Ordered droplets in quantum magnets with long-range interactions

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Abstract
A defect coupling to the square of the order parameter in a nearly quantum-critical magnet can nucleate an ordered droplet while the bulk system is in the paramagnetic phase. We study the influence of long-range spatial interactions of the form $r^{-(d+\sigma)}$ on the droplet formation. To this end, we solve a Landau-Ginzburg-Wilson free energy in saddle point approximation. The long-range interaction causes the droplet to develop an energetically unfavorable power-law tail. However, for $\sigma > 0$, the free energy contribution of this tail is subleading in the limit of large droplets; and the droplet formation is controlled by the defect bulk. Thus, for large defects, long-range interactions do not hinder the formation of droplets.

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When a magnetic system is close to its critical point, a local defect can induce the nucleation of a magnetic droplet in the nonordered bulk. In disordered systems, such droplets are responsible for Griffiths singularities [1] and other rare region effects [2]. At zero-temperature quantum phase transitions, droplets can lead to strong power-law quantum Griffiths singularities [3] or even to the smearing of the transitions [4]. A particularly interesting problem arises in metallic quantum ferromagnets because the coupling between magnetic modes and gapless fermions generates an effective long-range (power-law) spatial interaction between the magnetic fluctuations [5,6]. This interaction takes the form $r^{-(2d-1)}$ for clean electrons and $r^{-(2d-2)}$ for diffusive electrons, where $d \geq 2$ is the spatial dimensionality. Understanding disordered metallic quantum ferromagnets therefore requires analyzing the formation of magnetic droplets in the presence of long-range interactions.

In this paper, we consider this problem within a $d$-dimensional quantum Landau-Ginzburg-Wilson field theory with a general long-range attractive $1/r^{d+\sigma}$ interaction. The action reads $S = S_{\text{stat}} + S_{\text{dyn}}$ with the static part given by

$$S_{\text{stat}} = \int d\tau \int dxdy \varphi(x, \tau) \Gamma(x, y) \varphi(y, \tau) + \frac{u}{2} \int d\tau dx \varphi^4(x, \tau).$$

$\varphi(x, \tau)$ is the order parameter at position $x$ and imaginary time $\tau$. The two-point vertex, $\Gamma(x, y) = \Gamma_{\text{NI}}(x)\delta(x-y) + \Gamma_1(x, y)$, contains a non-interacting part and the attractive long-range interaction,

$$\Gamma_1(x, y) = -\gamma |\xi_0 + |x-y|^2|^{-(d+\sigma)/2}.$$  

(2)

Here, $\xi_0$ is a microscopic cutoff length scale. To ensure a proper thermodynamic limit, the range parameter $\sigma$ must be positive. The noninteracting part of the vertex reads

$$\Gamma_{\text{NI}}(x) = t_0 + \delta t(x) + \Gamma_0,$$

(3)

where $t_0$ is the bulk distance from criticality, and the constant $\Gamma_0$ cancels the $q = 0$ Fourier component of the interaction. $\delta t(x)$ is the defect potential. For definiteness we consider a single spherical defect of radius $a$ at the origin with potential $\delta t(x) = -V$ for $|x| < a$, and $\delta t(x) = 0$ otherwise. We are interested in defects that favor the ordered phase, i.e., $V > 0$.

The existence of locally ordered droplets at the defect can be studied within saddle-point approximation. Since time-independent saddle-point solutions give the lowest free energies, the static action [1] is sufficient for this purpose. To study the droplet quantum dynamics one has to specify the dynamical action $S_{\text{dyn}}$. This has been reported in Ref. [7].

To find saddle-point solutions we set $\varphi(x, \tau) = \phi(x)$ and minimize the total action with respect to $\phi(x)$. This leads to the saddle-point equation
The amplitudes of these power-law decays are analyzed in closed form. In Ref. [7], we have performed an asymptotic large-distance analysis based on the ansatz
\[
\phi(x) = \begin{cases} 
\phi_0 & (|x| < a) \\
C/|x|^{d-\sigma} & (|x| > a)
\end{cases}
\]
suggested by Griffiths’ theorem [8]. The parameters \(\phi_0\) and \(C\) follow from solving the linearized saddle-point equation and minimizing the action. For large defects \((a \gg \xi_0)\), they read
\[
C = \Omega_d \phi_0 \gamma a^d/(dt_0), \quad \phi_0 = \sqrt{(V - t_0)/u}.
\]
As a result, the saddle point action takes the form
\[
S_{SP} = \frac{\Omega_d}{d} \phi_0^2 \frac{a^d}{d} \left( t_0 - V + \frac{u}{2} \phi_0^2 \right) + \mathcal{O}(a^{d-1}, a^{d-2\sigma})
\]
to leading order in the defect size. This result, which is identical to the case of short-range interactions [9][10], implies that the droplet formation is dominated by the defect core. The power-law droplet tail thus does not hinder the droplet formation.

Here, we compare the results of the asymptotic analysis [7] with a numerically exact solution of the saddle-point equation. We focus on three space dimensions and the cases \(\sigma = 2\) and 3 not considered in [7]. The numerical procedure is as follows: For our spherical defect, the angular integrations on the right-hand side of (14) can be carried out analytically. The resulting one-dimensional equation is discretized (using 10^6 sites) and solved iteratively.

Figure 1 shows saddle-point solutions for \(d = 3, \sigma = 2\) and several \(t_0\). In agreement with (14), the droplet tail falls off like \(|x|^{-5}\). The solution for \(\sigma = 3\) is completely analogous, with the tail dropping off as \(|x|^{-6}\), as predicted. The amplitudes of these power-law decays are analyzed in Fig. 2. As predicted in (14), for small \(t_0\), \(C\) behaves like \(1/t_0\) (the small deviations at very low \(t_0\) are finite-size effects).

In summary, we have studied how a magnetic droplet nucleates at a defect coupling to the square of the order parameter in an quantum magnet with long-range interactions of the form \(r^{-(d+\sigma)}\) with \(\sigma > 0\). The droplet magnetization develops a long power-law tail, i.e., at large distances \(r\) from the defect, it decays like \(r^{-(d+\sigma)}\) in agreement with Griffiths’ theorem [8]. However, the droplet free energy is dominated by the core contribution while the tail contribution is subleading in the limit of large defects. Therefore, the droplet formation is analogous to the case of short-range interactions.

These results are potentially important for quantum phase transitions in disordered metallic ferromagnets such as Ni_{1-x}Pd_{x} [11], URu_{2-x}Re_{x}Si_{2} [12], or Fe_{1-x}Co_{x}S_{2} [13]. Our calculations show that in these systems, locally ordered droplets can form on rare (strongly coupled) spatial regions. We predict that the slow dynamics of these magnetic droplets leads to strong power-law quantum Griffiths effects close to the transition, (for Heisenberg symmetry) [14] or a destruction of the phase transition by smearing (for Ising symmetry) [4] just like for systems with short-range interactions [2]. provided the droplet-droplet interactions are negligible (see Ref. [7]).

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