Clarifications on the impact of theoretical uncertainties on the Tevatron Higgs exclusion limits*

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Abstract

In this note, we respond to the comments and criticisms made by the representatives of the CDF and D0 collaborations on our recent papers in which we point out that the theoretical uncertainties in the Higgs production cross section have been largely underestimated and, if properly taken into account, will significantly loosen the Tevatron Higgs exclusion bounds. We show that our approach to the theoretical uncertainties is reasonable and fully justified. In particular, we show that our procedure is not very different from that adopted by the LHC experiments and if the latter is used in the Tevatron case, one obtains much larger uncertainties that those assumed by the CDF and D0 collaborations. Furthermore, we provide additional details on our statistical analysis of the CDF and D0 exclusion limit and show that it is conceptually correct.

*Extended version of talks given at several winter conferences by the authors.
1. Introduction

In two earlier papers [1, 2], we updated the theoretical predictions for the production cross sections of the Standard Model Higgs boson at the Tevatron collider, focusing on the main search channel, the gluon–gluon fusion mechanism $gg \to H$ [3], including the relevant higher order QCD [4–9] and electroweak corrections [9, 10]. We then estimated the various theoretical uncertainties affecting these predictions: the scale uncertainties which are viewed as a measure of the unknown higher order effects, the uncertainties from the parton distribution functions (PDFs) and the related errors on the strong coupling constant $\alpha_s$, as well as the uncertainties due to the use of an effective field theory (EFT) approach in the determination of the radiative corrections in the process at next-to-next-to-leading order (NNLO). We found that contrary to the Higgs–strahlung processes [11], where the rates are well under control as the uncertainty is less than $\approx 10\%$, the theoretical uncertainties are rather large in the case of the gluon–gluon fusion channel, possibly shifting the central values of the NNLO cross sections by up to $\approx 40\%$. These uncertainties are thus significantly larger than the $\approx 10\%–20\%$ error assumed by the CDF and D0 experiments in their analysis that has excluded the Higgs mass range $M_H = 158–175$ GeV at 95% CL [12, 13, 14]. As $gg \to H$ is by far the dominant Higgs production channel in this mass range, we concluded that the above exclusion limit should be reconsidered in the light of these large theoretical uncertainties.

After our papers appeared, some criticisms have been made by the members of the CDF and D0 collaborations and of the Tevatron New Physics and Higgs working group (TevNPHWG) [15, 16] concerning the theoretical modeling of the $gg \to H$ production cross section that we proposed. This criticism was made more explicit in the Tevatron Higgs talks this winter at the La Thuile [17] and Moriond–QCD [18] conferences (where a long discussion on the theoretical uncertainties has been scheduled after the talk of one of the authors [19]).

In this note, we respond to this criticism point by point and show that that our approach to the theoretical uncertainties is fully justified. In particular, we will make use of of a recent collective effort [20] made by theorists along with experimentalists of the ATLAS and CMS collaborations to evaluate the Higgs cross section at the LHC, with a special attention to the gluon fusion mechanism which is also the process of interest here. Several issues discussed in our papers [1, 2] have been indeed addressed in the report of this working group. It turns out that many of the proposals that we put forward for the $gg \to H$ process are in fact similar to those adopted in this comprehensive LHC study. We will thus also use the conclusions of this report (together with other studies that appeared very recently) to strengthen some of our arguments even more.

Another criticism made by the CDF and D0 collaborations is on the statistical analysis of the exclusion limit that we performed in Ref. [2], using the detailed information and the multivariate analysis given in a CDF paper [14]. Apparently, there was a misunderstanding on what we actually did in our “emulation” of the CDF/D0 limit: we did not increase the theoretical uncertainty (or add an extra uncertainty) but simply changed the normalisation as if the cross section was evaluated using another set of PDFs (such as HERAPDF [21] or ABKM [22] rather than the adopted MSTW choice [23, 24]). In this case, using the neural network output of the CDF analysis to re-estimate the sensitivity and the exclusion limit is fully justified and our analysis is conceptually correct.

Finally, we take this opportunity to correct an error made in Ref. [2] in the numerical evaluation of the $gg \to H$ cross section using the HERAPDF [21] set. This error will only slightly change part of the discussion in Ref. [2] and will not alter our general conclusions.
2. Theoretical uncertainties on the $gg \to H$ cross section

A. The scale uncertainty

It is a known and well accepted fact that the choice for the domain of scale variation, which is supposed to account for the missing contributions at higher orders in perturbation theory, is subjective. This is true together with the fact the “scale variation only gives a lower limit on the true uncertainty from higher orders” (we use here the same words are those given in Ref. [20]). However, there are at least three arguments which make us believe that, in the particular case of the $gg \to H$ process at the Tevatron, the situation is really exceptional and the domain of scale variation should be extended from the usual choice (which, we stress again, is only a guess and by no means a dogma) $\frac{1}{2} \mu_0 \leq \mu_R, \mu_F \leq 2\mu_0$ (i.e. $\mu_0/\kappa \leq \mu_R, \mu_F \leq \kappa \mu_0$ with a factor $\kappa = 2$) [8, 9], with the central scale taken to be $\mu_0 = \frac{1}{2} M_H$, to at least the range $\frac{1}{3} \mu_0 \leq \mu_R, \mu_F \leq 3 \mu_0$ (i.e. with $\kappa = 3$). Here, we stick to a discussion of the QCD corrections up to NNLO only; additional QCD contributions beyond this perturbative order as well as the electroweak corrections will be addressed later.

1) The K factor, i.e. the effect of the higher order QCD corrections, in the $gg \to H$ process at the Tevatron is extraordinarily large. It increases the LO cross section by a factor of three and it is in fact this factor of three which makes the CDF/D0 experiments sensitive to the Standard Model Higgs boson with the presently collected data. There is basically no other electroweak process which receives such large contributions from higher orders (as long as no new coupling or a different type of contribution not present at LO does not appear present in these higher orders, as in fact is the case here). If we stick only to NNLO, then one should really be worried about the convergence of the perturbative series as the K factor is 2 at NLO and 3 at NNLO. A factor $\kappa = 2$ for scale variation to estimate the higher orders would thus be justified for any process where the QCD corrections are moderate (like most other processes that have been discussed so far) but not in this special case.

We note that the scale variation of the NLO cross section barely reaches the central value of the cross section at NNLO for $\kappa = 2$ (not to mention the LO cross section which needs a factor $\kappa = 4$ to contain the NNLO central value); to have a significant overlap of the NLO scale uncertainty band with the NNLO central value, the choice $\kappa = 3$ is more appropriate.

2) Another argument to increase the domain of scale variation from $\kappa = 2$, which leads to an uncertainty of $+10\%, -12\%$ for say $M_H = 160$ GeV, to $\kappa = 3$ which gives an uncertainty of $+15\%, -20\%$ for the same $M_H$ value, has been given in the addendum to Ref. [1] and is reproduced below. If the NNLO $gg \to H$ cross section is broken into the three pieces with 0, 1 and 2 jets, and one applies a scale variation for the individual pieces in the range $\frac{1}{2} \mu_0 \leq \mu_R, \mu_F \leq 2 \mu_0$, one obtains with selection cuts similar to those adopted by the CDF/D0 collaborations, a scale uncertainty on the “inclusive” cross section that is about $+20\%, -17\%$, when one averages over the various final states with their corresponding weights [25]. This is

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1. The choice $\mu_0 = \frac{1}{2} M_H$ for the central scale, instead of the (for a long time supposedly most natural) value $\mu_0 = M_H$, is motivated by the fact that it leads to a better convergence of the perturbative series; in addition, it implicitly takes into account the soft–gluon re-summation contributions which are at the level of $+15\%$ for $\mu_0 = M_H$. Note that a scale variation with $\kappa = 2$ around the value $\mu_0 = \frac{1}{2} M_H$, will bring us back only to the original central scale value at most, which we feel is rather optimistic.

2. We note that the original analyses of the higher order corrections to $gg \to H$ had mainly considered the case of the LHC (the extension for the Tevatron was made for completeness as it was not clear if the process was viable there) and, in this case, we fully support the choice of $\kappa = 2$ as the QCD corrections lead to only a total K factor of two with an acceptable convergence, as it goes from $\approx 1.7$ at NLO to $\approx 2$ at NNLO.

3. Note that in Ref. [20], the choice $\kappa = 3$ has been adopted for the Higgs-strahlung processes $pp \to HW/HZ$ despite of the fact that the QCD corrections are much smaller than in the $gg$ fusion channel.
very close to the result obtained in the CDF/D0 analysis [13] which quotes a scale uncertainty of \( \approx \pm 17.5\% \) on the total cross section, when the weighted uncertainties for the various jet cross sections are added. Thus, our supposedly “conservative” choice \( \frac{1}{3} \mu_0 \leq \mu_R = \mu_F \leq 3 \mu_0 \) for the scale variation of the total inclusive cross section \( \sigma_{\text{NNLO}}^{gg \rightarrow H} \) leads to a scale uncertainty that is very close to that obtained when one adds the scale uncertainties of the various jet cross sections for a variation around the more “consensual” range \( \frac{1}{2} \mu_0 \leq \mu_R, \mu_F \leq 2 \mu_0 \). We also note that when breaking \( \sigma_{\text{NNLO}}^{gg \rightarrow H} \) into jet cross sections, an additional error due to the acceptance of jets is introduced; the CDF and D0 collaborations, after weighting, have estimated it to be \( \pm 7.5\% \). This error, combined with the weighted uncertainty for scale variation, will certainly increase the total scale error in the CDF/D0 analysis, possibly (and depending on how the errors should be added) to the level where it almost reaches or even exceeds our own supposedly “conservative” estimate.

\( \text{iii) The above issue brings us to a last argument which appeared only very recently. In} \)

an analysis of the Higgs+0 jet cross section (which is the topology to which CDF and D0 are by far most sensitive), the authors of Ref. [26] show that imposing a tight jet veto to select this topology induces large double logarithms which significantly modify the Higgs production cross section. They calculate the Higgs+0 jet cross section from gluon fusion at next-to-next-to-leading-logarithmic (NNLL) order, fully incorporating the NNLO fixed order results and their conclusion is as follows: “At this order (NNLL), the scale uncertainty is 15–20\%, depending on the cut, implying that a larger scale uncertainty should be used in current Tevatron bounds on the Higgs”. This is a major issue which has to be addressed by CDF/D0 (also at the LHC where indeed an uncertainty of 15–20\% is adopted [27]) and before it is settled, it would be perhaps wise to increase the scale uncertainty to a level that is even larger than the one we are advocating. However, clearly we do not so at present.

Let us now briefly discuss the higher order corrections beyond NNLO to the \( gg \rightarrow H \) cross section, and justify why we do not take them into account (as is also done by the CDF/D0 and the LHC [20] collaborations in their analyses):

- As mentioned above, in the actual CDF/D0 analysis [13, 14], the \( gg \rightarrow H \) cross section has been broken into the three pieces pieces corresponding to the production of a Higgs with 0, 1 and 2 jets and only the NNLO QCD corrections to these jet cross sections have been taken into account. One should therefore stick, for consistency, to an inclusive cross section (which is the sum of these jet cross sections) at the same order, i.e. NNLO.

- For the consistency of the calculation, the corrections beyond NNLO (including soft–gluon resumation), need to be folded with PDFs that are at the same perturbative order. No PDF beyond NNLO is available at the moment and, thus, one should stick to NNLO.

- The soft–gluon resumation and the higher order effects beyond NNLO address especially the logarithmic corrections (as also do the scale uncertainties). However, it is well known that in the \( gg \rightarrow H \) process, very large contributions are coming from constant, in particular \( \pi^2 \), terms. These terms are not dealt with by the calculations beyond NNLO and their inclusion might in turn induce large corrections that are not accounted for by scale variation.

- An attempt to only partly re-sum these large \( \pi^2 \) terms has been made in Ref. [28]. This is the paper to which Refs. [18, 17] refer to as the study in which smaller uncertainties than those that are advocated by all analyses are claimed. For a discussion of this analysis, we simply refer to the report of the experts that contributed to the the Higgs cross section working group at the LHC (section 2.5 of Ref. [20]) in which arguments have been presented to show why the uncertainties advocated in this paper have been underestimated and reasons not to adopt them in the report [20] have been given.
B. Uncertainties from the use of an Effective Field Theory (EFT) approach

The uncertainties from the use of the EFT approach originate from three different sources and we summarize their impact below.

i) There are first the uncertainties from the exact NLO electroweak corrections and whether these corrections should be included in the complete or in the partial factorization approaches. This has been discussed in detail in Ref. [10] (where the full NLO electroweak correction itself has been derived) and it was advocated that an uncertainty which is the difference between the results obtained in the two approaches should be included. In our paper, we have simply followed this recommendation which, for a Higgs mass of 160 GeV, leads to an uncertainty of about 3%. For the three–loop mixed QCD–electroweak corrections (that, incidentally, are also included in our calculation) there is an additional problem: they have been calculated in the EFT approach in which $M_H \ll M_W$ which is obviously not a valid limit. These corrections should therefore be taken with care\(^4\). Nevertheless, first, we included this correction as stated before and second, we did not assign an uncertainty to this correction and stuck to the one advocated in Ref. [10], i.e. included only the difference between the complete and partial factorization approaches.

ii) The renormalisation scheme dependence for the $b$–quark mass in the $b$–quark loop contribution to the $ggH$ amplitude. Here, we indeed find a 1–2% uncertainty which is more or less equivalent to what has been discussed in Ref. [9], but which was not included in the final numbers for the total uncertainty on the $gg \to H$ cross section, as is discussed below.

iii) The EFT approach for the NNLO QCD contribution: we fully agreed with the experts that the infinite top mass limit is very good for the Higgs masses that are relevant at the Tevatron (but not at the LHC for $M_H \geq 2m_t$), so there is no problem here. The problem is the $b$–quark loop contribution for which the EFT approach is certainly not valid and, for instance, the omission of this contribution at LO leads to a 10 % difference compared to the exact case. Furthermore, the NLO K factor for the $b$–quark loop, which is about $K = 1.2–1.4$, is much smaller than that for the top–quark loop. In fact, the problem arises mainly because of the significant negative interference between the top and bottom loop contributions (the bottom quark contribution itself is rather small) up to NLO and the fact that the bottom contribution has been factorized out in the LO cross section. Thus, including the top quark contribution only using the EFT approach might overestimate the NNLO correction as this interference component is missing (and, since it is resulting simply from an approximation in the calculation it is not, in principle, accounted for by e.g. the scale variation that is performed at NNLO). We have estimated this uncertainty on the base of what is known at NLO (where both the exact and EFT results are available [5]) and taking into account the relative K factors for the $t$ and $b$ loops at this order and we obtain an uncertainty of a few percent. This uncertainty has not been discussed elsewhere and we believe that it should be definitely included as it is not accounted for by anything else.

Contrary to what is stated in Ref. [17], none of these uncertainties have been included in the analysis of Ref. [9] (nor in those of Ref. [8]) on which the CDF/D0 limits are based (some of these uncertainties have been indeed discussed in Ref. [9] but not included in the final

\(^4\)Indeed, if the NLO electroweak corrections have been calculated in the same limit, as was done in Ref. [29], one would obtain a result which is completely different: for instance a correction of less than 0.2% is obtained for the leading $m_t^2$ correction to the $gg \to H$ cross section [29] in the EFT approach $M_H \ll M_t$, compared to several percent for the electroweak correction in the exact case (another example is the two loop electroweak correction to the $\rho$ parameter where the leading $m_t^2$ correction in the EFT approach [30] is completely different from the result obtained with a more refined calculation as, for instance, in Ref. [31]).
numbers that have been given). In fact, all these uncertainties have been addressed in the LHC Higgs cross section report [20] (in which all the authors of Refs. [9, 8] have contributed) and we summarize their effect (see section 2.3 of [20]): 1% uncertainty for the electroweak contribution, 1 to 2% from the $b$-quark mass and 1% uncertainty from the EFT approach to the top quark contribution (the top–bottom quark loop interference has not been taken into account). This makes a total of up to 4% uncertainty which is not far from our 5% estimate (note that, unfortunately, these uncertainties have also not been included in the final numbers given in Ref. [20], but we have been told that it will be the case in an update of the analysis that will appear soon). We stress that these are only estimates and some arbitrariness will remain until further higher order or exact contributions are calculated.

C. The PDF uncertainties

Concerning the PDF uncertainties, there are two issues on which there is criticism: first, the way we estimate the uncertainties within the MSTW set of PDFs that is usually adopted and second, our later use of the ABKM09 and HERAPDF parameterizations to illustrate the impact of the PDF uncertainties on the $gg \rightarrow H$ cross section at the Tevatron. We will present a detailed discussion of the last issue in the next section of this note.

Let us begin by stressing the fact that our first and main approach was to use the (commonly adopted) MSTW2008 set of PDFs [23, 24] and their associated uncertainties. But first, we used the 90% CL combined PDF+$\Delta^{\text{exp}}\alpha_s$ uncertainty and, second, we added to that (in quadrature) an estimate of the theoretical uncertainty in $\alpha_s$ ($\Delta^{\text{th}}\alpha_s = 0.002$ at NNLO, given by the MSTW collaboration itself but not used in the final analysis) and obtained, for $M_H = 160$ GeV, a total of $\pm 15\%$ PDF+$\alpha_s$ uncertainty as stated in the paper. This is exactly what one obtains if the PDF4LHC recommendation [32] (which, incidentally, appeared after our first analysis was published) is used. Indeed, the PDF4LHC recommendation to estimate the uncertainties is that one takes the envelope of the PDF+$\Delta^{\text{exp}}\alpha_s$ uncertainties obtained using the three sets given by the MSTW, CTEQ [33] and NNPDF [34] collaborations. For the $gg \rightarrow H$ cross section at NNLO, to a very good approximation, this reduces to taking the PDF+$\Delta^{\text{exp}}\alpha_s$, MSTW uncertainty at the 68% CL and multiply it by a factor of two. One then obtains 15% PDF+$\alpha_s$ uncertainty for $M_H = 160$ GeV, i.e. almost exactly the value that we assume in the paper. So we are arriving, using two different ways, to the same result.

We nevertheless believe that the PDF4LHC recommendation, when taken literally (i.e. when using the prescription above), is not sufficient to account for all possible sources of uncertainties. First, the CTEQ and NNPDF groups have only parameterizations at NLO, while we are evaluating our process at NNLO and thus, only one PDF set can be used consistently in this particular case. Second, it would have been rather unfair to ignore in the final recipe to calculate the uncertainty the other PDF sets which, instead, are at NNLO. And indeed, contrary to statements that we heard, the PDF4LHC does not recommend to ignore totally the other PDF sets. The complete and correct statement of the PDF4LHC group is (see end section 2.2 of Ref. [32]): “Since there are NNLO PDFs obtained from fits by the ABM, GJR and HERAPDF groups, these should ideally be compared with the above procedure”. This is exactly what we did in our papers.

Thus, we end this discussion of PDF related uncertainties by noting that these uncertainties seem to be underestimated in the CDF/D0 analysis (and, in fact, also in our original paper Ref. [1]). The second issue of consistency of the predictions of the ABKM and HERAPDF sets with the Tevatron data and that of the large difference in the gluon densities between the various PDF sets on the other hand is deferred to section 3.
D. The combined scale+PDF uncertainty

Finally, we address the issue of combining the uncertainties which, as we stated in our answer to Ref. [16] in the addendum to Ref. [1], is the only relevant issue in this context and it explains the major part of the difference between the total uncertainties that we assume in our paper compared to that adopted by the CDF collaboration.

Let us start by an important comment: we do not add scale+EFT and the PDF uncertainties linearly as often stated. Our procedure for the combination has been presented in Ref. [1] and is the following: we calculate the maximal and minimal cross sections with respect to the scale variation and apply on these cross sections the PDF+\( \Delta^{\exp} \alpha_s + \Delta^{\text{th}} \alpha_s \) uncertainty. This procedure has been in fact already proposed, together with other possibilities which give similar results, in Ref. [35] where top quark pair production at hadron colliders was discussed. To this combined scale+PDF uncertainty, one can linearly add the small scheme/EFT uncertainty to obtain the overall theoretical error. Nevertheless, it turns out that our procedure for adding the scale and PDF uncertainties leads to a total uncertainty that is close but a little bit smaller than what we would get had one added them linearly.

Note that our procedure above would eventually take into account possible correlations between the scale and the PDFs (which are evaluated at a given factorization scale). Indeed, an argument for our approach put forward in Ref. [35] is that unknown high order effects also enter in the PDF determination and one needs to use the full information on the PDF uncertainties in the determination of the scale dependence. Nevertheless, we admit that the correlations between the PDF and the scales might be indeed small as stated in Ref. [16]. We believe, however, that the effects of this correlation cannot be reliably estimated at present; see also the discussion at the end of section 12.5.2 in Ref. [20].

Another important aspect is that the uncertainties in the PDFs discussed up to now are only those due to the errors in the data included in the fits and are thus of experimental origin (and indeed have a probabilistic meaning). However, there are many other possible sources of uncertainties which are, this time, of theoretical and not experimental nature: scale variation and higher order effects in the observables used in the fits, ambiguities in heavy quark flavor scheme definition, difference between the \( \alpha_s \) values that one obtains from DIS and LEP data, etc... These theoretical PDF uncertainties cannot be estimated within a given parametrisation, say the MSTW parametrisation. The only reasonable way to estimate them is to compare the predictions for the central values of the cross section given by different parameterizations (which mostly use the same data). The PDF theoretical uncertainties would be then equivalent to the spread that one observes when comparing different PDF sets. In this case, the PDF uncertainties should be considered as having no statistical ground and would for instance, simply change the normalisation of the cross section.

In our approach when dealing with the MSTW parametrisation only, since we cannot determine the pure theoretical uncertainties, we have simply interpreted the Hessian errors as a measure of the uncertainties due to the theoretical assumptions in the parametrisation of the quark and gluon densities (and, as stated before, equivalent to the spread that one observes when comparing different parameterizations). In this case, the PDF uncertainties cannot be combined in quadrature with the two other theoretical errors, namely the scale and the scheme/EFT uncertainties: all should be considered as pure theoretical uncertainties with a flat prior and added linearly.

In fact, even if the PDF+\( \alpha_s \) uncertainties are taken from the Hessian method and considered as experimental with a Gaussian behavior, it is still not obvious that they should be added quadratically to the scale uncertainties which have a flat distribution being purely
theoretical. The (not obvious) procedure of combining a Gaussian and a flat distribution has been addressed by the LHC Higgs cross section working group (see section 12 of Ref. [20]) and the conclusion was: “As a general rule that is sufficiently conservative only the linear combination of these (scale and PDF) errors can be recommended” (next to last paragraph of the concluding section 12.5.2). Thus, contrary to the statements of the CDF/D0 collaborations there is no consensus on how to add a flat and a Gaussian distribution.

In conclusion, we believe that our approach to combine the theoretical uncertainties is fully justified and, in fact, not the most conservative one (see the next point for a more radical but still justified possibility). If we follow our approach, the combined scale, EFT and PDF+αs uncertainties will lead to an ≈ +41%, −37% total uncertainty on the gg → H cross section at the Tevatron in the mass range M_H ≈ 160 GeV with almost the best sensitivity. This is to be compared to the ≈10% and ≈20% uncertainties assumed, respectively, by the D0 and CDF collaborations. For comparison, the result obtained when one adds linearly, i.e. as recommended by the LHC Higgs cross section working group, the uncertainties from scale (+20%, −17% on the sum of the jet cross sections) and PDFs (+16%, −15% when the MSTW 68%CL PDF+Δexpαs error is multiplied by a factor of two following the PDF4LHC recommendation) assumed by CDF, leading to a total of ≈+36%, −32% for M_H ≈ 160 GeV.

Thus, the uncertainty that we assume is comparable to the one obtained when the CDF scale and PDF uncertainties are combined using the LHC procedure [20], the difference being simply due to the additional O(5%) uncertainty from the use of the EFT approach that we also include. See also Ref. [36] for an independent discussion.

To highlight this very important point (which we hope will close this discussion), we have modified the original Fig. 2 of our recent most paper [2] to include the uncertainty as calculated using the LHC procedure; we append this figure below.

![Figure 1: The production cross section σ^{NNLO}_{gg→H} at the Tevatron using the MSTW PDFs, with the uncertainty band when all theoretical uncertainties are added as in Ref. [1]. It is compared the uncertainties quoted by the CDF and D0 experiments as well as the uncertainty when the LHC procedure [20] is adopted. In the insert, the relative size of the uncertainties compared to the central value are shown.](image)
3. The HERAPDF and ABKM PDF parameterizations

An important criticism made by the CDF/D0 collaborations concerns our choice of considering also the impact of other PDF sets. Let us say here at the outset that we are only “users” of the PDFs provided by different PDF groups. We do not wish to advocate any one determination to be better or worse. We simply wished to answer a simple question as to what would be the effect on the Higgs exclusion limit presented by the CDF/D0 collaborations, if one uses other PDF sets than MSTW. Please note that these issues and differences will be reasonably irrelevant in case of discovery, as there will be many other observations which one can then use to clarify the situation. The whole issue of assessing the theoretical uncertainty in the rates takes a special meaning when we talk of possible exclusion and associate a certain statistical significance with this exclusion. Given the fact that the experiments have not been able to give a limit on the Higgs production cross section multiplied by the branching ratio at a given level of statistical significance, the translation of the reduction in the normalisation on possible exclusion becomes a nontrivial task. We consider that our publication gives at least an indication of the answer, which many a theorists would like to have, independently of the recommendation by the PDF4LHC group.

With respect to the comments in Refs. [17, 18] regarding the use of HERAPDF and ABKM sets, we summarize different relevant facts which address those comments below.

i) An understanding of the difference in the gluon densities at NNLO obtained by MSTW on the one hand and ABKM/HERAPDF on the other, is still a matter under investigation and being studied by experts. For instance, the last word is still to be said about the recent ABM analysis of the possible large effect of the NMC data on the NNLO parameterizations [37]. This analysis shows that the constraints from fixed–target DIS data, in particular data from the NMC experiment, in the PDF fits at NNLO (the impact at NLO is expected to be small so that CTEQ and NNPDF might not be affected) can play a very important role in explaining the large differences between MSTW on the one hand and ABKM/HERAPDF on the other hand, both in the extracted values of the $\alpha_s$ coupling and of the gluon densities at high Bjorken–$x$. Both these effects could impact the Tevatron Higgs exclusion limits and, for instance, the conclusion of the ABM paper states: “[...] the current range of excluded Higgs masses at the Tevatron appears to be much too large”. A final word is still to be said and perhaps it is too early to draw definite conclusions from this new analysis (which the community is looking at very seriously as the consequences for Tevatron and LHC can be indeed far reaching), but this opens up the possibility that some effects which have now become important due to the increased precision of the theoretical calculations might have been overlooked in some of the analyses of the PDF determinations and hence the uncertainties might have been underestimated. Since we are not experts in the subject, we only take the observations of Ref. [37] to mean that the situation is not clear in this respect.

ii) We also have to realise that now with increasing accuracy of the theoretical calculations and data, the way certain corrections, such as target mass corrections to the Deuterium data, possible nuclear effects, were incorporated in the global fits have had to be revisited by the groups which make global fits and they have also observed certain tensions.

iii) It was mentioned in Refs. [17, 18] that the ABKM09 parametrization [22] should not be adopted as it uses only the DIS data in the determination of the gluon densities and that it does not give an acceptable fit of the Tevatron jet data. This is not entirely correct. Indeed, Alekhin, Blumlein and Moch provided very recently a new PDF set [38], in which the D0 Run II dijet data are included. It turns out that the impact of these data on the
ABKM fits is only marginal and that ABKM09 provides indeed a very good description of the Run II jet data. In fact this parametrization seems to perform satisfactorily [39] and for a detailed discussion, we refer to the two talks quoted in Ref. [38].

\textit{iv}) It is true that HERAPDF does not use any jet (either from the Tevatron or from their own DIS experiments) data so far in their determination of parton densities. The ZEUS experiment has demonstrated that the use of jets produced in the DIS process will in fact reduce the uncertainty in the knowledge of gluon density, but as things stand this has not been included so far. The interesting and special feature of the HERAPDF group is that they determine the flavour decomposition using only HERA data, without using the \(\nu\)-DIS data or the lower energy charged lepton DIS data, most of which is with nuclear targets. This makes them qualitatively different from the other data which use global fits. Hence, the predictions with HERAPDF for processes at the Tevatron or the LHC should be taken into account while studying the spread of the predictions due to the PDF uncertainties as this gives a better estimate of the uncertainties or our ignorance. In Moriond–QCD, a talk given by Voica Radescu [40] shows that HERAPDF performs very well in describing the Tevatron data. This is particularly true for the \(W/Z\) data which are described at NLO and, thus, have the gluon splitting there and give an indirect test of the gluon density.

The above points are made only to say that there are issues about the PDF fits that need more investigation and till the dust has settled on this, one should use ABKM and HERAPDF predictions as a reflection of the theoretical uncertainty in the game. As mentioned already, since apart from MSTW, ABKM and HERAPDF are the only parameterizations (together with GJR) which are available\(^5\) at NNLO, it becomes imperative that one assesses the impact of using these for the Higgs production cross section. Incidentally and as mentioned earlier, even the PDF4LHC recommendation asks that one should study the effect of using other parton densities. This is particularly important, for a crucial issue such as the exclusion of the Higgs boson in a certain mass range. In fact, by showing that the use of other PDF sets can have such a dramatic consequence for the (non)exclusion of the Higgs boson at the Tevatron, we had hoped that one could urge the community to understand the difference between the global fits and only the DIS fits, urgently.

Finally, let us take this opportunity to make a \textit{mea culpa}. After our paper [2] had appeared, we realized that an error occurred in the numerical analysis which had led to Fig. 1 of Ref. [2] for the \(gg \rightarrow H\) production cross section when the four NNLO PDF sets are adopted. In the plot with the two HERAPDF sets, the central scales at which \(\sigma_{gg \rightarrow H}^{\text{NNLO}}\) has been evaluated were not set to \(\mu_R = \mu_F = \frac{1}{2}M_H\) as it should have been, but at \(\mu_R = \mu_F = \frac{3}{2}M_H\) which gives the minimal cross section once the scale uncertainty is included. This explains the large difference in the cross section\(^6\), up to 40\%, between the MSTW and HERAPDF predictions\(^7\). We thus present our apologies and produce in Fig. 2 the correct figure where all scales are consistently set to \(\mu_R = \mu_F = \frac{1}{2}M_H\). The difference between the MSTW and HERAPDF predictions reduces now to \(\approx 20\%\) at most, which is indeed much more reasonable. In this case, the smallest value of the cross section is given when using the ABKM set and amounts to \(\approx 20\%-30\%\) in the considered Higgs mass range as noticed in Ref. [1] (this difference is slightly larger if the new ABM10 PDF set is used and for \(M_H \approx 160\) GeV, one has a \(\approx 30\%\) difference [39]).

\(^5\)The NNPDF collaboration is also releasing a PDF set a NNLO.

\(^6\)We thank Graham Watt for pointing out to us that his calculation of the \(gg \rightarrow H\) cross section with HERAPDF does not lead to such a large difference.

\(^7\)Note that the same analysis presented for the LHC in Ref. [41] is not affected by this problem.
Figure 2: The $gg \rightarrow H$ cross section as a function of $M_H$ when the four NNLO PDF sets, MSTW, ABKM, JR and HERAPDF, are used. In the inserts, shown are the deviations with respect to the central MSTW value.

This error does not affect the subsequent discussion and almost does not change our conclusions. Indeed, the main analysis which led to Fig. 1 (which, we believe, is the most important result of our papers) is still valid as we estimate the PDF uncertainties within the MSTW set and our conclusion, that the theoretical uncertainty on the $gg \rightarrow H$ cross section at the Tevatron is \( \approx 40\% \), still holds true.

4. Emulation of the Tevatron limit calculation

We come now to the specific issue of our emulation of the CDF sensitivity on the Higgs boson that we have performed in Ref. [2]. Let us first summarize our main result: by simply adopting a different choice of the PDF set for the signal and main background cross sections, we arrived at a needed luminosity which should be approximately a factor of two larger than the one assumed in the CDF paper (i.e. 5.9 fb\(^{-1}\)) in order to recover the present sensitivity.

A few important remarks can be made on our statistical treatment.

– First of all, one possible way to estimate the uncertainties related to the PDFs is to take the spread in the different predictions of the cross sections when evaluated with the various sets of NNLO PDFs as a measure of this uncertainty (this was how PDF uncertainties were estimated before the advent of the Hessian method). Since the maximal cross section for $gg \rightarrow H$ is obtained with the MSTW set and the minimal one was thought to be with the HERAPDF set which gave in our (incorrect) calculation a 40% lower rate, the PDF uncertainty when one takes the central value from MSTW will be $+0\%$, $-40\%$. If we now chose the parametrisation which gives the smallest $gg \rightarrow H$ cross section as a reference set for the normalisation\(^8\), one can ignore the PDF uncertainty when setting the Higgs exclusion

\(^8\)It was apparently understood by the CDF/D0 collaborations that we increased our PDF uncertainty which resulted in a larger theoretical uncertainty band and we considered the lower limit of this band to (re)estimate the sensitivity. This is not what we did in our paper: we are definitely not adding an extra systematic uncertainty but simply changing the normalisation of the cross section.
limit and the only uncertainties which remain are the scale+EFT uncertainties, which are at the minimum level of 20%. One can then simply use the CDF analysis to derive the Higgs exclusion limit but assuming a normalisation for $\sigma_{gg \rightarrow H}^{\text{NNLO}}$ that is lower compared to that used in the original study but with the same 20% uncertainty. Here, we assume indeed that only the normalisation of the rate will change and that the PDF effects on kinematical distributions will remain the same or small (this is, to our view, not a very bad approximation and we cannot, of course, estimate ourselves the impact on the experimental analysis).

Second, we do not pretend to have the cleanest and most complete treatment of systematic uncertainties and their correlations, which is simply beyond the scope of our paper and is definitely the job of the experimental collaborations. Our goal in Ref. [2] is different and more modest: it is to estimate the relative impact on sensitivity with the variation of the Higgs cross section that is obtained when a different choice of PDFs is made. The estimated value of the sensitivity itself is not very important; what is most important is its relative variation due to the different choice of PDFs. Thus, our estimate of the sensitivity is simply a starting point from which the relative variation has been evaluated. We made the effort of having this starting point as close as possible to the CDF sensitivity. The variation of the sensitivity is then evaluated relatively to our own estimate of the sensitivity and not relatively to the one of CDF. Therefore, the up to 30% difference in sensitivity between our estimate and the CDF one (in passing, it is less than 10% for the observed and less than 30% for expected) should have no impact on the relative variation of the sensitivity and the obtained necessary luminosity to recover the initial sensitivity. Note that CDF has used in the sensitivity estimate a Bayesian method of ratio of profile likelihood, while we use instead a simple frequentist ratio of log likelihood “à la LEP”; this is a possible source which could explain part of the difference in our results.

A final remark concerns the impact of systematic uncertainties and their correlations in estimating the relative variation of the sensitivity. A clean treatment of systematic uncertainties with correlations is indeed very important for a realistic estimate of the sensitivity and we have no doubt that this statistical treatment has been made in a complete way in the CDF analysis. However, since we are using the multivariate output plots of the CDF analysis to estimate the sensitivities and these output plots are supposed to include the entire information on the uncertainties and their correlations, we believe that our treatment is fully satisfactory contrary to the (surprising) statement of Refs. [17, 18]. In addition, as stated above, the (presently not large) systematic uncertainties will have no impact in evaluating the relative variation of the sensitivity when a different choice of PDF is made. Therefore, the entire discussion on including the correlations between the uncertainties in signal and background is not relevant in this context.

In fact, our main result of the needed luminosity to recover the present sensitivity can be checked with a back-of-the-envelope calculation by considering the counting sensitivity formula $S/\sqrt{S+B}$ and by equating $S$ and $B$. One then arrives at the very simple formula for the required luminosity $L'$ as a function of the initial luminosity $L$ and the signal cross section variation factor $f$, $L' = L \left(1 + f\right)/(2f^2)$. Setting the present luminosity to $L = 5.9 \text{ fb}^{-1}$ as in the CDF analysis, one obtains for a 20% and 40% reduction of the $gg \rightarrow H$ cross section, needed luminosities of $L' = 8.3 \text{ fb}^{-1}$ for $f = 0.8$ and $L' = 13.1 \text{ fb}^{-1}$ for $f = 0.6$. These are almost exactly the values that we obtain in our paper, respectively, 8 and 13 $\text{ fb}^{-1}$. This gives us further confidence that our analysis and our main result are correct.

These uncertainties are of course included in our analysis since we have used the multivariate output plots of the paper that have been obtained incorporating the full set of uncertainties.
Nevertheless, due to the error in the determination of the $gg \rightarrow H$ cross section using the HERAPDF set, the interpretation of the CDF/D0 limit when lowering the normalisation of the cross section, has to be modified. Instead of lowering the normalisation by 40%, one has to lower it by 30% which is the difference between the MSTW and ABKM predictions. The luminosity needed by the CDF experiment to recover the present sensitivity is shown in Fig. 3 in this case. With this normalisation and including the 10% uncertainty on the background rate, the needed luminosity to recover the present sensitivity will be slightly less than a factor of two\textsuperscript{10}.

![Graph showing needed L to recover sensitivity](image)

**Figure 3:** The luminosity needed by the CDF experiment to recover the current sensitivity (with 5.9 fb\textsuperscript{-1} data) when the $gg \rightarrow H \rightarrow \ell\ell\nu\nu$ signal rate is lowered by 20 and 30% and with a ±10% change in the $p\bar{p} \rightarrow WW$ dominant background.

### 4. Conclusion

To conclude this note, let us summarize the main points that we put forward in our analysis of the $gg \rightarrow H$ cross section at the Tevatron.

Concerning our discussion of the theoretical uncertainties in the NNLO production rate:

\textbf{i)} The scale uncertainty has not been overestimated in our analysis. We gave several arguments in favor of an extended domain for scale variation and in fact, it turns out that our uncertainty is comparable to that assumed by the CDF/DO collaborations when the $gg \rightarrow H$ cross section is broken into jet cross sections and to the uncertainty advocated in Ref. [26] when the impact of the jet veto is included in the Higgs+0 jet cross section alone.

\textsuperscript{10}Note, however, that the updated results given by the CDF/D0 experiments for the winter 2011 conferences with a luminosity of 7.1 fb\textsuperscript{-1} for CDF, lead to an exclusion limit that is slightly worse than the one quoted here and only the range $M_H = 158–173$ GeV is excluded. Thus, even for a 30% reduction of the production cross section only instead of the 40% used earlier, one still needs $\approx 13$ fb\textsuperscript{-1} data to recover the sensitivity obtained with 7.1 fb\textsuperscript{-1}.
ii) For the uncertainty from the EFT approach, many of its components have been discussed in other papers and we simply made the effort to estimate the overall impact.

iii) We do not believe that we are overestimating the PDF uncertainties. In fact the result that we quote within the MSTW set is exactly the one that is obtained used the PDF4LHC recommendation. In fact, we even believe that we are underestimating these PDF uncertainties, in particular if the analysis of Ref. [37] turns out to be correct.

iv) We do not add linearly the PDF and scale+EFT uncertainties. Our procedure, which has been also advocated in other analyses like Ref. [35], addressed also the theoretical part of the uncertainties. The result that we assume is indeed close to a linear sum (in fact slightly smaller), but a linear combination of scale+PDF uncertainties is exactly the one recommended in the LHC Higgs cross section working group report [20].

v) If the recommendations of the LHC Higgs cross section working group report [20] are adopted for the CDF uncertainties, one would obtain the same uncertainties as the ones that are advocating in the paper (modulo the small EFT uncertainties); see again Fig. 1 that was included here.

vi) The various issues discussed here appear also in the case of Higgs production in supersymmetric extensions of the Standard Model. The theoretical uncertainties turn out to be also rather large in the main production channels [42].

Concerning our emulation of the CDF limit calculation:

i) The PDF effect is not included as being a new source of systematic uncertainty but, rather, included as being a different choice for the PDF set from the one adopted in the CDF analysis, and which affects only the cross section normalisation.

ii) Our goal was not to re-estimate the CDF sensitivity but the relative variation of the sensitivity when the Higgs cross section is changed by a different PDF choice.

iii) Our results are robust regarding the systematic uncertainties and their correlations, since we are using the multivariate outputs of the CDF analysis that include them.

iv) Our main results for the needed luminosity to recover the present sensitivity agree with estimates obtained in a simple and heuristic way. We believe this agreement provides a nice check of our analysis.

v) It is highly desirable that the CDF and D0 collaborations provide us with a fully cut-based analysis which will be easier to follow and reinterpret; we will be more than happy if they could simply redo our analysis in Ref. [2], assume a different normalisation of the production cross section and reinterpret the Higgs mass limit.

Finally, concerning the discussions on the HERAPDF and ABKM parameterizations, let us stress again that they provide reasonable fits to the Tevatron jet data, contrary to an apparently common belief. There are issues about the PDF fits that need more investigation (in particular the point raised recently on the treatment of the NMC data which might lead to a significant impact) and until a better understanding of the large differences between the results of the various sets, one should use the ABKM and HERAPDF predictions as a reflection of the theoretical uncertainty in the game. This is very important since, except from MSTW, they are among the few other parameterizations which are available at NNLO, i.e. the order required to address Higgs production at hadron colliders. It is thus imperative that one assesses the impact of using these two sets for the Higgs production cross section. This is particularly important for a crucial issue such as the exclusion of the Higgs boson in a certain mass range.
In conclusion, and in view of the above arguments, we strongly believe that the analysis that we have developed in our paper [1, 2] scientifically sound (see also Ref. [36]). It could appear at first sight that we have been a little bit conservative in the estimate of the theoretical uncertainties (although recent analyses tend to show that it is far from being the case), but when it comes to a such a crucial issue as excluding the Higgs boson (which we believe is the most important issue in today high–energy physics), it is more recommended than, to the opposite, being too aggressive. A too optimistic analysis that excludes a possibility that can be discovered somewhere else, could affect the credibility of our field.

Acknowledgements: We thank the organizers of the Rencontres de la Vallée d’Aoste as well as Moriond QCD and electroweak in La Thuile this winter for their kind invitations to present our work and for making possible a critical debate with the experimentalists on the issues presented here. We acknowledge the projects SR/S2/JCB 64 DST (India) and ANR CPV-LFV-LHC NT09-508531 (FR) for support.

Note added

Very recently, a new analysis of the PDF and $\alpha_s$ dependence of the Higgs cross-section at the Tevatron and the LHC has been presented by members of the MSTW PDF group [43]. We would like to make here very brief preliminary remarks:

– Our analyses in Refs. [1, 2] should, by no means, be considered as a criticism of the work of the PDF fitters. The message that we wanted to convey was simple: there are other PDF sets than MSTW on the market and the spread between the various predictions is much larger than the estimate of the PDF uncertainty made within one given parametrization. As already mentioned, we are not PDF experts and we simply note that there is a problem (that Ref. [43] also acknowledges, although it is indeed less severe than initially thought by us and the spread in the predictions is only at the level of 25–30% and not 40%) which needs to be addressed by the experts.

– The analysis of Ref. [43] concludes that the PDF sets which do not take into account the Tevatron jet data fail to give a good description of the same. However, it is to be noted that this differs from the conclusions of Ref. [44] which ascribes this as possibly being remedied once the missing higher order corrections to the dijet calculations would be included. It is to be hoped that the data from LHC may help in increasing our understanding in the near future. But at present one can only conclude that the issue is being debated by experts.

– Ref. [43] does not recommend that one includes the theoretical uncertainty on $\alpha_s$ in addition to the PDF+$\alpha_s$ experimental uncertainty as we suggest in our studies. We insist again on the fact this suggestion was made before the PDF4LHC recommendation [32] became available and that the results for the PDF+$\alpha_s$ uncertainty that we obtain using our approach are the same as those obtained using the PDF4LHC recommendation.

– Ref. [43] focuses only on the PDF+$\alpha_s$ issue. This is, however, only part of the points that we raised in our studies. Indeed, besides the PDF+$\alpha_s$ problem, we have also addressed the issues with the scale and with the EFT uncertainties as well as the combination of all sources of uncertainties. We reiterate our main conclusion that if the approach adopted by the LHC Higgs cross section working group [20] is applied to the $gg \to H$ cross section at the Tevatron, one would find an uncertainty that is much larger than that assumed by the CDF/D0 collaborations (see again Fig. 1). We therefore still believe that the CDF/D0 Higgs exclusion limits are not as robust as claimed.
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