The watershed concept and its use in segmentation: a brief history

Fernand Meyer
Centre de Morphologie Mathématique
Département Maths et Systèmes
Mines-ParisTech
February 2, 2012

1 Introduction

The watershed is the principal tool of morphological segmentation. Its major advantages are the following:

• it produces closed contours: to each minimum or to each marker corresponds one region.

• flooding a topographic surface fills some minima, and the watershed of the flooded surface has less catchment basins. The catchment basins of successive floodings form a hierarchical segmentation.

• it is possible to flood a surface so as to impose minima at some predetermined places: this leads to marker based segmentation.

We try in this short paper to give an overview of the history of the watershed concept and analyze the influence of the technical possibilities to implement it on its development.

2 The history of the watershed

2.1 Thinnings, geodesic distances and skeletons by zone of influence

The history of the watershed for segmentation is linked with the technological development of the image processing devices. In the mid seventies, computer memory was expensive, and computers slow. At the CMM we developed the first image analyzers, subsequently commercialized by Leitz under the name TAS holding binary image memories [20]. The result of an image transform may be stored in a memory and become the source of a second transform. Chaining
operators permits new developments such as geodesic transforms, skeletons etc. The first watershed transform emerged from an alchemy mixing skeletons by zone of influence and binary thinning and thickening algorithms for constructing skeletons.

Christian Lantuejoul, in order to model a polycrystalline alloy, defined and studied the skeleton by zones of influence of a binary collection of grains in his thesis [21]; he studied the geodesic metric used for constructing a SKIZ in [22]. However, at that time, the binary operators of the TAS did not permit to construct geodesic distances and SKIZ directly. He used instead binary homotopic thinnings for the construction of the SKIZ.

For studying the drainage properties of a topographic surface, he had the idea to construct geodesic SKIZs of the minima, taking as masks the successive thresholds of the function. This gave the first algorithm for the construction of watersheds. With Serge Beucher, they applied the watershed transform to the gradient image of gas bubbles, yielding the first watershed application to segmentation [6] [7].

The same method, applied to the more complex image of electrophoretic gels highlighted the major drawback of watershed segmentation: a severe over-segmentation, due to the presence of multiple spurious minima in the gradient image. I proposed a slight modification of the thinning algorithm which solved the problem. Instead of performing successive geodesic thickenings of all regional minima, one performs a thickening of a set of markers, some of them inside the objects to segment and at least one of them in the background [31], [4], [8]. This method produces a coarse approximation of the contours, between the inside and outside markers of the objects, as starting point of the successive geodesic homothetic thinnings. For increasing thresholds of the gradient image, the contours narrow down and ultimately produce the correct result. Marker driven watershed became the dominant morphological segmentation paradigm for some time [4].

Homotopic thinnings peel off points of a thick contour until this contour becomes thin, producing a thin line between the various markers or minima. G. Bertrand defined destructible points whose grey tone may be lowered without connecting adjacent catchment basins, yielding a kind of thinning for gray tone images. As a result he got what he called the topological watershed [3] where a thin line separates grey tone flat zones containing each a regional minimum of the initial surface and having the same grey tone as this minimum.

As a matter of fact, in terms of geodesic distance, one may be interested by the set of points equidistant from two distinct seeds, and obtain a skeleton by zone of influence, in form of a thin line. One may also be interested by the points which are closer to one seed than to any other seed. On a digital grid, there exist pairs of neighboring pixels, such that one is closer to a seed and the other closer to another seed, without a third pixel separating them. Other pixels are at the same distance of two seeds. For this reason, it is often preferred to create a tessellation, i.e. a partition of the image, where each tile is made of all pixels closer to a seed than to any other, but also contains some pixels which are equidistant from two seeds. The price to pay is an arbitrary choice for a
assigning such pixels to one of the closest seeds. This phenomenon, which is true for the Voronoi tessellation of binary images directly translates to the watershed itself, as its construction is made by successive geodesic SKIZ. Such a partition is called watershed zone. If one consider a graph where the nodes are the pixels and the edges connect neighboring pixels, we obtain a partial graph connecting only pixels belonging to the same tile of the watershed partition. As there never exists an edge between two distinct tiles, this partial graph is a graph cut of the initial graph. Such partitions could not easily be constructed through thinnings but their construction became easy with the apparition of general purpose computers with cheap memories, able to hold complete images.

2.2 Random access memories and waiting queue driven algorithms

Random access memories permit simulating the progression of a flood in a much more efficient way, as on hardwired devices, where the whole image has to be processed for each step progression of the flood. The first development uses a hierarchical queue controlling the propagation of labels for constructing a skeleton by zones of influence. This method permits to construct ad libitum skeletons by zone of influence or Voronoi tessellations and by replacing the thinnings in the first generation algorithms produced efficient watershed algorithms on general purpose computers. A hardwired implementation of this algorithm has been proposed in [38]. In order to be able to rapidly generate the successive thresholds of a grey tone image, L.Vincent and P.Soille had the idea to produce a histogram of the image in a first run and then to order the addresses of each pixel in bins with the right size for this particular grey tone. With these innovations, the algorithm of Lantuejoul could be implemented and gained new speed.

The introduction of a hierarchical queue (HQ) for controlling the flood during the watershed construction presented a great advantage. It produces a correct flood not only from one grey tone to the next, but also within the flat-zones of the image. Furthermore, without modification, it is equally able to construct the watershed associated to all minima or to a set of markers.

2.3 The topographic distance and shortest path algorithms

This first period is dominated by algorithms and lack a precise definition of the watershed. Two independent papers introduced the topographic distance and defined the watershed as a SKIZ of the minima for this distance. The equidistant lines from the minima are the level lines of the topographic surface. This definition was thus compatible with the presentation of the watershed lines as dams to be erected for separating the floods from distinct minima during a flooding of a topographic surface. Furthermore, it can be shown that the HQ algorithm directly derives from this definition.

As the geodesic lines of the topographic surface follow lines of steepest descent, another type of algorithms has been developed, where a graph is con-
structured linking each node with its lowest neighbors. This graph is then pruned in order to keep only one lower neighbor, creating a forest, where each tree spans a region of the partition. This idea has been used for parallelization of the watershed between various processors [9], [48] and for a hardwired implementation of the watershed [23].

The construction of the watershed may be then be obtained as a shortest path problem on a graph for which many algorithms exist [33], [2]. In order to obtain higher precision on digital grids, G.Borgefors introduced chamfer distances [10]. The same type of neighborhoods, based on particular weights for first and second neighbors on a grid can also be adapted for the construction of chamfer topographic distances [29]. Using a hierarchical queue for controlling the Dijkstra-Moore algorithm furthermore permits a correct flooding of the plateaus. Shortest path algorithms lend themselves also very well to the implementation on graphics processors or GBU [19].

These watershed algorithms may be subdivided in two classes : the first class constructs a watershed line separating connected particles ; the second produces a partition of the image, where each region represents a catchment basin.

The definition of the watershed line leads to an eikonal equation, expressed as a PDE and may be solved as such. This leads to a continuous watershed algorithm. [24], [25].

The so-called watersnakes, which introduce some degree of viscosity in order to regularize the watershed contours are also based on the topographic distance [37]. J. Roerdink published a remarkable review on the various methods for constructing the watershed [39].

2.4 Minimum spanning trees and forests, marker based segmentation

The segmentation paradigm based on watershed and markers has proved to be robust and efficient for solving many segmentation tasks. Its strength lies in the decoupling between a loose localization of the objects of interest, detected as markers and the precise construction of the contours. This advantage is particularly true in 3D, where the construction of the contours is complex, whereas detecting the markers is often much simpler and may sometimes be done in 2D cuts of the 3D images.

Marker based segmentation is also ideal for interactive segmentation: a first set of markers obtained automatically or interactively introduced in the image produce a first segmentation. This segmentation may then be corrected by adding, modifying or suppressing markers. Adding a marker to an existing segmentation results in cutting a region of this segmentation in two parts. Suppressing a marker on the contrary results in merging two regions. As a matter of fact, marker based segmentation results in merging some of the catchment basins associated to the complete collection of minima of the image.

This leads to an approach where two scales are considered : for segmenting an image, the catchment basins of its gradient image are first constructed at the pixel level ; the final segmentation is then made at the level of regions. To this
effect one constructs the region adjacency graph, where nodes represent the regions and edges link neighboring nodes. The edges are furthermore weighted by a weight expressing the dissimilarity between regions. As the boundaries of the regions follow the crest lines of a gradient image, one often expresses this dissimilarity by the altitude of the pass point between adjacent regions. This weighting is coherent with the flooding paradigm underlying the watershed: the propagation of a flooding in a topographic surface crosses the boundaries between catchment basins through their pass points. Flooding a topographic surface creates lakes. The lowest level of a lake containing two regional minima \( m_1 \) and \( m_2 \) of a topographic surface constitutes an ultrametric distance between these minima. If \( m_1, m_2 \) and \( m_3 \) are three minima, then the lowest lake covering all three minima is higher or equal than the lowest lake covering only two minima, constituting the ultrametric inequality \( \max[d(m_1, m_2), d(m_2, m_3)] \geq d(m_1, m_3) \). The minimum spanning tree of the RAG is a tree spanning all nodes and whose total weight is minimal \( \text{[12]} \text{[13]} \text{[11]} \). If the edge weights are all distinct, the minimum spanning tree is unique; when several MSTs exist, they all have the same weight distribution.

MST constitute a sparse representation of a topographic surface as the number of edges equals to the number of nodes minus 1. Between any two nodes, there exists a unique path on the MST and the weight of the largest edge along this path is equal to the flooding ultrametric distance between these nodes (see the textbook \text{[17]}). Cutting all edges of the MST above some threshold produces a forest where each subtree spans a region of the domain. For higher thresholds, regions merge and coarser partitions produced. The series of nested partitions constitutes a hierarchy. If by cutting the edges above a given threshold produces \( n \) subtrees, they constitute a minimum spanning forest with \( n \) trees of the region adjacency graph. Marker based segmentation also produces minimum spanning forests with an additional constraint: each tree is rooted in a marker \text{[28]}. Marker based segmentation may also be formalized in terms of the SKIZ of the markers using a lexicographic distance \text{[32]}.

### 2.5 From connected operators and floodings to hierarchies

The partition obtained by cutting the edges of the MST or of the RAG above some threshold is often not very useful as long it only relies on local dissimilarities between regions. Better focused segmentations may be obtained if one selectively floods some catchment basins before constructing the watershed line. Floodings have been introduced as reconstruction closings \text{[16]} \text{[40]} and subsequently generalized as levelings \text{[30]}. The watershed partition of an image produces a first segmentation; flooding this image produces a coarser partition, where regions of the previous segmentation have merged. To each additional flooding of the preceding will correspond a coarser partition. The series of these partitions form a hierarchy. Such a hierarchy may be obtained in one run through the image, rather than repeating \( n \) increasing floodings and watershed basins detection.

M. Grimaud and L.Najman were the first to propose such a construction.
At the time, M.Grimaud tried to detect the microcalcification in breast X rays; they appear as small and contrasted bright dots. They appear on a fibrous substrate and coexist with noise particles. M.Grimaud wanted to rank all such events independently of the contrast of the image and measure additional features on the most contrasted ones. For this reason, he favoured a reconstruction closing sensitive to the contrast, where the marker is the function itself after the addition of a constant value $\lambda$. For increasing values of the constant $\lambda$, more and more basins will be filled and the subsequent watershed construction produce coarser segmentations. Each minimum can then be weighted by the parameter $\lambda$ for which it is completely filled; at the same time each contour can be weighted by the parameter $\lambda$ for which it disappears for the first time. M.Grimaud proposed an algorithm for weighting all minima, calling the contrast measure dynamics \[18\]; on the other hand, L. Najman weighted the contours and called the measure saliency \[34, 35\].

Other criteria than the contrast may be used for governing the flooding of the basins. If one uses as floodings the area closings introduced by L.Vincent \[45\], one obtains hierarchies governed by size criteria. More generally, one may flood the basins in such a way that the lakes which are created have in common, either the depth, or the area, or the volume of water \[41, 42\].

All these approaches have in common to use the same MST of the region adjacency graph. They take the MST with a given set of weights as input and output a new set of weights on the edges. Thanks to this common structure, efficient interactive segmentation toolboxes may be produced \[47\]. For instance minimum spanning forests with trees rooted in markers may be derived from the MST whatever its weight distribution.

In a hierarchy one goes from a fine to a coarse partition by merging adjacent regions. This operation is immediate if one deals with partitions: one assigns to all regions to be merged the same label. It is however more problematic if the contour is materialized between the regions and paradoxical situations may be met if one does not carefully chose the graph representing the images \[15\]. This is an additional reason why to prefer watershed zones without boundaries between regions; furthermore representing contours wastes space in the image and makes it impossible to segment adjacent small structures.

### 2.6 The waterfall hierarchy or graph cuts

S.Beucher introduced another type of hierarchy, expressing the nested structure of the catchment basins. In the RAG the edges are weighted but not the nodes. Marker based segmentations chooses a subset of the nodes and constructs a MSF where each tree is rooted in a node. S.Beucher considered the topography expressed by this graph and defined the regional minima as the maximal partial graphs whose internal edges have the same weight and whose adjacent edges have higher weights. Constructing a minimum spanning forest where each tree is rooted in one of these regional minima produces a coarser partition \[29\]. This partition itself may again be represented by a higher order RAG and MST on which the same procedure may be applied again. The corresponding hierarchy
is called waterfall hierarchy [5].

Later J.Cousty also considered the problem of an edge weighted graph. He called the resulting MSF graph cut and proposed an efficient algorithm for constructing it [14].

2.7 Viscous and stochastic watershed

The watershed, being based on floodings is extremely sensitive to leaks in the topographic surface. For this reason, some works have attempted to regularize the watershed by introducing some viscosity. We already quoted the watersnakes [37]. Another approach consists in applying to the topographic surface an adaptive closing in order to produce a new surface on which the ordinary watershed flooding would progress in the same way as a viscous fluid would propagate in the initial topographic surface [43].

The classical use of the watershed is to find the contours associated to all minima or to a set of markers in a topographic surface. J.Angulo had the idea to weight the contours of the watershed by the probability they appear when random markers are used for segmenting the image. He called it the stochastic watershed [1].

2.8 Watershed : a name put in all sauces

This brief history of the development of the watershed concepts, construction algorithms and its use in the segmentation shows a contrasted and confusing picture. Distinct algorithms claim to produce watersheds, although they clearly produce distinct objects. The watershed may be topographic, viscous, stochastic, with or without apparent contours, defined on pictures where the nodes are weighted or on graphs where the edges are weighed. A number of issues are often not clearly addressed. The most annoying is the fact that one always speaks of watershed lines, as if the watershed always is a line, at least in the continuous space. In fact, this is not at all the case, neither in images nor in the geology. There exist so called buttonholes which are large drainage zones whose outlet is a single point, at the same topographic distance of two minima. In this case, the complete buttonhole belongs to a thick watershed zone. If one decides to divide the buttonhole between these minima, is poses again the problem of the unicity of the watershed, as there are obviously many possibilities to perform this division ? There are objective reasons for the existence of multiple solutions. A drop of water falling inside a plateau has no clear indication in which direction to flow, if only local neighborhoods are considered. We also mentioned the non unicity of the MST of a RAG. What is the incidence of choosing one or another ?

Very often, definitions of watershed are given, without analyzing the unicity or multiplicity of solutions. Similarly does a particular algorithm give the same result if one changes the processing order. If several solutions may be produced by the same algorithm or be compatible with a given definition, are these solutions close one to another or in contrary extremely diverse ?
3 Conclusion

This short history of the birth of the watershed for segmentation is necessarily incomplete: Google finds 31,000,000 entries for watershed. I hope that it is not biased, despite the fact that it tells a story in which I was much involved as were my colleagues at the Centre of Mathematical Morphology, where many of the developments presented here had their origin.

My email address is: fernand.meyer@mines-paristech.fr and I am open to any discussion and suggestions for completing this history.

FURTHER READINGS

Besides the references given below, most concepts and algorithms discussed above may be found under the same hat, i.e. in the excellent book published by Wiley in 2010 (L. Najman and H. Talbot editors) with the title "Mathematical Morphology".

References

[1] J. Angulo and D. Jeulin. Stochastic watershed segmentation. ISMM07: Mathematical Morphology and its applications to Signal and Image Processing, pages 265–276, 2007.

[2] C. Berge. Graphs. Amsterdam: North Holland, 1985.

[3] G. Bertrand. On topological watersheds. Journal of Mathematical Imaging and Vision, 22(3):217–230, 2005. cited By (since 1996) 9.

[4] S. Beucher. Segmentation d’Images et Morphologie Mathématique. PhD thesis, E.N.S. des Mines de Paris, 1990.

[5] S. Beucher. Watershed, hierarchical segmentation and waterfall algorithm. ISMM94: Mathematical Morphology and its applications to Signal Processing, pages 69–76, 1994.

[6] S. Beucher and C. Lantuéjoul. Use of watersheds in contour detection. In Proc. Int. Workshop Image Processing, Real-Time Edge and Motion Detection/Estimation, 1979.

[7] S. Beucher and C. Lantuéjoul. Use of watersheds in contour detection. In Watersheds of Functions and Picture Segmentation, pages 1928–1931, Paris, May 1982.

[8] S. Beucher and F. Meyer. The morphological approach to segmentation: the watershed transformation. In E. Dougherty, editor, Mathematical morphology in image processing, chapter 12, pages 433–481. Marcel Dekker, 1993.
[9] Moga A. Bieniek, A. An efficient watershed algorithm based on connected components. *Pattern Recognition*, 33(6):907–916, 2000. cited By (since 1996) 78.

[10] Gunilla Borgefors. Distance transformations in digital images. *Comput. Vision Graph. Image Process.*, 34:344–371, June 1986.

[11] Otakar Boruvka. Otakar boruvka on minimum spanning tree problem: Translation of both the 1926 papers, comments, history. *DMATH: Discrete Mathematics*, 233.

[12] Otakar Boruvka. O jistm problmu minimlnm (about a certain minimal problem (in czech). *Prce mor. prrodoved. spol.*, 1926.

[13] Otakar Boruvka. Prspevek k reen otzky ekonomick stavby elektrovodnch st (contribution to the solution of a problem of economical construction of electrical networks) (in czech). *Elektronick Obzor 15*, 1926.

[14] Jean Cousty, Gilles Bertrand, Laurent Najman, and Michel Couprie. Watershed cuts: Minimum spanning forests and the drop of water principle. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 31:1362–1374, 2009.

[15] Jean Cousty, Michel Couprie, Laurent Najman, and Gilles Bertrand. Grayscale watersheds on perfect fusion graphs. In Ralf Reulke, Ulrich Eckardt, Boris Flach, Uwe Knauer, and Konrad Polthier, editors, *Combinatorial Image Analysis*, volume 4040 of *Lecture Notes in Computer Science*, pages 60–73. Springer Berlin / Heidelberg, 2006.

[16] J. Crespo, J. Serra, and R. Schafer. Image segmentation using connected filters. In *Workshop on Mathematical Morphology*, pages 52–57, Barcelona, May 1993.

[17] M. Gondran and M. Minoux. *Graphes et Algorithmes*. Eyrolles, 1995.

[18] M. Grimaud. New measure of contrast : dynamics. *Image Algebra and Morphological Processing III, San Diego CA, Proc. SPIE*, 1992.

[19] Piche N. Kauffmann, C. A cellular automaton for ultra-fast watershed transform on gpu. 2008.

[20] J. Klein. *Conception et Réalisation d’une unité logique pour l’analyse quantitative d’images*. PhD thesis, University of Nancy, 1976.

[21] C. Lantuéjoul. *La squelettisation et son application aux mesures topologiques de mosaïques poly cristallines*. PhD thesis, École nationale supérieure des mines de Paris, 1978.

[22] C. Lantuéjoul and S. Beucher. On the use of the geodesic metric in image analysis. *J. Microsc.*, 1981.
[23] F. Lemonnier. *Architecture Electronique Dediée aux Algorithmes Rapi-
des de Segmentation Basés sur la Morphologie Mathématique*. PhD thesis,
E.N.S. des Mines de Paris, 1996.

[24] Butt M.A. Maragos, P. Curve evolution, differential morphology, and dis-
tance transforms applied to multiscale and eikonal problems. *Fundamenta
Informaticae*, 41(1-2):91–129, 2000. cited By (since 1996) 17.

[25] P. Maragos and F. Meyer. Nonlinear PDEs and numerical algorithms for
modeling levelings and reconstruction filters. In *Scale-Space Theories in
Computer Vision*, Lecture Notes in Computer Science 1682, pages 363–
374. Springer, 1999.

[26] B. Marcotegui and S. Beucher. Fast implementation of waterfalls based on
graphs. *ISMM05: Mathematical Morphology and its applications to Signal
Processing*, pages 177–186, 2005.

[27] F. Meyer. Un algorithme optimal de ligne de partage des eaux. In *Proce-
dings 8ème Congrès AFCET, Lyon-Villeurbanne*, pages 847–857, 1991.

[28] F. Meyer. Minimal spanning forests for morphological segmentation.
*ISMM94: Mathematical Morphology and its applications to Signal Pro-
cessing*, pages 77–84, 1994.

[29] F. Meyer. Topographic distance and watershed lines. *Signal Processing*,
pages 113–125, 1994.

[30] F. Meyer. The levelings. In H. Heijmans and J. Roerdink, editors, *Math-
ematical Morphology and Its Applications to Image Processi ng*, pages 199–
207. Kluwer, 1998.

[31] F. Meyer and S. Beucher. Morphological segmentation. 1(1):21–46, Septem-
ber 1990.

[32] Fernand Meyer. Grey-weighted, ultrametric and lexicographic distances. In
Christian Ronse, Laurent Najman, and Etienne Decencire, editors, *Math-
ematical Morphology: 40 Years On*, volume 30 of *Computational Imaging
and Vision*, pages 289–298. Springer Netherlands, 2005.

[33] E.F. Moore. The shortest path through a maze. In *Proc. Int. Symposium
on Theory of Switching*, volume 30, pages 285–292, 1957.

[34] L. Najman. *Morphologie Mathématique: de la segmentation d’images à
l’analyse multivoque*. PhD thesis, Université Paris-Dauphine, 1994.

[35] L. Najman. Geodesic saliency of watershed edges and hierarchical segmen-
tation. *IEEE Trans. Pattern Anal. Machine Intell.*, 16(3):175–182, 1996.

[36] Laurent Najman and Michel Schmitt. Watershed of a continuous function.
*Signal Processing*, 38(1):99 – 112, 1994. Mathematical Morphology and its
Applications to Signal Processing.
[37] Hieu Tat Nguyen, Marcel Worringer, and Rein van den Boomgaard. Watersnakes: Energy-driven watershed segmentation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 25:330–342, 2003.

[38] D. Noguet. A massively parallel implementation of the watershed based on cellular automata. In *Application-Specific Systems, Architectures and Processors, 1997. Proceedings*, IEEE International Conference on, pages 42–52, jul 1997.

[39] Jos B. T. M. Roerdink and Arnold Meijster. The watershed transform: Definitions, algorithms and parallelization strategies. *Fundamenta Informaticae*, 41:187–228, 2001.

[40] P. Salembier and J. Serra. Flat zones filtering, connected operators and filters by reconstruction. *IEEE Transactions on Image Processing*, 3(8):1153–1160, August 1995.

[41] C. Vachier. *Extraction de Caractéristiques, Segmentation d’Image et Morphologie Mathématique*. PhD thesis, E.N.S. des Mines de Paris, 1995.

[42] C. Vachier and F. Meyer. Extinction values: A new measurement of persistence. In I. Pitas, editor, *1995 IEEE Workshop on Nonlinear Signal and Image Processing*, pages 254–257, 1995.

[43] Corinne Vachier and Fernand Meyer. The viscous watershed transform. *Journal of Mathematical Imaging and Vision*, 22:251–267, 2005.

[44] L. Vincent. *Algorithmes Morphologiques à Base de Files d’Attente et de Lacets. Extension aux Graphes*. PhD thesis, E.N.S. des Mines de Paris, 1990.

[45] L. Vincent. Morphological area openings and closings for grayscale images. *Shape in Picture, NATO Workshop, Driebergen*, Sept. 1992.

[46] Soille Pierre Vincent, Luc. Watersheds in digital spaces: An efficient algorithm based on immersion simulations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(6):583–598, 1991.

[47] F. Zanoguera, B. Marcotegui, and F. Meyer. A segmentation pyramid for the interactive segmentation of 3-d images and video sequences. In John Goutsias, Luc Vincent, and Dan S. Bloomberg, editors, *Mathematical Morphology and its Applications to Image and Signal Processing*, volume 18 of *Computational Imaging and Vision*, pages 223–232. Springer US, 2002.

[48] Iwanowski M. Aswiercz, M. Fast, parallel watershed algorithm based on path tracing. *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, 6375 LNCS(PART 2):317–324, 2010.