ON ELECTROSTRICTION OF A GRANULAR SUPERCONDUCTOR

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Zero-temperature field-induced polarization, supercurrent density, and the related electrostriction (ES) of a granular superconductor are calculated within a model of 3D Josephson junction arrays. Both the "bulk-modulus-driven ES" (the change of the sample’s volume in the free energy upon the applied stress) and the "change-of-phase ES" (due to the stress dependence of the weak-links-induced polarization) are considered. In contrast to magnetostriction of a granular superconductor, its electroelastic behavior is predicted to be dominated by the former contribution for all applied fields.

Some attention was given recently to rather peculiar electric-field induced phenomena, either observed experimentally (like a substantial critical current enhancement [1–3]) or predicted to occur (like a possibility of magnetoelectric effect due to the Dzyaloshinski-Moria type coupling between an applied electric field and an effective magnetic field of circulating Josephson currents [4]) in granular superconductors and attributed to their weak-link structure. At the same time, as compared to the magnetoelastic behavior of superconducting materials (dominated either by a vortex response [5–8] or by weak-links structure [9]), their electroelastic behavior still remains to be properly addressed.

In the present communication, another interesting phenomenon related to the modification of the sample’s weak-links structure in an applied electric field is discussed. Namely, we consider a possible role of Josephson junctions in low-temperature behavior of the field-induced polarization and the related electroelastic properties of granular superconductors.

As is well-known [10], the change of the free energy of a superconductor in the presence of an external electric field $E$ reads

$$
\Delta F(E) \equiv F(0) - F(E) = V \int_0^{E_i} dE \Delta P(E), \quad (1)
$$

where $P(E)$ is the electric polarization of a granular superconductor at zero temperature (see below), $V$ its volume, and the internal field $E_i$ is related to the applied field $E$ via an effective dielectric constant $\epsilon$, namely $E_i = E/\epsilon$. When a superconductor is under the influence of an external (homogeneous) stress $\sigma$, the above free energy results in the associated strain component (in what follows, we consider only a strain component $U$ normal to the applied electric field $E$)

$$
U = \frac{1}{V} \left( \frac{\partial \Delta F}{\partial \sigma} \right), \quad (2)
$$

Neglecting a possible change of the effective dielectric constant $\epsilon$ with the stress, Eqs. (1) and (2) give rise to the following two main contributions to the electrostrictive (ES) strains, namely

(a) the "bulk-modulus-driven ES" due to the change in the free energy arising from the stress dependence of the sample volume

$$
U_{BMD} \equiv \left( \frac{1}{V} \frac{\partial \Delta F}{\partial \sigma} \right)_p = \left( \frac{1}{V} \frac{\partial V}{\partial \sigma} \right) \int_0^{E_i} dE \Delta P(E); \quad (3)
$$

(b) the "change-of-phase ES" due to the stress dependence of the polarization via the Josephson junction effective surface (see below)

$$
U_{PH} \equiv \left( \frac{1}{V} \frac{\partial \Delta F}{\partial \sigma} \right)_V = \int_0^{E_i} dE \frac{\partial P(E)}{\partial \sigma}. \quad (4)
$$

To proceed, we need an explicit form of the induced polarization $P(E)$. And to this end, we employ the model of a granular superconductor based on the well-known tunneling Hamiltonian (see, e.g., Ref. [11])

$$
\mathcal{H} = \sum_{ij} N J_{ij} \left[ 1 - \cos \phi_{ij}(t) \right], \quad (5)
$$

where

$$
\phi_{ij}(t) = \phi_{ij}(0) + \omega_{ij}(\vec{E})t, \quad (6)
$$

with

$$
\omega_{ij}(\vec{E}) = \frac{2e}{\hbar} \vec{E} \vec{r}_{ij}, \quad (7)
$$

and

$$
\phi_{ij}(0) = \phi_i - \phi_j, \quad \vec{r}_{ij} = \vec{r}_i - \vec{r}_j, \quad (8)
$$
which describes an interaction between superconducting grains (with phases $\phi_i(t)$), arranged in a random three-dimensional (3D) lattice with coordinates $\vec{r}_i = (x_i, y_i, z_i)$. The grains are separated by insulating boundaries producing Josephson coupling with energy $J_{ij} = J$. The system is under the influence of an external electric field $\vec{E} = (E, 0, 0)$.

The corresponding pair polarization operator within the model reads

$$\vec{P} = \sum_i N q_i \vec{r}_i,$$

where $q_i = -2e n_i$ with $n_i$ the pair number operator, and $r_i$ is the coordinate of the center of the grain.

In view of Eqs.(5)-(9), and taking into account a usual "phase-number" commutation relation, $[\phi_i, n_j] = i \delta_{ij}$, the evolution of the polarization operator obeys the equation of motion

$$\frac{d\vec{P}}{dt} = \frac{1}{i\hbar} [\vec{P}, \mathcal{H}] = \frac{2e}{\hbar} \sum_{ij} J \sin \phi_{ij}(t) \vec{r}_{ij}.$$  \hspace{1cm} (10)

Resolving the above equation, we arrive at the following mean value of the field-induced polarization

$$\vec{P}(\vec{E}) = \frac{1}{V} \langle \vec{p}(t) \rangle = \frac{2eJ}{\hbar \tau V} \int_0^\tau dt \int_0^t dt' \sum_{ij} < \sin \phi_{ij}(t') \vec{r}_{ij} >,$$

where $<...>$ denotes a configurational averaging over the grain positions, while the bar means a temporal averaging with a characteristic time $\tau$ (see below).

To limit ourselves with field-induced polarization effects only, we assume that in a zero electric field, $\vec{P} = 0$, and thus $\phi_{ij}(0) \equiv 0$.

To obtain an explicit form of the field dependence of polarization, let us consider a site-type positional disorder allowing for weak displacements of the grain sites from their positions of the original 3D lattice, i.e., within a radius $d \approx \sqrt{S}$ ($S$ is an effective surface of grain-boundary Josephson junction) the new position is chosen randomly according to the normalized (probabilistic) distribution function $f(\vec{r}) = f(x)f(y)f(z)$. It can be shown that the main qualitative results of this paper do not depend on the particular choice of the probability distribution function. Hence, assuming, for simplicity, a normalized exponential distribution law, $f(x) = (1/d) e^{-x/d}$ (where $x > 0$ and $\int_0^\infty dx f(x) = 1$), we find that the electric field $\vec{E} = (E, 0, 0)$ (applied along the $x$-axis) will produce a non-vanishing longitudinal (along $x$-axis) polarization vector $\vec{P} = (P, 0, 0)$ with

$$P(E) = P_0 G(E/E_0),$$

where

$$G(z) = \frac{1}{z} \left( 1 - \frac{\arctan z}{z} \right)$$

Here $P_0 = 2JN/E_0V$, $E_0 = \hbar/ed\tau$, and $z = E/E_0$.

At the same time, as is well-known, the supercurrent density through the Josephson junction between grains $i$ and $j$ is related to the polarization operator $\vec{p}$ as follows (see Eq.(10))

$$j_s(\vec{E}) = \frac{1}{V} \left( \frac{d\vec{p}}{dt} \right).$$

Repeating the above-discussed averaging procedure, we find for the change of the longitudinal part of the supercurrent density in applied electric field

$$j_s(E) = j_0 D(E/E_0),$$

where

$$D(z) = \frac{z}{1 + z^2}$$

with $j_0 = 2eJNd/hV$, and $z = E/E_0$.

Figures 1 and 2 show the field-induced behavior of the normalized polarization $P(E)/P_0$ and supercurrent density $j_s(E)/j_0$, calculated according to Eqs.(12) and (15), respectively. Notice a rather pronounced peak at $E/E_0 \approx 2$ for both dependences. Assuming $d \approx 1 \mu m$ and $\tau \approx 10^{-15} s$ for an average grain size and a low-temperature estimate of the Josephson tunneling time of Cooper pairs through an insulating barrier in applied electric field $10^7 V/m$ for the estimate of the model characteristic field, which is very close to the typical applied field values where the critical current of ceramic samples was found to reach its maximum. Besides, assuming $V \approx N d^2$ and taking into account that the Josephson energy in $YBCO$ is $J/k_B \approx 90 K$, we obtain quite a reasonable estimate for the model characteristic critical current density $j_0$, typical for ceramic samples. Namely $j_0 \approx 2eJ/hd^2 \approx 10^3 A/m^2$. Let us briefly comment on the temporal averaging (used in Eq.(11)) and discuss the relationship between the characteristic time $\tau = \hbar/\epsilon_0 d$ and period of oscillations $T(E)$. The latter is defined via Eq.(7) as $T(E) = 2\pi/ < \omega_{ij}(E) >$, where $< \omega_{ij}(E) >= 2eEd/h$. Thus, depending on the strength of an applied electric field, the period of oscillations $T(E)$ can be larger or smaller than the tunneling time $\tau$. In particular, high-field region $E \geq E_0$ where the most interesting effects take place, is characterized by faster oscillations with the period $T(E) \leq \tau$, as compared with low-field behavior. In addition, we can compare $\tau$ with a zero-field (and low-temperature) Josephson tunneling time $\tau_0 \approx \hbar/J$ which in $YBCO$ gives $\tau_0 \approx 10^{-13} s$. Hence, at low applied fields (when $E \ll E_0$) $T(E) \approx \tau_0$ while in high-field regime (when $E \gg E_0$) $T(E) \ll \tau \ll \tau_0$. Furthermore, according to Eqs.(3) and (12), the explicit form of the "bulk-modulus-driven ES" contribution is as follows.
\[ U_{BMD}(E_i) = U_0 G_{BMD}(E_i/E_0), \quad (17) \]

where
\[ G_{BMD}(z) = zG(z) + \ln \sqrt{1 + z^2} \quad (18) \]

Here \( U_0 = \kappa E_0 P_0 \) with \( \kappa = -\partial \ln V/\partial \sigma \) being the compressibility coefficient.

To evaluate the "change-of-phase ES" contribution \( U_{PH}(E_i) \), we have to account for the stress dependence of the effective surface \( S(\sigma) \) of grain-boundary Josephson junctions which was found \([15]\) experimentally to decrease with the applied stress. Using the following chain of evident relations,
\[
\begin{align*}
\frac{\partial P}{\partial \sigma} &= \frac{\partial P_0}{\partial \sigma} G + P_0 \frac{\partial G}{\partial \sigma}, \\
\frac{\partial P_0}{\partial \sigma} &= \frac{\partial P_0}{\partial V} \frac{\partial V}{\partial \sigma} = P_0 \kappa, \\
\frac{\partial G}{\partial \sigma} &= \frac{\partial G}{\partial E_0} \frac{\partial E_0}{\partial \sigma}, \\
\frac{\partial E_0}{\partial \sigma} &= \frac{\partial E_0}{\partial S} \frac{\partial S}{\partial \sigma},
\end{align*}
\quad (19)
\]

and assuming that the sample volume \( V \) and the projected area \( S \) are related in a usual way, \( S \approx V^{2/3} \), we obtain from Eqs.(4), (12) and (19) for the "change-of-phase ES"
\[
U_{PH}(E_i) = -\frac{1}{3} U_0 G_{PH}(E_i/E_0), \quad (20)
\]

where
\[ G_{PH}(z) = zG(z) + 2G_{BMD}(z) \quad (21) \]

Figure 3 summarizes the predicted behavior of the two considered contributions (calculated according to Eqs.(17) and (20)), along with their total effect (solid line) on induced electrostriction. Notice that in contrast to the magnetostrictive behavior of a granular superconductor (considered in Ref. [4]), its electroelastic properties are completely dominated by the "bulk-modulus-driven" contribution over the whole region of applied fields. It would be interesting to verify the above-predicted behavior experimentally.

In conclusion, a low-temperature field-induced electroelastic behavior of a granular superconductor was considered within a model of 3D Josephson junction arrays. The "bulk-modulus-driven" contribution to the electrostriction (ES) of a granular superconductor (related to the change of the sample's volume in the free energy upon the applied stress) was shown to dominate over the "change-of-phase" ES (related to the stress dependence of the weak-links-induced polarization) for all applied fields.

ACKNOWLEDGMENTS

This work was financially supported by the Brazilian agency CNPq.

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FIG. 1. The behavior of the induced polarization $P/P_0$ in applied electric field $E/E_0$.

FIG. 2. The behavior of the induced supercurrent density $j_s/j_0$ in applied electric field $E/E_0$.

FIG. 3. The "bulk-modulus-driven" $U_{BMD}/U_0$ (dotted line), "change-of-phase" $U_{PH}/U_0$ (dashed line), and the total $(U_{PH} + U_{BMD})/U_0$ (solid line) contributions to the induced electrostriction vs internal electric field $E_i/E_0$. 