Constant-roll approach to non-canonical inflation

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Abstract

The scenario of constant-roll inflation is studied where the inflaton is a scalar field with modified kinetic term, known as non-canonical scalar field. This modification leads to some changes in the slow-roll parameters, and also by taking the second slow-roll parameter as a constant, the differential equation for the Hubble parameter is altered as well. Assuming $\eta = \beta$ and reconsidering the perturbation equations makes it clear that there should be some modification terms in the scalar spectral index and amplitude of scalar perturbations. After finding the exact solution, the main perturbation parameters are obtained at the horizon crossing time. Then by plotting the $r - n_s$ diagram it is shown that for some specific values of $\beta$ one could find a good agreement with observational data.

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I. INTRODUCTION

Since the first idea of Guth in 1981 about the very early universe [1], known as old inflation, many models of inflation have been put forward such as new inflation [2, 3], chaotic inflation [4], k-inflation [6, 7], brane inflation [8, 9], G-inflation [10–13], warm inflation [14–20] etc. Inflation is assumed to be a phase of acceleration expansion in very early times that the universe undergoes a huge expansion. This phenomenon is assumed to be caused by a scalar field.

The most of inflationary models are based on the slow-roll assumption. In this approach, the scalar field changes very slowly and rolls down slowly from the top of its potential. The flatness of the potential provide the situation for having a quasi-de Sitter expansion especially in the beginning of inflation. Such a model of inflation are describe by the slow-roll parameters that smallness of them is required to have enough amount of inflation to overcomes the problem of hot big bang theory [5]. The first slow-roll parameter is \( \epsilon = -\dot{H}/H^2 \), and the condition \( \epsilon < 1 \) is required for having an acceleration expansion phase \( (\ddot{a} > 0) \). The second slow-roll parameter is \( \eta = \ddot{\phi}/H\dot{\phi} \) that is the rate of time derivative of scalar field during a Hubble time [21]. Smallness of this parameter states that even \( \dot{\phi} \) changes very slowly too, and on the other hand guarantees occurring enough amount of inflation.

The simplest modification to the standard inflationary scenario might be a model with generalized kinetic term for the scalar field [22], known as the non-canonical scalar field. Such a models have a better consistency with observational data than canonical scalar field model. Some interesting feature of non-canonical scalar field model could be addressed as follows [23]

- Steep potential like \( v(\phi) \propto \phi^{-n} \) which are known as the potential dark energy of a canonical scalar field could give a good inflaton potential in non-canonical scalar field.

- The consistency relation \( r = -8\pi T \) is violated in non-canonical scalar field model of inflation.

- In canonical scalar field model of inflation, the exponential potential roughly stand in acceptable range of data. However, this type of potential in non-canonical model of inflation could have a good agreement with data.
The slow-roll feature of inflation is provided by a potential with almost a flat part. It rise to this question what happens if the potential is exactly flat? This question first was considered in [24]. From the equation of motion of scalar field, it is determined that for a flat potential the second slow-roll parameters becomes $\eta = -3$, which is not smaller than unity, actually it is of order one. After that, in [25] the author studied the non-Gaussianity of the case and it was specified that the non-Gaussianity is not ignorable anymore and it could be of order one. The idea of having flat potential was generalized in [26], where the authors assumed a constant for $\eta$. They find an approximate solution for the model and obtained a scalar perturbation amplitude that could even varies on superhorizon scale, and also for some choices it could be scale invariant. In [27], where for the first time the name "constant-roll" was addressed, the same model was reconsidered using Hamilton-Jacobi formalism [28–34] and they found an exact solution for the Hubble parameter which possesses the attractor behavior as well. Also, it was concluded that the power spectrum could remain scale invariant for specific choice of the constant. The scenario of constant-roll inflation in modified gravity is studied in [35–39], and the generalized version of this approach, known as smooth-roll inflation could be found in [40–42].

The interesting feature and application of non-canonical scalar field in slow-roll inflationary scenario motivates us to investigate this model in constant-roll inflation. The perturbation equations will be reconsidered again and we find the modified amplitude of scalar perturbations, and also the correction terms to the scalar spectral index are determined which are second order of $\eta$. Comparing the result with observational data illustrates that for some specific values of $\beta$ one could have an almost scale invariant perturbation on superhorizon scale.

The paper is organized as follow: In Sec.II, the general formula of the non-canonical scalar field is obtained and they are reexpressed for specific choice of the Kinetic term. The slow-roll parameters and the differential equation for the Hubble parameter are addressed in Sec.III, where the constant-roll approach is applied on the equations. The scalar and tensor perturbations are discussed in Sec.IV, and the power spectrum of perturbations are derived. And finally in Sec.V, the results of the work is summarized.
II. NON-CANONICAL SCALAR FIELD MODEL

The action is assumed to be

\[ S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R + \int d^4x \sqrt{-g} \mathcal{L}(\phi, X) \right) \]  

(1)

where \( X = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2 \), and \( \mathcal{L}(\phi, X) \) is the lagrangian of non-canonical scalar field that in general is an arbitrary function of scalar field \( \phi \) and \( X \). Variation of at action with respect to the metric comes to the field equation of the model

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left( \frac{\partial \mathcal{L}}{\partial X} \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L} \right) \]  

(2)

and also variation of the action with respect to the field come to the following equation of motion

\[ \frac{\partial \mathcal{L}}{\partial \phi} - \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \right) = 0. \]  

(3)

It is assumed that the geometry of the universe is describe by a spatially flat FLRW metric

\[ ds^2 = dt^2 - a^2(t) \left( dx^2 + dy^2 + dz^2 \right) \]

Since the term in parenthesis on the right hand of the field equation (1) is the energy-momentum tensor of scalar field, in a comparison to the energy-momentum of a perfect fluid \( T_{\mu\nu} = (\rho + p) u_\mu u_\nu - pg_{\mu\nu} \) it is concluded that the energy density, pressure and four velocity of the field are

\[ \rho = 2X \frac{\partial \mathcal{L}}{\partial X} - \mathcal{L}, \quad p = \mathcal{L}, \quad u_\mu = \frac{\partial_\mu \phi}{\sqrt{2X}}. \]  

(4)

Substituting this metric into the field equation (2), one arrives at the Friedmann equations

\[ H^2 = \frac{8\pi G}{3} \rho, \quad \dot{H} = -4\pi G \left( \rho + p \right). \]  

(5)

Also, the equation of motion (3) for this geometry is read as

\[ \left( \frac{\partial \mathcal{L}}{\partial X} + 2X \frac{\partial^2 \mathcal{L}}{\partial X^2} \right) \ddot{\phi} + \left( 3H \frac{\partial \mathcal{L}}{\partial X} + \dot{\phi} \frac{\partial \mathcal{L}}{\partial \phi} \right) \dot{\phi} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \]  

(6)

From now on, the lagrangian of the scalar field is assumed to be

\[ \mathcal{L} = X \left( \frac{X}{M^4} \right)^{\alpha-1} - V(\phi), \]
where $\alpha$ is a dimensionless constant and $M$ is a constant with mass dimension. Using this definition for scalar field Lagrangian, its energy density and pressure are expressed by

$$\rho = (2\alpha - 1)X \left( \frac{X}{M^4} \right)^{\alpha-1} + V(\phi), \quad p = X \left( \frac{X}{M^4} \right)^{\alpha-1} - V(\phi). \quad (7)$$

Then, the equation of motion (6) is reduced to

$$\ddot{\phi} + \frac{3H}{2\alpha - 1} \dot{\phi} + \left( \frac{2M^4}{\dot{\phi}^2} \right)^{\alpha-1} \frac{V_\phi(\phi)}{\alpha(2\alpha - 1)} = 0. \quad (8)$$

Introducing the Hubble parameters as a function of scalar field, and using Eq.(5), (7) and relation $\dot{H} = \dot{\phi}H_\phi(\phi)$, the time derivatives of the scalar field is given by

$$\dot{\phi}^{2\alpha-1} = \frac{-2^\alpha M_p^{2(2\alpha-1)} \mu^{4(\alpha-1)}}{\alpha} H_\phi(\phi), \quad (9)$$

in which $M_p^2 = 1/8\pi G$ and $\mu \equiv M/M_p$. Then substituting this in the Friedmann equation (5), and using Eq.(7) the potential of scalar field is read as

$$V(\phi) = 3M_p^2 H^2(\phi) - \frac{(2\alpha - 1) M_p^{4\alpha}}{2^\alpha M^{4(\alpha-1)}} \left[ \frac{-2^\alpha \mu^{4(\alpha-1)}}{\alpha} H_\phi(\phi) \right]^{\frac{2\alpha}{2\alpha - 1}}. \quad (10)$$

### III. NON-CANONICAL INFLATION

During the inflation, the universe undergoes an extreme expansion in short period of time. Here, it is assumed that the inflation is caused by a non-canonical scalar field. Usually, inflation is describe by using the slow-roll parameters. The first slow-roll parameter indicates the rate of the Hubble parameter during a Hubble time as \[ \epsilon = -\frac{\dot{H}}{H^2}, \quad (11) \]

so that to have a quasi-de Sitter expansion this parameter should be much smaller than unity \[.\] Using this equation and Eq.(5), one could obtain the time derivative of the scalar field in terms of the Hubble parameter and the slow-roll parameters $\epsilon$ as \[.\]

$$\dot{\phi}^{2\alpha} = \frac{2^\alpha}{\alpha} M_p^2 M^{4(\alpha-1)} \epsilon H^2. \quad (12)$$

Assuming the Hubble parameter as a function of scalar field, $H := H(\phi)$, one has $\dot{H} = \dot{\phi}H_\phi$. Then, the slow-roll parameter $\epsilon$ is read as

$$\epsilon = \left( \frac{2^\alpha}{\alpha} M_p^2 M^{4(\alpha-1)} \right)^{\frac{1}{2\alpha-1}} \frac{H_\phi^{\frac{1}{2\alpha-1}}}{H^2}. \quad (13)$$
The second slow-roll parameter is defined as \( \eta = \ddot{\phi} / \dot{\phi} H \), where in constant-roll inflation it is assumed as a constant \( \eta = \beta \). Using this definition and also Eq. (12) and (13), we arrive at the following differential equation for the Hubble parameter

\[
H_{,\phi}^{\frac{2\alpha-2}{2\alpha-1}} H_{,\phi\phi} = C_0 H, \quad C_0 \equiv \frac{\beta}{\left( \frac{2\alpha}{\alpha} M_p^2 M^4(\alpha-1) \right)^{\frac{1}{2\alpha-1}}},
\]

which comes to the same differential equation for the Hubble parameter as in \[?\], where the authors consider the constant-roll inflation for the canonical scalar field. Since \( H_{,\phi} = dH / d\phi \) there is \( d\phi = dH / H_{,\phi} \). Therefore, for the second derivative of the Hubble parameter in terms of the scalar field we have \( H_{,\phi\phi} = dH_{,\phi} / d\phi = H_{,\phi} dH_{,\phi} / dH \). Substituting this into the above differential equation, one arrives at

\[
H_{,\phi}^{\frac{2(\alpha-1)}{2\alpha-1}} = \frac{(3\alpha - 1)C_0}{2\alpha - 1} H^2 + C_1;
\]

where \( C_1 \) is the constant of integration. Taking another integration from Eq. (15), the scalar field could be expressed in terms of the Hubble parameter as

\[
\phi = \phi_0 + \frac{H}{C_1^{\frac{2\alpha-1}{2(\alpha-1)}}} \, 2F_1 \left[ \frac{1}{2}, \frac{2\alpha - 1}{2(3\alpha - 1)}; \frac{3}{2} \cdot \frac{-3(\alpha - 1)C_0}{(2\alpha - 1)C_1} H^2 \right],
\]

in which \( \phi_0 \) is constant of integration too. From Eqs. (15) and (16), it seems that every quantity could be expressed in terms of the Hubble parameter, such as the slow-roll parameter

\[
\epsilon = \left( \frac{2\alpha}{\alpha} M_p^2 M^4(\alpha-1) \right)^{\frac{1}{2\alpha-1}} \frac{(3\alpha - 1)C_0}{2\alpha - 1} H^2 + C_1 \right]^{\frac{\alpha}{2\alpha-1}},
\]

and also for the number of e-folds there is

\[
N = \int_{t_i}^{t_e} H dt = \int_{H_i}^{H_e} \frac{H}{H} dH = \int_{H_i}^{H_e} \frac{-1}{\epsilon H} dH
\]

\[
= \frac{-1}{\left( \frac{2\alpha}{\alpha} M_p^2 M^4(\alpha-1) \right)^{\frac{1}{2\alpha-1}}} \int_{H_i}^{H_e} \frac{H dH}{\left( \frac{(3\alpha - 1)C_0}{2\alpha - 1} H^2 + C_1 \right)^{\frac{\alpha}{2\alpha-1}}},
\]

and by taking integrate one arrives at

\[
N = \frac{-1}{2C_0 \left( \frac{2\alpha}{\alpha} M_p^2 M^4(\alpha-1) \right)^{\frac{1}{2\alpha-1}}} \left[ \frac{(3\alpha - 1)C_0}{2\alpha - 1} H^2 + C_1 \right]^{\frac{2\alpha-1}{2\alpha-1}} \bigg|_{H_i}^{H_e},
\]

where \( H_e \) is the Hubble parameter at the end of inflation, and \( H_i \) is the Hubble parameter at the horizon exit time. From Eq. (16) the potential of the scalar field could be derived only
as a function of the Hubble parameter too

\[ V = 3M_p^2 H^2 \left[ 1 - \frac{(2\alpha - 1)}{3\alpha} \left( \frac{2^\alpha}{\alpha} M_p^2 M^{4(\alpha - 1)} \right)^{\frac{1}{2\alpha - 1}} \left( \frac{C_0}{H^2} + C_1 \right)^{\frac{3\alpha - 1}{\alpha - 1}} \right] \]  

(20)

IV. PERTURBATIONS OF NON-CANONICAL SCALAR FIELD

Assume a small inhomogeneities of scalar field as \( \phi(t, x) = \phi_0(t) + \delta \phi(t, x) \). This perturbation with respect to the background \( \phi_0(t) \) (which from now on we omit the subscript ”0”) induce a perturbation to the metric because from the field equation it is clear that geometry and matter are tightly coupled together. The metric in longitudinal gauge is written as

\[ ds^2 = (1 + 2\Phi(t, x)) dt^2 - a^2(t) (1 - 2\Psi(t, x)) \delta_{ij} dx^i dx^j \]

where by assuming a diagonal tensor for the spatial part of energy-momentum tensor (i.e. \( \delta T_{ij} \propto \delta_{ij} \)) there is \( \Psi(t, x) = \Phi(t, x) \).

The action for linear scalar perturbation is derived as \[7, 43\]

\[ S = \frac{1}{2} \int \left( v'^2 + c_s^2 (\nabla v)^2 + \frac{z''}{z} v \right) d\tau d^3 x. \]  

(21)

in which \( v \equiv z \zeta \), and \( \zeta \) is the curvature perturbation given by \( \zeta = \Phi + H \frac{\dot{\phi}}{\phi} \), and prime denotes derivative with respect to the conformal time \( \tau \), \( a d\tau = dt \). The quantity \( z \) is known as the Mukhanov variable that for our model it is defined as \[7, 43\]

\[ z^2 \equiv \frac{2\alpha aX}{c_s^2 H^2} \left( \frac{X}{M^4} \right)^{\alpha - 1}. \]

From Eq.(21), the equation for perturbation quantity \( v \) becomes

\[ \frac{d^2}{d\tau^2} v(\tau, x) - c_s^2 \nabla^2 v(\tau, x) - \frac{z''}{z} v(\tau, x) = 0. \]  

(22)

and by utilizing the Fourier mode, one arrives at

\[ \frac{d^2}{d\tau^2} v_k(\tau) + \left( c_s^2 k^2 - \frac{1}{z} \frac{d^2 z}{d\tau^2} \right) v_k(\tau) = 0. \]  

(23)

Before discussing the solution of the above equation, we try to compute the term \( z''/z \). from the definition of \( z \) and the slow-roll parameters of the previous section, there is

\[ z \equiv \frac{\sqrt{\alpha}}{2^{2\alpha - 2} M^{2(\alpha - 1)} C_s H} \frac{\dot{\phi}^\alpha}{H}. \]
\[ \frac{dz}{d\tau} = z(aH) [1 + \alpha \eta + \epsilon] \]

To calculate the second derivative, we first need to obtain the derivative of the slow-roll parameters with respect to the conformal time
\[ \frac{de}{d\tau} = (aH) [2\alpha \eta \epsilon + \epsilon^2], \quad \frac{d\eta}{d\tau} = (aH) [2(1 - \alpha) \eta^2 + \epsilon \eta + \epsilon^2]. \]

Then, the second derivative of the quantity \( z \) with respect to the conformal time is read by
\[ \frac{1}{z} \frac{d^2 z}{d\tau^2} = 2(aH)^2 [1 + \epsilon + \frac{3}{2} \alpha \eta + \epsilon^2 + \frac{\alpha^2}{2} \eta^2 + \frac{3}{2} \alpha \epsilon \eta]. \]

At subhorizon scale, where \( c_s k \gg aH \) (i.e. \( c_s k \gg \frac{dz}{d\tau} \)), the quantity \( v_k \) is obtained as
\[ \frac{d^2}{d\tau^2} v_k(\tau) + c_s^2 k^2 v_k(\tau) = 0, \quad \Rightarrow \quad v_k(\tau) = \frac{1}{\sqrt{2c_s k}} e^{ic_s k \tau}. \]

In order to find the general solution of equation (23), we make the variable changes \( v_k = \sqrt{-\tau f_k} \) and \( x \equiv -c_s k \tau \). After some manipulation, the find the following Bessel’s differential equation
\[ \frac{d^2 f_k}{dx^2} + \frac{1}{x} \frac{df_k}{dx} + \left(1 - \frac{\nu^2}{x^2}\right) f_k = 0, \quad \Rightarrow \quad \frac{d^2}{d\tau^2} v_k(\tau) = \frac{1}{\sqrt{2c_s k}} e^{ic_s k \tau}. \]

The general solution for the above differential equation is the first and second kind of Henkel function. However to have a constancy with the solution in subhorizon scale, the second kind of Henkel function should be ignored, therefore the final solution is acquired as
\[ v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{\frac{\pi}{2}(\nu + \frac{1}{2})} \sqrt{-\tau} H^{(1)}_{\nu}(c_s k \tau). \]

The spectrum of curvature perturbation is defined as
\[ P_s = \frac{k^3}{2\pi^2} |\zeta|^2 = \frac{k^3}{2\pi^2} \left| \frac{v_k}{z} \right|^2. \]

On superhorizon scale, the asymptotic behavior of Henkel function is
\[ \lim_{-k \tau \to -\infty} H^{(1,2)}_{\nu}(x) = \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{2c_s k}} e^{\frac{i\pi}{2}(ic_s k \tau + \delta)}, \quad \delta = \frac{1}{2} (\nu + \frac{1}{2}). \]

Then, the spectrum of curvature perturbation on superhorizon scale will be
\[ P_s = \left( \frac{2^{\nu - \frac{3}{2}} \Gamma(\nu)}{\Gamma(3/2)} \right)^2 \left( \frac{H^2}{2\pi \sqrt{c_s (\rho + p)}} \right)^2 \left( \frac{c_s k}{aH} \right)^{3-2\nu}. \]

8
V. CONSISTENCY WITH OBSERVATION

An advantage of the inflationary scenario is the prediction of quantum perturbations in which scalar perturbations are the seeds for large scale structure of the universe and the tensor perturbation is known as the gravitational waves. During inflation these perturbations are stretched out the horizon and remain invariant. One the other hand the observational data indicates an almost scale invariant spectrum for curvature perturbations. This scale invariant feature is described by scalar spectral index so that it is defined from $P_s = A_s^2 \left( \frac{c_s k}{aH} \right)^{n_s - 1}$, where $A_s^2$ is the amplitude of scalar perturbation at horizon crossing $c_s k = aH$. For $n_s = 1$ the amplitude of scalar perturbation is exactly scale invariant, however that latest observational data implies that $\ln (P_s \times 10^{10}) = 3.062 \pm 0.029$ and $n_s = 0.9677 \pm 0.0060$ expressing an almost scale invariant perturbations [44]. The tensor perturbations usually measured indirectly through the parameter $r$ known as the tensor-to-scalar ratio, $r = P_t / P_s$. The data received from Planck shows there is only upper bound on this parameter, $r < 0.11$, and it is still not measured exactly [44].

Consistency of the presented model with observational data is the main goal of this section. At the end of inflation the first slow-roll parameter reaches unity [45] in which it happens for $H = H_e$

$$
\epsilon(H_e) = \left( \frac{2^\alpha}{\alpha} M_p^2 M^4(\alpha-1) \right) \frac{2 (3\alpha-1) C_0}{2^\alpha - 1} \frac{H_e^2 + C_1}{H_e^2} = 1. 
$$

(29)

At the horizon exit the slow-roll parameter $\epsilon$ is smaller than unity, and the corresponding Hubble parameter could be determine through the number of e-fold equation [18]

$$
H_i(N) = \left\{ \frac{2\alpha - 1}{(3\alpha - 1)C_0} \left[ 2C_0 A^{2\alpha - 1} N + \left[ \frac{(3\alpha - 1)C_0}{2\alpha - 1} H_e^2 + C_1 \right] \frac{3\alpha - 1}{2\alpha - 1} \right] - C_1 \right\}^{1/2}
$$

(30)

which is expressed in term of the number of e-fold parameter $N$. Note that, in order to overcome the horizon and flatness problems we need about $60 - 70$ number of e-fold [22].

Using Eq. (30), the main perturbations parameters such as the scalar spectral index, amplitude of scalar perturbation, and tensor-to-scalar ratio could be obtained in terms of $N$
as

\[ \epsilon(N) = \left( \frac{2^\alpha}{\alpha} M_p^2 M^{4(\alpha-1)} \right)^{2\alpha-1} \frac{\left( \frac{(3\alpha-1)C_0}{2\alpha-1} \frac{H_i^2(N)}{H_i(N)} + C_1 \right)}{H_i(N)} \]  

(31)

\[ n_s(N) = 4 - 2\nu(N) \]  

(32)

\[ \nu^2(N) = \frac{9}{4} + 3\alpha\beta + \alpha^2\beta^2 + \left( 6 + 9\alpha\beta + 2\alpha^2\beta^2 \right) \epsilon(N) \]  

(33)

\[ \mathcal{P}_s(N) = \left( \frac{2^{\nu(N) - \frac{5}{2}\Gamma(\nu(N))}}{\Gamma(3/2)} \right)^2 \left( \frac{H^2(N)}{2\pi \sqrt{c_s(\rho + p)}} \right)^2 \]  

(34)

\[ r(N) = \frac{2H_i^2(N)}{\pi^2 \mathcal{P}_s(N)} \]  

(35)

Fig. 1 the \( r - n_s \) diagram is plotted for different values of \( \alpha, \beta \) and \( M \) which have been placed on the \( r - n_s \) plot of Planck so that the blue era is the 68\% CL region and the light blue era is the 95\% CL region. The figures on the left hand side have been depicted for different values of \( \beta \), the right hand side figure related to the \( r - n_s \) diagram for different values of \( M \), and each line of figure belongs to an specific value of \( \alpha \). Fig. 1 displays \( r - n_s \) diagram for \( \alpha = 3/2, M = 3.5 \times 10^{-13} \) and different values of \( \beta \), where the point on each curve indicates the point \( (n_s, r) \) for \( N = 65 \). It is clear that by increasing \( \beta \) the curve recedes from our interested regime, and also the hight of the curves reduced as well. The same diagram for \( \alpha = 3/2, \beta = 0.00025 \) but different values of \( M \) have been illustrated in Fig. 1b. In this case all curves follows the same track however the point \( (r - n_s) \) point for \( N = 70 \) might end up in different place for values of \( M \). On the other side, variation of \( M \) has an significant effect on the magnitude of amplitude of scalar perturbations that should be noted so that enhancement of the \( M \) leads to bigger values of amplitude of scalar perturbations.

VI. CONCLUSION

This work wa related to the scenario of constant-roll inflation which inflation is derived by a non-canonical scalar field with modified kinetic term. In this scenario the second slow-roll parameter \( \eta = \ddot{\phi}/H\dot{\phi} \) is assumed as a constant. By calculating the slow-roll parameter in term of the Hubble parameter, we could derived a differential equation for the Hubble parameter which could leads to the corresponding differential equation for canonical scalar field model (namely taking \( \alpha = 1 \)). On the other side, by taking into account the assumption of constant slow-roll parameter, the perturbation equation should
be reconsidered again. Doing so make it clear that there are some modification terms for the amplitude of scalar perturbation and scalar spectral index where we could have second order of \( \eta \).

By finding an exact solution, it was realized that every parameter of the model could be expressed in terms of the Hubble parameter. Therefore, the Hubble parameter played the same roll that scalar field plays in Hamilton-Jacobi formalism of inflation. The condition \( \epsilon = 1 \) clarify the Hubble parameter at the end of inflation, i.e. \( H_e \), and using the number of e-fold the Hubble parameter was determined in terms of the number of e-fold and other

FIG. 1: \( r - n_s \) diagram
constant parameters of the model. Utilizing this result, we expressed the main perturbation parameters of the model in terms of the number of e-fold to have a better comparison with observational data.

The best comparison that one might perform is the $r - n_s$ diagram, and we displayed this diagram for various values of $\alpha$, $\beta$ and $M$ in Fig. The figure showed that for each value of $\alpha$ and $M$, the parameter $\beta$ plays an effective role so that by enhancement of this parameter the $r - n_s$ curve tends to the left hand side of Planck region. If we take $\alpha$ and $\beta$ as a constant and consider the diagram for various choice of $M$, the $r - n_s$ curve follow the same track for each choice however the point $(r, n_s)$ for $N = 70$ might be pushed to the left hand side of our interested era for bigger values of $M$. Final result that come from this diagram is that different choice of $\alpha$ for specific selection of $\beta$ and $M$ only lowers the height of $r - n_s$ curve. To sum up briefly, the results show that the model predictions could stand in 68% CL of Planck region which is a good agreement between model and observational data.

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[45] The condition $\epsilon = 1$ could be interpreted as the end of inflation like the standard inflationary scenario, or it could express the most initial time of the beginning of inflation like intermediate inflation.