Renormalization of the upper critical field by superconducting fluctuations

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We study the effect of superconducting fluctuations on the upper critical field of a disordered superconducting film at low temperatures. The first order fluctuation correction is found explicitly. In the framework of the perturbative analysis, superconducting fluctuations are shown to shift the upper critical field line toward lower fields and do not lead to an upward curvature. Higher order corrections to the quadratic term coefficient in the Ginzburg-Landau free energy functional are studied. We extract a family of the mostly divergent diagrams and formulate a general rule of calculating a diagram of an arbitrary order. We find that the singularity gets more severe with increasing perturbation theory order. We conclude that the renormalization of the Ginzburg-Landau coefficients by superconducting fluctuations is an essentially non-perturbative effect. As a result, the genuine transition line may be strongly shifted from the classical mean-field curve in a two-dimensional superconductor.

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I. INTRODUCTION

The magnetic field-temperature phase diagram of type II superconductors has been of experimental and theoretical interest for a very long time. Much interest has recently focused on the disordered two-dimensional superconductors partly motivated by the two-dimensional nature of the high-$T_c$ cuprates although disordered thin films of “ordinary” superconductors have been studied for years. The subject matter of this paper is the temperature dependence of the upper critical field, $H_{c2}(T)$, above which the system goes normal. In particular, we consider the problem of superconducting fluctuation effects on the upper critical field of a disordered superconducting film at low temperatures.

There have been a number of experiments studying the behavior of the upper critical field line as a function of temperature and disorder. Some of them have shown strong deviations from the classical mean-field theory such as an upward curvature in the $H_{c2}(T)$-line. These experiments have induced a considerable theoretical interest in the subject.

Let us start with recalling the previous theoretical studies. The first attempt to explain the observed effects has been made by Golubov and Dorin who considered the combined effects of disorder and Coulomb interaction in first-order perturbation theory. Golubov and Dorin’s calculations predicted some upward curvature in the upper critical field line. However, later re-examination of the Golubov and Dorin’s theory showed some very technical deficiencies in the original calculations. Namely, Smith et al. have performed both first-order perturbation theory calculations and Oreg and Finkelstein renormalization group treatments of the problem. Their careful analysis has not shown any anomalous upward curvature in either case. So, the final verdict was that disorder and Coulomb interaction together could not explain the observed deviations from the mean field results.

Zhou and Spivak have suggested a mechanism of $H_{c2}$ enhancement based on a possibility of the spontaneous formation of a granular-like superconducting structure in a disordered film at a low enough temperature. The idea is that optimal fluctuations in the distribution of impurities may lead to the formation of local regions where the local critical field exceeds the system-wide average value. At a low enough temperature, these disorder-induced superconducting droplets form a Josephson network allowing for the global superconductivity in the film. Recently, Galitski and Larkin have included the effects of quantum fluctuations into the picture and showed that the “mesoscopic disorder” (i.e., usual non-magnetic impurities) was very weak to produce a considerable effect on the critical field line and the global superconducting glassy state suggested by Zhou and Spivak should be destroyed by quantum fluctuations. However, the “pinning disorder” (grain boundaries, dislocation clusters, etc.) may be strong enough to compete with the quantum fluctuations. The prediction is that the anomalous upward curvature due to the optimal disorder fluctuations is possible but the corresponding effect is non-universal and depends on the pinning properties of the system. Let us mention a Comment on the paper by Ikeda, who emphasized a possible importance of superconducting fluctuations. The purpose of the present work, which is partially motivated by Ref. 8, is to explicitly evaluate fluctuation corrections to the upper critical field line at low temperatures.

Let us emphasize that the definition of the superconducting transition point itself is a very delicate issue in a two-dimensional system. Strictly speaking, at finite fields, superconductivity as a state with zero resistivity is never achieved which is basically a manifestation of the general Mermin-Wagner theorem. In real experiments, what is observed is a drop of the resistance to some very small but finite values. Such a point is usually defined to be the transition point. Further decrease of the temperature or external field yields a very slow decay of the resistivity. From the theoretical point of view, the def-
inition of the critical point is also non-trivial. One can define the transition point as a point at which the superfluid density $\rho_s$ becomes non-zero. Another approach is to study the behavior of the quantity $\langle \psi(0)\psi(r) \rangle = \Delta(r)$ by the use of Ginzburg-Landau type expansions and define the transition point as a point at which the quadratic term vanishes. There are no solid grounds to believe that these two approaches are equivalent. In the present paper, we follow the latter approach, which, as we think, should correspond to the point at which the critical drop in the resistance to some finite value takes place.

The question of $T_c$ renormalization in zero field has been recently addressed by Larkin and Varlamov in the Review [11]. The two-dimensional result for the shift of the transition temperature was found as $\delta T_c/T_c \sim [\ln G_i]/G_i$, where $G_i$ is the Ginzburg parameter, which in a disordered two-dimensional superconductor is of order of the inverse conductance. Thus, one concludes that the effect of fluctuations in zero field is quite small and the fluctuations can move the transition point only within an extremely narrow Ginzburg region. We will return to the discussion of the validity of this conclusion at the end of the paper.

Our paper is structured as follows: In Sec. II, we formulate the Ginzburg-Landau theory for the case of a strong external magnetic field. We emphasize that the Ginzburg-Landau expansion in this case can be made with respect to the modulus of the order parameter only and not on its spatial gradients. However, one can still write down a formal operator expansion. The vanishing of the lowest Landau level matrix element of the operator $A$ appearing in the quadratic term, corresponds to the transition point. In Sec. III, we study the effect of superconducting fluctuations on the upper critical field in first order perturbation theory. We calculate the first correction explicitly. We find that the first correction alone can not lead to a monotonous upward curvature of the $H_{c2}(T)$ line but can yield a decrease of the critical field compared to the mean-field value. In Sec. IV, we study the higher order perturbation theory corrections. We extract a family of the mostly divergent diagrams the effect of which parametrically exceeds the first and second order contributions. The first term in the family appears in third order perturbation theory only. We show that the singularity gets progressively stronger in higher orders. We conclude that the renormalization of the Ginzburg-Landau coefficients and, consequently, the transition line by superconducting fluctuations is an essentially non-perturbative effect.

II. GINZBURG-LANDAU EXPANSION IN STRONG FIELDS

It is well-known that a superconductor in a relatively weak magnetic field can be described with the aid of the Ginzburg-Landau theory. The Ginzburg-Landau expansion of the free energy has the following standard form:

$$\mathcal{F}[\Delta(r)] = \mathcal{F}_N + \int d^2 r \left\{ a |\Delta(r)|^2 + b |\Delta(r)|^4 \right\} + \frac{1}{4m} |\nabla - 2ieA(r)|^2 \Delta(r)^2,$$

where $\mathcal{F}_N$ is the free energy of a normal metal, $\Delta(r)$ is the superconducting order parameter, $a$, $b$, and $m$ are some coefficients which depend both on temperature and magnetic field. Coefficient $a$ vanishes at the transition point.

To derive Eq. (1) from the microscopic theory, i.e. from the initial BCS hamiltonian, one has to fulfill the following standard steps: First, the quartic interaction term should be decoupled with a Hubbard-Stratonovich field $\Delta$. Then, the functional integral on the electron degrees of freedom becomes Gaussian and can be easily evaluated. The resulting action $S(\Delta)$ is quite complicated, however in the vicinity of the superconducting transition, $\Delta$ is small and the action can be expanded on it. If one is interested in the equilibrium thermodynamic properties of a superconductor, the time-dependence of $\Delta$ may be suppressed and one can talk of deriving an effective free energy functional rather than of an effective action. However, another essential assumption has to be made in order to get Eq. (1). Namely, one has to suppose that the spatial variations of the order parameter are small enough so that term $|\nabla - 2ieA(r)|\Delta(r)|^2$ is small in some sense. Finally, expanding both on the order parameter and on its spatial gradients on can derive the Ginzburg-Landau free energy functional, the minimum of which yields the famous Ginzburg-Landau equations.

The above arguing partially fails in strong fields when the spatial dependence of the order parameter becomes important. In strong fields, the expansion on $|\nabla - 2ieA(r)|\Delta(r)|^2$ is no longer possible. However, one can still expand on the magnitude of the order parameter. The corresponding saddle point equation can be written as:

$$\frac{1}{\lambda} \Delta(r) = \int C(r, r') \Delta(r') d^2 r' - \int B(r, r_2, r_3, r_4) \Delta^*(r_2) \Delta^*(r_3) \Delta^*(r_4) d^2 r_2 d^2 r_3 d^2 r_4.$$

Here $\hat{C}$ is Cooperon (see Fig. 1) which is a linear operator in the case under consideration. In the presence of a magnetic field the operator can be written as:

$$C(r, r') = T \sum \xi \mathcal{G}_e(r) \mathcal{G}_{-\xi}(r'),$$

where $\mathcal{G}$ is the electron Green function in the magnetic field and $\xi$ is the fermion Matsubara frequency.

Non-linear operator $B$ is described by the square diagrams similar to the ones shown in Fig. 2. Let us note, that this operator is not singular at the transition point.
dependence in the Cooperon can be factored out: [15] below].

\[ \mathcal{L} = \lambda + \sum_{n} \lambda^{n} \mathcal{C}^{n} + \ldots \]

FIG. 1: Diagrammatic equation for the fluctuation propagator \( \mathcal{O} \) (curly line).

First order correction:

\[ \mathcal{C}(r, r') = C^{(0)}(r - r') \exp \left\{ -2ie \int_{r}^{r'} A(s) ds \right\}, \tag{3} \]

where \( C^{(0)}(r) \) is the Cooperon without magnetic field. Formally, operator \( \mathcal{C} \) can be written in the following form:

\[ \mathcal{C} = \int C^{(0)}(r) \exp \{-i\mathbf{r}\mathbf{\pi}\} d^{2}\mathbf{r}, \tag{4} \]

where \( \mathbf{\pi} = -i\nabla - 2eA(\mathbf{r}) \) is the operator of the kinetic momentum, which can be expressed in terms of the creation and annihilation operators in the Landau basis. Again, in small fields, one can expand the exponent in Eq. (4) which will result in the gradient term \( \mathbf{\pi}^{2} \) [see Eq. (1)]. In strong fields, we can not do such an expansion.

The transition point is defined as a point at which the coefficient in front of the \( \Delta^{2} \) term vanishes. The corresponding operator has the form:

\[ \hat{A} = \lambda^{-1} - \hat{C}. \tag{5} \]

The inverse operator can be easily recognized as the pairing vertex which corresponds to the ladder summation as shown in Fig. 1.

\[ \hat{L}(0) = \hat{A}^{-1} = \lambda \sum_{n=0}^{\infty} \lambda^{n} \hat{C}^{n}. \tag{6} \]

The divergence of this quantity at the transition point corresponds to the BCS instability.

It is possible to calculate pairing vertex \( \hat{L}(\Omega) \), which is also called the fluctuation propagator, as a function of the total energy in the Cooper channel \( \Omega \) at arbitrary magnetic fields. From Eq. (4), one can see that the Cooperon can be written as a series containing even powers of kinetic momentum \( \mathbf{\pi} \) only. Thus, the Cooperon is a diagonal operator in the Landau basis, which makes [see Eqs. (5,6)] the fluctuation propagator diagonal as well. The corresponding matrix elements have the following explicit form:

\[ \mathcal{L}_{n}(\Omega) = \frac{1}{N(0)} \left\{ \ln \frac{T}{T_{c0}} + \psi \left[ \frac{1}{2} + \frac{\Omega_{H} (n + \frac{1}{2}) + |\Omega|}{4\pi T} \right] - \psi \left[ \frac{1}{2} \right] \right\}^{-1}, \tag{7} \]

where \( N(0) \) is the density of states at the Fermi-line, \( T_{c0} \) is the transition temperature in zero field, \( \Omega = 2\pi nT \) is the Matsubara frequency, and \( \Omega_{H} = 4eDH \), with \( D \) being the diffusion coefficient and \( H \) the external magnetic field. The upper critical field line is defined by the condition:

\[ A_{0} = \mathcal{L}_{0}^{-1}(0) = 0, \tag{8} \]

where index “0” corresponds the lowest Landau level matrix element. The above equation yields the well-known Gor’kov’s mean-field curve for \( H_{c2}(T) \) (see solid line in Fig. 3).

In order to study the effect of superconducting fluctuations on the \( H_{c2} \) transition line, one has to go beyond the mean field theory. The first fluctuation correction is described by the diagrams shown in Fig. 2. The corresponding correction to the Ginzburg-Landau coefficient is

\[ \delta A^{(1)}(r_{1}, r_{2}) = T^{2} \sum_{\Omega, \xi} B_{\xi, \Omega}(r_{1}, r_{2}, r_{3}, r_{4}) \hat{L}_{\Omega}(r_{3}, r_{4}), \tag{9} \]

where \( B \) is the impurity box (see Fig. 2) and \( \hat{L}_{\Omega} \) is the fluctuation propagator introduced in the previous section.

Let us note that operator \( \delta \hat{A} \) in Eq. (9) is diagonal in the Landau representation, since it can be written in the form analogous to Eq. (5). The fluctuation propagator
can be expanded on the Landau wave-functions as follows (we use the Landau gauge here):

\[ \mathcal{L}(r_1, r_2) = \sum_n \mathcal{L}_n \int \frac{dp}{2\pi} \psi_{np}(r_1) \psi_{np}(r_2), \tag{10} \]

where

\[ \psi_{np}(r) = e^{ipy} \phi_n^{(osc)}(x-x_0), \quad x_0 = pL_H^2, \tag{11} \]

with \( \phi_n^{(osc)}(x) \) being the harmonic oscillator eigenfunction

\[ \phi_n^{(osc)}(x) = \frac{1}{\sqrt{2^{n!}n!}} \left[ \frac{1}{\pi L_H^2} \right]^{1/4} e^{-\pi x^2 L_H^2} H_n \left( \frac{x}{L_H} \right), \tag{12} \]

and \( L_H = (2eH)^{-1} \) the magnetic length for the fluctuating Cooper pairs.

Thus, the first fluctuation correction can be written as:

\[ \delta A_0^{(1)} = T^2 \sum_{\epsilon, \Omega} \mathcal{L}_n(\Omega) \int \prod_{i=1}^4 d^3r_i \ B_{\epsilon, \Omega}(r_1, r_2, r_3, r_4) \times \int \frac{dk dk'}{(2\pi)^2} \psi_{nk}(r_1) \psi_{n+k}(r_2) \times \psi_{0k'}(r_3) \psi_{0+k'}(r_4). \tag{13} \]

At the superconducting transition point, only the lowest Landau level matrix element of the fluctuation propagator is divergent. In the vicinity of the transition it has the simple form:

\[ \mathcal{L}_0(\Omega) = \frac{1}{N(0)} \left\{ \frac{H - H_{c2}(T)}{H_{c2}(0)} + \frac{2|\Omega|}{\Omega_H} \right\}^{-1}, \tag{14} \]

where \( H_{c2}(T) \) is the mean-field value of the upper critical field. Since only the lowest Landau level provides a divergent contribution, one can keep just one term in the sum over \( n \) in Eq. (13). With the same accuracy, one can put \( \Omega = 0 \) in the expression for the box diagrams as the divergence comes from the small values of \( \Omega \) in the sum. In this approximation, the ladder summation in diagram “c” in Fig. 1 is not necessary and we are left with the Hikami-box like diagram, the result for which is known [12]

\[ B_{\epsilon, \Omega=0}(r_1, r_2; r_3, r_4) = \frac{\pi N(0)}{2} \left( \prod_{k=1}^4 |\epsilon| + \frac{1}{2} D \partial^2 \right) \times \delta(r_1 - r_2) \delta(r_1 - r_3) \delta(r_1 - r_4) \times \left[ |\epsilon| + \frac{1}{8} D \left( \left[ \partial_{(1)} - \partial_{(2)} + \partial_{(3)} \right]^2 + \left[ \partial_{(2)} - \partial_{(4)} \right]^2 \right) \right], \tag{15} \]

where

\[ \partial_{(k)} = -i \nabla - 2e(-1)^k A(r). \]

In Eq. (15), the factors \( \left[ |\epsilon| + \frac{1}{2} D \partial^2 \right]^{-1} \) come from the four impurity vertices, three delta-functions correspond to the locality of the quantity in the dirty limit \( \tau T_{c0} \ll 1 \), where \( \tau \) is the scattering time), and the last factor comes from calculating the square diagrams themselves. Let us note that the leading frequency and momentum independent terms are cancelled out and one has to keep \( \epsilon \) and \( \mathbf{q}_0 \) finite to get a non-zero contribution. This property can be proven not only for box-like diagrams shown in Fig. 2, but for any 2n-sided polygon.

Acting by operator (15) on the lowest Landau level eigenfunctions in Eq. (7) and calculating the Matsubara

FIG. 3: Renormalization of the upper critical field by superconducting fluctuations. The solid line shows the mean-field Gor'kov-Helfand-Werthamer result. The dashed line is the upper critical field with the account for the superconducting fluctuations in first order perturbation theory.
sum, we derive the following expression:

$$\delta A_0^{(1)} = -\frac{1}{2\pi L_H^2 (4\pi T)^2} N(0) \left( \frac{\Omega_H}{8\pi T} + \frac{1}{2} \right) \left[ T \sum_\Omega \mathcal{L}(\Omega) \right]$$  \hspace{1cm} (16)

The last factor can be evaluated with the logarithmic accuracy as follows:

$$T \sum_\Omega \mathcal{L}(\Omega) = \frac{\Omega_H}{4\pi N(0)} \left[ \ln \frac{\Omega_H}{T} - \psi \left( \frac{H - H_{c2}(0)}{H_{c2}(0)} \right) \right] - \frac{T}{2\Omega_H} \frac{H_{c2}(0)}{H - H_{c2}} \right], \hspace{1cm} (17)$$

Let us remember that $N(0)$ is the density of states at the Fermi-line, $\Omega_H = 4eDH = \frac{2\pi}{\gamma}T_{c0}$, where $D$ is the diffusion coefficient, $e$ is the electron charge, $H$ is the external field, $T_{c0}$ is the transition temperature in zero field, and $\gamma \approx 1.78$ is Euler’s constant.

Eqs. (16,17) determine the first order correction to the upper critical field which is defined by the following equation:

$$A_0 + \delta A_0^{(1)} = 0.$$  

This is a non-linear algebraic equation, which can be easily solved by numerical means. As one can see in Fig. 3, the first correction yields a lower upper critical field compared to the mean field result. As expected, the perturbative analysis does not lead to an upward curvature in the $H_{c2}(T)$-line.

Let us study the asymptotic behavior of the obtained expressions. If the following inequality is satisfied

$$\frac{T}{T_{c0}} \ll \frac{H - H_{c2}(T)}{H_{c2}(0)} \ll 1$$

we have

$$\delta A_0^{(1)} = \frac{\gamma \Omega_H}{4\pi} \ln \left[ \frac{H}{H - H_{c2}(0)} \right], \hspace{1cm} (18)$$

where we have introduced the dirty limit Ginzburg parameter as $\Omega_H = (\varepsilon F)^{-1} \ll 1$. One can see that the first order correction is only logarithmic. In the case of small but finite temperatures $H - H_{c2}(T)/H_{c2}(0) \ll T/T_{c0} \ll 1$ we have

$$\frac{\delta A_0^{(1)}}{N(0)} = \frac{\gamma \Omega_H}{\pi^2} \left[ \frac{H}{H - H_{c2}(T)} \right] \frac{T}{T_{c0}}.$$  

We see that in this temperature domain, the effect of the superconducting fluctuations is more pronounced.

III. HIGHER ORDER CORRECTIONS

In this section, we will study higher order corrections to the mean field result. For the sake of technical sim-
licity, we will focus on the zero-temperature case which we will use to estimate the divergence rate of higher order diagrams. Generalization to non-zero temperatures is straightforward.

The simplest way to construct a higher order correction is to study a diagram made of a polygon with all possible impurity averagings and \( n \) curly lines connecting \( 2n \) vertices of the \((2n + 2)\)-sided polygon. In Fig. 4, all possible topologically non-equivalent second order diagrams are shown (see also Fig. 5, diagram "a"). Note that in the highly disordered limit, these polygons are local quantities which can be written in the form similar to \( \text{fig} \). The contributions due to such diagrams can be easily estimated. Expressions for these diagrams contain \( n \) integrals over the frequencies running through \( n \) fluctuation vertices, which leads to a contribution of order of \( \ln^n \left[ \frac{H_{c2}}{H - H_{c2}} \right] \). Therefore, we conclude that perturbation series order due to the single-polygon (irreducible) graphs has the form \( \sum_n C_n \ln^n \left[ \frac{H_{c2}}{H - H_{c2}} \right] \), where \( n \) is the perturbation series order and \( C_n \) are some combinatorial factors. Let us mention that similar diagrams have been studied by Kee \textit{et al.} who constructed a non-perturbative resummation technique to sum up the dominant contributions. In our problem, such a resummation is not possible since, as we shall see, another type of diagrams delivers the dominant contributions.

Problems start to appear in third order perturbation theory when one is able to construct a diagram containing several polygons connected by curly lines only (see, \textit{e.g.}, Fig. 4; diagram "b"). We will call a diagram reducible if it can be separated into several parts by cutting curly lines only. Otherwise, we will call a diagram irreducible. Let us consider a general case of a graph in \( n \) order perturbation theory which contains \( r \) irreducible parts. In this case, simple dimensional analysis of the problem yields the conclusion that we have \( (n + 1 - r) \) integrals running through \( n \) singular fluctuation propagators. Therefore, the contribution due to such a diagram can be estimated as follows:

\[
\frac{\delta A_0^{(n,r)}}{N(0)} \sim \ln^n \left( \frac{H_{c2}}{H - H_{c2}} \right)^{(r-1)} \left( \frac{H_{c2}}{H - H_{c2}} \right)^{(n-r).}
\]

One can easily see that in higher orders the singularity gets enhanced and contains a power low dependence on the closeness to the transition compared to only logarithmic dependence \( \ln^1 \left[ \frac{H_{c2}}{H - H_{c2}} \right] \) in first and second order perturbation theory. At low but finite temperatures \( H - H_{c2}(T)/H_{c2}(0) \ll T/T_{c0} \ll 1, \) we obtain the following estimate for the contribution of an \( n \) order graph built up of \( r \) irreducible fragments:

\[
\frac{\delta A_0^{(n,r)}}{N(0)} \sim \ln^n \left( \frac{T}{T_{c0}} \right)^{(n-r+1)} \left( \frac{H_{c2}(0)}{H - H_{c2}(T)} \right)^n.
\]

It is clear that the more irreducible parts contains a diagram, the more singular contribution it derives. Therefore, mostly divergent diagrams should contain as many cubic irreducible elements as possible. The corresponding subseries is built up of chain and ring-like diagrams (see Fig. 4, diagrams "b" and "c"). The contribution from a diagram which contains \( r \) Hikami boxes is found as \((T = 0)\):

\[
\frac{\delta A_0^{(2r-1,r)}}{N(0)} \sim \ln^{r-1} \left[ \frac{H_{c2}}{H - H_{c2}} \right].
\]

Such diagrams provide dominant contributions in the expansion. From Eq. 20, we see that although the singularity gets progressively stronger in the higher orders, it tends to a finite limit. Namely one can conclude (by taking the limit \( n \to \infty \)) that if \( \left( H - H_{c2} \right)/H_{c2} \ln^{-1} \left[ H_{c2}/(H - H_{c2}) \right] \approx H_{c2} \), each term in the perturbation series is small.

Let us mention that keeping any finite number of terms in the perturbation series does not make any qualitative changes to the shape of the renormalized \( H_{c2}(T) \)-curve compared to the first correction and to the mean-field results (see Fig. 3). However, increasing the number of terms in the series leads to a further decrease of the upper critical field. This gives a very tentative indication that superconducting fluctuations in a two-dimensional system can significantly change the upper critical field but, probably, should not yield an upward curvature.

\textbf{IV. CONCLUSION}

We have considered the effect of superconducting fluctuations on the upper critical field of a two-dimensional disordered superconductor at low temperatures. We have performed an exact calculation of the first perturbation correction and found the corresponding shift of the critical line. We have also estimated higher order corrections. The renormalized upper critical field was shown to be shifted toward lower fields compared to the classical mean field curve. However, no upward curvature was found in the framework of any finite-order perturbation theory expansion.

We have carefully studied higher order contributions and found that each consecutive order in the perturbation theory enhances the singularity. We formulated a general rule of constructing and calculating the dominant contributions in the perturbation series. One of our main conclusions is that the derivation of the Ginzburg-Landau functional in the case of strong fields is hardly possible in the framework of the conventional perturbation theory technique.

Indeed, our results and conclusions are directly applicable for the case of low temperatures and strong fields only. However, we feel that the issue of the perturbation theory applicability and convergence may be essential in the case of weak or even zero field as well. Certainly, in this case calculations are technically absolutely different from the case of strong fields, since one has to evaluate integrals over momenta instead of tracing over the Lan-
dau level indices. However, simple estimates show that reducible graphs bring up more singular contributions compared to the irreducible ones just like in the low-temperature case. Moreover, another problem of treating short-wavelength singularities comes out.14

Taking into account these considerations we conclude that the problem of the renormalization of Ginzburg-Landau coefficients by superconducting fluctuations and, therefore, the renormalization of the transition point itself, are essentially non-perturbative problems in a two-dimensional case. This makes the Ginzburg-Landau approach ill-justified. By saying this, we certainly do not challenge the commonly accepted and well-tested phenomenological form of the Ginzburg-Landau functional. However, it is possible that the coefficients in the expansion are quite different from the conventional mean-field prediction due to the non-perturbative renormalization by superconducting fluctuations.

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