Dual Vector Multiplet
Coupled to Dual N=1 Supergravity in 10D

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Abstract

We couple in superspace a ‘dual’ vector multiplet \((C_{m_1\cdots m_7}, \lambda^\alpha)\) to the dual version of \(N = 1\) supergravity \((e_{m^\alpha}, \psi_m^\alpha, M_{m_1\cdots m_6}, \chi_\alpha, \Phi)\) in ten-dimensions. Our new 7-form field \(C\) has its 8-form field strength \(H\) dual to the 2-form field strength \(F\) of the conventional vector multiplet. We have found that the \(H\)-Bianchi identity must have the form \(N \wedge F\), where \(N\) is the 7-form field strength in dual supergravity. We also see why only the dual version of supergravity couples to the dual vector multiplet consistently. The potential anomaly for the dual vector multiplet can be cancelled for the particular gauge group \(U(1)^{496}\) by the Green-Schwarz mechanism. As a by-product, we also give the globally supersymmetric Abelian Dirac-Born-Infeld interactions for the dual vector multiplet for the first time.

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1. Introduction

It is common wisdom in ten-dimensions (10D) that only the vector multiplet (VM) with a real vector $A_m$ and a Majorana spinor $\lambda$ [1] have interactions consistent with supersymmetry. For example, even though the tensor field $C_{m_1 \cdots m_7}$, which is Hodge dual to $A_m$, has the same eight degrees of freedom as the latter, there have been certain longstanding problems with constructing consistent interactions. For example, non-Abelian interactions with such a high-rank tensor have been known to be problematic. Additionally, this wisdom draws support from superstring theory [2], because it is the vector field $A_m$ instead of 7-form tensor $C_{\mu_1 \cdots \mu_7}$ that couples consistently to open or heterotic strings [1][3][2].

In component formulation in 10D [1], the main technical obstruction for the ‘dual’ VM multiplet $(C_{m_1 \cdots m_7}, \lambda)$ has been the necessity of an extra symmetry for the closure of supersymmetries $[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]$, in addition to the usual global Poincaré and $U(1)$ gauge symmetries. This seems to be a persistent problem, because such an extra symmetry does not seem to be maintained in the presence of interactions. In superspace language [4], this corresponds to the breakdown of $H$-Bianchi identities (BIs). Therefore, our two conventional tools for supersymmetry, i.e., component [1] and superspace [4] formulations seem to be of limited use to solve the problem. Naturally, so far very few papers have been devoted to solve this long-standing problem.

In this paper, we give the first solution to this long-standing problem, by coupling the dual VM $(C_{m_1 \cdots m_7}, \lambda^\alpha)$ to $N = 1$ dual supergravity in 10D with the field content $(e_m^a, \psi_m^\alpha, M_{m_1 \cdots m_6}, \chi_\alpha, \Phi)$ [5][6][7][8]. We will show that the $H$-BI for the superfield strength $H = dC$ is modified by a term $N \wedge F$ composed of the superfield strength $N = dM$ in the dual supergravity multiplet [7][8] and the $F$-superfield strength of the conventional vector multiplet.

As a technical tool, we use the so-called ‘beta-function-favored-constraints’ (BFFC) for the dual $N = 1$ supergravity in 10D. This is a very special set of constraints developed

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*3) We use $m, n, \cdots$ for curved coordinates in this paper.
in ref. [9]. Originally, the BFFC for usual version of $N = 1$ supergravity in 10D [10] was
developed in [11], in order to simplify $\beta$-function computations for the Green-Schwarz
$\sigma$-model, by simplifying the constraint system. It was found [9] that the BFFC set for the
dual version of $N = 1$ supergravity [7][8] has one of the simplest coupling structures for
our dual VM in 10D.

We also study possible anomalies for the Abelian dual VM. Due to the Abelian nature,
there can only be the gravitational anomalies. Moreover, we find that the gravitational
anomalies can be cancelled for the gauge group $U(1)^{496}$ by the Green-Schwarz mechanism
[12].

In a sense, our approach here is similar to the claim about the couplings of vector
multiplet to supergravity first presented by [5]. In that paper, it was claimed that a vector
multiplet can be coupled to supergravity only via dual formulation. However, this was
pointed out to be incorrect in [6], because the duality transformation [13] can connect the
dual formulation with the usual formulation. However, the difference in our formulation
seems to be that in the case of dual vector multiplet, due to its index structure, only the
dual formulation of $N = 1$ supergravity can be coupled to dual vector multiplet. This
point will be clarified in section 4.

As a by-product, we also show that an Abelian dual VM can have supersymmetric
Dirac-Born-Infeld (DBI) interactions [14][15][16] consistent with global supersymmetry.
We give the total lagrangian in component language at the order (length)$^2$ with supersym-
metry transformations. This lagrangian is derived by duality transformation [13] from the
conventional 2-form field strength $F$ into the dual 8-form strength $H$.

This paper is organized as follows: In the next section, we first clarify the reason why
we need the dual formulation [5][7][9] instead of the usual formulation [10][11] of $N = 1$
supergravity in 10D for our purpose. In section 3, we first review the BFFC set for the
superspace formulation of dual $N = 1$ supergravity in 10D [9]. In section 4, we first
review the coupling of the dual BFFC [9] to the usual VM. We next give our results of our
dual VM coupled to the dual $N = 1$ supergravity [9]. In section 5, we show that potential
anomaly can be cancelled for the gauge group $U(1)^{496}$, by applying the Green-Schwarz
mechanism [12] for the dual formulation [7][8]. In section 6, we give the supersymmetric Dirac-Born-infeld interaction with global supersymmetry in component language. Some concluding remarks are given in section 7.

2. Necessity of Dual Supergravity

Before going into the detailed computations, we first clarify why only the dual formulation of $N = 1$ supergravity in 10D [5][7][8][9] is needed for our purpose. In other words, we clarify why we can not use the usual formulation of supergravity [10] which is much simpler with fewer indices. Such a question is motivated by the expectation that a duality transformation [13] should be always performed between the usual and dual formulations of supergravity.

This question is also historically motivated. In the paper [5], where an $N = 1$ non-Abelian VM was first coupled to supergravity in 10D, only the dual formulation was claimed to be coupled to a VM. However, a later investigation [6] showed that such a conclusion in [5] was incorrect. Ref. [6] showed that a non-Abelian VM can be coupled not only to the dual formulation of $N = 1$ supergravity [10], but also to the usual formulation [10]. The crucial reason is that a duality transformation [13] can convert the dual formulation [5] into the usual formulation [10], even in the presence of a VM.

Based on such history, it is legitimate to expect a similar situation with the coupling of a dual VM $(C_{m_1\ldots m_7}, l)$, i.e., not only the dual [5] but also the usual [10] formulation of supergravity can be used. In this section, we explain that this is not the case for coupling the dual VM.

Consider the kinetic term for the 2-form potential $B_{mn}$ in the usual formulation [10]

\footnote{We have to distinguish two different dualities here. One between the usual supergravity [10] and dual supergravity [5][7][8], and another between the VM $(A_m, \lambda)$ and dual VM $(C_{m_1\ldots m_7}, \lambda)$.}
of $N = 1$ supergravity in 10D coupled to Abelian VM:

$$\frac{-1}{12} e G_{mnr}^2 \equiv \frac{-1}{12} e \left( \frac{1}{2} \partial_{[m} B_{nr]} + \frac{1}{2} F_{[mn} A_{r]} \right)^2 . \quad (2.1)$$

Because of the ‘bare’ potential field $A$ instead of its field strength $F$ in this Chern-Simons term in (2.1), we cannot perform the duality transformation [13] from the vector $A_m$ into its dual $C_{m_1 \cdots m_7}$. Notice also that the duality transformation that matters here is not between the dual version [5] and usual version [10] of supergravity, but that between the VM and dual VM.

On the contrary, in the dual formulation [5] of supergravity, the corresponding coupling to (2.1) is put into the form

$$\varepsilon^{m_1 \cdots m_6 nrs} M_{m_1 \cdots m_6} F_{nr} F_{st} , \quad (2.2)$$

where only the field strength $F$ with no ‘bare’ potential $A$ is involved. Therefore, the duality transformation from the usual VM into dual VM is possible. In the usual version [10], however, even if we try with partial integrations etc., we cannot convert the Chern-Simons term $F \wedge A$ in (2.1) into $F \wedge F$ only with the field strength $F$ with no bare potential $A$. This obstruction indicates that the dual VM cannot be coupled to the usual formulation [10] of supergravity.

These points clarify why the dual formulation [5] is needed for our purpose. It is now clear that we use the dual formulation [5][7][8] as the ‘necessity’ for our couplings of the dual VM, but not for simple ‘curiosity’.

3. Review of BFFC for Dual $N = 1$ Supergravity

We next review the main structure of BFFC superspace constraints [9] for the $N = 1$ dual supergravity in 10D [7], before coupling our new dual VM to supergravity, . The field content of the dual $N = 1$ supergravity is [9] $(e_m^\alpha, \psi_m^\alpha, M_{m_1 \cdots m_6}, \chi_\alpha, \Phi)$.

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*5) Our metric is $(\eta_{ab}) \equiv \text{diag. } (+,-,-,\cdots,-)$. Relevantly, $\varepsilon^{01 \cdots 9 \equiv 1}, \gamma_{11} \equiv \gamma_{(0)(1)\cdots(9)}$, where the indices $a, b, \cdots = (0), (1), \cdots, (9)$ are for local Lorentz indices. We use the antisymmetrization symbol normalized as $A_{[m} B_{n]} \equiv A_m B_n - A_n B_m$ without the factor of 1/2 in front, due to certain advantage in superspace computation [4].
The supertorsion $T_{AB}{}^{C}$, the supercurvature $R_{ABc}{}^{d}$ and, in particular, the 7-form superfield strength $N_{A_1\ldots A_7}$ of $M_{A_1\ldots A_6}$ characterize the superspace formulation of the dual supergravity [7][8][9]. The usual VM has the field content $(A_m{}^I, \lambda^{\alpha I})$ with the superfield strength $F_{AB}{}^{I}$, where $I$ is the adjoint index for a non-Abelian gauge group.

Accordingly, there are corresponding $T$, $N$ and $F$-BIAs in superspace *6)

\[
\frac{1}{2} \nabla_{[A T_{BC}]^D} - \frac{1}{2} T_{[A B]}^E T_{E(C)^D} - \frac{1}{4} R_{[A B]^e}^f (\mathcal{M}_{f}^e)^{C)^D} \equiv 0 \ ,
\]

\[
\frac{1}{7!} \nabla_{[A_1 A_2 \ldots A_8]} N_{A_3 \ldots A_7} - \frac{1}{6! 2} T_{[A_1 A_2]}^D N_{B[A_3 \ldots A_7]B} \equiv 0 \ ,
\]

\[
\frac{1}{2} \nabla_{[A F_{BC}]^I} - \frac{1}{2} T_{[A B]^D}^F F_{D(C)^I} \equiv 0 \ .
\]

Our superspace BFFC for $N = 1$ dual supergravity are [9] *7)

\[
T_{\alpha \beta}{}^{c} = +i(\gamma^c)_{\alpha \beta} \ , \quad N_{\alpha \beta c_1 \ldots c_5} = +\frac{i}{2}(\gamma_{c_1 \ldots c_5})_{\alpha \beta} \ ,
\]

\[
T_{ab}{}^{\gamma} = -\frac{i}{36}(\gamma^c)_{\alpha \beta} \tilde{N}_{\gamma}{}^{[c] [3]} \ , \quad T_{ab}{}^{c} = +2 \tilde{N}_{ab}{}^{c} \ ,
\]

\[
\nabla_{\alpha} \chi_{\beta} = -\frac{i}{12\sqrt{2}}(\gamma^c)_{\alpha \beta} \tilde{N}_{\gamma}{}^{[c] [3]} - \frac{i}{\sqrt{2}}(\gamma^c)_{\alpha \beta} \nabla_{\gamma} \Phi + \frac{i}{72\sqrt{2}}(\gamma^c)_{\alpha \beta} \chi_{[3]} + i(\gamma^c)_{\alpha \beta} A_{[3]} \ ,
\]

\[
R_{\alpha \beta c}{}^{d} = +\frac{5i}{3}(\gamma^c)_{\alpha \beta} \tilde{N}_{ee}{}^{d} + \frac{i}{18}(\gamma^c)_{\alpha \beta} \tilde{N}_{[3]} \ ,
\]

\[
F_{ab}{}^{I} = +\frac{i}{\sqrt{2}}(\gamma_{b})_{\lambda}{}^{I} \alpha \ ,
\]

\[
\nabla_{\alpha} \chi_{\beta} = -\frac{i}{2\sqrt{2}}(\gamma^c)_{\alpha \beta} F_{cd}{}^{I} \ ,
\]

\[
R_{abcd} = -\frac{i}{2}(\gamma_{[c} T_{d]} b - \gamma_{b} T_{cd})_{\alpha} + \frac{i}{4}(\gamma_{bcd} e f) T_{e f} \ , \quad \nabla_{\alpha} \tilde{N}_{bcd} = -\frac{i}{4}(\gamma_{bcd} e f) T_{e f} \alpha \ ,
\]

\[
\nabla_{\alpha} F_{bc}{}^{I} = -\frac{i}{\sqrt{2}}(\gamma_{b})_{\nabla_{c}} \lambda^{I} \alpha - \frac{5i}{3\sqrt{2}}(\gamma^c)_{\alpha \beta} \tilde{N}_{bc}{}^{d} \ 
\]

\[
+ \frac{i}{6\sqrt{2}}(\gamma_{[b}{}^{[2]} \lambda^{I} \alpha) \tilde{N}_{c][2]} - \frac{i}{18\sqrt{2}}(\gamma_{bc}{}^{[3]} \lambda^{I} \alpha) \tilde{N}_{[3]} \ ,
\]

\[
\nabla_{\gamma} T_{ab}{}^{\delta} = -\frac{i}{4}(\gamma_{cd}^c)_{\gamma} \delta R_{abcd} + \frac{1}{36}(\gamma^c)_{\gamma a} \delta \nabla_{[b} \tilde{N}_{cde} \ 
\]

\[
+ \frac{1}{18}(\gamma_{[a} \gamma_{c]} e \delta \tilde{N}_{ab} \epsilon \tilde{N}_{[3]} + \frac{1}{1296}(\gamma_{[a} \gamma_{c]} \gamma_{b]} (\lambda_{[3]} \lambda^{I} \alpha) \tilde{N}_{[3]} \tilde{N}_{[3]} \gamma^c)_{\gamma} \tilde{N}_{[3]} \ ,
\]

\[
\tilde{N}_{[3]} \equiv (1/7!)(\epsilon_{[3]}^{[7]} N_{[7]} \ , \quad \chi_{[3]} \equiv i(\bar{\chi}_{[3]} \lambda) \ , \quad A_{[3]} \equiv \frac{i}{32\sqrt{2}} e^{4\Phi/3}(\lambda_{[3]} \chi^{I} \lambda^{I}) \ ,
\]

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*6) For the superspace local Lorentz indices $A \equiv (a, \alpha), B \equiv (b, \beta), \ldots$, we use the indices $a, b, \ldots = (0), (1), \ldots, (9)$ for bosonic coordinates, while $\alpha, \beta, \ldots = 1, 2, \ldots, 16$ for chiral fermionic coordinates.

*7) We use the symbols like $\gamma^{[3]}$, where generally $[n]$ stands for totally antisymmetric $n$-indices.
As usual in superspace, all other independent components at \( d \leq 1 \), such as \( F_{\alpha\beta} \) or \( T_{ab}^c \) are zero. In order to save space, we use the simplified notations, such as \( (\gamma_b \chi)^\alpha = (\gamma_b)^\alpha \chi_{\beta} \), and \( (\gamma_{bcd}^e T_{ef})^\alpha = - (\gamma_{bcd}^e)^\alpha \beta T_{ef}^\beta \), etc., while the symbol \([n]\) is used for totally antisymmetric bosonic \( n \) indices e.g., \((\gamma^{[3]}_{\alpha\beta} \tilde{N})_{[3]} = (\gamma_{abc})^\alpha \beta \tilde{N}^{abc} \). The \( \tilde{N} \)'s is the Hodge dual of \( N_{[7]} \). We use \( A_{[3]} \) in (3.2j) as an important building block for coupling supergravity to the VM, because this \( A \)-tensor will accommodate higher-order superstring corrections [9].

The superfield equations are obtained from the BIs at the engineering dimensions \( d \geq 3/2 \), as

\[
i(\nabla \chi)^\alpha - \frac{2i}{9} (\gamma^{[3]} \chi)^\alpha \tilde{N}_{[3]} - \frac{4i}{3} (\gamma^a \chi)^\alpha \nabla_a \Phi
- \frac{i}{3} (\gamma^{[3]} \nabla)^\alpha A_{[3]} - \frac{2\sqrt{2}i}{9} (\gamma^{[3]} \chi)^\alpha A_{[3]} \doteq 0 \, ,
\]

\[
i(\gamma^b T_{ab})^\gamma + \frac{2\sqrt{2}}{3} \nabla_a \chi_a + \frac{1}{2\sqrt{2}} (\gamma^a \chi)^\gamma \tilde{N}_{[3]} - \frac{5}{\sqrt{2}} (\gamma_{abc})^\gamma \tilde{N}_{abc} - \frac{8\sqrt{2}}{3} \chi_a \nabla_a \Phi
+ \frac{\sqrt{2}}{189} (\gamma_{abc})^\gamma \chi_{abc} - \frac{20}{189} (\gamma_{[3]} \chi)^\gamma A_{[3]} + \frac{4}{9} (\gamma_{abc})^\gamma A_{abc}
- \frac{5\sqrt{2}}{63} (\gamma_{[3]} \nabla)^\gamma A_{[3]} + \frac{\sqrt{2}}{3} (\gamma_{abc})^\gamma A_{abc} \doteq 0 \, ,
\]

\[
(\gamma^{ab})^\gamma T_{ab}^\delta \doteq 0 \, ,
\]

\[
R_{ab} - \frac{4}{3} \nabla_a \nabla_b \Phi + \frac{16}{9} (\nabla_a \Phi)(\nabla_b \Phi) - \frac{1}{9} \eta_{ab} \tilde{N}_{[3]}^2 + \frac{4}{3} \tilde{N}_{ab} \nabla_c \Phi
+ \frac{i}{9} (\nabla_a \gamma_{[3]} \nabla_a \chi_a) + \frac{\sqrt{2}}{3} (\tilde{N}_{[3]} \gamma_a \nabla_a \Phi) + \frac{1}{2\sqrt{2}} (\nabla_a \gamma_{[3]} T_{bc}) - \frac{5}{6\sqrt{2}} (\nabla T_{ab})
+ \frac{1}{324} \eta_{ab} \chi_{[3]} \tilde{N}_{[3]} - \frac{1}{108} \chi_{a[2]} \tilde{N}_{b[2]} - \frac{5}{108} \chi_{b[2]} \tilde{N}_{a[2]} + \frac{5i}{252\sqrt{2}} \eta_{ab} (\nabla \gamma_{[3]} \nabla) A_{[3]}
- \frac{5i}{84\sqrt{2}} (\nabla \gamma_{[3]} A_{bcd}) - \frac{5i}{12\sqrt{2}} (\nabla \gamma_{bd} \nabla A_{ac}) + \frac{5\sqrt{2}}{3} \nabla_c \nabla_a \tilde{N}_{[3]} A_{[3]}
- \frac{26\sqrt{2}}{21} \tilde{N}_{[3]} A_{bcd} + \frac{38\sqrt{2}}{21} \tilde{N}_{bd} A_{ac} - \frac{5\sqrt{2}}{557} \eta_{ab} \chi_{[3]} A_{[3]} + \frac{5\sqrt{2}}{189} \chi_a \nabla_c A_{bd} + \frac{\sqrt{2}}{27} \chi_a \nabla_{ac} \tilde{N}_{[3]} A_{[3]}
- \frac{8\sqrt{2}}{9} (\nabla \Phi) A_{ab}^c - \frac{80}{63} \eta_{ab} A_{[3]}^2 + \frac{64}{7} A_{bd}^c A_{bc} + \frac{5i}{378} (\nabla \gamma_{ab} [3] \nabla) A_{[3]} + \frac{5i}{378} \eta_{ab} (\nabla \gamma_{[3]} \nabla) A_{[3]}
- \frac{5i}{126} (\nabla \gamma_{ab} \nabla A_{bd}) - \frac{1}{18} (\nabla \gamma_{bd} \nabla) A_{ac} + \frac{i}{9} (\nabla \gamma \nabla) A_{abc} \doteq 0 \, ,
\]

\[
R_{[ab]} \doteq 0 \, , \quad R - \frac{4}{3} \tilde{N}_{[3]}^2 \doteq 0 \, ,
\]

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\[ \nabla^2 \Phi - \frac{1}{6} \tilde{N}^2 - \frac{4}{3} (\nabla \Phi)^2 \]
\[ - \frac{i}{24 \sqrt{2}} (\nabla \gamma^{[3]} \nabla) A^{[3]} + \frac{17 \sqrt{2}}{9} \tilde{N} A^{[3]} + \frac{8}{3} A^{2} - \frac{i}{18} (\nabla \gamma^{[3]} \nabla) A^{[3]} \nabla \Phi^2 - \frac{i}{2} \sqrt{2} (\nabla \gamma \nabla \gamma) A^{[3]} + 12 \sqrt{2} (\nabla \gamma \nabla \gamma) A^{[3]} = 0, \quad (3.3f) \]
\[ \nabla_{[a} \tilde{N}_{bcd]} - 3 \tilde{N}_{[abc} e_{d]} = 4 \tilde{N}_{[abc} \gamma_{d]} - \frac{1}{\sqrt{2}} (\nabla \gamma \nabla) \Phi + \frac{i}{\sqrt{2}} (\nabla \gamma \nabla) A^{[3]} \]
\[ - 12 \sqrt{2} (\nabla \gamma \nabla) A^{[3]} + 34 \sqrt{2} \tilde{N}^{[3]} \gamma_{[abc} A^{cd]} e + \frac{40 \sqrt{2}}{3} (\nabla \gamma \nabla) A^{[3]} + 48 A^{[3]} e A^{cd]} e \]
\[ + \frac{i}{3} (\nabla \gamma \nabla) A^{[3]} + \frac{i}{3} (\nabla \gamma \nabla) A^{[3]} \gamma_{abcd} \nabla \Phi \nabla \Phi^2 - \frac{i}{2} \sqrt{2} (\nabla \gamma \nabla) A^{[3]} = 0, \quad (3.3g) \]
\[ i (\nabla \gamma) - \frac{i}{6} (\gamma^{[3]} \nabla) \tilde{N}^{[3]} = 0, \quad (3.3h) \]
\[ \nabla_b F_a^{\, b I} + \frac{i}{\sqrt{2}} (\nabla \gamma \nabla) A^{[3]} = \tilde{N} \nabla_b F_a^{\, b I} - \frac{i}{2 \sqrt{2}} (\nabla \gamma \nabla) A^{[3]} = 0. \quad (3.3i) \]

We use the symbol \( \equiv \) for a field equation, and \( (\nabla \gamma^{[3]} \nabla) \equiv (\gamma^{[3]})^{\alpha \beta} \nabla_\beta \nabla_\alpha \), etc. The last two superfield equations (3.3h,i) are for the usual VM.

As in the case of the usual BFFC in [11], all the exponential factors with the dilaton \( \Phi \) have disappeared, making the whole system drastically simplified [9]. We also remind the readers that this BFFC [9] is one of the simplest constraint sets among other possible sets connected by super-Weyl rescalings [17]. This leads to drastic simplification of the couplings of the dual VM in the next section. For example, the duality relationship (3.3c) between the fields strengths of usual and dual VMs will be clear with no additional fermionic terms.

Note also that it is only the dual version of \( N = 1 \) supergravity [7][8][9] that can couple to the dual VM consistently, as the presence of the \( N \wedge F \) -term reveals.

4. Dual VM Coupled to \( N = 1 \) Supergravity

Armed with the BFFC for \( N = 1 \) dual supergravity [9], we are now ready to consider the dual VM couplings. The VM up to now can carry non-Abelian adjoint index \( i \), but in this section, we consider no additional index on the dual VM. Our dual VM has the field content \( (C_{m_1 \cdots m_7}, \lambda^\alpha) \), where \( \lambda^\alpha \) is the same gaugino as in the last section, \( i.e., \) a
Majorana-Weyl spinor in 10D with positive chirality. In superspace, the 7-form potential $C_{A_1 \ldots A_7}$ has its 8-form superfield strength $H_{A_1 \ldots A_8}$. The spatial component $H_{a_1 \ldots a_8}$ is dual to $F_{ab}$, and the 7-form potential $C_{a_1 \ldots a_7}$ has the same 8 physical degrees of freedom as $A_a$.

The most important key ingredient for the coupling of the dual VM to $N = 1$ dual supergravity is the introduction of the $N \wedge F$-term in the $H$-BI:

$$\frac{1}{8!} \nabla_{[A_1} H_{A_2 \ldots A_9]} - \frac{1}{7!} T_{[A_1 A_2]} \, H_{B\, [A_3 \ldots A_9]} + \frac{1}{7!} N_{[A_1 \ldots A_7} F_{A_8 A_9]} \equiv 0 \quad (4.1)$$

Note that the coefficient of the last term is $1/7!$ instead of the commonly expected one $1/(7! \cdot 2)$. This is due to our notation, and nothing else. It is essential to consider this $H$-BI, in addition to the other BIs in (4.1) for our whole system.

Even though the presence of the 2-form superfield strength $F$ in (4.1), when dealing with the dual VM, is bizarre at first glance, this situation is similar to the duality symmetric formulation of 11D supergravity [18]. In the latter, both the usual 4-form superfield strength $F_{ABCD}$ and its dual 7-form superfield strength $G_{A_1 \ldots A_7}$ are present at the same time. To put it differently, we can maintain the constraints for each constraints of $F_{AB}$ related to (3.2e) or (3.2f) even in the formulation of dual VM. This is nothing unusual in superspace, when we deal with dual systems [18].

The necessity of the last $N \wedge F$-term in (4.1) is understood as follows. If we did not have this term, then a problem occurs at $d = 1/2$ with the $H$-BI: $(1/2) \nabla_{(\alpha} H_{\beta\gamma)} d_1 \ldots d_6 + \cdots \equiv 0$. A $\chi$-linear term develops with no counter-term to cancel. It is exactly the combination $N_{(\alpha\beta|[d_1 \ldots d_5 F_{d_6}]|\gamma)}$ from the $N \wedge F$-term that cancels this unwanted term. A similar cancellation is observed in the $H$-BI: $\nabla_{(\alpha} H_{\beta)} c_1 \ldots c_7 + \cdots \equiv 0$ at $d = 1$. The combination $N_{\alpha\beta[c_1 \ldots c_5 F_{c_6 c_7]}$ cancels the otherwise unwanted term linear in $\tilde{H}_{c_6 c_7}$, yielding also the on-shell duality relationship $F_{ab} \equiv \tilde{H}_{ab}$ as in (4.3c) below.

We emphasize that the modification of the $H$-BI is the solution to the problem of coupling the dual VM to supergravity. In particular, the structure of the $N \wedge F$-term is similar to the case of the duality-symmetric formulation of 11D supergravity [19], or the superspace formulation of the massive Type IIA supergravity in 10D [20]. Our $H$-BI
is analogous to [19], in the sense that the superfield strength is involved in the $H$-BI. However, it is more similar to [20], in the sense that the superfield strength $N$ is multiplied by $F$.

The new constraints related to the dual VM at $d \leq 1$ are

\begin{align}
H_{\alpha b_1 \cdots c_7} &= -\frac{i}{\sqrt{2}} (\gamma_{c_1 \cdots c_7} \lambda) \alpha , \quad (4.2a) \\
\nabla_\alpha \lambda^\beta &= - \frac{1}{2\sqrt{2}} (\gamma^{cd})_\alpha^\beta \tilde{H}_{cd} . \quad (4.2b)
\end{align}

For the reasons already mentioned, and to be clarified shortly, the fact that the gaugino $\lambda$ appears both at (4.2b) and (4.2f) does not pose any problem.

The dual VM superfield equations are our new ingredients here. They can be obtained from our new $H$-BIs at $d \geq 1$:

\begin{align}
i(\nabla \lambda) - \frac{i}{6} (\gamma^{[3]} \lambda) \tilde{N}_{[3]} \hat{=} 0 , \quad (4.3a) \\
\nabla_b \tilde{H}^b_a + \frac{i}{\sqrt{2}} (\lambda \gamma^b T_{ab}) \hat{=} \nabla_b \tilde{H}^b_a - \frac{i}{2\sqrt{2}} (\lambda \gamma^{bc} T_{bc}) \hat{=} 0 , \quad (4.3b) \\
F_{ab} \hat{=} \tilde{H}_{ab} \equiv \frac{1}{8!} \epsilon_{ab} [8] H[8] . \quad (4.3c)
\end{align}

Eq. (4.3a) is exactly the same as (4.3h), but is repeated here because $\lambda$ also belongs to the dual VM. Compared with the ordinary case, the on-shell duality relationship (4.3c) comes out of $H$-BI at $d = 1$. This result is common to supergravity theories involving dualities. Note that (4.3c) is also associated with the cancellation between the $N \wedge F$-term and $\tilde{H}$-term at $d = 1$ as mentioned before.

Under (4.3c), we can replace $F_{ab}$ everywhere by $\tilde{H}_{[8]}$, and vice versa everywhere in our system. For example, (4.2f) has $F$ on the r.h.s., but now it also implies (4.2b). This tells us that the $\lambda$-field belongs both to the original VM ($A_a, \lambda^\alpha$) and the dual VM ($C_{[7]}, \lambda^\alpha$) at the same time. Another example is (4.2h) with $F_{bc}$, now implying that

\begin{align}
\nabla_\alpha \tilde{H}_{bc} \hat{=} & - \frac{i}{\sqrt{2}} (\gamma_{[b} \nabla_{c]} \lambda) \alpha - \frac{3i}{2\sqrt{2}} (\gamma^{d} \lambda) \alpha \tilde{N}_{bcd} \\
&+ \frac{i}{6\sqrt{2}} (\gamma_{[b}^{[2]} \lambda) \alpha \tilde{N}_{c][2]} - \frac{i}{18\sqrt{2}} (\gamma_{bc}^{[3]} \lambda) \alpha \tilde{N}_{[3]} . \quad (4.4)
\end{align}
Accordingly, the couplings between the dual VM and $N = 1$ dual supergravity are through the $A_{[3]}$-tensor, i.e., the bilinear form in $\lambda$ exactly as in (4.2j), except that the $\lambda$'s now carries no adjoint index $I$.

There are several consistency checks to be performed. A typical one is the $H$-BI at $d = 2$ which yields immediately (4.3b). In particular, a possible $\tilde{N}^{acd}\tilde{H}_{cd}$-term in (4.3b) is cancelled by the peculiar $N \wedge F$-term in the $H$-BI. This makes (4.3b) equivalent to (4.3i) under $\tilde{H} = F$ (4.3c). We can also confirm the vanishing of the divergence of (4.3b), after using the relevant superfield equations.

The existence of the $N \wedge F$-term in our $H$-BI implies the necessity of the Chern-Simons term in the $H$-superfield strength itself:

$$H_{A_1\ldots A_8} = \frac{1}{7!} \nabla_{[A_1 C_{A_2\ldots A_8}] - \frac{1}{6!} T_{[A_1 A_2]} B_{C_{A_3\ldots A_8}}} - \frac{1}{6!} M_{[A_1\ldots A_8} F_{A_7 A_8]} \quad .$$

This implies, in particular, that the bosonic field strength with the Chern-Simons term

$$H_{m_1\ldots m_8} = \frac{1}{7!} \theta_{[m_1 C_{m_2\ldots m_8}] - \frac{1}{6!} M_{[m_1 m_6} F_{m_7 m_8]} \quad ,$$

which is a new result.

Notice that the ‘duality relationship’ exists not only between the component $F_{ab}$ and $H_{[8]}$, but also between $F_{ab}$ and $H_{[7]}$, as can be verified by (4.2e) and (4.2a):

$$F_{ab} = \frac{1}{9!} \epsilon_{b}^{cd[7]} (\gamma_{cd})_{\alpha}^{\beta} H_{[7]} \quad .$$

This suggests the existence of the superspace duality

$$F_{AB} = \frac{1}{8!} \mathcal{E}_{AB} C_{8\ldots C_1} H_{C_1\ldots C_8} \quad ,$$

with the generalized ‘$\epsilon$-tensor’ defined by

$$\mathcal{E}_{AB}^{C_1\ldots C_8} \equiv \begin{cases} \mathcal{E}_{ab}^{c_1\ldots c_8} & , \\
\mathcal{E}_{ab}^{c_1\ldots c_7} = \frac{1}{9} \epsilon_{b}^{dec_1\ldots c_7} (\gamma_{de})_{\alpha}^{\gamma} & , \\
\mathcal{E}_{ab}^{c_1\ldots c_6} = \frac{1}{10} \epsilon_{b}^{defg} c_1\ldots c_6 (\gamma_{de})_{\alpha}^{\gamma} (\gamma_{fg})_{\beta}^{\gamma_2} & . 
\end{cases}$$

In fact, a similar generalized duality relationship in 10D was found [8] between the 3-form superfield strength $G_{ABC}$ in usual supergravity [10] and the 7-form $N_{A_1\ldots A_7}$ in dual supergravity [7][8].
Some readers may wonder, whether it makes sense to have both the usual VM superfield strength $F$ and the dual VM superfield strength $H$ simultaneously. It looks bizarre at first glance, because it seems to contradict the counting of the physical degrees of freedom. However, this puzzle can be easily solved by the following points. First, note that there is a on-shell duality relation $F = \tilde{H}$ (4.3c). Therefore, even if we temporarily double the number of degrees of freedom from 8 of the usual vector $A$ into $8+8 = 16$ for $A$ and $C$, the duality relationship $F = \tilde{H}$ reduces it to $16/2 = 8$. This situation is not new, but similar to the case of duality symmetric formulation for 11D supergravity [19]. The simultaneous relationships (4.2f) and (4.2b) are also consistent with this counting, namely, the gaugino field $\lambda$ belongs both to the original VM and the dual VM simultaneously.

We mention an alternative answer to the question of degrees of freedom, based on the superspace duality (4.8). Namely, since all the $F_{AB}$ can be formally expressed in terms of $H_{A_1 \cdots A_8}$, there is no need to introduce $F_{AB}$ itself and $F$-BIIs themselves (4.1c), either. In other words, once we replace $F_{AB}$ everywhere by $H_{A_1 \cdots A_8}$ under (4.8), we can totally forget about the existence of $F_{AB}$, and therefore, of the usual VM. With this prescription, even the $N \wedge F$-term will contain only the $H$-superfield strength:

$$
\frac{1}{8!} \nabla [A_1 H_{A_2 \cdots A_9}] - \frac{1}{7!} 2 T_{[A_1 A_2]} B H_{B|A_3 \cdots A_9} + \frac{1}{7! 8!} N_{[A_1 \cdots A_7} E_{A_8 A_9} B_{1 \cdots B_8} H_{B_1 \cdots B_8} \equiv 0. \tag{4.10}
$$

Accordingly, the field strength $F$ in the Chern-Simons term $M \wedge F$ in (4.5) is replaced by $H$ itself. This reflects nothing but the non-linear nature of the duality transformation [13] for the dual VM in component language. In fact, in the dual version of supergravity [7], the $M \wedge F$-term in the lagrangian already suggests such a non-linearity for the duality transformation $F \rightarrow H$. Relevantly, such a duality transformation is possible only in the dual version [7], but not in the usual version [10] of supergravity. This is because the VM coupling in the latter is in the Chern-Simons term $F \wedge A$ in the $G$-field strength with the ‘bare’ potential $A$ preventing a duality transformation [13]. On the other hand, in the dual version [7], the corresponding term $M \wedge F \wedge F$ is only in terms of the field strength $F$, enabling the duality transformation $F \rightarrow H$ in a non-linear fashion.
5. Anomaly Cancellation

A pressing question to ask is whether our model makes sense at the quantum level, because it lacks a more fundamental formulation such as superstring theory [2]. In fact, this question can be rather easily answered by considering the following points. First, our gauge group is Abelian with no minimal couplings with fermions. As such, the purely gauge and mixed anomalies are absent.

As for purely gravitational anomalies, notice that the anomaly 12-form \( I_{12} \) is given by [12]

\[
I_{12} = \left( \frac{n - 496}{7560} \right) \left[ \text{tr} R^6 + \frac{21}{16} \text{tr} R^2 \text{tr} R^4 + \frac{35}{64} (\text{tr} R^2)^3 \right] \\
+ (\text{tr} R^2) \left[ \frac{1}{32} (\text{tr} R^2)^2 + \frac{1}{8} \text{tr} R^4 \right].
\] (5.1)

Compared with [12], due to the absence of pure gauge and mixed anomalies, no terms containing the gauge field strength are involved. The number \( n \) is the number of the \( C \)-fields. When \( n = 496 \), the leading term \( \text{tr} R^6 \) vanishes, while all the non-leading terms factorize as

\[
I_{12} = (\text{tr} R^2) X_8,
\]

\[
X_8 \equiv \frac{1}{8} \text{tr} R^4 + \frac{1}{32} (\text{tr} R^2)^2.
\] (5.2)

The consistent anomaly corresponding to \( I_{12} \) is [21]

\[
\Gamma = \int \left[ 2 \omega_{2L} \text{tr} X_8 + 4 (\text{tr} R^2) X_6^1 \right],
\] (5.3)

where the \( \omega \)'s and \( X \)'s are defined by

\[
d \omega_{3L} = \text{tr} R^2, \quad \delta \omega_{3L} = -d \omega_{2L}^1,
\]

\[
X_8 = d X_7, \quad \delta X_7 = -d X_6^1,
\] (5.4)

under a local Lorentz transformation \( \delta \). Now consider the counter-term

\[
S_c \equiv \int \left[ -6 M (\text{tr} R^2) - 2 \omega_{3L} X_7 \right],
\] (5.5)
where $M$ is the 6-form tensor present in the dual supergravity multiplet \[7\][8]. The anomaly $\Gamma$ is cancelled by the variation

$$\delta M = X_6^1,$$

(5.6)

because

$$\delta I(N \to \tilde{N}) + \delta S_c + \Gamma = 0,$$

(5.7)

for the the original action $I$, and $\tilde{N}$ is the modified field strength of $M$:

$$\tilde{N} \equiv dM + X_7, \quad \delta \tilde{N} = 0, \quad d\tilde{N} = X_8.$$

(5.8)

Thus we need 496 copies of the dual VMs for $U(1)^{496}$, where each multiplet has couplings to supergravity. Our total lagrangian is $\mathcal{L}_{\text{total}} = \sum_{i=1}^{496} \mathcal{L}_i$, where $\mathcal{L}_i$ has exactly the same coupling structure of section 3 for the $i$-th multiplet $(C_{m_1 \cdots m_7}, \lambda^i)$. Note that these 496 dual VMs form just a direct sum, with no mutual interaction among themselves.

The anomaly freedom indicates that despite the lack of a more fundamental theory such as superstring theory [2], the dual VM still has its proper raison d’etre as a consistent interacting theory in 10D, at least, within a point field theory formulation. It is imperative that dimensional reduction of our theory to lower-dimensions will generate a series of anomaly-free theories in those dimensions.

6. Supersymmetric Dirac-Born-Infeld Interactions for dual VM

We give here our result for an invariant action for the Abelian dual VM with globally supersymmetric DBI interactions [14][22][23][16]. Our action $I_{\text{SDBI}} \equiv \int d^{10}x \mathcal{L}_{\text{SDBI}}$ is given by the lagrangian

$$\mathcal{L}_{\text{SDBI}} = -\frac{1}{2[8]}(H_{[8]})^2 - \frac{i}{2}(\bar{\lambda}\gamma^a \partial a \lambda) - \frac{1}{4}\alpha^2 (\bar{H}^4)_a^a + \frac{1}{16}\alpha^2 [(\bar{H}^2)_a^a]^2$$

$$- \frac{i}{2}\alpha^2 (\bar{H}^2)^{ab}(\bar{\lambda}\gamma_a \partial b \lambda) + \frac{i}{8}\alpha^2 \bar{H}_a^d(\partial_d \bar{H}_{bc})(\bar{\lambda}\gamma^{abc}_d \lambda) + \frac{1}{12}\alpha^2 (\bar{\lambda}\gamma a \partial b \lambda)^2.$$  

(6.1)

The multiplication of the $\bar{H}$’s is defined, e.g., by $(\bar{H}^2)_a^b \equiv \bar{H}_a^c \bar{H}_c^b$, etc. The third and fourth terms in the first line are the DBI-interactions [14] at $O(\alpha^2)$, in terms of
our dual field strength $\tilde{H}$, where the constant $\alpha$ has the engineering dimension of (length) = (mass)$^{-1}$.

Our action $I_{SDBI}$ is invariant up to $O(\alpha^3)$ under supersymmetry

$$
\delta_Q C_7 = + \frac{i}{\sqrt{2}} (\tau^\gamma)_{[7]} \lambda + \frac{1}{6\sqrt{2}} \alpha^2 \epsilon_{[7]}^{bcd} \left[ - \frac{i}{8} (\tau^\gamma_{bcd}) \tilde{H}^2 e - 3i(\tau^\gamma e) \tilde{H}_{eb} \tilde{H}_{cd} \right. \\
- \frac{3i}{2} (\tau^\gamma_{ebc}) \tilde{H}^2_{ed} + \frac{i}{16} (\tau^\gamma_{bcd} e_{fh}) \tilde{H}_{ef} \tilde{H}_{gh} \\
+ \frac{3i}{4} (\tau^\gamma_{b ef}) \tilde{H}_{ef} \tilde{H}_{cd} - \frac{3i}{2} (\tau^\gamma_{e f}) \tilde{H}_{ce} \tilde{H}_{df} \left. \right] + O(\alpha^2 \lambda^2) \,, \quad (6.2a)$$

$$
\delta_Q \lambda = - \frac{1}{2\sqrt{2}} (\gamma^{ab} \epsilon) \tilde{H}_{ab} - \frac{3}{16\sqrt{2}} \alpha^2 (\gamma^{cd} \epsilon) \tilde{H}_{cd} (\tilde{H}^2 e) - \frac{3}{4\sqrt{2}} \alpha^2 (\gamma^{ab} \epsilon) (\tilde{H}^3)_{ab} \\
+ \frac{1}{96\sqrt{2}} \alpha^2 (\gamma^{abcdef}) \tilde{H}_{ab} \tilde{H}_{cd} \tilde{H}_{ef} + O(\alpha^2 \lambda \tilde{H}) \,. \quad (6.2b)
$$

The field equations for the $C$ and $\lambda$-fields are

$$
\frac{\delta}{\delta C_{[7]}} L_{SDBI} = - \frac{1}{12} \epsilon^{[7] abc} \partial_a \left[ \tilde{H}_{bc} - 2\alpha^2 (\tilde{H}^3)_{bc} + \frac{1}{2} \alpha^2 (\tilde{H}^2)_{e} \tilde{H}_{bc} - i\alpha^2 \tilde{H}^e_{b} (\bar{\lambda}_\gamma(e) \partial_c) \lambda \right. \\
+ \frac{i}{4} \alpha^2 (\partial_b \tilde{H}^{ef}) \bar{\lambda} \gamma_{c} \epsilon^{ef} \lambda + \frac{i}{2} \alpha^2 \tilde{H}^{ef} (\bar{\lambda} \gamma_{bce} \partial_f) \lambda \left. \right] \equiv 0 \,, \quad (6.3a)$$

$$
\frac{\delta}{\delta \lambda} L_{SDBI} = - i\partial \lambda - i\alpha^2 (\gamma_a \partial_b \lambda) (\tilde{H}^2)_{ab} - \frac{i}{2} \alpha^2 (\gamma_a \lambda) \partial_b (\tilde{H}^2)_{ab} \\
+ \frac{i}{4} \alpha^2 (\gamma^{abc}) \tilde{H}_a d \partial_d \tilde{H}_{bc} + \frac{1}{3} \alpha^2 (\gamma_a \partial_b \lambda) (\bar{\lambda} \gamma^a \partial^b \lambda) \equiv 0 \,. \quad (6.3b)
$$

The leading term in the $C$-field equation is like $\partial_{[a} \tilde{H}_{bc]} + \cdots$ which is dual to the Bianchi identity $\partial_{[a} F_{bc]}$ for the original $F$-field strength. On the other hand, the original $A$-field equation $\partial_a F^{ab} + \cdots \equiv 0$ becomes now an identity $\partial_a \tilde{H}^{ab} \equiv 0$.

Our lagrangian can be obtained by a direct construction, as we can perform for the conventional VM $(A_a, \lambda)$, with no essential obstruction, despite the subtlety about the extra symmetry. The extra symmetry at the free-field level arises in the commutator $[\delta_Q (\epsilon_1), \delta_Q (\epsilon_2)] C_{a_1 \cdots a_7}$ as

$$
\delta_E C_{a_1 \cdots a_7} = \frac{1}{n} \zeta_{a_1 a_2}^{[3]} H_{a_3 \cdots a_7} [3] \,, \quad (6.4)
$$
with the totally antisymmetric constant parameter $\zeta^5$, leaving the $H$-field strength invariant up to the $C$-field equation:

$$\delta E \tilde{H}_{ab} = \frac{1}{6} \zeta_{ab}^{cde} \partial_{c[\tilde{e}} \tilde{H}_{de]} \neq 0 \ , 
$$

before interactions are switched on. In the past, it looked very difficult to maintain this extra symmetry, once interactions are switched on. However, our system seems to have overcome this problem, because the supersymmetry commutator shows that (6.5) is modified to

$$\delta E \tilde{H}_{ab} = \frac{1}{6} \zeta_{ab}^{cde} \partial_{c[\tilde{e}} \tilde{H}_{de]} - \frac{1}{6} \zeta_{ab}^{cde} \partial_{c[\tilde{e}} \tilde{H}_{de]} + \frac{1}{6} \zeta_{ab}^{cde} \partial_{c[\tilde{e}} \tilde{H}_{de]} + \frac{1}{6} \zeta_{ab}^{cde} \partial_{c[\tilde{e}} \tilde{H}_{de]} \neq 0 \ . 
$$

This vanishes on-shell thanks to (6.3a). As is easily seen, the original free-field level relation (6.5) is now modified to include the supersymmetric DBI interactions, and it still vanishes upon using the $C$-field equation.

Our lagrangian has a structure very similar to the conventional VM in [22][23]. Namely, the difference in coefficients occurs only for the $H^2 \lambda^2$-terms, but not in the purely bosonic DBI-interactions or the quartic fermion term.

In the actual derivation of our lagrangian, we have used the duality transformation [13] from the conventional field strength $F_{ab}$ into the dual one $H_3$. However, our system exhibits subtlety compared with the usual case. In the usual duality transformation [13] performed in supergravity in $D$-dimensions, we replace the original $n$-form $F$ by a new independent $n$-form field $G$, so that the kinetic term of the $F$-field strength supplies the bilinear in $G$, while the interactions are rather simple, such as the Pauli or the Noether terms always linear in $G$. The constraint term $L_C = C \wedge dG$ [13] is also linear in $G$, supplying the desirable condition $dG \neq 0$. Therefore, in the usual case, the duality transformation is easy, because the $G$-field equation is simply algebraic $G = \tilde{H} + J$ with a $(D - n)$-form $H = dC$ and an $n$-form source $J$. If we try to apply this to the conventional VM with DBI interactions, the source $J$ now contains trilinear terms in $G$ due to the DBI interactions. Therefore we can no longer solve the $G$-field equation.
$$G = \tilde{H} + \mathcal{O}(G^3)$$ algebraically for $H$, and this was the major obstruction in the past. However, in the present case, this problem does not arise, because the action for (6.1) is invariant up to $\mathcal{O}(\alpha^3)$-terms.

7. Concluding Remarks

In this paper, we have shown how to couple the dual VM $(C_{m_1 \cdots m_7}, \lambda)$ to the dual $N = 1$ supergravity [5][7][8][9] in superspace in 10D. We have found that the peculiar new term $N \wedge F$ in the $H$-BI is the key ingredient for such couplings. Accordingly, the $H$-field strength should be modified by the Chern-Simons term $M \wedge F$.

Interestingly enough, our model is also free of anomalies for the particular group $U(1)^{496}$. In our theory, no gauge or mixed anomalies arise, while the purely gravitational anomaly has a vanishing leading term for $U(1)^{496}$, and all the non-leading terms are cancelled by the variation of the 6-form tensor $M$ in the supergravity multiplet by the Green-Schwarz mechanism [12][2][7][8]. Anomaly freedom is generally independent of the existence of superstring theory, but can be formulated within point-field theory. The anomaly freedom of our theory strongly suggests the deep significance of such interactions, and may lead to a more fundamental theory of extended objects which are not necessarily superstrings.

Subsequently, we have also given the component lagrangian for the dual VM with consistent supersymmetric DBI interactions [14][22][23][16]. We presented our lagrangian including the fermionic quartic terms. We have shown the parallel structures between the conventional VM [22] and the dual VM at the lagrangian as well as the transformation levels. The traditional problem with extra symmetry in the commutator algebra in component language has now been solved by the $\mathcal{O}(\alpha^2)$-modification of the extra symmetry itself consistent with the $C$-field equation.

It is to be stressed that the presence of the $N \wedge F$-term is very crucial for the dual VM to couple to supergravity. In particular, it is the dual supergravity [7][8][9] that can consistently couple to the dual VM. This is a solution to the long-standing problem with
formulating higher-rank tensors in superspace, in particular, with extra symmetries. The necessity of such a high-rank superfield strength as $N$ in superspace is also analogous to the superspace formulation of the massive Type IIA supergravity in 10D [20].

Once we have established dual VM interactions in 10D, we expect that dimensional reductions will generate descendant anomaly-free theories in lower-dimensions $D \leq 9$. Our Abelian DBI interactions for a dual VM may have interesting applications associated with non-linear supersymmetry with Nambu-Goldstone mechanism [24].

We emphasize that our formulation here is the first one giving non-trivial interactions between the dual VM and dual supergravity [5][7][8][9] in 10D, as well as the Born-Infeld type self-interactions [14][22][23][16] of the dual VM. Even though our dual VM has only Abelian symmetry, our results must be very crucial for future studies of dual VM in 10D. In fact, the anomaly freedom of our theory also suggests the existence of a more fundamental theory of extended objects, which is not necessarily superstring theory.

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