Modeling and Analysis of Tagging Networks in Stack Exchange Communities

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Abstract

Large Question-and-Answer (Q&A) platforms support diverse knowledge curation on the Web. While researchers have studied user behavior on the platforms in a variety of contexts, there is relatively little insight into important by-products of user behavior that also encode knowledge. Here, we analyze and model the macroscopic structure of tags applied by users to annotate and catalog questions, using a collection of 168 Stack Exchange websites. We find striking similarity in tagging structure across these Stack Exchange communities, even though each community evolves independently (albeit under similar guidelines). Using our empirical findings, we develop a simple generative model that creates random bipartite graphs of tags and questions. Our model accounts for the tag frequency distribution but does not explicitly account for co-tagging correlations. Even under these constraints, we demonstrate empirically and theoretically that our model can reproduce a number of statistical properties of the co-tagging graph that links tags appearing in the same post.

1 Introduction

Question-and-Answer (Q&A) platforms are now a standard context for social interaction on the Web with platforms such as Quora and Stack Exchange supporting large user bases. As a result, the social networks that these platforms support have undergone a great deal of study, including, for example, how people find interesting and popular questions on Quora (Wang et al. 2013), prediction of “best answer” selection on Yahoo Answers (Adamic et al. 2008), market design for knowledge base construction with Google Answers (Chen, Ho, and Kim 2010), and badge collection on Stack Overflow (Anderson et al. 2013). These studies have largely focused on models and analysis of the user behavior. However, the users also create other types of richly structured data. In this paper, we model and analyze the structure revealed by tags on Stack Exchange, which are used to annotate and catalog questions. Thus, our principal object of study is the tags (and their relationships through co-tagging), rather than the users; however, tags are still a by-product of user behavior since users apply the tags.

A Stack Exchange website is a Q&A forum for a particular community. The platform began with Stack Overflow, which is a community for computer programming. Stack Overflow is the the largest and arguably most well-known Stack Exchange community, but the Stack Exchange ecosystem supports a diverse set of communities ranging from pet ownership to coffee to philosophy. For the most part, these Stack Exchange communities evolve independently under the same Q&A format (Fig. 1). A linchpin of every Stack Exchange community is the tagging system. When posting a question, users are encouraged to apply a small number of tags (at least one and at most five) that provide a reasonable abstraction of the question’s topics. In addition to describing the question’s content, tags also serve users in information retrieval of similar questions as well as questions they might be able to answer. Tags on Stack Exchange are not taken lightly—users cannot immediately create new tags and are encouraged to use existing and popular tags (Fig. 1, bottom); moreover, there are also official tagging guidelines. Thus, tags on Stack Exchange are fundamentally different from, e.g., hashtags on social media platforms such as Twitter which are largely free from regulation. The value placed on tags means that they can contain rich information about the community. For example, tag frequencies can show popular topics and the change of tag frequency over time can reveal the change of a community’s interests over time.

Here, we provide the first large-scale study of the macroscopic structure of tagging behavior by analyzing a collection of 168 Stack Exchange communities. We frame our study through the lens of network analysis, focusing on two networks constructed from the tagging behavior of users. The first is the bipartite network of tags and questions, where there is an edge between a tag and all of the questions to which the tag was applied. The second is the co-tagging network, or the projection of the first network onto the tags; in this case, two tags are connected by an edge if the two tags jointly annotate at least one question. (We also consider a weighted version of the second network, where the weight is the number of questions containing the two tags.)

Oftentimes, network analyses suffer from the fact that there is only “one sample” of a social system to study. For example, there is only one Facebook friendship graph (Ugander et al. 2011) and one Twitter follower network (Kwak et al. 2010).
We further explore the Stack Exchange data by analyzing the “co-tagging network” induced by the bipartite tag-question network. Specifically, we analyze the graph where the nodes are tags and there is an edge connecting two tags if they are “co-tagged” on at least one question (with possible weighting on edges corresponding to the number of questions on which the two tags appear). Our analysis focuses on three macroscopic properties of the data. First, the weighted number of co-tags of a given tag is well-approximated by a linear function of the number of questions in which the tag appears. Second, the number of unique co-tags of a given tag is well-approximated by a simple third-degree polynomial of the number of the tag frequency. Qualitatively, as we increase the number of questions that a tag has appeared in, the number of unique co-tags will also increase; however, this growth tapers for popular tags, when it is difficult to accumulate more unique co-tags. Third, we measure three versions of the clustering coefficient for weighted and unweighted networks and find various levels of clustering and find that the unweighted clustering coefficient is only mildly correlated with the size of the Stack Exchange community (as measured by the number of questions), but two versions of the weighted versions both negatively correlate with size.

All three macroscopic properties are replicated by our model, which we validate with both empirical and theoretical analysis across the collection of 168 Stack Exchange networks. Importantly, the model does not bake in any notions of correlation or clustering in the co-tagging but can still replicate important co-tagging network properties. Thus, we can conclude that these network properties could actually be simply explained by our simple generative model that only makes a strong assumption on the frequency distribution of the tags. These findings contrast sharply with traditional social network analysis in measuring clustering. Standard random graph models for social networks that do not bake in clustering structure do not exhibit the same clustering levels as the real-world social system (Newman 2003). However, in our case, the co-tagging network constructed from our bipartite tag-question generative model matches the clustering levels in the empirical data.

Figure 1: Stack Exchange tagging. (Top) A question on the COFFEE Stack Exchange community with two tags: espresso and nespresso (https://coffee.stackexchange.com/q/1572). We study the tag frequency distributions across a large collection of Stack Exchange communities, as well as networks constructed from tags applied to the same questions. (Bottom) User interface of tagging guidelines on the COFFEE Stack Exchange (https://coffee.stackexchange.com/questions/ask). The last rule says that users cannot immediately create new tags without due process; thus, tagging is fundamentally different from hashtags on other social media platforms such as Twitter or Instagram.

2 Related Work

We summarize below how our research relates to several areas in social media, information retrieval, and network science.

Online Q&A platforms Question-and-Answer (Q&A) platforms have been a staple of online discussion for several years, involving major web companies such as Yahoo!, Google, and Quora. Research on these platforms has spanned a variety of topics, including reputation mechanisms (Bosu et al. 2013; Paul, Hong, and Chi 2012), answer quality measurement (Wang et al. 2013; Posnett et al. 2012; Anderson et al. 2012), network structure (Adamic et al. 2008; Paranjape, Benson, and Leskovec 2017); social behavior (Yang et al. 2011); answer prediction (Adamic et al. 2008; Tran, Zhang, and Li 2013); topic popularity (Marty, Sahni, and Mukherjee 2015); and expertise evaluation (MacLeod 2014; Posnett et al. 2012; Pal, Chang, and Konstan 2012). This research has largely focused on the questions, answers, and user behavior. Our paper, in contrast, treats tags as the
fundamental object of study. Furthermore, most prior work has only examined at most a few Q&A websites, whereas we study a large collection of Stack Exchange networks.

**Folksonomy** The tag-question network that we study is related to the idea of *folksonomy*, a term coined by Thomas Vander Wal to describe the practice of users tagging information for personal retrieval in an open social environment (Vander Wal, 2005). Folksonomy has been a lens for analysis on social media platforms such as CiteULike, del.icio.us, and BibSonomy (Cattuto et al. 2007), Capocci and Caldarelli 2008, and Cattuto et al. 2009). A major difference of these folksonomy studies and the present work is that folksonomies are much less restricted in the annotations—users can add many (possibly new) annotations freely—whereas the Stack Exchange system is restricted (between one and five tags with systematic vetting of new tags). And again, we analyze a large collection of Stack Exchange communities and not just a few folksonomies.

**Bipartite network models and co-tagging networks** Bipartite graph (network) models are employed across a broad range of scientific disciplines, including ecology (Bascompte, Jordano, and Olesen 2006), biomedicine (Goh et al. 2007), and information science (Akoglu, Chandy, and Faloutsos 2013). The model that we develop in this paper is a *generative (random) model* for a bipartite graph (network) between tags and questions. Other generative models for bipartite (or multipartite) graphs include the bipartite stochastic block model (Larremore, Clauset, and Jacobs 2014), evolutionary affiliation networks (Lattanzi and Sivakumar 2009), and generative models for folksonomy (Chojnacki and Kłopotek 2010). In contrast to prior research, the goal with our model is to develop a simple generative model that captures the empirical properties that we observe to persist across Stack Exchange communities. Our model is designed to capture the tag frequency distribution amongst questions, but we find that properties of the co-tagging network—where tags are connected if they have appeared in a question together—are still replicated with our model. Properties and statistics of co-tagging networks, such as clustering coefficients, characteristic path lengths, and number of co-tags have been used to analyze online communities such as del.icio.us and BibSonomy, have been studied (Cattuto et al. 2007, Halpin, Robu, and Shepherd 2007). Co-tagging networks have also been used for application on connecting users with similar interests (Wang, Liu, and Fan 2011).

3 **Data Description and Preliminary Analysis**

A Stack Exchange is a self-moderating online Q&A forum, and each Stack Exchange community centers on a different topic. Questions are annotated with at least one and at most five tags that serve as essential descriptors of the question (Fig. 1). Importantly, these platforms also largely evolve independently, allowing us to perform a better statistical analysis compared to analyzing a single Stack Exchange community. We now describe our dataset collection and provide preliminary statistical analyses that will serve the development of our generative model in the next section.
only a few times are much more common than tags used many times, and the distribution of tags is heavy-tailed. Many communities have tags appearing at much higher frequencies than most other tags; as an extreme example, the magic-the-gathering tag appears in more than 3000 questions in the BOARDGAMES community, while all other tags appear in fewer than 500 questions.

Such heavy-tailed distributions are common on the Web and other domains (Mitzenmacher 2004; Clauset, Shalizi, and Newman 2009). Here, we find that the tag frequencies are well-modeled by a lognormal distribution. Figure 3 illustrates four representative cases and also provides a comparison against other commonly-used heavy-tailed probability distributions such as a power law, truncated power law, and stretched exponential. (Fig. 3 shows four such cases). We find that a lognormal tends to match both the head and tail of the distribution, while other common heavy-tailed distributions can only capture either the head or tail of the distribution (e.g., in Fig. 3 the truncated power law captures the head of the tag frequency distribution in APPLE but not the tail and the tail of the COFFEE distribution but not the head). The lone outlier is the PATENT community, which does not seem to be well-approximated by any commonly-used heavy-tailed distribution.

More formally, we fit the parameters of a lognormal, power law, truncated power law, and stretched exponential distributions to the tag frequency of each Stack Exchange community using the powerlaw Python package (Alstott, Bullmore, and Plenz 2013). Figure 4 (top left) shows the fitted parameters, which are themselves approximately normally distributed. We use two standard procedures for evaluating the fit of the lognormal: the Kolmogorov-Smirnov (KS) statistic and the likelihood ratio test comparing the lognormal to other heavy-tailed degree distributions (Clauset, Shalizi, and Newman 2009). The distribution of the KS statistics is much smaller for the lognormal compared to the other distributions (Fig. 4 top right) and is less than 0.06 for 80% of the Stack Exchange communities. Furthermore, the p-values from the likelihood ratio test show that the power law, truncated power law, and stretched exponentials are not likely alternatives to the lognormal (Fig. 4 bottom).

To summarize, a lognormal distribution is an appropriate model for the distribution of tag frequencies. In the next section, we describe a simple generative model for random bipartite graphs of tags and questions based on this lognormal distribution. We will then later see that this model matches the real data in a number of characteristics related to the co-tagging, i.e., how multiple tags are used on the same question.

4 A Generative Model for Bipartite Tag-Question Networks

In this section, we propose a simple generative model for the bipartite tag-question network. Later, we will see that this model is able to recover many properties of the co-tagging network of Stack Exchange communities, i.e., the graph where nodes correspond to tags, and edges connect tags that have been applied to the same question. Formally, the bipartite tag-question graph $B$ consists of disjoint vertex sets $T$ and $Q$, each corresponding to the set of tags and questions, as well as a set of undirected edges $E$: where $(t, q) \in E$ with $t \in T$ and $q \in Q$ signifies that tag $t$ is applied to question $q$. The frequency, or number of occurrences, of a tag $t$ is then
Algorithm 1: Simple generative model for creating random bipartite graphs of tags and questions.

**Input:** number of tags $N_T$; number of questions $N_Q$; target number of tag occurrences $m$; $\mu$, $\sigma^2$  
**Output:** tag-question bipartite graph $B = (T \cup Q, E)$  

```plaintext
/* Sample tag occurrences and compute corrections. */
1 $x_t' \sim \text{LogNormal}(\mu, \sigma^2), t = 1, \ldots, N_T.$
2 $x_t \leftarrow \text{round}(m \cdot x_t'/\sum_{t=1}^{N_T} x_t'), t = 1, \ldots, N_T.$
3 Solve $\hat{N}_Q - \hat{N}_Q \exp(-m/N_Q) = N_Q$ for $\hat{N}_Q$.
4 $\hat{N}_Q \leftarrow \text{round}(\hat{N}_Q)$.
/* Construct bipartite graph */
5 $T \leftarrow \{1, \ldots, N_T\}, Q \leftarrow \{1, \ldots, \hat{N}_Q\}$.
6 for each tag $t \in T$ do
7     $Q_t \leftarrow \text{uniform sample of } x_t \text{ questions from } Q$.
8     for $q \in Q_t$ do add edge $(t, q)$ to edge set $E$.
9 end
10 $Q \leftarrow \{q \in Q \mid \exists t \in T \text{ for which } (t, q) \in E\}$
```

simply the degree of $t$ in the graph $B$.

Our random network model has two basic steps. First, given $N_T = |T|$, $N_Q = |Q|$, and the parameters $\mu$ and $\sigma$ of a lognormal distribution, we first generate a sequence of tag occurrence counts $x_t \sim \text{Lognormal}(\mu, \sigma^2)$. These samples are scaled by a constant so that $\sum_t x_t = m$ (where $m$ is the total number of tag occurrences in the original dataset) and then rounded to an integer. Since scaling a lognormal random variable by a constant is still lognormally distributed, we maintain this property of the tag distribution, and this preserves the total number of tag-question pairs in the dataset. Second, we assign tag $t$ to $x_t$ questions chosen uniformly at random without replacement. In this simplified version of the model, the output deviates from the Stack Exchange networks in two ways: (i) it is possible that a question has no tags and (ii) it is possible that a question is assigned more than five tags. We now show how to account for these deviations, and Algorithm 1 describes the full algorithm.

**Correction for question counts** To fix the problem where questions can have no tags, we make a “correction” in the number of questions. More specifically, we increase the number of questions from $N_Q$ to $\hat{N}_Q$ so that after the random assignment, the expected number of questions with at least one tag is close in expectation to $N_Q$, the number of questions in the empirical dataset. We then simply discard questions with no tags (Algorithm 1).

We approximate the expected number of questions with no tags under a simplification where tags can be duplicated in questions (the approximation is not necessary, but it makes the calculations simpler, has small variance theoretically, and provides good results empirically). Here, the probability that a question gets 0 tags is the same for each question—it is just the probability that all tags are assigned to the other $N_Q - 1$ questions:

$$\prod_{i=1}^{N_T} \prod_{j=0}^{x_t-1} \left[ 1 - 1/(\hat{N}_Q - j) \right] \approx (1 - 1/\hat{N}_Q)^m,$$

where $m$ is total number of tag occurrences. Thus, since $\hat{N}_Q$ and $m$ are generally large, when assigning tags uniformly at random to $\hat{N}_Q$ questions, the expected number of questions with 0 tags is

$$\hat{N}_Q (1 - 1/\hat{N}_Q)^m \approx \hat{N}_Q \exp(-m/\hat{N}_Q).$$

There are $N_Q$ questions if the following equation is satisfied:

$$\hat{N}_Q - \hat{N}_Q \exp(-m/\hat{N}_Q) = N_Q. \quad (1)$$

We claim that Eq. (1) has a unique positive solution $\hat{N}_Q > N_Q$. Since $m$ and $\hat{N}_Q$ are positive constants, the left hand side of Eq. (1) is a function $f$ of $\hat{N}_Q$. Moreover, the function $f$ is continuous and monotonically increasing in $\hat{N}_Q$, and $f(N_Q) = N_Q (1 - \exp(-m/N_Q)) < N_Q$. Therefore, the above equation has a unique positive solution for $\hat{N}_Q$ that is larger than $N_Q$. We can find the solution efficiently with binary search, and then round $\hat{N}_Q$ to the nearest integer.

In our experiments, using the corrected number of questions with our model is accurate, even with our approximations. Generating one sample for each dataset, the relative error between the number of questions with at least one tag in the model deviates from the true number of questions by 0.32% on average and by at most 3.75% across all datasets. While these statistics are for just one sample in each network, the variance in the number of questions with 0 tags is approximately $\hat{N}_Q p(1 - p)$. The ratio between the theoretical standard deviation and the corrected number of questions is small—less than 0.008 for 80% of the datasets (Fig. 5, left).

**Number of tags per question** We next justify our second model deviation, which is that questions can be assigned more than five tags. Our argument is that only a small fraction of questions are actually assigned more than five tags with our generative model. We generated tag-question bipartite graphs
with Algorithm 1 for each Stack Exchange community using the fitted lognormal parameters (Fig. 4 bottom). The mean fraction of questions with more than 5 tags in the generated networks across 168 Stack Exchange platforms is only 2.5% and more than 80% of datasets have less than 4.5% of questions with more than five tags (Fig. 5 right).

**Summary** Algorithm 1 is a simple generative model for bipartite tag-question networks that generates tag occurrences with the lognormal distribution that we found to be common across nearly all Stack Exchange communities. As a first look at how our model matches the empirical data, we consider the distribution of the number of tags per question. In the empirical data, this distribution tends to be uncorrelated with the size of the dataset (Fig. 5 right). We also find that the distribution of the number of tags per question in the model closely matches the empirical data (Fig. 6 right). In the next section, we analyze co-tagging, i.e., how tags jointly annotate questions. Our model has no built-in notion of correlations in co-tagging, yet we find that the model still matches macroscopic co-tagging properties in the data.

### 5 Co-tagging Analysis

In addition to the bipartite tag-question network, we also build a “co-tagging network” for each Stack Exchange community. Recall that the tag-question network \( B = (T \cup Q, E) \) is given by vertex sets \( T \) and \( Q \) corresponding to tags and questions and has edges \((t, q) \in E\) connecting tags to questions. The co-tagging network \( G \) is the projection of this graph onto the set of tags. Formally, \( G = (T, F) \), where \((s, t) \in F\) if and only if there is some question \( q \in Q\) such that \((s, q), (t, q) \in E\). In this case, we say that \( s \) and \( t \) co-tag with each other. We also associate a weight with each edge in \( G \) corresponding to the number of questions containing the two tags (the number of times that two nodes are co-tagged):\

\[
w_{s,t} = \left| \{ q \in Q \mid (s, q), (t, q) \in E \} \right|.
\]

In the rest of this section, we show that co-tagging networks constructed from samples of our generative model (Algorithm 1) match statistical properties of the co-tagging networks of empirical data, even though our model does not explicitly account for co-tagging behavior. Again, we use the lognormal parameters \( \mu \) and \( \sigma \) fitted for each dataset (Fig. 4) to generate a random graph for each Stack Exchange network. We focus our attention on three properties of the co-tagging network: (i) the expected number of co-tags (i.e., the weighted degree in \( G \)) as a function of tag frequency; (ii) the expected number of unique co-tags (i.e., the unweighted degree in \( G \)), again as a function of tag frequency; and (iii) weighted and unweighted versions of the clustering coefficient of the graph \( G \).

#### Weighted Co-tags and Tag Frequency

We first examine the relationship between the number of co-tags of a given tag as a function of its frequency (the number of questions in which it appears). Here, we consider the number of co-tags to be weighted, i.e., the number of co-tags of tag \( t \) is \( k_t = \sum_{s \in T} w_{s,t} \), following Eq. (2). In the empirical data, this relationship is essentially linear—a linear model of the number of co-tags in regressed on the number of questions containing the tag has a coefficient of determination \( (r^2 \text{ value}) \) greater than 0.95 in 95% of the Stack Exchange communities. Figure 7 (left) shows the distribution of slopes in the model across the Stack Exchange communities, which are strongly correlated.

| Slope (Data) | Slope (Model) |
|-------------|---------------|
| 0.00        | 0.25          |
| 0.50        | 0.75          |
| 1.00        | 1.25          |

| Frequency (Data) | Frequency (Model) |
|-----------------|-------------------|
| 0.00            | 0.25              |
| 0.50            | 0.75              |
| 1.00            | 1.25              |

Figure 6: Distributions of number of tags per post. Markers indicate number of tags: 1—blue triangle; 2—yellow ‘Y’; 3—green diamond; 4—red square; and 5—purple ‘+’.

Figure 7: (Left) The weighted number of co-tags is approximately a linear function of tag frequency. Here, we show the distribution of slopes from the linear regression over our collection of Stack Exchange communities. The regression has an \( r^2 \) value greater than 0.95 in 95% of the empirical datasets and greater than 0.97 in 97% of the generated datasets. (Right) The relationship between the fitted slope on the data and in the model across the Stack Exchange communities, which are strongly correlated.
Figure 8: (Left) The CDF of the mean-squared error in third-degree and first-degree (linear) polynomial models of the log number of unique co-tags in terms of the log of tag frequency in both the data and the samples from the generative model. The third-degree polynomial is a good approximation and matches the expected value of the model (Fig. 7). (Right) The CDF of the mean error in the expected number of unique co-tags in the model and the actual number of co-tags in the dataset. The error is less than 0.5 in 80% of the datasets. The model slightly over-estimates the number of unique co-tags by not taking into account tag correlations (see also Fig. 9).

tags). Although there is a quadratic relationship between \( k_t \) and \( x_t \), we know that \( x_t \) is typically small compared to \( n \). Thus, the gradient is well-approximated by the linear function \( m/N_Q \), i.e., \( \frac{d}{dx_t} k_t = m/N_Q \), independent of \( x_t \). Our analysis here is independent of the lognormal distribution of the tag frequency—we only relied on independence in the way that tags are assigned to questions.

In actual random samples, the linear relationship holds. We performed the same linear regression on random samples from our generative model using the fitted parameters in Fig. 4 as we did for the empirical datasets. In the model, 97% of the 168 datasets have a correlation coefficient \( r^2 > 0.97 \). Furthermore, the slopes from the regression on the generated data are highly correlated with the slopes on the empirical data (the correlation is 0.932; Fig. 7 right), and the mean squared error between the slope derived from a sample from the generative model and the computed slope on the empirical data across all Stack Exchange communities is just 0.10.

**Unique Co-tags and Tag Frequency**

In the above analysis, we saw that the number of co-tags of a given tag is approximately linear in the number of questions in which the tag appears—in both the empirical data and our model-generated data. In this section, we instead consider the number of *unique* co-tags of a given tag \( t \) as a function of the number of questions containing tag \( t \). In this case, the number of unique co-tags is equal to the unweighted degree of tag \( t \) in the co-tagging network \( G \) defined above.

We find that the log of the number of unique co-tags is well-approximated as a third-degree polynomial of the log of the number of question that contain the tag. Formally, let \( d_t \) denote the unweighted degree of tag \( t \) in the co-tagging network \( G \) and \( x_t \) the number of questions containing tag \( t \). We then fit a the following polynomial model:

\[
\log(d(t) + 1) = \sum_{i=0}^{3} a_i \log(x_t + 1)^i.
\]

Figure 9: Relationship between the number of unique co-tags and tag frequency on four Stack Exchange communities, which is well-approximated by a degree-three polynomial (see also Fig. 8). The model has the same shape, albeit slightly above the data.

Figure 8 (left) shows the CDF of the mean-squared error of the polynomial fit. The third-degree polynomial is a good fit for both the empirical data and the model across the collection of Stack Exchange communities. Figure 9 shows the distributions and fit of the third-degree polynomial for a few representative networks. In these cases, the polynomial fit is accurate and captures the fact that the number of unique co-tags does not grow linearly with tag frequency. Instead, the growth in unique co-tags tapers for some of the most frequently used tags. This happens because there is a limited total number of tags (Fig. 2), so tags that occur frequently have fewer options to increase the number of unique co-tags.

Interestingly, the fitted third-degree polynomial coefficients \( \{a_i\} \), when taken as a collection across the Stack Exchange communities, largely lie on a lower-dimensional subspace. In the empirical datasets, the first principal component explains 86% of the variability, and the second principal component explains an additional 13% of the variability. Similar results hold for the fitted coefficients in datasets generated with our model—89% of the variability is explained with the first principal component and an addition 10% is explained by the second principal component.

We can easily compute the expected number of unique co-tags with a simple summation. We argued in the previous section that the weighted number of co-tags between tags \( s \) and \( t \) is \( w_{s,t} \sim \text{Hypergeom}(N_Q, x_s, x_t) \). Thus, the expected number of unique co-tags \( d_t \) of tag \( t \) is

\[
E[d_t] = \sum_{s \neq t} P(w_{s,t} > 0) = \sum_{s \neq t} 1 - P(w_{s,t} = 0)
\]

\[
= \sum_{s \neq t} \left[ 1 - \frac{\binom{N_Q - x_t}{x_s}}{\binom{N_Q}{x_t}} \right],
\]

where \( x_s \) is the sampled number of questions for tag \( s \) in the generative model.
The log-weighted clustering coefficient, which is the same algorithm as the unweighted clustering coefficient:

\[
C = \frac{1}{|T|} \sum_{u \in T} \frac{2\Delta_u}{d_u(d_u - 1)}
\]

2. The weighted clustering coefficient:

\[
C_w = \frac{1}{|T|} \sum_{u \in T} \frac{1}{d_u(d_u - 1)} \sum_{v,z} \left( \hat{w}_{u,v} \hat{w}_{u,z} \hat{w}_{v,z} \right)^{1/3},
\]

where \( \hat{w}_{u,v} = w_{u,v}/\max_{x,y} w_{x,y} \) from Omenn et al. 2005.

We will analyze \( \log(C_w) \).

3. The log-weighted clustering coefficient, which is the same as the mean weighted clustering coefficient, except the weight \( w_{u,v} \) is replaced by \( w'_{u,v} = \log(w_{u,v} + 1) \):

\[
C_{lw} = \frac{1}{|T|} \sum_{u \in T} \frac{1}{d_u(d_u - 1)} \sum_{v,z} \left( \hat{w}_{u,v}' \hat{w}_{u,z}' \hat{w}_{v,z}' \right)^{1/3},
\]

where \( \hat{w}_{u,v}' = w_{u,v}'/\max_{x,y} w_{x,y} \) and summations over cases where \( w_{u,v}' = 0 \) (i.e., with no edge) are ignored.

Figure 10 (top row) shows that all three clustering coefficients are approximately normally distributed across the collection of Stack Exchange communities. Furthermore, the unweighted coefficients are only weakly correlated with the size of the community, measured by the log-number of questions on the Stack Exchange (Fig. 10 middle row). We conclude that the size of a Stack Exchange community is likely not a driving factor in the unweighted clustering of the network, which backs up conventional wisdom for the analysis of real-world networks (Newman 2003), differs from the behavior of random graph models that produce heavy-tailed degree distributions, where clustering decreases with size (Bolllobás and Riordan 2004). On the other hand, the weighted clustering coefficients tend to decrease with the size of the Stack Exchange community (Fig. 10 middle row).

The co-tagging networks derived from samples of our generative model reproduce these clustering coefficients remarkably closely and with strong positive correlations (Fig 10 bottom row). Again, we emphasize that our model does not bake in any explicit notion of clustering. Instead, our model only matches the lognormal distribution of the tag frequency and the total number of tags applied to all questions. Thus, clustering in the co-tagging in Stack Exchange communities could be explained simply by these simpler statistics. This finding contrasts sharply with typical (social) network analysis, where clustering is exhibited at a much higher level than is expected by random graph models (Newman 2003). The key difference is that our model is based on a projection of a bipartite tag-question graph rather than directly modeling the co-tagging network. This type of modeling has a long history in sociology (Breiger 1974) but has received relatively less theoretical attention in social network analysis (Lattanzi and Sivakumar 2009).

6 Discussion

In addition to providing answers to questions, the users of Q&A platforms create knowledge through annotation of questions. With their tagging system, Stack Exchange provides a unique opportunity to study these annotations for two main reasons. First, tags cannot be created freely and there are community guidelines for their application, which differ substantially from tagging norms on other social media platforms. Second, there is a collection of Stack Exchange communities that have largely evolved independently, enabling us to model and analyze tagging with more statistical evidence. And we indeed found similarities in macroscopic tagging structure—in terms of tag frequency and co-tagging network structure—across 168 Stack Exchange communities spanning a diverse range of topics. This contrasts from typical network analyses that study a single snapshot of a social network. Previously, researchers have circumvented this issue by looking at, for example, sets of disparate subgraphs from a larger graph (Traud, Mucha, and Porter 2012; Patania, Petri, and Vaccarino 2017); samples of ego networks (Gan- der, Backstrom, and Kleinberg 2013; Benson et al. 2018; McAuley and Leskovec 2014); and collections of snapshots.
of time-evolving networks (Yaveroglu et al. 2014).

One macroscopic property across communities is that the distribution of tag frequencies is well-modeled by a lognormal distribution. The fitted parameters of the lognormal distributions (Fig. 4) themselves are approximately normally distributed across our collection of Stack Exchange communities. Thus, one could incorporate this information as a simple prior in bayesian modeling of tag-question networks.

We used the tag frequency distribution to develop a simple generative model for random tag-question bipartite graphs, which was able to reproduce a number of the co-tagging and clustering properties of the datasets, without explicitly modeling correlations or clustering in the co-tagging process. Further understanding of the process producing this distribution is an avenue for future research. For example, multiplicative growth models are a well-known generative process for lognormal distributions (Mitzenmacher 2004). Although outside the scope of this paper, the availability of temporal information from Stack Exchange provides a path towards more robust understanding of the underlying processes of tag use, similar to other methods for estimating growth on the Web and in social networks (Huberman and Adamic 1999, Overgoor, Benson, and Ugander 2018).

Code and data. Code to reproduce our results, along with processed data, are available at https://github.com/yushangdi/stack-exchange-cotagging

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