CAN SYMMETRY NON-RESTORATION
SOLVE THE MONOPOLE PROBLEM?

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Abstract

We reexamine a recently proposed non-inflationary solution to the monopole problem, based on the possibility that spontaneously broken Grand-Unified symmetries do not get restored at high temperature. We go beyond leading order by studying the self-consistent one-loop equations of the model. We find large next-to-leading corrections that reverse the lowest order results and cause symmetry restoration at high temperature.

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1 Introduction

Since the pioneering works by Kirzhnits and Linde [1], Weinberg [2] and Dolan and Jackiw [3], the analysis of the phase diagrams of gauge theories at finite temperature has been object of intense research. Many of these studies are connected with spontaneously broken gauge theories where the typical picture which emerges is that symmetries get restored at high temperatures.

In standard early universe scenarios based on Grand Unified Theories (GUT’s), one then starts from the symmetric phase of the model, and as the temperature is lowered, several phase transitions occur until the broken phase described by the Standard Model is reached. A generic feature of this process is the unavoidable formation of topological defects during these phase transitions and, as it is well known, the over abundant production of monopoles at this stage represents one of the most serious drawbacks of GUT’s.

Different solutions to the monopole problem have been suggested along the years. Apart from inflation, which is the most popular one, several authors have proposed alternative scenarios. Among them, let us mention the work of Langacker and Pi [4], who have suggested that the existence of an intermediate phase, with no unbroken $U(1)$ symmetry, would cause a rapid annihilation of the monopoles produced during the GUT phase transition. Another and more radical possibility has been considered by Salomonson, Skagerstam and Stern [5] and very recently by Dvali, Melfo and Senjanović [6]. It is based on the attractive idea that the monopole producing phase transition could have not occurred at all. In other words, it is argued that, no matter how high the temperature was, the symmetric phase of $SU(5)$ was never realized.

Although not very popular and somehow counter-intuitive, the phenomenon of symmetry non-restoration has been present in the literature for quite a while. In fact, it dates back to the classic paper of Weinberg [2] and some interesting phenomenological applications have been discussed in the past [7].
The possibility of symmetry non-restoration in GUT’s is analyzed in ref. [5] for the minimal $SU(5)$ model, which has two Higgs fields in the representations $5$ and $24$. Neglecting the gauge couplings, it was shown there that there exists a range of parameters for the Higgs potential, leading to the symmetry of the Standard Model at low energies but such that the heavy Higgs responsible of the $SU(5)$ breaking keeps a non vanishing vev even at very high temperatures. Gauge couplings are considered, instead, in ref. [6], where it is shown that, due to the large value of the gauge-coupling constant at the scale of the $SU(5)$ phase transition, symmetry non-restoration in the minimal model requires such large values for the Higgs parameters that perturbation theory breaks down. It is subsequently observed that if the Higgs sector is enlarged with a scalar in $45$ representation of $SU(5)$ symmetry non-restoration can be achieved for values of the Higgs coupling constants which seem small enough as to make reliable the lowest order computation on which the whole analysis is based.

In this paper, we will reanalyze the phenomenon of symmetry non-restoration for the GUT models considered in ref. [6] by taking into account next-to-leading order corrections. The reasons for doing this stem from our study of the simpler $O(N_1) \times O(N_2)$ scalar model [8] (which mimics the scalar sector of GUT’s) where we found that the inclusion of sub-leading corrections reduces in a significant way the region of parameter space where symmetry non-restoration occurs. As we will show in this paper, the effect of including these corrections is even more dramatic here as they reverse the lowest order result and cause symmetry restoration in the entire range of parameters that was considered in ref. [6].

The paper is organized as follows. In Section 2 we present the model and discuss the phenomenon of symmetry non restoration to lowest order, while in Section 3 we present the results of our next-to-leading calculations. Finally, we leave for Section 4 some final remarks.
2 A review of symmetry non-restoration

In this Section, we shall briefly review the basic results about symmetry non-restoration in lowest order perturbation theory. For definiteness, we shall consider the GUT model of ref.[6], having the usual fermionic content, namely, three generations of fermions $(\Psi^f_L)_5^*, (\Psi^f_L)_10$, $f = 1, 2, 3$ in the $5^*$ and $10$ representations respectively and three Higgs multiplets, $\Phi_5$, $H_{24}$ and $\chi_{45}$ in the complex $5$, real $24$ and complex $45$ dimensional representations respectively.

The model is described by the Lagrangian

$$ L = -\frac{T_F}{2} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} (D_\mu H)_i (D^\mu H)_i + (D_\mu \Phi)^a_*(D^\mu \Phi)_a + (D_\mu \chi)^u_*(D^\mu \chi)_u + \frac{V(H_i, \Phi_a, \chi_u)}{2} + L_F, $$

(1)

where, in order to simplify the notation, we have called $\Phi_a, a = 1, \cdots 5$, $H_i, i = 1, \cdots 24$, and $\chi_u, u = 1, \cdots 45$, the independent components of the Higgs fields $\Phi_5$, $H_{24}$ and $\chi_{45}$ respectively\(^2\) and

$$ F_{\mu \nu} = (\partial_\mu A^i_\nu - \partial_\nu A^i_\mu - g f^{ijk} A^j_\mu A^k_\nu)T^i, $$

(2)

$$ [T^i, T^j] = i f^{ijk} T^k, \quad Tr(T^i T^j) = \frac{1}{2} \delta^{ij}, $$

(3)

$$ (D_\mu \Phi)_a = \partial_\mu \Phi_a - ig A^j_\mu (T^j)^a_b \Phi_b, $$

(4)

$$ (D_\mu H)_i = \partial_\mu H_i - ig A^j_\mu (T^j)^i_k H_k, $$

(5)

$$ (D_\mu \chi)_u = \partial_\mu \chi_u - ig A^j_\mu (T^j)^u_v \chi_v. $$

(6)

Here, $T^j_H$, $T^j_\chi$ are the $SU(5)$ generators in the $24$ and $45$ representations, satisfying the same commutation relations as in eq. (3) and such that:

$$ Tr(T^i_H T^j_H) = c_H \delta^{ij}, $$

(7)

\(^2\)Here and in what follows we shall adopt the following indices convention: Greek letters will be used for space-time indices, while Latin letters will denote group indices. Group indices will be denoted with the initial letters of the alphabet $a, b, c...$ when running from 1 to 5, with the middle letters $i, j, k...$ when running from 1 to 24 and with the last letters $u, v, w...$ when running from 1 to 45.
\[ Tr(T^i_i T^j_j) = c_X \delta^{ij}, \]

where \( c_H = 5 \) and \( c_X = 12 \). \( L_F \) stands for the fermionic Lagrangian, which we shall not write down explicitly as it will play a minor role in our discussion.

As it is well known, the starting \( SU(5) \) symmetry of the model can be broken down to the low-energy standard model \( SU(3)_c \times U(1)_{EM} \) symmetry in two stages. In the first, the field \( H_{24} \) gets a vev \( \langle H \rangle \) that breaks \( SU(5) \) to \( SU(3)_c \times SU(2)_L \times U(1)_Y \). In the second, the light fields \( \Phi_5 \) and/or \( \chi_{24} \) break this intermediate gauge symmetry down to \( SU(3)_c \times U(1)_{EM} \). It is during the first of these phase transitions that monopoles form with the Kibble mechanism. As we explained in the introduction, we are studying here the possibility that the phase transition leading to the formation of monopoles does not occur, namely that the heavy field \( H_{24} \) keeps a non-vanishing vev at arbitrarily large temperatures.

According to the mechanism discussed by Weinberg \[2\], this goal can be achieved if one lets the field \( H_{24} \) interact, with a negative coupling, with the other Higgs fields \( \Phi_5 \) and/or \( \chi_{24} \). Such couplings are naturally allowed in the model we are considering. If one writes the most general \( SU(5) \) invariant renormalizable potential \[3\] as:

\[
V(H_i, \Phi_a, \chi_u) = -\frac{1}{2}m_H^2 H_i H_i - m_a^2 \Phi_a^* \Phi_a - m_\chi^2 \chi_u^* \chi_u + \lambda_\chi (\chi_u^* \chi_u)^2 + \\
\frac{1}{4}\lambda_H (H_i H_i)^2 + \lambda_\Phi (\Phi_a^* \Phi_a)^2 - \alpha (\chi_u^* \chi_u)(H_i H_i) + v(H_i, \Phi_a, \chi_u),
\]

it is for example possible to obtain the desired symmetry breaking pattern at zero temperature with a \( v(H_i, \Phi_a, \chi_u) \) negligible compared to the other terms. Then the condition of boundedness from below on \( V \) reduces to:

\[
\lambda_\chi \lambda_H > \alpha^2, \quad \lambda_\chi > 0, \quad \lambda_H > 0.
\]

\[3\] We are using a normalization for the Higgs fields and coupling constants different from that of ref.\[6\]. The relation between the two is as follows: \( m_\chi^2 = 2m_{\chi'}^2, \quad m_H^2 = 4m_{H'}^2, \quad \lambda_\chi = 4\lambda_{\chi'}, \quad \lambda_H = 16\lambda_{H'}, \quad \alpha = 4\alpha' \), where the primed constants correspond to those used in \[6\].
The sign of $\alpha$ is thus unconstrained and one can choose it positive such as to give a negative interaction between $H_{24}$ and $\chi_{45}$. Obviously one may also consider the analogous case where $\chi_{45}$ is replaced by $\Phi_5$ in the $\alpha$ dependent interaction term in eq.(9) [5]. These are precisely the two cases considered in ref. [6], and on which we will concentrate our attention here.

Let us briefly review how symmetry non-restoration arises in the model. As it is well known, the basic tool to study the symmetry behavior of a theory at finite temperature is provided by the effective potential [3]. As we are only interested here in establishing the existence (or not) of a stable symmetric phase at arbitrarily high temperatures ($T \gg \langle \Phi_a \rangle, \langle H_i \rangle, \langle \chi_a \rangle$) and not in the details of the phase transition, we will only need to compute the leading terms, of order $T^2$, of the effective potential. This turns out to be equivalent to evaluating the leading $T^2$–contributions to the self-energies of the Higgs fields (Debye masses).

The result for the Debye masses to lowest order in the coupling constants can be obtained by adapting Weinberg’s formulae to this model and read [9]:

\[
m^2(\chi) = \Sigma(\chi , p = 0) = T^2 \left( \frac{1}{6} \lambda_{\chi} (1 + N_{\chi}) - \alpha \frac{N_H}{12} + \frac{1}{4} g^2 \frac{D c_{\chi}}{N_{\chi}} \right) \equiv \nu^2 T^2 , \tag{11}
\]

\[
m^2(H) = \Sigma(H , p = 0) = T^2 \left( \frac{1}{12} \lambda_H (2 + N_H) - \alpha \frac{N_H}{6} + \frac{1}{4} g^2 \frac{D c_{H}}{N_H} \right) \equiv \nu^2 T^2 , \tag{12}
\]

where $N_H = 24$ and $N_{\chi} = 45$ are the dimensions of the representations of the Higgs fields and $D = 24$ is the dimension of the group. The different numerical factors between eqs.(11) and (12) arise from the fact that $\chi_{45}$ is complex while $H_{24}$ is real (the analogous result for the case of a coupling between $H_{24}$ and $\Phi_5$ can be obtained by simply substituting in eqs.(11) and (12) to $N_{\chi}$ and $c_{\chi}$ the corresponding quantities $N_{\Phi}$ and $c_{\Phi}$).

These formulae exhibit the basic idea behind symmetry non-restoration: for positive $\alpha$’s the interaction $H_{24} - \chi_{45}$ term gives a negative contribution to the thermal masses and might make one of them negative (it is easy to see that condition (10) implies that...
they cannot be simultaneously negative). Once this happens the corresponding Higgs field maintains a vev at high temperatures and symmetry is never restored.

As explained earlier, we wish to have $\langle H_{24} \rangle \neq 0$ at high $T$ and according to eq.(12) this requires:

$$m_{H}^{2}(T) < 0 \Rightarrow \alpha > \lambda_{H} \frac{2 + N_{H}}{2 N_{\chi}} + \frac{3}{2} g^{2} \frac{D_{cH}}{N_{H} N_{\chi}} .$$

The set of coupling constants satisfying this inequality, together with the boundedness condition, eq.(10), thus represents, to lowest order, the region of parameter space in which symmetry non-restoration occurs.

The presence of gauge couplings plays an essential role: as it is evident from eqs.(11) and (12), it conspires against symmetry non-restoration. As a consequence, $\alpha$ has to be large enough to overcome their contributions to the thermal masses. But, on the other side, if $\alpha$ becomes too large, as a consequence of the boundedness condition eq.(10) $\lambda_{\chi}$ or $\lambda_{H}$ are pushed outside the range of applicability of perturbation theory and the whole analysis breaks down. In fact, combining eq.(13) with eq.(10) one gets:

$$\lambda_{\chi} > \frac{1}{\lambda_{H}} \left( \lambda_{H} \frac{2 + N_{H}}{2 N_{\chi}} + \frac{3}{2} g^{2} \frac{D_{cH}}{N_{H} N_{\chi}} \right)^{2} .$$

The r.h.s. has a minimum for

$$\hat{\lambda}_{H} = 3 g^{2} \frac{D_{cH}}{N_{H}(N_{H} + 2)} ,$$

which gives a lower bound for $\lambda_{\chi}$ and $\alpha$:

$$\lambda_{\chi} > \hat{\lambda}_{\chi} = \hat{\lambda}_{H} \left( \frac{N_{H} + 2}{N_{\chi}} \right)^{2} ,$$

$$\alpha > \hat{\alpha} = \hat{\lambda}_{H} \frac{2 + N_{H}}{N_{\chi}} .$$

Eqs.(16)(17) make it apparent that there are better chances of keeping $H_{24}$ broken at all temperatures with small values of the coupling constants by coupling it with Higgses
belonging to representations of large dimensions. This remark is relevant because of the large value of $g^2$, typically taken to be $g^2 \approx 1/4$. With this value for $g^2$, one has in fact:

$$\hat{\lambda}_H \approx 0.15 \ ,$$

(18)

$$\hat{\lambda}_\chi \approx 0.05 \ ,$$

(19)

$$\hat{\alpha} \approx 0.09 \ .$$

(20)

Had one coupled instead $H_{24}$ to $\Phi_5$, one would have found:

$$\hat{\lambda}_\Phi \approx 3.9 \ ,$$

(21)

$$\hat{\alpha} \approx 0.75 \ .$$

(22)

While it is more or less clear that for the $5$ representation one needs too large values of the coupling constants for perturbation theory to be reliable, there seems to be some hope with the $45$ representation. Nevertheless the reader should be aware that loop corrections may contain powers of $N_\chi\lambda_\chi$ and thus also for the case of a coupling between $H_{24}$ and $\chi_{45}$ an analysis of higher order corrections results unavoidable.

## 3 Next-to-leading order corrections

In this Section we will compute the next to leading order corrections to the thermal masses of the Higgs fields to determine if the results of the lowest order analysis of the previous Section are reliable, when the coupling constants are of the order of those of eqs. (18-20).

At this point, let us mention that some authors [10] have argued that Weinberg’s results on symmetry non-restoration are only an artifact of perturbation theory and that in reality symmetry is always restored a high temperatures. Their claims are based on the observation that non-perturbative calculations (such as $1/N$ expansion and Gaussian Effective Potential) when applied to $O(N_1) \times O(N_2)$ global scalar models give always
symmetry restoration. Even though non perturbative, the methods used in [10] are approximations, whose range of validity are not clear to us. We will follow a more standard route, by assuming that perturbation theory (conveniently improved) is valid and determining whether or not the lowest order results remain true after next-to-leading order corrections are included.

Due to the infrared characteristic behavior of field theories at high temperatures, the corrections to the self-energies are expected to be of order $\frac{e^3}{2}$ instead of $e^2$, where $e$ is the largest among $\lambda_\chi, \lambda_H, \alpha, g^2$. A simple way of obtaining these corrections is via the analysis of the so-called “gap equations”, which correspond to a one-loop truncation of the Schwinger-Dyson equations for the self-energy and which are equivalent to a resummation of the ring diagrams of the perturbative series. Alternatively, they can also be derived by adding and subtracting from the Lagrangian a temperature-dependent mass counter term [11], which is then determined self-consistently in such a way as to cancel $T^2$-divergent terms occurring in the self-energies. This kind of improvement of perturbation theory has been discussed recently in the context of studies on the Electroweak phase transition [12]. We will write the gap equations equations in the symmetric phase and we will identify the region of parameter space for which real solutions for the masses can be found as the region of symmetry restoration.

The one-loop gap equations we are considering are schematically represented in fig. (1). Notice that we have neglected the Yukawa couplings between the Higgs fields and the fermions. We will comment on this point below. The blobs in the internal lines of the diagrams shown there represent the complete thermal propagators of the fields. To the order we are interested in, they can be approximated with free-like propagators containing the thermal masses instead of the zero-temperature ones. For the Higgs fields this corresponds to using (in Euclidean space-time):

$$\langle H_i(-p)H_j(p) \rangle = \frac{\delta_{ij}}{p^2 + m_{H_i}^2(T)},$$  (23)

8
\[ \langle \chi_u(-p)\chi_v(p) \rangle = \frac{\delta_{uv}}{p^2 + m^2 \chi(T)} . \] (24)

As for the gauge fields \( A^i_\mu \), their propagators are written in the Landau gauge and read:

\[ \langle A^i_\mu(-p)A^j_\nu(p) \rangle = \frac{\delta^{ij}}{p^2 + m^2 \chi(T)}(P_L)_{\mu\nu} + \frac{\delta^{ij}}{p^2}(P_T)_{\mu\nu} , \] (25)

where

\[ (P_T)_{\mu\nu} = \delta_{\mu\nu} \left( \delta_{rs} - \frac{k_r k_s}{k^2} \right) \delta_{s\nu} . \] (26)

and

\[ (P_L)_{\mu\nu} = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - (P_T)_{\mu\nu} . \] (27)

(in eq.(24) \( r \) and \( s \) denote space indices \( r, s = 1, 2, 3 \)). Here \( m_L(T) \) stands for the longitudinal thermal mass of the gauge bosons. The standard high-temperature one-loop result for \( m_L(T) \) in an \( SU(N) \) gauge theory with \( F_i \) chiral fermions, \( h_j \) real Higgs and \( \Sigma_k \) complex Higgs in the representations \( R_i, R_j, R_k \) respectively is given by:

\[ \nu^2_g = \frac{m^2 L(T)}{T^2} = g^2 \left( \frac{N}{3} + \sum_{F_i} \frac{c_{R_i}}{6} + \sum_{h_j} \frac{c_{R_j}}{6} + \sum_{\Sigma_k} \frac{c_{R_k}}{3} \right) . \] (28)

Specializing this formula to our case, we get:

\[ \frac{m^2 L(T)}{T^2} = g^2 \left( \frac{5}{3} + \frac{1}{2}c_5 + \frac{1}{2}c_{10} + \frac{1}{6}c_H + \frac{1}{3}c_\Phi + \frac{1}{3}c_\chi \right) = \frac{23}{3} g^2 . \] (29)

( Remember that we have three generations of fermions and that \( c_5 = 1/2, \ c_{10} = 3/2. \)

In our calculation to order \( e^{3/2} \), this approximation for \( m_L(T) \) is good enough.

The gap equations determining \( m_\chi(T) \) and \( m_H(T) \) to order \( e^{3/2} \) can be obtained by evaluating the diagrams of fig.(1) at zero external momenta. A simple computation gives the high-\( T \) result:

\[ (\Sigma_\chi)_{uv} (p = 0, T) = \left( \Sigma^{a_1}_\chi + \Sigma^{a_2}_\chi + \Sigma^{a_3}_\chi + \Sigma^{a_4}_\chi \right) \delta_{uv} , \] (30)

where

\[ \Sigma^{a_1}_\chi = 4\lambda_\chi(1 + N_\chi)T^2 h \left( \frac{m^2 \chi(T)}{T^2} \right) , \] (31)
\[
\Sigma_{\chi}^{a2} = -2\alpha N_H T^2 h\left(\frac{m_H^2(T)}{T^2}\right),
\]
(32)

\[
\Sigma_{\chi}^{a3} = 2g^2 Dc \chi T^2 \left[ h\left(\frac{m_H^2(T)}{T^2}\right) + 2h(0) \right],
\]
(33)

\[
\Sigma_{\chi}^{a4} = 0,
\]
(34)

with a similar result for \( (\Sigma_H)_{ij} (p=0,T) \). Here \( h(y) \) represents the function:

\[
h(y^2) = \frac{1}{4\pi^2} \int_0^\infty dx \frac{x^2}{(x^2+y^2)^{1/2}(e^{(x^2+y^2)^{1/2}}-1)},
\]
(35)

which, for small values of \( y \) has the asymptotic expansion

\[
h(y^2) = \frac{1}{24} - \frac{1}{8\pi} \sqrt{y^2} - \frac{1}{16\pi^2} y^2 \left( \ln \frac{y^2}{8\pi} + \gamma - \frac{1}{2} \right) + \cdots.
\]
(36)

Upon using this expansion in eq.(39) and retaining up to the terms linear in \( x_\chi \equiv \sqrt{m_\chi^2(T)/T^2} \) and \( x_H \equiv \sqrt{m_H^2(T)/T^2} \) one gets the following gap equations:

\[
x_{\chi}^2 = \nu_{\chi}^2 - \frac{g^2}{4\pi} Dc \chi \nu_g - \lambda_{\chi} \frac{1 + N_{\chi}}{2\pi} x_\chi + \alpha \frac{N_H}{4\pi} x_H,
\]
(37)

\[
x_{H}^2 = \nu_{H}^2 - \frac{g^2}{4\pi} Dc H \nu_g - \lambda_{H} \frac{2 + N_{H}}{4\pi} x_H + \alpha \frac{N_{\chi}}{2\pi} x_\chi,
\]
(38)

where \( \nu_\chi, \nu_H \) are the same as in eqs.(11)(12). (including higher terms in the expansion would lead to corrections to the thermal masses of order \( e^2 \log e \), but then one would have to consider also two-loops contributions to the Schwinger-Dyson equations, which are of this order and that we have neglected in our one-loop computation).

To lowest order eqs.(37-38) reproduce the results of eqs.(11)(12) for the thermal masses, but the inclusion of the next-to-leading corrections modifies the equations in an important way as they introduce a coupling among the thermal masses of the Higgs fields. Notice that this coupling, represented by the term proportional to \( \alpha \), comes with a positive sign and then works against symmetry non-restoration.

Now, eqs.(37-38) represent a set of two parabolae \( x_H = P_H(x_\chi), \ x_\chi = P_\chi(x_H) \) in the \( x_\chi, x_H \) plane. When these parabolae intersect in the upper right part of this plane

10
(remember that \( x_H \) and \( x_\chi \) are defined to be positive), the mass terms of both Higgs fields have a positive sign at high temperature and thus symmetry is restored. The condition for the existence of such an intersection is:

\[
P_\chi(x_H = 0) < x_\chi^+ \tag{39}
\]

where \( x_\chi^+ \) is the positive root of the equation \( P_H(y) = 0 \). After some trivial algebra, this condition is found to be equivalent to

\[
\alpha < f(g^2, \lambda_\chi, \lambda_H, \alpha) , \tag{40}
\]

where

\[
f(g^2, \lambda_\chi, \lambda_H, \alpha) = \frac{2 + N_H}{2N_\chi} \lambda_H + \frac{3}{2N_\chi} \left[ g^2 \frac{D_{CH}}{N_H} \left( 1 - \frac{\nu_g}{\pi} \right) \right] + \frac{3}{\pi} \alpha \left\{ -\lambda_\chi \frac{1 + N_\chi}{4\pi} + \left[ \left( \lambda_\chi \frac{1 + N_\chi}{4\pi} \right)^2 + \nu_\chi^2 - g^2 \frac{D_{CH}}{4\pi N_\chi} \nu_g \right]^{1/2} \right\} . \tag{41}
\]

The effect of including next-to-leading corrections can be better visualized by fixing the coupling constant \( \lambda_\chi \) and showing how the region of symmetry non-restoration in the \((\lambda_H, \alpha)\) plane changes in comparison to the lowest order result. To lowest order, the values of coupling constants for which symmetry non-restoration occurs are those in the region enclosed by the curve \( c_1 \) which represents the bound \((40)\):

\[
c_1 : \quad \alpha = \sqrt{\lambda_\chi \lambda_H} \tag{42}
\]

and the curve \( c_2 \) which represents the condition \( m^2_H(T) = 0 \), see eq.\((13)\):

\[
c_2 : \quad \alpha = \lambda_H \frac{2 + N_H}{2N_\chi} + \frac{3}{2} g^2 \frac{D_{CH}}{N_H N_\chi} . \tag{43}
\]

To next-to-leading order, the curve \( c_2 \) is shifted to the curve \( c_3 \) which is obtained by solving with respect to \( \alpha \) the equation:

\[
c_3 : \quad \alpha = f(g^2, \lambda_\chi, \lambda_H, \alpha) . \tag{44}
\]
The choice of $\lambda_\chi$ is not completely free, since it has to be compatible with the bounds that come from an analysis of monopole production in charged particles collisions [13]. In ref. [6] it is shown that in order to fulfill this constraint one needs:

$$\lambda_\chi > \frac{71}{135} g^2$$

(45)

which is a stronger bound than that given in eq. (16). Figure (2), shows the curves $c_1$, $c_2$, $c_3$ for $g^2 = 1/4$ and $\lambda_\chi = 71g^2/135$. We see that the curve $c_3$ lies entirely above $c_1$, which means that for all values of coupling constants compatible with the bound (10) there is symmetry restoration.

For completeness, we have considered also other values of $\lambda_\chi$ and $g^2$. Keeping fixed $g^2 = 1/4$, we have found that symmetry restoration occurs for all values of $\lambda_\chi$ consistent with the bound (10). The results are instead more sensitive to the choice of $g^2$. As it is clear already from the lowest order result and our previous work on the ungauged model [8], smaller values of $g^2$ make symmetry non-restoration easier. As an indication, Figure (3) shows the situation when $g^2 = 1/16$ and $\lambda_\chi = 71/135 g^2$. We see that even with this unrealistically small value of $g^2$ symmetry non-restoration survives only in a tiny fraction of the lowest order region. On the other hand, even this region disappears for larger values of $\lambda_\chi$.

As we said earlier, in deriving our gap equations we neglected the Yukawa couplings between the Higgs fields and the fermions. However, as they always give a positive contribution to the Higgs self-energies, they conspire against symmetry non-restoration and so can only strengthen our conclusions.

4 Conclusions

In this paper we have analyzed the phenomenon of symmetry non-restoration in a Grand Unified Model, beyond leading order. We have used linearized gap equations to determine
$e^3$ corrections to the thermal masses of the Higgs fields at high temperature. For realistic values of the $SU(5)$ gauge coupling constant we have found that next-to-leading order corrections to the thermal masses of the Higgs fields completely overwhelm the lowest order result and cause symmetry restoration.

An interesting question would of course be to determine the critical temperature and the nature of the phase transition in this region of the parameter space. To address this issue one should compute the finite-temperature effective potential to an order consistent with the linearized gap equations. As it is known \cite{12, 14} a two-loop computation in the broken phase of the model is needed to perform this task. Notice that also the form of the linearized gap equations would change, due to the presence of additional couplings and to the fact that the zero-temperature masses of the fields could not be neglected.

We have limited our analysis to the particular potentials that were considered in ref.\cite{6}, which are clearly not the most general $SU(5)$ invariant renormalizable potentials. To our knowledge, the region of symmetry non-restoration has not been analyzed for the general case even to lowest order. In view of our results, which show that the mechanism of symmetry non-restoration does not work in the simple models discussed in \cite{6}, an analysis of the general case appears now to be necessary, before one can claim that this mechanism can provide a solution to the monopole problem. This analysis, although conceptually simple, becomes tedious due to the large number of free parameters (consider that the general potential, even in the case of the model containing only $H_{24}$ and $\chi_{45}$ depends on 16 real independent parameters). Moreover, as our work shows, it will have to take into account also the effects of subleading corrections.

In conclusion, we believe that the issue of whether the monopole problem can be solved in realistic GUT models via the mechanism of symmetry non-restoration remains an open and certainly interesting problem.
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FIGURE CAPTIONS

Figure 1. One-loop gap equations for the Higgs’ self-energies. Solid lines correspond to $\chi_{45}$, dashed lines to $H_{24}$ and wavy lines to gauge fields. The blobs represent insertions of full thermal propagators.

Figure 2. Plots of $c_1$ (the solid parabola), $c_2$ (the solid straight line) and $c_3$ (the dashed line), for $g^2 = 1/4$ and $\lambda_\chi = 71g^2/135$.

Figure 3. Plots of $c_1$ (the solid parabola), $c_2$ (the solid straight line) and $c_3$ (the dashed line), for $g^2 = 1/16$ and $\lambda_\chi = 71g^2/135$.
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\[ \begin{align*}
&= \quad a_1 + a_2 \\
&+ \quad a_3 + a_4 \\
&= \quad b_1 + b_2 \\
&+ \quad b_3 + b_4 \\
\end{align*} \]

*Figure 1*
Figure 2
