Adaptive control of non-affine MIMO systems with input non-linearity and unmodelled dynamics

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Abstract: In this study, an adaptive neural network control scheme is proposed for a class of multi-input multi-output (MIMO) non-affine systems with unmodelled dynamics and dead-zone non-linear input. This scheme solves the complexity of computation problem, broadens the variables of unmodelled dynamics and cancels the assumption of the neural network approximation error to be bounded. Using the mean value theorem and Young's inequality, only one adaptive parameter is tuned online for the whole MIMO systems, and the complexity of control system design is greatly reduced. The numerical simulation illustrates the effectiveness of the proposed scheme.

1 Introduction

Dead-zone non-linearity in actuators is the most common actuator non-linearities in many industrial processes, and many researchers are studying it in [1, 2], based on the dynamic surface control (DSC) technique and using radial basis function neural networks (RBFNNs), two adaptive schemes for a class of pure-feedback non-linear systems were proposed, respectively. While the uncertain multi-input multi-output (MIMO) non-linear time-varying delay systems with dead-zone were studied in [3]. In [4], a decentralised variable structure controller was presented for a class of system with dead-zone and time-delay. However, the dead-zone parameters and were needed to be known. The problem of over-parametrisation still existed in [5]. The DSC was first proposed to deal with the 'explosion of complexity' problem in [6], and [7, 8] expanded the DSC technology to backstepping design. It is well known that unmodelled dynamics exists in some non-linear systems, such as modelling errors, external disturbances, i.e. they may severely limit system performance. In [9], combining backstepping with DSC, a novel controller was used in the system with unmodelled dynamics. While in [10], the idea was used for the stochastic system with unmodelled dynamics. In [11], the main contributions lied in that a control strategy was provided for a class of strict-feedback non-linear systems with unmodelled dynamics and input saturation, and the scheme does not require any information for the parameter of input saturation non-linearity.

In practice, input non-linearity and unmodelled dynamics often appear simultaneously. Inspired by the previous articles, a class of MIMO non-affine systems with unmodelled dynamics and dead-zone non-linear input are discussed in this paper. The main contributions lie in:

(i) It is the first time to deal with such MIMO non-affine systems with unmodelled dynamics, non-linear input and dynamic disturbances.

(ii) Using Young's inequality, only one adaptive parameter needs to be tuned online for the whole MIMO systems, and the complexity of control system design is greatly reduced.

(iii) The assumptions of the unmodelled dynamics are broadened, and the functions in hypothesis 4 and hypothesis 6 are unnecessary to be known.

2 Problem description and assumptions

2.1 Problem description

A class of MIMO non-linear systems with unmodelled dynamics and dead-zone input are considered

$$
\begin{align*}
\dot{x}_i(t) &= F_i(x_i(t), x_{j\neq i}(t), u_i(t)) + d_i(z, x_i(t)), \\
\dot{x}_{ij}(t) &= F_{ij}(\bar{x}_{ij}, \bar{\hat{x}}(t)) + d_{ij}(z, x_i(t)), \\
y_j(t) &= x_j(t),
\end{align*}
$$

Dead-zone model is as follows:

$$
\begin{align*}
w_j(t) &= \begin{cases} 
    h_j(v_j(t)), & v_j(t) \geq v_{j*} \\
    0, & -v_{j*} < v_j(t) < v_{j*} \\
    h_j(v_j(t)), & v_j(t) \leq -v_{j*}
\end{cases}
\end{align*}
$$

where $\bar{x}_{ij} = [x_{i1}, \ldots, x_{ij}]^T \in \mathbb{R}^i$ is the state vector of the $j$th subsystem, $F_{ij}(\cdot, \cdot)$ and $m$ are positive constants. $i = 1, \ldots, \rho_i; j = 1, \ldots, m$. $x = (x_{i1}, \ldots, x_{im})^T \in \mathbb{R}^n$ is the state vector of the whole system. $N = \rho_1 + \cdots + \rho_m \in \mathbb{R}^n$ is the unmodelled dynamics, $F_{ij}(\cdot, \cdot)$ is the unknown smooth function; $d_{ij}(\cdot, \cdot)$ represents the unknown external disturbances; $d_{ij}(\cdot, \cdot)$ and $q(\cdot, \cdot)$ are uncertain and are continuous functions satisfying the Lipschitz condition. $y_j(t)$ is the output of the $j$th subsystem. $v_j(t)$ and $w_j(t)$ are the input and output of the dead-zone, respectively. $\bar{\hat{x}}(t) = [w_{j1}(t), \ldots, w_{j\rho_i}(t)]$, $v_{j*}$ and $v_{j-}$ are two unknown positive constants of the $j$th subsystem, $h_j(v_j(t))$ and $h_j(v_j(t))$ are smooth functions.

Assumption 1: $k_{j\rho_i}, k_{j\rho_i}$ are unknown positive constants, and hold

$$
\begin{align*}
\forall v_j(t) \in [v_{j*}, + \infty), & \quad 0 < k_{j*} \leq h_j'(v_j(t)) \leq k_{j+}; \\
\forall v_j(t) \in (-\infty, - v_{j*}], & \quad 0 < k_{j*} \leq h_j'(v_j(t)) \leq k_{j+}.
\end{align*}
$$

and $\beta \leq \min \{k_{j\rho_i}, k_{j\rho_i}\}$ is an unknown positive constant. For description, we will extend the definition for $h_j'(v_j(t))$ and $h_j'(v_j(t))$ as follows:
\[ h'_{j\rho}(v_j(t)) = h'_{j\rho}(v_j) \]
\[ h''_{j\rho}(v_j(t)) = h''_{j\rho}(-v_j) \]

According to Assumption 1, the dead-zone (2) can be rewritten as follows:

\[ w_j(t) = K_j(v_j(t))v_j(t) + \Delta_j(v_j(t)) \]

where

\[ K_j(v_j) = \begin{cases} h'_{j\rho}(v_j), & v_j > 0 \\ h'_{j\rho}(v_j), & v_j \leq 0 \\ -h'_{j\rho}(v_j)v_j, & v_j \geq 0 \\ -K_j(v_j)v_j, & -v_j < v_j < v_j \\ h''_{j\rho}(v_j), & v_j \leq -v_j \end{cases} \]

and

\[ \Delta_j(v_j(t)) = h''_{j\rho}(v_j) - v_j \]

Assumption 2: \( g_m \) and \( g_M \) are positive constants which satisfy \( g_m < g_B(\cdot) \leq g_M \).

Assumption 3: The expected trajectory vectors are continuous and available, let \( \tilde{\gamma} \in \{ y_{d1}, y_{d2}, \ldots, y_{dN} \} \) be a compact set, where \( Q_{d\rho} \neq 0 \) is known constant set.

Assumption 4: Unknown positive constant \( p_j^\rho \) is existed for all \( x(\cdot, t) \in \mathbb{R}^N \times \mathbb{R} \), and satisfies:

\[ d_j(\cdot, x, t) \leq p_j^\rho \zeta_j^\rho(\| x_j \|) + p_j^\rho \| z \| \zeta_j^\rho(\| \tilde{x}_j \|) \]

where \( \zeta_j^\rho(\| \cdot \|) \) and \( \zeta_j^\rho(\| \cdot \|) \) are known non-negative continuous functions.

Assumption 5: The equilibrium \( z = 0 \) of system \( \dot{z} = g(z, 0, t) - q(0, 0, t) \) is globally asymptotically stable, namely, there exists Lyapunov function \( W(z) \) such that the following inequalities hold:

\[ c_2 \| z \|^2 \leq W(z, t) \leq c_3 \| z \|^2 \]

\[ \frac{dW}{dt}(z, t) + \frac{dW}{dt}(z, t)(g(z, 0, t) - q(0, 0, t)) \leq -c_3 \| z \|^2 \]

where \( c_1, c_2, c_3, c_4 \) are positive constants. Also, there exists a constant \( c_1 > 0 \) satisfying \( \| q(0, 0, t) \| \leq c_1 \), \( \forall t \geq 0 \).

Assumption 6: There exist unknown positive constant \( p_0^\rho \) and unknown continuous function \( \psi_0 \in C^2(\| \psi_0(0) \| = 0) \) such that

\[ \| q(\cdot, x, t) - q(\cdot, 0, t) \| \leq p_0^\rho(\| x \|) \]

2.2 Radial basis function neural network

In this paper, we will use RBFNNs to approximate unknown smooth function. Suppose \( \Omega_{\hat{z}_j} \subset \mathbb{R}^{N+1} \) be a compact set, and \( \omega_{\hat{z}_j} \) be the approximation of the RBFNNs on the compact set \( \Omega_{\hat{z}_j} \) to \( L_{\hat{z}_j}(\hat{z}_j) \) where the unknown continuous function \( L_{\hat{z}_j}(\hat{z}_j) \) will be given later. Then, we have:

\[ L_{\hat{z}_j}(\hat{z}_j) = \omega_j^\rho \| \hat{z}_j \| + \epsilon_{\hat{z}_j}(\hat{z}_j) \]

where \( \hat{z}_j = [x_{j1}, x_{j2}, \ldots, x_{jN}]^T \), \( \epsilon_{\hat{z}_j}(\hat{z}_j) \) is the approximation error, \( \epsilon_{\hat{z}_j}(\hat{z}_j) \) is continuous function, the basis function vector \( \hat{z}_j(\hat{z}_j) = [\hat{z}_{j1}(\hat{z}_j), \ldots, \hat{z}_{jN}(\hat{z}_j)]^T \) with \( \hat{z}_{j1}(\hat{z}_j) \) being chosen as the commonly used Gaussian function, \( \hat{z}_{j1}(\hat{z}_j) = \exp(-|\hat{z}_j - \hat{z}_j^0|^2|\hat{z}_j - \hat{z}_j^0|\hat{z}_j^0) \), where \( j = 1, 2, \ldots, N + 1 \), \( \hat{z}_j^0 = [\hat{z}_j^1, \ldots, \hat{z}_j^N] \) is the centre of the receptive field with \( q_j = i + 2 \) and \( \hat{z}_j \) is the width of the Gaussian function.

To facilitate design, we agree on the following symbols:

\[ \hat{y}_{ji} = [y_{d1}, y_{d2}, \ldots, y_{dN}]^T, \quad \hat{y}_{ji} = [y_{d1}, y_{d2}, \ldots, y_{dN}]^T, \quad \theta = \max_{1 \leq i \leq N} \| \hat{y}_{ji} \|, \quad \hat{\theta} = \hat{\theta} - \theta \]

Define dynamic surfaces as:

\[ S_{ji} = x_{ji} - z_{ji}, \quad \bar{S}_{ji} = y_{dji} - z_{ji} \]

3 Adaptive DCs design

In the every step of n - 1 steps, we design virtual control, and we design the control law at the nth step.

Step 1: Consider the first dynamic surface of the jth subsystem, we have:

\[ S_{j1} = F_{j1}(\hat{x}_{j1}, 0) + g_1(\hat{x}_{j1}, \hat{\lambda}_{j1}, x_{j2}) + d_{j1}(\cdot, x, t) - z_{j1} \]

According to Assumption 4, and using Young's inequality, we obtain:

\[ S_{j1}d_{j1}(\cdot, x, t) \leq \| S_{j1} \| \| \Delta_j^\rho(\| \tilde{x}_j \|) \| \| z \| \| z \| \]

where \( \theta > 0 \) is a constant.

Substituting (6) into (5), we get:

\[ S_{j1}, S_{j1} = S_{j1}L_{ji}(\hat{z}_{j1}) + S_{j1}g_j(\hat{x}_{j1}, \hat{\lambda}_{j1}, x_{j2})x_{j2} \]

where

\[ L_{ji}(\hat{z}_{j1}) = F_{j1}(\hat{x}_{j1}, 0) + 0.5S_{j1}^\rho \| \Delta_{ji} \| + S_{j1}^\rho \| z \| \]

Using the RBFNNs to approximate the unknown \( L_{ji}(\hat{z}_{j1}) \), and Young's inequality, we obtain:

\[ S_{j1}S_{j1} \leq \frac{q_{\lambda_{j1}}}{2\sigma_{j1}} \| \Delta_{ji} \| + \| S_{j1} \| \| z \| \]

where

\[ \| \Delta_{ji} \| = S_{j1}^\rho \| \Delta_{ji} \| + \| z \| \]

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where \( \alpha_{j_i} \) is a positive design constant. 

\[
D_{j_i} = (\alpha_{j_i}/2\sigma_{\alpha}) + (1/2)p_{j_i}^2 + (\delta^2/4c_1)p_{j_i}^4.
\]

The virtual control is designed as follows:

\[
\phi_{j_i} = -\frac{1}{2\sigma_{\alpha}}\hat{\theta}\lVert \xi_{j_i} \rVert S_{j_i} - k_{j_i} S_{j_i,1}
\]

where \( k_{j_i} > 0 \) is a design constant.

Substituting (8) into (7), we get

\[
S_{j_i} S_{j_i,1} \leq (-k_{j_i}\sigma_{\alpha} + 3)S_{j_i} + \frac{\sigma_{\alpha}}{4}S_{j_i,1} + \frac{\hat{\theta}}{4}\lVert \xi_{j_i} \rVert S_{j_i,1} + \frac{\sigma_{\alpha}}{4}\lVert \xi_{j_i} \rVert^2 + D_{j_i}
\]

(9)

Step 2 (\( i \leq j \leq \rho_i - 1 \)): Consider the \( i \) th dynamic surface of the \( j \)th subsystem, we have

\[
\tilde{S}_{j_i} = F_{j_i}(\tilde{x}_{j_i,0}) + \frac{1}{2}g_{ji}(\tilde{x}_{j_i,0}, \tilde{\omega}_{j_i}, \tilde{\omega}_{j_i,1}) + d_{ji}(\tilde{x}, x, t) - \tilde{z}_{j_i,1}
\]

(10)

Similar to step 1, we have

\[
S_{j_i} S_{j_i,1}(\tilde{x}, x, t) \leq S_{j_i}^2 \lVert \tilde{x}_{j_i,0} \rVert + \frac{1}{2}p_{j_i}^2 + \frac{c_1}{2\sigma_{\alpha}^2} \lVert \xi_{j_i} \rVert^2 + \frac{4\delta^2}{c_1^2}p_{j_i}^4 + S_{j_i,1}^2 \xi_{j_i,1}(\tilde{x}_{j_i,0})
\]

and

\[
S_{j_i} S_{j_i,1} \leq S_{j_i} S_{j_i,1}(\tilde{x}_{j_i,0}) + S_{j_i} g_{ji}(\tilde{x}_{j_i,0}, \tilde{\omega}_{j_i}, \tilde{\omega}_{j_i,1}) + \frac{c_1}{2\sigma_{\alpha}^2} \lVert \xi_{j_i} \rVert^2 + \frac{4\delta^2}{c_1^2}p_{j_i}^4 + \frac{1}{2}p_{j_i}^2
\]

(11)

where

\[
L_{j_i}(\tilde{Z}_{j_i}) = F_{j_i}(\tilde{x}_{j_i,0}) + \frac{1}{2}g_{ji}(\tilde{x}_{j_i,0}, \tilde{\omega}_{j_i}, \tilde{\omega}_{j_i,1}) + \frac{1}{2}d_{ji}(\tilde{x}, x, t) - \tilde{z}_{j_i,1}
\]

Similar to step 1, we have

\[
S_{j_i} S_{j_i,1} \leq S_{j_i}^2 \lVert \tilde{x}_{j_i,0} \rVert + 3S_{j_i} + \frac{\hat{\theta}}{4}\lVert \xi_{j_i} \rVert S_{j_i,1} + \frac{\sigma_{\alpha}}{4}\lVert \xi_{j_i} \rVert^2 + D_{j_i}
\]

(12)

with design constant \( \alpha_{j_i} > 0 \) will be given later.

\[
D_{j_i} = (\alpha_{j_i}/2\sigma_{\alpha}) + (1/2)p_{j_i}^2 + (\delta^2/4c_1)p_{j_i}^4.
\]

The virtual control is designed as follows:

\[
\phi_{j_i} = -\frac{1}{2\alpha_{j_i}}\hat{\theta}\lVert \xi_{j_i} \rVert S_{j_i} - k_{j_i} S_{j_i,1}
\]

(13)

where \( k_{j_i} > 0 \) is a constant.

Substituting (13) into (12), we obtain

\[
S_{j_i} S_{j_i,1} \leq (-k_{j_i}\sigma_{\alpha} + 3)S_{j_i} + \frac{\sigma_{\alpha}}{4}S_{j_i,1} + \frac{\hat{\theta}}{4}\lVert \xi_{j_i} \rVert S_{j_i,1} + \frac{\sigma_{\alpha}}{4}\lVert \xi_{j_i} \rVert^2 + D_{j_i}
\]

(14)

Step \( \rho_i \): The control law will be designed in this step. Consider the \( \rho_i \) th dynamic surface, we have

\[
S_{j,\rho_i} = F_{j,\rho_i}(\tilde{x}_{j,\rho_i}, \tilde{\omega}_{j,\rho_i}, \tilde{\omega}_{j,\rho_i,1}) + g_{j,\rho_i}(\tilde{x}_{j,\rho_i}, \tilde{\omega}_{j,\rho_i}, \tilde{\omega}_{j,\rho_i,1}) + \frac{1}{2}[K_{j,\rho_i}(\tilde{\omega}_{j,\rho_i}) + \Delta(\tilde{\omega}_{j,\rho_i})] + d_{j,\rho_i}(\tilde{x}, x, t) - z_{j,\rho_i}
\]

Using the similar method, we can get

\[
S_{j,\rho_i} \leq S_{j,\rho_i}^2 \lVert \tilde{x}_{j,\rho_i} \rVert + \frac{1}{2}p_{j,\rho_i}^2 + \frac{c_1}{2\sigma_{\alpha}^2} \lVert \xi_{j,\rho_i} \rVert^2 + \frac{4\delta^2}{c_1^2}p_{j,\rho_i}^4 + \frac{1}{2}p_{j,\rho_i}^2
\]

and

\[
S_{j,\rho_i} S_{j,\rho_i,1} \leq S_{j,\rho_i} S_{j,\rho_i,1}(\tilde{x}_{j,\rho_i,0}) + g_{j,\rho_i}(\tilde{\omega}_{j,\rho_i}) + \frac{1}{2}d_{j,\rho_i}(\tilde{x}, x, t) - \tilde{z}_{j,\rho_i,1}
\]

where

\[
L_{j,\rho_i}(\tilde{Z}_{j,\rho_i}) = F_{j,\rho_i}(\tilde{x}_{j,\rho_i,0}) + \frac{1}{2}g_{j,\rho_i}(\tilde{x}_{j,\rho_i,0}, \tilde{\omega}_{j,\rho_i}, \tilde{\omega}_{j,\rho_i,1}) + \frac{1}{2}d_{j,\rho_i}(\tilde{x}, x, t) - \tilde{z}_{j,\rho_i,1}
\]

Using similar method, we obtain

\[
S_{j,\rho_i} S_{j,\rho_i,1} \leq \frac{\sigma_{\alpha}}{2\alpha_{j,\rho_i}} S_{j,\rho_i}^2 \lVert \tilde{x}_{j,\rho_i} \rVert + \frac{1}{2}p_{j,\rho_i}^2 + \frac{4\delta^2}{c_1^2}p_{j,\rho_i}^4 + \frac{1}{2}p_{j,\rho_i}^2
\]

where \( \alpha_{j,\rho_i} > 0 \) is a design constant

\[
D_{j,\rho_i} = \frac{\sigma_{\alpha}}{4\alpha_{j,\rho_i}^2} \lVert \tilde{x}_{j,\rho_i} \rVert^2 + \frac{4\delta^2}{c_1^2}p_{j,\rho_i}^4 + \frac{1}{2}p_{j,\rho_i}^2.
\]

The control law is designed as follows:

\[
v_{j,\rho_i}(t) = -\frac{1}{p_{j,\rho_i}}\lVert \tilde{x}_{j,\rho_i} \rVert S_{j,\rho_i} + \frac{1}{2\alpha_{j,\rho_i}} S_{j,\rho_i} \hat{\theta} \lVert \xi_{j,\rho_i} \rVert^2
\]

(15)

where \( k_{j,\rho_i} > 0 \) is a design constant.

For \( K_{j,\rho_i}(\tilde{\omega}_{j,\rho_i}) \geq \beta_{j,\rho_i} \) we have

\[
S_{j,\rho_i} K_{j,\rho_i}(\tilde{\omega}_{j,\rho_i}) \leq -k_{j,\rho_i} S_{j,\rho_i}^2 - \frac{1}{2\alpha_{j,\rho_i}} S_{j,\rho_i} \hat{\theta} \lVert \xi_{j,\rho_i} \rVert^2
\]

Further, we can obtain

\[
S_{j,\rho_i} S_{j,\rho_i,1} \leq (-k_{j,\rho_i}\sigma_{\alpha} + 2)S_{j,\rho_i} + \frac{\sigma_{\alpha}}{2\alpha_{j,\rho_i}} \lVert \xi_{j,\rho_i} \rVert^2 + D_{j,\rho_i} + \frac{c_1}{2\sigma_{\alpha}^2} \lVert \xi_{j,\rho_i} \rVert^2 + \frac{4\delta^2}{c_1^2}p_{j,\rho_i}^4 + \frac{1}{2}p_{j,\rho_i}^2
\]

(16)
The following adaptive law is used to update the unknown parameter $\theta$:

$$\dot{\theta} = \gamma \left( \sum_{j = 1}^{m} \sum_{i = 1}^{n} \| S_{i,j} \| T_{i,j} - \sigma \theta \right)$$  \hspace{1cm} (17)

with constants $\gamma$ and $\sigma$ positive.

### 4 Stability analysis

Define the first-order filter as follows:

$$\tau_{j,i+1} z_{j,i+1} + z_{j,i+1} = \phi_{j,i+1}, \quad z_{j,i+1}(0) = \phi_{j,i+1}(0)$$  \hspace{1cm} (18)

where $\tau_{j,i+1}$ is a design constant given later.

From (18), we have

$$\dot{z}_{j,i+1} = -y_{j,i+1}/\tau_{j,i+1}, \quad i = 1, \ldots, \rho_j - 1$$  \hspace{1cm} (19)

Noting Assumption 4, we have

$$\dot{y}_{j,i+1} + \frac{y_{j,i+1}}{\tau_{j,i+1}} = \frac{k_j}{\tau_{j,i+1}} S_{i,j} + \frac{1}{2\beta_j} d_j \| S_{i,j} \| + \frac{1}{2\beta_j} \| S_{i,j} \|^{1/2}$$  \hspace{1cm} (20)

and

$$\dot{y}_{j,i+1} + \frac{y_{j,i+1}}{\tau_{j,i+1}} \leq \Phi_{j,i+1}(S_{i,j}, \bar{y}_{j,i+1}, \dot{\theta}, y_{j,i+1}, \dot{y}_{j,i+1})$$  \hspace{1cm} (21)

where $\Phi_{j,i+1}(\cdot)$ is a continuous function.

Using (20) and (21), we get

$$\dot{y}_{j,i+1} \tilde{y}_{j,i+1} \leq - \frac{y_{j,i+1}}{\tau_{j,i+1}} + \Phi_{j,i+1} \left| y_{j,i+1} \right| \leq \frac{y_{j,i+1}}{\tau_{j,i+1}} + \frac{1}{4} B_{j,i+1}$$  \hspace{1cm} (22)

Let $V_0 = (m/\delta)W$. Differentiating $V_0$ with the time and using Young's inequality, we obtain

$$\dot{V}_0 \leq \frac{m}{\delta} \left[ -c_0 \| z \|^2 + c_0 \| z \| + c_0 \rho_i \| z \| \| \psi_{\text{max}}(t) \| \right] \leq \frac{m}{\delta} \| z \|$$  \hspace{1cm} (23)

And let

$$V(t) = \sum_{j = 1}^{m} \sum_{i = 1}^{n} \left( \frac{1}{2} S_{i,j}^{T} + \frac{1}{2} \sum_{i = 1}^{n} y_{j,i+1}^{T} \right) + V_W + \frac{g_0 \dot{\theta}^2}{2\gamma}$$  \hspace{1cm} (24)

Define a compact set as follows:

$$\Omega = \{ (S_{i,j}, \ldots, S_{j,i}, \tilde{y}_{j,i+1}, \bar{y}_{j,i+1}) \| z \|, \dot{\theta}^T; \ V \leq \chi \} \subset R^{m+1}$$

where $\chi > 0$ is a constant.

Let $\Phi_{j,i+1}$, $\bar{y}_{j,i+1}$ and $\psi_{\text{max}}(t)$ have maximum $M_{j,i+1}$, $N_{j,i+1}$ and $\psi_{\text{max}}$ on the compact set $\Omega$.

**Theorem 1:** Consider the closed-loop system shown as (1) under Assumptions 1–6, the controller (15) and adaptation law (17). For any bounded initial condition, $k_{j,i}$, $\tau_{j,i+1}$ and $\alpha_0$ are the existed positive constants, and satisfying $V(0) \leq \chi$, such that the overall closed-loop control system is semi-globally stable, namely, all the signals in the closed-loop system are bounded, and the tracking error can be more smaller, and $k_{j,i}$ and $\tau_{j,i+1}$ satisfy

$$\begin{align*}
{\frac{k_{j,i}}{\tau_{j,i+1}} &\geq 3 \left[ \frac{g_m}{4 \gamma} + \frac{1}{2} \alpha_0 \right] } 
& \hspace{1cm} (i = 1, \ldots, \rho_j) \\
{\frac{1}{\tau_{j,i+1}} &\geq 4 \gamma + \frac{1}{2} \alpha_0 } 
& \hspace{1cm} (i = 1, \ldots, \rho_j - 1) \\
\alpha_0 &= \min \left( \sigma, \sum_{j = 1}^{m} \left[ \frac{c_j}{2\gamma} - \epsilon_j \right] \right)
\end{align*}$$

**Proof:** Choose the Lyapunov function as follows:

$$V(t) = \sum_{j = 1}^{m} \left[ \frac{\rho_j}{2} S_{j,j}^{T} + \frac{1}{2} \sum_{i = 1}^{n-1} y_{j,i+1}^{T} \right] + V_W + \frac{g_0 \dot{\theta}^2}{2\gamma}$$  \hspace{1cm} (25)

Differentiating $V(t)$ to the time, we obtain

$$V(t) = \sum_{j = 1}^{m} \left[ \rho_j S_{j,j}^{T} + \sum_{i = 1}^{n-1} y_{j,i+1}^{T} \right] + V_W + \frac{g_0 \dot{\theta}^2}{2\gamma}$$  \hspace{1cm} (26)

Substituting (9, 13, 16, 22) and (23) into (26), and utilising (17), we get

$$V(t) \leq \sum_{j = 1}^{m} \left[ \rho_j \left( -k_j g_m + 3 \right) + \frac{g_m^2}{4 \gamma} \right] S_{j,j}^{T}$$
$$+ \sum_{i = 1}^{n-1} \left( \frac{1}{4} \tau_{j,i+1} + \frac{1}{4} \alpha_0 \right) \| y_{j,i+1}^{T} + \frac{1}{4} \Phi_{j,i+1} \right]$$
$$+ \sum_{i = 1}^{n-1} \left( \frac{\rho_i - 1}{4} \right) \| y_{j,i+1}^{T} + \frac{1}{4} \Phi_{j,i+1} \right]$$
$$+ m \delta^2 \sum_{i = 1}^{n-1} \left( c_i \epsilon_i + \frac{m \delta^2 c_i^2 \rho_i}{2c_i} + \frac{m \delta^2 c_i^2 \rho_i}{2c_i} \right)$$
$$- m \delta^2 \| z \|^2 + g_0 \dot{\theta}^2$$

For any specified positive constant $\chi$, if $V = \chi$, then $V_W \leq \chi$, further known $W/\delta \leq \chi$. According to Assumption 5, we have $c_i \| z \|^2 \leq \chi \delta_j g_{\bar{c}}$, so we know $\| z \|^2 \leq \delta_j c_i$. When $V \leq \chi$, we have $\Phi_{j,i+1} \leq M_{j,i+1}$, $\bar{y}_{j,i+1} \leq N_{j,i+1}$, $\psi_{\text{max}}(t) \leq \psi_{\text{max}}$, and then $(c_i/\delta_j) \| z \|^2 \leq (c_j/\chi)$. By completion of squares, the following inequality holds $-\sigma_{\text{max}} \dot{\theta} = -\sigma_{\text{max}} \dot{\theta} + \theta \leq \sigma_{\text{max}}, \| \dot{\theta}^2 + \theta^2/2 \|$. Let

$$\mu = \sum_{j = 1}^{m} \left[ \frac{\rho_j}{4} D_j + \frac{1}{4} \right] + \sum_{i = 1}^{n-1} \left[ \frac{M_i}{4} \right] + \frac{\psi_{\text{max}}}{2}$$

Substituting (24) and (28) into (27), we obtain

$$V(t) \leq -m \delta^2 \| z \|^2 + g_0 \dot{\theta}^2$$

If $V \leq \chi$, let $\alpha_0 > m \delta^2 \chi$, we have $V \leq 0$. It implies that $V(t) \leq \chi, \forall t \geq 0$ for $V(t) \leq \chi$. Multiplying (29) by $e^{-mt}$ yields

$$0 \leq V(t) \leq \frac{\mu}{\alpha_0} \left[ V(0) - \frac{\mu}{\alpha_0} e^{-mt} \right]$$

Therefore, all signals of the closed-loop system are uniformly ultimately bounded, $S_{j,j}, \bar{y}_{j,i+1}$ and $\| z \|$ are bounded.
So, we can obtain the tracking error that is arbitrarily small while the desired trajectories are also uniformly ultimately bounded. Using (27) and (30), we obtain

\[ S_{k+1} = y_j - y_{d|j} = x_{j|j} - y_{d|j} \leq \sqrt{2V} \leq \sqrt{\mu/\alpha_j + V(0)} \]  
(31)

So, we can obtain the tracking error that is arbitrarily small while the design parameters are chosen properly. □

5 Simulation results

To show the effectiveness of the proposed scheme, the following non-linear system is considered:

\[
\begin{aligned}
\dot{z} &= -z + 0.125x_1x_2x_3x_5 \sin t \\
x_{11} &= x_{11} + x_{12} + x_1^2 + 0.1\sin(x_1x_2x_3x_5) \\
x_{12} &= x_1x_2 + (1 + 0.1\sin(0.5x_1x_2))w_1 + 3 \frac{x_1}{\sigma} + 0.2\cos(0.5x_1x_2) \\
x_{21} &= x_2 + \frac{x_2^2}{\sigma} + 0.3\sin(x_1x_2x_3x_5) \\
x_{22} &= x_2x_3 + 3 \frac{x_2}{\sigma} + (1 + 0.1\sin(0.5x_2x_3))w_2 + 0.1\cos(0.5x_2x_3) \\
y_1 &= x_{11}, y_2 = x_{21}, \\
\end{aligned}
\]
(32)

The control objective is to make the system outputs \( y_1 \) and \( y_2 \) follow the desired trajectories \( y_{d|1} = 0.5 \sin t + 0.5\cos t \) and \( y_{d|2} = 0.2 \sin t + 0.3\cos t \). The initial conditions: \( z(0) = 0.5, x(0) = [0, 0, 0, 0, 0]^T, \theta(0) = 0.5, \varphi(0) = 0.1, \zeta(0) = 0.2, \beta_i = 0.5 \), \( \beta_i = 0.3, k_1 = 10, k_2 = 15, k_3 = 10, k_4 = 15, \gamma = 25, \sigma = 0.01, \tau_1 = \tau_2 = 0.001 \).

Furthermore, \( \tilde{S}_{j+1} \) and \( \tilde{z}_{j+1} \) are also uniformly ultimately bounded. Using (27) and (30), we obtain

\[ S_{k+1} = y_j - y_{d|j} = x_{j|j} - y_{d|j} \leq \sqrt{2V} \leq \sqrt{\mu/\alpha_j + V(0)} \]  
(31)

Remark 1: From (32), we have \( q(z, x, t) = -z + 0.125x_1x_2x_3x_5\sin t \). It yields \( q(0, 0, t) = 0, q(\varphi, 0, t) = -\varphi \). Thus, we obtain \( q(z, 0, t) - q(z, 0, 0) = -z \), \( z = -z \) is globally asymptotically stable. Let \( W(z, t) = (1/2)^2 \), then we get \( dW(z, t)/dt = \leq 0 \). Therefore, it is easy to know that the unmodelled dynamics \( z \) in (32) satisfies Assumptions 5 and 6.

6 Conclusion

An adaptive neural network control has been presented for a class of MIMO non-affine systems. Using the mean value theorem and Young's inequality, only one adaptive parameter is needed to be adjusted online for the whole MIMO system. The restrictions of unmodelled dynamics are relaxed by utilising RBFNNs approximating the unknown constructed function. The numerical simulation has verified the scheme effectiveness. In the future research work, we will extend the proposed results to a class of stochastic non-linear systems.

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8 References

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