Decoherence and disorder in quantum walks: From ballistic spread to localization

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We investigate the impact of decoherence and static disorder on the dynamics of quantum particles moving in a periodic lattice. Our experiment relies on the photonic implementation of a one-dimensional quantum walk. The pure quantum evolution is characterized by a ballistic spread of a photon’s wave packet along 28 steps. By applying controlled time-dependent operations we simulate three different environmental influences on the system, resulting in a fast ballistic spread, a diffusive classical walk and the first Anderson localization in a discrete quantum walk architecture.

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Random walks describe the probabilistic evolution of a classical particle in a structured space resulting in a diffusive transport. In contrast, endowing the walker with quantum mechanical properties typically leads to a ballistic spread of the particle’s wave function \( \psi \). The coherent nature of quantum walks has been theoretically explored, providing interesting results for a wide range of applications. They are not only a universal platform for quantum computing \( \psi \) but also constitute a powerful tool for modelling biological systems \( \psi \), thus hinting towards the mechanism of energy transfer in photosynthesis. Quantum walks of single particles on a line have been experimentally realized in several systems, e.g. with trapped atoms \( \psi \) and ions \( \psi \); energy levels in NMR schemes \( \psi \), photons in waveguide structures \( \psi \), a beam splitter array \( \psi \), and in a fiber loop configuration \( \psi \). Although these experiments opened up a new route to higher dimensional quantum systems, more sophisticated quantum walks need to be implemented to pursue the realm of real applications. A first step in this direction has been recently reported \( \psi \), in which two particles execute a simultaneous walk and display intrinsic quantum correlations.

One of the most important requirements for realizing quantum walk-based protocols is the ability to control the dynamics of the walk, that is to access and manipulate the walker’s state in a position dependent way \( \psi \). In this paper we present the first experimental realization of quantum walks with tunable dynamics. We investigate the evolution of quantum particles moving in a discrete environment presenting static and dynamic disorders.

As predicted by Anderson in 1958 \( \psi \), static disorder leads to an absence of diffusion and the wave function of the particle becomes localized, which, e.g. would render a conductor to behave as an insulator. Anderson localization has been experimentally investigated in different physical scenarios, e.g. employing photons moving in semiconductor powders \( \psi \) and photonic lattices \( \psi \), or even via Bose-Einstein condensates \( \psi \). However, although theoretically predicted in the context of quantum walks \( \psi \), the effect has never been observed in a discrete quantum walk scenario.

Furthermore, it is interesting to note that the energy transport in photosynthetic light-harvesting systems is influenced by both, static and dynamic disorders, and it is precisely the interplay between the two effects that lead to the highly efficient transfer in those molecular complexes \( \psi \). Thus, in order to simulate a realistic influence of the environment, we go further in our studies by investigating the effect of dynamical noise, which typically induces decoherence \( \psi \). Utilizing the ability to easily tune the conditions for the quantum walk, we demonstrate here the diverse dynamics of quantum particles propagating in these different systems.

In our experiment we realize the quantum walk of photons by employing a linear optical network. The evolution of the particle’s wave function \( \psi(x) \) is given by

\[
|\psi(x)\rangle \rightarrow \gamma_x|\psi(x)\rangle + \sum_{k \neq x} \beta_{x,k}|\psi(k)\rangle,
\]

with the position dependent amplitudes \( \gamma_x \) and \( \beta_{x,k} \) determining the probability of the particle to stay at the discrete position \( x \) or evolve to the adjacent sites \( k \), respectively.

We study the expansion of the particle’s wave packet in four different scenarios. (i) First of all we implement the quantum walk in a homogeneous lattice, showing that it presents an evolution that is free from decoherence. (ii) Next, we introduce static disorder by manipulating the lattice parameters \( \gamma_x \) and \( \beta_{x,k} \), thus observing Anderson localization. We then examine two scenarios leading to decoherence, which essentially differ in the time scales of the occurring dynamic perturbations. (iii) In this case a dynamic randomization of the lattice parameters \( \gamma_x \) and \( \beta_{x,k} \) simulates the evolution of a particle interacting with a fast fluctuating environment. The resulting dephasing suppresses the underlying interference effects and hence causes the particle to evolve just like in a classical ran-
dom walk \( n \geq 8 \). (iv) In the last scenario we simulate a slowly changing homogeneous environment. While \( \gamma_s \) and \( \beta_{s.x} \) are stable during a single realization, a slow drift leads to different conditions for subsequent particles, thus affecting results obtained in an ensemble measurement.

In a discrete quantum walk the position of a particle evolves according to its internal coin state \( |c\rangle \). For our photonic implementation we use the linear horizontal \( |H\rangle = (1,0)^T \) and vertical \( |V\rangle = (0,1)^T \) polarization of light. The state of the photon after \( N \) steps of the walk is found by applying the unitary transformation \( U = \prod_{n=1}^{N} S\hat{C}_n \) to the initial state \( |\psi(x)\rangle = |x_s\rangle \otimes |c_0\rangle \).

The coin operation \( \hat{C}_n(x) \) manipulates the polarization of the photon in dependence on the position \( x \) and the step number \( n \). In the basis \( \{ |H\rangle, |V\rangle \} \) the coin operator is given in matrix form by

\[
C(x) = \begin{pmatrix}
e^{i\phi_H(x)} & 0 \\
0 & e^{i\phi_V(x)}
\end{pmatrix} \begin{pmatrix}
\cos(2\theta) & \sin(2\theta) \\
\sin(2\theta) & -\cos(2\theta)
\end{pmatrix},
\]

with the diagonal matrix representing a phase shift \( \phi_H(x) \) for horizontal and \( \phi_V(x) \) for vertical polarizations, while the second matrix corresponds to a polarization rotation of \( 2\theta \). The step operation \( S \) shifts the position \( x \) of the photon by +1 if the polarization is horizontal and by −1 if it is vertical.

Following Eq. (11), the evolution of the wave function with the step number \( n \) is given by

\[
|\psi(x)_{n+1}\rangle = \gamma_s|\psi(x)_{n}\rangle + \beta_{s.x+1}|\psi(x+2)_{n}\rangle + |\psi(x-2)_{n}\rangle.
\]

Note that the transition coefficients \( \gamma_s \) and \( \beta_{s.x+1} \) are fully set by the coin operations \( C_{n+1}(x) \) and \( C_{n+1}(x) \). By changing the parameters \( \phi(x)_{H,V} \) and \( \theta \) in a controlled way we can alter the coefficients and hence create diverse types of physical conditions for a quantum walk scenario.

A simple measure to quantify the spread of the wave function in the different systems is provided by the variance \( \sigma^2 \) of the final spatial distribution. While the decoherence free quantum walk presents a ballistic spread, with \( \sigma^2 \propto n^2 \), the classical random walk is diffusive, characterized by \( \sigma^2 = n \). In contrast to both, in a one dimensional system with static disorder the wave packet shows exponential localization after a short initial expansion. The stagnation of the wave packet spread is thus evidenced by a constant variance.

The functional principle of our experimental setup is sketched in Fig. 1(a) and is discussed in detail in [13]. We generate the input photons with a pulsed diode laser with a central wavelength of 805 nm, a pulse width of 88 ps and a repetition rate of 110 kHz. The initial polarization state of the photons is prepared with polarizing beam splitters (PBS) and a fiber delay line, in which horizontally polarized light follows a longer path (Fig. 1(a)). The resulting temporal difference of 5.9 ns between both polarization components corresponds to a step in the spatial domain of \( x \pm 1 \). After a full evolution the photon wave packet is distributed over several discrete spatial positions or, equivalently, over respective time windows. For detection the photon gets coupled out of the loop by a beam splitter with a probability of 12\% per step. We employ two avalanche photodiodes (APD) to measure the photon’s time and polarization properties, which gives information about the number of steps, the specific position of the photon, as well as its coin state.
(0) \otimes |(H) + i|V\rangle⟩) and the Hadamard coin (\(\theta = \pi/8\)) at each position. The final state clearly shows the characteristic shape of a fully coherent quantum walk: the two pronounced side peaks and the low probability around the initial position. Moreover, the polarization analysis confirms the expected dependence of the particle’s final position on its coin state. An adapted theory including only small imperfections of the coin parameter \(\theta\), the initial coin state and differential losses between the two polarizations fully explains the final spatial and polarization distribution. The quality of the result can be quantified by the distance \(d(P_m, P_\text{rh}) = \frac{1}{2} \sum_x |P_m(x) - P_\text{rh}(x)|\) between the measured \(P_m\) and the theoretical \(P_\text{rh}\) probability distributions. It ranges between 0 for identical distributions and 1 for a complete mismatch. The distance of the measured walk to the adapted quantum theory is \(d(P_m, P_\text{rh}) = 0.052 \pm 0.015\). For comparison we calculated the distance to the fully decoherent (classical) scenario, obtaining \(d(P_m, P_\text{c}) = 0.661 \pm 0.015\). Hence, our result confirms an almost decoherence free evolution after 28 steps.

(ii) Static disorder.— We implemented the evolution of a particle in an environment with static disorder using a quantum walk with variable coin operation. To create a static disorder a coin operation is required, which is position and not step dependent. In our system this is realized by a controlled phase shift \(\phi_{V/H}(x)\), such that the photon acquires the same phase any instance it appears at position \(x\). To generate a random static phase pattern we applied a periodic noise signal to the EOM. The periodicity of the signal was carefully adjusted to ensure that the applied phase shift operation is strictly position dependent. Using different phase patterns at subsequent runs allows to average over various disorders, as considered in the model of Anderson. The strength of disorder is determined by the maximal applied phase shift \(\Phi_{\text{max}}\), which defines the uniform interval \(\phi_{V/H}(x) \in [-\Phi_{\text{max}}, \Phi_{\text{max}}]\), from which the phases are chosen. The probability distribution after eleven steps is shown in Fig. 2(a). We used the initial state \(|\psi_0\rangle = |0\rangle \otimes |H\rangle\), \(\theta = \pi/8\) and a high disorder strength \((\Phi_{\text{max}} = 1.14 \pm 0.05)\pi\). In contrast to the decoherence free quantum walk (\(\Phi_{\text{max}} = 0\), inset of Fig. 2(c)), in the disordered scenario the expansion of the wave packet is highly suppressed. We observe a strictly enhanced arrival probability around the initial position, which also displays the predicted exponential decay. This striking signature of Anderson localization is emphasized by linear fits in the semilog scaled plot (inset of Fig. 2(a)). Our results are in agreement with a theoretical model determined by a Monte Carlo simulation of \(10^4\) different phase patterns compatible with our experiment. Compared to (i), the number of steps is reduced due to the additional losses introduced by the EOM.

(iii) Fast fluctuations.— To generate a system with dynamic disorder we detuned the temporal length of the noise signal, thus eliminating position dependent phase correlations. Decoherence appears as a consequence of the dynamically varying phase suffered by the quantum particle during the evolution. As a result, the photon undergoes a classical random walk, revealing a binomial probability distribution (Fig. 2(b)). In contrast to the previous case, the spatial profile of the wave packet shows a parabolic shape in the semilog scale (inset, Fig. 2(b)).

A stepwise increase of the disorder strength \(\Phi_{\text{max}}\) nicely demonstrates the controlled transition of the system from the ballistic evolution (decoherence free quantum walk) towards the diffusive/ localized evolution due to dynamic (red squares) and static (green dots) disorder with increasing disorder strength \(\Phi_{\text{max}}\); dashed lines: theory without adaption for experimental imperfections. The solid red line marks the variance of a classical random walk. (Vertical error is smaller than the dotsize). (d) Relative frequency \(f(|\phi_\text{w}|\rangle)\) of the applied phases \(\phi_\text{w}\) for the signal with interval \(\Phi_{\text{max}} = (1.02 \pm 0.05)\pi\). The dashed line indicates the uniform distribution.

FIG. 2: Measured probability distribution (front) and respective theory (back, gray bars) of 11 steps of a quantum walk \((\theta = \pi/8)\) with static disorder (a), dynamic disorder (b) and in a decoherence free environment (Inset (c)). The insets in (a) and (b) show the measured distribution in semilog scale with linear (a) and parabolic fit (b). (c) Transition of the variance from ballistic quantum walk to diffusive/ localized evolution due to dynamic (red squares) and static (green dots) disorder with increasing disorder strength \(\Phi_{\text{max}}\); dashed lines: theory without adaption for experimental imperfections. The solid red line marks the variance of a classical random walk. (Vertical error is smaller than the dotsize). (d) Relative frequency \(f(|\phi_\text{w}|\rangle)\) of the applied phases \(\phi_\text{w}\) for the signal with interval \(\Phi_{\text{max}} = (1.02 \pm 0.05)\pi\). The dashed line indicates the uniform distribution.
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case of static disorder (green diamonds) they stagnate.

The agreement between theory and measurement in
the completely dephased scenario (Fig. 2(b)) confirms a
sufficient randomness of the applied noise signal. Fur-
more, an independent interferometric measurement
revealed the relative frequency of the used phases $f(|\phi_i|)$,
as can be seen in Fig. 2(d) with $\Phi_{max} = (1.02 \pm 0.05)\pi$.
However, small imperfections of the EOM lead to a dis-
tribution slightly off uniformity, which also influences the
measured variances shown in Fig. 2(c).

(iv) Slow fluctuations.— As the fourth scenario we sim-
ulated fluctuations in a homogeneous system, but with
parameters that change in a time scale much larger than
the full duration of a single quantum walk. Although the
individual evolution is not affected under these circum-
stances, an ensemble measurement of subsequent walks results
in an average over coherent evolutions in different
types of lattices. For this purpose we changed the pa-
rameter $\theta \in [0, \pi/4]$ in steps of $\pi/18$ for a quantum walk
with initial state $|\psi_0\rangle = |0\rangle \otimes (|H\rangle + i|V\rangle)$. An average
over the full range $\theta \in [0, \pi/4]$ exhibits a nearly uniform
spatial distribution of the wave packet with an enhanced
probability to arrive at its initial position $x = 0$ after 10
steps (Fig. 3(a)). Especially the high chance to reach the
outermost positions $x \approx \pm 10$ differs significantly from all
previous scenarios. This increases the variance of the
distribution ($\sigma_{x_0}^2 = 40.00 \pm 0.42$) to a level, which is
even higher than in the decoherence free quantum walk with
the Hadamard coin ($\sigma_0^2 = 31.27 \pm 0.19$). The result
demonstrates that special kinds of decoherences can even
speed up the expansion of wave packets in homogeneous
lattices.

Finally, the geometry of the setup allows to observe
easily the wave packet’s evolution step by step in all four
scenarios (Fig. 3(b)). For cases (i) and (iv) we observe a
ballistic spread, with an even faster expansion in a system
with slow fluctuations. The evolution with fast dynamic
order (iii) is clearly diffusive. Lastly, under the con-
dition of static disorder (ii) the variance saturates after
few steps and the dynamics is dominated by the effect of
Anderson localization. For comparison, we show in Fig.
3(c) a theoretical plot for the evolution of the variance
over fifty steps. The parameters used in simulation and
experiment are equivalent to the experimental settings
used for Figs. 1(b), 2(a-b) and 3(a).

In conclusion, we presented how disorder and fluctua-
tions in a periodic lattice can influence the evolution of a
traversing particle. We observed a fast ballistic spread for
slowly changing lattice parameters, a diffusive spread in
the case of dynamical disorder and Anderson localization
for lattices with static disorder. Furthermore, we showed
the controlled transition between the different regimes.
The high flexibility and control allows not only the study
of further decoherence phenomena in quantum walks but
also to simulate specific physical scenarios of interest for
the solid state and biophysics community. Moreover, the
possibility to manipulate quantum walks with time de-
dependent coin operations is a fundamental step towards
the realization of quantum walk-based protocols.

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[1] Y. Aharonov, L. Davidovich, and N. Zagury. Phys. Rev.
A 48, 1687 (1993).
[2] A. M. Childs, Phys.Rev. Lett. 102, 180501 (2009).
[3] M. Mohseni et al., J. Chem. Phys. 129, 174106 (2008).
[4] P. Rebentrost et al., New J. Phys. 11, 033003 (2009).
[5] S. Hoyer, M. Sarovar, and K. B. Whaley, New J. Phys.
12, 065041 (2010).
[6] M. Karski et al., Science 325, 174 (2009).
[7] H. Schmitz et al., Phys. Rev. Lett. 103, 009504 (2009).
[8] F. Zähringer et al., Phys. Rev. Lett. 104, 100503 (2010).
[9] J. Du et al., Phys. Rev. A 67, 042316 (2003).
[10] C. A. Ryan et al., Phys. Rev. A 72, 062317 (2005).
[11] H. B. Perets et al., Phys. Rev. Lett. 100, 170506 (2008).
[12] M. A. Broome et al., Phys. Rev. Lett. 104, 153602
(2010).
[13] A. Schreiber et al., Phys Rev. Lett. 104, 050502 (2010).
[14] A. Peruzzo et al., Science 329, 1500 (2010).
[15] N. Shenvi, J. Kempe and K. B. Whaley, Phys. Rev. A
[16] A. Ambainis, SIAM Journal on Computing, 37, 210-239 (2007).
[17] P. W. Anderson, Phys. Rev. 109, 1492 (1958).
[18] D. S. Wiersma et al., Nature 390, 671 (1997).
[19] Y. Lahini et al., Phys. Rev. Lett. 100, 013906 (2008).
[20] T. Schwartz et al., Nature 446, 52 (2007).
[21] G. Roati et al., Nature 453, 895 (2008).
[22] J. Billy et al., Nature 453, 891 (2008).
[23] P. Törmä, I. Jex, and W. P. Schleich, Phys. Rev. A 65, 052110 (2002).
[24] J. P. Keating et al., Phys. Rev. A 76, 012315 (2007).
[25] Yue Yin, D. E. Katsanos, and S. N. Evangelou, Phys. Rev. A 77, 022302 (2008).
[26] T. A. Brun, H. A. Carteret, and A. Ambainis, Phys. Rev. Lett. 91, 130602 (2003).
[27] V. Kendon, Math. Struct. Comp. Sci. 17, 1169 (2007).