Light-cone sum rules for the heavy-to-light transition in the
effective theory

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Abstract

Heavy-to-light weak form factor is calculated using the light-cone sum rule (LCSR) in the framework of soft-collinear effective theory (SCET). There are spin-symmetric and spin-nonsymmetric contributions. Leading order spin-symmetric contribution corresponds to the "soft overlap" where some of the partons carry very small momentum. The next-to-leading order spin-symmetric and spin-nonsymmetric parts are characterized by a collinear gluon exchange with the spectator quark. We reproduce the full theory LCSR results and give comments on recent LCSR in SCET.

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I. INTRODUCTION

Heavy-to-light decay processes become more and more important nowadays with copious data from the $B$ factories. Especially, $B \to \pi (\rho)$ decays provide information about the poorly known Cabibbo-Kobayashi-Maskawa (CKM) element $V_{ub}$. In a theoretical viewpoint, heavy-to-light transition shows interesting kinematical configurations which provoke new effective theories of strong interaction. Soft-collinear effective theory (SCET) is a useful framework to deal with light and energetic particles. 

But nonperturbative quantities are always involved in the analysis. The way how to deal with them is crucial and distinctively differs in various approaches. Among the nonperturbative calculational methods, light-cone sum rule (LCSR) has been very successful. Compared to the traditional sum rule by Shifman, Vainshtein, and Zakharov (SVZ), LCSR is more adequate for the heavy-to-light decays. In the SVZ sum rules, nonperturbative effects are encoded by the so-called vacuum condensates. But it is argued that the condensates exaggerate the end-point behavior of the final state meson distribution amplitudes (DA) in the heavy-to-light decays. The LCSR is based on an expansion of nonlocal operators in twist whose matrix elements between vacuum and the meson define the meson distribution amplitudes.

One of the most representative nonperturbative quantity for the heavy-to-light transition is the $B \to \pi$ form factor, $f_+$. It was already calculated by LCSR in. A new development of QCD factorization (QCDF) enlarged our understanding of various $B$ decays, and the spectator interactions are systematically examined. The advent of SCET enabled us to analyze $f_+$ more profoundly.

What makes the analysis more complicated is the possible existence of the Feynman mechanism, or soft overlap. In this situation, some of the partons carry very small momentum compared to others to make the final state meson. It is quite controversial how large the soft overlap is, or even how to define this soft contribution. For example, $\zeta^{B\pi}$ in includes only collinear quarks for the final pion, implicitly neglecting the soft-overlap configuration. That is the reason why the $\zeta^{B\pi}$ is power-counted from $\alpha_s(\sqrt{Q\Lambda})$, where $Q$ ($\Lambda$) is a large (hadronic) scale. On the other hand, the ”soft form factor” $\xi_\pi$ in includes a hard-collinear spectator interactions as its $\alpha_s(\sqrt{Q\Lambda})$ corrections which are not momentum asymmetric. And the authors of define the ”universal form factor” $\xi_M$.
in terms of soft-collinear messenger modes to describe the soft overlap. A clearer and more
definite construction will be needed to remove any confusions. We will use the terminology
of "spin-symmetric" and "spin-nonsymmetric" contributions to the form factors.

Recently, there was a try to establish the LCSR in SCET to calculate the heavy-to-light
form factors. Actually, there have been many efforts to construct the LCSR in the
effective theory, like HQET. In a conventional method, one constructs the correlation
function typically with the interpolating current for the initial state and the weak current.
After possible contractions, the remaining fields between the vacuum and the final state
define the distribution amplitudes. The authors of claim that when the momentum
configuration of the final partons is highly asymmetric, then the light-cone expansion of the
remaining fields is not guaranteed. They propose an alternative method where the final
state pion is described by the interpolating fields. But as we will see later, new method for
LCSR does not show a fundamental difference from the conventional one. Rather, the two
methods are equivalent under proper conditions.

In this paper, a natural extension of the conventional LCSR in the effective theory is
pursued. In other words, we describe the initial \( B \) meson in terms of the usual interpolating
fields. We reproduce the previous results for the form factors of LCSR.
As for the soft form factors, they will be still suffering from the criticism from. But
we argue that the defects are not fully overcome by a new method also. Still the essential
point is how to combine the momentum-asymmetric partons into the final state pion. In
spite of this difficulty, we would like to show that LCSR in SCET can be well established,
especially when the collinear gluon is exchanged with the spectator quark, hoping a deeper
understanding about SCET, much like the traditional SVZ sum rule in the HQET.

The paper is organized as follows. In the next section, the basic formalism of SCET
and LCSR is summarized. Sum rule calculations are given in Sec. III. We distinguish spin-
symmetric and spin-nonsymmetric contributions to the form factor, and reproduce the well
known results. Discussions and conclusions appear in Sec. IV.

II. SETUP

Effective fields in SCET include collinear quark fields \( \xi_{n,p} \), ultrasoft (usoft) heavy quark
fields \( h_v \), usoft light quark fields \( q_{us} \), collinear gluon fields \( A_{h,q}^\mu \), and usoft gluon fields \( A_{us}^\mu \).
These are the relevant degrees of freedom for the usoft-collinear interactions. Collinear covariant derivatives can be defined as

\[ i\bar{n} \cdot D_c = \bar{P} + g\bar{n} \cdot A_n, \quad iD_{c\perp}^{\perp\mu} = \mathcal{P}_{\perp}^{\mu} + gA_{n\perp}^{\perp\mu}, \]  

(1)

where \( \mathcal{P}^{\mu} \) is the usual label operator \[1, 26\], and usoft covariant derivatives are

\[ i\bar{n} \cdot D_{us} = i\bar{n} \cdot \partial + g\bar{n} \cdot A_{us}, \quad iD_{us\perp}^{\perp\mu} = i\partial_{\perp}^{\perp\mu} + gA_{us\perp}^{\perp\mu}. \]  

(2)

The \( n \)-components of the derivative appear as \[26\]

\[ in \cdot D = in \cdot \partial + gn \cdot A_n + gn \cdot A_{us}. \]  

(3)

We begin with the SCET Lagrangians for the usoft-collinear interactions \[26, 27\]:

\[ \mathcal{L}_{\xi q}^{(1)} = \bar{\xi}_n \frac{1}{i\bar{n} \cdot D_c} igB_{c\perp}^{\perp\mu} W_{q_{us}} + \text{h.c.}, \]

\[ \mathcal{L}_{\xi q}^{(2a)} = \bar{\xi}_n \frac{1}{i\bar{n} \cdot D_c} ign \cdot M W_{q_{us}} + \text{h.c.}, \]

\[ \mathcal{L}_{\xi q}^{(2b)} = \bar{\xi}_n \frac{ig}{2} i\bar{D}_{c\perp}^{\perp\mu} \frac{1}{(i\bar{n} \cdot D_c)^2} i\bar{P}^{\uparrow\mu} W_{q_{us}} + \text{h.c.}, \]  

(4)

where the field strength operators are defined as

\[ igB_{c\perp}^{\perp\mu} = [i\bar{n} \cdot D^\perp, iD_{c\perp}^{\perp\mu}], \]

\[ ign \cdot M = [i\bar{n} \cdot D^\perp, i\bar{n} \cdot D], \]  

(5)

and \( W \) is the collinear Wilson line,

\[ W = \left[ \sum_{\text{perm}} \exp \left( -\frac{g}{P} \bar{n} \cdot A_{n,q}(x) \right) \right]. \]  

(6)

The superscripts on \( \mathcal{L}_{\xi q} \) are the power suppression in \( \lambda = \Lambda_{QCD}/Q \).

Heavy-to-light currents are constructed in SCET by matching from full QCD. Up to the next-to-leading order (NLO) in \( \lambda \) at tree level, we have

\[ J^{(0)} = \bar{\xi}_n W \Gamma_{h_v}, \]

\[ J^{(1a)} = -\bar{\xi}_n \frac{ig}{2} i\bar{D}_{c\perp}^{\perp\mu} W \frac{1}{P} \Gamma_{h_v}, \]

\[ J^{(1b)} = -\bar{\xi}_n \Gamma \frac{ig}{2} i\bar{D}_{c\perp}^{\perp\mu} W \frac{1}{m_h} h_v. \]  

(7)
The essence of LCSR is to calculate the correlation function
\[ \Pi = i \int d^4x \ e^{-ip_B \cdot x} \langle \pi(p) | T[J(0) j_B^\dagger(x)] | 0 \rangle . \] (8)

The heavy-to-light current \( J(x) \) and the \( B \)-meson interpolating field \( j_B \) are
\[ J(x) = \bar{q}(x) \Gamma b(x) , \quad j_B^\dagger = \bar{b}(x) i \gamma_5 q(x) . \] (9)

Using the effective fields, the correlator \( \Pi \) can be written in the hadronic language as
\[ \Pi = \frac{F_B}{\sqrt{m_B}} \frac{\langle \pi | J_{\text{eff}} | B \rangle}{2m_B - \eta - \omega - i\epsilon} + (\text{res}) , \] (10)
where \( \eta \equiv 2v \cdot p \), \( \omega \equiv 2v \cdot q \), and \( J_{\text{eff}}(x) = \bar{\xi}(x) \Gamma h_v(x) \). \( B \)-meson effective decay constant \( F_B \) is defined by
\[ \langle B | \bar{h}_v i \gamma_5 q | 0 \rangle = \sqrt{m_B} F_B = \frac{m_B^2}{m_b} f_B . \] (11)

By the dispersion relation and quark-hadron duality, the LCSR is established as
\[ \frac{F_B}{\sqrt{m_B}} \frac{\langle \pi | J_{\text{eff}} | B \rangle}{2m_B - \eta - \omega - i\epsilon} = \frac{1}{\pi} \int_{m_b - \bar{\Lambda}}^{s_0} \frac{\text{Im} \Pi(s, \eta)}{s - \omega} ds , \] (12)
where \( \bar{\Lambda} = m_B - m_b \), and the Borel transformed version is
\[ \frac{F_B}{\sqrt{m_B}} \exp \left( -\frac{2m_B - \eta}{T} \right) \langle \pi(p) | J_{\text{eff}} | B \rangle = \frac{1}{\pi} \int_{m_b - \bar{\Lambda}}^{s_0} e^{-s/T} \text{Im} \Pi(s, \eta) \ ds , \] (13)
where \( T \) is the Borel parameter. Note that we use \( \omega = 2v \cdot q \) as a dynamical variable rather than the usual \( 2v \cdot k = 2v \cdot (p_B - m_w v) \), so the lower limit of the dispersion integral is \( m_b - \bar{\Lambda} \) which corresponds to \( 2v \cdot k = 0 \) when \( \eta = 2v \cdot p = m_B \). The matrix elements \( \langle \pi | J_{\text{eff}} | B \rangle \) are proportional to the effective form factors. In the next section, we evaluate \( \text{Im} \Pi(s, \eta) \) with the quark-gluon fields for various \( J^{(m)} \) to complete the sum rule.

III. SUM RULES IN THE EFFECTIVE THEORY

A. Spin-symmetric form factor : leading order

Spin-symmetric contributions come from the operators which preserve the spin-symmetric form factor relations [18]. After the field redefinition of \( \xi_n \to Y^\dagger \xi_n \), \( A_n \to Y^\dagger A_n Y \) where \( Y \) is the soft Wilson line, and scaling down to the SCET_{II}, their matrix elements are parameterized as [16]
\[ \langle \pi(p) | \bar{\xi}_n W Y_s h_v(0) | B(m_B v) \rangle = \bar{n} \cdot p \ z_{\text{sym}} . \] (14)
FIG. 1: Diagram for the leading order spin-symmetric form factor.

The leading order diagram is shown in Fig. 1. Sum rules for $\zeta_{sym}$ can be obtained via the correlation function $\Pi_{sym}$:

$$\Pi_{sym} = i \int d^4x \ e^{-ipB\cdot x} \langle \pi(p) | T \left[ J^{(0)}(0) j_B^+(x) \right] | 0 \rangle .$$  \hspace{1cm} (15)

Using the heavy quark propagator

$$\bar{h}_v(0) h_v(x) = \int \frac{d^4k}{(2\pi)^4} \ e^{ik\cdot x} \frac{i}{v \cdot k + i\epsilon} \frac{1 + \gamma^5}{2},$$  \hspace{1cm} (16)

we have

$$\Pi_{sym} = i \int d^4x \ \int \frac{d^4x}{(2\pi)^4} \ e^{-i(p_B - m_B v)\cdot x} \frac{i}{v \cdot k + i\epsilon}$$

$$\times \langle \pi(p) | \bar{\xi}_n(0) \gamma^\mu \left( \frac{1 + \gamma^5}{2} \right) i\gamma_5 q_s(x) | 0 \rangle .$$  \hspace{1cm} (17)

By expanding $\gamma^\mu$ and $\gamma^5$ into their light-cone components,

$$\bar{\xi}_n \gamma^\mu \left( \frac{1 + \gamma^5}{2} \right) i\gamma_5 q_s = \frac{1}{2} n^\mu \bar{\xi}_n \gamma^\gamma_5 q_s + \frac{1}{2} n^\nu \bar{\xi}_n i\gamma_5 q_s + \frac{1}{2} \bar{\xi}_n \gamma^\mu i\gamma_5 q_s + \frac{1}{2} \bar{\xi}_n \gamma^\mu \gamma_5 q_s .$$  \hspace{1cm} (18)

Here the terms containing $\gamma^\mu_\perp$ do not contribute, since in the phenomenological sector the matrix elements of the currents are proportional to $\sim p_B^\mu$ or $\sim p^\mu$ which has no perpendicular components. In the full theory, the matrix elements of nonlocal operators between vacuum and meson are described by the DAs. Up to the twist 3,

$$\langle \pi(p) | \bar{u}(0) i\gamma^\mu \gamma_5 d(x) | 0 \rangle = f_\pi p^\mu \int_0^1 du \ e^{i\bar{u}p\cdot x} \phi_\pi(u) ,$$

$$\langle \pi(p) | \bar{u}(0) i\gamma_5 d(x) | 0 \rangle = f_{\pi\mu} \int_0^1 du \ e^{i\bar{u}p\cdot x} \phi_\mu(u) ,$$

$$\langle \pi(p) | \bar{u}(0) \sigma^{\mu\nu} \gamma_5 d(x) | 0 \rangle = i f_{\pi\mu\pi} \int_0^1 du \ e^{i\bar{u}p\cdot x} \phi_\sigma(u) ,$$  \hspace{1cm} (19)
where \( \mu_x = m_x^2/(m_u + m_d) \).

One delicate point at this stage is how to describe the final state pion. In the literature, the energetic pion is constructed by two collinear quark fields in the effective theory. But this picture does not fully appreciate the soft contributions where the constituents’ momenta are asymmetric. For the leading order \( \zeta_{sym} \), we try to set the relevant matrix element as

\[
\langle \pi(p)|\tilde{\xi}_n(0)\gamma^\mu \left( \frac{1 + \gamma^5}{2} \right) i\gamma_5 q_s(x)|0 \rangle = \frac{n^\mu f_\pi}{4} \int_0^1 du \ e^{i\bar{u} \cdot \bar{p} \cdot x + 2\mu_\pi \phi_p(u)} ,
\]

and consequently,

\[
\Pi_{sym} = -\frac{n^\mu f_\pi}{2} \int_0^1 du \ \frac{\eta \phi_\pi(u) + 2\mu_\pi \phi_p(u)}{\omega + u\eta - 2m_b + i\epsilon} .
\]

If the collinear quark \( \xi_n \) and the soft quark \( q_s \) scale like

\[
\xi_n \sim Q(\lambda^2, 1, \lambda) , \quad \text{and} \quad q_s \sim Q(\lambda, \lambda, \lambda) ,
\]

then the combined momentum has a large virtuality \( Q^2\lambda \sim QA \). This is not desirable to form a pion in the final state. To reduce the large virtuality, it is assumed that the soft quark \( q_s \) participating in the final pion scales as \[15\]

\[
q_s \sim (\lambda^2, \lambda, \lambda) .
\]

The imaginary part of \( \Pi_{sym} \) is proportional to a delta function, which restricts the range of \( u \) after integrating over \( s \) in the Borel improved sum rule Eq. (13);

\[
u_0 < u < 1 , \quad 1 - u_0 \equiv \frac{s_0 - m_b + \bar{\Lambda}}{m_B} \approx \frac{2\bar{\Lambda}}{m_B} ,
\]

where we used the fact that \( \omega \) fluctuates around \( m_b \) with the amount of \( \sim \bar{\Lambda} \) and thus its maximum value \( s_0 \) is roughly \( \approx m_b + \bar{\Lambda} \). The constraint of (24) ensures the collinearity of a parton from the weak vertex, \( \tilde{\xi}_n(0) \), and the softness of the spectator quark \( q_s(x) \) in (20) [6, 22].

The final result for \( \zeta_{sym} \) is

\[
\frac{m_B}{m_b} f_B \eta \zeta_{sym} = \frac{f_\pi}{2} \int_{u_0}^1 du \left[ \eta \phi_\pi(u) + 2\mu_\pi \phi_p(u) \right] e^{(2\bar{\Lambda} - \bar{u}\eta)/T} ,
\]

where \( T \) is the Borel parameter. In the so called local duality limit where \( T \to \infty \),

\[
\frac{f_B}{f_\pi} m_b^2 \zeta_{sym} = -\left( \frac{m_b}{m_B} \right)^3 \left( \frac{s_0 - m_b + \bar{\Lambda}}{2} \right)^2 \phi_\pi'(1) .
\]
This is exactly the full theory LCSR result \[6, 22\]. To see this, note that \(\omega_0\) in \[6\] is given by
\[
\omega_0 \equiv v \cdot k_{\text{max}} = \frac{1}{2} (s_0 - m_b + \bar{\Lambda}) \approx \bar{\Lambda} .
\] (27)

It is quite interesting that Eq. (26) is also consistent with the result of \[23\]. In \[23\], the soft form factor \(\xi_\pi\) is given by
\[
\xi_\pi = \frac{m_B \omega_M}{f_\pi(\bar{n} \cdot p)} \left( 1 - e^{\omega_s/\omega_M} \right) f_B \phi_\perp^B(0) .
\] (28)

Then the Wandzura-Wilczek (WW) approximation,
\[
\phi_\perp^B(0) \simeq \int_0^\infty d\omega \frac{\phi_\perp^B(\omega)}{\omega} \equiv \frac{1}{\lambda_B} ,
\] (29)
and the sum rule results
\[
\bar{n} \cdot p \omega_M \left( 1 - e^{\omega_s/\omega_M} \right) \simeq 4\pi^2 f_\pi^2 ,
\] (30)
are used for their analysis. But if we combine the sum rule result for \(1/\lambda_B\) \[6, 22\],
\[
\frac{1}{\lambda_B} = \frac{3}{2\pi^2} \frac{\omega_0^2}{f_B^2 m_b} ,
\] (31)
we arrive at
\[
\frac{f_B}{f_\pi}(\bar{n} \cdot p m_b) \xi_\pi = \omega_0^2 \cdot 6 .
\] (32)
This is nothing but the result of \[26\] when \(\phi_\pi(u) = \phi_\pi^{\text{asy}}(u) \equiv 6u\bar{u}.

**B. Spin-symmetric form factor : NLO**

At NLO of \(\alpha_s\), \(\zeta_{\text{sym}}\) includes spectator interactions with collinear gluon exchanges. The spectator quark hit by the collinear gluon becomes collinear quark at this order, so the momentum configuration of the partons can be symmetric. Figure 2 shows some of them. As an illustration, we construct the correlation functions for these diagrams as

\[
\Pi_{\text{sym}}^{(0,2a)} = i \int d^4x \ e^{-ipB \cdot x}(\pi(p)|T \left[ J^{(0)}(0) j_B^\perp(x), \int d^4y \ L_{\xi_q}^{(2a)}(y) \right] |0 \rangle ,
\]
\[
\Pi_{\text{sym}}^{(n)} = i \int d^4x \ e^{-ipB \cdot x}(\pi(p)|T \left[ J^{(0)}(0) j_B^\perp(x), \int d^4z \ L_{\xi}^{(0)}(z), \int d^4y \ L_{\xi_q}^{(n)}(y) \right] |0 \rangle .
\] (33)
Note the presence of the vertex $\mathcal{L}_{\xi q}$ which converts the usoft spectator into a collinear quark.

With the dimensional regularization where the dimension $d = 4 - 2\epsilon$, we have

\[
\frac{1}{\pi} \text{Im} \left[ \Pi^{(0,2a)}_{\text{sym}} \right] = \frac{g^2 C_F}{16\pi^2} f_{\pi} n^\mu \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \left[ -\frac{1}{\epsilon} \frac{r}{\bar{u} - r} + \frac{2r}{\bar{u} - r} \ln \left( \frac{r\eta}{\mu} \right) - \frac{u}{\bar{u} - r} \ln \left( \frac{u}{\bar{u} - r} \right) \right],
\]

\[
\frac{1}{\pi} \text{Im} \left[ \Pi^{(1)}_{\text{sym}} \right] = \frac{g^2 C_F}{16\pi^2} f_{\pi} n^\mu \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \left[ u \left( -\frac{1}{\epsilon} \frac{r}{\bar{u} - r} - \frac{2r}{\bar{u} - r} \ln \left( \frac{r\eta}{\mu} \right) + \frac{u}{\bar{u} - r} \ln \left( \frac{u}{\bar{u} - r} \right) \right) \right],
\]

\[
\frac{1}{\pi} \text{Im} \left[ \Pi^{(2a)}_{\text{sym}} \right] = \frac{g^2 C_F}{16\pi^2} f_{\pi} n^\mu \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \left[ -\frac{1}{\epsilon} \frac{r(1 - r)}{\bar{u} - r} + \frac{2r(1 - r)}{\bar{u} - r} \ln \left( \frac{r\eta}{\mu} \right) - r - \frac{u\bar{u}}{\bar{u} - r} \ln \left( \frac{u}{\bar{u} - r} \right) \right],
\]

\[
\frac{1}{\pi} \text{Im} \left[ \Pi^{(2b)}_{\text{sym}} \right] = \frac{g^2 C_F}{16\pi^2} f_{\pi} n^\mu \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \left[ -\frac{1}{\epsilon} \frac{ur^2}{(\bar{u} - r)^2} + \frac{2ur^2}{(\bar{u} - r)^2} \ln \left( \frac{r\eta}{\mu} \right) \right] - \frac{ru\bar{u}}{(\bar{u} - r)^2} + \frac{\bar{u}}{(\bar{u} - r)^2} \left( -2ru + \bar{u} + \bar{u}^2 \right) \ln \left( \frac{\bar{u}}{\bar{u} - r} \right),
\]

where $r \equiv (\omega + \eta - 2m_b)/\eta$. Since $\omega$ describes a small fluctuation of order $\mathcal{O}(\bar{\Lambda})$ around $m_b$, $r = \mathcal{O}(\bar{\Lambda}/m_B) \ll 1$ when $\eta = \mathcal{O}(m_B)$. 

\[ \text{FIG. 2: Diagrams for the NLO spin-symmetric form factors.} \]
Summing up these terms gives the finite part as
\[
\frac{1}{\pi} \text{Im} \Pi_{\text{NLO}}^{\text{sym}} = \frac{1}{\pi} \text{Im} \left[ \Pi^{(\text{0},2a)}_{\text{sym}} + \Pi^{(1)}_{\text{sym}} + \Pi^{(2a)}_{\text{sym}} + \Pi^{(2b)}_{\text{sym}} \right] = \frac{g^2 C_F}{16\pi^2} f_\pi n^\mu \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \left[ -r + \frac{2r}{\bar{u} - r} \left\{ \bar{u} + (1 - r) + \frac{u r}{\bar{u} - r} \right\} \ln \left( \frac{r \eta}{\mu} \right) \right. \\
\left. - \frac{ru\bar{u}}{(\bar{u} - r)^2} + \frac{\bar{u}}{(\bar{u} - r)^2} \left\{ \bar{u}^2 + r(1 - 2u) \right\} \ln \left( \frac{\bar{u}}{\bar{u} - r} \right) \right].
\] (35)

In the limit of \( r \to 0 \),
\[
\frac{1}{\pi} \text{Im} \Pi_{\text{NLO}}^{\text{sym}} = \frac{g^2 C_F}{16\pi^2} f_\pi n^\mu \int_0^1 du \phi_\pi(u) \left[ 2r \left( \frac{1}{\bar{u}} + \frac{1}{\bar{u}^2} \right) \ln \left( \frac{r \eta}{\mu} \right) + \frac{r}{\bar{u}} \left( 1 - \frac{1}{\bar{u}} \right) \right].
\] (36)

The light-cone sum rule, Eq. (13), is now established as
\[
\frac{f_B}{f_\pi} m_{\text{NLO}}^2 \Pi_{\text{sym}} = \frac{g^2 C_F}{4\pi^2} \Lambda^2 \int_0^1 du \phi_\pi(u) \left[ \left( \frac{1}{\bar{u}} + \frac{1}{\bar{u}^2} \right) \ln \left( \frac{2\Lambda}{\mu} \right) - \frac{1}{\bar{u}^2} \right],
\] (37)
when \( T \to \infty \). This result must be compared with Eq. (14) of [6]. Terms which are not proportional to \( \phi_\pi'(1) \) are successfully reproduced.

C. Spin non-symmetric form factor

The matrix elements of power suppressed currents \( J^{(1a,1b)} \) are proportional to the spin non-symmetric form factors. Nonzero contributions involve a collinear gluon exchange between the weak vertex and the usoft spectator which becomes collinear after the interaction. Thus the correlation function which gives sum rules for the factorizable form factors can be written as \( m = 1a, 1b \)
\[
\Pi_{\text{ns}}^{(m,n)} = i \int d^4x e^{-ip_B \cdot x} \langle \pi(p) | T \left[ J^{(m)}(0) j_B^1(x), \int d^4y \mathcal{L}^{(n)}_{\xi q}(y) \right] | 0 \rangle.
\] (38)

Figure 3 shows the \( \Pi_{F}^{(m,n)} \) and kinematics. Using the Feynman rules for \( J^{(m)} \) and \( \mathcal{L}^{(n)}_{\xi q} \) from
FIG. 3: Diagram for the spin non-symmetric form factors where \( m = 1a, \ 1b \).

[26], we have

\[
\frac{1}{\pi} \text{Im} \left[ \Pi_{ns}^{(1a,1)} \right] = \frac{g^2 C_F}{16\pi^2} f_\pi n^\mu \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \ln \frac{1}{1 - r},
\]

\[
\frac{1}{\pi} \text{Im} \left[ \Pi_{ns}^{(1a,2a)} \right] = 0,
\]

\[
\frac{1}{\pi} \text{Im} \left[ \Pi_{ns}^{(1a,2b)} \right] = \frac{g^2 C_F}{16\pi^2} f_\pi n^\mu \int_0^1 du \frac{\phi_\pi(u)}{u} \left( \ln \frac{1}{1 - r} - \bar{u} \ln \frac{\bar{u}}{\bar{u} - r} \right),
\]

\[
\frac{1}{\pi} \text{Im} \left[ \Pi_{ns}^{(1b,1)} \right] = \frac{g^2 C_F}{16\pi^2} f_\pi n^\mu \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \frac{r\eta}{m_b},
\]

\[
\frac{1}{\pi} \text{Im} \left[ \Pi_{ns}^{(1b,2a)} \right] = \frac{g^2 C_F}{16\pi^2} f_\pi n^\mu \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \frac{\eta}{m_b} \left( r - \bar{u} \ln \frac{\bar{u}}{\bar{u} - r} \right),
\]

\[
\frac{1}{\pi} \text{Im} \left[ \Pi_{ns}^{(1b,2b)} \right] = -\frac{1}{\pi} \text{Im} \left[ \Pi_{ns}^{(1b,2a)} \right]. \tag{39}
\]

It is easy to check that the above results reproduce the previous calculations for the factorizable form factors by full theory LCSR and QCDF. To see this, first consider the sum rule for \( J^{(1b)} \). Since \( \text{Im} \left[ \Pi_{ns}^{(1b,2b)} \right] = -\text{Im} \left[ \Pi_{ns}^{(1b,2a)} \right] \), we have

\[
\frac{F_B}{\sqrt{m_B}} e^{-(2m_B - \eta)/T} \langle \pi | J^{(1b)} | B \rangle = \frac{g^2 C_F}{16\pi^2} f_\pi n^\mu \int_{m_B - \bar{\Lambda}}^{s_0} ds e^{-s/T} \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \left( s + \eta - 2m_b \right). \tag{40}
\]

In case of \( T \to \infty \) and \( \eta = \bar{n} \cdot p = 2E_\pi = m_B \) \( (q^2 = 0) \), the right-hand-side of sum rule becomes

\[
\text{(R.H.S)} = \frac{g^2 C_F}{16\pi^2} f_\pi n^\mu \langle \bar{u}^{-1} \rangle \bar{\Lambda} \frac{1}{2m_B} (s_0 - m_B + \bar{\Lambda})^2, \tag{41}
\]

11
where
\[ \langle \tilde{u}^{-1} \rangle_\pi = \int_0^1 du \frac{\phi_\pi(u)}{\tilde{u}}. \] (42)

On the other hand, the left-hand-side is
\[ (\text{L.H.S}) = \frac{F_B}{\sqrt{m_B}} \langle \pi \rangle \left( -\frac{1}{m_b} \right) \tilde{\xi}_n \gamma^\mu \frac{g}{2} i \not{D}_c W h_\nu |B \rangle. \] (43)

The matrix element is proportional to the factorizable form factor \( \Delta F_\pi \) in QCDF \[13, 16, 23\]:
\[ -\langle \pi | \bar{\xi}_n i \not{D}_c W h_\nu |B \rangle = \frac{m_b^2}{2} \frac{g^2 C_F}{16\pi^2} \Delta F_\pi. \] (44)

After contracting \( \bar{n}_\mu \) on both sides, combining Eqs. (41), (43), and (44) provides the sum rule for \( \Delta F_\pi \):
\[ \Delta F_\pi = \frac{m_b}{m_B^3 f_B} f_\pi \langle \tilde{u}^{-1} \rangle_\pi (s_0 - m_b + \bar{\Lambda})^2. \] (45)

Now that the well-known QCDF result for \( \Delta F_\pi \) is
\[ \Delta F_\pi = \frac{8\pi^2 f_B f_\pi}{3m_B^3} \lambda_B^{-1} \langle \tilde{u}^{-1} \rangle_\pi, \] (46)

Eq. (45) is consistent with QCDF provided that
\[ \lambda_B^{-1} = \frac{3}{2\pi^2} \frac{m_b}{m_B^2 f_B^2} \left( \frac{s_0 - m_b + \bar{\Lambda}}{2} \right)^2. \] (47)

This is indeed the case as already mentioned in Eq. (31).

Sum rules for \( J^{(1a)} \) also give the same result for \( \Delta F_\pi \). Since \( r \) is a small quantity in Eq. (39),
\[ \text{Im} \left[ \Pi_{ns}^{(1a,1)} \right] \sim \ln \frac{1}{1 - r} = r + \mathcal{O}(r^2) \approx \text{Im} \left[ \Pi_{ns}^{(1b,1)} \right]. \] (48)

As for \( \text{Im} \left[ \Pi_{ns}^{(1a,2b)} \right] \), we can estimate its size after integrating over \( u \) with the asymptotic form of \( \phi_\pi(u) \), \( \phi_\pi^{\text{asy}}(u) = 6u\tilde{u}: \)
\[ \text{Im} \left[ \Pi_{ns}^{(1a,2b)} \right] \sim \int_0^1 du \frac{\phi_\pi^{\text{asy}}(u)}{u\tilde{u}} \left( \ln \frac{1}{1 - r} - \tilde{u} \ln \frac{\tilde{u}}{u - r} \right) = \mathcal{O}(r^2). \] (49)

**IV. DISCUSSIONS AND CONCLUSIONS**

In the present analysis, \( \omega = 2v \cdot q \) describes a small fluctuation of order \( \mathcal{O}(\bar{\Lambda}) \) around \( m_b; \)
\[ m_b - \bar{\Lambda} \leq \omega \leq m_b + \bar{\Lambda}, \] (50)
so $s_\text{0} \approx m_b + \Lambda$. This parameterization is consistent with $\omega_\text{0}$ in \[6\] when $\omega_\text{0} = \Lambda$. For a numerical analysis one usually treats $s_\text{0}$ (or $\omega_\text{0}$) as a free parameter with some reasonable constraints, but the numerics will not be considered here.

As shown in the previous section, the sum rule result from \[23\] for the "soft form factor" is coincident with that of this work or the previous conventional LCSR. The reason is that when the final state pion is described by the interpolating current, only terms like

$$J_\pi(x) \sim \bar{\xi}_hcW_{hc}(x)\bar{\gamma}_sY_s^\dagger q_s(x) + h.c.$$ \(51\)

are contributing to the "soft form factor" (or, the leading spin-symmetric form factor) $\xi_\pi$. Here the soft field $q_s(x)$ should exist to define the $B$ meson distribution amplitude together with the heavy quark field $h_v$ from the weak current. In this picture, the presence of both $\bar{\xi}$ and $q_s$ at the point $x$ is a priori, without any dynamical explanations. The original problem of how to form an energetic pion in the final state with one soft and one collinear quark remains in principle unresolved. Descriptions of Eqs. \[20\] and \[21\] are equivalent in the sense that the pion is depicted by highly momentum-asymmetric configuration with the collinear field $\bar{\xi}$ and the soft field $q_s$. The potential problem of large nonlocality in Eq. \[20\] is alleviated by requiring an accidental smallness of the plus component of $q_s$, $q_s \sim (\lambda^2, \cdots)$ \[15\].

We classify the collinear gluon exchange diagrams for the spin-symmetric currents as the NLO correction to the spin-symmetric form factor. Note that the operators appearing in Eq. \[33\] are similar to the "non-factorizable" operators of \[18\]. But the non-factorizable operators contain the soft gluons in a nontrivial way, which makes it difficult to factorize their matrix elements. In general, the "non-factorizable" soft gluon effects can include the three-particle distribution amplitudes of the form \[28\]

$$\langle 0 | \bar{u}(z)\gamma_\mu \gamma_5 g G_{\alpha\beta} d(-z) | \pi \rangle , \quad \text{or} \quad \langle 0 | \bar{u}(z)\gamma_\mu ig \tilde{G}_{\alpha\beta} d(-z) | \pi \rangle ,$$ \(52\)

where $G_{\alpha\beta}(\tilde{G}_{\alpha\beta})$ is the gluon field strength. The importance of three-particle DAs was already pointed out in \[16\]. They introduce additional four distribution functions of twist 4. Thus it would be quite interesting to investigate the three-particle DAs in the full theory and in SCET and compare the results. The analysis will check if the operator set of \[18\] is complete at this accuracy. But as for the predictive power, there will be few improvements because three-particle DA analysis trades single nonperturbative parameter $\zeta^{B\pi}$ for four more DAs.
It must be emphasized that the NLO spin-symmetric contributions contain potential end-point singularity terms $\sim 1/\bar{u}^2$. In the full theory, these terms are combined with terms $\sim \phi'_\pi(1)/\bar{u}$ to regularize the end-point divergence \cite{6, 22}. We expect a similar situation in the effective theory. Terms like $\sim \phi'_\pi(1)/\bar{u}$ would appear for the diagrams of spectator interactions with soft gluons. Since the Feynman rules of the soft sector are the same as those in the full QCD, momentum-asymmetric features are exactly reproduced in the effective theory. And also, this should happen to satisfy the evolution equation of $\phi'_\pi(1, \mu)$ in the full theory \cite{6}.

Spin-nonsymmetric contributions are exactly the "factorizable" part of \cite{18, 13, 16}, or $\Delta F_\pi$ in \cite{13, 16}. In SCET, both $B$ and $\pi$ are described by the corresponding DAs in a convoluted manner:

$$\Delta F_\pi \sim \phi_B \otimes J \otimes \phi_\pi,$$

where $J$ is the jet function. But in LCSR, the initial $B$ meson is described by an interpolating current. Thus a comparison between LCSR and SCET will provide some relation for $\phi_B$, or the moment of $\phi_B$, more exactly. Actually, the relation is no other than Eq. (47). It is remarkable that $\lambda_B^{-1}$ from this work is coincident with that from LCSR in $B \rightarrow \gamma e\nu$ \cite{23}. On the other hand, a new approach of \cite{23} takes the pion to be described by an interpolating current. Consequently, a comparison between \cite{23} and SCET gives a relation for the moment of $\phi_\pi$, $\langle u^{-1} \rangle_\pi \approx 3$. In this sense, the present work and \cite{23} is complementary.

For the NLO spin-symmetric or spin-nonsymmetric contributions where the collinear gluon is exchanged, the main motivations of new sum rule of \cite{23} become weak since the final quarks are all collinear. Further, what we are mainly concerned about heavy-to-light decay is how to form an energetic light meson in the final state. In the new approach, the final state meson is described by a local interpolating current, leaving other information in the $B$ meson DA. Another merit of the conventional LCSR over the new one is that the variety of $B$ decays can be easily encapsulated by the final-state mesons’ DAs.

As for the numerical size of each contributions, present analysis provides nothing new. Relative size of each component of the form factor is still very disputable \cite{29}, and more efforts should be made in this direction.

In summary, heavy-to-light decay form factor is reexamined in the framework of LCSR in SCET. In this work, unlike a recent approach in \cite{23}, conventional LCSR is naturally extended for an effective theory. Establishments of the sum rules in the effective theory...
is useful in that different kinematic configurations are separated from the beginning at the operator level. After assembling all the pieces, we successfully reproduce the full theory results for the form factor. There still remain complicated and delicate problems of how to describe the Feynman mechanism clearly with operators in the effective theory, or how large each contributions is.

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