Estimating the detectable rate of capture of stellar mass black holes by massive central black holes in normal galaxies

Steinn Sigurdsson

Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, UK

(January 20, 2022)

The capture and subsequent inspiral of stellar mass black holes on eccentric orbits by central massive black holes, is one of the more interesting likely sources of gravitational radiation detectable by LISA. We estimate the rate of observable events and the associated uncertainties. A moderately favourable mass function could provide many detectable bursts each year, and a detection of at least one burst per year is very likely given our current understanding of the populations in cores of normal spiral galaxies.

I.

The proposed LISA observatory (1-3 and this volume) has optimal sensitivity to gravitational radiation in the $10^{-3}$–$10^{-2}$ Hz range. As such it is well matched to the expected frequency of gravitational radiation emitted by compact objects in the last stages of spiral–in to massive black holes with total masses of $M_{BH} \sim 10^6 M_\odot$ (see eg. 4). Previous papers 5,6 considered the “likely” signal from degenerate compact objects in the cusps of normal galaxies, captured by central black holes such as are inferred to be present in the Milky Way (5) and the nearby dwarf elliptical, M32 (6).

Here we consider the rate for capture of stellar mass black holes expected to be present in the cusps of normal galaxies, and estimate the detectable rate of events by LISA, under both very conservative assumptions and with more optimistic estimates of the black hole population (see also 1,2).

There are several large, systematic uncertainties in estimating the rate of black hole capture. The proportion of galaxies harbouring a central black hole is poorly constrained, although theoretical prejudices suggest central black holes may be ubiquitous 1,7. The mass function of any central black holes is highly uncertain; there are strong observational biases on detection of central black holes, and few galaxies for which there are strong observational constraints as yet 9. The mass function and total number of stellar mass black holes is also highly uncertain 2. Stellar mass black holes may form with masses as low as $2 M_\odot$ and go up to masses of 50–100 $M_\odot$. The range of masses of main sequence stars that form a black hole remnant and the resultant mass of the remnant is also uncertain, and binarity and metallicity may both affect the remnant mass. There are observational biases against detecting the higher mass range of stellar mass black holes as they may sustain higher accretion rates while in binaries, and be detectable in our galaxy for correspondingly shorter time.

We assume, conservatively, that masses of central black holes in galaxies are proportional to the mass of the luminous spheroid of the host galaxy 13, with a space density of $\sim 3 \times 10^{-3} \text{Mpc}^{-3}$ for the range of black hole masses of interest; and that stellar mass black holes are formed from main sequence stars with zero–age masses greater than $15 M_\odot$, drawn from a Salpeter mass function extending to $30 M_\odot$, with a total number fraction of $\sim 10^{-4}$ black holes per formed star. It is possible, if not likely, that this underestimates substantially the stellar mass black hole fraction.

In the presence of a central black hole, the stellar population assumes a self–consistent cusp profile, with the stellar density increasing like $r^{-3/2} - p(r)$, where $p(r)$ depends on the local relaxation time and the original phase space density of the stars 13,14. For (sub)populations of stars for which $p(r) > 0$, the time scale for two–body relaxation, $t_r$, decreases with $r$ (assuming $r > r_{coll}$, the collision radius, at which two–body scattering becomes ineffective, for black holes $r_{coll} = rs$).

In the cusp of stars around the central black hole, stars undergo diffusion in phase space due to three main processes: the perturbation of their orbits from the inhomogeneities of the potential, due simply to the finite number of stars contributing to the mass; large fractional changes in energy or angular momentum due to elastic scattering by close encounters with other stars; and, inelastic mergers with other (extended) stars. Due to the presence of an event horizon at $r_s$, diffusion in phase space is not symmetric, rather there is a sink at small radii, and stars with angular momentum below some critical value are captured by the central black hole (see 20 for discussion). The critical region of phase space for which compact stars are captured is well characterised by a loss–cone parameterised by a critical angle $\theta_c(r)$; here we consider the rate for stellar mass black holes in the cusp to enter $\theta < \theta_c$ through large angle scattering (ie $\delta \theta > \theta_c(r)$). The derivation is discussed in 20.

The rate of scattering, $R$, into the loss–cone from an initial orbit of period $P$, is given by 8

$$R(r) = \frac{N_s^2(r) \theta_c^2(r)}{P} \left(\frac{m_*}{M_{BH}}\right)^2. \quad (1.1)$$

Where $N_s$ is the number of scatterers interior to $r$. For a given $p(r)$ we can solve for the profile of each stellar component of the cusp and hence find the rate for capture. Typically the total rate is peaked at some radius
comparable to or larger than \( r_c \), where \( \theta_c(r_c) = 1 \), unless \( r_{\text{coll}} \gg r_c \). It is possible to do a more sophisticated analysis of angular momentum diffusion in the cusps, but the rate estimates are dominated by uncertainties in cusp parameters, composition of the stellar population and the black hole mass, not the details of the scattering.

Stellar mass black holes may be assumed to trace the light into the cusp, in proportion to their global number fraction. Inside the main–sequence collision radius, the black hole fraction increases, as relaxation remains effective for the compact stars at small radii. For the stellar mass black holes in normal nucleated spirals, \( p(r) \approx 0.7 \) at \( r \lesssim 10^9 r_S \) but may flatten to 0.3 inside \( 10^4 r_S \). Neglecting loss by capture, we expect high volume densities of black holes inside 0.001 pc, even though the total number of black holes is small.

The inferred rate of black hole capture from such a cusp is \( \gtrsim 10^{-5} \) yr \(^{-1} \). By assumption there are only a few hundred black holes in the cusp, and such a rate is not sustainable. Since the relaxation time for the black hole population increases with \( r \), we expect the inner cusp to be rapidly depleted of \( > 90\% \) of its low mass black holes, with the remainder establishing a steady state profile with capture rate of order \( 10^{-8} \) yr \(^{-1} \). Typically the black hole density profile then breaks at \( r \gtrsim 0.1 \) pc, with the local black hole relaxation time \( \gtrsim 10^7 \) years. This can provide a sustainable dribbling of stellar mass black holes from the outer cusp to the inner cusp, with capture dominated by scattering from \( \sim 10^{3-5} r_S \).

LISA strain sensitivity is \( \sim 10^{-23} \) between \( 10^{-3} \) and \( 10^{-2} \) Hz. The characteristic amplitude from a low mass particle, \( m_* \) of period \( P \) is

\[
h_c \sim 4 \times 10^{-24} \frac{m_* \Omega}{d/Gpc} (M_{BH}/10^6 M_\odot)^{2/3} (10^4 s/P)^{2/3}
\]

For \( m_* \sim 5 \sim 10 M_\odot \), assuming a space density of cuspy galaxies with \( M_{BH} \sim 10^6 M_\odot \) of 0.003 Mpc\(^{-3} \), as before, we conservatively expect to see about three captures per year out to 3 Gpc, if stellar mass black holes are present at all in cusps of normal nucleated galaxies.

The typical initial eccentricity of a stellar mass black hole committed to capture by the central black hole is \( e_i \sim 0.9995 \). By construction, we have considered the rate for orbits such that dynamical perturbations will not cause large fractional changes in the angular momentum of the star, after it is scattered into the loss–cone, but before it enters the central black hole. The lifetime to capture through gravitational radiation emission is well approximated by \( \tau_G \approx 10^7 (1 - e^2)^{7/2} (P/10^4 s)^{8/3} \) yr, or \( \sim 10^8 \) years for a stellar mass black hole with \( e_i = 0.9995 \) and apocentre at \( O(10^4 r_S) \). It is necessary that the period shrink to less than \( 10^4 \) seconds for the system to be detectable by LISA. As the orbit approaches \( r_S \), and the system departs from the classical regime, the eccentricity is not well defined (see eg. [1]). Detailed evolution of the orbit is complicated, but we can consider the qualitative behaviour of the orbit: assuming the peribothron \( r_p = (1 - e)a \) for classical semi–major axis \( a \) and some suitably defined semi–classical eccentricity \( e \). Until late stages of the capture, the orbit decay is well described by the post–Newtonian quadrupole decay formula, \( \dot{a} = -64 f(e)M_{BH} m_* / 5 a^3 \). Then \( \tau_D = (1 - e) \dot{a} - a \). The first term is negative and much smaller than \( \dot{a} \), while for the orbits we are interested in \( e < 0 \) [23], and thus the second term is positive. Hence evolution of peribothron will not lead to premature crossing of the event horizon by the star.

In general both the central black hole, and the stellar mass black holes will be rotating and the spins and orbital angular momentum of the system will be uncorrelated. Given high enough signal–to–noise, either from a system with \( m_* \sim 5 M_\odot \) and \( d < 1 \) Gpc, or \( m_* \sim 50 M_\odot \) at a redshift of \( \gtrsim 1 \), post 1.5 and higher order spin–orbit couplings may be detected in the frequency change of the orbit and the orbit precession with comparable contributions from both the stellar mass black hole spin and the central black hole spin (see eg. [24]), enabling measurements of general relativistic effects in the strong field regime, and correspondingly a unique test of the validity of general relativity as a theory of gravity in this regime. In addition, any such detection would provide important information about the masses and spin of both the central massive black hole, and the stellar mass black holes present in the centres of galaxies.

We have assumed that the proportion of stellar mass black holes in the high density cores of galaxies is comparable to that estimated for the total stellar population. If there is a systematic difference in initial mass function, theoretical considerations suggest the mass function should be biased towards more massive stars, and hence more massive remnants, in high density environments [23][24]. An initial higher fraction of black holes would lead to more rapid merger (commencing either a few million years after the first burst of star formation, if the central black hole formed before the stars; or, commencing after the central black hole came to dominate the dynamics of the core, if the central black hole arrived or formed after the initial stars formed), but the same sustainability problems exist independent of the number of black holes, provided there are fewer than \( 10^4 \) initial black holes, and more than 30 or so. Hence most of the stellar black holes are swallowed in a short time compared to the lifetime of the galaxy. There are two factors, however, which lead to a more favourable rate of bursts: with more initial stellar mass black holes, we expect more higher \(( \sim 30 \sim 100 M_\odot \) mass ones, and the capture of those is detectable by LISA anywhere in the universe; it also possible that high mass, low metallicity, stars are more likely to form massive black hole remnants. If there was even one 50 M_\odot black hole in the core of each galaxy now containing a \(( \sim 10^6 M_\odot \) central black hole, we expect several mergers per year at \( z > 1 \), all detectable by LISA. The redshifting of the gravitational radiation is not likely a concern, as central black holes with masses
now $> 10^6 \, M_\odot$ must have had somewhat smaller masses in the past.

In our own galaxy, there is now substantial circumstantial evidence that there was a recent starburst in the central region [27]. If such behaviour is typical of normal nucleated spirals, then every $\sim 10^8$ years, the cusp population of stellar mass black holes may be partially replenished. If this occurs commonly, rates of capture in the nearby universe may be an order of magnitude greater than we have estimated, and we may expect a strong detectable signal every few weeks.

Other favourable scenarios for providing detectable signals include the merger of two massive black hole of comparable mass, or a massive black hole and an “intermediate mass” black hole. It is possible that even more massive black holes form in galaxies ($M_{BH} \approx 10^2 - 10^3 \, M_\odot$), either during early stages of galaxy formation, as part of the ultra–low metallicity Population III stars, or in clusters of stars, growing from $< 10^2 \, M_\odot$ through mergers and accretion. If such black holes can reach the inner parsec of their host galaxy, relaxation will bring them to small radii and capture by the central black hole. The resultant burst of gravitational radiation would be easily detectable, but both the number of such objects and the rate at which they might reach centres of galaxies is highly uncertain (see eg. [28,29]).

It seems quite probable that LISA will detect signals from the capture of stellar mass compact objects by central massive black holes in normal galaxies, if our current understanding of the properties of the central stellar population is even marginally correct, and if the masses of the central black holes in nearby galaxies are at all typical. It is possible that the rate of detectable events is greater than one per month. The actual detection of such events, in addition to testing theories of gravity in the strong field regime, would provide the only data on the mass function of remnants stars in galactic centres, and would probe the low mass end of the range of massive central black holes in the distant universe.

ACKNOWLEDGMENTS

I would like to thank M. Rees, P. Bender and A.G. Polnarev for helpful discussions. The author gratefully acknowledges an EU DGXII TMR Category 30 Marie Curie personal Fellowship.

[1] Bender, P. et al., Pre–Phase A Report to ESA, Dec. 1995
[2] Hough J., et al., 1995, report to the Amaldi Meeting
[3] Danzmann, K., et al., 1993, Laser Interferometer Space Antenna for Gravitational Wave Measurements, Cornerstone Mission Concept submitted to ESA, Max Planck Institut für Quantenoptik
[4] Thorne, K.S., 1995, in van Paradijs, J., van den Heuvel, E., Kuulkers, E., eds., Proc. IAU Symp. 165, Compact Stars in Binaries. Kluwer Academic, p. 153
[5] Hils, D., Bender, P.L., 1995, ApJL, 445, L7
[6] Sigurdsson, S., Rees, M.J., 1996, MNRAS, 284, 318
[7] Eckart, A., Genzel, R., 1996, Nature, 383, 415
[8] Bender, R., Kormendy, J., Dehnen, W., 1996, ApJL, 464, L123
[9] van der Marel, R.P., Sigurdsson, S., Hernquist, L., 1997, in preparation
[10] van der Marel, R.P., de Zeeuw, T., Rix, H. W., Quinlan, G.D., 1996, Nature, submitted
[11] Shibata, M., 1994, Phys. Rev. D, 50, 6297
[12] Hils, D., Bender, P.L., 1997, in preparation
[13] Tremaine, S., 1995, in “Unsolved Problems in Astrophysics”, ed. J. Bahcall and J.P. Ostriker, Princeton University Press (Princeton), in press
[14] Haehnelt, M.G., Rees, M.J., 1993, MNRAS, 263, 168
[15] Rees M.J., 1990, Science, 247, 17
[16] Kormendy J., Richstone D., 1995, ARAA, 33, 581
[17] Timmes, F.X., Woosley, S.E., Weaver, T.A., 1996, ApJ, 457, 834
[18] Peebles, P.J.E., ApJ, 1972, 178, 371
[19] Young, P.J., 1980, ApJ, 242, 1232
[20] Shapiro S.L., 1985, in Goodman J., Hut P., eds, Proc. IAU Symp. 113. Reidel Dordrecht, p. 373
[21] Quinlan G.D., Hernquist L., Sigurdsson S., 1995, ApJ, 440, 554
[22] Frank J., Rees M.J., 1976, MNRAS, 176, 633
[23] Cutler, C., Kennefick, D., Poisson, E., 1994, Phys. Rev. D, 50, 3816
[24] Cutler, C., Flanagan, É. É., 1994, Phys. Rev. D, 50, 3816
[25] Larson R.B., Starrfield, S., 1971, A&A, 13, 190
[26] Larson R.B., 1992, MNRAS, 256, 641
[27] Krabbe, A., et al., 1996, ApJL, 447, L95
[28] Haehnelt, M.G., 1994, MNRAS, 269, 199
[29] Polnarev, A.G., Rees, M.J., 1994, A&A, 283, 301