Adaptive control systems with linear regressor and parameter identification algorithm applied to a magnetic levitation system

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Abstract. Conventional regulators or controllers are used regularly in control engineering, however, there are limitations to their use. In this document we develop the essential mathematical concepts and expressions in the study of adaptive control techniques. To do this, we will initially make a brief deduction of the equations that are identified in a linear regressor control system, and also for the case of the parameter identification algorithm. Next, we will apply this control technique in a non-linear state space system for a magnetic levitation system, allowing us to conclude, among other things, that this technique makes the control more robust.

1. Introduction
Conventional regulators or controllers are regularly used in control engineering as circuits that allow the introduction of active elements in the power circuits, however, we always find some limitation, such as interventions due to switching that introduce unbounded variations on the system in very short times, preventing the analysis through classical techniques such as the Laplace transform since the initial conditions could not be found due to the non-continuity of the state from zero minus to zero plus.

A particular case is the identification of the system and the calculation of the controller parameters that must be carried out offline, these parameters are fundamental since they oscillate in wide ranges which are highly elastic, so non-linearities in the system can cause instabilities in response, these types of models are the most representative in the industry. By using adaptive control, one of its advantages will be online operation. On many occasions, theoretical systems operate very well with the condition of operating offline, which is unlikely to happen in real systems, which is why more complex designs need to be implemented [1].

Adaptive control allows the automatic control system to be coupled to changing circumstances of behavior in the dynamics of a system and to turbulences [2], which is suitable for most industries since it allows operating on non-linear systems and allows separating the state of the process on two-time scales that evolve at different speeds [3]. A slow scale associated with the changes of the parameters and therefore with the speed with which the regulator parameters are modified, and a fast scale, which corresponds to the dynamics of the ordinary feedback loop [4].

Finally, the parameters are identified in an optimal way, thus avoiding the techniques based on heuristic or metaheuristic optimization, which can be computationally efficient but without guaranteeing global optimizations, which is inadequate since the answers found do not suggest being the most adequate.
2. Content
Adaptive control can be defined as the combination of a linear control law with an online identification algorithm as seen in the block diagram of Figure 1. To perform the analysis of this type of controllers it is necessary to initially study the concepts of linear regressor, identification algorithm and convergence test with stability [5].

![Figure 1. Block diagram.](image)

2.1. Linear return
It is considered a linear system, invariant in time, represented by the ordinary differential Equation (1).

\[ M \left( \frac{d}{dt} \right) y(t) = N \left( \frac{d}{dt} \right) u(t). \]  

(1)

The polynomials \( M(s), N(s) \in \mathbb{R}(s) \), are given by the Equation (2) and Equation (3).

\[ M(s) = s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n, \]  

(2)

\[ N(s) = b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1} s + b_m, \]  

(3)

with \( m \leq n \). The filters for the input and output of the system shown in Figure 2 and Figure 3, with their respective model, are considered Equation (4) and Equation (5).

![Figure 2. Filter input system.](image) ![Figure 3. Filter output system.](image)

\[ F \left( \frac{d}{dt} \right) u_f(t) = u(t), \]  

(4)

\[ F \left( \frac{d}{dt} \right) y_f(t) = y(t), \]  

(5)

where \( F(s) \in \mathbb{R}(s) \), is stable according to the Hurwitz stability criteria and has the form Equation (6).

\[ F(s) = s^n + f_1 s^{n-1} + \ldots + f_{n-1} s + f_n. \]  

(6)

Substituting of the filters, Equation (4) and Equation (5), in the differential Equation (1), you have to Equation (7).

\[ F \left( \frac{d}{dt} \right) \left[ M \left( \frac{d}{dt} \right) y_f(t) - N \left( \frac{d}{dt} \right) u_f(t) \right] = 0. \]  

(7)

You can enter the function \( \xi(t) \) Equation (8).

\[ F \left( \frac{d}{dt} \right) \xi(t) = 0. \]  

(8)
So, you can write the equation of the original function as Equation (9):

$$ M\left(\frac{d}{dt}\right)y_f(t) = N\left(\frac{d}{dt}\right)u_f(t) + \xi(t). \quad (9) $$

Since $F(s)$ is a stable Hurwitz polynomial, then it is said that $\exists k, \alpha \in \mathbb{R}^+$, such that Equation (10).

$$ |\xi(t)| \leq ke^{-\alpha t}, \forall t \geq 0. \quad (10) $$

For a sufficiently large $\alpha$, the behaviors of Equation (9) and Equation (10) will be approximately equal. Performing the development of Equation (9) the following linear regressor is obtained Equation (11).

$$ y_f^n(t) = \theta^T\phi(t) + \xi(t), \quad (11) $$

where $\theta^T$ and $\phi^T$ are defined by Equation (12) and Equation (13)

$$ \theta^T = (a_1 \cdots a_n b_0 \cdots b_m) \in \mathbb{R}^{n+m+1}, \quad (12) $$

$$ \phi^T = (-y_{f}^{(n-1)} \cdots -y_f \ u_t^{(m)} \cdots \ u_f) \in \mathbb{R}^{n+m+1}, \quad (13) $$

and are called the parameter vector $\theta^T$ and measurement vector $\phi^T$.

### 2.2. Online identification algorithm

The identification algorithm can be understood as the procedure for estimating the parameter vector $\theta$ by minimizing a predetermined criterion. The least squares identification algorithm is considered, which is presented below [6-7]. You want to find the parameter vector that minimizes the criterion of Equation (14):

$$ J\left(\hat{\theta}(t)\right) = \int_0^t \lambda(\tau)e^2(t, \tau)d\tau, \quad (14) $$

where $\lambda(\tau)$ and $e(t, \tau)$ are determinate by Equation (15) and Equation (16).

$$ 0 < \lambda(\tau) \leq 1, \quad (15) $$

$$ e(t, \tau) = \hat{\theta}^T(t)\phi(\tau) - y_f^{(n)}(\tau), \ \tau \in [0, t], \quad (16) $$

The criterion to minimize $J\left(\hat{\theta}(t)\right)$ is derived with respect to $\hat{\theta}(t)$ and is equal to zero obtaining Equation (17).

$$ \frac{\partial J\left(\hat{\theta}(t)\right)}{\partial \hat{\theta}(t)} = 2\int_0^t \lambda(\tau)\left(\hat{\theta}^T(t)\phi(\tau) - y_f^{(n)}(\tau)\right)\phi(\tau)d\tau = 0, \quad (17) $$

clearing $\hat{\theta}(t)$ is defined by Equation (18).

$$ \hat{\theta}(t) = \left(\int_0^t \lambda(\tau)\phi(\tau)\phi^T(\tau)d\tau\right)^{-1}\int_0^t \lambda(\tau)\phi(\tau)y_f^{(n)}(\tau)d\tau. \quad (18) $$

The Hessian for the minimization of $J\left(\hat{\theta}(t)\right)$ is Equation (19) [8,9].
\[
\frac{\partial^2 \dot{\theta(t)}}{\partial \theta(t) \partial \dot{\theta}(t)} = 2 \int_0^T \lambda(\tau) \phi(\tau) \phi^T(\tau) d\tau > 0. \tag{19}
\]

The matrix is defined Equation (20).

\[
P(t) = \left( \int_0^T \lambda(\tau) \phi(\tau) \phi^T(\tau) d\tau \right)^{-1}. \tag{20}
\]

Therefore, \( \dot{\theta}(t) \) is expressed by Equation (21).

\[
\dot{\theta}(t) = P(t) \int_0^T \lambda(\tau) \phi(\tau) y^{(n)}_\tau (\tau) d\tau. \tag{21}
\]

Deriving the previous Equation (21) with respect to time the following expressions are obtained Equation (22), Equation (23) and Equation (24).

\[
\frac{d\theta(t)}{dt} = -\lambda(t) P(t) \phi(t) e(t), \tag{22}
\]

\[
\frac{dP(t)}{dt} = -\lambda(t) P(t) \phi(t) \phi^T(t) P(t), \tag{23}
\]

\[
e(t) = \theta^T(t) \phi(t) - y^{(n)}_\tau(t). \tag{24}
\]

3. Application

The non-linear state space system is considered for a magnetic levitation system given by the Equation (25), Equation (26), Equation (27) and Equation (28) [10].

\[
\dot{x}_1 = x_2, \tag{25}
\]

\[
\dot{x}_2 = g - \frac{c}{m} \frac{x_3^2}{x_1}, \tag{26}
\]

\[
\dot{x}_3 = -\frac{R}{L} x_3 + \frac{1}{L} u, \tag{27}
\]

\[
\dot{y} = x_4. \tag{28}
\]

For which an approximate linearization model described by the matrices is obtained (Equation (29), Equation (30), Equation (31) and Equation (32)).

\[
A = \begin{bmatrix}
0 & 1 & 0 \\
\frac{g}{x} & 0 & -2 \frac{c}{\sqrt{mx}} \\
0 & 0 & -\frac{R}{L}
\end{bmatrix}, \tag{29}
\]

\[
B = \begin{bmatrix}
0 \\
0 \\
\frac{1}{L}
\end{bmatrix}, \tag{30}
\]

\[
C = [1 \ 0 \ 0], \tag{31}
\]
Using the linear regression method, and the estimation by means of the least squares algorithm [11,12], the following results are obtained for the proposed plant, which allow the estimation of the system parameters and the desired response to be appreciated Figure 4 and Figure 5.

Figure 4. Estimated parameters.

Figure 5. Estimated output.

4. Conclusions
The models obtained from parametric identifiers have great advantages, since they adapt better to the dynamics of the system, without neglecting any perceptible phenomenon, which makes the control more robust. Adaptive control seeks to improve the operation of a plant through continuous and automatic parameter estimation, making the controller adapt to changes produced by external or plant-related disturbances, which could eventually deteriorate its operation. This technique allows operate online while operating and avoid making simplifications such as operating offline which is highly inappropriate.
The implementation of the laws of electromagnetism to model a circuit is the most appropriate way to model a system compared to Kirchhoff's laws, which for ease pose systems of linear integrodifferential differential equations, but relaxing the real model to describe, the complexity generated through the equations of electromagnetism implies a complexity by itself non-linear but this detail does not prevent the control system from responding adequately to this type of model as long as the technique to be used foresees this type of model. The computational times may be longer than those used in other techniques, but this is explainable due to the on-line analysis while the plant is in operation than the one that is isolated, which is unreal.

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