SUPERCONDUCTIVITY IN A MESOSCOPIC DOUBLE SQUARE LOOP: EFFECT OF IMPERFECTIONS

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Abstract

We have generalized the network approach to include the effects of short-range imperfections in order to analyze recent experiments on mesoscopic superconducting double loops. The presence of weakly scattering imperfections causes gaps in the phase boundary $B(T)$ or $\Phi(T)$ for certain intervals of $T$, which depend on the magnetic flux penetrating each loop. This is accompanied by a critical temperature $T_c(\Phi)$, showing a smooth transition between symmetric and antisymmetric states. When the scattering strength of imperfections increases beyond a certain limit, gaps in the phase boundary $T_c(B)$ or $T_c(\Phi)$ appear for values of magnetic flux lying in intervals around half-integer $\Phi_0 = \hbar c/2e$. The critical temperature corresponding to these values of magnetic flux is determined mainly by imperfections in the central branch. The calculated phase boundary is in good agreement with experiment.
Early experiments\textsuperscript{1,2} have revealed that the effect of nonmagnetic impurities on the transition temperatures of bulk superconductors is very small. The critical temperature, $T_c$, changes by about 1% for 1% concentration of impurities. An interpretation of these observations was first proposed by Anderson\textsuperscript{3}. The only effect of the impurities is to change the energy of a free electron to eigenenergies determined by those impurities. This modifies the density of states in the integral equation for $T_c$. Therefore, scattering by non-magnetic impurities only slightly changes $T_c$ in bulk superconductors\textsuperscript{4}. Investigations extending Anderson’s work have been performed for multiband superconductors using the Abrikosov-Gor’kov approach\textsuperscript{5}. An interesting opportunity to intensify the effect of the imperfections occurs in mesoscopic superconducting structures where the confined condensate is much more sensitive to the action of impurities than in bulk structures.

Recently, the onset of the superconducting state has been studied\textsuperscript{6,7} in different mesoscopic structures of Al, comprising lines, dots, loops, double loops, microladders etc., with sizes smaller than the coherence length $\xi(T)$. Refs. \textsuperscript{8,9} show that experimentally observed phase boundaries for square loops with two attached leads are in excellent agreement with calculations based on Ginzburg-Landau (GL) theory\textsuperscript{10,11}.

In this present letter, we analyze the case of a superconducting In this present letter, we analyze the case of a superconducting mesoscopic double square loop (see the inset to Fig. 1a) where experiments\textsuperscript{6} have revealed the phase boundary shown in Fig. 1a. Using the micronet approach\textsuperscript{12–14} for a superconducting double loop, we obtain a phase boundary (cf. Ref. \textsuperscript{7}) which consists of the intersecting parabolas shown in Fig. 1b. One set (with minima at integer values of the magnetic flux $\Phi$ through one loop in units of the magnetic flux quantum $\Phi_0 \equiv hc/2e$: $\Phi/\Phi_0$) depends on the magnetic flux quanta penetrating each loop with $L = 0, 1, 2, \ldots$. The other set (with minima at half-integer values of $\Phi/\Phi_0$) depends on odd numbers of magnetic flux quanta penetrating the double loop as a whole with $L = 1/2, 3/2, \ldots$. Since the critical temperature corresponds to the lowest Landau level $E_{LLL}(\Phi)$, the way the parabolas intersect means that the minimum energy encounters a shift from one branch to another at certain values of magnetic flux. Moreover, since the derivative of the lowest Landau level with respect to magnetic flux is proportional to the persistent current (cf. Ref. \textsuperscript{15}, Eq. (4.5)) we arrive at the following paradox: At the intersections, the left and right derivatives of $E_{LLL}(\Phi)$ are different, the persistent current has a discontinuity and its value is consequently indefinite. In order to resolve this paradox, an analogy between superconducting loops and semiconducting quantum rings can be exploited. In such rings, in the presence of impurities and an Aharonov-Bohm magnetic field, a crossing is known to change into an anti-crossing. In this case, the gaps between the different eigenenergies as a function of the magnetic flux widen and hence the degeneracy of states, which contribute to the lowest level, is raised (see Ref. \textsuperscript{16}). In this letter we demonstrate that this also applies to the superconducting mesoscopic double square loop.
FIG. 1. Experimental phase boundaries for a double loop of Al (shown in the inset) with (panel a) and without (panel b) a parabolic background (the dashed line in panel a) due to the finite width of the loop. When transforming the phase boundary to the plane $T_c(\Phi)/T_c$ versus $\Phi/\Phi_0$, the value $Q=1302$ nm is taken in order to ensure that the maxima of $T_c(\Phi)/T_c$ are at integer values of $\Phi/\Phi_0$. Theoretical phase boundaries (solid lines in panel b) calculated for a perfect double loop with $Q=1302$ nm and $\xi_0 = 128$ nm.

The observed phase boundary for a superconducting mesoscopic double square Al loop is given in Fig. 1a. After subtraction of the parabolic background, related to the finite width, $w = 130$ nm, of the stripes (dashed line in Fig. 1a), at least 12 practically identical periods are seen. One such period is plotted in Fig. 1b (dots). From the period of the oscillations of the phase boundary with respect to the magnetic field, $\Delta B = 1.24$ mT, an effective loop side length $Q = 1.3 \mu m$ is obtained. This is close to the average loop size. For further experimental details, the reader is referred to Ref. 6. In order to understand the observed “anti-crossing” of the different elements of the experimentally observed phase boundary and, in particular, the smooth shape of the $T_c(\Phi)$ minima, we consider a superconducting mesoscopic double square loop with imperfections. These imperfections may be introduced during the fabrication of the mesoscopic structures. Most probably one of the sources of such imperfections is the inhomogeneity of the geometrical superconducting lines written by e-beam lithography.

Within the framework of the GL approach, the presence of imperfections in a superconducting structure may be modelled by a spatial inhomogeneity in the parameters $a$ and $b$ in the GL equation:

$$\frac{1}{2m} \left( -i\hbar \nabla - \frac{2e}{c} A(\mathbf{r}) \right)^2 \psi(\mathbf{r}) + a(\mathbf{r}) \psi(\mathbf{r}) + b(\mathbf{r}) \left| \psi(\mathbf{r}) \right|^2 \psi(\mathbf{r}) = 0. \tag{1}$$

Near the phase boundary, where the order parameter $\psi(\mathbf{r})$ is small, the system is adequately described by the linearized GL equation:

$$\frac{1}{2m} \left( -i\hbar \nabla - \frac{2e}{c} A(\mathbf{r}) \right)^2 \psi(\mathbf{r}) + a(\mathbf{r}) \psi(\mathbf{r}) = 0. \tag{2}$$

Moreover, the magnetic field may be assumed to be equal to the applied magnetic field. The vector potential $A$ of the uniform field $B \parallel e_z$ is taken in the symmetric gauge.

The presence of imperfections, localized in the loop around several points $\mathbf{r}_s$ over a distance which is much smaller than the coherence length or the typical loop size (“short range imperfections”), can be modelled by the following function:
\[ a(r) = a + \sum_s V_s \delta(r - r_s), \]  

(3)

where \( a \) is the GL coefficient of the substance, and the magnitudes \( V_s \) are determined by specific characteristics of the imperfections. Eq. (2) then becomes similar to the Schrödinger equation for a particle of mass \( m \) and charge \( 2e \) in the potential field described by the scalar form \( \sum_s V_s \delta(r - r_s) \) and by the vector potential \( \mathbf{A}(r) \), the quantity \(-a\) playing the role of the energy.

Short-range imperfections are assumed to be present in all three branches of the loop at the points characterized by the coordinates \( Q_s \) with \( s = L, M, R \) as shown in Fig. 2. Furthermore, taking \( \xi(T) \equiv \xi_0/\sqrt{(1 - T/T_c)} \) as a unit of length, we obtain the linearized GL equation in terms of dimensionless coordinates:

\[
\left\{ (i \nabla_x + \frac{2\pi}{\Phi_0} A_x)^2 + (i \nabla_y + \frac{2\pi}{\Phi_0} A_y)^2 + \sum_{s=L,M,R} \tilde{V}_s \delta(y - Q_s) - 1 \right\} \Psi = 0.
\]

(4)

It should be noted that the dimensionless scattering magnitude \( \tilde{V}_s = 2mV_s\xi(T)/\hbar^2 = C_s/\sqrt{(1 - T/T_c)} \) is temperature dependent.

![FIG. 2. A model configuration of imperfections (open circles) in the double loop.](image)

We have solved Eq. (4) following the micronet approach. A new conceptual feature compared with Refs. 12–14 is the use of additional nodal points at the positions of the imperfections. The presence of these imperfections implies an additional condition, which can be derived in the following way. In the vicinity of the point with \( y = Q_s \), Eq. (4) takes the form

\[
(i \nabla_y + \frac{2\pi}{\Phi_0} A_y)^2 \Psi - \Psi = -\tilde{V}_s \delta(y - Q_s) \Psi.
\]

(5)

We integrate both sides of this equation over \( y \) from \( Q_s - \epsilon \) to \( Q_s + \epsilon \), \( \epsilon \to +0 \). Taking into account continuity of the order parameter, \( \Psi(Q_s + \epsilon) = \Psi(Q_s - \epsilon) \), we obtain the additional constraint

\[
\frac{d\Psi}{dy} \bigg|_{y=Q_s+\epsilon} - \frac{d\Psi}{dy} \bigg|_{y=Q_s-\epsilon} = \tilde{V}_s \Psi(y = Q_s)
\]

(6)

for the derivatives to the left and to the right from the new nodal point \( y = Q_s \). Applying Eq. (4) to a single loop of length \( L \) with one imperfection leads to the following secular equation
\[ \cos(\varphi) = \cos(L) + \frac{1}{2} \tilde{V}_s \sin(L), \]  
(7)

where \(\varphi = 2\pi \Phi / \Phi_0\).

Analyzing the onset of superconductivity in a superconducting ring with a lateral arm of length \(\ell\), de Gennes derived an equation which differs from Eq. (7) by the substitution \(\tilde{V}_s = -\tan(\ell)/2\). Thus, from the point of view of our present analysis, a lateral arm may be considered of as a kind of imperfection in a ring.

After some lengthy, but straightforward algebra, we obtain the secular equation which determines the phase boundary:

\[
\begin{align*}
\cos^2 \varphi + \cos \varphi \frac{D_L + D_R}{2D_M} - \frac{D_LD_R}{4} \left[ \prod_{i=1}^{2} \left( \frac{N_{iM}}{D_M} + \frac{N_{iL}}{D_L} + \frac{N_{iR}}{D_R} \right) - \frac{1}{D_M^2} - \frac{1}{D_L^2} - \frac{1}{D_R^2} \right] - \frac{1}{2} &= 0 \quad (8)
\end{align*}
\]

with

\[
\begin{align*}
N_{1s} &= \cos 3Q + \tilde{V}_s \sin(2Q - Q_s) \cos(Q + Q_s), \\
N_{2s} &= \cos 3Q + \tilde{V}_s \cos(2Q - Q_s) \sin(Q + Q_s), \\
D_s &= \sin 3Q + \tilde{V}_s \sin(2Q - Q_s) \sin(Q + Q_s); \\
N_{1M} &= \cos Q + \tilde{V}_M \sin(Q - Q_M) \cos(Q_M), \\
N_{2M} &= \cos Q + \tilde{V}_M \cos(Q - Q_M) \sin(Q + Q_M), \\
D_M &= \sin Q + \tilde{V}_M \sin(2Q - Q_M) \sin(Q + Q_M),
\end{align*}
\]

(9)

where \(s = L, R\).

In Eq. (8), \(\varphi = 2\pi \Phi / \Phi_0\) with \(\Phi\), the magnetic flux through each of the loops. Here, we recall that lengths in the above equations are expressed in units of \(\xi(T)\) and are therefore functions of the temperature. Consequently, the secular equation establishes a relation between the magnetic flux \(\Phi\) and the temperature \(T\).

The phase boundaries obtained by solving Eq. (8), with an imperfection in only one branch of the double loop, are shown in Figs. 3 to 5 (\(L\) in Figs. 3 and 4, \(M\) in Fig. 5).

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**FIG. 3.** Theoretical phase boundaries for a double loop with an imperfection in one side branch at \(Q_L = 0.5Q\) for \(Q = 1302\) nm and \(\xi_0 = 128\) nm. (a) The dimensionless magnitude \(C_L = 0\) and 0.01 corresponds to dotted and solid lines, respectively. (b) The dimensionless magnitude \(C_L = 0, 0.5, 1.0\) corresponds to dotted, solid and dashed lines. The values \(\Phi_1\) and \(\Phi_2\) are discussed in the text.
As is seen in Fig. 3a, at small values $C_s$, an “energy” gap forms between solutions corresponding to different numbers of magnetic flux quanta penetrating each loop ($L = 0, 1/2, 1, \ldots$). The resultant phase boundary indicates that a continuous change takes place from a symmetric superconducting order parameter at integer values of $\Phi/\Phi_0$ to an antisymmetric state at half-integer values of the relative magnetic flux. It is also worthy of note that the presence of imperfections slightly diminishes the critical temperature of the double loop at zero magnetic field.

When increasing $C_s$ above a certain limit, the pattern of phase boundaries changes dramatically. Gaps appear in certain flux intervals (“flux gaps”) around half-integer $\Phi/\Phi_0$ values. This behavior is illustrated by the curves in Fig. 3b. For a given $T < T_c$, a superconducting state exists when $\Phi$ ranges from zero to a certain value of $\Phi_1$, after which the sample turns into the normal state. With a further increase of magnetic flux along a horizontal straight line shown in Fig. 3b, the sample remains in the normal state until a value of $\Phi_2$ is reached, at which the sample becomes again superconducting. This demonstrates a re-entrant behavior as a function of field. [When approaching in Fig. 3b the points where the phase boundary $\Phi(T)$ has its extrema and, consequently, the derivatives $\partial T/\partial \Phi$ would diverge, the superconducting state apparently becomes unstable.] It should be noted that the existence of such a regime, where the system is normal in a certain flux interval, was reported by de Gennes\textsuperscript{12} for a single superconducting ring with a lateral arm.

The trend of lowering of the critical temperature of a double loop at zero magnetic field with increasing $C_s$ is clearly seen by comparison of the curves in Fig. 3b which refer to different values of $V_L$.

![Theoretical phase boundaries](image)

**FIG. 4.** Theoretical phase boundaries (solid lines) for a double loop with an imperfection in one side branch at $Q_L = 0.5Q$ for $Q=1302$ nm and $\xi_0 = 128$ nm. The dimensionless magnitude is $C_L = 0.02$. Results obtained without and with renormalization of temperature are given in panels a and b, respectively. Experimental data are shown with points.

In regions adjacent to integer values of $\Phi/\Phi_0$, a good agreement with experiment of the calculated phase boundary $T_c(\Phi)$, is achieved for the zero-temperature coherence length $\xi_0 = 128$ nm. This value is in accordance with previous estimates (see Ref. [17]). The resulting set of phase boundaries is shown in Fig. 4a. For comparison with the experimental data, it is necessary to renormalize the temperature scale, taking as a unit temperature the specific critical value of a loop with imperfections (see Fig. 4b).

In Fig. 5, a plot of the phase boundary is shown for a double loop with an imperfection
in the middle branch ($M$ in Fig. 2). It is clear that by increasing $V_M$, one shifts a minimum of the $1-T_c(\Phi)/T_c$ curve (using a renormalized temperature), at half-integer values of $\Phi/\Phi_0$, to higher values, without modifying those parts of the phase boundary close to the integer values of $\Phi/\Phi_0$.

FIG. 5. Theoretical phase boundaries for a double loop with an imperfection in the middle branch at $Q_M = 0.1Q$ for $Q=1302$ nm and $\xi_0 = 128$ nm. The dimensionless magnitude $C_M = 0.11, 0.125$ corresponds to dashed and solid lines. Results are obtained with renormalization of temperature. Experimental data are shown with points.

The best agreement between the calculated phase boundary and experimental data is achieved for a configuration where imperfections are present on all three branches of the double loop ($L, M, R$ in Fig. 2). The corresponding curves are shown in Fig. 6. The main observation which follows from these figures is that imperfections in the central branch play a decisive role in determining the critical temperature of a double loop at the half-integer values of $\Phi/\Phi_0$.

FIG. 6. Theoretical phase boundaries (solid line) for a double loop with imperfections in all three branches at $Q_L = Q_R = Q_M = 0.5Q$ for $Q=1302$ nm and $\xi_0 = 128$ nm. The dimensionless magnitudes are: $C_L = -C_R = 0.017$ and $C_M = 0.090$. Results are obtained with renormalization of temperature. Experimental data are shown with points.

In conclusion, we have generalized the network approach to include the effects of short-range imperfections. The presence of weakly scattering imperfections leads to the formation of gaps between solutions corresponding to integer and half-integer numbers of magnetic flux quanta penetrating each loop. The phase boundary is characterized by a smooth tran-
sition between symmetric and antisymmetric states. For imperfections with relatively large magnitudes, gaps in the phase boundary $T_c(B)$ or $T_c(\Phi)$ appear when the magnetic flux lies in intervals around half-integer values. The critical temperature at the half-integer values of the relative magnetic flux has been shown to be determined mainly by imperfections in the central branch. The calculated phase boundary for a mesoscopic double square loop is in good agreement with experiment.

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