Branching ratio measurements of $B_s$ decays

Kristof De Bruyn,1 Robert Fleischer,1,2 Robert Knegjens,1 Patrick Koppenburg,1 Marcel Merk,1,2 and Niels Tuning1

1Nikhef, Science Park 105, NL-1098 XG Amsterdam, Netherlands
2Department of Physics and Astronomy, Vrije Universiteit Amsterdam, NL-1081 HV Amsterdam, Netherlands

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We have just entered an era of precision measurements for $B_s$-decay observables. A characteristic feature of the $B_s$-meson system is $B_s^0$-$B_s^0$ mixing, which exhibits a sizable decay width difference. The latter feature leads to a subtle complication for the extraction of branching ratios of $B_s$ decays from untagged data samples, leading to systematic biases as large as $O(10\%)$ that depend on the dynamics of the considered decay. We point out that this effect can only be corrected for using information from a time-dependent analysis and suggest the use of the effective $B_s$ decay lifetime, which can already be extracted from the untagged data sample, for this purpose. We also address several experimental issues that can play a role in the extraction of effective lifetimes at a hadron collider, and advocate the use of the $B_s$ branching ratios, as presented in this note, for consistent comparisons of theoretical calculations and experimental measurements in particle listings.

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I. INTRODUCTION

Weak decays of $B_s$ mesons encode valuable information for the exploration of the standard model. The simplest observables are branching ratios, which give the probability of the considered decay to occur. Measurements of $B_s$ branching ratios at hadron colliders, such as Fermilab’s Tevatron and CERN’s Large Hadron Collider (LHC), would require knowledge of the $B_s$ production cross-section, which presently makes absolute branching ratio measurements impossible. Hence, experimental control channels and the ratio of the $f_s/f_{u,d}$ fragmentation functions, describing the probability that a $b$ quark hadronizes as a $\bar{B}_q$ meson [1], are required for the conversion of the observed number of decays into the branching ratio. At $e^+e^-B$ factories operated at the $\Upsilon(5S)$ resonance, the total number of produced $B_s$ mesons is measured separately and subsequently also allows for the extraction of the $B_s$ branching ratio from the data [2].

A key feature of the $B_s$ mesons is $B_s^0$-$\bar{B}_s^0$ mixing, which leads to quantum-mechanical, time-dependent oscillations between the $B_s^0$ and $\bar{B}_s^0$ states. In contrast to the $B_d$ system, the $B_s$ mesons exhibit a sizable difference between the decay widths of the light and heavy mass eigenstates, $\Gamma_{L}^{(s)}$ and $\Gamma_{H}^{(s)}$, respectively [3]. Currently the most precise measurement is extracted from the $B_s^0 \to J/\psi \phi$ channel by the LHCb Collaboration [4]:

$$\frac{\Delta \Gamma_s}{\Gamma_s} = \frac{\Gamma_{L}^{(s)} - \Gamma_{H}^{(s)}}{2 \Gamma_s} = 0.088 \pm 0.014; \quad (1)$$

$$\tau_{B_s^{-1}} = \tau_s = \frac{\Gamma_{L}^{(s)} + \Gamma_{H}^{(s)}}{2} = 0.6580 \pm 0.0085 \text{ ps}^{-1}$$

is the inverse of the $B_s$ mean lifetime $\tau_{B_s}$.

In view of the sizable decay width difference, Eq. (1), special care has to be taken when dealing with the concept of a branching ratio. We shall clarify this issue and give an expression, allowing us to convert the experimentally measured $B_s$ branching ratio into the corresponding “theoretical” branching ratio. The latter is not affected by $B_s^0$-$\bar{B}_s^0$ mixing and encodes the information for the comparison with branching ratios of $B_s^0$ decays, where the relative decay width difference at the $10^{-3}$ level [3] can be neglected, or branching ratios of $B_s^0$ modes.

The difference between these two branching ratio concepts involves $y_s$ and is specific for the considered $B_s$ decay, thereby involving nonperturbative parameters. However, measuring the effective lifetime of the considered $B_s$ decay, the effect can be included in a clean way.

In experimental analyses, this subtle effect has so far been neglected or only been partially addressed; examples are the branching ratio measurements of the $B_s \to K^+K^-$ [5], $B_s \to J/\psi f_0(980)$ [6], $B_s \to J/\psi K_S$ [7], $B_s \to D^+_sD^-_s$ [8] and $B_s^0 \to D^- \pi^+$ [9] decays by the LHCb, CDF, D0 and Belle Collaborations.

II. EXPERIMENT VERSUS THEORY

What complicates the concept of a $B_s$ branching ratio is the fact that the untagged decay rate is the sum of two exponentials [10]:

$$\Gamma(B_s(t) \to f)) = \Gamma(B_s^0(t) \to f) + \Gamma(B_s^0(t) \to f) \quad = R^f e^{-\Gamma_{L}^{(s)} t} + R^f e^{-\Gamma_{H}^{(s)} t}$$

corresponding to two mass eigenstates with different lifetimes. Using Eq. (1), we write

$$\Gamma(B_s(t) \to f)) = (R^f + R^f) e^{-\Gamma_{st}} \cdot \left[ \cosh\left(\sqrt{\frac{\Delta \Gamma_s}{\tau_s}} t\right) + \mathcal{A}_{\Delta \Gamma} \sinh\left(\sqrt{\frac{\Delta \Gamma_s}{\tau_s}} t\right) \right]$$

where
\[ \mathcal{A}_{\Delta f} = \frac{R_H - R_L}{R_H + R_L} \]  

(4) is a final-state dependent observable.

In experiment, it is common practice to extract a branching ratio from the total event yield, ignoring information on the particles’ lifetime. The “experimental” branching ratio can thus be defined as follows [10]:

\[ \text{BR}(B_s \rightarrow f)_{\text{exp}} = \frac{1}{2} \int_0^\infty \langle \Gamma(B_s(t) \rightarrow f) \rangle dt = \frac{1}{2} \left[ \frac{R_H^f}{\Gamma_H^f} + \frac{R_L^f}{\Gamma_L^f} \right] \]

\[ = \frac{\tau_{B_s}}{2} (R_H^f + R_L^f) \left[ 1 + \mathcal{A}_{\Delta f} y_s \right]. \]

(5) Note that this quantity is the average of the branching ratios for the heavy and light mass eigenstates.

On the other hand, what is generally calculated theoretically are \( CP \)-averaged decay rates in the flavor-eigenstate basis, i.e.

\[ \langle \Gamma(B_s(t) \rightarrow f) \rangle|_{t=0} = \Gamma(B_s^0 \rightarrow f) + \Gamma(B_s^- \rightarrow f). \]

(6) This leads to the following definition of the theoretical branching ratio:

\[ \text{BR}(B_s \rightarrow f)_{\text{theo}} = \frac{\tau_{B_s}}{2} \langle \Gamma(B_s^0(t) \rightarrow f) \rangle|_{t=0} \]

\[ = \frac{\tau_{B_s}}{2} (R_H^f + R_L^f). \]

(7) By considering \( t = 0 \), the effect of \( B_s^0\bar{B}_s^0 \) mixing is “switched off.” The advantage of this \( B_s \) branching ratio definition, which has been used, for instance, in Refs. [11,12], is that it allows a straightforward comparison with branching ratios of \( B_s^0 \) or \( B_s^- \) mesons by means of the \( SU(3) \) flavor symmetry of strong interactions.

The experimentally measurable branching ratio, Eq. (5), can be converted into the theoretical branching ratio defined by Eq. (7) through

\[ \text{BR}(B_s \rightarrow f)_{\text{theo}} = \left[ \frac{1 - y_s^2}{1 + \mathcal{A}_{\Delta f} y_s} \right] \text{BR}(B_s \rightarrow f)_{\text{exp}}. \]

(8) In the case of \( y_s = 0 \), the theoretical and experimental branching ratio definitions are equal.

Inspection of Eq. (8) reveals that \( y_s \) and \( \mathcal{A}_{\Delta f} \) are required for the translation of the experimental branching ratios into their theoretical counterparts. Ideally, the latter quantities should eventually be used in particle compilations, in our opinion.

The decay width parameter \( y_s \) is universal and has already been measured, as summarized in Eq. (1). In Fig. 1, we illustrate Eq. (8) for a variety of values of \( \mathcal{A}_{\Delta f} \) and observe that differences between \( \text{BR}(B_s \rightarrow f)_{\text{theo}} \) and \( \text{BR}(B_s \rightarrow f)_{\text{exp}} \) as large as \( \mathcal{O}(10\%) \) may arise.

The simplest situation corresponds to flavor-specific decays such as \( B_s^0 \rightarrow D_s^- \pi^+ \), where \( \mathcal{A}_{\Delta f}^{FS} = 0 \) and the correction factor is simply given by \( 1 - y_s^2 \).

However, if both the \( B_s^0 \) and the \( B_s^- \) mesons can decay into the final state \( f \), the observable \( \mathcal{A}_{\Delta f} \) is more involved and depends, in general, on nonperturbative hadronic parameters, \( CP \)-violating weak decay phases and the \( B_s^0\bar{B}_s^0 \) mixing phase \( \phi_s \). Assuming the standard model structure for the decay amplitudes and using the \( SU(3) \) flavor symmetry to determine the hadronic parameters from relations to \( B_d \) decays, theoretical analyses of \( \mathcal{A}_{\Delta f}^{FS} \) were performed for the final states \( J/\psi \phi [12], K^+ K^- [13], J/\psi f_0(980) [14], J/\psi K_S [15] \) and \( D_s^+ D_s^- \) [16].

III. USING LIFETIME INFORMATION

The simplest possibility for implementing Eq. (8) is to use theoretical information about the \( \mathcal{A}_{\Delta f} \) observables. However, this input can be avoided once time information of the untagged \( B_s \) decay data sample becomes available. Then, the effective lifetime of the \( B_s \rightarrow f \) decay can be determined, which is theoretically defined as the time expectation value of the untagged rate [17]:

\[ \tau_f = \frac{\tau_{B_s}}{2} \left[ \frac{1 + \mathcal{A}_{\Delta f} y_s}{1 - y_s^2} \right] \]

(9) The advantage of \( \tau_f \) is that it allows an efficient extraction of the product of \( \mathcal{A}_{\Delta f}^{FS} \) and \( y_s \). Using the effective lifetime, Eq. (8) can be expressed as

\[ \text{BR}(B_s \rightarrow f)_{\text{theo}} = \left[ 2 - (1 - y_s^2) \frac{\tau_f}{\tau_{B_s}} \right] \text{BR}(B_s \rightarrow f)_{\text{exp}}. \]

(10)
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TABLE I. Factors for converting $\text{BR}(B_s \to f)_{\text{exp}}$ [see Eq. (5)] into $\text{BR}(B_s \to f)_{\text{theo}}$ [see Eq. (7)] by means of Eq. (8) with theoretical estimates for $A^{f}_{\Delta f}$. Whenever effective lifetime information is available, the corrections are also calculated using Eq. (10).

| $B_s \to f$ | $\text{BR}(B_s \to f)_{\text{exp}}$ (measured) | $A^{f}_{\Delta f}$ (SM) | $\text{BR}(B_s \to f)_{\text{theo}} / \text{BR}(B_s \to f)_{\text{exp}}$ |
|----------------|--------------------------------------|----------------------|--------------------------------------|
| $J/\psi f_3(980)$ | (1.29$^{+0.40}_{-0.28}$) $\times$ 10$^{-4}$ [18] | 0.9984 $\pm$ 0.0021 [14] | 0.912 $\pm$ 0.014 | 0.890 $\pm$ 0.082 [6] |
| $J/\psi K_S$ | (3.5 $\pm$ 0.8) $\times$ 10$^{-5}$ [7] | 0.84 $\pm$ 0.17 [15] | 0.924 $\pm$ 0.018 | N/A |
| $D^+_s \pi^+$ | (3.01 $\pm$ 0.34) $\times$ 10$^{-3}$ [9] | 0 (exact) | 0.992 $\pm$ 0.003 | N/A |
| $K^+ K^-$ | (3.5 $\pm$ 0.7) $\times$ 10$^{-5}$ [18] | $-0.972 \pm 0.012$ [13] | 1.085 $\pm$ 0.014 | 1.042 $\pm$ 0.033 [19] |
| $D^+_s D^-_s$ | (1.04$^{+0.29}_{-0.36}$) $\times$ 10$^{-2}$ [18] | $-0.995 \pm 0.013$ [16] | 1.088 $\pm$ 0.014 | N/A |

Note that on the right-hand side of this equation, only measurable quantities appear and that the decay width difference $\gamma_s$ enters at second order. The measurement of effective lifetimes is hence not only an interesting topic for obtaining constraints on the $B_s^0$, $\bar{B}_s^0$ mixing parameters [17], but an integral part of the determination of the theoretical $B_s$ branching ratios from the data. In Table I, we list the correction factors for converting the experimentally measured branching ratios into the theoretical branching ratios for various decays. Here, we have used theoretical information for $A^{f}_{\Delta f}$ and Eq. (8), or—if available—the effective decay lifetimes and Eq. (10).

The rare decay $B_s^0 \to \mu^+ \mu^-$, which is very sensitive to new physics [20], is also affected by $\Delta \Gamma_s$. In Ref. [21], we give a detailed discussion of this key $B_s$ decay, showing that the helicities of the muons need not be measured to deal with this problem, and that $\Delta \Gamma_s$ actually offers a new window for new physics in $B_s^0 \to \mu^+ \mu^-$.\index{CP violation!$B_s$ decay $B_s^0 \to \mu^+ \mu^-$}

IV. $B_s \to VV$ DECAYS

Another application is given by $B_s$ transitions into two vector mesons, such as $B_s \to J/\psi \phi$ [22], $B_s \to K^{0}\bar{K}^{*0}$ [23] and $B_s \to D^{*+} \bar{D}^{*-}$ [8]. Here, an angular analysis of the decay products of the vector mesons has to be performed to disentangle the CP-even and CP-odd final states, which affects the branching fraction determination in a subtle way, as recognized in Refs. [23,24]. Using linear polarization states $0, ||$ with CP eigenvalue $\eta_k = +1$ and $\perp$ with CP eigenvalue $\eta_k = -1$ [25], the generalization of Eq. (8) is given by

$$\text{BR}^{VV}_{\text{theo}} = (1 - y_s^2) \left[ \sum_{k=0, ||, \perp} \frac{f^{VV,k}_{\text{exp}}}{1 + \gamma_s \Delta A^{VV,k}_{\Delta f}} \right] \text{BR}^{VV}_{\text{exp}}, \quad (11)$$

where $f^{VV,k}_{\text{exp}} = \text{BR}^{VV,k}_{\text{exp}} / \text{BR}^{VV}_{\text{exp}}$ and $\Delta A^{VV,k}_{\Delta f} = \sum_k \text{BR}^{VV,k}_{\text{exp}}$ so that $\sum_k f^{VV,k}_{\text{exp}} = 1$. As discussed in Ref. [17], assuming the standard model structure for the decay amplitudes, we can write

$$A^{VV,k}_{\Delta f} = -\eta_k \sqrt{1 - C^{2}_{VV,k} \cos(\phi_s + \Delta \phi_{VV,k})}, \quad (12)$$

where $C_{VV,k}$ describes direct CP violation, $\phi_s$ is the $B_s^0$, $\bar{B}_s^0$ mixing phase and $\Delta \phi_{VV,k}$ is a nonperturbative hadronic phase shift. The expressions given in Ref. [23] for the $B_s \to K^{0}\bar{K}^{*0}$ decay take the leading order effect of $\gamma_s$ into account and assume $\phi_s = 0$ and negligible hadronic corrections.

The generalization of Eq. (10) is given by

$$\text{BR}^{VV}_{\text{theo}} = \text{BR}^{VV}_{\text{exp}} \sum_{k=0, ||, \perp} \left[ 2 - (1 - y_s^2) \frac{\tau_k}{\tau_{B_s}} \right] f^{VV,k}_{\text{exp}} \quad (13)$$

and does not require knowledge of the $A^{VV,k}_{\Delta f}$ observables.

V. EXPERIMENTAL ASPECTS

Additional subtleties arise in the experimental determination of $B_s$ branching ratios and effective lifetimes, in particular, at a hadron collider environment where many final-state particles are produced in the fragmentation.

Separating $B_s$ signal decays from the background typically involves selection criteria which use the flight distance of the $B_s$ meson or the impact parameter of its decay products, leading to a decay-time dependent efficiency. By rejecting short-living $B_s$ meson candidates, the relative amounts of $B_sL$ and $B_s\perp$ mesons in the remaining data sample are altered, resulting in a biased result for the branching ratio determination. The extrapolation of the event yield to full acceptance is usually obtained from simulation, but this requires a priori assumptions of the values for $y_s$ and $A^{f}_{\Delta f}$. For example, the dependence of the branching fraction correction on the value $A^{f}_{\Delta f}$ can be several percent if only decay times greater than 0.5 ps are considered. This systematic uncertainty is avoided by tuning the simulation using the measured value of the effective lifetime.

Furthermore, the presence of remaining background events with a different observed decay time distribution as the signal implies that it is experimentally unpractical to determine the time expectation value $\tau_f$ of the untagged rate as given in Eq. (9). Instead, the effective lifetime is commonly extracted by fitting a single exponential function to the untagged rate [6,19,26], which in general is described by two exponentials [see Eq. (2)]. In the Appendix, we demonstrate that such a fitting procedure leads to an unbiased determination of the effective lifetime in the case of a log likelihood fit and to a small bias for a $\chi^2$ minimization procedure.
VI. CONCLUSIONS

The established width difference of the $B_s$ mesons complicates the extraction of branching ratio information from the experimental data, leading to biases at the 10% level which depend on the specific final state. On the one hand, these effects can be included through theoretical considerations and phenomenological analyses. On the other hand, it is also possible to take them into account through the measurement of the effective $B_s \to f$ decay lifetimes, which is the preferred avenue. So far, these effects have not, or have only partially, been included, and we advocate to use the converted branching ratios for comparisons with theoretical calculations in particle listings.

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APPENDIX: EFFECTIVE LIFETIME FITS

An effective lifetime for a $B_s$ decay channel is obtained in practice by fitting a single exponential function to its untagged rate. As an untagged rate is in general described by two exponentials, corresponding to two mass eigenstates with different lifetimes, the single exponential fit is an approximation.

In order to find analytic expressions for the fitted effective lifetime $\tau_{eff}$, we let the untagged rate be the true probability distribution function, and the single exponential function the fitted probability distribution function, such that

\[ f_{\text{true}}(t) = \frac{A(t)\Gamma(t)}{\int_0^\infty A(t)\Gamma(t)dt}; \quad (A1) \]

\[ f_{\text{fit}}(t; \tau_{eff}) = \frac{A(t)e^{-t/\tau_{eff}}}{\int_0^\infty A(t)e^{-t/\tau_{eff}}dt}; \quad (A2) \]

where $A(t)$ is an acceptance efficiency function. The likelihood or $\chi^2$ function for the fit in question is then built using the above probability distribution functions, and maximized or minimized, respectively, in the limit of infinitesimally spaced bins. Specifically, for $n$ events, we minimize the functions:

\[ -\log L(\tau_{eff}) = -n \int_0^\infty f_{\text{true}}(t) \log[f_{\text{fit}}(t; \tau_{eff})]dt, \quad (A3) \]

\[ \chi^2(\tau_{eff}) = n \int_0^\infty \frac{[f_{\text{true}}(t) - f_{\text{fit}}(t; \tau_{eff})]^2}{f_{\text{fit}}(t; \tau_{eff})} dt, \quad (A4) \]

for a maximum likelihood and a least-squares fit, respectively. In a modified least-squares fit, where data is used to estimate the error, the denominator in the $\chi^2$ integrand should be replaced by $f_{\text{true}}(t)$. For the maximum likelihood fit, taking the infinitesimal bin limit is equivalent to an unbinned fit.

The effective lifetime $\tau_{eff}$ resulting from these fits is then given implicitly by the formula

\[ \frac{\int_0^\infty te^{-t/\tau_{eff}}A(t)dt}{\int_0^\infty e^{-t/\tau_{eff}}A(t)dt} = \frac{\int_0^\infty g(t; \tau_{eff})A(t)dt}{\int_0^\infty A(t)dt}; \quad (A5) \]

where

\[ g(t; \tau_{eff}) \equiv \begin{cases} (\Gamma(t)) : \text{maximum likelihood} \\ (\Gamma(t))^2 e^{t/\tau_{eff}} : \text{least squares} \\ (\Gamma(t))^{-1} e^{2t/\tau_{eff}} : \text{modified least squares}. \end{cases} \]

The effective lifetime definition given in Eq. (9) is reproduced for the untagged rate given in Eq. (3) if we assume a trivial acceptance function, $A(t) = 1$, and apply a maximum-likelihood fit. For nonzero values of $y_s$, the least-squares fits give different analytic expressions for the effective lifetime. Fortunately, for the current experimental range of $y_s$, the differences are of the order 0.1%.

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