A possible $\Omega \pi$ molecular state

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Abstract

The structure of $\Omega \pi$ state with isospin $I = 1$ and spin-parity $J^p = 3/2^-$ are dynamically studied in both the chiral SU(3) quark model and the extended chiral SU(3) quark model by solving a resonating group method (RGM) equation. The model parameters are taken from our previous work, which gave a satisfactory description of the energies of the baryon ground states, the binding energy of the deuteron, the nucleon-nucleon ($NN$) scattering phase shifts, and the hyperon-nucleon ($YN$) cross sections. The calculated results show that the $\Omega \pi$ state has an attractive interaction, and in the extended chiral SU(3) quark model such an attraction can make for an $\Omega \pi$ quasi-bound state with the binding energy of about several MeV.

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I. INTRODUCTION

The possible new resonance state is always an interesting topic in both the experimental and theoretical physics, and it can help us to catch more details about the hadron-hadron and quark-quark interaction. In 2001, Gao et al. followed the idea of Ref. [1] and estimated the QCD van der Waals attractive force of the $N\phi$ system [2]. They claimed that the QCD van der Waals attraction between $N$ and $\phi$, mediated by multi-gluon exchanges, can be strong enough to form a bound $N\phi$ state with a binding energy of about 1.8 MeV. At the same time they pointed out that it is possible to search for such a bound state using the $\phi$ meson below threshold quasi-free photo-production kinematics experimentally. Using a simple model, the authors calculated the rate for such subthreshold quasi-free production process using a realistic Jefferson Laboratory luminosity and a large acceptance detection system. They concluded that such an experiment is feasible. Recently, we have performed a dynamical study of the structures of $N\phi$ states with $J^P = 3/2^-$ and $J^P = 1/2^-$ in the chiral quark model by solving a resonating group method (RGM) equation [3]. Our results show that the $N\phi$ states have attractive interaction, and in the extended chiral SU(3) quark model such attraction, dominantly provided by the quark and $\sigma$ field coupling, can make $N\phi$ states to be bound with several MeV binding energies.

The above results about $N\phi$ excite our interest in its analogues, i.e. the systems of $\Omega\pi$, $\Omega\omega$ and $\Omega\rho$. Similar to the $N\phi$ system, all of them are the systems where two color singlet clusters have no common flavor quarks and there are no one-gluon exchange (OGE) interaction between these two hadrons, thus they are really the special two-hadron states to examine the quark-quark interactions and further the interactions between those two hadrons. Here we pay special attention to the $\Omega\pi$ system, which we think is the most interesting one. One knows that both $\Omega$ and $\pi$ have the traits of long life [in order of $10^{-10}$ and $10^{-8}$ seconds], and there is no coupling channel below the threshold of $\Omega\pi$. So, if there is really an $\Omega\pi$ bound state, it can not fall apart into $\Omega$ and $\pi$, and besides, it can not decay via strong interaction into any other channels, thus its width must be narrow. Furthermore, the $\Omega\pi^-$ state might be easily distinguished and detected in the experiments since it has two negative electronic charges.

It is a general consensus that QCD is the underlying theory of the strong interaction. However, as the non-perturbative QCD effect is very important for light quark systems
in the low energy region and it is difficult to be seriously solved, people still need QCD-inspired models to be a bridge connecting the QCD fundamental theory and the experimental observables. Among these phenomenological models, the chiral SU(3) quark model has been quite successful in reproducing the energies of the baryon ground states, the binding energy of the deuteron, the nucleon-nucleon ($NN$) scattering phase shifts, and the hyperon-nucleon ($YN$) cross sections [4]. In this model, the quark-quark interaction contains confinement, one-gluon exchange (OGE) and boson exchanges stemming from scalar and pseudoscalar nonets, and the short range quark-quark interaction is provided by OGE and quark exchange effects.

Actually it is still a controversial problem in the low-energy hadron physics whether gluon or Goldstone boson is the proper effective degree of freedom besides the constituent quark. Glozman and Riska proposed that the Goldstone boson is the only other proper effective degree of freedom [5, 6]. But Isgur gave a critique of the boson exchange model and insisted that the OGE governs the baryon structure [7, 8]. Anyway, it is still an open problem in the low-energy hadron physics whether OGE or vector-meson exchange is the right mechanism for describing the short-range quark-quark interaction, or both of them are important. Thus the chiral SU(3) quark model has been extended to include the coupling of the quark and vector chiral fields. The OGE that plays an important role in the short-range quark-quark interaction in the original chiral SU(3) is now nearly replaced by the vector meson exchanges. This model, named the extended chiral SU(3) quark model, has also been successful in reproducing the energies of the baryon ground states, the binding energy of the deuteron, and the nucleon-nucleon scattering phase shifts [9].

Recently, we have extended both the chiral SU(3) quark model and the extended chiral SU(3) quark model from the study of baryon-baryon scattering processes to the baryon-meson systems by solving a resonating group method (RGM) equation [10, 11, 12, 13, 14]. We found that some results are similar to those given by the chiral unitary approach study, such as that both the $\Delta K$ system with isospin $I = 1$ and the $\Sigma K$ system with $I = 1/2$ have quite strong attractions [12, 13, 14]. In the study of the $KN$ scattering [10, 11, 12], we get a considerable improvement not only on the signs but also on the magnitudes of the theoretical phase shifts comparing with other’s previous work. We also studied the phase shifts of $\pi K$ [15], and got reasonable fit with the experiments in the low energy region. All these achievements encourage us to investigate more baryon-meson systems by using these
two models.

In this paper, we dynamically study the interaction of the \( \Omega \pi \) state with \( I = 1 \) and \( J^p = 3/2^- \) in both the chiral SU(3) quark model and the extended chiral SU(3) quark model by resolving the RGM equation. The model parameters are taken from our previous works \cite{3, 13, 14}, which gave a satisfactory description of the energies of the baryon ground states, the binding energy of the deuteron, the \( NN \) scattering phase shifts and the hyperon-nucleon (\( YN \)) cross sections. The results show that in the extended chiral SU(3) quark model the \( \Omega \pi \) state can be weakly bound as a molecular state. Experimentally to search this state in the heavy ion collisions or \( e^+e^- \) collisions is worth trying in future.

In Sec. II, we give a brief introduction of the framework of these two chiral quark models. In Sec. III, the results for the \( \Omega \pi \) state and some discussions are presented. Finally, the summary is given in Sec. IV.

II. FORMULATION

The chiral SU(3) quark model and the extended chiral SU(3) quark model has been widely described in the literature \cite{10, 11, 12, 13, 14}, and we refer the reader to those works for details. Here we just give the salient features of these two models.

In these two models, the total Hamiltonian of baryon-meson systems can be written as

\[
H = \sum_{i=1}^{5} T_i - T_G + \sum_{i<j=1}^{4} V_{ij} + \sum_{i=1}^{4} V_{i\bar{5}},
\]

where \( T_G \) is the kinetic energy operator for the center-of-mass motion, and \( V_{ij} \) and \( V_{i\bar{5}} \) represent the quark-quark and quark-antiquark interactions, respectively,

\[
V_{ij} = V_{ij}^{OGE} + V_{ij}^{con} + V_{ij}^{ch},
\]

where \( V_{ij}^{OGE} \) is the OGE interaction, and \( V_{ij}^{con} \) is the confinement potential. \( V_{ij}^{ch} \) represents the chiral fields induced effective quark-quark interactions. In the chiral SU(3) quark model, \( V_{ij}^{ch} \) includes the scalar boson exchanges and the pseudoscalar boson exchanges,

\[
V_{ij}^{ch} = \sum_{a=0}^{8} V_{\sigma a}(r_{ij}) + \sum_{a=0}^{8} V_{\pi a}(r_{ij}),
\]

and in the extended chiral SU(3) quark model, the vector boson exchanges are also included,

\[
V_{ij}^{ch} = \sum_{a=0}^{8} V_{\sigma a}(r_{ij}) + \sum_{a=0}^{8} V_{\pi a}(r_{ij}) + \sum_{a=0}^{8} V_{\rho a}(r_{ij}).
\]
Here $\sigma_0, \ldots, \sigma_8$ are the scalar nonet fields, $\pi_0, \ldots, \pi_8$ the pseudoscalar nonet fields, and $\rho_0, \ldots, \rho_8$ the vector nonet fields. The expressions of these potentials can be found in the literature [10, 11, 12, 13, 14].

$V_{i5}$ in Eq. (1) includes two parts: direct interaction and annihilation parts,

$$V_{i5} = V_{i5}^{\text{dir}} + V_{i5}^{\text{ann}},$$

(5)

with

$$V_{i5}^{\text{dir}} = V_{i5}^{\text{conf}} + V_{i5}^{\text{OGE}} + V_{i5}^{\text{ch}},$$

(6)

and

$$V_{i5}^{\text{ch}} = \sum_j (-1)^{G_j} V_{i5}^{\text{ch},j}.$$  (7)

Here $(-1)^{G_j}$ represents the $G$ parity of the $j$th meson. The $q\bar{q}$ annihilation interactions, $V_{i5}^{\text{ann}}$, are not included in this work because they are assumed to be negligible, because it only acts in very short range region, thus it does not contribute significantly to a molecular state or a scattering process which is the subject of our present study.

All the model parameters are taken from our previous work [13, 14], which gave a satisfactory description of the energies of the baryon ground states, the binding energy of deuteron, and the $NN$ scattering phase shifts. Here we briefly give the procedure for the parameter determination. The three initial input parameters, i.e. the harmonic-oscillator width parameter $b_u$, the up (down) quark mass $m_{u(d)}$ and the strange quark mass $m_s$, are taken to be the usual values: $b_u = 0.5$ fm for the chiral SU(3) quark model and 0.45 fm for the extended chiral SU(3) quark model, $m_{u(d)} = 313$ MeV, and $m_s = 470$ MeV. The coupling constant for scalar and pseudoscalar chiral field coupling, $g_{ch}$, is fixed by the relation

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{NN\pi}^2}{4\pi} \frac{m_\sigma^2}{M_N^2},$$

(8)

with the empirical value $g_{NN\pi}^2/4\pi = 13.67$. The coupling constant for vector coupling of the vector-meson field is taken to be $g_{chv} = 2.351$, the same as used in the $NN$ case [9].

The masses of the mesons are taken to be the experimental values, except for the $\sigma$ meson. The $m_\sigma$ is adjusted to fit the binding energy of deuteron. The OGE coupling constants and the strengths of the confinement potential are fitted by baryon masses and their stability conditions. All the parameters are tabulated in Table I, where the first set is for the chiral
SU(3) quark model, the second and third sets are for the extended chiral SU(3) quark model by taking $f_{chv}/g_{chv}$ as 0 and $2/3$, respectively. Here $f_{chv}$ is the coupling constant for tensor coupling of the vector meson fields.

TABLE I: Model parameters. The meson masses and the cutoff masses: $m_{\sigma'} = 980$ MeV, $m_\kappa = 980$ MeV, $m_\epsilon = 980$ MeV, $m_\pi = 138$ MeV, $m_K = 495$ MeV, $m_\eta = 549$ MeV, $m_{\eta'} = 957$ MeV, $m_\rho = 770$ MeV, $m_{K^*} = 892$ MeV, $m_\omega = 782$ MeV, $m_\phi = 1020$ MeV, and $\Lambda = 1100$ MeV.

|          | $\chi$-SU(3) QM | Ex. $\chi$-SU(3) QM |
|----------|-----------------|---------------------|
|          | I               | II                  | III                 |
| $f_{chv}/g_{chv} = 0$ | $f_{chv}/g_{chv} = 2/3$ |
| $b_u$ (fm) | 0.5             | 0.45                | 0.45                |
| $m_u$ (MeV) | 313             | 313                 | 313                 |
| $m_s$ (MeV) | 470             | 470                 | 470                 |
| $g_u^2$   | 0.766           | 0.056               | 0.132               |
| $g_s^2$   | 0.846           | 0.203               | 0.250               |
| $g_{ch}$  | 2.621           | 2.621               | 2.621               |
| $g_{chv}$ |                 |                     |                     |
| $m_{\sigma'}$ (MeV) | 595             | 535                 | 547                 |
| $a^c_{uu}$ (MeV/fm$^2$) | 46.6            | 44.5                | 39.1                |
| $a^c_{us}$ (MeV/fm$^2$) | 58.7            | 79.6                | 69.2                |
| $a^c_{ss}$ (MeV/fm$^2$) | 99.2            | 163.7               | 142.5               |
| $a_{uu}^{c0}$ (MeV) | $-42.4$         | $-72.3$             | $-62.9$             |
| $a_{us}^{c0}$ (MeV) | $-36.2$         | $-87.6$             | $-74.6$             |
| $a_{ss}^{c0}$ (MeV) | $-33.8$         | $-108.0$            | $-91.0$             |

From Table I one can see that for both set II and set III, $g_u^2$ and $g_s^2$ are much smaller than the values of set I. This means that in the extended chiral SU(3) quark model, the coupling constants of OGE are greatly reduced when the coupling of quarks and vector-meson field is considered. Thus the OGE that plays an important role of the quark-quark short-range interaction in the original chiral SU(3) quark model is now nearly replaced by
the vector-meson exchange. In other words, the mechanisms of the quark-quark short-range interactions in these two models are quite different.

With all parameters determined, the Ωπ state can be dynamically studied in the framework of the RGM, a well established method for studying the interaction between two composite particles. The wave function of the Ωπ system is of the form

$$\Psi = \mathcal{A}[\hat{\psi}_Ω(\xi_1, \xi_2)\hat{\psi}_π(\xi_3)\chi(R_{Ωπ})],$$  \hspace{1cm} (9)

where $$\xi_1$$ and $$\xi_2$$ are the internal coordinates for the cluster Ω, and $$\xi_3$$ the internal coordinate for the cluster π. $$R_{Ωπ} = R_Ω - R_π$$ is the relative coordinate between the two clusters, Ω and π. The $$\hat{\psi}_Ω$$ is the antisymmetrized internal cluster wave function of Ω, as well as $$\hat{\psi}_π$$ is the internal wave function of π, and $$\chi(R_{Ωπ})$$ the relative wave function of the two clusters. The symbol $$\mathcal{A}$$ is the antisymmetrizing operator defined as

$$\mathcal{A} \equiv 1 - \sum_{i \in Ω} P_{i4} \equiv 1 - 3P_{34}. $$ \hspace{1cm} (10)

Expanding unknown $$\chi(R_{Ωπ})$$ by employing well-defined basis wave functions, such as Gaussian functions, one can solve the RGM equation for a bound-state problem or a scattering one to obtain the binding energy or scattering phase shifts for the two-cluster systems. The details of solving the RGM equation can be found in Refs. \[16, 17, 18\].

III. RESULTS AND DISCUSSIONS

As mentioned in the introduction, the Ωπ system is a very special two-hadron state. This is not only because these two color-singlet clusters have no common flavor quarks and there is no one-gluon exchange (OGE) interaction between these two hadrons, which makes this system an ideal place to examine the quark-quark interactions and further the interactions between those two hadrons, but also because both Ω and π have long life and narrow width, and there is no coupling channel below the threshold of Ωπ, which means that if there is really an Ωπ bound state, its width must be quite narrow. The narrow width and the two negative electronic charges of the possible Ωπ− bound state can make it easy to distinguish and detect this system in the experiment. Theoretically, a dynamical investigation of this interesting light quark system in the framework of the chiral quark model including the coupling between the quark and chiral fields seems to be necessary and essential.
FIG. 1: The GCM matrix elements of the Hamiltonian. The dotted, solid and dash-dotted lines represent the results obtained in model I, II and III, respectively.

Here we study the $\Omega\pi$ state in our chiral quark models by treating $\Omega$ and $\pi$ as two clusters and solving the corresponding RGM equation. Fig. 1 shows the diagonal matrix elements of the Hamiltonian for the $\Omega\pi$ system in the generator coordinate method (GCM) calculation, which can be regarded as the effective Hamiltonian of two clusters $\Omega$ and $\pi$ qualitatively. In Fig. 1, $\langle H_{\Omega\pi} \rangle$ includes the kinetic energy of the relative motion and the effective potential between $\Omega$ and $\pi$, and $s$ denotes the generator coordinate which can qualitatively describe the distance between the two clusters. From Fig. 1, one sees that the $\Omega\pi$ interaction, dominantly provided by the $\sigma$ field coupling, is attractive in the medium range. To study if such an attraction can make for a bound state of the $\Omega\pi$ system, we solve the RGM equation of bound state problem. The results show that in model I, i.e. the chiral SU(3) quark model, the $\Omega\pi$ state is unbound. However in model II and III, i.e. the extended chiral SU(3) quark model with $f_{chv}/g_{chv} = 0$ and $f_{chv}/g_{chv} = 2/3$, we get weakly bound states of $\Omega\pi$ with the binding energies of about 2 and 0.5 MeV, respectively. Actually, as seen in Fig. 1, the $\Omega\pi$ interaction in model II and III are more attractive than those in model I, thus in model II and III we can get the weakly $\Omega\pi$ bound states while in model I the $\Omega\pi$ state is unbound. These results are similar to those of the $N\phi$ systems [3].

As the same situation of the $N\phi$ state, in the $\Omega\pi$ system the OGE is not allowed between
TABLE II: The binding energy of $\Omega\pi$.

| $\chi$-SU(3) QM | Ex. $\chi$-SU(3) QM |
|------------------|----------------------|
| $f_{\text{chv}}/g_{\text{chv}} = 0$ | $f_{\text{chv}}/g_{\text{chv}} = 2/3$ |
| $B_{\Omega\pi}$ (MeV) | 2 | 0.5 |

$\Omega$ and $\pi$ since these two color-singlet clusters have no common flavor quarks, and the $\sigma$ exchange dominantly provides the $\Omega\pi$ attractive interaction in the chiral SU(3) quark model. At the same time, in the extended chiral SU(3) quark model there is no contribution from $\rho$, $\omega$ and $\phi$ exchanges, and the attraction in this special system also dominantly comes from $\sigma$ exchange. In our calculation, the model parameters are fitted by the $NN$ scattering phase shifts, and the mass of $\sigma$ is adjusted by fitting the deuteron’s binding energy, thus the value of $m_{\sigma}$ is somewhat different for the cases I, II and III. In the cases II and III the masses of the $\sigma$ meson are smaller than that in case I, which means that in cases II and III the $\Omega\pi$ states get more attractions than that in case I, thus much more binding energies of $\Omega\pi$ are obtained in the extended chiral SU(3) quark model than in the chiral SU(3) quark model.

We also study the structures of $\Omega\omega$ and $\Omega\rho$ in the chiral quark models by using the same method. Our results show that in the extended chiral SU(3) quark model with $f_{\text{chv}}/g_{\text{chv}} = 0$, there is a weakly bound state of $\Omega\rho$ with 0.9 and 2.6 MeV binding energy for $J^P = 3/2^-$ and $J^P = 5/2^-$, respectively. The same results can be obtained in the $\Omega\omega$ system. That is, there are also weakly $\Omega\omega$ and $\Omega\rho$ bound states with $J^P = 3/2^-$ and $J^P = 5/2^-$ in the extended chiral SU(3) quark model. But in these two cases the channel-coupling effects may be important and un-negligible since the threshold of $\Xi^*K^*$ is very near to those of $\Omega\rho$ and $\Omega\omega$ and the $\Xi^*K$ and $\Xi K^*$ channels also lie below the $\Omega\rho$ and $\Omega\omega$ thresholds. Hence $\Omega\rho$ and $\Omega\omega$ might decay via strong interaction into other channels and consequently, their widths will be considerably broad and it might be hard to measure these states experimentally.

Here we’d like to point out that all the results of $\Omega\pi$, $\Omega\omega$ and $\Omega\rho$ states are quite similar to that of $N\phi$ [3]. This is because all these states have the similar configurations, i.e. they are composed of two color-singlet hadrons with no common flavor quarks and no OGE, and their attractions are mostly provided by the $\sigma$ exchange. Presently the coupling strength of quark and $\sigma$-field is still an open problem, and people usually use different values in different cases in order to fit the observables. Experimentally, the measurement of the $\Omega\pi$ state to
examine whether it is bound or not would be very important for getting more knowledge of the coupling between quark and $\sigma$ chiral field. We strongly call for experimentalists to search for this interesting $\Omega\pi$ state in the $e^+e^-$ collisions or heavy ion collisions.

IV. SUMMARY

In this work, we dynamically studied the $\Omega\pi$ state in the chiral SU(3) quark model as well as in the extended chiral SU(3) quark model by solving the RGM equation. All the model parameters are taken from our previous work, which can give a satisfactory description of the energies of the baryon ground states, the binding energy of the deuteron, and the $NN$ scattering phase shifts. The calculated results show that the $\Omega\pi$ state has an attractive interaction, which is dominantly provided by the $\sigma$ exchange. In the extended chiral SU(3) quark model, such an attractive interaction can make for an $\Omega\pi$ quasi-bound state with several MeV binding energy. Experimentally whether there is an $\Omega\pi$ quasi-bound state or resonance state can help us to test the strength of the coupling between the quark and $\sigma$ chiral field.

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