Joint Upper & Lower Bound Normalization for IR Evaluation

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In this paper, we present a novel perspective towards IR evaluation by proposing a new family of evaluation metrics where the existing popular metrics (e.g., nDCG, MAP) are customized by introducing a query-specific lower-bound (LB) normalization term. While original nDCG, MAP etc. metrics are normalized in terms of their upper bounds based on an ideal ranked list, a corresponding LB normalization for them has not yet been studied. Specifically, we introduce two different variants of the proposed LB normalization, where the lower bound is estimated from a randomized ranking of the corresponding documents present in the evaluation set. We next conducted two case-studies by instantiating the new framework for two popular IR evaluation metric (with two variants, e.g., DCG^{UL}_{V_{1,2}} and MSP^{UL}_{V_{1,2}}) and then comparing against the traditional metric without the proposed LB normalization. Experiments on two different data-sets with eight Learning-to-Rank (LETOR) methods demonstrate the following properties of the new LB normalized metric: 1) Statistically significant differences (between two methods) in terms of original metric no longer remain statistically significant in terms of Upper Lower (UL) Bound normalized version and vice-versa, especially for uninformative query-sets. 2) When compared against the original metric, our proposed UL normalized metrics demonstrate higher Discriminatory Power and better Consistency across different data-sets. These findings suggest that the IR community should consider UL normalization seriously when computing nDCG and MAP and more in-depth study of UL normalization for general IR evaluation is warranted.

CCS Concepts: • Information systems → Evaluation of retrieval results.

Additional Key Words and Phrases: Information Retrieval, Evaluation, Upper Lower Bound, Normalization

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1 INTRODUCTION

Empirical evaluation is a key challenge for any information retrieval (IR) system. The success of an IR system largely depends on the user’s satisfaction, thus an accurate evaluation metric is crucial for measuring the perceived utility of a retrieval system by the real users. While original nDCG [19], MAP [11] etc. metrics are normalized in terms of their query-specific upper bounds based on an ideal ranked list, a corresponding query-specific LB normalization for them has not yet been studied. For instance, the normalization term in nDCG computation is the Ideal DCG at cut-off k, which converts the metric into the range between 0 and 1. On the other hand, MAP is

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normalized by the maximum possible Sum of Precision (SP) scores at cut-off $k$. Thus, Ideal DCG and Sum of Precision (SP) scores essentially serve as the query-specific upper-bound normalization factor for metric $nDCG$ and $MAP$, respectively.

Interestingly, above two popular metrics do not include a similar query-specific lower-bound (LB) normalization factor (the current widely used assumption for lower-bound is zero across all queries). However, each query is different in terms of its difficulty (informative/ uninformative/ distractive), user’s intent (exploratory/ navigational), distribution of relevance labels of its associated documents (hard/ easy) and user’s perceived utility at different cut-off $k$, essentially implying different low-bounds for each of them. Therefore, an accurate estimation of an evaluation metric should not only involve an upper-bound normalization (e.g., Ideal DCG, SP etc.), but also a proper query-specific lower-bound normalization.

Consider the case of re-ranking where an initial filtering has already been performed given a query and as expected, a large number of associated documents in the filtered set are highly relevant. In this case, even just a random ranking of those documents will yield a high accuracy as most of the documents are highly relevant anyway. This means that even if a ranker does not learn anything meaningful and merely ranks documents randomly, it can still achieve a very high score in terms of the original metric. In other words, the expected value/lower-bound of the original metric in this case is very high because of the skewed relevance label distribution of the associated documents and this factor should be accounted for while measuring the ranker’s quality. In summary, a proper lower-bound normalization is essential for IR evaluation metrics to accurately measure the quality of a ranker as well as for a fairer comparison across multiple ranking methods.

What does query-specific lower-bound normalization mean for an IR evaluation metric? How can we come up with a more realistic lower-bound for each query and include it with the original IR metric computation? One way to address this issue is to introduce a penalty term inside the formula of different IR evaluation metrics which will penalize queries with high expected values of the same metric. In other words, given a query, we propose to use the expected value of the particular evaluation metric as a query-specific lower-bound of the same metric for that query, which can yield customized lower-bounds for different queries and thus, ensure fairer treatment across all queries with different difficulty levels.

With the observation that both $nDCG$ and $MAP$ metrics only involve query-specific upper-bound normalization (e.g., normalization with ideal DCG for $nDCG$ computation, while MAP is normalized by the maximum possible Sum of Precision); none of them include a query-specific lower-bound normalization. In this paper, we proposed a new general framework for IR evaluation with both upper and lower-bound normalization and instantiated the new framework for two popular IR evaluation metric: $nDCG$ and $MAP$ by computing a more reasonable(non-zero) lower-bound. Specifically, we introduce two different variants of the framework, i.e., $V_1$, $V_2$, which are essentially two different ways to introduce penalty in terms of normalization with a query-specific upper and lower-bound of the metric (see section 4 for more details). We then show how we can compute a more realistic query-specific lower-bound for the two metrics by computing its expected value for each query in case of a randomized ranking of the corresponding documents, and then, use this lower-bound as a penalty term while computing the new metric. The intuition here is that an intelligent ranking method should perform at least as good as a random-ranking algorithm, which naturally inspired us to use the expected value in case of random ranking as our lower-bound. Finally, for each metric we also theoretically prove the correctness the expected lower-bound (Derivation details can be found in each case-study sections).

Next, we investigated the implications of upper lower-bound normalization on the original IR metric. How it may impact IR evaluation in general and more importantly, which metric is better? Why should we care? To answer these questions, we have conducted extensive experiments on two
popular Learning-to-Rank (LETOR) data-sets with eight LETOR methods including RankNet [9], RankBoost [15], AdaRank [48], Random Forest [5], LambdaMART [8], CoordinateAscent [30], ListNet [10] and L2 regularized Logistic Regression [14, 28]. Experimental results demonstrate that a significant portion of the queries in popular benchmark data-sets produced a high LB normalization factor, verifying that LB normalization can indeed alter the relative ranking of multiple competing methods (confirmed by Kendall’s τ tests [36, 38]) and thus, should not be ignored. At the same time, for a number of closely performing LETOR method-pairs, statistically significant differences in terms of original metric no longer remain statistically significant in terms of LB normalized metric and vice-versa, especially for uninformative query-sets (see section 3.3 for a concrete definition), suggesting LB normalization yields different conclusions than the original metric.

Next, we compare original metric against Upper lower bound normalized version from two perspectives: Distinguishability and Consistency. In case of discriminative power, we followed Sakai [36, 39] to use student’s t-test as well as computed "Percentage Absolute Differences" to quantify distinguishability and found that UL bound normalized version can better distinguish between two closely performing LETOR methods in case of uninformative queries. For consistency, we performed swap rate tests and and found that for MSPUL provide a better performance in terms of Consistency while DCGUL does not compromise in terms of Consistency.

These findings suggest that the community should rethink about IR evaluation and consider LB normalization seriously. In summary, we make the following contributions in the paper:

(1) We propose an extension of traditional IR evaluation metrics which includes a lower bound (LB) normalization term, and systematically perform two case-studies by showing how LB normalization can be materialized for nDCG and MAP.
(2) We propose two different variants of the proposed upper lower-bound normalized version for two popular IR evaluation metrics.
(3) We show how we can compute a more realistic query-specific lower-bound for two IR evaluation metrics by computing its expected value for each query in case of a randomized ranking of the document collection and also theoretically prove its correctness.
(4) We conducted extensive experiments to understand the implications of LB normalized metric and compared our proposed metric against the original metric from two important perspectives: Distinguishability and Consistency.
(5) Our proposed framework is very general and can be easily extended to other IR evaluation metrics or evaluation metrics in other domain.

The rest of the paper is organized as follows: Section 2 reviews related works from the past literature. Section 3 provides essential background about our two experimental metric computation and motivation for lower-bound normalization. In section 4, we first present the framework with query-specific upper and lower bound normalization. Section 4.3 presents the experiment details and results. Finally, section 5 concludes our paper with discussions and possible future directions.

2 RELATED WORK

Traditional IR evaluation metric: Many metrics have been introduced for IR system evaluation [22, 29] in recent years. Two most frequent and basic metrics for the performance evaluation of IR system are precision and recall, especially for extraction tasks [25, 41]. Empirical studies of retrieval performance have shown a tendency for precision to decline as recall increases [6]. Due to the trade-off between the two basic calculations, researchers also use other complex single metrics such as F-measure which can evenly weight the precision and recall. Other popular metrics such as MAP (Mean Average Precision) and Normalized Discounted Cumulative Gain (nDCG) are also
widely used as offline evaluation standards. Different metrics have different hyper-parameters for users to choose based on their own preferences.

**nDCG:** nDCG is the normalized version of Discounted Cumulative Gain (DCG), where the normalization term is essentially a query-specific upper-bound (i.e., normalization with Ideal DCG), which converts the metric into the range between 0 and 1 [19]. It has become one of the most important metrics because it can be applied to multi-level relevance judgments and is sensitive to small changes in a ranked list, and it has become the most popular measure for evaluating Web search and Learning-to-Rank algorithms [44]. Many researchers have investigated its properties (see, e.g., [34, 46, 50]). The fact that the general concept of nDCG can be implemented in a variety of ways was recognized in the previous work [23], where the authors scrutinized how to choose from a variety of discounting functions and different ways of designing the gain function to optimize the efficiency or stability of nDCG [26]. Previous research has also shown that with different gain functions, nDCG may lead to different results and the discounting coefficients do make a difference in evaluation results as compared to using uniform weights [45]. Regarding nDCG cutoff-depths, Sakai and others [37] have researched the reliability of nDCG by establishing that it is highly correlated with average precision if the cutoff-depth $k$ is big enough. According to a recent research [24], conventional nDCG score results in significant variance in response to the $k$ value and urged for query-specific customization of nDCG to acquire more trustworthy conclusions. Additionally, Lukas et.al [18] proposed a measure to explicitly reflect a system’s divergence by comparing the query-level nDCG with a randomized ranked nDCG, which they called $\text{RNDCG}$. They claimed that this measure can capture the general trend of query difficulty by the ratio-based score and further improving several issues such as selecting a specific set of query [18].

**MAP:** Average precision (AP) is one of most commonly used indicator for evaluating ranked output in IR experiments for a number of reasons as it already known to be stable [7] and and highly informative measure [2]. Whereas Mean Average Precision (MAP) [11] is the average of AP of each class which can reflect the overall performance among multiple topics. However, the main criticism to MAP is that it is based on the assumption that retrieved documents can be considered as either relevant or non-relevant to user’s information need, which is not accurate. Previous researchers have studied the properties of MAP in terms of different relevance judgement. Yilmaz et.al [49], for instance, proposed different variant of AP for addressing incomplete and imperfect relevance judgements, where they consider the document collection is dynamic, as in the case of web retrieval, and they use a expectation of randomly sample from the depth-100 pool. Furthermore, [35] proposed a extended Average Precision called Graded Average Precision (GAP) which can tackle the cases of multi-graded relevance.

**Query Specific Customization for General IR Evaluation:** Previous work has explored how to incorporate query specific customization for IR evaluation metrics in general. For example, Moffat et.al. [31] followed by Bailey et.al. [3] argued that user behavior varies on a per-topic basis depending on the nature of the underlying information need, and hence that it is natural to expect that evaluation parameterization should also be variable. Billerbeck et.al. studied the optimal number of top-ranked documents that should be used for extraction of terms for expanding a query [4]. Such work has shown the need to adapt a ranking function to each individual query. Egghe et.al. [13] demonstrated precision, recall, fallout and miss as a function of the number of retrieved documents and their mutual interrelations. Kuzi et.al. [27] presented a Best-Feature Calibration (BFC) strategy for analyzing learning to rank models and used this strategy to examine the benefit of query-level adaptive training, which demonstrated the importance of query-specific parameters in IR evaluation once again.
IR Evaluation with Variable Parameterization: Query specific customization can be viewed as a special case of variable parameterization for IR evaluation metrics, which has been explored previously. Webber et.al. [47] explored the role that the metric evaluation depth $k$ plays in affecting metric values and system-versus-system performances for two popular families of IR evaluation metrics: i.e., recall-based and utility-based metrics. Study by Jiang et.al. [21] showed that the adaptive effort metrics can better indicate user’s search experience compared with conventional metrics. Yilmaz et al. showed users are more likely to click on relevant results [51] and also examined the differences between searcher’s effort (dwell time) and assessor’s effort (judging time) on results, and features predicting such effort [52]. Sakai et.al. [40] modeled a user population to assess the appropriateness of different evaluation metrics. 

Distinction from prior work: Our work completely differs from the previous effort as our goal is to investigate the impact of lower-bound normalization on the prominent evaluation metrics. To the best of our knowledge, there has never been a systematic study of query-specific lower-bound normalization for IR evaluation metrics. Furthermore, our work is groundbreaking in that it proposes a generic upper and lower-bound (UL) normalization framework and effectively applies it to two prominent evaluation metrics. We additionally compute an expectation over a randomized ranked list to estimate a more realistic lower-bound and also give the derivation. Our research clearly articulates the effects of such lower-bound normalization on two popular evaluation metrics and lays the foundation for future research in this direction.

3 REVISITING ORIGINAL METRICS

In this section, we provide some essential background about $nDCG$ and $MAP$ computation and also provide our motivation of lower-bound normalization for the two metrics.

3.1 Computation of the standard $nDCG$

The principle behind Normalized Discounted Cumulative Gain ($nDCG$) is that documents appearing lower in a search result list should contribute less than similarly relevant documents that appear higher in the results [19]. This is accomplished by introducing a penalty term that penalizes the gain value logarithmically proportional to the position of the result [46]. Mathematically:

$$DCG@k = \sum_{i=1}^{k} \frac{2^{R_i} - 1}{\log_b (i + 1)}$$  \hspace{1cm} (1)

Here, $i$ denotes the position of a document in the search ranked list and $R_i$ is the relevance label of the $i$ – $th$ document in the list, cutoff $k$ means $DCG$ accumulated at a particular rank position $k$, the discounting coefficient is to use a log based discounting factor $b$ to unevenly penalize each position of the search result. $nDCG@k$ is $DCG@k$ divided by maximum achievable $DCG@k$, also called $DCG(IDCG@k)$, which is computed from the ideal ranking of the documents with respect to the query.

$$nDCG@k = \frac{DCG@k}{IDCG@k}$$  \hspace{1cm} (2)

3.2 Computation of the standard $MAP$

For our second case study, we selected another popular evaluation metric called Mean Average Precision ($MAP$). In the field of information retrieval, precision is the fraction of retrieved documents that are relevant to the query. The formula is given by: $Prec = TP/(TP + FP)$, where, $TP$ and $FP$ stands for True Positive and False Positive, respectively. Precision at cutoff $k$ is the precision calculated
by only considering the subset of retrieved documents from rank 1 through \( k \). However, the original precision metric is not sensitive to the relative order of the ranked documents, hence, we do not consider it for our exploration.

A related popular metric, which is sensitive to the relative order of the ranked documents, is **Average Precision**, which computes the sum of precision scores at each rank where the corresponding retrieved document is relevant to the query.

\[
AP@k = \frac{1}{k} \sum_{i=1}^{k} Prec(i) \cdot R_i
\]  

(3)

Here, \( R_i \) is an indicator variable that says whether \( i^{th} \) item is relevant (\( R_i = 1 \)) or non-relevant (\( R_i = 0 \)). From Formula 3, we can see \( AP@k \) is already normalized by the maximum possible \( Sum of Precision \) (SP), which is \( k \) in this case by assuming a precision value of 1.0 for every position from 1 to \( k \). Thus, \( AP@k \) is already upper-bound normalized version of \( SP@k \), like \( nDCG@k \) is for \( DCG@k \). Finally, Mean Average Precision (MAP) of a set of queries is defined by the following formula, where, \( |Q| \) is the number of queries in the set and \( AP(q) \) is the average precision (AP) for a given query \( q \).

\[
MAP = \frac{\sum_{q=1}^{|Q|} AP(q)}{|Q|}
\]

In summary, AP is essentially an upper-bound normalized version of Sum of Precision (SP), which is defined as follows:

**Sum of Precision (SP):** \( SP \) computes the summation of the precision scores at all ranks (from 1 to rank \( k \)), where the retrieved document is relevant to the query without any upper or lower bound normalization.

\[
SP@k = \sum_{i=1}^{k} Prec(i) \cdot R_i
\]  

(4)

### 3.3 Motivation for Lower-bound Normalization

A closer look into the formula of conventional \( nDCG \) and MAP shows that the two metrics incorporate only a query-specific upper-bound normalization (i.e., IDC is actually an upper-bound normalization term). However, as mentioned in section 1, each query is different in terms of difficulty (hard/ easy), informativeness (informative/ uninformative/ distractive), user’s intent (exploratory/ navigational); as such, they have different expected value for the lower-bound of different evaluation metric. Thus, an accurate estimation of average \( nDCG \) and MAP should include different lower-bounds for different queries.

The main motivation of our work is to relax the incorrect assumption of uniform lower-bound (of \( nDCG \) and MAP) across all queries while evaluating IR systems. We propose that an accurate evaluation metric should customize for each query and normalize with respect to both query-specific upper and lower-bound. A follow-up question that arises immediately is the following: How can we estimate a realistic lower-bound of an IR evaluation metric? While original implementation of above two metric assume zero as the lower bound, previous work proposed to use worst possible ranking score as the lower bound [17] to achieve a standardized range, we argue that this lower bound can be further constrained by using the score of a randomly ranked list for each query. The justification behind this choice is that a reasonable ranking function should be at least as good as the method that ranks documents merely randomly and should be penalized in cases where it performs worse than random.

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To better motivate LB normalization, we first define the following types of queries, which we will use throughout the rest of the paper:

(1) **Informative Queries:** These are queries where a reasonable ranking method performs significantly better than a pure random ranking system. Essentially, these are queries which contain the “right” keywords to find out the most relevant documents according to the user’s information need. Therefore, the actual evaluation metric scores are much higher than the expected lower-bound (the lower triangle region of the plot 1).

**Ideal Queries:** These are special cases of Informative queries where the difference between actual evaluation metric score and random ranked metric score (lower-bound) is the largest.

(2) **Uninformative Queries:** These are queries where a reasonable ranking method performs close to a pure random ranking system. In other words, these are queries which does not offer much value in finding out the most relevant documents. Therefore, the actual evaluation metric scores are similar to the expected lower-bound (region around the diagonal line). There are two special cases for Uninformative queries as defined below:

(a) **Hard Queries:** Hard queries are special cases of Uninformative queries, where both reasonable ranking methods as well as pure random ranking systems demonstrate poor performance. This usually happens in cases where there is no/very few relevant documents in the entire corpus.

(b) **Easy Queries:** Easy queries are special cases of Uninformative queries, where both reasonable ranking methods as well as pure random ranking systems demonstrate very high performance. This usually happens in cases where there is a lot of relevant documents in the corpus (for example, in case of re-ranking in multi-stage ranking systems[1, 12, 43]) and there is little room for improving beyond random ranking.

![Fig. 1. Query types with different Lower Bounds of evaluation metric.](image)
Figure 1 shows an illustration of different types of queries with different combinations of lower-bound evaluation metric and actual metric score. As apparent from Figure 1, the proposed LB normalization is expected to have large penalty on uninformative queries including special cases like hard queries (lack of relevant document scenarios) and easy queries (re-ranking scenarios). On the other hand, LB normalization will have minimal impact in case of ideal queries as the lower-bound tends to zero and actual metric score is very high. However, as demonstrated by our experiments, real-world queries are not ideal always and hence, a proper LB normalization is necessary while computing nDCG and MAP scores because: 1) It better captures the difficulty as well as variations across different queries. 2) It makes comparison and averaging across different queries fairer.

4 IR EVALUATION WITH JOINT UPPER & LOWER BOUND NORMALIZATION

Assume that $A@k$ is the standard evaluation metric and $k$ is the cutoff rank. Before introducing the generic IR evaluation framework with both upper & lower bound (UL) normalization, we first define the following terms.

- **IUB[A@k]**: Given a particular query and an associated collection of documents (each with a distinct relevance labels), $IUB[A@k]$ (Ideal Upper Bound for $A@k$) is the value that $A@k$ assumes in case of perfect ranking of the document collection.
- **RLB[A@k]**: Given a particular query and an associated collection of documents (each with a distinct relevance labels), $RLB[A@k]$ (Randomized Lower Bound for $A@k$) is the value that $A@k$ assumes in case of random ranking ($E[A@k]$) of the document collection.

**Upper-Bound Normalization**: Given a particular query and an evaluation metric $A@k$, Upper-bound normalization of the metric is defined as $[A@k]^U = \frac{A@k}{IUB[A@k]}$.

Now, we introduce two different variations of Joint Upper & Lower Bound Normalization, which is denoted by, $[A@k]^{UL}$. We call the two versions as $V_1$, $V_2$.

\[
[A@k]^{UL}_{V_1} = \left( \frac{A@k}{IUB[A@k]} \right) \left( \frac{A@k}{(A@k + RLB[A@k])} \right)
\]  

\[
[A@k]^{UL}_{V_2} = \begin{cases} 
\frac{A@k - RLB[A@k]}{IUB[A@k] - RLB[A@k]}, & \text{if } A \geq RLB \\
\frac{A@k - RLB[A@k]}{RLB[A@k]}, & \text{otherwise}
\end{cases}
\]

In the first Equation 5, we introduce a linear penalty term for Upper Lower Bound Normalization while in the second Equation 6 we introduce a non-linear penalty term. The intuition of above two Equation is that we want to penalize methods for queries where it performs close to a random ranking method, i.e., the difference between $A@k$ and $RLB[A@k]$ is minimal (the uninformative queries): $|A@k - A[A@k]| \equiv 0$. Even if a ranker achieves high $A@k$ in this case, it does not necessarily mean it is an “intelligent” ranker as the “vanilla” random ranking method can achieve similar performance as well. So, the reward for the method in this case should be discounted. Therefore, to truly distinguish between an “intelligent” and “vanilla” ranking method, it is important to penalize the traditional metric with a more realistic lower-bound, e.g., score w.r.t. a randomly ranked collection. In other words, for a ranking algorithm to claim a high $A@k$ score, it must perform significantly better than the random ranking baseline.
4.0.1 Range of LB normalized Metric: It should be noted that $V_1$ and $V_2$ are just two different ways to introduce the penalty for higher $RLB$ and obviously, more variants are possible while the basic idea remains the same. As can be seen from Equation 5, $V_1$ includes an additional multiplicative term that penalizes the original metric with the $RLB$ term in the denominator and the range of the metric is still bounded between 0 and 1. $V_2$ (Equation 6) works as follows: instead of range $[0, 1]$, it extends the range from negative to positive real numbers yielding negative numbers for a ranking method which performs worse than the random ranking baseline. In summary, for Equation 5, the range is still $[0, 1]$; while for Equation 6, the range of the metric is extended from $-1$ to $+1$ where, $+1$ means perfect ranking, $0$ means randomized ranking and $-1$ means all irrelevant results.

4.1 Data Set
We used two popular LETOR data-sets, i.e., ”MSLR-WEB30K” [33] and ”MQ2007” [32] for our experiments. The first and second data-set includes 30,000 and 1,700 queries respectively and have widely been used as benchmarks for LETOR tasks [16, 20, 42]. In these data-sets, each row corresponds to a query-document pair. The first column represents the relevance label of the pair, the second column is the query id, and the rest of columns represent features. The relevance scores are represented by an integer scale between 0 to 4 for ”MSLR-WEB30K” and between 0 to 2 for ”MQ2007”, where 0 means non-relevant and 4(2) means highly relevant. The larger the value of relevance label, the more relevant the query-document pair is. Features related to each query-document pair is represented by a 136 dimensional feature vector for ”MSLR-WEB30K” and 46 dimensional feature vector for ”MQ2007” data-set [24]. For more details on how the features were constructed, see [32] and [33].

We randomly sampled 10,000 queries from the “MSLR-WEB30K” and 1000 queries from “MQ2007” individually. For ”MSLR-WEB30K”, the average number of documents associated with each query was 119.06; while for ”MQ2007”, the number was 41.47. We kept all the features available (136 for ”MSLR-WEB30K” and 46 for ”MQ2007”) for all experiments conducted in this paper.

| Algorithm       | Short form | Algorithm       | Short form |
|-----------------|------------|-----------------|------------|
| RankNet [9]     | RNet       | LambdaMART [8]  | LMART      |
| RankBoost [15]  | RBoost     | CoordinateAscent [30] | CA |
| AdaRank [48]    | ARank      | ListNet [10]    | LNet       |
| Random Forest [5] | RF        | Logistic Regression [14] | L2LR |

Table 1. Popular learning to rank algorithms

4.2 Learning to Rank (LETOR) Methods
Table 1 contains eight prominent LETOR approaches along with popular classification and regression methods used for ranking applications. We also assign acronyms to each approach for notational convenience, which we will use throughout the rest of the paper.

4.3 Case Study 1: nDCG with Joint Upper & Lower Bound Normalization
In each case study sections we first describe how to compute a more realistic lower-bound for the corresponding metric, (nDCG for the first case study) i.e., the expected nDCG in case of random ranking. Although Lukas et.al [18] proposed to use the expectation to estimate this value, no derivation process provided. Note that, nDCG is already an upper-bound normalized version of DCG. Therefore, we start with the original metric DCG@k, where, $RLB[DCG@k]$ is the expected $DCG@k$ computed based on a randomly ranked list. Thus, we use the terms $E[DCG@k]$ and
RLB[DCG@k] interchangeably throughout the paper. Additionally, LB-normalized nDCG and upper lower bound(UL) normalized DCG also mean the same thing and we will use them interchangeably throughout the paper as well.

4.3.1 Expected DCG@k: Let R be a random variable denoting the relevance label of a query-document pair and R can assume values from a discrete finite set \( \phi = \{0, 1, 2, 3, \ldots, r\} \). Also let the current query be \( q \) and the total number of documents that needs to be ranked for the current query \( q \) is \( n \), let us denote this set by \( D_q \). To derive the formula of \( E[DCG@k] \), we start with the definition of expectation in probability theory.

\[
E[DCG@k] = E \left[ \sum_{i=1}^{k} \frac{2^R_i - 1}{\log_b(i + 1)} \right] = \sum_{i=1}^{k} \frac{E[2^R_i] - 1}{\log_b(i + 1)}
\]

So, the computation of \( E[DCG@k] \) is based on the computation of \( E[2^R_i] - 1 \), which is the expected relevance label of the retrieved document at position \( i \). Below we show how to estimate \( E[2^R_i] - 1 \) and first begin with the definition of expectation.

\[
E[2^R_i] - 1 = \sum_{j=0}^{r} (2^j - 1) \cdot Pr(R_i = j)
\]

Here, \( Pr(R_i = j) \) is the probability that the retrieved document at position \( i \) in a randomized ranking would assume a relevance label of \( j \) with respect to the current query. Let us assume that \( n_j \) be the number of documents with relevance label \( j \), where \( j \in \phi \), with respect to the current query. Thus, the constraint \( \sum_{j=1}^{r} n_j = n \) holds, where \( n \) is the total number of documents in \( D_q \). Thus, \( Pr(R_i = j) \) can essentially be computed by counting all the possible rankings which contain a document with relevance label \( j \) (with respect to the current query) at position \( i \) and dividing it by the total number possible rankings up-to position \( k \). Below we show the exact formula which is based on the permutation theory.

\[
E[2^R_i] - 1 = \sum_{j=0}^{r} (2^j - 1) \cdot \left[ \frac{n_j P_1 \cdot n_j 1 \cdot P_k - 1}{nP_k} \right] = \sum_{j=0}^{r} (2^j - 1) \cdot \left[ \frac{n_j!}{(n_j - 1)!} \cdot \frac{(n - 1)!}{(n - k)!} \cdot \frac{n!}{n!} \right]
\]

Note that, \( E[2^R] \) is different from \( E[2^R_i] - 1 \) because the former is independent of the position of a document in the ranked list, while later is dependent. However, the above derivation reveals that \( E[2^R_i] - 1 \) is indeed independent of the position \( i \) and equals to \( E[2^R - 1] \) for any \( i \). Thus, the final formula for computing \( E[DCG@k] \) boils down to the following formula:

\[
E[DCG@k] = E[2^R - 1] \cdot \sum_{i=1}^{k} \frac{1}{\log_2(i + 1)}
\]

(7)

4.4 Case-Study Observations

This section discusses some observed differences between the original \( nDCG \) and proposed \( DCG^{UL} \). For deeper analysis, we also created two special sub-sets of queries, i.e., 1) Uninformative query-set and 2) Ideal query-set, based on how close their average (of eight LETOR method and five cut-off k) Expected \( nDCG \) is to the average real \( nDCG \). To achieve this, we computed both average Expected
Joint Upper & Lower Bound Normalization for IR Evaluation

| Method | nDCG@ | 5   | 10  | 15  | 20  | 30  |
|--------|-------|-----|-----|-----|-----|-----|
| ARank  | 0.3218 | 0.3492 | 0.3704 | 0.3896 | 0.4237 |
| LNet   | 0.1534 | 0.1827 | 0.2066 | 0.2288 | 0.2686 |
| RBoost | 0.3062 | 0.3346 | 0.3578 | 0.3777 | 0.4141 |
| RF     | 0.3832 | 0.4118 | 0.4325 | 0.4493 | 0.4795 |
| RNet   | 0.154  | 0.1833 | 0.207  | 0.2292 | 0.269  |
| CA     | 0.3958 | 0.4138 | 0.4288 | 0.4428 | 0.4707 |
| L2LR   | 0.1977 | 0.2371 | 0.2696 | 0.2974 | 0.3444 |
| LMART  | 0.4365 | 0.4706 | 0.4856 | 0.513  |       |

Table 2. nDCG scores of different LETOR methods for variable k on MSLR-WEB30K data-set.

| Method | nDCG@ | 5   | 10  | 15  | 20  | 30  |
|--------|-------|-----|-----|-----|-----|-----|
| ARank  | 0.3881 | 0.4156 | 0.448 | 0.4797 | 0.5372 |
| LNet   | 0.3767 | 0.4035 | 0.4384 | 0.4687 | 0.5282 |
| RBoost | 0.3834 | 0.414  | 0.449 | 0.4807 | 0.5355 |
| RF     | 0.4035 | 0.4286 | 0.4609 | 0.4914 | 0.5476 |
| RNet   | 0.3809 | 0.4131 | 0.4451 | 0.4764 | 0.536  |
| CA     | 0.3928 | 0.4207 | 0.4544 | 0.4824 | 0.5399 |
| L2LR   | 0.3873 | 0.4159 | 0.4474 | 0.4779 | 0.538  |
| LMART  | 0.3931 | 0.4206 | 0.4535 | 0.4857 | 0.5441 |

Table 3. nDCG scores of different LETOR methods for variable k on MQ2007 data-set.

| Method | DCG_{V1}^{UL} | 5   | 10  | 15  | 20  | 30  |
|--------|---------------|-----|-----|-----|-----|-----|
| ARank  | 0.249         | 0.2616 | 0.2719 | 0.2818 | 0.2995 |       |
| LNet   | 0.0977        | 0.1124 | 0.1257 | 0.1384 | 0.1617 |       |
| RBoost | 0.2327        | 0.2474 | 0.2601 | 0.2708 | 0.2907 |       |
| RF     | 0.3043        | 0.3187 | 0.3289 | 0.3365 | 0.3505 |       |
| RNet   | 0.0982        | 0.113  | 0.126  | 0.1388 | 0.1621 |       |
| CA     | 0.3188        | 0.3208 | 0.3255 | 0.3306 | 0.3424 |       |
| L2LR   | 0.1373        | 0.1605 | 0.1809 | 0.1985 | 0.2278 |       |
| LMART  | 0.3549        | 0.3588 | 0.3648 | 0.3706 | 0.3818 |       |

| Method | DCG_{V2}^{UL} | 5   | 10  | 15  | 20  | 30  |
|--------|---------------|-----|-----|-----|-----|-----|
| ARank  |               | 0.2374 | 0.2531 | 0.2648 | 0.2761 | 0.2964 |
| LNet   |               | 0.0469 | 0.0606 | 0.0721 | 0.0837 | 0.1048 |
| RBoost |               | 0.2217 | 0.2374 | 0.2509 | 0.2626 | 0.2852 |
| RF     |               | 0.3086 | 0.3265 | 0.3385 | 0.3478 | 0.3652 |
| RNet   |               | 0.0476 | 0.0614 | 0.0727 | 0.0843 | 0.1054 |
| CA     |               | 0.3259 | 0.3286 | 0.3341 | 0.3401 | 0.3545 |
| L2LR   |               | 0.0982 | 0.1248 | 0.1475 | 0.1671 | 0.1996 |
| LMART  |               | 0.3677 | 0.3742 | 0.3824 | 0.3902 | 0.4056 |

Table 4. Upper & Lower Bound Normalized DCG (V1,V2) scores of different LETOR methods for variable k: Each cell shows a particular DCG_{V1}^{UL} score with a particular k on MSLR-WEB30K data-set.

nDCG and average real nDCG for eight LETOR method and five different cut-offs. Specifically, we followed the steps from Karmaker et.al. [24] to compute average real nDCG. Table 2 and 3 summarize the average (original) nDCG scores of different LETOR methods for different values of k, i.e., k = [5, 10, 15, 20, 30] for "MSLR-WEB30K" and "MQ2007" data-sets, respectively. One general observation from Table 2 and 3 is that average nDCG@k obtained by each method increases as we increase k and the extent of this change is indeed significant. For example, RankNet achieves nDCG value of 0.154 and 0.269 for k = 5 and k = 30 respectively with an increase of 74.6% (Table 2, "MSLR-WEB30K" data-set).

Next, we computed the expected nDCG score for each query according to equation 7. Figure 2 shows the histogram of expected nDCG scores of 10,000 queries from the “MSLR-WEB30K” data-set. It is interesting to note that, a large portion of “MSLR-WEB30K” queries indeed demonstrates a
large variance with high values in the ranges [0.5-0.6]. This justifies our position that lower-bound for each query can be very different and therefore, LB normalization should not be ignored while evaluating ranking performances.

Subsequently, we created two special sub-sets of queries based on the difference between their Expected nDCG and the average real nDCG obtained by eight LETOR methods, as defined below:

- **Uninformative Query-set**: These are the top 1,000 queries among the 10,000 “MSLR-WEB30K” pool (500 in case of MQ-2007 data-set), where difference between the Expected nDCG and the average real nDCG is minimal. In other words, these are the top 1,000 (500) queries where the LETOR methods struggle to perform better than the random baseline.

- **Ideal Query-set**: These are the top 1,000 queries among the 10,000 “MSLR-WEB30K” pool (500 in case of MQ-2007 data-set), where difference between the Expected nDCG and the average
real nDCG is maximal. In other words, these are the top 1,000 (500) queries where the LETOR methods outperforms the random baseline by the largest margin.

4.4.1 LB-normalized nDCG yields different rankings compare to Original nDCG for Uninformative query-set: We first test whether our proposed metrics generate different ranking results compared with the original nDCG or not. Table 6 shows the Kendall’s τ rank correlations between two rankings induced by nDCG and DCGUL scores in All, Uninformative or Ideal query collections from the two data-sets. We can notice that for both data-sets, DCGUL and nDCG generate different rankings for Uninformative queries resulting the Kendall’s τ less than 1 (i.e. 0.85 and 0.928). While for DCGULV, it generate different rankings for Uninformative queries in ‘MSLR-WEB30K” but not in “MQ2007”. Also, as expected in case of Ideal collections, there was no difference between nDCG and DCGUL in both data-sets(Kendall’s τ is 1). Another interesting observation is while we use all query collection, only DCGULV generate different ranking result in case of “MQ2007”.

| Data-set  | Kendall’s τ |   |   |   |
|-----------|-------------|---|---|---|
|           | Version     | All | uninorm. | Ideal |
| MSLR-WEB30K | nDCG vs V1  | 1  | 0.928 | 1  |
|           | nDCG vs V2  | 1  | 0.85  | 1  |
| MQ2007    | nDCG vs V1  | 1  | 1     | 1  |
|           | nDCG vs V2  | 0.785 | 0.928 | 1  |

Table 6. Kendall’s τ rank correlations between LETOR method ranks based on nDCG and two DCGUL on All, uninformative or ideal query sets from two data-sets.

4.4.2 Statistical Significance Test Yields Different Outcomes for Original nDCG Vs LB-normalized nDCG: Next we conducted statistical significance tests for every pair of LETOR methods based on their original nDCG and DCGUL scores to see how many times the two metrics disagree on the relative performance between two competing LETOR methods. Specifically, we followed the bootstrap Studentised Test (student’s t-test) from Sakai [36] to verify whether the observed difference has occurred due to mere random fluctuations or not for each pair of LETOR methods. Using the most widely used confidence value of 0.05 as the threshold, a p-value larger than 0.05 means the two distributions are statistically same, otherwise the pair of distributions are statistically different. More specifically, we compared each pair of LETOR methods (8C2 = 28 pairs in total) with respect to five cut-off k, i.e., k = [5, 10, 15, 20, 30]. Thus, the total number of comparison is 28 × 5 = 140.

Table 7 summarizes the number of disagreements between nDCG and DCGUL in two data-sets. For instance, based on student’s t-test, DCGULV disagreed with original nDCG on 46 (32%) pairs of LETOR methods for Uninformative query set from “MSLR-WEB30K”, while zero disagreements for Ideal query set. In “MQ2007”, we can also observe 24(17%) pairs of disagreements for Uninformative query set as well as there are 8 pairs of conflicts in Ideal query set. In particularly, we also see DCGULV disagreed with original nDCG on 6 pairs for all query set from “MQ2007”.

Given the difference in outcomes and disagreements between the original nDCG metric and it’s LB-normalized version, a natural follow-up question now is: which metric is better? To answer this question, we compared the nDCG and DCGUL metrics in terms of their Discriminative power and Consistency [36]. These are two popular methods for comparing evaluation measures.

4.4.3 Distinguishability.
We first focus on the implication of LB Normalization in terms of its capability to distinguish
Table 7. We used Student’s t-test to verify whether statistically significant difference occurred between a pair of LETOR methods while using $nDCG$ and $DCG^UL$ and counted the total number of disagreements on All, uninformative or ideal query sets from two data-sets.

Table 8. Student T-test induced total number of statistically significant differences detected based on $nDCG$ and $DCG^UL$ on All, uninformative or ideal query sets from two data-sets.

On “MSLR-WEB30K” Uninformative query set, $nDCG$ could detect only 33 (23%) significantly different pairs. In contrast, both two proposed $DCG^UL_{V_1}$ and $DCG^UL_{V_2}$ can detect more cases of significant differences. Additionally, $DCG^UL_{V_2}$ achieve the best performance which detected 78 (55%) significantly different pairs on the same set. On the other hand, on “MSLR-WEB30K” Ideal query-set, both $nDCG$ and two $DCG^UL$ detected 130 significantly different pairs. It is evident that, both two $DCG^UL$ can better distinguish between two LETOR methods than $nDCG$ on “MSLR-WEB30K” data-set, while not compromising distinguishability in case of Ideal queries, which is desired. We also observed similar improvements by $DCG^UL$ in case of “MQ2007” data-set. More importantly, $DCG^UL$ not only improve the distinguishability in case of uninformative query set, it can also detect more different cases while using All query set (for $DCG^UL_{V_2}$) and Ideal query set (for both $DCG^UL$), which is a bonus.

We also computed another metric to quantify distinguishability: Percentage Absolute Differences (PAD). More specifically, we computed the percentage absolute differences between pairs of LETOR
methods in terms of their original $nDCG$ and $DCG^{UL}$ scores, separately. The intuition here is that metrics with higher distinguishability will result in higher percentage absolute differences between pairs of LETOR methods. To elaborate, we first calculated the average value of both $nDCG$ and $DCG^{UL}$ with varying $k$ ($k = \{5, 10, 15, 20, 30\}$) for each LETOR method, and then, computed the percentage absolute difference between each pair of LETOR methods in terms of those two metrics separately (one percentage for $nDCG$ and another for $DCG^{UL}$), then we calculated the average of those percentage absolute differences. This experiment was performed on both data-sets. Mathematically, we used the following formula for percentage absolute differences (PAD) in terms of original $nDCG$:

$$PAD(nDCG) = \frac{|nDCG_{avg}^{M_1} - nDCG_{avg}^{M_2}|}{\max\left(nDCG_{avg}^{M_1}, nDCG_{avg}^{M_2}\right)} \times 100\%$$  \hspace{1cm} (8)

Here, $M_1$ and $M_2$ are two different LETOR methods and $nDCG_{avg}^{M_i}$ is the average $nDCG$ score obtained by method $M_i$ with respect to varying $k$. The equation for $PAD(DCG^{UL})$ is similar thus omitted. Besides, we use this equation for the PAD calculation of our second case-study. Table 9 shows these average percentage absolute differences of all possible LETOR method pairs in terms of original $nDCG$ and $DCG^{UL}$ scores on our two data-sets.

From this table, we can observe that while using $DCG^{UL}$, the PAD score of $DCG^{UL}$ is higher than the same for original $nDCG$ for all types of query collections, i.e., using All queries, Uninformative and Ideal query sub-sets. For instance, the average PAD of $nDCG$ on “MQ2007” is 1.74; while for $DCG^{UL}$, the score is 6.42 (using all query). Similarly, we discovered that for Uninformative query-set, $DCG^{UL}$ achieves a significant boost compared to the same in Ideal query-set in both data-sets.

These results show that the proposed LB normalization enhances the distinguishability of the original nDCG metric and can differentiate between two competing LETOR methods with a larger margin, which is a nice property of LB normalization.

| Metrics | PAD score | All Query | Ideal |
|---------|-----------|-----------|-------|
|         |           | MSLR | MQ2007 | MSLR | MQ2007 |
| $nDCG$  |           | 31   | 1.74   | 7.39 | 5.85   |
| $DCG^{UL}_{V_1}$ |       | 35.7 | 3.6    | 9.98 | 7.825  |
| $DCG^{UL}_{V_2}$ |       | 46.7 | 6.42   | 41.75 | 44.81 |

Table 9. Percentage Absolute Difference between pairs of LETOR methods in terms of average $nDCG$ and $DCG^{UL}$ scores on All, uninformative or ideal query sets from two data-sets.

4.4.4 Consistency.

This experiment focuses to compare the relative ranking of LETOR methods in terms of their $nDCG$ and $DCG^{UL}$ scores, separately, across different data-sets (“MQ2007” Vs “MSLR WEB30K”) as well as across Uninformative and Ideal query collections within the same data-set. The goal here is to see which metric yields a more stable ranking of LETOR methods across various types of documents and queries as well as across diverse set of data-sets. We computed swap rate [36] to quantify the consistency of rankings induced by $nDCG$ and $DCG^{UL}$ metrics across different data-sets. The essence of swap rate is to investigate the probability of the event that two experiments are contradictory given an overall performance difference.
Table 10 shows our swap rate results for $nDCG$ and $DCG_{UL}^V$ across the two data-sets, “MSLR-WEB30K” and “MQ2007”. Note that in our original setup, we selected Uninformative/ Ideal 1000 queries from “MSLR-WEB30K”. To make our results comparable, in this experiment we select 500 Uninformative/Ideal query from “MSLR-WEB30K” and compare the ranking result with the one from “MQ2007”. It can be observed that, both $nDCG$ and $DCG_{UL}^V$ share an identical swap rate probability when we conduct the experiment on the All/Uninformative/Ideal query collection (swap rate across data-sets is 0.107, 0.42 and 0.35 for both metrics).

| Metric   | Swap Rate | All | Uninform. | Ideal |
|----------|-----------|-----|-----------|-------|
| $nDCG$   |           | 0.107 | 0.42 | 0.35 |
| $DCG_{UL}^V_1$ | | 0.107 | 0.42 | 0.35 |
| $DCG_{UL}^V_2$ | | 0.107 | 0.42 | 0.35 |

Table 10. Swap rates between method ranks on All/ uniform/Ideal queries across “MSLR-WEB30K” and “MQ2007” data-sets.

Table 11 also shows our swap rate results for $nDCG$ and $DCG_{UL}^V$ across Uninformative Vs Ideal queries from the same data-set. We can still observe that both $nDCG$ and $DCG_{UL}^V$ generate the identical swap rate probability when we compare the ranking results across Uninformative and Ideal sets, except for $DCG_{UL}^V_1$ (generate a higher swap rate in “MSLR-WEB30K”).

| Metric   | Swap Rate | MSLR-WEB30K | MQ2007 |
|----------|-----------|-------------|--------|
| $nDCG$   |           | 0.21        | 0.5    |
| $DCG_{UL}^V_1$ | | 0.25 | 0.5 |
| $DCG_{UL}^V_2$ | | 0.21 | 0.5 |

Table 11. Swap rates between method ranks on MSLR-WEB30K/MQ2007 data-sets across “uninformative” and “Ideal” query collections.

4.5 Case Study 2: MAP with Joint Upper & Lower Bound Normalization

For our second case study, we selected another popular evaluation metric called Mean Average Precision ($MAP$). However, original $MAP$ computation needs binary label while our two data-sets are multi-relevance label. For consistency, in this paper, we only consider 0 relevance score as negative and others are positive for both two data-sets. Table 12 and 13 show the original $MAP$ scores from two data-sets. Below, we will first present how we can compute a realistic lower bound for $Sum Precision (SP)$ by computing its expected value in case of a randomly ranked list of documents. Then, demonstrate our findings of lower bound normalized $MAP$. Again, lower bound normalized $MAP$ essentially means upper lower bound normalized $MSP$.

First, we also show the histogram of expected AP score for 10,000 queries from “MSLR-WEB30K” data-sets. Figure 3 shows the histogram of expected AP scores of 10,000 queries from the “MSLR-WEB30K” data-set. We can still observe that a large variance of high expected AP appeared in this data-set, indicating that can not be ignored. Noted that we again created two special sub-sets of queries based on the difference between their Expected AP and and average real AP obtained by eight LETOR methods to define Uninformative query-set and Ideal Query-set (Details in 4.4).
4.5.1 Lower Bound of SP (SP for Random Ranking): Given a query \( q \), assume that \( N_p \) is the total number of relevant documents, \( N_n \) is the number of non-relevant document for query \( q \). Also, assume \( N_p > k \) and \( N_n > k \), \( k \) is the cutoff variable. \( \text{Prec}(i) \) is the precision at position \( i \) and \( R_i \) is the relevance at position \( i \). Then, expectation of \( SP@k \) in case of random ranking is the following:

\[
E[SP@k] = \sum_{i=1}^{k} E[\text{Prec}(i)] \cdot R_i
\]

Now assuming \( \text{Prec}(i) \) and \( R_i \) are independent, we have

\[
E[SP@k] = \sum_{i=1}^{k} E[\text{Prec}(i)] \cdot E[R_i], \text{ where,}
\]

\[
E[R_i] = P[R_i = 1] \cdot 1 + P[R_i = 0] \cdot 0 = P[R_r = 1] = \frac{N_p}{N_p + N_n}
\]

\[
E[\text{Prec}@i] = \frac{1}{i} \left[ P\left(\text{Prec}@i = \frac{1}{i}\right) \right] + \frac{2}{i} \left[ P\left(\text{Prec}@i = \frac{2}{i}\right) \right] + \ldots + \frac{i}{i} \left[ P\left(\text{Prec}@i = \frac{i}{i}\right) \right]
\]

\[
= \left( \frac{1}{i} \right) \left[ \frac{(N_p)(N_n)}{(N_p + N_n)} \right] + \left( \frac{2}{i} \right) \left[ \frac{(N_p)(N_n)}{(N_p + N_n)} \right] + \ldots + \left( \frac{i}{i} \right) \left[ \frac{(N_p)(N_n)}{(N_p + N_n)} \right] = \left( \frac{1}{i} \right) \frac{1}{(N_p + N_n)} \sum_{j=1}^{i} \binom{N_p}{j} \binom{N_n}{i-j}
\]

We will later prove that,
\[ \sum_{j=1}^{i} j \binom{N_p}{j} \binom{N_n}{i-j} = \frac{N_p}{N_p + N_n} i \binom{N_p + N_n}{i} \]

Thus, \( E[\text{Prec} @ i] = \frac{N_p}{N_p + N_n} \), Hence:

\[ E[SP @ k] = \sum_{i=1}^{k} E[\text{Prec}(i)] \cdot E[R_i] = \sum_{i=1}^{k} \left( \frac{N_p}{N_p + N_n} \right)^2 = k \left( \frac{N_p}{N_p + N_n} \right)^2 \]

Now, we will use induction to prove the following:

\[ \sum_{j=1}^{i} j \binom{N_p}{j} \binom{N_n}{i-j} = \left( \frac{N_p}{N_p + N_n} \right) i \binom{N_p + N_n}{i} \]  \hspace{1cm} (9)

**Base case:** For \( i = 1 \), L.H.S. = \( \binom{N_p}{1} \binom{N_n}{i-1} = N_p \)

\[ \text{R.H.S.} = \left( \frac{N_p}{N_p + N_n} \right) \binom{N_p + N_n}{1} = \frac{N_p}{N_p + N_n} (N_p + N_n) = N_p \]

So, equation 9 is true for \( i = 1 \)

**Induction step:** Now, let’s assume equation 9 is true for \( i = i-1 \), then we get the following:

\[ \sum_{j=1}^{i} j \binom{N_p}{j} \binom{N_n}{i-j} = \sum_{j=1}^{i-1} j \binom{N_p}{j} \binom{N_n}{i-j} + i \binom{N_p}{i} \]  \hspace{1cm} (10)

Now,

\[ \sum_{j=1}^{i} j \binom{N_p}{j} \binom{N_n}{i-j} = \sum_{j=1}^{i-1} j \binom{N_p}{j} \binom{N_n}{i-j} + i \binom{N_p}{i} \]

\[ = \sum_{j=1}^{i-1} j \binom{N_p}{j} \left[ \binom{N_n + 1}{i-j} - \binom{N_n}{i-j-1} \right] + i \binom{N_p}{i} \]

\[ = \sum_{j=1}^{i-1} j \binom{N_p}{j} \binom{N_n + 1}{i-j} + i \binom{N_p}{i} - \sum_{j=1}^{i-1} j \binom{N_p}{j} \binom{N_n}{i-j-1} \]

\[ = \sum_{j=1}^{i} j \binom{N_p}{j} \binom{N_n + 1}{i-j} - \left( \frac{N_p}{N_p + N_n} \right) (i-1) \binom{N_p + N_n}{i-1} \]  \hspace{1cm} [From (10)]

\[ = \sum_{j=1}^{i} N_p \binom{N_p - 1}{j-1} \binom{N_n + 1}{i-j} - \left( \frac{N_p}{N_p + N_n} \right) (i-1) \binom{N_p + N_n}{i-1} \]

\[ = N_p \sum_{j=1}^{i} \binom{N_p - 1}{j-1} \binom{N_n + 1}{i-j} - \left( \frac{N_p}{N_p + N_n} \right) (i-1) \binom{N_p + N_n}{i-1} \]

\[ = N_p \left( \binom{N_p + N_n}{i-1} - \binom{N_p}{i-1} \right) - \left( \frac{N_p}{N_p + N_n} \right) (i-1) \binom{N_p + N_n}{i-1} \]

\[ = \left( \binom{N_p + N_n}{i-1} \right) \left( \frac{N_p}{N_p + N_n} \right) \left( \binom{N_p + N_n}{i-1} - \binom{N_p}{i-1} \right) \]

\[ = \left( \binom{N_p + N_n}{i-1} \right) \left( \frac{N_p}{N_p + N_n} \right) \left( \binom{N_p + N_n}{i-1} - \binom{N_p}{i-1} \right) \]

\[ = \left( \binom{N_p + N_n}{i-1} \right) \left( \frac{N_p}{N_p + N_n} \right) \left( \binom{N_p + N_n}{i-1} - \binom{N_p}{i-1} \right) \]

\[ = \left( \binom{N_p + N_n}{i-1} \right) \left( \frac{N_p}{N_p + N_n} \right) \left( \binom{N_p + N_n}{i-1} - \binom{N_p}{i-1} \right) \]
Proof completed because

\[(n - r + 1) \binom{n}{r - 1} = r \binom{n}{r}\]

| Method  | MAP@ 5 | MAP@ 10 | MAP@ 15 | MAP@ 20 | MAP@ 30 |
|---------|--------|---------|---------|---------|---------|
| ARank   | 0.5414 | 0.4948  | 0.4724  | 0.4598  | 0.4493  |
| LNet    | 0.3203 | 0.2994  | 0.293   | 0.2911  | 0.2943  |
| RBoost  | 0.5449 | 0.4967  | 0.475   | 0.4618  | 0.452   |
| RF      | 0.6216 | 0.5717  | 0.5433  | 0.5244  | 0.5053  |
| RNet    | 0.3212 | 0.3008  | 0.2939  | 0.2919  | 0.2956  |
| CA      | 0.6235 | 0.5631  | 0.53    | 0.5107  | 0.4903  |
| L2LR    | 0.356  | 0.3353  | 0.333   | 0.3335  | 0.3457  |
| LMART   | 0.6487 | 0.5928  | 0.5613  | 0.5414  | 0.5198  |

Table 12. MAP scores of different LETOR methods for variable $k$ on 'MSLR-WEB30K' dataset.

| Method  | MAP@ 5 | MAP@ 10 | MAP@ 15 | MAP@ 20 | MAP@ 30 |
|---------|--------|---------|---------|---------|---------|
| ARank   | 0.3066 | 0.2923  | 0.302   | 0.3173  | 0.3624  |
| LNet    | 0.3379 | 0.3233  | 0.3328  | 0.3468  | 0.3905  |
| RBoost  | 0.3467 | 0.3366  | 0.3477  | 0.3636  | 0.4035  |
| RF      | 0.3674 | 0.352   | 0.3585  | 0.3736  | 0.414   |
| RNet    | 0.3281 | 0.3175  | 0.3275  | 0.3443  | 0.3878  |
| CA      | 0.3597 | 0.3457  | 0.356   | 0.3716  | 0.4127  |
| L2LR    | 0.3543 | 0.3386  | 0.3458  | 0.3607  | 0.404   |
| LMART   | 0.3582 | 0.3459  | 0.3539  | 0.3692  | 0.4101  |

Table 13. MAP scores of different LETOR methods for variable $k$ on 'MQ2007' dataset.

4.5.2 LB-normalized MAP yields different rankings compare to Original MAP for Uninformative query-set: Table 16 shows the Kendall’s $\tau$ rank correlations between two rankings induced by MAP and $MSP^{UL}$ scores in All, Uninformative or Ideal query collections for the two data-sets. Firstly, we can notice that for both data-sets, $MSP^{UL}$ and MAP generate identical rankings

| Method  | $MSP^{UL}_{V_1}$@ 5 | $MSP^{UL}_{V_1}$@ 10 | $MSP^{UL}_{V_1}$@ 15 | $MSP^{UL}_{V_1}$@ 20 | $MSP^{UL}_{V_1}$@ 30 | $MSP^{UL}_{V_2}$@ 5 | $MSP^{UL}_{V_2}$@ 10 | $MSP^{UL}_{V_2}$@ 15 | $MSP^{UL}_{V_2}$@ 20 | $MSP^{UL}_{V_2}$@ 30 |
|---------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| ARank   | 0.3856               | 0.3387               | 0.3156               | 0.3017               | 0.2868               | 0.3472               | 0.3055               | 0.2799               | 0.2617               | 0.2373               |
| LNet    | 0.1978               | 0.1732               | 0.1641               | 0.1597               | 0.1572               | -0.0721              | -0.0573              | -0.0508              | -0.0452              | -0.0389              |
| RBoost  | 0.3905               | 0.3422               | 0.3196               | 0.305                | 0.3031               | 0.3502               | 0.3019               | 0.2754               | 0.2567               | 0.2339               |
| RF      | 0.4579               | 0.4079               | 0.3791               | 0.3591               | 0.3363               | 0.4783               | 0.427                | 0.39                 | 0.3608               | 0.322                |
| RNet    | 0.1988               | 0.1745               | 0.1651               | 0.1606               | 0.1585               | -0.0718              | -0.0551              | -0.0498              | -0.0452              | -0.0388              |
| CA      | 0.4594               | 0.4001               | 0.3673               | 0.3471               | 0.3231               | 0.4836               | 0.4127               | 0.3676               | 0.3385               | 0.2977               |
| L2LR    | 0.226                | 0.2018               | 0.1963               | 0.1951               | 0.1982               | -0.0223              | -0.004               | 0.0149               | 0.0312               | 0.055                |
| LMART   | 0.482                | 0.4265               | 0.3948               | 0.3741               | 0.3488               | 0.5259               | 0.4632               | 0.4211               | 0.3896               | 0.3466               |

Table 14. Upper & Lower Bound Normalized $MSP (V_1, V_2)$ scores of different LETOR methods for variable $k$: Each cell shows a particular $MSP^{UL}$ score with a particular $k$. MSLR-WEB30K dataset.
for different query set which indicate that there is no difference between \( \text{MAP} \) with \( \text{MSP}^{UL} \) in terms of Kendall’s \( \tau \) rank test. While for \( \text{MSP}^{UL} \), it generate different rankings for all kinds of query collections in both two data-sets. For instance, in “MQ2007”, Kendall’s \( \tau \) correlation between MAP and \( \text{MSP}^{UL} \) are 0.785, 0.624 and 1 for all, uninformative and ideal query set, suggesting that \( \text{MSP}^{UL} \) achieves different outcomes. In addition, the impact is more prominent in case of uninformative compared with ideal.

| Data-set       | Kendall’s \( \tau \) |
|----------------|----------------------|
|                | Version | All | uninform. | Ideal |
| MSLR-WEB30K    | MAP vs V1 | 1   | 1         | 1     |
|                | MAP vs V2 | 0.928 | 0.857   | 0.928 |
| MQ2007         | MAP vs V1 | 1   | 1         | 1     |
|                | MAP vs V2 | 0.785 | 0.624   | 1     |

Table 16. Kendall’s \( \tau \) rank correlations between LETOR method ranks based on \( \text{MAP} \) and two \( \text{MSP}^{UL} \) on All, uninformative or ideal query sets from two data-sets.

4.5.3 **Statistical Significance Test Yields Different Outcomes for Original MAP Vs LB-normalized MAP.** We again conducted statistical significance tests for every pair of LETOR methods based on their original \( \text{MAP} \) and \( \text{MSP}^{UL} \) scores to see how many times the two metrics disagree on the relative performance between two competing LETOR methods. Table 17 summarizes the number of disagreements between \( \text{MAP} \) and \( \text{MSP}^{UL} \) in two data-sets. For instance, based on student’s t-test, \( \text{MSP}^{UL} \) disagreed with original \( \text{MAP} \) on 36 (26%) pairs of LETOR methods for Uninformative query set from “MSLR-WEB30K”, while 4 disagreements for Ideal query set. Although none of \( \text{MSP}^{UL} \) disagree with original \( \text{MAP} \) while using All query set from “MSLR-WEB30K”, there are still 1 and 8 conflicts appeared in “MQ2007” for two UL normalized version respectively.

Given the difference in outcomes and disagreements between the original \( \text{MAP} \) metric and it’s LB-normalized version, we still trying to compare these two metrics in terms of their Discriminative power and Consistency just like what we did in nDCG.

4.5.4 **Distinguishability.**

We again follow Sakai [36] to use student’s t-test to conduct this experiment and use 0.05 as our threshold. Using the aforementioned Uninformative and Ideal query collections, Table 18
shows some interesting results of these statistical tests for different query sets in ’MSLR-WEB10K’ and "MQ2007" data-sets.

On “MSLR-WEB30K” Uninformative query set, although MAP detect 61 (43%) significantly different pairs, both two proposed MSPULV1 and MSPULV2 can detect more cases of significant differences. What can be clearly seen is MSPULV2 still achieve the best performance which detected 81 (57%) significantly different pairs on the same set. On the other hand, on "MSLR-WEB30K" Ideal query set, both MAP and two MSPUL detected around 122 significantly different pairs. More interestingly, in “MQ2007”, while original MAP detect 45 cases of different pairs using all query set, MSPUL indeed improve this performance (for MSPULV1 is 50 and MSPULV2 is 59). Specifically in uninformative query set, MAP can not detect any significantly different pairs. However, MSPUL can detect 21 pairs of difference, which is very important. On the other hand, MSPULV2 can even detect more cases in ideal query set. It is evident that, both two MSPUL can better distinguish between two LETOR methods than MAP on two data-sets, while not compromising distinguishability in case of Ideal queries (even improve the distinguishability in “MQ2007”).

Again, we use the formula 8 to compute the percentage of absolute differences between pairs of LETOR methods in terms of their original MAP and MSPUL, separately. Here, X represents MAP and MSPULV1, MSPULV2, (Details of PAD can be found in 4.4.3).

Table 19 illustrates the PAD score in case of MAP and proposed two MSPUL from two data-sets for different query collections.

From this table, we can still observe that while using MSPUL can achieve higher PAD score than the same for original MAP for all types of query collections, i.e., using All queries, Uninformative and Ideal query sub-sets. For instance, the average PAD of MAP on "MSLR-WEB30K" is 25.57;
Table 19. Percentage Absolute Difference between pairs of LETOR methods in terms of average MAP and MSPUL scores on All, uninformative or ideal query sets from two data-sets.

| Metrics   | All Query | Uninform | Ideal  |
|-----------|-----------|----------|--------|
|           | MSLR | MQ2007 | MSLR | MQ2007 | MSLR | MQ2007 |
| MAP       | 25.57 | 5.91    | 12.28 | 5.89    | 30.18 | 6.77   |
| MSPULv1   | 31.84 | 6.86    | 16.0  | 7.19    | 35.53 | 8.04   |
| MSPULv2   | 97.63 | 20.01   | 25.65 | 28.27   | 48.29 | 13.49  |

while for MSPULv2, the score is 97.63 (using all query). Similarly, we can still discovered that for Uninformative query-set, both MSPUL versions achieve a significant boost compared to the same in Ideal query set in both data-sets.

These results show that the proposed LB normalization again improve the distinguishability of original MAP and can better differentiate between the quality of two LETOR methods with a larger margin.

4.5.5 **Consistency.**

This experiment again focuses to compare the relative ranking of LETOR methods in terms of their MAP and MSPUL scores, separately, across different data-sets (“MQ2007” Vs “MSLR-WEB30K”) as well as across Uninformative and Ideal query collections within the same data-set. We computed swap rate [36] to quantify the consistency of rankings induced by MAP and MSPUL metrics across different data-sets. Table 20 shows our swap rate results for MAP and MSPUL across the two data-sets, “MSLR-WEB30K” and “MQ2007”. In contrast to identical swap rate scores in nDCG and DCGUL, MSPULv2 can achieve a overall lower swap rate (swap rate of MAP is 0.25 while 0.178 for MSPULv2) across a data-sets comparison while considering all query set.

Table 20. Swap rates between method ranks on All/ uniform/Ideal queries across “MSLR-WEB30K” and “MQ2007” data-sets.

| Metric   | Swap Rate |
|----------|-----------|
|          | All | Uninform. | Ideal  |
| MAP      | 0.25 | 0.357    | 0.2857 |
| MSPULv1  | 0.25 | 0.321    | 0.25   |
| MSPULv2  | 0.178 | 0.25     | 0.321  |

Table 21 also shows our swap rate results for MAP and MSPUL across Uninformative Vs Ideal queries from the same data-set. Similarly, we can still observe that MSPULv2 can obtain a more consistent ranking results across different query collection, which is very useful for an evaluation metric.

5 **DISCUSSIONS AND CONCLUSION**

In this paper, we presented a novel perspective towards evaluation of Information Retrieval (IR) systems. Specifically, we performed two case-study on nDCG, and MAP both are widely popular metrics for IR evaluation, and started with the observation that, traditional nDCG and MAP computation does not include a query-specific lower-bound normalization although they include a query-specific upper-bound normalization. In other words, the current practice is to assume a
uniform lower bound (zero) across all queries while computing nDCG and MAP, an assumption which is incorrect. This limitation raises a question mark on the previous comparative studies involving multiple ranking methods where an average evaluation metric score is reported, because Uninformative vs. Informative vs. Ideal queries are rewarded equally in traditional IR evaluation metric computation and the expected lower-bound of the evaluation metric is ignored. How can we incorporate query-specific LB normalization into IR evaluation metrics and how will it impact IR evaluation in general? This is the central issue we investigated in this paper.

**Conceptual Leap:** To address the aforementioned issue, we proposed to penalize the traditional IR evaluation metric score of each query with a lower-bound normalization term specific to that query. To achieve this, we introduced a joint upper and lower bound normalization (UL-normalization) framework and instantiated two versions of the UL-normalization, \( V_1 \) \( V_2 \), for two popular IR evaluation metric \( nDCG \) and \( MAP \), essentially creating four new evaluation metrics.

The next challenge in our work was to estimate a more realistic query-specific lower-bound for above two metric. For this estimation, we argued that a reasonable ranking method should be at least as good as a random ranking method, so a more realistic lower-bound should be the score expected by mere random ranking of the document collection rather than the current practice of assuming zero as lower-bound across all queries. Using probability and permutation theory, we derived a closed-form formula to compute the expected \( DCG \) in case of random ranking. The proof was completed by showing that expected relevance label of a document at position \( i \) is actually independent of the position and can be replaced by the expected relevance label of the document collection associated with the particular query in the validation data-set. For expected \( SP \) we also use probability and induction to prove the correctness of our assumption. The derivation details can be found in each case study section.

**Depth of Impact:** Using two publicly available web search and learning-to-rank data-sets, we conducted extensive experiments with eight popular LETOR methods to understand the implications \( DCG^{UL} \) and \( MSP^{UL} \). The implications are briefly summarized as below:

1. Kendall’s \( \tau \) rank correlation coefficient test on two different rankings of multiple LETOR methods, where the ranks are induced by both traditional metric (i.e. \( nDCG \) and \( MAP \)) vs UL-normalized metrics (i.e. \( DCG^{UL} \) and \( MSP^{UL} \)) yields different conclusions regarding the relative ranking of multiple LETOR methods.
2. Statistical Significance tests can lead to conflicting conclusions regarding the relative performance between a pair of LETOR methods, when comparing them in terms of traditional metrics vs UL-normalized metrics scores.
3. The above two observations are more prominent in case of Uninformative query collection.

Next, we systematically compared the traditional evaluation metric and UL-normalized metrics from two important perspectives: distinguishability and consistency. The findings are briefly summarized below.

| Metric | Swap Rate |
|--------|-----------|
|        | MSLR-WEB30K | MQ2007 |
| MAP    | 0.1428 | 0.3928 |
| MSP_{UL}^{V_1} | 0.1428 | 0.3928 |
| MSP_{UL}^{V_2} | 0.1071 | 0.2857 |

Table 21. Swap rates between method ranks on MSLR-WEB30K/MQ2007 data-sets across “uninformative” and “Ideal” query collections.
Discriminative power analysis and PAD scores suggest that our metric can better distinguish between two closely performing LETOR methods. These results were confirmed through Student’s t-test and PAD score analysis.

For consistency, $MSP_{UL}^U$ achieves the lowest swap rate across a data-sets comparison as well as the lowest swap rate while we compare the ranking results from uninformative vs. ideal query sets. On the other hand, the proposed $DCG_{UL}^U$ metric is identical to the original $nDCG$ metric in terms of consistency across different data-sets as well as across Uninformative/ Ideal query sets within the same data-set.

All above experiments reveal that the impact of LB normalization is more substantial in case of “Uninformative” queries in comparison to “Ideal” queries, suggesting, LB normalization is crucial when the validation set contains a large number of Uninformative queries (i.e., the ranking methods fail to perform significantly better than the randomly ranked output).

Breadth of Impact: The proposed LB-normalization technique is very general and can be potentially extended to other IR evaluation metrics like ERR, which is an exciting future direction. Another direction can be to investigate such LB normalization for evaluation in domains other than IR, for example, ROUGE metric from the text summarization and NLP literature.

Final Words: The key take-away message from this paper is the following: The IR community should consider lower-bound (LB) normalization seriously while evaluating any IR system. Our work takes a first step towards this important direction and can serve as a pilot study to demonstrate the importance and implications of LB normalization.

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