Intense Cherenkov radiation from a charged particle revolving along a shifted equatorial orbit about a dielectric ball

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Abstract. The radiation from a charged particle uniformly revolving along a shifted equatorial orbit about a dielectric ball has been investigated. The results of numerical calculations based on exact solutions of Maxwell equations with due regard for dielectric losses of energy within the ball material testify that (a) for definite «resonant» values of the particle revolution frequency relativistic electron may generate microwave Cherenkov radiation at the first harmonic, that is approximately 900-1100 times more intensive than synchrotron radiation generated at the revolution of the same particle in vacuum and (b) in case of large values of permittivity of ball material (strontium titanate) the «resonant» radiation is generated by the revolving electron with energy of the order of 30 KeV.

1. Introduction and background
The operation of a number of devices intended for production of electromagnetic radiation is based on the interaction of relativistic electrons with matter [1–5]. Numerous applications motivate the importance of exploring different mechanisms of amplification and control of produced radiation. Specifically, one may use radiation of different kinds as well as reflecting surfaces for monitoring the flows of radiation.

A research of such a kind was carried out in [6–8], where the emission of radiation from a relativistic particle revolving about a dielectric ball has been investigated (figure 1). In such a geometry in addition to the synchrotron radiation (SR) the particle may generate the Cherenkov radiation (CR), since the field associated with the particle partially penetrates the ball depths and revolves together with the particle. In case of short distances of particle from the ball surface the velocity of its field displacement within the ball may exceed the phase velocity of light in the ball material and, hence, CR would be generated. Here the flow of produced CR may be controlled either by changing the radius of ball or the relative position of ball with respect to particle orbit.
As a result of the combination of SR and CR, as well as of the mechanism of radiation flow control (the ball-to-vacuum interface) the revolving particle may, at separate harmonic, generate CR tens of times more intense than CR and SR from the same particle generated in a continuous, infinite and transparent medium having the same permittivity as the ball material. The theoretical substantiation of this effect and its visual explanation are given in [6]. However, the case under consideration in [6] was confined to the simplest configuration of ball located in the centre of particle orbit (figure 1, \( d = 0 \)). In reality (electron beam), such a symmetry is difficult to provide.

This work is a continuation of papers [6–8], in particular [7, 8], where it was shown that the electron revolving along a non-equatorial [7] or shifted equatorial [8] orbits abound a dielectric ball can generate a "resonant" CR at the harmonics with the numbers \( k \geq 8 \), if its energy \( E_q \geq 2 \text{MeV} \).

In the present paper it is shown that a revolving non-relativistic electron (\( E_q \approx 30 \text{KeV} \)) also may generate such an intense radiation and not at the 8th, but at the first harmonic (\( k = 1 \)). New features of this radiation have been revealed. The possibility of practical applications of this phenomenon is discussed.

2. The basic formula

Now consider the uniform rotation of a charged particle in the magnetic field in vacuum about a dielectric ball in its equatorial plane under the assumption that the centers of ball and of particle orbit are shifted one with respect to the other (figure 1, \( d \) is the distance between the centers of the ball and particle orbit).

In spherical coordinates \( r, \theta, \phi \) with origin in the center of ball, the permittivity \( \varepsilon(r) \) of the substance is a step function of radial coordinate:

\[
\varepsilon(r) = \varepsilon_b \quad \text{for} \quad r \leq r_b \quad \text{and} \quad \varepsilon(r) = 1 \quad \text{for} \quad r > r_b,
\]

where \( \varepsilon_b \) is the permittivity of ball with radius \( r_b \). The magnetic permeability is taken to be 1. The current density may be written as

\[
j_r(t) = \frac{\alpha_q q(t) \hat{e}_r + v_p(t) \hat{e}_\phi}{r_0(t)} \delta(r - r_0(t)) \delta(\theta - \pi / 2) \delta(\phi - \phi_0(t)),
\]

where

\[
r_0(t) = \sqrt{d^2 + r^2 + 2dr \cos \omega_f t}, \quad v_r(t) = -\frac{\alpha_q q(t) r \sin(\omega_f t)}{r_0(t)},
\]

\[
v_\phi(t) = \frac{\alpha_q q(t) (r^2 + d \cos(\omega_f t))}{r_0(t)}, \quad \phi_0(t) = \Re \left\{ \frac{1}{i} \ln \left( \frac{d + r e^{i\omega f t}}{r_0(t)} \right) \right\}
\]

Figure 1. A relativistic charged particle rotating along a shifted equatorial orbit about a dielectric ball. \( \varepsilon_b, r_b \) are the permittivity of the ball material and its radius and \( q, r_q \) are the charge of particle and the radius of its orbit, \( d \) is the distance between the centres of the ball and of particle orbit.
and are the spherical Bessel and Hankel functions of the first kind respectively. 

\[ Y_l^m(\theta,\phi) \]

are the spherical harmonics. We have introduced the following notations:

\[ n_k = \frac{c}{\hbar \omega_k} \sum_{l,m} \left[ |a_{lm}^{(E)}|^2 + |a_{lm}^{(H)}|^2 \right] \]

based on exact solutions of Maxwell equations for the case under consideration. Here

\[ a_{lm}^{(E)} = \frac{4\pi}{cr_k^2(2l+1)} \left[ J_l(\beta) - Y_l^m(\theta,\phi) \right] \]

\[ a_{lm}^{(H)} = \frac{2q_0 \omega_q}{cr_k^2(2l+1)} \gamma J_l(\beta) \]

are amplitudes describing the contributions of electrical (E) and magnetic (H) type multipoles respectively, \( Y_{lm}(\theta,\phi) \) are the spherical harmonics. We have introduced the following notations:

\[ u_{lm}^{(E)} = \frac{q_0 \omega_q Y_{lm}(\pi / 2;0) \cdot (-i)^{l+1}}{2\pi} \int_0^\tau v(t) h_0(t) \exp[i(\omega_q t - m\phi(t))] dt \]

\[ u_{lm}^{(H)} = \frac{-imq_0 \omega_q Y_{lm}(\pi / 2;0) \cdot (-i)^{l+1}}{2\pi \sqrt{l(l+1)}} \int_0^\tau v(t) h_0(t) \exp[i(\omega_q t - m\phi(t))] dt \]

\[ B_{lm}^{(E)} = \frac{-imq_0 \omega_q Y_{lm}(\pi / 2;0) \cdot (-i)^{l+1}}{2\pi \sqrt{l(l+1)}} \int_0^\tau v(t) h_0(t) \exp[i(\omega_q t - m\phi(t))] dt \]

\[ B_{lm}^{(H)} = \frac{-imq_0 \omega_q Y_{lm}(\pi / 2;0) \cdot (-i)^{l+1}}{2\pi \sqrt{l(l+1)}} \int_0^\tau v(t) h_0(t) \exp[i(\omega_q t - m\phi(t))] dt \]

Here \( j_i(\tau) \) and \( h_i(\tau) \) are the spherical Bessel and Hankel functions of the first kind respectively. Eventually,

\[ \{ a_i(u) ; g_i(u) \} = u_i a_{i+1}(u) g_i(u) - a_i(u) u_{i+1}(u) \quad \text{and} \quad f_i(t) = f_i(t) / \{ j_i(u) ; h_i(u) \} . \]
It is worthwhile to mention that all foregoing expressions are valid in all the cases when a particle rotates uniformly in the equatorial plane of a dielectric ball under the condition that its trajectory doesn’t pass through the substance of ball. Two cases are possible here: the particle rotates around the ball \(0 < d < r_q - r_b\) and outside the ball \(d > r_q + r_b\). The main purpose of this paper is to study the first case.

3. Numerical results
We shall assume that the permittivity of the ball material is equal to \(\varepsilon_b = 232 \cdot (1 + 0.0001 \cdot i) = \varepsilon_b' + i\varepsilon_b''\) (strontium titanate in 10\(^8\) – 10\(^9\) Hz frequency range [9]). We shall also assume \(r_b = 1\, cm\) and confine ourselves to the consideration of electron radiation at a first harmonic: \(k = 1\). The radius of the particle orbit \(r_q = 1.1\, cm\).

First, let us consider the case of \(d = 0\). In figure 2 we give the number of electromagnetic field quanta \(n_i\) emitted at the first harmonics versus of the cyclic frequency \(\omega_q\). As is seen from the plot, the height of sharp peaks \(n_i^{(i)}\) observed at definite values of the cyclic frequency \(\omega_q = \omega_q^{(i)}\) («resonant» frequency) is \(\approx 1000\) times more intensive:

\[
\frac{n_i^{(i)}(\text{ball})}{n_i(\text{vac})} \approx 925; 1180, \quad i = 1; 2
\]

than SR generated at the revolution of the same particle in vacuum (the dashed line in figure 2, \(n_i(\text{vac}) \approx 0.003\)). At these peaks the electron energy is approximately 29 keV and nevertheless the Cherenkov condition for the particle speed and the ball material is satisfied:

\[
v > c/\sqrt{\varepsilon_b'}.
\]

Mentioned peaks witnesses to the strong effect that the ball-vacuum interface has on CR generated by the particle in the substance of ball.

![Figure 2. The number of electromagnetic field quanta \(n_i\) emitted at the first harmonic versus of the electron cyclic frequency \(\omega_q\), \(r_b = 1\, cm\), \(r_q = 1.1\, cm\), \(\varepsilon_b = 232 \cdot (1 + 0.0001 \cdot i)\).](image)
«resonant» values of the cyclic frequency \( \omega_q = \omega_q^{(i)} \) in figure 2) the particle may generate \( n_k \geq 1 \) quanta of electromagnetic field during one rotation period. Such high power radiation arises due to the fact that electromagnetic oscillations of CR induced by the particle along its entire trajectory are partially locked inside the ball and are superimposed in nondestructive way. More detailed visual explanation of this phenomenon is given in [6].

Now direct our attention to the case when the centers of the ball and of particle orbit do not coincide \( (d \neq 0) \). In figure 3 the dependence of the number \( n_1 \) of electromagnetic field quanta \( (6) \) generated per revolution of electron about a dielectric ball on the distance \( d \) between the centers of the ball and of particle orbit is shown. The rotation cyclic frequency of electron is equal to the first resonance value \( \omega_q^{(i)} = 8.803 \text{ GHz} \) (see figure 2). The corresponding kinetic energy of electron is \( \geq 30 \text{ KeV} \).

**Figure 3.** The dependence of \( n_1 \) on the distance \( d \) between the centres of the ball and particle orbit, \( \omega_q = \omega_q^{(i)} = 8.803 \text{ GHz} \). The remaining parameters are being the same as those in figure 2.

As is seen in figure 3, the radiation stays intensive when \( d \) increases from 0 to its maximum value:

\[
n_1(d) / n_1(\text{vac}) \approx 930 \quad \text{at} \quad 0 < d < (r_q - r_b) = 0.1 \text{cm}.
\]  

This fact simplifies the feasible applications of such a radiation.

At larger values of \( d \) \( (d > r_q + r_b) \) the particle rotates outside the ball in its equatorial plane. In figure 4 the dependence of \( n_1 \) on \( d \) is shown in this case. Numerical calculations were performed using formulae given in Section 2.
Figure 4. The dependence of $n_1$ on $d$, when the particle rotates outside the ball in its equatorial plane: $d > r_q + r_b$. The remaining parameters are being the same as those in figure 3. The dashed line represents SR generated at the revolution of the same particle in vacuum.

As is seen in figure 4 the function $n_1(d)$ decreases rapidly with increasing $d$ and tends to $n_1(\text{vac})$ when $[d-(r_q+r_b)] > r_{\text{eff}}$ ( $r_{\text{eff}} = \gamma r_q/2 \approx 0.5 \text{ cm}$ is the characteristic transverse dimension of the Coulomb field of a rotating charged particle, and $\gamma$ is the Lorentz factor of particle). This fact clearly indicates that the sharp peaks $n_1^{(i)}$ observed in figure 2 are due to the radiation generated inside the substance of dielectric ball.

As is shown in [6] the resonance frequencies

$$\omega_{q}^{(i)} = a_{\text{eff}}^{(i)} c \sqrt{\varepsilon_b}$$

(13)

are inversely proportional to the radius ($r_b$) of ball and do not depend on the radius ($r_q$) of particle orbit. The dimensionless proportionality coefficient in (13) is determined by the order number ($i$) of resonant frequency, the number ($k$) of radiated harmonic and the dielectric constant ($\varepsilon_b$) of ball: $a_{\text{eff}}^{(i)} = a^{(i)}(k, \varepsilon_b)$. Numerical results obtained in the framework of this paper indicate that $\omega_{q}^{(i)}$ are independent not only of $r_q$ but also of $d$ .

4. Conclusions

We have studied the radiation from a charged particle uniformly revolving along a shifted equatorial orbit about a dielectric ball. The results of numerical calculations based on exact solutions of Maxwell equations with due regard for dielectric losses of energy in the ball material testify that:

1) a non-relativistic electron revolving at resonance frequency about a strontium titanate ball generates the radiation at the first harmonic, that is $\approx 1000$ times more intensive than synchrotron radiation generated at the revolution of the same particle in vacuum. The intensity of radiation is practically unchanged for notable variations of the distance between the centres of the ball and particle orbit (see (12)).
2) High power radiation rises due to the fact that electromagnetic oscillations of Cherenkov radiation
induced by the particle along its entire trajectory are partially locked inside the ball and are
superimposed in nondestructive way [6].
The intensity of such radiation is highly sensitive to the values of $\omega_q$ and $\varepsilon_q$. This fact may be used
for accurate measurement of the permittivity of matter and detection of weak (e.g., ultrasonic) waves.

Acknowledgments
The authors are thankful to the anonymous reviewer, the valuable comments of which helped to
improve the statement of results obtained in this paper.

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