Effect of compressive stresses on P-wave velocity in samples of equivalent geomaterials

V P Kosykh, P V Kosykh and A F Revuzhenko*
Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences,
54 Krasny ave 54, Novosibirsk, 630091, Russia
E-mail: *revuzhenko@yandex.ru

Abstract. The authors have found experimentally that elastic wave velocity depends on compressive stresses in tubular samples made of equivalent geomaterials. The relation between the wave velocity and tangent modulus of elasticity is determined. The values of the dynamic modulus of elasticity as function of the compressive stress are obtained.

1. Introduction
In-situ rock masses experience constant or slowly varying static loads alongside multiple repeated weak variable loads (caused by tides, atmospheric variations, transport-induced vibrations, etc.) [1,2]. Rock is a medium having internal structure at different scales. Being subjected to deformation, rocks accumulate energy in the form of internal self-balancing stresses. Laboratory-scale and full-scale researches reveal that geomaterials can change their mechanical properties under the influence of cyclic loads [3–8] and release the accumulated energy under certain conditions [9,10].

Mechanical properties of rocks are currently determined using the static and dynamic methods. In the static tests, it is most often that a geomaterial sample is subjected to slowly varying compression by different loading programs, and the testing results are obtained as a set of deformation and strength characteristics of the material (static modulus of elasticity, modulus of deformation, limit strength, limit elasticity, etc.).

The dynamic tests most commonly use ultrasonic techniques. In this case, a sample is exposed to treatment by ultrasonic waves, and the velocities of the waves are used to determine dynamic modulus of elasticity and attenuation coefficient.

In a general case, the stress–strain curve is nonlinear, and the static modulus of elasticity $E$ is, therefore, found from averaging. The dynamic modulus $E_d$ is determined in an unloaded sample, as a rule. For this reason, $E$ and $E_d$ differ. Moreover, the difference between the elasticity moduli is influenced by an elastic after-effect.

There are a comparatively few known studies into the influence of cyclic long-term weak impact on the change of mechanical properties of geomaterials. The Institute of Mining, SB RAS, has designed a test bench for investigating effect of multiple weak impacts against static loading on evolution of properties of geomaterials; in particular, the test bench allows tracing the change in elastic wave velocity depending on loading conditions of samples [1].

This study focused on the elastic wave velocity in equivalent geomaterials depending on background level of static stresses of compression. The test bench enabled following up evolution of deformation and strength characteristics in geomaterial samples under weak impacts against a pre-set static load.
2. Research techniques
The test bench layout is shown in figure 1. A geomaterial sample is held in grippers 2 and 5. The right-hand gripper 5 is fixed on a vertical plate by means of membrane 4 and is aligned horizontally through its stem and a Teflon sleeve. The left-hand gripper 2 is also aligned horizontally by a Teflon sleeve and transmits compressive load from the screw 1 to the sample via a ball. The vertical plates are rigidly fixed on a solid base 7. Alongside the axial load, the sample is subjected to a dynamic force created by impacts of the known energy on the stem of the gripper 5. The measurement equipment of the test bench is the 8-channel LTR-EU-8-2 crate system manufactured by L-Card, Moscow, composed of 8 measurement modules. The system continuously takes data from 16 strain gauge transducers and records high-frequency (to 10 MHz) signals simultaneously in four channels. More details of the test bench are given in [11].

The hollow cylinder samples were made of the mixture of graded quartz sand particles 0.3 mm in size and a Plasticrete filler at a weight ratio of 1:1. The samples were 80 mm long and had external and internal diameters of 36 and 16 mm, respectively. After crystallizations the samples were dried at a temperature of 70°C for 6 hours. The prepared samples had density of 2.26 g/cm³.

To measure longitudinal and circumferential static deformation, constantan foil strain gages were glued on the samples and connected with the strain meter module of the crate system. The wave process was studied using four semi-conductor resistive-strain sensors with a spacing of 5 mm and strain conversion efficiency of 130. These sensors were glued on the sample surface along a line, at a distance of 21 mm from each other. The signals of the sensors, after amplification by wide-band amplifiers, were recorded in the form of oscillograms of axial dynamic strains.

![Figure 1. Test bench: 1 – compression load screw; 2 – left-hand gripper; 3 – geomaterial sample; 4 – membrane; 5 – movable gripper; 6 – strain sensors.](image)

3. Results and discussion
During the tests, a sample was subjected to static step-wise loading up to a pre-set axial compressive force. At the same time, the stress–axial/lateral strain curves were plotted. After each step of loading, for a certain time, deformation of a sample increased while the loading was decreased. Figure 2 illustrates the change in the axial stress $\sigma_1$ in a sample at a step of loading.

It is seen in figure 2 that attenuation of stresses (and strains) lasts for 1.5–2 hours. Therefore, a sample was held for 2 h after each loading step to complete stabilization of stresses. Then, the right-hand gripper stem was subjected to a series of 25 impacts, and oscillograms of dynamic strains were recorded.
Figure 3 shows a typical compressive stress–axial strain diagram. The values of stresses and strains at the moment of impacts are marked. The sample was not loaded to failure.

![Compressive stress–axial strain curve](image1)

**Figure 2.** Stress variation in time after stepwise increase in loading of a sample.

**Figure 3.** Compressive stress–axial strain curve in a sample made of equivalent geomaterial.

Apparentlly, the stress–strain curve is nonlinear. The average modulus of elasticity of the material was $2.31 \times 10^4$ MPa and Poisson’s ratio was 0.35.

The typical oscillograms are presented in figure 4. The impacts excite vibrations in the bench–sample system, and the frequency and attenuation rate of the vibrations are governed by the sample properties. The oscillatory process in the bench–sample system involves movable parts of the test bench; moreover, free vibrations are excited in the sample. As a result, the test material is exposed to variable stresses of different frequencies.

The typical oscillograms of incremental strains $\Delta \varepsilon_1$ after an impact on the sample are recorded by two sensors spaced at 63 mm. It is seen that the attenuating strains are the superposition of different frequency oscillations.

![Oscillograms of incremental axial strains](image2)

**Figure 4.** Oscillograms of incremental axial strains in a sample under impact.

Velocities of elastic waves in a sample were determined by correlation of oscillograms from different sensors. Figure 5 shows the start sections of oscillograms of incremental axial strains in a sample after impact after their normalization with respect to maximum amplitude.

![Start sections of oscillograms](image3)

It is seen in figure 5 that the second sensor signal is delayed by $\Delta t$ relative to the first sensor. This delay is governed by the elastic wave velocity in the sample. Considering that the distance $l$ between the first and second sensors is known, it is easy to find the elastic wave velocity $v = l/\Delta t$. 
Figure 5. Time of delay of elastic wave in a sample.

Figure 6a demonstrates the determined elastic wave velocity depending on the compressive force.

![Figure 6a](image)

**Figure 6.** Relations of (a) elastic wave and (b) tangent modulus of elasticity and compression stress in a sample.

As seen in figure 6a, the dependence of the elastic wave velocity on the compression stress is nonmonotonous. As the stress is increased, the elastic wave velocity decreases, passes a minimum and, then, grows. Such behavior correlates with the change in the tangent (local) modulus of elasticity $E_t$ under the increase in the compression stress (figure 6b).

It is well-known that the velocity of elastic waves in rods depends on the local modulus of elasticity and on the density of a rod material in accordance with the expression $v = \sqrt{E_t/\rho}$ where $E_t$ is the local (tangent) modulus of elasticity and $\rho$ is the density of material.

From this formula, we find the dynamic modulus of elasticity $E_d$ as function of the compression stress in a sample using the experimentally determined velocities of elastic waves (figure 6a). The results are depicted in figure 7 as the relation between the dynamic and tangent moduli of elasticity.

![Figure 7](image)

**Figure 7.** Relation of the dynamic and tangent moduli of elasticity.
Apparently, the experimental points group nearby the straight line. The coefficient of determination makes 0.7.

4. Conclusion
The authors have analyzed the influence of static stresses on the elastic wave velocities in samples made of equivalent geomaterials under uniaxial compression. It is found that the local deformation modulus and elastic wave velocities correlate. As the compressive stress is increased, the local modulus and elastic wave velocities nonmonotonously change. There exists a certain relation between the dynamic and tangent moduli of elasticity: as the tangent modulus increases so does the dynamic modulus of elasticity.

References
[1] Voznesensky E A 1999 Dynamic Instability of Soil (Moscow: Editorial URSS) p 264
[2] Sheinin V I, Blokhin D I, Maksimovich I B and Sarana E P 2016 Journal of Mining Science 52(6) 1039–1046
[3] Mokhnachev M P 1979 Fatigue of Rocks (Moscow: Nauka) p 152
[4] Fuenkajorn K and Phueakphum D 2009 Int. J. Rock Mech. &Min. Sci. 46 200–209
[5] Kocharyan G G, Kostyuchenko V N and Pavlov D V 2004 Fizich. Mezomekhanika 1 (7) 5–22
[6] Kurlenya M V, Adushkin V V, Garnov V V, Oparin V N, Revuzhenko A F and Spivak A A 1992 Doklady Akademii Nauk 323 (2) 263–265
[7] Kazantsev A A, Kosykh V P and Revuzhenko A F 2015 IOP Conf. Series: Materials Science and Engineering 91 DOI 10.1088/1757-899X/91/1/012089
[8] Klishin S V and Revuzhenko A F 2017 IOP Conf. Series: Earth and Environmental Science 84 012003
[9] Lavrikov S V and Revuzhenko A F 2017 Journal of Mining Science 53 (1) 12–20
[10] Lavrikov S V and Revuzhenko A F 2015 AIP Conference Proceedings DOI 10.1063/1.4932811
[11] Kosykh V P, Kosykh P V and Revuzhenko A F 2017 Journal of Mining Science 53 (6) 1147–1151