Charge density wave with meronlike spin texture induced by a lateral superlattice in a two-dimensional electron gas

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(Dated: February 23, 2018)

The combined effect of a lateral square superlattice potential and the Coulomb interaction on the ground state of a two-dimensional electron gas in a perpendicular magnetic field is studied for different rational values of \( \Gamma \), the inverse of the number of flux quanta per unit cell of the external potential, at filling factor \( \nu = 1 \) in Landau level \( N = 0 \). When Landau level mixing and disorder effects are neglected, increasing the strength \( W_0 \) of the potential induces a transition, at a critical strength \( W_0^{(c)} \), from a uniform and fully spin polarized state to a two-dimensional charge density wave (CDW) with a meronlike spin texture at each maximum and minimum of the CDW. The collective excitations of this “vortex-CDW” are similar to those of the Skyrme crystal that is expected to be the ground state near filling factor \( \nu = 1 \). In particular, a broken U(1) symmetry in the vortex-CDW results in an extra gapless phase mode that could provide a fast channel for the relaxation of nuclear spins. The average spin polarization \( S_z \) changes in a continuous or discontinuous manner as \( W_0 \) is increased depending on whether \( \Gamma \in [1/2, 1] \) or \( \Gamma \in [0, 1/2] \). The phase mode and the meronlike spin texture disappear at large value of \( W_0 \), leaving as the ground state partially spin-polarized CDW if \( \Gamma \neq 1/2 \) or a spin-unpolarized CDW if \( \Gamma = 1/2 \).

PACS numbers: 73.22Gk,73.43.Lp,73.43.-f

I. INTRODUCTION

The two-dimensional electron gas (2DEG) in a perpendicular magnetic field has a very rich phase diagram that includes several phases such as the Laughlin liquids that give rise to the integer and quantum Hall effects, the Wigner crystal at small filling factor in each Landau level, the bubble crystals and the stripe phase in higher Landau levels and the Skyrme crystal near filling factor \( \nu = 1 \) in the lowest Landau level. The phase diagram is even more complex when system with extra degrees of freedom such as double quantum wells (DQWs) are considered. In DQWs, the orientation of the pseudospin vector associated with the layer degree of freedom can be modified by changing the tunneling and electrical bias between the layers.

Another way to modify the properties of the 2DEG is by the addition of a lateral two-dimensional superlattice patterned on top of the GaAs/AlGaAs heterojunction hosting the 2DEG that creates a spatially modulated potential at the position of the 2DEG. The effect of a one-dimensional periodic potential on the Landau levels is particularly interesting since it leads to commensurability problems due to the presence of different lengths scales: the lattice constant of the external potential \( a_0 \), the magnetic length \( \ell = \sqrt{\hbar c/eB} \) (\( B \) is the magnetic field) and the Fermi wavelength. Novel magneto-oscillations resonances with period different than that of the well-known Shubnikov-de Haas oscillations have been detected in such systems. Even more interesting is the effect of a periodic two-dimensional potential on the band structure of the 2DEG. The intricate pattern of eigenvalues that results from such potential has been studied by many authors and is known as the Hofstadter butterfly spectrum. Its observation in GaAs/AlGaAs heterojunction is very difficult due to screening and disorder effects but experimental signature in magnetotransport experiments in 2DEGs with a lateral surface superlattice potential with period of the order of 100 nm and less have been reported. Interest in this problem has been revived recently by the experimental observations of the Hofstadter’s butterfly spectrum that use the moiré superlattices that arise from graphene or bilayer graphene placed on top of hexagonal boron nitride. Another interest of superlattice potentials is their use to create artificial lattices. For example, a lateral superlattice with a honeycomb crystal structure has recently been proposed to create an artificial graphenelike system in a GaAs/AlGaAs heterojunction.

In this work, we study theoretically the effect of a square lattice lateral potential with a period \( a_0 \) on the ground state of the 2DEG in GaAs/AlGaAs heterojunction at filling factor \( \nu = 1 \) and in Landau level \( N = 0 \). We include the spin degree of freedom and use the Hartree-Fock approximation to study the combined effects of the external potential and the Coulomb interaction. We assume that the potential is sufficiently weak so that Landau level mixing can be neglected. We also ignore disorder and work at zero temperature. We vary the potential strength \( W_0 \) and calculate the ground state for different rational values of \( \Gamma = \varphi_0/Ba_0^2 = q/p \in [0,1] \) (where \( \varphi_0 = \hbar c/e \) is the flux quantum and \( q,p \) are integers with no common factors) which is the inverse of the number of flux quanta per unit cell of the surface potential. Our formalism allows for the formation of uniform as well as spatially modulated ground states with or without spin texture. Our calculation indicates that, at a critical value, \( W_0^{(c)} \) of the external potential, there is a transition from a uniform fully spin polarized state to a charge density wave (CDW) with an intrinsic spin tex-
tecture. Each unit cell of this CDW contains two positive and two negative amplitude modulations and the vortex spin texture at each maximum/minimum resembles that of a positively/negatively charged meron. The two positively/negatively charged merons in each unit cell have the same vorticity but a global phase that differs by $\pi$. These meronlike textures, however, are not quantized since the amplitude of the CDW varies continuously with $W_0$. In the vortex-CDW, as we call it, the average spin polarization $S_z$ varies with $W_0$ in a continuous or discontinuous manner depending on whether $\Gamma \in [1/2, 1]$ or $\Gamma \in [0, 1/2]$ and saturates at a finite, positive, value of $S_z$ that depends only on $\Gamma$ in most cases. In the special case $\Gamma = 1/2$, the vortex-CDW phase is absent and the transition is directly from a fully spin polarized and uniform 2DEG to an unpolarized CDW. The phase diagram for $\Gamma \in [0, 1/2]$ is richer than that of $\Gamma \in [1/2, 1]$ as it involves the transition between the vortex-CDW and its conjugate phase, the anti-vortex CDW, obtained by reversing $S_z$ (r) and inverting the vorticity of all merons. This transition between the two CDWs is accompanied by a discontinuous change of $S_z$ that becomes continuous when the Zeeman coupling goes to zero.

We study the properties of the vortex-CDW at different values of $\Gamma$ and with a particular emphasis on its collective excitations which we derive using the generalized random-phase approximation (GRPA). The vortex-CDW has collective modes that have much in common with the collective excitations of the Skyrmee crystal\textsuperscript{22} that is expected to be the ground state near filling factor $\nu = 1$ in $N = 0$. Namely, the broken U(1) symmetry in the vortex-CDW phase leads to a new gapless mode that can provide a fast channel for the relaxation of nuclear spins\textsuperscript{23}. This mode and the meronlike spin texture disappear at larger values of the external potential leaving a ground state that is either unpolarized if $\Gamma = 1/2$ or partially polarized if $\Gamma \neq 1/2$.

Our paper is organized as follows. In Sec. II, we introduce the Hamiltonian of the 2DEG in the presence of the lateral square lattice potential and briefly review the Hartree-Fock and generalized random-phase approximation that we use to compute the density of states, the density and spin profiles and the collective excitations of the various phases. In Sec. III, we present our numerical results for the phase diagram of the 2DEG as a function of the potential strength $W_0$ and the inverse magnetic flux per unit cell $\Gamma$. We conclude in Sec. IV with a discussion on the experimental detection of the new vortex-CDW state.

II. HAMILTONIAN OF THE 2DEG IN AN EXTERNAL POTENTIAL

The system we consider is a 2DEG in a GaAs/AlGaAs heterojunction or quantum well submitted to a perpendicular magnetic field $B = B\hat{z}$ and to a lateral superlattice potential $V_x (r)$. The coupling of the electrons to this external potential is given by $H_c = -e \int dr V_x (r) n_e (r)$, where $n_e (r) = \sum_{\alpha = \pm} n_{e,\alpha} (r)$ is the total density operator including both spin states $\alpha = \pm 1$ (we take $e > 0$). We assume that only the Landau level $N = 0$ is occupied but our calculation can easily be generalized to any Landau level by changing the effective interactions $H (q)$ and $X (q)$ and the form factor $F (q)$. The Hamiltonian of the interacting 2DEG is given, in the Hartree-Fock approximation, by

$$H_{HF} = -N_e \frac{\Delta z}{2} \sum_{\alpha} \alpha \rho_{\alpha,\alpha} (0)$$

$$- eN_e S \sum_q \sum_{\alpha} V_e (-q) F (-q) \rho_{\alpha,\alpha} (q)$$

$$+ N_e \sum_{\alpha,\beta \neq 0} \sum_q H (q) \rho_{\alpha,\alpha} (-q) \rho_{\beta,\beta} (q)$$

$$- N_e \sum_{\alpha,\beta} \sum_q X (q) \rho_{\alpha,\beta} (-q) \rho_{\beta,\alpha} (q),$$

where $S$ is the 2DEG area, $N_e = S/2\pi\ell^2$ is the Landau level degeneracy, and the form factor for the $N = 0$ Landau level is $F (q) = e^{-q^2\ell^2/2}$, where $\ell = \sqrt{\hbar c/eB}$ is the magnetic length. The averages are over the Hartree-Fock ground state of the 2DEG. The non-interacting single-particle energies, measured with respect to the kinetic energy $h\omega_c/2$, are given by

$$E_{\alpha} = \frac{\Delta z}{2},$$

where the Zeeman energy $\Delta Z = |g^*| \mu_B B$, with $g^*$ the effective $g$–factor of bulk GaAs and $\mu_B$ the Bohr magneton. In some experiments on skyrmions, the effective $g$–factor was tuned in the range $-0.11$ to $0.065$ by applying hydrostatic pressure to a sample of GaAs/AlGaAs modulation doped quantum well\textsuperscript{24}. In our study, we will thus consider that $\Delta Z$ is not determined by the magnetic field, but is instead a parameter than can be adjusted.

The Hartree and Fock interactions in $N = 0$ are given by

$$H (q) = \left( \frac{\epsilon^2}{\kappa \ell} \right) \frac{1}{4\ell^2} e^{-q^2\ell^2/2},$$

$$X (q) = \left( \frac{\epsilon^2}{\kappa \ell} \right) \sqrt{2} \int_0^\infty dx e^{-x^2} J_0 \left( \sqrt{2} x q \ell \right),$$

where $\kappa = 12.9$ is the dielectric constant of GaAs. Finally, the operators $\rho_{\alpha,\beta} (q)$ are defined by

$$\rho_{\alpha,\beta} (q) = \frac{1}{N_e} \sum_{X, X'} e^{-\frac{q^2}{2} (X + X')}$$

$$\times \delta_{X, X' + q, \alpha} c_{X, \alpha}^\dagger c_{X', \beta}^\dagger,$$

where $c_{X, \alpha}^\dagger$ is the operator that creates an electron with guiding-center index $X$ (in the Landau gauge) and spin...
The four operators $\rho_{\alpha,\beta}(q)$ are related to the averaged electronic and spin densities in the $xy$-plane by

$$n_{\alpha}(r) = \frac{1}{2\pi^2} \sum_q \langle \rho_{\alpha,\alpha}(q) \rangle e^{-q^2r^2/4} e^{iq\cdot r},$$  

$$S_x(r) = \frac{1}{2\pi^2} \sum_q \text{Re} \left[ \langle \rho_{\alpha,-}(q) \rangle e^{-q^2r^2/4} e^{iq\cdot r} \right],$$

$$S_y(r) = \frac{1}{2\pi^2} \sum_q \text{Im} \left[ \langle \rho_{\alpha,-}(q) \rangle e^{-q^2r^2/4} e^{iq\cdot r} \right],$$

$$S_z(r) = \frac{h}{2} \left[ n_+(r) - n_-(r) \right].$$

The $\langle \rho_{\alpha,\beta}(q) \rangle$'s can be considered as the order parameters of an ordered phase of the 2DEG.

The averaged Hartree-Fock ground-state energy per electron at $\nu = 1$ is given by

$$\langle \langle H_{HF} \rangle \rangle = -\frac{\Delta Z}{2} \sum_{\alpha} \langle \rho_{\alpha,\alpha}(0) \rangle$$

$$-\frac{1}{8} \sum_{q,\alpha} V_{\alpha}(-q) F(q) \langle \rho_{\alpha,\alpha}(q) \rangle$$

$$+\frac{1}{2} \sum_{q,\alpha,\beta} H(q) \langle \rho_{\alpha,\alpha}(-q) \rangle \langle \rho_{\beta,\beta}(q) \rangle$$

$$-\frac{1}{2} \sum_{q,\alpha,\beta} \chi(q,\alpha,\beta) \langle \rho_{\alpha,\beta}(q) \rangle \langle \rho_{\alpha,\beta}(-q) \rangle,$$

where

$$\gamma_{q,q'} = e^{-i(q\cdot q')\pi^2/2},$$

and $\omega_n$ are fermionic Matsubara frequencies, $\mu$ is the chemical potential and we have defined the potentials

$$U^{H}(q) = \sum_{\alpha} H(q) \langle \rho_{\alpha,\alpha}(q) \rangle,$$

$$U^{F}_{\alpha,\beta}(q) = X(q) \langle \rho_{\beta,\alpha}(q) \rangle.$$

These potentials depend on the order parameters $\langle \rho_{\alpha,\beta}(q) \rangle$ that are unknown. The equation of motion for $G_{\alpha,\beta}(q,\omega_n)$ must thus be solved numerically by using a seed for the order parameters and then iterate Eq. (14) until a convergent solution is found. In case several solutions are found (corresponding to different choice for the initial seed), we choose the one with the lowest energy and compute the dispersion relation of its collective modes to make sure that it is a stable solution. We remark that this method does not guarantee that the true ground state is the solution that we keep.

The density of states $g(\omega)$ is obtained from the single-particle Green’s function by using the relation

$$g(\omega) = -\frac{N_{\varphi}}{\pi} \sum_{\alpha} \text{Im} [G_{\alpha,\alpha}(q = 0, \omega + i\delta)].$$

To find the dispersion relation of the collective modes, we derive the equation of motion in the generalized random-phase approximation for the two-particle Green’s function

$$\chi_{\alpha,\beta,\gamma,\delta}(q, q'; \tau) = -N_{\varphi} \langle T \rho_{\alpha,\beta}(q, \tau) \rho_{\gamma,\delta}(q', 0) \rangle$$

$$+ N_{\varphi} \langle \rho_{\alpha,\beta}(q) \rangle \langle \rho_{\gamma,\delta}(q') \rangle.$$

This equation is

$$\chi_{\alpha,\beta,\gamma,\delta}(q, q'; \tau) = \chi_{\alpha,\beta,\gamma,\delta}^{(0)}(q, q'; i\Omega_n)$$

$$+ \frac{1}{h} \sum_{q'} \sum_{\xi,\lambda} \chi_{\alpha,\beta,\xi,\delta}^{(0)}(q, q'; i\Omega_n) H(q'') \chi_{\lambda,\lambda,\gamma,\delta}(q'', q'; i\Omega_n)$$

$$- \frac{1}{h} \sum_{q'} \sum_{\xi,\lambda} \chi_{\alpha,\beta,\xi,\lambda}^{(0)}(q, q'; i\Omega_n) X(q'') \chi_{\lambda,\xi,\gamma,\delta}(q'', q'; i\Omega_n),$$

where $\Omega_n$ is a bosonic Matsubara frequency. Equation (20) represents the summation of bubble (polarization effects) and ladder (excitonic corrections) diagrams. The Hartree-Fock two-particle Green’s function (the single-bubble Feynman diagram with Hartree-Fock propagators) that enters this equation is obtained from the Hartree-Fock equation of motion for $\chi_{\alpha,\beta,\gamma,\delta}(q, q'; \tau)$ and is given by
\[ [i\hbar \Omega_n + (E_\alpha - E_\beta)] \chi^{(0)}_{\alpha, \beta, \gamma, \delta} (q, q', i\Omega_n) \]
\[ = \hbar [\gamma^*_q q' \langle \rho_{\alpha, \beta} (q - q') \rangle \delta_{\beta, \gamma} - \gamma q q' \langle \rho_{\gamma, \delta} (q - q') \rangle \delta_{\alpha, \beta}] \]
\[ - \frac{e}{\xi} \sum_{q''} V_e (q - q'') F(|q - q''|) \left[ \gamma^*_{q, q''} - \gamma_{q, q''} \right] \]
\[ \times \chi^{(0)}_{\alpha, \beta, \gamma, \delta} (q'', q', i\Omega_n) \]
\[ - \sum_{q'' \neq 0} U^H (q - q'') \left[ \gamma^*_{q, q''} - \gamma_{q, q''} \right] \]
\[ \times \chi^{(0)}_{\alpha, \beta, \gamma, \delta} (q'', q', i\Omega_n) \]
\[ + \sum_{q''} \sum_{q'''} U^F_{\alpha, \beta, \gamma, \delta} (q - q'', q''' - q'') \gamma^*_{q, q''} \chi^{(0)}_{\alpha, \beta, \gamma, \delta} (q', q''', i\Omega_n) \]
\[ - \sum_{q''} U^F_{\alpha, \beta, \gamma, \delta} (q - q'') \gamma_{q, q''} \chi^{(0)}_{\alpha, \beta, \gamma, \delta} (q', q'', i\Omega_n) . \]

By defining the super-indices \( I = (\alpha, \beta) \) and \( J = (\gamma, \delta) \), Eq. (20) can be rewritten as a \( 4 \times 4 \) matrix equation for the matrix of Green’s functions \( \chi_{IJ} \). This equation has the form \([i\hbar \Omega_n I - \chi] = \beta \). The matrix \( \chi \), which depends only on the \( \langle \rho_{\alpha, \beta} (q) \rangle \)'s is then diagonalized numerically to find \( \chi_{IJ} \). The retarded response functions are obtained with the analytic continuation \( \Omega_n \rightarrow \omega + i\delta \).

We compute the following density and spin responses:

\[ \chi^{R}_{\rho_{\alpha, \beta}} (q, q'; \omega) = -i N_0 \left[ \langle [\rho_i (q, t), \rho_j (-q', t')] \rangle \right] \theta (t - t') \omega , \]

where \( i = n, x, y, z \) and the operators

\[ \rho_x = \frac{1}{2} [\rho_{++} - \rho_{--}] , \]
\[ \rho_x = \frac{1}{2} [\rho_{+-} - \rho_{-+}] , \]
\[ \rho_z = \frac{1}{2} [\rho_{++} + \rho_{--}] , \]
\[ \rho_n = \rho_{++} + \rho_{--} . \]

In a uniform phase, the order parameters \( \langle \rho_{\alpha, \beta} (q) \rangle \) are finite only when \( q = 0 \) while in a two-dimensional CDW, they can be non zero each time \( q = G \), where \( G \) is a reciprocal lattice vector of the CDW. For the response function, we have to compute \( \chi^{R}_{\rho_{\alpha, \beta}} (q, q; \omega) \) in the uniform phase and \( \chi^{R}_{\rho_{\alpha, \beta}} (k + G, k + G'; \omega) \) in the CDW where \( k \) is, by definition, a vector in the first Brillouin zone of the CDW. In the CDW, the GRPA matrices \( F \) and \( B \) have dimensions \( 4n_R \times 4n_R \), where \( n_R \) is the number of reciprocal lattice vectors considered in the numerical calculation. We typically take \( n_R \approx 600 \).

The formalism developed in this section can also be applied to the 2DEG in graphene if the electrons are assumed to occupy only one of the two valleys. In Landau level \( N = 0 \) of graphene (and only in this level), the form factor \( F(q) \) and the Hartree and Fock interactions \( H(q) \) and \( X(q) \) are the same as those given in Eqs. (24).

### III. PHASE DIAGRAM FOR \( \nu = 1 \)

We study the phase diagram of the 2DEG at filling factor \( \nu = 1 \) in Landau level \( N = 0 \) and at temperature \( T = 0 \) K. For the external potential, we choose the simple square lattice form

\[ V_e (r) = 2V_e \left[ \cos \left( \frac{2\pi}{\sqrt{2a_0}} (x + y) \right) + \cos \left( \frac{2\pi}{\sqrt{2a_0}} (x - y) \right) \right] , \]

so that \( V_e (q) = SV_e \delta_{q, G} \) in Eq. (11) with the vectors \( G_1, G_2 \approx \{ (1, 1), (1, -1), (-1, 1), (-1, -1) \} \). This external potential tries to impose a two-dimensional density modulation of the 2DEG with a square lattice constant \( a_0 \). We allow the spin texture (if any) to have the bigger Coulomb interaction and the external potential can be neglected i.e. we work in the limit of a weak superlattice potential. We also neglect disorder effect and work at zero temperature.

The ratio \( \ell / a_0 \) that enters the Hartree-Fock energy and equation of motion for the single-particle Green’s function is given by

\[ \frac{\ell}{a_0} = \sqrt{\frac{1}{2\pi} \frac{\varphi_0}{B a_0}} = \sqrt{\frac{1}{2\pi}} , \]

where \( \varphi_0 = \hbar c / \ell \) is the flux quantum. The important parameter \( \Gamma^{-1} \) is the number of flux quanta piercing a "density" unit cell area. With this definition, the factor \( F (G_1) = e^{-G_1^2 \ell^2 / 4} = e^{-\pi \ell^2} \). We limit our analysis to \( \Gamma = q / p \) where \( q \) and \( p \) are integers with no common factors.

In Landau level \( N = 0 \), a Wigner crystal has a triangular lattice can form at sufficiently small filling factor \( \nu 20 \). At \( \nu = 1 \), however, the ground state of the 2DEG is a uniform electron liquid with full spin polarization i.e. a quantum Hall ferromagnet (QHF) whose energy per electron is given by

\[ \frac{H_{HF}}{N_e} = -\frac{\Delta \ell}{2} - \frac{1}{2} \sqrt{\frac{\pi}{2}} \left( \frac{e^2}{\kappa \ell} \right) \]

(neglecting the kinetic energy that is a constant in \( N = 0 \)). The QHF remains the ground state even when the Zeeman coupling goes to zero because a perfect alignment of the spins minimizes the Coulomb exchange energy [the second term on the right-hand side of Eq. (24)]. In a uniform state, the Coulomb Hartree energy is cancelled by the neutralizing uniform positive background.
A. Case $\Gamma \in [1/2, 1]$

We first consider the case $\Gamma \in [1/2, 1]$. Figure 1 shows the ground state energy and spin polarization $S_z/\hbar$ as a function of the potential $W_0$ for $\Gamma = 1/2, 2/3, 3/4, 4/5, 1$ and for a Zeeman coupling $\Delta_Z = 0.015$. The ground state is spatially uniform and has an energy $\langle H_{HF} \rangle$ and a spin polarization $S_z$ that remain constant until a critical field $W_0^{(c)} \approx 0.11$. This uniform state is described by only one order parameter, i.e., $\langle \rho_{z+} (0) \rangle = 1$, and is fully spin polarized i.e. the spin per electron is $S_z = \hbar/2$.

The corresponding change in the density of states (DOS) with $W_0$ is shown in Fig. 2. In the absence of external potential and Coulomb interaction, the DOS has two peaks at energies $E_{\pm} = \pm \Delta_Z/2$ corresponding to the two spin states. With Coulomb interaction, the Zeeman gap $\Delta_Z$ is strongly renormalized [see Fig. 2 (a)] as is well known. When the external potential is present, the DOS for each spin orientation, has $p$ peaks corresponding to the number of subbands expected when an electron is submitted to both a magnetic field and a weak periodic potential\(14\). This is clearly visible in Fig. 2 (a),(d),(e) for $\Gamma = 1, 2/3, 4/5$. The external potential increases the width of the peaks in the DOS and decreases the renormalized Zeeman gap (which is also the transport gap).

We remark that the rapid oscillations in some of the graphs at $\Gamma = 1$ are a numerical artefact. They depend strongly on the number of reciprocal lattice vectors kept in the calculation.

If we enforce the uniform solution beyond the critical value $W_0^{(c)} \approx 0.11$, we find that the transport gap closes at $W_0 \approx 0.15$ for $\Gamma = 1$ where the system becomes metallic. Our code no longer converges in this case. But, this transition to a metallic state does not occur because the uniform state becomes unstable at $W_0^{(c)}$. The stability of a state is evaluated by computing the dispersion relation of its collective modes. For the uniform state, the collective excitations reduce to a spin-wave mode. When $W_0 = 0$, the spin-wave dispersion is given by\(25\).

$$\omega_{SW} (\mathbf{k}) = \Delta_Z + \left( \frac{e^2}{\kappa \ell} \right) |X (0) - X (\mathbf{k})|. \tag{30}$$

This mode is gapped at the bare Zeeman energy and saturates at $\omega_{SW} (\mathbf{k} \to \infty) = \Delta_Z + \sqrt{\frac{\pi}{2}} \left( \frac{e^2}{\kappa \ell} \right)$. Figure 3 shows its dispersion for $\Gamma = 1$ and $W_0 = 0, 0.05, 0.10, 0.11$. The wave vector $\mathbf{k}$ runs along the path $\Gamma - X' - M' - \Gamma$ i.e. along the edges of the irreducible density Brillouin zone (with $\Gamma = (0, 0); M' = (1/\sqrt{2}, 0), X' = (1/2\sqrt{2}, 1/2\sqrt{2})$ in units of $2\pi/\mathbf{a}_0$). The spin-wave mode softens at a finite wave vector $\mathbf{k}$ as $W_0$ increases so that the uniform state becomes unstable at $W_0^{(c)} \approx 0.11$ which is also the value at which the 2DEG is seen to enter a new phase in Fig. 1(a). When plotted in the reduced zone scheme as in Fig. 3, the spin-wave mode is split into several branches that accumulate into a very dense manifold near $\omega_{SW} (\mathbf{k} \to \infty)$. Only some of these branches are shown in Fig. 3 since we are interested only in the low-energy sector. The spin-wave dispersion is obtained by following the pole of the response functions $\chi_{\rho_{z+} \rho_{z-}}^{R} (\mathbf{k}, \mathbf{q}; \omega)$ with $\rho_{z+} = \rho_{z} \pm i\rho_{y}$ for different values of $\mathbf{k}$. We remark that the softening of the spin-wave mode by a one-dimensional external potential was reported previously by Bychkov et al.\(29\). These authors suggested that the resulting condensation of the spin excitons at the softening wave vector would create a new spin density wave ground state. This is precisely what we find, but this time, for a two-dimensional surface potential.

The ground state in a small region of $W_0$ after $W_0^{(c)}$ is a charge density wave with a vortex spin texture. Hereafter, we refer to this state as the vortex-CDW. The range of $W_0$ where the vortex-CDW is the ground state depends on $\Gamma$ and $\Delta_Z$. The electronic density and spin texture of the vortex-CDW are shown in Fig. 4 for the parameters $\Gamma = 1$ and $W_0 = 0.12, \Delta_Z = 0.015$. The value $W_0 = 0.12$
and we are a numerical artefact. All energies are in units of $Z$.\(\Delta \) is close to 0.

FIG. 2: Density of states for the uniform fully spin polarized phase at \(\nu = 1\) in Landau level \(N = 0\) for Zeeman coupling \(\Delta_Z = 0.015\). (a)-(c) \(\Gamma = 1\) and \(W_0 = 0, 0.05, 0.1\) respectively; (d)-(e) \(\Gamma = 2/3\) and \(W_0 = 0.05, 0.1\) respectively; (f) \(\Gamma = 4/5\) and \(W_0 = 0.1\). The rapid oscillations in some of the graphs are a numerical artefact. All energies are in units of \(e^2/\kappa \ell\).

is close to \(W_0^{(c)}\) so that the amplitude of the CDW in this figure is small. The amplitude increases with \(W_0\) however. Minima and maxima of the CDW have the same amplitude and there is no net induced charge in a unit cell as expected. The spin density \(S_z(\mathbf{r})\) (not shown in the figure) varies only slightly.

The spin texture of the vortex-CDW is interesting.

There is a \(2\pi\) spin vortex at each positive and negative modulation of the density. Since the \(z\) component of the spin is everywhere positive and because of the spin-charge coupling inherent to a QHF, the positive and negative modulations have opposite vorticity. We could, loosely speaking, refer to the positive and negative modulations as merons and antimerons. A meron is an excitation of a unit vector field \(\mathbf{m}(\mathbf{r})\) that has \(m_z(0) = \pm 1\) at its center and \(m_z(\mathbf{r}) = 0\) far away from the center. The vectors lie in the \(xy\)-plane and form a vortex configuration with vorticity \(n_v = \pm 1\). As \(r\) increases from the meron core, the spins smoothly rotate up (if \(m_z(0) = -1\)) or down (if \(m_z(0) = +1\)) towards the \(xy\)-plane. There are four flavors of meron with a topological charge given by \(Q = \frac{1}{2} [m_z(\infty) - m_z(0)] n_v\). In a QHF, merons carry half an electron charge. In our vortex-CDW however, we are dealing with a spin density \(S(\mathbf{r})\) that does not have \(|S(\mathbf{r})| = \hbar/2\) everywhere in space (the vectors do not just rotate) so that our merons do not have a quantized charge.

In each magnetic unit cell of the vortex-CDW, there are two merons and two antimerons with the same vorticity but opposite global phase for two merons or antimerons. This bipartite meron lattice is similar to the square lattice antiferromagnetic state (SLA) of the Skyrme crystal that was predicted to occur in a 2DEG.
near (but not at) filling factor $\nu = 1$ in the absence of an external potential. In the Skyrme crystal, the electrons (or holes) added to the QHF state at $\nu = 1$ crystallize in the form of skyrmions for $\nu > 1$ or anti-skyrmions for $\nu < 1$. The vortex-CDW that we find here occurs at precisely $\nu = 1$.

Figure 1(b) shows that the average spin $S_z$ decreases with $W_0$ in the vortex-CDW and saturates at the precise value $S_z = \Gamma - 1/2$ for $\Gamma = 1/2, 2/3, 3/4$. In the saturation region for $S_z$, the spin texture has disappeared and the CDW has very little modulation in $S_z(r)$. We will call this phase, the normal-CDW. There is no saturation for the two cases $\Gamma = 4/5, 1$. We assume that this is due to the fact that there is another phase very close in energy that wins over the vortex-CDW for $W_0 \gtrsim 0.16$ in these two cases (and probably at a larger value of $W_0$ for the other cases) but we have not been able to stabilize this other phase. We thus limit our analysis to the range $W_0 \in [0, 0.16]$ for most values of $\Gamma$ in this work. The change in $S_z$ induced by the external potential should be detectable experimentally. In particular, the vortex-CDW is absent for $\Gamma = 1/2$ and thus the 2DEG makes a transition from a fully polarized to an unpolarized CDW with period $a_0$ instead of $\sqrt{2}a_0$.

The density of states for the vortex-CDW is shown in Fig. 5 for $\Gamma = 2/3$ and $W_0 = 0.12, 0.14, 0.18$. The subband structure gets more and more different from that of the uniform phase as $W_0$ increases [compare with Fig. 2(e)]. The electron-hole gap decreases slowly with $W_0$ in the vortex-CDW and normal-CDW phases.

Figure 6 shows the Hartree, Fock (or exchange), external potential and Zeeman contributions to the total energy of the vortex-CDW and uniform state for $\Gamma = 1$ and Zeeman coupling $\Delta_Z = 0.015$ and different values of $W_0$ (all energies are in units of $e^2/\kappa \ell$).

The energy of the vortex-CDW state does not depend on the global phase of its vortices. This $U(1)$ symmetry, which is broken in a particular realization of the vortex-CDW state, leads to a gapless phase mode (a Goldstone mode). This is clearly seen in Fig. 7(b) where the two modes for $\omega < 0.05e^2/\hbar \ell$ are the spin wave mode which is gapped at $\Delta_Z$ and the gapless phase mode. In Fig. 7, $\Gamma = 2/3$ and the wave vector $k$ now follows the path $\Gamma - M - X - \Gamma$ along the edges of the irreducible magnetic Brillouin zone of the square lattice [with $\Gamma = (0, 0); M = (1/2\sqrt{2}, 1/2\sqrt{2}), X = (1/2\sqrt{2}, 0)$ in units of $2\pi/a_0$]. To obtain the dispersions in the CDW phases, we have computed the response functions...
the phase mode is gapped as shown in Fig. 7(c). The ground state has transited to the normal-CDW and phase modes are stronger in \( \chi_{\nu, \omega}^{(\text{no summation over } G)} \) as comparison between the energy curves of these two phases. The summation allows the capture of modes that originate from a folding of the full dispersion into the first Brillouin zone. It also captures the electron-hole continuum that starts at the Hartree-Fock gap. In Fig. 7, we have cut the dispersions at a frequency corresponding to the onset of this continuum. Figure 7 shows the dispersion for: (a) the uniform phase, (b) the vortex-CDW, and (c) the normal-CDW.

The vortex and normal CDWs have a phonon mode gapped by the external potential. The branch we indicate as the gapped phonon mode in Fig. 7(b) has the strongest peak in the response function \( \chi^{R}_{\rho, \omega} (k, k, \omega) \) (no summation over \( G \)) as \( k \to 0 \) while the spin wave and phase modes are stronger in \( \chi^{R}_{\rho, \rho} (k, k, \omega) \) and \( \chi^{R}_{\rho, \omega} (k, k, \omega) \) respectively. At \( W_0 = 0.13 \) for \( \Gamma = 2/3 \), the ground state has transited to the normal-CDW and the phase mode is gapped as shown in Fig. 7(c).

**B. Case \( \Gamma \in [0, 1/2] \)**

The phase diagram for \( \Gamma \in [0, 1/2] \) is different from that of \( \Gamma \in [1/2, 1] \). For \( \Gamma \in [0, 1/2] \), we find a transition between two types of vortex-CDW phases. The first vortex-CDW is the one described in the previous section, the second one, the antivortex-CDW has the sign of all vortices and \( S_z \) inverted (but different amplitude for the charge and spin modulations). This antivortex-CDW evolves from a uniform state that has all spin down as shown in Fig. 8. At \( \Delta_Z = 0 \), these two CDW are degenerate in energy. At finite Zeeman coupling, there is a crossing between the energy curves of these two phases. The ground state thus evolves from the uniform state with all spins up, to the vortex-CDW, then to the antivortex-CDW and finally into the normal CDW. Figure 8 shows these transitions for the special cases of \( \Gamma = 1/3 \) and \( \Delta_Z = 0.015, 0.006, 0.002 \). The corresponding behavior of \( S_z \) is also shown. The region where \( S_z \) varies in each graph is where the vortex-(or antivortex)-CDW is the ground state. As the Zeeman coupling gets smaller, this region increases. The value of \( W_0 \) for the crossing be-
tween the two vortex-CDW states is shown by the dashed vertical line in the $S_z$ vs $W_0$ curves. The average spin $S_z$ changes discontinuously at this point but this discontinuity goes to zero as $\Delta Z \to 0$. The value of $S_z$ is always positive, however. For $\Gamma \in [1/2, 1]$, the energy curve for the anti-vortex CDW is above that of the vortex-CDW for all values of $W_0$. The two curves merge at $\Delta Z = 0$ but there is no crossing between the two solutions. If we take advantage of the possibility of changing the value of the $g$–factor independently of the magnetic field in GaAs/AlGaAs heterojunctions, then it is possible to reduce the Zeeman coupling and, as Fig. 9 clearly shows, to increase the transition region where the vortex-CDW is expected.

For $\Gamma = 1/2, 1/3, 1/4, 1/5$ and a very small Zeeman coupling $\Delta_Z = 0.001$. The vertical dashed lines indicate where the transition between the vortex and antivortex-CDW phases occurs for each value of $\Gamma$. The spin starts at $S_z/h = 1/2$ in the uniform state then decreases in the vortex-CDW (open symbols in Fig. 9). When the antivortex-CDW replaces the vortex-CDW as the ground state of the system, the value of $S_z$ changes discontinuously. This jump is more apparent for $\Gamma = 1/5$ in Fig. 9. After this discontinuity, $S_z$ increases (filled symbols in Fig. 9) until it reaches the finite value $S_z/h = 1/2 - \Gamma$ at large $W_0$, a value that is independent of the Zeeman coupling $\Delta_Z$. The behaviour of $S_z$ is not monotonic. It the limit $\Delta_Z \to 0$, the $S_z$ curves for the vortex- and antivortex-CDWs would cross at $S_z = 0$ and there would be no discontinuity. In the special case $\Gamma = 1/2$, the transition is directly from the uniform and fully polarized state with $S_z/h = 1/2$ to the normal CDW where $S_z = 0$. There is thus an important discontinuity in $S_z$ in this case.

![FIG. 8: Energy (left) and corresponding average spin $S_z$ per electron (right) of the vortex-CDW (squares) and antivortex-CDW (triangles) for $\Gamma = 1/3$ and Zeeman couplings : (a),(b) $\Delta_Z = 0.015$,(c),(d) $\Delta_Z = 0.006$,(e),(f) $\Delta_Z = 0.002$ in units of $e^2/\kappa l$. The vertical dashed lines indicate the value of the applied external potential $W_0$ at which the transition between the two vortex CDWs takes place.](image)

Figure 9 shows the behavior of $S_z$ in the ground state for $\Gamma = 1/2, 1/3, 1/4, 1/5$ and a very small Zeeman coupling $\Delta_Z = 0.001$. The vertical dashed lines indicate where the transition between the vortex and antivortex-CDW phases occurs for each value of $\Gamma$. The spin starts at $S_z/h = 1/2$ in the uniform state then decreases in the vortex-CDW (open symbols in Fig. 9). When the antivortex-CDW replaces the vortex-CDW as the ground state of the system, the value of $S_z$ changes discontinuously. This jump is more apparent for $\Gamma = 1/5$ in Fig. 9. After this discontinuity, $S_z$ increases (filled symbols in Fig. 9) until it reaches the finite value $S_z/h = 1/2 - \Gamma$ at large $W_0$, a value that is independent of the Zeeman coupling $\Delta_Z$. The behaviour of $S_z$ is not monotonic. It the limit $\Delta_Z \to 0$, the $S_z$ curves for the vortex- and antivortex-CDWs would cross at $S_z = 0$ and there would be no discontinuity. In the special case $\Gamma = 1/2$, the transition is directly from the uniform and fully polarized state with $S_z/h = 1/2$ to the normal CDW where $S_z = 0$. There is thus an important discontinuity in $S_z$ in this case.

![FIG. 9: (Color online) Spin polarization $S_z/h$ as a function of the applied external potential $W_0$ for several values of $\Gamma \leq 1/2$ and Zeeman coupling $\Delta_Z = 0.001e^2/\kappa l$. The vertical dashed lines indicate the potential strength $W_0$ for each value of $\Gamma$ where the transition from the vortex (filled symbols) to the anti-vortex CDW (open symbols) takes place. For $\Gamma = 1/2$, the transition is from the uniform (with $S_z/h = 1/2M$) to the normal CDW with $S_z = 0$.](image)

As we mentioned above, our formalism can equally well be used to discuss the energy of the electron gas in Landau level $N = 0$ in graphene if the electrons are assumed to occupy only one valley. An exact diagonalization study by Ghazaryan and Chakraborty for a 2DEG in graphene finds transition between unpolarized and partially polarized ground states induced by the external potential when $\Gamma = 1$ (their $\alpha = 1$). The equation was also used to study the effect of Coulomb interaction on the density of states for graphene in a modulated potential but the vortex-CDW state that we found
was not considered in that work.

IV. SUMMARY AND DISCUSSION

We have computed the phase diagram of the 2DEG at $\nu = 1$ in Landau level $N = 0$ in the presence of an applied external potential with a square lattice periodicity for several rational values of $\Gamma \in [0, 1]$. We restricted our analysis to $W_0 \in [0, 0.16]$. In this range, the 2DEG evolves first from a uniform state with full spin polarization to a vortex-CDW and finally into a normal CDW with no spin texture but with a finite spin polarization.

The change in the spin polarization $S_z$ with the applied field (smooth for $\Gamma > 1/2$ and abrupt for $\Gamma \leq 1/2$) is one feature of the phase transition described in this work that should be measurable experimentally. Another one is the gapless spin mode due to the broken U(1) symmetry in the vortex-CDW phase. The same mode occurs in a Skyrme crystal. In that system, it was shown that such mode could provide a fast channel for the relaxation of the nuclear spin in nuclear magnetic resonance experiments. Indeed, the bare Zeeman gap in the dispersion of the spin-wave mode is orders of magnitude larger than the nuclear spin splitting, impeding the creation of spin waves by nuclear spins. The softening of the spin wave mode in the uniform phase may also lead to an increase in nuclear spin-lattice relaxation time as suggested by Bychkov.

We have used $W_0$ for the external potential because the transition from the uniform to the vortex-CDW takes place at roughly the same value of $W_0$ when the potential is expressed in terms of $W_0$. The actual external potential however is $V_x = F^{-1}(G_1) W_0 = e^{i\Gamma} W_0$. This means that the critical field $W_0^{(c)} \approx 0.11$ translates into different real critical fields for different values of $\Gamma$ i.e. from $V_x^{(c)} = 0.21$ for $\Gamma = 1/5$ to $V_x^{(c)} = 2.5$ for $\Gamma = 1$. It is not clear, then if our assumption of neglecting Landau-level mixing can be justified for $\Gamma$ near unity. We assumed that the external lattice parameter $a_0$ is fixed experimentally. When $\Gamma$ is also given, all other parameters are determined: the magnetic field, the electronic density $n_e$ (at $\nu = 1$) and the ratio, $\alpha$, of the Coulomb interaction to the cyclotron energy:

$$B = \frac{\hbar e}{e a_0^2 \gamma} = \frac{413.57}{a_0} \frac{1}{\gamma} \text{T},$$

$$n_e = \frac{1}{e a_0^2 \gamma} = \frac{1}{a_0} \frac{10^{14}}{\gamma} \text{cm}^{-2},$$

$$\alpha = \frac{\hbar^2}{\omega_c^2} = \frac{a_0}{a_B^2} \sqrt{\frac{\Gamma}{2\pi}} = \frac{\pi_0}{10.2} \sqrt{\frac{\Gamma}{2\pi}},$$

where $a_B^* = \hbar/\epsilon e$ is the effective Bohr radius and $\pi_0$ is the lateral superlattice constant in nm. We used, for GaAs: $\kappa = 12.9$ and $m^* = 0.067m_e$ (where $m_e$ is the electron mass).

For $\Gamma = 1$, $\alpha = 0.03\pi_0$ so that with a very small (but physically feasible) superlattice period of $a_0 = 39$ nm, we get $\alpha = 1.5$, $B = 2.7$ T, $n_e = 0.65 \times 10^{11}$ cm$^{-2}$ while for $\Gamma = 1/5$, we get $\alpha = 0.68$, $B = 13.6$ T, $n_e = 3.29 \times 10^{11}$ cm$^{-2}$. The magnetic field and density pose no problem, but $\alpha$ is not small, especially when $\Gamma > 1/2$. Clearly, a more sophisticated calculation including a certain amount of Landau-level mixing and screening is required to confirm that the vortex-CDW phase is effectively the ground state in this system. We leave this to further work.

Acknowledgments

R. C. was supported by a grant from the Natural Sciences and Engineering Research Council of Canada (NSERC). Computer time was provided by Calcul Qu´ebec and Compute Canada.

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