Indication of a massive circumbinary planet orbiting the Low Mass X-ray Binary MXB 1658–298

Chetana Jain¹, Biswajit Paul², Rahul Sharma³, Abdul Jaleel³ and Anjan Dutta³

¹Hans Raj College, University of Delhi, Delhi 110007, India
²Raman Research Institute, Sadashivnagar, C. V. Raman Avenue, Bangalore 560080, India
³Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India

ABSTRACT

We present an X-ray timing analysis of the transient X-ray binary MXB 1658–298, using data obtained from the RXTE and XMM – Newton observatories. We have made 27 new mid eclipse time measurements from observations made during the two outbursts of the source. These new measurements have been combined with the previously known values to study long term changes in orbital period of the binary system. We have found that the mid-eclipse timing record of MXB 1658–298 is quite unusual. The long term evolution of mid-eclipse times indicates an overall orbital period decay with a time scale of \( -6.5(7) \times 10^7 \) year. Over and above this orbital period decay, the O-C residual curve also shows a periodic residual on shorter timescales. This sinusoidal variation has an amplitude of \( \sim 9 \text{ lt-sec} \) and a period of \( \sim 760 \text{ d} \). This is indicative of presence of a third body around the compact X-ray binary. The mass and orbital radius of the third body are estimated to lie in the range, 20.5–26.9 Jupiter mass and 750–860 lt-sec, respectively. If true, then it will be the most massive circumbinary planet and also the smallest period binary known to host a planet.

Key words: X-rays: binaries: eclipsing, Stars: neutron, individual: MXB 1658–298, planet-star interaction

1 INTRODUCTION

Low Mass X-ray Binary (LMXB) systems consist of a compact object accreting from a low mass companion star. The orbit of the LMXBs is expected to evolve due to mass transfer and redistribution of the angular momentum arising from the interaction of the binary components.

The orbital evolution of X-ray binaries can be measured by four different ways. When the compact object is a pulsating neutron star, the pulse arrival time delay over the binary period is used to determine the orbital parameters of the system and multiple measurements of the orbital epoch over a long period are used to determine the orbital evolution [Levine et al. 2000]. Orbital evolution in some black hole X-ray binaries (BHXBs), has been measured by constructing the radial velocity curve of the companion star from the doppler shifts of the spectral lines [González Hernández et al. 2014, 2017]. In the eclipsing binaries, connecting the mid eclipse time can give information on the long term evolution of the orbital period and an accurate determination of orbital period derivatives [Wolff et al. 2009, Jain, Paul & Dutta 2010, Falanga et al. 2015, Islam & Paul 2016]. The orbital evolution can also be measured from the stable orbital modulation of light curves [Chou & Grindlay 2001, Singh et al. 2002].

The orbital period of X-ray binaries can increase (for example, X 2127+119: Homer & Charles (1998); SAX J1808.4–3658: Jain, Dutta & Paul (2007); 4U 1822–37: Jain et al. (2010); 4U 1916–053: Hu, Chou & Chung (2008)) or decrease (for example, 4U 1820-30: Chou & Grindlay (2001); X-ray binary: Paul et al. (2004); Staubert, Klochkov & Wilms (2009); A 0620-00 & XTE J1118+480: González Hernández et al. (2014); AX J1745.6-2901: Ponti et al. (2014); Nova Muscae 1991 [González Hernández et al. (2017)] smoothly over several years of measurements. The orbital period can also undergo distinct epochs of sudden change, as observed in EXO 0748-676 (Wolff et al. 2009) and XTE J1710–281 (Jain & Paul 2011).

MXB 1658–298 is one of the rare LMXBs, that show X-ray eclipses in their light curves [Cominsky & Wood 1980]. It is a transient X-ray source which was discovered in 1976 [Lewin, Hoffman & Doty 1976] from observations made with the SAS-3 X-ray observatory. Through several follow-up observations, an orbital period of \( \sim 7.1 \) hr and an eclipse duration of \( \sim 15 \) min were determined.
2 Jain et al.

(Lewin et al. 1978; Cominsky & Wood 1984). About 2 years after its discovery, the X-ray intensity declined and the source was not detectable for the subsequent more than 20 years (‘t Zand et al. 1999). During another outburst and a renewed activity in 1999, burst oscillations with a period of ∼1.8 ms were reported (Wijnands, Strohmayer & Franco 2001), which could be the spin period of the neutron star. After being X-ray bright for about 2.5 years, the source went into quiescence near the beginning of 2001.

Comparing the orbital period of MXB 1658–298 measured from two eclipses during the first outburst and from four eclipses during the early part of the second outburst, Wachter, Smale & Bailyn (2000) reported an orbital period decay and determined an average decay timescale of 10^7 yr. But since this source was not detectable for a long time in between the two outbursts, there is no detailed record of the orbital period changes. Later, Oosterbroek et al. (2001) determined two more mid-eclipse times of MXB 1658–298, using the Beppo-SAX data during the second outburst. These measurements, along with the previous values, however, were not compatible with a simple orbital decay as was suggested earlier by Wachter et al. (2000). All the available eclipse measurements at this stage (eight), indicated some complexity in the orbital solution of this source.

Recently, MXB 1658–298 went into another outburst (Negoro et al. 2015), thus enabling a definitive study of its orbital evolution. In this work, we have determined mid-eclipse times using newer RXTE-PCA and XMM-Newton observations of this source, made during the second and the current outburst.

2 OBSERVATIONS AND ANALYSIS

The RXTE-PCA consists of an array of five collimated proportional counter units with a total photon collection area of 6500 cm^2 (Jahoda et al. 1996). We have analyzed 24 archived observations of MXB 1658–298 made with the RXTE observatory. The observation log is given in Table 1. The PCA data collected in the event mode was used to generate the light curves, using the FTOOL-SEEXTRACT from the astronomy software package HEASOFT-ver 6.10. The analysis was done in the energy band 2–20 keV. The background was estimated using the FTOOL-PCABACKEST. Faint source model was taken from the RXTE website. Thereafter, barycentric corrections were applied to all X-ray timings.

The XMM-Newton Observatory (Jansen et al. 2001) carries three X-ray mirrors and three focal plane instruments, each with a field of view of about 30' × 30'. Complete X-ray eclipses of MXB 1658–298 have been observed during two XMM-Newton observations. We have analyzed both of these archived observations. The first of these observations was made during the second outburst. It lasted for ∼31.5 ks and covered two complete eclipses. Another ∼42.9 ks long observation during the current outburst covered one complete eclipse. Observation details are summarized in Table 1. We have analyzed the 0.2-10 keV EPIC-PN data, using the XMM Science Analysis System (SAS version 8.0.0). Source counts were extracted from a circular region of radius 40″ centered on the position of the target. Background events were extracted from a similar source-free circular region. Background subtracted light curves were barycenter corrected using the SAS tool BARYCEN. Spectroscopic results from the February 2001 observation have been reported earlier by Sidoli et al. (2001). They have reported the presence of two eclipses. However, they did not report the mid-eclipse times for the purpose of orbital evolution measurement.

2.1 Eclipse Timing

Figure 1 shows two sample background subtracted light curves of MXB 1658–298 obtained from RXTE-PCA and XMM-Newton observations. The solid line in both the panels represents the best-fit model as described in the text.
mid-eclipse time measurements. Therefore, in order to estimate the additional random error in the mid-eclipse time measurements due to variability in the source, we simulated eclipses (with similar parameters as given in Table 1) at several positions in an RXTE-PCA light curve. Differences of the values of the mid-eclipse time that were used for simulation and those measured from the simulated data, were found to be about 1.3 seconds. Therefore, for further analysis, we have considered this value as an additional random error due to the intrinsic intensity variation of the LMXB, and have quadratically added it to the statistical error of each mid-eclipse time measurement.

2.2 Results

Since its discovery, MXB 1658–298 has undergone three outbursts. The first outburst lasted from 1976 to 1978, the second phase between 1999 - 2001; and the current phase of enhanced emission started around August 2015. Only two mid eclipse time measurements have been reported during the first active period. And during the current active phase of MXB 1658–298, so far we have only one measurement of the mid eclipse time. In contrast, from the second outburst, we have a total of 32 mid-eclipse time measurements with RXTE-PCA, Beppo-SAX and XMM-Newton.

We fitted a linear model to all the 35 mid eclipse time measurements. The best-fitting linear component was subtracted from the ephemeris history and the O-C residual curve is plotted in Figure 2. This curve hints at an orbital variation above a secular orbital period decay, this source shows a periodic residual at a shorter timescale. The pattern of the sinusoidal function to the residual curve (Equation 1).

$p_n = P_{\text{orb}} \times 10^{-7} \text{yr}$. It is larger than an earlier estimate using fewer mid-eclipse times (Wachter et al. 2000) and has quadratically added it to the statistical error of each mid-eclipse time measurement.

$T_n = T_0 + nP_{\text{orb}} + \frac{1}{2} n^2 P_{\text{orb}} \Delta P_{\text{orb}} + A_{\text{sin}} \sin \left( \frac{2\pi(n - n_0)}{P_{\text{sin}}} \right)$ (1)

In this equation, $P_{\text{orb}}$ is the orbital period at epoch $T_0$. The parameters, $A_{\text{sin}}$, $P_{\text{sin}}$ and $n_0$ are the amplitude, period and phase of the sinusoidal function, respectively. The best fit parameters are given in Table 2 and the best fit model is shown in Figure 2 after subtracting the best fit linear model. The timescale for evolution of the orbital period ($\tau = P_{\text{orb}}/\Delta P_{\text{orb}}$) is $6.5(7) \times 10^{-7}$ yr. It is larger than an earlier estimate using fewer mid-eclipse times (Wachter et al. 2000) by a factor of $\sim 6$. The sinusoidal variation in the O-C curve could be due to light travel time delay for motion of the X-ray binary in the presence of a third body.

3 DISCUSSION

We have determined 27 new mid-eclipse times of the X-ray binary MXB 1658–298 using data from RXTE and XMM-Newton observatories. These measurements have been used to determine the orbital evolution in this system.

Orbital evolution of LMXBs is complex and is known to display different trends. The orbital separation is known to increase in most of the LMXBs, at timescales which are shorter than that predicted by a conservative mass transfer or by gravitational wave radiation (Homer & Charles 1998; Hartman et al. 2000). A decreasing orbital period has been observed in a few LMXBs and some short period BHXBs. But the orbital decay in these systems is also unusual and is much faster than that predicted by conventional methods of gravitational wave radiation, magnetic braking and mass loss from the system (González Hernández et al. 2014; Ponti et al. 2016). Interaction with a third body could be responsible for a large orbital decay observed in two LMXBs (Peuten et al. 2014; Iaria et al. 2015). Orbital period glitches have been observed in a couple of LMXBs, and are attributed to be due to magnetic cycling of the secondary star (Wolff et al. 2009; Jain & Paul 2011). Even though most of the LMXBs show a reasonably good quadratic fit to the mid-eclipse time records, there are signatures of deviation from a constant orbital period derivative present on longer timescales (Iaria et al. 2015; Chou et al. 2014; Patrano et al. 2016).

The mid-eclipse time history of MXB 1658–298 seems to be quite unusual. The combined data spanning three outbursts and covering $\sim 40$ years of timeline, indicates an orbital decay. In addition, a large number of measurements in a two year period during the second outburst show sinusoidal variation in the eclipse timing residual, perhaps indicating the presence of a third body around this source.

We can consider the X-ray binary (the inner binary) to be a point mass in an approximate binary motion with this third body. In that case, the sinusoidal residual is due to orbital motion of the inner binary around the center of mass of the whole system. This is similar to the pulse arrival time delay of a binary X-ray pulsar, except that instead of a periodic pulse, we have a periodic eclipse.

For a 1.4 $M_\odot$ neutron star, the mass of the companion star lies between 0.25–0.9 $M_\odot$ (Cominsky & Wood 1984). Assuming the radius of the orbit of the X-ray binary (having an estimated total mass in the range 1.65–2.3 $M_\odot$) around the center of mass of the system to be same as amplitude of the sinusoidal residual, the third body (assumed to be co-planar with the inner binary) should have a mass range of 0.0195–0.0257 $M_\odot$ (i.e., 20.5–26.9 Jupiter mass) and an estimated range for orbital radius between 750–860 lt-sec. (Cominsky & Wood 1984).

The two extremes of this estimation are graphically shown in Figure 3. Taking $M_b$ and $R_b$ as the mass and radius of orbit of the binary; and $M_{1b}$ and $R_{1b}$ as the mass and orbital radius of the third object, we have drawn curves for the expressions below for two extremes of the mass of the inner binary, i.e., 1.65 $M_\odot$ (dashed line) and 2.3 $M_\odot$ (solid line).
An independent random error due to intrinsic source variability has been added quadratically for further analysis.

### Table 1. Measurements of the mid-eclipse times of MXB 1658–298.

| Date       | Mission/ID    | Cycle | MJD (d) | Observation time (sec) | Ingress/Egress (sec) |
|------------|---------------|-------|---------|------------------------|----------------------|
| 1976-10-07 | SAS-3         | –     | 43058.72595 (15) | –                      | –                    |
| 1978-03-07 | HEAO-1        | –     | 43574.64113 (15) | –                      | –                    |
| 1999-04-05 | RXTE          | 4014-01-01-00 | 27707 | 51273.9780792 (15) | –                    |
| 1999-04-06 | RXTE          | 40036-10-01-00 | 27709 | 51274.5711027 (37) | 903.10 (15)          |
| 1999-04-09 | RXTE          | 4014-01-02-00 | 27720 | 51277.832509 (37) | –                    |
| 1999-04-10 | RXTE          | 40050-04-02-00 | 27722 | 51278.4256585 (41) | 909.69 (15)          |
| 1999-04-13 | RXTE          | 4014-01-03-00 | 27733 | 51281.6871743 (37) | –                    |
| 1999-04-15 | RXTE          | 4014-01-04-00 | 27740 | 51283.7627259 (32) | –                    |
| 1999-04-17 | RXTE          | 40050-04-07-00 | 27747 | 51285.8382399 (21) | 899.24 (15)          |
| 1999-04-26 | RXTE          | 40050-04-15-00 | 27778 | 51295.0298420 (58) | 905.50 (15)          |
| 1999-04-29 | RXTE          | 40050-04-16-00 | 27787 | 51297.6984644 (58) | 908.24 (15)          |
| 1999-06-05 | RXTE          | 4014-01-05-00 | 27911 | 51334.469767 (23) | 898.51 (25)          |
| 1999-06-06 | RXTE          | 4014-01-06-00 | 27915 | 51335.6509729 (35) | 894.23 (20)          |
| 1999-06-08 | RXTE          | 4014-01-07-00 | 27920 | 51337.1334694 (42) | 898.72 (15)          |
| 1999-08-03 | RXTE          | 4014-01-09-00 | 28110 | 51393.4935853 (116) | 902.10 (80)          |
| 1999-08-06 | RXTE          | 4014-01-10-00 | 28119 | 51396.1375873 (41) | 898.76 (65)          |
| 1999-08-07 | RXTE          | 4014-01-11-00 | 28123 | 51397.3295883 (122) | 895.85 (70)          |
| 1999-09-15 | RXTE          | 4014-01-12-00 | 28355 | 51466.1195151 (23) | 904.77 (25)          |
| 1999-10-16 | RXTE          | 4014-01-14-00 | 28359 | 51467.2989903 (23) | 901.70 (15)          |
| 1999-10-18 | RXTE          | 4014-01-13-00 | 28368 | 51466.9675212 (23) | 898.99 (25)          |
| 2000-01-14 | RXTE          | 4014-01-15-00 | 28663 | 51557.4363548 (29) | 904.40 (20)          |
| 2000-01-18 | RXTE          | 4014-01-16-00 | 28676 | 51561.2909286 (12) | 909.02 (15)          |
| 2000-01-19 | RXTE          | 4014-01-17-00 | 28680 | 51562.4769833 (23) | 902.36 (35)          |
| 2000-05-13 | RXTE          | 50410-01-01-00 | 29069 | 51677.8173235 (23) | 901.50 (25)          |
| 2000-05-17 | RXTE          | 50410-01-02-00 | 29082 | 51681.6719031 (17) | 906.41 (15)          |
| 2000-05-18 | RXTE          | 50410-01-03-00 | 29086 | 51682.8579030 (58) | 902.26 (20)          |
| 2000-08-08 | RXTE          | 50410-01-06-00 | 29363 | 51764.9896893 (58) | 905.13 (20)          |
| 2000-08-12 | Beppo-SAX     | –     | 29376 | 51768.844257 (16) | –                    |
| 2000-08-13 | Beppo-SAX     | –     | 29376 | 51769.437259 (15) | –                    |
| 2000-10-18 | RXTE          | 50410-01-07-00 | 29600 | 51835.2612753 (58) | 909.21 (15)          |
| 2000-10-19 | RXTE          | 50410-01-08-00 | 29604 | 51836.472795 (51) | 907.53 (10)          |
| 2000-10-20 | RXTE          | 50410-01-09-00 | 29606 | 51837.0402811 (58) | 906.15 (15)          |
| 2001-02-20 | XMM-Newton    | 0008620701 | 30022 | 51960.386901 (23) | 903.13 (50)          |
| 2015-09-26 | XMM-Newton    | 0748391601 | 48004 | 57292.129569 (32) | 900.95 (40)          |

The numbers in brackets give the 1σ statistical error.

An independent random error due to intrinsic source variability has been added quadratically for further analysis.

These numbers are taken from Cominsky & Wood (1989).

These numbers are taken from Wachter et al. (2000).

These numbers are taken from Oosterbroek et al. (2003).

### Table 2. Orbital element s of MXB 1658–298.

| Parameter | Best fit value |
|-----------|----------------|
| $T_0$ (MJD) | 43058.72606 (23) |
| $P_{orb}$ (d) | 0.296504619 (11) |
| $P_{orb}$ | $-1.25(13) \times 10^{-11}$ |
| $A_{sin}$ (lt-sec) | 9.20 (75) |
| $P_{sin}$ (d) | 764 (37) |
| $i_0$ | 28846 (28) |
| $\chi^2$ (d.o.f) | 46.65 (29) |

The numbers in bracket indicate the 1σ errors

\[ R_{ch} = \frac{M_b R_b}{M_{ch}} \]

\[ R_{ch} = \left( \frac{G P_{sin}^2}{4 \pi^2} \right)^{1/3} \frac{M_b}{(M_b + M_{ch})^{2/3}} \]  

In Figure 3, the square markers indicate these estimated parameters for two extremes of the mass of the inner binary. Depending on the mass of the inner binary, the true mass and orbital radius of the third body will lie on the dotted line connecting these square markers. The additional errors in these two parameters due to the uncertainty in period and amplitude of the sinusoidal component have been estimated by a Monte Carlo simulation. The additional 1σ uncertainty is represented by two dash-dotted lines parallel to the diagonal (dotted) line.

The mass and orbital radius of the third body have been estimated assuming a circular and co-planar orbit. There is no evidence of circularity except that the residual shown in Figure 2 is consistent with being sinusoidal. Co-planarity is
a reasonable assumption as the circumbinary planets discovered with Kepler are nearly co-planar (Welsh et al. 2014).

Though rarer compared to planets around single stars, about 20 circumbinary planets are known among ~1000 eclipsing binaries observed with Kepler (Welsh et al. 2014). If true, the third body in the present system is the most massive circumbinary planet; exceeding Kepler-1647b by a factor of about 15 (Kostov et al. 2016). Simulations of planet formation and migration around binary stars show the most stable planets to be in the sub-Saturn mass range while more than Jupiter mass planets, if present, are likely to have large orbits around the binary (Pierens & Nelson 2008).

The binary period of MXB 1658-298 is also much shorter compared to the orbital period of all the binary stellar systems around which planets have been found, the shortest binary period being 7.4 days in Kepler-47 (Orosz et al. 2012). The lack of planets around short period binaries is believed to be related to the process of angular momentum loss that brings the two stars closer in the process of binary evolution (Welsh et al. 2014).

The system MXB 1658-298 gives important new input for the range of stellar configurations for which circumbinary planets may form and survive migration over several stages of binary evolution. In particular, the binary system being a low mass X-ray binary and having an age of several billion years is an important input for the study of planet formation and migration around binary stellar systems.

ACKNOWLEDGMENTS

This research has made use of data obtained from the High Energy Astrophysics Science Archive Research Center, provided by NASA’s Goddard Space Flight Center. The authors thank S. Sridhar for insightful discussions.

REFERENCES

Chou Y., Grindlay J. E., 2001, ApJ, 563, 934
Chou Y., Hsieh H.-E., Hu C.-P., Yang T.-C., Su Y.-H., 2016, ApJ, 831, 29
Cominsky L. R., Wood K. S., 1984, ApJ, 283, 765
Cominsky L. R., Wood K. S. 1989, ApJ, 337, 485
Falanga M., Bozzo E., Lutovinov A., Bonnet-Bidaud J. M., Fetisova Y., Puls J., 2015, A&A, 577, 130
González Hernández J. I., Rebolo R., Casares J., 2014, MNRAS, 438, L21
González Hernández J. I., Suárez-Andrés L., Rebolo R., Casares J., 2017, MNRAS, 465, L15
Hartman J. M., Patruno A., Chakrabarty D., Markwardt C. B., Morgan E. H., van der Klis M., Wijnands R., 2009, ApJ, 702, 1673
Homer L., Charles P. A., 1998, New Astron., 3, 435
Hu C.-P., Chou Y., Chung Y. Y., 2008, ApJ, 680, 1405
Iaria R., Di Salvo T., Gambino A. F., Del Santo M., Romanò P., Matranga M., Galiano C. G., Scarano F., Riggio A., Sanna A., et al., 2015, A&A, 582, 32
int Zand J., Heise J., Smith M. J. S., Cocchi M., Natalucci L., Celidonio G., 1999, IAUC, 7138
Islam N., Paul B., 2016, MNRAS, 461, 816
Jahoda K., Swank J. H., Giles A. B., Stark M. J., Strohmayer T., Zhang W., Morgan E. H., 1996, SPIE, 2808, 59
Jain C., Dutta A., Paul B., 2007, IAA, 28, 197
Jain C., Paul B., Dutta A., 2010, MNRAS, 409, 755
Jain C., Paul B., 2011, MNRAS, 413, 2
Jansen F. et al., 2001, A&A, 365, L1
Kostov V. B., Orosz J. A., Welsh W. F., Doyle L. R., Fabrycky D. C., Haghighipour N., Quarles B., Short D. R., Cochran W. D., Endl M., et al., 2016, ApJ, 827, 86
Levine A. M., Rappaport S. A., Zocchetti G. 2000, ApJ, 541, 194
Lewin W. H. G., Hoffmann J. A., Doty J., 1976, IAU Circ., 2994
Lewin W. H. G., Marshall H., Primini F., Wheaton W., Cominsky L., Jernigan G., Ossman W., 1978, IAUC, 3190
Negoro H., Furuya K., Ueno S., Tomida H., Nakahira S., Kimura M., Ishikawa M., Nakagawa Y. E., Mihara T., Sugizaki M., et al., 2015, Atel, 7943, 1
Oosterbroek T., Parmar A. N., Sidoli L., in’t Zand J. J. M., Heise J., 2001, A&A, 376, 532
Orosz J. A., Welsh W. F., Carter J. A., Fabrycky D. C., Cochran W. D., Endl M., Ford E. B., Haghighipour N., MacQueen P. J., Mazeh T., et al., 2012, Science, 337, 1511
Patruno A., Jaodand A., Kuiper L., et al., 2016 arXiv 1611.06023
Paul B., Naik S., Bhattacharya D., 2004, Nuclear Physics B Proc. Suppl., 132, 548
Peuten M., Brockamp M., Kupper A. H. W., Kroupa P., 2014, ApJ, 795, 116
Pierens A., Nelson R. P., 2008, A&A, 483, 633
Ponti G., De K., Munoz-Darias T., Stella L., Nandra K., 2017, MNRAS, 464, 840
Sidoli L., Oosterbroek T., Parmar A. N., Lumb D., Erd C., 2001, A&A, 379, 540
Staubert R., Klochkov D., Wilms J., 2009, A&A, 500, 883
Singh N. S., Naik S., Paul B., Agrawal P. C., Rao A. R., Singh K. Y., 2002, A&A, 392, 161
Wachter S., Smail A. P., Bailyn C., 2000, ApJ, 534, 367
Welsh W. F., Orosz J. A., Carter J. A., Fabrycky D. C., 2014, in IAU Symposium, 293, 125
Wijnands R., Strohmayer T., Franco L. M., 2001, ApJ, 549, L71
Wolf M. T., Ray P. S., Wood K. S., Hertz P. L., 2009, ApJS, 183, 156

Figure 3. Graphical representation of finding the mass and orbital radius of the third body. The square shaped markers indicate the mass and orbital radius of the third body for a binary mass of 1.65 $M_\odot$ (dashed line) and 2.3 $M_\odot$ (solid line), respectively.