Model Predictive Control Strategy  
Based on Improved Trajectory Extension  
Model for Deviation Correction in Vertical  
Drilling Process* 

Dian Zhang**, Min Wu**,† LuX Feng Chen**, Chengda Lu**, Weihua Cao**, Feng Wang**  

* School of Automation, China University of Geosciences,  
Wuhan 430074, China  
** Hubei Key Laboratory of Advanced Control and Intelligent  
Automation for Complex Systems, Wuhan 430074, China  

Abstract: Vertical drilling system is widely used in deep geological exploration. As only  
inclination angle is considered in conventional vertical drilling systems, which decreases the  
quality of drilling trajectory, especially in geological drilling. In this paper, a model predictive  
control strategy based on improved trajectory extension model is provided, and it aims to reduce  
the position deviation and inclination angle of the drilling trajectory in vertical drilling process.  
An improved trajectory extension model is established by considering both attitude dynamic  
and space movement of bottom hole assembly under ground in vertical drilling process; and  
then, in order to deal with control constraints directly, a model predictive controller is provided  
based on the improved trajectory extension model. Simulation results of deviation correction  
are presented for validating the proposed strategy.

Keywords: Deep geological exploration, vertical drilling, deviation correction, improved  
trajectory extension model, model predictive control.

1. INTRODUCTION

As shortage of shallow mineral resources and increase  
of energy demand, deep drilling has gradually been an  
important subject of geological exploration in the future  
(Mason, 2019). Vertical drilling occupies an important  
place in deep geological drilling due to economic and safety  
reasons (Ritesh et al., 2017).

The purpose of vertical drilling is to maintain the drilling  
trajectory to be straight along with the plumb line of  
wellhead. However, because of lithology characteristics,  
types of bottom hole assembly (BHA) et al., position  
deviation and inclination angle of drilling trajectory are  
easily increasing in practice (Wang et al., 2019). Large  
position deviation between actual drilling trajectory and  
the designed one will probably reduce the accuracy of  
target hitting. At the same time, too large inclination angle  
can also easily lead to a serious drilling accident such as  
stucking in vertical drilling process (Huang et al., 2018).  
Hence, reducing the position deviation and maintaining a  
small inclination angle are the most two important targets  
in vertical drilling. 

Conventional control strategy of vertical drilling adopt  
passive anti-deviation technologies and heavily rely on  
manual experience, and their correction performance is  
limit (Godhavn et al., 2011). Control strategy based on  
automatic vertical drilling tools is the development trend  
in vertical drilling process nowadays, they can improve  
the correction capacity significantly compared with conventional one (Zhao et al., 2014; Lu et al., 2015). However,  
only the inclination angle is taken into account in these  
strategies, which will easily cause position deviation of  
trajectory and reduce the quality of drilling trajectory  
especially in deep drilling.

Directional drilling control strategy can adjust both position  
development and inclination angle at the same time. As  
automatic vertical drilling process is similar to directional  
drilling, it could be expected that the control strategy of  
directional drilling will be effective when applied to the  
vertical drilling process as well. For directional drilling  
control strategy, Panchal et al. (2010) built an attitude  
dynamic model of BHA and designed an inclination and  
azimuth-hold controller. Bayliss et al. (2015) added uncertainty  
to attitude dynamic model and designed attitude-hold  
controllers, mixed uncertainty stability and performance  
analysis are also presented. Kremers et al. (2016)  
established a delay differential equations (DDE) trajectory  
tracking model and provided a model-based robust control  
strategy. As a fact that, in practice, the common way  
for steering is PID and fuzzy control method based on  
development vector theory (Xue et al., 2012). As for verti-
vertical drilling, there is still no effective model and control strategy considered correcting both position deviation and inclination angle at the same time, which motivates our work in this paper.

In this paper, in order to reduce both position deviation and inclination angle of the drilling trajectory in vertical drilling process, an improved trajectory extension model is established by considering both attitude dynamic and space movement of BHA under ground, and a model predictive control (MPC) strategy based on this model is proposed for deviation correction problem. Firstly, the control structure is proposed based on vertical drilling process analysis. Secondly, the trajectory extension model is established and revised for applying when inclination is small. Lastly, to deal with the constraints of vertical drilling directly, a MPC controller based on revised model is designed. The contributions of this work are that it establishes an improved trajectory extension model applied for deviation correction problem in vertical drilling process, and uses a MPC strategy to reduce both position deviation and inclination angle of drilling trajectory at the same time.

2. PROCESS DESCRIPTION AND CONTROL STRUCTURE

This section describes the vertical drilling process in detail, and the MPC control strategy for deviation correction is designed based on the improved trajectory extension model (Zhang et al., 2019).

2.1 Vertical Drilling Process

The vertical drilling system under consideration is equipped with the automatic vertical drilling tool, the drilling process is shown in Fig. 1 including measuring, calculation and BHA adjusting phases.

The trajectory parameters including inclination, azimuth angle and well depth are measured and sent to the computer on the ground though mud by measurement while drilling (MWD). After that, the computer on the ground calculates the control instructions, and sends them to downhole controller through MWD. Then the servo controller adjusts the servo system according to the control instructions, which changes the state of deflecting mechanism in real-time, and the drilling bit will follow the desired direction. At this time, one control cycle of vertical drilling process is completed.

For measurement constraints, only well depth, inclination angle and azimuth angle can be measured because of the limit of MWD. Meanwhile, geological drilling still adopts the static measurement method, which means these parameters are measured only when the drilling stops after drilling for a certain distance.

As for control constraints, it is generally necessary to keep the inclination angle less than 3° in vertical drilling process. At the same time, BHA has its tilting limit, conventional tools usually offer 6-7°/30m build rates (Wilson, 2016).

2.2 Control Structure

The control objective is to reduce the deviation while maintaining small inclination angle for vertical drilling. As system inputs are reference coordinates of trajectory and they are set according to the well plan, the structure of proposed control strategy and the corresponding parameters are given in Fig. 2. As it is seen that, the strategy mainly consists of two control loops.

The inner loop mainly adjusts servo system under control instructions named tool face angle and steering ratio in real time. Tool face angle always refers to rotation angle of the servo system. Steering ratio denotes the scaled magnitude of steering force applied in deflecting direction, and is always adjusted by changing the rotating time of servo system in one control cycle. A common control method that is widely used in servo system can be utilized to solve inner loop control problem.

Deviation correction control algorithm mainly implements in the outer loop. The scale of deviation can be calculated from coordinates of trajectory. Tool face angle and steering ratio are selected as control outputs. The control structure is mainly composed of the trajectory extension model and a MPC controller for deviation correction. A kinematics
3. TRAJECTORY EXTENSION MODEL FOR VERTICAL DRILLING

In this section, the kinematics model of BHA is built by combining attitude dynamics and circumferential movement formulas. Then the model is revised to be the improved trajectory extension model for vertical drilling.

3.1 Kinematics Model of BHA

In order to reduce both position deviation and inclination angle of the drilling trajectory at the same time, the model needs to consider both attitude dynamic and space movement of BHA under ground. A three-dimensional Cartesian reference system is created as shown in Fig. 3. As the Z-axis is pointing in the direction of gravity, the X-axis is due East, and the Y-axis is due North.

As seen in the figure, the downhole movement of BHA is divided into two parts, including the attitude dynamics and the circumferential movement with a certain curvature. For any point in the drilling trajectory, incidence angles are α and the azimuth angles is β. There are always transformations shown as (1):

\[
\begin{align*}
\frac{dS_x}{dS} &= \sin \alpha \cos \beta \\
\frac{dS_y}{dS} &= \sin \alpha \sin \beta \\
\frac{dS_z}{dS} &= \cos \alpha
\end{align*}
\]

Divide both sides of transformations by dt/dS. As the rate of penetration (ROP) \( \dot{S} = dS/dt \), velocity components of x-axis \( \dot{S}_x = dS_x/dt \), velocity components of y-axis \( \dot{S}_y = dS_y/dt \) and velocity components of z-axis \( \dot{S}_z = dS_z/dt \) are obtained.

In (2), \( r \) is the maximum deflection capability of BHA, and \( r \omega_{SR} \in [0, r] \) denotes the real deflection capability that BHA provided. Steering ratio \( \omega_{SR} \) and tool face angle \( \theta_{TF} \) are controllable variables. \( S_x \) and \( S_y \) are used to calculate the scale of position deviation, and \( \alpha \) and \( \beta \) are used to calculate the scale of angular deviation.

3.2 Improved Trajectory Extension Model

In vertical drilling process, inclination angle \( \alpha \) is generally less than 3°. It is easy to find in (2) that, when \( \alpha \) is small, the change rate of \( \dot{\beta} \) will be very large, which makes the scales of position deviation \( S_x \) and \( S_y \) hard to be controlled. Considering the worst situation that \( r = 6°/30m, \omega_{SR} = 100\% \) and \( \theta_{TF} = 0° \), the change rate of azimuth is shown in Fig. 4.

It can be seen that the azimuth change rate \( \dot{\beta} \) is larger than 20°/30m when inclination angle is less than 3°. The \( \dot{\beta} \) increases exponentially with the decrease of inclination angle, and it will be even too large when inclination is less than 1°, which makes it difficult to adjust the position deviation \( S_x \) and \( S_y \). So (2) is hard to apply to the deviation correction control in vertical drilling process directly. Meanwhile, tool face angle have two forms: high-edge tool face angle and magnetic tool face angle. In practical engineering, it is difficult to detect high-edge tool face angle when well inclination angle is less than 3°, while (2) can only be applicable with high-edge tool face angle. Therefore, it is necessary to adjust the model appropriately. The \( \alpha \) can be decomposed into angles in XOZ and YOZ planes, as shown in Fig. 5.
As $\alpha_x$ is the projection from $\alpha$ to XOZ plane, and $\alpha_y$ is the projection from $\alpha$ to YOZ plane. The relationship between angles and lengths is given as:

\[
\begin{align*}
    m \cos \alpha_y &= h \\
    l \cos a &= h \\
    d \cos \alpha_x &= h \\
    l \sin \alpha &= k \\
    m \sin \alpha_y &= p \\
    d \sin \alpha_x &= n \\
    k \cos \beta &= p \\
    k \sin \beta &= n
\end{align*}
\]

After simplifying (3), $\alpha_x$ and $\alpha_y$ are given in terms of azimuth and inclination:

\[
\begin{align*}
    \tan \alpha_x &= \tan \alpha \sin \beta \\
    \tan \alpha_y &= \tan \alpha \cos \beta
\end{align*}
\]

As inclination angles are less than $3^\circ$, $\sin \alpha \sin \beta \approx \tan \alpha$ and $\sin \alpha \cos \beta \approx \tan \alpha \beta = \tan \alpha_y$. Substituting (4) into (2) gives the improved trajectory extension model as:

\[
\begin{align*}
    \alpha &= \arctan \left( \sqrt{\tan^2 \alpha_x + \tan^2 \alpha_y} \right) \\
    \beta &= \arctan \left( \frac{\tan \alpha_x}{\tan \alpha_y} \right) \\
    \dot{S}_x &= \dot{S} \cos \alpha \\
    \dot{S}_y &= \dot{S} \tan \alpha_y \\
    \dot{\alpha}_x &= \omega_x = r \omega_{SR} \sin \tilde{\theta}_{eff} \\
    \dot{\alpha}_y &= \omega_y = r \omega_{SR} \cos \tilde{\theta}_{eff}
\end{align*}
\]

As $\alpha_x$ and $\alpha_y$ change more gently, so $S_x$ and $S_y$ are much easier to be controlled. Meanwhile it is closer for practical vertical drilling process to use magnetic tool face angle $\tilde{\theta}_{eff}$ to be the control input. In conclusion, improved trajectory extension model consider both attitude dynamic and moving law of BHA, and is also been revised to apply with small inclination angle, it provides convenience for designing deviation correction controller for vertical drilling.

4. DESIGN OF MPC CONTROLLER

In this section, in order to deal with the constraints of vertical drilling directly, a MPC strategy based on revised model is designed for vertical drilling. Firstly, an error system based on proposed trajectory extension model is constructed for predictive equation design of MPC controller. And then, the optimization problem of MPC is also discussed in this section.

4.1 Predictive equation

Linearizing and dispersing the improved trajectory extension model is the first step of building predictive equation. As inclination angles $\alpha_x$, $\alpha_y$ and the vertical deviation of BHA in X-axis and Y-axis $S_x$, $S_y$ are considered as the system’s state variables to describe the scale of deviation. Meanwhile, $\omega_x$, $\omega_y$ which can be acquired from steering ratio $\omega$ and tool face angle $\tilde{\theta}_{eff}$ are introduced as the control variables. In addition, as inclination angles are less than $3^\circ$, $\tan \alpha_x \approx \alpha_x$ and $\tan \alpha_y \approx \alpha_y$, converting (5) gives the linear model of BHA:

\[
\begin{align*}
    \begin{bmatrix}
        \dot{S}_x \\
        \dot{S}_y \\
        \dot{\alpha}_x \\
        \dot{\alpha}_y
    \end{bmatrix} &=
    \begin{bmatrix}
        0 & 0 & S_{x} & 10 & 0 \\
        0 & 0 & S_{y} & 0 & 0 \\
        0 & 0 & \alpha_y & 0 & 0 \\
        0 & 0 & \alpha_y & 0 & 0
    \end{bmatrix} +
    \begin{bmatrix}
        \omega_x \\
        \omega_y
    \end{bmatrix}
\end{align*}
\]

For dispersing, the sampling period is set to be $T$. In addition, linear model (6) can only describe the movement of BHA relative to Z-axis, it probably can not denote the deviation between real trajectory and the well plan. It notes that the horizontal coordinate $S_{ex}(k)$ and $S_{ey}(k)$, and inclination angles $\alpha_x(k)$ and $\alpha_y(k)$ of reference trajectory are constantly equal to zero in vertical drilling process, so the error state between real and reference $S_{ex}(k)$, $S_{ey}(k)$, $\alpha_x(k)$ and $\alpha_y(k)$ can be equal to the real state of BHA $S_{ex}(k)$, $S_{ey}(k)$, $\alpha_x(k)$ and $\alpha_y(k)$. Then the error system of BHA for vertical drilling in discrete time relative to reference trajectory can be written as follow:

\[
\begin{align*}
    \begin{bmatrix}
        S_x(k+1) \\
        a_x(k+1) \\
        S_y(k+1) \\
        a_y(k+1)
    \end{bmatrix} &=
    \begin{bmatrix}
        1 & ST & 0 & 0 & S_x(k) \\
        0 & 1 & 0 & 0 & a_x(k) \\
        0 & 0 & 1 & ST & S_y(k) \\
        0 & 0 & 0 & 1 & a_y(k)
    \end{bmatrix} +
    \begin{bmatrix}
        0 & 0 \\
        0 & 0 \\
        0 & 0 \\
        0 & T
    \end{bmatrix} \begin{bmatrix}
        \omega_x \\
        \omega_y(k)
    \end{bmatrix}
\end{align*}
\]

where $ST$ is equal to the length of a drill pipe, $S_x(k)$ and $S_y(k)$ are the displacements of BHA in x-axis and y-axis respectively relative to reference trajectory at time $k$, and $a_x(k)$ and $a_y(k)$ are projections from $\alpha$ to XOZ plane and YOZ plane relative to reference trajectory at time $k$ for vertical drilling. For parameter $ST$, as the trajectory parameters are measured after drilling for a certain distance, generally equal to the length of a drill pipe $L_{pipe}$, and $T$ is the sampling period $\bar{S}$ is the ROP, the $ST$ can be equal to $L_{pipe}$.

Using the above model (7) to be the basic prediction model, and $p$ is predictive window, $c$ is control window, the prediction equation of MPC controller can be established.
4.2 Optimization Problem

Assuming that $W(k)$ is a incremental control signal relative to the reference steering ratio, $Y(k)$ is a incremental state varieties relative to parameters of designed trajectory according to prediction equation, making $Y(k)$ tend to zero by adjusting the sequence $W(k)$ can ensure the BHA tracking reference trajectory. At the same time, in order to prevent drastic fluctuations of control signal, it is necessary to ensure that increment of control signal should be as small as possible. Combining with the constraints, the goal of our optimization is to minimize the error between the actual and the designed trajectory as well as the increment of control signal. As $\alpha$ is always small, $\alpha$ can be also equal to $\sqrt{\alpha_x^2 + \alpha_y^2}$. The deviation correction optimization problem for vertical drilling is given as:

$$\min J(Y(k), U(k)) = Y(k)^T Q Y(k) + W(k)^T R W(k)$$

subject to

$$\begin{align*}
(\alpha_x(k))^2 + (\alpha_y(k))^2 &\leq \alpha_{\text{max}}^2 \\
(\omega_x(k))^2 + (\omega_y(k))^2 &\leq r^2 \\
k &= 1, ..., n
\end{align*}$$

(8)

In the above equations, compared with the weight matrices of state and control signal respectively, larger $Q$ can make the tracking error to be smaller, but may cause oscillation. And larger $R$ can make the control signal changing more smoothly.

In addition, although the output of the controller is the increment of control signal relative to the reference control value, the actual control value is just equal to the control increment in vertical drilling because the reference control value is zero. At the same time, according to the model predictive control law, the actual control value should take the first value $\omega_x(k)$ and $\omega_y(k)$ of the optimized calculation sequence $W(k)$. So the actual control quantity can be obtained from (5).

5. SIMULATION AND RESULT ANALYSIS

Using the improved trajectory extension model to be the plant, simulations are carried out to test the deviation correction capacity for the case of vertical drilling. According to (Panchal et al., 2010), the parameters of simulation is selected as follow. ROP $S$ is 30 m/hr, the control cycle $T$ is 0.3 hr. For constraints the maximum deflection capability of BHA $r$ is $6^\circ/30$m, and $\alpha_{\text{max}}$ is $3^\circ$. Parameters of MPC is that: $p$ and $c$ is 5, $R$ is $[50000, 500000]$, $Q$ is $[0.1, 10, 0.1, 10]$.

According to data from an actual drilling site, the horizontal deviation between actual trajectory and the reference is 8.82m in XOZ plane at 600 meter measured depth (MD), meanwhile the horizontal deviation is 1.51m in the YOZ plane, the inclination angle is 1.5$^\circ$, the azimuth angle is 35.9$^\circ$ (He et al., 2012). As seen in Table 3, using conventional vertical drilling technology, the original trajectory of actual drilling site has a large position deviation, although the inclination angle is small. If applying proposed control strategy in this paper, the position deviation and inclination are eliminated to zero at nearly 800m MD.

In order to compared with the deviation correction performance of conventional control methods used in practice, simulations have been conducted with PID and fuzzy control method based on deviation vector control theory respectively. Fig. 6 shows the results of position and angle deviation correction. Fig. 7 shows the tool face angle and steering rate.

Here, the position deviation and inclination angle of two plane are eliminated to zero at nearly 800m MD, while the inclination $\alpha$ will not exceed the limit. PID and fuzzy control have long transient process when the deviation is small, and the real inclination $\alpha$ is greater than 3$^\circ$ as they can not deal with constraints directly. Adjusting parameters of PID or fuzzy controller will not improve performance significantly, because the cycle of measure-

| depth/m | original (He et al., 2012) | ours |
|---------|----------------------------|------|
| 600     | 8.82 1.51 1.5             | 8.82 1.51 1.5 |
| 700     | 11.61 1.41 1.6           | 4.35 0.93 3.0 |
| 800     | 15.26 1.14 2.1           | 0.11 0.02 0.38 |

Fig. 6. Deviation correction performance comparison

Fig. 7. Tool face angle and steering ratio

Table 1. Simulation results

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ment is too large. Besides, it is obvious that the change rate of $\omega_{SR}$ of proposed system is much smaller than the others, which means the MPC strategy makes deflecting mechanism easier to be controlled.

In conclusion, proposed strategy can efficiently correct the position deviation and inclination angle at the same time. And compared with PID and fuzzy control, it can efficiently deal with constraints, and its control output changes more gently.

6. CONCLUSION

In this paper, a model predictive control strategy based on improved trajectory extension model has been introduced to reduce both the deviation while maintaining and inclination angle for vertical drilling.

The deviation correction control structure has been mainly divided into two control loops: inner loop for adjusting state of the deflecting mechanism, and outer loop for evaluating state of drilling trajectory and calculating the control instructions with the MPC algorithm.

For trajectory extension model, the kinematics model of BHA has been developed by combining the attitude dynamics and the circumferential movement formulas for deviation correction problem. And then the model has been improved by introducing the magnetic tool face angle to meet the requirement of vertical drilling. Finally, the MPC-based controller has been proposed relying on this model. According to the simulation results, the proposed strategy effectively corrects the deviation during vertical drilling with satisfying constraints.

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