Active and reactive power in stochastic resonance for energy harvesting

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A power allocation to active and reactive power in stochastic resonance is discussed for energy harvesting from mechanical noise. It is confirmed that active power can be increased at stochastic resonance, in the same way of the relationship between energy and phase at an appropriate setting in resonance.

INTRODUCTION

Since noise appears everywhere in our surroundings, the energy conversion from noise to controlled motion is a key to come it to use. As an energy harvester from thermal noise, the molecular sized brownian ratchet was suggested, but this machine was verified its impossibility [1, 2]. On the other hand, stochastic resonance (SR) has been suggested as a method for energy harvesting from noise [3–5]. SR is a phenomena in which noise with moderate strength enhances SNR (Signal Noise Ratio) [6, 7]. SR is possibly to apply in biological machine was verified its impossibility [1, 2]. On the other hand, stochastic resonance (SR) has been suggested as a method for energy harvesting from noise [3–5]. SR is a phenomena in which noise with moderate strength enhances SNR (Signal Noise Ratio) [6, 7]. SR is possibly to apply in biological

Here, we discuss SR for harvesting energy from white noise and the method to enhance the energy. This letter develops a concept of power factor correction as same as electrical systems, and surveys a power allocation to active and reactive power. Finally it is clarified that active power of controlled motion is maximized at SR.

SYSTEM AND POWER EQUATIONS

Suggested energy harvesters by the method of SR possess bi-stable potential [3–5]. Here is assumed that dynamical equation of the SR harvesters are represented altogether by

The following formula:

\[
\frac{d\dot{x}}{dt} = -mg - \frac{\partial V(x, t)}{\partial x} + R(t),
\]

\[
-\frac{\partial V(x, t)}{\partial x} = -\frac{dU(x)}{dx} + h \cos \Omega t
\]

\[
= -(x^3 - x) + h \cos \Omega t
\]

where \( R(t) \) is given as zero-mean white Gaussian noise of auto-correlation function:

\[
< R(t)R(t+\Delta t) > = 2\nu m kT \delta(\Delta t).
\]

Where \( m \) denotes oscillator mass of a harvester, \( x \) displacement, \( \nu m \) damping constant, \( h \cos \Omega t \) sinusoidal external force, \( U(x) \) bi-stable potential, \( k \) Bolzmatan constant, and \( T \) noise temperature. \( \delta \) implies time differential \( d/dt \) and \( < > \) ensemble average operation. We take dissipative energy of controlled motion at frequency \( \Omega \) as harvested energy.

Without loss of generality, we can introduce the previously obtained theoretical relationship from [14–16]. In particular, \( Q \) and \( \phi \) are defined in [14] and [15].

\[
< x > = Q \cos(\Omega t - \phi)
\]

\[
Q = \frac{h}{kT} W \sqrt{W^2 + \omega^2}
\]

\[
\phi = \arctan \left( \frac{(\Omega/\Omega_r)(\Omega^2 W + \Omega^2 kT)}{\omega^2 W^2 + \Omega^2 kT} \right)
\]

\[
\frac{W}{2} = \frac{\omega R \Omega_r}{2\pi \nu m} \exp(\Delta U/kT)
\]

Response of amplitude \( Q \) and phase \( \phi \) for noise intensity \( kT \) are shown in Fig. 1. \( \Omega_r \) is a relaxation rate defined by \( d^2 U/dx^2 \lvert_{x=1}^{15} \). \( W/2 \) corresponds to Kramers rate [16] at \( \omega_0 = d^2 U/dx^2 \lvert_{x=1}^{15} \). \( \omega_0 = d^2 U/dx^2 \lvert_{x=0}^{15} \), and potential barrier \( \Delta U = 1/4 \). According to the reaction of \( Q \) in Fig. 1, SR appears around \( kT = 0.1 \). When SR appears, the dissipative energy becomes maximum as confirmed in [15]. The results explain that the external force frequency \( \Omega \) matches the time lag to overcome the potential barrier for the harvester.

On the other hand, when a harmonic oscillator has phase lag \( \pi/2 \) to sinusoidal external force, so called resonance appears, then dissipative energy becomes maximum. Similar phenomena is also confirmed in nonlinear oscillators [17]. In Fig. 1 phase lag \( \phi \) becomes maximum at SR [19, 15]. However, it has not been accounted for the relationship between phase lag and dissipative energy.

![FIG. 1: Behaviors of amplitude Q and phase \( \phi \) as functions of \( kT \), where \( m = 0.02, \gamma m = 3.00, h = 0.20 \), and \( \Omega = 0.04 \). Both Q and \( \phi \) show their single peaks.](image-url)
ACTIVE AND REACTIVE POWER

Here we focus on the relationship of phase lag $\phi$ and energy flow in system of Eqs. (1) and (3). Since there are two external forces; sinusoidal force and noise, it is difficult to decide each contribution. For energy harvesting by SR, it is necessary to see the energy flow. From Eq. (4), the following relationship is obtained.

$$\frac{m}{2} \frac{d}{dt} < \dot{x}^2 > = -m \gamma < \dot{x}^2 > - \frac{dU}{dx} \dot{x} > + < \dot{x} > h \cos \Omega t + < R(t) \dot{x} >. \quad (7)$$

Here allocates each term to input, active, and reactive power. Through the mechanical-electrical analogy [18], electrical contribution is explained as the mechanical energy flow. Generally active and reactive power are averaged over a period. On the other hand, instantaneous input, active, and reactive power are depicted as follows:

**Input power:**

$$< \dot{x} h \cos \Omega > + < R(t) \dot{x} >$$

$$= -\frac{1}{\gamma m} \left( \frac{\partial V}{\partial x} \right) h \cos \Omega t + y kT, \quad (8)$$

**Active power:**

$$< m \gamma \dot{x}^2 > = y kT + \frac{1}{\gamma m} \left( \frac{\partial V}{\partial x} \right)^2, \quad (9)$$

**Reactive power:**

$$\frac{m}{2} \frac{d}{dt} < \dot{x}^2 > + \left( \frac{dU}{dx} \dot{x} > \right)$$

$$= -\frac{1}{\gamma m} \left( \frac{\partial V}{\partial x} \right)^2 - \frac{1}{\gamma m} \left( \frac{\partial V}{\partial x} \right) h \cos \Omega t, \quad (10)$$

where the following two equations derived from the Fokker-Plank equation are substituted into Eq. (7).

$$< \dot{x}^2 > = \frac{kT}{m} + \frac{1}{\gamma^2 m^2} \left( \frac{\partial V}{\partial x} \right)^2 \quad (11)$$

$$< \dot{x} > = -\frac{1}{\gamma m} \left( \frac{\partial V}{\partial x} \right) \quad (12)$$

In Eqs. (8), (9), and (10), those power are consisted of following $P_i$ ($i = 1, 2, 3$):

$$P_1 = -\frac{1}{\gamma m} \left( \frac{\partial V}{\partial x} \right) h \cos \Omega t, \quad P_2 = \frac{1}{\gamma m} \left( \frac{\partial V}{\partial x} \right)^2, \quad P_3 = y kT.$$

Here we average $P_i$ ($i = 1, 2, 3$) over a period $2\pi/\Omega$, and express them as $\overline{P}_i$ ($i = 1, 2, 3$). Fig. 2 shows them as functions of noise strength $kT$. $\overline{P}_3$ is in proportion to $kT$. $\overline{P}_1$ and $\overline{P}_2$ reach their peak around SR.

**Numerical estimation**

Figure 3 shows time dependence of displacement $x$ at $kT = 0.03, 0.07,$ and $0.40$. SR appears around $kT = 0.07$. Fig. 4 shows numerically calculated $\overline{P}_1, \overline{P}_2$ from Eqs. (11) and (2). Blue (gray) bullets at each $kT$ are values of ten trials, and red (black) squares are ensemble averaged value from the ten trials. $\overline{P}_1$ and $\overline{P}_2$ become maximum locally around $kT = 0.07$ where SR appears. However the peaks are not clear as in Fig. 2. $\overline{P}_2$ increases at $kT = 0.5$. This is because of a reduction in calculation accuracy as we can see that the variance of blue (gray) bullets increases with noise intensity $kT$ growth.

**Fig. 2:** $\overline{P}_1$ (red solid line), $\overline{P}_2$ (blue dotted line), $\overline{P}_3$ (green broken line) as a function of $kT$ calculated theoretically.
FIG. 3: Time changes of displacement $x$ under the noise intensity $kT = 0.03$, 0.07, 0.40 with $h$ and $\Omega$ kept constant.

FIG. 4: Numerically estimated $P_1$ and $P_2$ under different noise strength.

CONCLUSION

In this letter, SR is investigated as a method for energy harvesting from noise. When SR appears, the phase lag and dissipative power of frequency $\Omega$ are maximized at an appropriate noise intensity. The result suggests the possibility of maximizing dissipative energy of controlled motion as same as the analogous to power factor correction.

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