Micro-motion parameter extraction of rotating target based on vortex electromagnetic wave radar

Hang Yuan | Ying Luo | Yi-Jun Chen | Jia Liang | Ying-Xi Liu

1Air Force Engineering University, Xi’an, Shaanxi, China
2Engineering University of AP, Xi’an, China

Correspondence
Ying Luo, Air Force Engineering University, Xi’an, Shaanxi, China.
Email: luoying2002521@163.com

Funding information
National Natural Science Foundation of China, Grant/Award Numbers: 61801516, 61971434

Abstract
The vortex electromagnetic (EM) wave radar has the potential to obtain more accurate micro-motion parameters for target recognition. However, with the existing algorithms of micro-motion parameter extraction it is difficult to obtain the real rotation radius and tilt angle of a rotational target in the presence of multiple scattering points in the radar beam. A micro-motion parameter extraction algorithm for rotating targets based on the vortex EM wave radar is proposed in this article. The angular Doppler is obtained from the dual-mode vortex EM echoes. The time interval between the maximum and minimum angular Doppler frequency is derived. The relationship between the time interval and micro-motion parameters is shown. By combining the linear Doppler and the angular Doppler, the micro-motion parameters are roughly estimated. Then, fine micro-motion parameters are obtained by using an iterative soft threshold algorithm. The proposed algorithm can extract the real rotation radius and tilt angle in the case of multiple scattering points. The performance and robustness of the algorithm are proved by simulations.

1 INTRODUCTION

Recently, due to its unique properties, the vortex electromagnetic (EM) wave [1] has attracted the attention of radar and communication researchers [2, 3]. Compared with the traditional EM wave [4], the vortex EM wave carries the orbital angular momentum (OAM), and the helical phase fronts structure [5] is generated in the beam. The vortex EM waves of different OAM modes are orthogonal to each other [6]. With traditional non-vortex EM waves, the relative velocity between the radar and the target leads to the linear Doppler effect [7]. The linear Doppler has been used to extract micro-motion parameters of targets [8–10]. Because the linear Doppler is induced by the relative movement along the radar line-of-sight (LOS), only the components of micro-motions projected onto the LOS can be observed from the radar echoes. When the vortex EM waves is used to illuminate the target, the relative velocity causes linear Doppler and angular Doppler effects [11]. By combining the linear Doppler and angular Doppler effects, the real rotation radius and the tilt angle of the target can be extracted [12].

So far, the application of the vortex EM wave has been studied by many researchers, including two-dimensional high-resolution imaging [13] and low signal-to-noise ratio (SNR) imaging [14]. However, the application of the vortex EM wave in extracting micro-motion parameters is still in the development stage. In [12], the Doppler effect and the micro-Doppler effect of a vortex EM wave are investigated and the angular Doppler frequency shift is deducted. The method of micro-motion parameter extraction is discussed under special circumstances in which the tilt angle is 0 or the rotation centre is on the Z-axis. This work can benefit the applications of vortex EM waves. However, the method of angular Doppler extraction and the angular Doppler frequency shift of the target composed of multiple scattering points are not discussed.

Due to the coupling relationship between the linear Doppler and the angular Doppler, the angular Doppler cannot be directly obtained. In [15], an effective linear and rotational Doppler separation method and a motion parameter estimation method are proposed. First, the time-frequency graph is obtained by performing a short-time Fourier transform (STFT) in the echo. Subsequently, the angular Doppler is...
extracted by performing Hough transform in the time-frequency graph. However, the presence of multiple scattering points in the radar beam is not considered. When the target contains multiple scattering points, the difference between the linear micro-Doppler of each scattering point makes it difficult to compensate them with a same compensation operation; therefore, the linear micro-Doppler cannot be removed and the angular micro-Doppler will be contaminated. As a result, it is difficult to estimate the micro-motion parameters accurately.

In this article, a micro-motion parameter extraction algorithm for a rotating target based on the vortex EM wave is studied to offer a solution to solve these problems. First, the extreme value of the angular Doppler frequency shift in which the target is located in an arbitrary position is derived. Subsequently, an extraction algorithm for the angular Doppler based on dual echoes is proposed. The range-slow-time profile can be obtained by performing fast Fourier transform (FFT) for the echo in the fast-time domain. The angular Doppler is extracted by conjugate multiplication for the dual-mode profiles, and the linear Doppler is extracted from the echo received at the centre of the antenna array. By combining the angular Doppler and the linear Doppler, the micro-motion parameters are estimated. Compared with the existing algorithms, the advantage of the proposed algorithm is that the real rotation radius and tilt angle can be estimated in the presence of multiple scattering points in the radar beam.

The article is organised as follows: In Section 2, the radar echo model and the position relationship between the target and the radar are displayed. The extreme value of the angular Doppler frequency shift is derived in Section 3. The method of extraction of micro-motion parameters is proposed in Section 4. In Section 5, the proposed algorithm is verified. Conclusions are given in Section 6.

## 2 | OBSERVATION MODEL

So far, many methods of generating vortex electromagnetic waves have been proposed [16–18]. The single-in-multiple-out model of generating vortex EM wave is used in this article. As shown in Figure 1, the linear frequency modulation (LFM) signal \( s_{\text{trans}}(t) \) emitted by an antenna located at the origin of the coordinate can be given as

\[
s_{\text{trans}}(t) = \text{rect}(t/T_p) e^{j2\pi(f_c t + 0.5f^2t^2)}
\]

where \( \text{rect}(t/T_p) \) is a rectangular window, \( T_p \) is the pulse duration, \( f_c \) is the carrier frequency, \( \gamma \) is the frequency modulation rate. The echo is received by the uniform circular array (UCA) with radius \( a \). The UCA is composed of \( N \) antennas, which are placed equidistant in a counter clockwise direction, and each antenna is multiplied by a phase term \( \exp(\alpha a (2\pi/N)n), n = 0, 1, ..., N-1 \), where \( \alpha \) is the OAM modes. Assuming that the targets are the ideal scattering point, the multi-target echo \( s(t,\alpha) \) received by the UCA can be expressed as shown in [16].

**FIGURE 1** Geometry of uniform circular array (UCA) radar and target

\[
s(t,\alpha) = \sum_{p=1}^{n} r^2 \sigma_p N F_s \left[ k(t - \tau_p(t)) \alpha \sin \theta_p(t) \right] \]

\[
\text{rect} \left( \left( t - \tau_p(t)/T_p \right) \right) e^{j2\pi\left( \left( f - f_c(t) \right)^2 + 0.5f^2(1-\tau_p(t))^2 \right)} e^{-j2\pi\tau(t)}
\]

where \( k(t) = 2\pi(f_c + \gamma t)/c, \tau(t) = 2r(t)/c \), and \( c \) is the light speed.

The geometric relationship between the target and the radar array is shown in Figure 1. \( O \) is the origin of the Cartesian coordinate system \( OXYZ \), located at \( (0,0,0) \). \( O' \) is the origin of the reference coordinate system \( O'X'Y'Z' \), located at \( (x_c, y_c, z_c) \). The reference coordinate system is parallel to the Cartesian coordinate system. The scattering point \( P \) is located in the target-local coordinate system \( O'X'Y'Z' \), rotating around the origin \( O' \). The location of \( P \) in the spherical coordinates can be written as \( (\phi_p, \theta_p, \varphi_p) \). The rotation frequency is \( f \) and the rotation radius is \( r_a \). Therefore, the coordinates of \( P \) in the target-local coordinate system are

\[
\left( x_p(t), y_p(t), z_p(t) \right)^T = \left( r_a \cos(2\pi ft), r_a \sin(2\pi ft), 0 \right)^T
\]

where \( \tau(t) \) is the transpose operation. Assume that the Euler angle between the reference coordinate system and the target-local coordinate system is \( (\phi_e, \theta_e, \varphi_e) \). The coordinates of \( P \) in the reference coordinate system are

\[
\left( x_p(t), y_p(t), z_p(t) \right)^T = R_{\text{init}} \left( x_p(t), y_p(t), z_p(t) \right)^T + \left( x_c, y_c, z_c \right)^T
\]

where
\[
R_{\text{init}} = \begin{bmatrix}
cos \phi_e & -sin \phi_e & 0 \\
sin \phi_e & cos \phi_e & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & cos \theta_e & -sin \theta_e \\
0 & sin \theta_e & cos \theta_e
\end{bmatrix}
\times
\begin{bmatrix}
cos \phi_e & -sin \phi_e & 0 \\
sin \phi_e & cos \phi_e & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\]
(5)

\[
\phi_e \text{ and } \phi_e \text{ are the degree of rotation of the target about the}
\]
\[
Z^* \text{-axis, and only the initial phase will be affected by them.}
\]
Since the micro-motion parameter extraction is independent of the initial phase of the target, the rotation matrix \( R_{\text{init}} \) can also be expressed as
\[
R_{\text{init}} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\]
(6)

It is assumed that the target is relatively stationary on the radar during the pulse duration. The slow time \( t_m \) is the pulse interval time sequence, and the fast time \( t \) is expressed as the pulse duration time sequence. \( s(t, \alpha) \) can be rewritten as
\[
s(t, t_m, \alpha) = \sum_{p=1}^{n} \sigma_p N_{j_p} \left(k(t-t_p(t_m)) \sin \theta_p(t_m)\right)
\]
\[
\times \text{rect}\left(\left(t-t_p(t_m)\right)/T_p\right) e^{j2\pi f(t-t_p(t_m))} e^{j\phi_p(t_m)}
\]
(7)

Finally, the echo \( s_o(t, t_m, \alpha) \) received by the antenna at the centre of the array can be expressed as
\[
s_o(t, t_m, \alpha) = \sum_{p=1}^{n} \sigma_p \text{rect}\left(\frac{t-t_p(t_m)}{T_p}\right) e^{2\pi f(t-t_p(t_m)) + j0.5} e^{j\phi_p(t_m)}
\]
(8)

### 3 | ANGULAR DOPPLER EFFECT

In this section, the extreme value and the extreme value point of the angular Doppler frequency shift are derived.

When the scattering point rotates in space, the azimuth angle \( \varphi_p(t_m) \) can be expressed as shown in [12].

\[
\varphi_p(t_m) = a \tan \left(\frac{y_p(t_m)}{x_p(t_m)}\right)
\]
(9)

\[
= a \tan \left(\frac{\cos \theta_a x_a + \sin(\frac{2\pi f t_m}{r_a}) + y_c}{r_a \cos(\frac{2\pi f t_m}{r_a}) + x_c}\right)
\]

The angular Doppler frequency shift \( f_A(t_m) \) is

\[
f_A(t_m) = \frac{a}{2\pi} \varphi_p^2(t_m)
\]
(10)

\[
= \frac{a}{2\pi} \left(\frac{\sqrt{r_f^2 + \cos^2 \theta_a x_a^2 + y_c^2 \cos(2\pi f t_m - \varphi_c)} + \frac{C_1 + 0.5r_f^2 \cos \theta_a \cos(4\pi f t_m + 2r_a \sqrt{x_a^2 + y_c^2 \cos(2\pi f t_m - \varphi_c)}}}{2}\right)
\]

where \( C_1 = x_a^2 + y_c^2 + 0.5r_f^2 \cos \theta_a \) and \( \varphi_c = \text{atan}(y_c/x_c) \), \( \varphi' = \text{atan}(y_c/x_c \cos \theta_c) \). Since \( \theta_c \neq 0 \) and the rotation centre is not on the Z-axis, the frequency shift of the angular Doppler is complicated. It can be found that the cosine functions with different initial phases exist in the numerator and denominator of Equation (8); as a result, the extreme value and the extreme value point of the angular Doppler frequency shift are difficult to obtain by derivation. The denominator of \( f_A(t_m) \) can be rewritten as

\[
A(t_m) = r_a^2 \left(1 - \sin^2 \theta_a \right) \sin(\frac{2\pi f t_m + x_a^2 + y_c^2}{r_a})
\]
(11)

\[
+ r_f^2 \cos(2\pi f t_m + 2r_a \sqrt{x_a^2 + y_c^2 \cos(2\pi f t_m - \varphi_c}})
\]

when \( 2r_a \sqrt{x_a^2 + y_c^2} = \varphi_c + \pi, A(t_m) \) can be expressed as

\[
A(t_m) = \frac{\varphi_c + \pi}{2\pi f t_m}
\]
(12)

\[
A(t_m) = r_a^2 \left(1 - \varphi_c^2 \sin^2 \theta_a \sin(2\pi f t_m)\right) > A(t_m)
\]

where \( r_a = \sqrt{x_a^2 + y_c^2} \) and \( A(t_m) \) is the minimum of \( A(t_m) \). When \( r_a \leq r_a \), the minimum of \( A(t_m) \) tends to zero. The interval of extremum of the denominator is related to the interval of extremum of \( f_A(t_m) \). The interval of extremum points of \( A(t_m) \) can be roughly estimated by the interval of extremum points of \( f_A(t_m) \). On ignoring the constant term in Equation (11), \( d(t_m) \) can be expressed as

\[
d(t_m) = 0.5r_f^2 \sin^2 \theta_a \cos(2\pi f t_m + 2r_a \sqrt{x_a^2 + y_c^2 \cos(2\pi f t_m - \varphi_c}})
\]
(13)

Due to the influence of the initial phase in the cosine function, the two cases \( \varphi_c = 0 \) and \( \varphi_c \neq 0 \) need to be discussed separately.

### A. \( \varphi_c \neq 0 \)

The cosine function is expressed as a Maclaurin series and \( d(t_m) \) can be rewritten as

\[
d(t_m) \approx 0.5r_f^2 \sin^2 \theta_a \left(1 - 0.5(2\pi f t_m)^2\right) + 2r_a \sqrt{x_a^2 + y_c^2 \sin(2\pi f t_m - \frac{2\pi f t_m}{6})}
\]
(14)
In Equation (10), $d_2(t_m)$ changes with $t_m$ and it can be expressed as

$$d_2(t_m) = 4r_a \pi f t_m \left(y_c - \pi f t_m(x_c + r_a \sin^2 \theta_c) - \frac{2y_c \pi f^2 t_m^2}{3} \right)$$

(15)

It is assumed that the superscript “$r$” is the derivation operation. $d_2'(t_m)$ can be obtained as

$$d_2'(t_m) = 4r_a \pi f \left(-2y_c \pi f t_m^2 - 2t_m \pi f(x_c + r_a \sin^2 \theta_c) + y_c \right)$$

(16)

The extreme value points $t_1$ and $t_2$ are presented as follows:

$$t_1 = \frac{r_a \sin^2 \theta_c + x_c + \sqrt{(x_c + r_a \sin^2 \theta_c)^2 + 2y_c^2}}{-2y_c \pi f}$$

$$t_2 = \frac{r_a \sin^2 \theta_c + x_c - \sqrt{(x_c + r_a \sin^2 \theta_c)^2 + 2y_c^2}}{-2y_c \pi f}$$

(17)

By substituting $t_1$ and $t_2$ into (10), the extreme values $f_1$ and $f_2$ can be given by

$$f_1 = f_2(t_1)$$

$$f_2 = f_2(t_2)$$

(18)

It can be seen that $f_1$ and $f_2$ will be affected by the rotation frequency $f$, the tilt angle $\theta_c$, the rotation radius $r_a$, and the rotation centre $(x_c, y_c, z_c)$. In this section, the time interval $\Delta t$ between the maximum value point and the minimum value point is used to extract the micro-motion parameters. $\Delta t$ is

$$\Delta t = |t_1 - t_2|$$

$$= \frac{\sqrt{(x_c + r_a \sin^2 \theta_c)^2 + 2y_c^2}}{y_c \pi f}$$

$$= \frac{1}{\pi f} \sqrt{\left(\frac{x_c + r_a \sin^2 \theta_c}{y_c} \right)^2 + 2}$$

(19)

If the target rotation centre $(x_c, y_c, z_c)$ is known, then $r_a \sin^2 \theta_c$ can be obtained.

$$r_a \sin^2 \theta_c = y_c \sqrt{(\pi f \Delta t)^2 - 2 - x_c}$$

(20)

when $y_c = 0$, the initial phase $\theta_c$ in Equation (10) is eliminated. $d(t_m)$ is rewritten as

$$d(t_m) = 0.5r_a^2 \sin^2 \theta_c \cos 4 \pi f t_m + 2r_a x_c \cos 2\pi f t_m$$

(21)

$d_2'(t_m)$ can be obtained as

$$d_2'(t_m) = -4r_a \pi f \sin(2\pi f t_m)(x_c + r_a \sin^2 \theta_c \cos 2\pi f t_m)$$

(22)

It can be found that the number of extreme points is affected by $x_c$ and $r_a \sin^2 \theta_c$ when $y_c = 0$. So, the two cases, $x_c \geq r_a \sin^2 \theta_c$ and $x_c < r_a \sin^2 \theta_c$, are discussed separately.

B. $y_c = 0, x_c \geq r_a \sin^2 \theta_c$

In this case, the extreme points $t_1$ and $t_2$ of Equation (22) can be expressed as

$$t_1 = 0$$

$$t_2 = \frac{1}{2f}$$

(23)

By substituting $t_1$ and $t_2$ into (8), $f_1$ and $f_2$ can be obtained

$$f_1 = \frac{r_a \cos \theta_c}{r_a + x_c}$$

$$f_2 = \frac{r_a \cos \theta_c}{r_a - x_c}$$

(24)

The ratio $y_1$ is

$$y_1 = \frac{f_1}{f_2} = \frac{x_c - r_a}{x_c + r_a}$$

(25)

when the value of $x_c$ is known, the radius of rotation $r_a$ can be obtained.

C. $y_c = 0, x_c < r_a \sin^2 \theta_c$

In this case, the extreme points $t_1, t_2, t_3$ of Equation (22) can be expressed as

$$t_1 = 0$$

$$t_2 = \frac{1}{2f}$$

$$t_3 = \pm \frac{1}{2\pi f} \cdot a \cos \frac{-x_c}{r_a \sin^2 \theta_c}$$

(26)

By substituting $t_1, t_2, t_3$ into Equation (10), the extreme values $f_1, f_2, f_3$ can be displayed as follows:

$$f_1 = \frac{r_a \cos \theta_c \cos \left(r_a \sin^2 \theta_c \cos \theta_c + x_c^2 \right)}{r_a + x_c}$$

$$f_2 = \frac{r_a \cos \theta_c}{r_a - x_c}$$

(27)

$$f_3 = \frac{r_a \cos \theta_c \left(r_a^2 \sin^2 \theta_c + x_c^2 \right)}{r_a \sin^2 \theta_c \left(x_c^2 + r_a^2 \right) + r_a \left(3x_c^2 - r_a^2 \sin^4 \theta_c \right)}$$
The micro-motion information can be obtained by the time interval $\Delta t$ of extreme points and $\Delta t$ is

$$\Delta t = |t_1 - t_2| = \frac{1}{2\pi f} \frac{a \cos \frac{-x_c}{r_a \sin \theta_c}}{r_a} = \frac{x_c}{\cos(2\pi f \Delta t)}$$

(29)

In this section, the extraction algorithm of micro-motion parameters is proposed. First, an extraction method of the angular Doppler based on dual echoes is proposed. Subsequently, rough micro-motion parameters are extracted by combining the linear Doppler and the angular Doppler. Lastly, the iterative soft thresholding (IST) algorithm [19] is used to estimate the fine micro-motion parameters.

First, an extraction method of the angular Doppler based on dual echoes is proposed. It is assumed that the reference signal is $s_{ref}(t) = \text{rect}(t/T_p)e^{j2\pi f (t+0.5\tau^2)}$. The echo signal $s(t, t_m, \alpha)$ is multiplied by the reference signal $s_{ref}(t)$, and then it leads to

$$s(t, t_m, \alpha) = \sum_{p=1}^{n} \varphi_s N e^{j\phi_s} \left(k(t - \tau_p(t_m))a \sin \theta_p(t_m)\right) e^{j2\pi(0.5\tau_p^2(t_m)-\tau_p(t_m)it)} (30)$$

It is assumed that FFT$\{\cdot\}$ is the fast Fourier transform. $S(f_r, t_m, \alpha)$ can be expressed as

$$S(f_r, t_m, \alpha) = \text{FFT}\{s(t, t_m, \alpha)\} = \sum_{p=1}^{n} \varphi_s N e^{j\phi_s} e^{-j2\pi(f_r+\tau_r(t_m))\tau_r(t_m)} e^{j2\pi(0.5\tau_p^2(t_m)-\tau_p(t_m)it)} (31)$$

and

$$H(f_r) = \text{FFT}\{f(t)\left|\text{rect}\left(\frac{t}{T_p}\right)\right.\} (32)$$

The amplitude spectrum of $H(f_r)$ is similar to that of the sinc function. After removing the RVP term, the dual-mode signals can be expressed as

$$S(f_r, t_m, \alpha) = \sum_{p=1}^{n} \varphi_s N e^{j\phi_s} e^{-j2\pi f_r \tau_r(t_m)} H(f_r + j\tau_r(t_m))$$

(33)

where $P_{j,k}(f_r, t_m)$ is $e^{-j2\pi(f_r+\tau_r(t_m))\tau_r(t_m)} \varphi_s e^{j2\pi(f_r+\tau_r(t_m))\tau_r(t_m)}$, and the superscript “*” represents the conjugate operation. Assuming that the rotating target is located in a range resolution unit $f_r = -\tau_r(t_m)$, the peak $S_{\text{peak}}(t_m, \alpha)$ of Equation (33) can be expressed as

$$S_{\text{peak}}(t_m, \alpha) = |H(i)|^2 \sum_{k=1}^{n} \sum_{j=1}^{n} \varphi_s N^2 e^{j\phi_s} e^{j\phi_k} (34)$$

The linear Doppler of Equation (34) is suppressed and the angular Doppler is extracted. Assuming that there is only a single scattering point in the range resolution unit, $S_{2}(f_r, t_m, \alpha)$ can be rewritten as

$$S_{2}(f_r, t_m, \alpha) = N^2 \sum_{k=1}^{n} \varphi_s N^2 e^{j\phi_k} e^{j2\pi f_r \tau_r(t_m)} H(f_r + j\tau_r(t_m))$$

(35)

The linear Doppler of Equation (35) is eliminated and the angular Doppler is extracted. In both cases, the angular Doppler can be extracted.
autocorrelation method on the time-frequency diagram. The maximum frequency shift \( f_{\text{max}} \) in the time-frequency diagram is

\[
f_{\text{max}} = \frac{4\pi r_s f_j \sin(\theta_e - \theta_j)}{c}
\]

and the radial length \( r' \) is

\[
r' = \frac{c f_{\text{max}}}{4\pi f_j} = r_s \sin(\theta_e - \theta_j)
\]

By combining \( r_s \sin^2 \theta_e \) and \( r' \), the rotation radius and tilt angle can be roughly estimated.

Lastly, the IST algorithm is used to estimate the fine micro-motion parameters. The optimisation goal of the IST algorithm can be expressed as

\[
X = \operatorname{argmin}_X \left\{ \| S_{\text{peak}}(t_m, \alpha) - DX \|^2_2 + \mu \| X \|_1 \right\}
\]

where \( D \) is the dictionary, \( \| \cdot \|_2 \) is 2-norm, \( \| \cdot \|_1 \) is 1-norm, and \( \mu \) is the regularisation coefficient. \( X \) is a column vector. To construct the dictionary of the IST algorithm, the parameters \( \sigma_j, n, \theta_e, r_s, f \) in \( S_{\text{peak}}(t_m, \alpha) \) need to be obtained. The number of scattering points \( n \) can be obtained by the time-frequency diagram of the linear Doppler. According to the peak ratio of different scattering points, the relative value \( \sigma_j \) of the normalised scattering coefficient can be obtained. Since the centre of rotation and \( r' \) are known, when \( \theta_e \) is determined, \( r_s \) is also determined. Ignoring the constant \( t_m \) and \( \alpha \) in the dictionary, \( S_{\text{peak}}(t_m, \alpha) \) under different tilt angles can be expressed as \( S_{p}(\theta) \).

\[
S_p(\theta) = |H(0)|^2 \sum_{k=1}^{n} \sum_{j=1}^{n} \sigma_j \theta_e \sum_{j=1}^{n} e^{i2\pi f_j (t_m \sin(\theta_e + \theta_j) + \phi_k + \theta_j)} e^{2\pi f_j (t_m \sin(\theta_e + \theta_j) - \theta_j)}
\]

\[
\text{TABLE 1: Parameters of radar and target}
\]

| Parameters         | Value | Unit |
|--------------------|-------|------|
| Carrier frequency  | 10    | GHz  |
| Pulse duration     | 1     | \( \mu s \) |
| Bandwidth          | 1     | GHz  |
| Pulse repetition frequency | 5000  | Hz    |
| The radius of the uniform circular array (UCA) | 0.1   | m     |
| Target rotation frequency | 20    | Hz    |
| Euler angle        | (0, 0.3491, 0) | rad   |
| Mode               | 1     |      |

\( S_p(\theta) \) is a column vector. A rough value is used to reduce the computation of the IST algorithm. The range of the inclination angle can be determined by the rough value. Then the dictionary \( D \) of the IST algorithm can be expressed as

\[
D = \left[ S_p(\theta_0), S_p(\theta_1), \ldots, S_p(\theta_k) \right]
\]

The target tilt angle can be obtained by the optimisation result of the IST algorithm. The maximum value in \( X \) is selected, and the corresponding \( \theta_k \) is the estimated value of the tilt angle.

The flow of the micro-motion parameter extraction method is shown in Figure 2.

5 | SIMULATION RESULTS AND ANALYSIS

In this section, the extreme value of the angular Doppler frequency in which the target is located in an arbitrary position and the extraction methods of the micro-motion parameters under different conditions are verified. In Section 1, the single-point target model is adopted so that the micro-Doppler shift
can be calculated precisely from the phase derivation of the echo. This will be helpful to better verify the analyses of Section III and IV. In Section 2, a target model with 2 scattering points is used in the simulation to investigate the performance of the method under the condition of multiple scattering points.

5.1 | Micro-Doppler effect

The parameters of the radar and target are listed Table 1. The rotation radius is 0.2 m. It is assumed that radial length \( r' \) of 0.0684 m and rotation centre are known.

(1) \( \gamma_c \neq 0 \)

When the rotation centre is \((0.1, 0.1, 50)\) m, the angular Doppler shift is as shown in Figure 3. The solid line is the theoretical value of the angular Doppler frequency, and the dashed line is the approximate value of the angular Doppler frequency after Maclaurin expansion. The maximum is 149.2 Hz at 0.0818s, and the minimum is 22.52 Hz at 0.0508s. It is found that the theoretical curve can be expressed accurately by the approximate curve.

According to Equation (17), the extreme values are 22.9 and 104.4 Hz, respectively. The minimum can be estimated accurately. Because the curve changes sharply at the maximum, it leads to high error. According to Equation (19), the time interval between the extreme points is \( \Delta t = 0.03s \). The time interval can be estimated accurately. The target tilt angle can be obtained by combining Equations (20) and (37). The tilt angle of the target is estimated as 0.3858 rad, and the radius of rotation is estimated as 0.1949 m. The micro-motion parameter can be reconstructed with high precision.

\[ \gamma_c = 0, x_c \geq r_\theta \sin^2 \theta_c \]

When the centre of rotation is \((0.1, 0, 50)\) m, the angular Doppler frequency is as shown in Figure 4. The maximum is 75.18 Hz at 0.025s, and the minimum is 25.06 Hz at 0.05s. According to Equation (24), the extreme values are 25.05 and 75.17 Hz, respectively. The maximum and minimum can be estimated accurately. According to Equation (25), the rotation radius can be estimated as 0.2 m. Then the tilt angle can be estimated as 0.3484 rad. The micro-motion parameters can be reconstructed with high precision.

\[ \gamma_c = 0, x_c \neq r_\theta \sin^2 \theta_c \]

When the rotation centre is \((0.01, 0, 50)\) m, the angular Doppler frequency is as shown in Figure 5. The maximum is 42.77 Hz at 0.0144 and 0.0356s. The local minimum is 39.57 Hz at 0.025s and the global minimum is 35.8 at 0.05s. According to Equation (27), the extreme values are estimated as 35.79, 39.56 and 43.93 Hz. The extreme values can be estimated accurately. The time interval is 0.0106s. Then the target tilt angle can be estimated as 0.3914 rad by combining Equations (20) and (37). The rotation radius is estimated as 0.1794 m. \( \Delta t \) cannot be accurately estimated by the interval of the extremum points; therefore, the micro-motion parameters cannot be accurately estimated.

The extreme value of the angular Doppler frequency is verified by simulation. If the linear Doppler and angular Doppler can be obtained, the micro-motion parameters can be estimated accurately. Compared with the method in [12], the tilt angle and rotation radius can be extracted, in which the target is located in an arbitrary position.

5.2 | Micro-motion parameter extraction

In this section, the effectiveness of the micro-motion parameter extraction method is verified. In practical
applications, the centre of rotation deviates from the Z-axis. The proposed algorithm can be applied, as estimated, in the presence of multiple scattering points in the radar beam. In this case, the performance of the algorithm is discussed when there are single or multiple scattering points in the range resolution unit. The radar and target parameters are shown in Table 1. The target consists of two scattering points symmetrical about the centre of rotation, and the centre of rotation is (0.1, 0.15, 50) m. The scattering coefficient of point 1 is 1 and that of point 2 is 0.8.

When the radius of rotation is 1 m, the range-slow-time image can be obtained by performing FFT for $s_0(t; t_m, \alpha)$, and the image is shown in Figure 6 (a). There is only one scattering point in a range resolution unit, and the radial length can be estimated as 0.45 m. Compared with the theoretical value 0.3387 m, the radial length is not accurately estimated. The time-frequency graph can be obtained by performing STFT for the echo of the red box in Figure 6a. The time-frequency graph of a single range unit is shown in Figure 6b. The maximum frequency is 2875 Hz. According to Equation (37), the radial length $r'$ can be estimated as 0.3432 m. Since the amplitude ratio of the curve peaks of the two scattering points is approximately 0.8, the normalised scattering coefficients of the two scattering points are estimated as 1 and 0.8. To separate the echoes of different scattering points, the target trajectory in the range-slow-time domain needs to be obtained. The trajectories of the scattering points are obtained and extracted by the skeleton. The range units on the trajectory are extracted, and the scattering points are separated. The time-frequency graph of scattering point 1 is shown in Figure 6c.
Subsequently, the time-frequency diagrams of the angular Doppler can be obtained by performing STFT for $S_{\text{peak}}(t_m, \alpha)$ and the results are shown in Figure 7. The time-frequency diagrams of scattering point 1 are shown in Figure 7a, and the time-frequency diagrams of scattering point 2 are shown in Figure 7b. By selecting the maximum frequency and the minimum frequency, the time interval $\Delta t = 0.036$s is obtained. According to Equations (20) and (37), the rough micro-motion parameters, including the rotation radius $\hat{r}_a = 0.9387m$ and tilt angle $\hat{\theta}_e = 0.3774$rad, are obtained.

The search range is set near the rough value. So, according to the obtained rough value, the parameters of the IST algorithm can be set as $\hat{\theta}_0 = 0.2264$rad, $\hat{\theta}_k = 0.5241$rad, and $\hat{\theta}_k - \hat{\theta}_{k-1} = 0.006$. The rotation radius $\hat{r}_a = 1.041m$ and tilt angle $\hat{\theta}_e = 0.3394$rad are precisely estimated by the IST algorithm.

When the radius of rotation is 0.2 m, the time-frequency graphs of the linear Doppler and angular Doppler are shown in Figure 8. In this case, the target motion is a line in the range-slow-time domain. The linear Doppler time-frequency diagram is shown in Figure 8a. According to Equation (37), the target radial length is calculated as $r' = 0.0671m$. The normalised scattering coefficient of the two scattering points are estimated as 1 and 0.8, respectively.

Subsequently, the time-frequency diagram of the angular Doppler is shown in Figure 8b. The time interval between the maximum frequency location and the minimum frequency location is 0.029s. Compared with the ideal time

![Figure 6](https://example.com/figure6.png)

**Figure 6** Imaging results when the rotation radius is 1 m. (a) The range-slow-time image, (b) The time-frequency graph of a single range unit, (c) The time-frequency graph of the scattering point 1

![Figure 7](https://example.com/figure7.png)

**Figure 7** The time-frequency diagram of the angular Doppler. (a) Scattering point 1, (b) Scattering point 2

![Figure 8](https://example.com/figure8.png)

**Figure 8** The time-frequency diagram of the linear Doppler and angular Doppler.
interval of 0.0266s, the time interval cannot be accurately estimated. According to Equations (20) and (37), the tilt angle is estimated as 0.8563 rad and the rotation radius is estimated as 0.0899 m. To obtain the micro-motion parameters, the IST algorithm is applied. The parameters of the IST algorithm can be set as $\theta_0 = 0.297\text{rad}$, $\theta_k = 1.412\text{rad}$, and $\theta_k - \theta_{k-1} = 0.022\text{rad}$. The rotation radius is estimated as $\hat{r}_n = 0.2022\text{m}$ and the tilt angle is estimated as $\hat{\theta}_c = 0.3451\text{rad}$. The target micro-motion parameters can be accurately estimated. Compared with the method in [15], the tilt angle and rotation radius can be extracted in the presence of multiple scattering points.

It is found that the proposed algorithm mainly consists of two parts: the micro-motion parameters, which are roughly estimated from the time-frequency diagrams of angular Doppler and the fine estimated value, which is further obtained by the IST algorithm. The first part does not require iteration and search operations, and so it takes a very short computation time, which is consistent with the traditional method. The second part contains iteration and search operations; however, because the micro-motion parameters have been roughly estimated in the first part, the computation time will not be very long. On our personal computer, with 16 GB memory and GeForce MX350 graphics card, it takes about 4s to run the programme.

5.3 | Robustness analysis

Since radar works in a noisy environment, the robustness of the algorithm to noise is demonstrated. In this section, the complex Gaussian white noise is added to the echo. The radar
parameters and the target parameters are consistent with those in section B.

First, the performance of the proposed algorithm is demonstrated when the rotation radius is 1 m. When the signal-to-noise ratio (SNR) is 0 dB, the time-frequency diagram of the linear Doppler and the angular Doppler are shown in Figure 9 (a) and (b), respectively. According to Figure 9b, the time interval between the maximum frequency and the minimum frequency is 0.035 s. The radial length $\gamma' = 0.3432 m$ is obtained by Figure 9a. The rotation radius $\tilde{\gamma}_e = 1.0811 m$ and tilt angle $\tilde{\theta}_e = 0.3266 rad$ are obtained. Even if the SNR is 0 dB, the micro-motion parameters can be accurately extracted.

To verify the robustness of the algorithm, the error curve of the tilt angle at different SNR is exhibited in Figure 10a, and the error curve of the rotation radius at different SNR is exhibited in Figure 10b. Because scattering points need to be separated, the operation is greatly affected by noise. With the decrease of SNR, the estimation accuracy of micro-motion parameters also decreases.

Finally, the performance of the proposed algorithm is demonstrated when the rotation radius is equal to 0.2 m. When the SNR is 0 dB, the time-frequency diagram of the linear Doppler is shown in Figure 11. The radial length $\gamma' = 0.0671 m$ can be estimated by Figure 11. The IST algorithm is applied, and the rotation radius $\tilde{\gamma}_e = 0.1996 m$ and the tilt angle $\tilde{\theta}_e = 0.3459 rad$ are estimated.

The error curve of the tilt angle at different SNR is exhibited in Figure 12a, and the error curve of the rotation radius at different SNR is exhibited in Figure 12b. When the rotation radius is equal to 0.2 m, there is no need to separate the scattering points, and the algorithm performance is less affected by noise. Compared with the case of 1 m rotation radius, the micro-motion parameters can be estimated more accurately in low SNR.

6 CONCLUSION

Vortex electromagnetic waves have attracted much attention in radar applications. In this article, the extraction algorithm of micro-motion parameters for rotating targets is studied. Compared with the existing algorithms, the advantage of the proposed algorithm is that it can be applied in the presence of
multiple scattering points in the radar beam. The algorithm takes the rotation centre as the known condition. But in practice, the rotation centre is not obtained. In the future, we will study the extraction of the micro-motion parameters when the rotation centre is unknown.

ACKNOWLEDGMENTS
This work was supported by the National Natural Science Foundation of China under grants 61971434 and 61801516.

CONFLICT OF INTEREST STATEMENT
None.

PERMISSION TO REPRODUCE MATERIALS FROM OTHER SOURCES
None.

ORCID
Ying Luo https://orcid.org/0000-0003-1460-4289
Yi-Jun Chen https://orcid.org/0000-0001-9264-7490

REFERENCES
1. Yuan, T., et al.: Beam steering for electromagnetic vortex imaging using uniform circular arrays. IEEE Antennas Wirel. Propag. Lett. 16, 704–707 (2017). https://doi.org/10.1109/lawp.2016.2600404
2. Chen, R., et al.: Orbital angular momentum waves: generation, detection, and emerging applications. IEEE Communications Surveys & Tutorials. 22(2), 840–868 (2020)
3. Liu, K., et al.: Generation of orbital angular momentum beams for electromagnetic vortex imaging. IEEE Antenn. Wireless Propag. Lett. 15, 1873–1876 (2016). https://doi.org/10.1109/lawp.2016.2542187
4. Liu, K., et al.: Generation of OAM beams using phased array in the microwave band. IEEE Trans. Antenn. Propag. 64(9), 3850–3857 (2016). https://doi.org/10.1109/tap.2016.2589960
5. Zhou, Y., et al.: Orbital angular momentum generation via a spiral phase microsphere. Opt. Lett. 43(1), 34–37 (2018)
6. Chen, R., et al.: A 2-D FFT-based transceiver architecture for OAM-OFDM systems with UCA antennas. IEEE Trans Veh Technol. 67(6), 5481–5485 (2018). https://doi.org/10.1109/rtv.2018.2817230
7. Su, N., et al.: Detection and classification of maritime target with micro-motion based on CNNs. Journal of Radars. 7(5), 565–574 (2018)
8. Hu, J., et al.: Three-dimensional interferometric imaging and micro-motion feature extraction of spinning space debris in low-resolution radar. J. Appl. Remote Sens. 12(4), 046013 (2018). https://doi.org/10.1117/1.jrs.12.046013
9. Luo, Y., et al.: Micro-doppler feature extraction for wideband imaging radar based on complex image orthogonal matching pursuit decomposition. IET Radar, Sonar Navig. 7(8), 914–924 (2013). https://doi.org/10.1049/iet-rsn.2012.0327
10. Zhao, F., et al.: Micro-motion feature extraction of a rotating target based on interrupted transmitting and receiving pulse signal in an anechoic chamber. Electronics. 8(9), 1028 (2019). https://doi.org/10.3390/electronics8091028
11. Mohammadi, S.M., et al.: Orbital angular momentum in radio—a system study. IEEE Trans. Antenn. Propag. 58(2), 565–572 (2010). https://doi.org/10.1109/tap.2009.2037701
12. Luo, Y., et al.: Doppler effect and micro-Doppler effect of vortex-electromagnetic-wave-based radar. IET Radar, Sonar Navig. 14(1), 2–9 (2020). https://doi.org/10.1049/iet-rsn.2019.0124
13. Bu, X., et al.: Implementation of vortex electromagnetic waves high-resolution synthetic aperture radar imaging. IEEE Antenn. Wireless Propag. Lett. 17(5), 764–767 (2018). https://doi.org/10.1109/lawp.2018.2814980
14. Li, R., et al.: Electromagnetic vortex imaging based on multiple measurement vectors in low SNR condition. In: 2019 IEEE International Conference on Computational Electromagnetics (ICCEM), pp. 1–3. (2019)
15. Wang, Y., et al.: Detection of rotational object in arbitrary position using vortex electromagnetic waves. IEEE Sensor. J. 21(4), 4989–4994 (2021). https://doi.org/10.1109/j sensors.2020.3032665
16. Liu, K., et al.: Passive OAM-based radar imaging with single-in-multiple-out mode. IEEE Microw. Wireless. Compon. Lett. 28(9), 840–842 (2018). https://doi.org/10.1109/lmwc.2018.2852146

17. Yuan, T., et al.: Electromagnetic vortex imaging using uniform concentric circular arrays. IEEE Antenn. Wireless. Propag. Lett. 15, 1024–1027 (2016). https://doi.org/10.1109/lawp.2015.2490169

18. Guo, Z., et al.: Advances of research on antenna technology of vortex electromagnetic waves. Journal of Radars. 8(5), 631–655 (2019)

19. Bi, H., Bi, G.: Performance analysis of iterative soft thresholding algorithm for L1 regularisation based sparse SAR imaging. In: 2019 IEEE Radar Conference (RadarConf), pp. 1–6. (2019)

20. Bai, X., et al.: Radar-based human gait recognition using dual-channel deep convolutional neural network. IEEE Trans. Geosci. Rem. Sens. 57(12), 9767–9778 (2019). https://doi.org/10.1109/tgrs.2019.2929096

How to cite this article: Yuan, H., et al.: Micro-motion parameter extraction of rotating target based on vortex electromagnetic wave radar. IET Radar Sonar Navig. 1–13 (2021). https://doi.org/10.1049/rsn2.12149