We present a QCD light-cone sum rule (LCSR) estimation of the $B \to \pi$, $B \to K$, and $B_s \to K$ form factors calculating gluon radiative corrections at next-to-leading order. The $\overline{MS}$ b-quark mass is used, instead of the one-loop pole mass employed in the previous analyses. For $B \to K$ and $B_s \to K$ form factors the SU(3)-symmetry breaking corrections are included, both in the hard-scattering kernels and in the distribution amplitudes (DAs). By combining the predicted value for $f_{B_s}^0(0)$ with the product $|V_{ub}| f_{B_s}^0$ extracted from the $B \to \pi l\nu_l$ measurement, we obtain the LCSR prediction for $V_{ub}$ CKM matrix element.

1. INTRODUCTION

Nowadays the measurements in the flavor physics are mostly dedicated to overdetermination of the unitarity triangle of the CKM matrix. One of the sides of the triangle is given by the CKM matrix element $V_{ub}$. It can be determined from inclusive or exclusive semileptonic $B$ decays, which are complementary in a sense that they involve different theoretical (and experimental) methods for $V_{ub}$ extraction. The inclusive $V_{ub}$ determination heavily relies on an accurate calculation of the decay spectrum under stringent kinematical cuts. On the other hand, the $V_{ub}$ extraction from the exclusive semileptonic $B \to \pi l\nu_l$ decay requires the knowledge of the $B \to \pi$ form factor, $f_{B\pi}^+$, which is determined by nonperturbative methods, either by lattice calculations, or by applying QCD sum rules.

Moreover, the $B \to \pi$ and $B_s \to K$ form factors serve as the main ingredients of different factorization models for calculating hadronic matrix elements in two-body nonleptonic $B$ decays, as one can see below:

$$\langle \pi\pi|O_1|\pi\rangle = \left. \langle \pi|\overline{u}\Gamma_{\mu u}|0\rangle \langle \pi|\overline{p}\Gamma_{\nu p}|B\rangle \right|_{\text{naive' factorization}} \left[ 1 + O(\alpha_s, \Lambda_{QCD}/m_b) \right] = \frac{m^2_B}{m^2_\pi} f_{\pi} f_{B\pi}^+ (m^2_\pi) \left[ 1 + O(\alpha_s, \Lambda_{QCD}/m_b) \right],$$

since they enter already at the leading level of a calculation.

The estimation of the SU(3) violation among the form factors is important for assessing the validity of various isospin and SU(3) relations applied to constrain new physics contributions. For example, in the relation

$$A(B^- \to \pi^- K^0) + \sqrt{2} A(B^- \to \pi^0 K^-) = \sqrt{2} \frac{V_{us}}{V_{ud}} A(B^- \to \pi^- \pi^0) (1 + \Delta_{SU(3)})$$

$\Delta_{SU(3)}$ measures the net SU(3) breaking effect which comes from ratio of the form factors $f_{BK}/f_{B\pi}$, the decay constants $f_K/f_\pi$, etc.

Therefore, here we intend to explore the $B_{(s)} \to \{\pi, K\}$ form factors in details by using the light-cone sum rules (LCSRs).

2. FORM FACTORS FROM LIGHT-CONE SUM RULES

Heavy-to-light vector, $f_{B_{(s)}^+}\mu$, and scalar $f_{B_{(s)}^0}\mu$, and form factors originate from the relation

$$\langle P(p)|\overline{q}\gamma_\mu b|\bar{B}_{(s)}(p+q)\rangle = 2f_{B_{(s)}^+}\mu (q^2) p_\mu + \left[ f_{B_{(s)}^+}\mu (q^2) + f_{B_{(s)}^0}\mu (q^2) \right] q_\mu ,$$

while the penguin form factor, $f_{B_{(s)}^T}\mu$, is defined as

$$\langle P(p)|\overline{q}\sigma_{\mu\nu}q^\nu b|\bar{B}_{(s)}(p+q)\rangle = \left[ q^2 (2p_\mu + q_\mu) - (m^2_{B_{(s)}} - m^2_T) q_\mu \right] \frac{f_{B_{(s)}^T}\mu (q^2)}{m_{B_{(s)}} + m_T}.$$
In the above relations $P = \pi$ or $K$, and $q = u$ for $B \to \pi$, $q = s$ for $B \to K$ and $q = d$ for $B_s \to K$ transitions. To obtain these form factors from the LCSR one introduces the correlator with the vector and the penguin current,

$$i \int d^4x \, e^{i q \cdot x} \langle P(p) | T \left\{ \bar{q}(x) \Gamma_\mu b(x), j_{B_i}(0) \right\} | 0 \rangle = \left\{ \begin{array}{l} F_o(q^2, (p + q)^2) p_\mu + \tilde{F}_o(q^2, (p + q)^2) q_\mu, \quad \Gamma_\mu = \gamma_\mu \\
F_T(q^2, (p + q)^2) [p_\mu q^2 - q_\mu (q p)], \quad \Gamma_\mu = -i \sigma_{\mu \nu} q^\nu 
\end{array} \right. \tag{5}$$

where the interpolating currents for the $B$ and $B_s$ mesons are $j_B = m_0 \bar{b} \gamma_5 d$ and $j_{B_s} = (m_0 + m_s) \bar{b} \gamma_5 s$, respectively. The light quark masses $m_{u,d}$ are neglected. For large virtualities of the currents in (5), the correlator is dominated by the light-cone distances, $x^2 \to 0$. Therefore one is allowed to perform the light-cone OPE, in terms of the light-cone DAs of increasing twist. By using dispersion relations and the quark-hadron duality assumption, the correlator is related to the sum over hadronic states which is proportional to $f_{B_i} f_{B_i}^+. \tag{6}$ For $B \to \pi$ vector form factor the final expression has the form

$$\frac{2m_B^2 f_B f^{+}_{B_\pi} (q^2)}{m_B^2 - (p + q)^2} = \frac{1}{\pi} \int_{m_b}^{\infty} \frac{ds}{s - (p + q)^2} \sum_{n = \text{twist}} \int_0^1 du \text{Im} \langle T_H^{(n)}(\Phi_\pi^{(n)}). \tag{6}$$

In addition, one needs the Borel transformation $1/(s - (p + q)^2)^n \Rightarrow 1/(M^2)^n e^{-s/M^2} / \Gamma(n)$ in order to suppress higher states and to enhance the ground state contribution. The Borel parameter $M^2$ and the effective continuum threshold parameter $s_0^B$ are parameters which have to be fixed by following certain criteria as explained in details in [1,2]. Since the decay constant $f_B$ can be also estimated by the sum rules, one can consistently perform estimation of the form factors and reduce the parameter uncertainties. In (6), $T_H^{(n)}$ is perturbatively calculable hard-scattering part and $\Phi_\pi^{(n)}$ is the light-cone distribution amplitude of twist $n$. The leading twist-2, two-particle DA $\phi_\pi$ is defined as

$$\langle \pi(q) | \bar{u}(x) \gamma_\mu \gamma_5 d(0) | 0 \rangle |_{x^2 = 0} = -i \mu \gamma_\mu \frac{f_\pi}{\sqrt{2}} \int_0^1 du e^{i u q \cdot x} \phi_\pi(u, \mu), \tag{7}$$

where $u$ is the fraction of the momentum carried by a meson’s constituent. In general there is a Gegenbauer polynomial expansion of the leading twist-2 DAs

$$\phi_{\pi, \chi}(u, \mu) = 6 u (1 - u) \left\{ 1 + \sum_{n = 0}^{\infty} a_\pi^{\chi}(\mu) C_n^{\chi/2} (2u - 1) \right\}^{\mu \to \infty} 6 u (1 - u) \tag{8}$$

with the well-known asymptotic behavior. The coefficients in the expansion are determined either by combining the sum rules for the pion form factors with experimental data or from lattice calculations. For the pion DA: $a_1^\pi(\mu) = 0$, $a_2^\pi(\mu) = 0.25 \pm 0.15$, $a_3^\pi(\mu) \simeq 0$, and for the kaon: $a_1^K(\mu) = 0.10 \pm 0.04$, $a_2^K \simeq a_2^{K^0}$. The structure of pseudoscalar and vector DAs is know to twist-4 accuracy. At the higher twists there exists two- and three-particle DAs. They are related by Wandzura-Wilczek-type relations and their parameters are obtained from the two-point sum rules or by some models.

3. $B \to \pi$ FORM FACTORS AND DETERMINATION OF $V_{ub}$ [1]

In the calculation [1] we employ the $\overline{MS}$ scheme, and use $m_b(\overline{m}_b) = 4.164 \pm 0.025$ GeV. The decay constants are calculated in the same scheme by using the two-point sum rule from [3] and at $O(\alpha_s)$ for our values of parameters we obtain

$$f_B = 214 \pm 18 \text{ MeV}, \quad f_{B_s} = 250 \pm 20 \text{ MeV}. \tag{9}$$

The sum rule parameters: the scale $\mu$, the Borel parameter $M$ and the threshold parameter $s_0^B$ are estimated by taking all other parameters, specified in [1,2], at their central values and allowing the coefficients of the leading
numerical analysis for the penguin form factor and adding all uncertainties in the quadratures we finally predict

\[ f_{B\pi}^+(q^2) \] (solid line), \[ f_{B\pi}^0(q^2) \] (dashed line) and \[ f_{B\pi}^T(q^2) \] (dash-dotted line) at \( 0 < q^2 < 12 \text{ GeV}^2 \) and for the central values of all input parameters.

twist-2 DA to vary within their intervals. In addition, we require that the subleading twist-4 terms in the LO are small, less than 3% of the LO twist-2 term, that the NLO corrections of twist-2 and twist-3 parts are not exceeding 30% of their LO counterparts, and that the subtracted continuum remains small, which fixes the allowed range of \( M^2 \).

The effective threshold parameters are fitted so that the derivative over \(-1/M^2\) of the expression of the complete LCSRs for a particular form factor reproduces the physical mass \( \mu = 3 \text{ GeV}, M^2 = 18 \text{ GeV}^2, s_0^B = 35.75 \text{ GeV}^2 \). The predicted vector \( B \rightarrow \pi \) form factor at zero momentum transfer then reads

\[
f_{B\pi}^+(0) = 0.263 \pm 0.004 \mid_{M,M} \pm 0.009 \mid_{\mu} \pm 0.02 \mid_{\text{shape}} \pm 0.02 \mid_{\mu} \pm 0.001 \mid_{m_b},
\]

while its \( q^2 \) dependence is depicted on Fig.1. The first error comes from the uncertainties in the Borel parameters for \( f_{B\pi}^+(M) \) and \( f_B (\bar{M}) \). The largest uncertainties are due to variation of the quark masses in \( \mu_\pi = m_\pi^2/(m_u + m_d) \) and due to the fitting of the experimental shape by varying of \( a_2^\pi \) and \( a_4^\pi \) twist-2 DA parameters. Making the same numerical analysis for the penguin form factor and adding all uncertainties in the quadratures we finally predict

\[
f_{B\pi}^0(0) = f_{B\pi}^+(0) = 0.263 \pm 0.04, \quad f_{B\pi}^T(0) = 0.255 \pm 0.035.
\]

Obtained results are close to other LCSR and lattice results on \( B \rightarrow \pi \) form factors.

The \( V_{ub} \) matrix element is extracted from the P.Ball’s fit \[ \text{[4]} \] of \( |V_{ub}f_{B\pi}| \) to BaBar data on \( B \rightarrow \pi l\nu_l \) and amounts to

\[
|V_{ub}| = \left( 3.5 \pm 0.4 \mid_{th} \pm 0.2 \mid_{\text{shape}} \pm 0.1 \mid_{BR} \right) \times 10^{-3},
\]

where the first error is due to the estimated uncertainty of \( f_{B\pi}^+(0) \) and the two remaining errors originate from the experimental errors of \( |V_{ub}f_{B\pi}| \). The prediction is in agreement with other recent determinations of \( V_{ub} \) from exclusive \( B \rightarrow \pi l\nu_l \) decay, see Table 1 in \[ \text{[1]} \].

4. \( B_{(s)} \rightarrow K \) FORM FACTORS AND SU(3) BREAKING EFFECTS \[ \text{[2]} \]

In the analysis of the \( B_{(s)} \rightarrow K \) form factors \[ \text{[2]} \] we include, apart from the SU(3) breaking effects of the parameters of the leading twist DAs, such as \( f_K/f_\pi \) and \( \mu_K/\mu_\pi \), the complete SU(3)-symmetry breaking corrections in the \( K \) meson DAs \[ \text{[5]} \] for all twist-3 and twist-4 two- and three-particle DAs. In the hard-scattering amplitudes at LO we
consider $p^2 = m_K^2$ corrections. At next-to-leading order (NLO) in the hard-scattering amplitudes, the mass effects cause nontrivial mixing between twist-2 and twist-3 DAs. Therefore, at NLO in the hard-scattering parts we set $p^2 = m_K^2 = 0$, and consistently use twist-2 and twist-3 two-particle kaon DAs without mass corrections. However, we analyze and include the kaon mass effects in the error estimates.

Since the LO hard-scattering amplitudes are already complicated when the twist-4 and three-particle DAs are included, and the mass effects make the calculation even more demanding, the work can be greatly simplified by applying the method of numerical integration of the amplitudes in the complex plane, making the usual analytical extraction of the imaginary parts of LCSR hard-scattering amplitudes obsolete. With the same conditions for the fitting of the sum rule parameters as those applied in the $B \to \pi$ calculation, we extract the central values, $\mu = 3$ GeV, $M^2 = 18.0$ GeV and $s_0^B = 38$ GeV and obtain

$$f_{BK}^+(0) = f_{BK}^-(0) = 0.36^{+0.05}_{-0.04}, \quad f_{BK}^T(0) = 0.38 \pm 0.05. \quad (13)$$

The $B_s \to K$ form factors are estimated at the default values $\mu = 3.4$ GeV, $M^2 = 19.0$ GeV and $s_0^B = 39$ GeV and the results are

$$f_{B_sK}^+(0) = f_{B_sK}^0(0) = 0.30^{+0.04}_{-0.03}, \quad f_{B_sK}^T(0) = 0.30 \pm 0.05. \quad (14)$$

Having the predictions for the $B \to \pi$ and $B_s(K)K$ form factors calculated in the same model, we are able to get the SU(3)-breaking corrections which amount to relatively large SU(3)-breaking corrections in $B \to K$ decays, and somewhat smaller in $B_s \to K$ decays:

$$\frac{f_{BK}^+}{f_{BK}}(0) = 1.38^{+0.11}_{-0.10}, \quad \frac{f_{B_sK}^+}{f_{B_sK}}(0) = 1.15^{+0.17}_{-0.09}, \quad (15)$$

$$\frac{f_{BK}^T}{f_{BK}}(0) = 1.49^{+0.18}_{-0.06}, \quad \frac{f_{B_sK}^T}{f_{B_sK}}(0) = 1.17^{+0.15}_{-0.11}. \quad (16)$$

By checking some of the SU(3) and U-spin relations in the factorization models for $B(s) \to K\pi, KK$ amplitudes like

$$\xi = \frac{f_K}{f_\pi} \frac{f_{B_sK}(m_K^2)}{f_{B_s}(m_\pi^2)} \frac{m_B^2 - m_\pi^2}{m_B^2 - m_K^2} = 1.01^{+0.07}_{-0.15}, \quad (17)$$

and

$$A_{fact}(B \to K^+K^-) \over A_{fact}(B_d \to \pi^+\pi^-) = \frac{f_K}{f_\pi} \frac{f_{B_sK}(m_K^2)}{f_{B_s}(m_\pi^2)} \frac{m_B^2 - m_\pi^2}{m_B^2 - m_K^2} = 1.41^{+0.20}_{-0.11}, \quad (18)$$

we have found that SU(3) and U-spin relations are case by case badly broken and therefore, for each particular case have to be carefully examined.

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