On the possibility of propagation of pipe and lemb waves in cylindrical wells filled with liquid

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Abstract. This work is devoted to the study of the propagation and dispersion of natural waves in oil-gas wells. A detailed analysis of well-known works devoted to this problem is given. In this work, a mathematical formulation and a methodology for studying the propagation, dispersion and attenuation of tube and Lamb waves in a well filled with liquid have been developed. To solve the problem and assess the damping properties of tube and Lamb waves in a well filled with liquid, the following methods were used: separation of variables, the theory of potential functions, an orthogonal sweep and central difference schemes. The complex roots (phase velocities) of the dispersion equation are determined by the methods of Mueller and Gauss. A number of new mechanical effects have been identified that have practical significance: the interference and dispersion of Lamb waves depends on the parameters of the well; the presence of a sliding contact between the pipe and the medium leads to the appearance of pipe and Lamb waves, and taking the viscoelastic properties of the pipe into account leads to a damping effect; with a system of zero frequency, despite the contact conditions between the elements of the system, both L and T waves have the same speed, and with an increase in the frequency of oscillations, the difference in the phase velocities of these waves increases.

Key words: natural waves, tube and Lamb waves, hydro wave, wavenumber, group and phase velocity, viscosity, dispersion, complex frequency.

1. Introduction

This article is devoted to the development of the acoustic attenuation method for observing the propagation of elastic waves in sections covered by a borehole, which is a vertical or inclined mine working (which is modelled as a pipe with a liquid of a relatively small cross-section and long length). Acoustic logging is, in essence, the application of seismic exploration methods to study the geological sections of wells [1,2]. Artificially excited elastic waves were first used to determine a
local geological structure by Fessenden [3]. The reflection, refraction and absorption of waves made it possible to draw conclusions about the structure of the medium between the wells. In the study of rocks by acoustic logging, the velocity of propagation of the longitudinal waves in the rock around the well was primarily used, and this changes continuously with depth. The obtained velocity data turned out to be very important information in the interpretation of seismic records in order to search for oil, and they also made it possible to evaluate the properties of oil-bearing structures. It was found that in addition to the velocities of longitudinal waves, the total wave field recorded by this type of down hole equipment contains much additional information. For example, the velocities of shear or other types of waves (i.e., T and L) obtained in the formation indicated the presence of faults crossing the well [4]. An important task of geophysical work in a well is the interpretation of the wave field that is excited and propagated in pipes in contact with a liquid or solid medium. Usually, at the border of a pipe with an elastic or viscoelastic medium, there is a continuity of stresses (i.e., conditions of rigid or sliding contact) [1-3]. The use of such a model in geophysical surveys allows one to assess the technical condition of the well.

Geophysical methods of studying layered wells (taking into account the properties of the material and the rheological features of both the well and the environment, as well as various contact conditions between the well and the environment, etc.) are used to study geological sections and extract minerals [4-6]. This refers mainly to the study of two types of normal waves (i.e., T and L waves), which largely determine the wave motion in low-frequency regions.

Usually, the speed of T waves is close to the speed of a longitudinal wave in the pipe material, and therefore, it is often called a pipe (T) wave. The second kind of wave has a speed lower than the speed of sound in a liquid and is called a Lamb wave (L) or hydro wave [7,8]. Both waves are axisymmetric waves, and their vibration modes differ from other vibration modes in that their spectrum starts from zero frequency. Tubular waves are used in cement metering, while Lamb waves provide information that is useful for interpretation.

Lamb waves began to be used to study the porosity and permeability of a medium. This aroused the interest of geophysicists regarding the physical properties of these waves and their relationship with the experimental conditions used in acoustics to study the medium [9, 10]. It should be noted that theoretical data, even with respect to the kinematics of T and L waves, are very limited. In addition, a number of works [11 - 16] are devoted to the dynamics of various systems, taking into account their features and operating conditions. This review concerns only some of the works devoted to the study of the dynamics of various inhomogeneous systems and media. Today, solving geophysical problems by studying the propagation of T and L waves in inhomogeneous systems, i.e., systems of wells with liquid, is an urgent task.

This work is devoted to the study of the damped properties of pipe and Lamb (T and L) waves in a well filled with fluid and to the determination of the criterion for the existence of normal waves depending on the parameters of the mechanical system.

2. Methods

2.1 Statement of the problem

The dynamics of an inhomogeneous deformable system consisting of a well located in an infinite environment are considered. The well is modelled as a three-layer structure located in an unbounded environment (figure 1).

In a cylindrical coordinate system $r, \theta, z$, the well model is considered to be a system consisting of an infinite pipe 1 filled with liquid ($r_1 \leq r \leq r_2$) separated from the environment 3 ($r > r_3$) and a liquid cylinder 2 ($r_2 \leq r \leq r_3$). It is necessary to study the propagation of natural waves in three-layer cylindrical bodies in an infinite deformable medium.

Let us denote by $\hat{\vartheta}_{p_k}, \hat{\vartheta}_{s_k}, \hat{\varrho}_{l_k}, \hat{\lambda}_{k}, \hat{\mu}_{k}$ ($k = 0,1,2,3$) the complex velocities of longitudinal and transverse waves, the density, and the complex modulus of elasticity [2,16]. For $k=0,2$, the corresponding equations describe the equations of fluid motion, and for $k=1,3$, they describe the equations of motion of a viscoelastic medium.
Let us consider the problem of the propagation of natural waves in such a system. The equations of motion of a viscoelastic medium for longitudinal $\varphi_k$ and transverse $\psi_{k\mathbf{r}}$ potentials at $k=1,3$ are represented in the form [17]:

$$\nabla^2 \varphi_k - \frac{1}{c_{pk}^2} \frac{\partial^2 \varphi_k}{\partial t^2} = 0;$$

$$\nabla^2 \psi_{k\mathbf{z}} - \frac{1}{c_{sk}^2} \frac{\partial^2 \psi_{k\mathbf{z}}}{\partial t^2} = 0;$$

$$\nabla^2 \psi_{k\mathbf{r}} - \frac{\psi_{k\mathbf{r}}}{r^2} + \frac{2}{r^2} \frac{\partial \psi_{k\mathbf{r}}}{\partial \theta} - \frac{1}{c_{sk}^2} \frac{\partial^2 \psi_{k\mathbf{r}}}{\partial t^2} = 0;$$

$$\nabla^2 \psi_{r\mathbf{r}} - \frac{\psi_{r\mathbf{r}}}{r^2} - \frac{2}{r^2} \frac{\partial \psi_{r\mathbf{r}}}{\partial \theta} - \frac{1}{c_{sk}^2} \frac{\partial^2 \psi_{r\mathbf{r}}}{\partial t^2} = 0. \tag{1}$$

**Figure 1.** A computational model of a well with a liquid in an infinite medium; 1 is a viscoelastic pipe, 0 and 2 are liquid media, and 3 is a surrounding viscoelastic medium.

Here,

$$c_{sk}^2 = c_{sk}^2 \Gamma_k', \ c_{sk}^2 = c_{sk}^2 \Gamma_k', \ \Gamma_k' = 1 - \Gamma_k^C (\omega_k) - i \Gamma_k^S (\omega_k), \ c_{pk}^2 = (\lambda_k + 2 \mu_k) / \rho_k; \ c_{rk}^2 = \mu_k / \rho_k.$$

When $k=0,2$, the corresponding equations describing the state of the liquid take the following form:

$$\nabla^2 \varphi_l - \frac{1}{c_{lk}^2} \frac{\partial^2 \varphi_l}{\partial t^2} = 0, \tag{2}$$

where $c_{lk}$ is the speed of propagation of the acoustic waves. At the interface between the viscoelastic medium and the liquid, the boundary conditions of the continuity of the normal components of the displacements and stresses and the equality of the tangential stresses in the solid
to zero are satisfied. In the presence of a liquid filler in the well space, the conditions are written as:

- at \( r=r_1 \):
  \[ G_r^{(0)} = \frac{\partial u_r^{(1)}}{\partial t}; \quad \tau_r^{(0)} = \frac{\partial \tau_{rr}^{(1)}}{\partial t}; \quad \tau_r^{(1)} = 0; \]  
  \[ \tau_r^{(0)} = 0; \]  
  \[ \tau_r^{(1)} = 0; \]  
  \[ \tau_r^{(2)} = 0. \]  

(3) \quad (4) \quad (5)

The voltages, displacements and displacement rates \( \tau_r^{(1)}, \tau_r^{(2)}, u_r^{(1)}, u_r^{(2)}, G_r^{(0)}, G_r^{(2)} \) are determined through the potentials of the displacements and speeds (at \( k = 1.3 \)) as

\[ \tau_r^{(k)} = 2\mu_k \frac{\partial^2 \psi^{(k)}}{\partial r \partial z} + \frac{\partial^2 \phi^{(k)}}{\partial r^2} + \frac{\partial^2 \psi^{(k)}}{\partial z^2} + \frac{\partial^2 \psi^{(k)}}{\partial r^2} - 2\mu_k \frac{\partial^2 \psi^{(k)}}{\partial z \partial r}, \]  

\[ u_r^{(k)} = \frac{\partial \psi^{(k)}}{\partial r}; \quad u_z^{(k)} = \frac{\partial \psi^{(k)}}{\partial z}; \quad u_r^{(k)} = \frac{\partial \psi^{(k)}}{\partial r} + \frac{\partial \psi^{(k)}}{\partial z}; \quad g_r^{(0)} = \frac{\partial \psi^{(0)}}{\partial r}; \quad g_r^{(2)} = \frac{\partial \psi^{(2)}}{\partial r}. \]  

(6)

Here,

\[ \lambda_k \left[ f(t) \right] = \lambda_{sk} \left[ 1 - \Gamma_{sk}^{(r)}(\omega_k) - i\Gamma_{sk}^{(s)}(\omega_k) \right] f(t), \]  

\[ \mu_k \left[ f(t) \right] = \mu_{sk} \left[ 1 - \Gamma_{sk}^{(s)}(\omega_k) - i\Gamma_{sk}^{(r)}(\omega_k) \right] f(t). \]

(7)

\[ \Gamma_{sk}^{(r)}(\omega_k) = \int_0^\infty R_{jk}(\tau) \cos \omega_k \tau d\tau, \]  

\[ \Gamma_{sk}^{(s)}(\omega_k) = \int_0^\infty R_{jk}(\tau) \sin \omega_k \tau d\tau. \]

are the cosine and sine Fourier images of the relaxation kernel, respectively; \( \omega_k \) is a real value. It is required to find a nontrivial solution of equations (1) and (2) that satisfy the boundary conditions (3) - (5) of the problem under consideration, taking into account (7).

2.2 Methods of solution

The solution to equations (1) and (2) satisfying the condition of the finiteness of the field on the axis \( r = 0 \) and the conditions for decreasing at infinity can be written in the form

\[ \varphi_0 (r, z, t) = a_0 J_0(\gamma_p r \alpha_0) \exp(i\gamma_p c_j t) \cos \gamma_p z. \]
\[
\varphi_1(r, z, t) = \left[ b H_0^{(1)}(r \rho r \alpha_1) + c H_0^{(2)}(r \rho r \alpha_1) \right] \exp(i \gamma \rho c_f t) \cos \gamma \rho z, \\
\psi_1(r, z, t) = \left[ d H_0^{(1)}(r \rho r \beta_1) + e H_0^{(2)}(r \rho r \beta_1) \right] \exp(i \gamma \rho c_f t) \sin \gamma \rho z, \\
\varphi_2(r, z, t) = \chi I_0(\gamma r \rho r \alpha_2) + \gamma K_0(\gamma r \rho r \alpha_2) \exp(i \gamma \rho c_f t) \cos \gamma \rho z, \\
\varphi_3(r, z, t) = \left[ h H_0^{(1)}(r \rho r \alpha_3) \right] \exp(i \gamma \rho c_f t) \cos \gamma \rho z, \\
\psi_3(r, z, t) = \left[ n H_0^{(2)}(r \rho r \beta_3) \right] \exp(i \gamma \rho c_f t) \sin \gamma \rho z, \\
\]

where \( I_0, K_0, H_0^{(1)}, H_0^{(2)} \) are modified Bessel and Hankel functions; \( c_f \) is the complex phase velocity; \( k = 2 \pi \omega / c_f \) is the wavenumber; \( a, b, c, \ldots, m \) are unknown constants; and \( \alpha_k = \sqrt{[1 - \gamma_k^2 r^2]}, \beta_k = \sqrt{[1 - \delta_k^2 r^2]}, \delta_k = c_{s1} / c_{sk} \). The substitution of expressions (8) into boundary conditions (3) - (5) yields a system of linear algebraic equations for determining the unknown constants \( a, b, c, \ldots, m \). The conditions for the existence of a nontrivial solution lead to a dispersion equation that determines the phase velocity of normal waves as a transcendental function of the complex frequency and parameters of the well model:

\[
C(\omega_R, \omega_p, c_{pv}, c_{sy}, \gamma_p, \gamma_k, D) - \omega^2 A = 0, \\
\]

where \( k = 0, 1, 2, 3 \), \( D \) is a set of geometric parameters, and \( A \) is a square matrix generally having a block-diagonal structure. Matrix \( C \) consists of matrices with a block structure, the elements of which are combined with the complex arguments of the modified Bessel and Hankel functions.

\[
C = \begin{pmatrix}
C_{1j} & \cdots & \cdots & C_{1n} \\
\cdots & \cdots & \cdots & \cdots \\
C_{nj} & \cdots & \cdots & C_{nn}
\end{pmatrix},
\]

Here, \( C \) is a complex matrix of dimensions \((6k * 6k)\). Residues in the roots of the dispersion equation (9) describe the field of normal waves arising in the viscoelastic well model (Fig. 1). Moreover, the complex roots \((\omega = \omega_R + i \omega_I)\) correspond to damped natural oscillations. The real parts \( \omega_R \) and complex frequency \((\omega = \omega_R + i \omega_I)\) express the natural frequencies of damped oscillations and imaginary \( \omega_I \)-attenuation coefficients.

If an elastic mechanical system is considered, then \( R_{sk} = 0, R_{pk} = 0 \); the natural oscillations will be undamped, and only the frequencies will be valid.

Dispersion equation (9), for the case of the propagation of free waves to a cylindrical cavity located in a viscoelastic medium and filled with a liquid, takes the following form:

\[
4(1 - \xi_1^2) \left[ \frac{1}{\gamma_p r_1} + \frac{H_1^{(1)}(\hat{\rho} r_1)}{H_1^{(0)}(\hat{\rho} r_1)} \right] - 2(1 - \xi_1^2)^2 (1 - \xi_2^2)^{1/2} \frac{m r_1}{\xi_1^d} J_0(\hat{\rho} r_1 (\xi_2^2 - 1)^{1/2}) J_1(\hat{\rho} r_1 (\xi_2^2 - 1)^{1/2}) = 0, \\
\]

where

\[
\hat{\rho} = \gamma_p (1 - (c_f / c_p)^2) \Gamma_p, \hat{\rho} = \gamma_p (1 - (c_f / c_s)^2 \Gamma_s)^{1/2}, \Gamma_p = 1 - \Gamma_p^C (\omega_R) - i \Gamma_p^S (\omega_R), \\
\Gamma_s = 1 - \Gamma_s^C (\omega_R) - i \Gamma_s^S (\omega_R), \quad \bar{\rho} = \rho_f / \rho_1, \xi = c_f / c_0 \xi_1 = c_0 \xi_1 / c_s, \xi_2 = c_s \xi_1 / c_p.
\]

When \( R_{sk} = 0, R_{pk} = 0 \), we obtain the dispersion equation given by Biot M.A. [18].
Let us consider low-frequency vibrations first. To do this, we pass to the limits of $\mathbf{\vec{r}}_i \to 0, \mathbf{\overrightarrow{m}r}}_i \to 0$ in equation (10), and at $R_{jk} = 0, R_{\mu k} = 0$, we obtain a biquadratic equation [18]. The result is the spectrum of the L wave, which starts at zero frequency. The phase velocity $c_L$ of wave L does not depend on the speed of longitudinal waves $c_{pl}$ in the environment. However, the phase speed $c_L$ of wave L depends on the density $\rho_1$, speed $c_{s1}$ and rheological properties of materials of deformable media

$$c_L = \frac{c_{s1}r_L}{\sqrt{[\rho_{01} + c_{s0}^2 r_L^2]}}$$

(11)

where $\rho_{01} = \rho_0/\rho_1, c_{s0} = c_s/c_0, \rho_0$ is the density of the liquid, and is the speed of sound in the liquid. Wave L undergoes exponential decay if its velocity $c_L$ is above speed $c_{s1}$. When $\omega = \omega_R + i\omega_I = 0$, the wave L becomes damped, and the condition $c_{s1} < c_0\sqrt{1 - \rho_{01}}$ holds. If the rheological properties of materials are taken into account, then $c_{s1} < c_0\sqrt{1 - \rho_{01}/r_L}$. The simplest model in which the T wave exists is a pipe in a void [18,19,20]. The spectrum of this wave begins at zero frequency, at which the phase velocity $c_T$ does not depend on the thickness of the walls and is equal to the velocity of the rod wave

$$c_T^2 = \left[ \frac{3 - 4\gamma_1^2}{1 - \gamma_1^2} \right] c_{s1}^2 c_{pl}, \quad \gamma_1 = c_{s1} / c_{pl} < 1.$$  

(12)

For a T wave to exist, the parameter $\gamma_1$ must be in the interval $\gamma_1 \in (3/4; 1)$. In the general case, the dispersion of hydraulic waves (for elastic or viscoelastic mechanical systems) can be normal or anomalous [21]. The corresponding group velocity is determined by the formula [21]

$$C_T = \frac{c_T^2}{c_T - \omega \frac{\partial c_T}{\partial \omega}}.$$  

(13)

Now let us consider a more complex model that is often used in practice [22,23]. To do this, consider, in an unbounded viscoelastic medium, a model of a well consisting of an unbounded pipe filled with fluid ($r_1 \leq r \leq r_2$), separated from the environment ($r > r_3$) in a liquid cylinder ($r_2 \leq r \leq r_3$). Under boundary conditions of the form (3) - (5) (at $r = r_1$ and $r = r_2$), frequency equation (9) for the axisymmetric well motion, after some simple transformations, takes the form

$$\Delta C_{kj} = 0(1 \leq k \leq 9, 1 \leq j \leq 9).$$  

(14)

The elements $C_{kj}$, which are nonzero, are modified Bessel and Hankel functions, as well as physico-mechanical and geometric parameters of a mechanical system. Equation (14) determines the phase velocity $c_f$ of damped oscillations as a function of the complex frequency $\omega = \omega_R + i\omega_I$.

3. Results and discussion

To calculate the complex roots of equation (14) corresponding to a damped wave along the well lining, a C++ program was compiled. The complex roots of equation (14) were found by the Muller method [24,25]. The frequencies of the corresponding elastic problem were used as an initial approximation. The largest root of equation (11) determines the speed of the T wave, which is less than the speed of the L wave [17,18]. In the presence of a viscoelastic medium surrounding the pipe, the T and L waves become damped ($R_{jk} \neq 0, R_{\mu k} \neq 0$) if the speed of their propagation is greater than the speed of the shear wave in the medium. Mathematically, this is manifested in the
fact that the roots of the dispersion equation become complex (or imaginary); their real part determines the phase velocity, and the imaginary part characterizes the absorption caused by radiation in the medium [26,27].

As an example of a viscoelastic pipe material, we take the three-parametric relaxation kernel of Koltunov-Rzhanitsyn [26, 27] \( R_k(t) = A_k e^{-\beta_k t / t^1 - \alpha_k} \), with parameters \( A_k = 0.048; \ \beta_k = 0.05; \ \alpha_k = 0.1, r_1 = 0.04 m, r_2 = 0.045 m, r_3 = 0.05 m \), and liquids \( c_{0i} = 1800 m / sek, \ \rho_0 = 1.2 g / sm^3 \). When obtaining the numerical results, three material options were used for the pipe material:

- **metal:** \( c_{p1} = 5300 m / sek, c_{i1} = 2800 m / sek, \ \rho_1 = 8 g / sm^3 \) (Fig. 2);
- **plastic:** \( c_{p1} = 2000 m / sek, c_{i1} = 890 m / sek, \ \rho_1 = 1.25 g / sm^3 \) (Fig.3);
- **rubber:** \( c_{p1} = 1400 m / sek, c_{i1} = 70 m / sek, \ \rho_1 = 0.90 g / sm^3 \) (Fig.4).

The calculation results are shown in figures 2-7. In a metal pipe \( c_T \) is close, in absolute value, to the speed \( c_{p1} \) in metal, but it has an imaginary part even at \( R_{Ak} = 0, R_{ik} = 0 \), which indicates absorption. When obtaining the numerical results, taking into account the viscoelastic properties of the pipe material, the above mentioned Koltunov-Rzhanitsyn kernels were used. The calculation results are shown in Figures 2-4 for metal, plastic and rubber pipes. It is seen that in the region of long waves \( (0 < \gamma < 1) \), the real and imaginary parts of tube waves, for metal pipes (Figure 2), gradually decrease, and in the region of short waves, they reach their minimum. This phenomenon was found only for metal pipes. For plastic (Figure 3) and rubber pipes (Figure 4), the real and imaginary parts of the tube waves decrease monotonically with increasing wavenumber. In plastic and rubber pipes, the T wave propagates, depending on the wavenumber, according to a weak exponential decay law.

![Diagram](image_url)

**Figure 2.** Phase (dashed line - 1(\( C_{TR} \)), 3(\( C_{TI} \))) and group (solid line - 2(\( C_{TR} \)), 4(\( C_{TI} \))) tube wave velocity (for metal pipes).
Figure 3. Phase (solid line: \(-2(c_{TR}), 4(c_{TI})\)) and group (dashed line: \(-1(C_{TR}), 3(C_{TI})\)) tube wave velocity (for plastic pipes).

Figure 4. Phase (solid line: \(-2(c_{TR}), 4(c_{TI})\)) and group (dashed line: \(-1(C_{TR}), 3(C_{TI})\)) tube wave velocity (for rubber pipes).
The velocity of a longitudinal wave in a rubber pipe is found to be an order of magnitude less than the velocity of the longitudinal wave in the pipe material. It was found that with an increase in the viscosity of the material, the intensity of the damping of vibrations of the mechanical system increases to 25%. For the damping speed of the tube (T) wave propagation, the condition \( c_{TI} < C_{LI} \) holds. Additionally, for the velocity of propagation of a transverse wave in a pipe, the condition \( c_{TR} > c_{SR1} \) holds. For the wave speed T, the elastic modulus \( \mu_3 \) of the deformable medium has little effect. Figure 5 shows similar results for \( c_{p3} = 1000 \) m/s. The solid lines correspond to rigid contact between the pipe and the medium, and the dashed lines correspond to incomplete contact. L waves are continuous for all pipe materials when \( R_{ijk} = 0, R_{ijk} = 0 \) \( (c_{LR} \neq 0, c_{LI} = 0 \) and \( C_{LR} \neq 0, C_{LI} = 0 \)). Taking into account the viscosity of the material, the phase velocity of the wave L \( (c_L = c_{LR} + iC_{LI}) \) and group speed \( C_L = C_{LR} + iC_{LI} \) become complex quantities. L waves are damped for all metal pipe materials \( (R_{ijk} \neq 0, R_{ijk} \neq 0) \) (Fig. 6) \[32\]. The speed of tube waves, depending on the frequency, obeys a nonlinear law (Figure 5).
Figure 6. Phase (solid line: -2($c_{LR}$), 4($c_{LI}$)) and group (dashed line: -1($C_{LR}$), 3($C_{LI}$)) speed of hydraulic waves (for metal pipes).

Figure 7. Phase (solid line: -2($c_{LR}$), 4($c_{LI}$)), group (dashed line: -1($C_{LR}$), 3($C_{LI}$)), and speed L (hydro waves) of the wave (for a plastic pipe).

The observed phenomena for the tube wave are not exemplified here. In a plastic pipe, the change in the dimensionless wavenumber $y_p$ from 0 to 3 leads to a decrease of almost half in the
values \( c_{LR} \) and \( c_{LI} \) (Figure 7). In a metal pipe, when the wavenumber \( \gamma_p \) changes from 0 to 3, this leads to a decrease \( c_L \) at approximately 500 m/s. It should be noted that the phase \( c_{LR} \) and group \( C_{LR} \) velocities, in the case of a rubber tube, take on low values. Taking into account the viscous properties of the pipe material has little effect (up to 5-7\%) on the attenuation of the L wave. The calculations show that the quantity \( c_{s3} \) has a significant effect on the velocity and damping of the L wave. The above results make it possible to estimate the influence of the viscoelastic parameters of the medium and the type of contact on the dispersion of the tube and Lamb waves.

Comparison of the results obtained with the numerical results obtained from the analytical solutions [28,29] of the system of integro-differential equations showed their difference up to 12\%.

The research results can also be used in the development of a new design for drying cotton seeds [30,31], as well as in improving the energy efficiency and reliability of power supply [32,33].

4. Conclusions
The research of this paper leads to the following conclusions:

1. A mathematical formulation has been developed for studying the decay properties of tube and Lamb waves in a well filled with fluid.

2. To study the damping properties of tube and Lamb waves in a well filled with fluid, an effective technique has been developed for solving dispersion problems using the method of separation of variables, the theory of potential functions, and the Mueller and Gauss methods.

3. The phase and group velocities of the propagation of tube and Lamb waves in a well filled with fluid for various geometric and physico-mechanical parameters of the system elements were investigated.

It was found that:

- the speed of propagation of longitudinal waves in the medium does not greatly affect the kinematic characteristics of the Lamb wave;

- Lamb waves have an interference property, the speed parameters of which depend on all parameters of the pipe, the fluid and the environment of the well;

- the dispersion of the Lamb waves is determined by the relationships among the speed of sound in the liquid and the speed of shear waves in the medium, the viscoelastic properties of the pipe material, and the thickness of the pipe (i.e., \( \Delta r_2 = r_2 - r_1 \));

- the presence of a sliding contact between the pipe and the medium leads to the appearance of pipe and Lamb waves. If the material of the pipe or the well environment has viscoelastic properties, then these waves are damped;

- if the system has zero frequency and the contact between the elements is rigid (or sliding), then both (L and T) waves have the same speed. In this case, with an increase in the frequency of the oscillations, the difference in the phase velocities of these waves increases.

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