We stabilize a chip-scale Si$_3$N$_4$ phase-locked Kerr frequency comb via locking the pump laser to an independent stable high-$Q$ reference microresonator and locking the comb spacing to an external microwave oscillator. In this comb, the pump laser shift induces negligible impact on the comb spacing change. This scheme is a step towards miniaturization of the stabilized Kerr comb system as the microresonator reference can potentially be integrated on-chip. Fractional instability of the optical harmonics of the stabilized comb is limited by the microwave oscillator used for comb spacing lock below 1 s averaging time and coincides with the pump laser drift in the long term.

OCIS codes: (140.3945) Microcavities; (140.3425) Laser stabilization; (190.4380) Nonlinear optics, four-wave mixing.

http://dx.doi.org/10.1364/OL.99.099999

An optical frequency comb is a powerful tool for high precision frequency measurements as an unprecedented frequency ruler [1,2]. Optical clock and metrology, direct broadband gas sensing, atomic-molecular spectroscopy, astronomical spectrograph calibration, light detection and ranging (LIDAR), and optical communication, photonic microwave generation benefit from the frequency comb. However, such innovations mainly remain in the laboratory environment because of its size, weight, and power consumption (SWaP). Power-efficient miniaturization of the combs will expand their versatility and deliver them as a field-usable device.

Parametric four-wave mixing combs in a high $Q$ microresonator driven by a continuous wave (cw) laser is a promising solution allowing on-chip integration [3-9]. Since the first demonstration of a Kerr nonlinearity frequency comb in a micro-scale resonator, low-noise Kerr comb formation, comb space uniformity, and its stability have been intensively studied and shown performances starting to approach that of the mode-locked laser frequency combs [4,10-15]. Currently, with the development of fabrication techniques for ultralow loss resonators and dispersion engineering, research focuses on the active stabilization of the Kerr frequency comb, examined in prior mode-locked laser frequency combs. This is important for applications such as atomic clock, optical to rf frequency division, and metrology. For the full comb self-referencing stabilization, the two degree of freedoms, namely the repetition frequency ($f_r$) and the carrier envelope offset (CEO) frequency, have to be detected and controlled. The repetition frequency is generally in the microwave domain such that it can be read out by a fast photodetector but the detection of CEO frequency needs an interferometric method, either $f-2f$ or $2f-3f$, which requires an octave-spanning spectrum or broad supercontinuum generation. The interferometric detection of CEO frequency also needs high power especially considering harmonic generation efficiencies for the comb teeth at the $f-2f$ or $2f-3f$. V. Brasch et al. reported a detection and control of CEO frequency using two transfer lasers from a two-third octave Kerr comb [16] and octave-spanning spectra were directly generated from the microresonators by P. Del’Haye et al. and Y. Oakwachi et al. although the comb repetition frequencies were hundreds of GHz [8,14], somewhat harder to detect the repetition frequency directly. Although the supercontinuum can be generated by an external highly nonlinear fiber with a cavity soliton pulse with tens of GHz repetition frequencies, it requires high power amplification owing to the Kerr comb’s low pulse energy in nature [17]. In addition, the pulse amplification and nonlinear spectral broadening may induce undesirable phase noise to the comb.

Alternatively, a fully stabilized Kerr comb can be achieved by using a stable optical reference to control the second degree of freedom without detecting CEO frequency and by locking its repetition frequency to a microwave oscillator. This has been realized and obtained a fractional instability of
7×10^{-13} at 1 s integration [4] assisted by an optical frequency comb reference. Stabilizations using all optical methods such as Rb clock transitions via transfer lasers [18,19] and a probe laser [18] have also been proposed and demonstrated. In this study, we utilize a high-Q MgF$_2$ whispering gallery mode (WGM) reference microresonator to stabilize the pump laser frequency and a microwave oscillator to stabilize the repetition frequency, which simplifies the system. Using this technique, we stabilize the pump laser frequency over a wide detuning range and achieve better than 5×10^{-11} at 1 s integration, limited by the local microwave oscillator used for the comb repetition frequency control.

The WGM microresonator from OEwaves is made of single crystalline MgF$_2$ by mechanical polishing and has a 3.45 mm in radius (r) and 25 μm rim thickness (L). The quality factor was measured to be 2.4×10^{13}. To reduce its thermal expansion sensitivity, the microresonator is sandwiched by a laminating Zerodur structure which reduces the phase noise at the low Fourier frequency regime by a factor of three. After the pump laser stabilization, we measure the frequency instability of both the pump laser and Kerr comb by beating them against a fiber laser reference comb at 1602.7 nm (f$_{1s}$) and at 1557.1 nm (f$_{2s}$) respectively. The fiber comb is filtered out by a reflection grating and combined with the cw pump laser. The generated Kerr comb and the fiber laser comb are mixed and then filtered by a fiber grating filter.

**Fig. 1**. Experimental setup. The cw pump is coupled into the Si$_3$N$_4$ microring and generates a low-noise phase-locked Kerr comb. The cw pump is stabilized to the high-Q WGM resonator reference via PDH lock. Both the cw pump and comb stabilities are measured by beating them against a fiber laser comb stabilized to a 1 Hz linewidth reference laser. IM: intensity modulator, G: reflection grating, LO: local oscillator, TEC: thermo-electric cooler, FBG: fiber Bragg grating.

**Fig. 2**. Optical Kerr comb spectrum after a bandpass filter at C-band. Inset (left) is a fundamental beat frequency. A clear 18 GHz peak with signal-to-noise ratio of 50 dB at the fundamental beat frequency without any noticeable noise peak around it implying a low noise phase-locked Kerr comb state. Another inset (right) is the Si$_3$N$_4$ microring on chip.

**Fig. 3** shows the single sideband (SSB) phase noise of the stabilized pump laser by beating it against the fiber comb (f$_{1s}$). We achieve -2.5 dBc/Hz at 10 Hz offset frequency in a simple aluminum box shown in Fig. 3 inset. The higher offset frequency noise is dominated by technical noise from the pump laser and the environmental perturbations of the WGM microresonator reference. Peaks in the phase noise measurement stem from the 60 Hz electric power line noise and acoustic noise which can be further reduced by vacuum and thermal isolation of the WGM microresonator. We also calculate and plot the thermorefractive noise limit of our WGM microresonator reference. Due to the small mode volume of the WGM microresonator, the thermorefractive noise is major noise source for our WGM microresonator determining the thermal noise limit. The thermorefractive noise for the cylindrical WGM microresonator geometry is derived and described by [22].

\[
S_r(f) = \frac{\kappa_0 g_0^2 \gamma}{\rho C_v} \left[ \frac{1}{1 + \left( \frac{2 \pi f}{\gamma \nu_D} \right)^{1/2}} \right]^{1/2} 
\]

The WGM microresonator from OEwaves is made of single crystalline MgF$_2$ by mechanical polishing and has a 3.45 mm in radius (r) and 25 μm rim thickness (L). The quality factor was measured to be 2.4×10^{13}. To reduce its thermal expansion sensitivity, the microresonator is sandwiched by a laminating Zerodur structure which reduces the phase noise at the low Fourier frequency regime by a factor of three. After the pump laser stabilization, we measure the frequency instability of both the pump laser and Kerr comb by beating them against a fiber laser reference comb at 1602.7 nm (f$_{1s}$) and at 1557.1 nm (f$_{2s}$) respectively. The fiber comb is filtered out by a reflection grating and combined with the cw pump laser. The generated Kerr comb and the fiber laser comb are mixed and then filtered by a fiber grating filter.
where $k_B$ is Boltzmann constant, $\alpha_\text{r}$ is the thermorefractive coefficient of the material, $\rho$ is the material density, $C$ is the specific heat capacity, $V_m$ is the mode volume of the WGM microresonator mode, $D$ is the temperature diffusion coefficient, and $m$ is the mode order defined by $m=2\pi n/\lambda$. The stabilized laser phase noise is close to the thermodynamic noise limit at the low offset frequency regime. The values of parameters used in the calculation is $\alpha_\text{r} = 6\times10^{-7}/K$, $T = 300 K$, $\rho = 3.18 \text{ g/cm}^3$, $C = 9.2\times10^6 \text{ erg/g} \cdot \text{K}^{-1}$, $V_m = 2.62\times10^{-6} \text{ cm}^3$, $D = 7.17\times10^{-2} \text{ cm}^2 \cdot \text{s}^{-1}$, $\lambda = 1602.7 \text{ nm}$, and $n = 1.37$. The phase noise of our stabilized pump laser is close to the thermorefractive noise limit near the carrier frequency regime.

Furthermore, we fully stabilize the Kerr comb by stabilizing the comb spacing via input power control using a fiber-coupled intensity modulator into the Si$_3$N$_4$ microring resonator. The power modulation provides the comb spacing change by 81 MHz/W. We stabilize the 18 GHz comb repetition frequency to an 18 GHz local oscillator with a feedback bandwidth of 125 kHz as shown in Fig 4. We measure the Kerr comb stability at 1557.1 nm ($f_{30}$), separated by 304 mode numbers from the pump laser frequency, and both pump laser and comb repetition frequency locked, respectively, illustrated in Fig 5. We also plot the fractional instability of the stabilized cw pump laser to see the difference with the Kerr comb stability. The brown circle shows the fractional instability ($\Delta f_{30}/\nu_{\text{pump}}$) of the cw pump laser at 1602.7 nm stabilized to the WGM microresonator. This shows that the fractional instability is 2.3$\times10^{-11}$ at 1 s averaging time and goes lower below 1 s averaging time (termed the short-term in this paper), which agrees with the low phase noise measured in Fig 3. The fractional instability increases with longer averaging times implying that the uncompensated thermo-mechanical loss degrades the quality of the long term stability of the WGM microresonator. The blue diamond shows the stability ($\Delta f_{30}/\nu_{1557.1}$) of the Kerr comb at 1557.1 nm with the pump laser stabilized to the WGM resonator. The stability is improved by two orders of magnitude at 1 s averaging time, compared with the free-running comb (black squares) to 1.29$\times10^{-10}$, which is limited by the Si$_3$N$_4$ microring cavity FSR drift. Above the 1 s averaging time, the comb stability follows the stability of the pump laser.
measurement shows $4.9 \times 10^{-11} + 0.5$ implying the random walk frequency noise caused by environmental factors such as vibration, and temperature fluctuations. The instability is slightly higher than that of the local oscillator used for the comb spacing lock, which is attributed to some residual noise of the repetition frequency stabilization near the carrier frequency regime as shown in Fig. 4.

The stability is improved by 3 times in 1 s averaging time resulting from the thermal expansion sensitivity, which could allow for improved comb referenced to an ultrastable laser possessing 1 Hz linewidth. In summary, we proposed and demonstrated a fully stabilized on-chip Si$_3$N$_4$ phase-locked comb using a stable high-Q WGM microresonator reference. This result shows any coherent comb can be similarly stabilized by controlling two degrees of freedom like soliton Kerr combs. We measured the stabilized pump laser phase noise and the Kerr comb stability by heterodyne beating the pump laser and a Kerr comb tooth against a mode-locked fiber laser, respectively. The stability is improved by 3 times in 1 s averaging time resulting from the thermal expansion sensitivity, which could allow for improved comb referenced to an ultrastable laser possessing 1 Hz linewidth.

In summary, we proposed and demonstrated a fully stabilized on-chip Si$_3$N$_4$ phase-locked comb using a stable high-Q WGM microresonator reference. This result shows any coherent comb can be similarly stabilized by controlling two degrees of freedom like soliton Kerr combs. We measured the stabilized pump laser phase noise and the Kerr comb stability by heterodyne beating the pump laser and a Kerr comb tooth against a mode-locked fiber laser comb referenced to an ultrastable laser possessing 1 Hz linewidth and 0.1 Hz/s drift. When we stabilized the pump laser to the WGM microresonator reference, the laser shows a phase noise of $-2.5$ dBc/Hz at 10 Hz and the Kerr comb stability at 1557.1 nm, separated by 304 mode numbers from the pump laser, is improved by two orders of magnitude in 1 s averaging time, to $1.29 \times 10^{-10}$ and $0.1$ Hz/s drift. When both pump laser and comb repetition frequency are stabilized simultaneously, the short-term stability is limited by the FSR drift and the pump laser frequency change (17 kHz/MHz at 1557.1 nm) is insignificant in the pump laser stabilization scheme. 

When both pump laser and comb repetition frequency are stabilized simultaneously, the short-term stability is limited by the FSR drift and the pump laser frequency change (17 kHz/MHz at 1557.1 nm) is insignificant in the pump laser stabilization scheme. When we stabilized the pump laser and comb repetition frequency are stabilized simultaneously, the short-term stability is limited by the FSR drift and the pump laser frequency change (17 kHz/MHz at 1557.1 nm) is insignificant in the pump laser stabilization scheme. When both pump laser and comb repetition frequency are stabilized simultaneously, the short-term stability is limited by the FSR drift and the pump laser frequency change (17 kHz/MHz at 1557.1 nm) is insignificant in the pump laser stabilization scheme.

When both pump laser and comb repetition frequency are stabilized simultaneously, the short-term stability is limited by the FSR drift and the pump laser frequency change (17 kHz/MHz at 1557.1 nm) is insignificant in the pump laser stabilization scheme.

Funding and acknowledgement. The authors acknowledge the funding support from the DARPA Direct On-Chip Digital Optical Synthesizer (DODOS) program under Dr. Robert Lutwark with contract HR0011-15-2-0014, the Air Force Young Investigator award (FA9550-15-1-0081 to S.W.H.), and the Office of Naval Research (N00014-14-1-0041). The authors acknowledge discussions with Jinghui Yang, Yongnan Li, and Hao Liu.

References

1. R. Holzwarth, T. Udem, T. W. Hansch, J. C. Knight, W. J. Wadsworth, and P. S. J. Russell, Phys. Rev. Lett. 85, 2264 (2000).
2. S. A. Diddams, D. J. Jones, J. Ye, S. T. Cundiff, and J. L. Hall, Phys. Rev. Lett. 84, 5102 (2000).
3. A. A. Savchenkov, A. B. Matsko, V. S. Ilchenko, I. Solomatine, D. Seidel, and L. Maleki, Phy. Rev. Lett. 101, 093902 (2008).
4. P. de'Haye, O. Arcizet, A. Schliesser, R. Holzwarth, and T. J. Kippenberg, Phys. Rev. Lett. 101, 053903 (2008).
5. J. S. Levy, A. Gondarenko, M. A. Foster, A. C. Turner-Foster, A. L. Gaeta, and M. Lipson, Nat. Photon. 4, 37 (2010).
6. T. J. Kippenberg, R. Holzwarth, and S. A. Diddams, Science 332, 555 (2011).
7. T. Herr, K. Hartinger, J. Riemenbarger, C. Y. Wang, E. Gavartin, R. Holzwarth, M. L. Grodetzky, and T. J. Kippenberg, Nat. Photon. 6, 480 (2012).
8. P. De’Haye, T. Herr, E. Gavartin, M. L. Gorodetsky, R. Holzwarth, and T. J. Kippenberg, Phys. Rev. Lett. 107, 063901 (2011).
9. J. Li, H. Lee, T. Chen, and K. J. Vahala, Phys. Rev. Lett. 109, 233901 (2012).
10. P. De’Haye, K. Beha, S. B. Papp, and S. A. Diddams, Phys. Rev. Lett. 112, 043905 (2014).
11. S. –W. Huang, H. Zhou, J. Yang, J. F. McMillan, A. Matsko, M. Yu, D. –L Kwong, L. Maleki, and C. W. Wong, Phys. Rev. Lett. 114, 033901 (2015).
12. S. –W. Huang, J. Yang, I. Lim, H. Zhou, M. Yu, D. –L Kwong, and C. W. Wong, Nat. Sci. Rep. 5, 13355 (2015).
13. X. Xue, Y. Xuan, Y. Liu, P.-H. Wang, S. Chen, J. Wang, D. E. Leaird, M. Qi, and A. M. Weiner, Nat. Photon. 9, 594 (2015).
14. Y. Okawachi, K. Saha, J. S. Levy, Y. H. Wen, M. Lipson, and A. L. Gaeta, Opt. Lett. 36, 3398 (2011).
15. S. –W. Huang, J. Yang, M. Yu, B. H. McGuiter, D. –L Kwong, T. Zelevinsky, and C. W. Wong, Sci. Adv. 2, e1501489 (2016).
16. V. Brusch, E. Lucas, J. D. Jost, M. Geiselmann, and T. J. Kippenberg, arXiv: 1605.02801v1 (2016).
17. P. De’Haye, A. Coilet, T. Fortier, K. Beha, D. C. Cole, K. Y. Yang, H. Lee, K. J. Vahala, S. B. Papp and S. A. Diddams, Nat. Photon. doi:10.1038/n photon.2016.105.
18. A. A. Savchenkov, D. Eliyahu, W. Liang, V. S. Ichenko, J. Byrd, A. B. Matsko, D. Seidel, and L. Maleki, Opt. Lett. 38, 2636 (2013).
19. S. B. Papp, K. Beha, P. De’Haye, F. Quinlan, H. Lee, K. J. Vahala, and S. A. Diddams, Optica, 1, 10 (2014).
20. J. D. Jost, E. Lucas, T. Herr, C. Lecaplain, V. Brusch, M. H. P. Pfeiffer, and T. J. Kippenberg, Opt. Lett. 40, 4723 (2015).
21. R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, Appl. Phys. B 31, 97 (1983).
22. A. A. Savchenkov, A. B. Matsko, V. S. Ilchenko, N. Yu, and L. Maleki, J. Opt. Soc. Am. B 24, 2988-2997 (2007).

References

1. R. Holzwarth, T. Udem, T. W. Hansch, J. C. Knight, W. J. Wadsworth, and P. S. J. Russell, “Optical frequency synthesizer for precision spectroscopy,” Phys. Rev. Lett. 85, 2264 (2000).
2. S. A. Diddams, D. J. Jones, J. Ye, S. T. Cundiff, and J. L. Hall, “Direct link between microwave and optical frequencies with a 300 THz femtosecond laser comb,” Phys. Rev. Lett. 84, 5102 (2000).
3. A. A. Savchenkov, A. B. Matsko, V. S. Ichenko, I. Solomatine, D. Seidel, and L. Maleki, “Tunable optical frequency comb with a crystalline whispering gallery mode resonator,” Phy. Rev. Lett. 101, 093902 (2008).

4. P. del’Haye, O. Arcizet, A. Schliesser, R. Holzwarth, and T. J. Kippenberg, “Full stabilization of a microresonator-based optical frequency comb,” Phys. Rev. Lett. 101, 053903 (2008).

5. J. S. Levy, A. Gondarenko, M. A. Foster, A. C. Turner-Foster, A. L. Gaeta, and M. Lipson, “CMOS-compatible multiple-wavelength oscillator for on-chip optical interconnects,” Nat. Photon. 4, 37 (2010).

6. T. J. Kippenberg, R. Holzwarth, and S. A. Diddams, “Microresonator-based optical frequency combs,” Science 332, 555 (2011)

7. T. Herr, K. Hartinger, J. Riemensberger, C. Y. Wang, E. Gavartin, R. Holzwarth, M. L. Grodetsky, and T. J. Kippenberg, “Universal formation dynamics and noise of Kerr-frequency combs in microresonators,” Nat. Photon. 6, 480 (2012)

8. P. Del’Haye, T. Herr, E. Gavartin, M. L. Gorodetsky, R. Holzwarth, and T. J. Kippenberg, “Octave Spanning Tunable Frequency Comb from a Microresonator,” Phys. Rev. Lett. 107, 063901 (2011)

9. J. Li, H. Lee, T. Chen, and K. J. Vahala, “Low-pump-power, low-phase-noise, and microwave to millimeter-wave repetition rate operation in microcombs,” Phy. Rev. Lett. 109, 233901 (2012)

10. P. Del’Haye, K. Beha, S. B. Papp, and S. A. Diddams, “Self-Injection Locking and Phase-Locked States in Microresonator-Based Optical Frequency Combs,” Phys. Rev. Lett. 112, 043905 (2014)

11. S. –W. Huang, H. Zhou, J. Yang, J. F. McMillan, A. Matsko, M. Yu, D. –L Kwong, L. Maleki, and C. W. Wong, “Mode-Locked Ultrashort Pulse Generation from On-Chip Normal Dispersion Microresonators,” Phys. Rev. Lett. 114, 053901 (2015).

12. S. –W. Huang, J. Yang, J. Lim, H. Zhou, M. Yu, D. –L Kwong, and C. W. Wong, “A low-phase-noise 18 GHz Kerr frequency microcomb phase-locked over 65 THz,” Nat. Sci. Rep. 5, 13355 (2015)

13. X. Xue, Y. Xuan, Y. Liu, P.-H. Wang, S. Chen, J. Wang, D. E. Leaird, M. Qi, and A. M. Weiner, “Mode-locked dark pulse Kerr combs in normal-dispersion microresonators,” Nat. Photon. 9, 594 (2015).

14. Y. Okawachi, K. Saha, J. S. Levy, Y. H. Wen, M. Lipson, and A. L. Gaeta, “Octave-spanning frequency comb generation in a silicon nitride chip,” Opt. Lett. 36, 3398 (2011)

15. S. –W. Huang, J. Yang, M. Yu, B. H. McGuyer, D. –L Kwong, T. Zelevinsky, and C. W. Wong, “A broadband chip-scale optical frequency synthesizer at 2.7 × 10^{-16} relative uncertainty,” Sci. Adv. 2, e1501489 (2016).

16. V. Brasch, E. Lucas, J. D. Jost, M. Geiselmann, and T. J. Kippenberg, “Self-referencing of an on-chip soliton Kerr frequency comb without external broadening,” arXiv: 1605.02801v1 (2016)

17. P. Del’Haye, A. Coilet, T. Fortier, K. Beha, D. C. Cole, K. Y. Yang, H. Lee, K. J. Vahala, S. B. Papp and S. A. Diddams, “Phase-coherent microwave-to-optical link with a self-referenced microcomb,” Nat. Photon. doi:10.1038/nphoton.2016.105

18. A. A. Savchenkov, D. Eliyahu, W. Liang, V. S. lichenko, J. Byrd, A. B. Matsko, D. Seidel, and L. Maleki, “Stabilization of a Kerr frequency comb oscillator,” Opt. Lett. 38, 2636 (2013)

19. S. B. Papp, K. Beha, P. Del’Haye, F. Quinlan, H. Lee, K. J. Vahala, and S. A. Diddams, “Microresonator frequency comb optical clock,” Optica, 1, 10 (2014)

20. J. D. Jost, E. Lucas, T. Herr, C. Lecaplain, V. Brasch, M. H. P. Pfeiffer, and T. J. Kippenberg, “All-optical stabilization of a soliton frequency comb in a crystalline microresonator,” Opt. Lett. 40, 4723 (2015)

21. R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, “Laser phase and frequency stabilization using an optical resonator,” Appl. Phys. B 31, 97 (1983)

22. A. A. Savchenkov, A. B. Matsko, V. S. lichenko, N. Yu, and L. Maleki, “Whispering-gallery-mode resonators as frequency references. II. Stabilization,” J. Opt. Soc. Am. B. 24, 2988-2997 (2007)