Is There a Universal Mass Function?

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ABSTRACT. Following an old idea of F. Zwicky, we make an attempt to establish a “universal” mass function for astronomical objects on all scales. The object classes considered are: solar system planets and small bodies, exoplanets, brown dwarfs, stars and stellar remnants, open and globular clusters, molecular clouds, galaxies, and groups and clusters of galaxies. For comparison, we also include CDM (cold dark matter) halos taken from numerical simulations. We show that the mass functions of individual object classes, when properly normalized, can indeed be concatenated to build a surprisingly continuous mass function of the universe, from \( M \approx 10^{-20} M_\odot \) (subkilometer size asteroids) up to \( M \approx 10^{16} M_\odot \) (rich clusters of galaxies), covering 36 orders of magnitude in mass. Most individual mass functions roughly follow a power law of the form \( \phi(M) \propto M^{-2} \). A notable exception are planets and small bodies, which seem to obey a flatter distribution. CDM halos from high-resolution numerical simulations show a very similar relation, again of “universal slope” of \( -2 \), from clusters of galaxies all the way down to the planetary mass scale. On the scale of stars and star clusters, this is a remarkable coincidence, as the formation processes involved are thought to be totally different (bottom-up gravitational clustering of DM halos vs. top-down gravoturbulent fragmentation of gas clouds).

1. INTRODUCTION

There is a great variety of astronomical objects in the universe; by increasing average size or mass: asteroids, planets, brown dwarfs, stars and stellar remnants, binary and multiple stars, star clusters, molecular clouds, galaxies, and groups and clusters of galaxies. Each of these classes of objects follows a certain (but generally not well known) distribution function in size, luminosity, or mass. The most fundamental characteristic property of all these distribution functions is that smaller or less massive things of a given kind are more abundant than larger or more massive ones. Presumably, the same would hold not only within a certain class of objects, but also if one were to glue together the individual distribution functions to build a “universal” distribution function, as there would be, per unit volume of space, more asteroids than planets, more planets than stars, more stars than galaxies, etc.

In this paper, we are doing just that: putting together all known distribution functions for different object classes to see whether there is such a thing as a “universal distribution function.” Universality has a twofold meaning here: we can ask (1) whether the all-object distribution function is sufficiently continuous to render the notion of universality useful in the first place, and (2) if yes, whether the slope of the individual distribution functions and that of the grand total is, to a certain degree, universally the same. For the present study, we choose mass as the independent parameter, as mass is probably the most fundamental—albeit difficult to determine—property of an astronomical object (luminosity is inappropriate because there are dark objects). We show that by proper normalization, we can indeed establish a continuous mass function of the universe from \( 10^{-20} M_\odot \) (100 m size asteroids) to at least \( 10^{16} M_\odot \) (the most massive clusters of galaxies), and that there is surprising similarity in the slope of the mass function over a large range of mass.

The question of a universal distribution function of astronomical objects has apparently been of interest only to F. Zwicky. In his 1942 Physical Review paper entitled “On the Large Scale Distribution of Matter in the Universe,” Zwicky argued on purely theoretical grounds that the luminosity function (LF) of galaxies had to be essentially exponential, i.e., ever fainter galaxies being increasingly more numerous, just as observed today (see below)—but in contradiction to the data available then, which showed a more Gaussian-type LF for galaxies. Zwicky’s vision was that galaxies constitute an ensemble of “particles” in a stationary state of statistical equilibrium. Frequent close encounters would dissolve larger aggregates of galaxies on the one hand and accumulate new ones on the other, resulting in a Boltzmann-type energy distribution for galaxies and groups and clusters of galaxies as a whole. Today, we know that the temperature and density of the “galaxy gas” is much too low, and thus the timescale of galaxy interactions much too long, for a statistical equilibrium to be established or maintained. But Zwicky was perfectly right not only in his prediction of an exponential LF (which he himself set out to prove; see Zwicky 1957, p. 220ff), but also in his view that galaxian systems, from dwarf galaxies up to the richest clusters of galaxies, follow one continuous distribution function.
function (which was observationally established for the first time by Bahcall 1979; see also below). This unity of objects on the scale of galaxies and systems of galaxies is now understood in terms of a general cosmological “bottom-up scenario” centered around the hierarchical gravitational clustering of dark halos in an expanding universe having formed from primordial density fluctuations, as described, e.g., by the Press-Schechter formalism (Press & Schechter 1974).

While Zwicky also considered stars in his 1942 paper, he was well aware that the mass distribution of stars must be shaped by totally different physical processes. Put in simple terms, the relation between stars and (small) galaxies is plausibly one of gas fragmentation (from large to small), rather than clustering (from small to large). So why should there be a universal mass function if there can hardly be universal (global) physics behind the frequency distribution of such things as asteroids, stars, and galaxy clusters? This must be the reason why the question of a universal mass distribution (beyond the scale of galaxies and systems of galaxies) has not been taken up since Zwicky (1942); it simply does not seem to make much sense from a physical point of view.

Conversely, the (surprising) degree of universality in the mass distribution of astronomical objects found in the present paper does not entitle us to conclude that there indeed is overarching physics at work, although this cannot be excluded either. While in the discussion section we briefly indulge in speculations on what it all could mean, we emphasize that our primary goal is not to find a sort of cosmological principle that governs the mass distribution function of the universe, but to provide a valuable piece of cosmography. A good knowledge of the frequency distribution of things in the universe is an end in itself.

There are conceptual difficulties with the definition of an astronomical object. What exactly is an astronomical object; i.e., what kinds of objects should we include in such a study? The most simple requirement would be that such an object is gravitationally bound. However, subkilometer asteroids (to say nothing of dust particles, if we were to push the distribution function to the extremes) are held together by nongravitational forces. In addition, on large scales, where gravity does dominate, we wish, for the sake of continuity, to include molecular clouds, which are in general unbound but are identifiable entities nonetheless.

Each class of objects has its own specific technical difficulties in establishing a distribution function. For example, there is a rich history of attempts to determine the stellar LF (and thus the initial mass function, IMF) or the LF and mass-to-light ratio (M/L) of galaxies. The present paper is not meant to be a metastudy in which all these attempts for the different object classes are critically assessed. Rather, we take “reasonable” distribution functions from the recent literature, with no consideration of the uncertainties, and put them together to construct a first-guess universal mass function. These distribution functions will partially overlap in mass and in some cases be separated by gaps in mass regions where we lack objects or data.

Another principal difficulty of such a study is the normalization. Everything has to be normalized to the same volume of space, but our objects are sampled in vastly different volumes: while clusters of galaxies are seen over a significant part of the observable universe, and the galaxian LF, although sampled in a fairly local volume, can still be regarded as universal, the stellar LF is simply that of the solar neighborhood, and the mass function of planets and small bodies is entirely drawn from our solar system (never mind the exponentially growing number of [nearby] exoplanets). So the smaller the scale, the larger and more uncertain the extrapolation to a universal mean. This kind of uncertainty will not soon go away. Rather, we have to rely, as all of cosmology does, on the cosmological principle; i.e., that we are living in a typical, average environment—all on all scales. All our mass functions will in the end be normalized to a volume of 1 Mpc$^3$ and given in units of solar masses ($\langle \delta(M) \rangle = M_{\odot}^{-1}$).

The paper is organized as follows. In § 2, we present the individual mass functions for the different object classes, starting with galaxies and going up first to supergalactic structures, then down to subgalactic structures. These functions are put together into a universal mass function in § 3, where the mass distribution of dark matter halos from theory is added for comparison. Finally, in § 4, we discuss possible meanings of the universality found and give our conclusions. A Hubble constant of 70 km s$^{-1}$ Mpc$^{-1}$ is used throughout.

### 2. OBSERVED MASS FUNCTIONS FOR DIFFERENT CLASSES OF OBJECTS

#### 2.1. Galaxies

While the galaxian LF is fairly well known, except at the very faintest end, the mass function (MF) of galaxies is very hard to determine, as galaxies are dominated by nonbaryonic dark matter (DM) of a principally unknown nature. The galaxian MF is reconstructed from individual or statistical mass-to-light ratio determinations (or rather estimates) that have to be convolved with the luminosity distribution, which is usually taken to be a Schechter (1976) function, of the form

$$\phi(L) \, dL = \phi_0(L/L_*)^b \exp(-L/L_*) \, dL. \quad (1)$$

Vale & Ostriker (2004) give the following mass-to-light function for galaxies:

$$L(M) = A \frac{(M/M')^c}{[c + (M/M')^d]^{1/a}}, \quad (2)$$

where the best-fitting parameters $A = 5.7 \times 10^9$, $M' = 10^{11}$, $b = 4$, $c = 0.57$, $d = 3.72$, and $k = 0.23$. At the low-mass end, this gives $L \propto M^2$; at the high-mass end, $L \propto M^{3.72}$. A total
galaxy MF is then obtained through

$$\phi(M) = \frac{dL(M)}{dM} \phi[L(M)]$$  \hspace{1cm} (3)$$

by inserting equation (2) and the Schechter function in equation (1), with parameters taken from the 2dF (Two Degree Field) galaxy LF (Norberg et al. 2002), as used also by Vale & Ostriker (2004): $\phi^* = 5.52 \times 10^{-3} \text{ Mpc}^{-3} \text{ L}_\odot^{-1}$, $L^* = 2.31 \times 10^{10} \text{ L}_\odot$ (in the $B$ band), and $\alpha = -1.21$. However, we use this Vale & Ostriker galaxy MF only for galaxy masses smaller than $10^{11} \text{ M}_\odot$, as the high-mass end (with its shallow slope; see above) would produce too many cD-type galaxies (as Vale & Ostriker note, the cD halos that went into their mass-to-light function are hosting the whole cluster, rather than the cD alone). A more appropriate MF at the high-mass end could be provided by van den Bosch et al. (2005), who work with the statistical distribution of satellite galaxies. Their mass-to-light function is given by

$$L(M) = \frac{2M}{(M/L)_{10}[(M/M_1)^{\gamma_1} + (M/M_2)^{\gamma_2}]^{1/2}},$$  \hspace{1cm} (4)$$

where the best-fitting parameters $(M/L)_{10} = 115$, $\log M_1 = 10.76$, $\log M_2 = 12.15$, $\gamma_1 = 2.92$, and $\gamma_2 = 0.29$ (for a $\Lambda$CDM clustering normalization of $\sigma_8 = 0.9$). This is somewhat steeper at the high-mass end ($L \propto M^{0.7}$ vs. $M^{0.3}$), giving a total MF that is falling down more rapidly now with increasing mass, while it is roughly compatible with Vale & Ostriker (2004) at the low-mass end. We therefore use the resulting van den Bosch et al. MF for galaxies with masses larger than $10^{11} \text{ M}_\odot$, keeping, however, the Vale & Ostriker MF for smaller masses, with continuity at $M = 10^{11} \text{ M}_\odot$. This total (DM-dominated) galaxy MF, as a combination of the two functions for larger and smaller masses as just described, is shown in Figure 1. The universal normalization (per Mpc$^3$) is of course provided by the adopted normalized LF.

For comparison, we also show the baryonic MF of galaxies in Figure 1 (stars, stellar remnants, and atomic and molecular gas), taken from Read & Trentham (2005). It is based on a host of near-infrared broadband as well as radio observations and is given in the form of a Schechter mass function

$$\phi(M) dM = \phi^* (M/M^*)^{\alpha} \exp(-M/M^*) dM,$$  \hspace{1cm} (5)$$

where the parameters $\phi^* = 2.5 \times 10^{-14} \text{ Mpc}^{-3} \text{ M}_\odot^{-1}$, $M^* = 1.31 \times 10^{11} \text{ M}_\odot$, and $\alpha = -1.21$. The baryonic galaxy mass function is indeed dominated by stars; the Schechter $\alpha$ is the same (see Read & Trentham 2005 for a detailed account of the contribution by the gas). We show the galaxy MFs down to a mass of $10^8 \text{ M}_\odot$. This is of course a bold extrapolation; the faint end of the LF, let alone the MF of galaxies, is not well constrained.

2.2. Groups and Clusters of Galaxies

The mass of groups and clusters of galaxies is dominated by DM as well and has to be estimated either from the kinematics of member galaxies or (for clusters only) from the X-ray intracluster gas properties or the gravitational lensing of background galaxies. There are several studies of group and cluster MF in the literature (e.g., Bahcall & Cen 1993; Biviano et al. 1993; Girardi et al. 1998). For our purposes, we take the MF of Girardi et al. (1998), which is based on optical virial mass estimates of a complete sample of 152 nearby clusters. It was derived only for masses larger than a few $10^{14} \text{ M}_\odot$, but we extrapolate it down to $10^{12} \text{ M}_\odot$ because it is compatible with the MF of Bahcall & Cen (1993), which also covers the low-mass end by including small groups. The adopted MF for groups and clusters, shown in Figure 2, is a Schechter-type MF of the form in equation (5), but fixing $\alpha = -1$ (flat end), with $\phi^* = 6.2 \times 10^{-20} \text{ Mpc}^{-3} \text{ M}_\odot^{-1}$ and $M^* = 3.7 \times 10^{14} \text{ M}_\odot$.

2.3. Star Clusters and Molecular Clouds

Next, we consider subgalactic structures: star clusters and molecular clouds, which can be as massive as extreme dwarf galaxies but in contrast do not contain dark matter.

2.3.1. Globular Clusters

The observed LF of globular clusters is typically bell-shaped (Gaussian). However, the decline toward fainter luminosities (masses) is probably an effect of dynamical evolution (dissolution of smaller clusters in the galactic tidal field) and may
not reflect the initial distribution (for a different view, however, see Parmentier & Gilmore 2007). As suggested by Surdin (1979), Racine (1980), and Harris & Pudritz (1994), the initial mass distribution of globular clusters can be modeled by a power law, of the form

$$\frac{dN}{H_{100}^2 b} \propto M,$$  \hspace{1cm} (6)

or in our MF notation,

$$\phi(M) \, dM = A M^{-b} \, dM.$$  \hspace{1cm} (7)

Such a distribution is indeed observed for young clusters in merging galaxies (e.g., Whitmore & Schweizer 1995), and the high-mass tails of the old globular cluster systems of nearby galaxies are still following this form very closely. For globulars with masses larger than $10^5 \, M_\odot$, Harris & Pudritz (1994) find a mean best-fitting $b = 1.7$, which we adopt for our globular cluster MF in the range $10^5$–$10^7 \, M_\odot$. For lower masses, we consider only open clusters (see below).

How can this MF now be normalized to a universal density? This is done with the “specific frequency”

$$S_N = 8.55 \times 10^7 \frac{N_f}{L_V/L_\odot},$$  \hspace{1cm} (8)

which is the number of globular clusters per “piece” of mother galaxy of absolute $V$ magnitude $-15$. The term $N_f$ is the estimated total number of globular clusters, and $S_N$ depends on the morphological type of the mother galaxy, but a reasonable mean value is $S_N = 3$ (see Harris 2001), which we adopt. With a universal $V$ luminosity density $\rho_V$, we have for the mean number of globular clusters per Mpc$^3$

$$A_{gc} \int_{10^5 M_\odot}^{10^7 M_\odot} M^{-1.7} \, dM = \frac{S_N (\rho_V/L_\odot)}{8.55 \times 10^3}.$$  \hspace{1cm} (9)

Norberg et al. (2002) give a $B$-band $\rho_L = 1.27 \times 10^8 \, L_\odot$ Mpc$^{-3}$. As the mean $B-V$ color of galaxies will not be far from the solar value of 0.65, we assume the same $\rho_V$ value in $V$. The resulting normalization constant from equation (9) is then $A_{gc} = 1.03 \times 10^4$ Mpc$^{-3} \, M_\odot^{-1}$.

### 2.3.2. Open Clusters

As shown, e.g., by Elmegreen & Efremov (1997) from their study of hundreds of open clusters in the LMC, the LF of open clusters systematically depends on age. Using the same power law for the LF,

$$\phi(L) \, dL \propto L^{-b} \, dL,$$  \hspace{1cm} (10)

$\beta$ is smaller (i.e., the LF is more flattened) the younger the cluster. Very young clusters (age of less than $10^7$ yr) have essentially a flat distribution ($\beta \approx 1$); somewhat older clusters up to an age of $10^8$ yr show $\beta \approx 1.5$; and for clusters older than $10^8$ yr, we finally have $\beta \approx 2$. The flat LFs found with young clusters is obviously due to the presence (and dominance, in terms of light) of short-lived OB stars. Clearly, then, for the mass distribution (eq. [7]) of open clusters, $\beta = 2$ is a reasonable choice. A normalization (fixing constant $A$ in eq. [7] for open clusters) is achieved by comparing the populations of open and globular clusters in the Milky Way galaxy. For open clusters, we rely on van den Bergh & Lafontaine (1984), who, aside from finding $\beta \approx 2$ as well, give a mean surface density of $N_o \approx 30$ kpc$^{-2}$ for open clusters in the solar neighborhood (see Piskunov et al. 2006 and references therein for more recent open cluster population studies). This is for clusters of absolute $V$ magnitude between $-2$ and $-10$, roughly corresponding to a mass range of $10^3$ to $10^7 \, M_\odot$. If we assume a more-or-less constant cluster density in the Galactic disk with a 10 kpc radius, giving a total disk surface of $\approx 300$ kpc$^2$, we expect a total number of approximately $30 \times 300 = 9000$ Galactic open clusters. As there are about 150 globular clusters in the Galaxy, we have

$$\frac{A_{gc}}{A_{gc}} \int_{10^5 M_\odot}^{10^7 M_\odot} M^{-1.7} \, dM = 9000 \times 150.$$  \hspace{1cm} (11)

With $A_{gc}$ from above, this gives $A_o = 1.89 \times 10^5$ Mpc$^{-3} \, M_\odot^{-1}$. 

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2.3.3. Molecular Clouds

Although molecular clouds are not gravitationally bound and are not even in a state of long-term equilibrium, being formed and dispersed on a timescale shorter than the dynamical timescale of the Galaxy, they are objects in the sense that one can measure their sizes and masses and thus determine a distribution function. Molecular clouds exhibit a fractal structure; i.e., smaller (and denser) clouds are embedded in larger ones. The cumulative size distribution for substructures inside a fractal is given by

$$N(S) \propto S^{-D},$$

or in differential form,

$$n(S) dS \propto S^{-(D+1)} dS,$$

where $D$ is the fractal dimension (Mandelbrot 1982; see Elmegreen & Falgarone 1996, whose formalism we follow here). On the other hand, there is a size-mass relation that can be assumed to be a power law again:

$$M \propto S^\kappa.$$  

The size distribution can now be transformed into a mass distribution:

$$n(M) dM = n(S) \frac{dS}{dM} M \propto M^{-(D+1)} dM.$$

Observations show that $\kappa \approx D \approx 2.3$ (see Elmegreen & Falgarone 1996). This is consistent with another definition of the fractal dimension: $M \propto S^D$ (Mandelbrot 1982), suggesting that the size and mass distribution of molecular clouds is, in fact, the result of fractal gas structure. The MF for molecular clouds can therefore be written again as

$$\phi(M) dM = A_{mc} M^{-2} dM.$$  

To fix the normalization constant $A_{mc}$, we assume that the mean density of molecular hydrogen in the universe is entirely made up of clouds with masses between $10^{-4}$ and $10^7 M_\odot$ (see Elmegreen & Falgarone 1996 for the mass range):

$$\Omega_{HI} \rho_{crit} = A_{mc} \int_{10^{-4} M_\odot}^{10^7 M_\odot} \phi(M) M dM = A_{mc} \int_{10^{-4} M_\odot}^{10^7 M_\odot} M^{-1} dM.$$

With $\rho_{crit} = 1.36 \times 10^{11} M_\odot \text{Mpc}^{-3}$ and $\Omega_{HI} = 0.00026$ from Read & Trentham (2005), we get $A_{mc} = 1.92 \times 10^6 \text{Mpc}^{-3} M_\odot^{-1}$.

The MFs we have derived so far are plotted together in Figure 3. We note, as has been noted before, that the MFs of molecular clouds and star clusters (and in fact of stars as well; see below) have very similar power laws. Such would be a natural outcome if stars and star clusters form(ed) by the fragmentation of turbulent cloud cores (e.g., Elmegreen & Efremov 1997).

2.4. Stars and Stellar Remnants

In the spectrum of cosmic objects, we would next have to deal with binary and (the much less frequent) multiple stars before coming to single stars. However, we are still lacking reliable binary statistics. A very rough estimate of the MF of binary stars is as follows. Consider the extreme case that every star is in a system of two stars of equal mass. Then the mass function of binaries per unit volume would obviously be lowered in amplitude by at least a factor of 2 compared to the MF of single stars, while otherwise being of similar shape and covering essentially the same mass range. This shows that in our first-guess universal mass function, binary and multiple star systems can be neglected.

In the following, we determine the MF of a universally averaged, evolved, present-day population of single stars and stellar remnants. Such a distribution function has never been dis-
discussed in the literature, precisely because it is interesting only in the present (new) context.

2.4.1. IMF and Final-to-Initial Mass Ratio

The mass function of single stars at their birth (i.e., the initial mass function, IMF) is an important datum for all of astronomy and is, of course, the backbone of our treatment. The original form given by Salpeter (1955),

\[ \phi(M) \, dM \propto M^{-2.35} \, dM, \quad (18) \]

is still in use for stars of higher mass. For low-mass stars, various modifications of the Salpeter function have been suggested (e.g., Kroupa 2001), but there is agreement that well below one solar mass, the power-law index is around \(-1.8\), rather than \(-2.35\). For our purposes, we adopt an index of \(-1.8\) for masses smaller than \(0.5 \, M_\odot\) (down to \(M = 0.1 \, M_\odot\)), and the Salpeter index for stellar masses above that limit (up to \(M = 120 \, M_\odot\)).

A second, even less well-known ingredient for our calculation is the relation between the initial mass of a star and the final mass of its remnant (white dwarf, neutron star, black hole). Stars with initial masses smaller than about \(8 \, M_\odot\) end up as white dwarfs; stars with initial masses above this critical limit end up as neutron stars or black holes. For the initial-mass range of \(1\) to \(7 \, M_\odot\) and solar metallicity (which we assume here for simplicity), we adopt the final white dwarf masses given by Weidemann (2000, Table 2.1); they range from \(0.55 \, M_\odot\) for a \(1 \, M_\odot\) star initially to \(1.02 \, M_\odot\) for a \(7 \, M_\odot\) star initially. For subsolar masses, where little is known about the initial-to-final mass ratio, we assume a simple linear relation from \((M_\text{if}, M_\text{f}) = (0.1, 0.1)\) to \((1, 0.55)\). The exact relation is nearly irrelevant for our purposes, because these stars do not evolve in a Hubble time anyway.

For high-mass stars (\(M > 8 \, M_\odot\)), the situation is very complex, as the evolution and therefore also the mass loss of these stars strongly depends on metallicity and other parameters, such as rotation, etc. (Woosley et al. 2002; Heger et al. 2003). We have constructed an initial-to-final mass function for these stars, drawing on the information contained in the papers just mentioned, by extending the low-mass relation continuously with a linear relation from \((M_\text{if}, M_\text{f}) = (7, 1.02)\) up to \((15, 1.4)\), and then again linearly from \((15, 1.4)\) all the way up to \((120, 25)\). This is a very rough approximation (for lack of better knowledge), and we assume solar metallicity (metal-poor stars would suffer less mass loss, resulting in higher-mass remnants). The adopted initial-to-final mass function is shown in Figure 4.

2.4.2. Stellar Lifetimes and Star Formation Histories

Two more ingredients for our calculation of the MF of stars and stellar remnants for an evolved stellar system are needed: the lifetime of a star as a function of its initial mass, and the star formation rate (SFR) as a function of time (i.e., the star formation history, SFH). The stellar lifetime (which is essentially the time spent on the main sequence) is also a function of metallicity; for simplicity, we again assume solar metallicity. A table with the stellar lifetimes for solar-metallicity stars with initial masses between \(0.8\) and \(120 \, M_\odot\) is given, e.g., in Sparke & Gallagher (2000); for stars with masses below \(0.8 \, M_\odot\), the lifetime is larger than a Hubble time. We have fitted a second-degree polynomial to the tabulated pairs \([\log (\text{stellar mass}), \log (\text{lifetime})]\) to get the estimated lifetimes for stars of any (nontabulated) mass.

Three types of star formation histories have been used as input: (1) an initial burst of star formation (i.e., all stars are born at the same instant), approximately applying to elliptical and S0 galaxies; (2) a constant star formation rate, simulating the case of spiral and irregular galaxies, and (3) a “cosmic” star formation history. The latter is inferred from high-redshift observations and is usually given as star formation rate per unit volume as a function of redshift. We have adopted the log (SFR)-\(z\) relation of Madau et al. (1998, Fig. 3) and turned it into a log (SFR)-\(t\) relation with the help of a “Cosmology Calculator,”\(^1\) using \(H_0 = 71\) km s\(^{-1}\) Mpc\(^{-1}\), \(\Omega_m = 0.270\), and \(\Omega_{\text{vac}} = 0.730\). The result is shown in Figure 5.

2.4.3. Mass Function of an Evolved Stellar Population

The present-day MF of stars and stellar remnants for an assumed star formation history is now calculated in the fol-

\(^1\) See Cosmology Calculator I at http://nedwww.ipac.caltech.edu/index.html.
Fig. 5.—Cosmic star formation history: the average SFR (the mass of stars born per unit volume per unit time) of the universe as a function of time (SFR in arbitrary units).

Fig. 6.—Present-day mass functions of stars and stellar remnants for a simple (coeval) stellar population of age 12 Gyr ("initial burst," dashed line), and a stellar population with a constant star formation rate (dotted line). The IMF (solid line) is shown for comparison. The staircase effect is a result of the binning (see text). The normalization is arbitrary.

The whole mass range from 0.1 to 120 $M_\odot$ is divided into 0.1 $M_\odot$ wide bins, giving 1200 mass points. The evolutionary time span considered is 12 Gyr; i.e., we assume that the first stars were created around 12 Gyr ago, in accordance with the typical age of Milky Way globular clusters. Dividing this time span into intervals of 0.1 Myr gives 12,000 time steps. Consider now that we start with a star formation event. The original mass function $\phi(M)$ is then simply the adopted IMF. After every time step, we compare the time elapsed since the birth event with the lifetime of a star of a given mass. Stars with lifetimes smaller than the population age are instantly lost; i.e., they are taken away from the mass function at their initial mass and added at a different place of the mass function at their much smaller remnant (final) mass. In this way, stars of subsequently smaller masses are lost from the population, leading to the well-known “burning down” of the IMF from high to low masses. Noncoeval stellar populations (constant SFR or cosmic SFH) can easily be calculated as a superposition of many little bursts distributed over the whole time span (one burst for a given population fraction at every time step).

The resulting present-day MFs of stars and stellar remnants for the three different star formation histories considered are shown, along with the IMF, in Figures 6 and 7. The MFs are normalized to the same integrated initial stellar mass as contained in the IMF; the normalization of the IMF itself is arbitrary. The MF would in principle be unchanged (i.e., remain
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identical to the IMF) for masses somewhat smaller than $1 \, M_\odot$. However, we note a small spike on top of the IMF around $\log M = -0.2$, or $M \approx 0.6 \, M_\odot$. This is due to the “graveyard” of white dwarfs with initial masses of $1 \, M_\odot$ or smaller. The depletion at masses larger than about 4 $M_\odot$ ($\log M > 0.6$) is independent of the star formation history—these are the black hole remnants of rapidly evolving high-mass stars (initial masses larger than $\sim 30 \, M_\odot$; see Fig. 4). In the intermediate-mass range of 1 to 4 $M_\odot$, we see the expected differences: the largest depletion of the MF at supersolar masses is found for the “initial burst” case (i.e., E and S0 galaxy stellar populations); the smallest depletion is for a constant star formation rate (spiral and irregular galaxy stellar populations); and the cosmic SFH case is lying somewhere between, as the two principal galaxy classes roughly contribute the same amount of stellar mass to the universal mean (see Read & Trentham 2005). The (artificial) cutoff at $\log M \approx 1.4$ corresponds to the maximum final black hole mass of $\sim 25 \, M_\odot$ originating from the initial-mass cutoff at 120 $M_\odot$. As the high-mass end of the MF is entirely shaped by the very poorly constrained final-to-initial mass function (Fig. 4), this part of the distribution function should not be taken at face value. Overall, from the viewpoint of a universal mass function of things, the present-day MF of stars and stellar remnants is not dramatically different from the stellar IMF; for subsolar masses it is essentially the same, while for supersolar masses it is down in amplitude by roughly 1 order of magnitude.

2.4.4. Brown Dwarfs

Brown dwarfs seem to cover a very narrow mass range, from the lower mass limit for hydrogen burning at $M = 0.08 \, M_\odot$, downward to about 0.01 $M_\odot$, where the realm of giant planets is entered (see below). The MF of brown dwarfs is much flatter than that of stars, even of low-mass stars. Luhmann et al. (2000) and Bejar et al. (2001), studying brown dwarfs in young open clusters, give $\alpha \approx 0.5 - 1$ for the usual power-law form $\phi(M) \propto M^{-\alpha}$. We adopt $\alpha = 0.8$ for objects with masses between 0.1 and 0.01 $M_\odot$ and attach this distribution continuously to the stellar one at 0.1 $M_\odot$.

2.4.5. Normalization

The normalization of the stellar MF is straightforward. Read & Trentham (2005) give a stellar contribution to the mean mass density of the universe of $\Omega_* = 0.0028$. Hence, we have

$$\Omega_* \rho_{\text{crit}} = \int_{0.1 \, M_\odot}^{25 \, M_\odot} \phi_*(M) M \, dM = A_* \int_{0.1 \, M_\odot}^{25 \, M_\odot} M^{-\alpha(M)} \, dM,$$

where $\phi_*(M)$ is the MF of stars and stellar remnants (from 0.1 to 25 $M_\odot$) as determined above for a cosmic star formation history (Fig. 7), and $x$ is the running (mass-dependent) power-law index of the same distribution. Brown dwarfs are not included; their contribution to the total mass is negligible. In fact, for the integrated MF we could also have neglected the stellar remnants. With $\rho_{\text{crit}} = 1.36 \times 10^{11} \, M_\odot \, Mpc^{-3}$, this gives a normalization constant of $A_* = 7.6 \times 10^7 \, M_\odot \, Mpc^{-3}$. The normalized MF of stars and stellar remnants and brown dwarfs appears in Figures 9 to 11.

2.5. Planets and Asteroids

In spite of the rapidly growing number of newly detected extrasolar planets, our knowledge of the mass function of planetary and small bodies must still rely entirely on the census of our solar system.

2.5.1. Solar System

But even for the solar system, we are far from having a complete census. The recent detection of several Pluto-sized objects in the Kuiper Belt means that completeness of the solar system member census can be claimed only to about a mass of $10^{-9} \, M_\odot$, or a diameter of 1000 km, which are roughly Pluto’s values. Taking data from the latest edition of Allen’s Astrophysical Quantities (Cox 2000) for planets, moons, and bright asteroids, and having turned the diameters of bright asteroids and of some “unweighted” moons into masses by assuming a reasonable mean mass density of 2 g cm$^{-3}$, we get a MF of solar system bodies as shown in Figure 8 (indicated as crosses, giving numbers of objects per unit solar mass). This distribution can be fitted by a straight line representing a rather shallow power-law index of $-0.96$.

As already mentioned, below $\approx 10^{-9} \, M_\odot$ (Pluto’s mass), this MF is bound to be incomplete. We construct a more reliable low-mass extension from faint asteroids in the following way. Ivezic et al. (2001) give a diameter distribution function for 13,000 asteroids found in the Sloan Digital Sky Survey (SDSS). The diameters had to be inferred from magnitudes by assuming an albedo. Ivezic et al. (2001) assumed an albedo of 0.14 for reddish asteroids (interpreted as mainly silicate, rocky) and 0.04 for bluish asteroids (mainly carbonaceous). The resulting diameter ($D$) distribution (see Ivezic et al., Fig. 25) can be fitted by a power law, of the form

$$N(D) \, dD \propto D^{-\gamma} \, dD,$$

where the parameter $\gamma$ is different, however, for different size regimes: for asteroids larger than 5 km, the distribution is very steep, with $\gamma = 4$; for smaller asteroids, it is a bit more gentle, with $\gamma = 2.3$. This seems to be in good agreement with other studies of the size distribution of asteroids (see, e.g., Tedesco et al. 2005; Bagatin 2005). The size distribution is easily turned
The new power-law index $H_{1100}$ higher masses (shown as a dotted line) up to $10^9$ inferred mass function of SDSS asteroids. This distribution is extrapolated to asteroids of km) up to $10^9$ inferred mass function of SDSS asteroids (i.e., more massive than $6.6 \times 10^9 \text{M}_\odot$) based on catalogs—deemed incomplete below a mass of $10^5 \text{M}_\odot$. The straight line fitted through the crosses corresponds to a power-law index of $-0.96$. The solid line at lower masses, with a knee around $10^{-16} \text{M}_\odot$ (diameter $\approx 5$ km), is the diameter-inferred mass function of SDSS asteroids. This distribution is extrapolated to higher masses (shown as a dotted line) up to $10^9 \text{M}_\odot$, where it is attached to the planetary distribution.

into a MF by again assuming a mean mass density $\rho = 2$ g cm$^{-3}$ for the asteroids. With

$$D = (\frac{6M}{\pi\rho})^{1/3},$$  \hspace{1cm} (21)

we have

$$\phi(M) \ dM = \frac{dD}{dM} N(D) \ dM = \frac{1}{M^{\gamma} \ dM},$$  \hspace{1cm} \text{(22)}$$

The new power-law index $-\gamma'$ for the MF is now $-2$ for asteroids larger than $5$ km (i.e., more massive than $6.6 \times 10^{-17} \text{M}_\odot$) and $-1.4$ for smaller (less massive) bodies. The mass range covered extends from $10^{-20} \text{M}_\odot$ (corresponding to asteroids of $D \approx 0.3$ km) up to $10^{-10} \text{M}_\odot$ ($D \approx 100$ km). This is the MF adopted for our study, shown in Figure 8. Lacking proper normalization, it is simply attached to the previously determined MF for planets, moons, and bright asteroids by extrapolating the steep, high-mass power law ($\gamma' = 2$) toward still higher masses, gluing the two distributions together at $10^{-9} \text{M}_\odot$, the assumed limit of completeness for the planetary MF. By doing this, we should more-or-less include the cataloged brighter (larger, more massive) asteroids missed by the SDSS-based work of Izević et al. (2001; see Tedesco et al. 2005). Kuiper Belt (trans-Neptunian) objects seem to follow a distribution similar to that of asteroids (e.g., Bagatin 2005, Fig. 2). By their exclusion, for lack of adequate data, we are likely underestimating the MF amplitude for small bodies, but probably not by more than a factor of 10, which is still not very relevant for our universal MF.

2.5.2. Extrasolar Planets

The exploration of extrasolar planets has only just begun. What we have learned so far is that the maximum planetary mass seems to be around 10 Jupiter masses ($M_J$), or 1/100 of a solar mass, which happens to coincide roughly with the minimum mass of brown dwarfs (see above). Most exoplanets detected so far have (by necessity, given the detection techniques) masses in the narrow range of $1M_J$ to $10M_J$ (in fact, these are minimum masses, due to the unknown inclination of the planetary orbits). So we can use the (minimum) mass distribution of the presently known exoplanets (around 200 as of late 2006) to extend the solar system MF toward $10M_J$. For this purpose, we have collected data published in the Web-based Interactive Extrasolar Planets Catalog by J. Schneider, 2 and glued the narrow mass distribution function directly to the high-mass end of the solar system MF (i.e., to Jupiter at 1/1000 of a solar mass); this is indicated by the dotted curve in Figure 9 labeled “Exoplanets.” Its exact form is irrelevant here, because it will change faster than we could publish it.

2.5.3. Normalization

The universal abundance of planetary systems is of course completely unknown. For solar-type (G-type) stars, the frequency of planetary systems is found to be in the 10% range (e.g., Quirrenbach 2006, p. 39). But the detection of extrasolar planets is still biased toward high masses and small periods, so that frequency could be significantly higher. On the other hand, the majority of stars are M dwarfs, for which planetary companions have yet to be discovered. The ubiquity of protoplanetary disks around low-mass stars seems to suggest the ubiquity of planetary systems, but we do not yet know whether this is generally true. Nor do we have a clue whether the abundance of planetary and small bodies scales with the mass of the star, or whether the shape of the planetary MF is always the same, etc. So for want of better knowledge, we make a very simple (solar-centric) assumption: we assume that on average in the universe, there is one solar system for every $10M_\odot$ (reproducing, of course, the 10% frequency for G-type stars). Since, according to Read & Trentham (2005), the mean

\footnote{See http://vo.obspm.fr/exoplanetes/encyclo/catalog.php.}
3. PUTTING IT ALL TOGETHER: A UNIVERSAL MASS FUNCTION

3.1. Mass Function of Astronomical Objects

We are now in a position to put together the MF for objects on superstellar scales (clusters and groups of galaxies, galaxies, star clusters, and molecular clouds; Fig. 3) with the MF for objects on the stellar and substellar scales (stars and stellar remnants, brown dwarfs, planets, and asteroids; Fig. 9) to get an overall “universal” MF that stretches from the most massive clusters of galaxies—the largest bound aggregates of matter in the universe, at $M \approx 10^{16} M_\odot$—all the way down to subkilometer-size bodies of the solar system, at $M \approx 10^{-20} M_\odot$ ($= 20$ million tons), thus covering 36 orders of magnitude in mass. This is shown in Figure 10, which constitutes the principal outcome of the present paper. It is gratifying that the two halves almost perfectly connect to each other around $1 M_\odot$. Remember that this normalization was achieved on the basis of the mean universal mass density carried by the stars, embodied in the galaxies on the one hand and comprised by the stars themselves on the other. Before discussing the meaning of this and the whole MF, we must add one more piece of information: the MF of dark matter halos as derived from theory and numerical simulations.

3.2. Mass Function of Dark Matter Halos from Numerical Simulations

A highly advanced theory of structure formation should one day be able to reproduce the observed universal MF. Of course, we are very far from having reached this point. However, on larger scales, the basic agent of structure formation is the dissipationless gravitational clustering of matter, a process that is fairly well understood and has been simulated, with ever increasing numerical resolution, for more than 30 years. In the context of a cold dark matter (CDM) scenario, starting with a Harrison-Zeldovich spectrum of primordial density fluctua-
tions, there is increasing fluctuation power with decreasing mass at recombination, getting asymptotically flat toward smaller masses (e.g., Longair 1998, p. 316). This means that CDM structure is forming first on small scales and is building up in a hierarchical way to ever larger scales. It is straightforward to show (e.g., Longair 1998, p. 373; see also below), by using the formalism of Press & Schechter (1974), that the flat low-mass end of the fluctuation spectrum at recombination results in a power-law MF with a slope of $-2$ at small masses; i.e.,

$$\phi(M)_{\text{DM}} \propto M^{-2} dM.$$  \hspace{0.5cm} (24)

At the high-mass end, the MF rapidly turns down to zero, giving the functional form a characteristic knee reminiscent of the Schechter luminosity function whose form was, in fact, motivated by the Press-Schechter result (Schechter 1976). The general form and low-mass slope of the DM mass function has been confirmed and strengthened by more recent semianalytic work (Sheth & Thormen 1999; see also Vale & Ostriker 2004) and by numerous numerical simulations of the hierarchical collapse and clustering of DM halos (e.g., Reed et al. 2003).

Recently, Diemand et al. (2005b) performed very high resolution numerical CDM simulations and found DM substructure right down to the theoretical limit, at $M \approx 10^{-6} M_\odot$, corresponding to the mass of Earth. The MF of these “microhalos” turns out to have the same slope of $-2$ as the MF of more massive halos, at least down to $M \approx 10^{-5} M_\odot$, where the function starts to bend down, depending on the nature of the CDM (neutralinos or axions). Moreover, in the mass range $10^{0.5} - 10^{15} M_\odot$, the normalization (absolute number of halos per solar mass and cubic Mpc) of Diemand et al.’s simulation is only a factor of 2 different from the one given by Reed et al. (2003) when the latter MF is extrapolated down by 10 to 15 orders of magnitude, with a forced slope of $-2$, to reach the mass range of Diemand et al. (2005b, see their Fig. 3).

For our study, in order to have a theoretical counterpart of the MF of galaxies and clusters of galaxies, we adopt a MF for DM halos of the form given in equation (24); i.e., a slope of $-2$ for the entire mass range of $10^{-5} - 10^{15} M_\odot$, with the normalization of the Reed et al. (2003) extrapolation given in Figure 3 of Diemand et al. (2005b). As mentioned earlier, below $10^{-5} M_\odot$, the MF is not known but will probably fall very rapidly. Above $10^{15} M_\odot$, the MF bends down in the usual “Schechter-Press” manner (see, e.g., Reed et al. 2003; Vale & Ostriker 2004); for this part of the MF, we can directly rely on the cluster data. The resulting very simple theoretical MF for DM halos is shown in Figure 11, along with all other MFs already shown in Figure 10. Obviously, there is good agreement on the scale of galaxies and clusters of galaxies. This was also to be expected, given the fact that with respect to mass, DM is the major constituent of these structures. In addition, the normalization of the simulated MF is of course not independent, but is drawn from observed quantities, such as the mean mass density of the universe and the density fluctuation amplitude encapsulated in $\sigma_8$. However, for galaxies, we note a difference: the observed slope is flatter than the simulated slope. This means that the simulations predict more substructure on galactic scales than is actually observed; i.e., most DM minihalos seem to lack a baryonic (dwarf galaxy) counterpart. This discrepancy has also manifested itself as the so-called “satellite catastrophe” (e.g., Moore et al. 1999). The relation of mini/microhalos on still smaller scales to astronomical objects is addressed below.

### 4. DISCUSSION AND CONCLUSIONS

Here we present first a recapitulation of our main findings:

1. The mass functions of individual classes of astronomical objects can be concatenated to build a nearly continuous, “universal” mass function that stretches from small asteroids all the way up to the richest clusters of galaxies, covering 36 orders of magnitude in mass.
2. Most individual MFs follow, in part or entirely, a power law with an index $\alpha \approx -2$. A notable exception is the MF of planetary and small bodies, which is significantly flatter. As a consequence, the combined MF from stars to clusters of galaxies shows a “universal” slope of $\alpha \approx -2$ as well.
3. CDM halos from numerical simulations follow a nearly identical MF, with $\alpha = -2$, holding even at the scale of stars and planets.

How should this be interpreted? What can be expected as a natural outcome of structure formation? What cannot and is therefore in need of a special explanation? Certainly the mass function(s) must somehow reflect the physical processes in...
volved in the formation of these objects. Three different regimes of astronomical objects that are known to have radically different formation histories can be distinguished:

1. **Galaxies and supergalactic structures** (from dwarf galaxies to rich clusters of galaxies).—Formed by gravitational clustering in a “bottom-up” manner. Their basic constituent is thought to be CDM, being clustered in underlying dark halos. These halos (according to the most recent simulations) have a rich substructure down to the scale of stars and planets. The standard formation scenario envisions the growth of primordial density fluctuations in CDM and the subsequent hierarchical gravitational instability and collapse of CDM halos from the smallest (planetary) scale to the largest (galaxy cluster) scale. The $M^{-2}$ mass distribution, which is roughly observed on galactic and supergalactic scales and is implied by numerical simulations to hold for all scales, is in principle well understood as a consequence of the power spectrum of primordial density fluctuations $P(k) \propto k^n$ and the “transfer function” describing the change of spectrum due to the damping of fluctuations in the prerecombination epoch, $T(k) \propto k^n$ in its most simple form. The resulting power spectrum at recombination is $P_{\text{rec}}(k) = P(k) T(k)^2 = k^{n-2}$, and the low-mass end ($k \to \infty$) of the mass function of DM halos at the present epoch becomes (as inferred, e.g., from Efstathiou 1990 and Longair 1998)

$$\phi_{\text{DM}} \propto M^{n+2} \propto k^2. \quad (25)$$

With a Harrison-Zeldovich spectrum ($n = 1$) and for adiabatic CDM fluctuations ($n' = -2$ in the limit for large $k$), this turns into the familiar $M^{-2}$ law. Clearly, the primordial spectral index is not as important as the transfer function. So the $M^{-2}$ behavior can essentially be viewed as a property of CDM. Hot dark matter (HDM), e.g., would behave very differently: all fluctuations on large scales would get completely damped out, and structure formation would (have to) turn into a top-down scenario.

2. **Stars and star clusters.**—Believed to have formed by the gravoturbulent fragmentation of molecular clouds in a “top-down” manner. It has been noted before that the mass functions of stars, star clusters, and molecular cloud cores are quite similar. For stars, we have the essentially still valid Salpeter IMF with a power-law slope of $-2.35$; star clusters have mass functions with a slope of $-2$ (see § 2.3); and molecular gas clouds show a fractal structure with fractal dimension $D \approx 2$, corresponding again to a MF slope of $-2$ (Testi & Sargent 1998; Elmegreen & Falgarone 1996). In fact, a fractal structure with a dimension between 1.5 and 2.5 is generally found for the interstellar medium, galactic star fields, and even dwarf irregular galaxies (Elmegreen & Elmegreen 2001; Parodi & Binggeli 2003). This MF universality has been interpreted as direct evidence for a universal formation mechanism for stars and star clusters in turbulent gas clouds (Elmegreen & Efremov 1997). The origin of the scale-invariant clumpy structure of the gas itself is not known; it could result from gravitational fragmentation, collisional agglomeration, or turbulence (see references in Elmegreen & Efremov 1997). Nor is it clear what physical parameters determine the slope of the MF; why it is $-2$; the physics involved is exceedingly complex (see, e.g., Jappsen et al. 2005).

In the realm of stars and star clusters, we should in principle also consider the question of lifetimes, in addition to formation processes (this point was brought to our attention by the referee of this paper). The lifetimes of stars were explicitly addressed in our calculation of the MF of an evolved stellar population. Low-mass stars can get older than a Hubble time anyway. The same is true for high-mass globular clusters. However, open clusters are known to have only a lifetime of a few 100 Myr (e.g., Piskunov et al. 2006), so one could indeed wonder why they match the extrapolated MF of stars so well (see Fig. 10). Apparently, their smaller lifetimes (as compared to the bulk of stars) are to a certain degree compensated by a higher formation efficiency. This is in accord with the basic fact that star formation efficiency is generally higher in high-density regions, which of course happen to be the loci of bound cluster formation (see Elmegreen 2007).

In any case, it is a remarkable coincidence that on the scale of stars and star clusters, the bottom-up CDM formation scenario should produce the same slope, and even similar amplitude, as the top-down star formation scenario. The underlying physical processes, even if not exactly known, are expected to be totally different. Or is it conceivable that the DM substructure in galaxies has some influence on the fragmentation of the gas, thereby imprinting on the cloud structure the same “universal” mass distribution? This is difficult to imagine but should not be excluded without further investigation. It was questioned whether microhalos could survive the tidal perturbations by other DM halos and, within the Galaxy, by stars and molecular clouds. But recent work (e.g., Goerdt et al. 2007) reaffirms that due to their high density (cuspy structure), most microhalos should have survived. If so, they might act on the gas in a manner that “conserves” the MF. Although different processes to explain the stellar MF are usually invoked (see above), minihalos (not microhalos) have already been causally connected with old stars and globular clusters, if only in terms of their distribution and kinematics, and not yet their MF (Die- 

3. **Planets, moons, and small bodies.**—Formed by the coagulation of dust particles and the subsequent accretional growth of small planetesimals (giant planets also, by the accretion of gas) in a bottom-up manner. However, there are also top-down processes on these scales. For instance, asteroids formed (and as a population continuously evolve) by the collisional fragmentation of large planetesimals. The formation processes for these objects are surely no less complex than for stars, and we cannot even try to explain the MF on the scale of planets and small bodies in detail. We also recall that the normalization of the planetary MF (their abundance per cubic
It would certainly be interesting to extend the MF of astronomical objects from asteroids further down in mass to meter-size and submeter-size bodies (which would have to include living things, such as the reader), to dust, molecules, atoms, nuclei, and finally elementary particles. Such an (even more speculative) extension was not aimed at here.

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REFERENCES

Bagatin, A. C. 2005, in IAU Symp. 229, Asteroids, Comets, Meteors, ed. D. Lazzaro et al. (Cambridge: Cambridge Univ. Press), 335

Bahcall, N. 1979, ApJ, 232, 689

Bahcall, N., & Cen, R. 1993, ApJ, 407, L49

Bejar, V. J., et al. 2001, ApJ, 556, 830

Biviano, A., Girardi, M., Giuricin, G., Mardirossian, F., & Mezzetti, M. 1993, ApJ, 411, L13

Cox, A. N., ed. 2000, Allen's Astrophysical Quantities (4th ed.; New York: Springer)

Diemand, J., Madau, P., & Moore, B. 2005a, MNRAS, 364, 367

Diemand, J., Moore, B., & Stadel, J. 2005b, Nature, 433, 389

Elmegreen, B. G. 2007, in ASP Conf. Ser. 13, Mass Loss from Stars and their Evolution of Stellar Clusters, ed. A. de Koter et al. (San Francisco: ASP), in press (astro-ph/0610679)

Elmegreen, B. G., & Elmegreen, D. M. 2001, AJ, 121, 1507

Elmegreen, B. G., & Falgarone, E. 1996, ApJ, 471, 816

Efremov, Y. N. 1979, ApL, 23, 13

Elmegreen, B. G., & Efremov, Y. N. 1997, ApJ, 480, 235

Elmegreen, B. G., & Elmegreen, D. M. 2001, AJ, 121, 1507

Elmegreen, B. G., & Falgarone, E. 1996, ApJ, 471, 816

Girardi, M., Borgani, S., Giuricin, G., Mardirossian, F., & Mezzetti, M. 1998, ApJ, 506, 45

Goerd, T., Genedin, O., Moore, B., Diemand, J., & Stadel, J. 2007, MNRAS, 375, 191

Harris, W. E. 2001, in Star Clusters, ed. L. Labhardt & B. Binggeli (Berlin: Springer), 223

Harris, W. E., & Pudrátz, R. E. 1994, ApJ, 429, 177

Heger, A., Fryer, C. L., Woosley, S. E., Langer, N., & Hartmann, D. H. 2003, ApJ, 591, 288

Ivezić, Ž., et al. 2001, ApJ, 122, 2749

Jappsen, A.-K., Klessen, R. S., Larson, R. B., Li Y., Mac Low, M.-M. 2005, A&A, 435, 611

Kroupa, P. 2001, MNRAS, 322, 231

Longair, M. S. 1998, Galaxy Formation (New York: Springer)

Luhmann, K. L., et al. 2000, ApJ, 540, 1016

Madau, P., Pozzetti, L., & Dickinson, M. 1998, ApJ, 498, 106

Mandelbrot, B. 1982, The Fractal Geometry of Nature (San Francisco: Freeman)

Mandl, P., Pozzetti, L., & Dickinson, M. 1998, ApJ, 498, 106

Moore, B., et al. 1999, ApJ, 524, L19

Norberg, P., et al. 2002, MNRAS, 336, 907

Parmentier, G., & Gilmore, G. 2007, MNRAS, 377, 352

Parodi, B. R., & Binggeli, B. 2003, A&A, 398, 501

Piskunov, A. E., Kharchenko, N. V., Röser, S., Schlabach, E., Scholz, R.-D. 2006, A&A, 445, 545

Press, W. H., & Schechter, P. 1974, ApJ, 187, 425

Quirrenbach, A. 2006, in Extrasolar Planets, ed. D. Queloz et al. (Berlin: Springer)

Racine, R. 1980, in Star Clusters, ed. J. E. Hesser (Dordrecht: Reidel), 369

Read, J. I., & Tremint, N. 2005, Phil. Trans. R. Soc. London A, 363, 2693

Reed, D., et al. 2003, MNRAS, 346, 565

Salpeter, E. E. 1955, ApJ, 121, 161

Schechter, P. 1976, ApJ, 203, 297

Sheth, R. K., & Tormen, G. 1999, MNRAS, 308, 119

Sparke, L. S., & Gallagher, J. 2000, Galaxies in the Universe (Cambridge: Cambridge Univ. Press), 10

Surdin, V. G. 1979, Soviet Astron., 23, 648

Tedesco, E. F., Cellino, A., & Zappala, V. 2005, ApJ, 129, 2869

Testi, L., & Sargent, A. I. 1998, ApJ, 508, L91

van der Waals, J. 1923, Phys. Rev., 6, 79

van der Waals, J. 1947, Phys. Rev., 69, 294

van den Bosch, F. C., Yang, X., Mo, H. J., & Norberg, P. 2005, MNRAS, 363, 2693

Whitmore, B. C., & Schweizer, F. 1995, ApJ, 429, 177

Whitmore, B. C., & Schweizer, F. 1995, ApJ, 429, 177

Widemann, V. 2000, A&A, 363, 647

Whitmore, B. C., & Schweizer, F. 1995, ApJ, 429, 177

Wolfson, B. S., Heger, A., & Weaver, T. A. 2002, Rev. Mod. Phys., 74, 1015

Zwicky, F. 1942, Phys. Rev., 61, 489

———. 1957, Morphological Astronomy (Berlin: Springer)