Using BBN in cosmological parameter extraction from CMB: a forecast for PLANCK

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Abstract. Data from future high-precision Cosmic Microwave Background (CMB) measurements will be sensitive to the primordial Helium abundance $Y_p$. At the same time, this parameter can be predicted from Big Bang Nucleosynthesis (BBN) as a function of the baryon and radiation densities, as well as a neutrino chemical potential. We suggest to use this information to impose a self-consistent BBN prior on $Y_p$ and determine its impact on parameter inference from simulated PLANCK data. We find that this approach can significantly improve bounds on cosmological parameters compared to an analysis which treats $Y_p$ as a free parameter, if the neutrino chemical potential is taken to vanish. We demonstrate that fixing the Helium fraction to an arbitrary value can seriously bias parameter estimates. Under the assumption of degenerate BBN (i.e., letting the neutrino chemical potential $\xi$ vary), the BBN prior’s constraining power is somewhat weakened, but nevertheless allows us to constrain $\xi$ with an accuracy that rivals bounds inferred from present data on light element abundances.
1. Introduction

Forthcoming experiments on Cosmic Microwave Background (CMB) anisotropies such as PLANCK‡ [1], combined with other astrophysical observations, are expected to provide detailed information on the cosmological model which describes the evolution of the Universe. Their data will allow us to constrain the parameters of these models with an unprecedented accuracy. In fact, to fully extract information from such precise experimental data, it is crucial to have detailed theoretical models to compare with, possibly using reliable “priors” on cosmological parameters or models which can be obtained by independent theoretical tools or experimental data. This helps in reducing the effect of parameter degeneracies which typically limits the amount of information which can be obtained from data.

In this paper we present one example of this kind, by considering the impact of a detailed estimate of the $^4$He mass fraction§ $Y_p = 4n_{He}/n_b$ on CMB data analyses, obtained from an accurate prediction of its value from Big Bang Nucleosynthesis (BBN). In this framework, $Y_p$ is given as a function of the baryon density and, in more exotic scenarios, extra relativistic degrees of freedom and/or non-zero neutrino chemical potentials $\mu_\nu$.

The amount of $^4$He nuclei produced during BBN plays a relevant rôle at the epoch of recombination, as it is one of the parameters controlling the evolution of the free electron fraction $\gamma$ [2, 3] (it also affects the later phase of reionisation of the Universe, but this effect is extremely small [4]). Thus, the CMB power spectrum which is observed today depends significantly on this parameter. Typically, in current CMB analyses (and in a number of forecasts), this parameter is fixed to a reference value $Y_p = 0.24$, suggested by independent measurements obtained by studying extragalactic HII regions in blue compact galaxies, which are however affected by quite a large systematic uncertainty [5, 6]. This approach is satisfactory with present CMB data, but it is not fully correct as it does not take into account the fact that indeed, $Y_p$ is strongly correlated to other cosmological parameters, in particular the baryon fraction $\Omega_b h^2$ and the energy density $\rho_R$ in the form of relativistic species, which is usually parameterised in terms of the effective neutrino number $N_{eff} = 3 + \Delta N$ such that

$$\rho_R = \frac{\pi^2}{15} T_\gamma^4 \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} (3 + \Delta N) \right], \quad (1.1)$$

with $T_\gamma$ the photon temperature. The standard scenario of three non-degenerate active neutrinos corresponds to $\Delta N = 0.046$ due to the effect of non-thermal corrections to neutrino distributions during the $e^+ - e^-$ annihilation phase [7].

A different (and more consistent) approach is taken in [8], where $Y_p$ is considered as a free parameter to be obtained as a result of a likelihood analysis along with the other

‡ ESA home page for the PLANCK project: http://astro.estec.esa.nl/SA-general/Projects/Planck/ and Planck-HFI web site: http://www.planck.fr/
§ We point out that as usual in the literature we refer to $Y_p$ as the ”$^4$He mass fraction”, though strictly speaking it is not correct at the percent level, for it neglects the contribution of nuclear binding energy.
relevant cosmological parameters, or rather fixed up to some given error by adopting a gaussian prior. This approach leads to a more correct estimate of the baryon density and, as we will also show in the following, avoids biasing the point estimates of some cosmological parameters, such as the scalar spectral index $n_s$.

In the present work we will consider yet a different approach previously suggested by Ichikawa and Takahashi [9], which explicitly takes into account the dependence of $Y_p$ on $\Omega_b h^2$ (and possibly also on $\Delta N$ and on the neutrino chemical potentials) as obtained from BBN. We use the scale-independent ratio $\xi \equiv \mu/\kappa T$, where $T$ is the neutrino temperature and $\mu$ is the chemical potential, which is assumed to be the same for all three active neutrino species, due to the effect of flavour oscillations [10, 11]. In particular, the Helium mass fraction is considered as a known function $Y_p^{BBN}(\Omega_b h^2, \Delta N, \xi)$ and is thus neither fixed a priori, nor left as a free independent parameter. This function can be obtained in the framework of standard BBN theory and is found to be a smooth monotonically increasing function of $\Omega_b h^2$ and $\Delta N$, and decreasing with $\xi$. In particular, the result adopted in the present paper has been computed by using the public BBN code PArthENoPE [12]. This tool provides a careful determination of light nuclei abundances, with very small uncertainties. PArthENoPE is the result of a large reanalysis, updating the whole nuclear reaction network, and also including the effect of radiative corrections to neutron/proton weak processes, and the detailed treatment of neutrino decoupling described in [7]. For a comprehensive discussion of these issues, see e.g. [13]. The theoretical error on $Y_p^{BBN}$ is of the order of 0.0002, thus at the level of per mille, over the relevant ranges of $(\Omega_b h^2, \Delta N, \xi)$ [13]. The accuracy is ultimately limited by the present experimental uncertainty on the neutron lifetime. This tiny error essentially amounts to imposing a consistency relation between $Y_p$, $\Omega_b h^2$, $\Delta N$ and $\xi$.

The goal of this paper is therefore to determine how the assumption of a well motivated BBN prior on $Y_p$ will affect the estimates of cosmological parameters from PLANCK data. Of course, the validity of this method is based on the assumption of standard BBN (or degenerate BBN (dBBN) in section 2.3, where $\xi \neq 0$ will be considered). It also implicitly assumes that the value of $N_{\text{eff}}$ does not change from the BBN epoch until last scattering. However, possible future evidence for non-standard BBN scenarios or a more involved evolution of relativistic degrees of freedom could similarly be accounted for by using a different – yet still theoretically calculable – functional form of $Y_p$.

2. Forecast for PLANCK data

In our analysis, we compare three different ways of treating the Helium mass fraction:

- **Fixed $Y_p$:**
  
  As in most recent analyses and forecasts of CMB data, $Y_p$ is fixed to a value of $Y_p = 0.24$. This is also the default value set in CosmoMC/CAMB [14, 15]. Note that if we set $N_{\text{eff}}$ to its standard value of 3.046 and assume no neutrino asymmetry, the value of 0.24 is not consistent with current bounds on the baryon density. At the
moment, CMB data are not very sensitive to $Y_p$, so the bias expected from fixing $Y_p$ in this way is negligible.

- **Free $Y_p$:**
  Here, no additional assumptions are made and the Helium mass fraction is kept completely free, with a top hat prior ranging from 0 to 1.

- **BBN prior on $Y_p$:**
  Under the assumption that BBN proceeded in the standard way, and that $N_{\text{eff}}$ remains constant between BBN and last scattering, we exploit the fact that $Y_p$ is related to $\Omega_b h^2$, $\Delta N$, and possibly $\xi$ if neutrinos have a sizable chemical potential. In a first-order approach, one could fix $Y_p$ to the BBN prediction $Y_p^{BBN}(\Omega_b h^2, \Delta N, \xi)$ calculated by PArthENoPE. That way, however, one would not take into account the theoretical uncertainty in $Y_p^{BBN}$. In order to treat the uncertainty properly, we keep $Y_p$ a free parameter, but, taking $\delta Y_p^{BBN}$ to be gaussian, add the following term to the negative logarithm of the likelihood $L$ of each point in parameter space:

$$\Delta(-\ln L) = \frac{1}{2} \left( \frac{Y_p - Y_p^{BBN}(\Omega_b h^2, \Delta N, \xi)}{\sigma(Y_p)} \right)^2.$$ (2.1)

Since recomputing $Y_p^{BBN}$ for each point would not be practical, we interpolate its value from a pre-computed grid. To account for errors introduced due to interpolation, we increase the absolute error on $Y_p^{BBN}$ to $\sigma(Y_p) = 0.0003$. Note that we do not employ any additional data here e.g., astrophysical measurements of primordial element abundances, that may be subject to large systematic errors.

### 2.1. Fiducial data and parameter inference

Following the method described in detail in [16, 17], one can generate a set of mock CMB data, using the projected specifications of the PLANCK satellite [1] (see table 1). However, for the purpose of forecasting errors, it is sufficient to replace the power spectrum of the mock data by that of the fiducial model, which stands for an average over many possible mock data sets [16]. The data set comprises the $TT$- and $EE$-auto-correlation spectra as well as the $TE$-cross-correlation spectrum for multipoles up to $\ell = 2500$, and we assume a sky coverage of $f_{\text{sky}} = 0.65$. In our fiducial model we impose the standard BBN consistency relation, spatial flatness, and ignore tensor modes; its parameter values are summarized in table 2.

We then perform the exercise of Bayesian parameter inference for a number of models, differing in the number of basic free parameters and the treatment of the primordial Helium fraction $Y_p$, as described above. We first work under the standard assumption that the chemical potential of neutrinos is negligible ($\mu_\nu \ll kT_\nu$), and consider two basic models:

- A minimal model, with six free parameters ($\Omega_b h^2$, $\Omega_{dm} h^2$, $H_0$, $z_{re}$, $\ln[10^{10} A_S]$, $n_S$), inspired by the current “vanilla” model.
An extended model, where in addition to the parameters of the minimal model we also vary the neutrino mass fraction $f_\nu$ and the number of extra relativistic degrees of freedom $\Delta N$. The introduction of $f_\nu$ is motivated by the observation of neutrino oscillations, implying a non-negligible effect of neutrino masses on cosmological perturbations [18]. The neutrino fraction and $\Delta N$ are known to be correlated in the analysis of CMB data [19–21].

In section 2.3, we will repeat the analysis with one extra free parameter which is known to modify the outcome of BBN predictions: a non-zero chemical potential for neutrinos.

We employ a modified version of the Markov-Chain-Monte-Carlo code CosmoMC [14] to infer the posterior probability density from the data. Eight Markov chains are generated in parallel; their convergence is monitored with the help of the Gelman-Rubin $R$-statistic [22], and our convergence criterion is $R - 1 \leq 0.02$.

When facing real data, one could be worried that the theoretical prediction for the anisotropy spectra might be insufficient. In particular, issues like recombination or foreground contamination need to be better understood. Since we use the same numerical code for generating and analysing the data, we implicitly assume in this forecast that all systematics are perfectly under control. Thus, our inferred parameter errors may be slightly optimistic.

### 2.2. Standard BBN

#### 2.2.1. Minimal model

The one-dimensional marginalised posterior probabilities for the parameters of the minimal model are presented in figure 1. A first striking observation is that fixing $Y_p$ “incorrectly” to 0.24 leads to a significant bias of up to one standard deviation in the point estimates for the baryon density, Hubble parameter, spectral index and primordial spectrum normalisation. This is particularly worrisome for $n_S$, since bounds on this parameter are often used to constrain inflationary models. We therefore strongly recommend not to fix $Y_p$ to some arbitrary value, such as 0.24, in any analysis of future data.

The reason for the bias in these parameters are degeneracies with the Helium mass fraction; we illustrate these degeneracies in figure 2. Of these four degeneracies, only

| $\nu$/GHz | $\theta_{beam}$ | $\Delta T/\mu K$ | $\Delta P/\mu K$ |
|-----------|----------------|-----------------|----------------|
| 100       | 9.5'           | 6.8             | 10.9           |
| 143       | 7.1'           | 6.0             | 11.4           |
| 217       | 5.0'           | 13.1            | 26.7           |
Table 2. In this table we show the free parameters of our model, their fiducial values used to generate the data set and the prior ranges adopted in the analysis.

| Parameter               | Fiducial Value | Prior Range     |
|-------------------------|----------------|-----------------|
| Dark matter density     | $\Omega_{\text{dm}} h^2$ | 0.11            | 0.01 → 0.99    |
| Baryon density          | $\Omega_{b} h^2$    | 0.022           | 0.005 → 0.1    |
| Hubble parameter        | $h$              | 0.7             | 0.4 → 1        |
| Redshift of reionisation| $z_{\text{re}}$   | 12              | 3 → 50         |
| Normalisation @ $k = 0.002$ Mpc$^{-1}$ | $\ln[10^{10} A_S]$ | 3.264           | 2.7 → 4        |
| Scalar spectral index   | $n_S$            | 0.96            | 0.5 → 1.5      |
| Helium fraction         | $Y_p$            | 0.2477          | 0 → 1          |
| Neutrino mass fraction  | $f_\nu$          | 0               | 0 → 1          |
| Number of extra rel. d.o.f. | $\Delta N$ | 0.046           | −3 → 4         |
| Neutrino chemical potential | $\xi$     | 0               | −1 → 1         |

Table 3. This table shows the projected absolute errors on the parameters of the two models with zero neutrino chemical potential. For all parameters except the neutrino fraction we quote the half width of the minimal 68% credible interval [23], for $f_\nu$ we give the values of the 68% upper limit (the lower limit being zero). The columns labelled “free” show the results when leaving the Helium fraction a free parameter, while those labelled “BBN” correspond to the results imposing our BBN prior. Our results are in very good agreement with those found in [9].

| Parameter               | Minimal model | Extended model | Minimal model | Extended model |
|-------------------------|---------------|----------------|---------------|----------------|
| $\Omega_b h^2$         | $2.2 \times 10^{-4}$ | $1.3 \times 10^{-4}$ | $2.3 \times 10^{-4}$ | $2.2 \times 10^{-4}$ |
| $\Omega_{\text{dm}} h^2$ | $1.4 \times 10^{-3}$ | $1.2 \times 10^{-3}$ | $4.9 \times 10^{-3}$ | $3.1 \times 10^{-3}$ |
| $h$                     | $7.9 \times 10^{-3}$ | $5.9 \times 10^{-3}$ | $28 \times 10^{-3}$ | $24 \times 10^{-3}$ |
| $z_{\text{re}}$        | 0.40           | 0.39           | 0.41          | 0.41           |
| $\ln[10^{10} A_S]$     | 0.024          | 0.015          | 0.024         | 0.021          |
| $n_S$                   | $7.2 \times 10^{-3}$ | $3.4 \times 10^{-3}$ | $8.2 \times 10^{-3}$ | $7.7 \times 10^{-3}$ |
| $Y_p$                   | 0.011          | 3.1 $\times 10^{-4}$ | 0.015         | 2.1 $\times 10^{-3}$ |
| $\Delta N$             | −              | −              | 0.26          | 0.15           |
| $f_\nu$                | −              | −              | ≤ 0.041       | ≤ 0.035        |

the one with $\omega_b = \Omega_b h^2$ is “physical”, as was already noticed in [8]. The redshift of decoupling $z_s$, for instance, crucially depends on the number density of free electrons $n_e$. Between helium and hydrogen recombination, this density is related to the baryon number density $n_b (\propto \omega_b)$ and Helium abundance by $n_e = n_b (1 - Y_p)$. So a shift in $z_s$ due to a change in $Y_p$ can be reversed by changing the baryon density. The dependence of $n_e$ on the Helium fraction also plays a rôle for the diffusion damping scale $d$, with $d \propto n_e^{-1/2}$. An increase in $Y_p$, for example, means smaller $n_e$ and hence a larger $d$, which leads to an additional suppression of power in the CMB temperature anisotropies on small scales. Phenomenologically, this signature is similar to tilting the spectrum.
of primordial anisotropies (i.e., lowering $n_S$), which explains the degeneracy with the spectral index.

The degeneracies with the Hubble parameter and the normalisation are only indirect ones, since these parameters are themselves correlated with the baryon density and the spectral index, respectively. Note that the degeneracy with $A_S$ is not invariant under a change of the pivot scale. Had we chosen the pivot at a small scale instead of a large scale, one would expect a positive correlation instead of an anticorrelation between the two parameters.

A comparison between the run with free $Y_p$ and that with a BBN prior shows that imposing standard BBN places an extremely strong constraint on the Helium fraction if we do not vary $\Delta N$. The width of the 68% credible interval on $Y_p$ is $6.2 \times 10^{-4}$, i.e., the error is dominated by the theoretical uncertainty in the prediction of $Y_p^{BBN}$ (see table 3). Essentially, the precise determination of the baryon density from PLANCK data will also nail down the Helium fraction. This should not be surprising, given that $Y_p^{BBN}$ is relatively flat in the direction of $\omega_b$. Consequently, the expected errors on the cosmological parameters hardly differ from the results of an analysis with fixed $Y_p$. 

Figure 1. Marginalised posterior probabilities for the parameters of the minimal model. The dashed purple curves correspond to the case where $Y_p$ is fixed to an “incorrect” value of 0.24, the red curves have $Y_p$ as a free parameter and the thick black curves correspond to the case with a standard BBN prior.
Figure 2. Two-dimensional marginalised joint posterior 68%- and 95%-credible contours for the minimal model. The thick red contours correspond to results with free $Y_p$, the thin black contours represent the result when standard BBN is imposed. This plot illustrates the degeneracies of $Y_p$ with other cosmological parameters and shows how they can be broken by imposing the BBN prior.

while the errors on the spectral index, the normalisation, the Hubble parameter and the baryon density are up to a factor two smaller than in the case with $Y_p$ as a free parameter.

2.2.2. Extended model  Allowing $\Delta N$ to vary in the extended model slightly weakens the constraining power of the standard BBN consistency relation. Since $Y_p^{BBN}$ is somewhat steeper in the direction of $\Delta N$, the Helium fraction will be allowed to vary over a wider range of values. Still, as we can see from figure 3, the error on $Y_p$ is still significantly improved (the width of the 68% credible interval is $4.1 \times 10^{-3}$ if we impose standard BBN, while for a free $Y_p$ it is larger by a factor of seven).

By adding $f_\nu$ and $\Delta N$ to the free parameters of the model, we naturally introduce new correlations which weaken the bounds on the other parameters. The new degeneracies (particularly those with $\Delta N$) turn out to be more serious than the degeneracies with the Helium fraction. In models with free $\Delta N$, constraints on the other cosmological parameters (apart from $\omega_{dm} = \Omega_{dm} h^2$ and $\Delta N$ itself) are essentially independent on how well the Helium fraction is constrained. Even fixing $Y_p$ to the
Figure 3. Marginalised posterior probabilities for the parameters of the extended model, still with zero neutrino chemical potential. The dashed purple curves correspond to the case where $Y_p$ is fixed to a value of 0.24, the red curves have $Y_p$ as a free parameter and the thick black curves correspond to the case with a standard BBN prior.

“wrong” value of 0.24 does not lead to a bias worth mentioning. The only exceptions are $\Delta N$ and the dark matter density (the latter due to a correlation with $\Delta N$, see, e.g. [23]), for which the error improves between “free $Y_p$” and “BBN prior on $Y_p$” by a factor of 1.7 and 1.5, respectively.

Note that in these runs, not all posteriors peak at their fiducial values. At first glance, this may sound strange since we use the fiducial spectra in place of mock data spectra. However, the mismatch just reflects the difference between the one-dimensional likelihood profile and the marginalised posterior when the posterior is far from gaussian in certain directions (like, in our case, the neutrino fraction).

2.3. Degenerate BBN

The effect of neutrino chemical potentials on BBN predictions is twofold. It contributes to the radiation energy density so the value of $N_{\text{eff}}$ for three neutrinos with a common

\begin{align*}
\omega_b & = 0.0215, & \omega_{\text{dm}} & = 0.11, & H_0 & = 66, \\
\omega_{\text{b}} & = 0.022, & \omega_{\text{dm}} & = 0.12, & H_0 & = 67, \\
\omega_{\text{b}} & = 0.0225, & \omega_{\text{dm}} & = 0.13, & H_0 & = 70, \\
z_{\text{re}} & = 11, & n_S & = 0.98, & \ln \left[ 10^{10} A_S \right] & = 3.2, \\
z_{\text{re}} & = 12, & n_S & = 0.94, & \ln \left[ 10^{10} A_S \right] & = 3.25, \\
z_{\text{re}} & = 13, & n_S & = 0.96, & \ln \left[ 10^{10} A_S \right] & = 3.3, \\
f_\nu & = 0, & \Delta N & = 0, & Y_p & = 0.24, \\
f_\nu & = 0.05, & \Delta N & = -0.5, & Y_p & = 0.26, \\
f_\nu & = 0.1, & \Delta N & = 0.5, & Y_p & = 0.28.
\end{align*}
value of $\xi$ becomes

$$N_{\text{eff}} = 3.046 + 3 \left( \frac{30}{7} \left( \frac{\xi}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi}{\pi} \right)^4 \right),$$

(2.2)

implying a larger expansion rate of the Universe, an earlier weak process freeze out with a higher value for the neutron to proton density ratio, and thus a larger value of $Y_p$. Subdominant effects on neutrino decoupling [24, 25] and their eventual distribution after the $e^+e^-$ annihilation phase [26] are indeed very small and can be neglected. On the other hand, a positive $\xi$ for the electron neutrino, i.e., a larger number of $\nu_e$ with respect to $\bar{\nu}_e$, enhances $n \rightarrow p$ weak processes compared to the inverse processes, lowering the number of neutrons per proton available at the onset of BBN.

Bounds on the value of the neutrino chemical potential from BBN data analyses have been considered by many authors, see e.g. [24–31]. Since flavour oscillations enforce the condition of equal chemical potentials for the three neutrino species [10], the (common) value of $\xi$ is strongly bounded by the neutron-proton beta equilibrium and the observed value of $^4\text{He}$ mass fraction. Adopting a conservative error analysis of primordial $Y_p$ as in [5], one gets $-0.04 \leq \xi \leq 0.07$ [31]. This bound can be evaded in non-standard scenarios with extra relativistic degrees of freedom contributing to $\Delta N$, which is strongly degenerate with $\xi$.

We now consider the effect of a dBBN prior on $Y_p$ in CMB data analysis, in order to see what is the impact of the extra parameter $\xi$ on future PLANCK estimates of cosmological parameters, including of course the value of $\xi$ itself. As for the previous standard BBN case, we do not use any direct experimental information on $Y_p$.

2.3.1. Minimal model plus $\xi$ We repeat our analysis for the minimal model with one extra free parameter $\xi$, leaving $Y_p$ either free, or imposing a dBBN prior on it. The constraint on cosmological parameters is essentially the same in the case “free $\xi \neq 0$ and BBN prior” as in the case “$\xi = 0$ and no BBN prior”. In other words, by assuming dBBN instead of standard BBN, the constraining power of the BBN prior on the six basic LCDM cosmological parameters disappears.

However, it is interesting to see how the degenerate BBN prior improves the constraint on $\xi$ itself. To this end one can compare the results of the “free” and “BBN” cases of table 4 (both for the minimal model and the extended one). The projected error on $Y_p$ does not change noticeably between the two cases, since now the $^4\text{He}$ mass fraction is not only determined by the baryon density and $\Delta N$, but also depends on the value of $\xi$. Even very small departure from zero allows quite a large variation of $Y_p$, therefore there is not much difference in this case between imposing the BBN relation and leaving the Helium mass fraction a free parameter. In turn, this strong dependence of $Y_p$ on $\xi$ is also responsible for the significant decrease of the error on $\xi$ in the BBN case. Indeed, the CMB alone is able to constrain $\xi$ only through its contribution to the total energy density during radiation domination, see equation (2.2). In this case the 68% error is $\sigma(\xi) = 0.34$, consistent with previous analyses based on the Fisher matrix.
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Table 4. This table shows the same quantities as in table 3 for the degenerate BBN scenario (with one extra parameter $\xi \equiv \mu_\nu/kT_\nu$). Note that $f_\nu = 0$ does not lie within the 68% credible interval for the extended model with free $Y_p$, so we quote the limits of the interval instead of an upper bound.

| Parameter     | Minimal model free | Minimal model BBN | Extended model free | Extended model BBN |
|---------------|--------------------|-------------------|---------------------|-------------------|
| $\Omega_b h^2$ | $2.1 \times 10^{-4}$ | $2.1 \times 10^{-4}$ | $2.2 \times 10^{-4}$ | $2.3 \times 10^{-4}$ |
| $\Omega_{dm} h^2$ | $2.1 \times 10^{-3}$ | $1.3 \times 10^{-3}$ | $5.0 \times 10^{-3}$ | $4.9 \times 10^{-3}$ |
| $h$           | $10. \times 10^{-3}$ | $7.3 \times 10^{-3}$ | $28 \times 10^{-3}$ | $27 \times 10^{-3}$ |
| $z_{re}$      | 0.41               | 0.40              | 0.43                | 0.41               |
| $\ln(10^{10} A_S)$ | 0.023            | 0.022             | 0.023               | 0.024              |
| $n_S$         | $7.4 \times 10^{-3}$ | $6.9 \times 10^{-3}$ | $8.0 \times 10^{-3}$ | $8.1 \times 10^{-3}$ |
| $Y_p$         | 0.012              | 0.010             | 0.016               | 0.016              |
| $\xi$         | 0.34               | 0.061             | 0.45                | 0.093              |
| $\Delta N$   | –                  | –                 | 0.27                | 0.27               |
| $f_\nu$       | –                  | –                 | $0.016 \rightarrow 0.063$ | $\leq 0.039$ |

approximation [32–34]. When including a BBN prior, the sensitivity of the CMB to $Y_p$ further reduces the error down to $\sigma(\xi) = 0.061$. Remarkably, this result is comparable to the error expected using observations of light element abundances, in case one adopts a conservative approach on the $Y_p$ error estimate to account for possible systematics. We conclude that future CMB data like that from PLANCK will be a very useful probe of the neutrino asymmetry. In case of a $\xi \neq 0$ detection, it would be of particular interest to compare this finding with constraints from primordial element data.

2.3.2. Extended model plus $\xi$  
Lastly, we analyse the extended model with a neutrino chemical potential. This requires some particular modifications of CAMB in order to explicitly include the chemical potential of neutrinos and anti-neutrinos in the expression of the massive neutrino phase-space distribution (as explained in [35]). The total density during radiation domination is now parameterised as

$$N_{\text{eff}} = 3 \left( 1 + \frac{30}{7} \left( \frac{\xi}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi}{\pi} \right)^4 \right) + \Delta N . \quad (2.3)$$

Our results are shown in table 4. Note that due to slow convergence of the chains for these models, we relaxed our convergence criterion slightly, demanding $R - 1 \leq 0.07$. As for the minimal model, the value of $Y_p$ is determined with the same (poor) accuracy regardless of whether one imposes the dBBN prior or leaves the Helium mass fraction as a free parameter, while the error on $\xi$ is strongly reduced in the first case, because of the strong dependence of $Y_p$ on this parameter. The only other parameter affected is the neutrino mass fraction $f_\nu$. Its degeneracy with $\xi$ is illustrated in figure 4. In fact, if one were to include large scale structure data the bounds on $\xi$ would likely be significantly further reduced, because the degeneracy existing between $f_\nu$ and $N_{\text{eff}}$. 

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Figure 4. Two-dimensional marginalised joint posterior 68%- and 95%-credible contours for $\xi$ and $f_\nu$ in the extended model with free neutrino chemical potential. Red lines correspond to “free $Y_p$”, the thin black lines are the results of imposing the BBN prior.

would be alleviated. Notice also that the error on $\Delta N$ is basically unchanged, as the dependence of $Y_p$ on this parameter is much weaker. Finally, it is also worth comparing the results for the extended models when imposing the standard or degenerate BBN priors, i.e. the last column of tables 3 and 4. With the exception of the dark matter density, the basic $\Lambda$CDM cosmological parameter errors do not undergo any substantial change, while we again see the effect of the introduction of the extra parameter $\xi$ in reducing the sensitivity to $Y_p$. Correspondingly, the error on $\Delta N$, and, due to the aforementioned correlation, also the error on $\Omega_{dm} h^2$, grow by almost a factor of two with respect to the standard BBN scenario, reaching the values which are obtained if $Y_p$ is taken as a free parameter and $\xi = 0$, see table 3.

3. Conclusions

In this paper we have considered in detail the effects of a careful calculation of the Helium mass fraction on future CMB experiments, such as PLANCK, using the known fact that $Y_p$ is not an independent free parameter, but can rather be fixed in the framework of Big Bang Nucleosynthesis as a function of the baryon density, the energy density during the relativistic dominated era, as well as, in more exotic scenarios, other physical inputs such as neutrino degeneracy. In view of the high precision in parameter estimates which is expected to be achieved by PLANCK, this method first adopted in [9] is more consistent than the current strategy of fixing by hand the value of $Y_p$. It is also worth stressing that it is completely independent from any astrophysical information on light
nuclei abundance, as $Y_p$ determination from low metallicity HII regions in blue compact galaxies. In this respect, this method is different from combined analyses of CMB and BBN data (as performed for example in [28, 30, 36–38]).

We have considered two different models, the standard BBN scenario, with possibly extra relativistic species in addition to three standard active neutrinos, and the case of degenerate BBN, with sizable neutrino chemical potentials.

For standard BBN, $Y_p$ only depends on the baryon fraction and $\Delta N$. Exploiting this functional dependence in a forecast for PLANCK data, we have shown that one can avoid a possible bias in the estimate of some cosmological parameters, i.e. the spectral index $n_S$ and $A_S$, which is instead present if $Y_p$ is fixed a priori to some reference value, usually given by $Y_p = 0.24$. Furthermore, this method allows for a better determination of various parameters (like e.g. the baryon density, the spectral index, the number of extra relativistic degrees of freedom and of course $Y_p$ itself, see table 3). With a BBN prior, the Helium mass fraction can be determined with an accuracy better than 1 %, at the level of statistical error of astrophysical determinations, which are however possibly plagued by a larger systematic error. On the other hand, without imposing the BBN prior, CMB data from PLANCK can determine $Y_p$ at the 5-6 % level only.

In the case of degenerate BBN, due to the strong dependence of $Y_p$ on one extra parameter – namely, the neutrino chemical potential parameter $\xi$ assumed to be flavour independent due to flavour oscillations – this result is no longer valid. Imposing the BBN prior, the value of $Y_p$ is only determined with an order 10 % uncertainty, just as if a flat prior was assumed over the whole range $0 \leq Y_p \leq 1$. Nevertheless, exploiting the dependence of the Helium mass fraction on $\xi$ has a big impact on the way this parameter can be determined by CMB anisotropy data. For a fiducial value $\xi = 0$, we found that the 68% absolute error on this parameter is 0.06 for the minimal model with no extra radiation, and 0.09 for the extended model where $\Delta N$ and the neutrino mass fraction $f_\nu$ are allowed to vary. In both cases, the result is comparable with the error obtained when using nuclei abundance data alone. If $Y_p$ is assumed to be an independent parameter, with no BBN prior, the effect of $\xi$ on the CMB power spectrum is only via its contribution to the relativistic energy density which shifts the matter-radiation equivalence point. Actually, in this case the 68% bound is up to one order of magnitude weaker, $|\xi| \leq 0.45$, again for our fiducial model with $\xi = 0$. Further spectroscopic measurement of $^4$He abundance or a better understanding of systematics effects would of course, provide a powerful way of independently constraining (or measuring) the lepton asymmetry in the neutrino sector. Yet comparison with future CMB data will represent an important consistency check.

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