Higher dimensional operators and their effects in (non)supersymmetric models

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Abstract

It is shown that a 4D N=1 softly broken supersymmetric theory with higher derivative operators in the Kahler or the superpotential part of the Lagrangian and with an otherwise arbitrary superpotential, can be re-formulated as a theory without higher derivatives but with additional (ghost) superfields and modified interactions. The importance of the analytical continuation Minkowski-Euclidean space-time for the UV behaviour of such theories is discussed in detail. In particular it is shown that power counting for divergences in Minkowski space-time does not always work in models with higher derivative operators.

Based on talk presented at
“Supersymmetry 2007” Conference, 26 July - 1 August 2007, Karlsruhe, Germany.

1 This is based on work done in collaboration with I. Antoniadis and E. Dudas.
1 Introduction

Higher dimensional operators play an important role in the study of physics beyond the Standard Model (SM) and its minimal supersymmetric version (MSSM). These operators are a generic presence in phenomenological studies and are also predicted in effective field theories of compactification, which attempt to recover in the low energy limit the SM or MSSM.

Higher dimensional operators, derivative or otherwise, can dramatically affect physics near the scale where they become relevant and also the UV regime of the theory. While non-derivative higher dimensional operators have been studied in the past, the case of higher derivative operators is less investigated. Such operators were studied in the context of Randall-Sundrum models [1], cosmology [2], phase transitions [3], supergravity [4], string theory [5], have applications to regularisation in 4D [6, 7], and effects in the UV [8], are generated by loop corrections in extra dimensions in 5D and 6D orbifolds [9]. In the following we would like to show how one can address the effects of higher derivative operators, while still including arbitrary higher dimensional (non-derivative) operators. To this purpose we show that one can re-write a theory with higher dimensional (derivative or otherwise) operators as a theory with only two-derivatives with additional superfields and higher dimensional non-derivative interactions. This will be done in the context of general 4D N=1 supersymmetric models and the case of softly broken supersymmetry will also be considered.

Before proceeding to the supersymmetric case which we discuss in Section 3, we review the case of non-supersymmetric higher derivative operators. This case has been investigated in [10] where a path integral quantisation of a theory with such operators was presented in an Euclidean formulation of the theory. Interacting theories with higher derivative operators involve the presence of ghost states, and this can bring in unitarity violation. However this presence is not something specific to such theories, and is actually quite common: for example 4D gauge theories regularised à la Pauli-Villars also have ghosts [6, 7]. Also in string theory the world-sheet theory of D-branes has non-renormalisability problems similar to gravity/supergravity, and allows the presence of higher derivatives (and thus ghosts). Returning to field theories with higher derivatives, it was shown in [10] that in such theories the vacuum to vacuum amplitude and therefore Green functions are well-defined and there is no exponential growth of the amplitude, provided that the ghost fields are not asymptotic states (they can still be present as loop states). The result is that one violates unitarity (at the scale of higher derivative operators) but one cannot create ghosts in the final state.
There is in general some ambiguity in theories with higher dimensional operators, associated with the analytical continuation form Minkowski to Euclidean space-time. The presence of higher derivative operators brings in additional poles in the complex plane, associated with ghost states, and this requires one define a contour of integration around these poles compatible with that of second order theories. To show the importance of the analytical continuation we provide examples where it dramatically alters the UV behaviour of the theory. As a result, different contours of integration give a different UV behaviour for the theory. One consequence is that, contrary to the naive expectation, the power counting of divergences in the Minkowski space-time does not always work in such theories (whether supersymmetric or not). This is then questioning, in some circumstances, the UV regularisation role usually attributed in the literature to higher derivative operators. For details on the results presented below see [8, 11].

2 Higher dimensional operators: non-supersymmetric case

Let us review the situation in the non-supersymmetric case. Consider the 4D Lagrangian

\[ \mathcal{L} = -\frac{1}{2} \phi (\xi \Box^2 + \Box + m^2 - i \epsilon) \phi - f(\phi), \tag{1} \]

with \( \xi \equiv 1/M_*^2 > 0 \) and \( M_* \) is the (high) scale of “new physics” where the higher derivative term becomes important; \( f(\phi) \) is a general function accounting for terms in the potential other than the mass term and can include higher dimensional (non-derivative) operators. In (1) one can change the basis to [10] (see also [8])

\[ \varphi_{1,2} = -\frac{(\Box + m_{\pm}^2 \pm i \epsilon^*) \phi \sqrt{\xi}}{(m_{\pm}^2 - m_2^2 + 2i\epsilon^*)^{1/2}}, \tag{2} \]

where we introduced

\[ m_{\pm}^2 = \frac{1}{2\xi} \left[ 1 \pm (1 - 4\xi m^2)^{1/2} \right], \quad \epsilon^* = \frac{\epsilon}{(1 - 4\xi m^2)^{1/2}} \tag{3} \]

Assume in the following that \( \Lambda^2 \gg M_*^2 \gg m^2 \), therefore \( \epsilon^* \approx \epsilon \ll 1 \) and also \( m_{\pm}^2 \approx 1/\xi, m_{-}^2 \approx m^2; \Lambda \) is the UV cut-off of the theory. With this, eq.(1) becomes

\[ \mathcal{L} = -\frac{1}{2} \varphi_1 (\Box + m_2^2 - i \epsilon^*) \varphi_1 + \frac{1}{2} \varphi_2 (\Box + m_{\pm}^2 + i \epsilon^*) \varphi_2 - f(\varphi_2 - \varphi_1) \tag{4} \]

\(^2\)The metric convention is \((+, -, -, -)\) and \( \Box = \partial_\mu \partial^\mu \).
The field $\varphi_2$ corresponds to a ghost field (has negative kinetic term). As seen from (2), original $\phi \propto \varphi_1 - \varphi_2$ is a mixing of a particle and a ghost-like degrees of freedom. The result is that the original Lagrangian with four derivatives is “unfolded” into a two-derivative Lagrangian. In Section 3 we shall see how to perform a similar procedure in the supersymmetric case [11].

Above we assumed a particle-like $i\epsilon$ prescription in the propagator of the 4-derivative theory of eq.(1), and obtained from this the corresponding propagators’ prescriptions in the two-derivative formulation of eq.(4). These prescriptions can also be seen from:

$$\frac{1}{-\xi \Box^2 - \Box - m^2 + i\epsilon} = \frac{1}{(1 - 4\xi m^2)^{1/2}} \left[ \frac{1}{-\Box - m^2_- + i\epsilon^*} - \frac{1}{-\Box - m^2_+ - i\epsilon^*} \right]$$

(5)

In the rhs we obtained the same propagators and prescriptions as in (4), for the particle ($m_-$) and ghost ($m_+$) fields. A natural assumption was made above that the familiar pole prescription for a particle field in the two-derivative theory remains true in the presence of the higher derivative term. This is shown in the lhs of (5) and in (11) by the sign in front of $i\epsilon$. This is certainly true at low $p^2$, but at higher scales this is less clear because there is no path integral formulation of (11) in the Minkowski space-time, to fully clarify which analytical continuation one should take for a higher derivative theory. In any case, with our prescription (11), one finds that the propagators in (11), (5) emerged with opposite (signs for the) $i\epsilon^*$ prescriptions.

In conclusion, once a particular prescription in the fourth order theory is made, the second order theory has a well-defined analytical continuation from Minkowski to Euclidean space-time. In principle, one could ask that the ghost propagator (second term) in the rhs in (5) have a similar sign in front of $i\epsilon^*$ as the particle field. That would give in the lhs a momentum dependent $\epsilon = \epsilon^*(1 + 2\xi \Box)$. Such analytical continuation can be considered and corresponds to different physics. Note that in this case, for fixed $\epsilon$, the relation $\epsilon^* \ll 1$ does not remain valid for $p^2 \sim 1/\xi$ and poles would move in the complex plane, far from the real axis.

To examine some of the consequences of (11) we calculate the one-loop correction $\delta m^2$ to the mass of $\phi$, for an interaction $f(\phi) = \lambda \phi^4/4!$ Using either formulation (11) or (11), one has

$$-i\delta m^2 = -i\lambda \mu^{4-d} \int \frac{d^dp}{(2\pi)^d} \frac{i}{-\xi p^4 + p^2 - m^2 + i\epsilon}$$

$$= \frac{-i\lambda \mu^{4-d}}{\sqrt{1 - 4\xi m^2}} \int \frac{d^dp}{(2\pi)^d} \left[ \frac{i}{p^2 - m^2_- + i\epsilon^*} - \frac{i}{p^2 - m^2_+ - i\epsilon^*} \right]$$

$$= \frac{-i\lambda \mu^{4-d}}{\sqrt{1 - 4\xi m^2}} \int d^dE \left[ \frac{1}{p^2 + m^2_-} + \frac{1}{p^2 + m^2_+} \right],$$

(6)
where the last integral is in the Euclidean space, showed by the index \( \mathbf{E} \); \( \mu \) is the standard mass scale introduced by the DR scheme, and \( d = 4 - \omega, \omega \to 0 \). The result for \( \delta m^2 \) is quadratically divergent; this is in contradiction to what power-counting suggests, when applied to the first line in (6), that the integral should only be logarithmic divergent. The difference is explained by the fact that the two propagators in the second line above are Wick-rotated in opposite directions (corresponding to \( \pm i \epsilon \)), bringing in a relative minus sign. Moreover, if regarded in two-dimensions \( (d \to 2) \) the above equation would appear as finite by power counting, while the result shows it is log divergent. The conclusion is simple: power counting in Minkowski space-time does not always work in effective field theories. This is an example which shows that higher derivative operators do not always improve the UV behaviour of the theory, as commonly stated in the literature.

To understand better the above results, it is useful to discuss the Euclidean picture. The Euclidean version of (1) is

\[
\mathcal{L}_E = \frac{1}{2} \phi \left( \xi \Box_E^2 - \Box_E + m^2 \right) \phi + f(\phi)
\]  

(7)

At one-loop, the Feynman rules of \( \mathcal{L}_E \) with \( f(\phi) = \lambda \phi^4/4! \) give

\[
- \delta m^2 = -\lambda \mu^{4-d} \int_{\mathbf{E}} \frac{d^d p}{(2\pi)^d} \frac{1}{\xi p^4 + p^2 + m^2} = -\lambda \mu^{4-d} \int_{\mathbf{E}} \frac{d^d p}{(2\pi)^d} \left[ \frac{1}{p^2 + m_-^2} - \frac{1}{p^2 + m_+^2} \right]
\]  

(8)

where all \( p^2 \) are evaluated in Euclidean space, as shown by subscript \( \mathbf{E} \). A similar result is found by working in \( \varphi_{1,2} \) basis. It is obvious that this result has for \( d = 4 \) no quadratic divergence but only a logarithmic one, in agreement with power-counting in (5). Also, eq. (8) is different from the last line in (6) which has an opposite sign between the contributions from \( m_- \) and \( m_+ \); the origin of this different result in Euclidean case is due to the analytical continuation from the Minkowski to the Euclidean space-time. To conclude, we provided an explicit example where the same operator brings a different UV of a one-loop correction in the Euclidean and Minkowski formulations of a theory. This discussion shows the importance of the analytical continuation in effective field theories with higher derivative operators.

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\(^3\) One could impose the prescription in (5) for the lhs be such as the ghost and particle in the rhs have identical prescriptions and thus similar Wick rotations when going to the Euclidean space-time. The result in (6) would then be as in Euclidean case (5). It would be interesting to know if such a (momentum dependent) prescription is preferred by a Minkowskian path integral formulation of the higher derivative theory.
From the above discussion one could conclude that the UV behaviour of $\delta m^2$ in the Euclidean formulation is improved compared to that in Minkowski space-time, and that it has no quadratic divergences. As discussed in [8] a counterexample shows that this is not true. Consider the addition to the Lagrangian in (7) of $\Delta L_E = -z \phi^2 \Box \phi^2$ which also has dimension six, just like $\xi \phi \Box \phi$, and should then be included. Its one-loop effect on the mass of $\phi$ is

$$
\left. \delta m^2 \right|_{\Delta L_E} \equiv -16 z \mu^{4-d} \frac{d}{(2\pi)^d} \int_E \frac{m^2}{\sqrt{1 - 4 \xi m^2}} \left[ \frac{m^2_+}{p^2 + m^2_+} - \frac{m^2_-}{p^2 + m^2_-} \right]
$$

This result is clearly quadratic divergent. The Minkowski counterpart to this result is also quadratic divergent, regardless of the signs $\pm i \epsilon^*$ of the prescriptions one adds to the above denominators for analytical continuation. Therefore the Euclidean version of a 4D theory with additional dimension-six operators has quadratic divergences. The effect of this dimension-six operator was not included in the analysis in [12].

From the previous discussion we can also conclude that in a 4D theory, in the presence of higher derivative operators, the mapping of the radiative corrections computed in Euclidean and Minkowski formulations is not always at the level operator-by-operator. Instead, as it was shown in [8], for a given order in perturbation theory, a set of operators in the Minkowski formulation can provide the same correction to the mass as in the Euclidean theory, for an appropriate choice of the couplings.

We conclude that a 4D theory with higher dimensional (derivative) operators can have a two-derivative description in Minkowski space-time, and the UV regime of such a theory depends strongly on the analytical continuation to the Euclidean space-time. In particular we saw that, contrary to common statements, theories with such operators do not always have an improved UV behaviour. As a result, the role of higher derivative operators as UV regulators in Minkowski space-time can be questioned, in the absence of a specific analytical continuation.

### 3 Higher dimensional operators: the supersymmetric case

In the following we extend the above discussion to the case of (softly broken) supersymmetric models [8, 11]. Consider for example the case of a Wess-Zumino model with an additional supersymmetric higher derivative operator:


\[ \mathcal{L} = \int d^4 \theta \Phi (1 + \xi \Box) \Phi + \left\{ \int d^2 \theta \left[ \frac{1}{2} m \Phi^2 + \frac{1}{3} \lambda \Phi^3 \right] + \text{c.c.} \right\} - m_0^2 \Phi \Phi^* \]

\[ = \ F^* (1 + \xi \Box) \Phi - \phi^* \Box (1 + \xi \Box) \phi + i \partial_{\mu} \bar{\psi} \tilde{\sigma}^{\mu} (1 + \xi \Box) \psi \]

\[ + \left[ \frac{1}{2} m (2 \phi F - \psi \psi) + \lambda \left( \phi^2 F - \phi \psi^2 \right) + \text{h.c.} \right] - m_0^2 \phi \phi^* \] (10)

Notice that in the presence of the higher derivative term, \( F \) and \( \Box \phi \) are dynamical degrees of freedom. Let us compute the one-loop correction to the mass of scalar field \( \phi \) after soft supersymmetry breaking by \( m_0^2 \phi \phi^* \). For simplicity we present the result for a massless case \( (m = 0) \). The one-loop scalar (fermionic) contributions \( \Delta m^2_b \) \( (\Delta m^2_f) \) to the mass of \( \phi \) are

\[ \Delta m^2_b = \frac{4 (i\lambda)^2}{(2\pi)^4} \int \frac{d^4 p}{(2\pi)^4} \left[ \frac{1}{p^2 (1 - \xi p^2) - m_0^2 + i\epsilon} \right] \left( p^2 - 1/\xi + i\tilde{\epsilon} \right), \]

\[ \Delta m^2_f = \frac{2 (-i\lambda)^2}{(2\pi)^4} \int \frac{d^4 p}{(2\pi)^4} \left[ \frac{2}{(1 - \xi p^2) + i\epsilon} \right] \left( p^2 - 1/\xi - i\tilde{\epsilon} \right) \] (11)

The first propagator under the integral of \( \Delta m^2_b \) is that of \( \phi \) in the presence of the higher derivative, in agreement with (1), while the second is that of the auxiliary field \( F \), and for which a prescription \( \tilde{\epsilon} = -\epsilon \) is fixed by supersymmetry. Adding the above contributions gives [8]

\[ \Delta m^2_\varphi = \frac{\lambda^2}{4\pi^2} \left\{ (3\xi m_0^2) 2\Lambda^2 - m_0^2 \ln(1 + \Lambda^2/m_0^2) + \cdots \right\} \] (12)

where the dots account for corrections which are at most logarithmic in \( \Lambda \) where \( \Lambda \) is the UV cutoff of the loop integrals. In the decoupling limit of higher derivative operators \( 1/\xi = M^2_* \gg \Lambda^2 \) one obtains the usual logarithmic correction for \( \Delta m^2_\varphi \), while for a mass \( M_* < \Lambda \) the one-loop correction is quadratic divergent; this could be somewhat surprising since power-counting in (11) would suggest the result is finite! As discussed in the previous section, here is another example that power-counting applied to (11) gives a misleading result, since such method ignores the poles prescriptions of the ghost degrees of freedom. The conclusion is that, for the analytical continuation considered here, the breaking of supersymmetry, although done by terms which are soft, does not remain soft in the presence of supersymmetric higher derivative terms. This discussion shows the importance of analytical continuation Minkowski-Euclidean space-time in solving the hierarchy problem in the presence of such (supersymmetric) operators. It is possible to find a continuation such as no quadratic divergence is present; this
happens if the factor in $\Delta m_{b,f}^2$ containing $p^4$ is a product of two propagators with similar Wick rotations (i.e. same sign for $i\epsilon$); this would then require a momentum-dependent prescription in the higher derivative theory with all its consequences (see non-susy discussion after eq. (5)). This discussion underlines the need for a path integral formulation of the higher derivative theories in Minkowski space-time from which one could obtain the prescriptions on rigorous mathematical grounds.

The next step is to extend to the supersymmetric level, the earlier finding that a non-supersymmetric theory with higher derivative operators can be reformulated as a theory with two derivatives only and modified interactions [11]. Consider for this a Lagrangian which generalises that in (10) (no susy breaking):

$$L = \int d^4\theta \left[ \Phi \, (1 + \xi \Box) \Phi + S \Phi^\dagger S \right] + \{ \int d^2\theta \ W(\Phi, S) + h.c. \}$$

(13)

where $S$ is some additional chiral superfield and $W$ is a superpotential whose exact form is not relevant in the following. This Lagrangian can be re-written in a new fields basis with only two-derivative operators present, similar to the non-supersymmetric case. To see how this works first notice that one can re-write $\xi \Phi \Box \Phi$ as $-\xi \left( D^2 \Phi^\dagger / 4 \right) \left( D^2 \Phi \right) / 4$. The idea is to replace somehow $D^2 \Phi$ (and its conjugate) by another chiral (anti-chiral) superfield. This can be done by using a $2 \times 2$ unitary transformation to a new basis $\Phi_{1,2}$, of the form: $\Phi = a_1 \Phi_1 + a_2 \Phi_2$ and $m^{-1} \tilde{D}^2 \Phi^\dagger = b_1 \Phi_1 + b_2 \Phi_2$ where $m$ is some mass scale in the theory (for example the mass of the field or a vev) and $a_1, a_2$ and $b_1, b_2$ form a unitary matrix. Since $\Phi, \tilde{D}^2 \Phi$ are not independent, one must also introduce a constraint to account for this. To this purpose, the Lagrangian is modified by adding $\Delta L$ where

$$\Delta L = \int d^2\theta \left[ m^{-1} \left( a_1^* D^2 \Phi^\dagger_1 + a_2^* D^2 \Phi^\dagger_2 \right) - (b_1 \Phi_1 + b_2 \Phi_2) \right] \Phi_3 m_* + h.c.,$$

$$= -4 \int d^4\theta \left[ \frac{m_*}{m} \left( a_1^* \Phi^\dagger_1 + a_2^* \Phi^\dagger_2 \right) \Phi_3 + h.c. \right] - \int d^2\theta \left( b_1 \Phi_1 + b_2 \Phi_2 \right) \Phi_3 m_* + h.c.$$ (14)

Here $m_* \equiv \sqrt{\xi} m^2/4$; its dependence on $\xi$ was determined by requiring the constraint be absent in the limit $\xi \to 0$ and by using that each squared (super)derivative comes with a $1/4$ factor. $\Phi_3$ is a new “constraint” superfield introduced by the holomorphic $\Delta L$. The new, total Lagrangian is $L' = L + \Delta L$ from which we recover the constraint via the eq of motion for $\Phi_3$.\footnote{No such formulation is currently available.}
The next step is to bring to a canonical form the kinetic terms in $L'$. This is done via a unitary transformation from $\Phi_{1,2,3}$ to new $\tilde{\Phi}_{1,2,3}$ and a subsequent rescaling of $\tilde{\Phi}_{1,2,3}$ to obtain canonically normalised Kahler terms. In all this the superfield $S$ is spectator. After some calculations one finds for $L'$ in basis $\tilde{\Phi}_{1,2,3}$ and for a small $\xi$ ($m^2 \ll 1/\xi$)

$$
L' = \int d^4\theta \left[ \tilde{\Phi}^\dagger_1 \tilde{\Phi}_1 - \tilde{\Phi}^\dagger_2 \tilde{\Phi}_2 - \tilde{\Phi}^\dagger_3 \tilde{\Phi}_3 + S^\dagger S \right] 
+ \left\{ \int d^2\theta \left[ -\frac{1}{\sqrt{\xi}} \tilde{\Phi}_2 \tilde{\Phi}_3 + W(\tilde{\Phi}_2 - \tilde{\Phi}_1; S) \right] + \text{h.c.} \right\},
$$

(15)

The above result shows that one can re-write the initial Lagrangian with higher derivative operators in the Kahler term, as a Lagrangian with two derivatives only. The difference to the original formulation (10), (13) is the appearance of extra (ghost) superfields $\tilde{\Phi}_{2,3}$, of negative kinetic terms. Their presence is easily understood - we initially had $F$ and $\Box\Phi$ as dynamical fields, which, by supersymmetry, require the presence of two extra superfields, in agreement with (15). The relation between the original $\Phi$ and $\tilde{\Phi}_{1,2,3}$, is found to be $\Phi = (\tilde{\Phi}_2 - \tilde{\Phi}_1)$. According to this the original superfield $\Phi$ contains a piece of “ghost”!

The result in (15) is the supersymmetric equivalent of (4) and “unfolds” a theory with higher dimensional (derivative) supersymmetric operators into a two-derivative theory, in a manifest supersymmetric way. The technique applied to obtain (15) is general and does not depend on the exact form of $W$ which can contain operators of $D > 4$. The method can also be generalised to the case when more higher derivative terms are present, provided the appropriate holomorphic constraints ($\Delta L$) are introduced. Under eventual iteration, we can map a theory with higher derivative terms in the Kahler potential, into one without such terms, but eventually with higher dimensional non-derivative operators.

The analysis can be extended to the case when higher derivative terms are present in the superpotential, following the same steps [11]. For example one can consider

$$
L = \int d^4\theta \left[ \Phi^\dagger \Phi + S^\dagger S \right] + \left\{ \int d^2\theta \left[ s \sqrt{\xi} \Phi \Box \Phi + W(\Phi, S) \right] + \text{h.c.} \right\}
$$

(16)

where $s = \pm 1$; $W$ is not specified, and can contain higher dimensional operators. One then follows the same procedure: first replace $D^2 \Phi$ by a chiral superfield, then re-write the higher derivative term as a D-term, and then follow the same steps as in the case of higher derivative Kahler terms. Since the auxiliary field of $\Phi$ is not dynamical, the form of the Lagrangian in
the new basis has only one additional (ghost) superfield. The equivalent Lagrangian of the second order theory is then obtained:

\[ \mathcal{L}' = \int d^4\theta \left[ \tilde{\Phi}^\dagger_1 \tilde{\Phi}_1 - \tilde{\Phi}^\dagger_2 \tilde{\Phi}_2 + S^\dagger S \right] + \left\{ \int d^2\theta \left[ \frac{\tilde{\Phi}^2_2}{4 s \sqrt{\xi}} + W(\tilde{\Phi}_2 - \tilde{\Phi}_1; S) \right] + h.c. \right\} \] (17)

This shows that one can indeed re-write a supersymmetric theory with higher derivative terms in the Kahler or superpotential part of the Lagrangian, as a two-derivative theory, eventually with additional higher dimensional (non-derivative) operators.

The above results can also be extended to the case of supersymmetry breaking. If one added soft breaking terms to the original Lagrangian of (10), they are easily re-written in the new basis by using the relation between the old and new fields \( \Phi = \tilde{\Phi}_2 - \tilde{\Phi}_1 \). This step raises the legitimate question whether the technique we employed, by introducing a holomorphic constraint (14), remains valid in the presence of soft breaking. We checked this is indeed the case for specific cases of soft breaking terms. To do so we computed the spectrum of the model in the old (10) and new basis (17) of fields, after soft supersymmetry breaking, to obtain the same result [11]. This was checked for a standard superpotential of the form given in eq. (10).

4 Conclusions

We have stressed the role that analytical continuation Minkowski-Euclidean space-time is playing in 4D models with additional higher dimensional (derivative) operators. To show this we provided detailed examples where it can dramatically alter the UV behaviour of the theory. These examples also showed that power-counting is not always a reliable tool in examining the UV of loop integrals in Minkowski space-time, in the presence of such operators. This questions the UV regularisation role of higher derivative operators in Minkowski formulations of such theories, in the absence of a rather specific analytical continuation.

In the supersymmetric case, it was discussed how a theory with higher derivative operators in the kinetic term or in the superpotential and with an otherwise arbitrary superpotential is equivalent to a theory with two-derivatives only, with additional superfields and modified interactions, and with higher dimensional (non-derivative) operators. The new formulation of the theory generalises a similar non-supersymmetric equivalence and can prove useful for applications.
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