Theoretical study of the flow of cement raw material sludge through pipes

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Abstract. The flow of fluids through pipes directly depends on the physicomechanical characteristics of the material. When designing technological equipment, it is assumed that the equipment operates with the limiting physical characteristics of the liquid. The study of the flow of non-Newtonian fluids is important for the development of energy-saving methods for the cement industry. In this paper, we study the flow of cement raw sludge through rigid pipes. It is shown that when moving in pipes near the pipe walls, the sludge moves like a normal Newtonian liquid, and near the pipe it moves like a solid. This indicates that when moving through a pipe with a prolonged non-stationary effect, the sludge behaves like a normal Bingham body. The article found the flow rate, the drag coefficient, the Reynolds number, and the hydraulic resistance when the sludge moves through the pipe. In this paper, the parametric law of resistance of sludge movement in pipes is defined, which can be considered as a convenient way to calculate pressure drop. It follows from the law that, when flowing through pipes, sludge now acquires the properties of a plastic body and behaves like Bingham plastic.

Key words: cement sludge, flow through pipe, Newtonian fluids, hydraulic resistance, Reynolds, Bingham plastic.

1. Introduction

In order to set in motion the thixotropic liquid, which had been at rest for a long time, the transfer pump must develop more power [1-7]. At the moment of the beginning of the movement, the structure of the liquid is destroyed. Therefore, further the pump load is reduced. The fluid, as a result of long-term destruction under the action of shear loads, acquires properties independent of time. The stationary movement of thixotropic and other liquids should differ little from each other.

2. Flow of cement slurries

In actual production, a sludge buffer storage device is involved in the sludge dosing process, in which, as a rule, an excess amount of sludge is transported from 15 to 50 percent of the total amount, while the excess sludge is returned to the sludge pool [8]. At the same time, pre-sludge from the sludge pool is fed to the storage tanks, and then distributed to the furnaces, and from the storage tank excess sludge is returned to the sludge pool. In this regard, there is a problem of increasing the efficiency of sludge
supply in cement kilns. Uneven feed of sludge into the rotary kiln with a wet production method can lead to a deterioration in the quality of the clinker firing, and sometimes to emergency operation of the unit [9].

3. Flow of the sludge through pipes

It is assumed that the equipment operates with the limiting physical characteristics of the liquid for designing technological equipment [10]. For stationary movements, the relationship between flow rate and pressure drop is set by the equation [11, 12]

\[ Q = \frac{\pi R^3}{\tau_w} \int_0^{\tau_w} \sigma^2 f(\sigma) d\sigma, \]

where \( R \) – pipe radius, \( \tau_w \) – shear stress on the pipe wall.

In the non-stationary case, the flow of fluid through the cross section according to [13-15]

\[ Q(t) = 2\pi \int_0^R r v_z(t, r) dr, \]

where \( v_z(t, r) \) - flow velocity along the pipe axis. We suppose that the dependence of speed \( v_z \) on \( r \) same as when Bingham’s body is moving [11]

\[ v_z = C_1 \left( R^2 (t, z) - r^2 \right) + C_2 \left( R(t, z) - r \right). \]

To find the constants we consider the balance of forces acting on the cylindrical element of the fluid (figure 1.)

**Figure 1.** Scheme of flow in pipe.

\[ \pi r^2 \delta p = 2\pi L \sigma, \]

From where

\[ \sigma = \frac{r \delta p}{2L}. \]

As mentioned in [16-20], the rheological relationship describing the relationship between stress and strain is
\[ \varepsilon = \frac{\sigma}{E_1} + \frac{\sigma}{E_2} \left( 1 - \exp\left( -\frac{E_2}{\eta_2} t \right) \right) + \frac{\sigma - \sigma_k}{\eta_1}. \]  

(5)

where \( \sigma \) - stress tensor, \( E \) - modulus of elasticity, \( \varepsilon \) - strain tensor, \( \eta \) - viscosity, \( \sigma_k \) - yield strength.

For flow in a cylindrical vessel

\[ -\frac{dv_z}{dr} = \frac{\sigma}{\eta_2} \exp\left( -\frac{E_2}{\eta_2} t \right) + \frac{\sigma - \sigma_k}{\eta_1}. \]

Consequently,

\[ -\int_{v_z}^0 dv_z = \int_r^R \left( \frac{\sigma}{\eta_2} \exp\left( -\frac{E_2}{\eta_2} t \right) + \frac{\sigma - \sigma_k}{\eta_1} \right) dr. \]

(6)

From equation (6), on condition that \( v_z = 0 \) for \( r = R \) we get after simple transformations

\[ v_z = -\frac{\partial \rho}{4\eta_2 L} \left( 1 + \exp\left( -\frac{E_2}{\eta_2} t \right) \right) \left( R^2(t, z) - r^2 \right) - \frac{\sigma_k}{\eta_1} \left( R(t, z) - r \right). \]

(7)

Near the axis of the pipe, where \( \sigma < \sigma_k \), the sludge will move like a solid rod with a radius

\[ r_0 = \frac{2\sigma_k L}{\partial \rho}. \]

(8)

Substituting \( r_0 \) from (8) into (7), we obtain the speed of quasi-solid motion

\[ v_z = -\frac{\partial \rho}{4\eta_2 L} \left( (R - r_0)^2 + (R^2 - r_0^2) \exp\left( -\frac{E_2}{\eta_2} t \right) \right). \]

(9)

The velocity diagram will consist of a paraboloid of rotation (from the wall to a cylindrical surface of radius \( r_0 \)) and a flat platform perpendicular to the axis of the pipe (figure 2).
Figure 2. Velocity diagram.

The velocity profile at different points in time in the plane of symmetry of the pipe is shown in figure 3.

![Velocity diagram](image)

Figure 3. The dimensionless velocity profile at different points of time.

Figure 3 shows that the sludge moves like a normal Newtonian fluid when moving in pipes near the pipe walls, and near the pipe it moves like a solid. Such a movement is characteristic of Bingham liquids. Moreover, if at the initial moment of movement, the velocity of the sludge is higher than the speed of movement of the Bingham body, then over time the velocity profile tends to the profile obtained during the flow of a visco-plastic fluid. Those with prolonged non-stationary exposure, the sludge behaves like a normal Bingham body.

4. Hydraulic resistance when moving through pipes

The hydraulic resistance for the pipe is determined by the expression

\[ \Delta p = \frac{\lambda}{d} \frac{L}{v_{cp}^2} \rho \delta \]

where \( \lambda \) - coefficient of resistance, \( d \) - pipe diameter, \( v_{cp} \) - average flow rate. The drag coefficient \( \lambda \) depends on the physical properties of the medium — density and viscosity, as well as the average velocity — and is a function of the Reynolds number \( Re \)

\[ \lambda = \lambda(Re) \]

The average velocity of the sludge through the pipe will be

\[ v_{cp} = \frac{Q}{\pi R^2} \]

We calculate the flow rate \( Q \) by substituting (7) into (1)
\[ Q = \frac{\pi R^2}{24\eta_2} \left( 3\delta p \left( 1 + e^{\frac{F_{w}}{\eta_2}} \right) - 8\sigma_k \right). \]  

(13)

Turning from an independent variable \( r \) to a variable \( \sigma \) by the formula (8), we obtain

\[ Q = \frac{\pi R^3}{\eta_2 \tau_w^3} \int_{\sigma_k}^{\sigma} \sigma^2 \left( \frac{e^{\frac{F_{w}}{\eta_2}}}{\eta_2} + (\sigma - \sigma_k) \right) d\sigma = \]

\[ = \frac{\pi R^3 \tau_w}{4\eta_2} \left[ 1 + \left( 1 - \frac{\sigma_k^4}{\tau_w^4} \right) e^{\frac{F_{w}}{\eta_2}} \frac{1}{3} \frac{\sigma_k^4}{\tau_w^4} - \frac{4}{3} \frac{\sigma_k}{\tau_w} \right], \]

(14)

or

\[ Q = \frac{\pi R^3 \tau_w}{4\eta_2} \cdot f \left( t, \frac{\sigma_k}{\tau_w} \right), \]

(15)

where

\[ f \left( t, \frac{\sigma_k}{\tau_w} \right) = \left[ 1 + \left( 1 - \frac{\sigma_k^4}{\tau_w^4} \right) e^{\frac{F_{w}}{\eta_2}} \frac{1}{3} \frac{\sigma_k^4}{\tau_w^4} - \frac{4}{3} \frac{\sigma_k}{\tau_w} \right]. \]

(16)

The dependence of the function \( f \left( t, \sigma_k / \tau_w \right) \) on time in coordinates \((\log t, \log f)\) is presented in figure.

**Figure 4.** The dependence of the function \( f \) on time.

From Figure 4 shows that over time, the function tends to a stationary value. This means that, rushing to the limit from formulas (14) - (16), we obtain the formula for the plastic body flow rate (the Buckingham formula).

Using (16) the average speed \( v_{cp} \), according to (12) and (14) will be equal to
Because the shear stress on the pipe wall \( \tau_w \), according to equation (8), will be equal to

\[
\tau_w = \frac{R \dot{\sigma}_p}{2L},
\]

formula (17) takes the form

\[
v_{wp} = \frac{R^2 \dot{\sigma}_p}{8 \eta_2 L} \left[ 1 + \left( 1 - \frac{2L \sigma_{k1}}{R \dot{\sigma}_p} \right)^4 \right] \frac{L}{\eta_2} + \frac{1}{3} \left( \frac{2L \sigma_{k1}}{R \dot{\sigma}_p} \right)^4 - \frac{4}{3} \frac{2L \sigma_{k1}}{R \dot{\sigma}_p} \right] =
\]

(18)

We rewrite equation (18)

\[
\dot{\sigma}_p = \frac{8 \eta_2 L v_{wp}}{R^3} f^{-1} \left( t, \frac{2L \sigma_{k1}}{R \dot{\sigma}_p} \right)
\]

(19)

After simple transformations we get

\[
\dot{\sigma}_p = \frac{64 \eta_2 \rho v_{wp} L}{\rho dv_{wp}^2} f^{-1} \left( t, \frac{2L \sigma_{k1}}{R \dot{\sigma}_p} \right)
\]

(20)

from where

\[
\lambda = \frac{64}{\text{Re}} f^{-1} \left( t, \frac{2L \sigma_{k1}}{R \dot{\sigma}_p} \right) \quad \text{Re} = \frac{\rho v_{wp} d}{\eta_2}
\]

(21)

Find from the equation (18) the Reynolds number is

\[
\text{Re} = \frac{\rho d^3 \dot{\sigma}_p}{32 \eta_2^2 L} f^{-1} \left( t, \frac{4L \sigma_{k1}}{d \dot{\sigma}_p} \right) - \frac{\rho d^3 \sigma_{k1}}{8 \eta_2^2} f_1^{-1} \left( t, \frac{4L \sigma_{k1}}{d \dot{\sigma}_p} \right)
\]

(22)

\[
f_1 \left( t, \frac{4L \sigma_{k1}}{d \dot{\sigma}_p} \right) = \frac{f \left( t, \frac{4L \sigma_{k1}}{d \dot{\sigma}_p} \right)}{4L \sigma_{k1}} \frac{d \dot{\sigma}_p}{d \dot{\sigma}_p}
\]

(23)

Number

\[
\rho d^3 \sigma_{k1} / \eta_2^2
\]

(24)
is dimensionless and characterizes the visco-plastic properties of the liquid [21, 22]. If in equality

\[ \lambda = \frac{64}{\text{Re}} \int_{t}^{t_{f}} \left( \frac{2L\dot{\sigma}_l}{R\bar{\rho}} \right) \, dt = \frac{1}{8} \frac{\rho d^2 \dot{\sigma}_l}{\bar{\eta}_l} f(t, \frac{4L\dot{\sigma}_l}{\partial \bar{\rho}}) \]

(25)

Is considered \(4L\dot{\sigma}_l/\partial \bar{\rho}\) as a parameter, these equations express the parametric law of resistance of sludge movement in pipes. Figure 5 shows the graphs of the Re number for the sludge and plastic body. It can be seen from the figure that the Reynolds slurry number tends to the Re number of the plastic body over time.

![Figure 5](image-url)

**Figure 5.** Dependence of the Reynolds sludge number on time: 1 – Re of sludge, 2 – Re of plastic body

Figures 6 and 7 shows the law of resistance for sludge and plastic bodies on a logarithmic scale with the same mechanical characteristics.
**Figure 6.** The law of resistance $\lambda(Re)$ for sludge  

**Figure 7.** The law of resistance $\lambda(Re)$ for the plastic body

Figure 8 shows the family of curves of the dependence of the resistance coefficient on the number $Re$. The parameter is a number $\rho d^2 \sigma_s / \eta^2$, called the Hedstrom criterion [23-28]. This figure can be considered as a convenient way to calculate the pressure drop.

**Figure 8.** shows that the resistance coefficient during the flow of sludge through the pipe tends to the coefficient of resistance of the plastic body over time.

5. **Conclusion**

Hydraulic resistance when moving sludge through the pipes is considered in the work. Parametric law of resistance to the movement of sludge in the pipes is found. This law can be used as a convenient way to calculate the pressure drop. It follows from the law that sludge now acquires the properties of a plastic body flowing through pipes and behaves like Bingham plastic.

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