A detailed study of guided wave propagation in a viscoelastic multilayered anisotropic plate

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Abstract. Guided waves (GW) are very attractive in nondestructive technique applications (eg. Structural Health Monitoring) because of their ability to propagate at long range. In a structure made of composite materials, their propagation is complex due to material anisotropy and to their dispersive and multi-modal nature. Interpreting measurements of GW in such a structure requires a sound grasp of their behaviour. Here, the Semi-Analytical Finite Element (SAFE) method is used for studying GW propagation in viscoelastic multilayered anisotropic plates. Beside classical post-processing techniques used to compute the displacement, dispersion and slowness curves, the Poynting vector is also obtained, allowing us to study energy propagation in complex plate structures. Then, GW propagation in multilayered viscoelastic composite (C-epoxy) plates is studied; different stacking sequences typical of those used to build aeronautical parts are considered. Phase, energy velocities and attenuation are studied for different propagation directions and frequencies. It appears that symmetries of GW behaviour are complex: the axes of symmetry depicting this behaviour do not coincide with those of stacking sequences and depend on frequency. Modes appearing above the first cut-off frequency have such a complex behaviour that they cannot be used in practical applications.

1. Introduction
Nowadays, composite materials are used to build many parts of aircrafts (structural components, wing and fuselage elements). Some can be subjected to impacts that must be both detected and characterized for operating the aircraft safely; nondestructive techniques are thus extensively used in aircraft maintenance. Structural Health Monitoring (SHM) is an alternative technique to classical non-destructive testing methods. It may be used either actively (for periodic examinations) or passively if operated while the component under examination is in-service. In the aeronautic industry, SHM could constitute a good choice of method to make the maintenance both easier and more efficient [1,2].

The present work is part of a more general study on the feasibility of SHM for composite parts using elastic guided waves led by EADS [2]. Guided waves (GW) can propagate at long range in plate-like geometries. Therefore, the number of transducers to generate or to measure such waves propagating in a plate could be limited in principle while an overall sensitivity to events (impact) or to the presence of a defect positioned arbitrarily in the structure could be ensured. Optimizing the number, type and position of permanently attached transducers to ensure full sensitivity requires...
simulation tools to deal with the variety of constituting materials, thicknesses involved etc. and to deal with the complex behaviour of GW in general (multimodal and dispersive natures) and specifically as they propagate in a multilayered viscoelastic anisotropic plate.

The present study aims at better understanding the behaviour of GW in multilayered composite plates typical of those used at EADS. At first, a simulation tool is developed; a model is first derived then validated. Then, this tool is used for a detailed study of GW behaviour in composite plates of different stacking sequences and at various frequencies. Special attention is paid for determining their degree of anisotropy as far as GW propagation is concerned.

2. Theoretical model for GW in composite plates and its implementation

There is a very large body of literature offering various theoretical or numerical methods to predict GW propagation in multilayered anisotropic plate-like structures, see [3] for a recent review. Among the methods available, the SAFE method (Semi-Analytical Finite Element, see [4] for the reference used in this paper for its specific relevance to our concerns) is rather easy to develop and to extend for addressing complex cases (propagation in guides of complex geometry and complex material structure). In plate geometry, the discretization by finite elements is limited solely to the thickness of the plate; the method is therefore very efficient computationally. A general tool is under development at CEA LIST for the simulation of NDT techniques based on the use of guided elastic waves [5], to be included in the CIVA software platform [6]. This tool uses the SAFE method for computing modes in.

2.1. Viscoelasticity of a multi-layered plate

Before deriving a SAFE model for the problem in hand, we have to decide how to model the viscoelastic behaviour of a multilayered composite plate. The viscoelastic model chosen in this study consists in writing elastic stiffness constants as complex numbers. Constants are defined for a uni-directional layer where Carbon fibres are aligned and randomly positioned in a viscous Epoxy matrix. Plates are modelled by stacking sequences of such layers (figure 1) defined by their orientation and thickness. Solid-solid boundary conditions at interfaces between layers are further assumed.

For one given layer, the real part of stiffness constants is assumed to be independent of the frequency. The imaginary part results from the use of a hysteretic model to account for losses in the layer. Other models together with this standard model are described in [7]; they could be equally used at no additional cost. Under this assumption, the imaginary part of stiffness coefficients is independent of frequency. According to the chosen sign convention, the complex-valued stiffness tensor is written

\[
\hat{C} = C + i\eta
\]

(1)

where \(C\) stands for the elastic tensor in a purely elastic medium and \(\eta\), for the viscosity tensor. The stiffness tensor of the layer is rotated by an angle \(\theta\) corresponding to the layer orientation in the stacking sequence, using a standard matrix calculation, \(R_1\) and \(R_2\) being defined in the Appendix.

\[
\hat{C}_\theta = R_1(\theta) \hat{C} R_2^{-1}(\theta)
\]

(2)

**Figure 1.** Plate geometry, constituting material and definition of notations. The stacking sequence shown for half the thickness is generally symmetrised relatively to the mid-plane, this being denoted by a subscript S. The sequence pictured here would be denoted by \([45^\circ,0^\circ,-45^\circ,90^\circ,45^\circ,0^\circ,-45^\circ,90^\circ]_S\).
2.2. SAFE method for GW propagation along an arbitrary direction in multi-layered plate

The SAFE method consists in the decomposition of elastodynamic fields which relies on the discretization of fields by finite elements in the guide cross section while the propagation in guiding directions is calculated analytically (see [4] for the treatment of cases similar to present ones).

Specific attention must be paid to deal with a multilayered structure and with anisotropic properties of plates considered here. Contrary to GW propagation in isotropic materials, the behaviour of GW in anisotropic materials depends on the direction of propagation. The angle $\beta$ shown on figure 1 stands for that direction relatively to the $z$-axis of the reference frame; it must be introduced in the SAFE system. For this, the wave number is written as

$$k = k \cos \beta z - k \sin \beta y,$$

so that the displacement $u$ at a point $(x,y,z)$ at time $t$ can be decomposed in the form

$$u(x,y,z,t) = U(x) \exp i(\omega t + k \sin \beta y - k \cos \beta z).$$

Standard formulae of the SAFE method must be slightly modified to account for the $\beta$-dependency. It is out of the scope of the present paper to give the detailed derivation of the SAFE method adapted with these modifications. This derivation is straightforward by following the various steps described in the literature (see [4] for example). Here, only specific formulae for the problem in hand are given.

The discretization is limited to the thickness of the plate ($x$-dependency) – this is the main reason for its high numerical performance –, while the propagation kernel depends on the two other space variables $y$ and $z$. The displacement at an arbitrary point in the plate is re-written as a function of the interpolation matrix $N$ (see Appendix), nodal displacements, and the propagation term. One has

$$u^{(e)}(x,y,z,t) = N d^{(e)} \exp i(\omega t + k \sin \beta y - k \cos \beta z),$$

where

$$d^{(e)} = \begin{bmatrix} d^{(e),1} \\ d^{(e),1} \\ . . . \\ d^{(e),n} \\ d^{(e),n} \\ d^{(e),n} \end{bmatrix}^{T}$$

and $d^{(e),j}$ denotes the $i$-th component of the nodal displacement at the $j$-th node of element $(e)$, the element $(e)$ involved in this formula being the element corresponding to the same $x$-value. In our implementation, finite elements considered are segments of three nodes; the interpolation matrix $N$ for this type of elements is given explicitly in the Appendix.

The strain vector $\varepsilon$ for one element is then given by

$$\varepsilon = \begin{bmatrix} B_1 - i k \frac{\partial}{\partial x} B_2 - i k \frac{\partial}{\partial y} B_3 \end{bmatrix} d^{(e)} \exp i(\omega t + k \sin \beta y - k \cos \beta z),$$

where

$$B_1 = L_x N_x, B_2 = L_y N, B_3 = L_z N.$$

The matrices $L$ being given in the Appendix. $N_x$ denotes the derivative of $N$ relatively to $x$.

Then, classical steps for obtaining the variational formulation and its discretized form are followed, as in [4]. The dependency on the direction of propagation $\beta$ makes it necessary to introduce the following stiffness matrices $K_{mn}$ (in the FE sense of the word) and the mass matrix $M$. These matrices are given here for each element along the thickness, assuming that one element does not belong to two different layers of the stacking sequence. For the element $(e)$, one has

$$k_{mn}^{(e)} = \int_{e} B_n^{T} \tilde{C}_{\theta} B_m dx \quad \text{and} \quad m^{(e)} = \int_{e} \rho N^{T} N dx.$$

The overall matrices are obtained by standard assembling techniques

$$\mathbf{K} = \bigcup_{e=1}^{n_e} k_{mn}^{(e)} \quad \text{and} \quad \mathbf{M} = \bigcup_{e=1}^{n_e} m^{(e)}.$$

Finally, a quadratic system of equations is obtained that is written...
\begin{equation}
-\omega^2 M d + [k^2 (\cos^2 \beta K_{33} + \sin^2 \beta K_{22} - \cos \beta \sin \beta K_{23}) \\
- i k (\cos \beta K_{13} - \sin \beta K_{12}) + K_{11}] d = 0.
\end{equation}

To simplify the resolution, the quadratic system is re-written as a linear system. Its eigenvalues are the mode wave numbers and its eigenvectors give corresponding modal displacement. This system is thus:

\begin{equation}
(A - kD) Q = 0,
\end{equation}

where

\begin{equation}
Q = \begin{bmatrix} d \\ kd \end{bmatrix},
\end{equation}

and

\begin{equation}
A = \begin{bmatrix} 0 & -\omega^2 M \\ K_{11} - \omega^2 M & -i(\cos \beta K_{13} - \sin \beta K_{12}) \end{bmatrix},
\end{equation}

\begin{equation}
D = \begin{bmatrix} K_{11} - \omega^2 M & 0 \\ 0 & -(\sin^2 \beta K_{22} + \cos^2 \beta K_{33} - \cos \beta \sin \beta K_{23}) \end{bmatrix}.
\end{equation}

This system explicitly depends on the angle $\beta$. Predicting the anisotropic behaviour of GW properties makes it necessary to loop on $\beta$, equation (12) being solved for each direction considered.

2.3. Energy velocity

For NDT applications operated for locating defects, the knowledge of the energy direction is crucial. In damping materials, it is well known that, due to viscosity, the standard derivative for defining the group velocity is no longer valid [8]; rather, the energy velocity must be considered which is defined as

\begin{equation}
v_e = \frac{P}{E_T}, \text{ with } E_T = E_C + E_P,
\end{equation}

where $E_T$ stands for the total energy (see Appendix for its expression). Computing the energy velocity allows us to represent the direction of energy propagation in the composite plate and to predict wave front curves. Each component of the Poynting vector is computed for one given phase velocity direction. The Poynting vector is obtained by post-processing the SAFE results; it is written [9],

\begin{equation}
P_j = \frac{1}{2} \text{Re} \ i \omega \sigma \mu_j u_j^*, \text{ } j \text{ and } k \in \{1, 2, 3\},
\end{equation}

where $\sigma$ is obtained through Hooke’s relations from the displacement computed by the SAFE method.

2.4. Convergence and validation studies

2.4.1. Convergence study

To study the convergence of the present SAFE model, wave numbers have been computed for various discretizations of the same multilayered viscoelastic plate. For example, results obtained with one element per layer were compared to results obtained with 10 elements per layer. Relative errors were computed and are in a range of $10^{-4}$ and $10^{-5}$. This accuracy was tested in a configuration typical of those of interest for the present study: overall plate thickness of a few mm and excitation frequencies less than 2 MHz were considered. More quantitative convergence studies have been conducted but these results are out of the scope of the present paper.

2.4.2. Validation studies

To validate the model, we have reproduced results of the literature obtained by means of methods which fundamentally differ from the SAFE method. Among many other authors, Wang and Yuan [10]
developed a transfer matrix method for predicting GW; they considered multilayered plates of various stacking sequences, each layer being assumed to be purely elastic. They show dispersion curves (phase and group velocities as functions of the frequency) and polar plots of slowness and group velocities as functions of the direction of propagation. Figure 2 successfully compares our predictions to published results for the slowness of propagative modes in a plate with stacking sequence of $[45°/−45°]_S$ [10].

![Figure 2](image_url)

**Figure 2.** Slownesses (polar plots) at 412.8 kHz for $[±45°/−45°]_S$ stacking sequence, of a) symmetric b) anti-symmetric modes from [10]; c) present results (S, A modes superimposed). All results at same scale.

Since our model allows us to consider viscoelastic plates, further comparisons were made with original results obtained by Neau [11] who derived an analytical formulation for GW propagation in a single viscoelastic layer. The comparisons concern now both the attenuation and the energy velocity in a unidirectional layer of Carbon fibres in Epoxy matrix. They are shown in figures 3 and 4.

Again, our predictions are in perfect agreement with Neau’s, while the two methods are radically different. Many similar comparisons have been made leading to the same conclusion of validity of our model and implementation. Future experiments will complete our validation work.
Figure 3. Attenuation polar plots (in Np/m) at 100 kHz in a unidirectional viscoelastic layer. Left: column: results in [11]. Right column: present model. Top: $S_0$ (green), $SH_0$ (blue): Bottom: $A_0$ mode.

Figure 4. $S_0$ energy velocity (polar plots) at four frequencies. Left: from [11], right: present model. Curves are displayed in coordinates $V_c \cos \psi$ and $V_c \sin \psi$ (in mm/µs), $\psi$ denoting the energy direction.
3. Detailed study in multilayered viscoelastic anisotropic plate

The present paragraph is now dedicated to studies of GW propagation in plates typical of those used in practice. Aeronautic engineers use such plates in the design of many parts of an aircraft and may specify which stacking sequence is to be used according to the required structural performance at each location.

As far as SHM applications are concerned, it is important to have an excellent grasp of GW behaviour in typical plates to be used and to be tested. Our validated model allows us to study in detail this behaviour.

A 16-layer plate is first considered (unidirectional Carbon fibre-Epoxy layer 0.133 mm-thick each). Viscoelastic properties used herein are given by (manufacturer for C, postulated from [4] for η)

Table 1. Viscoelastic properties of a Carbon-Epoxy layer (stiffness in GPa, ρ in kg.m⁻³).

| C_{11} | C_{12} | C_{13} | C_{22} | C_{23} | C_{33} | C_{44} | C_{55} | C_{66} | ρ     |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| 11.5   | 5.258  | 5.0    | 11.5   | 5.0    | 145.5  | 5.2    | 5.2    | 3.5    | 1494  |

| η_{11} | η_{12} | η_{13} | η_{22} | η_{23} | η_{33} | η_{44} | η_{55} | η_{66} |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.8    | 0.4    | 0.7    | 0.8    | 0.7    | 10     | 0.4    | 0.4    | 0.2    |

The stacking sequence of this plate is [45°/0°/135°/90°/45°/0°/135°/90°] as shown in figure 1. Structural engineers consider such sequences lead to static isotropic properties in the plate plane. Thus, one question is to know whether this property is shared by GW propagating in it. Before answering this question, it is important to study the GW behaviour for one given direction of propagation in the plate. Therefore, the first results concern the prediction of phase and energy velocities and attenuation in the reference direction 0° as functions of the frequency.

3.1. Dispersion curves for the anisotropic plate

Figure 5 shows the dispersive and multimodal behaviour of GW propagating in the 0° direction.

Several comments can be made at this stage. Below the first cut-off frequency (the frequency above which more than three modes exist), phase velocities vary much less than above this frequency. This is an interesting property since time-dependent waveforms of frequency content below the first cut-off will be less affected by dispersion. Another point is that in this frequency range, attenuation is at its...
lowest. Moreover, it varies linearly with the frequency. Above the first cut-off frequency, attenuation is strong and varies in a complicated manner with frequency. Using GW for the examination of such plates seems therefore far easier in the frequency range below the first cut-off frequency.

A new question arises from these comments: does the first cut-off frequency vary with the direction of propagation? This has been studied and the following table gives its value for four different directions. It appears that the variation is very small (less than 3 kHz, or 0.7%).

Table 2. 1st cut-off frequency for various directions of propagation.

| direction of propagation (°) | 0     | 45    | 90    | 135   |
|-----------------------------|-------|-------|-------|-------|
| 1st cut-off frequency (kHz) | 372.09| 372.24| 373.74| 374.77|

3.2. Study of wave propagation in the plate plane

It has been observed that dispersion curves depend on the direction of propagation. This effect is quantified now for every possible directions of propagation in the plate. Figure 6 displays at four frequencies (three below the first cut-off, one above) the slowness curves of the various modes.

![Figure 6](image)

For one of the three modes (in red) existing below the first cut-off frequency, the curve is almost circular. This almost isotropic mode is the symmetric $S_0$ which propagates with the smallest slowness (highest speed). The blue plot corresponds to the antisymmetric $A_0$ mode; it has the highest slowness but this slowness varies slightly with the direction of propagation. At 410 kHz, above the first cut-off frequency, new modes appear of higher speed, which are much more anisotropic.

In the presence of anisotropy, it is known that phase and energy velocities are no longer collinear. Figure 7 shows how the energy velocities vary with the direction of propagation (of energy).
At the lowest frequency considered, anisotropy is already visible for the slowest $A_0$ mode. Then, as the frequency increases, the anisotropy becomes more and more marked. Above the first cut-off frequency, higher modes are strongly anisotropic. It is noticeable that symmetries exist: curves are centro-symmetric due to the stacking sequence which makes the plate invariant by a $\pi$-rotation.

It is also interesting to measure how the symmetry varies. For this, we have determined the directions of highest slowness, of highest energy velocity and of lowest slowness, for the three modes below the first cut-off. Values are given in the following table.

Table 3. Directions (angle relative to $z$) of maximum slowness, maximum energy velocity, minimum slowness for $A_0$, $SH_0$ and $S_0$ modes below the first cut-off frequency, for various frequencies.

| f (kHz) | angle (°) of maximum slowness | A_0 | SH_0 | S_0 | angle (°) of maximum energy velocity | A_0 | SH_0 | S_0 | angle (°) of minimum slowness | A_0 | SH_0 | S_0 |
|--------|-------------------------------|-----|------|-----|-------------------------------------|-----|------|-----|--------------------------------|-----|------|-----|
| 100    | 110.8                         | 67.3| 29.3 |     | 47.5                                | 107.4| 151.5|     | 36.9                                | 107.2| 151.6|     |
| 200    | 116.2                         | 67.3| 29.2 |     | 55.1                                | 107.6| 150.4|     | 40.7                                | 107.4| 150.9|     |
| 350    | 123.0                         | 67.2| 29.0 |     | 58.9                                | 108.1| 145.9|     | 44.9                                | 107.7| 148.6|     |

Angles of maximal values of phase and energy velocity differ, due to the fact that phase and energy velocity are not aligned. Moreover, variations with frequency differ for angles of maximal and minimal values of slowness: for the $A_0$ mode, the angles of maximal slowness values vary from 110.8° to 123.0° ($\Delta_{\text{max}} = 12.2°$), whereas, for the minimal value, angles vary from 36.9° to 44.9° ($\Delta_{\text{min}} = 8.0°$).

Interestingly, the various angles considered depend on frequency and they are not aligned with fibre orientations. This latter point must be understood. For this, the particle displacement of the various modes propagating in the plate is studied in detail in the next paragraph.

3.3. Particle displacement in the thickness
This study aims at explaining why predicted anisotropy depends on frequency as observed in previous results. For this, the various components of the particle displacement of the three modes existing below the first cut-off frequency are computed and shown in figure 8 at two different frequencies (100 and 350 kHz). In both cases, the direction of phase velocity is taken in the 0° reference direction.
As commented above, the \( A_0 \) mode is the most anisotropic even at low frequency. Its \( u_\theta \) and \( u_z \) components at 100 kHz are almost constant in the thickness of the plate but its \( u_r \) component takes its highest values (anti-symmetrically) in the layers close to the outer surfaces. Interestingly, the inner multilayered structure is not perceptible in these results where the amplitude of all components varies smoothly in the thickness. At 350 kHz, this last remark is no longer true: the inner structure is clearly “seen” by the guided modes, especially for the \( A_0 \) one. The highest amplitude of \( u_r \) component is even more concentrated in the external layers. This influences directly the global anisotropy of the mode: each layer being unidirectional, it is strongly anisotropic and favours the propagation along fibres.

The lower anisotropy of the two other modes \( \text{SH}_0 \) and \( S_0 \) can be explained according to the same detailed study of amplitude variation of the various components of the particle displacement. Note again that the multilayered structure of the plate is not at all perceptible at the lowest frequency and is only slightly visible for a few components of the displacement at the highest frequency considered.

Symmetries observed in the various polar curves (slowness, energy velocity, and as it will be seen farther, attenuation) and their dependence upon the frequency clearly show that a plate made up of typical stacking sequences involving layers of \( 0^\circ, \pm 45^\circ, 90^\circ \) fibre orientations with equal partition and thickness cannot be modelled as a transversely isotropic plate in the plate plane, as far as guided waves are concerned. Moreover, even at relatively low frequencies (lower than the first cut-off frequency), the multilayered structure cannot be ignored and is actually responsible for both the global anisotropy observed and its dependency upon the frequency.

### 3.4. Comparison between two different stacking sequences

To demonstrate the importance of the inner structure of composite plates, we now compare guided wave propagation in two plates with two different stacking sequences: the same as that in previous results \([45^\circ/0^\circ/135^\circ/90^\circ/45^\circ/0^\circ/135^\circ/90^\circ]\) (16 layers) and the sequence \([45^\circ/0^\circ/135^\circ/90^\circ]\) (8 layers).

A first comparison concerns the angle of maximal slowness; the same product thickness \( x \) frequency is considered for easy comparisons. Results are shown in Table 4.

**Table 4.** Angles (°) of maximal slowness of \( A_0 \) for the 8-(left) and the 16-layer plate (right).

| Frequency (kHz) | Angle (°) | Frequency (kHz) | Angle (°) |
|----------------|----------|----------------|----------|
| 200            | 118.1    | 100            | 110.8    |
| 700            | 132.5    | 350            | 123.0    |
The maximal slowness values are significantly different for the 8-layer and 16-layer plates for the \( A_0 \) mode. Similar results were obtained for the two other modes, even if the difference is smaller. Likewise, the orientations of maximal energy velocity and minimal slowness differ in the two plates.

Now, energy velocity curves are plotted for the 8-layer sequence on Figure 9. Singularities in the form of loops appear below the 1st cut-off frequency at 700 kHz. Such singularities were not observed for the 16-layer plate at 350 kHz. The presence of loops in the energy velocity curves is a phenomenon which would make very difficult the use of such a guided mode in SHM application: it would generate complex time-dependent signals strongly varying in a limited range of propagation angles.

Above the first cut-off frequency (figure 9), new modes appearing have a particularly complex behaviour that prohibits their use in SHM applications.

![Energy velocity curves (polar plots) for the 8-layer plate at a) 200 kHz, b) 400 kHz, c) 700 kHz and d) 820 kHz.](image)

Other results for plates made of a larger number of layers have also been computed which are not shown here for conciseness. As the analysis of previous results suggested, it has been observed that for an increasing number of layers, energy velocity curves for modes below the first cut-off frequency eventually tend to become more and more isotropic (in the plate plane). At the lowest values of the frequency \( x \) thickness product, whatever the sequence considered, the energy velocities are roughly the same for a given mode in a given direction (same property for phase velocities). This is an interesting fact for practitioners.

### 3.5. Attenuation

For completeness, figures 10-11 show the variation of GW attenuation in the propagation plane. Curves are displayed in Np/m. As far as attenuation is concerned, the absolute values obtained depend on the frequency: this explains why keeping the same frequency \( x \) thickness product for computing results in the two plates leads to higher attenuations for the 8-layer thinner plate (about twice that of the 16-layer plate at half the frequencies).

Interestingly, attenuation below the first cut-off frequency is all the more anisotropic since the frequency increases; in the case of the 8-layer plate, some curves show strong concavities. At frequencies above the first cut-off frequency, attenuation curves of new modes become very complex. The report of such behaviour reinforces the conclusion already reached that the use of higher (than the fundamental ones) guided modes cannot be recommended for SHM applications.
4. Conclusions

A detailed study of elastic guided wave propagation in multilayered viscoelastic plates has been conducted. For this, simulation tools have been developed based on the well-established SAFE method which allows fast and accurate predictions; the underlying theory of the SAFE method has been given in some details specific to our implementation dealing with multilayered viscoelastic anisotropic plates.
and computing quantities of interest by post-processing eigenmode raw results. These tools have been used to depict the complex behaviour of GW in plate structures typical of some found in aircrafts.

It may be concluded that in the usual frequency range for using elastic GW in non-destructive testing, i) stacking sequences of anisotropic layers assumed to have isotropic (in the plane) static properties lead to anisotropic behaviours of GW, ii) anisotropy increases with increasing frequency, iii) the generally strong attenuation is at its lowest for modes below the first cut-off frequency and varies linearly with the frequency in this range, iv) modes appearing at frequencies above the first cut-off have such a complex behaviour that they cannot be used in practice, v) designing NDT methods based upon GW propagation for multilayered plates makes it necessary to study the detailed behaviour of these waves; vi) eventually, only appropriate simulation tools can handle the intrinsic complexity of GW in these structures and vii) the SAFE method is particularly well suited for such simulations, combining accuracy, numerical performance with easy adaptability to deal with complex cases.

In a future work, the present model will be hybridised with a FE model to compute GW scattering in multilayered plates by various discontinuities, this generalising the approach described in [5, 12].

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Appendix. Definition of terms appearing in the SAFE formulation

The matrices of rotations of an angel $\theta$ (one angle per layer), $R_1$ and $R_2$, are given by (with $c = \cos \theta$ and $s = \sin \theta$):
The matrices $L_x$, $L_y$ and $L_z$ are defined as:

$$
L_x = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix},
L_y = \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{bmatrix},
L_z = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix}
$$

(A2)

and $N$ is the interpolation matrix which is written:

$$
N(x) = \begin{pmatrix}
\varphi_1 & 0 & 0 & \varphi_2 & 0 & 0 & \varphi_3 & 0 & 0 \\
0 & \varphi_1 & 0 & 0 & \varphi_2 & 0 & 0 & \varphi_3 & 0 \\
0 & 0 & \varphi_1 & 0 & 0 & \varphi_2 & 0 & 0 & \varphi_3
\end{pmatrix}
$$

(A3)

with the following interpolation functions,

$$
\varphi_1 = 2(\xi(x) - 1)(\xi(x) - 0.5),
\varphi_2 = 4\xi(x)(1 - \xi(x)),
\varphi_3 = 2\xi(x)(\xi(x) - 0.5),
$$

(A4)

where

$$
\xi(x) = (x - x_1^{(e)}) / \Delta
$$

(A5)

and $x$ is the integration variable and $x_1^{(e)}$ denotes the coordinate of the first node of element $e$. $\Delta$ denotes the length of an element assuming a constant discretization step (not a necessary condition).

Finally, the total energy appearing in the expression of energy velocity (equation 16) is the sum of the two following terms (the detailed derivation of these terms may be found in [9], p. 52) where $h$ denotes the plate thickness:

$$
E_c = \frac{\rho \omega^2}{4h} \int_{-h/2}^{h/2} u^T u \, dx,
$$

(A6)

$$
E_p = \frac{1}{4h} \int_{-h/2}^{h/2} \varepsilon^T C \varepsilon \, dx.
$$

(A7)