New Predictions for Multiquark Hadron Masses

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Abstract

The recent reported charmed-strange resonance at 2.32 GeV/c suggests a possible multiquark state. Three types of multiquark bound states are reviewed. A previous model-independent variational approach considers a tetraquark with two heavy antiquarks and two light quarks as a heavy antidiquark bound to the two light quarks with a wave function like that of a heavy baryon. Results indicate that a charmed-strange tetraquark $\bar{c}\bar{s}ud$ or a bottom-strange tetraquark $\bar{b}\bar{s}ud$ with this “diquark-heavy-baryon” wave function is not bound, in contrast to “molecular-type” $D - K$ and $B - K$ wave functions. However, a charmed-bottom tetraquark $\bar{c}\bar{b}ud$ might be bound with a very narrow weak decay mode. A “molecular-type” $D - B$ state can have an interesting $B_c\pi$ decay with a high energy pion.

I. THREE TYPES OF MULTIQURK STATES

The recent observation [1] of a charmed-strange state at 2.32 GeV that decays into $D_s\pi^0$ suggests a possible four-quark state (tetraquark). [2–8]

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Three different mass scales are relevant to the description of multiquark hadrons, the nuclear-molecular scale, the hyperfine or color-magnetic scale and the diquark scale.

The nuclear scale is characterized by the deuteron, a bound state of two color singlet hadrons with a reduced mass of 500 MeV, a binding energy of several MeV and a radius of \( \approx M_\pi \). The underlying quark structure of the hadrons plays no role. The kinetic energy of the state confined to this radius is

\[
T_N = \frac{p^2}{M_N} \approx \frac{M_\pi^2}{M_N} \approx 20 \text{MeV}
\] (1.1)

No two-meson bound state containing a pion has been found. The reduced mass of any such state is too small to be bound in a radius of \( \approx M_\pi \) by a similar interaction; its kinetic energy would be too high.

\[
T_\pi \approx \frac{M_\pi^2}{M_\pi} \approx 140 \text{MeV}
\] (1.2)

The two-kaon system with a reduced mass of 250 MeV seems to be on the borderline,

\[
T_K = \frac{p^2}{M_K} \approx \frac{M_\pi^2}{M_K} \approx 40 \text{MeV}
\] (1.3)

Suggestions that the \( f_0 \) and \( a_0 \) mesons are deuteron-like \( K\bar{K} \) states or molecules are interesting, but controversial. There is no unambiguous signature because \( K\bar{K} \) couples to \( \pi - \pi \) and \( \eta - \pi \) and both states break up strongly.

The \( D - K \) system with a kinetic energy

\[
T_{DK} = \frac{p^2(M_D + M_K)}{2M_DM_K} \approx \frac{M_\pi^2(M_D + M_K)}{2M_DM_K} \approx 25 \text{MeV}
\] (1.4)

is therefore an attractive candidate for such a state [3–8]. The transition for the \( I = 0 \) \( DK \) state to \( D_S\pi \) is isospin forbidden; thereby suggesting a narrow width.

The color-magnetic scale is characterized by a mass splitting of the order of 400 MeV; e.g. the \( K^* - K \) splitting. Recoupling the colors and spins of a system of two color-singlet hadrons has been shown to produce a gain in color-magnetic energy [2–4]. However, whether this gain in potential energy is sufficient to overcome the added kinetic energy required for a bound state is not clear without a specific model.
The diquark scale arises when two quarks are sufficiently heavy to be bound in the well of the coulomb-like short-range potential required by QCD. A heavy antidiquark in a triplet of color SU(3) has the color field of a quark and can be bound to two light quarks with a wave function like that of a heavy baryon. Since the binding energy of two particles in a coulomb field is proportional to their reduced mass and all other interactions are mass independent, this diquark binding must become dominant at sufficiently high quark masses.

II. THE DIQUARK-HEAVY-BARYON MODEL FOR TETRAQUARKS

We now examine the diquark-heavy-baryon model for states containing heavy quarks. Our “model-independent” approach assumes that nature has already solved the problem of a heavy color triplet interacting with two light quarks and given us the answers; namely the experimental masses of the Λ, Λ_c and Λ_b. These answers provided by nature can now be used without understanding the details of the underlying theoretical QCD model. This approach was first used by Sakharov and Zeldovich [9] and has been successfully extended to heavy flavors [10–12].

The calculated mass can be interpreted as obtained from a variational principle with a particular form of trial wave function [5]. This model neglects the color-magnetic interactions of the heavy quarks, important for the charmed-strange four-quark system at the colormagnetic scale [2–4] and is expected to overestimate the mass of a ¯c¯sud state. Thus obtaining a model mass value above the relevant threshold shows only that this type of diquark-heavy-baryon wave function does not produce a bound state; i.e that the heavy quark masses are not at the diquark scale. The previous results [3,4] at the colormagnetic or nuclear-molecular scale should be better. However, the bc system may already be sufficiently massive to lead to stable diquarks and the model predictions for the ¯c¯bud state may suggest binding.

We first apply this model to a ¯c¯sud state with a light ud pair seeing the color field of the ¯c¯s antidiquark like the field of a heavy quark in a heavy baryon. The ¯c¯s antidiquark
differs from the $c\bar{s}$ in the $D_s$ by having a $QQ$ potential which QCD color algebra requires \[5\] to have half the strength of the $Q\bar{Q}$ potential in the $D_s$. The tetraquark mass is estimated by using the known experimental masses of the heavy baryons and heavy meson with the same flavors and introducing corrections for the difference between the heavy meson and the heavy diquark.

\[
M(\bar{c}sud) = m_c + m_s + m_u + m_d + \langle H_{udQ}\rangle + \langle H_{ud}\rangle + \langle T_{cs}\rangle_{cs} + \langle V_{cs}\rangle_{cs} \tag{2.1}
\]

\[
M(cs) = m_c + m_s + \langle T_{cs}\rangle_{cs} + \langle V_{cs}\rangle_{cs} \tag{2.2}
\]

\[
M(D_s) = m_c + m_s + \langle T_{cs}\rangle_{cs} + \langle V_{cs}\rangle_{cs} \tag{2.3}
\]

\[
M(\Lambda) = m_s + m_u + m_d + \langle H_{ud}\rangle + \langle H_{udQ}\rangle \tag{2.4}
\]

\[
M(\Lambda_c) = m_c + m_u + m_d + \langle H_{ud}\rangle + \langle H_{udQ}\rangle \tag{2.5}
\]

where $H_{ud}$ and $H_{udQ}$ respectively denote the Hamiltonians describing the internal motions of the $ud$ pair and of the three-body system of the $ud$ pair and the antidiquark which behaves like a heavy quark, $T_{cs}$ and $V_{cs}$ denote the kinetic and potential energy operators for the internal motion of a $cs$ diquark which is the same as that for a $\bar{c}s$ antidiquark. The expectation values are taken with the “exact” wave function for the model, with the subscript $cs$ indicating that it is taken with the wave function of a diquark and not of the $D_s$. The kinetic energy operator $T_{cs}$ is the same for the $cs$ diquark and the $D_s$ but the potential energy operators $V_{cs}$ and $V_{cs} = 2V_{cs}$ differ by the QCD factor 2. This difference between $cs$ diquark and $D_s$ wave functions is crucial to our analysis.

The quark masses $m_q$ are effective constituent quark masses and not current quark masses. We follow the approach begun by Sakharov and Zeldovich \[9\] who noted that both the difference $m_s - m_u$ between the effective masses of strange and nonstrange quarks and their ratio $m_s/m_u$ have the same values when calculated from baryon masses and meson masses.
\[ \langle m_s - m_u \rangle_{\text{Bar}} = M_\Lambda - M_N = 177 \text{ MeV} \quad (2.6) \]

\[ \langle m_s - m_u \rangle_{\text{Mes}} = \frac{3(M_{K^*} - M_\rho) + M_K - M_\pi}{4} = 180 \text{ MeV} \quad (2.7) \]

\[ \left( \frac{m_s}{m_u} \right)_{\text{Bar}} = \frac{M_\Delta - M_N}{M_{\Sigma^*} - M_\Sigma} = 1.53 \approx \left( \frac{m_s}{m_u} \right)_{\text{Mes}} = \frac{M_\rho - M_\pi}{M_{K^*} - M_K} = 1.61 \quad (2.8) \]

where the “Bar” and “Mes” subscripts denote values obtained from baryons and mesons, respectively. Similar results have since been found for hadrons containing heavy quarks along with many more relations using these same effective quark mass values for baryon magnetic moments and hadron hyperfine splittings [10–12]. We therefore assume that the values of the effective quark masses \( m_q \) remain the same for all meson and baryon states in our analysis.

Substituting eqs. (2.2 - 2.5) into eq. (2.1) gives

\[ M(\bar{c}\bar{s}ud) = (1/2) \cdot [M(D_s) + M(\Lambda) + M(\Lambda_c)] + \langle \delta H_{cs} \rangle \quad (2.9) \]

where \( \langle \delta H_{cs} \rangle \) expresses the difference between the \( D_s \) and the \( \bar{c}\bar{s} \) wave functions

\[ \langle \delta H_{cs} \rangle = \langle T_{cs} \rangle_{cs} + \langle V_{cs} \rangle_{cs} - (1/2) \cdot [\langle T_{cs} \rangle_{\bar{c}\bar{s}} + \langle V_{cs} \rangle_{\bar{c}\bar{s}}] \quad (2.10) \]

To calculate \( \langle \delta H_{cs} \rangle \) we first improve on the treatment of ref [5] and define the Hamiltonian

\[ H(\alpha) = \alpha T_{cs} + V_{cs} = \alpha T_{cs} + (1/2) \cdot V_{\bar{c}\bar{s}} \quad (2.11) \]

This Hamiltonian \( H(\alpha) \) is seen to describe both the \( cs \) diquark and the \( D_s \)

\[ M(cs) = m_c + m_s + \langle H(\alpha) \rangle_{\alpha=1} \quad (2.12) \]

\[ M(D_s) = m_c + m_s + 2 \cdot \langle H(\alpha) \rangle_{\alpha=(1/2)} \quad (2.13) \]

\[ \langle \delta H_{cs} \rangle = \langle H(\alpha) \rangle_{\alpha=1} - \langle H(\alpha) \rangle_{\alpha=(1/2)} \quad (2.14) \]

To evaluate \( \langle \delta H_{cs} \rangle \) we use the Feynman-Hellmann theorem and the virial theorem to obtain,
\[
\frac{d}{d\alpha} \langle H(\alpha) \rangle = \left\langle \frac{dH(\alpha)}{d\alpha} \right\rangle = \langle T_{cs} \rangle = \left\langle \frac{r}{2\alpha} \cdot \frac{dV_{cs}}{dr} \right\rangle_{\alpha} \tag{2.15}
\]

\[
\langle \delta H_{cs} \rangle = \int_{(1/2)}^{1} d\alpha \left\langle \frac{dH(\alpha)}{d\alpha} \right\rangle = \int_{(1/2)}^{1} d\alpha \left\langle \frac{r}{2\alpha} \cdot \frac{dV_{cs}}{dr} \right\rangle_{\alpha} \tag{2.16}
\]

This expression can be simplified by using the Quigg-Rosner logarithmic potential \[13\] with its parameter \(V_o\) determined by fitting the charmonium spectrum.

\[
V_{cs}^{QR} = \left(\frac{1}{2}\right) \cdot V_o \cdot \log \left(\frac{r}{r_o}\right) \tag{2.17}
\]

\[
\langle \delta H_{cs} \rangle_{QR} = \frac{V_o}{4} \int_{(1/2)}^{1} d\alpha \alpha = \frac{V_o}{4} \log 2 = 126 \text{ MeV} \tag{2.18}
\]

Substituting experimental values then shows \(M(\bar{c}\bar{s}ud)\) well above the \(DK\) threshold \[14\]

\[
M(\bar{c}\bar{s}ud) = \left(\frac{1}{2}\right) \cdot [M(D_s) + M(\Lambda) + M(\Lambda_c)] + \langle \delta H_{cs} \rangle = 2685 + 126 = 2811 \text{ MeV} \tag{2.19}
\]

\[
M(\bar{c}\bar{s}ud) = 2811 \text{ MeV} \gg M(D) + M(K) = 2361 \text{ MeV} \tag{2.20}
\]

In the limit of very high heavy quark masses this model must give a stable bound state. The \(cs\) diquark is evidently not heavy enough to produce a bound diquark-heavy-baryon state.

A similar calculation for \(\bar{b}\bar{s}ud\) indicates that the \(bs\) diquark is also not heavy enough.

\[
M(\bar{b}\bar{s}ud) = \left(\frac{1}{2}\right) \cdot [M(B_s) + M(\Lambda) + M(\Lambda_b)] + \langle \delta H_{bs} \rangle = 6180 \text{ MeV} \gg M(B) + M(K) = 5773 \text{ MeV} \tag{2.21}
\]

However, the \(bc\) diquark may be heavy enough to produce a bound four-quark state.

\[
M(\bar{c}\bar{b}ud) = \left(\frac{1}{2}\right) \cdot [M(B_c) + M(\Lambda_b) + M(\Lambda_c)] + \langle \delta H_{cs} \rangle = 7280 \pm 200 \text{ MeV} \tag{2.22}
\]

\[
M(D) + M(B) = 7146 \text{ MeV} \tag{2.23}
\]

Here the experimental error on the \(B_c\) mass is too large to enable any conclusions to be drawn. But if the bound state exists, it may produce striking experimental signatures.
A bound $\bar{c}b\bar{u}d$, $\bar{c}b\bar{u}u$ or $\bar{c}b\bar{d}d$ state would decay only weakly. either by $b$-quark decay into two charmed mesons (with the same sign of charm, so that there cannot be a $J/\psi$ decay mode), or a $c$-quark decay into a $b$ meson and a strange meson. The signature with a vertex detector will see a secondary vertex with a multiparticle decay and one or two subsequent heavy quark decays and either one track or no track from the primary vertex to the secondary.

On the other hand, if the 2.32 GeV state seen by BaBar is really a $DK I = 0$ molecule with an isospin violating $D_s - \pi$ decay, the analog for the $bc$ system is a $BD$ molecule with either $I = 1$ or $I = 0$ and a $B_c - \pi$ decay. which is isospin conserving for $I = 1$ or isospin violating for $I = 0$.

Here the masses are very different and give a completely different signature with a high energy pion. $M(B) = 5279$ MeV, $M(D) = 1867$ MeV. This gives $M(B) + M(D) = 7146$ MeV, while $M(B_c) = 6400 \pm 400$ MeV. So a molecule just below $BD$ threshold would just rearrange the four quarks into $B_c$-$\pi$ and fall apart, either with or without isospin violation, giving a neutral or charged pion having a well defined energy of $750 \pm 400$ MeV with the precision improved by better measurements.

In any case this is a striking signal which cannot be confused with a $q\bar{q}$ state. Experiments can look for a resonance with a pion accompanying any of $B_c$ states.

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