S wave $K\pi$ scattering and effects of $\kappa$ in $J/\psi \rightarrow \bar{K}^{*0}(892)K^+\pi^-$

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$K\pi$ S wave scattering is studied using a chiral unitary approach (ChUT) taking into account coupled channels. With the amplitudes derived from the lowest order chiral Lagrangian as the kernel of a set of coupled channel Bethe-Salpeter equations, the $I=1/2$ S wave $K\pi$ scattering phase shifts below 1.2 GeV can be fitted by one parameter, a subtraction constant, and a scalar resonance corresponding to the controversial $\kappa$ ($K^*_0(800)$) can be generated dynamically. A good description of the $I=3/2$ S wave $K\pi$ scattering phase shifts below 1.2 GeV can also be obtained. An artificial singularity in the conventional cut-off method of the 2-meson loop integral of the ChUT is found. The formalism is applied to deal with the S wave $K\pi$ final state interaction (FSI) in the decay $J/\psi \rightarrow \bar{K}^{*0}(892)K^+\pi^-$, and a qualitatively good fit to the data is achieved. The role of $\kappa$ in the decay is discussed.

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I. INTRODUCTION

The scattering between the lowest pseudoscalar mesons is significant in the understanding of low energy strong interaction and nonperturbative QCD. The most popular approach dealing with this problem is Chiral Perturbation Theory (ChPT) \[1, 2, 3, 4, 5\] (for a recent comprehensive review, see \[6\]). The lowest pseudoscalar meson octet \((\pi, K, \eta)\) is identified with the Goldstone bosons associated with the spontaneous breakdown of chiral symmetry of QCD. The most general chiral Lagrangian can be written in a perturbative manner to an order by expanding the QCD Lagrangian containing external sources in powers of the external momenta of the pseudoscalar mesons and of the light quark masses \(m_u, m_d\) and \(m_s\). However, a perturbative expansion to any finite order cannot describe the appearance of a resonance. For instance, the pure ChPT is not suitable for the physics of the scalar isoscalar \(\pi \pi\) scattering above 0.6 GeV where a broad resonance \(f_0(600)\) (or \(\sigma\)) is generally believed to exist. Furthermore, unitary relation of \(S\) matrix is only respected by ChPT in a perturbative sense.

Over the last few years, several nonperturbative methods were proposed to extend the chiral expansion to higher energies, such as resummation of loops using a Bethe-Salpeter equation (BSE) \[7, 8\], the inverse amplitudes method (IAM) \[9, 10, 11, 12\] and dispersion relation methods \[13, 14, 15\].

The existence of the scalar particle \(\kappa\) (i.e. \(K_0^*(800)\)) and the mass and width of this particle, if it exists, has been long controversial since the 1970’s \[16, 17, 18\]. The evidences of the \(\kappa\) has been observed in the analysis of the \(K\pi\) scattering phase shifts \[19\], the Dalitz Plot Analysis of the Decay \(D^+ \rightarrow K^-\pi^+\pi^+\) \[20\], and the BES data of the \(J/\psi\) decays \[21, 22, 23\].

In order to understand the \(\kappa\), a good understanding of \(S\) wave \(K\pi\) scattering is required. \(K\pi\) scattering has been investigated using the ChPT by V. Bernard \textit{et al.} at next-to-leading order \[24\] and a satisfactory description of the available data up to about 900 MeV was found, and by J. Bijnens \textit{et al.} at next-to-next-to-leading order \[25\]. The problem has also been investigated using various unitarisation approaches of the chiral Lagangian \[10, 12, 13, 15, 26\], and by other methods such as the interfering Breit-Wigner amplitude method \[27\], the unitarized meson model \[28\] and the \(K\)-matrix method \[29\]. A scalar meson corresponding to \(\kappa\) was found in all of the mentioned unitary chiral approaches. While Long Li \textit{et al.} found the \(K\pi\) data could be fitted without a pole around 900 MeV in their \(K\)-matrix approach and only one \(s\)-channel
resonance between the $K\pi$ threshold and 1.6 GeV, i.e., $K^0(1430)$ with a mass around 1438-1486 MeV and a width of about 346 MeV was found \cite{29}. Recently, an important work appeared. Some authors have performed a detailed analysis using a set of Roy-Steiner (RS) equations \cite{30}. The input was high-statistical data at $\sqrt{s} \gtrsim 1$ GeV for both $\pi K \to \pi K$ and $\pi\pi \to K\bar{K}$ amplitudes, then the amplitudes below 1 GeV were determined. The authors computed $S$ wave scattering lengths and the coupling constants $L_1$, $L_2$, $L_3$ and $L_4$ appearing in the $O(p^4)$ order chiral Lagrangian. The precise $S$ wave and $P$ wave phase shifts below about 1 GeV were also obtained by solving the RS equations.

In this paper, we use the on-shell coupled-channel BSE approach (it will be referred as ChUT below) proposed by J. A. Oller and E. Oset \cite{7} to study $S$ wave $K\pi$ scattering. In Ref. \cite{7}, the lowest order ChPT amplitudes are employed as the kernels of a set of coupled channel on-shell BSEs. The BSEs are solved to resum the contributions from the s-channel loops of the re-scattering between pseudoscalar mesons. The loop integral was calculated using a three-momentum cut-off, and this is the only free parameter in the approach. By properly choosing the cut-off, the ChUT can well-describe the data of the $S$ wave meson-meson interaction up to $\sqrt{s} \simeq 1.2$ GeV in the isoscalar and isovector channels. And more interestingly, the $\sigma$, $f_0(980)$ and $a_0(980)$ scalar mesons can be generated dynamically as the poles in the second Riemann sheet of the $T$ matrix. It was mentioned in Ref. \cite{31} that satisfactory fits to the $K\pi$ scattering phase shifts could be produced using a cut-off of 850 MeV and using $f = 100$ MeV (an average between $f_\pi$ and $f_K$). However, we find there is a severe problem in the cut-off method in calculating the loop integral, which makes it difficult to describe $K\pi$ data above 1 GeV, and we will discuss this in detail in the corresponding section.

On the other hand, $J/\psi$ radiative and hadronic decays provide a rather rich source of the information of light hadrons including $\sigma$ and $\kappa$, and hence a good understanding of the interaction between the final state hadrons is required. The ChUT has been used to dealing with the meson-meson $S$ wave FSI in $\gamma\gamma \to$ meson-meson reactions \cite{32}, in some heavy quarkonium decays \cite{33,34,35,36}, and in heavy meson decays \cite{37,38}.

The paper is organized as follows: The basic definitions and the lowest order chiral amplitudes to be used are given in Sec. \textbf{II}. In Sec. \textbf{III}, we give a simple review of the ChUT first, and then in Sec. \textbf{III A} we discuss an artificial singularity appeared in the conventional cut-off method used in the ChUT, and a very simple proof of the unitarity
is given in Sec. III B. The calculated $S$ wave $K\pi$ phase shifts are shown in Sec. IV A and IV B, and a good description of the experimental data below 1.2 GeV is achieved. The pole position associated to $\kappa$ is given in Sec. IV D. As an example of applying the ChUT formalism to dealing with the $S$ wave $K\pi$ FSI, the decay $J/\psi \rightarrow \bar{K}^0(892)K^+\pi^-$ is discussed in Sec. V. A brief summary is given in Sec. VI.

II. THE LOWEST ORDER CHPT AMPLITUDES

A. Definitions

The lowest order, i.e. $O(p^2)$, Lagrangian for SU(3) ChPT reads

$$\mathcal{L}^{(2)} = \frac{f^2}{4} (\partial_\mu U \partial^\mu U^\dagger + M(U + U^\dagger)),$$

(1)

where $f$ is the pion decay constant, the physical value of which is 92.4 MeV, and $\langle \rangle$ stands for the trace of matrices. The $3 \times 3$ special unitary matrix $U$ is defined as $U = \exp \frac{i\Phi}{f}$, and $\Phi$ is made of the pseudo Goldstone boson fields

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 \end{pmatrix}.$$

(2)

The pseudoscalar meson mass matrix $M$ is given by

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix},$$

(3)

where the isospin limit $m_u = m_d$ is assumed. The relevant part concerns four mesons and can be derived from Eq. (1) as

$$\mathcal{L}^{(2)} = \frac{1}{12 f^2} ((\partial_\mu \Phi \Phi - \Phi \partial_\mu \Phi)^2 + M\Phi^4).$$

(4)

Let $T^I_l(s, t, u)$ denotes the amplitude with total isospin $I$ and $T^I_l(s)$ its partial wave projection with angular momentum $l$. We take the normalization for partial wave amplitudes in such a way that

$$T^I_l(s) = \frac{1}{2} \int_{-1}^{1} d \cos \theta P_l(\cos \theta) T^I_l(s, t(s, \cos \theta), u(s, \cos \theta)).$$

(5)
where $P_l(\cos \theta)$ is the $l$-th order Legendre polynomial, $\theta$ is the scattering angle in the center of mass frame. The Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$ are bound to the identity $s + t + u = \sum_{i=1}^{4} m_i^2$ when the particles are on-shell, and

$$t(s, \cos \theta) = m_1^2 + m_3^2 - 2\sqrt{m_1^2 + \frac{\lambda(s, m_1^2, m_2^2)}{4s}}[m_3^2 + \frac{\lambda(s, m_3^2, m_4^2)}{4s}]$$

$$+ \frac{1}{2s} \sqrt{\lambda(s, m_1^2, m_2^2)\lambda(s, m_3^2, m_4^2)} \cos \theta$$

with $\lambda(s, m_i^2, m_j^2) = [s - (m_i + m_j)^2][s - (m_i - m_j)^2]$.

We shall use the phase convention $|\pi^+\rangle = -|1, 1\rangle$ and $|\bar{K}^0\rangle = -|1/2, 1/2\rangle$. The calculations of $I = 1/2$ scattering will be performed with single channel ($K\pi$), two channels ($K\pi$ and $K\eta$) and three channels ($K\pi$, $K\eta$ and $K\eta'$), respectively, and the single channel $I = 3/2$ $K\pi$ scattering will also be investigated. The lowest order chiral amplitudes to be used in our coupled channel calculations of the $S$ wave $K\pi$ scattering will be given in the following subsections. We will label them with $V$ and label the full amplitudes in the BSEs with $T$ to avoid confusion. Since we will restrict ourselves in the $S$ wave, the subscript $l$ will be neglected in the following and $l = 0$ will be implied everywhere.

### B. Two channels case

In this case, we take $\eta = \eta_8$ and the mixing between $\eta$ and $\eta'$ will not be considered. Let us label the channels $K\pi$ and $K\eta$ with 1 and 2 respectively, such that $V_{11}^I(s, t, u)$, $V_{12}^I(s, t, u)$ and $V_{22}^I(s, t, u)$ represent the amplitudes with total isospin $I$ for $K\eta \rightarrow K\eta$, $K\eta \rightarrow K\pi$ and $K\eta \rightarrow K\eta$. By time reversal invariance, one has $V_{1j}^I(s, t, u) = V_{j1}^I(s, t, u)$.

$$\pi^+K^+ \rightarrow \pi^+K^+$$ is a pure $I = 3/2$ process,

$$V^{3/2}(s, t, u) = V_{\pi^+K^+ \rightarrow \pi^+K^+}(s, t, u).$$

According to the crossing symmetry, the isospin relation for $V_{11}^{1/2}(s, t, u)$ and $V_{12}^{1/2}(s, t, u)$ can be interpreted as

$$V_{11}^{1/2}(s, t, u) = \frac{3}{2} V^{3/2}(u, t, s) - \frac{1}{2} V^{3/2}(s, t, u),$$

$$V_{12}^{1/2}(s, t, u) = \sqrt{3} V_{K^0\eta \rightarrow K^0\eta^0}(s, t, u).$$
Now the $O(p^2)$ order ChPT amplitudes can be derived from Eq. (4). One has a pure $S$ wave amplitude for $I = 3/2$ process

$$V^{3/2}(s,t,u) = \frac{1}{2f^2}(s - m_\pi^2 - m_\eta^2), \quad (10)$$

and for $I = 1/2$ the amplitudes contain both $S$ wave and $P$ wave

$$V^{1/2}_{11}(s,t,u) = -\frac{1}{4f^2}(4s + 3t - 4m_\pi^2 - 4m_K^2), \quad (11)$$

$$V^{1/2}_{12}(s,t,u) = -\frac{1}{4f^2}(-3t + \frac{1}{3}m_\pi^2 + \frac{8}{3}m_K^2 + m_\eta^2), \quad (12)$$

$$V^{1/2}_{22}(s,t,u) = -\frac{1}{4f^2}(3t - \frac{2}{3}m_\pi^2 - 2m_\eta^2). \quad (13)$$

Note that the calculations will be performed using on-shell BSE, so that the on-shell condition $p_i^2 = m_i^2$ has been used in writing the above amplitudes. The isospin phase convention of $K$ is different from the one in Ref. [10], such that there is an extra minus before Eq. (12) compared with $T_2(s,t,u)$ in Eq. (B14) therein, but the physical observables are not influenced by different phase conventions as they should be.

C. Three channels case

If one want to see the possible impacts of the $K\eta'$ channel, $\eta'$ should be included in the chiral Lagrangian. One way is extending the $SU(3)$ matrix to a $U(3)$ one

$$\Phi_{nonet} = \Phi + \frac{I}{\sqrt{3}}\eta_0$$

$$= \begin{pmatrix}
\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' \\
\pi^- \\
K^-
\end{pmatrix}
\begin{pmatrix}
\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' \\
\pi^+ \\
K^+
\end{pmatrix}
\begin{pmatrix}
\pi^0 \\
\eta \\
\eta'
\end{pmatrix}, \quad (14)$$

where the standard $\eta - \eta'$ mixing

$$|\eta\rangle = \frac{1}{3}|\eta_0\rangle + \frac{2\sqrt{2}}{3}|\eta_8\rangle,$$

$$|\eta'\rangle = \frac{2\sqrt{2}}{3}|\eta_0\rangle - \frac{1}{3}|\eta_8\rangle \quad (15)$$

is assumed. We label the channel $K\eta'$ with 3, such that $V^{1/2}_{13}(s,t,u)$, $V^{1/2}_{23}(s,t,u)$ and $V^{1/2}_{33}(s,t,u)$ represent the amplitudes for $K\eta' \to K\pi$, $K\eta' \to K\eta$ and $K\eta' \to K\eta'$,
respectively. The amplitudes concerning only $K$ and $\pi$ do not change while the others change to

$$V_{12}^{1/2}(s, t, u) = -\frac{\sqrt{2}}{6f^2}(-3t + 2m_K^2 + m_\eta^2),$$

$$V_{22}^{1/2}(s, t, u) = -\frac{2}{9f^2}(3t - m_K^2 - 2m_\eta^2).$$

The amplitudes involving $\eta'$ are

$$V_{13}^{1/2}(s, t, u) = \frac{1}{12f^2}(-3t + 3m_\pi^2 + 8m_K^2 + m_\eta^2),$$

$$V_{23}^{1/2}(s, t, u) = \frac{\sqrt{2}}{18f^2}(3t - 3m_\pi^2 + 2m_K^2 - m_\eta^2 - m_\eta'^2),$$

$$V_{33}^{1/2}(s, t, u) = -\frac{1}{12f^2}(t - 2m_\pi^2 + 12m_K^2 + \frac{2}{3}m_\eta^2).$$

### III. COUPLED-CHANNEL CHIRAL UNITARY APPROACH

In our normalization, the unitary relation for the partial amplitudes with isospin $I$ satisfies

$$\text{Im}T^I(s) = -T^{I\dagger}(s)\rho(s)T^I(s),$$

where $T^I(s)$ and $\rho(s)$ are $n \times n$ matrices for $n$ channel calculations. The matrix elements $T^I_{ij}(s)$ will be given by the on-shell BSEs. $\rho(s)$ is a diagonal matrix with

$$\rho_{ii}(s) = \frac{p_{cm}^2}{8\pi\sqrt{s}},$$

where $p_{cm_i}$ is the three-momentum of one meson in the center-of-mass (c.m.) frame for the $i$-th channel. This momentum is given by

$$p_{cm} = \frac{1}{2\sqrt{s}}\sqrt{[s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]}$$

in a channel with two mesons of masses $m_1$ and $m_2$ respectively. In what follows we will omit the label $I$ for simplicity.

Following Ref. [7], the amplitude of meson-meson scattering can be cast using the BSE

$$T = (1 - VG)^{-1}V,$$

where $V$ is the lowest order chiral amplitude and $G$ is the loop propagator. For coupled channel calculations, all of $T$, $V$ and $G$ are matrices. One important feature of this
approach is that $V$ can be factorized on shell in the BSEs, and so that the integral equations become algebraic equations which can be solved simply. This feature is justified using the $N/D$ method of dispersion relations [13] and by a comprehensive treatment of the BSE [8].

A. Artificial singularity in the cut-off method

The loop integral in the $i$-th channel is

$$G_{ii}(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_i^2 + i\varepsilon} \frac{1}{(p_1 + p_2 - q)^2 - m_2^2 + i\varepsilon},$$

(25)

where $p_1$ and $p_2$ are the four-momentum of the two initial particles respectively, $m_1$ and $m_2$ are the masses of the two particles appearing in the loop. Sometimes in ChUT the integral is calculated by using a three-momentum cut-off parameter $q_{\text{max}}$ in the c.m. frame [4]. The analytic expression can be worked out as

$$G(s) = \frac{1}{16\pi^2 s} \{\sigma(\arctan \frac{s + \Delta}{\sigma \lambda_1} + \arctan \frac{s - \Delta}{\sigma \lambda_2})$$

$$- [(s + \Delta) \ln \frac{q_{\text{max}}}{m_1} (1 + \lambda_1) + (s - \Delta) \ln \frac{q_{\text{max}}}{m_2} (1 + \lambda_2)]\}$$

(26)

where $\sigma = [- (s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)]^{1/2}$, $\Delta = m_1^2 - m_2^2$ and $\lambda_i = \sqrt{1 + \frac{m_i^2}{q_{\text{max}}}}$ ($i = 1, 2$), and the channel label $ii$ has been dropped.

However, there are some problems. If one goes to values of the on-shell momenta above the cut-off, one cannot use the cut-off formula [39]. On the other hand, we find that there is a severe problem using this cut-off method to calculate the loop integral. From Eq. (26), one can find there is a singularity above the threshold which is located at

$$(\sqrt{s})_{\text{sing}} = \sqrt{m_1^2 + q_{\text{max}}^2} + \sqrt{m_2^2 + q_{\text{max}}^2}.$$  

(27)

When $\sqrt{s}$ goes to $(\sqrt{s})_{\text{sing}}$, the real part of $G(s)$ will go to infinity, and when $\sqrt{s}$ is greater than $(\sqrt{s})_{\text{sing}}$ given below in Eq. (27), Eq. (26) gives a vanishing imaginary part. However, if momenta are large enough to bring the propagators on shell, the imaginary part should be given by $-p_{cm}/(8\pi \sqrt{s})$. We can give some remarks on the singularity. First, after some manipulations, the real part of the loop integral can be given as

$$\text{Re}G(s) = \frac{1}{4\pi^2} \text{P} \int_0^{q_{\text{max}}} d|q| \frac{q^2(\omega_1 + \omega_2)}{\omega_1 \omega_2 [s - (\omega_1 + \omega_2)^2]},$$

(28)
where $P$ represents principal integral, and $\omega_i = \sqrt{q^2 + m_i^2}$. One can see that the singularity comes from the possibility of the denominator in the integrand being zero. And when the cut-off $q_{\text{max}}$ runs to infinity, the singularity will disappear. Second, the singularity rises from the arctan $f(s)$ function. If we perform a series expansion at the singularity point, the first term should be $\arctan i$, or equivalently $\frac{1}{2i}\ln 0$ using the identity $\arctan z = \frac{1}{2i}\ln \frac{z-i}{z+i}$. It should be a logarithmic singularity. Third, when $\sqrt{s} = (\sqrt{s})_{\text{sing}}$, the three-momenta of the propagators equal to $q_{\text{max}}$. Then no momentum is available to bring $\sqrt{s}$ larger than $(\sqrt{s})_{\text{sing}}$, and the imaginary part of $G(s)$ should vanish as it appears in Eq. (26). In other words, $(\sqrt{s})_{\text{sing}}$ should be available limit of this method. On the other hand, there is still a region below $(\sqrt{s})_{\text{sing}}$ where the impact of the singularity is large, and one should be care of that. For the $S$ wave $K\pi$ scattering, a typical value of the $q_{\text{max}}$ is about 0.6 GeV from fitting to the data when the kernel of the BSE is of the $O(p^2)$ order. The corresponding position of the artificial pole will be about 1.4 GeV, and its impact will show up above 1 GeV apparently. Then it is difficult to describe $K\pi$ data above about 1 GeV in this formalism. Note that this cut-off method is different from the one used in Pauli-Villars regularization (see e.g. Ref. [40]). In this method, the cut-off is just a three-momentum cut-off.

The way to solve this problem is using a dimensional regularization method with a dispersion relation to deal with the loop integral. The analytic expression has been given as [13, 41]

$$G(s) = \frac{1}{16\pi^2} \left\{ a(\mu) + \log \frac{m_1^2}{\mu^2} + \frac{\Delta - s}{2s} \log \frac{m_2^2}{m_1^2} \right. $$

$$+ \frac{\sigma}{2s} \left[ \log (s - \Delta + \sigma) + \log (s + \Delta + \sigma) \\
- \log (-s + \Delta + \sigma) - \log (-s - \Delta + \sigma) \right] \right\},$$

(29)

where $a(\mu)$ is a subtraction constant, $\mu$ is the regularization scale. The result is independent of $\mu$ because the change causing by a change of $\mu$ can be cancelled by a change of the subtraction constant $a(\mu)$.

In order to show the impact of the artificial singularity when the cut-off method is used, we plot the real part of the loop integral with $m_1 = m_\pi$ and $m_2 = m_K$ in Fig. 1. In Fig. 1 the solid curve represents the result from the cut-off method with $q_{\text{max}} = 0.57$ GeV and the dashed curve represents the result from the dimensional regularization method calculated at $\mu = m_K$ with $a(m_k) = -1.41$. From Eq. (27), the singularity is located at $\sqrt{s} = 1.34$ GeV, and the values above 1 GeV are all affected significantly.
FIG. 1: The real part of the loop integral calculated using the cut-off method (solid curve) and the dimensional regularization method (dashed curve).

However, the physical results of Ref. [7] where the cut-off method was used will not be influenced, because with the parameter $q_{\text{max}} = 1.03$ GeV therein the singularity will be located 2.08 GeV, 2.29 GeV and 2.21 GeV for the $\pi\pi$, $K\bar{K}$ and $\pi\eta$ channels respectively, and the effect below 1.2 GeV is very small.

B. A simple proof of unitarity

It has been shown that by this coupled-channel BSE approach unitarity is ensured [10, 13]. In fact, it can be realized quickly as follows: The unitary relation Eq. (21) can be rewritten as

$$\frac{1}{T(s)^*} - \frac{1}{T(s)} = -2i\rho(s).$$

The general solution of Eq. (30) is of the form

$$T(s) = \frac{1}{K(s) + i\rho(s)},$$

where $K(s)$ is a real function of $s$. While by the Cutcosky rule [42], the imaginary part of the loop integral can be related to the phase space factor as

$$\text{Im}G(s) = -\rho(s).$$

So Eq. (24) can be rewritten as

$$T(s) = \frac{1}{V^{-1}(s) - G(s)}.$$
\[ K(s) = \frac{1}{V^{-1}(s) - \text{Re}G(s) - i\text{Im}G(s)} = \frac{1}{K(s) + i\rho(s)}, \quad (33) \]

where \( K(s) = V^{-1}(s) - \text{Re}G(s) \). In this approach, \( V(s) \) is the lowest order ChPT amplitude and so that is real. From the above equation one can see in this approach Eq. (24) ensures unitarity actually. Furthermore, if one use such a kernel that the BSE can be dealt with in an on-shell way, the unitary relation can be ensured if and only if the kernel is real.

IV. RESULTS AND DISCUSSION

A. S wave \( I = 1/2 \) \( K\pi \) phase shifts

Given the above knowledge, one can calculate the \( S \) wave \( K\pi \) scattering phase shifts numerically. The only parameter in our calculations is the subtraction constant \( a(\mu) \). In principle, it should be different for different channels and be same only under exact \( SU(3) \) symmetry. In practice, in order to reduce the number of free parameters, we take \( a(\mu) \) to be same for different channels. All of the results presented in this paper are calculated at \( \mu = m_K \). In order to get the most accurate description, we use the MINUIT function minimization and error analysis package from the CERN Program Library [43] to find the most appropriate value of the free parameter \( a(m_K) \).

The \( S \) wave \( I = 1/2 \) \( K\pi \) phase shift data are taken from Refs. [44, 45, 46]. The parameters can be determined from fitting to the data, and the values are

- for one channel : \( a(m_K) = -1.412 \pm 0.017 \),
- for two channels : \( a(m_K) = -1.278 \pm 0.014 \),
- for three channels : \( a(m_K) = -1.383 \pm 0.006 \). \quad (34)

The calculated phase shifts are plotted as solid curves in Fig. [2] where the dashed line depicts one-channel case, the dotted line depicts two-channel case, and the solid line depicts three-channel case, respectively. One can see the data below 1.2 GeV can be well described for one channel and two channels, and the data below 1.2 GeV can be well described for three channels. The fit from three-channel calculations is slightly better.
As mentioned in the previous section, if one use the three-momentum cut-off method to deal with the loop integral, there will be an artificial singularity in the loop function. The typical value of $q_{\text{max}}$ is 0.4-0.6 GeV from fitting the phase shift data below 1 GeV or 1.2 GeV, and the singularity is located at about 1.0-1.4 GeV. Then the effect of the singularity above 1 GeV is large, see Fig. 1. That is to say, one cannot get a satisfactory description of $K\pi$ $S$ wave scattering above 1 GeV using the three-momentum cut-off method. One way out is using the dimensional regularization to deal with the loop integral as illustrated above.

**B. $S$ wave $I = 3/2$ $K\pi$ phase shifts**

The experimental data of $S$ wave $I = 3/2$ $K\pi$ phase shifts are taken from Refs. [44, 46]. The calculated results are plotted in Fig. 3. From fitting to the data, the subtraction constant is determined to be $a(m_K) = -4.643 \pm 0.083$.

**C. Comparison with the results from Roy-Steiner equation analysis**

Very recently, some authors have performed a complicated analysis of $K\pi$ scattering using RS equations [30]. The experimental data above $s = 0.935$ GeV$^2$ were used as
input, and the solutions were obtained in the range $s < 0.935$ GeV$^2$. The results are very precise, and it is deserved for our results to be compared with the RS equation results. For illustration, we plot our results in the range $s < 1$ GeV$^2$ for $I = 1/2$ with two channels and for $I = 3/2$ as well as the RS equation results \cite{30} in Fig. 4. The upper half and the lower half of Fig. 4 are for $I = 1/2$ and $I = 3/2$, respectively. In this figure, the solid lines represent the ChUA results, and the dashed lines represent the results from the detailed analysis using the RS equations \cite{30}. From comparison, we find that our results are consistent with the precise results in Ref. \cite{30} below 1 GeV for both of $I = 1/2$ and $I = 3/2$.

D. Poles in the amplitudes

The physical resonances can be associated to the poles in unphysical Riemann sheets of the scattering amplitudes. After the same analytic continuation of the loop function $G(s)$ as in Refs. \cite{7,47}, one can find a pole corresponding to the long controversial scalar particle $\kappa$ which is called $K^*_0(800)$ in the PDG \cite{48}. The pole position for each calculation is given in Table \ref{table1}.

In order to see the stability of the pole position against different regularization schemes, the results from the cut-off method are also listed. For each calculation, the parameter $q_{max}$ is determined from fitting to the phase shift data below 1 GeV where the influence of the artificial singularity is small as pointed out in the previous section. In the first column of Table \ref{table1} DRM represents the dimensional regularization.
FIG. 4: Comparison of $S$ wave $K\pi$ phase shifts with the results from Roy-Steiner equation analysis. The upper half and the lower half are for $I = 1/2$ and $I = 3/2$, respectively. The solid and dashed lines depict our results and the RS equation results, respectively.

method and COM represents the cut-off method. In the second column of Table I, the parameter is $a(m_K)$ for DRM and is $q_{\text{max}}$ for COM. The pole positions are calculated using the central value of each parameter. One can see that the difference of pole position, especially the real part, is small using different regularization schemes.

TABLE I: Pole positions found in the second Riemann sheet of the $T$ matrices.

| Method | Number of channels | Parameter         | Pole position (GeV) |
|--------|-------------------|-------------------|---------------------|
| DRM    | 1                 | $-1.412 \pm 0.017$ | $0.736 - i0.273$    |
|        | 2                 | $-1.278 \pm 0.014$ | $0.725 - i0.297$    |
|        | 3                 | $-1.383 \pm 0.006$ | $0.742 - i0.273$    |
| COM    | 1                 | $(0.512 \pm 0.005)$ GeV | $0.729 - i0.244$    |
|        | 2                 | $(0.477 \pm 0.003)$ GeV | $0.730 - i0.244$    |
|        | 3                 | 0.486 GeV         | $0.728 - i0.247$    |

Because in the lowest order, the decay constants are same for $\pi$, $K$ and $\eta$, one can investigate some effects of higher order Lagrangian on the pole position by taking the physical values of the decay constants of $K$ and $\eta$, that is, $f_K = 113.0$ MeV and $f_\eta = 110.9$ MeV [48]. Let us study the case of two channels, and the results of the other cases are similar. The replacement can be done following the Appendix B of Ref. 10.
For example, $1/f_{\pi}^2$ in Eq. (11) will be replaced with $1/(f_K f_\pi)$. The pole positions are listed in Table II. The real and imaginary parts of the pole position are both slightly larger than the corresponding ones listed in Table I.

**TABLE II:** Pole positions found in the second Riemann sheet of the two channel $T$ matrices taking different decay constants for $\pi$, $K$ and $\eta$.

| Method | Parameter | Pole position (GeV) |
|--------|-----------|---------------------|
| DRM    | $-1.798 \pm 0.015$ | $0.790 - i0.320$  |
| COM    | $0.619 \pm 0.006$ GeV | $0.773 - i0.276$  |

These results can be compared with previous results in other chiral unitary approaches. The pole position appeared in the second Riemann sheet associated to $\kappa$ were found to be around $770 - i250$ MeV in Ref. [10] where COM was used and $779 + i330$ MeV in Ref. [13] where DRM was used. These results are obtained using the $O(p^4)$ chiral amplitudes as the four-meson contact terms. The position of the pole we found is similar to the one found in other approaches. Furthermore, from comparison, it seems that the higher order corrections from the decay constants give an important contribution to the $O(p^4)$ order corrections to the pole position.

**V. APPLICATION TO THE DECAY** $J/\psi \to \bar{K}^*0(892)K^+\pi^-$

In the preceding sections, we have constructed a formalism of the $S$ wave $K\pi$ scattering below 1.2 GeV. In this section, as an example of its applications, it will be applied to describe the $S$ wave $K\pi$ FSI in the decay $J/\psi \to \bar{K}^*0(892)K^+\pi^-$. We will calculate the invariant mass of $K^+\pi^-$ below 1.2 GeV and fit to the experimental data [23]. The relevant decay mechanisms are plotted in Fig. 4.

In Fig. 5 (a) represents the direct decay term, and (b) takes into account the $S$ wave $K\pi$ FSI with coupled channels. We are interested in the region below 1.2 GeV. In this region, only one resonance, $K^*0(982)$, can decay into $K^+\pi^-$. However, the branching ratio of $J/\psi \to K^*0(892)K^*0(892)$ is very small. As a simple consideration, no intermediate resonances will be considered.
FIG. 5: Diagrams for the decay $J/\psi \to \bar{K}^0(892)K^+\pi^-$.

Similar to Refs. [33, 35], the contact terms of the vertex $J/\psi V PP$ can be extracted from a Lagrangian of the form

$$\mathcal{L}_c = g\psi_\mu \langle V^\mu \Phi \Phi \rangle,$$

where $\psi_\mu$, $V^\mu$ and $\Phi$ are the fields of $J/\psi$, vector and pseudoscalar octets respectively. $V$ and $\Phi$ are both $3 \times 3$ matrices with

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{2}} \omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{2}} \omega & K^{*0} \\ K^{*-} & K^{*0} & \phi \end{pmatrix},$$

where ideal mixing between $\phi$ and $\omega$ is assumed, and $\Phi$ has been given in Eq. (2). The relevant terms in Eq. (35) can be written as

$$g\psi_\mu \bar{K}^{*0\mu}(K^+\pi^- - \frac{1}{\sqrt{2}}K^0\pi^0 - \frac{1}{\sqrt{6}}K^0\eta).$$

Let us denote the momenta of $J/\psi$, $K^+$, $\pi^-$ and $\bar{K}^0(892)$ with $p$, $p_1$, $p_2$ and $p_3$, respectively. From Eq. (37), the amplitude for Fig. 5(a) is of $S$ wave, and it is

$$t_0 = -g\varepsilon^{(\lambda)}_\mu(p)\varepsilon^{(\lambda)}(p_3),$$

where $\varepsilon^{(\lambda)}_\mu(p)$ and $\varepsilon^{(\lambda)}(p_3)$ are the polarization vectors of $J/\psi$ and $\bar{K}^0$.

From the preceding sections, the $S$ wave $I = 1/2 K\pi$ phase shift data below 1.2 GeV can be fitted well using two coupled channels. The coupled channel $S$ wave $K\pi$ FSI can be taken into account using the formalism. After that, the amplitude corresponding to Fig. 5(a) and (b) is

$$t_c = -g\varepsilon^{(\lambda)}_\mu(p)\varepsilon^{(\lambda)}(p_3)F(s_{12})$$

$$= -g\varepsilon^{(\lambda)}_\mu(p)\varepsilon^{(\lambda)}(p_3)(1 + G_{11}(K^+\pi^-|t|K^+\pi^-)$$

$$- \frac{1}{\sqrt{2}}G_{11}(K^+\pi^-|t|K^0\pi^0) - \frac{1}{\sqrt{6}}G_{22}(K^+\pi^-|t|K^0\eta)),$$
where \( \langle K^+\pi^-|t|P_1P_2 \rangle \) (\( P_1P_2 = K^+\pi^- \), etc.) represents the full amplitude of the process \( P_1P_2 \rightarrow K^+\pi^- \) calculated in the ChUT, and \( s_{12} = (p_1 + p_2)^2 = m_{12}^2 \). Because the isospin of \( J/\psi \) is 0 and of \( \bar{K}^*0 \) is 1/2, the FSI amplitudes must be of isospin 1/2. These amplitudes can be related to the amplitudes in the isospin basis as

\[
\langle K^+\pi^-|t|K^+\pi^- \rangle^{I=1/2} = \frac{2}{3} T_{11}^{1/2}(s),
\]

\[
\langle K^+\pi^-|t|K^0\pi^0 \rangle^{I=1/2} = -\frac{\sqrt{2}}{3} T_{11}^{1/2}(s),
\]

\[
\langle K^+\pi^-|t|K^0\eta \rangle^{I=1/2} = -\sqrt{\frac{2}{3}} T_{12}^{1/2}(s),
\]

where \( T_{ij}^{(1/2)}(s) \) is the full amplitude of the channel \( j \rightarrow i \) (\( i = 1 : K\pi, i = 2 : K\eta \)).

The invariant mass spectrum of \( K^+\pi^- \) can be calculated using the formula

\[
\frac{d\Gamma}{dm_{12}} = \frac{1}{(2\pi)^3} \frac{1}{16M^2} \sum_\lambda \sum_{\lambda'} \int |t_{\lambda}|^2 |\mathbf{p}_{1\lambda'}^*| |\mathbf{p}_{3\lambda}^*| d\Omega_{1\lambda'} d\Omega_3,
\]

where \( \sum_\lambda \sum_{\lambda'} \) describes the average over initial states and the sum over final states, \( (|\mathbf{p}_{1\lambda}^*|, \Omega_{1\lambda}^*) \) is the momentum of \( K^+ \) in the c.m. frame of \( K^+\pi^- \), and \( (|\mathbf{p}_{3\lambda}|, \Omega_3) \) is the momentum of \( \bar{K}^*0(892) \) in the rest frame of \( J/\psi \).

On the other hand, because the experimental data are without sideband subtraction, we will consider the sequential process \( J/\psi \rightarrow K^*0(892)K^-\pi^+ \rightarrow K^+\pi^-K^-\pi^+ \) as a sideband background.

For the process \( J/\psi \rightarrow K^*0K^-\pi^+ \), one can write an effective amplitude like the one in Eq. \((39)\),

\[
t_e = -g_\varepsilon^{(\lambda)}(p)\varepsilon^{(\lambda')}(p_{K^*0})F(s_{34}),
\]

where \( s_{34} = (p_{K^-} + p_{\pi^+})^2 = m_{34}^2 \).

The amplitude of the \( P \) wave decay \( K^*0(892) \rightarrow K^+\pi^- \) can be written as

\[
t_{K^*0K^+\pi^-} = g_{K^*K\pi} \varepsilon^{(\lambda'')}(p_4) \cdot (p_2 - p_1).
\]

From the width of \( K^*0(892) \), the dimensionless coupling constant can be determined to be \( g_{K^*K\pi} = 4.61 \). Now one can write the amplitude for the sideband background process as

\[
t_s = -gg_{K^*K\pi} F(s_{34})\varepsilon^{(\lambda)}(p) \frac{-g^{\mu\nu} + (p_1 + p_2)^{\mu}(p_1 + p_2)^{\nu}/s_{12}}{s_{12} - m_{K^*}^2 + i\sqrt{s_{12}}\Gamma_{K^*}(s_{12})} (p_2 - p_1)_\nu,
\]
where $\Gamma_{K^*(s_{12})}$ is the momentum-dependent width of $K^{*0}(892)$. The relation of $\Gamma_{K^*(s_{12})}$ and the on-shell width $\Gamma_{K^*}(m_{K^*}^2) = 50.7$ MeV is

$$\Gamma_{K^*(s_{12})} = \Gamma_{K^*}(m_{K^*}^2) \frac{m_{K^*}^2 q^3}{s_{12}} q_0^3,$$

where $q$ and $q_0$ is momentum of one meson in the c.m. frame

$$q = \frac{1}{2\sqrt{s_{12}}} \sqrt{(s_{12} - (m_K + m_\pi)^2)(s_{12} - (m_K - m_\pi)^2)},$$

$$q_0 = \frac{1}{2m_{K^*}} \sqrt{(m_{K^*}^2 - (m_K + m_\pi)^2)(m_{K^*}^2 - (m_K - m_\pi)^2)}.$$

Since $K^+$ and $\pi^-$ in their c.m. frame are in $S$ wave in the direct decay process and in $P$ wave in the sideband background process, there is no interference between the amplitudes $t_c$ and $t_b$. Hence the invariant mass of $K^+\pi^-$ can be calculated using the formula

$$\frac{d\Gamma}{dm_{K\pi}} = \frac{1}{(2\pi)^5} \frac{1}{16 M^2} \sum_{\lambda} \sum_{\lambda'} \int |t_c|^2 |p_1^*||p_3|d\Omega_1^*d\Omega_3$$

$$+ \frac{1}{(2\pi)^8} \frac{1}{32 M^2} \int \int dm_{34} \sum_{\lambda} \sum_{\lambda'} |t_s|^2 |p_1^*||p_3'|d\Omega_1^*d\Omega_3^*d\Omega_{12}$$

where $\sum_{\lambda} \sum_{\lambda'}$ describes the average over initial states and the sum over final states, $\langle |p_1^*|, \Omega_1^* \rangle$ is the momentum of $K^+$ in the c.m. frame of $K^+\pi^-$, $(|p_3|, \Omega_3)$ is the momentum of $K^{*0}(892)$ in the rest frame of $J/\psi$, and $(|p_3'|, \Omega_3')$ is the momentum of $K^-$ in the c.m. frame of $K^-\pi^+$. In the integral $\int dm_{34}$ in Eq. (50), $a = 0.812$ GeV and $b = 0.972$ GeV according to the experimental events selection criterion $0.812$ GeV $< m_{K^-\pi^+} < 0.972$ GeV.

There is only one parameter $g$. The program package MINUIT is used to make a fit and find the most appropriate value of the parameter. The BES data are taken from Ref. The results are plotted in Fig. and the fit is good qualitatively. The solid line represents the fit result, the dash-dotted and dashed lines represent the contributions of the direct term and the direct term + the $S$ wave $K\pi$ FSI, respectively. The dotted line represents the background or sideband contribution coming from $J/\psi \rightarrow K^{*0}K^-\pi^+ \rightarrow K^+\pi^-K^-\pi^+$ in the region $0.812$ GeV $\leq m_{K^-\pi^+} \leq 0.972$ GeV. The dash-dot-dotted line represents the contributions concerning $\kappa$ or of the FSI and the interference of the FSI and the direct term. One can see the wide bump below 1.1 GeV besides the peak of $K^{*0}(892)$ is due to $\kappa$. 
FIG. 6: The invariant mass spectrum of $K^+\pi^-$ below 1.2 GeV of the decay $J/\psi \rightarrow \bar{K}^*_{0}(892)K^+\pi^-$. The black dots with error bars are the experimental data. The solid line represents the fit result, the dash-dotted, dashed and dotted lines represent the contributions of the direct term, the direct term + the $S$ wave $K\pi$ FSI and the sideband background process $J/\psi \rightarrow K^{*0}(892)K^-\pi^+ \rightarrow K^+\pi^-K^-\pi^+$, respectively. The dash-dot-dotted line represents the total contributions of the FSI and the interference of the FSI and the direct term.

A decay amplitude can be expressed as a sum of a background and some Breit-Wigner amplitudes associated to the relevant resonances. From the experimental data plotted in Fig. 6 one can see that there is a wide bump below 1.1 GeV besides the peak of $K^*_{0}(892)$. Were there no $\kappa$ and were the background of no structure and just like a form of Eq. (38), this pump would be hardly understood. From our results plotted in Fig. 6 the wide bump is generated by the $S$ wave $K\pi$ FSI. Note that in our ChUT approach, the scalar particle $\kappa$ is generated dynamically by the $K\pi$ FSI. That is to see, the bump is generated by $\kappa$.

The value of the parameter determined from fitting to the spectrum of the number of events is $g = 1770.3$. However, it is not physical, and so far we cannot determine its physical value because the branching ratio $B(J/\psi \rightarrow \bar{K}^*_{0}(892)K^+\pi^-)$ has not been known yet.

VI. CONCLUSIONS

We have used a chiral unitary approach to study $S$ wave $K\pi$ scattering and gotten a satisfactory description of phase shift data below 1.2 GeV. On-shell BSE is used, and
the lowest order ChPT amplitudes are used as its kernel. One-channel, two-channel and three-channel calculations are performed for the $I = 1/2$ $K\pi$ scattering, and one-channel for $I = 3/2$. There is only one parameter $a(m_K)$ for each isospin in the model, and its value is determined from fitting to the data. A pole of the $I = 1/2$ $T$ matrix is found in the second Riemann sheet. We investigate the pole position with different number of channels with both cut-off and dimensional regularization schemes, and find that the pole position is stable against different regularization schemes. If we take into account some higher order corrections, the real and imaginary parts of the pole are both slightly enhanced. This pole corresponds to the long controversial broad resonance $\kappa$. In this way, the mass and width of $\kappa$ are estimated to be about $757 \pm 33$ MeV and $558 \pm 82$ MeV. It is pointed out that there would be a singularity in the loop function if the conventional cut-off method were used. The singularity comes from that the three-momentum cut-off set a limit for the applicable region.

One advantage of this approach is that there is only one parameter. Once this parameter is determined from the phase shift data, it can be used to other processes such as heavy particle decays involving $S$ wave $K\pi$ or $K\eta$ FSI. In recent years, there are other excellent works on $K\pi$ scattering, such as Refs. [26, 30]. In Ref. [26], the resonance chiral Lagrangian together with a unitarization was used, and the data of $S$ wave $K\pi$ scattering below 2 GeV can be fitted well. However, the number of free parameters is large. The pole position corresponding to the $\kappa$ is about $700 - i300$ MeV. It is similar to the one found in our work. The difference of the mass is large slightly. However, the width of the resonance is about 600 MeV, and this makes a small difference of the mass and width is of little importance. In Ref. [30], the authors solved a set of six RS equations for the $S$ and $P$ waves of the $K\pi$ scattering amplitudes. The input was high-statistical data at $\sqrt{s} \gtrsim 1$ GeV for both $\pi K \to \pi K$ and $\pi \pi \to K\bar{K}$ amplitudes, then the amplitudes near threshold were determined. Certainly, this is an important work, and we compared our results for $S$ wave $K\pi$ scattering phase shifts below 1 GeV with the results in this work. From comparison, we find that our results are consistent with the precise results in Ref. [30]. Furthermore, the one-parameter model can be used easily to deal with $S$ wave $K\pi$ FSI appeared in other processes below $m_{K\pi} = 1.2$ GeV. As an example, we calculated the invariant mass spectrum of $K^+\pi^-$ of the decay $J/\psi \to K^{*0}(892)K^+\pi^-$, and the fit to the experimental data is qualitatively good below 1.1 GeV. From the analysis of different contributions from different processes,
we found that $\kappa$ is important to produce the wide bump below 1.1 GeV.

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[1] S. Weinberg, Physica 96A (1979) 327.
[2] J. Gasser, H. Leutwyler, Ann. Phys. (N.Y.) 158 (1984) 142.
[3] J. Gasser, H. Leutwyler, Nucl. Phys. B 250 (1985) 465.
[4] Ulf-G. Meißner, Rep. Prog. Phys. 56 (1993) 903.
[5] A. Pich, Rep. Prog. Phys. 58 (1995) 563.
[6] S. Scherer, in Advances in Nuclear Physics, Vol.27, edited by J. W. Negele and E. W. Vogt (Kluwer Academic/Plenum Publishers, New York, 2003), 277-538 [arXiv: hep-ph/0210398].
[7] J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438; (Erratum) ibid. 652 (1999) 407.
[8] J. Nieves, E. Ruiz Arriola, Phys. Lett. B 455 (1999) 30; Nucl. Phys. A 679 (2000) 57.
[9] T. N. Truong, Phys. Rev. Lett.61, (1988) 2526.
[10] J. A. Oller, E. Oset, J. R. Peláez, Phys. Rev. Lett. 80 (1998) 3452; Phys. Rev., D 59 (1999) 074001.
[11] F. Guerrero, J. A. Oller, Nucl. Phys. B 537 (1999) 459; (Erratum) ibid. B 602 (2001) 641.
[12] A. Gómez Nicola, J. R. Peláez, Phys. Rev. D 65 (2002) 054009.
[13] J. A. Oller, E. Oset, Phys. Rev. D 60 (1999) 074023.
[14] Z. G. Xiao, H. Q. Zheng, Nucl. Phys. A 695 (2001) 273.
[15] H. Q. Zheng, Z. Y. Zhou, G. Y. Qin et al., Nucl. Phys. A 733 (2004) 235.
[16] E. van Beveren, T.A. Rijken, K. Metzger, C. Dullemond, G. Rupp, J.E.Ribeiro, Z. Phys. C 30 (1986) 615.
[17] N.A. Törnqvist, M. Roos, Phys. Rev. Lett. 76 (1996) 1575.
[18] E. van Beveren, G. Rupp, Eur. Phys. J. C 10 (1999) 469.
[19] D. Aston et al., Nucl. Phys. B 296 (1988) 253.
[20] E791 Collaboration, E. M. Aitala et al., Phys. Rev. Lett. 89 (2002) 121801.
[21] N. Wu, International Symposium on Hadron Spectroscopy, Chiral Symmetry and Relativistic Description of Bound Systems, Tokyo, Japan, February 24-26, 2003.
[22] D. V. Bugg, Phys. Rept. 397 (2004) 257.
[23] BES Collaboration, M. Ablikim et al., Phys. Lett. B 633 (2006) 681.
[24] V. Bernard, N. Kaiser, Ulf-G. Meißen, Nucl. Phys. B 357 (1991) 129; ibid. B 364 (1991) 283.
[25] J. Bijnens, P. Dhonte, P. Talavera, JHEP 0405 (2004) 036; http://www.thep.lu.se/~bijnens/chpt.html.
[26] M. Jamin, J. A. Oller, A. Pich, Nucl. Phys. B 587 (2000) 331.
[27] S. Ishida, M. Ishida, T. Ishida, K. Takamatsu, T. Tsuru, Prog. Theor. Phys. 98 (1997) 621.
[28] E. van Beveren, G. Rupp, Eur. Phys. J. C 22 (2001) 493.
[29] Long Li, B.-S. Zou, G.-L. Li, Phys. Rev. D 67 (2003) 034025.
[30] P. Büttiker, S. Descotes-Genon, B. Moussallam, Eur. Phys. J. C 33 (2004) 409.
[31] F. J. Llanes-Estrada, E. Oset, V. Mateu, Phys. Rev. C 69 (2004) 055203.
[32] J. A. Oller, E. Oset, Nucl. Phys. A 629 (1998) 739.
[33] U.-G. Meißen, J. A. Oller, Nucl. Phys. A 679 (2001) 671.
[34] Chiangbing Li, E. Oset, M. J. Vicente Vacas, Phys. Rev. C 69 (2004) 015201.
[35] L. Roca, J. E. Palomar, E. Oset, H. C. Chiang, Nucl. Phys. A 744 (2004) 127.
[36] F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, Nucl. Phys. A 761 (2005) 269.
[37] S. Gardner, Ulf-G. Meißen, Phys. Rev. D 65 (2002) 094004.
[38] J. A. Oller, Phys. Rev. D 71 (2005) 054030.
[39] Private communication from E. Oset.
[40] M. E. Peskin, D. V. Schroeder, An Introduction to Quantum Field Theory (Addison-Wesley, New York, 1995).
[41] E. Oset, A. Ramos, C. Bennhold, Phys. Lett. B 527 (2002) 99.
[42] C. Itzykson, J. B. Zuber, Quantum Field Theory (McGraw-Hill, New York, 1980).
[43] F. James, CERN Program Library Long Writeup D506, Version 94.1, CERN Geneva, Switzerland (1998).
[44] R. Mercer et al., Nucl. Phys. B 32 (1971) 381.
[45] H. H. Bingham et al., Nucl. Phys. B 41 (1972) 1.
[46] P. Estabrooks et al., Nucl. Phys. B 133 (1978) 490.
[47] L. Roca, E. Oset, J. Singh, Phys. Rev. D 72 (2005) 014002.
[48] Particle Data Group, S. Eidelman et al., Phys. Lett. B 592 (2004) 1.