How to make a bilayer exciton condensate flow

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Bose condensation is responsible for many of the most spectacular effects in physics because it
can promote quantum behavior from the microscopic to the macroscopic world. Bose condensates
can be distinguished by the condensing object; electron-electron Cooper-pairs are responsible
for superconductivity, Helium atoms for superfluidity, and ultracold alkali atoms in vacuums for coherent
matter waves. Electron-hole pair (exciton) condensation has maintained special interest because it
has been difficult to realize experimentally, and because exciton phase coherence is never perfectly
spontaneous. Although ideal condensates can support an exciton supercurrent, it has not been
clear how such a current could be induced or detected, or how its experimental manifestation
would be altered by the phase-fixing exciton creation and annihilation processes which are inevitably
present. In this article we explain how to induce an exciton supercurrent in separately contacted
bilayer condensates, and predict electrical effects which enable unambiguous detection.

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The order parameter of an exciton condensate is

$$\Psi(\vec{r}) = |\Psi(\vec{r})| \exp(i\phi(\vec{r})) = \langle \psi^\dagger_e(\vec{r}) \hat{\psi}_h(\vec{r}) \rangle \rho(h, \vec{r}; e, \vec{r})$$

where $\psi^\dagger$ and $\hat{\psi}$ are electron creation and annihilation operators, $\rho(\vec{r})$ is the condensate phase, the labels $e$ (electron) and $h$ (hole) refer to the states between which phase
coherence is established (nearly) spontaneously, and $\rho(h, \vec{r}; e, \vec{r})$ is the anomalous density matrix. Microscopic
considerations suggest that spontaneous coherence is likely between a conduction band with occupied states
inside a Fermi surface and a valence band with occupied states outside a nearly identical Fermi surface. Part of
the reason that exciton condensation has not been easy to realize is that sufficiently perfect nesting between conduction
and valence bands is unlikely to occur naturally. The systems of interest here are artificially fabricated bilayers in which the electrons and holes are in well separated two-dimensional electron systems (2DESs), either
semiconductor quantum wells or graphene layers separated by a dielectric barrier. The dielectric barrier
reduces the strength of exciton creation and annihilation processes, and gate control of the density in each layer allows electron and hole band Fermi surfaces to be tuned to the same area. Although simple to describe, this quantum engineering is difficult to execute successfully. Exciton condensation has so far been realized only in the quantum Hall regime in which band dispersion is irrelevant allowing spontaneous coherence to occur between spatially separated conduction bands, or spatially separated valence bands, under circumstances that
are achieved routinely. The considerations explained in this article apply to quantum Hall exciton condensates in the Corbino geometry, in which current flows across the 2DES bulk, but not directly to the Hall bar geometry in which current flows along the 2DES edge.

In their pioneering work on exciton Bose condensation Blatt et al. argued that because an exciton is neutral, condensation cannot lead to spectacular electrical effects. Experimental studies of quantum Hall exciton condensates have already made it clear that this pessimistic conclusion is not valid. The key technical capability not anticipated in 1962 is the possibility of making independent electrical contact to the electron and hole parts of the condensate. In this article we explain how condensation leads to a reorganization of the low-energy charged fermion degrees of freedom which is responsible for dramatic changes in the transport properties of separately contacted condensed bilayers.

Spectacular electrical effects in exciton condensates are enabled by the possibility of exciton superflow. The key issue which arises in addressing these phenomena theoretically is understanding how a supercurrent of neutral excitons can be driven by electrochemical potential differences. We consider a bilayer with contacts to both left (L) and right (R) ends of the separate quantum wells (top ($T$) and bottom ($B$)) between which coherence is established, as illustrated schematically in Fig. 1. In the case of the Corbino geometry transport of quantum Hall condensates L and R refer to the outer and inner edges of an annular 2DES.) For the sake of definiteness we focus our attention on voltage biased transport; our analysis is easily extended to the case of current biased transport and to systems with additional voltage probes along the sample length. The observables in this transport geometry are the currents and voltages in the leads. Exciton condensation alters transport properties by introducing an anomalous symmetry-breaking field in the fermion quasiparticle Hamiltonian (more technically in the one-particle Greens function) which enforces inter-layer phase coherence, opens up a gap at the Fermi level, and allows charge to move freely between layers. In mean-field-theory, the quasiparticle amplitude for tunneling between top (T) and bottom (B) layers is:

$$\langle T, \vec{r}_T | H_{HF} | B, \vec{r}_B \rangle = V_c(|\vec{r}_T - \vec{r}_B|) \rho(T, \vec{r}_T; B, \vec{r}_B)$$

where $V_c(r) = e^2/\epsilon \sqrt{\vec{r}^2 + d^2}$ is the inter-layer Coulomb interaction and $\rho(T, \vec{r}_T; B, \vec{r}_B)$ is the inter-layer component of the density matrix. $\rho(T, \vec{r}_T; B, \vec{r}_B)$ is non-zero
The total current in each layer can be expressed as the sum of the condensate current and a quasiparticle current (QC) driven by lead voltage differences. The QC conduction is related via the Landauer-Büttiker formula to the transmission coefficients of quasiparticle waves incident from the various leads. In order to drive steady state condensate current, the gate voltages must be chosen so that the counter-flow component of the QC is not spatially constant:

\[ \partial_x (J_{T}^{CC}(x) - J_{B}^{CC}(x)) = -\partial_x (J_{T}^{QC}(x) - J_{B}^{QC}(x)) \]  

Condensate currents are induced by space-dependent quasiparticle counterflow currents. This relationship between quasiparticle (QC) and condensate (CC) counterflow currents follows from the separate conservation of charge in each layer. Since condensate currents cannot enter the leads, they must be present in the bulk only when all four lead currents are possible only when these two quantities are equal:

\[ I_{TL} + I_{BR} = I_{TR} + I_{BL}. \]  

In the linear response regime, the four lead currents are proportional to the three independent electrochemical potential differences. Eq. \((1)\) places a restriction on the two independent difference ratios.

One convenient way to reduce the lead-voltage-space dimension experimentally is to connect two contacts with a resistor \(R\), for example in either the loop geometry or the load geometry illustrated in Fig. \((2)\). In the load geometry, the contact voltages are limited to the surface on which \(I_{BL} = I_{BR} = (V_{BL} - V_{BR})/R\). The consequences of this resistive link between contacts are easily anticipated in the most common limit in which the gap due to condensation is large enough or disorder strong enough to prevent quasiparticle current conduction across the
space. Bulk quasiparticle currents are expected to be negligible for Corbino geometry transport in the quantum Hall regime, and we anticipate that they will also be negligible in zero-field bilayer exciton condensates when these are ultimately realized. When quasiparticles cannot flow across the sample, $I_{TL} = I_{BL}$ and $I_{TR} = I_{BR}$. It follows that Eq. (5) is always satisfied in the load geometry. Because it is possible to induce an exciton supercurrent in the bulk of the bilayer, a large charge current can flow through the circuit with a resistance due only to the load resistor and contact resistances at the two ends of the sample. In the case of quantum Hall bilayers, we predict a Corbino resistance for the load geometry which remains constant as temperature $T \to 0$ on bilayer quantum Hall plateaus which are due to exciton condensation [14]. This behavior is in stark contrast with the dramatically increasing resistance expected at low temperatures on plateaus not associated with exciton condensation.

Although the loop geometry of Fig. (2) is naively compatible with steady state counterflow currents, we predict that it cannot support steady state exciton superflow because it does not guarantee that the the exciton supercurrent emitted at the left end of the sample is identical to that absorbed at the right end. Indeed for $I_{TR} > 0$ the loop geometry will lead to $V_{TR} > V_{BR}$ and therefore to quasiparticle current flowing in the wrong direction from top to bottom.

In the load geometry, the effective two-probe quasiparticle conductances at left and right $G_{L,R} = (e^2/h)T_{L,R}$ depend on the details of these contacts and on the quasiparticle Hamiltonian near the sample ends. Taking the $TR$ lead as ground, the voltages on the other leads in this circumstance are $V_{TL} = I ((T_L^{-1} + T_R^{-1})(h/e^2) + R)$, $V_{BL} = I (R + T_R^{-1}(h/e^2))$, and $V_{BR} = I (h/e^2)T_R^{-1}$ where $I_{TL} = I_{BL} = I_{BR} = I_{TR} = I$. When the load resistor $R$ is small, the current which flows through the bilayer systems is limited only by the quantum contact resistances $(h/e^2)T_R^{-1}$. Even when the microscopic electron tunneling amplitude is zero, quasiparticles move freely between layers allowing charge to be conducted across the highly resistive bulk by exploiting the parallel load resistor channel.

In Fig. 3 we plot numerical results for the self-consistently calculated quasiparticle and condensate current distributions in a one-dimensional tight-binding model bilayer system with an electron band in the top layer and an equal density hole band in the bottom layer. The inter-layer tunneling amplitude in the quasiparticle Hamiltonian of this model is

$$\langle T, I' | H_{HF} | B, I \rangle = \delta_{T, I} V \rho(T, I'; B, I)$$

(6)

where $V$ is the short-range inter-layer interaction strength and $I, I'$ is the site indices. Because of the energy gap induced in the quasiparticle Hamiltonian by this tunneling term, the quasiparticle current is deflected to flow between $T$ and $B$ leads at both left and right. The numerical results in Fig. 3 have been obtained by using mean-field theory (Eq. (6)) to calculate the quasiparticle Hamiltonian given the density matrix, and the NEGF formalism [16] to evaluate the non-equilibrium density matrices given the lead voltages and the quasiparticle Hamiltonian. In the top panel of Fig. 3, $V_{BL} = ((V_{TL} + V_{TR}) \pm V')/2$. Because the left and right sides of our model system are identical this choice produces $I_{BL} = I_{BR}$, corresponding to the load geometry. $V'$ is related to the load wire resistance by $V'/(V_{TL} - V_{TR}) = R/(R + T_L^{-1} + T_R^{-1})$. When the non-equilibrium mean-field calculation is carried to self-consistency, the phase of the order parameter develops a spatial gradient and the system carries a steady state
condensate current.

The total current is constant in each layer as required by charge conservation. In the ordered state, the low-energy charged degrees of freedom are reorganized from bare electrons which cannot transfer between layers to exciton-condensate quasiparticles which occupy the two-layers simultaneously and coherently. This profound reorganization of low-lying charged excitations decreases the resistance exponentially compared to the case in which the gap in the quasiparticle spectrum is not due to exciton condensation. In the quantum Hall case for example, we predict load geometry resistances that are orders of magnitude smaller at $\nu = 1$ (the exciton condensation case) than at $\nu = 2$. For the loop geometry, no self-consistent steady-state solutions of the non-equilibrium self-consistent field equations exist. In this case we anticipate enhanced noise due to order-parameter time dependence and dramatically higher steady state time-averaged resistance.

As emphasized in a fundamental early paper by Kohn and Sherrington[1], phase coherence between different bands in a solid can never be completely spontaneous. In the case of bilayer condensates, inter-layer electron tunneling, which creates or annihilates excitons, is expected to be the dominant process which fixes a preferred phase and violates separate charge conservation in each layer. As illustrated in panel b in Fig.(3), however, weak inter-layer electron tunneling has little effect on transport properties. Most experimental anomalies associated with bilayer exciton condensation require only that the quasiparticle tunneling amplitude be dramatically enhanced compared to its bare values. As we now explain the main consequence of bare-electron inter-layer tunneling is partial relaxation of the current-conservation condition, Eq.(1).

The role of a bare electron inter-layer tunneling amplitude is most simply described using the minimal field theory model[2] of a bilayer exciton condensate:

$$E = \int dr \left[ \frac{\hbar^2 \rho_s}{2m^*} |\nabla \theta|^2 - M \Delta_t \cos \theta \right]$$

(7)

where $\theta$ is the inter-layer coherence phase, $\Delta_t$ is the bare-electron tunneling amplitude, $m^*$ is the effective mass in the Landau-Ginzburg type theory, $M = 2|\rho(T, B)|$ is the condensate order parameter, and $\rho_s$ is its superfluid density. In this description $j = (h\rho_s \partial_x \theta)/m^*$ is the exciton condensate supercurrent. Minimizing this energy functional leads to a sine-Gordon equation from which it follows that

$$\partial_x \left[ \frac{1}{2} \lambda^2 (\partial_x \theta)^2 + \cos \theta \right] = 0.$$

(8)

where $\lambda^2 = (\rho_s/M)(\hbar^2/m^* \Delta_t)$ is the model’s Josephson length. From the constant-of-motion in square brackets in Eq.(5), we obtain an explicitly expression for the difference between the condensates currents at opposite ends of the sample:

$$j_R^2 - j_L^2 = (\cos \theta_L - \cos \theta_R) \frac{2}{\lambda^2} \left( \frac{h\rho_s}{m^*} \right)^2$$

(9)

Defining $J = (J_R + J_L)/2$ and $\delta j = j_R - j_L$, we conclude that steady state collective currents are possible provided that the exciton currents injected at left and absorbed at right differ by less than

$$\delta j_{\text{max}} \leq \Delta_t \frac{2\rho_s M^*}{h e^2} \frac{1}{J}.$$  

(10)

The limit on $\delta j$ is closely analogous to the critical current predicted[13] for inter-layer tunneling. For contact resistances are $\sim h/e^2$ we predict that large conductances will occur when $V_{BR}$ is varied by less than $\sim (h/e)\delta j_{\text{max}}$ from its load resistance value. In our one-dimensional toy model for example ($J_L = I_{TL}/e$, $J_R = I_{TR}/e$), Eq.(10) implies that $\delta j_{\text{max}} \sim 0.1$ Rydberg/h for $\Delta_t = 0.005$ (Rydberg) and total current $J = 0.5$ (Rydberg/h). Indeed, our NEGF calculations find a self-consistent solution, illustrated in Fig.(3c), for the voltage configuration $V_{TL} = 0.5$, $V_{BL} = 0.0$, $V_{TR} = -0.5$, $V_{BR} = -0.1$ (in (Rydberg/e) units) (corresponding to $\delta j \sim 0.1$ (Rydberg/h)) but not for larger $\delta j$ in qualitative agreement with this prediction.

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