An improved micro-thermo-mechanics model for shape memory alloys: analysis and applications

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Abstract

In this paper, micro-mechanics of shape memory alloys (SMA) is investigated for design of smart structure and devices, based upon three dimensional constitutive model. In this work, a micro-mechanics based approach is presented. A modification in temperature field of the above model is proposed, where temperature is taken as function of time. Thermal parameters corresponding to transformation temperatures are introduced here. The model solution is presented for the of a set of initial and boundary conditions based upon any type of thermal parameters. Three particular cases for different thermal and structural loading-stresses are presented here. The solution given here expresses the response of smart dampers for various thermal and structural loading conditions. Moreover, the applications of this system were also investigated.

1. Introduction

Micromechanics is important field which deals with the particulars of any field either thermal, mechanical or both. Micromechanics study becomes more important for smart and functional materials like shape memory alloys (SMA). In materials, where transformation temperature levels are important in determining some specific levels of properties, micro-thermo-mechanics is important field of study. SMA is one those materials which are based on transformation temperature levels and these transformation temperature levels are highly important in its functioning. Micromechanics study is important for any system, because it deals with micro-level of relationships of characteristics of systems and materials.

SMA is one of functional materials that retain 'shape recovery' characteristics because of phase transformation [1–6]. It so happens in SMA, thermal levels and parameters are strongly linked to applied external strain. SMA inherently has two different thermal approaches because of; one lower temperature phase, martensite, generally labeled by M and second higher temperature phase, austenite labeled by A. Therefore, 04 thermal states, the austenitic finish and start, labelled as As and the martensitic start and finish, Ms and Mf are foundations of thermo-elastic characteristics of SMA. Higher thermal austenite phase converts to martensitic phase from higher thermal level A to Ms [5–9].

The shape memory alloys (SMA) have been sensitive to thermal changes and their transformation temperatures play significant role in their behavior [1–7]. The properties of SMA fit well within the needs and extreme conditions based structural designs [5–8] in micro-devices [7–11], smart and intelligent structures
in the ancient and modern seismic resistant structures and becoming stronger candidates for the
Earthquake engineering and super elastic vibration dampers [3–8]. The effects of thermal energy on the critical
stress of SMA are studied in [3, 4]. Further studies about the relation of thermal changes near transformation
temperatures and the super elastic behavior have been conducted [16–18]. It is studied that temperature affects
hysteresis loops as shown in [3–6]. Recently the need for the model linking the working of SMA with dissimilar
applications. So, here exact technique is chosen because of their precision and accurateness [18, 19], whereas
concerns about the health and accuracy of numerical schemes are also discussed in [20].

Here, an effort is made to present a model that is intended to help in the design the SMA based systems for
smart structural applications. Furthermore, we opted to devise some changes in the model, that we have
modified temperature field, and have taken temperature as function of time. This modification would be
affecting the utility and scope of model. This modified model provides a handy tool in illustrating that how
vibration is minimized when temperature field is affecting the phase transition.

The distribution of the paper is like that in main results and modification section we find the two
dimensional analytical solution of selected model, which was not previously presented. In the section 3, we
present results of two different examples for different loadings. Lastly in the summary section, we would present
the summary our work.

2. Formulation of oberaigner, fischer and tanaka model

The constitutive law discussed here relates stresses and alteration of martensitic-austenitic phases. The
mathematical model [1] statuses to analyze the structural response of SMA; in one condition when temperature
is changing and in second when temperature is always constant.

The constitutive law discussed here relates stresses and transformation of phases with heat transfer
Model consists of three sections

1. Thermodynamical part
2. Mechanics related part
3. Phase transformation part

3. Three-dimensional behavior of oberaigner, fischer and tanaka model

The three-dimensional form of the Oberaigner, Fischer and Tanaka (OFT) model [1] is given as;

\[
\frac{\rho}{E} \left( \frac{\partial^2 U}{\partial t^2} \right) - \nabla^2(U) = \left( \frac{aE\Omega}{1 + aE\Omega} \right) \nabla^2(U) - \left( \frac{b\Omega + \theta}{1 + aE\Omega} \right) (Q(x, y, z, t))
\]

\[
\frac{\rho}{E} \left( \frac{\partial^2 U}{\partial t^2} \right) - \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) = \left( \frac{aE\Omega}{1 + aE\Omega} \right) \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) - \left( \frac{b\Omega + \theta}{1 + aE\Omega} \right) \left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} \right)
\]

(1)

The constants \(\rho\), \(E\), \(U\), \(t\), \(\Omega\), \(T\), \(a\), \(b\), \(\theta\) and \(\xi\) used in equation (1) are same as described earlier.

Three-dimensional SMA is depicted in figure 1.

3.1. Solution of three-dimensional oberaigner, fischer and tanaka model

If we substitute here \(\frac{\partial}{\partial t} = \sqrt{E} \left( \frac{\partial^2 U}{\partial t^2} \right) = A_0, \left( \frac{b\Omega + \theta}{1 + aE\Omega} \right) = B_0, v^2(1 - A_0) = \alpha^2, v^2B_0 = \beta^2\) and

\[Q(x, y, t) = \beta^2 \left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} \right)\]
in equation (2), we get
\[
\begin{align*}
\left( \frac{\partial^2 U}{\partial t^2} \right) - \alpha^2 \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) &= -Q(x, y, z, t) \quad 0 < x < L, \, 0 < y < H, \, 0 < z < W, \, t > 0
\end{align*}
\] (2)

Here, it is assumed \( \nu \neq 0 \) and \( \lambda_0 \neq 1 \). Boundary conditions chosen here are \( U(x, 0, z, t) = 0 \), \( U(x, y, 0, t) = 0 \), \( U(x, y, z, 0) = 0 \), and initial conditions are \( \frac{\partial U(x, y, z, t)}{\partial t} = 0 \), \( U(x, y, z, t) = k(x, y, z) \), where \( f \) is the final time.

Analytical solution of equation (2) is presented here according to the methodology discussed in [1]. It is known that general solution includes complementary solution and particular solutions. For solving the complementary part, the non-homogenous part is taken as zero. The method of separation of variables is adopted for the solution of equation (2) discussed in [1]. The equation (2) is separated into two kinds of variables: spatial and time-dependent variables, we get \( \varphi_{\text{amp}}(x, y, z) = \cos \phi(x, y, z) + \sin \phi(x, y, z) \), where \( \phi \) is no-where zero for the equation (2).

Then from the above information,

\[
\varphi_{\text{amp}}(x, y, z) = \sum_{m=1}^{n} \sum_{n=1}^{m} \sum_{p=1}^{m} \sum_{q=1}^{n} \frac{(-1)^{m+n+p+q}}{L^m M^n W^p} \sin \frac{m\pi}{L} \sin \frac{n\pi}{H} \sin \frac{p\pi}{W}, \quad \text{for} \quad m, n, p = 1, 2, 3...
\] (3)

From the above information, it is derived

\[
\sum_{m=1}^{n} \sum_{n=1}^{m} \sum_{p=1}^{m} \sum_{q=1}^{n} \left( \frac{d^2 A_{\text{amp}}(t)}{dt^2} + \alpha^2 \lambda_{\text{amp}} A_{\text{amp}}(t) \right) \varphi_{\text{amp}}(x, y, z) = -Q(x, y, z, t)
\] (4)

\[
\frac{d^2 A_{\text{amp}}(t)}{dt^2} + \alpha^2 \omega_{\text{amp}}^2 A_{\text{amp}}(t) = -\int_0^W \int_0^H \int_0^L \varphi_{\text{amp}}^2(x, y, z) \left( Q(x, y, z, t) \right) dx dy dz
\]

\[
= -q(t)
\] (5)

here, applying the method of variation of parameter for equation (4.25), we get the solution

\[
A_{\text{amp}}(t) = C_{11} \cos \alpha \sqrt{\omega_{\text{amp}}} t + C_{12} \sin \alpha \sqrt{\omega_{\text{amp}}} t + \left( \cos \alpha \sqrt{\omega_{\text{amp}}} t \right) \left( \frac{1}{\alpha \omega_{\text{amp}}} \right)
\]

\[
\int \sin \alpha \sqrt{\omega_{\text{amp}}} t \times q(t) dt = -\left( \cos \alpha \sqrt{\omega_{\text{amp}}} t \right) \left( \frac{1}{\alpha \omega_{\text{amp}}} \right) \left( \int \cos \alpha \sqrt{\omega_{\text{amp}}} t \times q(t) dt \right)
\] (6)

Applying boundary conditions \( \frac{\partial U(x, y, z, t)}{\partial t} = 0 \), the constant has values \( C_{11} = 0 \)

and \( C_{12} = \left( \frac{1}{\alpha \omega_{\text{amp}}} \right) \left( \int \cos \alpha \sqrt{\omega_{\text{amp}}} t \times q(t) dt \right) \)

}\( |_{t=0} \).
Therefore the solution becomes

\[
U(x, y, z, t) = \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \sin \frac{n \pi}{L} x \sin \frac{m \pi}{H} y \sin \frac{p \pi}{W} z \right) \times \left( C_{\text{amp}} \cos \alpha \sqrt{\omega_{\text{amp}}} t + \frac{B_{\text{amp}}}{\alpha \sqrt{\omega_{\text{amp}}}} \left( \int \cos \alpha \sqrt{\omega_{\text{amp}}} t \times q(t) \, dt \right)_{t=0} \right) \times \sin \alpha \sqrt{\omega_{\text{amp}}} t + \frac{B_{\text{amp}}}{\alpha \sqrt{\omega_{\text{amp}}}} \left( \int \sin \alpha \sqrt{\omega_{\text{amp}}} t \times q(t) \, dt \right)_{t=0} \right) - \frac{B_{\text{amp}}}{\alpha \sqrt{\omega_{\text{amp}}}} \left( \int (\sin \alpha \sqrt{\omega_{\text{amp}}} t \times q(t) \, dt \right)_{t=0} \right)
\]

(7)

In order to get \( C_{\text{amp}} \) and \( B_{\text{amp}} \), the conditions \( U(x, y, z, 0) = k_a(x, y, z) \) and \( U(x, y, z, f) = k_f(x, y, z) \) must be applied.

Now for the values of \( C_{\text{amp}} \) and \( B_{\text{amp}} \), we put \( U(x, y, z, 0) = k_a(x, y, z) \) and \( U(x, y, z, f) = k_f(x, y, z) \) in equation (4.27).

3.2. Damping response of Three-Dimensional SMA with Different Thermal and Mechanical Loadings on prepared alloys

In this section, thermal and mechanical loading conditions are selected; taking into the consideration on the basis of different types of forces may have to encounter SMA during service life as damper [21–27].

Before presenting some examples, it is imperative here to mention the possibilities in eigenvalues. There are many possibilities for the eigenvalues \( \omega_{\text{amp}} \). They may be used in singular or in added way jointly (all combined) there may be five different solutions possible; either \( \omega_{\text{nn}} \), \( \omega_{\text{mm}} \), \( \omega_{\text{mp}} \), \( \omega_{\text{np}} \) and (all combined) \( \omega_{\text{nm}} + \omega_{\text{mn}} + \omega_{\text{mp}} + \omega_{\text{pm}} + \omega_{\text{np}} \).

If \( n \), \( m \) and \( p \) are designated as some positive real numbers, then, there may be five different solutions possible; either \( \omega_{11} \), \( \omega_{12} \), \( \omega_{13} \), \( \omega_{21} \), and (all combined) \( \omega_{11} + \omega_{12} + \omega_{13} + \omega_{21} \).

Here, for the sake of first set of conditions, now, \( Q(x, y, z, t) = \beta^2 (\eta_{Af} + \eta e^{-\omega t}) (x + y + z) \) and temperature field is

\[
q(t) = \beta^2 (\eta_{Af} + \eta e^{-\omega t}) \int_0^W \int_0^H \int_0^L (x + y + z) \sin \frac{n \pi}{L} x \sin \frac{m \pi}{H} y \sin \frac{p \pi}{W} z \, dx \, dy \, dz
\]

(9)
Simplifying, we get

\[ q(t) = \beta^2 (\eta_{\text{lf}} + \eta e^{-\omega t}) \left[ \frac{L^2}{n^2} (\sin n\pi/n\pi - \cos n\pi) + \frac{H^2}{m^2} (\sin m\pi/m\pi - \cos m\pi) + \frac{W^2}{p^2} (\sin p\pi/p\pi - \cos p\pi) \right] \]

where

\[ q_{\text{amp}} = \left[ \frac{L^2}{n^2} (\sin n\pi/n\pi - \cos n\pi) + \frac{H^2}{m^2} (\sin m\pi/m\pi - \cos m\pi) + \frac{W^2}{p^2} (\sin p\pi/p\pi - \cos p\pi) \right] \]

That implies \( q(t) = \beta^2 (\eta_{\text{lf}} + \eta e^{-\omega t}) q_{\text{amp}} \). Putting this value in equation (7)

\[ U(x, y, z, t) = \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{L} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{W} z}{\sin \frac{n\pi}{L} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{W} z} \times \left\{ C_{\text{amp}} \cos \alpha \sqrt{\omega_{\text{amp}}} t + B_{\text{amp}} \frac{\beta^2 q_{\text{amp}}}{\alpha \sqrt{\omega_{\text{amp}}}} \left( \frac{\beta^2 q_{\text{amp}}}{\alpha \sqrt{\omega_{\text{amp}}}} \right) \times \sin \alpha \sqrt{\omega_{\text{amp}}} t \right\} \]

(10)

Now for the values of \( C_{\text{amp}} \) and \( B_{\text{amp}} \), applying conditions,

\[ k_0(x, y, z) = \frac{H}{100} \cos \frac{m\pi}{H} y \cos \frac{n\pi}{L} x \text{ and } k_f(x, y, z) = \frac{H}{100} \cos \frac{m\pi}{H} y \cos \frac{n\pi}{L} x \cos \frac{p\pi}{W} z \rho \]

Putting \( U(x, y, z, 0) = k_0(x, y, z) \) in equation (4.30), the solution is:

\[ U(x, y, z, t) = \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{L} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{W} z}{\sin \frac{n\pi}{L} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{W} z} \times \left\{ C_{\text{amp}} \cos \alpha \sqrt{\omega_{\text{amp}}} t + B_{\text{amp}} \frac{\beta^2 q_{\text{amp}}}{\alpha \sqrt{\omega_{\text{amp}}}} \left( \frac{\beta^2 q_{\text{amp}}}{\alpha \sqrt{\omega_{\text{amp}}}} \right) \times \sin \alpha \sqrt{\omega_{\text{amp}}} t \right\} \]

(11)

Applying \( U(x, y, z, f) = k_f(x, y, z) \):

\[ \frac{2}{n^2} \frac{2}{p^2} \frac{2}{m^2} \times (1 - \cos n\pi)(1 - \cos m\pi)(1 - \cos p\pi) \]

\[ \left[ \frac{H}{100} \cos \frac{m\pi}{H} y \cos \frac{n\pi}{L} x \cos \frac{p\pi}{W} z dxdydz - \frac{H}{100} \cos \frac{m\pi}{H} y \cos \frac{n\pi}{L} x \cos \frac{p\pi}{W} z dxdydz \right] \cos \alpha \sqrt{\omega_{\text{amp}}} \]

\[ = B_{\text{amp}} \]

where\( (\beta_1 + \beta_2 - \beta_3) \) are

\[ \beta_1 = \frac{1}{\alpha \sqrt{\omega_{\text{amp}}} \times \beta^2 q_{\text{amp}}} \left[ \left( \frac{\eta_{\text{lf}} + \eta e^{-\omega t}}{\eta_{\text{lf}} + \eta e^{-\omega t}} \right) \times \sin \alpha \sqrt{\omega_{\text{amp}}} dt \right]_{t=0} \]

\[ \beta_2 = (\cos \alpha \sqrt{\omega_{\text{amp}}} \times \beta^2 q_{\text{amp}}) \left[ \frac{1}{\alpha \sqrt{\omega_{\text{amp}}} \times \beta^2 q_{\text{amp}}} \left( \frac{\eta_{\text{lf}} + \eta e^{-\omega t}}{\eta_{\text{lf}} + \eta e^{-\omega t}} \right) \times \sin \alpha \sqrt{\omega_{\text{amp}}} dt \right]_{t=1} \]

\[ \beta_3 = (\sin \alpha \sqrt{\omega_{\text{amp}}} \times \beta^2 q_{\text{amp}}) \left[ \frac{1}{\alpha \sqrt{\omega_{\text{amp}}} \times \beta^2 q_{\text{amp}}} \left( \frac{\eta_{\text{lf}} + \eta e^{-\omega t}}{\eta_{\text{lf}} + \eta e^{-\omega t}} \right) \times \sin \alpha \sqrt{\omega_{\text{amp}}} dt \right]_{t=1} \]

Table 1 gives us chemical composition of alloys prepared for data purpose. Table 2 provides mechanical properties. Table 3 provides thermal properties to be used in model solution. Table 4 provides data set for modified OFT model. Table 5 provides transformation temperatures of synthesized alloys are tabulated here.

Total strain is entirety of the strain due to phase alteration, thermal straining and the elastic straining. As term \( A_3 \), is connected to stress-related phase alteration, their value generally has a range \( 0 < A_3 < 1 \). The value \( a \phi \Omega \) of has range \( 0 < a \phi \Omega < \infty \) \[28-30\], as \( 'a' \) and \( '\Omega' \) are strain related function. The value of for modified OFT model is \( A_3 = 0.5 \). Term \( B_3 \) is related to temperature-induced phase transformation. As thermal strain is connected to thermal straining and transformation temperatures, its values has range \( 0 < B_3 < \theta \) \[30\]. The relation for \( A_3 \) and \( B_3 \) are given in appendix A.
In order to get simulation responses, \( \eta = 50 \) degrees is operating temperature for simulation. By using MATLAB code, the response at \( v = 0.52 \) is presented in Figure 2.

For second case, the temperature field is changing, so

\[
Q(x, y, z, t) = \beta^2 \left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} \right).
\]

| Table 1. Chemical compositions of the synthesized alloys. |
|---------------------------------|
| Contents element percent (wt %) | Al  | Mn  | Ni  | Cu  |     |
| Alloy 1       | 8.91 | 6.65 | 2.62 | Bal.|     |
| Alloy 2       | 8.71 | 6.15 | 3.03 | Bal.|     |
| Alloy 3       | 8.91 | 6.19 | 3.32 | Bal.|     |
| Alloy 4       | 8.42 | 6.41 | 3.54 | Bal.|     |
| Alloy 5       | 8.82 | 6.46 | 3.89 | Bal.|     |
| Alloy 6       | 8.44 | 6.16 | 4.26 | Bal.|     |

| Table 2. Table of mechanical properties. |
|---------------------------------|
| Alloys | Young’s modulus (GPa) | Density (gm/mm³) |
| Alloy 1       | 84.22 | 0.00869 |     |
| Alloy 2       | 84.7  | 0.00873 |     |
| Alloy 3       | 85.4  | 0.00879 |     |
| Alloy 4       | 86.5  | 0.00884 |     |
| Alloy 5       | 87.3  | 0.00887 |     |
| Alloy 6       | 88.2  | 0.00891 |     |

| Table 3. Thermal properties to be used in model solution. |
|---------------------------------|
| Alloy number | M_f (K) | M_s (K) | A_s (K) | A_f (K) |
| Alloy 1       | 251    | 270    | 275     | 281     |
| Alloy 2       | 259    | 279    | 282     | 287     |
| Alloy 3       | 269    | 284    | 291     | 296     |
| Alloy 4       | 271    | 285    | 296     | 315     |
| Alloy 5       | 273    | 291    | 299     | 328     |
| Alloy 6       | 279    | 297    | 318     | 334     |

| Table 4. Data set for modified OFT model. |
|---------------------------------|
| Alloys | Young’s modulus to density ratio (GPa/ gm/mm³) | B_0 | A_0 |
| Alloy 1       | 9666.28 | 0.55 | 0.5 |
| Alloy 2       | 9702.18 | 0.55 | 0.5 |
| Alloy 3       | 9715.8  | 0.55 | 0.5 |
| Alloy 4       | 9742.2  | 0.55 | 0.5 |
| Alloy 5       | 9785.2  | 0.55 | 0.5 |
| Alloy 6       | 9892.3  | 0.55 | 0.5 |

| Table 5. Transformation temperatures of synthesized alloys are tabulated here. |
|---------------------------------|
| Alloy number | M_f (K) | M_s (K) | A_s (K) | A_f (K) |
| Alloy 1       | 251    | 270    | 275     | 281     |
| Alloy 2       | 259    | 279    | 282     | 287     |
| Alloy 3       | 269    | 284    | 291     | 296     |
| Alloy 4       | 271    | 285    | 296     | 315     |
| Alloy 5       | 273    | 291    | 299     | 328     |
| Alloy 6       | 279    | 297    | 318     | 334     |
where \( Q(x, y, z, t) = \beta^2(\eta_{Af} + (\gamma - \omega_1 t))(x + y + z) \) and temperature field is
\[
q(t) = \beta^2(\eta_{Af} + (\gamma - \omega_1 t)) \int_0^W \int_0^H \int_0^L (x + y + z) \sin \frac{n\pi}{W} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{L} z \, dx \, dy \, dz
\]
\[
q(t) = \beta^2(\eta_{Af} + (\gamma - \omega_1 t)) \left[ \frac{L^2}{n^2 \pi} \sin(n \pi / n_{123}) - \frac{m^2 \pi}{m^2 \pi} \sin(m \pi / m_{123}) - \frac{p^2 \pi}{p^2 \pi} \sin(p \pi / p_{123}) \right]
\]
provides \( q(t) = \beta^2(\eta_{Af} + (\gamma - \omega_1 t))q_{\text{cmp}} \) and
\[
q_{\text{cmp}} = \left[ \frac{L^2}{n^2 \pi} \sin(n \pi / n_{123}) - \frac{m^2 \pi}{m^2 \pi} \sin(m \pi / m_{123}) - \frac{p^2 \pi}{p^2 \pi} \sin(p \pi / p_{123}) \right]
\]

And our solution transforms into the form of
\[
U(x, y, z, t) = \frac{2}{\pi^2} \sum_{n=1}^\infty \sum_{m=1}^\infty \sum_{p=1}^\infty \left[ \sin \frac{n\pi}{W} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{L} z \right] \left[ \frac{C_{\text{cmp}} \cos \alpha \sqrt{\omega_{\text{cmp}}} t + B_{\text{cmp}} q_{\text{cmp}} \alpha \sqrt{\omega_{\text{cmp}}}^2}{\alpha \sqrt{\omega_{\text{cmp}}}^2} \right] \left[ -\sin \alpha \sqrt{\omega_{\text{cmp}}} t - (\eta_{Af} + (\gamma - \omega_1 t)) + \frac{1}{\alpha \sqrt{\omega_{\text{cmp}}}^2} \sin 2\alpha \sqrt{\omega_{\text{cmp}}} t \right]
\]

For dynamic mechanical loading conditions, the boundary conditions chosen are
\[
k_f(x, y, z) = \frac{H}{50} \cos \frac{m\pi}{H} x \cos \frac{p\pi}{L} z \quad \text{and} \quad k_f(x, y, z) = \frac{H}{100} \cos \frac{m\pi}{H} x \cos \frac{p\pi}{L} \cos \frac{q\pi}{W} t \bigg|_{t=0}
\]

The value of \( C_{\text{cmp}} \) is:
\[
C_{\text{cmp}} = \frac{1}{n^2 \pi} \int_0^L (k_f(x, y, z)) \, dx \, dy \, dz \, \frac{2}{m^2 \pi} \int_0^W \int_0^H (k_f(x, y, z)) \, dx \, dy \, dz \, \frac{2}{p^2 \pi} \int_0^W \int_0^H (k_f(x, y, z)) \, dx \, dy \, dz
\]
\[
- B_{\text{cmp}} \left( \int n^2 \pi \int m^2 \pi \int p^2 \pi \sin \alpha \sqrt{\omega_{\text{cmp}}} t \times q(t) \bigg|_{t=0} \right)
\]

Applying \( U(x, y, z, t) = k_f(x, y, z) \), we get
\[
\frac{2}{\pi^2} \frac{2}{m^2 \pi} \frac{2}{p^2 \pi} \frac{1}{n^2 \pi} \left[ (1 - \cos \pi n)(1 - \cos \pi m)(1 - \cos \pi p) \right] \left[ \int n^2 \pi \int m^2 \pi \int p^2 \pi \left( \frac{1}{\alpha \sqrt{\omega_{\text{cmp}}}^2} \int q(t) \bigg|_{t=0} \right) \times \cos \alpha \sqrt{\omega_{\text{cmp}}} t \right] = B_{\text{cmp}}
\]

where \( \beta_1 + \beta_2 - \beta_3 \) are
\[
\beta_1 = \left( \frac{1}{\alpha \sqrt{\omega_{\text{cmp}}}^2} \right)^2 \left[ \int \left( \eta_{Af} + (\gamma - \omega_1 t) \right) \times \cos \alpha \sqrt{\omega_{\text{cmp}}} t \, dt \right] \bigg|_{t=0} \times \sin \alpha \sqrt{\omega_{\text{cmp}}},
\]
\[
\beta_2 = \left( \cos \alpha \sqrt{\omega_{\text{cmp}}} \times \beta_2^2 q_{\text{cmp}} \right) \left( \frac{1}{\alpha \sqrt{\omega_{\text{cmp}}}^2} \right) \left[ \int \left( \eta_{Af} + (\gamma - \omega_1 t) \right) \times \sin \alpha \sqrt{\omega_{\text{cmp}}} t \, dt \right] \bigg|_{t=0},
\]
\[
\beta_3 = \left( \sin \alpha \sqrt{\omega_{\text{cmp}}} \times \beta_3^2 q_{\text{cmp}} \right) \left( \frac{1}{\alpha \sqrt{\omega_{\text{cmp}}}^2} \right) \left[ \int \left( \eta_{Af} + (\gamma - \omega_1 t) \right) \times \cos \alpha \sqrt{\omega_{\text{cmp}}} t \, dt \right] \bigg|_{t=0},
\]

with the help of MATLAB code, damping response for the dynamic mechanical loading at \( \omega_1 = 1 \) is given in figure 3.

The factors \( \eta - \omega_1 t \) and \( \eta e^{-\omega_1 t} \) are used here for control purpose. According to figures 2 and 3 it takes nearly 50 s for a wave to diminish to minimize the effect of dynamic mechanical loading, but damping pattern is different.

For third case, \( Q(x, y, z, t) = \beta^2(\eta_{Af} + (\gamma))(x + y + z) \), we get
\[
q(t) = \beta^2(\eta_{Af} + (\gamma)) \int_0^W \int_0^H \int_0^L (x + y + z) \sin \frac{n\pi}{W} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{L} z \, dx \, dy \, dz
\]
\[
\int_0^W \int_0^H \int_0^L \sin \frac{n\pi}{W} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{L} z \, dx \, dy \, dz
simplifying

\[
q(t) = (\beta^2(\eta_{AF} + \eta)) \left[ \frac{U_{pt}}{n\pi} (\sin \frac{n\pi}{n\pi} - \cos \frac{n\pi}{n\pi}) + \frac{H^2}{m\pi} (\sin \frac{m\pi}{m\pi} - \cos \frac{m\pi}{m\pi}) + \frac{W^2}{p\pi} (\sin \frac{p\pi}{p\pi} - \cos \frac{p\pi}{p\pi}) \right] \\
L_{nt}(1 - \cos \frac{n\pi}{n\pi}) \frac{H_{mt}}{n\pi} (1 - \cos \frac{m\pi}{m\pi}) \frac{W_{pt}}{p\pi} (1 - \cos \frac{p\pi}{p\pi})
\]

if we substitute

\[
q_{nmp} = \left[ \frac{U_{pt}}{n\pi} (\sin \frac{n\pi}{n\pi} - \cos \frac{n\pi}{n\pi}) + \frac{H^2}{m\pi} (\sin \frac{m\pi}{m\pi} - \cos \frac{m\pi}{m\pi}) + \frac{W^2}{p\pi} (\sin \frac{p\pi}{p\pi} - \cos \frac{p\pi}{p\pi}) \right] \\
L_{nt}(1 - \cos \frac{n\pi}{n\pi}) \frac{H_{mt}}{n\pi} (1 - \cos \frac{m\pi}{m\pi}) \frac{W_{pt}}{p\pi} (1 - \cos \frac{p\pi}{p\pi})
\]

then \( q(t) = \beta^2(\eta_{AF} + \eta)q_{nmp} \) and our solution transforms into the form of

\[
U(x, y, z, t) = \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \sin \frac{n\pi}{L} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{W} z \right) \times \left( C_{nmp} \cos \frac{\omega_{nmp} t}{t} - \frac{B_{nmp}}{\alpha \omega_{nmp} t} (\beta^2(\eta_{AF} + \eta))q_{nmp} \right)
\]

Summarizing, we get

\[
U(x, y, z, t) = \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \sin \frac{n\pi}{L} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{W} z \right) \times \left( C_{nmp} \cos \frac{\omega_{nmp} t}{t} - \frac{B_{nmp}}{\alpha \omega_{nmp} t} (\beta^2(\eta_{AF} + \eta))q_{nmp} \right)
\]

For dynamic loading conditions, the values of \( C_{nmp} \) and \( B_{nmp} \). Applying

\[
k_0(x, y, z) = \frac{H}{50} \cos \frac{m\pi}{H} y \cos \frac{p\pi}{W} z \]

and

\[
k_0(x, y, z) = \frac{H}{100} \cos \frac{m\pi}{H} y \cos \frac{p\pi}{W} z \cos \frac{p\pi}{L} t \]

For \( U(x, y, z, 0) = k_n(x, y, z) \), we have

\[
U(x, y, z, 0) = \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \sin \frac{n\pi}{L} x \sin \frac{m\pi}{H} y \sin \frac{p\pi}{W} z \right) \times \left( C_{nmp} \cos \frac{\omega_{nmp} t}{t} - \frac{B_{nmp}}{\alpha \omega_{nmp} t} (\beta^2(\eta_{AF} + \eta))q_{nmp} \right)
\]

(13)

\[
\begin{align*}
C_{nmp} &= \int_0^W \int_0^L \int_0^H \left( \frac{H}{5} \cos \frac{m\pi}{H} y \frac{W}{5} \cos \frac{p\pi}{W} z \right) \frac{2}{n\pi} \frac{2}{m\pi} \frac{2}{p\pi} (1 - \cos \frac{n\pi}{n\pi})(1 - \cos \frac{m\pi}{m\pi}) \\
&\quad \times (1 - \cos \frac{p\pi}{p\pi}) - B_{nmp} \left( \frac{1}{\alpha \omega_{nmp}} \right) \left( \frac{2}{n\pi} \int_0^L \sin \frac{\omega_{nmp} t}{t} \sin(q(t)) dt \right) \quad t=0
\end{align*}
\]

(14)
and

\[
\frac{2}{\pi} \frac{2}{\pi} \frac{2}{\pi} \times (1 - \cos n \pi) (1 - \cos m \pi) (1 - \cos p \pi) \\
\left[ \int_0^W \int_0^W \int_0^H \cos \frac{m \pi}{W} \cos \frac{p \pi}{W} \cos \frac{z \pi}{H} \sin \beta \left( \eta_{Af} \sin t + \eta \right) \eta_{amp} \cos \frac{\alpha \sqrt{\omega_{amp}}}{\beta_{1} \beta_{3} - \beta_{2}} \right]_t^{t=1} = B_{amp}
\]

where \((\beta_1 + \beta_2 - \beta_3)\) are

\[
\beta_1 = 0, \quad \beta_2 = \cos \alpha \sqrt{\omega_{amp}} \frac{1}{\alpha \sqrt{\omega_{amp}}} \left( \beta_2^2 (\eta_{Af} + \eta) \eta_{amp} \sin \alpha \sqrt{\omega_{amp}} t \right)_t^{t=1},
\]

\[
\beta_3 = \sin \alpha \sqrt{\omega_{amp}} \frac{1}{\alpha \sqrt{\omega_{amp}}} \left( \beta_2^2 (\eta_{Af} + \eta) \eta_{amp} \cos \alpha \sqrt{\omega_{amp}} t \right)_t^{t=1}
\]

The equation (14) is a mathematical formulation of SMA based passive device with \((\eta_{Af} + \eta)\) as a thermal input. The responses are controlled with \((\eta_{Af} + \eta)\) only. In this case, the quantity of thermal input given to the system is fixed damping completes in 25 s in given in figure 4. Literature review reveals that the intrinsic damping of SMA in austenitic phase is low, if the external stress is enough to lead of stress-induced phase transformation; here stress-induced martensite causes higher strain energy dissipation \[31–43\]. Damping response of this condition is established and given in figure 4 using MATLAB codes.

According to the figure 4, the damping ratio is higher relatively, because of its temperature more than \(A_t\) the SMA displays pseudo-elastic behavior due to formulation of stress-induced-martensite, that have strain energy degeneracy more here than the response of SMAs in the figures 2 and 3. In the figures 2 and 3, the responses of SMA is based on SME characteristics or partial pseudo-elasticity and hence strain energy dissipation is lower, thus damping is slower, as also reported in \[43–48\].

In the case 3, the applied loads creates stress-induced transformations and transformation stresses are higher leading to higher damping capacity \[43–48\]. In case 3, the evidence of over-damped SMA system or critical-damped system is observed, whereas in the cases 1 and 2, under-damped type of systems is perceived \[27, 41\]. In above discussed, the case 1 and case 2, the active control performance of SMA.

The mathematical correlation developed in equations (10) and (11) represents active passive based model of SMA. Results of equations (12) and (13) show that SMA can act as active device and actuator with temperature control module, where \((\eta_{Af} + (\eta - \omega_1 t))\) and \((\eta_{Af} + (\eta \omega_{amp} + \eta_{amp} \sin \alpha \sqrt{\omega_{amp}} t))\) is a control module. .

4. Comparison at different thermal states with respect to transformation levels

The modified OFT models have a capacity to provide understanding about minor changes with respect to thermal variations. This effect is also modeled on the changes based on transformation levels; \(A_t, A, M_s\) and \(M_f\) and simulated as well.

Here, investigation the effect of different thermal states \((\eta_{Af} + (\eta e^{-\omega_1 t}))\), \((\eta_{Af} + (\eta e^{-\omega_1 t}))\), \((\eta_{Af} + (\eta e^{-\omega_1 t}))\) and \((\eta_{Af} + (\eta e^{-\omega_1 t}))\) on damping response, the thermal states for alloy 3,
\( \eta_{AF} = 296, \ \eta_{At} = 291, \ \eta_{Ms} = 284, \ \text{and} \ \eta_{bf} = 269 \) are used in modified OFT model. Here \( \eta = 50 \) and \( \omega_2 = 0.7 \) is taken for simulation purposes. The response is given in figures 5 and 6. Referring to the figure 6, the damping responses of above thermal conditions clearly establish that study of damping behavior of SMA under modified OFT model for different thermal scenarios are elaborate and more effective. The damping ratios of above illustrated responses in figure 7 are also calculated and given in figure 8.

With reference to figure 7, for condition (a), damping ratio is more than other states, as here it tends to confirm the pseudoelastic behavior based damping response. Whereas in condition (b), the damping is based upon still pseudoelastic behavior and in (c) and (d) conditions, partial pseudoelastic and shape memory behavior are main contributors.

The damping behavior is further illustrated in figure 8. The damping factor of case (a) is more than others, because SMA behaves as perfectly super elastic material and in case (d) SMA is under shape memory effect.

### 4.1. Significance of modified OFT model and comparison with OFT model

The scope of the modified OFT model has certainly been enhanced. The significance of modified OFT model is two-fold; first different thermal effects like \((\eta_{AF} + (\eta e^{-\frac{t}{\rho}})) \), \((\eta_{At} + (\eta e^{-\frac{t}{\rho}})) \), \((\eta_{bf} + (\eta e^{-\frac{t}{\rho}})) \) and

---

**Figure 5.** Comparative damping behavior (a) alloy 1, (b) alloy 2, (c) alloy 3, (d) alloy 4, (e) alloy 5, (f) alloy 6. Damping in alloys 1–6 increases as Ni contents increase.
and secondly different mechanical loadings are incorporated. The modified OFT has a capacity to demonstrate damping behavior of SMA on micro-transformation temperature stages, like at \(A_f\) and \(M_s\). The equations (7), (8), and (9) represent the modified one-dimensional OFT model. Whereas, equations (16), (19) and (20) are solutions in modified form of modified two-dimensional and equations (27), (30) and (32) are solutions for the modified three-dimensional OFT models. The two-dimensional and three-dimensional models are presented here for the most general cases for mechanical and thermal conditions. The set of initial and boundary conditions taken is \(U(x, y, z, 0) = m(x, y), U(x, y, 0) = k(x, y)\) for modified two-dimensional and \(U(x, y, z, 0) = k_0(x, y, z), U(x, y, z, f) = k_f(x, y, z)\) for modified three-dimensional models.
respectively. Hence, modified OFT model incorporates more areas than the OFT model. The conditions

\[ U(x, y, 1) = m(x, y), \quad U(x, y, 0) = k(x, y), \quad U(x, y, z, 0) = k_0(x, y, z) \text{ and } U(x, y, z, f) = k_f(x, y, z) \]

show the mechanical loadings.

Referring to the figures 2–5, the damping capacity is greater, because of its thermal state that displays pseudo-elastic behavior due to formulation of stress-induced-martensite. At thermal state higher than \( A_f \), SMA have strain energy dissipation ability more than at lower than \( A_f \), as evident from the response of SMA in the figures 2–4. With reference to the figures 2 and 3, SMA displays SME characteristics or partial pseudo-elasticity and hence strain energy degeneracy is lower, thus damping is slow [24–28]. The study shows that variations in thermal states would lead to alter the damping performance.

In this investigation, the analytical solution of the most general form one, two and three dimensional Oberaigner, Fischer and Tanaka model for SMA. The solutions of the above models are valid for any value of temperature and any rectangular geometry of SMAs.

**4.2. Importance of choosing \( A_f \) and \( A_s \) and \( A_m \)**

According to SMA phenomenon, a pseudoelastic loading scenario happens at a thermal management that is at higher level than its transformation temperature [1, 16, 17]. In the above case, as in equations (11), (12), (13) and (14) of constant temperature condition, the value of \( A_f \) would be greater than the transformation temperature range, so it would show pseudoelastic behavior as expressed in figure 9. Moreover, it is studied that even pseudoelasticity is not completely an isothermal phenomenon due to the latent heat of phase transformation. During application of load, some latent heat is generated and leads to self-heating of the material and slightly alters the shape of the hysteresis curves [3, 4]. The phenomenon of ‘partial pseudoelasticity’ has also been reported in SMA [3] and the pseudoelastic behavior exists at some range of temperatures rather than any specific value [4].
Table 6. Comparison of application of SMAs with respect to constant temperature and variable temperature change of above Studies.

| Temperature                      | Provision of Thermal Energy | Type of Damper  | Type of Control | Uses                                      | References         |
|----------------------------------|-----------------------------|-----------------|-----------------|-------------------------------------------|--------------------|
| Constant Temperature             | Constant                    | Passive Damper  | Passive Control | Large Scale Dampers, Structural Applications | [43–47]            |
| Variable Temperature Change      | Variable and controlled     | Controllable Damper | Active Control | Active Control Systems, Actuators and MEMS | [45–48]            |
4.3. Proposed theoretical design of damper with temperature settings
Shape memory alloys have many structural applications; both passive and active [48–53]. Table 6 gives us insight about different thermo-mechanical properties for different usages.

5. Conclusions

- The compositional variation influenced the mechanical properties of alloys, especially tensile behavior of the alloys. There is an increasing trend in UTS of alloys with increase in the Ni contents. Addition of Ni results in improvement in shape memory effect. Increase in Ni contents has elevated the transformation temperatures. It is observed that there is increase in c/a ratio of alloys as Ni increases. A slight decrease in SME of alloy 6 is due to an increase in the stacking fault magnitude in the alloy.
- The energy dissipated by SMA per cycle is studied at different strains. The capacity to dissipate strain is more in alloys 1 and 2 and they also show perfectly pseudoelastic behavior. Whereas in alloys 3 and 4, their A_s is near to room temperature, so they show partial-pseudoelastic behavior. In alloys 5 and 6, and recovery is through shape memory effect. So, all three phenomena pseudoelasticity, partial-pseudoelasticity and SME have been observed.
- The solutions of modified OFT models justify its effectiveness for any value of temperature and any geometry of SMA. Acknowledging the properties of SMA, a modification in the above model has been proposed; the temperature is taken as function of time. The results presented here illustrate the scope of temperature regimes and how nonlinear active and semi-active energy dissipation in SMA may be realized in practical settings. The above solution is presented for different type of loadings.
- This proposed modification has certainly enhanced the scope of model and the modified OFT has a capacity to address static as well as dynamic thermal states. The presented mechanical loading scenarios in modified OFT are basically general set of conditions, illustrating that solutions based on modified OFT are more helpful in addressing any type of mechanical forces.
- Mainly two types of thermal states, one constant temperature and other decreasing temperature, are taken to illustrate the damping behavior of SMA on the basis of modified OFT. The damping responses presented above are based on data of developed alloy 3. The damping capacity on different thermal states (e_Mf–eMs), (e_Mf + (ψe–1)), (ψ_Mf + (ψe–1)) and (ψ_Mf + (ψe–1)) have also been investigated and depict how different thermal levels affect the damping response. The modified OFT has a capacity to show damping behavior of SMAs on transformation temperature levels.
- The effect of compositional variation on damping behavior has also been studied. Comparative damping response of alloys samples 1–6, show that Ni contents increase the damping capacity of developed alloys.

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