Ginsburg-Landau Expansion in a non-Fermi Superconductor

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January 18, 2022

Abstract

We study the Ginsburg-Landau expansion for the non-Fermi model proposed by Anderson. We analyze the deviations of the main properties of a non-Fermi superconductor from the isotropic s-wave bidimensional superconductor.

Keywords: non-Fermi superconductor, critical temperature, Ginsburg-Landau coefficients

1 Introduction

Since the discovery of high temperature superconductivity (HTcS) the microscopic description of the HTcS was not a simple problem. It is generally accepted that the normal and superconducting phase are not describe by a Fermi theory. There are a lot of microscopical models describing the normal and superconducting phase. The nearly antiferromagnetic model proposed by Millis, Monien and Pines [1], non-Fermi Anderson model [2], marginal Fermi liquid proposed by C.M. Varma et.al. [3] are phenomenological models used to explain the normal state physical properties of high temperature superconductors. Using the BCS-like model, the Gorkov equations have been applied to describe the superconducting state in a non-Fermi liquid, proposed by the Anderson model [2]. The superconducting state properties have been studied by different authors [4, 5]. The idea of this model is the existence of a state in bidimensional systems similar with the state described by the Luttinger liquid for one-dimensional systems. In such a system the spectral function \( A(k, \omega) = -ImG(k, \omega) \) satisfies the homogeneity relation \( A(\Lambda k, \Lambda \omega) = \Lambda^{-1+\alpha}A(k, \omega) \) with an exponent \( \alpha \) greater than zero [5]. This spectral anomaly implies the break down of the Fermi liquid theory, the limit \( \alpha = 0 \) is a special choice leading to the Fermi liquid model. Wen [12] has demonstrated that the exponent \( \alpha \) is non-universal depending on coupling between the fermions.

In this paper we analyze the properties of the non-Fermi superconductor using the method developed in Ref.[3]. We calculate the Ginsburg-Landau coefficients (Sec. III) and analyze the main physical properties of the non-Fermi superconductor (Sec. IV). Finally we compare the results with other theoretical models (Sec. V) for the cuprates superconductors.

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2 Model

The non-Fermi behavior of the normal state of cuprates superconductors has been discussed by different authors \[4,5\] in order to explain the physical properties of the cuprate superconductors. In the following we consider the superconducting phase which is described by the BCS-like order parameter $\Delta_k$ which can be calculated from the Gorkov equations. In the superconducting phase the electrons are described by the normal and anomalous Green functions:

$$G(k, \omega) = e^{-i\pi(1-\frac{\alpha}{2})}\frac{(g(\alpha))^{-1}}{(\omega \omega_c^2)^{-\frac{1}{2}}(\varepsilon_k^2 - \omega^2 - i\delta)^{1-\alpha} + |\Delta_k|^2},$$

$$F(k, \omega) = \frac{-\Delta_k}{(\omega \omega_c^2)^{-\frac{1}{2}}(\varepsilon_k^2 - \omega^2 - i\delta)^{1-\alpha} + |\Delta_k|^2},$$

where $\omega_c$ is a cut-off energy, $0 < \alpha < 1$, $g(\alpha) = \pi \alpha / 2 \sin(\pi \alpha / 2)$. The equation for the transition temperature $T_c$ can be obtained from the equation for the Cooper instability:

$$1 - \Pi(0, 0) = 0,$$

where the Cooper susceptibility $\Pi(0, 0)$ can be calculated following Ref.[6]. As a result the equation for $T_c$ is:

$$\frac{1}{V} = T_c \sum_n \int \frac{d^3k}{(2\pi)^3} \left[ G(k, i\omega_n)G(-k, -i\omega_n) + F(k, i\omega_n)F(-k, -i\omega_n) \right],$$

where $V$ is the attractive interaction between the electrons in the superconducting state. The critical temperature $T_c$ was also calculated in Ref.[8] and the final result is:

$$T_c = \omega_D \left( \frac{D(\alpha)}{C(\alpha)} \right) \exp \left[ -\frac{1}{V} \frac{1}{2\alpha A(\alpha)D(\alpha)} \left( \frac{\omega_c}{\omega_D} \right)^\alpha \right],$$

where $A(\alpha), C(\alpha), D(\alpha)$ are functions of $\alpha$ given by Ref.[7]. The critical temperature $T_c$ was calculated also in Ref.[8]. The small difference from the result obtained in Ref.[8] has the origin in the mathematical approximations made in the calculations. In deriving eq. (5) the sum over Matsubara frequencies is calculated first and then the integration over the momentum.

The result (3) gives in the limit $\alpha \to 0$ the well known result for $T_c$:

$$T_c = 1.13\omega_D \exp \left[ -\frac{1}{N(0)V} \right],$$

which make us to believe that eq.(3) may be correct. The result given in Ref.[8] also in the limit $\alpha \to 0$ gives the BCS result for the critical temperature.

3 Ginsburg-Landau Expansion

The difference between the superconducting and normal free energy densities is written in the usual Ginsburg-Landau expansion:

$$F_S - F_N = a |\Delta_k|^2 + \frac{b}{2} |\Delta_k|^4 + c k^2 |\Delta_k|^2,$$
where \(a, b, c\) are the Ginzburg-Landau coefficients. The calculation of the critical coefficients is done following Ref.\[8\]\[13\]. The coefficient \(a\) is given by the relation:

\[
a = \frac{1}{V} - T \sum_n \int \frac{d^2k}{(2\pi)^2} \left[ G(k, i\omega_n)G(-k, -i\omega_n) + F(k, i\omega_n)F(-k, -i\omega_n) \right]. \tag{8}
\]

Evaluation in eq. (8) was done using eq. (4) for the attractive potential between the fermions. The integration over \(k\) is transformed using the relation \(\int \frac{d^2k}{(2\pi)^2} \rightarrow N(0) \int d\varepsilon\) where \(N(0)\) is the density of states. After a short calculation we get for \(a\) the following expression:

\[
a = 2\pi^{2\alpha-1}N(0) \left( \frac{g(\alpha)}{\omega_c} \right)^2 \left( T^{2\alpha} - T_c^{2\alpha} \right) \int_0^\infty \frac{dx}{(1 + x^2)^{1-\alpha}} \sum_n (2n + 1)^{2\alpha-1}, \tag{9}
\]

where \(x = \varepsilon/\omega_n\) is a new parameter and \(\omega_n = (2n + 1)\pi T\) are the fermionic Matsubara frequencies. \(N\) is connected with the cutoff \(\omega_D\) by the relation \(N = \omega_D/\pi T_c\). After performing the integration over \(x\) and the summation over \(n\) we get the final result:

\[
a = N(0) \left( \frac{T - T_c}{T_c} \right) \left( \frac{\omega_D}{\omega_c} \right)^{2\alpha} g^2(\alpha) \frac{B \left( \frac{1}{2}, \frac{1}{2} - \alpha \right)}{\pi}, \tag{10}
\]

where \(T_c\) is the critical temperature given by eq. (5) and \(B(x, y) = \Gamma(x) \Gamma(y)/\Gamma(x + y)\), and \(\Gamma(x)\) are Euler functions. In the limit \(\alpha \rightarrow 0\) we obtain for the coefficient \(a\):

\[
a_0 = N(0) \left( \frac{T - T_{co}}{T_{co}} \right), \tag{11}
\]

which is the expression for the case of a two-dimensional isotropic superconductor. The coefficient \(b\) can be written as \[8\]\[13\]:

\[
b = T_c \sum_n \int \frac{d^2k}{(2\pi)^2} \left[ G(k, i\omega_n)G(-k, -i\omega_n) + F(k, i\omega_n)F(-k, -i\omega_n) \right]^2. \tag{12}
\]

The calculation for evaluating \(b\) follows the same procedure described for the calculation of \(a\), the only differences being the appearance of different exponents in the \(x\) integration and summation over frequencies. After performing the calculations in eq. (12) we obtain for \(b\) the result:

\[
b = N(0) \frac{7\zeta(3)}{8\pi^2T_c^2} g^4(\alpha) \left( \frac{2\pi T_c}{\omega_c} \right)^{4\alpha} \frac{2B \left( \frac{3}{2}, \frac{3}{2} - 2\alpha \right)}{\pi} \frac{2^{3-4\alpha} - 1}{7} \frac{\zeta(3 - 4\alpha)}{\zeta(3)}, \tag{13}
\]

where \(\zeta(x)\) is Riemann function. In the limit \(\alpha \rightarrow 0\) we obtain the well known result:

\[
b_0 = N(0) \frac{7\zeta(3)}{8\pi^2T_{co}^2}. \tag{14}
\]

In order to calculate \(c\) we must perform a Taylor expansion in powers of \(q\) of the expression:

\[
- T_c \sum_n \int \frac{d^2k}{(2\pi)^2} \left[ G(k + \frac{q}{2}, i\omega_n)G(-k + \frac{q}{2}, -i\omega_n) + F(k + \frac{q}{2}, i\omega_n)F(-k + \frac{q}{2}, -i\omega_n) \right] \tag{15}
\]
The integration over $k$ is now transformed using the relation $\int \frac{d^2k}{(2\pi)^2} \rightarrow N(0) \int d\varepsilon \frac{d\theta}{2\pi}$ because of the appearance of the $\cos(\theta)$ term where $\theta$ is the angle between $k$ and $q$. We have to consider the first and the second orders of Taylor expansion in order not to lose some important contributions. After performing the calculations in eq.(15) we get for $c$ the result:

$$c = N(0) \frac{v_F^2 \zeta(3)}{32\pi^2 T_c^2} \frac{g^2(\alpha)}{\omega_c} \left( \frac{2\pi T_c}{\omega_c} \right)^{2\alpha} \frac{2^{3-2\alpha} - 1}{7} \zeta(3 - 2\alpha)$$

$$\times \frac{(3 - \alpha)(1 - \alpha) B \left( \frac{1}{2}, \frac{5}{2} - \alpha \right) - (1 - \alpha^2) B \left( \frac{3}{2}, \frac{3}{2} - \alpha \right)}{\pi}. \quad (16)$$

In the limit $\alpha \rightarrow 0$, $c$ becomes:

$$c_0 = N(0) \frac{v_F^2 \pi \zeta(3)}{32\pi^2 T_{co}^2}. \quad (17)$$

c$$_0$ is the coefficient for the s-wave isotropic superconductor. From eqs. (11), (14), (17) we can see that we reobtain the results for the ordinary superconductors, for the Ginsburg-Landau coefficients in the limit $\alpha = 0$.

### 4 Physical Characteristics of a non-Fermi Superconductor

In this section we present the behavior of the coherence length, the penetration depth of the magnetic field and the jump in the heat capacity at the transition point for a non-Fermi superconductor. The coherence length at a given temperature is given by the following expression between the Ginsburg-Landau coefficients:

$$\xi^2(T) = -\frac{c}{\alpha}. \quad (18)$$

In the ordinary isotropic superconductor $\xi_0(T) = \sqrt{-c_0/a_0} = 0.74\xi_0/\sqrt{1-T/T_{co}}$ where $\xi_0 = 0.18v_F/T_{co}$ [9]. Using eqs. (10), (16) we obtain for the coherence length the expression:

$$\xi^2(T) = \xi_0^2(T) \left( \frac{T_{co}}{T_c} \right)^{2(1 - \frac{T}{T_{co}})} \left( \frac{2\pi T_c}{\omega_D} \right)^{2\alpha} \frac{2^{3-2\alpha} - 1}{7} \zeta(3 - 2\alpha) \zeta(3)$$

$$\times \frac{(3 - \alpha)(1 - \alpha) B \left( \frac{1}{2}, \frac{5}{2} - \alpha \right) - (1 - \alpha^2) B \left( \frac{3}{2}, \frac{3}{2} - \alpha \right)}{B \left( \frac{1}{2}, \frac{1}{2} \right)}. \quad (19)$$

From eq. (19) we find the same temperature dependence of the coherence length $\xi(T) = \xi/\sqrt{1-T/T_{c}}$. At a given temperature $T$ the coherence length is a function of $\alpha$. The dependence of $\xi(T)$ as a function of $\alpha$ is presented in Fig.1. The general expression for the penetration depth in terms of Ginsburg-Landau coefficients is given by:

$$\lambda^2(T) = -\frac{\tau^2}{32\pi e^2} \frac{b}{ac}. \quad (20)$$
The general expression (20) can be simplified if we introduce the \( \lambda_0^2(T) \) as the penetration depth in an ordinary superconductor \[9\] as:

\[
\lambda_0(T) = \left( \frac{\lambda_0}{\sqrt{1 - T/T_{co}}} \right),
\]

where \( \lambda_0^2 = m\bar{c}^2/4\pi ne^2 \) is the penetration depth at \( T=0 \). Using eqs. (20) and (21) we have for \( \lambda^2(T) \):

\[
\lambda^2(T) = \lambda_0^2(T) \left( \frac{1 - \frac{T}{T_{co}}}{1 - \frac{T}{T_c}} \right) \left( \frac{2\pi T_c}{\omega_D} \right)^{2\alpha} \frac{2^{3-4\alpha} - 1 - \zeta(3 - 4\alpha)}{2^{3-2\alpha} - 1 - \zeta(3 - 2\alpha)}
\]

\[
\times \frac{2\pi B \left( \frac{1}{2} \left( \frac{3}{2} - 2\alpha \right) \right)}{B \left( \frac{1}{2} \left( \frac{3}{2} - \alpha \right) \right)} \left[ (3 - \alpha)(1 - \alpha) B \left( \frac{1}{2} \left( \frac{3}{2} - \alpha \right) \right) - (1 - \alpha^2) B \left( \frac{3}{2} \left( \frac{3}{2} - \alpha \right) \right) \right].
\]

The temperature dependence of the penetration depth is the same as that in ordinary superconductors, but we have an \( \alpha \) dependence of \( \lambda^2(T) \) as in Fig.2. Using eqs. (19) and (22) it is possible to calculate the Ginsburg-Landau parameter \( k = \lambda(T)/\xi(T) \). The dependence of \( k \) as function of \( \alpha \) is the same with the \( \lambda(T) \) dependence. We can also calculate the size of the discontinuity in the heat capacity at the transition point. The jump in the heat capacity is given by:

\[
\frac{C_S - C_N}{\Omega} = \frac{T_c}{b} \left( \frac{a}{T - T_c} \right)^2,
\]

where \( C_S \) and \( C_N \) represent the superconducting and normal heat capacities, and \( \Omega \) is the volume of the system. If we consider the ratio between the jump in the heat capacity of a non-Fermi superconductor and the jump of a normal bidimensional superconductor we obtain:

\[
\frac{(C_S - C_N)_{T_c} T_{co}}{(C_S - C_N)_{T_{co} T_c}} = \left( \frac{T_c}{T_{co}} \right)^2 \left( \frac{\sqrt{\omega_c \omega_D}}{2\pi T_c} \right)^{4\alpha} g^{-2}(\alpha) \frac{B^2 \left( \frac{1}{2} \left( \frac{1}{2} - \alpha \right) \right)}{2\pi B \left( \frac{1}{2} \left( \frac{3}{2} - 2\alpha \right) \right)} \frac{7}{2^{3-4\alpha} - 1 - \zeta(3 - 4\alpha)}
\]

The \( \alpha \) dependence is presented in Fig.3. As we can see in Fig. 3 the jump in \( C_V \) decreases with increasing \( \alpha \).

## 5 Conclusions

We have calculated the critical temperature and the Ginsburg-Landau coefficients for a non-Fermi superconductor \[4\]. We have found a similar behavior of the Ginsburg-Landau coefficients with those in the BCS case. The results are in agreement with previous calculations \[14\]. A similar model was used by Sudbo \[4\] and Muthukumar \[11\] for the calculation of critical temperature. Using these results we calculated the coherence length, the penetration depth and the jump in the specific heat at the transition point. We have to mention that from a non-Fermi superconductor we can obtain the results for the BCS case by putting \( \alpha = 0 \) in eqs. (19), (22), (24). Taking some reasonable value of the specific heat jump \[14, 13, 10\] \[ see Fig. 3 \] we get that \( \alpha \) should satisfy the condition 0.2 < \( \alpha \) < 0.4.

In high temperature superconductivity the overdoped region is well described by the standard Fermi liquid theory. Our theory is applicable in the underdoped and slightly overdoped regions where the Fermi liquid theory breaks down. In these regions there is another problem which should be taken into consideration, the opening of a pseudogap in the normal state of these materials.
Finally we mention that these calculations give a qualitative idea on the superconductivity in the non-Fermi superconductor. A quantitative comparison with the experiment seems difficult because of the existence of many parameters in the theory.

Acknowledgments

The author is grateful to Prof. M. Crisan for his comments on the physical aspects of the model.

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Figure Captions

Fig. 1 The $\alpha$ dependence of the coherence length for $T=0.1T_c$ and $T=0.8T_c$ given by eq. (19).
Fig. 2 The $\alpha$ dependence of the penetration depth for $T=0.1T_c$ and $T=0.8T_c$ given by eq. (22).
Fig. 3 The $\alpha$ dependence of the specific heat jump at the critical point given by eq. (24).
