EVOLUTION OF LOW-MASS HELIUM STARS IN SEMIDETACHED BINARIES

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We present results of a systematic investigation of the evolution of low-mass (0.35\(M_\odot\), 0.40\(M_\odot\) and 0.65\(M_\odot\)) helium donors in semidetached binaries with accretors – white dwarfs. In the initial models of evolutionary sequences abundance of helium in the center \(0.1 \leq Y_c \leq 0.98\). Results of computations may be applied to the study of the origin and evolutionary state of AM CVn stars. It is shown that the minimum orbital periods of the systems only weakly depend on the total mass of the system and evolutionary state of the donor at RLOF and are equal to 9-11 min. The scatter in the mass-exchange rates at given \(P_{\text{orb}}\) in the range \(P_{\text{orb, min}} < P_{\text{orb}} \lesssim 40\) min. does not exceed \(\sim 2.5\). At \(P_{\text{orb}} \gtrsim 20\) min mass-losing stars are weakly degenerate homogeneous cooling objects and abundances of He, C, N, O, Ne in the matter lost by them depends on the extent of He-depletion at RLOF. For the systems which are currently considered as the most probable model candidates for AM CVn stars with helium donors these abundances are \(Y \gtrsim 0.4, X_C \lesssim 0.3, X_O \lesssim 0.25, X_N \lesssim 0.5 \times 10^{-2}\). At \(P_{\text{orb}} \gtrsim 40\) min. the timescale of mass-loss begins to exceed thermal time-scale of the donors, the latter begin to contract, they become more degenerate and, apparently, “white-dwarf” and “helium-star” populations of AM CVn stars merge.

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1 Introduction

Non-degenerate helium stars in close binaries (CB) form in so-called “case B” of mass-exchange, when stars with mass $M \gtrsim (2.3 - 2.5) M_\odot$ overflow their Roche lobes in the hydrogen-shell burning stage (Kippenhahn and Weigert, 1967; Paczyński, 1967b). Masses of helium stars are $\gtrsim 0.32 M_\odot$ (Iben, 1990; Han et al., 2002).

For the current notions on stellar evolution, importance of low-mass helium stars is defined by the possibility of formation of pairs containing white dwarfs or neutron stars accompanied by helium stars in the course of evolution of CB. Orbital periods of such systems may be so short, that angular momentum loss (AML) via gravitational wave radiation (GWR) enables Roche-lobe overflow (RLOF) by helium star before helium exhaustion in the core of the latter and, under proper conditions, stable mass-transfer is possible (Savonije et al., 1986; Iben and Tutukov, 1987; see also Yungelson, 2005a and Postnov and Yungelson, 2006).

Semidetached pairs of neutron stars with helium-star companions were not observed as yet, though, the first computation of the evolution of a CB with a non-degenerate helium donor was performed just for such a pair (Savonije et al. 1986). On the other hand, it is assumed that in the case when in a semidetached binary helium star is accompanied by a white dwarf, such a system may be observed as an ultra-short period cataclysmic variable of the AM CVn type (Savonije et al., 1986). Other hypothetical scenarios for formation of AM CVn stars assume stable mass-exchange between white dwarfs (Paczyński, 1967a) or mass-loss by a remnant of low-mass ($\lesssim 1.5 M_\odot$) main-sequence star which overflowed its Roche lobe after exhaustion of a significant fraction of the hydrogen in its core ($X_c \lesssim 0.4$) or even immediately after formation of a low-mass ($\sim 0.01 M_\odot$) helium core (Tutukov et al., 1985; Podsiadlowski et al., 2003).

AM CVn stars are important for the physics in general, since it is expected that, thanks to their extremely short orbital periods, they, along to the close detached short-period white dwarf pairs, will be among the first objects which will be able to detect space-born gravitational wave antenna LISA (Evans et. al., 1987; Nelemans et al., 2004). More, since the distance to some of the AM CVn stars is known with sufficient accuracy, they could be used as verification binaries for LISA (Stroeer and Vecchio, 2006).

AM CVn stars, despite small number of identified and candidate objects ($\sim 20$, see the list with parameters of the systems and references at

www.astro.ru.nl/~nelemans/dokuwiki/doku.php?id=verification_binaries:am_cvn_stars),

are of great interest not only because they are potential sources of detectable gravitational waves, but also, for instance, because their formation involves evolution in common envelopes, their accretion

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1 For realization of this scenario angular momentum loss by magnetically coupled stellar wind is also necessary.
disks may consist of helium or helium-carbon-oxygen mixture. AM CVn stars are potential progenitors of SN Ia (Tutukov and Yungelson, 1979; see also Solheim and Yungelson, 2005), of still hypothetical SN Ia (Bildsten et al., 2007), and of other explosive phenomena associated with accumulation of helium at the surface of white dwarfs (see, e.g., Tutukov and Yungelson, 1981; Iben and Tutukov, 1991). Note also, that it is expected that ∼ 100 most tight Galactic AM CVn stars might be observed both in gravitational waves and in the electromagnetic spectrum (Nelemans et al., 2004).

It is still unclear which of the above-mentioned scenarios for formation of AM CVn stars acts in the Nature. All scenarios for formation of AM CVn stars imply that their progenitors pass through one or two common-envelope stages, but since clear understanding of the processes occurring in the latter is absent, the efficiency of common envelopes ejection remains a crucial parameter of scenarios that defines final separation of components and, hence, possibility of formation of a semidetached system. Also, conditions for stable mass transfer after RLOF by a white dwarf are not clear (see, e.g., Marsh et al., 2004; Gokhale et al., 2007; Motl et al., 2007). On the other hand, for instance, if AM CVn stars descend from strongly evolved hydrogen-rich cataclysmic variables, only a minor fraction of them evolves to $P_{\text{orb}} < 25$ min. which are typical for a considerable number of the AM CVn stars; more, hydrogen which has to be present in the spectra of accretion disks of most of such systems is not observed as yet. Nevertheless, it is possible that all formation channels contribute to the population of AM CVn stars. Resolution of the problem of the origin of AM CVn stars is also important because observational estimates of their Galactic population are by an order of magnitude below model estimates based on the above-described models (Roelofs et al., 2007a,b). This may point to certain flaws in the understanding of evolution of close binaries. Note, however, that the “deficit” of observed AM CVn stars may be due to numerous selection effects (see detailed discussion in Roelofs et al., 2007a,c and Anderson et al. 2005, 2008).

Nelemans and Tout (2003) noticed that the chemistry of accretion disk in an AM CVn type star may serve as an identifier of its origin. If a helium white dwarf serves as the donor, the disk has to contain He and other H-burning products. In the case of helium-star donor, the disk, can, along to He, contain CNO-cycle and He-burning products. If the donor is an evolved main-sequence star, the disk may contain H and H-burning products. In the first case abundances depend on the mass of the white dwarf progenitor, while in other cases it depends also on how far was the star evolved prior to RLOF.

Evolution of CB with low-mass He-donors was computed by Savonije et. al. (1986), Tutukov and Fedorova (1989), Ergma and Fedorova (1990). These papers were focused, mainly, on mass-transfer rates, ranges of orbital periods, since results of computations were applied to X-ray systems and SN Ia progenitors. In the present paper we carry out a systematic investigation of the evolution of helium stars in semidetached binaries as a function of their mass, evolutionary state at RLOF, and the parameters of a binary that contains helium star. Special attention is paid to the chemical composition of the matter lost by helium stars. In the forthcoming study (Nelemans and Yungelson, in prep.) these results will be applied for the analysis of the origin of observed AM CVn stars.
2 Method of computations

For our computations we have used a specially adapted version of P.P. Eggleton evolutionary code (1971; private comm. 2006). Equation of state, opacity tables, and other input data are described by Pols et al. (1995). Information on the latest modifications of the code may be found at www.ast.cam.ac.uk/stars/. In the code, abundances of $^1$H, $^4$He, $^{12}$C, $^{14}$N, $^{16}$O, $^{20}$Ne, $^{24}$Mg are computed. The nuclear reactions network is given in Table 1 of Pols et al. (1995). Nuclear reactions rates are taken after Caughlan and Fowler (1988), with exception of $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction for which the data of Caughlan et al. (1985) are used. Homogeneous helium models have mass fractions of helium $Y_{\text{He}} = 0.98$, carbon $X_{\text{C}} = 0.00019$, nitrogen $X_{\text{N}} = 0.01315$, oxygen $X_{\text{O}} = 0.00072$, neon $X_{\text{Ne}} = 0.00185$, magnesium $X_{\text{Mg}} = 0.00068$.

It was assumed that mass-exchange is conservative. Angular momentum loss via GWR was taken into account using standard Landau and Lifshitz (1971) formula

$$\left(\frac{\dot{J}}{J}\right)_{\text{GWR}} = -\frac{32}{5} \frac{G^3 M_{\text{He}} M_{\text{wd}} (M_{\text{He}} + M_{\text{wd}})}{a^4}.$$ (1)

Here $M_{\text{He}}$ and $M_{\text{wd}}$ are the masses of components, $a$ is their separation.

Mass loss rate $\dot{M}_{\text{He}}$ is related to $\dot{J}/J$ as

$$\frac{\dot{M}_{\text{He}}}{M_{\text{He}}} = \left(\frac{\dot{J}}{J}\right)_{\text{GWR}} \times \left[\frac{\zeta(M_{\text{He}})}{2} + \frac{5}{6} - \frac{M_{\text{He}}}{M_{\text{wd}}}\right]^{-1},$$ (2)

where $\zeta(M_{\text{He}}) = d \ln R / d \ln M_{\text{He}}$. The term in brackets is $\sim 1$.

3 Results

3.1 Evolution of helium stars

Following the model of the Galactic population of AM CVn stars (Nelemans et al., 2001), we have considered as typical progenitors of helium-donor AM CVn stars the systems with masses $M_{\text{He}} + M_{\text{wd}} = (0.35 + 0.5) M_\odot$, $(0.4 + 0.6) M_\odot$, and $(0.4 + 0.8) M_\odot$. In addition, we have considered a pair $(0.65+0.8) M_\odot$. If helium-accreting white dwarfs avoid double- (or “edge-lit-”) detonation which may destroy the dwarf, such a system may belong to progenitors of AM CVn stars, but not to the most “fertile” of them. As the second parameter of computations we considered initial orbital period of the system, i.e., the period of the system immediately after completion of the common envelope stage that resulted in formation of a helium star. The range of $P_{\text{orb},0}$ for every system was chosen in such a way that the set of computed models included both stars that filled their Roche lobe virtually unevolved and stars that had at the instant of RLOF radii close to the maximum of the radii of low-mass helium stars which are attained at $Y_c \approx 0.1$ (see Table 1 and Fig. 1). The maximum period in the set of initial orbital periods for every computed $M_{\text{He}} + M_{\text{wd}}$ pair is, in fact, the limiting initial period which still
Table 1. Parameters of calculated evolutionary tracks. $M_d, M_a$ – initial masses of donor and accretor, $P_0$ – initial orbital period, $t_c$ – time to RLOF, $P_c$ – orbital period at RLOF, $Y_c$ – central abundance of He at RLOF, $t_f$ – the age of the last computed model, $P_f$ – final orbital period, $M_{df}$ – mass of the last computed model of the donor, $Y_{sf}$ – surface He-abundance of the last computed model.

| No. | $M_d$, $M_a$ | $P_0$, min. | $t_c$, $10^6$ yr | $P_c$, min. | $Y_c$ | $t_f$, $10^6$ yr | $P_f$, min. | $M_{df}$, $M_\odot$ | $Y_{sf}$ |
|-----|--------------|-------------|-----------------|-------------|-------|-----------------|-------------|-------------------|--------|
| 1   | 0.35         | 0.5         | 20              | 1.29        | 15.96 | 0.977           | 427.00      | 42.06             | 0.027  | 0.976             |
| 2   | 0.35         | 0.5         | 40              | 15.99       | 16.24 | 0.936           | 400.99      | 41.52             | 0.028  | 0.935             |
| 3   | 0.35         | 0.5         | 60              | 50.80       | 17.02 | 0.871           | 426.05      | 41.18             | 0.028  | 0.870             |
| 4   | 0.35         | 0.5         | 80              | 110.71      | 18.14 | 0.774           | 492.51      | 40.81             | 0.027  | 0.723             |
| 5   | 0.35         | 0.5         | 100             | 202.32      | 19.71 | 0.642           | 615.18      | 40.35             | 0.025  | 0.640             |
| 6   | 0.35         | 0.5         | 120             | 332.36      | 22.46 | 0.435           | 689.67      | 38.30             | 0.025  | 0.428             |
| 7   | 0.35         | 0.5         | 140             | 502.68      | 34.76 | 0.178           | 874.27      | 37.05             | 0.022  | 0.166             |
| 8   | 0.35         | 0.5         | 144             | 557.78      | 35.00 | 0.118           | 840.34      | 35.31             | 0.024  | 0.104             |
| 9   | 0.40         | 0.6         | 20              | 0.14        | 19.51 | 0.979           | 348.53      | 41.90             | 0.027  | 0.978             |
| 10  | 0.40         | 0.6         | 40              | 11.68       | 19.94 | 0.923           | 338.05      | 41.50             | 0.028  | 0.920             |
| 11  | 0.40         | 0.6         | 60              | 50.78       | 17.02 | 0.871           | 426.05      | 41.18             | 0.028  | 0.825             |
| 12  | 0.40         | 0.6         | 80              | 84.52       | 22.42 | 0.704           | 476.41      | 41.63             | 0.025  | 0.698             |
| 13  | 0.40         | 0.6         | 100             | 154.47      | 25.57 | 0.509           | 733.73      | 41.96             | 0.020  | 0.495             |
| 14  | 0.40         | 0.6         | 120             | 252.55      | 29.97 | 0.228           | 652.76      | 38.52             | 0.021  | 0.194             |
| 15  | 0.40         | 0.6         | 130             | 315.30      | 29.71 | 0.066           | 528.00      | 34.08             | 0.026  | 0.035             |
| 16  | 0.40         | 0.8         | 20              | 0.07        | 19.71 | 0.976           | 323.79      | 42.55             | 0.027  | 0.977             |
| 17  | 0.40         | 0.8         | 40              | 9.14        | 20.61 | 0.933           | 330.17      | 42.61             | 0.027  | 0.923             |
| 18  | 0.40         | 0.8         | 60              | 30.21       | 21.67 | 0.854           | 358.96      | 42.51             | 0.027  | 0.849             |
| 19  | 0.40         | 0.8         | 80              | 66.95       | 22.85 | 0.751           | 431.17      | 42.60             | 0.025  | 0.744             |
| 20  | 0.40         | 0.8         | 100             | 122.98      | 25.40 | 0.601           | 198.19      | 30.51             | 0.043  | 0.587             |
| 21  | 0.40         | 0.8         | 120             | 201.61      | 28.65 | 0.376           | 568.45      | 40.12             | 0.023  | 0.353             |
| 22  | 0.40         | 0.8         | 140             | 306.39      | 30.63 | 0.090           | 551.02      | 36.14             | 0.024  | 0.057             |
| 23  | 0.65         | 0.8         | 35              | 0.17        | 34.54 | 0.976           | 324.85      | 43.96             | 0.027  | 0.856             |
| 24  | 0.65         | 0.8         | 40              | 2.06        | 35.29 | 0.928           | 340.97      | 44.04             | 0.026  | 0.806             |
| 25  | 0.65         | 0.8         | 60              | 14.76       | 38.74 | 0.708           | 443.33      | 43.96             | 0.022  | 0.547             |
| 26  | 0.65         | 0.8         | 80              | 36.29       | 44.87 | 0.396           | 353.64      | 39.99             | 0.022  | 0.129             |
| 27  | 0.65         | 0.8         | 85              | 42.83       | 47.07 | 0.286           | 324.29      | 38.23             | 0.023  | 0.0135            |
| 28  | 0.65         | 0.8         | 90              | 50.86       | 48.56 | 0.186           | 69.36       | 4.31              | 0.378  | 0.00              |

allows RLOF and formation of an AM CVn star. Nuclear evolution of He-stars with mass $M \lesssim 0.8 M_\odot$ terminates after helium burning stage and they evolve directly into white dwarfs (Paczyński, 1971).
The lifetime of helium stars is short. According to our computations for (0.35 - 0.65) $M_\odot$ mass range

$$t_{\text{He}} \approx 10^{6.95} M_{\text{He}}^{-4.1}(1 + M_{\text{He}}^{3.74}),$$

where time is in yr, masses in $M_\odot$. During He-burning, the radii of helium stars increase by $\sim 30\%$ only. Therefore, the crucial factor which defines the range of the orbital periods of the precursors of AM CVn stars is AML. In the Nelemans et. al. (2001) model, in $\approx 90\%$ of the systems helium stars overflow their Roche lobes during first $\approx 50\%$ of their lifetime. Figure 1 shows that this time-span corresponds to the reduction of $Y_c$ to $\approx 0.5$. Thus, most of the AM CVn systems might have initial orbital periods up to $100 - 120$ min.

Evolution of mass-losing stars is followed to $M \approx (0.02 - 0.03) M_\odot$. Continuation of computations is hampered by the absence of adequate EOS and low-temperature opacity tables in our code.
Figure 2. Dependence of mass-loss rate on the period for computed evolutionary sequences for the systems with initial mass of components $0.35 \, M_\odot + 0.5 \, M_\odot$, $0.40 \, M_\odot + 0.8 \, M_\odot$, $0.65 \, M_\odot + 0.8 \, M_\odot$ (top to bottom).

However, for the analysis of the scenarios of formation and evolution of AM CVn stars which involves information on the elemental abundances in the donor-star this is not important, since the chemical composition of the stars in our case becomes “frozen” even before the minimum of $P_{\text{orb}}$ is reached, because of the drop of the temperature in the core and switch-off of nuclear burning (see below).

Figure 2 shows dependence of the mass-loss rate by Roche-lobes filling helium stars on $P_{\text{orb}}$. The behavior of low-mass components of CB evolving under the influence of AML via GWR, like for “usual” hydrogen-rich cataclysmic variables, is defined by the relation between the timescale of stellar evolution $\tau_{\text{ev}}$, thermal timescale of the star $\tau_{\text{KH}}$ and AML timescale $\tau_{\text{GW}}$ (Faulkner, 1971; Paczyński, 1981; Savonije et al., 1986). The relation between the timescales itself depends on the mass of the star, its evolutionary state at RLOF and total mass of the binary (see Fig. 3).

In the system $(0.35+0.5) \, M_\odot$, $P_{\text{orb,0}}=20$ min. with initially virtually unevolved donor, $\tau_{\text{KH}} > \tau_{\text{GW}}$ after the RLOF. Due to AML separation of components and Roche-lobe radius decrease. The star which is not in thermal equilibrium reacts to the mass loss by an increase of mass-loss rate. But since mass-transfer acts in opposite direction (moves components apart), $\dot{M}$ does not change significantly. A considerable fraction of the energy generated by nuclear burning is absorbed in the envelope of the star, resulting in decrease of luminosity and further growth of $\tau_{\text{KH}}$. With decrease of the stellar mass the significance of nuclear burning is rapidly diminishing. convective core disappears, while
a surface convective zone appears; during this stage $\tau_{KH} \gg \tau_{GW}$. As $P_{\text{orb}}$ drops, $\tau_{GW}$ continues to decrease, resulting in increase of $\dot{M}$. At certain instant the effect of mass-transfer starts to dominate, orbital period reaches the minimum, while $\dot{M}$ – the maximum. With increase of $P_{\text{orb}}$ the timescale of AML increases, but since the star becomes more degenerate and has convective envelope, the power of $M - R$ relation is negative and mass loss continues, but $\dot{M}$ rapidly drops.

If the donor is initially more massive and more evolved at the instant of RLOF, it retains thermal equilibrium for a longer time and $\dot{M}$ is initially completely defined by AML. But $\tau_{KH}$ increases with decrease of $M_{\text{He}}$, while $\tau_{GW}$ decreases and, starting from a certain moment, evolution proceeds like in the lower-mass and less-evolved system described above. Evolution of characteristic timescales for the system $(0.65+0.8)M_\odot$, $P_{\text{orb},0}=60$ min. is similar to that described by Savonije et al. (1986) for a $(0.6+1.4)M_\odot$ system with $Y_c = 0.28$ at RLOF which is usually quoted as “typical”.

The minimum periods of the systems with helium donors are confined to a narrow range of $P_{\text{orb}} = (9.3 - 10.9)$ min, masses of stars at $P_{\text{orb},\text{min}}$ are $0.20 M_\odot - 0.26 M_\odot$. The models that overfilled their Roche lobes in the advanced stages of evolution reach lower periods and have at minimum period lower masses than initially less evolved donors. This may be related to the lower abundance of He and enhanced abundance of heavy elements.

### 3.2 Mass-radius relation

All existing computations of the evolution of initially non-degenerate helium donors are discontinued at $M_{\text{He}} \simeq 0.02 M_\odot$. However, starting from $P_{\text{orb}} \simeq 20$ min. these stars are, in fact, weakly degenerate homogeneous objects (the parameter of degeneracy $\psi \approx 8.33 \times 10^7 \rho_c T_c^{-3/2} \sim 10$). This allows to consider qualitatively their further evolution using results of computations for mass-losing
Figure 4. Dependence of model radii on stellar mass for the systems $0.35 \, M_{\odot} + 0.5 \, M_{\odot}$ (upper panel), $0.40 \, M_{\odot} + 0.6 \, M_{\odot}$ (middle panel), $0.65 \, M_{\odot} + 0.8 \, M_{\odot}$ (lower panel). Less evolved systems are at the top of every panel. In the upper panel upper dashed line shows $M - R$ relation (4), lower dashed line shows $M - R$ relation for $0.3 \, M_{\odot}$, log $\psi_0 = 1.1$ helium white dwarf (Deloye et al., 2007) and dash-dotted line shows the fit to $M - R$ relation for zero-temperature He white dwarfs suggested in the latter paper.

arbitrary degenerate helium dwarfs (Deloye et al., 2007). Deloye and coauthors have shown that, when the mass of the dwarf decreases to $\simeq 0.01 - 0.03 \, M_{\odot}$, their $\tau_{KH}$ becomes comparable to the timescale of mass loss $\tau_m$ and further $\tau_{KH} < \tau_m$. The stage of adiabatic expansion terminates, the donor gradually cools, becomes more degenerate and $M - R$ relation approaches relation for fully degenerate configurations. Expected picture of evolution is confirmed by Fig. 4 which shows variation of stellar radii with decrease of mass. In addition to our data, we show in Fig. 4 mass-radius relation for degenerate helium white dwarf with initial mass of $0.3 \, M_{\odot}$ and initial degeneracy parameter log $\psi = 1.1$ (the least degenerate initially model, C. Deloye priv. comm.) and $M - R$ relation for zero-temperature He white dwarfs (Deloye et al. 2007). It is seen that the morphology of $M - R$ curves for helium stars and helium dwarfs is similar and, hence, one may expect that the remnants of helium stars will gradually approach the curve for the remnants of helium dwarfs. Note, Deloye et al. in fact predicted this effect, based on consideration of the physics of cooling objects and they

\footnote{When comparing our results with those of Deloye et. al. one must have in mind the differences in initial masses of models and degeneracy parameters, as well as in the input parameters of different codes.}
noticed also that some of the evolutionary tracks published by Tutukov and Fedorova (1989) have features described above.

Our results, combined with results of Deloye et al. (2007) suggest that at $P_{\text{orb}} \gtrsim 40$ min. two families of AM CVn stars – the “white dwarf” one and the “helium star” one – merge and the origin of the donors may be then identified by their chemical composition only (see §4).

Our conclusions concerning $M - R$ relation may have certain implications for population models of AM CVn stars. Existing models (Tutukov and Yungelson, 1996; Hils and Bender 2000; Nelemans et al., 2001, 2004) were carried out under assumption that the power of $M - R$ relation is constant, keeps also for $M_{\text{He}} < 0.02 M_\odot$ and that the evolution of stars is restricted by the Hubble time only. This assumption allowed to use for modeling Eq. (2) with constant $\zeta(M_{\text{He}})$. Such an approach is, apparently, not justified and one has to use an $M - R$ relation that takes into account growing degeneracy of donors with mass loss.

Figure 4 shows that the $M - R$ relation (in solar units)

$$R \approx 10^{-1.367} M_{\text{He}}^{0.062},$$

(4)

suggested by Nelemans et al. (2001) using results of Tutukov and Fedorova (1989) does not agree well with the results of more modern computations and is not valid for $P_{\text{orb}} \gtrsim 35$ min.

In the period range $P_{\text{orb}} \approx 10 - 35$ min which hosts 10 out of 17 AM CVn stars with estimated orbital periods, the radii of the models with initial masses $(0.35-0.40) M_\odot$ may be approximated as

$$R \approx 10^{-1.478} \left(\frac{P_{\text{orb,0}}}{20 \text{min}}\right)^{-0.05} M_{\text{He}}^{-0.16} \left(\frac{0.35}{M_{\text{He,0}}}\right)^{0.345},$$

(5)
Figure 6. Orbital period – donor mass relations. For every given combination of initial masses of components we show $P_{\text{orb}} - M$ relation for the system with initially least evolved and most evolved donor. $0.35 \, M_\odot + 0.5 \, M_\odot$ – thin solid lines, $0.40 \, M_\odot + 0.6 \, M_\odot$ – dashed lines, $0.65 \, M_\odot + 0.8 \, M_\odot$ – dotted lines. For initially evolved donors the lines practically coincide. Dash-dot-dot-dot line shows $P_{\text{orb}} - M$ relation for $0.35 \, M_\odot + 0.5 \, M_\odot$, $P_{\text{orb},0}$ system based on approximation (4). Dot-dash line shows $P_{\text{orb}} - M$ relation for $M_0 = 0.3 \, M_\odot$, $\log \psi_0 = 1.1$ helium white dwarf from Deloye et al. (2007). Vertical bars show the ranges of donor-mass estimates for AM CVn, HP Lib, CR Boo, V803 Cen, SDSS J0926+3624, and GP Com (see references in the text).

where $P_{\text{orb},0}$ and $M_{\text{He},0}$ are initial orbital period of the system and initial mass of the donor and $M$ and $R$ are in solar units, $P_{\text{orb}}$ - in min.

The time spent by a star in the certain range of orbital periods is proportional to $P_{\text{orb}}/\dot{P}_{\text{orb}}$. In Fig. 5 we compare dependencies of $P_{\text{orb}}/\dot{P}_{\text{orb}}$ on $P_{\text{orb}}$ for the system $(0.35+0.50) \, M_\odot$, $P_{\text{orb},0} = 20$ obtained in evolutionary computations and obtained by means of Eqs. (4) and (5). It is evident that in the $P_{\text{orb}} \approx (10-35) \, \text{min.}$ range Eq. (4) results in the 20-25% difference in the number of systems. Note, however, that this discrepancy is not very significant if one takes into account all uncertainties involved in the modeling of the population of AM CVn stars.

### 3.3 Masses of donors in AM CVn stars

Relatively accurate masses of components are derived only for SDSS J0926+3624 with $P_{\text{orb}}=28.3 \, \text{min}$, a unique partially eclipsing AM CVn type system (Marsh et al., 2007): $M_{\text{wd}} = 0.84 \pm 0.05 \, M_\odot$, $M_{\text{He}} = 0.029 \pm 0.002 \, M_\odot$. In the $P - M$ diagram (Fig. 6) the donor of SDSS J0926+3624 is located both below the curve corresponding to the initially most evolved nondegenerate helium donors and the curve that describes the least degenerate white dwarfs. Apparently, this system belongs to the latter family of stars (see also Deloye et al., 2007). Note however, that at the moment when $P_{\text{orb}}=28.3 \, \text{min}$,
the remnants of initially substantially evolved donors become oxygen-neon white dwarfs (see Figs. 8–10). Though existing models of the population of AM CVn stars suggest that the probability of RLOF by a far-evolved helium star is low, only analysis of chemical composition of the donor will be able to identify the scenario of the origin of SDSS J0926+3624.

We show in Fig. 6 also mass estimates for several other AM CVn stars after Roelofs et al. (2006; 2007b). For AM CVn the masses of components are based on the kinematical data: $M_{\text{wd}} = (0.68 \pm 0.06) M_\odot$, $M_{\text{He}} = 0.125 \pm 0.012 M_\odot$. For HP Lib, CR Boo, and V803 Cen masses of the donors are derived indirectly, assuming that for these systems one may use the relation between superhump period excess and mass ratio of components derived by Roelofs et al. (2006) for AM CVn: $\varepsilon(q) = 0.12q$. For GP Com the range of possible donor mass is based on the limits for its luminosity and assumptions about the nature of the star. The estimated masses of the donors agree with assumption that they descend from nondegenerate helium stars (see also Roelofs et al., 2007a). Note however, that there are no traces of He-burning products in the spectra of these stars.

Spectral lines of nitrogen, neon, and helium were discovered in GP Com (Lambert and Slovak, 1981; Marsh et al., 1991, 1995; Strohmayer, 2004). In particular, Strohmayer found the following abundances: $Y = 0.977 \pm 0.002$, $X_N = 1.7 \pm 0.1 \times 10^{-2}$, $X_O = 2.2 \pm 0.3 \times 10^{-3}$, $X_{Ne} = 3.7 \pm 0.2 \times 10^{-3}$, $X_C < 2 \times 10^{-3}$, $X_{Mg} < 1.6 \times 10^{-4}$. Considerable excess of nitrogen compared to oxygen and carbon points to the enrichment of stellar matter by the products of CNO-cycle. Such abundances may be typical for the remnants of the least massive possible progenitors ($M_{\text{He},0} \sim 0.3 M_\odot$) which overfilled Roche lobe soon after TAMS, before helium started to burn in their interiors. In such stars abundances in the matter lost by them is virtually the same during whole course of evolution and close to the ones found by Strohmayer.

3.4 The system (0.65+0.8) $M_\odot$, $P_{\text{orb},0} = 90$ min.

The system with initial parameters (0.65+0.8) $M_\odot$, $P_{\text{orb},0} = 90$ min. is of special interest because of its “nonstandard” evolution which may illustrate transitional behavior between “white dwarf” and “helium star” scenarios of formation of AM CVn stars. At the instant of RLOF abundances in the center of the donor are $Y \approx 0.19$, $X_C \approx 0.45$, $X_O \approx 0.34$, nitrogen is destroyed already. At difference to lower mass stars in which nuclear burning terminates after loss of several hundredth of $M_\odot$, in this more massive and hotter star it lasts longer. When the mass decreases to $M_2 \approx 0.52 M_\odot$ and $Y$, becomes $\approx 0.008$, the star begins to contract. Mass-exchange interrupts for $\approx 2.2$ Myr and resumes in the helium-shell burning stage (Fig. 7). After the loss of additional $\approx 0.1 M_\odot$ nuclear burning ceases and the star is now a “hybrid” white dwarf with $\approx 0.3 M_\odot$ carbon-oxygen-neon core and $\approx 0.1 M_\odot$ helium envelope with an admixture of C and O. The timescale of AML $\tau_{GW}$ continues to decrease, while $\tau_{KH}$ increases. Evolution of the donor is accompanied by its cooling and decrease of radius. The dwarf experiences a stage of adiabatic contraction similar to the one that is typical for the initial stages of mass-exchange in the systems with degenerate donors (Deloye et al., 2007). In the last
computed model of the sequence, $P_{\text{orb}} = 4.3 \text{ min}$, $M_{\text{He}} = 0.375 M_{\odot}$, degeneracy parameter of the center of the star is $\psi \approx 14$. One may expect that further evolution will be qualitatively similar to the evolution of helium white dwarf-donors: contraction phase will be followed by expansion which will continue until mass will decrease to $\approx 0.01 M_{\odot}$ and thermal timescale will become comparable to the mas-loss timescale and a stage of contraction to the fully degenerate configuration will ensue. The minimum $P_{\text{orb}}$ of the system will be several minutes. Note, that at difference to He white-dwarf donors which retain their original chemical composition during evolution, in the case of He-star descendant under consideration, the donor will become an oxygen-carbon white dwarf, with an admixture of neon ($X_O \approx 0.27$, $X_C \approx 0.71$, see Fig. 1).

Total mass of this system is $1.45 M_{\odot}$. If unstable He-burning at the surface of white dwarf does not decrease its mass below Chandrasekhar limiting mass, such a system may be a SN Ia progenitor.

4 Evolution of stellar chemical composition

Figures 8–10 show the evolution of chemical composition of matter lost by stars upon RLOF. As we mentioned before, in the stars with initial masses 0.35 and 0.40 $M_{\odot}$ nuclear burning termi-
Figure 8. Variations of He, C, N, and O abundances in the accreted matter in the system with initial masses of components 0.35 $M_\odot$ and 0.5 $M_\odot$. Solid lines are for systems with initial orbital periods $P_{\text{orb},0} = 20$, 40 and 60 min., dashed lines – for $P_{\text{orb},0} = 80$ min., dash-dot lines – for $P_{\text{orb},0} = 100$ min., dotted line – for $P_{\text{orb},0} = 120$ min., dash-dot-dot-dot line – for $P_{\text{orb},0} = 140$ and 144 min. For the system with $P_{\text{orb},0} = 20$ min. He-abundance virtually does not change. All abundances do not change any more at $P_{\text{orb}} \gtrsim 25$ min.

nates soon after RLOF. Simultaneously, disappears the convective core. When $P_{\text{orb}}$ is still larger than $P_{\text{orb, min}}$, abundances in the lost matter are virtually constant and correspond to the abundances in the matter that experienced hydrogen burning via CNO-cycle. In the period range from $P_{\text{orb, min}}$ to $P_{\text{orb}} \approx (15 - 20)$ min. the convective core of initial model is uncovered. The donors in initially most wide systems are an exception, since they had large cores and chemical composition of the lost matter starts to change already before $P_{\text{orb, min}}$. But this change happens over such a short time that it is highly unlikely to observe a star in this “transition” state. Thus, in the range of $P_{\text{orb}}$ that harbors almost all known AM CVn stars chemical composition of the matter accreted by white dwarf is defined by the extent of helium exhaustion at the instant of RLOF (compare Fig. 1 and Figs. 8 – 10).

As we mentioned above, it is expected that the overwhelming majority of precursors of AM CVn stars in the “helium star” channel of formation were formed with $P_{\text{orb}} < 100 – 120$ min. Then, typical abundances in the transferred matter are $Y \gtrsim 0.4$, $2 \times 10^{-4} \lesssim X_C \lesssim 0.3$, $7 \times 10^{-4} \lesssim X_O \lesssim 0.25$, $5 \times 10^{-4} \lesssim X_N \lesssim 0.5 \times 10^{-2}$. However, one cannot exclude that abundance of nitrogen is much lower or it is absent at all; diminished (or zero) $X_N$ has to correlate with relatively low $Y$ and enhanced $X_C$. 
Figure 9. Same as in Fig. 8 for the system with initial masses of components $0.4 \, M_{\odot}$ and $0.6 \, M_{\odot}$. Solid lines show abundances for the systems with $P_{\text{orb},0} = 20, 40$ and $60 \, \text{min.}$, dashed lines – for $P_{\text{orb},0} = 80 \, \text{min.}$, dash-dot lines – for $P_{\text{orb},0} = 100 \, \text{min.}$, dashed line – for $P_{\text{orb},0} = 120 \, \text{min.}$, dash-dot-dot-dot line – for $P_{\text{orb},0} = 130 \, \text{min.}$ and $X_O$.

We did not plot in Figs. 8–10 variations of the Ne abundance. It also changes rapidly by $P_{\text{orb},\text{min}}$ and at $P_{\text{orb}} > 20 \, \text{min.}$ neon abundance $2 \times 10^{-3} \lesssim X_{\text{Ne}} \lesssim 2 \times 10^{-2}$. Variation of $X_{\text{Ne}}$ is, evidently, too small to serve as identifier of the scenarios of AM CVn stars formation.

If the donor in an AM CVn type system was initially a white dwarf, there are no reasons to expect that abundances in the matter lost by it vary in the course of evolution. They must correspond to the “standard” chemical composition of a stellar core that experienced hydrogen burning, with small variations due to the differences in the masses of precursors of white dwarfs. Similar abundances will have matter lost by He-stars that filled Roche lobes soon after TAMS. However, since initial $Y \approx 1$, while initial $X_C \sim 10^{-4}$, $X_N \sim 10^{-2}$, $X_O \sim 10^{-3}$, detection of even slight enrichment in C and O or impoverishment in N may indicate possibility of formation of AM CVn stars with He-star donors.

### 5 Conclusion

Above, we presented results of evolutionary computations for semidetached low-mass helium components in the systems with white dwarf accretors. Evolution was considered as conservative in mass, but accompanied by angular momentum loss via gravitational waves radiation. We may
summarize our main results as follows.

Evolution of binaries under consideration only weakly depends on the mass of helium star, total mass of the system, and evolutionary state of the donor at the instant of RLOF. The minimum $P_{\text{orb}} = 9.3 - 10.9$ min, the masses of the donor at $P_{\text{orb, min}}$ are $0.20 M_\odot - 0.26 M_\odot$ (with exception of initially very far-evolved system $(0.65 + 0.8) M_\odot$, $P_{\text{orb}, 0} = 90$ min.) which, in fact, represents a transitional case between systems with initially nondegenerate and degenerate donors). In the period range $20 - 40$ min, which may be adequately described by our models and which hosts a significant fraction of all observed AM CVn stars, the scatter of $\dot{M}$ for a given $P_{\text{orb}}$ does not exceed a factor $\sim 2.5$.

Our computations are limited by $M_{\text{He}} \gtrsim 0.02 M_\odot$. With further decrease of stellar mass thermal timescale of the donor becomes shorter than the angular momentum loss timescale, the stage of adiabatic donor expansion comes to the end, and the matter of the donor becomes more degenerate as it cools down. Qualitative comparison with the computations for initially arbitrary degenerate helium white dwarfs allows to suggest that at $P_{\text{orb}} \gtrsim (40-45)$ min “white-dwarf” and “helium-star” families of AM CVn stars merge and only analysis of the chemical composition of the matter lost by the donor is able to discriminate between different scenarios of formation. Such an analysis is carried out in continuation of the present study. This result also shows that existing theoretical models of the pop-
ulation of AM CVn stars might need certain revision, since they were carried out under assumption that helium stars evolve till Hubble time with the same $M - R$ relation.

Initial periods of the most typical, according to current understanding, progenitors of AM CVn stars with initially nondegenerate helium donors are confined to about $(20 - 120)$ min. This implies a very narrow range of initial separations of components (for instance, $2R_\odot - 7R_\odot$ for $M_{\text{He}} + M_{\text{wd}} = 0.35 M_\odot + 0.5 M_\odot$ pair). This interval is defined by a combination of different parameters. A binary passes through two common envelope stages. The “standard” equation for the variation of orbital separation of components based on energy balance between binding energy of the mass-losing star and orbital energy of the system (Webbink, 1984; de Kool et al., 1987) is

$$
\frac{a_f}{a_i} = \frac{M_{1,c}}{M_1} \left[ 1 + \left( \frac{2}{\alpha \lambda r_{1,L}} \right) \left( \frac{M_1 - M_{1,c}}{M_2} \right) \right]^{-1}, \tag{6}
$$

where the indexes $i$ and $f$ label initial and final separation of components, $\alpha$ – is the parameter of common envelope efficiency, $\lambda$ – is the parameter of the binding energy of the stellar envelope, $M_1$ and $M_{1,c}$ are initial mass of mass-losing star and the mass of its remnant, $r_{1,L}$ is dimensionless radius of the star at the beginning of mass transfer, $M_2$ is the mass of companion.

For every common-envelope stage the values of $\alpha$ and $\lambda$ are, most probably, different and depend on $a_i$ at the beginning of the stage; the value of $a_f$ after the second common envelope depends on “initial-final” mass relations for progenitors of white dwarf and helium star, which, in their own turn, depend on their evolutionary state at the instant of RLOF. This suggests that formation of AM CVn stars needs a very fine “tuning” of evolutionary parameters. The treatment of stellar evolution incorporated in the population synthesis codes that predict existence of AM CVn stars with initially nondegenerate He-donors is, at the moment, inevitably too crude for account of all subtleties of parameters. If AM CVn stars with confidently established presence of He-burning products signatures in their spectra will not be detected, this may mean that the “necessary” combination of parameters is not realized in the Nature or this combination is such, that all He-donors overflow Roche lobes prior to ignition of He or when He burns very weakly. Such a possibility is suggested by positions of AM CVn and V803 Cen in the period-mass diagram (Fig. 6). Since initial abundance of He in stars is close to 1, while abundances of C, O, N are by 2 to 4 orders of magnitude lower, even slight enhancements of $X_C$ and $X_O$ or reduction of $X_N$ may indicate possibility of formation of AM CVn systems with He-star donors.

Above-discussed scenario of evolution needs some caveats. Evolution of every considered system has two phases – prior and after period minimum. In the systems with $M_{2,0} \approx (0.35 - 0.40)$ and $P_{\text{orb,0}} \leq 120\text{ min.}$ which we consider as typical, in the first stage of mass-exchange $2.5 \times 10^{-8} \lesssim \dot{M} \lesssim 10^{-7} M_\odot \text{ yr}^{-1}$. Livne (1990), Livne and Glasner (1991), Livne and Arnett (1995) and other authors have shown that for accretion rate $\dot{M} \sim 10^{-8} M_\odot \text{ yr}^{-1}$ accumulation of $\sim 0.1 M_\odot$ of He at the surface of CO white dwarf may result in detonation of He which may initiate detonation
of carbon in the center of white dwarf and, presumably, SN Ia\textsuperscript{3}. Modeling by means of population synthesis has shown that ELDs may be dominating mechanism of SN Ia in young ($\lesssim$ 1 Gyr) populations (see, e.g., Branch et al., 1995; Hurley et al., 2002; Yungelson, 2005b). However, models of light-curves (Hoeflich and Khokhlov, 1996; Hoeflich et al., 1996) and spectra (Nugent et al., 1997) of objects experiencing ELDs revealed that they disagree with observations. But this conclusion is by no means final in the absence of more detailed calculations. Double-detonation, if it occurs, may prevent transformation into AM CVn stars of some (\sim 40\%) of their potential precursors (Nelemans et al., 2001). Note, for instance, that double-detonation may prevent formation of an AM CVn star with initial parameters $M_{\text{He}} + M_{\text{wd}} = (0.65+0.80) M_\odot$. It is possible that instead of a detonation of a massive He-layer recurrent Nova-scale helium flashes occur (see, e.g., Bildsten et al., 2007 and references therein). The flashes may be accompanied by mass and momentum loss from the system and change the behavior of evolutionary sequences. The problem of unstable He-burning at the surface of accreting white dwarfs and its possible impact on the evolution of AM CVn stars needs additional investigation.

\textit{Availability of results:} Detailed results of computations may be found at \url{www.inasan.ru/~lry/HELIUM_STARS/}.

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