Calls continue for teacher education in the United States to better prepare prospective teachers (PSTs) to effectively teach the increasingly culturally, linguistically, and socio-economically diverse student population (Hollins & Guzman, 2005; Nieto, 2000). PSTs—largely White, middle class, female, and monolingual—are typically underprepared to address diverse students’ learning needs (Sleeter, 2001). Specific to mathematics, this reflects both a lack of preparation to teach mathematics in ways that support children’s understanding (Hiebert et al., 1997) and more specifically to connect to students’ diverse cultural, linguistic, and community-based knowledge and experiences in ways that support learning (Gay, 2009). If these understandings are absent, PSTs often reproduce their own mathematics learning experiences (Lortie, 1975), drawing on traditional, teacher-centered pedagogies and decontextualized curricula that research has shown to be ineffective for many children (Boaler & Greeno, 2000).

Although the field of mathematics teacher education has begun to address this challenge, many efforts have focused on particular aspects of PSTs’ development, rather than attending to the integrated understandings needed to effectively teach mathematics to diverse groups of children. For example, research has examined how PSTs learn about children’s mathematical thinking (Jacobs, Lamb, & Philipp, 2010; Jenkins, 2010; Vacc & Bright, 1999) and to a lesser extent, PSTs’ perspectives on children’s home and community-based knowledge and experiences relevant to mathematics (Lipka et al., 2005). Yet as a field, we have tended to isolate these two constructs. This separation is evident in research, as demonstrated by the studies noted above and in teacher education pedagogy (e.g., some class sessions focused on children’s thinking and others focused on supporting English learners [ELs]). As a result, we know little about how PSTs make connections to children’s mathematical funds of knowledge in their teaching, a practice shown to be effective for teaching diverse groups of children.

### Abstract

This study examines the ways prospective elementary teachers (PSTs) made connections to children’s mathematical thinking and children’s community funds of knowledge in mathematics lesson plans. We analyzed the work of 70 PSTs from across three university sites associated with an instructional module for elementary mathematics methods courses that asks PSTs to visit community settings and develop problem solving mathematics lessons that connect to mathematical practices in these settings (Community Mathematics Exploration Module). Using analytic induction, we identified three distinct levels of connections to children’s mathematical thinking and their community funds of knowledge evidenced in PSTs’ work (emergent, transitional, and meaningful). Findings describe how these connections reflected different points on a learning trajectory. This study has implications for understanding how PSTs begin to connect to children’s mathematical funds of knowledge in their teaching, a practice shown to be effective for teaching diverse groups of children.

### Keywords

prospective mathematics teacher education, equity, elementary mathematics education, children’s mathematical thinking, culturally relevant mathematics

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Making Connections in Practice: How Prospective Elementary Teachers Connect to Children’s Mathematical Thinking and Community Funds of Knowledge in Mathematics Instruction

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learn to integrate these various and important understandings in their practice.

In the TEACH MATH (Teachers Empowered to Advance CHange in Mathematics) project, our goal is to support preK-8 mathematics teachers to develop understandings and practices that connect to children’s multiple mathematical knowledge bases (MMKB), specifically children’s mathematical thinking and children’s community/cultural funds of knowledge (Aguirre et al., 2012; Turner et al., 2012). One way the TEACH MATH project supports PSTs in making connections to children’s MMKB is through the development and implementation of instructional modules for elementary mathematics methods courses.

In this study, we examine PST work from the Community Mathematics Exploration Module (CME), in which PSTs visit locations in the community surrounding their field-placement school and then draw on what they learn to design a problem solving–based mathematics lesson. Given our focus on understanding how PSTs learn to connect to children’s MMKB in their teaching, our analysis focused on the following research question.

*Research Question 1*: In what ways do PSTs make connections to children’s multiple mathematical knowledge bases in the lessons they design?

**Relevant Research**

Given the limited research on how PSTs learn to connect to children’s MMKB in their teaching, we first discuss studies that have addressed the knowledge bases separately.

**Learning to Attend to Children’s Mathematical Thinking**

Research has documented the effectiveness of instruction that centers on children’s mathematical thinking (CMT), which includes attention to such things as children’s solution strategies, common understandings and misconceptions, and number choices and problem structures that would support an appropriate level of cognitive demand (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Stein, Smith, Henningsen, & Silver, 2000). This research has linked teachers’ understanding of children’s mathematical thinking to productive changes in teachers’ knowledge and beliefs, classroom practices, and student learning (Fennema et al., 1996). One way that teachers evidence understanding of CMT is by selecting and implementing high cognitive demand tasks (i.e., requiring complex thinking and reasoning strategies such as conjecturing, justifying, and interpreting) and using questioning to elicit, connect to, and support students’ strategies and reasoning (Sleep & Boerst, 2012; Stein et al., 2000).

Specific to PSTs, experiences such as conducting or analyzing problem-solving interviews with children supports increased awareness of the diversity of student ideas and strategies, and an enhanced capacity to interpret children’s reasoning, including relating children’s strategies and misconceptions to frameworks outlined in research (Jenkins, 2010; Philipp et al., 2007). Other studies have argued that PSTs need sustained and scaffolded opportunities to learn about CMT (Sleep & Boerst, 2012), as shifts in beliefs and understandings about CMT are not always accompanied by parallel shifts in practice (Vacc & Bright, 1999).

One routine instructional practice that has the potential to evidence PSTs’ understandings of CMT is designing, adapting, and planning mathematics lessons. Parks (2008) documented that PSTs in a lesson study project attended to CMT as they planned, taught, and reflected on lessons, albeit inconsistently. Two of four groups of PSTs developed what Parks referred to as a mathematical lens, which included attention to how students might reason about key mathematical ideas in the lesson. Similarly, Vacc and Bright (1999) documented that even when PSTs evidence knowledge about CMT, they often struggle to use this knowledge in their planning and instruction. Collectively, these studies suggest that understanding PSTs’ learning about CMT requires attention to their emerging teaching practices.

**Learning to Attend to Children’s Linguistic, Cultural, and Community-Based Funds of Knowledge in Mathematics Instruction**

Historically underrepresented groups benefit from instruction that draws upon their cultural, linguistic, and community-based knowledge (Ladson-Billings, 1994; Lipka et al., 2005). This research has argued that teachers need to understand how children’s cultural funds of knowledge (CFOK)—the knowledge, skills, and experiences found in students’ homes and communities—can support their mathematical learning (Civil, 2007; Foote, 2009; González, Moll, & Amanti, 2005). Furthermore, PSTs need opportunities to develop instructional practices for eliciting and incorporating students’ funds of knowledge in their instruction. As noted by Grossman, McDonald, Hammerness, and Ronfeldt (2008),

> It is not enough to prepare teachers with . . . knowledge of their students’ cultural and linguistic resources. Teachers need to know how to use such knowledge in order to help students develop intellectual skills and to succeed academically. (p. 244)

Research that examines how PSTs learn to connect to CFOK in their mathematics teaching is limited. In a study of two novice teachers using a culture-based mathematics curriculum developed for Alaskan Yup’ik communities, Lipka and colleagues (2005) found that teachers implemented the lessons in ways that connected to students’ out-of-school experiences and maintained a focus on rigorous mathematics. However, these teachers had the benefit of materials designed...
by local mathematics educators and community elders that included connections between familiar cultural activities (i.e., designing a fish drying rack) and conceptually demanding mathematics (e.g., relationships between perimeter and area). In the absence of such curriculum materials, connecting to CFoK in school mathematics lessons is challenging (e.g., Civil, 2007; Gonzalez, Andrade, Civil, & Moll, 2001).

Other studies have noted that teachers do draw on children’s home and community experiences as they are planning mathematics lessons, but the connections they make are often superficial ones, changing names in problems, and adjusting contexts to reflect students’ interests (Nicol & Crespo, 2006). That said, PSTs seem interested in lessons that connect to CFoK but often express uncertainty about the sustainability of this kind of teaching and whether such lessons will help students learn the mathematics they need to learn (Andrews, Yee, Greenhough, Hughes, & Winter, 2005; Leonard, Brooks, Barnes-Johnson, & Berry, 2010).

In summary, although separate bodies of research have documented teachers’ understandings and practices related to CMT and CFoK, little is known about how teachers learn to connect to children’s multiple mathematical knowledge bases (CMT and CFoK) in mathematics instruction. In the next section, we describe how PSTs might develop these practices via a conjectured teacher learning trajectory.

**Theoretical Perspectives**

**Teacher Learning**

We understand teacher learning as a sociocultural activity wherein PSTs develop understandings and practices as they interact over time in multiple communities, such as mathematics methods courses, elementary classrooms, and children’s homes and communities (Lerman, 2001; Wenger, 1998). A sociocultural lens on teacher learning draws our attention to the nature of teachers’ participation in activities such as planning problem solving–based mathematics lessons, including how they draw upon artifacts, tools, and others to make sense of their experiences (Putnam & Borko, 2000). As PSTs interact with participants across multiple spaces, we consider how they develop understandings and practices that position them as effective teachers of mathematics for diverse students.

**Learning to Connect to Children’s MMKB: A Trajectory**

In prior work (Turner et al., 2012), we drew on existing theory and research (Jacobs et al., 2010; Mason, 2008) as well as our own empirical analyses to conceptualize and represent, via a conjectured teacher learning trajectory, how PSTs develop understandings and practices for connecting to children’s MMKB in instruction. We understand a learning trajectory to include goals for participants’ learning, as well as conjectures about progressions in relevant understandings and practices (Stevens, Shin, & Krajcik, 2009). An important aspect of learning trajectories is the connection between learning and tasks (Clements & Sarama, 2004). We assume that PSTs are at different points along the trajectory as they engage in mathematics methods course activities. Thus, in contrast to Clements and Sarama (2004), we designed tasks (such as the Community Mathematics Exploration) not to correspond to particular points on the trajectory, but to allow for multiple points of access and to support collaboration among participants. We see a teacher learning trajectory as describing development both within and across different levels of understanding and practices. We understand this development to be dynamic, and assume that PSTs’ understandings and practices will reflect different points on the trajectory at different times. In this way, we use the learning trajectory as a tool to conceptualize general progressions in PSTs’ understandings and practices, rather than as a linear model of individual PST learning.

The first phase of our conjectured learning trajectory (Initial Practices, see Figure 1) includes three interrelated practices that we found to be foundational to integrating children’s MMKB: attention, awareness, and eliciting. Drawing on Mason (2008), attention refers to what teachers attend to, including what they notice, whereas awareness refers to how teachers interpret, or assign meaning, to their observations. Eliciting includes questioning and other strategies for interacting with children and families to elicit children’s multiple mathematical knowledge bases (CMT and CFoK). This study focused on the second phase of the trajectory, Making Connections, wherein PSTs begin to make connections to children’s MMKB in their teaching. A final phase of the trajectory includes ongoing and purposeful Incorporation of MMKB in instruction.

Previously, we conjectured two levels of connections within the Making Connections phase of the trajectory. We used emergent connections to describe superficial attempts to connect to CFoK and/or CMT, such as changing names or contexts of word problems to be more relevant to children. We hypothesized that PSTs begin to make meaningful connections as they deepen their understanding of children’s MMKB, and learn to draw upon what they know about children’s mathematical reasoning and children’s out-of-school mathematical practices to design rich problem-solving experiences. Our prior study broadly described PSTs’ attempts at Making Connections and was not focused on variations and patterns; we stopped short of outlining a possible progression from emergent to meaningful connections. In the present analysis, we focused on understanding the patterns and progression of connections that PSTs made as they designed mathematics lessons grounded in community contexts. This is important because it provides insights into different entry points into the practice of making connections to children’s MMKB, which may help mathematics teacher educators to support PST learning.
Method

Research Setting

Data collected for this analysis occurred at three TEACH MATH project sites reflecting a diversity of both geographic contexts (e.g., suburban, borderland, and a mix of urban and suburban) and program contexts (e.g., the nature of field placements). Although these differences are important with respect to PST development, this article reports on a cross-site analysis and thus does not disaggregate data by site.

Participants

Participants included 113 PSTs whom, overall, reflected the PST population nationwide—predominantly White, middle-class females in their early 20s (Hollins & Guzman, 2005). However, one site enrolled PSTs who were more racially and ethnically diverse than national trends might predict.

Community Mathematics Exploration

We analyzed PST work from the Community Mathematics Exploration Module (CME). The goal of this module was to engage PSTs with the local community, and to increase their knowledge of students’ out-of-school activities and practices, particularly those that might relate to mathematics instruction. Some participants conducted multiple visits to the same community locations, and others conducted their visits with a parent or community member as a guide. Next, PSTs designed a problem solving–based mathematics lesson that built upon what they learned about students and their communities, and then reflected on their experiences, describing the benefits and challenges of mathematics teaching that connects to community contexts. (See appendix and math-connect.hs.iastate.edu for a description of the CME assignment.)

The majority of the CME projects (n = 51) reflected the work of pairs or small groups of PSTs. Connected to our theoretical framework that asserts teacher learning is a socio-cultural activity that we constructed to maximize PST interaction. However, due to logistical issues with field placements, some PSTs (n = 19) elected to complete the CME project individually. As our goal was not to make claims about individual PSTs’ understanding, or to place individual PSTs on the learning trajectory, but rather to use overall patterns and variations in PSTs’ work to outline a possible progression within the Making Connections phase of the trajectory, we included both single and coauthored projects in our analysis.

Data Collection and Analysis

We analyzed 70 CME projects (reflecting the work of 113 PSTs), including lesson plans, reflection papers, and/or presentations. Data analysis consisted of analytic induction (Bogdan & Biklen, 1992) based on the theoretical framework informing our conjectured learning trajectory (Turner et al., 2012). Each round of coding analysis began by marking evidence of connections to each knowledge base, CMT and CFoK. Drawing on key ideas from the literature, evidence of attention to CMT included descriptions of (a) students’ prior knowledge, (b) possible solution strategies, (c) possible student confusions or misconceptions, and (d) tasks requiring a high level of cognitive demand. Evidence of attention to CFoK included (a) connections to community mathematical practices (e.g., mathematical practices that

![Diagram](image-url)}
occur in the setting) and (b) authentic tasks (e.g., tasks connected to practices students or adults might use in the setting).

Next, we drew on these codes to categorize each CME project as either Emergent or Meaningful. Emergent projects were those that reflected little attention to CFoK, or CMT, or both. For example, emergent lessons often included typical school mathematics tasks that connected superficially to the community context (little/no attention to CFoK), and/or the lesson lacked specific and consistent attention to children’s prior knowledge, possible solution strategies and misunderstandings (little/no attention to CMT). Some emergent lessons connected to one dimension of children’s mathematical knowledge but not the other. Other emergent lessons lacked attention to both CMT and CFoK. However, meaningful projects connected to both CMT and CFoK throughout the lesson plan and across project components. Lesson tasks reflected authentic mathematical practices grounded in a community setting, required a high level of cognitive demand, and included opportunities to elicit and build on CMT.

In the initial round of coding, researchers individually coded 12 CME projects selected from across sites. Disagreements were discussed and codes were refined based on these discussions (e.g., additional CMT and CFoK coding categories emerged). In the second round of coding, each of the remaining 58 CME projects was coded and categorized by several researchers. In cases of disagreement, discussion ensued and when needed, code descriptions were elaborated.

This second round of coding resulted in the creation of a third category of connections to children’s MMKB that we refer to as Transitional. This category became necessary when we encountered examples that did not align with our descriptions and examples of emergent and meaningful projects. Transitional connections were defined as those where PSTs made an explicit attempt to connect to both CMT and CFoK, but the connections to either or both constructs were underdeveloped. Transitional projects included those where (a) attention to CMT was present but lacked specificity (i.e., general discussion about mathematical confusions and strategies not tied to specific math content) or tasks posed were mixed level of cognitive demand and (b) attention to CFoK was present, but inconsistent (e.g., lesson connected to family practices more generally but not mathematical practices that families might use in a specific setting).

With the addition of this category, in a third round of analysis at least two researchers recoded each of the 70 CME projects, first with respect to the project’s connections to CMT and CFoK, and then with respect to the overall project categorization (emergent, meaningful, or transitional). To test for reliability, subsets of previously coded projects that were similar in nature and thus might suggest refinements to our coding scheme (e.g., emergent and transitional projects that both attended to CMT) were recoded by all researchers until agreement was achieved.

Findings

Summary Across Categories

Our analysis of CME projects resulted in three types of connections made to CMT and CFoK—emergent, transitional, and meaningful—that supported, and expanded, the Making Connections progression outlined in our conjectured learning trajectory (Turner et al., 2012). Specifically, 53% (n = 37) of CME projects evidenced emergent connections, 30% (n = 21) evidenced transitional connections, and 17% (n = 12) demonstrated meaningful connections (see Table 1). In the sections that follow, we use representative examples to demonstrate variation and patterns within each category. We begin with the two endpoints (meaningful connections and emergent connections) and continue with the intermediate category of transitional connections. We conclude with a discussion of patterns across categories.

Meaningful Connections

Although fewer in number (17%, n = 12), CME projects that evidenced meaningful connections reflected consistent attention to CMT and CFoK throughout the lesson plan and across project components. The following sections discuss two meaningful CME projects in detail.
Shopping at Las Socias/Abuela’s Shopping List, third grade. A group of PSTs visited “Las Socias,” a neighborhood food market frequented by Latino families and interviewed the co-owner about the mathematics she used in her work. In addition, PSTs drew on their knowledge of children’s experiences, specifically that children in their third grade class shopped at Las Socias with their families and sometimes talked with family members about shopping lists, the cost of different items, and the total amount of money needed for a grocery purchase. PSTs planned a lesson that asked students to calculate the cost of items on “Abuela’s (grandma’s) shopping list” and determine whether a given amount of money (US$40 or US$70) would be “enough.” This was an authentic task, based on PSTs’ consideration of children’s and families’ experiences shopping for groceries on a budget. In addition, the shopping list and accompanying photos and prices reflected items that could be actually purchased at Las Socias. The lesson connected to mathematical practices that children and families may engage in at Las Socias, including adding and multiplying prices, estimating total costs, and subtracting from available funds. Moreover, the lesson was situated in the context of shopping for a grandmother visiting from Mexico (a reality for many students), thus incorporating children’s connections to family. The PSTs noted,

[The] background story, parts of list/instructions, and grocery items are intended to link to the Hispanic population of the classroom. A local market (Las Socias), one block from school, is mentioned and items purchased there. Also, shopping for “grandma” is a personal link for most children.

Consistent attention to CFoK was also evidenced in how the project positioned Latino/a families and the use of Spanish as a resource. For example, the launch of the lesson involved a scenario role-play by students. Two Spanish-speaking children were selected to play an “Abuela” (grandma) and her grandchild having a conversation about a grocery list in Spanish. Although the language of instruction in most of PSTs’ field applications classrooms was English, the lesson launch and translation of mathematical tasks positioned Spanish as a resource.

Not only did the PSTs incorporate CFoK in meaningful ways throughout the lesson, the Las Socias lesson also evidenced attention to CMT, via an emphasis on encouraging multiple strategies, and supporting and extending all students’ reasoning. For example, the main mathematical task—determine if you have enough money to purchase the groceries for grandma’s list—required multiple solutions (e.g., “Explain your reasoning in at least two ways.”). In addition, the lesson included focus questions that anticipated possible student strategies and ways of extending students’ reasoning (i.e., “Can you use multiplication to explain this in another way?”). The PSTs also expressed a strong intent to provide multiple entry points for students with different mathematics backgrounds and minimize status issues in the lesson. For example, they attended carefully to number choices to ensure access for all students. They explained,

We liked the idea of creating two shopping lists with different items to be bought. This enabled one list with “easy” price points of round numbers to multiply, and another with more “challenging” price points and letting students choose their list/level of difficulty. We kept the items on the list distinctly different so it would not appear as if one were an “easy” list for “low” kids, as two lists with [the] same goods but different quantities surely would have.

The PSTs made deliberate number choices in an effort to open up access to high cognitive demand tasks for students with a wide range of prior mathematical experiences (Stein et al., 2000). In summary, the Las Socias project reflected in multiple ways PSTs making meaningful connections to children’s MMKB in their planned mathematics instruction.

Church Carnival, second grade. In another example, PSTs designed a problem solving–based lesson around a local church carnival that many students planned to attend. While planning, PSTs elicited students’ prior experiences with this particular carnival (and carnivals in general). They discovered which rides students preferred and that students were often unable to ride all of the rides because they ran out of tickets. PSTs then drew on this knowledge to design a series of open-ended, high cognitive demand tasks about different ways to spend a given amount of tickets at the carnival. For example, “You must figure out two different ways you would want to spend all of your 25 tickets at the Maple Lane Carnival. What rides do you go on? How many times did you go on each ride? What food did you eat? Did you eat more than once?” Students were provided with the number of tickets needed for each ride or food item.

The PSTs anticipated strategies that students in their classrooms might use to solve this task, such as guess-and-check methods, repeated addition (when students wanted to ride the same ride multiple times), incremental subtraction, and partitioning methods (dividing the set of 25 tickets into small sets and then deciding how to spend each group of tickets, that is, some on food, some on rides). They were also aware of potential struggles students might have, such as students who are not able to find a combination that exhausts the entire set of 25 tickets. In addition, they planned to provide students with concrete materials (i.e., sample tickets, paper for drawing) because they recognized that some students would need to physically model possible combinations. This detailed attention to children’s ways of reasoning about the content was evident throughout the lesson plan.
The project also evidenced consistent attention to CFoK. For example, the launch of the lesson invited students to share carnival experiences, including how they made decisions about rides and food, and whether they ever ran out of tickets. In addition, the first time that the “figure out two ways to spend 25 tickets” task was posed, students “enacted” spending tickets by visiting different carnival ride stands placed throughout the classroom and leaving the required number of tickets at each stand. This enactment, which PSTs anticipated would elicit students’ prior experiences of spending and potentially “running out of tickets” before riding all the favorite rides, was followed by an opportunity to plan out different ways to spend the 25 tickets. In this way, the lesson not only leveraged students’ prior experiences but also engaged them in mathematical activity that had the potential to enrich future, similar experiences related to this annual community event.

In summary, as in many of the meaningful CME projects, in Las Socias and the Church Carnival, PSTs made consistent connections both to CMT and CFoK. They did so by attending carefully to such things as number choice and children’s strategies, through linguistic and cultural connections, and noticing authentic ways children and families might use mathematics in their out-of-school activity. It is important to note that while CME projects evidencing meaningful connections made up the smallest proportion of the data set, their presence confirms that PSTs can learn to connect to children’s MMKB as they develop their instructional practice.

Emergent Connections

In contrast, 53% (n = 37) of all CME projects evidenced emergent connections. The majority of emergent connections projects (n = 16, 43% of projects in the emergent category) reflected limited attention to both CMT and CFoK. More specifically, these projects included limited or no attention to possible student strategies, the mathematical tasks were typically low in cognitive demand (e.g., emphasizing procedures without understanding) and tended to resemble traditional school-based tasks that vaguely connected to the community context. For example, one second-grade CME project used the monthly tribal pow-wow (familiar to many of their students) as the context for a typical school mathematics task—determine the best way to purchase 25 pounds of salmon and other meal items such as salad and cake for the monthly tribal school pow-wow. The lesson did not elicit student knowledge and experiences related to the pow-wow (e.g., how they or their families use math in the pow-wow setting), and no attention was paid to possible student strategies for comparing prices and “calculating savings,” or to potential challenges that the numbers included in the task might pose for second-grade students (e.g., 15.99 per ⅛ sheet of cake).

However, other emergent connections CME projects evidenced explicit attention to either CMT or CFoK (but not both). For example, we identified projects where PSTs paid attention to CMT while superficially drawing on CFoK (n = 13, 35% of emergent category). Yet even these rather superficial connections reflected an attempt by PSTs to learn about students’ out-of-school lives and to connect to students’ experiences in their instruction. In other emergent CME projects, PSTs drew upon children’s out-of-school experiences but evidenced little attention to how children might reason about the mathematics and/or include appropriately demanding tasks (n = 8, 22% of emergent category). The following two examples illustrate these emergent entry points into the practice of making connections.

Military Time, fourth grade. In this lesson, PSTs visited a military base adjacent to the school, and observed how time was measured and represented on base versus in civilian settings. PSTs then interviewed their students, many of whom had parents serving in the military, to gather information about their current understandings and uses of military time in their homes and family interactions. To the PSTs’ surprise, students they interviewed had general knowledge and familiarity with military time, but often could not articulate how to convert back and forth between military and civilian time (e.g., adding 12 or 1,200 to 1:00 p.m. to get 1,300 [military time]). PSTs also interviewed school personnel and learned that no attention had been given to strengthening students’ understanding of military time. From the PSTs’ perspective, this skill needed to be addressed to affirm mathematical resources embedded in the military community and help students better understand the need for different systems of measuring and representing time. In these ways, the lessons’ connections to community-based knowledge and experiences were quite rich.

However, the PSTs highly scaffolded the mathematical tasks and directed students to use specific step-by-step procedures to convert between military time and civilian time. For example, the main mathematical task was described as follows:

Model for students how to convert standard time to military time and vice versa. This is done by adding 12 hours from standard time or subtracting 12 hours from military time. Lead students through multiple examples of both types of conversions. When students are feeling comfortable, ask for volunteers to do some conversions in front of the class.

The lesson’s emphasis on reproducing teacher-modeled step-by-step procedures generated a low cognitive demand task. In addition, no attention was paid to alternate strategies to convert between military and civilian time or to ways that children might reason about or explain the procedure that was modeled. Furthermore, possible confusions related to
conversions were not considered (e.g., following the procedure above, students might subtract 12 from 1,300 and get 1,288.) In summary, although the Military Time lesson evidenced connections to CFoK, in that it reflected an important mathematical practice within the community, the lesson lacked attention to CMT. The fact that the lesson only connected to CFoK, versus to both CMT and CFoK indicated emergent connections.

**Family Dollar Store, first grade.** In a contrasting, but still emergent CME example, PSTs designed a problem solving–based lesson about purchasing toys at Family Dollar. PSTs noticed the store was close to the school and knew that some students had mentioned going to this store with their parents. The PSTs used this knowledge to design a series of tasks about purchasing toys at Family Dollar:

You have saved money to buy toys at Family Dollar. You can only spend 12 dollars. The toys the store has for you to choose from are the following:

- A coloring book for US$1
- A water squitter for US$2
- A Phineas and Ferb doll for US$3
- A Mickey Mouse doll for US$4
- Toy trucks for US$5
- A checkers game for US$6
- A Rapunzel doll for US$8

What are the different ways you can spend all of your US$12 on these toys?

PSTs felt this task would interest students because the problem mentioned toys and because it built on “knowledge they have about money.” Although the task reflected a familiar setting and an imaginable scenario, the lesson did not attend to how children and families might use mathematics at the Family Dollar Store (i.e., PSTs did not describe asking students about their experiences shopping at the store, making purchasing decisions, estimating total costs, etc.) or to the potential math practices of store employees. Instead, PSTs based the task on their own trip to the store “looking for the best items [from their perspective] that would engage all students in the class.” The result was a rather contrived, textbook-like task that while connected to students’ interests and based in a familiar setting, did not draw on students’ or families’ experiences using mathematics in out-of-school contexts.

At the same time, the Family Dollar lesson evidenced attention to CMT. Given that the lesson was for first graders, who generally are developing their understanding that a number can be made up of a variety of two-addend (or in this case, multiple addend) combinations, the task was a high cognitive demand task. In addition, the lesson allowed for multiple solution strategies and asked students to explain and justify their strategies, including identifying the meaning of any numbers used or referenced. PSTs anticipated a range of different strategies that students might use to solve this task, including using either trial and error or known facts to find two numbers that add up to 12 dollars or using subtraction to work from 12 dollars down to 0. The PSTs also considered how students’ knowledge of double facts (i.e., $6 + 6 = 12$) and multi-addend number sentences might relate in that students might look for two items that each equal 6 dollars. In addition, PSTs considered prior mathematical knowledge relevant to this task, such as an understanding of addition and subtraction in the context of part-part-whole problem structures. Although in the Family Dollar lesson PSTs evidenced attention to CMT, they made superficial connections to CFoK that did not tap into specific ways children, families, or other community members might engage mathematics outside of school.

In summary, the emergent CME projects evidenced various entry points to the practice of making connections to children’s MMKB in instruction. These variations suggest that while some PSTs struggle to attend to both CMT and CFoK in their lessons, others take up the practice of connecting to children’s knowledge bases by emphasizing one over the other. Knowing these potential orientations and entry points provides insights on supporting PSTs movement along the trajectory, a point we return to in the discussion.

**Transitional Connections**

Another outcome of this analysis was the construction of a new category of Transitional connections. In all, 30% (n = 21) of the CME projects reflected notable attempts to connect to both CMT and CFoK in more than superficial ways. In terms of CFoK, transitional lessons clearly connected to contexts and experiences observed in the community and reflected general attempts to elicit students’ knowledge and experiences related to these contexts. In terms of CMT, transitional projects included at least general, but often not specific, discussions about possible mathematical confusions and student solutions strategies and/or the task required high cognitive demand for some components but not others. In this way, these projects made moderate attempts to connect to both CMT and CFoK, but opportunities were missed in the lessons to make deeper connections. The following two CME projects represent transitional connections that reflect movement along the trajectory away from superficial connections and toward more meaningful integration of children’s MMKB.

**Panadería (Mexican Bakery), second/third grade.** One group of PSTs working in neighborhood schools with predominantly Mexican American/Mexican immigrant student populations visited a local Mexican Bakery known throughout the city for its selection of sweet breads (pan dulce), fresh tortillas, cakes, and other desserts. PSTs learned through student interviews that children in their field-placement classrooms were familiar with the bakery and that many families...
shopped there for traditional Mexican treats. PSTs felt that designing a mathematics lesson around the bakery would help them “to relate math in class to something the students are familiar with outside of class, something important and valuable in their own community.” In addition, the PSTs noted that the lesson would help them, as teachers, to become “familiar with students’ community, and a part of their culture.” By talking with bakery employees and the owner, PSTs discovered various ways that mathematics was used including, “[how] the cost of rent, utilities, and wages is factored into the cost of each [bakery] item” and that items were categorized (i.e., filling or no filling, sweet or savory, seasonal or year round) and arranged in display cases according to this categorization. The PSTs decided to design mathematics tasks around this categorization practice, so that students could recognize how bakery employees used mathematics in their work, which would “create positive math role models [for students] in their own community.” For example, the main task of the lesson asked students to use pictures of bakery items and a blank Venn diagram (representing display cases in the bakery) to “categorize bakery items in a way that they [the students] think would be most effective.” PSTs noted that students would need to “evaluate what characteristics of a bakery item are most important to take into consideration when categorizing them . . . and then categorize the items correctly according to the limitations they set themselves.”

While the Panadería project connected in general to a mathematical practice used by bakery employees (categorization), the specific way this practice was represented in the lesson (i.e., using a Venn diagram to categorize sweet breads and identify areas of overlap) did not reflect how categorization was actually used in the bakery. Bakery employees did sort items into distinct categories (i.e., sweet and savory, traditional, and seasonal), but were not concerned by the potential overlap between categories (e.g., a seasonal item might also be savory but was still placed on the seasonal tray). In this way, while the project reflected an explicit attempt to honor and connect to community-based mathematical funds of knowledge, the authenticity of the mathematical practice used by bakery employees was diminished when it was translated into a school mathematics task on Venn diagram categorization. We see this missed opportunity to develop a more authentic connection to how community members use mathematics in the bakery as evidence of transitional connections.

The Panadería project also evidenced attention to CMT, albeit inconsistent. The task had the potential to support an appropriate level of cognitive demand (i.e., students generate categorization schemes themselves, the Venn diagram representation pushes students not just to sort items into distinct categories, but to identify similarities and differences among categories). However, the PSTs did not anticipate specific strategies or solutions that students might generate or potential confusions that might arise. For example, the PSTs did not anticipate that students might sort items by a single attribute, such as color, and then struggle to identify overlapping features. In addition, although the lesson plan listed possible questions to probe, support, and extend student thinking, these questions were general (e.g., “Can you explain what you have done so far?”) rather than specific to this particular task (i.e., “Explain what the overlap between the two circles represents.”). In summary, the Panadería project evidenced general connections to ways that mathematics was used in a familiar community business, but not to the ways that children and families might use mathematics in the setting. In addition, the lesson reflected moderate, but not consistent attention to CMT, as evidenced by the high cognitive demand task coupled with the lack of attention to specific strategies and confusions. Taken together, this moderate but not consistent attention to both CMT and CFoK, evidenced transitional connections.

**Shopping for Dinner, fourth grade.** In this project, a group of PSTs surveyed students in their field-experience classrooms about common out-of-school activities and found that many students accompanied family members to one of the neighborhood grocery stores on a regular basis. Based on this information, PSTs decided to design a mathematics lesson around shopping for items to make a favorite family dinner. Rather than basing the lesson on a standard, predetermined “grocery list,” PSTs decided that students should choose a recipe from home with the help of a family member. The recipe may be a favorite recipe, one of cultural significance, or otherwise one that a student may want to share with the class. This decision to build the lesson around shopping for authentic family recipes was a purposeful one, aimed at incorporating “each student’s family values and familiar ways of doing/thinking.” Once students selected a recipe, they were asked to “find out the total ingredients needed for the number of people in your family” and then “find prices for the ingredients needed [at] two different grocery stores, consider[ing] sales offers in weekly store ads.”

While the *Shopping for Dinner* lesson connected to children’s experiences shopping, planning meals, and cooking with their families, the lesson emphasized connections to general family practices (i.e., select a recipe with your family), rather than connections to explicit mathematical practices (i.e., measuring ingredients, scaling recipes up or down, estimating quantities, estimating total costs of items while in grocery store, etc.). In other words, the lesson attended to families’ practices but not to specific ways that children or families might engage in mathematics while shopping or preparing food, which we took as a missed opportunity.

The *Shopping for Dinner* project also evidenced moderate attention to CMT. For example, PSTs designed a task requiring high levels of cognitive demand for some components but not others. The initial task was not very challenging for fourth grade, requiring students to
perform a simple cost comparison between a variety of items found at the local Safeway store and another local store, Sam’s Market. This is done using the Safeway coupons and teacher provided prices for items at the market. Students add up the cost of their items for each location and decide which provides what they need for the lowest overall cost.

Moreover, the planned lesson gave students an example “using two lists of the same ingredients but with different prices to explicitly show the format that you want the students to use when computing the prices,” at best engaging students in computational procedures. However, the next sequence of tasks asked students to factor in travel expenses.

Given the cost for a bus fare (both child and adult) and/or the cost of a gallon of gas, they are asked to figure out if the saving would still be worth the trip if they had to also factor the cost of transportation to the alternative store location.

This latter activity allowed for multiple entry points (i.e., students may begin working on the task using a variety of approaches and techniques) and required students to connect their mathematical calculations to real-world experiences. The lesson also encouraged justifications of mathematical reasoning (e.g., “How do you factor [in] travel time and cost [when you need to] pick up a single, particular item?”).

In summary, we found that some PSTs entered the practice of making connections to children’s MMKB in transitional ways, making notable connections to both CMT and CFoK, albeit with missed opportunities. This transitional category is important because it marks ways that PSTs begin to connect to children’s MMKB that are clearly beyond emergent, superficial connections and that reflect a movement toward more meaningful connections. Understanding different ways that PSTs move beyond superficial connections is important because it suggests specific ways of focusing PSTs’ attention and provides insights about particular perspectives or lenses they might find useful as they advance toward making meaningful connections. We elaborate this point in the discussion.

Discussion
Prevailing views of mathematics teacher preparation emphasize the importance of PSTs attending to CMT (e.g., Jacobs et al., 2010). Similarly, culturally responsive pedagogies emphasize the importance of tapping into CFoK to support student learning (Aguirre, 2009; Leonard et al., 2010). Our project emphasizes the importance of both constructs to help PSTs develop robust teaching practices. Specifically, the goal of the CME module was to help PSTs learn to make connections to CMT and CFoK in the context of a problem solving–based mathematics lesson. Our approach in the module aligns with Grossman and colleagues’ (2009) framework for the teaching of practice in professional education programs. The CME module engages PSTs in “approximations of practice” (Grossman et al., 2009) that are proximal to classroom teaching practices, such as interactions with children and families and the design of problem solving–based lessons, but with the support of carefully designed instructional scaffolds.

Patterns and Variation
Our findings demonstrate that PSTs entered the practice of making connections to children’s MMKB in various ways and with varying specificity. For example, over half of the emergent lessons evidenced clear attention to CMT or CFoK (but not both), suggesting that PSTs may have different entry points into this practice. Some PSTs attempted to connect to CFoK but in ways that simply used familiar community contexts as the motivation for traditional school mathematics tasks (i.e., the Family Dollar Store example). Perhaps these PSTs follow a trajectory in which they begin with less demanding components of the practice, such as incorporating community contexts in problems, and advance to identifying mathematical practices in the setting and considering ways to connect to those practices in their instruction. Other PSTs’ projects connected to children and family practices in the community and yet included problem-solving tasks that required little mathematical reasoning (i.e., Military Time), suggesting another possible path. These PSTs may begin by noticing how children and families use mathematics outside of school, and advance to designing high cognitive demand tasks that connect to these practices and to anticipating students’ reasoning about such tasks. An implication of this work is to consider, given a specific entry point, how to help PSTs progress and strengthen the practice of making connections to both CMT and CFoK. In addition, the existence of different entry points suggests maintaining an integrated focus on children’s MMKB, as this may support PSTs’ multiple entry points into the learning trajectory.

Furthermore, we found an almost even distribution of emergent (53%) and nonemergent (i.e., transitional and meaningful connections; 47%) projects. This is notable given that we expect PSTs entering our mathematics methods courses to be building the foundations of attending to CMT and CFoK, and that prior research with PSTs and even practicing teachers has repeatedly shown that drawing on CMT or CFoK in one’s teaching is a challenging practice that develops over time (Civil, 2007; Jacobs et al., 2010; Sleep & Boerst, 2012). The fact that almost half of our projects reflected connections that went beyond superficial attention to both CMT and CFoK suggests that with scaffolded opportunities to learn about children’s MMKB, such as those provided in the CME module, PSTs can make substantive strides in developing an integrated practice that connects to both CMT and CFoK. In fact, although small in number, the
meaningful connections projects provide a powerful response to research that has tended to highlight the challenges and difficulties that PSTs face as they learn to teach mathematics effectively to diverse groups of children (Villegas & Lucas, 2002).

Specific to the transitional category, we see PSTs entering the trajectory with clear attempts to connect to both CMT and CFoK, yet these connections remain underdeveloped or lacking in specificity. This distinction of the transitional category is important because it reflects a key juncture in the learning trajectory where PSTs begin to make connections to both CMT and CFoK in their instruction. Projects in the transitional category also highlight the varied ways that PSTs begin to connect to children’s MMKB, further emphasizing the need for learning experiences that support multiple pathways through these practices. Moreover, the transitional category points to the complexity of this teaching practice. Similar to other aspects of teaching mathematics (Ball, Sleep, Boerst, & Bass, 2009), learning to connect to children’s MMKB is complex. PSTs need repeated and scaffolded opportunities to try on these practices, to wrestle with ideas and make them their own. The transitional CME projects exemplify PSTs engaged in learning this complex teaching practice.

Finally, we found that similar contexts supported all three categories of connections, which suggests that choosing the “right site” is not critical to a PST’s ability to make meaningful connections to CMT and CFoK. For example, in the context of planning a party, which required trips to local grocery stores or restaurants, we had projects that reflected emergent, transitional, and meaningful connections. In addition, sites that we initially thought might be less productive (e.g., chain restaurants, grocery stores, movie theater) because of their ubiquitous presence and strong emphasis on money transactions ended up supporting meaningful and transitional lessons. The reverse was also true—some contexts that we thought would likely lead to meaningful connections because they were at the core of the social and economic life of a specific community did not yield mathematics lessons that deepened students’ mathematical understanding. More important than choosing a particular site was attending to the ways that children, families, and community members engaged mathematics at that site, and considering mathematical tasks that drew upon these authentic practices.

**Implications for Supporting PST Learning**

An important implication of this work is to identify specific leverage points that help PSTs develop the practice of connecting to children’s MMKB. For example, a prominent pattern that emerged from the meaningful and transitional CME projects was that the lessons emphasized connections to children’s and families’ experiences and mathematical practices in the community setting (vs. connections to the mathematical practices of employees or shop owners, for example, see the Las Socias and Church Carnival projects). These lessons generated opportunities for children to leverage their community experiences as resources for engaging and making sense of the task. This pattern suggests that PSTs need increased opportunities to learn about CFoK and to explicitly identify children’s out-of-school experiences as resources to support mathematics learning. The fact that mathematical activity in home and community settings is often implicit and harder to recognize (Civil, 2007) suggests that PSTs need support as they explore children’s and families’ mathematical practices, including opportunities to develop skills for both observing and interacting with participants. Moreover, PSTs need support in recognizing the tensions inherent in translating authentic, everyday mathematical practices into school mathematics lessons (Civil, 2002; Foote, 2009). This translation, particularly when it involves representing authentic practices in the form of a school mathematics-like word problem, often involves reducing the complexity (and therein the authenticity) of the practice. As mathematics teacher educators, we need to both increase PSTs’ opportunities to interact with children and families about their out-of-school mathematics practices and also help PSTs consider ways to respond to these inherent tensions.

Alternatively, lessons that attended to CFoK but evidenced limited attention to CMT suggest other strategies for helping PSTs learn to connect to children’s MMKB. For example, teacher educators might draw on meaningful projects that reflect connections to CMT to focus PSTs’ attention on things such as anticipating children’s strategies and possible confusions (e.g., Church Carnival example), and selecting numbers that create multiple entry points and reflect ways to minimize status in the classroom (e.g., Las Socias example). This would enable PSTs to analyze and decompose the practice in ways that maintain the integrated and complex nature of making connections to children’s MMKB (Grossman et al., 2009).

An important contribution of this study is that we focus on how PSTs develop robust instructional practices that involve connections to children’s multiple mathematical knowledge bases. In this way, we extend current research on high-leverage mathematics teaching practices (Ball et al., 2009) to include an explicit focus on connecting to both CMT and CFoK that together contribute to effective mathematics instruction for culturally and linguistically diverse students. Furthermore, we describe learning experiences, like the CME, that have the potential to support PSTs both in approximating complex instructional practices as well as decomposing those practices in ways that emphasize important teacher moves (Grossman et al., 2009). Our goal is to help mathematics teacher educators and PSTs to explore the practice of making connections to children’s MMKB in ways that do not restore the historical separation of CMT and CFoK in the field, but retain the complexity.
By illustrating the various ways PSTs begin to connect to both CMT and CFoK in their teaching, we contribute important understandings related to how PSTs attend to children’s MMKB and use what they learn to build problem-solving–based mathematics lessons. Our findings suggest that the CME is one activity that supported PSTs in making these connections. We would argue that the field needs more research on other learning experiences that emphasize this practice to better understand how to prepare effective mathematics teachers. Furthermore, our findings indicate specific leverage points to support PSTs learning. Future studies might explore how prospective and early career teachers develop the practice of making connections to children’s MMKB over time, and how these practices support and strengthen the mathematics learning of their students. Starting from wherever PSTs are in the practice of making connections and supporting them to make forward progress toward making meaningful connections is our call.

Appendix

Community Mathematics

Exploration Assignment

COMMUNITY MATHEMATICS EXPLORATION

PURPOSE:
To deepen your knowledge about math teaching, your students’ out-of-school experiences, and the local community(ies) that your school serves by closely examining and documenting mathematical resources within the local community and creating a math lesson/problem-solving task that deepens student mathematical understanding(s) and explicitly connects to family/community experiences, practices, and knowledge.

PREP-WORK:
Talk to students and adults (e.g., family members, community members, etc.) to get a sense of local social “hubs”/meeting places, events, organizations, parks, business, and so forth that students/families frequent or are important to a specific community.

Assignment

PART 1: COMMUNITY WALK & DATA COLLECTION:
Visit at least two different sites in the community around your practicum placement. Some examples might include parks, stores, community health centers, cultural centers, churches, fire/police stations, banks, military base, factories/processing plants, bakeries, construction sites, restaurants, corner stores, and so forth.

A. Look or/and document evidence of mathematics.
This could include people using mathematics, mathematical concepts or principles “in action,” mathematical practices, and so on. Talk to individuals who work/play/shop in the setting about how they use mathematics. Take/draw pictures and take field notes. Identify how each picture or experience you document provides evidence of mathematics. In addition, take note of things in the neighborhood that you hadn’t noticed before or that surprise you. Refrain from making judgments about the neighborhood because this is an opportunity for you to identify resources for your lesson planning and to build relationships with community members.

B. Formulate a series of questions about the contexts you visited that could be investigated mathematically. Start by reviewing the pictures, artifacts, and notes that you took. Then brainstorm a list of possible questions and data sources that could be used to answer those questions. Try to generate questions that “matter” or are authentic—in other words, these should be questions that you, students, or someone else who works, visits, and/or plays in the setting actually encounters and would want answered (i.e., authentic questions). You might consider questions that involve comparisons or that relate to issues of equity and/or fairness.

PART 2: MATHEMATICS LESSON DEVELOPMENT

Meet with your group/partner to identify the mathematical concepts, procedures, reasoning processes, and representations used in your investigation. Design a problem solving–based math lesson or problem-solving task that draws upon what you learned about your students’ communities, including the questions you asked and the data you collected. The lesson could invite students to do a similar activity (e.g., adapting a community walk) or could include tasks that deepen their understanding of a specific mathematical concept or practice you identified in this assignment. Be creative. Use the institution/school lesson plan template. Include any activity sheets you develop.

PART 3: FINAL WRITE-UP

A. Group Commentary about your community math walk experience that
   • Summarizes the mathematical question(s) you investigated, and the mathematical knowledge and practices you encountered on your community math walk. Be sure to discuss who was involved in these mathematical practices (employees, families, children, etc.) and team insights gained from the experience.
   • Describes how the math lesson explicitly connects to your team’s community math walk experience.

B. Individual reflection: Write a critical reflection on this experience using the prompts below:
   • What did you learn about the community/neighborhood, including the mathematical activity occurring in the
community/neighborhood? What was new? Surprising?

• How, if at all, did investigating the mathematical resources of the community inform your vision for teaching math and engaging families/communities in mathematics learning?

• What did you learn about yourself (e.g., challenging/affirming assumptions or stereotypes; views of yourself as a math teacher; views of students, community members, mathematics)?

• What do you think might be the benefits and challenges of this kind of math teaching for you or your students? How might you respond to the challenges?

Authors’ Note

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