Regularizing method for the analysis of CT images of welded joints of pipelines

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Abstract. The aim of the paper is to improve the accuracy of the estimation of the shape and size of cracks in the welded seams of pipelines from their X-ray tomograms. Based on the results of previous studies, the procedure of deconvolution of projection data before reconstruction is considered. The influence of the value of the regularization parameter in the deconvolution algorithm based on the Fourier transform on the quality of the tomograms is investigated. In the numerical simulations, the estimates of the optimal values of the regularization parameter for different data recording conditions are derived.

1. Introduction

One of the possible tools of monitoring the condition of engineering construction being in operation is X-ray sensing [1–3]. With this diagnostics, cracks, cavities and other internal defects can be detected and localized. Their size and shape can be estimated with a high degree of accuracy by applying tomography techniques to the sensing data [4–6].

The mathematical basis of tomographic methods is the two-dimensional Radon transform [7, 8]. This transformation matches a function of two variables \(g(x, y)\) with a set of its integrals along straight lines crossing its support. It is written as follows:

\[
f(p, \varphi) = \int_{-\infty}^{\infty} g\left(\sqrt{p^2 + l^2}, \varphi + \arctan\left(\frac{l}{p}\right)\right)dl,
\]

where \(f(p, \varphi)\) is the value of the integral from \(g(x, y)\) along the line, which is specified by the parameters \(p\) and \(\varphi\) – which are the distance to the origin and the angle of inclination of the normal to the abscissa axis, see Fig. 1, on the left. The left part of the expression (1.1) in tomography applications is usually called projection data from \(g(x, y)\), and the function \(f_\varphi(p)\) obtained from \(f(p, \varphi)\) if one fixes the angle \(\varphi\), is a parallel projection.

Suppose that the function \(g(x, y)\) describes the X-ray attenuation coefficient distribution in the weld cross section under study. Under certain assumptions about the propagation of X-rays in a substance, it can be shown that the intensity of the radiation measured by the detector at point \(D\) from the source at point \(S\) is shown in Fig. 1, left, given by the formula:
\[ I = I_0 \exp \left( - \int_{S}^{D} g(l) \, dl \right), \quad (1.2) \]

where \( I_0 \) is source intensity. Integration is carried out on the segment connecting the source and the detector. It is easy to see that the expression (1.2) is reduced to the integral from along the line. Thus, as a result of the sensing of the sample, the projection data from the distribution of the X-ray attenuation coefficient are measured.

Figure 1. Left: Full set of projection data measuring system. Right: Planar registration scheme: 1 – X-ray source; 2 – object under study; 3 – detector; L – the line along which the source moves between the points \( S_1 \) and \( S_2 \).

The Radon transform (1.1) is invertible. This means that the function \( g(x, y) \) can be reconstructed (restored) from the projection data \( f(p, \phi) \). In particular, if there is a complete set of one-dimensional parallel projections (i.e. a set of projections registered by all angles), then the following inversion formula takes place:

\[ g(x, y) = \frac{1}{8\pi^2} \int_{0}^{2\pi} \int_{-\infty}^{\infty} |\phi| \hat{f}_\phi(\omega) \exp(i\omega(x\cos\phi + y\sin\phi)) \, d\omega \, d\phi. \quad (1.3) \]

In formula (1.3) and further the tilde sign above means the Fourier transform. As can be seen from (1.3), in the process of reconstruction the Fourier image of each projection is multiplied by the frequency module (this procedure is called ramp filtering). After that, the integration of the angle is performed.

In principle, a complete set of one-dimensional projections can be measured in the registration system schematically presented in Fig. 1, on the left. The radiation source and detector move synchronously in the direction indicated by the arrows, scanning the entire object. After that, the system rotates around the Z axis, and the next projection is measured at the following angle, and so on. Of course, in reality, a complete set of data cannot be obtained. However, as practice shows, to get a high-quality tomogram, it is sufficient to register projections in the range of angles from \( 0^\circ \) to \( 360^\circ \) in steps of \( 0.5^\circ \) – \( 1^\circ \). Unfortunately, in the considered case such registration scheme often cannot be implemented. Access to many types of pipelines, in particular, in heat exchange devices is limited. Therefore, the registration system cannot make a complete turnover around the pipe. Moreover, sometimes a turn in this direction is impossible.
A different scanning geometry, called planar geometry, was developed for pipeline inspection. It is presented in Fig. 1, right. The X-ray source 1 moves along the line segment \( L \), bounded by points \( S_1 \) and \( S_2 \). The radiation transmitted through object 2 is recorded by a fixed line or matrix of detectors 3. In the latter case, the distribution of the attenuation coefficient can be obtained in some volume, which significantly increases the information content of the diagnosis. But this raises the problem of scattered radiation, which distorts the projection data, which in turn leads to incorrect reconstruction results. The use of a line of detectors weakens the influence of the scattering process, but the image of only one section is reconstructed in the plane passing through the line of detectors and the source. In particular, [9] describes a portable equipment that implements the described registration scheme.

For planar geometry of projection data collection, the inversion formula (1.3) cannot be used directly. In [9] its following modification is offered:

\[
g(x, y) = \frac{1}{2\pi} \int_\infty^{S_1} \int_{-\infty}^\infty \omega f(x, y, \omega) \exp\left(i\omega \left(r_g(s-x)/y\right)\right) \mathrm{d}\omega \mathrm{d}s.
\]

(1.4)

Here it is assumed that the line \( L \) coincides with the abscissa axis; \( r_g \) is the distance between the line \( L \) and detectors, \( s \) is the current coordinate of the source. The function \( \tilde{f}_g(s, \omega) \) is the Fourier transform from the function \( f_g(x) \) representing the fan projection, i.e. the data measured by the line of detectors at a certain position of the source.

Projection data obtained by planar registration, or measured during the rotation of the recording system around the object, in the range of angles less than 180°, is called incomplete. The papers [10 – 12] are devoted to tomography from such kind of data. There special procedures are developed to improve the quality of reconstruction. The present study also relates to this area. It takes into account the features of the diagnosed object and the specifics of the measurement process.

2. Deconvolution of projection data
2.1 Overview of previous results

It is known that in the images reconstructed from incomplete data, there are characteristic distortions: the structures are stretched in the directions from which the object is probed. In particular, in the planar registration scheme presented in Fig. 1, on the right, such a stretching will be observed mainly along the \( Y \) axis. Thus, cracks perpendicular to the pipe surface will be longer in the image than they actually are. Such erroneous diagnostics can lead to the dismantling of the pipeline section suitable for operation.

![Figure 2](image)

**Figure 2.** Left: The effect of increasing the projection of a small object: 1 – source; 2 – object; 3 – ray projection; 4 – projection observed in real measurements. Right: Explanation of the method of experimental determination of the hardware function: 1 – source, 2 – object, 3 – projection, 4 – derivative of the projection.
In [9] it was shown that the stretching of the structure element is stronger the larger its size on the projection. It was also found that for small elements, this size on real projections is several times greater than the size they have on model projections calculated under the assumption that the equation (1.2) is satisfied; see Fig. 2, on the left. This anomaly is due to the imperfect properties of the recording system: the finite size of the focal spot of the X-ray source, the presence of scattered radiation, the inertia of the detectors, etc. We denote by \( f_s^{(0)}(x) \) a model fan projection, which is also referred to hereinafter ray projection. We will consider all effects related to the effect of the equipment in a linear approximation. Then the registered projection can be expressed as follows:

\[
f_s(x) = \int_{-\infty}^{\infty} f_s^{(0)}(x')K(x-x')dx' + \xi(x).
\]  

(2.1)

The first term in the right part (2.1) is a convolution of the ray projection with the so-called hardware function \( K(x) \), which is determined by the properties of the equipment used, but does not depend on the projection itself. The second term is a random function describing uncontrolled distortions – noises – always present in the results of real measurements.

If the hardware function \( K(x) \) is known, the ray projection can be approximated from (2.1). According to [13], the following estimation takes place:

\[
f_s^{(0)}(x) \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}_s(\omega) \hat{K}(\omega) + \alpha(\omega) \exp(i\omega x) d\omega.
\]  

(2.2)

Here \( \alpha \) is the regularization parameter, which is generally determined from the statistical properties of the noise component \( \xi(x) \). In practice, the function \( K(x) \) can be determined experimentally. In particular, in [9] a method was proposed, the essence of which is explained in Fig. 2 (right side). Suppose that sample 2, made of a homogeneous material, is a step, and the X-ray source 1 is located exactly above its cut. When this occurs, the projection 3 is measured. As a mathematical model of the attenuation coefficient distribution in the \( XY \) plane, we take the \( \theta \) function. Then the measured projection in the absence of noise will be determined by the equation

\[
f_s(x) = \int_{-\infty}^{\infty} \theta(x')K(x-x')dx'.
\]  

(2.3)

Differentiating the expression (2.3), and given that the generalized derivative of \( \theta \) function is \( \delta \) function we come to expression \( K(x) = df_s(x)/dx \). Thus, differentiating certain projections it is possible to obtain an estimation of the hardware function of the recording system. According to the results of [9] it turned out that it is well approximated by the Gauss function. Numerical simulations, as well as experiments carried out in [9], showed that the utilization of the deconvolution procedure to projection data reduces the error in determining the crack length by 1.5 – 3.5 times depending on its width, viewing angle, noise level and other factors.

2.2. Investigation of dependence on the value of the regularization parameter

The most important point of the theory of solving ill-posed problems is the choice of the value of the regularization parameter. For the deconvolution problem there are a number of methods for determining the value of \( \alpha \) in the formula (2.2), see [13]. Their implementation requires nontrivial a priori information on the noise power spectral density \( W(\omega) \) as well as on the analytical properties of the Fourier transform of convolution kernel \( \hat{K}(\omega) \). In the case of X-ray diagnostics, the available estimates of the statistical characteristics of noise and the hardware function of the equipment do not provide the accuracy required for the effective operation of the known methods for calculating the
regularization parameter. In addition, the solution of the convolution equation (2.1) is only a part of the problem under study, therefore, the optimal value of \( \alpha \) obtained for it may be suboptimal, or even unacceptable, in terms of reconstruction of the image of the attenuation distribution. In this situation, it seems appropriate to use numerical modeling to find the optimal values of \( \alpha \) by brute force.

The paper deals with the planar registration scheme. The two-dimensional section of the sample with a crack is modeled by a rectangular area. The sides of the rectangle are parallel to the coordinate axes, its size along the \( X \) axis is much larger than the distance \(|S_1S_2|\), see Fig. 1, right. A crack is simulated also as a rectangle with a large aspect ratio \( a:b \), located symmetrically relative to axis \( Y \). It is assumed that it starts from the surface of the material facing the source. In Fig. 4,a the model of the cross section of the sample is presented. In the selected conventional units \( a = 0.06 \), \( b = 0.002 \). The size of the shown area is \( 0.2 \times 0.2 \), the sampling of the image is \( 1025 \times 1025 \) pixels.

The error of the crack length estimation is calculated by the formula

\[
\delta_l = \left( \frac{a - a_r}{a} \right) \times 100\%,
\]

where \( a_r \) – the length of the crack in the reconstructed image, \( a \) – its real length (i.e. the length of the rectangle modeling the crack). As in previous studies, the value of \( a \) defined as the length of the axis \( Y \) segment bounded by the points separating the regions where the attenuation coefficient is higher and lower than \( g_{\text{min}} + k(g - g_{\text{min}}) \). Here \( g_{\text{min}} \) is the minimum value of attenuation coefficient in the reconstructed profile along the \( Y \) axis; \( g \) is the average value of the restored image; the coefficient \( k \) takes values from the range from 0 to 1. For the results below, \( k \) is 0.8.

The number of projections (source positions) is 500. The ray projections are convolved with the kernel

\[
K(x) = \exp \left( -x^2 \ln 2/\beta^2 \right)
\]

which effective width \( \beta \) is 0.004, i.e. twice the width of the crack. Uncorrelated centered Gaussian noise with constant variance \( \sigma^2 = \xi^2 \bar{f}^2 \) (here \( \xi \) is a positive number and \( \bar{f} \) is the average value of the projection data in the absence of noise) is added to each projection. Tomographic reconstruction was carried out according to the formula (1.4). The value \( \varphi_0 = \arctan \left( |S_1S_2|/r_0 \right) \) is taken as the angle of view of the study area. In the process of modeling the parameters \( \varphi_0 \) and \( \xi \) are varied.

![Figure 3](image)

**Figure 3.** Dependences of the \( \delta_l \) on the regularization parameter \( \alpha \). (a) Viewing angle 30°; (b) viewing angle 60°. Curves 1 - \( \xi = 0 \) (no noise); curves 2 - \( \xi = 0.01 \); curves 3 - \( \xi = 0.02 \).

In Fig. 3 the dependences of the \( \delta_l \) on the value of the regularization parameter \( \alpha \) in the formula (2.2) are presented. For Fig. 3,a the viewing angle is 30°, and for Fig. 3,b it is 60°. In Fig. 3,a,b for curves 1, 2, and 3 the values of the parameter \( \xi \), which determines noise level, are 0 (noise is missing), 0.01 and 0.02, respectively. You can see that all the curves in Fig. 3 have a
minimum. Its presence in the dependences of errors on a regularization parameter is typical for solving ill-posed problems by regularizing algorithms [13]. In the case under consideration, this is explained by the fact that for small $\alpha$, the deconvolution procedure, even in the absence of noise in the original data, generates high-frequency distortions in the estimation of ray projections, which are then amplified by the reconstruction algorithm. On the other hand, the growth of $\alpha$ not only suppresses noise, but also blurs the projections of the crack, which, as mentioned above, leads to an increase in the length of the crack in the reconstructed image. From the analysis of the results presented in Fig. 4, the following conclusion was obtained: the values of the regularization parameters providing the least standard deviation for the estimation of the ray projection data were on average 1.2 – 1.4 times lower than those that provide the least error in determining the crack length. This is explained by the fact that the reconstruction of the image from projections is also an ill-posed problem, the solution of which, generally speaking, also needs to be regularized.

![Figure 4](image)

**Figure 4.** (a) Mathematical phantom modeling a crack in the material; (b) – (f) tomograms: (b) the ray projection data; (c) projections convoluted with a Gaussian kernel; (d) – (f) results after deconvolution of the data: (d) $\alpha=9\cdot10^{-6}$; (e) $\alpha=1.4\cdot10^{-5}$; (f) $\alpha=2\cdot10^{-5}$

Examples of the obtained tomograms are shown in Fig. 4,b – f. The noise in the projection data corresponds to the parameter $\xi=0.01$. This means that the average signal to noise ratio is 100. The viewing angle is 60°. In Fig. 4,b the reconstruction result from the ray data is presented. In this case $\delta_j = 4.5\%$. The other tomograms are reconstructed from the projections convoluted with the Gaussian kernel. For the tomogram in Fig. 4,c deconvolution was not carried out. For Fig. 4,d – f data deconvolution was carried out by the formula (2.2) with the values of the parameter $\alpha$ equal to $9\cdot10^{-6}$,
\(10^5\) and \(2 \cdot 10^{-5}\), respectively. It is seen that, using deconvolution, it is possible with the correct choice of the value of the regularization parameter to significantly improve the estimation of the crack length. In particular, for Fig. 4.e \(\delta = 11.2\%\) vs \(27.4\%\) for Fig. 4.c. Thus, the error decreased by almost 2.5 times. According to the tomogram in Fig. 4.d, it can be concluded that the value \(\alpha = 9 \cdot 10^{-6}\) is underestimated: the noise level in the image is too high, which prevents a good visualization of the crack. Value \(\alpha = 2 \cdot 10^{-5}\), by contrast, seems overestimated, the cracks are too blurry, so determining its true parameters is difficult, see Fig. 4.f. However, for Fig. 4.d and Fig. 4.f, error values are \(18.1\%\) and \(18.9\%\), respectively, i.e. less than for Fig. 4.c. Thus, the utilization of the deconvolution procedure was expedient even with the nonoptimal choice of \(\alpha\).

3. Conclusion

The actual problem of determining the size of the crack in the weld by X-ray tomography was considered. In previous studies, the authors found that the size of the crack in the image exceeds its true size, and this excess is much larger than its estimate, which is obtained within the framework of the ray approximation. The presumable reason for this is the distortion of projection data by the hardware function of the recording system, which in the linear approximation can be represented by a convolution operation. To improve the estimation of the crack depth, the procedure of projection data deconvolution was used. Since deconvolution, as well as tomographic reconstruction, is an ill-posed problem, the influence of the regularization parameter value on the estimation accuracy by means of numerical modeling was investigated in this paper. The dependences of the error in determining the crack depth on the regularization parameter were obtained for different values of the parameters characterizing the registration conditions.

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