Intensional RDB for Big Data Interoperability

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Abstract. A new family of Intensional RDBs (IRDBs), introduced in [1], extends the traditional RDBs with the Big Data and flexible and 'Open schema' features, able to preserve the user-defined relational database schemas and all preexisting user’s applications containing the SQL statements for a deployment of such a relational data. The standard RDB data is parsed into an internal vector key/value relation, so that we obtain a column representation of data used in Big Data applications, covering the key/value and column-based Big Data applications as well, into a unifying RDB framework. Such an IRDB architecture is adequate for the massive migrations from the existing slow RDBMSs into this new family of fast IRDBMSs by offering a Big Data and new flexible schema features as well. Here we present the interoperability features of the IRDBs by permitting the queries also over the internal vector relations created by parsing of each federated database in a given Multidatabase system. We show that the SchemaLog with the second-order syntax and ad hoc Logic Programming and its querying fragment can be embedded into the standard SQL IRDBMSs, so that we obtain a full interoperability features of IRDBs by using only the standard relational SQL for querying both data and meta-data.

1 Introduction

Current RDBMSs were obsolete and not ready to accept the new Big Data (BD) social-network Web applications in the last 10 years, so that the isolated groups of developers of these ad-hoc systems (e.g., Google, Amazon, LinkedIn, Facebook, etc.) could use only the ready old-known technics and development instruments in order to satisfy the highly urgent business market requirements. In an article of the Computerworld magazine [2], June 2009, dedicated to the NoSQL meet-up in San Francisco is reported the following: "NoSQLers came to share how they had overthrown the tyranny of slow, expensive relational databases in favor of more efficient and cheaper ways of managing data”. Moreover, the NoSQL movements advocate that relational fit well for data that is rigidly structured with relations and are designated for central deployments with single, large high-end machines, and not for distribution. Often they emphasize that SQL queries are expressed in a sophisticated language.

However, we can provide the BD infrastructure and physical level in a form of simpler structures, adequate to support the distributive and massive BigData query computations, by preserving the logically higher level interface to customer’s applications. That is, it is possible to preserve the RDB interface to data, with SQL query languages for the programmers of the software applications, with the "physical” parsing of data in more
simple structures, able to deal with Big Data scalability in a high distributive computation framework.

The first step to maintain the logical declarative (non-procedural) SQL query language level, is obtained by a revision of traditional RDBMSs is provided by developing H-store (M.I.T., Brown and Yale University), a next generation OLTP systems that operates on distributed clusters of shared-nothing machines where the data resides entirely in main memory, so that it was shown to significantly outperform (83 times) a traditional, disc-based DBMS. A more full-featured version of the system [3] that is able to execute across multiple machines within a local area cluster has been presented in August 2008. The data storage in H-store is managed by a single-thread execution engine that resides underneath the transaction manager. Each individual site executes an autonomous instance of the storage engine with a fixed amount of memory allocated from its host machine. Multi-side nodes do not share any data structures with collocated sites, and hence there is no need to use concurrent data structures (every read-only table is replicated on all nodes and other tables are divided horizontally into disjoint partitions with a k-safety factor two). More recently, during 2010 and 2011, Stonebraker has been a critic of the NoSQL movement [4,5]: "Here, we argue that using MR systems to perform tasks that are best suited for DBMSs yields less than satisfactory results [6], concluding that MR is more like an extract-transform-load (ETL) system than a DBMS, as it quickly loads and processes large amounts of data in an ad hoc manner. As such, it complements DBMS technology rather than competes with it." After a number of arguments about MR (MapReduction) w.r.t. SQL (with GROUP BY operation), the authors conclude that parallel DBMSs provide the same computing model as MR (popularized by Google and Hadoop to process key/value data pairs), with the added benefit of using a declarative SQL language. Thus, parallel DBMSs offer great scalability over the range of nodes that customers desire, where all parallel DBMSs operate (pipelining) by creating a query plan that is distributed to the appropriate nodes at execution time. When one operator in this plan send data to next (running on the same or a different node), the data are pushed by the first to the second operator (this concept is analog to the process described in my book [7]. Section 5.2.1 dedicated to normalization of SQL terms (completeness of the Action-relational-algebra category RA)), so that (differently from MR), the intermediate data is never written to disk. The formal theoretical framework (the database category DB) of the parallel DBMSs and the semantics of database mappings between them is provided in Big Data integration theory as well [7].

One step in advance in developing this NewSQL approach [1] is to extend the "classic" RDB systems with both features: to offer, on user’s side, the standard RDB database schema for SQL querying and, on computational side, the "vectorial" relational database able to efficiently support the low-level key/value data structures together, in the same logical SQL framework. A new family of Intensional RDBs (IRDBs), introduced in [1], which extends the traditional RDBs with the Big Data and flexible and ‘Open schema’ features, able to preserve the user-defined relational database schemas and all preexisting user’s applications containing the SQL statements for a deployment of such a relational data. The standard RDB data is parsed into an internal vector key/value relation, so that we obtain a column representation of data used in Big Data applications, covering the key/value and column-based Big Data applications as well, into a unifying
RDB framework. The idea of having relational names as arguments goes back to [8] where the authors describe an algebraic operator called SPECIFY that converts a single relation name into its relation, but it is too far from our work by considering the Codd’s normal forms with their concept of ”aggregation” and their concept of ”generalization” (as in Quillian’s semantic networks) and trying to mix both the database and AI areas.

The version of the relational model in which relation names may appear as arguments of other relations is also provided in [9]. In such an approach has been proposed an extension of the relational calculus using HiLog as logical framework rather than FOL, and they call this extension ”relational HiLog”.

Thus, it seems syntactically to be a FOL but it has the particular semantics of the Second Order Logic. In our approach we remain in the FOL syntax with only new terms for the intensional elements (n-ary concepts) and with only simple intensional extension of the standard Tarski’s semantics for the FOL. The extension of the relational algebra in [9] is very different from our standard SQL algebra framework: instead of this standard relational SQL algebra in [9] are provided two relational algebra extensions: “E-relational algebra” which extends standard relational algebra with a set of expansion operators (these operators expand a set of relation names into the union of their relations, with the relational name as an extra argument, and hence not to a key/value representations in Big Data used in our vector relation $r_V$); The second is ”T-relational algebra” which extends e-relational algebra by a set of “totality” operators (to allow the access to the names of all nonempty relations in the relational database).

Thus, both from the algebraical, structural and logical framework this approach is very different from our GAV Data integration model and a minimal conservative intensional extension of the Tarski’s FOL semantics.

Another approaches in which relation and attribute names may appear as arguments of other relations are provided in the area of integration of heterogeneous databases. In [10] a simple Prolog interpreter for a subset of F-logic was presented, but the negation in Prolog is not standard as in FOL, and such an approach is far from SQL models of data and querying of RDB databases. Also in [11] is demonstrated the power of using variables that uniformly range over data and meta-data, for schema browsing and interoperability, but their languages have a syntax closer to that of logic programming languages, and far from that of SQL. The more powerful framework (where the variables can range over the following five sets: (i) names of databases in a federation; (ii) names of the relations in a database; (iii) names of the attributes in the scheme of a relations; (iv) tuples in a given relation in a database; and (v) values appearing in a column corresponding to a given attribute in a relation) is presented in SchemaSQL [12] where the SQL is extended in order to be able to query metadata, which in our case is not necessary because we preserve the original RDB SQL in order to be able to migrate from the RDB models into IRDB models with Big Data vector relation without unnecessarily complications. In fact, the extended relational algebra in SchemaSQL would be an non desirable complication in order to obtain the flexible schema and Big Data RDB features.
The interoperability is the ability to share, interpret and manipulate the information across the heterogeneous database systems supported by Multidatabase systems (MDBS) in a distributed network by encompassing a heterogeneous mix of local database systems. Languages based on higher-order logic have been used for the interoperability by considering that the schematic information should be considered as part of a database’s information content. The major advantage associated with such approaches, used in SchemaLog [13,12], is the declaratively they derive from their logical foundation. The weak points of the SchemaLOG is that it uses the second-order logic syntax and an ad-hoc Prolog-like fixpoint semantics. However, both of them are not necessary, as we will show by using the IRDBs, just because the ordinary RDBs have the FOL syntax and do not need fixpoint semantics but ordinary FOL semantics.

In what follows, we denote by $B^A$ the set of all functions from $A$ to $B$, and by $A^n$ a n-folded cartesian product $A \times \ldots \times A$ for $n \geq 1$, we denote by $\neg, \land, \lor, \Rightarrow$ and $\Leftrightarrow$ the logical operators negation, conjunction, disjunction, implication and equivalence, respectively. For any two logical formulae $\phi$ and $\psi$ we define the XOR logical operator $\lor$ by $\phi \lor \psi$ logically equivalent to $(\phi \lor \psi) \land \neg(\phi \land \psi)$.

1.1 Syntax and semantics of SchemaLog

The SchemaLog is syntactically higher-order clausal logic, and is based on the technical benefits of soundness, completeness and compactness by a reduction to first-order predicate calculus. It has a strictly higher expressive power than first-order logic based on this syntax, differently from IRDB which have the standard FOL syntax but reacher intensional conservative Tarski’s semantics.

The vocabulary of the SchemaLog language $\mathcal{L}_S$ consists of the disjoint sets: $G$ of k-ary ($k \geq 1$) functional symbols, $S$ of non-functional symbols (language constants, i.e., nullary functional symbols), $V$ of variables and usual logical connectives $\neg, \lor, \land, \exists$ and $\forall$.

Every symbol in $S$ and $V$ is a term of the language, i.e., $S \cup V \subseteq T$. If $f \in G$ is a $n$-ary function symbol and $t_1, \ldots, t_n$ are terms in $T$ then $f(t_1, \ldots, t_n)$ is a term in $T$.

An atomic formula of $\mathcal{L}_S$ is an expression (note that it is not a predicate-based atom of the FOL, that is, in SchemaLog we do not use the predicate letters) of the following forms [13]:

(i) $\langle \text{db} \rangle :: \langle \text{rel} \rangle[[\langle \text{tid} \rangle : \langle \text{attr} \rangle \rightarrow \langle \text{val} \rangle]]$;
(ii) $\langle \text{db} \rangle :: \langle \text{rel} \rangle[\langle \text{attr} \rangle]$;
(iii) $\langle \text{db} \rangle :: \langle \text{rel} \rangle$;
(iv) $\langle \text{db} \rangle$;

where $\langle \text{db} \rangle$ (the database symbols or names), $\langle \text{rel} \rangle$ (the relational symbols or names), $\langle \text{attr} \rangle$ (the attribute symbols or names), $\langle \text{tid} \rangle$ (the tuple-ids) and $\langle \text{val} \rangle$ are the sorts in $S$ of $\mathcal{L}_S$.

The well-formed formulae (wff) of $\mathcal{L}_S$ are defined as usual: every atom is a wff; $\neg \phi$, $\phi \lor \psi$, $\phi \land \psi$, $(\exists x) \phi$ and $(\forall x) \phi$ are wffs of $\mathcal{L}_S$ whenever $\phi$ and $\psi$ are wffs and $x \in V$ is a variable.

A literal is an atom or the negation of an atom. A clause is a formula of the form $\forall x_1, \ldots, \forall x_m (L_1 \lor \ldots \lor L_n)$ where each $L_i$ is a literal and $x_1, \ldots, x_m$ are the variables occurring in $L_1 \lor \ldots \lor L_n$. A definite clause is a clause in which one positive literal is
present and represented as \( A \leftarrow B_1, ..., B_n \) where \( A \) is called the head and \( B_1, ..., B_n \) is called the body of the definite-clause. A unit clause is a clause of the form \( A \leftarrow \), that is, a definite clause with an empty body.

Let \( D \) be a nonempty set of elements (called “intensions”). A semantic structure of the language \( L_S \) is a tuple \( M = \langle D, I, I_{\text{fun}}, F \rangle \) where:

1. \( I : S \rightarrow D \) is a function of non-function symbols in \( S \);
2. \( I_{\text{fun}}(f) : D^n \rightarrow D \) is an interpretation of the functional symbol \( f \in G \) of arity \( n \);
3. \( F : D \rightarrow |D| \leftarrow |D| \rightarrow D \), where \( [A \leftarrow B] \) denotes the set of all partial functions from \( A \) to \( B \).

To illustrate the role of \( F \), consider the atom \( d :: r \). For this atom to be true, \( F(I(d))(I(r)) \) should be defined in \( M \). Similarly, for the atom \( d :: r[t : a \rightarrow v] \) to be true, \( F(I(d))(I(r))(I(a))(I(t)) \) should be defined in \( M \) and \( F(I(d))(I(r))(I(a))(I(t)) = I(v) \).

A variable assignment \( g \) is a function \( g : V \rightarrow D \) (i.e., \( g \in D^V \)). We extend it to all terms in \( T \) as follows:

\[
g(\overline{s}) = I(\overline{s}) \quad \text{for every } \overline{s} \in S;
\]

\[
g(f(t_1, ..., t_n)) = I_{\text{fun}}(f)(g(t_1), ..., g(t_n)) \quad \text{where } f \in G \text{ is a functional symbol of arity } n \text{ and } t_i \text{ are terms.}
\]

For a given set of terms \( t_i \in T, i = 1, 2, ... \) and the formulae \( \phi \) and \( \psi \), we define the satisfaction relation \( M \models g \) for a given assignment \( g \) and the structure \( M \) as follows:

1. \( M \models g \ t_1 \) iff \( F(g(t_1)) \) is defined in \( M \);
2. \( M \models g \ t_1 :: t_2 \) iff \( F(g(t_1))(g(t_2)) \) is defined in \( M \);
3. \( M \models g \ t_1 :: t_2[t_3] \) iff \( F(g(t_1))(g(t_2))(g(t_3)) \) is defined in \( M \);
4. \( M \models g \ t_1 :: t_2[t_4 : t_3 \rightarrow t_5] \) iff \( F(g(t_1))(g(t_2))(g(t_3))(g(t_4)) \) is defined in \( M \) and \( F(g(t_1))(g(t_2))(g(t_3))(g(t_4)) = g(t_5) \);
5. \( M \models g \ \phi \lor \psi \) iff \( M \models g \phi \) or \( M \models g \psi \);
6. \( M \models g \ \phi \) iff not \( M \models g \ \phi \);
7. \( M \models g \ (\exists x) \phi \) iff for some valuation \( g' \), that may differ from \( g \) only on the variable \( x \), \( M \models g' \phi \).

The specification of an extension of a RDB in this logic framework can be done by specification of the Logic program with the (high) number of unit and definite ground clauses (for each tuple in some relational table of such an RDB), which renders it useless for the Big Data applications, because such a Logic Program would be enormous. Moreover, we do not need to use the fixpoint semantics of Logic programming for the definition of the extension of the RDBs instead of the standard Tarski’s semantics of the FOL. Thus, the SchemaLog framework, defined for the Multidatabase interoperability, cannot be used for the interoperability in Big Data applications, and hence we will show that SchemaLog can be reduced to intentional RDB (IRDB) which are designed for Big Data NewSQL applications.

The plan of this paper is the following:

In Section 2 we introduce the parsing of the RDBs into the Big Data vector relations, and then we present in Section 3 the intentional semantics for this new data structures, that is, of the intentional RDBs (IRDBs). The main Section 4 is dedicated to the Multidatabase IRDBs and we explain how a meta-data interoperability SchemaLog framework is embedded into the IRDBs Big Multidatabase systems.
2 Vector databases of the IRDBs

We will use the following RDB definitions, based on the standard First-Order Logic (FOL) semantics:

- A database schema is a pair \( \mathcal{A} = (S_A, \Sigma_A) \) where \( S_A \) is a countable set of relational symbols (predicates in FOL) \( r \in \mathbb{R} \) with finite arity \( n = ar(r) \geq 1 \) (\( ar(r) : \mathbb{R} \rightarrow \mathbb{N} \)), disjoint from a countable infinite set \( \text{att} \) of attributes (a domain of \( a \in \text{att} \) is a nonempty finite subset \( \text{dom}(a) \) of a countable set of individual symbols \( \text{dom} \)). For any \( r \in \mathbb{R} \), the sort of \( r \), denoted by tuple \( a = atr(r) = (atr_r(1), \ldots, atr_r(n)) \) where all \( a_i = atr_r(m) \in \text{att} \), \( 1 \leq m \leq n \), must be distinct: if we use two equal domains for different attributes then we denote them by \( a_i(1), \ldots, a_i(k) \) (\( a_i \) equals to \( a_i(0) \)). Each index ("column") \( i, 1 \leq i \leq ar(r) \), has a distinct column name \( nr_r(i) \in SN \) where \( SN \) is the set of names with \( nr(r) = (nr_r(1), \ldots, nr_r(n)) \). A relation symbol \( r \in \mathbb{R} \) represents the relational name and can be used as an atom \( r(x) \) of FOL with variables in \( x \) assigned to its columns, so that \( \Sigma_A \) denotes a set of sentences (FOL formulae without free variables) called integrity constraints of the sorted FOL with sorts in \( \text{att} \).

- An instance-database of a nonempty schema \( \mathcal{A} \) is given by \( A = (\mathcal{A}, I_T) = \{ R = \|r\| = I_T(r) \mid r \in S_A \} \) where \( I_T \) is a Tarski’s FOL interpretation which satisfies all integrity constraints in \( \Sigma_A \) and maps a relational symbol \( r \in S_A \) into an n-ary relation \( R = \|r\| \in A \). Thus, an instance-database \( A \) is a set of n-ary relations, managed by relational database systems.

- We consider a rule-based conjunctive query over a database schema \( \mathcal{A} \) as an expression \( q(x) \leftarrow r_1(u_1), \ldots, r_n(u_n) \), with finite \( n \geq 0 \), \( r_i \) are the relational symbols (at least one) in \( \mathcal{A} \) or the built-in predicates (e.g. \( \leq, = \)), \( q \) is a relational symbol not in \( \mathcal{A} \) and \( u_i \) are free tuples (i.e., one may use either variables or constants). Recall that if \( v = (v_1, \ldots, v_m) \) then \( r(v) \) is a shorthand for \( r(v_1, \ldots, v_m) \). Finally, each variable occurring in \( x \) is a distinguished variable that must also occur at least once in \( u_1, \ldots, u_n \). Rule-based conjunctive queries (called rules) are composed of a subexpression \( r_1(u_1), \ldots, r_n(u_n) \) that is the body, and the head of this rule \( q(x) \).

The deduced head-facts of a conjunctive query \( q(x) \) defined over an instance \( A \) (for a given Tarski’s interpretation \( I_T \) of schema \( \mathcal{A} \)) are equal to \( \|q(x_l, \ldots, x_k)\|_A = \{ < v_1, \ldots, v_k > \in \text{dom}^k \mid \exists y(r_1(u_1) \land \ldots \land r_n(u_n))[x_l/v_l]_{1 \leq l \leq k} \text{ is true in } A \} = I_T^* (\exists y(r_1(u_1) \land \ldots \land r_n(u_n))) \), where the \( y \) is a set of variables which are not in the head of query, and \( I_T^* \) is the unique extension of \( I_T \) to all formulae. Each conjunctive query corresponds to a "select-project-join" term \( t(x) \) of SPRJU algebra obtained from the formula \( \exists y(r_1(u_1) \land \ldots \land r_n(u_n)) \).

- We consider a finitary view as a union of a finite set \( S \) of conjunctive queries with the same head \( q(x) \) over a schema \( \mathcal{A} \), and from the equivalent algebraic point of view, it is a "select-project-join-rename + union" (SPJRU) finite-length term \( t(x) \) which corresponds to union of the terms of conjunctive queries in \( S \). A materialized view of an instance-database \( A \) is an n-ary relation \( R = \bigcup_{q(x) \in S} \|q(x)\|_A \).

Recall that two relations \( r_1 \) and \( r_2 \) are union-compatible iff \( \{ atr(r_1) \} = \{ atr(r_2) \} \). If a relation \( r_2 \) is obtained from a given relation \( r_1 \) by permutating its columns, then we tell that they are not equal (in set theoretic sense) but that they are equivalent. Notice that
in the RDB theory the two equivalent relations are considered equal as well. In what follows, given any two lists (tuples), \( d = \langle d_1, \ldots, d_k \rangle \) and \( b = \langle b_1, \ldots, b_m \rangle \) their concatenation \( \langle d_1, \ldots, d_k, b_1, \ldots, b_m \rangle \) is denoted by \( d \& b \), where ‘&’ is the symbol for concatenation of the lists.

The set of basic relation algebra operators are:

1. Rename is a unary operation written as \( \text{name} \) RENAME \( \text{name}_1 \) AS \( \text{name}_2 \) where the result is identical to input argument (relation) \( r \) except that the column \( i \) with name \( nr_r(i) = \text{name}_1 \) in all tuples is renamed to \( nr_r(i) = \text{name}_2 \).

2. Cartesian product is a binary operation \( \text{TIMES} \), written also as \( \text{\&} \), such that for the relations \( r_1 \) and \( r_2 \), first we do the rename normalization of \( r_2 \) w.r.t. \( r_1 \).

3. Projection is a unary operation written as \( \text{\{} \text{S} \text{\}} \), where \( S \) is a tuple of column names such that for a relation \( r_1 \) and \( S = \langle nr_{r_1}(i_1), \ldots, nr_{r_1}(i_k) \rangle \), with \( k \geq 1 \) and \( 1 \leq i_m \leq ar(r_1) \) for \( 1 \leq m \leq k \), and \( i_m \neq i_j \) if \( m \neq j \), we define the relation \( r \) by: 
   \[
   r_1[S] = \{ \langle r_{1}(i_1), \ldots, r_{1}(i_k) \rangle \mid \|r_{1}\| = \|r_{1}[S]\| \text{ if } \exists \text{name} \in S. \text{name} \notin nr(r_1); \text{otherwise } \|r_{1}\| = \pi_{<1,\ldots,i_k>}(\|r_{1}\|),
   \]
   where \( nr_{r_1}(m) = nr_{r_1}(i_m), atr_{r_1}(m) = atr_{r_1}(i_m), \) for \( 1 \leq m \leq k \).

4. Selection is a unary operation written as \( \text{\{} \text{WHERE} \text{C} \text{\}} \), where a condition \( C \) is a finite-length logical formula that consists of atoms ‘\( \text{name}_i \text{\theta name}_j \)’ or ‘‘\( \text{name}_i \text{\theta \bar{d}} \)’’, with built-in predicates \( \theta \in \Sigma \), and the logical operators \( \land \) (AND), \( \lor \) (OR) and \( \neg \) (NOT), such that for a relation \( r_1 \) and \( \text{name}_i, \text{name}_j \) the names of its columns, we define the relation \( r \) by:
   \[
   r_1 \text{WHERE } C,
   \]
   as the relation with \( atr(r) = atr(r_1) \) and the function \( nr_r \) equal to \( nr_{r_1} \), where \( \|r\| \) is composed by the tuples in \( \|r_1\| \) for which \( C \) is satisfied.

5. Union is a binary operation written as \( \text{\{} \text{UNION} \text{\}} \), such that for two union-compatible relations \( r_1 \) and \( r_2 \), we define the relation \( r \) by:
   \[
   r_1 \text{UNION} r_2,
   \]
   where \( \|r\| \triangleq \|r_1\| \cup \|r_2\| \), with \( atr(r) = atr(r_1) \), and the functions \( atr_r = atr_{r_1} \), and \( nr_r = nr_{r_2} \).

6. Set difference is a binary operation written as \( \text{\{} \text{MINUS} \text{\}} \), such that for two union-compatible relations \( r_1 \) and \( r_2 \), we define the relation \( r \) by:
   \[
   r_1 \text{MINUS} r_2,
   \]
   where \( \|r\| \triangleq \{ t \mid t \in \|r_1\| \text{ such that } t \notin \|r_2\| \} \), with \( atr(r) = atr(r_1) \), and the functions \( atr_r = atr_{r_1} \), and \( nr_r = nr_{r_1} \).

Natural join \( \bowtie \) is a binary operator, written as \( r_1 \bowtie r_2 \), where \( r_1 \) and \( r_2 \) are the relations. The result of the natural join is the set of all combinations of tuples in \( r_1 \) and \( r_2 \) that are equal on their common attribute names. In fact, \( r_1 \bowtie r_2 \) can be obtained by creating the Cartesian product \( r_1 \bowtie r_2 \) and then by execution of the Selection with the condition \( C \) defined as a conjunction of atomic formulae \( (nr_{r_1}(i) = nr_{r_2}(j)) \) with \( (nr_{r_1}(i), nr_{r_2}(j)) \in S \) (where \( i \) and \( j \) are the columns of the same attribute in \( r_1 \) and \( r_2 \), respectively, i.e., satisfying \( atr_{r_1}(i) = atr_{r_2}(j) \)) that represents the equality of the common attribute names of \( r_1 \) and \( r_2 \).

We are able to define a new relation with a single tuple \( \langle d_1, \ldots, d_k \rangle, k \geq 1 \) with the given list of attributes \( \langle a_1, \ldots, a_k \rangle, \) by the following finite length expression,

\[
\text{EXTEND} (\ldots (\text{EXTEND } r_0 \text{ ADD } a_1, \text{name}_1 \text{ AS } \bar{d}_1) \ldots) \text{ ADD } a_k, \text{name}_k \text{ AS } \bar{d}_k, \text{ or equivalently by } r_0 (a_1, \text{name}_1, d_1) \bowtie \ldots \bowtie r_0 (a_k, \text{name}_k, d_k),
\]

where \( r_0 \) is the empty type relation with \( \|r_0\| = \{<>\}, ar(r_0) = 0 \).
inition5 and empty functions atr_{t_R} and nr_{t_R}. Such single tuple relations can be used for an insertion in a given relation (with the same list of attributes) in what follows.

**Update operators.** The three update operators, 'UPDATE', 'DELETE' and 'INSERT' of the Relational algebra, are derived operators from these previously defined operators in the following way:

1. Each algebraic formulae 'DELETE FROM r WHERE C' is equivalent to the formula 'r MINUS (r WHERE C)'.
2. Each algebraic expression (a term) 'INSERT INTO r[S] VALUES (list of values)', 'INSERT INTO r[S] AS SELECT...', is equivalent to 'r UNION r_1', where the union compatible relation r_1 is a one-tuple relation (defined by list) in the first, or a relation defined by 'SELECT... in the second case.
3. Each algebraic expression 'UPDATE r SET [nr_r(i_1) = e_{i_1},...,nr_r(i_k) = e_{i_k}] WHERE C', for n = ar(r), where e_{i_m}, 1 ≤ i_m ≤ n for 1 ≤ m ≤ k are the expressions and C is a condition, is equal to the formula '(r WHERE ¬C) UNION r_1', where r_1 is a relation expressed by (EXTEND(...(EXTEND (r WHERE C) ADD att_r(1),name_1 AS e_1)...) ADD att_r(n),name_n AS e_n)[S], such that for each 1 ≤ m ≤ n, if m /∈ {i_1,...,i_k} then e_m = nr_r(m), and S = < name_1,...,name_n >.

Consequently, all update operators of the relational algebra can be obtained by addition of these 'EXTEND _ ADD a,name AS e' operations.

Let us define the \( \Sigma_R \)-algebras sa follows ([7], Definition 31 in Section 5.1):

**Definition 1.** We define the algebra of the set of operations, introduced previously in this section (points from 1 to 6 and EXTEND _ ADD a,name AS e) with additional nullary operator (empty-relation constant) \( \perp \), by \( \Sigma_{RE} \). Its subalgebra without _ MINUS_ operator is denoted by \( \Sigma_{RE}^+ \), and without \( \perp \) and unary operators EXTEND _ ADD a,name AS e is denoted by \( \Sigma_R \) (it is the "select-project-join-rename+union" (SPIRU) subalgebra). We define the set of terms \( T_PX \) with variables in \( X \) of this \( \Sigma_R \)-algebra (and analogously for the terms \( T_P^+X \) of \( \Sigma_{RE}^+ \)-algebra), inductively as follows:

1. Each relational symbol (a variable) \( r \in X \subseteq \mathbb{R} \) and a constant (i.e., a nullary operation) is a term in \( T_PX \);
2. Given any term \( t_R \in T_PX \) and an unary operation \( o_i \in \Sigma_R, o_i(t_R) \in T_PX \);
3. Given any two terms \( t_R,t'_R \in T_PX \) and a binary operation \( o_i \in \Sigma_R, o_i(t_R,t'_R) \in T_PX \).

We define the evaluation of terms in \( T_PX \), for \( X = \mathbb{R} \), by extending the assignment \( \llbracket \cdot \rrbracket : \mathbb{R} \rightarrow \mathcal{L} \) which assigns a relation to each relational symbol (a variable) to all terms by the function \( \llbracket \cdot \rrbracket : T_P \mathbb{R} \rightarrow \mathcal{L} \) (with \( \llbracket r \rrbracket \# = \llbracket r \rrbracket \)), where \( \mathcal{L} \) is the universal database instance (set of all relations for a given universe \( D \)). For a given term \( t_R \) with relational symbols \( r_1,...,r_k \in \mathbb{R} \), \( \llbracket t_R \rrbracket \# \) is the relational table obtained from this expression for the given set of relations \( \llbracket r_1 \rrbracket \# ,...,\llbracket r_k \rrbracket \in \mathcal{L} \) with the constraint that \( \llbracket t_R \rrbracket \# \perp \llbracket t'_R \rrbracket \# \) if the relations \( \llbracket t_R \rrbracket \# \) and \( \llbracket t'_R \rrbracket \# \) are union compatible; \( \perp = < > \) = \( \llbracket r_0 \rrbracket \# \) (empty relation) otherwise.

We say that two terms \( t_R,t'_R \in T_PX \) are equivalent (or equal), denoted by \( t_R \approx t'_R \), if for all assignments \( \llbracket t_R \rrbracket \# = \llbracket t'_R \rrbracket \#. \)
The principal idea for the IRDBs (intensional RDBs) introduced in [1] is to use an analogy with a GAV Data Integration [14,7] by using the database schema \( \mathcal{A} = (S_A, \Sigma_A) \) as a global relational schema, used as a user/application-program interface for the query definitions in SQL, and to represent the source database of this Data Integration system by parsing of the RDB instance \( A \) of the schema \( \mathcal{A} \) into a single vector relation \( \bar{A} \).

Thus, the original SQL query \( q(x) \) has to be equivalently rewritten over (materialized) source vector database \( \bar{A} \).

In fact, each \( i \)-th column value \( d_i \) in a tuple \( d = \langle d_1, ..., d_i, ..., d_{ar(r)} \rangle \) of a relation \( R_k = \|r_k\|, r_k \in S_A \), of the instance database \( A \) is determined by the free dimensional coordinates: relational name \( nr(r) \), the attribute name \( nr_a(i) \) of the \( i \)-th column, and the tuple index \( \text{Hash}(d) \) obtained by hashing the string of the tuple \( d \). Thus, the relational schema of the vector relation is composed by the four attributes, relational name, tuple-index, attribute name, and value, i.e., \( x\text{-name}, t\text{-index}, a\text{-name} \) and \( \text{value} \), respectively, so that if we assume \( r_V \) (the name of the database \( \mathcal{A} \)) for the name of this vector relation \( \bar{A} \) then this relation can be expressed by the quadruple \( r_V(x\text{-name}, t\text{-index}, a\text{-name}, \text{value}) \), and the parsing of any RDB instance \( A \) of a schema \( \mathcal{A} \) can be defined as:

**Definition 2. Parsing RDB instances:**

Given a database instance \( A = \{R_1, ..., R_n\} \), \( n \geq 1 \), of a RDB schema \( \mathcal{A} = (S_A, \Sigma_A) \) with \( S_A = \{r_1, ..., r_n\} \) such that \( R_k = \|r_k\|, k = 1, ..., n \), then the extension \( \bar{A} = \|r_V\| \) of the vector relational symbol (name) \( r_V \) with the schema \( r_V(x\text{-name}, t\text{-index}, a\text{-name}, \text{value}) \), and NOT NULL constraints for all its four attributes, and with the primary key composed by the first three attributes, is defined by:

- we define the operation \( \text{PARSE} \) for a tuple \( d = \langle d_1, ..., d_{ar(r)} \rangle \) of the relation \( r_k \in S_A \) by the mapping
  \[
  (r_k, d) \mapsto \{\langle r_k, \text{Hash}(d), nr_{r_a}(i), d_i \rangle \mid \text{NOT NULL, } 1 \leq i \leq ar(r_k)\},
  \]
- Based on the vector database representation \( \|r_V\| \) we define a GAV Data Integration system \( \mathcal{I} = (\mathcal{A}, S, \mathcal{M}) \) with the global schema \( \mathcal{A} = (S_A, \Sigma_A) \), the source schema \( S = \{\{r_V\}, \emptyset\} \), and the set of mappings \( \mathcal{M} \) expressed by the tgds (tuple generating dependencies)

\[
(2) \quad \forall y, x_1, ..., x_{ar(r)}(\langle (r_V(r_k, y, nr_{r_a}(1), x_1) \lor x_1 \text{NULL} \rangle \land \ldots \land (r_V(r_k, y, nr_{r_a}(ar(r_k)), x_{ar(r_k)}) \lor x_{ar(r_k)} \text{NULL}) \Rightarrow r_k(x_1, ..., x_{ar(r_k)})),
\]

for each \( r_k \in S_A \).

The operation \( \text{PARSE} \) corresponds to the parsing of the tuple \( v \) of the relation \( r_k \in S_A \) of the user-defined database schema \( \mathcal{A} \) into a number of tuples of the vector relation \( r_V \).

In fact, we can use this operation for virtual inserting/deleting of the tuples in the user defined schema \( \mathcal{A} \), and store them only in the vector relation \( r_V \). This operation avoids to materialize the user-defined (global) schema, but only the source database \( S \), so that each user-defined SQL query has to be equivalently rewritten over the source database (i.e., the big table \( \bar{A} = \|r_V\| \)) as in standard FOL Data Integration systems.

Notice that this parsing defines a kind of GAV Data Integration systems, where the source database \( S \) is composed by the unique vector relation \( \|r_V\| = \bar{A} \) (Big Data) which does not contain NULL values, so that we do not unnecessarily save the NULL
values of the user-defined relational tables $r_k \in S_A$ in the main memories of the parallel RDBMS used to horizontal partitioning of the unique big-table $\overrightarrow{A}$. Moreover, any adding of the new columns to the user-defined schema $A$ does not change the table $\overrightarrow{A}$, while the deleting of a $i$-th column of a relation $r$ will delete all tuples $r_V(x, y, z, v)$ where $x = nr(r)$ and $z = nr_r(i)$ in the main memory of the parallel RDBMS. Thus, we obtain very schema-flexible RDB model for Big Data.

The intensional Data Integration system $\mathcal{I} = (A, S, M)$ in Definition 2 is used in the way that the global schema is only virtual (empty) database with a user-defined schema $A = (S_A, \Sigma_A)$ used to define the SQL user-defined query which then has to be equivalently rewritten over the vector relation $r_V$ in order to obtain the answer to this query. Thus, the information of the database is stored only in the big table $\| r_V \|$. Thus, the materialization of the original user-defined schema $A$ can be obtained by the following operation:

**Definition 3. Materialization of the RDB**

Given a user-defined RDB schema $A = (S_A, \Sigma_A)$ with $S_A = \{r_1, \ldots, r_n\}$ and a big vector table $\| r_V \|$, the non SQL operation $\text{MATTER}$ which materializes the schema $A$ into its instance database $A = \{R_1, \ldots, R_n\}$ where $R_k = \| r_k \|$, for $k = 1, \ldots, n$, is given by the following mapping, for any $R \subseteq \| r_V \|$: 

$$
(r_k, R) \mapsto \{ \langle v_1, \ldots, v_{ar(r_k)} \rangle \mid \exists y \in \pi_2(R)(r_V(r_k, y, nr_{r_k}(1), v_1) \lor v_1 \text{NULL}) \land \ldots \land (r_V(r_k, y, nr_{r_k}(ar(r_k)), v_{ar(r_k)}) \lor v_{ar(r_k)} \text{NULL}) \}.
$$

so that the materialization of the schema $A$ is defined by $R_k = \| r_k \| \overset{\text{MATTER}}{\Rightarrow} (r_k, \| r_V \|)$ for each $r_k \in S_A$.

The canonical models of the intensional Data Integration system $\mathcal{I} = (A, S, M)$ in Definition 2 are the instances $A$ of the schema $A$ such that $\| r_k \| = \text{MATTER}(r_k, \bigcup_{r \in \| r_k \|} \text{PARSE}(r_k, v))$, that is, when $A = \{ \text{MATTER}(r_k, \overrightarrow{A}) \mid r_k \in S_A \}$.

We say that an extension $\| t_R \|_\#$, of a term $t_R \in TP(X)$, is vector relation of the vector view denoted by $t_R$ if the type of $\| t_R \|_\#$ is equal to the type of the vector relation $r_V$.

Let $R = \| t_R \|_\#$ be the relational table with the four attributes (as $r_V$) $x$-name, $t$-index, $a$-name and value, then its used-defined view representation can be derived as follows:

**Definition 4. View Materialization:** Let $t_R \in TP(X)$ be a user-defined SPJU (Select-Project-Join-Union) view over a database schema $A = (S_A, \Sigma_A)$ with the type (the tuple of the view columns) $\mathcal{G} = ((r_{k_1}, \text{name}_{k_1}), \ldots, (r_{k_m}, \text{name}_{k_m}))$, where the $i$-th column $r_{k_i}$ of the schema $A$ is the column with name $\text{name}_{k_i}$ of the relation name $r_{k_i} \in S_A$, $1 \leq i \leq m$, and $\overrightarrow{t_R}$ be the rewritten query over $r_V$. Let $R = \| \overrightarrow{t_R} \|_\#$ be the resulting relational table with the four attributes (as $r_V$) $x$-name, $t$-index, $a$-name and value. We define the operation $\text{VIEW}$ of the transformation of $R$ into the user defined view representation by:

$$
\text{VIEW}(\mathcal{G}, R) = \{ (d_1, \ldots, d_m) \mid \exists ID \in \pi_3(R), \forall_1 \leq i \leq m ((r_{k_i}, ID, \text{name}_{k_i}, d_i) \in R) \text{; otherwise set } d_i \text{ to } \text{NULL} \}.
$$

Notice that we have $\| r_k \| = \text{VIEW}(\mathcal{G}, R) = \text{MATTER}(r_k, R)$ for each $r_k \in S_A$ with $R = \bigcup_{d \in \| r_k \|} \text{PARSE}(r_k, d)$, and $\mathcal{G} = ((r_{k}, nr_{r_k}(1)), \ldots, (r_{k}, nr_{r_k}(ar(r_k))))$, and...
hence the nonSQL operation MATTER is a special case of the operation VIEW.

For any original user-defined query (term) $t_R$ over a user-defined database schema $\mathcal{A}$, by $\tilde{t}_R$ we denote the equivalent (rewritten) query over the vector relation $r_V$. We have the following important result for the IRDBs:

**Proposition 1** There exists a complete algorithm for the term rewriting of any user-defined SQL term $t_R$ over a schema $\mathcal{A}$, of the full relational algebra $\Sigma_{RE}$ in Definition 1, into an equivalent vector query $\tilde{t}_R$ over the vector relation $r_V$.

If $t_R$ is a SPJU term (in Definition 4) of the type $\mathcal{S}$ then $\|\tilde{t}_R\|_\# = \text{VIEW}(\mathcal{S}, \|\tilde{t}_R\|_\#)$.

The proof can be found in [1]. This proposition demonstrates that the IRDB is full SQL database, so that each user-defined query over the used-defined RDB database schema $\mathcal{A}$ can be equivalently transformed by query-rewriting into a query over the vector relation $r_V$. However, in the IRDBMSs we can use more powerful and efficient algorithms in order to execute each original user-defined query over the vector relation $r_V$.

Notice that this proposition demonstrates that the IRDB is a kind of GAV Data Integration System $\mathcal{I} = (\mathcal{A}, \mathcal{S}, \mathcal{M})$ in Definition 2 where we do not materialize the user-defined schema $\mathcal{A}$ but only the vector relation $r_V \in \mathcal{S}$ and each original query $q(x)$ over the empty schema $\mathcal{A}$ will be rewritten into a vector query $\tilde{q}(x)$ of the type $\mathcal{S}$ over the vector relation $r_V$, and then the resulting view $\text{VIEW}(\mathcal{S}, \|\tilde{q}(x)\|_\#)$ will be returned to user’s application. The operators PARSE, MATTER and VIEW can be represented [15] as derived algebraic operators of the (UN)PIVOT operators (introduced in ) and of the relational operators in Definition 1.

Thus, an IRDB is a member of the NewSQL, that is, a member of a class of modern relational database management systems that seek to provide the same scalable performance of NoSQL systems for online transaction processing (read-write) workloads while still maintaining the ACID guarantees of a traditional database system.

We can easy see that the mapping tgd's used from the Big Data vector table $\tilde{\mathcal{A}}$ (the source schema in Data Integration) into user-defined RDB schema $\mathcal{A}$ (the global schema of this Data Integration system with integrity constraints) is not simple FOL formula. Because the same element $r_k$ is used as a predicate symbol (on the right-side of the tgd’s implication) and as a value (on the left side of the implication as the first value in the predicate $r_V$). It means that the elements of the domain of this logic are the elements of other classes and are the classes for themselves as well. Such semantics is not possible in the standard FOL, but only in the intensional FOL.

### 3 Intensional semantics for IRDBs

More about relevant recent works for intensional FOL can be found in [16,17] where a new conservative intensional extension of the Tarski’s semantics of the FOL is defined. Intensional entities are such concepts as propositions and properties. The term ’intensional’ means that they violate the principle of extensionality; the principle that extensional equivalence implies identity. All (or most) of these intensional entities have been classified at one time or another as kinds of Universals [18].

We consider a non empty domain $\mathcal{D} = D_{-1} \cup D_I$, where a subdomain $D_{-1}$ is made of
particulars (extensional entities), and the rest \( D_1 = D_0 \cup D_1 \cup ... \cup D_n \) is made of universals (\( D_0 \) for propositions (the 0-ary concepts), and \( D_n, n \geq 1 \), for \( n \)-ary concepts).

The fundamental entities are intensional abstracts or so called, 'that'-clauses. We assume that they are singular terms; Intensional expressions like 'believe', 'mean', 'assert', 'know', are standard two-place predicates that take 'that'-clauses as arguments. Expressions like 'is necessary', 'is true', and 'is possible' are one-place predicates that take 'that'-clauses as arguments. For example, in the intensional sentence "it is necessary that \( \phi \)" where \( \phi \) is a proposition, the 'that \( \phi \)' is denoted by the \( \langle \phi \rangle \), where \( \langle \cdot \rangle \) is the intensional abstraction operator which transforms a logic formula into a term. Or, for example, "x believes that \( \phi \)" is given by formula \( p_i(x, \langle \phi \rangle) \) (\( p_i \) is binary 'believe' predicate). We introduce an intensional FOL [17], with slightly different intensional abstraction operator which transforms a logic formula into a term.}

**Definition 5.** The syntax of the First-order Logic (FOL) language \( L \) with intensional abstraction \( \langle \cdot \rangle \) is as follows:

1. All variables and constants are terms. All propositional letters are formulae.
2. If \( t_1, ..., t_k \) are terms then \( r_i(t_1, ..., t_k) \) is a formula for a \( k \)-ary predicate letter \( r_i \).
3. If \( \phi \) and \( \psi \) are formulae, then \( (\phi \land \psi), \neg \phi, (\exists x) \phi \) are formulae.
4. If \( \phi(x) \) is a formula (virtual predicate) with a list of free variables in \( x = (x_1, ..., x_n) \) (with ordering from-left-to-right of their appearance in \( \phi \)), and \( \alpha \) is its sublist of distinct variables, then \( \langle \phi \rangle_\alpha^\beta \) is a term, where \( \beta \) is the remaining list of free variables preserving ordering in \( x \) as well. The externally quantifiable variables are the free variables not in \( \alpha \). When \( n = 0, \langle \phi \rangle \) is a term which denotes a proposition, for \( n \geq 1 \) it denotes a \( n \)-ary concept.

An occurrence of a variable \( x_i \) in a formula (or a term) is bound (free) iff it lies (does not lie) within a formula of the form \((\exists x_i) \phi\) (or a term of the form \( \langle \phi \rangle_\alpha^\beta \) with \( x_i \in \alpha \)).

A variable is free (bound) in a formula (or term) iff it has (does not have) a free occurrence in that formula (or term). A sentence is a formula having no free variables.

An interpretation (Tarski) \( I_T \) consists of a nonempty domain \( \mathcal{D} = D_{-1} \cup D_1 \) and a mapping that assigns to any predicate letter \( r_i \in \mathcal{R} \) with \( k = ar(r_i) \geq 1 \), a relation \( \| r_i \| = I_T(r_i) \subseteq \mathcal{D}^k \); to each individual constant \( \overline{c} \) one given element \( I_T(\overline{c}) \in \mathcal{D} \), with \( I_T(\overline{0}) = 0, I_T(\overline{1}) = 1 \) for natural numbers \( N = \{0, 1, 2, ...\} \), and to any propositional letter \( p \in \mathcal{P} \) one truth value \( I_T(p) \in \{f, t\} \), where \( f \) and \( t \) are the empty set \( \{\} \) and the singleton set \( \{< >\} \) (with the empty tuple \( < > \in D_{-1} \)), as those used in the Codd’s relational-database algebra [20] respectively, so that for any \( I_T \), \( I_T(\langle \phi \rangle) = \{< >\} \) (i.e., \( r_0 \) is a tautology), while \( Truth \in D_0 \) denotes the concept (intension) of this tautology.

Note that in the intensional semantics a k-ary functional symbol, for \( k \geq 1 \), in standard (extensional) FOL is considered as a \((k + 1)\)-ary predicate symbols: let \( f \) be such a \((k + 1)\)-ary predicate symbol which represents a k-ary function denoted by \( f \) with stan-
standard Tarski’s interpretation \( I_T(f) : D^k \rightarrow D \). Then \( I_T(f) \) is a relation obtained from its graph, i.e., \( I_T(f) = R = \{ (d_1, ..., d_k, I_T(f)(d_1, ..., d_k)) \mid d_i \in D \} \).

The universal quantifier is defined as usual by \( \forall = \neg \exists \neg \). Disjunction \( \phi \lor \psi \) and implication \( \phi \Rightarrow \psi \) are expressed by \( \neg \neg \phi \land \neg \psi \) and \( \neg \phi \lor \psi \), respectively. In FOL with the identity \( = \), the formula \( (\exists x)\phi(x) \) denotes the formula \( (\exists x)\phi(x) \land (\forall x)(\forall y)(\phi(x) \land \phi(y) \Rightarrow (x = y)) \). We denote by \( R_\exists \) the Tarski’s interpretation of \( = \).

In what follows any open-sentence, a formula \( \phi \) with non empty tuple of free variables \( (x_1, ..., x_m) \), will be called a \( m \)-ary virtual predicate, denoted also by \( \phi(x_1, ..., x_m) \). This definition contains the precise method of establishing the ordering of variables in this tuple: such an method that will be adopted here is the ordering of appearance, from left to right, of free variables in \( \phi \). This method of composing the tuple of free variables is the unique and canonical way of definition of the virtual predicate from a given formula.

An intensional interpretation of this intensional FOL is a mapping between the set \( \mathcal{L} \) of formulae of the logic language and intensional entities in \( D \), \( I : \mathcal{L} \rightarrow D \), is a kind of “conceptualization”, such that an open-sentence (virtual predicate) \( \phi(x_1, ..., x_k) \) with a tuple \( x \) of all free variables \( (x_1, ..., x_k) \) is mapped into a \( k \)-ary concept, that is, an intensional entity \( u = I(\phi(x_1, ..., x_k)) \in D_k \), and (closed) sentence \( \psi \) into a proposition (i.e., logic concept) \( v = I(\psi) \in D_0 \) with \( I(\top) = Truth \in D_0 \) for a FOL tautology \( \top \). This interpretation \( I \) is extended also to the terms (called as denotation as well). A language constant \( \pi \) is mapped into a particular (an extensional entity) \( a = I(\pi) \in D_{-1} \) if it is a proper name, otherwise in a correspondent concept in \( D \). A variable \( x_i \) is mapped into the attribute-name \( I(x_i) \in D_{-1} \) of this variable, while for each \( k \)-ary atom \( r_i(x) \), \( I(<r_i(x)>)_x \) is the relation-name (symbol) \( r_i \in \mathbb{R} \) (only if \( r_i \) is not defined as a language constant as well). The extension of \( I \) to the complex abstracted terms is given in [17] (in Definition 4).

An assignment \( g : \mathcal{V} \rightarrow D \) for variables in \( \mathcal{V} \) is applied only to free variables in terms and formulae. Such an assignment \( g \in D^\mathcal{V} \) can be recursively uniquely extended into the assignment \( g^* : T \mathcal{X} \rightarrow D \), where \( T \mathcal{X} \) denotes the set of all terms with variables in \( X \subseteq \mathcal{V} \) (here \( I \) is an intensional interpretation of this FOL, as explained in what follows), by:

1. \( g^*(t_k) = g(x) \in D \) if the term \( t_k \) is a variable \( x \in \mathcal{V} \).
2. \( g^*(t_k) = I(\pi) \in D \) if the term \( t_k \) is a constant \( \pi \).
3. if \( t_k \) is an abstracted term \( \theta_\gamma^\alpha \), then \( g^*(\theta_\gamma^\alpha) = I(\theta_\gamma^\alpha) \in D_k \), \( k = |\alpha| \) (i.e., the number of variables in \( \alpha \)), where \( g(\beta) = g(y_1, ..., y_m) = (g(y_1), ..., g(y_m)) \) and \( \theta_\gamma^\alpha \) is a uniform replacement of each \( i \)-th variable in the list \( \beta \) with the \( i \)-th constant in the list \( g(\beta) \). Notice that \( \alpha \) is the list of all free variables in the formula \( \theta_\gamma^\alpha \).

We denote by \( t_k/g \) (or \( \phi/g \)) the ground term (or formula) without free variables, obtained by assignment \( g \) from a term \( t_k \) (or a formula \( \phi \)), and by \( \phi[x/t_k] \) the formula obtained by uniformly replacing \( x \) by a term \( t_k \) in \( \phi \).

The distinction between intensions and extensions is important especially because we are now able to have an equational theory over intensional entities (as \( \theta_\gamma^\alpha \)), that is predicate and function “names”, that is separate from the extensional equality of relations and functions. An extensionalization function \( h \) assigns to the intensional
elements of $D$ an appropriate extension as follows: for each proposition $u \in D_0$, $h(u) \in \{f,t\} \subseteq \mathcal{P}(D_0)$ is its extension (true or false value); for each n-ary concept $u \in D_n$, $h(u)$ is a subset of $D^n$ (n-th Cartesian product of $D$); in the case of particulars $u \in D_{-1}$, $h(u) = u$.

We define $D^0 = \{< >\}$, so that $\{f,t\} = \mathcal{P}(D^0)$, where $\mathcal{P}$ is the powerset operator. Thus we have (we denote the disjoint union by '+'):

$h = (h_{-1} + \sum_{i \geq 0} h_i) : \sum_{i \geq 1} D_i \rightarrow D_{-1} + \sum_{i \geq 0} \mathcal{P}(D^i)$,

where $h_{-1} = id : D_{-1} \rightarrow D_{-1}$ is identity mapping, the mapping $h_0 : D_0 \rightarrow \{f,t\}$ assigns the truth values in $\{f,t\}$ to all propositions, and the mappings $h_i : D_i \rightarrow \mathcal{P}(D^i)$,

$i \geq 1$, assign an extension to all concepts. Thus, the intensions can be seen as names of abstract or concrete entities, while the extensions correspond to various rules that these entities play in different worlds.

**Remark:** (Tarski’s constraints) This intensional semantics has to preserve standard Tarski’s semantics of the FOL. That is, for any formula $\phi \in L$ with a tuple of free variables $(x_1, \ldots, x_k)$, and $h \in \mathcal{E}$, the following conservative conditions for all assignments $g, g' \in D^l$ has to be satisfied:

$$(T) \quad h(I(\phi/g)) = t \quad \text{iff} \quad (g(x_1), \ldots, g(x_k)) \in h(I(\phi));$$

and, if $\phi$ is a predicate letter $p$, $k = ar(p) \geq 2$ which represents a (k-1)-ary functional symbol $f^{k-1}$ in standard FOL,

$$(TF) \quad h(I(\phi/g)) = h(I(\phi/g')) = t \quad \text{and} \quad \forall_{1 \leq i \leq k-1}(g'(x_i) = g(x_i)) \quad \text{implies} \quad g'(x_{k+1}) = g(x_{k+1}).$$

Thus, intensional FOL has a simple Tarski’s first-order semantics, with a decidable unification problem, but we need also the actual world mapping which maps any intensional entity to its actual world extension. In what follows we will identify a possible world by a particular mapping which assigns, in such a possible world, the extensions to intensional entities. This is direct bridge between an intensional FOL and a possible worlds representation \[21,22,23,24,16]\, where the intension (meaning) of a proposition is a function, from a set of possible worlds $W$ into the set of truth-values. Consequently, $\mathcal{E}$ denotes the set of possible extensionalization functions $h$ satisfying the constraint $(T)$. Each $h \in \mathcal{E}$ may be seen as a possible world (analogously to Montague’s intensional semantics for natural language \[23,25]\,), as it has been demonstrated in \[26,27\,], and given by the bijection $\text{is} : W \simeq \mathcal{E}$.

Now we are able to formally define this intensional semantics \[16\,]

**Definition 6.** Two-step Intensional Semantics:

Let $\mathcal{R} = \bigcup_{k \in \mathbb{N}} \mathcal{P}(D^k) = \sum_{k \in \mathbb{N}} \mathcal{P}(D^k)$ be the set of all k-ary relations, where $k \in \mathbb{N} = \{0, 1, 2, \ldots\}$. Notice that $\{f,t\} = \mathcal{P}(D^0) \in \mathcal{R}$, that is, the truth values are extensions in $\mathcal{R}$. The intensional semantics of the logic language with the set of formulae $L$ can be represented by the mapping $\mathcal{L} \xrightarrow{I} D \xrightarrow{\text{is}} \mathcal{W} \xrightarrow{\text{is}} \mathcal{R}$

where $\xrightarrow{I}$ is a fixed intensional interpretation $I : L \rightarrow D$ and $\xrightarrow{\text{is}} \mathcal{W} \xrightarrow{\text{is}} \mathcal{R}$ is the set of all extensionalization functions $h = \text{is}(w) : D \rightarrow \mathcal{R}$ in $\mathcal{E}$, where $\text{is} : \mathcal{W} \rightarrow \mathcal{E}$ is the mapping from the set of possible worlds to the set of extensionalization functions.

We define the mapping $I_0 : L_{op} \rightarrow \mathcal{R}^W$, where $L_{op}$ is the subset of formulae with free
variables (virtual predicates), such that for any virtual predicate \( \phi(x_1, ..., x_k) \in \mathcal{L}_{op} \) the mapping \( I_n(\phi(x_1, ..., x_k)) : \mathcal{V} \rightarrow \mathfrak{R} \) is the Montague’s meaning (i.e., intension) of this virtual predicate \( \{27\,22\,23\,24\,25\} \), that is, the mapping which returns with the extension of this (virtual) predicate in each possible world \( w \in \mathcal{V} \).

Another relevant question w.r.t. this two-step interpretations of an intensional semantics is how in it is managed the extensional identity relation \( \equiv \) (binary predicate of the identity) of the FOL. Here this extensional identity relation is mapped into the binary concept \( Id = I(\equiv (x, y)) \in D_2 \), such that \( (\forall w \in \mathcal{V})(is(w)(Id) = R_{\equiv}) \), where \( \equiv (x, y) \) (i.e., \( p_1^2(x, y) \)) denotes an atom of the FOL of the binary predicate for identity in FOL, usually written by FOL formula \( x \equiv y \).

Note that here we prefer to distinguish this formal symbol \( \equiv \in \mathfrak{R} \) of the built-in identity binary predicate letter in the FOL, from the standard mathematical symbol ‘\( = \)’ used in all mathematical definitions in this paper.

In what follows we will use the function \( f_{<\rangle} : \mathfrak{R} \rightarrow \mathfrak{R} \), such that for any relation \( R \in \mathfrak{R} \), \( f_{<\rangle}(R) = \{<\rangle \} \) if \( R \neq \emptyset \); \( \emptyset \) otherwise. Let us define the following set of algebraic operators for relations in \( \mathfrak{R} \):

1. binary operator \( \bowtie_S : \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R} \), such that for any two relations \( R_1, R_2 \in \mathfrak{R} \), the \( R_1 \bowtie_S R_2 \) is equal to the relation obtained by natural join of these two relations \( I \notin S \) is a non empty set of pairs of joined columns of respective relations (where the first argument is the column index of the relation \( R_1 \) while the second argument is the column index of the joined column of the relation \( R_2 \)); otherwise it is equal to the cartesian product \( R_1 \times R_2 \).

For example, the logic formula \( \phi(x_1, x_2, x_3, x_4, x_5, x_6) \land \psi(x_1, y_1, x_2, y_2) \) will be traduced by the algebraic expression \( R_1 \bowtie_S R_2 \) where \( R_1 \in \mathcal{P}(D^5) \), \( R_2 \in \mathcal{P}(D^4) \) are the extensions for a given Tarski’s interpretation of the virtual predicate \( \phi, \psi \) relatively, so that \( S = \{\{(4, 1), (2, 3)\}\} \) and the resulting relation will have the following ordering of attributes: \( (x_1, x_2, x_3, x_4, x_5, y_1, y_2) \).

2. unary operator \( \sim : \mathfrak{R} \rightarrow \mathfrak{R} \), such that for any k-ary (with \( k \geq 0 \)) relation \( R \in \mathcal{P}(D^k) \subset \mathfrak{R} \) we have that \( \sim(R) = D^k \setminus R \in D^k \), where \( \setminus \) is the substraction of relations.

For example, the logic formula \( \neg\phi(x_1, x_2, x_3, x_4, x_5, x_6) \) will be traduced by the algebraic expression \( D^5 \setminus R \) where \( R \) is the extensions for a given Tarski’s interpretation of the virtual predicate \( \phi \).

3. unary operator \( \pi_{-m} : \mathfrak{R} \rightarrow \mathfrak{R} \), such that for any k-ary (with \( k \geq 0 \)) relation \( R \in \mathcal{P}(D^k) \subset \mathfrak{R} \) we have that \( \pi_{-m}(R) \) is equal to the relation obtained by elimination of the\( m \)-th column of the relation \( R \) \( \forall m \leq 1 \leq m \leq k \) and \( k \geq 2 \); equal to \( f_{<\rangle}(R) \) \( \forall m = k = 1 \); otherwise it is equal to \( R \).

For example, the logic formula \( \exists x_k \phi(x_1, x_2, x_3, x_4, x_5) \) will be traduced by the algebraic expression \( \pi_{-3}(R) \) where \( R \) is the extensions for a given Tarski’s interpretation of the virtual predicate \( \phi \) and the resulting relation will have the following ordering of attributes: \( (x_1, x_2, x_3, x_4, x_5) \).

Notice that the ordering of attributes of resulting relations corresponds to the method used for generating the ordering of variables in the tuples of free variables adopted for virtual predicates.
Definition 7. Intensional algebra for the intensional FOL in Definition 3 is a structure \( \mathcal{A}_{int} = (\mathcal{D}, \mathcal{F}, t, \mathcal{I}, \text{Id}, \text{Truth}, \{\text{conj}_S\} \in \mathcal{P}[\mathcal{D}], \text{neg}, \{\text{exists}_n\} \in \mathcal{D}) \), with binary operations \( \text{conj}_S : \mathcal{D} \times \mathcal{D} \rightarrow \mathcal{D} \), unary operation \( \neg : \mathcal{D} \rightarrow \mathcal{D} \), unary operations \( \text{exists}_n : \mathcal{D} \rightarrow \mathcal{D} \), such that for any extensionalization function \( h \in \mathcal{E} \), and \( u \in D_k, v \in D_j, k, j \geq 0 \),

1. \( h(\text{Id}) = R_\mathcal{E} \) and \( h(\text{Truth}) = \{< >\} \).
2. \( h(\text{conj}_S(u, v)) = h(u) \triangleright_S h(v) \), where \( \triangleright_S \) is the natural join operation defined above and \( \text{conj}_S(u, v) \in D_m \) where \( m = k + j - |S| \) if for every pair \((i_1, i_2) \in S\) it holds that \( 1 \leq i_1 \leq k, 1 \leq i_2 \leq j \) (otherwise \( \text{conj}_S(u, v) \in D_{k+j} \)).
3. \( h(\neg(u)) = \approx (h(u)) = \mathcal{D}^k \setminus (h(u)), \) where \( \approx \) is the operation defined above and \( \neg(u) \in D_k \).
4. \( h(\text{exists}_n(u)) = \pi_{-n}(h(u)), \) where \( \pi_{-n} \) is the operation defined above and \( \text{exists}_n(u) \in D_{k-1} \) if \( 1 \leq n \leq k \) (otherwise \( \text{exists}_n \) is the identity function).

Notice that for \( u, v \in D_0 \), so that \( h(u), h(v) \in \{t, f\}, h(\neg(u)) = \approx (h(u)) = \mathcal{D}^0 \setminus (h(u)) = \{< >\}\{h(u)\} \in \{t, f\} \), and \( h(\text{conj}_S(u, v)) = h(u) \triangleright_S h(v) \in \{t, f\} \).

Intensional interpretation \( I : \mathcal{L} \rightarrow \mathcal{D} \) satisfies the following homomorphic extension:

1. The logic formula \( \phi(x_1, x_2, x_3, x_4, x_m) \land \psi(x_1, y_1, y_2, y_3) \) will be intensionally interpreted by the concept \( u_1 \in D_7 \), obtained by the algebraic expression \( \text{conj}_S(u, v) \) where \( u = I(\phi(x_1, x_2, x_3, x_4, x_m)) \in D_5, v = I(\psi(x_1, y_1, y_2, y_3)) \in D_4 \) are the concepts of the virtual predicates \( \phi, \psi \), respectively, and \( S = \{(1, 4), (2, 3)\} \). Consequently, we have that for any two formulae \( \phi, \psi \in \mathcal{L} \) and a particular operator \( \text{conj}_S \) uniquely determined by tuples of free variables in these two formulae, \( I(\phi \land \psi) = \text{conj}_S(I(\phi), I(\psi)) \).
2. The logic formula \( \neg \phi(x_1, x_2, x_3, x_4, x_m) \) will be intensionally interpreted by the concept \( u_1 \in D_5 \), obtained by the algebraic expression \( \neg u \) where \( u = I(\phi(x_1, x_2, x_3, x_4, x_m)) \in D_5 \) is the concept of the virtual predicate \( \phi \). Consequently, we have that for any formula \( \phi \in \mathcal{L} \), \( I(\neg \phi) = \neg u \).
3. The logic formula \( \exists x_1 \phi(x_1, x_2, x_3, x_4, x_m) \) will be intensionally interpreted by the concept \( u_1 \in D_4 \), obtained by the algebraic expression \( \text{exists}_n(u) \) where \( u = I(\phi(x_1, x_2, x_3, x_4, x_m)) \in D_5 \) is the concept of the virtual predicate \( \phi \). Consequently, we have that for any formula \( \phi \in \mathcal{L} \) and a particular operator \( \text{exists}_n \) uniquely determined by the position of the existentially quantified variable in the tuple of free variables in \( \phi \) (otherwise \( n = 0 \) if this quantified variable is not a free variable in \( \phi \)), \( I((\exists x_1)\phi) = \text{exists}_n(I(\phi)) \).

We can define the derived intensional disjunctions \( \text{disj}_S \) in a standard way as, \( \text{disj}_S(I(\phi), I(\psi)) \triangleq I(\phi \lor_S \psi) = I(\neg(\neg \phi \land_S \neg \psi)) = \neg u(\neg \phi \land \neg \psi), \) and \( \neg u(\neg \phi_1 \land \neg \psi_1) \).

Once one has found a method for specifying the interpretations of singular terms of \( \mathcal{L} \) (take in consideration the particularity of abstracted terms), the Tarski-style definitions of truth and validity for \( \mathcal{L} \) may be given in the customary way. What is proposed specifically in [17] is a method for characterizing the intensional interpretations of singular terms of \( \mathcal{L} \) in such a way that a given singular abstracted term \( < \phi > \) will denote an appropriate property, relation, or proposition, depending on the value of \( m = |\alpha| \).

Notice that if \( \beta = \emptyset \) is the empty list, then \( I(< \phi > \emptyset) = I(\phi) \). Consequently, the denotation of \( < \phi > \) is equal to the meaning of a proposition \( \phi \), that is, \( I(< \phi >) = I(\phi) \in \mathcal{D} \).
In the case when $\phi$ is an atom $p_i(x_1, \ldots, x_m)$ then $I(\langle p_i(x_1, \ldots, x_m) \rangle_{x_1, \ldots, x_m}) = I(p_i(x_1, \ldots, x_m)) \in D_m$, while
$\forall \alpha \in D^m, \exists \beta \in \mathcal{D}^m$I($\langle p_i(x_1, \ldots, x_m) \rangle_{x_1, \ldots, x_m}) = \text{union}\{ I(p_i(x_1, \ldots, g(x_m))) \mid g \in D^m \} \in D_0$, with $h(I(\langle p_i(x_1, \ldots, x_m) \rangle_{x_1, \ldots, x_m})) = h(I((\exists x_1)(\exists x_m)p_i(x_1, \ldots, x_m))) \in \{ f, t \}$.

For example,
$h(I(\langle p_i(x_1) \land \neg p_i(x_1) \rangle_{x_1})) = h(I((\exists x_1)(\langle p_i(x_1) \land \neg p_i(x_1) \rangle_{x_1}))) = f$.

The interpretation of a more complex abstract $<\phi>^\beta_\alpha$ is defined in terms of the interpretations of the relevant syntactically simpler expressions, because the interpretation of more complex formulae is defined in terms of the interpretation of the relevant syntactically simpler formulae, based on the intensional algebra above. For example, $I(p_i(x) \land p_k(x)) = \text{conj}_I(I(p_i(x)), I(p_k(x)))$, $I(\neg \phi) = \neg I(\phi)$,
$I(\exists x_1 \phi(x_1, x_j, x_i, x_k) = \exists x_1 \text{ conj}_I(I_p(x_1), I_p(x_j), I_p(x_i), I_p(x_k))$. Consequently, based on the intensional algebra in Definition and on intensional interpretations of abstracted terms, it holds that the interpretation of any formula in $L$ (and any abstracted term) will be reduced to an algebraic expression over interpretations of primitive atoms in $L$. This obtained expression is finite for any finite formula (or abstracted term), and represents the meaning of such finite formula (or abstracted term).

Let $\mathcal{A}_{FOL} = (L, \ll, \ll, \land, \lor, \exists, \forall)$ be a free syntax algebra for "First-order logic with identity $\approx$", with the set $L$ of first-order logic formulae, with $\top$ denoting the tautology formula (the contradiction formula is denoted by $\neg \top$), with the set of variables in $V$ and the domain of values in $D$.

Let us define the extensional relational algebra for the FOL by,$\mathcal{A}_\bowtie = (\mathcal{A}, R, \ll, \ll, \land, \lor, \exists, \forall, \top, \bot, \approx)$, where $\ll \in \bowtie$ is the algebraic value correspondent to the logic truth, and $R_\bowtie$ is the binary relation for extensionally equal elements. We use $\bowtie \bowtie = \bowtie$ for the extensional identity for relations in $\bowtie$.

Then, for any Tarski’s interpretation $I_T$ its unique extension to all formulae $I_T^+: L \to \bowtie$ is also the homomorphism $I_T^+ : \mathcal{A}_{FOL} \to \mathcal{A}_\bowtie$ from the free syntax FOL algebra into this extensional relational algebra.

Consequently, we obtain the following Intensional/extensional FOL semantics $[16]$:

For any Tarski’s interpretation $I_T$ of the FOL, the following diagram of homomorphisms commutes,

\[ \text{intensional interpret. } I \xrightarrow{\mathcal{A}_{FOL} \text{ (syntax)}} \text{Frege/Russell semantics} \xrightarrow{h \text{ (extensionalization)}} \mathcal{A}_\bowtie \text{ (denotation)} \]

where $h = \text{is}(w)$ where $w = I_T \in \mathcal{W}$ is the explicit possible world (extensional Tarski’s interpretation).

This homomorphic diagram formally expresses the fusion of Frege’s and Russell’s semantics $[28, 29, 30]$ of meaning and denotation of the FOL language, and renders mathematically correct the definition of what we call an “intuitive notion of intensionality”, in terms of which a language is intensional if denotation is distinguished from sense: that
is, if both a denotation and sense is ascribed to its expressions. In fact there is exactly one sense (meaning) of a given logic formula in $\mathcal{L}$, defined by the uniquely fixed intentional interpretation $I$, and a set of possible denotations (extensions) each determined by a given Tarski’s interpretation of the FOL as follows from Definition[6].

$$\mathcal{L} \xrightarrow{I} \mathcal{D} \implies h=\text{is}(I_T), I_T \in \mathcal{W}_\mathcal{A}.$$ 

Often 'intension' has been used exclusively in connection with possible worlds semantics, however, here we use (as many others; as Bealer for example) 'intension' in a more wide sense, that is as an algebraic expression in the intensional algebra of meanings (concepts) $\mathcal{A}_{int}$ which represents the structural composition of more complex concepts (meanings) from the given set of atomic meanings. In fact, in our case, the meaning of the database concept $u_{DB}$ is expressed by an algebraic expression in what follows. Consequently, not only the denotation (extension) is compositional, but also the meaning (intension) is compositional.

The application of the intensional FOL semantics to the Data Integration system $\mathcal{I} = (\mathcal{A}, \mathcal{S}, \mathcal{M})$ in Definition[3] with the user defined RDB schema $\mathcal{A} = (\mathcal{S}_\mathcal{A}, \Sigma_\mathcal{A})$ and the vector big table $v_T$ can be summarized in what follows:

- Each relational name (symbol) $r_k \in \mathcal{S}_\mathcal{A} = \{r_1, \ldots, r_n\}$ with the arity $m = ar(r_k)$, is an intensional m-ary concept, so that $r_k = I(<r_k(x)>_x) \in D_m$, for a tuple of variables $x = \langle x_1, \ldots, x_m \rangle$ and any intensional interpretation $I$.

For a given Tarski’s interpretation $I_T$, the extensionalization function $h$ is determined by $h(r_k) = \|r_k\| = \{\langle d_1, \ldots, d_m \rangle \in D_m | I_T(r_k(d_1, \ldots, d_m)) = t\} = I_T(r_k) \in \mathcal{A}$. The instance database $A$ of the user-defined RDB schema $\mathcal{A}$ is a model of $\mathcal{A}$ if it satisfies all integrity constraints in $\Sigma_\mathcal{A}$.

- The relational symbol $r_T$ of the vector big table is a particular (extensional entity), defined also as a language constant, that is, a term for which there exists an intentional interpretation with $I(r_T) = r_T \in D_{-1}$, so that $h(r_T) = r_T$ (the name of the database $\mathcal{A}$). We define the intensional concept of the atom $r_T(y_1, \ldots, y_4)$ of the relational table $r_T$ as $u_{r_T} = I(<r_T(y_1, \ldots, y_4)>_{y_1, \ldots, y_4}) \in D_4$, such that for a given model $A = \{\|r_1\|, \ldots, \|r_n\|\}$ of the user-defined RDB schema $\mathcal{A}$, corresponding to a given Tarski’s interpretation $I_T$, its extension is determined by $h(u_{r_T}) = I_T(r_T) = \|r_T\| = \overrightarrow{A}$.

- The database unary concept of the user-defined schema $\mathcal{A}$ is defined by the intensional expression $u_{DB} = \text{exists}_{2,3,4}(u_{r_T}) \in D_1$, so that its extension is equal to $h(u_{DB}) = h(\text{exists}_{2,3,4}(u_{r_T})) = \pi_1(\|r_T\|) = \pi_1(\|r_T\|) \subseteq S_\mathcal{A}$, that is, to the subset of the nonempty relations in the instance database $A$.

- Intensional nature of the IRDB is evident in the fact that each tuple $\langle r_k, \text{Hash}(d_1, \ldots, d_m), \text{nr}_r(i), d_i \rangle \in \overrightarrow{A}$, corresponding to the atom $r_T(y_1, y_2, y_3, y_4)$ (with $I(y_1) = \text{x-name} \in D_{-1}, I(y_3) = \text{a-name} \in D_{-1}, I(y_2) = \text{t-index} \in D_{-1}$ and $I(y_4) = \text{value} \in D_{-1}$ for an assignment $g$ such that $g(y_1) = r_k \in D_m, g(y_3) = \text{nr}_r(i) \in D_{-1}, g(y_2) = \text{Hash}(d_1, \ldots, d_m) \in D_{-1}$ and $g(y_4) = d_i \in D$, is
equal to the intensional tuple $\langle I(\ll r_k(\mathbf{x}) \gg \mathbf{x}), \text{Hash}(d_1, ..., d_m), I(x_i), d_i \rangle$.

Notice that the intensional tuples are different from ordinary tuples composed by only particulars (extensional elements) in $D_{-1}$, what is the characteristics of the standard FOL (where the domain of values is equal to $D_{-1}$), while here the "value" $r_k = I(\ll r_k(\mathbf{x}) \gg \mathbf{x}) \in D_m$ is an m-ary intensional concept, for which $h(r_k) \neq r_k$ is an m-ary relation (while for all ordinary values $d \in D_{-1}$, $h(d) = d$).

Based on the intensional interpretation above, we are able to represent any instance user-defined database $A$ as an intensional hierarchy system of concepts, presented in the next diagram, where for each tuple of data $d_i = (d_{i1}, ..., d_{im}) \in D_{-1}, 1 \leq i \leq N$, of the relation $h(r_k) = \|r_k\|$, we have that $h(I(r_V(r_k, \text{Hash}(d_i), nr_{\mathbf{r}}(j), d_{ij}))) = t$, for $j = 1, ..., m = ar(r_k)$.

The canonical models of such intensional Data Integration system $I = \langle A, S, M \rangle$ can be provided in a usual logical framework as well [1]:

**Proposition 2** Let the IRDB be given by a Data Integration system $I = \langle A, S, M \rangle$ for a used-defined global schema $A = (S_A, \Sigma_A)$ with $S_A = \{r_1, ..., r_n\}$, the source schema $S = (\{r_V, \emptyset\})$ with the vector big data relation $r_V$ and the set of mapping tgsds $M$ from the source schema into the relations of the global schema. Then a canonical model of $I$ is any model of the schema $A^+ = (S_A \cup \{r_V\}, \Sigma_A \cup M \cup M^{OP})$ where $M^{OP}$ is an opposite mapping tgsds from $A$ into $r_V$ is given by the following set of tgsds: $M^{OP} = \{\forall x_1, ..., x_{ar(r_k)}(r_k(x_1, ..., x_{ar(r_k)}) \land x_i \text{ NOT NULL}) \Rightarrow r_V(r_{\mathbf{r}}(x_1, ..., x_{ar(r_k)}), nr_{\mathbf{r}}(i), x_i) \mid 1 \leq i \leq ar(r_k), r_k \in S_A\}$.

The proof can be found in [1]. The fact that we assumed $r_V$ to be only a particular (a language constant, i.e., an extensional entity) is based on the fact that it always will be materialized (thus non empty relational table) as standard tables in the RDBs. The other reason is that the extension $h(r_V)$ has not to be equal to the vector relation (the set of tuples) $\|r_V\|$ because $r_V$ is a name of the database $A$ composed by the set of relations in the instance database $A$. Consequently, we do not use the $r_V$ (equal to the name of the database $A$) as a value in the tuples of other relations and we do not use the parsing.
used for all relations in the user-defined RDB schema \( \mathcal{A} \) assumed to be the intensional concepts as well. Consequently, the IRDB has at least one relational table which is not an intensional concept and which will not be parsed: the vector big table, which has this singular built-in property in every IRDB.

4 Reduction of SchemaLog into IRDB

From the introduction of SchemaLog, we can deduce that the definition of the Multidatabases has to be obtained mainly by the following unit clauses (the "facts" in Logic programming) of the following forms \([13]\):

(i) \((\langle db \rangle :: \langle rel \rangle | \langle\langle tid \rangle : \langle attr \rangle \rightarrow \langle val \rangle \rangle) \leftarrow \);

(ii) \((\langle db \rangle :: \langle rel \rangle | \langle\langle attr \rangle \rangle) \leftarrow ; \)

(iii) \((\langle db \rangle :: \langle rel \rangle) \leftarrow ; \)

(iv) \(\langle db \rangle \leftarrow ; \)

where the clause (i) correspond to the SQL-like operation of inserting the value \(\langle val \rangle\) into the attribute \(\langle attr \rangle\) of the relation \(\langle rel \rangle\) of the database \(\langle db \rangle\) while other cases correspond to the DDL-like operations of definitions of the attributes of the relations, the relations of the databases and the databases. From the fact that we are interested in the operations over the vector relations \(r_{V_1}, \ldots, r_{V_n}\) each one dedicated to a single database of the given Multidatabase system, the unit clause \(\langle db \rangle \leftarrow\) corresponds to the RDB DDL of the creation of the vector relation with the name \(r_{V_i} = \langle db \rangle\) with the four fixed attributes \(r\text{-name}, t\text{-index}, a\text{-name} \) and \(value\). We do not use the clauses (ii) and (iii) for vector relations, and the only interesting clause is (i). In fact, the clause (i) corresponds to the SQL statement 'INSERT INTO \(r_{V_i}\) VALUES (\(\langle rel \rangle, \langle tid \rangle, \langle attr \rangle, \langle val \rangle\))'.

However, here we can see why SchemaLog cannot be used for real Multidatabase systems, because each insertion, deletion or update must be realized by the updating of the whole Logic program \(P\) which defines the extension of the databases, and then for such a modified program \(P\) to compute its least fixpoint. It is not only a hard computational process (to rebuild the complete extension of all databases of a given Multidatabase system by the fixpoint semantics, but also very complicated task of the concurrent updates of these databases by different users. This is the common problem and weak point for almost all AI logic-programming approaches to big databases, and explains why they can not replace the concurrent RDBMSs and why we intend to translate the SchemaLog framework into the concurrent and Big Data IRDBMSs and then to show that IRDBMs can support the interoperability for the Multidatabase systems.

Remark (*): We will consider only the meaningful cases of the SchemaLog used for Multidatabases, when each relation \(r\) of any database \(\mathcal{A}\) is not empty and for each attribute of such a relation there is at least one value different from NULL, that is, when every relation and its attributes are really \textit{used} in such a database to contain the information.

□

Consequently, we consider the IRDB interoperability with a set of relational databases \(S_{DB} = \{u_{DB_1}, \ldots, u_{DB_n}\}\), where each \(u_{DB_i} = \exists_{s_{2,3,4}}(u_{r_{V_i}}) \in D_4\), for \(i = 1, \ldots, n\), is the intensional DB concept of the \(i\)-th RDB parsed into the vector relation with the name \(r_{V_i}\) (with \(u_{r_{V_i}} = I(<r_{V_i}(y_1, \ldots, y_4)\rangle_{y_1, \ldots, y_4}) \in D_4\) for a given inten-
Thus, in this interoperability framework, we will have $n \geq 1$ tree-systems of concepts (provided in previous section) with the top Multidatabase intensional concept $u_{mdb} = I(call_1(x)) \in D_1$ (where $call_1$ is the unary predicate letter introduced for this concept introduced for SchemaLog reduction in [13]) such that $h(u_{mdb}) = S_{DB}$ is the set of database names in a given Multidatabase system, represented in the next figure:

Thus, we can introduce the following intensional concepts (the sorts of relations, tuples, attributes and values):

1. $u_{rel} = disj_s(u_{DB_1}, disj_s(..., disj_s(u_{DB_{n-1}}, u_{DB_n})...)) \in D_1$;
2. $u_{tid} = disj_s(exists_{1,3,4}(u_{r_1}), disj_s(..., disj_s(exists_{1,3,4}(u_{r_{n-1}}), exists_{1,3,4}(u_{r_n})...)) \in D_1$;
3. $u_{attr} = disj_s(exists_{1,2,4}(u_{r_1}), disj_s(..., disj_s(exists_{1,2,4}(u_{r_{n-1}}), exists_{1,2,4}(u_{r_n})...)) \in D_1$;
4. $u_{val} = disj_s(exists_{1,2,3}(u_{r_1}), disj_s(..., disj_s(exists_{1,2,3}(u_{r_{n-1}}), exists_{1,2,3}(u_{r_n})...)) \in D_1$;

where $S_i \{ (i,i) \}$ for $i = 1, 2, 3, 4$.

Notice that these intensional unary concepts above are derived from the FOL formulae, as follows:

\[
\begin{align*}
u_{rel} &= I(\exists x_2, x_3, x_4 r_1(x_1, x_2, x_3, x_4) \lor (... \lor \exists x_2, x_3, x_4 r_n(x_1, x_2, x_3, x_4)));
\end{align*}
\]

\[
\begin{align*}
u_{tid} &= I(\exists x_1, x_3, x_4 r_1(x_1, x_2, x_3, x_4) \lor (... \lor \exists x_1, x_3, x_4 r_n(x_1, x_2, x_3, x_4)));
\end{align*}
\]

\[
\begin{align*}
u_{attr} &= I(\exists x_1, x_2, x_4 r_1(x_1, x_2, x_3, x_4) \lor (... \lor \exists x_1, x_2, x_4 r_n(x_1, x_2, x_3, x_4)));
\end{align*}
\]

\[
\begin{align*}
u_{val} &= I(\exists x_1, x_2, x_3 r_1(x_1, x_2, x_3) \lor (... \lor \exists x_1, x_2, x_3 r_n(x_1, x_2, x_3)));
\end{align*}
\]
Then, given a SchemaLog formula $\phi$, its encoding in the intensional FOL of the IRDB is determined by the recursive transformation rules given below. In this transformation $\overline{\pi} \in S \subseteq T$, $f \in G$, $t_1, t_{rel}, t_{attr}, t_{id}, t_{val} \in T$, $t_{db} \in \{r_{V_1}, ..., r_{V_n}\} \subset S \subseteq T$, are the SchemaLog terms, and $\phi, \psi$ are any formulae:

1. $encode(\overline{\pi}) = \overline{\pi}$
2. $encode(f) = f$
3. $encode(f(t_1, ..., t_m)) = encode(f)(encode(t_1), ..., encode(t_m))$
4. $encode(t_{db} :: t_{rel}[t_{id} : t_{attr} \rightarrow t_{val}]) =$
   $= encode(t_{db})(encode(t_{rel}), encode(t_{id}), encode(t_{attr}), encode(t_{val}))$
5. $encode(t_{db} :: t_{rel}[t_{attr}]) = (\exists x_2, x_4)encode(t_{db})(encode(t_{rel}), x_2, encode(t_{attr}), x_4)$
6. $encode(t_{db} :: t_{rel}) = (\exists x_2, x_3, x_4)encode(t_{db})(encode(t_{rel}), x_2, x_3, x_4)$
7. $encode(t_{db}) = callt_1(t_{db})$
8. $encode(\phi \land \psi) = encode(\phi) \land encode(\psi)$
9. $encode(\phi \lor \psi) = encode(\phi) \lor encode(\psi)$
10. $encode(\neg \phi) = \neg encode(\phi)$
11. $encode(\phi \rightarrow \psi) = encode(\phi)$
12. $encode((Qx)\phi) = (Qx)encode(\phi)$, where $Q \in \{\exists, \forall\}$.

In the case of the intensional FOL defined in Definition 5 without Bealer’s intensional abstraction operator $\langle \rangle$, we obtain the syntax of the standard FOL but with intensional semantics as presented in [16]. Such a FOL has a well known Tarski’s interpretation, defined as follows:

- An interpretation (Tarski) $I_T$ consists in a non empty domain $D$ and a mapping that assigns to any k-ary predicate letter $p_i$ a relation $R = I_T(p_i) \subseteq D^k$, to any k-ary functional letter $f_i$, a function $I_T(f_i) : D^k \rightarrow D$, or, equivalently, its graph relation $R = I_T(f_i) \subseteq D^{k+1}$ where the $k+1$-th column is the resulting function’s value, and to each individual constant $\overline{c}$ one given element $I_T(\overline{c}) \in D$. Consequently, from the intensional point of view, an interpretation of Tarski is a possible world in the Montague’s intensional semantics, that is $w = I_T \in W$. The correspondent extensionalization function is $h = is(w) = is(I_T)$.

- For a given interpretation $I_T$, we define the satisfaction $I_T \models_g \phi$ of a logic formulae in $L$ for a given assignment $g : V \rightarrow D$ inductively, as follows:

If a formula $\phi$ is an atomic formula $p_i(t_1, ..., t_k)$, then this assignment $g$ satisfies $\phi$, denoted by $I_T \models_g \phi$, iff $(g'(t_1), ..., g'(t_k)) \in I_T(p_i)$; $g$ satisfies $\neg \phi$ iff it does not satisfy $\phi$; $g$ satisfies $\phi \land \psi$ iff $g$ satisfies $\phi$ and $g$ satisfies $\psi$; $g$ satisfies $\exists x_i \phi$ iff $g$ exists an assignment $g' \in D^V$ that may differ from $g$ only for the variable $x_i \in V$, and $g'$ satisfies $\phi$.

A formula $\phi$ is true $I_T \models \phi$ is satisfied by every assignment $g \in D^V$. A formula $\phi$ is valid (i.e., tautology) iff $\phi$ is true for every Tarski’s interpretation $I_T$. An interpretation $I_T$ is a model of a set of formulae $\Gamma$ iff every formula $\phi \in \Gamma$ is true in this interpretation.

**Semantics:** Given a SchemaLog structure $M = \langle D, I, \mathcal{I}_{fun}, \mathcal{F} \rangle$ we construct a corresponding Tarski’s interpretation $I_T = encode(M)$ on the domain $D$ as follows:
\( I_T(\varpi) \equiv \cal I(\varpi) \), for each \( \varpi \in \cal S \);
\( I_T(f(d_1, \ldots, d_k)) \equiv \cal I_{\text{fun}}(f)(d_1, \ldots, d_k) \), for each \( k \)-ary functional symbol \( f \in \cal G \) and \( d_1, \ldots, d_k \in \cal D \);

Note that the \( \text{Hash} \) functional symbol has to be inserted into \( \cal G \), so that the built-in function on strings \( \cal I_{\text{fun}}(\text{Hash}) \) satisfies the condition:

If \( (\cal F(r_{V_i})(r)(id)(nr_1(1))) = v_1 \land \ldots \land (\cal F(r_{V_i})(r)(id)(nr_1(ar(r)))) = v_{ar(r)} \) for \( r_{V_i}, r, id, ar_1(k), v_k \in \cal D \), for \( k = 1, \ldots, ar(r) \), then \( id = \cal I_{\text{fun}}(\text{Hash})(v_1, \ldots, v_{ar(r)}) \), where some of \( v_i \) can be equal to the value \( \text{NULL} \in \cal D_{-1} \).

We recall that, for intensional FOL, each \( k \)-ary functional symbol \( f \) is considered as a \((k + 1)\)-ary relational concept, so that \( I_T(f) \in \cal D_{k+1} \) with

\( I_T(f) = h(I(f)) = \{(d_1, \ldots, d_k, \cal I_{\text{fun}}(f)(d_1, \ldots, d_k)) \in h(I(f))|d_1, \ldots, d_k \in \cal D\} \).

The unique relations that are materialized in IRDBs are the vector relations, so we will consider only the relations \( r_{V_1}, \ldots, r_{V_n} \) (corresponding to databases \( A_1, \ldots, A_n \) of this Multidatabase interoperability system), so that the Tarski’s interpretation for them is constructed in the following way:

1. Let \( r_{V_i}, r, id, a, v \in \cal D \), then \( \{r, id, a, v\} \in I_T(r_{V_i}) \iff \cal F(r_{V_i})(r)(a)(id) \) is defined in \( M \) and \( \cal F(r_{V_i})(r)(a)(id) = v \).
2. For the unary predicate \( \text{call}_1 \), such that from the Tarski’s constraints \( u_{md_b} = I_T(\text{call}_1(x)) \), we have that \( I_T(\text{call}_1) = h(I(\text{call}_1(x))) = h(u_{md_b}) = S_{DB} \) (the set of intensional DB concepts in the figure above). Then, \( r_{V_i} \in I_T(\text{call}_1) \iff \cal F(r_{V_i}) \) is defined in \( M \).

**Proposition 3** Let \( \phi \) be a SchemaLog formula, \( M \) be a SchemaLog structure, and \( g \in \cal D^\forall \) an assignment. Let \( \text{encode}(\phi) \) be the first-order formula corresponding to \( \phi \) and \( I_T = \text{encode}(M) \) the corresponding Tarski’s interpretation.

Then, \( M \models_g \phi \iff I_T^g \models_g \text{encode}(\phi) \).

Proof: Let us show that it holds for all atoms of SchemaLog:

1. Case when \( \phi \) is equal to an atom \( (t_1 :: t_2[t_4 : t_5 \rightarrow t_3]) \). Then, \( M \models_g (t_1 :: t_2[t_4 : t_3 \rightarrow t_5]) \)
   - \( \cal F(g(t_1))(g(t_2))(g(t_3))(g(t_4)) \) is defined in \( M \) and \( \cal F(g(t_1))(g(t_2))(g(t_3))(g(t_4)) = g(t_5) \)
   - \( \langle g(t_2), g(t_4), g(t_3), g(t_5) \rangle \in I_T(g(t_2)) \)
   - \( I_T^g =_g (t_1 : t_2)[t_4 : t_3 \rightarrow t_5] \)
   - \( I_T^g =_g \text{encode}(t_1 : t_2)[t_4 : t_2 \rightarrow t_5] \).

2. Case when \( \phi \) is equal to an atom \( (t_1 :: t_2[t_3]) \). Then, \( M \models_g (t_1 :: t_2[t_3]) \)
   - \( \cal F(g(t_1))(g(t_2))(g(t_3)) \) is defined in \( M \)
   - \( I_T^g =_g (t_1 : t_2)[t_3 : t_4 \rightarrow t_5] \)
   - \( I_T^g =_g \text{encode}(t_1 :: t_2[t_3]) \).

3. Case when \( \phi \) is equal to an atom \( (t_1 :: t_2) \). Then, \( M \models_g (t_1 :: t_2) \)
   - \( \cal F(g(t_1))(g(t_2)) \) is defined in \( M \)
iff $I_T^g \models (\exists x_2, x_3, x_4)(t_1/g)(t_2, x_2, x_3, x_4)$
iff $I_T^g \models (\exists x_2, x_3, x_4)(\text{encode}(t_1)/g)(\text{encode}(t_2), x_2, x_3, x_4)$
iff $I_T^g \models_g \text{encode}(t_1 :: t_2)$.

3. Case when $\phi$ is equal to an atom $t_{db}$. Then,
   \[ M \models_g t_{db} \iff \mathcal{F}(g(t_{db})) \text{ is defined in } M \]
   iff $g(t_{db}) \in I_T^{\text{call}_1}$ iff $I_T^g \models \text{call}_1(t_{db})$ iff $I_T^g \models_g \text{encode}(t_{db})$.

4. For the composed formulae, we can demonstrate by induction. Let us suppose that
   this property holds for $\phi$ and for $\psi$. Then
\[ M \models_g \phi \lor \psi \]
iff $M \models_g \phi$ or $M \models_g \psi$.
iff $I_T^g \models \text{encode}(\phi)$ or $I_T^g \models \text{encode}(\psi)$
iff $I_T^g \models \text{encode}(\phi \lor \psi)$,
and analogously for all other cases.

\[ \Box \]

Note that w.r.t. the Remark (*) above, the relations (predicates) $\text{call}_1, \text{call}_2, \text{call}_3$ and $\text{call}_4$ (obtained by a similar reduction of SchemaLog in FO Logic Programs in [13]), can be defined by the vector relations in IRDBs (see [1] for the syntax-semantics if the relational algebra operators used in next expressions) as follows:
\[
\text{call}_4 = (\text{EXTEND } r_{V_1} \text{ ADD } a, \text{db-name, } r_{V_1}) \text{ UNION } (...) \text{ UNION } (\text{EXTEND } r_{V_n} \text{ ADD } a, \text{db-name, } r_{V_n})...,
\]
where $a$ is the attribute used for the database names;
\[
\text{call}_1 = \text{call}_4[\text{db-name}];
\]
\[
\text{call}_2 = \text{call}_4[\text{db-name, } r\text{-name, } a\text{-name}];
\]
(note that we also have $\text{call}_1 = \text{call}_2[\text{db-name}]$, with $h(u_{rel}) = \|\text{call}_4[r\text{-name}]\|\#$,
\[
h(u_{attr}) = \|\text{call}_4[a\text{-name}]\|\#$, and $h(u_{val}) = \|\text{call}_4[value]\|\#$.
Thus, based on [13], we obtain the result that technically SchemaLog has no more expressive power than the intensional first-order logic used for IRDBs.
However, differently from the SchemaLog that needs a particular extension $\mathcal{ERA}$ of the conventional (standard) relational algebra with the new operations, $\delta, \rho, \alpha$ and $\gamma$ [13] (so that the resulting algebra is capable of accessing the database names, relational names and attribute names besides the values in a federation of database), here we can use the conventional (standard) SQL over the vector relations $r_{V_1}, ..., r_{V_n}$ and $\text{call}_1$.

These new operators $\beta, \rho, \alpha$ are defined in IRDBs by the following SQL expressions:
\[
\delta(S) = \text{call}_2 \text{ WHERE } nr_{\text{call}_2}(1) \text{ IN } S, \text{ for each } S \subseteq \|\text{call}_1\|\#;
\]
\[
\rho(S) = \text{call}_3 \text{ WHERE } (nr_{\text{call}_4}(1), nr_{\text{call}_2}(2)) \text{ IN } S, \text{ for each } S \subseteq \|\text{call}_2\|\#
\]

Only the operation $\gamma$ is a more complex, defined in [13] as follows:

A pattern is a sequence $(p_1, ..., p_k), k \geq 0$, where each $p_i$ is one of the forms $'a_i \rightarrow v_i'$,
\['a_i \rightarrow t', 'v_i' \rightarrow t'.\]

Here $a_i$ is called the attribute component and $v_i$ is called the value component of $p_i$. Let $r$ be any relation name, then
\['a_i \rightarrow v_i' is satisfied by a tuple $tid$ in relation $r$ if $tid[a_i] = v_i$;
\['a_i \rightarrow t' is satisfied by a tuple $tid$ in relation $r$ if $a_i$ is an attribute name in $r$;
\['v_i' \rightarrow t' is satisfied by a tuple $tid$ in relation $r$ if there exists an attribute $a_i$ in the scheme of $r$ such that $t[a_i] = v_i$;
\['t' \rightarrow t' is trivially satisfied by every tuple $tid$ in relation $r$.

A pattern $(p_1,...,p_k)$ is satisfied by a tuple $tid$ in relation $r$ if every $p_i, i = 1, ..., k,$ is
satisfied by \( \text{tid} \).

Operator \( \gamma \) takes a binary relation \( S \) as input, and a pattern as a parameter and returns a relation that consists of tuples corresponding to those parts of the database where the queried pattern is satisfied. That is, let \( S \) be a binary relation and \( (p_1, ..., p_k) \) be a pattern, then [13].

\[
\gamma(p_1, ..., p_k)(S) \triangleq \{ (d, r, a_1, v_1, ..., a_k, v_k) \mid (d, r) \in S \text{ and } d \text{ is a database in the federation, and } r \text{ is a relation in } d, \text{ and } a_i \text{'s are attributes in } r, \text{ and there exists a tuple } \text{tid} \text{ in } r \text{ such that } \text{tid}[a_1] = v_1, ..., \text{tid}[a_k] = v_k, \text{ and } \text{tid} \text{ satisfies } (p_1, ..., p_k) \}.
\]

Note that when the pattern is empty \((k = 0)\), \( \gamma_0(S) \) would return the set of all pairs \((d, r) \in S \text{ such that } r \text{ is a nonempty relation in the database } d \text{ in the federation.}

**Theorem 1.** All new relational operators introduced in SchemaLog extended relational algebra \( \mathcal{ERA} \) can be equivalently expressed by standard SQL terms in IRDBs.

**Proof:** The \( \text{call}_1 \) and \( \text{call}_2 \) are SQL terms (over the vector relations \( r_{V'} \) of the federated databases) defined previously, so that the definition of the SchemaLog operators \( \delta, \rho, \text{ and } \alpha \), given above, are the standard SQL terms as well. It is enough to demonstrate that each \( \gamma(p_1, ..., p_k) \) operator defined above, can be equivalently represented by a standard SQL term in the IRDBs as follows:

(i) Case when \( k = 0 \). Then \( \gamma_0(S) = \rho(S) \) (because from Remark (*) we are dealing with the databases with all nonempty relations);

(ii) Case when \( k = 1 \). Then for the SQL term \( t = \text{call}_4 \gamma(p_1)(S) = (t \text{ WHERE } C_{p_1}(\text{db-name}, \text{r-name}, \text{a-name}, \text{value}), \)

where the condition \( C_{p_1} \) is defined by (here \( \top \) is a tautology, for example \( \top = \top \)),

\[
C_{p_1} = \begin{cases} 
(nr_1(4) = a_i \land (nr_1(5) = v_i) && \text{iff } p_1 = \rightarrow a_i \rightarrow v_i' \\
(nr_1(4) = a_i && \text{iff } p_1 \rightarrow a_i \rightarrow r \\
(nr_1(5) = v_i && \text{iff } p_1 \rightarrow v_i' \\
\top && \text{otherwise}
\end{cases}
\]

(iii) Case when \( k \geq 2 \). Let us define the SQL Cartesian product \( t = \text{call}_4 \times \ldots \times \text{call}_4 \).

Then

\[
\gamma(p_1, ..., p_k)(S) = (t \text{ WHERE } (nr_1(1) = ... = nr_1(5k - 4)) \land (nr_1(2) = ... = nr_1(5k - 3)) \land (nr_1(3) = ... = nr_1(5k - 2)) \land (\text{C}_{p_1} \land ... \land \text{C}_{p_k})[nr_1(1), nr_1(2), nr_1(4), nr_1(5), nr_1(9), nr_1(10), ..., nr_1(5k - 1), nr_1(5k)]
\]

where the conditions \( C_{p_m} \), for \( m = 1, ..., k \), are defined by:

\[
C_{p_m} = \begin{cases} 
(nr_1(5m - 1) = a_i \land (nr_1(5m) = v_i) && \text{iff } p_m = \rightarrow a_i \rightarrow v_i' \\
(nr_1(5m - 1) = a_i && \text{iff } p_m = \rightarrow a_i \rightarrow r \\
(nr_1(5m) = v_i && \text{iff } p_m = \rightarrow v_i' \\
\top && \text{otherwise}
\end{cases}
\]

\( \square \)
Example 1. Let us consider the Multidatabase (federated) system given in Example 2.1 in [13], consisting of RDB univ_A, univ_B and univ_C corresponding to universities A, B and C. Each database maintains information on the university’s departments, staff, and the average salary in 1997, as follows:

1. The RDB univ_A has the following single relation pay-info which has one tuple for each department and each category in that department:

| category | dept | avg-sal |
|----------|------|---------|
| Prof     | CS   | 70,000  |
| Assoc. Prof | CS | 60,000  |
| Secretary | CS  | 35,000  |
| Prof     | Math | 65,000  |

2. The RDB univ_B has the single relation, (also pay-info), but in this case, department names appear as attribute names and the values corresponding to them are the average salaries:

| category | pay-info | dept | avg-sal |
|----------|----------|------|---------|
| CS       | Prof     | 80,000| 65,000  |
|          | Assoc. Prof | 65,000| 55,000  |
|          | Assist. Prof | 45,000| 42,000  |

3. The RDB univ_C has as many relations as there are departments, and has tuples corresponding to each category and its average salary in each of the dept_i relations:

| category | avg-sal |
|----------|---------|
| Prof     | 65,000  |
| Assist. Prof | 40,000  |

| category | avg-sal |
|----------|---------|
| Secretary | 30,000  |
| Prof     | 70,000  |

By parsing of these three RDBs, we obtain the three vector relations $r_{V_1} = \text{univ}_A$, $r_{V_2} = \text{univ}_B$ and $r_{V_3} = \text{univ}_C$.

Let us consider the tuple $ID_1 = Hash(Secretary CS 35,000)$ of the database univ_A, so that

| r-name | t-index | a-name | value |
|--------|---------|--------|-------|
| pay-info | $ID_1$ | category | Secretary |
| pay-info | $ID_1$ | dept | CS |
| pay-info | $ID_1$ | avg-sal | 35,000 |

and consider the tuple $ID_2 = Hash(Secretary 30,000)$ of the relation ece of the database univ_C, so that

| r-name | t-index | a-name | value |
|--------|---------|--------|-------|
| ece | $ID_2$ | category | Secretary |
| ece | $ID_2$ | avg-sal | 30,000 |

so that the following set of tuples are the part of the relation obtained from the SQL algebra term $call_4$:
\[ \text{db-name} \quad r-name \quad t-index \quad a-name \quad \text{value} \]

|           |           |        |        |        |
|-----------|-----------|--------|--------|--------|
| univ_A pay-info | ID\(_1\) | category | Secretary |        |
| univ_A pay-info | ID\(_1\) | dept    | CS      |        |
| univ_A pay-info | ID\(_1\) | avg-sal | 35,000  |        |
| univ_C ece     | ID\(_2\) | category | Secretary |        |
| univ_C ece     | ID\(_2\) | avg-sal | 30,000  |        |

Then the operation \(\gamma(\rightarrow\text{Secretary}, \rightarrow)(S)\) against the university databases above is equivalent to the SQL term (for \(t = \text{call}_4 \otimes \text{call}_4\))

\( \left( t \right. \left. \text{WHERE} \right. \left( \text{nr}_t(1) = \text{nr}_t(6) \right) \land \left( \text{nr}_t(2) = \text{nr}_t(7) \right) \land \left( \text{nr}_t(3) = \text{nr}_t(7) \right) \land \left( \text{nr}_t(5) = \text{Secretary} \right) \mid \text{nr}_t(1), \text{nr}_t(2), \text{nr}_t(4), \text{nr}_t(5), \text{nr}_t(9), \text{nr}_t(10) \right), \)

will yield for

\[ S = \text{univ}_A \quad \text{pay-info} \quad \text{univ}_B \quad \text{pay-info} \quad \text{univ}_C \quad \text{CS} \quad \text{univ}_C \quad \text{ece} \]

the relation

|           |           |        |        |        |
|-----------|-----------|--------|--------|--------|
| univ_A pay-info | category | Secretary | dept    | CS      |
| univ_A pay-info | category | Secretary | avg-sal | 35,000  |
| univ_C ece     | category | Secretary | avg-sal | 30,000  |

Thus, we obtain the following completeness result for the SQL in the IRDBs w.r.t. the Querying Fragment \(L_Q\) of SchemaLog (provided in Definition 6.6 in [13]):

**Corollary 1** Let \(DB\) be a relational Multidatabase system with nonempty relations and attributes (Remark (*)), \(P\) be a set of safe rules in the Querying Fragment \(L_Q\) of SchemaLog and \(p\) any (virtual) predicate defined by \(P\). Then there exists a standard SQL expression \(t\) such that the computed relation \(\|t\|_\#\) in the IRDB obtained by parsing of this Multidatabase system is equal to the relation corresponding to \(p\) computed by SchemaLog.

**Proof:** From Lemma 6.1 in [13] for such a query \(p \in L_Q\) there is an expression \(E\) of the extended relational algebra \(ERA\), such that the relation corresponding to \(p\) is equal to the relation obtained by computing the relational expression \(E\). From Theorem , we are able to translate this expression \(E \in ERA\) into an equivalent standard SQL term \(t\) whose extension \(\|t\|_\#\) in the IRDB obtained by parsing of this Multidatabase system is equal to the relation corresponding to \(p\) computed by SchemaLog.

\[ \square \]

Consequently, any querying of data and metadata information (of nonempty relations and nonempty attributes, as explained in Remark (*)) of the federated relational database system \(DB\) provided by the interoperability framework of the SchemaLog can be done in the IRDBs framework by the standard SQL.

**Remark**(***)**: If we need to use the interoperability framework also for the empty
database schemas or empty relations, in that case we need to create the relation table \textit{call}$_3$ not by deriving it as a particular projections from \textit{call}$_4$ (SQL term) but directly from the RDB dictionary of the Multidatabase system.

Consequently, by permitting the SQL querying over the vector relations in the IRBDs we can obtain the answers (see \cite{13} for more useful cases) like, for example,:

\((Q_4) "\text{Find the names of all the relations in which the token 'John' appears}"; \)

\((Q_5) "\text{Given two relations } x \text{ and } y \text{ (in database } \text{db}), \text{ whose schemas are unknown, compute their natural join}"; \)

\text{etc.}

\section{Conclusion}

The method of parsing of a relational instance-database \(A\) with the user-defined schema \(\mathcal{A}\) into a vector relation \(\vec{\mathcal{A}}\), used in order to represent the information in a standard and simple key/value form, today in various applications of Big Data, introduces the intensional concepts for the user-defined relations of the schema \(\mathcal{A}\). Moreover, we can consider the vector relations as the concept of \textit{mediator}, proposed by Wiederhold \cite{31}, as means for integrating data from also non-relational heterogeneous sources. The expressive power of IRDB which includes the expressive power of SchemaLog and its ability to resolve data/meta-data conflicts suggests that it has the potential for being used in the interoperability frameworks for the Multidatabase systems and as a platform for developing mediators. This new family of IRDBs extends the traditional RDBS with new features. However, it is compatible in the way how to present the data by user-defined database schemas (as in RDBs) and with SQL for management of such a relational data. The structure of RDB is parsed into a vector key/value relation so that we obtain a column representation of data used in Big Data applications, covering the key/value and column-based Big Data applications as well, into a unifying RDB framework. The standard SQL syntax of IRDB makes it possible to express powerful queries and programs in the context of component database interoperability. We are able to treat the data in database, the schema of the individual databases in a Multidatabase (a federation) system, as well as the databases and relations themselves as first class citizens, without using higher-order syntax or semantics.

Note that the method of parsing is well suited for the migration from all existent RDB applications where the data is stored in the relational tables, so that this solution gives the possibility to pass easily from the actual RDBs into the new machine engines for the IRDB. We preserve all metadata (RDB schema definitions) without modification and only dematerialize the relational tables, of a given database \(\mathcal{A}_i\), by transferring their stored data into the vector relation \(r_{\text{V}_i}\) (possibly in a number of disjoint partitions over a number of nodes). From the fact that we are using the query rewriting IDBMS, the current user’s (legacy) applications does not need any modification and they continue to "see" the same user-defined RDB schema as before. Consequently, this IRDB solution is adequate for a massive migration from the already obsolete and slow RDBMSs into a new family of fast, NewSQL schema-flexible (with also 'Open schemas') and Big Data scalable IRDBMSs.
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