Enhancing the Jet Quenching Parameter from Marginal Deformations

J.F. Vázquez-Poritz

Department of Physics
University of Cincinnati
Cincinnati OH 45221-0011, USA

jporitz@physics.uc.edu

ABSTRACT

A number of recent papers have applied the AdS/CFT correspondence to a strong-coupling calculation of the medium-induced radiative parton energy loss in nucleus-nucleus collisions at RHIC. The predicted value of the “jet quenching parameter” $\hat{q}$, however, is rather small compared to the experimental results. For hot $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, certain marginal deformations can have the effect of enhancing $\hat{q}$. This result is highly sensitive to the location of the fundamental string’s endpoints in the internal space.
1 Introduction

Collisions of nuclei at RHIC may shed light on the properties of hot and dense QCD matter. In particular, the plasmic medium modifies the fragmentation of partons that are produced with high transverse momentum \[1, 2, 3, 4\]. This medium-dependent effect is described by the “jet quenching parameter” \(\hat{q}\). The so-called dipole approximation which is frequently used in jet quenching calculations is given by

\[
\langle W^A(C) \rangle \approx \exp[-\frac{1}{4} \hat{q} L^{-} L^{+}],
\]

where \(\langle W^A(C) \rangle \) is the expectation value of the light-like Wilson loop in the adjoint representation whose contour \(C\) is a rectangle with large extension \(L^{-}\) in the \(x^{-}\) direction and small extension \(L^{+}\) in a transverse direction \([5]\). It has been proposed that (1) can be taken to be a nonperturbative definition of \(\hat{q}\) \([6]\).

Furthermore, since the quark-gluon plasma produced at RHIC is believed to be strongly-coupled, it has been argued that the AdS/CFT correspondence \([7]\) is a suitable framework with which to calculate the energy loss of quarks \([8, 6, 9, 10, 11, 12, 13]\). On the supergravity side, it is more convenient to calculate the thermal expectation value of a Wilson loop in the fundamental representation \(\langle W^F(C) \rangle\). In the planar limit, \(\langle W^A(C) \rangle\) and \(\langle W^F(C) \rangle\) are related by

\[
\langle W^A(C) \rangle = \langle W^F(C) \rangle^2.
\]

According to the AdS/CFT correspondence, \(\langle W^F(C) \rangle\) is given by

\[
\langle W^F(C) \rangle = \exp[-S(C)],
\]

where \(S\) is the regularized action of the extremal surface of a fundamental string whose large-distance boundary is the contour \(C\) in Minkowski spacetime \(\mathbb{R}^{3,1}\) \([14, 15]\). \([16, 17, 18]\). In principle, the \(\langle W^A(C) \rangle\) found with \([2]\) and \([3]\) via the AdS/CFT correspondence can then be equated with its proposed nonperturbative definition in \([1]\) in order to give a theoretical prediction for \(\hat{q}\) which can then be compared the RHIC measurements.

Although we do not yet have a supergravity description of strongly-coupled QCD, we are able to study various gauge theories at large \(N_c\) from the supergravity point
of view. The most-studied example is the \( \mathcal{N} = 4 \) \( SU(N_c) \) supersymmetric Yang-Mills theory which in the planar limit at large 't Hooft coupling is described by type IIB supergravity on \( AdS_5 \times S^5 \) \[7\]. In fact, it is for this case that the calculation of the jet quenching parameter was done in \[6\]. However, the predicted value is rather small compared to the experimental results. This need not be worrisome, since this calculation was done for a superconformal field theory as a preliminary demonstration.

In fact, it has been found that the jet quenching parameter is gauge theory specific and not universal \[10\]. Therefore, it is of interest to consider the predictions for the value of \( \hat{q} \) in theories which are more realistic, such as those that are non-conformal at zero temperature and have less supersymmetry. This was done in \[10\], for example, in the case of a strongly-coupled non-conformal gauge theory plasma. However, the resulting \( \hat{q} \) was found to decrease, rather than increase, as one goes away from the conformal gauge theory.

One way in which the supersymmetry of the theory in the zero temperature limit can be reduced to \( \mathcal{N} = 1 \) is to replace the 5-sphere with other Sasaki-Einstein spaces, such as \( T^{1,1} \) \[25\] or one of the countably infinite \( Y^{p,q} \) \[26\] or \( L^{p,q,r} \) spaces \[27\]. One could also replace the 5-sphere by an Einstein space which does not support Killing spinors, such as the \( T^{pq} \) spaces, in order to have a nonsupersymmetric theory at zero temperature \[28\]. However, since the Wilson loop of a purely radial string configuration is independent of the internal space, this will not affect the calculation for \( \hat{q} \).

We will consider an alternative way by which the supersymmetry can be reduced down to \( \mathcal{N} = 1 \), which is to have the theory undergo marginal deformations. Since the theory has an isometry group which includes \( U(1) \times U(1) \), one to use U-duality to find the gravity dual of the deformed theory. The deformation on the gravity side can be matched to an exactly marginal operator in the field theory, providing a holographic test of the methods of Leigh and Strassler \[30\]. In particular, the deformed superpotential is given by

\[
W = Tr(e^{i\pi\beta}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\beta}\Phi_1\Phi_3\Phi_2),
\]

where \( \beta = \gamma - \tau_s\sigma \), \( \gamma \) and \( \sigma \) are real deformation parameters and \( \tau_s \) is a complex structure parameter related to the gauge coupling and theta parameter of the dual gauge
theory. The resulting geometry of the gravity dual is a warped product of AdS$_5$ with the deformed internal space, where the warp factor depends on the internal directions. As shown in Figure 1, when temperature is added to the theory, these deformations can ultimately have the effect of enhancing the “jet quenching parameter” $\hat{q}$.

![Diagram](image.png)

Figure 1: Marginal deformations of conformal field theories can change the properties of the theories at finite temperature. In particular, $\sigma$ deformations can lead to an enhancement of the ”jet quenching parameter” $\hat{q}$.

This is actually one of the less dramatic effects that marginal deformations can have on a theory as it is moved away from its conformal limit. For example, $\sigma$ deformations of the Coulomb branch of $\mathcal{N} = 4$ super Yang-Mills theory can induce a transition from Coulombic attraction between quarks and anti-quarks to that of linear confinement. This was studied from the point of view of type IIB supergravity in [33, 34, 35]. Similar phenomena were discussed on the gauge theory side in [37, 38, 39].

This paper is organized as follows. In section 2, we will review the gravity dual of marginal deformations of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory which reduce the supersymmetry down to $\mathcal{N} = 1$. We will show how $\sigma$ deformations can enhance the $\hat{q}$ parameter, depending on the location of the fundamental string endpoints in the internal space. In section 3, we will generalize this to a more general class of marginal deformations which do not preserve any supersymmetry. We discuss various issues and further directions in section 4.
2 From $\mathcal{N} = 1$ supersymmetric deformations

Figure 2 shows the steps involved in a solution-generating technique which can be employed to find the type IIB supergravity duals of marginally-deformed field theories. The procedure can be outlined as follows. First, T-dualize along one of the $U(1)$ directions to type IIA theory. Lifting the solution to eleven dimensions provides a third direction which is associated with a $U(1)$ symmetry. One can now apply an $SL(3, R)$ transformation along these $U(1)^3$ directions. Dimensionally reducing and T-dualizing along shifted directions yields a new type IIB solution. Of course, instead of lifting to eleven dimensions, one could apply an $SL(2, R)$ transformation to the type IIA solution. However, this corresponds to $\gamma$ deformations with $\sigma = 0$ which, as we will see, do not effect the Wilson loop calculation for purely radial strings.

![Diagram](image.png)

Figure 2: Generating new solutions via U-dualities.

This procedure can be applied to any solution that has an isometry group which contains $U(1) \times U(1)$. If in addition to this symmetry there is a $U(1)$ R-symmetry, then the deformed solution preserves $\mathcal{N} = 1$ supersymmetry. This has been applied to finding the type IIB supergravity background corresponding to marginal deformations of $\mathcal{N} = 4$ super Yang-Mills theory [29], as well as the marginal deoformations of the superconformal field theories associated with the $Y^{p,q}$ spaces [29] and the $L^{p,q,r}$ spaces [36].

One can also apply this solution-generating technique to the gravity duals of non-supersymmetric field theories. Here we will consider the near-horizon of a nonextremal
D3-brane. This corresponds to a Yang-Mills theory at finite temperature, which is an $\mathcal{N} = 4$ superconformal field theory in the zero temperature limit \cite{7}. The metric can be expressed in light-cone coordinates\footnote{We are using light-cone coordinates because this will lead to a light-like string configuration, which ensures that the corresponding quarks are ultrarelativistic.} as

\[ ds_{10}^2 = ds_5^2 + R^2 d\Omega_5^2, \]

where

\[ ds_5^2 = \frac{r^2}{R^2} \left[ -(1 + f)dx^+ dx^- + dx_2^2 + dx_3^2 + \frac{1}{2}(1 - f)((dx^+)^2 + (dx^-)^2) \right] + \frac{R^2}{r^2} f dr^2. \]

where $f = 1 - \frac{r_0}{r}$ and $R^4 = \alpha'^2 \alpha_{SYM} N_c$. The metric on the unit 5-sphere can be written as

\[ d\Omega_5^2 = \sum_{i=1}^{3} (d\mu_i^2 + \mu_i^2 d\phi_i^2), \quad \text{with } \sum_i \mu_i^2 = 1. \]

We can parameterize the 5-sphere metric as

\[ \mu_1 = \cos \alpha, \quad \mu_2 = \sin \alpha \cos \theta, \quad \mu_3 = \sin \alpha \cos \theta. \]

The temperature of the dual field theory is equal to the Hawking temperature of the black hole, which is $T = \frac{r_0}{\pi R^2}$.

The $\beta$ deformation of nonextremal D3-branes in the near-horizon region has the string-frame metric

\[ ds_{str}^2 = \sqrt{H} (ds_5^2 + R^2 d\tilde{\Omega}_5^2), \]

where

\[ d\tilde{\Omega}_5^2 = \sum_i (d\mu_i^2 + G \mu_i^2 d\phi_i^2) + |\hat{\gamma}|^2 \mu_1^2 \mu_2^2 \mu_3^2 (\sum_i d\phi_i)^2. \]

The functions $G$ and $H$ are given by

\[ G^{-1} = 1 + |\hat{\gamma}|^2 s_\alpha^2 (c_\alpha^2 + s_\alpha^2 s_\theta^2 c_\theta^2), \]

\[ H = 1 + \hat{\sigma}^2 s_\alpha^2 (c_\alpha^2 + s_\alpha^2 s_\theta^2 c_\theta^2), \]

where $\hat{\gamma} \equiv \gamma R^2$, $\hat{\sigma} \equiv \sigma R^2$ and $\hat{\beta} \equiv \beta R^2$.

If we begin with $N$ D3-branes, then the deformed solution includes $\gamma N$ D5-branes and $\sigma N$ NS5-branes wrapped on a two-torus. Charge quantization implies that $\gamma$ and
σ must take on rational values. There are also flux and scalar fields turned on by the deformations but these are not important for our purposes. What is important is the conformal factor $H$, which renders the solution implicitly higher dimensional. Note that $H$ depends on $σ$ but not $γ$. Also, note that $H \geq 1$ which, as we will see shortly, is why the value of the jet quenching parameter $\hat{q}$ is enhanced rather than diminished.

The classical supergravity description is valid as long as the curvature is small relative to the string scale, the two-torus corresponding to the $U(1)^2$ global symmetry is larger than the string scale, and the metric does not degenerate at arbitrary points. These conditions can be met for

\[ R \gg 1, \quad \dot{γ} \ll R, \quad \dot{σ} \ll R. \]  

The action for the Euclideanized string worldsheet is given by

\[ S = \frac{1}{2\pi\alpha'} \int d\tau dσ \sqrt{\det G_{MN} \partial_α X^M \partial_β X^N}. \]  

We will set $τ = x^-$ and $σ = x_2$, so that the string lies at constant $x_3$ and $x^+$. Also, we are interested in a purely radial string which is lies on a single point in the internal space. Therefore, the effective metric is

\[ ds^2_{\text{eff}} = \sqrt{H} \left[ \frac{r^2}{R^2} dσ^2 + \frac{r^2}{2R^2} (1 - f) dτ^2 + \frac{R^2}{r^2 f} dr^2 \right]. \]  

In order for such a string configuration to solve the equations of motion, it is required that $\partial_α H = \partial_θ H = 0$. This only occurs at several particular points in the internal space. The solutions for which $H \geq 1$ are:

| $\sin^2 α$ | $\sin^2 θ$ | $H$       |
|------------|------------|-----------|
| $1$        | $\frac{1}{2}$ | $1 + \frac{1}{4} \dot{σ}^2$ |
| $\frac{1}{2}$ | $0, 1$       |           |
| $\frac{2}{3}$ | $\frac{1}{2}$ | $1 + \frac{1}{3} \dot{σ}^2$ |

We are interested in the latter cases for which $H > 1$, since only then will the thermal expectation value for purely radial string configurations differ from that on
the undeformed background. Then

\[ S = \sqrt{H} \frac{r_0^2 L^-}{\sqrt{2\pi \alpha' R^2}} \int_0^{L/2} d\sigma \sqrt{1 + \frac{(\partial_\sigma r)^2 R^4}{f r^4}}. \]  

(15)

Since \( H \) is a constant, the Wilson loop calculation proceeds as in [6]. For completeness, we will show some of the details. The radial equation of motion is

\[ (\partial_\sigma r)^2 = c^2 \frac{r^4 f}{R^4}, \]  

(16)

where \( c \) is an integration constant. For the nontrivial solution with nonvanishing \( c \), the turning point occurs at the black hole horizon where \( f = 0 \), for all values of \( L \). A rough picture of this is given by Figure 3. This is to be expected, since \( \hat{q} \) describes the thermal medium and not the ultraviolet physics [6]. The previous condition \( R >> 1 \) ensures that the horizon is far enough from the black hole singularity so that the curvature is small and classical gravity can be trusted there.

![Figure 3: The thermal expectation value of a Wilson loop can be calculated on the supergravity side by considering a fundamental string in the background of an AdS black hole. The string endpoints lie on a probe brane at large distance and the string’s turning point is on the black hole horizon. However, this picture is slightly misleading because the directions along the probe brane are actually orthogonal to those along the horizon.](image)

Integrating (16) yields \( L = \frac{2}{\sqrt{\pi}} \Gamma(\frac{5}{4})/(\Gamma(\frac{3}{4}) R^2 c r_0) \) and

\[ S = \sqrt{H} \frac{\pi \sqrt{\alpha_{\text{SYM}}} N_c L^- L T^2}{2\sqrt{2} \pi \Gamma(\frac{5}{4})/\Gamma(\frac{3}{4}) L^2 T^2} \sqrt{1 + \frac{4\Gamma^2(\frac{5}{4})}{\pi \Gamma^2(\frac{3}{4}) L^2 T^2}} - \sqrt{H} \frac{\pi \Gamma(\frac{5}{4})}{\sqrt{2} \Gamma(\frac{3}{4})} \sqrt{\alpha_{\text{SYM}}} L^- T. \]  

(17)
where we have subtracted the “self-energy” of the quark and antiquark. We can now use (1), (2) and (3) to obtain an expression for the jet quenching parameter $\hat{q}$ from the above string action in the limit $LT \ll 1$. The $\hat{q}$ for the marginally deformed theory can be written in terms of that for the undeformed theory:

$$\hat{q}_{\text{deformed}} = \sqrt{H} \hat{q},$$

where $\hat{q}$ was found in [6] to be

$$\hat{q} = \frac{\pi^{3/2} \Gamma\left(\frac{3}{4}\right)}{\sqrt{2} \Gamma\left(\frac{5}{4}\right)} \sqrt{\alpha_{\text{SYM}} N_c T^3}.$$  

Therefore, the value of the jet quenching parameter is increased by a factor of $\sqrt{H}$ by the $\sigma$ deformations. The $\gamma$ deformations do not have any effect in this regard.

One can also consider the $\beta$ deformations of finite-temperature theories which reduce, in the zero temperature limit, to the theories associated with the $Y^{p,q}$ spaces and the $L^{p,q,r}$ spaces. The corresponding gravity dual has the string-frame metric of the form (9) with $d\tilde{\Omega}_5^2$ replaced by the deformed $L^{p,q,r}$ metric. As before, for a purely radial string configuration it is the warp factor $H$ which is important, rather than the exact form of the deformed internal metric. Consider the theory associated with $T^{1,1}$ as an example. After the $\beta$ deformations, the resulting $H$ is given by

$$H = 1 + \hat{\sigma}^2 \left( \frac{1}{54} \left( \cos^2 \theta_2 \sin^2 \theta_1 + \cos^2 \theta_1 \sin^2 \theta_2 \right) + \frac{1}{36} \sin^2 \theta_1 \sin^2 \theta_2 \right),$$

where $\theta_1$ and $\theta_2$ are two coordinates in $T^{1,1}$. For a purely radial string configuration, we must have $\partial_{\theta_1} H = \partial_{\theta_2} H = 0$. The solutions for which $H \geq 1$ are:

| $\sin^2 \phi_1$ | $\sin^2 \phi_2$ | $H$  |
|-----------------|-----------------|------|
| 1               | 0               | $1 + \frac{2}{3} \hat{\sigma}^2$ |
| 0               | 1               | $1 + \frac{2}{3} \hat{\sigma}^2$ |
| 1               | 1               | $1 + \frac{1}{3} \hat{\sigma}^2$ |
3 From nonsupersymmetric deformations

In the previous section, we considered marginally deformed theories which were supersymmetric in their zero temperature limits. We found that, for the $\sigma$ deformations, purely radial string configurations can only exist at particular points in the internal space. Since there are just a handful of such points for which there is an enhancement of the $\hat{q}$ parameter, it is important to see if more general backgrounds can yield more examples for which this enhancement occurs.

We will now consider dropping the condition that our deformed backgrounds preserve supersymmetry. We will focus on the case in which the undeformed theory is $\mathcal{N} = 4$ super Yang-Mills. In particular, since the 5-sphere of the initial geometry has three directions which associated with a $U(1)$ isometry, the steps described in Figure 2 can be applied three consecutive times to three different pairs of $U(1)$ directions. The resulting 6-parameter deformed metric has the form (21) with (31) (32)

$$d\tilde{\Omega}_5^2 = \sum_i (d\mu_i^2 + G\mu_i^2 d\phi_i^2) + G\mu_1^2 \mu_2^2 \mu_3^2 \sum_i \beta_i R^2 d\phi_i^2,$$  

(21)

where

$$G^{-1} = 1 + |\hat{\beta}_1|^2 \mu_3^2 + |\hat{\beta}_2|^2 \mu_1^2 \mu_2^2 + |\hat{\beta}_3|^2 \mu_1^2 \mu_2^2,$$

$$H = 1 + \tilde{\sigma}_1^2 \mu_2 \mu_3 + \tilde{\sigma}_2^2 \mu_1 \mu_3 + \tilde{\sigma}_3^2 \mu_1 \mu_2,$$  

(22)

Various tests of the AdS/CFT correspondence have been successfully carried out for this background [31] [32].

The conditions for a purely radial string, namely $\partial \alpha H = \partial \theta H = 0$, imply that

$$0 = \left[\tilde{\sigma}_1^2 s_\alpha s_{2\theta} + 2c_{2\alpha}(\tilde{\sigma}_2^2 s_\theta^2 + \tilde{\sigma}_3^2 s_\theta^2)\right] s_{2\alpha},$$

$$0 = \left[\tilde{\sigma}_1^2 c_\alpha c_{2\theta} + (\tilde{\sigma}_2^2 - \tilde{\sigma}_3^2)c_{2\alpha}^2\right] s_\alpha s_{2\theta}.$$  

(23)

It is straightforward but not very illuminating to solve for the most general solutions. Here, we present some simple solutions for which $H > 1$, along with the conditions imposed on $\sigma_i$: 
The reality conditions on $\alpha$ and $\theta$ put further constraints on the $\hat{\sigma}_i$ which, in turn, ensure that $H > 1$. As can be seen in the above table, dropping the supersymmetry condition leads to an increase in the number of string endpoints in the internal space which lead to an enhancement of the jet quenching parameter $\hat{q}$. For the case of equal $\hat{\sigma}_i$, $\mathcal{N} = 1$ supersymmetry is preserved and the above solutions degenerate to those of the first example that was considered in the previous section.

| conditions       | $\sin^2 \alpha$ | $\sin^2 \theta$ | $H$          |
|------------------|------------------|------------------|--------------|
| none             | 1                | $\frac{1}{2}$   | $1 + \frac{1}{4} \hat{\sigma}_1^2$ |
| none             | $\frac{1}{2}$   | 1                | $1 + \frac{1}{4} \hat{\sigma}_2^2$ |
| $\hat{\sigma}_1 = 0$, $\hat{\sigma}_2 = \hat{\sigma}_3$ | $\frac{1}{2}$ | not specified | $1 + \frac{1}{4} \hat{\sigma}_3^2$ |
| none             | $\frac{1}{2}$   | 0                | $1 + \frac{1}{4} \hat{\sigma}_3^2$ |
| $\hat{\sigma}_2 = \hat{\sigma}_3$ | $\frac{2\hat{\sigma}_2^2}{4\hat{\sigma}_1^2 - \hat{\sigma}_3^2}$ | $\frac{1}{2}$ | $1 + \frac{\hat{\sigma}_1^2}{4\hat{\sigma}_1^2 - \hat{\sigma}_2^2}$ |
| $\hat{\sigma}_1 = \hat{\sigma}_2$ | $\frac{3\hat{\sigma}_1^2 - \hat{\sigma}_3^2}{4\hat{\sigma}_1^2 - \hat{\sigma}_3^2}$ | $\frac{2\hat{\sigma}_2^2 - \hat{\sigma}_1^2}{3\hat{\sigma}_1^2 - \hat{\sigma}_3^2}$ | $1 + \frac{\hat{\sigma}_3^2}{4\hat{\sigma}_1^2 - \hat{\sigma}_3^2}$ |
| $\hat{\sigma}_1 = \hat{\sigma}_3$ | $\frac{3\hat{\sigma}_1^2 - \hat{\sigma}_3^2}{4\hat{\sigma}_1^2 - \hat{\sigma}_3^2}$ | $\frac{\hat{\sigma}_3^2}{3\hat{\sigma}_1^2 - \hat{\sigma}_3^2}$ | $1 + \frac{\hat{\sigma}_1^2}{4\hat{\sigma}_1^2 - \hat{\sigma}_3^2}$ |

4 Discussion

We have considered particular marginal deformations of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory which, at the level of supergravity, decrease the supersymmetry down to $\mathcal{N} = 1$ or $\mathcal{N} = 0$. When temperature is turned on, we have demonstrated that the $\sigma$ deformations can have the effect of enhancing the jet quenching parameter $\hat{q}$ as follows:

$$\hat{q} \to \sqrt{1 + c^2 \hat{\sigma}^2} \hat{q},$$

(24)

where $c$ is a constant which depends on the location of the fundamental string’s endpoints in the internal space. In particular, $c$ is nonzero only when the endpoints are at particular points in the internal space. This demonstrates the that value of the jet quenching parameter $\hat{q}$ can be highly sensitive to certain deformations of the theory. Probing more realistic theories with less degrees of freedom might lead to greater predictive power.

For instance, one can generalize the results of [6] to theories which exhibit a confining phase [40, 41, 42] or contain fundamental matter [43, 44]. A step in the
former direction was done in [10]. However, the jet quenching parameter was found to decrease as one goes from the conformal gauge theory to a confining gauge theory, thereby increasing the discrepancy between the predicted value and the experimental measurements at RHIC. One might wonder if $\sigma$ deformations could help solve this problem. Unfortunately, such deformations would not lead to a well-defined supergravity solution, due to the presence of the RR-flux on the 3-sphere [29]. On the other hand, the background used in [10] is perturbative in the deformation away from the conformal gauge theory [45, 46]. In fact, there is no clear reason to expect that the $\hat{q}$ parameter monotonically increases as one goes from the conformal phase to QCD. Recently, a numerical solution was found which might include a nonextremal generalization of the Klebanov-Strassler solution [47]. Since this solution works for all temperatures, it would be interesting to see if the jet quenching parameter increases as one gets closer to QCD.

We have only considered purely radial string configurations. One could also consider strings whose endpoints lie at two different locations within the internal space. Then the corresponding quarks will have different scalar charges. One might naively expect that the string action, and therefore the parameter $\hat{q}$, increases if the string endpoints move away from each other within the internal space. This is because there will be additional positive terms in the action of the form $(\partial_\sigma \phi)^2$, where $\phi$ is an internal coordinate and $\sigma$ is the spatial coordinate of the string worldsheet. However, at least for the simplest scenario of an (undeformed) AdS black hole background, this is more than compensated for by a decrease in the $(\partial_\sigma r)^2$ term. Therefore, moving the string endpoints away from each other serves to decrease the value of $\hat{q}$. The result might be different in more complicated backgrounds, such as with the addition of marginal deformations.

The marginally deformed solutions discussed in this paper may provide new backgrounds on which to investigate transport phenomena such as diffusion and sound propagation. It would be interesting to see if the universal ratio between the shear viscosity and the entropy density [19, 20, 21] is obeyed. It would also be interesting to calculate the speed of sound in this background, as was done for the gravity duals of other nonconformal field theories [22, 23, 24]. Unfortunately, it might be rather difficult to apply the techniques of holographic renormalization to a background whose
asymptotic geometry has a warp factor which depends on the internal directions.

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