Energy-angular distributions of electron-positron pair creation in collisions of a laser beam and a nonlaser photon are calculated using the S-matrix formalism. The laser field is modeled as a finite pulse, similar to the formulation introduced in our recent paper in the context of Compton scattering [Phys. Rev. A 85, 062102 (2012)]. The nonperturbative regime of pair creation is considered here. The energy spectra of created particles are compared with the corresponding spectra obtained using the modulated plane wave approximation for the driving laser field. A very good agreement in these two cases is observed, provided that the laser pulse is sufficiently long. For short pulse durations, this agreement breaks down. The sensitivity of pair production to the polarization of a driving pulse is also investigated. We show that in the nonperturbative regime, the pair creation yields depend on the polarization of the pulse, reaching their maximal values for the linear polarization. Therefore, we focus on this case. Specifically, we analyze the dependence of pair creation on the relative configuration of linear polarizations of the laser pulse and the nonlaser photon. Lastly, we investigate the carrier-envelope phase effect on angular distributions of created particles, suggesting the possibility of phase control in relation to the pair creation processes.

PACS numbers: 12.20.Ds, 12.90.+b, 42.55.Vc, 13.40.-f

I. INTRODUCTION

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I. INTRODUCTION

Due to the recent advances in laser technology, ultra-high intense laser pulses of short durations are routinely produced in contemporary laboratories. This has inspired the fast growth of high field physics with a renewed interest in studying quantum electrodynamic (QED) processes in powerful laser fields (for recent reviews, see, for instance, Refs. [1, 2]). A series of laser-based QED experiments were performed at the Stanford Linear Accelerator Center (SLAC) [3–6], reporting on nonlinear Compton scattering and nonlinear electron-positron pair creation processes (with the latter being of particular importance in light of this paper). These first experiments have triggered numerous related theoretical studies. In particular, various proposals for the pair production through the laser-matter [7–11], laser-laser [12–17], or laser-photon [18–23] interactions have been reconsidered, setting the grounds for predictions of avalanchelike QED cascades [24–28].

Generally speaking, strong-field QED processes fall into two categories as they can be either induced or modified by the laser field. While the laser-induced processes do not occur in the absence of the field, the laser-modified processes do, even though their properties can change dramatically under the influence of the field. By way of illustration, the Feynman diagrams for the laser-induced and the laser-modified Breit-Wheeler processes [29], in which the electron-positron pairs are created when a laser beam collides with nonlaser photons, are presented in Fig. 1. The double solid lines are the fermion lines representing the Volkov states [30] for electrons and positrons in the background laser field. It is anticipated that for not too intense laser pulses, as compared to the Sauter-Schwinger unit of intensity, the laser-induced process, being the lowest order perturbation process, dominates over the laser-modified processes. Exceptions can occur for parameters such that the internal fermion line in the bottom diagrams of Fig. 1 exhibits Oleinik resonances [31]. A similar situation is met, for instance, in the Bethe-Heitler process represented by the trident diagram, in which pairs are created by virtual photons [32–38] (see, 

FIG. 1. (Color online) The Feynman diagrams for the laser-induced (top diagram) and the laser-modified (bottom diagrams) Breit-Wheeler processes [29]. The fermion lines (double solid lines) represent the Volkov states for electrons and positrons in the background laser field.
also Refs. [39][12]). In the current paper, we study exclusively the laser-induced Breit-Wheeler process.

Originally, the Breit-Wheeler process consisted in a collision of two energetic photons, which combined their energies to produce the $e^+e^-$ pair [29]. The generalization of this process for the case when one of the photon sources is a strong laser beam was considered in Refs. [43–45]. In these early studies, the laser field was treated as a monochromatic plane wave (see, also Refs. [19][21]). The process in a weakly nonmonochromatic laser field was investigated in [18] by means of the slowly-varying envelope approximation. The results presented in Ref. [18] were obtained for an intense laser pulse that includes several oscillations of the field. The Breit-Wheeler process induced by a few-cycle pulse but in the perturbative regime was considered in [23]. In both these studies, the driving laser pulse was circularly polarized, which considerably simplifies the treatment of the problem. It is also worth mentioning that the linearly polarized strong laser pulse driving the pair creation was considered in Ref. [22], however with the focus on the mass shift effects. What is important in the context of the current paper is that in neither of the aforementioned works was the carrier-envelope phase (CEP) effect on the yields of produced pairs analyzed.

In this article, we give a complete description of laser-induced Breit-Wheeler process by finite ultrastrong laser pulses, which is along the lines formulated for the Compton scattering in Ref. [46]. Our current numerical results are for different pulse durations (i.e., for few- and many-cycle laser pulses) in the nonperturbative regime of laser-matter interaction. It is expected that in this regime, the yields of produced pairs strongly depend on a polarization of the driving laser field, being dominant for the linear polarization. This is the case, for instance, for the Bethe-Heitler process [33], and it is also the main focus in this paper. In our numerical illustrations, we choose the colliding extra photon to be linearly polarized as well. Although we refer to it as the $\gamma$-photon, we would like to point out that it has not necessary to be the high-energy photon (as it follows from the discussion in Sec. II D).

This paper is organized as follows. A detailed theoretical description of the laser-induced Breit-Wheeler process by a single pulse and by an infinite sequence of such pulses is provided in Secs. II A and II B respectively, with the latter being the modulated plane wave field. We discuss the general kinematic conditions for the Breit-Wheeler process by the finite pulse in Sec. II C whereas we recapitulate our formulation of this process (given in Sec. II A) to account for the relativistic invariance of QED in Sec. II D. The numerical results presenting the energy-angular spectra of created particles and their sensitivity to the pulse duration of the driving laser field are studied in Sec. III. The validity of the modulated plane wave approximation is also tested in Sec. III. The CEP effects in the pair creation spectra are demonstrated in Sec. IV. We summarize our results and give prospects for our further investigations in Sec. V.

II. THEORY

We use the notation and mathematical convention introduced in our recent publication [46]. In particular, in formulas we keep $\hbar = 1$ but our numerical results are presented in relativistic units such that $\hbar = c = m_e = 1$, where $m_e$ is the electron mass. We write $a \cdot b = a^{\mu}b_{\mu}(\mu = 0, 1, 2, 3)$ for a product of any two four-vectors $a$ and $b$, and $\varphi = \gamma \cdot a = \gamma^{\mu}a_{\mu}$ where $\gamma^{\mu}$ are the Dirac gamma matrices. Here, the Einstein summation convention is used. We use the so-called light-cone variables defined in Ref. [46]. Namely, for a given space direction determined by a unit vector $n$ and for an arbitrary four-vector $a$ we keep the following notations: $a^\parallel = n \cdot a$, $a^- = a^0 - a^\parallel$, $a^\perp = (a^0 + a^\parallel)/2$, and $a^\perp = a - a^\parallel n$. For the four-vectors, we use both the contravariant ($a^0, a^1, a^2, a^3$) and the standard $(a_0, a_x, a_y, a_z)$ notations.

Using the $S$-matrix formalism, we find that in the lowest order of perturbation theory (represented in the upper diagram of Fig. I), the probability amplitude for the Breit-Wheeler process, $\gamma K \sigma \rightarrow e^- p_- \lambda_- + e^+ p_+ \lambda_+$, with electron and positron momenta and spin polarizations $p_\sigma, \lambda_-$ and $p_\sigma, \lambda_+$, respectively, equals

$$A(\gamma K \sigma \rightarrow e^- p_- \lambda_- + e^+ p_+ \lambda_+) =$$

$$-ie \int d^4x J_{p_\sigma - \lambda_-, p_\sigma + \lambda_+}(x) \cdot A_{K \sigma}^{(+)}(x),$$

where $K \sigma$ denotes the initial nonlinear photon momentum and polarization. Here,

$$A_{K \sigma}^{(+)}(x) = \sqrt{\frac{1}{2\epsilon_0 c^2 |K| V}} \epsilon_{K \sigma} e^{-iK \cdot x},$$

where $V$ is the quantization volume, $\epsilon_0$ is the vacuum electric permittivity, $\omega_K = cK^0 = c|K| (K \cdot K = 0)$, and $\epsilon_{K \sigma} = (0, \epsilon_{K \sigma})$ is the polarization four-vector satisfying the conditions,

$$K \cdot \epsilon_{K \sigma} = 0, \quad \epsilon_{K \sigma} \cdot \epsilon_{K' \sigma'} = -\delta_{\sigma \sigma'},$$

for $\sigma, \sigma' = 1, 2$. The matrix element of the pair current operator, $J_{p_\sigma - \lambda_- - p_\sigma + \lambda_+}(x)$, is defined as

$$[j_{p_\sigma - \lambda_- - p_\sigma + \lambda_+}(x)]^{(+)} = \psi^{(+)}_{p_\sigma - \lambda_-}(x) \gamma^\nu \psi^{(+)}_{p_\sigma + \lambda_+}(x).$$

Moreover, $\psi^{(\beta)}_{p \lambda}(x)$ (with $\beta = +1$ for electron and $\beta = -1$ for positron) is the Volkov solution of the Dirac equation coupled to the electromagnetic field [31][32]

$$\psi^{(\beta)}_{p \lambda}(x) = \sqrt{\frac{m_e c^2}{V E_p}} \left(1 - \frac{\beta \cdot e}{2k \cdot p} A \right) u_p^{(\beta)} p \lambda e^{-i \beta S^{(\beta)}(x)},$$

where $A = \frac{1}{m_e c^2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} E(\omega) e^{-i \omega t}$.
with the phase \( S_p^{(\beta)}(x) \) having the meaning of the classical action of an electron (positron) in a laser field,

\[
S_p^{(\beta)}(x) = p \cdot x + \int_{-x}^{x} \left[ \frac{eA(\phi, p)}{k \cdot p} - \frac{e^2A^2(\phi)}{2k \cdot p} \right] d\phi.
\]

(6)

Here, \( E_p = cp^0 \geq m_e c^2, p = (p^0, p) \), and \( u_p^{(\beta)} \) are the free-electron (positron) bispinors normalized such that

\[
\bar{u}^{(\beta)}_p u^{(\beta)}_p = \beta \delta_{\beta\beta} \delta_{\lambda\lambda}.
\]

(7)

The four-vector potential \( A(k \cdot x) \) in Eq. (6) represents an external electromagnetic radiation generated by lasers such that \( k \cdot A(k \cdot x) = 0 \) and \( k \cdot k = 0 \). Henceforth, we consider the Coulomb gauge for the radiation field which means that the four-vector \( A(k \cdot x) \) has a vanishing zero component and that the electric and magnetic fields are equal to

\[
\mathcal{E}(k \cdot x) = -\partial_t A(k \cdot x) = -ck^0 A'(k \cdot x),
\]

(8)

\[
\mathcal{B}(k \cdot x) = \nabla \times A(k \cdot x) = -k \times A'(k \cdot x),
\]

(9)

where ‘prime’ stands for the derivative with respect to the argument \( k \cdot x \). In addition, the electric field generated by lasers has to fulfill the following condition

\[
\int_{-\infty}^{\infty} \mathcal{E}(ck^0 t - k \cdot r) dt = 0,
\]

(10)

which is equivalent to

\[
\lim_{t \to -\infty} A(ck^0 t - k \cdot r) = \lim_{t \to \infty} A(ck^0 t - k \cdot r).
\]

(11)

In the next sections, two cases will be considered: when the vector potential \( A(k \cdot x) \) describes a single laser pulse or an infinite sequence of such pulses; the latter being a modulated plane wave. Note that each of these situations requires a separate theoretical treatment.

### A. Single laser pulse

In this Section, we formulate theory for the nonlinear Breit-Wheeler process by an isolated laser pulse. Assume that a pulse lasts for time \( T_p \), which defines its fundamental frequency \( \omega = 2\pi/T_p \). The laser field wave four-vector is \( k = k^0 (1, \mathbf{n}) \), where \( k^0 = \omega/c \) and \( \mathbf{n} \) is a unit vector determining the propagation direction of the laser pulse. In accordance with the condition \( (11) \), we define the laser pulse such that

\[
A(k \cdot x) = 0 \quad \text{for} \quad k \cdot x < 0 \quad \text{and} \quad k \cdot x > 2\pi.
\]

(12)

With this definition of the four-vector potential, it is justified to interpret the momentum \( p \) present in the Volkov solution \( (5) \) as an asymptotic momentum of a free electron (positron).

Keeping this in mind, we consider the most general form of the four-vector potential,

\[
A(k \cdot x) = A_0 \left[ \varepsilon_1 f_1(k \cdot x) + \varepsilon_2 f_2(k \cdot x) \right],
\]

(13)

in which two real four-vectors \( \varepsilon_i \) describe two linear polarizations and satisfy the relations,

\[
\varepsilon_i^2 = -1, \quad \varepsilon_1 \cdot \varepsilon_2 = 0, \quad k \cdot \varepsilon_i = 0.
\]

(14)

Note that these conditions do not uniquely determine \( \varepsilon_i \). As it can be easily checked, the transformation:

\[
\varepsilon_i \rightarrow \varepsilon_i + a_i(k \cdot x) k,
\]

(15)

with arbitrary differentiable functions \( a_i(k \cdot x) \), does not violate Eqs. \( (14) \) provided that \( k \cdot k = 0 \), which is indeed the case. Note that Eq. \( (15) \) is a special case of the gauge transformation. The observable quantities, like the probability distributions that we derive below, should be invariant with respect to such a transformation. We will use this fact as a test for our numerical software. The invariance with respect to the gauge transformation, Eq. \( (15) \), permits us to choose \( \varepsilon_i \) as the spacelike vectors, since their zeroth-components can be nullified by the gauge transformation,

\[
\varepsilon_i \rightarrow \varepsilon_i - \frac{\varepsilon_i^0}{k^0} k.
\]

(16)

The same applies to \( \varepsilon_{K\sigma} \) in Eq. \( (3) \). For this reason, in our presentation we choose \( \varepsilon_i \) and \( \varepsilon_{K\sigma} \) as the spacelike four-vectors.

The probability amplitude for the laser-induced Breit-Wheeler process \( (1) \) can be rewritten as

\[
A(\gamma K\sigma \rightarrow \varepsilon_{p^-} \lambda_- + \varepsilon_{p^+} \lambda_+) = i \sqrt{\frac{2\pi \alpha m_e c^2}{4\pi \varepsilon_0 c}} K V_{\gamma\Gamma} A^{BW}_\gamma
\]

(17)

with \( \alpha = e^2/(4\pi \varepsilon_0 c) \) is the fine-structure constant and

\[
A^{BW}_\gamma = \int d^4x \ e^{-i(K \cdot x - S_{p^-}^{(+)}(x) - S_{p^+}^{(-)}(x) + \varepsilon_{p^-}^{(+)} u_{p^-}^{(-)} \lambda_-}
\]

(18)

\[
\times \left( \frac{\mu m_e c}{2k \cdot p_{\gamma}} \right) \left\{ f_1(k \cdot x) \eta_1(k) + f_2(k \cdot x) \eta_2(k) \right\} \right|_{K\sigma}
\]

\[
\times \left( \frac{\mu m_e c}{2k \cdot p_{\sigma}} \right) \left\{ f_1(k \cdot x) \eta_1(k) + f_2(k \cdot x) \eta_2(k) \right\} \right|_{\sigma} u_{p^+}^{(-)} \lambda_+.
\]

Here, we have introduced a relativistically invariant parameter, \( \mu = |e A_0|/(m_e c) \), which measures the intensity of the laser field. After some algebraic manipulations, we find that a phase present in Eq. \( (15) \) equals

\[
K \cdot x - S_{p^-}^{(+)}(x) - S_{p^+}^{(-)}(x) = (K - \vec{p}_- - \vec{p}_+) \times x + G(k \cdot x),
\]

(19)

with electron and positron laser-field-dressed momenta:

\[
\vec{p}_e = \vec{p}_e + \frac{\mu m_e c}{k \cdot p_{e^+}} \left( \frac{\varepsilon_1 \cdot \vec{p}_e}{k \cdot p_{e^+}} \langle f_1 \rangle + \frac{\varepsilon_2 \cdot \vec{p}_e}{k \cdot p_{e^+}} \langle f_2 \rangle \right) k
\]

\[
+ \frac{1}{2} \left( \frac{\mu m_e c}{k \cdot p_{e^+}} \right)^2 \frac{\langle f_1^2 \rangle + \langle f_2^2 \rangle}{k \cdot p_{e^+}} k.
\]

(20)
Here, we understand that
\[
(f_i^j) = \frac{1}{T_p} \int_0^{T_p} \int_0^{2\pi} \int_0^{k \cdot x} \left[ f_i (ck^0 t - k \cdot r) \right]^j = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{k \cdot x} \left[ f_i (\phi) \right]^j,
\]
for \(i, j = 1, 2\). In Eq. (19), we have also introduced,
\[
G(k \cdot x) = \int_0^{k \cdot x} \int_0^{2\pi} \int_0^{k \cdot x} \left[ f_i (\phi) - \langle f_i \rangle \right] - \mu mc \left( \frac{\varepsilon_i \cdot p_{e+}}{k \cdot p_{e+}} - \frac{\varepsilon_i \cdot p_{e-}}{k \cdot p_{e-}} \right) \times \left( f_1 (\phi) - \langle f_1 \rangle \right) - \mu mc \left( \frac{\varepsilon_2 \cdot p_{e+}}{k \cdot p_{e+}} - \frac{\varepsilon_2 \cdot p_{e-}}{k \cdot p_{e-}} \right) \times \left( f_2 (\phi) - \langle f_2 \rangle \right) + \frac{1}{2} \mu mc \left( \frac{1}{k \cdot p_{e+}} + \frac{1}{k \cdot p_{e-}} \right) \times \left( f_1 (\phi) - f_1^2 (\phi) + f_2 (\phi) - f_2^2 (\phi) \right),
\]
which is a periodic function of the argument \(k \cdot x\), meaning that \(G(0) = G(2\pi) = 0\). Using the Fourier expansion,
\[
[f_1 (k \cdot x)]^n \cdot [f_2 (k \cdot x)]^n e^{-iG(k \cdot x)} = \sum_{N=-\infty}^{\infty} G_N^{\langle n, m \rangle} e^{-iNK \cdot x},
\]
we can perform the space-time integration in Eq. (19), provided that \(n\) and \(m\) are not simultaneously equal to 0.
As we have already argued in Ref. [46] when studying the Compton scattering, the case when \(n = m = 0\) requires a separate treatment. This becomes clear when the light-cone coordinates are used. The point being that, only if \(N \neq 0\) or \(M \neq 0\), the integral with respect to \(x^\perp\) becomes limited to the finite region, \(0 \leq x^\perp \leq 2\pi/k_0\). In order to treat the case when both \(n\) and \(m\) are 0, we use the Boca-Florescu transformation (see, Ref. [46, 47]) which states that
\[
\int d^4x \exp \left( -iQ \cdot x - i \int_{-\infty}^{k \cdot x} d\phi h(\phi) \right) = -\frac{k_0}{Q^3} \int d^4x h(k \cdot x) \exp \left( -iQ \cdot x - i \int_{-\infty}^{k \cdot x} d\phi h(\phi) \right),
\]
if \(Q_0 \neq 0\). In our case,
\[
Q = K - p_{e-} - p_{e+}, \\
h(\phi) = a_1 f_1 (\phi) + a_2 f_2 (\phi) + b [f_1^2 (\phi) + f_2^2 (\phi)],
\]
with
\[
a_1 = -\mu mc \left( \frac{\varepsilon_1 \cdot p_{e+}}{k \cdot p_{e+}} - \frac{\varepsilon_1 \cdot p_{e-}}{k \cdot p_{e-}} \right) = -Q_0 \tilde{a}_1 / k_0^3, \\
a_2 = -\mu mc \left( \frac{\varepsilon_2 \cdot p_{e+}}{k \cdot p_{e+}} - \frac{\varepsilon_2 \cdot p_{e-}}{k \cdot p_{e-}} \right) = -Q_0 \tilde{a}_2 / k_0^3, \\
b = \frac{1}{2} \mu mc \left( \frac{1}{k \cdot p_{e+}} + \frac{1}{k \cdot p_{e-}} \right) = -Q_0 \tilde{b} / k_0^3,
\]
Using the Fourier decomposition (23) and applying the Boca-Florescu transformation (24), we arrive at
\[
A_{BW} = \sum_N D_N \int d^4x e^{-i(K \cdot p_{e-} - p_{e+} + N k \cdot x)},
\]
where
\[
D_N = \left[ \tilde{u}_{E+} \cdot K \sigma \cdot \tilde{u}_{E+} \right] G_N^{(0,0)} \\
- \frac{1}{2} \mu mc \left( \frac{1}{k \cdot p_{e+}} \tilde{u}_{E+} \cdot K \sigma \cdot \tilde{u}_{E+} \right) G_N^{(0,0)} \\
+ \frac{1}{k \cdot p_{e+}} \tilde{u}_{E+} \cdot K \sigma \cdot \tilde{u}_{E+} G_N^{(0,0)} \\
+ \frac{1}{k \cdot p_{e+}} \tilde{u}_{E+} \cdot K \sigma \cdot \tilde{u}_{E+} G_N^{(0,0)} \\
+ \frac{1}{4(k \cdot p_{e+})(k \cdot p_{e-})} \tilde{u}_{E+} \cdot K \sigma \cdot \tilde{u}_{E+} G_N^{(0,0)} \\
+ \tilde{u}_{E+} \sigma \cdot \tilde{u}_{E+} + \tilde{u}_{E+} \cdot \tilde{u}_{E+} G_N^{(0,0)} \\
+ \tilde{u}_{E+} \cdot \tilde{u}_{E+} + \tilde{u}_{E+} \cdot \tilde{u}_{E+} G_N^{(0,0)}
\]
and
\[
G_N^{(0,0)} = \tilde{a}_1 G_N^{(1,0)} + \tilde{a}_2 G_N^{(0,1)} + \tilde{b} G_N^{(2,0)} + G_N^{(0,2)},
\]
with \(\tilde{a}_1, \tilde{a}_2,\) and \(\tilde{b}\) defined by Eqs. (27).
Performing the space-time integration in Eq. (29) and keeping in mind that \(0 \leq x^\perp \leq 2\pi/k_0\), we obtain
\[
A_{BW} = \sum_N \left( 2\pi \right)^3 \delta^{(1)} (P_N) \delta^{(2)} (P_N) D_N \int \frac{1 - e^{-2\pi p_{e+} / k_0}}{iP_N^0},
\]
where we define
\[
P_N = K - p_{e-} - p_{e+} + N k.
\]
To solve the momentum conservation conditions imposed by the delta functions in (21), we assume that the final positron momentum is known. This allows us to introduce the four-vector \(w = K - p_{e+}\), so that
\[
\nu_{e-} = \nu_{e-} + w^{-}, \quad \nu_{e+} = w^{+}.
\]
Since the electron mass is different from zero, it follows from the first of these equations that \(w^{-} > 0\), and
\[
p_{e-} = \frac{(m_e c)^2 + (w^+)^2 - (w^-)^2}{2w^-} = K \cdot p_{e+} / w^- + w^{+},
\]
which means that
\[
Q_0 = K^0 - p_{e-} - p_{e+} = K^0 - p_{e-} - p_{e+} = -K \cdot p_{e+} / w^- < 0.
\]
This is the applicability condition for the Boca-Florescu transformation (46, 47). It also shows that this transformation, Eq. (21), is relativistically invariant, as the ratio
\[
Q_0 / k_0 = -K \cdot p_{e+} / k \cdot w = -K \cdot p_{e+} / k \cdot p_{e+} = -K \cdot p_{e+} / k \cdot p_e
\]
does not depend on the reference frame. Note that \( P_N \) and \( P_{\tilde{N}} \) do not depend explicitly on \( N \), and

\[
\int d^4p_e^- \delta^{(1)}(P_{0N})\delta^{(2)}(P_N^{\perp}) = \frac{k^0 p_{e-}^0}{k \cdot p_{e-}}.
\]

Taking this into account, we derive that the differential probability distribution of created positrons in the Breit-Wheeler process induced by a finite laser pulse is

\[
\frac{d^3P(p)}{dE_{p+}d^2\Omega_{p+}} = \sum_{\lambda_-,\lambda_+ = \pm} |p_{e+}|^{-2} \frac{\alpha(m_e c)^2 k^0}{(2\pi)^2 \omega \epsilon(k \cdot p_{e-})} \times \sum_N D_N \frac{1 - e^{-2\pi i P_N^{\perp}/k^0}}{P_N^{\perp}}^2,
\]

with the electron four-momentum \( p_{e-} \) equal to

\[
\begin{align*}
    p_{e-}^\perp &= w^\perp, \\
    p_{e-}^\parallel &= \frac{(m_e c)^2 + (w^\perp)^2 - (w^-)^2}{2w^-}, \\
    p_{e-}^0 &= \frac{(m_e c)^2 + (w^\perp)^2 + (w^-)^2}{2w^-}.
\end{align*}
\]

One can check that the above solution of the momentum conservation conditions, \( P_N^{\perp} = 0 \) and \( P_{\tilde{N}}^{\perp} = 0 \), satisfies the on-mass shell relation, \( p_{e-} \cdot p_{e-} = (m_e c)^2 \).

If an incident laser pulse is long enough, terms such as \( p_{e-} \cdot p_{e-} \) contribute the most in Eq. (31). Hence, it is possible to define the effective energy \( N_{\text{eff}} c^0 \) absorbed from the laser pulse such that \( P_{\tilde{N}}^{\perp} = 0 \). In this case,

\[
N_{\text{eff}} = \frac{p_{e-}^0 + p_{e+}^0 - K^0}{k^0} = cT_p \frac{p_{e-}^0 + p_{e+}^0 - K^0}{2\pi}.
\]

This allows us to define the differential probability of pair creation,

\[
\frac{d^3P(p)}{dN_{\text{eff}} d^2\Omega_{p+}} = \frac{dE_{p+} d^3P(p)}{dN_{\text{eff}} dE_{p+} d^2\Omega_{p+}},
\]

which will have its analog with the case when a train of pulses, instead of a single pulse, is used. In a very similar way we derive the energy-angular distribution for created electrons, the explicit form of which is not presented here.

Note that the laser-dressed momenta \( P_{\tilde{N}} \) are gauge-dependent, as they change their values if the gauge transformation, Eq. (13), with a constant \( a_i \) is applied. For the first time, such a definition of the laser-dressed momenta has been introduced in the context of Compton scattering in Ref. [40]. A similar definition of the laser-dressed momenta has been used recently by Harvey et al. [48] for the classical analogue of this process, i.e., Thompson scattering. At the same time, we would like to note that in the formulation presented above only the sum \( p_{e-} + \tilde{p}_{e+} \) appears. The point being that this sum and, hence, also \( N_{\text{eff}} \) [Eq. (42)] are gauge-invariant. We can redefine the laser-dressed momenta such that

\[
\tilde{p}_{e+} \rightarrow \tilde{p}_{e+} \mp s_0 k \mp s_1 e_1 \mp s_2 e_2,
\]

with, in principle, arbitrary real constants \( s_i \) \( (i = 0, 1, 2) \). These constants can still depend on different kinds of time-averaged shape functions, like \( \langle f_1 \rangle \), \( \langle f_2 \rangle \), \( \langle f_1 f_2 \rangle \), etc., however they should vanish for vanishing laser field in order to keep the correspondence to the free particle case. Note that the new definition \( \tilde{p}_{e+} \) preserves the sum \( \tilde{p}_{e-} + \tilde{p}_{e+} \). Therefore, one can use this arbitrariness to define the gauge-invariant laser-dressed momenta. By making the substitutions \( \varepsilon_i \rightarrow \varepsilon_i + a_ik \), we find that the dressed momenta do not depend on \( a_i \) provided that \( s_1 = -\mu m_e c \langle f_1 \rangle \) and \( s_2 = -\mu m_e c \langle f_2 \rangle \), with still an arbitrary \( s_0 \). Let us stress, however, that such modifications of the momentum dressing do not affect any observable quantity, and should be considered only as a mathematical operation. In closing this Section, let us also emphasize that \( N_{\text{eff}} \) [Eq. (42)] is not only gauge-invariant but it is also relativistic-invariant, since

\[
N_{\text{eff}} = \frac{Q_0}{k^0} + \mu m_e c \langle f_1 \varepsilon_1 + \langle f_2 \varepsilon_2 \rangle \left( \frac{p_{e+}^0}{k \cdot p_{e+}} - \frac{p_{e-}^0}{k \cdot p_{e-}} \right) + \frac{1}{2} (\mu m_e c)^2 \left( \langle f_1^2 \rangle + \langle f_2^2 \rangle \right) \left( \frac{1}{k \cdot p_{e+}} + \frac{1}{k \cdot p_{e-}} \right),
\]

where \( Q_0/k^0 \) is defined by Eq. (38).

### B. Train of laser pulses

In this Section, we consider the Breit-Wheeler process by a train of laser pulses. We assume that the duration of a single pulse within such a field is \( T_p \). This defines the fundamental frequency of the laser field, \( \omega = 2\pi/T_p \), and also the four-vector \( k = k^0(1, n) \) with \( k^0 = \omega/c \) and the propagation direction vector \( n \). We consider a laser field described by the four-vector potential \( A_\mu \), with the polarization four-vectors as before. This time the shape functions \( f_i(\phi) \) \((i = 1, 2)\) are periodic functions of their argument \( \phi \), with the period of \( 2\pi \). This means that \( \langle f_i \rangle = 0 \), and the laser-dressed four-momenta of created particles read now

\[
\tilde{p}_{e+} = p_{e+} + \frac{1}{2} (\mu m_e c)^2 \left( \langle f_1^2 \rangle + \langle f_2^2 \rangle \right) k, \quad \tilde{p}_{e-} = \tilde{p}_{e-} + s_0 k
\]

again up to terms \( \mp s_0 k \) [see, Eq. (44) and the following discussion].

When calculating the probability rate for the nonlinear Breit-Wheeler process by a laser pulse train, we proceed along the same lines as in Sec. 1A. This leads us to expression (28), with Eq. (29) defining the Fourier components \( D_N \) where the substitution (30) no longer applies. Performing the space-time integration in Eq. (28), we end up with

\[
A_{\text{BW}} = (2\pi)^4 \sum_N D_N \delta^{(4)}(K - \tilde{p}_{e-} - \tilde{p}_{e+} + Nk),
\]

where the \( \delta \)-function determines the four-momenta conservation condition. Hence, the probability of pair cre-
ation by a single pulse from the train can be calculated,

\[
P^{(1)} = \frac{\alpha T_p}{2\pi} \sum_{\lambda_+ - \lambda_- = \pm} \sum_{m=-\infty}^{\infty} \int d^3 p_e - d^3 p_{e^+} 
\times \frac{(m_e c^2)^2}{E_{p_e} - E_{p_{e^+}} + \omega_k} |D_N|^2 \delta^{(4)}(K - p_{e^-} - p_{e^+} + N k).
\]  
(48)

Due to the conservation condition, four integrations can be carried out in Eq. (48). Let us assume that we know the ejection angles of positrons. This means that we can define the angular distribution of positrons created in the process accompanied by the absorption \( (N > 0) \) or emission \( (N < 0) \) of the laser radiation energy \( N c^2 \),

\[
\frac{d^2 P^{(1)}}{d\Omega_{p_{e^+}}} = \frac{\alpha T_p}{2\pi} \sum_{\lambda_+ - \lambda_- = \pm} \int dE_{p_{e^+}} d^3 p_{e^-} 
\times |p_{e^+}| \frac{(m_e c^2)^2}{E_{p_{e^+}} - \omega_k} |D_N|^2 \delta^{(4)}(K - p_{e^-} - p_{e^+} - N k).
\]  
(49)

The integration over the momentum \( p_{e^-} \) leads to

\[
\int d^3 p_{e^-} \delta^{(3)}(K - p_{e^-} - p_{e^+} + N k) = \left[ \frac{\partial p_{e^-}}{\partial p_{e^+}} \right] \left[ \frac{p_{e^-}^0}{p_{e^+}^0} \right] = \frac{p_{e^-}^0}{p_{e^+}^0}.
\]  
(50)

For the remaining integral, we obtain

\[
\int dE_{p_{e^+}} \delta^{(1)}(K_0 - p_{e^+}^0 - p_{e^+}^0 + N k_0) = \sum_{\ell} d^{(0)}(p_{e^+})
\]

where

\[
D^{(0)}(p_{e^+}) = \left[ -1 + \frac{p_{e^-}^0}{p_{e^+}^0} \frac{p_{e^-} \cdot p_{e^+}}{|p_{e^+}|^2} + (p_{e^+}^0 - p_{e^+}^0) \right] 
\times \frac{k \cdot p_{e^-}}{p_{e^+}^0 (k \cdot p_{e^+})} \left( 1 - \frac{p_{e^-}^0}{k_0} \frac{k \cdot p_{e^+}}{|p_{e^+}|^2} \right).
\]  
(51)

and \( \ell \) labels all possible solutions of the conservation condition \( 0 \). Finally, we obtain

\[
\frac{d^2 P^{(1)}}{d\Omega_{p_{e^+}}} = \frac{\alpha (m_e c^2)^2}{k_0^3 K_0} \sum_{\lambda_+ - \lambda_- = \pm} \sum_{\ell} \frac{|p_{e^+}|}{D^{(0)}(p_{e^+})} |D_N|^2.
\]  
(52)

(53)

In order to be able to compare this result with the case of a single laser pulse discussed in Sec. II A, let us define symbolically the triple differential probability distribution of pair creation,

\[
\frac{d^3 P^{(1)}}{dN d\Omega_{p_{e^-}} d\Omega_{p_{e^+}}} = \frac{d^2 P^{(1)}}{d\Omega_{p_{e^+}}}.
\]  
(54)

This quantity corresponds to Eq. (48) defined in Sec. II A.

C. Remarks on kinematics

Going back to Sec. II A, where the e\(^-\)e\(^+\) pair creation induced by a finite laser pulse was considered, we note that the condition \( w^- > 0 \) imposes some constraints on possible values of the positron final momenta and the geometry of the process. Let us choose the system of coordinates such that the z-axis is determined by the propagation direction of the laser field, \( n = e_z \). Therefore, the inequality \( w^- > 0 \) can be put down as

\[
|p_{e^-}|^2 \sin^2 \theta_{e^-} - 2K^- |p_{e^+}| \cos \theta_{e^+} + (m_e c^2)^2 - (K^-)^2 < 0,
\]  
(55)

where \( \theta_{e^+} \) is the positron polar angle measured with respect to the z-axis. This inequality has real solutions for \( |p_{e^+}| \) provided that

\[
\sin \theta_{e^+} < \sqrt{\frac{(K^-)^2}{m_e c^2}}.
\]  
(56)

We conclude from here that \( K^- > 0 \), which excludes the situation when the incident photon and the laser field propagate in the same direction. Defining

\[
p_{\text{min}} = K^- \cos \theta_{e^+} - \sqrt{(K^-)^2 - (m_e c^2)^2 \sin^2 \theta_{e^+}},
\]  
(57)

\[
p_{\text{max}} = K^- \cos \theta_{e^+} + \sqrt{(K^-)^2 - (m_e c^2)^2 \sin^2 \theta_{e^+}},
\]  
(58)

we find that

\[
\max(0, p_{\text{min}}) < |p_{e^-}| < p_{\text{max}},
\]  
(59)

provided that \( p_{\text{max}} > 0 \); otherwise, the \( \gamma \)-photon cannot create the e\(^-\)e\(^+\) pair. In particular, if \( \cos \theta_{e^+} \geq 0 \) the pair can always be created [under the circumstances that the inequality \( 56 \) still remains valid], whereas for \( \cos \theta_{e^+} < 0 \) the pair can be formed only for a sufficiently energetic nonlaser photon,

\[
\omega_k > \frac{m_e c^2}{1 - n \cdot n_K}.
\]  
(60)

It follows from this condition that the least energetic incident photon is required for the head-on collision of the \( \gamma \)-photon with the laser pulse.

For a fixed energy of the created positron, we find the constraint for its direction:

\[
\cos \theta_{e^+} > \sqrt{\frac{|p_{e^-}|^2 + (m_e c^2)^2 - K^-}{|p_{e^+}|}}.
\]  
(61)

Hence, it follows that a pair can be created in any space direction if the \( \gamma \)-photon energy is sufficiently large, i.e.,

\[
\omega_k > \frac{c |p_{e^+}| + \sqrt{(c |p_{e^+}|)^2 + (m_e c^2)^2}}{1 - n \cdot n_K}.
\]  
(62)

On the other hand, the pair creation will not occur at all if the incident photon energy \( \omega_k \) is too small,

\[
\omega_k < -c |p_{e^+}| + \sqrt{(c |p_{e^+}|)^2 + (m_e c^2)^2}.
\]  
(63)
D. Relativistic invariance

The formulas derived above for the differential probability distributions of the laser-induced Breit-Wheeler process [Eqs. (38) or (54)] are not relativistically invariant. We have presented them in a noninvariant form in order to analyze below the validity of an approximation of a laser pulse by a modulated plane wave. Since quantum electrodynamics is the relativistic quantum theory, therefore, it is appropriate to use the relativistic invariance in order to simplify the geometry of the Breit-Wheeler process. In particular, we can show that from the theoretical point of view the head-on collision of the $\gamma$-photon with the laser pulse is the generic configuration for this process, in the sense that there exists the Lorentz transformation that transforms an arbitrary laboratory geometry to the head-on configuration. To this end, let us recall that the pair creation is forbidden if the laser beam and the $\gamma$-photon propagate in the same direction (see, Sec. [14]). If so, we can always choose the space coordinates such that the vectors $\mathbf{k}$ and $\mathbf{K}$ are coplanar with the $(xz)$-plane, with the axes chosen such that $\mathbf{k}$ and $\mathbf{K}$ form the same angle $\theta$ with the $z$-axis, as presented in Fig. 2. We shall assume, without losing the generality, that $0 \leq \theta < \pi/2$, so that $k_z > 0$.

Next, let us apply the Lorentz boost, $L_x$, in the direction of the $x$-axis with the parameters $\beta_x$ and $\gamma_x$ such that

$$L_x : \quad \beta_x = \sin \theta, \quad \gamma_x = 1/\cos \theta,$$

which transforms the laser and the $\gamma$-photon wave four-vectors, $k$ and $K$, to

$$k' = k'_0(1, 0, 0, 1) = k'_0n',\quad K' = K'_0(1, 0, 0, -1) = K'_0nK',$$

where $k'_0 = k_0 \cos \theta$ and $K'_0 = K_0 \cos \theta$ (we see, that this boost lowers the frequencies of both the laser field and the $\gamma$-photon). Also, the created electron and positron four-momenta are transformed accordingly,

$$p'_0 = (p_0 - px \sin \theta) / \cos \theta, \quad p'_x = (px - p_0 \sin \theta) / \cos \theta, \quad p'_y = p_y, \quad p'_z = p_z.$$

Moreover, the real polarization vectors for the laser field, $\varepsilon_1$ and $\varepsilon_2$, as well as the polarization vector of the $\gamma$-photon, $\varepsilon_{K\sigma}$, are transformed appropriately. However, if we choose them as being either parallel or perpendicular to the $(xz)$-plane, then they preserve these properties under the Lorentz boost (64).

We can proceed even further and apply the Lorentz boost in the $z$-direction, $L_z$, with the parameters:

$$L_z : \quad \beta_z = \frac{k_0^2 - k_0'^2}{k_0^2 + k_0'^2}, \quad \gamma_z = \frac{k_0^2 + k_0'^2}{2k_0 k_0'},$$

and an arbitrary $k_0'^2 > 0$. One can check that the laser four-vector $k'$ and the $\gamma$-photon four-vector $K'$ transform as

$$k'' = k''_0(1, 0, 0, 1) = k'_0n',\quad K'' = K''_0(1, 0, 0, -1) = K'_0nK',$$

where

$$K''_0 = \frac{k_0'}{k_0} K'_0.$$

This means that such a boost keeps the product of both frequencies invariant; by increasing the laser field frequency we decrease the $\gamma$-photon energy, and vice-versa. In other words, the laser field cannot be described as low-frequency and the $\gamma$-photons cannot be thought of as the high-energy radiation, as compared to the electron rest energy, since these concepts depend on the choice of the reference frame.

For the head-on geometry, we can define the analogue of the energy-angular differential probability distribution given by Eq. (38). For this geometry and for the Lorentz boost (67), the linear polarization vectors $\varepsilon'_1$, $\varepsilon'_2$, and $\varepsilon_{K\sigma}$ do not change and the total pair creation probability, $P^{(p)}(k' \cdot K')$, depends only on the product of $k'$ and $K'$ (note that all the scalar products of the wave vectors with the polarization vectors vanish for this configuration). Hence, as the relativistically invariant quantity, $P^{(p)}(k' \cdot K')$ fulfills the equation

$$P^{(p)}(k' \cdot K') = P^{(p)}(k'' \cdot K'').$$

Equivalently, instead of $k' \cdot K'$ one can use the $s$-invariant, $s = (k' + K')^2 = 2k' \cdot K'$, as discussed in Ref. [23].
Now, the total probability of pair creation can be expressed in one of the equivalent forms,
\[
P^{(p)}(k' \cdot K') = \int dE_p d^2\Omega_p |\frac{p^\prime}{c^2}|^2 D^{(p)}_e(k', p'_e) - D^{(p)}_e(k', K', p'_e) = \int dE_p d^2\Omega_p |\frac{p^\prime}{c^2}|^2 D^{(p)}_e(k', K', p'_e) + D^{(p)}_e(k', K', p'_e),
\]
where
\[
D^{(p)}_e(k', K', p'_e) = \frac{\alpha(m_e c^2)^2}{(2\pi)^3 k' \cdot p'_e} D^{(p)}_e(k', K', p'_e),
\]
and
\[
D^{(p)}_e(k', K', p'_e) = \frac{k^0}{\omega K'} \sum_{\lambda_{e+}, \lambda_{e-}} \left| \sum N \frac{1 - e^{-2\pi i p^\prime N / k^0}}{p^\prime N} \right|^2.
\]
The reason for writing Eqs. (71) and (72) in their current forms is that the integration measure
\[
dE_p d^2\Omega_p |\frac{p}{c^2}|^2 = \frac{d^3p}{E_p}
\]
is relativistically invariant. Hence, the energy-angular distributions for electrons, \(D^{(p)}_e\), and positrons, \(D^{(p)}_{e'}\), are also relativistic invariant with respect to the Lorentz boost \((67)\). This invariance, together with the gauge invariance discussed above, has been used for testing our numerical code.

In closing this Section, let us stress that the introduction of the second Lorentz boost was motivated mainly by two reasons. First, in the real experimental setup the laser field frequencies are much smaller than the electron rest energy. This might cause some numerical problems. To avoid them, it is better to analyze numerically this process in the reference frame where the frequency of the laser field is of the order of the relativistic unit of energy or it is comparable to the frequency of the \(\gamma\)-photon. Second, the relativistic invariance of the Breit-Wheeler process shows that, irrespectively of the value of the laser field frequency in the laboratory frame, it is forbidden to consider the laser field as a slowly changing function on the electron’s Compton wavelength scale; in particular, it is not permitted to approximate the action of the laser field by a constant or by a slowly-varying time-dependent electric fields.

III. SINGLE PULSE VS. TRAIN OF PULSES

Consider a single laser pulse described by the following four-vector potential,
\[
A(k \cdot x) = A_0 B [\varepsilon_1 g_1(k \cdot x) \cos \delta + \varepsilon_2 g_2(k \cdot x) \sin \delta],
\]
which propagates along the z-axis and lasts for time \(T_p\). Its fundamental frequency equals \(\omega = 2\pi / T_p\) and the wave four-vector is \(k = k^0(1, e_z) = (\omega / c)(1, e_z)\). We choose two linear polarization vectors, \(\varepsilon_1 = (0, e_z)\) and \(\varepsilon_2 = (0, e_y)\). In Eq. (77), we keep the parameter \(\delta\) which describes the ellipticity of the laser field. The pulse shape functions \(g_1(k \cdot x)\) and \(g_2(k \cdot x)\) are chosen such that
\[
g_1(k \cdot x) = -\frac{N_f}{2} \left[ \frac{k \cdot x}{2} \right] \sin \left( \frac{N_{osc} k \cdot x + \chi}{2} \right),
\]
and
\[
g_2(k \cdot x) = \frac{N_f}{2} \left[ \frac{k \cdot x}{2} \right] \cos \left( \frac{N_{osc} k \cdot x + \chi}{2} \right),
\]
for \(0 \leq k \cdot x \leq 2\pi N_{osc}\), and they are 0 otherwise. Here, \(\chi\) is the carrier-envelope phase, whereas \(k_{L} = N_{osc} k = (\omega_l / c)(1, e_z)\) with the carrier frequency of the laser pulse, \(\omega_l\). The normalization constant \(N_f\) in the above equations is chosen such that
\[
\frac{1}{2\pi} \int_{0}^{2\pi} d\phi \left[ \left| g_1' (\phi) \right|^2 \cos^2 \delta + \left| g_2' (\phi) \right|^2 \sin^2 \delta \right] = \frac{1}{2}.
\]
The pair creation process induced by pulses of different duration but carrying out the same energy will be compared. This condition is satisfied if the peak value of the vector potential \((77)\) is scaled as \(\sqrt{N_{osc}}\) when changing the number of laser field oscillations within the pulse, \(N_{osc}\). If we keep,
\[
\mu = \frac{|eA_0|}{m_e c},
\]
then \(B = \sqrt{N_{osc}}\) has to be chosen in Eq. (77). Note that Eq. (77) is a particular realization of a more general vector potential \((13)\), with \(f_1(k \cdot x) = B g_1(k \cdot x) \cos \delta\) and \(f_2(k \cdot x) = B g_2(k \cdot x) \sin \delta\).

Up till now, we have specified a single laser pulse containing the fixed energy but with a varied duration. Next, let us define a train of such pulses as their infinite sequence \(\{E_1, E_2, \ldots\}\) with the zero harmonic subtracted in Eqs. (73) and (74). This does not change the value of the electric and the magnetic fields characterizing the pulse. At the same time, the energy within one pulse from the train is fixed, irrespectively of its duration.

For a numerical illustration, we consider a head-on collision of the laser pulse introduced above with a nonlaser photon. As argued in Sec. 11C any kinematic configuration of a laser and a nonlaser fields can be transformed, by a suitable Lorentz transformation, to a reference frame in which they become counterpropagating. This excludes the case when both fields propagate in the same direction, which however is irrelevant to the present problem (see, Sec. 11C). For the chosen head-on configuration, we take the laser field parameters such that the carrier frequency \(\omega_{L}\) is 0.1mc2 and \(\mu = 1\). The carrier-envelope
FIG. 3. (Color online) Comparison of the positron energy-angular distributions for a single laser pulse (blue solid line) and for a train of pulses (green dots) [Eqs. (45) and (46), respectively], in the case when a different number of laser field oscillations, $N_{osc}$, is contained in the pulse. The positron polar and azimuthal angles are $\theta_{+} = \pi/2$ and $\varphi_{+} = 0$, respectively. The results are for the parameters of the laser pulse such that $\mu = 1$ and $\omega_{L} = 0.1m_{e}c^{2}$, and for the energy of the $\gamma$-photon $\omega_{K} = 2m_{e}c^{2}$. Both, the $\gamma$-photon and the laser field are linearly polarized, with $\varepsilon_{K\sigma} = \varepsilon_{1} = e_{x}$.

The phase of the laser field is $\chi = 0$ and the parameter $\delta = 0$, unless otherwise stated. The incident nonlaser photon is linearly polarized, with the polarization direction specified below, and the energy $\omega_{K} = 2m_{e}c^{2}$. As explained in Sec. II B due to relativistic invariance, the presented results are valid in an arbitrary reference frame transformed according to (67), if the following condition for $\omega_{L}$ and $\omega_{K}$ is kept: $\omega_{L}\omega_{K} = 0.2(m_{e}c^{2})^2$, and if the electron and the positron momenta are changed accordingly. Since $s = (k + K)^2 = 4\omega_{L}\omega_{K}/c^{2} < 2(m_{e}c)^2$, hence, we consider the subthreshold pair creation, according to the nomenclature introduced in 23.

In Fig. 3 we demonstrate the energy distributions for positrons created in the direction of the laser field polarization ($\theta_{+} = \pi/2$ and $\varphi_{+} = 0$), in the case when the linear polarization vector of the colliding $\gamma$-photon is in the same direction as that of the laser field, $\varepsilon_{K\sigma} = \varepsilon_{1} = e_{x}$.

The solid blue line corresponds to the Breit-Wheeler process by a single laser pulse, whereas the green dots are for the Breit-Wheeler process by a train of pulses [let us remind that in the latter case, the respective distribution is defined per pulse from a train; for more details, see Secs. II A and II B, respectively]. The spectra in Fig. 3 relate to different pulse durations, characterized by a different number of field oscillations, $N_{osc}$, as specified in each panel. While for long pulses (including 32 and 16 oscillations of the laser field) a very good agreement between the results for an individual pulse and a pulse from a train of pulses is observed, this agreement decreases with decreasing the pulse duration. The same was observed for the energy spectra of Compton photons [46], showing the necessity to account for an actual temporal structure of ultrashort laser pulses when studying processes in strong laser fields. Moreover, our results show that for long laser pulses it is allowed to investigate the process numerically by treating a single laser pulse as a pulse from the train of pulses, which significantly speeds up the computations.

The aforementioned feature can also be noted in Fig. 4. This figure illustrates the exact same physical situation as Fig. 3 but for a different linear polarization of the $\gamma$-photon, when $\varepsilon_{K\sigma} = -e_{y}$. It is particularly well seen for $N_{osc} = 4$, that the results in Figs. 3 and 4 for an individual laser pulse and a pulse from a sequence of pulses do not only differ in magnitude but they are also shifted with respect to each other. This is caused by a different energy threshold in both of these cases, that is related to a different laser-field dressing, as defined by Eqs. (20) and (46). For long laser pulses [when $\langle f_{i} \rangle \rightarrow 0$ ($i=1,2$) in Eq. (20)], the energy threshold is basically the same for both the Breit-Wheeler process induced by a single laser pulse and by a laser pulse train. With decreasing the pulse duration, the threshold energy starts to differ for both cases which results in a respective shift of the energy spectra of created particles.

Let us note that the probability distributions of created positrons, shown in Figs. 3 and 4 increase in magnitude when decreasing the pulse duration: for the parameters considered in Figs. 3 and 4, one observes roughly four orders of magnitude increase when the driving laser pulse changes from 32 to four cycles. Once we keep the energy contained in a laser pulse fixed, the maximum value of the electric and magnetic fields must be effectively increased when decreasing the pulse duration. Consequently, a significant enhancement of the probability of pair produc-
tion induced by shorter laser pulses is observed.

Although our main objective in this paper is to present the results for the linearly polarized driving field, in Fig. 5 we show the angular spectra of positrons created by a circularly polarized laser field \( \delta = \pi/4 \) whereas the \( \gamma \)-photon is linearly polarized along the \( x \)-axis, i.e., \( \varepsilon_{K\sigma} = \varepsilon_x \).

In Fig. 6, we present angular distributions for positrons created with momenta \( |p_{\perp}| = m_e c \) in the \((xz)\)-plane, i.e., for \( \varphi_{\perp} = 0 \) or \( \pi \), in the case when a 32-cycle pulse collides with a countermoving \( \gamma \)-photon. The nonlaser photon is also linearly polarized, with the polarization vector \( \varepsilon_{K\sigma} = \varepsilon_x \). We have also checked that for \( \varepsilon_{K\sigma} = -\varepsilon_y \), the results hardly change. As expected, when comparing Fig. 6 and 3, the probabilities of pair creation are smaller for the circularly polarized laser field than for the linearly polarized field. Similar strong dependence of pair production on the polarization of a driving laser pulse was observed in Ref. [33] for the nonlinear Bethe-Heitler process.

In Fig. 6, we present angular distributions for positrons produced with momentum \( |p_{\perp}| = m_e c \) by a single laser pulse including 32 field oscillations \( (N_{osc} = 32) \). The other parameters are \( \mu = 1 \), \( \omega_m = 0.1 m_e c^2 \), \( \omega_K = 2 m_e c^2 \), \( \varphi_{\perp} = 0 \) for \( \Theta_{\perp} \leq \pi \) (or, \( \varphi_{\perp} = \pi \) for \( \Theta_{\perp} > \pi \)), and \( \chi = 0 \). The upper panel relates to the case when the \( \gamma \)-photon is linearly polarized such that \( \varepsilon_{K\sigma} = \varepsilon_x \), whereas the lower panel is for \( \varepsilon_{K\sigma} = -\varepsilon_y \). The peaks appear when \( N_{eff} \) changes by multiples of \( N_{osc} \). The spectra are symmetric with respect to the propagation direction of the incident \( \gamma \)-photon.

### IV. CARRIER-ENVELOPE PHASE EFFECTS

In this Section, we investigate the effect of the carrier-envelope phase of a few-cycle pulse on angular spectra of created positrons in the nonlinear Breit-Wheeler process. This is done for different configurations of linear polarizations of the colliding laser pulse (i.e., with \( \delta = 0 \)) and the nonlaser photon.

In Figs. 7 and 8 we demonstrate the angular distributions of positrons with momenta \( |p_{\perp}| = m_e c \) created in a head-on collision of a two-cycle pulse \( (N_{osc} = 2) \) with a \( \gamma \)-photon. As before, the spectra are for the positron azimuthal angle, \( \varphi_{\perp} = 0 \) for \( 0 \leq \Theta_{\perp} < \pi \) and \( \varphi_{\perp} = \pi \) otherwise. The upper panels in both figures relate to the collinear configuration of the linearly polarized vectors \( \varepsilon_{K\sigma} = \varepsilon_z = \varepsilon_x \), whereas the lower panels are for the perpendicular configuration, when \( \varepsilon_{K\sigma} = -\varepsilon_y \) and \( \varepsilon_1 = e_x \). While the results presented in Fig. 7 are for the carrier-envelope phase \( \chi = 0 \), the results in Fig. 8 correspond to \( \chi = \pi/2 \). For such choices of \( \chi \), the CEP effect is clearly visible.

One sees from Fig. 7 that for \( \chi = 0 \), positron angular distributions are asymmetric with respect to the propagation direction of the \( \gamma \)-photon, with its maximum shifted towards angles \( \Theta_{\perp} \) smaller than \( \pi \). On the other hand, the positron angular spectra are symmetric for \( \chi = \pi/2 \), as demonstrated in Fig. 8. These features are independent of the respective polarization directions of both the laser pulse and the nonlaser photon. What does depend on their polarization configuration is the
oscillatory pattern shown by the positron angular distributions. In particular, for the collinear polarizations the angular spectra of positrons exhibit stronger interferences than for the perpendicular polarizations. However, the envelope of these oscillatory patterns stays roughly the same. Such a difference in the oscillatory behaviors can be intuitively understood by noticing that the positron energy-angular distributions depend, in principle, on the positron momentum $p_+$ through the following scalar products: $k \cdot p_+$, $K \cdot p_+$, $p_+ \cdot p_+$, $p_+ \cdot \varepsilon_1 \cdot p_+$ and $\varepsilon_k \cdot p_+$. For the $\gamma$-photon polarization vector perpendicular to the $(xz)$-plane (bottom panels of Figs. 7 and 8) the last scalar product vanishes. Hence, such a behavior has a purely geometrical origin, as our numerical investigations for other configurations show.

In closing this Section, let us compare Figs. 6 and 7, which are for the same CEP ($\chi = 0$) but for different incident pulse durations (32- and two-cycle laser pulses, respectively). One can see that the angular distributions of created positrons are very sensitive not only to a change of the carrier-envelope phase, which is particularly important for ultrashort laser pulses, but also to a change of the driving pulse duration. For the fixed CEP, the positron angular spectra are symmetric for very long laser pulses driving the pair creation (Fig. 6), whereas they become asymmetric with decreasing the pulse duration (except the cases when $\chi = \pi/2$ or $3\pi/2$, as illustrated in Fig. 8). The same was observed in Ref. [46] when analyzing the angular spectra of Compton photons by finite laser pulses. It can be considered, therefore, a general feature of QED processes induced by laser pulses of finite durations. The reason being that while for many-cycle laser pulses the vector potential is symmetric, meaning that $\langle f_i \rangle \sim 0$ for $i = 1, 2$ [see, Eq. (13)], thus for few-cycle laser pulses it is asymmetric (for almost all values of $\chi$).

In addition, we observe that by decreasing the pulse duration the angular spectra of positrons presented in Figs. 6 and 7 exhibit weaker interferences.

V. CONCLUSIONS

In the present article, the laser-induced Breit-Wheeler scenario of electron-positron pair production in which the pairs are created in collisions of a laser field with a nonlaser photon has been investigated. The driving laser field has been modeled as a finite laser pulse, along the lines developed in Ref. [40] for the Compton scattering. In order to check the validity of our theory, a comparison with a modulated plane wave approximation for the laser field has been performed. Let us stress that we have developed a general formulation of the laser-induced Breit-Wheeler process, which accounts for arbitrary parameters of both the colliding laser pulse (e.g., arbitrary pulse shapes, pulse durations, strengths, etc.) and the nonlaser photon (e.g., arbitrary polarization and energy).

Dependence of energy-angular distributions of created particles on parameters of the driving laser pulse has been studied in this paper in great detail. Contrary to the most recent work by Titov et al. [23], we have focused here on the nonperturbative regime of pair creation. In this regime, a sensitivity of the process to the polarization of the colliding laser pulse has been demonstrated, with the highest probabilities in the case of linear polarization. Therefore, our main focus in this paper has been on analyzing the case when a linearly polarized laser pulse collides with a $\gamma$-photon. Moreover, for numerical illustrations, we have chosen the head-on configuration of the colliding beams. As we have also shown, any other configuration can be obtained by a suitable Lorentz transformation from the head-on setup, and vice-versa.

We have demonstrated that for sufficiently long laser
pulses driving the Breit-Wheeler process, the energy-angular spectra of created particles agree very well with the spectra calculated for an infinite train of laser pulses, as we refer to using the modulated plane wave approximation. This agreement breaks down, however, for short pulse durations (see, also Ref. [10] for the related analysis of Compton scattering). For short pulses, a careful treatment of their actual temporal structure must be carried out. To demonstrate this, we have studied the carrier-envelope phase effects in the signal of pair creation. We have observed that by changing the CEP, the angular distributions of positrons (electrons) vary dramatically from being symmetric to asymmetric. This general feature does not depend on the relative linear polarizations of the colliding laser pulse and the nonlaser photon. Indeed, only a very detailed pattern of positrons (electrons) spectra depend on their relative space configuration. We believe that the sensitivity of the energy-angular spectra of created particles to a variation of the CEP can be used as means of phase control. In this context, it would be further interesting to analyze the CEP effects in total probabilities of pair creation, or in the electron-positron correlations that have been defined for the nonlinear Bethe-Heitler process in Ref. [33]. These tasks, when performed in the nonperturbative regime, require very demanding numerical calculations. For this reason, we will perform them separately from this paper and present their results in due course.

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