Self-excited periodic oscillations in a supersonic laminar flow past a blunt-fin body mounted on a plate

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Abstract. Results of numerical simulation of supersonic gas flow past a blunt fin mounted on a flat plate with developing laminar boundary layer are presented. The calculations cover flow cases with the freestream Mach number of 6.7 and three different Reynolds numbers. A detailed analysis of flow characteristics such as temperature and surface heat flux are presented. In the smaller Reynolds number case, solution is steady-state and agrees very well with known numerical and experimental data. In case of larger Re values, the flow becomes periodic. These regimes are characterized by self-excited non-linear oscillations with almost the same fundamental frequency.

1. Introduction
Supersonic flow past a body mounted on a plate is one of the generic cases where the effects of the viscous-inviscid interaction play important role [1]. The flows of such type arise in many practical applications, for example, in designing such aircraft elements as the junction of the fuselage and the wing, tail assembly and others [2]. Relevant problems, important both theoretically and for practical applications, have been intensively studied by many researchers [3-11]. It was shown, in particular, that interaction of the boundary layer with an obstacle leads to a strongly nonuniform heat flux distribution in the region near the junction. Peak values of heat flux can be higher than the heat flux rate in the undisturbed boundary layer. Obtaining detailed information on flow structure in relevant cases and analysis of the effects of various parameters on heat transfer is of great interest.

The stationary laminar flow regimes take place at relatively low Reynolds numbers. An increase in the Reynolds number leads to extension of the separation region, and an unsteady flow regime can occur. Non-stationary effects can contribute to a further growth of local heat flux. For large Reynolds numbers, the unsteady flow nature can be quite complicated [7], while for moderate Reynolds numbers the flow is quasi-periodic [8].

In this study, we consider a laminar supersonic flow past a blunt fin mounted on a plate along which the boundary layer evolves (see figure 1). The problem definition is based on [5], where the flow with the freestream Mach number of 6.7 is considered in the range of Reynolds numbers, \( \text{Re}_D \), based on the leading-edge diameter \( D \) of the fin from \( 1.25 \times 10^4 \) to \( 3.75 \times 10^4 \). In [5], results of both experiments and numerical simulations were reported, and the authors interpreted the obtained numerical solutions as stationary for all \( \text{Re}_D \)-values considered. However, it was shown later, [8], that the flow simulated at \( \text{Re}_D = 2.5 \times 10^4 \) is unsteady. Below we present refined numerical solutions...
obtained for three values of $\text{Re}_D$: $1.25 \times 10^4$, $1.56 \times 10^4$ and $1.7 \times 10^4$. The solution got for the smallest $\text{Re}_D$ is steady, while self-excited oscillations occur for two other values of $\text{Re}_D$. Note also that numerical results for the lower Reynolds number case with the same geometric configuration were reported in our previous paper, [10], where the influence of the Mach number, plate length, and temperature factor on the flow structure and heat transfer was studied.

![Figure 1](image.png)

**Figure 1.** Flow structure scheme and computational domain.

2. Problem definition and numerical aspects

The computational domain for the problem under consideration is shown in figure 1 (dashed lines). The flow is assumed to be symmetric, therefore only half of the configuration is considered. The flow structure and heat transfer is determined by the following dimensionless parameters: geometric factor $L/D$, freestream Mach number $M = U_\infty/a_\infty$, Reynolds number $\text{Re}_D = U_\infty D/\nu_\infty$, Prandtl number $\text{Pr}$, temperature factor $T_\infty/T_w$, and the ratio of the specific heats $\gamma$. According to [5], the following set of unchanged parameters is considered: $M = 6.7$, $T_\infty/T_w = 4.75$, $\text{Pr} = 0.7$, $\gamma = 1.4$. As mentioned above, the simulation has been performed for three values of $\text{Re}_D$: $1.25 \times 10^4$, $1.56 \times 10^4$, and $1.7 \times 10^4$. It should be noted that, following [5], a desired change in $\text{Re}_D$ was achieved by modifying the diameter of the fin $D$ from 2.5 mm to 3.4 mm, whereas the distance $L$ between the leading edge of the plate and the body was fixed and equal to 145 mm. Thus, the boundary layer evolves on the plate in the same manner for all considered cases (the unit Reynolds number $\text{Re} = U_\infty/\nu_\infty = 5 \times 10^6$ $1/m$).

At the inlet boundary of the computational domain, a homogeneous flow is specified; the no-slip boundary conditions are specified at the fin surface and at the plate. At the lateral and upper boundaries, the non-reflecting boundary conditions are specified, while the zero-gradient condition is specified at the outlet. The fin and plate surfaces are maintained at constant temperature $T_w$.

The equations solved are the three-dimensional compressible Navier-Stokes equations with the perfect-gas equation of state. The viscosity is evaluated using the Sutherland formula. The conductivity of the gas is related to the viscosity through a constant Prandtl number. Numerical solutions were obtained using finite-volume unstructured code SINF/Flag-S developed at the Peter the Great St. Petersburg Polytechnic University. Details of numerical method are described in [11]. To get unsteady solutions, iterative time advancing scheme of second order of accuracy (backward scheme) was used. Steady-state solution at the lowest $\text{Re}_D$ was also obtained using the unsteady solver, to ensure that all oscillations disappear in this case (as initial data, an instant of a solution obtained at a larger $\text{Re}_D$ was used). The calculations were performed using a quasi-structured grid consisting of 10 mln cells; to check the solution sensitivity to grid refinement, a grid consisting of 25 mln cells was also used. More information on grid parameters ensuring solution grid-independency is given in [9].

The results of the work were obtained using computational resources of Peter the Great Saint-Petersburg Polytechnic University Supercomputing Center (scc.spbstu.ru).
3. Results and discussion

3.1. Case of $Re=1.25 \times 10^4$. Steady-state solution and comparison with experimental data

For the smallest Reynolds number considered, the solution is steady-state with flow structure similar to the ones reported in the literature [5-6, 8]. Structure of the flow is illustrated in figure 2a, where a surface Stanton number distribution and a Mach number map in symmetry plane is presented. Here, the Stanton number is evaluated using the adiabatic wall temperature $T_{aw}$:

$$
St = q_w / \left( \rho_w V_w C_p (T_{aw} - T_w) \right) \quad T_{aw} = T_\infty \left( 1 + r \left( \gamma - 1 \right) / 2M^2 \right)
$$

$r = \sqrt{Pr}$ – recovery factor.

One can see a separation region in front of the blunt fin and horseshoe vortices expanding around the side of the fin. Within the separation region, there are zones of supersonic flow including shock waves. High pressure gradients near the plate lead to re-separation of the near-wall flow. As a result, the separation zone formed in front of the body is quite wide. The limiting streamlines on the plate (obtained using the wall shear stress vectors) allow to identify clearly the regions occupied by horseshoe vortices. Forming in front of the body bow shock interacts with additional shock wave emanating from the separated-flow region, this shock-shock intersection causes bend of the bow shock towards the body.

In figure 2a one can see also that the largest increase in heat flux is observed at the saddle stagnation point located on the leading edge of the body, where incoming flow is divided into three parts: one part flows up, another part spreads over the body, and the third part of the flow turns towards the plate. The latter part is responsible for formation of the main horseshoe-shaped vortex and a sharp increase in plate-surface heat transfer in the region between the main horseshoe vortex and the corner vortex.

Figure 2b illustrates non-dimensional heat flux distribution along the plate centerline; the experimental and numerical data from [5], as well as the solution obtained with the refined mesh, are also shown. At each point of the centerline, the heat flux is referenced to the local value of the heat flux in the undisturbed boundary layer on the flat plate, $q_{wp}$. Local peaks on the Stanton number distribution are associated with formation of vortices in the separation region: local intensification of heat transfer is observed in the region between vortices, where the flow is “pressed” to the plate.

The steady-state numerical solution obtained in the present simulation is almost grid-independent and agrees well with the steady-state numerical solution reported in [5]. A slight difference between the solutions can be attributed to different numerical methods used in [5] and in the present study. Compared with experiment, there is a notable distinction with the numerical solutions: the experiment also predicts two local peaks of the heat flux, but one of them is slightly overestimated in calculation (experimental data in the region near the body, where the most intense heat flux takes place, are not reported in [5]).

![Figure 2](image_url)

*Figure 2.* (a) Flow structure in the junction region: Stanton number distribution over the plate and fin, Mach number map on a symmetry plane and streamlines; (b) heat flux distributions along symmetry line.
3.2. Cases of $Re=1.56\times10^4$ and $Re=1.7\times10^4$. Self-excited oscillations

A notable increase in the Reynolds number leads to appearing of self-excited temporal oscillations. In the present calculations, non-dimensional time step $\Delta tU_\infty/L = 3.67\times10^{-4}$ was used. To check the influence of time step on unsteady solutions, we performed also test simulations with twice reduced time step. The results did not change any noticeably.

For two Reynolds numbers, figure 3 illustrates time histories of the normalized heat flux at different points on the plate centerline (time scale is defined as $t_{ref} = L/U_\infty$). Note, that point $X/D = -0.21$ is located near the position of the highest heat flux peak. One can see that the increase in the Reynolds number leads to increasing in both peak heat flux value and the oscillation amplitude. However, the oscillation frequency remains almost the same.

The plate-centerline distributions of the averaged in time heat flux predicted for $Re_D = 1.7\times10^4$ (unsteady solution) and for $Re_D = 1.25\times10^4$ (steady-state solution) are shown in figure 4. The bars in figure 4 show the root mean square (RMS) fluctuations of the heat flux. It is seen that, in comparison with the steady-state solution, the unsteady regime is characterized by a slightly larger length of the separation region; peaks of heat flux are located further from the body. Such behaviour of the solution due to increasing in the Reynolds number is in agreement with our previous results reported in [10]. As mentioned above, the most intense oscillations occur in the region of the highest heat flux peak. The mean heat flux distribution computed for the intermediate Reynolds number, $Re_D = 1.56\times10^4$, is very close to that for $Re_D = 1.7\times10^4$ and is not shown here.

![Figure 3](image1.png)  
**Figure 3.** Time histories of heat flux at some points of the plate centerline.

![Figure 4](image2.png)  
**Figure 4.** Heat flux distributions along the centerline: comparison between steady and unsteady solutions.

For two Reynolds numbers, figure 5 shows distributions of RMS of heat flux oscillations over the plate ($q_{wp}L$ is the heat flux value in the undisturbed boundary layer at $X = L$). One can see that oscillations are observed in the whole region occupied by the horseshoe vortices but the most intense fluctuations occur in front of the body.

Figure 6 shows distribution of RMS of temperature fluctuations in a symmetry plane. The largest values of RMS are observed in the region where the bow shock interacts with oblique waves emanated from the separation region. Also, intensive fluctuations take place in the regions of supersonic flow within the separation region. In general, one can conclude that the external “inviscid” part of the flow is practically not involved in unsteady oscillations. Comparing distributions for two different Reynolds numbers, one can see that, as expected, the oscillations are more intensive in the larger Reynolds number case.
To reveal the spectral properties of the oscillations, the FFT analysis of the heat flux signals at $X/D = -0.21$ was made. For the lowest frequency in the spectra obtained, the normalized frequency $F = \omega v_\infty / U_\infty^2$ (known in the literature as frequency parameter; $\omega$ is the angular frequency) is evaluated as $F \approx 1.7 \times 10^{-4}$ for both values of the Reynolds numbers. It has been revealed also that the spectra contain only multiple harmonics of the fundamental (lowest) frequency. Table 1 presents squared magnitude of Fourier transform divided by the one for the fundamental frequency. One can conclude that in both cases, the oscillations are strongly nonlinear, and in case of the greater Reynolds number, the second harmonic has a bigger amplitude than the fundamental frequency oscillations. Most likely, a decrease of the Reynolds number to a value less than $1.56 \times 10^4$ would lead to weakening of nonlinearity and to reduction of the higher harmonics amplitudes, thus leading to the case of linear oscillations, but the accurate numerical simulation of such small linear oscillations is very challenging.

Low sensitivity of the fundamental frequency $F$ to the Reynolds number $Re_D$ based on the body leading-edge diameter suggests that the primary nature of the arising unsteadiness is in instabilities of the incoming laminar boundary layer. It is well known [12, 13] that there is a family of instability modes (the Mack modes) in supersonic boundary layers. In case of $M > 4$, the Mack second mode becomes the dominant instability. Based on data presented in the literature [13, 14], we can conclude that the discovered fundamental frequency $F \approx 1.7 \times 10^{-4}$ is close to the one of the Mack second mode. It should be noted that spectral characteristics of the revealed self-oscillations can be sensitive to the grid and numerical scheme used; therefore, further investigations are required.
4. Conclusions
Numerical study of supersonic laminar air flow past a blunt-fin/plate junction has been carried out for the case of the freestream Mach number of 6.7 and varying the Reynolds number $Re_D$ from $1.25 \times 10^4$ to $1.7 \times 10^4$. The computations have been performed assuming the plate and fin surface isothermal. The essentially three-dimensional flow predicted is characterized by clearly manifested effects of the viscous–inviscid interaction. The separation region emerging in front of the fin has a complex vortex structure: a family of horseshoe-shaped vortices is formed around the body, which leads to a strongly nonuniform distribution of the plate-surface heat flux.

In the smallest Reynolds number case, the solution predicted is steady-state and agrees very well with known numerical and experimental data. Increasing in the Reynolds number leads to occurrence of self-excited periodic oscillations in the flow. Concerning the averaged flow parameters, it has been revealed that the length of the separation region increases and local peaks of heat flux become higher and shift from the body. An analysis of RMS of fluctuations has shown that the most intensive oscillations of the plate-surface heat flux are observed in the region occupied by horseshoe vortices, especially in front of the body, where the vortices are most intense. Intensive temperature fluctuations are concentrated in the region where the bow shock bent towards the body due to interaction with oblique waves. Oscillations also occurs in the supersonic flow zones within the separation region.

Based on the FFT analysis of the time history of the plate-surface heat flux it has been revealed that at $Re_D = 1.56 \times 10^4$ and $1.7 \times 10^4$ periodic flow regimes are characterized by non-linear oscillations with similar fundamental normalized frequency, which, apparently, corresponds to the Mack second mode instability peculiar to supersonic boundary layers. Further studies of unsteady regimes are of undoubted interest, both at larger Reynolds numbers, when the oscillations spectra become multi-frequency/quasi-periodic, and at smaller $Re_D$, when the oscillations are close to linear.

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Table 1. The spectral properties of the heat flux oscillations.

| Frequency parameter | $Re_D = 1.56 \times 10^4$ | $Re_D = 1.7 \times 10^4$ |
|---------------------|--------------------------|--------------------------|
| $F$                 | 1                        | 1                        |
| $2F$                | 0.71                     | 1.19                     |
| $3F$                | 0.27                     | 0.54                     |
| $4F$                | 0.04                     | 0.13                     |