Tunneling “zero-bias” anomaly in the quasi-ballistic regime

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For the first time, we study the tunneling density of states (DOS) of the interacting electron gas beyond the diffusive limit. A strong correction to the DOS persists even at electron energies exceeding the inverse transport relaxation time, which could not be expected from the well-known Altshuler-Aronov-Lee (AAL) theory. This correction originates from the interference between the electron waves scattered by an impurity and by the Friedel oscillation this impurity creates. Account for such processes also revises the AAL formula for the DOS in the diffusive limit.

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A number of experiments [1] on tunneling in disordered conductors consistently reveal a suppression of the differential conductance at low biases, commonly referred to as zero-bias anomaly. This anomaly has found a coherent explanation in the well-known Altshuler-Aronov-Lee (AAL) theory [2,3] of one-particle density of states. AAL have shown that the electron-electron interaction in the presence of disorder results in a negative correction to the density of states (DOS), which is singular at Fermi energy. In the case of a two-dimensional conductor, the AAL result reads:

$$\frac{\delta \nu^{AAL}(\epsilon)}{\nu} = A \ln(|\epsilon| \tau / \hbar), \quad (1)$$

where the parameter \(A > 0\) depends on the details of the inter-electron interaction, and is inversely proportional to the transport relaxation time \(\tau\). Correction (1) diverges if the electron energy approaches the Fermi level, \(\epsilon \to 0\). The AAL theory assumes the diffusive motion of electrons, which constrains the electron energy to the interval \(\epsilon < h/\tau\). Clearly, in the case of a strong disorder, \(\epsilon \tau \sim \hbar\), this condition is not restrictive. However in cleaner samples (e.g., heterostructures with tunable disorder potential) the energy domain \(\epsilon > \hbar/\tau\) becomes accessible, while the region of applicability of Eq. (1) shrinks.

One may infer from Eq. (1) that the correction to the DOS vanishes at energies larger than \(h/\tau\), i.e., in the quasi-ballistic regime. The main purpose of this Letter is to show that, to the contrary, interaction does lead to a significant correction to the DOS even for electron energies exceeding the inverse transport relaxation time. This correction arises from the interference of scattering on an impurity and on the Friedel oscillation it creates. Account for such processes also revises the original AAL formula for the DOS in the diffusive limit, adding a non-singular however large contribution to Eq. (1).

Electron density of states for energies larger than the inverse transport relaxation time is associated with the electron dynamics on the time scale shorter than \(\tau\). During such a small time an electron does not experience a large number of scattering events, i.e., the scattering on disorder potential can be treated in the lowest order of the perturbation theory, in contrast to the diffusive limit. This approximation accounts only for the trajectories of electrons which were scattered only once. We will show that the logarithmically divergent correction to the density of states appears already in this approximation.

We start with the most instructive case of the finite-range interaction potential, and will calculate the correction to the one-particle DOS in the quasi-ballistic limit due to a single short-range scatterer. Consider an impurity at the origin; its potential \(U_{\text{imp}}(r)\) induces a modulation of electron density around the impurity. In the Born approximation, one can find the oscillating correction to the electron density, \(\delta n(r)\), which is known as the Friedel oscillation:

$$\delta n(r) = -\frac{\nu g \sin(2k_F r)}{2\pi r^2}. \quad (2)$$

Here \(r\) is the distance from the impurity, \(\nu = m/\pi \hbar^2\) is the free-electron density of states, \(m\) is the electron mass, \(k_F\) is the Fermi wave vector, and \(g = \int U_{\text{imp}}(r) dr\). In the presence of interaction \(V(r - r')\) between electrons, this density oscillation produces an additional scattering potential, which can be presented as a sum of Hartree and exchange (Fock) terms:

$$V_H(r) = \int V(r - r') \delta n(r') dr' \quad (3a)$$

$$V_{ex}(r_1, r_2) = V(r_1 - r_2) \frac{\delta \rho(r_1, r_2)}{2}, \quad (3b)$$

where \(\delta \rho(r_1, r_2) \approx \delta n[(r_1 + r_2)/2]\) is the perturbation of the density matrix by the impurity. The exchange interaction occurs only with the electrons of the same spin, which is reflected by the factor \(1/2\) in Eq. (3b). The Hartree-Fock energy \((3a), (3b)\) oscillates as a function of distance from the impurity in the same manner as \(\delta n(r)\) does.

The local density of states is related to the retarded Green function, \(\nu(\epsilon, r) = -(2/\pi) \text{Im} G_R^R(r, r)\). Let us find now the correction \(\delta G_R^R(r, r)\) due to a coherent process, which includes a scattering on the impurity potential itself, and a scattering on the potential \(\delta n(r)\) formed by the Friedel oscillation:
δG_{\epsilon}^R(r, r) = 2g \left\{ G_{\epsilon}^R(r) \int G_{\epsilon}^R(r_1) V_H(r_1) G_{\epsilon}^R(r_1 - r) dr_1 - G_{\epsilon}^R(r) \int G_{\epsilon}^R(r_1) V_{ex}(r_1, r_2) G_{\epsilon}^R(r_2 - r) dr_1 dr_2 \right\} \quad (4)

The Green function $G_{\epsilon}^R(r)$ for a free electron at large distances, $k_F r \gg 1$, and small energies, $\epsilon \ll \epsilon_F$, is

$$G_{\epsilon}^R(r) = \frac{m e^{i\pi/4}}{\hbar^2} \sqrt{2\pi k_F r} e^{i(k_F + \epsilon/\hbar v_F) r} \quad (5)$$

in two dimensions; $\epsilon$ is measured from the chemical potential.

Below we will be interested in the density of states averaged over the spatial scales much larger than the Fermi wave length $\lambda_F \equiv 2\pi/k_F$. Therefore, we should retain only those corrections $\delta\nu(\epsilon, r)$, which are smooth functions of $r$. Let us show now, using the Hartree contribution as an example, that Eq. (4) indeed yields such a correction. This contribution corresponds to the following process. Electron starts motion in the point $r$, then experiences two scatterings, (1) on the impurity potential $V_H(r_1)$, and (2) on the potential $V_{ex}(r_1, r_2)$ formed by the Friedel oscillation in point $r_1$, and finally returns to point $r$, see Fig. 1. Motion along this closed contour is represented in Eq. (3) by the product $G_{\epsilon}^R(r) G_{\epsilon}^R(r_1) G_{\epsilon}^R(r_1 - r) \propto \exp[i\phi(r, r_1)]$, where

$$\phi(r, r_1) = (r + r_1 + |r_1 - r|)(k_F + \epsilon/\hbar v_F) \quad (6)$$

is the geometric phase acquired by the electron. There is another strongly oscillating factor in the integrand of Eq. (4) – the scattering potential $V_H(r_1) \propto \sin(2k_F r_1)$. Obviously, the result of integration is determined by the domain in space where the total phase of the integrand, $\phi(r, r_1) - 2k_F r_1$, is a slow function of $r_1$. The corresponding electron trajectories are those close to the straight line, see trajectory $A$ on Fig. 1. At $r_1 > r$, Eq. (3) yields the total phase of the integrand $2(\epsilon/\hbar v_F) r_1$. Remarkably, this phase does not depend on $r$. As a result, the Hartree correction to the Green function, Eq. (3), becomes a non-oscillating function of $r$. Similar arguments can be applied to the evaluation of the exchange correction to the Green function. The resulting expression for the interaction correction to the local DOS is

$$\delta\nu(\epsilon, r) \approx \frac{|V(0) - 2V(2k_F)| \nu g^2}{8k_F^2 r^2}, \quad (7)$$

if the distance from the impurity lies within the interval $\max\{\lambda_F, d\} \lesssim r \lesssim \hbar v_F/\epsilon$, drops rapidly ($\propto 1/r^3$) at $r \approx \hbar v_F/\epsilon$, and saturates at $r \lesssim \max\{\lambda_F, d\}$. Here $d$ is the characteristic spatial scale of the interaction potential, and $V(0)$ and $V(2k_F)$ are the Fourier components of the interaction potential appearing from the exchange and the Hartree terms respectively.

In order to find the averaged density of states one should sum up contributions of the type given by Eq. (7) from all the impurities and then average over point $r$ where the correction is measured. Introducing the concentration of impurities $n_i$ and using $\hbar/\tau = 2\pi \nu n_i g^2$, we arrive to the following expression for the interaction correction to the averaged DOS in quasi-ballistic ($\epsilon \gg \hbar/\tau$) limit:

$$\frac{\delta\nu(\epsilon)}{\nu} \equiv \frac{\langle \delta\nu(\epsilon, r) \rangle}{\nu} = \frac{|V(0) - 2V(2k_F)| \nu \hbar}{4\pi \epsilon_F \tau} \ln \frac{\epsilon}{\Delta} \quad (8)$$

with $\Delta = \min\{\epsilon_F, \hbar v_F/\tau\}$.

![FIG. 1. Two typical trajectories (A, B) of an electron scattered by an impurity (I) and by the corresponding Friedel oscillation (concentric arcs). The correction $\delta\nu(\epsilon, r)$ is dominated by the trajectories of the type A, for which the electron is almost scattered back at I and $r_1$.](image)

In principle, the correction can have any sign depending on the relation between $V(0)$ and $V(2k_F)$. However, in any realistic system the inter-electron interaction is sufficiently smooth, $d \gtrsim \lambda_F$, and $V(0) \gg V(2k_F)$. Therefore, in the following we will concentrate on the exchange contribution, which always dominates in the correction to the DOS.

The derivation of Eq. (8) is valid for energies $\epsilon$ exceeding $\hbar/\tau$, which is the high-energy cutoff in the AAL theory, see Eq. (1). Quasi-ballistic formula (8) at the boundary of the region of its applicability, $\epsilon \sim \hbar/\tau$, does not match the AAL result (1). The reason of this mismatch is the choice of the high-energy cutoff. Physically, AAL cutoff means taking into account only electron states within a narrow energy strip, $-\hbar/\tau \lesssim \epsilon < 0$, below the Fermi level, in the formation of the Friedel oscillation. This cut-off was dictated by the range of applicability of the diffusion approximation for the electron dynamics AAL used [23]. On the other hand, our analysis leading to Eq. (8) demonstrates that electron states within a much wider strip, $-\Delta \lesssim \epsilon < 0$, are important for the correction. As it turns out, this wider strip is important for the calculation of the DOS at $\epsilon \lesssim \hbar/\tau$ as well. To show this and to remedy the mismatch, below we calculate the DOS without using the diffusion approximation.

We are interested in the spatially averaged density of states, which makes it possible to use the standard di-
agrammatic techniques. The correction to the averaged one-particle density of states has the form:

\[ \delta \nu(\epsilon, T) = -\frac{2}{\pi} \Im \int \frac{dp}{(2\pi)^2} \delta G(ie_n - \epsilon + i0, p), \]  

where \( \epsilon_n = \pi(2n + 1)T \) is Matsubara frequency, \( T \) is the temperature. (For brevity we omit the Planck constant in all the intermediate formulas.) We will calculate the correction to the electron propagator, \( \delta G(ie_n, p) \), in the first order in the screened electron-electron interaction \( V_{se}(i\Omega_l, Q) \). In the metallic regime \( (\varepsilon_F \tau \gg 1) \) the exchange contribution to the propagator is:

\[ \delta G(ie_n, p) = -\frac{[G(ie_n, p)]^2 T}{\Omega_l} \int \frac{dQ}{(2\pi)^2} \theta(\epsilon_n - \epsilon_n) \times [\Gamma(i\Omega_l, Q)]^2 V_{se}(i\Omega_l, Q) G(ie_n - \Omega_l, p - Q). \]  

(10)

Here \( G(ie_n, p) = [ie_n - \xi_p + (i/2\tau) \text{sign} \epsilon_n]^{-1} \) is the electron Green’s function in the dirty conductor, and \( \Gamma \) is the vertex function calculated in the ladder approximation. As long as we are developing theory applicable for any relation between electron energy and \( \hbar/\tau \), we cannot use the usual diffusion form for vertex function. The formula valid for an arbitrary momentum \( Q \) and energy \( \Omega_l \) transfer is:

\[ \Gamma(i\Omega_l, Q) = \left( 1 - \frac{1/\tau}{\sqrt{|\Omega_l| + 1/\tau)^2 + (v_F Q)^2}} \right)^{-1}. \]  

(11)

Note, that in the limit \( \Omega_l, v_F Q \ll \tau^{-1} \), Eq. [1] reduces to the standard diffusion expression.

Integration over \( p \) in Eq. [3] gives

\[ \frac{\delta \nu(\epsilon, T)}{\nu} = -\text{Re} \lim_{\epsilon_n \to \epsilon + i0} T \sum_{\Omega_l} \int \frac{dQ}{(2\pi)^2} \theta(\epsilon_n - \epsilon_n) \times [\Gamma(i\Omega_l, Q)]^2 V_{se}(i\Omega_l, Q) G(ie_n - \Omega_l, p - Q). \]  

(12)

The case of a finite-range electron-electron interaction is especially simple, because we can replace \( V_{se}(i\Omega_l, Q) \) in Eq. [2] by the Fourier component of the unscreened interaction potential \( V(Q) \). In this case the correction derived from Eq. [12] coincides with the exchange term in formula [5]. It means that the exchange correction to the DOS

\[ \frac{\delta \nu(\epsilon)}{\nu} = \frac{V(0)\hbar}{4\pi \varepsilon_F \tau} \ln \left| \frac{\epsilon}{\Delta} \right| \]  

(13)

is universal, i.e., is valid for the energies both larger and smaller than \( \hbar/\tau \).

For the long-range Coulomb interaction, \( V(Q) = 2\pi e^2/Q \), the screening should be taken into account, \( V_{se}(i\Omega_l, Q) = V(Q)/[1 + V(Q)\Pi(i\Omega_l, Q)] \). Here the polarization operator

\[ \Pi(i\Omega_l, Q) = \nu \left( 1 - \frac{\Gamma(i\Omega_l, Q)\Omega_l}{\sqrt{|\Omega_l| + 1/\tau)^2 + (v_F Q)^2}} \right) \]  

(14)

is derived in the random phase approximation.

Straightforward evaluation of Eq. [2] with the account for Eqs. [1] and [4] yields

\[ \frac{\delta \nu(\epsilon, T)}{\nu} = -\frac{1}{8\pi \varepsilon_F \tau} \int_{\epsilon}^{\Delta} d\Omega \ln \left( \frac{v^2_{F}\alpha_B^2}{\Omega^2 + (1/\tau)^2} \right), \]  

(15)

where \( \alpha_B = 1/me^2 \) is the Bohr radius, \( \varepsilon \equiv \max\{|\epsilon|, T\} \), and the cut-off energy \( \Delta \) is given now by \( \Delta = \hbar v_F/\alpha_B \).

Equation [15] gives the correction to the one-particle DOS due to the Coulomb interaction and is the main quantitative result of this Letter. The integral in Eq. [15] cannot be expressed in terms of elementary functions. However, the diffusive \( (\varepsilon \ll \hbar/\tau) \) and quasi-ballistic \( (\varepsilon \gg \hbar/\tau) \) asymptotic behaviors are easily found.

In the diffusive limit exchange correction to the one-particle DOS has the form:

\[ \frac{\delta \nu_{dif}(\epsilon)}{\nu} = -\frac{\hbar}{16\pi \varepsilon_F \tau} \left\{ \ln \left( \frac{\bar{\varepsilon}}{\hbar D^2 a_B^2 \tau} \right) \ln(\bar{\varepsilon}/\hbar) \right\} - 2 \left\{ \ln(\tau \Delta/\hbar) \right\}, \]

(16)

where \( D = v^2_F \tau/2 \) is the diffusion coefficient. The first term of the sum in [14] is the famous result of the Altshuler-Aronov-Lee theory [2]. The second, new term is not singular. This part of the correction represents the contribution of electrons deep in the Fermi sea, with energies below the "\( \hbar/\tau \) strip".

**FIG. 2.** The crossover function \( f(x) \), see Eq. [8].

In the quasi-ballistic limit exchange correction to the one-particle DOS is:

\[ \frac{\delta \nu_{bal}(\epsilon)}{\nu} = -\frac{\hbar}{8\pi \varepsilon_F \tau} \left\{ \ln(\bar{\varepsilon}/\Delta) \right\}^2. \]

(17)

The leading term in the energy dependence of the correction is \( \propto (\ln \bar{\epsilon})^2 \) at any relation between \( \bar{\epsilon} \) and \( \tau \). In
the crossover region, the correction is described by the formula
\[
\frac{\delta \nu(\bar{\epsilon})}{\nu} = \frac{\delta \nu_{\text{bal}}(\bar{\epsilon})}{\nu} = -\frac{\hbar}{8\pi \epsilon F \tau} f(\bar{\epsilon} \tau), \tag{18}
\]

The dimensionless function \( f(x) \) is plotted in Fig. 2. Its asymptotes, \( f(x \to \infty) = 0 \) and \( f(x \to 0) = -(1/2)|\ln(x)|^2 \), allow one to obtain from (18) the limits (17) and (18) respectively.

In the practically important case of a gated semiconductor structure, the Coulomb potential is suppressed at all distances exceeding the separation \( d \) between the two-dimensional electron gas and the gate. If \( d > a_B \), then the correction to the density of states coincides with Eq. (3) at energies \( \bar{\epsilon} > h c_F / \sqrt{a_B d} \), and becomes logarithmic (18) at smaller energies,
\[
\frac{\delta \nu(\bar{\epsilon})}{\nu} = -\frac{\hbar}{8\pi \epsilon F \tau} \ln(d/a_B) \ln(\bar{\epsilon}/\Delta), \tag{19}
\]
with \( \Delta = h c_F / a_B \). This formula is applicable for an arbitrary relation between \( \epsilon \) and \( \hbar / \tau \).

The density of states Eq. (3) describes adequately the electron tunneling without the lateral momentum conservation, such as tunneling through an inhomogeneous barrier. However, the electron-electron interaction affects the tunneling through a homogeneous barrier as well. We will consider below tunneling between two identical quantum wells (QW), assuming (in accordance with the experiments [6]) the lateral momentum conservation in the course of tunneling. The correction to the conductance we find, has a logarithmic zero-bias anomaly.

In the absence of disorder and of the electron-electron interaction, the conservation of in-plane momentum implies that an electron can tunnel only if the levels of spatial quantization in the wells line up precisely. This makes differential tunneling conductance of a symmetrical double-QW system have a singular peak at zero bias. Disorder alone smears the singularity, leading to the \( I-V \) characteristic (18)
\[
I_0(V) = G_0 \frac{eV (h/\tau_s)^2}{(eV)^2 + (h/\tau_s)^2}, \tag{20}
\]
with the width given by the inverse quantum lifetime of electrons in the wells, \( 1/\tau_s \). Here \( V \) is the bias applied to the contact, and \( G_0 \) is the zero bias conductance.

Electron-electron interaction adds a singular at zero bias, negative logarithmic correction to the current. Details of the calculations will be published elsewhere; the result for the interaction correction to the tunneling current is:
\[
\frac{\delta I(V)}{I_0(V)} \approx \frac{\hbar}{\pi \epsilon F \tau} \ln(d/a_B) \ln(eV/\Delta). \tag{21}
\]
Here \( eV \equiv \max\{eV, T\} \) is assumed to satisfy (18) the conditions \( eV \ll h/\tau_s, v_F / \sqrt{a_B d} \), and \( d \) is the separation between wells. Qualitatively Eq. (21) and Eq. (19) have the same feature: the upper cut-off for the correction is \( \Delta = h c_F / a_B \), and by no means \( h / \tau \). The \( \ln(eV) \) bias dependence, instead of \( \ln(eV)^2 \) one, appears in Eq. (21) because of a partial cancellation (18) of the corrections coming from the intra- and inter-well electron-electron interaction. In the absence of interaction, Eq. (22) would lead to a peak in the differential conductance \( dI/dV \) at zero bias. Negative diverging correction (21) splits this peak in two. The separation between the maxima of these two sub-peaks is:
\[
eV_{sp} = \frac{\hbar}{\tau_s \sqrt{8\pi \epsilon F \tau / \hbar}}. \tag{22}
\]
The sub-peaks should be resolved at sufficiently low temperatures, \( T \ll eV_s \). Estimate for \( V_{sp} \) for the data of Turner et al., Ref. [3], gives \( V_{sp} \approx 0.05mV \). It is important, that Eqs. (21) and (22) are valid at any relation between \( eV, eV_{sp} \), and the energy \( \hbar / \tau \).

In summary, we studied the tunneling density of states of interacting two-dimensional electron gas beyond the diffusive limit. A significant interaction-induced suppression of the density of states persists at electron energies even larger than the inverse transport relaxation time, which could not be expected from the well-known Altshuler-Aronov-Lee theory [3]. The AAL formula for the density of states at low energies is also revised, and an additional non-singular, however large, contribution was found.

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larger than the previously known result, see A. Yu. Zuzin, Pis’ma Zh. Eksp. Teor. Phys. 33, 377 (1981) [JETP Lett. 33, 360 (1981)] and Ref. 3.

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