Axino as a sterile neutrino and $R$ parity violation

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Abstract

We point out that axino can be a natural candidate for sterile neutrino which would accommodate the LSND data with atmospheric and solar neutrino oscillations. It is shown that the so called 3+1 scheme can be easily realized when supersymmetry breaking is mediated by gauge interactions and also $R$-parity is properly broken. Among the currently possible solutions to the solar neutrino problem, only the large angle MSW oscillation is allowed in this scheme. The weak scale value of the Higgs $\mu$ parameter and the required size of $R$-parity violation can be understood by means of spontaneously broken Peccei-Quinn symmetry.
Current data from the atmospheric and solar neutrino experiments are beautifully explained by oscillations among three active neutrino species [1]. Another data in favor of neutrino oscillation has been obtained in the LSND experiment [2]. Reconciliation of these experimental results requires three distinct mass-squared differences, implying the existence of a sterile neutrino $\nu_s$ [3]. In the four-neutrino oscillation framework, there are two possible scenarios: the $2+2$ scheme in which two pairs of close mass eigenstates are separated by the LSND mass gap $\sim 1$ eV and the $3+1$ scheme in which one mass is isolated from the other three by the LSND mass gap. It has been claimed that the LSND results can be compatible with various short-baseline experiments only in the context of the $2+2$ scheme [4]. However, according to the new LSND results [2], the allowed parameter regions are shifted to smaller mixing angle, accepting the $3+1$ scheme [5, 6, 7]. Although it can be realized in a rather limited parameter space, the $3+1$ scheme is attractive since the fourth (sterile) neutrino can be added without changing the most favorable picture that the atmospheric and solar neutrino data are explained by the predominant $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\mu, \nu_\tau$ oscillations, respectively. In particular, the $3+1$ scheme with the heaviest $\nu_s$ would be an interesting explanation of all existing neutrino data. On the other hand, it is rather difficult to find a well-motivated particle physics model which would yield the desired four-neutrino masses and mixing in a consistent way [3].

In this paper, we show that the $3+1$ scheme can be easily realized in supersymmetric model with $U(1)$ Peccei-Quinn (PQ) symmetry when supersymmetry (SUSY) breaking is mediated by gauge interactions. In this model, axino can be as light as 1 eV, so plays the role of sterile neutrino [4]. A proper axino-neutrino mixing can be induced by $R$-parity violating couplings which appear as a consequence of spontaneously broken $U(1)_{PQ}$. It turns out that only the large angle MSW solution to the solar neutrino problem is allowed in our model. The weak scale value of the Higgs $\mu$-parameter and the required size of $R$-parity violation can be understood by means of the Frogatt-Nielsen mechanism [10] of spontaneously broken $U(1)_{PQ}$.

The model under consideration includes three sectors: the observable sector, the SUSY-breaking sector, and the PQ sector. The first contains the usual quarks, leptons, and two Higgs superfields, i.e. the MSSM superfields. The second contains a gauge-singlet Goldstino superfield $X$ and the gauge-charged messenger superfields $Y, Y^c$. Finally the third contains gauge-singlet superfields $S_k$ which break $U(1)_{PQ}$ by their vacuum expectation values (VEV), as well as gauge-charged superfields $T, T^c$ which have the Yukawa coupling with some of $S_k$. The superpotential of the model is given by

$$W = hS_3(S_1S_2 - f_{PQ}^2) + \kappa S_1TT^c + \lambda XYY^c + W_{\text{MSSM}} + W_{\text{SB}}$$

where $W_{\text{MSSM}}$ involves the MSSM fields, and $W_{\text{SB}}$ describes SUSY breaking dynamics enforcing $X$ develop a SU(5) breaking VEV: $\lambda X = M_X + \theta^2 F_X$. This VEV generates soft masses of the MSSM fields [11],

$$m_{\text{soft}} = \frac{\alpha}{2 \pi} F_X / M_X,$$

which are assumed to be of order the weak scale. One can easily arrange the symmetries of the model, $U(1)_{PQ}$ and an additional discrete symmetry to get

$$W_{\text{MSSM}} = y_{ij}^E H_1 L_i E_j^c + y_{ij}^D H_1 Q_i D_j^c + y_{ij}^U H_2 Q_i U_j^c + \frac{y_0}{M_s} S_1^2 H_1 H_2$$

$$+ \frac{y'}{M_s} S_3^2 L_i H_2 + \frac{\gamma_{ijk}}{M_s} S_1 L_i L_j E_k^c + \frac{\gamma'_{ijk}}{M_s} S_1 L_i Q_j D_k^c,$$

(2)
where the Higgs, quark and lepton superfields are in obvious notations. Here \( M_* \) denotes the fundamental scale of the model which will be taken to be the grand unification scale, \( M_{\text{GUT}} \approx 10^{16} \) GeV. Full details of the model will be presented elsewhere [12].

Let \( A = (\phi + ia) + \vec{b}a + \theta^2 F_A \) denote the axion superfield where \( a, \phi \) and \( \vec{a} \) are the axion, saxion and axino, respectively. It is then convenient to parameterize \( S_1 \) and \( S_2 \) as
\[ S_1 = S e^{A/f_{PQ}} \quad \text{and} \quad S_2 = S e^{-A/f_{PQ}}. \]
Note that the VEV of \( S \) is uniquely determined to be \( \langle S \rangle = f_{PQ} \) which would correspond to the axion decay constant if the saxion is stabilized at \( e^{\phi/f_{PQ}} \approx 1 \). After integrating out the SUSY-breaking sector as well as the heavy fields in the PQ sector, the low energy effective action includes the Kähler potential and the superpotential of \( A \) in the PQ sector, the low energy effective action includes the Kähler potential and the superpotential of \( A \),

\[
K_{\text{eff}} = f_{PQ}^2 \{ e^{(A + A^*)/f_{PQ}} + e^{-(A + A^*)/f_{PQ}} \} + \Delta K_{\text{eff}}
\]

\[
W_{\text{eff}} = \mu_0 e^{2A/f_{PQ}} H_1 H_2 + \mu'_i e^{3A/f_{PQ}} L_i H_2
\]

\[
+ e^{A/f_{PQ}} (\lambda_{ijk} L_i E_k^c + \lambda'_{ijk} Q_j D_k^c),
\]

where \( \Delta K_{\text{eff}} \) is \( A \)-dependent loop corrections involving the SUSY-breaking effects and

\[
\mu_0 = y_0 f_{PQ}^2 / M_*, \quad \mu'_i = y'_i f_{PQ}^2 / M_*^2,
\]

\[
\lambda_{ijk} = \gamma_{ijk} f_{PQ} / M_*, \quad \lambda'_{ijk} = \gamma'_{ijk} f_{PQ} / M_*.\]

This shows that in our framework the weak-scale value of \( \mu_0 \) and also the smallness of \( R \)-parity violating couplings can be understood by means of the Froggatt-Nielsen mechanism of \( U(1)_{PQ} \) with \( f_{PQ} \ll M_* \) [11]. Although not written explicitly, the coefficients of \( B \)-violating operators \( U^c D_j^c D_k^c \) can be easily arranged to be small enough to avoid too rapid proton decay, including the decays into axino or gravitino [13]. The best lower bound on \( f_{PQ} \) is from astrophysical arguments implying \( f_{PQ} \gtrsim 10^9 \) GeV [14]. To accommodate the LSND data, we need the axino-neutrino mixing mass of order 0.1 eV. It turns out that this value is difficult to be obtained for \( f_{PQ} > 10^{10} \) GeV. We thus assume \( f_{PQ} = 10^9 - 10^{10} \) GeV with \( M_* = M_{\text{GUT}} \) for which \( \mu_0 \) takes an weak scale value (with appropriate value of \( y_0 \)) and all \( R \)-parity violating couplings are appropriately suppressed.

An important issue here is the saxion stabilization. One dominant contribution to the saxion effective potential comes from \( \Delta K_{\text{eff}} \) which is induced mainly by the threshold effects of \( T, T^c \) having the \( A \)-dependent mass \( M_T = \kappa f_{PQ} e^{A/f_{PQ}} \). If \( M_T \lesssim \lambda X \), one finds

\[
\Delta K_{\text{eff}} \approx \frac{N_T}{16\pi^2} \frac{M_T M_*^2}{Z_T Z_{T^c}} \ln \left( \frac{\Lambda^2 Z_T^2 Z_{T^c}}{M_T M_*^2} \right),
\]

where \( N_T \) is the number of superfields in \( T \), \( Z_T \) is the Kähler metric of \( T \), and \( \Lambda \) is a cutoff scale which is of order \( M_X \). If \( M_T \gtrsim \lambda X \), \( \Delta K_{\text{eff}} \) would be enhanced by \( |\lambda X / M_T|^2 \); however then the resulting saxion effective potential can not stabilize \( \phi \) at the desired VEV [12]. We thus assume \( M_T \lesssim \lambda X \). With \( Z_T |\theta|^2 \approx -m_{\text{soft}}^2 \), the above \( \Delta K_{\text{eff}} \) gives a negative-definite saxion potential

\[
V_\phi^{(1)} \approx -\frac{N_T}{16\pi^2} m_{\text{soft}}^2 |\kappa f_{PQ}|^2 e^{2\phi / f_{PQ}}.
\]

There is another (positive-definite) potential from the \( A \)-dependent \( \mu \)-parameter:

\[
V_\phi^{(2)} \approx e^{4\phi / f_{PQ}} |\mu_0|^2 (|H_1|^2 + |H_2|^2).
\]
If $\kappa f_{PQ}$ is of order few TeVs, $\phi$ is stabilized by $V^{(1)}_{\phi} + V^{(2)}_{\phi}$ at the desired value $\phi^{f_{PQ} \phi} \approx 1$. This requires a rather small Yukawa coupling $\kappa \sim 10^{-6}$ which may be a consequence of some flavor symmetry. For $\phi^{f_{PQ} \phi} \approx 1$, the saxion and axino masses are estimated to be

\[ m_{\phi}^2 = (10 - 10^2 \text{ keV})^2 + \Delta m_{\phi}^2, \]

\[ m_{\tilde{a}} = (10^{-4} - 10^{-2} \text{ eV}) + \Delta m_{\tilde{a}}, \]  

(8)

where the numbers represent the gauge-mediated contributions for $f_{PQ} = 10^9 - 10^{10} \text{ GeV}$, and $\Delta m_{\phi}$ and $\Delta m_{\tilde{a}}$ are the supergravity-mediated contributions which are generically of order $m_{3/2}$ [13]. In gauge-mediated SUSY breaking models [11], the precise value of $m_{3/2}$ depends on the details of SUSY breaking sector. However most of models give $m_{3/2} \gtrsim 1 \text{ eV}$, implying $m_{\tilde{a}}$ is dominated by supergravity contribution. In this paper, we assume

\[ m_{\tilde{a}} \approx \Delta m_{\tilde{a}} \sim 1 \text{ eV} \]

(9)

which would allow the axino to be a sterile neutrino for the LSND data. We note that although it is generically of order $m_{3/2}$, $\Delta m_{\tilde{a}}$ can be significantly smaller than $m_{3/2}$ when the supergravity Kähler potential takes a particular form, e.g. the no-scale form [13].

Having defined our supersymmetric axion model, we discuss the full $4 \times 4$ axino-neutrino mass matrix:

\[ \frac{1}{2} m_{AB} \nu_A \nu_B \]

(10)

where $A, B = s, e, \mu, \tau$ and $\nu_s \equiv \tilde{a}$ with $m_{ss} = m_{\tilde{a}}$. We will work in the field basis in which $\mu'_i L_i H_2$ ($i = e, \mu, \tau$) in $W_{\text{eff}}$ are rotated away by an appropriate unitary rotation of $H_1$ and $L_i$. It is straightforward to see that the axino-neutrino mixing mass is given by

\[ m_{is} = \frac{\epsilon_i \mu_0 \langle H_2 \rangle}{f_{PQ}} \approx 0.1 \left( \frac{\epsilon_i}{10^{-5}} \right) \left( \frac{\mu_0}{600 \text{ GeV}} \right) \left( \frac{10^{10} \text{ GeV}}{f_{PQ}} \right) \text{ eV}, \]

(11)

where $\epsilon_i = \mu'_i / \mu_0$ for $\mu'_i$ in $W_{\text{eff}}$ before $\mu'_i L_i H_2$ are rotated away. Note that this axino-neutrino mixing survives under the unitary rotation eliminating $\mu'_i L_i H_2$ since $L_i$ and $H_1$ have different $U(1)_{PQ}$-charges. This charge difference is also responsible for the suppression of $R$-parity violating couplings as well as the weak-scale value of $\mu_0$. The $3 \times 3$ mass matrix of active neutrinos is induced by $R$-parity violating parameters [13]. At tree-level,

\[ m_{ij} \approx g_a^2 \langle \tilde{\nu}_i \rangle \langle \tilde{\nu}_j \rangle M_a, \]

(12)

where $M_a$ denote the gaugino masses. The sneutrino VEV’s $\langle \tilde{\nu}_i \rangle$ are determined by the bi-linear $R$-parity violations in the SUSY-breaking scalar potential: $m_{L_i H_1}^2$ and $B'_i L_i H_2$. In our model, nonzero values of $m_{L_i H_1}^2$ and $B'_i$ at the weak scale arise through renormalization group evolution (RGE), mainly by the coupling $\lambda'_{333} y_b$ where $y_b$ is the $b$-quark Yukawa coupling [14]. Moreover, $B H_1 H_2$ arises also through RGE which predicts a large $\tan \beta \approx 40 - 60$ [15]. We then find

\[ m_{ij} \approx 10^{-2} t^4 \left( \frac{\lambda'_{333} y_b}{10^{-6}} \right) \left( \frac{\lambda'_{333} y_b}{10^{-6}} \right) \text{ eV}. \]

(13)
where \( t = \ln(M_X/m_i)/\ln(10^3) \) for the slepton mass \( m_i \). Here we have taken \( m_i \approx 300 \) GeV and \( \mu_0 \approx 2m_i \) which has been suggested to be the best parameter range for correct electroweak symmetry breaking [13]. There are also bunch of loop corrections to \( m_{ij} \) from \( R \)-parity violating couplings [19], however in our case they are too small to be relevant.

Let us now see how nicely all the neutrino masses and mixing parameters are fitted in our framework. The analysis of Ref. [7] leads to the four parameter regions, R1–R4 of Table 1, accommodating the LSND with short baseline results. In our model, Eqs. (9) and (11) can easily produce these LSND values of the largest mass eigenvalue \( m_4 \approx m_{ss} = m_s \sim 1 \) eV and the mixing matrix elements \( U_{ij} \approx m_{is}/m_{ss} \approx 0.1 \). The effective \( 3 \times 3 \) mass matrix of active neutrinos is then given by

\[
m_{ij}^{\text{eff}} = m_{ij} - m_{is}m_{js}/m_{ss}. \tag{14}
\]

Upon ignoring the small loop corrections, this mass matrix has rank two, and can be written as

\[
m_{ij}^{\text{eff}} = m_x\hat{x}_i\hat{x}_j + m_y\hat{y}_i\hat{y}_j \tag{15}
\]

where \( \hat{x}_i \) and \( \hat{y}_j \) are the unit vectors in the direction of \( m_{is} \) and \( \langle \hat{\nu}_j \rangle \), respectively. Remarkably, the mass scale \( m_x \sim (m_{is}/m_{ss})^2m_{ss} \approx 10^{-2} \) eV gives the right range of the atmospheric neutrino mass. Eq. (13) shows that \( m_y \) is also in the range of \( 10^{-2} \) eV, so \( m^{\text{eff}} \) would be able to provide the right solar neutrino mass unless \( \Delta m_{\text{sol}}^2 \ll 10^{-4} \) eV^2. Note from Eq. (4) that the typical size of \( \epsilon_i, \lambda_{ijk}, \lambda^\prime_{ijk} \) is around \( 10^{-6} \) for \( f_{PQ} \approx 10^{10} \) GeV and \( M_s \approx 10^{16} \) GeV. To discuss the consequences of the mass matrix (15) on the atmospheric and solar neutrino oscillations of three active neutrinos, we diagonalize it to find the mass eigenvalues

\[
m_{2,3} = \frac{1}{2} \left( m_x + m_y \pm \sqrt{(m_x + m_y \cos 2\xi)^2 + m_y^2 \sin^2 2\xi} \right) \tag{16}
\]

and the \( 3 \times 3 \) mixing matrix

\[
U = (\hat{z}^T, \hat{\nu}^T c_\theta - \hat{x}^T s_\theta, \hat{w}^T s_\theta + \hat{x}^T c_\theta). \tag{17}
\]

where \( \hat{z} \equiv \hat{x} \times \hat{y}/|\hat{x} \times \hat{y}|, \hat{w} \equiv \hat{x} \times \hat{z}/|\hat{x} \times \hat{z}|, \) and \( \tan 2\theta = m_x \sin 2\xi/(m_x + m_y \cos 2\xi) \) for \( \cos \xi = \hat{x} \cdot \hat{y}. \) The Super-Kamiokande data [20] combined with the CHOOZ result [21] imply that \( U^2_{\mu3} \approx U^2_{\tau3} \approx 1/2 \) and \( U^2_{e3} \ll 1 \). The solutions to the solar neutrino problem can have either a large mixing angle (LA); \( U^2_{e1} \approx U^2_{e2} \approx 1/2 \), or a small mixing angle (SA): \( U^2_{e1} \approx 1 \). This specify the first column \( \hat{z}^T \) of \( U \) as \( \hat{z} \approx (1/\sqrt{2}, -1/2, 1/2) \) for LA and \( \hat{z} \approx (1, 0, 0) \) for SA up to sign ambiguities. Since \( \hat{x} \cdot \hat{z} = 0 \), the pattern \( \hat{z} \approx (1, 0, 0) \) implies \( \hat{x}_e \approx 0 \). This leads to a too small \( U_{e4} \approx m_{es}/m_{ss} \approx 10^{-2} \), so the SA solution is not allowed within our model. Among various LA solutions to the solar neutrino problem, only the large-angle MSW solution with \( \Delta m_{\text{sol}}^2 \sim 10^{-4} \) eV^2 can be naturally fitted since \( m_x \approx m_y \sim 10^{-2} \) eV in our scheme. It is remarkable that \( f_{PQ} \approx 10^{10} \) GeV and \( M_s \approx M_{GUT} \) lead to the right size of \( R \)-parity violation yielding the desired values of \( m_{is} \) and \( m_{ij} \) also for the atmospheric and solar neutrino masses.

To see the feasibility of our whole scheme, we scanned our parameter space to reproduce the allowed LSND islands R1–R4 of Table 1 together with the following range of atmospheric and solar neutrino parameters: \( \Delta m^2_{31} = (1 - 8) \times 10^{-3} \) eV^2, \( \Delta m^2_{21} = (0.1 - 8) \times 10^{-4} \) eV^2 and
\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|c|c|}
\hline
 & $\Delta m_{41}^2$ (eV$^2$) & $|U_{e4}|$ & $|U_{\mu 4}|$ \\
\hline
R1 & 0.21-0.28 & 0.077-0.1 & 0.56-0.74 \\
R2 & 0.88-1.1 & 0.11-0.13 & 0.15-0.2 \\
R3 & 1.5-2.1 & 0.11-0.16 & 0.09-0.14 \\
R4 & 5.5-7.3 & 0.13-0.16 & 0.12-0.16 \\
\hline
\end{tabular}
\end{center}
\caption{Allowed regions for the LSND oscillation.}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{scatter_plots}
\caption{Scatter plots on (a) $m_y$ vs. $m_x$, (b) $\cos \xi$ vs. $m_x$ reproducing the correct oscillation parameters for R2.}
\end{figure}

\[ \tan^2 \theta_{23} = 0.33 - 3.8, \ \tan^2 \theta_{12} = 0.2 - 3.0, \ \tan^2 \theta_{13} \lesssim 0.055 \]  
Our parameter space consists of $m_{ss}, m_{is}, \lambda_{i33}^{\prime} y_b$ whose values are centered around 1 eV, 0.1 eV, $10^{-6}$, respectively. For R1 and R4, we could find some limited parameter space, however they need a strong alignment between $\hat{x}$ and $\hat{y}$ and also a large cancellation between $m_x$ and $m_y$. On the other hand, R2 and R3 do not require any severe fine tuning of parameters, although the bi-maximal mixing is obtained by some accident. We provide in Fig. 1 the scatter plots on the planes of $(m_y, m_x)$ and $(\cos \xi, m_x)$ for the LSND island R2. The scatter plots for R3 have similar shape. The full details of the analysis will be presented elsewhere [12].

To conclude, we have shown that the 3+1 scheme of four-neutrino oscillation can be nicely obtained in supersymmetric model endowed with the PQ solution to the strong CP problem and supersymmetry breaking mediated by gauge interactions.

\textbf{Acknowledgement:} This work is supported by BK21 project of the Ministry of Education, KOSEF through the CHEP of KNU and KRF Grant No. 2000-015-DP0080.
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