MOMENTUM DISTRIBUTIONS IN $t\bar{t}$ PRODUCTION AND DECAY NEAR THRESHOLD (II):
MOMENTUM DEPENDENT WIDTH
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Abstract

We apply the Green function formalism for $t - \bar{t}$ production and decay near threshold in a study of the effects due to the momentum dependent width for such a system. We point out that these effects are likely to be much smaller than expected from the reduction of the available phase space. The Lippmann–Schwinger equation for the QCD chromostatic potential is solved numerically for $S$ partial wave. We compare the results on the total cross section, top quark intrinsic momentum distributions and on the energy spectra of $W$ bosons from top quark decays with those obtained for the constant width.

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1 Introduction

There are accumulating indications that the long awaited top quark will be discovered soon. The value of its mass $m_t$ preferred by the indirect LEP measurements \[1\] implies that the $t$ quark is a short–lived particle, and its width $\Gamma_t$ is of the order of several hundreds MeV. As a consequence the cross section for $t\bar{t}$ pair production near energy threshold has a rather simple and smooth shape. In particular, it is likely that in $e^+e^-$ annihilation only the 1S peak survives as a remnant of toponium resonances \[2, 3\]. Nevertheless, as pointed out by Fadin and Khoze \[4\] the excitation curve $\sigma(e^+e^- \rightarrow t\bar{t})$ allows a precise determination of $m_t$ as well as of other physical quantities such as $\Gamma_t$ and the strong coupling constant $\alpha_s$. These results, derived analytically for the Coulomb chromostatic potential, have been confirmed by Strassler and Peskin \[5\] who studied numerically a more realistic QCD potential. The idea \[4, 5\] to use Green function instead of summing over overlapping resonances has been also applied in calculations of differential cross sections: energy spectra of $W$’s from decays \[6\], and intrinsic momentum distributions of top quarks in $t\bar{t}$ systems \[6, 7\].

Pandora’s box was opened when the authors of \[7\] found that the effects of the momentum dependent width may show up in the annihilation cross section $\sigma(e^+e^- \rightarrow t\bar{t})$. Despite the fact that the calculation is plagued by unpleasant and unphysical gauge dependence, one has to consider their conclusions as an indication that the accuracy of the calculations \[4, 5, 6\] assuming a momentum independent decay rate should be reconsidered. One cannot use the results of \[7\] for such an estimation, because some potentially relevant effects are not taken into account there.

In the present paper we extend the calculation of \[6\] by including a momentum dependent width. Our work is based on an observation \[8\] that relatively simple semiclassical arguments reproduce well the results of a complete calculation of the lifetimes and the energy spectra for decays of muons bound in light and medium nuclei. Since the strength of interaction is comparable for $t\bar{t}$ and $\mu^–$–nucleus for the nucleus charge $Z \sim 10–20$, one may expect that analogous considerations closely approximate the results of the complete QCD calculations for top quark pair production near threshold.

One of the most important future applications of this calculation is the determination of $m_t$ and $\alpha_s$ from the combined measurements of the total and the differential cross sections in $e^+e^- \rightarrow t\bar{t}$. We study the theoretical uncertainties related to model assumptions on the momentum dependent width. These result from $\mathcal{O}(\alpha_s^2)$ corrections to the process $e^+e^- \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$ which have been not calculated yet. Some corrections due to reduction of the available phase space \[6\] have been considered in \[7\]. However, as we already
pointed out in our previous paper [6] one should expect large cancellations between the corrections due to the phase space reduction and the Coulomb enhancement of the $b$ and $\bar{b}$ wave functions. In order to determine theoretical uncertainties related to the momentum dependent width we evaluate predictions of a model (Model I) which overestimates the effects on $\sigma(e^+e^- \rightarrow tt)$ and the differential cross sections. Nevertheless, the resulting theoretical uncertainties in determination of $m_t$ and $\alpha_s$ are quite small.

We also work out the predictions of a more realistic model (Model II) for the momentum dependent width and find out that for this model the results are close to those obtained assuming constant width.

Our paper is organized as follows: In sect.2 we describe the physics of bound $t\bar{t}$ decays and formulate our models of the momentum dependent width. In sect.3, which we add for the sake of completeness, the Green function method and our QCD potential are briefly described. In sect.4 the results of the numerical calculations are presented, and in sect.5 our conclusions are given.

2 Models for the width of $t - \bar{t}$ system

In our earlier paper [4] we followed [4, 5] and confined the discussion to the non–relativistic approximation and the constant decay rate. The basic reason was that numerous effects related to the intrinsic momentum in the $t - \bar{t}$ system were known to be small and they were contributing with opposite signs to the width. Let us list now the main physical effects which result in the momentum dependent width. The width of the $t - \bar{t}$ system depends on the intrinsic momentum, of say $t$ quark, because both the matrix element and the phase space available for the decay products depend on it. The phase space effect tends to reduce the decay rate of bound top quarks relative to free ones [4], and the effect is enhanced, because for short–lived particles the distribution of intrinsic momentum is broad. However, for the same reason the decays take place at short relative distances, where the wave functions of $b$ and $\bar{b}$ quarks originating from the decays are distorted (enhanced) by Coulomb attraction. Therefore, when calculating the amplitude of $t \rightarrow bW$ transition, one should use Coulomb wave functions rather than plane waves for $b$ quarks. This effect clearly increases the rate. A third factor is the time dilatation: a top quark moving with velocity $v$ lives longer in the center–of–mass laboratory frame. Let us consider now QCD corrections to the decay

\[ \Gamma = \Gamma_{\text{free}} \left[ 1 - 5(Z\alpha)^2 \right] \left[ 1 + 5(Z\alpha)^2 \right] \left[ 1 - (Z\alpha)^2 / 2 \right] \]

where the first correction factor comes from the phase space suppression, the second from the Coulomb enhancement, and the third one from time dilatation.
The calculations of the corrections to the total rate \([10]\), the energy spectrum of the charged lepton \([11]\) and the energy of \(W\) \([12]\) do not include two effects which may be non-negligible for \(t - \bar{t}\) production near threshold: 
a) the QCD Chudakov effect, i.e. the suppression of soft gluon emission from a small and colour singlet \(t \bar{t}\) source, and 
b) the additional suppression of the transition from a colour singlet into a colour octet \(t \bar{t}\) system due to the related change of its potential energy\([1]\). In the following we ignore all these effects, as well as analogous effects in transverse gluon QCD correction to the production vertex. Since \(O(\alpha_s)\) QCD corrections to the decay \([10, 11, 12, 13]\) are \(\sim 10\%\), we hope that the accuracy of our approximation is of the order of \(1\%\), i.e. that it is comparable to the size of \(\alpha_s^2\) effects which are also neglected. In particular we neglect all relativistic and spin corrections but the time dilatation in the momentum dependent width. QCD corrections to the decay rate \(\Gamma_{t - \bar{t}}\) are included as an overall factor \(f\) modifying the width \(\Gamma_t\) for single top decay:

\[
\Gamma(p, E) = \frac{1}{2} \Gamma_{t - \bar{t}} = C(p, E) \Gamma_t
\]

where

\[
\Gamma_t = \frac{G_F m_t^3}{8\sqrt{2}\pi} \left( 1 - \frac{m_W^2}{m_t^2} \right)^2 \left( 1 + 2\frac{m_W^2}{m_t^2} \right) \left[ 1 - \frac{2}{3} \frac{\alpha_s}{\pi} f\left( \frac{m_W^2}{m_t^2} \right) \right]
\]

and \([11]\)

\[
f(y) = \pi^2 + 2Li_2(y) - 2Li_2(1 - y) + [4y(1 - y - 2y^2) \ln y + 2(1 - y)^2] \cdot \frac{(5 + 4y) \ln(1 - y) - (1 - y)(5 + 9y - 6y^2)}{[2(1 - y)^2(1 + 2y)]}
\]

The correction factor \(C(p, E)\) depends on the intrinsic momentum \(\vec{p}\) and the energy \(E = \sqrt{s} - 2m_t\) as explained at the beginning of this section. Some effects such as the phase space reduction or the time dilatation can be easily implemented. In fact these effects have been already included in the calculations performed in \([7]\). The Coulomb enhancement cannot be easily taken into account. In principle one has to replace the plane wave functions for \(b\) quarks by relativistic Coulomb functions when evaluating the amplitude for \(t \rightarrow bW\) transition. One may hope, however, that the following observation, valid for muons bound in nuclei \([8]\), holds also for chromostatic attraction in \(t - \bar{t}\) systems: the phase space suppression and the Coulomb enhancement nearly cancel each other. For light nuclei the result is well described by the time dilatation suppression alone. For heavier nuclei there is a further reduction in the decay rate because some of the decay electrons will not have sufficient kinetic energy to escape to infinity from the region of the electrostatic potential of the nucleus. We think that for \(t - \bar{t}\) it is

\footnote{For a recent discussion of the effect of \(\Gamma_t\) on soft gluon radiation see \([14]\).}
reasonable to replace this ‘escape into infinity’ condition by a condition on the effective mass $\mu_{b\bar{b}}$ of the $b - \bar{b}$ system resulting from the decays of $t$ and $\bar{t}$.

At first we discuss a Model I which overestimates the effects of the momentum dependent width. Comparing the suppression factors corresponding to Model I with the ones discussed in [7], one observes that they are not very different, see Fig. 1b in the present paper and Fig. 6 in [7]. Thus, the results derived from Model I should resemble the ones obtained in [7]. We assume that for a $t - \bar{t}$ system the $t$ quark with the intrinsic three-momentum $\vec{p}$ decays in its rest frame as a free particle of mass $m_t$. $W^+$ originates from this decay with energy fixed by two-body kinematics. Then its four-momentum in the $t - \bar{t}$ rest frame is obtained from the Lorentz transformation. The same procedure is applied to $W^-$ from the decay of $\bar{t}$. $W$’s from the decays are colour singlets, so they escape without any final state interaction. On the other hand the $b$ and $\bar{b}$ interact and their momenta are changed by final state interactions. The four-momentum of the $b - \bar{b}$ system is fixed by energy-momentum conservation. It is clear that for such a model the energy spectra of $W$’s are broadened due to the Fermi motion. The effective mass $\mu_{b\bar{b}}$ can be smaller than for decays of unbound $t$ and $\bar{t}$. For some configurations of the momenta of $W$’s the resulting $\mu_{b\bar{b}}^2$ can even be negative. These configurations must be rejected. We impose a stronger condition on $\mu_{b\bar{b}}$ and require that $\mu_{b\bar{b}}$ must be larger than $\mu_0 = 2m_b + \Delta$, where $\Delta = 2$ GeV. It is plausible that if this condition is fulfilled the $b - \bar{b}$ system decays into hadrons with probability one. For $\mu_{b\bar{b}} < \mu_0$ we assume complete suppression. Thus, our Model I for the momentum dependent width is defined by the following formula:

$$d\Gamma(\vec{p}, E) = \Gamma_t \sqrt{1 - \vec{p}^2/(\vec{p}^2 + m_t^2)} \Theta(\mu_{b\bar{b}} - \mu_0) dL_{2\otimes2}$$

(4)

where the volume of the phase space $L_{2\otimes2}$ for $(bW^+)(\bar{b}W^-)$ is normalized to one. Thus, if the suppression factors were absent the width $\Gamma_{t - \bar{t}}$ of the $t - \bar{t}$ system would be equal to the sum of the widths of free $t$ and $\bar{t}$. The first suppression factor in (4) is due to the time dilatation, and the step $\Theta$ function describes the additional suppression. Depending on the total energy and the intrinsic momentum of the decaying top quarks, the above-mentioned effects result in a decrease of the width $\Gamma_{t - \bar{t}}$. In Figs. 1a–b the correction factors $C(p, E)$ are plotted for the top masses 120 and 150 GeV, respectively, and for three energies near the threshold. The phase space reduction is larger for energies below threshold and leads together with the relativistic time dilatation factor to a strong suppression of decays for large values of the intrinsic momentum. Both effects also depend on the top mass: the larger $m_t$, the smaller the reduction of $\Gamma_{t - \bar{t}}$. The energy dependence is also weaker for smaller values of $\Delta$. However, the latter effect is rather small.

Let us describe now Model II which we consider to be more realistic. Instead of (4) the momentum dependent width is given by the formula:
Figure 1: The suppression factors $C(p, E)$ for the decays of $t - \bar{t}$ system in comparison to the single quark decays of $t$ and $\bar{t}$: a) $m_t = 120$ GeV and b) $m_t = 150$ GeV. Model I: the energies correspond to $1S$ peak — dashed, $E = 0$ — dotted, and $E = 2$ GeV — dashed–dotted lines. Model II: energies of $1S$ peak — solid lines.

\[
d\Gamma(\vec{p}, E) = \Gamma_t \sqrt{1 - \frac{\vec{p}^2}{(\vec{p}^2 + \mu^2)}} \Theta(\mu_{bb} - \mu_0) \, dL_{2\bar{2}} \tag{5}\]

where

\[
\mu^2 = p_\alpha p^\alpha
\]

and $p_\alpha$ is the four–momentum of the off–shell top quark. In its rest frame the energy of $W$ originating from the decay is given by the two–body kinematics:

\[
E^*_W = \frac{\mu^2 + m_W^2 - m_b^2}{2\mu} \tag{6}\]

The correction factor $C(p, E)$ evaluated from Model II depends weakly on energy and is close to 1, see solid lines in Figs. 1a–b. Therefore, the corresponding results are much closer to those derived assuming constant width. Two more remarks are in order here.

The models which we consider are inspired by the observation [15] that for short–lived top quarks the final states hadrons originate from two–jet $b - \bar{b}$ configurations. The lower the value of top mass and the longer its lifetime, the less realistic become our assumptions. For small $\Gamma_t$ the decays take place in a well–defined sequence, and the treatment of the recoil effects given in [9] is more appropriate.

The width for off–shell top quarks is a gauge dependent object. The results for physical quantities do not depend on the choice of gauge because the gauge dependent contributions from propagators, vertex corrections, wave function renormalization and non–resonant graphs cancel order by order in perturbation theory. The problem of gauge dependence in $Z^0$ and $W$ pair production has been recently discussed in the literature [16]. The case of $t\bar{t}$ production near threshold is even more complicated because of multiple gluon exchange and resonance formation due to Coulomb–like strong attraction. In our opinion the only way to avoid gauge dependence in final results is to solve the bound state problem including the constant on–shell width, and to treat the difference between on–shell and off–shell widths in the framework of perturbation theory. However, such a complete approach is beyond...
the scope of the present paper. Our main goal is to estimate the importance of the effects related to the momentum dependent width. Therefore, we follow [7] and include the momentum dependent width but ignore all other contributions. We argue that due to the cancellations between phase space suppression and Coulomb enhancement the difference between the on–shell and off–shell widths is strongly reduced. Thus, the related differences in the cross sections are small, and their gauge dependence may be unimportant from the practical point of view.

3 Green function for $t$ -- $\bar{t}$ system

In this section we briefly describe the Green function method for $e^+ e^- \rightarrow t \bar{t}$ annihilation and the numerical solution of the Lippmann–Schwinger equation near the energy threshold; see [8] for details. In our discussion we neglect $Z^0$ contribution and transverse gluon correction to the production vertex. The differential cross section for top quark pair production in electron positron annihilation reads

\[
\frac{d\sigma}{d^3 p} (\vec{p}, E) = \frac{3\alpha^2 Q_t^2}{\pi s m_t^2} \Gamma(p, E) |\mathcal{G}(\vec{p}, E)|^2 .
\]

(7)

The Green function $\mathcal{G}(\vec{p}, E)$ is the solution of the Lippmann-Schwinger equation

\[
\mathcal{G}(\vec{p}, E) = \mathcal{G}_0(\vec{p}, E) + \mathcal{G}_0(\vec{p}, E) \int \frac{d^3 q}{(2\pi)^3} \tilde{V}(\vec{p} - \vec{q}) \mathcal{G}(\vec{q}, E)
\]

(8)

where $\tilde{V}(\vec{p})$ is the potential in momentum space. The free Hamiltonian that is used to define the Green function $\mathcal{G}_0$ includes the momentum dependent width:

\[
\mathcal{G}_0(\vec{p}, E) = \frac{1}{E - \frac{p^2}{m_t} + i\Gamma(p, E)}
\]

(9)

In our numerical calculations we use the two–loop QCD potential [17] for $n_f = 5$ quark flavours, and join it smoothly to a Richardson–like phenomenological potential [18] for intermediate and small momenta. This potential gives a very good description of $J/\psi$ and $\Upsilon$ families.

We define the potential $\tilde{V}(p)$, where $p = |\vec{p}|$ as follows:

\[
\tilde{V}_{JKT}(p) = -\frac{16\pi}{3} \frac{\alpha_{\text{eff}}(p)}{p^2} + V_0 \delta(p)
\]

(10)

where

\[
\alpha_{\text{eff}}(p) = \begin{cases} 
\alpha_{\text{pert}}(p) & \text{if } p > p_1 , \\
\alpha_R(p) + (p - p_2)\frac{\alpha_{\text{pert}}(p_1) - \alpha_{\text{pert}}(p_2)}{p_1 - p_2} & \text{if } p_1 > p > p_2 , \\
\alpha_R(p) & \text{if } p < p_2 .
\end{cases}
\]

(11)

\footnote{In (7) of [8] a factor $(2\pi)^{-3}$ is omitted. The results, plots and conclusions presented there are not affected by this omission.}
Figure 2: $\alpha_{\text{eff}}(q)$ for different values of $\alpha_s(M_Z)$: solid: 0.12, dashed: 0.11, dashed–dotted: 0.13, dotted: 0.10 and 0.14.

Figure 3: QCD–Potential in the position space $V_{\text{JKT}}(r)$ for different values of $\alpha_s(M_Z)$: solid: 0.12, dashed: 0.11, dashed–dotted: 0.13, dotted: 0.10 and 0.14.

with

$$\alpha_{\text{pert}}(p) = \frac{4\pi}{b_0 \ln \left(p^2/\Lambda_{\text{MS}}^2\right) + b_1 \ln \ln \left(p^2/\Lambda_{\text{MS}}^2\right)} \left[1 + \left(\frac{31}{3} - \frac{10}{9} n_f\right) \frac{\alpha_{\text{MS}}(p^2)}{4\pi}\right]$$

(12)

and

$$\alpha_{\text{MS}}(p^2) = \frac{4\pi}{b_0 \ln \left(p^2/\Lambda_{\text{MS}}^2\right) + b_1 \ln \ln \left(p^2/\Lambda_{\text{MS}}^2\right)}$$

(13)

$$b_0 = 11 - \frac{2}{3} n_f, \quad b_1 = 102 - \frac{38}{3} n_f.$$  

The Richardson–like part is given by

$$\alpha_R(p) = \frac{4\pi}{9} \left[\frac{1}{\ln(1 + p^2/\Lambda_R^2)} - \frac{\Lambda_R^2 q_{\text{cut}}^2}{p^2(p^2 + q_{\text{cut}}^2)}\right].$$

(14)

Throughout our calculations we take $p_1 = 5$ GeV, $p_2 = 2$ GeV, $\Lambda_R = 400$ MeV and $q_{\text{cut}} = 50$ MeV. The constant $V_0$ is fixed by the requirement that after Fourier transform the potential in the position space

$$V_{\text{JKT}}(r = 1\text{GeV}^{-1}) = -1/4\text{GeV}.$$  

In Fig. 2 we plot $\alpha_{\text{eff}}$ for $\alpha_s(M_Z)$ varying between 0.10 and 0.14 whereas in Fig. 3 our potential $V_{\text{JKT}}(r)$ in the position space is shown.

Near the energy threshold one can neglect all but $S$ partial waves and numerically solve the corresponding one–dimensional integral equation. The spherically symmetric solution fulfills the unitarity condition

$$\int \frac{d^3p}{(2\pi)^3} \Gamma(p, E) |G(p, E)|^2 = -\text{Im}G(\vec{x} = 0, \vec{x}' = 0, E)$$

(15)

which for the constant decay rate reduces to the formula for the total cross section derived in [4].
Figure 4: Comparison of the differential cross sections $\frac{d\sigma}{dp}$ evaluated for the momentum dependent $\Gamma(p,E)$ (Model I: dotted, Model II: solid) and constant $\Gamma_t$ widths (dashed lines) for $m_t = 120$ GeV; and a) $E = -2.3$, b) $E = 0$ and c) $E = 2$ GeV, respectively.

Figure 5: Comparison of the annihilation cross sections $\sigma(e^+e^- \rightarrow t\bar{t})$ (in units of $R$) evaluated for the momentum dependent $\Gamma(p,E)$ (Model I: dotted, Model II: solid) and constant $\Gamma_t$ widths (dashed lines).

4 Results

As described in the last section, the momentum dependent width $\Gamma(p,E)$ enters our numerical calculation of the Green function $\mathcal{G}(p,E)$. The differential cross sections $\frac{d\sigma}{dp}$ derived from (7) are different from the analogous cross sections obtained assuming constant width. In the latter case $\frac{1}{\Gamma_t} \frac{d\sigma}{dp}$ is proportional to $|pG(p,E)|^2$, and we show these distributions as dashed lines in Figs. 4a–c for $m_t = 120$ GeV and $\Gamma_t = 0.3$ GeV. For a comparison we plot also analogously normalized cross sections $\frac{\Gamma(p,E)}{\Gamma_t} |pG(p,E)|^2$ for the momentum dependent width (dotted lines for Model I and solid for Model II).

The main effect of the varying width $\Gamma(p,E)$ in Model I is a change in the normalization of the distribution ($\approx 15\%$ for the $1S$ peak) accompanied by the expected suppression of the large momentum tail. Thus the Green functions are slightly shifted to smaller momenta. However, it is remarkable that the position of the maximum is not much affected. The energy dependence of the annihilation cross section in the threshold region can be obtained by integration of the differential cross section over the intrinsic momentum:

$$\sigma(e^+e^- \rightarrow t\bar{t})|_E = \frac{16\alpha^2_{\text{QED}}}{3s_{m_t}^2} \int_0^\infty dp \, p^2 \Gamma(p,E)|G(p,E)|^2$$  \hspace{1cm} (16)

In Figs. 5a–b we show $\sigma(e^+e^- \rightarrow t\bar{t})$ as functions of energy for two values of $m_t$: 120 and 150 GeV. Once again we observe a stronger effect of the momentum dependent width for smaller $m_t$: while for a top mass of 120 GeV an effective decrease of the width in Model I leads to the increase of the cross section at $1S$ resonance by more than 15%, the total cross section is much
Figure 6: Normalized energy spectra of $W$ bosons originating from the decays of $t - \bar{t}$ systems at 1S peaks (narrower) and for $E = 2$ GeV (broader). The dashed lines correspond to the momentum dependent width evaluated in Model I and solid ones to the constant widths. The $W$ spectra obtained from Model II are very close to the solid curves (constant width case).

less affected for top masses as high as 150 GeV and larger. Above the 1S peaks, and in particular above the threshold, the effects of the momentum dependent width are small, typically of the order of few percent. We observe that also for the total cross section the position of the 1S peak is not shifted, despite the fact that in Model I the normalization is significantly changed. It can be shown \[19\] that the position of the peaks in the total $\sigma(e^+e^- \rightarrow t\bar{t})$ and differential $d\sigma/dp$ cross sections are essential for a precise simultaneous determination of $m_t$ and $\alpha_s$ from the threshold region. Thus we conclude that the precision of such an analysis is not affected by theoretical uncertainties related to the momentum dependent width.

As expected, the results obtained from Model II are close (within a few percent) to the results corresponding to the constant width, c.f. solid and dashed lines in Fig. 4 and 5.

The top momentum distributions themselves cannot be measured experimentally. They can be reconstructed kinematically from the four–momenta of the final state particles, but, as should be clear from our discussion, those are sensitive to model assumptions. In particular the four–momenta of $b$ and $\bar{b}$ quarks are distorted by final state interactions. The final state interactions, however, do not change the four–momenta of $W$'s which are colourless. Thus their distributions contain important information on the dynamics of production and decay of top quarks. In Figs. 6a–b we compare the normalized energy spectra of $W$'s calculated for the momentum dependent $\Gamma(p,E)$ in Model I and constant $\Gamma_t$ widths. The calculations have been performed for the non–zero width of $W$ bosons, $\Gamma_W = 2.2$ GeV. Since the distributions of the intrinsic momentum are narrower for $\Gamma(p,E)$, c.f. Figs. 4 and 5, the resulting energy spectra of $W$'s are also narrower than the corresponding distributions for the constant width. We show these distributions as dashed and solid lines for the varying and constant width, respectively. The sharper spectra are obtained for the energies corresponding to 1S peaks and the broader ones for $E = 2$ GeV. The spectra of $W$'s obtained from Model II are
close to those for the constant width.

It is self-evident that for narrow resonances the effects related to changes of the width become enhanced. Thus the observed mass dependence is understandable. It is clear also that the initial state radiation, not included in our calculations, dilutes sharp resonances and significantly reduces effects due to changes of the width. Therefore, in our opinion, for $m_t$ around 150 GeV and larger theoretical uncertainties introduced by model assumptions of the present paper, or those introduced in [7], may be neglected. In particular the position of $1S$ peak is rather insensitive to model assumptions on the width. The normalization of the cross section at the peak is sensitive to these assumptions if the top mass is in the lower range of the allowed region. Thus a rigorous (not model) calculation including the effects discussed in this paper may prove to be indispensable if the top mass is around 120 GeV and one aims at a high precision study based on the absolute normalization of the cross section at $1S$ peak.

However, we would like to stress once again that considering only phase space suppression one overestimates effects of the momentum dependent width. It is likely that rigorous calculations will give results similar to those of Model II, which are quite close to the predictions obtained using the constant width.

5 Summary

In this paper we have studied the influence of the energy and momentum dependent width $\Gamma(p, E)$ on $t - \bar{t}$ production near the energy threshold. The position of the $1S$ peak in the total cross section as well as the maximum of $d\sigma/dp$ are rather insensitive to model assumptions related to $\Gamma(p, E)$. The total cross section $\sigma(e^+e^- \to t\bar{t})$ around the $1S$ peak reflects the effective change of the width introduced by $\Gamma(p, E)$. The effects of the momentum dependent width are likely to be much smaller than expected from the arguments based on the reduction of the available phase space. Further dilution of the $1S$ peak due to initial state radiation results in additional reduction of the differences between the results for the momentum dependent and constant widths. The energy spectra of $W$ bosons are slightly narrower, because the momentum dependence in $\Gamma(p, E)$ results in suppression of the large $p$ components of $t - \bar{t}$ wave functions.
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