Trapping of Quantized Electrical Charge in Superfluid $^3$He-B via the Electrodynamics of Spin-orbit Coupled Systems

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Exploiting analogies between spin-orbit coupled spin superfluids and non-Abelian Yang-Mills theory, we argue that machines can be built capable of trapping a quantized amount of electric linear charge density, while the line charge quantum itself is surprisingly large. The required conditions might well hold in superfluid $^3$He-B, for which we propose an experimental realization of this phenomenon.

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The subject of spintronics in semiconductors recently received new impetus in the form of the spin Hall effect arising from spin-orbit coupling[1]. This is governed by an elegant macroscopic transport equation,

$$j^a_i = \sigma_{S,H} \epsilon_{ial} E_l$$

(1)

where $\epsilon_{ial}$ is the 3-dimensional antisymmetric tensor, $E_l$ the electrical field and $j^a_i$ the spin-current (a internal spin- and i embedding space direction). Since both $j^a_i$ and $E_l$ are even under time reversal, the transport coefficient $\sigma_{S,H}$ is also even under time reversal, indicating that it is fundamentally a dissipationless transport phenomenon. According to the present consensus, the spin-Hall effect of the semiconductors can be viewed as a genuine quasi-classical transport phenomenon[2]. This has its drawbacks since Eq. (1) does not have truly hydrodynamic status because classical spin currents are not conserved in the presence of spin orbit coupling.

Is there another, in a way deeper meaning to Eq. (1)? Like matter fluids, also spin fluids can occur in quantum-coherent incarnations as spin superfluids. The B superfluid phase of $^3$He is of this kind as it breaks spin rotational invariance[3, 4, 5]. In the presence of spin-orbit coupling, these order-parameter theories take the form of non-Abelian Yang-Mills theories with Higgs fields, albeit with the specialty that the gauge fields occur in a ‘fixed-frame’: these actually are set by the physical electromagnetic fields. The spin-Hall equation Eq. (1) acquires now a most special meaning: it is nothing else than the non-Abelian London equation ‘in the fixed frame’, the constituent equation catching the essence of the quantum hydrodynamics associated with the non-Abelian generalizations of the Meissner effect.

The relation between spin-orbit coupling and Yang-Mills gauge structures in the fixed frame is rooted in the Pauli equation[6]. This was realized by Mineev and Volovik in the context of $^3$He-B[4]. Some of the physical ramifications of these theoretical observations have been explored by Balatsky and Altshuler[7] who made predictions of Aharonov-Casher[8] spin-orbit phase interferences in $^3$He-A$\lambda$ driven by a fixed external electric field. We expand on their work and obtain the other side of the coin. We predict that the persistent currents of a spin superfluid like $^3$He-B are capable to trap a quantized charge.

We follow the experimental set-up proposed by Balatsky and Altshuler[7] and consider a cylindrical glass (or plastic) container of inner radius $R_1$ and outer radius $R_2$ filled with $^3$He-B, threaded by a metal wire of radius $a$ (Fig. 1). The outside cylindrical surface is plated with a grounded metal. The wire and the outer metal cylinder form a capacitor and a bias is switched to charge the superfluid $^3$He-B. We quickly remove the bias source. Upon attempting to discharge the wire by touching it to ground, it will not discharge: the
charge per unit length left in the wire will be \( N\lambda_0 \), where \( N \) is an integer, while the ‘fat’ elementary linear charge density quantum amounts to,

\[
\lambda_0 = \left( \frac{\hbar e^2}{4\mu} \right) \sim 3.5 \times 10^{-7} \text{Coulomb/meter}.
\]  

(2)

\( \mu \) is the magnetic moment of \(^3\)He atoms and \( c \) is the speed of light. A quantized charge is trapped! The only way to get the charge out is to heat the Helium and right at the superfluid transition, the quantized charge will be released. This quantization is due to the constructive interference of the Aharonov-Casher phase of the \(^3\)He-B order parameter, in close analogy to magnetic flux-quantization and -trapping in normal superconductors.

We first consider a hypothetical pure spin superfluid with an \( SU(2) \) spin degree of freedom – the real life case of \(^3\)He-B is qualitatively the same but complexer in detail due to the mixed spin-orbital nature of its order parameter. The starting point is the Pauli equation, containing the leading relativistic corrections, which can be casted in the form of a \( SU(2) \) Yang-Mills Schrödinger equation\[4, 8\],

\[
ihD_0 \psi = -\frac{\hbar^2}{2m_0} D_i^2 \psi + V \psi
\]  

(3)

with \( D_i = \partial_i - i \frac{q}{\hbar c} A_i^a \frac{\tau^a}{2} \), \( D_0 = \partial_0 + i \frac{q}{\hbar} A_0^a \frac{\tau^a}{2} \) where \( q \) is the magnetic moment of particle. This is true as long as one takes a particular ‘gauge-fix’ which actually amounts to identifying the Yang-Mills gauge fields with the physical electrical (E) and magnetic (B) fields,

\[
A_i^a = \epsilon_{ial} E_l, \quad A_0^a = B^a
\]  

(4)

where \( a \) and \( 0 \) denote space and time directions, respectively. The Maxwell equation \( \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \) implies \( \partial^\mu A_\mu^0 = 0 \), thus forcing the Lorentz gauge condition on the non-Abelian fields.

The important point is that spin-orbit coupling leads to parallel spin transport governed by the Yang-Mills connection. One easily infers the Ginzburg-Landau-Wilson action describing the spin superfluid order parameter dynamics. The non-Abelian order parameter is \( \Psi = q|\Psi| \) with \( q = \exp(i\varphi^a \tau^a / 2) \) and \( \varphi_a \) the non-Abelian phase. Fixing the gauge \( a \),

\[
F = |(\partial_\tau - i \frac{q}{\hbar c} A_\tau^a \tau^a)\Psi|^2 + \cdots = \rho_s (\omega^a - \frac{q}{\hbar c} A_\tau^a \tau^a)^2 + \cdots
\]  

(5)

with non-Abelian phase velocity

\[
\omega_i^a = -i Tr[g^{-1} \partial_\tau g^{\tau a}] = (1/2)\epsilon^{abc} \tilde{R}_{ba} \partial_\tau \tilde{R}_{cd}
\]  

(6)

where \( \tilde{R}_b^a (\vec{z}) \frac{s^a}{2} = e^{-i \psi^a \tau^a / 2} \frac{s^a}{2} e^{i \psi^a \tau^a / 2} \). This ‘spin’ phase velocity associated with Abelian superfluids and superconductors. The fact that in the latter the velocity can be written as the gradient of a scalar function just implies that contrary to classical fluids, quantum hydrodynamics is irrotational. For the non-Abelian fluid this is not true generally as it obeys a Mermin-Ho type relation, \( \nabla \times \vec{\omega} = \epsilon_{abc} \vec{\omega}^b \times \vec{\omega}^c \).

Lacking genuine conservation laws, the non-Abelian classical fluid is not governed by a universal hydrodynamic behavior. However, when the fluid turns quantum coherent this changes drastically: Eq. (4) is an order parameter theory as well behaved as any other and there is a true sense of quantum hydrodynamics. After all, in the full Yang-Mills theory, the phase currents are supposedly responsible for giving mass to nature through the Higgs mechanism. The non-Abelian London equation responsible for the Higgs mechanism is obtained by varying to \( \vec{\omega}^a \) in Eq. (6): \( \vec{\omega}^a = \frac{\epsilon_a}{\hbar c} \vec{A}^a \). In the spin-orbit coupled spin-superfluid \( A^a_i = \epsilon_{ial} E_l \) (the fixed frame, Eq. (4)) and we directly recover the spin Hall relation, Eq. (1)!

This reveals the great depth of the spin-Hall relation, dealing with quantum-coherent matter: the only meaningful currents in the spin superfluid are spin-Hall currents!

The ramifications of superfluid hydrodynamics have to do with quantized topological numbers. In the full \( SU(2) \) scalar Higgs Yang-Mills theory these are the ’t Hooft-Polyakov monopoles\[10\], point-like analogs of the magnetic flux lines of normal superconductors, characterized by \( \omega^a_i = A^a_i = \epsilon_a x_i / r^2 \). The ‘fixed frame’ point charge electric field is \( A^a_i = \sim \epsilon_a x_i / r^3 \). In contrast to the ’t Hooft-Polyakov monopole, it corresponds to a field strength with a wrong radial dependence for a nontrivial topology. On the other hand, the electric field associated with the wire configuration in Fig.1 is \( E_i = 2\lambda x_i / r^2 \), \( E_3 = 0 \), where \( x_{i,1} \) is \( x \) or \( y \) and \( r \) is the radial direction in the \( xy \) plane. The non-zero fixed frame gauge fields become,

\[
A_1^a = -2\lambda x_2 / r^2, \quad A_2^a = 2\lambda x_1 / r^2, \quad A_3^a = -A_2^a, \quad A_1^a = -A_3^a.
\]  

(7)

These gauge field configurations are a special case of the general \( SU(2) \) BPST-type line textures discovered by Witten in the 1970’s\[11\], obtained by choosing the gauge fix \( A_0 = A_1 = \phi_1 = 0 \) and \( \phi_2 = \lambda - 1 \). The relevant symmetry for the topological stability is \( U(1) \), allowing for stable quantized vortices. The full order parameter \( (i\tau_a A_i^a dx_i) \in SU(2) \cong S^3 \), but with the ‘wired in’ topology it becomes \( U(1) \). Indeed, if we construct a vector \( \alpha \) with elements \( \alpha_k = \epsilon_{ijk} A_j \), we obtain a \( U(1) \)-subgroup of the rotations \( SO(3) \) \( \exp(i\tau_1 \alpha_1) = \exp(\epsilon \frac{\alpha_1}{2} (x_1, x_2) \cdot (\tau_1, \tau_2)) \), i.e., \( U(1) \)-rotations about an axis.

In standard Higgs-Yang-Mills theory the above would be a sufficient condition for spin-superfluid vorticity quantization, which leads to line charge trapping. Here we are dealing with physical fields playing the role of gauge fields. Does that make a difference? Fibre bundles are topologically classified by Chern classes, which are properties of the gauge group, i.e., transition functions.
on the bundle, and not of the special connection (gauge field) or standard fibre (Higgs field) chosen. Our fixed frame fields just correspond to a particular gauge. Regardless of the fact that they are actually physical fields, they ‘impose’ the topological invariants on the matter sector as well. Hence, spin superfluid vorticity is quantized. At least for topological purposes one can rely on the spin-Hall relation and infer that the spin superflow is of the Abelianized Aharonov-Casher form, \( v_1^L = -2\lambda \frac{\delta \alpha}{\delta \rho} \), \( v_2^L = 2\lambda \frac{\delta \alpha}{\delta \rho} \) and it is a straightforward exercise to show that this leads to the quantization condition Eq. (2). This completes our proof of principle.

The central observation of Mineev and Volovik[4] is that the breaking of spin- and orbital rotational invariance\[3, 4, 5\]. We will analyze this in great detail elsewhere\[4\] and let us present here a sketch addressing a simplified version of the B phase. Let us first summarize the basics, due to Mineev and Volovik[4]. The \(3\)He-B order parameter is described by, \(3\)He-B order parameter: 

\[
A^{B}_{\alpha\beta} = \Delta_B \Phi^{\alpha\beta} R_{\alpha\beta}
\] 

where \(\Delta_B\) is the amplitude, \(\Phi\) the phase associated with number, and \(R_{\alpha\beta}(\hat{n}, \theta)\) is a matrix associated with the spin- \((S, \alpha)\) and orbital \((L, j)\) degrees of freedom such that it describes a rotation by an angle \(\theta\) about the arbitrary ordering direction \(\hat{n} \propto (\hat{L} \times \hat{S})\). This describes the breaking of spin- and orbital rotational invariance separately while the total angular invariance \(L + S\) is unbroken. The order parameter matrix is \(R_{\alpha\beta} = R^S_{\alpha\beta} R^L_{\alpha\beta}\) where \(R^S\) and \(R^L\) describe pure spin- and orbital rotations, respectively.

From the comparison with Eq. (6) it follows that one can identify a quantity which is uniquely associated with the spin-only phase velocity field \(\omega_{a\beta}\) and magnetization density \(\omega_{a}\): 

\[
\omega_{a\beta} = \frac{1}{2} \epsilon_{\alpha\beta\gamma} R_{\beta\gamma} \partial_t R_{\alpha\beta}, \quad \omega_{a} = \frac{1}{2} \epsilon_{\alpha\beta\gamma} R_{\beta\gamma} \partial_0 R_{\alpha\beta}.
\] 

The central observation of Mineev and Volovik[4] is that electrical fields as mediated by spin-orbit coupling enter the \(3\)He-B superfluid hydrodynamics exclusive by their coupling to the above spin superfluid velocity field. This is exactly equivalent to how spin orbit coupling enters the ideal spin superfluid, Eq. (5). The \(3\)He-B order parameter “Higgs” Lagrangian is 

\[
L(R_{\alpha\beta}, A_0, A) = L_{\text{kin}}(R_{\alpha\beta}) + F_{\text{grad}}(R_{\alpha\beta}) + \frac{1}{8\pi} \left( E^2 - B^2 \right)
\]

where \(L_{\text{kin}}(R_{\alpha\beta}) = -\frac{n_s \hbar^2}{2mc^2} \left( \omega^2 + \frac{4\mu}{\hbar} \nabla \cdot \vec{B} \right)\)

\[
F_{\text{grad}}(R_{\alpha\beta}) = \frac{1}{2} \rho_{\alpha\beta} \left( \omega_{\alpha\beta} \omega_{\alpha\beta} - \frac{4\mu}{\hbar^2} \omega_{\alpha\beta} \epsilon_{\alpha\beta\gamma} E_k \right)
\]

\[
\rho_{\alpha\beta} = \frac{n_s \hbar^2}{mc^2} \left[ (\epsilon_{\alpha\beta\gamma}^2 \delta_{\alpha\beta}\delta_{\gamma\delta} - (\epsilon_{\alpha\beta\gamma}^2 - \epsilon_{\alpha\beta\gamma}^2)(R_{\alpha\beta}\gamma + R_{\beta\gamma}R_{\alpha\delta}) \right].
\]

with \(n_s\) the superfluid density, \(m\) the mass of the \(3\)He atoms, \(\epsilon_{\alpha\beta\gamma}^2 = 2\) and \(\epsilon_{\alpha\beta\gamma}^2 = (1/2) \left( \epsilon_{\alpha\beta\gamma}^2 + \epsilon_{\alpha\beta\gamma}^2 \right)\), while \(c_{||}\), \(c_{\perp}\) and \(c_s\) are the longitudinal-, transversal- and average spin wave velocities.

As we show elsewhere[4], the spin wave velocity anisotropy cannot change the topological structure of the system. Hence we will neglect it suppose[3] He-B to be spin isotropic, \(\rho_{\alpha\beta} = \frac{n_s \hbar^2}{mc^2} \delta_{\alpha\beta}\delta_{\gamma\delta}\). Let us now depart from Mineev and Volovik, to find out the equations of motions associated with the Lagrangian Eq. (10). By varying it with respect to scalar- and vector potentials respectively, we obtain the pair of Maxwell equations, 

\[
\partial_\mu E_\mu = 4\pi \partial_\mu \left( \frac{2n_s \mu}{mc^2} \epsilon_{\alpha\beta\gamma} \omega_{\alpha\beta} \right)
\]

\[
(\nabla \times \vec{B})_\alpha = -4\pi \left( \nabla \times \frac{2n_s \mu}{mc^2} \omega_{\alpha} + \partial_\beta D_\alpha \right)
\]

These are just the usual Maxwell equations associated with the electric displacement \(\nabla \cdot \vec{D} = 0\) where \(\vec{D} = \vec{E} + 4\pi \vec{P}\) with 

\[
P_k = -\frac{2n_s \mu}{mc^2} \epsilon_{\alpha\beta\gamma} \omega_{\alpha\beta}.
\]

The spin current acts as a polarization for the medium because, upon Lorentz transforming, the magnetic field in the frame of the moving spin turns into the lab frame in an electric field. This is of course well known, but different from the semiconductors[12] \(3\)He is an electrically very quiet environment where the only other source of electrical fields comes from deformation of the electronic shells of the \(3\)He atoms caused by gradients of the order parameter[4]. It appears to us that by electrical measurements much can be learned about spin superflow in \(3\)He.

Let us now focus on the equations of motion of the spin superfluid associated with Eq. (10), 

\[
0 = \partial_\beta \left[ \frac{1}{2} \epsilon_{\alpha\beta\gamma} R_{\gamma\alpha} \frac{n_s \hbar^2}{mc^2} \left( \omega_{\alpha\beta} + \frac{2\mu}{\hbar^2} B_\alpha \right) \right] + \partial_t \left[ -\frac{1}{2} \epsilon_{\alpha\beta\gamma} R_{\gamma\alpha} \frac{n_s \hbar^2}{m} \left( \omega_{\alpha\beta} - \frac{2\mu}{\hbar^2} \epsilon_{\alpha\beta\gamma} E_k \right) \right] + \frac{1}{8\pi} \epsilon_{\alpha\beta\gamma} \left( \partial_t R_{\gamma\alpha} \frac{n_s \hbar^2}{mc^2} \left( \omega_{\alpha\beta} + \frac{2\mu}{\hbar^2} B_\alpha \right) \right) - \frac{1}{2} \epsilon_{\alpha\beta\gamma} \left( \partial_t R_{\gamma\alpha} \frac{n_s \hbar^2}{mc^2} \left( \omega_{\alpha\beta} - \frac{2\mu}{\hbar^2} \epsilon_{\alpha\beta\gamma} E_k \right) \right).
\]

This equation is quite a bit more involved than the simple spin-Hall relation Eq. (11) of the \(SU(2)\) superfluid.
with the genuine B-phase order parameter! The single
the gradient of the relative angle, a quantity associated
Remarkably, the spin current is now also associated with
Upon integrating Eq. (15) we obtain
4
a quantized line charge,
with Eq. (17) this implies that the wire will be left with
we have to take for the orbital rotation matrix
the spin rotation matrix
should be in the z-direction always and this implies that
the order parameter. To satisfy this constraint, the spins
that the spin current is,
This reflects the fundamental difference between B-phase
order parameter and a pure spin condensate: in the B-
phase, the order parameter is the ‘relative spin-orbital’
SO(3) while only the spin-transport is ‘gauged’ by spin-
orbital coupling. We nevertheless managed to find a solution of Eq. ’s
for the geometry in Fig. 1, demonstrating that despite these complications the B-
phase does quantize the line charge density in exactly
the same way as the SU(2) superfluid.
This solution is as follows. As before, the electrical
field will be radial ($E = \frac{2\pi}{r}\hat{r}$) and we can invoke the same
argument as we did for the SU(2) superfluid, which
becomes even simpler since the symmetry of the Higgs field
is SO(3). The cylindrical symmetry of the electrical field
may now be interpreted as an SO(2) = U(1) gauge field
directly. It carries an integer topological charge, and
as before the Higgs field will inherit this charge as well.
Hence, topologically the spin sector is vortex like. As we
will discuss in a moment, the electrical field strength is
not affected by the presence of the Helium and it follows
that the spin current is,
\[ \vec{w}_z = \frac{4\lambda\mu}{\hbar c^2 r} \hat{\phi} . \] (15)
The charge per unit length, $\lambda$, in the wire is given by the
potential difference $V_{\text{battery}} = 2\lambda\ln\frac{d_2}{a}$.
We have now to reconcile this constraint coming from
the gauge side with the relative spin-orbital nature of the
order parameter. To satisfy this constraint, the spins
should be in the z-direction always and this implies that
the spin rotation matrix $R_{a\beta}^S$ should be taken to be the
identity matrix $\delta_{a\beta}$. Since the superfluid is flowing in the
azimuthal direction, and since the spin part is diagonal,
we have to take for the orbital rotation matrix $R_{ij}^L$ and the Helium order parameter $R_{ij}$
in terms of the relative angle $\theta$. Using the above in the
definition of the spin current (Eq. 9) we find: $\vec{w}_z = -\nabla \theta$.
Remarkably, the spin current is now also associated with
the gradient of the relative angle, a quantity associated
with the genuine B-phase order parameter! The single
valuedness of the order parameter implies the quantization
condition upon going around the cylinder ($N$ is the winding number),
\[ \oint \vec{w}_z \cdot dl = 2\pi N \] , (17)
Upon integrating Eq. (15) we obtain $\frac{4\lambda\mu}{\hbar c^2}2\pi$. Together
with Eq. (17) this implies that the wire will be left with a quantized line charge,
\[ \lambda' = N \left( \frac{\hbar c^2}{4\mu} \right) . \] (18)
Therefore, if we measure the potential difference be-
tween the wire and the outside cylinder, it will be nonzero
even after shorting out. It will be
\[ V = 2N \left( \frac{\hbar c^2}{4\mu} \right) \ln \frac{R_2}{a} . \] (19)
This potential is much larger than atomic potentials
because of the smallness of the spin orbit coupling constant
$\mu/\hbar c^2$ to which it is inversely proportional. We notice
that this requires that the dimensions of the vessel should
be less than the dipolar length $\sim 10\mu m$ because at larger
distances a ‘soliton tail’ develops because $\theta$ pins to the
Leggett angle[13].
In conclusion, for quite non-trivial reasons$^3$He B-phase
mimics perfectly the charge quantization effect of the ide-
alized spin-orbit coupled spin superfluid, and we leave it
to the experts to find out if this device is technically
feasible. To stress how remarkable this effect is, let us
consider what actually happens with the electrical field
in the presence of the spin-fluid or B-phase. In anal-
ogy with normal superconductors, the spin Hall-
Eq. (11) and Maxwell (Eq. (11) equations take the role of the
London- and magnetic ($\epsilon_{a\beta}\partial_k B_c \sim J_a$) Maxwell equations. However, instead of the Meissner effect, if follows
that $(1-\text{const.})\partial_k E_k = 0$ showing that the electrical
field is not at all affected by the presence of spin-matter!
This actually makes sense: the appropriate analogy is
that the electrical charge in the wire takes the role of
magnetic flux, and the electrical field that of the vector
potential, and we encounter a quite ‘material’ version of
the ghostly action on a distance discovered by Aharonov
and Bohm. It is ‘material’ at least in the sense that the
effect lowers the free energy. For a constant electrical
field it follows from Eq. (11) that the spin currents lower
the total energy by $\Delta E = 2n_s\mu^2/mc^2E^2$.

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