OPTIMAL CONTROL PROBLEM
FOR A CONVEYOR-TYPE PRODUCTION LINE

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Abstract. A method is developed for the optimal control of parameters of a conveyor-type production line. The model of the conveyor line is presented by the partial differential equation, which allows taking into account the distribution of products along the technological route as a function of time. Various variants of stepped speed control of the conveyor belt are investigated. The features of stepped control are described. The divergence of the rate of output by the production line from the given demand is determined for different parameters of stepped control.

Keywords: conveyor, subject of labor, production line, parameters of the state of production line, technological position, transition period, production management systems.

INTRODUCTION

Competitive capacity of a flow production depends on the system of control of parameters of the production line. To design management systems for modern flow production lines, discrete-event models (DES models) [1], queueing theory models (QN models) [2], fluid models [3], and models with application of partial differential equations (PDE models) [4, 5] are most often used. The class of PDE models of production lines was generated due to the global tendency of the development of flow production systems, which implies reduction of production cycle under nonstationary demand for the goods [6]. A considerable part of the life cycle, flow production lines operate with a variable in time productivity of the output. The manufacture development factors listed above substantially limited the possibility of using the well proved DES, QN, and fluid models as basic ones for design of flow production line management systems.

PROBLEM STATEMENT

Theoretical calculations and experimental studies of the operation of such production systems have shown that the rate of production output from the final technological operation of flow production line $[\chi](t, S_d)$ depends both on the total number of subjects of labor $W(t)$

$$[\chi](t, S_d) = CL(W(t)), \quad W(t) = \int_0^{S_d} [\chi]_0(t, S) dS, \quad (1)$$

being in progress at instant of time $t$ ($W(t) =$ WIP: Work-in-Progress; $CL(W(t))$ is clearing function of the production system) [7] and on their distribution with respect to technological operations along the technological route with the linear density $[\chi]_0(t, S)$ [6, 8, 9]. The coordinate $S \in [0, S_d]$ specifies the technological position, or the place of

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processing of the subject of labor in the technological route, where \( S = S_0 \) and \( S = S_d \) are the values of the coordinate of technological position for the first and final operations, respectively. The dependence between processing the rate \([\chi]_1(t, S)\) and density function of the subjects of labor \([\chi]_0(t, S)\) along the technological route in single-moment description has the following form [4–6]:

\[
\frac{\partial[\chi]_0(t, S)}{\partial t} + \frac{\partial[\chi]_1(t, S)}{\partial S} = 0, \tag{2}
\]

\[
[\chi]_0(0, S) = \Psi(S), \tag{3}
\]

where \( S_d \) is the coordinate of the technological position for the final operation; \([\chi]_0(t, S)\) and \([\chi]_1(t, S)\) are the density function and rate of processing of the subjects of labor at the instant of time \( t \) at the technological position characterized by the coordinate \( S \); \( \Psi(S) \) is the initial distribution of the subjects of labor along the technological route. Occurrence of a new class of models of production systems (PDE models) has posed the problem of designing systems of control of parameters of flow production lines with regard for the factors of work in progress distributed along the technological route.

**REVIEW OF THE CONDUCTED RESEARCH**

The study [10] considers the problem of optimal control of a production line whose model is presented by the system of equations

\[
\frac{\partial[\chi]_0(t, S)}{\partial t} + \frac{\partial([\chi]_0(t, S)V(W(t)))}{\partial S} = 0,
\]

\[
V(W(t)) = \frac{V_{\max}}{1 + W(t)}, \quad \lambda(t) = V(W(t))[\chi]_0(t, 0),
\]

\[
[\chi]_0(0, S) = \Psi(S).
\]

The speed of the motion of products along the technological route \( V(W(t)) \) depends on the total amount of work in progress \( W(t) \) varying in time as a result of the production activity; \( \lambda(t) \) is the flow of products arriving at the first technological operation.

In the present paper, we will present a numerical method to construct the optimal control and will analyze the obtained results. The method of orthogonal functions is used in the problem solution. The theoretical base of application of the method of conjugate operators in constructing the optimal controls of parameters of the flow production line is generated in [11]. The numerical method (IMEX-Runge–Kutta Discretization) of the design of optimal control of flow production with application of the small parameter method is considered in [12]. Using PDE models to construct optimal operation modes for steel-rolling manufactures is described in [13]. Application of PDE models in problems of design of management systems for production lines on semiconductor manufacturing is considered in [5, 6, 9, 11, 14, 15]. The problem of optimal control of parameters of flow production lines in mechanical engineering is considered in [8, 16–18]. Solution of the system of equations (2), (3) for the case of constant speed of the motion of products along the flow production line or its separate part in the absence of control is presented in [16, 19]. The system of equations (2), (3) allows describing the operation of a wide class of production systems [9–11], among which conveyor-type enterprises are noteworthy [20–24]. Adjustment of the speed of conveyor belt changes the channel capacity of the production line. Non-uniform loading of rock along belt conveyor influences energy consumption in rock transportation [21–23], determines the dynamics of system’s operation as a whole. The value of the rock flow sent to the conveyor permanently varies in time. The requirement occurs that the speed of conveyor belt should correspond to the value that ensures optimal conveyor system operation mode. Non-uniform distribution of the loaded rock along the conveyor significantly influences the principle of conveyor speed regulation and in turn the production cost [21, 23]. Figure 1 shows the daily share of time during which the conveyor operates in one of the speed modes determined by the production technology \((g = v/V_n\) is relative velocity; \(v\) is actual speed of the conveyor; \(V_n = 3.8\) (m/sec) is the maximum admissible speed of the conveyor) [24].
In the present paper, we will propose a method for the optimal control of conveyor belt speed depending on the current demand for production.

**PDE MODEL OF THE CONVEYOR BELT LINE**

A specific feature of modeling a conveyor line for an industrial enterprise is that production loaded to conveyor belt moves with identical speed. For mine conveyors, rock transportation speed at an arbitrary point is equal to the belt speed. The system of equations for production line parameters in single-step approximation [8, 16], which describes the motion of rock along the conveyor line has the form

\[ \frac{\partial x_0(t, S)}{\partial t} + \frac{\partial x_1(t, S)}{\partial S} = \delta(S) \lambda(t), \quad \int_{-\infty}^{\infty} \delta(S) dS = 1, \tag{4} \]

under the initial conditions

\[ [x_0](0, S) = H(S) \Psi(S), \quad H(S) = \begin{cases} 0 & \text{if } S < 0, \\ 1 & \text{if } S \geq 0, \end{cases} \quad S \in [0, S_d]. \]

Parameters \([x_0](t, S)\) and \([x_1](t, S)\) are related by the coefficient \(a = a(t)\) (m/h), which defines the speed of the conveyor belt. The right-hand side \(\delta(S) \lambda(t)\) of Eq. (4) specifies the source of the material for the first technological operation \((S = 0\) (m)), \(\delta(S)\) is delta function. The intensity of rock inflow at the conveyor is expressed by the function \(\lambda(t)\), which characterizes the capacity of the line (t/h). At the initial instant of time \(t = 0\) (h), on the conveyor there is a material distributed along the belt with linear density \([x_0](0, S)\) (t/m). Suppose that demand for production [5, 10, p. 2513] is defined and is presented by time function \(\sigma(t)\) (t/h).

We will describe the parameters of the conveyor by the dimensionless variables:

\[ \tau = \frac{t}{T_d}, \quad \xi = \frac{S}{S_d}, \tag{5} \]

\[ \theta_0(\tau, \xi) = \frac{x_0(t, S)}{\Theta}, \quad \psi(\xi) = \frac{\Psi(S)}{\Theta}, \quad \gamma(\tau) = \lambda(t) \frac{T_d}{S_d} \theta, \quad \vartheta(\tau) = \sigma(t) \frac{T_d}{S_d}, \tag{6} \]

\[ g(\tau) = a(t) \frac{T_d}{S_d}, \quad \Theta = \max \left\{ \Psi(S), \frac{\lambda(t)}{a(t)} \right\}, \quad \delta(\xi) = S_d \delta(S), \quad H(\xi) = H(S). \tag{7} \]
Taking into account the notation introduced in (5)–(7), we write the equation of balance of flow parameters of the conveyor in the dimensionless form

\[
\frac{\partial \theta_0(\tau, \xi)}{\partial \tau} + g(\tau) \frac{\partial \theta_0(\tau, \xi)}{\partial \xi} = \delta(\xi) \gamma(\tau),
\]

(8)

\[\theta_0(0, \xi) = H(\xi) \psi(\xi).\]  

(9)

The system of equations (8), (9) is associated with the system of characteristics

\[
\frac{d\xi}{d\tau} = g(\tau), \quad \xi|_{\tau=0} = \beta,
\]

(10)

\[
\frac{d\theta_0(\tau, \xi)}{d\xi} = \delta(\xi) \frac{\gamma(\tau)}{g(\tau)}, \quad \theta_0(0, \beta) = H(\beta) \psi(\beta).
\]

(11)

We will write the solution of Eq. (10) as

\[
\xi = G(\tau) + C_1 = G(\tau) - G(0) + \beta, \quad C_1 = \text{const}, \quad G(\tau) = \int g(\tau) \, d\tau,
\]

(12)

where the integration constant \(C_1 = \beta - G(0)\) is defined from the condition \(\xi|_{\tau=0} = \beta\) in formula (10).

Integrating Eq. (11) yields the relation

\[
\theta_0(\tau, \xi) = \int \delta(\xi) \frac{\gamma(\tau)}{g(\tau)} \, d\xi + C_2, \quad C_2 = \text{const}.
\]

(13)

From formula (12), we express \(\tau = G^{-1}(\xi - \beta + G(0))\), substitute the result into the integral expression (13), and obtain

\[
\theta_0(\tau, \xi) = \int \delta(\xi) \frac{\gamma(G^{-1}(\xi - \beta + G(0)))}{g(G^{-1}(\xi - \beta + G(0)))} \, d\xi + C_2 = H(\xi) \frac{\gamma(G^{-1}(G(\tau) - \xi))}{g(G^{-1}(G(\tau) - \xi))} + C_2.
\]

We will find the integration constant \(C_2\) from the boundary condition (11)

\[
\theta_0(0, \beta) = H(\beta) \frac{\gamma(G^{-1}(G(0) - \beta))}{g(G^{-1}(G(0) - \beta))} + C_2 = H(\beta) \psi(\beta),
\]

\[C_2 = H(\beta) \psi(\beta) - H(\beta) \frac{\gamma(G^{-1}(G(0) - \beta))}{g(G^{-1}(G(0) - \beta))},
\]

which allows writing the solution of Eq. (8) under the initial conditions (9) as

\[
\theta_0(\tau, \xi) = H(\xi) \frac{\gamma(G^{-1}(G(\tau) - \xi))}{g(G^{-1}(G(\tau) - \xi))} + H(\xi - G(\tau)) \psi(\xi - G(\tau)) - H(\xi - G(\tau)) \frac{\gamma(G^{-1}(G(\tau) - \xi))}{g(G^{-1}(G(\tau) - \xi))}.
\]

(14)

Let us introduce variable \(\tau_\xi\) defined by the relation \(G(\tau_\xi) = G(\tau) - \xi\) and represent solution (14) as

\[
\theta_0(\tau, \xi) = [H(\xi) - H(-G(\tau_\xi))] \frac{\gamma(\tau_\xi)}{g(\tau_\xi)} + H(-G(\tau_\xi)) \psi(-G(\tau_\xi)), \quad G^{-1}(G(\tau) - \xi) = \tau_\xi.
\]

(15)

Let us consider the steady state mode of operation of the conveyor. This mode begins at the instant of time \(\tau > \tau_\psi\), i.e., in the time interval from the beginning of operation of the conveyor during which all the materials being on the
conveyor belt at the initial instant of time are processed at the final operation, and their distribution at the subsequent instants of time is defined by the intensity of input of materials \(\gamma(\tau)\) at the conveyor. We will find the value of the time interval \(\Delta \tau_p > (\tau_p - \tau_0)\), \(\tau_0 = 0\), necessary to pass to steady state operation mode, from the inequality

\[ G(\tau_p) - 1 > 0. \]

For the speed of the conveyor belt \(a(t) = 3.8\) (m/sec) \([14]\) and conveyor length \(S_d = 13500\) (m) \([20]\), estimated value of the duration of transient mode is \(\Delta \tau_p \approx 1\) (h). For motor industry, this quantity is several days, for enterprises on manufacturing semiconducting production it is several months. For the steady state mode \((\tau > \tau_p)\), we will write solution (15) as

\[ \theta_0(\tau, \dot{\xi}) = \frac{\gamma(\tau \xi)}{g(\tau \xi)}. \]  

(16)

We obtain the value of the output flow of production \(\theta_1(\tau, 1) = \theta_0(\tau, 1)g(\tau)\) from the conveyor for the steady state mode from the expression

\[ \theta_1(\tau, 1) = \frac{\gamma(\tau \xi)}{g(\tau \xi)}g(\tau), \quad \tau_1 = G^{-1}(G(\tau) - 1). \]

(17)

The dimensionless density of subjects of labor at the given technological position is defined by the value of function \(\theta_0(\tau, \dot{\xi})\). For two technological positions, which characterize input to the conveyor and output from it, respectively, the relation \(\theta_0(\tau, 1) = \theta_0(\tau, 0)\) is true. Let us introduce delay time \(\Delta \tau_1 = \tau - \tau_1\), which characterizes lag factor for the technological positions being at the distance \(\Delta \xi = 1\). The subject of labor processed at the instant of time \(\tau\) and arrived for processing at the instant of time \(\tau_1\) will be in processing during time \(\Delta \tau_1\). Taking this into account, we can rearrange expression (17) for the output flow of product from the conveyor with the use of delay time \(\Delta \tau_1\):

\[ \theta_1(\tau, 1) = \frac{\gamma(\tau \xi - \Delta \tau_1)}{g(\tau \xi - \Delta \tau_1)}g(\tau), \quad \Delta \tau_1 = \tau - \tau_1. \]

(18)

Thus, if the intensity of input of materials at the conveyor \(\gamma(\tau)\) and conveyor belt speed \(g(\tau)\) are known, then formulas (16) and (18) uniquely define distribution of materials along the conveyor and output flow of production from the conveyor at the instant of time \(t\).

PROBLEM OF THE OPTIMAL CONTROL OF THE SPEED OF CONVEYOR BELT

Let us formulate the problem of constructing the optimal program to control the speed of conveyor belt for steady state operation mode of the conveyor line (16), (18): determine production output \(\theta_1(\tau, 1)\) from the conveyor during time interval \(\tau = [0, \tau_k]\) under stepped control of conveyor belt speed \(u(\tau) = (u_1, u_2)\), \(0 < u_1 < u_2 < \infty\), \(u_j = \text{const}\), which minimizes the functional

\[ \int_0^{\tau_k} |\theta_1(\tau, 1) - \theta(\tau)| \, d\tau \rightarrow \min \]

(19)

under the differential relations based on Eqs. (8)

\[ \frac{\partial \theta_0(\tau, \dot{\xi})}{\partial \tau} + u(\tau) \frac{\partial \theta_0(\tau, \dot{\xi})}{\partial \dot{\xi}} = \delta(\dot{\xi})\gamma(\tau), \quad g(\tau) = u(\tau), \]

(20)

constraints

\[ \theta_0(\tau, \dot{\xi}) \geq 0, \]

(21)

and initial conditions (9)

\[ \theta_0(0, \dot{\xi}) = H(\dot{\xi})\psi(\dot{\xi}). \]

(22)
Let us reformulate problem (19)–(22) taking into account the system of characteristic equations (10), (11): find the product output 
\[ \int_0^{\tau_k} |\theta_1(\tau, 1) - \bar{\theta}(\tau)| \, d\tau \to \min \]
under the differential relations based on the system of characteristic equations (10), (11)
\[ \frac{d\xi}{d\tau} = u(\tau), \quad \frac{d\theta_0(\tau, \xi)}{d\xi} = \delta(\xi) \frac{\gamma(\tau)}{g(\tau)} \quad \text{or} \quad \theta_0(\tau, \xi) = \frac{\gamma(\tau \xi)}{g(\tau \xi)}, \]
constraints
\[ \theta_0(\tau, \xi) \geq 0, \quad 0 \leq \xi \leq 1, \]
and initial conditions
\[ \xi|_{\tau=0} = \beta, \quad \theta_0(0, \beta) = H(\beta)\psi(\beta). \]

Assume that prior to introducing the control, the conveyor operated in the steady state mode with constant speed 
\[ g(\tau)|_{\tau=0} = u_1. \]
The Pontryagin function and the conjugate system have the form
\[ H = -\left[ \frac{\gamma(\tau - \Delta \tau_1)}{u(\tau - \Delta \tau_1)} u(\tau) - \bar{\theta}(\tau) \right] + \psi_1 u(\tau) \to \max, \quad \int_{\tau - \Delta \tau_1}^{\tau} u(\tau) \, d\tau = 1, \]
\[ \frac{d\psi_1}{d\tau} = \frac{\partial H}{\partial \xi} = 0. \]

Since the right-hand end of the phase trajectory is free, \( \psi_1(\tau_k) = 0 \) and hence, \( \psi_1(\tau) \equiv 0 \), which allows writing the Pontryagin function (25) as
\[ H = -\left[ \frac{\gamma(\tau - \Delta \tau_1)}{u(\tau - \Delta \tau_1)} u(\tau) - \bar{\theta}(\tau) \right] \to \max. \]

Let us construct the optimal control of the conveyor belt speed for the case of constant intensity \( \gamma(\tau) = 1 \) of the input of raw materials to the conveyor, under the current demand for production, which is defined by the function 
\( \bar{\theta}(\tau) = 1 + \sin(\omega \tau) \) [10, p. 2517]. We suppose that the conveyor can operate in one of the following speed modes: with the belt speed \( u_1 \) or \( u_2 \), 
\[ u(\tau) = (u_1, u_2) . \]
The results of calculations are presented in Fig. 2 for different values of stepped belt speed control \( u_1 \) or \( u_2 \). Maximization of function (24) defines control \( u(\tau) \) that provides production output with the minimum deviation from current demand \( \bar{\theta}(\tau) \).

**ANALYSIS OF THE RESULTS**

Figure 2 shows the calculation of the optimal control \( u(\tau) \) of the speed of conveyor belt depending on the demand \( \bar{\theta}(\tau) \). The graphs on the left-hand side represent the dependence of optimal control \( u(\tau) \) on the value of demand \( \bar{\theta}(\tau) \); on the right, for each variant of dependence \( u(\tau) \) and \( \bar{\theta}(\tau) \), the graph of the family of characteristics (23) with knees at the instants of speed mode switching is presented. To each control mode in Figs. 2a–2e there corresponds time dependence of the output flow of production \( \theta_1(\tau, 1) \) from the conveyor (Fig. 3) according to the current demand \( \bar{\theta}(\tau) \). For the variant of velocities of stepped control \( u(\tau) = (0.5, 2.0) \), Fig. 2a, there are no conveyor belt speed switching modes. The rate of output of production from the conveyor \( \theta_1(\tau, 1) \) is constant and does not depend on the current demand (Fig. 3a). Such behavior is somewhat caused by the fact that the initial speed of conveyor operation without control is assumed to be the smaller one out of two velocities, \( g(\tau)|_{\tau=0} = u_1 \), and by a considerable scatter between the
stepped values of the velocities of control, which is characterized by the coefficient \( k_u = \frac{u_2}{u_1} = 4.0 \) (Fig. 2a). When \( k_u \) is reduced by increasing the value of \( u_1 \), the conveyor begins operating in double-speed mode (Fig. 2b), the family of characteristics at the switching moments has knees. The rate of output of production from the conveyor \( \theta_1 (\tau, 1) \) (Fig. 3b) is defined by the behavior of demand \( \theta (\tau) \) and takes one of the three values \{0.4; 1.0; 2.5\}, which is defined by the relation

\[
\frac{u(\tau)}{u(\tau - \Delta \tau_1)} = \begin{bmatrix} 2.0 & 0.8 & 0.8 \\ 0.8 & 0.8 & 2.0 \end{bmatrix}
\]

in (26). The non-uniformity of the density of production \( \theta_0 (\tau, \xi) \) (16) is specified by the conveyor speed mode and takes one of the two values \( \theta_0 (\tau, \xi) = \frac{\gamma (\tau, \xi)}{g (\tau, \xi)} = \begin{bmatrix} 1.0 & 1.0 \\ 0.8 & 2.0 \end{bmatrix} \). The production

Fig. 2. Graphs of the optimal control of the speed and families of characteristics for operating modes: a — \( u(\tau) = (0.5, 2.0) \); b — \( u(\tau) = (0.8, 2.0) \); c — \( u(\tau) = (1.0, 2.0) \); d — \( u(\tau) = (1.2, 2.0) \); e — \( u(\tau) = (1.8, 2.0) \).
density function along the conveyor \( \theta_0(0.5, \xi) \) for the instant of time \( \tau = 0.5 \) is reflected in Fig. 3b. Note that the duration of operation of the conveyor in control mode \( u(\tau) = u_2 \) increases with each switching and attains the steady state mode (Fig. 2b). Subsequent reduction of \( k_u \) adds short-term peak switchings (Figs. 2c, d), which disappear with further reduction of \( k_u \) (Figs. 2e, 3e). Occurrence of these peaks is shown on the graphs of the distribution of density of production \( \theta_0(\tau, \xi) \) (16) (Figs. 3c, d). Such behavior of the control function can be partially explained by the fact that control of the conveyor speed at the current instant of time \( u(\tau) \) depends on the control accepted at the instant of time \( u(\tau - \Delta \tau_1) \), where \( \Delta \tau_1 \) is a time-dependent quantity defined by relation (24).
CONCLUSIONS

In the paper, we have considered the method of constructing the optimal control of the speed of a conveyor for the PDE model of a conveyor-type production line. We have used a model example of two-step adjustment of the speed of conveyor belt, which is a widespread variant of operation of modern production lines, and have analyzed a stepped speed control of the conveyor belt of a production system. We have investigated various modes of control of conveyor speed. We have determined short-term peak switchings of the conveyor speed and found the reasons of their occurrence. Of practical interest are dependences of the rate of production output from the conveyor on the given demand for it for the optimal mode of stepped speed control of the conveyor [24].

The results presented in the paper expand the perspectives of the analysis of flow production systems and can be used to design a production management system with multiple cyclic transition of details through the production line or its separate section (re-entrant factory) [6, 14]. Interest to this field of studies is due to the high rates of the development of semiconductor industry. The technological process of manufacture of semiconductor elements requires multiple transition of repeated stages of manufacture, which include a great number of technological operations specified in a certain order [14]. Such principle of construction of production process leads to a queue at the input of production line, which is formed by product batches that have completed different numbers of stages of technological process due to the recurrence of production phases. Thus, there arises the problem of establishing the necessary size of batches of semiconductor products and sequence of their processing. Solution of the problem of flow-shop scheduling with multiple cyclical transition of details through its sections (re-entrant factory) implies constructing the optimal feasible schedule within the limits of the given optimality criterion and finding the optimal control of parameters of the production line, which is premises for the further studies.

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