Stability on Quinquevigintic Functional Equation in Different Spaces

Ramdoss Murali, Sandra Pinelas and Veeramani Vithya

Abstract. In this work, we determine the general solution of the quinquevigintic functional equation and also investigate its stability of this equation in the setting of matrix normed spaces and the framework of matrix non-Archimedean fuzzy normed spaces by using the fixed point method.

1 Introduction

In 1940, S. M. Ulam [36] initiated the study of stability problems for various functional equations. He raised a question related to the stability of homomorphism. In the following year, D. H. Hyers [8] was able to give a partial solution to Ulam’s question in the context of Banach spaces. This was the first significant breakthrough and a step towards more studies in this domain of research. In 1978, Th. M. Rassias [25] succeeded in extending the Hyers theorem by considering an unbounded Cauchy difference. He was the first to prove the stability of the linear mapping in Banach spaces. In 1950, T. Aoki [1] had provided a proof of a special case of the Rassias result when the given function is additive. In 1994, Gavruta [7] provided a further generalization of Rassias theorem in which he replaced the bound by a general control function for the existence of a unique linear mapping. Since then, a large number of papers have been published in connection with various generalizations of Ulam problem and Hyers theorem.

MSC 2020: AMS classification 46S40, 46S50, 47L25, 47H10, 54C30, 54E70
Keywords: Generalized Hyers-Ulam stability, fixed point, quinquevigintic functional equation, matrix normed spaces, matrix non-Archimedean fuzzy normed spaces
Affiliation:
Ramdoss Murali – PG and Research Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur – 635 601, TamilNadu, India
E-mail: shcmurali@yahoo.co.in
Sandra Pinelas – Departamento de Ciências Exatas e Engenharia, Academia Militar, A. Conde Castro Guimarães, 2720-113 Amadora, Portugal
E-mail: sandra.pinelas@gmail.com
Veeramani Vithya – PG and Research Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur – 635 601, TamilNadu, India
In 1991, the fixed point method was used for the first time by J. A. Baker [3], who applied a variant of Banach’s fixed point theorem to obtain the Hyers-Ulam stability of a functional equation in a single variable (for more applications of this method [4], [5], [9], [24]).

During the last eight decades, the stability problems of various functional equations such as additive [14], [18], [26], quadratic [22], [31], cubic [10], [28], [33], quartic [11], [15], [27] and mixed types [12], [37] have been investigated by many mathematicians.

In the recent years, the stability problems of higher degree functional equations (like quintic, sextic, septic, octic up to trevigintic and quattuorvigintic) have been broadly investigated by a number of mathematicians [2], [19], [20], [21], [23], [29], [30], [32], [34], [35], [38]. So, we are interested to introducing the new functional equation of the form

\[
\varsigma (u + 13v) - 25\varsigma (u + 12v) + 300\varsigma (u + 11v) - 2300\varsigma (u + 10v) + 12650\varsigma (u + 9v) \\
- 53130\varsigma (u + 8v) + 177100\varsigma (u + 7v) - 480700\varsigma (u + 6v) + 1081575\varsigma (u + 5v) \\
- 2042975\varsigma (u + 4v) + 3268760\varsigma (u + 3v) - 4457400\varsigma (u + 2v) \\
+ 5200300\varsigma (u + v) - 5200300\varsigma (u) + 4457400\varsigma (u - v) - 3268760\varsigma (u - 2v) \\
+ 2042975\varsigma (u - 3v) - 1081575\varsigma (u - 4v) + 480700\varsigma (u - 5v) - 177100\varsigma (u - 6v) \\
+ 53130\varsigma (u - 7v) - 12650\varsigma (u - 8v) + 2300\varsigma (u - 9v) - 300\varsigma (u - 10v) \\
+ 25\varsigma (u - 11v) - \varsigma (u - 12v) = 25!\varsigma (v),
\]

where the above functional equation (1) is said to be the quinquevigintic functional equation if the function \(\varsigma (u) = au^{25}\) is its solution.

The concept of non-Archimedean fuzzy normed spaces and matrix normed spaces has been introduced by Mirmostafafe et al. [17] and Effors et al. [6] respectively. Quite recently, the new results on stability of functional equations in non-Archimedean fuzzy normed spaces and matrix normed spaces studied in [16], [33], [34] and [13], [14], [15] respectively. The first result on the stability of functional equation in the setting of matrix non-Archimedean fuzzy normed spaces has been given in [22].

In this paper, we determine the general solution of the functional equation (1) and we also prove the stability of the functional equation (1) in matrix normed spaces and matrix non-Archimedean fuzzy normed spaces by using fixed point approach.

### 2 The General Solution of Functional Equation (1)

In this part, we provide the general solution of the quinquevigintic functional equation (1). For this, let us consider \(\mathcal{D}\) and \(\mathcal{E}\) real vector spaces.

**Theorem 2.1.** If \(\varsigma : \mathcal{D} \rightarrow \mathcal{E}\) is a mapping satisfying equation (1) for all \(x, y \in \mathcal{D}\), then \(\varsigma (2u) = 2^{25}\varsigma (u)\) for all \(u, v \in \mathcal{D}\).

**Proof.** Letting \(u = v = 0\) in (1), one gets \(\varsigma (0) = 0\). Substituting \(u = 0, v = u\) and \(u = u, v = -u\) in (1) and adding the two resulting equations, we get \(\varsigma (-u) = -\varsigma (u)\). Hence, \(\varsigma\) is an odd mapping.
Substituting $u = 0$, $v = 2u$ and $u = 12u$, $v = u$ in (1) and subtracting the two resulting equations, one gets

$$25\zeta(25u) - 324\zeta(24u) + 2300\zeta(23u) - 12375\zeta(22u) + 53130\zeta(21u) - 179100\zeta(20u)
+ 480700\zeta(19u) - 1071225\zeta(18u) + 2042975\zeta(17u) - 3309240\zeta(16u) + 4457400\zeta(15u)
- 5076330\zeta(14u) + 52000300\zeta(13u) - 4761000\zeta(12u) + 3268760\zeta(11u)
- 1442100\zeta(10u) + 1081575\zeta(9u) - 1442100\zeta(8u) + 177100\zeta(7u) + 1172655\zeta(6u)
+ 12650\zeta(5u) - 1190940\zeta(4u) + 300\zeta(3u) - 25!\zeta(2u) - 25!\zeta(u) = 0$$

(2)

for all $u \in \mathcal{D}$.

Substituting $(u, v)$ by $(12u, u)$ in (1), and increasing the resulting equation by 25, and then subtracting the resulting equation from (2), we get

$$301\zeta(24u) - 263120\zeta(21u) + 1149150\zeta(20u) - 3946800\zeta(19u) + 10946275\zeta(18u)
- 24996400\zeta(17u) + 47765135\zeta(16u) - 77261600\zeta(15u) + 106358670\zeta(14u)
- 124807200\zeta(13u) + 125246500\zeta(12u) - 108166240\zeta(11u) + 80276900\zeta(10u)
- 49992800\zeta(9u) - 25597275\zeta(8u) - 11840400\zeta(7u) + 5600155\zeta(6u) - 5200\zeta(23u)
- 1315600\zeta(5u) + 874690\zeta(4u) - 57200\zeta(3u) - 25!\zeta(2u) + 25!(26)\zeta(u) + 45125\zeta(22u) = 0$$

(3)

for all $u \in \mathcal{D}$.

Substituting $(u, v)$ by $(11u, u)$ in (1), and increasing the resulting equation by 301, and then subtracting the resulting equation from (3), we have

$$2325\zeta(23u) - 45175\zeta(22u) + 429180\zeta(21u) - 2658500\zeta(20u) + 12045330\zeta(19u)
- 42360825\zeta(18u) + 119694300\zeta(17u) - 277788940\zeta(16u) + 537673875\zeta(15u)
- 1440043800\zeta(14u) + 1457124060\zeta(13u) - 1261400500\zeta(12u) + 933903960\zeta(11u)
- 589338200\zeta(8u) + 313713675\zeta(7u) - 139090545\zeta(6u) + 51991500\zeta(5u)
- 877538090\zeta(14u) + 1216870200\zeta(13u) - 16866820\zeta(4u)
+ 3750450\zeta(3u) - 25!\zeta(2u) + 25!(327)\zeta(u) = 0$$

(4)

for all $u \in \mathcal{D}$.

Substituting $(u, v)$ by $(10u, u)$ in (1), and increasing the resulting equation by 2325, and then subtracting the resulting equation from (4), we have

$$12950\zeta(22u) - 268320\zeta(21u) + 2689000\zeta(20u) - 17365920\zeta(19u) + 81166425\zeta(18u)
- 292063200\zeta(17u) + 83938560\zeta(16u) - 197698800\zeta(15u) + 3872378785\zeta(14u)
- 6382996800\zeta(13u) + 8923411200\zeta(12u) - 1.063357344\times10^{10}\zeta(11u)
+ 1.0829297\times10^{10}\zeta(10u) - 9429551040\zeta(9u) + 7010528800\zeta(8u)
- 4436203200\zeta(7u) + 2375571330\zeta(6u) - 1065636000\zeta(5u) + 394890680\zeta(4u)
- 119776800\zeta(3u) - 25!\zeta(2u) + 25!(2652)\zeta(u) = 0$$

(5)
for all $u \in \mathcal{D}$.

Substituting $(u, v)$ by $(9u, u)$ in (1), and increasing the resulting equation by 12950, and then subtracting the resulting equation from (5), we arrive at

\[
55430\varsigma(21u) - 1196000\varsigma(20u) + 12419080\varsigma(19u) - 82651075\varsigma(18u) + 395970300\varsigma(17u) \\
- 1453606440\varsigma(16u) + 4248077000\varsigma(15u) - 1.013401747x10^{10}\varsigma(14u) \\
+ 2.007352945x10^{10}\varsigma(13u) - 3.34070308x10^{10}\varsigma(12u) + 4.708975656x10^{10}\varsigma(11u) \\
- 5.6514588x10^{10}\varsigma(10u) + 5.791433396x10^{10}\varsigma(9u) - 5.07128012x10^{10}\varsigma(8u) \\
+ 3.78942388x10^{10}\varsigma(7u) - 2.4080954x10^{10}\varsigma(6u) + 1.294076025x10^{10}\varsigma(5u) \\
- 5830174320\varsigma(4u) + 2173655250\varsigma(3u) - 25\varsigma(2u) + 25!(15602)\varsigma(u) = 0
\]  
(6)

for all $u \in \mathcal{D}$.

Substituting $(u, v)$ by $(8u, u)$ in (1), and increasing the resulting equation by 55430, and then subtracting the resulting equation from (6), we arrive at

\[
189750\varsigma(20u) - 4209920\varsigma(19u) + 44837925\varsigma(18u) - 305219200\varsigma(17u) + 1491389460\varsigma(16u) \\
- 5568576000\varsigma(15u) + 1.651118354x10^{10}\varsigma(14u) - 3.98781728x10^{10}\varsigma(13u) \\
+ 7.983507345x10^{10}\varsigma(12u) - 1.340976102x10^{11}\varsigma(11u) + 1.90559094x10^{11}\varsigma(10u) \\
- 2.30338295x10^{11}\varsigma(9u) + 2.375398278x10^{11}\varsigma(8u) - 2.091794432x10^{11}\varsigma(7u) \\
- 2.091794432x10^{11}\varsigma(7u) + 1.571064119x10^{11}\varsigma(6u) - 1.00301344x10^{11}\varsigma(5u) \\
+ 5.41214725x10^{10}\varsigma(4u) - 2.447016x10^{10}\varsigma(3u) - 25!\varsigma(2u) + 25!(71032)\varsigma(u) = 0
\]  
(7)

for all $u \in \mathcal{D}$.

Substituting $(u, v)$ by $(7u, u)$ in (1), and increasing the resulting equation by 189750, and then subtracting the resulting equation from (7), we have

\[
533830\varsigma(19u) - 12087075\varsigma(18u) + 131205800\varsigma(17u) - 908948040\varsigma(16u) \\
+ 4512841500\varsigma(15u) - 1.709354147x10^{10}\varsigma(14u) + 5.13346522x10^{10}\varsigma(13u) \\
- 1.25393782x10^{10}\varsigma(12u) + 2.53556896x10^{10}\varsigma(11u) - 4.29688116x10^{11}\varsigma(10u) \\
+ 6.15453355x10^{11}\varsigma(9u) - 7.492170972x10^{11}\varsigma(8u) + 7.75774818x10^{11}\varsigma(7u) \\
- 6.886852381x10^{11}\varsigma(6u) + 5.199456763x10^{11}\varsigma(5u) - 3.3352829x10^{11}\varsigma(4u) \\
+ 1.807017713x10^{11}\varsigma(3u) - 25!\varsigma(2u) + 25!(260782)\varsigma(u) = 0
\]  
(8)

for all $u \in \mathcal{D}$.

Substituting $(u, v)$ by $(6u, u)$ in (1), and increasing the resulting equation by 533830, and then subtracting the resulting equation from (8), we have

\[
1258675\varsigma(18u) - 28943200\varsigma(17u) + 318860960\varsigma(16u) + 1.630276745x10^{12}\varsigma(8u) \\
- 2240108000\varsigma(15u) + 1.126884644x10^{10}\varsigma(14u) - 4.32066408x10^{10}\varsigma(13u) \\
+ 1.312182982x10^{11}\varsigma(12u) - 3.238202862x10^{11}\varsigma(11u) + 6.609132283x10^{11}\varsigma(10u) \\
- 1.129508796x10^{12}\varsigma(9u) - 1.998498667x10^{12}\varsigma(7u) + 2.087390377x10^{12}\varsigma(6u) \\
- 1.85953482x10^{12}\varsigma(5u) + 1.411273712x10^{12}\varsigma(4u) \\
- 9.08671764x10^{11}\varsigma(3u) - 25!\varsigma(2u) + 25!(794612)\varsigma(u) = 0
\]  
(9)
for all \( u \in \mathcal{D} \).

Substituting \((u, v)\) by \((5u, u)\) in (1), and increasing the resulting equation by 1258675, and then subtracting the resulting equation from (9), we have

\[
2523675\zeta(17u) - 58741540\zeta(16u) + 654844500\zeta(15u) - 4653392315\zeta(14u) \\
+ 2.366676195x10^{10}\zeta(13u) - 9.169304432x10^{10}\zeta(12u) + 2.812247863x10^{11}\zeta(11u) \\
- 7.004381849x10^{11}\zeta(10u) + 1.441932762x10^{12}\zeta(9u) - 2.484029748x10^{12}\zeta(8u) \\
+ 3.61198019x10^{12}\zeta(7u) - 4.458065759x10^{12}\zeta(6u) + 4.68557518x10^{12}\zeta(5u) \\
- 4.196249281x10^{12}\zeta(4u) - 3.18971249x10^{12}\zeta(3u) - 25!\zeta(2u) + 25!(2053287)\zeta(u) \\
= 0,
\]

(10)

for all \( u \in \mathcal{D} \).

Substituting \((u, v)\) by \((4u, u)\) in (1), and increasing the resulting equation by 2523675, and then subtracting the resulting equation from (10), we have

\[
4350335\zeta(16u) - 102258000\zeta(15u) + 1151060185\zeta(14u) \\
- 8257726800\zeta(13u) + 4.238980845x10^{10}\zeta(12u) - 1.657180562x10^{11}\zeta(11u) \\
+ 5.126923876x10^{11}\zeta(10u) - 1.287611026x10^{12}\zeta(9u) + 2.671772661x10^{12}\zeta(8u) \\
- 4.63730678\zeta(7u) + 6.790260684x10^{12}\zeta(6u) - 8.43248747x10^{12}\zeta(5u) \\
+ 8.895693333x10^{12}\zeta(4u) - 7.925233602x10^{12}\zeta(3u) - 25!\zeta(2u) + 25!(4576962)\zeta(u) = 0
\]

(11)

for all \( u \in \mathcal{D} \).

Substituting \((u, v)\) by \((3u, u)\) in (1), and increasing the resulting equation by 4350335, and then subtracting the resulting equation from (11), we have

\[
6500375\zeta(15u) - 154040315\zeta(14u) + 1748043700\zeta(13u) - 1.26419293x10^{10}\zeta(12u) \\
+ 6.54152423x10^{10}\zeta(11u) - 2.577519409x10^{11}\zeta(10u) + 8.035906583x10^{11}\zeta(9u) \\
- 2.033332158x10^{12}\zeta(8u) - 4.249013764x10^{12}\zeta(7u) - 7.41998918x10^{12}\zeta(6u) \\
+ 1.090366402x10^{13}\zeta(5u) - 1.349622047x10^{13}\zeta(4u) \\
+ 1.392736917x10^{13}\zeta(3u) - 25!\zeta(2u) + 25!(8927297)\zeta(u) = 0
\]

(12)

for all \( u \in \mathcal{D} \).

Substituting \((u, v)\) by \((2u, u)\) in (1), and increasing the resulting equation by 6500375, and then subtracting the resulting equation from (12), we have

\[
8469060\zeta(14u) - 202068800\zeta(13u) + 2308933200\zeta(12u) \\
- 1.681450144x10^{10}\zeta(11u) + 8.760648248x10^{10}\zeta(10u) - 3.474632448x10^{10}\zeta(9u) \\
+ 1.089447992x10^{12}\zeta(8u) - 2.766678464x10^{12}\zeta(7u) + 5.777884692x10^{12}\zeta(6u) \\
- 9.99913684x10^{12}\zeta(5u) + 1.432733464x10^{13}\zeta(4u) \\
- 1.675180068x10^{13}\zeta(3u) - 25!\zeta(2u) + 25!(15427672)\zeta(u) = 0
\]

(13)
for all $u \in \mathcal{D}$.

Substituting $(u, v)$ by $(u, u)$ in (1), and increasing the resulting equation by $8469060$, and then subtracting the resulting equation from (13), we have

\[
9657700\zeta(13u) - 231784800\zeta(12u) + 2655867500\zeta(11u) - 1.931540001x10^{10}\zeta(10u) \\
+ 9.9957195x10^{10}\zeta(9u) - 3.90943696x10^{11}\zeta(8u) + 1.197265069x10^{12}\zeta(7u) \\
- 2.93207772x10^{12}\zeta(6u) + 5.803070488x10^{12}\zeta(5u) - 9.28491278x10^{12}\zeta(4u) \\
- 1.183826379x10^{13}\zeta(3u) - 25\zeta(2u) + 25!(23896732)\zeta(u) = 0
\]  
(14)

for all $u \in \mathcal{D}$.

Substituting $(u, v)$ by $(0, u)$ in (1), and increasing the resulting equation by $2496144$, and then subtracting the resulting equation from (14), we have $\zeta(2u) = 2^{25}\zeta(u)$ for all $u \in \mathcal{D}$. Thus $\zeta : \mathcal{D} \to \mathcal{E}$ is a quinquevigintic mapping.

\[\square\]

3 Generalized Hyers-Ulam Stability in Matrix Normed Spaces for Functional Equation (1)

In this section, we prove the generalized Hyers-Ulam stability for the functional equation (1) in matrix normed spaces by using the fixed point method.

Throughout this section, let us consider $(X, \|\cdot\|_n)$ a matrix normed spaces, $(Y, \|\cdot\|_n)$ a matrix Banach spaces and let $n$ be a fixed non-negative integer.

For a mapping $\zeta : X \to Y$, define $\mathcal{H}_\zeta : X^2 \to Y$ and $\mathcal{H}_n : M_n(X^2) \to M_n(Y)$ by

\[
\mathcal{H}_\zeta(c, d) = \zeta(c + 13d) - 25\zeta(c + 12d) + 300\zeta(c + 11d) - 2300\zeta(c + 10d) \\
+ 12650\zeta(c + 9d) - 53130\zeta(c + 8d) + 177100\zeta(c + 7d) - 480700\zeta(c + 6d) \\
+ 1081575\zeta(c + 5d) - 2042975\zeta(c + 4d) + 3268760\zeta(c + 3d) - 4457400\zeta(c + 2d) \\
+ 5200300\zeta(c + d) - 5200300\zeta(c) + 4457400\zeta(c - d) - 3268760\zeta(c - 2d) \\
+ 2042975\zeta(c - 3d) - 1081575\zeta(c - 4d) + 480700\zeta(c - 5d) - 177100\zeta(c - 6d) \\
+ 53130\zeta(c - 7d) - 12650\zeta(c - 8d) + 2300\zeta(c - 9d) - 300\zeta(c - 10d) \\
+ 25\zeta(c - 11d) - \zeta(c - 12d) - 25\zeta(d)
\]

for all $c, d \in X$.

\[
\mathcal{H}_\zeta(x_{rs}, y_{rs}) = \zeta(x_{rs} + 13y_{rs}) - 25\zeta(x_{rs} + 12y_{rs}) + 300\zeta(x_{rs} + 11y_{rs}) \\
- 2300\zeta(x_{rs} + 10y_{rs}) + 12650\zeta(x_{rs} + 9y_{rs}) - 53130\zeta(x_{rs} + 8y_{rs}) \\
+ 177100\zeta(x_{rs} + 7y_{rs}) - 480700\zeta(x_{rs} + 6y_{rs}) + 1081575\zeta(x_{rs} + 5y_{rs}) \\
- 2042975\zeta(x_{rs} + 4y_{rs}) + 3268760\zeta(x_{rs} + 3y_{rs}) - 4457400\zeta(x_{rs} + 2y_{rs}) \\
+ 5200300\zeta(x_{rs} + y_{rs}) - 5200300\zeta(x_{rs}) + 4457400\zeta(x_{rs} - y_{rs}) \\
- 3268760\zeta(x_{rs} - 2y_{rs}) + 2042975\zeta(x_{rs} - 3y_{rs}) - 1081575\zeta(x_{rs} - 4y_{rs}) \\
+ 480700\zeta(x_{rs} - 5y_{rs}) - 177100\zeta(x_{rs} - 6y_{rs}) + 53130\zeta(x_{rs} - 7y_{rs}) \\
- 12650\zeta(x_{rs} - 8y_{rs}) + 2300\zeta(x_{rs} - 9y_{rs}) - 300\zeta(x_{rs} - 10y_{rs}) \\
+ 25\zeta(x_{rs} - 11y_{rs}) - \zeta(x_{rs} - 12y_{rs}) - 25\zeta(y_{rs})
\]
Stability on Quinquevigintic Functional Equation in Different Spaces

for all \( x = [x_{rs}], y = [y_{rs}] \in M_n(X) \).

**Theorem 3.1.** Let \( q = \pm 1 \) be fixed and \( \sigma : X^2 \to [0, \infty) \) be a function such that there exists a \( \kappa < 1 \) with

\[
\sigma(c, d) \leq 2^{25q} \kappa \sigma\left(\frac{c}{2^q}, \frac{d}{2^q}\right) \quad \forall \ c, d \in X.
\]

Let \( \varsigma : X \to Y \) be a mapping satisfying

\[
\|H_\varsigma([x_{rs}, y_{rs}])\| \leq \sum_{r,s=1}^n \sigma(x_{rs}, y_{rs}) \quad \forall \ x = [x_{rs}], y = [y_{rs}] \in M_n(X).
\]

Then there exists a unique quinquevigintic mapping \( \Psi : X \to Y \) such that

\[
\|\varsigma_n([x_{rs}]) - \Psi_n([x_{rs}])\|_n \leq \sum_{r,s=1}^n \frac{\kappa \frac{1-2^q}{2^{25}}}{2^{25}(1-\kappa)} \sigma^*(x_{rs}) \quad \forall \ x = [x_{rs}] \in M_n(X),
\]

where

\[
\sigma^*(x_{rs}) = \frac{1}{25!} [\sigma(0, 2x_{rs}) + \sigma(13x_{rs}, x_{rs}) + 25\sigma(12x_{rs}, x_{rs}) + 301\sigma(11x_{rs}, x_{rs}) \\
+ 2325\sigma(10x_{rs}, x_{rs}) + 12950\sigma(9x_{rs}, x_{rs}) + 55430\sigma(8x_{rs}, x_{rs}) \\
+ 189750\sigma(7x_{rs}, x_{rs}) + 533830\sigma(6x_{rs}, x_{rs}) + 1258675\sigma(5x_{rs}, x_{rs}) \\
+ 2523675\sigma(4x_{rs}, x_{rs}) + 4350335\sigma(3x_{rs}, x_{rs}) + 6500375\sigma(2x_{rs}, x_{rs}) \\
+ 8469060\sigma(x_{rs}, x_{rs}) + 9657700\sigma(0, x_{rs})].
\]

**Proof.** Setting \( n = 1 \) in (16), we get

\[
\|H_\varsigma(c, d)\| \leq \sigma(c, d).
\]

Utilizing Theorem 2.1, we get

\[
\|-\varsigma(2c) + 2^{25}\varsigma(c)\| \leq \frac{1}{25!} [\sigma(0, 2c) + \sigma(13c, c) + 25\sigma(12c, c) + 301\sigma(11c, c) \\
+ 2325\sigma(10c, c) + 12950\sigma(9c, c) + 55430\sigma(8c, c) \\
+ 189750\sigma(7c, c) + 533830\sigma(6c, c) + 1258675\sigma(5c, c) \\
+ 2523675\sigma(4c, c) + 4350335\sigma(3c, c) + 6500375\sigma(2c, c) \\
+ 8469060\sigma(c, c) + 9657700\sigma(0, c)].
\]

Therefore,

\[
\|\varsigma(2c) - 2^{25}\varsigma(c)\| \leq \sigma^*(c) \quad \forall \ c \in X.
\]

Hence

\[
\|\varsigma(c) - \frac{1}{2^{25q}}\varsigma(2^q c)\| \leq \frac{\kappa^{\frac{1-2^q}{2^{25}}}}{2^{25}} \sigma^*(c) \quad \forall \ c \in X.
\]
Taking \( S = \{ f : X \to Y \} \) and the generalized metric \( \rho \) on \( S \) as follows:
\[
\rho(f, g) = \inf \{ \tau \in \mathbb{R}^+ : \|f(c) - g(c)\| \leq \tau \sigma^*(c), \forall c \in X \},
\]
it is easy to check that \((S, \rho)\) is a complete generalized metric (see also [18]). Define the mapping \( \mathcal{P} : S \to S \) by
\[
\mathcal{P} f(c) = \frac{1}{2^{25q}} f(2^q c) \quad \forall f \in S \text{ and } c \in X.
\]
Let \( f, g \in S \) and \( \nu \) an arbitrary constant with \( \rho(f, g) = \nu \). Then
\[
\|f(c) - g(c)\| \leq \nu \sigma^*(c)
\]
for all \( c \in X \). Using (15), we find that
\[
\|\mathcal{P} f(c) - \mathcal{P} g(c)\| = \left\| \frac{1}{2^{25q}} f(2^q c) - \frac{1}{2^{25q}} g(2^q c) \right\| \leq \kappa \nu \sigma^*(c)
\]
for all \( c \in X \). Hence we have
\[
\rho(\mathcal{P} f, \mathcal{P} g) \leq \kappa \rho(f, g)
\]
for all \( f, g \in S \). By (20), we have
\[
\rho(\varsigma, \mathcal{P} \varsigma) \leq \frac{\kappa^{\frac{1-q}{q}} - 1}{2^{25q}}.
\]
By Theorem 2.2 in [4], there exists a mapping \( \mathbb{V} : X \to Y \) which satisfies:

1. \( \mathbb{V} \) is a unique fixed point of \( \mathcal{P} \), which satisfies \( \mathbb{V}(2^q c) = 2^{23q} \mathbb{V}(c) \) \( \forall c \in X \).
2. \( \rho(\mathcal{P}^k \varsigma, \mathbb{V}) \to 0 \) as \( k \to \infty \). This implies that \( \lim_{k \to \infty} \frac{1}{2^{25q}} \varsigma(2^k c) = \mathbb{V}(c), \forall c \in X \).
3. \( \rho(\varsigma, \mathbb{V}) \leq \frac{1}{1 - \kappa} \rho(\varsigma, \mathcal{P} \varsigma) \), which implies the inequality
\[
\|\varsigma(c) - \mathbb{V}(c)\| \leq \frac{\kappa^{\frac{1-q}{q}} - 1}{2^{25}(1 - \kappa)} \sigma^*(c) \quad \forall c \in X.
\]
It follows from (15) and (16) that
\[
\|\mathcal{H} \mathbb{V}(c, d)\| = \lim_{k \to \infty} \frac{1}{2^{25kq}} \|\mathcal{H} \varsigma(2^{kq} c, 2^{kq} d)\|
\leq \lim_{k \to \infty} \frac{1}{2^{25kq}} \sigma(2^{kq} c, 2^{kq} d)
\leq \lim_{k \to \infty} \frac{2^{kq} \kappa q}{2^{25kq}} \sigma(c, d)
= 0,
\]
for all \( c, d \in X \). Therefore, the mapping \( \mathbb{V} : X \to Y \) is a quinquevigintic mapping. By Lemma 2.1 in [14] and (21), we get (17). Hence \( \mathbb{V} : X \to Y \) is a unique quinquevigintic mapping satisfying (17). \( \square \)
Corollary 3.2. Let \( q = \pm 1 \) be fixed and let \( l, \omega \) be non-negative real numbers with \( l \neq 25 \). Let \( \varsigma : X \to Y \) be a mapping such that

\[
\|H^n_{\varsigma}([x_{rs}], [y_{rs}])\|_n \leq \sum_{r,s=1}^{n} \omega \left(\|x_{rs}\|^l + \|y_{rs}\|^l\right) \quad \forall \ x = [x_{rs}], y = [y_{rs}] \in M_n(X). \tag{22}
\]

Then there exists a unique quinquevigintic mapping \( \mathcal{V} : X \to Y \) such that

\[
\|\varsigma_n([x_{rs}]) - \mathcal{V}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^{n} \frac{\omega_0}{|2^{25} - 2^l|} \|x_{rs}\|^l \quad \forall x = [x_{rs}] \in M_n(X),
\]

where

\[
\omega_0 = \frac{\omega}{25!} \left[ 8469060 + 6500375(2^a) + 4350335(3^a) + 2523675(4^a) + 1258675(5^a) + 533830(6^a) + 189750(7^a) + 55430(8^a) + 12950(9^a) + 2325(10^a) + 301(11^a) + 25(12^a) + (13^a) \right].
\]

Proof. The proof is similar to the proof of Theorem 3.1 by taking \( \sigma(c, d) = \omega(\|c\|^l + \|d\|^l) \) for all \( c, d \in X \). Then we can choose \( \kappa = 2^{(l-25)} \), and we obtain the required result. \( \Box \)

4 Quinquevigintic Functional Equation (1) and its Ulam-Gavruta-Rassias Stability

In this section, we investigate the Ulam-Gavruta-Rassias stability for the functional equation (1) in matrix normed spaces by using the fixed point method.

Theorem 4.1. Let \( q = \pm 1 \) be fixed and let \( l, \omega \) be non-negative real numbers such that \( l = a + b \neq 25 \). Let \( \varsigma : X \to Y \) be a mapping such that

\[
\|H^n_{\varsigma}([x_{rs}], [y_{rs}])\|_n \leq \sum_{r,s=1}^{n} \omega \left(\|x_{rs}\|^a \cdot \|y_{rs}\|^b\right) \quad \forall x = [x_{rs}], y = [y_{rs}] \in M_n(X). \tag{23}
\]

Then there exists a unique quinquevigintic mapping \( \mathcal{V} : X \to Y \) such that

\[
\|\varsigma_n([x_{rs}]) - \mathcal{V}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^{n} \frac{\omega_0}{|2^{25} - 2^l|} \|x_{rs}\|^l \quad \forall x = [x_{rs}] \in M_n(X),
\]

where

\[
\omega_0 = \frac{\omega}{25!} \left[ 8469060 + 6500375(2^a) + 4350335(3^a) + 2523675(4^a) + 1258675(5^a) + 533830(6^a) + 189750(7^a) + 55430(8^a) + 12950(9^a) + 2325(10^a) + 301(11^a) + 25(12^a) + (13^a) \right].
\]

Proof. The proof is similar to the proof of Theorem 3.1. \( \Box \)
5 Quinquevignic Functional Equation (1) and its J. M. Rassias Stability

In this section, we establish the J. M. Rassias stability for the functional equation (1) in matrix normed spaces by using the fixed point method.

**Theorem 5.1.** Let \( q = \pm 1 \) be fixed and let \( l, \omega \) be non-negative real numbers such that \( l = a + b \neq 25 \). Let \( \varsigma : X \to Y \) be a mapping such that

\[
\|H_{\varsigma_n}([x_{rs}], [y_{rs}])\|_n \leq \sum_{r,s=1}^{n} \omega (\|x_{rs}\|^{a} \cdot \|y_{ij}\|^{b} + \|x_{rs}\|^{a+b} + \|y_{rs}\|^{a+b}) \tag{24}
\]

for all \( x = [x_{rs}], y = [y_{rs}] \in M_n(X) \). Then there exists a unique quinquevignic mapping \( \mathcal{V} : X \to Y \) such that

\[
\|\varsigma_n([x_{rs}]) - \mathcal{V}_n([x_{rs}])\|_n \leq \sum_{r,s=1}^{n} \frac{\omega_0}{|2^{25} - 2^l|} \|x_{rs}\|^l
\]

for all \( x = [x_{rs}] \in M_n(X) \), where

\[
\omega_0 = \frac{\omega}{2^{25}}[50492552 + 6500375(2^a) + 6500376(2^b) + 4350335(3^a + 3^b)
+ 2523675(4^a + 4^b) + 1258675(5^a + 5^b) + 533830(6^a + 6^b)
+ 189750(7^a + 7^b) + 55430(8^a + 8^b) + 12950(9^a + 9^b)
+ 2325(10^a + 10^b) + 301(11^a + 11^b) + 25(12^a + 12^b) + (13^a + 13^b)].
\]

**Proof.** The proof is identical to the proof of Theorem 3.1. \( \square \)

6 Generalized Hyers-Ulam Stability in Matrix Non-Archimedean Fuzzy Normed Spaces for (1)

In this section, we investigate the generalized Hyers-Ulam stability for the functional equation (1) in matrix non-Archimedean fuzzy normed spaces by using the fixed point method.

Throughout this section, we assume that \( \mathbb{K} \) is a non-Archimedean field, \( X \) is a vector space over \( \mathbb{K} \) and \( (Y, N_n) \) is a complete matrix non-Archimedean fuzzy normed spaces over \( \mathbb{K} \), and \( (Z, N') \) is an Archimedean or a non-Archimedean fuzzy normed spaces.

**Theorem 6.1.** Let \( q = \pm 1 \) be fixed and let \( \sigma : X \times X \to Z \) be a mapping such that for some \( \kappa \neq 2^{25} \) with \( \left( \frac{\kappa}{2^{25}} \right)^t < 1 \) we have

\[
N'(\sigma(2^a c, 2^b d) \geq N' (\sigma(c, d), \kappa^{-qt}) \tag{25}
\]
for all \(c, d \in X\) and \(t > 0\), and \(\lim_{k \to \infty} \mathcal{N}(2^{-25kq}\mathcal{H}_\varsigma(2^{kq}c, 2^{kq}d), t) = 1\) for all \(c, d \in X\) and \(t > 0\). Suppose that an odd mapping \(\varsigma : X \to Y\) with \(\varsigma(0) = 0\) satisfies the inequality

\[\mathcal{N}(\mathcal{H}_\varsigma([x_\nu], [y_\nu]), t) \geq \sum_{i,j=1}^{n} \mathcal{N}'(\sigma(x_\nu, y_\nu), t)\]  

(26)

for all \(x = [x_\nu], y = [y_\nu] \in M_n(X)\), and \(t > 0\). Then there exists a unique quinquevigintic mapping \(\mathcal{V} : X \to Y\) such that

\[\mathcal{N}_n(s_n([x_\nu])) - \mathcal{V}_n([x_\nu]), t) \geq \min \left\{ \Gamma \left( x_\nu, \frac{|c - 2^{25}|}{n^2} : r, s = 1, 2, ..., n \right) \right\}\]  

(27)

for all \(x = [x_\nu] \in M_n(X)\) and \(t > 0\), where

\[\Gamma (x_\nu, t) = \min \left\{ \mathcal{N}'(\sigma(0, 2x_\nu), |25!|t), \mathcal{N}'(\sigma(13x_\nu, x_\nu), |25!|t) \right\}, \mathcal{N}'(\sigma(12x_\nu, x_\nu), \left| \frac{25!}{25} \right| t), \mathcal{N}'(\sigma(10x_\nu, x_\nu), \left| \frac{25!}{2325} \right| t)
\]

\[\mathcal{N}'(\sigma(9x_\nu, x_\nu), \left| \frac{25!}{12950} \right| t), \mathcal{N}'(\sigma(8x_\nu, x_\nu), \left| \frac{25!}{55430} \right| t), \mathcal{N}'(\sigma(7x_\nu, x_\nu), \left| \frac{25!}{189750} \right| t), \mathcal{N}'(\sigma(6x_\nu, x_\nu), \left| \frac{25!}{533830} \right| t)
\]

\[\mathcal{N}'(\sigma(5x_\nu, x_\nu), \left| \frac{25!}{1258675} \right| t), \mathcal{N}'(\sigma(4x_\nu, x_\nu), \left| \frac{25!}{2523675} \right| t), \mathcal{N}'(\sigma(3x_\nu, x_\nu), \left| \frac{25!}{4350335} \right| t), \mathcal{N}'(\sigma(2x_\nu, x_\nu), \left| \frac{25!}{6500375} \right| t)
\]

\[\mathcal{N}'(\sigma(x_\nu, x_\nu), \left| \frac{25!}{8469060} \right| t), \mathcal{N}'(\sigma(0, x_\nu), \left| \frac{25!}{9657700} \right| t), \mathcal{N}'(\sigma(11x_\nu, x_\nu), \left| \frac{25!}{301} \right| t)\].

Proof. For the cases \(q = 1\) and \(q = -1\), we consider \(\kappa < 2^{25}\) and \(\kappa > 2^{25}\), respectively. Substituting \(n = 1\) in (26), we obtain

\[\mathcal{N}(\mathcal{H}_\varsigma(c, d), t) \geq \mathcal{N}'(\sigma(c, d), t)\]  

(28)
for all \( c, d \in X \) and \( t > 0 \). Utilizing Theorem 1, we arrive at

\[
\mathcal{N}(-25|\varsigma(2c) + 33554432(25!)\varsigma(c), t) \\
\geq \min \left\{ \mathcal{N}'(\sigma(0, 2c), |25!|t), \mathcal{N}'(\sigma(13c, c), |25!|t), \right. \\
\mathcal{N}'(\sigma(10c, c), \frac{|25!|t}{2325}), \mathcal{N}'(\sigma(9c, c), \frac{|25!|t}{12950}), \\
\mathcal{N}'(\sigma(8c, c), \frac{|25!|t}{55430}), \mathcal{N}'(\sigma(7c, c), \frac{|25!|t}{189750}), \\
\mathcal{N}'(\sigma(6c, c), \frac{|25!|t}{533830}), \mathcal{N}'(\sigma(5c, c), \frac{|25!|t}{1258675}), \\
\mathcal{N}'(\sigma(4c, c), \frac{|25!|t}{2523675}), \mathcal{N}'(\sigma(3c, c), \frac{|25!|t}{4350335}), \\
\mathcal{N}'(\sigma(2c, c), \frac{|25!|t}{6500375}), \mathcal{N}'(\sigma(c, c), \frac{|25!|t}{8469060}), \\
\mathcal{N}'(\sigma(0, c), \frac{|25!|t}{9657700}), \mathcal{N}'(\sigma(12c, c), \frac{|25!|t}{25}), \\
\mathcal{N}'(\sigma(11c, c), \frac{|25!|t}{301}) \right\},
\]

for all \( c \in X \) and \( t > 0 \). Thus

\[
\mathcal{N} \left( \varsigma(c) - \frac{1}{2^{25q}} \varsigma(2^q c), \frac{\kappa(\frac{q-1}{2})}{2^{25q}(\frac{14^q}{2})} t \right) \geq \Gamma(c, t)
\]

(29)

for all \( c \in X \) and \( t > 0 \). We consider the set \( S_1 = \{ f_1 : X \rightarrow Y \} \) and introduce the generalized metric \( \rho \) on \( S_1 \) as follows:

\[
\rho(f_1, g_1) = \inf \{ \mu \in \mathbb{R}_+ : \mathcal{N}(f_1 - g_1, t) \geq \mu \Gamma(c, t), \forall c \in X, t > 0 \}.
\]

It is easy to check that \((S_1, \rho)\) is a complete generalized metric (see Lemma 3.2 in [18]).

Define the mapping \( \mathcal{J} : S_1 \rightarrow S_1 \) by

\[
\mathcal{J}f_1(c) = \frac{1}{2^{25q}} f_1(2^q c)
\]

for all \( f_1 \in S_1 \) and \( c \in X \).

Let \( f_1, g_1 \in S_1 \) and \( \nu \) be an arbitrary constant with \( \rho(f_1, g_1) \leq \nu \).

Then

\[
\mathcal{N} \left( f_1 - g_1, \nu t \right) \geq \Gamma(c, t)
\]

for all \( c \in X \) and \( t > 0 \).

Therefore, using (25), we get

\[
\mathcal{N} \left( \mathcal{J}f_1(c) - \mathcal{J}g_1(c), \nu t \right) = \mathcal{N} \left( f_1(2^q c) - g_1(2^q c), 2^{25q} \nu t \right) \geq \Gamma \left( c, \frac{2^{25q}}{\kappa^q} t \right)
\]
for all $c \in X$ and $t > 0$. Hence by definition $\rho(Jf_1, Jg_1) \leq \left(\frac{n}{225}\right)^q \nu$. This means that $J$ is a contractive mapping with Lipschitz constant $L = \left(\frac{n}{225}\right)^q < 1$. It follows from (29) that

$$\rho(\varsigma, J\varsigma) \leq \frac{\kappa(\frac{g-1}{r})}{|225(\frac{1}{2} + \nu)|}.$$ 

Therefore according to Theorem 2.2 in [5], there exists a mapping $V : X \to Y$ which satisfies:

1. $V$ is a unique fixed point of $J$, which satisfies $V(2^q c) = 2^{25q} V(c)$ $\forall c \in X$.
2. $\rho(J^k \varsigma, V) \to 0$ as $k \to \infty$, which implies that $\lim_{k \to \infty} 2^{25kq} \varsigma(2^{kq} c) = V(c)$ $\forall c \in X$.
3. $\rho(\varsigma, V) \leq \frac{1}{1-\kappa} \rho(\varsigma, J\varsigma)$, which implies that $\rho(\varsigma, V) \leq \frac{1}{|225-\kappa|}$.

So,

$$N\left(\varsigma(a) - V(c), \frac{1}{225-c}t\right) \geq \Gamma(c, t)$$

for all $c \in X$ and $t > 0$. By (28),

$$N(\mathcal{H}V(c, d), t) = \lim_{k \to \infty} N(2^{25kq} \mathcal{H}\varsigma(2^{kq} c, 2^{kq} d), t) \geq \lim_{k \to \infty} N'(2^{25kq} \sigma(2^{kq} c, 2^{kq} d), t) = 1.$$ 

Hence, $\mathcal{H}V(c, d) = 0$. Thus, the function $V$ satisfies quinquevigintic.

We note that $e_s \in M_{1,n}(\mathbb{R})$ means that the sth component is 1 and the others are zero, $E_{rs} \in M_{n}(X)$ means that $(r, s)$-component is 1 and the others are zero, and $E_{rs} \otimes x \in M_{n}(X)$ means that $(r, s)$-component is $x$ and the others are zero. Since $N(E_{kl} \otimes x, t) = N(x, t)$, we have

$$N_n([x_{rs}], t) = \sum_{r,s=1}^n E_{rs} \otimes x_{rs}, t \geq \min \{N_n(E_{rs} \otimes x_{rs}, t_{rs}) : r, s = 1, 2, ..., n\}$$

$$= \min \{N(x_{rs}, t_{rs}) : r, s = 1, 2, ..., n\}.$$ 

where $t = \sum_{r,s=1}^n t_{rs}$.

So, $N_n([x_{rs}], t) \geq \min \{N(x_{rs}, \frac{t_{rs}}{n^2}) : r, s = 1, 2, ..., n\}$. By (30), we get (27). Thus $V : X \to Y$ is a unique quinquevigintic mapping satisfying (27). □

References

[1] Aoki T.: On the stability of the linear transformation in Banach spaces. J. Math. Soc. Japan 2 (1950) 64–66.

[2] Arunkumar M., Bodaghi A., Rassias J. M., Sathiya E.: The general solution and approximations of a decic type functional equation in various normed spaces. J. Chungcheong Math. Soc. 29 (2) (2016) 287–328.

[3] Baker J. A.: The stability of certain functional equations. Proc. Amer. Math. Soc. 112 (3) (1991) 729–732.
[4] Cadariu L., Radu V.: Fixed points and the stability of Jensen’s functional equation. *J. Inequal. Pure Appl. Math.* 4 (1) (2003) 1–7.

[5] Diaz J. B., Margolis B.: A fixed point theorem of the alternative, for contractions on a generalized complete metric space. *Bull. Amer. Math. Soc.* 74 (1968) 305–309.

[6] Effros E. G., Ruan Z. J.: On Matricially Normed Spaces. *Pacific J. Math.* 132 (2) (1988) 243–264.

[7] Gavruta P.: A generalization of the Hyers-Ulam Rassias stability of approximately additive mappings. *J. Math. Anal. Appl.* 184 (1994) 431–436.

[8] Hyers D. H.: On the stability of the linear functional equation. *Proc. Natl. Acad. Sci. USA* 27 (1941) 222–224.

[9] Isac G., Rassias T. M.: Stability of \( \varphi \)-additive mappings: Applications to nonlinear analysis. *Internat. J. Math. Math. Sci.* 19 (1996) 219–228.

[10] Jun K. W., Kim H. M.: The generalized Hyers-Ulam-Rassias stability of a cubic functional equation. *J. Math. Anal. Appl.* 274 (2002) 867–878.

[11] Lee S. H., Im S. M., Hwang I. S.: Quartic Functional Equations. *J. Math. Anal. Appl.* 307 (2005) 387–394.

[12] Lee J., Shin D., Park C.: An AQCQ- functional equation in matrix Banach spaces. *Adv. Difference Equ.* 146 (2013) 1–15.

[13] Lee J., Shin D., Park C.: An additive functional inequality in matrix normed spaces. *Math. Inequal. Appl.* 16 (4) (2013) 1009–1022.

[14] Lee J., Shin D., Park C.: Hyers-Ulam stability of functional equations in matrix normed spaces. *J. Inequal. Appl.* 22 (2013) 1–11.

[15] Lee J., Shin D., Park C.: Functional equations in matrix normed spaces. *Proc. Indian Acad. Sci.* 125 (3) (2015) 399–412.

[16] Mihet D.: Fuzzy \( \varphi \)- contractive mapping in non- Archimedean fuzzy metric spaces. *Fuzzy Set Sys.* 159 (2008) 739–744.

[17] Mirmostafaei A. K., Moslehian M. S.: Fuzzy versions of Hyers-Ulam Rassias theorem. *Fuzzy Set Sys.* 159 (2008) 720–729.

[18] Mihet D., Radu V.: On the stability of the additive Cauchy functional equation in random normed spaces. *J. Math. Anal. Appl.* 343 (2008) 567–572.

[19] Murali R., Pinelas S., Antony Raj A.: General solution and a fixed point approach to the Ulam Hyers stability of viginti duo functional equation in multi-Banach spaces. *IOSR J. Math.* 13 (4) (2017) 48–59.

[20] Murali R., Pinelas S., Vithya V.: Stability of tredecic functional equation in matrix normed spaces. *J. Adv. Math.* 13 (2) (2017) 7135–7145.

[21] Murali R., Pinelas S., Vithya V.: The stability of vigintiumus functional equation in various spaces. *Global J. Pure Appl. Math.* 13 (2) (2017) 5735–5759.

[22] Murali R., Thandapani E., Vithya V.: A new quadratic functional equation in two spaces. *Asian J. Math. Comput. Res.* 20 (3) (2017) 169–178.

[23] Murali R., Vithya V.: Trevigintic and quattuorvigintic functional equations in matrix normed spaces. *Malaya J. Mathematik* 7 (2) (2019) 251–258.
Stability on Quinquevigintic Functional Equation in Different Spaces

[24] Radu V.: The fixed point alternative and the stability of functional equations. Fixed Point Theory 4 (1) (2003) 91–96.

[25] Rassias T. M.: On the stability of the linear mapping in Banach spaces. Proc. Am. Math. Soc. 72 (1978) 297–300.

[26] Rahimi A., Najafzadeh S.: Hyers-Ulam-Rassias stability of additive type functional equation. General Mathematics 17 (4) (2009) 45–55.

[27] Rassias J. M.: Solution of the Ulam stability problem for quartic mappings. Glasnik Matematicki Series III 34 (54) (1999) 243–252.

[28] Rassias J. M.: Solution of the Ulam stability problem for cubic mappings. Glasnik Matematicki Series III 36 (56) (2001) 63–72.

[29] Rassias J. M., Ravi K., Senthil Kumar B. V.: A fixed point approach to Ulam-Hyers stability of duodecic functional equation in quasi-β normed spaces. Tbil. Math. J. 10 (4) (2017) 83–101.

[30] Ravi K., Rassias J. M., Senthil Kumar B. V.: Ulam-Hyers stability of undecic functional equation in quasi-β normed spaces: Fixed point method. Tbil. Math. J. 9 (2) (2016) 83–103.

[31] Ravi K., Murali R., Arunkumar M.: The generalized Hyers-Ulam-Rassias stability of quadratic functional equation. J. Inequal. Pure Appl. Math. 9 (1) (2008) 1–5.

[32] Ravi K., Rassias J. M., Pinelas S., Suresh S.: General solution and stability of quattuordecic functional equation in quasi-β normed spaces. Adv. Pure Math. 6 (2006) 921–941.

[33] Renu C., Sushma L.: A fixed point approach to Ulam stability problem for cubic and quartic mapping in non-archimedean fuzzy normed spaces. Proc. WCE 2010 3 (2010) 1931–1934.

[34] Senthil Kumar B. V., Ashish K., Narasimman P.: Estimation of approximate nonic functional equation in non-Archimedean fuzzy normed spaces. Int. J. Pure Appl. Math. Tech. 1 (2) (2016) 1–29.

[35] Shen Y., Chen W.: On the stability of septic and octic functional equations. J. Comput. Anal. Appl. 18 (2) (2015) 277–290.

[36] Ulam S. M.: Problems in Modern Mathematics, Science Editions. Wiley, NewYork (1964).

[37] Wang Z., Sahoo P. K.: Stability of an ACQ- functional equation in various matrix normed spaces. J. Nonlinear Sci. Appl. 8 (2015) 64–85.

[38] Xu T. Z., Rassias M. J., Xu W. X., Rassias J. M.: A fixed point approach to the stability of quintic and sextic functional equations in quasi-β normed spaces. J. Inequal. Appl. 2010 (2010) 1–23.

Received: July 05, 2018
Accepted for publication: April 30, 2021
Communicated by: Karl Dilcher