The CKMT model for the nucleon structure function $F_2$ is in good agreement with the HERA data at low and moderate $Q^2$. The fit to the same data obtained with a modified version of the model in which a logarithmic dependence on $Q^2$ has been included is also discussed. Finally, we show how the parametrization of the CKMT model for the nucleon structure function $F_2$ describes the HERA data when presented in the Caldwell-plot as the behavior of the logarithmic slopes of $F_2$ vs $x$ and $Q^2$.

1 The CKMT model

The CKMT model for the parametrization of the nucleon structure function $F_2$ is a theoretical model based on Regge theory which provides a consistent formulation of this function in the region of low $Q^2$, and can be used as a safe and theoretically justified initial condition in the perturbative QCD evolution equation to obtain the structure function at larger values of $Q^2$.

The CKMT model proposes for the nucleon structure functions

$$F_2(x,Q^2) = F_S(x,Q^2) + F_{NS}(x,Q^2),$$

the following parametrization of its two terms in the region of small and moderate $Q^2$. For the singlet term, corresponding to the Pomeron contribution:

$$F_S(x,Q^2) = A \cdot x^{-\Delta(Q^2)} \cdot (1 - x)^{\alpha(Q^2)+4} \cdot \left(\frac{Q^2}{Q^2 + a}\right)^{1+\Delta(Q^2)}.$$

$a$ Contribution to the Proceedings of the XVII Autumn School on QCD: Perturbative or Non-perturbative?, 29 September-4 October 1999, IST, Lisbon (Portugal), edited by L.S. Ferreira, P. Nogueira, and J.I. Silva-Marcos, World Scientific.
where the $x \to 0$ behavior is determined by an effective intercept of the Pomeron, $\Delta$, which takes into account Pomeron cuts and, therefore (and this is one of the main points of the model), it depends on $Q^2$. This dependence was parametrized as:

$$\Delta(Q^2) = \Delta_0 \cdot \left(1 + \frac{\Delta_1 \cdot Q^2}{Q^2 + \Delta_2}\right).$$  (3)

Thus, for low values of $Q^2$ (large cuts), $\Delta$ is close to the effective value found from analysis of hadronic total cross-sections ($\Delta \sim 0.08$), while for high values of $Q^2$ (small cuts), $\Delta$ takes the bare Pomeron value, $\Delta \sim 0.2-0.25$. The parametrization for the non-singlet term, which corresponds to the secondary reggeon ($f, A_2$) contribution, is:

$$F_{NS}(x, Q^2) = B \cdot x^{1-\alpha_R} \cdot (1-x)^n(Q^2) \cdot \left(\frac{Q^2}{Q^2 + b}\right)^{\alpha_R},$$  (4)

where the $x \to 0$ behavior is determined by the secondary reggeon intercept $\alpha_R$, which is in the range $\alpha_R=0.4-0.5$. The valence quark contribution can be separated into the contribution of the $u, B_u$, and $d, B_d$, valence quarks, the normalization condition for valence quarks fixing both contributions at one given value of $Q^2$ (we use $Q_0^2 = 2 GeV^2$ in our calculations). For both the singlet and the non-singlet terms, the behavior when $x \to 1$ is controlled by $n(Q^2)$, with $n(Q^2)$ being

$$n(Q^2) = \frac{3}{2} \left(1 + \frac{Q^2}{Q^2 + c}\right),$$  (5)

so that, for $Q^2=0$, the valence quark distributions have the same power, given by Regge intercepts, as in Dual Parton Model: $n(0)=\alpha_R(0)-\alpha_N(0) \sim 3/2$, and the behavior of $n(Q^2)$ for large $Q^2$ is taken to coincide with dimensional counting rules.

The total cross-section for real ($Q^2=0$) photons can be obtained from the structure function $F_2$ using the following relation:

$$\sigma_{\gamma p}^{tot}(\nu) = \left[\frac{4\pi^2\alpha_{EM}}{Q^2} \cdot F_2(x, Q^2)\right]_{Q^2=0}.$$  (6)

The proper $F_2(x, Q^2) \sim Q^2$ behavior when $Q^2 \to 0$, is fulfilled in the model due to the last factors in equations (3) and (4). Thus, the $\sigma_{\gamma p}^{tot}(\nu)$ has the following form in the CKMT model:

$$\sigma_{\gamma p}^{tot}(\nu) = 4\pi^2\alpha_{EM} \cdot \left(A \cdot a^{-\Delta_0} \cdot (2m\nu)^{\Delta_0} + (B_u + B_d) \cdot b^{-\alpha_R} \cdot (2m\nu)^{\alpha_R-1}\right).$$  (7)
The parameters were determined from a joint fit of the $\sigma_{\gamma p}^{tot}$ data and the NMC data on the proton structure function in the region $1 GeV^2 \leq Q^2 \leq 5 GeV^2$, and a very good description of the experimental data available was obtained.

2 Structure functions at high $Q^2$

The next step in this approach is to introduce the QCD evolution in the partonic distributions of the CKMT model and thus to determine the structure functions at higher values of $Q^2$. For this, the evolution equation in two loops in the $\overline{\text{MS}}$ scheme with $\Lambda = 200 MeV$ was used.

As starting point for the QCD evolution, the value $Q_0^2 = 2 GeV^2$, where the logarithmic derivative in $Q^2$ of $F_2(x, Q^2)$ in the model is very close to that obtained from the QCD evolution equation, was chosen.

The results obtained by taking into account the QCD evolution in this way are in a very good agreement with the experimental data on $F_2(x, Q^2)$ at high values of $Q^2$.

Although the fit of the NMC data was restricted to the region $1 GeV^2 \leq Q^2 \leq 5 GeV^2$, it is interesting to mention that a good fit can also be obtained with the model up to $Q^2 \sim 10 GeV^2$. This allows to start the perturbative QCD evolution at larger values of $Q^2$.

3 Description of the HERA data on $F_2$ at low $Q^2$

When the CKMT model was first used to fit the available experimental data on $F_2$, the lack of experimental data at low and moderate $Q^2$ limited the accuracy in the determination of the values of the parameters in the model. Later on, the publication of the new experimental data on $F_2$ from HERA at low and moderate $Q^2$ provided the opportunity to include in the fit of the parameters of the model experimental points from the kinematical region where the parametrization should give a good description without need of any perturbative QCD evolution.

Thus, one proceeded as one had done in the previous fit, but by adding the above mentioned experimental data on $F_2$ from H1 and ZEUS at low and moderate $Q^2$, to those from NMC and E665 collaborations, and to data on cross-sections for real photoproduction, into a global fit which allowed the test of the model in wider regions of $x$ and $Q^2$. One took as initial condition for the values of the different parameters those obtained in the previous fit. The result of the new common fit to $\sigma_{\gamma p}^{tot}$ and $F_2$ is presented in figures and the final values of the parameters can be found in Table. The quality of the description of all the experimental data provided by the CKMT model,
and, in particular, of the the new experimental data from HERA is very good, with a value of $\chi^2/d.o.f.$ for the global fit, $\chi^2/d.o.f. = 106.95/167$, where the statistical and systematic errors have been treated in quadrature, and where the relative normalization among all the experimental data sets has been taken equal to 1. Moreover, since the small-$x$ HERA experiments allowed for the first time the experimental study of the question of the interplay between soft and hard physics, the model, which basically has only power dependence on $Q^2$, was modified to include a logarithmic dependence on $Q^2$ as the one predicted asymptotically by perturbative QCD.

To include the logarithmic dependence on $Q^2$ in our model, we take into account...
account that the behavior of $F_2$ at small-$x$ in QCD is given by the singularities of the moments of the structure functions, the rightmost singularity giving the leading behavior. Thus, the following factors, which correspond to the moments of the structure functions in the language of the OPE expansion, and can be calculated by the convolution in rapidity of the hard-upper part with the soft-lower part of the leptoproduction diagram, were introduced in the expression that the CKMT model gives for $F_2$:

$$
\left( \frac{\alpha_s(Q^2_0)}{\alpha_s(Q^2)} \right)^{d_i(n_i)}, i = S, NS,
$$

(8)

where the strong coupling constant is taken as

$$
\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \cdot \log \left( \frac{Q^2 + M^2}{\Lambda^2_{QCD}} \right)},
$$

(9)

with $M \sim 1$ GeV, a hadronic mass included in $\Lambda_{QCD}$ to avoid the singularity in $\alpha_s$ when $Q^2 \rightarrow \Lambda^2_{QCD}$, $\Lambda_{QCD}$=0.2 GeV, and $\beta_0=11-\frac{2}{3}n_f$ (in the calculations a number of flavors $n_f=3$ was used). The exponents $d_S(n_S)$ and $d_{NS}(n_{NS})$ in $\alpha_s(Q^2)$ are proportional to the largest eigenvalue of the anomalous dimension matrix, and to the anomalous dimension, respectively:

$$
d_S(n_S) \sim \frac{d_0}{4(n_S - 1)} - d_1,
$$

(10)

with

$$
d_0 = \frac{48}{\beta_0}, d_1 = \frac{11 + \frac{2}{3}n_f}{\beta_0},
$$

(11)

and

$$
d_{NS}(n_{NS}) = \frac{16}{33 - 2n_f} \left( \frac{1}{2n_{NS}(n_{NS} + 1)} + \frac{3}{4} - S_1(n_{NS}) \right),
$$

(12)

with

$$
S_1(n_{NS}) = n_{NS} \cdot \sum_{k=1}^{\infty} \frac{1}{k(k + n_{NS})}.
$$

(13)

Thus, we modify $\alpha_s(Q^2)$ in the following way:

$$
F_2(x, Q^2) = \left( \frac{\alpha_s(Q^2_0)}{\alpha_s(Q^2)} \right)^{d_S(n_S)} \cdot F_S(x, Q^2) + \left( \frac{\alpha_s(Q^2_0)}{\alpha_s(Q^2)} \right)^{d_{NS}(n_{NS})} \cdot F_{NS}(x, Q^2).
$$

(14)
Figure 2: $F_2(x,Q^2)$ vs $Q^2$ (in GeV$^2$) for different values of $x$. Theoretical fits have been obtained with the CKMT model (full line) and the modified version of the CKMT model (dashed line). Experimental points at (a), from left to right, $x=0.42\cdot10^{-5}$, $x=0.44\cdot10^{-5}$, and $x=0.46\cdot10^{-5}$ (*8.); (b), from left to right, $x=0.85\cdot10^{-5}$, $x=0.84\cdot10^{-5}$, and $x=0.86\cdot10^{-5}$ (*6.); (c), from left to right, $x=0.14\cdot10^{-4}$ (*5.); (d), $x=0.5\cdot10^{-4}$ (*4.); (e), $x=0.8\cdot10^{-4}$ (*3.); (f), $x=0.2\cdot10^{-3}$ (*2.); (g), $x=0.5\cdot10^{-3}$ (*1.); (h) $x=0.52\cdot10^{-2}$ (*1.); (i) $x=0.13\cdot10^{-1}$ (*1.). Experimental points for $F_2$ are from references (3), (black circles), (4), (crosses), and (6), (black squares for the E665 points, and black diamonds for the NMC points).

The exponents of the new factors in $d_S(n_S)$ and $d_{NS}(n_{NS})$, give us the singularities in $n_i$, $i=S,NS$, of the momenta, which, as we mentioned above, control the QCD small-$x$ behavior of $F_2$. Therefore, in the CKMT model, these exponents have to be evaluated (see 2 and 4), at $n_S=1+\Delta(Q^2\rightarrow\infty)=1+\Delta_0(1+\Delta_1)$, and at $n_{NS}=\alpha_{Ri}$, respectively.

Then, this modified version of the CKMT parametrization of $F_2$ was used to repeat the fit of the same experimental data, including the HERA data on $F_2$ at small and moderate $Q^2$. As starting point for the QCD evolution, one takes the same value that $Q^2_0=2\text{ GeV}^2$ was used to fix the normalization of the valence component. The result of this second fit is also presented in figures 3 and 4, and the final values of the parameters in the model are given.
in Table 1(c).

As it can be seen in the figures 1 and 2, the quality of this second fit is also reasonable, although the value of $\chi^2/d.o.f.$ ($\chi^2/d.o.f.=453.19/167$), is now appreciably higher than in the fit obtained with the non-modified version of the CKMT model.

Table 1: Values of the parameters in the CKMT model obtained in former fits, (a), in the fit in which also the low $Q^2$ HERA data have been included, (b), and in the fit to the same data obtained with the modified version of the CKMT model in which a logarithmic dependence of $F_2$ on $Q^2$ has been taken into account, (c). All dimensional parameters are given in GeV$^2$.

The valence counting rules provide the following values of $B_u$ and $B_d$, for the proton case, when fixing their normalization at $Q^2_0=2\,\text{GeV}^2$: (a) $B_u=1.2064$, $B_d=0.1798$; (b) $B_u=1.1555$, $B_d=0.1722$; (c) $B_u=0.6862$, $B_d=0.09742$. In previous fits, (a), the parameter $\Delta_1$ had been fixed to a value $\Delta_1=2$.

| CKMT model | (a)     | (b)     | (c)     |
|------------|---------|---------|---------|
| A          | 0.1502  | 0.1301  | 0.1188  |
| $a$        | 0.2631  | 0.2628  | 0.07939 |
| $\Delta_0$ | 0.07684 | 0.09663 | 0.1019  |
| $\Delta_1$ | 2.0 (fixed) | 1.9533 | 1.2527  |
| $\Delta_2$ | 1.1170  | 1.1606  | 0.1258  |
| $c$        | 3.5489  | 3.5489 (fixed) | 3.5489 (fixed) |
| $b$        | 0.6452  | 0.3840  | 0.3194  |
| $\alpha_R$ | 0.4150  | 0.4150 (fixed) | 0.5872  |

4 Description of the Caldwell-plot

The so-called Caldwell-plot shows the logarithmic slope of the structure function $F_2$, $dF_2/d\ln Q^2$, derived from the ZEUS data, as a function of $x$, by fitting $F_2 \sim a + b \ln Q^2$ in bins of fixed $x$, using only statistical errors. This plot allows the study of the QCD scaling violations of $F_2$, and, in particular, in the small-$x$ domain now accessible at HERA, where $dF_2/d\ln Q^2$ is directly related to the gluon density, can be an useful tool to investigate down to which value of $Q^2$ the perturbative NLO DGLAP QCD predictions give a good description of the $F_2$ data. Thus, the logarithmic slope $dF_2/d\ln Q^2$ can be used to investigate the fundamental question of the interplay between the soft and the hard physics.

7
As it can be seen in reference (10), for values of \( x \) down to \( 3 \cdot 10^{-4} \), the slopes are increasing as \( x \) decreases, but at lower values of \( x \) and \( Q^2 \) the slopes decrease, what seems to indicate a deviation from the perturbative behavior of the hard regime.

Also in the reference (10) one can see how both Regge based parametrizations with a constant effective value for the Pomeron intercept, like the Donnachie-Landshoff Regge fit or the ZEUSREGGE fit, and pure perturbative NLO QCD predictions, like GRV94 NLO QCD fit or the ZEUSQCD fit, fail in describing correctly the experimental data in the whole kinematical region. While the Donnachie-Landshoff and the ZEUSREGGE fits do not describe the data for values of \( x \) larger than \( \sim 10^{-5} \), GRV and ZEUSQCD do not follow the data when one goes to values of \( x \) smaller than \( \sim 6 \cdot 10^{-5} \) (see (10) and references therein for more details on this discussion).

On the other side, the CKMT model described above, based on the Regge behavior, but with a \( Q^2 \)-dependent Pomeron intercept, describes the data in the region of low \( Q^2 \), and when taken as the initial condition for the QCD evolution equations, provides a complete description of the experimental results in the whole ranges of \( x \) and \( Q^2 \).

As a matter of fact, by using the formulae of the pure CKMT model in section 1, when \( x \) is kept fixed one can write for the CKMT model the slope \( dF_2/d\ln Q^2 \) as:

\[
\frac{dF_2(x,Q^2)}{d\ln Q^2} = F_S(x,Q^2)[\frac{\Delta_2}{Q^{2+\Delta_2}^2} (\Delta(Q^2) - \Delta_0) \ln \frac{Q^2}{2(Q^2+a)} + \frac{c}{Q^{2+c}} (n(Q^2) - \frac{3}{2}) \ln(1-x) + \frac{b\alpha_R(0)}{Q^{2+c}}]\]

\[ + F_{NS}(x,Q^2)[\frac{c}{Q^{2+c}} (n(Q^2) - \frac{3}{2}) \ln(1-x) + \frac{b\alpha_R(0)}{Q^{2+c}}], \]

that in the limit \( Q^2 \to 0 \) takes the form

\[
\frac{dF_2(x,Q^2)}{d\ln Q^2} \sim (1 + \Delta_0) F_S(x,Q^2) + \alpha_R(0) F_{NS}(x,Q^2). \]

Also, if one considers the case when \( W \) is fixed one can take \( x \sim cte \cdot Q^2 \), and
then, up to constant factors, one gets:

\[
\frac{dF_2(x, Q^2)}{dlnQ^2} = F_S(x, Q^2)[-\frac{\Delta(Q^2)}{Q^2 + \Delta_a} (\Delta(Q^2) - \Delta_0) \ln(Q^2 + a) -\Delta(Q^2) + \frac{a(1 + \Delta(Q^2))}{Q^2 + a} n(Q^2) - \frac{a}{2}) \ln(1 - Q^2) -Q^2n(Q^2) + \frac{a}{1 - Q^2} + \frac{b \alpha R(0)}{Q^{2+6}} + (1 - \alpha_R(0)) - Q^2n(Q^2)\]

(17)

Now, if one takes W fixed with \(Q^2 \sim x \to 0\), one can easily see that this equation simply reduces to:

\[
\frac{dF_2(x, Q^2)}{dlnQ^2} \sim F_2(x, Q^2).
\]

(18)

One has to note that both equations 16 and 18 are valid for any well-behaved parametrization of \(F_2\) (i.e., any parametrization fulfilling the relation in equation 3).

Taking into account the general features of the CKMT model described above, we use the pure CKMT model to describe the experimental data in the region of low \(Q^2\) \((0 < Q^2 < Q_0^2 = 2. GeV^2)\), and then we take this parametrization as the initial condition at \(Q_0^2 = 2. GeV^2\), to be used in the QCD evolution equation to obtain a description of the experiment at values of \(Q^2\) higher than \(Q_0^2 = 2. GeV^2\). We present our results in the shape of both the \(dF_2/dlnQ^2\) and the \(dlnF_2/dln(1/x)\) slopes in order to compare with the experimental data presented in the so-called Caldwell-plot.

The way we proceed to calculate \(F_2\), and the derivatives \(dF_2/dlnQ^2\) and \(dlnF_2/dln(1/x)\) is the following (see reference (12) and the appendix there for all the technical details on how the QCD evolution has been performed):

- In the region \(0 < Q^2 \leq Q_0^2 = 2. GeV^2\) we use the pure CKMT model for \(F_2\).
- For \(Q_0^2 < Q^2 \leq charm\ threshold\) we make the QCD evolution of \(F_2\) at NLO in the \(\overline{\text{MS}}\) scheme for a number of flavours \(n_f = 3\), and we take as the starting parametrization the CKMT one at \(Q_0^2 = 2. GeV^2\).
- When \(charm\ threshold < Q^2 \leq Q_0^2 = 50. GeV^2\), also the QCD evolution of \(F_2\) is implemented at NLO in the \(\overline{\text{MS}}\) scheme for a number of flavours \(n_f = 3\), using the parton distribution functions for the \(u, d, s\) quarks, and by including the charm contribution via photon-gluon fusion.
For values of $Q^2 > \bar{Q}^2$, QCD evolution is computed at NLO in the $\overline{\text{MS}}$ scheme, but now with a number of flavours $n_f = 4$, and by using the parton distribution functions for the $u, d, s$, and $c$ quarks.

One has to note that in the treatment of the charm contribution we have followed reference (13).

The results we have obtained are presented in figures 3 to 6. In figure 3 (Caldwell-plot), the slope $dF_2/d\ln Q^2$ is shown as a function of $x$, and compared with the $a + b\ln Q^2$ fit to the ZEUS $F_2$ data in bins of $x$.

![Graph showing $dF_2/d\ln Q^2$ as a function of $x$](image)

Figure 3: $dF_2/d\ln Q^2$ as a function of $x$ computed in the CKMT model (see reference (12) for details on the calculation), compared with the fit of the ZEUS $F_2$ data in bins of $x$ to the form $a + b\ln Q^2$ (see reference (10) and references therein for more details on the data and the experimental fit).

Figures 4 and 5 show the slope $d\ln F_2/d\ln(1/x)$ as a function of $Q^2$ compared to the fits $F_2 = A x^{-\lambda_{eff}}$ of the the ZEUS and H1 data, respectively. In Figure 4, as the $x$ range of the BPC95 data is restricted, also the E665 $F_2$ data were included in (10), and are now also taken into account. The interest of these figures is clear, since this slope can be interpreted as the effective $\lambda$ of
the Pomeron exchange, $\lambda_{\text{eff}} = \frac{d\ln F_2}{d\ln (1/x)}$. In the experimental fits, each $Q^2$ bin corresponds to an average value of $x$, $<x>$, calculated from the mean value of $\ln(1/x)$ weighted by the statistical errors of the corresponding $F_2$ values in that bin. Even though we can proceed as in the experimental fits, and we get a very good agreement with the data, since the estimation of $<x>$ is in some sense artificial and arbitrary, and introduces unphysical wiggles when drawing one full line connecting the different bins, we preferred to make for all the $Q^2$ bins in this figures the choice of the smallest $x$ in the data instead of considering a different $<x>$ for each $Q^2$. This choice is based on the fact that the ansatz $\lambda_{\text{eff}} = \frac{d\ln F_2}{d\ln (1/x)}$ is actually valid for small $x$, and results in a smooth curve except for the jump in the region around $Q^2 \sim 50 GeV^2$, where the evolution procedure changes (again, see reference (12) for more details).

Finally, figure 5 is the compilation of the behavior of $F_2$ as a function of $x$ for twelve different values of $Q^2$ (from $Q^2 = 0.6 GeV^2$ to $Q^2 = 17 GeV^2$),
Figure 5: $\frac{d\ln F_2}{d\ln (1/x)}$ as a function of $Q^2$ calculated in the CKMT model, and compared to the fit $F_2 = A x^{-\lambda_{eff}}$ of the H1 data (11). For details on the CKMT calculation, see reference (12).

corresponding to the values presented by the ZEUS Collaboration in reference (10).

A very good agreement with the experiment is obtained for all the $x$ and $Q^2$ values, showing that the experimental data can be described by using as initial condition for the QCD evolution equation a model of $F_2$ where the shadowing effects which are important at low values of $Q^2$ are included.

5 Conclusions

The CKMT model for the parametrization of the nucleon structure functions provides a very good description of all the available experimental data on $F_2(x, Q^2)$ at low and moderate $Q^2$, including the more recent small-$x$ HERA points. A second fit to the same data obtained with a modified version of the model in which a logarithmic dependence on $Q^2$ is included, has been also presented. Even though the quality of this second description is reasonable,
its $\chi^2/d.o.f.$ is appreciably higher than that corresponding to the fit obtained with the non-modified version of the CKMT model.

Finally, the CKMT model for $F_2$ has been used as the initial condition in the QCD-evolution equation to describe the HERA experimental data presented in the so-called Caldwell-plot, where the $x$-dependence of the logarithmic slope of the structure function, $dF_2/d\ln Q^2$, is shown for different $Q^2$ bins, and in the plot of the $Q^2$-dependence of the $\lambda_{eff}$, i.e., of the $Q^2$ behavior of the slope $d\ln F_2/d\ln(1/x)$, now for different bins of $x$. The obtained results show that the available experimental data can be described by performing the QCD evolution of a model of $F_2$ where the shadowing effects which are important at low values of $Q^2$ are included.

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References

1. A. Capella, A.B. Kaidalov, C. Merino, and J. Tran Than Van, Phys. Lett. B 337, 358 (1994).
2. A. Capella, U. Sukhatme, C.-I. Tan, and J. Tran Than Van, Phys. Rep. 236, 225 (1994).
3. C. Adloff et al (H1 Collaboration), Nucl. Phys. B 497, 3 (1997).
4. J. Breitweg et al (ZEUS Collaboration), Phys. Lett. B 407, 432 (1997).
5. A.B. Kaidalov and C. Merino, [hep-ph/9806367] and Eur. Phys. J. C 10, 153 (1999).
6. P. Amaudruz et al (New Muon Collaboration), Phys. Lett. B 259, 159 (1992). M.R. Adams et al (E665 Collaboration), FERMILAB-Pub 1995/396, and PRD 54, 3006 (1996).
7. D.O. Caldwell et al, Phys. Rev. Lett. 40, 1222 (1978). M. Derrick et al (ZEUS Collaboration), Phys. Lett. B 293, 465 (1992), and Z. Phys. C 63, 391 (1994). S. Aid et al (H1 Collaboration), Z. Phys. C 69, 27 (1995).
8. A. de Rújula et al, Phys. Rev. D 10, 1649 (1974).
9. K. Adel, F. Barreiro and F.J. Ynduráin, Nucl. Phys. B 495, 221 (1997).
10. A. Caldwell, DESY Theory Workshop, DESY, Hamburg (Germany), October 1997. J. Breitweg et al (ZEUS Collaboration), DESY-98-121, [hep-ex/9809003] and Eur. Phys. J. C 7, 609 (1999).
11. S. Aid et al (H1 Collaboration), DESY-96-039, [hep-ex/9603004], Proceedings of the XXXI Rencontres de Moriond: QCD and High Energy Hadronic Interactions, March 1996, Les Arcs (France), edited by J. Tran Thanh Van, Editions Frontières, Gif-sur-Yvette (France), 1996 M93, pages 349-355, and Nucl. Phys. B 470, 3 (1996).

12. A.B. Kaidalov, C. Merino, and D. Pertermann, to appear.

13. M. Glück, E. Reya, and A. Vogt, ZPC 67, 433 (1995).

14. L.P.A. Haakman, A.B. Kaidalov, and J.H. Koch, [hep-ph/9704203], and Eur. Phys. J. C 1, 547 (1999).
Figure 6: $F_2$ as a function of $x$ computed in the CKMT model (12) for twelve different values of $Q^2$, and compared with the following experimental data (see (10) for the experimental references): ZEUS SVX95 (black circles), H1 SVX95 (white triangles), ZEUS BPC95 (white squares), E665 (white diamonds), and ZEUS 94 (white circles). The dotted line is the theoretical result obtained with the pure CKMT model, and the solid line is the result obtained with the QCD-evoluted CKMT model.