1/4} \) when the iteration process is completed.
References

[1] C. G. Callan, R. Dashen and D. J. Gross, Phys. Rev. D 17, 2717 (1978);
   C. G. Callan, R. Dashen and D. J. Gross, Phys. Lett. B 66, 375 (1977);
   E.-M. Ilgenfritz and M. Müller-Preussker, Nucl. Phys. B 184, 443 (1981);
   E. V. Shuryak, Nucl. Phys. B 203, 93, 116, 140 (1982); B 328, 85, 102 (1989);
   T. Schäfer and E. V. Shuryak, Rev. Mod. Phys. 70, 323 (1998).

[2] D. I. Diakonov and V. Yu. Petrov, Nucl. Phys. B 245, 259 (1984);
   D. I. Diakonov and V. Yu. Petrov, in Hadronic Matter under Extreme Conditions,
   edited by V. Shelest and G. Zinovjev (Naukova Dumka, Kiev, 1986) p. 192;
   D. I. Diakonov, V. Yu. Petrov and P. V. Pobylitsa, Phys. Lett. B 226, 471 (1989).

[3] J.W. Negele, hep-lat/9902032.

[4] G. ’t Hooft, Phys. Rev. D14, 3432 (1976).

[5] L. S. Brown, R. D. Carlitz, D. B. Creamer and C. Lee, Phys. Rev. D 17, 1583 (1979).

[6] C. Lee and W. A. Bardeen, Nucl. Phys. B153, 210 (1979).

[7] M. Musakhanov, Eur. Phys. J. C9, 235 (1999);
   V. F. Tokarev, Theor. Mat. Fiz. (in Russian) 73, 223 (1987).

[8] S. V. Molodtsov, A. M. Snigirev and G. M. Zinovjev, Phys. Rev. D 60 056006 (1999);
   Proceedings of NATO Advanced Workshop ”Lattice fermions and the structure of the vacuum”,
   1999, Dubna, Russia.

[9] N. S. Manton, Phys. Lett. B110, 54 (1982); B154, 397 (1985);
   G. W. Gibbons and N. S. Manton, Nucl. Phys. B274, 183 (1986).

[10] A. G. Sergeev and S. V. Chechin, Theor. Math. Phys. (in Russian) 85, 397 (1990).

[11] D. I. Diakonov, M. V. Polyakov and C. Weiss, Nucl. Phys. B461, 539 (1996);
    D. I. Diakonov, hep-ph/9802298.

[12] A. B. Migdal, N. O. Agasyan and S. B. Khohlachev, Pis'ma JETP. 41, 405 (1985);
    N. O. Agasyan and S.B. Khohlachev, Yad. Fiz. 55, 1116 (1992);
    N. O. Agasyan and Yu. A. Simonov, Mod. Phys. Lett. 10A, 1755 (1995).