The impossible regularization of the Nambu Jona-Lasinio model with vector interactions.

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Abstract

We show that the procedure of regularizing the real part of the euclidean action, while leaving the imaginary part unregularized, leads to a non-analytic and highly singular functional of the fields. It is customary to work with an imaginary time component of the vector field, in order to avoid regularization of the anomalous processes. We show that this procedure is flawed by the fact that a stationary point of the action occurs for a real, not imaginary, time component of the vector field. Furthermore the action in the vicinity of the stationary point is singular. The regularized action is thus not suitable for an evaluation of the partition function using a saddle point method. We discuss proposed solutions to this problem, as well as other regularizations. They all lead to practical problems.
1 Introduction.

There have recently been several attempts to formulate a theory of solitons composed of quarks interacting with both scalar and vector fields [1,2]. A problem has been raised in connection with the treatment of the time component of the vector field in the Euclidean action. It is convenient to work with a pure imaginary time component \(iV_4\) of the vector field, with \(V_4\) Hermitian. This yields a classical action for the soliton, which has both real and imaginary parts. The usual regularization procedure consists in regularizing the real part. The imaginary part, which is finite, is left unregularized. The reason for doing this is phenomenology. The imaginary part gives rise to anomalous processes which are known to fit experiment when left unregularized. Suitable counterterms may need to be added to account for anomalous processes such as \(\gamma \rightarrow 3\pi\) [3].

We show that the resulting action is a non-analytic and highly singular functional of the fields. We show that the regularization described above is flawed by the fact that the stationary point of the action occurs for real time components of the vector fields (as in the Hartree approximation) and that the action is in fact singular in the vicinity of the stationary point. As a result, the regularized action is not suitable for a saddle point evaluation of the partition function. We discuss some of the proposed solutions of this problem. We show that all regularizations are fraught with problems.

2 The partition function expressed in terms of quark fields.

We consider a Nambu Jona-Lasinio lagrangian density involving scalar and vector interactions:

\[
L = \overline{q} (i\partial_\mu \gamma^\mu - m) q + \frac{g_s^2}{2} \left[ (\overline{q}q)^2 + (\overline{q} i \gamma_5 \tau_a q)^2 \right] - \frac{g_v^2}{2} (\overline{q} \gamma_\mu q) (\overline{q} \gamma^\mu q) \quad (2.1)
\]

where \(m\) is a mass matrix and where the coupling constants \(g_s^2\) and \(g_v^2\) have the dimension \(E^{-2}\) of an inverse energy squared. More general 4-fermion interactions can be included without altering the argument presented in this paper.
To avoid any possible confusion with later notation, we give explicitly the Minkowski 4-vectors in (2.1):

\[ x^\mu = (t, \vec{r}) \quad \gamma^\mu = (\beta, \vec{\gamma}) \quad \gamma_\mu = (\beta, -\vec{\gamma}) \]

Using the notation \( K_a = (\beta, i\beta\gamma^5 \tau_a) \), the action of the system, described by the lagrangian (2.1), can be written as follows:

\[ I = \int_M d^4x \left( q^\dag (i\partial^\mu \gamma^\mu - m) q \right) - \int_M d^4x \left( -\frac{g_5^2}{2} (q^\dag K_a q)^2 - \frac{g_5^2}{2} (q^\dag \vec{\alpha} q)^2 + \frac{g_5^2}{2} (q^\dag q)^2 \right) \]

where \( \int_M d^4x \equiv \int dt \int d^3r \). This action (2.3) is often referred to as the Minkowski action.

By canonical quantization of quark fields in (2.3) we obtain the hermitian hamiltonian of the system:

\[ H (q, q^\dag) = \int d^3r q^\dag \left( \frac{\vec{\alpha} \cdot \vec{\nabla}}{i} + \beta m \right) q + \int d^3r \left( -\frac{g_5^2}{2} (q^\dag K_a q)^2 - \frac{g_5^2}{2} (q^\dag \vec{\alpha} q)^2 + \frac{g_5^2}{2} (q^\dag q)^2 \right) \]

The partition function of the system can be expressed in terms of the path integral [4, 5]:

\[ T \text{re}^{-\beta H} \equiv \int D(q) D(q^\dag) e^{-I(q^\dag, q)} \quad I (q, q^\dag) = \int_0^\beta d\tau \left( q^\dag \partial_\tau q + H (q^\dag, q) \right) \]

where the integration variables \( q^\dag \) and \( q \) are Grassman variables, where \( H (q, q^\dag) \) is the form (2.3) and where \( \beta \) is the inverse temperature. The action \( I (q, q^\dag) \) is the Euclidean action. Explicitly, the path integral (2.5) is:

\[ T \text{re}^{-\beta H} \equiv \int D(q) D(q^\dag) e^{-\int d^4x q^\dag \left( \partial_\tau + \frac{\vec{\alpha} \cdot \vec{\nabla}}{i} + \beta m \right) q} \]

\[ \quad - \int d^4x \left( -\frac{g_5^2}{2} (q^\dag K_a q)^2 - \frac{g_5^2}{2} (q^\dag \vec{\alpha} q)^2 + \frac{g_5^2}{2} (q^\dag q)^2 \right) \]

and \( \int d^4x \equiv \int_0^\beta d\tau \int d^3r \).
3 Bosonized form of the Euclidean action.

From the identity 
\[ \int D(S) e^{-\frac{1}{2g^2}(S-g^2(q\Gamma q))^2} = \int D(S) e^{-\frac{1}{2g^2}S^2} \]
we deduce the identity:
\[ e^{-\frac{g^2(q\Gamma q)^2}{2}} = \frac{\int D(S) e^{-\frac{S^2}{2g^2}+S(q\Gamma q)}}{\int D(S) e^{-\frac{S^2}{2g^2}}} \] (3.1)

The integration variable \( S \) may be chosen to be real. This identity can be used for the first two (attractive) quartic interactions of the Hamiltonian (2.4). For the third (repulsive) quartic interaction we can use the identity:
\[ e^{-\frac{g^2(q\Gamma q)^2}{2}} = \frac{\int D(S) e^{-\frac{S^2}{2g^2}+iS(q\Gamma q)}}{\int D(S) e^{-\frac{S^2}{2g^2}}} \] (3.2)

which follows trivially from the identity \( \int D(S) e^{-\frac{1}{2g^2}(S-ig^2(q\Gamma q))^2} = \int D(S) e^{-\frac{1}{2g^2}S^2} \).

Proceeding this way, we associate the fields \( S_a, \vec{V} \) and \( V_4 \) respectively to the quark bilinear forms \( q^i K_a q, q^i \alpha q \) and \( q^i q \) and we write the partition function (2.5) in the form:
\[ T e^{-\beta H} = \frac{1}{N} \int D(q) D(q^i) D(S) D(\vec{V}) D(V_4) e^{-I(q,q^i,S,\vec{V},V_4)} \] (3.3)

with
\[ I(q,q^i,S,\vec{V},V_4) = \int d_4x q^i \left( \partial_\tau + \frac{\tilde{\alpha} \cdot \vec{V}}{\alpha} + \beta m + K_a S_a + \tilde{\alpha} \cdot \vec{V} + iV_4 \right) q \\
+ \int d_4x \left( \frac{1}{2g_a^2} S_a^2 + \frac{1}{2g_v^2} \left( \vec{V}^2 + V_4^2 \right) \right) \] (3.4)

and:
\[ N = \int D(S) D(\vec{V}) D(V_4) e^{-\int d_4x \left( \frac{1}{2g_a^2} S_a^2 + \frac{1}{2g_v^2} \left( \vec{V}^2 + V_4^2 \right) \right)} \] (3.5)

Integration of the quark fields, which now appear in quadratic form, yields a partition function in the form:
\[ T e^{-\beta H} = \frac{1}{N} \int D(S) D(\vec{V}) D(V_4) e^{-I(S,\vec{V},V_4)} \] (3.6)
where the Euclidean action has the familiar form:

\[
I (S, \vec{V}, V_4) = -Tr \ln D + \int d^4 x \left( \frac{1}{2g_s^2} S_a^2 + \frac{1}{2g_v^2} (\vec{V} + V_4^2) \right)
\]

(3.7)

In (3.7) the Dirac operator \(D\) and the Dirac hamiltonian \(h\) are:

\[
D = \partial_\tau + h \quad h = \frac{\vec{\alpha}.\vec{V}}{i} + \beta m + K_a S_a + \vec{\alpha}.\vec{V} + iV_4 \neq h^\dagger
\]

(3.8)

Because of the appearance of the term \(iV_4\), the Dirac hamiltonian in (3.8) is not hermitian. We note that we have not derived (3.7) by a Wick rotation \(V_0 \rightarrow -iV_4\) of the time component of a vector field. Nor is any “inverse” Wick rotation called for.

The Euclidean action (3.7) is usually expressed in covariant form by redefining the following Euclidean 4-vectors:

\[
x^\mu = x_\mu = (\vec{r}, \tau) \quad \gamma^\mu = \gamma_\mu = (\vec{\gamma}, i\beta) \quad V^\mu = V_\mu = (\vec{V}, V_4)
\]

\[
\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = -2\delta_{\mu\nu}
\]

(3.9)

With these definitions, we have \(\beta D = -i\partial_\mu \gamma_\mu + m + \Gamma_a S_a + V_\mu \gamma_\mu\) and the effective Euclidean action (3.7) acquires the familiar covariant form:

\[
I (S_a, V_\mu) = -Tr \ln D + \int d^4 x \left( \frac{1}{2g_s^2} S_a^2 + \frac{1}{2g_v^2} V_\mu^2 \right)
\]

(3.10)

where the Dirac operator is defined to be:

\[
D = -i\partial_\mu \gamma_\mu + m + \Gamma_a S_a + V_\mu \gamma_\mu
\]

(3.11)

The equivalent actions (3.7) and (3.10) are the forms used in Refs. [1, 2].

4 The regularized Euclidean effective action.

We consider the proper time regularization of the fermion determinant because most calculations of solitons performed so far have used this regularization. The proper time regularization consists in separating the fermion determinant \(-Tr \ln D\) into real and imaginary parts \(-\frac{1}{2} Tr \ln D^T D\) and \(-\frac{1}{2} Tr \ln D/D^T\),
and in regularizing only the real part. Thus the proper time regularized action is:

\[ I_\Lambda \left( S, \vec{V}, V_4 \right) = \frac{\text{Tr}}{2} \int_1^{\infty} \frac{ds}{s} e^{-sD^\dagger D} - \frac{1}{2} \text{Tr} \ln D / D^\dagger \]

\[ + \int d^4x \left( \frac{1}{2g_s^2} S_a^2 + \frac{1}{2g_v^2} (\vec{V} + V_4^2) \right) \]

where the Dirac operator $D$ is defined by (3.8). Whereas the Dirac operator is a simple linear analytic function of the fields, the operator $D^\dagger D$ is not. It is therefore not surprising to find that different analytic continuations of the real part of the effective action have been considered in the literature [1, 2].

The problems we shall discuss are caused by the separation of the action into a regularized real part and an unregularized imaginary part. They are not resolved by regularizing the real part by another method.

Suppose that electroweak gauge fields are added to the Dirac operator and that we make the time component of the gauge fields pure imaginary. In the expansion of the fermion determinant $\text{Tr} \ln D$, the odd powers of the imaginary time component $iV_4$ of the gauge fields arise in conjunction with the antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$. Such terms give rise to the usual anomalous processes [6], which are known to fit experiment when they are calculated without regularization. Indeed in the case of $\pi \rightarrow 2\gamma$, for example, it is known that cut-offs, which can be as low as 800 MeV, can induce up to 40% discrepancies between calculated and observed values [7]. Thus the use of an imaginary time component of the electroweak gauge field in the action (4.1) is a convenient way to ensure that the amplitudes for anomalous processes are not cut off.

5 A stationary point of the action.

We will show that a stationary point of the actions exists, in which the fields $S$ and $\vec{V}$ are real, where $V_4$ is pure imaginary and where, in addition, the fields are time independent:

\[ S_a (\vec{r}) , \vec{V} (\vec{r}) : real \quad V_4 (\vec{r}) = iV_0 (\vec{r}) \quad V_0 (\vec{r}) : real \]  

This is in fact the form which the mean fields would have in the Hartree approximation applied to the hamiltonian (2.4). In the vacuum the stationary
point occurs for vanishing vector fields. However in solitons, the vector field at the stationary point does not vanish. It has a shape which is similar to the vector density $\langle q^\dagger (\vec{r}) q (\vec{r}) \rangle$.

At the point (5.1), the Dirac operator (3.8) is:

$$D = \partial \tau + h = \frac{\vec{\alpha}.\vec{V}}{i} + \beta m + K_a S_a + \vec{\alpha}.\vec{V} - V_0 = h^\dagger$$

(5.2)

The Dirac hamiltonian $h$ in (5.2) is hermitian. It has real eigenvalues:

$$h |\lambda\rangle = e^{\lambda} |\lambda\rangle$$

(5.3)

Because the fields $S, \vec{V}$ and $V_0$ are time independent, the Dirac operators $D$ and $D^\dagger$ can be diagonalized simultaneously in the basis:

$$D |\lambda, \omega\rangle = (i\omega + e^{\lambda}) |\lambda\omega\rangle$$

$$D^\dagger |\lambda, \omega\rangle = (-i\omega + e^{\lambda}) |\lambda\omega\rangle$$

(5.4)

It is easy to check that the imaginary part of the action vanishes in this case. Using the basis (5.4), the action (4.1), at the time independent point (5.1), is finite and it can be written in the form:

$$I_A \left( S, \vec{V}, V_4 = iV_0 \right) = \frac{1}{2} \sum_{\lambda, \omega} \int_0^\infty \frac{ds}{s} e^{-s(\omega^2 + e^{2\lambda})} + \int d^4 x \left( \frac{1}{2g_s^2} S^2_a + \frac{1}{2g_v^2} (\vec{V} - V_0^2) \right)$$

(5.5)

We now consider the expansion of the action (4.1) in powers of the fields, about the point (5.1). The field variations are:

$$S_a \rightarrow S_a + \delta S_a \quad \vec{V} \rightarrow \vec{V} + \delta \vec{V} \quad V_4 \rightarrow V_4 + \delta V_4$$

(5.6)

where, for now, the increments $\delta S_a, \delta \vec{V}$ and $\delta V_4$ are all real. The corresponding variation of the Dirac operator (3.8) is:

$$D \rightarrow D + \delta S + \delta V \quad D^\dagger \rightarrow D^\dagger + \delta S^\dagger + \delta V^\dagger$$

(5.7)

where:

$$\delta S \equiv K_a \delta S_a \quad \delta V \equiv \vec{\alpha}.\delta \vec{V} + i\delta V_4$$

(5.8)

If we use the basis (5.4) to evaluate the traces, a straightforward calculation will show that the first order variation of the action (4.1) is:

$$\delta I_A \left( S, \vec{V}, V_4 \right) \equiv I_A \left( S + \delta S, \vec{V} + \delta \vec{V}, iV_0 + \delta V_4 \right) - I_A \left( S, \vec{V}, iV_0 \right)$$

7
\[
\begin{align*}
- \sum_{\lambda \omega} \int_0^\infty dse^{-s(\omega^2+\varepsilon^2)}e_\lambda \left( \lambda \omega \right) \beta \delta S & + i \beta \gamma_5 \tau_3 \delta P_a + \bar{\alpha} \delta \vec{V} \left| \lambda \omega \right) \\
+ \sum_{\lambda \omega} \frac{1}{i \omega + e_\lambda} \langle \lambda \omega | i \delta V_4 | \lambda \omega \rangle
\end{align*}
\]

The first and second terms are respectively the contributions of the real and imaginary parts of the action. The regularized real part contributes to the first order variations of the fields \( S \) and \( \vec{V} \), whereas the imaginary part contributes to the first order variation of \( V_4 \). The first order variation can be further reduced to the form:

\[
\delta I_A (S, \vec{V}, V_4) = \int d_4 x \delta S_a (x) \left( \bar{\rho}_a (\vec{r}) + \frac{1}{g_s^2} S_a (x) \right) + \int d_4 x \delta \vec{V} \cdot \left( \bar{\rho} (\vec{r}) + \frac{1}{g_v^2} \vec{V} (x) \right)
\]

\[
\int d_4 x \delta V_4 (x) \left( i \rho (\vec{r}) + \frac{1}{g_v^2} V_4 (x) \right)
\]

In the expression (5.10), \( \bar{\rho}_a (\vec{r}) \) and \( \bar{\rho} (\vec{r}) \) are the regularized densities:

\[
\bar{\rho}_a (\vec{r}) = \frac{1}{2} \sum_\lambda \langle \lambda | \vec{r} \rangle K_a \langle \vec{r} | \lambda \rangle \frac{e_\lambda}{|e_\lambda|} n \left( \frac{e_\lambda}{\Lambda} \right)
\]

\[
\bar{\rho} (\vec{r}) = \frac{1}{2} \sum_\lambda \langle \lambda | \vec{r} \rangle \bar{\alpha} \langle \vec{r} | \lambda \rangle \frac{e_\lambda}{|e_\lambda|} n \left( \frac{e_\lambda}{\Lambda} \right)
\]

and \( \rho (\vec{r}) \) is the unregularized density:

\[
\rho (\vec{r}) = \sum_{e_\lambda < 0} \langle \lambda | \vec{r} \rangle \langle \vec{r} | \lambda \rangle
\]

The densities have been expressed in terms of the occupation probability \( n \left( \frac{e_\lambda}{\Lambda} \right) \) which is particular to the proper time regularization:

\[
n \left( \frac{e_\lambda}{\Lambda} \right) = \frac{1}{\sqrt{\pi}} \int_2^\infty \frac{1}{\sqrt{x}} x^{-\frac{3}{2}} e^{-\frac{x}{2}} dx
\]

After subtracting the vacuum density, the vector density (5.12) turns out to be finite in the case of a soliton. The equations (5.10) show a posteriori that a time independent stationary point of the form (5.1) does indeed exist.
6 The non-analytic and singular nature of the regularized action.

Consider a pure imaginary variation of the field $S$:

$$S \rightarrow S + i\delta S \quad D \rightarrow D + iK_\alpha \delta S_\alpha \quad i\delta S : \text{imaginary} \quad (6.1)$$

and let us calculate the corresponding variation of the action (4.1):

$$\delta I_\Lambda = I_\Lambda \left( S + i\delta S, \vec{V}, V_4 = iV_0 \right) - I_\Lambda \left( S, \vec{V}, V_4 = iV_0 \right) \quad (6.2)$$

We then find that the variation (6.2) of the action comes from the unregularized imaginary part and that it is infinite. Thus to a real variation of the field $S$ corresponds a finite variation of the action, whereas to an imaginary variation of the field $S$ corresponds an infinite variation of the action. Likewise, if we make an imaginary variation of the field $V_4$, the corresponding variation of the action would come from the regularized real part and the result would not be equal to the last term of (5.10) multiplied by $i$. These examples show that the regularized action (4.1) is a non-analytic functional of the fields $S$ and $V_4$ and that furthermore, it is a singular functional of the field $S$. In the next section, we show that it is also a singular functional of the fields $\vec{V}$ and $V_4$. This is why the discussions of its analytic properties are hazardous to say the least.

7 A problem connected to the second order variation of the action.

Consider the second order variation of the action about the point (5.1). The regularized real part of the action makes a finite contribution. The unregularized imaginary part, however, makes an infinite second order contribution. To see how this comes about, and to discuss a possible solution to this problem, let us write out explicitly the second order contribution of the unregularized imaginary part:

$$I_{\text{imag}}^{(2)} \left( S, \vec{V}, V_4 \right) =$$
\[-\frac{1}{4} \text{Tr} \frac{1}{D^\dagger} (\delta S^\dagger + \delta V^\dagger) \frac{1}{D^\dagger} (\delta S^\dagger + \delta V^\dagger) + \frac{1}{4} \text{Tr} \frac{1}{D} (\delta S + \delta V) \frac{1}{D} (\delta S + \delta V) \]

(7.1)

The terms which are second order in \(\delta S = \delta S^\dagger\) cancel out. This is not so for the terms which are second order in \(\delta V \neq \delta V^\dagger\). A straightforward calculation will show that a non-vanishing term proportional to \(\delta V_i \delta V_4\) remains which is unregularized and infinite.

Thus, although the regularized action (4.1) is finite at the stationary point (5.1), variations of the action around the stationary point are infinite. This is yet another illustration of the singular nature of the regularized action (4.1). The problem remains when imaginary variations of the vector fields are performed. As a result, the regularized action (4.1) is not suitable for a saddle point evaluation of the partition function. It is this problem which led the authors of Refs. [1, 2] to modify the action (4.1). It is not some fundamental necessity of transforming the Euclidean action back to a Minkowski action.

8 A proposed solution to the problem.

Arriola and coworkers have proposed a modification to the regularized action which can avoid this problem, while maintaining the anomalous processes unregularized. The proposed solution consists in replacing, in (4.1), \(D^\dagger\) by a suitably modified Dirac operator \(\bar{D}\). The regularized action is then defined to be:

\[
I_{\Lambda} (S, \vec{V}, V_4) = \frac{T r}{2} \int_{-\infty}^{\infty} \frac{ds}{s} e^{-s\bar{D}D} + \frac{1}{2} Tr \ln \frac{D}{\bar{D}}
\]

\[
+ \int d_4x \left( \frac{1}{2g_s^2} S_a^2 + \frac{1}{2g_v^2} (\vec{V} + V_4^2) \right)
\]

(8.1)

where the Dirac operators \(D\) and \(\bar{D}\) are:

\[
D = \partial_t + h \quad h = \frac{\alpha.\vec{V}}{i} + \beta m + K_a S_a + \alpha.\vec{V} + iV_4
\]

\[
\bar{D} = -\partial_t + \bar{h} \quad \bar{h} = \frac{\bar{\alpha}.\vec{\bar{V}}}{i} + \beta m + K_a S_a + \bar{\alpha}.\vec{\bar{V}} - iV_4
\]

(8.2)
Of course, $D$ is the same as before. As long as the field $V_4$ remains real, we have $\bar{D} = D^\dagger$ and the regularized actions (4.1) and (8.1) are the same. However, when $V_4$ is pure imaginary, that is, when $V_4 = iV_0$, then:

$$D = \partial_{\tau} + h$$

$$h = \frac{\alpha}{i} \vec{\nabla} + \beta m + K_a S_a + \vec{\alpha} \vec{V} - V_0$$

$$\bar{D} = -\partial_{\tau} + \bar{h}$$

$$\bar{h} = \frac{\bar{\alpha}}{i} \vec{\nabla} + \beta m + K_a S_a + \bar{\vec{\alpha}} \vec{V} + V_0$$

and $\bar{D} \neq D^\dagger$. This choice has several advantages and also disadvantages.

The operator $\bar{D}D$ is an analytic function of the fields, whereas $D^\dagger D$ was not. The first term of the action (8.1) is even in $V_4$ or $V_0$, whereas the second term is odd. This ensures that anomalous processes remain unregularized in the vacuum, where a stationary point exists with vanishing vector fields. The action (8.1) has also disadvantages. When the time component $V_4$ of the vector field is pure imaginary, the Dirac hamiltonians become hermitian, however they do not commute. As a result the Dirac operators $D$ and $\bar{D}$ can no longer be simultaneously diagonalized when the fields are time independent. Soliton calculations become more complicated, because they require the diagonalization of the three non commuting hamiltonians $h$, $\bar{h}$ and $h\bar{h}$. But, foremost, the argument $s\bar{D}D$ in the exponential of the expression (8.1) is not necessarily positive definite. It would be for small enough vector fields. Provisional estimates [1] indicate that the operator $\bar{D}D$ acquires negative eigenvalues for model parameters which are quite close to the ones required to form a hedgehog soliton.

In Ref[2] it was proposed to solve this problem by using an effective action which includes only the second order expansion of the action (4.1) in powers of the vector field. This is the same as the solution proposed in Ref.[1], except that it is carried out only to second order in the vector field. The validity of such an expansion will obviously break down in the vicinity of the region where the operator $\bar{D}D$ develops a negative eigenvalue. This would not be felt by a second order expansion in powers of the field.

9 Discussion of other regularizations.

The problems we have encountered above are essentially due to the separation of the action into real and imaginary parts which are not regularized in the
same way. Some regularization procedures do not require such special action to be taken “by hand” to deal with electro-weak anomalies. For example, in some effective theories, the fermion determinant is regularized by the appearance of non-local fields [9, 10, 11]. In such cases, when the electro-weak current is coupled to a conserved vector current in a gauge invariant way, the anomalous processes turn out to be naturally independent of the particular form of the cut-off function, provided that the corrections to the currents, calculated by a Noether construction, are included [12]. They are therefore correctly calculated without recourse to an extra phenomenological ansatz. This should also be true of regularizations involving 4-momentum cut-offs [13, 14], although this point has not been thoroughly investigated. However there are other problems with such regularizations. The calculation of solitons with non-local fields is more complicated and longer. Furthermore, analytic continuation of the calculated meson propagators from euclidean space-like momenta to on-shell time-like momenta, where the theory can really be confronted with experiment, are flawed with ambiguities, so that non-local fields can only reliably be used in the euclidean region.

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