Exploiting CP–asymmetries in rare charm decays

Rigo Bausch, Hector Gisbert, Marcel Golz, and Gudrun Hiller
Fakultät für Physik, TU Dortmund, Otto-Hahn-Str. 4, D-44221 Dortmund, Germany

We analyze patterns from CP–violating new physics (NP) in hadronic and semileptonic rare charm $|Δc| = |Δu| = 1$ transitions. Observation of direct CP–violation in hadronic decays, as in $ΔA_{CP}$, provides opportunities for $c → uℓ⁺ℓ^–$, $ℓ = e, \mu$ transitions, and vice versa. For the concrete case of flavorful, anomaly-free $Z'$–models a NP–interpretation of $ΔA_{CP}$ suggests measurable CP–asymmetries in semileptonic decays such as $D → πℓ⁺ℓ^–$ or $D → ππℓ⁺ℓ^–$. Conversely, an observation of CP–violation in $c → u e^±e^−$ or $c → u μ^±μ^−$ decays supports a NP–interpretation of $ΔA_{CP}$. Flavorful $U(1)'$–extensions provide explicit $U$–spin and isospin breaking which can be probed in patterns of hadronic decays of charm mesons. We work out signatures for CP–asymmetries in $D^0 → π^±π^−$, $D^0 → K^±K^−$ and $D^0 → π^±π^0$, $D^+ → π^+π^0$ decays, which can be probed in the future at LHCb and Belle II and provide further informative cross checks.

I. INTRODUCTION

Suppressions of standard model (SM) amplitudes due to accidental symmetries provide useful directions for searches to new physics (NP). Among the salient features of $|Δc| = |Δu| = 1$ transitions within the SM are a strong Glashow–Iliopoulos–Maiani (GIM)–suppression and small CP–violation. Hierarchies of the Cabibbo–Kobayashi–Maskawa (CKM) matrix $V$ suggest SM CP–violation at the order of $\text{Im}(V^*_{cb}V_{ub}/(V^*_{ub}V_{cb})) ~ \approx 7 \cdot 10^{-4}$, somewhat below LHCb’s observation of CP–violation in charm $\Gamma(D^0 \rightarrow π^0\ell\nu) / \Gamma(D^0 \rightarrow π^0\ell\nu)$ = $ΔA_{CP}(K^+K^-) - ΔA_{CP}(π^+π^-)$ = $(-15.4 \pm 2.9) \cdot 10^{-4}$, where $ΔA_{CP}(f) = \Gamma(D^0 \rightarrow f) - \Gamma(D^0 \rightarrow f)$ / $\Gamma(D^0 \rightarrow f) + \Gamma(D^0 \rightarrow f)$ = $ΔA_{CP}^{HFLAV} = (-16.4 \pm 2.8) \cdot 10^{-4}$.

While this leaves room for NP, due to the sizable uncertainties of hadronic $D$–decays, Eqs. (1) and (3) provide no clear-cut sign of NP. On the other hand, $ΔA_{CP}$ as large as the permille level is non-trivial to achieve in concrete models of NP. Correlations with other observables in charm and the down-quark sector exist, which are subject to partly very strong flavor constraints. For recent works, see Refs. [3,12]. Turning this around, the study of patterns using different sectors can hence disfavor or support a particular $ΔA_{CP}$ interpretation, and vice versa.

In this work we pursue a global analysis of CP–asymmetries in rare hadronic and semileptonic charm decays. Our focus is on NP patterns induced by four-fermion operators. Links via dipole operators between hadronic and semileptonic $ΔA_{CP}$–asymmetries in $D \rightarrow πℓ⁺ℓ^−$ decays have been pointed out by Ref. [13]. We work out predictions and correlations for anomaly-free $Z'$–extensions of the SM with generation-dependent $U(1)'$–charges, see Refs. [13,21] for recent phenomenological works. Flavorful charges can give rise to explicit isospin and $U$–spin breaking effects. It is our goal to work out corresponding experimental signatures for hadronic charm decays, exploiting yet another SM null test strategy in charm [21].

This paper is organized as follows: In Section II we briefly review CP–violation in hadronic $D$–decays, $D$–mixing and semileptonic $c → uℓ⁺ℓ^−$ transitions. In Section III we analyze effects of anomaly-free $U(1)'$–extensions with generation-dependent charges in hadronic 2-body $D$–decays and how $D$–mixing constraints can be evaded to address $ΔA_{CP}$. Patterns among CP–asymmetries in $D^0 \rightarrow π^±π^−$, $D^0 \rightarrow K^±K^−$, $D^0 \rightarrow π^±π^0$, $D^0 \rightarrow π^0π^0$, and $D^+ \rightarrow π^+π^0$ decays are worked out in Section IV. Correlations with CP–asymmetries in rare semileptonic decays are studied in Section V. We conclude in Section VI. Auxiliary information is given in several appendices.

II. CP–PHENOMENOLOGY IN CHARM

We review CP–violation in hadronic $D$–decays (Section II A), $D$–mixing (Section II B) and semileptonic $c → uℓ⁺ℓ^−$ processes (Section II C).
A. Direct CP-violation in $D^0 \rightarrow \pi^+\pi^-, K^+K^-$

The single-Cabibbo-suppressed (SCS) $D^0(\bar{D}^0)$ decay amplitudes $A_f$ ($\bar{A}_f$) to CP–eigenstates $f$ can be written as

$$A_f = A_f^T e^{i\phi_f^T} \left[ 1 + r_f e^{i(\delta_f + \phi_f)} \right] ,$$

$$\bar{A}_f = \eta_{\text{CP}} A_f^T e^{-i\phi_f^T} \left[ 1 + r_f e^{i(\delta_f - \phi_f)} \right] ,$$

where $\eta_{\text{CP}} = \pm 1$ is the CP–eigenvalue of $f$. The dominant SCS “tree” amplitude in the SM is denoted by $A_f^T e^{i\phi_f^T}$, and $r_f$ parametrizes the relative magnitude of all subleading amplitudes. Inserting Eqs. (4) into Eq. (2), in the limit of $r_f \ll 1$, yields

$$A_{\text{CP}}(f) = -2 r_f \sin \delta_f \sin \phi_f + O(r_f^2) ,$$

requiring both strong ($\delta_f$) and weak ($\phi_f$) relative phases for a non-vanishing direct CP–asymmetry. Beyond the SM the SCS $D^0$ decay amplitude can be written as

$$A_f = \sum_{q=d,s,b} \lambda_q (A_f^q)_{\text{SM}} + A_f^{\text{NP}} ,$$

where the first term corresponds to the SM contribution with CKM–factors $\lambda_q = V_{q\mu} V_{q\nu}^*$ made explicit, and the second term accounts for NP. Using CKM unitarity $\lambda_d + \lambda_s + \lambda_b = 0$ and writing for the final states $K^+K^-$ and $\pi^+\pi^-$ in the subscripts $f = K$ and $f = \pi$, respectively, one finds

$$A_{K(\pi)} = \lambda_s(d) (A_{K(\pi)}^{d(s)} - A_{K(\pi)}^{d(s)})_{\text{SM}}$$

$$+ \lambda_b (A_{K(\pi)}^{b(s)} - A_{K(\pi)}^{b(s)})_{\text{SM}} + A_{K(\pi)}^{\text{NP}} .$$

Here, the first term is the SCS contribution and the second one corresponds to “penguin” contributions with small Wilson coefficients which are strongly CKM–suppressed with respect to the SCS one by $\lambda_b/\lambda_{s,d}$. The last term $A_{K(\pi)}^{\text{NP}}$ encodes NP contributions. Using Eqs. (4), (5) and (7), we obtain

$$\Delta A_{\text{CP}} = \Delta A_{\text{CP}}^{\text{SM}} - \frac{2}{|\lambda_{s,d}|} \Delta r^{\text{NP}} ,$$

where

$$\Delta r^{\text{NP}} = r_K \sin \delta_K \sin \phi_K + r_\pi \sin \delta_\pi \sin \phi_\pi ,$$

and

$$r_K = \frac{A_{K(\pi)}^{\text{NP}}}{(A_{K(\pi)}^{d(s)} - A_{K(\pi)}^{s(d)})_{\text{SM}}} ,$$

$$r_\pi = \frac{A_{\pi}^{\text{NP}}}{(A_{\pi}^{d(s)} - A_{\pi}^{s(d)})_{\text{SM}}} .$$

and $r_{\pi,K} \ll 1$. The strong phases $\delta_{\pi,K}$ are associated with the NP amplitudes. Since we are interested in maximal NP contributions, we employ in our numerical analysis sin $\delta_{\pi,K} \sim 1$. Note, there is a priori no information on the sign of $\Delta r^{\text{NP}}$ as it depends on products of strong and weak phases. The branching ratios of the $D \rightarrow f$ modes are dominated by their respective SM contributions. We can therefore extract $|A_{K(\pi)}^{s(d)} - A_{K(\pi)}^{d(s)}|_{\text{SM}}$ from data, see Appendix A for details.

B. CP-violation in $D^0 - \bar{D}^0$ mixing

Here we consider constraints from charm meson mixing. The $D^0 - \bar{D}^0$ transition amplitude can be written as

$$\langle D^0 | H_{\text{eff}} | \bar{D}^0 \rangle = M_{12} - \frac{i}{2} \Gamma_{12} ,$$

which can be parametrized in terms of the following physical quantities

$$x_{12} = 2 \frac{|M_{12}|}{\Gamma} ,$$

$$y_{12} = \frac{|\Gamma_{12}|}{\Gamma} ,$$

$$\phi_{12} = \arg \left( \frac{M_{12}}{\Gamma_{12}} \right) .$$

Here, $x_{12}$ and $y_{12}$ are CP–conserving, while $\phi_{12}$ is a phase difference that results in CP–violation in mixing. A global fit from the HFLAV collaboration [2] results in

$$x_{12} \in [0.22, 0.63] % ,$$

$$y_{12} \in [0.50, 0.75] % ,$$

$$\phi_{12} \in [-2.5^\circ, 1.8^\circ] .$$

In absence of a sufficiently controlled SM prediction of the mixing parameters, we require the NP contributions to saturate the current world averages [13],

$$x_{12}^{\text{NP}} \leq x_{12} ,$$

$$x_{12}^{\text{NP}} \sin \phi_{12}^{\text{NP}} \leq x_{12} \sin \phi_{12} .$$

C. CP–violation in $c \rightarrow u \ell^+ \ell^-$

CP–violation in semileptonic rare charm decays arises from complex-valued Wilson coefficients $C_i^{\ell\ell}$, $C_i^{\ell\ell'}$ in the effective Hamiltonian [18],

$$\mathcal{H}_{\text{eff}} \supset - \frac{4 G_F}{\sqrt{2}} \frac{\alpha_e}{4\pi} \sum_{i=9,10} (C_i^{\ell\ell} O_i^{\ell\ell} + C_i^{\ell\ell'} O_i^{\ell\ell'}) + \text{h.c.} ,$$

with the operators

$$O_9^{\ell(i)} = \langle \pi_L(R) \gamma_\mu C_L(R) \rangle (\bar{\ell}\gamma^\mu \ell) ,$$

$$O_10^{\ell(i)} = \langle \pi_L(R) \gamma_\mu C_L(R) \rangle (\bar{\ell}\gamma^\mu \gamma_5 \ell) .$$

Here, $\alpha_e$ denotes the fine structure constant, $G_F$ is Fermi’s constant and $L = (1 - \gamma_5)/2, R = (1 + \gamma_5)/2$ are chiral projectors. CP–violation has not been observed.
in semileptonic $|\Delta c| = |\Delta u| = 1$ decays yet. Available measurements for CP–asymmetries in rare semileptonic charm decays are at the level of few to $\mathcal{O}(10)\%$ \cite{22}, which is close to possible NP effects \cite{13,18,21.}

Braching ratio and high–$p_T$ data imply the following constraints, barring cancellations \cite{23,24}:

$$|C_{9,10}^{\mu}(t)| \lesssim 1, \quad |C_{9,10}^{e}(t)| \lesssim 3,$$

(18)

stronger for muons than for electrons.

### III. A FLAVORFUL Z' IN CHARM

We work out NP–effects in charm from anomaly-free $U(1)'$–extensions of the SM with fermion charges $F_{\psi_i}$ that depend on the generation, $i = 1, 2, 3$. Specifically, SM fermion multiplets plus possibly right-handed neutrinos $\psi = Q, u, d, L, e, \nu$ in representations of $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ can be characterized, in that order, as

$$Q_i = (3, 2, 1/6, F_{Q_i}), \quad u_i = (3, 1, 2/3, F_{u_i}),$$

$$d_i = (3, 1, -1/3, F_{d_i}), \quad L_i = (1, 2, -1/2, F_{L_i}),$$

$$e_i = (1, 1, -1, F_{e_i}), \quad \nu_i = (1, 1, 0, F_{\nu_i})$$

(19)

Concrete models with $F_{\psi_i}$–assignments that fulfill the anomaly-cancellation conditions and induce $c \to u$ flavor changing neutral currents (FCNCs) are given in TABLE \[1\]. Related models (models 1 to 8) have been studied previously in the context of semileptonic rare charm decays in Ref. \[18\], to which we refer for further details. The models in TABLE \[1\] satisfy $\sum_{i=1}^3 (F_{Q_i} - F_{L_i} + 2 F_{u_i} - F_{d_i} - F_{\nu_i}) = 0$ and therefore avoid kinetic mixing at one-loop \cite{25}.

In Section IIIA we discuss couplings of the fermions to the $Z'$-boson, which arises from the $U(1)'$–group. We assume the $Z'$ to have a mass $M_{Z'}$ of the electroweak scale or heavier. We discuss the induced $c \to u$ four-quark operators and Wilson coefficients in Section IIIB.

In Section IIIC we discuss how to bypass constraints from $D^0 \to \bar{D}^0$ mixing. We work out predictions for $\Delta A_{\text{CP}}$ in Section IIID.

#### A. Z’–FCNCs

The $Z'$–couplings relevant to charm FCNCs can be written as

$$\mathcal{L}_{Z'} = (g_{L,R}^{u,c} \bar{u}_L \gamma^\mu c_L Z'_\mu + g_{L,R}^{u,c} \bar{u}_R \gamma^\mu c_R Z'_\mu + \text{h.c.})$$

$$+ g_{L,R}^{\ell_L} \bar{\ell}_L \gamma^\mu \ell_L Z'_\mu + g_{L,R}^{\ell_R} \bar{\ell}_R \gamma^\mu \ell_R Z'_\mu$$

$$+ g_{L,R}^{e_L} \bar{e}_L \gamma^\mu e_L Z'_\mu + g_{L,R}^{e_R} \bar{e}_R \gamma^\mu e_R Z'_\mu$$

(20)

with $\ell = e, \mu, \tau$. The flavor diagonal couplings $g_{L,R}^{d,s}$ and $g_{L,R}^{d,s}$ are given as the $U(1)'$–gauge coupling $g_4$ times the associated charge $F_{\psi_i}$.

The $|\Delta c| = |\Delta u| = 1$ FCNC couplings $g_{L,R}^{u,c}$ are generated via rotations from the gauge to the mass basis, and are in general complex-valued. Four different unitary rotations exist in the quark sector, corresponding to the left-handed (LH) and right-handed (RH) ones both for up- and down-type quarks. The product of LH up- and down-type rotations gives the CKM–matrix. In order to evade the severe constraints in the kaon sector, we assume the CKM–matrix to predominantly stem from the LH up-type rotation, implying

$$g_{L}^{u,c} \approx g_4 \lambda_{\text{CKM}} \Delta F_L, \quad \Delta F_L = F_{Q_2} - F_{Q_1},$$

(21)

where $\lambda_{\text{CKM}} \sim 0.2$ denotes the Wolfenstein parameter and we used $\lambda_{\text{CKM}} \ll 1$. In contrast, the RH rotation is a priori unconstrained and induces

$$g_{R}^{u,c} \approx g_4 \sin \theta_u \cos \theta_u e^{i \phi_R} \Delta F_R,$$

(22)

where $\theta_u$ is the up-charm mixing angle for the up-quark singlets, $\Delta F_R = F_{u_2} - F_{u_1}$ and $\phi_R$ the corresponding CP–phase.

#### B. Four-fermion operators and matching

Generation-dependent quark-couplings result in additional operators in the effective weak Hamiltonian beyond the ones considered usually, i.e. Ref. \[20\]. At the scale $m_5 < \mu < m_{\text{EWK}}$,

$$\mathcal{H}_{\text{eff}}^{\Delta c = 1} \supset \frac{G_F}{\sqrt{2}} \sum_q (\bar{c}_i^{(t)} \cdot \bar{G}_i^{(t)} + \text{h.c.}),$$

(23)

with the new operators

$$\bar{Q}_7 = (\bar{u}c)_V - A \sum_q F_{u_i,d_i} (\bar{q}q)_{V+A},$$

(24)

$$\bar{Q}_7^c = (\bar{u}c)_V + A \sum_q F_{Q_i} (\bar{q}q)_{V-A},$$

(25)

$$\bar{Q}_8 = (\bar{u}c)_V - A \sum_q F_{u_i,d_i} (\bar{q}q_{d})_{V+A},$$

(26)

$$\bar{Q}_8^c = (\bar{u}c)_V + A \sum_q F_{Q_i} (\bar{q}q_{d})_{V-A},$$

(27)

$$\bar{Q}_9 = (\bar{u}c)_V - A \sum_q F_{Q_i} (\bar{q}q_{V-A}),$$

(28)

$$\bar{Q}_9^c = (\bar{u}c)_V + A \sum_q F_{Q_i} (\bar{q}q_{V+A}),$$

(29)

$$\bar{Q}_{10} = (\bar{u}c)_V - A \sum_q F_{Q_i} (\bar{q}q_{V-A}),$$

(30)

$$\bar{Q}_{10}^c = (\bar{u}c)_V + A \sum_q F_{Q_i} (\bar{q}q_{V+A}),$$

(31)

where $(V \pm A)$ refers to the Dirac structures $\gamma_{\mu}(1 \pm \gamma_5)$, $q = u, c, d, s, b$ and $\alpha, \beta$ are the color indices. The strength of these operators is given by their respective
Wilson coefficients $\tilde{C}_i$, $\tilde{C}'_i$ which depend on both heavy masses and weak phases responsible for CP–violating phenomena. The Wilson coefficients induced by the Lagrangian [20] read

$$\tilde{C}_7(M_{Z'}) = \tilde{C}_9(M_{Z'}) = \frac{\sqrt{2}}{G_F} g_L^{uc} \frac{g_4}{4 M_{Z'}^2},$$

$$\tilde{C}'_7(M_{Z'}) = \tilde{C}'_9(M_{Z'}) = \frac{\sqrt{2}}{G_F} g_R^{uc} \frac{g_4}{4 M_{Z'}^2},$$

$$\tilde{C}_{10}^{(i)}(M_{Z'}) = \tilde{C}_{10}^{(i)}(M_{Z'}) = 0.$$  

They are evolved from $M_{Z'}$ to $m_e$ using the renormalization group equations (RGEs) with top and bottom quarks integrated out at their respective threshold scales. Finite values of $\tilde{C}_{10}^{(i)}$ and $\tilde{C}_{10}^{(i)}'$ arise from the RGE mixing at the charm mass scale, see Appendix B for details.

### C. $D^0 - \bar{D}^0$ mixing constraints

Rare $|\Delta c| = |\Delta u| = 1$ decays are induced in the $Z'$-models by operators with coefficients proportional to $g_L^{uc}$ or $g_R^{uc}$ in Eq. (32). These couplings induce at second order $D^0 - \bar{D}^0$ mixing [13], and are constrained as

$$|(g_L^{uc})^2 + (g_R^{uc})^2 - X g_L^{uc} g_R^{uc}| \lesssim 6 \cdot 10^{-7} \left( \frac{M_{Z'}}{\text{TeV}} \right)^2,$$  

with $X \sim 20$ for $M_{Z'}$ in the TeV range [13]. This constraint on $x_{12}$ can be evaded if both $g_L^{uc}$ and $g_R^{uc}$ are present, for either $g_L^{uc} \sim X g_R^{uc}$ or $g_L^{uc} \sim 1/X g_R^{uc}$. However, in these cases the CP-phases have to be aligned $\text{Arg}(g_L^{uc}) \sim \text{Arg}(g_R^{uc})$ to fulfill Eq. (33). As kaon constraints force $\text{Arg}(g_R^{uc})$ to be SM–like, CP–violating effects in charm become negligible.

We therefore choose $g_L^{uc} = 0$, which can be achieved with $\Delta F_L = 0$. The models in TABLE I satisfy for this reason $F_{Q_1} = F_{Q_2}$. Consequently, we focus on FCNCs in the up-singlet sector [22], that is, $g_R^{uc} \neq 0$ and complex. If there is a single coupling only, the above mixing constraint on $x_{12}$ becomes

$$|g_A^{uc}| \lesssim 8 \cdot 10^{-4} \left( \frac{M_{Z'}}{\text{TeV}} \right), \quad A = L, R.$$  

The even tighter constraint [14] for CP–violating couplings on $x_{12}$ can be bypassed for $\text{Arg}(g_R^{uc}) = \phi_R$ around $\pi/2$ (or $3\pi/2$), as the CP–phase of the mixing amplitude is twice the one of the $|\Delta c| = |\Delta u| = 1$ FCNC [26]. The contributions to $\Delta A_{\text{CP}}$ become maximal while simultaneously mixing constraints are satisfied. This interplay of $\phi_R$ versus the coupling $g_4/M_{Z'}$ (TeV$^{-1}$) for model 2 and fixed $\theta_e = 1 \cdot 10^{-4}$ is illustrated in FIG. 1. The red (hatched) area corresponds to the $D^0 - \bar{D}^0$ mixing constraints on the imaginary part $x_{12} \sin \phi_{12}$ (red area) and the absolute value $x_{12}$ (red hatched area) in the $\phi_R$–$g_4/M_{Z'}$ (TeV$^{-1}$) plane for $\theta_e = 1 \cdot 10^{-4}$. $F_{e_i}$-charges are as in model 2, see TABLE I. The golden star indicates a benchmark point [10], see text for details.

---

**TABLE I**: Sample solutions of an anomaly-free $U(1)'$–extension of the SM+$3\nu_R$ with $F_{Q_1} = F_{Q_2}$. Models 2, 4 and 5 are taken from Ref. [18]. Models 9 and 10 feature $F_{Q_3} = 0$. In general, the ordering of generations is arbitrary due to permutation invariance. However, our analysis explicitly uses the ordering stated here, that is, the $i$th entry corresponds to the $i$th generation. Model $10\mu$ is the same as model 10 with the smallest lepton-coupling to muons.

| model | $F_{Q_i}$ | $F_{u_i}$ | $F_{d_i}$ | $F_{L_i}$ | $F_{e_i}$ | $F_{\nu_i}$ |
|-------|----------|----------|----------|----------|----------|-----------|
| 2     | 3        | 3        | -6       | -10      | 0        | 0         |
| 4     | -1       | -1       | 2        | 0        | 0        | 0         |
| 5     | -1       | -1       | 2        | -1       | -1       | 0         |
| 9     | 0        | 0        | -11      | 7        | -8       | -6        |
| 10    | 0        | 0        | -13      | -14      | -14      | -14       |
| $10\mu$ | 0        | 0        | 0        | 0        | 0        | 0         |

**FIG. 1**: $|\Delta A_{\text{CP}}^{(A)}|$ (green bands) versus $D^0 - \bar{D}^0$ mixing exclusion regions [14] on the imaginary part $x_{12} \sin \phi_{12}$ (red area) and the absolute value $x_{12}$ (red hatched area) in the $\phi_R$–$g_4/M_{Z'}$ (TeV$^{-1}$) plane for $\theta_e = 1 \cdot 10^{-4}$. $F_{e_i}$-charges are as in model 2, see TABLE I. The golden star indicates a benchmark point [10], see text for details.
D. \(Z'\)-effects for \(\Delta A_{\text{CP}}\)

Taking into account the running from \(M_{Z'}\) to \(m_c\), details of which are given in Appendix \(3\), we find that \(\Delta A_{\text{CP}}\) can be written as

\[
\Delta A_{\text{CP}}^{\text{NP}} = A_{\text{CP}}^{\text{NP}}(K^+K^-) - A_{\text{CP}}^{\text{NP}}(\pi^+\pi^-),
\]

with

\[
A_{\text{CP}}^{\text{NP}}(K^+K^-) \sim \frac{g_2^2}{M_{Z'}^2} \theta_u \Delta F_R [c_K F_{Q_2} + d_K F_{d_2}],
\]

\[
A_{\text{CP}}^{\text{NP}}(\pi^+\pi^-) \sim \frac{g_3^2}{M_{Z'}^2} \theta_u \Delta F_R [c_\pi F_{Q_1} + d_\pi F_{d_1}],
\]

where

\[
c_K = \frac{\chi_K}{a_K} r_1(m_c, M_{Z'}), \quad c_\pi = -\frac{\chi_\pi}{a_\pi} r_1(m_c, M_{Z'}),
\]

\[
d_K = \frac{1}{a_K} r_2(m_c, M_{Z'}), \quad d_\pi = -\frac{1}{a_\pi} r_2(m_c, M_{Z'}).
\]

As explained in the previous Section \(\text{III}\), we analyze models with \(g_3^{\mu e} = 0\) and \(\text{Im}(g_2^{\mu e})\) large. In Eq. \(36\) we use \(\sin \delta_{12} \sim 1\) and \(\theta_u \ll 1\). The parameters \(c_{K,\pi}\) and \(d_{K,\pi}\) depend on the chiral factors \(\chi_{K,\pi}\) at the charm scale, the LO QCD running functions \(r_{1,2}(m_c, M_{Z'})\) and the tree-level contributions \(a_{K,\pi}\), which are determined experimentally. Further details can be found in Appendices \(\text{A, D}\). Numerical values of \(c_{K,\pi}\) and \(d_{K,\pi}\) for different \(Z'\) masses are displayed in Table II.

| \(M_{Z'}\) [TeV] | 2   | 4   | 6   | 8   | 10  |
|------------------|-----|-----|-----|-----|-----|
| \(c_K\)          | 1.133 | 1.217 | 1.266 | 1.302 | 1.330 |
| \(d_K\)          | -0.046 | -0.054 | -0.058 | -0.061 | -0.063 |
| \(c_\pi\)        | -1.446 | -1.553 | -1.616 | -1.661 | -1.698 |
| \(d_\pi\)        | 0.058 | 0.068 | 0.074 | 0.077 | 0.080 |
| \(d_{o'}\)       | 0.071 | 0.083 | 0.090 | 0.094 | 0.098 |
| \(d_{o''}\)      | 0.077 | 0.090 | 0.097 | 0.102 | 0.106 |

In Fig. 2 we show sizable \(Z'\)-contributions to \(\Delta A_{\text{CP}}^{\text{NP}}\) and \(D^0 \to D^+\) mixing constraints (red area) in the plane of \(g_4/M_{Z'}\) (TeV\(^{-1}\)) and the parameter \(\Delta F_R = \Delta F_R \theta_u\) for models 2, 5, 9 and 10(\(\mu\)). The corresponding plot of model 4 is not given in Fig. 2 because it exhibits very similar bands as model 5 due to identical \(F_{Q_1,2}\) and \(\Delta F_R\), as shown in Table II. Constraints from branching ratios of (semi-)muonic \(D\)–decays (dash-dotted and dotted lines), here for \(g_2^{\mu e} = 0\),

\[
|\frac{g_R}{\sqrt{1 + (g_4^{\mu e})^2}}| \leq \frac{0.04}{M_{Z'}^{\text{TeV}}},
\]

\[
|\frac{g_R^{\mu e}}{(g_4^{\mu e})^2} + (\frac{g_R^{\mu e}}{M_{Z'}^{\text{TeV}}})^2| \leq 0.03\frac{M_{Z'}^{\text{TeV}}}{|\frac{g_R}{\sqrt{1 + (g_4^{\mu e})^2}}|},
\]

start to be competitive with mixing constraints close to the non-perturbativity region (black region). This is particularly relevant for model 9 and 10, which exhibit large couplings to leptons. To evade the muon constraints and allow for slightly larger values of \(\Delta A_{\text{CP}}\) we also consider model 10(\(\mu\)), which is the same as model 10 with the lepton-charges ordered in such a way that the smallest ones are for muons, stressing the interplay between hadronic and leptonic sectors; model 10 can accommodate \(\Delta A_{\text{CP}}^{\text{NP}}\) up to \(1.5 \times 10^{-3}\), while model 10(\(\mu\)) can reach \(1.8 \times 10^{-3}\). Fig. 2 shows the stronger bound for each model, i.e., Eq. \(38\) for models 2, 5, 9 and 10(\(\mu\)) (dash-dotted) and Eq. \(38\) for model 10 (dotted).

In Figs. 1 and 2 we show benchmark points. They pass constraints from \(D\)-mixing and semi-(muonic) decays, while giving \(\Delta A_{\text{CP}}^{\text{NP}}\) \(\sim 10^{-3}\). The golden star corresponds to model 2 with \(\Delta F_R = 12\) and

\[
\phi_R \sim \pi/2, \quad g_4/M_{Z'} \sim 0.38/\text{TeV}, \quad \theta_u \sim 1 \times 10^{-4}.
\]

The pink diamond corresponds to model 10 with \(\Delta F_R = 19\) and

\[
\phi_R \sim \pi/2, \quad g_4/M_{Z'} \sim 2.3/\text{TeV}, \quad \theta_u \sim 1.7 \times 10^{-5}.
\]

We learn that \(Z'\)-models with charges as in Table II can provide concrete NP–interpretations of \(\Delta A_{\text{CP}}\) of the order of \(10^{-3}\). \(D^0 \to D^+\) mixing provides upper limits on the achievable \(\Delta A_{\text{CP}}^{\text{NP}}\). To distinguish the different model scenarios we explore correlations of \(\Delta A_{\text{CP}}\) with other sectors, hadronic 2-body \(D\)–decays in Section IV and semileptonic \(c \to u \ell^+ \ell^-\) transitions in Section IV C.

IV. PATTERNS IN HADRONIC DECAYS

\(Z'\)-models with non-universal charges \(F_\psi\) can give rise to large flavor-breaking effects which could explicitly violate relations between hadronic charm decays \(28, 31\). We study signatures of \(Z'\)-induced U–spin and isospin breaking in Section IV A and Section IV B, respectively. \(A_{\text{CP}}\) in \(D^0 \to \pi^0\pi^0\) is studied in Section IV C.

A. U–spin patterns in \(D^0 \to \pi^+\pi^-, K^+K^-

U–spin breaking arises for \(F_{Q_1} \neq F_{Q_2}\) or \(F_{d_1} \neq F_{d_2}\), and can upset the U–spin sum rule \(28\),

\[
A_{\text{CP}}(D^0 \to K^+K^-) + A_{\text{CP}}(D^0 \to \pi^+\pi^-) = 0.
\]

\[
(42)\]
FIG. 2: $|\Delta A_{\text{NP}}|$ for different $Z'$-models (2 upper left, 5 upper right, 9 lower left and 10($\mu$) lower right) in the plane of $g_4/M_{Z'}$ (TeV$^{-1}$) and $\Delta F_R = \Delta F_R - \theta_a$, together with the excluded region from $D^0 - D^0$ mixing (red). Light green, dark green, blue and cyan bands correspond to $|\Delta A_{\text{CP}}| = (4.0 \pm 0.2) \cdot 10^{-3}$, $|\Delta A_{\text{CP}}| = (1.5 \pm 0.2) \cdot 10^{-3}$, $|\Delta A_{\text{CP}}| = (8 \pm 2) \cdot 10^{-4}$ and $|\Delta A_{\text{CP}}| = (3 \pm 1) \cdot 10^{-4}$, respectively. The black region indicates the upper bound coming from perturbativity and direct searches in dimuon and dielectron spectra [27], which read $g_4 \leq 4\pi$ and $M_{Z'} \geq 4.5$ TeV, respectively. The magenta dash–dotted and dotted lines show the stronger (if any) of the bounds from Eqs. (38) and (39). In the lower right plot the dotted line corresponds to model 10, and the dash–dotted to model 10($\mu$). The golden star and pink diamond are benchmark points [40] and (41). See text for details.

To quantify deviations from this relation we define \(^2\)

$$U_{\text{break}}^{\text{tot}} = \left| 1 + \frac{A_{\text{CP}}(D^0 \to K^+ K^-)}{A_{\text{CP}}(D^0 \to \pi^+ \pi^-)} \right|.$$ \hspace{1cm} (43)

In the U–spin limit $U_{\text{break}}^{\text{tot}} = 0$.

\(^2\) For model 10($\mu$) we use instead $\left| 1 + \frac{A_{\text{CP}}(D^0 \to \pi^+ \pi^-)}{A_{\text{CP}}(D^0 \to K^+ K^-)} \right|$ to avoid $U_{\text{break}}^{\text{tot}} > 1$. It is tacitly understood that $K, Q_2, d_2$ and $\pi, Q_1, d_1$–indices in Eq. (44) and following need to be swapped in this case.

Using Eqs. (36), $U_{\text{break}}^{\text{tot}}$ can be written as

$$U_{\text{break}}^{\text{tot}} = \left| 1 + \frac{c_K F_{Q_2} + d_K F_{d_2}}{c_\pi F_{Q_1} + d_\pi F_{d_1}} \right|. \hspace{1cm} (44)$$

In TABLE [II] we give $U_{\text{break}}^{\text{tot}}$ for models 2, 4, 5, 9 and 10($\mu$), for $M_{Z'} = 6$ TeV. The variation of $U_{\text{break}}^{\text{tot}}$ with $M_{Z'}$ in the range shown is within a few percent.

Taking advantage of the smallness of the parameters $d_{K, \pi}$ relative to $c_{K, \pi}$, we perform a Taylor expansion in Eq. (44) up to $O(d_{K, \pi}^2)$ to qualitatively understand how U–spin breaking in our models emerges. This leads to

$$U_{\text{break}}^{\text{tot}} \approx \left| 1 + \frac{c_K d_{K} F_{d_1}}{c_\pi^2 F_{Q_1} + d_\pi F_{Q_1}} \right|,$$ \hspace{1cm} (45)
for $F_{Q_1} = F_{Q_2} \neq 0$ (models 2, 4 and 5), while for $F_{Q_1} = F_{Q_2} = 0$ (models 9 and 10(µ)) Eq. (44) simply becomes

$$U_{\text{tot\ break}} = \left| 1 + \frac{d_K F_{d_1}}{d_\pi F_{d_1}} \right|. \quad (46)$$

For models with $F_{Q_1} = F_{Q_2} \neq 0$ different sources of U–spin breaking exist. The second term in Eq. (45) accounts for effects originating from interference between the SM–amplitude and the $F_{Q_1,2}$ charges. This contribution is responsible for 22 % U–spin breaking, which is of the same order of magnitude as the expected U–spin breaking uncertainty of the SM. In contrast, the last two terms in Eq. (45) are pure NP U–spin breaking effects. Eq. (45) can further be simplified with $d_K \approx \frac{c_K}{c_\pi} d_\pi$ due to $\chi_\pi \approx \chi_K$, which holds numerically at the level of $\mathcal{O}(0.1-1)\%$. It follows that

$$U_{\text{tot\ break}} \approx \left| 1 + \frac{c_K}{c_\pi} + \frac{d_K}{c_\pi} \left( \frac{F_{d_2} - F_{d_1}}{F_{Q_1}} \right) \right|, \quad (47)$$

highlighting that pure NP U–spin breaking effects are
induced by

$$U^\text{NP}_{\text{break}}(F_{Q_1,2} = 0) \approx 0.78 \left| \frac{F_{d_2} - F_{d_1}}{F_{d_1}} \right|, \quad (49)$$

which, unlike in Eq. (48), is unsuppressed. Models with $F_{Q_1} = F_{Q_2} = 0$ are therefore prime candidates for sizable NP U–spin breaking effects. Models 9 and 10(µ) have been constructed for this purpose. However, in model 9 $F_{d_1} = F_{d_1}$ and U–spin breaking arises from $d_K \neq -d_\pi$ only, and is SM-like.

Note, the strong phases associated with NP are assumed to be similar, $\sin \delta^* \simeq \sin \delta_K$, and order one; violation of Eq. (42) can be suppressed or even further enhanced by U–spin breaking in the strong phases. While this is an uncertainty on the NP interpretation, $Z'$–signals could even be more striking.

In FIGs. 3 and 4 we show the contributions of models 2, 5, 9 and 10(µ) to the individual CP–asymmetries $A_{\text{CP}}(K^+K^-)$ and $A_{\text{CP}}(\pi^+\pi^-)$ in blue, magenta, yellow and cyan, respectively. The U–spin limit is given by the red dashed line with 30% U–spin breaking indicated by the red contour. Present experimental bounds from TABLE IV are shown in FIG. 3 as 1σ regions in gray for the individual asymmetries and in green for $\Delta A_{\text{CP}}$. The future sensitivities are indicated in light (dark) gray and green bands in FIG. 4 for LHCb Run 1-3 (1-5). We use the following central values for the plot to the left (right)

$$A^\text{CP}(K^+K^-) = -0.6 \cdot 10^{-3} \left( -1.45 \cdot 10^{-3} \right),$$

$$A^\text{CP}(\pi^+\pi^-) = 1.0 \cdot 10^{-3} \left( 0.15 \cdot 10^{-3} \right). \quad (50)$$

The orange error ellipses illustrate the NP sensitivity of the projected uncertainties of $A_{\text{CP}}(K^+K^-)$ and $A_{\text{CP}}(\pi^+\pi^-)$ assuming no correlations. A future database analysis which takes into account correlations between the individual asymmetries and $\Delta A_{\text{CP}}$ can be expected to be more powerful.

U–spin symmetry within the SM is broken at the level of 30%. We find that flavorful $Z'$–models can exceed this by far (model 10(µ)), or moderately (model 2), which makes the measurements of $A_{\text{CP}}(K^+K^-)$ and $A_{\text{CP}}(\pi^+\pi^-)$ smoking guns for NP, within reach of Belle II and LHCb with the projected sensitivities.

### B. Isospin breaking patterns in $D^+ \to \pi^+\pi^0$

Isospin breaking arises in $Z'$–models if $F_{u_1} \neq F_{d_1}$. In charm physics, the hadronic decay $D^+ \to \pi^+\pi^0$ represents a formidable candidate to study these effects, because the CP–asymmetry $A_{\text{CP}}(\pi^+\pi^0)$, defined by

$$A_{\text{CP}}(\pi^+\pi^0) = \frac{\Gamma(D^+ \to f^+) - \Gamma(D^- \to f^-)}{\Gamma(D^+ \to f^+) + \Gamma(D^- \to f^-)}, \quad (51)$$

with $f^\pm = \pi^\pm\pi^0$ is a clean SM null test [35]. Following the same procedure as in Section III D for $\Delta A_{\text{CP}}^\text{NP}$ we obtain, using $\theta_u \ll 1$,

$$A_{\text{CP}}^{\text{NP}}(\pi^+\pi^0) \sim \frac{\theta_1}{M_{Z'}} \Delta F_R d_\pi' (F_{d_1} - F_{u_1}), \quad (52)$$

with

$$d_\pi' = -1 \frac{1}{a_\pi'} \frac{1}{r_2(m_c, M_{Z'})}. \quad (53)$$

Here, $a_{\pi'}$ denotes the tree-level contribution to $D^+ \to \pi^+\pi^0$ whose modulus has been fixed experimentally, see Appendix A for details. Numerical values of $d_\pi'$ for different values of $M_{Z'}$ are given in TABLE III. Inserting Eq. (53) into Eq. (52), we obtain

$$A_{\text{CP}}^{\text{NP}}(\pi^+\pi^0) \sim \beta_\pi' \Delta A_{\text{CP}}^{\text{NP}}, \quad (54)$$
where
\[ \beta_{\pi'} = \frac{d_{\pi'} (F_{d_3} - F_{u_3})}{c_K F_{Q_2} + d_K F_{d_2} - c_{\pi} F_{Q_1} - d_{\pi} F_{d_3}}. \] (55)

Values of \( \beta_{\pi} \) for \( M_{Z'} = 6 \) TeV and different \( Z' \)-models can be seen in TABLE III. Since we have lost information about the signs of the leading SM decay amplitudes with which NP is interfering, we cannot predict the relative sign between the CP-asymmetries in Eq. (57) without relying on assumptions on the strong interaction. Note, unlike for \( A_{CP}(K^+ K^-) \) and \( A_{CP}(\pi^+ \pi^-) \), there is no SM flavor symmetry here at work.

We find that model 9 and 10(\( \mu \)) induce values near
\[ A_{NP}^{\pi^+ \pi^-} \sim (1 - 2) \cdot \Delta A_{NP}^{\pi \pi}, \] (56)
which for \( \Delta A_{NP}^{\pi \pi} \sim 10^{-3} \) is within the projected sensitivity of Belle II with 50 ab\(^{-1} \) [34], see TABLE IV. Model 2, 4 and 5 induce \( A_{NP}^{\pi^+ \pi^-} \lesssim 0.1 \cdot \Delta A_{NP}^{\pi \pi} \sim 10^{-4} \), beyond the reach of current facilities.

This behavior can be understood by expanding Eq. (55) in the \( d_1 \) up to \( O(d_1) \). For \( F_{Q_1} = F_{Q_2} = 0 \) (model 9 and 10(\( \mu \))), we find that \( \beta_{\pi} \) scales with \( d_{\pi'/d_K} \approx -1.6 \) times a combination of charges \( (F_{d_1} - F_{u_1})/F_{d_2}(1 + ...) \sim O(1) \) resulting in \( O(1) \) isospin breaking effects. For models \( F_{Q_1} = F_{Q_2} \neq 0 \) instead a suppression factor \( d_{\pi'}/(c_K - c_{\pi}) \approx 0.03 \) exists from the chiral enhancement of the \((V - A) \times (V + A) \) operators, leading to \( \beta_{\pi'} \) of \( O(10^{-2} - 10^{-1}) \).

\[ \text{V. SEMILEPTONIC DECAYS VS. } \Delta A_{CP} \]

The dominant Wilson coefficients in \( c \to \ell \ell \ell^{-} \) transitions are \( C_{9/10}^{\ell \ell} \), defined in Eq. (15). In flavorful \( Z' \)-models [18]
\[ C_{9/10}^{\ell \ell} (M_{Z'}) = -\frac{\pi}{\sqrt{2} G_F} \alpha_e \frac{g_{L \ell}^{u,c}}{M_{Z'}} \left( g_{R \ell}^{u,c} \pm g_{L \ell}^{\ell \ell} \right), \] (60)
\[ C_{9/10}^{\ell \ell} (M_{Z'}) = -\frac{\pi}{\sqrt{2} G_F} \alpha_e \frac{g_{R \ell}^{u,c}}{M_{Z'}} \left( g_{L \ell}^{\ell \ell} \pm g_{R \ell}^{\ell \ell} \right), \] (61)
where \( g_{R \ell}^{u,c} = g_4 F_c \), and \( g_{L \ell}^{\ell \ell} = g_4 F_L \), with in general different couplings for muons and electrons. As explained in Section III C we analyze in this work \( Z' \)-models with \( g_{L \ell}^{u,c} = 0 \) and \( \text{Im}(g_{R \ell}^{u,c}) \) large.

CP-asymmetries in the branching ratios are induced by interference of NP, here through \( g_{R \ell}^{u,c} \), with \( C_{9/10}^{u,c} \), the effective coefficient of \( O_9 \) present in the SM, which is lepton universal, depends on the dilepton invariant mass and has sizable hadronic contributions and provides sizable strong phases. This interference term is sensitive to \( C_{9/10}^{\ell \ell} \) only. Angular analysis offers further opportunities.

An interesting recent example for the latter is \( D^0 \to \pi^+ \pi^- \mu^+ \mu^- \) decays [21][22][37]. Notably, the angular observables \( I_{5,6,7} \) are GIM–protected in the SM and clean null tests [21]. In the \( Z' \)-models under consideration, \( I_{5,6} \) are induced by \( \text{Re}(C_{9}^{\ell \ell} \cdot C_{10}^{\ell \ell}) \) and \( \text{Im}(C_{9}^{\ell \ell} \cdot C_{10}^{\ell \ell}) \), whereas \( I_7 \) is induced by \( \text{Re}(C_{9}^{\ell \ell} \cdot C_{10}^{\ell \ell} \cdot C_{10}^{\ell \ell}) \), CP-asymmetries in angular asymmetries, on the other hand, can stem from naive \( T \)-odd observables and do not rely on strong phases (\( I_{7,8,9} \)). CP–odd ones (\( I_{5,6,8,9} \)) provide CP–asymmetries that can be measured without tagging, see Ref. [21] for details. A complete and detailed analysis of angular asymmetries in \( Z' \)-models is beyond the scope of this work. What we do want to point out here is that a global analysis of angular and CP-asymmetries can probe both \( C_{9}^{\ell \ell} \) and \( C_{10}^{\ell \ell} \) for electrons, \( \ell = e \) and muons, \( \ell = \mu \) separately, and therefore can distinguish different \( U(1)' \)-charge assignments.

Taking the imaginary part of Eq. (61) and employing Eq. (35), we obtain
\[ \text{Im}(C_{9/10}^{\ell \ell}) \sim \frac{\pi}{\sqrt{2} G_F} \alpha_e \beta_{9/10}^{\ell \ell} \cdot \Delta A_{CP}^{\ell \ell}, \] (62)
where
\[ \beta_{9/10}^{\ell \ell} = \frac{F_{e,\ell} \pm F_{e,\ell}}{c_K F_{Q_2} + d_K F_{d_2} - c_{\pi} F_{Q_1} - d_{\pi} F_{d_3}}. \] (63)
Values of \( \beta_{9/10}^{\ell \ell} \) for \( \ell = \mu, e \) in (TeV)\(^{-2} \) are given in TABLE III. For \( \Delta A_{CP}^{\ell \ell} \sim 10^{-3} \) we find
\[ \text{Im}(C_{9/10}^{\ell \ell}) \sim 0.03 \text{(TeV)}^2 \cdot \beta_{9/10}^{\ell \ell}, \] (64)
consistent with \( C_{9/10}^{\ell \ell} \sim O(10^{-2}) \) for \( g_{L}^{\ell \ell} = 0, g_{R}^{u,c} \neq 0 \) [18] and for \( \beta_{9/10}^{\ell \ell} = O(1/\text{TeV}^2) \) (models 2, 4 and 5). Models 9 and 10(\( \mu \)) have sizable couplings to leptons, and in addition \( F_{Q_2,z} = 0 \), which bring a factor...
of $c_{\pi,K}/d_{\pi,K}$, see Eq. (63), score $\beta_{YY}^{\ell\ell} = \mathcal{O}(10/\text{TeV}^2)$ and sizable $C_{9/10}^{\ell\ell} = \mathcal{O}(10^{-1})$. As values of $\text{Im}(C_{9/10}^{\ell\ell}) \gtrsim \mathcal{O}(10^{-2} - 10^{-1})$ suffice to induce CP-asymmetries beyond the SM in semileptonic $D$-decays at the few percent level and above [13, 18, 21, 23], all models can simultaneously lead to $|\Delta A_{\text{NP}}^{\text{CP}}| \sim 10^{-3}$ with NP patterns in $c \to u \ell^+\ell^-$ decays.

In FIG. 5 we show the imaginary part of Wilson coefficients with di-electrons (upper plots) and di-muons (lower plots) for different models as in Eq. (62). Plots to the left show lepton vector couplings versus lepton axial vector couplings, $\text{Im}(C_9^{\ell\ell'})$ vs. $\text{Im}(C_{10}^{\ell\ell'})$, respectively. Also given is $\text{Im}(C_{9/10}^{\ell\ell}) = -\text{Im}(C_{9/10}^{\ell\ell'})$ (thin gray line). The lines corresponding to model 2, 4, and 5 end when the corresponding $|\Delta A_{\text{NP}}^{\ell\ell'}| > 10^{-3}$. Results are lepton non-universal as anticipated and sensitive to the lepton doublet and singlet charges. In the plots to the right the correlation (62) between $\text{Im}(C_9^{\ell\ell'})$ (solid) and $\text{Im}(C_{10}^{\ell\ell'})$ (dashed) and $|\Delta A_{\text{NP}}^{\ell\ell'}|$ in the $Z'$-models 2, 9, 10 and 10$\mu$ is made explicit. Curves for models 4 and 5 are only in mild excess of those for model 2, or smaller, see TABLE and are not shown to avoid clutter.

As couplings to electrons and muons differ, lepton non-universality in charm [13, 21, 38] is induced, for example in the ratio of branching ratios of $D \to \pi\mu^+\mu^-$ and $D \to \pi e^+e^-$ using identical kinematic cuts, $R_{\mu}^D$. To better control SM backgrounds from intermediate resonances $R = \phi, \eta, \rho, \ldots$, via $D \to \pi R(\to \ell^+\ell^-)$, interesting regions are for low (high) dilepton mass, below the $\eta$-mass (above the $\phi$-mass), see Ref. for details. We focus on the high mass region as it has fewer sensitivity to unknown strong phases from the resonances.

Using $\beta_{YY}^{\ell\ell}$ from TABLE and Eq. (64) we find that all models yield order one deviations from the universality limit $R_{\mu}^D = 1$. Except for model 10$\mu$, which has smaller couplings to muons by construction, all models can induce significant enhancements or suppressions from the SM. In particular, in the high mass region, for $\phi_R = \pi/2$ and varying strong resonance phases, see Ref. for
details,
\begin{align*}
R^D_\pi & \sim [0.6 \ldots 1.5] \quad \text{(model 2, 4, 5)}, \\
R^D_\eta & \sim [0.2 \ldots 70] \quad \text{(model 9)}, \\
R^D_\eta & \sim [0.2 \ldots 70] \quad \text{(model 10)}, \\
R^D_{\eta'} & \sim [0.03 \ldots 0.8] \quad \text{(model 10\mu)},
\end{align*}
allowing to signal NP.

VI. CONCLUSIONS

Patterns of observables are indispensable for pinning down an underlying NP–dynamics. We looked globally into hadronic and semileptonic charm decays and their respective CP–asymmetries. We find that there is strong benefit in doing so.

Most important, all flavorful, anomaly-free Z′–models in TABLE VI can simultaneously accommodate \(\Delta A_{CP}^{NP} \sim 10^{-3}\) and induce measurable CP–asymmetries in the semileptonic \(c \rightarrow u \ell^+\ell^-\) modes for \(\ell = e\) or \(\ell = \mu\) above the SM. An observation of CP–violation in, for instance, \(D \rightarrow \pi^+\pi^-\) or \(D \rightarrow \pi^0\pi^0\) decays supports a NP–interpretation of \(\Delta A_{CP}\), Eqs. (3) and (4), see FIG. 5.

Additional cross checks are provided by CP–asymmetries in \(D^0 \rightarrow \pi^+\pi^-,\) where probe for U–spin breaking NP, see FIGS. 3 and 4 for present data and future sensitivities, respectively. In addition, isospin violating NP can be observed with projected sensitivities at Belle II in \(D^0 \rightarrow \pi^0\pi^0,\) which CP–asymmetries can exceed \(\Delta A_{CP}\), Eqs. (54) and (57). In the Z′–models lepton non–universality is generic, and observable in the ratio of branching fractions of \(D \rightarrow \pi^+\pi^-\) and \(D \rightarrow \pi^0\pi^0\) decays, as briefly discussed in Section V.

The Z′–model 9 with order one enhancement over the universality limit, \(R^D_\pi \gg 1\), also induces \(A_{CP}^{NP}(\pi^+\pi^-) \sim A_{CP}^{NP}(\pi^0\pi^0) \lesssim 2 \cdot \Delta A_{CP}^{NP}\). Z′–model 10\mu with order one suppression of the universality limit, \(R^D_\pi \ll 1\) exhibits sizable NP U–spin breaking \(A_{CP}^{NP}(\pi^+\pi^-) \ll A_{CP}^{NP}(K^+K^-) \sim \Delta A_{CP}\).

Checking correlations pin down models. Improved data and sensitivities from LHCb and Belle II are important in this program. We encourage and look forward to further CP–studies of rare semileptonic and hadronic charm decays.

Acknowledgments

This work is supported by the Studienstiftung des Deutschen Volkes (MG) and the Bundesministerium für Bildung und Forschung – BMBF (HG).

Appendix A: Experimental input

We extract the modulus of the dominant, SM decay amplitudes from data on branching ratios [27] given in TABLE VI. We use
\begin{equation}
\text{BR}(D \rightarrow P_1 P_2) = \frac{|A_P|^2}{16\pi m_D} \sqrt{1 - \frac{4m^2_P}{m_D^2}} \tau_D, \quad (A1)
\end{equation}
where [29]
\begin{equation}
A_P = \eta_P \lambda_P \eta_P \frac{G_F}{\sqrt{2}} \left(m_P^2 - m_D^2\right) f_0^{D \rightarrow P} m_D P f_P, \quad (A2)
\end{equation}
\(P = \pi, \pi^0, K, \lambda_\pi = \lambda_d = \lambda_K = \lambda_s\) and
\begin{equation}
\eta_P = \begin{cases} 
1 & P = \pi, \pi^0, K \\
\frac{1}{\sqrt{2}} & P = \pi'.
\end{cases} \quad (A3)
\end{equation}
The subscript \(\pi'\) corresponds to the \(D^+ \rightarrow \pi^+\pi^0\) channel, and \(\pi^0 \rightarrow D^0 \rightarrow \pi^0\pi^0\). Relevant form factors \(f_0^{D \rightarrow P}\) and decay constants \(f_P\) are taken from Ref. [40] and [27], respectively. Resulting values of \(a_P > 0\) are given in TABLE VI.

Appendix B: Evolution of Wilson coefficients

The Wilson coefficients \(\tilde{C}_{7,8,9,10}(t)\) at the Z′ mass scale [32] are evolved to the charm mass scale at LO in \(\alpha_s\). The requisite anomalous dimension matrix for the operators \(Q_{7,8,9,10}\) can be inferred from Ref. [41]. We obtain
\begin{equation}
\gamma^0_F = \begin{pmatrix} 6 & 0 & 0 \\ 0 & \frac{6}{N_C} & 0 \\ 0 & 0 & \frac{6}{N_C} \end{pmatrix}, \quad (B1)
\end{equation}
where \(N_C = 3\) is the number of colors. Since QCD conserves parity, \(\gamma^0_F\) is identical for \(Q_1\) and \(Q'_1\). Using Eq. (B1), the Wilson coefficients are evolved to the charm scale, integrating out degrees of freedom at the \((Z', t, b)\)–scales,
\begin{equation}
\tilde{C}(\mu) = U_4(\mu, m_b) \tilde{U}_5(m_b, m_t) \tilde{U}_6(m_t, M_{Z'}) \tilde{C}(M_{Z'}) ,
\end{equation}
where \(\tilde{U}_f(m_1, m_2) \equiv M_f(m_1) U_f(m_1, m_2)\) and \(U_f(m_1, m_2)\) is the evolution matrix from scale \(m_2\) to scale \(m_1\) in an effective field theory with \(f\) active

| mode | BR (mode) | \(a_P\) |
|---|---|---|
| \(D^0 \rightarrow K^+ K^-\) | \((4.08 \pm 0.06) \cdot 10^{-3}\) | 1.19 ± 0.04 |
| \(D^0 \rightarrow \pi^+ \pi^-\) | \((1.455 \pm 0.024) \cdot 10^{-3}\) | 0.94 ± 0.07 |
| \(D^0 \rightarrow \pi^0 \pi^0\) | \((8.26 \pm 0.25) \cdot 10^{-4}\) | 0.71 ± 0.05 |
| \(D^+ \rightarrow \pi^0 \pi^+\) | \((1.247 \pm 0.033) \cdot 10^{-3}\) | 0.77 ± 0.05 |

TABLE V: Measured branching ratios [27] and \(a_P\) parameters from Eq. (A2) for different decay modes.
flavors; $M_f$ is the threshold matrix that matches the two effective theories with $f – 1$ and $f$ active flavors. At LO in $\alpha_s$, the $M_f$ matrices are equal to the identity matrix. For $\mu = m_c$ and $M_Z' = 6$ TeV, one finds
\[
\begin{align*}
\tilde{C}^7_7(m_c) & = 0.829 \tilde{C}^7_7(M_z') , \\
\tilde{C}^8_7(m_c) & = 1.224 \tilde{C}^8_7(M_z') + 4.502 \tilde{C}^8_7(M_z') , \\
\tilde{C}^9_7(m_c) & = 1.404 \tilde{C}^9_7(M_z') - 0.718 \tilde{C}^{10}_7(M_z') , \\
\tilde{C}^{10}_7(m_c) & = -0.718 \tilde{C}^{10}_7(M_z') + 1.404 \tilde{C}^{10}_7(M_z') .
\end{align*}
\]
We use $m_c = (1.280 \pm 0.013)$ GeV, $m_b = (4.198 \pm 0.012)$ GeV \cite{42} and $m_t = (165.9 \pm 2.1)$ GeV \cite{43,44} and central values for the thresholds.

**Appendix C: Hadronic matrix elements**

In order to estimate the NP decay amplitudes, we need to determine the hadronic matrix elements for each operator given by Eqs. \textit{24}–\textit{31}. For that purpose, we employ factorization of currents, $P = \pi, K$,
\[
\langle P^+ P^- | Q_i | D^0 \rangle \tag{C1}
\]
\[
= \langle P^+ | \langle q_1 \Gamma_1 q_2 | 0 \rangle \langle P^- | \langle q_3 \Gamma_2 q_4 | D^0 \rangle B_i^{P^+ P^-} ,
\]
where $Q_i = \langle \bar{q}_1 \Gamma_1 q_2 | q_3 \Gamma_2 q_4 \rangle$ is a 4-quark operator and $\Gamma_{1,2}$ represent possible Dirac and color structures while $q_i$ denote quarks. The factor $B_i^{P^+ P^-}$ parametrizes the deviation of the true hadronic matrix element from its naïve approximation, $B_i^{P^+ P^-}|_{\text{naïve}} = 1$. For the NP contributions we work in this approximation. After employing Fierz identities in the flavor and color space, we find for $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ decays
\[
\langle \bar{Q}_7 \rangle_{K, \pi} = \frac{1}{N_C} \langle \bar{Q}_8 \rangle_{K, \pi} ,
\]
\[
\langle \bar{Q}_8 \rangle_{K, \pi} = F_{d_2, d_1} \chi_{K, \pi}(\mu) \langle Q_1^{d_1} \rangle_{K, \pi} ,
\]
\[
\langle \bar{Q}_9 \rangle_{K, \pi} = \frac{1}{N_C} \langle \bar{Q}_{10} \rangle_{K, \pi} ,
\]
\[
\langle \bar{Q}_{10} \rangle_{K, \pi} = F_{Q_2, q_1} \langle Q_1^{d_1} \rangle_{K, \pi} ,
\]
where $\langle \ldots \rangle_P = \langle P^+ P^- | \ldots \rangle | D^0 \rangle$, $Q_1^{d_1} = \langle \bar{u} \bar{p} \rangle \nu - A \langle \bar{p} \bar{c} \rangle \nu - A$ and $\chi_{K, \pi}(\mu)$ are the usual chiral enhancements generated by $(V - A) \times (V + A)$ operators,
\[
\chi_K(\mu) = \frac{2 M_K^2}{m_c(\mu) m_s(\mu)} ,
\]
\[
\chi_\pi(\mu) = \frac{2 M_\pi^2}{m_c(\mu) (m_d + m_u)(\mu)} ,
\]
with values $\chi_K(m_c) \approx 3.626$ and $\chi_\pi(m_c) \approx 3.655$ at the charm mass scale. For the $\bar{Q}_i$ operators the same relations hold but with the proper exchange of charges $F_{Q_1} \leftrightarrow F_{d_1}$.

For $D^+ \rightarrow \pi^0 \pi^+$ decays we find
\[
\langle \bar{Q}_7 \rangle_{\pi^+ \pi^-} = \frac{1}{N_C} \langle \bar{Q}_8 \rangle_{\pi^+ \pi^-} ,
\]
\[
\langle \bar{Q}_8 \rangle_{\pi^+ \pi^-} = \frac{\chi_\pi(\mu)}{\sqrt{2}} (F_{u_1} - F_{d_1}) \langle Q_1^{d_1} \rangle_{u} ,
\]
\[
\langle \bar{Q}_9 \rangle_{\pi^+ \pi^-} = \frac{1}{N_C} \langle \bar{Q}_{10} \rangle_{\pi^+ \pi^-} = 0 ,
\]
and for the corresponding $\bar{Q}_i$ operators
\[
\langle \bar{Q}_7 \rangle_{\pi^0 \pi^0} = \frac{1}{N_C} \langle \bar{Q}_8 \rangle_{\pi^0 \pi^0} = 0 ,
\]
\[
\langle \bar{Q}_8 \rangle_{\pi^0 \pi^0} = \frac{1}{N_C} \langle \bar{Q}_{10} \rangle_{\pi^0 \pi^0} ,
\]
\[
\langle \bar{Q}_9 \rangle_{\pi^0 \pi^0} = \frac{1}{N_C} \langle \bar{Q}_{10} \rangle_{\pi^0 \pi^0} = 0 ,
\]
For $D^0 \rightarrow \pi^0 \pi^0$ decays we obtain
\[
\langle \bar{Q}_7 \rangle_{\pi^0 \pi^0} = \frac{1}{N_C} \langle \bar{Q}_8 \rangle_{\pi^0 \pi^0} ,
\]
\[
\langle \bar{Q}_9 \rangle_{\pi^0 \pi^0} = \frac{1}{N_C} \langle \bar{Q}_{10} \rangle_{\pi^0 \pi^0} ,
\]
and for the corresponding $\bar{Q}_i$ operators
\[
\langle \bar{Q}_7 \rangle_{\pi^0 \pi^0} = \frac{1}{N_C} \langle \bar{Q}_8 \rangle_{\pi^0 \pi^0} = 0 ,
\]
\[
\langle \bar{Q}_9 \rangle_{\pi^0 \pi^0} = \frac{1}{N_C} \langle \bar{Q}_{10} \rangle_{\pi^0 \pi^0} ,
\]
\[
\langle \bar{Q}_9 \rangle_{\pi^0 \pi^0} = \frac{1}{N_C} \langle \bar{Q}_{10} \rangle_{\pi^0 \pi^0} = 0 ,
\]
where $\langle \ldots \rangle_{\pi^0 \pi^0} = \langle \pi^+ \pi^- | \ldots \rangle | D^0 \rangle$, $\langle \ldots \rangle_q = \langle \bar{q} q | \ldots \rangle | D^0 \rangle$. Eqs. \textit{24}–\textit{31} are obtained in the isospin limit, $m_u = m_d$ and $\epsilon = 0$, since these isospin breaking corrections from within the SM are negligible with respect to the NP ones, $F_{u_1, d_1, q_i} \neq 0$.

**Appendix D: RGE functions**

Eq. \textit{35} for $\Delta A^\text{NP}_{\chi}$ accounts for the running and mixing of operators through the functions $r_{1,2}$. The latter can be obtained from the evolution of the Wilson coefficients described in Appendix B. We obtain
\[
r_1(m_c, M_{Z'}) = \frac{R^{-2}}{3 \sqrt{2} G_F \lambda_s} ,
\]
\[
r_2(m_c, M_{Z'}) = \frac{2 R^{1/2} - R^{-1}}{3 \sqrt{2} G_F \lambda_s} ,
\]
where
\[
R = \left( \frac{\alpha_s^{(4)}(m_b)}{\alpha_s^{(4)}(m_c)} \right)^{\frac{12}{5}} \left( \frac{\alpha_s^{(5)}(m_t)}{\alpha_s^{(5)}(m_b)} \right)^{\frac{2}{5}} \left( \frac{\alpha_s^{(6)}(M_{Z'})}{\alpha_s^{(6)}(m_t)} \right)^{\frac{4}{5}} .
\]

[1] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 122, no. 21, 211803 (2019) doi:10.1103/PhysRevLett.122.211803 arXiv:1903.08726 [hep-ex].

[2] Y. S. Anghis et al. [HFLAV Collaboration], arXiv:1909.12294 [hep-ex].

[3] M. Chala, A. Lenz, A. V. Rusov and J. Scholtz, JHEP 1907, 161 (2019) doi:10.1007/JHEP07(2019)161 arXiv:1903.10490 [hep-ph].

[4] A. Dery and Y. Nir, JHEP 1912, 104 (2019) doi:10.1007/JHEP12(2019)104 arXiv:1909.11242 [hep-ph].

[5] F. Buccella, A. Paul and P. Santorelli, Phys. Rev. D 99 (2019) no.11, 113001 doi:10.1103/PhysRevD.99.113001 arXiv:1902.05545 [hep-ph].

[6] H. N. Li, C. D. Liu and F. S. Yu, arXiv:1903.10638 [hep-ph].

[7] A. Soni, arXiv:1905.00957 [hep-ph].

[8] H. Y. Cheng and C. W. Chiang, Phys. Rev. D 100 (2019) no.9, 093002 doi:10.1103/PhysRevD.100.093002 arXiv:1909.03063 [hep-ph].

[9] A. Khodjamirian and A. A. Petrov, Phys. Lett. B 774 (2017) 235 doi:10.1016/j.physletb.2017.09.070 arXiv:1706.07780 [hep-ph].

[10] A. L. Kagan and L. Silvestrini, arXiv:2001.07207 [hep-ph].

[11] U. Nierste, arXiv:2002.06686 [hep-ph].

[12] A. Pich, PoS LHC 2019 (2019) 078 doi:10.22323/1.350.0078 arXiv:1911.06211 [hep-ph].

[13] S. Fabjan and N. Kosnik, Phys. Rev. D 87, no. 5, 054026 (2013) doi:10.1103/PhysRevD.87.054026 arXiv:1208.0759 [hep-ph].

[14] J. Ellis, M. Fairbairn and P. Tunney, Eur. Phys. J. C 78 (2018) no.3, 238 doi:10.1140/epjc/s10052-018-5725-0 arXiv:1705.03447 [hep-ph].

[15] B. C. Allanach, J. Davighi and S. Melville, JHEP 1902 (2019) 082 doi:10.1007/JHEP02(2019)082 arXiv:1812.04602 [hep-ph].

[16] J. Rathmann and F. Tellander, arXiv:1902.08529 [hep-ph].

[17] D. B. Costa, B. A. Dobrescu and P. J. Fox, arXiv:1905.13729 [hep-th].

[18] R. Bause, M. Golz, G. Hiller and A. Tayduganov, Eur. Phys. J. C 80, no. 1, 65 (2020) doi:10.1140/epjc/s10052-020-7621-7 arXiv:1909.11108 [hep-ph].

[19] J. Aebischer, J. A. Buras, M. Cerdà-Sevilla and F. De Fazio, JHEP 2002 (2020) 183 doi:10.1007/JHEP02(2020)183 arXiv:1912.09308 [hep-ph].

[20] D. Choudhury, K. Deka, T. Mandal and S. Sadhukhan, arXiv:2002.02349 [hep-ph].

[21] S. De Boer and G. Hiller, Phys. Rev. D 98, no. 3, 035041 (2018) doi:10.1103/PhysRevD.98.035041 arXiv:1805.08516 [hep-ph].

[22] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 121 (2018) no.9, 091801 doi:10.1103/PhysRevLett.121.091801 arXiv:1806.10793 [hep-ex].

[23] S. de Boer and G. Hiller, Phys. Rev. D 93, no. 7, 074001 (2016) doi:10.1103/PhysRevD.93.074001 arXiv:1510.00311 [hep-ph].

[24] J. Fuentes-Martin, A. Greljo, J. Martin Camalich and J. D. Ruiz-Alvarez, arXiv:2003.12421 [hep-ph].

[25] S. Fajfar and N. Kosnik, Phys. Rev. D 87, no. 5, 054026 (2013) doi:10.1103/PhysRevD.87.054026 arXiv:1208.0759 [hep-ph].

[26] J. Ellis, M. Fairbairn and P. Tunney, Eur. Phys. J. C 78 (2018) no.3, 238 doi:10.1140/epjc/s10052-018-5725-0 arXiv:1705.03447 [hep-ph].

[27] B. C. Allanach, J. Davighi and S. Melville, JHEP 1902 (2019) 082 doi:10.1007/JHEP02(2019)082 arXiv:1812.04602 [hep-ph].

[28] J. Rathmann and F. Tellander, arXiv:1902.08529 [hep-ph].

[29] D. B. Costa, B. A. Dobrescu and P. J. Fox, arXiv:1905.13729 [hep-th].

[30] R. Bause, M. Golz, G. Hiller and A. Tayduganov, Eur. Phys. J. C 80, no. 1, 65 (2020) doi:10.1140/epjc/s10052-020-7621-7 arXiv:1909.11108 [hep-ph].

[31] J. Aebischer, J. A. Buras, M. Cerdà-Sevilla and F. De Fazio, JHEP 2002 (2020) 183 doi:10.1007/JHEP02(2020)183 arXiv:1912.09308 [hep-ph].

[32] D. Choudhury, K. Deka, T. Mandal and S. Sadhukhan, arXiv:2002.02349 [hep-ph].

[33] S. De Boer and G. Hiller, Phys. Rev. D 98, no. 3, 035041 (2018) doi:10.1103/PhysRevD.98.035041 arXiv:1805.08516 [hep-ph].

[34] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 121 (2018) no.9, 091801 doi:10.1103/PhysRevLett.121.091801 arXiv:1806.10793 [hep-ex].

[35] S. de Boer and G. Hiller, Phys. Rev. D 93, no. 7, 074001 (2016) doi:10.1103/PhysRevD.93.074001 arXiv:1510.00311 [hep-ph].

[36] J. Fuentes-Martin, A. Greljo, J. Martin Camalich and J. D. Ruiz-Alvarez, arXiv:2003.12421 [hep-ph].