Experimental validation and refinement of mixture optimization for a mixed gas Joule-Thomson cycle

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Abstract. An initial prototype of a mixed gas Joule-Thomson (JT) cryocooler was constructed and installed in a test facility to experimentally validate and refine a computational tool developed to optimize the gas mixture composition for a Joule-Thomson cycle with specific operating parameters. The mixture optimization model determines an optimal three-component mixture based on the analysis of the maximum value of the minimum isothermal enthalpy change, $\Delta h_T$, that occurs over the operating temperature range coupled with an evaluation of the percent of the heat exchanger that exists in a two-phase state within that range. The heat exchanger performance model has been refined and expanded to determine both the steady-state and transient operation of the system. The initial prototype of the JT cryocooler was installed in a test facility capable of providing and measuring a range of gas composition, molar flow rates, and pressures. The JT cryocooler has been operated while charged with several gas mixtures over a range of operating pressures. The pressure drop and temperature at the outlet of the JT valve were compared to the expected values based on the mixture optimization model. Results were used to refine the model, particularly the heat exchanger performance model, and gain confidence in its ability to steer future experimental iterations. A second prototype of decreased size with increased heat exchanger conductance, minimized parasitic heat loss and improved instrumentation is being developed to further investigate optimal mixture selection, particularly in the 120-150K cold head temperature range.

1. Introduction

The advantages of a small closed-cycle JT cryocooler in many applications are their small size and fast cool-down time. [1] When compared specifically to Stirling-cycle coolers, they offer flexible heat rejection and low vibration. Nonetheless, the use of small closed-cycle JT cryocoolers has been limited due to the low thermodynamic efficiency of the cycle at typical cryogenic device temperatures of 80K. Recent advances in mid-wave infrared focal plane technology allow operation in the increased temperature range of 125 to 150K. Within this temperature range, the thermodynamic efficiency of JT cryocoolers has the potential of competing with the efficiency of Stirling-cycle coolers. [2]

In order for the efficiency to be competitive, the JT cryocooler must provide cooling at low pressure ratios and low values of operating pressures. Under these conditions, research has shown that using a suitable mixed gas fluid can provide a greater cooling capacity than is possible with a pure fluid. [3] However, properly selecting a suitable mixture is a challenging design problem. A MATLAB program was previously developed to determine an optimal three-component mixture based on the analysis of the maximum value of the minimum $\Delta h_T$ coupled with an evaluation of the percent of the heat exchanger that exists in a two-phase range. [4, 5, 6] The model has been improved, particularly with regards to the
heat exchanger performance, and a JT cryocooler prototype has been assembled to experimentally validate the modeling results.

2. Heat exchanger performance model

The previous heat exchanger performance model split the heat exchanger into many sub heat exchangers and used the effectiveness-NTU method to match a conductance assigned by the user to the cycle. There are issues with calculating the cycle performance in this manner. Using this method, the conductance and therefore the heat transfer coefficient is assumed based on previous experience or tuned to match data rather than calculated from first principles. The heat transfer coefficient varies with changes in operating parameters and gas mixture composition. Furthermore, research has shown that in the enhanced heat transfer region, between 15 and 85 percent quality, the heat transfer coefficient for gas mixtures can be up to three times higher than it is in the single-phase regions. [7] Since the heat transfer coefficient varies with operating parameters, mixture composition and quality, the conductance of the heat exchanger must also vary and this should be accounted for in simulation and design.

The heat exchanger performance model has been updated to consider variations in conductance due to changes in operating parameters, mixture composition and quality. The model utilizes a conductance and pressure drop estimate to predict the steady-state cold-end temperature given a three-component mixture, supply temperature, suction and discharge pressures, and heat exchanger geometry and material. A flow diagram of the approach is shown below in Figure 1.

![Flow diagram for heat exchanger performance model.](image)

2.1. Estimate of conductance

Given a three-component mixture, load and supply temperatures, suction and discharge pressures, and heat exchanger geometry and material, the model determines the conductance of the heat exchanger by applying the process described below:

1. Discretize the heat exchanger into control volumes of equal size and solve energy balances to obtain the spatial temperature distribution within the heat exchanger
2. Call the property routine REFPROP to determine the quality and other properties of the fluid within the control volumes from the temperature distribution [8]
3. Correlate the properties to heat transfer coefficients using the correlations and experimental results from Barazza [7]
4. Solve for the conductance of each control volume using resistance networks and add them together to obtain the total conductance of the heat exchanger

Estimating the heat transfer coefficients associated with two-phase, multi-component mixtures at cryogenic temperatures is challenging due to the scarcity of data currently available. The process outlined above uses the work of Barazza to correlate quality to heat transfer coefficient as the operating parameters and mixture compositions that were used in his experiments are similar to those under investigation.

2.2. Pressure drop estimate

Given the fluid composition, suction and discharge pressures and geometry of the heat exchanger, the model predicts the pressure drop in the low and high pressure streams. The pressure drop for single-
phase flow through the high pressure stream is calculated assuming a straight tube and the results are modified to account for the curvature of the helical pipe. [9] As the fluid within the heat exchanger is typically two-phase, the Müller-Steinhagen and Heck correlations for two-phase fluid flow are used to calculate the pressure drop. The equation used is as follows:

\[
\frac{dP}{dz} = \left\{ \left( \frac{dP}{dz} \right)_L + 2 \left( \frac{dP}{dz} \right)_G - \frac{dP}{dz} \right\} x (1 - x)^3 + \left( \frac{dP}{dz} \right)_G \cdot x^3
\]

where \(\frac{dP}{dz}_L\) and \(\frac{dP}{dz}_G\) are the liquid and gas only pressure drop per length and \(x\) is the quality of the fluid. [10] To calculate the liquid and gas only pressure drop per length, the molecular weight, dynamic viscosity, density, and Prandtl number are obtained from REFPROP. Additionally, through experimental analysis, the flow in the heat exchanger was shown to be turbulent, even at low molar flow rates. The model was adapted to reflect this.

The pressure drop of the low pressure stream is calculated approximately using an approximate flow area determined from the cryocooler geometry. With the pressure drops of the high and low pressure streams calculated, the pressure drop across the JT valve is known which for design calculations would allow the sizing of an appropriate orifice.

2.3. Cooling capacity prediction

The conductance and pressure drop estimated in the first two stages of the model are used to predict the cooling capacity of the JT cryocooler as a function of cold-end temperature. The process applied is described below:

1. Set the specified cooling capacity (i.e. 0 W)
2. Guess the cold-end temperature
3. Discretize the heat exchanger into \(N\) sub-heat exchangers with equal heat transfer and pressure drop [11]
4. Solve for the conductance of each sub-heat exchanger using the effectiveness-NTU method and add them together to obtain the total conductance of the heat exchanger
5. Compare the conductance of the heat exchanger calculated in step 4 with the conductance estimated in stage 1 of the model
   a. Move forward to step 6 if the calculated and estimated conductance are within a user-specified error tolerance
   b. Repeat steps 2-4 by altering the guess value of the cold-end temperatures until the calculated and estimated conductance agree to within tolerance
6. Repeat steps 1-5 for the next incremental cooling capacity until the cold-end temperature is at or above the supply temperature in order to generate an entire load curve
7. Subtract the measured parasitic heat loss from the simulated cooling capacity to generate the corrected cooling capacity as a function of cold-end temperature
These results provide the lowest cold-end temperature the JT cryocooler can achieve with the given input parameters and current cryocooler design.

Figure 2 shows a visualization of the cooling capacity prediction process. The model predicts the cooling capacity as a function of cold-end temperature (dashed line) and subtracts the measured parasitic heat loss (dotted line). This generates a corrected cooling capacity as a function of cold-end temperature (solid line). The results shown are for 50% R14 and 50% R23 on a molar basis with suction and discharge pressure of 63 and 138 PSIA, respectively, and a molar flow rate of 0.95 mmol/s for geometry that is consistent with prototype 1 (discussed in Section 3).

2.4. Cool-down curve
The corrected cooling capacity can be integrated through time using an estimate of the thermal mass of the system in order to predict a cool-down curve for the cold-end of the cryocooler. From this result, a steady-state cold-end temperature and cool-down time for the cryocooler can be predicted.

3. Comparison of model predictions and experimental data
An initial prototype (prototype 1) of a mixed gas JT cryocooler was constructed and installed in a test facility capable of providing and measuring a range of gas composition, molar flow rates, and pressures. The design of the cryocooler and test facility is outlined in previously published works. [5]

3.1. Pressure drop
To investigate the accuracy of the pressure drop prediction, the pressure drop of the high and low pressure streams and the JT valve was measured for both pure argon and a mixed synthetic refrigerant of 16% R23, 20% R14 and 64% R134a on a molar basis. The predicted pressure drop for the low pressure stream and JT valve for both the pure and mixed fluid matched to within 10%.

A comparison of the predicted and measured pressure drop of the high pressure stream for both pure argon and the mixture is shown in Figure 3. The comparison for pure argon shows that the model accurately predicts the pressure drop for molar flow rates between 0.75 and 2.75 mmol/s. For the synthetic refrigerant mixture, the model is most accurate between 0.7 and 0.9 mmol/s. Increasing or decreasing the molar flow rate from this range reduces the accuracy of the pressure drop estimate for the high pressure stream.

It is important to note that these measurements were taken at room temperature when both the pure fluid and synthetic refrigerant mixture are single-phase. Further investigation must be pursued to ensure the accuracy of the pressure drop estimates for two-phase flow.
Figure 3. Comparison of the predicted (solid line) and measured (cross) pressure drop of the high pressure stream for argon (shown on left) and mixture of 20% R14, 16% R23 and 64% R134a on a molar basis (shown on right).

3.2. Steady-state cold-end temperature

The JT cryocooler has been operated while charged with several synthetic refrigerant mixtures of R23, R14, R134a and argon over a range of operating pressures from 25-150 PSIA with a JT orifice size of 0.0091". The measured cold-end temperature from twelve runs was compared to the predicted cold-end temperature calculated by the heat exchanger performance model and the results are summarized in Table 1. This comparison is robust as the measured cold-end temperature is influenced by all aspects of the behaviour including the pressure drop of both the high and low pressure streams and the conductance.

Table 1. Comparison of measured and predicted cold-end temperatures for prototype 1. The mixture composition is stated on a molar basis. LP and HP represent low and high pressure, respectively. \( T_{\text{cold}} \) represents cold-end temperature.

| Mixture Composition | LP (PSIA) | HP (PSIA) | Molar Flow Rate (mmol/s) | Measured \( T_{\text{cold}} \) (K) | Predicted \( T_{\text{cold}} \) (K) | % Error |
|---------------------|-----------|-----------|--------------------------|-------------------------------|---------------------------------|---------|
| 0.16 R23 + 0.20 R14 + 0.64 R134a | 54        | 142       | 1.59                     | 231                           | 232                             | 1%      |
|                      | 50        | 136       | 1.45                     | 232                           | 230                             | 3%      |
|                      | 45        | 123       | 0.88                     | 220                           | 210                             | 13%     |
|                      | 64        | 143       | 1.83                     | 231                           | 251                             | 28%     |
|                      | 38        | 106       | 0.76                     | 245                           | 205                             | 72%     |
|                      | 33        | 96        | 0.36                     | 254                           | 212                             | 92%     |
|                      | 27        | 75        | 0.41                     | 282                           | 209                             | 408%    |
| 0.375 R23 + 0.375 R14 + 0.25 R134a | 50        | 133       | 0.80                     | 256                           | 246                             | 22%     |
| 0.25 R23 + 0.25 R14 + 0.5 R134a | 42        | 116       | 0.87                     | 241                           | 193                             | 80%     |
| 0.5 R23 + 0.5 R14   | 63        | 138       | 0.95                     | 274                           | 250                             | 90%     |
| 0.09 Argon + 0.41 R14 + 0.5 R23 | 60        | 143       | 1.00                     | 273                           | 251                             | 82%     |
|                      | 52        | 132       | 0.84                     | 283                           | 257                             | 153%    |
The metric used to compare the results is a relative error based on the measured and predicted cold-end temperature and is calculated as

\[
\% \text{ Error} = \left| \frac{\Delta T_{th} - \Delta T_{exp}}{\Delta T_{exp}} \right| \times 100
\]

where \(\Delta T_{th}\) and \(\Delta T_{exp}\) is the difference in temperature from the supply temperature to the predicted and measured steady-state cold-end temperature, respectively. The supply temperature was approximately 300K for all runs.

From the analysis above, it is seen that the heat exchanger performance model generally over-predicts the cooling (i.e., under-predicts the cold head temperature) achieved by the JT cryocooler. There are only two runs where the cooling achieved is under-predicted. The accuracy of the prediction is positively correlated to the molar flow rate and negatively correlated to the measured cold-end temperature. In the first two rows of Table 1, the model is able to predict the cold-end temperature of a synthetic refrigerant mixture to within a few degrees of the measured cold-end temperature. For these two runs, the molar flow rates were large, 1.59 and 1.45 mmol/s, and the measured cold-end temperatures were low, 231 and 232K, yielding cold-end temperature predictions within 1% and 3% relative error.

Moving down the table, the accuracy of the cold-end temperature prediction decreases as the molar flow rate decreases and the measured-cold end temperature increases. The relative error in the cold-end temperature prediction becomes particularly large for an experimental trial with both a low molar flow rate of 0.41 mmol/s and a high measured cold-end temperature of 282K. All the mixtures tested exhibited this trend.

Future work will continue to investigate the cooling capacity of a JT cryocooler at low pressure ratios, low values of operating pressures and cold-end temperatures in the 120-150K range. From the above trends, it is expected that the heat exchanger performance model will be able to predict the cold-end temperature well enough to be used as a design tool in this temperature range. The molar flow rates expected for the second prototype are between 0.5-1.5 mmol/s.

4. Cryocooler design

A second prototype of decreased size with increased heat exchanger conductance, minimized parasitic heat loss and improved instrumentation is under construction to further investigate optimal mixture selection, particularly in the 120-150K cold-end temperature range. A Solidworks CAD software rendering of the cryocooler is shown in Figure 4.

![Figure 4](https://example.com/figure4.png)

**Figure 4.** A SolidWorks CAD software rendering of the JT cryocooler prototype 2. Not shown for rendering simplicity in the software is the winding technique performed to construct the heat exchanger.
The high pressure stream enters the cryocooler and splits into three stainless steel tubes with copper fins that are helically coiled around a hollow G10 mandrel filled with polyurethane foam. The finned heat exchanger tubing has inner, outer, and finned diameter of 0.012”, 0.02” and 0.04”, respectively. The length of each finned heat exchanger tube is 60”. The hollow G10 mandrel has inner and outer diameter of 0.187” and 0.25”, respectively, and is closed at both ends by stainless steel plugs. The plugs provide structure for the assembly of the heat exchanger while eliminating the flow path through the G10 mandrel. The wiring for the cold end platinum resistance thermometer (PRT) runs through the plugs and G10 mandrel to the outlet of the ConFlat. The three finned heat exchanger tubes merge back into one flow path at the JT valve. The JT orifice consists of a 1/8” diameter copper blank with a small diameter hole in its center and it is held in position by 1/8” VCR female and male nuts. The low pressure stream then returns to the outlet of the ConFlat by flowing over the fins on the outside of the high pressure stream finned tubing. The heat exchanger and JT valve are enclosed by a 5/8” Kovar-to-7052 glass domed adapter. The overall length of the heat exchanger and cryocooler are 2.6” and 6”, respectively.

The design of the three finned heat exchanger tubes wound around the G10 mandrel was given much consideration. To reduce the possibility of flow maldistribution of the high pressure stream and any associated performance deterioration, the length of flow passage for each finned heat exchanger tube is 60”. To maintain uniform flow of the low pressure stream, the radial spacing of the finned heat exchanger tube and monofilament line is uniform. The length of each finned heat exchanger tube when wound is 2.6” to avoid flow maldistribution due to unequal temperature distributions within the wound finned tubes. To maintain equal flow passage length, wound length and radial spacing, the inner and outer layers of finned heat exchanger tubing are swapped three times on the G10 mandrel during construction as shown in Figure 5. Additionally, each layer of finned heat exchanger tubing is wound in opposite directions (clockwise, then counterclockwise, etc) with a layer of Teflon tape in between to assist with monofilament line placement on each layer. Using the heat exchanger performance model, the conductance of the heat exchanger is predicted to be around 3 W/K, about twice that of prototype 1.

![Figure 5. Visualization of the winding technique used to construct the helically coiled heat exchanger is shown on the left. Each layer of the heat exchanger is wound in opposite directions. The inner layer (solid line) and outer layer (dotted line) are swapped three times along the G10 mandrel. A layer of Teflon tape is placed between each layer. A picture of the winding practiced for assembly is shown on the right.](image-url)
5. Conclusions
This paper describes further development of a computational tool designed to optimize the gas mixture composition for a JT cycle with specific operating parameters. Further development of the mixture optimization model focused on the heat exchanger performance addressing the variation in conductance due to operating parameters, mixture composition and quality. From analysis of the predicted and measured cold-end temperatures for prototype 1, the accuracy of the prediction is positively correlated to the molar flow rate and negatively correlated to the measured cold-end temperature. It is expected that the heat exchanger performance model will be able to predict the cold-end temperature with greater accuracy for cold-end temperatures of 120-150K. Further investigation is needed to increase the accuracy of the cold-end temperature prediction as the molar flow rates fall below 0.8 mmol/s.

A design was presented for a glass encased JT cryocooler prototype 2. This device is currently being developed and will be installed in the same testing facility as the first prototype. This prototype will allow further investigation of optimal mixture selection in the 120-150K cold-end temperature range.

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