The role of bulk energy in nuclear multifragmentation.

N. Buyukcizmeci\textsuperscript{1}, A.S. Botvina\textsuperscript{2,3}, I.N. Mishustin\textsuperscript{3,4} and R. Ogul\textsuperscript{1}

\textsuperscript{1}Department of Physics, University of Selcuk, 42070 Konya, Turkey
\textsuperscript{2}Institute for Nuclear Research, Russian Academy of Sciences, 117312 Moscow, Russia
\textsuperscript{3}Frankfurt Institute for Advanced Studies, J.W. Goethe University, D-60438 Frankfurt am Main, Germany
\textsuperscript{4}Kurchatov Institute, Russian Research Center, 123182 Moscow, Russia

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Because of thermal expansion and residual interactions, hot nuclear fragments produced in multifragmentation reactions may have lower nucleon density than the equilibrium density of cold nuclei. In terms of liquid-drop model this effect can be taken into account by reducing the bulk energy of fragments. We study the influence of this change on fragment yields and isotope distributions within the framework of the statistical multifragmentation model. Similarities and differences with previously discussed modifications of symmetry and surface energies of nuclei are analyzed.

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I. INTRODUCTION

Multifragmentation has been observed in nearly all types of high energy nuclear interactions induced by hadrons, photons, and heavy ions (see a review \cite{1}). This is an universal phenomenon occurring when a large amount of energy is deposited in a nucleus, and a hot blob of nuclear matter is formed. At low excitation energies the nuclear system can be treated as a compound nucleus of nuclear matter is formed. At low excitation energies the nuclear system can be treated as a compound nucleus \cite{2}, which decays later via evaporation of light particles or fission. However, at high excitation energy, or possibly, compression during the initial dynamical stage of the reaction, this matter blob will expand to the sub-saturation densities, where it becomes unstable and breaks up into many fragments. As is well known (see e.g. Refs. \cite{3,4}) multifragmentation is a fast process, with a characteristic time around 100 fm/c. Nevertheless, as shown by numerous analysis of experimental data, a high degree of equilibration can be reached in these reactions, and statistical models are very suitable for description of fragment yields \cite{2,6,7,8,9,10,11}. We believe that taking multifragmentation into account is crucially important for correct description of fragment production in high energy reactions. On the other hand multifragmentation opens a unique possibility for investigating the phase diagram of nuclear matter at temperatures $T \approx 3 - 8$ MeV and densities around $\rho \approx 0.1 - 0.3\rho_0$ ($\rho_0 \approx 0.15$ fm$^{-3}$ is the normal nuclear density). These conditions are typical for the liquid-gas coexistence region. It is interesting that similar conditions are realized in stellar matter during the supernova explosions \cite{12,13}.

In the course of nuclear disintegration hot primary fragments are first formed in close vicinity to each other, and, therefore, they are still subject to Coulomb and, possibly, residual nuclear interactions. It is commonly accepted that the liquid-drop description of individual nuclei is very successful in nuclear physics. However, in a multi-fragment system in the freeze-out volume the parameters of the liquid-drop model may change as compared with those for isolated nuclei. An obvious example is the reduction of the fragment Coulomb energy due to the presence of other fragments. This effect can be reasonably evaluated within the Wigner-Seitz approximation \cite{3}. The Coulomb interaction between the fragments can also influence the proton and neutron distributions in hot fragments \cite{14}. Moreover, nuclear interactions parameterized as bulk, surface and symmetry energy terms in the liquid-drop description of nuclei may change too. Possible modifications of surface and symmetry energies of primary fragments, and constraints from relevant experimental data, were analyzed in the previous works \cite{15,16,17,18,19}. It was found that the symmetry energy of hot fragments drops significantly in the freeze-out volume, and the surface energy can be considerably modified at high temperatures. In this paper we investigate possible changes of the nuclear bulk energy in primary fragments, and how this may affect their production in multifragmentation reactions. We will show that these effects may be quite essential for explaining some key observables, and they should be included in realistic models.

2. Statistical description of nuclear multifragmentation

All dynamical models used for description of the initial stage of the reaction lead to the conclusion that after a time interval of few tens fm/c, when fast particles leave the system, the evolution of the remaining nuclear system changes its character. Because of intensive interactions between nucleons the system evolves toward statistical equilibrium. At later times the hot nuclear residue expands and breaks-up into hot primary fragments. The Statistical Multifragmentation Model (SMM) is based on the assumption of statistical equilibrium between produced fragments in a low-density freeze-out volume \cite{1}. We believe that at this point the chemical equilibrium is established, i.e., the baryon composition (mass and charge) of primary fragments is fixed. However, the fragments can still interact with other nuclear species via the Coulomb and nuclear mean fields. Hence their energies...
and densities may be affected by these residual interactions. All breakup channels composed of nucleons and excited fragments are considered, and the conservation of mass, charge, momentum and energy is taken into account. An advantage of the model is that the formation of a compound nucleus is included as one of the channels. This allows for a smooth transition from the decay via evaporation and fission at low excitation energies [2] to the multifragmentation at high excitations. In the microcanonical treatment [1, 20] the statistical weight of the mutation at high excitations. In the microcanonical treatment [1, 20] the statistical weight of the decay channel $j$ is given by $W_j \propto \exp S_j$, where $S_j$ is the entropy of the system in channel $j$ which is a function of the excitation energy $E_x$, mass number $A_0$, charge $Z_0$ and other global parameters of the source. After formation in the freeze-out volume, the fragments propagate independently in their mutual Coulomb field and undergo secondary decays. The deexcitation of the hot primary fragments proceeds via evaporation, fission, or Fermi-breakup [21].

In the SMM light fragments with mass number $A \leq 4$ and charge $Z \leq 2$ are considered as structure-less particles (nuclear gas) with masses and spins taken from the nuclear tables. Only translational degrees of freedom of these particles contribute to the entropy of the system. Fragments with $A > 4$ are treated as heated nuclear liquid drops, and their individual free energies $F_{AZ}$ are parameterized as a sum of the bulk, surface, Coulomb and symmetry energy terms:

$$F_{AZ} = F^B_{AZ} + F^S_{AZ} + E^C_{AZ} + E^{sym}_{AZ}.$$ (1)

In this standard expression $F^S_{AZ} = (-W_0 - T^2/\epsilon_0)A$ is the bulk energy term including contribution of internal excitations controlled by the level-density parameter $\epsilon_0$, and $W_0 = 16$ MeV is the binding energy of infinite nuclear matter. $F^S_{AZ} = B_0 A^{2/3}((T^2 - T^3)/T_0^2 + T_0^2)\gamma^{3/4}$ is the surface energy term, where $B_0 = 18$ MeV is the surface coefficient at $T = 0$, and $T_0 = 18$ MeV is the critical temperature of infinite nuclear matter. The Coulomb energy is $E^C_{AZ} = cZ^2/A^{1/3}$, where $c$ is the Coulomb parameter obtained in the Wigner-Seitz approximation, $c = (3/5)(e^2/r_0)(1 - (\rho/\rho_c)^{1/3})$, where $e$ is the proton charge, $r_0 = 1.17$ fm, and the last factor describes the screening effect due to presence of other fragments. $E^{sym}_{AZ} = \gamma(A - 2Z^2)/A$ is the symmetry energy term, where $\gamma = 25$ MeV is the symmetry energy coefficient. These parameters are taken from Bethe-Weizsäcker formula and correspond to the isolated fragments with normal nuclear density. This assumption has been proven to be quite successful in many applications. However, a realistic treatment of primary fragments in the freeze-out volume may require certain modifications of the liquid-drop parameters as indicated by experimental data.

In the grand canonical treatment of the SMM [22], after integrating out translational degrees of freedom, one can write the mean multiplicity of nuclear fragments with $A$ and $Z$ as

$$\langle N_{AZ} \rangle = g_{AZ} \frac{V_f}{\lambda_T} A^{3/2} \exp \left[ -\frac{1}{T} (F_{AZ}(T, \rho) - \mu A - \nu Z) \right].$$

Here $g_{AZ}$ is the ground-state degeneracy factor of species $(A, Z)$, $\lambda_T = (2\pi\hbar^2/m_N T)^{1/2}$ is the nucleon thermal wavelength, and $m_N \approx 939$ MeV is the average nucleon mass. $V_f$ is the free volume available for the translational motion of fragments. The chemical potentials $\mu$ and $\nu$ are found from the mass and charge constraints:

$$\sum_{(A,Z)} \langle N_{AZ} \rangle A = A_0, \quad \sum_{(A,Z)} \langle N_{AZ} \rangle Z = Z_0.$$ (2)

As was demonstrated by numerous comparisons of the SMM with various experiments, the model describes data very well (see, e.g., Refs. [5, 6, 7, 8, 9, 10, 11]). This confirms that the statistical approach with liquid-drop description of individual fragments provides adequate treatment of the multifragmentation process. This also justifies the application of the statistical approach for investigating the liquid-gas phase transition in nuclear systems [23, 24].

### 3. Modifications of properties of primary fragments

In recent years several new analyzes of experimental data with statistical models, related to the nuclear isospin in multifragmentation reactions have been performed [15, 17, 18, 19]. They conclude that modifications of the liquid-drop parameters of hot fragments produced in the freeze-out volume are needed to explain the data. It was suggested that this can happen because of a new physical condition where fragments are formed, in particular, since they are surrounded by nucleons and other hot fragments. The residual interactions may lead to energy and density changes which can effectively be explained by a modification of the macroscopic nuclear parameters. The symmetry energy coefficient $\gamma$ was investigated in several independent experiments [17, 18, 19], which used both the isoscaling phenomenon [23] and isotope distributions of fragments. It is important that all experiments come to the conclusion that the coefficient $\gamma$ drops from about 25 MeV, known for isolated cold nuclei, down to $\approx 15$ MeV for hot primary fragments at multifragmentation. The same results were extracted also from analysis of mean neutron content of fragments [13, 19]. One of the aims of future experiments is to verify this conclusion.

A recent analysis of the ALADIN data has revealed modifications in the nuclear surface properties too [15]. There the neutron-to-proton $(N/Z)$ dependence of the surface energy was analyzed for different event classes corresponding to different excitation energies. At low excitation energies, corresponding to the onset of multifragmentation, the surface energy follows the trend predicted by the standard liquid-drop model, i.e., it decreases with $N/Z$. This trend is usually explained by the contribution of the surface part of the symmetry energy. In the region of developed multifragmentation (temperatures $T \approx 5$–6 MeV), where intermediate-mass fragments...
densities and Coulomb fluctuations at the subnuclear average density of fragments. Due to the spinodal function of the nucleon density:

\[ W(\rho) = W_0(\rho_0) - \frac{K}{18} \left( \frac{\rho_f - \rho_0}{\rho_0} \right)^2, \]

where \( K \approx 260 \text{ MeV} \) is the nuclear compressibility modulus. There are natural limits for possible reduction of the average density of fragments. Due to the spinodal instabilities and Coulomb fluctuations at the subnuclear densities \( \rho < (0.6 - 0.7)\rho_0 \), uniform nuclear matter becomes unstable, and the nuclear "pasta" phases are produced \[29, 30, 51\]. By this reason, we take \( \rho_f \approx 0.6\rho_0 \) as the minimum possible nuclear density in individual fragments. One can see from eq. (3), that this corresponds to decreasing \( W_0 \) from 16 MeV at \( \rho_f = \rho_0 \) down to \( W_0 \approx 14 \) MeV. In the following analysis we allow for even smaller \( W_0 \), in order to perform a complete investigation of this effect.

At the last stage of the multifragmentation process hot primary fragments undergo deexcitation and propagate in the mutual Coulomb field. As was demonstrated in many works (see, e.g., \[1\]) this stage is very important for correct calculations of final yields of fragments. In the beginning of the deexcitation the hot fragments are still surrounded by other species, and, therefore, their modified properties should be taken into account. As far as we know, only one evaporation code was designed, which takes into account the modified properties of fragments in their de-excitation. It was developed in Refs. \[16, 32\], where modifications of symmetry energy were explicitly considered. In the present analysis we use the same prescription. Namely, we start from the modified bulk energies of hot nuclei and restore their normal properties by the end of the evaporation cascade. In actual calculations we have used a simple interpolation between these two limiting states. The energy and momentum conservation laws were fulfilled in the course of this evaporation process.

We emphasize that the treatment of this de-excitation stage should be consistent with the physical properties of primary fragments. For example, a failure to describe the experimental isoscaling data \[33\], by using a dynamical model for primary fragment formation, may be related to the fact that the sequential evaporation from primary fragments was included without paying attention to their reduced densities and, consequently, to their lower symmetry energies. As was demonstrated in Refs. \[16, 18\], this effect can influence essentially the observed isotope distributions, since separation energies of neutrons and protons are changed when compared to the isolated low-excited nuclei. The transition to the secondary de-excitation can be consistently controlled in statistical description of fragment formation, by using a generalized evaporation prescription \[16\]. Moreover, we think that the final conclusions about properties of primary fragments can be obtained only after a many-component analysis of experimental data. Besides the isotope information and isoscaling observable this should include the corresponding fragment charge distributions, IMF multiplicities, temperatures, and the other relevant characteristics (examples of such an approach are presented in Refs. \[6, 7, 8, 13, 34\]).

4. Influence of the bulk energy on multifragmentation characteristics

We have used the SMM to simulate multifragmentation of the gold source with excitation energies in the range of 2–12 MeV per nucleon at different values of the bulk energy coefficient \( W_0 \). For simplicity, we fix the freeze-out density at \( 1/3\rho_0 \) as was done also in many previous SMM studies. In Figure 1 we show some characteristics of the hot fragments: effective temperatures, mass number of the largest fragment, average multiplicity of IMFs produced in the freeze-out volume. The effective temperature \( T_{eff} \) is calculated from the energy balance in the
system, in the same way as it was done in Ref. \cite{15}.

One can see that decreasing the bulk energy does not change qualitatively the general picture of multifragmentation. Nevertheless, some new important features appear. For example, the back-bending of the caloric curve become more pronounced, i.e., multifragmentation reactions become more endothermic. Such behavior of $T_{\text{eff}}$ can be interpreted as a manifestation of the negative heat capacity. An interesting result is that the number of IMFs and the mass of large fragments at freeze-out become smaller, while the number of light particles (Fig. 2) is increased considerably. From Fig. 2 one can see a drastic rise of the yields of $\alpha$-particles and clusters with $A < 4$ at reduced $W_0$. The reason is that we take into account fragments with $A \leq 4$ as 'gas' particles with their table binding energies, while binding energies of other fragments become smaller. We believe that $\alpha$-particles have normal properties in the low density freeze-out volume. This assumption is consistent with many experimental data. For example, by interpretation of fragment yields in direct knock-out reactions on nuclei, $\alpha$-clusters are assumed to pre-exist even at the normal nuclear density, which is much higher than the freeze-out density at multifragmentation \cite{35}. We suggest that this effect may explain an excess of $\alpha$-particle yields over SMM predictions observed in emulsion data \cite{36}.

It is important that all new trends in the fragment yields caused by changing the bulk energy survive after the secondary deexcitation. Figs. 3 and 4 show the final yields of cold fragments. The number of emitted $\alpha$-particles even increases, since their evaporation becomes more probable at the first stages of deexcitation, in addition to the increased production at the break-up of the source (see Fig. 4). This fact leads to the decrease of the so-called He–Li temperature (see Fig. 3), which is one of the useful experimental observable for the calorimetry of multifragmentation processes \cite{37,38}. On the other hand, decreasing $W_0$ causes a drop of neutron and proton yields, since they are now bound in the light clusters. For the same reason, the maximum IMF multiplicity becomes lower.

In order to give a detailed picture of nuclear break-up,
in Figs. 5 and 6 we present the distributions of produced fragments in the full mass range, before and after the secondary deexcitation. We choose the range of excitation energies where the transition from the 'U-shaped' to the power-law distributions is taking place. It is obvious that decreasing the bulk energy favours multifragmentation, since less bound largest clusters are destroyed in favor of light clusters and nucleons. However, the yields of IMFs in this crucial region change very little with $W_0$ (as one can also see from Figs. 1 and 3), and the general shape of the IMF distributions does not change. By using a well-known power-law parametrization $A^{-\tau}$ of IMF yields we obtain that the $\tau$ remains practically the same for the considered variations of $W_0$. This result is quite understandable: since the $W_0$ parameter enters binding energies of all IMF, their relative yields do not change.

For this reason, the previously reported results concerning evolution of the isospin-dependent contribution to the surface energy with excitation energy of the system \[13\], which are based on consideration of the IMF yields, remain valid in the case of reduced bulk energy too.

Much attention is now paid to the isotope production in multifragmentation reactions, because of its connection to properties of neutron-rich nuclear matter and to astrophysical applications \[13\]. We have investigated this problem, and Fig. 7 demonstrates isotope distributions of fragments with $Z = 10$ before (in the freeze-out volume) and after the secondary deexcitation (at infinity). At smaller bulk energies the hot fragments become a little bit more neutron rich. This is explained by enhanced production of the symmetric $\alpha$ clusters, leading to a neutron enrichment of the remaining nuclear matter. However, this trend is more evident for the cold fragments, and we have two contributions to this effect. The first one is caused by the considerable probability of $\alpha$ particles emission in the beginning of the evaporation cascade, due to the modified bulk energy of heavier nuclei. The second one is related to the fact that the initial temperature is lower, and, therefore, the evaporational evolution of nuclei towards the $\beta$-stability line is ceased at relatively large neutron excess. This possibility for obtaining neutron-rich isotopes should be considered in future
FIG. 5: Mass distributions of hot fragments produced from Au sources at excitation energies of 4, 5 and 6 MeV per nucleon for different bulk energy coefficients. 

FIG. 6: Mass distributions of the cold fragments produced from Au sources at excitation energies of 4, 5 and 6 MeV per nucleon.

analyzes of data alongside with possible reduction of the symmetry energy of fragments [14, 17, 18, 19, 25].

On the other hand, these two possibilities may complement each other, since an expansion (i.e., a reduced density) of hot fragments should lead to decreasing their symmetry energy too [39]. The bulk symmetry energy as a function of density is usually parameterized as

$$\gamma(\rho_f) \propto \gamma(\rho_0) \left(\frac{\rho_f}{\rho_0}\right)^n, \quad (4)$$

where the exponent $n$ ranges between 0.5 and 1.5 depending on the model assumptions on isospin-dependent nuclear interactions. Recent analyses of experimental data have shown that around $\rho_0$, the values of $n$ may lie in the interval between 0.7 [40] and 0.9 [41]. Thus, in order to explain a considerable drop of 40% in $\gamma$, which fits experimental observations, we need to decrease $\rho_f$ to $(0.45 - 0.55)\rho_0$. As we have noted previously, this is already lower than the threshold for appearance of different “pasta” phases [29, 30, 31]. Also, the realistic Hartree–Fock and Thomas–Fermi calculations for hot isolated compound nuclei predict a rather moderate expansion at temperatures of 5 – 6 MeV, not more than 10 – 20% [42, 43], depending on the nuclear forces used. Therefore, the maximum reduction of the fragment density, which can be approximately considered, is around 30-40%. In this situation one should find additional mechanisms to explain the observed modification of the symmetry energy.

5. Conclusions

In this paper we proceed further with the investigation of possible modifications of nuclear properties in surrounding of other nuclei, which can still interact with nuclear and electromagnetic forces. This subject is relevant for nuclear matter at low density, which breaks-up into fragments. This state of matter can also be considered as a mixed phase of the nuclear liquid-gas phase transition. It is expected that this transition takes place in such astrophysical processes as collapses of massive stars and supernova explosions. In terrestrial laboratories this physical phenomenon can experimentally be studied in multifragmentation reactions induced by hadrons and heavy
mass distributions of hot and cold fragments with the charge number $Z=10$ at the excitation energy of 5 MeV per nucleon.

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