An Improved Method for Dynamic Behaviour Prediction of Carbon Fibre Reinforced Epoxy (CFRE) using Finite Element Model Updating

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Abstract. The dynamic behaviour prediction scheme of carbon fibre composite structures are extremely challenging to be performed due to the geometrical ply-by-ply properties and the ply orientation issues. In this paper, a laminated carbon fibre reinforced epoxy plate with elastic and linear layers was used with aim to investigate the dynamic behaviour of the carbon fibre reinforced epoxy using practical modelling scheme via finite element modelling and updating method. A simplified FE model of the CFRE plate was developed using shell element properties known as PSHELL to represent geometrical laminated properties. Subsequently, the dynamic behaviour of the FE model was calculated using Nastran SOL103. Potential updating parameters of the FE model was analysed and identified using the sensitivity analysis. The model updating method was then used to update the initial FE model of the CFRE plate based on the experimental modal analysis (EMA) result. The comparison of the results were used in verifying the accuracy of the updated FE model of the CFRE plate. The result suggested that the PSHELL properties can be used efficiently to represent the geometrical laminated properties of the CFRE plate without considering the geometrical effect of ply-by-ply properties.

1. Introduction
Carbon fibre composite materials are increasingly used in replacing the conventional materials such as steel and aluminium in many engineering supplications such as in the automotive industries for boosting the fuel economy while maintaining the safety and performance. Carbon fibre structures are usually made up from two or more constituent materials and laminated together to produce anisotropic properties and allows the manufactures to produce more complex shapes of with a higher strength-to-weight ratios, corrosion resistance, and workability [1]. However, the anisotropic material behaviour of the laminated carbon fibre complicates the structural dynamic behaviour prediction in which the information of the direction-dependent mechanical properties of the composite materials are often limited [2]. Furthermore, the end-product of the carbon fibre structure is always found not precise as the mould shape because the manufacturing process of the structure induced distortion to the structure and causing the geometrical variations. Therefore, to analyse the dynamics behaviour of the laminated composites are complicated and computationally expensive due to the material behaviour and geometrical variations such as a thermal expansion mismatch between the fibres and resin, chemical shrinkage of the resin during polymerisation, and moisture absorption [3].
There are several approaches has been suggested by researchers in predicting the dynamic behaviour of carbon fibre structure [4–6]. Different type of FE models to represent carbon fibre structure have been studied with various modelling schemes from detailed to simplified models. Previously, detailed modelling using solid (3D) elements has found to be the most schemed to be used by research as presented by [7,8]. There are models in which thin layer elements are used with considering the ply thickness, orientations and individual ply material properties [9]. Most of the researchers modelled composite structures using PCOMP method and PCOMPS method in which the layers properties and detailed meshing predominant [10]. However, a simple and practical modelling has been preferred in which most the previous studies developed a computationally expensive and time consuming modelling schemes for predicting the dynamic behaviour of composite structure.

In this study, the dynamics behaviour of carbon fibre reinforced epoxy is investigated numerically and experimentally. The finite element method is used to calculate the natural frequencies and mode shapes of the initial model of carbon fibre reinforced epoxy and experimental modal analysis (EMA) is used to obtain the experimental result. The low stiffness suspension spring was used to perform the EMA to separate between rigid body and deformation modes of the composite structure because the stiffness and damping of the suspension cord may have significantly altered the modal parameters of the elastic modes of the test structure, if the separation between rigid body and deformation modes was not achieved [11]. In modelling the carbon composite structure, material properties of fibre and matrix of the carbon fibre composite structure are simplified as an orthotropic material and they are modelled as PSHELL element. The validity of the finite element modelling using the PSHELL element was compared with the experimental model. The finite element model updating was used to improve the accuracy of the initial finite element model of the carbon composite structure as close as the experimental model.

2. Materials and methods

2.1. Finite element modelling of the carbon fibre reinforced epoxy (CFRE) plate

The natural frequencies and mode shapes of the composite plate were calculated by using the finite element (FE) method. In the FE modelling, the eight layers of the composite were modelled as a single layer 2D element as shown in figure 1. The composite plate was discretised into 1658 2D shell elements with 2mm uniform thickness profile. The 2D shell elements were defined as orthotropic material properties. The material properties in table 1, which were obtained in [12], were used as the material properties of the FE model. It is worth noting that there are no boundary conditions applied to the FE model.

![Figure 1. FE model of the CFRE plate.](image)
Table 1. Material and geometrical properties of the CFRE plate [12].

| Property                        | Value  | Unit     |
|---------------------------------|--------|----------|
| Young’s Modulus in longitudinal direction, $E_{11}$ | 40776.5 | MPa      |
| Young’s Modulus in longitudinal direction, $E_{22}$ | 40776.5 | MPa      |
| In-plane shear modulus, $G_{12}$ | 14197.9 | MPa      |
| Poisson’s ratio, $v_1$          | 0.436  | Dimensionless |
| Density                         | 1421.9 | kg/m$^3$            |
| Thickness                       | 2      | mm        |

MSC NASTRAN SOL103 normal modes solution was used in calculating the natural frequencies and mode shapes of the carbon fibre reinforced epoxy. The calculated results were then compared and validated with the corresponding experimental results prior to performing the model updating method.

2.2. Experimental modal analysis (EMA) of the carbon fibre reinforced epoxy (CFRE) plate

In this work, the natural frequencies and mode shapes of the composite plate were measured for validation and model updating purposes. The measured data was obtained by performing EMA. An impact hammer, accelerometer, and Leuven Measurement System (LMS) data acquisition were used in the EMA. The sensitivity of the impact hammer and accelerometer were 11.76mV/N and 10mV/g respectively. The frequency range for the experimental work was between 0 and 1000 Hz. Roving hammer was selected as the excitation method in order to minimise the mass loading issues in the physical structure of the composite plate as the weight of accelerometers might significantly influenced the measured data as reported in [13] and [14]. The test structure was suspended with two soft rubber bands to simulate free-free boundary conditions. Figure 2 presents EMA set-up for the CFRE plate.

2.3. Finite element model updating of the carbon fibre reinforced epoxy (CFRE) plate

The main goal of optimization scheme using model updating method is to improve the correlation results between experimental data and predicted data [15–17]. The discrepancies between the experimental and
predicted eigenvalue data is used as a penalty function in the updating procedure. This method also allows a wide choice of the updating parameters to be corrected using the sensitivity analysis method.

In the model updating method, mode pairing algorithm is mandatory to be conducted to ensure that the predicted and experimental eigenvalues correspond to the same physical mode. However, it is difficult to pairing the mode shapes and natural frequencies correctly especially in the presence of close modes or repetitive roots in the structure under study [18]. Moreover, inaccurate FE modelling with highly uncertain parameters may lead to the inaccurate predicted mode shapes and thus contribute to the high complexity in the mode pairing process. Failure to follow the procedure of pairing the modes will lead to questionable updated results and the worst case is that the updated results do not reflect any physical meaning. One way to avoid the issue is to use the modal assurance criterion (MAC) which is one of the best methods of verifying the mode shapes are paired correctly [19]. The MAC is normally presented in matrix form is used by calculating experimental ($\phi_a$) and finite element ($\phi_m$) mode shapes using Eq. (1).

$$MAC = (\Phi_m\Phi_a) = \frac{|\Phi_m^T\Phi_a|^2}{(\Phi_m^T\Phi_m)(\Phi_a^T\Phi_a)}$$

In this study, the updating work for the CFRE plate were conducted by studying two different input properties namely, geometrical and material properties. In the geometrical properties, thickness of the plate was used as potential updating parameters meanwhile, for the material properties, Young’s modulus in the longitudinal direction, Young’s modulus in the transverse direction, the in-plane shear modulus, Poisson’s ratio, and density were used as potential updating parameters.

In the sensitivity analysis, the sensitivity of the mentioned potential updating parameters to the structural responses which are the natural frequencies of the CFRE plate were study and identified. Hence, the updating procedure was used to correct the structural properties with high sensitivity values by using the Nastran SOL 200 script which can be expressed as follow:

$$S_i = \frac{\partial \lambda_i}{\partial \theta} = \Phi_i^T \left[\frac{\partial K}{\partial \theta} - \lambda_i \frac{\partial M}{\partial \theta}\right] \Phi_i$$

where $S_i$ is the $m \times n$ sensitivity matrix at the $i^{th}$ iteration which represent as rates of change of the structural eigenvalues $\lambda_i$ with respect to the changes in the parameters $\partial \Theta$. Meanwhile the objective function can be expressed as

$$J = \min \sum_{j=1}^{m} \left(\frac{\lambda_i^{FE}}{\lambda_i^{EXP}} - 1\right)^2$$

where $\lambda_i^{FE}$ is the $i^{th}$ predicted natural frequency and $\lambda_i^{EXP}$ represents the $i^{th}$ experimental natural frequency.

### 3. Results and discussion

In this study, the dynamic behaviour of composite structure using carbon fibre reinforced epoxy (CFRE) were obtained experimentally using experimental modal analysis (EMA) and numerically by using finite element modelling (FEM). These works were conducted as an attempt to predict the natural frequencies and mode shapes of the CFRE and reconcile the predicted data in the light of the EMA data. After that, finite element model updating method was used to improve the accuracy of the predicted model.

The result tabulated in table 1 comprises the comparison of first 10 natural frequencies of initial finite element model for CFRE with EMA data. It was found that the total error of the first frequencies modes recorded is 42.61 percent (Column III.). Accordingly, the largest errors recorded is on $3^{rd}$ mode and $9^{th}$ mode with error of 7.30 percent and 7.54 percent. In Table 2, the comparisons of mode shapes between initial FE with experimental counterpart in form of MAC matrix have indicate that, there are 2 swap modes in initial FE which are between mode 3 and mode 4, mode 9 and mode 10. The swap modes have
contributed to the large errors in the predicted frequencies, specifically in mode 3 and mode 9. Moreover, the result in table 2 shows that the predicted mode shapes of initial FE were not in good agreement with EMA data. It was found that the MAC values recorded in mode 4, mode 7, mode 9 and mode 10 were below 70 percent. Therefore, it is essential to improve the accuracy of the initial FE in light of the EMA data using model updating prior to be used for subsequence analysis [20].

In model updating method, it is important to identify updated parameters that are only sensitive to the structural response [21]. In this work, several potential updated parameters were used in sensitivity analysis such as plate thickness \( T \), Young’s modulus in longitudinal direction \( E_{11} \), Young’s Modulus in transverse direction \( E_{22} \), Shear Modulus in -plane, \( G_{12} \), Poisson’ Ratio, \( v_{12} \) and density, \( P \). MSC Nastran SOL 200 was used to calculate the sensitivity of the response toward potential updated parameters. Table 1 (Column VI.) shows the result of the updated FE in comparison with EMA data.

A reasonable achievement is seen in the attempt with the total error managed to reduce from 42.61 percent to 25.05 percent with no swapping mode. The updated values of the updating parameters are listed in Table 4. However, The MAC matrix in table 3 indicated that, predicted mode 4, mode 9 and mode 10 still failed to be in good agreement with EMA data.

### Table 2. Comparison of experimental and FE analysis of CFRE.

| Mode | Experimental (Hz) | Initial FE (Hz) | Error (I-II)/I (%) | Updated FE (Hz) | Error (I-IV)/I (%) |
|------|------------------|----------------|-------------------|----------------|-------------------|
| 1    | 129.71           | 125.29         | 3.41              | 126.46         | 2.50              |
| 2    | 161.99           | 166.18         | 2.58              | 163.82         | 1.13              |
| 3    | 339.92           | 364.74         | 7.30              | 350.86         | 3.22              |
| 4    | 376.19           | 351.15         | 6.66              | 358.86         | 4.61              |
| 5    | 597.02           | 628.11         | 5.21              | 617.24         | 3.39              |
| 6    | 674.91           | 647.10         | 4.12              | 663.67         | 1.67              |
| 7    | 732.65           | 750.11         | 2.38              | 723.18         | 1.29              |
| 8    | 753.05           | 763.85         | 1.43              | 766.30         | 1.76              |
| 9    | 933.48           | 1003.91        | 7.54              | 965.96         | 3.48              |
| 10   | 964.26           | 983.38         | 1.98              | 983.62         | 2.01              |
|      | Total Error      |                |                   | 42.61          | 25.05             |

### Table 3. MAC Matrix for initial FE model of CFRE.

| Mode | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1    | 91  | 3.8 | 0.4 | 0.4 | 0.3 | 1.4 | 0.3 | 6.8 | 9.5 |     |
| 2    | 7.7 | 94.6| 1   | 1.5 | 0.2 | 1   | 0.1 | 0.3 |     |     |
| 3    | 0   | 0   | 14.7| 64.9| 0.3 | 0   | 0.5 | 1   | 0.9 | 0.7 |
| 4    | 0   | 0   | 81.2| 34.5| 1.6 | 0   | 0.4 | 0.3 | 0.4 | 0.9 |
| 5    | 0.3 | 2.2 | 0.1 | 0   | 88.7| 0.7 | 8.8 | 0.2 | 1.2 | 0.7 |
| 6    | 0   | 0   | 0   | 0   | 0   | 0   | 0.3 | 91.8| 0.8 | 3.0 |
| 7    | 3.5 | 0   | 0.1 | 0   | 4.6 | 25.4| 59.4| 7.1 | 0.5 | 1.2 |
| 8    | 0.1 | 0   | 0.2 | 0.1 | 0.5 | 0.1 | 19.3| 88.5| 3.2 | 0.2 |
| 9    | 0   | 0   | 3.5 | 2.6 | 0.2 | 0   | 0.2 | 0.1 | 31.6| 50.4|
| 10   | 24.3| 0.6 | 0   | 2.2 | 2.7 | 0.3 | 0   | 54.1| 45.1|     |
The results of updated natural frequencies in table 1 (Column IV.) and updated mode shapes in table 3 strongly suggest that the updated parameters used in model updating method as listed in table 4 are still not good enough to improving the accuracy of the predicted FE of the CFRE. These results suggest that there is structural uncertainty particularly in the experimental work that might need to be considered in the modelling process.

| Mode | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| 1    | 91 | 1.3 | 0.4 | 0 | 0.6 | 0.3 | 1.1 | 0.4 | 7 | 9.3 |
| 2    | 3.6 | 93.1 | 0.7 | 0 | 1.5 | 0.2 | 1.4 | 0.1 | 0 | 0.7 |
| 3    | 0.2 | 0.3 | 81.6 | 4.6 | 0.9 | 0 | 0.7 | 1 | 1.3 | 1.6 |
| 4    | 0.2 | 0.1 | 30.7 | 65.3 | 0.3 | 0 | 0 | 0 | 0.1 |
| 5    | 0.2 | 0.7 | 0.4 | 0.1 | 93.1 | 3.1 | 1.3 | 0.4 | 0.4 | 0 |
| 6    | 0 | 0.1 | 0.1 | 0 | 2.1 | 93.5 | 0.1 | 0 | 2.5 | 4.7 |
| 7    | 1.1 | 0.7 | 0 | 0.4 | 0 | 7.1 | 87.6 | 18.4 | 0.3 | 0.5 |
| 8    | 1.7 | 0.1 | 0.3 | 0 | 0.8 | 10.4 | 0.3 | 89.6 | 0.8 | 0.8 |
| 9    | 7.7 | 1.1 | 0.9 | 2.9 | 0.1 | 0.1 | 0.4 | 0.6 | 68.7 | 0.5 |
| 10   | 14.4 | 0.3 | 1.5 | 0.9 | 0.4 | 1.2 | 0 | 0.2 | 5.8 | 67.2 |

Table 5. Updated parameters of CFRE model.

| Property | Parameter | Nominal value | Updated value | Unit |
|----------|-----------|---------------|---------------|------|
| PSHELL   | Thickness, T | 2.00 | 2.03 | mm |
|          | Young’s Modulus in transverse direction, $E_{22}$ | 40776.6 | 40601.26 | MPa |
| MAT8 (Material) | Shear Modulus in-plane, $G_{12}$ | 14198 | 13326.24 | MPa |
|          | Poisson’s Ratio, $\nu_{12}$ | 0.436 | 0.291 | Unitless |

4. Conclusions

A practical modelling scheme for investigating the dynamic behaviour of the carbon fibre reinforced epoxy (CFRE) plate using finite element modelling and updating method is presented in this study. The presented study suggest that the proposed scheme improve the accuracy of the dynamic behaviour prediction for the CFRE plate in light with experimental (EMA) data. The biggest achievement in the proposed scheme is the scheme abilities in improving the mode swapping issue and reducing the natural frequencies total error up to 40 percent. The advantage of the proposed scheme over other published methods are the abilities in minimising the complexity in finite element modelling of composite structure by implemented the PSHELL properties method coupled with model updating method.

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