Gravitationally Coupled Electroweak Monopole

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We present a family of gravitationally coupled electroweak monopole solutions in Einstein-Weinberg-Salam theory. Our result confirms the existence of globally regular gravitating electroweak monopole which changes to the magnetically charged black hole as the Higgs vacuum value approaches to the Planck scale. Moreover, our solutions could provide a more accurate description of the monopole stars and magnetically charged black holes.

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Ever since Dirac has proposed the Dirac monopole generalizing the Maxwell’s theory, the monopole has become an obsession theoretically and experimentally [1]. After Dirac we have had the Wu-Yang monopole [2], the ’t Hooft-Polyakov monopole [3], the grand unification (Dokos-Tomaras) monopole [4], and the electroweak (Cho-Maison) monopole [5–7]. But none of them except the electroweak monopole might become realistic enough to be discovered.

Indeed the Dirac monopole in electrodynamics should transform to the electroweak monopole after the unification of the electromagnetic and weak interactions, and Wu-Yang monopole in QCD is supposed to make the monopole condensation to confine the color. Moreover, the ’t Hooft-Polyakov monopole exists in an unphysical theory, and the grand unification monopole which could have existed at the grand unification scale probably has become completely irrelevant at present universe after the inflation.

This makes the electroweak monopole the only realistic monopole we could ever hope to detect, which has made the experimental confirmation of the electroweak monopole one of the most urgent issues in the standard model after the discovery of the Higgs particle at LHC. In fact the newest MoEDAL (“the magnificent seventh”) detector at LHC is actively searching for the monopole [8]. But to detect the electroweak monopole at LHC, we have to ask the following questions.

First, does the electroweak monopole really exist? This, of course, is the fundamental question. As we know, the Dirac monopole in electrodynamics does not have to exist, because there is no reason why the electromagnetic U(1) gauge group has to be non-trivial. So we must know if the standard model predicts the monopole or not.

Fortunately, unlike the Dirac monopole, the electroweak monopole must exist. This is because the electromagnetic U(1) in the standard model is obtained by the linear combination of the U(1) subgroup of SU(2) and the hypercharge U(1), but it is well known that the U(1) subgroup of SU(2) is non-trivial. In this case the mathematical consistency requires the electromagnetic U(1) non-trivial, so that the electroweak monopole must exist if the standard model is correct [9, 10]. But this has to be confirmed by experiment. This makes the discovery of the monopole, not the Higgs particle, the final (and topological) test of the standard model.

Second, what (if any) is the characteristic feature of the electroweak monopole which is different from the Dirac monopole? This is an important question for us to tell the monopole (when discovered) is the Dirac monopole or the electroweak monopole. The characteristic difference is the magnetic charge. The electroweak monopole has the magnetic charge twice bigger than that of the Dirac monopole.

On the other hand, the magnetic charge of the electroweak monopole becomes a multiple of 4\pi/e, because the period of the electromagnetic U(1) becomes 4\pi [5, 9, 10]. The reason is that this U(1) is given by the linear combination of the U(1) subgroup of SU(2) and the hypercharge U(1), but the period of the U(1) subgroup of SU(2) is well known to be 4\pi.

Third, can we estimate the mass of the electroweak monopole? This is the most important question from the experimental point of view. There was no way to predict the mass of the Dirac monopole theoretically, which has made the search for the monopole a blind search in the dark room without any theoretical lead.
Remarkably the mass of the electroweak monopole can be predicted. Of course, the Cho-Maison monopole has a singularity at the origin which makes the energy divergent [5]. But there are ways to regularize the energy and estimate the mass, and all point consistently to 4 to 10 TeV [9,11]. This, however, is tantalizing because the upgraded 14 TeV LHC can produce the electroweak monopole pairs only when the monopole mass becomes below 7 TeV. So we need a more accurate estimate of the monopole to see if LHC can actually produce the monopole.

The purpose of this Letter is discuss how the gravitational interaction affects the electroweak monopole. We show that, when the gravity is turned on, the monopole becomes a globally regular gravitating electroweak monopole which looks very much like the non-gravitating monopole, but turns to the magnetic black holes as the Higgs vacuum value approaches to the Planck scale. This confirms that the change of the monopole mass due to the gravitational interaction is negligible, which assures that the present LHC could produce the electroweak monopole.

Before we discuss the modification of the monopole induced by the gravitation, we briefly review the non-gravitating electroweak monopole and explain how we can estimate the monopole mass. Consider the following effective Lagrangian of the standard model,

\[
\mathcal{L}_{\text{eff}} = \frac{1}{2} |D_\mu \phi|^2 - \frac{\lambda}{2} \left( \phi^2 - \frac{\mu^2}{\lambda} \right)^2 - \frac{\epsilon(\phi)}{4} F_{\mu\nu}^2,
\]

\[
D_\mu \phi = (\partial_\mu - i \frac{g}{2} \vec{r} \cdot \vec{A}_\mu - i \frac{g'}{2} B_\mu) \phi, \tag{1}
\]

where \(\epsilon(\phi)\) is a positive dimensionless function of the Higgs doublet which approaches to unit asymptotically. Obviously when \(\epsilon = 1\), the Lagrangian reproduces the standard model. In general \(\epsilon\) modifies the permeability of the hypercharge U(1) gauge field, but the effective Lagrangian still retains the SU(2) \(\times\) U(1) gauge symmetry.

When \(\epsilon = 1\), we can obtain the Cho-Maison monopole with the ansatz [5]

\[
\phi = \frac{1}{\sqrt{2}} \rho \xi, \quad \rho = \rho(r), \quad \xi = i \left( \sin \frac{\theta}{2} e^{-i\varphi} \right),
\]

\[
\vec{A}_\mu = \frac{1}{g} \left( f(r) - 1 \right) \vec{r} \times \partial_\mu \vec{r},
\]

\[
B_\mu = -\frac{1}{g'} \left( 1 - \cos \theta \right) \partial_\mu \varphi. \tag{2}
\]

Notice that \(\vec{A}_\mu\) has the structure of the ’t Hooft-Polyakov monopole, but \(B_\mu\) has the structure of the Dirac monopole. This tells that the Cho-Maison monopole is a hybrid between ’t Hooft-Polyakov and Dirac.

The ansatz clearly shows that the U(1) point singularity in \(B_\mu\) makes the energy of the Cho-Maison infinite, so that classically the monopole mass is undetermined.

But, unlike the Dirac monopole, here we can estimate the mass of the electroweak monopole. A simplest way to do so is to realize that basically the monopole mass comes from the same mechanism which generates the weak boson mass, i.e., the Higgs mechanism, except that here the coupling becomes magnetic (i.e., \(4\pi/e\)) [9,10]. This must be clear from (2). So, from the dimensional reasoning the monopole mass \(M\) should be of the order of \(M \approx (4\pi/e^2) \times M_W\), or roughly about 10 TeV.

A better way to estimate the mass is to notice that the Cho-Maison monopole energy consists of four parts, the SU(2) part, the hypercharge U(1) part, the Higgs kinetic part, and the Higgs potential part, but only the U(1) part is divergent [5,9]. Now, assuming that this divergent part can be regularized by the ultra-violet quantum correction, we can derive a constraint among the four parts which minimizes the monopole energy using the Derrick’s theorem. From this we can deduce the monopole energy to be around 3.96 TeV [9,10].

Moreover, we can regularize the Cho-Maison monopole introducing non-vacuum permeability \(\epsilon\) which can mimic the quantum correction. This is because, with the rescaling of \(B_\mu\) to \(B_\mu/g', g'\) changes to \(g'/\sqrt{\epsilon}\), so that \(\epsilon\) changes the U(1) gauge coupling \(g'\) to the “running” coupling \(g' = g'/\sqrt{\epsilon}\). So, by making \(g'\) infinite (requiring \(\epsilon\) vanishing) at the origin, we can regularize the monopole. For example, with \(\epsilon = (\rho/\rho_0)^8\), the regularized monopole energy becomes 7.19 TeV [9,10].

The monopole energy, of course, depends on the functional form of \(\epsilon\), so that we could change the monopole energy changing \(\epsilon\). Recently Ellis et al. pointed out that \(\epsilon = (\rho/\rho_0)^8\) is unrealistic because it makes the Higgs to two photon decay rate larger than the experimental value measured by LHC. Moreover, they have argued that the monopole mass can not be larger than 5.5 TeV if we choose a more realistic \(\epsilon\) which reproduces the experimental value of the Higgs to two photon decay rate [11]. The effective couplings induced by two different \(\epsilon\) are shown in Fig. 1.

Now we discuss the gravitational modification of the

![FIG. 1: The running coupling \(g'\) of the hypercharge U(1) induced by \(\epsilon\). The dotted (blue) curve is obtained with \(\epsilon = (\rho/\rho_0)^8\), and the solid (red) curve is obtained with \(\epsilon\) proposed by Ellis et al. The vertical line indicates the Higgs mass scale.](image-url)
monopole. Intuitively, the gravitational modification is expected to be negligible. But there is the possibility that the gravitational attraction could change the monopole to a black hole and make the monopole mass arbitrary \[12\]. We show that this happens only when the Higgs vacuum value approaches to the Planck scale.

Consider the effective Lagrangian \([1]\) minimally coupled to Einstein’s theory. For the sake of simplicity, we focus on the static spherically symmetric solutions and assume the space-time metric to be

\[
ds^2 = -N^2(r)A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2),
\]

\[A(r) = 1 - \frac{2GM(r)}{r}. \quad (3)
\]

Now, with the ansatz \([2]\) the reduced Einstein-Weinberg-Salam action becomes

\[
S = \int \left[ \frac{1}{4\pi} m - AK - U \right] N dr,
\]

\[K = \frac{f^2}{g^2} + \frac{r^2}{2} \dot{\rho}^2,
\]

\[U = \frac{(1-f^2)^2}{2g^2r^2} + \frac{\lambda}{8} r^2 (\rho^2 - \rho_0^2)^2 + \frac{\epsilon(r)}{2g^2r^2} + \frac{1}{4} f^2 \rho^2, \quad (4)
\]

where the dot represents \(d/dr\).

From this we have the following equations of motion

\[\frac{\dot{N}}{N} = 8\pi G \frac{K}{r}, \quad \dot{m} = 4\pi (AK + U),
\]

\[A\dot{f} + \left( \dot{A} + A \frac{\dot{N}}{N} \right) f + \frac{1}{2} f^2 - \frac{f}{4} g^2 \rho^2 f = 0,
\]

\[A\dot{\rho} + \left( \frac{2A}{r} + \dot{A} + A \frac{\dot{N}}{N} \right) \dot{\rho} - \frac{f}{2r^2} \rho
\]

\[-\frac{\lambda}{2} (\rho^2 - \rho_0^2) \rho - \frac{1}{2g^2r^2} \frac{d\epsilon(r)}{d\rho} = 0. \quad (5)
\]

This has two limiting solutions. First, when gravitational field decouples (i.e., when \(G \rightarrow 0\)) we have the non-gravitating electroweak monopole solution \([3]\). Second, when \(f = 0, \rho = \rho_0, \) and \(\epsilon = 1\), we have the magnetically charged Reissner-Nordstrom black hole solution with the magnetic charge \(4\pi/e \langle e = gg' / \sqrt{g^2 + g'^2} \rangle \[12\]. But in general the solution depends on three parameters

\[\alpha = \sqrt{G}\rho_0 = \rho_0/M_p, \quad \beta = M_H/M_W, \quad (6)
\]

and the Weinberg angle \(\theta_W\).

Depending on the boundary conditions the entire solutions of \([3]\) can be classified into two categories: the globally regular solutions and the black holes. Asymptotic flatness of the space-time requires that both \(N(r)\) and \(m(r)\) become constants at spatial infinity, which requires the following boundary conditions on the W-boson and Higgs field,

\[f(\infty) = 0, \quad \rho(\infty) = \rho_0. \quad (7)
\]

We fix the scale of the time coordinate adopting

\[N(\infty) = 1. \quad (8)
\]

Notice that \(m(\infty)\) which determines the total mass of the monopole is not constrained.

For the regular monopole solutions we require the regularity at the origin,

\[f(0) = 1, \quad \rho(0) = 0, \quad m(0) = 0. \quad (9)
\]

Now, choosing \(\epsilon = (\rho/\rho_0)^8\) for simplicity, we find that the solutions have the following expansions near \(r = 0\),

\[f(r) = 1 - f_1 x^2 + ..., \quad \rho(r) = h_1 \rho_0 x^{\delta_1} + ..., \quad (\delta_1 = \sqrt{3} - 1)
\]

\[m(r) = \frac{2\pi \alpha^2 h_1^2 \delta_1^2}{GM_W \delta_2} x^{\delta_2} + ..., \quad (\delta_2 = \sqrt{3}), \quad (10)
\]

where \(x = M_W r\). From this we can obtain the solutions by the standard shooting method with \(f_1\) and \(h_1\) as the shooting parameters. The result is shown in Fig. 2. Notice that, except for the metric, the gravitating monopole looks very much like the non-gravitating monopole. In particular, the size of the monopole remains roughly about \(1/M_W\).

To find the mass of the monopole, first note that with \([9]\) we have

\[m(r) = 4\pi \epsilon^{P(r)} \int_0^r (K + U)e^{-P(r')} dr', \quad (11)
\]

\[P(r) = 8\pi G \int_r^\infty \frac{K}{r'} dr'. \quad (12)
\]

So, with \([7]\) we have the total mass given by

\[M = m(\infty) = 4\pi \int_0^\infty (K + U)e^{-P(r')} dr, \quad (13)
\]
which assures the positivity of the total mass. Moreover, this confirms that the gravitation reduces the monopole mass, which is expected from the attractive nature of the gravitational interaction [13]. In reality, of course, the gravitational interaction becomes negligible because $\alpha$ is very small (of $10^{-16}$). The $\alpha$-dependence of the mass is calculated numerically in Table 1.

| $\alpha = \rho_0/M_P$ | $M$         |
|-----------------------|-------------|
| 0 (non-gravitating)   | 7.19 TeV    |
| 0.10                  | 7.15 TeV    |
| 0.20                  | 6.97 TeV    |
| 0.38                  | 6.34 TeV    |
| $\alpha_{\text{max}} \simeq 0.39$ | black hole |

This shows that (even for the globally regular solutions) the metric for the monopole becomes asymptotically Reissner-Nordstrom type, with ADM mass equal to $M$ and the magnetic charge $Q_m = 4\pi/e$.

Our result shows that the generic feature of the gravitating electroweak monopole solution is quite similar to the gravitating 't Hooft-polyakov monopole solution obtained by Breitenlohner, Forgacs, and Maison [14]. In fact mathematically our solution could be viewed as the electroweak generalization of their solution. From the physical point of view, however, they are totally different. Theirs is hypothetical, but ours describes the real monopole dressed by the physical W-boson and Higgs field.

Clearly our solutions have the dyonic generalization, and have mathematically very interesting properties to study. Moreover, they must have important applications on the monopole stars and the magnetically charged black holes. The details of our solutions and their physical applications, in particular the dyonic generalization and the comparison between our solutions and the Breitenlohner-Forgacs-Maison solutions, will be discussed in a separate paper [15].

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